Privacy Assured Recovery of Compressively Sensed ECG signals

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Abstract—Cloud computing for storing data and running complex algorithms have been steadily increasing. As connected IoT devices such as wearable ECG recorders generally have less storage and computational capacity, acquired signals get sent to a remote center for storage and possible analysis on demand. Recently, compressive sensing (CS) has been used as secure, energy-efficient method of signal sampling in such recorders. In this paper, we propose a secure procedure to outsource the total recovery of CS measurement to the cloud and introduce a privacy-assured signal recovery technique in the cloud. We present a fast, and lightweight encryption for secure CS recovery outsourcing that can be used in wearable devices, such as ECG Holter monitors. In the proposed technique, instead of full recovery of CS-compressed ECG signal in the cloud, to preserve privacy, an encrypted version of ECG signal is recovered by using a randomly bipolar permuted measurement matrix. The user with a key, decrypts the encrypted ECG from the cloud to obtain the original ECG signal. We demonstrate our proposed method using the ECG signals available in the MITBIH Arrhythmia Database. We also demonstrate the strength of the proposed method against partial exposure of the key.

Index Terms—Compressive sensing, ECG signal, Privacy preserving outsourcing, IoT, Connected health.

I. INTRODUCTION

In biomedical area, there are equipment and devices that produce huge amount of data. As an example, Holter monitor, a wearable device, is used for continuous monitoring of the electrical activity of the heart, namely electrocardiogram (ECG). Holter monitor is used by cardiology patients for several days to capture events such as cardiac arrhythmia. Even in cases, where physiological signals are recorded only intermittently, say by devices such as Empatica watch that have limited on the device storage memory, the amount of data produced is large that an external storage solution is in order. The ECG data produced by devices that continuously monitor, need to be stored for analysis and tracking improvements in the physiology when medical interventions are undertaken. Compression can be used for efficient use of communication channel bandwidth and storage.

Recently compressive sensing (CS) has been used as a fast and energy-efficient algorithm for simultaneous sampling and compressing of potentially sparse signals [1, 2]. CS has wide variety of applications in signal processing such as biomedical signal compression, enhancement, and recovery [3, 4]. The applications of CS has also been extended to ECG signal under the assumption that ECG signals are compressible signal [5–12]. In [5], CS-based compression was shown to present the best overall energy efficiency due to its lower complexity and also reduced CPU execution time. Since compression phase in CS is simple, fast, and energy efficient, CS has been chosen for compression in many sensing applications. However, recovery phase which is non-linear and complex in terms of computations demands use of processors that have speed, large on-board memory and computational capability. Currently wearable devices do not have such capabilities and storage and recovery need to be outsourced. In addition, clinics and hospitals, usually generate enormous amount of data, thereby requiring a place to store the data. Cloud environments generally provide “unlimited” resources and facilities. Hence, cloud can be used for storing CS-based compressed ECG signal, and based on user request ECG signal can be recovered. However, the cloud as a third party between real user (patient) and clinic should not be permitted to have access to the recovered ECG signal which will be referred to as the plaintext in this paper.

There are numerous works that have proposed different procedures for secure CS recovery outsourcing [13–15]. There are some research papers that specifically focus on methods for assuring the secrecy of ECG data in communication [16–18]. Recently, T.Y. Liu et al proposed a new encryption-then-compression (ETC) method for the ECG signal [16]. In this work, authors encrypt ECG signal with a symmetric key. Their proposed key is a square orthogonal random matrix that changes after sending every encrypted ECG signal (ciphertext). Hence, their method can be classified as a one-time-pad cryptosystem. For compression, they apply a transformation based algorithm in which singular value decomposition (SVD) technique is used [19]. However, it just focuses on compression and does not consider the security of the data. In [16], a modification has been made on SVD technique to provide secrecy as well. SVD-based methods bring higher compression ratio (CR). They are computationally intensive and may introduce delay in the system. In contrast, at expense of having lower compression ratio, CS-based methods are linear, faster, and energy efficient [20].

This paper presents a novel fast, and light-weight encryption for secure CS recovery outsourcing that can be used in resource constrained devices, such as wearable ECG recorders. The paper is arranged in the following manner. In next
section, CS is introduced and presented as a cryptosystem, and followed by the background research in secure CS recovery outsourcing. Section III contains the proposed method followed by security analysis and experimental results to verify the secrecy of the method in section IV. Finally Section V concludes the paper.

II. BACKGROUND

A. Compressive sensing

Compressive sensing (CS) is a sampling technique for efficiently sampling a signal by solving under-determined linear systems [1], [2]. It takes advantage of the signal’s sparsity, and the signal can be effectively represented by fewer measurements than the Nyquist rate. For instance, given an ECG signal $x \in \mathbb{R}^N$ and an orthogonal basis $\Psi \in \mathbb{R}^{N \times N}$, then one can map the ECG signal to sparse domain via, $x = \Psi s$, where $s \in \mathbb{R}^N$ is sparse vector with $k$ ($k << N$) nonzero entries. In other words, $s$ is a sparse representation of $x$ under the chosen predefined dictionary. Compression phase in CS provides the measurement vector through a linear operation as given below:

$$y = \Phi x = \Phi \Psi s \quad (1)$$

where $y \in \mathbb{R}^M$ is the measurement vector and $\Phi \in \mathbb{R}^{M \times N}$ is the measurement matrix. For simplicity, let $A = \Phi \Psi$. $A \in \mathbb{R}^{M \times N}$ is a rectangular matrix, sometimes referred to as “total” dictionary in the CS literature. For exact and stable recovery of sparse signal, restricted isometry property (RIP) is a sufficient condition [21]. RIP is satisfied if there exists a restricted isometry constant (RIC) $\delta_K$, $0 < \delta_K < 1$ such that

$$(1 - \delta_K) ||s||_2^2 \leq ||As||_2^2 \leq (1 + \delta_K) ||s||_2^2 \quad (2)$$

where $\delta_K$ denotes isometry constant of a matrix $A$, and its value belongs to a set of real numbers between zero and one. But, checking the RIP condition of a matrix or calculating the value of its isometry constant is difficult to verify. Hence, conditions that lead to RIP were proposed [22], [23]. Another condition, which is easier to verify in practice, is the requirement that measurement matrix $\Phi$ must be incoherent with the sparsity basis $\Psi$. Mutual coherence $\mu$ between $\Phi$ and $\Psi$ is defined as follow:

$$\mu(\Phi, \Psi) = \frac{\sqrt{N} \max_{i,j}}{1} \left| \left\langle \phi_i, \psi_j \right\rangle \right| \frac{1}{\|\phi_i\|_2 \|\psi_j\|_2} \quad (3)$$

where $\phi_i \in \{1, \ldots, M\}$ and $\psi_j \in \{1, \ldots, N\}$ respectively represent the row vectors of $\Phi$ and the column vectors of $\Psi$. The coherence measures the maximum correlation between the two matrices. Smaller coherence can lead to better signal reconstruction performance [24]. Since $\mu \in [1, \sqrt{N}]$, the matrices $\Phi$ and $\Psi$ are incoherent if $\mu(\Phi, \Psi)$ is closer to one, which corresponds to the lower bound of $\mu$.

A step called the recovery process reconstructs the input signal $x$ from the measurement vector $y$ by solving the equation (1). Since $A$ is a rectangular matrix ($M < N$), the problem formulated in equation (1) is ill-posed and has infinite solutions. However, based on the knowledge that $x$ has a sparse representation with respect to a basis $\Psi$, the recovery process can be performed in two steps [21]. The first step finds the sparse vector $\tilde{s}$ by solving the following equation:

$$\min_{\tilde{s}} \|\tilde{s}\|_0 \text{ such that } A\tilde{s} = y. \quad (4)$$

Once the vector $\tilde{s}$ has been obtained, the second step reconstructs the original signal as follows:

$$\hat{x} = \Psi \tilde{s}. \quad (5)$$

Various methods have been proposed to find an appropriate solution to equation (4) leading to numerous recovery algorithms such as Basic Pursuit [25], [26], StOMP [27], OMP [28], CoSAMP [22], Belief Propagation [29] and SL0 [30].

B. CS as cryptosystem

From a different viewpoint, CS can be assumed as a cryptosystem [31]. Since CS can map every sparse signal from $N$ dimensional space to $M$ dimensional space, where $M \ll N$, numerous researchers have considered CS as a strong cryptosystem [32]–[43]. In [43], it was proved that under certain conditions, CS can even meet the perfect secrecy as defined by Shannon.

In [44], [45], Linear Feedback Shift Registers have been used to generate CS measurement matrix as a key. In [46], a low-complexity approach for Privacy-Preserving Compressive Analysis based on subspace-based representation has been proposed to preserve privacy from an information theoretic perspective.

One important issue in almost every previous work is that the CS-recovery is assumed to be done by the real user. In other words, by having both the key (measurement matrix) and the ciphertext (measurement vector), a real user would be able to reconstruct the plaintext (initial signal). As mentioned works suppose that decryption can be done at the user end. But, in many contexts, devices at the user end do not have enough computational resources; hence such complex decryption is not feasible. A powerful remote server or cloud can be used for doing recovery process of CS problem. However, the third party would have access to the plaintext after the recovery. In addition, while transmitting the recovered signal from the third party back to the user, the data need to undergo encryption again. Privacy preserving outsourcing techniques can be used to overcome these concerns when the recovered signal contains private information of an individual. For example, ECG signal contains information that enable unique identification of an individual. Recently, there have been increased interest on ECG for biometric recognition [37]. Temporal features, amplitude features and morphological features of an ECG signal have been used for ECG-based biometric. As ECG signal contains biometrics of the person, privacy is exposed when ECG signals are fully recovered in the cloud. Hence, a solution that avoids complete recovery to preserve privacy is in order. Cloud environments can be a good option to store the compressed data. But on-demand through CS-recovery, the plaintext should not be exposed in the cloud.
This paper provides a privacy assured CS outsourced recovery. Researchers have proposed different methods to shift away the recovery phase of CS in a secure manner [13]–[15]. In this paper, we propose a fast and lightweight privacy-preserved CS recovery approach for ECG signal.

C. Privacy-preserved CS-Recovery Outsourcing

In privacy-preserved CS-recovery outsourcing, there are three levels of data to be considered: the cipher, the intermediate cipher, and the plaintext. The cipher is the measurement vector which is compressed or encrypted. Since cloud should not obtain the plaintext, recovery in the cloud yields an encrypted signal. This encrypted signal is called intermediate cipher. Intermediate cipher is sent to the real user, and real user (let us say Alice) with a private key can decrypt this intermediate cipher to obtain the plaintext. Plaintext is the original raw signal that got compressed initially.

Recently, "Outsourced Image Recovery Service (OIRS)" was proposed by Cong Wen et al [13], where a technique to securely shift away the recovery of CS in cloud environment is presented. However, the proposed method in [20] requires the cloud to solve linear programing (LP) problem to reconstruct the CS-encrypted image (the cipher). In other words, OIRS requires the cloud to use LP method to convert cipher to an intermediate cipher. But, LP is only one of the CS recovery methods and its order of complexity is \( O(N^3) \). However, there are other efficient and faster algorithms from greedy algorithms such as OMP with \( O(kMN) \), or SL0 with \( O(MN) \) that can be used instead of LP method. In addition, OIRS uses multiple keys for assuring privacy. This leads to heavy computation and consumes large time for processing. Such algorithms may not be appropriate for simultaneous encryption and compression of wearable ECG recorders where we have limited power and computational capability. "Kryptein" is another CS-based encryption scheme for the internet of things (IoT) that has been proposed by Xue et al [14]. In this work, CS has been used as compression and encryption algorithm. The secrecy of proposed cryptosystem mainly revolves around the sparsifying dictionary. However, it limits CS by choosing the adaptive sparsifying dictionaries. In other words, it uses an adaptive dictionary learning to generate sparsifying dictionary and also use this learnt dictionary as part of the key. This learnt sparsifying dictionary along with a perturbation matrix are used for designing their secret key. In [15], a secure reconstruction of image from CS in cloud was introduced. It assumes CS as a compression algorithm, and not as a sampling method. The pre-processing used in this work led to delays in generating compressed and encrypted signal. This method mapped the initial signal to a sparse domain and then put a threshold to force negligible coefficients to be zero. This method, thus, required all the components of sparse vector to be checked for zeroing, which may not be an efficient way of utilizing the limited computational capability and the power of weak devices such as ECG wearable recorders.

In this paper a simple, light-weight encryption is applied to map the initial sparse signal to another sparse signal. Considering the fact that sparsity is a required condition for CS, we are limited in options as we cannot violate this condition. In order to maintain sparsity and still achieve lightweight encryption, two keys are used: a random square matrix and a random bipolar permutation matrix. The former encrypts the measurement matrix uniquely for each wearable recorder, and the latter encrypts signal after reconstruction in the cloud for secure transmission back to the user.

III. PROPOSED METHOD

Consider a common scenario where an ECG sensor sends \( y = \Phi x = \Phi \Psi s \) to cloud environment for storage. For simplicity, let us suppose \( A = \Phi \Psi \). On demand for recovery, cloud can reconstruct the sparse signal \( s \) if the cloud is supplied with both \( y \) and \( A \). Cloud can choose any CS recovery algorithm to solve the following \( \ell_1 \) minimization problem:

\[
\min_s \|s\|_1 \quad \text{s.t.} \quad As = y. \tag{6}
\]

Once \( s \) is obtained, the initial ECG signal can be generated using \( \Psi \), i.e; \( x = \Psi s \). In order to securely shift away the full CS-recovery task, the use of two keys is proposed. The first key is used to encrypt \( \Phi \). Because, measurement matrix is a specific information of every CS-based sensing device, it should not be shared with the third party. In addition, besides random class of measurement matrices that preserve RIP condition, there is also a deterministic approach to generate a measurement matrix [48]–[50]. Since such matrices have defined structures, if we encode these structures, then we may increase the secrecy of cryptosystem. To do so, we use a random measurement matrix \( Q_{M \times M} \) to encrypt initial measurement matrix. Then, instead of sending \( A \) to the cloud, \( \hat{A} = Q(\Phi \Psi) = QA \) will be sent. If we multiply \( Q \), the recovery relation is changed as follows:

\[
\min_s \|s\|_1 \quad \text{s.t.} \quad \hat{A}s = Qy = \hat{y}. \tag{7}
\]

When \( \hat{A} \) and \( \hat{y} \) are provided to the cloud, the cloud can reconstruct the sparse vector \( s \). This level of encryption just hides the measurement matrix but the secrecy of reconstructed signal is still not preserved. To further maintain secrecy, a second key is used in the following manner. We multiply the encrypted measurement matrix with a random bipolar permutation matrix \( P \), an invertible matrix that contains either "\( +\alpha \) or "\( -\alpha \)" in each row and column at random positions, where \( \alpha \) is a random scalar number. For example, a \( 5 \times 5 \) \( P \) may be as follows

\[
P_{5 \times 5} = \begin{bmatrix}
0 & -\alpha & 0 & 0 & 0 \\
+\alpha & 0 & 0 & 0 & 0 \\
0 & 0 & +\alpha & 0 & 0 \\
0 & 0 & 0 & -\alpha & 0 \\
0 & 0 & 0 & +\alpha & 0 
\end{bmatrix} \tag{8}
\]

By multiplying \( \hat{A} \) with \( P \), a resultant new matrix \( A^* = \hat{AP} \) results. The effect of \( P \) is to map the reconstructed sparse signal into a random permuted sparse signal and to randomly change the sign of the sparse components. After this multiplication, the recovery in cloud becomes:
\[
\min_{\mathbf{P}^{-1}s} \|\mathbf{P}^{-1}s\|_1 \text{ s. t. } (\mathbf{A}\mathbf{P})(\mathbf{P}^{-1}s) = \mathbf{A}^*(\mathbf{P}^{-1}s) = \hat{\mathbf{y}} \tag{9}
\]

Sending the \(\mathbf{A}^*\) and \(\hat{\mathbf{y}}\) to the cloud, the cloud would be able to recover the intermediate cipher, \(\mathbf{P}^{-1}s\). Since the inverse of a permutation matrix is also a permutation matrix, the recovered signal from cloud is still sparse. Note that \(\mathbf{P}\) changes the position of components and does not change the order of sparsity. Therefore, we can guarantee that the sparsity of signal is preserved. Sparsity is a required condition for CS recovery, and without it, recovery cannot be done accurately. In words, in our proposed method, the original sparse vector is now mapped into another sparse vector; this mapping is done in the sparse domain and not in the domain in which signal is acquired. In the proposed privacy-assured recovery, the cloud after recovery yields \(\mathbf{P}^{-1}s\), which is a mapped sparse vector or encrypted sparse vector. Cloud may then send the encrypted sparse vector, \(\mathbf{P}^{-1}s\), to the real user, and the user would be able to reconstruct initial signal by using corresponding key, \(\mathbf{P}\) as follows:

\[
\mathbf{P}^\star \mathbf{P}^{-1}s = s; \quad \Psi s = x \tag{10}
\]

Note that \(\mathbf{P} = \alpha \mathbf{P}'\) where \(\mathbf{P}'\) is an orthonormal matrix. Multiplication of an orthonormal matrix with a measurement matrix does not affect the RIP condition. Multiplying \(\mathbf{A}\) by \(\mathbf{P}\) would still preserve the RIP inequality:

\[
(1 - \delta_K)\|\mathbf{Ps}\|_2^2 \leq \|\hat{\mathbf{A}}\mathbf{Ps}\|_2^2 \leq (1 + \delta_K)\|\mathbf{Ps}\|_2^2. \tag{11}
\]

As \(\mathbf{P} = \alpha \mathbf{P}'\), the above equation can be rewritten as follows:

\[
(1 - \delta_K)\|\mathbf{P}'s\|_2^2 \leq \|\hat{\mathbf{A}}\mathbf{P}'s\|_2^2 \leq (1 + \delta_K)\|\mathbf{P}'s\|_2^2 \tag{12}
\]

Note that \(\|s\|_2^2 = \|\mathbf{P}'s\|_2^2\), that is, \(\mathbf{P}'\) does not change the norm of a sparse vector. In this case, left and right sides of inequality shown in equation \([12]\) would be same as without encryption mode which means, \(\mathbf{P}\) does not affect the RIP condition.

IV. RESULTS

In order to assess the quality of reconstruction, appropriate metrics need to be considered. There are a few metrics proposed in the literature to measure the quality of reconstructed signal. Three such metrics that are commonly used for assessing the quality of recovered ECG signals are percentage root-mean-square difference (PRD), the normalized version of PRD namely PRDN, and signal to noise ratio (SNR),

\[
PRD[\%] = 100 \sqrt{\frac{\sum_{n=0}^{N-1} (x(n) - \hat{x}(n))^2}{\sum_{n=0}^{N-1} x^2(n)}}, \tag{13}
\]

\[
PRDN[\%] = 100 \sqrt{\frac{\sum_{n=0}^{N-1} (x(n) - \hat{x}(n))^2}{\sum_{n=0}^{N-1} (x(n) - \tilde{x}(n))^2}}, \tag{14}
\]

\[
SNR[\text{dB}] = -20 \log_{10} \left( \frac{PRD}{100} \right), \tag{15}
\]

where \(x(n)\) is the original signal, \(\tilde{x}(n)\) is the recovered signal, \(\hat{x}(n)\) is the mean of original ECG signal (uncompressed), and \(N\) denotes the length of ECG signal. In \([51]\), Zigel et al. established a link between the PRD and the diagnostic distortion. In \([51]\), different values of PRD for the reconstructed ECG signals were considered and a qualitative assessment as perceived by the specialist was given. Table \([IV]\) shows the classified quality and corresponding PRD and SNR.

| PRD     | SNR     | Quality          |
|---------|---------|------------------|
| 0 < PRD < 2% | SNR > 33 dB | "Very Good"      |
| 2% < PRD < 9%  | 20 dB < SNR < 33 dB | "Good"          |
| PRD ≥ 9%      | SNR ≤ 20 dB      | "undetermined"   |

A. Analysis of attacks on intermediate cipher

Cloud should have a pair of \((\mathbf{A}^*, \hat{\mathbf{y}})\) to conduct the recovery process. Two scenarios, one obtaining \(\mathbf{P}\) based on \(\mathbf{A}^*\) and the other obtaining \(\mathbf{A}^*\) based on the recovered signal or intermediate ciphertext are considered. In the first scenario, it is statistically impossible to separate \(\mathbf{P}\) from \(\mathbf{A}^*\). To prove this, we consider a simpler condition where there is no first key. The measurement matrix assumed to be an i.i.d. Gaussian matrix with \(\mu_{ij} = 0\) and \(\sigma_{ij} = 1/M\), where \(\mu_{ij}\) and \(\sigma_{ij}\) are the mean and standard deviation of the i.i.d. Gaussian matrix entries, respectively. As the distribution of the linear combination of multiple independent random variables having a normal distribution is also a normal distribution, \(\mathbf{A}^* = \Phi \Psi P\) is also a Gaussian matrix. The entries of \(\mathbf{A}^*\) and the mean and variance of its entries are obtained as follows:

\[
\mathbf{A}_{ij}^* = (\Phi \Psi \mathbf{P})_{ij} = \sum_{k=1}^{N} \Phi_{ik}(\Psi \mathbf{P})_{kj} \tag{16}
\]

\[
E(\mathbf{A}_{ij}^*) = \sum_{k=1}^{N} \mu_{ij}(\Psi \mathbf{P})_{kj} = 0 \tag{17}
\]

\[
Var(\mathbf{A}_{ij}^*) = \sum_{k=1}^{N} \sigma_{ij}^2(\Psi \mathbf{P})_{kj}^2 = (\alpha/M)^2 \sum_{k=1}^{N} (\Psi \mathbf{P})_{kj}^2 \tag{18}
\]

where subscript \(ij\) refers to the element of \(i\)th row and \(j\)th column of the matrix, \(\beta\) is the Euclidean norm of the rows of sparsifying dictionary, and \(E(.)\) and \(Var(.)\) are the mean and variance of random variable. Almost all sparsifying dictionaries are orthonormal, \(\beta = 1\); then, the resultant matrix
in the cloud side is a Gaussian matrix with zero mean and variance \( \alpha^2/M^2 \). Also, the covariance of \( A^* \) can be calculated as follow,

\[
\text{Cov}(A^*) = \mathbb{E}(\Phi \Psi P)(\Phi \Psi P)^T = \mathbb{E}(\Phi \Psi PP^T \Psi^T \Phi^T) = \mathbb{E}(\Phi (\alpha \mathbb{I}) \Phi^T)
\]

\[= \alpha^2 \mathbb{E}(\Phi \Phi^T) = \alpha^2 \text{Cov}(\Phi) = \frac{\alpha^2}{M^2} \mathbb{I}
\]

(19)

where superscript \( T \) denotes the matrix transpose, and \( \mathbb{I} \) is the identity matrix. Since the entries of \( \Phi \) were chosen from an i.i.d Gaussian distribution, the covariance matrix of \( A^* \) is a diagonal matrix which shows its entries are i.i.d as well. Therefore, the statistical distance of \( A^* \) in cloud and any Gaussian matrix \( \mathcal{N}(0, \alpha/M) \) is zero. In other words, given \( A^* = \Phi \Psi P \), cloud cannot reveal any information about \( P \), and there is no statistical difference between \( \Phi \Psi P \) and any random Gaussian matrix \( \mathcal{N}(0, \alpha/M) \).

In the second scenario, a "curious" cloud or an attacker tries to discover \( P \) based on intermediate ciphertext. Given the intermediate ciphertext, \( P^{-1}s \), the initial ECG signal cannot be obtained by \( \Psi s \), because \( x = \Psi s \) and not \( \Psi P^{-1}s \).

Meanwhile, an attacker or curious cloud may try to find the bipolar matrix and reconstruct the plaintext or initial uncompressed ECG signal. To do this, attacker should exactly detect the bipolar permutation key. Any change in original key will be completely propagated into the actual time domain values of the signal and corrupt the signal. In other words, as bipolar permutation matrix is applied in sparse domain, the position and sign of elements of sparse vector are changed arbitrarily. After transforming back into the time domain, the recovered signal will be totally different from the original signal. Hence, a small change in permutation matrix can lead to a small change in sparse domain, but a major change in the domain in which signal acquired (generally time domain).

To demonstrate the role of bipolar permutation matrix in maintaining the secrecy, recovery was tested with a number of estimated bipolar permutation matrices with different levels of similarity with the original key. Let the estimated bipolar permutation matrix be \( E' \) which \( E \) contains exactly \( r\% \) of the columns of \( P \) and only \((100 - r)\% \) of its columns is unclear or unknown for the attacker. Intermediate ciphertext with different estimated permutation matrices (estimated key) were decrypted and the similarity of the estimated key with the actual key, were measured using Frobenius norm. Given a \( M \times N \) matrix \( A \), its Frobenius norm is defined as the square root of the sum of the absolute squares of its elements as follow,

\[
\|A\|_F = \sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} |A_{ij}|^2}
\]

(20)

where the \( A_{ij} \) is the element of ith row and jth column of \( A \). Accordingly, the Frobenius norm of the difference between the true key and the estimated key can be obtained as follow,

\[
\|P - E'\|_F = \sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} |P_{ij} - E'_{ij}|^2}
\]

(21)

where the \( P_{ij} \) and \( E'_{ij} \) are the elements of ith row and jth column of \( P \) and \( E' \), respectively. Equation (21) was considered as a metric to show similarity of the estimated key to the actual key. Three estimated matrices were generated by copying 99%, 98% and 97% of the columns of the \( P \) into 3 estimated matrices \( E^{99} \), \( E^{98} \), and \( E^{97} \), respectively. Then, the elements of rest of 1%, 2%, and 3% columns of these estimated matrices were randomly generated. One ECG signal, record number 101 was selected from the MIT Arrhythmia database [52]. First 1000 samples of this ECG signal was selected plaintext \( x \), and the orthogonal Daubechies wavelets (db 10) was considered as sparsifying dictionary. Daubechies wavelet (db 10) is the most popular wavelet basis that used in ECG transform-based compression techniques [53]. A random bipolar permutation matrix of size \( P_{1000 \times 1000} \) was chosen, and the estimated keys were generated accordingly. The simulation results are available in Table II and it shows that a small difference in permutation matrix (or a small dissimilarity) leads to a major difference in decrypted ECG signal. For instance, the \( E^{99} \) contained 99% of the columns of actual key, and just 1% of its columns were chosen randomly. In other words, 990 columns were the exact replica of the main key, and just 10 columns were randomly estimated. The simulation results show that these 10 columns contributed to a totally different decrypted signal from the originally considered plain text ECG signal.

\begin{table}[h]
\centering
\caption{The strength of bipolar permutation key}
\begin{tabular}{|c|c|c|c|}
\hline
Key & \( \|P - E\|_F \) & PDRN(\%) \\
\hline
\( P \) & 0 & 30 \\
\( E^{99} \) & 4.47 & 249 \\
\( E^{98} \) & 6.32 & 370 \\
\( E^{97} \) & 7.73 & 450 \\
\hline
\end{tabular}
\end{table}

Table II shows that the permutation key is very sensitive and a small change in its elements can fail to provide exact decryption. Also, consider the scenario that, for instance \( E^{99} \), 99% of its columns are truly estimated. However, in practice such estimation demands heavy computational resources and time. Because, there are \( 2^N \times N! \) bipolar permutation matrices of size \( N \), where \( ! \) represents the factorial operation, need to be tried. For the case of \( N = 1000 \), 512, 256, or 128, the number of permutation matrices is a very huge number\(^1\) and the probability of estimating actual key is negligible. However, considering systematic attack scenario, an unfaithful cloud or eavesdropper may try to employ the order of sparsity (\( k \)) as a side information and then estimate the initial sparse vector. Although employing the order of sparsity might decrease the search space for attacker, but the attacker still faces a problem of searching for a solution, that is nearly not feasible, that is, for a large \( N \) and for a certain given \( k \), the search requires

\(^1\)1000! \( \cong 4 \times 10^{2567} \), 128! \( \cong 3.8 \times 10^{215} \)
non-polynomial (\(NP\)) time to solve. To estimate the initial sparse vector attacker will have to do \(2^k (N^E)\) times exhaustive search. To clarify the complexity of breaking the cipher, if the recovered sparse signal has 1024 components, and if the sparse vector has at least 64 nonzero elements (our experiments show more than 64); then it requires \(2^{64} (1024) \simeq 4.8 \times 10^{102}\) trials for the attacker to guess the plaintext. Moreover, the diagnostic information of an ECG signal is very sensitive, and a bit change in recovered signal can disturb the real information within the signal. The ultimate goal of the proposed method was to provide a simple and secure outsourcing method that is robust to the aforementioned issues. In comparison to one of the strongest outsourced CS-recovery service proposed in [13], this method of encryption demands less computational resources. In [13], cloud had to do CS-recovery based on LP method, however, in the proposed method cloud is free to choose any CS-recovery algorithm. For instance, through faster and simpler algorithms such as SL0, cloud can recover intermediate cipher three times faster than LP method [30].

Also, in [13], five keys were used which led to a further computational burden in ECG recorders. In comparison with the very recent work, “Kryptein”, in which adaptive dictionary learning was used for generating sparsifying dictionary, in the current proposed work, any dictionary either fixed or adaptive dictionaries can be used. By selecting fixed dictionaries, such as Wavelet transform family or discrete cosine transform (DCT), the heavy task of training dictionary can be removed [14].

\[\text{B. Experimental results}\]

The records from MIT-BIH Arrhythmia Database is used as ECG signal resource, [52]. As ECG signal is used for diagnosis, the morphology and relative time positions of the various morphological features are important. We considered the diagnostic needs and proposed our method of encryption. Figure 1 shows the initial ECG signal (record number 105), recovered in cloud and then decrypted with and without the actual key. In this simulation, 2000 samples of data record 105 were chosen. Also, the deterministic binary block diagonal (DBBD) sensing matrix of size \(A_{128 \times 512}\) as suggested in [48] was used. DCT was used as sparsifying basis, and the first key \(Q\) was randomly chosen from a normal distribution \(N(0,1/128)\) to encrypt the measurement matrix. The second key \(P\) was randomly selected as a bipolar permutation key. According to the size of measurement matrix, the compression ratio is \(N/M \approx 4\). The important portion of diagnostic information of an ECG signal lies between its two consecutive QRS complexes, Fig. 1 shows that bipolar permutation conceals the diagnostic information, and without having the actual key, the decrypted signal is totally wrong.

To evaluate the effectiveness of this approach on front of attacks, we simulated the process of estimating the bipolar permutation key. Four keys were chosen: one of them was the actual key and the others were \(90\%\), \(80\%\), and \(70\\%\) replica of actual key, i.e. \(P, E^{90}, E^{80}\) and \(E^{70}\). For instance, for the case of \(90\%, 90\%\) of the actual key’s components were copied into another matrix as estimated key and the remaining \(10\%\) columns were randomly guessed. The simulation results verify that the bipolar permutation is strong enough for the application of ECG signal. The proposed approach was tested with DCT and orthogonal Daubechies wavelets (\(db10\)) dictionaries as these two are the fixed sparsifying dictionaries commonly used in the compressive sensing studies using ECG signals. In this simulation, 1024 ECG samples of five different signals and a compression ratio, \(CR = M/N = 1/8\) were chosen. The results are shown in Table III.

In the aforementioned simulations, the fixed sparsifying dictionary was assumed to be available on the cloud side. On the other hand, if we employ adaptive dictionary learning, beside bipolar permutation matrix, sparsifying dictionary will also be unknown for the “curious” cloud or attacker. Hence, if adaptive dictionary learning were used, the secrecy of the
TABLE III
RECOVERY BY DIFFERENT KEYS AND SPARSIFYING BASIS (PRD%)

| Records | DCT | Wavelet |
|---------|-----|---------|
|         | P   | E<sup>90</sup> | E<sup>80</sup> | E<sup>70</sup> | P   | E<sup>90</sup> | E<sup>80</sup> | E<sup>70</sup> |
| 100     | 1.8 | 30.3    | 54.2    | 97.2    | 1.6 | 54.8    | 77.1    | 89.9    |
| 101     | 1.4 | 30.2    | 68.0    | 105     | 1.3 | 54.0    | 76.3    | 93.6    |
| 102     | 1.2 | 2.1     | 16.8    | 36.6    | 1.2 | 51.7    | 73.6    | 90.4    |
| 103     | 2.3 | 7.5     | 41.6    | 61.0    | 2.3 | 52.7    | 76.3    | 90.2    |
| 104     | 1.3 | 16.8    | 58.7    | 68.7    | 1.3 | 56.7    | 76.6    | 93.2    |
| 105     | 0.6 | 7.2     | 54.9    | 84.0    | 0.6 | 55.3    | 75.8    | 91.6    |
| 106     | 2.6 | 1.7     | 41.1    | 64.9    | 1.8 | 49.4    | 72.2    | 88.0    |
| 107     | 1.5 | 21.2    | 55.8    | 95.3    | 2.2 | 53.3    | 71.8    | 87.6    |
| 108     | 0.4 | 15.4    | 59.3    | 63.7    | 0.4 | 55.0    | 73.6    | 88.8    |
| 109     | 0.6 | 17.6    | 42.0    | 84.5    | 0.8 | 52.1    | 71.9    | 86.0    |

TABLE IV
IMPACT ON THE QUALITY OF RECONSTRUCTION

| Records | CR=50% SNR(dB) | CR=75% SNR(dB) | CR=87.5% SNR(dB) |
|---------|----------------|----------------|------------------|
|         | Ordinary Secure | Ordinary Secure | Ordinary Secure |
| 100     | 57.02          | 57.02          | 35.28            |
| 101     | 58.35          | 58.35          | 35.81            |
| 102     | 56.21          | 56.21          | 38.24            |
| 103     | 60.08          | 60.08          | 33.23            |
| 104     | 52.15          | 52.15          | 37.31            |

system can be increased. Adaptive dictionaries usually yield higher quality in reconstruction at the expense of computational burden to the system. Since, the learning process needs to be executed only once for a subject, this complexity may be conveniently ignored. There are numerous adaptive dictionary learning methods such as method of optimal direction MOD [54], and K-singular value decomposition (K-SVD) [55]. In order to demonstrate the proposed method with the adaptive dictionary learning, MOD, which is one of the fastest method to learn sparsifying dictionary was chosen. Figure 2 shows the result obtained using the adaptive sparsifying dictionary while using the ECG signal (record number 101) from the MIT-BIH Arrhythmia database. It is evident that recovery without key, or recovery of encrypted signal leads to totally wrong recovery where no information of the original ECG signal can be seen. Proposed method does not affect the quality of the reconstruction. In section III it was shown that after recovery in cloud, end user should be able to exactly recover the initial signal at their end. This aspect of the proposed method was also tested for a number of ECG signals from the database, (records no. 100 – 104). Table IV shows that there is no difference in the quality of the reconstructed signal with and without proposed encryption system.

Also, any change in mutual coherence can be reflected to the quality of reconstruction [11], [24], [48]. With this regard, we checked the effect of proposed method on the mutual coherence. Figure 3 shows this effect for random and deterministic measurement matrices as a function of number of measurements. For the class of random measurement matrices, we generated by a zero-mean and variance $1/M$ i.i.d. Gaussian process, denoted by $\Phi_{\text{Gaussian}}$. For the class of deterministic measurement matrices, DBBD matrix $\Phi_{\text{DBBD}}$ was used. Deterministic DCT matrix $\Psi_{\text{DCT}}$ and encrypted DCT matrix $\Psi_{\text{DCT}} \ast P$ were used as sparsifying and encrypted sparsifying dictionary, respectively. In this simulation, $N = 500$ and the number of measurements was changed to check the effect of mutual coherence on different sizes of matrices. The results show the bipolar permutation matrix does not affect the mutual coherence.

To the best of knowledge of the authors, there has been no specific work on secure CS recovery outsourcing for the ECG signal. However, there are two methods in the literature, namely OIRS and Kryptein, that are related to the proposed
method for secure CS recovery outsourcing. The proposed work was compared with these related methods under the following considerations: recovery algorithms, sparsifying bases and computational complexity. Table V shows comparison of the proposed method with the ORIS and Kryptein. The proposed method can be applied for any CS recovery algorithm and has low overload both on the user side and the cloud.

![Fig. 3. Impact of proposed encryption on mutual coherence.](image)

**TABLE V**

| Functionality                          | OIRS [13] | Kryptein [14] | Proposed |
|----------------------------------------|-----------|---------------|----------|
| Recovery based on LP methods           | Yes       | Yes           | Yes      |
| Recovery based on Matching pursuit, Belief Propagation, and SIO | No       | Yes           | Yes      |
| Using DCT/Wavelet sparsifying dictionaries | Yes     | No            | Yes      |
| Using adaptive sparsifying dictionary  | Yes       | Yes           | Yes      |
| Complexity in User end (multiplication operation) | $4N^2$  | $N^2$         | $N^2$    |

**C. Complexity of the proposed method**

The proposed method can be categorized as a fast and energy efficient method of encryption. Proposed method requires two keys; first key is a random square matrix and the second key is a bipolar permutation matrix. The first key is used to encrypt the measurement matrix. As sensors might use deterministic or structural measurement matrices in certain applications, attacker may use the structure in measurement matrix and consequently detect the bipolar permutation matrix. When using deterministic measurement matrices for the recovery service, cloud has $A^*$, where $A^* = AP = QAP = QΨΦP$. Without the first key, for the case where $Φ$ is deterministic, attacker can separate $ΨP$ from $ΨΦP$ and since $Ψ$ are known-such as DCT or wavelet dictionary- the permutation matrix may be revealed. But, if the first key is applied in addition, this attack can be avoided. Suppose the first key to be chosen from Gaussian distribution as it has maximum entropy that causes maximum diffusion. The overload of the first key is just $M \times M$ multiplications and $M \times (M - 1)$ addition operations for sending each measurement vector. The measurement matrix that is shared with the cloud would be $QΦΨP$ instead of $QΨΦP$. It leads to $N$ random shift in the columns of $QΦΨP$ and its components are randomly multiplied by $−α$ or $+α$. The matrix $QΦΨP$ must be available in cloud to do the recovery process. To further enhance privacy, every individual user would have a unique key. Also, after certain number of queries, to prevent potential known plaintext attack (KPA), the key can be updated.

**V. Conclusion**

For doing CS-recovery service in cloud environment, the secrecy of information should be preserved. When ECG measurements are transmitted to the cloud, the cloud with its strong resources can do the CS recovery for the client. Through the proposed method, not only does the cloud conduct the CS-recovery, but post recovery it also delivers an encrypted version of signal, thereby preserving the privacy of patient’s information during the entire process. The proposed encryption is carried out in the sparse domain. Through a bipolar permutation matrix, the initial sparse vector (plaintext) is mapped into to another sparse vector (cipher). The cloud after recovery presents a permuted sparse vector to the user and without knowing the key, it would be very difficult to guess the original signal as the degree of freedom for this guess is small. In other words, with respect to the ECG signal where small changes might distort the signal, it is practically difficult to guess the information contained in the signal for “curious” cloud or eavesdropper. The role of the sparsifying basis in improving the secrecy of information is also demonstrated in this study. Appropriate choice of adaptive sparsifying basis can also provide additional secrecy.

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