Implementation of linear programming for optimizing on productions and profits simplex algorithm

J Jumadi*, A Wahana, D P Utami and C Slamet
UIN Sunan Gunung Djati Bandung, Bandung, Indonesia

*jumadi@uinsgd.ac.id

Abstract. Every factory based on production, always try to find the best way to get high profit from their capital that spent on production processes. The Konveksi Zom T-Shirt is a factory that produces jacket, shirt and t-shirt. This factory, use simplex method to predict composition and number of items to get maximum profits from available capital. Based on simplex method with parameters; Rp. 2,000,000 as capital, 3,000 minutes as time production, and 6 pcs as minimum productions. This method get result with composition and number of items; 15 pcs of jacket, 6 pcs of shirts, and 22 pcs of t-shirts. The profit of production uses this method can increase 3.5% of profits from conventional method. Finally, this method can be used to support on deciding composition and number of item in productions to get a maximum profit.

1. Introduction
In order to be able to adjust to market demand, the quality of production produced by the company must be truly considered, consumer needs, limited resources, and capital issued by the company, the company requires one of the objectives of production management in a company [1]. The problem that must be faced by these companies is how to combine the factors of production or resources that are owned appropriately in order to obtain maximum profits at the lowest possible cost by using linear programming to achieve the desired goals or objectives optimally by involving linear variables with simplex method [2]. The simplex method is divided into two, namely, the maximized simplex method to look for maximum profits and minimization simplex methods to find minimal costs [3]. As with the algebraic method, the simplex method must also be standardized in formulating the model, before the initial completion stage is done. The constraints functions that are still inequality form must be changed to form equations and the prerequisites of the simplex method are Gauss elimination [4]. The object of in this research is the Zom T-shirt convection in the city of Bandung. This convection is a business that is engaged in the convection of clothes, jackets, sweaters, shirts, shirts, etc. Capital, processing time and maximum production limits are often obstacles in a production. Besides that capital is sometimes the biggest obstacle in the progress of a business. So that the resulting profits can be maximized by using the simplex method to define as a way to solve problems that have at least two decision variables using a table tool.

2. Methodology
Decision Support System (DSS) is an interactive information system that provides on modelling and manipulating data to get alternative decisions [5]. DSS uses the linear programming to compile a model that can be used to help make decision in determining the optimal allocation of company resources to
various alternatives [6]. The Simplex method is used to change linear programming problem in a standard form before it problems is summarized in the simplex table. Steps of simplex method to solve the problem, are [7]:

- Make a change of variables and normalize the sign of the independent term.
- Normalize restrictions.
- Match the objective function to zero.
- Write the initial tableau of Simplex method.
- Stopping condition.
- Choice of the input and output base variables.
- Update tableau.
- When checking the stop condition is observed which is not fulfilled since there is one negative value in the last row, -1. So, continue iteration steps 6 and 7 again.
- Checking again the stop condition reveals that the pivot row has one negative value, -1. It means that optimal solution is not reached yet and we must continue iterating (steps 6 and 7).
- End of Algorithm. It is noted that in the last row, all the coefficients are positive, so the stop condition is fulfilled.

3. Results

The research use the Zom T-shirt Convection data, show in Table 1.

| Table 1. Data of the Zom T-shirt convection. |
|---------------------------------------------|
| **Production Cost** | **T-Shirt (x₁)** | **Shirt (x₂)** | **Jacket (x₃)** | **Total** |
|----------------------|------------------|----------------|-----------------|----------|
| 20000                | 20000            | 61000          | 73000           | 200000   |
| **Processing time** |                  | 60 minutes     |                 |          |
| 50 minutes           | 50 minutes       | 90 minutes     |                 | 3000 minutes |
| **Minimum production limit** | 6 pcs | 6 pcs | 6 pcs | |
| **Selling price** | 25000            | 70000          | 85000           | |

The linear programming with simplex method is used after all data are enough. The next a step is making linear programming equation, result function of linear programming is maximum profit, but factors should be defined.

- Capital obstacles
  
  \[21000x₁ + 61000x₂ + 73000x₃ \leq 2000000\]

- Time obstacles
  
  \[50x₁ + 60x₂ + 90x₃ \leq 3000\]

- Minimum limitation obstacles of products
  
  \[x₁ \geq 6\]
  
  \[x₂ \geq 6\]
  
  \[x₃ \geq 6\]

- Profit (price sell – production cost)
  
  \[x₁: 25000 - 20000 = 5000\]
  
  \[x₂: 70000 - 61000 = 9000\]
  
  \[x₃: 85000 - 73000 = 12000\]

The next process is making linear programming model,

\[Max \ Z = 5000x₁ + 9000x₂ + 12000x₃\]

Become,
With limitation:

\[ \begin{align*}
20000x_1 + 61000x_2 + 73000x_3 & \leq 2000000 \\
50x_1 + 60x_2 + 90x_3 & \leq 3000 \\
x_1 & \geq 6 \\
x_2 & \geq 6 \\
x_3 & \geq 6
\end{align*} \]

Become:

\[ \begin{align*}
20000x_1 + 61000x_2 + 73000x_3 + s_1 & \leq 2000000 \\
50x_1 + 60x_2 + 90x_3 + s_2 & \leq 3000 \\
x_1 - s_3 & \geq 6 \\
x_2 - s_4 & \geq 6 \\
x_3 - s_5 & \geq 6
\end{align*} \]

Table 2 showed value of the initial variables of simplex method.

| Variable | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( s_1 \) | \( s_2 \) | \( s_3 \) | \( s_4 \) | \( s_5 \) | \( z \) | \( nk \) |
|----------|---------|---------|---------|-------|-------|-------|-------|-------|-----|-------|
| \( S_1 \) | 20000   | 61000   | 73000   | 1     | 0     | 0     | 0     | 0     | 0   | 2000000 |
| \( S_2 \) | 50      | 60      | 90      | 0     | 1     | 0     | 0     | 0     | 0   | 3000   |
| \( S_3 \) | 1       | 0       | 0       | 0     | 0     | -1    | 0     | 0     | 0   | 6      |
| \( S_4 \) | 0       | 1       | 0       | 0     | 0     | 0     | -1    | 0     | 0   | 6      |
| \( S_5 \) | 0       | 0       | 1       | 0     | 0     | 0     | 0     | -1    | 0   | 6      |
| \( z \)   | -5000   | -9000   | -12000  | 0     | 0     | 0     | 0     | 0     | 1   | 0      |

The value of goal function is \( Z - ((5000 \times 0 + (9000 \times 0) + (12000 \times 0) + (0 \times 2000000) + (0 \times 3000) + (0 \times 36) + (0 \times 30) + (0 \times 21)) = 0 \) that showed in NK in Table 2.

Optimum solution from \( Z \) equation, and non-based variables \( x_1, x_2, \) and \( x_3, \) are have a negative coefficient, it means have coefficient on original goal function. Because, maximum result is, so \( Z \) value get is updated by fixing and increasing of the value greater than 0. The sign of “\( \geq \)” indicate non-basis variable has smallest value. Optimum condition of simplex method on maximum, when non-basis variable has coefficient is positive or non-negative on \( Z \) equation. It means, the maximum solution is achieved. After simplex table is created, the next process is iteration.

| Variable | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( S_1 \) | \( S_2 \) | \( S_3 \) | \( S_4 \) | \( S_5 \) | \( z \) | \( nk \) | Ratio |
|----------|---------|---------|---------|-------|-------|-------|-------|-------|-----|-------|-------|
| \( S_1 \) | 20000   | 61000   | 73000   | 1     | 0     | 0     | 0     | 0     | 0   | 2000000 | 100   |
| \( S_2 \) | 50      | 60      | 90      | 0     | 1     | 0     | 0     | 0     | 0   | 3000   | 60    |
| \( S_3 \) | 1       | 0       | 0       | 0     | 0     | -1    | 0     | 0     | 0   | 6      | 0     |
| \( S_4 \) | 0       | 1       | 0       | 0     | 0     | 0     | -1    | 0     | 0   | 6      | 0     |
| \( S_5 \) | 0       | 0       | 1       | 0     | 0     | 0     | -1    | 0     | 0   | 6      | 0     |
| \( z \)   | -5000   | -9000   | -12000  | 0     | 0     | 0     | 0     | 0     | 1   | 0      | 0     |

Key column is determined by negative \( z \) value base that located on non-based variable \( x_1 \). Key line is determined from smallest ratio value, where ratio value is obtained from NK\( \div \)key column.

The calculation result of ratio value on first iteration:

\[
\begin{align*}
2000000 \div 20000 &= 100 \\
3000 \div 50 &= 60 \\
6 \div 1 &= 6 \\
6 \div 0 &= 0 \\
6 \div 0 &= 0 \\
0 \div -5000 &= 0
\end{align*}
\]
Key line is line $x_1$, key column is $S_3$ because it have smallest ratio value (as long ratio value $> 0$) and it became key element is 1. Next, determining new key line. Determination is old key value $\div$ key element.

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 6 \\
\end{bmatrix} / 1
$$

New key line is obtained with equation:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
\end{bmatrix} - \begin{bmatrix}
1 & 0 & 0 & 0 & 6 \\
\end{bmatrix}
$$

The next process, determine new equation except key line include $Z$. New equation = old equation minus (new key line $\times$ old key column). The calculation of $S_1$ line is:

**Old equation**

| Variable | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $z$ | nk | Ratio |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-----|-----|-------|
| $S_1$    | 0     | 61000 | 73000 | 1     | 0     | 20000 | 0     | 0     | 0   | 200000 | 1880000 | 30,8196  |
| $S_2$    | 0     | 60    | 90    | 0     | 1     | 50    | 0     | 0     | 0   | 2700   | 45       |
| $S_3$    | 1     | 0     | 0     | 0     | 0     | -1    | 0     | 0     | 0   | 6      |
| $S_4$    | 0     | 1     | 0     | 0     | 0     | 0     | -1    | 0     | 0   | 6      |
| $S_5$    | 0     | 0     | 1     | 0     | 0     | 0     | 0     | -1    | 0   | 6      |
| $z$      | 0     | -9000 | -12000| 0     | 0     | -5000 | 0     | 0     | 1   | 3000   | -3,3333 |

Repeat until the last line and, after calculation process finish, before create result table of the first iteration, change first $S_4$ Based column become base on key column $x_1$. So that, it is obtained result on Table 4.

**Table 4. 2nd iteration.**

**Table 5. 3rd iteration.**

X$_2$ and S$_4$ become key column at 2nd iteration as key line and one key element. This iteration loop until z based have positive value or non-negative. However non-based variable $x_1$, $x_2$ and $x_3$ at equation $z$ have not positive value, and other value variable cannot be changed.

The smallest value is determined to get key column on $z$ equation at 4th iteration, after all non-based variables on $z$ equation has positive value or non-negative to determine key column with provisions negative that biggest one on $z$ equation is $S_5$ and counting ratio. The smallest Ratio value (ratio value $> 0$) will be key line, it is $S_1$ and become key line $s_1$ and key element with value 73.000.
Table 6. 4th iteration.

| Variable | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $z$ | $nk$ | Ratio |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-----|------|-------|
| $S_1$    | 0     | 0     | 0     | 0     | 0,27  | 0,83  | 1     | 0     | 14,73 | 54,55 |
| $S_2$    | 0     | 0     | 0     | 0     | 25,34 | -15,21| 0     | 0     | 473,42| 18,68 |
| $S_3$    | 1     | 0     | 0     | 0     | -1    | 0     | 0     | 0     | 6    | -6   |
| $S_4$    | 0     | 1     | 0     | 0     | 0     | -1    | 0     | 0     | 6    | 0    |
| $S_5$    | 0     | 0     | 1     | 0     | 0,27  | 0,84  | 0     | 0     | 20,73 | 24,67 |
| $z_{nk}$ | 0     | 0     | 0,16  | 0,01  | -1712 | 1027,39| 0     | 1     | 332876,71 | -194,43 |

Iteration do loop until $z$ based in positive value or non-negative. $S_3$ become line column because it has the biggest negative value. Key line is defined by the smallest ratio value, where its value from NK = key column that id explained before, its $S_3$ and become key element is 25,432.

Table 7. The result of 5th iteration ke-5 (optimum result).

| Variable | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $z$ | NK  |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-----|-----|
| $S_1$    | 0     | 0     | 0     | -0,01 | 0     | 1     | 1     | 0     | 9,62|     |
| $S_2$    | 0     | 0     | 0     | 0,04  | 1     | -0,6  | 0     | 0     | 18,68|     |
| $S_3$    | 1     | 0     | 0     | 0,04  | 0     | -0,6  | 0     | 0     | 24,68|     |
| $S_4$    | 0     | 1     | 0     | 0     | 0     | -1    | 0     | 0     | 6    |     |
| $S_5$    | 0     | 0     | 1     | -0,01 | 0     | 1     | 0     | 0     | 15,62|     |
| $z$      | 0     | 0     | 0,08  | 67,57 | 0     | 0     | 0     | 1     | 364864,8|     |

The last iteration is known on iteration process table or on goal iteration, because $z$ based already completed, because no negative value any more. This case, known as optimum. The calculation result of process on Table 7 are:

$x_1$: 24 pcs, $x_2$: 6 pcs, $x_3$: 15 pcs

Profit calculation:

$x_1$ (t-shirt): 24 x 5000 = 120.000 rupiah

$x_2$ (shirt): 6 x 9000 = 54.000 rupiah

$x_3$ (jacket): 15 x 12000 = 180.000 rupiah

Maximum profit is obtained Rp 354.000. The result of calculation test show on Table 8.

Table 8. Comparison between system test result and manual test system.

| Limitation     | System Result | Testing Result | Limitation |
|----------------|---------------|----------------|------------|
| Goal function (profit) | 354.000       | 374.000        |            |
| Time limitation | 2910          | 1960           | 3000       |
| Production Limitation | T-Shirt ($x_1$) ≥ 24 | T-Shirt ($x_1$) ≥ 25 | T-Shirt ($x_1$) ≥ 6 |
| Capital Limitation | 1.941.000     | 1.961.000      | 2.000.000  |

According calculation test, if t-shirt product is added count of its production, all limitation still fulfilled. This condition happened because calculation factor that use decimal number, so that on final result calculation rounding down is done. Because if rounding up is done, will not fulfilled existence limitation.

Table 9. Comparison between before and after production result use simplex method.

| T-Shirt ($x_1$) | Shirt ($x_2$) | Jacket ($x_3$) | Profit      |
|----------------|--------------|---------------|-------------|
| Before use the method | 16           | 8             | 15          | 342.000   |
| After use the method  | 22           | 6             | 15          | 354.000   |
Based on Table 9, known comparation of result precaution profit, before and after used simplex method are Rp 342.000 obtained from (18 pcs of t-shirt x Rp5000) + (8 pcs of shirt x Rp 9000) + (15 pcs jacket x Rp12.000) and the result of profit after used simplex method is Rp 354.000 that obtained from (24 pcs t-shirt x Rp5000) + (6 pcs shirt x Rp9000) + (16 pcs jacket x Rp 12.000).

The final result, simplex method influenced profit in The Konveksi Zom T-Shirt of the conventional calculation before. Profit use simplex method in this case, Rp. 354.000 and profit without use it, Rp. 342.000.

\[
\frac{\text{profit difference}}{\text{profit without use simplex method}} \times 100\% = \frac{78000}{276000} \times 100\% = 3.5\%
\]

4. Conclusion

The Konveksi Zom T-Shirt is a factory that produces jacket, shirt and t-shirt. This factory, use simplex method to predict composition and number of items to get maximum profits from available capital. Based on simplex method with parameters; Rp. 2.000.000 as capital, 3.000 minutes as time production, and 6 pcs as minimum productions. This method get result with composition and number of items; 15 pcs of jacket, 6 pcs of shirts, and 22 pcs of t-shirts.

The profit of production uses this method can increase 3.5% of profits from conventional method. Finally, this method can be used to support on deciding composition and number of item in productions to get a maximum profit.

Acknowledgments

This research work was partially supported by Lembaga Penelitian dan Pengabdian kepada Masyarakat (LP2M) and Informatics Dept. of Sciences and Technology Faculty UIN Sunan Gunung Djati Bandung.

References

[1] Ari I 2016 Perancangan Aplikasi Optimasi Produksi Pada CV. Indahserasi Menggunakan Metode Simpleks Jurnal INFOTEK 1 3 ISSN 2502-6968 (Medan: STMIK Budidarma Medan)

[2] Husni A 2013 Program Linier untuk Perencanaan Sumber Daya Manusia [Online] Retrieved from: https://boomershusni.wordpress.com/2013/12/03/program-linier/

[3] Anisa, Risya H A, Tajul A, Titi N H, Widiyaningsih and Yuyun Y 2017 Primal-Dual (Dualitas) (Jurusan Matematika, Fakultas Tarbiyah: IAIN Syekh Nurjati Cirebon)

[4] Risnawati I 2014 Optimalisasi Kasus Pemrograman Linear dengan Metode Grafik dan Simpleks (Makasar: UIN Alauddin)

[5] Yayuk R, Eka S and Nurul H 2016 Optimasi Produksi dan Keuntungan Pada Industri Kerupuk dengan Metode Linear Programming Simpleks dan Branch and Bound di CV KYRIA REZEKI (Tanjungpinang: Universitas Maritim Raja Ali Haji)

[6] Muhammad M 2009 Metode Pengambilan Keputusan Kuantitatif (Jakarta: Bumi Aksara)

[7] Aminudin 2005 Prinsip-Prinsip Riset Operasi (Jakarta: Erlangga)