Rush the inspiral: efficient Effective One Body time-domain gravitational waveforms

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Computationally efficient waveforms are of central importance for gravitational wave data analysis of inspiralling and coalescing compact binaries. We show that the post-adiabatic (PA) approximation to the effective-one-body (EOB) description of the binary dynamics, when pushed to high order, allows one to accurately and efficiently compute the waveform of coalescing binary neutron stars (BNSs) or black holes (BBHs) up to a few orbits before merger. This is accomplished bypassing the usual need of numerically solving the relative EOB dynamics described by a set of ordinary differential equations (ODEs). Under the assumption that radiation reaction is small, Hamilton’s equations for the momenta can be solved analytically for given values of the relative separation. Time and orbital phase are then recovered by simple numerical quadratures. For the least adiabatic BBH case, equal-mass, quasi-extremal spins anti-aligned with the orbital angular momentum, 6PA/8PA orders are able to generate waveforms that accumulate less than $10^{-3}$ rad of phase difference with respect to the complete EOB ones up to $\sim 3$ orbits before merger. Analogous results hold for BNSs. The PA waveform generation is extremely efficient: for a standard BNS system from 10 Hz, a nonoptimized Matlab implementation of the TEOBResumS EOB model in the PA approximation is almost 100 times faster ($\sim 0.09$ s) than the corresponding C++ code based on a standard ODE solver. Once optimized further, our approach will allow us to (i) avoid the use of the fast, but often inaccurate, post-Newtonian inspiral waveforms, drastically reducing the impact of systematics due to inspiral waveform modelling, and (ii) alleviate the need of constructing EOB waveform surrogates to be used in parameter estimation codes.

I. INTRODUCTION

Analytical waveform models informed by (or calibrated to) numerical relativity (NR) simulations are essential for the analysis of gravitational wave (GW) events \cite{1-7}. The effective-one-body (EOB) approach to the general relativistic two-body problem \cite{8-11} is currently the only available analytical tool that reliably describes both the dynamics and the gravitational waveform through inspiral, merger and ringdown for binary black holes (BBHs) \cite{12-14}, and up to merger for binary neutron stars (BNSs) \cite{15}. The analytical model is crucially improved in the late-inspiral, strong-field, fast-velocity regime by numerical relativity (NR) information, which allows one to properly represent the merger and ringdown part of the waveform \cite{12,13,16}. The synergy between EOB and NR creates EOBNR models, whose more recent avatars are SEOBNRv4/SEOBNRv4T \cite{12,14,17,18} and TEOBResumS\cite{13,19}, which describe nonprecessing binaries and SEOBNRv3\cite{20}, which incorporates precession for BHB. However, it has to be understood that, though the framework is analytical, the Hamiltonian equations of motion have to be solved numerically with standard techniques. The computational cost of computing an EOB waveform is then mostly due to the solution of Hamilton’s equations: the longer the waveform, the more expensive is its generation. For low-mass binaries, with long inspirals within the sensitivity band of the detector, the computational cost of generating an EOB waveform with TEOBResumS\cite{21} is $\sim$ a few seconds, such that the model cannot be directly used for data analysis purposes (see, however, Refs. \cite{22,23}; by contrast, it was possible to use SEOBNRv4 explicitly on high-mass binaries \cite{7,21}). This prompted several recent efforts to optimize EOB codes \cite{25,26} or to compute surrogate EOB waveforms based on reduced-order modeling (ROM) techniques \cite{12,27,31}. Building such surrogates is currently an obliged path to use EOB models in standard data analysis pipelines. In addition, closed-form frequency-domain phenomenological (Phenom) waveform models offer a valid alternative \cite{32,33,34,35,36}. These models are obtained by first joining together EOB-based inspirals with NR simulations describing the last orbits through merger and ringdown and then building suitable interpolating fits all over the parameter space. Though both EOB (surrogate) and Phenom proved comparatively good from the BBH waveform generation point of view, EOB models are physically richer because of the built-in description of the relative dynamics. As a drawback, the construction of these models is time consuming and not very flexible. For example, if the original model is changed, the surrogate has to be rebuilt. Thus, an intermediate step (i.e., construction of a surrogate) is always needed before new theoretical ideas can be tested on GW experimental data. The detection and subsequent analysis of GW170817 \cite{5} illustrated that the current status is far from optimal: the most developed models with tidal interactions could not be immediately

1. It is of the order of hours with SEOBNRv4\cite{12}.
used on the data because of their computational inefficiency; tidal EOB surrogates were not available except for the special nonspinning case [31]; PN-based and/or Phenom-like tidal models [19, 64] were available, but they might be plagued by systematic effects that have to be understood thoroughly [19], as they may result in biases in the measured parameters. In this paper we follow a different route. Focusing on spin-aligned (nonprecessing) binaries, we use the post-adiabatic approximation to the EOB inspiral (using the TEOBResum5 model [21]) to obtain the gauge-invariant dynamics analytically, without the need of solving ODEs. Through two additional quadratures, we then obtain computationally inexpensive, though robust and accurate, EOB inspiral waveforms up to a few orbits before merger. This is expected to tame, if not completely remove, most of the problems mentioned above.

II. THE POST–ADIABATIC APPROXIMATION TO EOB INSPIRAL

The circularized binary dynamics of two objects with masses \((m_1, m_2)\) evolves quasi-adiabatically under the action of a gravitational radiation reaction. Long ago, Ref. [35] constructed resummed inspiral waveforms based on the adiabatic approximation (i.e., the dynamics is represented by a sequence of circular orbits). When compared to state-of-the-art 3PN EOB dynamics, they proved to be more reliable and robust than the corresponding PN approximants. However, the system inspirals inward because of the presence of a small, but non-negligible, radial momentum \(P_r\), and the dynamics becomes progressively less adiabatic as the merger is approached. The need of analytically computing post-adiabatic (PA) corrections was pointed out as early as in Refs. [2, 3], so to provide low-eccentricity initial data to the EOB dynamics when it is started at relatively close \((r = 15M)\) separations. When high-accuracy, low-eccentricity NR data came into play [36], EOB/NR comparisons prompted the need of post-post-adiabatic (2PA) initial data [37, 38], so as to reduce the EOB eccentricity below the NR one. The PA approximation to EOB dynamics is built as follows. We use phase-space dimensionless variables \((r, p_r, \varphi, p_{\varphi})\), related to the physical ones by \(r = R/GM\), \(p_r = P_r/\mu\), \(p_{\varphi} = P_{\varphi}/(\mu GM)\), and \(t = T/(GM)\), where \(\mu = m_1 m_2/M\), with \(M = m_1 + m_2\). The radial momentum \(p_r\) is defined as \(p_r = (A/B)^{1/2} r\), in which \(A\) and \(B\) are the EOB potentials. Following Ref. [39], the EOB Hamiltonian is

\[
\hat{H}_{\text{EOB}} = \left(1 + 2\nu(\hat{H}_{\text{eff}} - 1)\right)/\nu, \quad \text{where} \quad \nu = \mu/M
\]

and \(\hat{H}_{\text{eff}} = \hat{G}p_\varphi + \hat{H}_{\text{orb}}\), with \(\hat{G}p_\varphi\) being the spin-orbit sector, while \(\hat{H}_{\text{orb}}\) looks formally like the orbital (nonspinning) effective Hamiltonian, though it actually incorporates in special resummed form spin-spin effects through the use of the concept of centrifugal radius \(r_c\) [39]. Explicitly, it reads

\[
\hat{H}_{\text{orb}} = \left(1 + p_\varphi^2/r_c^2 + \frac{z_3}{3} p_r^3/r_c^2\right), \quad \text{with} \quad z_3 = 2\nu (4 - 3\nu).
\]

In addition, \(\hat{G} \equiv G_S \hat{S} + G_S \hat{S}_z\), where \(\hat{S} \equiv (S_A + S_B)/M^2\), \(\hat{S}_z \equiv ((m_B/m_A) S_A + (m_A/m_B) S_B)/M^2\), in which \((S_A, S_B)\) are the individual spins, and \((G_S, G_{S_z})\) are the gyromagnetic functions considered at (next-to)-leading order. The spin gauge is fixed so that they only depend on \((r, p_r)\) [39, 41]. An effective (next-to)-leading-order parameter \(c_3\) [39, 42] is included in \((G_S, G_{S_z})\) and informed by NR simulations as in Ref. [21]. The EOB Hamiltonian’s equations for spin-aligned binaries that are usually solved numerically read

\[
\frac{d\varphi}{dt} = \frac{1}{\nu H_{\text{EOB}} H_{\text{eff}}^2} \left[A p_\varphi^2/r_c^2 + \hat{H}_{\text{orb}} \hat{G}\right] - \frac{\nu H_{\text{EOB}} H_{\text{eff}}^2}{1 + 2\nu H_{\text{EOB}} H_{\text{eff}}^2} (1 + \hat{G} p_\varphi \hat{p}_r) (A p_\varphi^2/r_c^2) + \hat{H}_{\text{orb}} \hat{p}_r \frac{\partial \hat{G}}{\partial p_r},
\]

\[
\frac{dp_r}{dt} = -\frac{(A/B)^{1/2}}{2\nu H_{\text{EOB}} H_{\text{eff}}^2} \left[A' + p_\varphi^2 \left(\frac{A'}{r_c^2}\right)' + z_3 p_r^3 \left(\frac{A'}{r_c^2}\right)' + 2 H_{\text{orb}} \hat{p}_r \hat{G}\right],
\]

where \((\cdot)' \equiv \partial_r (\cdot)\), and we fix \(\hat{F}_r = 0\) in Eq. (4). The \(A\) function incorporates an effective 5PN parameter \(a_3(\nu)\) informed by NR simulations [34]. Tidal effects, as well as spin-induced quadrupole-moment effects, are also included in the formalism [17, 18, 21, 43, 48]. The adiabatic approximation assumes no radiation reaction, \(\hat{F}_r = 0\), so that \(p_r = 0\) and \(p_r = j_0\), obtained imposing \(\partial_r \hat{H}_{\text{EOB}} = 0\) at a given radius \(r\) [see Eq. (C1) of [21]]. The PA approximation [9] assumes \(\hat{F}_r\) to be small and then consistently calculates \(p_r\), combining Eqs. (2) and (3) as \(dp_r/dt = \hat{F}_r/(dr/dt)^{-1}\). At the 2PA level [37], one obtains the additional correction to \(p_r\) as \(p_r \neq 0\) using Eq. (2) and (4). The procedure can then be iterated further. At a formal level, one is assuming that, for each \(r\), \(\hat{F}_r(r) = \sum_{n=0}^{\infty} F_{2n+1}(r) \varepsilon^{2n+1}\), where \(\varepsilon\) is a formal bookkeeping parameter that is eventually set to 1. We can hence write the solution of the EOB equations of motion as a formal expansion in powers of \(\varepsilon\) as \(p_r^\varepsilon_r (r) = j_0^\varepsilon_r (r) (1 + \sum_{n=0}^{\infty} k_{2n+1}(r) \varepsilon^{2n})\), and \(p_r (r) = \sum_{n=0}^{\infty} \pi_{2n+1}(r) \varepsilon^{2n+1}\). To finally obtain the corrections \((k_{2n+1}, \pi_{2n+1})\) to the circular solution \((j_0, 0)\), one has to iteratively solve the two equations \(dp_r/dt = (dp_r/dr)/(dr/dt)\) and \(dp_\varphi/dt = (dp_\varphi/dr)/(dr/dt)\), where the radial derivatives of the momenta are replaced by their series expansion above, and the time-derivatives are substituted by Hamilton’s equations. We solve these equations alternatively power by power in \(\varepsilon\), obtaining the corrections to the angular and radial momentum respectively. In general, every coefficient depends on the
lower-order ones and their radial derivatives. This procedure can be iterated as many times as one likes. We call \( n \)-post-adiabatic (\( nPA \)) a quantity calculated up to \( \varepsilon^n \). Computing the \( nPA \) approximation is then straightforward, since only linear equations are involved, though tedious at high order, as it involves many terms. We can obtain the same results following a mathematically less rigorous but quicker path. We can express the two equations above as

\[
\left( \frac{A}{r^2} \right) \theta + \frac{2 \dot{H}_{\text{orb}}}{H} \frac{\partial G}{\partial p_r} \frac{\partial p_r}{\partial t} + \frac{\partial G}{\partial p_r} \frac{\partial p_r}{\partial r} \right) p_r + A' \left( 2 + 2 z \frac{A}{r^2} p_r^2 \right) \frac{\partial p_r}{\partial r} + z_3 \left( \frac{A}{r^2} \right) p_r^* = 0,
\]

We treat the explicit \( p_r \) in the first equation and the \( p_r \) in the left-hand side of the second as the only unknown variables. All other \((p_r, p_r, \ldots)\) that appear, also within \((H_{\text{EOB}}, \dot{H}_{\text{eff}}, G, \ldots)\), are kept at previously known order. This is to ease implement, as the same equations must be solved at each order. Since we did not find any significant discrepancy with the rigorous PA approximation, we only present here results with Eqs. (5) and (6). The cant discrepancy with the rigorous PA approximation, we be solved at each order. Since we did not find any signifi-

\[
\frac{\partial H_{\text{EOB}}}{\partial p_r} \frac{\partial^2 p_r}{\partial t^2} + \left( 1 + 2 z \frac{A}{r^2} p_r^2 \right) \frac{\partial p_r}{\partial r} = 0,
\]

We present here results with Eqs. (6). The \( nPA \) dynamics is then obtained via a three-step procedure: (i) A radial grid is built between \( r_{\text{max}} \) and \( r_{\text{min}} \) with spacing \( \Delta r \) chosen uniform for simplicity. (ii) For each grid point, \((p_r, p_r, \ldots)\) are obtained from Eqs. (5) and (6) at a given iteration order. (iii) The time \( t \) and the orbital phase \( \phi \) are recovered by quadratures as \( t = \int_{r_{\text{max}}}^r \text{d}r (\partial p_r \dot{H})^{-1} \) and \( \phi = \int_0^t \text{d}t \partial p_r \dot{H} = \int_{r_{\text{max}}}^{r_{\text{min}}} \text{d}r \partial p_r \dot{H} (\partial p_r \dot{H})^{-1} \). This way, one obtains \((t, \phi, p_r, p_r, \ldots)\) on a given grid. Since the \( r \) grid is evenly spaced, \( t \) is not, with time steps becoming progressively smaller as \( r \) decreases. Finally, the (r_{\text{max}}, r_{\text{min}}) grid is built with r_{\text{min}} near to the EOB LSO, the physically meaningful range of the PA approximation is only up to a given \( r_{\text{impl-end}} > r_{\text{min}} \) where \( r_{\text{impl-end}} \) corresponds to the first inflection point of \( p_r \), i.e., where it starts decreasing more slowly instead of keeping on accelerating, as is the case for the complete EOB. This point depends on the PA order and will be explicitly marked when discussing results below. The waveform is built upon the PA dynamics \((t, \phi, p_r, p_r)\) using the standard EOB prescription [49, 50, 49]. The multipolar strain waveform \( h_{\text{in}} \) is defined as \( h_{\text{in}} - h_{\text{in}} = \mathcal{R}^{-1} \sum_{\ell m} h_{\text{in} - 2Y_{\ell m}}(\theta, \Phi) \), where \( \mathcal{R} \) is the distance from the source and \( 2Y_{\ell m}(\theta, \Phi) \) is the \( s = -2 \) spin-weighted spherical harmonics. Though we obtain all inspiral multipoles at once up to \( \ell = 8 \), we only discuss \( h_{22} = \mathcal{R}e^{-i\phi} \). We refer to the PA approximated EOB as EOB_{PA,III. RESULTS

The quality of the EOB_{PA} model is assessed on a few relevant cases involving spin-aligned BBHs and BNSs. Figure 1 illustrates the phasing performance for three fiducial BBHs with \((q, \chi_2, \chi_B)\) equal to \((1, -0.99, -0.99), (1, +0.99, +0.99)\) and \((2, +0.8, -0.2)\). The first configuration allows us to test the approximation in the most difficult regime i.e., when the inspiral is less adiabatic due to the strong, attractive, spin-orbit interaction exerted by the two spins anti-aligned with the orbital angular momentum. The various PA approximations are contrasted with the corresponding time-domain TEOBResumS waveform [13, 19]. The top row shows the relative amplitude difference \( \Delta_{\text{EOBPA}}/\Delta_{\text{EOB}} \equiv 1 - (\frac{\text{EOB}}{\Delta_{\text{EOB}}} - \Delta_{\text{EOB}}) \) and the middle panel shows the phase difference \( \Delta_{\text{EOBPA}} = \phi_{\text{EOB}} - \phi_{\text{PA}} \), while the bottom panel depicts the real part of the waveform. The black line is the pure inspiral EOB waveform, without the NR-informed next-to-quasi-circular (NQC) correction parameters or the ringdown [19]. For completeness, we also added the full EOB waveform with merger and ringdown (gray line). For this specific comparison, we did not iterate on NQC parameters [19]. The plot focuses on the last few cycles of inspirals that began at initial separation \( r_{\text{max}} = 20 \). The time-evolution of TEOBResumS was initiated with 2PA initial data. To orient the reader, the orange vertical line marks the location of the adiabatic EOB last stable orbit (LSO) for \((1, -0.99, -0.99) \) (LSO _{\text{EOB}} = 6.69) and \((3, +0.80, -0.20) \) (LSO _{\text{EOB}} = 4.30), while it corresponds to the \( r_{\text{Schwarzschild}} = 6 \) crossing for \((1, +0.99, +0.99) \) (the adiabatic TEOBResumS dynamics does not have an LSO when the spins are large and aligned [50]). The PA approximation converges very fast, and moving from 2PA to 4PA is already sufficient to obtain phase differences \( \lesssim 0.05 \) rad up to \( 3 \) orbits before merger. When pushed to higher order (notably 5PA) the phase difference is \( \lesssim 10^{-3} \) rad up to \( 3 \) orbits before merger. Note that we did not perform any additional phase or time alignment between the waveforms. The PA waveforms were obtained with resolution \( \Delta t = 0.1 \) and \( r_{\text{min}} \sim (8.8, 4.2, 4.1) \) respectively, with \( N_{\text{PA}} = (112, 159, 159) \) points. We verified that, thanks to the crucial fact that we use a third-order integration routine to recover \([\phi(r), \dot{r}(r)]\), the waveform temporal length is insensitive to the choice of resolution, which can be safely increased up to \( \Delta t = 0.4 \). For illustrative purposes in Fig. 1 the original, sparse and nonuniform, temporal grid corresponding to (\( r_{\text{max}}, r_{\text{impl-end}} \)) is interpolated on a uniform grid with \( \Delta t = 0.5M \). The end of the 8PA-inspiral corresponds to \( r_{\text{impl-end}} = (10.55, 4.1, 5.9) \). Figure 2 compares the binding energy per reduced mass, \( E_b = (E - M)/\mu \), where \( E = \nu \dot{H} \) computed along the EOB and EOBPA dynamics. The agreement is excellent nearly up to the LSO and, notably, within the expected uncertainty on this quantity computed from NR simulations [49, 19, 51]. Figure 3 illustrates the similar behavior for a fiducial, equal-mass, BNS system, with tidal polarizabilities \( \Lambda_A = \)
\[ \Lambda_B = \frac{(2/3) k_2}{C^5} = 392, \] where \( k_2 \) is the quadrupolar relativistic Love number [52, 50], and \( C \) is the star compactness. This picture is stable when changing equation of state (EOS) and/or incorporating the spins. For simplicity, the EOB\(_{PA}\) model we discuss here was implemented in Matlab without any optimization strategy. Table I contrasts the performance of such Matlab implementation with the C++ version of TEOBResumS [21] for a few long inspirals. The radial PA grid has \( \Delta r = 0.15 M \), but, as before, the EOB\(_{PA}\) waveform remains stable even with coarser grids up to \( \Delta r = 0.4 \). We note in passing that the 8PA running time is comparable to (and actually smaller than) the one provided by the TEOBResum\_ROM model for nonspinning BNSs of Ref. [61].

**FIG. 1.** Waveform comparison, \( \ell = m = 2 \) strain mode: EOB\(_{PA}\) inspiral (colors) versus EOB inspiral obtained solving the ODEs (black). The light-gray curve also incorporates the EOB merger and ringdown. The orange vertical line marks the EOB LSO crossing for \((1, -0.99, -0.99)\) and \((3, +0.80, -0.20)\), while it corresponds to \( r = 6 \)-crossing for \((1, +0.99, +0.99)\). The filled markers highlight the end of the PA inspirals. The 4PA approximation already delivers an acceptable EOB/EOB\(_{PA}\) agreement for both phase, \( \phi \), and amplitude, \( A \). This is improved further by the successive PA approximations. At 8PA, the GW phase difference is \( \lesssim 10^{-3} \) rad up to \( \sim 3 \) orbits before merger.

**FIG. 2.** Illustrative comparison between energies versus orbital angular momentum curves. The black vertical line marks the EOB merger, while the orange one marks the EOB LSO crossing. The EOB/EOB\(_{PA}\) agreement is excellent up to the range of validity of each PA order (filled markers).

**TABLE I.** Illustrative performance comparison between the C++ implementation of TEOBResumS with ODE solver and the corresponding 8PA Matlab implementation for the fiducial BNS system of Fig. 1. The sampling rate of the ODE is 4096 Hz, corresponding to \( \Delta t \approx 18 M \). The EOB\(_{PA}\) waveform is obtained from \( N_{\ell m}^{\text{PA}} \) radial points. The runs are done on a MacBook Pro, with Intel Core i7, 3.5GHz, 16GB RAM.

| \( f_0 \) [Hz] | \( R_{\text{max}}/M \) | \( N_{\ell m}^{\text{PA}} \) | \( C_{\text{max}} \) [s] | \( \tau_{\text{trans}} \) [s] |
|----------------|-----------------|-----------------|-----------------|-----------------|
| 10             | 178.73          | 1143            | 6.12            | 0.09            |
| 20             | 112.73          | 702             | 1.17            | 0.08            |
| 30             | 86.029          | 524             | 0.49            | 0.065           |
IV. CONCLUSIONS

We showed that the post-adiabatic analytic approximation to the EOB dynamics, when pushed to high order, is a useful tool to compute approximate, though reliable, EOB inspiral waveforms. These EOBPA waveforms well reproduce ($\Delta \phi \lesssim 0.001$ rad) the non-approximated ones, obtained numerically solving Hamilton’s equations, up to $\sim 3$ orbits before merger, independently of the (ilustrative) binary configurations considered. For BNS inspirals, Table I showed that even a largely non-optimized Matlab infrastructure can generate waveforms whose computational cost is orders of magnitude smaller than the dedicated TEOBResum$^S$ C++ numerical code. Though the PA approximation is not reliable in the last $\sim 3$ orbits, nonetheless it can be used to start ODE-based EOB evolutions from a radius larger than $r_{\text{inspiral}}$ (e.g., twice as large, to be conservative), so to reduce the computational cost of a complete EOB waveform. For a BBH event, the C++ typically takes $\lesssim 16$ ms to ODE-evolve the last seven orbits through merger and ringdown. We expect that our approach, once properly implemented in data-analysis pipelines, will allow one to avoid the use of PN-based inspiral waveform models, so as to drastically reduce the impact of systematics due to waveform modeling on GW data analysis of long-inspiral coalescing compact binaries, like GW170817. One could also combine the PA approximation with the stationary phase approximation to directly compute the Fourier-domain inspiral phase \cite{57, 58}. The need of EOB waveform surrogates might then be reduced. Due to its flexibility, the EOBPA model could also be directly used to perform tests of general relativity. Finally, we expect the PA approximation to be useful for circularized precessing binaries, as well as for the EOB inspiral of eccentric binaries \cite{59–62}, although the impact of the radial part of the radiation reaction needs to be properly evaluated.

A C implementation of EOBPA is publicly available as a stand-alone code (see Ref. \cite{21}, Appendix E). Its performances are better than those of the Matlab one discussed here and will be detailed elsewhere.

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