Proposal of Specific Impulse Prediction Method for Bipropellant Thrusters*

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For attitude control and orbital transfer of satellites, bipropellant thrusters are crucial components in space propulsion systems. The overall performance of the thruster is evaluated by the specific impulse ($I_{SP}$), which directly determines the lifetime and propellant mass of satellites. Therefore, in the present study, a new theoretical framework is firstly proposed to predict $I_{SP}$ directly from injection conditions and nozzle configurations by considering the distribution of mixture and mass flow rates in the thruster chamber. As the performance index of the combustion chamber, the characteristic velocity is formulated. The frozen flow assumption is applied to the nozzle internal flow to calculate the thrust coefficient. The analytical results of $I_{SP}$ are compared to the corresponding combustion test of a 10 N bipropellant thruster using a propellant combination of mixed nitrogen oxides with 3% nitric oxide and monomethyl hydrazine, which validates the prediction model proposed.

Key Words: Bipropellant Thruster, Performance Prediction, Mixing Model, Specific Impulse

Nomenclature

\[A: \text{area [m}^2\text{]}\]
\[c^*: \text{characteristic velocity [m/s]}\]
\[C_t: \text{thrust coefficient}\]
\[D: \text{injector diameter [m]}\]
\[F_{fc}: \text{film-cooling ratio}\]
\[F_R: \text{mass flow rate ratio to total injection flow rate}\]
\[g: \text{gravitational acceleration [m/s}^2\text{]}\]
\[I_{SP}: \text{specific impulse [s]}\]
\[m: \text{injection mass flow rate [kg/s]}\]
\[M_R: \text{mixture ratio}\]
\[N: \text{number of fuel nozzles per oxidizer nozzle in an injector element}\]
\[p: \text{pressure [Pa]}\]
\[V: \text{injection velocity [m/s]}\]
\[y: \text{specific heat ratio}\]
\[\Lambda: \text{nondimensional length scale}\]
\[\eta_e*: \text{c}^*\text{efficiency}\]
\[\rho: \text{density [kg/m}^3\text{]}\]

Subscripts

\[a: \text{ambient}\]
\[c: \text{combustion chamber}\]
\[e: \text{nozzle exit}\]
\[f: \text{fuel}\]
\[fc: \text{film-cooling fuel}\]
\[i: \text{value at each stream tube}\]
\[o: \text{oxidizer}\]
\[t: \text{throat}\]

1. Introduction

Bipropellant thrusters that use hypergolic liquid propellants have been widely utilized in propulsion systems for orbital transfer and attitude control.\cite{1} They have continuously demonstrated high reliability and achieved good performance in missions for several decades.\cite{1,2} They will continue to play an important role in orbits, as well as planet landing and takeoff modules.\cite{3,4} Therefore, continual improvements in performance and reliability are expected.

The overall performance of a thruster is evaluated using the specific impulse ($I_{SP}$) given by the product of characteristic velocity as the combustion chamber performance, and the thrust coefficient as the nozzle performance. Because it is difficult to optimize the design of thruster components through many combustion tests in terms of cost and time,\cite{6,7,9} an \textit{a priori} prediction of $I_{SP}$ is expected to determine the injection conditions that achieve the most efficient operation for the mission requirements, which could realize a more efficient thruster design procedure while minimizing cost.

The phenomena in a combustion chamber of a bipropellant thruster consist of a complex multiphase flow with chemical reactions in a condensed phase and gas phase. Pressurized liquid propellants of fuel and oxidizer are ejected into the combustion chamber as liquid jet streams, and their impingement creates a sheet and a spray while mixing. With instantaneous condensed phase reactions followed by evaporation and gas phase reactions, which all occur in a sequential fashion. Several studies have been devoted to predicting $c^*$ since the 1950s. A practical indicator, the so-called Rupe factor,\cite{5,6} was derived as a function of the momentum flux ratio and nozzle diameter ratio using comprehensive cold-flow tests for unlike-doublet injectors\cite{6,7} and triplet injectors.\cite{8} Wrobel\cite{9} theoretically derived $c^*$ using the stratification

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model, namely stream tube analysis, by imposing an adequate local mixture ratio for each stream tube to take into account the distributions of mixture ratio inside the combustion chamber. Subsequently, the cold-flow-based approach was validated as a practical method for the prediction of $c^*$ for various injector types to reduce the number of expensive combustion tests. Recently, Inoue et al. were the first to achieve the straightforward formulation of $c^*$ as a function of injection conditions in a unified manner independent of injector types considering the effect of film-cooling fuel.

There have also been some attempts to obtain $I_{SP}$. Fujii et al. reported combustion test results. Rollbuhler measured $I_{SP}$ at several injectors using combustion tests. However, in past studies, a direct formulation of $I_{SP}$ as a function of injection conditions and nozzle configurations has not been derived yet. Therefore, in the present study, a direct prediction method of $I_{SP}$ by extending the $c^*$ prediction framework combined with a thrust coefficient calculation is proposed. The theoretical values of $I_{SP}$ over a wide range of injection mixture ratios are compared with corresponding combustion test results to confirm the validity of the original formulation.

2. Formulation of Specific Impulse

$I_{SP}$ is generally defined by Eq. (1). In the following, the formulations of $c^*$ as a combustion chamber performance and $C_t$ as a nozzle performance are explained considering the mixing states inside the thrust chamber.

$$I_{SP} = \frac{c^* C_t}{g} \tag{1}$$

2.1. Modeling of mixture ratio distribution

Bipropellant thrusters often use unlike impinging injectors, such as unlike doublet and triplet types. As shown in Fig. 1, there are the main spray and film-cooling fuel inside the combustion chamber. The main spray region is produced by impinging the fuel and oxidizer jet streams. The film-cooling fuel protects the chamber wall from the high-temperature combustion core, as visible during the combustion test in Fig. 2. The film-cooling fuel evaporates and finally self-decomposes.

The nondimensional length scale of $A$ for the impinging spray structure is given by Eq. (2), using propellant density, injection velocity, and injector diameter.

$$A = \sqrt{\frac{N \rho_f V_f^2 D_f}{\rho_o V_o^2 D_o}} \tag{2}$$

It is found that $A$ is equivalent to the spray width ratio of oxidizer to fuel in a physical sense (see Appendix), and $A$ of unity achieves the optimal mixing condition. However, since the value of $A$ inherently changes corresponding to the injection conditions, a nonuniform local mixture ratio as $A$ of non-unity is realized. Considering the film-cooling fuel, there are several stream tubes with respect to the local mixture ratio. The overall performance consists of the summation of the performance in each stream tube, which is determined by the local mixture ratio. Therefore, it is mandatory to evaluate the distributions of mixture ratio inside the thrust chamber to predict the performance with high accuracy.

In the present study, four types of mixture ratio distributions in the thrust chamber are investigated, as shown in Fig. 3, based on the stream tube approach. Case 1: A uniform mixture ratio is ideally realized inside the thrust chamber, the same as the injection mixture ratio. The main spray and the film-cooling fuel perfectly mix. Case 2: There are nonuniform mixture ratio distributions inside the thrust chamber. Two stream tubes are allocated for the main spray, as indicated by “Main-1” and “Main-2”, and one stream tube for the film-cooling fuel, as depicted by “Film”. All stream tubes have their own mixture ratios, which remain up to the nozzle exit.

Fig. 1. Schematic of a bipropellant thruster using an FOF-type triplet injector with film-cooling fuel on the wall.

Fig. 2. Visualization result inside a combustion chamber.

Fig. 3. Mixture ratio distribution models (each stream tube has a mixture ratio).
Case 3: There are mixture ratio distributions inside the combustion chamber. The main spray of Main-1 and Main-2 instantaneously mixes to be the uniform mixture ratio at the nozzle throat, indicated as mixing plane. Inside the nozzle, two stream tubes of “Mixed main” and “Film” exist.

Case 4: There are mixture ratio distributions inside the combustion chamber. The combustion gases in all of the stream tubes of Main-1, Main-2, and Film mix at the nozzle throat, being the “Mixed flow”. Inside the nozzle, a uniform mixture ratio is achieved.

2.2. Modeling of characteristic velocity

The total mixture ratio is defined as the flow rate ratio of the injected oxidizer to fuel, including film-cooling fuel.

\[ M_R = \frac{m_o}{m_f + m_{fc}} \]  \( \text{(3)} \)

The film-cooling ratio is expressed by the flow rate ratio of film-cooling fuel to the injected fuel in total.

\[ F_C = \frac{m_{fc}}{m_f + m_{fc}} \]  \( \text{(4)} \)

In Cases 2–4, the values of local mixture ratio and local flow rate ratio for each stream tube inside the combustion chamber are consistently given using \( A, M_R, \) and \( F_C \), as depicted in Table 1.\(^{14}\)

The characteristic velocity at each stream tube of \( c_i^* \), corresponding to \( M_{Ri} \), is calculated from the chemical equilibrium analysis.\(^{20}\) By integrating \( c_i^* \) for all stream tubes of \( i = \text{Main-1, Main-2, and Film} \) weighted-average by the flow rate ratio in respective stream tubes of \( F_{Ri} \), \( c^* \) is deduced as the performance of the combustion chamber.

\[ c^* = \sum c_i^* \cdot F_{Ri} \]  \( \text{(5)} \)

The ratio of \( c^* \) to the maximum value, \( c^*_{\text{ideal}} \), at given \( M_R \) is defined by \( \eta_{c^*} \) as follows.

\[ \eta_{c^*} = c^*/c^*_{\text{ideal}} \]  \( \text{(6)} \)

Obviously, \( c^* \) of Case 1 coincides with \( c^*_{\text{ideal}} \).

2.3. Modeling of thrust coefficient and specific heat ratio

As the combustion gas expands in the supersonic nozzle, the pressure and temperature decrease downstream. In the present study, a frozen flow is assumed, in which the chemical components inside the nozzle remain the same as in the combustion chamber, while \( \gamma \) changes inside the nozzle as the pressure and temperature change. For each stream tube inside the nozzle, the value of \( \gamma_i \) is used as the mean value of \( \gamma \) at the throat and nozzle exit. In Cases 3 and 4, some stream tubes inside the combustion chamber mix at the throat, and \( \gamma_i \) at the mixed stream tube is obtained by the weighted-average of \( F_{Ri} \) in the mixed regions. For instance in Case 3, \( \gamma_i \) in the stream tube of Mixed main is obtained applying the mean values of \( \gamma_i \) in Main-1 and Main-2 weighted by each \( F_{Ri} \).

The pressure ratio of \( p_{th} / p_e \), and \( \gamma_i \) is iteratively calculated using Eq. (7) and the isentropic relationships inside the nozzle at the given value of \( A_e/A_t \).

\[ \frac{A_e}{A_t} = \left( \frac{2}{\gamma_i + 1} \right)^{1/2} \sqrt{\frac{p_e}{p_{th}}} \left( \frac{\gamma_i - 1}{\gamma_i + 1} \right) \left( 1 - \left( \frac{p_e}{p_{th}} \right)^{\frac{\gamma_i - 1}{\gamma_i}} \right)^{-1} \]  \( \text{(7)} \)

The thrust coefficient in each stream tube of \( C_{fi} \) is obtained depending on the ambient pressure of \( p_a \).

\[ C_{fi} = \sqrt{\frac{2\gamma_i^2}{\gamma_i - 1} \left( \frac{2}{\gamma_i + 1} \right)^{1/2} \left( 1 - \left( \frac{p_e}{p_{th}} \right)^{\frac{\gamma_i - 1}{\gamma_i}} \right)} \] + \left( \frac{p_e}{p_{th}} \right) \frac{A_e}{A_t} \]  \( \text{(8)} \)

As the nozzle performance, \( C_f \) is calculated as follows.

\[ C_f = \sum C_{fi} \cdot F_{Ri} \]  \( \text{(9)} \)

2.4. Modeling of \( IS_{P} \)

Finally, the total performance of \( IS_{P} \) is deduced using \( c_i^* \) and \( C_{fi} \) in Eq. (10).

\[ IS_{P} = \sum \frac{c_i^* \cdot C_{fi} \cdot F_{Ri}}{g} \]  \( \text{(10)} \)

The value of \( IS_{P} \) is available directly from the injection conditions of \( A, M_R, F_C, \) and nozzle configurations.

In Case 1, in which a uniform \( M_{Ri} \) is realized equivalent to \( M_R \) in the thrust chamber, \( F_{Ri} \) is unity, and \( c_i^*, C_{fi} \) and \( IS_{P} \) are all simply calculated using the chemical equilibrium analysis\(^{20}\) at given \( M_R \).

3. Analysis Results and Discussion

The calculation results and the past combustion test results regarding \( c^* \) and \( IS_{P}\)\(^{16}\) are compared to confirm the validity of the prediction model proposed. The analysis conditions are shown in Table 2. The propellant is a combination of a mixture of nitrogen tetroxide and 3% nitric oxide (MON3) as the oxidizer and monomethyl hydrazine (MMH) as the fuel. The injector type is fuel–oxidizer–fuel triplet, which has two fuel injection ports for one oxidizer injection port as \( N = 2 \). The combustion test was performed in a vacuum.
3.1. Characteristic velocity

Figure 4 shows the results $\eta_{\text{c}}$ for Case 1 and Cases 2–4 with the combustion test results against $M_R$. Because the distributions of mixture ratio inside the combustion chamber in Cases 2–4 are equivalent, $c^*$ and $\eta_{\text{c}}$ of the three cases are the same. At the ideal mixing condition of Case 1, $\eta_{\text{c}}$ achieves unity, and overestimates the combustion test results. By considering the distributions of mixture ratio inside the combustion chamber, $\eta_{\text{c}}$ of Cases 2–4 reproduce the experimental results well, in a quantitative sense, indicating that the values of $c^*$ and $F_{\text{Ri}}$ in the three kinds of stream tubes (i.e., Main-1, Main-2, and Film) are adequately calculated. The prediction models, however, do not consider the factors that cause $c^*$ to deteriorate, such as heat loss from the combustion chamber wall. Therefore, the analytical results overestimate the experimental value of maximum $\eta_{\text{c}}$.

3.2. Thrust coefficient

Figure 5 presents the results calculated for $F_{\text{Ri}}$ in Case 2. It is clear that $F_{\text{Ri}}$ of Main-1 is dominant for all $M_R$. The values of $\gamma_i$ and $C_{i\text{f}}$, which are weighted-averages using $F_{\text{Ri}}$, are expected to be determined by those of Main-1.

Figure 6 shows the results calculated for $\gamma_i$ and $C_{i\text{f}}$ at each stream tube for Cases 2–4. Throughout all $M_R$ conditions, $\gamma_i$ and $C_{i\text{f}}$ at the stream tubes of the mixed main in Case 3 and mixed flow in Case 4 are almost equivalent to the values of Main-1, because $F_{\text{Ri}}$ of Main-1 is dominant. At the stream tube of Main-1, $C_{i\text{f}}$ varies corresponding to the change in $\gamma_i$, in which a larger $\gamma_i$ results in a smaller $C_{i\text{f}}$, consistent with Eq. (8). At Main-2, $\gamma_i$ changes at $M_R = 1.2$, equivalent to $\Lambda$ of unity, and $M_{\text{Ri}}$ changes from 0 to infinity, as shown in Table 1; therefore, $C_{i\text{f}}$ changes discontinuously. The film-cooling fuel self-decomposes and the products are frozen. Hence, the values of $\gamma_i$ and $C_{i\text{f}}$ are constant against $M_R$ in the film stream tube.

Following Eq. (9), we obtain $C_{i\text{f}}$ as shown in Fig. 7. We confirm the results calculated for $C_{i\text{f}}$ of Cases 2–4 are almost the same, independent of $M_R$. However, $C_{i\text{f}}$ of Case 1 shows a different trend. Therefore, it is important to consider the nonuniform distribution of $M_{\text{Ri}}$ inside the combustion cham-

Table 2. Calculation conditions.

| Oxidizer/Fuel | MON3/MMH |
|--------------|-----------|
| $\rho_{\text{i}}/\rho_{\text{f}}$ | 1.6 |
| $D_{\text{i}}/D_{\text{f}}$ | 1.5 |
| $N$ | 2 (FOF-triplet) |
| $F_{\text{C}}$ | 0.3 |
| $A_{\text{i}}/A_{\text{f}}$ | 180 |
| $p_{\text{i}}$ | 0.8 MPa |
| $p_{\text{a}}$ | 0 MPa |

Fig. 4. $\eta_{\text{c}}$ during the combustion test and result calculated.

Fig. 5. Mass flow rate ratio of each stream tube in Case 2.

Fig. 6. Values calculated for $\gamma_i$ and $C_{i\text{f}}$ in each stream tube in Cases 2–4.

Fig. 7. Comparison of total thrust coefficient for Cases 1–4.
ber, while the distribution inside the nozzle is negligible, attributed to the dominant value of $F_Ri$ in the Main-1 stream tube.

3.3. Specific impulse

Figure 8 compares $I_{SP}$ and $M_R$. The combustion test results reach the maximum value of $I_{SP}$ at $M_R = 1.3$. The results calculated for $I_{SP}$ of Case 1 present a different trend from the results of the combustion test, while the results of $I_{SP}$ for Cases 2–4 reproduce the combustion test results with a relative difference of less than 10% over a wide range of total mixture ratios. The same values of $e^*$ and $C_1$ throughout Cases 2–4 provide the same $I_{SP}$. The validity of $e^*$ (see also Inoue et al.14) and $I_{SP}$ using the model proposed proves that present $C_1$ model expressed by Eq. (9) is reasonable. The relative variance of $C_1$ against $M_R$ is less than 5% in Fig. 7, and $I_{SP}$ is strongly affected by $e^*$, which must be accurately calculated. One can recognize the importance of considering the distribution of $M_R$ to predict the $I_{SP}$ of Eq. (10), a direct function of injection conditions, as in Table 1, which is proposed here for the first time.

4. Conclusions

The present paper proposes a straightforward formulation of $I_{SP}$ to evaluate the overall performance of bipropellant thrusters. The conclusions are summarized as follows:

(1) By calculating adequate distributions of mixture and mass flow rate ratios in a thrust chamber directly using the injection conditions, the $I_{SP}$ is formulated as a function of the injection condition and nozzle shape.

(2) The present analytical results of $I_{SP}$ reproduce the combustion test results with a relative error of less than 10% using a wide range of mixture ratios.

(3) The thrust coefficient at the main spray determines the overall thrust coefficient. Hence, the nonuniformity of the mixture ratio inside the nozzle is insignificant to $I_{SP}$.

In the future, we need to continuously further validate the framework proposed to predict $I_{SP}$ by making comparisons with other experimental results in order to utilize the bipropellant design procedure.

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Appendix

Figure A1 schematically shows a single element of an injector as unlike-doublet, FOF triplet, and OFO triplet. The origin of the coordinate system is defined at the stagnation point produced by the collision of the fuel and oxidizer jets. The symbol $\mathbf{O}$ and $\mathbf{Y}$ denote the location of “absolute” central gravity in the $x$–$y$ plane at a distance of $z$, and the length, respectively. By identifying the flux of the oxidizer and fuel towards the $z$-direction defined as $\dot{q}$ [kg/s/m$^2$], we obtain the ratio of $Y_o$ to $Y_f$ as follows:

$$
\frac{Y_o}{Y_f} = \frac{\sum (y|\dot{q}_o|)}{\sum \dot{q}_f}
$$

(11)

Independent of the liquid species and injection conditions, the following relationship was confirmed.$^{14,15}$

$$
\frac{Y_o}{Y_f} = \Lambda
$$

(12)

A simple one-dimensional mixing model is assumed, in which the spray uniformly distributes along the $y$-direction in the $x$–$y$ plane, as illustrated in Fig. A2. Here, the respective values of $\dot{q}_o$ and $\dot{q}_f$ are constant in respect to the $y$-direction. The oxidizer and main fuel sprays spread with mixing; however, they are schematically shown separately. The half-spread width of the oxidizer and fuel sprays along the $y$-direction corresponds to $2Y$, at twice the absolute central gravity of $Y$. The film cooling fuel spreads on the chamber wall without mixing with the oxidizer.

For $\Lambda \leq 1$ in Fig. A2(a), the oxidizer spray extends more narrowly compared to the fuel spray, and the oxidizer and fuel can mix in the area of $|y| < 2Y_o$ defined as the stream tube of Main-1. Only the fuel exists in $2Y_o < |y| < 2Y_f$ defined as the stream tube of Main-2.

For $\Lambda > 1$ in Fig. A2(b), the oxidizer and fuel can mix in the area of $|y| < 2Y_f$ (Main-1); however, the oxidizer exists in $2Y_f < |y| < 2Y_o$ (Main-2) without mixing with the fuel. The spread widths are equivalent at $\Lambda$ of unity, at which time the sprays mix perfectly in the Main-1 region.

For each stream tube, labeled Main-1, Main-2, and Film, the local mixture ratio of $M_R$ and local flow rate ratio of $F_R$ are consistently provided using $\Lambda$, $M_R$, and $F_C$, as depicted in Table 1. The derivation is described in detail in Inoue et al.$^{14}$

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