Formation of Cosmic String network from black holes: Implications from liquid crystal experiments

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We present observation of large, expanding string loops forming around a heated wire tip embedded in a nematic liquid crystal sample. Loops expand due to convective stretching. This observation leads to a new insight into phenomena which could occur in the early universe. We show that local heating of plasma in the early universe by evaporating primordial black holes can lead to formation of large, expanding cosmic string loops, just as observed in the liquid crystal experiment. Intercommutation of string loops from neighboring black holes can lead to percolation, thereby forming an infinite string network. This is remarkable as such an infinite string network is thought to arise only when the entire universe undergoes phase transition.

Condensed matter physics experiments can provide very fruitful analogues of phenomena thought to have occurred in the early universe. An important example of this is provided by the realization that the theories of formation of exotic topological defects, such as cosmic strings in the early universe [1], can be tested in condensed matter systems [2]. Many experimental studies have been carried out in liquid crystals [3], superfluid helium [4], and superconductors [5], by looking for analogies of cosmic phenomenon, as well as by focusing on universal properties for rigorous quantitative tests of cosmic defect theories [6, 7], in these systems. Here, we demonstrate the tremendous potential of this correspondence by showing that a completely new insight into the phenomena in the early universe is obtained from a liquid crystal experiment, namely, the formation of large string loops due to convective stretching in a liquid crystal sample with a heated wire tip. We show that the same physics implies that local heating of plasma in the early universe by evaporating primordial black holes [8] can lead to formation of large cosmic string loops which can percolate to form an infinite string network (even when ambient temperature is far below the cosmic string scale). This is remarkable as such an infinite string network is thought to arise only when the entire universe undergoes phase transition.

Formation of cosmic strings is of great importance as they can provide a direct possible window to the physics of ultra-high energy scales. Recent realization that superstring theories can also lead to existence of cosmic strings [9] has further added to the motivation of understanding various mechanisms of formation of cosmic strings. In the conventional picture, strings are produced when the universe goes through a symmetry breaking phase transition [1]. This may not always be possible, for example, in models of inflation which have low reheat temperature, or in models of TeV scale gravity [10]. Our results are therefore important as they provide a new possibility where an infinite string network can form without an over all phase transition and when the ambient temperature is much lower than the cosmic string scale.

Our experimental setup consists of a droplet (size ∼ 2-3 mm) of uniaxial nematic liquid crystal (NLC) 4′-Penty1-4-biphenyl-carbonitrile (98% pure, purchased from Aldrich Chem.) placed on a clean, untreated glass slide. For uniaxial nematic liquid crystals the order parameter is called the director which is a vector with opposite directions identified. This arises due to rod like nature of the molecules in this case (or, if the molecules are disk like, they are stacked to form a rod like structure). In the low temperature phase (the nematic phase), molecules tend to align locally, leading to non zero magnitude of the order parameter with the director representing the average orientation of the molecules in a local region. In the high temperature phase (the isotropic phase), which occurs above a critical value of the temperature, $T_c$, molecules are randomly oriented leading to vanishing magnitude of the order parameter. This leads to the order parameter space being $RP^2(\equiv S^2/Z_2)$ which allows for string defects with winding 1/2. The director rotates by $\pi$ around this string defect.

The isotropic-nematic (I-N) transition temperature $T_c$ for the NLC we have used is about 35.3 °C. The sample was kept at the ambient temperature less than $T_c$ so it was in the nematic phase. A copper wire of about 100 micron thickness, with shape $\mathbb{C}$ was connected to a resistor for heating (using voltage supply) such that the tip of the wire could be lowered into the sample drop from the top. The wire tip was in between the objective of the microscope and the sample. By increasing the current, the wire tip could be heated to a temperature much higher than $T_c$ (upto about 50 - 60 ° C). A Leica DMRM microscope with 20x objective, and a CCD camera was used for observations, at the Institute of Physics, Bhubaneswar. The string formation was recorded using a camcorder (for larger field of view).

When the current is increased with the wire tip inside the NLC drop then a small region around the tip heats up above $T_c$ to the isotropic phase. When the current is low such that this isotropic region is small then small string loops form and shrink near the wire tip, as seen in Fig.1a. As convection currents take the liquid (crystal) elements
away from the heated tip, the temperature of the liquid element drops and falls below \( T_c \) at a certain distance from the tip. Resulting I-N phase transition can lead to string formation via the standard Kibble mechanism \[1\]. Another way strings can form here is because of turbulence generated by the heated tip due to temperature gradients in both vertical and horizontal directions. This can lead to formation of string defects as the order parameter (director) undergoes non-trivial rotations due to turbulent vorticity.

Fig. 1b-d show large string loops further away from the wire tip as the wire tip is heated further. (Tip appears bigger in the picture when it is embedded deeper in the sample.) Larger heating increases string formation. At the same time we see that string loops, instead of shrinking, move away from the tip and stretch to large sizes. This is surprising, as one expects string loops in liquid crystal to always shrink due to string tension, (i.e. due to excess free energy). This is because, unlike superfluid vortices, there are no velocity fields associated with the winding of the liquid crystal string defects. What happens here is that strings are swept away from the wire tip by the radial convection currents which force strings to stretch and become larger. Eventually string loops shrinks, presumably when the fluid velocity subsides and is unable to compensate for the string tension (i.e., the tendency for string length to shrink due to excess free energy). Due to vertical temperature gradient, the convection currents also force string loops to return towards the wire tip. (Interestingly, confirmation for this pattern of convection currents is provided by observation of dirt particles embedded in the sample which are seen to follow the motion of string loops.) Often we see that string loops get trapped in cyclic motions, repeatedly swept away from the tip and becoming larger, then shrinking while returning towards the tip. We have checked that when the tip is kept just outside the sample (so there are no boundary effects due to the tip inside the sample) then string loops are still produced, though lesser in number (which, at least partly is due to lesser heating).

There is a natural analog of this phenomenon in the early universe, where the Hawking evaporation of primordial black holes \[8\] can lead to local heating of the ambient plasma \[11,12\] (see, also, \[13\]), leading to restoration of various symmetries. An investigation of this local heating was carried out in \[11\] utilizing known results about the energy loss of quarks and gluons traversing a region of quark-gluon plasma \[14\]. (We mention here that the results of ref. \[14\] are certainly applicable for high energy partons (say several 100 GeV), it is not clear whether they can be extrapolated to the case of Hawking radiation with energies almost near the GUT scale. Still, the overall picture of pbh heating the local plasma to very high temperatures may remain valid even for such large Hawking temperatures). It was shown in \[11\] that this energy loss leads to rapid heating of the plasma near the black hole. Further, resulting temperature gradients lead to large pressure gradients such that plasma develops radial flow.

Possibility of formation of primordial black holes, and various observational constraints on them are discussed in the literature, see refs. \[8\] for early discussions. In first order phase transitions, particularly in first order inflationary scenarios, primordial black holes can be produced by collapsing regions of false vacuum \[15\] or due to inhomogeneities formed during bubble wall collisions \[16\]. They could also form by large amplitude density perturbations produced due to fluctuations in the inflaton field \[17\]. By shrinking cosmic string loops \[18\].

For primordial black holes produced from large amplitude density fluctuations produced by the inflation, one might ask whether CMBR data puts any constraints. However, as discussed in ref. \[19\], one should note that CMBR data imposes restrictions on the power spectrum at large scales (of order 1 - \( 10^3 \) Mpc), whereas, the primordial black holes form due to large amplitude and much smaller scale density fluctuations produced by the inflation. Hence they do not affect CMBR results at all but imposes constraints on the spectral index and consequently puts restrictions on certain models of inflation.

The constraints on primordial black hole formation come from other sources like nucleosynthesis, gamma ray background etc. \[19\]. However, all these phenomenon essentially imposes constraints on primordial black holes with a mass range which is much larger than the ones relevant here. We consider primordial black holes with masses at most few tonnes and such black holes evaporate away much above the QCD scale, without causing any conflict with current observations. Our main aim is to illustrate a completely novel possibility of forming infinite string networks via back holes. Hence, we will simply assume the required masses and number densities of black holes, without discussing any specific mechanism which could give rise to the formation of such black holes.

Consider the case when the plasma heated by the black hole leads to local restoration of a gauge symmetry (with an energy scale and associated phase transition temperature of order \( \eta \)) which allows for the existence of cosmic strings \[1\]. As the plasma flows out radially away from the black hole, its temperature will decrease and fall below \( T_c (= \eta) \) at some distance \( r_\eta \) where the symmetry will be broken. Near that region, again, string defects will form either via the Kibble mechanism, or due to turbulence. (For internal symmetries, spatial variations of the order parameter could lead to formation of non-trivial windings when turbulent motion of the plasma folds up extended spatial regions, essentially compactifying parts of spatial regions, i.e. plasma). If the friction forces dominate over the string tension so that strings are effectively frozen locally in the plasma then the string loops will be carried out by the radially expanding plasma. They will then stretch out to large sizes instead of shrinking due to tension, just as for the liquid
By equating its temperature is \( T \) due to large temperature gradients near the black hole. Assuming a steady state situation where the luminosity using the Euler equation for a relativistic fluid, it was shown in [11] that plasma velocity rapidly becomes relativistic taken to be the final mass of the black hole when its evaporation becomes effective.

If there is a uniform density of similar primordial black holes, then each black hole will emit string loops which will be stretched to large sizes as they are carried away by convective flow, and may intersect each other. It is well established that string defects when crossing each other, intercommute [20] (unless they have ultra-relativistic velocities, which is not the case here). Intercommutation of string loops from different black holes will result in formation of larger string loops, leading to the possibility that these string loops may percolate and form an infinite string network.

Let us discuss the conditions for percolation of strings. A black hole of mass \( M_{bh} \) evaporates by emitting Hawking radiation with an associated temperature \( T_{bh} = \frac{M_{bh}^2}{8 \pi M_{Pl}^2} \) where \( M_{Pl} = 1.2 \times 10^{19} \text{GeV} \) is the Planck mass. We use natural units with \( h = c = 1 \). The rate of loss of mass by the evaporating black hole is given by

\[
\frac{dM_{bh}}{dt} = -\frac{\alpha M_{Pl}^4}{M_{bh}^2}.
\]

Here, \( \alpha \) accounts for the scattering of emitted particles by the curvature and depends on \( T_{bh} \). For values of \( T_{bh} \) relevant for our model, we take \( \alpha \) to be \( \sim 3 \times 10^{-3} \) (see, [11, 21]). The lifetime \( \tau_{bh} \) of the black hole obtained from Eq.(1) is \( \tau_{bh} \sim 10^2 M_{Pl}^4 M_0^4 \), where \( M_0 \) is the initial mass of the black hole. Eq.(1) implies that very little energy is emitted until time of the order of \( \tau_{bh} \) which is when most of the energy gets emitted. The age of the Universe \( t_U \) when its temperature is \( T_U \) is [22] \( t_U \sim 0.3 g_{\ast}^{-1/2} \frac{M_{Pl} T_U^2}{M_{bh}^2} \), where \( g_{\ast} (\approx 100) \) is the number of relevant degrees of freedom [23]. By equating \( \tau_{bh} = t_U \), we can get the mass \( M_0 \) of the black hole such that its evaporation becomes effective when the temperature of the Universe is \( T_U \),

\[
M_0 = 0.07 M_{Pl}^{5/3} T_U^{-2/3}.
\]

For example, with \( T_U = 1 \text{ GeV} \) we get \( M_0 = 4 \times 10^{11} M_{Pl} \), \( T_{bh} = 10^6 \text{ GeV} \) and \( \tau_{bh} = 5 \times 10^{17} \text{ GeV}^{-1} = 3 \times 10^{-7} \) s. (We mention here that the mass of a primordial black hole can also increases due to accretion of background plasma particles. However, for relevant ranges of black hole masses here, all the dominant particle species are ultra relativistic, so only the geometric cross-section of black hole is relevant which is not very effective [22]. Some slow growth of the mass of the black hole could occur because of this, and the black hole masses we use here should be taken to be the final mass of the black hole when its evaporation becomes effective.)

In ref. [11], conditions for the equilibration of the Hawking radiation in the ambient plasma were discussed. Further, using the Euler equation for a relativistic fluid, it was shown in [11] that plasma velocity rapidly becomes relativistic due to large temperature gradients near the black hole. Assuming a steady state situation where the luminosity \( L(r) \) is independent of the distance \( r \) from black hole center, and is equal to \(-dM_{bh}/dt \) (Eq.(1)), the temperature profile \( T(r) \) of the expanding plasma was obtained in [11] to be,

\[
T(r) = \left[ \frac{L}{(8\pi^3 g_{\ast}/45)r^2 v_p} \right]^{1/4}.
\]

This was valid for distances where bulk plasma flow dominates over diffusion of particles [11], and will be applicable for distances relevant here. We will take the plasma velocity \( v_p \) to be of order of sound velocity \( v_p \approx 1/\sqrt{3} \). We are neglecting shock formation here for simplicity, though shocks may be more favorable for string percolation.

With Eq.(1) we see that it is the last stages of black hole evaporation which will lead to highest temperatures for the plasma in the nearby region, and will be most relevant for our model. Thus, we consider the situation when a black hole of initial mass \( M_0 \) (Eq.(2)) has reduced by evaporation to a much smaller black hole with a mass \( M_x \equiv M_0/x \), whose life time is \( \tau_x \) and temperature is \( T_x \). Here \( x > 1 \), and is constrained by requiring that the thickness of the
heated plasma region (which is of the order of \( r_x \) for plasma expanding with sound velocity) should be greater than \( \eta^{-1} \) (for self consistency of string formation at scale \( \eta \)). We thus require:

\[
100 M_{\text{pl}}^3 \frac{M_0^3}{x^3} \geq \eta^{-1} .
\] (4)

Temperature profile of the plasma around this remaining black hole is obtained from Eq.(3), and using Eq.(1) with \( M_{bh} = M_x = \frac{M_0}{x} \). Even though steady state is not a good approximation for late stages of black hole evaporation, we will take this profile for our case. In our case it will not represent the entire temperature profile at a given time. Rather one can think of this as temperature of plasma shells which expand and cool. Assumption of constancy of luminosity may then be valid for these plasma shells. In this respect it is important to note that essentially the same profile is obtained if we take \( T = T_x \) effectively at a distance of order of the Schwarzhild radius \( r_x = \frac{2}{M_{\text{pl}}^2} \frac{M_0}{x} \) along with the condition \( T^4 v^2 = \text{constant} \) (which essentially means conservation of energy-momentum of the expanding shells).

Temperature will drop to the value \( \eta \) at a distance \( r_\eta \) which is obtained by setting \( T(r_\eta) = \eta \) for \( T(r) \) (obtained from Eq.(3)). String loops will be produced at this distance and will be dragged (and stretched) by expanding plasma. The plasma flow should stop at a distance of order \( r_U \) where the temperature drops to the ambient value \( T_U \). \( r_U \), therefore, sets an upper limit on the distance \( R_{\text{stretch}} \) up to which strings can be carried out by plasma flow, and is determined by setting \( T(r_U) = T_U \). Using Eqs.(2),(3), we get, \( r_U \simeq 0.04 x M_{\text{pl}}^{1/3} T_U^{-4/3} \). To determine \( R_{\text{stretch}} \) we need to find when string motion is dominated by friction forces arising due to the scattering of plasma particles from the string. The dominant contribution to this comes from Aharanov-Bohm scattering (of particles with appropriate fractional charges) for gauge strings [24].

The plasma flow should stop at a distance of order \( \eta \) near the black hole (as also seen in Fig.1 for the liquid crystal case). String tries to collapse due to its tension. For a string loop of radius \( R \), one can estimate [24] the tension force to be of order \( F_{\text{tension}} \sim \frac{\mu}{R} = \frac{x^2}{R} \).

At the formation stage, near \( r \simeq r_\eta \), the string will be at rest in the local (expanding) plasma frame with \( v = 0 \) in Eq.(5), and hence \( F_{\text{frict}} = 0 \). String loop will then start collapsing under its tension (in the local rest frame of the plasma). This will lead to a non-zero \( v \), and consequently a non-zero friction force via Eq.(5). String loop will expand in the rest frame of the black hole, due to drag of the plasma expanding with sound velocity, when \( F_{\text{frict}}(v = 1/\sqrt{3}) > F_{\text{tension}} \). This condition gives the upper limit for \( R_{\text{stretch}} \) as,

\[
R_{\text{stretch}} < 10^{-8} \times \frac{x^3 M_{\text{pl}}^6}{M_0^3 \eta^4} \equiv R_{\text{max}} .
\] (6)

As plasma flow stops beyond a distance of order \( r_U \), we conclude that plasma flow can stretch string loops to sizes of order \( R_{\text{stretch}} \simeq \min(R_{\text{max}}, r_U) \). We should also consider the effect of black hole gravity on the string loops. Gravitational force per unit length on the string due to the black hole of mass \( M_{bh} \) can be roughly estimated as \( \frac{G M_{bh} \mu}{R^2} \) where \( R \) is the separation of the string segment from the black hole. (We are taking string as a simple gravitating system, as for a string loop. For special geometries, such as for a straigher string the gravitational force will be different, and will be typically smaller). We see that, per unit length, this gravitational force becomes much less.
than the force \( f_{\text{tension}} \sim \frac{R}{r^2} \) due to string tension when \( R > GM_{\text{bh}} \). Thus for any distances much larger than the Schwarzschild radius of the black hole, gravity of the black hole is subdominant compared to string tension forces. We are considering the situation when plasma drag forces completely dominate over the string tension forces, leading to stretching of string to large distances (where black hole gravity becomes even more negligible). Thus black hole gravity is not relevant for string dynamics in our case.

If black hole number density in the universe is such that inter-black hole separation \( d_{\text{bh}} \) is smaller than \( R_{\text{stretch}} \) then loop intersections will be frequent. This will lead to intercommutation of strings with probability (almost) 1, implying percolation of strings, resulting in an infinite string network. By assuming that energy density of black holes then loop intersections will be frequent. This will lead to intercommut ation of strings with probability (almost) 1, to stretching of string to large distances (where black hole gravity becomes even more negligible). Thus black hole

\[ d_{\text{bh}} \simeq \frac{M_{\text{pl}}^{5/9}}{8 f^{1/3} T_U^{14/9}} \]  

\[ \text{TABLE I: Sample parameter values for percolation of strings} \]

| \( f \) | \( T_U \) | \( \eta \) | \( M_0/M_{\text{pl}} \) | \( M_2/M_{\text{pl}} \) | \( r_\eta \) | \( R_{\text{stretch}} \) |
|-------|-------|-------|------------|------------|------|------------|
|       | GeV   | GeV^-1| GeV-1      | GeV-1      |      |            |
| 1     | \( 2 \times 10^7 \) | \( 10^6 \) | \( 5 \times 10^6 \) | 4000 | \( 10^{-7} \) | 0.02 |
| 10^{-3} | \( 10^7 \) | \( 10^8 \) | \( 8 \times 10^8 \) | 550 | \( 6 \times 10^{-7} \) | 0.7 |
| 1.0   | \( 10^6 \) | \( 10^6 \) | \( 10^6 \) | 42000 | \( 8 \times 10^{-3} \) | 87 |
| 10^{-3} | \( 10^4 \) | \( 10^8 \) | \( 8 \times 10^8 \) | 12000 | \( 3 \times 10^{-4} \) | 30000 |
| 10^{-7} | \( 10^2 \) | \( 10^8 \) | \( 4 \times 10^9 \) | 1500 | \( 2 \times 10^{-3} \) | \( 2 \times 10^7 \) |
| 1     | \( 10^4 \) | \( 2 \times 10^{10} \) | \( 10^6 \) | 400 | \( 4 \times 10^6 \) |
| 10^{-7} | \( 1.0 \) | \( 10^4 \) | \( 4 \times 10^{11} \) | 33000 | \( 10^5 \) | \( 10^{12} \) |
| 1     | \( 1.0 \) | \( 100 \) | \( 4 \times 10^{11} \) | \( 7 \times 10^6 \) | \( 5 \times 10^7 \) | \( 5 \times 10^9 \) |
| 10^{-7} | \( 1.0 \) | \( 100 \) | \( 4 \times 10^{11} \) | 33000 | \( 10^8 \) | \( 10^{12} \) |

Percolation of loops (forming an infinite string network) will happen when \( d_{\text{bh}} \leq R_{\text{stretch}} \). With \( R_{\text{stretch}} = r_U \) (when \( r_U < R_{\text{max}} \)), this gives the minimum allowed value of \( x \) as \( x_{\text{min}} \simeq 3 f^{-1/3} (M_{\text{pl}}/T_U)^{2/9} \) (consequently, maximum value of \( M_2 \)). We will give results with \( x = x_{\text{min}} \) (so that \( d_{\text{bh}} = R_{\text{stretch}} \)). We find that for string percolation, with \( f = 1, 10^{-3}, 10^{-5}, \) and \( 10^{-7}, \eta \) must be be below about \( 2 \times 10^{14}, 10^{12}, 2 \times 10^{10}, \) and \( 5 \times 10^8 \) GeV respectively, while \( T_U \) must be less than about \( 2 \times 10^{13}, 10^9, 3 \times 10^6, \) and \( 5 \times 10^3 \) GeV respectively. (Note that \( M_2 \) should be large enough to create many strings of sizes \( \sim R_{\text{stretch}} \)). This is always true with our parameters.) When \( r_U > R_{\text{max}} \) then \( R_{\text{stretch}} = R_{\text{max}} \). \( \eta \) can be larger in this case (e.g., up to \( 4 \times 10^{15} \) for \( f = 1 \)). However, for very large \( \eta, M_2 \) cannot be much bigger than \( M_{\text{pl}} \). Use of Eq.(1) may be suspect for such small black holes. We will quote results for parameter values when \( r_U < R_{\text{max}} \), and will not consider very large values of \( \eta \).

In Table 1 we have given several sets of values for various parameters for which string percolation happens. We see that \( R_{\text{stretch}} \gg r_\eta \), meaning that the strings are dragged (via friction forces) and stretched by the plasma flow up to very large distances. We also see that the percolation of strings is not a rare occurrence, but happens for rather generic set of parameter values. For example, even with almost negligible fraction of energy density in black holes, \( f = 10^{-7} \), string percolation is achieved.

We conclude by emphasizing that our results present first possible scenario where string loop generation by localized sources (heated wire tip for the liquid crystal experiment, and primordial black holes for the early universe) can lead to the formation of an extended string network without an overall phase transition, and even when the ambient temperature is much below the string scale. During scaling regime, there may not be any difference in the properties of string networks formed via the mechanism discussed here, and those arising from the conventional mechanism during a phase transition. These results illustrate the tremendous potential of correspondence between phenomenon observed in condensed matter systems and those which could have occurred in the early universe.

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Figure Caption

Fig.1: Production of string loops by heated wire tip embedded inside NLC droplet. Strings are carried out by convection, and are stretched to large sizes.