Low Energy Implications of Minimal Superstring Unification

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ABSTRACT

We study the phenomenological implications of effective supergravities based on string vacua with spontaneously broken N=1 supersymmetry by dilaton and moduli F-terms. We further require Minimal String Unification, namely that large string threshold corrections ensure the correct unification of the gauge couplings at the grand unification scale. The whole supersymmetric mass spectrum turns out to be determined in terms of only two independent parameters, the dilaton-moduli mixing angle and the gravitino mass. In particular we discuss the region of the parameter space where at least one superpartner is “visible” at LEPII. We find that the most likely candidates are the scalar partner of the right-handed electron and the lightest chargino, with interesting correlations between their masses and with the mass of the lightest higgs. We show how discovering SUSY particles at LEPII might rather sharply discriminate between scenarios with pure dilaton SUSY breaking and mixed dilaton-moduli breaking.
The stunning experimental confirmations of the Standard Model (SM) that have kept accumulating along these last years make it mandatory for any new physics to exactly reproduce the SM at the Fermi scale.

Given that the two major open questions of SM concern the incorporation of gravity among elementary interactions and the origin or naturalness of the electroweak scale ($M_W \ll M_{\text{Planck}}$), it is likely that this new physics might be based on a locally supersymmetric quantum field theory that contains the SM, gravity and, may be, some additional interactions in which known particles do not take part. The hierarchy of mass scales may then naturally result from the spontaneous breaking of local supersymmetry above $M_W$ by some non-perturbative mechanism. The decoupling of this mechanism at low energy would imply that the new physics at the Fermi scale should be describable by an effective lagrangian with an N=1 global supersymmetry (SUSY) explicitly broken by a set of soft terms [1]. Clearly, these terms as seen from an $M_{\text{Planck}}$ point of view, are entirely calculable in terms of the supergravity couplings.

On the other hand, the non-renormalizability of supergravity strongly favours the view that supergravity itself has ultimately to be considered as an effective theory valid only at $E \leq M_{\text{Planck}}$. The best candidate we have for a description of physics at or above $M_{\text{Planck}}$ is a heterotic superstring theory. At the perturbative level, it possesses a large class of vacua that lead to effective N=1 supergravities below $M_{\text{Planck}}$, however the implementation of the above mentioned programme to finally derive an effective theory that at $M_W$ reproduces the SM is still far from being realized. The major obstacle is the still large ignorance of how to handle the crucial non-perturbative properties of string theory. Most believe that the non-perturbative breakdown of SUSY or the selection of the true string vacuum or the determination of the gauge couplings indeed result from “stringy” mechanism whose non-perturbative nature prevents us from a deeper comprehension.

This major difficulty has prompted several authors to parametrize the effects of this unknown non-perturbative physics into a set of arbitrary parameters of the low energy effective theory. Such a set comprises couplings which are calculable in string perturbation theory and couplings which genuinely depend on the non-perturbative aspects. Remarkably enough, even with such a general parameterization, several features which
are common to the whole class of low energy effective supergravities emerge. In the work of Kaplunovski and Louis \[2\] along these lines, the properties of non-perturbative couplings were constrained making some rather general assumption on the non-perturbative dynamics\[†\]. In particular, it was assumed that the flatness of moduli and dilaton directions of the effective potential was lifted by such non-perturbative dynamics and that SUSY breaking arises from the non-vanishing vacuum expectation values (VEV) of the \( F \)-terms of the moduli \( T_i \), and/or dilaton \( S \) supermultiplets. We follow here the approach of Brignole, Ibañez and Muñoz where local SUSY breaking with vanishing cosmological constant is assumed to be saturated by the dilaton and moduli auxiliary fields. Within a specified compactification scheme the soft terms become function of the gravitino mass and of the so-called goldstino angle, \( \text{i.e.} \) the angle which accounts for the relative magnitude of the \( T_i \) and \( S \) \( F \)-terms VEV’s in the SUSY breaking \[4\].

In this letter we make use of the above general frame to study the implications of effective supergravities which emerge in the low-energy limit of superstring theory for LEPII physics. In particular, we will discuss the three distinct situations which can be encountered with dilaton \( \langle F^S \rangle \) dominance (\( \langle F^S \rangle \gg \langle F^T \rangle \)), moduli \( \langle F^T \rangle \) dominance (\( \langle F^T \rangle \gg \langle F^S \rangle \)) or comparable role of dilaton and moduli (\( \langle F^S \rangle \simeq \langle F^T \rangle \)) in SUSY breaking. The phenomenological implications of SUSY breaking solely due to the dilaton \( F \)-term were discussed in \[3\]. A further specification that we adopt for the class of superstring theories under analysis is related to the well-known problem of gauge couplings unification. We assume the so-called Minimal Superstring Unification \[6\], \( \text{i.e.} \) that the only light particles with SM gauge couplings are just those of the minimal SUSY SM (MSSM) and no partial (field theoretical) unification occurs below the string scale. Then one has to rely on the string threshold contribution to cover the gap between the unification scale and the string scale. In orbifold compactification this was shown to be possible under rather constrained circumstances, \( \text{i.e.} \) with a particular choice of the modular weights of the matter fields. This point will be further discussed below.

Assuming the presence of one dominant modulus \( T \), the orbifold compactification and

\[†\]The analysis of ref. \[2\] finds its ground in previous extensive work on gaugino condensation and duality-invariant effective lagrangians \[5\].
target space modular invariance, and the minimal matter and Higgs content \((Q, U^c, D^c, L, E^c \text{ and } H_1 \text{ and } H_2 \text{ superfields})\), the soft breaking terms can be expressed in terms of the modular weights and only two other parameters at the compactification scale: the gravitino mass \(m_{3/2}\) and the goldstino angle \(\theta\), defined by \(\tan \theta = \langle F^T \rangle / \langle F^S \rangle\) (\(\theta = \pi/2\) corresponds to pure dilaton scenarios).

The scalar masses squared \(m_i^2\) read [4]:

\[
m_i^2 = \frac{m_{3/2}^2}{2} \left(1 + n_i \cos^2 \theta\right),
\]

where \(n_i\) are integer numbers, known as modular weights. A possible way to constrain the modular weights is provided by the demand to have minimal string unification. As discussed above this constraint entails a severe limitation on the available values of the modular weights. Some time ago Ibañez, Lust and Ross [3] showed that assuming generation independence for the modular weights as well as \(-3 \leq n_i \leq 1\), minimal string unification could be achieved for

\[
\begin{align*}
n_L &= -3, \quad n_{E^c} = -3, \\
n_Q &= -1, \quad n_{D^c} = -1, \quad n_{U^c} = -2, \\
n_{H_1} &= -2, \quad n_{H_2} = -3,
\end{align*}
\]

or also the same as above with the replacement \(n_{H_1} \leftrightarrow n_{H_2}\).

Obviously, the choice of the values of the modular weights has a major impact on the phenomenological implications that we wish to study here. It might turn out that the \(M_X - M_{\text{String}}\) discrepancy will be finally overcome in schemes (intermediate GUT, extra light states between \(M_X\) and \(M_{\text{String}}\), etc.) other than in the minimal string unification approach that we follow here. In any case we find it interesting to adopt this promising solution taking it seriously and trying to fully explore its impact on the coming LEP II physics.

In the soft sector of the trilinear scalar couplings we focus only on the \(A\)-term which is related to the top quark Yukawa coupling, \(A_t\). The reason is that we consider only \(A_t\) as a relevant trilinear term for the electroweak radiative breaking. Its expression is:

\[
A_t = -m_{3/2} \left(\sqrt{3} \sin \theta - 3 \cos \theta\right),
\]
For the gaugino masses $M_i$, taking the Green-Schwarz parameter $\delta_{GS} = -10$ we obtain:

$$
M_3 = \sqrt{3} \frac{m_3}{2} (\sin \theta + 0.12 \cos \theta),
$$

$$
M_2 = \sqrt{3} \frac{m_3}{2} (\sin \theta + 0.06 \cos \theta),
$$

$$
M_1 = \sqrt{3} \frac{m_3}{2} (\sin \theta - 0.02 \cos \theta).
$$

Finally we have to deal with the scalar bilinear soft breaking term $B \mu H_1 H_2$ (where $H_1$ and $H_2$ denote the scalar doublets), which strictly depends on the origin of the $\mu$-term in the superpotential. The smallness of $\mu$ in comparison with some typical superlarge scale (in our case the string scale) finds a natural explanation if $\mu$ arises solely from couplings in the Kähler potential [7]. Since these couplings are indeed there in string theory it becomes appealing to view them as the only source of the $\mu$-term [4]. In this case $B$ takes the form [4]:

$$
B_Z = m_3/2 \left(2 + 5 \cos \theta + 3 \cos^2 \theta\right).
$$

(5)

A second option pointed out by [4] is that $\mu$ arises solely from the $S$ and $T$ sector. Then:

$$
B_{\mu} = m_3/2 \left(-1 - \sqrt{3} \sin \theta + 2 \cos \theta\right).
$$

(6)

(We recall that, in the formula of $A_t$, $B_{\mu}$ and $B_Z$, we have used the above values of the modular weights).

Obviously it might well be the case that the mechanism originating $\mu$ is kind of admixture of the two above possibilities and then $B$ would be some combination of $B_{\mu}$ and $B_Z$ and we should consider it as an additional free parameter in the determination of the SUSY mass spectrum. For definiteness, in this work we will concentrate on the case of $B$ being $B_{\mu}$. This option for $B$ allows for a larger region of SUSY parameter space available for electroweak radiative breaking, although it is maybe less attractive in the string context. A more general analysis including the $B_Z$ option as well as the case of $B$ as an additional independent parameter will be presented elsewhere [8].

Given the boundary conditions in equations (1), (3), (4), (6) at the compactification scale $M_S = 3.6 \times 10^{17}$ GeV, we have to determine the evolution of the couplings according to $\delta_{GS}$ is shown to vary in the range $-5 \leq \delta_{GS} \leq -10$ for orbifold compactification [4]. Changing $\delta_{GS}$ in this range does not significantly affect the results of our analysis.
to their renormalization group equation (RGE) to finally compute the mass spectrum of the SUSY particles at the weak scale. In using the RGE’s we keep only the top Yukawa coupling $\lambda_t$, i.e. we assume that $\lambda_t \gg \lambda_b, \lambda_{\tau}$. In so doing we are automatically leaving aside of our discussion those large values of $\tan \beta$ for which $\lambda_t \simeq \lambda_b$. We postpone the discussion of the large $\tan \beta$ regime to the abovementioned longer analysis [8].

First we impose the condition of electroweak symmetry breaking. The potential for the two neutral components $H_1^0$ and $H_2^0$ of the higgs doublets reads [1]:

\[
V = (m_{H_1}^2 + \mu^2) |H_1^0|^2 + (m_{H_2}^2 + \mu^2) |H_2^0|^2 - B\mu (H_1^0 H_2^0 + \text{h.c.}) + \frac{1}{8} (g_1^2 + g_2^2) (|H_1^0|^2 - |H_2^0|^2). \quad (7)
\]

$(m_{H_1}^2$ and $m_{H_2}^2$ satisfy the boundary condition at $M_{\text{String}}[1]$, with the modular weights for $H_1$ and $H_2$ as in eq. (2); we recall that the product $B\mu$ can be assumed to be non-negative by appropriate choice of the Higgs field phases). As usual the electroweak symmetry breaking requires the following conditions among the renormalized quantities:

\[
m_{H_1}^2 + m_{H_2}^2 + 2 \mu^2 > 2B\mu, \quad (8)
\]

and

\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \quad (9)
\]

where $\tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$. A further constraint on the parameter space is entailed by the demand of colour and electric charge conservation. In particular, the latter conservation yields a powerful constraint [1], that has been taken into account in our analysis.

Since $\mu^2$ and $\tan \beta$ can be expressed in terms of $m_{H_1}^2, m_{H_2}^2$ and $B$, all quantities depend in last analysis on the boundary conditions at $M_{\text{String}}$. To be sure, in the evolution of the parameters entering equation (3) there is also a dependence on the top Yukawa coupling. However we take the mass of the top, $m_t = 174$ GeV [1], as an experimental input and, then, using $\lambda_t = m_t / (v \sin \beta)$, with $v = \sqrt{\langle H_1^0 \rangle^2 + \langle H_2^0 \rangle^2} = 174$ GeV, we can express $\lambda_t$ in terms of $\sin \beta$. In conclusion all low energy quantities are just functions of $m_{3/2}$, $\theta$ and

\[\text{A discussion of the dependence of our results on the experimental error in the determination of } m_t \text{ will be provided in [8].} \]
sign of \( \mu \). Obviously if instead of considering the definite situation for \( B \) in which \( B = B_\mu \) or \( B = B_Z \) one does not make any commitment on \( B \), then also this parameter should be added to the above list. As we said, in the present analysis we consider only the case of \( B = B_\mu \). For actual computation we take into account also the one-loop corrections to the scalar potential due to the top-stop exchange which are known to affect the Higgs masses in a relevant way.

We now come to the main bulk of our analysis. The allowed region in the parameter space has to satisfy the usual requirement that chargino and sfermion masses are \( \geq M_Z/2 \) and we further demand that the lightest SUSY particle (LSP) be a neutralino or sneutrino, but should not carry electric or colour charge. Actually, this constraint is automatically fulfilled once electric charge conservation is implemented. As we mentioned above, we want to focus our analysis on the implications for LEPII physics. We will briefly denote by “LEPII available region” those points of the parameter space for which at least one of the charginos, sleptons or squarks is lighter than \( M_Z \).

As we have seen, the whole low energy spectrum is determined in terms of \( m_{3/2} \) and \( \theta \). In fig. 1 we show the LEPII available region in the \((\theta, m_{3/2})\) plane. The excluded area at the bottom part of the figure corresponds to points where some SUSY particle is too light. The vast dotted area occupying the center of the figure represents a region of the parameter space which is unavailable to LEPII physics according to our previous definition. In conclusion the LEPII available region which is not already experimentally excluded corresponds to the blank area. The vertical solid line denotes the value of the goldstino angle which corresponds to the pure dilaton case \((\theta = \pi/2)\).

A remarkable feature of the figure is that in the pure or almost pure dilaton case the gravitino mass is constrained to be below 80 GeV or so to warrant LEPII discovery of some SUSY particle, while rather higher value of \( m_{3/2} \) are available when we move to situations of significant admixture of \( \langle F^S \rangle \) and \( \langle F^T \rangle \) contributions to the SUSY breaking. Also we can see that the available range for \( \theta \), where the correct electroweak breaking takes place and all the experimental bounds on the SUSY particles are satisfied, is very limited (approximately \( \theta \in [0.98, 2] \) rad\(^1\)). These values of \( \theta \) strictly depends on the

\(^1\)There is an analogous region in the neighborhood of \( \theta = 3\pi/2 \), for which an analogous discussion...
values of modular weights given in equation (2).

The next relevant question becomes: which SUSY particle(s) is (are) most likely to be seen at LEPII in the minimal string unification scenario that we are studying here? The answer is provided in figs. 2 and 3 where we plot the values of the mass of the right-handed selectron and lightest chargino, respectively, as a function of the goldstino angle corresponding to the LEPII available region in fig. 1. The dots constitute kind of iso-gravitino mass curves. The value of $m_{3/2}$ increases going from the lower to the upper part of the figure. The message that the two figures convey is the following: considering the LEPII available region, for non-negligible dilaton-moduli admixture ($\theta < 1.2$ rad.) the mass of $\tilde{e}_R$ is always within the LEPII discovery reach, while for $1.2 \text{ rad.} < \theta < 2 \text{ rad.}$, i.e. for a case closer to the pure dilaton situation, the lightest chargino is always lighter than 90 GeV. Notice also that finding a right selectron with mass lighter than 70 GeV would be a signal for a departure from the pure dilaton scenario.

In fig. 4 we plot the mass of the lightest chargino vs. the mass of $\tilde{e}_R$ for the points of the LEPII available region varying $m_{3/2}$ and $\theta$. The dilaton case corresponds to the highest allowed values of $m_{\tilde{e}_R}$ for chargino masses in the range 45-90 GeV. While there are several points corresponding to a lightest chargino mass $> 90$ GeV, few points with $m_{\tilde{e}_R} > 90$ GeV are present. If a chargino is seen at LEPII it is very likely that also $\tilde{e}_R$ is visible there.

In fig. 5 and 6 we show the correlation between the mass of the lightest higgs and the mass of the lightest chargino and $\tilde{e}_R$, respectively. From fig. 5 we gather that the “visibility” of the lightest chargino at LEPII implies that also the lightest higgs is visible. This does not hold true in the case of visibility of $\tilde{e}_R$, since fig. 6 shows that for $m_{\tilde{e}_R} < 90$ GeV there exist several points corresponding to a mass of the lightest higgs above 90 GeV. Indeed, it might be interesting to ask about the “visibility” of the lightest higgs at LEPII if some SUSY particle (essentially, the $\tilde{e}_R$ or the lightest chargino, in our analysis) is discovered there. Fig. 7 shows that in correspondence to the “LEPII available region” the mass of the lightest higgs is always below 90 GeV for the case of almost pure dilaton, while it can grow above 90 GeV when $\theta < 1.2$ rad., i.e. with conspicuous admixture of applies.
\langle F^S \rangle \text{ and } \langle F^T \rangle .

Finally, we notice that the sneutrino tends to be heavier than the \( \tilde{e}_R \) or the lightest chargino. There exist vast areas of the parameter space region available at LEPII where \( \tilde{e}_R \) and/or the lightest chargino have mass < 90 GeV, while the sneutrino mass is above 90 GeV.

In conclusion in this letter we have shown that schemes of minimal string unification provide interesting and rather detailed implications on physics to be tested in coming machines, in particular LEPII. Clearly many results we reached in our analysis are tightly related to the choice of modular weights or the origin of the \( \mu \)-parameter or some other assumption we make. We plan to provide a more exhaustive analysis of the general phenomenological features of effective supergravities in a forthcoming work [8].

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Figure Captions

**Fig. 1** The plane \((\theta, \text{m}_{3/2})\). The crossed region is excluded by direct mass searches; the one with dots cannot be accessed by LEPII. The rest is the region which will be probed by LEPII direct searches. The solid vertical line \(\theta = \pi/2\) corresponds to the pure dilaton case.

**Fig. 2** The right selectron mass in the LEPII available region (see the text for definition) as a function of the goldstino angle. The horizontal lines correspond to the visibility at LEPII. Notice, in comparison with next figure, the smaller vertical range.

**Fig. 3** The lightest chargino mass in the LEPII available region, as a function of the goldstino angle. Horizontal lines as before.

**Fig. 4** The right selectron mass versus the lightest chargino mass. The vertical (horizontal) lines enclose the region in which the chargino (right selectron) is visible at LEPII energies.

**Fig. 5** The lightest Higgs particle mass versus the lightest chargino mass. Vertical lines as before.

**Fig. 6** The lightest Higgs particle mass versus the right selectron mass. Vertical lines correspond to visibility of right selectron at LEPII.

**Fig. 7** The lightest Higgs particle mass as a function of the goldstino angle. The vertical line corresponds to pure dilaton case.
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right selectron mass [GeV]
goldstino angle [rad.]
lightest Higgs mass [GeV]

lightest chargino mass [GeV]
