Analysis of the Distance Scales by Cepheids from the Gaia EDR3 Catalogue Data

V. V. Bobylev* and A. T. Bajkova

aPulkovo Astronomical Observatory, Russian Academy of Sciences, St. Petersburg, 196140 Russia
*e-mail: vbobylev@gaoran.ru

Received March 8, 2022; revised April 3, 2022; accepted May 16, 2022

Abstract—We study the kinematics of a sample of classical Cepheids younger than 120 Myr. For these stars, the estimates of distances taken from Skowron et al., which are based on the period–luminosity relation, and the line-of-sight velocities and the proper motions from the Gaia catalog are available. There are also distance estimates derived from the trigonometric parallaxes contained in the Gaia EDR3 catalog. A method, which relies on comparison of the first-order derivative of the Galactic rotation angular velocity, showed the need to lengthen the distance scales determined by Skowron et al. by about 10%. This conclusion was confirmed by direct comparison to the distances predicted on using the trigonometric parallaxes. With taking into account this result, we obtained new estimates of the Galactic rotation parameters and the parameters of a spiral density wave.

Keywords: Cepheids, kinematics, distance scale, Galactic rotation

DOI: 10.1134/S1063772922080029

1. INTRODUCTION

Cepheids are stars, the brightness variability of which is caused by their radial pulsations. By means of the period–luminosity relation [1, 2], it is possible to estimate the distances to these stars with high accuracy. Ultimately, this allows one to establish an independent Cepheid distance scale and to cover it with a substantial area of the Universe. Classical Cepheids are those exhibiting pulsation periods of 1 to 100 days. Their age does not exceed approximately 400 Myr. These stars are important for studying the properties of the thin disk of the Galaxy, its spiral structure, rotation, evolution, etc.

For example, with the use of 220 classical Cepheids with the proper motion values from the Hipparcos catalog [3], the Galactic rotation parameters were estimated with higher precision [4, 5]. Different samples of classical Cepheids were used to estimate the Galactic spiral structure parameters [6–10]. Since Cepheids are seen from large distances, these stars are used to determine the distance from the Sun to the Galactic center [11–13]. They also serve to study the Galactic disk warp [14–16] and the features in the distribution of various chemical elements in the Galactic disk [17–19].

New data on more than 2200 classical Cepheids were presented in the catalog by Skowron et al. [20]. Distances to these stars were determined from the period–luminosity relation with errors of 5–10%. Mróz et al. [21] provided more than 800 Cepheids from these catalog with the line-of-sight velocities from the Gaia DR2 catalog [22] and determined the Galactic rotation parameters from these data. In the paper by Brown et al. [22], it was concluded that the distances to Cepheids obtained by Skowron et al. [20] should be made longer by 9%, while the kinematic parameters were estimated without correcting the distance scale [20].

When analyzing the kinematics of stars, it is important to have in hand high-accuracy values for their trigonometric parallaxes and proper motions. At present, there is a version of the Gaia EDR3 catalog [23] containing the trigonometric parallax values for 1.5 billion stars specified more accurately by approximately 30%, as compared to those in the previous Gaia DR2 version; and the accuracy of the proper motions of these stars were improved roughly twice. The trigonometric parallaxes of about one third of stars from the Gaia EDR3 catalog were measured with errors smaller than 0.2 milliarcsecond (mas). The proper motions of about half of stars from this catalog were measured with an error less than 10%.

We note that the distances to stars calculated through trigonometric parallaxes are reliable only if the errors in parallaxes are small (less than 10%). If the errors in parallaxes are high, the techniques, which are similar to the method developed by Lutz and Kelker [24], are used. With these techniques, the distances to about 1.47 billion stars from the Gaia EDR3 catalog, which had been calculated through trigonometric par-
allaxes, were improved by Bailer-Jones et al. [25]. In this study, we use the distances to Cepheids from the paper [25] as one of the sources of the distances.

The purpose of this study is to compare the distances to Cepheids, which were obtained from the period–luminosity relations in a paper [20], with the distances based on trigonometric parallaxes from the Gaia catalog. It would be also of interest to redefine the Galactic rotation parameters and the parameters of a spiral density wave with the use of a more accurate Cepheid distance scale.

2. METHOD

From observations, we have three components of the velocity of a star: the line-of-sight velocity \( V_r \) and two projections of the tangential velocity \( V_l = 4.74 r \mu_\ell \cos b \) and \( V_b = 4.74 r \mu_b \) directed along the Galactic longitude \( l \) and latitude \( b \), respectively. Each of these three velocities is expressed in kilometers per second; the coefficient 4.74 is a dimension ratio, and \( r \) is the heliocentric distance of a star expressed in kiloparsecs. The proper motion components \( \mu_\ell \cos b \) and \( \mu_b \) are expressed in milliarcseconds per year (mas/yr). The components \( V_r, V_l, \) and \( V_b \) are used to calculate the velocities \( U, V, \) and \( W \) directed along the axes of the Galactic rectangular coordinate system:

\[
U = V_r \cos l \cos b - V_l \sin l - V_b \cos l \sin b,
\]
\[
V = V_r \sin l \cos b + V_l \cos l - V_b \sin l \sin b,
\]
\[
W = V_r \sin b + V_b \cos b,
\]

where the velocities \( U, V, \) and \( W \) are oriented from the Sun to the Galactic center, along the Galactic rotation direction, and to the Northern Galactic pole, respectively. Two velocities—\( V_R \)—directed radially from the Galactic center and the orthogonal one \( V_{\text{circ}} \) directed along Galactic rotation—can be determined from the following relationships

\[
V_{\text{circ}} = U \sin \theta + (V_0 + V) \cos \theta,
\]
\[
V_R = -U \cos \theta + (V_0 + V) \sin \theta,
\]

where the position angle \( \theta \) satisfies the relationship \( \tan \theta = y/(R_0 - x) \); \( x, y, \) and \( z \) are the rectangular heliocentric coordinates of a star (the velocities \( U, V, \) and \( W \) are directed along the corresponding axes \( x, y, \) and \( z \)); and \( V_0 \) is the linear velocity of Galactic rotation at the solar distance \( R_0 \).

To determine the parameters of the Galactic rotation curve, we use the equations derived from Bottlinger’s formulas, in which the angular velocity \( \Omega \) was expanded in a series to terms of the second order of smallness in \( r/R_0 \):

\[
V_r = -U \cos b \cos l - V_0 \cos b \sin l - W_0 \sin b + R_0 (R - R_0) \sin l \cos b \Omega^2_0 + 0.5 R_0 (R - R_0) \sin l \cos b \Omega^4_0,
\]
\[
V_l = U \sin l - V_0 \cos l - r \Omega_0 \cos b + (R - R_0) (R_0 \cos l - r \cos b) \Omega^2_0 + 0.5 (R - R_0) (R_0 \cos l - r \cos b) \Omega^4_0,
\]
\[
V_b = U \cos l \sin b + V_0 \sin l \sin b - W_0 \cos b - R_0 (R - R_0) \sin l \sin b \Omega^2_0 - 0.5 R_0 (R - R_0) \sin l \sin b \Omega^4_0,
\]

where \( R \) is the distance from the star to the Galactic rotation axis and \( R^2 = r^2 \cos^2 b - 2 R_0 r \cos b \cos l + R_0^2 \). The velocities \( (U, V, W)_\odot \) are the mean group velocity of a sample; since they are representative of the peculiar motion of the Sun, they are taken with opposite signs. \( \Omega_0 \) is the angular velocity of Galactic rotation at the solar distance \( R_0 \); and the parameters \( \Omega^2_0 \) and \( \Omega^4_0 \) are the corresponding derivatives of the angular velocity \( \Omega_0 = R_0 \Omega^2_0 \). In this paper, the value of \( R_0 \) is assumed to be \( 8.1 \pm 0.1 \) kpc according to a review [26], where it was derived as a weighted average value from a large number of up-to-date individual estimates.

By solving conditional equations of form (3)—(5) with the least squares method (LSM), we can find the following six unknown quantities: \( (U, V, W)_\odot, \Omega_0, \) and \( \Omega^2_0 \). When solving only one of the conditional equations of form (3) with the LSM, we may find only five unknown quantities: \( (U, V, W)_\odot, \Omega_0, \) and \( \Omega^2_0 \).

The impact of a spiral density wave on the radial \( V_R \) and residual tangential \( \Delta V_{\text{circ}} \) velocities is periodic, and its amplitude is about 10 km/s. According to the linear theory of density waves by Lin and Shu [27], this influence is described by the relationships of the following form:

\[
V_R = -f_R \cos \chi,
\]
\[
\Delta V_{\text{circ}} = f_\theta \sin \chi,
\]

where

\[
\chi = m (\cot(i) \ln(R/R_0) - \theta) + \chi_\odot
\]

is the spiral wave phase; \( m \) is the number of spiral arms; \( \cot(i) \) is the cotangent of a pitch angle \( i \) of the spiral pattern \( i < 0 \) for a coiling spiral); \( \chi_\odot \) is the radial phase of the Sun in a spiral wave; while \( f_R \) and \( f_\theta \) are the amplitudes of perturbations in the radial and tan-


To study the periodicities in the velocities $V_R$ and $\Delta V_{\text{circ}}$, we use the spectral (periodogram) analysis. The wavelength $\lambda$ (the distance between neighboring segments of spiral arms counted along the radial direction) is calculated from the relationship

$$2\pi R_0/\lambda = m \cot(i).$$

Let us consider a number of the measured velocities $V_{R_k}$ (they may be radial $V_R$, tangential $\Delta V_{\text{circ}}$, or vertical $V_T$ velocities), where $n = 1, \ldots, N$ and $N$ is the number of objects. The purpose of the spectral analysis is to reveal the periodicity from a data set in accordance with the assumed model describing a spiral density wave with the parameters $f$, $\lambda$ (or $i$), and $x_0$.

Due to accounting for the logarithmic character of a spiral wave and the position angles of objects $\theta_i$, our spectral analysis of the velocity perturbation series is reduced to calculations of the square of the amplitudes (the power spectrum) of the standard Fourier transformation [28]:

$$V_{\lambda_k} = \frac{1}{N} \sum_{n=1}^{N} V'_n(R_n) \exp \left( -j \frac{2\pi R'_n}{\lambda_k} \right),$$

where $V_{\lambda_k}$ is the $k$th harmonic of the Fourier transformation with the wavelength $\lambda_k = D/k$; $D$ is the period of the analyzed series; and

$$R'_n = R_0 \ln(R_n/R_0),$$

$$V'_n(R_n) = V_n(R_n) \exp(jm\theta_n).$$

The required wavelength $\lambda$ corresponds to a peak value of the power spectrum $S_{\text{peak}}$. The pitch angle of a spiral density wave is calculated from Eq. (8). The amplitude and the phase of perturbations are obtained from fitting the measured data by the harmonic with the wavelength determined.

3. DATA

In this study, we use the data on classical Cepheids reported by Skowron et al. [20], who estimated the distances and the ages of 2431 Cepheids. These Cepheids were observed in the frames of the fourth stage of the Optical Gravitational Lensing Experiment (OGLE) program [29]. Skowron et al. [20] calculated the distances to Cepheids on the base of the period–luminosity relation from the light curves in the mid-IR range. According to these authors, in their catalog, random errors in determining the distances to Cepheids are within an interval of 5–10%. In a paper [20], the age of Cepheids was estimated by the technique proposed by Anderson et al. [30] with accounting for the axial rotation of stars and their metallicity.

We used the stars from the paper by Mróz et al. [21], who provided a number of Cepheids from the list [20] with the line-of-sight velocities and the proper motions taken from the Gaia DR2 catalog. For the kinematic analysis, we selected the Cepheids younger than 120 Myr, which are located no further than 5 kpc from the Sun, and identified them with objects from the Gaia EDR3 catalog and the list by Baier-Jones et al. [25].

In a paper [25], for approximately 1.47 billion stars from the Gaia EDR3 catalog, the distances were more accurately calculated through trigonometric parallaxes. The systematic correction to the Gaia EDR3 parallaxes, the value of which, $\Delta \pi = -0.017$ mas, had been determined by Lindegren et al. [31], was considered. The authors used two techniques, purely geometric (geom) and photogeometric (phgeom) ones, to calculate two versions of the distances. According to their opinion, the photogeometric technique results in more precise distances. We note that the both methods are model-dependent, since they use the referencing to the Galaxy model. As the authors themselves notice, these distances are useful when the errors in determining the trigonometric parallaxes of stars from the Gaia EDR3 catalog are high.

Thus, we have 363 Cepheids. For them, there are distances obtained from the period–luminosity relation, as well as distances based on trigonometric parallaxes from the Gaia EDR3 catalog. At the same time, a number of distance scales can be used. First, these are the distances calculated directly from the Gaia EDR3 parallaxes. Second, these are two kinds of the distances to Cepheids taken from a paper [25].

4. RESULTS AND DISCUSSION

We solved conditional equations of form (3)–(5) in three ways. First, three velocities $V_r$, $V_t$, $V_{z}$ were jointly used. Second, only the line-of-sight velocities $V_r$ of Cepheids were used. Third, only the velocities $V_t$ were used.

In the upper part of Table 1, we show the kinematic parameters obtained by using the distances to Cepheids, which were calculated on the base of the period–luminosity relation in a paper [20].

According to papers [32, 33], the ratio of the first-order derivative of the Galactic rotation angular velocity obtained only from the proper motions to that obtained only from the line-of-sight velocities yields the coefficient of the distance scale $p = (\Omega'_r)/(\Omega'_t)$. This coefficient serves as a correction factor of the type $p = r/r_{\text{true}}$, where $r$ and $r_{\text{true}}$ stand for the used and actual distances, respectively; hence, $r_{\text{true}} = r/p$. The error in the coefficient $p$ was calculated from the relationship $\sigma_p^2 = (\sigma_{\Omega'_r}/\Omega'^2_r)^2 + (\Omega'^2_t/\sigma_{\Omega'_t})^2$. According to the data from Table 1, we find...
Table 1. Kinematic parameters found by Cepheids with the use of the distances from a paper [20]

| Parameters | $V_r + V_l + V_b$ | $V_r$ | $V_l$ |
|------------|------------------|------|------|
| $U_0$, km/s | 8.15 ± 0.63 | 9.57 ± 1.25 | 8.08 ± 0.94 |
| $V_0$, km/s | 12.87 ± 0.80 | 15.53 ± 1.13 | 7.47 ± 1.77 |
| $W_0$, km/s | 7.25 ± 0.61 | | |
| $\Omega_0$, km/s/kpc | 29.03 ± 0.25 | 29.17 ± 0.35 | |
| $\Omega'_0$, km/s/kpc^2 | −4.015 ± 0.066 | −4.216 ± 0.101 | −3.747 ± 0.123 |
| $\Omega''_0$, km/s/kpc^3 | 0.639 ± 0.052 | 0.700 ± 0.106 | 0.417 ± 0.087 |
| $\sigma_0$, km/s | 11.60 | 14.12 | 13.30 |
| $U_0$, km/s | 9.03 ± 0.65 | 9.55 ± 1.24 | 8.77 ± 1.02 |
| $V_0$, km/s | 11.58 ± 0.81 | 12.78 ± 1.12 | 6.89 ± 1.91 |
| $W_0$, km/s | 8.01 ± 0.62 | | |
| $\Omega_0$, km/s/kpc | 28.87 ± 0.23 | 29.25 ± 0.35 | |
| $\Omega'_0$, km/s/kpc^2 | −3.894 ± 0.063 | −3.893 ± 0.093 | −3.887 ± 0.126 |
| $\Omega''_0$, km/s/kpc^3 | 0.602 ± 0.044 | 0.593 ± 0.088 | 0.519 ± 0.081 |
| $\sigma_0$, km/s | 11.89 | 13.99 | 14.48 |

The parameters obtained on the base of the period—luminosity relation (see [20]) are in the upper part of the table. The parameters obtained from the distances to Cepheids, which were lengthened by 10%, are in the lower part of the table.

$p = 0.89 ± 0.04$. This means that the distances to Cepheids calculated on the base of the period—luminosity relation in a paper [20] should be lengthened approximately by 10%.

In the lower part of Table 1, we present the kinematic parameters obtained by using the distances already lengthened by 10%. The results of comparing different scales of distances to Cepheids are shown in Figs. 1 and 2. In these diagrams, the distances from a paper [20] are shown in dependence on those calculated from the Gaia EDR3 trigonometric parallaxes ($r = 1/\pi$), the geometric and photogeometric distances from a paper [25], as well as the distances calculated from the Gaia EDR3 trigonometric parallaxes with accounting for the correction $\Delta \pi = −0.017$ mas ($r = 1/(\pi + 0.017)$).

It is seen from Fig. 1 that the scale of distances to Cepheids obtained through the Gaia EDR3 trigonometric parallaxes is substantially longer than the scale by Skowron et al. [20]. Moreover, even for small distances, the agreement between these scales is not quite good.

From a comparison of the distances to Cepheids shown in Fig. 2, we may conclude the following: (1) in general, the distances from a paper [20] and the scales based on the Gaia EDR3 trigonometric parallaxes well agree; (2) the worst agreement is observed in panel (a) of this figure, and this is most evident when the distances from the Sun exceed 4 kpc; (3) the distances shown in panels (b, c, d) almost completely agree; (4) after correcting the distance scales shown in Fig. 2d, they become well consistent with each other; this circumstance is especially important for the scale by Skowron et al. [20], with which the kinematic parameters are estimated in the present study.

Figure 3 shows the distribution of Cepheids younger than 120 Myr in projection onto the Galactic plane $XY$. In this coordinate system, the axis $X$ is directed from the Galactic center to the Sun, while the direction of the axis $Y$ coincides with the Galactic rotation direction. Hereinafter, the distances to Cepheids from the list [20] were made longer by 10%.

Figure 4 shows the circular rotation velocities of Cepheids $V_{\text{circ}}$ in dependence on the distance $R$. We present the Galactic rotation curve, which is constructed with the parameters given in the lower part of Table 1 (second column). This Galactic rotation curve was used to calculate the residual circular rotation velocities of Cepheids $\Delta V_{\text{circ}}$.

The Galactic rotation parameters determined in the present study well agree with their estimates obtained by using the other Galactic sources. For example, from the data on 130 galactic masers with the measured trigonometric parallaxes, the authors of a paper [33] found the velocity components for the Sun $(U_0, V_0) = (11.40, 17.23) ± (1.33, 1.09)$ km/s and the following values for the Galactic rotation curve parameters: $\Omega_0 = 28.93 ± 0.53$ km/s/kpc, $\Omega'_0 = −3.96 ± 0.07$ km/s/kpc$^2$, $\Omega''_0 = 0.87 ± 0.03$ km/s/kpc$^3$, and $V_0 = 243 ± 10$ km/s for a value of $R_0 = 8.40 ± 0.12$ kpc obtained.

We note that the catalog [20] contains Cepheids exhibiting pulsations in both the fundamental mode and the first overtone. For each of these pulsation modes, slightly differing calibrations were used, which may influence the distance scales.
Among 363 Cepheids selected for this analysis, those with the fundamental pulsation mode strongly dominate: their number is 308, while there are only 55 Cepheids with pulsations on the first overtone. In Table 2, we present the kinematic parameters obtained by 308 Cepheids with the fundamental pulsation mode. The values of the kinematic parameters from Table 2 should be compared to those in the lower part of Table 1. We note that this comparison reveals no significant differences between these parameters. The most important fact is that, according to the data in Table 2, the value of the scaling factor $p$ is $1.01 \pm 0.04$. This suggests that these Cepheids are completely consistent with the correction of the scale by Skowron et al. [20] performed here. Thus, there are no differences between the results for the most homogeneous (in terms of a pulsation mode) sample of Cepheids and the mixed sample.

For the velocities $V_r$, $\Delta V_{\text{circ}}$, and $W$, a spectral analysis was conducted. The results—the velocities $V_r$, $\Delta V_{\text{circ}}$, and $W$ versus the distance $R$ and the corresponding power spectra—are presented in Fig. 5. From the velocities $V_r$ and $\Delta V_{\text{circ}}$, the following estimates were obtained: $f_R = 5.5 \pm 2.0$ km/s and $f_0 = 7.1 \pm 2.0$ km/s; $\lambda_R = 1.9 \pm 0.5$ kpc ($i_R = -8.3^\circ \pm 2.5^\circ$.

**Table 2.** Kinematic parameters determined by 308 Cepheids with the fundamental pulsation mode

| Parameters | $V_r + V_l + V_b$ | $V_r$ | $V_l$ |
|------------|------------------|------|------|
| $U_\odot$, km/s | 8.77 ± 0.69 | 9.54 ± 1.34 | 8.34 ± 1.08 |
| $V_\odot$, km/s | 12.68 ± 0.88 | 13.81 ± 1.23 | 7.98 ± 2.01 |
| $W_\odot$, km/s | 8.27 ± 0.79 | | |
| $\Omega_0$, km/s/kpc | 28.52 ± 0.25 | | 28.99 ± 0.37 |
| $\Omega_0^\prime$, km/s/(kpc)^2 | $-3.877 \pm 0.068$ | $-3.870 \pm 0.099$ | $-3.911 \pm 0.136$ |
| $\Omega_0^\prime\prime$, km/s/(kpc)^3 | 0.641 ± 0.047 | 0.619 ± 0.093 | 0.575 ± 0.085 |
| $\sigma_0$, km/s | 11.84 | 14.00 | 14.27 |

The distances from a paper [20] were lengthened by 10%.
for \( m = 4 \) and \( \lambda_\theta = 2.6 \pm 0.5 \text{ kpc} \ (i_\theta = -11.4^\circ \pm 2.8^\circ \) for \( m = 4 \); and \( \chi_\odot_R = -208^\circ \pm 16^\circ \) and \( \chi_\odot_0 = -185^\circ \pm 18^\circ \). The vertical velocities (Figs. 5e and 5f) also exhibit periodicity, though with a smaller amplitude: \( f_W = 3.9 \pm 2.0 \) km/s, \( \lambda_W = 2.4 \pm 0.5 \) kpc \( (i_W = -10.7^\circ \pm 2.7^\circ \) for \( m = 4 \) \), and \( \chi_\odot_W = -138^\circ \pm 18^\circ \). It should be noted that there is a noticeable diversity between the values of the wavelength \( \lambda \) determined by the velocities \( V_R \) and \( \Delta V_{\text{circ}} \).

We also performed a joint spectral analysis of the velocities \( V_R \) and \( \Delta V_{\text{circ}} \). Here, for both kinds of velocity, we assume that the wavelength and the phase of the Sun in a density wave take the same values. The results are shown in Fig. 6. This approach yielded the following estimates: \( f_R = 2.7 \pm 1.7 \) km/s, \( f_\theta = 4.4 \pm 1.7 \) km/s, and \( \lambda_R = 2.6 \pm 0.5 \) kpc.

Our preference is that the separate solutions yield more reliable results; this especially concerns the results of the separate solution in the radial velocities. Thus, for the joint solution (Fig. 6), the significance of a peak in the power spectrum is 0.986. At the same time, the analysis performed separately in the tangential velocity (Fig. 5d) and the radial one (Fig. 5b) yields the values 0.985 and 0.990, respectively, for the significance of a peak in the power spectrum. By the way, the significance of the peak in the power spectrum in a separate analysis of vertical velocities (Fig. 5f) is 0.616. This is a formal point of view.

On the other hand, as is seen from Figs. 5 and 6, the difference in the phase of the Sun in a wave is strong. The wave, which is most close to the expected one, is a wave in the radial velocities found in the separate analysis (Fig. 5a). Indeed, from Fig. 3 we can see that...
Fig. 3. The distribution of the considered sample of Cepheids in projection onto the Galactic $XY$ plane.

Fig. 4. The circular rotation velocities of Cepheids $V_{\text{circ}}$ in dependence on the distance $R$. The Galactic rotation curve is shown with the confidence region bordered at the $1\sigma$ level, and the position of the Sun is marked by a vertical line.
Cepheids are strongly concentrated toward a segment of the Carina–Sagittarius spiral arm at a distance of $R \approx 7$ kpc. According to the model by Lin and Shu [27], the velocity perturbation $f_0$ in the center of the spiral arm (at $R = 7$ kpc) should be directed to the Galactic center (i.e., it should take negative values in the diagram), which can be observed in Fig. 5a rather than Fig. 6a. We note that the radial velocities are most important for this problem, since they are independent of the accounting for the rotation curve.

The parameters found in this study may be compared to the estimates obtained in a paper [13] from a sample of Cepheids younger than 90 Myr (their distances are also from the paper by Skowron et al. [20], but the proper motions are from the Gaia DR2 catalog): $f_R = 12.0 \pm 2.6$ km/s, $f_0 = 8.9 \pm 2.5$ km/s, $\lambda_R = 2.5 \pm 0.3$ kpc ($i_R = -10.8^\circ \pm 3.1^\circ$ for $m = 4$), and $\lambda_0 = 2.7 \pm 0.5$ kpc ($i_0 = -11.8^\circ \pm 3.1^\circ$ for $m = 4$). This comparison reveals a complete agreement between the estimates of $f_0$, $\lambda_0$, and $i_0$ in the tangential direction and a less agreement between the estimates in the radial direction. The difference is caused by the samples’ arrangement. In our case, the sample is limited by the distance and dominated by Cepheids from the Carina–Sagittarius spiral arm segment (Fig. 3).
ANALYSIS OF THE DISTANCE SCALES BY CEPHEIDS

It is also worth mentioning the paper by Loktin and Popova [34], where the authors analyzed the kinematics of roughly 1000 open star clusters (OSCs) of different ages with the proper motions of stars from the Gaia DR2 catalog, which yielded the following estimates: \( f_R = 4.6 \pm 0.7 \text{ km/s} \) and \( f_\theta = 1.1 \pm 0.4 \text{ km/s} \).

In a paper [35], a spectral analysis was applied to the spatial velocities of 233 young OSCs with the proper motions and parallaxes from the Gaia EDR3 catalog. It was shown that the values for the wavelength and the velocity perturbation determined independently by each of the velocity kinds generally agree, and the following estimates were obtained: \( \lambda_R = 3.3 \pm 0.5 \text{ kpc} \) and \( \lambda_\theta = 2.6 \pm 0.6 \text{ kpc} \), where \( i_R = -14.5^\circ \pm 2.1^\circ \) and \( i_\theta = -11.4^\circ \pm 2.6^\circ \) for \( m = 4 \) and \( R_0 = 8.1 \pm 0.1 \text{ kpc} \) assumed. The amplitudes of the radial and tangential velocity perturbations amounted to \( f_R = 9.1 \pm 0.8 \text{ km/s} \) and \( f_\theta = 4.6 \pm 1.2 \text{ km/s} \), respectively.

5. CONCLUSIONS

We compared the distances to Cepheids obtained from the period–luminosity relation to the distances based on the trigonometric parallaxes of the Gaia catalog. For this purpose, we used the sample of young Cepheids from the paper by Skowron et al. [20]. It contains 363 Cepheids younger than 120 Myr, which are located no further than 5 kpc from the Sun. It has been shown that the scale by Skowron et al. [20] should be lengthened by approximately 10%.

It has been also shown that the corrected scale by Skowron et al. [20] well agrees with the scale that is based on the trigonometric parallaxes from the Gaia EDR3 catalog and contains the systematic correction \( \Delta \pi \). The corrected scale by Skowron et al. [20] is also well consistent with the scales from the paper by Bailer-Jones et al. [25].

By correcting the scale of Skowron et al. [20], we found new estimates for the group velocity of the Sun and the Galactic rotation parameters. To estimate the parameters of the Galactic spiral density wave, a spectral analysis of the velocities \( V_R \) and \( \Delta V_{\text{circ}} \) was performed.

When jointly solving the kinematic equations, we obtained the following estimates: \((U_0, V_0, W_0) = (9.03, 11.58, 8.01) \pm (0.65, 0.81, 0.62) \text{ km/s} \), \( \Omega_0 = \)
\[
28.87 \pm 0.23 \text{ km/s/kpc}, \omega_0 = -3.894 \pm 0.063 \text{ km/s/kpc}^2, \text{ and } \omega_0'' = 0.602 \pm 0.044 \text{ km/s/kpc}^3, \text{ where an error in the unit of weight was } \sigma_0 = 11.89 \text{ km/s and } V_0 = 233.9 \pm 3.4 \text{ km/s (for the assumed distance } R_0 = 8.1 \pm 0.1 \text{ kpc). Here, we used the distances to Cepheids from a paper [20] multiplied by a coefficient of 1.1. In the considered sample, Cepheids with the fundamental pulsation mode amount to 85%. It has been shown that there are no significant differences between the estimates of the kinematic parameters found with the most homogeneous (in terms of a pulsation mode) sample and the mixed sample of Cepheids.}

A separate spectral analysis of the radial \( V_R \), residual tangential \( \Delta V_{circ} \), and vertical velocities yielded the following estimates: \( f_R = 5.5 \pm 2.0 \text{ km/s}, \ f_\theta = 7.1 \pm 2.0 \text{ km/s}, \ f_V = 3.9 \pm 2.0 \text{ km/s}, \ \lambda_R = 1.9 \pm 0.5 \text{ kpc} (i_R = -8.3^\circ \pm 2.5^\circ \text{ for } m = 4), \ \lambda_\theta = 2.6 \pm 0.5 \text{ kpc} (i_\theta = -11.4^\circ \pm 2.8^\circ \text{ for } m = 4), \ \lambda_V = 2.4 \pm 0.5 \text{ kpc} (i_V = -10.7^\circ \pm 2.7^\circ \text{ for } m = 4), \ (\chi_0)_R = -208^\circ \pm 16^\circ, \ (\chi_0)_\theta = -185^\circ \pm 18^\circ, \text{ and } \ (\chi_0)_V = -138^\circ \pm 18^\circ. \text{ This analysis was also performed with the distances to Cepheids from a paper [20] multiplied by a coefficient of 1.1.}

ACKNOWLEDGMENTS

The authors are grateful to the referee for useful comments, which resulted in a much improved manuscript.

REFERENCES

1. H. S. Leavitt, Ann. Harvard College Observ. 60, 87 (1908).
2. H. S. Leavitt and E. C. Pickering, Harvard College Observ. Circ. 173, 1 (1912).
3. The HIPPARCOS and Tycho Catalogues, ESA SP-1200 (1997).
4. M. Feast and P. Whitelock, Mon. Not. R. Astron. Soc. 291, 683 (1997).
5. A. M. Mel’nik, P. Rautiainen, L. N. Berdnikov, A. K. Dambis, and A. S. Rastorguev, Astron. Nachr. 336, 70 (2015).
6. A. M. Mel’nik, A. K. Dambis, and A. S. Rastorguev, Astron. Lett. 25, 518 (1999).
7. V. V. Bobylev and A. T. Bajkova, Astron. Lett. 38, 638 (2012).
8. A. K. Dambis, L. N. Berdnikov, Yu. N. Efremov, A. Yu. Kniazev, et al., Astron. Lett. 41, 489 (2015).
9. I. I. Nikiforov and A. V. Veselova, Astron. Lett. 44, 81 (2018).
10. A. V. Veselova and I. I. Nikiforov, Res. Astron. Astrophys. 20, 209 (2020).
11. X. Chen, S. Wang, L. Deng, and R. de Grijs, Astrophys. J. 859, 137 (2018).
12. D. Kawata, J. Bovy, N. Matsunaga, and J. Baba, Mon. Not. R. Astron. Soc. 482, 40 (2019).
13. V. V. Bobylev, A. T. Bajkova, A. S. Rastorguev, and M. V. Zabolotskikh, Mon. Not. R. Astron. Soc. 502, 4377 (2021).
14. J. D. Fernie, Astron. J. 73, 995 (1968).
15. L. N. Berdnikov, Astron. Lett. 13, 45 (1987).
16. V. V. Bobylev, Astron. Lett. 39, 753 (2013).
17. S. M. Andrievsky, J. R. D. Lépine, S. A. Korotin, R. E. Luck, V. V. Kovtyukh, and W. J. Maciel, Mon. Not. R. Astron. Soc. 428, 3252 (2013).
18. V. A. Marsakov, V. V. Koval’, V. V. Kovtyukh, and T. V. Mishenina, Astron. Astrophys. Trans. 28, 367 (2014).
19. V. V. Kovtyukh, S. M. Andrievsky, R. P. Martin, S. A. Korotin, et al., Mon. Not. R. Astron. Soc. 489, 2254 (2019).
20. D. M. Skowron, J. Skowron, P. Mróz, A. Udalski, et al., Science (Washington, DC, U. S.) 365, 478 (2019).
21. P. Mróz, A. Udalski, D. M. Skowron, J. Skowron, et al., Astrophys. J. 870, L10 (2019).
22. A. G. A. Brown, A. Vallenari, T. Prusti, J. H. J. de Bruijine, et al., Astron. Astrophys. 616, 1 (2018).
23. G. A. Brown, A. Vallenari, T. Prusti, J. H. J. de Bruijine, et al., Astron. Astrophys. 649, 1 (2021).
24. T. E. Lutz and D. H. Kelker, Publ. Astron. Soc. Pacif. 85, 573 (1973).
25. C. A. L. Bailier-Jones, J. Rybicki, M. Fouesneau, M. Dem-leitner, and R. Andrae, Astron. J. 161, 147 (2021).
26. V. V. Bobylev and A. T. Bajkova, Astron. Rep. 65, 498 (2021).
27. C. C. Lin and F. H. Shu, Astrophys. J. 140, 646 (1964).
28. A. T. Bajkova and V. V. Bobylev, Astron. Lett. 38, 549 (2012).
29. A. A. Udalski, M. K. Szymański, and G. Szymański, Acta Astron. 65, 1 (2015).
30. R. I. Anderson, H. Saio, S. Ekström, C. Georgy, and G. Meynet, Astron. Astrophys. 591, A8 (2016).
31. L. Lindegren, S. A. Klioner, J. Hernández, A. Bombrun, et al., Astron. Astrophys. 649, A2 (2021).
32. M. V. Zabolotskikh, A. S. Rastorguev, and A. K. Dambis, Astron. Lett. 28, 454 (2002).
33. A. S. Rastorguev, N. D. Utkin, M. V. Zabolotskikh, A. K. Dambis, A. T. Bajkova, and V. V. Bobylev, Astrophys. Bull. 72, 122 (2017).
34. A. V. Loktin and M. E. Popova, Astrophys. Bull. 74, 270 (2019).
35. V. V. Bobylev and A. T. Bajkova, Astron. Lett. 48, 9 (2022).

Translated by E. Petrova