What Bandwidth Do I Need for My Image?

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ABSTRACT. Computer representations of real numbers are necessarily discrete, with some finite resolution, discreteness, quantization, or minimum representable difference. We perform astrometric and photometric measurements on stars and co-add multiple observations of faint sources to demonstrate that essentially all of the scientific information in an optical astronomical image can be preserved or transmitted when the minimum representable difference is a factor of 2 finer than the root variance of the per-pixel noise. Adopting a representation this coarse reduces bandwidth for data acquisition, transmission, or storage, or permits better use of the system dynamic range, without sacrificing any information for downstream data analysis, including information on sources fainter than the minimum representable difference itself.

1. INTRODUCTION

Computers operate on bits and collections of bits; the numbers stored by a computer are necessarily discrete; finite in both range and resolution. Computer-mediated measurements or quantitative observations of the world are therefore only approximately real-valued. This means that choices must be made, in the design of a computer instrument or a computational representation of data, about the range and resolution of represented numbers.

In astronomy this limitation is keenly felt at the present day in optical imaging systems, where the analog-to-digital conversion of CCD or equivalent detector read-out happens in real time and is severely limited in bandwidth; often there are only 8 bits readout pixel−1. This is even more constrained in space missions, where it is not just the bandwidth of real-time electronics but the bandwidth of telemetry of data from space to ground that is limited. If the “gain” of the system is set too far in one direction, too much of the dynamic range is spent on noise, and bright sources saturate the representation too frequently. If the gain is set too far in the other direction, information is lost about faint sources.

Fortunately, the information content of any astronomical image is limited naturally by the fact that the image contains noise. That is, tiny differences between pixel values—differences much smaller than the amplitude of any additive noise—do not carry very much astronomical information. For this reason, the discreteness of computer representations of pixel values do not have to limit the scientific information content in a computer-recorded image. All that is required is that the noise in the image be resolved by the representation. What this means, quantitatively, for the design of imaging systems is the subject of this article; we are asking this question: What bandwidth is required to deliver the scientific information content of a computer-recorded image?

This question has been asked before, using information theory, in the context of telemetry (Gaztañaga et al. 2001) or image compression (Watson 2002), treating the pixels (or linear combinations of them) as independent. Here we ask this question, in some sense, experimentally, and for the properties of imaging on which optical astronomy depends, where groups of contiguous pixels are used in concert to detect and centroid faint sources. We perform experiments with artificial data, varying the bandwidth of the representation—the size of the smallest representable difference Δ in pixel values—and measuring properties of scientific interest in the image. We go beyond previous experiments of this kind (White & Greenfield 1999; Pence et al. 2010) by measuring the centroids and brightnesses of compact sources, and sources fainter than the detection limit. The higher the bandwidth, the better these measurements become, in precision and in accuracy. We find, in agreement with previous experiments and information-theory–based results, that the smallest representable difference Δ should be on the order of the root variance sigma (σ) of the noise in the image. More specifically, we find that the minimum representable difference should be about half the per-pixel noise σ, or that about 2 bits should span the FWHM of the noise distribution if the computer representation is to deliver the information content of the image.

Of course, tiny mean differences in pixel values, even differences much smaller than the noise amplitude, do contain extremely valuable information, as is clear when many short exposures (for example) of one patch of the sky are co-added or analyzed simultaneously. “Blank” or noise-dominated parts of the individual images become signal-dominated in the co-added image. In what follows, we explicitly include this

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“below-the-noise” information as part of the information content of the image. Perhaps surprisingly, all of the information can be preserved, even about sources fainter than the discreteness of the computer representation, provided that the discreteness is finer than the amplitude of the noise. This result has important implications for image compression, but our main interest here is in the design and configuration of systems that efficiently take or store raw data, using as much of the necessarily limited dynamic range on signal as possible.

Our results have some relationship to the study of stochastic resonance, where it has been shown that signals of low dynamic range can be better detected in the presence of noise than in the absence of noise (see Gammaitoni et al. 1998 for a review). These studies show that if a signal is below the minimum representable difference $\Delta$, it is visible in the data only when the digitization of the signal is noisy. A crude summary of this literature is that the optimal noise amplitude is comparable to the minimum representable difference $\Delta$. We turn the stochastic resonance problem on its head: The counterintuitive result (in the stochastic resonance context) that weak signals become detectable only when the digitization is noisy becomes the relatively obvious result (in our context) that so long as the minimum representable difference $\Delta$ is comparable to or smaller than the noise, signals are transmitted at the maximum fidelity possible in the data set.

What we call here the “minimum representable difference” has also been called by other authors the “discretization” (Gaztañaga et al. 2001) or “quantization” (Watson 2002; White & Greenfield 1999; Pence et al. 2009).

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**Fig. 1.**—Starting at top left and moving to bottom right, we show $16 \times 16$ images of increasing bit depth. The original images are identical but snapped to integer as described in the text. The images are labeled by the ratio $|\sigma/\Delta|$ of noise root variance $\sigma$ to the minimum representable difference $\Delta$. At ratios $|\sigma/\Delta| > 2^6$, the images become virtually indistinguishable from the high-bandwidth images.

**Fig. 2.**—Measurement of image noise variance as a function of bit depth or minimum representable difference $|\Delta/\sigma|$ for images with a randomly chosen mean level and gaussian noise with true variance $\sigma^2 = 1.0$. Each data point has been dithered by a small amount horizontally to make the distribution visible. Circles show medians (of samples of 1024) for each value of the multiplicative factor. The variance is well measured as long as the noise root variance $\sigma$ is twice the minimum representable difference $\Delta$. 

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2. METHOD

The artificial images we use for the experiments that follow are all made with the same basic parameters and processes. The images are square $16 \times 16$ pixel images, to which we have added Gaussian noise to simulate sky (plus read) noise. A random number generator chooses a mean sky level $\nu$ for this Gaussian noise within the range 0 to 100 but the variance $\sigma^2$ is fixed for all experiments at $\sigma^2 = 1$. For most experiments we also add a randomly placed "star" with a Gaussian point-spread function at a location $(x_0, y_0)$ within a few pixel radius of the center of the image. The intensity of the star is given by a circular two-dimensional Gaussian function. The FWHM of the star point-spread function is set to 2.35 pixels for convenience, and the total flux of the star—total counts above background after integration over the array—is a variable. In the experiments to follow we set this total flux $S$ to 2.0, 64.0, and 2048.0. Given the FWHM setting of 2.35 pixels and the sky noise setting of $\sigma^2 = 1$, the peak intensities corresponding to these fluxes are $0.32\sigma, 10\sigma$, and $320\sigma$.

When we add the star, we do not add any Poisson or star-induced noise contribution to the images. That is, the images are "sky-dominated" in the sense that the per-pixel noise is the same in the center of the star as it is far from the star. In the context of setting the minimum representable difference, this choice is conservative, but it is also slightly unrealistic.

The method for setting the minimum representable difference $\Delta$ for the artificial images is used extensively in the experiments to follow. We define an array of factors $\Delta$ ranging from $2^4\sigma$ to $2^{-8}\sigma$, which we use to scale the data. We divide the image data by each value of $\Delta$, round all pixels to their nearest-integer values, and then multiply back in by $\Delta$. For convenience, we will call this "scale, snap to integer, unscale" procedure "SNIP". Figure 1 shows identical images SNIPped at different multiplicative factors; each panel shows the same original image data but with a different minimum representable difference $\Delta$.

In the first experiments we determine the effect of the minimum representable difference on the measurement of the variance of the noise in the image, which we generated as pure Gaussian noise. For this experiment we created empty images, added Gaussian noise with mean $\nu$ (a real value selected in the range 0 to 100) and variance $\sigma^2 = 1$, and applied the SNIP procedure. Figure 2 shows the dependence of the measured variance on the minimum representable difference $[\Delta/\sigma]$. As expected, the measured variance increases in accuracy as the minimum representable difference decreases; the accuracy is good when $\Delta < 0.5\sigma$.
In the second experiments we add a star to each image and ask how well we can measure its centroid. We added the randomly placed “star” before applying the SNIP procedure. The star is given a set integrated flux, a FWHM of 2.35 pixels, and a randomly selected true centroid \((x_0, y_0)\) within the central few pixels of the image. Our technique for measuring the centroid of the star involves fitting a quadratic surface to a \(3 \times 3\) section of the image data with the center of this array set on a first-guess value for the star position. It recenters the \(3 \times 3\) array around the highest-value pixel in the neighborhood of the first-guess value. We perform a simple least-squares fit to these data, using \(I(x, y) = a + bx + cy + dx^2 + exy + fy^2\) as our surface model, where \(x\) and \(y\) are pixel coordinates in the \(3 \times 3\) grid, and \(a, b, c, d, e,\) and \(f\) are parameters. Our centroid measurement \((x_s, y_s)\) is then computed from the best-fit parameters by

\[
(x_s, y_s) = \left(\frac{ce - bf}{2df - 2e^2}, \frac{be - cd}{2df - 2e^2}\right).
\]

The offset of this measurement from the true value (in pixels) is then \(\sqrt{(x_s - x_0)^2 + (y_s - y_0)^2}\). For sources strongly affected by noise, this fitting method sometimes returns large offsets; we artificially cap all offsets at 2 pixels.

In the third experiments we consider the effect of quantization on the photometric properties of the star. We have now centroided the star, and so we use the position of the star as found in the preceding paragraph along with the known variance \(\sigma^2\) to do a Gaussian fit to the point-spread function of the star. We only allow the height \(A\) to vary for fitting the star, which is related to the total flux \(S\) of the star by \(S = A \times 2\pi \sigma^2\). The fit is therefore really just a linear fit which is represented by the model

\[
Ae^{\frac{(x - x_0)^2 + (y - y_0)^2}{2\sigma^2}} + \mu_{\text{sky}},
\]

where \(\mu_{\text{sky}}\) is the “sky level.”

Figure 3 shows the dependence of the measured centroids and brightnesses of the stars relative to the known values as a function of minimum representable difference \(\Delta\). We find (not surprisingly) that the accuracy by which we measure the centroid and brightness increases as \(\Delta\) decreases; the accuracy saturates at \(\Delta < 0.5\sigma\). The expectation is that a star of high flux compared to the noise level will be very accurately measured, even at the highest minimum representable difference \(\Delta = 16\sigma\). At lower fluxes the offset is expected to be larger. Figures 3 and 4 confirm these intuitions. In each of these Figures, the experiment is performed on 1024 independent trials—each trial image has a unique sky level, noise sample, and star position—and each trial image has been SNIPped at each value of \(\Delta\).

Tiny variations in mean pixel values, even those smaller than the noise amplitude, do contain valuable information. In the fourth experiments we investigate this by co-adding noise-dominated images of the same region of the sky to reveal sources too faint to be detected in any individual image. The test we perform is—for each trial—to take 1024 images, add a faint source (fainter than any detection limit) to each image at a common location \((x_0, y_0)\), generate independent sky level \(\nu\) and sky noise for each image, apply the SNIP method for the
same range of minimum representable differences $\Delta$, co-add the images, and measure the star offsets in the SNIPped co-added images. The star position remains constant, but each image has independent sky properties. Just to reiterate, we co-add after applying the SNIP procedure, but given enough images we can measure astrometric and photometric properties of the extremely faint star with good accuracy. The co-adding procedure is illustrated in Figure 5.

Figure 6 shows that for a total star flux of 2.0, if we co-add 1024 images with independent sky properties, we can centroid and photometer the source with similar quality to the “single exposure” Figure 3 with a total star flux of 64.0. This is expected when the minimum representable difference $\Delta$ is small. What may not be expected is that even for sources in which every pixel is fainter than the minimum representable difference $\Delta$ in any individual image pixel, we are able to detect, centroid, and photometer accurately by co-adding images together; that is, the imposition of a large individual-image minimum representable difference does not distort information about exceedingly faint sources. Figure 7 shows the same for total flux 64.0; the trend is similar to that in Figure 4.

![Fig. 5.—Four 16 × 16 pixel images that demonstrate the co-adding procedure. The top left image shows a single image with noise variance $\sigma^2 = 1.0$ and an (extremely faint) Gaussian star with a total flux of 2.0 and FWHM of 2.35 pixels. The top right image is the same as the top left, but with the pixel values snapped to finite resolution $|\sigma/\Delta| = 2$ or minimum representable difference $\Delta = 0.5\sigma$. The bottom left image shows the result of co-adding 1024 images without snapping to finite resolution. The bottom right image is the same but co-adding after snapping each individual image’s data to $|\sigma/\Delta| = 2$. The similarities of the images indicates that information has been preserved. The peak per-pixel intensity of the star is $0.32\sigma$; this star is not visible in any of the individual images, but appears in the co-added images.](image)

In the co-add experiments, we have made the optimistic assumption that the sky level will be independent in every image that contributes to each co-add trial. To test the importance of varying the sky among the co-added exposures, we made a version in which we did not vary the sky. That is, we made each individual image not just with a fixed star flux and location but also a fixed sky level—different for each trial, but the same for each co-added exposure within each trial. The differences between Figures 6 and 8 are substantial when the minimum representable difference $\Delta$ is significantly larger than the per-pixel noise level $\sigma$.

### 3. Discussion

Because of finite noise, the information content in astronomical images is finite, and can be captured by a finite numerical resolution. In § 2, we scaled and snapped-to-integer real-valued images by a SNIP procedure such that in the SNIPped image, the minimum representable difference $\Delta$ between pixel values was set to a definite fraction of the Gaussian noise root variance $\sigma$. We found with direct numerical experiments that the SNIP procedure introduces essentially no significant error in estimating the variance of the image, or in centroiding or photometering stars in the image, when the minimum representable difference is set to any value $\Delta \leq 0.5\sigma$. In addition, we showed that all the information about sources fainter than the per-pixel noise level is preserved by the quantization (SNIP) procedure, again provided that $\Delta \leq 0.5\sigma$. This is somewhat remarkable because at $\Delta = 0.5\sigma$ the faintest sources in our experiments were fainter than the minimum representable difference.

Although it is somewhat counterintuitive that integer quantization of the data does not remove information about sources fainter than the quantization level, it is perhaps even more counterintuitive how well photometric measurements perform in our co-add tests. For example, in Figures 6 and 7, the photometric measurements are relatively accurate even when the data are quantized at minimum representable difference $\Delta = 16\sigma!$. The quality of the measurements can be understood in part by noting that the co-added images have a per-pixel noise $\sigma = \sqrt{1024\sigma} = 32\sigma$, which is once again larger than the minimum representable difference, and in part by noting that each image has a different sky level, so each individual image is differently “wrong” in its photometry; many of these differences average out in the co-add. When the sky level is held fixed across co-added images, photometric measurements become inaccurate again—as seen in Figure 8—because individual-image biases caused by the coarse quantization no longer “average out.”

Our fundamental conclusion is that all of the scientifically relevant information in an astronomical image is preserved as long as the minimum representable difference $\Delta$ in pixel values is smaller than or equal to half the per-pixel root-variance $\sigma$ in the image noise. This confirms previous results based on information-theory arguments (for example, Gaztañaga et al.
Fig. 6.—Same as Fig. 3, except with a star of total flux 2.0, and co-adding sets of 1024 exposures after snap to integer to make the extremely faint source detectable. In this experiment we give each of the co-added images a different sky level (see text).

Fig. 7.—Same as Fig. 6, except with a star of total flux 64.0.
2001), and extends previous experiments on bright-source photometry (White & Greenfield 1999; Pence et al. 2010) to astrometry and to sources fainter than the noise.

Our experiments were performed on images with pure Gaussian noise; of course many images contain significant non-Gaussianity in their per-pixel noise so the empirical variance will depart significantly from the true noise variance (White & Greenfield 1999). The conservative approach for such images is to take not the true variance for $\sigma^2$ but rather use for $\sigma^2$ something like the minimum of the straightforwardly measured variance and a central variance estimate, such as a sigma-clipped variance estimate, an estimate based on the curvature of the central part of the noise value frequency distribution function, or the median absolute difference of nearby pixels (Pence et al. 2009). With this redefinition of the root variance $\sigma$, the condition $\Delta \leq 0.5\sigma$ represents a conservative setting of the minimum representable difference.

The fact that a $\Delta = 0.5\sigma$ representation preserves information on the faint sources—even those fainter than $\Delta$ itself—has implications for the design of data-taking systems, which are necessarily limited in bandwidth. If the system is set with $\Delta$ substantially smaller than $0.5\sigma$, then bright sources will saturate the representation more frequently than necessary, while no additional information is being carried about the faintest sources. Any increase in $\Delta$ pays off directly in putting more of the necessarily limited system dynamic range onto bright sources, so it behooves system designers to push as close to the $\Delta = 0.5\sigma$ limit as possible.

To put this in the context of a real data system, we looked at a “DARK” calibration image from the Hubble Space Telescope Advanced Camera for Surveys (ACS). The dark image should have the lowest per-pixel noise of any ACS image, because it has only dark and read noise. We chose image set $jbanbea2q$, and measured the median noise level in the raw DARK image with the median absolute difference between values of nearby pixels (for robustness). The ACS data system is operating with a minimum representable difference $0.25\sigma < \Delta < 0.33\sigma$, comfortably within the information-preserving range and close to the minimum-bandwidth limit of $\Delta = 0.5\sigma$. Of course this is for a dark frame; sky exposures (especially long ones) could have been profitably taken with a larger $\Delta$ (because $\sigma$ will be greater); this would have preserved more of the system dynamic range for bright sources. If the ACS took almost exclusively long exposures, the output would contain more scientific information with a larger setting of the minimum representable difference.

In some sense, the results of this article recommend a “lossy” image compression technique, in which data are scaled by a factor and snapped-to-integer values such that the minimum representable difference $\Delta$ is made equal to or smaller than $0.5\sigma$. Indeed, when typical real-valued astronomical images are converted to integers at this resolution, the integer versions compress far better with subsequent standard file compression.
techniques (such as gzip) than do the floating-point originals (Gaztañaga et al. 2001; Watson 2002; White & Greenfield 1999; Pence et al. 2009; Bernstein et al. 2009). In the $\Delta = 0.5\sigma$ representation, after lossless compression, storage and transmission of the image “costs” only a few bits per noise-dominated pixel. Because the snap-to-integer step changes the data, this overall procedure is technically lossy, but we have shown here that none of the scientific information in the image has been lost.

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