Four-Dimensional Einstein Yang-Mills De Sitter Gravity
From Eleven Dimensions

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ABSTRACT

We obtain \( D = 4 \) de Sitter gravity coupled to \( SU(2) \) Yang-Mills gauge fields from an explicit and consistent truncation of \( D = 11 \) supergravity via Kaluza-Klein dimensional reduction on a non-compact space. The “internal” space is a smooth hyperbolic 7-space (\( H^7 \)) written as a foliation of two 3-spheres, on which the \( SU(2) \) Yang-Mills fields reside. The positive cosmological constant is completely fixed by the \( SU(2) \) gauge coupling constant. The explicit reduction ansatz enables us to lift any of the \( D = 4 \) solutions to \( D = 11 \). In particular, we obtain dS\(_2\) in M-theory, where the nine-dimensional transverse space is an \( H^7 \) bundle over \( S^2 \). We also obtain a new smooth embedding of dS\(_3\) in \( D = 6 \) supergravity.

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1 Introduction

The embedding of Anti-de Sitter (AdS) spacetimes in M-theory and string theories is rather straightforward. In fact, gauged supergravities in diverse dimensions with AdS vacuum have either been shown or are expected to be obtainable from consistent Kaluza-Klein sphere reductions of $D = 11$ or $D = 10$ supergravities. Notable examples include the simple embedding of $D = 4, \mathcal{N} = 2, SU(2)$ Yang-Mills AdS supergravity in $D = 11$ [1], the significantly more complicated $S^7$ [2, 3] and $S^4$ [4, 5, 6] reductions of M-theory, and the warped $S^4$ reduction of massive type IIA theory [7]. Although the $S^5$ reduction of type IIB theory has yet to be fully established, the reduction of a certain truncation of the theory has been constructed [9, 10].

On the other hand, there is less known regarding embedding de Sitter (dS) spacetime in M-theory or string theories, mostly because this is quite a bit more complicated than the case of AdS. With recent experimental evidence suggesting that our universe might be de Sitter [11, 12], there is increasing interest in de Sitter gravity in cosmology and the dS/CFT correspondence [13, 14, 15, 16, 17]. Thus, it is of importance to obtain the embedding of a non-trivial de Sitter gravity theory in M-theory or string theories.

The first de Sitter solution within the context of an extended supergravity theory was found in [18]. While no-go theorems [19, 20] imply that de Sitter spacetime cannot arise from a compactification of a supergravity theory, it can arise from a supergravity theory with a non-compact “internal” space [21]. Explicit embeddings of dS$_4$ and dS$_5$ in M-theory and type IIB supergravity, respectively, were obtained in [22]. These arise as ten or eleven-dimensional solutions that have a non-compact hyperbolic internal space.

In this paper, we obtain four-dimensional de Sitter gravity with $SU(2)$ Yang-Mills gauge fields from a less constrained truncation of $D = 11$ supergravity via Kaluza-Klein dimensional reduction on the non-compact space. In this construction, a consistent truncation of the higher-dimensional theory is required in which there are no modes which depend on the internal space. The $SU(2)$ fields arise from modes on the $S^3$ portions of the internal space. The Yang-Mills gauge coupling constant

\footnote{The full metric ansatz was conjectured in [8].}
is completely fixed by the cosmological constant. The kinetic terms for the $SU(2)$
gauge fields have the correct sign, implying that the theory is not merely an analytical
continuation of $SU(2)$ AdS supergravity.

This paper is organized as follows. In section 2, we rederive the embedding of dS$_4$
spacetime in M-theory, with the transverse space being an $H^7$ written as a foliation
of two 3-spheres. In section 3, we propose a reduction ansatz for obtaining $D = 4$
$SU(2)$ Yang-Mills de Sitter gravity from $D = 11$. We show that the reduction ansatz
is indeed consistent with the $D = 11$ equations of motion, and hence obtain the
Lagrangian for $D = 4$ $SU(2)$ Yang-Mills de Sitter gravity. The consistency of the
reduction ansatz enables us to lift any $D = 4$ solution back to $D = 11$. In section 4,
we discuss this in detail. In particular, we embed dS$_2$ and (Minkowski)$_2$ spacetimes in
$D = 11$. The internal space is an $H^7$ bundle over $S^2$. We also embed a cosmological
solution which smoothly interpolates between dS$_2 \times S^2$ at the infinite past in the
co-moving time to a dS$_4$-type geometry at the infinite future. In section 5, we obtain
an embedding of dS$_3$ in $D = 6$ supergravity with the transverse space being an $H^3$
written as a foliation of two circles. We conclude our paper in section 6.

2 Embedding dS$_4$ in $D = 11$

We now show how the dS$_4$ embedding in $D = 11$ supergravity found in [22] can be
obtained directly from the eleven-dimensional equations of motion. We start with the
Lagrangian of the bosonic sector of $D = 11$ supergravity, given by

$$\mathcal{L} = R \ast 1 - \frac{1}{2} \ast F_4 \wedge F_4 - \frac{1}{6} A_3 \wedge F_4 \wedge F_4,$$

where $F_4 = dA_3$. We consider the ansatz

$$ds^2 = H^2 ds^2_4 + d\rho^2 + a^2 d\Omega^2_3 + b^2 d\tilde{\Omega}^2_3,$$  

$$F_4 = q \epsilon_4,$$

where $H$, $a$ and $b$ are functions of $\rho$, $ds^2_4$ is four-dimensional de Sitter spacetime
with cosmological constant $\Lambda = 6\lambda^2$, i.e. $R_{\mu\nu} = 3\lambda^2 g_{\mu\nu}$, and $\epsilon_4$ is the corresponding
volume-form. $d\Omega^2_3$ and $d\tilde{\Omega}^2_3$ are the metrics of the two unit 3-spheres. The Einstein
equations of motion are given by

\[
\begin{align*}
\frac{4\ddot{H}}{H} + \frac{3\ddot{a}}{a} + \frac{3\ddot{b}}{b} &= -\frac{q^2}{6H^8}, \\
\frac{\ddot{H}}{H} + \frac{3\dot{H}^2}{H^2} + 3\ddot{H} \left(\ddot{a} + \frac{\dot{b}}{b}\right) &= \frac{q^2}{3H^8} + \frac{3\lambda^2}{H^2}, \\
\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{3\dddot{b}}{ab} + 4\dot{H} \frac{\dot{a}}{a} - \frac{2}{a^2} &= -\frac{q^2}{6H^8}, \\
\frac{\ddot{b}}{b} + \frac{2\dot{b}^2}{b^2} + \frac{3\dddot{a}}{ab} + 4\dot{H} \frac{\dot{b}}{b} - \frac{2}{b^2} &= -\frac{q^2}{6H^8},
\end{align*}
\]

(3)

where a dot represents a derivative with respect to \(\rho\). If the metric \(ds_4^2\) is AdS instead of dS, then one can have a solution with \(H\) being a constant. In this case, \(\rho\) becomes an angular coordinate with \(a \sim \cos \rho\) and \(b \sim \sin \rho\), so that the metric becomes the direct product of AdS\(_4\) and \(S^7\), with the seven sphere written as a foliation of two three spheres. Inspired by the sphere reduction ansatz [3], we consider the following redefinition of variables

\[
H^2 = \Delta^{2/3}, \quad a^2 = \Delta^{-1/3} \tilde{a}^2, \quad b^2 = \Delta^{-1/3} \tilde{b}^2.
\]

(4)

Following the analogous relation in the case of AdS\(_4\) \(\times S^7\), we have

\[
a^2 = \frac{1}{2} \ell^2 (\Delta + 1), \quad b^2 = \frac{1}{2} \ell^2 (\Delta - 1),
\]

(5)

where \(\ell\) is a constant scale parameter. Substituting (4) and (5) into (3), we find that the constants \(q\) and \(\lambda\) must satisfy

\[
q^2 \ell^2 = 4, \quad \lambda^2 \ell^2 = \frac{4}{3}.
\]

(6)

The equations (3) reduce to a single first-order differential equation

\[
\ell^2 \Delta^{\frac{2}{3}} \dot{\Delta}^2 - 4\Delta^2 + 4 = 0.
\]

(7)

Making a coordinate change \(d\rho = \ell \Delta^{1/3} d\theta\), we can easily solve for \(\Delta\), which is given by

\[
\Delta = \cosh (2\theta).
\]

(8)

Thus we have an explicit embedding of dS\(_4\) in \(D = 11\) given by

\[
\begin{align*}
\Delta &= \cosh (2\theta) , \\
F_4 &= \frac{2}{\ell} \epsilon_{(4)} .
\end{align*}
\]

(9)
This solution was obtained in [22].

Each $S^3$ in [9] can be replaced by a three-dimensional lens space. A Kaluza-Klein reduction and Hopf T-duality transformation on the fibre coordinate of the two lens spaces yields an embedding of $dS_4$ in type IIB theory. This procedure was used for the case of AdS solutions in [23].

If we consider (9) as a reduction ansatz from $D = 11$ to $D = 4$, then the resulting four-dimensional theory is Einstein gravity with a positive cosmological constant, and with a corresponding Lagrangian given by

$$e^{-1}L = R - \frac{8}{\ell^2}.$$  \hspace{1cm} (10)

### 3 Embedding Yang-Mills de Sitter gravity

The metric of the 3-spheres in (9) can be written as

$$d\Omega_3^2 = \frac{1}{4} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) , \quad d\tilde{\Omega}_3^2 = \frac{1}{4} (\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2 + \tilde{\sigma}_3^2) ,$$  \hspace{1cm} (11)

where $\sigma_i$ and $\tilde{\sigma}_i$ are $SU(2)$ left-invariant 1-forms satisfying

$$d\sigma_i = -\frac{1}{2} \epsilon_{ijk} \sigma_j \wedge \sigma_k , \quad d\tilde{\sigma}_i = -\frac{1}{2} \epsilon_{ijk} \tilde{\sigma}_j \wedge \tilde{\sigma}_k .$$  \hspace{1cm} (12)

Thus, we can introduce $SU(2)$ Yang-Mills fields $A^i_{(1)}$ to the vielbein

$$h^i = \sigma_i - g A^i_{(1)} , \quad \tilde{h}^i = \tilde{\sigma}_i - g A^i_{(1)} .$$  \hspace{1cm} (13)

With these preliminaries, we propose the reduction ansatz

$$ds_{11}^2 = \Delta^2 ds_4^2 + g^{-2} \Delta^2 d\theta^2 + \frac{1}{4} g^{-2} \Delta^{-\frac{1}{3}} \left[ c^2 \sum_i (h^i)^2 + s^2 \sum_i (\tilde{h}^i)^2 \right] ,$$  \hspace{1cm} (14)

$$F_{(4)} = 2g \epsilon_{(4)} - \frac{1}{4} g^{-2} \left( s c d\theta \wedge h^i \wedge *F^i_{(2)} - s c d\theta \wedge \tilde{h}^i \wedge *F^i_{(2)} - \frac{1}{4} c^2 \epsilon_{ijk} h^i \wedge h^j \wedge *F^k_{(2)} + \frac{1}{4} s^2 \epsilon_{ijk} \tilde{h}^i \wedge \tilde{h}^j \wedge *F^k_{(2)} \right) ,$$  \hspace{1cm} (15)

where $c = \cosh \theta$, $s = \sinh \theta$, $\Delta = \cosh(2\theta)$, and $*$ denotes the four-dimensional Hodge dual. Note that we have rewritten the scale parameter $\ell$ of section 2 in terms of $g = \ell^{-1}$. The $SU(2)$ Yang-Mills field strengths $F^i_{(2)}$ are given by

$$F^i_{(2)} = dA^i_{(1)} + \frac{1}{2} g \epsilon_{ijk} A^j_{(1)} \wedge A^k_{(1)} .$$  \hspace{1cm} (16)
The $D = 11$ Hodge dual of the 4-form is given by

\[
\hat{F}_4 = \frac{1}{32} g^{-6} \Delta^{-2} c^3 s^3 d\theta \wedge \epsilon_{(3)} \wedge \tilde{\epsilon}_{(3)} \\
-\frac{1}{128} g^{-5} \Delta^{-1} c^4 s^4 \epsilon_{ijk} h^i \wedge h^j \wedge F_{(2)}^k \wedge \tilde{\epsilon}_{(3)} \\
-\frac{1}{128} g^{-5} \Delta^{-1} s^2 c^4 \epsilon_{ijk} h^i \wedge \tilde{h}^j \wedge F_{(2)}^i \wedge \epsilon_{(3)} \\
+ \frac{1}{32} g^{-5} c s^3 d\theta \wedge h^i \wedge F_{(2)}^i \wedge \epsilon_{(3)} + \frac{1}{32} g^{-5} s c^3 d\theta \wedge \tilde{h}^i \wedge F_{(2)}^i \wedge \epsilon_{(3)}.
\]

(17)

It is straightforward to verify that the Bianchi identity $dF_{(4)} = 0$ is satisfied provided that the $SU(2)$ Yang-Mills fields $A_{(2)}^i$ satisfy the lower-dimensional equations of motion

\[
D^* F_{(2)}^i = 0,
\]

(18)

where the covariant derivative $D$ is defined by $DV^i = dV^i + g \epsilon_{ijk} A^j \wedge V^k$, for any vector $V^i$. The following identities are useful in verifying the equations of motion

\[
DF_{(2)}^i = 0, \quad Dh^i = -\frac{1}{2} \epsilon_{ijk} h^j \wedge h^k - g F_{(2)}^i, \quad D\tilde{h}^i = -\frac{1}{2} \epsilon_{ijk} \tilde{h}^j \wedge \tilde{h}^k - g F_{(2)}^i.
\]

(19)

The following formulae are also useful

\[
d(h^i \wedge F_{(2)}^i) = Dh^i \wedge F_{(2)}^i - h^i \wedge D^* F_{(2)}^i
\]

\[
= -\frac{1}{2} \epsilon_{ijk} h^j \wedge h^k \wedge F_{(2)}^i - g F_{(2)}^i \wedge F_{(2)}^i,
\]

\[
\epsilon_{ijk} d(h^i \wedge h^j \wedge F_{(2)}^k) = \epsilon_{ijk} D(h^i \wedge h^j) \wedge F_{(2)}^k = 0.
\]

(20)

The verification of $d\hat{F}_4 = \frac{1}{2} F_4 \wedge F_4$ requires the following identity

\[
\Delta^{-2} c^3 s^3 - \frac{1}{2}(\Delta^{-1} c^2 s^4)' + c s^3 = 0,
\]

\[-\Delta^{-2} c^3 s^3 - \frac{1}{2}(\Delta^{-1} s^2 c^4)' + s c^3 = 0.
\]

(21)

The evaluation of the $D = 11$ Einstein equations of motion are much more complicated, and we have not performed the calculation in full detail. However, we have verified that the equations of motion work for the $U(1)$ subsector of the $SU(2)$ gauge fields. Combining the result, the lower-dimensional equations of motion can be obtained from the Lagrangian

\[
e^{-1} \mathcal{L} = R - \frac{1}{4}(F_{(2)}^i)^2 - 8g^2.
\]

(22)
The cosmological constant $8g^2$ is totally fixed by the gauge coupling constant $g$. Thus, we have obtained four-dimensional Einstein $SU(2)$ Yang-Mills de Sitter gauged gravity from $D = 11$ by consistent Kaluza-Klein reduction on a hyperbolic 7-space.

It is worth remarking that $D = 11$ supergravity can also give rise to $D = 4 \ SU(2)$ AdS supergravity [1]. Also, de Sitter gravity with the wrong sign in the kinetic terms for gauge fields can arise from hyperbolic reduction of * variations of M-theory, type IIB or massive type IIA theories [27, 24]. In our reduction of M-theory, however, the sign of the kinetic term for $A_{(1)}^i$ is the right one.

4 Lifting of solutions

The four-dimensional Lagrangian [22] admits a large class of solutions, including multi-center black holes [25, 26, 27]. Using our reduction ansatz (14) and (15), all of these solutions can be lifted to eleven dimensions. We will first explicitly lift the case of $dS_2 \times S^2$, which is supported by one of the $SU(2)$ gauge fields, e.g. $F_{(2)}^3$. This solution is given by

$$
\begin{align*}
    ds_4^2 &= -d\tau^2 + e^{\frac{\tau}{\ell}} d\sigma_2 + 4\ell^2 d\Omega_2^2, \\
    F_{(2)}^3 &= \frac{1}{\ell} e^{\frac{\tau}{\ell}} d\tau \wedge d\sigma_2, \quad *F_{(2)}^3 = 2\ell\Omega_{(2)},
\end{align*}
$$

(23)

where $\ell^2 = 3/(64g^2)$. Note that the role of $F_{(2)}^3$ and $*F_{(2)}^3$ are interchangeable, giving rise to either the electric or magnetic solutions, with the metric unchanged. Lifting this solution back to $D = 11$ yields a smooth and regular embedding of $dS_2$ given by

$$
\begin{align*}
    ds_{11}^2 &= \Delta^\frac{1}{3} \left( -d\tau^2 + e^{\frac{\tau}{\ell}} d\sigma_2 + 4\ell^2 d\Omega_2^2 \right) + g^{-2} \Delta^\frac{2}{3} d\theta^2 \\
    &\quad + \frac{1}{4} g^{-2} \Delta^{-\frac{1}{3}} \left[ e^2 \left( d\omega_2^2 + (\sigma_3 - \sqrt{2} g e^{\frac{\tau}{2\ell}} d\sigma_2) \right) \\
    &\quad + s^2 \left( d\omega_2^2 + (\sigma_3 - \sqrt{2} g e^{\frac{\tau}{2\ell}} d\sigma_2) \right) \right], \\
    F_{(4)} &= 8g^2 e^{\frac{\tau}{2\ell}} e^{\frac{1}{2}} d\tau \wedge d\sigma_2 \wedge \Omega_{(2)} \\
    &\quad - \frac{1}{2} g^{-2} \ell \left( s c d\theta \wedge (\sigma_3 - \bar{\sigma}_3) - \frac{1}{2} e^2 \omega_{(2)} \wedge + \frac{1}{2} s^2 \bar{\omega}_{(2)} \right) \wedge \Omega_{(2)},
\end{align*}
$$

(24)

We explicitly verified that the above solution satisfies the equations of motion of $D = 11$ supergravity, which serves as a double check of our reduction ansatz (14) and (15). In this smooth embedding of $dS_2$ in M-theory, the metric can be viewed as a
rotating brane. Of course, if we interchange the role of $F^3_{(2)}$ and $\ast F^3_{(2)}$ in (23), then the transverse space is a nine-dimensional non-compact space which can be viewed as an $H^7$ bundle over $S^2$.

We will now consider a regular cosmological solution of (22) given by

$$ds^2_4 = H^2 \left( - f^{-1} dt^2 + f \, dx^2 + t^2 \, d\Omega_2^2 \right)$$

$$F^3_{(2)} = \frac{2\ell}{(t \, H)^2} \, dt \wedge dx, \quad \ast F^3_{(2)} = 2\ell \, \Omega_{(2)},$$

$$H = 1 + \frac{\ell}{t}, \quad f = \frac{4}{3} g^2 t^2 H^4 - 1. \quad (25)$$

This solution is, in fact, nothing but the BPS AdS Reissner-Nordstrøm black hole with $g \to i \, g$ [28]. When $g^2 \ell^2 = \frac{3}{64}$, the solution interpolates between dS$_2 \times S^2$ at the infinite past in the co-moving time to a dS$_4$-type geometry at the infinite future with the boundary of $S^2 \times S^1$ [28]. It is straightforward to lift the solution back to $D = 11$ and obtain a regular cosmological solution in M-theory. The corresponding metric is given by

$$ds^2_{11} = \Delta^2 H^2 \left( - f^{-1} dt^2 + f \, dx^2 + t^2 \, d\Omega_2^2 \right) + g^{-2} \Delta^2 \, dt^2$$

$$+ \frac{1}{4} g^{-2} \Delta^{-\frac{1}{2}} \left[ c^2 \left( d\omega_2^2 + (\sigma_3 - \frac{2g}{H} dx)^2 \right) + s^2 \left( d\bar{\omega}_2^2 + (\bar{\sigma}_3 - \frac{2g}{H} dx)^2 \right) \right]. \quad (26)$$

As a final example, we will lift (Minkowski)$_2 \times S^2$ to eleven dimensions. This four-dimensional solution is given by

$$ds^2_4 = -d\tau^2 + dx^2 + \frac{1}{8g^2} \, d\Omega_2^2,$$

$$F^3_{(2)} = 4g \, d\tau \wedge dx, \quad \ast F^3_{(2)} = \frac{1}{2g} \Omega_{(2)}. \quad (27)$$

Lifting this solution back to $D = 11$ yields a smooth and regular embedding of M$_2$, whose metric is given by

$$ds^2_{11} = \Delta^2 \left( - d\tau^2 + dx^2 + \frac{1}{8g^2} \, d\Omega_2^2 \right) + g^{-2} \Delta^2 \, dt^2$$

$$+ \frac{1}{4} g^{-2} \Delta^{-\frac{1}{2}} \left[ c^2 \left( d\omega_2^2 + (\sigma_3 - 4g^2 dx)^2 \right) + s^2 \left( d\bar{\omega}_2^2 + (\bar{\sigma}_3 - 4g^2 dx)^2 \right) \right]. \quad (28)$$

As in the previous examples, one can interchange the role of $F^3_{(2)}$ and $\ast F^3_{(2)}$ in (27).
5 Further embeddings of dS spacetime

In [22], there are further embeddings of dS spacetime in M-theory or type IIB supergravities. In our example of a dS₄ embedding in M-theory, the $H^7$ is a foliation of $S^3 \times S^3$. The embeddings of dS₄ in M-theory with a squashed $H^7$ being a foliation of $S^2 \times S^4$ and dS₅ in type IIB theory with the $H^5$ being a foliation of $S^2 \times S^2$ were obtained in [22]. In this section, we obtain an new embedding of dS₃ in $D = 6$ supergravity. The relevant Lagrangian is given by

$$e^{-1} \mathcal{L} = R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{12} e^{\sqrt{2} \phi} F^2_{(3)}.$$  \hspace{1cm} (29)$$

The solution is given by

$$ds^2_6 = \Delta ds^2_3 + g^{-2} \Delta d\theta^2 + g^{-2} \Delta^{-1} (c^2 d\phi_1^2 + s^2 d\phi_2^2),$$

$$F_{(3)} = 2g (e_{(3)} + * \text{psilon}_{(3)}), \quad \phi = 0,$$ \hspace{1cm} (30)$$

where $ds^2_3$ is dS₃ spacetime with cosmological constant $\Lambda = 2g^2$. Note that the transverse space is an $H^3$ written as a foliation of two circles. This theory can be reduced to pure de Sitter gravity in three dimensions, with the corresponding Lagrangian given by

$$e^{-1} \mathcal{L}_3 = R - 2g^2.$$ \hspace{1cm} (31)$$

6 Conclusions

We have obtained $D = 4$ de Sitter gravity coupled to $SU(2)$ Yang-Mills fields from a consistent Kaluza-Klein reduction and truncation of $D = 11$ supergravity on a hyperbolic 7-space. The hyperbolic space is written as a foliation of two 3-spheres, on which the $SU(2)$ fields are embedded. Unlike in the case of * theories, the kinetic terms of the gauge fields have the correct sign. The four-dimensional cosmological constant is completely fixed by the gauge coupling constant. Although our reduction procedure fits within the general pattern of non-compact gaugings and their higher-dimensional origins, described in [21], our result provides the first explicit embedding of a non-trivial de Sitter gauge gravity in M-theory.

The reduction ansatz enables us to lift any solution of the four-dimensional theory to eleven dimensions. We discuss the embeddings of smooth cosmological solutions.
In particular, we obtain the embeddings of $dS_2$ and $M_2$ in M-theory, as well as that of a cosmological solution which smoothly interpolates between $dS_2 \times S^2$ at the infinite past in the co-moving time to a $dS_4$-type geometry at the infinite future. We also obtain a new embedding of $dS_3$ in $D = 6$ supergravity. Our results provide important tools with which to study the $dS$/CFT correspondence from the point of view of string and M-theory.

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