Calculation of the optical variasystems with tunable optical power lenses

P A Nosov, D E Piskunov, M A Vinogradov, V O Tigaev and A A Yablokova

Bauman Moscow State Technical University, ul. Baumanskaya 2-ya, 5/1, Moscow, 105005, Russia

Abstract. Modern optical variasystems include lenses with a fixed and tunable focal length, which can also move relative to each other according to a certain law. The method of dimensional synthesis of the variasystems converting incoherent radiation and laser Gaussian beams is considered. Examples of the calculation of optical variasystems are presented.

1. Introduction
Optical vario- or zoom-systems have become widespread in various fields of science and technology. In their development, different approaches are used to allow a required change in the optical characteristics of the system. The most common approach is to move the components of the optical system according to a certain law [1-3]. Recently lenses with tunable optical power have begun to be used [4-6]. To change the optical power of the lens, it is necessary to change either its shape or the index of refraction. Liquid optical media are used for these purposes, therefore in most cases lenses with variable optical power are meant to be liquid lenses. The shape of the lens surface can be changed both mechanically and electrically. Today, there are several technologies for creating liquid lenses, two of them are based on the effect of electrowetting, as a result of which the curvature of the contact surface of two immiscible liquids changes, and on the basis of a change in the curvature of the elastic membrane limiting volume. These technologies have been commercialized by Varioptic and Optotune. Liquid crystal lenses are also perspective. The tunable refractive index of the liquid crystal lens allows to create the necessary phase profile and adjust the optical power of the lens.

The calculation of various systems with liquid lenses includes the same steps as for optical systems with moving lenses [1, 3]. The main feature of the calculation of an optical variosystem based on liquid lenses appears at the stage of structural and dimensional synthesis. Moreover, the development of optical systems for the transformation of laser Gaussian beams should be carried out on the basis of the laser optics theory [7-9]. Let’s consider the features of the development of optical various systems of different types and methods for their automated calculation.

2. Optical various systems calculation
In the general case, the problem of calculating of a various system is reduced to solving a system of nonlinear equations [3, 10]:

\[ P(d, \varphi) = P_t, \] (1)
where $P$ is the vector of paraxial values, which should be equal to the required values (e.g. the focal length and the length of the system), $d$ is the vector of the distances between the components, $\varphi$ is the vector of optical powers of the components, $P_k$ is the vector of required values of the paraxial values for the $k$-th position.

For a two-component optical variosystem that converts incoherent homocentric radiation, the system (1) has the following form (figure 1) [10]:

\[
\begin{align*}
   d_1 + d_2 + d_3 &= L, \\
   1 - d_2\varphi_1 - d_1\varphi_1 - d_2\varphi_2 + d_2d_1\varphi_1\varphi_2 &= \beta, \\
   1 - d_1\varphi_1 - d_3\varphi_2 - d_2\varphi_2 + d_2d_1\varphi_1\varphi_2 &= \frac{1}{\beta}, \\
\end{align*}
\]

where $\varphi_i$ is the optical power of the component ($i=1,2$), $d_1$ and $d_3$ are the distances from the object plane to the first component and from the second component to the image plane, respectively, $d_2$ is the distance between the components, $L$ and $\beta$ are the length and transverse magnification of the system, respectively (they form the vector of the paraxial values $P_k$).

For incoherent homocentric radiation in the analytical solution of the system of equations (2) in the case of a variosystem with moving components, it is assumed that the vector $\varphi$ is known, i.e. the problem is reduced to determining the laws of components moving. One of approaches to solving the system (1) with the known vector $\varphi$ is its transformation to a power equation. From (2) it is possible to obtain the law of displacement of the components of a two-component variosystem [6, 11]:

\[
d_2 = \frac{L}{2} \left[ 1 \pm \sqrt{1 - 4\left(\frac{\varphi_1 + \varphi_2}{L\varphi_1\varphi_2} + \frac{(\beta - 1)^2}{L^2}\beta\varphi_1\varphi_2\right)} \right].
\]

Substituting the solution of (3) to the system of equations (2), one can determine the distances $d_1$ and $d_3$.

If the change in the magnification in the variosystem is carried out using liquid lenses, then it is assumed that the separations are known and it’s necessary to determine the laws of change in the optical powers. In this case, for a two-component system, the solution of the system (2) is [6]:

\[
\varphi_1 = \frac{\beta(d_1 - d_2) - d_3}{\beta d_1 d_2}, \quad \varphi_2 = \frac{d_1 + d_3 - d_2\beta}{d_1 d_2^2}.
\]
In [3], it was shown that for convenience of numerical calculations, the system of equations (1) could be converted to a system of the form:

\[ P(a, b) = P(m), \]

where \( m \) is the argument of the basis function that determines the position of the components or optical power; \( a, b \) are vectors of coefficients of expansion into basis functions.

The laws of displacement \( d(m) \) and changes in the optical power \( \varphi(m) \) of individual components, which are presented in the form of expansion into basis functions, are written as follows:

\[ d(m) = \sum_{i=0}^{N} a_i F_{dl}(m), \quad \varphi(m) = \sum_{i=0}^{N} b_i F_{\varphi i}(m), \tag{4} \]

where \( a_i, b_i \) are expansion coefficients from \( a, b \) vectors; \( N \) is the number of expansion members.

As a result, the calculation of a variosystem is reduced to the determination of the expansion coefficients \( a_i, b_i \). Such an approach makes it possible to calculate both traditional systems with moving components and with liquid lenses, as well as combined ones, i.e. including either moving components or optical components with tunable optical power.

As a rule, the system of equations (1) has several solutions. For example, in the case of a two-component system, we have two solutions (3). This means that two paths of movement of the components are possible. In numerical calculations, the solution is defined as individual points on the curves (3), which determine the corresponding position of the components. A problem may arise when these points belong to different curves. This means that the magnification of the system will change with a jump while the movement of the components is smooth. If we define a solution in the form of the expansion (4), then the problem of matching the roots for a smooth change in magnification does not arise, since the solution is defined as a curve (of the law of displacement), but not a single point on it.

Now let’s consider the calculation of optical variosystems for the transformation of Gaussian laser beams. Let the variosystem be designed to form a waist of a Gaussian beam of a constant diameter at different distances from the input waist (of the laser). This problem can be solved by an optical system in which components move along the optical axis [12], or can change their optical power. Figure 2 shows an optical system with a lens of variable optical power and its longitudinal movement to form a waist of a Gaussian beam of a constant diameter at different distances from the input waist.

\[ \text{Figure 2. Single-component laser variosystem with tunable optical power lens and its longitudinal movement: } d \text{ is the distance from the waist of the input beam to the lens, } L \text{ is the distance from the waist of the beam at the lens input to the output waist, } f' \text{ is the lens focal length.} \]
When the Gaussian beam is transformed by the optical system, its invariant is performed for its spatial parameters [7-9, 12]: 

\[ 2h_w \theta = 2h'_w \theta' = \text{const} = 4M^2 \lambda / \pi \]

and 

\[ (2h_w)^2 / z_R = (2h'_w)^2 / z'_R = \text{const} = 4M^2 \lambda / \pi, \]

where \( h_w \), \( \theta \) and \( z_R \) are the diameter, angular divergence and Rayleigh range of the beam, \( \lambda \) is a beam wavelength, \( M^2 \) is the quality factor of the beam. The parameters of the transformed beam are denoted with a prime symbol. From this expression it follows that when the Gaussian beam is formed with a constant waist diameter at different distances from the input waist, the angular divergence of the output beam also remains constant.

Analysis of the transformation of the Gaussian beam of the laser optical system gives the following result [7]. The angular divergence of the beam on the optical system output is equal to 

\[ 2\theta'_k = 2h'_k / f'_k, \]

where \( 2h'_k \), the diameter of the beam at the input of the \( k \)-th optical system component in its front part, \( f'_k \) is the back focal length of the \( k \)-th component of the optical system.

Then the parameters of the beam and variousystem for forming the waist of a constant diameter at different distances from the input waist are connected by the expression:

\[ \frac{2h_f}{f'} = \text{const} = \frac{2M^2 \lambda}{\pi h'_u} = 2 \sqrt{\frac{M^2 \lambda}{\pi z'_R}}. \]

From this condition, one can obtain the law of change in the focal distance of the lens from its position.

The position of the waist of the Gaussian beam transformed by the lens and the distance from the input to the output waist are determined by longitudinal magnification in the near zone \( \alpha_G \) [7-9, 12]:

\[ \alpha_G = \frac{f'^2}{z'_w + z'_c}, \quad z'_w = -z_w \alpha_G, \quad L = -z_w \left(1 + \alpha_G \right) + 2f', \]

where \( z_w \) and \( z'_w \) are the distances that determine the position of the waist of the input and transformed beam relative to the front and rear foci of the optical system, respectively. For the distances \( z_w \) and \( z'_w \), the following rule of signs holds true: to the right of the foci – positive, to the left – negative.

3. Examples of variousystem calculation

**Example 1.** Let’s consider as an example the results of calculating of a 20⁴ variousystem with a focal lengths range from 15 to 300 mm, consisting of four components. The change in focal length is provided by moving of the 2nd, 3rd and 4th components of the optical system along the optical axis. As the basis functions in the formula (4) we choose power functions. Then the distances between the components of the variousystem can be represented as an expansion in the power series of the following form:

\[ d(f') = a_0 + a_1 f' + a_2 f'^2 + a_3 f'^3, \quad (5) \]

where as a parameter \( m \) from (4) the focal length \( f' \) of the variousystem can be chosen.

Table 1 presents the paraxial parameters of the components of the calculated variousystem, and table 2 presents the expansion coefficients for the corresponding air distances. The laws of the moving components are shown in figure 3.
Table 1. Paraxial parameters of the variolens.

| Component # | Focal length (mm) | Distance | Relative aperture |
|-------------|------------------|----------|------------------|
| 1           | 78.700           | $d_1$    | 0.600            |
| 2           | -18.850          | $d_2$    | 0.900            |
| 3           | 32.170           | $d_3$    | 0.700            |
| 4           | -44.000          | $d_4$    | 0.200            |

Table 2. Expansion coefficients of the air distances of the variolens in (5).

| $d_1$  | $a_0$ (mm) | $a_1$ (mm) | $a_2$ (mm$^{-1}$) | $a_3$ (mm$^{-2}$) |
|--------|-------------|------------|------------------|------------------|
| 18.893 | 0.18064     | -0.00030755| 1.95E-07         |
| 85.994 | -0.22193    | 8.37E-05   | -5.51E-07        |
| 41.88  | -0.053718   | 5.21E-05   | 2.13E-07         |
| 3.2334 | 0.095008    | 0.00017175 | 1.44E-07         |

Table 3. The expansion coefficients of focal lengths of the components of the variolens in (6).

| $f'_i$ (f') | $a_0$ (mm$^{-2}$) | $a_1$ (mm$^{-2}$) | $a_2$ (mm$^{-1}$) | $a_3$ (mm$^{-2}$) | $a_4$ (mm$^{-3}$) |
|-------------|------------------|------------------|------------------|------------------|------------------|
| $f'_1$      | -0.0011          | 0.1051           | -3.4208          | 35.6882          | 100.0000         |
| $f'_4$      | 0.0003           | -0.0083          | -0.0845          | 5.1280           | -11.8967         |

Figure 3. The laws of displacement of the components of the variolens.

The apertures of the components (table 1) were obtained with the following system parameters: a linear field in the image space is 15 mm, the f-number at the minimum focal length is 2.8, and 6 at the maximum focal length. The resulting system can be used for subsequent aberration synthesis.

Example 2. Let us now consider the results of calculating a $2^2$ variolens with liquid lenses with a focal length range of 17..34 mm (figure 4). The variolens consists of 6 components, two of which, the second and fifth are liquid lenses. The laws of change of the focal length of the $i$-th component $f'_i$ are determined as an expansion in powers of the focal length $f'$ of the entire lens:

$$f'_i(f') = a_0 + a_1 f' + a_2 f'^2 + a_3 f'^3 + a_4 f'^4.$$  \hspace{1cm} (6)

Table 3 presents the expansion coefficients, and Figure 5 shows the laws of change in the focal length of liquid lenses (figure 4).
The parameters of a variolens after the transformation to a system of lenses of finite thickness are presented in tables 4 and 5.

**Figure 4.** The scheme of the variolens with liquid lenses for 3 positions.

**Figure 5.** The laws of the focal length change of the tunable components.
Table 4. The optical characteristics of the components.

| Component # | \(f'_1\) (mm) | \(s'_f\) (mm) | \(s_f\) (mm) | \(s'_r\) (mm) | \(s_r\) (mm) | \(d\) (mm) |
|-------------|----------------|---------------|---------------|---------------|---------------|------------|
| 1           | 82.5           | 78.88         | -82.17        | -3.63         | 0.34          | 6.36       |
| 2*          | 80.6           | 76.79         | -80.64        | -3.85         | 0             | 10.42      |
| 3           | -6.5           | -6.73         | 7             | -0.21         | 1.08          | 6.93       |
| 4           | 13.6           | 13.98         | -11.34        | 0.42          | 2.22          | 17.63      |
| 5*          | 56.5           | 53.67         | -56.47        | -2.8          | 0             | 5.88       |
| 6           | 83.5           | 73.31         | -93.04        | -10.17        | -9.56         | 5          |

Note: * tunable components, \(s_f\) and \(s'_f\) are the front and rear focal distances of the component, \(s_r\) and \(s'_r\) are positions of the front and rear principal points of the component, \(d\) is the air distance.

Table 5. Parameters of the tunable components.

| Focal length of the variolens (mm) | Focal length of the component (mm) |
|-----------------------------------|-----------------------------------|
|                                   | \#2                               | \#5                             |
| 17.0                              | 120.67                            | 35.76                           |
| 24.0                              | 80.64                             | 56.47                           |
| 34.0                              | 52.84                             | 122.28                          |

The field of view of the calculated system of the visible spectrum range varies in the range of 17°…33°. The f-number at the minimum focal length is 2.8, at the maximum is 4. The illumination drop across the field does not exceed 25%. The resulting system is a good initial approximation for subsequent optimization.

Example 3. Let laser radiation have the following parameters: wavelength 1.03 \(\mu\)m, waist diameter \(2h_w = 150 \mu\)m, beam quality factor \(M^2 = 1.05\). It is required to calculate the parameters of the laser variosystem, which forms a beam with a waist diameter of \(2h'_w = 75 \mu\)m and provides a change of \(L > 50\) mm. The results of calculating of the variosystem for the initial, intermediate, and extreme positions of the lens with tunable optical power are presented below (for the notation, see figure 2):

| \(f''\) (mm) | \(d\) (mm) | \(L\) (mm) |
|--------------|------------|------------|
| 17.06        | 47.0       | 71.54      |
| 25.08        | 72.5       | 109.43     |
| 33.34        | 98.0       | 147.51     |

4. Conclusion
The considered methods and the above expressions make it possible to carry out an automated synthesis of optical variosystems of different types. Examples of calculating the variosystems show the effectiveness of the considered methods, and the resulting systems are a good initial approximation for subsequent optimization. The use of lenses with tunable optical power made it possible to abandon the mechanical movements in classical systems and reduce the number of moving components of the laser variosystem. Variosystems can be used in video camera lenses, computer vision systems with a rapid change in magnification, in laser systems for manipulating micro-objects (optical tweezers), and other areas [13-20].

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