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To cite this article: Xue Chen and Helong Yang 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **472** 012069

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Full Waveform Inversion Based on Wavefield Correlation

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Abstract. Low frequencies and initial model play a crucial role in full waveform inversion (FWI). However, in traditional seismic exploration, the frequency range of signals recorded by receivers is limited, and a good initial model for FWI is not available easily, leading to FWI algorithms easily fall into local minima. We propose a wavefield correlation method to improve cycle skipping issue of FWI. Test of 2D frequency domain FWI based on Marmousi model shows that the result of inversion has a better result than conventional method.

1. Introduction

Full waveform inversion (FWI) is an optimization procedure of gradually minimizes the waveform difference between modeled data and observed data. The wellknown advantage of FWI is the ability to obtain high resolution and accurate velocity model in complex structure areas, and has recently gained wide interests with specific application in exploration and producing.

As FWI algorithms are a procedure of local optimization, both frequency domain FWI and time domain FWI will be easily trapped in local minima when cycle skipping occurs, leading to the failure of obtaining an accurate velocity model. Therefore, many efforts have been made to solve cycle skipping issue. The most straightforward strategy is the two-step velocity model building [1], that is, first obtain a smooth velocity model with a traditional traveltime tomography, and then FWI is applied on this smooth model to get a more accurate velocity model. Another strategy is the combination of wave equation tomography and full waveform inversion [2-3], but huge computational cost is the drawback of this method. Shin and Cha (2008) proposed Laplace domain FWI algorithms[4], which can recover large scale structures without low frequency data, but this method is sensitive to signal-to-noise ratio and the accuracy of source estimation. Bi and Lin  (2014) developed an effective cycle skipping reduction strategy through adaptive data selection for full waveform inversion[5], which assures all input data for FWI is within a half-cycle difference compared with the predicted data by discarding bad traces and muting bad data.

In this paper, we proposed a novel FWI method based on wavefield correlation to overcome cycle skipping issue. We perform conventional FWI method and wavefield correlation FWI method on Marmousi model. Numerical tests show that the new FWI method can obtain a higher resolution result without low frequencies in seismic data, while conventional FWI is trapped in local minima due to cycle skipping issue.

2. Full Waveform Inversion Theory

Seismic forward modeling is an important part of FWI. FWI algorithm is an iterative process, and each iteration of inversion should carry out the forward modeling at least twice (for the incident wavefield and the back-propagated residual wavefield), so that the accuracy and costs of the inversion highly depend on the forward modeling. In this paper, we choose the finite difference method based on the two dimensional frequency-domain acoustic wave equation [6]:
\[
\frac{\omega^2}{\kappa(x,z)} p(x,z,\omega) + \frac{\partial}{\partial x} \left( \frac{1}{\rho(x,z)} \frac{\partial p(x,z,\omega)}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\rho(x,z)} \frac{\partial p(x,z,\omega)}{\partial z} \right) = -s(x,z,\omega),
\]

where \( p(x,z,\omega) \), \( \kappa(x,z) \), and \( s(x,z,\omega) \) denote the wavefield, bulk modulus, and source term, respectively, \( \omega \) is angle frequency and \( \rho(x,z) \) denotes the density.

In the inversion problem, misfit function \( C(m) \) is expressed as the difference between real seismic data \( d^{\text{obs}} \) and simulated seismic data \( d^{\text{cal}} \):

\[
C(m) = \frac{1}{2} (d^{\text{obs}} - d^{\text{cal}})^T (d^{\text{obs}} - d^{\text{cal}}) = \frac{1}{2} \|d^{\text{obs}} - d^{\text{cal}}\|^2,
\]

where \( ^T \) denotes conjugate transpose; \( \| \| \) denotes \( l_2 \) norm, and \( m \) is geological model, such as subsurface velocity model.

3. Wavefield Correlation Full Waveform Inversion Method

The process of local optimization is to find the minimum value of the misfit function around the starting model \( m_0 \), which requires \( m_0 \) is in the vicinity of true model. If not, or seismic data lacks low frequency components, FWI algorithms will be easily trapped in local minima and yield a bad result. However, low frequency components and good starting model are not always available in the application of exploration and producing. In this paper we propose a novel method to mitigate the effect of cycle skipping issue, named wavefield correlation full waveform inversion. Here we assume the model is elastic and with no attenuation, and the phase don’t change along wave propagation:

(1). Compute cross-correlation coefficients of the phase between calculated data and observed data trace by trace before inversion:

\[
C_r(\tau) = \frac{1}{N} \sum_{i=1}^{N} \phi_i^{\text{obs}}(x,t)\phi_i^{\text{cal}}(x,t+\tau), i=1,2,...,N,
\]

where \( C_r \) is cross-correlation coefficients, \( \phi \) denotes phase, \( i \) denotes trace number, and \( N \) denotes total trace number.

(2). Select the value of \( \tau \) when \( C_r(\tau) \) is maximum. And shift the modeled data as follow:

\[
d_i^{\text{cal}}(x,t) = d_i^{\text{cal}}(x,t + \tau).
\]

where \( d_i^{\text{cal}} \) is the shifted calculated data. In frequency domain, equation (3) can be written as:

\[
D_i^{\text{cal}} = D_i^{\text{cal}} \cdot e^{i\omega\tau},
\]

where \( D_i^{\text{cal}} \) is frequency domain calculated data, and \( D_i^{\text{cal}} \) is the shifted frequency domain calculated data.

Thus, the deformation between calculated data and observed data caused by model perturbation is reduced after shifting, which mitigate the probability of cycle skipping.

4. Numerical Test

We demonstrate our new method on a modified 2D Marmousi P-wave velocity model. We add a water layer above and don’t update the water layer during the nonlinear procedure. Assuming the model is elastic and with no attenuation. Figure 1(a) and 1(b) are true model and starting model, respectively. The starting velocity model is a smoothed version of true model. We use a Ricker wavelet with peak frequency of 12Hz. The source is located on the surface with the interval 24m. The grid size is 24m and time step is 4ms. All the grid points on the surface acts as receivers. The inversion frequency band is 7~30Hz. Figure 2(a) is the result of conventional FWI method, and figure 2(b) is the result of wavefield correlation FWI method. Figure 3 shows the 1D vertical velocity curves extracted from inversion results (location at receiver No: 241) by different methods. As we start the inversion at a
relatively high frequency, conventional FWI method fails to obtain a reliable result due to cycle skipping issue, while our new method obtains a higher resolution result.

![Figure 1](image1.png)

**Figure 1.** FWI model: (a) True velocity model; (b) Starting velocity model for FWI.

![Figure 2](image2.png)

**Figure 2.** Inversion result of FWI: (a) conventional FWI method; (b) wavefield correlation FWI method.
Figure 3. 1D vertical velocity curves extracted from inversion models (location at receiver No: 241) by different methods (Black line represents the starting model, green line represents the real model, and red line and blue line represent the conventional FWI model and wavefield correlation FWI model respectively).

5. Conclusion
The advantage of FWI is the high resolution of the inversion result by using a large quantity of waveform information. But many weaknesses are still existed in FWI, such as easy to get stuck in local minima and cycle-skipping if the starting model is too far from the actual model or seismic data lacks of low frequencies.

In this paper we propose a wavefield phase correlation shifting FWI method to mitigate the effect of cycle skipping issue on FWI. Seismic wavefield perturbation caused by inaccurate velocity model can be regarded as a ‘deformation’ deviating from the observed wavefield, and FWI inversion will fall into local minima if the deformation is too large and seismic data without low frequencies. So we can estimate the position of this deformation by correlation algorithm and shift the simulated data to reduce the deformation before inversion. Numerical examples on Marmousi model show that the new
method can mitigate the effect of cycle skipping issue and obtain a better result with the absence of low frequencies than conventional FWI method.

Acknowledgments
We would like to thank Scientific Research Foundation of Hebei GEO University (BQ 2017026) for the funding support.

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