Speed Estimation of Asynchronous Motor Based on Model Reference Adaptive Method and On-line Identification of Rotor Time Constant

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Abstract. In this paper, the principle of model reference adaptive method (MRAS) is analysed and studied. Aiming at the shortcomings of installing speed sensor to collect the speed signal and feedback to the control system, a method for speed estimation and parameter identification of asynchronous motor without speed sensor is presented, And taking the model reference adaptive method (MRAS) as the main line, the Popov super stability theory is used to derive and analyse the adaptive law. Finally, the proposed method is verified by MATLAB software and hardware circuit experiments.

1. Introduction

Nowadays, electric motors play an important role in national production. Whether it is mining or transportation, they are inseparable from motors. Such as coal mining machines, high-speed rail and airplane and so on. This makes it particularly important to improve the performance of the motor. Since the advent of the electric motor, many scientists have been working on the structure of the motor and the control method in order to make the motor perform better in practical applications. The birth of the first DC motor in the 19th century opened the door to motor applications. Due to its good working performance, simple mathematical model, and simple operation, it was active in the industrial field until the middle of the 20th century. In the 1970s, the industrial field put forward higher requirements for the use environment of the motor, single-unit capacity, and voltage level. The DC motor could not be satisfied [1], which promoted the rapid development of the asynchronous motor, making the AC electric drive have a wide and stable speed range. High accuracy, safety and stability, high reliability, easy maintenance and many other advantages. In some industrial and mining enterprises, mining locomotives often use slip frequency control with speed closed loop in order to get better working efficiency and control performance. The detection of speed becomes an indispensable part.

The core of the speed estimation is the value of some parameters of the asynchronous motor. For example, the stator rotor resistance of the motor, self-inductance, mutual inductance. The acquisition
of these parameter values is difficult to obtain directly, so it is often obtained by parameter identification.

The asynchronous motor speed estimation and parameter identification proposed in this paper are based on the Model Reference Adaptive Method (MRAS). Because the model reference adaptive method has the characteristics of simple algorithm and high accuracy compared with the traditional speed estimation and parameter identification method, so it is favored by researchers at home and abroad.

2. The basic principle of model reference adaptive method

The model reference adaptive method can be divided into three structures according to the output connection method. Among them, the output parallel model reference adaptive method (also commonly referred to as output error method) is most common, and the basic structure is shown in Figure 1.

![Figure 1. basic structure.](image)

The reference model is an already determined value, equivalent to a reference value. The adjustable model is related to the parameter to be estimated, the output value is uncertain, and it needs to be adjusted continuously. The outputs of the two models are y(t) and respectively, which have the same physical meaning. The difference between the two outputs is the error signal e, which is fed back to the adjustable model by adaptive law. When the output of the two models is the same, the error signal e tends to 0. At this time, it is considered that the parameter to be estimated converges to the true value, and the identification result is obtained.\(^2\)\(^3\)

It can be seen from Fig. 1 that the design of the adaptive law is related to the whole system, so the design of the adaptive law becomes the focus. The design of the adaptive law in this paper is based on the Popov hyper stability theory. The parameter optimization method was the first method used to design the MRAS system. The MIT method has a simple structure and does not need to set state variables. However, the MIT solution cannot guarantee the stability of the system, therefore, the stability check must be performed at the end. Currently, there are few applications. So the Popov method is used in this paper.

3. MRAS method based on rotor flux linkage

Analytical derivation of reference models, tunable models, and adaptive laws \(^4\).

Equation (1) is the voltage equation of the asynchronous motor in the two-phase stationary \(\alpha-\beta\) coordinate system:

\[
\begin{bmatrix}
    u_{\alpha a} \\
    u_{\beta a} \\
    u_{\alpha r} \\
    u_{\beta r}
\end{bmatrix}
=
\begin{bmatrix}
    R_s + L_s p & 0 & L_m p & 0 \\
    0 & R_r + L_r p & 0 & L_m p \\
    L_m p & \omega_r L_m & R_r + L_r p & \omega_r L_r \\
    -\omega_r L_m & L_m p & -\omega_r L_r & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
    i_{\alpha a} \\
    i_{\beta a} \\
    i_{\alpha r} \\
    i_{\beta r}
\end{bmatrix}
\]  \hspace{1cm} (1)

The flux linkage equation is:
\[
\begin{bmatrix}
\psi_{sa} \\
\psi_{sb} \\
\psi_{ra} \\
\psi_{rb}
\end{bmatrix} = \begin{bmatrix}
L_s & 0 & L_m & 0 \\
0 & L_s & 0 & L_m \\
L_m & 0 & L_r & 0 \\
0 & L_m & 0 & L_r
\end{bmatrix} \begin{bmatrix}
i_{sa} \\
i_{sb} \\
i_{ra} \\
i_{rb}
\end{bmatrix}
\] (2)

From \(u_{sa}, u_{sb}\) of formula (1) and \(\psi_{ra}, \psi_{rb}\) of formula (2):

\[
\begin{align*}
\psi_{ra} &= \frac{L_r}{L_m} \left[ \int (u_{sa} - R_s i_{sa}) dt - \sigma L_s i_{sa} \right] \\
\psi_{rb} &= \frac{L_r}{L_m} \left[ \int (u_{sb} - R_s i_{sb}) dt - \sigma L_s i_{sb} \right]
\end{align*}
\] (3)

In the formula, \(\sigma\) is the magnetic flux leakage coefficient, and \(\sigma = 1 - L_s / (L_m L_r)\). Equation (3) describes the relationship between the rotor flux change rate and the induced electromotive force, called the voltage model.

The rotor side of the asynchronous motor forms a closed loop, which can be regarded as a short circuit.

Let \(u_{ra} = u_{rb} = 0\), from \(u_{ra}, u_{rb}\) of equation (1) and \(\psi_{ra}, \psi_{rb}\) of equation (2) of flux linkage we can obtain:

\[
\begin{align*}
p\psi_{ra} &= -\frac{1}{T_r} \psi_{ra} - \omega_r \psi_{rb} + \frac{L_m}{T_r} i_{sa} \\
p\psi_{rb} &= -\frac{1}{T_r} \psi_{rb} + \omega_r \psi_{ra} + \frac{L_m}{T_r} i_{sb}
\end{align*}
\] (4)

Equation (4) represents the functional relationship between \(\psi_{ra}, \psi_{rb}\) and \(i_{sa}, i_{sb}\), which is the current model.

Substituting the updated value of the correlation amount in the formula (4) into the following:

\[
\begin{align*}
p\hat{\psi}_{ra} &= -\frac{1}{T_r} \hat{\psi}_{ra} - \hat{\omega}_r \hat{\psi}_{rb} + \frac{L_m}{T_r} i_{sa} \\
p\hat{\psi}_{rb} &= -\frac{1}{T_r} \hat{\psi}_{rb} + \hat{\omega}_r \hat{\psi}_{ra} + \frac{L_m}{T_r} i_{sb}
\end{align*}
\] (5)

Where \(\hat{\omega}_r\) is the speed estimate, \(\hat{\psi}_{ra}\) and \(\hat{\psi}_{rb}\) are the rotor flux estimates.

The error formula for defining the rotor flux linkage is:

\[
\begin{bmatrix}
e_{ra} \\
e_{rb}
\end{bmatrix} = \begin{bmatrix}
\psi_{ra} \\
\psi_{rb}
\end{bmatrix} - \begin{bmatrix}
\hat{\psi}_{ra} \\
\hat{\psi}_{rb}
\end{bmatrix}
\] (6)

Substituting \(p\psi_{ra}, p\psi_{rb}, p\hat{\psi}_{ra}, p\hat{\psi}_{rb}\) and respectively into the rotor flux error formula can be obtained (7):

\[
\dot{e} = p \begin{bmatrix}
e_{ra} \\
e_{rb}
\end{bmatrix} = Ae - W_\omega
\] (7)

It is obtained by Popov's hyper stability theory, \(V = Ge\), where the value of \(G\) is related to system stability. In order to simplify the calculation, take \(G = I\), then \(V = e\)

\[
\hat{\omega}_r = \int_0^t f_1 (e, t, \tau) d\tau + f_2 (e, t)
\] (8)

Substituting \(V\), \(W_\omega\) and \(\hat{\omega}_r\) into Popov integral inequality and decomposing:
\[
\begin{align*}
\eta_1(0, t_0) &\geq -\gamma_1^2 \\
\eta_2(0, t_0) &\geq -\gamma_2^2
\end{align*}
\]

Where \(\gamma_1^2\) and \(\gamma_2^2\) are finite positive numbers with the same properties as \(\gamma^2\).

When \(k_i > 0\), inequality (10) is always established.

\[
\int_0^t k_i x(t) x(t) dt = \frac{k_i}{2} [x^2(t_0) - x^2(0)] \geq -\frac{k_i}{2} x^2(0)
\]

For \(\eta_1(0, t_0)\), the value is as follows:

\[
\gamma^2 = \frac{k_i}{2} x^2(0)
\]

\[
\dot{x}(t) = e_{rp} \dot{\psi}_{ra} - e_{ra} \dot{\psi}_{rp}
\]

\[
k_j x(t) = \int_0^t f_1(e, t, \tau) d\tau - \omega_r
\]

Then \(\eta_1(0, t_0) \geq -\gamma_1^2\) can be guaranteed. Calculated by equations (12) and (13) we can get:

\[
f_1(e, t, \tau) = k_j (e_{rp} \dot{\psi}_{ra} - e_{ra} \dot{\psi}_{rp})
\]

For \(\eta_2(0, t_0)\), if the integrand is greater than zero, then inequality \(\eta_2(0, t_0) \geq -\gamma_2^2\) must be true, so take:

\[
f_2(e, t) = k_p (e_{rp} \dot{\psi}_{ra} - e_{ra} \dot{\psi}_{rp})
\]

Substituting \(f_1(e, t, \tau)\) and \(f_2(e, t)\) into equation (8), the rotating speed adaptation law is

\[
\dot{\omega}_r = k_j \left[ (\psi_{rp} \dot{\psi}_{ra} - \psi_{ra} \dot{\psi}_{rp}) \right] + k_p \left[ (\psi_{rp} \dot{\psi}_{ra} - \psi_{ra} \dot{\psi}_{rp}) \right]
\]

Switch to the S domain, which is:

\[
\dot{\omega}_r = \left( k_p + \frac{k_j}{s} \right) (\psi_{rp} \dot{\psi}_{ra} - \psi_{ra} \dot{\psi}_{rp})
\]

4. On-line identification of rotor time constant

The online identification of the rotor time constant is based on the MRAS method, and the commonality of the rotational speed adaptive law derivation is that the reference model and the adjustable model are the same. While the difference is that the two adaptive laws are different.

Error equation of state related to the rotor time constant:

\[
\dot{e} = \begin{bmatrix} e_{\alpha} \\ e_{\beta} \end{bmatrix}
\]

\[
= \begin{bmatrix} \frac{1}{T_r} - \omega_r \\ \omega_r - \frac{1}{T_r} \end{bmatrix} e_{\alpha} - \begin{bmatrix} \frac{1}{T_r} & 0 \\ 0 & \frac{1}{T_r} \end{bmatrix} \begin{bmatrix} \dot{\psi}_{ra} \\ \dot{\psi}_{gb} \end{bmatrix} - \frac{1}{T_r} \begin{bmatrix} l_{p} l_{ra} \\ l_{r} l_{gb} \end{bmatrix}
\]

\[
= \Lambda e - x_r
\]

If the nonlinear time-varying link satisfies the Popov inequality, the system will approach stability. For the sake of convenience, directly ask for the adaptive law of \(\frac{1}{T_r}\):
\[
\frac{1}{T_r} = \int f_1(e, t, \tau) d\tau + f_2(e, t)
\] (19)

Substituting \(V = e, \omega_r\) and \(\frac{1}{T_r}\) into the Popov integral inequality and decomposing, calculated:

\[
f_1(e, t, \tau) = f_1^0(e, t)
\] (20)

Substituting \(f_1(e, t, \tau)\) and \(f_2(e, t)\) into \(\frac{1}{T_r}\) and switching to the S domain, will get the following formula:

\[
\frac{1}{T_r} = \left(k_p + \frac{k_i}{s}\right) \left[ (L_a i_{sa} - \dot{\psi}_{r\alpha}) (\psi_{r\alpha} - \dot{\psi}_{r\alpha}) + (L_a i_{sb} - \dot{\psi}_{r\beta}) (\psi_{r\beta} - \dot{\psi}_{r\beta}) \right]
\] (21)

5. Simulation and experimental verification

In order to verify the feasibility and accuracy of the method proposed for motor speed estimation and parameter identification, a simulation model is built on the basis of MATLAB software. System simulation diagram shown in Figure 2.

![System simulation diagram](image)

Figure 2. system simulation diagram.

Asynchronous motor was used for the simulation, and the relevant parameters are as follows:

| Parameter                  | Value       |
|----------------------------|-------------|
| Rated power \(P_N\)        | 1.1KW       |
| Stator resistance \(R_s\)  | 5.27 \(\Omega\) |
| Rated speed \(n_N\)        | 2850 r/min  |
| Rotor resistance \(R_r\)   | 5.07 \(\Omega\) |
| Rated current \(I_N\)      | 2.5A        |
| Stator leakage inductance \(L_{ls}\) | 0.0304H |
| Rated voltage \(U_N\)      | 380V        |
| Rotor leakage inductance \(L_{lr}\) | 0.0298H |
| Rated frequency \(f_N\)    | 50Hz        |
| Mutual sense \(L_m\)       | 0.394H      |
| Number of pole pairs \(P\) | 1           |
| Moment of inertia \(J\)    | 0.02 kg·m\(^2\) |
5.1. MRAS method based on rotor flux linkage

In the simulation, according to the change trend of the whole speed under different values of kp and ki, as shown in Fig. 3, the PI parameter value is determined after multiple adjustments: kp=100, ki=22000, and the simulation results under no load are shown in Fig. 4. It can be seen that the estimated rotational speed waveform and the actual waveform are basically coincide, and the rotational speed of the motor is relatively large when the motor is started, when the motor is fully started, the error is basically zero.

Figure 3. Comparison of estimated waveforms with actual waveforms in three different cases.

Figure 4. No-load speed waveform.

5.2. Rotor time constant online identification simulation
The simulation module constructed according to the derived rotor time constant adaptive law[6] is shown in Fig. 5.

![Simulation Diagram](image)

**Figure 5.** Rotor time constant adaptive law module simulation diagram.

Figure 6 shows the simulation results of online identification. Similarly, take different values for kp and ki, observe the trend, and adjust repeatedly to get kp=0.7, ki=39. The reciprocal of the rotor time constant calculated according to Equation 1/Tr= Rr/Lr is 11.958 s. It can be clearly seen from Fig. 6(a) that 1/Tr can be stabilized to the actual set value in 1s~2s, and the convergence speed is fast. It can be seen from Fig. 6(b) that the difference between the actual value and the estimated value is small, which proves that the online identification result is in good agreement with the actual one.

![Waveform](image)

**Figure a.** 1/Tr online identification waveform.

![Waveform](image)

**Figure b.** 1/Tr online identification of steady-state amplified waveform.

**Figure 6.** A online identification results.

Based on the above theoretical basis and simulation analysis, a related hardware experiment platform was built. The motor parameters used in the test platform are shown in Table 1. The motor is started under no-load conditions. The voltage and current waveforms are shown in Figure 7. The three curves represent the voltage rms, the current rms, and the current instantaneous value.
Figure 7. No-load starting voltage and current waveform. Figure 8 shows the comparison between the actual speed and the estimated speed waveform. The actual speed of the asynchronous motor is indicated by the blue line, the actual speed is collected by the installation speed sensor and obtained after conversion; the red line indicates the estimated speed.

Figure 8. Comparison of actual speed and estimated speed waveform.

6. Conclusion
Based on the model reference adaptive method, this paper deeply studies the method of asynchronous motor speed estimation and parameter identification. The adaptive laws of rotational speed and rotor time constant are deduced respectively, and the relevant simulation models are built. Compared with the actual values, the estimated values of rotational speed and rotor time constant are more accurate, and the feasibility of the method is verified. Good results have also been achieved on the hardware experimental platform, providing a basis for practical application in the future.

Acknowledgments
At the completion of this thesis, I would like to express my sincere gratitude and most sincere respect to the instructor, Teacher Tong Jun. The unswerving scientific spirit of the teacher, the profound knowledge, the work attitude of the facts, the generous and modest spirit, and the spirit of selfless dedication have always inspired me to constantly learn knowledge and pursue the realm of life. Inexhaustible, there are too many people who have helped me, sincerely thank all those who care and help me, thank them for giving me the power to overcome myself, transcend myself, and constantly pursue, thank you.

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