Connection between In Medium Nucleon Form Factors and Deep Inelastic Structure Functions

S. Liuti

1Physics Department, University of Virginia, 382 McCormick Rd., Charlottesville, Virginia 22904, USA.

We present a connection between the modifications induced by the nuclear medium of the nucleon form factors and of the deep inelastic structure functions, obtained using the concept of generalized parton distributions. Generalized parton distributions allow us to access elements of the partonic structure that are common to both the hard inclusive and exclusive scattering processes in nuclei.

PACS numbers: 13.60.Hb, 13.40.Gp, 24.85.+p

An important question that remains currently unsolved in Quantum Chromodynamics (QCD), is the determination of the quark and gluon structure of nuclei. Theoretical efforts have so far concentrated on two apparently distinct areas. Many studies were dedicated on one side to pinning down the mechanisms producing nuclear medium modifications of quark and gluon momentum distributions from inclusive (and semi-inclusive) deep inelastic scattering experiments. Sizable $A$-dependent effects have in fact been observed during the past two decades (see the review in Ref. [1]), clearly indicating a non-trivial deep inelastic structure of the nucleus beyond its naive description as a collection of weakly bound nucleons whose quark and gluon structure is unaffected by the nuclear forces. On the other side, a number of quasi-elastic both inclusive and exclusive electron-nucleus scattering experiments allow for investigations of the nucleon form factors of bound nucleons. Their initial outcome is also suggestive of non trivial deformations of the charge and magnetic current distributions of nucleons inside the nuclear medium.

Recently, a more comprehensive object, the Generalized Parton Distribution (GPD) was introduced that interpolates between the Parton Distribution Functions (PDFs) from Deep Inelastic Scattering (DIS), and the hadronic form factors. A connection was subsequently unraveled between GPDs and the impact parameter dependent parton distributions, which allows us to study simultaneously both the light cone momentum and transverse spatial distributions of partons. At leading order, GPDs in nuclei are best visualized in terms of the soft parts in the Deeply Virtual Compton Scattering (DVCS) process described in Fig. 1.

GPDs allow us to make the connection, in this paper, between the in medium modifications of the nucleon form factor, and the modifications of the deep inelastic structure functions of bound nucleons. We will consider $^4$He as an ideal nuclear target for our calculations, since its binding energy per nucleon is strong compared to other few nucleon systems, and at the same time, by avoiding extra complications due to non zero spin components it allows us to focus more directly on nuclear medium modifications. Evaluations of GPDs in spin zero nuclei were performed in [2, 3, 4, 5]. To calculate the amplitude for nuclear DVCS we define (see Fig. 1): $P_A$, $P$, $k$, representing the nuclear, the active nucleon’s and active quark’s four-momenta; $P_A' = P_A - \Delta$, $P' = P - \Delta$, and $k' = k - \Delta$ are the final nuclear, nucleon’s and quark momenta, respectively; $q$ is the virtual photon momentum, and $q' = q + \Delta$ is the final photon’s momentum. The relevant invariants are: $X = k^+/(P_A^+/A)$, $Y = P^+/P_A^+/A)$, $X_N = X/Y \equiv k^+/P^+$, $\zeta = -\Delta^+/(P_A^+/A)$, $\zeta_Y = \zeta/Y \equiv -\Delta^+/P^+$, and $q^2 = -Q^2$, $t = -\Delta^2$ (we use the notation: $p^+ = 1/\sqrt{\sigma}(p_0 \pm p_3)$, with $(pk) = p^+k^− + p^−k^+ - p_1k_1$). In this paper we focus on the quark region defined in a nucleus by $\zeta < X < A$, whereby the GPD for a spin zero nucleus reads [5, 6].

\[
H^A(X, \zeta, t) = \int \frac{dY d^2 P_\perp}{2(2\pi)^3} \frac{A}{(A-Y)} \rho_A (Y, P^2, t) \sqrt{\frac{Y - \zeta}{Y}} \left[ \hat{H}^N \left( \frac{X}{Y}, \zeta, P^2, t \right) - \frac{1}{4} \frac{(\zeta/Y)^2}{1 - \zeta/Y} \hat{E}^N \left( \frac{X}{Y}, \zeta, P^2, t \right) \right].
\]

(1)
\[ \rho_A(Y, P^2, t) \] is the light cone (LC) nucleon momentum distribution which depends explicitly on the nucleon's virtuality, \( P^2 \equiv P^2(Y, P^2) \neq M_N^2; A \) is a kinematical factor. \( \hat{H}^N \) and \( \tilde{E}^N \) are the GPDs for the bound nucleon which are intrinsically modified with respect to the free nucleon ones. They are assumed to have a flavor decomposition similar to the off-shell case:

\[ \hat{H}^N = \frac{2}{3} \hat{H}_u - \frac{1}{3} \hat{H}_d - \frac{1}{3} \hat{H}_s \]  

(2)

The nuclear PDFs, \( q^A(X) \), and form factor, \( F^A(t) \), are defined from Eq. (1) by respectively taking the limit \((\zeta, t) \to 0\) and by integrating \(H^A\) over \(X\):

\[ q^A(X) = H^A_u(X, 0) = q = u, d, s \]  

(3)

\[ F^A(t) = \int_0^A dX H^A(X, t). \]  

(4)

Since we are interested in the connection with the form factor, we consider \( \zeta = 0 \), so that Eq. (1) reduces to:

\[ H^A(X, t) = \int_0^A dX d^2 \rho_A(Y, P^2, t) \hat{H}^N \left( \frac{X}{Y}, P^2, t \right), \]  

(5)

If the intrinsic modifications of the nucleon GPD are disregarded, i.e. by assuming \( \hat{H}^N(X_N, P^2, t) \approx H^N(X_N, t) \), \( H_A \) is further reduced to a LC longitudinal convolution:

\[ H_{\text{LC}}^A(X, t) = \int_0^A dY f_A(Y, t) H^N \left( \frac{X}{Y}, t \right), \]  

(6)

\[ f_A(Y, t) = \int d^2 \rho_A(Y, P^2, t). \]  

(7)

Deviations from LC convolution are crucial for the description of both the “forward” and “off-forward” EMC effects. The approach summarized in Eqs. (6) in fact fails to describe nuclear DIS and in medium form factors. A number of mean field based calculations have instead been put forth that take into account modifications of nucleon structure due to the joint effect of attractive scalar and repulsive vector interactions generated in the average field of the nucleus. These impact Eq. (6) through the replacement: \( H^N(X_N, t) \to H^N[X_N, t, P^2(\rho)] \), \( \rho \) being the nuclear density. Based on precisely this type of off-shell effect the authors of [14, 15] provide an explanation of both nuclear DIS and in medium form factors.

In our approach, we propose that transverse degrees of freedom, that are introduced in a nucleus along with off-shell effects, do not decouple as in Eqs. (6) and (7) but are being merely integrated over, but they play an explicit role.

We illustrate the mechanism for obtaining what we define as “active-\( k_{\perp} \)” effects, within Hard Scattering Factorization where the free nucleon GPD, \( H_q \), can be written as:

\[ H_q = \frac{X}{1 - X} \int dk_X^2 \int \frac{d^2 k_{\perp}}{(2\pi)^2} \rho_q(k^2, k'^2, k_X^2) \]  

(8)

\( k^2 \) and \( k'^2 \) are the initial and final quarks virtualities related to \( k^2_+ \), and \( (k_{\perp} + (1 - X)\Delta)^2 \), respectively; \( k_X = P - k; \rho_q \propto \text{Tr} \{ \gamma^0 M \} \), \( M \) being the Fourier transform of the correlator at the nucleon blob.

We assume that nuclear medium modifications of \( H_q \) originate similarly to the forward case, with a few important differences that we discuss below. At large \( X \) (\( X \geq 0.2 \)) we adopt a quark-diquark model for the in medium \( H_q \), the diquark being a scalar with fixed mass, \( k_X^2 \equiv M_X^2 \). The relationship between the quarks virtualities and transverse momenta is shifted in a nucleus with respect to the free nucleon one in an \( A \)-dependent way:

\[ k^2 = X_N P^2 - \frac{X_N}{1 - X_N} M^2_X - \frac{(k_{\perp} - X_N P_{\perp})^2}{1 - X_N}, \]

with \( P^2 = [(Y/A)M_A^2 - (M_{A-1} + P_{\perp}^2)Y/(A - Y) - P_{\perp}^2], \) \( M_A \) and \( M_{A-1} \), being the masses of the initial nucleus and of the spectator \( A - 1 \) nucleons system, respectively (similar relations hold for \( k'^2 \)). This generates a further “rescaling” in the \( X \) dependence of \( H_q \) in a bound nucleon that adds on to the relatively small rescaling effect implicit in the LC convolution. As a result, active-\( k_{\perp} \) kinematical effects enhance the so-called binding correction to the bound nucleon structure functions.

Nuclear modifications are best presented in the off-forward case by plotting the ratio of the nuclear GPD over the nucleon one, normalized by their corresponding form factors:

\[ R_A(X, t) = \frac{H^A(X, t)}{H^N(X, t)} / \frac{F^A(t)}{F^N(t)} \]  

(9)

The kinematical effects are visible as a dip in the ratios plotted in Fig.2. The increase of the dip with \( P \) can be explained by observing that the off-shellness+binding correction, \( B_A(t) \):

\[ B_A(t) = \frac{\int_A dY \rho_A(Y, \langle P^2 \rangle, t)(1 - Y)}{\int_0^A dY \rho_A(Y, \langle P^2 \rangle, t)} \]  

(10)

grows in our model almost linearly with \( t \). When active-\( k_{\perp} \) effects are considered, i.e. \( \langle P^2 \rangle \neq M_N^2 \), one obtains a bigger shift in \( X \), and therefore an enhanced effect displayed in Fig.2.

Intrinsic dynamical modifications allow us in principle, to access the nucleon’s spatial deformations inside the nuclear medium. The latter have been so far taken into account in a variety of models by shifting the parameters that either represent directly, or are related to the confinement size in hadrons [14]. GPDs because of their interpretation in coordinate space, supersede ad hoc models by allowing for a direct evaluation of the in
medium spatial distributions of hadrons \[17\]. In the approach presented here, the origin of dynamical modifications is associated with the \(k_\perp\) dependent parton re-interactions generating shadowing (suppression of the in medium structure function with respect to the free nucleon one) and antishadowing (enhancement), at low \(X\) (\(X \lesssim 0.2\)). By inspecting the energy denominators dominating the lepton-nucleus scattering process, one sees that the mass of the spectator component is in this case, \(M \approx \frac{s}{1/X} \[18\]. \(\rho_1(k^2, k_\perp^2, k_X^2)\) in Eq.\[15\] is calculated in a Regge theory based model for the quark-nucleon amplitude, \(M \approx T_{qN} \[19\] \[20\]. In forward lepton-nucleus scattering \(T_{qN}\) is, in fact, assumed to have similar analytical properties as the proton-proton (proton-antiproton) amplitude. One can express it therefore in terms of Pomeron (P), Odderon (O), and other Reggeon (R) exchanges. As shown in \[11\] \[20\] nuclear shadowing and antishadowing arise because at low \(X\) the struck partonic configuration lives over several intra-nuclear distances and, as a consequence, it undergoes multiple scattering. The nuclear amplitude \(T_{qN}\) reflects a rather complicated combination of destructive (shadowing) and constructive (antishadowing) contributions from the different exchange terms entering Glauber’s multiple scattering series with same or opposite phases, respectively. In this paper we model the nuclear GPDs at low \(X\) and \(\zeta = 0\) by directly extrapolating from Ref.\[20\] \[i.e.\] by replacing the forward \(qN\) amplitudes used in \[20\] with the \(t \neq 0\) ones. Because of the absence of skewedness, we expect in fact the longitudinal distances that constitute the driving mechanism behind shadowing/antishadowing models, to behave similarly to the forward case. As for the \(t\) dependence, we obtain an increase with \(t\) in the antishadowing that tends to balance the effect at large \(X\) (Fig.\[2b\]), or a continuation of the trend set by fixing the parameters for \(t = 0\). The interplay between kinematical and dynamical effects might lead to a dominance of either the large \(X\) part (depletion) or the low \(X\) part (enhancement) of the structure function, lacking, in this case the constraint from baryon number conservation. Finally, we point out the difference between our approach and the one in Ref.\[22\] where nuclear modifications were estimated in a completely different situation, namely at \(t = 0\), and \(\zeta \neq 0\).

The modifications thus obtained by explicitly considering active-\(k_\perp\) effects impact directly the in medium form factor. We define it as:

\[
F_1(t) = \left[ \frac{F^A(t)}{F_{LC}^A(t)} \right] F_1^N(t) \tag{11}
\]

where:

\[
F_{LC}^A(t) = \int_0^A H_{LC}^A(X, t) dX = F_1^N(t) \int_0^A f_A(Y, t) dY. \tag{12}
\]

\(F^A(t)\) in Eq.\[11\] is given by Eqs.\[11\] and \[15\]; \(F_{LC}^A\) is calculated using Eq.\[3\]: \(F_1^N\) is the free nucleon form factor. According to our description of the \(t\) dependence of GPDs (Fig.\[2b\]), at \(t \approx 0.5\), the antishadowing enhancement starts dominating over the depletion given by both shadowing and the EMC effect, resulting in larger values for the in medium form factor \((F_1^N > F_1^N, \text{Fig.}2b)\). The dominance of antishadowing can also be understood by analyzing the \(X\) values that govern the form factor. As shown in \[23\] \[24\], the average value, \(X(t)\), increases with \(t\) with a slope such that at \(t \lesssim 4 \text{ GeV}^2\) the antishadowing region \((0.02 < X < 0.2)\) is dominating. In Fig.\[3\] we show a comparison with the results obtained in Ref.\[21\] using a Nambu-Jona-Lasinio (NJL) based effective quark theory (notice that we appropriately rescaled
the curves shown in [21] for a consistent comparison). Although there is agreement between the two models at low $t$, a discrepancy occurs around $t = 0.5$, perhaps due to the usage of the quark-diquark model in [21], as opposed to the shadowing/antishadowing description in our calculation.

We are, therefore, able to unravel a common origin between the modifications of the deep inelastic structure and the form factor of the proton. In our interpretation a key role is played by $k_{1\perp}$-dependent, leading order parton re-interactions in a nucleus, governed by Regge-type exchanges, at variance with the meson exchanges of Refs. [14, 17, 21]. Our findings point to a form of duality perhaps in a similar direction as recent phenomenological studies performed in [27].

What are the consequences of our results for phenomenology? In medium effects have been investigated in quasi-elastic electron scattering, both through the measurement of the Coulomb Sum Rule [2], and in polarization transfer experiments yielding the ratio of the nucleon’s electric and magnetic form factors through its proportionality to the transverse to longitudinal transfer experiments yielding the ratio of the nucleon’s Dirac form factor, are so far consistent with these findings. We furthermore predict the onset of saturation and an enhancement at larger values of $t$ of interest for future proposals [27].

In conclusion, we presented a connection using GPDs, between the in medium nucleon form factors, and the nuclear deep inelastic structure functions. The in medium form factor deformation follows a pattern in four-momentum transfer, $t$, with an initial suppression, up to $t \approx 0.5$, and a subsequent enhancement. This pattern is a direct consequence of the behavior with $X$ of the zero skewedness GPD, $H_A(X,t)$ which, because of the coupling in our model to transverse degrees of freedom in a nucleus ($k_{1\perp}$), displays enhanced antishadowing and EMC effects with increasing $t$. This reflects on the form factor’s behavior that, being given by the integral of the nuclear GPD, results from a balance of the same physical effects which are known to be present in deep inelastic scattering from nuclei.

I am indebted to Z.E. Meziani, N. Nikolaev, J. Ralston and A. Thomas for critical comments and discussions. I also thank W. Bentz for providing me with his calculations of the in medium form factors. This work is supported by the U.S. Department of Energy grant no. DE-FG02-01ER41200.