Completing the Square to Find the Supersymmetric Matter Effective Action Induced by Coupling to Linearized $N = 1$ Supergravity

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ABSTRACT

We consider generic $N = 1$ supersymmetric matter coupled to linearized $N = 1$ supergravity through the multiplet of currents. By completing the square, we find the effective action giving the leading supergravity induced correction to the matter dynamics, expressed explicitly as a quadratic form in the components of the current multiplet. The effective action is supersymmetry invariant through an interplay of the local terms arising from the auxiliary field couplings, and the nonlocal terms arising from graviton and gravitino exchange, neither of which is separately invariant. Having an explicit form for the supergravity induced effective action is a first step in studying whether supergravity corrections can lead to dynamical supersymmetry breaking in supersymmetric matter dynamics. In Appendices we give explicit expressions for the currents, in our notational conventions, in the Wess-Zumino and supersymmetric Yang Mills models.
1. Introduction

Supersymmetry, if it is to be relevant to physics, must be broken, and mechanisms for supersymmetry breaking have been a subject of ongoing study. While the familiar O’Raifeartaigh and Fayet-Iliopoulos mechanisms [1] rely on the presence of scalar components of chiral matter supermultiplets, or $U(1)$ gauge supermultiplet auxiliary fields, respectively, there still exists the possibility that supersymmetry may be dynamically broken in theories lacking these fields. In particular, for supersymmetric non-Abelian gauge theories, while there are general index theorem arguments which show that such theories cannot break supersymmetry dynamically when considered in isolation [1], such theorems do not apply to supersymmetric non-Abelian theories coupled to supergravity. Thus, there remains the interesting possibility that supergravity couplings may trigger dynamical symmetry breaking in supersymmetric gauge theories.

The aim of this paper is to carry out the first technical step needed in a study of whether supergravity couplings can induce matter supersymmetry breaking, by integrating out the supergravity dynamics to leading order to give a supersymmetric effective matter action, which incorporates the effects of the supergravity couplings. Although calculating the supergravity induced effective action involves no fundamentally new concepts, it appears not to have been done before. Since the results are elegant, and illustrate the intimate connection between the supercurrent multiplet and the linearized supergravity multiplet including auxiliary fields, we present a detailed account of them here. We carry out our calculations in terms of component fields, with careful attention to such issues as the phases appearing in the supercurrent transformation, the effect of gauge invariances of linearized supergravity on the inversions of the kinetic terms needed to isolate the effective action, and the independence of
the results of the choice of gauge fixing. In future work, we plan to study the effective action derived in this paper, to see whether it permits evasion of the classical “no-go” theorems restricting dynamical symmetry breaking in supersymmetric theories. Well-known examples in non-supersymmetric theories where analogous effective actions lead to symmetry breaking are the BCS theory of superconductivity, where the phonon exchange effective action is responsible for symmetry breaking, and models for chiral symmetry breaking, dynamical electroweak symmetry breaking, and “color superconductivity” in non-Abelian gauge theories, where the one gluon exchange effective action leads to symmetry breaking.

In Sec. 2 we review the transformation properties of the linearized supergravity multiplet formulated using the minimal auxiliary fields, as well as the analogous transformation properties of the supercurrent multiplet, and give the standard interaction Lagrangian which is invariant under simultaneous supersymmetry transformations of the supergravity and supercurrent multiplets. We also summarize the Abelian gauge invariances which incorporate the effect of general coordinate invariance in the linearized theory, and the closely related conservation laws for the supercurrent multiplet, which render the interaction Lagrangian gauge invariant. In Sec. 3 we consider, as an analog, the familiar case of quantum electrodynamics, and show that current conservation permits one to complete the square to find the effective action giving the effects of the photon dynamics on the charged fermions, independently of the choice of gauge fixing action. In Sec. 4 we show how to carry out the analogous calculation in the linearized supergravity case, by using energy momentum tensor and supersymmetry current conservation to complete the square in the graviton and gravitino kinetic terms, independently of the choice of the supergravity gauge fixing action. In Sec. 5 we summarize the resulting formula for the effective action, which is explicitly in-
variant under the supersymmetry transformation of the supercurrent multiplet. In Appendix A we give our metric and gamma matrix conventions, in Appendix B we give formulas for the supercurrent components and the related “supercurrent anomalies” in the Wess-Zumino model, and in Appendix C we give analogous formulas for the supersymmetric Yang-Mills model. These currents obey the transformation formulas given in Sec. 2 (and were used as a check on the phases appearing in these transformation formulas), and will be needed in applications of the effective action to the study of the possibility of supergravity induced dynamical symmetry breaking in the respective models to which they pertain.

2. The linearized supergravity multiplet and the supercurrent multiplet

In linearized general relativity, the spacetime metric $g_{\mu\nu}$ is assumed to deviate from the Minkowski metric $\eta_{\mu\nu}$ by only a small perturbation proportional to $h_{\mu\nu}$, 

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \quad .$$

(2.1)

The constant $\kappa$ appearing in Eq. (2.1) is defined by

$$\kappa = (8\pi G)^{\frac{1}{2}} = M_{\text{Planck}}^{-1} \quad ,$$

(2.2)

where $G$ is Newton’s constant, and so the perturbation $h_{\mu\nu}$ has dimension one, as is usual for a bosonic field. In linearized supergravity, one adjoins to the spin 2 graviton field $h_{\mu\nu}$ a spin 3/2 Rarita-Schwinger Majorana field $\psi_\mu$, which describes the fermionic gravitino partner of the graviton. Although one can write down a supersymmetry algebra based on just the graviton and gravitino fields, it closes only “on shell”, that is, with use of the equations of motion.

To get a supergravity multiplet for which the supersymmetry algebra closes without use of the equations of motion, it is necessary to add auxiliary fields which vanish on shell in
the absence of matter source couplings. The minimal set of auxiliary fields for supergravity has been shown [2] to be an axial vector $b_{\mu}$, a scalar $M$, and a pseudoscalar $N$; the supersymmetry variations of these fields and of $h_{\mu\nu}$ and $\psi_{\mu}$, which represent the supersymmetry algebra without invoking the equations of motion, are

\[
\delta h_{\mu\nu} = \frac{1}{2} \bar{\epsilon} (\gamma_{\mu} \psi_{\nu} + \gamma_{\nu} \psi_{\mu}) ,
\]

\[
\delta \psi_{\mu} = [-\sigma^{\kappa\nu} \partial_{\kappa} h_{\mu\nu} - \frac{1}{3} \gamma_{\mu} (M + i \gamma_{5} N) + (b_{\mu} - \frac{1}{3} \gamma_{\mu} \gamma \cdot b) i \gamma_{5}] \epsilon ,
\]

\[
\bar{\delta} \psi_{\mu} = \bar{\epsilon} [\sigma^{\kappa\nu} \partial_{\kappa} h_{\mu\nu} + \frac{1}{3} (M + i \gamma_{5} N) \gamma_{\mu} + i \gamma_{5} (b_{\mu} - \frac{1}{3} \gamma \cdot b \gamma_{\mu})] ,
\]

\[
\delta b_{\mu} = \frac{3}{2} \bar{\epsilon} \gamma_{5} (R_{\mu} - \frac{1}{3} \gamma_{\mu} \gamma \cdot R) ,
\]

\[
\delta M = - \frac{1}{2} \bar{\epsilon} \gamma \cdot R ,
\]

\[
\delta N = - \frac{1}{2} \bar{\epsilon} \gamma_{5} \gamma \cdot R .
\]

Here $\epsilon$ is a constant Grassmann supersymmetry parameter (in linearized supergravity, the supersymmetry transformations represent a global supersymmetry), the $\gamma_{\mu}$ are the usual Dirac matrices and $\sigma_{\mu\nu}$ are proportional to their commutators (see Appendix A for details of our conventions), and $R_{\mu}$ is defined by

\[
R^{\nu} = i \epsilon^{\nu\mu\kappa\rho} \gamma_{5} \gamma_{\mu} \partial_{\kappa} \psi_{\rho} .
\]

A straightforward calculation [1] now shows that the transformations of Eq. (2.3) are an invariance of the linearized supergravitational action

\[
S_{\text{grav}} = \int d^{4}x [E^{\mu\nu} h_{\mu\nu} - \frac{1}{2} \bar{\psi}_{\mu} R_{\mu} - \frac{1}{3} (M^{2} + N^{2} - b_{\mu} b_{\mu})] ,
\]

with $E^{\mu\nu}$ the linearized Einstein tensor defined by

\[
E_{\mu\nu} = \frac{1}{2} \left( \partial_{\mu} \partial_{\nu} h^{\lambda}_{\lambda} + \Box h_{\mu\nu} - \partial_{\mu} \partial^{\lambda} h_{\lambda\nu} - \partial_{\nu} \partial^{\lambda} h_{\lambda\mu} - \eta_{\mu\nu} \Box h^{\lambda}_{\lambda} + \eta_{\mu\nu} \partial^{\lambda} \partial^{\rho} h_{\lambda\rho} \right) .
\]
Linearized supergravity couples to supersymmetric matter through a real supermultiplet of currents \([3, 4]\), consisting of the energy momentum tensor \(\theta^{\mu\nu}\), the supersymmetry current \(j_\mu\), an axial vector current (the \(R\) symmetry current) \(j_\mu^{(5)}\), a scalar density \(P\) and a pseudoscalar density \(Q\). These transform under supersymmetry variations as

\[
\delta \theta^{\mu\nu} = \frac{1}{4} \epsilon (\sigma^{\kappa\mu} \partial_\kappa j^\nu + \sigma^{\kappa\nu} \partial_\kappa j^\mu),
\]

\[
\delta j_\mu = [2 \gamma^\nu \theta_{\mu\nu} - i \gamma_5 \gamma_\nu \partial_\mu j_\nu^{(5)} + i \gamma_5 \gamma_\mu \partial_\nu j^{(5)} + \frac{1}{2} \epsilon_{\mu\nu\rho\kappa} \gamma^\rho \partial^\kappa j^{(5)} + \frac{1}{3} \sigma_{\mu\nu} \partial^\rho (P + i \gamma_5 Q)] \epsilon,
\]

\[
\delta j_\mu^{(5)} = i \epsilon \gamma_5 j_\mu - \frac{1}{3} i \epsilon \gamma_5 \gamma_\mu \gamma_\cdot j,
\]

\[
\delta P = \epsilon \gamma_\cdot j,
\]

\[
\delta Q = i \epsilon \gamma_5 \gamma_\cdot j.
\]

Calculating from Eq. (2.7), one finds that the “anomaly” chiral supermultiplet consisting of \(\theta_\mu, \gamma_\cdot j, \partial_\cdot j^{(5)}, P,\) and \(Q\), has the supersymmetry variations

\[
\delta \theta_\mu = \frac{1}{2} \epsilon \gamma_\cdot j \cdot \partial j,
\]

\[
\delta \gamma_\cdot j = [2 \theta_\mu^{(5)} - 3 i \gamma_5 \partial_\nu j_\nu^{(5)} + \gamma_\cdot \partial (P + i \gamma_5 Q)] \epsilon,
\]

\[
\delta j^{(5)} = [-2 \theta_\mu^{(5)} + 3 i \gamma_5 \partial_\nu j_\nu^{(5)} + \gamma_\cdot \partial (P - i \gamma_5 Q)],
\]

\[
\delta P = \epsilon \gamma_\cdot j,
\]

\[
\delta Q = i \epsilon \gamma_5 \gamma_\cdot j.
\]

A direct calculation now verifies that the matter interaction action that is invariant under the simultaneous supersymmetry variations of Eqs. (2.3) and (2.7) is

\[
S_{\text{int}} = \kappa \int d^4 x [h_{\mu\nu} \theta^{\mu\nu} + \frac{1}{2} \bar{\psi}_\mu j^\mu - \frac{1}{2} b_\mu j_\mu^{(5)} - \frac{1}{6} (MP + NQ)].
\]
The overall normalization of Eq. (2.9) is determined by the requirement that Eqs. (2.5) and (2.9) give the correct Newtonian potential in the static limit; this will be verified explicitly in Sec. 4. Since the auxiliary fields \( b_\mu, M, \) and \( N \) enter Eqs. (2.5) and (2.9) with no differential operators acting on them, their equations of motion are simply the algebraic relations

\[
M = -\frac{1}{4} \kappa P, \quad N = -\frac{1}{4} \kappa Q, \quad b_\mu = \frac{3}{4} \kappa j^{(5)}_\mu.
\] (2.10)

Using these relations to eliminate the auxiliary fields, we get

\[
S_{\text{tot}} = S_{\text{grav}} + S_{\text{int}}
\]

\[
= \int d^4x [E^{\mu\nu} h_{\mu\nu} - \frac{1}{2} \psi_\mu R^\mu + \kappa (h_{\mu\nu} \theta^{\mu\nu} + \frac{1}{2} \psi_\mu j^{(5)}_\mu) - \frac{3}{16} \kappa^2 j^{(5)}_\mu j^{(5)}_\mu + \frac{1}{48} \kappa^2 (P^2 + Q^2)] .
\] (2.11)

Our aim in this paper will be to integrate out the dynamical graviton and gravitino fields, thus transforming Eq. (2.11) into the complete order \( \kappa^2 \) effective action giving the effect of supergravity couplings on the supersymmetric matter fields.

The reason this integration is nontrivial is that the actions of Eqs. (2.5), (2.9), and (2.11) have two Abelian gauge invariances, which are the reflection in the linearized theory of general coordinate invariance [1]. These invariances are

\[
h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \Phi_\nu + \partial_\nu \Phi_\mu ,
\]

\[
\psi_\mu \rightarrow \psi_\mu + \partial_\mu \Psi ,
\] (2.12)

with \( \Phi_\nu \) and \( \Psi \) arbitrary real four vector and Majorana spinor gauge functions, respectively. The invariance of \( S_{\text{grav}} \) follows directly from Eqs. (2.4)–(2.6) and (2.12), while the invariance of \( S_{\text{int}} \) is a consequence of Eqs. (2.9) and (2.12), together with the energy momentum tensor and the supersymmetry current conservation relations

\[
\partial_\mu \theta^{\mu\nu} = \partial_\nu \theta^{\mu\nu} = \partial_\mu j^{(5)}_\mu = 0 .
\] (2.13)
In order to integrate out the graviton and gravitino fields to obtain the complete effective action, we will have to take the existence of these Abelian gauge invariances into account.

3. Quantum electrodynamics as an instructive analog

Since the Abelian gauge invariances of Eq. (2.12) resemble the familiar gauge invariance of quantum electrodynamics (QED), it is instructive to consider the calculation of the effective action in QED as an example. We start from the action

\[ S_{\text{QED}} = \int d^4x (A_{\alpha} \frac{1}{2} P_{\alpha\beta} A^{\beta} + j_{\alpha} A^{\alpha}) , \]  

with \( A^{\alpha} \) the Abelian gauge potential, with \( P_{\alpha\beta} \) the differential operator

\[ P_{\alpha\beta} = \square \eta_{\alpha\beta} - \partial_{\alpha} \partial_{\beta} , \]  

and with \( j_{\alpha} \) a conserved source current,

\[ \partial^{\alpha} j_{\alpha} = 0 . \]

By virtue of the structure of \( P_{\alpha\beta} \) and the conservation of the source current \( j_{\alpha} \), the action of Eq. (3.1) is invariant under the Abelian gauge transformation

\[ A_{\alpha} \rightarrow A_{\alpha} + \partial_{\alpha} \Phi , \]

with \( \Phi \) an arbitrary real gauge function. As a result of this gauge invariance, \( P_{\alpha\beta} \) is not an invertible operator, necessitating the inclusion of a gauge fixing term when the action of Eq. (3.1) is used in a Feynman path integral. However, because the current \( j_{\alpha} \) is conserved, we can nonetheless complete the square in the kinetic term of the action by writing

\[ A_{\alpha} \frac{1}{2} P_{\alpha\beta} A^{\beta} + j_{\alpha} A^{\alpha} = \frac{1}{2} (A^{\alpha} + j^{\alpha} \frac{1}{\square}) P_{\alpha\beta} (A^{\beta} + j^{\beta} \frac{1}{\square}) - \frac{1}{2} j^{\alpha} \frac{1}{\square} j_{\alpha} . \]
When Eqs. (3.1) and (3.5) are inserted in a functional integral, one can use the
translation invariance of the functional measure to define a new integration variable

\[ A'^\alpha \equiv A^\alpha + \frac{1}{\Box} j^\alpha , \] (3.6)

and so the \( P_{\alpha\beta} \) term on the right hand side of Eq. (3.5) contributes a \( j^\alpha \) independent constant factor to the functional integral. This argument is independent of the choice of gauge fixing action, since the Faddeev-Popov gauge fixing procedure is simply a method for isolating the integral over the gauge orbit [5], which in the Abelian case is unchanged by a constant translation. In the special case in which one employs the “standard” covariant gauge fixing action

\[ S_{\text{fix}} = \frac{1}{2} \alpha_s (\partial \cdot A)^2 , \] (3.7)

the gauge fixing action itself is unchanged in form by the substitution of Eq. (3.6), independent of the value of the gauge parameter \( \alpha_s \), because of current conservation. For generic gauge fixings, the \( P_{\alpha\beta} \) term on the right hand side of Eq. (3.5) makes a contribution that is independent of the source current only after the functional integral has been carried out.

Thus, by completing the square and making the change of variable of Eq. (3.6), we have learned that, after doing the gauge field functional integral, the source current dependence is contained solely in the second term of Eq. (3.5), giving

\[ S_{\text{eff}} = -\int d^4x \frac{1}{2} j^\alpha \frac{1}{\Box} j_\alpha . \] (3.8)

Explicitly indicating the space time arguments, this takes the form

\[ S_{\text{eff}} = \int d^4x d^4y \frac{1}{2} j^\alpha(x) \Delta_F(x - y) j_\alpha(y) , \] (3.9)
where we have introduced the Feynman propagator $\Delta_F(x - y)$ given by

$$
\Delta_F(x - y) = \frac{1}{(2\pi)^4} \int d^4q \frac{e^{iq(x-y)}}{q^2 - i0^+},
$$

which obeys

$$
\Box \Delta_F(x - y) = -\delta^4(x - y).
$$

(The use of the Feynman $i0^+$ contour prescription in inverting $\Box$ is dictated, as usual, by the requirements of relativistic invariance and causality.) We note that Eq. (3.9) is just the result that would be obtained from the usual covariant Feynman rules, since the propagator corresponding to the standard gauge fixing of Eq. (3.7) is proportional to $(\delta_{\alpha\beta} + ...)\Delta_F$, with ... indicating $\alpha_s$ dependent derivative terms that vanish when acting on the conserved source currents.

As a check on normalizations, let us verify that Eq. (3.9) leads to the correct Coulomb force law in the static limit when $j^0(x) = -j_0(x) = \rho(\vec{x})$, $\vec{j} = 0$. Writing

$$
\int dx^0 dy^0 = \int dt d(x^0 - y^0),
$$

with $t = \frac{1}{2}(x^0 + y^0)$, and using

$$
\int d(x^0 - y^0) \Delta_F(x - y) = \frac{1}{4\pi|x - y|},
$$

the static limit of Eq. (3.9) becomes

$$
S_{\text{eff}} = \int dt \int d^3x d^3y (-\frac{1}{2} \frac{\rho(\vec{x})\rho(\vec{y})}{4\pi|x - y|}).
$$

This agrees in both sign and magnitude with the action contribution $S = -\int dt V_{\text{Coulomb}}$ arising from a static charge distribution $\rho(\vec{x})$. 

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We are now ready to return to the gravitational case, where we will see that every aspect of the familiar QED calculation that we have just reviewed has a direct analog.

4. Completing the square for the graviton and gravitino

We now apply the lessons learned in the preceding section to the gravitino and graviton terms in Eq. (2.11). Beginning with the gravitino, let us introduce the invertible kernel \( M^{\nu \kappa} \) defined by

\[
M^{\nu \kappa} = - (\gamma^\nu \partial^\kappa + \gamma^\kappa \partial^\nu) + \eta^{\nu \kappa} \gamma \cdot \partial + \frac{1}{2} \gamma^\nu \gamma \cdot \partial \gamma^\kappa
\]

which obeys

\[
M_{\mu \nu} M^{\nu \kappa} = \square \delta^\kappa_\mu .
\]

In terms of \( M_{\mu \nu} \), we define a second kernel \( N_{\mu \nu} \) by

\[
N_{\mu \nu} = - \frac{1}{2} i \epsilon_{\mu \nu} \eta^\rho \partial^\eta \gamma^\rho \gamma^5 = - \frac{1}{2} \left( M_{\mu \nu} + \frac{1}{2} \gamma^\mu \gamma \cdot \partial \gamma^\nu \right)
\]

which obeys

\[
N_{\mu \nu} M^{\nu \theta} = - \frac{1}{2} \square \delta^\theta_\mu + \frac{1}{2} \gamma^\mu \gamma \cdot \partial \delta^\theta
\]

Since the kinetic term for the gravitino in Eq. (2.11) can be expressed in terms of the kernel \( N_{\mu \nu} \),

\[
\int d^4x \left( - \frac{1}{2} \overline{\psi}_\mu R^\mu \right) = \int d^4x \overline{\psi}^\mu N_{\mu \nu} \psi^\nu ,
\]

and since the second term on the right of Eq. (4.4) contains a factor \( \partial^\theta \) that gives zero when acting on the conserved supersymmetry current \( j_\theta \), we can use Eq. (4.4) to complete the
square for the gravitino. Using Eq. (A.7) to relate the adjoint of $M^{\mu \nu}$ to $M^{\tau \mu}$, we get

$$
\bar{\psi}^\mu N_{\mu \nu} \psi^\nu + \frac{1}{2} \kappa \bar{\psi}^\mu j^\mu = \left( \psi^\mu - \frac{1}{2} \kappa M^{\nu \mu} \frac{1}{\Box} j_\tau \right) N_{\mu \nu} \left( \psi^\nu - \frac{1}{2} \kappa M^{\nu \theta} \frac{1}{\Box} j_\theta \right) + \frac{1}{8} \kappa^2 \bar{j}_\tau M^{\tau \nu} \frac{1}{\Box} j_\nu .
$$

(4.6)

When Eqs. (2.11), (4.5), and (4.6) are inserted in a functional integral, we can use translation invariance of the functional measure to define a new integration variable

$$
\psi^\nu = \psi^\nu - \frac{1}{2} \kappa M^{\nu \theta} \frac{1}{\Box} j_\theta ,
$$

(4.7)

and so the $N_{\mu \nu}$ term on the right hand side of Eq. (4.6) contributes a $j_\nu$ independent constant factor to the functional integral. As in the QED case this argument is independent of the choice of gauge fixing. In the special case in which one employs the “standard” Rarita-Schwinger gauge fixing action

$$
S_{\text{fix}} = \frac{1}{2} \alpha_{\text{RS}} \int d^4x \bar{\psi}^\mu \frac{1}{2} \gamma_\mu \gamma^\nu \partial_\nu \psi^\nu ,
$$

(4.8)

the fact that

$$
\gamma_\nu M^{\nu \theta} \propto \partial^\theta
$$

(4.9)

implies that by current conservation, the gauge fixing action of Eq. (4.8) is unchanged in form by the change of variable of Eq. (4.7).

We conclude that by completing the square and making the change of variable of Eq. (4.7), after doing the gravitino functional integral the supersymmetry current dependence is contained solely in the second term on the right hand side of Eq. (4.6), giving

$$
S_{\text{eff}} = \frac{1}{8} \kappa^2 \bar{j}_\tau M^{\tau \nu} \frac{1}{\Box} j_\nu
$$

$$
= -\frac{1}{8} \kappa^2 \int d^4x d^4y \bar{j}_\tau (x) \left( \eta^{\nu \gamma} \gamma^\tau \gamma^\theta + \frac{1}{2} \gamma^{\tau \gamma} \gamma^\theta \partial_\nu \gamma^{\nu \gamma} \right) \Delta_F (x - y) j_\nu (y) ,
$$

(4.10)
where we have dropped terms that vanish by conservation of the supersymmetry current $j_\nu$. This is just the result that would be obtained by the usual covariant Feynman rules, since the gravitino propagator corresponding to the gauge fixing of Eq. (4.8) is proportional to $M^{\tau\nu}\Delta_F + ...$, with ... indicating $\alpha_{RS}$ dependent derivative terms that vanish when acting on the conserved supersymmetry currents.

We turn next to the analogous completion of square argument for the graviton. We rewrite the graviton kinetic term of Eq. (2.11) in the form

$$
\int d^4 x E^{\mu\nu} h_{\mu\nu} = \int d^4 x h^{\alpha\beta} P_{\alpha\beta,\mu\nu} h^{\mu\nu} ,
$$

with $P_{\alpha\beta,\mu\nu}$ the differential operator

$$
P_{\alpha\beta,\mu\nu} = \frac{1}{2}(\eta_{\alpha\beta}\partial_\mu \partial_\nu + \eta_{\mu\nu}\partial_\alpha \partial_\beta) + \frac{1}{4}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu}) - \frac{1}{4}(\partial_\mu \partial_\alpha \eta_{\beta\nu} + \partial_\nu \partial_\alpha \eta_{\beta\mu} + \partial_\mu \partial_\beta \eta_{\alpha\nu} + \partial_\nu \partial_\beta \eta_{\alpha\mu}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}\Box .
$$

Introducing now the projector $Q^{\mu\nu,\gamma\delta}$ defined by

$$
Q^{\mu\nu,\gamma\delta} = \eta^{\mu\gamma}\eta^{\nu\delta} + \eta^{\mu\delta}\eta^{\nu\gamma} - \eta^{\mu\nu}\eta^{\gamma\delta} ,
$$

we find that

$$
P_{\alpha\beta,\mu\nu} Q^{\mu\nu,\gamma\delta} = \frac{1}{2}\Box(\delta_\alpha^\gamma \delta_\beta^\delta + \delta_\beta^\gamma \delta_\alpha^\delta) + \frac{1}{2}z_{\alpha\beta}^\gamma \partial^\delta + \frac{1}{2}z_{\alpha\beta}^\gamma \partial^\delta ,
$$

with

$$
z_{\alpha\beta}^\gamma \partial^\delta = \eta^{\alpha\beta} \partial^\delta - (\eta^{\delta\beta} \partial^\alpha + \eta^{\delta\alpha} \partial^\beta) .
$$

Since the $z_{\alpha\beta}^\gamma \partial^\delta$ terms in Eq. (4.14) contain derivatives $\partial^\gamma$, $\partial^\delta$ that give zero when acting on the conserved energy momentum tensor $\theta_{\gamma\delta}$, we can complete the square for the graviton
as follows,

\[ h^{\alpha\beta} P_{\alpha\beta,\mu\nu} h^{\mu\nu} + \kappa h_{\alpha\beta} \theta^{\alpha\beta} \]

\[ = \left( h^{\alpha\beta} + \kappa \theta^{\nu\tau} \frac{1}{\Box} Q_{\nu\tau,\alpha\beta} \theta^{\alpha\beta} \right) P_{\alpha\beta,\mu\nu} \left( h^{\mu\nu} + Q^{\mu\nu,\gamma\delta} \frac{1}{\Box} \kappa \theta^{\gamma\delta} \right) \]

(4.16)

\[- \frac{1}{4} \kappa^2 \theta^{\nu\tau} \frac{1}{\Box} Q_{\nu\tau,\alpha\beta} \theta^{\alpha\beta} . \]

Again, when Eqs. (2.11), (4.11), and (4.16) are inserted in a functional integral, one can use translation invariance of the functional measure to define a new integration variable

\[ h'^{\mu\nu} = h^{\mu\nu} + Q^{\mu\nu,\gamma\delta} \frac{1}{\Box} \kappa \theta^{\gamma\delta} , \]

(4.17)

and so the \( P_{\alpha\beta,\mu\nu} \) term on the right hand side of Eq. (4.16) contributes an energy momentum tensor independent constant factor to the functional integral. As before, this argument is independent of the choice of gauge fixing action. Again, in the special case that one makes the “standard” gauge fixing choice*

\[ S_{\text{fix}} = \frac{\alpha_G}{\kappa^2} \int d^4 x \eta_{\mu\nu} \partial_\alpha (\sqrt{g} g^{\alpha\mu}) \partial_\beta (\sqrt{g} g^{\beta\nu}) , \]

(4.18)

which after linearization is proportional to

\[ \frac{1}{2} \alpha_G \int d^4 x \left[ (\partial_\alpha h^{\alpha\mu})^2 + h_\theta \partial_\alpha \partial_\mu h^{\alpha\mu} + \frac{1}{4} (\partial^\mu h_\theta)^2 \right] \]

\[ = \frac{1}{2} \alpha_G \int d^4 x h^{\alpha\beta} G_{\alpha\beta,\mu\nu} h^{\mu\nu} , \]

(4.19)

the gauge fixing action is unchanged in form by the change of variables of Eq. (4.17). This

* We remark that there is no linear combination of the gauge fixing actions of Eq. (4.8) and Eq. (4.18) which is supersymmetric, even “on shell” when the equations of motion are used.
follows from the fact that the projector introduced in Eq. (4.19),
\[
G_{\alpha\beta,\mu\nu} = \frac{1}{2} (\eta_{\alpha\beta} \partial_\mu \partial_\nu + \eta_{\mu\nu} \partial_\alpha \partial_\beta) - \frac{1}{4} \eta_{\mu\nu} \eta_{\alpha\beta} \Box
\]
\[- \frac{1}{4} (\partial_\mu \partial_\alpha \eta_{\beta\nu} + \partial_\nu \partial_\alpha \eta_{\beta\mu} + \partial_\mu \partial_\beta \eta_{\alpha\nu} + \partial_\nu \partial_\beta \eta_{\alpha\mu})
\]
\[= P_{\alpha\beta,\mu\nu} - \frac{1}{4} \Box Q_{\alpha\beta,\mu\nu},
\]
obeys
\[
G_{\alpha\beta,\mu\nu} Q^{\mu\nu,\gamma\delta} \propto z_{\alpha\beta} \partial^\gamma + z_{\alpha\beta} \partial^\delta,
\]
and hence vanishes when acting on the conserved energy momentum tensor.

We again conclude that by completing the square and making the change of variable of Eq. (4.17), after doing the graviton functional integral the energy momentum tensor dependence is contained solely in the second term on the right hand side of Eq. (4.16), giving
\[
S_{\text{eff}} = - \frac{1}{4} \kappa^2 \theta^{\nu\tau} \frac{1}{\Box} Q_{\nu\tau,\alpha\beta} \theta^{\alpha\beta}
\]
\[
= -\frac{1}{4} \kappa^2 \int d^4x d^4y \theta^{\nu\tau}(x)(\eta_{\nu\alpha} \eta_{\tau\beta} + \eta_{\nu\beta} \eta_{\tau\alpha} - \eta_{\nu\tau} \eta_{\alpha\beta}) \Delta_F(x - y) \theta^{\alpha\beta}(y).
\]
This is again the result that would be obtained by using the standard covariant Feynman rules, since the graviton propagator corresponding to the gauge fixing of Eq. (4.18) is proportional to $Q_{\nu\tau,\alpha\beta} \Delta_F + ..., \text{ with ... indicating } \alpha_G \text{ dependent derivative terms that vanish when acting on the conserved energy momentum tensor.}$

As a check on the normalization, let us verify that the static limit of Eq. (4.22) agrees with the Newtonian potential. Considering the case when $\theta_{00} = \rho(\vec{x}), \theta_{0j} = \theta_{ij} = 0$, and using $Q^{00,00} = 1$ and $\kappa^2/(8\pi) = G$ together with Eqs. (3.12)-(3.13), we get
\[
S_{\text{eff}} = \int dt \int d^3x d^3y \frac{1}{2} G \frac{\rho(\vec{x}) \rho(\vec{y})}{|\vec{x} - \vec{y}|},
\]
agreeing with the action contribution $S = -\int dt V_{\text{Newton}}$ arising from a static mass distribution $\rho(\vec{x})$.

5. The effective action and its supersymmetry invariance

We are now ready to assemble our final result. Combining the order $\kappa^2$ terms in Eq. (2.11) with the gravitino and graviton effective actions of Eqs. (4.10) and 4.22, we get for the complete order $\kappa^2$ effective action the result

$$\kappa^{-2} S_{\text{eff}} = \int d^4 x \left[ -\frac{3}{16} j^{(5)}_\mu j^{(5)}_\mu + \frac{1}{48} (P^2 + Q^2) \right]$$

$$+ \int d^4 x d^4 y \left[ \frac{1}{4} \theta^{\nu\tau}(x) (\eta_{\alpha\beta} \eta_{\tau\nu} + \eta_{\nu\beta} \eta_{\tau\alpha} - \eta_{\nu\tau} \eta_{\alpha\beta}) \Delta_F(x - y) \theta^{\alpha\beta}(y) \right.$$  

$$- \frac{1}{8} j_\tau(x) \left( \eta^{\tau\nu} \gamma \cdot \partial_x + \frac{1}{2} \gamma^{\tau} \gamma \cdot \partial_x \gamma^\nu \right) \Delta_F(x - y) j_\nu(y) \left. \right] .$$

(5.1)

By use of the current multiplet supersymmetry transformation given in Eq. (2.7), it is straightforward to verify that the effective action of Eq. (5.1) is supersymmetry invariant when the conservation relations of Eq. (2.13) are used, even though neither the local terms coming from eliminating the auxiliary fields, nor the nonlocal terms arising from graviton and gravitino exchange, is separately supersymmetry invariant. Equation (5.1) gives the leading order effects of supergravity coupling on the dynamics of the supersymmetric matter fields which give rise to the multiplet of currents, and can be used as a starting point for studying whether the supergravitational coupling can lead to dynamical symmetry breaking in the matter field dynamics. This topic will be the subject of a future investigation. Explicit expressions for the components of the current and anomaly multiplets, in the notation used here, are given in Appendices B and C for the Wess-Zumino and supersymmetric Yang-Mills models, respectively.

Throughout this paper, we have used the notational conventions for the current
multiplet employed in the text of West [1]. For completeness, we remark that in the notation employed in the text of Weinberg [1], Eq. (5.1) reads

\[ \kappa^{-2} S_{\text{eff}} = \int d^4x \left[ -\frac{3}{16} R_\mu R^\mu + \frac{3}{4} (M^2 + N^2) \right] \]

\[ + \int d^4x d^4y \left[ \frac{1}{4} T^{\nu\tau}(x) (\eta_{\nu\alpha} \eta_{\tau\beta} + \eta_{\nu\beta} \eta_{\tau\alpha} - \eta_{\nu\tau} \eta_{\alpha\beta}) \Delta_F(x - y) T^{\alpha\beta}(y) \right. \]

\[ - \frac{1}{8} \overline{S}_F(x) \left( \eta^{\nu\gamma} \partial_\nu + \frac{1}{2} \gamma^\nu \gamma \partial_\nu \right) \Delta_F(x - y) S_\nu(y) \right]. \]

(5.2)

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**Appendix A. Metric and Gamma Matrix Conventions**

We work with the metric convention

\[ \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \quad , \]

(A.1)

and we use the usual summation convention that repeated Greek indices are summed from 0 to 3, together with the abbreviations for four vector inner products

\[ (a_\mu)^2 = a_\mu a^\mu \quad , \]

\[ a \cdot b = a_\mu b^\mu \quad , \]

(A.2)

\[ \Box = \partial_\mu \partial^\mu \quad . \]

Our convention for the four index antisymmetric tensor is

\[ \epsilon^{0123} = -\epsilon_{0123} = 1 \quad . \]

(A.3)
Our Dirac gamma matrices $\gamma_\mu$ obey the anticommutation relations

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}, \quad (A.4)$$

and we define

$$\hat{\gamma}^0 = i\gamma^0, \quad (A.5)$$

$$\gamma_5 = i\gamma^1 \gamma^2 \gamma^3 \gamma^0 = i\gamma_0 \gamma_1 \gamma_2 \gamma_3,$$

$$\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu].$$

In carrying out the gamma matrix algebra, we have found it convenient to use Majorana representation gamma matrices for which $\hat{\gamma}^0 \gamma^\mu T \hat{\gamma}^0 = -\gamma^\mu$, with the superscript $T$ denoting the transpose. A convenient explicit representation for these gamma matrices in terms of Pauli matrices $\rho_{1,2,3}$ and $\tau_{1,2,3}$ is

$$\gamma^0 = -\gamma_0 = -i\rho_2 \tau_1,$$

$$\hat{\gamma}^0 = \rho_2 \tau_1,$$

$$\gamma^1 = \gamma_1 = \rho_3,$$

$$\gamma^2 = \gamma_2 = -\rho_2 \tau_2,$$

$$\gamma^3 = \gamma_3 = -\rho_1,$$

$$\gamma_5 = -\rho_2 \tau_3.$$

With Majorana representation gamma matrices the role of the adjoint is generally played by the transpose; in particular, we have

$$\hat{\gamma}^0 (\gamma^\tau \gamma \gamma^\mu \gamma^\tau) T \hat{\gamma}^0 = -\gamma^\mu \gamma \cdot \overrightarrow{\partial} \gamma^\tau,$$

which becomes $\gamma^\mu \gamma \cdot \overrightarrow{\partial} \gamma^\tau$ after integration by parts in the derivation of Eq. (4.6), and the conjugate Rarita-Schwinger spinor $\overline{\psi}^\mu$ is related to $\psi^\mu$ by

$$\overline{\psi}^\mu = \psi^\mu T \hat{\gamma}^0.$$
Appendix B. Currents and anomalies in the Wess-Zumino model

We summarize here the formulas for the current and anomaly multiplets in the Wess-Zumino model [1]. The action for this model is

\[
S = \int d^4x \mathcal{L} = \int d^4x \left[ -\frac{1}{2} (\partial_{\mu} A)^2 - \frac{1}{2} (\partial_{\mu} B)^2 - \frac{1}{2} \chi \gamma \cdot \partial \chi + \frac{1}{2} (F^2 + G^2) - m \left( AF + BG - \frac{1}{2} \chi \right) - \lambda \left[ (A^2 - B^2)F + 2GAB - \overline{\chi}(A - i\gamma_5 B)\chi \right] \right],
\]

with \(\chi\) a Majorana spinor for which \(\overline{\chi} = \chi^T \gamma^0\), with the superscript \(T\) again denoting the transpose.

The Euler-Lagrange equations for this action give the equations of motion

\[
\begin{align*}
\Box A = & mF + \lambda(2AF + 2BG - \overline{\chi}\chi) , \\
\Box B = & mG + \lambda(-2BF + 2AG + i\overline{\chi}\gamma_5 \chi) , \\
\gamma \cdot \partial \chi = & [m + 2\lambda(A - i\gamma_5 B)]\chi , \\
-\overline{\chi} \gamma \cdot \partial = & \overline{\chi}[m + 2\lambda(A - i\gamma_5 B)] , \\
F = & mA + \lambda(A^2 - B^2) , \\
G = & mB + 2\lambda AB .
\end{align*}
\]

The action of Eq. (B.1) is invariant under the supersymmetry transformations

\[
\begin{align*}
\delta A = & \overline{\epsilon} \chi , \\
\delta B = & i\overline{\epsilon} \gamma_5 \chi , \\
\delta \chi = & [F + i\gamma_5 G + \gamma \cdot \partial(A + i\gamma_5 B)] \epsilon , \\
\delta \overline{\chi} = & \overline{\epsilon}[F + i\gamma_5 G - \gamma \cdot \partial(A - i\gamma_5 B)] , \\
\delta F = & \overline{\epsilon} \gamma \cdot \partial \chi , \\
\delta G = & i\overline{\epsilon} \gamma_5 \gamma \cdot \partial \chi .
\end{align*}
\]
The current multiplet in the Wess-Zumino model, which obeys the variations of Eq. (2.7), is given by

\[
\begin{align*}
\theta_{\mu\nu} & = - \partial_\mu A \partial_\nu A - \partial_\mu B \partial_\nu B + \frac{1}{8} [\mp \gamma_{\mu} \partial_{\nu} \chi - \mp \gamma_{\nu} \partial_{\mu} \chi + \mp \chi \partial_{\mu} \chi - \mp \chi \partial_{\nu} \chi] \\
& - \eta_{\mu\nu} L + \frac{1}{6} (\partial_\mu \partial_\nu - \Box \eta_{\mu\nu}) (A^2 + B^2) , \\
\gamma \cdot j & = 2m (A - i\gamma_5 B) \chi , \\
\partial \cdot j^{(5)} & = - \frac{2}{3} m \left( B \partial_\mu A - A \partial_\mu B + \frac{1}{4} \mp \chi \gamma_5 \gamma_\mu \chi \right) , \\
P & = m (A^2 - B^2) , \\
Q & = 2mAB .
\end{align*}
\]

The corresponding anomaly multiplet, which obeys the variations of Eq. (2.8), is given by

\[
\begin{align*}
\theta^\mu & = - \frac{1}{2} m \mp \chi \chi + m (AF + BG) , \\
\gamma \cdot j & = 2m (A - i\gamma_5 B) \chi , \\
\partial \cdot j^{(5)} & = - \frac{2}{3} m \left( \frac{1}{2} \mp \chi \gamma_5 \chi + AG - BF \right) .
\end{align*}
\]

Appendix C. Currents and anomalies in the supersymmetric Yang-Mills model

We summarize here the formulas for the current and anomaly multiplets in the supersymmetric Yang-Mills model [1]. The action for this model is

\[
S = \int d^4 x \mathcal{L} = \text{Tr} \left[ \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \mp \gamma_\mu D_\mu \chi + \frac{1}{2} D^2 \right] ,
\]

with the field strength \( F_{\mu\nu} \) and the covariant derivative \( D_\mu \) defined by

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] , \\
D_\mu \mathcal{O} = \partial_\mu \mathcal{O} + [A_\mu, \mathcal{O}] .
\]
and with $\chi$ again a Majorana spinor. Note that we have defined both the gauge potential $A_\mu$ and the field strength $F_{\mu\nu}$ to be anti-self-adjoint, which is why the kinetic term for the gauge field in Eq. (C.1) has the opposite sign to that of Eq. (3.1), where we took the QED gauge potential to be self-adjoint. All fields $O = A_\mu, F_{\mu\nu}, \chi, D$ appearing in Eq. (C.1) transform according to the adjoint representation of a compact Lie group, with the generator expansion

$$O = \sum_a \frac{1}{2} \lambda_a O_a \quad .$$

(C.3)

The trace with an upper case T is defined by

$$\text{Tr} = 2\text{tr} \quad ,$$

(C.4)

with tr the usual trace for which

$$\text{tr} \lambda_a \lambda_b = 2\delta_{ab} \quad ,$$

(C.5)

so that

$$\text{Tr} \frac{1}{2} \lambda_a \frac{1}{2} \lambda_b = \delta_{ab} \quad .$$

(C.6)

The Euler-Lagrange equations of Eq. (C.1) imply the equations of motion

$$\gamma^\mu D_\mu \chi = 0 \quad ,$$

$$\overline{D}_\mu \chi \gamma^\mu = 0 \quad ,$$

$$D_\mu F^{\mu\nu} = g^2 \chi \gamma^\nu \chi \quad .$$

(C.7)

The action of Eq. (C.1) is invariant under the supersymmetry transformations

$$\delta A_\mu = ig \overline{\epsilon} \gamma_\mu \chi \quad ,$$

$$\delta \chi = \left( \frac{i}{2g} \sigma_{\mu\nu} F^{\mu\nu} + ig_5 D \right) \epsilon \quad ,$$

$$\delta \overline{\chi} = \overline{\epsilon} \left( -\frac{i}{2g} \sigma_{\mu\nu} F^{\mu\nu} + ig_5 D \right) \quad ,$$

$$\delta D = i \overline{\epsilon} \gamma_5 \gamma^\mu D_\mu \chi \quad .$$

(C.8)
The current multiplet in the supersymmetric Yang-Mills model, which obeys the variations of Eq. (2.7), is given by

$$\theta_{\mu \nu} = -\eta_{\mu \nu} \mathcal{L} + ...$$

$$= \text{Tr} \left( -\eta_{\mu \nu} \frac{1}{4g^2} F_{\alpha \beta} F^{\alpha \beta} + \frac{1}{g^2} F^\alpha \_ \mu F_{\mu \alpha} 
+ \frac{1}{8} \left[ \overline{D}_\mu \gamma_\nu \chi - \overline{\chi} \gamma_\nu D_\mu \chi + \overline{D}_\nu \gamma_\mu \chi - \overline{\chi} \gamma_\mu D_\nu \chi \right] \right) ,$$

$$j_\mu = \text{Tr} \left( -\frac{i}{2g} F_{\mu \sigma} \sigma^{\nu \sigma} \gamma_\mu \chi \right) , \quad (C.9)$$

$$\overline{j}_\mu = \text{Tr} \left( -\frac{i}{2g} \overline{\chi} \gamma_\mu \sigma^{\nu \sigma} F_{\mu \sigma} \right) ,$$

$$j^{(5)}_\mu = -\frac{1}{2} \text{Tr} (\overline{\chi} \gamma_\mu \gamma_5 \chi) ,$$

$$P = 0 , \quad Q = 0 .$$

Since the supersymmetric Yang-Mills model is classically conformally invariant, the tree level anomalies are zero. The anomaly multiplet which obeys Eq. (2.8) arises as a one loop radiative correction [7], and is given by

$$\theta^\mu = -2f \text{Tr} \left( \frac{1}{4g^2} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} \overline{\chi} \gamma^\mu D_\mu \chi + \frac{1}{2} D^2 \right) ,$$

$$\gamma \cdot j = 2f \text{Tr} \left( -\frac{i}{2g} \sigma_{\mu \nu} F^{\mu \nu} + i \gamma_5 D \right) \chi ,$$

$$\overline{j} \cdot \gamma = 2f \text{Tr} \overline{\chi} \left( -\frac{i}{2g} \sigma_{\mu \nu} F^{\mu \nu} - i \gamma_5 D \right) , \quad (C.10)$$

$$\partial \cdot j^{(5)} = -\frac{2}{3} f \text{Tr} \left( \frac{1}{4g^2} \epsilon_{\mu \nu \rho \sigma} F^{\mu \nu} F^{\rho \sigma} + \frac{i}{2} \partial^\mu (\overline{\chi} \gamma_\mu \gamma_5 \chi) \right) ,$$

$$P = f \text{Tr} \overline{\chi} \chi ,$$

$$Q = if \text{Tr} \overline{\chi} \gamma_5 \chi ,$$

with $f$ related to the beta function of the theory by

$$f = \frac{\beta(g)}{g} . \quad (C.11)$$
References

[1] For excellent surveys of standard supersymmetry and supergravity topics, see S. Weinberg, *The Quantum Theory of Fields, Volume III Supersymmetry*, Cambridge University Press, Cambridge, 2000; P. West, *Introduction to Supersymmetry and Supergravity*, Extended Second Edition, World Scientific, Singapore, 1990.

[2] K. Stelle and P. West, Phys. Lett. **B74** (1978), 330; S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. **B74** (1978), 333.

[3] S. Ferrara and B. Zumino, Nucl. Phys. **B87** (1975), 207.

[4] P. West, Ref. 1, gives the current multiplet transformation in the form used here in his Eq. (20.43), which differs by some phases from our Eq. (2.7).

[5] For an exposition, see S. Weinberg, *The Quantum Theory of Fields, Volume II Modern Applications*, Cambridge University Press, Cambridge, 1996; T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particle Physics*, Oxford University Press, Oxford, 1988, pp. 250-254.

[6] Some formulas relevant for this section are given in L. Baulieu, A. Georges, and S. Ouvry, Nucl. Phys. **B273** (1986), 366; H. A. Weldon, “Thermal Green Functions in Coordinate
Space for Massless Particles of Any Spin,” hep-ph/0007138.

[7] See P. West, Ref. 1, Sec. 20.4, for a discussion and further references. Note that Eq. (C.10) differs by phases from West’s Eq. (20.71).