Analytical solutions of pure-spinor superstring field theory

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ABSTRACT: We examine the possibility of constructing analytical solutions describing marginal deformations in the open superstring field theory that is based on the non-minimal pure-spinor formalism. It is found out that some methods used for constructing solutions of bosonic and RNS string field theories do not seem to generalize to the pure-spinor case, while other methods do lead to reliable analytical solutions.

KEYWORDS: String Field Theory, Pure-Spinors, Marginal Deformations
1 Introduction

Most of the attempts towards a covariant open superstring field theory [1–14] are based on the Ramond-Neveu-Schwarz (RNS) world-sheet formulation. Up to some subtleties, most of these formulations seem to be classically equivalent when restricted to the NS sector [12, 15–17]. In the Ramond sector, on the other hand, these theories suffer from various problems [16, 19]. While it seems that these issues can be overcome using a new (democratic) RNS formulation [20, 21], it might also be useful to study superstring field theory in a formulation that treats the NS and Ramond sectors on the same footing from the beginning, such as the Green-Schwarz (GS) formalism.

It is hard to use the GS formalism covariantly due to complications related to kappa-symmetry. A non-trivial modification of the GS formalism, in which these issues are resolved, is the pure-spinor formulation of string theory [23–27]. A superstring field theory based on the (non-minimal) pure-spinor formulation exists [26]. Since its construction this theory was largely ignored, presumably because most of the string field theory research following Sen’s conjectures [28–30] concentrated around tachyon condensation. Moreover, it was shown that the current pure-spinor formulations cannot be considered complete and cannot lead to a viable quantum string field theory, due to problems related to the definition of the space of string fields [31, 32]. Nonetheless, one would still expect to be able to obtain classical solutions within this framework, at least solutions that are not related to a condensation of a tachyon, which is absent. It is the purpose of this paper to look for analytical solutions of this theory. Specifically, we study solutions that represent marginal deformations.

1See [18] for a review of recent developments in string field theory.
2Another possibility would be to generalize the novel closed superstring field theory of Jurčo and Münster [22] to the open string case.
The rest of the paper is organized as follows. In section 2, we briefly review the non-minimal pure-spinor formalism and the open string field theory built upon it. Then, in section 3, we attempt the construction of solutions describing marginal deformations in this theory. Some concluding remarks are presented in section 4.

2 The non-minimal pure-spinor formalism

The pure-spinor formalism extends the GS formalism, so let us begin by a brief reminder of the building blocks of the latter. In this formalism the open string is defined in flat space in terms of the bosonic coordinates \( X^\mu(z) (\mu = 0..9) \) and the Grassmann odd, space-time chiral spinor \( \theta^\alpha(z) (\alpha = 1..16) \). We follow the common practice of the pure-spinor literature and work in the \( \alpha' = 2 \) convention,

\[
\partial X^m(z)\partial X^n(0) \sim -\frac{\eta_{mn}}{z^2}.
\]

The \( \partial X^\mu \) are weight one conformal primaries and the \( \theta^\alpha \) are conformal primaries of weight zero. Central to the theory are the GS constraints. One can formulate the theory in first-order form with respect to \( \theta^\alpha \) [33]: A spinor \( p_\alpha \), whose chirality is opposite to that of \( \theta^\alpha \) is added to the system. This spinor describes the (fermionic) momentum conjugate to \( \theta^\alpha \).

Then, the GS constraints take the form

\[
d_\alpha = p_\alpha - \frac{1}{2}\partial X^m \gamma_{m\alpha\beta} \theta^\beta - \frac{1}{8}(\theta^\gamma \gamma^m \partial \theta^\delta) \gamma_{m\alpha\beta} \theta^\beta.
\]

For the bosonic variables we do not add explicit momenta variables. The (supersymmetric) bosonic momenta are given by,

\[
\Pi^m = \partial X^m + \frac{1}{2} \theta^\alpha \gamma_{m\alpha\beta} \partial \theta^\beta.
\]

The minimal pure-spinor formalism was constructed in [23], by postulating that the ghost system that should be added to the GS variables consists of an even, zero-weight chiral spinor \( \lambda^\alpha \) together with its conjugate momentum \( w_\alpha \). The spinor \( \lambda^\alpha \) is constrained by the pure-spinor condition,

\[
\lambda^\alpha \gamma^m \lambda^\beta = 0 \quad \forall m.
\]
This ad-hoc ghost system together with the GS constraints (2.2) define the following ad-hoc BRST operator (the subscript \( m \) on \( Q \) is for “minimal”),

\[
Q_m = \frac{1}{2\pi i} \oint dz \lambda^\alpha d_\alpha .
\]  

(2.6)

The problems caused by the constraints (2.2) in the GS formalism do not occur here due to the pure-spinor condition (2.5).

There are several reasons for modifying the minimal pure-spinor formalism, e.g., one cannot devise in this formalism a covariant, composite \( b \)-ghost field (except in specific backgrounds [34]). Such a field is important for defining scattering amplitudes and other objects, in light of the expected identity,

\[
Q_b(z) = T(z) .
\]  

(2.7)

Here, \( Q \) is the BRST charge and \( T \) is the energy-momentum tensor. A formalism in which this issue is resolved was proposed by Berkovits [26]. In this formalism a non-minimal sector is added to the conformal fields of the minimal formalism. The quartet mechanism [35] guarantees that this non-minimal sector does not modify the cohomology of the minimal formalism, which is isomorphic to the cohomology of the RNS string [25]. Thus, the non-minimal sector consists of a conjugate pair of even fields \( \bar{\lambda}_\alpha \) and \( \bar{w}^\alpha \) together with a conjugate pair of odd fields \( r_\alpha \) and \( s^\alpha \). The field \( \bar{\lambda}_\alpha \) can be thought of as the opposite chirality partner of \( \lambda^\alpha \). Hence, \( \bar{\lambda}_\alpha \) and \( r_\alpha \) must also be zero-weight primaries. Moreover, they have to obey the pure-spinor constraints,

\[
\bar{\lambda}_\alpha \gamma^\alpha_\mu \bar{\lambda}_\beta = 0 \quad \forall \mu, \\
\bar{\lambda}_\alpha \gamma^{\alpha}_\mu r_\beta = 0 \quad \forall \mu.
\]  

(2.8) 

(2.9)

Note, that imposing the condition \( r_\alpha \gamma^\alpha_\mu r_\beta = 0 \) would have no consequences, since it must hold in any case, in light of the odd character of \( r \) and the symmetry property of the \( \gamma_\mu \). Hence, (2.9) is used in order to reduce the amount of degrees of freedom of \( r \) to the desired amount. The decoupling of the quartet \((\bar{\lambda}_\alpha, \bar{w}^\alpha, r_\alpha, s^\alpha)\) is achieved, as usual, by adding to the minimal BRST charge \( Q_m \) the following non-minimal component,

\[
Q_{nm} = \frac{1}{2\pi i} \oint dz \bar{w}^\alpha r_\alpha .
\]  

(2.10)

The total BRST charge is defined to be the sum of (2.6) and (2.10),

\[
Q = Q_m + Q_{nm} .
\]  

(2.11)

The pure-spinor constraints on \( \lambda^\alpha, \bar{\lambda}_\alpha \) and \( r_\alpha \) lead to gauge symmetries of the respective conjugate momenta. Observables must depend only on gauge invariant combinations of these momenta such as

\[
J = w\lambda, \quad N_{nm} = \frac{1}{2} w\gamma_{nm}\lambda, \quad T_\lambda = w\partial\lambda .
\]  

(2.12)
Here, we omitted the contracted spinor indices. We follow this convention below, e.g., we write $\bar{\lambda}\lambda$ instead of $\bar{\lambda}_\alpha \lambda^\alpha$. The gamma matrices with multiple indices, such as the $\gamma_{nm}$ above are defined as the odd part (i.e., repeated vector indices lead to identically vanishing expressions) of products of gamma matrices of alternating chirality, i.e.,

\[
\begin{aligned}
(\gamma_{nm})^\alpha_\beta &= \gamma^{n}_\gamma \gamma^{m}_\gamma \gamma^\beta_\gamma \ (n \neq m), \\
(\gamma_{nm})^\beta_\alpha &= \gamma^{\alpha}_\alpha \gamma^{n}_\gamma \gamma^{m}_\gamma \ (n \neq m).
\end{aligned}
\] (2.13)

When indices are omitted we infer the location of the indices from the fields with which the matrices are contracted, e.g., in the definition of $N_{nm}$ in (2.12), $(\gamma_{nm})^\alpha_\beta$ should be used.

A composite and covariant $b$ field can now be defined,

\[
b = s\partial\bar{\lambda} + \frac{\lambda(2\Pi^m\gamma^m d - N_{mn} \gamma^{mn} \partial \theta - J \partial \bar{\theta} - \partial^2 \theta)}{4\lambda\lambda} + \frac{(\bar{\lambda}\gamma^m p r) (d\gamma_{mp} d + 24 N_{mn} \Pi_p)}{192(\lambda\lambda)^2} \frac{(r \gamma_{mp} r) N_{np}}{16(\lambda\lambda)^3} + \frac{(r \gamma_{mp} r) (\lambda\gamma^{pqr} r) N^{mn} N_{qr}}{128(\lambda\lambda)^4}.
\] (2.14)

An interesting characteristic of this expression is its dependence on inverse powers of $\lambda$ and $\bar{\lambda}$. While this is not a-priori wrong, due to the fact that $\bar{\lambda}\lambda$ is a zero-weight scalar, one may wonder whether arbitrary negative powers of $\lambda$ should be allowed in defining states (and string fields). In fact the answer to this question is negative. The simplest way to see this is to note the existence of the state $\xi(z)$ [26],

\[
\xi = \frac{\bar{\lambda}\theta}{\lambda\lambda + r\theta} = \frac{\bar{\lambda}\theta}{\lambda\lambda} \left( 1 - \frac{r\theta}{\lambda\lambda} + \left( \frac{r\theta}{\lambda\lambda} \right)^2 - \ldots \right).
\] (2.15)

The sum above terminates after the eleventh power of $r$, due to its odd nature and the pure-spinor constraint (2.9). Hence, it has a finite degree of singularity with respect to $\lambda$. A direct calculation shows that $\xi$ is a contracting homotopy operator for $Q$, i.e.,

\[
Q\xi = 1.
\] (2.16)

This implies that the cohomology of $Q$ is trivial. Thus, the requirement of a physical cohomology for $Q$ implies that $\xi$ and similar states should be kept outside the space of allowed states. On the other hand, some sort of a singularity with respect to powers of $\lambda$ should be allowed if $b$, as well as some other ghost states related to the pure-spinor constraint [31], are to be included. It is not quite clear which condition exactly should one impose on the space of allowed states.

We are now ready to define the pure-spinor open string field theory action. This action takes a form, which is very similar to that of the modified and democratic RNS theories,

\[
S = - \int N \left( \frac{1}{2} \Psi^\dagger Q \Psi + \frac{1}{3} \Psi \Psi \Psi \right).
\] (2.17)

Here, the string fields are multiplied, as usual, using the star product, which we keep implicit and the regularization factor $N$ is a mid-point insertion of the following zero-weight primary field,

\[
N = e^{Q\chi} = e^{-\lambda\lambda - r\theta}, \quad \chi = -\bar{\lambda}\theta.
\] (2.18)
While we do not believe that this really makes a difference, we note that this insertion has a trivial kernel. The insertion tempers the singularities that originate from zero modes and enables the derivation from the action of the equations of motion

$$Q\Psi + \Psi \Psi = 0,$$

for string fields with an arbitrary number of $\theta$ insertions.

One could think of candidates for $\chi$ other than the one of (2.18). The question then would be whether the off-shell physics depends on the choice of $\chi$. In [26], it was argued that this is not the case at least for the simplest possible modification $\chi \rightarrow \rho \chi$, with $\rho \in \mathbb{R}_+$. We continue for the rest of this paper with the simple choice for $\chi$ (2.18).

3 Marginal deformations

In this work we study open string field theory. Hence, the marginal deformations we are interested in are boundary marginal deformations [36]. Marginal deformations are induced by their vertex operators. As usual, each vertex operator has two variants, integrated and unintegrated. There is no $c$-ghost in the pure-spinor formalism and the two types of vertex operators are related by

$$QU = \partial V,$$

where $U$ is the integrated vertex operator (the integrand to be precise) and $V$ is the unintegrated vertex. The massless vertex operators take the form,

$$V = \lambda^\alpha A_\alpha,$$

$$U = \partial \theta^\alpha A_\alpha + \Pi^m B_m + d_\alpha W^\alpha + \frac{1}{2} N_{mn} F^{mn}.$$  

Here $A_\alpha = A_\alpha(x, \theta)$ is a superfield obeying the SYM equations and $B_m, W^\alpha$ and $F_{mn}$ are derived from it in the usual way. The case of a constant $A_\alpha$ can be written as,

$$A_\alpha = \frac{i}{2}(\gamma^m \theta)_\alpha a_m + \frac{i}{12}(\theta \gamma^{mnp}\theta)(\gamma_{mnp}\chi)_\alpha = \frac{i}{2}(\gamma^m \theta)_\alpha a_m + 2i(\chi \gamma^m \theta)(\gamma_m \theta)_\alpha,$$

where $a_m$ are the constants parameterizing the photon deformations and the (Grassmann-odd) $\chi^\alpha$ are constants that parametrize the photino deformations ($\chi^\alpha$ should not be confused with the $\chi$ of (2.18)). A Fierz identity can be used to relate the two representations of (3.4). Plugging (3.4) into (3.3) leads to,

$$U = U_{NS} + U_R,$$

$$U_{NS} = ia_m \partial X^m,$$

$$U_R = 6i\chi^\alpha q_\alpha,$$

where $q_\alpha$ are the the supersymmetry currents,

$$q_\alpha = p_\alpha + \frac{1}{2} \partial X^m (\gamma_m \theta)_\alpha + \frac{1}{24} (\theta \gamma^m \partial \theta)(\gamma_m \theta)_\alpha.$$  

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It is straightforward to see that (3.2) and (3.5) indeed obey (3.1). In what follows we focus on the NS states (3.5b). We defer the study of Ramond solutions to future work [37].

The action (2.17) is very similar to that of the bosonic and the cubic RNS string field theories. Hence, it seems sensible to look for solutions of a similar form to those found in these other theories. In the bosonic theory there are several forms of solutions for marginal deformations, which we describe below. All these solutions are most simply represented using insertions over wedge states in the cylinder coordinates [38, 39]. The wedge states $W_n$ ($0 \leq n \leq \infty$) form a one-parameter family of zero-ghost-number states. This family interpolates between the identity string field $W_0$ and the sliver state $W_\infty$ [38, 40, 41]. The state $W_1$ is the perturbative vacuum and the wedge states obey the simple algebra

$$W_n W_m = W_{n+m}.$$  

(3.7)

We refer the reader to the literature (e.g., section 2 of [42], or section 4 of [18]) for more details on these states, the insertions over them and the cylinder coordinates.

The first type of analytical solutions describing marginal deformations was presented in [43, 44]. There, solutions are defined in terms of the non-integrated vertex. They include integration over surface size and it is known how to write them down explicitly only for vertex operators whose self OPE is regular. These solutions were extended to cover the NS case in [45–47].

A second type of solutions can be used also for vertex operators whose self-OPE is singular. These solutions do not include integration over surface size. The solutions can be constructed using one of two approaches. In the first one, solutions are defined in terms of a formal primitive of the unintegrated vertex [48], while in the second one they are defined in terms of the integrated vertex [49]. The former approach has the advantage of representing the solution as a (formal) gauge solution, while the latter is more systematic and might also improve our general understanding of marginal deformations. The two approaches can be shown to agree when there are no OPE singularities as well as for some other solutions [18, 49]. Nonetheless, there is no clear proof that this is always the case. The problem in constructing such a proof is that both approaches depend on objects that are not defined a-priori. In the first approach one has to specify the primitive and its OPE with all other fields, while in the second approach one has to specify the recipe for renormalizing exponents of integrated vertex operators. Both these concepts rely on the details of the marginal deformation. Presumably they both hold the same information and can be properly defined simultaneously. These solutions were also extended to the NS case [50, 51].

A third type of analytical solutions was presented in [52]. There, the building blocks of the solutions are boundary changing operators. These solutions are very elegant. However, as in the case of the first type of solutions, this construction is defined only for the case of a regular OPE. The extension of this construction to the NS case was achieved in [53].

We now wish to examine the possibility to generalize these three methods to the pure-spinor theory for the test case of the photon marginal deformation. First, we attempt, in 3.1, to generalize the solutions related to regular marginal deformations and show that they fail to be reliable. Next, in 3.2 we examine the generalization of the solutions that
can describe general marginal deformations and show that a regular generalization does exist. Then, in 3.3 we examine the case of solutions that are based on boundary condition changing operators and illustrate that these solutions also seem not to generalize to the pure-spinor theory.

3.1 The method for regular marginal deformations

In this method solutions are obtained from the unintegrated vertex. The photon vertex operator can be read from (3.2) and (3.4),

\[ V = \frac{ia_m}{2}(\lambda \gamma^m \theta). \] (3.8)

This vertex has regular OPE with itself. The fact that the OPE of the unintegrated photon vertex operator with itself is regular, might seem a bit strange, since this is quite unlike the bosonic case. OPE singularities might have physical significance, e.g., a linear divergent term in the self-OPE implies that the deformation cannot be exactly marginal. However, in the case at hand this obstacle is absent as the photon deformation is indeed an exactly marginal deformation.

The second building block of the solution is an integral over the b-ghost. As mentioned above, this field is not an “elementary field” in the pure-spinor formalism. Nonetheless, we can attempt to construct the solution using the composite b field (2.14). Specifically, we define, in the cylinder coordinates,

\[ B = -\frac{1}{2\pi i} \int_{-i\infty}^{i\infty} b(z) dz. \] (3.9)

Being a line integral, B can be deformed, as long as it does not pass by any other insertion.

The solution of [43, 44] depends on a marginality parameter \( \mu \) and is given by,

\[ \Psi = \sum_{n=1}^{\infty} \mu^n \Psi_n, \] (3.10a)

\[ \Psi_1 = V(0)W_1, \] (3.10b)

\[ \Psi_n = \int_0^1 dt_1 \ldots \int_0^1 dt_{n-1} V(0)BV(t_1)B \ldots V(t_1 + \ldots t_{n-1})W_1 + \sum t_n. \] (3.10c)

It is easy to note that this defines a solution also in the pure-spinor case, as long as,

\[ V(z)V(0) = O(z). \] (3.11)

This is the same condition as in the bosonic case of [43, 44], since our \( V \) is the analogue of \( cV \) in these papers. Using (3.8), one can see that this condition indeed holds.

The solution (3.10) includes an unbounded power of the b ghost from its appearance in \( B \). However, the b ghost (2.14) includes negative powers of \( \lambda \) and \( \bar{\lambda} \). Recall that states including too high negative powers of \( \lambda \) and \( \bar{\lambda} \) should be discarded from the space of string
fields. Hence, the $\Psi_n$ above are potentially singular. In the bosonic case, one can eliminate the $B$’s, using the properties of the commutator\(^3\),

$$\tilde{V}_{\text{bos}} \equiv [B_{\text{bos}}, V_{\text{bos}}].$$

(3.12)

The operator $\tilde{V}_{\text{bos}}$ is the matter part of the unintegrated vertex,

$$V_{\text{bos}} = c \tilde{V}_{\text{bos}},$$

(3.13)

and is also equal to the integrated vertex,

$$\tilde{V}_{\text{bos}} = U_{\text{bos}}.$$  

(3.14)

Then, using the fact that,

$$B^2_{\text{bos}} = 0,$$

(3.15)

which follows from the regularity of the $bb$ OPE, one can rewrite the solution using no more than a single $B_{\text{bos}}$ line integral,

$$\Psi_{n, \text{bos}} = \int_0^1 dt_1 \ldots \int_0^1 dt_{n-1} V_{\text{bos}}(0) U_{\text{bos}}(t_1) U_{\text{bos}}(t_1 + t_2) \ldots B V_{\text{bos}}(t_1 + \ldots t_{n-1}) W_{1+\sum t_n}. \tag{3.16}$$

If we could claim that something similar happens in the pure-spinor case, the solution (3.10) could be saved.

The $bb$ OPE is regular also for the composite $b$ (2.14) [54, 55]. This implies that an analogue of (3.15) holds also in the pure-spinor case. On the other hand, there is no analogue of (3.13) in this case. In fact, even for the RNS string, the analogues of (3.13) hold only for specific pictures [56]. Hence, we have to examine the commutation relation of $B$ and $V$. Let us decompose,

$$b = \sum_{k=-1}^{3} b_k,$$

(3.17)

where the $\bar{\lambda}$ dependence of $b_k$ is $b_k \sim \bar{\lambda}^{-k}$. Acting on $V$ (3.8), the number of $\bar{\lambda}$’s does not change. Hence, the condition that $[B, V]$ has no $\bar{\lambda}$ poles is equivalent to the condition,

$$[B_k, V] = 0 \quad \forall k > 0. \tag{3.18}$$

Here, we decomposed

$$B = \sum_{k=-1}^{3} B_k,$$

(3.19)

and $B_k$ is the component of $B$ for which $b_k$ is the integrand. However, a direct calculation reveals that,

$$\tilde{V}_3 \equiv [B_3, V] = i \frac{(\nu_{mnpr}^r)(\bar{\lambda}\gamma^{pq}r)(w_{\gamma^{mn}}\lambda)(\theta\gamma_{qr}^\lambda) + (\theta\gamma_{n}^m\lambda)(w_{\gamma_{qr}}\lambda))a^s}{1024(\lambda\bar{\lambda})^2}. \tag{3.20}$$

\(^3\)Here and elsewhere in this paper the commutator is a graded one, i.e., for two odd objects, as we have here, it represents their anticommutator.
We verified that this expression is non-zero for some specific cases that obey the pure-spinor constraints (2.5), (2.8) and (2.9). This suggests that $\Psi_5$, which has four $B$’s in its definition, includes a factor of $\lambda^{-12}$, which cannot be cancelled by any other factor. This factor is too singular. Hence $\Psi_5$ does not belong to the space of string fields. This observation renders the solution itself unphysical.

One could have objected the observation above on several grounds. First, one might hope that while the $\Psi_n$’s are “bad”, the resulting $\Psi$ is still a legitimate string field. This probably cannot be the case, since $\Psi$ depends on the free parameter $\mu$ (3.10). It is unlikely that the “magic” of obtaining a legitimate $\Psi$ from a sum of the $\Psi_n$’s could occur for all values of $\mu$. Moreover, in the limit $\mu \to 0$, the contribution of $\Psi_5$ should become dominant over that of all the other singular $\Psi_n$’s and as such could not be cancelled. Alternatively, one could have suggested that we should have considered not (3.10c), but rather (3.16) for generalizing the solution in a way that avoids multiple $B$ insertions. The problem with this would be that (3.16) does not generalize to a solution in the pure-spinor case. A direct proof that this expression defines a solution relies on (3.14), which does not hold in the pure-spinor case. The closest we can get in the general case to proving (3.14) is

$$Q(\tilde{V} - U) = 0,$$

which follows from (3.1) and

$$[K, V] = \partial V.$$

Here we defined, as usual, the string field $K$ to be an insertion over the identity string field of the following state, which we also denote by $K$,

$$K = QB = -\frac{1}{2\pi i} \int_{-i\infty}^{i\infty} T(z)dz.$$  

Unfortunately, this is not enough, as we explicitly illustrated above. Yet another reservation might arise from counting the total power of $r$ insertion. For the $\Psi_5$ above it exceeds eleven. Hence, one might think that while $[B_3, V]$ is non-zero, the part of $\Psi_5$, which is obtained exclusively from it, is zero, due to the fact that $r$ is an odd field that obeys the pure-spinor constraint (2.9). Such a claim would, however, ignore the fact that the insertions are located at different points. This means that parts of the expression can have the non-zero modes of $r$ multiplying the inverse power of $\lambda$ and $\bar{\lambda}$ zero modes at a too-high power. Moreover, we can add one more objection to the solution. The fact that the vertex operators had regular OPE with themselves was enough in the bosonic construction, due to the matter-ghost factorization. Here, on the other hand, it is not clear whether $\tilde{V}$ has regular OPE with itself, which might imply that the solution is unreliable even if we do not care about the issue of $\lambda$ zero-modes. In any case, we conclude that solutions of this type do not carry over in general from the bosonic and NS cases to the pure-spinor case.

3.2 The method for general marginal deformations

The second type of solutions can be used even in the case of a marginal deformation with a singular OPE. While the unintegrated vertex operator that we consider has regular OPE,
the integrated vertex operator (3.5b), which is the one relevant for this method, has indeed singular OPE with itself. As with the previous method (3.10), the solution is given as a power series in $\mu$, with the same initial condition,

$$\Psi = \sum_{n=1}^{\infty} \mu^n \Psi_n,$$

$$\Psi_1 = V(0)W_1.$$  

The integrated vertex operators can be written in terms of their primitives $\hat{U}$. As a result of (3.1), these primitives obey,

$$\partial \hat{U} = U,$$

$$Q \hat{U} = V.$$  

In terms of the primitives the solution takes a simple form as a formal gauge solution [48],

$$\Psi_L = Q\Lambda_L \frac{1}{1 - \Lambda_L},$$

where $\Lambda_L$ is the formal gauge string field that depends on $\hat{U}$ and the subscript $L$ stands for “left”. There is also a “right” solution, which is gauge equivalent to the left one. A real solution can be obtained by going along this gauge trajectory to half the way between the two solutions [49]. The main issue with this representation is the normal ordering of the solution [49] (see also [18]). However, for the photon deformation this issue was resolved already in [48]. It is obvious that any expression of the form (3.26) is a solution also in the case of the pure-spinor theory and the main challenge, as in the other cases, would be to distinguish genuine solutions from pure-gauge and singular ones.

For the photon, $\hat{U}$ is the holomorphic half of the $X$ field,

$$\hat{U} = ia_mX^m(z).$$

The gauge string field is,

$$\Lambda_L = \sum_{n=1}^{\infty} \mu^n \Lambda_n,$$

$$\Lambda_n = -\left(\frac{iX^n(0)}{n!}\right)W_n,$$

$$X \equiv a_mX^m,$$

where the insertions are implicitly normal-ordered. Normal ordering $X^n$ can be achieved by point-splitting. The same results, however, follow from the definition,

$$X^n = \partial_p e^{pX}|_{p=0},$$

with the familiar normal ordering of the exponent. In the pure-spinor case, this relation implies that,

$$QX^n = \frac{n(\lambda\gamma\theta)}{2}X^{n-1},$$
where we defined,\[\gamma \equiv a_m \gamma^m.\] (3.31)

From (3.26) and (3.28) we see that the first few terms in the expansion (3.24a) of the photon solution are,
\[
\Psi_1 = Q\Lambda_1 = \frac{i}{2}(\lambda\gamma\theta)(0)W_1,\]
\[
\Psi_2 = QA_2 + QA_1A_1 = -\frac{(\lambda\gamma\theta)(0)}{2}(X(1) - X(0))W_2,\]
\[
\Psi_3 = QA_3 + QA_2A_1 + QA_1(A_2 + A_1A_1)
= -\frac{i(\lambda\gamma\theta)(0)}{4}(X(0)^2 - 2X(0)X(2) + 2X(1)X(2) - X(1)X(1))W_3.\]

First, we note that \(\Psi_1\) indeed agrees with (3.24b). Next, we verify that \(\Psi_2\) can be written in terms of the integrated vertex, i.e., in terms of \(\partial X\),
\[
\Psi_2 = -\frac{(\lambda\gamma\theta)}{2} \int_0^1 dz \partial X(z)W_2.\]

With \(\Psi_3\) a new complication appears that stems from the fact that insertions at the same point are normally ordered, but insertions at different points are not, e.g., by \(X(0)^2\) we mean \(X(0)X(0)\), but \(X(0)X(2) \neq X(0)X(2)\). Normal ordering the whole expressions, we can write the result in terms of \(\partial X\) as (implicit normal ordering),
\[
\Psi_3 = -\frac{i(\lambda\gamma\theta)(0)}{4} \left( C + \int_0^1 \partial X(w)dw \left( \int_0^2 + \int_1^2 \right) \partial X(z)dz \right)W_3,\]

where \(C\) is a known constant coming from normal ordering. This constant depends on the details of the conformal transformation between the upper half-plane and \(W_3\) with the parametrization used for the location of the insertions. At higher orders various such constants multiply various powers of \(X\), but as explained in [48], the result can always be written in terms of integrals of a normal-ordered polynomial in \(\partial X\). Employing the method of [49] would have lead us, as usual, exactly to the same results. We have found out that no pure-spinor-specific complication arise for this type of solutions. We conclude that this type of solutions adequately generalizes to the pure-spinor theory.

**3.3 The method based on boundary changing operators**

In [52], solutions were defined in terms of the boundary changing operators that change the original BCFT to the one that should be described by the solution. This is a desirable description both, because it might be possible to generalized it to non-marginal deformations and because it relates the space of possible string field theory solutions to the space of possible BCFTs, which for the bosonic case should be the same as the space of reliable string backgrounds. The drawback of this method is that it is, again, appropriate only for the case of a regular OPE. Since the objects defining the solution are the boundary changing operators, the regularity condition is defined in terms of them,
\[
\sigma_L(z)\sigma_R(0) \sim 1.\] (3.35)
Here, $\sigma_L$ is the operator that changes the boundary conditions from those of the original theory to those of the deformed BCFT and $\sigma_R$ is the operator that changes the boundary conditions back to the original ones. The fact that the solutions of [52] have properties similar to those that we studied in 3.1, is not a coincidence. Indeed, these two types of solutions are closely related. However, the functional form of the solution, as introduced in [52], suggests a different type of extension to the pure spinor case, which we now explore.

Let us illustrate how the regularity condition (3.35) is related to the previously discussed regularity conditions (3.11). In the case of a general marginal deformation, the boundary changing operators are defined by

$$
\sigma_L(a)\sigma_R(b) = \exp \left( \mu \int_a^b U(z) dz \right).
$$

Here, we kept the dependence of the boundary changing operators on the marginality parameter $\mu$ implicit. This expressions should be regularized if $U$ has singularities in its OPE, since the exponent leads to collisions of the $U$ operators. In the case of a regular OPE there is no such issue and we can write,

$$
\sigma_L(a)\sigma_R(b) = \exp \left( \mu (\hat{U}(b) - \hat{U}(a)) \right) = e^{\mu \hat{U}(b)} e^{-\mu \hat{U}(a)}.
$$

From here we deduce that the boundary changing operators are the exponents of the primitives,

$$
\sigma_L = \sigma_{-\mu}, \quad \sigma_R = \sigma_\mu, \quad \sigma_\mu \equiv e^{\mu \hat{U}}.
$$

This relation might also hold in the general case of singular OPE, in light of (3.25).

Consider now, for simplicity the case of the photon marginal deformation (3.5b). The integration is immediate and gives,

$$
\int_a^b i a_m \partial X^m(z) dz = i a_m (X^m(b) - X^m(a)).
$$

As long as $a_m$ is not time-like, one cannot exponentiate this result, due to OPE singularities. Nonetheless, one can easily guess the properly regularized form of the boundary changing operators,

$$
\sigma_\mu(z) =: e^{i \mu X(z)} :.
$$

Here, we wrote the normal ordering explicitly. However, while we can define localized boundary changing operators using normal ordering, the OPE of such objects is non-trivial,

$$
\sigma_{\mu_1}(z)\sigma_{\mu_2}(0) = z^{a_m a_m \mu_1 \mu_2} (\sigma_{\mu_1 + \mu_2}(0) + \mathcal{O}(z)),
$$

and the construction of [52] cannot be used.

Note, that the problem we are facing now is quite different from what we had in 3.1. There, the problem seemed to be inherent and unrelated to the choice of the vector $a_m$. Here, on the other hand, we can try to continue with the construction, albeit only for the case of a light-like $a_m$. Let us assume then that $a_m$ defines a light-like direction. In split-string conventions [57, 58], the solution of [52] takes the form,

$$
\Psi = -\frac{1}{\sqrt{1-K}} Q\sigma_L \frac{1}{1-K}\sigma_R (1-K) Be^{-\frac{1}{\sqrt{1-K}}}.
$$
The form (3.42) is obviously inadequate for generalization to the pure-spinor case, since it depends on the $c$ ghost, which is absent in this formalism. Luckily, it was also shown in [52], that the solution can be rewritten in a $c$-independent way,

$$
\Psi = \frac{1}{\sqrt{1-K}} Q\sigma_L \left( \sigma_R + \frac{B}{1-K} Q\sigma_R \right) \frac{1}{\sqrt{1-K}} .
$$

(3.43)

In order to obtain (3.43) from (3.42), one has to use peculiarities of the bosonic theory. However, the end result (3.43) is more general, e.g., it can constitute a solution also in theories for which (3.13) does not hold.

An important question one should examine at this stage is whether (3.43) constitutes a legitimate string field, i.e., whether it is independent of the $X$ zero-mode. While this is obvious in the bosonic case, in light of the matter-ghost factorization, the presence of $B$ in the current case can potentially lead to problems. Thus, we should think of (3.43) as a formal expression that is defined in an extended space that includes the zero-mode. Then, we should verify that it is indeed independent of this zero-mode. This resembles the logic we used in 3.2, following [48]. In order to prove the zero-mode-independence of (3.43), let us introduce the operator

$$
P \equiv \oint \frac{dz}{2\pi i} \partial X .
$$

(3.44)

This operator is the integral of a weight-one primary and is therefore a derivation of the star product. A string field is zero-mode-independent if and only if it is annihilated by this operator. This is indeed the case for (3.43), in light of the fact that $[P, Q] = 0$.

In [53], it was directly shown that (3.43) obeys the equation of motion, using only the relations (3.35), (3.15), (3.23) and

$$
[B, \sigma_{L,R}] = 0 ,
$$

(3.45)

as well as the identities that follow from these relations when they are acted upon by $Q$. Another advantage of (3.43) over (3.42) is that it manifestly obeys the reality condition of the string field. One can simplify (3.43) slightly using a gauge transformation to the form,

$$
\Psi = \frac{1}{1-K} Q\sigma_L \left( \sigma_R + \frac{B}{1-K} Q\sigma_R \right) ,
$$

(3.46)

which does not obey the reality condition. However, the fact that this expression is gauge equivalent to (3.43) guarantees that it is also physical.

The expression (3.43) is linear with respect to $B$. Hence, powers of $\lambda$ and $\bar{\lambda}$ that are smaller than $-4$ do not appear in it. Thus, it does not suffer from the problems of the solutions discussed in 3.1 and could be considered a legitimate string field. The only thing one still has to show in order to infer that (3.43) is a genuine solution in the case of the pure-spinor theory is to prove that (3.45) holds in this case, assuming that $\sigma_{L,R}$ is given by (3.38) and $a_m$ a light-like vector.

In the bosonic case the relation (3.45) holds since the integrated vertex operator is a pure matter state. The integrated vertex operator is related to the unintegrated vertex operator via (3.13). Thus, the latter has ghost dependence. However, the boundary changing

\footnote{I am grateful to the referee for suggesting this approach.}
operator is defined in terms of the integrated vertex and is, therefore, also a pure-matter state. In the RNS case (3.13) holds only for specific pictures. Nonetheless, the integrated vertex operator and the boundary changing operators can be assumed to live in the matter sector and the relation (3.45) still holds. Unfortunately, it seems that (3.45) does not hold in general in the pure-spinor case. The reason for that is the fact that the $b$ field (2.14) is a composite field, which has a non-trivial dependence on the matter fields via its dependence on $d_\alpha$ and $\Pi^m$.

Let us check then, whether (3.45) holds in the specific case of the photon deformation. Recall the decomposition of $B$ (3.19). Inspecting the expressions for $d_\alpha$ (2.2) and $\Pi^m$ (2.3), we conclude that each of the $B_k$ should separately commute with the boundary changing operators. For $B_{-1}$ and $B_3$ commutation is trivial, since they do not depend on $d_\alpha$ and $\Pi^m$. The other three components do depend on $d_\alpha$ and $\Pi^m$. Of these, the dependence of $B_2$ is the simplest and leads to

$$[B_2, \sigma_\mu] = -\mu \sigma_\mu \frac{(r^p \gamma_{mnp}) (\lambda \gamma^m \gamma^\theta) N^{np}}{32 (\lambda \lambda)^3} \neq 0. \quad (3.47)$$

Hence, we conclude that in the case of the photon deformation this type of solutions cannot be generalized to the pure-spinor case. Moreover, devising a deformation that commutes with $B$ is probably a non-trivial task. Thus, the problems that we face in the concrete example we examined are probably quite general. Note, that similar problems could have occurred also with the first type of solutions had we been trying to generalize other standard representations of that approach. In that case we managed to overcome this difficulty only to get to the other problem that we described. Here, we did not manage to pass even the first obstacle. Of course, this does not mean that it is impossible to find other generalizations of these solutions that do work. We are currently studying such possibilities [37].

4 Conclusions

In this paper we examined the possibility of generalizing known string field theory solutions to the pure-spinor string field theory, focusing for simplicity on specific marginal deformations. While it was found that some solutions seem not to generalize properly, others had no such problems. It would be interesting to find more solutions to the pure-spinor string field theory as well as to further study solutions of this theory by, e.g., constructing their boundary states, along the lines of [59, 60].

We believe that the existence of solutions to the pure-spinor string field theory suggests that a reliable pure-spinor string field theory might exist. Such a theory would generalize the current formalism and would also support the current solutions. Furthermore, the results obtained here point towards a specific problem with the current formulation of pure-spinor string field theory, namely the definition of the $b$-ghost. Indeed, the solution that generalized nicely to the pure-spinor case is $b$-ghost independent, while the solutions that do depend on the $b$-ghost had problems with their generalizations. Moreover, the problems we faced did not depend on regularity of the OPE or on any other property that
was unrelated to the nature of the composite $b$-ghost. We can identify the two problematic characteristics of this field as its dependence on inverse powers of the pure-spinors $\lambda$ and $\bar{\lambda}$ and its dependence on the matter sector, i.e., the lack of matter-ghost factorization in the pure-spinor formalism.

Of these two problems, the first one stood already behind the claim that the space of string fields cannot be defined in this framework [31]. This is not the “standard problem” with defining this space, which is related to the infinite summation of basis states and the lack of a natural norm on this space that leads, e.g., to the introduction of phantom terms [39, 42, 61, 62]. Rather, the problem is with defining the basis states themselves. This problem is present already at the level of closing the algebra of vertex operators. More precisely, it was shown in [31] that one has to include vertex operators with negative powers of $\bar{\lambda}\lambda$, in order to properly obtain all the needed ghost fields. While this might be acceptable from the point of view of the first quantized theory, the requirement of closure under the star product seems to imply that arbitrarily negative powers of $\bar{\lambda}\lambda$ exist as basis elements of the space of string fields [32]. The resolution of this problem presumably calls for a further refinement of the definition of the pure-spinor worldsheet theory.

We should recall at this point that a-priori we do not really need a $b$-ghost for the study of marginal deformations. Indeed, we did manage to see that the most general form of solutions, in which the $b$-ghost is absent, does generalize to the pure-spinor case. Moreover, these general solutions do not depend on the non-minimal sector in an essential way. Their only dependence on the non-minimal sector comes from their dependence on the energy-momentum tensor. While we do not have a string field theory action for the minimal pure-spinor theory, the equations of motion are known and are identical to those of the non-minimal theory with which we worked so far. Thus, the solutions that do generalize are automatically solutions for the minimal case as well. The fact that the solutions do not depend on the non-minimal sector could have been expected. One could further wonder whether the solutions that did not generalize properly could be gauge transformed to a regular form that does not depend on the non-minimal sector. On the one hand, this is problematic, since these solutions are improper from the point of view of the pure-spinor theory. On the other hand, in the case of the bosonic theory, the solutions that did generalize are gauge equivalent to the ones that did not. It is not that we are missing some solutions, rather we found out that some of the approaches towards the construction of solutions do not work in the pure-spinor theory. The solutions that did not work just happened to depend on the $b$-ghost, which is problematic for other reasons as well. A regularization of the $b$-ghost was proposed in the context of scattering amplitudes [63]. It would be interesting to examine the possibility of using this regularized $b$-ghost for constructing solutions, as well as for evaluating amplitudes within string field theory. However, even if this is possible, it still seems to us that a modified world-sheet formulation is in order.

Several attempts have been carried out towards the refinement of the pure-spinor formalism. In particular, Berkovits showed that an elementary $bc$ ghost system can be introduced into the pure-spinor formalism, by adding some non-minimal sectors [27]. In this formulation the study of tachyon condensation (and more generally the study of the
GSO(\(-\)) sector) can be attempted. Such a formulation could also potentially resolve the problems we faced with the solutions that did not generalize nicely. The non-minimal sector of [26], considered also in the current work, is a different one. It would be interesting to combine the non-minimal sectors of [26] and [27] and to extend the pure-spinor string field theory to this case. One would then face the problem of having both a composite and an elementary \(b\) field. This difficulty could be avoided by declaring that negative powers of \(\lambda\) and \(\bar{\lambda}\) should not be allowed. Such a definition could also resolve those problems with defining the space of string fields that are peculiar to the pure-spinor formalism. The non-minimal sector of [26] would still be useful for obtaining a non-degenerate inner product and for the regularization and definition of a string field theory action. It might also help with obtaining a manifestly Lorentz invariant formulation of the theory. While we feel that such a combined formulation would still be too naive, it might shed some light on the nature of the sought after proper extension of the pure-spinor formalism.

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