Efficiency and Consistency Assessment of Value at Risk Methods for Selected Banks Data

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Abstract

The study assesses Value at Risk (VaR) methods with respect to their efficiency and consistency in selected banks of the Nigeria Stock Market. The daily data on share prices of each bank was used from 2006 to 2018. The Value at Risk of each bank was estimated and the predictive performance of each method was assessed using the Failure Ratio and the Confidence Interval. The quality of each method was assessed based on the efficiency and consistency of the estimates. The VaR of each bank was estimated using Historical Simulation, Kernel Estimator, Empirical Estimator and Weighted Mean methods. The weighted mean method had the least estimates while Kernel estimator method had the highest estimates. The Failure Ratio and Confidence Interval show that Historical and Empirical methods had the least number of rejections at both confidence levels. The efficiency and consistency of the various methods shows the Historical Simulation and Weighted mean method had the minimum mean square errors (MSE). The Banks A, D and E gives an efficient and consistent result with Historical Simulation while B and C, is more efficient and consistent with weighted mean method.

Keywords: VaR; weighted mean; stock market; Nigeria.
1 Introduction

A commercial bank is an institution that provides financial services, including issuing money in various forms, receiving deposits of money, lending money and processing transactions and the creating of credit [1]. Over different periods, there has been a lot of crises that affects both banks and other financial markets such as; the stock market crash (1987), the financial crisis (1997-1998), Global financial crisis (2007-2008), Venezuelan banking crisis (2009-2010) and Irish banking crisis (2008-2011). These issues of market crash, financial crisis and bank crisis has lead to bank runs, banking panics and systemic banking crises. A lot of bank failures were attributed to the inappropriate use of derivatives and lack of sufficient internal controls. Banks were weakened and some regulations were put in place so that they have sufficient capital reserve based on the risk structure. The need for improved risk management, especially for financial organizations, became clear at that time.

Value at Risk is risk measures which calculate the total capital a firm needs to cater for risk. Banks and financial organizations need to keep certain amount of money in order to cater for risk in their organizations. Not just keeping of capital, but adequate capital to meet the adverse movements of the market.

It is a matter of concern in practice whether the reported VaR is truly in line with the actual level of risks, estimated by the banks. Moreover, research shows that different techniques of calculating Value at Risk (VaR) have the tendency of providing varying results. The study by Dargiri [2], described and assessed the accuracy of predicted Value at Risk by applying parametric and nonparametric approaches using Malaysia industries. The nature of any Economy depends on the activities of that country, since VaR measures and quantifies level of financial risk within a firm.

VaR has gained rapid acceptance as a valuable approach to address and measure market risk because of its ability to quantify risk in a single number. Authors, among others (Jadhav and Ramanathan [3]; Rodrigues [4]; Guhary [5]; Cerovic [6]; Vladimir [7] and Ringqvist [8]) have estimate risk using parametric methods and nonparametric methods, in parametric a specified distribution is fitted to the observed returns by calibrating the parameters. This method is, of course, very sensitive to the assumption of distribution.

According to Jadhav and Ramanathan [3], which stated that, the correct estimation of VaR is essential for any financial institution, in order to arrive at the accurate capital requirements and to meet the adverse movements of the market. They gave a brief review of some of the existing parametric and non-parametric methods of estimating VaR. Comparison between the estimators were made using in-sample and out-of-sample back-testing techniques of Kupiec likelihood test. The extreme value theory and observation closest to \( n\alpha \) was seen to perform well compared with all other methods.

The study by Ringqvist [8], investigates several models that estimate the financial risk measure with the objective to find the best model for the Swedish stock market. Using 1-day forecasted VaR at 95% and 99% level the following VaR models were compared: Basic Historical Simulation (HS), age weighted HS (AWHS), volatility weighted HS (VWHS) using a GARCH model, Normal VaR and t-distributed VaR. The study was performed on the Swedish stock exchange data OMXS and on the single stock series Boliden for the years 2005-2013. Running a backtest of the models it was found that the VWHS, where the volatility is modeled with a GARCH(1:1) model, estimates 1-day 95% and 99% VaR most accurately on the Swedish stock market and is therefore preferred to the other models.

Etuk et al. [9] estimated Value at Risk and Expected Shortfall in the presence of fat tails in returns using historical data of five selected banks in Nigeria First Bank, Zenith Bank, UBA, Guaranty Trust Bank and Access bank, using the following methods; GARCH(1,1) dynamics with Extreme Value Theory, Cauchy and Burr XII distributions. The authors evaluate Value at Risk and Expected Shortfall forecasting performance using various backtesting approaches. It was shown that models with Extreme Value Theorem and Cauchy Distribution give better fit than Burr. This implies that GARCH-Cauchy can calculate the minimum required capital to cover the market risks of the banks.
Etuk et al. [10] estimated Value at Risk and Expected Shortfall in Nigerian banks using Normal, Lognormal, Weibull and Extreme Value Theorem to assess the efficiency of the various methods. It was discovered that First, Access bank gives more efficient estimate with EVT while Zenith, UBA and GTB is efficient with Weibull distribution.

This work will assess the performance of some nonparametric Value at Risk techniques based on their efficiency and consistency properties, using information from five (5) major banks in Nigeria.

2 Materials and Methods

This section briefly discusses the some nonparametric methods for the estimation of Value at Risk: Historical Simulation, Kernel Estimator, Empirical Estimator and Weighted Mean.

2.1 Data used for the study

The study used share price of five major banks listed in the Nigerian Stock Market (First Bank, UBA, GTB Bank, Zenith Bank and Access Bank). The closing price at each trading day will be used covering the period, 3rd January 2006 to 31st December 2018.

2.2 Value at risk

Value-at-risk is defined as the maximum potential loss in the value of a portfolio of financial instruments with a given probability over a certain horizon. VaR is the $100(1 - \alpha)$th quantile of the loss function, where $\alpha$ is the upper tail probability. It is the possible maximum loss over a given holding period within a fixed confidence level. Mathematically, VaR confidence level $\alpha$ is given by the smallest number $1$ such that the probability of loss $L$ to exceed $1$ is not greater than $1 - \alpha$, as follows:

$$ VaR_\alpha = \inf \{l \in R : P(L > l) \leq 1 - \alpha \} $$

(1)

$$ VaR_\alpha = \inf \{l \in R : F_L(l) \geq \alpha \} $$

(2)

The lowest value of the portfolio return at the chosen time horizon “t” with a certain probability “$\alpha$” is determined from the distribution of return.

$$ 1 - \alpha = P(X < R_\alpha) = \int_{-\infty}^{R_\alpha} f(x)dx $$

(3)

2.3 Historical simulation method

Let $X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \cdots \leq X_{(n)}$ denote the order statistics in ascending order corresponding to the original financial returns $X_1, X_2, \cdots X_n$. The historical method suggests to value at risk estimate by

$$ VaR_\alpha(X) = X_{[np]} $$

(4)

where $p$ is the upper tail probability.

2.4 Kernel estimator

The kernel estimator of a density $f$, based on a sample $X_1, ..., X_n$, is given by
\[ f(x) = \frac{1}{nh} \sum_{j=1}^{n} K\left( \frac{x - X_j}{h} \right) \]  

where \( X_j \) is the \( j^{th} \) observed return, \( K \) is a kernel function, \( h \) is a bandwidth and \( n \) is the size of sample. The kernel function \( K \) is defined to be a symmetric and continuous probability density function, and the bandwidth \( h \) controls the smoothness of the estimated density, and so it affects the bias of the estimated density. Equation above shows that the kernel density estimator is an equally weighted linear combination of the kernel function \( K \) evaluated at each observation, with weight \( i/n \) at each kernel function or observation. Accordingly, a kernel estimator of \( F(x) \) is:

\[ \hat{F}_{n,h}(x) = \frac{1}{n} \sum_{j=1}^{n} G\left( \frac{x - X_j}{h} \right) \]  

where \( G(x) = \int_{-\infty}^{x} K(u)du \). Where \( u = \frac{x - X_j}{h} \), the kernel function used is normal. An estimator of the VaR can be obtained by solving the following equation:

\[ \frac{1}{n} \sum_{j=1}^{n} G\left( \frac{VaR_{\alpha,h}(x) - X_j}{h} \right) = \alpha \]  

for a given value of \( \alpha = 0.05 \), [11].

2.5 Empirical estimator

Let \( X_1, X_2, \ldots, X_n \) be a random sample from a return distribution \( F(.) \), with \( X_{(1)} \leq \cdots \leq X_{(n)} \) as the corresponding order statistics. For given \( \alpha \), define \( j = \lfloor n\alpha \rfloor \) and \( g = n\alpha - j \)

By standard result on empirical distribution ([12]), the \( p^{th} \) quantile can be estimated by

\[ VaR_p(x) = F^{-1}(1 - p) = X_{(i)}, \quad 1 - p \in \left[ \frac{j-1}{n}, \frac{j}{n} \right] \]  

2.6 Weighted estimator of value at risk

In this section, we proposed a new VaR model known as the weighted estimator. Let \( x_1, x_2, x_3, \ldots, x_n \) be a set of random variables with the weighted mean given by

\[ \mu^* = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \]  

For the normalized weight

\[ \sum_{i=1}^{n} w_i = 1 \]

The weighted variance given by

\[ \sigma_{\text{weighted}}^2 = \frac{\sum_{i=1}^{n} w_i(x_i - \mu^*)^2}{\sum_{i=1}^{n} w_i} \]

\[ VaR_{(\alpha)} = \mu^* + z_\alpha \sigma_{\text{weighted}} \]
2.7 Predictive performance procedure

In-sample VaR computation and backtesting allow us to examine only the past performance of the VaR models. The real contribution of VaR computation is its forecasting ability, which provides investors or financial institutions with the information about the largest loss they may incur. The rolling window length (the observation period) used to estimate the model parameters is also an important factor. The rolling window of 25, 50, 100, 200, 250 and 500 are tested to estimate of VaR based on all the methods.

Forecasting quality of the estimated methods was estimated using the Violation Ratio, where the number of the Observed Violation is compared to the number of predicted violations, [13].

\[
\text{Violation Ratio} = \frac{\text{Number of observed violations}}{\text{Number of predicted violations}} \quad (11)
\]

Thus, the confidence interval by Alexander (2009) is adopted due to sampling error;

\[
na + Z_\alpha \sqrt{na(1-a)}, \quad na - Z_\alpha \sqrt{na(1-a)}
\]

The Null hypothesis is accepted if the cumulative number of violations falls within confidence interval.

2.8 Comparison of estimators

The Mean Squared Error for Value at Risk (VaR) by [14] is given;

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} [\hat{F}(x_i) - VaR]^2 \quad (12)
\]

where \(\hat{F}(x_i)\) is the share price of returns for each day, while VaR is the computed estimate for each methods. The method with the minimum mean squared error (MSE) becomes the best method for the estimation of VaR.

3 Results and Discussion

The results were obtained from the use of Historical Simulation, Kernel density, Empirical Quantile and Weighted mean estimator. The share price was divided into two sections; the in-sample which was the share price from 2006 – 2018 and the out-of-sample represent the rolling window of 2/3 of the original sample [3]. The variables are represented by letters A, B, C, D and E (the identity of each bank is hide).

Table 1. Estimation of value at risk with in-sample, \(\alpha = 0.05\) and \(0.01\)

| Variable | Historical simulation | Kernel estimator | Empirical estimator | Weighted estimator |
|----------|-----------------------|------------------|---------------------|-------------------|
|          | \(\alpha=0.05\)      | \(\alpha=0.01\)  | \(\alpha=0.05\)    | \(\alpha=0.01\)  |
| A        | 45.1                  | 56.0206          | 52.1840             | 62.4428           |
| B        | 48.5                  | 63.05            | 54.9925             | 64.7207           |
| C        | 50.9                  | 55.44            | 54.3871             | 58.6393           |
| D        | 33.83                 | 36.91            | 35.7106             | 37.6883           |
| E        | 19.32                 | 24.00            | 22.3453             | 24.7097           |
|          | \(\alpha=0.05\)      | \(\alpha=0.01\)  | \(\alpha=0.05\)    | \(\alpha=0.01\)  |
| A        | 45.1                  | 56.81            | 45.13               | 56.81             |
| B        | 48.66                 | 63.22            | 48.66               | 63.22             |
| C        | 50.9                  | 55.50            | 50.9                | 55.50             |
| D        | 33.84                 | 36.95            | 33.84               | 36.95             |
| E        | 19.32                 | 24.01            | 19.32               | 24.01             |

Table 1 shows the estimate of the various in sample VaR estimate using the various methods at \(\alpha = 0.05\) and \(\alpha = 0.01\) significance levels. Historical Simulation estimates at averages at 39.53 and 47.08412, Kernel method at 43.9239 and 49.64016, Empirical Method had 39.57 and 47.298 while Weighted Mean had
33.43962 and 40.41168. The overall averages shows that Weighted Mean had the least VaR estimate at $\alpha = 0.05$ while Kernel had the highest VaR estimate. At $\alpha = 0.01$, Historical Simulation had the least VaR estimate while Kernel had the highest.

Table 2. Estimation of value at risk with 500 out-sample, $\alpha = 0.05$ and 0.01

| Variable | Historical simulation | Kernel estimator | Empirical estimator | Weighted estimator |
|----------|----------------------|------------------|--------------------|-------------------|
|          | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ |
| A        | 19.9923    | 21.4019 | 20.8476   | 21.9907 | 20.0216  | 21.5734  | 20.8334  | 24.2410   |
| B        | 24.9575    | 25.9492 | 26.1663   | 27.7985 | 24.9982  | 27.1951  | 23.9636  | 26.7089   |
| C        | 14.6553    | 16.8918 | 16.0253   | 17.9638 | 14.6989  | 17.1865  | 13.5699  | 16.0720   |
| D        | 29.2438    | 30.1612 | 29.7916   | 30.3883 | 29.2671  | 30.2431  | 29.5524  | 33.3039   |
| E        | 11.0024    | 11.3188 | 11.1914   | 11.3936 | 11.0106  | 11.3462  | 11.1731  | 12.6037   |

Table 2 shows the estimate of the various 500 out sample VaR estimate using the various methods at $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. Historical Simulation estimates at averages at 19.97026 and 21.14458, Kernel method at 20.80444 and 21.90698, Empirical Method had 19.99928 and 21.50886 while Weighted Mean had 19.81848 and 22.5859. The overall averages shows that Weighted Mean had the least VaR estimate at $\alpha = 0.05$ while Kernel had the highest VaR estimate. At $\alpha = 0.01$, Weighted Mean had the least VaR estimate while Kernel had the highest.

Table 3. Estimation of value at risk with 250 out-sample, $\alpha = 0.05$ and 0.01

| Variable | Historical simulation | Kernel estimator | Empirical estimator | Weighted estimator |
|----------|----------------------|------------------|--------------------|-------------------|
|          | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ |
| A        | 19.7355    | 20.9503 | 20.5802   | 21.5705 | 19.7573  | 21.4019  | 20.4768  | 23.8626   |
| B        | 24.6001    | 26.3052 | 25.7891   | 27.1937 | 24.6313  | 26.9492  | 20.2061  | 21.4772   |
| C        | 14.2770    | 16.1421 | 15.5969   | 17.2092 | 14.3089  | 16.8918  | 13.0966  | 15.4869   |
| D        | 29.0317    | 29.9141 | 29.6176   | 30.2140 | 29.0502  | 30.1612  | 29.1018  | 32.7513   |
| E        | 10.9276    | 11.2350 | 11.1311   | 11.3357 | 10.9342  | 11.3188  | 11.0055  | 12.3982   |

Table 3 shows the estimate of the various 250 out sample VaR estimate using the various methods at $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. The following results were obtained with the respective $\alpha$. Historical Simulation estimates at averages at 19.71438 and 20.90934, Kernel method at 20.80444 and 21.90698, Empirical Method had 19.73638 and 21.34458 while Weighted Mean had 19.81848 and 22.5859. The overall averages shows that Weighted Mean had the least VaR estimate at $\alpha = 0.05$ while Kernel had the highest VaR estimate. At $\alpha = 0.01$, Weighted Mean had the least VaR estimate while Kernel had the highest.

Table 4. Estimation of value at risk with 200 out-sample, $\alpha = 0.05$ and 0.01

| Variable | Historical simulation | Kernel estimator | Empirical estimator | Weighted estimator |
|----------|----------------------|------------------|--------------------|-------------------|
|          | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ | $\alpha=0.05$ | $\alpha=0.01$ |
| A        | 19.5718    | 20.9503 | 20.3337   | 21.3184 | 19.7355  | 21.4019  | 20.5770  | 23.6187   |
| B        | 24.3752    | 26.3052 | 25.4411   | 26.8306 | 24.6011  | 26.9492  | 23.4835  | 26.0830   |
| C        | 14.0400    | 16.1421 | 15.4411   | 16.7573 | 14.2770  | 16.8918  | 13.0402  | 15.3862   |
| D        | 28.8896    | 29.9141 | 29.4578   | 30.1106 | 29.0317  | 30.1612  | 29.1613  | 32.7899   |
| E        | 10.8773    | 11.2350 | 11.0759   | 11.3015 | 10.9276  | 11.3188  | 11.0295  | 12.4146   |

Table 4 shows the estimate of the various 200 out sample VaR estimate using the various methods at $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. The following results were obtained with the respective $\alpha$. Historical
Simulation estimates at averages at 19.55078 and 20.90934, Kernel method at 20.34992 and 21.26368, Empirical Method had 19.71458 and 21.34458 while Weighted Mean had 15.4183 and 22.05848. The overall averages shows that Weighted Mean had the least VaR estimate at $\alpha = 0.05$ while Kernel had the highest VaR estimate. At $\alpha = 0.01$, Weighted Mean had the least VaR estimate while Kernel had the highest.

Table 5 shows the estimate of the various 100 out sample VaR estimate using the various methods at $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. The following results were obtained with the respective $\alpha$.

| Variable | Historical simulation $\alpha=0.05$ | Historical simulation $\alpha=0.01$ | Kernel estimator $\alpha=0.05$ | Kernel estimator $\alpha=0.01$ | Empirical estimator $\alpha=0.05$ | Empirical estimator $\alpha=0.01$ | Weighted estimator $\alpha=0.05$ | Weighted estimator $\alpha=0.01$ |
|----------|----------------------------------|----------------------------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|-------------------------------|----------------------------------|
| A        | 19.7573                          | 19.8992                          | 20.2165                       | 20.6456                       | 19.8174                          | 21.4019                          | 20.4119                       | 23.6745                          |
| B        | 24.6313                          | 24.8142                          | 25.2773                       | 25.8817                       | 24.7145                          | 26.9492                          | 23.2977                       | 25.8294                          |
| C        | 14.3089                          | 14.5025                          | 15.0250                       | 15.6971                       | 14.3968                          | 16.8918                          | 12.8949                       | 15.1891                          |
| D        | 29.0502                          | 29.1603                          | 29.3681                       | 29.6607                       | 29.1008                          | 30.1612                          | 29.1059                       | 32.7399                          |
| E        | 10.9342                          | 10.9730                          | 10.9738                       | 11.1459                       | 10.9276                          | 11.3188                          | 11.0221                       | 12.4126                          |

Table 6 shows the estimate of the various 50 out sample VaR estimate using the various methods at $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. The following results were obtained with the respective $\alpha$.

| Variable | Historical simulation $\alpha=0.05$ | Historical simulation $\alpha=0.01$ | Kernel estimator $\alpha=0.05$ | Kernel estimator $\alpha=0.01$ | Empirical estimator $\alpha=0.05$ | Empirical estimator $\alpha=0.01$ | Weighted estimator $\alpha=0.05$ | Weighted estimator $\alpha=0.01$ |
|----------|----------------------------------|----------------------------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|-------------------------------|----------------------------------|
| A        | 19.7573                          | 19.8992                          | 20.2165                       | 20.6456                       | 19.8174                          | 21.4019                          | 20.4119                       | 23.6745                          |
| B        | 24.6313                          | 24.8142                          | 25.2773                       | 25.8817                       | 24.7145                          | 26.9492                          | 23.2977                       | 25.8294                          |
| C        | 14.3089                          | 14.5025                          | 15.0250                       | 15.6971                       | 14.3968                          | 16.8918                          | 12.8949                       | 15.1891                          |
| D        | 29.0502                          | 29.1603                          | 29.3681                       | 29.6607                       | 29.1008                          | 30.1612                          | 29.1059                       | 32.7399                          |
| E        | 10.9342                          | 10.9730                          | 10.9738                       | 11.1459                       | 10.9276                          | 11.3188                          | 11.0221                       | 12.4126                          |

Table 7 shows the estimate of the various 25 out sample VaR estimate using the various methods at $\alpha = 0.05$ and $\alpha = 0.01$ significance levels. The following results were obtained with the respective $\alpha$.
Simulation estimates at averages at 19.73638 and 19.7963, Kernel method at 19.80552 and 19.838, Empirical Method had 19.7963 and 19.86784 while Weighted Mean had 20.90616 and 23.4719. The overall averages shows that Weighted Mean had the least VaR estimate at $\alpha = 0.05$ while Kernel had the highest VaR estimate. At $\alpha = 0.01$, Weighted Mean had the least VaR estimate while Kernel had the highest.

### 3.1 Comparative of the predictive performance of the VaR methods

The predictive performance was carried out using the Violation Ratio by [13].

#### Table 8. Summary of Backtesting with $\alpha = 0.05$

| Method      | Sample size | In Sam | 500 | 250 | 200 | 100 | 50 | 25 | Total |
|-------------|-------------|--------|-----|-----|-----|-----|----|----|-------|
| Historical  |             | 1      | 0   | 0   | 0   | 0   | 0  | 0  | 1     |
| Kernel      |             | 5      | 3   | 1   | 0   | 0   | 0  | 0  | 14    |
| Empirical   |             | 1      | 0   | 0   | 0   | 0   | 0  | 0  | 1     |
| Weighted    |             | 5      | 1   | 0   | 0   | 0   | 0  | 0  | 10    |

Table 8 present the number of rejections on the different methods with the different rolling windows. The method with the least number of rejections gives a better fit. Historical Simulation and Empirical Estimator have the minimum number of rejections at $\alpha = 0.05$.

#### Table 9. Summary of backtesting with $\alpha = 0.01$

| Method      | Sample size | In Sam | 500 | 250 | 200 | 100 | 50 | 25 | Total |
|-------------|-------------|--------|-----|-----|-----|-----|----|----|-------|
| Historical  |             | 0      | 1   | 0   | 0   | 0   | 0  | 2  | 3     |
| Kernel      |             | 3      | 0   | 0   | 0   | 0   | 0  | 0  | 3     |
| Empirical   |             | 0      | 0   | 0   | 0   | 0   | 0  | 0  | 0     |
| Weighted    |             | 4      | 0   | 1   | 0   | 0   | 0  | 0  | 5     |

Table 9 present the number of rejections on the different methods with the different rolling windows. The method with the least number of rejections gives a better fit. Historical Simulation and Empirical Estimator have the minimum number of rejections at $\alpha = 0.01$.

#### Table 10. Summary of MSE of VaR , $\alpha = 0.05$ and $0.01$

| Bank | $\alpha = 0.05$ | $\alpha = 0.01$ |
|------|-----------------|-----------------|
|     | IS 500 250 100 50 25 IS 500 250 100 50 25 | IS 500 250 100 50 25 |
| A   | W   H   H   H   H   H   W   H   H   H   H   H | W   H   H   H   H   H   W   H   H   H   H   H |
| B   | W   W   W   W   W   W   W   H   W   W   H   K | W   W   W   W   W   W   W   H   W   W   H   H |
| C   | W   W   W   W   W   W   W   W   W   W   W   H | W   W   W   W   W   W   W   W   W   W   W   H |
| D   | W   H   H   H   H   H   H   W   H   H   H   H | W   H   H   H   H   H   W   H   H   H   H   H |
| E   | W   H   H   H   H   H   H   H   W   H   H   H | W   H   H   H   H   H   W   H   H   H   H   H |

Table 10 is the summary of the mean square error of VaR with different methods at $\alpha = 0.05$ and 0.01 with the different rolling windows. The method with the highest numbers of Mean Square error gives an efficient result compare to other ones. Where H(Historical), K(Kernel), E(Empirical Estimator) and W(Weighted Mean). The result shows that Weighted Mean and Historical Simulation have the highest numbers of MSE.

The result of Table 11 shows the summary of the frequency of occurrence of different methods of estimation of Value at Risk at both confidence levels. The results show that Historical and Weighted Mean had the highest number of Mean Square error.
Table 11. Summary of MSE of VaR, $\alpha = 0.05$ and $0.01$

| Method     | VaR 0.05 | VaR 0.01 |
|------------|----------|----------|
| Historical | 21       | 20       |
| Kernel     | 0        | 0        |
| Empirical  | 0        | 0        |
| Weighted   | 14       | 10       |

3.2 Discussion

The study assesses the efficiency and consistency of Value at Risk (VaR) techniques in some 5 banks of the Nigeria Stock market. The results show the in-sample estimation of VaR and the out-of-sample estimation with different banks and with an average of 2678 sample size on each bank and 500, 250, 200, 100, 50 and 25 for the out-of-sample. The weighted mean had the least VaR estimate across all the sample sizes while Kernel had the highest VaR estimate across all rolling windows.

The work combines the Violation Ratio and the Confidence Interval measure to test for the predictive performance of VaR. It is expected that if the estimator is well specified that the violation ratio should be as close to one or a bit above one, the exception must fall within the specified confidence interval. Values above the confidence interval, underestimate risk while below the confidence interval means overestimation of risk. The predictive performance of Historical Simulation at various sample size shows that number of rejections of 1 at 95% confidence level, while the number of rejections of 3, it underestimate at B bank (500, 25), E bank at 25 sample point at 99% confidence level, For Kernel Estimator at various sample sizes, the number of rejections of 14 at 95% confidence level whiles the number of rejections of 3, at 99% confidence level. The Empirical Estimator at various sample size show the number of rejections of 1 and underestimate at E bank(in sample) at 95% confidence level, while the numbers of rejections was 0 at 99% confidence level. It was shown that the various sample size of Weighted Estimator show that the number of rejections of 10 with overestimations at A bank and E bank (500) and at B bank (250) at 95% confidence level. while the number of rejections of 5 at 99% confidence level.

The efficiency of the various methods of VaR estimation were assess using the Mean Squared Error(MSE). The mean squares error which measure the average squared difference between the estimated values and the actual value and the result that is close to zero, give the best result. The most efficient point estimator is the one with the smallest mean square error. The methods with representation H(Historical), K(Kernel), E(Empirical Estimator) and W(Weighted Mean) shows that Weighted Mean and Historical Simulation have the highest numbers of MSE.

The consistent assessment of Value at Risk with the different methods assessed the different sample sizes using the mean square error. The results show that the consistent estimator requires a large sample size for it to be more consistent and accurate. Weighted mean distribution gives a more consistent results compare to their counterpart in the estimation of Value at Risk.

4 Conclusion

The assessment of Value at Risk methodologies, with respect to their efficiency and consistency in selected banks of the Nigeria Stock Market shows that A bank, D bank and E bank gives an efficient and consistent result with Historical Simulation while B bank and C bank, is efficient and consistent in Estimating Value at Risk with Weighted mean. This useful information can facilitate Bank risk manager to measure firm-level market risk and Bank Executives to set limits of risk and Regulators to determine capital requirements.
5 Implication of the Findings

(a) Historical Simulation method gives the best fit method in the estimation of the minimum capital requirement of A, D, and E banks.
(b) Weighted Mean give the best fit in the estimation of the minimum capital requirement of B and C banks.
(c) The market volatilities are not properly captured by Kernel methods.
(d) It was also found that when the sample size is large, Historical Simulation and Empirical methods give an efficient and consistent result.

Competing Interests

Authors have declared that no competing interests exist.

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