Neutron matter - Quark matter phase transition and Quark star

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We consider the neutron matter quark matter phase transition along with possible existence of hybrid quark stars. The equation of state for neutron matter is obtained using a nonperturbative method with pion dressing of the neutron matter and an analysis similar to that of symmetric nuclear matter. The quark matter sector is treated perturbatively in the small distance domain. For bag constant $B^{1/4}=148$ MeV, a first order phase transition is seen. In the context of neutron quark hybrid stars, Tolman-Oppenheimer-Volkoff equations are solved using the equations of state for quark matter and for neutron matter with a phase transition as noted earlier. Stable solutions for such stars are obtained with the Chandrasekhar limit as 1.58 $M_\odot$ and radius around 10 km. The bulk of the star is quark matter with a thin crust of neutron matter of less than a kilometer.
I. INTRODUCTION

It is widely believed that neutron matter undergoes a phase transition to quark matter at high densities and/or high temperatures. The high temperature limit is expected to have interesting consequences in heavy ion collision and/or cosmology, whereas high baryon density behaviour is important for the study of neutron stars.

The problem here for the treatment of nuclear matter or neutron matter equation of state is basically nonperturbative. A treatment of the same was developed by Walecka [1] which consists of interactions of nucleons with a neutral scalar field $\sigma$ as well as a neutral vector meson $\omega$. These calculations however use meson fields as classical, and, use a $\sigma$-field which is not observed. An alternative model for infinite nuclear matter consisting of interacting nucleons and pions was considered [2,3] recently where scalar isoscalar pion condensates simulate [4,5] the effects of $\sigma$ mesons with a phenomenological short distance repulsion arising from composite structure of nucleons or through vector meson exchanges. We extend this nonperturbative technique to neutron matter with the same parameters as for symmetric nuclear matter [3], and use this for hybrid stars.

We first consider neutron matter equation of state at finite temperature through thermofield dynamics [6] as earlier [3]. A first order phase transition between neutron matter and quark matter seems to be indicated. As stated, neutron matter is treated in a nonperturbative manner. Quark matter is treated perturbatively for high densities or short distances [7,8]. Solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations yield, with the above equations of state, a hybrid star having a quark core and a crust of neutron matter, the two states of matter being in equilibrium at the interface [9]. We obtain the Chandrasekhar limit for such hybrid stars as 1.58 $M_\odot$.

The outline of the paper is as follows. In Sec. II we shall consider neutron matter with pion condensates at zero temperature as well as finite temperature. In Sec. III we shall consider a standard form for quark matter with pressure given in terms of chemical potential and discuss about the phase transition from neutron matter to quark matter. Section IV
II. NEUTRON MATTER EQUATION OF STATE

We shall consider here neutron matter equation of state at finite temperatures similar to the treatment of the same for symmetric nuclear matter [3]. We start with the effective Hamiltonian for pion nucleon interactions [3,4] as

\[ H_N(\vec{x}) = H_N^0(\vec{x}) + H_{int}(\vec{x}) \]  

(1)

where the free nucleon part \( H_N^0 \) is given as

\[ H_N^0(\vec{x}) = \psi_I(\vec{x}) \epsilon_x \psi_I(\vec{x}) \]  

(2)

and the effective pion nucleon interaction part is given as

\[ H_{int}(\vec{x}) = \psi_I(\vec{x}) \left[ -iG^2 \frac{\vec{\sigma} \cdot \vec{p}}{2\epsilon_x} \phi + \frac{G^2}{2\epsilon_x} \phi^2 \right] \psi_I(\vec{x}). \]  

(3)

In the above, \( \epsilon_x = (M^2 - \nabla_x^2)^{1/2} \) where \( M \) denotes the nucleon mass. Furthermore the free meson part of the Hamiltonian is given as

\[ H_M(\vec{x}) = \frac{1}{2} \left[ \dot{\phi}_i^2 + (\vec{\nabla} \phi_i) \cdot (\vec{\nabla} \phi_i) + m^2 \phi_i^2 \right] \]  

(4)

Clearly in the above \( \psi_I \) refers to the large component of the nucleon spinor and \( \phi = \tau_i \phi_i \).

We expand the field operator \( \phi_i(\vec{z}) \) in terms of the creation and annihilation operators of off-mass shell mesons satisfying equal time algebra as

\[ \phi_i(\vec{z}) = \frac{1}{\sqrt{2\omega_z}} (a_i(\vec{z}) + a_i(\vec{z})) \]  

(5)

and

\[ \dot{\phi}_i(\vec{z}) = i \frac{\sqrt{\omega_z}}{2} (a_i(\vec{z}) - a_i(\vec{z})) \]  

(6)

will consist of calculations of the properties of hybrid stars such as total mass, radius and moment of inertia. In section V we summarise the results.
where we take with the perturbative basis $\omega_z = (m^2 - \vec{\nabla}^2_z)^{1/2}$, with $m$ denoting the mass of the meson.

We shall now consider finite temperature neutron matter. For this purpose we shall use the methodology of thermofield dynamics [3]. We shall have the temperature dependent background off-shell pion pair configuration given as

$$ |f,\beta> = U_I(\beta)|f> = U_I(\beta)|\text{vac}> $$  \hspace{1cm} (7)

where $U$ is given as

$$ U = \exp \left( \frac{1}{2} \int \tilde{f}(\vec{k}) a_i(\vec{k})^\dagger a_i(-\vec{k})^\dagger d\vec{k} - \text{h.c.} \right) $$  \hspace{1cm} (8)

and $U_I(\beta)$ describes the effect of temperature. The expression for this in terms of ordinary and thermal modes is given as [3,4]

$$ U_I(\beta) = \exp(B_I^\dagger(\beta) - B_I(\beta)) $$  \hspace{1cm} (9)

with

$$ B_I(\beta)^\dagger = \frac{1}{2} \int \theta_B(\vec{k},\beta) b_i(\vec{k})^\dagger b_i(-\vec{k})^\dagger d\vec{k} $$  \hspace{1cm} (10)

In the above,

$$ b_i(\vec{k})^\dagger = U a_i(\vec{k}) U^\dagger $$  \hspace{1cm} (11)

so that $b_i(\vec{k})|f> = 0$ and $\theta_B(\vec{k},\beta)$ is a function to be determined later. The parallel unitary transformation as in Eq.(9) for the temperature dependance in neutron sector is given as [3]

$$ U_{II}(\beta) = \exp(B_{II}^\dagger(\beta) - B_{II}(\beta)) $$  \hspace{1cm} (12)

with

$$ B_{II}(\beta)^\dagger = \frac{1}{2} \int \theta_F(\vec{k},\beta) \psi_I(\vec{k})^\dagger \psi_I(-\vec{k})^\dagger d\vec{k} $$  \hspace{1cm} (13)

where $\theta_F(\vec{k},\beta)$ will be determined later. Thus we have the neutron matter density
\[ \rho(\beta) = \langle \text{vac}|U_{II}(\beta)\psi_\alpha(\vec{x})\psi_\alpha(\vec{x})U_{II}(\beta)|\text{vac} \rangle \]
\[ = \gamma(2\pi)^{-3} \int d\vec{k} \sin^2 \theta_F \]  

(14)

where the degeneracy factor \( \gamma = 2 \) for neutron matter [3]. \( \sin^2 \theta_F \) is the distribution function for the fermions. Clearly, with \( \sin^2 \theta_F = \Theta(k_f - k) \), the step function, Eq. (14) gives \( \rho = \gamma k_f^3/6\pi^2 \) at zero temperature. For the interacting system \( \sin^2 \theta_F \) will be determined from the minimisation of the thermodynamic potential, as is done later.

We shall first calculate different contributions to the energy expectation values corresponding to the Hamiltonian as in Eq. (1) and (4). We thus have for the nucleon kinetic term

\[ h_f(\beta) = \langle \text{vac}|U_{II}(\beta)\psi_I(\vec{x})\psi_I(\vec{x})U_{II}(\beta)|\text{vac} \rangle \]
\[ = \gamma(2\pi)^{-3} \int d\vec{k} \frac{k^2}{2M} \sin^2 \theta_F \]  

(15)

The kinetic energy due to the mesons is given by

\[ h_k(\beta) = \langle f, \beta|\mathcal{H}_M(\vec{x})|f, \beta \rangle \]
\[ = 3(2\pi)^{-3} \int d\vec{k} \omega(\vec{k}) \left[ \sinh^2 \tilde{f}(\vec{k}) \cosh 2\theta_B(\vec{k}, \beta) + \sinh^2 \theta_B(\vec{k}, \beta) \right] \]  

(16)

where \( \omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2} \). We next derive the interaction energy density from Eq. (3) as [3]

\[ h_{int}(\beta) = \langle f, \beta|U_{II}(\beta)\psi_I(\vec{x})\psi_I(\vec{x})G^2 \phi^2(\vec{x})U_{II}(\beta)|f, \beta \rangle \]
\[ \simeq \frac{G^2}{2M} \rho(\beta) \langle f, \beta|: \phi_i(\vec{x})\phi_i(\vec{x}) : |f, \beta \rangle \]
\[ = \frac{G^2 \rho(\beta)}{2M} I_{2M} \]  

(17)

where

\[ I_{2M} = \frac{3}{(2\pi)^3} \int \frac{d\vec{k}}{\omega(\vec{k})} \left( \frac{\sinh 2\tilde{f}(\vec{k}) \cosh 2\theta_B}{2} + \sinh^2 \tilde{f}(\vec{k}) \cosh 2\theta_B \right) \sinh^2 \theta_B \]  

(18)

We shall now assume a phenomenological term corresponding to meson repulsion due to composite structure of mesons which is given as [3].
\[ h^R_m(\beta) = 3a(2\pi)^{-3} \int \left( \sinh^2 \tilde{f}(\vec{k}) \cosh^2 \theta_B + \sinh^2 \theta_B \right) e^{R_x k^2} d\vec{k}, \]  

(19)

where \( a \) and \( R_\pi \) are constants. Finally, the nucleon repulsion term is taken as

\[ h_R = \lambda \rho^2(\beta) \]  

(20)

where \( \rho(\beta) \) is as given in Eq. (14) and \( \lambda \) is another constant. This repulsion term could arise from \( \omega \)-exchange or otherwise.

Finally, the energy density is given as

\[ E(\beta) = (h_f(\beta) + h_m(\beta) + h_R(\beta))/\rho(\beta) \]  

(21)

where

\[ h_m(\beta) = h_k(\beta) + h^R_m(\beta) + h_{\text{int}}(\beta). \]

The thermodynamic potential density \( \Omega \) is given by

\[ \Omega(\beta) = E(\beta)\rho - \frac{\sigma}{\beta} - \mu_B \rho \]  

(22)

where the last term corresponds to nucleon number conservation with \( \mu_B \) as the chemical potential. We may note that we shall be considering temperatures much below the nucleon mass so that in the expression for \( \rho(\beta) \) we do not include antiparticle channel. The entropy density above is \( \sigma = \sigma_F + \sigma_B \) with \( \sigma_F \) being the entropy in fermion sector given as

\[ \sigma_F = -\frac{\gamma}{(2\pi)^3} \int d\vec{k} \left[ \sin^2 \theta_F(\vec{k},\beta) \ln(\sin^2 \theta_F(\vec{k},\beta)) + \cos^2 \theta_F(\vec{k},\beta) \ln(\cos^2 \theta_F(\vec{k},\beta)) \right]. \]

and similarly the meson sector contribution \( \sigma_B \) is given as

\[ \sigma_B = -\frac{3}{(2\pi)^3} \int d\vec{k} \left[ \sinh^2 \theta_B(\vec{k},\beta) \ln(\sinh^2 \theta_B(\vec{k},\beta)) - \cosh^2 \theta_B(\vec{k},\beta) \ln(\cosh^2 \theta_B(\vec{k},\beta)) \right]. \]

Thus the thermodynamic potential density \( \Omega \) is now a functional of \( \theta_F(\vec{k},\beta), \theta_B(\vec{k},\beta) \) as well as the pion dressing function \( \tilde{f}(\vec{k}) \) which will of course depend upon temperature. Extremisation of Eq (21) with respect to \( \tilde{f}(\vec{k}) \) yields
\[
tanh^2 \tilde{f}(\vec{k}) = - \frac{G^2 \rho}{2M} \cdot \frac{1}{\omega^2(\vec{k}) + \frac{G^2 \rho}{2M} + a \omega(\vec{k}) e^{R_\pi^2 k^2}}.
\]

Similarly minimising the thermodynamic potential with respect to \( \theta_B(\vec{k}, \beta) \) we get

\[
\sinh^2 \theta_B = \frac{1}{e^{\beta \omega'} - 1}
\]

where

\[
\omega' = (\omega + \frac{G^2 \rho}{2M \omega} + ae^{R_\pi^2 k^2}) \cosh 2 \tilde{f}(\vec{k}) + \frac{G^2 \rho}{2M \omega} \sinh 2 \tilde{f}(\vec{k})
\]

Once we substitute the optimised dressing as in Eq. (23), the above simplifies to

\[
\omega' = (\omega + \frac{G^2 \rho}{M \omega} + ae^{R_\pi^2 k^2})^{1/2} (\omega + ae^{R_\pi^2 k^2})^{1/2}
\]

which is different from \( \omega \) due to interactions. Further, minimising the thermodynamic potential with respect to \( \theta_F(\vec{k}, \beta) \) we have the solution

\[
\sin^2 \theta_F = \frac{1}{e^{\beta (\epsilon_F - \mu_B)} + 1}
\]

with

\[
\epsilon_F = \frac{G^2}{2M I_{2M}} + 2 \lambda \rho + \frac{k^2}{2M}
\]

where \( I_{2M} \) is given in equation (18). We may note that the change in \( \epsilon_F \) above from \( k^2/2M \) is also due to interaction.

To calculate different thermodynamic quantities as functions of baryon number density we first use Eq. (14) to calculate the chemical potential \( \mu_B \) in a self consistent manner with \( \rho \) and \( \mu_B \) occuring also inside the integrals through \( \sin^2 \theta_F \) as in Eq. (27). Thus for each \( \rho \), we determine \( \mu_B \) so that Eq. (14) is satisfied. The ansatz functions \( \theta_B, \theta_F \) and \( \tilde{f}(\vec{k}) \) get determined analytically through the extremisation of the thermodynamic potential. The parameters \( a, \lambda \) and \( R_\pi \) are fixed so as to reproduce the ground state properties of zero temperature nuclear matter as is done earlier [3]. The parameter values \( \lambda = 0.54 \text{ fm}^2 \), \( R_\pi = 1.18 \text{ fm} \) and \( a = 0.12 \text{ GeV} \) give the nuclear matter single particle energy as \(-15.03\)
MeV at the saturation density of 0.19 fm$^{-3}$. We shall also use here the same values for $a$, $R_\pi$ and $\lambda$.

With the thermodynamic potential determined as above, we calculate different thermodynamic quantities. Pressure is calculated using the thermodynamics relation \[ P(\beta) = -\Omega(\beta) \] (29)

Pressure as a function of neutron matter density is plotted in Fig. 1 at temperatures of 0, 10, 15 and 20 MeV. We note that as earlier [2,3] the equation of state is quite soft. In Fig 2 we plot pressure versus baryon chemical potential at temperatures of 0, 5 and 10 MeV. We note from Fig. 1 that at zero temperature pressure for neutron matter vanishes at number density of about 0.1/fm$^3$ so that the pressure at the surface for a neutron star can be zero with a finite number density. This unusual feature is due to the presence of nonperturbative contributions in equation (28).

In the next section we shall consider quark matter as well as the transition from neutron phase to quark phase. The zero temperature version of equation (29) shall be used for phase transition studies.

**III. QUARK MATTER EQUATION OF STATE AND PHASE TRANSITION**

Existence of quark matter in the core of neutron stars/pulsars is an exciting possibility [11–13]. Densities of these stars are expected to be high enough to force the hadron constituents or nucleons to overlap thereby yielding quark matter. Since the distance involved is small, perturbative QCD is used to derive quark matter equation of state. We take the quark matter equation of state as in Ref [7] in which u,d and s quark degrees of freedom are included in addition to electrons. As here [7] we set the electron, up and down quark masses to zero and the strange quark mass is taken to be 180 MeV. In chemical equilibrium $\mu_d = \mu_s = \mu_u + \mu_e$. In terms of baryon and electric charge chemical potentials $\mu_B$ and $\mu_E$, one has
\[
\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_E, \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_E, \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_E. \quad \text{(30)}
\]

The pressure contributed by the quarks is computed to order \(\alpha_s = g^2/4\pi\) where \(g\) is the QCD coupling constant. Confinement is simulated by a bag constant \(B\). The electron pressure is [7]

\[
P_e = \frac{\mu_e^4}{12\pi^2}. \quad \text{(31)}
\]

The pressure for quark flavor \(f\), with \(f=u,d\) or \(s\) is [7]

\[
P_f = \frac{1}{4\pi^2} \left[ \mu_f k_f (\mu_f^2 - 2.5m_f^2) + 1.5m_f^4 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right] - \frac{\alpha_s}{\pi^3} \left[ \frac{3}{2} \left( \mu_f k_f - m_f^2 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right)^2 - k_f^4 \right]. \quad \text{(32)}
\]

The Fermi momentum is \(k_f = (\mu_f^2 - m_f^2)^{1/2}\). The total pressure, including the bag constant \(B\), is

\[
P = P_e + \sum_f P_f - B. \quad \text{(33)}
\]

There are only two independent chemical potentials \(\mu_B\) and \(\mu_E\). \(\mu_E\) is adjusted so that the matter is electrically neutral, i.e. \(\partial P/\partial \mu_E = 0\). The baryon number density is given by \(\rho = \partial P/\partial \mu_B\).

We now consider the scenario of phase transition from cold neutron matter to quark matter. As usual, the phase boundary of the coexistence region between the neutron and quark phase is determined by the Gibbs criteria. The critical pressure and critical chemical potential are determined by the condition

\[
P_{nm}(\mu_B) = P_{qm}(\mu_B). \quad \text{(34)}
\]

The r.h.s of equation (34) is the same as \(P\) in equation (33), where as the l.h.s is the zero temperature limit of equation (29). We take \(\alpha_s = 0.4\) and the bag constant \(B = (148 \text{ MeV})^4\), which is a reasonable value to calculate pressure in the quark sector. In Fig 3 we plot pressure versus chemical potential \(\mu_B\) for cold neutron matter and quark matter as considered here.
The \((P, \mu)\) curves for quark matter and neutron matter yield the critical parameters as \(P_{cr} = 4\ \text{MeV/fm}^3\) and \((\mu_B)_{cr} = 960\ \text{MeV}\). The corresponding critical energy densities for neutron matter and quark matter respectively are given as \(\epsilon_{cr}^{nm} = 226\ \text{MeV/fm}^3\) and \(\epsilon_{cr}^{qm} = 284\ \text{MeV/fm}^3\). The corresponding baryon number densities are \(\rho_B^{nm} = 0.24/\text{fm}^3\) and \(\rho_B^{qm} = 0.33/\text{fm}^3\). Thus there is a first order phase transition. We also note that the phase transition seems to occur around number density of only about one and half times the nuclear matter density. There is a second crossing point in \((P, \mu)\) curves not seen in Fig.3. However, it occurs at higher densities where we do not expect neutrons to exist because of overlap of quark wave functions so that neutron matter does not exist.

The phase transition as above was possible due to the first two terms in equation (28). These terms arise in a nontrivial manner from interactions, with \(I_{2M}\) given in equation in (18). We note that here we are using the same parameters for neutron matter as earlier for symmetric nuclear matter, giving rise to a soft equation of state [3].

The early phase transition from neutron matter to quark matter obviously implies that the interior of “neutron star” will usually consists of quark matter. We investigate this possibility in the next section.

\section*{IV. HYBRID STARS}

For the description of neutron star, which is highly concentrated matter so that the metric of space-time geometry is curved, one has to apply Einstein’s general theory of relativity. The space-time geometry of a spherical neutron star described by a metric in Schwarzschild coordinates has the form [14,15]

\[ ds^2 = -e^{\nu(r)}dt^2 + [1 - 2M(r)/r]^{-1}dr^2 + r^2[d\Theta^2 + \sin^2\Theta d\phi^2] \] (35)

The equations which determine the star structure and the geometry are, in dimensionless forms [14]

\[ \frac{d\dot{P}(\dot{r}_0)}{d\dot{r}} = -G\frac{[\dot{\epsilon}(\dot{r}_0) + \dot{P}(\dot{r}_0)][\dot{M}(\dot{r}_0) + 4\pi a^2\dot{r}_0^3 \dot{P}(\dot{r}_0)]]}{\dot{r}^2[1 - 2GM(\dot{r}_0)/\dot{r}]}, \] (36a)
\[ \dot{M}(\dot{r}r_0) = 4\pi a \int_0^{\dot{r}r_0} d\dot{r}' \dot{r}'^2 \dot{\epsilon}(\dot{r}'r_0), \quad (36b) \]

and the metric function, \( \nu(r) \) is given by

\[ \frac{d\nu(\dot{r}r_0)}{d\dot{r}} = 2\dot{G}\frac{[\dot{M}(\dot{r}r_0) + 4\pi a\dot{r}^3 \dot{P}(\dot{r}r_0)]}{\dot{r}^2 \left[ 1 - 2G\dot{M}(\dot{r}r_0)/\dot{r} \right]}, \quad (36c) \]

In equations (36) the following substitutions have been made.

\[ \dot{\epsilon} \equiv \epsilon/\epsilon_c, ~ \dot{P} \equiv P/\epsilon_c, ~ \dot{r} \equiv r/r_0, ~ \dot{M} \equiv M/M_\odot, \quad (37a) \]

where, with \( f_1 = 197.329 \text{ MeV fm} \) and \( r_0 = 3 \times 10^{19} \text{ fm} \), we have

\[ a \equiv \epsilon_c r_0^3/M_\odot, ~ \dot{G} \equiv Gf_1 M_\odot/r_0 \quad (37b) \]

In the above, quantities with hats are dimensionless. \( G \) in equation (37b) denotes the gravitational constant with \( G = 6.707934 \times 10^{-45} \text{ MeV}^{-2} \).

In order to construct a stellar model, one has to integrate equations (36a) to (36c) from the star’s center at \( r=0 \) with a given central energy density \( \epsilon_c \) as input until the pressure \( P(r) \) at the surface vanishes. As stated in the last section, with any reasonable central density, we expect that at the center we shall have quark matter, and not neutron matter. Hence we shall be using here the equation of state for quark matter through equations (32) and (33) with \( \dot{P}(0) = P(\epsilon_c) \). We then integrate the TOV equations until the pressure and density decrease to their critical values, so that there is a first order phase transition from quark matter to neutron matter at radius \( r = r_c \). For \( r > r_c \), we shall have equation of state for neutron matter where pressure will change continuously but the energy density will have a discontinuity at \( r = r_c \). TOV equations with equation of state for neutron matter shall be continued until we reach the density around 0.1 particles/fm\(^3\) when the pressure vanishes (see Fig 1). This will complete the calculations for stellar model for hybrid “neutron” star, whose mass and radius can be calculated for different central densities.

We now plot in Fig 4 the mass of a star as a function of central energy density to examine the stability of such stars. As may be seen from the figure \( dM/d\epsilon_c \) becomes negative around
1540 MeV/fm$^3$ after which they become unstable and may collapse into black holes with the Chandrasekhar limit as $1.58 M_\odot$ [14,16]. The dashed part of the curve indicates the instability region. Fig 5 shows the mass as a function of radius for such stars obtained for different central densities varying in the range 900 to 1500 MeV/fm$^3$. This yields stable hybrid stars of masses $M \simeq 1.49$ to 1.58 $M_\odot$ with radii $R \simeq 9.85$ to 9.4 km respectively, similar to the results of Ref. [13].

The energy density profile obtained from (36a) to (36c) are plotted in Fig 6 for central densities $\epsilon_c = 900$ MeV/fm$^3$ and $\epsilon_c = 1500$ MeV/fm$^3$. As we go away from the core towards the surface through TOV equations, when the critical pressure is reached, the density drops discontinuously indicating the first order phase transition. Thus e.g. for central density of 900 MeV/fm$^3$ such a star has a quark matter core of radius 9 km with a crust of neutron matter of about 0.85 km, whereas for $\epsilon_c=1500$ MeV/fm$^3$, the quark matter core radius is 8.9 km with neutron matter crust of 0.5 km.

It may be interesting to consider smaller central densities, when neutron matter is expected to be more abundant. For example, we have noted that at central densities of $\epsilon_c=700$ MeV/fm$^3$ to 350 MeV/fm$^3$, $R$ changes from 9.87 km to 7.3 km, whereas the quark core changes from 9 km to 5.1 km. The neutron crust here increases from 0.87 km to 2.2 km, and the mass of the star decreases to as low as $0.32 M_\odot$. It is not clear to us whether, such small hybrid stars or for that matter neutron stars can get formed.

We may also calculate the surface gravitational red shift $Z_s$ of photons to see the possibility of distinguishing such a hybrid star from a neutron star or a quark star. It is given by

$$Z_s = \frac{1}{\sqrt{1 - 2GM/R}} - 1. \quad (38)$$

In Fig 7 we plot $Z_s$ as a function of $M/M_\odot$. As may be seen, $Z_s$ as related to mass and radii of hybrid stars is similar to that of pure neutron stars and hence a measurement of the same will not be an evidence for the existence of such a star. It is however possible that the discrete slowing down of pulsars due to the presence of two states of matter with various
radii shall throw some light on the above structure. For a quantitative estimate of the same, however, we shall need interactions at the interface, which at present looks impossible to tackle. This will need for example the expression for the moment of inertia of pulsars as well as the equilibrium for the rotating hybrid stars given as \[ I = \frac{8\pi}{3} \epsilon c_0^5 \int_0^{R_0} d\hat{r} \hat{r}^4 \left[ \hat{\epsilon}(\hat{r}r_0) + \hat{P}(\hat{r}r_0) \right] e^{-\nu(\hat{r}r_0)/2} \sqrt{\left[ 1 - 2GM(\hat{r}r_0)/\hat{r} \right]} \] (39)

We may further take the relativistic Keplerian angular velocity \( \Omega_k \) to be given as \[ \frac{\Omega_k}{10^{4} s^{-1}} = 0.72 \sqrt{\frac{M/M_\odot}{(R/10 km)^3}} \] (40)
as for neutron stars and estimate the same. Fig 8 shows the variation of \( \Omega_k \) as a function of \( M/M_\odot \) for such a star.

In Fig 9 we plot the fraction of the quark matter in a star as a function of the central density. As can be seen from the figure quark matter constitutes the dominant contribution to the mass of such stars. The internal quark matter structure of such hybrid stars could influence neutrino emissivity due to larger number of \( \beta \)-decay channels in quark matter phase which may be relevant for the dynamics of supernova explosions giving rise to pulsars, now expected to be really hybrid stars.

**V. CONCLUSIONS**

Let us summarise the findings of the present paper.

Using a *nonperturbative* equation of state for neutron matter and a usual *perturbative* equation of state for quark matter we saw that a first order phase transition exists between the neutron phase and quark phase. We note that neutron matter equation of state practically has no free parameters since the parameters are fixed \[ \] to yield the standard nuclear matter properties. Quark matter equation of state has the parameters : \( m_s, \alpha_s \) and \( B \). The bag constant \( B \), turns out to be a very sensitive parameter. We found, for example, for \( \alpha_s = 0.4 \), neutron matter quark matter transition occurs for \( B \approx (148MeV)^4 \). As can be seen in Fig. 3 there is not much freedom for \( B \).
The phase transition from neutron matter to quark matter seems to indicate that the core of a “neutron star” shall consist of quark matter. To study the hybrid star, namely a star consisting of both quark matter and neutron matter, we applied the TOV equation to the appropriate equations of state with a given central energy density $\epsilon_c$. It turns out that stable hybrid stars with a core of quark matter and a crust of neutron matter can exist up to $\epsilon_c \simeq 1500$ MeV/fm$^3$ beyond which instability is indicated. The bulk of the hybrid star is provided by the quark matter, the neutron matter providing only a thin crust. It is thus the quark matter equation of state and the corresponding parameters which shall play a dominant role in the formation of hybrid stars. Pulsars are expected to be stars of this type, but the gross properties appear to be similar to what we believe regarding neutron stars. We may recall that the nuclear matter equation of state used here is rather soft with compressibility of about 150 MeV.

We have also calculated other gross properties of such hybrid stars like moment of inertia, surface gravitational red shift and Keplerian rotation period. The results are similar to those obtained by others [13].

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FIGURES

FIG. 1. Pressure $P$ as a function of neutron matter density $\rho$ is plotted for temperatures $T=0$ MeV, 10 MeV, 15 MeV and 20 MeV.

FIG. 2. Pressure $P$ of neutron matter as a function of baryon chemical potential is plotted for temperatures $T=0$ MeV, 5 MeV and 10 MeV.

FIG. 3. We plot here pressure versus baryon chemical potential for neutron matter and for quark matter with $\alpha_s=0.4$ and $B^{1/4}=148$ MeV at zero temperature. The critical parameters may be seen as $P_{cr}=4$ MeV/fm$^3$ and $(\mu_B)_{cr}=960$ MeV.

FIG. 4. We plot mass of the hybrid star as a function of central density. The instability region is shown as dashed curve.

FIG. 5. We plot mass as a function of radius for hybrid stars here. The dashed line here indicates the instability of the solutions.

FIG. 6. We plot here energy density profile for central densities $\epsilon_c=900$ MeV/fm$^3$ and $\epsilon_c=1500$ MeV/fm$^3$. The discontinuities at the ends indicate a first order phase transition at the interface of quark matter and neutron matter.

FIG. 7. We plot here surface gravitational red shift $Z_s$ versus the star mass.

FIG. 8. We plot here Keplerian angular velocity $\Omega_k$ versus star mass.

FIG. 9. We plot here the ratio between the mass in the quark matter phase and the total mass of the star as a function of its central density. The dashed curve indicates the instability region.