We identify the recently observed charmonium-like structure \( Z_c^+(3900) \) as the charged partner of the \( X(3872) \) state. Using standard techniques of QCD sum rules, we evaluate the three-point function and extract the coupling constants of the \( Z_c^+ \) decay widths. In the present case, models it is relatively easy to reproduce the masses of meson molecule \([4]\), tetraquark \([5]\), hadro-charmonium \([6]\) and charmonium-molecule mixture \([7]\). For a comprehensive review of the theoretical and experimental status \([1]\) and \([7]\). For a comprehensive review of the theoretical and experimental status of these states we refer the reader to \([8]\). In most of these models it is relatively easy to reproduce the masses of the states. It is however much more difficult to reproduce their measured decay widths. In the present case, the \( Z_c(3900) \) decay width possesses an additional challenge to theorists. Its mass is very close to the \( X(3872) \), which may be considered its neutral partner. However, while the \( Z_c(3900) \) decay width is in the range 40 – 60 MeV, the \( X(3872) \) width is smaller than 2.3 MeV. A possible reason for this difference is the fact that the \( X(3872) \) may contain a significant \(|c\bar{c}|\) component \([7]\), which is absent in the \( Z_c(3900) \). Probably for this same reason the \( Z_c \) was not observed in \( B \) decays, as pointed out in \([9]\).

In this work we present a calculation of the \( Z_c(3900) \) decay width into \( J/\psi \pi^+ \), \( \eta_c \rho^+ \) and \( Z_c^+ D^+ D^{*0} \).

If the \( Z_c \) is a real \( D^* - \bar{D} \) molecular state its decay into \( J/\psi \pi^+ \) (or \( \eta_c \rho^+ \)) must involve the exchange of a charmed meson. Since the exchange of heavy mesons is a short range process, when the distance between \( D^* \) and the \( \bar{D} \) is large it becomes more difficult to exchange mesons. Using the expression of the decay width obtained with the one boson exchange potential (OBEP), we can relate the decay width with the effective radius of the state. In \([10]\) it was shown that, in order to reproduce the measured width, the effective radius must be \( r_{eff} \approx 0.4 \) fm. This size scale is small and pushes the molecular picture to its limit of validity. In another work \([11]\) the new state was again treated as a charged \( D^* - \bar{D} \) molecule, in which the interaction between the charm mesons is described by a pionless effective field theory. Introducing electromagnetic interactions through the minimal substitution in this theory, the authors of \([11]\) were able to study the electromagnetic structure of the \( Z_c \) and, in particular, its charge form factor and charge radius, which turned out to be \( (r^2) \approx 0.11 \) fm². Taking this radius as a measure of the spatial size of the state, we conclude that it is more compact than a \( J/\psi \) for which \( (r^2) \approx 0.16 \) fm². We take the combined results of \([10]\) and \([11]\) as an indication that the \( Z_c \) is a compact object, which may be better understood as a quark cluster, such as a tetraquark. Therefore in this work we explore this possibility.

As the number of new states increases, a new question arises concerning their grouping in families: which ones belong together? Which ones are groundstates and which are excitations? A possible organization of the charmonium and bottomonium new states was suggested in \([12]\) and it is summarized in Fig. 1. In the figure we compare the charm and bottom spectra in the mass region of interest. On the left (right) we show the charm (bottom) states with their mass differences in MeV. The comparison between the two left lines with the two lines on the right emphasizes the similarity between the spectra. In the bottom of the second column we have now the newly found \( Z_c(3900) \). In \([12]\) there was a question mark in this position. In fact, the existence of a charged partner of the \( X(3872) \) was first proposed in \([3]\). A few years later \([13]\) the same group proposed that the \( Z^+(4430) \), observed by BELLE \([14]\), would be the first radial excitation of the charged partner of the \( X(3872) \). This suggestion was
based on the fact that the mass difference corresponding to a radial excitation in the charmonium sector is given by $M_{\psi(2S)} - M_{\psi(1S)} = 590$ MeV. This number is close to the mass difference $M_{Z^+}^{(4430)} - M_{X^+(3872)} = 560$ MeV. The very same connection between $Z^+$ close to the mass difference given by a radial excitation in the charmonium sector is based on the fact that the mass difference corresponds to a radial excitation in the charmonium sector.

In this work we use the method of QCD sum rules (QCDSR)\(^\text{[19,21]}\) to study some hadronic decays of $Z_c(3900)$, considering $Z_c$ as a four-quark state.

**II. $Z_c^+(3900) \to J/\psi \pi^+$ DECAVy WIDTH**

The QCDSR were used in ref.\(^\text{[22]}\) to study the $X(3872)$ meson considered as a $I^G(J^{PC}) = 0^+(1^{++})$ four-quark state, and a good agreement with the experimental mass was obtained. The $Z_c(3900)$ is interpreted here as the isospin 1 partner of the $X(3872)$. As in \(^{[12,17]}\) we assume the quantum numbers for the neutral state in the isospin multiplet to be $I^G(J^{PC}) = 1^+(1^{+-})$. Therefore, the interpolating field for $Z_c^+(3900)$ is given by:

$$j_\alpha = \frac{i \epsilon_{abc}}{\sqrt{2}} [ (u_\alpha^T C_\gamma c_b) (\bar{d}_\gamma a c_c) - (u_\alpha^T C_\gamma a c_b) (\bar{d}_\gamma c c_c) ],$$

where $a$, $b$, $c$, ... are color indices, and $C$ is the charge conjugation matrix. Considering $SU(2)$ symmetry, the mass obtained in QCDSR for the $Z_c$ state is exactly the same one obtained for the $X(3872)$, as it happens in the case of $\rho$ and $\omega$ states. There are also QCDSR calculations for the $Z_c$ state considered as a $DD^*$ molecular state\(^\text{[23,24]}\). These calculations only confirm the results presented in refs.\(^\text{[22,23]}\). Therefore here we evaluate only the decay width.

We start with the $Z_c^+(3900) \to J/\psi \pi^+$ decay. The QCDSR calculation of the vertex $Z_c(3900) J/\psi \pi$ is based on the three-point function given by:

$$\Pi_{\mu\nu\alpha}(p,p',q) = \int d^4x \; d^4y \; e^{ip' \cdot x} e^{iq \cdot y} \; \Pi_{\mu\nu\alpha}(x,y),$$

(2)

with $\Pi_{\mu\nu\alpha}(x,y) = \langle 0| [j^\psi_\mu(x) j^\pi_\nu(y)] j^\alpha_\mu(0)|0\rangle$, where $p = p' + q$ and the interpolating fields for $J/\psi$ and $\pi$ are given by:

$$j^\psi_\mu = \bar{c}_a \gamma_\mu c_a,$$

(3)

$$j^\pi_{5\nu} = \bar{d}_a \gamma_\nu \gamma_5 u_a,$$

(4)

In order to evaluate the phenomenological side of the sum rule we insert intermediate states for $Z_c$, $J/\psi$ and $\pi$ into Eq. (2). We get:

$$\Pi^{\text{phen}}_{\mu\nu\alpha}(p,p',q) = \frac{\lambda_{Z_c} m_{\psi} m_{\pi} F_\pi}{(p^2 - m_{\psi}^2)} \frac{g_{Z_c \psi \pi}(q^2) q_\nu}{(q^2 - m_{\psi}^2)}$$

$$\left( -g_\alpha + \frac{p_\mu p_\lambda}{m_{\psi}^2} \right) \left( -g_\alpha + \frac{p_\mu p_\lambda}{m_{Z_c}^2} \right) + \cdots,$$

(5)

where the dots stand for the contribution of all possible excited states. The form factor, $g_{Z_c \psi \pi}(q^2)$, is defined as the generalization of the on-mass-shell matrix element, $\langle J/\psi \pi | Z_c \rangle$, for an off-shell pion:

$$\langle J/\psi(p') \pi(q) | Z_c(p) \rangle = g_{Z_c \psi \pi}(q^2) \varepsilon_\pi^\dagger(p') \varepsilon_\lambda(p),$$

(6)

where $\varepsilon_\alpha(p)$, $\varepsilon_\mu(p')$ are the polarization vectors of the $Z_c$ and $J/\psi$ mesons respectively. In deriving Eq. (5) we
have used the definitions:

\[ (0| j_{\psi}^{\alpha}(p)| \psi(p')) = m_{\psi} f_{\psi} \varepsilon_{\mu}(p'), \]
\[ (0| j_{\pi}^{\alpha}(q)| \pi(q)) = i q_{\nu} F_{\pi}, \]
\[ (Z_{c}(p)| j_{\alpha}(0) = \lambda_{Z_{c}} \varepsilon_{\alpha}(p). \]

(7)

To extract directly the coupling constant, \( g_{Z_{c}\psi\pi} \), instead of the form factor, we can write a sum rule at the pion-pole \([26]\), valid only at \( Q^{2} = 0 \), as suggested in \([20]\) for the pion-nucleon coupling constant. This method was also applied to the nucleon-hyperon-kaon coupling constant \([27, 28]\) and to the nucleon–Δc, Δ–D coupling constant \([29]\). It consists in neglecting the pion mass in the denominator of Eq. (5) and working at \( q^{2} = 0 \). In the OPE side only terms proportional to \( 1/q^2 \) will contribute to the sum rule. Therefore, up to dimension five the only diagrams that contribute are the quark condensate and the mixed condensate.

\[ \begin{align*}
\text{Fig. 2. CC diagram which contributes to the OPE side of the sum rule.}
\end{align*} \]

As discussed in refs. \([30, 31]\), large partial decay widths are expected when the coupling constant is obtained from QCDSR in the case of multiquark states. By multiquark states we mean that the initial state contains the same number of valence quarks as the number of valence quarks in the final state. This happens because, although the initial current, Eq. (1), has a non-trivial color structure, it can be rewritten as a sum of molecular type currents with trivial color configuration through a Fierz transformation, Tab. to avoid this problem we follow refs. \([30, 31]\), and consider in the OPE side only the diagrams with non-trivial color structure, which are called color-connected (CC) diagrams. In the present case the CC diagram that contributes to the OPE side at the pion pole is shown in Fig. 2. Possible permutations (not shown) of the diagram in Fig. 2 also contribute.

The diagram in Fig. 2 contributes only to the structures \( q_{\nu} p_{\mu} \) and \( q_{\nu} p_{\mu} p_{\alpha} \) appearing in the phenomenological side. Since structures with more momenta are supposed to give better results, we choose to work with the \( q_{\nu} p_{\mu} \) structure. Therefore in the OPE side and in the \( q_{\nu} p_{\mu} p_{\alpha} \) structure we obtain:

\[ \Pi^{\text{(OPE)}} = \langle \bar{q} q G(q) \rangle \frac{1}{12 \sqrt{2} \pi^2} \frac{1}{q^2} \int_{0}^{1} d \alpha \frac{\alpha(1-\alpha)}{m_{c}^2 - \alpha(1-\alpha)p^2}. \]

(8)

Isolating the \( q_{\nu} p_{\mu} \) structure in Eq. (5) and making a single Borel transformation to both \( P^2 = P_{\nu}^2 \rightarrow M^2 \), we finally get the sum rule:

\[ A \left(e^{-m_{c}^2/M^2} - e^{-m_{c}^2/m_{c}^2} \right) + B \ e^{-s_{0}/M^2} = \]
\[ = \frac{\langle \bar{q} q G(q) \rangle}{12 \sqrt{2} \pi^2} \int_{0}^{1} d \alpha \frac{\alpha}{m_{c}^2 - \alpha(1-\alpha)p^2}, \]

(9)

where \( s_{0} \) is the continuum threshold parameter for \( Z_{c} \), and \( B \) is a parameter introduced to take into account single pole contributions associated with pole-continuum transitions, which are not suppressed when only a single Borel transformation is done in a three-point function sum rule \([30, 32, 34]\). In the numerical analysis we use the following values for quark masses and QCD condensates \([22, 53]\):

\[ m_{c}(m_{c}) = (1.23 \pm 0.05) \text{ GeV}, \]
\[ \langle \bar{q} q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3, \]
\[ \langle \bar{q} q G(q) \rangle = m_{c}^2 \langle \bar{q} q \rangle, \]
\[ m_{c}^2 = 0.8 \text{ GeV}^2. \]

(11)

For the meson masses and decay constants we use the experimental values \([36]\): \( m_{\psi} = 3.1 \text{ GeV}, m_{\pi} = 138 \text{ MeV}, f_{\psi} = 0.405 \text{ GeV} \) and \( F_{\pi} = 131.52 \text{ MeV} \). For the \( Z_{c} \) mass we use the value measured in \([1]\): \( m_{Z_{c}} = (3899 \pm 6) \text{ MeV} \). The meson-current coupling, \( \lambda_{Z_{c}} \), defined in Eq. (7), can be determined from the two-point sum rule \([22]\): \( \lambda_{Z_{c}} = (1.5 \pm 0.3) \times 10^{-2} \text{ GeV}^5 \). For the continuum threshold we use \( s_{0} = (m_{Z_{c}} + \Delta s_{0})^2 \), with \( \Delta s_{0} = (0.5 \pm 0.1) \text{ GeV} \). We evaluate the sum rule in the range \( 2.0 \leq M^2 \leq 3.0 \text{ GeV}^2 \), which is the range where the two-point function

\[ \text{Fig. 3. Dots: the RHS of Eq. (9), as a function of the Borel mass for } \Delta s_{0} = 0.5 \text{ GeV. The solid line gives the fit of the QCDSR results through the LHS of Eq. (9).} \]
for $X (3872)$ (which is the same for $Z_c(3900)$) shows good OPE convergence and where the pole contribution is bigger than the continuum contribution \cite{22}. In Fig. 3 we show, through the circles, the right-hand side (RHS) of Eq. 9, as a function of the Borel mass.

To determine the coupling constant $g_{Z_c\psi\pi}$ we fit the QCDSR results with the analytical expression in the left-hand side (LHS) of Eq. 9, and find (using $\Delta s_0 = 0.5$ GeV): $A = 1.46 \times 10^{-4}$ GeV$^5$ and $B = -8.44 \times 10^{-4}$ GeV$^5$. Using the definition of $A$ in Eq. 9, the value obtained for the coupling constant is $g_{Z_c\psi\pi} = 3.89$ GeV, which is in excellent agreement with the estimate made in \cite{17}, based on dimensional arguments. Considering the uncertainties given above, we finally find:

$$g_{Z_c\psi\pi} = (3.89 \pm 0.56) \text{ GeV.} \quad (12)$$

The decay width is given by \cite{17}:

$$\Gamma(Z_c^+(3900) \to J/\psi \pi^+) = \frac{p^*(m_{Z_c}, m_\psi, m_\pi)}{8\pi m_{Z_c}^2} \times \frac{1}{3 \langle g_{Z_c\psi\pi}^2 \rangle} \left(3 + \frac{(p^*(m_{Z_c}, m_\psi, m_\pi))^2}{m_\psi^2}\right), \quad (13)$$

where

$$p^*(a, b, c) = \sqrt{a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2} / 2a.$$ \quad (14)

Therefore we obtain:

$$\Gamma(Z_c^+(3900) \to J/\psi \pi^+) = (29.1 \pm 8.2) \text{ MeV.} \quad (15)$$

**III. $Z_c^+(3900) \to \eta_c \rho^+$ DECAY WIDTH**

Next we consider the $Z_c^+(3900) \to \eta_c \rho^+$ decay. The three-point function for the corresponding vertex is obtained from Eq. (2) by using

$$\Pi_{\mu\alpha}(x, y) = \langle 0 | T \left[ j_5^\rho(x) j_5^\rho(y) j_6^\mu(0) \right] | 0 \rangle, \quad (16)$$

with

$$j_5^\rho = i c_{\gamma\gamma} c_a, \text{ and } j_6^\rho = \bar{d}_a \gamma_\mu u_a. \quad (17)$$

In this case the phenomenological side is

$$\Pi_{\mu\alpha}^{(\text{phen})}(p, p', q) = \frac{-i \lambda g_{Z_c \eta_c \rho} f_{\eta_c} f_{\rho} m_{Z_c}^2 g_{Z_c \eta_c \rho} (q^2)}{2m_c (p^2 - m_{Z_c}^2)(p'^2 - m_{\rho}^2)(q^2 - m_{\rho}^2)} \times \left(-g_{\mu\rho} + \frac{q_\mu q_\rho}{m_{\rho}^2}\right) \left(-g_{\alpha\gamma} + \frac{p_\alpha p_\gamma}{m_{Z_c}^2}\right) + \cdots, \quad (18)$$

where now we have used the definitions:

$$\langle 0 | j_6^\rho \gamma_\mu(q) \rangle = m_\rho f_{\rho} e_\mu(q), \quad \langle 0 | j_5^\rho \gamma_\alpha \eta_c(p') \rangle = \frac{f_{\eta_c} m_{\eta_c}^2}{2m_c}. \quad (19)$$

In the OPE side we consider the CC diagrams of the same kind of the diagram in Fig. 2. In the $p'_\alpha q_\mu$ structure we have:

$$\Pi^{(\text{OPE})} = \frac{-i m_c \langle q g_\sigma G q \rangle}{48\sqrt{2} \pi^2} \frac{1}{q^2} \int_0^1 d\alpha \frac{1}{m_c^2 - \alpha(1-\alpha)p'^2}. \quad (20)$$

Remembering that $p = p' + q$, isolating the $g_{\mu\alpha}q_\mu$ structure in Eq. (18) and making a single Borel transformation on both $P^2 = p'^2 = M^2$, we finally get the sum rule:

$$C \left(e^{-m_{\eta_c}^2/M^2} - e^{-m_{\rho}^2/M^2}\right) + D e^{-s_0/M^2} =$$

$$Q^2 + m_\rho^2 f_{\rho} f_{\eta_c} m_{\eta_c}^2 \frac{1}{2m_\rho m_{\eta_c}(m_{Z_c}^2 - m_{\eta_c}^2)} \int_0^1 d\alpha \frac{e^{\alpha(1-\alpha)M^2}}{\alpha(1-\alpha)}. \quad (21)$$

with $Q^2 = -q^2$ and

$$C = \frac{g_{Z_c \eta_c}(Q^2)}{2m_c m_{\eta_c}} \frac{1}{m_{\eta_c}^2}. \quad (22)$$

We use the experimental values for $m_\rho$, $f_\rho$ and $m_{\eta_c}$ \cite{36} and we extract $f_{\eta_c}$ from ref. \cite{37}:

$$m_\rho = 0.775 \text{ GeV, } m_{\eta_c} = 2.98 \text{ GeV, } f_\rho = 0.157 \text{ GeV, } f_{\eta_c} = 0.35 \text{ GeV.} \quad (23)$$

One can use Eq. (21) and its derivative with respect to $M^2$ to eliminate $D$ from Eq. (21) and to isolate $g_{Z_c \eta_c}(Q^2)$. In Fig. 4 we show $g_{Z_c \eta_c}(Q^2)$ as a function of both $M^2$ and $Q^2$. A good Borel window is determined when the parameter to be extracted from the sum rule is as much independent of the Borel mass as possible. Therefore, from Fig. 4 we notice that the Borel window
FIG. 5. QCDSR results for $g_{Z_{c}n_{c}p}(Q^2)$, as a function of $Q^2$, for $\Delta s_0 = 0.5$ GeV (squares). The solid line gives the parametrization of the QCDSR results through Eq. (24). The cross gives the value of the coupling constant.

where the form factor is independent of $M^2$ is in the region $4.0 \leq M^2 \leq 10.0$ GeV$^2$. The squares in Fig. 5 show the $Q^2$ dependence of $g_{Z_{c}n_{c}p}(Q^2)$, obtained for $M^2 = 5.0$ GeV$^2$. For other values of the Borel mass, in the range $4.0 \leq M^2 \leq 10.0$ GeV$^2$, the results are equivalent. Since the coupling constant is defined as the value of the form factor at the meson pole: $Q^2 = -m_p^2$, we need to extrapolate the form factor for a region of $Q^2$ where the QCDSR are not valid. This extrapolation can be done by parametrizing the QCDSR results for $g_{Z_{c}n_{c}p}(Q^2)$ with the help of an exponential form:

$$g_{Z_{c}n_{c}p}(Q^2) = g_1 e^{-g_2 Q^2}, \quad (24)$$

with $g_1 = 4.83$ GeV and $g_2 = 5.6 \times 10^{-3}$ GeV$^{-2}$. We also show in Fig. 5 through the line, the fit of the QCDSR results for $\Delta s_0 = 0.5$ GeV, using Eq. (24). The value of the coupling constant, $g_{Z_{c}n_{c}p}$, is also shown in this figure through the cross. We obtain:

$$g_{Z_{c}n_{c}p} = g_{Z_{c}n_{c}p}(-m_p^2) = (4.85 \pm 0.81) \text{ GeV}. \quad (25)$$

The uncertainty in the coupling constant given above comes from variations in $s_0$, $\lambda_{Z_c}$ and $m_c$ in the ranges given above. This value for the coupling is bigger than the estimate presented in [17]. Inserting this coupling and the corresponding masses into Eq. (13) we find

$$\Gamma(Z_{c}^+(3900) \rightarrow \eta_{c}p^+) = (27.5 \pm 8.5) \text{ MeV}. \quad (26)$$

IV. $Z_{c}^+(3900) \rightarrow D^+\bar{D}^{*0}$ DECAY WIDTH

Finally we consider the $Z_{c}^+(3900) \rightarrow D^+\bar{D}^{*0}$ decay. In this case we use in Eq. (2)

$$\Pi_{\mu\alpha}(x, y) = \langle 0| T[j_\mu^D(x)j_\alpha^D(y)]|0\rangle, \quad (27)$$

where

$$j_\mu^D = i\bar{d}_a \gamma_\mu c_a, \text{ and } j_\alpha^D = \bar{c}_a \gamma_\alpha u_a. \quad (28)$$

Using the definitions

$$\langle 0| j_\mu^D |D(q)\rangle = f_D m_D^2, \quad (29)$$

the phenomenological side is given by

$$\Pi_{\mu\alpha}^{\text{phen}}(p, p', q) = \frac{-i\lambda_{Z_c} m_D f_D m_D^2 g_{Z_{c}DD^*}(q^2)}{m_c (p^2 - m_{Z_c}^2)(p'^2 - m_{D^*}^2)(q^2 - m_{D^*}^2)} \times \left( -g_{\mu\lambda} + \frac{p'_\mu p'_\lambda}{m_{D^*}^2} \right) \left( -g_{\alpha\lambda} + \frac{p_\alpha p_\lambda}{m_{Z_c}^2} \right) + \cdots. \quad (30)$$

In the OPE side we consider again only the CC diagrams. In the $p'_\alpha p'_\mu$ structure we have:

$$\Pi^{\text{OPE}} = \frac{-im_c (q g q G)}{48\sqrt{2}\pi} \left[ \frac{1}{m_c^2 - q^2} \int_0^1 da \frac{\alpha(2 + \alpha)}{m_c^2 - (1 - \alpha)q^2} \right]. \quad (31)$$

Isolating the $p'_\alpha p'_\mu$ structure in Eq. (30) and making a

FIG. 6. QCDSR results for the form factor $g_{Z_{c}DD^*}(Q^2)$ as a function of $Q^2$ and $M^2$ for $\Delta s_0 = 0.5$ GeV.
single Borel transformation on both $P^2 = P'^2 \to M^2$, we get:

\[
\frac{1}{Q^2 + m_D^2} \left[ E \left( e^{-m_D^2/M^2} - e^{-m_c^2/M^2} \right) + F e^{-s_0/M^2} \right] =
\frac{1}{48\sqrt{2}\pi^2} \left[ \frac{1}{m_c^2 + Q^2} \int \frac{d\alpha}{1 - \alpha} \frac{\alpha(2 + \alpha)}{\alpha e^{-\alpha(1 - \alpha)M^2}} \right. - e^{-m_c^2/M^2} \left. \int \frac{d\alpha}{1 - \alpha} \frac{\alpha(2 + \alpha)}{m_c^2 + (1 - \alpha)Q^2} \right],
\]

with

\[
E = \frac{g_{Z,DD^*}(Q^2)\lambda_{Zc} f_{D^*} f_{D} m_D^2}{m_{Zc}(m_{Zc}^2 - m_{D^*}^2)}.
\]

We use the experimental values for $m_D$ and $m_{D^*}$ and we extract $f_D$ and $f_{D^*}$ from ref. [20]:

\[
m_D = 1.869 \text{ GeV}, \ f_D = (0.18 \pm 0.02) \text{ GeV},
\]

\[
m_{D^*} = 2.01 \text{ GeV}, \ f_{D^*} = (0.24 \pm 0.02) \text{ GeV}.
\]

In Fig. 6 we show $g_{Z,DD^*}(Q^2)$, as a function of both $M^2$ and $Q^2$, from where we notice that we get a Borel stability in the region $2.2 \leq M^2 \leq 2.8$ GeV$^2$.

\[
FIG. 7. QCDSR results for $g_{Z,DD^*}(Q^2)$, as a function of $Q^2$, for $\Delta s_0 = 0.5$ GeV (squares). The solid line gives the parametrization of the QCDSR results through Eq. (24).
\]

Fixing $M^2 = 2.6$ GeV$^2$ we show in Fig. 7 through the squares, the $Q^2$ dependence of the $g_{Z,DD^*}(Q^2)$ form factor. Again, to extract the coupling constant we fit the QCDSR results using the exponential form in Eq. (24) with $g_1 = 1.733$ GeV and $g_2 = 0.076$ GeV$^{-2}$. The line in in Fig. 7 shows the fit of the QCDSR results for $\Delta s_0 = 0.5$ GeV, using Eq. (24). We get for the coupling constant:

\[
g_{Z,DD^*} = g_{Z,DD^*}(-m_D^2) = (2.5 \pm 0.3) \text{ GeV}.
\]

The uncertainty in the coupling constant comes from variations in $s_0$, $\lambda_{Zc}$, $f_D$, $f_{D^*}$ and $m_c$. This value for this coupling is again in excellent agreement with the estimate presented in [17]. Using again Eq. (12) with this coupling, the decay width in this channel is

\[
\Gamma(Z_c^+ \to D^+ D^{*0}) = (3.2 \pm 0.7) \text{ MeV}.
\]

V. CONCLUSIONS

In conclusion, we have used the three-point QCDSR to evaluate the coupling constants in the vertices $Z_c^+(3900)J/\psi\pi^+$, $Z_c^+(3900)\eta_{c}\rho^+$ and $Z_c^+(3900)D^+ D^{*0}$. In the case of the $Z_c^+(3900)J/\psi\pi^+$ vertex, we have used the sum rule at the pion pole, and the coupling was extracted directly from the sum rule. In the cases of $Z_c^+(3900)\eta_{c}\rho^+$ and $Z_c^+(3900)D^+ D^{*0}$ vertices, we have extracted the form factors, and the couplings were obtained with a fit of the QCDSR results. In the three cases we have only considered the color connected diagrams, since we expect the $Z_c(3900)$ to be a genuine tetraquark state with a non-trivial color structure. The obtained couplings, with the respective decay widths, are given in Table I. We have also included in this table the results for the vertex $Z_c^+(3900)\bar{D}^0 D^{*+}$, since it is exactly the same result as in the $Z_c^+(3900)D^+ D^{*0}$ vertex.

| Vertex                              | coupling constant (GeV) | decay width (MeV) |
|-------------------------------------|-------------------------|-------------------|
| $Z_c^+(3900)J/\psi\pi^+$            | 3.89 ± 0.56             | 29.1 ± 8.2        |
| $Z_c^+(3900)\eta_{c}\rho^+$        | 4.85 ± 0.81             | 27.5 ± 8.5        |
| $Z_c^+(3900)\bar{D}^0 D^{*+}$      | 2.5 ± 0.3               | 3.2 ± 0.7         |
| $Z_c^+(3900)D^+ D^{*0}$            | 2.5 ± 0.3               | 3.2 ± 0.7         |

Considering these four decay channels we get a total width $\Gamma = (63.0 \pm 18.1)$ GeV for $Z_c(3900)$ which is in agreement with the two experimental values: $\Gamma = (46 \pm 22)$ MeV from BESIII [1], and $\Gamma = (63 \pm 35)$ MeV from BELLE [2].

Acknowledgments

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