Performance Analysis of the Two-Piecewise Linear Companding Technique on Filtered-OFDM Systems

MISHAL AL-GHARABALLY, (Member, IEEE),
ALI F. ALMUTAIRI, (Senior Member, IEEE),
AND APARNA KRISHNA
Electrical Engineering Department, College of Engineering and Petroleum, Kuwait University, Kuwait 13020, Kuwait
Corresponding author: Mishal Al-Gharabally (m.algharabally@ku.edu.kw)

ABSTRACT Filtered orthogonal frequency division multiplexing (F-OFDM) is one of the most prominent waveform candidates when it comes to fifth-generation (5G) wireless communication and beyond. This paper analyzes the performance of the F-OFDM system in terms of the peak-to-average power ratio (PAPR) and bit error rate (BER). The F-OFDM system exhibits a high PAPR, which introduces earnest deterioration in its performance. Consequently, an efficient PAPR-reduction technique has been recommended to reduce the effect of the high value of PAPR of the F-OFDM system. The prospective two-piecewise companding (TPWC) scheme effectively reduces the peak power by analyzing large and small amplitudes individually, so its outcome has both piecewise linear and continuous characteristics. The performances of different PAPR reduction techniques are compared in terms of PAPR complementary cumulative distribution functions and BERs. It is shown that the proposed TPWC transform can significantly reduce PAPR with reduced computational complexity compared with conventional companding techniques.

INDEX TERMS Peak-to-average power ratio (PAPR), companding techniques, Filtered-OFDM, two-piecewise linear companding technique, 5G.

I. INTRODUCTION
Fifth generation (5G) wireless communication systems and beyond are expected to handle innovative applications such as enhanced mobile broadband (eMBB), massive machine-type communications (mMTC), and ultra-reliable, low-latency communications (URLLC) [1], [2]. They should also provide higher data rate transmission and user-friendly resource utilization. Several multicarrier modulation (MCM) techniques have been proposed to handle these applications. Among them, filtered orthogonal frequency division multiplexing (filtered OFDM or F-OFDM) has attracted attention because of its advantages in efficiency and system flexibility [3]–[5]. F-OFDM utilizes a sub-band filtering approach that can be obtained by passing the standard OFDM signal through a spectrum shaping filter. Hence, this technique exploits the advantages of OFDM with cyclic prefix (CP) in terms of robustness to multipath fading and implementation simplicity [6]. The expediency of F-OFDM over other 5G waveform techniques, such as universal filtered multicarrier (UFMC), filter-bank multicarrier (FBMC), generalized frequency division multiplexing (GFDM), and standard OFDM, has made it an outstanding candidate for future communication applications [7]–[11]. F-OFDM and UFMCs are two forms of sub-band filtering multicarrier waveforms that have been developed for 5G wireless communication applications. To avoid inter-symbol interference (ISI), the filter length is limited to the length of the CP in the UFMC. However, the length of the sub-band filter is allowed to outpace the CP length in the F-OFDM system for a better balance between the frequency and time localization [3]. However, if the filter length outreaches the CP length, there will be certain forms of ISI, even for flat channels. When the filter length is shorter, the ISI can be avoided by zero padding without losing too much information. However, the shorter filter impulse response leads to higher out-of-band (OoB) radiation [6], [12]. At the same time, for a longer filter impulse response,
the ISI cannot be completely avoided but can reduce the OoB radiation. Thus, there is a tradeoff between time and frequency localization in a F-OFDM system [6], [12]. Moreover, the implementation of the CP in F-OFDM helps reduce the complexity of channel equalization. F-OFDM depends on the orthogonal frequency division multiple access (OFDMA), which divides the bandwidth into several sub-bands, each of which has sub-carriers. The sub-bands are designated for different users and different services. Then, sub-band filters are used to filter the sub-bands before transmission. Notably, F-OFDM uses much longer filters with tails to enlarge the neighboring symbols, allowing it to retain the same system overhead as the CP-OFDM system [13], [14]. Furthermore, by adjusting the filter attributes and system specifications, the F-OFDM system can contain the requests of respective users and service types. On the other hand, its high PAPR rate is a major shortcoming because it degrades the high power amplifier efficiency.

A variety of techniques are recommended to cut back the large PAPR value in MCM techniques, such as coding, partial transmit sequence (PTS), precoding, clipping, selective mapping, and companding techniques [15]–[20]. The iterative clipping and filtering method is a simple technique that truncates the peak amplitude of the signal to a particular limit. Furthermore, the in-band and out-of-band interferences and the noise introduced because of the clipping technique will result in performance deterioration in the F-OFDM system [21], [22]. To reduce the PAPR value, a simple companding technique was recommended in [23]. The PTS technique is another effective method for improving PAPR reduction performance. However, the benefit of PAPR reduction due to the PTS technique will be at the cost of system complexity [24].

Recently, a number of methods have been proposed in the literature for reducing the PAPR for 5G multicarrier systems [25]–[33]. To enhance the performance of a F-OFDM system, a DFT-precoding technique was evaluated in [25]. The proposed method preserved the spectral efficiency and the BER performance of the F-OFDM system [25]. A comparison of various forms of clipping methods adapted to limit of the PAPR was discussed in [26], [30]. In [30], the authors proposed an amplitude clipping and sub-band filtering approach to reduce the PAPR of a F-OFDM system. They utilized sub-band filters to mitigate the shortcomings of amplitude clipping. This method reduced the PAPR effectively and maintained good BER performance [30]. Another study proposed an efficient Gray-PF-PTS C algorithm to reduce the PAPR in both OFDM and F-OFDM systems with low computational complexity; it proved that the proposed algorithm reduced the computational complexity by 52.06% compared with the PR-PTS method [32]. In [33], PAPR reduction techniques were studied for the mixed-numerology transmissions using iterative clipping and filtering (ICF) and optimization methods. The proposed PAPR reduction methods are also used in F-OFDM and W-OFDM systems.

A. CONTRIBUTIONS

A detailed analysis of a single rate (the baseband sampling rate for entire sub-bands are same) F-OFDM transceiver system and a PAPR-reduction technique based on a two-piecewise companding transform is provided in this paper. The TPWC transform uses a particular scaling factor to linearly transform small signal amplitudes. However, large amplitudes are linearly transformed with a shift that occurs along with the scaling factor. It is inferred from the analysis that an effective trade-off among PAPR performance and BER can be obtained by selecting the scaling parameters carefully. The numerical results exhibit the proposed method’s capability to reduce the PAPR with low computational complexity.

B. PAPER ORGANIZATION

This paper is constructed as follows: An introduction of the standard F-OFDM signal is given in Section II. We first summarize the working of the F-OFDM transmitter model and generation of F-OFDM symbol. Then, we discuss the working of the F-OFDM receiver. Section III briefly explains the characteristics of the PAPR of the F-OFDM system. Section IV explains the proposed TPWC scheme to minimize the PAPR. A brief analysis of companding parameters are also provided in this section. A simulation analysis and comparison of result with other PAPR reduction techniques are laid out in Section V; finally, the conclusions are provided in Section VI.

NOTATIONS

Throughout this work, vectors and matrices are represented by lowercase and uppercase bold letters; and regular small letters are used to represent scalars. Time and the sub-carrier index are represented using \( n \) and \( k \). The companding and decompanding operators are denoted as \( C[.] \), \( C^{-1}[.] \), respectively. The superscripts \( T \), and \( H \) indicate transpose, and Hermitian transpose, respectively. \( E[.]\) and \( E^* \) represents the expectation and convolution operation, respectively, and \( j = \sqrt{-1} \).

II. F-OFDM SYSTEM MODEL

Figure 1 shows the transmitter of an F-OFDM system that divides the entire bandwidth into several sub-bands. Each sub-band of the F-OFDM system is like the CP-OFDM system, generating filtered OFDM signals by passing them through a spectrum-shaping filter. The sub-band filter helps remove the interference from adjacent sub-bands by reducing the out-of-band emission. On the receiver side, a matched filter is used, followed by CP removal and FFT processing. In F-OFDM, the sub-band filtering operation causes performance loss because of the filter ramping up and ramping down at the edges of each sub-band. Thus, the power is unevenly allocated between the sub-carriers in each sub-band [5], [13]. Hence, to compensate for the power allocation among the sub-carriers and achieve optimal performance, a power compensation matrix can be used.
of each sub-band filter should be the same as that of the other window functions [4]. Notably, the center frequency meets the flexibility requirement of the F-OFDM better than [9], [34]. The rooted raised cosine (RRC) window function response, which ensures smooth transitions for both ends [4], in the sub-band is a filter with a rectangular frequency F-OFDM system to have a flat passband over the sub-carriers.

The transmitted symbol can be represented in vector form as: \( \mathbf{d} = [d_1; d_2; \cdots; d_B] \in \mathbb{C}^M \times 1 \), where, \( d_k \) is the signal transmitted in the \( k \)th sub-band represented in (1).

As illustrated in (2), the QAM modulated data symbols of individual sub-bands are then passed to N-point IDFT blocks to obtain time-domain modulated signal that has a length of N samples. However, before doing the IDFT operation, a power compensation procedure can be performed to avoid the slight power imbalance caused due to the sub-band filtering operation at the transmitter and receiver [13].

\[
x_{Bi}(n) = IDFT [X_{Bi}] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{Bi} e^{j2\pi nk/N}
\]  

(2)

Later, the CP of length \( L_{CP} \) will be added to the time-domain signal followed by filter \( f_i \) of length \( L_F \). Thus, the length of the signal after the CP insertion procedure can be represented as \( L_s = N + L_{CP} \). The impulse response of the filter at the k-th sub-band can be written as follows:

\[
f_k = [f_k(0), f_k(1), \cdots, f_k(L_F - 1)]
\]

(3)

where, the sub-band filtering operation matrix is represented by \( F_k \). The ideal low pass filter that can be used in an F-OFDM system to have a flat passband over the sub-carriers in the sub-band is a filter with a rectangular frequency response, which ensures smooth transitions for both ends [4], [9], [34]. The rooted raised cosine (RRC) window function meets the flexibility requirement of the F-OFDM better than other window functions [4]. Notably, the center frequency of each sub-band filter should be the same as that of the corresponding sub-band. Then, the filtered signal of length \( L_{tr} = L_s + L_F - 1 \) is given by (4), and the transmitted vector can be represented as (5).

\[
y_{Bi} = X_{Bi} \ast f_i
\]

(4)

\[
y_{tr,k} = E_k d_k x_k R F_k
\]

(5)

where, \( E_k \in \mathbb{C}^{M_B \times M_B} \) represents the power compensation matrix, whose value depends on the filter frequency response. However, in this analysis no power compensation is applied. Thus, \( E_k = I_{M_B} \), \( R \), \( d_k \), and \( X_k \) represent the CP insertion matrix, the data vector, and N-point IDFT matrix, respectively. Finally, these time-domain output signals from different sub-bands can be added together to generate the F-OFDM base-band signal \( y \), as in (6).

\[
y = \sum_{i=1}^{B} y_{Bi}
\]

(6)

**B. RECEIVER**

Figure 2 shows the structure of the receiver for the \( k \)th sub-band. To recover the desired information, each sub-band filtering of the received signal is performed using a matched filter \( (M_k) \), where \( M_k \) is the matrix form of the receiver filter at the \( k \)th sub-band and its elements are \( m_k = [m_k(0), m_k(1), \cdots, m_k(L_F - 1)] \). Because both the transmitter and receiver use a matched filter, \( M_k = F_k^H \) The resulting signal is then applied to a conventional OFDM receiver, so the filtered signal undergoes CP removal and an N-point DFT operation. Sub-carrier extraction, linear detection, and demodulation are then performed on the frequency domain signal. Thus, the received signal at the \( k \)th sub-band is written as in (7) [35],

\[
r_k = r_{des,k} + r_{ISI,k} + r_{ACL,k} + \eta_k
\]

(7)

where, \( r_{des,k} \) contains the desired signal information as in (8) and the ISI component (\( r_{ISI,k} \)) represents a combination of the previous and next signal. This can be written as in (9),

\[
r_{des,k} = F_k^H T_k^H X_k C H_k z_{tr,k}
\]

(8)

where, \( T \) represents the CP removal operation and \( H_k \) represents the channel impulse response Toeplitz matrix.

\[
r_{ISI,k} = r_{prev,k} + r_{next,k}
\]

(9)

The term \( r_{ACL,k} \) represents the adjacent carrier interference caused by the non-orthogonality between the sub-bands, and \( \eta_k \) represents the white Gaussian noise with zero mean and variance \( \sigma^2 \).

1Note that, we consider only the standard F-OFDM transmitter and receiver blocks for the analysis.
III. PEAK TO AVERAGE POWER RATIO
As explained before, F-OFDM is a multicarrier modulation (MCM) technique with several sub-bands, each comprising many modulated signals in each sub-carrier. Thus, the peak value of the F-OFDM signal can be much higher than the average value. The PAPR of the F-OFDM signal, defined as the maximum peak power of the F-OFDM signal to the mean power, which can be expressed as shown in (10):

\[ \text{PAPR} = \frac{\max(|y(n)|^2)}{E[|y(n)|^2]}, \quad (10) \]

where, \( y(n) \) represents the standard F-OFDM transmitted signal. Furthermore, the transmitter filter in the F-OFDM system increases both the system complexity and the PAPR because the filter situated at the transmitter increases the power distribution among the samples, reducing the average power of the signal. Therefore, the F-OFDM system has a PAPR value higher than that of the OFDM system [32], [36].

The high PAPR increases the analog to digital (A/D) and digital to analog (D/A) converter complexity and OoB radiation. Moreover, it degrades the efficiency of power amplifiers. To increase the efficiency of the F-OFDM system, researchers have proposed many PAPR-reduction techniques. To evaluate efficiency of PAPR reduction technique, the complementary-cumulative-distribution-function (CCDF) is used, which represents the probability of the PAPR value exceeding a certain threshold value (\( \gamma \)). The CCDF of the PAPR for the generalized waveforms for the multicarrier (GWMC) signal can be expressed as follows:

\[ Pr(\text{PAPR} > \gamma) \approx 1 - \prod_{k \in [0,MN-1]} (1 - e^{(-c_k \gamma)}), \quad (11) \]

where,

\[ c_k = \frac{\sum_{n \in \mathbb{Z}} \sum_{m=0}^{M-1} |g_{m,n}[k]|^2}{N \sum_{m=0}^{M-1} |g_{m,n}[k]|^2}, \quad (12) \]

and \( g_{m,n}[k] \) is the waveform filter [25], [37].

In the following section, we introduce a new, efficient TPWC transform to reduce the PAPR of the F-OFDM system, hence enhancing the performance of F-OFDM systems.

IV. PAPR REDUCTION TECHNIQUE
As illustrated in Fig. 1, the proposed PAPR reduction technique is implemented after the sub-band filtering technique. The proposed companding technique operates on the F-OFDM signal after the filter operation introduces non-linear distortion. Therefore, companded signals undergo higher BER and out-of-band interference (OBI) than the original signals. The crucial challenge in designing companding transforms is keeping this distortion within allowable limits. Toward this end, we propose a design criterion for TPWC transform companding parameters: the non-linear distortion should have a smaller effect on the system performance for a given amount of PAPR reduction.

A. TWO-PIECE-WISE COMPANDING TRANSFORM
Assuming the complex-valued F-OFDM input data symbols are statistically independent and identically distributed (i.i.d) with zero mean and variance \( \sigma_x^2 \), based on the central limit theorem, the amplitude of the F-OFDM samples have a Rayleigh distribution with the probability density function (PDF), as shown in (13).

\[ f(x) = \frac{2x}{\sigma_x^2} e^\left(-\frac{x^2}{\sigma_x^2}\right) \quad (13) \]

Based on the probability distribution function of the F-OFDM signal, small amplitudes clearly occur with high probability and large amplitudes with low probability. Thus, by preserving the average power and compressing large amplitudes, we reduce the PAPR. Moreover, expanding smaller amplitudes significantly affects BER performance. Thus, both small and large amplitudes of the F-OFDM signal must be analyzed independently for a suitable balance between BER and PAPR reduction characteristics, so we introduce the TPWC transform to treat the small and large amplitudes individually [38]. The companding transform of the TPWC is illustrated in Fig. 3, which shows piece-wise linear and continuous characteristics [38], [39].

![Figure 3. Transfer characteristics of the TPWC transform.](image)
where, \( u_1 > 1 \) is the first slope, \( 1 > u_2 > 0 \) is the second slope, and \( S = \nu(u_1 - u_2) \) is the intercept of \( u_2 \) slope with the Y-axis and \( \nu = m\sigma_\nu \) is the cut-off point of the amplitude, where \( m > 0 \).

**B. DESIGN OF PROPOSED TPWC PARAMETERS**

The most important feature of a TPWC transform is that the transform parameters and can be represented as in (15). That means,

\[
\sigma_x^2 = \int_0^\infty x^2f(x)dx - \int_0^\infty C^2[x]f(x)dx \tag{14}
\]

Equation (14) can be modified by substituting the TPWC transform parameters and can be represented as in (15).

\[
\int_0^\infty C^2[x]f(x)dx = \int_0^\nu u_1x^2f(x)dx + \int_\nu^\infty (u_2x + S)^2f(x)dx = \sigma_x^2. \tag{15}
\]

where, \( f(x) \) is the PDF, as shown in (13).

Equation (15) can be expanded as in (16).

\[
\sigma_x^2 = u_1^2 \int_0^\nu x^2f(x)dx + \int_\nu^\infty (u_2x)^2f(x)dx + S^2 + 2u_2S \int_\nu^\infty xf(x)dx \tag{16}
\]

To simplify (16), we assume \( \sigma_x^2 = I \).

That is,

\[
I = u_1^2 \int_0^\nu x^2f(x)dx + u_2^2 \int_\nu^\infty x^2f(x)dx + S^2 \int_\nu^\infty f(x)dx + 2u_2S \int_\nu^\infty xf(x)dx \tag{17}
\]

Then, we represent (17) as in (18),

\[
I = u_1^2I_1 + u_2^2I_2 + S^2I_3 + 2u_2SI_4 \tag{18}
\]

where,

\[
I_1 = \int_0^\nu x^2f(x)dx \tag{19}
\]

We substitute the value of \( f(x) \) from (13) into (19), and this can be expressed as:

\[
I_1 = \int_0^\nu x^2 \frac{2x}{\sigma_x^2} \exp \left( -\frac{x^2}{\sigma_x^2} \right) dx. \tag{20}
\]

Equation (20) can be solved using integration by the parts method, and the solution can be represented as in (22).

\[
I_1 = \sigma_x^2 \left[ -e^{-x^2/\sigma_x^2} \left( \frac{x^2}{\sigma_x^2} \right) - e^{-x^2/\sigma_x^2} \right] + 1 \tag{21}
\]

Later, we can substitute the value of \( \nu = m\sigma_\nu \) into (22), and we get,

\[
I_1 = \sigma_x^2 \left[ 1 - (m^2 + 1)e^{-m^2} \right]. \tag{22}
\]

Similarly,

\[
I_2 = \int_\nu^\infty x^2f(x)dx \tag{23}
\]

Again, we can substitute the value of \( f(x) \) from (13) into (23) and solve it by using the integration by parts method. Then, we can substitute the value of \( \nu \), and we get the solution of \( I_2 \), as in (24).

\[
I_2 = \nu^2e^{-\nu^2/\sigma_\nu^2} + \sigma_\nu^2e^{-\nu^2/\sigma_\nu^2} = \sigma_x^2(m^2 + 1)e^{-m^2} \tag{24}
\]

We repeat the same procedure for \( I_3 \) and \( I_4 \) and get solutions as in (27) and (26).

\[
I_3 = \int_0^\infty f(x)dx = e^{-\sigma_\nu^2/2} = \left[ e^{-m^2/2} \right], \tag{25}
\]

\[
I_4 = \int_0^\infty xf(x)dx = \frac{\sqrt{\pi}\sigma_\nu - \sqrt{\pi}\sigma_\nu\text{erf} \left( \frac{\sigma_\nu}{\nu} \right)}{2} + \nu e^{-\frac{\nu^2}{2}} \tag{26}
\]

where, \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}dy \) is the error function.

We can substitute (20)-(26) into (18). Then,

\[
I = u_1^2 \left[ \sigma_x^2 \left[ 1 - (m^2 + 1)e^{-m^2} \right] \right] + u_2^2 \left[ \sigma_x^2 \left[ (m^2 + 1)e^{-m^2} \right] \right] + (u_1 - u_2)m\sigma_x^2 \left[ e^{-m^2} \right] + 2u_2(u_1 - u_2)m\sigma_x^2 \sigma_x^2 \left[ \frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2}\text{erf}(m) + me^{-m^2} \right] = \sigma_x^2. \tag{27}
\]

Later, we can expand and simplify (27) to get (28).

\[
u_1^2 - (u_2^2)e^{-m^2} + (u_2^2)e^{-m^2} + m(1 - \text{erf}(m))\sqrt{\pi} = 1 \tag{28}
\]

\[
u_1^2 - u_2^2(1 - e^{-m^2}) + u_2^2 + u_2u_1m(1 - \text{erf}(m))\sqrt{\pi} - u_2^2(m(1 - \text{erf}(m))\sqrt{\pi}) = 1 \tag{29}
\]

Then, we substitute \( 1 - e^{-m^2} = A \) and \( m(1 - \text{erf}(m))\sqrt{\pi} = B \). Finally, the exact equation for the TPWC can be given as in (30) [39].

\[
u_1^2 - u_2^2A + u_2^2 + u_2u_1B - u_2^2B = 1 \tag{30}
\]

As mentioned in [38], when \( m > 1.2, \text{erf}(m) \approx 1 \). Thus, (30) can be further simplified into (31).

\[
u_1^2A + (u_2^2)e^{-m^2} = 1 \tag{31}
\]

It is clear from (31) that the value of \( \nu_1 \) can be evaluated based on the values of \( u_2^2 \) and \( m \) and that there can be several values of the companding parameters to satisfy the equation. It is also important to mention that the PAPR and BER characteristics depend on the combination of the values of \( u_2 \) and \( m \). Thus, we assume the values of \( u_2 \) and \( m \); then the value of \( \nu_1 \) will be evaluated based on that. Optimization can
be done to select the optimum value for $u_2$ and $m$ such that it provides better PAPR reduction along with good BER characteristics. A detailed analysis of effect of these parameters on the F-OFDM characteristics are available in V-A1.

V. RESULTS AND DISCUSSIONS
To evaluate the performance of the F-OFDM transceiver and the proposed TPWC technique, we conduct MATLAB simulations. The signal is modulated using the M-QAM modulation technique. A matched filter (root raised cosine with sinc windowing) is used at the transmitter and receiver for the analysis. To compare the performance of the conventional F-OFDM system and the proposed technique, we exploit the CCDF of the PAPR and BER vs. $E_b/N_0$ performance. We utilize the 3GPP LTE-10 MHz radio frame structure with 1024 sub-carriers, a sub-carrier spacing of 15 KHz, and 600 occupied sub-carriers divided into sub-bands of 12 sub-carriers each. Table 1 summarizes the F-OFDM parameters derived from the LTE system parameters. We assume an AWGN channel in all the simulations and compare the results with OFDM and UFMC to evaluate the advantages of F-OFDM over them.

TABLE 1. Simulation parameters of the F-OFDM system.

| Parameter                  | Value |
|---------------------------|-------|
| Size of PFT (N)           | 1024  |
| Total number of sub-carriers ($M$) | 600   |
| Modulation method (g)     | M-QAM (4 QAM, 16 QAM) |
| Number of sub-bands ($B$) | 50    |
| The number of sub-carriers in each sub-band ($M_B$) | 12    |
| Filter                    | Root Raised Cosine |
| CP length                 | 72    |
| Filter length             | 513   |
| Channel                   | AWGN  |

A. PERFORMANCE ANALYSIS OF THE F-OFDM SYSTEM
This section analyzes the performance of the F-OFDM system and compares it with the OFDM and UFMC systems. Figure 4 compares the power spectral density (PSD) of the F-OFDM signal with the standard OFDM and UFMC systems. In the figure, F-OFDM clearly has lower side lobes than OFDM and UFMC, so it exhibits better frequency localization and increased spectral efficiency than its counterparts.

1) PEAK TO AVERAGE POWER RATIO
As mentioned in the previous section, the dependency of TPWC parameters on the PAPR is analyzed using simulation software. We evaluate different sets of parameters and analyze the effect of these parameters on F-OFDM characteristics. From Fig. 5 it is evident that for a particular $m$ value, the PAPR characteristics degrade as $u_2$ increases. Furthermore, for a particular $u_2$ value, the PAPR characteristics deteriorate as $m$ increases. Similarly, Fig. 6 shows the relation of $u_2$ and $m$ on BER characteristics. It can be inferred from the figure that as $u_2$ or $m$ increase, the system provides better BER performance. Thus, to have better agreement of PAPR reduction performance and BER characteristics, the value of

$u_2$ and $m$ must be selected carefully. We evaluated different sets of TPWC parameters and analyzed their effect on F-OFDM system. Table 2 shows the sets of parameters that give better performance.

Figure 7 shows the CCDF characteristics of the F-OFDM system using the proposed TPWC technique for 4-QAM modulation. The figure compares the proposed TPWC technique with the original OFDM, F-OFDM, and UFMC systems. The simulation results show that the PAPR of the
TABLE 2. Parameters of the TPWC.

| m     | U₂ | U₁   |
|-------|----|------|
| 1.2   | 0.1, 0.13, 0.2 | 1.1434, 1.1425, 1.1393 |
| 2     | 0.1, 0.15, 0.2  | 1.0092, 1.0091, 1.0089 |

F-OFDM system is higher than the OFDM and UFMC systems, but the proposed TPWC technique reduces its PAPR considerably. In Fig. 7, when \( m = 1.2 \), the PAPR clearly decreases to 5.5 dB when CCDF \( = 10^{-3} \). Furthermore, when \( m = 2 \), the PAPR decreases to 8 dB at CCDF \( = 10^{-3} \). The CCDF plot of the proposed TPWC technique for 16-QAM modulation is shown in Fig. 8. From the analysis, it is clear that the TPWC technique significantly reduces the PAPR but deteriorates the BER performance. When compared with the standard-F-OFDM system, the TPWC-F-OFDM with the compression parameter \( m = 1.2 \), 2 achieves PAPR gains of just about 7 dB and 5 dB, respectively, at CCDF \( = 10^{-3} \) for 16-QAM modulation.

Figure 9 shows the comparison of the CCDFs of the PAPRs of a standard F-OFDM signal with that of the TPWC signal, and two non-linear companding techniques (NLCT), such as with exponential companding (EXP) and tangent rooting companding techniques (tanhR). It can be easily found that both the tanhR with \( R = 0.5 \) and the TPWC technique with \( m = 2 \) can reduce the corresponding PAPR almost by 5 dB at CCDF \( = 10^{-3} \). Meanwhile, both the exponential transform (EXP) and the TPWC technique \( (m = 1.2) \) can also obtain almost 7 dB PAPR reduction. Here, the effectiveness of the TPWC transform in PAPR reduction has been explained. However, NLCT has the best PAPR reduction performance compared with a linear technique because of its non-linear characteristics.

2) FILTER LENGTH AND PAPR

As mentioned before, the length of the sub-band filter is allowed to outpace the CP length in the F-OFDM system to obtain a better balance between the frequency and time localization [3]. However, the order of the filter increase the system complexity. Hence, the F-OFDM filter length should be restricted to an explicit order.

Figure 10 illustrates a comparison of the PAPR of the F-OFDM system with different filter lengths. It is clear from the figure that the length of the filter affects the PAPR characteristics in the F-OFDM system. As the filter order increases, the PAPR also increases. Consequently, it is also clear from the simulation result that the PAPR of the F-OFDM system is higher than that of the OFDM and UFMC systems. Both the UFMC and F-OFDM systems use a sub-band filtering procedure to achieve a low OoB emission. However, the main contrast between UFMC and F-OFDM lies in the length of the filter and its flexibility [3], [7]. A F-OFDM system uses a long filter with the filter length exceeding the CP, while UFMC signals use short fixed length FIR filters for each sub-band. The transmitter filter is the major reason for the increment in the PAPR of the F-OFDM system. This is because the filter causes the power distribution among the samples to be wider than that in the CP-OFDM system, which results in a decrease in the mean signal power, thereby causing degradation of the PAPR performance [7], [12]. Hence, CP-OFDM will
exhibit the best PAPR performance when compared with the F-OFDM and UFMC systems [36], [40], [41].

3) BIT ERROR RATE
The Bit error rate of the F-OFDM system has been analyzed over an AWGN channel. As mentioned in Section II, the signal progresses through AWGN channel to reach the receiver. Then, (32) shows the theoretical representation of the received signal.

\[ r_k = C^{-1}[y_k + \eta_k] = C^{-1}\left[\sum_{i=1}^{B} y_{Bi} + \eta_k\right] \]  

(32)

The analytical expression for the BER of the standard OFDM using M-array QAM modulation in AWGN is represented as shown in (33) by [42], [43],

\[ B_e = \frac{2(\sqrt{M} - 1)\sqrt{M\log_2\sqrt{M}}}{\sqrt{M\log_2\sqrt{M}}\sqrt{N_0}} Q\left(\frac{6E_b \log_2\sqrt{M}}{N_0} \right) \]  

(33)

where, \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \) is known as the Q-function and \( E_b/N_0 \) is the signal to-noise ratio.

As mentioned before, the F-OFDM system is related to a conventional OFDM system, contradicting only in the inclusion of filters. F-OFDM systems are based on sub-band filtering operation, and hence, at the receiver, individual sub-band must execute the inverse filtering operation. Accordingly, the variance of the noise on the \( k^{th} \) sub-carrier in the \( n^{th} \) sub-band is \( \sigma^2 = \sigma_k^2/F_k^2 \), where \( F_k \) specifies the corresponding filter response and \( \sigma_k \) represents the variance of the Gaussian white noise. Thus, the BER of a conventional F-OFDM system can be written as:

\[ S_F = \frac{1}{KB} \sum_{i=1}^{B} \sum_{k=1}^{K} \frac{2(\sqrt{M} - 1)\sqrt{M\log_2\sqrt{M}}}{\sqrt{M\log_2\sqrt{M}}\sqrt{N_0}} Q\left(\frac{6E_b \log_2\sqrt{M}}{N_0} \right) \times \left(\frac{6E_b}{N_0}\sqrt{\frac{|M_k|^2 \log_2\sqrt{M}}{(M+1)}}\right) \]  

(34)

From (34), it is clear that the BER of the F-OFDM system is mainly influenced by the filter response. Hence, to improve the performance of F-OFDM, the noise amplification generated due to the filtering operation needs to be minimized.

Figure 11 compares the BER performance of the proposed TPWC F-OFDM system with the standard F-OFDM, UFMC, and OFDM systems for the 4-QAM modulation technique over AWGN channel. For the sake of simplicity, we use only a single sub-band for the receiver analysis. In the figure, as \( m \) increases, the F-OFDM system clearly achieves better BER performance. Figure 12 shows the comparison of the BER performance of the proposed TPWC F-OFDM system with a standard F-OFDM for the 16-QAM modulation technique. From both figures, it is clear that the TPWC technique declines the BER characteristics as the PAPR is reduced. Furthermore, from the simulation results, it is evident that the modulation index has an effect on BER performance as QPSK modulation provides improved BER characteristics compared with a 16-QAM.

Comparison of the BER of the proposed TPWC F-OFDM system with NLCTs is presented in Fig. 13. The analysis
shows that, NLCT affects the BER characteristics of the F-OFDM system compared with the TPWC technique. However, it is inferred from the analysis that the proposed TPWC technique with $m = 2$ not only has good PAPR minimization performance with lower computational complexity, but also contains good BER performance among all the other NLCTs and the conventional F-OFDM method.

B. DISCUSSION

It can be seen from the analysis that, the proposed TPWC technique has been impressive in reducing the PAPR for F-OFDM. Furthermore, the BER, which is analyzed under an AWGN channel, also indicates very good performance. However, when compared with the conventional F-OFDM system, the PAPR is reduced significantly by an amount of $6$ dB at CCDF $= 10^{-3}$ for $m = 1.2$. On the other hand, the BER characteristics deteriorates by about $7.2$ dB compared with a conventional-F-OFDM. Hence, we can observe from the analysis that, at a fixed value of $u_2 = 0.2$ and $m = 2$, the proposed TPWC technique attains improved PAPR and BER characteristics. From Fig. 13, it is clear that when $u_2 = 0.2$ and $m = 2$, the PAPR is reduced by a significant amount of $4.6$ dB, while BER at $10^{-3}$ deteriorates by only $0.2$ dB compared with a conventional F-OFDM. Better BER performance can thus be obtained when the value of either the parameters $m$ or $u_2$ increases. Adjusting these two parameters carefully yields an effective solution for a trade-off between PAPR performance and BER characteristics.

VI. CONCLUSION

In this article, a simple TPWC technique has been implemented to depreciate the PAPR of an F-OFDM system. A comparative analysis of non-linear companding techniques and the proposed technique shows that the latter reduces the PAPR compared with a normal F-OFDM system. Based on the simulation analysis it is clear that companding distortion affects the BER performance. However, by properly designing the companding parameters, the proposed TPWC technique can reduce the companding distortions and can improves the BER performance compared with other NLCTs. The simulation results indicate that the TPWC technique can reduce the PAPR effectively with lower computational complexity while providing a better BER performance. In future work, we will investigate characteristics of an F-OFDM system on a flat fading channel. In addition, a modified and straightforward PAPR reduction technique will be analyzed to acquire an improved bit error rate while reducing the PAPR effectively on this channel.

REFERENCES

[1] I. F. Akyildiz, S. Nie, S.-C. Lin, and M. Chandrasekaran, “5G roadmap: 10 key enabling technologies,” Comput. Netw., vol. 106, pp. 17–48, Sep. 2016.
[2] IMT Vision—Framework and Overall Objectives of the Future Development of IMT for 2020 and Beyond, Standard I-R. Recommendation ITU-R M. 2083-0, 2015.
[3] J. Abdoli, M. Jia, and J. Ma, “Filtered OFDM: A new waveform for future wireless systems,” in Proc. IEEE 16th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC), Jun. 2015, pp. 66–70.
[4] X. Zhang, M. Jia, L. Chen, J. Ma, and J. Qiu, “Filtered-OFDM—enabler for flexible waveform in the 5th generation cellular networks,” in Proc. IEEE Global Commun. Conf. (GLOBECOM), Dec. 2014, pp. 1–6.
[5] L. Zhang, A. Ijaz, P. Xiao, and R. Tafazolli, “Multi-service system: An enabler of flexible 5G air interface,” IEEE Commun. Mag., vol. 55, no. 10, pp. 152–159, Oct. 2017.
[6] J. Li, E. Bala, and R. Yang, “Resource block filtered-OFDM for future spectrally agile and power efficient systems,” Phys. Commun., vol. 11, pp. 36–55, Jun. 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S187449071300061X
[7] F. Schaich and T. Wild, “Waveform contenders for 5G—OFDM vs. FBMC vs. UFMC,” in Proc. 6th Int. Symp. Commun., Control Signal Process. (ISCSP), May 2014, pp. 457–460.
[8] P. Banelli, S. Buzzi, G. Colavolpe, A. Modenini, F. Rusek, and A. Ugolini, “Modulation formats and waveforms for 5G networks: Who will be the heir of OFDM?: An overview of alternative modulation schemes for improved spectral efficiency,” IEEE Signal Process. Mag., vol. 31, no. 6, pp. 80–93, Nov. 2014.
[9] Y. Liu, X. Chen, Z. Zhong, B. Ai, D. Miao, Z. Zhao, J. Sun, Y. Teng, and H. Guan, “Waveform design for 5G networks: Analysis and comparison,” IEEE Access, vol. 5, pp. 19282–19292, 2017.
[10] Y. Cai, Z. Qin, F. Cui, G. Ye Li, and J. A. McCann, “Modulation and multiple access for 5G networks,” 2017, arXiv:1702.07673. [Online]. Available: http://arxiv.org/abs/1702.07673
[11] R. Guerraguet, N. Bartoswilk, L. C. Bai, Y. Berg, J.-B. Doré, D. Kičans, O. Font-Bach, X. Mestre, M. Payaró, M. Färber, and K. Roth, “The 5G candidate waveform race: A comparison of complexity and performance,” EURASIP J. Wireless Commun. Netw., vol. 2017, no. 1, p. 13, Jan. 2017.
[12] P. Wittmer, J. Bazzi, K. Kusume, A. Benjebbour, and Y. Kishiyama, “Adaptive filtered OFDM with regular resource grid,” in Proc. IEEE Int. Conf. Commun. Workshops (ICC), May 2016, pp. 462–467.
[13] L. Zhang, A. Ijaz, P. Xiao, M. M. Mola, and R. Tafazolli, “Filtered OFDM systems, algorithms, and performance analysis for 5G and beyond,” IEEE Trans. Commun., vol. 66, no. 3, pp. 1205–1218, Mar. 2018.
[14] W. Zhang, J. Chen, M. Cao, and J. Dong, “Subband frequency-domain filtering for filtered OFDM,” in Proc. 11th Int. Conf. Graph. Image Process. (ICGIP), vol. 11373, Jan. 2020, Art. no. 113732.
[15] R. van Nee and A. D. Wild, “Reducing the peak-to-average power ratio of OFDM,” in Proc. IEEE Veh. Technol. Conf. (VTC), May 1998, pp. 2072–2076.
[16] S. A. Aburakhia, E. F. Badran, and D. A. E. Mohamed, “Linear companding transform for the reduction of peak-to-average power ratio of OFDM signals,” IEEE Trans. Broadcast., vol. 55, no. 1, pp. 155–160, Mar. 2009.
[17] I. Baig and V. Jeciti, “PAPR analysis of DHT-precoded OFDM system for M-QAM,” in Proc. Int. Conf. Intell. Serv., Jun. 2010, pp. 1–4.
[18] V. Tabatabavakili and A. Zahedi, “Reduction in peak to average power ratio of OFDM signals using a new continuous linear companding transform,” in Proc. 5th Int. Symp. Telecommun., Dec. 2010, pp. 426–430.
[19] J. Mountassir, A. Isar, and T. Mountassir, “Precoding techniques in OFDM systems for PAPR reduction,” in Proc. 16th IEEE Medit. Electrotech. Conf., Mar. 2012, pp. 728–731.

[20] W. Wang, M. Hu, Y. Li, and H. Zhang, “A low-complexity tone injection scheme based on distortion signals for PAPR reduction in OFDM systems,” IEEE Trans. Broadcast., vol. 62, no. 4, pp. 948–956, Dec. 2016.

[21] X. Li and L. J. Cimini, “Effects of clipping and filtering on the performance of OFDM,” in Proc. IEEE 47th Veh. Technol. Conf. Technol. Motion, vol. 3, May 1997, pp. 1634–1638.

[22] L. Yao, J. He, and X. Xu, “Analysis and comparison of two clipping methods in PAPR reduction for OFDM system,” in Proc. 5th Int. Conf. Biomed. Eng. Informat., Oct. 2012, pp. 1435–1438.

[23] X. Wang, T. T. Tjhung, and C. S. Ng, “Reduction of peak-to-average power ratio of OFDM system using a companding technique,” IEEE Trans. Broadcast., vol. 45, no. 3, pp. 303–307, Sep. 1999.

[24] Y. J. Jawhar, L. Audah, M. A. Taher, K. N. Ramli, N. S. M. Shah, M. Musa, and M. S. Ahmed, “A review of partial transmit sequence PAPR reduction in the OFDM systems,” IEEE Access, vol. 7, pp. 18021–18041, 2019.

[25] M. B. Mabrouk, M. Chafii, Y. Louet, and F. Bader, “A precoding-based PAPR reduction technique for UF-OFDM and filtered-OFDM modulations in 5G systems,” in Proc. 25th Eur. Wireless Conf., May 2017, pp. 1–6.

[26] M. N. Tápin, J. Cáceres, M. N. Jiménez, I. N. Cano, and G. Arévalo, “Comparison of clipping techniques for PAPR reduction in UFMC systems,” in Proc. IEEE 9th Latin-Am. Conf. Commun. (LATINCOM), Nov. 2017, pp. 1–4.

[27] W. Rong, J. Cai, and X. Yu, “Low-complexity PTS PAPR reduction scheme for UFMC systems,” Cluster Comput., vol. 20, no. 4, pp. 3427–3440, Dec. 2017.

[28] S. K. Bandari, V. M. Vakamulla, and A. Drosopoulos, “Novel hybrid PAPR reduction schemes for the MGFDM system,” Phys. Commun., vol. 31, pp. 69–78, Dec. 2018.

[29] I. Shaheen, A. Zekry, F. Newagy, and R. Ibrahim, “PAPR reduction for FBMC/OQAM using hybrid scheme of different precoding transform and mu-law companding,” Int. J. Eng. Technol., vol. 6, no. 4, p. 154, Nov. 2017.

[30] C. Wang and Y. Bai, “PAPR reduction with amplitude clipping and subband filter in filtered-OFDM system,” in 5G for Future Wireless Networks (Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering), vol. 211, K. Long, V. Leung, H. Zhang, Z. Feng, Y. Li, and Z. Zhang, Eds. Cham, Switzerland: Springer, 2018, doi: 10.1007/978-3-319-72823-0_21.

[31] A. F. Almutairi, M. Al-Gharaibally, and A. Krishna, “Performance analysis of hybrid peak to average power ratio reduction techniques in 5G UFMC systems,” IEEE Access, vol. 7, pp. 80651–80660, 2019.

[32] Y. A. Al-Jawhar, K. N. Ramli, A. Mustapha, S. A. Mostafa, N. S. M. Shah, and M. A. Taher, “Reducing PAPR with low complexity for 4G and 5G waveform designs,” IEEE Access, vol. 7, pp. 97673–97688, 2019.

[33] X. Liu, X. Zhang, L. Zhang, P. Xiao, J. Wei, H. Zhang, and V. C. M. Leung, “PAPR reduction using iterative clipping/filtering and ADMM approaches for OFDM-based mixed-numerology systems,” IEEE Trans. Wireless Commun., vol. 19, no. 4, pp. 2586–2600, Apr. 2020.

[34] J. Wang, A. Jin, D. Shi, L. Wang, H. Shen, D. Wu, L. Hu, L. Gu, L. Lu, Y. Chen, J. Wang, Y. Saito, A. Benjebbour, and Y. Kishiyma, “Spectral efficiency improvement with 5G technologies: Results from field tests,” IEEE J. Sel. Areas Commun., vol. 35, no. 8, pp. 1867–1875, Aug. 2017.

[35] A. Baghaki and B. Champagnie, “Channel estimation for filtered OFDM transceiver systems,” in Proc. 53rd Asilomar Conf. Signals, Syst., Comput., Nov. 2019, pp. 2132–2138.

[36] F. A. P. D. Figueredo, N. F. T. Aniceto, J. Seki, I. Moerman, and G. Fraidenreich, “Comparing f-OFDM and OFDM performance for MIMO systems considering a 5G scenario,” in Proc. IEEE 2nd 5G World Forum (5GWF), Sep. 2019, pp. 532–535.

[37] M. Chafii, J. Palicot, and R. Gribonval, “Closed-form approximations of the peak-to-average power ratio distribution for multi-carrier modulation and their applications,” EURASIP J. Adv. Signal Process., vol. 2014, no. 1, pp. 1–13, Dec. 2014.

[38] P. Yang and A. Hu, “Two-piecewise companding transform for PAPR reduction of OFDM signals,” in Proc. 7th Int. Wireless Commun. Mobile Comput. Conf., Jul. 2011, pp. 619–623.

[39] H. A. Patel and D. J. Shah, “Cellular range extension by optimized two piecewise companding technique for PAPR reduction with M-ary modulation in OFDM system,” Int. J. Eng. Technol., vol. 7, no. 4, pp. 5736–5740, 2019.

[40] Y. A. Al Jawhar, K. N. Ramli, M. A. Taher, N. S. Shah, S. A. Mostafa, and B. A. Khalaf, “Improving PAPR performance of filtered OFDM for 5G communications using PTS,” ETRI J., Oct. 2020.

[41] Y. Qiu, Z. Liu, and D. Qu, “Filtered bank based implementation for filtered OFDM,” in Proc. 7th IEEE Int. Conf. Electron. Inf. Emergency Commun. (ICEIEC), Jul. 2017, pp. 15–18.

[42] S. Wei, H. Li, W. Zhang, and W. Cheng, “A comprehensive performance evaluation of universal filtered multi-carrier technique,” IEEE Access, vol. 7, pp. 81429–81440, 2019.

[43] M. Agrawal and Y. Raut, “BER analysis of MIMO OFDM system for AWGN & Rayleigh fading channel,” Int. J. Comput. Appl., vol. 34, no. 9, pp. 33–37, 2011.

MISHAL AL-GHARABALLY (Member, IEEE) was born in Kuwait, in 1976. He received the B.S. degree in electrical engineering from Kuwait University, in 1998, and the M.S. and Ph.D. degrees in electrical engineering from the University of California, San Diego, in 2004 and 2007, respectively. Since 2007, she has been an Assistant Professor with the Electrical Engineering Department, Kuwait University. He is currently serving as the Vice Dean of admission and registration. His research interests include communications theory, wireless communications, signal processing, and channel coding.

ALI F. ALMUTAIRI (Senior Member, IEEE) received the B.S. degree in electrical engineering from the University of South Florida, Tampa, FL, USA, in 1993, and the M.S. and Ph.D. degrees in electrical engineering from the University of Florida, Gainesville, FL, in 1995 and 2000, respectively. He served as the Dean of admission and registration with Kuwait University, from 2018 to 2020. He served as the Vice Dean of academic affairs with the College of Engineering and Petroleum, Kuwait University, from 2016 to 2018. He served as the Chairperson for the Electrical Engineering Department, Kuwait University, from 2007 to 2011, and as the Graduate Program Director for the Electrical Engineering Department, Kuwait University, from 2015 to 2016. He is currently a Professor with the Electrical Engineering Department. His current research interests include multiuser detection, wireless networks, antenna design, and current and future cellular networks performance issues. He is a member of other professional societies. In 1993, he received a Full Scholarship from Kuwait University to pursue his graduate studies. He served as an associate editor and a reviewer for many technical publications.

APARNA KRISHNA received the B.Tech. degree in electronics and communication engineering from Calicut University, Kerala, India, in 2006, the M.E. degree in communication systems from Anna University, Chennai, India, in 2008, and the Ph.D. degree in electrical engineering from Qatar University, Doha, Qatar, in 2018. She has been working as a Lecturer with SBCE, Kerala, since 2009. From 2011 to 2013, she worked as a Research Assistant with the Department of Electrical Engineering, Qatar University, where she worked on the application of statistical electromagnetics on harsh environments. From 2013 to 2016, she was a Teaching Assistant with the Electrical Engineering Department, Qatar University. She is currently working as a Research Associate with the Electrical Engineering Department, Kuwait University, Kuwait. Her research interests include the analysis and design of microstrip antennas, statistical electromagnetics, and broad area of 5G wireless communication systems.

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