A primordial feature at the scale of superclusters of galaxies

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ABSTRACT

We investigate a spatially flat cold dark matter model (with the matter density parameter $\Omega_m = 0.3$) with a primordial feature in the initial power spectrum. We assume that there is a bump in the power spectrum of density fluctuations at wavelengths $\lambda \sim 30-60 h^{-1} Mpc$, which corresponds to the scale of superclusters of galaxies. There are indications for such a feature in the power spectra derived from redshift surveys and also in the power spectra derived from peculiar velocities of galaxies. We study the mass function of clusters of galaxies, the power spectrum of the cosmic microwave background (CMB) temperature fluctuations, the rms bulk velocity and the rms peculiar velocity of clusters of galaxies. The baryon density is assumed to be consistent with the big bang nucleosynthesis value. We show that with an appropriately chosen feature in the power spectrum of density fluctuations at the scale of superclusters, the mass function of clusters, the CMB power spectrum, the rms bulk velocity and the rms peculiar velocity of clusters are in good agreement with the observed data.

Key words: galaxies: clusters: general – cosmic microwave background – cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

The spatially flat cold dark matter (CDM) model (matter density parameter $\Omega_m = 0.3$, with flatness being restored by a contribution from a cosmological constant $\Omega_k = 0.7$) with scale-invariant initial conditions has been a standard model in cosmology over the last 5 years (see, e.g., Ostriker & Steinhardt 1995). This model successfully explains many large- and small-scale structure observations including the mass function and the peculiar velocities of clusters of galaxies. The flat cosmological model with the matter density parameter $\Omega_m = 0.3$ is also consistent with the observations of type Ia supernovae at redshift $z \sim 1$ (Perlmutter et al. 1999; Riess et al. 1998).

In this paper we investigate the CDM model with a primordial feature in the initial power spectrum of density fluctuations. Adams, Ross & Sarkar (1997) have noted that according to our current understanding of the unification of fundamental interactions, there should have been phase transitions associated with spontaneous symmetry breaking during the inflationary era. This may have resulted in the breaking of the scale-invariance of the initial power spectrum. Chung et al. (2000) studied an alternative mechanism that can alter the classical motion of the inflation and produce features in the initial power spectrum. They showed that if the inflation is coupled to a massive particle, resonant production of the particle during inflation modifies the evolution of the inflation, and may leave an imprint in the initial power spectrum. The spectral features in the initial spectrum may also be generated if the inflation evolves through a kink in its potential (Starobinsky 1992; Lesgourgues, Polarski & Starobinsky 1998; Gramann & Hütsi 2000).

A number of different non-scale-invariant initial conditions have been recently used to analyse the cosmic microwave background (CMB) data (see, e.g., Wang & Mathews 2000; Hannestad, Hansen & Villante 2000; Griffiths, Silk & Zaroubi 2001; Atrio-Barandela et al. 2000; Kanasawa et al. 2000; Barriga et al. 2001). Griffiths et al. (2001) and Hannestad et al. (2000) showed that the CMB data favour a bump-like feature in the power spectrum around a scale of $k = 0.004 h Mpc^{-1}$. Barriga et al. (2001) studied the step-like spectral feature in the range $k \sim (0.06-0.6) h Mpc^{-1}$ and found that such a spectral break enables a good fit to both the APM and CMB data. Atrio-Barandela et al. (2000) investigated the temperature power spectrum in the CDM models, where the power spectrum of density fluctuations at $z \sim 10^3$ was in the form $P(k) \sim k^{-1.9}$ at wavenumbers $k > 0.05 h Mpc^{-1}$. This power spectrum of density fluctuations was derived by Einasto et al. (1999) by analysing different observed power spectra of galaxies and clusters of galaxies. Atrio-Barandela et al. (2000) found that this form of the power spectrum of density fluctuations is consistent with the recent CMB data. However, in this paper we examine the mass function of clusters of galaxies in the same model and find that for $\Omega_m = 0.3$, the number density of clusters is significantly smaller than observed.
Suhhonenko & Gramann (1999; hereafter SG) studied properties of clusters of galaxies in two cosmological models that rely on the observed power spectra of the distribution of galaxies. In the first model (hereafter model 1), the power spectrum of density fluctuations at $z \sim 10^3$ was in the form $P(k) \sim k^{-2}$ at wavenumbers $k > 0.05h\,\text{Mpc}^{-1}$. In the second model (hereafter model 2), the power spectrum contained a feature (bump) at wavenumbers $k \approx 0.1–0.2h\,\text{Mpc}^{-1}$ ($\lambda \sim 30–60h^{-1}\,\text{Mpc}$), which correspond to the scale of superclusters (see, e.g., Einasto et al. 1997). SG examined the mass function, peculiar velocities, the power spectrum and the correlation function of clusters in both models and found that in many aspects the power spectrum of density fluctuations in model 2 fits the observed data better than the simple power-law model 1. This study suggested that probably at wavenumbers $k \sim 0.05–0.2h\,\text{Mpc}^{-1}$, the power spectrum of density fluctuations is not a featureless simple power law.

Fig. 1 shows the power spectrum of density fluctuations in the CDM model examined in this paper (see equation 5 below). We assume that there is a feature (bump) in the power spectrum at wavenumbers $k = 0.1–0.2h\,\text{Mpc}^{-1}$. The power spectrum in the CDM model with a scale-invariant initial power spectrum is also plotted. It is assumed that the density parameter $\Omega_m = 0.3$ and the normalized Hubble constant $h = 0.65$. For comparison, we show in Fig. 1 the observed power spectra derived from the distribution of galaxies in the APM, Stromlo–APM and Durham/UKST surveys (Baugh & Efstathiou 1993; Tadros & Efstathiou 1996; Hoyle et al. 1999). For the Stromlo–APM and Durham/UKST surveys, we present estimates for the flux-limited sample with $P(k) = 8000h^{-3}\,\text{Mpc}^3$ in the weighting function (see Tadros & Efstathiou 1996; Hoyle et al. 1999 for details). There are indications of a similar bump in the power spectrum derived from the Stromlo–APM survey. On the other hand, there is no similar feature in the APM and Durham/UKST power spectrum. Hoyle et al. (1999) analysed the power spectrum for different volume-limited and flux-limited samples drawn from the Durham/UKST redshift survey. There are indications of a similar bump in the power spectrum measured in a volume-limited sample with $z_{\text{max}} = 0.08$ (see Hoyle et al. 1999 for details).

![Figure 1](https://example.com/figure1.png)

**Figure 1.** The power spectrum of density fluctuations in the CDM model examined in this paper (full curve) and in the standard CDM model with a scale-invariant initial power spectrum (dotted curve). In the models studied, $\Omega_m = 0.3$ and $h = 0.65$. Filled circles, open circles and crosses show the power spectrum of the galaxy distribution in the APM, Stromlo–APM and Durham/UKST surveys, respectively. For clarity, error bars are only shown for the APM data.

We also examined the power spectrum derived from the distribution of galaxies in the SSRS2 + CfA2 redshift survey (da Costa et al. 1994). There is a feature in the SSRS2 + CfA2 power spectrum at wavenumbers $k = 0.1–0.2h\,\text{Mpc}^{-1}$. Silberman et al. (2001) studied the power spectrum of peculiar velocities of galaxies and found an indication for a wiggle in the power spectrum: an excess near $k \sim 0.05h\,\text{Mpc}^{-1}$ and a deficiency at $k \sim 0.1h\,\text{Mpc}^{-1}$. This wiggle in the power spectrum is similar to the spectral feature studied in this paper. Most recently, there are indications for such a feature in the preliminary power spectrum derived from part of the 2dF redshift survey (Percival et al. 2001).

SG studied the power spectrum of clusters using $N$-body simulations and showed that the power spectrum of clusters in model 2 is in good agreement with the observed power spectrum of the APM clusters determined by Tadros, Efstathiou & Dalton (1998). SG also investigated the relation between the power spectrum of clusters and the power spectrum of matter fluctuations and found that in this model the relation between the cluster power spectrum and matter power spectrum is not linear at wavenumbers $k > 0.1h\,\text{Mpc}^{-1}$ (see also Gramann & Suhhonenko 1999).

In this paper we investigate the mass function of clusters of galaxies and the temperature power spectrum in the model with a bump in the power spectrum of density fluctuations at the scale of superclusters of galaxies. We also study the rms bulk velocity of galaxies and the rms peculiar velocity of clusters of galaxies in this model. The results are compared with observations. We examine the flat cosmological model with the density parameter $\Omega_m = 0.3$, the baryon density $\Omega_b h^2 = 0.019$ and the normalized Hubble constant $h = 0.65$ and 0.70. These values are in agreement with measurements of the density parameter (e.g. Bahcall et al. 1999), with measurements of the baryon density from abundances of light elements (O’Meara et al. 2001; Tytler et al. 2000) and with measurements of the Hubble constant using various distance indicators (Freedman et al. 2001; see also Parodi et al. 2000). The Hubble constant is given as $H_0 = 100h\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. To restore the spatial flatness in the low-density model, we assume a contribution from a cosmological constant $\Omega_A = 0.7$.

To study the mass function of clusters we use the Press–Schechter (Press & Schechter 1974, hereafter PS) approximation. The transfer functions $T(k)$ and the temperature power spectra are calculated using the fast Boltzmann code CMBFAST developed by Seljak & Zaldarriaga (1996). The code CMBFAST has been modified to incorporate a primordial feature in the initial power spectrum. We assume that the initial fluctuations are adiabatic and that the initial density fluctuation field is a Gaussian field. In this case, the power spectrum provides a complete statistical description of the field.

This paper is organized as follows. In Section 2 we study the mass function of clusters and the temperature power spectrum in our model, and compare the results with observations. In Section 3 we examine peculiar velocities of galaxies and clusters of galaxies. A discussion and summary are presented in Section 4.

## 2 THE MASS FUNCTION OF CLUSTERS AND CMB ANISOTROPIES

Let us first consider the CDM model, where the power spectrum of
density fluctuations at $z \sim 10^3$ is in the form

$$P(k) = A k^7 T^2(k), \quad \text{if} \quad k < k_0;$$

$$P(k_0)(k/k_0)^{-1.9}, \quad \text{if} \quad k > k_0,$$

(1)

where $k_0 = 0.05h$ Mpc$^{-1}$. Here, the initial power spectrum is defined as $P_{in}(k) = A k^7 T^2(k)$ and $T(k)$ is the transfer function, which describes the modification of the initial power spectrum during the era of radiation domination. The function $S(k)$ describes the deviation of the initial power spectrum from a scale-invariant form $P(k) \sim k$. The normalization constant $A$ is determined by the large-scale CMB anisotropy. This form of the power spectrum at wavenumbers $k > 0.05h$ Mpc$^{-1}$ was derived by Einasto et al. (1999) by analysing different observed power spectra of galaxies and clusters of galaxies. We denote the CDM model, where the power spectrum is in the form (1), as model 1.

Fig. 2 (a) shows the power spectrum of density fluctuations in model 1 for $h = 0.65$ and 0.70. We also show the power spectrum in the CDM model with a scale-invariant spectrum [$S(k) = 1$]. In comparison with the standard model, the power spectrum of density fluctuations in model 1 is depressed at wavenumbers $k > 0.05h$ Mpc$^{-1}$. A similar break in the power spectrum of density fluctuations was analysed by Atrio-Barandela et al. (2000).

To calculate the angular power spectrum of the CMB temperature fluctuations, we used the code CMBFAST, which was modified to incorporate the function $S(k)$ in the initial power spectrum. We examined the models with no reionization (optical depth $\tau = 0$). Fig. 2 (b) shows the CMB power spectrum, $\Delta T^2_l = l(l+1)C_l/2\pi$, predicted in model 1. Here $C_l = \langle a_{lm}^2 \rangle$ and $a_{lm}$ are the coefficients of the spherical harmonic decomposition of the CMB temperature field: $\Delta T(\theta, \phi) = \sum \hat{a}_{lm} Y_{lm}(\theta, \phi)$. Fig. 2 (b) also shows the temperature power spectrum predicted in the standard CDM model. We see that lowering the amplitude of the CMB power spectrum at wavenumbers $k > 0.05h$ Mpc, also lowers the amplitude of the temperature power spectrum at multipoles $l > 400$. The height of the first peak is different in the models in Fig. 2 (b) owing to the change in the Hubble constant.

Fig. 2 (b) also shows the CMB power spectrum derived from the Boomerang (Netterfield et al. 2001) and from the Maxima-1 (Hanany et al. 2000) experiments. We see that the temperature power spectrum in model 1 is consistent with the observed temperature power spectrum. The amplitude of the second acoustic peak at $l \sim 500$ in the observed temperature power spectrum is smaller than that predicted in the standard CDM model with $\Omega_m = 0.3$. However, in the analyses we have assumed that the spectral index $n = 1$ and the optical depth $\tau = 0$. Also, both CMB data sets have a calibration uncertainty and a beam uncertainty that are not included in the errors plotted in Fig. 2 (b).

Netterfield et al. (2001) analysed the CMB power spectrum in the standard CDM model in more detail and showed that this model is consistent with the observed CMB data, once the parameters $n \neq 1$ and $\tau \neq 0$, and beam and calibration uncertainties are taken into account.

To study the mass function of clusters we use the Press–Schechter (PS) approximation. The PS mass function has been compared with N-body simulations (Efstathiou et al. 1988; White, Efstathiou & Frenk 1993; Lacey & Cole 1994; Eke, Cole & Frenk 1996) and has been shown to provide an accurate description of the abundance of virialized cluster-size haloes. In the PS approximation the number density of clusters with the mass between $\Sigma$ and $M + dM$ is given by

$$n(M) dM = - \frac{2}{\pi^2} \rho_b \frac{\delta}{\bar{\sigma}} \frac{d\sigma(M)}{dM} \exp \left[ - \frac{\delta^2}{2\bar{\sigma}^2(M)} \right] dM. \quad (2)$$

Here $\rho_b$ is the mean background density and $\delta_i$ is the linear theory
overdensity for a uniform spherical fluctuation which is now collapsing; $\delta_c = 1.675$ for $\Omega_0 = 0.3$ (Eke et al. 1996). The function $\sigma(M)$ is the rms linear density fluctuation at the mass scale $M$. We will use the top-hat window function to describe haloes. For the top-hat window, the mass $M$ is related to the window radius $R$ as $M = 4\pi R^3/3$. In this case, the number density of clusters of mass larger than $M$ can be expressed as

$$n_c(> M) = \int_M^{\infty} n(M') dM' = -\frac{3}{(2\pi)^{5/2}} \frac{2 \delta_c}{\sigma^2(r)} \frac{d\sigma(r)}{dr} \times \exp \left[ -\frac{2 \delta_c^2}{\sigma^2(r)} \right] \frac{dr}{r^2}. \quad (3)$$

Fig. 2(c) shows the cluster mass function predicted in model 1 for $h = 0.65$ and 0.70. The mass function in the standard CDM model is also plotted. We investigated the cluster masses within a 1.5 $h^{-1}$Mpc radius sphere around the cluster centre. This mass $M_{1.5}$ is related to the window radius $R$ as

$$R = 8.43\Omega_{0.7}^{1/2} \frac{M_{1.5}}{6.99 \times 10^{14} h^{-1} M_\odot} \left( h^{-1} \text{Mpc} \right). \quad (4)$$

Here the parameter $\alpha$ describes the cluster mass profile, $M(r) \sim r^n$, at radii $r \sim 1.5 h^{-1}$Mpc. Numerical simulations and observations of clusters indicate that the parameter $\alpha \approx 0.6-0.7$ for most of clusters (e.g. Navarro, Frenk & White 1996; Carlborg, Yee & Ellingson 1997). In this paper we use the value $\alpha = 0.65$.

Fig. 2(c) also shows the mass function of clusters derived by Bahcall & Cen (1993; hereafter BC) and by Girardi et al. (1998; hereafter G98). BC used both optical and X-ray observed properties of clusters to determine the mass function of clusters. The function was extended towards the faint end using small groups of galaxies. G98 determined the mass function of clusters by using virial mass estimates for 152 nearby Abell –ACO clusters including the ENACS data (Katgert et al. 1998). The mass function derived by G98 is somewhat larger than the mass function derived by BC, the difference being larger at larger masses (see Fig. 2c). Reiprich & Böhringer (1999) determined the cluster mass function using X-ray flux-limited sample from ROSAT All-Sky survey. They determined the masses for different outer radii of the clusters and for a radius $r = 1.5 h^{-1}$Mpc their mass function agree with that determined by BC.

Let us consider the amplitude of the mass function of galaxy clusters at $M_{1.5} = 4 \times 10^{14} h^{-1} M_\odot$. For this mass, the cluster abundances derived by BC and G98 are $n(> M) = (2.0 \pm 1.1) \times 10^{-6} h^{-1} \text{Mpc}^{-3}$ and $n(> M) = (6.3 \pm 1.2) \times 10^{-6} h^{-1} \text{Mpc}^{-3}$, respectively. By analysing X-ray properties of clusters, White et al. (1993) found that the number density of clusters with the mass $M_{1.5} \sim 4.2 \times 10^{14} h^{-1} M_\odot$ is $n(> M) = 4 \times 10^{-6} h^{-1} \text{Mpc}^{-3}$.

Fig. 2(c) shows that the cluster mass function in the standard CDM model is in good agreement with the observed data. However, the number density of clusters in model 1 is substantially lower than is observed; for the mass $M_{1.5} = 4 \times 10^{14} h^{-1} M_\odot$, the cluster abundance $n(> M) = 1.5 \times 10^{-7} h^{-1} \text{Mpc}^{-3}$ and $n(> M) = 4.1 \times 10^{-7} h^{-1} \text{Mpc}^{-3}$ for $h = 0.65$ and 0.70, respectively.

In model 1, the amplitude of the power spectrum of density fluctuations is depressed with respect to the standard CDM model for $k > 0.05 h^{-1} \text{Mpc}^{-1}$. Lowering the amplitude of the power spectrum of density fluctuations at wavenumbers $k > 0.05 h^{-1} \text{Mpc}^{-1}$ also lowers the CMB power spectrum at multipoles $l > 400$. However, lowering the amplitude of the power spectrum of density fluctuations also lowers the cluster mass function, and as a result in model 1 the number density of clusters is smaller than observed. One possibility to get rid of the last effect is to consider a bump in the power spectrum of density fluctuations at wavenumbers $k \sim 0.1-0.2 h^{-1} \text{Mpc}$. The cluster mass function for masses $M \sim 10^{14}-10^{15} h^{-1} M_\odot$ is sensitive to the amplitude of the power spectrum at wavenumbers $k \sim 0.2 h^{-1} \text{Mpc}$, while the temperature anisotropy at the second acoustic peak is sensitive to the amplitude of the power spectrum at wavenumbers $k \sim 0.05-0.1 h^{-1} \text{Mpc}$.

Thus let us now study a CDM model, where the power spectrum of density fluctuations at $z \sim 10^2$ contains a specific feature at wavenumbers $k \sim 0.1-0.2 h^{-1} \text{Mpc}^{-1}$ $(\lambda \sim 30-60 h^{-1} \text{Mpc})$ which correspond to the scale of superclusters:

$$P(k) = AKS(k)T^2(k) = \begin{cases} \frac{AKT^2(k)}{P(k_0)k/k_0}, & k < k_0; \\ \frac{P(k_1)}{k/k_0}, & k_0 < k < k_1; \\ BKT^2(k), & k > k_2, \end{cases} \quad (5)$$

where $k_0 = 0.05 h^{-1} \text{Mpc}^{-1}$, the spectral index $m = \log[P(k_1)/P(k_0)]/\log[k_1/k_0]$ and $B = P(k_1)/[k_1T^2(k_1)]$. This form of the power spectrum contains three free parameters, which describe the bump in the power spectrum: $k_1$, $k_2$ and $P(k_1)$. The parameter $k_1$ determines the beginning of the bump, the parameter $k_2$ – the end of the bump, and the parameter $P(k_1)$ – the amplitude of the power spectrum for the bump. In this paper we examine the models where $k_1 = 0.1 h^{-1} \text{Mpc}^{-1}$, $k_2 = 0.2 h^{-1} \text{Mpc}^{-1}$ and $P(k_1) = 2500-3500 h^{-3} \text{Mpc}^3$. We denote the CDM model, where the power spectrum is in the form (5), as model 2.

Fig. 3(a) shows the power spectrum of density fluctuations in model 2 for different values of the parameter $P(k_1)$. In the models studied, $P(k_1) = 2500, 3000$ and $3500 h^{-3} \text{Mpc}^3$. The normalized Hubble constant $h = 0.65$. In Fig. 3(b), we show the function $S(k)$, which describes the deviation of the initial power spectrum from the scale-invariant form. The function $S(k) = 1$ at wavenumbers $k < 0.05 h^{-1} \text{Mpc}^{-1}$, reaches the minimum at $k = 0.1 h^{-1} \text{Mpc}^{-1}$ and then increases up to the wavenumber $k_2 = 0.2 h^{-1} \text{Mpc}^{-1}$. At the minimum, $S(k_1) = 0.48$ and $S(k_1) = 0.67$ for $P(k_1) = 2500 h^{-3} \text{Mpc}^3$ and $P(k_1) = 3500 h^{-3} \text{Mpc}^3$, respectively. For wavenumbers $k > 0.2 h^{-1} \text{Mpc}^{-1}$, we find that $S(k) = 1.3$ and 1.9, respectively. We also investigated the CDM model with $h = 0.70$ assuming that $P(k_1) = 2500, 3000$ and $3500 h^{-3} \text{Mpc}^3$. In this model, $S(k_1) = 0.38$, $S(k_2) = 1.0$ and $S(k_1) = 0.52$, $S(k_2) = 1.4$ for the parameter $P(k_1) = 2500 h^{-3} \text{Mpc}^3$ and $P(k_1) = 3500 h^{-3} \text{Mpc}^3$, respectively.

Fig. 4 shows the variance, $\sigma^2(R)$, in the standard CDM model and in model 2 for different values of the parameter $P(k_1)$. The variance is given as a function of top-hat window radius $R$. For the masses $M_{1.5} = 10^{14} h^{-1} M_\odot$ and $M_{1.5} = 10^{15} h^{-1} M_\odot$, the window radius $R = 5.8 h^{-1} \text{Mpc}$ and $R = 15.3 h^{-1} \text{Mpc}$, respectively (see equation 4).

Fig. 5 shows the cluster mass function as predicted in model 2 for the parameter $P(k_1) = 2500, 3000$ and $3500 h^{-3} \text{Mpc}^3$. The mass function of clusters in the standard CDM model is also plotted. Fig. 5(a) shows the results for $h = 0.65$ and Fig. 5(b) for $h = 0.70$. In model 2, the mass function is steeper than that in the standard CDM model. At the same value of $P(k_1)$, the cluster mass function for $h = 0.65$ and 0.70 is similar. For comparison, we also show in Fig. 5 the observed mass function of clusters of galaxies derived by BC and G98. In the models studied, the cluster mass function is consistent with the observed data. If $h = 0.65$, then for the mass $M_{1.5} = 4 \times 10^{14} h^{-1} M_\odot$, the cluster abundance

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Figure 6 demonstrates the angular power spectrum of the CMB temperature fluctuations as predicted in model 2. We also show the temperature power spectrum in the standard CDM model with a scale-invariant power spectrum. Fig. 6(a) demonstrates the results for $h = 0.65$ and Fig. 6(b) for $h = 0.70$. In model 2, the amplitude of the temperature power spectrum at multipoles $l > 400$ is smaller than that predicted in the standard CDM model. Fig. 6 also shows the CMB power spectrum derived from the Boomerang (Netterfield et al. 2001) and from the Maxima-1 (Hanany et al. 2000) experiments. In the models studied, the temperature power spectrum is consistent with the observed temperature power spectrum. Therefore, in the model with a bump in the power spectrum of density fluctuations at the scale of superclusters, the mass function of clusters and the temperature power spectrum are in good agreement with the observed data.

3 PECULIAR VELOCITIES

The observed rms peculiar velocity of galaxy clusters has been studied in several papers (e.g. Bahcall, Gramann & Cen 1994; Bahcall & Oh 1996; Borgani et al. 1997; Watkins 1997; Dale et al. 1999). Watkins (1997) developed a likelihood method for...
estimating the rms peculiar velocity of clusters from line-of-sight velocity measurements and their associated errors. This method was applied to two observed samples of cluster peculiar velocities: a sample known as the SCI sample (Giovanelli et al. 1997) and a subsample of the Mark III catalogue (Willick et al. 1997). Watkins (1997) found that the rms one-dimensional cluster peculiar velocity is 265$^{+15}_{-18}$ km s$^{-1}$, which corresponds to the three-dimensional rms velocity 459$^{+184}_{-130}$ km s$^{-1}$. Dale et al. (1999) obtained Tully–Fisher peculiar velocities for 52 Abell clusters distributed throughout the sky between $\sim 50$ and 200 h$^{-1}$ Mpc. They found that the rms one-dimensional cluster peculiar velocity is 341 $\pm$ 93 km s$^{-1}$, which corresponds to the three-dimensional rms velocity 591 $\pm$ 161 km s$^{-1}$.

To investigate peculiar velocities of clusters in our models, we use the linear theory predictions for peculiar velocities of peaks in the Gaussian field. The linear rms velocity fluctuation on a given scale $R$ at the present epoch can be expressed as

$$\sigma_v(R) = H_0 f(\Omega_0) \sigma_8(R),$$

where $f(\Omega_0)$ is the linear velocity growth factor in the flat models and $\sigma_8$ is defined for any integer $j$ by

$$\sigma_j^2 = \frac{1}{2\pi^2} \int P(k) W^2(kR) k^{2j+2} dk.$$  

Bardeen et al. (1986) showed that the rms peculiar velocity at peaks of the smoothed density field differs systematically from $\sigma_v(R)$, and can be expressed as

$$\sigma_o(R) = \sigma_v(R) \sqrt{1 - \frac{\sigma_j^2}{\sigma_8^2 \sigma_7^2}}.$$  

SG examined the linear theory predictions for the peculiar velocities of peaks and compared these to the peculiar velocities of clusters in N-body simulations. The N-body clusters were determined as peaks of the density field smoothed on the scale $R \sim 1.5 h^{-1} \text{Mpc}$. The numerical results showed that the rms peculiar velocity of small clusters is similar to the linear theory expectations, while the rms peculiar velocity of rich clusters is higher than that predicted in the linear theory.

The rms peculiar velocity of clusters with a mean cluster separation $d_{cl}$ = 30$h^{-1}$ Mpc was $\sim 18$ per cent higher than that predicted by the linear theory. We assume that the observed cluster samples studied by Watkins (1997) and Dale et al. (1999) correspond to the model clusters with a separation $d_{cl}$ = 30$h^{-1}$ Mpc ($n_{cl}$ = $3.7 \times 10^{-3} h^{3}$ Mpc$^{-3}$) and determine the rms peculiar velocity of the clusters, $v_{cl}$, as

$$v_{cl} = 1.18 \sigma_p(R),$$

where the radius $R = 1.5 h^{-1}$ Mpc.

In the standard CDM model with $\Omega_m = 0.3$, we found that $v_{cl}$ = 582 and 635 km s$^{-1}$ for $h = 0.65$ and 0.70, respectively.

Table 1 lists the rms peculiar velocity of clusters, $v_{cl}$, in our models. The rms peculiar velocity of clusters is $\sim 555$--$620$ km s$^{-1}$, which is consistent with the observed rms peculiar velocity of clusters derived by Watkins (1997) and Dale et al. (1999).

We also studied the rms bulk velocity for a radius $r = 60 h^{-1}$ Mpc, $V_{60}$. The rms bulk velocity was determined by using equation (6). In the standard $\Omega_m = 0.3$ model, $V_{60} = 273$ and 285 km s$^{-1}$ for $h = 0.65$ and 0.70, respectively. Table 1 lists the rms bulk velocity, $V_{60}$, in our models. The bulk velocity is similar to that in the standard model. The observed bulk velocities are determined in a sphere centred on the Local Group and represent a single measurement of the bulk flow on large scales.

### Table 1. Peculiar velocities in the different models.

| $h$   | $P(k_0)$ ($h^{-3} \text{Mpc}^{-3}$) | $v_{cl}$ (km s$^{-1}$) | $V_{60}$ (km s$^{-1}$) |
|-------|----------------------------------|------------------------|------------------------|
| 0.65  | 2500                            | 557                    | 273                    |
| 3000  | 578                             | 273                    |
| 3500  | 598                             | 273                    |
| 0.70  | 2500                            | 579                    | 284                    |
| 3000  | 599                             | 284                    |
| 3500  | 619                             | 284                    |

The observed bulk velocity derived from the Mark III catalogue of peculiar velocities for $r = 60 h^{-1}$ Mpc is 370 $\pm$ 110 km s$^{-1}$ (Kolatt & Dekel 1997). Giovanelli et al. (1998) studied the bulk velocity in the SCI sample and estimated that the bulk flow of a sphere of radius $r = 60 h^{-1}$ Mpc is between 140 and 320 km s$^{-1}$. In the models studied, the rms bulk velocity is $\sim 275$--$285$ km s$^{-1}$, which is consistent with the observed data.

### 4 DISCUSSION AND SUMMARY

In this paper we have examined a CDM model, where the power
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