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Low-Complexity Multi-User Parameterized Beamforming in Massive MIMO Systems

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Abstract: In this paper, we design a complexity-reduced transmission scheme in massive antenna environments. To reduce the implementation complexity for the generation of beam weight, we design a multi-user parameterized beamforming (MUPB) scheme that can control the beam direction using a single parameter with combined use of maximum ratio transmission and partial zero-forcing scheme that partially nulls out interference. We design the MUPB to maximize the signal-to-leakage plus noise ratio (SLNR). To further reduce the implementation complexity, we optimize the MUPB based on approximated SLNR instead of accurate SLNR. Finally, the performance of the proposed MUPB is verified by computer simulation.

Keywords: massive MIMO; MRT; ZF; complexity-reduced beamforming

1. Introduction

The use of massive multi-input multi-output (m-MIMO) techniques has widely been applied to advanced wireless communication systems [1–3]. A base station (BS) equipped with an m-MIMO configuration can serve a large number of users by means of beamforming. The channel becomes asymptotically orthogonal to each other as the number of antennas increases to infinity [4]. In this case, we may achieve optimum performance by using a maximum ratio transmission (MRT) technique with affordable implementation complexity. However, the MRT may suffer from inter-user interference in practical m-MIMO operation environments mainly due to the presence of insufficient orthogonality [5].

The use of zero-forcing (ZF) beamforming can be a practical choice in m-MIMO environments since it may easily null out inter-user interference [6–8]. However, it may require large computational complexity for the generation of beam weight in m-MIMO environments. A number of approaches have been proposed to reduce the computational complexity of ZF [9–17]. The complexity can be reduced by using an approximation technique such as Neumann series (NS) expansion [9], truncated polynomial expansion (TPE) [10], Kapteyn series expansion [11], and eigen-based approximation [12]. The complexity can also be reduced by using an iteration method such as Gauss-Seidel (GS) method [13], symmetric successive over relaxation (SSOR) method [14], Newton iteration (NI) method [15], Jacobi algorithm [16], and weighted two stage (WTS) method [17]. These approaches may somewhat reduce the computational complexity, but they may suffer from performance degradation associated with the approximation in matrix inversion. Moreover, when they are applied to a large number of users, they may not be effective since their complexity increases in proportion to the number of users.

The complexity for the generation of ZF beam weight can be reduced as the dimension of nulling subspace decreases [18,19]. The lower the dimension of nulling subspace, the larger the spatial degrees of freedom (DoF), making it possible to enhance desired signal power while marginally increasing interference. It may be effective in low signal-to-noise ratio (SNR) environments (e.g., cell boundary region). A ZF technique can decrease the dimension of nulling subspace by only considering intra-cell...
interfering channel [18]. A cell-edge-aware (CEA)-ZF technique can selectively null out interference to inter-cell users based on the path loss between the BS and inter-cell users [19]. However, these approaches may not be effective in low SNR environments since they decrease the dimension of nulling subspace without careful consideration of transmission environments.

In this paper, we design a complexity-reduced multi-user beamforming scheme in m-MIMO environments. The beam weight can be determined to maximize the signal-to-interference plus noise ratio (SINR), which may require large computational complexity mainly due to non-convexity property of the SINR [20,21]. Instead, we consider the signal-to-leakage plus noise ratio (SLNR) for the generation of beam weight. The SLNR is defined by the ratio between the desired signal power to a target user and the total interference power to other users (i.e., the leakage power) plus noise. The Max SLNR technique maximizes the SLNR by applying Rayleigh–Ritz quotient theorem [22,23]. However, it may require computational complexity in cubic proportion to the number of antennas, which may be unaffordable in m-MIMO environments. To further reduce the computational complexity, we employ a parameterized beamforming (PB) technique that employs MRT and ZF beamforming techniques, where large-scale beam weight can be determined by a single parameter [24,25]. As the number of users increases, the ZF may experience the presence of insufficient spatial DoF, yielding poor performance of PB in multi-user environments. To alleviate this problem, we consider the use of a partial ZF (PZF) beamforming technique that selectively nulls out interference, making it possible to reduce the dimension of nulling subspace (i.e., to exploit more spatial DoF). We may alleviate the performance degradation of ZF in multi-user environments by increasing the spatial DoF, while reducing the computational complexity. With combined use of MRT and PZF, we design a multi-user parameterized beamforming (MUPB) scheme that maximizes the SLNR. To further reduce the computational complexity, we approximate the SNLR by exploiting the deterministic equivalent property [10,26,27]. We analytically design the MUPB by applying the zero-gradient condition [28] to the approximated SNLR. The MUPB provides performance marginally poorer than the Max SLNR, while significantly reducing the computational complexity. We analyze the computational complexity in terms of real-floating point operation (flop) [29,30]. Finally, we verify the performance of the proposed MUPB by computer simulation.

The rest of this paper is organized as follows. The system model is described in Section 2. The proposed beamforming scheme is described in Section 3. The performance of the proposed scheme is verified by computer simulation in Section 4. Finally, conclusions are given in Section 5.

Notations: Boldface letter X and x denote a matrix and a column vector, respectively. XH and tr(X) denote the conjugate transpose and the trace of X, respectively. |x| denotes the two-norm of x and IM denotes an (M × M) identity matrix. x ∼ CN(m, C) refers to that x is a circularly-symmetric complex Gaussian random vector with mean m and covariance C. CM×1 denotes all sets of M dimensional complex column vectors. The notation “a.s. → M→∞” refers to almost sure convergence.

2. System Model

We consider a cellular system comprising B BSs with an m-MIMO configuration, where BS i serves Kj users and 1 ≤ i ≤ B. We assume that each BS and users are equipped with M antennas and a single antenna, respectively, and that each BS has perfect channel state information (CSI) of all users. Let Ω be the set of BSs and Ui be the set of users in BS i. Then, Ui can be represented as

\[ U_i = \left\{ k \left| \sum_{j=1}^{i-1} K_j < k \leq \sum_{j=1}^{i} K_j + K_i \right. \right\}. \]

The signal received by user k can be represented as

\[ y_k = \sqrt{p_i} \alpha_{ik} h_{ik}^H v_k s_k + \sum_{l \in U_i \setminus k} \sqrt{p_{il}} \alpha_{ik} h_{il}^H v_l s_l + \sum_{j \in \Omega \setminus i} \sum_{l \in U_j} \sqrt{p_{jl}} \alpha_{ik} h_{jl}^H v_l s_l + n_k \]

where p_i denote the transmitted power, α_{ik} are the channel coefficients, and n_k denotes an additive white Gaussian noise vector.

In this model, we assume that the BSs have perfect CSI and users have perfect channel estimation information. The objective of the beamforming scheme is to maximize the SLNR for the target user k by designing the beamforming vector v_k = \frac{h_{ik}}{\| h_{ik} \|_2}.

3. Proposed Beamforming Scheme

The proposed beamforming scheme is described in Section 3. The performance of the proposed scheme is verified by computer simulation in Section 4. Finally, conclusions are given in Section 5.

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where $p_k$ is the transmit power to user $k$, $\alpha_{i,k}$ is the path loss between BS $i$ and user $k$, $h_{i,k} \sim \mathcal{CN}(0, I_M)$ is the channel between BS $i$ and user $k$, $v_k \in \mathbb{C}^{M \times 1}$ is the beam weight for user $k$ ($||v_k||^2 = 1$), $s_k$ is the data for user $k$, and $n_k$ is zero-mean complex circular-symmetric additive white Gaussian noise (AWGN). The SINR of user $k$ can be represented as

$$\text{SINR}_k = \frac{S_k}{I_k + \sigma_n^2}$$

where $S_k$ is the desired signal power to user $k$ and can be represented as

$$S_k = p_k \alpha_{i,k} ||h_{i,k} v_k||^2$$

and $I_k$ is the interference power to user $k$ and can be represented as

$$I_k = \sum_{l \in U \backslash \{k\}} I_{l,k} + \sum_{j \in \Omega \backslash \{i\}} \sum_{l \in U_j} I_{l,k}$$

here, $I_{l,k}$ is the interference power to user $k$ generated by signal transmission to user $l$, represented as

$$I_{l,k} = \begin{cases} p_l \alpha_{i,k} ||h_{i,k} v_{l,k}||^2, & \text{for } l \in U \backslash \{k\} \\ p_l \alpha_{j,k} ||h_{j,k} v_{l,k}||^2, & \text{for } l \in U_j, j \in \Omega \backslash \{i\} \end{cases}$$

and $\sigma_n^2$ is the noise power. The corresponding achievable transmission rate of user $k$ can be represented as

$$r_k = \log_2 (1 + \text{SINR}_k)$$

The corresponding sum rate can be represented as

$$r_{\text{sum}} = \sum_{i \in \Omega} \sum_{k \in U_i} r_k$$

3. Proposed Beamforming Scheme

3.1. Proposed Multi-User Parameterized Beamforming (MUPB)

As $M$ increases to infinity, the MRT can provide optimum transmission performance [4], where the beam weight for user $k$ can be determined by

$$v_{\text{MRT}}^k = \frac{h_{i,k}}{||h_{i,k}||}$$

However, the MRT may suffer from interference when $M$ is not sufficiently large [5]. The ZF can be a practical choice in m-MIMO environments since it can simply null out interference [6–8]. The ZF beam weight for user $k$ can be determined by

$$v_{\text{ZF}}^k = \frac{w_k}{||w_k||}$$

where $w_k$ is the $k$-th column vector of $H_i (H_i^H H_i)^{-1}$. Here, $H_i$ can be represented as

$$H_i = [H_{i,1} \cdots H_{i,B}]$$

where $H_{i,j}$ is the channel from BS $i$ to users served by BS $j$ and can be represented as

$$H_{i,j} = \begin{bmatrix} h_{i,j,l_1} & \cdots & h_{i,j,l_K} \end{bmatrix}$$

(11)

here, $l_n$ denotes the $n$-th user in $U_j$.

When the beam weight is determined to maximize the SINR, it may require large computational complexity [20,21]. To reduce the complexity, we determine the beam weight to maximize the SLNR. The SLNR of user $k$ can be represented as

$$\text{SLNR}_k = \frac{S_k}{L_k + \sigma_n^2}$$

(12)

where $L_k$ is the leakage power to the other users by $v_k$ and can be represented as

$$L_k = \sum_{l \in U \backslash \{k\}} L_{k,l} + \sum_{j \in \Omega \backslash \{i\}} \sum_{l \in U_j} L_{k,l}$$

(13)

here, $L_{k,l}$ is the leakage power to user $l$ by $v_k$, represented as

$$L_{k,l} = \begin{cases} p_k \alpha_{i,l} |h_{i,l}v_k|^2, & \text{for } l \in U \backslash \{k\} \\ p_k \alpha_{j,l} |h_{j,l}v_k|^2, & \text{for } l \in U_j, j \in \Omega \backslash \{i\} \end{cases}$$

(14)

It can be seen that the SLNR of user $k$ is a function of $v_k$. We can determine the beam weight for user $k$ so as to maximize the SLNR by

$$\mathbf{v}_k = \arg \max_{\|\mathbf{v}_k\|^2 = 1} \text{SLNR}_k, k \in U_i, i \in \Omega$$

(15)

It can be shown from Rayleigh–Ritz quotient theorem that the beam weight (15) can be represented as [22,23]

$$\mathbf{v}_k = \left( \sum_{i \in \Omega} H_i H_i^H + \frac{\sigma_n^2}{p_k} I_M \right)^{-1} h_{i,k}$$

(16)

It may require a computational complexity of $O(M^3 + K_{\text{total}}M^2)$ for the calculation of (16), where $K_{\text{total}} = \sum_{i \in \Omega} K_i$, which may not be affordable in m-MIMO environments.

We consider a sub-optimal solution of (15) to reduce the computational complexity. We employ a parameterized beamforming (PB) scheme that determines the beam weight for user $k$ by [24,25]

$$\mathbf{v}_k^{\text{PB}} = \beta_k \left[ \theta_k \mathbf{v}_k^{\text{MRT}} + (1 - \theta_k) \mathbf{v}_k^{\text{ZF}} \right]$$

(17)

where $\theta_k (\in [0, 1])$ is a scalar parameter and $\beta_k$ is a coefficient for normalization. The beam weight can be determined by optimizing the scalar parameter as

$$\theta_k^* = \arg \max_{\theta_k \in [0,1]} \text{SLNR}_k, k \in U_i, i \in \Omega$$

(18)

Figure 1 illustrates the behavior of PB according to parameter $\theta_k$. It can be seen that as $\theta_k$ increases, $S_k$ increases and $L_{k,l}$ may increase as well. On the other hand, as $\theta_k$ decreases, $L_{k,l}$ and $S_k$ may also decrease. Thus, it may be desirable to determine $\theta_k$ by taking consideration of the desired signal power and the leakage power as well.
As the number of users increases, the ZF may not provide desired signal power mainly due to the absence of sufficient spatial DoF [6–8]. As a result, the PB may not provide desired performance in multi-user environments. To alleviate this problem, we consider the use of a PZF scheme that selectively nulls out interference. The PZF can reduce the dimension of nulling subspace, making it possible to exploit more spatial DoF. Moreover, the computational complexity for the generation of beam weight can be reduced as the dimension of nulling subspace decreases.

For BS $i$ $U$ can be partitioned into two sets, $U_i^o$ and $U_i^{−o}$, where $U_i^o$ denotes a set of users whose channel vector lies in the nulling subspace by the PZF and $U_i^{−o} = U − U_i^o$. Let user $n_i^o$ be the $n$-th user in $U_i^o$. When user $k$ is the $u$-th user in $U_i^o$, the PZF beam weight for user $k$ can be determined by

$$v_k^{PZF} = \frac{w_{u^o}^o}{\|w_{u^o}^o\|} \quad (19)$$

where $w_{u^o}^o$ is the $u$-th column vector of $H_i^o\left(\left(H_i^o\right)^H H_i^o\right)^{-1}$. Here,

$$H_i^o = \left[\begin{array}{c} h_{i,n^o_1} \\
... \\
\cdot \\
\cdot \\
h_{i,n^o_{K^o_i}} \end{array}\right] \quad (20)$$

where $K^o_i$ is the number of users in $U_i^o$. The PZF beam weight for user $k$ in $U_i^{−o}$ can be determined by

$$v_k^{PZF} = \frac{w_{1^{−o}}^{−o}}{\|w_{1^{−o}}^{−o}\|} \quad (21)$$

where $w_{1^{−o}}^{−o}$ is the first column vector of $H_{i,k}^{−o}\left(\left(H_{i,k}^{−o}\right)^H H_{i,k}^{−o}\right)^{-1}$. Here,

$$H_{i,k}^{−o} = \left[\begin{array}{c} h_{i,k}^H \end{array}\right] \quad (22)$$

It can be conjectured that, when $K^o_i$ is small, the desired signal power may increase. However, users in $U_i^{−o}$ may experience severe interference since their channel does not lie in the nulling subspace. It may be desirable to determine $U_i^{−o}$ so that total interference-to-noise ratio (INR) is lower than a certain level.

When user $k$ is included in the non-nulling user set of all BSs, the INR of user $k$ can be represented as

$$\delta_k = \frac{\sum_{l \in U_i^{−o}} p_l|\alpha_{l,k}|^2 |h_{i,k}^H v_l^{PZF}|^2 + \sum_{l \in (U_i^{−o})} \sum_{j \in U_i} p_l|\alpha_{l,k}|^2 |h_{i,k}^H v_j^{PZF}|^2}{\sigma_n^2} \quad (23)$$
The INR can be calculated using the PZF beam weight. However, the PZF beam weight has not been determined since the nulling user set is yet to be determined. As $M$ goes to infinity, the channel becomes deterministic [10, 26, 27], making it possible to calculate the INR without knowing the PZF beam weight. To this end, we approximate the INR assuming that the channel is deterministic. For ease of description, we introduce the following lemmas [26].

**Lemma 1.** Let $A \in \mathbb{C}^{M \times M}$ and $x \sim \mathcal{CN}(0, \frac{1}{M}I_M)$. When the spectral norm of $A$ is uniformly bounded and $x$ is independent of $A$, it can be shown that [26]

$$x^H Ax - \frac{1}{M} \text{tr}(A) \xrightarrow{a.s.} 0 \quad (24)$$

**Lemma 2.** Let $A$ be in Lemma 1, and $x$ and $y \sim \mathcal{CN}(0, \frac{1}{M}I_M)$. When $x$ and $y$ are independent of $A$, it can be shown that [26]

$$x^H Ay \xrightarrow{a.s.} 0 \quad (25)$$

It can be shown from Lemma 1 that

$$h_j^H v_{PZF}^l (v_{PZF}^l)^H h_j - \text{tr}(v_{PZF}^l (v_{PZF}^l)^H) \xrightarrow{a.s.} 0 \quad (26)$$

Note that

$$v_{PZF}^l = \frac{P_{o,j}^l h_j}{\|P_{o,j}^l h_j\|}$$

where $P_{o,j}^l$ is the projection onto the null space by the PZF of BS $j$ for user $l$ [7]. It can be shown that

$$\text{tr}(v_{PZF}^l (v_{PZF}^l)^H) = \frac{\text{tr}(P_{o,j}^l h_j^H (P_{o,j}^l)^H)}{\|P_{o,j}^l h_j\|^2} = 1$$

It can also be shown that

$$\delta_k - \tilde{\delta}_k \xrightarrow{a.s.} 0 \quad (29)$$

where $\tilde{\delta}_k$ is a deterministic equivalent of $\delta_k$ and can be represented as

$$\tilde{\delta}_k = \frac{\sum_{l \in U \setminus \{k\}} p_l \alpha_{i,k} + \sum_{l \in U \setminus \{k\}} \sum_{j \in \Omega} p_l \alpha_{j,k}}{\sigma_n^2} \quad (30)$$

If $\tilde{\delta}_k < \delta_{th}$, where $\delta_{th}$ is a threshold to be determined, user $k$ can be included in the non-nulling user set of all BSs. Otherwise, the BS yielding the highest interference to user $k$ excludes user $k$ from its non-nulling user set and updates $\delta_k$. This process is repeated until $\delta_k < \delta_{th}$ or all BSs exclude user $k$ from their non-nulling user set. Finally, we can generate a PZF beam weight that yields interference to users in the non-nulling user set lower than a certain level.

With combined use of MRT and PZF, we determine the beam weight for user $k$ by

$$v_k^{MUPB} = \beta_k \left[ \theta_k v_k^{MRT} + (1 - \theta_k) v_k^{PZF} \right]$$

(31)
The beam weight $v_k^{\text{MUPB}}$ can provide user $k$ SLNR represented as

$$\text{SLNR}_k = \frac{S_k}{L_k^o + \sigma_n^2}$$

where $L_k^o$ is the leakage power to the other users in $U_i^o$. Note that the parameter $\theta_k$ only affects the leakage power to users in the nulling user set.

Since $P_{ik}^o$ is a Hermitian and idempotent matrix (i.e., $(P_{ik}^o)^H = P_{ik}^o$ and $(P_{ik}^o)^2 = P_{ik}^o$) [31], it can be shown that

$$h_{ik}^Hv_k^{\text{MRT}} = \frac{b_{ik}^Hb_{ik}}{\|b_{ik}\|} = \|h_{ik}\| \geq 0$$

$$h_{ik}^Hv_k^{\text{PZF}} = \frac{b_{ik}^HP_{ik}b_{ik}}{\|P_{ik}b_{ik}\|} = \|P_{ik}^o h_{ik}\| \geq 0$$

(33)

It can be shown that

$$S_k = \rho_k \alpha_{ik} \left| h_{ik}^Hv_k^{\text{MUPB}} \right|^2$$

$$= \rho_k \alpha_{ik} \beta_k^2 \left| \theta_k h_{ik}^Hv_k^{\text{MRT}} + (1-\theta_k)h_{ik}^Hv_k^{\text{PZF}} \right|^2$$

$$= \rho_k \alpha_{ik} \beta_k^2 \left( \theta_k h_{ik}^Hv_k^{\text{MRT}} + (1-\theta_k)h_{ik}^Hv_k^{\text{PZF}} \right)^2$$

$$= \frac{\rho_k \alpha_{ik} \left( \sqrt{G_k} + \sqrt{P_k} \right)^2}{2(1-\theta_k)\beta_k(\theta_k-1)+1}$$

(34)

where $\Delta G_k = \left( \sqrt{G_k} - \sqrt{G_k} \right)^2$, $G_k^{\text{MRT}} = \left| h_{ik}^Hv_k^{\text{MRT}} \right|^2$, $G_k^{\text{PZF}} = \left| h_{ik}^Hv_k^{\text{PZF}} \right|^2$, and $q_k = (v_k^{\text{MRT}})^Hv_k^{\text{PZF}}$.

Here, (a) and (b) follow from the fact that $h_{ik}^Hv_k^{\text{MRT}}$ and $h_{ik}^Hv_k^{\text{PZF}}$ are non-negative.

Let $U_i^o$ be the set of users whose interference is nulled out by $v_k^{\text{PZF}}$ and user $n_{ik}^o$ be the $n$-th user in . . . Then, $U_i^o$ and $L_k^o$ can be represented as, respectively,

$$U_i^o = \begin{cases} 
L_i^o \setminus \{k\} & \text{for } k \in L_i^o \\
L_i^o & \text{for } k \notin L_i^o 
\end{cases}$$

(35)

$$L_k^o = \sum_{n_{ik}^o \in U_i^o} L_k^{n_{ik}^o}$$

(36)

where $L_k^{n_{ik}^o}$ is the leakage power to user $n_{ik}^o$. It can be shown that

$$L_{k,n_{ik}^o} = \rho_k \alpha_{ik} \left| h_{ik}^Hv_k^{\text{MUPB}} \right|^2$$

$$= \frac{\rho_k \alpha_{ik} \beta_k^2 \left( \theta_k h_{ik}^Hv_k^{\text{MRT}} + (1-\theta_k)h_{ik}^Hv_k^{\text{PZF}} \right)^2}{2(1-\theta_k)\beta_k(\theta_k-1)+1}$$

(37)
where the last equality comes from $h_{i,n_o,k}^H v_{k}^{PZF} = 0$ and $L_{k,n_o,k}^{MRT} = p_k \alpha_{i,n_o,k} |h_{i,n_o,k}^H v_{k}^{MRT}|^2$.

It may still require large computational complexity to determine the parameter $\theta_k$ maximizing the SLNR. We consider the approximation of SLNR to further reduce the complexity.

**Lemma 3.** A deterministic equivalent of MRT gain can be represented as

$$G_{k}^{MRT} - G_{k}^{\overline{MRT}} \xrightarrow{a.s.} 0 \quad \text{as} \quad M \to \infty \quad (38)$$

where $G_{k}^{\overline{MRT}} = M$.

**Lemma 4.** A deterministic equivalent of PZF gain can be represented as

$$G_{k}^{PZF} - G_{k}^{\overline{PZF}} \xrightarrow{a.s.} 0 \quad \text{as} \quad M \to \infty \quad (39)$$

where $G_{k}^{\overline{PZF}} = M - K_{i,k}^{o}$.

**Proof.** Please refer to Appendix A. □

**Lemma 5.** A deterministic equivalent of $q_k$ can be approximated to 1.

**Proof.** Please refer to Appendix B. □

It can be shown from Lemma 5 that

$$S_{k} - S_{k}^\overline{ } \xrightarrow{a.s.} 0 \quad \text{as} \quad M \to \infty \quad (40)$$

where $S_{k}^\overline{ }$ is a deterministic equivalent of $S_{k}$ and can be approximated as

$$S_{k}^\overline{ } \approx p_k \alpha_{i,k}(\sqrt{G_{k}^{MRT} - \frac{\overline{G_{k}^{PZF}}}{G_{k}^{PZF}}} + \sqrt{G_{k}^{PZF}})^2 \quad (41)$$

Here, $\Delta G_k = \left(\sqrt{G_{k}^{MRT}} - \frac{\overline{G_{k}^{PZF}}}{G_{k}^{PZF}}\right)^2$. It can be shown from Lemma 1 that

$$L_{k,n_o,k}^{MRT} - L_{k,n_o,k}^{\overline{MRT}} \xrightarrow{a.s.} 0 \quad \text{as} \quad M \to \infty \quad (42)$$

where $L_{k,n_o,k}^{\overline{MRT}}$ is a deterministic equivalent of $L_{k,n_o,k}^{MRT}$ and can be represented as

$$L_{k,n_o,k}^{\overline{MRT}} = p_k \alpha_{i,n_o,k}^o \quad (43)$$

It can also be shown from Lemma 5 that

$$L_{k,n_o,k}^o - L_{k,n_o,k}^{\overline{o}} \xrightarrow{a.s.} 0 \quad \text{as} \quad M \to \infty \quad (44)$$
where $L_{k,n_{ik}}$ is a deterministic equivalent of $L_{k,n_{ik}}$ and can be approximated as

$$L_{k,n_{ik}} \approx \sum_{n_{ik} \in U_{ik}^P} L_{k,n_{ik}}^{MRT} \theta^2_k$$  \hspace{1cm} (45)

It can be seen that

$$\text{SLNR}_k - \text{SLNR}_k \xrightarrow{a.s.} 0 \text{ as } M \to \infty$$  \hspace{1cm} (46)

where $\overline{\text{SLNR}}_k$ is a deterministic equivalent of $\text{SLNR}_k$ and can be approximated as

$$\overline{\text{SLNR}}_k \approx p_k \alpha_{i,k} \left( \frac{\Delta G_k \theta_k + \sqrt{\text{PZF}}}{L_{ik}^{MRT} \theta^2_k + \sigma^2_n} \right)^2$$  \hspace{1cm} (47)

We can determine the parameter $\theta_k$ maximizing (47) by invoking the zero-gradient condition [28]. The gradient of $\overline{\text{SLNR}}_k$ with respect to $\theta_k$ can be represented as

$$\nabla_{\theta_k} \overline{\text{SLNR}}_k = \Phi_k \Gamma_k$$  \hspace{1cm} (48)

where

$$\Phi_k = \frac{2 p_k \alpha_{i,k} \left( \sqrt{\Delta G_k \theta_k} + \sqrt{\text{PZF}} \frac{\Delta G_k}{\sqrt{G_k}} \Omega^2 \right)}{L_{ik}^{MRT} \theta^2_k + \sigma^2_n}$$  \hspace{1cm} (49)

$$\Gamma_k = \sqrt{\Delta G_k \sigma^2_n - L_{ik}^{MRT} \sqrt{\text{PZF}} G_k \theta_k}$$  \hspace{1cm} (50)

It can be seen from $\Phi_k > 0$ that $\Gamma_k$ should be zero to satisfy the zero-gradient condition. The parameter $\theta_k$ can optimally be determined by

$$\theta_k^* = \frac{\sigma^2_n}{L_{ik}^{MRT} \frac{\Delta G_k}{\sqrt{G_k}}}$$  \hspace{1cm} (51)

Note that $\theta_k^*$ is unique and globally optimal since $\overline{\text{SLNR}}_k$ is a strict quasi-concave function of $\theta_k$ [28]. For the proof, refer to Appendix C.

It can be shown that

$$\theta_k^* = \frac{\sigma^2_n}{L_{ik}^{MRT} \frac{\Delta G_k}{\sqrt{G_k}}} \frac{M \left( 1 - \frac{K_{ik}^o}{M} - 1 \right) - K_{ik}^o}{M - K_{ik}^o}$$  \hspace{1cm} (52)

Let $\overline{\alpha}_{i,k}$ be the average path loss between BS $i$ and users in $U_{ik}^P$. It can be shown that

$$\overline{\alpha}_{i,k} = \frac{\sum_{n_{ik} \in U_{ik}^P} \alpha_{i,n_{ik}}}{K_{ik}^o}$$  \hspace{1cm} (53)

$$L_{ik}^{MRT} = \sum_{n_{ik} \in U_{ik}^P} p_k \alpha_{i,n_{ik}} = K_{ik}^o \overline{\alpha}_{i,k}$$  \hspace{1cm} (54)
It can also be shown from the first-order Taylor approximation [28] that

\[ \theta_k^* \approx \frac{\sigma_n^2}{2p_k^*K_n^0h_{ik}} \]  

(55)

where \( \eta_{ik}^0 \) is the effective spatial DoF of BS \( i \) for user \( k \). It can be seen that as \( \eta_{ik}^0 \) increases, \( \theta_k^* \) decreases. When the effective spatial DoF is large, the PZF may yield marginal performance deterioration, implying marginal improvement of desired signal power with the use of a large value of \( \theta_k \). In this case, the use of a small \( \theta_k \) may be effective to maximize the approximated SLNR, while decreasing the leakage power. On the other hand, as \( \eta_{ik}^0 \) decreases, \( \theta_k \) increases, increasing the desired signal power. In this case, the use of a large \( \theta_k \) may be effective to maximize the approximated SLNR, while avoiding large performance degradation by the PZF.

3.2. Computational Complexity

We measure the computational complexity in terms of real-floating point operation (flop). We assume that a multiplication of two \((p \times q)\) and \((q \times r)\) complex matrices requires \(8pqr\) flops, an inner product of two \((p \times 1)\) complex vectors requires \(2p\) flops, a calculation of \((p \times q)\) complex Gram matrix requires \(pq^2\) flops, and an inversion of \((q \times q)\) Hermitian matrix requires \(4q^3\) flops [29,30].

It can be shown that BS \( i \) requires \( \psi_i^{MRT} = 4K_iM \) flops to generate the MRT beam weight, approximately \( B(B - 1)K_{total} \) flops to determine the nulling and the non-nulling user sets, \( \psi_i^{ZF} = K_i^0M^2 + \left(8(K_i^0)^2 + 4K_i^0\right)M + \frac{3}{2}(K_i^0)^3 \) flops to generate the PZF beam weight for users in \( U_i^0 \), and \( \psi_{i,U_i^o}^{PZF} = \left(K_i^0 + 1\right)M^2 + \left(8(K_i^0 + 1)^2 + 4M + \frac{3}{2}(K_i^0 + 1)^3\right)Q_i^o \) flops to generate the PZF beam weight for users in \( U_i^o \), where \( Q_i^0 \) and \( Q_i^o \) are the number of users served by BS \( i \) in \( U_i^0 \) and \( U_i^o \), respectively. Thus, BS \( i \) requires \( \psi_i^{PZF} = \psi_{i,U_i^0}^{PZF} + \psi_{i,U_i^o}^{PZF} \) flops to generate the PZF beam weight. It may approximately require a computational complexity of \( 10K_{total}M \) flops to generate the normalized beam weight using MRT and PZF beam weight. Finally, the MUPB requires a computational complexity of \( \psi_i^{MUPB} = \sum_{i=1}^{B}(\psi_i^{MRT} + \psi_i^{PZF}) + [B(B - 1) + 10M]K_{total} \) flops. The computational complexity of beamforming schemes is summarized in Table 1, where \( K_i^o \) is the number of users in other cells \((j \in \Omega, j \neq i)\) to be nulled out.

| Scheme       | Computational Complexity |
|--------------|--------------------------|
| MRT          | \(4K_{total}M\)          |
| ZF           | \(BK_{total}M^2 + (8BK_{total} + 4)K_{total}M + \frac{3}{2}BK^3_{total}\) |
| CEA-ZF       | \(\sum_{i=1}^{B}\left[K_i + K_i^o\right]M^2 + \left(8\left(K_i + K_i^o\right)^2 + 4K_i\right)M + \frac{3}{2}\left(K_i + K_i^o\right)^3\) |
| Max SLNR     | \(\frac{3}{2}K_{total}M^2 + (8K_{total} + 2(B - 1))M^2 + 6K_{total}M\) |
| PB           | \(BK_{total}M^2 + (8BK_{total} + 18)K_{total}M + \frac{3}{2}BK^3_{total}\) |
| MUPB         | \(\sum_{i=1}^{B}(\psi_i^{MRT} + \psi_i^{PZF}) + [B(B - 1) + 10M]K_{total}\) |

4. Performance Verification

We verify the performance of the proposed MUPB by computer simulation. We consider a three-cell cellular system as illustrated in Figure 2 [7,30], where each cell equally serves \( K \) (i.e., \( K_i = K, \forall i \)) users randomly distributed in the cell boundary region. The other simulation parameters are summarized in Table 2 [32,33].
Table 2. Simulation parameters.

| Simulation Parameters | Value |
|-----------------------|-------|
| Number of BSs $B$     | 3     |
| Cell radius $R_{\text{max}}$ | 300 m |
| Cell center radius $R_c$ | 200 m |
| Path loss $\alpha_{i,k}$, $\forall i$ and $k$ | $148.1 + 37.6 \log_{10}(d_{i,k})$ dB, where $d_{i,k}$ is the distance between BS $i$ and user $k$ in km |
| Maximum transmit power $P_T$ | 30 dBm |
| Power allocation $P_{k}$, $\forall k$ | $P_T/K$ |
| Noise power $\sigma_n^2$ | $-92$ dBm |

Figure 2. A three-cell cellular system.

Figure 3 depicts the sum rate and the number of users in the nulling user set of the MUPB according to the threshold $\delta_{th}$ when $M = 64$, $K = 12$, 16, and the INR is 0 dB, 3 dB, 6 dB. It can be seen that as $K$ increases, $\delta_{th}$ maximizing the sum rate and the number of users in the nulling user set increase as well. That is, when $K$ is large, the number of users in the nulling user set needs to be decreased to avoid performance degradation by the PZF. Otherwise, the MUPB may suffer from performance degradation mainly due to the absence of sufficient spatial DoF for the PZF. It can also be seen that when $\delta_{th}$ is large, the performance severely degrades even for large $K$ due to the presence of large interference caused by a small number of users in the nulling user set. This implies that the number of users in the nulling user set should properly be adjusted for the generation of MUPB beam weight. It can also be seen that the number of users in the nulling user set should be large to reduce interference in multi-user environments.

Figure 4 depicts the normalized mean square error (NMSE) of SLNR approximation in the MUPB according to the number of antennas $M$ when $K = 4$, 8, 16. We define the NMSE by

$$NMSE = \frac{1}{K_{\text{total}}} \sum_{k=1}^{K_{\text{total}}} \left( \frac{\text{SLNR}^*_k - \text{SLNR}^{\text{MUPB}}_k}{\text{SLNR}^*_k} \right)^2$$

where $\text{SLNR}^*_k$ is the optimum SLNR of user $k$ obtained by Rayleigh–Ritz quotient theorem, $\text{SLNR}^{\text{MUPB}}_k$ is the approximated SLNR of user $k$ in the MUPB, and the threshold $\delta_{th}$ is determined to minimize the NMSE. It can be seen that the validity of the proposed approximation increases as $M$ increases. This is mainly because the deterministic equivalent property, applied to the approximation of SLNR, becomes
accurate as $M$ increases. As a consequence, the proposed MUPB can provide performance slightly worse than the Max SLNR, while significantly reducing the computational complexity.

![Graph showing sum rate and number of nulling users](image)

**Figure 3.** Performance of multi-user parameterized beamforming (MUPB) according to $\delta_{th}$.

Figure 5 depicts the parameter $\theta$ according to $K$ when $M = 64$, 96, 128 and $\delta_{th} = -6$ dB. It can be seen that as $K$ increases, $\theta$ also increases. When $K$ is large, the PZF may experience performance deterioration mainly due to the absence of sufficient spatial DoF. In this case, $\theta$ needs to be increased so as to maximize the approximated SLNR. It can be seen that as $M$ increases, $\theta$ decreases. When $M$ is large, the approximated SLNR can be maximized by reducing the leakage power with the use of small $\theta$.

Figure 6 depicts the performance of the proposed MUPB in terms of the sum rate and the computational complexity according to $K$ when $M = 64$ and $\delta_{th} = -6$ dB. For comparison, we also...
evaluate the performance of MRT [4], ZF [6], CEA-ZF [19], Max SNLR [23], and PB that maximizes the approximated SLNR by (51) with combined use of MRT and ZF. It can be seen that the MUPB provides performance slightly worse than the Max SLNR, while outperforming the other schemes. It can also be seen that the MRT and the ZF may be effective in certain operation environments, and the CEA-ZF can work better than the ZF since it does not require full dimensional nulling subspace. However, since the CEA-ZF determines the beam weight without consideration of SINR or SLNR, it may provide marginal performance improvement. The PB and the MUPB work better than the CEA-ZF since their beam weight is determined to maximize the approximated SLNR. However, as $K$ increases, the performance gap between the PB and the MUPB increases since the PB may suffer from the absence of sufficient spatial DoF due to the use of ZF. It can be seen that the MUPB requires the computational complexity much lower than the Max SLNR, the ZF, and the PB. The MUPB can outperform the CEA-ZF, while requiring the computational complexity similar to the CEA-ZF.

![Figure 4](image4.png)

**Figure 4.** Normalized mean square error (NMSE) performance of MUPB according to $M$.

![Figure 5](image5.png)

**Figure 5.** Parameter $\theta$ according to $K$ when $\delta_{th} = -6$ dB.
Figure 5. Parameter $\theta$ according to $K$ when $\delta = -6$ dB.

(a) Sum rate.

(b) Computational complexity.

Figure 6. Performance according to $K$ when $M = 64$ and $\delta_{th} = -6$ dB.

Figure 7 compares the sum rate and the computational complexity according to $M$ when $K = 16$ and $\delta_{th} = -6$ dB. It can be seen that the MUPB provides performance improvement over the other schemes other than the Max SLNR which requires unaffordable computational complexity.
In this paper, we have proposed a low-complexity multi-user beamforming scheme MUPB in m-MIMO environments. To reduce the computational complexity, we have determined the beam weight of MUPB to maximize approximated SLNR with combined use of MRT and partial ZF. The simulation results show that the MUPB provides performance slightly worse than the Max SLNR, while significantly reducing the computational complexity.

5. Conclusions

Figure 7. Performance according to $M$ when $K = 16$ and $\delta_\text{th} = -6$ dB.
Author Contributions: G.-W.J. conceived the main idea of this paper and designed the proposed scheme. The mathematical analysis and the experiments were performed by G.-W.J.; Y.-H.L. directed the academic research. G.-W.J. wrote the paper and Y.-H.L. revised the manuscript. All authors have read and agreed to the published version of the manuscript.

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Appendix A

Proof of Lemma 4. Let user \( k \) be the \( u \)-th user in \( U_0 \). \( G_k^{\text{PZF}} \) can be represented as [6]

\[ G_k^{\text{PZF}} = G_k^{\text{MRT}} - D_k^{\text{PZF}} \]  
(A1)

where

\[ D_k^{\text{PZF}} = \sum_{n \neq u} D_{u,n}^{\text{PZF}} \]  
(A2)

here, \( D_{u,n}^{\text{PZF}} \) can be represented as

\[ D_{u,n}^{\text{PZF}} = (-1)^{u+n+1} h_u^H h_n \Theta_{u,n} \]  
(A3)

where

\[ \Theta_{u,n} = \frac{\det(R_{(u,n)^T})}{\det(R_{(u,u)^T})} \]  
(A4)

here, \( R = (H^o)^H H^o \) and \( R^{(u,u)} \) denotes \( R \) without the \( u \)-th row and the \( n \)-th column. It can be shown that [34]

\[ \Theta_{u,n} = \sum_{p=1}^{K_o} (-1)^{p+u+n} h_{u,p}^H h_n \det\left( \begin{bmatrix} R_{(u,u)}^{(p,p)} & R_{(u,u)}^{(p,l)} \\ R_{(u,u)}^{(l,l)} & R_{(l,l)}^{(p,l)} \end{bmatrix} \right) \]  
(A5)

where

\[ l_{u,n} = \begin{cases} u & \text{for } u < n \\ u - 1 & \text{for } u > n \end{cases} \]  
(A6)

\[ z_{u,n} = \begin{cases} u - n + 1 & \text{for } u < n \\ u - n - 1 & \text{for } u > n \end{cases} \]  
(A7)

\[ b = \begin{cases} p & \text{for } p < u \\ p + 1 & \text{for } p > u \end{cases} \]  
(A8)

\[ w_{u,n}^{(p)} = (-1)^p \det\left( R_{(u,u)}^{(p,p)} \right) \]  
(A9)
Since \((-1)^{u+n+1}(-1)^{\omega_{n}} = 1\) irrespective of \(u\) and \(n\), it can also be shown that

\[
D_{u,n}^{PZF} = \frac{K_{0}^{u}}{\sum_{p=1}^{K_{0}^{u}} f_{u,n}^{(p)}} - \frac{K_{0}^{u} f_{u,n}^{(p)}}{\sum_{p=1}^{K_{0}^{u}} f_{u,n}^{(p)}} - \frac{K_{0}^{u}}{\sum_{p=1}^{K_{0}^{u}} g_{u,n}^{(p)}} + \frac{K_{0}^{u} g_{u,n}^{(p)}}{\sum_{p=1}^{K_{0}^{u}} g_{u,n}^{(p)}} \tag{A10}
\]

where \(f_{u,n}^{(p)} = w_{u,n}^{(p)} h_{i,j}^{H} h_{i,j}^{H} h_{i,j}^{H} h_{i,j}^{H} \) and \(g_{u,n}^{(p)} = w_{u,n}^{(p)} h_{i,j}^{H} h_{i,j}^{H} h_{i,j}^{H} h_{i,j}^{H} \). It can be seen from Lemma 2 that for \(b \neq n\)

\[
h_{b}^{H} h_{b}^{H} h_{b}^{H} h_{b}^{H} \xrightarrow{a.s.} 0 \quad \text{and} \quad h_{n}^{H} h_{n}^{H} h_{n}^{H} h_{n}^{H} \xrightarrow{a.s.} 0,
\]

and that \(\sum_{p=1}^{K_{0}^{u}} f_{u,n}^{(p)} \xrightarrow{a.s.} 0\) and \(\sum_{p=1}^{K_{0}^{u}} g_{u,n}^{(p)} \xrightarrow{a.s.} 0\). It can be shown that

\[
\frac{f_{u,n}^{(p)} g_{u,n}^{(p)}}{\sum_{p=1}^{K_{0}^{u}} f_{u,n}^{(p)} g_{u,n}^{(p)}} \xrightarrow{a.s.} 0. \tag{A11}
\]

It can be seen from Lemma 1 that \(h_{b}^{H} h_{b}^{H} h_{b}^{H} h_{b}^{H} \xrightarrow{a.s.} 0\) and \(h_{n}^{H} h_{n}^{H} h_{n}^{H} h_{n}^{H} \xrightarrow{a.s.} 0\), and thus \(f_{u,n}^{(p)} g_{u,n}^{(p)} \xrightarrow{a.s.} 0\). Finally, it can be shown that \(G_{k}^{PZF} - K_{0}^{u} \xrightarrow{a.s.} 0\), where \(G_{k}^{PZF} = G_{k}^{PZF} \xrightarrow{a.s.} 0\). Appendix B

**Proof of Lemma 5.** It can be shown from Lemma 1 that

\[
h_{i,k}^{H} p_{i,k}^{0} h_{i,k} - \text{tr}(p_{i,k}^{0}) \xrightarrow{a.s.} 0. \tag{A12}
\]

It can be seen from Lemma 3 and 4 that

\[
q_{k} - \bar{q}_{k} \xrightarrow{a.s.} 0 \tag{A13}
\]

where \(\bar{q}_{k}\) is a deterministic equivalent of \(q_{k}\) and can be represented as

\[
\bar{q}_{k} = \frac{\text{tr}(p_{i,k}^{0})}{\sqrt{M} \sqrt{M - K_{0}^{u}_{i,k}}} \tag{A14}
\]

Since \(\text{tr}(p_{i,k}^{0}) = M - K_{0}^{u}_{i,k}\), it can be seen that

\[
\bar{q}_{k} = \frac{M - K_{0}^{u}_{i,k}}{M \sqrt{1 - \frac{K_{0}^{u}_{i,k}}{M}}} \tag{A15}
\]

Thus, it can be seen that \(\lim_{M \to \infty} \bar{q}_{k} \approx 1. \square\)

**Appendix C**

**Proof of strict quasi-concavity of \(\tilde{\text{SLNR}}_{k}\).** We prove that \(\tilde{\text{SLNR}}_{k}\) is a strict quasi-concave function of \(\theta_{k}\) by using the following Theorem A1 [28].
Theorem A1. A continuous function \( f : \mathbb{R} \rightarrow \mathbb{R} \) is strictly quasi-concave if and only if there is a point \( c \in \text{dom } f \) such that \( f \) is an increasing function for \( x < c \) (\( x \in \text{dom } f \)), and \( f \) is a decreasing function for \( x > c \) (\( x \in \text{dom } f \)). The point \( c \) can be chosen by any point that globally maximizes \( f \).

Let \( c_k \) be a global maximizer of \( \text{SLNR}_k \). Without loss of generality, we can assume that \( c_k \) can be set to \( \frac{\sigma^2}{L_{\text{I}_{PZF}} L_{\text{MRT}}^2} \sqrt{\frac{\Delta G_k}{\theta_k}} \) since the global optimum occurs at which the gradient vanishes [28]. For \( \theta_k < c_k \), \( \theta_k \) can be set to \( \theta^*_k - \epsilon \), where \( \epsilon \) is an arbitrary positive real number satisfying \( \theta^*_k - \epsilon \in [0, 1] \). For \( \theta_k < c_k \), \( \Phi_k \) in (49) and \( \Gamma_k \) in (50) can be represented as, respectively,

\[
\Phi_k = \frac{2p_k \alpha_{i,k}}{L_{\text{I}_{PZF}} L_{\text{MRT}}^2 (\theta^*_k + \sigma^2 G_k)} \left[ \sqrt{G_k (\theta^*_k + \sqrt{\frac{\Delta G_k}{\theta_k}})} - \sqrt{\frac{\Delta G_k}{\theta_k}} \right] \tag{A16}
\]

\[
\Gamma_k = \frac{L_{\text{I}_{PZF}} L_{\text{MRT}}^2 \theta_k \epsilon}{G_k (\theta^*_k + \sqrt{\Delta G_k})} \tag{A17}
\]

It can be shown that

\[
\nabla_{\theta_k} \text{SLNR}_k = \frac{N_k}{D_k} \tag{A18}
\]

where

\[
N_k = 2p_k \alpha_{i,k} L_{\text{I}_{PZF}} L_{\text{MRT}}^2 (\theta^*_k + \sqrt{\Delta G_k}) \tag{A19}
\]

\[
D_k = \left( L_{\text{I}_{PZF}} L_{\text{MRT}}^2 \theta^*_k + \sigma^2 G_k \right)^2 \tag{A20}
\]

It can be seen that \( N_k > 0 \) since \( \theta_k \geq 0 \) and \( D_k > 0 \). Then, it can be shown for \( \theta_k < c_k \) that \( \text{SLNR}_k \) is a strictly increasing function since \( \nabla_{\theta_k} \text{SLNR}_k > 0 \)

For \( \theta_k > c_k \), \( \theta_k \) can be set to \( \theta^*_k + \epsilon \), where \( \epsilon \) is an arbitrary positive real number satisfying \( \theta^*_k + \epsilon \in [0, 1] \). It can be shown for \( \theta_k > c_k \) that

\[
\nabla_{\theta_k} \text{SLNR}_k = -\frac{N_k}{\epsilon D_k} \tag{A21}
\]

It can be seen for \( \theta_k > c_k \) that \( \text{SLNR}_k \) is a strictly decreasing function since \( \nabla_{\theta_k} \text{SLNR}_k < 0 \). Finally, it can be shown from Theorem A1 that \( \text{SLNR}_k \) is a strict quasi-concave function of \( \theta_k \).

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