Abstract

The group theoretic method is extended to include fields with a background charge. This formalism is used to compute the tree level scattering for $W_3$ strings. The scattering amplitudes involve Ising model correlation functions. A detailed study of the four tachyon amplitude shows that the $W_3$ string must possess additional states in its spectrum associated with intercept $1/2$ and the energy operator of the Ising model.
In this letter we first explain how to use the group theoretic approach to string theory [1] to calculate scattering for a scalar field with a background charge. Such a field \( \phi \) possesses the energy momentum tensor \( T = -\frac{1}{2}(\partial \phi)^2 - Q \partial^2 \phi \), which under a conformal transformation \( z \to f(z) \) induces the transformation

\[
\phi(z) \to \phi(f(z)) + Q \ln \frac{\partial f}{\partial z}.
\] (1)

The central object in the group theoretic approach is the \( N \) string vertex, \( V^N \), which corresponds to a Riemann surface with \( N \) marked points (Koba-Nielsen coordinates), \( z_i \). The vertex is specified by relations called unintegrated overlap equations, which fall into two types; one states how conformal operators of a given weight acting on different legs of the vertex are related, while the other relates the transformation around the non-trivial homology cycles of such operators when acting on the vertex. To specify the overlap relations we must specify a coordinate system \( \xi_i, i = 1, \ldots, N \) in the neighbourhood of each marked point, \( z_i, i = 1, \ldots, N \) of the Riemann surface which is constrained to have its origin at its respective point. The vertex \( V \) is such, that for any conformal operator \( R(z) \) of conformal weight \( d \), it obeys the relation

\[
VR^i(\xi^i)(d\xi^i)^d = VR^i(\xi^j)(d\xi^j)^d.
\] (2)

For a scalar field with a background charge, we take its corresponding vertex to satisfy the unintegrated overlap equation

\[
V\phi^i(\xi^i) = V\{\phi^i(\xi^i) + Q \ln \frac{d\xi^j}{d\xi^i}\}.
\] (3)

Differentiating with respect to \( \xi^i \) we find that \( V \) satisfies the relation

\[
VP^i(\xi) = V\{P^i(\xi^j)\frac{d\xi^j}{d\xi^i} + iQ \frac{\partial}{\partial \xi^i} \ln \frac{d\xi^j}{d\xi^i}\}
\] (4)

for the operator \( P(z) = i\frac{d\phi}{dz} \).

The vertex is completely determined up to a constant by these unintegrated equations; however, it is often quicker to derive the integrated overlap equations and derive most of the vertex from these. These equations are found by considering the contour integral of \( V\Phi^i(\xi^i) \) around the point \( z^i \) and then deforming the contour. The function \( \Phi \) is such that it should be analytic except that it may have poles only at the points \( z_i \). As the
contour approaches another Koba-Nielson point \( z_j \), we must change to the coordinates \( \xi^j \) associated with this point. The inhomogeneous term in the transformation of \( \phi \) of equation (1) does not lead to any singularities and so contribute at this new point; however, it does provide a contribution as we change coordinates to include the point at infinity, whose local coordinates we denote by \( u \). This latter step is necessary even if we have only one leg, since we require two coordinate patches to cover the Riemann surface. The net result of these manoeuvres is the equation

\[
\sum_{j=1}^{N} V \oint d\xi^j P^j(\xi^j) f = \oint_0 du V^2 iQ f_u. \quad (5)
\]

Taking \( \Phi \) to be a constant we find the well known momentum conservation condition in the presence of a background charge, namely

\[
\sum_{j=1}^{N} \alpha_0^j = 2iQ. \quad (6)
\]

Taking \( f \Phi = 1/(\xi^j)^n \) for \( n > 0 \) we find no contribution at infinity and as a consequence the \( \alpha_n^i \alpha_m^j \) and \( \alpha_n^i \alpha_0^j \) terms in the vertex are the same as the case for when the background charge is zero. To determine the \( \alpha_0^i \alpha_0^j \) terms we must examine the unintegrated overlap of equation (3). Following the discussion of section (3) of reference [2], we find that for tree scattering the vertex is of the form

\[
V = \{ \prod_{i=1}^{N} \langle 0 \rangle \} \exp \left\{ - \sum_{1 \leq i < j \leq N} \left\{ \sum_{n,m=1}^{\infty} \frac{\alpha_n^{(i)} \alpha_m^{(j)}}{\sqrt{n}} \frac{(n+m-1)!(-1)^m}{(m-1)!(n-1)!(z_j - z_i)^{n+m+1}} \frac{\alpha_0^{(j)}}{\sqrt{m}} \right. \right.
\]

\[
\left. + \sum_{n=1}^{\infty} \left\{ \frac{\alpha_n^{(i)} \alpha_0^{(j)}}{\sqrt{n}} \frac{\alpha_0^{(j)}}{\sqrt{n}} \frac{(z_j - z_i)^n}{(z_i - z_j)^n} \right\} + \alpha_0^{(i)} \alpha_0^{(j)} m(z_j - z_i) \right\} \delta(\sum \alpha_{\mu} \alpha_0^\mu + 2iQ) \quad (7)
\]

for the simplest possible relation between the coordinate patches, namely \( \xi^i = \xi^j + z^j - z^i \). This result is in agreement with reference [3]. In fact for this cycling transformation the additional term in the overlap of equation (3) vanishes for transitions between the patches for the marked points. However, one can verify for any \( sl(2,R) \) transition functions that the vertex, when written in a form in which it contains in the exponential no terms bilinear in the same species of oscillator, has a \( Q \)-dependence only in the momentum conservation \( \delta \)-function. The vertex for any transition functions can then be easily read
off from past papers. To verify this statement we must redo the calculation of reference [2] to find the \( Q \)-dependence we must include the term \( 2iQ \) whenever we use momentum conservation and take into account the additional terms in the overlap equation. In fact momentum conservation is used twice; to reprocess the second term of equation (3.9) and when cancelling the last two terms of equation (3.10). This leads to an additional term \((-i)(-2iQ)\ln(c^{ij} - a^{ij} - \xi^i)\) in the notation of that paper, except that we take \( z = \xi^i \) and include a factor of \(-i\) to gain a more usual definition of \( \phi \). This term is precisely cancelled by the additional \(-Q \ln d\xi^j/d\xi^i\) term in the overlap.

Loops are also easily incorporated, but now the transformations around the non-trivial homology cycles reflect the presence of the additional term in the transformation of \( \phi \). For example, for the Schottkey representation, we would implement the equation

\[
P^i(\xi^i) = V \left( P^i_n(\xi^i) \frac{d(P^i_n(\xi^i))}{d\xi^i} + iQ \frac{\partial}{\partial \xi^i} \ln \frac{d(P^i_n(\xi^i))}{d\xi^i} \right)
\]

for the transformation \( P_n \) between the two circles associated with the \( n \)'th loop.

The changes of the moduli dependence follow the usual path, but now we must take into account equation (1). We illustrate it for the simplest case, namely changing the position of a Koba-Nielsen coordinate. Under \( z \to z + a \), which is induced by \( e^{aL_{-1}} \), equation (1) takes the form \( \phi(z) \to \phi(z + a) \). Acting with \( e^{aL_{-1}} \) on equation (3), we find an integrated overlap for the vertex \( V e^{aL_{-1}} \), which we readily interpret as being the old vertex, but with \( z^i \to z^i + a \). Infinitesemally, we have the usual result

\[
\frac{\partial V}{\partial z^i} = VL_{-1}.
\]

The energy momentum tensor \( T(z) \) includes the background charge, but obeys the same conformal transformation law as the more usual case, and as such it will obey the usual overlap [1]

\[
V \sum_{j=1}^{N} \oint d\xi^j T(\xi^j)\phi = 0.
\]

We may have to, as usual, add constants [1], although this is not the case for tree level vertices.

In the group theoretic method the scattering amplitude is given by integrating the vertex over the moduli space weighted with a function of the moduli which is determined by
demanding that zero norm physical states decouple. Let us illustrate the procedure for a D scalar string,\(\mu = 0, 1, \ldots, D - 1\), which has a background charge, \(\alpha^\mu\), adjusted so that it is critical, i.e. \(\alpha \cdot \alpha = (26 - D)/12\). The physical states of such a string satisfy \((L_n - \delta_{n,0})|\psi\rangle = 0, n \geq 0\), and it has a level 1 null state, i.e. \(L_{-1}|\psi\rangle\) where \(L_n|\Omega\rangle = 0, n \geq 0\).

The N string scattering amplitude is given by

\[
\int \prod_{i=1}^{N} \prod d^i z^i V(z^i)f
\]

Demanding that the above null states decouple implies that \(\frac{\partial f}{\partial z_i} = 0\) or \(f\) is a constant.

The formalism developed in this paper can also be applied to \(W_3\) string scattering. A \(W_3\) string was constructed [6,7] by utilizing the Miura transformation and obeying the consistency conditions arising from the BRST charge[8]. This can be extended, by generalizing the Miura transformation, to have a \(W_3\) string for any number of scalar fields [7]. It has been noticed as a phenomenological observation that the central charges arising in \(W_N\) strings are related to those of the minimal models[6,7]. It has been found [4] by explicitly solving the physical state conditions at low levels, that the \(W_3\) string theory has one of the oscillators suppressed and that physical, positive definite norm, states are contained in a subspace of the original oscillator space that obeys \(L_n|\psi\rangle = 0, n \geq 1\) and \((L_0 - a_{eff})|\psi\rangle = 0\) for \(a_{eff} = 1\) or \(15/16\). In these equations the \(L_n\)’s are the Virasoro operators for \(D + 1\) scalar fields (the original \(W_3\) string having \(D + 2\) scalar fields) with a background charge tuned to give a central charge of \(51/2\). The lowest level null states in the \(a_{eff} = 1\) sector is \(L_{-1}|\psi\rangle\), where \(L_n|\psi\rangle = 0\) for \(n \geq 0\), and so scattering in this sector is given by \(\int \prod d z_i V^N f\). By applying our previous argument we find that \(\partial_i f = 0\) for \(i = 1, \ldots, N\).

The lowest null state in the \(a_{eff} = 15/16\) sector, however, is given by \((L_{-2} + 4/3L_{-1}^2)|\Omega'\rangle\), where \(L_n|\Omega'\rangle = 0\) for \(n \geq 1\) and \(L_0|\Omega'\rangle = -17/16|\Omega'\rangle\). The scattering is given by \(\int \prod d z_i V^N f\); applying the above null state on leg \(j\) and physical states \(|\chi\rangle_i\) on the other legs we find that physical null states decouple if

\[
\int \prod d z_i f V^N (L_{-2} + 4/3L_{-1}^2) \prod |\chi\rangle_i |\Omega'\rangle_j = 0.
\]

Using equation (9) with \(f = 1/\xi^i\) we can replace \(L_{-2}\) by \(L_n, n \geq -1\), on the other legs.
Carrying out this, using equation (7) and the physical state conditions we find that
\[
\int \prod_i dz_i f \left\{ \frac{4}{3} \frac{\partial^2 V}{\partial z_j^2} + \frac{\partial V}{\partial z_i} \frac{1}{(z_j - z_i)} - \frac{17}{16} \frac{L_i}{(z_j - z_i)^2} \right\} \prod |\chi_i \rangle \langle \Omega'_j| = 0. \tag{13}
\]
Integrating by parts we conclude that the measure \( f \) satisfies
\[
\left\{ \frac{4}{3} \frac{\partial^2 f}{\partial z_j^2} - \sum \left( \frac{\partial f}{\partial z_j} \frac{1}{(z_j - z_i)} + \frac{1}{16} \frac{f}{(z_j - z_i)^2} \right) \right\} = 0. \tag{14}
\]
This equation for \( f \), however, is none other than the equation obeyed by the correlation function for \( N \) primary fields of conformal weight 1/16 of the Ising model. In fact the measure in the \( a_{eff} = 1 \) sector can also be given such an interpretation since the correlation function for \( N \) primary fields of weight \( h = 0 \) is independent of \( z_i \). The correlation function for four weight 1/16 Ising states was found in reference [5]. Thus we find an interesting connection between the \( W_3 \) string scattering and the Ising model correlation functions.

We could also consider the scattering of strings from the two sectors; let \( i = 1, \ldots, M \) be from the \( a_{eff} = 1 \) sector and \( i = M + 1, \ldots, M + N \) be from the \( a_{eff} = 15/16 \) sector. The vertex satisfies the overlaps of equations (3) and (4) for any \( i, j \in [1, \ldots, M + N] \) as well as the integrated overlap of equation (5). As a result the vertex is of the same form as equation (5) but with the sums over \( i = 1, \ldots, M + N \). It is straightforward to extend the above arguments for the decoupling of null states to show that the corresponding measure \( f \) obeys the equation for the Ising correlation function for \( N \) fields of weight 1/16 and \( M \) fields of weight 0. We could rewrite the vertex so as to absorb the measure by including the Ising operators \( \Xi = \{1, \sigma\} \), where 1 is the identity operator and \( \sigma \) the weight 1/16 field, in the scattering vertex:
\[
V' = V \prod_{i=1}^N \Xi'(z_i), \tag{15}
\]
where \( \Xi' \) is valued according to whether the external state has \( a_{eff} = 1 \) or 15/16 respectively. With such a vertex the measure can be taken to be one. One could also work in the more standard, but less powerful, conformal field theory approach to string theory. The corresponding vertex operators can be extracted from the three-vertex by putting on one leg the appropriate external state and identifying the oscillators of the remaining two legs. The tachyon vertex operators are \( e^{ip \cdot \phi}, p^2 = 2 \) and \( \sigma e^{ip \cdot \phi}, p^2 = 15/16 \) in the two sectors respectively.

It will prove instructive to work out in detail the scattering of four tachyons in the \( a_{eff} = 1/2 \) sector. Putting four such states on the vertex of equation (5) we find, after
choosing $z_1 = \infty$, $z_2 = 1$, $z_3 = x$ and $z_4 = 0$ the result

$$F(s, t) = \int_0^1 dx x^{-\alpha' s - 15/8} (1 - x)^{-\alpha' t - 15/8} f(x) z_1^{1/8}. \quad (16)$$

The function $f(x)$ is the four point $\sigma$ Ising correlation function which is of the form [5]

$$f(x) = [(z_1 - z_3)(z_2 - z_4)]^{-1/8} Y(x), \quad (17)$$

where

$$x = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_3)(z_2 - z_4)} \quad (18)$$

and

$$Y(x) = \frac{1}{[x(1 - x)]^{1/8}} (a \cos \theta + b \sin \theta), \quad (19)$$

with $x = \sin^2 \theta$. The two constants $a$ and $b$ correspond to the existence of two solutions to the differential equation (13). Taking the above choice of $z$’s we find that

$$F(s, t) = \int_0^1 dx x^{-\alpha' s - 2} (1 - x)^{-\alpha' t - 2} (a \cos \theta + b \sin \theta), \quad (20)$$

which is finite as $z_1 \to \infty$. In the above we have introduced the slope $\alpha'$, which is usually taken to be $1/2$ for the open string.

The values of the constants $a$ and $b$ are to be chosen by a physical requirement, which in our case is crossing. The open string amplitude $T^{(4)}(p_i)$ is as usual the sum of three terms

$$T^{(4)}(p_i) = F(s, t) + F(t, u) + F(u, s). \quad (21)$$

Crossing for four identical particles means that $T^{(4)}(p_i)$ should be symmetric under the exchange of any two legs or equivalently momenta. This means for example that it should be symmetric under $s \leftrightarrow t$. This follows provided that $F$ itself is a symmetric function of its arguments. This property is in turn guaranteed if the integrand is symmetric under $x \leftrightarrow 1 - x$, while at the same time we interchange $s$ and $t$, or $p_2$ and $p_4$. The transformation $x \to 1 - x$ can be written as $\theta \to \pi/2 - \theta$, whereupon it is obvious that we should take

$$F(s, t) = \int_0^1 dx x^{-\alpha' s - 2} (1 - x)^{-\alpha' t - 2} \cos(\theta/2 - \pi/8)$$

$$= \frac{1}{\sqrt{2}} \int_0^1 dx x^{-\alpha' s - 2} (1 - x)^{-\alpha' t - 2} \{\cos \pi/8 \sqrt{1 + \sqrt{1 + x}} + \sin \pi/8 \sqrt{1 - \sqrt{1 - x}}\}. \quad (22)$$
Having found the expression for four tachyon scattering we can examine the particles exchanged in a given channel. Let us consider the s-channel. Expanding the factor \((1 - x)^{-\alpha't^2}\) we find that

\[
F(s,t) = \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \cos \frac{\pi}{8}\sqrt{2}a_p \left(\alpha't + 2\right)\left(\alpha't + 3\right)\ldots, \alpha't + n + 1 \right)
\]

\[
+ \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \sin \frac{\pi}{8}\sqrt{2}b_p \left(\alpha't + 2\right)\left(\alpha't + 3\right)\ldots, \alpha't + n + 1 \right)
\],
\]

where

\[
\sqrt{1 + \sqrt{1 + x}} = \sqrt{2} \sum_{p=0}^{\infty} a_px^p
\]

and

\[
\sqrt{1 - \sqrt{1 - x}} = \sqrt{\frac{x}{2}} \sum_{p=0}^{\infty} b_px^p.
\]

The states exchanged in the first and second terms have masses which satisfy \(\alpha'm^2 = p + n - 1\) and \(\alpha'm^2 = p + n - 1/2\) respectively. While the former states are contained in the \(a_{\text{eff}} = 1\) sector the latter are in neither the \(a_{\text{eff}} = 1\) nor the \(a_{\text{eff}} = 15/16\) sector. Consequently we find that the \(W_3\) string requires the existence of additional states in its spectrum. These states correspond to an intercept of 1/2, and so it is very natural, given the above pattern, to associate them with the weight \(1 - 1/2 = 1/2\) operator \(\epsilon\) of the Ising model. As such, all the operators of the Ising model appear in the \(W_3\) string! the tachyon in the new sector should correspond to the vertex operator \(\epsilon(z)e^{ik\phi}\), with \(k^2 = 1\). The existence of these new intermediate states is to be expected in view of the fusion rule \(\sigma\sigma = 1 + \epsilon\).

We now repeat the above discussion for the closed \(W_3\) string. The closed string overlaps are the same as those of equation (3), except that now \(\phi\) involves the left and right oscillators. Tracing through the argument following this equation we conclude that the closed string vertex is the product of two open string vertices, one containing left oscillators and the other right oscillators, with a zero mode term in common with appropriate normalization. The decoupling of null states implies that the measure \(f\) satisfies the appropriate Ising model equation, and so for four tachyons in the \(a_{\text{eff}} = 15/16\) sectors for left and right it is of the form

\[
f(z_i, \bar{z}_i) = \left[(z_1 - z_3)(z_2 - z_4)(\bar{z}_1 - \bar{z}_3)(\bar{z}_2 - \bar{z}_4)\right]^{-1/8}\left[x\bar{x}(1 - x)(1 - \bar{x})\right]^{-1/8}
\]

\[
\{u_{11} \cos \theta/2 \cos \bar{\theta}/2 + u_{12} \cos \theta/2 \sin \bar{\theta}/2 + u_{21} \sin \theta/2 \cos \bar{\theta}/2 + u_{22} \sin \theta/2 \sin \bar{\theta}/2\},
\]
where \( \bar{x} \) is the cross-ratio of equation (16) but with \( \bar{z}'s \) instead of \( z's \), the \( u_{ij}'s \) are constants and \( x = \sin^2 \theta, \bar{x} = \sin^2 \bar{\theta} \).

The amplitude for four tachyon closed string scattering is

\[
T^{(4)}(p_i) = \int d^2 z_3 |z_1 - z_2|^2 |z_1 - z_4|^2 |z_2 - z_4|^2 \prod_{i<j} |z_i - z_j|^{2\alpha'k_i k_j} f(z_i, \bar{z}_i). \tag{25}
\]

Taking \( z_1 = \infty, z_2 = 1, z_3 = x \) and \( z_4 = 0 \), this amplitude is finite and becomes

\[
T^{(4)}(p_i) = \int d^2 x |x|^{-\alpha's-4}|1 - x|^{-\alpha't-4}\{u_{11}\cos \theta/2 \cos \bar{\theta}/2 + \ldots u_{22}\sin \theta/2 \sin \bar{\theta}/2\}. \tag{26}
\]

The constants are again determined by requiring crossing. However, for the closed string we only have one term and so in addition to the symmetry arising from exchanging particles 2 and 4 which induces \( s \leftrightarrow t, x \leftrightarrow 1 - x, \bar{x} \leftrightarrow 1 - \bar{x} \) we have that arising from exchanging particles 4 and 1 which induces \( s \leftrightarrow u, x \leftrightarrow 1/x \) and \( \bar{x} \leftrightarrow 1/\bar{x} \). There is only one choice of the \( u_{ij}'s \), namely \( u_{11} = u_{22} \) and \( u_{12} = u_{21} = 0 \), and so the final amplitude is

\[
T^{(4)}(p_i) = \int d^2 x |x|^{-\alpha's-4}|1 - x|^{-\alpha't-4}\cos \left( \frac{\theta - \bar{\theta}}{2} \right). \tag{27}
\]

The last part of this expression is none other than the usual Ising result. It is clear that if we factorize this amplitude we will find the closed string analogue of the above new states.

It would be interesting to confirm the above scattering results by working in the full Fock space including all the oscillators in the \( W_3 \) string and not just those that occur in the states in the \( a_{eff} = 1 \) and \( 15/16 \) sectors. The starting point in this calculation is the vertex \( \hat{V} \) which obeys the overlaps of equation (1.3) for the \( D+2 \) string coordinates. This vertex is then given by equation (1.7) with the appropriate background charges encoded in the \( \delta \)-function, and it obeys in addition to the T overlap of equation (1.10) an overlap for \( W \) of the form

\[
\hat{V} \sum_j \int d\xi^j W^j(\xi^j)\Phi = 0, \tag{28}
\]

where \( \Phi \) is now a second rank tensor. There are, on the sphere, 5 such analytic tensors which we may take to be \( (\xi^j + z^j)^n \) for \( n = 0, 1, \ldots, 4 \). To find the scattering we must take account of the \( W \) moduli. This would be best achieved by incorporating a geometric
understanding of the $W$ algebra analogous to the use of superspace for the super Virasoro algebra. However, one could also proceed using the method of the third paper of reference [1], which regarded the vertex as an induced representation. Its isotropy group is generated by the overlap identities for the generators, and two vertices which differ by transformations that annihilate on-shell states are considered equivalent. Hence for an $N$-string tree level vertex we have for the Virasoro part of the algebra one modulus (i.e. $L_{-1}$) associated with each string, subject to 3 identities corresponding to 3 analytic vector fields, giving in all $N - 3$ moduli. For the $W$ part of the algebra we have 2 moduli (i.e. $W_{-2}$ and $W_{-1}$) associated with each string, subject to the 5 identities corresponding to the 5 analytic second-rank tensors. This gives in all $2N - 5$ $W$ moduli.

For a genus $g$ surface ($g \geq 2$) with $N$ strings we find, by the same argument, $N + 3(g - 1)$ Virasoro moduli and $2N + 5(g - 1)$ $W$ moduli.

We can regard the vertex $\hat{V}$ as being evaluated at zero moduli and we can introduce by a boost the $W$ moduli by the action of $W_{-n}$’s on the vertex. While this is straightforward for an infinitessimal $W$ moduli change, i.e. for tree level scattering

$$V(z_j)(1 + \sum'(w_{2j}W_{-2}^j + w_{1j}W_{-1}^j)) = V(z_j, w_{1j}, w_{2j}),$$

(29)

where the prime on the sums indicates the absence of 5 terms, a difficulty can occur for finite $W$ moduli as one must know how to move from the $W$ algebra to the $W$ group. It would be most interesting to confirm the presence of the Ising correlation functions from this viewpoint.

In this paper we have shown that the “group theoretic method” naturally encompasses the possibility of fields with a background charge. This formalism was then used to compute the scattering in $W_3$ string theory. The scattering amplitudes were found to involve Ising model correlation functions containing the identity and spin operator. Examining the intermediate states in the four tachyon amplitude showed that the $W_3$ string must possess new states in its spectrum associated with a 1/2 intercept and the energy operator of the Ising model. This connection between the Ising model and the $W_3$ string will extend to higher minimal models and $W_N$ strings. It would be interesting to find the fundamental reason why the Ising model plays such a crucial role in the $W_3$ string. A more complete treatment of the $W_3$ string from a path integral viewpoint may be instructive in this respect.
Note added

While this work was being written up it was found that the required additional states in the $W_3$ string were indeed contained in the cohomology of $Q$, but in non-standard ghost-number sectors $[9]$.

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