Composite Controller Design for Flywheel Based on Disturbance Observer and Robust PI Control

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Abstract. Flywheel is widely applied in the attitude control system (ACS) of the spacecraft as actuator, but friction torque caused by flywheel has been a difficult problem reducing the accuracy and stability of the ACS. This paper concerns the disturbance attenuation and rejection problem for flywheel system, and a new composite control scheme is presented. A disturbance observers (DO) is designed to reject the friction torque by feedforward compensation, and PI controller optimized by H∞ method is designed to control the wheel speed and attenuate other unmodeled disturbances, furthermore, input saturation problem is considered also. Finally, simulations for flywheel control system show that by combining the DO with conventional control law, disturbances can be attenuated and rejected and the dynamic performances are enhanced.

Introduction

Flywheel has been widely applied in the ACS of the spacecraft as key actuator to perform attitude control, but it is also a major source of disturbances, especially friction torque between the wheel body and the bearing reduces the pointing accuracy and stabilization of ACS. when the flywheel speed crosses zero, the static friction will dominate the relationship between commanded and actual control torque, and the relationship between the wheel speed and the friction torque is complex nonlinearity [1]. Then we should decrease friction torque caused by flywheel, and disturbances caused by flywheel imbalance and bearing imperfections should be taken into consideration also [2].

Friction is a complex nonlinear phenomenon that exists in mechanical systems. In various motion control applications, solid friction is one of dominant nonlinearities limiting the control performance. The problems caused by friction such as limit cycles, tracking error and stick-slip motion pose a problem in the control of mechanisms that require a high degree of dynamic accuracy and performance [3]. Many studies have been reported, which formulated a friction model, identified its parameters, and compensated for friction. In compensator design, the role of friction modeling can be categorized according to whether or not the friction compensation is model-based. Model-based compensators perform feedforward cancellation of the friction force. Their success depends on knowledge of the model structure, its observability, and knowledge of the model parameters [4,5].

In past years, disturbance-observer-based control (DOBC) has attracted considerable attention and many different schemes have been suggested [6,7]. Compensation through feedforward for the modeling error or the exogenous disturbance has been considered as a robust control scheme when the error or disturbance can be observed. In recent years, many available approaches based on DO are presented for different situations [8].

For flywheel system, PI control is a classic control law for its simplicity and reliability. H∞, which has the merit of keeping system robust stable in the presence of disturbance and model uncertainty, has attracted especial attention. In this paper, we designed the composite controller based on DOBC and PI control, and H∞ was adopted to optimize the parameters of PI controller; in which, DOBC can mitigate the fiction torque, and robust PI control can guarantee the robust stability against unmodeled disturbances.
Problem Formulation and Preliminaries

The Flywheel Dynamic Model

In general, there are two working mode for flywheel, that is, torque mode and angular speed mode. In this paper, we selected the angular speed mode, and seek the way how to decrease the friction torque. The dynamic model of flywheel is

\[ a\dot{\omega} + b\omega + cM_f(\omega) + cM_d(t) = U_{PWM} \]  

(1)

Where \( a = \frac{JR}{K_m}, b = K_e, c = \frac{R}{K_m} \); and \( R_u \) represents the resistance of motor armature, \( K_m \) represents the torque coefficient, \( K_e \) represents the anti-voltage coefficient, \( J \) represents the inertia of flywheel, \( M_f(\omega) \) represents the friction torque, \( U_{PWM} \) represents the control voltage input, \( \omega \) represents the angular speed of flywheel, \( M_d(t) \) represents the other disturbances except friction torque. To system (1), we design the PI controller \( u(t) = L_x \int_0^t s(t) dt + L_u s(t) \). Denoting \( s(t) = \omega_d(t) - \omega(t) \), \( x(t) = [\int_0^t \alpha c \sigma \tau d\tau, s(t)]^T \), and \( L = [L_x, L_u] \), then system(1) can be changed to the state-space format as follows

\[
\begin{align*}
\dot{x}(t) &= A_0 x(t) + B_0 u(t) + B_f d_0(t) + B_d d_1(t) \\
z(t) &= C_0 x(t)
\end{align*}
\]

where \( d_0 \in R^m; \quad d_1 \in R^m; \quad u \in R^m; \quad \) and \( z \in R^p \) are the friction torque, the unknown disturbance, the control input and the measurement output, respectively. \( d_1(t) \) represents unmolded disturbance such as random disturbance, measurement noise and system uncertainty, and it is supposed to be bounded by \( \tilde{d}_1(t) \) in Euclidean norm. Where \( A_0 = \begin{bmatrix} 0 & 1 \\ 0 & -a^{-1}b \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ -a^{-1} \end{bmatrix}, \quad B_f = \begin{bmatrix} 0 \\ a^{-1}c \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ -a^{-1}c \end{bmatrix} \).

Friction Torque Model

The behavior of friction has been extensively examined, and a lot of friction models are presented, such as Striebeck model, Dahl model, LuGre model, Maxwell model, etc. When we survey these models, we could find that the change rate of the friction torque are bounded unless the angular speed of the mechanism is zero, for instance, Striebeck model are widely used in the friction compensation, it can be described by: \( d_0(t) = M_c \text{sgn}(\dot{\theta}) + (M_m - M_c) e^{-a[\dot{\theta}]} \text{sgn}(\dot{\theta}) + k_1 \dot{\theta} \).

![Figure 1. Striebeck friction model.](image1)

Figure 1. Striebeck friction model.

![Figure 2. Friction torque and observed friction torque.](image2)

Figure 2. Friction torque and observed friction torque.
Where $M_c$ represents Coulomb friction, $M_m$ represents static friction, $\dot{\theta}$ represents angular speed, and $k_v$ represents viscous friction coefficient. Figure 1 described the probable changing tendency of friction torque, then $\frac{d\ddot{d}}{d\theta} \leq \alpha(M_m - M_c)$, that we can get $|\ddot{d}| \leq \alpha(M_m - M_c)\dot{\theta}$ unless the angular speed of the mechanism is zero.

**Disturbance Observer Design**

**Disturbance Observer**

According to system equation (2), the disturbance observer is formulated as:

$$
\dot{e}(t) = -N(x)B_f(t + p(x)) - N(x)(A_vx(t) + B_uu(t))
$$

$$
\ddot{d}(t) = \tau + p(x)
$$

(3)

Where $N(x)$ is the gain of the observer, defined by $N(x) = \frac{\partial p(x)}{\partial x}$. The error of disturbance observer is defined as $e(t) = \ddot{d}(t) - \dot{\ddot{d}}(t)$; then $\ddot{e}(t) = \dot{\ddot{d}}(t) - N(x)B_f e(t) - N(x)B_u d_1(t)$. To make $e(t) \to 0$, we should design appropriate $N(x)$.

**Controller Saturation Problem**

For the output voltage of the PWM circuit, which is the input of the controller, is restricted by its own physical configuration, then the input saturation which limits the controller gain must be considered. For this purpose, we consider the saturating controller as $\hat{u}(t) = \text{sat}(u(t))$, which is a nonlinear function whose element is defined by:

$$
\text{sat}(u(t)) = \begin{cases} 
  u & \text{if } |u| \leq u_{\text{lim}} \\
  \text{sign}(u)u_{\text{lim}} & \text{if } |u| > u_{\text{lim}} \end{cases} \quad (i = 1, 2, 3);
$$

where $u_{\text{lim}}$ is the maximum output torque of the actuator. Here, we designed the composite controller $u(t) = -\dot{\ddot{d}}(t) + \text{sat}(2Lx(t))$. Combined with equation (1), we can get the composite system:

$$
\begin{bmatrix} x(t) \\ \dot{e}(t) \end{bmatrix} + D_0\ddot{d}(x, t) = \begin{bmatrix} A_0 + B_vL & B_v \\ 0 & -NB_f \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B_v \\ -NB_f \end{bmatrix} d_1(t) + \begin{bmatrix} B_v \\ 0 \end{bmatrix} v(t);
$$

where $v(t) = \text{sat}(2Lx(t)) - Lx(t)$ is an auxiliary vector. It is easily to verify that it satisfies $v^T(t)v(t) \leq x^T(t)L^TLx(t)$. Denote $\bar{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$, then the composite system can be described as:

$$
\begin{bmatrix} \dot{x}(t) + D_0\ddot{d}(\bar{x}, t) = A\bar{x}(t) + B_2d_1(t) + B_2v(t) \\
\hat{x}(t) = C\bar{x}(t) \end{bmatrix}
$$

(4)

where

$$
A = \begin{bmatrix} A_0 + B_vL & B_v \\ 0 & -NB_f \end{bmatrix}; D_0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; B_1 = \begin{bmatrix} B_v \\ -NB_f \end{bmatrix}; B_2 = \begin{bmatrix} B_v \\ 0 \end{bmatrix}; C = [C_1, C_2]; W_0 = [W, 0]
$$

(5)

When stibec model is selected as system friction model, where $W = [0 \quad \alpha(M_m - M_c)]$, and $\|\ddot{d}(\bar{x}, t)\| \leq \|W_0\|\dot{x}$.
Stability of the Composite Systems

In this section, the stability of the composite system is proved, simultaneously we can get the solution of the system by LMI (linear matrix inequality).

**Theorem 1**: For some parameters \( \lambda_1 > 0 \), \( \lambda_2 > 0 \) and \( \gamma > 0 \), suppose that there exists matrix \( Q_1 > 0 \), \( P_2 > 0 \), \( R_1 \) and \( R_2 \), that satisfying

then the closed loop system (4) is stable and satisfies \( H_\infty \) performance, the controller gain is given by \( L = R_1 Q_1^{-1} \), and \( N = P_2^{-1} R_2 \).

\[
\Psi < 0
\]

\[
\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22} \end{bmatrix}; \Psi_{11} = \begin{bmatrix} \text{sym}(A_0 Q_1 + B_u R_1) & Q C_1^T & B_0 & 0 \\ * & -I & 0 & 0 \\ * & * & -\lambda_1 I & 0 \\ * & * & * & -\lambda_2^2 I \end{bmatrix}; \Psi_{12} = \begin{bmatrix} \lambda_0 Q_1 L^T & B_0 \\ 0 & 0 & 0 & C_2 \\ 0 & \lambda_2 B_2^T W T & 0 \\ 0 & 0 & 0 & P_2 \end{bmatrix}; \Psi_{22} = \begin{bmatrix} -\gamma^2 I & \lambda_2 B_2^T W T & -B_2^T R_2^T \\ * & -I & 0 & 0 \\ * & * & -I & \lambda_2 WB_2 \\ * & * & * & \text{sym}(-R_1 B_1) \end{bmatrix}
\]

Proof: See [7].

**Simulations on Flywheel**

**Simulation Parameter**

Here, the parameters of flywheel are selected as \( R_0 = 2 \Omega \), \( K_n = 0.036 \text{NM} / \text{A} \), \( K_c = 0.0283 \text{V} / (\text{rad} / \text{s}) \), \( J = 0.0038 \text{kgm}^2 \), \( U_{PWM} = -28 \text{V} \leftrightarrow +28 \text{V} \), \( k_v = 0.00006 \text{NM} / (\text{rad} / \text{s}) \), \( M_\omega = 0.0015 \text{NM} \), \( M_m = 0.005 \text{NMS} \).

We select \( \lambda_0 = 0.01 \), \( \lambda_2 = 0.01 \) and \( \gamma = 0.7 \), then applying Theorem 1, by LMIs, we can get the anticipated controller and observer parameter: \( L = -3.6256 \times 10^4 \); \( N = -27.4285 \).

Applying the flywheel to satellite with the nominal moment-of-inertia as below:

\[
I_x = 18.4; I_y = 18.2; I_z = 6.8;
\]

The satellite is designed to move in a circular orbit with the altitude of 500km, then the orbit rate \( n = 0.0011 \text{rad/s} \), and the initial attitude of the satellite are:

\[
\phi = 0.05 \text{rad}; \theta = 0.03 \text{rad}; \psi = 0.05 \text{rad}; \dot{\phi} = 0.001 \text{rad} / \text{s}; \dot{\theta} = 0.001 \text{rad} / \text{s}; \dot{\psi} = 0.0015 \text{rad} / \text{s}.
\]

**Simulation Analysis**

Figure 2 shows that the friction torque is precisely observed by DO, and then decreased by DOBC. When we adopt the flywheel controlled by composite controller as the actuator of the satellite, the precision and stabilization of ACS is improved, which is testified by figure 3 and figure 4, owing to feedforward compensation of the friction torque.
Conclusion
In this paper, composite hierarchical controller consisting of DO and PI controller optimized by $H_{\infty}$ is designed for system with multi-source disturbances. We apply this approach to a flywheel speed control system. It is shown that the composite controller can reject friction torque and attenuate other un-modeled disturbances; when we employ the improved flywheel as the actuator of a small satellite, the precision and stabilization of ACS is enhanced.

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