A minimum dilution scenario for supernovae and consequences for extremely metal-poor stars

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Accepted 2020 August 24. Received 2020 August 13; in original form 2020 June 17

ABSTRACT
To date no metal-free stars have been identified by direct observations. The most common method of constraining their properties is searching the spectra of the most metal-poor stars for the chemical elements created in the first stars and their supernova (SN). In this approach, modelled SN yields are compared to the observed abundance patterns in extremely metal-poor stars. The method typically only uses the abundance ratios, i.e. the yields are diluted to the observed level. Following the usual assumption of spherical symmetry we compute a simple lower limit of the mass an SN can mix with and find that it is consistent with all published simulations of early chemical enrichment in the interstellar medium. For three different cases, we demonstrate that this dilution limit can change the conclusions from the abundance fitting. There is a large discrepancy between the dilution found in simulations of SN explosions in minihaloes and the dilution assumed in many abundance fits. Limiting the dilution can significantly alter the likelihood of which supernovae are possible progenitors of observed CEMP-no stars. In particular, some of the faint, very low yield SNe, which have been suggested as models for the abundance pattern of SMSS0313−6708, cannot explain the measured metal abundances, as their predicted metal yields are too small by two orders of magnitude. Altogether, the new dilution model presented here emphasizes the need to better understand the mixing and dilution behaviour of aspherical SNe.

Key words: stars: luminosity function, mass function – stars: Population II – stars: Population III – ISM: supernova remnants – dark ages, reionization, first stars – early Universe

1 INTRODUCTION
The first stars, so-called Population III (Pop III) stars form in the absence of heavy elements in the early Universe. Due to the lack of metal\(^1\) cooling, they are expected to be drastically different from the stars found in our vicinity at the present day (Bromm 2013; Glover 2013; Greif 2015). Initially, Pop III stars were thought to be very massive (e.g. Bromm, Coppi & Larson 1999; Omukai & Palla 2001), but later it was found that their protostellar discs may fragment, leading to the formation of clusters of low-mass metal-free stars (Clark et al. 2011; Greif et al. 2011). Whereas it is clear that Pop III stars may form over a wide range of masses, until today simulations are unable to constrain well the metal-free initial mass functions (IMFs). The results depend significantly on the physics employed, the choice of numerical method, and the resolution (see e.g. Hosokawa et al. 2016; Stacy, Bromm & Lee 2016; Susa 2019).

There are so far no direct detections of metal-free stars. Pop III stars are expected to form in high-redshift, relatively low-mass mini- and atomic-cooling haloes. Therefore, ‘Pop III galaxies’ are most likely not bright enough to be detected today (Xu et al. 2016; Hartwig 2015), which is one of the main reasons why these first stars have proven so difficult to observe. Most evidence on the properties of the first stars comes from the study of extremely metal-poor stars (EMP stars; e.g. Hosokawa et al. 2016; Stacy, Bromm & Lee 2016; Susa 2019). EMP stars are the most metal-poor stars that have been discovered so far, but there is still significant diversity in their properties.

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\(^2\) Throughout this study, we use the term ‘metals’ to refer to all elements heavier than helium.

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et al. 2016b; Visbal, Bryan & Haiman 2017). Having no direct observations, there are several indirect methods that allow us to gain observational constraints on the IMF of Pop III stars. Direct detection of supernovae (SNe) (Hummel et al. 2012; Hartwig, Bromm & Loeb 2018; Rydberg et al. 2020) or gravitational waves (Kinugawa et al. 2014, 2016; Hartwig et al. 2016a) from the first stars are challenging, but may provide constraints on the high-mass end of the Pop III IMF in the coming decade. The 21 cm absorption feature, as reported by the EDGES experiment (Bowman et al. 2018), can constrain the timing of the first star formation and the star formation efficiency, but it is not very sensitive to the assumed IMF (Schauer, Liu & Bromm 2019).

There are two remaining methods to constrain the pristine IMF that are feasible at present. Both are related to the observations of ancient metal-poor stars in the Milky Way and its satellites. The first one is constraining the low-mass end of the IMF with the current non-detection of metal-free stars (Salvadori, Schneider & Ferrara 2007; Hartwig et al. 2015; Ishiyama et al. 2016; Magg et al. 2018, 2019). The second method, which is our focus here, is comparing the abundance patterns observed in metal-poor stars to simulated SN yields in Pop III stars. It was found that the most metal-poor stars, often called extremely metal-poor (EMP) stars, can be made by one of these SNe, they form with a very small iron abundance. Notably, these SNe do not produce particularly large absolute amounts of carbon compared to the well-fitting ones are interpreted as likely progenitor. For example, the STARFIT pipeline can be used for such an analysis.

In this study, we argue that this standard approach has to be amended because the amount of ambient gas into which the metals from a Pop III supernova are mixed cannot be assumed to be arbitrarily small. We derive a simple analytical model for the lower limit of the mass an SN remnant has to mix with before it can collapse. We find that this limit is consistent with the results from 3D hydrodynamical simulations. In many cases, there are large differences between the halo-scale mixing found in hydrodynamical simulations and the mixing implicitly assumed by fitting abundance ratios with arbitrary dilution. We show how the dilution limit can be applied in abundance fitting methods. Finally, we investigate examples of the impact this dilution limit has on the conclusions drawn from fitting observed abundances with modelled SN yields.

2 THE MINIMUM MIXING MASS

2.1 Analytic estimate

As outlined before, abundance fitting usually employs the observed ratios of abundances of certain metals and compares these to the ratios found in theoretical SN models. Of particular importance is, e.g. the [C/Fe] ratio. This method, however, typically neglects the actual abundance value (i.e. [Fe/H] or [C/H]) and treats them as an arbitrary normalization factor. Conceptually, this normalization can be achieved by diluting the SN yields with the correct amount of metal-free gas. As in published works usually only single SNe are fitted to observed abundance patterns, we only consider single, isolated SNe in this work.

We consider SN explosions as well as their subsequent expansion into the ambient medium and the corresponding mixing processes in spherical symmetry. Simulations carried out in two (Tominaga 2009) and three (Chan et al. 2020) dimensions, however, show that Pop III SNe can be strongly aspherical. In this context, we note that even when considering anisotropic SNe, the observed abundances in most published studies are compared to angle-integrated yields. This means that the problem considered is effectively spherically

\[ X/H = \log_{10}(N_X/N_H) - \log_{10}(N_{X,\odot}/N_{H,\odot}) \]

where \( N_X \) and \( N_H \) are the fractional abundances of any element \( X \) and hydrogen, and \( N_{X,\odot} \) and \( N_{H,\odot} \) are the corresponding solar abundances.

2 The explosion energy and progenitor star mass are not necessary larger than those for Pop II SNe (Kobayashi et al. 2014).

\[ \text{http://starfit.org} \]
symmetric, as the angular average implies that different elements ejected in different directions become well mixed before the second generation stars form. An exception to this may be, if the abundances are distributed more spherically than the energy input, such as seen in some of the models in Tominaga (2009). A critical analysis of the validity of this approximation is one of the primary motives for the study presented here. We argue that properly accounting for the asymmetries expected in Pop III SNe requires both detailed three dimensional explosion models as well as high-resolution simulations of the expansion of the resulting anisotropic shock wave into an inhomogeneous ambient medium that are able to adequately follow the chemical mixing process.

Since there is no analytic model for such a small-scale inhomogeneous mixing, however, we follow the bulk of the existing literature and approximate the SN as a spherical explosion inside a homogeneous ambient medium. As SNe are very energetic events, a large amount of gas is required to confine the metals and thus not all dilution masses are physically plausible. The lowest limit for this mass is the mass enclosed in the final radius of the SN remnant. Analytical solutions to spherical blast waves of SNe can be derived under a variety of assumptions (e.g. Ostriker & McKee 1988), with the expansion of the remnant stalling at the end of the momentum-driven snowplough phase. In this phase, the expansion velocity reaches the speed of sound in the ambient medium. As shocks cannot be subsonic, the shock wave transforms into a sound wave and dissipates. This occurs at the fade-away radius \( R_{\text{fade}} \) which is

\[
R_{\text{fade}} \approx 2.07 \times 10^{20} \text{ cm} \left( \frac{E_{51}}{10^{51} \text{ erg}} \right) \left( \frac{c_s}{10 \text{ km s}^{-1}} \right)^{-2/5},
\]

where \( n_0 \) is the nucleon number density of the ambient medium in units of \( \text{cm}^{-3} \), \( E_{51} \) is the explosion energy in units of \( 10^{51} \text{ erg} \) and \( c_s \) is the ambient medium speed of sound (e.g. Draine 2011). We assume the ambient medium is ionized, i.e. that it has a speed of sound of \( c_s = 18 \text{ km s}^{-1} \), for a metal-free H II region (see e.g. Abel, Wise & Bryan 2007). In case the medium is actually neutral, the speed of sound would be lower and the stalling radius larger. Thus, this is a conservative assumption. In the homogeneous mixing case, the minimum mass with which the ejecta are mixed is the mass that is enclosed in the stalling radius, i.e.

\[
M_{\text{dil, min}} = \frac{4}{3} \pi n_0 \mu m_{\text{H}} R_{\text{fade}}^3 = 1.9 \times 10^4 M_{\odot} E_{51}^{0.96} n_0^{-0.11},
\]

where \( m_{\text{H}} \) is the mass of a hydrogen nucleus and where we assumed a mean molecular weight of \( \mu = 1.22 \). The fade-away radius used here is for gas cooling rates of solar metallicity gas. We discuss the assumption of solar metallicity and dependence of the dilution mass on the metallicity in Appendix A. As we aim at computing a lower limit for the mixing mass, the reduced cooling can be neglected. By definition the SN remnant expands faster than the speed of sound in the ionized medium. As we consider haloes below the atomic cooling limit the escape velocity from the haloes is much smaller than this speed of sound. Therefore, SN remnants expand much faster than the escape velocity and the effect of gravity can be neglected.

This result is very similar to the one obtained through numerical simulations by Thornton et al. (1998). While it has been widely used in the discussion of stellar feedback, it is often neglected when fitting abundance patterns of individual stars. For example, Tominaga, Iwamoto & Nomoto (2014) note that the minimum dilution mass obtained by Thornton et al. (1998) is not a binding limit, as metal mixing is highly inhomogeneous (Ritter et al. 2012). We will later see that our derived limit holds even in cases of inhomogeneous mixing.

We assume an ambient density of \( n_0 = 1 \text{ cm}^{-3} \), which should be the typical case for the ionized regions around massive Pop III stars (Whalen, Abel & Norman 2004). We note that the density dependence of the minimum mixing mass (equation 2) is very weak, so it would need to be higher by several orders of magnitude to affect our conclusions. If the density is this much higher than the assumed value, the free-fall time of the ambient gas is smaller than the lifetime of the star, and thus it should form stars already before the SN explodes or while the remnant expands. Furthermore, simulations show that it is difficult to mix metals into gas that is already very dense when the SN explodes (Ritter et al. 2016; Chiaki, Susa & Hirano 2018).

Under the assumptions outlined above, the dilution mass is a lower limit for two main reasons:

(i) We assume a homogeneous medium. If the medium is not homogeneous the denser gas will be less enriched but form stars first. This effect is discussed further below.

(ii) We assume no further mixing. Realistically further mixing with additional pristine gas should occur during recollapse, rather than the stalled SN remnant monolithically collapsing back on itself. This effect would further increase the dilution mass.

We note that we assume all SNe are able to produce second generation stars. Very energetic explosions may actually disrupt their host haloes, which suppresses or delays second generation star formation (Whalen et al. 2008). This effect is difficult to quantify without hydrodynamical simulations in cosmological context, and is therefore neglected here.

2.2 Consistency with simulations

To see whether sub-galactic-scale inhomogeneous mixing can lead to higher metallicities than predicted by the minimum dilution we will compare it to the dilution found in all suitable published simulations of inhomogeneous mixing and the formation of second generation stars which we are aware of. For comparison with our limit, we use an ambient density of \( n_0 = 1 \text{ cm}^{-3} \) in all cases but take the explosion energies used in the simulations to compute the minimum mixing. Simulations are included if they

(i) are three dimensional hydrodynamical simulations of the expansion of Pop III SN remnants into their ambient medium,

(ii) model individual, isolated, Pop III SNe, and not groups or clusters of stars,

(iii) follow the enriched gas until it re-collapses, and

(iv) provides the output needed for our comparison.

We cannot compare our model to the simulations of Greif et al. (2007) and Chen et al. (2015) because they simulate only the initial expansion of the enriched material, but not its re-collapse. Larger scale simulations, such as the ones from Wise et al. (2012), Johnson, Dalla & Khochfar (2013), or Tarumi, Yoshida & Inoue (2020a), are not considered, because they do not follow individual isolated SNe. The simulations of Whalen et al. (2008) are not included here because they are one-dimensional. Nevertheless, we note that their metal-enriched gas masses are consistent with our upper limit in most cases. Only in one of their models is the enriched gas mass they find smaller than our prediction in equation (2). In this case, the star completely fails to create an ionized region, and the ability to model an off-centre re-collapse would be crucial to make accurate predictions for the metallicity of the second generations star.
We begin with the dilution found in Ritter et al. (2012, 2015, 2016). In all three simulations, the SNe considered are core collapse (CC) SNe with $E_{51} = 1$. They eject $M_{\text{max}} = 4 M_\odot$ of metals in Ritter et al. (2012, 2015) and $M_{\text{max}} = 6 M_\odot$ in Ritter et al. (2016). Thus, according to equation (2) the maximum final metallicity we should expect is

$$Z_{\text{max}} = \frac{M_{\text{met}}}{M_{\text{dil, min}}} \approx 10^{-3.6} \approx 10^{-1.7} Z_\odot$$

where $Z_\odot = 0.0142$ is the solar metallicity (Asplund et al. 2009).

While the mixing is highly inhomogeneous, and orders of magnitude of spread in metallicity can be seen, the newly collapsing cores always show metallicities below this value. All simulations also contain gas at higher metallicities than predicted by the minimum dilution. While from Ritter et al. (2012) it is unclear in which phase this gas is contained, in Ritter et al. (2015, 2016) only some of the very diffuse gas has metallicites above the dilution limit.

Chiaki et al. (2018, 2020) and Chiaki & Wise (2019) have metallicities in several different haloes. The simulations cover a wide range of different environments in which SNe can explode. For massive stars, haloes are often completely photoevaporated, whereas, for the lowest mass stars that they investigate, with $M = 13 M_\odot$, the gas in the stellar birth-cloud remains dense throughout the lifetime of the star. The results show large variations between the mixing behaviour and the metallicities of the second-generation stars. Chiaki et al. (2018) distinguish between three separate enrichment channels:

(i) Internal enrichment: in this case, the SN expands efficiently and the metals mix well with the surrounding gas before the halo collapses back on itself.

(ii) External enrichment: the metals escape from the halo in which the SN explodes and mix with the gas in a different halo that has not formed stars yet. This type of enrichment is also found in Smith et al. (2015).

(iii) Inefficient internal enrichment: dense structures remain in the halo. When the SN explodes these structures are only enriched to very low metalicities and proceed to form stars with metallicities much lower than the average gas metallicity in the halo.

None of these simulations, however, show the formation of second generation stars that violate our dilution limit. According to equation (2) the predicted maximum metallicity ranges between $10^{-2.6} Z_\odot < Z_{\text{max}} < 10^{-1.6} Z_\odot$. All second generation stars from Chiaki et al. (2018) and Chiaki & Wise (2019) have metallicities in the range $10^{-6.3} Z_\odot < Z < 10^{-2.2} Z_\odot$. The second-generation stars in Chiaki et al. (2020) have much lower metallicities, even reaching $Z < 10^{-9.2} Z_\odot$. The re-collapsing region has a metallicity of 40 per cent of our computed upper limit. In these simulations, there are several cases of stars with much lower metallicities than predicted by the minimum dilution model. These are the cases in which the surroundings of the SNe are the most dense and the mixing seen in the simulations is very inhomogeneous. The second-generation stars form in clumps that already exist when the SN ejects and only the outer layers of these clumps are enriched with metals. Thus, the enrichment proceeds in what Chiaki et al. (2018) label the ‘inefficient internal enrichment’ channel.

Greif et al. (2010) simulate the explosion of a single PISN with $E_{51} = 10$ and $100 M_\odot$ of metal ejecta. According to our model the maximum metallicity in this extreme case should be below $Z = 10^{-1.8} Z_\odot$. They find metallicities in the recollapsing galaxy that are around $Z = 10^{-3} Z_\odot$. As Greif et al. (2010) note, the average metallicities are initially much higher but they decrease to this low value during the recollapse of the halo, which takes around 300 Myr.

The simulations by Jeon et al. (2014) include several SNe exploding in three different haloes. The authors provide information on the metallicity of recollapsing regions in three cases: a 15, 25 and $40 M_\odot$ star exploding in their ‘halo1’. They all explode as $E_{51} = 1$ CCSN and eject 5 per cent of their stellar mass as metals. According to our model, this should lead to metallicities of $Z < 10^{-2.1} Z_\odot$. Their reported metallicities are all below $Z = 10^{-3.5} Z_\odot$.

Smith et al. (2015) highlight the external enrichment channel. Their SN is an $E_{51} = 1$ CCSN which ejects $11.19 M_\odot$ of metals, leading us to predict a maximum metallicity of $Z_{\text{max}} = 10^{-1.4} Z_\odot$. Only a very small fraction of gas is found at such high metallicities, and none of it is in the re-collapsing region. The metal-enriched star-forming gas in this case has a metallicity of $Z = 10^{-4.7} Z_\odot$.

We convert the metallicities found in the simulations back to an ‘effective dilution mass’ with equation (3) and summarize the simulations in Fig. 1. None of the simulations of inhomogeneous mixing show inconsistencies with the minimum dilution mass derived from the spherically symmetric case. In some of the simulations, there is gas above the derived upper limit for the metallicity, but it tends to

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5We only consider the $I_{1\text{SN}}$ model from Ritter et al. (2015) as the $T_{1\text{SN}}$ model deals with enrichment by multiple SNe, which is not the topic of our analysis.

6For comparison with simulations we generally use the central metallicities of the metal-enriched gravitationally unstable regions, either as reported by the authors of the respective studies or as read from their figures.
be diffuse and hot. This can be understood intuitively: as thermal energy and metals are ejected together, more metal-rich gas tends to be hotter. It is important to note that there is significant scatter in the simulation results: even for similar exploding stars the effective dilution mass can vary by many orders of magnitude. The cases with the largest effective dilution masses are usually external or inefficient internal enrichment. We conclude that, to the best of our knowledge and the current state of modelling, our estimate provides a useful limit on the mixing and dilution of metals even in the presence of inhomogeneous mixing.

2.3 Bayesian fitting

We will here briefly discuss how the derived limit on mixing can be implemented in abundance fitting codes. For this purpose, we create an algorithm that fits observed abundances by comparing to them to the modelled SN yields from Heger & Woosley (2010). The yields of the SNe generally depend on the progenitor mass (\(M_{\text{prog}}\)), the explosion energy (\(E_{\text{51}}\) in units of \(10^{51}\) erg) as well as a mixing factor (\(f_{\text{mix}}\)). This mixing factor is the mass scale over which abundances in the SN yields are averaged before part of the SN ejecta fall back, expressed as fraction of the mass of the He core of the star. For matching observed and modelled abundances, we use the SN yields and analysis tools provided with STARFIT and supplement them with a Gaussian error function. The theoretical models are combined by multiplication:

\[
L(x|M) = \prod_i L_i(x_i|M). 
\]

The same approach to compute fit likelihoods was also used in, e.g. Fraser et al. (2017). In cases where there are only detections and no upper limits, maximizing this likelihood is equivalent to minimizing \(\chi^2\). This way of combining likelihoods implicitly assumes that the errors of all abundance determinations are uncorrelated. Especially for errors from uncertainties in the determination of stellar parameters, this may not be true (McWilliam et al. 1995). This is because all low-excitation lines arising from neutral minority species tend to have similar sensitivity to the effective temperature, which typically dominates the error budget. However, we only aim at showing the importance of constraining the dilution of SN ejecta, and a complete treatment of the error distributions and dependencies of abundance determinations exceeds the scope of the current investigation.

If we assign each model in the SN library the same prior probability, we can further compute the probability of each model \(M\) given the observations \(x\) by

\[
P(M|x) = N \times L(x|M),
\]

where \(N\) is a normalization constant chosen such that

\[
\sum_M P(M|x) = 1.
\]

3 APPLICATION TO OBSERVATIONS

In this section, we demonstrate in three cases why it is important to consider the dilution when fitting abundances of metal-poor stars. First, we will show that it can help to break degeneracies in a fit; secondly, that it may systematically change properties of large fitted samples of stars; and thirdly, that for some stars there may not be a viable single-progenitor scenario to explain the observed abundance patterns.

3.1 Example 1: the progenitor of HE 0020–1741

To investigate the impact of the minimum dilution mass on abundance fitting, we first fit the CEMP-no star HE 0020–1741 ([Fe/H] = −3.6). Hansen et al. (2019) have determined abundances for 13 elements (C, N, O, Mg, Ca, Sc, Ti, Cr, Mn, Ni, Fe, Sr, Ba). We chose this star as it is an extremely metal-poor CEMP-no star and therefore thought to carry the chemical signature of a Pop III SN and being unaffected by binary interactions. As the yields from Heger & Woosley (2010) do not include r- and s-process elements, Sr and Ba are excluded from the fits.
The low-mass progenitors do not produce enough metals to explain the constrained dilution case, only the high-mass progenitors still fit. However, if we consider only the high-mass progenitors. If only the abundance ratios are considered, both give an equally plausible fit, yet constraining the dilution rules out a single low-mass star as a progenitor. Sc is shown in grey to indicate that it is treated as an upper limit in the fits in order to account for the fact that theoretical models are known to underpredict Sc (Section 2.3). The fits shown are therefore not expected to match the observed value for Sc.

We show the prior and the posterior distribution of stellar masses in Fig. 2. The prior is bottom heavy, as there are many more models of low-mass SNe in the libraries than there are models of high-mass SNe. This could potentially bias fitting results towards lower masses. We perform the fits with unconstrained and with constrained dilution factors. In the former case, we chose the dilution factors to maximize the combined likelihoods as defined in equation (7). In the latter case, we only allow dilution factors above our analytical limit. Notably there is no apparent correlation between the corresponding dilution mass for each fitted star in their sample. In Fig. 4, we show that this re-fitting leads to significant changes in the mean (median) $\chi^2$ from 16 (13) in the unconstrained case to 24 (15) in the constrained case. For many stars, the best fit with the dilution constraint becomes worse than that without it. In Fig. 5, we show that this re-fitting leads to significant changes in the ratio of the minimum dilution mass and the dilution mass derived in Ishigaki et al. (2018) as function of the reduced $\chi^2$, as well as histograms of both values. We show the original fits from Ishigaki et al. (2018) (orange) as well as re-fits in which the minimum dilution limit is enforced (green). In some of the original fits, the dilution ratio is very low (down to $\sim 10^{-2}$) and therefore outside of the boundaries of this figure. These stars are included in the lowest bin of the histogram.

Thus, if HE 0020–1741 is to be explained with a single progenitor SN, it should be a massive star with $70 M_\odot < M_* < 80 M_\odot$ for the Heger & Woosley (2010) yields. Note, however, that it may be possible to find additional or better fits with different yield sets (e.g. Limongi & Chieffi 2012; Grimmett et al. 2018; Ishigaki et al. 2018). However, as the aim here is to show the usefulness of the dilution limit to constrain fits, a comparison of these different yield set exceeds the scope of this study.

### 3.2 Example 2: large sample fitting

Ishigaki et al. (2018) fitted the abundances of 201 EMP stars by picking the best-fitting SN model for each of these stars. The compiled sample of stars has been selected to consist only of stars with determined abundances for the elements C, N, O, Na, Mg, Al, Si, Ca, Sc, Ti, Cr, Mn, Fe, Co, Ni, and Zn based on spectroscopic data with a resolution of at least $R = 28,000$. These observed abundances were compared to SN models which were computed over a grid of stellar masses, explosion energies as well as three parameters that quantify the properties of the mixing-and-fallback process. Details on the sample selection and the SN modelling can be found in Ishigaki et al. (2018). For each star, they selected a best-fitting SN model through a $\chi^2$ minimization, where the predicted abundance yields of each model in the grid was compared to the observed abundances. Using the explosion energy of these best-fitting models, we compute the corresponding dilution mass for each fitted star in their sample. In Fig. 4, we show the ratio between these minimum dilution masses and the dilution masses from the fits for all 201 stars. Of these 201 best-fitting models, 128 violate our derived limit and 43 do so by more than a factor of four. Notably there is no apparent correlation between $\chi^2$ and the dilution mass. Thus, whether the dilution factor found by fitting is consistent with our limit is unrelated to the goodness of fit.

We have replicated the fitting procedure from Ishigaki et al. (2018), and added a criterion based on the dilution mass. In this approach, we reject all fits in which the dilution is inconsistent with the minimum dilution mass described in equation (2). This leads to a significant increase in the mean (median) $\chi^2$ from 16 (13) in the unconstrained case to 24 (15) in the constrained case. For many stars, the best fit with the dilution constraint becomes worse than that without it. In Fig. 5, we show that this re-fitting leads to significant changes in...
the distribution of best-fitting progenitor masses. The most notable difference is that progenitors with a stellar mass of 25 and 40 M\(_{\odot}\) are now much rarer and progenitors with 15 M\(_{\odot}\) more common. The reason for this is that many of the previously common 25 and 40 M\(_{\odot}\) models were hypernovae (HNe) with a high explosion energy and a large fallback fraction. These stars have relatively low absolute yields, but due to their large explosion energies, we predict large dilution masses in spherical symmetry. Therefore, such models are not able to reproduce the relatively large carbon abundances of many CEMP-no stars, when taking the dilution constraint into account.

We note that the prescription of faint SNe used in Ishigaki et al. (2018) is chosen to reproduce the angle-averaged yields of aspherical jet SNe (Tomimaga 2009). Our dilution model, however, does not apply to such SNe if their asphericity is preserved. In the used prescription only the total yields are considered. Even if the abundance distribution in the ejecta is strongly aspherical, this approximation assumes that the SN yields are mixed and the angular variations are washed out during later phases of the expansion of the SN. In principle, the mixing behaviour in aspherical SNe can be very different from our approximation if the metal yield per unit energy shows strong angular variations. Additionally, aspherical SNe from Ishigaki et al. (2018) have systematically larger (and in some cases much larger) explosion energies, which are used in equation (2), than their 2D counterparts with similar yields (Tomimaga 2009). This further limits the applicability of our model to these SNe. Our results here suggest that developing realistic models for the dilution of heavy elements produced in aspherical SNe is of vital importance for fitting large samples of stars, not just individual cases.

3.3 Example 3: no spherical progenitor for SMSS0313–6708

As we realized previously that stars with high carbon and low iron abundances are particularly strongly affected by applying the dilution criterion, we will look in more detail at a pathological example of such a star, i.e. SMSS0313–6708 (Keller et al. 2014). The star is known for having no detected iron abundance with an upper limit of [Fe/H] \(< -7.1\). We here use abundances that are based on 3D atmospheric models that do not assume local thermodynamical equilibrium (3D, NLTE) for Na, Mg, AI, Ca, and Fe from Nordlander et al. (2017). For these elements statistical and systematic errors are provided which we add with a quadratic sum. The systematic errors are typically on a level of 0.1 dex. The remaining abundances are taken from Bessell et al. (2015) and are based on 3D LTE models for C, N, and O and on 1D LTE models for Si, Sc, Ti, V, Cr, Mn, Co, Ni, and Cu. Notably, most of these elements are not detected and only upper limits on their abundance have been derived. Bessell et al. (2015) only give statistical but no systematical errors for their abundance determinations. Because we want to avoid biasing our results towards abundances with an unaccounted source of error, we add a systematical error of 0.1 dex to the abundance determinations from Bessell et al. (2015).

We fit the abundances with the same procedure as described in Section 3.1. The best constrained and unconstrained models are shown in Fig. 6. The upper limits shown here are at a 84 per cent confidence level. This implies that a value 1\(\sigma\) above the upper limit corresponds to a 98 per cent significant discrepancy. Even with unconstrained dilution, we find no model that produces a convincing fit of the abundance patterns. The best-fitting model overproduces Na, C and Si. There are three features in the abundances that are difficult to fit simultaneously:

(i) the CNO pattern with high C and O but very low N,
(ii) the low upper limit on Na with the detection of a large amount of Mg, and
(iii) the detection of Ca in conjunction with the low upper limits on Al and Si.

The difficulty involved in reproducing all three of these features may partially be related to the grid of models not containing a sufficiently large variety of SN explosion energies. We note that none of the elemental abundances that have been derived in 1D LTE play a critical role in constraining the models. All 1D LTE abundances are only upper limits that lie well above the best-fitting models. It is still unclear how different the C and O abundances would be in a 3D NLTE analysis, but they would need to differ by approximately 1 dex from 3D LTE in order for us to be able to find SNe with matching abundances. Nordlander et al. (2017) were able to fit the abundance patterns by interpolating the abundance patterns as function of the explosion energy. However, the result of such a
procedure is potentially sensitive to the way the interpolation is done. We therefore decided against interpolating to a finer grid here.

Both the unconstrained best-fitting model and the best-fitting model from Keller et al. (2014) violate the dilution limit by around two orders of magnitude. They would require the SN ejecta to be diluted with less than 500 $M_\odot$ of pristine material. The best-fitting model that fulfills the dilution limit is clearly inconsistent with the observed upper limits of N and Na. In Ishigaki et al. (2014) this star was best witted with 25 and 40 $M_\odot$ SN or HNe (jet-induced, aspherical, and energetic SN), where Ca is produce by static/explosive O burning and incomplete Si burning in contrast to the explanation in Keller et al. (2014). Of these models, the SNe are consistent with our dilution limit and the HNe are neither compatible with the dilution criterion nor with the updated upper limit on Si that we use. The fits presented in Ishigaki et al. (2018) are compatible both with the abundance pattern we use and with our dilution limit. Chen et al. (2017) model potential progenitor SNe for SMSS0313−6708 in one and two dimensions. Assuming our dilution limit equally applies to 2D SNe, we find that only the two dimensional model of the SN of a 60 $M_\odot$ Pop III star is consistent with our dilution limit. As these elements are not modelled, however, it is unclear whether this model is able to reproduce the observed upper limits of the Na and Al abundances. Chan et al. (2020) performed full 3D SNe simulations of the progenitor of SMSS0313−6708 using a 40 $M_\odot$ star, as suggested by Bessell et al. (2015), with asymmetric explosion of low and high energy. Their nucleosynthesis has the same constraints as those by Chen et al. (2017), and the explosion was not followed beyond shock breakout. The low-energy model does not produce any significant metals, the high-energy model too much iron – if spherically averaged.

4 DISCUSSION AND SUMMARY

It is common practice to fit observed stellar abundance by comparing them to modelled SN yields that have been diluted with a freely chosen dilution mass. We argue that this factor cannot be chosen freely, however, as SNe generally inject large amounts of energy into their surroundings and dilution must be large enough to contain the SN remnant until it recollapses. Therefore, we have introduced an analytical limit for the dilution of metals produced by a single SN with the following three assumptions:

(i) the SN being alone and isolated,
(ii) the explosions being spherical and well mixed, and
(iii) the surrounding medium being homogeneous.

The first two assumptions are commonly made when comparing observed abundance patterns to SN yields in previous works, because if these are not fulfilled the total elemental yields from a single SN cannot be representative stellar abundance pattern. For the last assumption we compared this limit to all hydrodynamical simulations of metal enrichment in high-redshift minihaloes which we are aware of and which included the needed details and resolution for a comparison. We found that, despite assuming homogeneity, the limit is consistent with all of these simulations.

We demonstrate that previous fits were often inconsistent with our understanding of metal dilution and mixing on the scale of minihaloes. Including our dilution criterion into fitting procedures for abundance patterns can have important consequences for the conclusions drawn:

(i) Considering the dilution can help to break degeneracies in progenitor models of individual stars.
(ii) The limit does not just affect individual stars but it can also change the properties of large samples of progenitor models. In particular, low-yield SNe are disfavoured if constraints on the dilution are taken into account.
(iii) It may be difficult to explain certain stars, such as SMSS0313−6708 by enrichment from a single, spherical SN if the dilution is taken into account. The best-fitting models that have been put forward by Keller et al. (2014) and Bessell et al. (2015) explain the rough shape of the observed abundance ratios, but with an implicit dilution mass that is too small by approximately two orders of magnitude, the yield from the SNe are too small to explain the absolute metal abundances. Ishigaki et al. (2014, 2018) find fits to the abundance pattern that are consistent with our dilution criterion.

During the preparation of this manuscript, Komiy et al. (2020) derived a similar estimate for the minimal dilution and implemented it into a semi-analytical model of the formation of the Milky Way. While we apply this estimate to the exploration of progenitor scenarios of individual stars, Komiy et al. (2020) focus on the chemical evolution of the Milky Way and in particular on whether the overall population of CEMP-no stars can be reproduced. They find it difficult to reproduce the prevalence of large carbon abundances in the lowest metallicity stars with faint SNe. This tension between the mixing-and-fallback SN model and the large observed carbon abundances is consistent with our findings.

A recent study by Ji et al. (2020) adds a dilution limit into their fitting procedure in a very similar way as we do here. This is done in the context of the observation of metal-poor stars in the ultra-faint dwarf galaxies Carina II and III. They find that applying the dilution limit moves their fit systematically to higher explosion energies and stellar masses. Ji et al. (2020) further point out that the dilution mass can be a helpful criterion in deciding whether the metals originate from the galaxy itself or from external enrichment.

The minimum dilution estimate can serve for evaluating whether a single, spherical SN is a viable progenitor scenario for a certain star. This test may be less reliable, if applicable at all, for asymmetric SNe or cases with several SNe in one halo. In asymmetric SNe, a large fraction of the metals can be ejected along jets (Tominaga 2009). Evidence for such SNe has recently been found by Ezzeddine et al. (2019). The dilution and recollapse occurring after such SNe are yet to be explored by numerical simulations.

Altogether, we conclude that for the adequate astrophysical interpretation of the observed elemental abundances in EMP stars, both the relative abundance patterns as well as the absolute abundance values need to be taken into account. Only then reliable and well founded constraints on the properties of the preceding generation of stars can be derived. Given the fact that simple spherically symmetric models often fail to match the dilutions mass constraint introduced here, we furthermore conclude that the effects of aspherical SNe, the impact of inhomogeneous mixing in a highly structured interstellar medium, and the combined yields of multiple SNe requires further investigation.

ACKNOWLEDGEMENTS

We thank the anonymous referee for useful comments and insights. The authors would like to thank Nozomu Tomiaga for very productive discussions and comments. We also thank Alex Ji and Gen Chiaki for their helpful remarks. In preparation of this manuscript, the software packages F2PY (Peterson 2009), NUMPY (Oliphant 2006), MATPLOTLIB (Hunter 2007), and SCIPY (Virtanen et al. 2020) were used.
MM was supported by the Max-Planck-Gesellschaft via the fellowship of the International Max Planck Research School for Astronomy and Cosmic Physics at the University of Heidelberg (IMPRS-HD). SCOG and RS acknowledge funding from the Deutsche Forschungsgemeinschaft (DFG) – Project-ID 138713538 – SFB 881 (‘The Milky Way System’, sub-projects A1, B1, B2 and B8). Further financial support was provided by the DFG via the Heidelberg Cluster of Excellence STRUCTURES in the framework of Germany’s Excellence Strategy (grant EXC-2181/1-390900948). AH was supported in part by the National Science Foundation under grant no. PHY-1430152 (JINA Center for the Evolution of the Elements), the Australian Research Council Centre of Excellence for Gravitational Wave Detection (OzGrav), through project number CE170100004, the Australian Research Council Centre of Excellence for All Sky Astrophysics in 3 Dimensions (ASTRO 3D) through project number CE170100013, and by a grant from Science and Technology Commission of Shanghai Municipality (grant no. 16DZZ2260200) and National Natural Science Foundation of China (grant no. 11655002). CK acknowledges funding from the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan, and by the Japan Society for the Promotion of Science (JSPS) KAKENHI grant numbers JP17K05382 and JP20K04024.

DATA AVAILABILITY

No new data were generated in support of this research. The SN models from Heger & Woosley (2010) are available on http://2sn.org. For the availability SN models or the sample of stars used in Section 3.2, please inquire with the authors of Ishigaki et al. (2018).

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APPENDIX A: METALLICITY DEPENDENCE OF THE MINIMUM DILUTION MASS

In our derivation of the minimum dilution mass we used the common description for the expansion of SN remnants in the solar metallicity case. In nature, however, the situation is more complicated. The metallicity within the SN remnant is expected to be considerably lower than solar metallicity. The exact value will depend on the yields of the SN and will evolve as the remnant expands. Lower metallicities mean less efficient cooling and therefore a longer Sedov–Taylor phase. Consequently, the final mass enclosed within the stalling radius will be larger at sub-solar metallicity than at solar metallicity. Adopting the solar metallicity result therefore gives us a conservative lower limit on the minimum mixing mass. Nevertheless, it is useful to estimate how large the effect of this assumption is by comparing the solar metallicity result with the analogous result in the limit of zero metallicity.

To estimate the time at which the SN remnant leaves the Sedov–Taylor phase, we follow the procedure outlined in Draine (2011, chapter 39.1.2). We calculate the radiative cooling rate of the remnant by integrating the cooling function $\Lambda$ over the volume of the remnant:

$$\frac{dE}{dr} = -\int_0^{R_s} 4\pi r^2 \Lambda_0 dr.$$  \hfill (A1)

For metal-free gas, we can approximate the cooling rate as

$$\Lambda_0 = C n_H^2 T_6^{-1.2}$$  \hfill (A2)

where $T_6$ is the temperature in units of $10^6$ K, $n_H$ is the hydrogen number density and $C \approx 4.5 \times 10^{-24}$ ergs$^{-1}$ cm$^{-3}$. This simple power-law fit is a good approximation of the zero-metallicity cooling rate in the temperature range $0.1 < T_6 < 1.0$ and underestimates it at higher temperatures (e.g. fig. 34.2 of Draine 2011). The energy loss rate of the remnant therefore becomes

$$\frac{dE}{dr} = -1.2 \times 4\pi \frac{R_s}{3} C n_H^2 T_6^{-1.2} \left( \frac{\rho}{\rho_0} \right)^2 \left( \frac{T}{T_6} \right)^{-1.2},$$  \hfill (A3)

where $R_s$ is the radius of the SN remnant and $\rho$ is the mass density. Variables subscripted with s refer to values just inside the blast wave and variables subscripted with 0 refer to quantities in the ambient medium. The $\langle \cdots \rangle$ brackets denote a volume-weighted average over the blastwave. While the energy loss is small, we can use the density and temperature profiles from the Sedov–Taylor solution for a constant density ambient medium, and $\langle (\rho/\rho_0)^2 (T/T_6)^{-1.2} \rangle = 1.674$. The total energy loss can be computed by integrating the energy loss rate:

$$\Delta E = -1.2 \times \frac{4\pi}{3} \times 1.674 n_H^2 C \int R_s^3 T_6^{-1.2} \, dr.$$  \hfill (A4)

The values of the blast-wave radius and the temperature just behind the shock front are given as a function of time by the Sedov–Taylor solution:

$$R_s = 1.54 \times 10^{19} \text{ cm} \, E_5^{1/5} n_H^{-1/5} T_5^{2/5},$$  \hfill (A5)

$$T_5 = 52.5 E_5^{1/5} n_H^{-2/5} T_0^{-6/5},$$  \hfill (A6)

where $T_5$ is the time since the explosion in units of 1000 yr. Therefore we can simplify equation (A4) to

$$\Delta E = -3.76 \times 10^{43} \text{ erg s}^{-1} E_5^{12/5} n_H^{-10/5} \int_0^{t_5} t_5^{2.64} \, dt_5$$

$$=-1.03 \times 10^{43} \text{ erg s}^{-1} E_5^{12/5} n_H^{-10/5} t_5^{3.64}.$$  \hfill (A7)

Finally, if we follow Draine (2011) and assume that the remnant leaves the Sedov–Taylor phase when $\Delta E/E = -1/3$, we can compute the time $t_{rad}$ at which the Sedov–Taylor phase ends.

$$t_{rad} = 115 \times 10^3 \text{ yr} \, E_5^{24/25} n_H^{-1/5}.$$  \hfill (A8)

For comparison, the calculation given by Draine (2011) for the solar metallicity case yields

$$t_{rad} = 49 \times 10^3 \text{ yr} \, E_5^{-0.22} n_H^{-0.55}.$$  \hfill (A9)

Therefore, despite the much smaller volumetric cooling rate in the zero metallicity case, the cooling time of the remnant only changes by a factor of about 2, thanks to the strong time dependence of the total energy loss rate of the remnant. We can also consider the dependence of the dilution mass on the time at which the Sedov–Taylor phase ends. The fade-away radius is

$$R_{fade} = R_{rad} \left( \frac{t_{fade}}{t_{rad}} \right)^{2/7},$$  \hfill (A10)

where

$$t_{fade} = \left( \frac{2 R_{rad}}{7 t_{rad} c_s} \right)^{7/5} t_{rad},$$  \hfill (A11)

and where $c_s$ is the ambient medium speed of sound. The radius at the end of the Sedov–Taylor phase can be obtained as

$$R_{rad} = R(t_{rad}) \propto t_{rad}^{2/5}.$$  \hfill (A12)

This leads to

$$R_{fade} = R_{rad} \left( \frac{t_{fade}}{t_{rad}} \right)^{2/7} \propto R_{rad}^{2/5} t_{rad}^{-2/5} t_{rad}^{4/25}.$$  \hfill (A13)

Since $M_{dil} \propto R_{fade}^3$, the dilution mass is related to the duration of the Sedov–Taylor phase via

$$M_{dil} \propto t_{fade}^{-12/25}.$$  \hfill (A14)

Therefore the reduction of the cooling rate in zero metallicity gas would increase the minimum dilution mass by around 50 per cent. All simulations presented in Section 2.2 are consistent with this limit too. As the metallicity of the SN remnant depends on the SN yields, we use the more conservative solar metallicity value for the dilution mass in this study.