IN MEDIUM MODIFICATIONS TO $\rho$ FROM FINITE PION DENSITY EFFECTS AND DILEPTON SPECTRUM

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The behavior of the pion dispersion relation in a pion medium is strongly modified by the introduction of a finite chemical potential associated to the finite pion number density. Such behavior is particularly important during the hadronic phase of a relativistic heavy-ion collision, between chemical and thermal freeze-out, where the pion number changing processes, driven by the strong interaction, can be considered to be frozen. We make use of an effective Lagrangian that explicitly respects chiral symmetry through the enforcement of the chiral Ward identities. The pion dispersion relation is computed through the computation of the pion self-energy in a non-perturbative fashion by giving an approximate solution to the Schwinger-Dyson equation for this self-energy. Given the strong coupling between $\rho$ vectors and pions, we argue that the modification of the pion mass due to finite pion density effects has to be taken into account self-consistently for the description of the in-medium modifications of $\rho$’s. We finally study some possible consequences of finite pion density effects for the low-mass dilepton spectrum produced in relativistic heavy-ion collisions.

1 Introduction

Dileptons are a prime tool to study the evolution of the dense and hot hadronic region formed in relativistic nuclear collisions. Dilepton final states are mediated by electromagnetic currents which in turn are connected to vector mesons. For low invariant masses, the vector mesons involved are the $\rho$, $\omega$ and $\phi$. Among these, $\rho$ plays an special role since its lifetime is smaller than the expected lifetime of the interacting region and thus is able to probe different stages during the collision of heavy systems.

Though it is true that a great deal of the features of the measured low-mass dilepton spectra at SPS energies can be ascribed to finite baryon density effects at relativistic and even more at ultrarelativistic energies, there is a large amount of pions produced in the central rapidity region. Consequently, it is important to include in the calculation of the dilepton spectrum the proper treatment of the large pion density, particularly during the hadronic phase of the collision.

Strictly speaking, pion number is not a conserved quantity. However, the characteristic time for electromagnetic and weak pion number-changing reactions, is very large compared to the lifetime of the system created in relativistic heavy-ion collisions and therefore, these processes are of no relevance for the propagation properties of pions within the lifetime of the collision. As for the case of strong processes, it is by now accepted that they drive pion number toward chemical freeze-out at a temperature considerably higher than the thermal freeze-out temperature and therefore, that from chemical to thermal freeze-out, the pion system evolves with the pion abundance held fixed. Under these circumstances, it is possible to ascribe to the pion density a chemical potential and consider the pion number as conserved.

The description of hadronic degrees of freedom relies on effective approaches that implement the dynamical symmetries of QCD. In a series of recent papers it has been shown that the linear sigma model can be used as one of such effective approaches to describe the pion propagation properties within a pion medium at energies, temperatures and densities small compared to the sigma mass. The sigma degree of freedom can be integrated out in a systematic expansion to obtain an effective theory of like-isospin pions interacting among themselves through an effective quartic term with coupling $\alpha = 6(m_{\pi}^2/2f_{\pi}^2)$, where $m_{\pi}$ and $f_{\pi}$ are the vacuum pion mass and decay constant, respectively.
In this work we report on the use of such effective description to study the interaction of pions with the \( \rho \) vector, paying particular attention to the effects that a finite pion density introduce on the propagation properties of \( \rho \) at finite temperature. By invoking vector dominance, we also study the modifications introduced on the production of \( e^+ e^- \) pairs near the \( \rho \) peak. As a preliminary result, we compare the theoretical spectrum obtained to SPS data on \( S + Au \) collisions at 200 GeV/c A. A detailed account of this approach can be found in Ref. 10.

2 Effective Lagrangian

In Refs. 8 it has been shown that starting from the linear sigma model Lagrangian, including only meson degrees of freedom and working in the kinematical regime where the pion momentum, mass and temperature are small compared to the sigma mass, it is possible to construct an effective theory that encodes the dynamics of low energy pion interactions and reproduces the leading order modification to the pion mass at finite temperature obtained from chiral perturbation theory 11. By gauging the theory 12, replacing the derivative \( \partial^\mu \) by the covariant derivative \( D^\mu = (\partial^\mu - ig\rho^\mu) \), where the constant \( g \) represents the \( \pi - \rho \) coupling, and introducing the mass term and kinetic energy for the \( \rho \) field, the effective Lagrangian for the theory is

\[
L \to L' = \frac{1}{2}(D^\mu \phi)^2 - \frac{1}{2}m_\pi^2 \phi^2 - \frac{\alpha}{4!(\phi\phi)^2} + \frac{1}{2}m_\rho^2 p^\mu p_\mu - \frac{1}{4}p_\mu p_\nu p^{\mu\nu}.
\] (1)

To introduce a finite chemical potential associated to a conserved pion number, let us further modify the Lagrangian in Eq. (1), writing it in terms of a complex scalar field and regarding \( \phi \) and \( \phi^* \) as independent fields

\[
L' \to L'' = (D^\mu \phi)(D^\mu \phi^*) - m_\pi^2 \phi \phi^* - \frac{\alpha}{4!}(\phi\phi^*)^2 + \frac{1}{2}m_\rho^2 p^\mu p_\mu - \frac{1}{4}p_\mu p_\nu p^{\mu\nu}.
\] (2)

It is easy to show that the Lagrangian in Eq. (2) leads to a conserved Noether current and to the conserved charge

\[
N = i \int d^3x (\phi^* \partial^0 \phi - \phi \partial^0 \phi^* + 2i g \rho^0 \phi^* \phi).
\] (3)

that can be identified with the particle number which in turn allows to introduce a chemical potential \( \mu \) conjugate to \( N \) which modifies the grand partition function and translates into a modification of the Matsubara pion propagator and the \( \pi - \rho \) vertex which now read as (hereafter capital letters are used to denote four-vectors whereas lower case letters are used do denote the components)

\[
\Delta(i\omega_n, p; \mu) = \frac{1}{-(i\omega_n + \mu)^2 + p^2 + m_\pi^2},
\] (4)

and

\[
\Gamma_{\pi\rho}(P_\mu, P'_\mu; \mu) = -ig\{[-(i\omega_n + \mu), p] + [-(i\omega_n' + \mu), p']\},
\] (5)

respectively.

3 Self-energies

At one loop level, the leading contribution to the pion self-energy \( \Pi_0 \), including the effects of resummation, corresponds to the tadpole approximation 14 and can be obtained from the solution to the transcendental equation

\[
\Pi_0 = \left( \frac{\alpha T}{4\pi^2} \right) \sqrt{m_\pi^2 + \Pi_0} \sum_{n=1}^{\infty} K_1 \left( \frac{n\sqrt{m_\pi^2 + \Pi_0}}{T} \right) \cosh(n\mu/T) \frac{1}{n}.
\] (6)
On the other hand, the explicit expression for the one-loop $\rho$ self-energy at finite temperature in the imaginary-time formalism is given by

$$\Pi^{\mu\nu} = -g^2 T \sum_n \int \frac{d^3 p}{(2\pi)^3} \frac{(2P^\mu - K^\mu)(2P^\nu - K^\nu)}{[(K - P)^2 + \tilde{m}_\pi^2]} + \delta^{\mu\nu} g^2 T \sum_n \int \frac{d^3 p}{(2\pi)^3} \frac{1}{P^2 + \tilde{m}_\pi^2}.$$  (7)

For a massive vector field, the tensor structure of its self-energy can be written in terms of the longitudinal $P^{\mu\nu}_L$ and transverse $P^{\mu\nu}_T$ projection tensors

$$\Pi^{\mu\nu} = F(K) P^{\mu\nu}_L + G(K) P^{\mu\nu}_T.$$  (8)

The $\rho$ propagator is thus given by

$$-i\mathcal{D}^{\mu\nu} = \frac{P^{\mu\nu}_L}{K^2 - m_\rho^2 - F} + \frac{P^{\mu\nu}_T}{K^2 - m_\rho^2 - G} + \frac{K^\mu K^\nu}{m_\rho^2 K^2}.$$  (9)

In order to identify the coefficients $F$ and $G$, we take $k$ along the $z$-axis. Thus, in Minkowski space, their expressions are

$$F(K) = -\frac{K^2}{k_0 k} \Pi^{03},$$

$$G(K) = \Pi^{11},$$  (10)

where $\Pi^{03}$ and $\Pi^{11}$ are obtained from Eq. (7) with the analytical continuation

$$i\omega \rightarrow k_0 + i\epsilon,$$  (11)

that give the retarded functions and that can be performed after carrying out the sum over the Matsubara frequencies.

4 Dilepton rate

Under the assumption of vector dominance and $\rho$ saturation, the expression for the rate of dilepton production per unit space-time volume and unit pair four-momentum is given by

$$\frac{dN}{d^4x d^4k} = \frac{1}{3(2\pi)^5} \left( e^4 g^4 \right) \frac{m_\rho^4}{M^4} \left\{ \frac{-\text{Im} F}{(M^2 - m_\rho^2 - \text{Re} F)^2 + \text{Im} F^2} \left( \frac{1}{e^{\beta\omega_L} - 1} \right) 
+ \frac{-2 \text{Im} G}{(M^2 - m_\rho^2 - \text{Re} G)^2 + \text{Im} G^2} \left( \frac{1}{e^{\beta\omega_T} - 1} \right) \right\}.$$  (12)

In order to illustrate the result in Eq. (12), let us take as a model for the space-time history of the collision the Bjorken model taking as the initial hadron proper-time formation $\tau_0 = 1$ fm, an initial temperature $T_0 = 210$ MeV and a final freeze-out temperature $T_f = 130$ MeV. To compute the sound velocity, we use an equation of state for a hadron gas comprising pions, nucleons, kaons and $\Delta(1232)$. Figure 1 shows the computed dilepton spectrum for a chemical potential $\mu = 100$ MeV and $T = 130$ MeV compared to data on $S + Au$ collisions at 200 GeV/c.

5 Conclusions

Under the assumption of VDM and $\rho$ saturation, we have computed the dilepton production rate as a function of the pair invariant mass taking as a model for the space-time history of the collision the Bjorken model with an equation of state appropriate for a hadron gas. We have found that the finite pion density produces a moderate broadening of the distribution and a moderate increase of the position of the peak. The finite pion density also produces a decrease of the distribution at the position of the peak compared to the $\mu = 0$ case.
Figure 1: Dilepton spectrum around the \( \rho \) peak for a chemical potential \( \mu = 100 \) MeV and \( T = 130 \) MeV compared to data on S + Au collisions at 200 GeV/c A.

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