Determining the gluon condensate with DIS experiments with holographic approach

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Abstract

Using gauge/gravity duality we study deep inelastic scattering (DIS) of a lepton with a proton target in presence of gluon condensation. We adopt a modified $\text{AdS}_5$ background in which the modification parameter $c$ corresponds to the gluon condensation in the boundary theory. Firstly, in study of electromagnetic field, we find that non-zero $c$ can strengthen magnitude of the field. In the next step, we compute baryonic states wave function equations in which mass of the proton target demands contribution of value of $c$ as $c = 0.0120 \pm 0.0005 \text{ GeV}^4$. Proceeding by electromagnetic field and baryonic states, we derive the holographic interaction action that is related to amplitude of the scattering. Eventually we numerically compute the corresponding structure functions as functions of $x$ and $q$ which are Bjorken variable and the momentum lepton transfers respectively. Comparing Jlab Hall C data with our theoretical results, we find that there is a good agreement between theory and phenomenology.

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1 Introduction

Deep inelastic scattering (DIS) process has the usage of studying proton internal structure which is a puzzle in quantum chromodynamic (QCD). The main difficulty comes from the fact that massless gluons and nearly massless quarks give rise to the mass of the proton $M \sim 1 \text{GeV}$. One may say that from special relativity point of view the missing mass could be considered initiating from kinetic energy of quarks and gluons inside the proton, but it turns out that the mentioned kinetic energy is not sufficient to describe the total vanishing mass. To explore more on this issue, breaking of conformal symmetry in QCD is another reason which should be considered. It leads to an anomalous dimension contributes in the DIS. Let us recall that QCD coupling is large at low energies, hence, perturbative calculations can not be used for studying many properties of hadrons. The alternative way for this purpose, is using holographic approach to study DIS.

In AdS/CFT description a strongly coupled field theory on the boundary of the AdS space could be described by a weakly coupled gravity theory in the bulk of the AdS. In other words a ten dimensional geometry on the corresponding boundary is an exact dual of a supersymmetric $SU(N)$ gauge theory with large $N$ in the bulk. Specifically, string theory in $AdS_5 \times S^5$ space is a dual of a four dimensional gauge theory. AdS/CFT duality is for conformal theories originally, but fortunately it has been generalized to non-conformal theories like QCD, so in any case of interest by adopting an appropriate AdS background one can study a QCD problem. In current holographic study a DIS process with a proton target will be considered by using a modified $AdS_5$ background. Generally in a phenomenological holographic approach, AdS/QCD tries to fit a five-dimensional effective field theory to QCD as much as possible. Doing so, mass gap, confinement and supersymmetry breaking are included by considering some modifications in gravity duals. To break conformal symmetry one may modify radial coordinate, the 5th dimension of space-time, as it was done in the reference.
We should adopt a modified AdS which introduces the gluon condensation in QCD side of duality. Originally gluon condensation was a measure for nonperturbative physics in QCD at zero temperature [5]. Later it was identified as an order parameter for confinement to study some nonperturbative phenomena [6–9].

Well-known modified holographic model which introduces gluon condensation in the boundary theory is given by the following action in Minkowski spacetime [10],

\[ S = -\frac{1}{2k^2} \int d^5x \sqrt{g} \left( R + \frac{12}{L^2} - \frac{1}{2} \partial_\lambda \varphi \partial^\lambda \varphi \right), \]  

where \( k \) is the gravitational coupling in 5-dimensions, \( R \) is Ricci scalar, \( L \) is the radius of the asymptotic \( AdS_5 \) spacetime, and \( \varphi \) is a massless scalar which is coupled with the gluon operator on the boundary. To solve the Einstein equation and the dilaton equation of motion one needs a suitable ansatz as follows,

\[ ds^2 = \frac{L^2}{z^2}(\sqrt{1 - c^2 z^8}(-dt^2 + \sum_i dx_i^2) + dz^2), \]  

\[ \varphi(z) = \sqrt{\frac{3}{2}} \ln \frac{1 + cz^4}{1 - cz^4} + \varphi_0. \]  

In the above dilaton-wall solution, \( i = 1, 2, 3 \) are orthogonal spatial boundary coordinates, \( z \) denotes the 5th dimension, radial coordinate and \( z = 0 \) sets the boundary. \( \varphi_0 \) is a constant, \( c = \frac{1}{z_\text{c}^4} \) and \( z_\text{c} \) denotes the IR cutoff. It shows that \( z \) is defined from zero to IR limit as usual. So to clarify, parameter \( c \) does not bound upper limit of \( z \) to values less than cutoff! in fact it should be interpreted as always \( c < \frac{1}{z_\text{c}^4} \). In other words, at very large value of \( z \), parameter \( c \) tends to infinitesimal value, based on the domain \( 1 - c^2 z^8 > 0 \). Also we compute in the unit where \( L = 1 \). To investigate how the forth correction of radial coordinate (with the coefficient denoted by \( c \)) appears in the metric, let us mention that the dilaton field is dual to a scalar operator and the metric is dual to the energy-momentum tensor of the dual field theory [11] (for more discussions see [12–14]). Expanding the dilaton profile near \( z = 0 \) will give,

\[ \varphi(z) = \varphi_0 + \sqrt{6}cz^4 + .... \]  

According to the holographic dictionary \( \varphi \) and \( c \) are the source and the parameter associated with the confinement respectively. Obviously \( c \) in the background metric breaks the conformal symmetry so the gluon condensation in the boundary theory appears. The relevant phenomenological information show its value generally lie in the range \( 0 < c \leq 0.9 \text{GeV}^4 \) [15, 17], however we are interested to find the exact value of \( c \) in a DIS process with proton target.

Holographic description of gluon condensation allows one to explore many physical quantities in this context. Firstly in order to get familiar with its phenomenological aspects notice that the dilaton wall solution represented by (2), (3) is related to the zero temperature case, thus this is
appropriate for studying DIS and its physics. As it must be, one can readily check that in the limit \( c \to 0 \), (2) reduces to \( \text{AdS}_5 \) which does not present mass gap, while by modifying it on radial coordinate more phenomenological results would turn out. In fact it has become an approach to discuss more phenomenological aspects by using modified AdS \[18–20\].

In the most related work to our case in [21], such a scattering has been studied by using a deformed AdS. Since models with anomalous dimension in AdS/QCD lead to generation of a mass scale of fermionic fields many works have used them to deal with DIS [21, 54]. With regard to all the mentioned motivations, in the current work we will use holographic model of gluon condensation [2] and [3] to study DIS with proton target. This paper is organized as follows, after brief review of DIS properties via holography in section 2 we will study electromagnetic interactions and baryonic states in deep inelastic scattering in sections 3 and 4 respectively. Proceeding by these results, interaction action will be given in section 5 then according to the relation between such action and scattering amplitude we will study structure functions. In section 6 we briefly review and discuss our results.

2 DIS parametrization and holography

We start this section with a brief review of deep inelastic scattering, to explicit our motivation and aims. The main usage of DIS in particle physics is exploring the inner hadronic structure and strong interactions. Let us consider a DIS process in which a lepton scattered off a proton target. During this scattering a virtual photon is exchanged. Proton fragmentation produces a lepton and some final hadronic states. It should be noted that production of final hadronic states depends on four momentum the initial lepton transfers. Therefore, the four momentum causes the inner quarks and gluons of proton expelled out, eventually in the next step quark anti-quark pairs hadronize. According to [55] DIS is parametrized by Bjorken dynamical variable which is defined as,

\[
x = \frac{-q^2}{2P \cdot q},
\]

where \( q \) is the momentum lepton transfers to the proton target via a virtual photon and \( P \) is the initial proton momentum. We adopt the method has been explained in [55] and rederived in [56, 57].

Doing so, hadronic transition amplitude is given as,

\[
W^{\mu \nu} = F_1(\eta^{\mu \nu} - \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{q^2} F_2(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x}),
\]

where \( F_{1,2} = F_{1,2}(x, q^2) \) are some structure functions.

Now, let us relate the above matrix to holography. From AdS/QCD dictionary, elements of (6) in QCD side are connected to the interaction action in AdS side as [22],

\[
\eta_{\mu < P + q, sX | J^\mu(0) | P, s_i >} = K_{eff} S_{int},
\]
where $\eta_\mu$ is polarization of virtual photon, $|P, s_i>$ represents a normalizable proton state with spin $s_i$, $J^\mu$ is the electromagnetic quark current and $s_X$ denotes the final state. It is worth to mention that $K_{eff}$ is an effective factor that adjusts the bulk supergravity quantities to the boundary phenomenologically. This is based on a different point of view in reference [41] that bulk/boundary quantities of (7) are proportional, rather than necessarily equal.

The interaction action is written as,

$$S_{int} = g_V \int d^4y e^{-\varphi} \sqrt{-g} \phi^\mu \Psi_X \Gamma_\mu \Psi_i,$$

and $g_V$ is a coupling constant related to the electric charge of the baryon, $\varphi$ is the dilaton field and $\sqrt{-g}$ is given by the metric, $\phi^\mu$ is the electromagnetic gauge field, $\Psi_i$ and $\Psi_X$ are the initial and final state spinors for the baryon respectively and $\Gamma_\mu$ are Dirac gamma matrices in the curved space. By computing all above quantities in accordance with (2) and (3), we study interaction action of DIS.

3 Electromagnetic interactions in deep inelastic scattering

During the scattering photon is exchanged, so we study the electromagnetic interactions in the bulk. It can be described as the presence of photon in modified AdS. The action for a five dimensional massless gauge field $\phi^n$ is given by,

$$S = -\frac{1}{4} \int d^5x e^{-\varphi} \sqrt{-g} F^{mn} F_{mn},$$

where $F^{mn} = \partial^m \phi^n - \partial^n \phi^m$, and $m,n$ refer to the 5-dimensional space includes Minkowski spacetime coordinates, $\mu, \nu$ and $z$, and $\varphi$ is the dilaton field given by (3). Note that $\varphi$ and $\phi$ should be differentiated. In fact (9) is an action showing the gauge field $\phi$ on a background coupled to a dilaton field $\varphi$. From (9) the equation of motion of such an electromagnetic field is derived as,

$$\partial_m [e^{-\varphi} \sqrt{-g} F^{mn}] = 0.$$

Considering $m, n \equiv \mu, \nu, z$ the relation (10) leads to,

$$\partial_\mu [\frac{1}{z}(1 + cz^4)^{-1} \sqrt{\frac{2}{3}} (1 - cz^4)^{1+\sqrt{\frac{2}{3}} F^{\mu z}}] = 0,$$

$$\partial_z [\frac{1}{z}(1 + cz^4)^{-1} \sqrt{\frac{2}{3}} (1 - cz^4)^{1+\sqrt{\frac{2}{3}} F^{z \mu}}] = 0.$$ (11)

To solve the equations of motion of the gauge field in (11), first we should fix the gauge. let us consider there is an electromagnetic field in the bulk with the geometry metric (2) defines. This obeys the 5–dimensional Maxwell equation supplemented by a gauge condition which we take to be,

$$e^{-\varphi} \sqrt{-g} \partial_\mu \phi^\mu + \partial_z (e^{-\varphi} \sqrt{-g} \phi_z) = 0.$$ (12)
From (12) one can write,

\[ \partial_\mu \phi^\mu + \frac{z}{(1 + cz^4)^{1 - \sqrt{\frac{3}{2}}} (1 - cz^4)^{1 + \sqrt{\frac{3}{2}}}} \partial_z \left( (1 + cz^4)^{1 - \sqrt{\frac{3}{2}}} (1 - cz^4)^{1 + \sqrt{\frac{3}{2}}} \phi_z \right) = 0, \]  

(13)

so,

\[ \Box \phi^\mu + \partial_\mu \partial_z \phi_z - \frac{1 + 4\sqrt{6}cz^4 + 7c^2 z^8}{z(1 - c^2 z^8)} \partial_\mu \phi_z = 0. \]  

(14)

Using the gauge (14) together with (11) leads to the following equations,

\[ \Box \phi_z - \partial_\mu \partial_z \phi^\mu = 0, \]  

(15)

\[ \Box \phi^\mu + \partial_z^2 \phi^\mu - \frac{1 + 4\sqrt{6}cz^4 + 7c^2 z^8}{z(1 - c^2 z^8)} \partial_\mu \phi_z = 0. \]  

(16)

At this point one could continue by considering a photon with a particular polarization as \( \eta_\mu q^\mu = 0 \) for simplicity, hence only the \( \phi^\mu \) component contributes in the scattering [22, 25, 56]. In the latter case we need to solve only (10). This equation can not be solved analytically and we need to use numerical methods. Let us consider \( \phi_\mu(z, q, y) = \eta_\mu e^{iq.y} \phi_1(z, q) \), and the boundary condition \( \phi_\mu(z, q, y)|_{z=0} = \eta_\mu e^{iq.y} \) in (16). Numerically, we can obtain \( \phi_1(z, q) \) versus \( z \) that describes the behaviour of electromagnetic field in the bulk. Figure 1 shows \( \phi_1(z, q) \) for different values of \( q^2 \) and \( c \). Plot a) shows that at large values of \( q^2 \) increasing of parameter \( c \) does not affect \( \phi_1(z, q) \) significantly. Also near boundary \( (z = 0) \), \( \phi_1 \) has its maximum magnitude. In plot b) at small values of \( q^2 \), increasing of parameter \( c \), increases magnitude of \( \phi_1 \) which is more visible at large values of \( z \). Plot c) is a comparison of \( \phi_1 \) at different values of \( q^2 \) and a fixed \( c \). Obviously the smaller \( q^2 \) is, the stronger \( \phi_1 \) is. In other words the magnitude of electromagnetic field is larger at small values of \( q^2 \).
Figure 1: Electromagnetic field in the bulk with a) large value of $q^2$ and two different values of $c$, b) small value of $q^2$ and two different values of $c$, c) two different values of parameter $q^2$ at fixed value of $c$.

4 Baryonic state equations in deep inelastic scattering

In this section, we study the baryonic initial and final states for further requirements of the interaction action (8). The equation of motion of fermionic states are,

$$(\not{D} - m_5)\Psi = 0,$$  \hspace{1cm} (17)

where $m_5$ is the baryon bulk mass and the operator $\not{D}$ is defined as,

$$\not{D} = g^{mn}e^a_n\gamma_a(\partial_m + \frac{1}{2}\omega_{bc}^{mn}\Sigma_{bc}),$$  \hspace{1cm} (18)

in which $\gamma_\alpha = (\gamma_\mu, \gamma_5), \{\gamma_a, \gamma_b\} = 2\eta_{ab}$ and $\Sigma_{\mu 5} = \frac{1}{2} [\gamma_\mu, \gamma_5]$ \cite{58, 62}. $\gamma_\mu$ are Dirac’s gamma matrices. a, b, c are flat space and, m, n, p, q are AdS space indices respectively. As before $\mu, \nu$ represent
the Minkowski space. With the metric (2) Vielbein are computed as,

\[ e^a_n = \frac{(1 - c^2 z^8)^{\frac{1}{4}}}{z} \delta^a_n, \]

\[ a = t, x_1, x_2, x_3, \]

\[ e^b_n = \frac{1}{z} \delta^b_n, \]

\[ b = z. \]  (19)

The above terms will give us first term of (18). Now we should compute the second term of that. Spin connection is given by,

\[ \omega^{ab}_m = e^a_n \partial_m e^{nb} + e^a_n e^{pb} \Gamma^m_{pn}, \]  (20)

where the Christoffel symbols are,

\[ \Gamma^p_{mn} = \frac{1}{2} g_{pq} \left( \partial_n g_{mq} + \partial_m g_{nq} - \partial_q g_{mn} \right). \]  (21)

From the metric (2), one may write,

\[ g_{\mu\nu} = \sqrt{1 - c^2 z^8} \eta_{\mu\nu}, \]

\[ g_{zz} = \frac{1}{z^2}. \]

So the only non vanishing terms are, \( \Gamma^z_{\mu\nu}, \Gamma^z_{zz}, \Gamma^\mu_{\nu z} \). After computation they are written as,

\[ \Gamma^z_{\mu\nu} = \frac{1}{z^2} \eta_{\mu\nu}, \]

\[ \Gamma^z_{zz} = 1, \]

\[ \Gamma^\mu_{\nu z} = \frac{1}{z^2} \delta^\mu_\nu. \]  (22)

Also from (2) together with (19) and (21) the relation (20) turns to,

\[ \omega^{z\nu}_\mu = -\omega^{\nu z}_\mu = -\frac{(1 + c^2 z^8)}{z^2} \delta^\mu_\nu, \]  (23)

hence other components of \( \omega^{ab}_m \) are zero. Using these solutions, (18) is given by,

\[ \mathcal{D} z = z \gamma^5 \partial_z + \frac{z}{(1 - c^2 z^8)^{\frac{3}{4}}} \gamma^\mu \partial_\mu - 2 \frac{(1 + c^2 z^8)}{z(1 - c^2 z^8)^{\frac{1}{4}}} \gamma^5, \]  (24)

and the EOM (17) is written as,

\[ [z \gamma^5 \partial_z + \frac{z}{(1 - c^2 z^8)^{\frac{3}{4}}} \gamma^\mu \partial_\mu - 2 \frac{(1 + c^2 z^8)}{z(1 - c^2 z^8)^{\frac{1}{4}}} \gamma^5 - m_5] \Psi = 0. \]  (25)

According to the fact that spinor is either left handed or right handed, and since Kaluza-Klein modes are dual to the chirality spinors we decompose these components and expand as,

\[ \Psi_{L/R}(x^\mu, z) = \sum_n f_{L/R}^n(x^\mu) \chi_{L/R}^n(z), \]  (26)
applying (26) in the equation of motion (25) we find the coupled equations as,

\[
(\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)} + \frac{m_5}{z}) \chi_L(z) = \frac{M_n}{(1 - c^2 z^8)^{\frac{3}{4}}} \chi_R(z),
\]

\[
(\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)} - \frac{m_5}{z}) \chi_R(z) = \frac{-M_n}{(1 - c^2 z^8)^{\frac{3}{4}}} \chi_L(z).
\]

(27)

(28)

Decoupling (27) and (28) leads to the following equation which describes both left handed and right handed sectors as,

\[
-(1 - c^2 z^8)^{\frac{1}{4}} \left( \partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)} \pm \frac{m_5}{z} \right) \left( \partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)} \mp \frac{m_5}{z} \right) \chi_{R/L}(z) = M_n^2 \chi_{R/L}(z).
\]

(29)

In continue we make a Schrödinger-like equation by applying a transformation as follows,

\[
\chi_{R/L}(z) = e^{-2(1 - c^2 z^8)^{\frac{1}{4}}} \frac{z}{(1 - c^2 z^8)^{\frac{3}{8}}} \psi_{R/L}(z),
\]

(30)

so the equation (29) is written as,

\[
\sqrt{1 - c^2 z^8} \left( - \psi''_{R/L}(z) + \frac{m_5(m_5 \pm 1) - c^2 z^8(2m_5^2 + 7) + e^4 z^{16} m_5(m_5 \pm 1)}{z^2(1 - c^2 z^8)^2} \psi_{R/L}(z) \right) = M_n^2 \psi_{R/L}(z)
\]

(31)

In (31), \(m_5\) is a parameter in AdS side of gauge/ gravity duality and related to the baryon mass in the gauge side, so the normalizable solutions of the above equations are dual to the states in the boundary theory. In pure AdS space, the bulk mass is related to the canonical conformal dimension \(\Delta_{can}\) of a boundary operator as,

\[
|m_5^{AdS}| = \Delta_{can} - 2.
\]

(32)

Remind that QCD is not a conformal field theory since it has mass gap. Therefore the gravity side should be modified somehow and then it is not pure AdS any more. Modifying AdS, the canonical dimension \(\Delta_{can}\) of an operator has an anomalous contribution of \(\gamma\) which implies an effective scaling dimension,

\[
|m_5| = \Delta_{can} + \gamma - 2.
\]

(33)

Contribution of anomaly is related to how one modifies the theory. For example in modification of the scale introduces the mass gap in the theory. Therefore the anomalous contribution represents the energy scale in the theory and leads to the mass spectra. Hence, the main task is finding value of bulk mass in (33). In AdS/CFT dictionary, the bulk mass is related to the dimension, means the energy scale of the boundary theory is holographically related to the localization in the z coordinate,
therefore we have $z$ dependent mass in the bulk. Let us focus on $m_5$. Numerically, one may fit $m_5$ as the equations (31) have normalizable solutions. Fixing $M$ as proton mass, we should find suitable values for $c$ and $m_5$ which give us well defined answers.

Figures 2 and 3 show initial (n=1) and final (for two excited states as n=2,3) chiral components of the wave function respectively. To solve the equations (31) numerically, we fix proton mass $M$ as eigenvalue of equation, therefore $m_5$ and $c$ are found as $c = 0.0120 \pm 0.0005 \text{GeV}^4$ and $m_5 = 0.081 \text{GeV}$. Interestingly, the value of parameter $c$ in AdS side is very close to the phenomenological GC value of QCD as it has been found $G_2 = 0.010 \pm 0.0023 \text{GeV}^4$ in the reference [18]. Another consequence of presence of $c$ is that the anomaly $\gamma$ in (33) intensively affects the bulk mass. After

![Figure 2](image1)

Figure 2: Left handed (dashed) and right handed (solid) sectors of wave function from (31) for the initial state (target proton), by considering $c = 0.0120 \pm 0.0005 \text{GeV}^4$ and $m_5 = 0.081 \text{GeV}$.

![Figure 3](image2)

Figure 3: Left handed (dashed) and right handed (solid) sectors of wave function from (31) for the final state a) n=2 and b) n=3, by considering $c = 0.0120 \pm 0.0005 \text{GeV}^4$ and $m_5 = 0.081 \text{GeV}$.
finding both left handed and right handed modes from (31) we may consider wave functions,

$$\Psi_i = e^{\frac{2(1-c^2)z^4}{2}} \psi_i^[(1 + \frac{\gamma_5}{2})\psi_i^L + (1 - \frac{\gamma_5}{2})\psi_i^R]u_s_i(p),$$  \hspace{1cm} (34)

as initial wave function for the target proton and,

$$\Psi_X = e^{\frac{2(1-c^2)z^4}{2}} \psi_i^[(1 + \frac{\gamma_5}{2})\psi_i^L + (1 - \frac{\gamma_5}{2})\psi_i^R]u_s_X(p),$$  \hspace{1cm} (35)

as final wave function for hadronic state. These will be used later.

5 Modified geometry and action of deep inelastic scattering

According to (7) and (8) we find interaction action with electromagnetic field and baryonic states obtained from (19) and (31) to (35) respectively. The interaction action (8) is written as,

$$S_{int} = g_V \int dzd^4y e^{-\phi} g_\mu \hat{\Psi}_X \gamma_\mu \Psi_i$$

$$= g_V \int dzd^4y \frac{1}{z}(1-cz^4)^{1+\sqrt{2}}(1+cz^4)^{1-\sqrt{2}} \phi g_\mu \Psi_X \frac{1}{z}(1-c^2z^8)^{1+\sqrt{2}} \delta_\mu \gamma_\alpha \Psi_i$$

$$= g_V \int dzd^4y \frac{1}{z}(1-cz^4)^{1+\sqrt{2}}(1+cz^4)^{1-\sqrt{2}} \phi g_\mu \Psi_X \gamma_\mu \Psi_i,$$  \hspace{1cm} (36)

and from (35) one may write,

$$\Psi_X = e^{\frac{2(1-c^2)z^4}{2}} e^{-iP_X \cdot y} \bar{u}_{sX}(p)[(1 + \frac{\gamma_5}{2})\psi_i^X + (1 - \frac{\gamma_5}{2})\psi_i^R].$$  \hspace{1cm} (37)

Therefore (36) is given by,

$$S_{int} = \frac{g_V}{2} \int dzd^4y e^{-i(P_X - P_y) \cdot y} \phi_\mu \frac{1}{z}(1-cz^4)^{\frac{5}{2}+\sqrt{2}}(1+cz^4)^{\frac{5}{2}-\frac{\sqrt{2}}{2}}$$

$$\left[ \bar{u}_{sX}(\hat{P}_L \psi_i^L + \hat{P}_R \psi_i^R) \gamma_\mu (\hat{P}_L \psi_i^L + \hat{P}_R \psi_i^R)u_{sX} \right]$$

$$= \frac{g_V}{2} (2\pi)^4 \delta_\mu (P_X - P_y) \eta^\mu \int dz \frac{1}{z}(1-cz^4)^{\frac{5}{2}+\sqrt{2}}(1+cz^4)^{\frac{5}{2}-\frac{\sqrt{2}}{2}}$$

$$\left[ \bar{u}_{sX} \gamma_\mu \hat{P}_R u_{sX} \psi_i^X + \bar{u}_{sX} \gamma_\mu \hat{P}_L u_{sX} \psi_i^R \right].$$  \hspace{1cm} (38)
By defining the following integral,

\[ B_{R,L} = \int dz \frac{\sqrt{\frac{(1-cz^2)^2}{(1-c^2z^2)^2} + \frac{1}{2z^2}(1-cz^2)^2 + \sqrt{\frac{1}{z^2}}(1+cz^2)^2}}{\phi_1^R \psi^L_{R,L} \psi^L_{R,L}}, \]  

(39)

is written as,

\[ S_{int} = \frac{g_v}{2}(2\pi)^4 \delta^4(P_X - \mu - \nu)\eta^\mu [\bar{u}_s \gamma_\mu \hat{P}_R u_s B_L + \bar{u}_s \gamma_\mu \hat{P}_L u_s B_R], \]  

(40)

and (7) is written as,

\[ \eta^\mu \eta^\nu W^{\mu\nu} = \eta^2 F_1(q^2, x) + \frac{2x}{q^2} (\eta P)^2 F_2(q^2, x), \]  

(42)

where \( F_1 \) and \( F_2 \) are,

\[ F_1(q^2, x) = \frac{g^2_{eff}}{4} \left[ M_0 M_X B_L B_R + (B_L^2 + B_R^2) \frac{q^2}{4x} + \frac{M^2_X}{2} \right] \frac{1}{M_X}, \]  

(43)
Numerical strategy

As we mentioned in section 4, in the equation of states the eigenvalue of the ground state should be close to the square of the mass of the proton. So, we consider ranges $0 < m_5 < 1 \text{GeV}$ and $0.001 < c < 1 \text{GeV}^4$, while the eigenvalue of the equation of ground state, is in the range from $0.876 \text{GeV}$ to $1 \text{GeV}$ ($M_{\text{proton}} = 0.938 \text{GeV}$). Accordingly we obtain a series of suitable values of parameters $m_5$ and $c$. Their approximate ranges are $0.001 < m_5 < 0.2 \text{GeV}$ and $0.006 < c < 0.02 \text{GeV}^4$. In the next step we look for the appropriate values of $K_{\text{eff}}^2$ as $m_5$ and $c$ satisfy their ranges and our theoretical calculations can be fitted with the experimental data for $F_2$. We continue by focusing on only order $0.01 - 0.04$ of $x$ and small $q^2$. In the form of sweep spectrum, we determine a set of $m_5$ and $c$ and then for each set, we fit the experimental data for $F_2$ to obtain $K_{\text{eff}}^2$ and the error bar. What needs to be mentioned here is that for $m_5$, the search step is 0.01, and for $c$, the search step is 0.001. The optimal values of parameter with smallest error are $m_5 = 0.081 \text{GeV}$, $c = 0.0120 \pm 0.0005 \text{GeV}^4$, $K_{\text{eff}}^2 = 37.3259$. Using this set of parameters, we will get the proton structure function $F_2$ as function of $q^2$.

| $x$    | $m_5/\text{GeV}$ | $c/\text{GeV}^4$ | $K_{\text{eff}}^2$ |
|--------|------------------|------------------|--------------------|
| 0.015  | 0.081            | $0.012 \pm 5 \times 10^{-4}$  | 37.3259            |
| 0.025  |                  |                  |                    |
| 0.04   |                  |                  |                    |

Table 1: Adjustment of parameter $K_{\text{eff}}^2$ at different $x$ with $c = 0.0120 \pm 0.0005 \text{GeV}^4$. Remind that the value of parameter $c$ is demanded by phenomenological value of proton mass.

Figure 4 is a comparison between Jlab Hall C data [63] and our theoretical results. In plots a), b), c) our results have good agreements with experimental data at $x = 0.015$, $x = 0.025$ and $x = 0.04$. Plot d) is the ratio of structure functions $\frac{F_2(1+2x\frac{M_0^2}{q^2})}{2F_1}$ from (46) as it should be, is near one especially at small $q^2$ and for the smallest value of Bjorken variable.
In a holographic description of DIS we found effects of parameter $c$ which appears in the background metric and represents gluon condensation in the boundary theory. Since there is proton target in the scattering, mass of proton and value of parameter $c$ both play important role in this study. One of our main aim was determining the value of $c$ from experimental data. Solving the equation of baryonic wave function numerically, we set the proton mass as eigenvalue to find best values of bulk mass and parameter $c$. In accordance with it, only small values of $c$ lead to well defined answer of the equation, or proton target demands small value of $c$. It could be suggested that since parameter $c$ breaks the conformal symmetry, it’s value represents the confinement. So in our case of study, confinement is not strong.

Electromagnetic field was another quantity we discussed during the mentioned DIS. We showed that parameter $c$ can strengthen the magnitude of this field, especially at small values of $q^2$.

Proceeding by above results we discussed structure functions in the scattering at in comparison with Jlab Hall C data (with order $0.01 - 0.04$ of $x$ and small $q^2$). Numerically, in the form of sweep spectrum we considered suitable set of bulk mass and $c$ already were determined to fit experimental data and the structure function. Then we showed the ratio of structure functions. Our theoretical

6 Conclusions

Figure 4: a), b), c) Comparison between Jlab Hall C data and our theoretical results. Dashed lines are theoretical results and square dots are experimental data. d) Ratio of structure functions.
results are in a good agreement with data, means our model works better for non-large $x$, weak confinement and small momentum.

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