Hadronic Electroweak Spin-Torsion Interactions

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Abstract

In a previous paper we considered matter field equations with torsional contribution giving rise to special interactions which have eventually been shown to have the same structure of the electroweak forces; the results obtained in that paper are extended in this paper to the whole first generation of fermions. As a possible outlook the extension to all generations is discussed and consequently some speculations about the value for the mass of the Higgs field are presented.

Introduction

In the structure of the matter field equation given by the Dirac fermionic field equation, the most general spinorial derivative contains torsion; because torsion is a tensor then the torsional contribution can be separated apart without spoiling the covariance of the whole derivative: after torsion has been separated away what remains is the simplest spinorial derivative plus terms given by torsion and representing additional interactions. Eventually field equations coupling torsion to the spin distribution of the spinorial field are taken; when these field equations are plugged into the matter field equation, the additional interactions will turn out to be spinorial autointeractions.

In the case in which many spinorial fields are considered, then the spin distribution is the total spin distribution given by the sum of the spin distributions of each and every single spinor involved; in each matter field equations, the additional interactions turns out to be spinorial autointeraction of the spinor with itself as well as spinorial interactions with the spinor with all other spinors that take place in the dynamics of the process. Then it is possible to consider these interactions in order to study the form they may possibly have.

In a previous paper [1] we have considered the simplest case given by two spinors of which one is a spinor and the other is a single-handed spinor showing that the matter field equation in the torsional free case is formally equivalent to the matter field equation without torsion but with the electroweak gauge interactions; in the final remarks that paper raised the question about the possible extension to more general situations in which the torsional interactions may take place for two spinors with both projections although one does not interact. In this paper we consider this extension by studying the case in which torsional interactions take place for two spinors with both projections with the two left-handed projections able to mix but with the two right-handed projection unable to mix showing that the matter field equation in the free case is
formally equivalent to the matter field equation in the torsionless case with the electroweak gauge interaction.

1 Torsional interaction

As in the previous paper, we consider a set of $k$ matter fields labeled with the indices in parentheses each of which governed by the matter field equation

$$i\gamma^\mu D_\mu \phi^a = 0, \quad a = 1 \ldots k$$

(1)

given in the massless case, and these equations come along with the background field equations that are given for the combination of the Ricci tensor and scalar and for the Cartan tensor in terms of the energy and the spin distribution of the matter field as

$$G_{\alpha\beta} = \frac{1}{2}g_{\alpha\beta} G = \frac{1}{4} \sum_a \left[ \bar{\psi}^a \gamma_\alpha D_\beta \psi^a - D_\beta \bar{\psi}^a \gamma^\alpha \psi^a \right]$$

(2)

and

$$Q_{\mu\alpha\beta} = -\frac{i}{4} \sum_a \bar{\psi}^a \{ \gamma_\mu, \sigma_{\alpha\beta} \} \psi^a$$

(3)

according to the prescription of the Einstein–Sciama-Kibble scheme. Then it is possible to use the torsion as given by the field equations (3) in order to substitute torsion with the spin of the spinor fields as

$$i\gamma^\mu \nabla_\mu \psi^a + \frac{1}{16} \sum_\delta \bar{\psi}^b \gamma_\mu \gamma^b \gamma^\mu \gamma^\delta \psi^a = 0, \quad a = 1 \ldots k$$

(4)

in which we see that spinorial bilinears appear.

1.1 Torsional interaction: spin coupling of spinor and single-handed spinor

In this paper we will consider the case given when only two spinor fields are present and although both are taken to be the full spinor their two right-handed projections are not allowed to mix.

So in the case we have two spinors the matter field equations are given by the matter field equations (4) with $k = 2$ and they can be explicitly written as

$$i\gamma^\mu \nabla_\mu \psi^1 + \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \gamma^1 + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \gamma^1 = 0$$

(5)

$$i\gamma^\mu \nabla_\mu \psi^2 + \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \gamma^2 + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \gamma^2 = 0$$

(6)

as it can be seen by separating the fields.

Moreover, we can separate the right-handed and left-handed projections as

$$i\gamma^\mu \nabla_\mu \psi^1_L + \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^1_L + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^1_L - \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_L - \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_L = 0$$

(7)

$$i\gamma^\mu \nabla_\mu \psi^1_R + \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^1_R + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^1_R - \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_R - \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_R = 0$$

(8)

$$i\gamma^\mu \nabla_\mu \psi^2_L + \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_L + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_L - \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_R - \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_R = 0$$

(8)

$$i\gamma^\mu \nabla_\mu \psi^2_R - \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_R + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_R - \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_R + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_R = 0$$

(9)

$$i\gamma^\mu \nabla_\mu \psi^1_L + \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^1_L + \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^1_L - \frac{3}{16} \bar{\psi}^1 \gamma_\mu \gamma^1 \gamma^\mu \psi^2_L - \frac{3}{16} \bar{\psi}^2 \gamma_\mu \gamma^2 \gamma^\mu \psi^2_L = 0$$

(10)
in which all spinors are semi-spinors in single-handed irreducible representation.

By using the Fierz identities we can write these field equations in formally equivalent ways that will be more suited for the task we want to pursue: the first insight will be to try to follow (11) in order to reproduce the field equations for the hadron fields before the symmetry breaking occurring in the standard model and in doing so we will see that we will obtain two different composite scalar fields corresponding to the two different fermions in the family; yet another possibility is look for a different way in which supplementary interactions will appear in the field equations for the hadron fields before the symmetry breaking occurring in the standard model but for which there will only be one composite scalar field written as a combination of all fermion fields. However we also know that in the first case each fermion undergoes mass generation provoked by the corresponding Higgs in such a way that the fermion mass is equal to the corresponding Higgs mass, with the consequence that for a general family of hadrons the Linde-Weinberg bound is not achieved and the stability of the vacuum configuration is not accomplished; thus we have to turn to the second case in which all fermions undergo mass generation provoked by a single Higgs in such a way that it is in terms of a combination of all fermion masses that the Higgs mass is written, so that if only one family has only one hadron whose mass is higher than the Linde-Weinberg bound then the stability of the vacuum configuration is ensured for the entire system under consideration.

Massless fundamental hadrons and composite scalar and vector fields.

In order to see that what we have discussed above may be obtained we see that the field equations may be rearranged in a form that is the same as

\[ i \gamma^\mu \nabla_\mu L - \frac{1}{2} g \vec{A} \cdot \vec{A} \gamma^\mu L - \frac{1}{6} g' B_\mu \gamma^\mu L - G_\mu i \sigma^2 \phi^* u - G_d \phi d = 0 \]

\[ i \gamma^\mu \nabla_\mu u - \frac{1}{2} g' B_\mu \gamma^\mu u + G_\mu i \phi^T \sigma^2 L = 0 \]

\[ i \gamma^\mu \nabla_\mu d + \frac{1}{16} g' B_\mu \gamma^\mu d - G_d \phi^* L = 0 \]

in which all supplementary interactions appeared as three-field interactions and therefore extra terms are present but negligible leaving a system of equations in a known form. In fact in the leading order of approximation we have that this form is that of the field equations for the hadron fields before the symmetry breaking occurring within the standard model of elementary fields.

In order to be able to get this form we have to rename the spinor fields

\[ (\psi^I_R) = u \quad (\psi^J_R) = d \quad \left( \begin{array}{c} \psi^I_L \\ \psi^J_L \end{array} \right) = L \]

as new hadron fields: then we have to consider their bilinear fields defining

\[ \frac{3}{32 G_d G_u} \left( \begin{array}{c} 5 G_u \psi^I_R \psi^J_L - 2 G_d \psi^I_R \psi^J_L \\ 5 G_u \psi^I_R \psi^J_L + 2 G_d \psi^I_R \psi^J_L \end{array} \right) = \phi \]

for the scalar field; and finally we have

\[ \frac{3}{32} \left[ \frac{1}{2} (\psi^I_L \gamma_\mu \psi^J_L + \psi^J_L \gamma_\mu \psi^I_L) + 2 \psi^I_L \gamma_\mu \psi^J_R - \psi^J_R \gamma_\mu \psi^I_R \right] = g' B_\mu \]

\[ \frac{3}{32} \left[ \frac{1}{2} (\psi^I_L \gamma_\mu \psi^J_L + \psi^J_L \gamma_\mu \psi^I_L) \right] = g A^3_\mu \]

\[ \frac{3}{32} \left[ \frac{1}{2} (\psi^I_L \gamma_\mu \psi^J_L - \psi^J_L \gamma_\mu \psi^I_L) \right] = g A^2_\mu \]

\[ \frac{3}{32} \left[ \frac{1}{2} (\psi^I_L \gamma_\mu \psi^J_L + \psi^J_L \gamma_\mu \psi^I_L) \right] = g A_\mu \]

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for the vector fields. Transformation laws for the hadrons and the scalar and vector fields are assigned as before the symmetry breaking occurring within the standard model of elementary fields.

Massless fundamental hadrons and composite scalar and vector fields: structure of $U(1) \times SU(2)_L$ local electroweak gauge interaction. Finally we consider the field equations written as

\begin{align}
\bar{u}\gamma_\mu D_\mu u &- G_u \sigma^2 \phi^* u - G_d \phi d = 0 \quad (17) \\
\bar{u}\gamma_\mu D_\mu u + G_u \phi^T \sigma^2 L & = 0 \quad (18) \\
\bar{d}\gamma_\mu D_\mu d - G_d \phi^d L & = 0 \quad (19)
\end{align}

in which the derivatives are in compact form.

This form is obtained by defining the derivatives

\begin{align}
\mathbb{D}_\mu L &= \nabla_\mu L + \frac{i}{2} (g \bar{\sigma} \cdot \vec{A}_\mu + \frac{1}{3} g' B_\mu) L \\
\mathbb{D}_\mu u &= \nabla_\mu u + \frac{i}{2} g' B_\mu u \\
\mathbb{D}_\mu d &= \nabla_\mu d - \frac{i}{2} g' B_\mu d
\end{align}

covariant for general $U(1) \times SU(2)_L$ local transformations. This generalization is possible since the massless fundamental hadrons $u$ and $d$ and also $L$ are functions of the spacetime position and so their mixing may take place with coefficients depending on the spacetime position themselves.

By following the same procedure we have followed in [1] it is possible to see that a stable vacuum configuration is assumed in which in the same order of approximation discussed above it is in terms of the sum of the squared fermion masses that the Higgs mass is given, and so the mass of the Higgs depends on the masses of all leptons and hadrons within a specific family; if we speculate that the procedure in [1] applied here can be fully extended then it would be possible to have a stable vacuum configuration in which in the same order of approximation discussed above it might be in terms of the sum of squared fermion masses that the Higgs mass would be given, and so the mass of the Higgs would depend on the masses of all leptons and hadrons in all families: therefore if such generalization were possible in the leading order of approximation such a mass relation would read

\begin{align}
m_H^2 &\approx m_t^2 + m_b^2 + m_s^2 + m_c^2 + m_u^2 + m_d^2 + m_e^2 + m_\mu^2 + m_\tau^2 \quad (23)
\end{align}

in which we see that because the heaviest hadron in the last family is more massive than the Linde-Weinberg bound then the stability of the vacuum configuration would be ensured. Under these assumptions in the case here presented it would be possible to obtain an approximated mass of

\begin{align}
m_H &\approx 173 \text{ GeV} \quad (24)
\end{align}

and it might be possible to reproduce the correct dynamics for the Higgs field.

Conclusion

In this paper we have proved that the matter field equations in the most general torsional case for spinors with both projections although one does not interact
are formally equivalent to the matter field equations in the simplest torsionless case plus the electroweak gauge interactions for massless quarks; to proceed in logical order next step would be to know whether this derivation can be extended to more general situations in which the torsional interactions take place for both spinors having both projections in interaction thus formally obtaining the strong gauge interaction of massless quarks.

Should all this be done the result would be that matter field equations with torsion for spinors are formally equivalent to matter field equations for gauge invariant massless fermions implying a connection between torsion and gauge interactions; as already mentioned in the previous paper the gauge interactions arise from gauging internal transformations whereas torsion generates spacetime translations (as discussed in [3]); so spacetime translations are related to internal transformations like for supersymmetric transformations and a link between the torsion tensor and supersymmetry may be established whenever the hypotheses of the Coleman-Mandula theorem are satisfied.

Differently from the previous paper however we have here more drastic issue related to the fact that what we have obtained are matter field equations for gauge invariant massless fermions, that is before the breakdown of the gauge symmetry generating the mass of fermions themselves; in fact a symmetry breaking for the mass generation may be assigned as usual by the introduction of the Higgs field and, although in the previous paper this could be done by defining the Higgs field to be a composite state of fermion fields, in this paper a single Higgs field has to be a combination of all fermion fields: this is after all to be expected, since the initial field equations are formally equivalent to the well-known Nambu-Jona–Lasinio field equations, which produce spontaneous chiral symmetry breaking with the consequent mass generation for the fields (as discussed for nuclear interactions with mesons in [2]).

In the case in which these extensions were to be achieved then we would have that the initial work started in with the Nambu-Jona–Lasinio model of nuclear interactions mediated by mesons here extended to the model of electroweak interactions mediated by bosonic states of bound fermions will be enlarged to all nuclear interactions mediated by bosonic states of bound fermions; the symmetry breaking for the mass generation would be accomplished again by bosonic states of bound fermions. Therefore this would mean that the idea put forward by Hehl, Von Der Heyde, Kerlick and Nesterin for which the nuclear interactions arise from the torsional interactions might be justified.

References

[1] Luca Fabbri, arXiv:1006.4025 [hep-ph].

[2] Y. Nambu and G. Jona–Lasinio, Phys. Rev. 124, 246 (1961).

[3] F. Hehl, P. Von Der Heyde, G. Kerlick and J. Nester, Rev. Mod. Phys. 48, 393 (1976).