About the possibility of creating a stable transonic region in supersonic flow in the channel

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Abstract. The work is devoted to the mathematical modeling of the influence of transversal jet and the near-wall energy sources on the shock wave structure of supersonic flow in channel with variable cross section. Stable regimes with the region of transonic velocities are obtained. Their stability is confirmed by the width of the corridor of the input power in the region of existence of the regime modes.

1. Introduction
The reduction of total pressure losses in combustion chamber is one of the main issues that defines the effectiveness of high-speed ramjet [1]. The technologies designed to implement reliability of ignition and stability of combustion also lead to total pressure losses. The gas-dynamic and kinetic methods are applied to solve the problem. In particular, jets for flow control are currently widely used. The injection of oncoming jet or co-current jet is considered [2]. A rotating detonation wave created with a jet [3] is also used. Subsurface sources of energy are of interest as well [4]. All the possible ways, active and passive, are considered as shown in the review section of the paper [5]. These methods either require a significant investment of energy, or do not give a stable result. In [1] a new method of combustion chamber starting is proposed. It enables the transition to supersonic flow in the combustion chamber by a jet and distributed fuel injection into the chamber. Experimental data confirm its effectiveness. The proposed work presents an analytical and numerical analysis of control of supersonic flow into a channel with both the jet and the surface energy supply to identify relatively stable regimes with an extended transonic region.

2. Mathematical formulation
Numerical simulation is carried out for unsteady flow in a channel of variable cross section. Plane channel consists of a nozzle with sound speed flow at the inlet, the section being of constant cross section into which the jet flows and near-wall energy supply is carried out (the Mach number at the entrance to this section is approximately equal to M = 2). The expanding section ends with a constant cross section. Two-dimensional unsteady Euler equations in a dimensionless form for an ideal gas with a constant ratio of the specific heats γ are solved. They can be used to solve such problems, as it is shown, for example, in [6]. All lengths are related to the size d of the input section of the channel. The ratio of this size to the sound speed at the entrance of the channel is used as the time scale. The velocity components relate to the critical speed in the throat of
the nozzle, the pressure and temperature relate to the corresponding values in the input hole. On the right border “soft” boundary conditions are used; impermeability conditions are used on the walls of the channel. The jet is supplied from a gas generator with a set pressure, and the temperature is maintained equal to its value at the entrance to the channel. Impulse energy supply is made instantly, but the gas density and its velocity are not changed. The gas energy density in the energy supply zones increases (correspondingly, the temperature and pressure increase). In the source model used the power density is proportional to the local density of the gas. The intensity of the supplied energy is determined from the condition of equivalence of the energy released during the combustion of hydrogen at a temperature of 1000 K with the excess air ratio \( \alpha \) in the range of 2.6 to 10. Numerical integration of the differential equations is carried out in the intervals between the moments of energy supply. For the numerical solution to the problem the finite-volume scheme that reduces the total variation is used.

3. The parameters of the simulated system.

The calculations were performed for a plane channel with variable cross section. The shape and basic sizes of the channel are shown in the upper part of Figure 1, a fragment of the first section of channel constant cross section with the positions of the transverse jet and the zones of energy release indicated by rectangles, is shown in the lower part.

![Figure 1](image)

**Figure 1.** The channel scheme. The fragment of the Mach numbers distribution before the energy input in the zones.

The Mach number at the entrance to the channel is assumed to be \( M = 1 \). The nozzle creates in the first part of constant cross section (width is equal 1.66) a stream with Mach number \( M \approx 2 \) that corresponds to the conditions in the path of high-speed ramjet. At the exit of the channel, the Mach number varies from 2.4 to 2.9. The jet of width \( \Delta x = 0.104 \) is injected into the part with constant cross section. The hole size is selected from the following requirement: the jet energy should be negligibly small in comparison to the energy of the entire flow in the channel. The pressure in the gas generator, which forms the jet, is \( p = 2 \), that is significantly greater than the pressure in the undisturbed flow. The magnitude of pressure in the gas generator is based on the requirement of approximate equality of pressure drop between the gas generator and the excess pressure, which can create an instantaneous thermal pulse in the channel. The thermal
pulse energy corresponds to the instantaneous reaction in the hydrogen–air mixture with the range of values of the excess air factor $2.6 < \alpha < 10$. This energy is supplied upstream from the jet in two subsurface zones which are placed opposite each other. The length of the zones of 1.7 along the channel is greatly larger than the perpendicular size of 0.16. As can be seen from Figure 1, the distance from the zones to the jet is comparable to the characteristic size of the zones along the channel axis, whereby the intensity of the pressure pulse will decrease slightly to the moment of its convective drift to the jet site. The jet flow leads to a compression of gas in front of it. The system can serve as a very approximate physical model of the experiment [1]. The energy supply period $\Delta t = 2.16$. The value of the period is chosen from the condition that the gas portion heated by an energy pulse has left the energy supply zone to the next energy supply. At the same time almost every portion of gas passing through the energy supply zone should have received energy.

4. The numerical simulation results

The impact of the considered surface energy supply only (without jet) on the flow does not lead to the formation of the extensive area with transonic speeds localized in the constant cross section. It is caused by demolition of the heated gas in the widening part of the channel (channel locking is not taking place). The jet without heat pulses does not cause locking of channel. The Mach number distribution in Figure 1 is obtained when the gas flow in the channel is preliminary heated to 1000 K (this corresponds to $\alpha = 2.9$). This periodic solution is shown at a time immediately before the supply of energy. The flow with $M \approx 2$ is braked to subsonic speed ($M \approx 0.8$) in the shock wave, which in the period examined is carried out by the stream beyond the zone of energy release. Further subsonic flow is accelerated to the speed of sound, and then broadens and becomes supersonic. The formation of modes is shown in Figure 2 ($I$ denotes the coordinate $x_{sh}$ of the shock wave propagating upstream; $t$ is the time, 2 and 3 are the wall coordinates of the shock wave caused by the jet in the absence of energy supply, 4 is the boundary of the energy supply zone). The front moves upstream, but remains in the constant cross section of the channel.

![Figure 2](image-url)

**Figure 2.** The process of formation of mode with transonic region ($I$ is the coordinate $x_{sh}$ of the shock wave propagating upstream; $t$ is the time, 2 and 3 are the wall coordinates of the shock wave caused by the jet in the absence of energy supply, 4 is the boundary of the energy supply zone).
At values of excess air factor $\alpha < 2.9$ the combined action of the sources and the jet leads to the locking of channel; for $\alpha > 4$ the flow remains supersonic. The interval $2.9 \leq \alpha \leq 4$ is quite wide. The significant interval width means the stability of this mode in relation to the fluctuating of parameters. Thus, the combined effect of surface energy supply and the jet leads to the formation of subareas with transonic velocities with the length along the channel which is comparative to the size of the channel inlet. This creates favorable conditions for effective supply and combustion of fuel in a widening channel section. The Table 1 shows the coordinates restricting the channel subsonic region ($x_{up}$ and $x_{sk}$ are the minimum and maximum coordinates of the subsonic region along the center line; $<x_{up}>$ and $<x_{sk}>$ are the minimum and maximum coordinates of the subsonic region averaged over the cross section of the channel), and corresponding sizes of the subsonic area depending on the excess air factor $\alpha$.

Table 1. The coordinates of the boundaries of the subsonic region and its size along the center line and the values averaged over the cross section of the channel depending on the excess air factor $\alpha$.  

| $\alpha$ | $x_{up}$ | $x_{sk}$ | $\Delta x$ | $<x_{up}>$ | $<x_{sk}>$ | $<\Delta x>$ |
|---------|----------|----------|-------------|------------|------------|-------------|
| 2.9     | 6.744    | 7.222    | 0.478       | 6.754      | 7.902      | 1.148       |
| 3.0     | 6.804    | 7.265    | 0.461       | 6.815      | 7.872      | 1.057       |
| 3.5     | 7.279    | 7.311    | 0.032       | 7.113      | 7.689      | 0.576       |
| 4.0     | 7.524    | 7.560    | 0.036       | 7.786      | 8.096      | 0.310       |

The values given correspond to the minimum values of the width of this region because they are obtained at the moment of time before the supply of energy. With decrease of excess air ratio the transonic speed region becomes wider. The ability to create such regimes is ensured by the use of negative feedback mechanism on the pressure. Its implementation is shown in Figure 3 which demonstrates the dependence of the pressure at the exit of the jet from the hole (curve 1) and the corresponding jet consumption $m$ (curve 2). These dependencies have an oscillatory character and are in opposite phase. When the high pressure area which was created by surface energy sources is drifting into the region of the inflow jet, the pressure difference decreases which leads to decrease of the flow rate in the jet and vice versa. This mechanism works quite effectively, if the pressure difference generated by the energy release, in the order of magnitude, is comparable to the pressure drop between the gas generator (which implies the jet) and the flow in the channel. Figure 4 demonstrates the chart of "normalized power $N$" – "period $\Delta t$ of energy supply." Power $N$ is normalized to the product $3\rho a^3 S$, where $S$ is the cross section area of constant cross-section, $\rho$ and $a$ are the density and sound velocity at the channel inlet. Curves 1–5 and the dashed horizontal line correspond to the transversal source of energy. These curves are described in [7]. The dashed lines correspond to an analytical assessment for the transverse sources corridor of values when a disturbance moving upstream stopped in a channel part of constant cross section. The dot-dash line demonstrates the same assessment for the near-wall sources. The squares and diamonds present a numerical solution without locking for transverse and near-wall sources, respectively, the triangles (with a base at the bottom) and circles correspond to the solution with locking. An important result revealed by the chart is a very narrow corridor formed by lines 1–2, which are obtained by numerical calculations for the transverse source, and the almost complete absence of similar corridor for near-wall sources. The small width of the corridors means that a small perturbation will result in either locking of channel or demolition of a shock wave downstream. The solid lines with triangles in Figure 4 marks the boundaries of this corridor for the joint...
Figure 3. The dependence of the pressure $1$ and the gas consumption from gas generator $2$ at the exit of the jet from the hole.

action of the jet and the near-wall surface sources which is obtained numerically. The diagram shows that its width is almost 40% of the lower limit of the corridor. A sufficiently large width of the corridor means that the regime will be more resistant to disturbances than the regimes with energy sources only. This is provided by the mechanism of negative feedback on pressure. The mechanism of interaction between the flow and heat sources is a positive argument for the justification of the method for starting the combustion process in the combustion chamber. This mechanism explains why the intensification of combustion in the experiment [1] does not lead to locking up the channel, but forms a continuous mode of combustion.
5. Conclusion

Thus, the possibility of realization of the stable modes with transonic region in supersonic flow is shown. Stability is achieved under the condition of approximate equality of pressure drop between the gas generator and the maximum additional pressure resulting from the near-wall energy. The significant wide corridor for input power in the mode chart diagram confirms the regime stability. Such modes may be used as the initial conditions for a realization of combustion in the combustion chambers.

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