Plasmonic solitons and dromions via plasmon-induced transparency

Zhengyang Bai and Guoxiang Huang

1State Key Laboratory of Precision Spectroscopy, School of Physical and Material Sciences, East China Normal University, Shanghai 200062, China
2NYU-ECNU Joint Institute of Physics at NYU-Shanghai, Shanghai 200062, China
E-mail: gxhuang@phy.ecnu.edu.cn

Abstract. We propose a method to enhance Kerr nonlinearities and realize low-power gigahertz solitons via plasmon induced transparency (PIT) in a new type of metamaterial, which is constructed by an array of unit cell consisting of a cut-wire and a pair of varactor-loaded split-ring resonators. We show that the PIT in such metamaterial can not only mimic the electromagnetically induced transparency in coherent three-level atomic systems, but also exhibit a crossover from PIT to Autler-Townes splitting. We further show that the system suggested here also possess giant second- and third-order nonlinear optical susceptibilities, which may be used to create plasmon solitons and dromions with extremely low power. Our studies raise the possibility for obtaining strong nonlinear effects of gigahertz radiation at very low intensity based on room temperature metamaterials.

1. Introduction

Recently, considerable attention has been paid to the investigation on plasmon-induced transparency (PIT) [1, 2, 3, 4], an interesting destructive interference effect arising from the strong coupling between wide-band bright oscillatory mode and narrow-band dark oscillatory mode of the meta-atoms in plasmonic metamaterials. PIT, a plasmonic analog of the atomic electromagnetically induced transparency (EIT) [5], can not only be used to significantly suppress the radiative loss of metamaterials, but also bring steep dispersion and hence large reduction of the propagation velocity of plasmonic polaritons.

Because of highly resonant (and hence dispersive) character inherent in PIT metamaterials, linear plasmon polaritons inevitably undergo a significant distortion during propagation. Furthermore, due to diffraction effect, which can not be neglected for the cases of small transverse size or long propagation distance, a large deformation of linear plasmon polaritons is unavoidable. Thus it is necessary to extend the PIT to nonlinear propagation regime[6, 7, 8, 9].

Here, we give a report of our recent studies [10, 11, 12] on this subject. We elucidate that the PIT in such metamaterial can not only mimic the EIT in coherent three-level atomic systems, but also exhibit a crossover from PIT to Autler-Townes splitting. We show that the system suggested here also possess giant second-order and third-order nonlinear optical susceptibilities, which may be used to create plasmon solitons and dromions with extremely low generation power. Our studies raise the possibility for obtaining nonlinear plasmon polaritons at very low intensity based on room-temperature metamaterials.
2. Model

The metamaterial structure considered here is an array of PIT unit cell [13] consisting of a CW and two SRRs with a nonlinear varactor inserted into the slits of the SRRs [see Fig. 1(a) and Fig. 1(c)]. We assume that an incident gigahertz radiation is collimated on the metamaterial array with the electric field parallel to the CW, as illustrated in Fig. 1(b) [11, 12, 13]. Normalized absorption spectrum of the sole-CW (red), SRR-pair (blue) and the unit cell of PIT metamaterial (green) are shown in Fig. 1(d). The CW array shows a typical localized surface plasmon resonance, while the SRRs support an inductive-capacitive (LC) resonance at the same frequency. The CW is directly excited by the incident electric field along the CW, while the SRRs are weakly coupled to the incident field due to the perpendicular orientation of the field. The near field coupling between the CW and SRRs excites the LC resonance in the SRRs, and hence the CW and SRRs serve respectively as the bright and dark modes in such excitation scheme, which leads to a dip at the center of the broad peak for the absorption spectrum.

The dynamics of the bright and dark modes in the unit cell at the position \( \mathbf{r} = (x, y, z) \) can be described by the Lorentz equations for two coupled oscillators [1, 13]

\[
\ddot{q}_1 + \gamma_1 \dot{q}_1 + \omega_0^2 q_1 - \kappa^2 q_2 = g E(\mathbf{r}, t),
\]

\[
\ddot{q}_2 + \gamma_2 \dot{q}_2 + (\omega_0 + \Delta)^2 q_2 - \kappa^2 q_1 + \alpha q_2^2 + \beta q_2^3 = 0,
\]

where \( q_1 \) and \( q_2 \) are respectively amplitudes of the bright and dark modes (the dot over \( q_j \) denotes time derivative), with \( \gamma_1 \) and \( \gamma_2 \) respectively their damping rates; \( \omega_0 = 2\pi \times 32 \text{ GHz} \) and \( \omega_0 + \Delta \)
are respectively linear natural frequencies of the bright and dark modes \((\gamma_2 \ll \gamma_1 \ll \omega_0)\); parameter \(\kappa\) denotes the coupling strength between the CW and SRR-pair; \(g\) is the parameter indicating the coupling strength of the bright mode with the incident radiation \(E\). The last two terms on the left hand of Eq. (2) are provided by the hyperabrupt tuning varactors mounted onto gaps of the SRRs \([7, 9]\). Thus the metamaterial structure suggested here is a coupled anharmonic oscillator system with the nonlinear coefficients \(\alpha\) and \(\beta\) and driven by the electric field \(E\).

The equation of motion for the electric field \(E\) is governed by the Maxwell equation

\[
\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 P}{\partial t^2},
\]

with the electric polarization intensity given by \(P = \varepsilon_0 \chi_D^{(1)} E + Neq_1\), where \(N\) the density of unit cells, \(e\) is the unit charge, and \(\chi_D^{(1)}\) is the optical susceptibility of the background material (which is assumed to be liner). In terms of the relation \(P = \varepsilon_0 \chi E\), the electric susceptibility \(\chi\) can be obtained by the formula

\[
\chi = \chi_D^{(1)} + \frac{Ne}{\varepsilon_0 E} q_1.
\]

We assume the incident radiation has frequency \(\omega_f\), which is near \(\omega_0\). Thus there is resonant interaction between the electric field \(E\) and the oscillators \(q_1\) and \(q_2\). To treat such resonant, nonlinear problem analytically, we assume \(q_j = q_{dj} + (q_{fj} e^{i(k_0 z-\omega_0 t)} + c.c.) + (q_{sj} e^{2i(k_0 z-\omega_0 t)} + c.c.)\), \(E = E_d + (E_f e^{i(k_f z-\omega_f t)} + c.c.) + (E_s e^{2i(k_f z-2\omega_f t)} + c.c.)\). Here \(q_{dj}, q_{fj},\) and \(q_{sj}\) are respectively amplitudes of the longwave (rectification field or mean field), shortwave (fundamental wave), and second harmonic wave of the electric field; \(k_f (\omega_f)\) is the wavenumber (frequency) of the fundamental wave; \(E_d, E_f,\) and \(E_s\) are respectively amplitudes of the longwave, shortwave, and second harmonic wave of the electric field; \(k_j (\omega_j)\) the wavenumber (frequency) of the fundamental wave, and \(\Delta k\) is a detuning. From the Maxwell-Lorentz (ML) equations (1) and (3) and using rotating-wave and slowly varying envelope approximations, we can obtain a series of equations for the motion of \(q_{fj}\) and \(E_{\mu} (\mu = d, f)[11, 12]\).

In panels (a), (b), (c) of Fig. 2 we show respectively the numerical result of normalized absorption spectrum of the system versus frequency for separation \(d = 0.38, 0.24,\) and \(0.02\) (in unit mm), obtained by using the commercial finite difference time domain soft-ware package (CST Microwave Studio). We observe that a PIT transparency window in the absorption spectrum opens; furthermore, the transparency window becomes wider and deeper as \(d\) is reduced. These phenomena are clear indications of PIT-ATS crossover and will be analyzed in detail below.

3. PIT-ATS crossover

We now solve the equations for \(q_{fj}\) and \(E_{\mu}\) by using the method of multiple scales. Take the asymptotic expansion \(q_{fj} = \epsilon q_{fj}^{(1)} + \epsilon^2 q_{fj}^{(2)} + \cdots, q_{dj} = \epsilon^2 q_{dj}^{(2)} + \cdots, q_{sj} = \epsilon^2 q_{sj}^{(2)} + \cdots\), \(E_f = \epsilon E_f^{(1)} + \epsilon^2 E_f^{(2)} + \cdots, E_d = \epsilon^2 E_d^{(2)} + \cdots\), where \(\epsilon\) is a dimensionless small parameter characterizing the amplitude of the incident electric field. All quantities on the right-hand side of the expansion are assumed as functions of the multi-scale variables \(x_1 = \epsilon x, y_1 = \epsilon y, z_j = \epsilon^j z\) \((j = 0, 1, 2)\) and \(t_j = \epsilon^j t\) \((j = 0, 1)\). Substituting this expansion into the the equations for \(q_{fj}\) and \(E_{\mu}\) and comparing the expansion parameter of each power \(\epsilon\), we obtain a chain of linear but inhomogeneous equations, which can be solved order by order.

At the first-order we obtain the solution for the shortwave field \(E_f^{(1)} = F \exp[i(Kz_0 - \delta t_0)]\) where \(F\) is a yet to be determined envelope function depending on the “slow” variables \(x_1, y_1,\)
Figure 2. Numerical results of the normalized absorption spectrum of the metamaterial (Fig. 1) for (a) \(d = 0.38\), (b) \(d = 0.24\), and (c) \(d = 0.02\) (in unit mm), respectively. Analytical results given in (d), (e), and (f) are obtained from solving the model Eqs. (1) and (3) in linear regime.

\[ z_1, z_2, \text{ and } t_1, \delta = \omega_f - \omega_0 \text{ is frequency detuning, and } K \text{ is the linear dispersion relation given by} \]

\[ K = \frac{nD}{c} \delta + \frac{\kappa_0 g D_2(\delta)}{D_1(\delta)D_2(\delta) - \kappa^4}. \]  

(5)

Here \(D_j(l\delta) = \omega_0^2 - l^2(\omega_0 + \delta)^2 - il\gamma_j(\omega_0 + \delta)\) \((j, l = 1, 2)\) and \(\kappa_0 = (N\omega_0)/(2\varepsilon_0cn_D)\).

Shown in panels (d), (e), (f) of Fig. 2 is the absorption spectrum \(\text{Im}(K)\) (the imaginary part of \(K\)) as a function of frequency. When plotting the figure we used the damping rates \(\gamma_1 \approx 60\) GHz and \(\gamma_2 \approx 10\) GHz, which are nearly independent of \(d\), whereas \(\kappa\) decreases from 152.5 GHz at \(d = 0.02\) mm to 69 GHz at \(d = 0.38\) mm. We see that the analytical result (the lower part of Fig. 2) fits well with the numerical one (the upper part of Fig. 2).

To obtain a clear physical explanation on the variation of the PIT transparency window for different \(d\) shown in Fig. 2, we make a detailed analysis on the linear dispersion relation. Near the resonance point (i.e. \(\delta \ll \omega_0\)), we have \(\omega_0^2 - (\omega_0 + \delta)^2 \approx -2\omega_0\delta\), and hence

\[ K = \frac{nD}{c} \delta + \frac{A(\delta + i\frac{\omega_2}{2})}{B^2 - (\delta + i\frac{\omega_2}{2})(\delta + i\frac{\omega_2}{2})}, \]  

(6)

with \(A = \kappa_0 g/(2\omega_0)\) and \(B = \kappa^2/(2\omega_0)\).

The linear dispersion relation (6) is similar to that obtained in \(\Lambda\)-type three-level atomic systems [5]. The PIT condition of the system reads \(B^2 \gg \gamma_1\gamma_2/4\). To illustrate the interference effect between the bright and dark modes apparently, we make a spectrum decomposition on \(\text{Im}(K)\) using the technique developed recently in Refs. [14, 15] for different coupling constant \(\kappa\).

(i). Weak coupling region \((\kappa^2 < (\gamma_1 - \gamma_2)\omega_0/2)\): In this region, the absorption spectrum can be expressed as

\[ \text{Im}(K) = -\frac{AC_+W_+}{\delta^2 + W_+^2} - \frac{AC_-W_-}{\delta^2 + W_-^2}, \]  

(7)

where \(W_\pm = -\gamma_1 + \gamma_2)/4 \pm [(\gamma_1 - \gamma_2)^2/16 - B^2]^{1/2}\) and \(C_\pm = \pm(\gamma_2/2 + W_\pm)/[(\gamma_1 - \gamma_2)^2/4 - 4B^2]^{1/2}\).

We see that \(\text{Im}(K)\) consists of two Lorentzians terms. Fig. 3(a) shows \(\text{Im}(K)\) as a function of \(\delta\)
Figure 3. (a) The dashed (dotted-dashed) line is the result of the first (second) Lorentzian term in Eq. (7). The solid line is the sum of the two Lorentzian terms, giving the absorption spectrum Im(\(K\)) in the weak coupling (PIT) region \((d = 0.38\, \text{mm})\). (b) The dashed-dotted lines denote the first two Lorentzian terms in Eq. (8); the dashed lines denote the third and fourth terms in Eq. (8). The solid line is the sum of all four terms, giving Im(\(K\)) in the intermediate coupling (PIT-ATS crossover) region \((d = 0.24\, \text{mm})\). (c) The same as (b) but with \(d = 0.02\, \text{mm}\), giving Im(\(K\)) in the strong coupling (ATS) region. (d) Transition from PIT to ATS when \(\kappa\) changes.

for \(\kappa = 69\, \text{GHz}\). The dashed (dotted-dashed) line is the result of the first (second) Lorentzian term in Eq. (7), which is negative (positive). Because the two Lorentzians have the same center position but opposite sign, their superposition gives a destructive interference between CW and the SRR-pair. As a result, a small dip in Im(\(K\)) curve (i.e. the solid line) appears. Such phenomenon belongs PIT in nature.

(ii). Intermediate coupling region \((\kappa^2 > (\gamma_1 - \gamma_2)\omega_0/2))\): In this case, we obtain

\[
\text{Im}(K) = \frac{A(\gamma_1 + \gamma_2)/8}{(\delta - \delta_0)^2 + (\gamma_1 + \gamma_2)^2/16} + \frac{A(\gamma_1 + \gamma_2)/8}{(\delta + \delta_0)^2 + (\gamma_1 + \gamma_2)^2/16} \\
- \frac{Ah(\delta - \delta_0)}{(\delta - \delta_0)^2 + (\gamma_1 + \gamma_2)^2/16} + \frac{Ah(\delta + \delta_0)}{(\delta + \delta_0)^2 + (\gamma_1 + \gamma_2)^2/16},
\]

with \(\delta_0 = \sqrt{B^2 - (\gamma_1 - \gamma_2)^2/16}\) and \(h = (\gamma_2 - \gamma_1)/(8\delta_0)\). We see that the absorption spectrum is made of four terms. The first two terms are Lorentzians with the same width but different center position, which are two resonance peaks belonging respectively to the CW and the SRR-pair. The dip between the two Lorentzians can be interpreted as a gap between two resonances, which is a typical character of ATS. The next two terms are interference terms. Because they
lowers the dip formed by the first two terms, a destructive interference (i.e. PIT) occurs. Since both PIT and ATS occur simultaneously in the system, we assign such phenomenon as PIT-ATS crossover. Shown in Fig. 3(b) is the result of various terms in Eq. (8) and the total absorption spectrum \( \text{Im}(K) \) for \( \kappa = 82 \text{ GHz} \).

(iii). Strong coupling region \( (\kappa^2 \gg (\gamma_1 - \gamma_2 |\omega_0|/2)) \): The spectrum decomposition of \( \text{Im}(K) \) is still given by Eq. (8), but the destructive interference effect contributed by the last two terms plays a negligible role. Fig. 3(c) shows the result for \( \kappa = 152.5 \text{ GHz} \). We see that in the strong coupling region the absorption spectrum of the electric field displays mainly the character of ATS.

In Fig. 3(d) we show the “phase diagram” of the system, where three regions (i.e. PIT, PIT-ATS crossover, and ATS) are indicated clearly. We see that a transition from PIT to ATS indeed occurs when \( \kappa \) changes from small to large values.

4. Enhanced Kerr effect

We now extend the equations for \( q_{\mu} \) and \( E_\mu \) to higher order to demonstrate enhanced Kerr effect due to the resonant interaction.

At the second-order, a divergence-free condition requires \( \partial F/\partial \tau_1 + (1/V_p)\partial F/\partial t_1 = 0 \), where \( V_p = (\partial K/\partial \delta)^{-1} \) is the group velocity of the shortwave envelope \( F \). The solution for the longwave (rectification) field reads \( E_d^{(2)} = G \). With the above results we proceed to the third order. The solvability condition at this order yields the nonlinear equation

\[
\frac{i}{\tau_2} \frac{\partial F}{\partial \tau_2} - \frac{1}{2} K_2 \frac{\partial^2 F}{\partial \tau_1^2} + \frac{c}{2 \omega_0 n_D} \left( \frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial y_1^2} \right) + \frac{\omega_0}{2cn_D} \chi^{(3)} |F|^2 F e^{-2a_{z2}} + \frac{m_1 \omega_0}{2cn_D} \chi^{(2)} G F = 0. \quad (9)
\]

Here \( \tau_1 = \tau \tau_p (\tau \equiv t - z/V_p) \), \( \bar{\alpha} = \epsilon^{-2} \text{Im}(K) \) is the coefficient describing linear absorption, \( K_2 = \partial^2 K/\partial \delta^2 \) is the coefficient describing group-velocity dispersion, \( m_1 \equiv |D_1(\delta)D_2(\delta) - \kappa^4|/|D_1(\delta)D_2(\delta) - \kappa^4|^2 \), \( \chi^{(2)} \) and \( \chi^{(3)} \) are respectively the second-order and third-order nonlinear susceptibilities with the form

\[
\chi^{(2)} = \frac{-2Ne^2 \kappa^6 \alpha}{\epsilon_0 (\omega_0^4 - \kappa^4)|D_1(\delta)D_2(\delta) - \kappa^4|^2},
\]

\[
\chi^{(3)} = \left( \frac{4 \alpha^2 \omega_0^2}{\omega_0^3 - \kappa^4} + \frac{2 \alpha^2 D_1(2\delta)}{D_1(2\delta)D_2(2\delta) - \kappa^4 - 3\beta} \right) \times \frac{g^3 \kappa^8 N e}{\epsilon_0 (D_1(\delta)D_2(\delta) - \kappa^4)|D_1(\delta)D_2(\delta) - \kappa^4|^2}.
\]

We see that \( \chi^{(2)} \) is proportional to the parameter \( \alpha \), i.e. it is contributed by the quadratic nonlinearity in Eq. (2); \( \chi^{(3)} \) is proportional to the parameters \( \alpha \) and \( \beta \), which means that it comes from the contributions by the quadratic and cubic nonlinearities in Eq. (2).

Furthermore, the nonlinear equation for the longwave (rectification) field \( G \) arises at the fourth-order approximation, which reads

\[
\left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} \right) G - \left( \frac{1}{V_p^2} - \frac{1}{V_g^2} \right) \frac{\partial^2 G}{\partial \tau_1^2} - \frac{\chi^{(2)}}{c^2} \frac{\partial^2 |F|^2}{\partial \tau_1^2} e^{-2a_{z2}} = 0, \quad (12)
\]

where \( V_g \) is the phase velocity of the longwave field \( G \), defined by \( 1/V_p^2 = (n_D/c)^2 + (Ne\omega_0^2)/[\epsilon_0 e^2 (\omega_0^4 - \kappa^4)] \). The last term on the left-hand of Eq. (12) corresponds to a second-order plasmonic rectification. Above results tell us that the self-interaction of the shortwave
Figure 4. Nonlinear susceptibilities of the PIT metamaterial. (a) Real and imaginary parts of the third-order susceptibility $\chi^{(3)}$ [i.e. Re($\chi^{(3)}$) and Im($\chi^{(3)}$)] as functions of the frequency detuning $\delta$. (b) Real and imaginary parts of the second-order susceptibility $\chi^{(2)}$ [i.e. Re($\chi^{(2)}$) and Im($\chi^{(2)}$)] as functions of $\delta$. System parameters used are given in the text.

(with the envelope $F$) can stimulate the generation of the longwave field $G$ [Eq. (12)], and at the same time longwave field $G$ has a back-action to the shortwave field $F$ [Eq. (9)]. Equations (9) and (12) are the general form of Davey-Stewartson (DS) equations describing the propagation of high-dimensional nonlinear plasmonic polaritons in the system.

In Fig. 4(a) and Fig. 4(b) we show respectively curves of $\chi^{(3)}$ and $\chi^{(2)}$ as functions of the frequency detuning $\delta$. We observe that $\chi^{(2)}$ is nearly real and has the order of magnitude $10^{-3}$ mV$^{-1}$; The real part of the third-order susceptibility, Re($\chi^{(3)}$), has the order of magnitude $10^{-6}$ m$^2$V$^{-2}$. The physical reason for such large second- and third-order nonlinearities predicted here is due to the fact that the incident radiation $E$ is resonant with the oscillators $q_1$, $q_2$ and the system works under PIT condition. We also show the imaginary part of $\chi^{(3)}$, which contributes a nonlinear absorption to the radiation field, is much less than the real part Re($\chi^{(3)}$) when the system works in PIT transparency window. Such suppression of the nonlinear absorption is also due to the PIT effect. Interestingly, the third-order nonlinear susceptibility $\chi^{(3)}$ can be further enhanced by using longwave-shortwave resonance. For simplicity, we assume that the transverse spatial distribution of the radiation is large enough so that the diffraction effect (i.e. its dependence on the transverse coordinates $x$ and $y$) of the system can be neglected. Then from Eq. (12) we obtain $G = \chi^{(2)} |E_f|^2/|c|^2(1/V_p^2 - 1/V_g^2)$. Plugging this result into Eq. (9), we obtain an effective third-order nonlinear susceptibility

$$\chi^{(3)}_{\text{eff}} = \chi^{(3)} + \chi^{(3)}_{\text{SL}},$$ (13)

$$\chi^{(3)}_{\text{SL}} = m_1 \frac{(\chi^{(2)})^2}{c^2 \left( \frac{1}{V_p^2} - \frac{1}{V_g^2} \right)},$$ (14)

where the subscript “SL” means that the corresponding term is due to the longwave-shortwave interaction. Equation (13) tells us that if some region of system parameters can be found where $V_p \approx V_g$, in addition to the PIT enhancement, the effective third-order nonlinear susceptibility
Figure 5. (a) The denominator $1/V_p^2 - 1/V_g^2$ of $\chi_{SL}^{(3)}$ as functions of frequency detuning $\delta$ and the coupling coefficient $\kappa$. The rectangle enclosed by purple dashed lines shows the region where the longwave-shortwave resonance occurs (i.e. $V_g \approx V_p$). (b) Real part Re($\chi_{eff}^{(3)}$) (orange solid line) and imaginary part Im($\chi_{eff}^{(3)}$) (green dashed line) of the effective third-order nonlinear susceptibility $\chi_{eff}^{(3)}$ as functions of frequency detuning $\delta$ for $\kappa = 180$ GHz.

$\chi_{eff}^{(3)}$ can be further enhanced because of the drastic enhancement of $\chi_{SL}^{(3)}$.

In the situation $V_p \approx V_g$, the system undergoes a resonant interaction between the longwave $G$ and the shortwave $F$, a special form of three-wave resonance satisfying the conditions $\omega_1 + \omega_2 = \omega_3$ and $k_f(\omega_1) + k_f(\omega_2) = k_f(\omega_3)$. This point can be illustrated clearly if we choose $\omega_1 = \omega - \varepsilon \delta$, $\omega_2 = 2\varepsilon \delta$, and $\omega_3 = \omega + \varepsilon \delta$. Then one has $\omega_1 + \omega_2 = \omega_3$, and $k_f(\omega_1) + k_f(\omega_2) - k_f(\omega_3) = k_f(\omega - \varepsilon \delta) + k_f(2\varepsilon \delta) - k_f(\omega + \varepsilon \delta) = -2\varepsilon \delta \partial k_f/\partial \omega + k_f(2\varepsilon \delta) + O(\varepsilon^3)$ which is zero to order $\varepsilon^3$ if $\partial k_f/\partial \omega \approx k_f(2\varepsilon \delta)/2\varepsilon \delta$, i.e. $V_g \approx V_p$ [16].

In our system, a broad parameter region for $V_p \approx V_g$ exists. Illustrated in Fig. 5(a) is the denominator $1/V_p^2 - 1/V_g^2$ of $\chi_{SL}^{(3)}$ as functions of the frequency detuning $\delta$ and the coupling coefficient $\kappa$. The rectangle enclosed by purple dashed lines in upper part of the figure illustrates the parameter region where $V_g \approx V_p$. Consequently, the longwave-shortwave resonance can indeed occur in the present PIT metamaterial, and based on this the effective third-order nonlinear susceptibility $\chi_{eff}^{(3)}$ of the system can be enhanced greatly.

Fig. 5(b) shows curves of the real part and the imaginary part of the effective third-order nonlinear susceptibility $\chi_{eff}^{(3)}$ as functions of $\delta$ for $\kappa = 180$ GHz. We observe that Re($\chi_{eff}^{(3)}$) (which is much larger Im($\chi_{eff}^{(3)}$) near $\delta = 0$) is enhanced one order of magnitude (up to the value $6.64 \times 10^{-5}$ m²V⁻²), which is contributed by the longwave-shortwave resonance interaction. When plotting the figure, other system parameters used are $\gamma_1 = 60$ GHz, $\gamma_2 = 10$ GHz, $g = 1.79 \times 10^{11}$ C·kg⁻¹, $\epsilon_0 = 2.8 \times 10^{-8}$ kg·C⁻¹cm⁻¹GHz², obtained by fitting the numerical result given in Fig. 1(d); parameters $\alpha = -1.27 \times 10^{15}$ cm⁻¹GHz² and $\beta = 2.26 \times 10^{25}$ cm⁻²GHz² are derived from the result given in Ref. [7].
5. Plasmonic solitons and dromions

5.1. Plasmonic solitons

In this section, we focus on the regime that transverse spatial distribution of the radiation is large enough so that the diffraction effect (i.e., its dependence on the transverse coordinates \( x \) and \( y \)) of the system can be neglected. Based on such condition and returning to the original variables, Eq. (9) returns to the nonlinear Schrödinger (NLS) equation

\[
i \left( \frac{\partial}{\partial z} + \alpha_1 \right) U - \frac{1}{2} K_2 \frac{\partial^2 U}{\partial \tau^2} - W |U|^2 U = 0,
\]

where \( W = -|\omega_0/(2cn_D)|^3 \), \( \tau = t - z/V_g \), and \( U = e^{F \exp(-\alpha_1 z)} \).

Generally, Eq. (15) has complex coefficients and hence it is a Ginzburg-Landau equation. However, due to the PIT effect the imaginary part of the complex coefficients can be made much smaller than their real part. Thus Eq. (15) can be converted into the dimensionless form

\[
i \frac{\partial}{\partial s} u + \frac{1}{2} \frac{\partial^2 u}{\partial \sigma^2} + |u|^2 u = i \frac{d_0}{2} u,
\]

with \( z = -L_{\text{Disp}} s \), \( \tau = \tau_0 \sigma \), \( U = U_0 u \), and \( d_0 = L_{\text{Disp}}/L_A \). Here \( \tau_0 \), \( L_{\text{Disp}} \), \( \tau_0^2/\tilde{K}_2 \), \( L_A = 1/\alpha_1 \), and \( \tilde{K}_2 = (1/\tau_0) \sqrt{\tilde{K}_2/W} \) are typical pulse length, dispersion length, absorption length, and amplitude of the pulse, respectively (the tilde above the quantity means taking real part). Notice that for forming solitons \( L_{\text{Disp}} \) has been set to equal typical nonlinearity length \( L_{\text{Nonl}} (\equiv 1/(U_0^2 W)) \).

If \( d_0 \) is small, the term \( d_0 u \) can be taken as a perturbation. One can use the perturbation method for solitons to solve the Eq. (16) to obtain a single-soliton solution under the perturbation. The result is given by \( u = \eta e^{d_0 s} \text{sech}[2\eta e^{d_0 s} (\sigma - \sigma_0 + 4\varsigma s)] e^{-2\varsigma \sigma - 4\varsigma^2 (\varsigma^2 - \varsigma^2) s - i\phi_0} \), where \( \eta \), \( \varsigma \), \( \sigma_0 \) and \( \phi_0 \) are real free parameters determining the amplitude (as well as the width), propagating velocity, initial position, and initial phase of the soliton, respectively. When taking \( \eta = 1/2 \), \( \varsigma = \sigma_0 = \phi_0 = 0 \) and noting that \( s = -z/L_{\text{Disp}} \), we obtain the electric field corresponding to the above single-soliton solution

\[
E = \frac{1}{\tau_0} \sqrt{\frac{\tilde{K}_2}{W}} \text{sech} \left[ \frac{1}{\tau_0} \left( t - \frac{z}{V_g} \right) \right] e^{-z/L_A} e^{i \Phi(z,t) - z/L_A + c.c.}
\]

with \( \Phi(z,t) \equiv [\tilde{K} + k_f - 1/(L_D)] z - \omega_f t \), which describes a damped bright soliton traveling with velocity \( V_g \).

For \( \delta = 15 \text{ GHz} \) and with other parameters given above, we obtain numerical values of the coefficients in Eq. (15), given by \( \alpha_1 = 0.0278 \text{ cm}^{-1} \), \( K_2 = (5.02 + 0.57 i) \times 10^{-23}\text{cm}^{-1}\text{s}^2 \), \( W = (4.62 + 0.72 i) \times 10^{-2} \text{C}^2\text{kg}^{-2}\text{cm}^{-3}\text{s}^4 \). We see that, as expected, the imaginary part of these coefficients are indeed much smaller than their real part. By taking \( \tau_0 = 1.5 \times 10^{-11} \text{s} \), we obtain \( U_0 = 2.2 \text{ V cm}^{-1} \), \( L_{\text{Disp}} = L_{\text{Nonl}} = 4.48 \text{ cm} \), and \( L_A = 36 \text{ cm} \). Thus one has \( d_0 = 0.12 \) and hence the dissipation in the system can be indeed taken as a perturbation.

The power associated with the soliton is given by Poynting’s vector integrated over the cross-section of the radiation in the transverse directions, i.e. \( P = \int \hat{E} \times \hat{H} \cdot dS \), where \( \hat{e}_z \) is the unit vector in the propagation direction. We obtain the peak power of the soliton \( P_{\text{max}} = 2\epsilon_0 cn_p S_0 \tilde{K}_2/(W \tau_0^2) \), here \( n_p = 1 + e\tilde{K}/\omega_0 \) is the refractive index and \( S_0 \) is the area of the intensity distribution of the radiation field in the transverse directions. Using the above
parameters and taking $S_0 = 6 \times 10^{-4}$ m$^2$, the peak power for generating the soliton is found to be $P_{\text{max}} = 568$ mW, which means that to generate the soliton in present system a very low input power is needed. This is a drastically contrast to the case in conventional media such as optical fibers, where pico- or femto-second laser pulses are needed to reach a very high peak power (usually at the order of several hundred kW) to stimulate enough nonlinearity for the formation of solitons.

The stability of the gigahertz soliton is tested by using numerical simulation. Fig. 6(a) shows the result of the radiation intensity $|E/U_0|^2$ of the soliton as a function of $t/\tau_0$ and $z/L_D$. The solution is obtained by numerically solving Eq. (15) with the complex coefficients taken into account, with the initial condition given by $E(0,t)/U_0 = \text{sech}(t/\tau_0)$. We see that the shape of the soliton undergoes no apparent deformation during propagation.

The collision between two solitons is also studied numerically, with the result shown in Fig. 6(b). The initial condition is given by $E(0,t)/U_0 = \text{sech}[(t-3.0)/\tau_0]+1.2\text{sech}[1.2(t+3.0)/\tau_0]$. We see that the both solitons can resume their original shapes after the collision, indicating that solitons in the PIT-metamaterial are robust during interaction.

5.2. Plasmonic dromions

We will take the diffraction effect into consideration in this section. Now we turn to the formation and propagation of high-dimensional nonlinear plasmon polaritons in the system by examining possible dromion solutions of the Eq. (9) and Eq. (12). After returning to original variables and converting them into dimensionless forms, Eq. (9) and Eq. (12) become

$$i \frac{\partial u}{\partial s} + \left( \frac{\partial^2}{\partial \xi^2} + g_y \frac{\partial^2}{\partial \eta^2} + g_{d1} \frac{\partial^2}{\partial \sigma^2} \right) u + 2g_1 |u|^2 u + g_2 v u = -i d_0 u, \quad (18)$$

$$g_{d2} \frac{\partial^2 v}{\partial \sigma^2} - \left( \frac{\partial^2}{\partial \xi^2} + g_y \frac{\partial^2}{\partial \eta^2} \right) v + g_3 \frac{\partial^2 |u|^2}{\partial \sigma^2} = 0, \quad (19)$$

where $u = \epsilon F \exp(-\alpha z)/U_0$, $v = \epsilon^2 G/V_0$, $s = z/(2L_{\text{Diff}})$, $\sigma = (t-z/V_g)/\tau_0$, $\xi = x/R_x$, $\eta = y/R_y$, $g_y = (R_x/R_y)^2$, $g_{d1} = L_{\text{Diff}}/L_{\text{Disp}}$, $g_{d2} = R_x^2(1/V_p^2 - 1/V_g^2)/\tau_0^2$, $g_1 = L_{\text{Diff}}/L_{\text{Nonl}}$, $g_2 = L_{\text{Diff}}^2 n_1 \omega_0 V_0 X/(cn_D)$, $g_3 = \chi^{(2)} R_x^2 U_0^2/(\epsilon^2 \tau_0^2 V_0)$, and $d_0 = 2L_{\text{Diff}}/L_A$. Here $R_x$ ($R_y$) is typical radius of the incident pulse in $x$ ($y$) direction; $U_0$ and $V_0$ are respectively amplitudes of the shortwave envelope and the longwave; $L_{\text{Disp}} = -\tau_0^2 / \bar{K}_2$, $L_{\text{Diff}} = \omega_0 n_D R_x^2/c$, $L_{\text{Nonl}} = \omega_0 n_D R_x^2/c$. $\bar{K}_2$ is the effective nonlinearity coefficient...
\[ \phi \text{ and } \sigma \text{ neglected the imaginary parts of } \text{length, nonlinear length and absorption length. Note that when obtaining Eq. (18) we have} \]

\[ \text{here is the shortwave envelope } |u| = |U|/U_0 \text{ (the hump at the center) and the longwave (rectification) field } v = V/U_0 \text{ [the two crossed tracks (plane solitons)] as functions of } \xi = x/R_x \]

\[ \text{and } \sigma = (t - z/	ilde{V}_b)/\tau_0. \text{ System parameters are chosen as } \mu = 1, \lambda = 1, a_1 = 0, p = 0, \varphi_\rho = 0, \varphi_\gamma = 0 \text{ at } s = 0. \]

\[ 2\epsilon_{nD}/[\omega_0\chi^{(3)}U_0^2] \text{ and } L_A = 1/\text{Im}(K) \text{ are respectively typical dispersion length, diffraction length, nonlinear length and absorption length. Note that when obtaining Eq. (18) we have neglected the imaginary parts of } K_2 \text{ and } \chi^{(3)} \text{ under the PIT condition } \kappa^2 \gg \omega_0\sqrt{\gamma_1/\gamma_2}. \]

\[ \text{Expressions (18) and (19) are coupled (3+1)D nonlinear partial differential equations including effects of dispersion, diffraction, nonlinearity, and a small loss caused by } \text{Im}(K). \]

\[ \text{Because a general consideration to obtain stable high-dimensional nonlinear solutions of such equations is not available yet, here we consider only a specific case to seek possible (2+1)D dromion solutions by using some assumptions for simplification. First, we assume } \]

\[ L_{\text{Disp}}, L_{\text{Diff}} \text{ and } L_{\text{Nonl}} \text{ have the same order of magnitude, which can be achieved by taking } \]

\[ \tau_0 = \sqrt{-\omega_0\epsilon_{nD}K_2/cR_x} \text{ and } U_0 = \sqrt{2\epsilon_{nD}/(\omega_0^2\epsilon_{nD}R_x^2\chi^{(3)})}. \]

\[ \text{Second, we choose realistic system parameters } \delta = -10 \text{ GHz}, R_x = 0.47 \text{ cm}, R_g = 2.5 \text{ cm}, \tau_0 = 2.21 \times 10^{-11} \text{ s}, V_0 = 0.51 \text{ V cm}^{-1}, \]

\[ \alpha = -3.99 \times 10^{14} \text{ cm}^{-1} \text{GHz}^2, \beta = 4.36 \times 10^{25} \text{ cm}^{-2} \text{GHz}^2, \kappa = 198 \text{ GHz}. \]

\[ \text{Then we obtain } L_{\text{Diff}} (= L_{\text{Disp}} = L_{\text{Nonl}}) = 4.99 \text{ cm}, d_0 = 0.064, U_0 = 4.57 \text{ V cm}^{-1}, \text{ and } g_{d1} = g_1 = g_2 = g_{d2} = 1, g_3 = 4, g_\eta \approx 0. \]

\[ \text{Because } d_0 \text{ is small, the term on the right hand side of Eq. (18) can also be taken as a perturbation. As a first step, we neglect such perturbation and hence Eq. (18) and Eq. (19) are simplified into standard Davey-Stewartson-I (DSI) equations} \]

\[ i \frac{\partial u}{\partial s} + \frac{\partial^2 u}{\partial \sigma^2} + \frac{\partial^2 u}{\partial \xi^2} + 2|u|^2u + vu = 0, \quad (20) \]

\[ \frac{\partial^2 v}{\partial \sigma^2} - \frac{\partial^2 v}{\partial \xi^2} + 4 \frac{\partial^2 |u|^2}{\partial \sigma^2} = 0. \quad (21) \]

\[ \text{which are completely integrable and can be solved exactly by the use of inverse scattering transform. One of remarkable properties of the DSI equations is that they allow various dromion solutions. A single dromion solution of the DSI equations reads } u = Q/P, v = 4\partial^2 \ln P/\partial \sigma^2, \]

\[ \text{where } P = 1 + \exp(\eta_1 + \eta_1^*) + \exp(\eta_2 + \eta_2^*) + \gamma \exp(\eta_1 + \eta_1^* + \eta_2 + \eta_2^*) \text{ and } Q = \rho \exp(\eta_1 + \eta_2), \text{ with} \]

\[ \text{Figure 7. A single dromion excitation in the PIT metamaterial, which consists of a localized envelope for the shortwave component and two “tracks” for the longwave component. Plotted here is the shortwave envelope } |u| = |U|/U_0 \text{ (the hump at the center) and the longwave (rectification) field } v = V/U_0 \text{ [the two crossed tracks (plane solitons)] as functions of } \xi = x/R_x \]

\[ \text{and } \sigma = (t - z/	ilde{V}_b)/\tau_0. \]
\[ \eta_1 = (k_r + ik_i)(\xi + \sigma)/\sqrt{2} + (\Omega_r + i\Omega_i)s, \]
\[ \eta_2 = (l_r + il_i)(\xi - \sigma)/\sqrt{2} + (\omega_r + i\omega_i)s, \Omega_r = -2k_r k_i, \]
\[ \omega_r = -2l_r l_i, \Omega_i + \omega_i = k_r^2 + l_r^2 - k_i^2 - l_i^2, \rho = |\rho| \exp(i \varphi_\rho), \] and \[ |\rho| = 2[2k_r l_r(\gamma - 1)]^{1/2}. \] Here, \( k_r, k_i, \]
\[ l_r, l_i, |\rho|, \varphi_\rho, \) and \( \gamma \) are real integration constants. If we choose \( k_r l_r > 0, \) we have \[ \gamma = \exp(2 \varphi_\gamma) \]
with \( \varphi_\gamma > 0. \) By taking \( k_r = \sqrt{2} \mu, \]
\[ k_i = \sqrt{2} a_1, l_r = \sqrt{2} \lambda, l_i = \sqrt{2} \rho (\lambda \rho \geq 0), \Omega_i = 2(\mu^2 - a_1^2), \]
\[ \omega_i = 2(\lambda^2 - \rho^2), \Omega_r = -4a\mu, \) and \( \omega_r = -4\lambda \rho, \) we obtain explicit expressions
\[
\begin{align*}
u &= \frac{2\mu \exp(i \eta)}{m \cosh f_1 + n \cosh f_2}, \quad (22a) \\
v &= \frac{4(m^2 + n^2)(\mu^2 + \lambda^2) - 8\mu^2}{(m \cosh f_1 + n \cosh f_2)^2} \\
&\quad + \frac{8m n[(\mu^2 + \lambda^2) \cosh f_1 \cosh f_2 - (\mu^2 - \lambda^2) \sinh f_1 \sinh f_2]}{(m \cosh f_1 + n \cosh f_2)^2}, \quad (22b)
\end{align*}
\]
where \( m = \{\mu/\lambda(\gamma - 1)\}^{1/2}, n = \{\lambda \gamma/\lambda(\gamma - 1)\}^{1/2}, \)
\[ h = a_1(\sigma + \xi) + p(\sigma - \xi) + 2(\mu^2 + \lambda^2 - a_1^2 - \rho^2)s + \varphi_\rho, f_1 = \mu(\sigma + \xi) - \lambda(\sigma - \xi) - 4(a_1 \mu + \lambda \rho)s, f_2 = \mu(\sigma + \xi) + \lambda(\sigma - \xi) - 4(a_1 \mu + \lambda \rho)s + \varphi_\gamma. \]
Obviously, the dromion given above consists of a localized envelope \( u \) for the shortwave component, which decays exponentially in all spatial directions (shown by the hump at the center of Fig. 7), and two plane solitons for the longwave component \( v, \) in which each plane soliton decays in its traveling direction (shown by the two “tracks” in Fig. 7).

The result presented above is the dromion solution based on Eq. (20) and Eq. (21) without considering the loss in the system. It is necessary to investigate the spatiotemporal evolution of the dromion and its stability starting directly from Eq. (9) and Eq. (12). We make a numerical simulation on Eq. (9) and Eq. (12) by taking the dromion solution (22) with a random disturbance as an initial condition. Shown in Fig. 8 is the evolution of the dromion pulse as a function of \( \xi = x/R_x \) and \( \sigma = (t - z/\tilde{V}_g)/\tau_0 \) by taking \( \varepsilon = 0.1. \) The profiles from (a) to (e) in the figure are respectively for the propagation distance \( z = 0, 0.5L_{\text{Diff}}, L_{\text{diff}}, 1.5L_{\text{diff}}, \) and \( 2L_{\text{diff}} \), with \( L_{\text{Diff}} = 4.99 \) cm. We see that the shape of the dromion undergoes no apparent change, but its amplitude is reduced a little during propagation due to the small loss inherent in the dark oscillator \( q_2. \)

With our system parameters, the average peak power of the plasmon dromion is \( \bar{P}_{\text{peak}} = 814 \) mW, which corresponds average peak intensity \( \bar{I}_{\text{peak}} = 361 \) mW/cm². We see that in the PIT metamaterial extremely low generation power is needed for generating (2+1)D spatiotemporal dromions.
6. Conclusions

We have proposed a scheme to realize giant Kerr nonlinearities and create low-power gigahertz solitons via PIT in a metamaterial. We have demonstrated that the PIT in such system can not only mimic the electromagnetically induced transparency in coherent three-level atomic systems, but also exhibit a crossover from PIT to Autler-Townes splitting. We have also shown that the system suggested here possess very large second-order and third-order nonlinear susceptibilities, which may abe used to create plasmonic solitons and dromions with extremely low generation power. Our studies raise the possibility for obtaining strong nonlinear effects for gigahertz radiations at very low intensity based on PIT metamaterials.

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