Isospin analysis of $D^0$ decay to three pions

M. Gaspero, B. Meadows, K. Mishra, and A. Soffer

$^1$Universit`a di Roma La Sapienza, Dipartimento di Fisica and INFN, I-00185 Roma, Italy
$^2$University of Cincinnati, Cincinnati, Ohio 45221, USA
$^3$Tel Aviv University, Tel Aviv, 69978, Israel

(Dated: May 27, 2008)

Abstract

The final state of the decay $D^0 \rightarrow \pi^+\pi^-\pi^0$ is analyzed in terms of isospin eigenstates. It is shown that the final state is dominated by the isospin-0 component. This suggests that isospin considerations may provide insight into this and perhaps other $D^0$-meson decay. We also discuss the isospin nature of the nonresonant contribution in the decay, which can be further understood by studying the decay $D^0 \rightarrow \pi^0\pi^0\pi^0$.

*Now at Fermi National Accelerator Laboratory, Batavia, IL, USA
I. INTRODUCTION

An analysis of the resonant sub-structure in the decay \( D^0 \rightarrow \pi^+\pi^-\pi^0 \) was recently performed by the \textit{BABAR} collaboration \[1\]. The Dalitz-plot distribution of the \( D^0 \rightarrow \pi^+\pi^-\pi^0 \) events (Fig. 1) shows a clear six-fold symmetry, with the probability density function vanishing along three axes. As first described by Zemach \[2\] and noted in Ref. \[1\], this behavior is indicative of a final state with isospin \( I = 0 \).

\[
\psi(s_+, s_-) = \sum_r B_r g_r(s_+, s_-),
\]

where \( s_+ \equiv (p_{\pi^+} + p_{\pi^0})^2 \) and \( s_- \equiv (p_{\pi^-} + p_{\pi^0})^2 \) are the squared invariant masses of the \( \pi^+\pi^0 \) and \( \pi^-\pi^0 \) pairs, respectively, \( B_r \) is a complex coefficient, and \( g_r(s_+, s_-) \) is the distribution of contribution \( r \), whose functional form is outlined in Ref. \[1\]. The definitions of \( g_r(s_+, s_-) \) used here differ from that of Ref. \[1\], in that we define these functions to be normalized over
the Dalitz plot,

\[ \int ds_+ ds_- |g_r(s_+, s_-)|^2 = 1. \]  

(2)

The values for the \( B_r \) coefficients consistent with Eqs. (1) and (2) are reproduced in Table I.

TABLE I: Amplitude coefficients \( B_r = |B_r| e^{i \phi_r} \) of the contributing final states of the decay \( D^0 \rightarrow \pi^- \pi^+ \pi^0 \), adapted from Ref. [1]. The \( f_0(400) \) was labeled \( \sigma(400) \) in Ref. [1].

| Final state \( r \) | Amplitude \( |B_r| \) | Phase \( \phi_r (^\circ) \) |
|---------------------|-----------------|-----------------|
| Nonresonant         | 0.106 ± 0.013 ± 0.014 | −11±4±2         |
| \( \rho(770)^+ \pi^- \) | 0.588 ± 0.006 ± 0.002 | 10±8±13         |
| \( \rho(770)^0 \pi^0 \) | 0.714 ± 0.008 ± 0.002 | −2.0±0±0.6      |
| \( \rho(450)^+ \pi^- \) | 0.154 ± 0.010 ± 0.007 | 16±3±3          |
| \( \rho(450)^0 \pi^0 \) | 0.062 ± 0.012 ± 0.007 | −146±18±24      |
| \( \rho(450)^- \pi^- \) | 0.040 ± 0.011 ± 0.024 | −17±2±3         |
| \( \rho(1700)^+ \pi^- \) | 0.236 ± 0.019 ± 0.014 | −17±2±2         |
| \( \rho(1700)^0 \pi^0 \) | 0.267 ± 0.016 ± 0.014 | −50±3±3         |
| \( \rho(1700)^- \pi^- \) | 0.210 ± 0.012 ± 0.007 | 156±9±6         |
| \( f_0(980) \pi^0 \) | 0.056 ± 0.005 ± 0.006 | 12±9±4          |
| \( f_0(1370) \pi^0 \) | 0.074 ± 0.007 ± 0.007 | 51±8±7          |
| \( f_2(1270) \pi^0 \) | 0.072 ± 0.010 ± 0.011 | −171±3±4        |
| \( f_0(400) \pi^0 \) | 0.130 ± 0.005 ± 0.026 | 7±8±8           |

The goal of this paper is to quantify the extent to which the \( I = 0 \) component dominates the final state and learn about the contributions of the other isospin eigenstates. In Section II we perform an isospin analysis of the \( \pi^+ \pi^- \pi^0 \) final state. The observed dominance of the \( I = 0 \) component suggests that isospin considerations are more useful for developing an understanding of this decay. In Section III we discuss our results, the nature of the nonresonant contribution to the decay, a possible mechanism for the observed \( I = 0 \) dominance, and further measurements that will help clarify outstanding questions.
II. ISOSPIN DECOMPOSITION

Next, we analyze the decay $D^0 \rightarrow \pi^+\pi^-\pi^0$ in terms of isospin eigenstates. The 3-pion final state can be described in terms of the total isospin $I$, the isospin $I_{12}$ of two of the three pions, and the $z$-projection $I^z$, which is always 0 for this final state. The seven eigenstates $|I(I_{12})\rangle$ of these quantum numbers that also satisfy $I^z = 0$ can be written as a linear combination of the three-pion final states using the appropriate Clebsh-Gordan coefficients:

$$|3(2)\rangle = \frac{1}{\sqrt{10}}\left( |+0-\rangle + |0+-\rangle + |+-0\rangle + |-+\rangle + |0-+\rangle + |-0+\rangle + 2|000\rangle \right)$$

$$|2(2)\rangle = \frac{1}{2}\left( |+0-\rangle + |0+-\rangle - |0-+\rangle - |-0+\rangle \right),$$

$$|1(2)\rangle = \frac{1}{\sqrt{60}}\left[ 3\left( |+0-\rangle + |0+-\rangle + |0-+\rangle + |-0+\rangle \right) - 2\left( |+0-\rangle + |0+-\rangle \right) - 4|000\rangle \right],$$

$$|2(1)\rangle = \frac{1}{\sqrt{12}}\left[ |+0-\rangle - |0+-\rangle + 2\left( |+0-\rangle - |0+-\rangle \right) + |0-+\rangle - |-0+\rangle \right],$$

$$|1(1)\rangle = \frac{1}{2}\left( |+0-\rangle - |0+-\rangle - |0-+\rangle - |-0+\rangle \right),$$

$$|0(1)\rangle = \frac{1}{\sqrt{6}}\left( |+0-\rangle - |0+-\rangle - |+0-\rangle + |0-+\rangle + |0-0\rangle - |-0+\rangle \right),$$

$$|1(0)\rangle = \frac{1}{\sqrt{3}}\left( |+0-\rangle - |000\rangle + |0-0\rangle \right),$$

(3)

where we have used the notation

$$|+0-\rangle = |1, 1\rangle |1, 0\rangle |1, -1\rangle = |\pi^+\rangle |\pi^0\rangle |\pi^-\rangle,$$

$$|000\rangle = |1, 0\rangle |1, 0\rangle |1, 0\rangle = |\pi^0\rangle |\pi^0\rangle |\pi^0\rangle,$$

(4)

eetc., and it is implied that the first two pions are in an isospin eigenstate whose eigenvalue is indicated by the bracketed number $I_{12}$.

The three states in Eq. (3) for which $I_{12} = 1$ are identified as those with a $\rho(770)$, $\rho(1450)$, or $\rho(1700)$. We denote these states as $\rho_n\pi$ according to their radial excitation quantum number $n \in \{1, 2, 3\}$, and use $\rho^+$, $\rho^0$, and $\rho^-$ to indicate any linear combination of these states with specific electric charge. We define the $\rho$ states to be

$$|\rho^+\rangle = |1, 1\rangle = \frac{1}{\sqrt{2}}\left( |+0-\rangle - |0+-\rangle \right),$$

$$|\rho^0\rangle = -|1, 0\rangle = \frac{1}{\sqrt{2}}\left( |-+\rangle - |+-\rangle \right),$$

$$|\rho^-\rangle = |1, -1\rangle = \frac{1}{\sqrt{2}}\left( |0-\rangle - |-0\rangle \right),$$

(5)
where the minus sign in the $|\rho^0\rangle$ definition implies that there is no sign change under cyclic permutations of the three pions, maintaining consistency with the definitions used in Ref. [1].

Given Eq. (5), the $I_{12} = 1$ states in Eq. (3) can be written as

$$
|2(1)\rangle = \frac{1}{\sqrt{6}} \left( |\rho^+\pi^-\rangle - 2|\rho^0\pi^0\rangle + |\rho^-\pi^+\rangle \right),
$$

$$
|1(1)\rangle = \frac{1}{\sqrt{2}} \left( |\rho^+\pi^-\rangle - |\rho^-\pi^+\rangle \right),
$$

$$
|0(1)\rangle = \frac{1}{\sqrt{3}} \left( |\rho^+\pi^-\rangle + |\rho^0\pi^0\rangle + |\rho^-\pi^+\rangle \right),
$$

(6)

where the sign of each $|\rho\pi\rangle$ state is such that it is symmetric under cyclic permutations of the three pions and anti-symmetric under the exchange of any pair of pions.

The $\pi^+\pi^-\pi^0$ part of the state $|1(0)\rangle$ is identified as the sum of the contributions involving the two-body, $I = 0$ resonances $f_i$, with $i = 0, 2$. We therefore write

$$
|1(0)\rangle = \frac{1}{\sqrt{3}} \left( \sqrt{2} |f\pi^0\rangle - |000\rangle \right).
$$

(7)

Since there are no $I = 2$ resonances in Table I, the $I_{12} = 2$ states in Eq. (3) have no resonant contributions. However, the symmetry of the $\pi^+\pi^-\pi^0$ components of $|3(2)\rangle$ indicates that it may be identified with the nonresonant contribution of Table I. Alternatively, it may constitute the $\pi^+\pi^-\pi^0$ component of the symmetric $I = 1$ state

$$
|1(S)\rangle \equiv \frac{2}{3} |1(2)\rangle + \frac{\sqrt{5}}{3} |1(0)\rangle
$$

$$
= \frac{1}{\sqrt{15}} \left( |+0-\rangle + |0+-\rangle + |+0-\rangle + |0-0\rangle + |0+-\rangle + |000\rangle \right).
$$

(8)

In principle, the observed nonresonant state may be a superposition of $|1(S)\rangle$ and $|3(2)\rangle$. However, the $|1(S)\rangle$ state is expected to dominate, due to the following argument. The four-quark final state produced by the weak decay $c \rightarrow d\bar{u}u$, shown in Fig. 2, cannot have $I = 3$. Since production of the third $q\bar{q}$ pair will be dominated by the strong-interaction, it will not change the total isospin. Therefore, $I = 3$ is disfavored. It is also possible that a very broad, $\pi^+\pi^- S$-wave resonance is present in these decays, and that it was partly described by the constant nonresonant term in the fit in Ref. [1]. In that case, it would contribute only to the $|1(0)\rangle$ isospin eigenstate.

In what follows, we take the nonresonant contribution $|NR\rangle$ to be due only to $|1(S)\rangle$. Then Eqs. (7) and (8) yield the relation

$$
|1(2)\rangle = \frac{3}{\sqrt{10}} |NR\rangle - \frac{\sqrt{5}}{6} |f\pi^0\rangle - \frac{2}{\sqrt{15}} |000\rangle.
$$

(9)
We now reorder the terms of Eq. (1) according to their \( I_{12} \) eigenvalues:

\[
\psi(s_+, s_-) = B_{NR} g_{NR}(s_+, s_-) \\
+ B_{\rho^+\pi^-} g_{\rho^+\pi^-}(s_+, s_-) \\
+ B_{\rho^0\pi^0} g_{\rho^0\pi^0}(s_+, s_-) \\
+ B_{\rho^-\pi^+} g_{\rho^-\pi^+}(s_+, s_-) \\
+ B_{f\pi^0} g_{f\pi^0}(s_+, s_-),
\]

(10)

where the first term is the nonresonant term, the last is a sum over the six final states with \( I_{12} = 0 \) resonances listed at the bottom of Table I, and each of the second, third, and fourth terms is a sum over the three \( I_{12} = 1 \) \( \rho\pi \) states. For example,

\[
g_{\rho^+\pi^-}(s_+, s_-) \equiv \frac{S_{\rho^+\pi^-}}{N_{\rho^+\pi^-}} \exp \left[ -i\delta_{\rho^+\pi^-} \right],
\]

(11)

where

\[
S_{\rho^+\pi^-} \equiv \sum_{n=1}^{3} B_{\rho_n^+\pi^-} g_{\rho_n^+\pi^-}(s_+, s_-),
\]

\[
\delta_{\rho^+\pi^-} \equiv \arg (S_{\rho^+\pi^-}),
\]

\[
N_{\rho^+\pi^-} \equiv \sqrt{\int ds_+ ds_- |S_{\rho^+\pi^-}|^2},
\]

(12)

and \( \rho_n \ (n = 1, 2, 3) \) indicates the three \( \rho \) resonances of Table I. With these definitions, the wave function \( g_{\rho^+\pi^-}(s_+, s_-) \) is explicitly normalized and has vanishing average phase. Requiring that Eq. (10) be identical to (1) leads to the following values for the coefficients of Eq. (10):

\[
B_{NR} = 0.1066 e^{-i11.4^\circ},
\]

\[
B_{\rho^+\pi^-} = N_{\rho^+\pi^-} \exp [i\delta_{\rho^+\pi^-}] = 1.1976 e^{-i4.3^\circ},
\]

\[
B_{\rho^0\pi^0} = N_{\rho^0\pi^0} \exp [i\delta_{\rho^0\pi^0}] = 0.8867 e^{i6.3^\circ},
\]

\[
B_{\rho^-\pi^+} = N_{\rho^-\pi^+} \exp [i\delta_{\rho^-\pi^+}] = 1.0077 e^{-i8.2^\circ},
\]

\[
B_{f\pi^0} = N_{f\pi^0} \exp [i\delta_{f\pi^0}] = 0.0700 e^{40.0^\circ},
\]

(13)

where the symbols \( N_s \) and \( \delta_s \) for final state \( s \) are defined analogously to Eq. (12). The value of \( B_{NR} \) is taken from Table I, and the rest are calculated numerically as in Eqs. (11) and (12). The phase convention is that of Table I, namely, \( \delta_{\rho_n^+\pi^-} = 0 \).
Next, we write the wave function of Eq. (10) as a sum over the Dalitz-plot representations of the eigenstates of $I$ and $I_{12}$ of Eq. (3):

$$\psi(s_+, s_-) = C_{1(2)} M_{1(2)}(s_+, s_-)$$
$$+ C_{2(1)} M_{2(1)}(s_+, s_-)$$
$$+ C_{1(1)} M_{1(1)}(s_+, s_-)$$
$$+ C_{0(1)} M_{0(1)}(s_+, s_-)$$
$$+ C_{1(0)} M_{1(0)}(s_+, s_-),$$  \hspace{1cm} (14)

where $M_{I(I_{12})}(s_+, s_-)$ is the normalized distribution function of the eigenstate $|I(I_{12})\rangle$, obtained by linearly combining the functions $g_x(s_+, s_-)$ of Eq. (10) with the coefficients of either Eq. (6), (7), or (9). Terms for $|3(2)\rangle$ and $|2(2)\rangle$ were not included in Eq. (14), as reasoned earlier. Then from the definition of $M_{I(I_{12})}(s_+, s_-)$ follows the desired transformation between the resonance-based fit coefficients and the isospin coefficients:

$$C_{1(2)} = \frac{\sqrt{10}}{3} B_{NR},$$
$$C_{2(1)} = \frac{1}{\sqrt{6}} (B_{\rho^+\pi^-} - 2B_{\rho^0\pi^0} + B_{\rho^-\pi^+}),$$
$$C_{1(1)} = \frac{1}{\sqrt{2}} (B_{\rho^+\pi^-} - B_{\rho^-\pi^+}),$$
$$C_{0(1)} = \frac{1}{\sqrt{3}} (B_{\rho^+\pi^-} + B_{\rho^0\pi^0} + B_{\rho^-\pi^+}),$$
$$C_{1(0)} = \sqrt{\frac{3}{2}} B_{f\pi^0} + \sqrt{\frac{5}{6}} C_{1(2)},$$  \hspace{1cm} (15)

where the expressions for $C_{1(0)}$ and $C_{1(2)}$ were chosen so as to satisfy the $\pi^+\pi^-\pi^0$ projection of Eqs. (7) and (9).

Taking the numerical values of the $B_r$ coefficients from Eq. (13) and Table I, Eq. (15) gives

$$C_{1(2)} = (0.0629 \pm 0.0028) \exp \left[ i \left( -8.9 \pm 2.6 \right)^{\circ} \right],$$
$$C_{2(1)} = (0.1395 \pm 0.0016) \exp \left[ i \left( -42.5 \pm 0.7 \right)^{\circ} \right],$$
$$C_{1(1)} = (0.0814 \pm 0.0023) \exp \left[ i \left( 18.0 \pm 2.0 \right)^{\circ} \right],$$
$$C_{0(1)} \equiv 1,$$
$$C_{1(0)} = (0.0954 \pm 0.0052) \exp \left[ i \left( 14.5 \pm 2.4 \right)^{\circ} \right],$$  \hspace{1cm} (16)
where we have normalized the coefficients so that $C_{0(1)} = 1$. The errors reflect the full error matrix of the results presented in Table I [4]. The correlation matrix for these coefficients are given in Table II.

Eq. (16) quantifies the observation, made qualitatively in Ref. [1] on the basis of the symmetry exhibited by the Dalitz-plot distribution, that the final state of the decay $D^0 \rightarrow \pi^+ \pi^- \pi^0$ is dominated by an $I = 0$ component.

### TABLE II: Correlation matrix for the $C_{I(12)}$ amplitude coefficients of Eq. (16).

|       | $|C_{1(2)}|$ | $\arg(C_{1(2)})$ | $|C_{2(1)}|$ | $\arg(C_{2(1)})$ | $|C_{1(1)}|$ | $\arg(C_{1(1)})$ | $|C_{1(0)}|$ | $\arg(C_{1(0)})$ |
|-------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|
| $|C_{1(2)}|$ | 1           | -0.12           | 0.105       | -0.018           | 0.631       | 0.110           | 0.279       | 0.657            |
| $\arg(C_{1(2)})$ | -0.120      | 1.000           | 0.106       | -0.211           | 0.539       | -0.760          | 0.136       |                  |
| $|C_{2(1)}|$ | 0.105       | 0.062           | 1.000       | 0.008            | 0.179       | 0.029           | -0.017      | 0.078            |
| $\arg(C_{2(1)})$ | -0.018      | 0.106           | 0.008       | 1.000            | 0.333       | 0.110           | 0.151       |                  |
| $|C_{1(1)}|$ | 0.631       | -0.211          | 0.179       | 0.148            | 1.000       | 0.050           | 0.259       | 0.288            |
| $\arg(C_{1(1)})$ | 0.110       | 0.539           | 0.029       | 0.333            | 0.050       | 1.000           | -0.296      | 0.097            |
| $|C_{1(0)}|$ | 0.279       | -0.760          | -0.017      | 0.110            | 0.259       | -0.296          | 1.000       | 0.077            |
| $\arg(C_{1(0)})$ | 0.657       | 0.136           | 0.078       | 0.151            | 0.288       | 0.097           | 0.077       | 1.000            |

### III. DISCUSSION AND CONCLUSIONS

We have analyzed the relative contributions of different components to the decay $D^0 \rightarrow \pi^+ \pi^- \pi^0$ using results published by BABAR [1]. It appears that isospin considerations may form a solid basis for understanding the observed decay pattern, as the amplitude of the $|0(1)\rangle$ final state dominates by factors of seven or more over the other isospin components. This dominance has no natural explanation in the decay mechanisms suggested by the factorization-motivated diagrams of this decay, shown in Fig. 2. While factorization is useful in predicting the behavior of $B$-meson decays, it is not as successful when applied to the lighter $D$ mesons. The observed $|0(1)\rangle$ dominance in the decay $D^0 \rightarrow \pi^+ \pi^- \pi^0$ may lead to a better general understanding of charmed meson decays. Alternatively, perhaps the $I = 0$ component is enhanced by the presence of a yet-unknown and possibly broad state with this quantum number, which couples strongly to three pions. An inclusive search for
FIG. 2: Feynman diagrams for the decay $D^0 \rightarrow \pi^+\pi^-\pi^0$. With curly brackets indicating a resonance, the diagrams correspond to the decays (a) $D^0 \rightarrow \rho^+\pi^-$, (b) $D^0 \rightarrow \pi^+\rho^-$, and (c,d) $D^0 \rightarrow \rho^0\pi^0$ or $D^0 \rightarrow f\pi^0$.

such a state may answer this question.

In conducting the isospin analysis, we took only the $\pi^+\pi^-\pi^0$ projections of the isospin-eigenstates $|1(2)\rangle$ and $|1(0)\rangle$. The CLEO collaboration \cite{5} has set an upper limit of $3.4\times10^{-4}$ on the branching fraction $\cal{B}(D^0 \rightarrow \pi^0\pi^0\pi^0)$. Together with the BABAR \cite{6} measurement of $\cal{B}(D^0 \rightarrow \pi^+\pi^-\pi^0) = (1.493 \pm 0.057)\%$, this implies an upper limit on the amplitude ratio $A(D^0 \rightarrow \pi^0\pi^0\pi^0)/A(D^0 \rightarrow \pi^+\pi^-\pi^0) < 0.15$, consistent with the suppression seen in the coefficients $C_{1(2)}$ and $C_{1(0)}$, and the expectation from Eqs. (7) and (9).

As discussed above, the $\pi^+\pi^-\pi^0$ nonresonant amplitude may be a combination of $|3(2)\rangle$, $|1(S)\rangle$, and a broad $\pi^+\pi^-$ resonance term in $|1(0)\rangle$. If it is due only to the $|3(2)\rangle$, Eq. (3) predicts the ratio between the nonresonant $\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$ amplitudes to be $R_{NR} = \sqrt{2}/3$. By contrast, $|1(S)\rangle$-dominance leads to $R_{NR} = \sqrt{3}/2$, from Eq. (8). In the $|1(0)\rangle$ case, the ratio between the nonresonant $\pi^0\pi^0\pi^0$ amplitude and the sum of the $f\pi^0$ and nonresonant $\pi^+\pi^-\pi^0$ amplitudes should be $1/\sqrt{2}$. We note that the ratio $R_{NR} = \sqrt{1.556 \pm 0.012}$ is observed in $K_L$ decays to three pions, where the nonresonant contribution accounts for over
95% of the branching fractions. The same situation exists in the decay $\eta \to \pi^+\pi^-\pi^0$. This strengthens the justification of our choice to identify the nonresonant contribution with the $|1(S)\rangle$ state. In any case, the arguments given here demonstrate that a measurement of the branching fraction $\mathcal{B}(D^0 \to \pi^0\pi^0\pi^0)$ and, possibly, an analysis of this mode’s Dalitz-plot distribution should shed more light on the role of isospin symmetry in $D^0$ decays to three-pion final states.

Acknowledgments

This research was supported by INFN, Italy; by grant number 2006219 from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel; and by the United States National Science Foundation grant number 0457336. The authors thank Y. Grossman, J. Silva, and L. Wonfenstein for useful suggestions.

[1] The BABAR Collaboration (B. Aubert et al.), Phys. Rev. Lett. 99, 251801 (2007).
[2] C. Zemach, Phys. Rev. 133, B1201 (1964).
[3] Particle Data Group, Y.-M. Yao et al., J. Phys. G 33, 1 (2006).
[4] K. Mishra, Ph.D. Thesis, University of Cincinnati, SLAC-Report-893, 72-77 (2008).
[5] The CLEO Collaboration (P. Rubin et al.), Phys. Rev. Lett. 96, 081802 (2006).
[6] The BABAR Collaboration (B. Aubert et al.), Phys. Rev. D 74, 091102 (2006).