New estimation for neutron-neutron scattering length: charge symmetry and charge independence breaking revisited

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Abstract

Recent experimental results for neutron-neutron scattering length are reanalyzed from the point of view of three-nucleon force contribution. We found that the limiting value of $a_{nn} = -15.8 \pm 0.5$ fm must be free of any implicit three-body force contribution. We have also shown that the difference between the above experimental value of $a_{nn}$ and the well established value of neutron-proton scattering length $a_{np}$ can be explained by differences in the one-pion exchange potentials.

Keywords: neutron-neutron scattering length, charge independence

1. Introduction

The correct value of neutron-neutron scattering length $a_{nn}$ has a fundamental importance for nuclear physics at whole as well as for numerous particular problems like existence of multinucleons, size of charge-symmetry and charge-independence breaking effects etc. Unfortunately, up to date there is no clear knowledge on exact value of $a_{nn}$. Many different values for $a_{nn}$ (in interval from -16 up to -19 fm) which have been extracted from many different-type experiments must be considered almost on equal footing (see, e.g., reviews [1], [2], [3]). The main experimental results for $a_{nn}$ have been found from two different types of experiments:

(i) the final-state $nn$ interaction in three-body breakup $n + d \rightarrow nn + p$;

(ii) the final-state $nn$ interaction in $d(\pi^-, \gamma)nn$ in stopped pion radiation capture on deuteron.

The second-type experiments are considered now as most accurate ones due to absence of three-body rescattering and three-body force effects in the final state. These experiments have lead to the value $a_{nn} = -18.9 \pm 0.4$ fm (corrected for the magnetic-moment interaction of the two neutrons) [3] and this value (within the limits) is accepted for majority of modern realistic $NN$ potentials [1].

Alternatively, the $a_{nn}$ values extracted from the first-type experiments can also be divided into two categories: the first, extracted from the Migdal-Watson approximation (MWA) for the final-state interaction (FSI) of two neutrons, and the second, extracted from the exact solution of Faddeev equations for $3N$ breakup reaction. The values of $a_{nn}$ extracted from the old experiments done until 1973 with usage of MWA have been well summarized in the review [4] and the averaged $a_{nn}$ value in [4] was $-16.61 \pm 0.54$ fm. It can be compared with another averaged value of $a_{nn} = -15.4 \pm 0.3$ fm found from reanalysis of data of kinematically incomplete experiments [2].

On the other hand, the values of $a_{nn}$ extracted with usage of the Faddeev treatment for whole process are varying for different experiments from $a_{nn} = -16.2 \pm 0.3$ fm [5] or $a_{nn} = -16.5 \pm 0.9$ fm [6] until $a_{nn} = -18.8 \pm 0.5$ fm [7]. It is very likely that the rather large difference between the above values of $a_{nn}$ extracted from the same type of experiments but using different initial energies and different kinematical conditions is due to different contribution of three-body forces. This contribution is not established very reliably though the authors of the experimental results claimed that they have chosen the three-body kinematics in such a way to minimize three-body force effects.

However, the true origin of three-body force is still obscure and the above requirement can depend upon the three-body force operator structure which is in general still unknown. We can add to this that the result of the Faddeev equation solution is still sensitive to the three-body force contribution (because just this contribution explains the proper binding energies for $^3\!H$ and $^3\!He$ nuclei) while the results of FSI using MWA should be much less sensitive to the $3N$-force contribution. Thus, this difference can be a reason for difference in sensitivity to $3N$ force contribution between the MWA and Faddeev results for $a_{nn}$.
Thus, to extract the proper value of $a_{nn}$ from the breakup experiments one needs to elaborate a specific treatment which makes it possible to remove (or to minimize) the three-body force contribution objectively, i.e. independently upon particular structure of three-body force operator. We suggest such a method in the present paper.

2. New measurements for $a_{nn}$ in $dd$ breakup reactions

To get a new improved estimation for $a_{nn}$ from breakup reaction with $nn$ pair near threshold in the final state, some of the present authors made recently a novel measurement \([8]\) using $d + d \rightarrow 2n_0 + 2p_0 \rightarrow n + n + p + p$ reaction, where $2n_0$ and $2p_0$ mean the near pairs of two neutrons and two protons respectively, from whose momentum distribution the values of $a_{nn}$ can be extracted.

The $dd$ breakup experiment was performed using a 15 MeV deuterion beam in the Skobeltsyn Institute for Nuclear Physics in Moscow State University \([9]\). In the measurement the CD$_2$-target with thickness of 2 mg/cm$^2$ was used. Two protons were detected by a $\Delta E − E$ telescope at the angle of 27° while a single neutron was detected at 36° with time-of-flight technique using the distance of 0.79 m. In more detail the experimental setup scheme has been described in \([9]\).

The resulting time-of-flight neutron spectrum was compared with the kinematic simulation results for various values of the energy $\varepsilon_{nn}$ of the $nn$ virtual state. Figure 1 shows the results of such a comparison for three values of $\varepsilon_{nn}$.

![Figure 1: Experimental and simulated neutron TOF spectra for various $\varepsilon_{nn}$: 40 keV (violet), 76 keV (red), and 160 keV (blue).](image)

The $\varepsilon_{nn}$-dependence of $\chi^2$ was approximated by a quadratic polynomial, the minimum of which is achieved at $\varepsilon_{nn} = 76$ keV, $\Delta \varepsilon = \pm 6$ keV. Then the energy of virtual level $\varepsilon_{nn}$ is related to the scattering length $a_{nn}$ and the effective range $r_{nn}$ by the well-known equation

$$
\frac{1}{a_{nn}} = \left( \frac{m_n \varepsilon_{nn}}{\hbar^2} \right)^{1/2} \frac{1}{2} m_n \varepsilon_{nn} \frac{r_{nn}}{\hbar^2},
$$

from which one gets $a_{nn} = -22.2 \pm 0.6$ fm. This value very likely does include somehow the three-body force contribution.

3. Unified analysis for $a_{nn}$ values

Now we use this new $a_{nn}$ value together with previous results for $a_{nn}$ extracted from three-nucleon breakup reaction $n + d \rightarrow nn + p$ at different energies in kinematically complete experiments (except the result \([10]\)) for our analysis. All the $a_{nn}$ values corresponding to different energies (presented in Table 1) have been taken from the dedicated experiments with full three-body kinematics done after 1999 with usage of fully realistic Faddeev equations, except the results \([10, 8]\).

| $E_n$, MeV | $R$, fm | $a_{nn}$, fm | Ref. |
|-----------|--------|-------------|------|
| 15 (dd)   | 2.38   | $-22.2 \pm 0.6$ | \([8]\) |
| 13        | 4.25   | $-18.7 \pm 0.6$ | \([11]\) |
| 13        | 4.25   | $-18.8 \pm 0.5$ | \([7]\) |
| 16.6      | 5.93   | $-16.2 \pm 0.3$ | \([5]\) |
| 17.4      | 5.16   | $-16.5 \pm 0.9$ | \([6]\) |
| 19        | 5.44   | $-17.6 \pm 0.2$ | \([12]\) |
| 25.3      | 6.44   | $-16.1 \pm 0.4$ | \([5]\) |
| 40        | 8.35   | $-16.6 \pm 1.0$ | \([10]\) |

- Here we used the corrected value of $a_{nn}$ instead of the value in \([10]\) $a_{nn} = -17.9 \pm 1.0$ fm which was obtained in the approximation of zero-range nuclear forces ($r_{nn} = 0$).

Then we analyzed all these $a_{nn}$ values from a unified point of view of possible impact of three-nucleon force which must be excluded. The criterion for the exclusion is following: we have chosen some fixed time interval $\tau$ which corresponds to a characteristic time for possible three-body force (e.g. which corresponds to the average energy-exchange value $\Delta E$ in three-nucleon force operator using the relation: $\Delta E \tau \sim \hbar$). The exact value of $\tau$ is no matter due to evident scaling. So, if the distance $R$ between $nn$ pair and proton (or $2p_0$ pair in $dd$ experiment) in intermediate $3N$ (or $4N$) state corresponding to the incident energy $E_n$ and the time interval $\tau$ is much larger than the characteristic range of $3N$ force $r_{3N}$, i.e. if $R \gg r_{3N}$, one can ignore the $3N$-force contribution in interpretation of the result for $a_{nn}$ in the given experiment.

We displayed on Fig. 2 the $a_{nn}$ values extracted from experimental data for breakup reactions $n + d \rightarrow nn + p$ (seven experiments) and $d + d \rightarrow 2n_0 + 2p_0$ (one experiment) versus distance $R$ between two-neutron pair and proton (or $2p_0$ pair in $dd$ experiment) corresponding to the initial energy for particular experiment.

It was a big surprise for us to see that all eight values of $a_{nn}$ measured in the kinematically complete three-body
experiments lie on some curve (see Fig.2) so that extrapolation of this curve to infinity \((R \to \infty)\) gives the asymptotic value of \(a_{nn}\) which looks to be free of any three-body force contribution: \(a_{nn}^{\text{asym}} = -15.5 \pm 0.5\). This asymptotical value for \(a_{nn}\) still includes some small magnetic interaction which is repulsive and leads to some minor correction of \(a_{nn}\). According to [1] this repulsive effect can be estimated as \(\Delta a_{nn} \sim -0.3\) fm. This leads to the extrapolated \(a_{nn}\) value corrected to magnetic interaction as \(a_{nn}^{\text{corr}} = -15.8 \pm 0.5\).

This value is very close to the \(a_{nn}\) values, extracted for this type experiments by authors of [5] and our group at highest incident energies 25 and 40 MeV respectively. It is extremely interesting also that this value of \(a_{nn}\) is rather close to the average values summarized in [1, 2] from the old breakup experiments with usage of MWA (which is not sensitive to the 3N force contribution).

Thus, the asymptotic value \(a_{nn} = -15.8 \pm 0.5\) found here gives a new impetus for further reconsideration of the whole problem of neutron-neutron scattering length.

4. Charge independence symmetry in NN interaction

Using our novel estimations for \(nn\) scatterings length \(a_{nn}^{\text{corr}} = -15.8\) fm, it would be interesting to reanalyze the neutron-proton scattering length, \(a_{np}\), and put the question: what are the real charge independence breaking (CIB) effects and can one explain the difference between \(a_{np}\) and \(a_{nn}^{\text{corr}}\) only by the difference in masses of charged and neutral pions and the corresponding coupling constants.

To do this, we start with well accepted value \(a_{np} = -23.74\) fm and then derive the \(a_{nn}\) value using the difference between one-pion exchange potential (OPEP) in \(nn\) and \(np\) pairs [1]. In the first case \((nn)\) it is pure \(\pi^0\)-exchange while in the second case \((np)\) it is a combination of two OPEP values: \(2V_{\text{OPEP}}(\pi^\pm) - V_{\text{OPEP}}(\pi^0)\) [1]. We employ the following values for the pion masses: \(m_{\pi^0} = 134.977\) MeV, \(m_{\pi^\pm} = 139.570\) MeV and respective coupling constants recommended in [1]: \(f^{2}_{\pi^0} = 0.075\) \((g^{2}_{\pi^0}/4\pi = 13.959)\) for neutral pions and \(f^{2}_{\pi^\pm} = 0.079\) \((g^{2}_{\pi^\pm}/4\pi = 14.30)\) for charged pions.

We use the dibaryon-model \(np\) potential (with proper OPEP) fitted to exact value \(a_{np} = -23.74\) fm and then replace \(np\) OPEP with \(nn\) OPEP as noted above and also replace the masses of colliding nucleons. This leads immediately to the value \(a_{np} = -15.75\) fm which is in a very good agreement with the value \(a_{nn}^{\text{corr}} = -15.8\) fm obtained from extrapolation of experimental values (with correction for magnetic interaction). Thus, we see that the CIB effects in \(NN\) scattering can be explained only by differences in the potentials of one-pion exchange.

Keeping in mind the importance of this conclusion we made still another test to remove as far as possible a model dependence of this conclusion. For this we used the old Reid soft core (RSC) potential for singlet \(np\) s-wave scattering. We replaced the term in it corresponding to the averaged OPEP with the above correct OPEP for \(np\) interaction and fitted the exact value \(a_{np} = -23.74\) fm using small variation of the attractive Yukawa term in RSC potential. Then we replaced the \(np\) OPEP with the proper \(nn\) OPEP and have obtained as the result \(a_{nn}^{\text{RSC}} = -15.57\) fm, in good agreement with the value for dibaryon model. Thus, our above conclusion seems to be insensitive to the underlying \(NN\)-force model (but still dependent on the \(\pi NN\) form factor which regularizes the short-range behavior of OPEP [1]).

The difference between the neutron-neutron scattering length \(a_{nn}\) and the “purely nuclear” or strong-interaction proton-proton scattering length \(a_{pp}^{\text{nuc}}\) is considered as the measure of charge-symmetry breaking. The value \(a_{np}^{\text{nuc}} = -17.3 \pm 0.3\) fm [13] is now generally accepted. So, the large value \(a_{nn} = -18.9\) fm is greater than \(a_{pp}^{\text{nuc}}\) and their difference is 1.6 fm. However, the value \(a_{nn} = -15.8\) fm found here leads to the opposite sign of charge-symmetry breaking effects \(a_{nn} - a_{pp}^{\text{nuc}} = -1.5\) fm.

5. Further implications for nuclear physics

One of the arguments against the low value of \(a_{nn} \approx -16\) fm is the Coulomb displacement energy between binding energies of \(^3\text{H}\) and \(^3\text{He}\) nuclei \(E_{C} = E_{B}(^3\text{H}) - E_{B}(^3\text{He})\). Here the well assumed point of view is that the alternative “large” value \(a_{nn} = -18.9 \pm 0.4\) fm is necessary to get the proper value for the Coulomb displacement energy \(\Delta E_{C} \approx 740\) keV. However, this conclusion is valid only for conventional \(NN\)- and 3N-force models. In paper [14] we have shown that within the dibaryon model [15] for \(NN\) and 3N forces one gets the proper value for \(\Delta E_{C} = 740\) keV using just the “small” value for \(a_{nn} = -16.5\) fm without any free parameters. Thus, from this alternative picture one can conclude that, at least, the one-to-one correspondence between the values \(a_{nn}\) and
$\Delta E_C$ is not valid and the problem should be considered as model dependent one.

6. Problem with description of $d(\pi^-, \gamma)nn$ reaction

Still another argument against the low value of $a_{nn}$ is the results for $a_{nn}$ extracted from the stopped pion radiation capture in deuteron, $d(\pi^-, \gamma)nn$, where two-neutron pair in $^1S_0$ final state appears at very low energy $[3]$. This reaction is considered to be free from the 3N-force ambiguities and due to this as a process giving the most reliable estimation for the $a_{nn}$ value. Unfortunately, this conclusion is also not free from the serious doubts. The detailed discussion will be presented in our next paper, so that here we present only short outline of our arguments. In fact, the general mechanism for the above pion reaction on deuteron can be schematically described as following stages (see Fig. 3a): (i) the excited two-neutron state ($C'$) is emerged after the stopped $\pi^-$-absorption on deuteron; (ii) by subsequent emission of $\gamma$-quantum this intermediate state goes to the final $(nn)_0$ singlet state.

![Figure 3: Two-stage mechanism for the pion radiation capture in deuteron (left) and pp bremsstrahlung process (right).](image)

There is rather similar process $pp \rightarrow (pp)_0\gamma$ where the singlet $(pp)_0$ pair at very low energy is emerged in the final state (see Fig 3b). It is easy to see that the final stages of both processes, starting from the intermediate excited two-neutron $P$-wave state ($C$) and the excited two-proton $P$-wave state ($C'$) look to be very similar. However, the best for today theoretical description for the reaction displayed in Fig. 4b leads to very serious disagreement (about 40% for cross section) with the accurate experimental data, the strongest disagreement appears just in the case when the two final protons occur in near-threshold $^1S_0$ singlet resonance state, i.e. fully similar to the pion capture process displayed in Fig. 4a. Moreover, the disagreement degree is increasing with approaching the final two proton energy to the $^1S_0$ resonance region.

The true reasons for such a disagreement are still unknown but one suggests the very probable reason is manifestation of the of the $p$-wave dibaryon state in both $pp$ and $nn$ excited intermediate states discovered recently $[17]$. 

7. Conclusion

In the paper we suggested some new method to reanalyze the $a_{nn}$ values extracted from kinematically complete breakup experiments at different energies with aim to remove somehow the three-body force contribution to the final result. From our reanalysis we extracted the limiting value for $a_{nn} = 15.8 \pm 0.5$ which corresponds to higher incident energy and very large separation distance between $nn$ pair and the proton. It should be emphasized that this $a_{nn}$ value is close to the magnitude of $a_{nn}$ extracted from the very numerous old breakup experiments using MW approach. This finding leads immediately to the important conclusion that the charge symmetry breaking effects in $NN$ interaction have the opposite sign as compared to the conventional treatment.

Our further analysis for difference between $a_{nn}$ and $a_{np}$ has shown that the difference can be almost entirely explained by difference in OPE interaction for $nn$ and $np$ systems. This means that the charge dependence of the nuclear force can be attributed almost fully to the difference in OPE interaction.

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