Dissipation-induced geometric phase for an atom in cavity QED

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We present a feasible scheme to investigate the geometric phase for an atom trapped in an optical cavity induced by the effective decay process due to cavity photon loss. The cavity mode, together with the external driving fields, acts as the engineered environment of the atom. When the parameters of the reservoir is adiabatically and cyclically changed, the system initially in the nontrivial dark state of the effective Lindblad operator undergoes a cyclic evolution and acquires a geometric phase. The geometric phase can be observed with the atomic Ramsey interference oscillation in the decoherence-free subspace.

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I. INTRODUCTION

In 1984 Berry discovered that a quantum system in a pure state picks up an extra phase when the parameters of the Hamiltonian make an adiabatic and cyclic evolution [1,2]. Such a phase, which depends only on the geometry of the cyclic path traced out by the system and has no time and energy dependence as compared with the dynamical phase, has been generalized to the nonadiabatic [3], noncyclic [4], and mixed state [5,6] cases. The Berry phase produced by unitary evolution has been experimentally tested in various two-state systems [7-12] and in a resonator [13].

In recent years, the geometric phase has received renewed interest due to its use in implementation of quantum computation [14-21]. Compared with schemes relying on dynamical effects, geometric quantum gates have potential fault tolerance since the geometric phase is robust against any fluctuation in the control parameters that preserves the area enclosed by the path [22]. For the use in quantum information processing, it is important to understand the effect of decoherence on the geometric phase due to unavoidable interaction with the environment. Several papers have investigated the behavior of geometric phases of a spin-1/2 system [22-26] and a quantum harmonic oscillator [27,28] in the presence of decoherence. Remarkably, Carollo et al. showed that the geometric phase of a quantum system can be generated through slow variation of the parameters of the engineered reservoir [29]. There exists a decoherence-free subspace (DFS) that is not affected by interaction with the engineered environment. A state lying in the DFS is decoupled from states outside the DFS and does not evolve in time if the reservoir is time independent. However, when the environment depends upon some parameter that is varied in time, the DFS becomes time dependent. Under the adiabatic condition, the system initially lying in the DFS coherently evolves, adiabatically following the DFS. When the DFS undergoes a cyclic evolution, the system returns to the initial state but acquires a geometric phase. This is an example of generating parallel transport by means of an irreversible quantum evolution. To generate this geometric effect a nontrivial DFS is required. Carollo et al. suggested a way to observe this effect using a multilevel atom interacting with a broadband squeezed vacuum with adiabatically changing squeezing parameter [30]. However, how to implement the scheme in a realistic physical system is experimentally challenging.

In this paper we investigate geometric phases for an atom trapped in an optical cavity through engineering decay in the framework of cavity QED. When the cavity decay rate is much larger than the Raman coupling strengths, the cavity mode can be adiabatically eliminated and the dynamics of the atomic degree of freedom is modeled by a master equation in Lindblad form. The effective Lindblad operator, which depends upon the parameters of the external classical fields, has a nontrivial dark state. When the parameters of the Lindblad operator are slowly and cyclically varied, this dark state undergoes a cyclic evolution and acquires a geometric phase. This phase can be manifested in the interference between this state and the other dark state that is not affected by the dissipative quantum dynamical process.

II. THEORETICAL MODEL

We consider a four-level atoms, having three ground states $|e\rangle$, $|g\rangle$, and $|f\rangle$ and an excited state $|r\rangle$, trapped in a single-mode optical cavity. The transition $|e\rangle \rightarrow |r\rangle$ ($|f\rangle \rightarrow |r\rangle$) is driven by two classical laser fields with the same Rabi frequency $\Omega_1$ ($\Omega_2$) and phase $\varphi_1$ ($\varphi_2$), but with opposite detunings $\Delta$ and $-\Delta$, as shown in Fig. 1. The transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant $g$ and detuning $\Delta$. In the interaction
engineered reservoir parameters is manifested in the phase change of the coherence between the two states. If the change rate is small enough the leakage probability from $|e\rangle \rightarrow |r\rangle$ and all of the other states coupled to the squeezed vacuum reservoir would decay to it. This model is distinguished from that described in Ref. [30] in which there is only one decoherence-free state process. Unlike the usual Berry’s adiabatic phase, this phase is associated with the incoherent process, other than the unitary dynamics. The parallel transport is generated via decay engineering. To show how the adiabatic manipulation of the engineered dissipative quantum dynamical process induced by cavity loss. The bright state of the engineered Lindblad term is $|\psi_d\rangle$, where $\kappa = \frac{\Omega g}{\Delta}$.

In the limit $\kappa \gg \frac{g^2}{\Delta}$, $\lambda_j$, where $\kappa$ is the cavity decay rate, the cavity mode is only virtually populated and can be adiabatically eliminated. Then the dynamics of the atom is described by the master equation

$$\dot{\rho} = 2L\rho L^\dagger - \rho L^\dagger L - L^\dagger L\rho,$$

where $L = \sqrt{\Gamma}(\sin \frac{\theta}{2} |g\rangle \langle e| + e^{-i\varphi} \cos \frac{\theta}{2} |g\rangle \langle f|)$, $\Gamma = \lambda^2/\kappa$, $\lambda = |\lambda_1 + \lambda_2|^2/2$, $\sin \frac{\theta}{2} = \lambda_1/\lambda$, and $\varphi = \varphi_1 - \varphi_2$. It can be easily shown that the state $|\psi_d\rangle = \cos \frac{\theta}{2} |e\rangle - e^{i\varphi} \sin \frac{\theta}{2} |f\rangle$ is the dark state of the engineered Lindblad operator $L$, i.e., $L^\dagger |\psi_d\rangle = 0$. In terms of the basis $\{|e\rangle, |f\rangle, |g\rangle\}$, $|\psi_d\rangle$ can be expressed as $|\psi_d\rangle = (\cos \frac{\theta}{2}, -e^{i\varphi} \sin \frac{\theta}{2}, 0)^T$. This state and the trivial ground state $|g\rangle$ form the DFS. States lying in such a subspace remain unaffected by the dissipative quantum dynamical process induced by cavity loss. The bright state of the engineered Lindblad term $L^\dagger L$ is $|\psi_g\rangle = (e^{-i\varphi} \sin \frac{\theta}{2}, \cos \frac{\theta}{2}, 0)^T$. Such a state would decay to the ground state $|g\rangle$ due to the engineered dissipative process. This model is distinguished from that described in Ref. [30] in which there is only one decoherence-free state and all of the other states coupled to the squeezed vacuum reservoir would decay to it.

### III. DISSIPATION-INDUCED GEOMETRIC PHASE

Since the state $|\psi_d\rangle$ depends upon the parameters of the Lindblad operator $L$, one can adiabatically follow the dark state in the DFS by slowly changing the parameters $\theta$ and $\varphi$ of $L$. On one hand, the adiabatic manipulation of the decay parameters leads to the parallel transport of the dark state $|\psi_d\rangle$. On the other hand, the change of the dark state in time causes the transition $|\psi_d\rangle \rightarrow |\psi_b\rangle$ followed by a rapid decay to the ground state $|g\rangle$. The transition to the state $|g\rangle$ is allowed, but the transition away from it is forbidden since it is not pumped. Therefore, $|g\rangle$ is the steady state, whose population is increased during the variation of the decay parameters. However, when the change rate is small enough the leakage probability from $|\psi_d\rangle$ to $|g\rangle$ through $|\psi_b\rangle$ is negligible and the pure effect is the adiabatic coherent evolution of the nontrivial dark state in the DFS. It should be noted that the states $|e\rangle$ and $|f\rangle$ have a nonvanishing probability to be excited by the pump fields during the adiabatic evolution. In experimental realizations, one must consider the influence of this excitation as a source of systematic errors. After a cyclic evolution of the parameters, the dark state would traverse a circuit in the projected Hilbert space and gain a geometric phase. Unlike the usual Berry’s adiabatic phase, this phase is associated with the incoherent process, other than the unitary dynamics. The parallel transport is generated via decay engineering. To show how the adiabatic manipulation of reservoir can take place, consider the unitary transformation

$$U = \begin{pmatrix}
\cos \frac{\theta}{2} e^{i\varphi/2} & -\sin \frac{\theta}{2} e^{-i\varphi/2} & 0 \\
\sin \frac{\theta}{2} e^{i\varphi/2} & \cos \frac{\theta}{2} e^{-i\varphi/2} & 0 \\
0 & 0 & 1
\end{pmatrix}.$$  

Under this change the states $|\psi_d\rangle$ and $|\psi_b\rangle$ are transformed to $(e^{i\varphi/2}, 0, 0)^T$ and $(0, e^{i\varphi/2}, 0)^T$, respectively. On the other hand, $|g\rangle$ remains unchanged, i.e., $U |g\rangle = |g\rangle = (0, 0, 1)^T$. This transformation corresponds to a reference frame whose basis states $|e\rangle = (1, 0, 0)^T$ and $|f\rangle = (0, 1, 0)^T$ respectively coincide with the time-dependent states $e^{-i\varphi/2} |\psi_d\rangle$ and $e^{-i\varphi/2} |\psi_b\rangle$ in the original frame, and the geometric effect arising from the cyclic evolution of the engineered reservoir parameters is manifested in the phase change of the coherence between the two states $|e\rangle$ and $|f\rangle$. 

$$H_e = \frac{\gamma^2}{\Delta} a^+ a |g\rangle \langle g| + [\lambda_1 e^{-i\varphi} a |e\rangle \langle g| + \lambda_2 e^{-i\varphi^2} a |f\rangle \langle g|] + H.c.,$$

where $\lambda_j = \Omega_j g$.
\[ |g\> \]. After the transformation the master equation becomes
\[ \dot{\rho} = 2L' \rho' L'\dagger - \rho' L'\dagger L' - L'\dagger \rho' L' + \dot{U} U\dagger \rho' + \rho' U \dot{U}\dagger, \]
where \( \rho' = U \rho U\dagger \) and \( L' = ULU\dagger \). In this frame the Lindblad term \( L'\dagger L' \) has the diagonal form:
\[ L'\dagger L' = \Gamma \langle f' \rangle \langle f' |. \]
The last two terms of the master equation arise from the time-dependent unitary transformation which is determined by the decay parameters. In the rotating frame the three eigenstates of the Lindblad operator \( L'\dagger L' \) corresponds to the three basis states so that it is convenient to calculate the evolution of the coherence between these states.

Suppose that the parameter \( \theta \) is kept constant and \( \phi \) is slowly changed from 0 to \( 2\pi \). In terms of the basis \( \{ |e\> \}, \langle f' \rangle, |g\> \} \) in the rotating frame the evolution of the system is described by the following coupled differential equations for the density matrix elements:
\[
\begin{align*}
\dot{\rho}'_{e,e} &= \frac{i}{2} \sin \theta \dot{\varphi} (\rho'_{f,e} - \rho'_{e,f}), \\
\dot{\rho}'_{g,g} &= 2\Gamma \rho'_{f,f}, \\
\dot{\rho}'_{f,f} &= -2\Gamma \rho'_{f,f} + \frac{i}{2} \sin \theta \dot{\varphi} (\rho'_{e,f} - \rho'_{f,e}), \\
\dot{\rho}'_{e,g} &= \frac{i}{2} \dot{\varphi} (\cos \theta \rho'_{e,g} + \sin \theta \rho'_{f,g}), \\
\dot{\rho}'_{f,g} &= -\Gamma \rho'_{f,g} + \frac{i}{2} \dot{\varphi} (\sin \theta \rho'_{e,g} - \cos \theta \rho'_{f,g}), \\
\dot{\rho}'_{e,f} &= -\Gamma \rho'_{e,f} + \frac{i}{2} \sin \theta \dot{\varphi} (-\rho'_{e,e} + \rho'_{f,f}) + i \cos \theta \dot{\varphi} \rho'_{e,f},
\end{align*}
\]
where \( \rho'_{j,k} = \langle j | \rho' | k\rangle \) \( (j,k = e,f,g) \). The motion of the matrix elements \( \dot{\rho}'_{e,g}(t) \) and \( \dot{\rho}'_{f,g}(t) \) are decoupled from other elements. Suppose that \( \rho'_{f,g}(0) = 0 \). Then we have
\[
\begin{align*}
\rho'_{e,g}(t) &= \frac{\rho'_{e,g}(0)}{\lambda_+ - \lambda_-} \left[ (\lambda_- - \frac{i}{2} \cos \theta \dot{\varphi}) e^{\lambda_+ t} - (\lambda_+ - \frac{i}{2} \cos \theta \dot{\varphi}) e^{\lambda_- t} \right], \\
\rho'_{f,g}(t) &= \frac{2\rho'_{f,g}(0)}{i(\lambda_- - \lambda_+)} \sin \theta \dot{\varphi} \left( \lambda_- - \frac{i}{2} \cos \theta \dot{\varphi} \right) (\lambda_- - \frac{i}{2} \cos \theta \dot{\varphi}) e^{\lambda_- t} - e^{\lambda_+ t},
\end{align*}
\]
where \( \lambda_{\pm} = \frac{1}{2}(\Gamma \pm \sqrt{\Gamma^2 + 2\Gamma \cos \theta \dot{\varphi} - \dot{\varphi}^2}) \). We here have assumed that \( \dot{\varphi} \ll \Gamma \)
we can retain these matrix elements to the first order in \( \dot{\varphi} / \Gamma \). At the time \( T = 2\pi / \dot{\varphi} \) when the parameter \( \varphi \) makes a cyclic evolution, these two matrix elements are approximately given by
\[
\begin{align*}
\rho'_{e,g}(T) &\approx \rho'_{e,g}(0) e^{i\pi \cos \theta - \pi \sin^2 \theta \varphi / 2\Gamma}, \\
\rho'_{f,g}(T) &\approx \rho'_{e,g}(0) \frac{i \sin \theta \dot{\varphi}}{2\Gamma} e^{i\pi \cos \theta - \pi \sin^2 \theta \varphi / 2\Gamma}.
\end{align*}
\]
We here have discarded the terms decaying at the rate \( \Gamma \). This result shows that the coherence \( \rho'_{e,g}(t) \) is well preserved during the engineered dissipative process. To the first order in \( \dot{\varphi} / \Gamma \) the other matrix elements are
\[
\begin{align*}
\rho'_{e,e}(T) &\approx (1 - \frac{\pi \sin^2 \theta \dot{\varphi}}{\Gamma}) \rho'_{e,e}(0), \\
\rho'_{g,g}(T) &\approx \rho'_{g,g}(0) + \frac{\pi \sin^2 \theta \dot{\varphi}}{\Gamma} \rho'_{e,e}(0), \\
\rho'_{f,f}(T) &\approx 0, \\
\rho'_{e,f}(T) &\approx -\frac{i \sin \theta \dot{\varphi}}{2\Gamma} \rho'_{e,e}(0).
\end{align*}
\]
Reversing the unitary transformation $U(T)$ we obtain the density matrix elements in the original frame

$$\rho_{d,d}(T) \simeq (1 - \frac{\pi \sin^2 \theta \dot{\varphi}}{\Gamma}) \rho_{d,d}(0),$$

$$\rho_{d,b}(T) \simeq \frac{-i \sin \theta \dot{\varphi}}{2\Gamma} \rho_{d,d}(0),$$

$$\rho_{b,b}(T) \simeq 0,$$

$$\rho_{g,g}(T) = \rho_{g,g}(0) + \frac{\pi \sin^2 \theta \dot{\varphi}}{\Gamma} \rho_{d,d}(0),$$

$$\rho_{d,g}(T) \simeq \rho_{d,g}(0)e^{i\beta - \pi \sin^2 \theta \dot{\varphi}/2\Gamma},$$

$$\rho_{b,g}(T) \simeq \rho_{d,g}(0)\frac{i \sin \theta \dot{\varphi}}{2\Gamma}e^{i\beta - \pi \sin^2 \theta \dot{\varphi}/2\Gamma},$$

(11)

where $\beta = (cos \theta - 1)\pi$, $\rho_{g,g}(T) = \langle g | \rho(T) | g \rangle$, $\rho_{j,g}(T) = \langle \psi_j | \rho(T) | g \rangle$, and $\rho_{j,k}(T) = \langle \psi_j | \rho(T) \psi_k \rangle$ ($j, k = d, b$). The geometric phase induced by the steering process is encoded in the phase of the coherence $\rho_{d,g}(T)$ in the DFS. The adiabatic manipulation of the decay dynamics drives the atom to undergo a parallel transport if it is initially in the dark state $|\psi_d\rangle$. Due to the time evolution of the DFS, the atom has a probability of leaking out of this subspace to the bright state $|\psi_b\rangle$, which rapidly decays to $|g\rangle$. This causes the damp of the coherence in the DFS. In the limit $\dot{\varphi} \ll \Gamma$ the main effect of the incoherent process is just a phase factor $e^{i\beta}$. Since the state $|g\rangle$ is not affected by the steering process, the phase $\beta$ is associated with the cyclic evolution of the dark state $|\psi_d\rangle$ due to adiabatic variation of the parameters of the Lindblad operator. The acquired phase also has a geometric interpretation in the parameter space of the Lindblad operator $L$: $\beta = -\Theta/2$, where $\Theta$ is the solid swept by the vector always pointing to $(\theta, \varphi)$ on the Poincaré sphere. It is worth noting that this phase coincides with the usual Berry geometric phase acquired by the dark state $|\psi_d\rangle |0\rangle$ of the Hamiltonian (2) when the Hamiltonian parameters are dragged along a closed loop in the absence of decoherence, where $|0\rangle$ is the vacuum state of the cavity mode. This is because in both processes the dark state traverses the same closed path in the projected Hilbert space as long as the change rate of the parameters is sufficiently slow.

IV. MEASUREMENT OF THE GEOMETRIC PHASE

Now we show that such a phase can be measured by the Ramsey interference. Suppose that the atom is initially prepared in the superposition state $\frac{1}{\sqrt{2}}(|\psi_d(0)\rangle + |g\rangle)$. After the cyclic evolution of the Lindblad operator the state of the system is

$$\rho \simeq (1 - \frac{\pi \sin^2 \theta \dot{\varphi}}{2\Gamma}) |\phi\rangle \langle \phi| + 3\frac{\pi \sin^2 \theta \dot{\varphi}}{4\Gamma} |g\rangle \langle g|$$

$$- \frac{\pi \sin^2 \theta \dot{\varphi}}{4\Gamma} |\psi_d(0)\rangle \langle \psi_d(0)|, $$

(12)

where

$$|\phi\rangle = \frac{1}{\sqrt{2} + \sin \theta \dot{\varphi}^2} [e^{i\beta} |\psi_d(0)\rangle + |g\rangle + \frac{\sin \theta \dot{\varphi}}{2\Gamma} e^{i\alpha} |\psi_b(0)\rangle]$$

(13)

and $\alpha = (cos \theta + 3/2)\pi$. After the steering process, we apply two classical pulses to the atom. The first classical pulse produces the transformation $|e\rangle \rightarrow \cos \frac{\theta}{2} |e\rangle + \sin \frac{\theta}{2} |f\rangle$ and $|f\rangle \rightarrow \cos \frac{\theta}{2} |f\rangle - \sin \frac{\theta}{2} |e\rangle$, while the second leads to $|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$ and $|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle)$. Then the state of the atom evolves to

$$\rho' \simeq (1 - \frac{\pi \sin^2 \theta \dot{\varphi}}{2\Gamma}) |\phi'\rangle \langle \phi'| + 3\frac{\pi \sin^2 \theta \dot{\varphi}}{8\Gamma} ((|g\rangle + |e\rangle)(|g\rangle + |e\rangle)$$

$$- \frac{\pi \sin^2 \theta \dot{\varphi}}{8\Gamma} ((|e\rangle - |g\rangle)(|e\rangle - |g\rangle), $$

(14)

where

$$|\phi'\rangle = \frac{1}{\sqrt{2} + \sin \theta \dot{\varphi}^2} [e^{i\beta} (|e\rangle - |g\rangle)/\sqrt{2} + (|g\rangle + |e\rangle)/\sqrt{2} + \frac{\sin \theta \dot{\varphi}}{2\Gamma} e^{i\alpha} |f\rangle].$$

(15)
Finally, the probabilities of finding the atom in the states \( |g\rangle \) and \( |e\rangle \) are given by

\[
P_{g,e} \simeq \frac{1}{2} \left[ 1 \mp \left( 1 - \frac{\pi \sin^2 \theta \varphi}{2\Gamma} \right) \cos \beta \right],
\]

which is independent of the atom-field interaction time. By varying the geometric phase \( \beta \) the probability of finding the atom in the state \( |g\rangle \) or \( |e\rangle \) exhibits Ramsey interference fringes, with the visibility shrunk by a factor

\[
V = 1 - \frac{\pi \sin^2 \theta \varphi}{2\Gamma}.
\]

It should be noted that the reduction of the visibility is unavoidable since the acquired geometric phase is associated with the incoherent dynamics.

We proceed to consider the effect of atomic spontaneous emission. The classical fields do not induce the atomic transition from the dark state \( |\psi_d\rangle \) to the excited state \( |r\rangle \) due to destructive quantum interference between the transition paths. The atomic spontaneous emission arises from the coupling of the state \( |\psi_d\rangle \) to the excited state with the effective decay rate \( \gamma_e \sim \gamma P_b \Omega^2 / \Delta^2 \), where \( P_b = \frac{\sin^2 \theta \varphi^2}{8T} \) and \( \gamma \) is the atomic spontaneous emission rate. This further reduces the atomic coherence by a factor \( V' \sim e^{-\gamma_e T} \). The required atomic level configuration can be achieved in Cs. The Zeeman sublevels \( |F = 4, m = -4\rangle, |F = 4, m = -3\rangle, \) and \( |F = 3, m = -3\rangle \) of \( 6^2 S_{1/2} \) can act as the ground states \( |g\rangle, |e\rangle, \) and \( |f\rangle \), respectively, while the Zeeman sublevel \( |F' = 4, m' = -4\rangle \) of \( 6^2 P_{3/2} \) can act as the excited state \( |e\rangle \). In cavity QED experiments with Cs atoms trapped in an optical cavity, the corresponding atom-cavity coupling strength is \( g = 2\pi \times 34 \text{ MHz} \) \( [31] \). The decay rates for the atomic excited state and the cavity mode are \( \gamma = 2\pi \times 2.6 \text{ MHz} \) and \( \kappa = 2\pi \times 4.1 \text{ MHz} \), respectively. Setting \( \Omega = g, \Delta = 100g, \theta = \pi/4, \) and \( \varphi = 10^{-4}g \), we have \( \Gamma \simeq 2\pi \times 0.0282 \text{ MHz}, V \simeq 0.91, \) and \( V' \sim e^{-4\times10^{-4}} \), which means that the Ramsey oscillation is almost not affected by the atomic spontaneous emission and the visibility of the interference fringes is about 91%.

V. CONCLUSION

In conclusion, we have proposed a scheme for generating and measuring geometric phases based on reservoir engineering in the framework of cavity QED. The decaying cavity mode and the external driving fields form the engineered environment for the atom. By adiabatically changing the environment parameters the atom can be driven to undergo a coherent cyclic evolution in DFS, picking up a phase that depends upon the geometry of the path executed in the parameter space of the engineered Lindblad operator. The acquired phase is purely geometrical since there is no dynamical evolution during the manipulation of the engineered environment. This phase can be measured through the Ramsey interference in DFS. Based on cavity QED techniques presently or soon to be available the proposed scheme might be realizable.

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Fig. 1 (color online). The atomic level configuration and excitation scheme. The transition $|e\rangle \rightarrow |r\rangle$ ($|f\rangle \rightarrow |r\rangle$) is driven by two classical laser fields with the same Rabi frequency $\Omega_1$ ($\Omega_2$) and phase $\phi_1$ ($\phi_2$) but with opposite detunings $\Delta$ and $-\Delta$. The transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant $g$ and detuning $\Delta$. 

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FIG. 1: