Generalized Lyapunov Demodulator for Amplitude and Phase Estimation by the Internal Model Principle

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Outline

Introduction

Generalized Lyapunov demodulator

Filter Design

Simulation Results

Conclusion
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Conclusion
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— Some applications require high-bandwidth demodulation.
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Atomic force microscopy (AFM) in dynamic mode.
Introduction

— The Lyapunov demodulator already achieves a very high demodulation bandwidth.
— However, compares unfavorably in terms of off-mode rejection.
  • Requirement in e.g. multi-frequency AFM.
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The Lyapunov demodulator already achieves a very high demodulation bandwidth.

However, compares unfavorably in terms of off-mode rejection.

• Requirement in e.g. multi-frequency AFM.

Here we propose a generalized Lyapunov demodulator, enabling direct filtering design.
⇒ Achieves increased off-mode rejection by employing higher-order filters.
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Filter Design

Simulation Results

Conclusion
Standard Lyapunov demodulator

Demodulation problem

\[ r(t) = a(t) \sin(\omega c t + \varphi(t)) \]  \hspace{1cm} (1)
Standard Lyapunov demodulator

Demodulation problem

\[ r(t) = a(t) \sin(\omega_c t + \varphi(t)) \quad (1) \]

The standard Lyapunov demodulator can be written as

\[ \dot{x} = \gamma c \varepsilon, \]
\[ y = c^T x, \quad (2) \]

where \( \varepsilon = r - y \) and

\[ c = [\cos(\omega_c t), \sin(\omega_c t)]^T. \quad (3) \]

Amplitude and phase can be recovered from

\[ \hat{a} = \sqrt{x_1^2 + x_2^2}, \quad \hat{\varphi} = \text{atan2} \left( x_1, x_2 \right). \quad (4) \]
Generalized Lyapunov demodulator

Sinusoidal signal $r(t)$ generated by the output of:

$$\dot{w} = Sw$$
$$w(0) = w_0$$
$$r(t) = \Gamma^T w$$

with $\Gamma = [1, 0]^T$ and

$$S = \begin{bmatrix} 0 & \omega_c \\ -\omega_c & 0 \end{bmatrix}.$$
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$$S = \begin{bmatrix} 0 & \omega_c \\ -\omega_c & 0 \end{bmatrix}.$$  \hfill (6)

Standard Lyapunov demodulator equivalently recast by the change of coordinates $\mathbf{v} = e^{St}\mathbf{x}$, which gives

$$\dot{\mathbf{v}} = S\mathbf{v} + \gamma \Gamma \epsilon$$
$$y = \Gamma^T \mathbf{v}. \hfill (7)$$
Generalized Lyapunov demodulator

Sinusoidal signal \( r(t) \) generated by the output of:

\[
\dot{w} = Sw \\
w(0) = w_0 \\
r(t) = \Gamma^T w
\]

with \( \Gamma = [1, 0]^T \) and

\[
S = \begin{bmatrix}
0 & \omega_c \\
-\omega_c & 0
\end{bmatrix}.
\]

Standard Lyapunov demodulator equivalently recast by the change of coordinates \( \nu = e^{St} x \), which gives

\[
\dot{\nu} = Sv + \gamma \Gamma \epsilon \\
y = \Gamma^T \nu.
\]

Replace \( \epsilon \) by a filtered version, for additional design degrees-of-freedom ⇒
Generalized Lyapunov demodulator

\begin{equation}
\begin{aligned}
\dot{\eta} &= A\eta + B\varepsilon \\
\dot{v} &= S\eta + \Gamma C\eta \\
y &= \Gamma^T v.
\end{aligned}
\end{equation}

where $A$, $B$, $C$ can be freely chosen to meet some design specifications.
Indirect filter design

— Design $K(s)$ such that the demodulator loop $T(s)$ becomes a desired bandpass shape.
— Perfect tracking is guaranteed for any stable $K(s)$. 
Direct filter design
Direct filter design

- Design $T(s)$ directly as a bandpass filter.
- Perfect tracking is guaranteed by the condition $T(j\omega_c) = 1$.

⇒ Approach taken in this work.
Outline

Introduction

Generalized Lyapunov demodulator

Filter Design

Simulation Results

Conclusion
Filter design considerations

- Bandwidth.
- Relative filter order.
- Phase delay.
- Group delay.
Example filter implementations I

Higher-order Lyapunov demodulators

The standard Lyapunov demodulator represented in the generalized scheme:

\[ T_1(s) = \frac{\gamma s}{s^2 + \gamma s + \omega_c^2}. \]  

(9)
Example filter implementations I

Higher-order Lyapunov demodulators

The standard Lyapunov demodulator represented in the generalized scheme:

\[ T_1(s) = \frac{\gamma s}{s^2 + \gamma s + \omega_c^2}. \]  

(9)

The higher order Lyapunov demodulators are then formulated as

\[ T_i(s) = T_1(s)^i \]  

(10)

where \( i \) represents the relative order of the filter.
Example filter implementations II

Bandpass form of the standard filters

— Butterworth filter
— Bessel filter
— Chebyshev type-I filter
Outline

Introduction

Generalized Lyapunov demodulator

Filter Design

Simulation Results

Conclusion
Simulation Procedure

Compare 3 kHz and 30 kHz bandwidth settings of:

— Relative order 1 Lyapunov filter (standard).
— Relative order 3 Lyapunov filter (higher-order).
— Relative order 3 Butterworth, Bessel, Chebyshev filters.

With carrier frequency 50 kHz.
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In terms of:
- Off-mode rejection.
- Transient tracking performance.
Bode plot $T(s)$

3 kHz bandwidth
Tracking frequency response

![Frequency response diagram](image)

- Lyap1
- Lyap3
- Butter
- Bessel
- Cheby

3 kHz bandwidth

30 kHz bandwidth
Step response

![Graph showing step response with different filter types and bandwidths.](image)
Off-mode rejection

![Error norm, $\|a_0 - a\|$ vs. Filters](chart.png)

Attenuation of harmonic frequency components outside the tracking bandwidth
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Simulation Results

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  ● high bandwidth
  ● large off-mode rejection
  ● simplicity of implementation
suitable for applications such as multifrequency AFM.
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— Highly flexible. LTI filters can be designed to meet application’s demands.
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Questions?
Bibliography

Ragazzon, Michael R P et al. (2019). “Generalized Lyapunov Demodulator for Amplitude and Phase Estimation by the Internal Model Principle”. In Proc. IFAC Mechatronics. Vienna, Austria.