Damage Constitutive and Failure Prediction of Artificial Single-Joint Sandstone Based on Acoustic Emission

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Received: 15 August 2021 / Accepted: 28 June 2022 / Published online: 16 July 2022
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Abstract The effect law of deformation and failure of a jointed rock mass is essential for underground engineering safety and stability evaluation. In order to study the evolution mechanism and precursory characteristics of instability and failure of jointed rock masses, uniaxial compression and acoustic emission (AE) tests are conducted on sandstones with different joint dip angles. To simulate the mechanical behavior of the rock, a jointed rock mass damage constitutive model with AE characteristic parameters is created based on damage mechanics theory and taking into account the effect of rock mass structure and load coupling. To quantify the mechanism of rock instability, a cusp catastrophe model with AE characteristic parameters is created based on catastrophe theory. The results indicate that when the joint dip angle increases from 0° to 90°, the failure mechanism of sandstone shifts from tensile to shear, with 45° being the critical failure mode. Sandstone’s compressive strength reduces initially and subsequently increases, resulting in a U-shaped distribution. The developed damage constitutive model’s theoretical curve closely matches the test curve, indicating that the model can reasonably describe the damage evolution of sandstone. The cusp catastrophe model has a high forecast accuracy, and when combined with the damage constitutive model, the prediction accuracy can be increased further. The research results can provide theoretical guidance for the safety and stability evaluation of underground engineering.

Keywords Rock mechanics · Jointed rock · Acoustic emission · Damage constitutive · Cusp catastrophe

1 Introduction
As the main research object of underground engineering, rock mass has produced discontinuous surfaces such as joints, bedding and cracks under the long geological evolution and complex crustal stress conditions, which display complex mechanical properties such as heterogeneity, discontinuity and anisotropy (Bahaaddini et al. 2013; Kulatilake et al. 2006). The presence of these discontinuities threatens the stability and safety of underground engineering in rock mass. Numerous examples demonstrate that the deformation and failure of the rock mass of the discontinuous structural surface is directly related to engineering geological disasters such as deep rock burst, stratum subsidence, and stope collapse, which pose a great threat to the safety of construction.
personnel and equipment and seriously affect the progress of the project (e.g., Mehranpour et al. 2018; Chen et al. 2017; Zhao et al. 2016; Wasantha et al. 2015). Therefore, it is of great theoretical and practical significance to study the deformation and failure evolution characteristics of jointed rock mass.

The influence of the joint surface on the deformation and failure characteristics of rock mass mainly involves joint dip angle (Son and Adedokun 2016b), roughness (Zhao et al. 2021), filling material (Yang et al. 2016) and so on. Among the above factors, joint dip angle is directly related to underground engineering excavation. In the process of compressive failure of jointed rock mass, micro cracks grow and coalesce into macro cracks and converge into large cracks along a certain direction of joint dip angle, resulting in the whole deformation failure of rock mass (Sun et al. 2019; Liu et al. 2022a, b). Many scholars have studied the mechanical properties of rock mass affected by joints (Chen et al. 2021; Fan et al. 2015). Li et al. (2014) carried out a triaxial unloading test on a single-joint specimen with different angles, and found that the deformation modulus of the joint specimen showing a U-shaped change with the joint dip angle. To study the mechanical properties of jointed rock mass, Zhao et al. (2020) and Kulatilake et al. (2001a, b) made jointed rock samples for uniaxial compression test, and summarized several failure modes of jointed rock mass. Mou et al. (2020) studied the influence of the loading direction and the joint dip angle on the mechanical properties of the coal sample through uniaxial compression tests. The results show that with the increase of joint dip angle, the peak load and failure time decrease slightly at first and then increase, and the failure time decreases sharply from 0° to 30°. Yuan et al. (2020) used uniaxial compression test and particle flow code (PFC2D) to analyze the influence of crack dip angle on the mechanical properties, and concluded that the joint dip angle has a significant impact on the mechanical strength of the rock mass. AE is a transient elastic wave generated in rock due to the rapid release of local strain energy. It can reflect the micro-failure characteristics of rock, infer the change of internal behavior of rock mass and invert the failure mechanism of rock (He and Ping 2014). The constitutive relation is one of the most significant rock mechanical laws. On the one hand, using these relations, we can know the actual behavior of rock deformation under a given state and environment, and provide a link between stress and strain; on the other hand, because we typically observe the strain or stress rate of the subsurface medium, this structural relationship is incredibly helpful for comprehending the stress state and environmental conditions of subsurface reservoirs. How to include rock acoustic emission parameters into a rock constitutive model is one of the pressing issues in rock mechanics. Yang et al (2021) used time as an intermediate variable, fit the connection between sandstone strain and cumulative acoustic emission count considering the loading rate, and produced a damage constitutive model for sandstone based on the loading rate and cumulative acoustic emission ringing count. Wu et al. (2015) established the relationship model between the cumulative count of AE ringing and the stress and damage variables. Based on Lemaitre’s equivalent strain principle and continuous damage theory, Zhang et al. (2019) constructed a three-point bending damage constitutive model for limestone and verified the rationality of the model. Bruning et al. (2019) constructed a hard rock constitutive model based on stress-strain-damage correlation theory by obtaining the test results of total stress, strain and acoustic emission damage. For rock mass, how to establish the relationship between AE parameters and mechanical parameters and predict instability and failure is still a hot and difficult point in rock mechanics research. From the perspective of energy, the essence of rock failure is a sudden change of state driven by energy (Liu et al. 2015a, b; Dui and Ren. 1999). In several actual applications of geotechnical engineering, such as coal mine (Liu et al. 2022a, b), rock slope (Zhou et al. 2020; Hou et al. 2020), and underground surrounding rock (Fu and Chen 2008; Zhao et al. 2014) in goaf, the construction will encounter unstable elements resulting from rock’s nonlinear mutation properties. Catastrophe theory, as a powerful tool to study the phenomenon of system state catastrophe instability (Xu et al. 2020), can well reflect the characteristics of material instability, reveal the essence of catastrophe and deepen the existing knowledge. It has been applied in dynamic risk prediction of pressure bump (Qiao et al. 2021; Odintsev 1995), rock burst (Guo and Chen 2017; Jin et al. 2013), water inrush (Tang et al. 2018; Zhu and Li 2020) and rock damage...
mechanism analysis (Liang et al. 2016; Liu et al. 2015a, b) in deep mining engineering.

There are some studies on the deformation and failure characteristics of jointed rock masses. However, due to the complexity of the jointed rock mass, it is necessary to further explore its deformation and failure characteristics and instability mechanism. In this paper, uniaxial compression and AE tests of single-jointed sandstone with different dip angles are carried out. The mechanical properties and AE damage evolution characteristics of rocks with different joint dip angles are analyzed. A damage constitutive model and a cusp catastrophe model with AE characteristic parameters have been developed based on damage mechanics theory and catastrophe theory. The research results are useful in improving the monitoring capability of AE technology for rock mass stability, as well as providing certain guidance for safe operation and maintenance of practical projects.

2 Experimental Procedure

2.1 Preparation of Specimens

The specimens of yellow sandstone were derived from a construction site in Hunan Province, China. Standard cylindrical specimens of Φ50 × 100 mm were manufactured following the standards of the International Society of Rock Mechanics (ISRM). To reduce the influence of rock dispersion on the test results, ultrasonic testing was carried out on all specimens, and rock specimens with similar wave velocities and good uniformity were selected. In order to meet the test requirements, a hole with a diameter of 1–2 mm was drilled in the center of the height of the specimen. Then, 0.3 mm of emery steel wire was used to go through the small hole, and the joint with a length of 20 mm and an average thickness of 0.5–0.7 mm was sawed out. Joint dip angle β references (Feng et al. 2021) select 0°, 30°, 45°, 60° and 90°. After the test specimens are made, the test specimens shall be numbered in groups of 3 test specimens each. The specimen groups are: A, B2, C2, D2, E2, F2 (The numbering rules are: A, B, C, D, E, and F represent intact rock, 0°, 30°, 45°, 60°, and 90° joint, respectively. The number 2 represents the length of the joint, which is 20 mm. The specimen preparations are shown in Fig. 1.

2.2 Experiment Methods

The compression tests were carried out on the RMT150-B rock mechanical test system, developed by the Wuhan Institute of Rock and Soil Mechanics, Chinese Academy of Sciences. According to the ISRM, the loading rate of the specimen was set to 1 kN/s. A DS5-8B AE test apparatus was applied to
collect the AE signals. In order to make the surface of the cylindrical test specimen better fit the surface of the AE sensor, one end of the test piece was attached to the sensor, and the other end was attached to the small transition device on the surface of the sensor. Each sensor was equipped with a preamplifier. A 45 dB threshold was selected for all sensors, and the preamplifier gain was 40 dB. The equipment is shown in Fig. 2. To reduce the friction effect and the noise of the end face, butter was applied to the upper and lower ends of the specimens. Before the test, the response of the sensor was measured to detect its response amplitude to the analog signal source, and the test could be started when it was normal.

3 Test Results and Analysis

3.1 Characteristic Analysis of Stress–strain Curve

Stress–strain characteristics are the basic physical and mechanical properties of rock materials. The stress–strain curve can not only reflect the basic mechanical parameters such as strength and elastic modulus of rock materials, but also reflect the response of external load. Specimens of sandstones with various dip angles was prepared, three samples per group, for a total of eighteen specimens (Fig. 1(b)). According to the elastic modulus and strength of the three stress–strain curves obtained in a group of rock specimens, one of the stress–strain curves that was most compatible with the predicted was chosen as the test result. This was done to reduce the impact of the rock specimen discreteness and test errors. The stress–strain curves of the tested specimens is shown in Fig. 3. The stress strain of yellow sandstone specimens experiences the following stages: compaction stage, linear elastic stage, plastic stage, and failure stage. (The different stages of the stress–strain curve of the intact specimen are shown in Fig. 3). When the stress level of the specimen is about 0 to 24% of its peak strength,
the specimen is in the compaction stage. The yellow sandstone belongs to coarse sandstone with more voids inside, so the stress level is low and the strain increases quickly. When the stress level is about 24 to 95% of the peak strength, the specimen is in the line elastic deformation stage. At this stage, the stress–strain curve is approximately linear. With each successive increase in stress, the strain increases uniformly, and the internal micro-cracks of the specimen are continuously initiated and develop stably. When the stress level reaches more than 95%, the slope of the curve is significantly reduced, and the specimen enters the stage of plastic deformation. At this time, the micro-cracks inside the specimen rapidly expand and penetrate to form a macro-crack until the specimen is destroyed. By comparing the four stages in the failure process of the specimen, it can be found that the compaction stage of the yellow sandstone specimen is more obvious, while the plastic stage is not. This phenomenon is consistent with the deformation characteristics of coarse sandstone, which has more voids and strong brittleness.

### 3.2 Influence of Joint Dip Angle on Peak Strength and Elastic Modulus

The peak strength and elastic modulus changes of the specimens are shown in Fig. 4. It can be seen from the figure analysis that the existence of joints has a great weakening effect on the strength and elastic modulus of the specimen, and the joint dip angle is a key factor affecting the strength of the specimen. With the increase in joint dip angle, the change trend of peak strength and elastic modulus is consistent, showing a U-shaped distribution with the opening upward. When the joint dip angle is 45°, the peak strength and elastic modulus are the lowest, which decrease by 32.3% and 17.4%, respectively, compared with the intact specimen. This is because the joint dip angle of 45° is the closest to the failure angle in the Moiré strength theory.

### 3.3 The Effect of Joint Dip Angle on Failure Mode

The joint dip angle affects the failure mode of rock. By analyzing the failure modes of the specimens, we can know which forces are responsible for which ones controlling the failure process of the specimens. The failure mode of the specimen is shown in Fig. 5. The failure modes of joint specimens are mainly tension failure through the joint surface and shear failure along the joint surface. As the joint dip angle increases from 0° to 90°, the failure mode of specimens gradually changes from tension failure to shear failure. The joint dip angle of 45° is the critical angle for the two failure modes. For specimens with joint dip angles of 0° and 30°, due to the large angle between the joint surface and the direction of
loading, the specimen is mainly subjected to the normal stress perpendicular to the joint surface, resulting in more wing cracks. The failure mode is tension failure through the joint surface. For specimens with joint dip angles of 60° and 90°, because the angle between the joint surface and the direction of loading is small, the shear stress on the joint surface is large. There are many secondary cracks along the joint direction in the specimens, which show shear failure along the joint surface. For the specimen with a joint dip angle of 45°, the normal stress on the joint surface is equivalent to the shear stress. Wing cracks and secondary cracks are found in the specimen, which shows the mixed mode of failure of tension and shear. From the perspective of fracture mechanics, the fracture toughness along a certain direction of joints with different inclination angles is lower, which will cause the internal original or newly generated microcracks to preferentially expand and connect along this direction, resulting in different types of crack propagation (wing cracks and secondary cracks), and finally forming jointed sandstone with different failure modes.

3.4 AE Response to Different Joint Dip Angles

As shown in Fig. 6, the AE ringing count rate of yellow sandstone has a good correspondence with its corresponding strain stage. This shows that it is very suitable to study the mechanical properties and deformation characteristics of rock mass with an AE ringing count rate. The stress–strain curve can be divided into four stages: the compaction stage (OA), the linear elastic stage (AB), the plastic stage (BF), and the failure stage (FS).

OA: The AE ringing count rate is at a low level. The whole specimen shows a gradual increase, while AE in the joint specimen shows a small jump phenomenon, which indicates that the micro-cracks and voids in the rock began to close and develop along the direction of the joint under the condition of compression.

AB: The AE ringing count rate is stable without obvious fluctuations and oscillations. When comparing the joint specimens with the intact specimens, the AE signal of the joint specimens at this stage is significantly reduced, the internal damage is reduced, and no new cracks occur.

BF: In specimens with a joint dip angle of >45°, the AE ringing count rate has a significant step or steep increase before reaching the failure stage, which indicates that the sudden increase in AE ringing count rate can be used as a precursor of rock failure and instability. With the increase of joint dip angle, the plastic deformation stage of rock increases obviously. When the joint dip angle is less than 45°, the rock mass is mainly tension failure. Joint dip angle >45° is mainly shear failure, and the plastic deformation time becomes longer. It shows that the increase in joint dip angle will cause an increase in the plasticity of rock, which makes the AE signal better separated.

FS: With the peak stress as the boundary, the AE ringing count rate increases sharply and reaches its maximum, showing strong synchronization. The cracks inside the specimen rapidly propagate and penetrate, and then the specimen is instability and failure. At this time, the AE signal drops sharply or even disappears.

4 Damage Constitutive Model with AE Characteristic Parameters

4.1 Establishment of A Damage Constitutive Model

The damage of rock mass results in changes in microstructures and fracture of bearing surfaces under the loading of rock materials. According to the concept of damage factor and effective stress proposed by Kachanov and Krajcinovic (1986), the continuous damage variable D of rock can be defined as

\[ D = \frac{A - \tilde{A}}{A} \]  

(1)

where \( \tilde{A} \) is the effective bearing area of the material after damage, \( A \) is the cross-sectional area of the material without damage.

The loading force \( F \) on rock specimens can be expressed as

\[ F = \sigma \times A = \tilde{\sigma} \times \tilde{A} \]  

(2)

where \( \sigma \) is the initial stress without damage, \( \tilde{\sigma} \) is the effective stress corresponding to the effective area \( \tilde{A} \) after damage.

Combine Eq. (1) and Eq. (2), get Eq. (3).

\[ \tilde{\sigma} = \frac{\sigma}{1 - D} \]  

(3)
Fig. 6 Specimen stress-strain-AE ringing count rate
According to the relation between deformation component and stress component of ideal elastomer in theory of elasticity
\[ \varepsilon_1 = \frac{\sigma}{E} = \frac{1}{E} \left[ \sigma_1 - \mu (\sigma_2 + \sigma_3) \right] \] (4)
where \( E \) is the elastic modulus of the material, \( \varepsilon \) is the strain.

According to the equivalent strain hypothesis, get Eq. (5)
\[ \varepsilon_1 = \frac{\tilde{\sigma}}{E} = \frac{\sigma}{(1 - D)E} = \frac{[\sigma_1 - \mu (\sigma_2 + \sigma_3)]}{(1 - D)E} \] (5)

The constitutive model of rock mass damage is obtained after deformation as shown in Eq. (6).
\[ \sigma_1 = E \varepsilon_1 (1 - D) + \mu (\sigma_2 + \sigma_3) \] (6)

During the process of applying external load to a rock mass, various randomly distributed micro-damages occur inside the rock mass, so that the damage under load can be studied from a statistical point of view and the corresponding statistical damage constitutive model can be established. In this paper, the Weibull distribution model is used for constitutive analysis of AE damage in jointed rock mass.

The probability density function is
\[ \varphi(\varepsilon) = \frac{m}{\alpha} \left( \frac{\varepsilon}{\alpha} \right)^{m-1} \exp \left[ -\left( \frac{\varepsilon}{\alpha} \right)^m \right] \] (7)
where \( \varphi(\varepsilon) \) is the distribution function of rock micro element strength, \( \varepsilon \) is rock micro-element strain, \( m \) and \( \alpha \) are distribution parameters.

It is assumed that under certain loading conditions the material strain is \( \varepsilon \), the probability of failure of the micro-elements in the material section is obtained as
\[ \begin{cases} F(\varepsilon) = P(\varepsilon \geq 0) \\ P(\varepsilon \geq 0) = \int_0^\infty \varphi(x)dx \end{cases} \] (8)

The upper and lower expressions in Eq. (8) are derived from \( \varepsilon \).
\[ \frac{dF(\varepsilon)}{d\varepsilon} = \varphi(\varepsilon) \] (9)
where \( \varphi(\varepsilon) \) is the probability density of failure of the material element.

The effective area of the specimen can be expressed as
\[ \tilde{A} = A[1 - F(\varepsilon)] \] (10)

The damage variable \( D \) based on Weibull distribution statistical model is Eq. (11).
\[ D = F(\varepsilon) = \int_0^\varepsilon \varphi(x)dx \] (11)

When rock mass is loaded to the strain level \( \varepsilon \), its damage variable can be expressed as
\[ D = \int_0^\varepsilon \varphi(x)dx = 1 - \exp \left[ -\left( \frac{\varepsilon}{\alpha} \right)^m \right] \] (12)

In the process of damage, AE cumulative relationship is
\[ N = \frac{N_f}{A} \cdot \Delta A \] (13)
where \( A \) is the cross-sectional area of the whole specimen. \( N_f \) is the accumulation of AE when the whole section is completely destroyed. \( \Delta A \) is a micro area element. \( N \) is the accumulation of AE when the compressive strain of the rock specimen increases to \( \varepsilon \).

Based on the strength distribution of the element, it is assumed that when the strain of the specimen increases \( \Delta \varepsilon \), the increment of the section area resulting in failure is
\[ \Delta A = A \cdot \varphi(\varepsilon) \cdot \Delta \varepsilon \] (14)

By simultaneous Eq. (13) and (14), get Eq. (15), it can be obtained that the AE accumulation of rock specimen when the compressive strain increases to \( \varepsilon \).
\[ N = N_f \int_0^\varepsilon \varphi(x)dx \] (15)

Substituting Eq. (12) into Eq. (15), the load rock damage variable based on AE characteristic parameters is obtained as
\[ D_s = \frac{N}{N_f} \] (16)

The damage of the jointed rock mass under load can be equivalent to the coupling of two damage states, one is the initial damage caused by the initial defect of the joint surface, and the other is the
damage caused by the loading of the rock mass, then the jointed rock mass. The internal damage constitutive relationship (Kachanov and Krajcinovic 1986) can be expressed as

$$
\sigma = (1 - D_s)E_0\epsilon
$$  \hspace{1cm} (17)

where $E_\phi$ is the elastic modulus of the rock mass with the joint dip angle $\phi$, $D_s$ is the damage variable of the rock mass under load.

The presence of complicated discontinuity patterns, the inherent statistical nature of their geometrical parameters, and the uncertainties involved in the estimation of their geomechanical and geometrical properties make accurate initial damage of jointed rock masses difficult (Kulatilake et al. 2001a, b; Wu and Kulatilake 2012). The response of the macro-physical properties of the rock can represent the degree of internal deterioration of the material. Since the elastic modulus of jointed rock masses with different dip angles is easier to analyze and measure, the degree of deterioration of the elastic modulus of the jointed rock mass can be used to characterize the initial joint damage value, so the initial damage variable $D_\phi$ of jointed rock mass can be expressed as (Wang et al. 2018).

$$
D_\phi = 1 - \frac{E_\phi}{E_0}
$$  \hspace{1cm} (18)

where $E_0$ is the initial elastic modulus of the complete rock mass.

From Eqs. (17) and (18), the stress–strain relationship of the jointed rock mass expressed by the initial joint damage variable and the loaded damage variable is

$$
\sigma = (1 - D_\phi)(1 - D_s)E_0\epsilon
$$  \hspace{1cm} (19)

From Eqs. (16), (18) and (19), the total damage variable of rock mass joints and load coupling based on the characteristic parameters of AE is

$$
D = 1 - \frac{E_\phi}{E_0}(1 - \frac{N}{N_f})
$$  \hspace{1cm} (20)

By substituting Eq. (20) into Eq. (6), the statistical constitutive relation of jointed rock damage based on AE characteristic parameters is

$$
\sigma_1 = \frac{E_\phi}{E_0}^2 \epsilon_1 \left(1 - \frac{N}{N_f}\right) + \mu(\sigma_2 + \sigma_3)
$$  \hspace{1cm} (21)

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ is the stress component in three directions of the specimen, $\epsilon_1$ is the strain in the direction $\sigma_1$ of the specimen, $E$ is the elastic modulus of the material, $\mu$ is Poisson’s ratio of material, $N$ is the AE accumulation when the strain reaches $\epsilon_1$, $N_f$ is the AE accumulation when the specimen is completely destroyed.

4.2 Verification of The Damage Constitutive Model

Based on the AE damage constitutive Eq. (21) and AE ringing count, the AE constitutive curve is plotted. As shown in Fig. 7.

When comparing the damage constitutive curves of different joint dip angles with the complete specimens in the Fig. 7, the evolution law is basically the same, which indicates that the joint dip angle does not affect the damage evolution law of rock mass. There is a certain difference between the theoretical curve and the experimental curve, and there is an obvious compaction stage in the initial loading stage of the experimental curve, but it is not obvious in the theoretical curve. This may be because the acoustic emission signal at this stage is difficult to detect, causing the theoretical damage value to be smaller than the actual damage value. Therefore, there is a great difference between the theoretical curve and the test curve in the compaction stage, which changes the subsequent trend of the theoretical curve, resulting in the low fitting degree of the two curves as a whole. In the line plastic and failure stages, the AE activity is obvious and easy to detect. So, the theoretical damage value is gradually getting close to the actual damage value, and the theoretical curve is in good agreement with the experimental curve. The stress peak value is also very close. The above analysis shows that the AE damage constitutive curve is basically consistent with the experimental curve, which indicates that the AE ringing count has good consistency with the rock damage failure. The damage constitutive model is appropriate and reasonable for the damage evolution of rocks.
(a) Intact specimen

(b) 0° joint

(c) 30° joint

(d) 45° joint

(e) 60° joint

(f) 90° joint
Catastrophe theory is a mathematical theory that studies the discontinuous change of a system state due to continuous change under the influence of external parameters (Huang et al. 2016). The process of rock deformation and failure is the process of the initiation, expansion and penetration of internal cracks. This process is essentially a discontinuous and abrupt phenomenon. Therefore, there is a theoretical basis for applying the cusp catastrophe model to rock mass failure.

The potential function of cusp catastrophe model is

\[ v(x) = x^4 + px^2 + qx \] (22)

where \( x \) is the state variable, \( p \) and \( q \) are the control variables.

When the system is in equilibrium, the corresponding equilibrium surface equation of the potential function is

\[ v(x) = 4x^3 + 2px + q = 0 \] (23)

The bifurcation set equation corresponding to the stable critical state in the equilibrium state of the system is

\[ v(x)'' = 6x^2 + p = 0 \] (24)

The simultaneous (22) and (23) eliminates \( x \) and obtains the bifurcation equation is

\[ F = 8p^3 + 27q^2 = 0 \] (25)

When the control variable satisfies the bifurcation equation, the system can be transferred from one equilibrium state to another. According to catastrophe theory, a system catastrophe occurs when \( F < 0 \). When \( F = 0 \), the system enters an unstable critical state. When \( F > 0 \), the system is stable and does not change abruptly. Equation (25) can be used to predict rock failure.

AE parameters (ring counting rate, energy counting rate, etc.) can be considered as a continuous variable function \( x(t) \) of the time variable \( t \). In the Taylor series expansion, the first four items can be intercepted to meet the accuracy requirements. As shown in Eq. (26).

\[ y = x(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 \] (26)

Let \( x = t - n \), convert Eq. (26) into the standard form (22) of the cusp catastrophe model, and obtain the expressions of \( n \), \( p \) and \( q \). The state of the system can be determined according to the decision rule in Eq. (25).

Rock Failure Prediction Based on Cusp Catastrophe Model

The ringing count of AE characteristic parameters is the oscillation times of the waveform signal that has crossed the threshold value, which can better reflect the frequency and intensity of AE events, and can also be used to identify and locate the type of wave source. Therefore, the AE ringing count rate is selected in this section to analyze the failure process of the specimens. Time series of AE ringing count rate is substituted into Eq. (27), where \( n \), \( p \) and \( q \) and other parameters are calculated, and then the value of discriminant \( F \) is calculated to predict specimen failure. The failure prediction of specimens is shown in Table 1.

Figure 8 shows the curve of stress and ringing count rate with time. When the rock stress reaches its peak strength, it breaks down and loses its bearing capacity. At the moment when \( F < 0 \), the cusp catastrophe model has just entered a instability state, it has been marked with a blue line. Figure 8(a) when the stress reaches 81.6% of the peak stress, the intact specimen is loaded to \( F < 0 \) in the 102th second, and then the specimen fails in the 125th second. Figure 8(c) and
(f) show that the value $F$ is less than 0 at the time the specimen reaches the peak stress. Thus, it can be seen that the prediction effect of Fig. 8(a) is the best, while the prediction effect of Fig. 8(c) and Fig. 8(f) is poor. In addition, in Fig. 8(b), (d), (E), when the stress reaches 86.8%, 95.5% and 86.7% of the peak stress respectively, $F < 0$. Therefore, failure prediction of jointed rock mass can be carried out by combining the parameters of peak stress and cusp catastrophe model.

Firstly, the AE signal of rock mass is collected and analyzed, and then the damage constitutive model is used to estimate the stress level of jointed rock mass and make mutation prediction. In all specimens, when the $F$ value is less than 0, the minimum stress of the specimen is 81.6% of the peak stress. Therefore, when the $F$ value of the cusp catastrophe model is less than 0 or the stress level reaches 80% of the peak stress, the imminent failure of the rock mass can be predicted.

### 6 Conclusions

(1) When the joint dip angle is $0^\circ$~$30^\circ$, the main mode of failure is tensile; when the joint dip angle is $60^\circ$~$90^\circ$, the main failure mode is shear; and $45^\circ$ is the critical angle value for both tension and shear failure modes. The specimen’s peak strength and elastic modulus change U-shaped with joint dip angle, reaching their minimum values at $45^\circ$. Peak strength and elastic modulus are reduced by 32.3% and 17.4%, respectively, compared to the intact specimen.

(2) The AE ringing counting rate can better reflect the damage evolution process of rock. During the plastic stage, a dramatic increase in the ring count rate of rock specimens with a joint dip angle $\geq 45^\circ$ can be used as precursory information of rock failure. However, no abrupt rise in the ringing counting rate is detected in specimens with a joint dip angle $< 45^\circ$, which may be owing to the increased plasticity of the rock mass caused by the increased joint dip angle.

(3) It is constructed a damage constitutive model with acoustic emission characteristic parameters that takes into account the structural effects of jointed rock masses and load coupling. In the plastic and failure stages, the model curves agree closely with the experimental curves. There is a difference between the model and experimental

| Joint length/mm | Joint dip angle/° | Time/s | n   | p   | q   | F               | State             |
|-----------------|------------------|--------|-----|-----|-----|-----------------|-------------------|
| 0               | 90               | 123    | 125 | 102 | 6908 | 251,100         | $-9.35E + 11$ Undestroyed state |
| 20              | 0                | 90     | 103 | 70.5| 7192.2 | 299,120.0      | $-5.60E + 11$ Undestroyed state |
| 30              | 94               | 95     | 96  | 43.2| 1733.0 | 10,207.0       | $-3.88E + 10$ Failure state    |
| 45              | 81               | 83     | 85  | 39.1| 1536.2 | 5813.0         | $-2.81E + 10$ Failure state    |
| 60              | 89               | 99     | 101 | 44.4| 1715.6 | 12,233.0       | $-3.64E + 10$ Failure state    |
| 90              | 111              | 112    | 113 | 54.0| 1706.3 | 27,983.0       | $-1.86E + 10$ Failure state    |
Fig. 8 Specimen stress and AE ringing count rate variation curve with time
curves in the compression and elastic stages, but this difference can reflect the mechanical behavior of jointed rock as a whole. This demonstrates that the AE signal is consistent with sandstone damage. The model is capable of accurately describing the process of damage evolution in joint sandstone subjected to uniaxial compression.

(4) By combining AE characteristic parameters with catastrophe theory, it is possible to forecast the failure of joint specimens, and the prediction results are relatively accurate. To improve the prediction results, the AE damage constitutive model is used to estimate the stress level of a joint rock mass. When the estimated stress value reaches 80% of the peak stress or the F value is less than 0, it can be determined that the common rock mass is about to be destroyed.

Acknowledgements This work was supported by the Natural Science Foundation of Hunan Province, China (Grant No.2021JJ30575), and National Natural Science Foundation of China (Grant No. 51204098)

Author’s Contribution The experiments were designed and performed by BS and SZ and YL. Data processing was performed by HY. The manuscript was prepared by HY. All authors reviewed the manuscript.

Data Availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Conflict of interest All authors declare that they have no conflict of interest.

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