Leptogenesis with heavy neutrino flavours: from density matrix to Boltzmann equations

Steve Blanchet\textsuperscript{a}, Pasquale Di Bari\textsuperscript{b}, David A. Jones\textsuperscript{b}, Luca Marzola\textsuperscript{b}

\textsuperscript{a} Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

\textsuperscript{b} School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, U.K.

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Abstract

Leptogenesis with heavy neutrino flavours is discussed within a density matrix formalism. We write the density matrix equation that describes the generation of the matter-antimatter asymmetry, for an arbitrary choice of the right-handed (RH) neutrino masses. For hierarchical RH neutrino masses lying in the fully flavoured regimes, the density matrix equation reduces to multiple-stage Boltzmann equations. In this case we recover and extend results previously derived within a quantum state collapse description. We confirm the generic existence of phantom terms, which are not washed out at production and contribute to the flavoured asymmetries proportionally to the initial RH neutrino abundances. Even in the $N_1$-dominated scenario they can give rise to lepton flavour asymmetries much larger than the baryon asymmetry with potential applications. We also confirm that there is a (orthogonal) component in the asymmetry produced by the heavier RH neutrinos which completely escapes the washout from the lighter RH neutrinos and show that phantom terms additionally contribute to it. The other (parallel) component is washed out with the usual exponential factor, even for a weak washout. Finally, as an illustration, we study the two RH neutrino model in the light of the above findings, showing that phantom terms can contribute in this case as well.
1 Introduction

Leptogenesis \cite{1} is a direct cosmological application of the see-saw mechanism \cite{2} for the explanation of neutrino masses and mixing and it realises a highly non trivial link between cosmology and neutrino physics. The discovery of neutrino masses and mixing in neutrino oscillation experiments \cite{3} has drawn great attention on leptogenesis that became the most attractive model of baryogenesis for the explanation of the matter-antimatter asymmetry of the Universe.

In most cases, classical Boltzmann equations are sufficient for the calculation of the final asymmetry \cite{4, 5, 6, 7, 8, 9}. However, when lepton flavour effects are taken into account \cite{10, 11}, different set of classical Boltzmann equations apply depending whether the asymmetry is generated in the one-flavour regime, when the mass of the decaying RH neutrinos $M_i$ is much above $10^{12}$ GeV, in the two-flavour regime, for $10^{12}$ GeV $\gg M_i \gg 10^9$ GeV, or in the three-flavour regime for $M_i \ll 10^9$ GeV. Moreover classical Boltzmann equations fail in reproducing the correct result in the transition regimes for $M_i \sim 10^9$ GeV and for $M_i \sim 10^{12}$ GeV. However, in the case that just the lightest RH neutrino species is assumed to be responsible for the generation of the asymmetry, classical Boltzmann equations provide quite a convenient description in phenomenological investigations, since the final asymmetry can be expressed in terms of simple analytical expressions that well approximate the numerical solutions \cite{9, 12}.

On the other hand, the contribution from heavier RH neutrinos can also relevantly contribute to the final asymmetry (heavy neutrino flavour effects) and has, therefore, consistently to be taken into account in general \cite{13}. When lepton flavour effects are considered \cite{14}, a reliable calculation of the asymmetry cannot neglect the contribution from the heavier RH neutrinos even in the two RH neutrino model \cite{15} usually considered as a paradigmatic case for the validity of the traditional $N_1$-dominated scenario. It has also been shown that a successful $N_2$-dominated scenario is naturally realised in an interesting class of $SO(10)$ inspired models \cite{16}.

When heavier RH neutrinos are included, one has to distinguish quite a large number of possible mass patterns with different corresponding sets of classical Boltzmann equations for the calculation of the final asymmetry. For example, in the typical case of three RH neutrinos one has ten different possible mass patterns \cite{17} (see Fig. 1). In addition, the requirement that all RH neutrino masses do not fall in a transition regime becomes clearly much more restrictive.

Moreover new effects arise when heavy neutrino flavours are taken into account. First, part of the asymmetry generated by a heavier RH neutrino, the flavour orthogonal com-
ponent, escapes the washout from a lighter RH neutrino. Second, parts of the flavour asymmetries (phantom terms) produced in the one or two flavour regimes are not washed out at the production.

Therefore, it is necessary to extend the density matrix formalism beyond the traditional $N_1$-dominated scenario and account for heavy neutrino flavours effects in order to calculate the final asymmetry for an arbitrary choice of the RH neutrino masses. This is the main objective of this paper. At the same time we want to show how Boltzmann equations can be recovered from the density matrix equations for the hierarchical RH neutrino mass patterns shown in Fig. 1 allowing an explicit analytic calculation of the final asymmetry. In this way we will confirm and extend results that were obtained within a simple quantum state collapse description. For illustrative purposes, we proceed in a modular way, first discussing the specific effects in isolation within simplified cases and then discussing the most general case that include all effects. The paper is organised in the following way.

In Section 2 we discuss the derivation of the kinetic equations for the $N_1$-dominated scenario in the absence of heavy neutrino flavours. This is useful both to show the extension from classical Boltzmann to density matrix equations and to highlight some features that will prove to be quite important when, in the next Section, we include
heavy neutrino flavour effects. In particular we show the existence of phantom terms and how the expression for the CP asymmetry matrix can be unambiguously derived from the flavoured CP asymmetries, taking into account the different flavour compositions of lepton and anti-lepton quantum states. We also interestingly point out that, though phantom terms cannot affect the final asymmetry in the $N_1$-dominated scenario, they can give rise to lepton asymmetries much larger than the observed baryon asymmetry with potential applications. In Section 3 we start discussing the case where two heavy RH neutrino flavours are involved directly in the generation of the asymmetry, considering a simplified case with only two charged lepton flavours. In this way we simplify the notation and we better highlight the main results. In this Section we are particularly interested to show two effects that specifically arise when the interplay between heavy neutrino and lepton neutrino flavours is considered. The first one is phantom leptogenesis [18]. The second, that we call projection effect, is how part of the asymmetry generated by a heavy RH neutrino, the component orthogonal to the heavy neutrino flavour associated to a lighter RH neutrino, is not washed out by the inverse processes of the latter [6, 20]. We also show that these two effects, phantom leptogenesis and projection effect, in general combine with each other. In Section 4 we extend the discussion to the general case with three heavy neutrino flavours and three charged lepton flavours. In this section we finally obtain general density matrix equations for the calculation of the asymmetry for an arbitrary choice of the RH neutrino masses. From these equations we derive the classical Boltzmann equations for a particularly interesting case: the two RH neutrino model. The derivation can be easily extended to all 10 hierarchical RH neutrino mass patterns shown in Fig. 1. In Section 5 we draw the conclusions.

2 Kinetic equations for the $N_1$ dominated scenario

We discuss leptogenesis within a minimal type I seesaw mechanism with three RH neutrino species, $N_1$, $N_2$ and $N_3$, with masses $M_1 \leq M_2 \leq M_3$ respectively.

In this section we review the main steps underlying the derivation of the kinetic equations in leptogenesis when heavy neutrino flavours are neglected, assuming that only the lightest RH neutrino decays and inverse processes contribute to the final asymmetry: the traditional $N_1$-dominated scenario.

We first derive the Boltzmann (rate) equations and then we extend them writing the density matrix equations, accounting for quantum decoherence and flavour oscillations. This derivation will prove to be useful not only to setup the notation but also to highlight some basic features of the kinetic equations in leptogenesis that will be relevant when we
will include heavy neutrino flavour effects in the next section.

We will neglect different effects, processes and corrections that have been studied during the last years and that will not play a relevant role in our discussion. These include for example $\Delta L = 2$ washout [4, 8], $\Delta L = 1$ scatterings [5], momentum dependence [21], thermal corrections [8, 22], flavour coupling from the Higgs and quark asymmetries [6, 18], quantum kinetic effects [23].

We will moreover always assume vanishing initial asymmetry though notice that the results that we will obtain in Section 3 when we discuss the projection effect, are also important in order to describe the evolution of a non-vanishing pre-existing asymmetry [20, 17].

2.1 Boltzmann equations

If we indicate with $\Gamma_1$ the decay rate of the lightest RH neutrinos into leptons, $N_1 \to \ell_1 + \Phi^\dagger$, and with $\bar{\Gamma}_1$ the decay rate into anti-leptons, $N_1 \to \bar{\ell}_1 + \Phi$, we can introduce the decay term $D_1$ and the washout term $W_1$ given respectively by

$$D_1(z) \equiv \frac{\Gamma_1 + \bar{\Gamma}_1}{Hz} = K_1 z \left\langle \frac{1}{\gamma_{11}} \right\rangle$$

and

$$W_1(z) \equiv \frac{1}{2} \frac{\Gamma_{ID}^f + \bar{\Gamma}_{ID}^f}{Hz} = \frac{1}{4} K_1 K_1(z) z^3,$$

where $z \equiv M_1/T$, $K_1 \equiv (\Gamma_1 + \bar{\Gamma}_1)_{T=0}/H_{T=M_1}$ is the decay parameter, $H$ is the expansion rate and the averaged dilution factor, in terms of the Bessel functions, is given by $\langle 1/\gamma_{11} \rangle = K_1(z)/K_2(z)$. Under the afore mentioned assumptions and approximations and considering the unflavoured regime, the calculation of the asymmetry is described by the most traditional set of kinetic equations for leptogenesis from the decays of the lightest RH neutrinos $N_1$ [6, 9]

$$\frac{dN_{N_1}}{dz} = -D_1 \left( N_{N_1} - N_{N_1}^{\text{eq}} \right),$$

$$\frac{dN_{B-L}}{dz} = \varepsilon_1 D_1 \left( N_{N_1} - N_{N_1}^{\text{eq}} \right) - W_1 N_{B-L},$$

where with $N_X$ we indicate any particle number or asymmetry $X$ calculated in a portion of co-moving volume containing one heavy neutrino in ultra-relativistic thermal equilibrium in a way that $N_{N_1}^{\text{eq}}(T \gg M_i) = 1$. In this way baryon-to-photon number ratio at recombination is related to the final $B - L$ asymmetry by

$$\eta_B = a_{\text{sph}} \frac{N_{B-L}^f}{N_{\gamma}^{\text{rec}}} \simeq 0.01 N_{B-L}^f,$$

where

$$\eta_B^{\text{CMB}} = (6.2 \pm 0.15) \times 10^{-10}. $$

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Let us very shortly recall the basic steps for the derivation of the Eq. (3) for the $B - L$ asymmetry. Ignoring the reprocessing action of sphalerons, we can write
\[
\frac{dN_{B-L}}{dz} = \frac{dN_{\ell_1}}{dz} - \frac{dN_{\bar{\ell}_1}}{dz}.
\] (6)
The net production rate of leptons and anti-leptons is then given by the difference between the production rate due to decays and the depletion rate due to inverse decays, for leptons
\[
\frac{dN_{\ell_1}}{dz} = \frac{\Gamma_1}{Hz} N_{N_{\ell_1}} - \frac{\Gamma^{ID}_1}{Hz} N_{\ell_1},
\] (7)
and for anti-leptons
\[
\frac{dN_{\bar{\ell}_1}}{dz} = \frac{\bar{\Gamma}_1}{Hz} N_{N_{\bar{\ell}_1}} - \frac{\bar{\Gamma}^{ID}_1}{Hz} N_{\bar{\ell}_1}.
\] (8)
The inverse decay rates are related to the decay rates by \footnote{This expression directly accounts for the resonant $\Delta L = 2$ contribution that is needed not to violate the Sakharov condition on the necessity of a departure from thermal equilibrium for the generation of an asymmetry \cite{4}. Here we are just interested in showing the separate Boltzmann equations for lepton anti-lepton numbers that we will use in the next subsection to derive the $CP$ violating term in the density matrix equation.}
\[
\Gamma^{ID}_1 = \Gamma_1 \frac{N_{N_{\ell_1}}^{eq}}{N_{\ell_1}^{eq}} \quad \text{and} \quad \bar{\Gamma}^{ID}_1 = \bar{\Gamma}_1 \frac{N_{N_{\bar{\ell}_1}}^{eq}}{N_{\bar{\ell}_1}^{eq}},
\] (9)
where $N_{\ell_1}^{eq} = N_{\bar{\ell}_1}^{eq} = 1$ is the number of leptons $\ell_1$ and of anti-leptons $\bar{\ell}_1$ in thermal equilibrium for vanishing asymmetry. The number of leptons and anti-leptons can be then recast as
\[
N_{\ell_1} = \frac{1}{2} (N_{\ell_1} + N_{\bar{\ell}_1}) + \frac{1}{2} (N_{\ell_1} - N_{\bar{\ell}_1})
= N_{\ell_1}^{eq} - \frac{1}{2} N_{B-L} + \mathcal{O}(N_{B-L}^2)
\] (10)
and
\[
N_{\bar{\ell}_1} = \frac{1}{2} (N_{\ell_1} + N_{\bar{\ell}_1}) - \frac{1}{2} (N_{\ell_1} - N_{\bar{\ell}_1})
= N_{\bar{\ell}_1}^{eq} + \frac{1}{2} N_{B-L} + \mathcal{O}(N_{B-L}^2).
\] (11)
Inserting these last expressions into the Eq. (6) and neglecting terms $\mathcal{O}(N_{B-L}^2)$, the Eq. (3) is obtained, with $D_1$ and $W_1$ given by the Eqs. (1).

The solution for the final asymmetry has a very simple analytical expression \footnote{This expression directly accounts for the resonant $\Delta L = 2$ contribution that is needed not to violate the Sakharov condition on the necessity of a departure from thermal equilibrium for the generation of an asymmetry \cite{4}. Here we are just interested in showing the separate Boltzmann equations for lepton anti-lepton numbers that we will use in the next subsection to derive the $CP$ violating term in the density matrix equation.}
\[
N_{B-L}^{f} = \varepsilon_1 \kappa(K_1), \quad \text{with} \quad \kappa(x) \equiv \frac{2}{xz_B(x)} \left[ 1 - \exp \left( -\frac{1}{2} x z_B(x) \right) \right],
\] (12)
where $\kappa(K_1)$ is the final efficiency factor that here we have written in the simple case of initial thermal $N_1$-abundance. The asymmetry is generated in a quite narrow interval of temperatures around $T_{B1} \equiv M_1/z_{B1}$, where $z_{B1} \equiv z_B(K_1)$ [9].

The unflavoured assumption, underlying the Eqs. (2) and (3), proves to describe the correct final asymmetry only for masses $M_1 \gtrsim 10^{13}$ GeV [19, 25]. In this range of masses, during all the relevant period of the asymmetry production, the lepton and anti-lepton quantum states produced from the decays of the $N_1$, that we will indicate respectively simply with $|1\rangle$ and $|\bar{1}\rangle$, can be treated, in flavour space, as pure states between their production at decay and their absorption at a subsequent inverse decay. They can be expressed as a linear combination of flavour eigenstates ($\alpha = e, \mu, \tau$)

$$|1\rangle = \sum_{\alpha} C_{1\alpha} |\alpha\rangle, \quad C_{1\alpha} \equiv \langle \alpha |1\rangle \quad \text{and} \quad |\bar{1}\rangle = \sum_{\alpha} \bar{C}_{1\alpha} |\bar{\alpha}\rangle, \quad \bar{C}_{1\alpha} \equiv \langle \bar{\alpha} |\bar{1}\rangle,$$

(13)

where notice that in general the final anti-lepton states produced by the $N_2$ decays are not the $CP$ conjugated of the final lepton states and therefore, in general, $C_{1\alpha} \neq \bar{C}_{1\alpha}$ [10].

It will prove useful to introduce the branching ratios $p_{1\alpha} \equiv |C_{1\alpha}|^2$ and $\bar{p}_{1\alpha} \equiv |\bar{C}_{1\alpha}|^2$ giving respectively the probabilities that a lepton $\ell_1$ or an anti-lepton $\bar{\ell}_1$ is found in a flavour eigenstate $\alpha$ or $\bar{\alpha}$ in a flavour measurement process. It is also useful to recast the branching ratios as

$$p_{1\alpha} = p_{1\alpha}^0 + \frac{\Delta p_{1\alpha}}{2}, \quad \bar{p}_{1\alpha} = p_{1\alpha}^0 - \frac{\Delta p_{1\alpha}}{2},$$

(14)

where the average $p_{1\alpha}^0 = (p_{1\alpha} + \bar{p}_{1\alpha})/2$ are the tree level contributions, while the differences $\Delta p_{1\alpha} = p_{1\alpha} - \bar{p}_{1\alpha}$ describe the $CP$ violating contributions from the interference with loop diagrams.

If the unflavoured regime holds during the period of generation of the asymmetry, for $z \lesssim z_B$, the flavour compositions of the leptons and of the anti leptons do not play any role since the only relevant interactions, the neutrino Yukawa interactions, are flavour blind. On the other hand for masses $10^{12}$ GeV $\gg M_1 \gg 10^9$ GeV, the coherent evolution of the $|1\rangle$ and $|\bar{1}\rangle$ quantum states breaks down before they inverse decay interacting with the Higgs bosons, due to collisions with right-handed taunos. At the inverse decays, lepton quantum states can then be described as an incoherent mixture of tauon eigenstates $|\tau\rangle$ and of $|\tau_1^\perp\rangle$ quantum states. These second ones are a coherent superposition of muon and electron eigenstates that can be regarded as the projection of the lepton quantum states $|1\rangle$ on the plane orthogonal to the tauon flavour (see Fig. 2). In this two fully flavored regime, classical Boltzmann equations can be still used as in the unflavored regime, with the difference, in general, that now the flavour compositions of leptons and anti-leptons do play a role in the generation of the asymmetry. In this case each single flavour asymmetry
Figure 2: For $10^{12}$ GeV $\gg M_1 \gg 10^9$ GeV, the lepton quantum states $|1\rangle$ can be treated as an incoherent mixture of a $\tau$ and of a $\tau_1^\perp$ component during the generation of the asymmetry and a two fully flavoured regime applies.

has to be tracked independently and the total final $B - L$ asymmetry has to be calculated after freeze-out as the sum of the two flavoured asymmetries, a $\tau$ asymmetry and a $\tau_1^\perp$ asymmetry. To this extent, we have to introduce the flavoured $CP$ asymmetries

$$\varepsilon_{i\alpha} \equiv - \frac{p_{i\alpha} \Gamma_i - \bar{p}_{i\alpha} \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} = p_{i\alpha}^0 \varepsilon_i - \frac{\Delta p_{i\alpha}}{2}. \quad (15)$$

Since sphaleron processes conserve the quantities $\Delta_\alpha \equiv B/3 - L_\alpha$ ($\alpha = e, \mu, \tau$) [6], these are the convenient independent variables to be used in the set of Boltzmann equations that can be written as

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}),$$

$$\frac{dN_{\Delta_\tau}}{dz} = \varepsilon_{1\tau} D_1 (N_{N_1} - N_{N_1}^{eq}) - p_{1\tau}^0 W_1 N_{\Delta_\tau},$$

$$\frac{dN_{\Delta_{\tau_1^\perp}}}{dz} = \varepsilon_{1\tau_1^\perp} D_1 (N_{N_1} - N_{N_1}^{eq}) - p_{1\tau_1^\perp}^0 W_1 N_{\Delta_{\tau_1^\perp}},$$

where $p_{1\tau}^0 \equiv p_{1e}^0 + p_{1\mu}^0$ and $\varepsilon_{1\tau_1^\perp} \equiv \varepsilon_{1e} + \varepsilon_{1\mu}$.

Using the decomposition of the flavoured $CP$ asymmetries in terms of $p_{i\alpha}^0$ and $\Delta p_{i\alpha}$ (cf. Eq. (15)) and assuming strong washout for both flavours (i.e. $K_{1\tau}, K_{1\tau_1^\perp} \gg 1$), the final asymmetry is approximated by the expression

$$N_{B-L}^f \simeq 2 \varepsilon_1 \kappa(K_1) + \frac{\Delta p_{1\tau}}{2} \left[ \kappa(K_{1\tau_1^\perp}) - \kappa(K_{1\tau}) \right], \quad (17)$$
where $K_{i\alpha} \equiv p_{i\alpha}^0 K_i$. This approximated expression shows how, compared to the expression obtained in the unflavoured case, large lepton flavour effects can arise only when leptons and anti-leptons have a different flavour composition, for non-vanishing $\Delta p_{1\tau}$. The most extreme case is realized when $\varepsilon_1 = 0$ [10]. In the unflavoured case this would imply a vanishing final asymmetry but in the flavoured case it does not. It should be indeed noticed that when flavour effects are considered, $B-L$ violation is not a necessary condition for the generation of a baryon asymmetry via leptogenesis, it is sufficient to have a $\Delta_\alpha$ violation accompanied by an asymmetric washout between the two flavours, that in this context corresponds to the requirement of departure from thermal equilibrium.

For $M_1 \ll 10^9$ GeV muon interactions are able to break the coherent evolution also of the $|\tau_1\rangle$ quantum states between decays and inverse decays during the period of the generation of the asymmetry. In this case a three-fully flavoured regime is realised and the set of classical Boltzmann equations is a straightforward generalisation of that one written in the two fully flavoured regime. In the $N_1$-dominated scenario with hierarchical RH neutrinos, because of the lower bound $M_1 \gtrsim 10^9$ GeV for successful leptogenesis [26, 7], a three fully flavoured regime is not relevant for the calculation of the final asymmetry. On the other hand, in a $N_2$-dominated scenario, a three flavoured regime has to be considered in the calculation of the lightest RH neutrino washout [14].

2.2 Density matrix equations

Within a density matrix formalism [11, 27, 19], the description of leptogenesis is more general than with classical Boltzmann equations, since it makes possible to describe those intermediate regimes where lepton quantum states interact with the thermal bath via charged lepton interactions between decays and inverse decays but not so efficiently that a quantum collapse approximation can be applied in a statistical description. In this case the lepton quantum states cannot be described neither in terms of pure states nor as an incoherent mixture. Yukawa interactions and charged lepton interactions compete with each other in the determination of the lepton flavour. A statistical quantum-mechanical description of lepton flavour cannot treat leptons as decoupled from the thermal bath. Therefore, the concept of lepton quantum states itself is blurred since one should consistently describe together leptons and thermal bath. A density matrix formalism [28] is then particularly convenient since it still allows to describe the leptonic subsystem in a

\textsuperscript{2}Notice that relaxing the assumption of strong washout for both flavours one can only get an asymmetry that is even closer to the unflavoured calculation. Indeed in the limit of no washout ($K_{1\tau}, K_{1\tau}^\dagger \ll 1$) one exactly recovers the unflavoured expression for the final asymmetry.
separate way, neglecting back-reaction effects and encoding the coupling with the thermal bath in the evolution of the off-diagonal terms of the lepton density matrices.

Let us see how a density matrix equation for the $B - L$ asymmetry can be obtained starting first from the case where charged lepton interactions are negligible. In this case we just expect to reproduce the Eq. (3).

Let us consider a simple two lepton flavour case able to describe the intermediate regime between the unflavoured case and the two fully flavoured regime that are recovered as asymptotic limits. The two relevant flavours are then $\tau$ and $\tau_1^+$ (see Fig. 2). In this two flavour space the flavour composition of the lepton quantum states produced by the $N_1$ decays can be written as $(\alpha = \tau, \tau_1^+)$

$$|1\rangle = C_{1\tau} |\tau\rangle + C_{1\tau_1^+} |\tau_1^+\rangle, \quad C_{1\alpha} \equiv \langle \alpha | 1 \rangle,$$

$$|\bar{1}\rangle = C_{1\bar{\tau}} |\bar{\tau}\rangle + C_{1\bar{\tau}_1^+} |\bar{\tau}_1^+\rangle, \quad C_{1\bar{\alpha}} \equiv \langle \bar{\alpha} | \bar{1} \rangle.$$  

At three level, the amplitudes $C_{i\alpha}$ and $\bar{C}_{i\bar{\alpha}}$ are given by

$$C_{i\alpha}^0 = \frac{h_{\alpha i}}{\sqrt{(h^\dagger h)_{ii}}} \quad \text{and} \quad \bar{C}_{i\bar{\alpha}}^0 = \frac{h_{\alpha i}^*}{\sqrt{(h^\dagger h)_{ii}}}.$$  

Including one-loop CP-violating corrections, these amplitudes become

$$C_{i\alpha} = \frac{1}{\sqrt{(h^\dagger h)_{ii}} - 2 \text{Re}(h^\dagger h \xi_u)_{ii}} (h_{\alpha i} + (h \xi_u)_{\alpha i}),$$

$$\bar{C}_{i\bar{\alpha}} = \frac{1}{\sqrt{(h^\dagger h)_{ii}} - 2 \text{Re}(h^\dagger h \xi_v)_{ii}} (h_{\alpha i}^* + (h^* \xi_v)_{\alpha i}).$$

We are following here notation and formalism introduced in [29] and more recently in [30]. The one-loop corrections are included in the $\xi_u$ and $\xi_v$ functions, which are given by

$$[\xi_u(M_2^2)]_{ki} \equiv \left[ u^T(M_2^2) + Mb(M_2^2)(h^\dagger h)^T M_k \right]_{ki},$$

$$[\xi_v(M_2^2)]_{ki} \equiv \left[ v^T(M_2^2) + Mb(M_2^2)(h^\dagger h)^T M_k \right]_{ki}.$$  

Note that the mass matrix being diagonal, we simply have $M_{ki} = M_i \delta_{ki}$. The first term on the right-hand side describes the self-energy correction, whereas the second one is the vertex correction. The imaginary part of the loop factor $b(M_2^2)$ evaluated on mass shell for the RH neutrino $N_i$ is given by

$$b_{ki}(M_2^2) = \frac{1}{16\pi M_i M_k} f(M_k^2/M_i^2),$$  

where $f(x) = \sqrt{x} \left( 1 - (1 + x) \log \left( \frac{1 + x}{x} \right) \right)$. As for the $u$ and $v$ terms in Eq. (23), they are given by

$$u_{ik}(M_2^2) = \omega_{ik}(M_i^2) \left[ M_i \Sigma_{N,ik}(M_i^2) + M_k \Sigma_{N,ki}(M_k^2) \right],$$

$$v_{ik}(M_2^2) = \omega_{ik}(M_i^2) \left[ M_i \Sigma_{N,ki}(M_i^2) + M_k \Sigma_{N,ik}(M_k^2) \right].$$
They depend on the self-energy $\Sigma_{N,ki}(M_i^2) = a(M_i^2)(h^\dagger h)_{ki}$, where the loop factor $a$, whose imaginary part is Im$(a) = -1/(16\pi)$, is evaluated again on mass-shell for the RH neutrino $N_i$. Finally, the propagator $\omega$, evaluated on shell, can be found to be

$$\omega_{ik}(M_i^2) = \frac{M_i(M_i^2 - M_k^2)}{(M_i^2 - M_k^2)^2 + (M_k\Gamma_i - M_i\Gamma_k)^2}.$$  \hfill (28)

In the two flavour bases $\ell_1-\ell^\perp_1$ and $\bar{\ell}_1-\bar{\ell}^\perp_1$, the lepton and anti-lepton density matrix are respectively simply given by $\rho_{ij}^\ell = \text{diag}(1,0)$ and $\rho_{ij}^{\bar{\ell}} = \text{diag}(1,0)$, where $i,j = 1,1^\perp$ and $\bar{i},\bar{j} = \bar{1},\bar{1}^\perp$. Because of the different flavour composition of leptons and anti-leptons, the two bases are not CP conjugated of each other.

If we introduce the lepton and anti-lepton number density matrices, respectively $N_{ij}^\ell \equiv N_{\ell_i}\rho_{ij}^\ell$ and $N_{ij}^{\bar{\ell}} \equiv N_{\ell_i}\rho_{ij}^{\bar{\ell}}$, their evolution at $T \sim T_L$ is given by

$$\frac{dN_{ij}^\ell}{dz} = \frac{\Gamma_1}{Hz} N_N - \frac{\Gamma_{ID}^1}{Hz} N_{\ell_1} \rho_{ij}^\ell, \quad \frac{dN_{ij}^{\bar{\ell}}}{dz} = \frac{\bar{\Gamma}_1}{Hz} N_N - \frac{\bar{\Gamma}_{ID}^1}{Hz} N_{\ell_1} \rho_{ij}^{\bar{\ell}}.$$ \hfill (29)

In order to obtain an equation for the total $B - L$ asymmetry $N_{B-L}$, we have first to write these two equations in the same flavour basis, for convenience the lepton flavour basis $\tau-\tau^\perp_1$, and then subtract them. The rotation matrices are then given by

$$R_{i\alpha}^{(1)} = \begin{pmatrix} C_{1\tau} & -C_{1\tau^\perp_1} \\ C_{1\tau^\perp_1}^* & C_{1\tau}^* \end{pmatrix} \quad \text{and} \quad \bar{R}_{i\alpha}^{(1)} = \begin{pmatrix} C_{1\tau} & -C_{1\tau^\perp_1} \\ C_{1\tau^\perp_1}^* & C_{1\tau}^* \end{pmatrix}.$$ \hfill (30)

for leptons and anti-leptons respectively, where notice that for matrices we indicate the heavy neutrino flavour index with an superscript in round brackets. Also notice that at tree level, corresponding to neglect CP violation, they would simply be conjugated of each other, i.e.

$$R_{i\alpha}^{0(1)} = \begin{pmatrix} C_{01\tau} & -C_{01\tau^\perp_1} \\ C_{01\tau^\perp_1}^* & C_{01\tau}^* \end{pmatrix} \quad \text{and} \quad \bar{R}_{i\alpha}^{0(1)} = \begin{pmatrix} C_{01\tau} & -C_{01\tau^\perp_1} \\ C_{01\tau^\perp_1}^* & C_{01\tau}^* \end{pmatrix}.$$ \hfill (31)

In the charged lepton flavour basis one can finally write the equation for the $B - L$ asymmetry matrix as

$$\frac{dN_{\alpha\beta}^{B-L}}{dz} = \bar{R}_{i\alpha}^{(1)} \frac{dN_{ij}^\ell}{dz} \bar{R}_{ij}^{(1)} + R_{i\alpha}^{(1)} \frac{dN_{ij}^{\bar{\ell}}}{dz} R_{ij}^{(1)},$$ \hfill (32)

\begin{footnote}{For the sake of brevity in the following we will say ‘same (or different) flavour composition’ and ‘same (or different) flavour basis’ for leptons and anti-leptons meaning that they are (are not) CP conjugated of each other.}

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whose trace gives the $B - L$ asymmetry $N_{B-L}$. Notice that we are now identifying the $\alpha$ and $\bar{\alpha}$ indexes since the lepton and anti-lepton flavor bases are CP conjugated of each other. We will find useful to define the following two matrices:

\[
P_{\alpha\beta}^{(1)} \equiv R_{\alpha i}^{(1)\dagger} \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) R_{j\beta}^{(1)} = \left( \begin{array}{cc} p_{1\tau} & -C_{1\tau}^* C_{1\tau}^+ \\ -C_{1\tau} C_{1\tau}^* & p_{1\tau}^\dagger \end{array} \right), \tag{33} \]

\[
P_{\alpha\beta}^{(1)} \equiv \bar{R}_{\alpha i}^{(1)} \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \bar{R}_{j\beta}^{(1)} = \left( \begin{array}{cc} \bar{p}_{1\tau} & -C_{1\tau}^* C_{1\tau}^+ \\ -C_{1\tau} C_{1\tau}^* & \bar{p}_{1\tau} \end{array} \right), \tag{34} \]

which, at tree level, are simply given by

\[
\mathcal{P}_{\alpha\beta}^{(1)} = R_{\alpha i}^{(1)\dagger} \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) R_{j\beta}^{(1)} = \left( \begin{array}{cc} p_{1\tau}^0 & -C_{1\tau}^0 C_{1\tau}^* \\ -C_{1\tau} C_{1\tau}^* & p_{1\tau}^{0\dagger} \end{array} \right) = \frac{1}{(h^\dagger h)_{11}} \left( \begin{array}{cc} |h_{\tau 1}|^2 & -h_{\tau 1}^* h_{\tau 1} \cr -h_{\tau 1} h_{\tau 1}^* & |h_{\tau 1}|^2 \end{array} \right). \tag{35} \]

Using these results, we can now rewrite Eq. (32) as

\[
\frac{dN_{B-L}^{(1)}}{dz} = \varepsilon_{\alpha\beta}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 (\mathcal{P}_{\alpha\beta}^{(1)} N_{B-L})_{\alpha\beta}. \tag{36} \]

The CP asymmetry matrix for the lightest RH neutrino $N_1$ is given by

\[
\varepsilon_{\alpha\beta}^{(1)} = -\frac{\mathcal{P}_{\alpha\beta}^{(1)} \Gamma_1 - \mathcal{P}_{\alpha\beta}^{(1)} \Gamma_1}{\Gamma_1 + \Gamma_1}, \tag{37} \]

where the diagonal terms simply correspond to the flavoured CP asymmetries, $\varepsilon_{\alpha\alpha}^{(1)} = \varepsilon_{1\alpha}$, while the off-diagonal terms obey $\varepsilon_{\alpha\beta}^{(1)} = (\varepsilon_{\beta\alpha}^{(1)})^*$ and are not not necessarily real. This expression can be generalised to the CP asymmetry matrix $\varepsilon_{\alpha\beta}^{(i)}$, of any RH neutrino species $N_i$ that in terms of the Yukawa couplings can be written as

\[
\varepsilon_{\alpha\beta}^{(i)} = \frac{3}{32\pi(h^\dagger h)_{ii}} \sum_{j \neq i} \left\{ i \left[ h_{\alpha i} h_{\beta j} (h^\dagger h)_{ji} - h_{\beta i}^* h_{\alpha j} (h^\dagger h)_{ij} \right] \frac{\xi(x_j/x_i)}{x_j/x_i} + i \frac{2}{3(x_j/x_i - 1)} \left[ h_{\alpha i} h_{\beta j}^* (h^\dagger h)_{ji} - h_{\beta i}^* h_{\alpha j} (h^\dagger h)_{ij} \right] \right\}. \tag{38} \]

This expression slightly differs from that one in [11, 19] (simply the first term there is minus the imaginary part of the first term written here, so that the off-diagonal terms are real) while it coincides with the expression given in [25]. Notice that we have neglected terms $\mathcal{O}(\Delta \rho N_{B-L})$ so that

\[
(\mathcal{P}^{0(1)} N_{B-L})_{\alpha\beta} \approx \left[ R_{\alpha i}^{0(1)\dagger} \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) R_{j\beta}^{0(1)} \right]_{\alpha\beta} = N_{B-L} \mathcal{P}_{\alpha\beta}^{0(1)}. \tag{39} \]
In the case we are discussing, when charged lepton interactions are negligible, $\mathcal{P}^{0(1)}$ and $N^{B-L}$ commute. However, when charged lepton interactions become effective, $\mathcal{P}^{0(1)}$ and $N^{B-L}$ do not commute in general and the term $(\mathcal{P}^{0(1)} N^{B-L})_{\alpha\beta}$ is replaced by the anti-commutator \[11\]: $(\mathcal{P}^{0(1)} N^{B-L})_{\alpha\beta} \rightarrow \{\mathcal{P}^{0(1)}, N^{B-L}\}_{\alpha\beta}/2$.

The diagonal components of the Eq. (32) can be explicitly written as

$$\frac{dN^{B-L}_{\tau\tau}}{dz} = \varepsilon^{(1)}_{\tau\tau} D_1 (N_{N_1} - N_{N_1}^{eq}) - p_{1\tau}^0 W_1 N_{B-L},$$  

(40)

$$\frac{dN^{B-L}_{f_{\tau\tau} f_{\tau\tau}^+}}{dz} = \varepsilon^{(1)}_{f_{\tau\tau} f_{\tau\tau}^+} D_1 (N_{N_1} - N_{N_1}^{eq}) - p_{f_{\tau\tau} f_{\tau\tau}^+}^0 W_1 N_{B-L}.$$  

(41)

Summing these two equations, one finally recovers the usual Eq. (3) for the total $B-L$ asymmetry $N_{B-L} = \text{Tr}[N_{\alpha\beta}]$, which is washed out in the usual way at the production.

On the other hand, from Eqs. (40) and (41), one finds the relation

$$\frac{1}{p_{1\tau}^0} \frac{dN^{B-L}_{\tau\tau}}{dz} - \frac{1}{p_{f_{\tau\tau} f_{\tau\tau}^+}^0} \frac{dN^{B-L}_{f_{\tau\tau} f_{\tau\tau}^+}}{dz} = -\frac{\Delta p_{1\tau}^0}{2} \left( \frac{1}{p_{1\tau}^0} + \frac{1}{p_{f_{\tau\tau} f_{\tau\tau}^+}^0} \right) D_1 (N_{N_1} - N_{N_1}^{eq})$$  

(42)

which, together with Eq. (2), forms a system of equations that can be solved analytically. At low temperatures $T \ll T_{B1} = M_1/z_{B1} \ll M_1$, the final values are then found to be

$$N^{B-L, f}_{\tau\tau} \approx p_{1\tau}^0 N_{f_{\tau\tau}}^{B-L} - \frac{\Delta p_{1\tau}^0}{2} N_{N_1}^{eq},$$  

(43)

$$N^{B-L, f}_{f_{\tau\tau} f_{\tau\tau}^+} \approx p_{f_{\tau\tau} f_{\tau\tau}^+}^0 N_{f_{\tau\tau}}^{B-L} + \frac{\Delta p_{1\tau}^0}{2} N_{N_1}^{eq}.$$  

(44)

This solution shows that the flavoured asymmetries contain terms that escape the washout at the production: these are the phantom terms \[18\]. If one only considers the unflavoured regime, where charged lepton interactions can be neglected as we have done so far, the flavoured asymmetries are not themselves measured and the phantom terms cannot give any physical effect, in particular they cannot affect the baryon asymmetry.

On the other hand, at $T \sim 10^{12}$ GeV, tauon lepton interactions start to be in equilibrium breaking the coherence of the lepton quantum states. These are described by additional terms in Eq. (32), which then generalises into \[11\] \[31\] \[25\]

$$\frac{dN^{B-L}_{\alpha\beta}}{dz} = \varepsilon^{(1)}_{\alpha\beta} D_1 (N_{N_1} - N_{N_1}^{eq}) - \frac{1}{2} W_1 \{\mathcal{P}^{0(1)}, N^{B-L}\}_{\alpha\beta}$$  

(45)

$$+ \frac{i}{H z} \left[ \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), N^{f_{\tau\tau} f_{\tau\tau}^+} \right]_{\alpha\beta} - \frac{\text{Im}(\Lambda)}{H z} \left[ \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), N^{B-L} \right]_{\alpha\beta},$$

where we defined $N^{f_{\tau\tau} f_{\tau\tau}^+}_{\alpha\beta} \equiv N^{f_{\tau\tau}}_{\alpha\beta} + N^{f_{\tau\tau}^+}_{\alpha\beta}$. The real and imaginary parts of the tau-lepton self-energy are respectively given by \[32\] \[33\]

$$\text{Re}(\Lambda) = \frac{f_{\tau\tau}^2 T}{64}$$  

and  

$$\text{Im}(\Lambda) = 8 \times 10^{-3} f_{\tau\tau}^2 T,$$  

(46)
where $f_\tau$ is the tauon Yukawa coupling. The commutator structure in the third term on the RHS of Eq. (45) accounts for oscillations in flavor space driven by the real part of the self energy, and the double commutator accounts for damping of the off-diagonal terms driven by the imaginary part of the self energy. These effects can also be conveniently expressed in terms of the Pauli matrices $\sigma_i$, $i = 1, 2, 3$, so that the evolution equation reduces to

$$
\frac{dN_{\alpha\beta}^{B-L}}{dz} = \varepsilon_{\alpha\beta}^{(1)} D_1 (N_N - N_{eq}) - \frac{1}{2} W_1 \{ \mathcal{P}^{0(1)}, N^{B-L} \}_{\alpha\beta} - \frac{\text{Re}(\Lambda_\tau)}{Hz} (\sigma_2)_{\alpha\beta} N_{\alpha\beta}^{\ell+\bar{\ell}} - \frac{\text{Im}(\Lambda_\tau)}{Hz} (\sigma_1)_{\alpha\beta} N_{\alpha\beta}^{B-L},
$$

(47)

In order to close the system of equations, we also need a Boltzmann equation for the $N_{\ell+\bar{\ell}}$, which is given by

$$
\frac{dN_{\alpha\beta}^{\ell+\bar{\ell}}}{dz} = -\frac{\text{Re}(\Lambda_\tau)}{Hz} (\sigma_2)_{\alpha\beta} N_{\alpha\beta}^{B-L} - S_g \left( N_{\alpha\beta}^{\ell+\bar{\ell}} - 2 N_{eq}^{\ell+\bar{\ell}} \delta_{\alpha\beta} \right),
$$

(48)

where $S_g \equiv \Gamma_g / (Hz)$ accounts for gauge interactions. As shown in [25], this term has the effect of damping the flavor oscillations. This can be understood by noticing that gauge interactions force $N_{\alpha\beta}^{\ell+\bar{\ell}} = 2 N_{eq}^{\ell+\bar{\ell}} \delta_{\alpha\beta}$, which in turn renders the oscillatory term Eq. (45) negligible.

Let us indicate with $T_* \ll 10^{12}$ GeV that value of the temperature below which one can approximate, with the desired precision, the lepton quantum states as a fully incoherent mixture of $|\tau\rangle$ and $|\tau_1\rangle$ quantum states corresponding to a complete damping of the off-diagonal terms in the lepton density matrix (analogously for anti-leptons). This means that for $T \lesssim T_*$ the $\tau$ and $\tau_1$ lepton asymmetries, given by the diagonal entries of $N_{\alpha\beta}^{B-L}$, are fully measured by the thermal bath and reprocessed by sphaleron processes conserving the $\Delta_\tau$ and $\Delta_{\tau_1}$ asymmetries, so that at $T^* \equiv N_{\Delta_\tau}^T = N_{\tau\tau}^{B-L}(z_B)$ and $N_{\Delta_{\tau_1}}^T = N_{\tau_1\tau_1}^{B-L}(z_B)$. However, notice that since, for $M_1 \gg 10^{12}$ GeV, the total $B - L$ asymmetry has been already produced and got frozen in the unflavoured regime, this fully flavoured regime stage does not affect the final total $B - L$ asymmetry. Even with $N_{N_1}^{in} \neq 0$, the phantom terms do not contribute to the final asymmetry because they cancel with each other. In other words, within the $N_1$-dominated scenario, phantom terms have no effect on the final asymmetry. Phantom terms are therefore a consequence of charged

$\footnote{On the other hand remember that if one considers $10^9$ GeV $\ll M_1 \ll 10^{12}$ GeV, the two fully flavoured regime holds during the period of leptogenesis and the density matrix equations reduce to the set of classical Boltzmann equations Eqs. (16). The terms in the flavoured asymmetries coming from CP violating terms due to a different flavour composition are still present but they are not phantom,}$
lepton flavour effects but they do not have any consequence on the final asymmetry in the $N_1$-dominated scenario in the absence of heavy neutrino flavour effects.

Our results show explicitly the presence of the phantom terms. They extend previous results [19, 25] where the lepton and anti-lepton density matrices were, implicitly or explicitly, assumed to be diagonalizable in bases that are CP conjugated of each other and this precludes the derivation of the phantom terms. However, as far as the $N_1$-dominated scenario is considered, phantom terms can be safely neglected in the calculation of the final asymmetry, and therefore there is no contradiction between our results and those of [19, 25].

On the other hand, since we are interested in accounting for heavy neutrino flavour effects, we cannot neglect phantom terms. Indeed, as we will discuss, they can in this case affect the total final asymmetry giving rise to the possibility of phantom leptogenesis [18].

It is also quite interesting to notice that, even in the $N_1$-dominated scenario, the phantom terms can produce lepton flavour asymmetries much larger than the observed baryon asymmetry, though depending on the initial RH neutrino abundances. These large cancelling flavour asymmetries can be as large as $10^{-5} - 10^{-4}$ for initial thermal $N_1$ abundance and in this case they could have potential effects at lower temperatures $T \lesssim 100 \text{ MeV}$ when the thermal bath is flavour sensitive. They can very unlikely have direct effects on primordial nuclear abundances since an impact on BBN requires much larger asymmetries ($\gtrsim 0.01$ [34]). However, asymmetries as small as $\sim 10^{-5}$ could for example be relevant for active-sterile neutrino oscillations [35].

3 A simplified case with two charged lepton flavours

In this section we account for heavy neutrino flavour effects considering a simplified two charged lepton flavour case. This will greatly simplify the notation making the new results more easily readable. It will be then quite straightforward in the next Section to generalise all the equations to a realistic three lepton flavour case. For definiteness we consider masses $M_i \gg 10^9 \text{ GeV}$, when only tauon lepton interactions have to be taken into account.

We also assume that the heaviest RH neutrinos $N_3$ do not contribute to the final asymmetry. This is in any case a valid assumption if $M_3 \gg T_{RH} \gg M_2$, since in this way since they are measured directly at production and undergo washout. Therefore, if there is a flavour-asymmetric production, they contribute to the final asymmetry, yielding the second term in the Eq. (17), and can even dominate as we discussed.
Figure 3: Flavour configuration of the two heavy neutrino lepton flavours, $\ell_1$ and $\ell_2$, leading to the simplified two charged lepton neutrino flavour case considered in this section.

the $N_3$’s would not thermalise. As for lepton flavours, we will extend the results to the three heavy neutrino flavour case in the next Section.

Notice that with these assumptions, the two charged lepton flavour case can be regarded as a special case where the two heavy neutrino lepton flavours, $\ell_1$ and $\ell_2$, lie on the same plane orthogonal to the $e - \mu$ plane and therefore $\tau_2^\perp = \tau_1^\perp = \tau^\perp$ (see Fig. 3). Correspondingly the two anti-lepton flavours, $\bar{\ell}_1$ and $\bar{\ell}_2$, also lie on the same plane orthogonal to the $e - \mu$ plane and therefore $\bar{\tau}_2^\perp = \bar{\tau}_1^\perp \equiv \bar{\tau}^\perp$ with $\bar{\tau}^\perp$ that is now assumed to be $CP$ conjugated of $\tau^\perp$. In this way, in the whole following discussion, we will have only two charged lepton flavours, $\tau$ and $\tau^\perp$.

The density matrix equation Eq. (45) for $N_{\alpha\beta}^{B-L}$, valid for $N_1$ leptogenesis, gets then generalised into

$$\frac{dN_{\alpha\beta}^{B-L}}{dz} = \varepsilon^{(1)}_{\alpha\beta} D_1 (N_{N_1} - N_{N_1}^{eq}) - \frac{1}{2} W_1 \{\mathcal{P}^{0(1)}, N_{B-L}^{B-L}\}_{\alpha\beta}$$

$$+ \varepsilon^{(2)}_{\alpha\beta} D_2 (N_{N_2} - N_{N_2}^{eq}) - \frac{1}{2} W_2 \{\mathcal{P}^{0(2)}, N_{B-L}^{B-L}\}_{\alpha\beta}$$

$$+ i \text{Re}(\Lambda_r) \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N_{\ell^+\bar{\ell}}\right]_{\alpha\beta} - \text{Im}(\Lambda_r) \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N_{B-L}^{B-L}\right]_{\alpha\beta}.$$

We now discuss the three asymptotic cases, the first one for $M_2 \gg 10^{12}$ GeV $\gg M_1$, the second for $M_2, M_1 \gg 10^{12}$ GeV and the third for $M_1, M_2 \ll 10^{12}$ GeV, where Boltzmann equations are recovered. In this way we will derive, within a density matrix formalism,
results that were already obtained within an instantaneous collapse of the quantum state formalism: the first is phantom leptogenesis [18], the second is the heavy neutrino flavour projection [6, 20].

3.1 Case \( M_2 \gg 10^{12} \text{GeV} \gg M_1 \): three stages phantom leptogenesis

Let us consider the asymmetry production from the \( N_2 \)'s at \( T \sim M_2 \). This is basically described by the same equations that we wrote in the previous section for the \( N_1 \)-dominated scenario where now simply all quantities have to be relabelled in a way that \( 1 \to 2 \). Since charged lepton interactions are negligible, we can use the Eq. (36) for the calculation of \( N_{\alpha \beta}^{B-L} \), that now simply becomes

\[
\frac{dN_{\alpha \beta}^{B-L}}{dz} = \varepsilon_{\alpha \beta}^{(2)} D_2 (N_{N_2} - N_{N_2}^{eq}) - W_2 N_{B-L} \mathcal{T}_{\alpha \beta}^{0(2)},
\]

with an obvious re-definition of all quantities that now refer to \( N_2 \). At \( T \simeq T_{B2} \equiv M_2/z_{B2} \), the \( \tau \) and \( \tau^\perp \) asymmetries are described by the Eqs. (43) with \( 1 \to 2 \),

\[
N_{\tau \tau}^{B-L}(T \simeq T_{B2}) \simeq p_{2\tau}^0 N_{B-L}^{T \simeq T_{B2}} - \frac{\Delta p_{2\tau}}{2} N_{N_2}^{\text{in}},
\]

\[
N_{\tau^\perp \tau^\perp}^{B-L}(T \simeq T_{B2}) \simeq p_{2\tau^\perp}^0 N_{B-L}^{T \simeq T_{B2}} + \frac{\Delta p_{2\tau}}{2} N_{N_2}^{\text{in}}.
\]

Again, at temperatures below \( T^* \ll 10^{12} \text{GeV} \), the \( N_{\alpha \beta}^{B-L} \) off-diagonal terms have been fully damped by the tauon charged interactions, so that the \( N_{\Delta \tau} \) and \( N_{\Delta \tau^\perp} \) asymmetries, corresponding to the diagonal terms, can be treated as measured quantities.

At \( T \sim T_{B2} \), the phantom terms in the Eqs. (52) cancel with each other and they do not contribute to the total asymmetry. Therefore, so far, the description of the asymmetry evolution is completely analogous to that one in the \( N_1 \)-dominated scenario.

However, there is still a third stage to be taken into account: the lightest RH neutrino washout. For \( T \sim M_1 \), the tauon and the \( \tau^\perp \) asymmetries are washed out by the lightest RH neutrino inverse processes. At \( T \simeq T_{B1} = M_1/z_{B1} \), they get frozen to their final values

\[
N_{\Delta \tau}^f \simeq \left[ p_{2\tau}^0 N_{B-L}^{T \simeq T_{B2}} - \frac{\Delta p_{2\tau}}{2} N_{N_2}^{\text{in}} \right] e^{-\frac{3\pi}{8} K_{1\tau}},
\]

\[
N_{\Delta \tau^\perp}^f \simeq \left[ p_{2\tau^\perp}^0 N_{B-L}^{T \simeq T_{B2}} + \frac{\Delta p_{2\tau}}{2} N_{N_2}^{\text{in}} \right] e^{-\frac{3\pi}{8} K_{1\tau^\perp}},
\]

(54)
so that the final total asymmetry $N_{B-L}^f \simeq N_{\Delta\tau}^f + N_{\Delta\tau\perp}^f$. If for the flavour $\alpha = \tau (\tau^\perp)$ one has $K_{1\alpha} \lesssim 1$, while for the other flavour $\beta = \tau (\tau^\perp)$ one has $K_{1\beta} \gg 1$, the final asymmetry will be dominated by the $\alpha$ asymmetry,

$$N_{B-L}^f \simeq p_{2\alpha}^0 N_{B-L}^{T \simeq T_{B2}} = \frac{\Delta p_{2\alpha}}{2} N_{N_2}^{\text{in}}.$$  \hfill (56)

The interesting thing is that the phantom term is not affected by the washout at the production, in a way that, if this is very strong, the phantom term can dominate the final asymmetry if $N_{N_2}^{\text{in}}$ is non-vanishing and sufficiently large (phantom leptogenesis). This result has different implications. As noticed [18, 17], phantom terms introduce a further source of dependence of the final asymmetry on the initial conditions. Therefore, phantom leptogenesis scenarios necessary rely on a description of the initial conditions. On the other hand, as we already noticed in the case of the $N_1$-dominated scenario, large phantom terms could also have some relevance at lower temperatures when the thermal bath is flavour sensitive.

Phantom leptogenesis has been first discussed within an instantaneous quantum state collapse description [18], here we have re-derived it within a density matrix formalism. Notice that there are three well separated stages: $N_2$ asymmetry production at $T \simeq T_{B2}$, decoherence at $T \sim T_*$ and asymmetric $N_1$ washout at $T \sim M_1$.

Notice also that phantom leptogenesis has some analogies with the scenario of $N_1$-leptogenesis with $\varepsilon_1 = 0$ [10] that we discussed in the previous section. In both cases the final asymmetry originate from the $CP$ violating terms $\propto \Delta p_{i\alpha}$ due to a different flavour composition of leptons and anti-leptons. In both cases a non-vanishing final asymmetry relies on an asymmetric washout acting on the two flavour asymmetries. There are however important differences. In the case of $N_1$ leptogenesis with $\varepsilon_1 = 0$ one has that production, decoherence and washout occur simultaneously, while in the case of phantom leptogenesis they occur at different stages and between the production and the $N_1$ washout stage the phantom terms they cancel in the final asymmetry. Another important difference is that in the case of phantom leptogenesis one has not to assume the special assumption $\varepsilon_1$ or $\varepsilon_2 = 0$ ($B-L$ conservation): it is enough that the washout at the production is sufficiently strong that phantom terms can naturally dominate. On the other hand phantom leptogenesis rely on non-vanishing and sufficiently large initial $N_2$ abundances.

As we are going to show, phantom leptogenesis is even more general and it does not necessarily require that the $N_2$ production and the $N_1$ washout stages occur in two different fully flavoured regimes.
3.2 Case $M_2 \gtrsim 3 M_1 \gg 10^{12}$ GeV: heavy neutrino flavour projection and two stages phantom leptogenesis

Let us now consider the case when both heavy neutrino masses $M_2, M_1 \gg 10^{12}$ GeV and charged lepton interactions do not affect the final asymmetry. This can be called the heavy flavoured scenario [17] since the only lepton flavours that affect the final asymmetry are those produced from the heavy RH neutrinos.

3.2.1 Projection effect (in isolation)

For illustrative purposes, we want first to describe just the $N_1$ washout of the asymmetry produced by the $N_2$ decays, without any additional effect. Therefore, we first neglect the different flavour compositions of leptons and anti-leptons assuming that $\Delta p_{1\alpha} = \Delta p_{2\alpha} = 0$.

With such a simplifying assumption, the lepton quantum states are given by

$$|1\rangle = C_{1\tau} |\tau\rangle + C_{1\tau^\perp} |\tau^\perp\rangle \quad \text{and} \quad |\bar{1}\rangle = C^*_{1\tau} |\bar{\tau}\rangle + C^*_{1\tau^\perp} |\bar{\tau}^\perp\rangle,$$

$$|2\rangle = C_{2\tau} |\tau\rangle + C_{2\tau^\perp} |\tau^\perp\rangle \quad \text{and} \quad |\bar{2}\rangle = C^*_{2\tau} |\bar{\tau}\rangle + C^*_{2\tau^\perp} |\bar{\tau}^\perp\rangle.$$  \hspace{1cm} (57)  

(58)

Assuming the hierarchical limit, $M_2 \gtrsim 3 M_1$ [36], there are two well distinguished different stages. In a first stage at $T \sim M_2$, an asymmetry is produced from $N_2$ decays. The lepton density matrix is then given by $\rho_{ijk}^L = \text{diag}(1, 0)$ in the basis $\ell_2 - \ell_2^\perp$. Analogously the anti-lepton density matrix is given by $\rho_{ijk}^L = \text{diag}(1, 0)$ in the basis $\bar{\ell}_2 - \bar{\ell}_2^\perp$ that for the moment we are assuming to be $CP$ conjugated of $\ell_2 - \ell_2^\perp$. As in the previous subsection, the asymmetry production from $N_2$ decays is again described by the Eq. (51) with vanishing phantom terms so that we simply have

$$N_{\tau\tau}^{B-L}(T \simeq T_{B2}) \simeq p_{2\tau}^0 N_{B-L}^{T\simeq T_{B2}}, \quad N_{\tau^\perp\tau^\perp}^{B-L}(T \simeq T_{B2}) \simeq p_{2\tau}^0 N_{B-L}^{T\simeq T_{B2}}, \quad (59)$$

where $N_{B-L}^{T\simeq T_{B2}} \simeq \varepsilon_2 \kappa(K_2)$. We have now to consider the $N_1$ washout stage at $T \sim M_1$. Since at the moment we are just interested in describing the $N_1$ washout, we also neglect the $N_1$ asymmetry production assuming a vanishing $\varepsilon_{(1)}^{(1)}$. Moreover let us first further assume $|1\rangle = |\tau\rangle$ and correspondingly $|\bar{1}\rangle = |\bar{\tau}\rangle$.

In this way, at $T \sim M_1$, the Eq. (50) for the asymmetry evolution in the charged lepton flavour basis simply reduces to $(\alpha, \beta = \tau, \tau^\perp)$

$$dN_{\alpha\beta}^{B-L} \over dz = -W_1 \left( \begin{array}{cc} \frac{1}{2} N_{\tau\tau}^{B-L} & \frac{1}{2} N_{\tau^\perp\tau^\perp}^{B-L} \\ \frac{1}{2} N_{\tau^\perp\tau\tau}^{B-L} & 0 \end{array} \right), \quad (60)$$

and, at the end of the $N_1$-washout at $T \simeq T_{B1}$, one simply finds

$$N_{\tau\tau}^{B-L}(T \simeq T_{B1}) \simeq e^{-\frac{3}{8}} K_1 \bar{p}_{2\tau}^0 N_{B-L}^{T\simeq T_{B2}}, \quad N_{\tau^\perp\tau^\perp}^{B-L}(T \simeq T_{B1}) \simeq \bar{p}_{2\tau}^0 N_{B-L}^{T\simeq T_{B2}}. \quad (61)$$
Finally, at $T \sim 10^{12}$ GeV, the charged lepton interactions damp the off-diagonal terms measuring the tauon and the ‘non-tauon’ (i.e. the $\tau^\perp$) asymmetries.

This result can be easily generalised. Let us, first of all, allow an arbitrary $|1\rangle$ flavour composition but continuing, for the time being, to neglect the $N_1$ asymmetry production, at $T \sim T_{B1}$. The Eq. (60) has now to be written in the basis $\ell_1 - \ell_1^\perp$:

$$\frac{dN_{B-L}^{\ell_1\ell_1}}{dz} = -W_1 \begin{pmatrix} N_{11}^{B-L} & \frac{1}{2} N_{11}^{B-L} \\ \frac{1}{2} N_{11}^{B-L} & 0 \end{pmatrix} \left( i_1, j_1 = 1, 1^\perp \right).$$

The solution is again quite trivial in this basis: the $11$ term is washed out,

$$N_{11}^{B-L}(T \sim T_{B1}) = e^{-\frac{3 \pi}{8} K_1} N_{11}^{B-L}(T \sim T_{B2}),$$

while all the other terms are unwashed. The asymmetry matrix at $T \sim T_{B2}$, in the $\ell_1 - \ell_1^\perp$ basis, can now be calculated in terms of the rotation matrices (cf. Eq. (31)) as

$$N_{i_1j_1}^{B-L}(T \sim T_{B2}) = N_{B-L}^{T_{B2}^\perp} R_{i_1\alpha}^0 R_{\alpha i_2}^0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} R_{j_2\beta}^0 R_{\beta j_1}^0.$$  

In a more compact way, considering that $N_{B-L}^{T\sim T_{B2}}(T \sim T_{B2}) = N_{B-L}^{T\sim T_{B2}}(2\langle 2 | 2 | 1 \rangle)$, this can be more conveniently written as

$$N_{i_1j_1}^{B-L}(T \sim T_{B2}) = N_{B-L}^{T_{B2}^\perp} \begin{pmatrix} p_{12} & \langle 1 | 2 \rangle \langle 2 | 1^\perp \rangle \\ \langle 1^\perp | 2 \rangle \langle 2 | 1 \rangle & 1 - p_{12} \end{pmatrix},$$

where

$$p_{12} \equiv |\langle \ell_1 | \ell_2 \rangle|^2 = \frac{|(\hbar^\dagger \hbar)_{12}|^2}{(\hbar^\dagger \hbar)_{11} (\hbar^\dagger \hbar)_{22}}.$$  

The final asymmetry can then be calculated as

$$N_{B-L}^f = \text{Tr}[N_{i_1j_1}^{B-L}(T \sim T_{B1})] = e^{-\frac{3 \pi}{8} K_1} p_{12} N_{B-L}^{T_{B2}^\perp} + (1 - p_{12}) N_{B-L}^{T_{B2}^\perp}.$$

The asymmetry can be also rotated in the charged lepton flavour basis,

$$N_{\alpha\beta}^{B-L}(T \sim T_{B1}) = R_{i_1\alpha}^{0(1)} N_{i_1j_1}^{B-L}(T \sim T_{B1}) R_{j_1\beta}^{0(1)}.$$  

At $T \approx 10^{12}$ GeV the charged lepton interactions just damp the off-diagonal terms without affecting the total asymmetry given by the trace and for this reason we could directly write the Eq. (67).

This result fully confirms what one expects within an instantaneous quantum state collapse description. It is not only confirmed that just the orthogonal component (in our
simplified case the $\tau^+$ component) undergoes the $N_1$ washout while the parallel component (the $\tau$ component) completely escapes it \[6, 20\], but also that the washout of the parallel component is exactly described by the factor $\exp[-(3\pi K_1/8)]$, independently of the value of $K_1$ \[17\]. Notice that in an intermediate regime $K_1 \sim 1$, the quantum states at $T \simeq T_{B1}$ are left in a sort of partially incoherent mixture, with some residual flavour oscillations that however do not affect the total asymmetry.

Notice that this result also applies to a possible pre-existing asymmetry produced by some other external mechanism \[20, 17\]. Therefore, the conclusions of \[17\], employing this result in various situations, are also confirmed.

One can then easily further generalise this result accounting also for a possible $N_1$ asymmetry generation, simply obtaining for the final asymmetry

$$N^f_{B-L} = \varepsilon_1 \kappa(K_1) + \left( e^{-\frac{3\pi}{8} K_1} p_{12} + 1 - p_{12} \right) \varepsilon_2 \kappa(K_2) .$$  \[(69)\]

### 3.2.2 Projection effect in combination with phantom leptogenesis

We still miss the (usual) last step. We have so far assumed that the flavour compositions of the $\ell_2$ and $\bar{\ell}_2$ quantum states are the same. We want now to show that when this additional flavoured CP violation contribution is taken into account, again phantom terms appear in the expression Eq. \[(69)\].

Therefore, the quantum states $|2\rangle$ and $|\bar{2}\rangle$ have now to be written more generally as

$$|2\rangle = |1\rangle |2\rangle + |1\perp\rangle |2\rangle \quad \text{and} \quad |\bar{2}\rangle = |\bar{1}\rangle |\bar{2}\rangle + |\bar{1}\perp\rangle |\bar{2}\rangle .$$  \[(70)\]

Notice that this time the role played by the charged lepton flavour basis in the previous subsection, is replaced by the heavy neutrino lepton basis $\ell_1-\ell_1^\perp$. Therefore, writing the Eq. \[(51)\] in this basis, we obtain

$$\frac{dN^B_{i_{j1}}}{dz} = \varepsilon_{i_{j1}}^{(2)} D_2 \left( N_{N_2} - N_{N_2}^{eq} \right) - W_2 N_{B-L} \left( \begin{array}{c} p_{12}^0 \\ \langle 1\perp |2\rangle \langle 2|1\rangle^0 \\ 1 - p_{12}^0 \end{array} \right) ,$$  \[(71)\]

where the superscript “0” indicates, as usual, the tree level quantities that can be approximately employed in the calculation of the washout term. In this way we obtain expressions for the heavy neutrino flavoured asymmetries, that are analogous to the Eqs. \[(52)\] for the charged lepton flavoured asymmetries,

$$N^B_{11} (T \simeq T_{B2}) \simeq p_{12}^0 \varepsilon_2 \kappa(K_2) - \frac{\Delta p_{12}}{2} N_{N_2}^{in} ,$$  \[(72)\]

$$N^B_{11\perp} (T \simeq T_{B2}) \simeq (1 - p_{12}^0) \varepsilon_2 \kappa(K_2) + \frac{\Delta p_{12}}{2} N_{N_2}^{in} .$$  \[(73)\]
Finally, taking into account the lightest RH neutrino washout, we obtain for the final asymmetry

\[ N_{B-L}^f = \varepsilon_1 \kappa(K_1) + \left[ p_{12}^0 e^{-\frac{3\pi}{8} K_1} + (1 - p_{12}^0) \right] \varepsilon_2 \kappa(K_2) + \left( 1 - e^{-\frac{3\pi}{8} K_1} \right) \frac{\Delta p_{12}}{2} N_{N_2}^{\text{in}}. \] (74)

Therefore, the phantom terms give an additional contribution to both components and in particular to the orthogonal component. As usual this contribution is \((N_2)\) unwashed at the production and proportional to the \(N_2\) initial abundance. If \(K_1 \ll 1\), both the parallel and the orthogonal components are unwashed and the phantom terms cancel with each other. On the other hand, in the opposite case, for \(K_1 \gg 1\), the parallel component is completely washed out so that only the orthogonal one survives together with the additional \((N_1)\) unwashed phantom term contribution.

This result shows that phantom leptogenesis goes even beyond the case where the two RH neutrinos masses fall into two different flavour regimes [18].

Finally, it should be clear that an account of the different flavour compositions of the \(\ell_1\) and \(\bar{\ell}_1\) quantum states, leads to additional phantom terms. These, however, cancel with each other and do not contribute to the final asymmetry, as already discussed in section 2.

### 3.3 Case \(10^{12} \text{ GeV} \gg M_1, M_2\)

When \(M_1, M_2 \ll 10^{12} \text{ GeV}\) both RH neutrinos produce their asymmetry in the two-flavour regime. The production from the heavier RH neutrino is given by the usual result

\[ N_{\tau \tau}^{B-L}(T \simeq T_{B2}) = \varepsilon_{2\tau} \kappa(K_{2\tau}), \quad N_{\tau^\perp \tau^\perp}^{B-L}(T \simeq T_{B2}) = \varepsilon_{2\tau^\perp} \kappa(K_{2\tau^\perp}). \] (75)

In the strong washout regime for both flavours, \(K_{2\tau}, K_{2\tau^\perp} \gg 1\), the sum, the total asymmetry, can be approximated by the Eq. (17) rewritten for the heavier RH neutrino. When the temperature drops down to \(T \sim T_{B1}\), the washout from the lighter RH neutrino starts to act. Similarly to the previous cases, this washout factorizes from the general expression and can be expressed as a simple exponential pre-factor. We have

\[ N_{\tau \tau}^{B-L}(T \simeq T_{B2}) = \varepsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{3\pi}{8} K_{1\tau}}, \] (76)
\[ N_{\tau^\perp \tau^\perp}^{B-L}(T \simeq T_{B2}) = \varepsilon_{2\tau^\perp} \kappa(K_{2\tau^\perp}) e^{-\frac{3\pi}{8} K_{1\tau^\perp}}. \] (77)

The production of the asymmetry from the \(N_1\) decays is then added to what is left from the \(N_2\) production, so that we finally obtain

\[ N_{\tau \tau}^{B-L}(T \simeq T_{B1}) = \varepsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{3\pi}{8} K_{1\tau}} + \varepsilon_{1\tau} \kappa(K_{1\tau}), \] (78)
\[ N_{\tau^\perp \tau^\perp}^{B-L}(T \simeq T_{B1}) = \varepsilon_{2\tau^\perp} \kappa(K_{2\tau^\perp}) e^{-\frac{3\pi}{8} K_{1\tau^\perp}} + \varepsilon_{1\tau^\perp} \kappa(K_{1\tau^\perp}). \] (79)
It should be noted that there are no phantom terms in this formula because of the assumption made at the beginning of the Section that $\tau_2^1 = \tau_1^1 \equiv \tau^1$. In this case, we have an effective two-flavour problem and there is no hidden asymmetry left unwashed at production and, therefore, no phantom terms. If we relax this assumption and have $\tau_2^1 \neq \tau_1^1$, we have to work in a full three-flavour basis, and, as we will see, phantom terms will appear again in the final asymmetry. We will study such a case in the next Section.

4 General case with three charged lepton flavours and three heavy neutrino flavours

If we consider the general realistic case where the full three lepton flavour space is considered, the density matrix equation has to be written in term of $3 \times 3$ matrices. In general the three heavy neutrino flavours have no particular flavour orientations in the three charged lepton flavour space (see Fig. 4). If we also consider generic three RH neutrinos mass patterns with masses $M_i \gg 10^6$ GeV, the density matrix equations (50) further generalise into $(\alpha, \beta = \tau, \mu, e)$.
\[
\frac{dN_{B-L}^{\alpha \beta}}{dz} = \varepsilon_{\alpha \beta}^{(1)} D_1 (N_{N_1} - N_{eq}^{N_1}) - \frac{1}{2} W_1 \{ P^{0(1)}_{0}, N^{B-L}_{B-L} \}_{\alpha \beta} \\
+ \varepsilon_{\alpha \beta}^{(2)} D_2 (N_{N_2} - N_{eq}^{N_2}) - \frac{1}{2} W_2 \{ P^{0(2)}_{0}, N^{B-L}_{B-L} \}_{\alpha \beta} \\
+ \varepsilon_{\alpha \beta}^{(3)} D_3 (N_{N_3} - N_{eq}^{N_3}) - \frac{1}{2} W_3 \{ P^{0(3)}_{0}, N^{B-L}_{B-L} \}_{\alpha \beta} \\
+ i \text{Re}(\Lambda_{\tau}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}}_{B-L} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, [\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L}_{B-L}]_{\alpha \beta} \\
+ i \text{Re}(\Lambda_{\mu}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}}_{B-L} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, [\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L}_{B-L}]_{\alpha \beta}.
\]

If one of the three masses is lower than \( \sim 10^6 \text{ GeV} \), electron flavour interactions terms have to be included as well, though they have no real impact, within this framework, on the final asymmetry. This is because the electron asymmetry is in any case already measured as a ‘neither-muon-nor-tauon’ asymmetry.

This master equation can now be used to calculate the final asymmetry not only for all the 10 mass patterns shown in Fig. 1, but also when the \( M_i \)’s fall in one of the flavour transition regimes.

Notice that, though in this paper we are only considering hierarchical RH neutrino mass patterns, this equation can also be used to calculate the asymmetry beyond the hierarchical limit [36] and even in the resonant case [37]. In this latter case, however, many different effects can become important and should be included [38].

Solutions of this set of equations are particularly difficult when at least two of the five kinds of interactions are simultaneously effective, something that goes beyond our objectives. Here, as an example with three flavours, we want to show a particularly interesting asymptotic limit that could not be described within the simplified two-flavour case discussed in the previous section: the two RH neutrino model [15]. We will show that, even in this case, phantom terms have in general to be taken into account.

### 4.1 Boltzmann equations for the two RH neutrino model

We consider a two RH neutrino model [39] corresponding to a situation where \( M_3 \) is sufficiently large (\( M_3 \gg 10^{14} \text{ GeV} \)) to decouple in the seesaw formula for the calculation of the neutrino masses [40]. In order to reproduce the observed baryon asymmetry one has to impose \( M_1 \gg 10^9 \text{ GeV} \) so that the muon interactions can be neglected in the Eq. (80).
On the other hand, in order to have $M_1$ and $M_2$ as low as possible, it is interesting to consider the case $10^9 \text{GeV} \ll M_1 \ll M_2 \ll 10^{12} \text{GeV}$ in a way to obtain a RH neutrino mass spectrum corresponding to the third panel (starting from upper left) in Fig. 1.

This model has been recently revisited in [15]. We want here to re-derive, starting from the density matrix equation Eq. (80), the Boltzmann kinetic equations and the consequent formula for the final asymmetry that in [15] has been used to calculate the value of $M_1$ necessary to reproduce the observed baryon asymmetry.$^5$

Thanks to the hierarchical limit, we can again introduce different simplifications. First of all we can impose the complete damping of the $\tau \alpha$ and $\alpha \tau$ ($\alpha \neq \tau$) off-diagonal terms in the asymmetry matrix.

Second, we can consider the $N_2$ production at $T \simeq T_{B2}$. With these assumptions, only the $N_2$-terms can be considered in the Eq. (80) and the asymmetry matrix can be treated as a $2 \times 2$ matrix in $\tau - \tau_2^\perp$ flavour space. In this way the density matrix equation reduce to a set of two Boltzmann equations in an effective full two flavour regime,

$$\frac{dN_{\tau \tau}^{B-L}}{dz} = \varepsilon_{\tau \tau}^{(2)} D_2 (N_{N_2} - N_{N_2}^{\text{eq}}) - p_{1\tau}^0 W_2 N_{\tau \tau}^{B-L},$$  \hspace{1cm} (81)$$

$$\frac{dN_{\tau_2^\perp \tau_2^\perp}^{B-L}}{dz} = \varepsilon_{\tau_2^\perp \tau_2^\perp}^{(2)} D_2 (N_{N_2} - N_{N_2}^{\text{eq}}) - p_{1\tau_2^\perp}^0 W_2 N_{\tau_2^\perp \tau_2^\perp}^{B-L}.$$  \hspace{1cm} (82)

As usual, assuming first for simplicity that the $|\tau_2^\perp\rangle$ and the $|\bar{\tau}_2^\perp\rangle$ quantum states have the same flavour compositions, one finds

$$N_{\tau \tau}^{B-L}(T \simeq T_{B2}) = \varepsilon_{2\tau} \kappa(K_{2\tau}) \text{ and } N_{\tau_2^\perp \tau_2^\perp}^{B-L}(T \simeq T_{B2}) = \varepsilon_{2\tau_2^\perp} \kappa(K_{2\tau_2^\perp}).$$  \hspace{1cm} (83)

These values of the asymmetries at the end of the $N_2$ production stage have to be used as initial values in the set of equations describing the evolution of the asymmetries during the $N_1$ production,

$$\frac{dN_{\tau \tau}^{B-L}}{dz} = \varepsilon_{\tau \tau}^{(1)} D_1 (N_{N_1} - N_{N_1}^{\text{eq}}) - p_{1\tau}^0 W_1 N_{\tau \tau}^{B-L},$$  \hspace{1cm} (84)$$

$$\frac{dN_{\tau_1^\perp \tau_1^\perp}^{B-L}}{dz} = \varepsilon_{\tau_1^\perp \tau_1^\perp}^{(1)} D_1 (N_{N_1} - N_{N_1}^{\text{eq}}) - p_{1\tau_1^\perp}^0 W_1 N_{\tau_1^\perp \tau_1^\perp}^{B-L}.$$  \hspace{1cm} (85)

The $\tau_2^\perp$ component of the asymmetry at the end of the $N_2$ production has to be decomposed into a $\tau_1^\perp$ parallel component and into a $\tau_1^\perp$ orthogonal component that we indicate with the symbol $\tau_1^\perp$. In this way one finds that the final asymmetry is the sum of three flavour components (see Fig. 5),

$^5$It has been shown in [15] that even the $N_2$ production depends just on $M_1$ and not on $M_2$, provided that this is much smaller than $10^{12} \text{GeV}$.
Figure 5: Relevant lepton flavours in the two RH neutrino model.

\[ N^f_{B-L} = N^B_{\tau\tau}(T \simeq T_{B1}) + N^B_{\tau_1^+\tau_1^-}(T \simeq T_{B1}) + N^B_{\tau_1^+\tau_1^-}(T \simeq T_{B1}), \]  

where

\[ N^B_{\tau\tau}(T \simeq T_{B1}) = \varepsilon_{1\tau} \kappa(K_{1\tau}) + \varepsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{3\pi}{8} K_{1\tau}}, \]  

\[ N^B_{\tau_1^+\tau_1^-}(T \simeq T_{B1}) = \varepsilon_{1\tau_1^+} \kappa(K_{1\tau_1^+}) + p^0_{\tau_1^+\tau_2} \varepsilon_{2\tau_2} \kappa(K_{2\tau_2}) e^{-\frac{3\pi}{8} K_{1\tau_1^+}}, \]  

\[ N^B_{\tau_1^+\tau_1^-}(T \simeq T_{B1}) = (1 - p^0_{\tau_1^+\tau_2}) \varepsilon_{2\tau_2} \kappa(K_{2\tau_2}). \]

This expression coincides with the result found in [15] and is strictly valid for vanishing initial \( N_2 \) abundance.

More generally, if \( N_{N_2} \neq 0 \), one has also to include the phantom terms allowing for different flavour compositions between the \( |\tau_1^\pm\rangle \) and the \( |\bar{\tau}_2^\mp\rangle \) quantum states. The procedure is essentially the same as in Section 3.2, with the only difference that now the phantom terms will appear only in the \( \tau_1^+ \) and \( \tau_1^- \) components but not in the measured tauon component. We can therefore directly write the final result,

\[ N^f_{B-L} = \varepsilon_{1\tau} \kappa(K_{1\tau}) + \varepsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{3\pi}{8} K_{1\tau}}, \]  

\[ \quad + \varepsilon_{1\tau_1^+} \kappa(K_{1\tau_1^+}) + \left(p^0_{\tau_1^+\tau_2} \varepsilon_{2\tau_2} \kappa(K_{2\tau_2}) - \frac{\Delta p_{\tau_1^+\tau_2}}{2} N_{N_2}^{in} \right) e^{-\frac{3\pi}{8} K_{1\tau_1^+}}, \]  

\[ \quad + \left(1 - p^0_{\tau_1^+\tau_2}\right) \varepsilon_{2\tau_2} \kappa(K_{2\tau_2}) + \frac{\Delta p_{\tau_1^+\tau_2}}{2} N_{N_2}^{in}, \]  

26
where each of the three lines corresponds respectively to the $\tau$, $\tau_1^\perp$ and $\tau_1^{\perp\perp}$ components. This last example shows, once more, how phantom terms are present all times that the production occurs either in a two or in a three flavour regime though only those generated by the heavier RH neutrinos can be afterwards asymmetrically washed out by the lighter RH neutrinos and contribute to the final asymmetry without cancelling with each other.

5 Final discussion

Within a Boltzmann classical kinetic formalism one has to distinguish the ten different RH neutrino mass patterns shown in Fig. 1. These are obtained in the limits where the masses $M_i$ are hierarchical and do not fall in the transition regimes. We have extended the density matrix formalism for the calculation of the matter-anti matter asymmetry in leptogenesis including heavy neutrino flavours. In this way we obtained a unique density matrix equation describing the asymmetry for any choice of the RH neutrino masses, even beyond the hierarchical limit.

Within this more general description, the ten hierarchical RH neutrino mass patterns of Fig. 1 correspond to those cases where the (five) different interactions are only singularly effective within a given range of temperatures. In this way the evolution of the asymmetry can be described in well separated stages where the density matrix equations greatly simplify reducing to multiple sets of Boltzmann equations, one for each stage. In this cases we recovered and extended results that were already derived within a description based on an instantaneous collapse of lepton quantum states.

The flavour projection effect, where the orthogonal component of a previously produced asymmetry escapes the RH neutrino washout, is fully confirmed. We have also shown that the washout of the parallel component is exactly described by the usual exponential washout factor independently of the washout regime.

Phantom terms emerge as quite a generic feature of flavoured leptogenesis and have to be taken into account for non-vanishing initial RH neutrino abundances. We have indeed shown how their presence goes beyond the $N_2$-dominated scenario where they were originally discussed [18]. They can contribute to the final asymmetry even if the production from an heavier RH neutrino species and the washout from a lighter RH neutrino species occur in the same flavour regime. We have also noticed that, even though in the $N_1$-dominated scenario they exactly cancel with each other and do not contribute to the final asymmetry, they can give rise to flavoured asymmetries much larger than the observed baryon asymmetry with potential interesting applications, for example in active-sterile neutrino oscillations.
Even though we have explicitly calculated the final asymmetry only in one of the ten asymptotic limits RH neutrino mass patterns shown in Fig. 1, in the case of the two RH neutrino model, the procedure can be easily extended to all others neutrino mass patterns. For example one can easily show the expression for the final asymmetry in the $N_2$-dominated scenario, when $M_1 < 10^{9}$ Gev [13, 14, 41, 18].

It would be desirable in future to calculate the asymmetry beyond these ten asymptotic limits, solving the full density matrix equation. In this way the calculation of the matter-anti matter asymmetry would be extended to a generic RH neutrino mass pattern, including the cases where the RH neutrino masses fall in the transition regimes between two fully flavoured regimes and quantum decoherence from charged lepton interactions acts simultaneously with asymmetry generation and wash-out. This would make possible to interpolate between the asymptotic limits, finding the exact conditions on the RH neutrino masses for the validity of the solutions that we have discussed here.

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