Ultrafast Dynamics of Intrinsic Anomalous Hall Effect in the Noncollinear Antiferromagnet Mn₃Sn

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Recently the large anomalous Hall effect (AHE) in noncollinear antiferromagnets at room temperature has been drawing much attention for fundamental interests in multipole magnetism and topological electronic structure as well as for potential application in terahertz (THz) spintronics. To realize the high-speed electrical readout of the antiferromagnetic information storage by using the AHE, sub-picosecond dynamics of the AHE out of equilibrium must be clarified but has evaded experimental detection. In this work, we conducted optical pump-THz probe spectroscopy of a Weyl antiferromagnet Mn₃Sn thin film and observed ultrafast reduction of the AHE within 100 fs. Intriguingly, even in the helical phase where the AHE increases with increasing temperature, the ultrafast reduction of the AHE also emerged and subsequently the pump-induced increase of the AHE was observed within several picoseconds. We clarified that the ultrafast dynamics of the AHE is dominated by the photoinduced change in the electron distribution rather than that of the spin moment and texture. This work unveils the timescale necessary for measuring magnetism by using the AHE based on the intrinsic Berry-curvature mechanism. Furthermore, our study presents a new approach to identify the microscopic origin of AHE from a perspective of nonequilibrium dynamics.
As a consequence of the broken time-reversal ($\mathcal{T}$) symmetry in ferromagnets, a current flow is deflected perpendicularly to the bias electric field, which is known as the anomalous Hall effect (AHE)\(^1\). Recent theories\(^2\)–\(^6\) and experiments\(^7\)–\(^9\) demonstrated that the AHE in noncollinear Weyl antiferromagnet Mn$_3$X compounds ($X$=Sn, Ge) can be as large as that in ferromagnets despite the absence of net magnetization, which is in stark contrast to the conventional antiferromagnets. As shown in Fig. 1a, the antiferromagnetic phase in Mn$_3$Sn near room temperature is characterized by the cluster octupole moment on the $a$-$b$ plane which explicitly breaks the $\mathcal{T}$ symmetry\(^4\), resulting in the topologically nontrivial band structure with Weyl points near the Fermi energy\(^10\). The large AHE in topological antiferromagnets has opened a new avenue for realizing novel antiferromagnetic spintronics\(^11\)–\(^13\) with less stray field and fast spin rotation at terahertz (THz) frequencies. A recent study on the THz time-domain spectroscopy (THz-TDS) of Mn$_3$Sn revealed that the AHE at THz frequencies is in accordance with the DC AHE and nearly dissipationless at room temperature\(^14\), revealing its potential utility for clarifying the dynamics in magnetic and electronic structures as well as for the high-speed electrical readout of the magnetic order. For implementation in spintronic devices, it would be necessary to determine the amount of time the AHE requires to reflect the magnetic order when the spin configuration is modulated by external stimuli. Therefore, a sub-picosecond time-resolved study of the AHE out of equilibrium is highly demanded but has been still lacking even for any other magnets\(^15\),\(^16\).

The AHE has also attracted interest in terms of its fundamental characteristics because of their topological nature characterized by the Berry phase\(^1\). The intrinsic AHE is solely determined by electron band structure\(^17\), and the intrinsic anomalous Hall conductivity (AHC) $\sigma_{xy}^{\text{int}}$ in the DC limit is obtained by integrating the Berry curvature for the occupied states as

$$
\sigma_{xy}^{\text{int}} = \frac{e^2}{h} \sum n \int \frac{dk}{(2\pi)^3} f(\epsilon_n(k)) B_{n,z}(k),
$$

(1)

where $n$ is a band index, $f(\epsilon_n(k))$ is an electron distribution function, and $B_{n,z}(k)$ is the Berry curvature in momentum space\(^1\). In contrast, the extrinsic AHE originating from the skew scattering\(^18\) or side-jump mechanism\(^19\) is induced by impurities. The origin of the intrinsic or extrinsic effect can be distinguished by the universal scaling law of conductivity owing to their different dependences on scattering\(^20\), whereas certain materials deviate from the scaling\(^21\). Polarization-resolved infrared spectroscopy is another tool to identify the origin of AHE, as the intrinsic AHC spectrum, $\sigma_{xy}^{\text{int}}(\omega)$, can be calculated in theory for comparison with the experiments\(^22\). However, the comparison between the experiment and the first-principle calculation is difficult at the low energy scale of THz frequencies. As an alternative approach, we study the large anomalous response to the photoexcitation change in the electron distribution and the scattering rate. The nonequilibrium dynamics of the AHE may provide valuable information regarding the microscopic view.
In this work, we conduct optical pump-THz probe spectroscopy for a Mn$_3$Sn thin film to investigate the nonequilibrium dynamics of the AHE out of equilibrium. THz-TDS can be used to observe the AHE by detecting a polarization rotation of the THz field$^{14,15,23-29}$ and this can be extended to pump-probe spectroscopy with sub-picosecond time resolution. We show that the AHE is largely suppressed by photoexcitation with a timescale of 100 fs whereas the scattering rate remains almost unchanged. Our experiments clarify that this ultrafast reduction of the AHE is not ascribed to simple demagnetization by heating the spin system but can be accounted for by the instantaneous change in the electron distribution based on a microscopic view with the intrinsic mechanism. Our findings provide deep insights into the nonequilibrium dynamics of the AHE and opens a path for its potential use in applications.

The sample is a polycrystalline Mn$_3$Sn thin film deposited on SiO$_2$ substrates by DC magnetron sputtering$^{30}$. The film thickness of 20 nm is less than the penetration depth (22 nm) of the optical pump so that inhomogeneous excitation can be avoided. The characterization of the sample is described in the Supplementary Information (SI). The temperature dependence of the Hall resistivity $\rho_{xy}$ in DC transport in Fig. 1b shows the antiferromagnetic phase below $T_N \sim 430$ K. By using polarization-resolved THz-TDS$^{14}$, we evaluated the longitudinal and anomalous Hall conductivity spectra $\bar{\sigma}_{xx}(\omega)$ and $\bar{\sigma}_{xy}(\omega)$, as respectively shown in Figs. 1d and 1e. $\bar{\sigma}_{xx}(\omega)$ exhibits a typical Drude response with a scattering rate of $\sim$10 fs$^{14,31}$. Re$\bar{\sigma}_{xy}(\omega)$ is flat and as large as 10 $\Omega^{-1}$cm$^{-1}$ and Im$\bar{\sigma}_{xy}(\omega)$ is negligibly small, showing that the THz AHE is also within the DC limit$^{14}$. The circles on the left axes are the data obtained in the DC transport with values lower than the THz conductivity. This can be attributed to the effects of grain boundaries or surface roughness in the DC transport in such a very thin sample.

Figure 1c shows a schematic of the pump-probe setup. The THz pulse was linearly polarized along the x-direction before entering the sample and detected by the THz-TDS with the transmission geometry. By changing the configuration of a wire-grid polarizer (WGP2), the x- and y-components of the THz field after transmittance by the sample can be obtained separately$^{32}$. By scanning the THz pulse delay time $t_{\text{probe}}$, the THz pulse waveform was evaluated and converted to the frequency domain. The optical pump pulses with 1.55-eV photon energy and 40-fs pulse duration irradiate the sample with a controllable time delay $t_{\text{pump}}$. We performed the experiments under the magnetic field of +2 and -2 T and obtained $E_x$ and $E_y$ of the THz field as even and odd components for the magnetic field, respectively, and evaluated the longitudinal and Hall conductivity spectra $\bar{\sigma}_{xx}(\omega)$ and $\bar{\sigma}_{xy}(\omega)$. More details are provided in SI.

First, we conducted the pump-probe experiment at 220 K at which the AHE is maximized. Figure 2a shows $E_x$ and $E_y$ of the THz pulse waveform after transmittance by the sample without the pump. To investigate the ultrafast dynamics, we set the probe delay equal to the
peak \((t_{\text{probe}} = t_0 \text{ in Fig. } 2a)\) and scanned the pump delay \(t_{\text{pump}}\) to detect the change in \(E_y\) induced by the pump, \(\delta E_y\). Figure 2b shows the results of \(\delta E_y\) as a function of \(t_{\text{pump}}\) with pump fluences \(I_p=300\) and 500 \(\mu\)J cm\(^{-2}\). Immediately after the pump, \(E_y\) rapidly decreased, and subsequently relaxed within a few picoseconds. We fitted the data and evaluated the rising times of \(E_y\) as 50 and 120 fs for \(I_p=300\) and 500 \(\mu\)J cm\(^{-2}\), respectively, which are comparable to the time resolution limited by the pulse duration of 40 fs. To obtain the spectral information, we set \(t_{\text{pump}}\) to 0.52 ps and scanned \(t_{\text{probe}}\). Figure 2c shows \(\text{Re} \tilde{\sigma}_{xy}(\omega)\) for different pump fluences. We found that \(\text{Re} \tilde{\sigma}_{xy}(\omega)\) decreases by 40% for \(I_p=500\) \(\mu\)J cm\(^{-2}\). Moreover, we also investigated the change in \(E_x\) and found the change in \(\tilde{\sigma}_{xx}(\omega)\) to be negligibly small (see SI). A fitting with the Drude model suggested that the change in the scattering time by the pump is only 3% at most.

Here we discuss the results from the viewpoint of the spin dynamics. Previously ultrafast dynamics of magnetism has been studied by using time-resolved magneto-optical Kerr effect (TR-MOKE) or magnetic circular dichroism, etc\(^{33}\). Since the discovery of ultrafast demagnetization within 100-300 fs in ferromagnets such as Ni\(^{34,35}\), many efforts have been devoted to clarifying its microscopic behavior with possible spin-flip scattering mechanisms\(^{36,37}\). Phenomenologically it was described by heating the spin temperature\(^{34}\). In addition, superdiffusive transport could be another origin of ultrafast demagnetization\(^{38}\). Our experiment, however, was performed in transmission geometry for a single-layer film that is thinner than the pump penetration depth and the probe size is as large as 6 mm so that the spin transport away from the probe spot is hardly anticipated. Therefore, the spin-transport mechanism was excluded in this work. A key factor to be considered for ultrafast demagnetization is (i) the way in which the spin angular momentum is transferred to other degrees of freedom\(^{39,40}\). In addition, (ii) the itinerant or localized nature of electrons is also crucial as the demagnetization of localized magnetism can be slower than itinerant magnets by a few orders of magnitude\(^{37}\). From these perspectives, Mn\(_3\)Sn is a unique and nontrivial system: (i) the net magnetization is fully compensated and the magnetic order is determined by the cluster octupole, and (ii) Mn\(_3\)Sn hosts itinerant \(d\) electrons but the spin moments are strongly localized at the Mn sites, which positions the system in the intermediate regime between the local spin moment and itinerant electrons\(^{41}\). A recent experiment for Mn\(_3\)Sn has reported an ultrafast build-up of the MOKE signal\(^{42}\), but the occurrences within 1 ps remained unexplored. It is also to be noted that an ultrafast TR-MOKE signal does not necessarily correspond to a change in magnetism because a change in the electron system can contribute to the TR-MOKE\(^{43-46}\).

In an attempt to gain further insights for nonequilibrium dynamics in Mn\(_3\)Sn, we considered the peculiar temperature dependence of magnetism. Below 220 K, a spin reorientation phase transition occurs from the cluster octupole phase to the helical phase\(^{47}\) where the cluster moment gradually rotates along the \(c\)-axis with mitigating a degree of the
broken $T$ symmetry. As shown in Fig. 2d, the AHE near 150 K increases with increasing temperature, a trend that is opposite to that in usual magnets. Therefore, if the photoexcitation at 150 K heats the spin system, not the photo-induced demagnetization but photo-induced “magnetization” is expected to occur with an increase in $\sigma_{xy}$. To examine it, we conducted the pump-probe experiment at 150 K. The result of $\delta E_y$ as a function of $t_{\text{pump}}$ is shown in Fig. 2e. Interestingly, we observed that $\delta E_y$ first became negative with the timescale of ~100 fs similarly to the result at 220 K. A few picoseconds later, the signal in turn became positive by flipping its sign. By fixing $t_{\text{pump}}$ to 0.52 and 12 ps and by scanning $t_{\text{probe}}$, we obtained Re$\Delta\tilde{\sigma}_{xy}(\omega)$ in Figs. 2f and 2g, respectively. The results clearly showed that the AHE first decreases, whereupon it changes direction to increase. At $t_{\text{pump}}$=12 ps, the AHE was increased by ~6 $\Omega^{-1}$cm$^{-1}$ for $I_p$=500 $\mu$J cm$^{-2}$, corresponding to an increase in the temperature by ~40 K in thermal equilibrium.

We used the two-temperature model to evaluate photoinduced heating. Figure 3a shows the simulation results at the original temperatures of 220 and 150 K and $I_p$=500 $\mu$J cm$^{-2}$ (see SI). The optical pump excites electrons into higher energy bands, and these electrons are expected to immediately form a quasi-thermalized distribution with an electron temperature $T_e$. The maximum $T_e$ is as high as 700 and 630 K for the pump at 220 and 150 K, respectively. Subsequently, electron-phonon coupling elevates the lattice temperature $T_L$ such that the two systems are equilibrated within several picoseconds with an increase in the temperature of ~40 K, which is in good agreement with the increase in AHE observed in Fig. 2g. Therefore, the positive sign of $\delta E_y$ around ~10 ps in Fig. 2e is attributed to pump-induced “magnetization” as a result of heating the sample from 150 K. In contrast, the ultrafast reduction of Re$\tilde{\sigma}_{xy}(\omega)$ within 100 fs in Fig. 2f cannot be explained by the change in the spin configuration. The same argument also holds at 220 K, at which the pump would induce “demagnetization” that should appear around 10 ps in Fig. 2b. However, the increase in the temperature by 40 K has a minor effect on the AHE at 220 K as shown in Fig. 2d and therefore the demagnetization is not clearly identified in Fig. 2b. Meanwhile, the ultrafast reduction of Re$\tilde{\sigma}_{xy}(\omega)$ within 100 fs is not explained by the demagnetization and requires an analysis on a more microscopic level.

Because the spin is a degree of freedom of electrons, an ultrafast change in the electron distribution itself simultaneously may alter the spin moment and texture. To evaluate it, we calculated the band structure of Mn$_3$Sn by using density functional theory (DFT), as shown in Fig. 3b. By considering finite electron temperatures, we also calculated the local spin moment which is proportional to the cluster octupole moment$^4$. The result is shown in Fig. 3c. Note that a previous angular-resolved photoemission spectroscopy in a comparison with the DFT calculation revealed that the band structure of Mn$_3$Sn in the vicinity of $E_F$ is renormalized by a factor of 5 owing to strong many-body correlation$^{10}$. Therefore, $T_e$=700 K corresponds to $T_{\text{DFT}}$=3,500 K in the calculation. Figure 3c shows that the increase from $T_e$=220 to 700 K ($T_{\text{DFT}}$=1,100 to 3,500 K) is accompanied by a decrease in the local spin moment, although only
by 6%. Such insensitivity of the local spin moment to \( T_e \) can be explained by the projected density of states for the cluster octupole moment as shown in the right panel of Fig. 3b\(^4\). The result shows that the spin configuration in the cluster octupole is determined mostly by the electrons of which the energy is a few eV below \( E_F \). Therefore, the change in the electron distribution of the scale of \( T_{\text{DFT}}=3,500 \) K (\(-0.3 \) eV) does not immediately alter the spin moment.

Based on the above arguments, we consider the origin of the ultrafast reduction of the AHE within 100 fs. We focus on the well-defined cluster octupole phase at 220 K and its pumped effect because it is difficult to reproduce the band structure in the helical phase at 150 K in the calculation. We consider the temperature of 220 K of the electron system to be elevated to 700 K by the pump whereas the lattice and spin systems remain almost unperturbed. This means that photoexcitation does not modify the band structure and only broadens the electron distribution, which allows us to calculate the intrinsic AHE by using Eq. (1). The curves in Fig. 3d show the results of the calculated \( \sigma_{xy}^{\text{int}} \) normalized at the value of 220 K as a function of \( T_e=T_{\text{DFT}}/5 \). The different colors correspond to different chemical potentials. The composition of Mn in our sample indicates that the chemical potential is close to zero. The markers in Fig. 3d show the experimental results of \( \sigma_{xy} \) at 220 K for \( I_p=160, 300, \) and 500 \( \mu \)J cm\(^{-2}\), corresponding to \( T_e=430, 560, \) and 700 K, respectively. The result shows that the photoinduced ultrafast decrease of AHE is reasonably accounted for by the calculated intrinsic AHE with the elevated electron temperatures. The experimental data in Fig. 3d show larger reductions in the AHE than those in the calculation. This result suggests that even the electron system might not be fully thermalized within 100 fs, which can further suppress the AHE.

The results show that the intrinsic AHE is sensitive to the rise of \( T_e \), as can be understood from Eq. (1). Nominally all the electron states below \( E_F \) can contribute to the AHE. However, the Berry curvature are mainly located at around the (anti)crossing points in the band structure and the upper and lower branches of each (anti)crossing point have Berry curvatures with opposite signs. Therefore, the contribution of the states much below \( E_F \) is mostly cancelled by the integration in Eq. (1), and the (anti)crossing points near \( E_F \) play a dominant role in the AHE as schematically shown in Fig. 3e. Therefore, the rise in \( T_e \) to several hundred K can largely modify the intrinsic AHE and dominates its ultrafast dynamics. Because the sensitiveness of the intrinsic AHE to \( T_e \) is expected to generally occur in other magnets, this work opens a new approach to distinguish the microscopic origins of AHE from the nonequilibrium dynamics. Importantly, the photoexcitation in Mn\(_3\)Sn largely decreased the AHE whereas the scattering rate was almost unchanged, which also excludes the extrinsic mechanism by impurity scattering.

It should be emphasized that the pump-probe result in Fig. 2e does not mean that the magnetic order changes slowly over several picoseconds. The magnetic order might change in as short time as 1 ps, but we could not determine it because the AHE is dominated by the effect
due to the elevated $T_e$ in the fast time scale. The spin dynamics must be carefully decomposed using other methods such as two-color TR-MOKE\textsuperscript{45}. The present work found that the AHE reflects the magnetic order soon after $T_e$ equilibrate with $T_L$. Therefore, to use the AHE for electrical readout of the magnetic order, suppressing the rise in $T_e$ during the writing process is crucial to overcome the limit of speed. Recently current-induced spin reversal has been realized in Mn$_3$Sn\textsuperscript{48}, and another control method with a reduced current is also proposed\textsuperscript{49}. These methods, which involve rotating the spin moment by current-induced torque, should suppress the increase in $T_e$, which may realize high-speed electrical control and readout of magnetic information.

**Methods**

All information for our methods can be found in the Supplementary Information.

**Data Availability**

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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**Author contributions**

R.M. conceived this project. T.H., H.P., T. Matsuo, and S.N. fabricated the sample and characterized it in the DC measurement. T. Matsuda performed the THz spectroscopy experiments and analysis with helps of N.K., Y.H., N.Y., R.S., and R.M. T.K. conducted the DFT calculation. All the authors discussed the results. T. Matsuda and R.M. wrote the manuscript with substantial feedbacks from all the coauthors.

**Additional Information**
Supplementary Information is available in the online version of the paper.

Figure captions

Fig. 1 | Schematics and characterization of the Mn₃Sn thin film. a, Schematics of the atom configuration (left) and the spin texture on the Kagome bilayer (right) in Mn₃Sn. The hexagon shows the cluster octupole moment. b, Temperature dependence of the anomalous Hall resistivity $\rho_{xy}$ in DC transport. c, A schematic of the optical pump and polarization-resolved terahertz (THz) probe setup. WGP: wire-grid polarizer. d,e, THz longitudinal and anomalous Hall conductivity spectra, $\tilde{\sigma}_{xx}(\omega)$ and $\tilde{\sigma}_{xy}(\omega)$, respectively, at 280 K. The circles on the left axes are the values in DC.

Fig. 2 | Ultrafast dynamics of the AHE studied by optical pump-THz probe spectroscopy. a, x- and y-components of the THz pulse waveforms ($E_x$ and $E_y$) after transmittance by the sample at 220 K. The vertical line defines $t_{\text{probe}} = t_0$ at the peak of $E_y$. b, Pump-induced change of $E_y$, $\delta E_y$, at $t_{\text{probe}} = t_0$ as a function of $t_{\text{pump}}$ at 220 K. The data for different pump fluences are shown with offset. The solid curves show results of fitting. c, The Hall conductivity spectra $\text{Re}\tilde{\sigma}_{xy}(\omega)$ at $t_{\text{pump}}=0.52$ ps after the pump at 220 K for different pump fluences. d, Temperature dependence of $\text{Re}\tilde{\sigma}_{xy}(\omega)$ measured by polarization rotation of the THz pulse. The open circles (ZFC) are the results in zero-field cooling. The closed circles (FC) are obtained by the results at ±2 T. e, Pump-induced change of $E_y$ at $t_{\text{probe}} = t_0$ as a function of $t_{\text{pump}}$ at 150 K. The data for different pump fluences are shown with offset. The solid curves show results of fitting. f,g, The change of $\text{Re}\tilde{\sigma}_{xy}(\omega)$ at $t_{\text{pump}}=0.52$ and 12 ps, respectively, after the pump at 150 K. The error bars in c,f,g indicate the standard deviations for the statistical fluctuation after repeating the measurements.

Fig. 3 | Two-temperature model analysis and DFT calculation. a, Expected temperatures of the electron and lattice systems ($T_e$ and $T_L$) in the two-temperature model analysis at the original temperatures of 220 and 150 K and $I_p=500$ μJ cm⁻². b, (left) The band structure of Mn₃Sn at the cluster octupole phase obtained by the DFT calculation. (right) The projected density of states for the spin configuration of the cluster octupole. c, The local spin moment in the DFT calculation with finite temperatures. d, The circles show the Hall conductivity $\text{Re}\tilde{\sigma}_{xy}(\omega)$ observed in the pump-probe experiment at $t_{\text{pump}}=0.52$ ps at 220 K as a function of the maximum $T_e$. The curves show the calculated $\tilde{\sigma}_{xy}^{\text{int}}$ in Eq. (1) with various chemical potentials as a function of $T_{\text{DFT}}$. $T_{\text{DFT}}$ is reduced to $T_e$ by a factor of 5 for considering the band renormalization. The data are normalized at the value of 220 K. e, (left) A schematic of the Berry curvature around anticrossing points. (right) Occupation of electrons with low (blue) and high (red) temperatures. The error bars in d indicate the standard deviations for the statistical fluctuation after repeating the measurements.
Figure 1
Figure 2
Figure 3
References

1. Nagaosa, N., Sinova, J., Onoda, S., MacDonald, A. H. & Ong, N. P. Anomalous Hall effect. *Rev. Mod. Phys.* **82**, 1539-1592 (2010).

2. Chen, H., Niu, Q. & MacDonald, A. H. Anomalous Hall Effect Arising from Noncollinear Antiferromagnetism. *Phys. Rev. Lett.* **112**, 017205 (2014).

3. Kübler, J. & Felser, C. Non-collinear antiferromagnets and the anomalous Hall effect. *EPL (Europhysics Letters)* **108**, 67001 (2014).

4. Suzuki, M. T., Koretsune, T., Ochi, M. & Arita, R. Cluster multipole theory for anomalous Hall effect in antiferromagnets. *Phys. Rev. B* **95**, 094406 (2017).

5. Zhang, Y. *et al.* Strong anisotropic anomalous Hall effect and spin Hall effect in the chiral antiferromagnetic compounds Mn$_3$X (X=Ge, Sn, Ga, Ir, Rh, and Pt). *Phys. Rev. B* **95**, 075128 (2017).

6. Hao, Y. *et al.* Topological Weyl semimetals in the chiral antiferromagnetic materials Mn$_3$Ge and Mn$_3$Sn. *New J. Phys.* **19**, 015008 (2017).

7. Nakatsuji, S., Kiyohara, N. & Higo, T. Large anomalous Hall effect in a non-collinear antiferromagnet at room temperature. *Nature* **527**, 212-215 (2015).

8. Kiyohara, N., Tomita, T. & Nakatsuji, S. Giant Anomalous Hall Effect in the Chiral Antiferromagnet Mn$_3$Ge. *Phys. Rev. Appl.* **5**, 064009 (2016).

9. Nayak, A. K. *et al.* Large anomalous Hall effect driven by a nonvanishing Berry curvature in the noncolinear antiferromagnet Mn$_3$Ge. *Sci. Adv.* **2**, e1501870 (2016).

10. Kuroda, K. *et al.* Evidence for magnetic Weyl fermions in a correlated metal. *Nat. Mater.* **16**, 1090-1095 (2017).

11. Jungwirth, T., Marti, X., Wadley, P. & Wunderlich, J. Antiferromagnetic spintronics. *Nat. Nanotech.* **11**, 231-241 (2016).

12. Smejkal, L., MacDonald, A. H., Sinova, J., Nakatsuji, S. & Jungwirth, T. Anomalous Hall antiferromagnets. *Nat. Rev. Mater.* (2022).

13. Nakatsuji, S. & Arita, R. Topological Magnets: Functions Based on Berry Phase and Multipoles. *Ann. Rev. Condens. Matter. Phys.* **13**, 119-142 (2022).

14. Matsuda, T. *et al.* Room-temperature terahertz anomalous Hall effect in Weyl antiferromagnet Mn$_3$Sn thin films. *Nat. Commun.* **11**, 909 (2020).

15. Huisman, T. J. *et al.* Sub-100-ps dynamics of the anomalous Hall effect at terahertz frequencies. *Phys. Rev. B* **95**, 094418 (2017).

16. Sala, G., Krizakova, V., Grimaldi, E., Lambert, C.-H., Devolder, T., & Gambardella, P., Real-time Hall-effect detection of current-induced magnetization dynamics in ferrimagnets. *Nat. Commun.* **12**, 656 (2021)

17. Karplus, R. & Luttinger, J. M. Hall Effect in Ferromagnetics. *Phys. Rev.* **95**, 1154-1160 (1954).

18. Smit, J. The spontaneous hall effect in ferromagnetics II. *Physica* **24**, 39-51 (1958).
Berger, L. Side-Jump Mechanism for the Hall Effect of Ferromagnets. *Phys. Rev. B* **2**, 4559-4566 (1970).

Onoda, S., Sugimoto, N. & Nagaosa, N. Intrinsic Versus Extrinsic Anomalous Hall Effect in Ferromagnets. *Phys. Rev. Lett.* **97**, 126602 (2006).

Fujishiro, Y. *et al.* Giant anomalous Hall effect from spin-chirality scattering in a chiral magnet. *Nat. Commun.* **12**, 317 (2021).

Fang, Z. *et al.* The Anomalous Hall Effect and Magnetic Monopoles in Momentum Space. *Science* **302**, 92-95 (2003).

Shimano, R. *et al.* Terahertz Faraday rotation induced by an anomalous Hall effect in the itinerant ferromagnet SrRuO$_3$. *EPL (Europhysics Letters)* **95**, 17002 (2011).

Okamura, Y. *et al.* Giant magneto-optical responses in magnetic Weyl semimetal Co$_3$Sn$_2$S$_2$. *Nat. Commun.* **11**, 4619 (2020).

Khadka, D. *et al.* Kondo physics in antiferromagnetic Weyl semimetal Mn$_{3+x}$Sn$_{1-x}$ films. *Sci. Adv.* **6**, eabc1977 (2020).

Bhandia, R. *et al.* THz-range Faraday rotation in the Weyl semimetal candidate Co$_2$TiGe. *J. Appl. Phys.* **128**, 244303 (2020).

Hayashi, Y. *et al.* Magneto-optical spectroscopy on Weyl nodes for anomalous and topological Hall effects in chiral MnGe. *Nat. Commun.* **12**, 5974 (2021).

Seifert, T. S. *et al.* Frequency-Independent Terahertz Anomalous Hall Effect in DyCo$_5$, Co$_{32}$Fe$_{68}$, and Gd$_{27}$Fe$_{73}$ Thin Films from DC to 40 THz. *Adv. Mater.* **33**, 2007398 (2021).

Han, X., Markou, A., Stensberg, J., Sun, Y., Felser, C., & Wu, L., Giant intrinsic anomalous terahertz Faraday rotation in the magnetic Weyl semimetal Co$_2$MnGa at room temperature,. *Phys. Rev. B* **105**, 174406 (2022).

Higo, T. *et al.* Anomalous Hall effect in thin films of the Weyl antiferromagnet Mn$_3$Sn. *Appl. Phys. Lett.* **113**, 202402 (2018).

Cheng, B. *et al.* Terahertz conductivity of the magnetic Weyl semimetal Mn$_3$Sn films. *Appl. Phys. Lett.* **115**, 012405 (2019).

Kanda, N., Konishi, K. & Kuwata-Gonokami, M. Terahertz wave polarization rotation with double layered metal grating of complimentary chiral patterns. *Opt. Express* **15**, 11117-11125 (2007).

Kirilyuk, A., Kimel, A. V. & Rasing, T. Ultrafast optical manipulation of magnetic order. *Rev. Mod. Phys.* **82**, 2731-2784 (2010).

Beaurepaire, E., Merle, J. C., Daunois, A. & Bigot, J. Y. Ultrafast Spin Dynamics in Ferromagnetic Nickel. *Phys. Rev. Lett.* **76**, 4250-4253 (1996).

Stamm, C. *et al.* Femtosecond modification of electron localization and transfer of angular momentum in nickel. *Nat. Mater.* **6**, 740-743 (2007).

Koopmans, B., Ruigrok, J. J. M., Longa, F. D. & de Jonge, W. J. M. Unifying Ultrafast Magnetization Dynamics. *Phys. Rev. Lett.* **95**, 267207 (2005).
Koopmans, B. et al. Explaining the paradoxical diversity of ultrafast laser-induced demagnetization. *Nat. Mater.* 9, 259-265 (2010).

Battato, M., Carva, K. & Oppeneer, P. M. Superdiffusive Spin Transport as a Mechanism of Ultrafast Demagnetization. *Phys. Rev. Lett.* 105, 027203 (2010).

Dornes, C. et al. The ultrafast Einstein–de Haas effect. *Nature* 565, 209-212 (2019).

Tauchert, S. R. et al. Polarized phonons carry angular momentum in ultrafast demagnetization. *Nature* 602, 73-77 (2022).

Battiato, M., Carva, K. & Oppeneer, P. M. Superdiffusive Spin Transport as a Mechanism of Ultrafast Demagnetization. *Phys. Rev. Lett.* 105, 027203 (2010).

Dornes, C. et al. The ultrafast Einstein–de Haas effect. *Nature* 565, 209-212 (2019).

Tauchert, S. R. et al. Polarized phonons carry angular momentum in ultrafast demagnetization. *Nature* 602, 73-77 (2022).

Cable, J. W., Wakabayashi, N. & Radhakrishna, P. Magnetic excitations in the triangular antiferromagnets Mn₃Sn and Mn₃Ge. *Phys. Rev. B* 48, 6159-6166 (1993).

Miwa, S. et al. Giant Effective Damping of Octupole Oscillation in an Antiferromagnetic Weyl Semimetal. *Small Sci.* 1, 2000062 (2021).

Koopmans, B., van Kampen, M., Kohlhepp, J. T. & de Jonge, W. J. M. Ultrafast Magneto-Optics in Nickel: Magnetism or Optics? *Phys. Rev. Lett.* 85, 844-847 (2000).

Kampfrath, T. et al. Ultrafast magneto-optical response of iron thin films. *Physical Review B* 65, 104429 (2002).

Kojima, E. et al. Observation of the spin-charge thermal isolation of ferromagnetic Ga₀.⁹₄Mn₀.₀₆As by time-resolved magneto-optical measurements. *Phys. Rev. B* 68, 193203 (2003).

Oppeneer, P. M. & Liebsch, A. Ultrafast demagnetization in Ni: theory of magneto-optics for non-equilibrium electron distributions. *J. Phys.: Condens. Matter* 16, 5519-5530 (2004).

Ohmori, H., Tomiyoshi, S., Yamauchi, H. & Yamamoto, H. Spin structure and weak ferromagnetism of Mn₃Sn. *J. Mag. Mag. Mater.* 70, 249 (1987).

Tsai, H. et al. Electrical manipulation of a topological antiferromagnetic state. *Nature* 580, 608-613 (2020).

Takeuchi, Y. et al. Chiral-spin rotation of non-collinear antiferromagnet by spin–orbit torque. *Nat. Mater.* 20, 1364-1370 (2021).