Reverse-loading coefficients identification for updated homogeneous anisotropic hardening model

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Abstract. The homogeneous anisotropic hardening (HAH) model formulation, which is solely based on distortional plasticity, was improved recently. In this study, a robust coefficient identification scheme was developed for this updated model. Tension-compression tests were conducted on a mild steel sheet sample to assess the hardening behavior in the forward-reverse loading mode. An inverse identification procedure was employed to calibrate the coefficients of the model. The Nelder-Mead simplex and genetic optimization methods were investigated and the results regarding the accuracy of the calibration were compared. As a result, both optimization methods led to reasonable coefficients for the new model.

1. Introduction

The effect of a strain path change is one of the important factors to consider for the prediction of accurate sheet metal forming simulation results. For example, many researchers have shown that the modeling of the Bauschinger and related effects after load reversal is essential for an accurate prediction of springback [1]. Although kinematic hardening is a classical approach to capture the Bauschinger effect, a recently proposed distortional plasticity concept [2] was shown to describe strain path change effects reasonably well. The homogeneous anisotropic hardening model, so-called HAH, is formulated based on pure distortional plasticity without back-stress. The application of the HAH model in finite element simulations of forming processes has led to a good prediction of springback in the U-draw bending test [3,4]. However, some researchers found out that the HAH model exhibits a singularity for strain path changes close to cross-loading [5]. In addition, the permanent softening behavior under cross-loading conditions could not be predicted satisfactorily in the original HAH model.

Recently, Barlat et al [6] proposed a revised version of the HAH model to mitigate these issues. The updated HAH model, called HAH20, showed indeed a much-improved capability to predict the material behavior after strain path change between pure reverse loading and pure cross-loading. In addition, the singularity problem was virtually eliminated by updating the evolution equations of the so-called microstructure deviator, a tensorial state variable.

However, a robust calibration method should be established in order to proliferate the use of this updated HAH model adequately. Some researchers have already developed various calibration methods for the original HAH model [7,8] but the robustness of which was not demonstrated. The calibration of the coefficients is an important aspect of material modeling because an unsuitable coefficient set could lead to problems in numerical simulations [4].

The aim of this work is to investigate optimization methods for the calibration of the HAH20 model. Since five coefficients are available to describe the reverse loading effects, including permanent softening, the possibility to end up the calibration at the local minima of the objective function is likely.
Therefore, an adequate optimization scheme is necessary to avoid these traps. Two optimization methods, the Nelder-Mead simplex and genetic algorithm are considered in this work.

2. Experiments

2.1. Tension-compression test
A custom-made tension-compression testing machine was used to characterize the material behavior under reverse loading cycles. A 1.2 mm thick EDDQ (Extra Deep Drawing Quality) mild steel was investigated. Details about the specimen size and testing equipment are given elsewhere [9]. The holding force preventing buckling was set to 1.96 kN and the displacement rate of the moving grip to 0.03 mm/s. The stress state was reversed when the engineering strain reached the values of 2, -2, 4, -4 and 4 %. Therefore, the stress-strain curve was composed of 2 full cycles after a pre-strain. The final accumulated strain was approximately 0.28, which is considered large enough for various sheet metal forming simulations. After an experiment, a correction scaling factor was multiplicatively applied to the measured stress to compensate for the friction effect. The biaxial stress effect was neglected as the normal force was sufficiently small. Figure 1 shows the true stress-true strain and absolute true stress-accumulated true strain curves.
Figure 1. (a) True stress-strain curve and (b) Absolute true stress-Accumulated strain curve in the tension-compression test for EDDQ steel

3. Inverse identification

Two constitutive models are required before the identification of the HAH20 model. First, an isotropic hardening equation that represents monotonic loading is calibrated using the combination of uniaxial tension and bulge test data. The combination curve was made by the same method in reference [10]. This experimental data was able to cover an effective strain range of up to 0.9. The Swift-modified Voce hardening law, defined as

\[ \sigma = K (\varepsilon_0 + \varepsilon)^n + \rho \varepsilon + \sigma_y + \sigma_b (1 - \exp(-\eta \varepsilon)) \]  

was selected for the isotropic hardening model. The inverse method [11] was adopted to determine the Swift-modified Voce coefficients using the Nelder-Mead simplex optimization method. Table 1 shows the coefficients of the isotropic hardening model. The other is yield function that could represent material anisotropy. The von-Mises isotropic yield function is selected in this study.

| Swift-modified Voce | \( K \) | \( \varepsilon_0 \) | \( n \) | \( \rho \) | \( \sigma_y \) | \( \sigma_b \) | \( \eta \) |
|---------------------|--------|--------|--------|--------|--------|--------|--------|
|                     | 326.2  | 0.013  | 0.313  | 7.526  | 55.14  | 89.45  | 12     |

The new model, HAH20, employs enhanced evolution equations for the state variables, including a second microstructure deviator which delays the evolution of the original deviator. In addition, the influence of the hydrostatic pressure on plasticity was incorporated in the yield condition, allowing the occurrence of a strength-differential (SD) effect between tension and compression. In this work, the pressure coefficient was set to the constant value of 20 TPa, as recommended in [12], which corresponds to flow stress about 5% higher in compression compared to tension. The formulations of the new model, HAH20, are not described in this article but the reader is referred to Reference [6] for details. The five reverse loading coefficients, which include permanent softening,
are denoted $k_1$, $k_2$, $k_3$, $k_4$ and $k_5$ as in the original paper. The role of these five coefficients is similar to that of the former model [2], although the state variable evolution equations were modified. $k_1$ controls the transient change of the hardening rate just after reversal. $k_2$ and $k_3$ controls the reloading stress after unloading and reversal. $k_4$ and $k_5$ are related to the rate and limit of permanent softening. In this work, these five coefficients were calibrated using inverse identification. The experimental tension-compression curve was compared with the simulated curve obtained from a stand-alone FORTRAN code containing the constitutive model to evaluate an error function. This error (or objective) function was minimized by an optimization method.

Two optimization methods were adopted to use inverse identification [11]. One is the Nelder-Mead simplex method (NM) and the other is the genetic algorithm (GA) both of which built-in MATLAB software. Both NM and GA algorithms require a search domain, which is set by selecting a large range of suitable coefficients. In contrast, only the NM algorithm requires an initial set of coefficients, which corresponds to the middle of the range for each coefficient. Table 2 lists the search ranges and initial values. Figure 2 shows the experimental and optimized curves from both optimization methods. Table 2 provides detailed values of the coefficients, mean error and computation time. Fig. 2 shows that the approximated curves using both optimization methods are in excellent agreement with the experimental data for this EDDQ steel sheet sample although both optimization methods lead to somewhat different coefficients, especially $k_2$. A sensitivity study is needed to assess the validity of this result. Table 2 indicates that the error percentage obtained between these methods is very similar but the computation time is much larger for the GA algorithm. This is not a significant issue because the computation time is still acceptable for calibration and, more importantly, because the GA algorithm is expected to converge towards the global minimum of the objective function.

![Comparison by optimization methods](image)

**Figure 2.** Comparison by optimization methods
### Table 2. Coefficients by optimization methods

| Tension-compression in EDDQ steel | NM | \(k_1\) | \(k_2\) | \(k_3\) | \(k_4\) | \(k_5\) | Error | Time |
|----------------------------------|----|--------|--------|--------|--------|--------|-------|------|
|                                  |    | 399.99 | 100.15 | 0.69   | 0.78   | 28.1   | 1.68 % | 10 min |
| Genetic                          |    | 391.12 | 158.87 | 0.75   | 0.78   | 27.9   | 1.52 % | 312 min |

| Search range (Initial value)     |    | 50–400 (225) | 50–200 (125) | 0.1–0.8 (0.45) | 0.7–1 (0.85) | 5–30 (17.5) || |

4. Conclusion

The coefficients of the updated HAH model, HAH20, related to reverse loading were calibrated from a tension-compression test data obtained on EDDQ mild steel. Two optimization methods, the Nelder-Mead simplex and genetic algorithms were used to calibrate the coefficient. Both optimizations led to similar curves, in excellent agreement with the experiment, but one coefficient, \(k_2\) was significantly different from one model to the other. Additional research is required to understand this result.

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