ON SPECIAL CASES OF GENERAL GEOMETRY:
geometries with changing length of vectors

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Abstract

We find relations between quantities defining geometry and quantities defining
the length of a curve in geometries underlying Electromagnetism and unified
model of Electromagnetism and Gravitation. We show that the length of a
vector changes along a curve in these geometries.

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1 Introduction

In paper [1] a new geometry called General Geometry is formulated and it is shown that its the most simplest case is geometry underlying electromagnetism. However, relation between quantities defining geometry $F^\sigma_\lambda$ and the length of a curve $A_\mu$ was assumed. Next, in paper [2] it is shown that geometry underlying unified model of electromagnetism and gravitation is also a special case of General Geometry. There, relations between quantities defining geometry $F^\sigma_\lambda$, $\Gamma^\sigma_{\mu\nu}$ and the length of a curve $g_{\mu\nu}$, $A_\mu$ were also assumed.

In the present paper, it is shown that relations assumed in [1] and [2] hold to be true provided that the length of a vector changes along a curve in both geometries.

In Riemannian geometry the length of a vector does not change and this makes it be an underlying geometry for Gravitation. If the length of a vector changes in Riemannian geometry then it fails to be an underlying geometry for Gravitation. This failure has been demonstrated by H. Weyl [3] who investigated Riemannian geometry with changing length of a vector in an attempt to unify electromagnetism and gravitation (for discussion see [4]). However, we show that the length of a vector changes along a curve in the presence of electromagnetic field in a geometry completely different from Riemannian one.

In summary, Geometry of Electromagnetism [1] with changing length of a vector has physical interpretation as geometry underlying Electromagnetism. Riemannian geometry with constant length of a vector has physical interpretation as geometry underlying Gravitation. Combination of Geometry of Electromagnetism and Riemannian geometry with changing length of a vector is geometry underlying unified model of Electromagnetism and Gravitation.

In the next section we prove relations assumed in [1] and [2] and show that the length of a vector changes along a curve in these geometries.

2 On Special Cases of General Geometry

We recall that Geometry of Electromagnetism [1] is defined by

$$\frac{d\xi}{du} = -F^\sigma_\lambda(x)\xi^\lambda.$$  \hspace{1cm} (1)

We consider the following metric

$$ds = \sqrt{\eta_{\mu\nu}dx^\mu dx^\nu + \frac{q}{cm}A_\mu(x)dx^\mu}, \quad \eta_{\mu\nu} = diag(1-1-1-1).$$ \hspace{1cm} (2)

Accordingly, the length of a vector $V = \xi^\lambda\frac{\partial}{\partial x^\lambda}$ is

$$dl = \sqrt{\eta_{\mu\nu}\xi^\mu\xi^\nu + \frac{q}{cm}A_\mu(x)\xi^\mu}.$$ \hspace{1cm} (3)

And we assume that

$$\frac{dl}{du} = \Phi_\nu(A_\lambda, F_{\mu\nu})\xi^\nu.$$ \hspace{1cm} (4)
where $A_\mu$ are some functions of $x$, $\Phi_\nu$ are functions of $A_\mu$ and $F_{\mu\nu}$, and $q$, $c$, $m$ are some parameters. Equation (4) means that the length of a vector changes along a curve due to $\Phi_\nu$. Substitution of $dl$ in (4) by (3) leads to equations

$$\xi^\mu \xi^\nu (F_{\mu\nu} + F_{\nu\mu}) = 0, \quad \frac{q}{cm} (\partial_\mu A_\sigma x^\mu_\sigma - A_\mu F^\mu_\sigma) \xi^\sigma = \Phi_\nu (A_\lambda, F_{\mu\sigma}) \xi^\nu. \quad (5)$$

The most general solution to the first one is any antisymmetric tensor

$$F_{\mu\nu} = -F_{\nu\mu}.$$  

We choose $\Phi_\nu$ such that the second equation has solution

$$F_{\mu\nu} = \frac{q}{cm} (\partial_\mu A_\nu - \partial_\nu A_\mu).$$

As it is shown in [1], curvature vector $R_\lambda$ is equal to $R_\lambda = \partial^\mu F_{\mu\lambda}$. Equation $R_\lambda = 0$ coincides with Maxwell equation for electromagnetic field $A_\mu$ and equation for geodesics coincides with the equation for a particle interacting with electromagnetic field $A_\mu$. This allows us to interpret $A_\mu$ as electromagnetic field and geometry defined by (1) with (2) and (4) as geometry underlying electromagnetism. $q$ is identified with charge, $m$ with mass of a particle interacting with electromagnetic field $A_\mu$, $c$ is the speed of the light.

If we choose $\Phi_\nu = 0$ then the second equation in (5) reduces to

$$\partial_\mu A_\sigma x^\mu_\sigma - A_\mu F^\mu_\sigma = 0.$$  

Multiplication by $A^\sigma$ gives

$$A^\sigma \partial_\mu A_\sigma = 0.$$  

This equation is a constraint for $A_\mu$. Therefore in order to consider general functions $A_\mu$ of $x$ we have to allow $\Phi_\nu \neq 0$. Hence, the length of a vector must change along a curve in Geometry of Electromagnetism.

Next we consider geometry underlying unified model of electromagnetism and gravitation [2] defined by

$$\frac{d\xi^\sigma}{du} = -(F^\sigma_\lambda (x) + \Gamma^\sigma_\lambda_\mu (x)x^\mu_\nu)\xi_\lambda, \quad (6)$$

and choose metric as

$$ds = \sqrt{g_{\mu\nu}dx^\mu dx^\nu} + \frac{q}{cm} A_\mu (x) dx^\mu,$$

where $g_{\mu\nu} (x)$ is a metric tensor and the length of a vector $V$ is

$$dl = \sqrt{g_{\mu\nu} \xi^\mu \xi^\nu} + \frac{q}{cm} A_\mu \xi^\mu, \quad (7)$$

If we choose $ds = \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}$ and $\frac{d\xi^\sigma}{du} = 0$ we obtain that $F_{\mu\nu}$ is an arbitrary antisymmetric tensor and electromagnetic field $A_\mu$ has to be introduced artificially.
and it changes as
\[ \frac{dl}{du} = \Phi'_\nu(A_\lambda, F_{\mu\nu})\xi^\nu. \tag{8} \]

Note that in this geometry \( \xi_\rho = g_{\rho\mu}\xi^\mu \). Substitution of \( dl \) in (8) by (7) gives rise to
\[ \Gamma_{\nu,\sigma\lambda} + \Gamma_{\lambda,\sigma\nu} = \partial_\sigma g_{\lambda\nu}, \quad \frac{q}{cm}(\partial_{\mu}A_{\sigma}x_{\mu}^\sigma - A_{\mu}F_{\mu\sigma}) = \Phi'_{\sigma}. \]

Solutions to the first equation are
\[ 2\Gamma_{\lambda,\mu\nu} = \frac{\partial g_{\lambda\nu}}{\partial x^\mu} + \frac{\partial g_{\lambda\mu}}{\partial x^\nu} - \partial g_{\mu\nu}. \]

We choose \( \Phi' \) so that the second equation solves as
\[ F_{\mu\nu} = \frac{q}{cm}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}). \]

According to the results obtained in [2] we interpret \( g_{\mu\nu} \) as gravitational field and \( A_{\mu} \) as electromagnetic field.

### 3 Conclusion

In this paper we considered only two special cases of General Geometry [1]. Resuming, geometries discussed in this paper, with appropriate metrics are underlying geometries for physical theories. The most simplest case of General Geometry
\[ \frac{d\xi^\sigma}{du} = -F_{\sigma}^\lambda(x)\xi^\lambda, \]
with metric
\[ ds = \sqrt{\eta_{\mu\nu}dx^\mu dx^\nu + \frac{q}{cm}A_{\mu}(x)dx^\mu} \]
is geometry underlying Electromagnetism. Next order in \( x_u \), Riemannian geometry,
\[ \frac{d\xi^\lambda}{du} = -\Gamma^\sigma_{\lambda\nu}(x)x_u^\nu\xi^\lambda \]
with metric
\[ ds = \sqrt{g_{\mu\nu}dx^\mu dx^\nu} \tag{9} \]
is geometry underlying Gravitation. Combination of two previous geometries
\[ \frac{d\xi^\sigma}{du} = -(F_{\sigma}^\lambda(x) + \Gamma^\sigma_{\lambda\mu}(x)x_u^\mu)\xi^\lambda \]
with metric
\[ ds = \sqrt{g_{\mu\nu}dx^\mu dx^\nu + \frac{q}{cm}A_{\mu}(x)dx^\mu} \]
is geometry underlying unified model of Electromagnetism and Gravitation [2].
We do not discuss the other special cases in this paper. Riemannian Geometry with metric \( ds = \sqrt{g_{\mu \nu}dx^\mu dx^\nu} \) without parameters instead of (9) has been considered in [5] and applied to Kaluza-Klein theory. As we demonstrated in [1] any attempt to geometrize electromagnetism in geometries like Riemannian, (for example in the so called Finsler geometry) independent of the chosen metric must fail [6]. By choosing different metrics we do not change geometry [5], [7], [8].

4 Remarks

Thanks to moderators of [9] and [10] almost all attempts to sabotage these serious papers are made available online.

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\(^{2}\)I thank Prof. M. Anastasiei for informing me about [5] after [1], [2] and this paper have been posted on the Internet. It is surprising that Prof. M. Anastasiei recommended to publish [6] although its results contradicts those obtained in [1] (also, see [8]).