Universal Dark Halo Scaling Relation for the Dwarf Spheroidal Satellites

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Abstract

Motivated by a recently found interesting property of the dark halo surface density within a radius, $r_{\text{max}}$, giving the maximum circular velocity, $V_{\text{max}}$, we investigate for dark halos of the Milky Way’s and Andromeda’s dwarf satellites based on cosmological simulations. We select and analyze the simulated subhalos associated with Milky-Way-sized dark halos and find that the values of their surface densities, $\Sigma_{V_{\text{max}}}$, are in good agreement with those for the observed dwarf spheroidal satellites even without employing any fitting procedures. Moreover, all subhalos on the small scales of dwarf satellites are expected to obey the universal relation, irrespective of differences in their orbital evolutions, host halo properties, and observed redshifts. Therefore, we find that the universal scaling relation for dark halos on dwarf galaxy mass scales surely exists and provides us with important clues for understanding fundamental properties of dark halos. We also investigate orbital and dynamical evolutions of subhalos to understand the origin of this universal dark halo relation and find that most subhalos evolve generally along the $r_{\text{max}} \propto V_{\text{max}}$ sequence, even though these subhalos have undergone different histories of mass assembly and tidal stripping. This sequence, therefore, should be the key feature for understanding the nature of the universality of $\Sigma_{V_{\text{max}}}$.

Key words: dark matter – galaxies: dwarf – galaxies: kinematics and dynamics – methods: simulation

1. Introduction

The $\Lambda$-dominated cold dark matter ($\Lambda$CDM) theory is most successful in explaining cosmological and astrophysical observations on spatial scales larger than about 1 Mpc, including the temperature fluctuations of the cosmic microwave background (e.g., Komatsu et al. 2011; Planck Collaboration et al. 2016) and the clustering of galaxies (e.g., Tegmark et al. 2004), which plays a crucial role in the formation of structure in the universe.

Nevertheless, on spatial scales smaller than 1 Mpc, i.e., galactic and sub-galactic mass scales, there are outstanding issues regarding the predictions from this standard theory that are significantly in disagreement with some observational facts. These representative issues include the so-called missing satellite (Klypin et al. 1999; Moore et al. 1999), core/cusp (Moore 1994; Burkert 1995; de Blok et al. 2001; Gilmore et al. 2007), and too-big-to-fail problems (Boylan-Kolchin et al. 2011, 2012). While there are two main aspects that could resolve these issues, baryonic physics and alternative dark matter theories, there still remain uncertainties in actual dark halo structure on small mass scales that have been inferred from currently available data.

In this context, dwarf spheroidal (dSph) galaxies in the Milky Way (MW) and Andromeda (M31) galaxies are ideal sites for studying the nature of dark matter through internal stellar kinematics. This is because these galaxies are the most dark-matter-dominated systems, and line-of-sight velocities for their resolved stars can be observed precisely by high-resolution spectroscopy (e.g., Walker et al. 2009a, 2009b). Moreover, these satellites have drawn special attention as the most promising targets in the indirect detection for particle dark matter though $\gamma$-ray stemmed from dark matter annihilation (e.g., Geringer-Sameth et al. 2015; Hayashi et al. 2016). For this reason, implementing extensive dynamical analysis for those galaxies should be of great importance for shedding light on the nature of dark matter on small mass scales.

From the dynamical analysis of the galaxies including the dSphs, several studies have derived dark halo structures for various kinds of galaxies with luminosities over ~14 magnitudes based on core dark matter density profiles with core radii ($r_c$) and densities ($\rho_0$), such as the Burkert and pseudo-isothermal profiles, and found that the central surface density, $\rho_0 r_0$, is nearly constant for all of these galaxies (e.g., Donato et al. 2009; Gentile et al. 2009; Salucci et al. 2012; Kormendy & Freeman 2016). By contrast, Boyarsky et al. (2010) defined the average column density of a dark halo derived by integrating line-of-sight, and estimated it for about 300 objects ranging from dSphs to galaxy clusters. Then they concluded that the column density weakly depends on dark halo mass except for dSphs, and that this trend is in agreement with the prediction of $\Lambda$CDM $N$-body simulations. Several subsequent studies have clearly shown that the dark halo surface density is not constant (Cardone & Del Popolo 2012; Del Popolo et al. 2013; Saburova & Del Popolo 2014).

More recently, Hayashi & Chiba (2015a, hereafter HC15a) proposed another common scale for dark halos. They defined the surface density inside a radius of the maximum circular velocity, $\Sigma_{V_{\text{max}}}$, and found that $\Sigma_{V_{\text{max}}}$ shows a very weak trend or is almost constant over a wide range of $V_{\text{max}}$ from dwarf galaxy to giant spiral elliptical galaxy scales, irrespective of different dark halo profiles and galaxy types. In addition, Hayashi & Chiba (2015b) showed that $\Sigma_{V_{\text{max}}}$ is also constant with respect to
$B$-band luminosity of galaxies over $\sim 14$ magnitudes from $M_B = -8$ to $-22$ mag.

Following this work, Okayasu & Chiba (2016) investigated the evolution of baryonic components of the dSphs in the MW and M31, under the constraint of a constant $\Sigma_{V_{\text{max}}}^\text{le}$ for their dark halos, and found that the models well reproduce star formation histories as derived from both observations and simulations. Thus, the properties of the surface density $\Sigma_{V_{\text{max}}}^\text{le}$ may play an important role in understanding star formation histories of low-mass galaxies.

In this work, we adopt the mean surface density of a dark halo de$\bar{e}$ned by HC15a to compare this density estimate from observations still largely remain. In particular, calculating $\Sigma_{V_{\text{max}}}^\text{le}$ on the dwarf galaxy mass scales in HC15a merely extrapolates from the mass-concentration relation estimated by using massive dark halos with heavier than $\sim 10^{10}M_\odot$ in the cosmological simulations (Klypin et al. 2016). Thus, in this work, we investigate the evolution of dark matter subhalos associated with an MW-like dark halo from cosmological $N$-body simulations to understand the properties of this surface density in more detail. In particular, we focus on less-massive subhalos associated with MW-sized dark halos because the dark halo surface density at these halo mass scales plays a key role in understanding the nature of dark matter and formation histories of low-mass galaxies.

This paper is organized as follows. In Section 4, we briefly introduce a dark halo surface density de$\bar{e}$ned by HC15a. In Section 3, we describe the properties of our cosmological simulations and selection criteria of MW-sized host halos and their subhalos in this work. In Section 4, we compare dark matter surface densities calculated from observations and simulations and then present the universal scaling relation of subhalos. In Section 5, we discuss the origin of this universal relation by analyzing orbital evolutions of subhalos. We summarize our results in Section 6.

2. Dark Halo Surface Density within a Radius of Maximum Circular Velocity

In this work, we adopt the mean surface density of a dark halo de$\bar{e}$ned by HC15a to compare this density estimate from observational data with those from pure $N$-body simulations. Given any of the parameters of a dark halo (e.g., scale length, scale density, and any slopes of dark matter density pro$\ddot{o}$les), this surface density within a radius of the maximum circular velocity, $V_{\text{max}}$, is given as

$$
\Sigma_{V_{\text{max}}} = \frac{M(r_{\text{max}})}{\pi r_{\text{max}}^2},
$$

where $r_{\text{max}}$ denotes a radius of maximum circular velocity of a dark halo, and its enclosed mass within $r_{\text{max}}$ is given as

$$
M(r_{\text{max}}) = \int_0^{r_{\text{max}}} 4\pi \rho_{\text{dm}}(r')r'^2 dr',
$$

where $\rho_{\text{dm}}$ denotes a dark matter density pro$\ddot{o}$le. Under the axisymmetric assumptions, the variables of the spherical radius, $r'$, are changed to those of the elliptical radius, $m'$, and then

Equation (2) can be rewritten by

$$
M(m_{\text{max}}) = \int_0^{m_{\text{max}}} 4\pi \rho_{\text{dm}}(m')Q'^2m'^2 dm',
$$

where $m'$ is described by cylindrical coordinates $(R, \varphi)$ and the dark halo axial ratio, $Q$, respectively. This surface density is fundamentally proportional to the product of a scale density, $\rho_{\text{le}}$, and radius, $r_{\text{le}}$, for any of the density pro$\ddot{o}$les of a dark halo, where the definition of $\rho_{\text{le}}$ and $r_{\text{le}}$ depends on an assumed density pro$\ddot{o}$le (in contrast to $\Sigma_{V_{\text{max}}}^\text{le}$), such as the so-called Navarro–Frenk–White (NFW) profile (Navarro et al. 1996) and Burkert profile (Burkert 1995).

Calculating $\Sigma_{V_{\text{max}}}$ and $V_{\text{max}}$ of dark halos in the dSph galaxies, we apply our axisymmetric mass models (e.g., Hayashi & Chiba 2012, 2015b) to their available kinematic data to obtain more reliable and realistic limits on their dark halo structures. This is motivated by the fact that the observed light distributions of the dSphs are actually non-spherical shapes (e.g., McConnachie 2012) and that $\Lambda$CDM theory predicts non-spherical virialized dark subhalos (e.g., Jing & Suto 2002; Kuhlen et al. 2007; Vera-Ciro et al. 2014). Thus, relaxing the assumption of spherical symmetry in the mass modeling should be valuable in evaluating the dark halo structures of the dSphs. In the axisymmetric mass models we use here, we employ the axisymmetric Jeans equations for the velocity anisotropy of tracer stars, $\beta_\varphi = \frac{\sqrt{v_\varphi^2} - \sqrt{v_z^2}}{\sqrt{v_r^2} + \sqrt{v_\varphi^2} + v_z^2}$, where $v_r$, $v_\varphi$, and $v_z$ are the velocity second moments toward $R$ and $Z$ directions, respectively. We should bear in mind that there is a degeneracy between the shape of the dark halo $Q$ and $\beta_\varphi$, as shown in Cappellari (2008) and Hayashi & Chiba (2015b). The stellar surface densities of dSphs are assumed by an oblate Plummer profile (Plummer 1911) with an inclination angle of a SP. For the dark matter distribution, on the other hand, we adopt an inner slope of the dark halo density pro$\ddot{o}$le as a free parameter as well as the parameterization of the central density and scale length of a dark halo (Hayashi & Chiba 2015b; Hayashi et al. 2016).

Finally, we adopt six free parameters: the axial ratio, the central density, the scale length and the inner slope of a dark halo, the stellar velocity anisotropy, and the inclination angle. Then these parameters are determined by fitting the observed line-of-sight velocity distribution.

In this work, we re-estimate $\Sigma_{V_{\text{max}}}$ and $V_{\text{max}}$ for the seven MW (Carina, Fornax, Sculptor, Sextans, Draco, Leo I, and Leo II) and the five M31 (And I, And II, And III, And V, and And VII) dSphs from those estimated by HC15a and add the results of the Ursa Minor dSph. This is because we have access to the latest stellar kinematic data for Draco (taken from Walker et al. 2015) and Ursa Minor, kindly provided by M. G. Walker (private communication), and because the procedure of fitting axisymmetric mass models to their kinematic data has changed slightly from previous axisymmetric works. Namely, in this work, we adopt an unbinned analysis for the comparison between data and models; this is unlike previous studies because an unbinned analysis is a more robust method for constraining dark halo parameters rather than a binned analysis (the detailed descriptions are presented in Hayashi et al. 2016). From this unbinned fitting procedure, we find that seven dSphs (Carina, Draco, Leo I, Leo II, Ursa Minor, Andromeda V, and VII) have an NFW or steeper inner density slope, while the
other ones show a cored or shallower cusped dark halo. In addition, most of the dSphs with \( V_{\text{max}} \geq 25 \text{ km s}^{-1} \) are settled in extended dark halos with scale lengths larger than 1.5 kpc, especially Draco and Andromeda I which have the largest scale length (\( \sim 6.0 \) kpc). Estimating \( \Sigma_{\text{max}} \) and \( V_{\text{max}} \) for these dSphs, we calculate an enclosed mass within \( r_{\text{max}} \) along the major axis of the mass distributions from estimated dark halo parameters described above. In order to determine \( r_{\text{max}} \), we calculate a circular velocity in gravitational potentials stemmed from axisymmetric mass distributions (Binney & Tremaine 2008)

\[
V^2_{\text{circ}}(R) = |R| - 2\Phi,
\]

where \( \Phi \) indicate gravitational force. We estimate circular velocity along the major axis, \( R \), using

\[
g(R, z) = -\frac{\partial \Phi}{\partial R} = -2\pi G a_0^3 R \int_0^\infty d\tau \frac{\rho(m^2)}{\tau^2 + Q^2 a_0^2},
\]

where \( \tau \) is a new variable of integration \( \tau = a_0^2 (1 - Q^2) \sin^2 u_m - ((1 - Q^2) - 0.5 - 1) a_0^2 \) in the spheroidal coordinate \( (u_m, v_m) \), and \( m^2 \) is defined by

\[
m^2(a_0^2) = \frac{R^2}{\tau + a_0^2} + \frac{z^2}{\tau + Q^2 a_0^2}.
\]

The estimates of \( \Sigma_{\text{max}} \) and \( V_{\text{max}} \) for the 13 dSphs are listed in Table 1 and plotted in Figure 1.

Here, we note that the observational results from Sextans and Andromeda II and VII dSphs are significantly affected by the sizes of data samples and their peculiar stellar kinematics. Since Sextans might have a very large tidal radius, the currently available spectroscopic data for this galaxy are quite incomplete in the outer region. Moreover, Hayashi & Chiba (2015b) demonstrated that the lack of kinematic data samples in the outer region of a stellar system has a large impact on determining dark halo parameters, especially the axial ratio of a dark halo and its velocity anisotropy (see Figure 12 in their paper). For Andromeda II, Ho et al. (2012) first inspected the kinematical properties of this galaxy and suggested that this is a prolate rotating system, that is, their stars rotate with respect to their stellar minor axis. This is a peculiar system, to which our models cannot be properly applied. For Andromeda II, although there is no observational evidence due to lack of a data sample, Hayashi & Chiba (2015b) suggested from the fitting results that the three-dimensional stellar density distribution of this galaxy would be preferable to the prolate system. Thus, this galaxy could also have a peculiar system like Andromeda II. Therefore, there is the possibility that the obtained parameters for these dark halos may contain large systematics.

### 3. Dark Matter Simulations

#### 3.1. Cosmological Simulation

In this work, we utilize the two cosmological simulations. One is the high-resolution simulation performed by Ishiyama et al. (2016), the other is the Cosmogrid simulation performed by Ishiyama et al. (2013). The former provides the higher resolution, and the latter enables us to collect more dark halo samples as it has a larger simulation box.

The run parameters in these simulations are listed in Table 2, and the best-fit cosmological parameters are consistent with the cosmic microwave background obtained by the Planck satellite (Planck Collaboration et al. 2014), namely \((\Omega_m, \Omega_{\Lambda}, h, n_s, \sigma_8) = (0.31, 0.69, 0.68, 0.96, 0.83)\) for the high-resolution simulation, and \((\Omega_m, \Omega_{\Lambda}, h, n_s, \sigma_8) = (0.30, 0.70, 0.70, 1.0, 0.8)\) for the Cosmogrid simulation, respectively.

To identify halos and subhalos and their merger trees, we used the ROCKSTAR (Robust Overdensity Calculation using K-Space Topologically Adaptive Refinement) halo finder\(^8\) (Behroozi et al. 2013) and CONSISTENT TREES (Behroozi et al. 2013). In this paper we use the physical values of each dark halo computed by ROCKSTAR and CONSISTENT TREES analysis, namely virial mass \( (M_{\text{vir}}) \), virial radius \( (r_{\text{vir}}) \), maximum circular velocity \( (V_{\text{max}}) \), and radius \( (r_{\text{max}}) \).

In the high-resolution simulation, the four MW-sized dark halos are identified, and the total mass of the halos ranges from \( \sim 0.8 \) to \( \sim 3.0 \times 10^{12} M_{\odot} \) as estimated from the dynamical analysis of blue horizontal branch stars or/and dwarf satellites (e.g., Sakamoto et al. 2003; Deason et al. 2012; Kafle et al. 2014). On the other hand, in order to get a number of dark halo samples, we identify the 56 MW-sized halos with a wider mass range from \( \sim 1.0 \) to \( \sim 6.0 \times 10^{12} M_{\odot} \) and select the 18 MW-sized halos that possess over 10 subhalos that satisfied some criteria (as described in the next section) among the 56 identified dark halos (as seen in Figure 2).

In Table 3, we summarize the virial mass, virial radius, concentration indicator (as described below) and number of subhalos that satisfied the criteria (as described below) for the four MW-sized dark halos in each simulation. For the Cosmogrid simulation, we demonstrate that, out of 18 MW-like dark halos, the four host halos show relatively rapid growth at a high redshift; that is, the subhalos associated with these parent halos did fall in at some earlier time and thus might have passed through the pericenter of the host several times. Using these subhalos, we will discuss in detail their orbital evolutionary histories in Section 5.

| Object          | \( V_{\text{max}} \) (km s\(^{-1}\)) | \( \Sigma_{\text{max}} \) (\( M_{\odot} \) pc\(^{-2}\)) |
|-----------------|------------------------------------|------------------------------------------|
| Milky Way       | 27.9\(^{+10.3}_{-5.7}\)            | 10.9\(^{+5.9}_{-3.7}\)                   |
| Carina          | 23.3\(^{+10.3}_{-5.7}\)            | 21.0\(^{+4.6}_{-2.4}\)                   |
| Fornax          | 27.3\(^{+10.3}_{-5.7}\)            | 28.1\(^{+9.8}_{-4.4}\)                   |
| Sculptor        | 27.3\(^{+10.3}_{-5.7}\)            | 38.6\(^{+8.3}_{-3.2}\)                   |
| Sextans         | 25.7\(^{+10.3}_{-5.7}\)            | 25.5\(^{+2.5}_{-0.2}\)                   |
| Draco            | 76.4\(^{+10.3}_{-5.7}\)            | 6.1\(^{+6.1}_{-0.2}\)                   |
| Leo I           | 22.7\(^{+10.3}_{-5.7}\)            | 13.4\(^{+6.1}_{-3.2}\)                   |
| Leo II          | 27.9\(^{+10.3}_{-5.7}\)            | 16.8\(^{+6.1}_{-3.2}\)                   |
| Ursa Minor      | 19.7\(^{+10.3}_{-5.7}\)            | 18.4\(^{+6.1}_{-3.2}\)                   |
| Andromeda       | 61.3\(^{+10.3}_{-5.7}\)            | 27.7\(^{+6.1}_{-5.2}\)                   |
| Andromeda I     | 44.3\(^{+10.3}_{-5.7}\)            | 11.9\(^{+6.1}_{-5.2}\)                   |
| Andromeda II    | 47.7\(^{+10.3}_{-5.7}\)            | 21.3\(^{+6.1}_{-5.2}\)                   |
| Andromeda III   | 27.3\(^{+10.3}_{-5.7}\)            | 31.1\(^{+6.1}_{-5.2}\)                   |
| Andromeda V     | 27.3\(^{+10.3}_{-5.7}\)            | 31.1\(^{+6.1}_{-5.2}\)                   |
| Andromeda VII   | 29.4\(^{+10.3}_{-5.7}\)            | 54.1\(^{+6.1}_{-5.2}\)                   |

\(^8\) This finder identifies dark halos based on the adaptive hierarchical refinement of friends-of-friends groups of particles in six phase-space dimensions and time.
In what follows, we investigate the properties of the dark halo surface densities of subhalos within the selected host halos in each simulation.

3.2. Subhalo Criteria

We extract the catalog data of subhalos within these hosts and impose criteria to avoid several numerical influences on small mass dark halos.

First, in order to avoid the effects caused by the limited spatial resolution of the simulation, we select the halos having a scale radius ($r_s$) of a dark matter profile that is larger than twice the softening length of this simulation.

Second, the virial mass needs to be larger than $\sim 10^7M_\odot$ ($\sim 10^8M_\odot$) in the high-resolution (the Cosmogrid) simulations, so that a halo has at least around 1000 dark matter particles. Finally, we select a subhalo that settles within the radius of $r_{\text{vir}}$ of a host halo at redshift $z=0$.

From the above three criteria with respect to the simulated dark halos, we extract the selected subhalos from each host dark halo. The number of these subhalos is listed in the fifth column of Table 2. In the next section we demonstrate the subhalo distributions on the $V_{\text{max}}$ vs. $V_{\text{max}}$ plane from this dark matter simulation, and compare it with the observed relation. Moreover, tracking the evolution history of subhalos, we investigate the origin of the observed $\Sigma V_{\text{max}}$ versus $V_{\text{max}}$ relation and whether this relation depends on the differences in the properties of each host halo.

4. The Universal Dark Halo Relation for the Satellite Galaxies

Using the $V_{\text{max}}$ and $r_{\text{max}}$ of subhalos, which fulfill the above three criteria, we calculate their spherical-averaged $\Sigma V_{\text{max}}$ derived from Equation (1). First of all, as a indicator representing the compactness of a dark halo, we utilize the mean physical density within the radius of the maximum circular velocity in units of the critical density ($\rho_{\text{crit},0}$) as defined

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**Table 2**

| Number of Particles, Box Length, Mass Resolution, Softening Length, and Initial Redshift of the High-resolution and Cosmogrid Simulations |
|-------------|-------------|-------------|-------------|-------------|
| $N$         | $L$ (Mpc)  | $m$ ($M_\odot$) | $\rho_{\text{DM}}$ (pc) | $z_{ini}$   |
| High-resolution | 2048$^3$  | 11.8        | $7.54 \times 10^3$ | 176.5       | 127         |
| Cosmogrid   | 2048$^3$  | 30.0        | $1.28 \times 10^5$ | 175.0       | 65          |

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*We also perform the same calculations of dark halo properties such as $\Sigma V_{\text{max}}$, in which case $r_s$ is larger than thrice the softening length, and confirm that our conclusions in this work are not largely influenced by this criterion.*

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**Figure 1.** $\Sigma V_{\text{max}}$ as a function of $V_{\text{max}}$ focusing on dark subhalos have $V_{\text{max}} \leq 120$ km s$^{-1}$. The colored dots depict the simulated subhalos associated with each MW-sized halo. The color difference indicates $\rho_{\text{max}}$. The magenta diamonds with error bars indicate the MW and M31 luminous dwarf spheroidals. The dashed lines are the fitting results of $\Sigma V_{\text{max}} \propto V_{\text{max}}^\alpha$ relation (see the text for more details).
by Diemand et al. (2007, hereafter D07),
\[
\rho_{V_{\text{max}}} = \frac{\tilde{\rho}(\leq r_{\text{max}})}{\rho_{\text{crit},0}} = 2\left(\frac{V_{\text{max}}}{H_0 r_{\text{max}}}\right)^2,
\]
where \(H_0\) denotes the Hubble constant at \(z = 0\). This corresponds to the physical concentration indicator of a dark halo. The concentration indicator, which is commonly used in \(N\)-body simulations, is defined by the ratio between a virial and a scale radii of a halo, assuming an NFW profile. However, because density profiles of less-massive subhalos, especially those affected by tidal forces from a massive host, should deviate from their initial mass profiles (Aguilar & White 1986; Hayashi et al. 2003; Peñarrubia et al. 2008), the usually defined concentration indicator is not necessarily appropriate. Thus, we adopt an alternative \(\rho_{V_{\text{max}}}\) as the concentration indicator of subhalos in this work.

4.1. Comparison between the Observations and the Simulations

Figures 1 and 2 show the results derived from both the simulations and the observations on the \(\Sigma_{V_{\text{max}}} - V_{\text{max}}\) plane,
focusing on mass scales of dSph-sized dark halos. From these figures, we demonstrate that pure dark matter simulations reproduce the dark halo surface densities derived from observations for dSphs. This means that $\Sigma_{V_{\text{max}}}$ can be unaffected by net effects of baryon physics and be dependent only on the intrinsic properties of dark halos. To confirm this accordance, we also calculate $\Sigma_{V_{\text{max}}}$ using the available catalog in the Illustris Project, a series of N-body and hydrodynamics simulations on cosmological volume. We use the results from the highest mass-resolution run, Illustris-1, and select the MW-sized halos and their subhalos though the same criteria as above (i.e., $r_\text{min} > 2 r_{\text{DM}}$ and $M_{\text{vir}} \geq 10^{9.8} M_\odot$). Figure 3 shows the results of the comparison with the Illustris simulation. From this figure, the effect of mass resolution emerges at low mass scales ($V_{\text{max}} \leq 30$ km s$^{-1}$), and it seems that these $\Sigma_{V_{\text{max}}}$ are slightly declined by the baryonic effects at higher mass scales ($V_{\text{max}} \geq 30$ km s$^{-1}$). However, the dark halo surface densities calculated by the Illustris simulation do not significantly differ from those observations, as do the results of the pure N-body simulations.

This is perhaps no surprise: indeed, the radius of the maximum circular velocity of a dark halo should be much larger than the star-forming regions, which settle in the center part of a dark halo, and thus this radius would be influenced only by gravitational effects such as mass assembly history and tidal force from their host. Therefore, even if inner dark matter profiles at dwarf-galaxy scales can be transformed from cusped to cored due to energy feedback from gas outflows driven by the star formation activity of galaxies, the expelling dark matter particles from the center of a dark halo cannot escape beyond the radius of the maximum circular velocity. In other words, the cusp-to-core transformation, which is a possible solution to the core-cusp and the too-big-to-fail problem, might not affect the $\Sigma_{\text{vmax}}-V_{\text{max}}$ relation. These results are shown by some N-body and hydrodynamical simulations of dwarf galaxies (e.g., Madau et al. 2014; Ogiya et al. 2014; Ogiya & Burkert 2015; Sawala et al. 2016). However, it is notable that using even observational results, we present and confirm the maximum circular velocity, and the radius of a dark halo is not sensitive to baryon feedback.

### 4.2. The Universal $\Sigma_{V_{\text{max}}}-V_{\text{max}}$ Relation

Intriguingly, comparing each panel in Figures 1 and 2, all of the host halos have a similar relation between $\Sigma_{V_{\text{max}}}$ and $V_{\text{max}}$. To confirm this quantitatively, we perform a least-squares fitting method to the relation $\alpha$ and $\beta$ from $\Sigma_{V_{\text{max}}} = \beta V_{\text{max}}^{\alpha}$ with respect to the subhalos associated with the four host halos in the high-resolution simulations. As a result, from the gray dashed lines in each panel of Figure 1, we obtain $(\alpha, \beta) = (0.89 \pm 0.02, 0.70 \pm 0.05), (1.09 \pm 0.02, 0.39 \pm 0.03), (0.80 \pm 0.02, 0.80 \pm 0.07), \text{and} (1.12 \pm 0.04, 0.33 \pm 0.04)$ for H1, H2, H3, and H4; that is, $\Sigma_{V_{\text{max}}}-V_{\text{max}}$ might have a similar trend. Also, we redo the same fitting procedure with all of the subhalos from each simulation and conclude that these subhalos reside along the relation $\Sigma_{V_{\text{max}}} = (0.80 \pm 0.02)V_{\text{max}}^{0.88 \pm 0.01}$ (as seen in the top-left panel of Figure 5). Therefore, we suggest that the dark halos associated with a host halo may exhibit the universal relation, irrespective of the difference in the host halo’s properties including mass, radius, and concentration. H15a proposed that the properties of dark halo surface densities with a wide dark halo mass range of $V_{\text{max}} \sim 10-400$ km s$^{-1}$, corresponding to a galaxy mass range from dwarf to elliptical galaxies, $\Sigma_{V_{\text{max}}}$ are generally constant. This means that the value of $\Sigma_{V_{\text{max}}}$ is “universal” with respect to $V_{\text{max}}$ of each dark halo. In this work, on the other hand, focusing only on low-mass scales with $V_{\text{max}} \leq 100$ km s$^{-1}$, especially the satellite galaxies, it becomes apparent that this surface density is actually dependent on $V_{\text{max}}$. Also, the new perspective is that this dependence is “universal” with respect to some of the properties of host halos.

### Table 3

| Name       | $M_{\text{vir}}$ ($\times 10^{12} M_\odot$) | $r_{\text{vir}}$ (kpc) | $\rho_{\text{vir}}$ ($\times 10^4$) | $N_{\text{subhalos}}$ |
|------------|------------------------------------------|-----------------------|----------------------------------|-----------------------|
| High-resolution |                                          |                       |                                  |                       |
| H1         | 2.88                                     | 374                   | 0.36                             | 187                   |
| H2         | 2.40                                     | 352                   | 0.35                             | 146                   |
| H3         | 2.31                                     | 348                   | 1.49                             | 118                   |
| H4         | 1.11                                     | 272                   | 0.61                             | 48                    |
| Cosmogrid  |                                          |                       |                                  |                       |
| CG1        | 4.28                                     | 420                   | 0.13                             | 41                    |
| CG2        | 2.87                                     | 368                   | 0.44                             | 19                    |
| CG3        | 3.36                                     | 388                   | 0.77                             | 20                    |
| CG4        | 3.08                                     | 377                   | 0.75                             | 18                    |

<figure>

![Figure 3](image.png)

Figure 3. Same as Figure 1, but for the Illustris simulations with the highest mass resolution. The different colors of points depict the subhalos associated with the different host halos.

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10. [http://www.illustris-project.org](http://www.illustris-project.org)

11. The run parameters of the Illustris-1 simulation are $m_{\text{DM}} = 6.3 \times 10^9 M_\odot$ and $r_{\text{DM}} = 1.4$ kpc for a dark matter particle, and $m_{\text{baryon}} = 1.3 \times 10^8 M_\odot$ and $r_{\text{baryon}} = 0.7$ kpc for a baryonic particle, respectively. Therefore, this simulation has $\sim 830$ ($\sim 50$) times lower dark matter mass resolution than the high-resolution (the Cosmogrid) simulations. The cosmological parameters are adopted by WMAP-9 results (Hinshaw et al. 2013), namely $(\Omega_\Lambda, \Omega_m, h, n_s, \sigma_8) = (0.2726, 0.7274, 0.704, 0.963, 0.809)$.

12. In this fitting, we utilize the subhalos with $V_{\text{max}} \geq 10$ km s$^{-1}$. This is because all $V_{\text{max}}$ derived from the observational data are much larger than 10 km s$^{-1}$, and the $\Sigma_{V_{\text{max}}}-V_{\text{max}}$ distribution of subhalos with smaller than $V_{\text{max}} = 10$ km s$^{-1}$ would be somewhat different from those with $V_{\text{max}}$ larger than 10 km s$^{-1}$. To confirm this, we perform the Chow test between these two subsamples of each host halo, showing that most of samples have two subsamples that are statistically different.

13. We also investigate whether there is a close relation between $\rho_{\text{vir}}$ and $V_{\text{max}}$, and find that $\rho_{\text{vir}}-V_{\text{max}}$ largely depends on host halo properties and has a much larger scatter than $\Sigma_{V_{\text{max}}}-V_{\text{max}}$.}
5. What is the Origin of the Universal $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ Relation?

To understand the origin of the universality of the $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ relation found for dark halos on the scales of dwarf galaxies, we investigate the evolutionary histories of subhalos in detail, especially the link between orbital evolution and $r_{\text{max}}$ and $V_{\text{max}}$ evolution in the Cosmogrid simulations. The time resolutions of the high-resolution simulations are not enough to investigate the orbital evolutions of their subhalos, therefore we focus only on the subhalos in the Cosmogrid simulations. Using the merger tree data of subhalos computed by CONSISTENT TREES, we trace the evolution of the mass distribution in satellite halos undergoing tidal stripping from a host halo.

5.1. Evolutionary History of Subhalos

The left four panels in Figure 4 show the orbital evolutionary tracks of ten representative subhalos associated with four parent dark halos derived from the Cosmogrid simulations. Out of 18 MW-like dark halos, we choose the four host halos showing relatively rapid growth at a high redshift, because subhalos associated with these parent halos did fall in at some earlier time and thus might have passed through the pericenter of the host a number of times. The basic properties of the four Cosmogrid host halos are listed in Table 3. In the left panels of Figure 4, we preferentially select subhalos that have undergone at least one pericenter passage with respect to a host halo. Eventually, however, the majority of subhalos have little experiences of pericenter passage. This implies that the subhalos, which have already passed through pericenters many times and thus have been disturbed strongly by tidal effects, have possibly been disrupted or lost their masses significantly by the present day. Such subhalos do not fulfill our subhalo criteria. Therefore, most of selected subhalos in each panel show the recent infall into their hosts.

The right four panels in Figure 4 display the evolution in the $r_{\text{max}} - V_{\text{max}}$ plane of subhalos and correspond to those in the left panels. On this plane, the halo tracks start at the lower-left corner because dark halos are still small at a high redshift. After the subhalos move toward the upper right during their mass-growth phase, both their $r_{\text{max}}$ and $V_{\text{max}}$ start decreasing because of their infall into a host halo and associated tidal stripping. This means that tidal stripping is likely to stop the inside-out passage and some important results for the properties of the satellite dark halos, as follows.

5.2 A Possible Solution to the Origin of the Universal Relation

The evolutionary sequence on the $r_{\text{max}} - V_{\text{max}}$ plane described above and the deviation from it are basically consistent with the results shown in D07 (see their Figure 13). Therefore, it is suggested that subhalos within their own host halos have a common dynamical evolution, regardless of the properties of host halos. Moreover, as mentioned in D07, this sequence in all host halos can be expressed simply as the linear relation, $r_{\text{max}} \propto V_{\text{max}}^{-\mu}$, and we confirm their suggested relation (dashed lines in the four right panels in Figure 4). Thus, this relation plays a key role in untangling the origin of the universal $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ relation. Based on $r_{\text{max}} \propto V_{\text{max}}$, the mean physical dark halo density defined by Equation (8) becomes $\rho_{\text{V}_{\text{max}}} \propto (V_{\text{max}}/r_{\text{max}})^{\mu} \approx \text{const}$. On the other hand, $\Sigma_{\text{V}_{\text{max}}}$ can be rewritten by $\Sigma_{\text{V}_{\text{max}}} \approx \rho_{\text{V}_{\text{max}}} r_{\text{max}}$. Consequently, we arrive at $\Sigma_{\text{V}_{\text{max}}} \propto V_{\text{max}}$ using the $r_{\text{max}} - V_{\text{max}}$ relation.

Furthermore, subhalos evolve generally along $r_{\text{max}} \propto V_{\text{max}}$, although the relation holds some dispersion because the dark halo structures in these subhalos are affected by a change in mass accretion rate. Thus, the $\Sigma_{\text{V}_{\text{max}}}$ versus $r_{\text{max}}$ relation that we have derived here can be nearly constant with time. Following this hypothesis, in Figure 5 we plot the redshift evolution of all of the subhalos associated with the different host halos on the $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ plane. Since the virial radius of a subhalo decreases naturally with redshift, the lower limit of $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ goes up and gets close to $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ relation at $z = 0$. Although the amplitude of the $\Sigma_{\text{V}_{\text{max}}} - V_{\text{max}}$ relation at $z = 5$ is thus shifted upward, its slope does not change significantly. Therefore, we find that irrespective of some scatters, the relation has indeed been kept roughly invariant since its emergence.

6. Summary and Conclusions

We have investigated a dark halo surface density inside a radius at the maximum circular velocity, $\Sigma_{\text{V}_{\text{max}}}$, first introduced by HC15a, based on the comparison between the results from the observations and those from cosmological $N$-body simulations. While the $\Sigma_{\text{V}_{\text{min}}}$ versus $V_{\text{max}}$ relation is semi-analytically inferred from the assumption of an NFW profile combined with an empirical mass concentration relation (HC15a), this can be affected by some environmental effects, such as tidal stripping and heating, as well as halo-to-halo scatters. Therefore, in this work, we have scrutinized dark matter halos, especially subhalos associated with a MW-like dark halo, taken from cosmological pure dark matter simulations performed by Ishiyama et al. (2013, 2016) in order to understand the properties of the surface density in more detail. We have found several important results for the properties of the satellite dark halos, as follows.
1. Dark halo surface densities derived from our simulated subhalos associated with MW-sized hosts are in remarkable agreement with those from the observations of the MW and M31 dSph satellites. This implies that the surface density is only weakly modified by baryonic feedback. Therefore, the surface density provides us with a clue for understanding the fundamental dark halo properties of the Local Group.

2. Even if host dark halos have different masses, compactness, assembly history, and properties than their subhalos, all subhalos are found to obey the universal $\Sigma V_{\text{max}}$ relation. Furthermore, this universality appears to be sustained even at high redshifts.

3. In order to understand the origin of this universal dark halo relation, we have investigated the orbital and dynamical evolutions of subhalos. It is found that most
of the subhalos have several or little experiences of pericenter passage with respect to a host halo. Analyzing such a subhalo evolution, we have confirmed that most of the subhalos evolve along the \( V_{\text{max}} \) sequence, whereas more disturbed subhalos, which have undergone pericenter passage a number of times and are very close to the center of a host halo, show somewhat of a deviation from this sequence. This suggests that the tidal force from host halos mainly removes only the outer parts of subhalos, which are accreted from the outside, so that this \( r_{\text{max}} \propto V_{\text{max}} \) sequence remains preserved. Since the \( \Sigma_{\text{Vmax}} \) is derived from this sequence, \( r_{\text{max}} \propto V_{\text{max}} \), it is found to be a basic property for understanding the common dark halo surface density scales.

Following the results obtained here, there is the possibility that this universal relation for dark halos exists not only for the MW and M31, but also for larger gravitational systems such as galaxy groups and clusters. For instance, recently discovered ultra-diffuse galaxies in clusters (e.g., Koda et al. 2015; van Dokkum et al. 2015a, 2015b; Yagi et al. 2016) might be affected by strong tidal disturbances, and if so, these galaxies may show some characteristic trends in the \( \Sigma_{\text{Vmax}} \) versus \( V_{\text{max}} \) relation. Also, it remains yet unclear whether the ultra-faint galaxies in the MW indeed obey this universal relation, because of the paucity of observable bright stars in such faint systems. We thus require high-quality photometric and spectroscopic data of these galaxies. Current and future facilities such as the Hyper Suprime Cam (Miyazaki et al. 2012) and the Prime Focus Spectrograph (Tamura et al. 2016), to be mounted on the Subaru Telescope, and high-precision spectroscopy the Thirty Meter Telescope (Simard et al. 2016) will have the capability of obtaining more severe constraints on dark halo structures of these galaxies and enable us to understand the basic properties of dark matter in the universe.

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