Magnetoresistance of composite fermions at \( \nu = 1/2 \)

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Abstract

We have studied temperature dependence of both diagonal and Hall resistivity in the vicinity of \( \nu = 1/2 \). Magnetoresistance was found to be positive and almost independent of temperature: temperature enters resistivity as a logarithmic correction. At the same time, no measurable corrections to the Hall resistivity has been found. Neither of these results can be explained within the mean-field theory of composite fermions by an analogy with conventional low-field interaction theory. There is an indication that interactions of composite fermions with fluctuations of the gauge field may reconcile the theory and experiment.

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Experimentally, it has been known for some time that in low disorder two-dimensional electron systems (2DES) at filling factor $\nu = 1/2$ the diagonal resistivity $\rho_{xx}$ remains finite at low temperatures and exhibits a shallow minimum, while the Hall resistivity $\rho_{xy}$ is nearly linear in magnetic field and does not form a plateau. An understanding of the phenomenon came with the theory of composite fermions (CFs), where weakly interacting new particles – composite fermions – were proposed to form a metallic Fermi liquid-like state near $\nu = 1/2$. In the mean-field approximation CFs experience a reduced effective magnetic field $B_{cf} = B - 2n\phi_0$, where $n$ is the electron (and CF) concentration, and $\phi_0 = h/e$ is the flux quantum. At $\nu = 1/2$ the external magnetic field is fully cancelled and $B_{cf} = 0$; it has been shown experimentally that some properties of a Fermi liquid are preserved for CFs, in particular, a reasonably well defined Fermi surface.

Despite some similarity between $\nu = 1/2$ and $B = 0$ phenomenology, there are apparent differences in transport properties. For example, magnetoresistance is negative near $B = 0$, while it is positive near $\nu = 1/2$. Magnetoresistance at low $B$ has been a powerful tool in the study of weak localization and electron interaction effects. This method relies on the prediction of the classical Drude model that $\rho_{xx}$ is not affected by magnetic field, while $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$ displays negative magnetoconductance via $\rho_{xy} \propto B$. Any magnetoresistance then results from quantum corrections to the conductivity tensor, which, in general, have different $B$- and $T$-dependence than Drude $\sigma^0_{xx}$ and $\sigma^0_{xy}$ and, thus, can be separated. Altshuler–Aronov quantum correction to conductivity $\Delta \sigma^{AA}_{xx}$ due to interaction effects has a logarithmic temperature dependence and is field independent at low $B$ because the correction to Hall conductivity $\Delta \sigma^{AA}_{xy} = 0$. Neglecting the weak localization contribution, for electrons at low $B$ the resulting quantum magnetoresistance $\Delta \rho_q = \rho_{xx}(B) - \rho_{xx}(0) \approx \rho_{xy}^2 \Delta \sigma^{AA}_{xx}$ (for $\Delta \sigma^{AA}_{xx} \ll \sigma^0_{xx}$) is negative, because $\Delta \sigma^{AA}_{xx} < 0$.

We have reported recently observation of a logarithmic correction to the conductivity of CFs $\sigma^{cf}_{xx}$ at $\nu = 1/2$ and attributed it to the short-range interaction between CFs. An enhancement of the coupling constant, compared to the low-field regime, was found recently to be a result of an interaction between CFs via the gauge field fluctuations. Naively, one
may also expect that this effect should lead to a negative magnetoresistance, in analogy to the low-$B$ case. However, experimentally positive magnetoresistance and no correction to the Hall resistivity are measured near $\nu = 1/2$. Thus, non–zero correction $\Delta \sigma_{xy}^{cf} \neq 0$, in addition to $\Delta \sigma_{xx}^{cf} \neq 0$, both $B$-dependent, is required to reconcile measured corrections to $\rho_{xx}$ and $\rho_{xy}$ with the constrains imposed by the matrix inversion of transport coefficients.

We have studied several samples fabricated from high mobility ($\mu \approx 2 \times 10^6$ cm$^2$/Vs) GaAs/Al$_x$Ga$_{1-x}$As heterojunction wafers. The wafers have double Si $\delta$-doping, the first layer is separated from the 2DES by a $d_s = 120$ nm thick spacer. 2DES with densities $0.4$ and $1.2 \times 10^{11}$ cm$^{-2}$ were prepared by illuminating a sample with red light. The temperature was measured with a calibrated Ruthenium Oxide chip resistor. Measurements were done in a top-loading into a mixture dilution refrigerator using a standard lock-in technique. Samples were patterned in either Corbino or Hall bar geometry.

Representative magnetoresistivity data $\rho_{xx}(B_{cf}, T)$ near $\nu = 1/2$ are plotted in Fig. (a) (note that $\rho_{xx}^{cf} = \rho_{xx}$). Magnetoresistance is positive near $\nu = 1/2$ and depends on temperature weakly. A remarkable result is that $\rho_{xx}$ at a given $B_{cf}$ changes logarithmically with temperature for $13$ mK $< T < 1000$ mK. A simple function

$$\frac{[\rho_{xx}(B_{cf}, T) - \rho_{xx}(B_{cf}, T_1)]}{\ln(T_1/T)}$$

collapses $\rho_{xx}$ vs $B_{cf}$ traces at different temperatures $T$ into a single curve [Fig. (b)]. Such a scaling requires that both, the $B_{cf} = 0$ part of resistivity $\rho_{xx}(0, T)$, and the part responsible for the magnetoresistance, have terms proportional to $\log T$. We fit the data with a polynomial

$$\rho_{xx}(B_{cf}, T) = \rho_{xx}(0, T) + \alpha(T)B_{cf} + \beta(T)B_{cf}^2$$

(1)

dashed lines in Fig. a) in a classically weak-field region for CFs $\rho_{xy}^{cf} \leq \rho_{xx}^{cf}$ (corresponds to $|B_{cf}| \leq 0.12$ T for the sample in Fig. ). Value of $\alpha \neq 0$ corresponds to a known term in $\rho_{xy}$ proportional to $B \frac{dp_{xy}}{dT}$. As it is expected from the above analysis, both $\rho_{xx}(0, T)$ and $\beta(T)$ change logarithmically with temperature (Fig. ). Zero–field CF conductivity
$\sigma_{xx}^{cf}(0, T) = 1/\rho_{xx}(0, T)$ has a negative logarithmic $T$-dependent correction, which has been attributed to interaction effects between CFs, analogous to the Altshuler-Aronov type localization correction for electrons at low magnetic field\textsuperscript{2}. However, as apparent from Fig., there is a positive magnetoresistance near $B_{cf} = 0$, in a stark contrast to the negative magnetoresistance near $B = 0$.

In contrast to the low-field regime, we have found no deviation of Hall resistivity $\rho_{xy}$ from its free-electron value $\rho_{xy}^0 = B/en$ near $\nu = 1/2$ (electron concentration $n$ is determined from Shubnikov-de Haas oscillations with 2% accuracy). A direct comparison of $\rho_{xy}$ at 35 and 560 mK shows (Fig.) that there is no $T$-dependent correction to $\rho_{xy}$ within experimental error of 0.1% in the range $|\omega_{c}^{cf}\tau| < 3$. This value should be contrasted with $\approx 15\%$ change of $\rho_{xx}$. Thus, we conclude that $\Delta \rho_{xy} = 0$ near $\nu = 1/2$.

Within the mean field theory transport properties of 2DES near $\nu = 1/2$ closely resemble those near $B = 0$. Let us examine mechanisms which may lead to the positive magnetoresistance within the $\{\nu = 1/2\} \leftrightarrow \{B = 0\}$ analogy. At low $B$, there are no corrections to $\rho_{xy}$ due to weak localization\textsuperscript{1}. Near $\nu = 1/2$, the disorder–induced fluctuations of electron density $\delta n$ produce static fluctuations of the gauge field $\delta B_{cf} = 2\delta n\phi_0$, and the first–order correction to $\rho_{xx}$ is suppressed\textsuperscript{2}. The second–order correction is $\sim 100$ times less than the measured logarithmic term in $\rho_{xx}(0, T)$\textsuperscript{23}. Also, static fluctuations of the gauge field would suppress quantum interference at $\omega_{c}^{cf}\tau \approx 1$, although the positive magnetoresistance is observed to much higher effective magnetic fields.

Another possible source for positive magnetoresistance is a classical correction to the Drude resistivity $\rho_{xx}^0$, which results from the fact that an average size of potential fluctuations is larger than the Fermi wavelength. Simple arguments\textsuperscript{27} lead to the following positive quadratic in $B_{cf}$ correction to $\rho_{xx}^0$: 

$$
\Delta \rho_{cl} \propto \rho_{xx}^0 \left(\frac{d_s}{r_c}\right)^2,
$$

(2)

where $d_s$ is the spacer thickness and $r_c = \frac{\hbar v_F}{eB_{cf}}$ is the cyclotron radius. Recent experiments\textsuperscript{18} show that, in the presence of a spatially non-uniform magnetic field, a positive magnetoresis-
tance is observed in 2DES at low magnetic fields. However, the classical magnetoresistance has been calculated for \( T = 0 \) and thus does not have any temperature dependence. We do not expect appreciable temperature dependence for this scattering mechanism, at least for \( T < 0.5 \) K, when phonon scattering is negligible, inconsistent with the observed \( \log T \) dependence of resistivity. Thus, the classical correction alone cannot explain the experimental results.

The logarithmic temperature dependence of \( \beta(T) \) strongly suggests that the positive quadratic magnetoresistance originates from the interaction effects between CF’s. This conclusion is further supported by the observation that both \( \rho_{xx}(0,T) \) and \( \beta(T) \) deviate from \( \log T \) dependence at about the same \( T \). However, matrix inversion of transport coefficients, combined with Onsager relations and experimental observations that (i) \( \Delta \rho_{xy}/\rho_{xx}^0 \ll \Delta \rho_{xx}/\rho_{xx}^0 \) (Fig. ), and (ii) both \( \rho_{xx} \) and \( \rho_{xy} \) are non-singular near \( \nu = 1/2 \), impose certain constraints on the corrections to the Drude conductivity tensor. Assuming that both corrections are small (\( \Delta \sigma_{xx} \ll \sigma_{xx} \) and \( \Delta \sigma_{xy} \ll \sigma_{xy} \)) they can be expressed in the following form:

\[
\Delta \sigma_{xx}^{cf}(B_{cf}, T) \approx f(\gamma)(1 - \gamma^2) \Delta \sigma_{xx}^{cf}(0, T) \tag{3a}
\]

\[
\Delta \sigma_{xy}^{cf}(B_{cf}, T) \approx 2\gamma f(\gamma) \Delta \sigma_{xx}^{cf}(0, T) \tag{3b}
\]

where \( \gamma = \rho_{xy}^{cf}/\rho_{xx}^0 \propto B_{cf} \) (\( \gamma = \omega_{cf}^{cf} \tau \) in the Drude model), \( f(\gamma) \) is an even smooth function of \( B_{cf} \) and \( f(0) = 1 \). Note, that the \( B \)– and \( T \)–dependencies are separated, and \( T \) enters only through the zero-field correction to diagonal conductivity \( \Delta \sigma_{xx}(0, T) \). Indeed, experimentally determined \( \Delta \sigma_{xx} \) and \( \Delta \sigma_{xy} \) are both \( B \)-dependent and \( \Delta \sigma_{xx} \) changes sign at \( \rho_{xx} \approx \rho_{xy}^{cf} \) (Fig. ).

All these findings contradict the results of the conventional low-field interaction theory, which predicts \( \Delta \sigma_{xy} = 0 \) and a field independent \( \Delta \sigma_{xx} \). A recent theory investigated interaction effects between CFs in the presence of disorder beyond the mean-field approximation. The calculated corrections are in agreement with the above qualitative analysis [Eqs. (3)] with \( f(\gamma) \equiv 1 \) and \( \gamma \equiv \omega_{cf}^{cf} \tau \). These corrections to conductivity lead to
the following corrections to the resistivity tensor:

\[
\Delta \rho_{xx}(B_{cf}, T) \approx \Delta \rho_{xx}(0, T)[1 + (\omega_{cf}^2 \tau)^2] \tag{4a}
\]

\[
\Delta \rho_{xy}(B_{cf}, T) \approx -\rho_{0}^{xy}[\Delta \rho_{xx}(0, T)/\rho_{0}^{xx}]^2[1 + (\omega_{cf}^2 \tau)^2] \tag{4b}
\]

Qualitatively, Eqs. (4) predict a positive magnetoresistance and a vanishing term linear in \( \Delta \rho_{xy} \). However, thus calculated \( \Delta \rho_{xx} \) overestimates \( \beta \) from Eq. (3) by a factor of 20, if we use \( \omega_{cf}^2 \tau = \rho_{xy}^c / \rho_{0}^{xx} \), with \( \rho_{0}^{xx} = 0.65 \text{ k\Omega} \). Also, a large quadratic correction to the Hall resistivity, \( \Delta \rho_{xy}/\rho_{0}^{xy} > 2.5\% \), estimated from Eq. (3b), is inconsistent with experiment (< 0.1\%, see Fig. ).

Our main results can be summarized as follows: (i) experimentally, the resistivity has a logarithmic temperature dependence near \( \nu = 1/2 \), which implies that both \( B \)-independent resistivity and magnetoresistance have \( \log T \) dependence, and (ii) there is no measurable correction to the classical Hall resistivity near \( \nu = 1/2 \). From analysis of possible mechanisms which may lead to a positive magnetoresistance, we conclude that the observed \( T \)-dependencies cannot be explained within the mean-field theory of CFs. The similar \( \log T \) dependence of resistivity at \( \nu = 1/2 \) and of magnetoresistance suggests that both corrections have the same physical origin, namely, interactions between CFs.

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14. Drude field-independent resistivity is obtained under the assumption that a scattering event is unaffected by magnetic field. This assumption is justified for a short range scattering, i.e., if the size of the scatterer is less than the inverse Fermi wavevector $1/k_F$. In high mobility 2DES the relevant spatial scale is determined by the spacer thickness $d_s \gg 1/k_F$. 

7
During the time $t_{sc} \approx d_s/v_F$, which takes for an electron to traverse a scatterer of the size $d_s$, its trajectory is curved by magnetic field and the electron spends longer time within the potential of the scatterer compared to the motion along a straight line. Simple geometrical arguments give an estimate of the increase of $t_{sc}$ as $(\Delta t_{sc})/t_{sc} \approx (d_s/r_c)^2$, $r_c$ is the cyclotron radius, which leads to a corresponding increase of the transport cross section. As a result, resistivity acquires a positive correction quadratic in the magnetic field Eq. (2). The correction can be obtained from the solution of the Boltzmann equation in a random magnetic field\(^3\): the result agrees with the estimate Eq. (2) with a numerical coefficient $\approx 0.1$. Calculations reproduce magnitudes of both resistivity and magnetoresistance within a factor of 3 for studied samples.

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FIGURES

FIG. 1. a) Magnetoresistivity data $\rho_{xx}$ vs $B_{cf}$ near $\nu = 1/2$ for $T = 13, 77, 260$ and 810 mK (from top to bottom). Dashed lines are polynomial fits Eq. (1) in the range $|B_{cf}| < 0.12$ Tesla. Resistivity in a larger field range is shown in the inset. b) The scaling of the difference between $\rho_{xx}$ at 13 mK and other temperatures, normalized by the log of the ratio of temperatures.

FIG. 2. The $\nu = 1/2$ resistivity $\rho_{xx}(0,T)$ plotted as a function of temperature. The coefficient $\beta$, defined in Eq. (1), is obtained from the fits in Fig. 1.

FIG. 3. Relative change of $\rho_{xx}$ and $\rho_{xy}$ with temperature. Note that the change in $\rho_{xy}$ is multiplied by a factor of 5.

FIG. 4. Deviation of $\sigma_{xx}^{cf}$ from the Drude value is shown near $\nu = 1/2$ for $T = 13, 77, 260$ and 810 mK. Note the change of sign of $\Delta \sigma_{xx}^{cf}$ at $\rho_{xx}^{cf} = \rho_{xy}^{cf}$. 
FIG. 4.
FIG. 3.
FIG. 2.
FIG. 1.