CLASSICAL EFFECTIVE THEORY FOR HOT QCD

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Abstract

In high temperature QCD, the perturbation theory is plagued with infrared divergences which reflect long-range non-perturbative phenomena. I argue that it is possible to study such phenomena within a classical thermal field theory which can be put on a three-dimensional lattice. The classical theory is an effective theory for the soft, non-perturbative modes, as obtained after integrating out the hard modes in perturbation theory. It is well suited for numerical studies of the non-perturbative real-time dynamics, which cannot be studied within the standard, imaginary-time formulations of lattice QCD.

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CLASSICAL EFFECTIVE THEORY FOR HOT QCD

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In high temperature QCD, the perturbation theory is plagued with infrared divergences which reflect long-range non-perturbative phenomena. I argue that it is possible to study such phenomena within a classical thermal field theory which can be put on a three-dimensional lattice. The classical theory is an effective theory for the soft, non-perturbative modes, as obtained after integrating out the hard modes in perturbation theory. It is well suited for numerical studies of the non-perturbative real-time dynamics, which cannot be studied within the standard, imaginary-time formulations of lattice QCD.

1 Introduction

There is currently a considerable interest in the physics of ultrarelativistic plasmas, by which we mean plasmas in, or near, thermal equilibrium at very high temperature, much larger than any mass scale: $T \gg m$. This interest, which has triggered many theoretical advances, is mainly motivated by two important applications: The first is to the deconfined phase of QCD, the celebrated quark-gluon plasma (QGP), which is expected to be found in the heavy ion collisions at RHIC and LHC. The second is to the high-temperature, symmetric, phase of the electroweak theory, which is relevant for baryogenesis in the early Universe.

If the temperature is high enough, the physics of non-Abelian gauge theories is expected to be simple. This common wisdom, which we shall see below to be a little too optimistic, is motivated by asymptotic freedom: as the temperature increases, the coupling “constant” becomes small, $g(T) \ll 1$, and perturbation theory becomes applicable. That is, the high-$T$ plasma can be treated as a gas of weakly interacting quarks and gluons.

By using perturbation theory, significant progress has been indeed achieved in understanding the long-wavelength excitations of the plasma and the related screening phenomena (see Refs. for a summary and more references, and Sec. 2 below for a brief account). Furthermore, the free energy in hot QCD has been computed up to the order $g^5$, which is the highest accuracy permitted

\footnote{For definiteness, I shall mostly use a QCD-inspired terminology. Note, however, that most of the present considerations do also apply to the electroweak plasma, provided the temperature is high enough ($T \gg T_c$).}
by perturbation theory (the correction $O(g^6)$ turns out to be non-perturbative; see Sec. 3 below).

At the same time, lattice simulations have been used to compute thermodynamic quantities (like the free energy and the entropy) and to study static aspects of the deconfining and the electroweak phase transitions. Besides providing “exact” numerical results, the lattice calculations allow us to explore non-perturbative physics like the strong coupling regime in QCD at intermediate temperatures ($T \gtrsim T_c \sim 200$ MeV). As the temperature increases, the comparison with perturbation theory becomes possible: it has been found, for instance, that the lattice estimates for the energy density approach rather quickly the Stefan-Boltzmann limit above $T_c$, thus comforting the picture of hot QCD as a weakly interacting gas.

It thus may come as a surprise that, in some cases, lattice simulations cannot be avoided not even in the study of the high temperature limit, where the coupling constant is arbitrarily small. This is so since perturbation theory is plagued with infrared divergences which reflect large collective effects (cf. Sec. 3). Moreover, such divergences affect not only static characteristics, like the free energy, but also dynamical quantities, that is, real-time correlation functions, for which the standard lattice simulations — as formulated in imaginary time — are not applicable. An important example, to which I shall return later, is the rate of baryon number violation at high temperature. To compute such quantities, we need new, non-perturbative, methods which allow for numerical studies of the real-time dynamics. It is my purpose in this talk to present such a method which is based upon a semiclassical approximation (cf. Sec. 4).

2 Collective behaviour and screening

I consider a purely Yang-Mills plasma in thermal equilibrium at a temperature $T$ which is high enough for the coupling constant to be small: $g \ll 1$. In the absence of interactions, this would be simply the “black body radiation”, that is, a collection of free, massless gluons with typical energies of the order $T$ and Bose-Einstein occupation numbers: $N_0(E) = 1/(e^{\beta E} - 1)$, with $\beta \equiv 1/T$. However, the gauge interactions — although weak — do significantly change this picture. They give rise to an hierarchy of scales:

$$T \gg gT \gg g^2T \gg g^3T \cdots,$$

Rather surprisingly, it turns out that the perturbative expansion for the free energy is poorly convergent up to temperatures as high as 3 GeV. Still, as shown in Refs. the convergence can be greatly improved by using Padé approximants.

At high-$T$, quarks are not important for the infrared physics to be discussed here.
with the various scales corresponding to different physical phenomena.

Thus, the typical excitations of the plasma are “hard” gluons, with momenta \( k \sim T \). Such gluons can develop a collective behaviour over a typical space-time scale \( \sim 1/gT \), which is large as compared to the mean interparticle distance \( \sim 1/T \). This results in long-wavelength (\( \lambda \sim 1/gT \gg 1/T \)) oscillations of the average colour density which are most economically described in terms of kinetic equations. In these equations, the hard \((k \sim T)\) gluons are represented by their average colour density \( \delta N_a(k, x) \) to which couple the soft (i.e., long-wavelength) colour fields \( A_{\mu}^a(x) \). The relevant equations read:

\[
(D_\nu F^{\nu\mu})_a(x) = 2gC_A \int \frac{d^3k}{(2\pi)^3} v^\mu \delta N_a(k, x),
\]

\[
(v \cdot D_x)_{ab} \delta N^b(k, x) = -g v \cdot E_a(x) \frac{dN_0}{dk},
\]

where \( D^\mu = \partial^\mu + igA^\mu_a T_a \) is the covariant derivative, \( E^i_a \equiv F^{i0}_a \) is the chromoelectric field, \( C_A = N \) for \( SU(N) \), and \( v^\mu = (1, v) \) with \( v = k/k \) denoting the velocity of the hard particles \( (k = |k|, \text{so that } |v| = 1) \).

The first equation above is the Yang-Mills equation for the soft fields \( A_{\mu}^a \). It involves, in its r.h.s., the colour current induced by the collective motion of the hard particles:

\[
\tilde{j}_a^\mu(x) = 2gC_A \int \frac{d^3k}{(2\pi)^3} v^\mu \delta N_a(k, x).
\]

In turn, the soft colour wave \( A^\mu_a(x) \) acts as a driving force for the collective behaviour, and this is described by the second Eq. (2) (which may be seen as a non-Abelian generalization of the familiar Vlasov equation). By solving this equation, we can express the current \( \tilde{j}_a^\mu \) in terms of the gauge fields \( A^\mu_a \) and thus obtain an effective Yang-Mills equation which involves the soft fields alone:

\[
D_\nu F^{\nu\mu} = m_D^2 \int d\Omega \frac{v^\mu v^i}{4\pi} v \cdot D E_i.
\]

Here, the angular integral \( \int d\Omega \) runs over the orientations of the unit vector \( v \), and \( m_D \) is the Debye mass:

\[
m_D^2 = -\frac{g^2C_A}{\pi^2} \int_0^\infty dk k^2 \frac{dN_0}{dk} = \frac{g^2C_AT^2}{3}.
\]

Eq. (4) describes the propagation of long-wavelength colour waves in the high-\( T \) plasma. The induced current in its r.h.s. is the result of the wave scattering.
off the hard thermal particles. In general, this current is non-local, and also
non-linear in the gauge fields (note the covariant derivative in the denomina-
tor). Still, for time-independent fields, it reduces to a very simple expression:

\[ j^\mu_a(x) = \delta^\mu_0 m_D^2 \partial_\mu A^0_a(x) \]

which defines a screening “mass” for the Coulomb field \( A^0_a \). Indeed, for such static fields, the \( \mu = 0 \) component of Eq. (1) simplifies
to:

\[ D \cdot E + m_D^2 A_0(x) = 0, \]

which is the non-Abelian generalization of the Poisson-Debye equation:

\[ (\Delta + m_D^2)A_0(x) = 0, \]

and implies the screening of any electrostatic fluctuation \( A_0(x) \) over distances
\( r \sim 1/m_D \). In other terms, the static \( (k_0 \to 0) \) limit of the Coulomb propagator
— as following from Eq. (3) — reads:

\[ D_{00}(k_0 = 0, k) = \frac{1}{k^2 + m_D^2}, \]

so that the Debye mass acts as an infrared cutoff \( \sim gT \) in the electric sector.

The situation in the magnetic sector is more complex: in the static limit the
vector current \( j_a \) vanishes, as alluded to before, so that the time-independent
magnetic fields are not screened. The analogue of Eq. (8) reads then:

\[ D_{ij}(k_0 = 0, k) = \frac{\delta_{ij} \hat{k}_i \hat{k}_j}{k^2}, \]

which is the same as at tree level. For time-dependent magnetic fields, however,
screening does occur, through the mechanism of Landau damping. This too
can be studied on Eq. (3), with the result that, for small but non-vanishing
frequency \( k_0 \), the magnetic piece of the gluon propagator reads:

\[ D_{ij}(k_0, k) \simeq \frac{\delta_{ij} \hat{k}_i \hat{k}_j}{k^2 - i(\pi k_0/4k)m_D^2}. \]

Note the purely imaginary character of the self-energy in the denominator:
this is a dissipative effect, describing the absorption of the magnetic field by
the plasma constituents. Eq. (10) also shows that, for large enough frequencies
\( k_0 \sim k \), the magnetic fields are screened as efficiently as the electric ones.

The above kinetic-theory picture of the screening (which is actually equiv-
alent to the one-loop picture in the “hard thermal loop” approximation)
turns out to be further complicated by non-perturbative phenomena. There
are indeed theoretical arguments, which are also supported by lattice simulations, and which suggest that screening should occur also for the static magnetic fields, but only at the softer scale $g^2T$ (see Sec. 4 below). In practical calculations, this is often parametrized by introducing an infrared cutoff $\mu \sim g^2T$ in the magnetic sector (“magnetic mass”). But it is fair to say that the corresponding physical mechanism is not yet fully understood. This is so since, as we shall see shortly, $g^2T$ is precisely the scale where perturbation theory breaks down.

3 The breakdown of the perturbation theory

The screening phenomena greatly improve the infrared behaviour of the perturbation theory (PT), thus allowing for many interesting calculations. Still, when going to higher orders in PT, one is often confronted with severe infrared (IR) divergences, which signal a breakdown of the perturbation theory at the scale $g^2T$. There are essentially two reasons for that:

The first is the singularity in the magnetostatic propagator as $k \equiv |k| \to 0$ (cf. Eq. (9)). Although this might be screened, as alluded to before, at the scale $g^2T$, nevertheless such a screening will not be enough to restore perturbation theory (see below).

The second reason is the Bose-Einstein amplification of the soft modes with momenta $k \ll T$: such modes have large thermal occupation numbers,

$$N_0(k) = \frac{1}{e^{\beta k} - 1} \simeq \frac{T}{k} \gg 1,$$

and therefore give large “radiative” corrections in higher orders. Specifically, by adding a new loop to a preexistent Feynman graph, we generate a correction of relative order $g^2N_0(k)$, where $k$ is the momentum carried by the added gluon propagator. If $k \sim g^2T$, then $N_0(k) \sim T/k \sim 1/g^2$ and $g^2N_0(k) \sim 1$: that is, by adding more soft ($k \sim g^2T$) loops, we remain at the same order in $g$, and the loop expansion breaks down.

To see a specific example of this difficulty, let’s follow Linde and consider the higher order corrections to the free energy: a typical $n$-loop (with $n \geq 4$) diagram has the “ladder” topology in Fig. and gives a contribution $F^{(n)} \sim g^nT^4 (g^2T/\mu)^{n-4}$ (this is a simple power counting estimate). In this equation, $\mu$ is an infrared cutoff which has been introduced by hand to give a meaning to an otherwise IR-divergent loop integral. As we know by now, such a cutoff is indeed generated via the screening effects. In the electric sector, we have Debye screening and therefore $\mu \sim m_D \sim gT$ (cf. Eq. (8)); with $\mu \sim gT$, $F^{(n)} \sim g^{n+2}T^4$, and higher loops contribute to higher orders in $g$, as it should
for PT to make sense. In the magnetic sector, on the other hand, we have at most \( \mu \sim g^2 T \), in which case all the diagrams with four or more loops contribute to the same order in \( g \) (namely, to the order \( g^6 \)). We thus face a breakdown of PT in the magnetic sector which, unlike the electric sector, is not protected by Debye screening.

For static quantities like the free energy, the non-perturbative corrections can be estimated, at least in principle, via lattice QCD. But there are also time-dependent correlation functions which appear to be non-perturbative and for which the standard lattice calculations (as formulated in imaginary time) are not applicable. Let me give you some examples in this sense:

In relation to baryogenesis, one is interested in the rate for anomalous baryon number violation in the high-\( T \), symmetric, phase of the electroweak theory\(^7\). This is a genuinely non-perturbative phenomenon where the variation \( \Delta B \) of the baryon number is tied up — via the chiral anomaly — to the transitions between topologically inequivalent vacua:

\[
\Delta B(t) \propto \int_0^t dx_0 \int d^3 x \, F_\mu^a \tilde{F}^\nu_{\mu a},
\]

(12)

with \( \tilde{F}^\mu_{\nu a} = (1/2) \epsilon^{\mu \nu \rho \lambda} F_\rho^a \). At high temperature, such transitions will necessarily involve very soft \( (k \sim g^2 T) \) magnetic fields, the only ones to be responsible for non-perturbative phenomena. Still, it has been recently recognized\(^9\) that the topological transitions are also sensitive to the hard \( (k \sim T) \) plasma modes, via the Landau damping alluded to before (cf. Eq. (10)).

But non-perturbative effects are also met in the study of simpler correlation functions, like the gluon, or quark, propagator: e.g., when studying the spectrum of the elementary excitations (the plasma “quasiparticles”), we consider the large time behaviour of the corresponding 2-point functions. On general grounds, one expects an exponential decay of the quasiparticles, as coming from their scattering off the plasma constituents. For instance, if \( S(t, p) = \langle \bar{\psi}(t, p) \psi(t, -p) \rangle \) denotes the quark propagator, then one ex-
pects $S(t \to \infty, \mathbf{p}) \propto e^{iE(p)t} e^{-\gamma(p)t}$, where $E(p)$ is the mass-shell energy and $\gamma(p)$ is the damping rate, i.e., the total interaction rate of the quark in the plasma. Still, explicit calculations of $\gamma(p)$ show a logarithmic infrared divergence already in leading order, as coming from collisions with the exchange of soft magnetic gluons (cf. Fig. 2). Similar divergences occur for gluons, and also for electrons in a hot QED plasma. Thus, the lifetime of the quasiparticles turns out not to be computable in perturbation theory.

4 Classical effective theory for real-time processes

I now present a method which allows us, at least in principle, to compute the non-perturbative, real-time, correlation functions alluded to before.

The basic idea is not new: since the non-perturbative phenomena are associated with soft ($k \sim g^2 T$) magnetic fields which have large thermal occupation numbers (cf. Eq. (11)), some semi-classical approximation should be applicable. To see this in a simple way, let’s put back Planck’s constant $\hbar$ in Eq. (11) and compute the average energy per mode in thermal equilibrium:

$$\varepsilon(k) = \frac{\hbar k}{e^{\beta \hbar k} - 1} \simeq T \quad \text{as} \quad \hbar k \ll T. \quad (13)$$

As $\hbar \to 0$, we recover the classical equipartition theorem, as expected. But the above example shows that the relevant inequality is $\hbar k \ll T$, so that the classical limit ($\hbar \to 0$ at fixed $k$ and $T$) is actually equivalent to the soft momentum limit ($k \to 0$ at fixed $\hbar$ and $T$). This observation is useful since we know how to perform real-time lattice simulations for a classical thermal field theory: All we have to do is to solve the classical equations of motion for given initial conditions, and then average over the classical phase space with the Boltzmann weight $\exp(-\beta H)$. Since the initial conditions (say $\phi(x)$ and $\dot{\phi}(x)$ for a scalar theory) depend only on the spatial coordinate $x$, the phase
space integration is actually a *three-dimensional* functional integral, which can be implemented on a lattice in the standard way (and actually, with even less numerical effort than in four dimensions!). The only question is, what is the correct classical theory?

It has been originally assumed that, in order to compute the hot baryon number violation (cf. Eq. (12)), it should be enough to consider the classical Yang-Mills theory at finite temperature. This hypothesis, which led to the lattice calculations in Ref. 12, is highly non-trivial: it assumes that the topological transitions are totally insensitive to the hard ($k \sim T$) plasma modes, which are not properly described by the classical theory (since the approximation in Eq. (13) fails at momenta $k \sim T$). And indeed, the classical theory is well-known to run into ultraviolet (UV) problems, like the famous “ultraviolet catastrophe” of Rayleigh and Jeans: The classical estimate for the energy density of the black body radiation, namely ($\Lambda$ is an ad-hoc UV cutoff):

$$E_{cl}/V = \int \frac{d^3k}{(2\pi)^3} \varepsilon_{cl}(k) = T \int \frac{d^3k}{(2\pi)^3} \propto T \Lambda^3,$$

(14)
is obviously wrong (since UV divergent), in contrast to the quantum result:

$$E/V = \int \frac{d^3k}{(2\pi)^3} \frac{k}{e^{\beta k} - 1} \propto T^4,$$

(15)
which is finite since the large momenta $k \gg T$ are exponentially suppressed by the Bose-Einstein distribution function.

Now, as already argued in Sec. 3, the topological transitions in hot QCD are driven by soft ($k \sim g^2T$) field configurations, and this is the main justification for using the classical Yang-Mills theory in Refs. 7, 12. Still, it has been also argued in Sec. 2 that the dynamics of the soft modes is strongly modified by the hard particles, which generate screening. Thus, even though a soft process, the baryon number violation might still be sensitive to the hard particles, via the screening effects. And actually we have both theoretical and numerical evidence that this is indeed the case.

Thus, in order to properly compute the non-perturbative correlation functions of interest, we need to correct the classical Yang-Mills theory by including the screening effects. That is, the relevant classical theory should be an *effective* theory which applies only to the soft modes, but where the hard modes have been integrated over to generate screening. From Sec. 2, we have a candidate for such a theory: Eq. (4) includes indeed the screening effects, via the colour current in its r.h.s. Still, there are a few “technical” complications associated with this equation: First, Eq. (4) is non-local (and also dissipative:
recall the imaginary part in the denominator of Eq. (10), so it is not a priori clear how to construct the thermal phase-space and the Hamiltonian. Second, in order to avoid overcounting, we need a precise, and gauge-invariant, separation between hard and soft degrees of freedom.

The discussion in Sec. 2 suggests a solution to the first problem above: rather than working with the non-local equation 4, we can conveniently replace it with the coupled system of local equations in Eq. (2). There is a price to be paid for that: in addition to the gauge fields $A^a_\mu(x)$, the local description in Eq. (2) also involves the average colour density $\delta N^a_0(x,k)$, which can be seen as an “auxiliary field”. Still, when working with a local theory, we are in a better position to look for a Hamiltonian formulation, as I discuss now.

The first step is to recognize, on the second Eq. (2), that the $v$ and $k$-dependence can be factorized in $\delta N^a_0(x,k)$ by writing:

$$\delta N^a_0(x,k) \equiv -gW^a(x,v) (dN_0/dk).$$

The new functions $W^a(x,v)$ satisfy the equation:

$$(v \cdot D_x)_{ab} W^b(x,v) = v \cdot E^a(x),$$

which is independent of $k$ since the hard particles move at the speed of light: $|v| = 1$. By using Eq. (16), the induced current can be written as:

$$j^\alpha_0(x) = m_D^2 \int \frac{d\Omega}{4\pi} v^\mu W^a(x,v),$$

where the radial integration (i.e., the integration over $k \equiv |k|$) has been explicitly worked out, and the Debye mass $m_D$ is defined in Eq. (5).

The Hamiltonian formulation of the effective theory involves the auxiliary fields $W^a(x,v)$ together with the soft gauge fields $A^a_\mu(x)$. In the temporal gauge $A^a_0 = 0$, the independent degrees of freedom are $E^a_i$, $A^a_\mu$, and $W^a$, and the corresponding equations of motion read

$$E^a_i = -\partial_0 A^a_i,$$

$$-\partial_0 E^a_i + \epsilon_{ijk} (D_j B_k)^a = m_D^2 \int \frac{d\Omega}{4\pi} v_i W^a(x,v),$$

$$(\partial_0 + v \cdot D)^{ab} W_b = v \cdot E^a,$$

(together with Gauss’ law which in this gauge must be imposed as a constraint:

$$G^a(x) \equiv (D \cdot E)^a + m_D^2 \int \frac{d\Omega}{4\pi} W^a(x,v) = 0.$$\(^4\)

\(^4\)Note that Eqs. (14) are not in canonical form: this is already obvious from the fact that we have an odd number of equations.
Eqs. (19) are conservative; the corresponding, conserved energy functional (which also acts as a Hamiltonian) has the following simple form:

\[ H = \frac{1}{2} \int d^3x \left\{ \mathbf{E}_a \cdot \mathbf{E}_a + \mathbf{B}_a \cdot \mathbf{B}_a + m_D^2 \int d\Omega \frac{1}{4\pi} W_a(x, \mathbf{v}) W_a(x, \mathbf{v}) \right\}, \quad (21) \]

which is manifestly gauge invariant.

We are now in position to write down the classical partition function and compute (generally time-dependent) thermal expectation values. As discussed at the end of Sec. 3, we are interested in correlation functions of the magnetic fields \( A^i_a \). These can be obtained from the following generating functional:

\[ Z_{cl}[J^a_i] = \int \mathcal{D}\mathbf{E}^a_i \mathcal{D}A^a_i \mathcal{D}W^a \delta(G^a) \exp \left\{ -\beta H + \int d^4x J^a_i(x) A^a_i(x) \right\}, \quad (22) \]

where \( A^i_a(x) \) is the solution to Eqs. (19) with the initial conditions \{\( E^a_i, A^a_i, W^a \)\} (that is, \( E^a_i(t, x) = E^a_i(x) \), etc., with arbitrary \( t_0 \)), and \( G^a \) and \( H \) are expressed in terms of the initial fields (cf. Eqs. (20) and (21)).

It can be verified that the phase-space measure \( \mathcal{D}\mathbf{E}^a_i \mathcal{D}A^a_i \mathcal{D}W^a \) in Eq. (22) is invariant under the time evolution described by eqs. (19), so that \( Z_{cl}[J] \) is independent of the (arbitrary) initial time \( t_0 \), as it should. (This point is not trivial because of the non-canonical structure of the equations of motion.)

There is another essential — but technically quite involved — point that I am currently glossing over: this is the intermediate cutoff separating hard from soft degrees of freedom, and which should appear as an ultraviolet cutoff in Eq. (22) (without such a cutoff, the effective theory would develop linear UV divergences to one loop order). As discussed in Ref. 11, this cutoff can be indeed introduced in such a way to make the effective theory UV finite, but cancel — in the calculation of physical quantities — against appropriate “counterterms” in the Hamiltonian. Moreover, the cutoff procedure proposed in Ref. 11 can be also implemented on a lattice; this is important since it allows one to take the continuum limit in the lattice calculations.

For illustration, let me finally consider two simple, yet non-trivial, applications of the effective theory. The first is the \( J^a_i = 0 \) limit of Eq. (22) which yields, after some simple algebra,

\[ Z_{cd} = \int \mathcal{D}A^a_0 \mathcal{D}A^a_0 \exp \left\{ -\frac{\beta}{2} \int d^3x \left( B^a_i B^a_i + (D_i A_0)^2 + m_D^2 A^a_0 A^a_0 \right) \right\}, \quad (23) \]

where the \( A^a_0 \) components of the gauge fields have been reintroduced as Lagrange multipliers to enforce Gauss’ law. This is “almost” the thermal partition function for classical Yang-Mills theory: the only new feature is the screening
mass for the electric fields, which is the only trace of the screening effects in the static limit (cf. Eq. (6)).

The expression in Eq. (23) also coincides with the first order result of the “dimensional reduction” method which consists in integrating out the non-static Matsubara modes to obtain an effective 3-dimensional theory for the static one. By putting this theory on a lattice, one has been able to perform accurate studies of the electroweak phase transition and also to compute the non-perturbative contributions to the free-energy (cf. Sec. 3) and to the non-Abelian Debye mass.

Note also that the magnetic sector of Eq. (23) (that is, the sector involving only the vector fields $A^i(x)$) is formally the same as 3-dimensional Euclidean QCD with dimensionful coupling constant $g_3 = g\sqrt{T}$. This theory is ill-behaved in perturbation theory (the IR divergences in Fig. 1 may be seen as an illustration of such a bad IR behaviour), but it is generally believed to generate a dynamical mass gap $\propto g_3^2 = g^2T$, which is the only mass scale in the problem. This is the celebrated “magnetic mass” alluded to in Sec. 3.

As a second application, consider the large-time behaviour of the electron, or quark, propagator, which we have seen to be ill defined in perturbation theory (cf. Sec. 3). Fermions have not been yet included in the effective theory, but it is easy to compute the fermion propagator in a soft background field $A^i$ (in the eikonal approximation), and then average over the thermal fluctuations of the background field as shown in Eq. (22). In QED, all these calculations can be done explicitly (the corresponding functional integral is Gaussian), with the striking result that, at very large times, the electron propagator shows a non-exponential decay $S(t) \propto \exp\{-\alpha T t \ln(m_D t)\}$, with $\alpha = e^2/4\pi$. In terms of diagrams, this calculation corresponds to a resummation of all the “quenched” self-energy corrections illustrated in Fig. 3.
In QCD, the corresponding calculation can be performed only numerically, which requires a lattice implementation of the effective theory. More generally, such an implementation would allow for systematic studies of the real-time non-perturbative dynamics in hot gauge theories.

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