MeqSilhouette v2: Spectrally-resolved polarimetric synthetic data generation for the Event Horizon Telescope

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ABSTRACT

We present MeqSilhouette v2.0 (MeqSv2), a fully polarimetric, time-and frequency-resolved synthetic data generation software for simulating millimetre (mm) wavelength very long baseline interferometry (VLBI) observations with heterogeneous arrays. Synthetic data are a critical component in understanding real observations, testing calibration and imaging algorithms, and predicting performance metrics of existing or proposed sites. MeqSv2 applies physics-based instrumental and atmospheric signal corruptions constrained by empirically-derived site and station parameters to the data. The new version is capable of applying instrumental polarization effects and various other spectrally-resolved effects using the Radio Interferometry Measurement Equation (RIME) formalism and produces synthetic data compatible with calibration pipelines designed to process real data. We demonstrate the various corruption capabilities of MeqSv2 using different arrays, with a focus on the effect of complex bandpass gains on closure quantities for the EHT at 230 GHz. We validate the frequency-dependent polarization leakage implementation by performing polarization self-calibration of synthetic EHT data using PoSolve. We also note the potential applications for cm-wavelength VLBI array analysis and design and future directions.

Key words: techniques: interferometric – techniques: high angular resolution

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1 INTRODUCTION

Very long baseline interferometry (VLBI) enables the highest angular resolution in astronomy, on the order of milli-arcseconds (mas) to micro-arcseconds (μas), by operating radio antennas separated by thousands of kilometres synchronously using atomic clocks. The Event Horizon Telescope (EHT, Event Horizon Telescope Collaboration et al. 2019b) is a global mm-VLBI array whose principal goal is to spatially resolve the supermassive black holes at the cores of the Milky Way galaxy (Sgr A*) and M87, and image their shadows, the depression in observed intensity inside the apparent boundary of the black hole (e.g. Falcke et al. 2000; Dexter et al. 2012; Psaltis et al. 2015; Mościbrodzka et al. 2016), together with a bright crescent-shaped emission ring (e.g. Bromley et al. 2001; Broderick & Loeb 2009; Kamruddin & Dexter 2013; Lu et al. 2014).

The EHT 2017 campaign has yielded total intensity images of the shadow of the black hole at the centre of M87 at 230 GHz (Event Horizon Telescope Collaboration et al. 2019a,d,f). Assuming statistical and interstellar scattering using simple Gaussian sources and narrow-field complex skymodels and introduce physically-motivated tropospheric phase and amplitude corruptions, interstellar scattering using scatterbranes (Johnson & Gwinn 2015), and time-variable antenna pointing errors. It has been significantly enhanced since then, first in step with the publication of the first results from 2017 EHT observations, and then with the development of SYMB (Roelofs et al. 2020) and the first polarimetric results of the 2017 M87 observations (Event Horizon Telescope Collaboration et al. 2021a,b).

The code has been refactored to be fully compatible with the same pipelines used for the analysis of real EHT data. The pointing and atmospheric models have been rewritten to include more sophisticated effects. Source and instrumental polarization simulation capabilities have been introduced. meosv2 also accounts for the effects of bandwidth on various propagation path effects at mm-wavelengths. These features facilitate a variety of studies for both the EHT and upcoming VLBI arrays, such as performing rotation measure (RM) synthesis studies (Brentjens & de Bruyn 2005) and multi-frequency synthesis imaging with the increasing fractional bandwidth of the EHT, as well as the envisioned multi-band imaging at 230 GHz and 345 GHz for the ngEHT. The ability to vary instrumental polarization across the receiver bandwidth is crucial to take full advantage of ultra-wideband receivers and high dynamic-range polarimetric imaging. With the polarimetric primary beam module, full Stokes primary beam modelling for upcoming arrays such as the ngEHT can be undertaken. Roelofs et al. (2020) seamlessly combines the functionality of meosv2 and rpicard (Janssen et al. 2019), the casa-based VLBI pipeline for calibrating data from the EHT and other VLBI facilities. Synthetic data generated by both eht-imaging and SYMB, representing complementary approaches, have been found to be consistent with each other (Event Horizon Telescope Collaboration et al. 2019d).

In this paper, we present the components of meosv2 and illustrate its simulation capabilities. In particular, we illustrate the new polarimetric and spectral resolution capabilities using synthetic data and validate them. We study the effects of bandpass gains on closure quantities, which can limit or bias constraints on intrinsic structure asymmetry, by generating synthetic data with realistic bandpasses. The polarimetric capabilities of meosv2 are

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1. https://github.com/krosenfeld/scatterbrane.

2. “Measurement EQuation” (see section 2) + “Silhouette” (referring to the black hole “shadow”).
validated using polsolve, a casa task developed for polarization calibration of VLBI observations (Martí-Vidal et al. 2021). Unlike conventional VLBI polarimetric calibration software packages such as LPCAL (Leppanen et al. 1995) that use a linear approximation to model polarization leakage, polsolve uses a non-linear model derived from the full RIME to handle high leakage values for specialised cases such as the EHT. In addition, polsolve uses a combined multi-source model when the parallactic angle coverage for individual calibrators is limited and can model the frequency-dependence of the leakage terms for calibrating large fractional bandwidths.

This paper is organized as follows: Section 2 provides a brief overview of the RIME formalism that forms the basis of how meqs2 models visibilities to make this a self-contained publication. Section 3 provides a detailed account of the control flow of meqs2, its signal corruption capabilities, and their Jones matrix implementations. Section 4 describes the casa polsolve tool for estimating polarization leakage in heterogenous VLBI arrays. Section 5 demonstrates the synthetic data generation capabilities of meqs2 for three different mm-wave telescopes. Section 6 quantifies the effect of complex bandpass gains on EHT observations at 230 GHz and section 7 demonstrates and validates the polarimetric simulation capabilities of meqs2 using multiple polarized source models. Section 8 provides a general discussion on the potential uses of meqs2 and section 9 summarises the results and future outlook.

2 THE RADIO INTERFEROMETER MEASUREMENT EQUATION

Hamaker et al. (1996) originally developed the mathematical formalism for describing radio polarimetry using the Jones (Jones 1941) and Mueller (Mueller 1948) calculi from optics. Smirnov (2011a) extended this formulation to incorporate direction-dependent effects (DDEs) in calibration. Here we provide only a brief summary of the relevant aspects of this formalism and, given the range of notations in use, establish the notation used in this paper.

An interferometer produces four pairwise correlations between the voltage vectors from two stations $p$ and $q$ (each with two feeds $x$ and $y$), that can be arranged into the 2×2 visibility matrix:

$$
\mathbf{V}_{pq} = 2 \begin{pmatrix}
(v_{px}v_{qx}^*) & (v_{px}v_{qy}^*) \\
(v_{py}v_{qx}^*) & (v_{py}v_{qy}^*)
\end{pmatrix},
$$

(1)

where the angled brackets denote averaging over some small time and frequency bin, based on considerations of smearing and decorrelation, field of interest, and processing requirements. In terms of the voltage two-vectors $\mathbf{v}_p$, equation (1) can be represented as

$$
\mathbf{V}_{pq} = 2 \begin{pmatrix}
\langle v_{px} \rangle & \langle v_{px}^* \rangle \\
\langle v_{py} \rangle & \langle v_{py}^* \rangle
\end{pmatrix} = 2 \langle \mathbf{v}_p \mathbf{v}_q^H \rangle,
$$

(2)

where $\langle \rangle$ is the incoming electromagnetic wave, $\mathbf{J}_p$ are the 2x2 Jones matrices that describe any linear transformation acting on the incoming wave, and $\mathbf{H}$ is the Hermitian conjugate. The matrix product $\mathbf{e} \mathbf{e}^H$ in equation (2) is related to the four Stokes parameters $I$, $Q$, $U$, and $V$ that describe the polarization state of electromagnetic radiation (Hamaker et al. 1996; Thompson et al. 2017) by the following relation:

$$
2 \begin{pmatrix}
\langle e_x e_x^* \rangle & \langle e_x e_y^* \rangle \\
\langle e_y e_x^* \rangle & \langle e_y e_y^* \rangle
\end{pmatrix} = \begin{pmatrix}
I + Q & U + iV \\
U - iV & I - Q
\end{pmatrix} = \mathbf{B}.
$$

(3)

$\mathbf{B}$ is the brightness matrix that describes the intrinsic source brightness. $e_x$ and $e_y$ are the orthogonal polarizations as measured by the two feeds $x$ and $y$.

In the ideal case of corruption-free reception, the phase delay associated with signal propagation, denoted by the scalar $K$-Jones matrix, is always present, giving rise to the source coherency, $\mathbf{X}_{pq}$:

$$
\mathbf{X}_{pq} = K_p \mathbf{B} K_q^H.
$$

(4)

in which $\phi_p$ denotes the phase delay between the antenna $p$ and the reference antenna. In the presence of multiple discrete sources in the sky, taking into account the direction-dependence of the source coherency and some Jones matrices, the RIME generalizes to

$$
\mathbf{V}_{pq} = G_p \sum_s E_{sp} \mathbf{X}_{spq} E_{sq}^H G_q^H,
$$

(5)

where the summation is carried out over all the sources and $E_{sp}$ and $G_p$ denote generic direction-dependent effects (DDEs) and direction-independent effects (DIEs) respectively. meqs2 does not simulate any DDEs for EHT observations, since, aside from scattering, there are no major DDEs that occur along the signal path.

3 THE MEQSILHOUETTE FRAMEWORK

meqs2 was designed to use the Measurement Set (MS), a database format designed to store radio astronomical data in next-generation facilities such as JVLA, ALMA, MeerKAT, and SKA. Fig. 1 shows the basic layout and the components of a typical meqs2 run. Scattering by the ISM is not applied to the input sky models and is assumed to have been applied externally, simplifying the user interface compared to vl. meqs2 uses a driver script to set up the sequence of steps to be executed to generate the synthetic data. The framework module contains the various functions necessary to create synthetic data, corrupt them, and optionally generate additional data products. The inputs to the driver script are presented as attribute-value pairs in a file in the rason format (Crockford 2006) containing information on the source, weather, and antenna parameters necessary for computing various components of the RIME. The Jones matrices are applied to the uncorrupted visibilities in the order in which they occur along the signal path (Noordam & Smirnov 2010), unless they are scalar, in which case they can be applied anywhere along the signal chain.

Advanced users may write their own driver scripts to tailor the basic strategy provided by meqs2 for their own needs. For instance, in symba (Roelofs et al. 2020), we use this framework to create synthetic data that follow real EHT observing schedules using input VEX files (the scheduling protocol for VLBI experiments) and

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3 The factor of 2 is introduced in this equation to ensure that the brightness matrix (introduced shortly) becomes 1 for a 1 Jy unpolarized source (Smirnov 2011a).

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4 This equation is valid for linear (XY) feeds. See section 3.1 for circular (RL) feeds.

5 https://casa.nrao.edu/Memos/229.html.

6 https://vlbi.org/vlbi-standards/vex.
Figure 1. Flowchart showing the basic components of synthetic data generation with msqsv2. The inputs and outputs are shaded orange, the processes are shaded blue, and the decision boxes (diamonds) are shaded green. Multiple input configuration files are used and various output data products are produced. The input values are determined from empirical values obtained from the individual observations themselves, as well as \textsc{vlrmonitor}. Each component in the diagram is explained in section 3.

Table 1. Physical sizes and mount specifications of EHT2017 stations participating in the simulations.

| Station  | Effective Diameter (m) | Mount type                |
|----------|------------------------|---------------------------|
| ALMA     | 70                     | Alt-az                    |
| APEX     | 12                     | Alt-az+Nasmyth-Right      |
| LMT      | 32                     | Alt-az+Nasmyth-Left       |
| PV       | 30                     | Alt-az+Nasmyth-Left       |
| SMT      | 10                     | Alt-az+Nasmyth-Right      |
| JCMT     | 15                     | Alt-az                    |
| SMA      | 16                     | Alt-az+Nasmyth-Left       |

\(^{\dagger}\) Single-antenna equivalent of phased arrays.

extend the pointing offset module to compute short and long-term pointing offsets mimicking the behaviour of EHT stations. \textsc{symba} also performs additional a priori calibration so that the output more closely resembles the uncalibrated EHT data. \textsc{msqsv2} can just as easily be used for simulating observations with other VLBI arrays (see section 5), although other propagation path effects that become significant at longer wavelengths may need to be implemented.

The following subsections explain the various modules of \textsc{msqsv2}. The plots shown are obtained using a hypothetical 12-hour observing run using the EHT2017 array listed in Table 1 at an observing frequency of 230 GHz towards M87 (\(\alpha_{2000} = 12^h30^m49^s.42, \delta_{2000} = 12^\circ23'28''.04\)). The SPT station from EHT2017 has been excluded since M87 is always below the horizon from the south pole.

3.1 Input sky models

\textsc{msqsv2} introduces the capability to generate synthetic visibilities from wide-field non-parametric images and retains the ability to input simple parametric source models in the form of ASCII text files describing point or Gaussian source models. Since images are gridded representations of the sky, we use \textsc{wsclean}, a fast generic widefield imager (Offringa et al. 2014), to Fourier-invert the model image to the \(uv\)-plane. Each polarized, time and frequency variable image frame is Fourier-inverted into the appropriate subset of the generated MS\(^7\). Parametric sky models are handled by \textsc{msts} (Noordam & Smirnov 2010), which performs a direct Fourier transform of the sky model into the MS.

For the EHT, the visibilities are always computed in the circular polarization basis (RR, RL, LR, and LL) except in the case of ALMA which records signals in linear basis. In \textsc{msqsv2}, we assume that the ALMA visibilities have been perfectly converted to circular polarization (Mari-Vidal et al. 2015) so that the basis is uniform across all stations. Then, equation (3) takes the form

\[
B = \begin{bmatrix}
I + V \\
Q - iU \\
Q + iU \\
I - V
\end{bmatrix} \cdot \begin{bmatrix}
\langle e_1^x e_2^x \rangle \\
\langle e_1^y e_2^y \rangle \\
\langle e_1^x e_2^y \rangle \\
\langle e_1^y e_2^x \rangle
\end{bmatrix},
\]

where ‘\(\circ\)’ indicates circular polarization (Smirnov 2011a). Figure 2 shows the Stokes I visibility amplitudes for all baselines for a single channel of the data set generated using a point source sky model with intrinsic time variability. The scatter of the visibilities is due to thermal noise. The capability to simulate time-varying sources is particularly useful for simulating sources exhibiting variability on timescales of minutes to hours, such as the radio source associated with the supermassive black hole Sagittarius A\(^*\) (or Sgr A\(^*\)) located at the Galactic centre (e.g. Lu et al. 2016). In addition, studies on decoupling time-varying instrumental effects from source evolution could be undertaken.

Figure 3 shows the Stokes I visibilities of a point source with a steep spectral index, across a 2 GHz bandwidth divided into 64 channels, centred at 227 GHz. The data shown correspond to a 5-minute subset of the entire observation for all channels. As before, the scatter seen is due to thermal noise.

\(^7\) The naming conventions for the image frames are explained in the official documentation.
3.2 Antenna pointing errors

Several factors cause antennas to mispoint and modify its gain response, such as differential flexure of the antenna, wind pressure, and thermal expansion. Moreover, errors in drive mechanics or the telescope control software also introduce pointing errors, which attenuate the measured visibility amplitudes, \( |V_{pq}| \). Even small offsets in antenna pointing can cause significant reduction in visibility amplitudes at mm-wavelengths.

Formulated in terms of the RIME, this effect is represented by a time and direction-dependent antenna-based E-Jones matrix (Smirnov 2011b):

\[
E_p(l,m) = E(l + \delta l_p, m + \delta m_p),
\]

where \( \delta l \) and \( \delta m \) are the time-variable offsets in the \( lm \)-plane. \texttt{MeqSilhouette} provides a WSRT-derived analytic \( \cos^3 \) beam (from the previous version) and a Gaussian primary beam profile. The Gaussian profile has the advantage that it can be conveniently described by a single parameter, the full width at half maximum (FWHM), and does not deviate from the more complex Bessel function from the centre up to very close to the first null (Middelberg et al. 2013). This assumption is justified in the context of EHT, since, in most cases, the field of interest is much smaller than the location of the first null.

The offsets per pointing epoch (or scan) for station \( p \), \( \rho_p \), are drawn from the normal distribution \( N(0, \mathcal{P}_{\text{rms}} \cdot \rho_p) \). \( \mathcal{P}_{\text{rms}} \) values depend on station and weather characteristics and are determined based on empirical measurements. The full width at half maximum (FWHM) of the primary beam \( \mathcal{P}_{\text{FWHM}} \) of each antenna at 230 GHz is scaled to the centre frequency of the observation. The beam model with the pointing errors is given by

\[
E_P = \exp \left( -\frac{1}{2} \left( \frac{\rho_p}{\mathcal{P}_{\text{FWHM}} \cdot 2 \sqrt{2 \ln 2}} \right)^2 \right),
\]

where \( \rho_p = \sqrt{\delta l_p^2 + \delta m_p^2} \).

The \( 2 \sqrt{2 \ln 2} \) factor arises due to the relationship FWHM = \( 2 \sqrt{2 \ln 2} \sigma \), where \( \sigma \) is the standard deviation. Updating equation (8) is all that is required to implement more complex, asymmetric primary beam patterns. The corresponding E-Jones matrix is given by

\[
E_P = \begin{pmatrix} E_{pa}(l,m) & 0 \\ 0 & E_{pb}(l,m) \end{pmatrix}
\]

This is implemented as a scalar term \( (E_{pa} = E_{pb}) \) that can commute with other components of the RIME.

Figure 4 shows an example simulation of pointing offsets, \( \rho_p \), and how they affect the primary beam response of the EHT2017 antennas. SMA is the least affected owing to its large beam size, while LMT is the most affected due its narrow primary beam.

3.3 Tropospheric effects

The troposphere has a significant effect on signal propagation at mm wavelengths (Carilli & Holdaway 1999). \texttt{MeqSilhouette} models three significant tropospheric effects – (i) the signal attenuation due to atmospheric opacity \( \tau \), (ii) the increased system temperature due to atmospheric emission at the sky brightness temperature \( T_{\text{sky}} \), and (iii) the phase fluctuations due to atmospheric turbulence. These effects are separated into mean and turbulent components, with the mean component further divided into wet (due to water vapour) and dry components. A detailed treatment of the propagation fundamentals can be found in Blecher et al. (2017). Here we provide a summary along with the updated equations and algorithms implemented in \texttt{MeqSilhouette}.
the thermal noise component, is accounted for in the total noise budget used for visibility weighting in m{eq}o}\text{qs}v2 (see section 3.6).

### 3.3.2 Turbulent troposphere

Turbulence in the troposphere introduces phase instabilities in the measured visibilities. The spatial distribution of water vapour in the troposphere evolves rapidly, reducing the coherence timescale to \(\sim 10 \text{s} \) (Thompson et al. 2017). This limits the coherent averaging time (and hence the S/N) of uncalibrated visibilities, and renders conventional calibration procedures (with interleaved observations of calibrator and target source) ineffective. Self-calibration is also limited by the S/N required for fringe-fitting.

\text{m\text{eq}}v2 models these phase variations, \(\delta \phi(t, v)\), by assuming a thin, Kolmogorov-turbulent phase screen moving with constant velocity (e.g. Johnson & Gwinn 2015). This method is applicable to any situation in which the troposphere induces delays in electromagnetic wave propagation (e.g. Treuhaft & Lanyi 1987). The phase offsets introduced can be described by the phase structure function given by (Carilli & Holdaway 1999)

\[ D_\phi(x, x') = \langle (\phi(x + x') - \phi(x))^2 \rangle, \]

where \(x\) and \(x'\) are points on the screen and the angled brackets denote ensemble average. This equation can be reasonably approximated by a power law (e.g. Armstrong et al. 1995):

\[ D_\phi(r) = \left( \frac{r}{r_0} \right)^\beta, \]

where \(r^2 = (x - x')^2\) and \(r_0\) is the phase coherence scale at which \(D_\phi(r_0) = 1 \text{rad}\).

Scattering can be classified as strong or weak based on the relationship between \(r_0\) and the Fresnel scale, \(r_F\), defined as \(\sqrt{D_{\text{os}}/2\pi}\), where \(D_{\text{os}}\) is the distance between the observer and the scattering screen. The radiative power measured at a given point originates from a single region of area \(A_{\text{weak}} = \pi r_F^2\) when \(r_F \ll r_0\) and from multiple disconnected zones each of area \(A_{\text{strong}} = \pi r_0^2\) when \(r_F \gg r_0\) (Narayan 1992). Empirical estimates of \(r_0\) fall in the range of \(\sim 50 - 500 \text{m}\) above Mauna Kea (Masson 1994) and \(\sim 90 - 700 \text{m}\) above Chajnantor (Radford & Holdaway 1998). At \(\lambda = 1.3\) mm and \(D_{\text{os}} = 2 \text{km}\) (the water vapour scale height), \(r_F \approx 0.64 \text{m} \approx r_0\), and hence scattering falls under the weak regime.

Equation (12) may be rewritten with explicit time-dependence as \(D_\phi(t) = D_\phi(r)|_{r=vt}\), where \(v\) is the bulk transverse velocity of the phase screen (Coulman 1985). Assuming that the coherence timescale when observing towards the zenith, \(\tau_c = r_0/|v|\) (Treuhaft & Lanyi 1987; Blecher et al. 2017), we get

\[ D_\phi(t) = \left( \frac{t}{\tau_c} \right)^\beta. \]

Both Kolmogorov theory and empirical measurements show that the exponent \(\beta\) should be equal to \(5/3\) when \(r < \Delta h\), where \(\Delta h\) is the thickness of the turbulent layer which is \(\sim 1 \text{km}\) (Carilli & Holdaway 1997). The processed Field-of-View (FoV) of the EHT is about \(100 \times 100 \text{mas}^2\) and is much smaller than \(r_0\), allowing us to represent it as a diagonal Jones matrix in the RIME.

The antenna-based turbulent phase errors manifest as a series of correlated, normally distributed random variables in time. This time-series, \{\(\delta \phi'(t)\}\), can be constructed as follows (e.g. Rasmussen & Williams 2006). From the structure function \(D\) we construct the covariance matrix \(\Sigma\). Since \(\Sigma\) is symmetric and positive definite,
the lower triangular matrix, $L$, resulting from the Cholesky decomposition of $\Sigma$ (where $\Sigma = LL^T$) can be applied to a time-series of the desired length with zero mean and unit variance to arrive at correlated random samples.

We assume a simple linear scaling with frequency across the bandwidth since the wet dispersive path delay is not more than a few per cent of the non-dispersive component at mm-wavelengths (Curtis et al. 2009). Also taking into account the airmass towards the horizon when observing away from the zenith, the phase error time-series for antenna $p$ becomes

$$\{\delta \phi_p(t, \nu)\} = \frac{1}{\sqrt{\sin(\theta_\ell(t))}} \{\delta \phi'_p(t)\} \{\nu\} / \nu_0,$$  

(14)

where $\nu$ is the list of channel frequencies, $\nu_0$ is the reference frequency, taken to be the lowest channel frequency, and $\theta_\ell$ is the elevation angle.

Since VLBI stations are typically located hundreds or thousands of kilometres from each other, the tropospheric corruptions over individual stations are uncorrelated with each other and must be simulated independently. This is not strictly true for the short-baseline pairs of ALMA-APEX (2.6 km) and JCMT-SMA (160 m) in the EHT, for which the turbulence may be correlated since the baseline lengths are so short as to be comparable to $r_0$ (e.g. Carilli & Holdaway 1997). Currently, intra-site baseline correlations are not simulated and the phase errors are generated independently for these stations.

### 3.4 Instrumental polarization

The two polarization feeds nominally measure two orthogonal polarization states of the incoming wave, in either circular (RL) or linear (XY) bases. In practice, mechanical imperfections in the feed or electronic imperfections in the signal path cause signals from each independent signal path to leak into the other (Hamaker et al. 1996; Sault et al. 1996). Additionally, the possible rotation of the polarization states of the incoming wave, in either circular (RL) or linear (XY) bases. In practice, mechanical imperfections in the feed or electronic imperfections in the signal path cause signals from each independent signal path to leak into the other (Hamaker et al. 1996; Sault et al. 1996). Additionally, the possible rotation of the feed matrix may be decomposed as $\Sigma = \sum_{\text{feed}} \sum_{\text{feed}}$ (D-Jones frame). This feed matrix may be decomposed as

$$\begin{bmatrix} P'_{pR} & d'_{pR} \\ d_{pL} & D'_{pL} \end{bmatrix} = \begin{bmatrix} P & 0 \\ 0 & e^{j\chi_p} \end{bmatrix} G'_p \begin{bmatrix} 1 & d_{pR}d_{pL} \\ d_{pL} & 1 \end{bmatrix} \begin{bmatrix} P & d_{pR} \\ d_{pL} & 0 \end{bmatrix},$$  

(18)

where $R$ and $L$ denote the two feed receptors (here, in the circular frame). This feed matrix may be decomposed as

$$\begin{bmatrix} D'_{pR} & 0 \\ 0 & D'_{pL} \end{bmatrix} \begin{bmatrix} 1 & d_{pR}d_{pL} \\ d_{pL} & 1 \end{bmatrix} = G'_p D_p,$$  

(19)

where $d_{pR}$ is the feed error or the leakage $D$-Jones term, with the complex numbers $d_{pR}$ and $d_{pL}$ representing the fractional leakage from either feed.

### 3.4.2 Polarization leakage

The two receptors in the feed are designed to be sensitive to orthogonal polarization states. Ideally, the $D$-Jones terms (or the $D$-terms) corresponding to polarization leakage is represented by a unit matrix, expressed in the appropriate coordinate system (i.e. that of the two polarization states measured by the feed receptors) (Hamaker et al. 1996). In practice, the non-diagonal terms of this matrix are non-zero, due to the fact that each receptor is sensitive to the opposite polarization state due to electronic or mechanical imperfections in the feed. Hence, the feed matrix is given by

$$\begin{bmatrix} D'_{pR} & 0 \\ 0 & D'_{pL} \end{bmatrix} \begin{bmatrix} 1 & d_{pR}d_{pL} \\ d_{pL} & 1 \end{bmatrix} = G'_p D_p,$$  

where $d_{pR}$ is the feed error or the leakage $D$-Jones term, with the complex numbers $d_{pR}$ and $d_{pL}$ representing the fractional leakage from either feed.

### 3.4.3 Implementation

MeqSilhouette v2 can introduce station-based, frequency-dependent, complex-valued instrumental polarization. The per station complex $D$-term values for the two polarization feeds are sampled independently for each frequency channel from the normal distribution.
\( \mathcal{N}(p_{\mu}, p_{\sigma}) \); \( p_{\mu} \) denotes the characteristic empirical leakage value and \( p_{\sigma} \) denotes the scatter for station \( p \). The visibilities may be written to the MS in either the sky or the antenna coordinate system (i.e. without or with parallactic angle rotation correction respectively). The visibilities in the two coordinate systems are related by

\[
\mathbf{V}_{\text{ant}} = P_p \mathbf{V}_{\text{sky}} P_p^H.
\]  

(20)

where \( P_p \) are given by equation (17). This rotation also includes a constant offset in the feed angle per station. Once this rotation has been applied, the D-terms can be applied to the visibilities in the antenna frame,

\[
\mathbf{V} = D_p(P_p)\mathbf{V}_{\text{sky}}(P_p^H)D_p^H.
\]

(21)

If the visibilities are required to be written in the sky frame, then \( \mathbf{V}_{\text{sky}} \) is converted to \( \mathbf{V}_{\text{ant}} \) so that the D-terms may be applied, before being converted back to the sky frame,

\[
\mathbf{V} = (P_p^H)D_p(P_p)\mathbf{V}_{\text{sky}}(P_p^H)D_p^H(\mathbf{P}_q).
\]

(22)

The product \( P_p^H D_p P_p \) evaluates to a rotation of the D-terms in the antenna frame by twice the feed angle:

\[
P_p^H D_p P_p = \left( \frac{1}{\exp(-2j\chi_p)} \exp(2j\chi_p) \right).
\]

(23)

The visibilities are usually generated in this frame to correspond to the EHT data.

### 3.5 Temporal and spectral variability in receiver gains

The antenna-based gain terms are represented by the complex-valued G-Jones matrices which are a generalization of simple antenna gains to polarimetry (e.g. Smirnov 2011a). The electronic gains of a pair of circular receptors are given by a diagonal G-Jones matrix in the circular coordinate frame,

\[
G_p(t) = \begin{pmatrix}
\bar{g}_{pR}(t) & 0 \\
0 & \bar{g}_{pL}(t)
\end{pmatrix}.
\]

(24)

Different antenna-based factors such as the \( G' \) term in equation (19) may be subsumed into the G-Jones terms. The time-variable complex gains are generated by sampling from the normal distribution \( \mathcal{N}(g_{p \mu}, g_{p \sigma}) \) per timestamp, for both the polarization feeds for each station \( p \), where \( g_{p \mu} \) denotes the characteristic station-dependent gain and \( g_{p \sigma} \) denotes the scatter.

Various components along the signal path and the bandpass filters used at each station determine the frequency response of the receiving channels over the observing bandwidth (Taylor et al. 1999). This response is not constant over the entire bandwidth and usually falls to about half of the maximum towards both edges of the passband. This results in a frequency-dependent component in the antenna gains which is captured using complex-valued B-Jones matrices that vary as a function of frequency. This effect is usually corrected for by observing a bandpass calibrator source whose behaviour across the observing bandwidth is well-known.

As with the G-Jones terms, the B-Jones matrices for circular receptors are diagonal in a circular coordinate frame,

\[
B_p(v) = \begin{pmatrix}
\bar{b}_{pR}(v) & 0 \\
0 & \bar{b}_{pL}(v)
\end{pmatrix}.
\]

(25)

For each station, mqsxv2 accepts nominal gain values at a few representative frequencies within the bandwidth of the observation, and performs spline interpolation of the bandpass amplitudes for each frequency channel. To these amplitudes, random phases are generated and added to produce the complex quantities. The G-Jones and B-Jones terms are simulated separately to provide independent, fine-grained variability of gains along the time and frequency axes.

### 3.6 Noise considerations

#### 3.6.1 Thermal noise

Noise due to various factors such as receiver electronics, atmospheric emission, background radiation and ground (i.e. spillover) radiation affect the system sensitivity adversely. The system temperature, \( T_{\text{sys}} \), equivalent to the power per unit frequency due to the noise, \( P_N \), accounts for this noise and is given by

\[
T_{\text{sys}} = \frac{P_N}{k_B}.
\]

(26)

where \( k_B \) is the Boltzmann constant\(^8\). The system temperature is often expressed in terms of the system equivalent flux density (SEFD), which is defined as the flux density of a source that would deliver the same amount of power as the system

\[
\text{SEFD} = \frac{2k_B T_{\text{sys}}}{\eta A_e}.
\]

(27)

where \( \eta \) is the antenna efficiency and \( A_e \) is the effective area of the antenna. mqsxv2 accepts the station-dependent SEFD values as inputs from which the per-baseline rms uncertainty, \( \sigma_{pq} \), on a visibility in units of Jy is computed (Thompson et al. 2017):

\[
\sigma_{pq} = \frac{1}{\eta} \sqrt{\frac{\text{SEFD}_{p} \cdot \text{SEFD}_{q}}{2 \Delta f \tau}},
\]

(28)

where \( A_e \) denotes the effective area of the telescope and \( \eta \) comprises any relevant efficiency terms, such as the antenna aperture efficiency, \( \eta_{ap} \), the correlator efficiency, \( \eta_{corr} \), \( \Delta f \) is the bandwidth and \( \tau \) the integration time. For standard 2-bit quantization, \( \eta \) is set to 0.88.

Since the system noise is broadband and mostly stationary, it can be described using a Gaussian distribution and the uncertainty in their measurements can be reduced by increasing the number of independent measurements \( N = 2\Delta f \tau \).

In the RIME implementation, this thermal noise becomes an additive term per polarization, distributed normally with zero mean and a variance of \( \sigma_{pq}^2 \) per visibility:

\[
\mathbf{V}_{pq} = \mathbf{V}_{pq} + N(0, \sigma_{pq}^2).
\]

(29)

This additive thermal noise matrix has the same dimensionality as the data, varying with time, baseline, frequency, and polarization.

#### 3.6.2 Visibility weighting

The MS format allows for the estimated rms noise values and visibility weights to be recorded alongside the data. The \( \sigma_{pq} \) values computed above are used to generate per-visibility baseline-dependent thermal noise. These terms are added with the increase in the sky brightness temperature due to the troposphere (section 3.3.1) and are used to fill the SIGMA and SIGMA_SPECTRUM columns in the MS. Inverse-variance weighting is used to fill in the visibility weights columns WEIGHT and WEIGHT_SPECTRUM.

\(^8\) The tropospheric contribution to the increase in \( T_{\text{sys}} \) (section 3.3.1) is added to the noise budget as mentioned below.
4 POLSOLVE: POLARIZATION LEAKAGE ESTIMATION

We use POLSOLVE to validate the instrumental polarization capabilities of MeqSilhouette v2. POLSOLVE is part of poltools\(^9\), the polarimetry toolbox developed for CASA, aimed at the simulation, calibration, and basic analysis of polarimetric VLBI observations (Martí-Vidal et al. 2021). It uses the full RIME, simplified for the case of narrow-field observations, to estimate and correct for instrumental polarization using observations of spatially-resolved polarization calibrators.

POLSOLVE has many advantages compared to the aips\(^10\) task LPCAL (Leppanen et al. 1995), the main algorithm used by the VLBI community for polarimetric calibration. It uses a non-linear model of polarization leakage derived from the full RIME for handling high leakage values. POLSOLVE can also model the frequency-dependent variations in D-terms to enable calibration of wide fractional bandwidths and perform D-term estimates based on cross-polarization self-calibration. We take advantage of many of these features in performing the validation tests for this paper.

The aips-based gpcal\(^11\) pipeline also addresses many of the shortcomings of LPCAL (Park et al. 2021) and has been shown to be consistent with POLSOLVE (Event Horizon Telescope Collaboration et al. 2021a). A detailed discussion of the impact of these features on the quality of VLBI polarimetry are discussed in Martí-Vidal et al. (2021). Below, we give a brief description of the procedure and equations used by POLSOLVE.

4.1 The POLSOLVE fitting parameters

POLSOLVE uses the Levenberg-Marquardt algorithm (Press 2007) to minimize the error function \(\chi^2(x)\), where \(x\) consists of parameters that model both source and instrumental polarization, divided into two subsets. It uses two different approaches to account for the polarization structure of a source, both of which assume that the source brightness distribution can be described as a linear combination of source components (e.g., point sources, for the case of CLEAN deconvolution, Hogbom 1974).

The first approach divides the set of CLEAN components \(\{I_i\}\) into \(N_s\) subsets, also called source "subregions", for which a constant fractional polarization is assumed. The corresponding model visibility functions for Stokes \(Q\) and \(U\) are given by

\[
V_Q = \sum_{s} \left( q_s \sum_{k} f(\tau^s_k) \right) \quad \text{and} \quad V_U = \sum_{s} \left( u_s \sum_{k} f(\tau^s_k) \right),
\]

(30)

where \(N_s\) is the number of subregions with constant fractional polarization, \(n_s\) is the number of CLEAN components inside the \(s\)-th subregion, and \(I^s_k\) is the \(k\)-th CLEAN component of Stokes \(I\) in the \(s\)-th subregion. The vectors \(q = (q_s)\) and \(u = (u_s)\) are the parameters that POLSOLVE fits for.

The second approach uses independent, and fixed \((N_s = 0)\), source models for the brightness distributions of \(Q\) and \(U\). The model visibilities for \(Q\) and \(U\) then take the simple form

\[
V_Q = \sum_{i} f(Q_i) \quad \text{and} \quad V_U = \sum_{i} f(U_i),
\]

(31)

where \(\{Q_i\}\) and \(\{U_i\}\) are the (e.g. CLEAN) components used to model the polarization source structure. In this case, POLSOLVE performs a cross-polarization self-calibration estimate of the D-terms.

Instrumental polarization is represented by two complex quantities per antenna, corresponding to each polarization. The leakage terms obey the equation

\[
\begin{pmatrix}
V^R_{\ell R} \\
V^R_{\ell L} \\
V^L_{\ell L}\end{pmatrix} = \begin{pmatrix} 1 & D^P_{\ell} & V^M_{\ell R} \end{pmatrix} \begin{pmatrix} (D^P_{\ell})^2 & \gamma^P_{\ell} \gamma^I_{\ell} & \gamma^I_{\ell} \end{pmatrix}.
\]

(32)

where \(V^k_{\ell}\) is the \(i\)-th observed visibility of polarization product \(k\) (where \(k\) is one of \(RR, RL, LR,\) or \(LL\)) and \(u\) and \(v\) are the coordinates in Fourier space. We construct the functions for the complex polarization vector, \(P\), in the form

\[
V_P = V_Q + j V_U \quad \text{and} \quad V_P = V_Q - j V_U.
\]

(33)

These visibilities are assumed to be fully calibrated for atmospheric effects and electronic antenna gains (computed in the frame of the antenna mounts).

5 SIMULATING MM-WAVE OBSERVATIONS

MeqSilhouette v2 was developed primarily for generating synthetic data for the EHT at mm-wavelengths, but it can equally well be applied to any array, including proposed arrays such as ngEHT and ngVLA. For instance, MeqSilhouette v2 can be used to perform a more elaborate exploration of the VLBI capabilities of MeerKAT for performing extragalactic surveys presented in Deane (2016). MeqSilhouette v2 has also been used to generate 5 GHz synthetic EVN observations for simulating phase corruptions affecting astrometric uncertainties (van Langevelde et al. 2019c). Roelofs et al. (2020) use MeqSilhouette v2 for generating uncalibrated synthetic data observed using an extended EHT array with additional stations located at potential future sites.

We generate synthetic data with MeqSilhouette v2 for three mm-wavelength arrays: EHT2017 array, ngVLA in its long baseline array configuration, and ALMA array in its extended configuration. The EHT2017 array consists of the stations shown in Table 1 observing at a frequency of 230 GHz. The ngVLA array consists of all 18 stations in the Long Baseline Array (LBA) and the central core reduced to a single high-sensitivity site, observing at 86 GHz. ALMA consists of 43 12-metre diameter antennas in its most extended configuration, observing at 230 GHz. An asymmetric crescent (Kanrruddin & Dexter 2013) created with EHT IMAGING, with the inner radius offset by 3 μas in the horizontal direction was used as the source model (Figure 7, right).

Various propagation path effects described in the previous section such as pointing offsets, tropospheric effects, receiver gains, polarization leakage, and thermal noise are introduced. Nominal pointing offsets based on a priori station information are used (e.g., Roelofs et al. 2020). The aperture efficiencies were determined using targeted observations of planets of known brightness temperatures as calibrators (Event Horizon Telescope Collaboration et al. 2019c). The individual station SEFDs are determined by extrapolating measured system temperatures to zero airmass (Janssen et al. 2019; Roelofs et al. 2020).

We adopt the median values measured by VLBI monitor\(^12\) at the individual sites during the 2017 EHT observing campaign (Event Horizon Telescope Collaboration et al. 2019c).

\(^9\) https://launchpad.com/casa-poltools.
\(^10\) http://www.aips.nrao.edu.
\(^11\) https://github.com/jhparkastro/gpcal.
\(^12\) https://bitbucket.org/vlbi.
Horizon Telescope Collaboration et al. 2019b) for the weather parameters. For stations for which this information does not exist, it was obtained through climatological modelling using the data sets from Modern-Era Retrospective Analysis for Research and Applications, version 2 (MERRA-2) from the NASA Goddard Earth Sciences Data and Information Services Center (GES DISC; Gelaro et al. 2017) and the AM atmospheric model software (Paine 2019). The atmospheric conditions over all the ALMA antennas are the same, though independently simulated for each antenna.

The top row of Figure 8 shows the $uv$-coverages for the three arrays in the direction of M87 at their respective observed frequencies, showing how the different arrays complement each other. The bottom row of Figure 8 shows both the corrupted and uncorrupted Stokes $I$ visibility amplitudes for each array as a function of $uv$-distance. The 60 $\mu$m crescent is fully resolved by EHT2017 as expected. At 86 GHz, ngVLA resolves it partially, clearly seeing the source as having some extended structure. ALMA, with its longest baselines of only 12 km, sees the crescent as a point source at 230 GHz and the model visibilities show up as a straight line at 5 Jy in the figure. In the following sections, we concentrate only on the EHT2017 array.

### 6 EFFECT OF BANDPASS ON CLOSURE QUANTITIES

In this section, we estimate the magnitude of systematic non-closing errors introduced by complex bandpass gains in EHT observations at 230 GHz using synthetic data. We introduce station-based bandpasses of varying shapes, spline-interpolated to all frequency channels as shown in Figure 9. The phases are sampled conservatively within the uniform range of $\pm 30^\circ$ (Michael Jansen, private comm). Two different sky models from (Kamruddin & Dexter 2013), generated using HEY-IMAGING were used: a symmetric ring with outer radius 30 $\mu$m and width 3 $\mu$m, and an asymmetric crescent with the inner radius offset by 3 $\mu$m in the horizontal direction (Figure 7). The total flux of the ring was set to 5 Jy to ensure that the S/N of the data was high so that systematic non-closing errors could be unambiguously detected. Thermal noise generated using equation (28) was applied to all datasets. All synthetic data sets used in this subsection are generated with a time resolution of 1 s and a frequency resolution of 31.25 MHz per frequency channel, with 64 channels, spanning a bandwidth of 2 GHz. These data are then band-averaged to a single channel of width 2 GHz. We generated four categories of synthetic data, with 10 data sets to each category, differing in the realisation of thermal noise:

1. symmetric ring with only thermal noise corruption,
2. symmetric ring with thermal noise and bandpass corruption,
3. asymmetric crescent with only thermal noise corruption, and
4. asymmetric crescent with thermal noise and bandpass corruption.

Closure quantities are formed around a closed loop of stations to eliminate the effects of station-based gain errors (e.g. Blackburn et al. 2020). Closure phases are formed over a closed triangle $pqrs$ as

$$\phi_{C,pqrs} = \arg \left( V_{pq} V_{rs} V_{rp} \right),$$

and closure amplitudes are formed over a quadrangle of stations $pqrs$:

$$\ln A_{C,pqrs} = \ln \frac{V_{pq} V_{rs}}{V_{pr} V_{qs}}.$$  

where “$\ln$” is the natural logarithm. The closure phases (CP) and the log closure amplitudes (LCA) are susceptible to systematic non-closing errors. The intra-site baselines ALMA-APEX and JCMT-SMA enable one to form “trivial” closure quantities, between different combinations of these four stations. The trivial CP and LCA so formed should ideally be zero, but factors such as band-averaging without correcting for bandpass errors and intrinsic large-scale asymmetry in source structure can break the assumptions of trivial closure.

We evaluate the magnitude of systematic non-closing errors in trivial closure quantities following the EHT data validation procedures outlined in Event Horizon Telescope Collaboration et al. (2019c) and in Wielgus et al. (2019). We employ the MAD$_0$ (median absolute deviation from zero) statistic, which, for a normally distributed variable $Y$ with zero mean takes the form

$$\text{MAD}_0(Y) = 1.4826 \text{median}(|Y'|),$$

where the subscript 0 denotes that the raw moment is estimated. The scaling factor 1.4826 ensures that MAD$_0$ acts as an unbiased estimator of standard deviation for normally distributed data. The total uncertainty, $\sigma$, associated with the trivial closure quantities defined above is given by

$$\sigma^2 = \sigma^2_{\text{th}} + s^2,$$

where $\sigma_{\text{th}}$ is the known thermal component and $s$ denotes the systematic non-closing error, modeled as a constant. For a trivial closure quantity $X$, we solve for the characteristic value of $s$ by enforcing a following condition:

$$\text{MAD}_0(X) = \text{MAD}_0 \left( \frac{X}{\sqrt{\sigma^2_{\text{th}} + s^2}} \right) = 1.$$  

The MAD$_0$ values estimated using equation (36) and the characteristic magnitude of systematic errors $s$ calculated with equation (38) for the four data sets are given in Table 2. The uncertainties are estimated by obtaining these values for 10 data sets with different thermal noise realisations.

If the error budget is exactly accounted for, then the MAD$_0$ estimator is equal to 1. This is evident from the estimated MAD$_0$ values being very close to unity when thermal noise is the only corruption introduced. The corresponding systematic errors are about 0.08 for trivial closure phases and less than 0.04 per cent for trivial
Figure 8. *Top row:* The $uv$-coverages of EHT2017, ngVLA, and ALMA towards M87; note that the ALMA baselines are three orders of magnitude shorter than the other two. *Bottom row:* Model (black) and corrupted Stokes $I$ visibility amplitudes as a function of $uv$-distance, as observed by each array. ALMA, due to its relatively short baselines, does not resolve the source and the model visibilities correspond to a horizontal line at 5 Jy.

Table 2. Estimated MAD$_0$ values and characteristic magnitudes of systematic errors in the trivial closure quantities in the synthetic data sets described in section 6.

| Type of data set | Trivial CP | Trivial LCA |
|------------------|------------|-------------|
|                  | $\text{MAD}_0$ (deg.) | $\text{MAD}_0$ (%) |
| (i) Symmetric ring (thermal noise only) | 1.012 ± 0.042 | 0.041 ± 0.011 | 1.148 ± 0.051 | 0.4 ± 0.1 |
| (ii) Symmetric ring (thermal noise + bandpass gains) | 2.76 ± 0.408 | 0.681 ± 0.008 | 4.074 ± 0.361 | 4.7 ± 0.5 |
| (iii) Asymmetric crescent (thermal noise only) | 1.118 ± 0.067 | 0.077 ± 0.016 | 0.984 ± 0.04 | 0.08 ± 0.02 |
| (iv) Asymmetric crescent (thermal noise + bandpass gains) | 3.862 ± 0.363 | 0.684 ± 0.071 | 7.484 ± 0.829 | 3.8 ± 0.5 |

log closure amplitudes. In the presence of bandpass gains that are not accounted for in the error budget, the MAD$_0$ values and the systematic errors increase noticeably. For the trivial closure phases, the MAD$_0$ values indicate that the reported errors are too small by factors of 2.76 and 3.86, for the symmetric ring and asymmetric crescent models respectively; for trivial log closure amplitudes, these factors increase to about 4 and 7.5 respectively. The estimated systematics are about 0.7° for trivial closure phases and up to 5 per cent for trivial log closure amplitudes.

7 SIMULATING INSTRUMENTAL POLARIZATION

7.1 Simulating polarized sources with leakage

To validate the implementation of instrumental polarization in MeqSilhouette using polsolve, we generate six synthetic data sets, corrupted with instrumental polarization, time-varying complex gains, and thermal noise. The accuracy to which D-terms can be estimated depends on various factors such as station parameters and intrinsic source polarization structure. The source models we use are shown in the first column of Figure 10. The truth images convolved with the nominal EHT beam are shown in the second column. Source model 1 is a highly polarized ring while model 2 is an offset crescent with low polarization. In both models, the polarized structure closely follows the Stokes I distribution. Models 3-5 are ring sources with differing EVPA structures. Model 6 represents the characteristic source structure of any VLBI calibrator source at cm and mm-wavelengths. We choose models differing in EVPA structures and fractional polarization. All source model images are generated using EHT-IMAGING.

For the scale of bandpass gains assumed here, these errors are comparable to the magnitude of the systematic non-closing errors for EHT 2017 observations of 3C279 and M87 ($2^\circ$ for closure phases and less than 4% for log closure amplitudes) quantified by Event Horizon Telescope Collaboration et al. (2019c). Alongside the many factors such as intrinsic source structure or polarisation leakage, this indicates how unaccounted-for bandpass errors can have a similar effect on the closure quantities. The spectral capabilities of MeqSilhouette will enable us to perform a thorough study of EHT bandpass characteristics and their effect on the observed data.
model and are fixed for the rest of the calibration process. A model of the source is generated using clean and multiple iterations of phase and amplitude self-calibration are performed with decreasing solution intervals. Finally, several iterations of polsolve and clean are performed, by fixing the full polarization model to the output of clean at each iteration.

The reconstructed images convolved with the clean beam are shown in the third column of Figure 10. The polarization peaks displayed in each panel are found to correlate well between the ground-truth images and the reconstructed images, as are the Stokes I distribution and the polarization structure. Figure 11 shows the D-terms recovered by polsolve for each data set for all stations, along with the ground-truth D-term values. The recovered D-terms correlate highly with the ground-truth values. The recovered D-terms for model 1 are the least accurate owing to its high intrinsic fractional polarization. D-terms of similar magnitude applied to models with low fractional polarization are estimated more accurately. The D-term estimates for the stations participating in intra-baseline fitting (ALMA-APEX and JCMT-SMA) are the most tightly constrained and those corresponding to the station with the longest baselines on a short \((u, v)\) track, PV, have the largest dispersion.

### 7.2 Frequency-dependent polarization leakage

\textsc{meqsv2} can also introduce frequency-dependent variations in the simulated D-terms. The D-terms were generated independently for each channel by sampling them from a normal distribution. We use polsolve in iterative self-calibration mode to solve for these D-terms, treating them as independent complex numbers per frequency channel (see section 4).

We use Model 3 from Figure 10 to generate synthetic data with 2 GHz bandwidth divided into 32 spectral windows, with frequency-varying D-terms introduced alongside complex gains and thermal noise. Figure 12 shows the simulated D-terms and the D-terms recovered by polsolve with error bars. The estimated D-terms correlate remarkably well with the simulated values. The magnitude of the errors are similar to that corresponding to model 3 in Figure 11.

### 8 DISCUSSION

Synthetic observations provide uniform data sets with known source and instrumental properties for performing various kinds of feasibility studies. Since \textsc{meqsv2} aims to compute data corruptions from first principles, it can be used to perform a systematic exploration of realistic site and antenna parameters, which helps in commissioning new sites and optimizing observing schedules (e.g. Blackburn et al. 2020). Conversely, with known site and antenna characteristics, the synthetic data can be used to quantitatively compare different astrophysical source models that characterize the observed emission.

The realistic synthetic data provided by \textsc{meqsv2} can be used to validate new imaging, calibration, and parameter estimation techniques. Synthetic data generated with \textsc{meqsv2} have been used to test \textsc{zagros}, a fully-Bayesian fringe-fitting framework for analysing VLBI observations (Natarajan et al. 2020). These data can potentially be used to compare different parameter estimation frameworks currently being used for analysing EHT data, such as \textsc{themis} (Broderick et al. 2020), \textsc{march} (Psaltis et al. 2020), and \textsc{dmc} (Pesce 2021). Fully end-to-end pipelines starting from synthetic data generation with \textsc{meqsv2}, followed by a priori calibration and posterior estimation using statistical visibility analysis packages are under active development to enable large-scale feasibility studies. The modular nature of the software lends itself easily to constructing such end-to-end pipelines that allow calibration to start further up the signal chain (e.g. Natarajan et al. 2020). The computational requirements for such large-scale studies necessitate adapting these software packages to a High Performance Computing (HPC) environment.

The experiments performed in this paper are intended to illustrate the capabilities of \textsc{meqsv2}. More elaborate studies of the various propagation path effects on calibration and imaging must be performed for future site selection and testing the capabilities and limitations of new algorithms. More exhaustive exploration of frequency-dependent gain variations and their effects on closure quantities (see section 6) need to be performed to fully understand the bandpass characteristics of the EHT array and solve for them.

Section 7 illustrates how fractional polarization intrinsic to the source and the lack of short baselines to a station can affect the recovery of station-based instrumental polarization. The accuracy of the measured D-terms can be improved by performing a self-consistent multi-source fit to the D-terms, if observations of multiple sources are available for the same epoch (e.g. Martí-Vidal et al. 2021). More complex experiments devised to study this effect will help to fully exploit the full-polarization data products generated by EHT observations.

### 9 SUMMARY AND OUTLOOK

We present \textsc{meqsv2}, a synthetic data generation software package capable of performing synthetic VLBI observations with full polarimetric, time and frequency-resolved astrophysical models and realistic propagation path effects. It can also be seamlessly integrated with calibration tools such as the EHT software packages to a High Performance Computing (HPC) environment.

EHT observations.

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**Figure 9.** Station-based bandpass amplitudes used in generating synthetic data for section 6.

**Figure 10.** Gain amplitude vs. channel for station-based bandpass amplitudes used in generating synthetic data.

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https://github.com/dpesce/eht-dmc
Figure 10. From left to right: source model used in the simulations, model convolved with the nominal EHT beam of FWHM 25 μas, and CLEAN model obtained from the synthetic data convolved with the same 25 μas beam. Contours indicate total intensity and are logarithmically spaced between 1 and 100 per cent of the intensity peak; grayscale images show polarization intensity; lines indicate the EVPA distribution, with lengths proportional to the local fractional polarization.
Figure 11. D-term estimates for all stations from POLSOLVE in iterative self-calibration mode for synthetic data sets described in section 7.1. Each square with error bar corresponds to one data set. The filled circles denote the ground-truth D-term values.

Figure 12. POLSOLVE estimates of per-channel D-terms in polarization self-calibration mode. The solid lines represent the ground-truth values, while the circles and asterisks represent the POLSOLVE estimates.

MEQSV2 by generating synthetic EHT observations of polarized geometric source models with instrumental polarization and estimating the D-terms using POLSOLVE, showing that the reconstructed images correlate very well with input source models.

Future versions will be able to simulate more polarimetric effects such as Faraday rotation in the ISM and ionospheric phase corruption effects for simulating cm-VLBI radio observations. The ability to input more time and frequency-variable weather parameters will also be introduced. We also plan to take full advantage of new distributed and parallel computing algorithms and software packages for generating synthetic data sets on a large scale much faster.

MEQSV2 currently optionally splits large data sets into subsets for processing, to accommodate systems with low memory specifications. Such features can easily be ported to forward-modelling software packages such as CODEX-AFRICANUS (Perkins et al. in prep) which provide distributed CPU and GPU computing functionality. For handling large data sets, we plan to use DASK-MS (Perkins et al. in prep), which uses DASK, an open source library for parallel programming in Python (Rocklin 2015), to scale computations. DASK-MS can convert MS v2.0 data between the CASA table format, and other high performance, cloud-native formats such as PARquet (Vohra 2016) and ZARR (Miles et al. 2021). These new formats are explicitly designed for parallel, distributed processing and offer superior disk I/O performance, which will accelerate the synthetic data generation process. Finally, we also plan to set up a publicly available online interface to MEQSV2, which can be of use to students and teachers alike without them having to invest in the requisite computing power.

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DATA AVAILABILITY

MeqSilhouette v2 is open source and publicly available on GitHub at https://github.com/rdeane/MeqSilhouette. Its documentation can be found at https://meqsilhouette.readthedocs.io. Bugs and other issues can be reported by creating an issue on the repository.

The data underlying this article will be shared on reasonable request to the corresponding author.
Natarajan et al.

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