The refractive scattering of $^{17}$F+$^{12}$C

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Abstract. The experimental data of the elastic scattering angular distribution of $^{17}$F+$^{12}$C at 170 MeV is analyzed by the continuum-discretized coupled channels (CDCC) method and the optical model (OM). In the CDCC calculation, the unambiguous optical potential of $^{16}$O+$^{12}$C is used as the input to give the coupling potentials. A very refractive feature is found and two evident Airy minima are predicted at large angles. The one-channel calculation is also performed and gives nearly the same result. In the OM calculations, this optical potential of $^{16}$O+$^{12}$C is used again and adjusted to reproduce the angular distribution of $^{17}$F+$^{12}$C. The Airy oscillation appears again in the calculated angular distribution. These results indicate that the elastic scattering of $^{17}$F+$^{12}$C at 170 MeV has the possibility of the nuclear rainbow phenomenon, which is probably due to the contribution from the $^{16}$O core.

1 Introduction

The atmospheric rainbow is one of the most spectacular phenomena in nature. Airy has proven that it is caused by the self-interference of the light wavefront during the refraction and reflection processes [1]. Similar phenomenon has been also observed in the nuclear scattering of light heavy ions at tens MeV per nucleon and is called the nuclear rainbow [2]. The nuclear rainbow manifests itself as a huge hump (the primary Airy maximum) followed by a structureless exponential falloff in the elastic scattering angular distribution. From the semiclassical viewpoint, it is caused by the interference of the farside scattering trajectories at the same scattering angle. It can happen when the absorption in the colliding process is weak enough (in other words, the system is transparent enough). The first observation of the nuclear rainbow is in the elastic scattering measurements of $\alpha$ particle on different targets at intermediate energies [3–5]. And successively, the nuclear rainbow has been also observed in the elastic scattering of heavier tightly bound nuclei such as $^{12}$C and $^{16}$O [6–8]. The experimental data at the nuclear rainbow region is very precious in constraining the optical potential (OP) between the colliding pair at the small distances (i.e. eliminating the discrete ambiguities) and extracting the information about the internal nuclear structure [2].

For the elastic scattering of exotic and/or loosely bound nuclei, due to the extra absorption caused by the stronger breakup channels, it is not easy to judge whether the rainbow features exist or not [9]. Nevertheless, the existent experimental and theoretical results show that the elastic scattering in this case can be very refractive (such as $^6$Li and $^9$Be) [2]. For exotic nuclei (such as halo and skin nuclei), if the nuclear rainbow pattern can be observed in the elastic scattering, the internal interaction and structure information could be probed. But a firm conclusion about its existence has not been established up to now due to the lack of the experimental differential cross sections at large angles.

It deserves attentions that these loosely bound nuclei are usually assumed to be constructed by clusters (i.e. a tightly bound core plus one or several valence nucleons) due to the small separation energies. For example, it is believed that $^4$He=$\alpha+n+n$, $^6$Li=$\alpha+d$, $^7$Li=$\alpha+t$ and so on. Although the experiments of the refractive elastic scattering of exotic nuclei are not sufficiently performed, there are usually abundant experimental data accumulated for the cores and valences. Therefore, it is possible to investigate the nuclear rainbow patterns of the elastic scattering angular distributions of exotic nuclei based on the cluster description. The cluster folding model (CFM) can be used to obtain the OP between the loosely bound projectile and the target at the ground states. It folds the OPs of core+target and valence+target with the square of the relative wave function between the clusters. This model originates from the calculations of the deuteron OP, where the deuteron is treated as a neutron plus a proton [10]. And then it is applied to calculate the OP for more complex nuclei such as $^6$Li [11]. Due to the low breakup threshold of loosely bound nuclei, the continuum-discretized coupled channels (CDCC) method is developed to consider the breakup coupling effects on the elastic scattering [12, 13]. In the CDCC framework, the CFM potential extends to the coupling potentials between any initial and final states (including bound and unbound states).

The exotic nucleus $^{17}$F is on the proton drip line with a very small valence proton separation energy (0.600 MeV). Hence it can be treated as an $^{16}$O core plus a valence proton. Besides, its first excited state (1/2^+ at the excit-
tion energy of 0.495 MeV has been proved to be a halo state [14], which arises widespread attentions. Blackmon et al. has measured the elastic scattering of $^{17}$F+$^{12}$C at 170 MeV [15]. In their OM calculations, they find that the nominal absorption completely damps the Airy patterns. However, only the diffractive part of the elastic scattering angular distribution is measured in the experiment, which does not eliminate the discrete ambiguities of the OP. The elastic scattering differential cross sections at larger angles is hard to be determined by only fitting the experimental data of the diffractive part.

In the present work, we have analyzed the elastic scattering data of $^{17}$F+$^{12}$C at 170 MeV within the CDCC and OM frameworks, respectively. In the CDCC calculation, the unambiguous OP of $^{16}$O+$^{12}$C extracted from its elastic scattering data at 170 MeV given by Ref. [8] is and used. The calculated angular distribution exhibits a very refractive feature. Two Airy minima are predicted at large angles, which is quite different from the previous calculated results [15]. Meanwhile, the one-channel (1-ch) calculation without including the first excited state and the continuum of $^{17}$F is also performed, which gives the similar refractive pattern. In the OM calculations, an OP between $^{17}$F and $^{12}$C at 170 MeV is obtained by adjusting that of $^{16}$O+$^{12}$C. The Airy oscillation is observed again. The CDCC and the OM calculations are performed with the code FRESCO [16].

2 Methods

The present CDCC calculation includes both coupling effects from the breakup channels and the inelastic scattering to the first excited state of $^{17}$F. It is assumed that the nucleus $^{17}$F is constructed by an $^{16}$O core plus a valence proton. At its ground state ($5/2^+$), the valence proton is at the the 1d$_{5/2}$ sub-shell. And the binding potential obtained from Ref. [17] is adopted to describe the relative motion between two clusters. For the continuum states of $^{17}$F, the relative energy between $^{16}$O and $p$ is truncated at 24 MeV and discretized into 12 bins equally in the momentum space. The quantum number of the relative orbital angular momenta included in the calculation is up to 5.

To give the coupling potentials of $^{17}$F+$^{12}$C, the OPs of $^{16}$O+$^{12}$C and $p+$^{12}C (i.e. $V_{16O}$+$^{12}$C and $V_{p}$+$^{12}$C) are folded:

$$V_{ij}(r) = \int \psi_i(r') (V_{16O}$+$^{12}$C + $V_{p}$+$^{12}$C)$\psi_j(r') dr,$$  

(1)

where $R$ and $r$ are the relative coordinates of $^{17}$F+$^{12}$C and $^{16}$O+$p$, respectively. $\psi_i(r)$ is the relative wave function between the clusters of $^{17}$F at the (bound or unbound) state denoted as $i$. In the present calculations, the OP of $^{16}$O+$^{12}$C is chosen carefully. Because if it is not chosen well, the discrete ambiguity might be introduced to the interaction between $^{17}$F and $^{12}$C. The elastic scattering angular distribution data of $^{16}$O+$^{12}$C at 170 MeV shows an evident nuclear rainbow feature with two Airy minima at $\theta_{c.m.} \approx 57^\circ$ and $80^\circ - 120^\circ$, respectively [8]. From this dataset, the authors of Ref. [8] derive an OP denoted as “WS1” which belongs to the same potential family of the double folding potential. Therefore the OP “WS1” is away from the discrete ambiguities and is used in the present calculation. While for the interaction between the valence and the target, a global proton OP is used [18]. Besides, to investigate the coupling effects from the inelastic scattering and breakup channels, the 1-ch calculation is also performed with the CFM potential between $^{17}$F and $^{12}$C. The CFM potential is calculated with Eq. (1) by setting $i$=$j$=$0$, which means that the initial and final states of $^{17}$F are both at the ground state.

3 Results and discussions

The CDCC calculation result is plotted as the solid curve in Fig. 1. It can be seen that the experimental data is reproduced very well. And the diffractive oscillation structure is successfully described. What is more amazing is that two Airy minima appear at about 70° and 100°, respectively. The latter is the first Airy minimum (A1) and the former is the second one (A2). This calculated Airy oscillation is a significant feature of the nuclear rainbow. Due to the very low breakup threshold of $^{17}$F, it seems that the nuclear rainbow phenomenon is not easy to occur during the elastic scattering. However, it should be noted that this angular distribution pattern is very similar to that of the elastic scattering of its core $^{16}$O on the $^{12}$C target at 170 MeV [8]. This indicates that the rainbow feature of $^{17}$F+$^{12}$C is probably due to contribution from its strongly bound core $^{16}$O. This idea is an extension of that in Ref. [2] where the authors believe that the nuclear rainbow of $^6$Li scattering is due to the strong contribution from the $^4$He core.
The 1-ch calculated result is plotted as the dashed curve in Fig. 1. One can see that experimental data is still described well. But the differential cross sections are enhanced at large angles, which reveals the loss of flux due to the considered coupling channels in the CDCC calculations. On the other hand, the positions of A1 and A2 are nearly unchanged, which may be interpreted as due to the weak breakup coupling [19, 20]. This implies that the 1-ch calculation with the CFM potential could be a good tool for the investigation of the nuclear rainbow phenomenon in the elastic scattering of exotic nuclei. At least for the $^{17}$F scattering, the 1-ch approach can provide a similar result to that of CDCC with much less computing time.

In the CDCC and 1-ch calculations, the proton OP adopted is the global version. The accuracy of using this proton global OP should be verified. Therefore, we fit the elastic scattering data of $p+$^{15}C at 9.95 MeV extracted from Ref. [21] with the global OP. The result is shown in Fig. 2. One can see that the global OP failed to describe the experimental data in a large angular range, while the fitted one improves the agreement between the theory and experiment much. However, when adopting the fitted proton OP in the 1-ch calculation of $^{17}$F+$^{15}$C, we find that the difference between the 1-ch calculations with global or fitted proton OP is very small (Fig. 1). This reveals that the contribution of the proton OP to the interaction of $^{17}$F+$^{15}$C is very small, which may support the opinion that the $^{16}$O core plays a major role for the formation of the nuclear rainbow.

Now let us talk about the OM calculations of $^{17}$F+$^{12}$C. The original OP of $^{16}$O+$^{12}$C is in the Woods-Saxon form with the following parameters. For the real part, the depth $V_0=255.5$ MeV, the reduced radius $r_V=0.629$ fm and the diffuseness $a_V=0.967$ fm. For the imaginary part, the depth $W_0=16.27$ MeV, the reduced radius $r_W=1.245$ fm and the diffuseness $a_W=0.514$ fm. The Coulomb reduced radius $r_C$ is set as 1.25 fm. These reduced radii are defined as $r_i=R_i/(16i^{1/3}+12^{1/3})$, where $i=V, W$ or C. In the fitting process, only $r_V$ and $a_V$ are adjusted and the other parameters are fixed. The obtained values of $r_V$ and $a_V$ are 0.610 fm and 0.985 fm, respectively. The OM calculation result with the new OP is plotted in Fig. 3. One can find that two Airy minima appear again and their positions are nearly the same as those calculated by the CDCC method. The farside/nearside decomposition is performed following the approach proposed by Fuller [22]. To realize the decomposition, the numerical method given in Ref. [23] is adopted. It can be seen that the angular distribution at large angles is dominated by the farside scattering component and the minima are truly the Airy minima. The OM calculations with the OPs given in Ref. [15] are performed as well and compared with the present one (Fig. 3). Instead of diffractive oscillation, an A1 minimum and an exponential falloff structure are predicted at large angles with both OPs. And one can find that the positions of these A1 minima are very close to that calculated with the present OP.

![Figure 2](image2.png)

**Figure 2.** The elastic scattering angular distribution of proton on $^{12}$C target at 9.95 MeV. The circles denote the experimental data extracted from Ref. [21]. The curves are the OM calculation with different OPs.

![Figure 3](image3.png)

**Figure 3.** The elastic scattering angular distribution of $^{17}$F+$^{12}$C at 170 MeV. The solid squares denote the experimental data extracted from Ref. [15]. The curves represent the result of the OM calculations. The present OM calculated angular distribution is decomposed into the farside and nearside component. The OM calculations with the OPs from Ref. [15] are also plotted for comparison.

### 4 Conclusions

In the present work, we have analyzed the elastic scattering angular distribution data of $^{17}$F+$^{12}$C at 170 MeV. Through the CDCC, the 1-ch and the OM calculations, we find that the elastic scattering of $^{17}$F+$^{12}$C can be very refractive and exhibit an evident rainbow feature with two Airy minima in its angular distribution. These predicted Airy oscillatory structures are very similar to that of the elastic scattering of its core $^{16}$O observed experimentally. This implies that the strongly bound core could have a strong contribution on the formation of the nuclear rainbow structure. We also find that the 1-ch approach with the CFM potential can be a very effective tool to investigate the refractive scattering of the exotic nuclei. The investigation of the energy dependence of the evolution of the rainbow pattern of the $^{17}$F+$^{12}$C elastic scattering is in progress.

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