Evaluation of the fatigue crack growth rate models under the action of tensile-compressive cyclic loading

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Abstract: In order to study the influence of compressive stress in cyclic load on the crack growth of the rib-to-deck joint, this paper analyzes the surface crack growth at the rib-to-deck joint under the action of overloaded vehicles based on the local three dimensional fracture mechanics model of the weld. The results show that the reverse plastic zone model is a better model to account for the effect of compressive stress in cyclic loading on the crack growth compared with the Paris Law either using the maximum tensile stress or using the full stress amplitude in cyclic loading.

1. Introduction
The orthotropic steel deck is generally welded by bridge deck, longitudinal rib and beam. The weld of orthotropic steel deck is prone to fatigue cracks due to its complex structure, the residual stress caused by welding, the defects of the structure itself, and the repeated action of the wheel load (Tong and Shen, 2000; Fricke, 2003; Zhang et al., 2013). The weld joint of longitudinal rib and bridge deck is most easily to appear fatigue cracks, and is the most dangerous weld crack (Kiss and Dunai, 2002; Xiao et al., 2008; Shi et al., 2013). In China, the overload phenomenon on highway bridges is commonly existing because of the relatively lagging design standards and the pursuit of maximizing economic benefits. The stress and stress amplitude at the rib-to-deck joint would increase significantly under the action of overload vehicles, thus accelerating the fatigue damage of the steel deck weld. Therefore, the crack growth analysis of rib-to-deck joint of orthotropic steel deck under the action of overloaded vehicles is particularly important.

The rib-to-deck joint of orthotropic steel deck is subjected to tension and compression cyclic load dominated by compressive stress, the fatigue cumulative damage of which is a complicated process. Present fatigue analysis of the rib-to-deck joint based on fracture mechanics mainly uses the Paris formula to calculate the crack growth rate, and calculates the stress intensity factor based on the stress amplitude of tension and compression cyclic stress (Xiao et al., 2008). Shi et al. (2013) pointed out the problem of using present method to evaluate the fatigue life of rib-to-deck joint of orthotropic steel deck under cyclic load dominated by compressive stress. When \( \sigma_{\text{min}} < 0 \), the stress intensity factor at the rib-to-deck joint cannot be simply regarded as proportional to the stress amplitude. The influence of compressive stress on crack growth has been analyzed from the perspective of elastoplastic mechanics. Silva et al. (2004) found that the compressive stress in cyclic stress can drive crack growth under the negative stress ratio condition due to the influence of plastic properties of material under cyclic load. Therefore, the inherent properties of the material should be included in the stress intensity factor. In a tension and compression load cycle, when the tension load is reduced to zero and enters the compression load, there is stress concentration at the crack tip since the crack is not completely closed. The squeezing effect of compression load leads to reverse yielding, forming a reverse plastic zone. Sha and Zhang
(2012) pointed out that under the cyclic load of negative stress ratio, the reverse plastic zone generated in the last cycle can make the crack tip more severe sharpening and passivation. Meanwhile, they proposed the crack growth formula including the influence of maximum compressive stress.

In order to study the influence of compressive stress in cyclic load on the crack growth of the rib-to-deck joint, this paper analyzes the surface crack growth at the rib-to-deck joint under the action of overloaded vehicles based on the local three dimensional fracture mechanics model of the weld. We analyze the crack growth of rib-to-deck joint through the Paris formula and Zhang et al. (2012)’s crack growth model based on the crack tip reverse plastic zone. Through comparative analysis, the crack growth rate model of rib-to-deck joint considering the influence of compressive stress is preliminarily explored in this paper.

2. The finite element model of stress analysis for fatigue assessment of rib-to-deck joint

2.1. The local finite element model of steel deck

This study is based on a bridge in North China. The roof thickness of the orthotropic steel deck of this bridge is 14 mm and the diaphragm density is 3200 mm. The height of the beam is 700 mm and the thickness is 10 mm. The thickness of the U-shaped ribs is 6 mm and the distance between the U-shaped ribs is 300 mm. The length of bottom is 184 mm, and the height is 260 mm. The steel type used for the orthotropic steel deck is Q345q, its elastic modulus is 206 GPa and Poisson’s ratio is 0.3. Ignoring the bridge deck pavement, a local finite element model of the orthotropic steel deck with four diaphragms and five longitudinal ribs is established by using the shell 181 element for grid partition. The total number of elements is 121120. The boundary condition of the model is to restrain the displacement of x-axis at the bridge deck end, the displacement of y-axis at the outer beam end and the displacement of z-axis at the bottom of the beam.

2.2. The fatigue load selection

As shown in Fig. 1, according to the standard fatigue load vehicle model specified in the “General code for design of highway bridges and culverts” (P. R. China, 2004), the first condition selects the five axle standard fatigue load vehicle with total weight of 55t as calculation load.

![Figure 1](image1)

Figure 1 The axle loads of standard five-axle vehicle. (the length unit is in meter)

The vehicle load is statistically analyzed based on the Weigh-In-Motion (WIM) system. According to the statistical results, the overloaded vehicle load can be calculated as 1.8 times of the standard load. As shown in Fig. 2, the second condition selects the five axle overload vehicle with total weight of 99t as calculation load.

![Figure 2](image2)

Figure 2 The axle loads of overloaded five-axle vehicle. (the length unit is in meter)
As shown in Fig. 3, this study adopts a 45° diffusion angle due to the diffusion effect of the 60 mm bridge pavement on the wheel load. The wheel load area is 320 mm 720 mm. The calculation takes into account the influence of the dynamic load factor of the vehicle load. According to the American AASHTO specification (AASHTO LRFD, 2008), the dynamic load factor is 0.15.

**Figure 3**  Schematic of rib-to-deck joint and the over-rib loading of wheels

### 3. The stress analysis of rib-to-deck joints

We calculate the process of a five-axle standard fatigue load vehicle with total weight of 55 t from one longitudinal end the local finite element model of orthotropic steel deck to the other end. The vehicle moves 150 mm in each load step. Then we obtain the transverse stress history of the outer node A (see Fig. 3) at the weld joint between the U rib and the bridge deck in the middle span.

The results show that, among three common lateral loading positions of wheels (In-between-rib loading, Riding-rib wall loading, Over-rib loading), the Over-rib loading generates the maximum transverse tensile stress at rib-to-deck joints. Therefore, the Over-rib loading is the most unfavorable loading position among three lateral loading positions of wheels.

After obtaining the most unfavorable lateral loading position of the wheel, we can further analyze the influence of overload vehicles on the stress state of rib-to-deck joints. The vehicle load is statistically analyzed based on the WIM system. According to results, a five axle overload vehicle with total weight of 99t is selected for analysis. Fig. 4 shows the transverse stress history of the five axle overload vehicle at the outer node A (see Fig. 3) of weld joint between the U rib and the bridge deck in the middle span under selected the most unfavorable lateral loading position of the wheel. The vehicle speed is 45 km/h.

**Figure 4** The time history of the transverse stress at the rib-to-deck joints under overloaded five-axle vehicle
As shown in Fig. 4, the transverse stress peak value and stress amplitude at the rib-to-deck joint increased significantly under the action of overloaded vehicles, and the maximum transverse tensile stress increased by 80%. This may significantly accelerate the fatigue damage of the rib-to-deck joint and greatly reduce the fatigue life of the crack at the rib-to-deck joint. In addition, the rib-to-deck joint of orthotropic steel deck is subjected to cyclic loading dominated by compressive stress. The stress ratio R=-1.3.

4. The three dimensional (3D) fracture mechanics analysis of rib-to-deck joint

Current methods to solve stress intensity factors include finite element method, boundary element method, finite difference method and so on. However, these methods have not been properly used in three-dimensional crack problems due to complexity of the calculation model. Based on the Schwartz-Neuman alternating method, Han and Atluri (2002) proposed a symmetric Galerkin boundary element-finite element (SGBEM-FEM) alternating method based on to solve the three-dimensional crack problem. As shown in Fig. 6, the method’s essence is to obtain the true solution of the cracked object through superposition and iteration of the finite element solution and the boundary element solution. The numerical results show that the alternating method of finite element and boundary element could be used to analyze the 3D fracture mechanics accurately and efficiently.

We adopt MSC.Patran to establish a local 3D fracture mechanics model of the rib-to-deck joint. The transverse width of the deck is 35 mm, the height of U-shaped rib is 20 mm, and the longitudinal length of the section is 25 mm. The position of the model corresponds to the position in the local finite element model of the steel bridge deck. The weld joint details is designed referring to the requirements of European codes according to the actual situation of the weld joint (Jia, 2016). The details of the local 3D fracture mechanics model of the rib-to-deck joint is shown in Fig. 5.

We select two positions of the rib-to-deck joint which are most prone to fatigue failure, i.e. weld toe and weld heel, and add the semi elliptical surface crack with the long semi-axis of 1.5 mm and the short semi-axis of 1 mm. Fig. 6 shows the stress intensity factors along the crack tips obtained from an improved Schwartz Neuman alternating method based on Han and Atluri. As shown in Fig. 6, compared with the weld heel, the weld toe is more prone to fatigue damage.
5. The fatigue crack growth rate model

This section adopts the Paris model and the crack growth formula including the influence of the maximum compressive stress proposed by Sha et al. (2012) to analyze the weld joint crack growth. The stress intensity factor amplitude in the Paris model is calculated by considering only tensile stress part and considering only full stress amplitude in the cyclic load respectively. Through comparison of the two models, we preliminary explore the crack growth rate model of the rib-to-deck joint of the orthotropic steel deck that considering the influence of compressive stress.

5.1. The Paris formula

The Paris formula is the most common empirical formula for calculating the crack growth under cyclic stress (Cheng, 2006). Formula (1) shows the relationship between the fatigue crack growth rate \( \frac{da}{dN} \) and the stress intensity factor amplitude \( \Delta K \) is as follows:

\[
\frac{da}{dN} = C (\Delta K)^m
\]  

Where \( a \) is the crack length, \( N \) is the number of cycles, \( C \) and \( m \) are material constants. \( K_{max} \) is the maximum stress intensity factor and \( K_{min} \) is the minimum stress intensity factor. For commonly used bridge steel models \( C = 2.7 \times 10^{-11}, \ m = 2.75 \) (Xiao et al., 2008).

At present, there are two main methods to calculate the stress intensity factor amplitude. The first method does not consider compressive stress in the cyclic load, i.e. \( \Delta K = K_{max} \). The other is to calculate the stress intensity factor amplitude based on the full stress amplitude in the cyclic load adopted by Xiao et al. (2008), i.e. \( \Delta K = (1 - R) K_{max} \).

Taking into account the two calculation methods of the stress intensity factor amplitude, we substitute the crack tip’s stress intensity factor of the weld toe and weld heel in Fig. 6 \( K_{max} = 2.38 \text{MPa}\sqrt{m} \) and \( R = -1.3 \) into formula (1). The results show that the crack growth rate of the rib-to-deck joint increases by 8.9 times when considering the full stress amplitude compared with the condition only considering the tensile stress in the cyclic load.

Taking the Paris model considering the full stress amplitude as an example, the semi-elliptical surface crack’s growth rate along the crack tips at weld toe and weld heel under the five-axle overloaded vehicle is shown in Fig. 7.
5.2. The reverse plastic zone model

From the transverse stress history at the rib-to-deck joint as shown in Fig. 4, it can be seen that the rib-to-deck joint of steel bridge deck is subjected to the cyclic load dominated by compressive stress. Sha and Zhang (2012) proposed the relationship equation between the maximum reverse plastic zone and the parameters of tension compression cyclic load based on the elastic-plastic finite element method. They further derived the crack growth rate formula under tension and compression cyclic load by combining the incremental plastic damage theory.

The relationship between the maximum reverse plastic zone size \( R_{rp,max} \), the stress intensity factor amplitude corresponding to the maximum stress \( K_{max} \) and the maximum compressive stress under cyclic load \( \sigma_{max,con} \) is as follows:

\[
R_{rp,max} = (1 - \gamma \sigma_{max,con} ) R_{rp0}
\]

(2)

Where \( \gamma \) is the material constant in unit of MPa\(^{-1}\), \( R_{rp0} \) is the size of reverse plastic zone at crack tip when tensile load is unloaded to zero.

The crack growth rate formula under tension and compression cyclic load derived based on the incremental plastic damage theory is:

\[
\frac{da}{dN} = C (1 - \gamma \sigma_{max,con} )^\beta ( K_{max} )^\mu
\]

(3)

\( C, m, \beta \) and \( \gamma \) in the formula are material constants. In this paper, \( C = 2.7 \times 10^{-11}, m = 2.75 \) (Xiao et al., 2008). \( \beta \) can be obtained by two sets of Paris formula coefficients with different stress ratios, i.e. \( R1, C1, m1 \) and \( R2, C2, m2 \). In this paper \( \beta = 0.6 \) (Zong et al., 2015). If the rib-to-deck joint is assumed to be in a plane strain state, then \( \gamma = 0.14 \) MPa\(^{-1}\) (Du and Shi, 2017). By introducing the above parameters into formula (2), we obtain the crack growth formula considering the influence of the maximum compressive stress at the crack tip under the plane strain condition:

\[
\frac{da}{dN} = 2.7 \times 10^{-11} \times (1 - 0.14 \sigma_{max,con} )^{0.68} ( K_{max} )^{2.75}
\]

(4)

Substituting the maximum compressive stress \( \sigma_{max,con} = -61.79 \) MPa in Fig. 4 and the crack tip stress intensity factor \( K_{max} = 2.38 \) MPa\(\sqrt{m}\) in Fig. 6 into formula (4), we obtain distributions of the semi-elliptical surface crack’s growth rate along the crack tips at weld root and weld toe under five-axle overloaded vehicle. The result are shown in Fig. 8.
5.3. Evaluation of crack growth rate model

The peak growth rates along the crack tip under different crack growth rate models are obtained through above crack growth analysis of rib-to-deck joints based on different crack growth formulas. The comparison of the results is shown in Table 1.

| Crack growth rate model | Stress amplitude | Driving force of crack growth | Growth rate (mm/cycle) |
|-------------------------|------------------|-------------------------------|------------------------|
| Paris 1                 | $\sigma_{\text{max}}$ | $\Delta K=K_{\text{max}}$ | $2.95 \times 10^{-7}$ |
| Paris 2                 | $\sigma_{\text{max}}-\sigma_{\text{min}}$ | $\Delta K=(1-R)K_{\text{max}}$ | $2.92 \times 10^{-6}$ |
| Reverse plastic zone model | $\sigma_{\text{max}}$ | $\Delta K=K_{\text{max}}$ and $\sigma_{\text{max,com}}$ | $1.55 \times 10^{-6}$ |

The results in Table 1 show that the growth rate of rib-to-deck joint along the crack tip calculated by the Paris formula considering the full stress amplitude in the cyclic load is the largest. The growth rate is 9.9 times that of the Paris model that only considers the tensile stress amplitude under cyclic load, and 1.9 times that of the reverse plastic zone model proposed by Sha and Zhang (2012). Firstly, the result of Paris formula which only considers the tensile stress amplitude under cyclic load is not credible. Because it does not consider the effect of pressure stress on crack growth, the crack growth rate of rib-to-deck joint is too conservative. Secondly, some scholars have pointed out the irrationality of the Paris model that simply assume the stress intensity factor amplitude is proportional to the stress amplitude (Shi et al., 2013). The crack growth rate of rib-to-deck joint obtained by this method is too dangerous due to not consistent with actual expansion mechanism and ignores the crack closure effect under compressive stress. Third, the crack growth formula propose by Zhang et al. (2013) based on the crack tip reverse plastic zone simultaneously considered the influence of tensile stress and compressive stress under cyclic load. The final crack growth rate of rib-to-deck joint is between the two conditions, which is applicable to the crack growth of rib-to-deck joint of orthotropic steel deck considering influence of compressive stress.

As a preliminary exploration of crack growth analysis of rib-to-deck joint considering the influence of compressive stress, we find that, when only consider the tensile stress under cyclic load and the stress intensity factor amplitude is considered to be proportional to the stress amplitude, the Paris model is not reasonable for evaluating crack growth of rib-to-deck joint of orthotropic steel deck under cyclic load dominated by compressive stress. The model proposed by Sha and Zhang (2012) indicates that the compressive stress concentration at the crack front will increase the reverse plastic zone of crack tip and
produce plastic damage. They further derived the crack growth rate formula under tension and compression cyclic load based on the incremental plastic damage theory. The model of Sha and Zhang (2012) is in good agreement with the experimental results on the fatigue crack growth rate of aluminum alloy under tension-compression cyclic load. It can be preliminarily considered that the reverse plastic zone model proposed by Sha and Zhang (2012) can give more reasonable results for crack growth of rib-to-deck joint of orthotropic steel deck that considering the effect of compressive stress.

Theoretically, the Walker and Forman formulas can consider the influence of the negative stress ratio R on the crack growth of the rib-to-deck joint of the orthotropic steel deck, however, the constant is difficult to determine (Zhang et al., 2015). Without accurate and reliable constants, it is difficult to use the two models to analyze the crack growth of the rib-to-deck joint of the orthotropic steel deck under tension-compression cyclic load.

6. Conclusion

This paper analyzes the crack growth of the rib-to-deck joint of the orthotropic steel deck under overload vehicles based on the local 3D fracture mechanics model of weld. The crack growth of the rib-to-deck joint is analyzed through the Paris model and crack growth rate formula including the influence of maximum compressive stress proposed by Sha and Zhang (2012). We attempt to preliminary explore the crack growth rate model of rib-to-deck joint considering the influence of compressive stress. Through the analysis, we can draw the following conclusions.

By adopting different crack growth formulas to analyze the crack growth of the rib-to-deck joint, the Paris formula, we find that the Paris formula which only considers the tensile stress amplitude under cyclic load underestimates the crack growth rate of the rib-to-deck joint. While the Paris formula which only considers the full stress amplitude under cyclic load overestimates the influence of compressive stress on the crack growth of the rib-to-deck joint. The crack tip reverse plastic zone model considers both the tensile stress and the compressive stress under cyclic load. The final crack growth rate of rib-to-deck joint is between the two conditions. Theoretically, the reverse plastic zone model gives more reasonable results than the above two conditions.

The compressive stress under cyclic load has a great driving effect on the fatigue crack growth. However, there is no recognized crack growth rate model that can accurately consider the influence of compressive stress. Therefore, it is necessary to establish a crack growth rate model that can accurately describe the influence of compressive stress under cyclic load.

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