TASI lectures: weak scale supersymmetry
— a top-motivated-bottom-up approach

Gordon L. Kane
Michigan Center for Theoretical Physics, Randall Physics Lab,
University of Michigan
E-mail: gkane@umich.edu

CONTENTS

◦ Introduction, Perspective – since particle physics beyond the SM is presently in an incoherent state, with lots of static, a long introduction is needed, including some history of the supersymmetry revolutions, physics not described by the SM, indirect evidence for low energy supersymmetry and how flavor physics should be approached.

◦ Derive the supersymmetric Lagrangian – the superpotential $W$

◦ Soft supersymmetry breaking – underlying physics – $L_{soft}$ – the MSSM

◦ The $\mu$ opportunity – R-parity conservation

◦ Count of parameters – constraints – measuring the parameters

◦ Connecting the weak and unification scale

◦ Derivation of the Higgs mechanism – in what sense does supersymmetry explain the Higgs physics

◦ The Higgs spectrum – $\tan \beta$, Yukawa couplings, constraints

◦ LEP Higgs physics – Tevatron Higgs physics can confirm the Higgs mechanism and coupling proportional to mass – Higgs sector measurements

◦ $\tilde{g}$, $\tilde{N}$, $\tilde{C}$ – cannot in general measure $\tan \beta$ at hadron colliders

◦ Effects of soft phases – all observables, not only CPV ones, $g_\mu - 2$, EDMs, $\tilde{g}$ phase, LSP CDM, possible connections to stringy physics

◦ Phase structure of simple D-brane models

◦ Tevatron superpartner searches, signatures

◦ Extensions of the MSSM

◦ The importance of low scale supersymmetry is not only that we learn of another profound aspect of our world, but also to provide a window to Planck scale physics, in order to connect string theory and our world
1 Introduction

For about 400 years we have improved our understanding of the physical world until we discovered and tested the Standard Model (SM) of particle physics, which provides a complete description of our world, of all that we see. We know that the basic constituents are quarks and leptons, and we have a complete theory of the strong, weak, and electromagnetic forces.

We also know that much is unexplained, such as what is the cold dark matter (CDM) of the universe, and why is the universe made of matter rather than being an equal mixture of matter and antimatter, neither of which can be explained by the SM. Below I will make a longer list of questions the SM cannot answer. And there are conceptual reasons also why we expect to find new physics beyond the SM.

While a number of approaches to physics beyond the SM have been worth considering, only one so far has actually explained and predicted phenomena beyond the SM, the supersymmetric extension of the SM, which will be the focus of these lectures. As we will see, the supersymmetric SM has a number of successes, and as yet no failures. It is not yet a complete theory in the sense that we do not yet understand fully the physics of all of its parameters, but it is a complete effective theory because we can write the full effective Lagrangian of the theory. One of its important successes is that it can be a valid theory to very high energy scales or very short distances, near to the Planck scale. Another is that it is not sensitive to new physics at some high scale.

Superstring theories have become very attractive in recent years as well. They are formulated near the Planck scale with ten dimensions and presumably unbroken supersymmetry. String theories only predict or explain that there is a gravitational force, and that we live in at most 10 dimensions. What is exciting about them from our point of view is that they seem to be able to accommodate the SM forces and quarks and leptons, and possibly explain how these forces and particles originate. So in these lectures we will assume that the basic theory is a string theory at the Planck scale (loosely speaking). We do not distinguish string theory from M-theory for our top-motivated purposes, since the effective low scale theory from both will be parameterized by the same Lagrangian.

Historically, physics has progressed by one basic method, with experiment and theory intermingling as each level of the world was understood. That method will continue to work as supersymmetry is established experi-
perimentally, as its parameters (masses, flavor rotation angles, phases, vacuum expectation values) are measured, and as supersymmetry breaking is studied. But the historical approach can only take us to a broken supersymmetric theory near the unification and string scales. It cannot be used to learn the form of the 10D supersymmetric string theory. There is a barrier that can only be crossed by human imagination. To cross it we must know the Lagrangian of the broken supersymmetric theory near the unification and string scales, and we must understand the 10D supersymmetric string theory very well. Then it will be possible to guess how to jump the barrier. In my view it will not be possible to do that until the broken supersymmetric theory near the Planck scale is known. No amount of thinking will tell us how to compactify the string theory, or to break supersymmetry, or to recognize the vacuum of the theory, because there is no practical way to recognize if one has it right.

Let’s pursue this in a little more detail. Sometimes people argue that calculating fermion masses will be a convincing way to learn when a compactification is correct. But the hierarchy of fermion masses implies that will be very hard to do. The small masses are unlikely to arise at the tree level, but rather depend on non-renormalizable operators and possibly on supersymmetry breaking effects. So perhaps the large masses can be calculated, but not the smaller ones, and if the large ones have Yukawa couplings of order unity that will be common to many theories. It is of course known that huge numbers of manifolds give three families of chiral fermions. A little thought suggests very strongly that most of the usual “string phenomenology” is of a similar nature, and is very unlikely to point toward the correct vacuum. Indeed, suppose some string theorist already knew how to compactify and to break supersymmetry and to find the correct vacuum. How would they convince themselves, or anyone else?

However, the supersymmetry soft-breaking Lagrangian, $L_{\text{soft}}$, may offer more hope for testing theory. The parameters of $L_{\text{soft}}$ are measurable, though little has been known until recently about how to measure most of them, and much of these lectures will be about how to measure them. If a theorist has an approach to compactification and to breaking supersymmetry, then $L_{\text{soft}}$ is likely to be calculable more easily than the full Yukawa matrix in that approach, and thus knowledge of $L_{\text{soft}}$ may test ideas better than knowledge of the fermion masses. The parameters of $L_{\text{soft}}$ may be less sensitive to uncertain higher order corrections (unless the leading term vanishes in which case the one-loop radiative correction is usually not hard to work out). The flavor structure of $L_{\text{soft}}$ depends on the flavor structure of the Yukawas and
may help untangle that. Progress will come from measuring $L_{soft}$ at the weak scale, and extrapolating it to the unification scale. The patterns of the soft parameters may be typical of one approach or another to compactification or supersymmetry breaking or the vacuum structure, so the measured $L_{soft}$ may focus attention toward particular solutions to these problems. Superpartners should be directly detected in the next few years, and once the initial excitement is past we will turn to the challenging and delightful opportunity to untangle the data and measure the Lagrangian.

Supersymmetry is an idea as old as the SM, and it has not been the most fashionable way to describe the real world in recent years. Consequently many students have not become familiar with supersymmetry as a practical theory, nor have they seen the arguments for its validity. Once superpartners are directly observed it will not be necessary to include these arguments, but at the present time there is some static in the messages most students get, so it is worthwhile to include some tables summarizing why classic supersymmetry is the best approach. In these largely pedagogical lectures I will also not focus on extensive referencing, with apologies to many authors. Some references are given to help the reader find the relevant additional papers. Many topics are integrated into these lectures, and most have been worked on by many authors, so I either have to provide extensive referencing or little referencing, and the latter seems reasonable here in a pedagogical treatment. For thorough referencing to the literature before the past three years the chapters in ref.[1] are useful. I will in places follow the approach of Martin in ref. [1], and he has very good referencing; the larger version of his chapter on the web is more valuable than the printed chapter [2].

It is good to recall some of the history of supersymmetry. We can basically split it into five “revolutions”:

**HISTORY OF SUPERSYMMETRY REVOLUTIONS**

- **1$^{st}$** 1970-72 The idea
- **2$^{nd}$** 1974 Supersymmetric relativistic quantum theory
- **3$^{rd}$** 1975 Local supersymmetry, supergravity
- **4$^{th}$** 1979-83 Supersymmetry solves many problems
- **5$^{th}$** 2000-03 Higgs boson and superpartners observed

Next let us consider a list of important questions that the SM does not deal with. Consequently, these can point the way beyond the SM.
2  Physics not described by the Standard Model

- Gravity
- Cosmological Constant
- Dark Energy
- What is (are) the inflation(s)?
- Strong CP problem
- Hierarchy problem
- How is the electroweak symmetry broken (EWSB)?
- Gauge coupling unification
- Matter asymmetry of the universe
- Cold dark matter
- 3 families
- Neutrino masses
- Values of quark and charged lepton masses
- Approximate Yukawa unification of bottom, tau, and perhaps top
- The value of the Higgs boson mass 115 GeV if the LEP signal is confirmed

The SM cannot account for or explain any of these. It can accommodate some of them. Any approach that claims to be making any progress (such as large extra dimension ideas) should be able to deal with some or most of these simultaneously. Where do they lead us? Supersymmetry of the form we are focusing on in these lectures is relevant to most or all of these. (There are additional reported deviations from the SM that could be relevant and arise from superpartner loops (a) the condition for charged current universality, or unitarity of the CKM matrix [3], and (b) the number of neutrinos is slightly less than 3 [4].)

3  The Hierarchy Problem

The hierarchy problem is the SM problem that quantum corrections raise the Higgs boson mass up to the highest mass scale there is. It is a serious problem — as someone said, the quantum corrections are not only infinite, they are large. The high mass scales do not have to couple directly to the Higgs boson; the coupling can be through several loops, as Martin explains
in some detail. All SM masses (W and Z and quarks and charged leptons) are proportional to \( m_h \) so if \( m_h \) is large they are too.

Supersymmetry was not invented or designed to solve this problem (contrary to what is often stated), but it did. If supersymmetry is unbroken then loops with particles cancel loops with their superpartners in general. For broken supersymmetry the effect is proportional to a power of some couplings times the square of the difference of the masses of superpartner pairs, and a log of mass ratios. Any solution of the hierarchy problem must be insensitive to high scales, and to higher order corrections. If an approach is claimed to deal with the hierarchy problem it must explain how the weak and gravitational scales are determined. Later when we discuss EWSB we examine in what sense supersymmetry provides this explanation. Sometimes people make connections between the cosmological constant problem and the Higgs hierarchy problem, but they are not the same because the calculation of the cosmological constant sums over all states, while the calculation of the Higgs mass only sums over states with SM gauge quantum numbers. Another way to think of the supersymmetry solution is that the Higgs doublet becomes a chiral supermultiplet so \( h \) and its superpartner have the same mass, and the fermion masses are not quadratically divergent so its superpartner mass is not quadratically divergent.

4 Gauge Coupling Unification

One of the most important things we have learned from LEP is that the gauge couplings unify at an energy above about \( 10^{16} \) GeV in a world described by a supersymmetric theory, though not in the SM. Further, where they meet points toward a unification with gravity near the Planck scale. Together these make one of the strongest indications of the validity of the view of physics at the foundation of these lectures. Any other view has to claim this unification is a coincidence! The gauge coupling unification implies two important results:

(1) The underlying theory is perturbative up to the unification scale. Sometimes it is said there should be a desert (apart from the superpartners) but that is not so — only that whatever is in that range (such as right handed neutrinos) does not destroy the perturbativity of the theory.

(2) Physics is simpler at or near the unification scale. That need not have happened — nothing in the SM implies such a result.
There is another important clue. The supersymmetric gauge coupling unification misses by about 10%. More precisely, the experimental value of the strong coupling $\alpha_3$ is about 10-15% lower than the value computed by running down theoretically from the point where the SU(2) and U(1) couplings meet. The details are interesting here — the one-loop result is somewhat small because of a cancellation, and the two-loop contribution therefore not negligible. If one only took into account the one-loop effect the theoretical value would be close to the experimental one but the two-loop effect increases the separation. Nature is kind here, on the one hand giving us information about the need for supersymmetric unification, and on the other giving us a further clue about the physics near the unification scale, or about particles that occur in the “desert” and change the running somewhat.

5 (Indirect) Evidence for weak scale supersymmetry

We have described two of the strongest pieces of evidence for weak scale supersymmetry. The third and to some the strongest is that this approach can explain the central problem of how the electroweak symmetry is broken — we will consider that in great detail after we derive the supersymmetric Lagrangian. First we list here additional evidence for weak scale supersymmetry. Sometimes people wrongly imagine that supersymmetry was invented to explain some of what it explains so the approximate date when it was realized that each of these pieces of evidence existed is listed. Of course the theory existed even before it was realized that it solved these problems — it was not invented for any of them. For completeness we include the evidence we have already examined.

1980 — Can stabilize hierarchy of mass scales.
1982 — Provides an explanation for the Higgs mechanism.
1982 — Gauge coupling unification.
1982 — Provides cold dark matter candidate.
1982 — Heavy top quark predicted.
1992 — Can explain the baryon asymmetry of the universe.
1993 — Higgs boson must be light in general supersymmetric theory.
1990 — Realization that either superpartners are light enough to find at LEP, or their effects on precision data must be very small and unlikely
to be observed at LEP/SLC. Supersymmetry effects at lower energies arise only from loops, which explains why the SM works so well even though it is incomplete.

1982/1995 — Starting from a high scale with a value for $\sin^2 \theta_W$ of 3/8, which arises in any theory with a unified gauge group that contains SU(5), and also in a variety of string-based theories, the value for $\sin^2 \theta_W$ at the weak scale is 0.2315 and agrees very accurately with the measured value.

Some of these successes are explanations, and some are correct predictions. It is also very important that all of them are simultaneously achieved — often efforts to deal with the real world can apparently work for one effect, but cannot describe the range of phenomena we know.

There are theoretical motivations for low energy supersymmetry too. It is the last four-dimensional space-time symmetry not yet known to be realized in some way in nature, it adds a fermionic or quantum structure to space-time, it allows theories to be extrapolated to near the Planck scale where they can be related to gravity, local supersymmetry is supergravity which suggests a connection of the supersymmetric SM to gravity, it allows many problems in string theory and string field theory to be solved, including stabilizing the string vacuum. It is expected, though not yet demonstrated, that low energy supersymmetry is implied by string theory. Not all of these necessarily require low energy supersymmetry. In any case, improving the theory is nice but is not strong motivation for something to exist in nature, so we have emphasized the evidence that actually depends on data.

6 Current limits on superpartner masses

The general limits from direct experiments that could produce superpartners are not very strong. They are also all model dependent, sometimes a little and sometimes very much. Limits from LEP on charged superpartners are near the kinematic limits except for certain models, unless there is close degeneracy of the charged sparticle and the LSP, in which case the decay products are very soft and hard to observe, giving weaker limits. So in most cases charginos and charged sleptons have limits of about 95 GeV. Gluinos and squarks have typical limits of about 250 GeV, except that if one or two squarks are lighter the limits on them are much weaker. For stops and sbottoms the limits are about 85 GeV separately.
There are no general limits on neutralinos, though sometimes such limits are quoted. It is clear no general limits exist — suppose the LSP was pure photino. Then it could not be produced at LEP through a $Z$ which does not couple to photinos, and suppose selectrons were very heavy so it’s production via selectron exchange is very small in pair or associated production. Then no cross section at LEP is large enough to set limits. There are no general relations between neutralino masses and chargino or gluino masses, so limits on the latter do not imply limits on neutralinos. In typical models the limits are $M_{\text{LSP}} \gtrsim 40$ GeV, $M_{\tilde{N}_2} \gtrsim 85$ GeV. Superpartners get mass from both the Higgs mechanism and from supersymmetry breaking, so one would expect them to typically be heavier than SM particles. All SM particles would be massless without the Higgs mechanism, but superpartners would not. Many of the quark and lepton masses are small presumably because they do not get mass from Yukawa couplings of order unity in the superpotential, so one would expect naively that the normal mass scale for the Higgs mechanism was of order the $Z$ or top masses. In models chargino and neutralino masses are often of order $Z$ and top masses, with the colored gluino mass a few times the $Z$ mass.

There are no firm indirect limits on superpartner masses. If the $g_\mu - 2$ deviation from the SM persists as the data and theory improve the first such upper limits will be deduced. If in fact supersymmetry explains all that we argue above it is explaining, particularly the EWSB, then there are rather light upper limits on superpartner masses, but they are not easily made precise. Basically, what is happening is that EWSB produces the $Z$ mass in terms of soft-breaking masses, so if the soft-breaking masses are too large such an explanation does not make sense. The soft parameters that are most sensitive to this issue are $M_3$ (basically the gluino mass) and $\mu$ which strongly affects the chargino and neutralino masses. Qualitatively one therefore expects rather light gluino, chargino, and neutralino masses. If one takes this argument seriously one is led to expect $M_3 \lesssim 500$ GeV; $M_{\tilde{N}_2}, M_{\tilde{C}} \lesssim 250$ GeV; and $M_{\tilde{N}_1} \lesssim 100$ GeV. These are upper limits, seldom saturated in models. There are no associated limits on sfermions. They suggest that these gaugino states should be produced in significant quantities at the Tevatron in the next few years.

There are some other clues that some superpartners may be light. If the baryon number is generated at the EW phase transition then the lighter stop and charginos should be lighter than the top. If the LSP is indeed the cold dark matter, then at least one scalar fermion is probably light enough
to allow enough annihilation of relic LSPs, but there are loopholes to this argument.

7 What can supersymmetry explain?

Supersymmetry can explain much that the SM cannot, as described above, particularly the Higgs physics as we will discuss in detail below. Sometimes people who do not understand supersymmetry say it can “explain or fit anything”. In fact it is the opposite. Supersymmetry is a full theory, and all that is unknown is the masses (which are matrices in flavor space) and the vacuum expectation values, exactly as for the SM. There are many conceivable phenomena that supersymmetry could not explain, including sharp peaks in spectra at colliders, a world with no Higgs boson below about 200 GeV, a top quark lighter than the W, deviations from SM predictions greater than about 1% for any process with a tree-level SM contribution (including Z decay to cc), leptoquarks, wide WW or ZZ resonances, excess high-Pt leptons at HERA, large violations of μ/e universality, and much more. None of these has occurred, consistent with supersymmetry, but a number of them have been reported and then gone away, and supersymmetry did not “explain” them while they were around. Supersymmetry alone also cannot explain some real questions such as why there are three families or the μ - τ mass ratio.

8 How does flavor physics enter the theory?

The “flavor problem” is one of the most basic questions in physics. By this usually three questions are intended. First, why are there three families of quarks and leptons, and not more or less? Second, why are the symmetry eigenstates different from the mass eigenstates? Third, why do the quarks and leptons have the particular mass values they do? Supersymmetry does not provide the answers to those questions directly, though it will affect the answers. The second and third questions are of course related, but different. We could know the answer to the second question but not the third. For example, the actual values of masses of the lighter quarks and leptons could depend on operators beyond the tree level in the superpotential. The u,d,e masses are so small that they could get large corrections from a number of
Where to look for those answers is not something that is agreed on — many people have tried to understand flavor physics at the TeV scale. Supersymmetry does suggest where to find the answers. Supersymmetry is like the SM in that it accommodates the three families and the flavor rotations but does not explain them. It clearly suggests that the flavor physics has basically entered once the superpotential is determined, i.e. when the Yukawa couplings in the superpotential are fixed. That occurs as soon as a 4D theory is written and depends in a basic way on the string physics and on the compactification and on the determination of the string vacuum. Since the superpotential of the observable sector does not know about supersymmetry breaking, the basic flavor physics probably does not depend on supersymmetry breaking either, though how the flavor physics manifests itself in $L_{\text{soft}}$ may. That in turn suggests that learning the Yukawa couplings and the off-diagonal structure of the trilinears and squark and slepton mass matrices can guide us to the formulation of how to compactify and how to find the string vacuum, and can test ideas about such physics. The role of supersymmetry breaking is unclear. For example, the structure of the trilinear soft-breaking terms can be calculated in terms of the Yukawa couplings and their derivatives, but may depend on how supersymmetry is broken as well.

An important point is that we are likely to learn more from data on the superpartner masses than we did from the quark masses (as we will discuss later). That is because the parameters of $L_{\text{soft}}$ are rather directly related to an underlying theory, while the quark and lepton masses probably are not. Probably what we learned from the fermion masses is that some Yukawa couplings are of order unity while others are small at tree level, arising from non-renormalizable operators and/or breaking of discrete symmetries and supersymmetry. The masses of the first and second family quarks and leptons are probably determined by or very sensitive to small effects that are hard to calculate (the first family masses are in the MeV range, while the theory makes sense for the 100 GeV range), while the squark and slepton masses, and probably the phases, and the approximate size of off-diagonal flavor dependent squark and slepton masses and trilinears all generally emerge from the theory at leading order and are thus much more easily interpretable than the fermion masses.

The next question is how to measure the flavor-dependent elements of $L_{\text{soft}}$, which has 112 flavor-dependent parameters not counting neutrino physics. Although certain combinations of them affect collider physics, and the masses
of the mass eigenstates can be measured at colliders, most of them affect rare
declays, mixing, and CP violation experiments. Collider studies of superpar-
tners may tell us little about flavor physics directly. If they are to have an
observable effect, of course, the supersymmetric contributions to the decays
and mixing and CP violation must be significant, which is most likely for
processes that are forbidden at tree level such as $b \rightarrow s + \gamma$, mixing, penguin
diagrams, $\mu \rightarrow e + \gamma$, etc.

The absence of flavor-changing decays for many systems puts strong con-
straints on some soft parameters. If the off-diagonal elements of the squark
or slepton mass matrices and trilinears were of order the typical squark or
slepton masses then in general there would be large flavor mixing effects,
since the rotations that diagonalize the quarks and charged leptons need not
diagonalize the squarks and sleptons. However, many of the constraints from
flavor-changing processes in the literature have been evaluated with assump-
tions that may not apply, so people should reevaluate them for any approach
they find attractive for other reasons. Much effort has gone into construct-
ing models of $L_{soft}$ that guarantee without tuning the absence of FCNC, and
several approaches exist. If one of them is confirmed when data exists it will
be a major clue to the structure of the high energy theory. Our view that
the flavor physics is determined at the high scale implies that the resulting
structure of the squark and slepton mass matrices, and the trilinear coeffi-
cients, is also determined at the high scale and not by TeV-scale dynamics.
Thus the absence of FCNC is not and should not be explained by an effec-
tive supersymmetric theory. Rather, the pattern of soft-breaking terms that
is measured and gives small FCNC will help us learn about the underlying
(presumably string) theory. Similar remarks could be made about proton
decay.

Once the soft flavor parameters are measured it is necessary to deduce
their values at the unification or string scales in order to compare with the
predictions of string-based models, or to stimulate the development of string-
based models. There are two main issues that arise. One is how to relate
measured values of the CKM matrix and soft parameters to the values of
Yukawa matrices and soft parameters at the unification scale, assuming no
other physics enters between the scales. This is subtle because the number
of independent parameters is considerably less than the number of apparent
parameters in $L_{soft}$ and the superpotential Yukawas, as discussed in Section
17, and the RGE running will for a generic procedure involve non-physical
parameters. This problem has recently been solved [3], giving a practical
technique to convert measurements into the form of the high scale theory.

The second issue is that presumably there is not a desert between the high and low scales. Both gauge coupling unification and radiative electroweak symmetry breaking imply that no part of the theory becomes strongly interacting below the unification scale. But we expect heavy RH neutrinos, axion physics, and perhaps “exotic” states such as those often generated in stringy models, e.g. vector multiplets, fractionally charged uncolored fermions, etc. This issue has not been studied much [6]. Perhaps by examining appropriate combinations of quantities for the RGE running, and by imposing appropriate conditions, it will be possible to use consistency checks to control the effects of intermediate scale physics.

9 Derivation of the supersymmetry Lagrangian

In order to understand the predictions and explanations of supersymmetry, particularly for the Higgs sector, we must learn the derivation of the supersymmetry Lagrangian. I will present the arguments fully though not all the algebra. I will largely follow the approach of Martin.

Consider a massless and therefore two-component fermion, $\psi$ whose superpartner is a complex scalar $\phi$. Both have two real degrees of freedom. But in the off-shell field theory the fermion is a four-component field with four degrees of freedom, and we want supersymmetry to hold for the full field theory. So we introduce an additional complex scalar $F$ so that there are four scalar degrees of freedom also. $F$ is called an auxiliary field. The combined fields $(\psi, \phi, F)$ are called a chiral superfield or chiral supermultiplet. I will not be systematic or careful about the two-component vs. four-component notation since the context usual is clear. The Lagrangian can be taken to be

$$-L_{\text{free}} = \sum_i (\partial^\mu \phi^*_i \partial_\mu \phi_i + \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + F^*_i F_i).$$

The sum is over all chiral supermultiplets in the theory. Note that the dimensions of $F$ are $[F] = m^2$. The Euler-Lagrange equations of motion for $F$ are $F = F^* = 0$, so on-shell we revert to only two independent degrees of freedom. One can define supersymmetry transformations that take bosonic degrees of freedom into fermionic ones; we will look briefly at them later.
The supersymmetry transformations can be defined so that $L_{\text{free}}$ is invariant. Next we write the most general set of renormalizable interactions,

$$L_{\text{chiral}} = L_{\text{free}} + L_{\text{int}}$$ (2)

$$L_{\text{int}} = -\frac{1}{2} W^{ij} \bar{\psi}_i \psi_j + W^i F_i + c.c.$$ (3)

Here $W^{ij}$ and $W^i$ are any functions of only the scalar fields, remarkably, and $W^{ij}$ is symmetric. If $W^{ij}$ or $W^i$ depended on the fermion or auxiliary fields the associated terms would have dimension greater than four, and would therefore not be renormalizable. There can be no terms in $L_{\text{int}}$ that depend on $\phi_i^*$ or $\phi_i$ since such terms would not transform into themselves under the supersymmetry transformations.

Now imagine supersymmetry transformations that mix fermions and bosons, $\phi \rightarrow \phi + \varepsilon \psi, \psi \rightarrow \psi + \varepsilon \phi$. We should go through these transformations in detail with indices, but one can see the basic argument simply. Here $\varepsilon$ must be a spinor so each term behaves the same way in spin space, and we can take $\varepsilon$ to be a constant spinor in space-time, and infinitesimal. Then the variation of the Lagrangian (which must vanish or change only by a total derivative if the theory is invariant under the supersymmetry transformation) contains two terms with four spinors:

$$\delta L_{\text{int}} = -\frac{1}{2} \frac{\delta W^{ij}}{\delta \phi_k} (\varepsilon \psi_k) \bar{\psi}_i \psi_j - \frac{1}{2} \frac{\delta W^{ij}}{\delta \phi_k^*} (\varepsilon^* \psi_k^*) \bar{\psi}_i \psi_j + c.c.$$ (4)

Neither term can cancel against some other term. For the first term there is a Fierz identity $(\varepsilon \psi_i)(\psi_j \psi_k) + (\varepsilon \psi_j)(\psi_k \psi_i) + (\varepsilon \psi_k)(\psi_i \psi_j) = 0$, so if and only if $\delta W^{ij}/\delta \phi_k$ is totally symmetric under interchange of i,j,k the first term vanishes identically. For the second term the presence of the hermitean conjugation allows no similar identity, so it must vanish explicitly, which implies $\delta W^{ij}/\delta \phi_k^* = 0$, and thus $W^{ij}$ cannot depend on $\phi^*$! $W^{ij}$ must be an analytic function of the complex field $\phi$.

Therefore we can write

$$W^{ij} = M^{ij} + y^{ijk} \phi_k,$$ (5)
where $M^{ij}$ is a symmetric matrix that will be the fermion mass matrix, and $y^{ijk}$ can be called Yukawa couplings since it gives the strength of the coupling of boson $k$ with fermions $i, j$; $y^{ijk}$ must be totally symmetric. Then it is very convenient to define

$$ W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k $$

and $W^{ij} = \delta^2 W / \delta \phi_i \delta \phi_j$. $W$ is the “superpotential”, an analytic function of $\phi$, and a central function of the formulation of the theory. $W$ is fully supersymmetric and gauge invariant and Lorentz invariant, and an analytic function of $\phi$ (i.e. it cannot depend explicitly on $\phi^*$), so it is highly constrained. It determines the most general non-gauge interactions of the chiral superfields.

A similar argument for the parts of $\delta L_{int}$ which contain a spacetime derivative imply that $W^i$ is determined in terms of $W$ as well,

$$ W^i = \frac{\delta W}{\delta \phi_i} = M^{ij} \phi_j + \frac{1}{2} y^{ijk} \phi_j \phi_k. $$

Because interactions are now present, the equations for $F$ are non-trivial,

$$ F_i = -W_i^*. $$

The scalar potential is related to the Lagrangian by $L = T - V$, so

$$ V = \sum_i |F_i|^2 $$

This contribution is called an “F-term”, and is automatically bounded from below, an important improvement.

The above analysis was appropriate for chiral superfields, which will contain the fermions and their superpartners. Now we repeat the logic for the gauge supermultiplets that contain the gauge bosons and their superpartners. Initially they are massless gauge bosons, like photons, $A_\mu^a$, with gauge index $a$, and two degrees of freedom. Their superpartners are two-component spinors $\lambda^a$. But as above, off shell the fermion has four degrees of freedom, while the massive boson has three, the two transverse polarizations and a
longitudinal polarization. So again it is necessary to add an auxiliary field, a real one since only one degree of freedom is needed, called $D^a$. Then the Lagrangian has additional pieces

$$L_{\text{gauge}} = -\frac{1}{4} F^a_{\mu \nu} F^a_{\mu \nu} - i \lambda^a \gamma^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a,$$

(10)

where

$$F^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$

(11)

and the covariant derivative is

$$D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c.$$

(12)

Note that the notation is unfortunate, with both the covariant derivative and the new field being denoted by the standard “$D$”. Also, I have not been careful about two component vs. four component spinors. It is crucial for gauge invariance that the same coupling $g$ appears in the definition of the tensor $F$ and in the covariant derivative. Lagrangians always have to contain all of the terms allowed by gauge invariance, etc., and here we can see another term to add,

$$\langle \phi_i^* T^a \phi_i \rangle D^a. \quad (13)$$

There is one more term that can be added that mixes the fields, $\lambda^a (\psi^i T^a \phi)$, and its conjugate, with an arbitrary dimensionless coefficient. Requiring the entire Lagrangian to be invariant under supersymmetry transformations determines the arbitrary coefficient and gives a resulting Lagrangian

$$L = L_{\text{gauge}} + L_{\text{chiral}} + g_a (\phi^* T^a \phi) D^a - \sqrt{2} g_a [\langle \phi^* T^a \psi \rangle \lambda^a + \lambda^a (\psi^i T^a \phi)] \quad (14)$$

where all derivatives in earlier forms are replaced by covariant ones. Remarkably, the requirement of supersymmetry fixed the couplings of the last terms to be gauge couplings even though they are not normal gauge interactions!
The equations of motion for $D^a$ give $D^a = -g(\phi^* T^a \phi)$, so the scalar potential is

$$V = F^* F_i + \frac{1}{2} \sum_a D^a D^a = |\partial W/\partial \phi_i|^2 + \sum_a g^2_a (\phi^* T^a \phi)^2. \quad (15)$$

The sum is over $a = 1, 2, 3$ for the three gauge couplings. The two terms are called “F-terms” and “D-terms”. Remarkable, the unbroken supersymmetric theory gives a scalar potential bounded from below. On the one hand that is good since unbounded potentials are a problem, but it also implies that the Higgs mechanism cannot happen for unbroken supersymmetry since the potential will be minimized at the origin. In the above,

$$L_{chiral} = D^\mu \phi^*_i D_\mu \phi_i + \bar{\psi}_i \gamma^\mu D_\mu \psi_i$$

$$(16) + (\frac{1}{2} M_{ij} \psi_i \psi_j + \frac{1}{2} y^{ijk} \phi_i \psi_j \psi_k + c.c.) + F^*_i F_i.$$  

This completes the derivation of the unbroken supersymmetry Lagrangian.

### 10 Non-renormalization theorem

For unbroken supersymmetry there is a very important result, called the non-renormalization theorem, that is very useful for building models to relate the theory to the real world. Because of this result the supersymmetry fields get a wave function renormalization only, so they have the familiar log running of couplings and masses, but no other renormalizations. Consequently the parameters of the superpotential $W$ are not renormalized, in any order of perturbation theory. In particular, terms that were allowed in $W$ by gauge invariance and Lorentz invariance are not generated by quantum corrections if they are not present at tree level, so no F-terms are generated if they are initially absent. If there is no $\mu$ -term in the superpotential (see below), none is generated. The non-renormalization theorem is difficult to probe without extensive formalism, so I just state it here. References and a pedagogical derivation are given in reference 7.
11 Toward softly-broken supersymmetry with a toy model

Consider the Wess-Zumino model, with,

\[ W = \frac{m}{2} \phi \phi + \frac{g}{6} \phi \phi \phi, \]  

(17)

and

\[ L = (\partial \phi)^2 + i \Psi^\dagger \sigma^\mu \partial_\mu \Psi - F^*_\phi F_\phi + \left( \frac{1}{2} W_\phi \Psi \Psi - W_\phi F_\phi + \text{c.c.} \right). \]  

(18)

This is written in two component notation. \( W_\phi = -F^*_\phi = m\phi + \frac{g}{2} \phi \phi \) is the derivative of the superpotential with respect to \( \phi \), and \( W_{\phi \phi} \) the second derivative. We put \( \phi = (A + iB)/2 \) and \( F_\phi = (F + iG)/2 \), where \( A, B, F, G \) are real scalars, and switch to four component notation. Under the supersymmetry transformations, with \( \varepsilon \) a constant spinor,

\[ \delta A = \bar{\varepsilon} \gamma_5 \Psi, \]  

(19)

\[ \delta B = -\bar{\varepsilon} \Psi, \]  

(20)

\[ \delta \Psi = F \varepsilon - G \gamma_5 \varepsilon + \gamma^\mu \partial_\mu \gamma_5 A \varepsilon + \gamma^\mu \partial_\mu B \varepsilon; \]  

(21)

\[ \delta F = -\bar{\varepsilon} \gamma^\mu \partial_\mu \Psi, \]  

(22)

\[ \delta G = -\bar{\varepsilon} \gamma_5 \gamma^\mu \partial_\mu \Psi, \]  

(23)

the Lagrangian changes by a total derivative, so the action is invariant with the usual assumptions.

Now substitute for \( W_\phi \) and \( W_{\phi \phi} \) etc. Then the Lagrangian is

\[ L = \frac{1}{2} (\partial A)^2 + \frac{1}{2} (\partial B)^2 + \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} m \bar{\psi} \psi \]

+ \( \frac{g}{\sqrt{2}} A \bar{\psi} \psi - \frac{ig}{\sqrt{2}} B \bar{\psi} \gamma_5 \psi - \frac{1}{2} (F^2 + G^2) \)

- \( \frac{m}{2} (2AF - 2BG) - \frac{g}{2\sqrt{2}} (F(A^2 - B^2) - 2GAB) \).  

(24)
Now the equations of motion for $F, G$ are

$$F = -mA - \frac{g}{2\sqrt{2}}(A^2 - B^2), \quad G = mB + \frac{g}{\sqrt{2}}AB. \quad (25)$$

Substituting these gives interaction vertices $\frac{mg}{2\sqrt{2}}A(A^2 - B^2)$.

With one coupling strength $g$ and one mass $m$ the full Lagrangian is supersymmetric. (Note that without supersymmetry there can be four different masses and four different couplings, so there are six relations predicted by supersymmetry which only allows one mass and one coupling.) But when supersymmetry is broken we expect the masses to differ. Suppose we allow four different masses, $m_A, m_B, m_\psi, m_g$, where the last is the mass that is needed in some terms to give each term dimension four, so it multiplies $g$. It’s clear how to rewrite the Lagrangian with these separate masses. There are four three-particle vertices, $A\psi\bar{\psi}$, $A^3, AB^2, B\bar{\psi}\psi$. Now if we write the expression for a tadpole graph,

$$\langle 0 | L | A \rangle =$$

$$\frac{g}{\sqrt{2}} \left\{ 4m_\psi \int \frac{d^4p}{p^2 - m_\psi^2} - mg \int \frac{d^4p}{p^2 - m_B^2} - 3mg \int \frac{d^4p}{p^2 - m_A^2} \right\}, \quad (26)$$

we see that in general this has a quadratic divergence, which cancels in the supersymmetry limit as expected. The fermion loop gives a minus sign, the factor of 4 in the first term arises from $Tr(\gamma^\mu p_\mu + m_\psi) = Tr m_\psi = 4m_\psi$, and the 3 in the last from the $A^3$. But — and here is the important point — the divergence still cancels if $m_A \neq m_B \neq m_g$, but not if $m_\psi \neq m_g$. Thus extra contributions to boson masses do not reintroduce quadratic divergences — they are called “soft” supersymmetry breaking. But extra contributions to fermion masses do lead to quadratic divergences, “hard” supersymmetry breaking. This result is true to all orders in perturbation theory, though this pedagogical argument does not show it. Some of the results are obvious since couplings proportional to masses will not introduce quadratic divergences, but it is still helpful to see the supersymmetry structure. After the supersymmetry breaking there are three masses and one coupling, so there are still four tests that the theory is a broken supersymmetric one.
To understand the general structure of supersymmetry breaking better, recall how symmetry breaking works in the SM. It is not possible to break the $SU(2) \times U(1)$ symmetry from within the SM. So a new sector, the Higgs sector is needed. Interactions are assumed in the Higgs sector that give a potential with a minimum away from the origin, so the Higgs field gets a vev which breaks the symmetry. To generate mass for $W, Z, q, l$ an interaction is needed to transmit the breaking to the “visible” particles $W, Z, q, l$. For fermions this interaction is $L_{\text{fermion}} = g_\epsilon \bar{e}_L \epsilon_R h + cc \rightarrow g_e \nu \bar{e} e$ after $h$ gets a vev for the fermions, and we can identify $m_e = g_e v$. Similarly, for the gauge bosons the Lagrangian term $(D^\mu h)(D_\mu h) \rightarrow g^2 hhW^\mu W_\mu \rightarrow g^2 v^2 W^\mu W_\mu$ giving $W, Z$ masses. The fundamental symmetry breaking is spontaneous ($h$ gets a vev), but the effective Lagrangian appears to have explicit breaking.

The situation is very similar for supersymmetry. It is not possible to break supersymmetry in the “visible” sector, i.e. the sector containing the superpartners of the SM particles. A separate sector is needed where supersymmetry is broken. Originally it was called the “hidden” sector, but that is not a good name since it need not be really hidden. Then there must be some interaction(s) to transmit the breaking to the visible sector. Since the particles of both sectors interact gravitationally, gravity can always transmit the breaking. Other interactions may as well. We will have to find out how the breaking is transmitted from data on the superpartners, their masses and decays and phases and flavor rotations. Different ways of transmitting the breaking give different patterns of the soft parameters that we discuss below. A significant complication is that the effects of the supersymmetry breaking are mixed up with effects of the transmission. All the effects of the supersymmetry breaking and of the way it is transmitted, for any theory, are embedded in the soft-breaking Lagrangian that we turn to studying.

12 The soft-breaking Lagrangian

The (essentially) general form of $L_{\text{soft}}$ is

$$L_{\text{soft}} = \frac{1}{2}(M_\lambda \lambda^a \lambda^a + c.c.) + m_{ij}^2 \phi_i^* \phi_j$$

$$+ \left( \frac{1}{2} b_{ij} \phi_i \phi_j + \frac{1}{6} a_{ijk} \phi_i \phi_j \phi_k + c.c. \right)$$

20
This obviously breaks supersymmetry since only scalars and gauginos get mass, not their superpartners. It is soft as in our example above because it can be proved to not introduce any quadratic divergences. Models for supersymmetry breaking, however they originate, in string theory or supergravity or dynamically, all lead to this form. We will write it for the SSM shortly.

If all fields carry gauge quantum numbers there are terms that could be added to this without generating quadratic divergences, such as $\phi^* i \phi \phi_k$, but such terms seldom arise in models so they are usually ignored \[9\]. If such terms are truly absent once measurements are analyzed, their absence may be a clue to how supersymmetry is broken and transmitted.

### 13 The Minimal Supersymmetric Standard Model

To write the supersymmetric SM we first take all of the quarks and leptons and put them in chiral superfields with superpartners. For each set of quantum numbers, such as up quarks or electrons, the scalar, fermion, and auxiliary fields ($\phi, \psi, F$) form a supermultiplet in the same sense as ($n, p$) form a strong isospin doublet or ($\nu_e, e$) form an electroweak doublet. All superpartners are denoted with a tilde, and there is a superpartner for each spin state of each fermion — that is important since the SM treats fermions of different chirality differently. The gauge bosons are put in vector superfields with their fermionic superpartners. Since $W$ is analytic in the scalar fields, we cannot include the complex conjugate of the scalar field as in the SM to give mass to the down quarks, so there must be two Higgs doublets (or more) in supersymmetry, and each has its superpartners. The requirement that the trace anomalies vanish so that the theories stay renormalizable, $\text{TR}(Y^3) = \text{TR}(T_3^2 L Y) = 0$, also implies the existence of the same two Higgs doublets. (The relevance of anomalies may seem unclear since we are only writing an effective theory, while anomaly conditions only need to be satisfied for the full theory. But if the anomaly conditions are not satisfied it may introduce a sensitivity to higher scales that the effective theory should not have.)

We proceed by first constructing the superpotential so we can calculate the F-terms, and then writing the Lagrangian, following equation 6 and summing over all the particles. The most general superpotential, if we don’t
extend the SM and don’t include RH neutrinos, is

\[
W = \bar{u}Y_uQH_u - \bar{d}Y_dQH_d - \bar{e}Y_eLH_d + \mu H_uH_d.
\]  

(28)

All the fields are chiral superfields. The bars over \( u, d, e \) are in the sense of Martin’s notation, specifying the conjugate fields. The signs are conventional so that masses later are positive. Indices are suppressed — for example, the fourth and first terms are

\[
\mu(H_u)_\alpha(H_d)_\beta \varepsilon_{\alpha \beta} \text{ and } \bar{u}_{ai}(Y_u)_{ij}Q^a(H_u)_j \varepsilon_{\alpha \beta}.
\]

(29)

The Yukawa couplings \( Y_u \) etc. are dimensionless \( 3 \times 3 \) family matrices that determine the masses of quarks and leptons, and the angles and phase of the CKM matrix after \( H^0_u \) and \( H^0_d \) get vevs. They also contribute to the squark-quark-higgsino couplings etc. since the fields in \( W \) are superfields containing all the components. This is the most general superpotential for the SSM if we assume baryon and lepton number are conserved (we’ll return to this question). To see the structure more explicitly we can use the approximations

\[
Y_u \approx \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y_t
\end{pmatrix}, \quad
Y_d \approx \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y_b
\end{pmatrix}, \quad
Y_e \approx \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y_\tau
\end{pmatrix},
\]

(30)

which gives

\[
W = Y_t(\bar{t}tH^0_u - \bar{b}bH^+_u) - Y_b(\bar{b}tH^-_d - \bar{\nu}_\tau H^0_\nu) \\
- Y_\tau(\bar{\nu}_\tau H^-_d - \bar{\nu}_\tau H^0_\nu) + \mu(H^+_uH^-_d - H^0_uH^0_d)
\]

(31)

There are also other interactions from \( W \) such as vertices \( H^0_u\bar{t}tL, \tilde{H}^0_u\bar{t}tL, \tilde{H}^0_u\tilde{t}tL \), etc., all with the same strength \( Y_t \). All of them are measurable, and it will be an important check of supersymmetry to confirm they are all present with the same strength. All are dimensionless, so supersymmetry-breaking will only lead to small radiative corrections to these coupling strengths. In general one goes from one to another of these by changing any pair of particles into superpartners.

Before we turn to writing the full soft-breaking Lagrangian, we first look at two significant issues that depend on how supersymmetry is embedded in a more basic theory.
14 The $\mu$ opportunity

The term $\mu H_u H_d$ in the superpotential leads to a term in the Lagrangian

$$L = \ldots + \mu (\bar{H}_u^+ \bar{H}_d^- - \bar{H}_u^0 \bar{H}_d^0) + \ldots \quad (32)$$

which gives mass terms for higgsinos in the chargino and neutralino mass matrices, so $\mu$ enters there. This term also contributes to the scalar Higgs potential from the F-terms,

$$V = \ldots |\mu|^2 (|H_u^0|^2 + |H_d^0|^2 + \ldots) + \ldots \quad (33)$$

so these terms affect the Higgs mass, and F-terms also give contributions to the Lagrangian that affect the squark and slepton mass matrices,

$$L = \ldots \mu^* (\bar{u}Y_u \bar{u}H_d^0 + \ldots) + \ldots \quad (34)$$

Thus phenomenologically $\mu$ must be of order the weak scale to maintain the solutions of the hierarchy problem, gauge coupling unification, and radiative electroweak symmetry breaking. The naive scale for any term in the superpotential is one above where the supersymmetry is broken, e.g. the string scale or unification scale, and since $\mu$ occurs in $W$ one would naively expect $\mu$ to be of order that scale, far above the weak scale. In the past that has been called the “$\mu$ problem”. But actually it is a clue to the correct theory and is an opportunity to learn what form the underlying theory must take. For example, in a string theory we expect all the mass terms to vanish since the SM particles are the massless modes of the theory, so in a string theory $\mu$, which is a mass term, would naturally vanish. That could be a clue that the underlying theory is indeed a string theory. In the following we will view $\mu = 0$ as a “string boundary condition”. Older approaches added symmetries to require $\mu = 0$. Note that because of the non-renormalization theorem once $\mu$ is set to zero in $W$ it is not generated by loop corrections.

We also know phenomenologically that the $\mu$ contribution to the chargino and neutralino masses and the Higgs mass cannot vanish, or some of them would be so light they would have been observed, so we know that somehow a piece that plays the same role as $\mu$ is generated. We will call it $\mu_{\text{eff}}$, but
whenever there is no misunderstanding possible we will drop the subscript and just write $\mu$ for $\mu_{\text{eff}}$. Different ways of generating $\mu_{\text{eff}}$ give different relations to the other soft-breaking parameters, a different phase for $\mu_{\text{eff}}$, a characteristic size for $\mu_{\text{eff}}$, etc. Once it is measured we will have more clues to the underlying theory. Any top-down approach must generate $\mu_{\text{eff}}$ and its phase correctly.

15  R-parity conservation

The $\mu$ opportunity looks like the $\mu$ problem if one views supersymmetry as an effective low energy theory without seeing it as embedded in a more fundamental high scale theory. Similarly, if we view supersymmetry as only a low energy effective theory there is another complication that arises. There are additional terms that one could write in $W$ that are analytic, gauge invariant, and Lorentz invariant, but violate baryon and/or lepton number conservation. No such terms are allowed in the SM, which accidentally conserves $B$ and $L$ to all orders in perturbation theory, though it does not conserve them non-perturbatively. These terms are

$$W_R = \lambda_{ijk} L_i L_j e_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \lambda''_{ijk} \bar{u}_i \bar{d}_j d_k.$$  \hspace{1cm} (35)

The couplings $\lambda, \lambda', \lambda''$ are matrices in family space. Combining the second and third one can get very rapid proton decay, so one or both of them must be required to be absent. That is not the way one wants to have a theory behave. Rather, $B$ and $L$ conservation consistent with observation should arise naturally from the symmetries of the theory. Most, but not all, theorists expect that an underlying symmetry will be present in the broader case to forbid all of the terms in $W_R$.

There are two approaches to dealing with $W_R$. We can add a symmetry to the effective low energy theory, called R-parity or a variation called matter parity, which we assume will arise from a string theory or extended gauge group. R-parity is multiplicatively conserved,

$$R = (-1)^{3(B-L)+2S}$$  \hspace{1cm} (36)

where $S$ is the spin. Then SM particles and Higgs fields are even, superpartners odd. This is a discrete $Z_2$ symmetry. Such symmetries that treat
superpartners differently from SM particles and therefore do not commute with supersymmetry are called R-symmetries. Equivalently, one can use “matter parity”,

\[ P_m = (-1)^{3(B-L)}. \]  

A term in \( W \) is only allowed if \( P_m = +1 \). Gauge fields and Higgs are assigned \( P_m = +1 \), and quark and lepton supermultiplets \( P_m = -1 \). \( P_m \) commutes with supersymmetry and forbids \( W_R \). Matter parity could be an exact symmetry, and such symmetries do arise in string theory. If R-parity or matter parity holds there are major phenomenological consequences,

- At colliders, or in loops, superpartners are produced in pairs.
- Each superpartner decays into one other superpartner (or an odd number).
- The lightest superpartner (LSP) is stable. That determines supersymmetry collider signatures, and makes the LSP a good candidate for the cold dark matter of the universe.

The second approach is very different, and does not have any of the above phenomenological consequences. One arbitrarily sets \( \lambda' \) or \( \lambda'' = 0 \) so there are no observable violations of baryon number or lepton number conservation. Other terms are allowed and one sets limits on them when their effects are not observed, term by term. In the MSSM itself R-parity must be broken explicitly if it is broken at all. If it were broken spontaneously by a sneutrino vev there would be a Goldstone boson associated with the spontaneous breaking of lepton number (called a Majaron), and some excluded Z decays would have been observed.

We will not pursue this ad hoc approach, because we do not like arbitrarily setting some terms to zero, and we do not like giving up the LSP as cold dark matter if we are not forced to. Further, large classes of theories conserve R-parity or matter parity [10]. Often theories have a gauged \( U(1)_{B-L} \) symmetry that is broken by scalar vevs and leaves \( P_m \) automatically conserved. String theories often conserve R-parity or \( P_m \). Often theories conserve R-parity at the minimum of the Higgs potential. Baryogenesis via leptogenesis probably requires R-parity conservation because the usual \( B + L \) violation plus \( L \) violation would allow the needed asymmetries to be erased. The lepton number needed for \( \nu \) seesaw masses violates \( L \) by two units and does not violate R-parity conservation. In general, when supersymmetry is viewed as
embedded in a more fundamental theory, R-parity conservation is very likely and easily justified. Ultimately, of course, experiment will decide, but we will assume R-parity conservation in the rest of these lectures.

16 Definition of MSSM

At this stage we can define the effective low energy supersymmetry theory, which we call the MSSM, as the theory with the SM gauge group and particles, and the superpartners of the SM particles, and conserved R-parity, and two Higgs doublets. Perhaps it would be better to include right handed neutrinos and their superpartners as well, but that is not yet conventional.

17 The MSSM soft-breaking Lagrangian

We can now write the general soft-breaking Lagrangian for the MSSM,

\[-L_{\text{soft}} = \frac{1}{2} (M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + c.c.) + \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{u}^\dagger m_u^2 \tilde{u} + \tilde{d}^\dagger m_d^2 \tilde{d} + \tilde{L}^\dagger m_L^2 \tilde{L} + \tilde{e}^\dagger m_e^2 \tilde{e} + (\tilde{Q} H_u \tilde{Q} - \tilde{Q} H_d \tilde{Q} - \tilde{L} H_d \tilde{L} + c.c.) + m_{H_u}^2 H_u^* H_u + m_{H_d}^2 H_d^{2*} + (bH_u H_d + c.c.)\]

For clarity a number of the indices are suppressed. $M_{1,2,3}$ are the complex bino, wino, and gluino masses, e.g. $M_3 = |M_3| e^{i\phi_3}$, etc. In the second line $m_Q^2$, etc, are squark and slepton hermitean 3x3 mass matrices in family space. The $a_{u,d,e}$ are complex 3x3 family matrices, usually called trilinear couplings. $b$ is sometimes written as $B\mu$ or as $m_3^2$ or as $m_{12}^2$. Additional parameters come from the gravitino complex mass and from $\mu_{\text{eff}} = \mu e^{i\phi_{\mu}}$; we will usually risk writing the magnitude of $\mu_{\text{eff}}$ as just $\mu$ assuming the context will distinguish this from the original $\mu$ of the superpotential. This may seem to involve a lot of parameters, but all the physical parameters are observable from direct production and study of superpartners and their
effects. The absence of observation of superpartners and their effects already gives us useful information about some of the parameters. It is important to understand that all of these parameters are masses or flavor rotation angles or phases or Higgs vevs, just as for the SM. If we had no measurements of the quark and lepton masses and interactions there would be even more parameters for the SM than here.

With this Lagrangian we can do general, useful, reliable phenomenology, as we will see. For example, in the SM we did not know the top quark mass until it was measured. Nevertheless, for any chosen value of the top mass we could calculate its production cross section at any collider, all of its decay BR, its contribution to radiative corrections, etc. Similarly, for the superpartners we can calculate expected signals, study any candidate signal and evaluate whether it is consistent with the theory and with other constraints or data, and so on. A possible signal might have too small or large a cross section to be consistent with any set of parameters, or decay BR that could not occur here. Many examples can be given. We can also study whether superpartners can be studied at any proposed future facility. Further, most processes depend on only a few of the parameters — we will see several examples of this in the following.

Now let us count the parameters of the broken supersymmetric theory relative to the SM. There are no new gauge or Yukawa couplings, and still only one strong CP angle $\tilde{\vartheta}$, so that is already rather economical. Then

- $m_Q^2$, etc are 5 $3 \times 3$ hermitean matrices $\to$ 9 real parameters each $\to$ 45
- $a_{u,d,e}$ are 3 $3 \times 3$ complex matrices $\to$ 18 real parameters each $\to$ 54
- $M_{1,2,3}, \mu, b$ are complex $\to$ 10
- $m^2_{H_{u,d}}$ are real by hermiticity $\to$ 2

giving a total of 111 parameters. As for the CKM quark matrix it is possible to redefine some fields and absorb some parameters. Baryon and lepton number are conserved, and there are two U(1) symmetries that one can see by looking at the Lagrangian. One arises because if $\mu$ and $b$ are zero there is a symmetry where $H_{u,d} \to e^{i\alpha}H_{u,d}$ and the combinations $L\bar{e}, Q\bar{u}, Q\bar{d} \to e^{-i\alpha}L\bar{e}, Q\bar{u}, Q\bar{d}$. For example, one can take $Q \to e^{-i\alpha}Q$, $L \to e^{-i\alpha}L$, and $\bar{e}, \bar{u}, \bar{d}$ invariant. Such a symmetry is called a Peccei-Quinn symmetry if it holds for $\mu = 0$ but is broken when $\mu \neq 0$. The other arises because if $M_i, a_i, b = 0$ there is a continuous R-symmetry, e.g. the Higgs fields can have charge 2, the other matter fields charge 0, and the superpotential charge 2. Symmetries are called R-symmetries whenever members of a supermultiplet are treated differently.
With these four symmetries, four parameters can be absorbed. Also, the SM has two parameters in the Higgs potential, \( \mu^2 \phi^2 + \lambda \phi^4 \), so to count the number beyond the SM we subtract those 2. Then there are 111-4-2=105 new parameters. The SM itself has 3 gauge couplings, 9 quark and charged lepton masses, 4 CKM angles, 2 Higgs potential parameters, and one strong CP phase → 19. So there are 124 parameters altogether. When massive neutrinos are included one has RH \( \nu \) masses, and the angles of the flavor rotation matrix (which has 3 real angles and 3 phases for the \( \nu \) case since the Majorana nature of the neutrinos prevents absorbing two of the phases). In the following we will discuss how to measure many of the parameters. All are measurable in principle. Once they are measured they can be used to test any theory. In practice, as always historically, some measurements will be needed to formulate the underlying theory (e.g. to learn how supersymmetry is broken and to compactify) and others will then test approaches to doing that.

Only 32 of these parameters are masses of mass eigenstates! There are four neutralinos, two charginos, four Higgs sector masses, three LH sneutrinos, six each of charged sleptons, up squarks, and down squarks, and the gluino. We will examine the connections between the soft masses and the mass eigenstates below. Of the 32 masses, only the gluino occurs directly in \( L_{\text{soft}} \) — the rest are all related in complicated ways to \( L_{\text{soft}} \)! One could add the gravitino with its complex mass to the list of parameters. Even the gluino mass gets significant corrections that depend on squark masses.

Some of the ways these parameters contribute is to determining the breaking of the EW symmetry and therefore to the Higgs potential, and the masses and cross sections and decays of Higgs bosons, to the relic density and annihilation and scattering of the LSP, to flavor changing transitions because the rotations that diagonalize the fermion masses will not in general diagonalize the squarks and sleptons, to baryogenesis (which cannot be explained with only the CKM phase), to superpartner masses and signatures at colliders, rare decays with superpartner loops (e.g. \( b \rightarrow s + \gamma \)), electric dipole and magnetic dipole moments, and more.

18 Connecting high and low scales

Two of the most important successes of supersymmetry depend on connecting the unification and EW scales. We will not study this topic in detail here.
since Martin covers it thoroughly, but we will look at the aspects we need, particularly for the Higgs sector. The connection is through the logarithmic renormalization and running of masses and couplings, with RGEs. In general we imagine the underlying theory to be formulated at a high energy scale, while we need to connect with experiment at the EW scale. We can imagine running the theory down (top-down) or running an effective Lagrangian determined by data up (bottom-up). It is necessary to calculate for all the parameters of the superpotential and of the soft-breaking Lagrangian. The RGEs are known for gauge couplings and for the superpotential couplings to three loops, and to two loops for other parameters, for the MSSM and its RHν extension. We will only look at one-loop results since we are mainly focusing on pedagogical features. An interesting issue is that calculations must be done with regularization and renormalization procedures that do not break supersymmetry, and that is not straightforward. How to do that is not a solved problem in general, but it is understood through two loops and more loops in particular cases, so in practice there is no problem.

Since our ability to formulate a deeper theory will depend on deducing from data the form of the theory at the unification scale, learning how to convert EW data first into an effective theory at the weak scale, and then into an effective theory at the unification scale, is in a sense the major challenge for particle physics in the coming years. There are of course ambiguities in running to the higher scales. Understanding the uniqueness of the resulting high scale theory, and how to resolve ambiguities as well as possible, is very important.

For the Higgs sector we need to examine the running of several of the soft masses, whose RGEs follow. The quantity \( t = \ln(Q/Q_0) \), where \( Q \) is the scale and \( Q_0 \) a reference scale.

\[
16\pi^2 dM^2_{H_u}/dt \approx 3X_t - 6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2
\]  

(39)

\[
16\pi^2 dM^2_{H_d}/d \approx 3X_b + X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5}|M_1|^2
\]  

(40)

where

\[
X_{t,b} \approx 2 |Y_{t,b}|^2 (M^2_{H_u,d} + m^2_{Q_3} + m^2_{\tilde{u}_3,\tilde{d}_3}) + 2 |a_{t,b}|^2
\]  

(41)
Note that $X_{t,b}$ are positive so $M_{H_{u,d}}^2$ decrease as they evolve toward the EW scale from a high scale, and unless $\tan \beta$ is very large, $X_t$ is larger than $X_b$. We also need to look at just the leading behavior of the squark running,

$$16\pi^2 dM_{Q_3}^2 / dt = X_t + X_b + ...$$  \hspace{1cm} (42)

$$16\pi^2 dM_{\tilde{u}_3}^2 / dt = 2X_t + ...$$  \hspace{1cm} (43)

$$16\pi^2 dM_{\tilde{d}_3}^2 / dt = 2X_b + ...$$  \hspace{1cm} (44)

Think back to the SM, where the coefficient (usually called $\mu^2$ there but remember that $\mu$ is not the same as our $\mu$) of $\phi^2$ in the Higgs potential must be negative to lead to spontaneous symmetry breaking with the minimum of the potential away from the origin. Here $M_{H_u}^2$ plays the role, effectively, of the SM $\mu^2$. We see that because of the large $X_t$ the right hand side of the equation for $M_{H_u}^2$ is indeed the largest, and not only does $M_{H_u}^2$ decrease as it runs but the other quantities run slower so they do not get vevs at the same time. Thus the theory naturally can lead to a derivation of the Higgs mechanism! This is extremely important. The theory could easily have had a form where no Higgs vev formed, or where a Higgs vev could only form if some squark also got a vev, which would violate charge and color conservation. The precise conditions for REWSB are somewhat more subtle in supersymmetry — $M_{H_u}^2$ does not actually need to be negative, just smaller than $M_{H_d}^2$, as we will see next.

### 19 Radiative electroweak symmetry breaking (REWSB)

The Higgs sector is the natural domain of supersymmetry. The Higgs mechanism occurs as the scale decreases from the more symmetric high scale, with vacuum expectation values becoming non-zero somewhat above the EW scale. As we will see, the Higgs mechanism is intricately tied up with supersymmetry and with supersymmetry breaking — there is no Higgs mechanism
unless supersymmetry is broken. This should be contrasted with the other big issue of flavor physics, the origin of the number of families and the differences between the flavor and mass eigenstates, which is already in the structure of the theory at the unification scale, as discussed above. Supersymmetry accommodates the flavor issues, and allows data to constrain them, but supersymmetry can explain the Higgs physics with string boundary conditions (we’ll be more precise about that later).

Once we have the superpotential and $L_{\text{soft}}$ we can calculate the scalar potential that determines the Higgs physics — that is very different from the SM case where one adds the scalar potential in by hand. The result is for the electrically neutral fields,

$$
V = |\mu_{\text{eff}}|^2 (|H_u|^2 + |H_d|^2) \quad F \quad (45)
$$

$$
+ \frac{1}{8}(g_1^2 + g_2^2)(|H_u|^2 - |H_d|^2) \quad D \quad (46)
$$

$$
+ m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 - (bH_u H_d + \text{c.c.}) \quad \text{soft} \quad (47)
$$

From now on again we will just write $\mu$ for $\mu_{\text{eff}}$. Now we want to minimize this. If it has a minimum away from the origin vevs will be generated. If we had included the charged scalars we could use gauge invariance to rotate away any vev for (say) $H_u^+$. Then we would find that the minimization condition $\partial V / \partial H_d^- = 0$ implied that $\langle H_d^- \rangle = 0$, so at the minimum electromagnetism is an unbroken symmetry. The only complex term in $V$ is $b$. We can redefine the phases of $H_u, H_d$ to absorb the $b$ phase, so we can take $b$ as real and positive. Then by inspection we will have a minimum when the term with $b$ subtracts the most it can, so $\langle H_u \rangle \langle H_d \rangle$ will be real and positive. Since $H_u, H_d$ have hypercharge $\pm \frac{1}{2}$, we can use a hypercharge gauge transformation to take the two vevs separately real and positive. Therefore at the tree level CP is conserved in the Higgs sector and we can choose the mass eigenstates to have definite CP.

Writing $\partial V / \partial H_u = \partial V / \partial H_d = 0$ one finds that the condition for a minimum away from the origin is
\[ b^2 > (|\mu|^2 + M_{H_u}^2)(|\mu|^2 + M_{H_d}^2). \] (48)

So \( M_{H_u}^2 < 0 \) helps to generate EWSB but is not necessary. There is no EWSB if \( b \) is too small, or if \( |\mu|^2 \) is too large. For a valid theory we must also have the potential bounded from below, which was automatic for the unbroken theory but is not when the soft terms are included. The quartic piece in \( V \) guarantees \( V \) is bounded from below except along the so-called D-flat direction \( \langle H_u \rangle = \langle H_d \rangle \), so we need the quadratic terms positive along that direction, which implies

\[
2b < 2|\mu|^2 + M_{H_u}^2 + M_{H_d}^2. \quad (49)
\]

Remarkably, the two conditions cannot be satisfied if \( M_{H_u}^2 = M_{H_d}^2 \), so the fact that \( M_{H_u}^2 \) runs more rapidly than \( M_{H_d}^2 \) is essential. They also cannot be satisfied if \( M_{H_u}^2 = M_{H_d}^2 = 0 \), i.e. if supersymmetry is unbroken!

We write \( \langle H_{u,d} \rangle = v_{u,d} \). Requiring the \( Z \) mass be correct gives

\[
v_u^2 + v_d^2 = v^2 = \frac{2M_Z^2}{g_1^2 + g_2^2} \approx (174 \text{GeV})^2 \quad (50)
\]

and it is convenient to write

\[
\tan \beta = v_u/v_d. \quad (51)
\]

Then \( v_u = v \sin \beta, \ v_d = v \cos \beta, \) and with our conventions \( 0 < \beta < \pi/2 \).

With these definitions the minimization conditions can be written

\[
|\mu|^2 + M_{H_d}^2 = b \tan \beta - \frac{1}{2} M_Z^2 \cos 2\beta \quad (52)
\]

\[
|\mu|^2 + M_{H_u}^2 = b \cot \beta + \frac{1}{2} M_Z^2 \cos 2\beta.
\]

These satisfy the EWSB conditions. They can be used (say) to eliminate \( b \) and \( |\mu|^2 \) in terms of \( \tan \beta \) and \( M_Z^2 \). Note the phase of \( \mu \) is not determined.
These two equations have a special status because they are the only two equations of the entire theory that relate a measured quantity \( M_2^2 \) to soft parameters. If the soft parameters are too large, these equations would require very precise cancellations to keep the \( Z \) mass correct.

We have two Higgs fields, each an SU(2) doublet of complex fields, so 8 real scalars. Three of them are Nambu-Goldstone bosons that are eaten by \( W^\pm, Z \) to become the longitudinal states of the vector bosons, just as in the SM, so 5 remain as physical particles. They are usually classified as 3 neutral ones, \( h, H, A \), and a charged pair, \( H^\pm \). The mass matrix is calculated from \( V \) with \( M_{ij}^2 = \frac{1}{2} \partial^2 V / \partial \phi_i \partial \phi_j \) where \( \phi_{i,j} \) run over the 8 real scalars. Then the eigenvalue equation \( \det | \lambda - M_{ij}^2 | = 0 \) determines the mass eigenstates. This splits into block diagonal 2x2 factors. The factors for the charged states and the neutral one in the basis \( (Im H_u, Im H_d) \) each have one zero eigenvalue, the Nambu-Goldstone bosons. The two CP even neutrals can mix, with mixing matrix

\[
\begin{pmatrix}
h \\
H
\end{pmatrix} = \sqrt{2} \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
Re H_u - v_u \\
Re H_d - v_d
\end{pmatrix}. \tag{53}
\]

The resulting tree level masses are

\[
m_{h,H}^2 = \frac{m_A^2 + M_Z^2}{2} \pm \frac{1}{2} \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}, \tag{54}
\]

\[
m_A^2 = 2b/ \sin 2\beta, \tag{55}
\]

\[
m_{H^\pm}^2 = m_A^2 + M_{W^\pm}^2. \tag{56}
\]

From eq. 54, one can see that if \( m_A^2 \to 0 \) then \( m_h^2 \to 0 \), and if \( m_A^2 \) gets large then \( m_h^2 \to 0 \), so \( m_h^2 \) has a maximum. A little algebra shows the maximum is

\[
m_h^{\text{tree}} \leq |\cos 2\beta| M_Z, \tag{57}
\]

33
where we have emphasized that this maximum does not include radiative corrections. This important result leads to the strongest quantitative test of the existence of supersymmetry, that there must exist a light Higgs boson. If the gauge theory is extended to larger gauge groups there are additional contributions to the tree level mass, but they are bounded too.

There are also significant radiative corrections \[12\]. The Higgs potential has contributions to the $h^4$ term from loops involving top quarks and top squarks. These are not small because the top Yukawa coupling is of order unity and the top-Higgs coupling is proportional to the top mass. To include the effect one has to calculate the contribution to the Higgs potential, reminimize, and recalculate the mass matrix eigenvalues. The result is

$$m_h^2 \lesssim \cos^2 2\beta M_Z^2 + \frac{3\alpha_2}{2\pi} \frac{m_t^4}{m_W^2} \ln \frac{\tilde{m}_t^2}{M_Z^2} \approx M_Z^2 (1 + \frac{1}{4} \ln \frac{\tilde{m}_t^2}{M_Z^2}) \quad (58)$$

where the last equality uses $|\cos 2\beta| = 1$, which is true for $\tan \beta \gtrsim 4$. The contributions from two loops have mainly been calculated and are small but not negligible. This result shows that if $m_h \approx 115$ GeV, it is necessary that the tree level term give essentially the full $M_Z$ contribution, so $\cos^2 2\beta \approx 1$.

If $\tan \beta$ is large the REWSB situation is more complicated. Then the top and bottom Yukawa couplings are approximately equal, so from the RGEs [equations 39-41] we see that $M_H^2, M_{H_d}$ run together, and both can go negative, or the conditions [equations 48,49] may not be satisfied. The EWSB conditions can be rewritten [using equation 55] so one condition is that

$$2m_A^2 \approx M_H^2 - M_{H_u} - M_Z^2. \quad (59)$$

Experimentally, $m_A^2 \gtrsim M_Z^2$ (or $A$ would have been observed at LEP or the Tevatron), so the EWSB condition is that $M_H^2$ must be smaller than $M_{H_u}^2$ by an amount somewhat larger than $M_Z^2$. That allows a narrow window, and preferably the theory would not have to be finely adjusted to allow the REWSB to occur. Also, in this situation the other condition can be written

$$b \approx \frac{M_{H_d}^2 - M_{H_u}^2}{\tan \beta} \sim \frac{M_Z^2}{\tan \beta} \ll M_Z^2 \quad (60)$$
when \( \tan \beta \) is large, and this is a clear fine tuning since the natural scale for \( b \) is of order the typical soft term, presumably of order or somewhat larger than \( M_Z^2 \). So REWSB is possible with large \( \tan \beta \) but it is necessary to explain why this apparent fine tuning occurs. The actual effects of increasing \( \tan \beta \) are complicated. The \( b \) and \( \tau \) Yukawas get larger, so the top and stop and \( m_{H_u,d}^2 \) RGEs change. \( m_{H_u,d}^2 \) get driven more negative, but the larger Yukawas decrease the stop masses, which makes \( m_{H_u}^2 \) less negative, etc.

If \( \tan \beta \) is large, theories with \( M_{H_u}^2 \) and \( M_{H_d}^2 \) split are then favored. That could occur in the unification scale formulation of the theory. One possible way to get a splitting even if \( M_{H_u}^2, M_{H_d}^2 \) start degenerate is via D-terms from extending the gauge theory. D-terms arise whenever a \( U(1) \) symmetry is broken. Under certain circumstances their magnitude may be of order the weak scale even though the \( U(1) \) symmetry is broken at a high scale, and they can contribute if the superpartners are charged under that \( U(1) \) symmetry. If one looks at SO(10) breaking to SU(5) \( \times U(1) \) and the breaking of this \( U(1) \), the soft masses are

\[
\begin{align*}
m_Q^2 &= m_e^2 = m_u^2 = m_{10}^2 + m_D^2 \\
m_L^2 &= m_d^2 = m_3^2 - 3m_D^2 \\
m_{H_d,u}^2 &= m_{10}^2 \pm 2m_D^2.
\end{align*}
\]

The main point for us is that \( m_{H_u}^2 \) and \( m_{H_d}^2 \) are split. The splitting affects the other masses, so in principle \( m_D^2 \) is accessible experimentally if sufficiently many scalar masses can be measured.

Note that because \( b \) is in \( L_{soft} \) it is not protected by a non-renormalization theorem. So to have \( b \) small at the weak scale does not mean it is small at the unification scale. It’s RGE is

\[
16\pi^2 db/dt = b(3Y_{t}^2 - 3g_2^2 + ...) + \mu(6a_t Y_t + 6g_2^2 M_2 + ...)
\]

so if it starts out at zero it is regenerated from the second term, or alternatively cancellations can make it small at the weak scale. Such cancellations would look accidental or fine tuned if one did not know the high scale theory, but the appropriate way to view them would be as a clue to the high scale theory. Similarly, large \( \tan \beta \) would presumably mean that one vev is approximately zero at tree level and a small value is generated for it by
radiative corrections. No theory is currently known that does that, but if an appropriate symmetry can be found that does it will be a clue to the high scale theory.

Before we leave Higgs physics we will derive one Feynman rule to illustrate how that works. From above we write

\[
H_d = v \cos \beta + \frac{1}{\sqrt{2}} (-h \sin \alpha + H \cos \alpha + iA \sin \beta)
\]  
(61)

\[
H_u = v \sin \beta + \frac{1}{\sqrt{2}} (h \cos \alpha + H \sin \alpha + iA \cos \beta).
\]

Then from the covariant derivative term there is the Lagrangian contribution

\[
\frac{g_2^2}{\cos^2 \theta_W} (|H_u|^2 + |H_d|^2) Z^\mu Z_\mu
\]  
(62)

so substituting this gives the hZZ vertex

\[
\frac{g_2^3 v}{2 \cos^2 \theta_W} Z^\mu Z_\mu h (\sin \beta \cos \alpha - \cos \beta \sin \alpha) = \frac{g_2 M_Z}{\cos \theta_W} \sin (\beta - \alpha) Z^\mu Z_\mu h.
\]  
(63)

Similar manipulations give the couplings

|       | h   | H         | A        |
|-------|-----|-----------|----------|
| \(\bar{t}t\), \(\bar{c}c\), \(\bar{u}u\) | \(\cos \alpha / \sin \beta\) | \(\sin \alpha / \cos \beta\) | \(\cot \beta\) |
| \(\bar{b}b\), \(\bar{\tau}\tau\)     | \(-\sin \alpha / \cos \beta\) | \(\cos \alpha / \sin \beta\) | \(\tan \beta\) |
| WW, ZZ            | \(\sin (\beta - \alpha)\) | \(\cos (\beta - \alpha)\)  | 0              |
| ZA     | \(\cos (\beta - \alpha)\)  | \(\sin (\beta - \alpha)\)  | 0              |

The ZAh and ZHA vertices are non-zero, while the Zhh and ZHH vertices vanish; there is no tree level ZW±H± vertex.

Finally, we note that in the supersymmetric limit where the soft parameters become zero one has

\[
V = |\mu|^2 (|H_u|^2 + |H_d|^2) + \frac{g_1^2 + g_2^2}{2} (|H_u|^2 - |H_d|^2)
\]  
(65)

so the minimum is at \(\mu = 0\), \(H_u = H_d\); the latter implies \(\tan \beta = 1\).
20 Yukawa couplings, $\tan \beta$, and theoretical and experimental constraints on $\tan \beta$

It’s important to understand how $\tan \beta$ originates, and what is known about it. At high scales the Higgs fields do not have vevs, so $\tan \beta$ does not exist. The superpotential contains information about the quark and lepton masses through the Yukawa couplings. As the universe cools, at the EW phase transition vevs become non-zero and one can define $\tan \beta = v_u/v_d$. Then quark and lepton masses become non-zero, $m_{q,l} = Y_{q,l}v_{u,d}$.

There are two values for $\tan \beta$ that are in a sense natural. As pointed out just above, the supersymmetric limit corresponds to $\tan \beta = 1$. Typically in string theories some Yukawa couplings are of order gauge couplings, and others of order zero. The large couplings for each family are interpreted as the top, bottom, and tau couplings. If $Y_t \approx Y_b$ then $\tan \beta \sim m_t/m_b$. Numerically this is of order 35, but a number of effects could make it rather larger or smaller, e.g. the values of $m_t$ and $m_b$ change considerably with scale, and with RGE running so $m_t(M_Z)/m_b(M_Z) \sim 50$. Finally $\tan \beta$ is determined at the minimum of the Higgs potential, and can be driven smaller.

There are theoretical limits on $\tan \beta$ arising from the requirement that the theory stay perturbative at high scales (remember, the evidence that the entire theory stays perturbative is both the gauge coupling unification and the radiative EWSB). Requiring that $Y_t = g_2 m_t/\sqrt{2} M_W \sin \beta$ not diverge puts a lower limit on $\sin \beta$ which corresponds to $\tan \beta \gtrsim 1.2$ when done in the complete theory, and similarly $Y_b = g_2 m_b/\sqrt{2} M_W \cos \beta$ leads to $\tan \beta \lesssim 60$. This upper limit is probably reduced by REWSB.

There are no measurements of $\tan \beta$, and as I emphasize below it is not possible to measure $\tan \beta$ at a hadron collider in general. Perhaps we will be lucky and find ourselves in a part of parameter space where such a measurement is possible, or more likely, a combination of information from (say) $g_\mu - 2$ and superpartner masses will lead to at least useful constraints on $\tan \beta$. LEP experimental groups have claimed lower limits on $\tan \beta$ from the absence of superpartner signals, but those are quite model dependent and do not hold if phases are taken into account. Similarly, there is a real lower limit on $\tan \beta$ from the absence of a Higgs boson below 115 GeV, as explained above and in Section 22. That limit is about 4 if phases are not included, but lower when they are.
21 In what sense does supersymmetry explain EWSB?

Understanding the mechanism of EWSB, and its implications, is still the central problem of particle physics. Does supersymmetry indeed explain it? If so, the explanation depends on broken supersymmetry, and we have seen that in the absence of supersymmetry breaking the EW symmetry is not broken. That’s OK. An explanation in terms of supersymmetry moves us a step closer to the primary theory. Historically we have learned to go a step at a time, steadily moving toward more basic understanding. If we think of supersymmetry as an effective theory at the weak scale only, then we would expect the sense in which it explains EWSB to be different from that we would find if we think of low energy supersymmetry as the low energy formulation of a high scale theory. That is, top-motivated bottom-up is different from bottom-up. It should be emphasized that one could have supersymmetry breaking without EWSB, but not EWSB without supersymmetry breaking.

It may clarify the issues to first ask what needs explanation. We can explicitly list

1. Why are there Higgs scalar fields, i.e. scalars that carry SU(2)\times U(1) quantum numbers, at all?
2. Why does the Higgs field get a non-zero vev?
3. Why is the vev of order the EW scale instead of a high scale?
4. Why does the Higgs interact differently with different particles, in particularly different fermions?

Let us consider these questions.

At least scalars are naturally present in supersymmetric theories, and generally carry EW quantum numbers, whereas in the SM scalars do not otherwise occur. If we connect to a high scale theory, some (most) explicitly have SM-like Higgs fields, e.g. in the E_6 representation of heterotic string theories. Basically as long as we view supersymmetry as embedded in a high scale theory we will typically have Higgs scalars present, though not in all possible cases. That in turn can point to the correct high scale theory.

We have seen that the RGE running naturally does explain the origin of the Higgs vev if the soft-breaking terms and $\mu_{\text{eff}}$ are of order the weak scale, and if one Yukawa coupling is of order the gauge couplings. If we view the theory as a low energy effective theory we have seen that we do not know why $\mu$ in the superpotential is zero, but if we view the theory as embedded
in a string theory then it is natural to have $\mu = 0$ in the superpotential. We referred to this as string boundary conditions. Then how $\mu_{\text{eff}}$ is generated points toward the correct high scale theory. If $\mu_{\text{eff}}$ is of order the weak scale then it is appropriate to explain the Higgs mechanism and gauge coupling unification. Similarly, the mechanism of supersymmetry breaking has to give soft masses of order the weak scale if supersymmetry explains (or, as some prefer to say, predicts) gauge coupling unification.

In a string theory, for example, we expect some Yukawa couplings to be of order the gauge couplings. We identify one of those with the top quark. Then the running of $M_{H_u}^2$ is fast and it is driven negative, or decreases sufficiently, to imply the non-zero Higgs vev. The relevant soft-breaking terms, particularly $M_{H_u}^2$ and $M_{H_d}^2$, must be of order the weak scale. The theory accommodates different couplings for all the fermions. It does not explain the numerical values of the masses, but allows them to be different — that is non-trivial.

So a complete explanation requires thinking of supersymmetry as embedded in a deeper theory such as string theory (so scalar fields exist in the theory, and $\mu \approx 0$, and the top Yukawa is of order 1), and requires that the soft terms are of order the weak scale after supersymmetry is broken. If we only think of supersymmetry as a low energy effective theory not all of these elements are present, so the explanation is possible but incomplete. It is not circular to impose soft-breaking parameters of order the weak scale to explain the EWSB since one is using supersymmetry breaking to explain EW breaking, which is important progress — that is how physics has increased understanding for centuries.

It is also very important to note that the conditions on the existence of Higgs and on $\mu$ and on the soft parameters are equally required for the gauge coupling unification — if they do not hold in a theory then it will not exhibit gauge coupling unification. The explanation of EWSB requires in addition to the conditions for gauge coupling unification only that there is a Yukawa coupling of order the gauge couplings, i.e. a heavy top quark, which is a fact.

Perhaps it is amusing to note that two families are needed to have both a heavy fermion so the EW symmetry is broken, and light fermions that make up the actual world we are part of. No reasons are yet known why a third family is needed — it is clear that CP violation could have arisen from soft phases with two families, and does not require the three family SM.

Now that we have developed some foundations we turn to applications in several areas.
Current and forthcoming Higgs physics

There are two important pieces of information about Higgs physics that both independently suggest it will not be too long before a confirmed discovery. But of course it is such an important question that solid data is needed.

The first is the upper limit on $m_h$ from the global analysis of precision LEP (or LEP + SLC + Tevatron) data [14]. Basically the result is that there are a number of independent measurements of SM observables, and every parameter needed to calculate at the observed level of precision is measured except $m_h$. So one can do a global fit to the data and determine the range of values of $m_h$ for which the fit is acceptable. The result is that at 95% C.L. $m_h$ should be below about 200 GeV. The precise value does not matter for us, and because the data really determines $\ln m_h$, the sensitivity is exponential so it moves around with small changes in input. What is important is that there is an upper limit. The best fit is for a central value of order 100 GeV, but the minimum is fairly broad. The analysis is done for a SM Higgs but is very similar for a supersymmetric Higgs over most of the parameter space.

In physics an upper limit does not always imply there is something below the upper limit. Here the true limit is on a contribution to the amplitude, and maybe it can be faked by other kinds of contributions that mimic it. But such contributions behave differently in other settings, so they can be separated. If one analyzes the possibilities [15] one finds that there is a real upper limit of order 450 GeV on the Higgs mass, if (and only if) additional new physics is present in the TeV region. That new physics or its effects could be detected at LHC and/or a 500 GeV linear electron collider, and/or a higher intensity $Z$ factory ("giga-$Z$") that accompanies a linear collider. So the upper limit gives us very powerful new information. If no other new physics (besides supersymmetry) occurs and conspires in just the required way with the heavier Higgs state, the upper limit really is about 200 GeV.

The second new information is a possible signal from LEP [16] in its closing weeks for a Higgs boson with $m_h=115$ GeV. The ALEPH detector was the only group to do a blind analysis, and it is technically a very strong detector, so its observation of about a 3σ signal is important information. It was not possible to run LEP to get enough more data to confirm this signal. Fortunately, its properties are nearly optimal for early confirmation at the Tevatron, since its mass is predicted, and cross section and branching ratio to $b\bar{b}$ are large. Less is required to confirm a signal in a predicted mass bin than to find a signal of unknown mass, so less than 10 $fb^{-1}$ of integrated
luminosity will be required if the LEP signal is indeed correct. If funding and the collider and the detectors all work as planned confirming evidence for $h$ could come in 2004.

Suppose the LEP $h$ is indeed real. What have we learned [17]? Most importantly, of course, that a point-like, fundamental Higgs boson exists. It is point-like because its production cross section is not suppressed by structure effects. It is a new kind of matter, different from the century old matter particles and gauge bosons. It completes the SM, and points to how to extend the SM. It confirms the Higgs mechanism, since it is produced with the non-gauge-invariant $ZZh$ vertex, which must originate in the gauge-invariant $ZZhh$ vertex with one $h$ having a vev.

The mass of 115 GeV also tells us important information. First, the Higgs boson is not a purely SM one, since the potential energy would be unbounded from below at that mass. Basically the argument is that one has to write the potential with quantum corrections, and the corrections from fermion loops dominate because of the heavy top and can be negative if $m_h$ is too small. The SM potential is

$$V(h) = -\mu^2 h^2 + \left\{ \lambda + \frac{3M_Z^4 + 6M_W^4 + m_h^4 - 12m_t^4}{64\pi^2 v^4} \ln( ) \right\} h^4, \quad (66)$$

where the argument of the ln is some function of the masses larger than one. In the usual way $\lambda = m_h^2/2v^2$. The second term in the brackets is negative, so $\lambda$ and therefore $m_h$ has to be large enough. The full argument has to include higher loops, thermal corrections, a metastable universe rather than a totally stable one, etc., and requires $m_h$ to be larger than about 125 GeV if $h$ can be a purely SM Higgs boson.

Second, 115 GeV is an entirely reasonable value of $m_h$ for supersymmetry, but only if $\tan\beta$ is constrained to be larger than about 4. That is because as described above, the tree level contribution is proportional to $|\cos 2\beta|$ and to get a result as large as 115 it is necessary that $|\cos 2\beta|$ be essentially unity, giving a lower limit on $\tan\beta$ of about 4. Even then the tree level can only contribute a maximum of $M_Z$ to $m_h$. The rest comes from the radiative corrections, mainly the top loop. Numerically one gets

$$m_h^2 \approx (91)^2 + (40)^2 \left\{ \ln \frac{m_t^2}{m_t^2} + \ldots \right\} \quad (67)$$
where $m_t^2$ is an appropriate average of the two stop mass eigenstates. The second term must supply about 25 GeV, which is quite reasonable.

The LEP signal, assuming it is correct, can only provide us a limited amount of information since it only supplies two numbers, $m_h$ and $\sigma \times BR$. The full Higgs potential depends on at least 7 parameters \[\text{[8]},\] so none of them can be explicitly measured. Because the potential depends on the stop loops, it depends on the hermitean stop mass matrix (equation 69 below).

Since the elements are complex, in general the loop contributions to the Higgs potential will be complex, so the potential will have to be re-minimized taking into account the possibility of a relative phase between the Higgs vevs. One can write

$$H_d = \frac{1}{\sqrt{2}} \left( v_d + h_d + i a_d \right), \quad H_u = e^{i \theta} \frac{1}{\sqrt{2}} \left( v_u + h_u + i a_u \right).$$ \hspace{1cm} (68)

At the minimum of the potential it turns out that $\theta$ cannot be set to zero or absorbed by redefinitions. The resulting $\theta$ is a function of the phase of $\mu$, $\phi_\mu$, and of the phase(s) in $a_t$ (and of course of other parameters). Thus $m_h$, and $\sigma_h \times BR(b\bar{b})$ are functions of the magnitudes of $\mu$ and $a_t$, $m_Q^2$, $m_u^2$, $b$, $\tan \beta$, and the physical phase(s) $\phi_\mu + \phi_{a_t}$ at least. Since some of these are matrices they can involve more than one parameter. Also, if $\tan \beta$ is large there will be important sbottom loops, and chargino and neutralino loops can contribute. So only in special cases can data about the Higgs sector be inverted to measure $\tan \beta$ and the soft parameters, and only then if there are at least 7 observables.

If $\theta$ is significant then even and odd CP states mix and there are 3 mixed neutral states which could all show up in the $b\bar{b}$ or $\gamma\gamma$ spectrum, and those spectra could show different amounts of the three mass eigenstates. Both cross section and BR for the lightest state can be different from the SM and from the CP conserving supersymmetry case.

One can check that the phase can be very important. For example, if a Higgs is observed at LEP and the Tevatron one can ask what region of parameter space is consistent with a given mass and $\sigma_h \times BR(b\bar{b})$. The answer is significantly different, for example for $\tan \beta$, if the phase is included. Or if no Higgs is observed one can ask what region of parameters is excluded. If the phase is included the actual limit on $m_h$ is about 10% lower than the published limits from LEP, below 100 GeV. Similarly, lower values of $\tan \beta$ are allowed if phases are included than those reported by LEP experimenters.
23 The stop mass matrix

Arranging the stop mass terms from the Lagrangian in the form

\[(\bar{t}_L^* t_R^*) m_i^2 \left( \begin{array}{c} \bar{t}_L \\ t_R \end{array} \right),\]

the resulting Hermitian stop mass matrix is

\[m_i^2 = \left( \begin{array}{cc} m_{Q_3}^2 + m_t^2 + \Delta_u & v(a_t \sin \beta - \mu Y_t \cos \beta) \\ m_u^2 + m_t^2 + \Delta_u & m_{Q_3}^2 + m_t^2 + \Delta_u \end{array} \right). \quad (69)\]

The \(\Delta's\) are D-terms, from the \((\phi^* T \phi)^2\) piece of the Lagrangian — \(\Delta_u = (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) \cos 2\beta M_Z^2\), \(\Delta_u = \frac{2}{3} \sin^2 \theta_W \cos 2\beta M_Z^2\). These EW D-terms are proportional to the \(T_3\) and hypercharge charges. The pieces proportional to \(\sin^2 \theta_W\) come from the breaking of the \(U(1)\) symmetry and vanish if \(\sin^2 \theta_W \rightarrow 0\). The \(m_t^2\) comes from the F-terms in the scalar potential, \(Y_t^2 H_0^* H_0 \bar{t}_L^* \bar{t}_L\) and a similar term for \(\bar{t}_R\), when the Higgs get vevs. F-terms in \(V\) also give the term \(-\mu Y_t \bar{t}_R^* H_0^*\) which gives the second term in the 12 position when \(H_0^d\) gets a vev. The soft term \(a_t \bar{t}_R H_0^* Q_3^* H_0\) gives the first 12 term when the Higgs gets a vev. Similar mass matrices are written for all the squarks and sleptons. For the lighter ones the Yukawas and possibly the trilinears are small, and the fermion masses are small, so only the diagonal elements are probably large. Each of the elements above is a \(3 \times 3\) matrix, so \(m_t^2\) is a \(6 \times 6\) matrix. \(a_t\) and \(\mu\) and even \(v\) are in general complex.

24 What can be measured in the Higgs sector?

Assuming the LEP signal is indeed valid, as suggested particularly by the ALEPH blind experiment, and it is confirmed at the Tevatron, what can we eventually learn? I will focus on the Tevatron and LHC since they will be our only direct sources of Higgs information in the next decade. The Tevatron can use the \(WW h, ZZ h\) channels. In addition once \(m_h\) is known the inclusive channel, with about a \(pb\) cross section, can be used. If the total cross section at the Tevatron for Higgs production is 1.5 \(pb\), and each
detector gets 15 fb$^{-1}$ of integrated luminosity, the total number of Higgs bosons produced is about 45,000 in a known mass bin. At some level it will be possible to measure $g_{WW}g_{bb}$ and $g_{ZZ}g_{bb}$ from $\sigma x BR$ for the WW$h$ and ZZ$h$ channels, so their ratio tests whether $h$ couples to gauge bosons proportional to mass. Once $m_h$ is known it will be possible to see $h \rightarrow \tau \bar{\tau}$ in both inclusive production and associated production with a $W$, and test if the coupling to fermions is proportional to mass. A similar test comes from not seeing $h \rightarrow \mu \bar{\mu}$ (or seeing a few events of this mode since it should occur a bit below the $10^{-3}$ level). The inclusive production is dominantly via a top loop so it measures $g_{th}$ indirectly, and this is complicated since superpartner loops contribute as well as SM ones. It may be possible to see the $ttth$ final state directly [19]. Since $BR(\gamma\gamma)$ is at the $10^{-3}$ level an observation or useful limit will be possible here if the resolution is good enough. All of these can give very important tests of what the Higgs sector is telling us.

It is also interesting to ask if data can distinguish a SM Higgs from a supersymmetric one, though most likely there will be signals of superpartners as well as a Higgs signal so there will not be any doubt. If $\tan \beta$ is large the ratio of $bb$ to $\tau \bar{\tau}$ is sensitive to supersymmetric-QCD effects and can vary considerably from its tree level value [20]. The ratio of top to bottom couplings is sensitive to ways in which the supersymmetric Higgs sector varies from the SM one. If $\tan \beta$ is large and $m_A$ is less than about 150 GeV it is possible $A$ can be observed at the Tevatron. Altogether, the Tevatron may be a powerful Higgs factory if it takes full advantage of its opportunities. It is still unlikely that there will be enough independent measurements at the Tevatron to invert the equations relating the soft parameters and $\tan \beta$ to observables. The lighter stop mass eigenstate $\tilde{t}_1$ may be observable at the Tevatron, and provide another observable for the Higgs sector.

At LHC it is very hard to learn much about the lightest Higgs $h$ if its mass is of order 115 GeV. It will most likely be observed in the inclusive production and decay to $\gamma\gamma$, but observation in the $\gamma\gamma$ mode does not tell us much about the Higgs physics once the Higgs boson has been discovered, which will have occurred if indeed $m_h \approx 115$ GeV. The $\gamma\gamma$ mode does not demonstrate the Higgs mechanism is operating since it occurs for any scalar boson. The SM does have a definite prediction for $BR(\gamma\gamma)$ from the top and $W$ loops, and superpartner loops can be comparable, so a measurement would be very interesting. Note that one cannot assume the $\gamma\gamma$ BR is known.

Maybe it will be possible to detect the $\tau \bar{\tau}$ mode at LHC using WW fusion to produce h and tagging the quarks [21]. This mode also confirms
the non-gauge-invariant $WWh$ vertex. Considerable additional information about the Higgs sector may come from observing the heavier Higgs masses and $\sigma \times BR$, and the heavier stop. Since $A \to \gamma \gamma$ but not to $ZZ, WW$ it may be possible to see $A$ if it is not above the $t\bar{t}$ threshold. Decays of the heavy Higgs to $\tau'$s are enhanced if $\tan \beta$ is large. Note that one cannot assume only SM decays of $h$ in analysis since channels such as $h \to \text{LSP} + \text{LSP}$ are potentially open and can have large BR since they are not suppressed by factors such as $m_b^2/M_W^2$. The combined data from the Tevatron and LHC may provide enough observables to invert the Higgs sector, at least under certain reasonable and checkable assumptions.

25 Charginos

The lightest superpartners are likely to be the neutralinos and charginos, possibly the lighter stop, and the gluino. Their mass matrices have entries from the higgsino-gaugino mixing once the SU(2)$\times$U(1) symmetry is broken, so the mass eigenstates are mixtures of the symmetry eigenstates. When phases are neglected these matrices are described in detail in many places so I will not repeat that here. However, it is worth looking at the most general case including phases for several instructive reasons. The chargino mass matrix follows from the $L_{soft}$, in the wino-higgsino basis:

$$M_{\tilde{C}} = \begin{pmatrix}
M_2 e^{i\phi_2} & \sqrt{2}M_W \sin \beta \\
\sqrt{2}M_W \cos \beta & \mu e^{i\phi_\mu}
\end{pmatrix}. \quad (70)$$

The situation is actually more subtle — this is a submatrix of the actual chargino mass matrix, but this contains all the information — and the reader should see Martin or earlier reviews for details. Also, the off-diagonal element can be complex too since it arises from the last term in eq.14 when the Higgs gets a vev, and the vev can be complex as explained above; I will just keep the phases of $M_2$ and $\mu$ here. The masses of the mass eigenstates are the eigenvalues of this matrix. To diagonalize it one forms the hermitean matrix $M^\dagger M$. The easiest way to see the main points are to write the sums and products of the mass eigenstates,

$$M_{\tilde{C}_1}^2 + M_{\tilde{C}_2}^2 = Tr M_{\tilde{C}}^\dagger M_{\tilde{C}} = M_2^2 + \mu^2 + 2M_W^2, \quad (71)$$

45
\[
M_{C_1}^2 M_{C_2}^2 = \text{det} M_C^\dagger M_C
\]

\[
= M_2^2 \mu^2 + 2M_W^4 \sin^2 2\beta - 2M_W^2 M_2 \mu \sin 2\beta \cos(\phi_2 + \phi_\mu)
\]

Experiments measure the masses of the mass eigenstates. One thing to note is that the masses depend on the phases \(\phi_2\) and \(\phi_\mu\), even though there is no CP violation associated with the masses. Often it is implicitly assumed that phases can only be measured by observing CP-violating effects, but we see that is not so. The combination \(\phi_2 + \phi_\mu\) is a physical phase, invariant under any reparameterization of phases, as much a basic parameter as \(\tan \beta\) or any soft mass.

If one wants to measure the soft masses, \(\mu\), \(\tan \beta\), \(\phi_2 + \phi_\mu\) it is necessary to invert such equations. Since there are fewer observables than parameters to measure, additional observables are needed. One can measure the production cross sections of the mass eigenstates. But then additional parameters enter since exchanges of sneutrinos (at an electron collider) or squarks (at a hadron collider) contribute. One can decide to neglect the additional contributions, but then one is not really doing a measurement. If one “measures” \(\tan \beta\) from the above equations by setting the phase to zero, as is usually done, the result is different from that which would be obtained if the phase were not zero. When the phases are present the phenomenology, and any deduced results, can be quite different. We saw that for the Higgs sector above. It is studied for the chargino sector in ref. [22]. Similar arguments apply for the neutralino mass matrix.

One implication of this analysis is that \(\tan \beta\) is not in general measurable at a hadron collider — there are simply not enough observables [23]. One can count them, and the equations never converge. Depending on what can be measured, by combining observables from the chargino and neutralino sectors, and the Higgs sector, it may be possible to invert the equations. This is a very strong argument [23] for a lepton collider with a polarized beam, where enough observables do exist if one is above the threshold for lighter charginos and neutralinos, because measurements with different beam polarizations (not possible at a hadron collider) double the number of observables, and measurements with different beam energies (not possible at a hadron collider) double them again. The precise counting has to be done carefully, and quadratic (and other) ambiguities and experimental errors mean that one must do a thorough simulation [24] to be sure of what is needed, but there
appear to be sufficient observables to measure the relevant parameters. The issue of observing the fundamental parameters of $L_{soft}$ is of course broader, as discussed in Section 17. There are 33 masses in the MSSM including the gravitino, but 107 new parameters in $L_{soft}$ (including the gravitino). The rest are flavor rotation angles and phases. Many can be measured by combining data from a linear electron collider above the threshold for a few superpartners and hadron colliders. It is also necessary to include flavor changing rare decays to measure the off-diagonal elements of the sfermion mass matrices and the trilinear couplings.

26 Neutralinos

In a basis $\Psi^0 = (\tilde{B}, \tilde{W}_3, \tilde{H}_d, \tilde{H}_u)$ terms in the Lagrangian can be rearranged into $-\frac{1}{2}(\Psi^0)^T M_{\tilde{N}} \Psi^0$ with the symmetric

$$M_{\tilde{N}} = \begin{pmatrix}
M_1 e^{i\phi_1} & 0 & -\frac{g_1}{\sqrt{2}} H_d^0 & \frac{g_2}{\sqrt{2}} H_u^0 \\
0 & M_2 e^{i\phi_2} & \frac{g_2}{\sqrt{2}} H_d^0 & -\frac{g_1}{\sqrt{2}} H_u^0 \\
-\frac{g_1}{\sqrt{2}} H_d^0 & \frac{g_2}{\sqrt{2}} H_d^0 & \mu & 0 \\
\frac{g_2}{\sqrt{2}} H_u^0 & -\frac{g_1}{\sqrt{2}} H_u^0 & 0 & -\mu e^{i\phi_\mu}
\end{pmatrix}.$$ 

Although the elements are complex, this matrix can still be diagonalized by a unitary transformation. Its form in a basis $\Psi' = (\tilde{\gamma}, \tilde{\tilde{Z}}, \tilde{h}_s, \tilde{h}_a)$ is sometimes useful:

$$M_{\tilde{N}} = \begin{pmatrix}
M_1 s^2_W + M_2 c^2_W & (M_1 - M_2) s_W c_W & 0 & 0 \\
(M_1 - M_2) s_W c_W & M_1 s^2_W + M_2 c^2_W & M_Z & 0 \\
0 & M_Z & \mu \sin 2\beta & -\mu \cos 2\beta \\
0 & 0 & -\mu \sin 2\beta & -\mu \sin 2\beta
\end{pmatrix}.$$ 

If $M_1 \approx M_2$ and/or if $\tan \beta$ is large (so $\sin 2\beta \approx 0$) this takes a simple form.

The lightest neutralino is the lightest eigenvalue of this, and may be the LSP. Its properties then determine the relic density of cold dark matter (if the LSP is indeed the lightest neutralino). It also largely determines the collider signatures for supersymmetry. It will be a linear combination of the basis states,

$$\tilde{N}_1 = \alpha \tilde{B} + \beta \tilde{W}_3 + \gamma \tilde{H}_d + \delta \tilde{H}_u$$
with $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$.

An interesting limit that is at least pedagogically instructive arises if we take $M_1 \approx M_2$ (at the EW scale) and $\tan \beta \approx 1$, and $\mu < M_Z$. Then $\tilde{N}_1 \approx \tilde{h}$, where $\tilde{h} = \tilde{h}_d \sin \beta + \tilde{h}_u \cos \beta$, and $M_{\tilde{N}_1} \approx \mu$. $\tilde{N}_2 \approx \tilde{\gamma}$, with $M_{\tilde{N}_2} \approx M_2$, $\alpha \approx -\beta \approx -45^\circ$ so $\cos(\alpha - \beta)$ and $\cos 2\beta \to 0$. At tree level the $Z\tilde{\gamma}\tilde{h}$, $h\tilde{\gamma}\tilde{h}$, and $Z\tilde{h}\tilde{h}$ vertices vanish, and the dominant decay of the second neutralino is $\tilde{N}_2 \to \tilde{N}_1 + \gamma$. $M_{\tilde{C}_1} \gtrsim M_{\tilde{N}_2}$.

\section{27 Effects of phases}

The effects of phases have been considered much less than the masses. As we saw above for charginos and the Higgs sector they affect not only CP-violating observables but essentially all observables. They can have significant impacts in a variety of places, including $g_\mu - 2$, electric dipole moments (EDMs), CP violation in the K and B systems, the baryon asymmetry of the universe, cold dark matter, superpartner production cross sections and branching ratios, and rare decays. We do not have space to give a complete treatment, but only to make some points about the importance of the observations and what they might teach us about physics beyond the SM in general; while we focus to some extent on the phases because they are usually not discussed, our concern is relating them to the entire $L_{\text{soft}}$.

There are some experiments that suggest some of the phases are small, mainly the neutron and electron EDMs. On the other hand, we know that the baryon asymmetry cannot be explained by the quark CKM phase, so some other phase(s) are large, and the soft phases are good candidates. Recently it has been argued that very large phases are needed if baryogenesis occurs at the EW phase transition [23]; see also [24]. Further, there is no known symmetry or basic argument that the soft phases in general should be small. If the outcome of studying how to measure them was to demonstrate that some were large that could be very important because both compactification and supersymmetry breaking would have to give such large phases. The phase structure of the effective soft Lagrangian at the weak scale and at the unification scale are rather closely related, so it may be easier to deduce information about the high scale phases from data than about high scale parameters in general. If the outcome of studying how to measure the phases was to demonstrate that the phases were small that would tell us different but very important results about the high scale theory. It would also greatly

48
simplify analyzing weak scale physics, but that is not sufficient reason to assume the phases are small.

\[ g_\mu - 2 \]

In early 2001 it was reported that the anomalous magnetic moment of the muon was larger than the SM prediction by a significant amount. The experiment is now analyzing several times more data than the original report was based on, and the SM theory is being reexamined.

Even if the effect disappears, it is worth considering \( g - 2 \) experiments, because in a supersymmetric world the entire anomalous moment of any fermion vanishes if the supersymmetry is unbroken, so magnetic moments are expected to be very sensitive to the presence of low energy supersymmetry, and particularly of broken supersymmetry. The analysis can be done in a very general and model independent manner [27], and illustrates nicely how one can say a great deal with supersymmetry even though it seems to have a number of parameters. So it is also pedagogically interesting. There are only two supersymmetric contributions, a chargino-sneutrino loop and a smuon-neutralino loop. One can see that starting from the complete theory, with no assumptions beyond working in the MSSM, there are only 11 parameters that can play a role out of the original set of over 100,

\[
|M_2|, |M_1|, |\mu|, |A_\mu|, m_{\tilde{\mu}_L}, m_{\tilde{\mu}_R}, m_{\tilde{\nu}}, \tan \beta, \phi_2 + \phi_\mu, \phi_1 + \phi_\mu, \phi_A + \phi_\mu.
\]

In the general case all 11 of them can be important, and the experimental result will give a complicated constraint among them. But if we ask about putting an upper limit on superpartner masses, which would be of great interest, we can say more. For larger masses one can see that the chargino-sneutrino diagram dominates, and in addition that it is proportional to \( \tan \beta \); The \( \tan \beta \) factor arises from the needed chirality flip on a chargino line. Thus only the magnitudes of \( M_2 \), and \( \mu \), \( \tan \beta \), \( m_{\tilde{\nu}} \) and the phase enter in this limit. If we illustrate the result by assuming a common superpartner mass \( \tilde{m} \) (just for pedagogical reasons, not in the actual calculations), we find that

\[
a^{\text{susy}}_\mu / a^{\text{SM}}_\mu \approx \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \tan \beta \cos(\phi_2 + \phi_\mu). \tag{73}
\]

Further, to put upper limits on the masses we can take the phase to be zero since it turns out to enter only in the above form under these assumptions.
And if we express results in terms of the lighter chargino mass rather than $M_2$ and $\mu$ we can eliminate one parameter; for a given chargino mass there will be ranges of $M_2$ and $\mu$. So we are down to three parameters, with no uncontrolled approximations or assumptions. We will not focus on details of the data here since the new data in 2002 will in any case require a new analysis. If the effect persists there will be significant upper limits on the superpartner masses. Note the relevant physical phase here is $\phi_2 + \phi_\mu$.

It is interesting to consider the supersymmetry limit so the supersymmetric SM contribution vanishes. In that limit the two lighter neutralino masses vanish, and their contribution cancels the photon contribution, the two heavier neutralino masses become $M_Z$ and their contribution cancels that of the $Z$, and the two charginos have $M_W$ and cancel the $W$ contribution. Since the chargino has a sign opposite to that of the $W$ in the supersymmetric limit but the same sign for the broken supersymmetry physical situation it is important to check that indeed the piece proportional to $\tan \beta$ does change sign as needed.

29 Electric dipole moments

In the SM electric dipole moments are unobservably small, of order $10^{-33}e$ cm. That is basically because they are intrinsically CP-violating quantities, and for CP violation to occur in the SM it is necessary for all three families to affect the quantity in question. Otherwise one could rotate the CKM matrix in such a way that the phase did not occur in the elements that contributed. So it must be at least a two-loop suppression. There must also be a factor of the electron or quark mass because of a chirality flip, with the scale being of order $M_W$. In addition there is a GIM suppression. Interpreting results will be complicated because the neutron, and any nuclear EDM, can have a contribution from strong CP violation, while the electron can only feel effects from EW interactions.

Naively, the EDM is the imaginary part of a magnetic moment operator, and the real part is the magnetic moment. So EDMs can arise from the same diagrams as $g_\mu - 2$, but for the electron and for quarks (in neutrons). It is more complicated in reality because the part of the amplitude that has an imaginary part may not give the dominant contribution to the magnetic moment. It has been known for a long time that if the soft phases were of or-
under unity and if all contributions were independent, then the supersymmetry contributions to EDMs are too large by a factor of order 50. However, over a significant part of parameter space various contributions can cancel. Some of that cancellation is generic, e.g. between chargino and neutralino in the electron EDM because of the relative minus sign in eq.32. The smallness of EDMs may be telling us that the soft phases are small. Then we need to find out why they are small. Or it may be telling us that cancellations do occur. Cancellations look fine tuned from the point of view of the low energy theory, but small phases look fine tuned too. Relations among soft parameters in the high scale theory will look fine-tuned in the low scale theory if we do not know the origin of those relations. If $\tan \beta$ is very large cancellations become unlikely since the chargino contribution will dominate the eEDM just as it does for $g_\mu - 2$, but if $\tan \beta$ is of order 4-5 the situation has to be studied carefully.

30 Measuring phases at hadron colliders

Phases, as well as soft masses, can affect distributions at colliders. We briefly illustrate that here for an oversimplified model [20]. Consider gluino production at a hadron collider. The Lagrangian contains a term

$$M_3 e^{i\phi_3} \lambda_\tilde{g} \lambda_\tilde{g} + c.c. \quad (74)$$

It is convenient to redefine the fields so the phase is shifted from the masses to the vertex, so one can write $\psi_{\tilde{g}} = e^{i\phi_3/2} \lambda_{\tilde{g}}$. Then writing the Lagrangian in terms of $\psi$ the vertices $q\bar{q}\tilde{g}$ get factors $e^{\pm i\phi_3/2}$. The production cross sections for gluinos, for example from $q + \bar{q} \to \tilde{g} + \tilde{g}$ by squark exchange, have factors $e^{+i\phi_3/2}$ at one vertex and $e^{-i\phi_3/2}$ at the other, so they do not depend on the phase. That is clear from general principles since $\phi_3$ is not by itself a physical, reparameterization-invariant phase. But gluinos always decay, and for example in the decay $\tilde{g} \to q + \bar{q} + \tilde{B}$ mediated by squarks there is a factor of $e^{i\phi_3/2}$ at the $q\bar{q}\tilde{g}$ vertex and a factor $e^{i\phi_1/2}$ at the $q\bar{q}\tilde{B}$ vertex, so the rate depends on the physical relative phase $\phi_3 - \phi_1$. In general it is more complicated with all the relative phases of the neutralino mass matrix entering. In this simple example the experimental distribution in Bino energy is
\[ d\sigma/dx \sim m_\tilde{g}^4 \left( \frac{1}{\tilde{m}_L^4} + \frac{1}{\tilde{m}_R^4} \right) \times \]

\[ [x - 4x^2/3 - 2y^2/3 + y(1 - 2x + y^2) \cos(\phi_3 - \phi_1)] \]

where \( x = E_{\tilde{B}}m_\tilde{g} \) and \( y = m_{\tilde{B}}m_\tilde{g} \). Other distributions are also affected. If one tries to obtain information from gluino decay distributions without taking phases into account the answers will be misleading if the phases are not small. The same result is of course true for many superpartner decays. It is important to realize that the same phases are appearing here as appear for example in studying \( \varepsilon \) and \( \varepsilon' \) in the kaon system or in \( b \rightarrow s + \gamma \).

### 31 LSP cold dark matter

If it is stable, the LSP is a good candidate for the cold dark matter of the universe. Historically, it is worth noting that this was noticed before we knew that non-baryonic dark matter was needed to understand large scale structure. It is a prediction of supersymmetry. We discussed above why we expected R-parity or a similar symmetry to hold, with the stability of the LSP as one of its consequences. Then the basic argument is simple. As the universe cools, soon after the EW phase transition all particles have decayed except photons, \( e^\pm, u^\pm, d^\pm, \) neutrinos, and LSPs. The quarks form baryons, which join with electrons to make atoms. The relic density of all but LSPs is known to be \( \Omega_{SM} < 0.05 \), while \( \Omega_{\text{matter}} \approx 1/3 \). The LSPs annihilate as the universe cools, with a typical annihilation cross section \( \sigma_{\text{ann}} \sim \rho_{\text{LSP}} G_F^2 E^2 \), and in the early universe \( E \sim T \). The expansion rate is governed by the Hubble parameter \( H \sim T^2/M_{Pl} \). The LSPs freeze out and stop annihilating when their mean free collision path is of order the horizon, so \( \sigma_{\text{ann}} \sim H \). This gives a density \( \rho_{\text{LSP}} \sim 1/M_{Pl} G_F^2 \sim 10^{-9} \text{ GeV}^3 \). At freeze-out \( T \sim 1 \text{ GeV} \), and \( \rho_s \) is of order the entropy \( S \sim T^3 \sim 1 \text{ GeV}^3 \); so \( \rho_{\text{LSP}}/\rho_s \sim 10^{-9} \), similar to the density of baryons. Thus \( \Omega_{\text{LSP}} \sim (M_{\text{LSP}}/M_{\text{proton}}) \Omega_{\text{baryon}} \). Quantitative calculations in many models confirm this.

But the actual calculations of the relic density depend on several soft parameters such as masses of sleptons and gauginos, and also on \( \tan \beta \) and on soft phases. In the absence of measurements or a theory that can convincingly
determine all of these, we cannot in fact say more than that qualitatively the LSP is a good candidate, even if WIMPs are apparently discovered. Since we have argued above that in practice it is unlikely that \( \tan \beta \) will be measured accurately at hadron colliders (though we may be lucky with \( g_\mu - 2 \) plus hadron colliders), it may be difficult to compute \( \Omega_{\text{matter}} \) accurately even after LSPs are detected. It should be emphasized that detection of LSPs is not sufficient to argue they are actually providing the cold dark matter — LSPs could be detected in direct experiments scattering off nuclei, and in space based searches, and at colliders even if \( \Omega_{\text{LSP}} \lesssim 0.05 \). Alternatively, they could make up the CDM even though they were not detected in direct and space based experiments.

Further, in recent years it has come to be understood that LSPs may be produced dominantly by processes that are not in thermal equilibrium rather than the equilibrium process described above. In that case the relic density is not so simply connected to the LSP nature.

\section*{32 Comments on relating CP violation and string theory; could the CKM phase be small?}

Where does CP violation originate? Can the pattern of CP phenomena give us important clues to formulating and testing string theory? Very little work has been done about the fundamental origins of CP violation. In 1985 Strominger and Witten discussed how to define CP transformations in string theory, and in 1993 Dine, Leigh, and McIntyre argued that CP was a gauge symmetry in string theory, for both strong and EW CP violation. As a gauge symmetry it could not be broken explicitly, perturbatively, or non-perturbatively. More recently Bailin et al, Dent, Geidt, and Lebedev have discussed aspects of this question. Little thought has been given to CP violation in D-brane worlds, Type IIB theories with SM particles as Type I open strings, and so on.

From the point of view of connecting to the observable world, however the CP violation originates it will appear as phases in either the Yukawa couplings in the superpotential, or as phases in \( L_{\text{soft}} \). Any theory for CP violation will produce characteristic patterns of such phases. So if we could measure those phases perhaps we would have rather direct information about
such questions as moduli dependence of Yukawas, supersymmetry breaking and transmission, vevs of moduli and the dilaton, and the compactification manifold.

If one begins with a string theory including proposed solutions to how to compactify, and to break supersymmetry, the connection to the observable world is first made by writing down the Kahler potential, gauge kinetic function, and superpotential, $W = Y_{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma$. Then $L_{\text{soft}}$ is calculated for the assumed approach to supersymmetry breaking, etc. The trilinear terms, for example, are linear combinations of the Yukawas and derivatives of the Yukawas with respect to moduli fields. So if the Yukawas have large phases it seems likely the trilinear terms will also have large phases. On the other hand, phases could enter the trilinears through the Kahler potential even if they were not present in the Yukawas. Recalling that the quark CKM phase is unable to provide the CP violation needed for the baryon asymmetry, it is interesting to consider the possibility that all CP violation originates in the soft phases. It is possible to describe CP violation in the kaon and B systems with only soft phases \[31\].

Phenomenologically there are a number of ways that soft phases could be shown to be large. One is observing an eEDM. The nEDM is not so simple to interpret since it could arise from strong CP violation, but perhaps the relative size of the nEDM and HgEDM could show the effect of soft phases. The Higgs sector could show phase effects, as could superpartner masses, production cross sections, and decay BR. The size of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ could deviate from the SM prediction. It is much harder to demonstrate that $\delta_{\text{CKM}} \neq 0$.

33 Phases (and flavor structure) of $L_{\text{soft}}$

The soft-breaking Lagrangian has, as we have seen, many phases, and interesting and potentially important flavor structure. Few top-down models, e.g. string based models, have studied or even looked at the phase and flavor structure. There is and will be much more data on these topics, and there should be much more theoretical analysis of them. We have looked a little at string motivated models that give interesting phase structure. There is some work on this by Bailin et al for the heterotic string. Following the framework of Ibanez, Munoz, and Rigolin \[39\], we have looked at how the phases emerge in some D-brane models \[33\].
If one embeds the MSSM on one brane, usually the gaugino masses $M_i$ all have the same phase, and using the freedom from a U(1) symmetry as one can rotate that phase away. An interesting structure emerges if one embeds the SM gauge groups on two intersecting branes. We studied the simplest case with SU(2) on one brane, and SU(3)×U(1) on the other. While we did not try to derive such a structure from an actual compactification, it is known that explicit compactifications of intersecting branes exist, and that open strings connecting D-branes intersecting at non-vanishing angles lead to theories with chiral fermions, so it is plausible that such a model can exist. We follow Ibanez et al in assuming the supersymmetry breaking occurs in a hidden sector, and is transmitted by complex F-term moduli vevs to the superpartners. Then this model gives for soft terms

$$M_1 = M_3 = -A_t \sim e^{i\alpha_1}, \quad M_2 \sim e^{-i\alpha_2}, \quad (76)$$

and all the other soft terms are real. One important lesson is that such a theory has only 9 parameters — the many parameters of $L_{soft}$ have been reduced by the theory down to this number. They are

$$\alpha_2 - \alpha_1, m_{3/2}, \tan \beta, |\mu|, |A_t|, \phi_\mu, X_1, X_2, X_3. \quad (77)$$

Here only the relative phase of the moduli vevs enters, $m_{3/2}$ is the gravitino mass and sets the overall mass scale, and the $X_i$ are measures of the relative importance of different moduli. The $X_i$ could be measured, in which case they would tell us about the structure of the theory, and/or they could be computed in a good theory. Measuring the string-based parameters here would teach us about formulating and testing string theory. Any theory will have relations among the soft parameters so the actual number of parameters is far smaller than the full number of $L_{soft}$. This number could be reduced further by some assumptions. Also, not all of them will contribute in any given process, as we have seen. The resulting theory can be used to simultaneously study collider physics and LSP cold dark matter as is usual, and also CP violation. An extended version of the model \[ can also address flavor issues.

In this model one can illustrate how results of the low energy theory can appear fine-tuned and somewhat arbitrary because they are not apparently due to a symmetry when they originate in dynamics that occur at the high
scale and are hidden at the low energy scale. If the gluino-gluon box diagram indeed explains direct CP violation in the kaon system, then one needs a certain phase relation to hold,

\[ \arg (\phi_{A_{ud}} M_3^* ) \approx 10^{-2}, \]

which seems fine-tuned. But as we saw in eq.76, \( M_3 \) and the elements of \( A \) have the same phase in this D-brane based theory, and so the quantity in eq.78 is zero at the high scale. Since the phases of \( M_3 \) and of \( A \) run differently, a small phase is generated at the low scale. While we are not arguing this is the actual explanation for \( \varepsilon_K' \), it does nicely illustrate how such phases could be related by an underlying theory yet not follow from any low energy symmetry.

### 34 Direct evidence for superpartners? — at the tevatron?

So far all the evidence for low energy supersymmetry is indirect. Although the evidence is strong, it could in principle be a series of coincidences. More indirect evidence could come soon from improved \( g\mu - 2 \), other rare decays, b-factories, proton decay, CDM detection. But finally it will be necessary to directly observe superpartners, and to show they are indeed superpartners. That could first happen at the Tevatron collider.

Indeed, as we discussed earlier, if supersymmetry is really the explanation for EWSB then the soft masses should be of order \( M_Z \), and the cross sections for their production are typical EW ones, or larger for gluinos, so superpartners should be produced in significant quantities at the Tevatron collider that has just begun to take data after a six year upgrade in luminosity. Assuming the luminosity and the detectors are good enough to separate signals from backgrounds, if direct evidence for superpartners does not emerge at the Tevatron then either nature does not have low energy supersymmetry or there is something completely missing from our understanding of low energy supersymmetry. There is no other hint of such a gap in our understanding. Thus if superpartners do not appear at the Tevatron many people will wait until LHC has taken data to be convinced nature is not supersymmetric, but it is unlikely that superpartners could be found at the LHC if they are not
first found at the Tevatron. So let us examine how they are likely to appear at the Tevatron.

Accepting that supersymmetry explains EWSB, we expect the gluinos, neutralinos, and charginos to be rather light. The lighter stop may be light as well. Sleptons may also be light though there is somewhat less motivation for that. We can list a number of channels and look at the signatures for each of them. Almost all cases require a very good understanding of the SM events that resemble the possible signals, both in magnitude (given the detector efficiencies) and the distributions. Missing transverse energy will be denoted by $\not{E}_T$. It is reasonable to expect the Tevatron to have an integrated luminosity of $2 \, fb^{-1}$ per detector by sometime in 2004, and $15 \, fb^{-1}$ by sometime in 2007. Until we know the ordering of the superpartner masses we have to consider a number of alternative decays of $\tilde{N}_2, \tilde{C}_1, \tilde{t}_1, \tilde{g}$, etc. [35].

- $\tilde{N}_1 + \tilde{N}_1$
  This channel is very hard to tag at a hadron collider.

- $\tilde{N}_1 + \tilde{N}_{2,3}$
  These channels can be produced through an s-channel $Z$ or a t-channel squark exchange. The signatures depend considerably on the character of $\tilde{N}_2, \tilde{N}_3, \tilde{N}_1$ escapes. If $\tilde{N}_2$ has a large coupling to $\tilde{N}_1 + Z$ (for real or virtual $Z$) then the $\tilde{N}_1$ will escape and the $Z$ will decay to $e$ or $\mu$ pairs each $3\%$ of the time, so the event will have missing energy and a prompt lepton pair. There will also be tau pairs and jet pairs, but those are somewhat harder to identify. Or, perhaps $\tilde{N}_2$ is mainly photino and $\tilde{N}_1$ mainly higgsino, in which case there is a large BR for $\tilde{N}_2 \rightarrow \tilde{N}_1 + \gamma$ and the signature of $\tilde{N}_2$ is one prompt $\gamma$ and missing energy. The production cross section can depend significantly on the wave functions of $\tilde{N}_1, \tilde{N}_2$. If the cross section is small for $\tilde{N}_1 + \tilde{N}_2$ it is likely to be larger for $\tilde{N}_1 + \tilde{N}_3$. Most cross sections for lighter channels will be larger than about $50 \, fb$, which corresponds to 200 events (not including BR) for an integrated luminosity of $2 \, fb^{-1}$ per detector.

- $\tilde{N}_1 + \tilde{C}_1$
  These states are produced through s-channel $W^\pm$ or t-channel squarks. The $\tilde{N}_1$ escapes, so the signature comes from the $\tilde{C}_1$ decay, which depends on the relative sizes of masses, but is most often $\tilde{C}_1 \rightarrow l^\pm + \not{E}_T$. This is the signature if sleptons are lighter than charginos ($\tilde{C}_1 \rightarrow \tilde{l}^\pm + \nu$, followed by $\tilde{l}^\pm \rightarrow l^\pm + \tilde{N}_1$), or if sneutrinos are lighter than charginos by a similar chain, or by a three-body decay ($\tilde{C}_1 \rightarrow \tilde{N}_1 + \text{virtual } W, W \rightarrow l^\pm + \nu$). But it is not guaranteed — for example if stops are lighter than charginos the
dominant decay could be $\tilde{C}_1 \rightarrow \tilde{\ell} + b$. In the case where the lepton dominates the event signature is then $l^\pm + E_T$, so it is necessary to find an excess in this channel. Compared to the SM sources of such events the supersymmetry ones will have no prompt hadronic jets, and different distributions for the lepton energy and for the missing transverse energy.

- $N_2 + \tilde{C}_1$

  If $N_2$ decays via a Z to $\tilde{N}_1 + l^+ l^-$ and $\tilde{C}_1$ decays to $\tilde{N}_1 + l^\pm$, this channel gives the well-known “tri-lepton” signature, three charged leptons, $E_T$, and no prompt jets, which may be relatively easy to separate from SM background. But it may be that $N_2 \rightarrow \tilde{N}_1 + \gamma$, so the signature may be $l^\pm + \gamma + E_T$.

- $\tilde{l}^\pm + \tilde{l}^\mp$

  Sleptons may be light enough to be produced in pairs. Depending on masses, they could decay via $\tilde{l}^\pm \rightarrow l^\pm + \tilde{N}_1, \tilde{C}_1 + \nu, W + \tilde{\nu}$. If $N_1$ is mainly higgsino decays to it are suppressed by lepton mass factors, so $\tilde{l}^\pm \rightarrow l^\pm + N_2$ may dominate, followed by $N_2 \rightarrow \tilde{N}_1 + \gamma$ [36].

  For a complete treatment one should list all the related channels, and combine those that can lead to similar signatures. The total sample may be dominated by one channel but have significant contributions from others, etc. It should also be emphasized that the so-called “backgrounds” are not junk backgrounds that cannot be calculated, but from SM events whose rates and distributions can be completely understood. Determining these background rates is essential to identifying a signal and to identifying new physics, and requires powerful tools in the form of simulation programs, which in turn require considerable expertise to use correctly. The total production cross section for all neutralino and chargino channels at the Tevatron collider is expected to be between 0.1 and 10 pb, depending on how light the superpartners are, so even in the worst case there should be several hundred events in the two detectors. If the cross sections are on the low side it will require combining inclusive signatures to demonstrate new physics has been observed.

- gluinos can be produced via several channels, $\tilde{g} + \tilde{g}, \tilde{g} + \tilde{C}_1, \tilde{g} + \tilde{N}_1$, etc.

  If supersymmetry indeed explains EWSB it would be surprising if the gluino were heavier than about 500 GeV, as argued above. Then the total cross section for its production should be large enough to observe it at the Tevatron. If all its decays are three-body, e.g. $\tilde{g} \rightarrow q + \tilde{q}$ followed by $\tilde{q} \rightarrow q + C_1$, etc, then the signature has energetic jets, $E_T$, and sometimes charged jets.
leptons. There are two channels that are particularly interesting and not
unlikely to occur — if $t + \tilde{t}$ or $b + \tilde{b}$ are lighter than $\tilde{g}$ then they will dominate
because they are two-body. The signatures can then be quite different, with
mostly $b$ and $c$ jets, and smaller multiplicity.

Gluinos and neutralinos are normally Majorana particles. Therefore they
can decay either as particle or antiparticle. If, for example, a decay path
$\tilde{g} \rightarrow \tilde{t}(\rightarrow W^- b) + \tilde{t}$ occurs, with $W^- \rightarrow e^- \nu$, there is an equal probability for
$\tilde{g} \rightarrow e^+ + ...$. Then a pair of gluinos can with equal probability give same-sign
or opposite sign dileptons! The same result holds for any way of tagging the
electric charge — we just focus on leptons since their charge is easiest to
identify. The same result holds for neutralinos. The SM allows no way to
get prompt isolated same-sign leptons, so any observation of such events is a
signal beyond the SM, and very likely a strong indication of supersymmetry.

• Stops can be rather light, so they should be looked for very seriously.
They can be pair-produced via gluons, with a cross section that is about
$1/8$ of the top pair cross section. It is smaller because of a $p$-wave threshold
suppression for scalars, and a factor of 4 suppression for the number of spin
states. They could also be produced in top decays if they were lighter than
$m_t - M_{\tilde{N}_1}$, and in gluino decays if they are lighter than $m_{\tilde{g}} - m_t$, which is
not at all unlikely. Their obvious decay is $\tilde{t} \rightarrow \tilde{C} + b$, which will indeed
dominate if $m_{\tilde{t}} > m_{\tilde{C}}$. If this relation does not hold, it may still dominate
as a virtual decay, followed by $\tilde{C}$ real or virtual decay (say to $W + \tilde{N}_1$), in
which case the final state is 4-body after $W$ decays, and suppressed by 4-body
phase space. That may allow the one-loop decay $\tilde{t} \rightarrow c + \tilde{N}_1$ to dominate
stop decay. As an example of how various signatures may arise, if the mass
ordering is $t > C_1 > \tilde{t} > \tilde{N}_1$ and $t > \tilde{t} + \tilde{N}_1$, then a produced $t\tilde{t}$ pair will
sometimes (depending on the relative branching ratio, which depends on the
mass values) have one top decay to $W + b$ and the other to $c + \tilde{N}_1$, giving
a $W + 2$ jets signature, with the jets detectable by $b$ or charm tagging, and
thus an excess of such events.

• An event was reported by the CDF collaboration from Tevatron Run
1, $p\bar{p} \rightarrow ee\gamma\gamma E_T$ , that is interesting for several reasons, both as a possible
signal and to illustrate some pedagogical issues. That such an event might
be an early signal of supersymmetry was suggested in 1986. It can arise
\[36, 37\] if a selectron pair is produced, and if the LSP is higgsino-like, in
which case the decay of the selectron to $e + \tilde{N}_1$ is suppressed by a factor
of $m_e$. Then $\tilde{e} \rightarrow e + \tilde{N}_2$ dominates, followed by $\tilde{N}_2 \rightarrow \tilde{N}_1 + \gamma$. The only
way to get such an event in the SM is production of $WW\gamma\gamma$ with both $W \to e + \nu$, with an overall probability of order $10^{-6}$ for such an event in Run 1. Other checks on kinematics, cross section for selectrons, etc., allow a supersymmetry interpretation, and the resulting values of masses do not imply any that must have been found at LEP or as other observable channels at the Tevatron, though over some of the parameter space some associated signal could have been seen. There are many consistency conditions that must be checked if such an interpretation is allowed, and a number of them could have failed but did not. Indeed, a related interpretation that had the decay of the selectron to electron plus very light gravitino is excluded by the absence of a signal at LEP for events with two photons and large missing energy. If this event were a signal additional ones would soon occur in Run 2. Because of the needed branching ratios there would be no trilepton signal at the Tevatron, since $\tilde{N}_2$ decays mainly into a photon instead of $l^+l^-$, and the decay of $\tilde{N}_3$ would be dominated by $\tilde{\nu}$.

Although it might look easy to interpret any non-standard signal or excess as supersymmetry, in fact a little thought shows it is very difficult. As illustrated in the above examples, a given signature implies an ordering of superpartner masses, which implies a number of cross section and decay branching ratios. All must be right. All the couplings in the Lagrangian are determined, so there is little freedom once the masses are fixed by the kinematics of the candidate events. To prove a possible signal is indeed consistent with supersymmetry one has also to check that relations among couplings are indeed satisfied. Such checks will be easy at lepton colliders, but difficult at hadron colliders, so we do not focus on them here. There can of course be alternative interpretations of any new physics, but in all cases it will be possible to show the supersymmetry one is preferred (if it is indeed correct) — that is a challenge we would love to have.

35 After the first celebration

Once a signal is found, presumably at the Tevatron, there will of course be a lot of checking required to confirm it because it will not be dramatic, as discussed above, but rather excesses in a few channels that slowly increase with integrated luminosity. Deducing even the masses of mass eigenstates may be difficult if more than one channel contributes significantly to a topological excess. Nevertheless, it will be possible to very quickly deduce some general
results about supersymmetry breaking and how it is transmitted.

For example, one of the key questions is the nature of the LSP \[38\]. That can immediately exclude some ways to transmit supersymmetry breaking and favor others, and constrain ideas about how supersymmetry breaking occurs. From the discussion above we can make a table whose columns are various forms the LSP can take and whose rows are qualitative signatures that do not require complete studies, though they still require an understanding of the backgrounds:

|        | \(\tilde{B}\) | \(\tilde{h}\) | \(\tilde{G}\) | unstable |
|--------|----------------|----------------|----------------|-----------|
| prompt \(\gamma/\)s | no | some | yes | no |
| trileptons | yes | no | no | no |
| large \(E_T\) | yes | yes | yes | no |

The table can be extended to other and more detailed LSP descriptions such as wino LSP, degenerate LSP and NLSP, etc. It can be extended to a number of additional signatures and made more quantitative. There are some caveats that can be added — e.g. for the gravitino case it can happen that long lifetimes for the lightest neutralino change the signature. But the basic point that qualitative features of the excess events will tell us a considerable amount remains. An unstable LSP implies that R-parity (or matter parity) is not conserved, a gravitino LSP implies gauge mediation for the way the supersymmetry breaking is transmitted, and the bino and higgsino LSP’s suggest gravity mediation, with different consequences for cold dark matter.

36 Extensions of the MSSM

I want to emphasize that it may be very important to not restrict analysis of data by over constraining the MSSM with additional assumptions. Also I have focused on the MSSM for pedagogical simplicity, but nature could define simplicity differently. Surely the neutrino sector must be added, and that affects RGE’s for the sectors we have examined. There is good motivation for extra U(1) symmetries, which may lead to extra D-terms and to extra neutralinos that mix to affect the neutralino mass eigenstates behavior and the CDM physics. There will be Planck-scale suppressed operators that may be crucial for flavor physics and for understanding the fermion masses and for precise calculations of gauge coupling unification. There may be extra scalars related to inflation, and axions, which affect cosmology and CDM physics.
By using the MSSM without assuming relations among parameters man y of these affects can be allowed for, while if parameters are related by ad hoc assumptions the extensions could only appear if inconsistencies appeared in the analysis — that is hard to see because of the initially large experimental uncertainties. For example, extra D-terms shift various scalar masses and separate $M^2_{H_u}$ and $M^2_{H_d}$, so assuming all the scalars masses are degenerate does not allow the D-term contributions to appear.

37 Concluding remarks

These lectures have emphasized how to construct a supersymmetric description of nature at the weak scale based on forthcoming data from colliders, rare decays, static properties, cold dark matter studies, and more, and how to connect that to a unification scale description, so that we can eventually learn a complete effective Lagrangian near the Planck scale. That is the most that can be achieved by the traditional approach of science. If we also understand string theory (and we do not distinguish here between string theory and M-theory) well enough, possibly we will be able to bridge the gap to the Planck scale in 10 dimensions and formulate a fundamental theory. If so a number of features of the effective theory will be able to test ideas about the fundamental theory. The most important features of the experimental discovery of supersymmetry will be threefold: we will understand the natural world better; we will know we are on the right track to make more progress; and we will be opening a window to see physics at the Planck scale, which makes immensely more likely that we will be able to formulate and test a fundamental theory at the Planck scale.

Sometimes I am asked “what is left to compute” by students or postdocs looking for interesting projects, and interested workers in related areas. Much is indeed already known about supersymmetry from over two decades of work by a number of good people. But in fact we have just gotten to the stage where the most important problems can be addressed!. Little is known about how to relate data to the parameters of $L_{soft}$ in practice, little is known about the flavor properties of $L_{soft}$ and how to compute them theoretically or extract them from data uniquely, and little is known about how to relate data at the weak scale to an effective Lagrangian at the string scale. There is much to understand and to compute. The third of those issues will be the main focus of supersymmetry research once superpartners are being directly
studied.

There are several practical features that should be emphasized. Unless we are missing important basic ideas, a Higgs boson and superpartners will be produced at the Tevatron collider. Supersymmetry signals have two escaping LSP’s, so they are never dramatic or obvious or easy to interpret. They will appear as excesses in several channels, where channels are labeled by numbers of leptons and jets, and missing transverse energy. Once superpartners are found, the entire challenge to experimenters is to measure the parameters of $L_{soft}$, which has been the main subject of these lectures. The relations of the parameters of $L_{soft}$ to data is complicated, and it is easy to get the wrong answers if care is not taken. Although there seem to be a large number of parameters, any given measurement depends only on a few, and most parameters enter in a number of places, so using information from one place in other analysis will greatly facilitate progress. Interpreting the data and learning its implications will be challenging, and it is a challenge we are eager to have.

Acknowledgments

These lectures have benefited greatly from my collaborations and discussions with Mike Brhlik, Dan Chung, Lisa Everett, Steve King, and Lian-Tao Wang.

References

[1] A Supersymmetry Primer, S.P.Martin, hep-ph/9709356

[2] Perspectives on Supersymmetry, G.L.Kane (ed.), Singapore, World Scientific (1998)

[3] I.S.Towner and J.C.Hardy, nucl-th/9809087

[4] O.Lebedev and W.Loinaz, hep-ph/0106056; G.Altarelli, F.Caravaglions, G.F.Giudice, P.Gambino, and G.Ridolfi, hep-ph/0106029; G.-C.Cho and K.Hagiwara, hep-ph/0105037

[5] L.Everett, G.L.Kane, and L.-T. Wang, in preparation
[6] S.P. Martin and P. Ramond, Phys. Rev. D51(1995)6515; hep-ph/9501244; G. Kribs, Nucl. Phys. B535(1998)41; hep-ph/9803259; K. Dienes, Physics Reports 287 (1997) 447

[7] The Supersymmetric Soft-Breaking Lagrangian: Theory and Applications, D.J.H. Chung, L. Everett, G.L. Kane, S.F. King, J. Lykken, and L.-T. Wang, in preparation.

[8] S. Dimopoulous and H. Georgi, Nucl. Phys. B193(1981)150; L. Girardello and M. Grisaru, Nucl. Phys. B194(1982)65

[9] The literature can be traced from I. Jack and D.R.T. Jones, Phys. Lett. B457(1999)101; hep-ph/9903365; J. L. Diaz-Cruz, Proceedings Merida 1999 “Particles and Fields”, 299.

[10] L. Ibanez and G. G. Ross, Nucl. Phys. B368(1992)3; S. P. Martin, Phys. Rev. D54(1996)2340; hep-ph/9602349

[11] L. Ibanez and G. G. Ross, Phys. Lett. B110(1982)215; L. Alvarez-Gaume, M. Claudsen, and M. Wise, Nucl. Phys. B221(1983)495; K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. 68(1982)927

[12] See H. Haber in Perspectives on Supersymmetry, G. L. Kane (ed.), Singapore, World Scientific (1993)

[13] For some review and references see Martin, [1].

[14] LEP Electroweak Working Group, LEPEWWG/2001-01

[15] G. L. Kane and J. Wells, hep-ph/0003249; see also the elaboration by M. Peskin and J. Wells, hep-ph/0101342

[16] LEP Higgs Working Group for Higgs boson searches hep-ex/0107029; ALEPH-2001-066; ALEPH Collaboration hep-ph/0201014

[17] G. L. Kane, S. F. King, and L.-T. Wang, Phys. Rev. D64(2001)095013; hep-ph/0010312

[18] G. L. Kane and L.-T. Wang, Phys. Lett. B488(2000)383, hep-ph/0003198; M. Carena, J. Ellis, A. Pilaftsis, and C. Wagner, Phys. Lett. B495(2000)155, hep-ph/0009212; T. Ibrahim and P. Nath, Phys. Rev. D63(2001)035009, hep-ph/0008237
[19] J.Goldstein, C.Hill, J.Incandela, S.Parke, D.Rainwater, D.Stuart, Phys. Rev. Lett. 86(2001)1694; L.Reina, S.Dawson, and D.Wackeroth, hep-ph/0109060; W.Beenakker, S.Dittmaier, M.Kramer, B.Plumber, M.Spira, P.Zerwas, hep-ph/0107081

[20] The literature can be traced from M.Carena, H.Haber, H.Logan, S.Mrenna, hep-ph/0106116

[21] D.Rainwater, D.Zeppenfeld, K.Hagiwara, Phys. Rev. D59(1999)014037; hep-ph/9808468

[22] N.Ghodbane, S.Katsanevas, I.Laktineh, and J.Rosiek, hep-ph/0012031

[23] M.Brhlik and G.L.Kane, Phys. Lett. B437(1998)331; hep-ph/9803391

[24] S.Y.Choi, J.Kalinowski, G.Moortgat-Pick, P.M.Zerwas, hep-ph/0202039

[25] J.Cline, M.Joyce, and K.Kainulainen, hep-ph/0110031

[26] M.Carena, J.M.Morena, M.Quiros, M.Seco, and C.Wagner, Nucl. Phys. B599(2001)158; hep-ph/0011055

[27] Lisa L.Everett, G.L.Kane, S.Rigolin, and L.-T. Wang, Phys. Rev. Lett 86(2001)3484; hep-ph/0102145

[28] T.Ibrahim and P.Nath, hep-ph/9908443

[29] S.Mrenna, G.L.Kane, and L.-T.Wang, Phys. Lett. B483(2000)175; hep-ph/9910477

[30] M.Brhlik, D.Chung, and G.L.Kane, Int. J. Mod. Phys. D10(2001)367; hep-ph/0005158

[31] M.Brhlik, L.Everett, G.L.Kane, S.F.King, and O.Lebedev, Phys. Rev. Lett. 84(3041)2000; hep-ph/9909480

[32] A.Nelson and L.Randall, Phys. Lett. B316 (1993) 516; hep-ph/9308274

[33] M.Brhlik, L.Everett, G.L.Kane, and J.Lykkken, Phys. Rev. Lett. 83(1999)2124, hep-ph/9905215. Phys. Rev. D62(2000)035005, hep-ph/9908326
[34] L.Everett, G.L.Kane, S.King, S.Rigolin, and L.-T.Wang, hep-ph/0202100; See also S.Abel, S.Khalil, and O.Lebedev, hep-ph/0103031 and references therein

[35] See chapters by J.-F.Grivaz; M.Carena, R.L.Culbertson, S.Eno, H.J.Frisch, and S.Mrenna; J.F.Gunion and H.Haber in ref.2.

[36] See chapter by G.L.Kane in ref.2.

[37] S.Ambrosanio, G.L.Kane, G.Kribs, S.Martin, and S.Mrenna, Phys. Rev. Lett. 76(1996)3498

[38] G.L.Kane, Proceedings of Supersymmetry 1997, ed. M.Cvetic and P.Langacker.

[39] L.Ibanez, C.Munoz, and S.Rigolin, hep-ph/9812397, Nucl. Phys. B553(1999)43.