THE STAR FORMATION LAW IN ATOMIC AND MOLECULAR GAS

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ABSTRACT

We propose a simple theoretical model for star formation in which the local star formation rate in a galaxy is determined by three factors. First, the interplay between the interstellar radiation field and molecular self-shielding determines what fraction of the gas is in molecular form and thus eligible to form stars. Second, internal feedback determines the properties of the molecular clouds that form, which are nearly independent of galaxy properties until the galactic ISM pressure becomes comparable to the internal GMC pressure. Above this limit, galactic ISM pressure determines molecular gas properties. Third, the turbulence driven by feedback processes in GMCs makes star formation slow, allowing a small fraction of the gas to be converted to stars per free-fall time within the molecular clouds. We combine analytic estimates for each of these steps to formulate a single star formation law, and show that the predicted correlation between star formation rate, metallicity, and surface densities of atomic, molecular, and total gas agree well with observations.

Subject headings: galaxies: ISM — ISM: clouds — ISM: molecules — stars: formation

1. INTRODUCTION

The last decade has seen a revolution in our understanding of star formation in galaxies, driven by the advent of spatially resolved multi-wavelength surveys. Prior to this work, our observational constraints on the star formation process were largely limited to low resolution surveys that characterized entire galaxies using only a handful of observable quantities, e.g. the mean surface density of star formation averaged over a whole disk. While these surveys yielded a number of intriguing results – most famously the Kennicutt (1998) star formation law – they left unanswered many basic questions about the physics of star formation. For example, they could not clearly determine whether star formation correlates more strongly with the molecular or total gas content of a galaxy (Kennicutt 1998; Wong & Blitz 2002), or whether star formation is regulated primarily by local processes within individual star-forming clouds (e.g. Krumholz & McKee 2005, hereafter KM05; Shu et al. 2007) or by galactic-scale processes such as spiral shocks, supernovae, or cloud-cloud interactions (e.g. Wyse 1986; Silk 1997; Tan 2000; Li et al. 2005).

Now, however, emission maps at 24 μm from the Spitzer Infrared Nearby Galaxy Survey (SINGS) provide us with accurate estimates of the rate of dust-enshrouded star formation at resolutions of better than a kpc in nearby galaxy disks (Kennicutt et al. 2003, 2007; Calzetti et al. 2007), while ultraviolet observations from the GALEX Nearby Galaxy Survey (NGS) reveal the rates of non-obscured star formation with comparable resolution and accuracy (Gil de Paz et al. 2007). Observations of H i emission from the VLA as part of The H i Nearby Galaxy Survey (THINGS; Walter et al. 2008) and of CO emission by the BIMA Survey of Nearby Galaxies (BIMA SONG; Helfer et al. 2003) and the HERA CO-Line Extragalactic Survey (HERACLES; Leroy et al. 2009) using the 30 m IRAM telescope provide maps of the gas content of galaxies at comparable resolutions. Combining these data sets leads to two particularly important conclusions that our theoretical models of star formation must incorporate and explain.

The first observational result is that star formation is a direct product of the molecular gas in a galaxy, not of all the gas. Across a wide range of galactic environments the star formation rate correlates well with molecular gas, and poorly or not at all with the atomic gas measured on sub-kpc spatial scales (Wong & Blitz 2002; Kennicutt et al. 2007; Leroy et al. 2008; Bigiel et al. 2008, hereafter B08). The correlation between star formation rate and molecular gas surface density is super-linear in samples that extend to starbursts with gas surface densities ∼ 10^3 M⊙ pc^-2 (B08; however, see [Kennicutt et al. 2007] for a different interpretation). Observations of the low density outskirts of galactic disks hint that the linearity may break down there (Gardan et al. 2007; Fumagalli & Gavazzi 2008), but it is unclear if this indicates a change in the star formation process or a change in the CO to H2 conversion factor.

The second observational result is that giant molecular clouds (GMCs) have remarkably similar properties in all...
nearby galaxies. Across the Local Group GMCs appear to have the same surface density, roughly $85 \ M_\odot\ pc^{-2}$, and to obey the same linewidth-size relation (Blitz et al. 2007, Bolatto et al. 2008, Hever et al. 2009). Together these two observations imply that all observed molecular clouds are not far from virial balance between gravity and internal turbulence. There is of course considerable uncertainty in the GMC surface density, arising mostly from the CO-to-H$_2$ conversion factor, but it is striking that there is no clear evidence of a systematic trend in GMC properties across a sample of galaxies ranging from H I-dominated dwarfs to molecule-rich giant spirals. This seems to be a critical clue to the physics of molecular clouds.

Our goal in this paper is to develop a model for the star formation law that is based on the insights provided by these new surveys, and that incorporates the theoretical understanding that has developed around them.

2. THE STAR FORMATION LAW

Given that star formation occurs in molecular gas, we formulate our theoretical law for the local star formation rate (SFR) surface density $\Sigma_*$ in a galaxy as a product of three factors:

$$\Sigma_* = \Sigma_g f_{H_2} \frac{\text{SFR}_{H_2}}{t_{ff}}.$$  \hspace{1cm} (1)

Here $\Sigma_g$ is the total gas surface density at some point in the galaxy. In practice this will always be an average over some size scale, determined by the resolution of observations (or simulations). This determines the total available “raw material” for star formation. The factor $f_{H_2}$ is the fraction of this mass in molecular form; atomic gas does not participate in star formation. The molecular component of the gas is organized into clouds which have some mean volume density $\rho_{H_2}$, and $t_{ff} = [3\pi/(32G\rho_{H_2})]^{1/2}$ is the free-fall time at this mean density. The quantity $\text{SFR}_{H_2}$ is the dimensionless star formation rate; it is the fraction of the gas transformed into stars per free-fall time. Alternately, one may think of it as the star formation efficiency over one free-fall time. The third factor, $\text{SFR}_{H_2}/t_{ff}$, is simply the SFR per free-fall time divided by the free-fall time, which is the inverse of the time required to convert all of the gas into stars. To make a model for the star formation law, we must estimate $f_{H_2}$, $t_{ff}$, and $\text{SFR}_{H_2}$ in terms of the observable quantities for a galaxy.

2.1. The Molecular Fraction

The molecular mass fraction $f_{H_2}$ is determined by the balance between dissociation of molecules by the far ultraviolet (FUV) interstellar radiation field in the Lyman-Werner bands and formation of molecules on the surfaces of dust grains. Krumholz et al. (2008, 2009, hereafter KMT08 and KMT09) show that to good approximation $f_{H_2}$ within a single atomic-molecular complex is a function of the gas surface density of the complex $\Sigma_{\text{comp}}$ and the metallicity $Z$. We will not repeat the full derivation of this result here, but a summary of the calculation that produces it is that one first solves the idealized problem of finding where the transition between the atomic envelope and the molecular interior occurs within a uniform sphere of hydrogen gas and dust embedded in an isotropic dissociating radiation field. This analysis shows that the fraction of the complex in molecular form depends on the dust optical depth of the complex $\Sigma_{\text{comp}} \sigma_d$ and on the dimensionless ratio $\chi \propto \sigma_d G_0/(n_{\text{CNM}} R)$, where $\sigma_d$ is the dust cross section per hydrogen nucleus, $G_0$ is the intensity of the dissociating radiation field, $n_{\text{CNM}}$ is the number density of gas in the cold atomic medium that surrounds the molecular part of the cloud, and $R$ is the rate coefficient for H$_2$ formation on the surfaces of dust grains. Since $\sigma_d$ and $R$ are both, to first order, simply measures of the total amount of dust in a galaxy, their ratio should not vary widely between galaxies. Similarly, in a galaxy with a two-phase atomic medium the cold atomic gas density $n_{\text{CNM}}$ is determined by thermal pressure balance between the two phases, which in turn depends on the balance between heating by FUV photons and atomic line cooling in the atomic gas. Analysis of these processes implies that the ratio $G_0/n_{\text{CNM}}$ is a weak function of metallicity and is otherwise independent of galaxy properties (Wolfire et al. 2003). Thus $\chi$ varies little between galaxies, and this result enables us to write the molecular fraction for a given atomic-molecular complex as a function solely of its gas surface density $\Sigma_{\text{comp}}$ and its metallicity $Z$:

$$f_{H_2}(\Sigma_{\text{comp}}, Z') \approx 1 - \left[1 + \left(\frac{3}{41 + \delta}\right)^s\right]^{-1/5}$$  \hspace{1cm} (2)

where $s = \ln(1 + 0.6\chi)/(0.04 \Sigma_{\text{comp}} Z')$, $\chi = 0.77(1 + 3.1 Z_{0.365}^\delta)$, $\delta = 0.0712(0.1s^{-1} + 0.675)$, $\Sigma_{\text{comp}} = \Sigma_{\text{comp}}/(1M_\odot\ pc^{-2})$, and $Z'$ is the metallicity normalized to the solar value. Note that this approximation is slightly different with the one given in KMT09, the two agree to within a few percent for clouds that are substantially molecular, but this one is more accurate at small molecular fractions (McKee, Krumholz, & Tumlinson 2009, in preparation).

Here $\Sigma_{\text{comp}}$ is the surface density of a $\sim 100$ pc-sized atomic-molecular complex. However, extragalactic observations generally measure a gas surface density $\Sigma_g$ that is averaged over a much larger scale. Since $f_{H_2}$ increases super-linearly with $\Sigma_{\text{comp}}$, clumping of the gas on scales below the observational resolution would lead us to underpredict $f_{H_2}$ if we were simply to use the large-scale-averaged value of $\Sigma_g$ in place of $\Sigma_{\text{comp}}$ equation 2. Since we wish to propose a model that is applicable to data and simulations at a range of resolutions, it is convenient to approximately correct for this effect by letting $\Sigma_{\text{comp}} = c \Sigma_g$, where $c \geq 1$ is a clumping factor and $c \to 1$ as the resolution approaches $\sim 100$ pc.

As a final caveat, it is important to point that our calculation of $f_{H_2}$ in KMT09 assumes that the Wolfire et al. (2003) semi-analytic model for the atomic ISM is applicable, and the model begins to break down at metallicities below roughly 5% of solar (see Figure 13 of Wolfire et al. 2003) because dust grains and polycyclic aromatic hydrocarbons begin to be neutral rather than positively charged, as the model assumes. Turbulent heating of the cold H I phase (Pan & Padoan 2009), which is not included in the Wolfire et al. models, is also likely to be important at low metallicity. Thus, although our general method of calculating molecular fractions will apply even at low metallicities, the relationship between $n_{\text{CNM}}$ and $G_0$ which is used to derive equation 2 is not valid at metallicities $Z' < 0.05$.\[\]
2.2. Giant Molecular Cloud Properties

Next we must compute $t_{ff}$ and SFR$_{ff}$, which will depend on the properties of the star-forming GMCs in a galaxy. Before proceeding with such a calculation, we note that observations of Local Group galaxies indicate that in galaxies ranging from metal-poor dwarfs to molecule-rich spirals, the molecular cloud surface density $\Sigma_{\text{cl}} \approx 85 \, M_\odot \, \text{pc}^{-2}$ and the molecular cloud virial ratio $\alpha_{\text{vir}} \approx 2$ independent of galactic environment (Blitz et al. 2007, Bolatto et al. 2008). This invariance is reasonably easy to understand on theoretical grounds. The virial theorem implies that the mean pressure within the cloud is $P_{\text{cl}}/k_B = 0.7 \times 10^5 \alpha_{\text{vir}} \Sigma_{\text{cl}}^2 \ \text{K cm}^{-3}$ (KM05), where $\Sigma_{\text{cl}} = \Sigma_{\text{cl}}/(85 \, M_\odot \, \text{pc}^{-2})$. In comparison, Boulacher & Cox (1990) find that the mean kinetic pressure in the ISM of a Milky Way-like galaxy is $1.4 \times 10^4 \, \text{K cm}^{-3}$, an order of magnitude lower, although the pressure may be higher than average in spiral arms where GMCs form. The pressure is almost certainly lower in low surface-density dwarfs. The mismatch between the pressures in GMCs and the pressures in their environments indicates that external pressure is at most marginally important in determining the properties of molecular clouds, and that GMCs must instead be internally regulated; in effect, a GMC forgets about its mean galactic magnetic field and cosmic rays pervade GMCs and the intercloud medium equally, and therefore provide neither support nor confining pressure.

The constant surface densities and virial ratios of GMCs provide a natural way to estimate $t_{ff}$ and SFR$_{ff}$. Consider a GMC of mass $M$, surface density $\Sigma_{\text{cl}}$, and virial ratio $\alpha_{\text{vir}}$. The volume density in this cloud is $\rho_{\text{H}_2} \approx (3\pi^{1/2}/4)\Sigma_{\text{cl}}^{-3/2}M^{-1/2}$, so the free-fall time is

$$t_{ff} = 8 \Sigma_{\text{cl}}^{-3/4} M_6^{1/4} \, \text{Myr}.$$  \hspace{1cm} (3)

where $M_6 = M/10^6 M_\odot$. Similarly, KM05 show that the star formation rate per free-fall time in a turbulent medium is approximately

$$\frac{\text{SFR}_{{ff}}}{t_{ff}} \approx 0.15 \epsilon_{\text{core}} \alpha_{\text{vir}}^{-0.68} M^{-0.32},$$  \hspace{1cm} (4)

where $M$ is the 1-D Mach number of the turbulence and $\epsilon_{\text{core}}$ is the fraction of the mass in a gravitationally-bound prestellar core that is incorporated into a star rather than being ejected by protostellar outflows. KM05 adopt $\epsilon_{\text{core}} = 0.5$ based on analytic models showing $\epsilon_{\text{core}} \approx 0.25 \pm 0.75$ (Matzner & McKee 2000), more recent work suggests the true value is $\epsilon_{\text{core}} \approx 0.3$ (Alves et al. 2007), so we adopt $\epsilon_{\text{core}} = 0.3$.

The virial ratio is related to the one-dimensional velocity dispersion $\sigma$ in a GMC by $\alpha_{\text{vir}} \approx 5 \sigma^{-1/2}(M\Sigma_{\text{cl}})^{-1/2}G$ (Bertoldi & McKee 1992), so $\sigma = 3.7 \alpha_{\text{vir}}^{1/2} \Sigma_{\text{cl}}^{1/4} M_6^{-1/4} \, \text{km s}^{-1}$. For a molecular cloud temperature of 10 K the corresponding Mach number is $M = 20\alpha_{\text{vir}}^{1/2} \Sigma_{\text{cl}}^{1/4} M_6^{-1/4}$, so

$$\text{SFR}_{{ff}} \approx 0.017\alpha_{\text{vir}}^{-0.84} \Sigma_{\text{cl}}^{-0.08} M_6^{-0.08}. \hspace{1cm} (5)$$

This is consistent with the observed value $\text{SFR}_{{ff}} \approx 0.01$ (Krumholz & Tan 2007).

Combining equations (3) and (4), and adopting a fiducial value of $\alpha_{\text{vir}} = 2$, gives

$$\frac{\text{SFR}_{{ff}}}{t_{ff}} = \frac{\Sigma_{\text{cl}}^{0.67} M_6^{-0.33}}{0.8 \, \text{Gyr}}.$$  \hspace{1cm} (6)

The invariance of molecular cloud properties that we observe in nearby galaxies must break down in galaxies with sufficiently high surface densities, where the external pressure is no longer negligible compared to a GMC's internal pressure. Since pressure varies as $P \propto \Sigma^2$ for both molecular clouds and galactic disks (KM05), the galactic environment will become significant in determining molecular cloud properties once the galactic surface density averaged over large scales becomes comparable to the surface density of an individual GMC. In this case the GMC surface density must increase in order to maintain pressure balance with the rest of the galaxy's ISM, which simply requires that $\Sigma_{\text{g}} \approx \Sigma_{\text{g}}$ for $\Sigma_{\text{g}} > 85 \, M_\odot \, \text{pc}^{-2}$. Observations are consistent with this hypothesis: in the central kpc of M64, where the galactic surface density runs from $\sim 50 - 1000 \, M_\odot \, \text{pc}^{-2}$, the GMC surface density is not constant, and instead rises with galactic pressure. Averaged over the entire galaxy the mean GMC surface density is $250 \, M_\odot \, \text{pc}^{-2}$ (Rosolowsky & Blitz 2005). Thus, the free-fall time in GMCs in high surface density galaxies varies as $\Sigma_{\text{g}}^{-3/4}$. If we adopt a column density of $\Sigma_{\text{cl}} = 85 \, M_\odot \, \text{pc}^{-2}$ for all GMCs in normal surface density galaxies and $\Sigma_{\text{cl}} = 2 \Sigma_{\text{g}}$ at higher surface densities, then we have

$$\frac{\text{SFR}_{{ff}}}{t_{ff}} \approx \frac{M_6^{-0.33}}{0.8 \, \text{Gyr}} \max \left[1, \left(\frac{\Sigma_{\text{g}}}{85 \, M_\odot \, \text{pc}^{-2}}\right)^{0.67}\right]. \hspace{1cm} (7)$$

Equation (7) gives an estimate for $\text{SFR}_{{ff}}/t_{ff}$ in a molecular cloud of a known mass. To complete the calculation, we must estimate the characteristic molecular cloud mass in a galaxy. We follow KM05 to estimate that this will be determined by the Jeans mass in the galaxy, which is

$$M \approx \frac{\sigma_g^4}{G^2 \Sigma_g Q^4} = \frac{\pi^4 G^2 \Sigma_g^3 Q^4}{4R^4},$$  \hspace{1cm} (8)

where $\sigma_g$ is the gas velocity dispersion, $Q$ is the Toomre $Q$ of the galactic disk, and $\Omega$ is the angular velocity of its rotation. If we can directly measure $\Sigma_{\text{cl}}$, $\Omega$, and $Q$, or $\Sigma_{\text{g}}$ and $\sigma_g$ for a galaxy, then we can solve for $M$ directly and substitute into equation (7) to obtain a characteristic value of $\text{SFR}_{{ff}}/t_{ff}$ for that galaxy. However, often one or more of the quantities are unknown.

1 Note that this value of $\Sigma_{\text{cl}}$ is lower than the 170 $M_\odot \, \text{pc}^{-2}$ for Galactic GMCs found by Solomon et al. (1985), but is consistent with the lower value determined by the more recent survey of Heyer et al. 2008.

2 We consider only turbulent and thermal pressure because the mean galactic magnetic field and cosmic rays pervade GMCs and the intercloud medium equally, and therefore provide neither support nor confining pressure.

3 For our fiducial $\Sigma_{\text{g}} = 1$ and $\alpha_{\text{vir}} = 2$, this agrees with the observed linewidth-size relation $\sigma = 0.44^{+0.18}_{-0.13} (R/\text{pc})^{0.60^{+0.10}_{-0.04}}$ km s$^{-1}$ (Bolatto et al. 2008) to within the error bars.
and even when they are known it is useful to have a rough estimate in terms of a single quantity such as \( \Sigma_g \) rather than three quantities \( \Sigma_{\text{cl}}, \Omega, \) and \( Q \). Since \( M_6 \) enters the star formation rate only to the 0.33 power, any errors we make in this approximation are unlikely to have strong effects. We therefore follow KM05 in assuming that all galaxies will be marginally Toomre stable, \( Q \approx 1 \), and noting that there is broad statistical correlation \( \Omega/\text{Myr}^{-1} \approx 0.054(\Sigma_g/85 \, M_{\odot} \, \text{pc}^{-2})^{0.49} \). If we use this correlation in (8) then we obtain

\[
M_6 \approx 37 \left( \frac{\Sigma_g}{85 \, M_{\odot} \, \text{pc}^{-2}} \right)^{1.0}.
\]

Finally, it is worth noting here that our estimate of the molecular cloud volume density, which depends on \( \Sigma_{\text{cl}} \) and \( M_6 \), is somewhat different than that of KM05. They assumed that GMC surface densities were set largely by external pressure in a galaxy, and computed the density based on this assumption. As discussed above, more recent observational and theoretical work suggests that instead GMC densities are primarily set by internal feedback processes and do not vary significantly with galactic conditions, at least in Milky Way-like galaxies. Our model in this paper takes this result into account.

### 2.3. The Full Star Formation Law

We have now derived the major components of our star formation law (equation 1). The molecular fraction \( f_{\text{H}_2} \) depends only on gas surface density \( \Sigma_g \), metallicity \( Z' \), and the clumping of the gas \( c \) on scales unresolved in a given observation or simulation (equation 2). It increases with \( \Sigma_g \), becoming fully molecular at \( \approx 10/cZ' \, M_{\odot} \, \text{pc}^{-2} \). We have also derived an analytic relation for the inverse star formation timescale \( f_{\text{H}_2}/t_{\text{ff}} \) in two regimes. Where internal GMC pressure far exceeds the ambient ISM gas pressure and GMCs “forget” their environment – as typically occurs in nearby galaxies with \( \Sigma_g < 85 \, M_{\odot} \, \text{pc}^{-2} \) – this timescale does not depend on \( \Sigma_g \) except indirectly through the molecular cloud mass (equation 4). Above \( \Sigma_g = 85 \, M_{\odot} \, \text{pc}^{-2} \), ambient pressure becomes comparable to the GMC internal pressure and the star formation timescale depends on \( \Sigma_g \) (equation 7). In neither case does the timescale depend on either the metallicity or the clumping, so the star formation rate in molecular gas does not depend on either of these quantities. Only the star formation rate in total gas does.

We are now ready to combine these pieces into our single star formation law:

\[
\dot{\Sigma}_* = f_{\text{H}_2}(\Sigma_g, c, Z') \frac{\Sigma_g}{2.6 \, \text{Gyr}} \times \begin{cases} \frac{\Sigma_g}{85 \, M_{\odot} \, \text{pc}^{-2}} &< 1 \\ \frac{\Sigma_g}{85 \, M_{\odot} \, \text{pc}^{-2}} &> 1 \end{cases}.
\]

### 3. COMPARISON TO OBSERVATIONS

We compare our proposed star formation law, equation (10), to the observed relationship between star formation, atomic gas, and molecular gas in Figures 4 and 2. The majority of the observations come from the THINGS sample. The full sample covers metallicities from \( \log Z' = -1.22 \) to 0.49 ( Walter et al. 2008, KMT09), but only four of the thirty-four galaxies have metallicities below \( \log Z' = -1.0 \), and these are all dwarfs with such low star formation rates that they contribute negligibly to the total star formation rate in the sample. Moreover, the molecular gas masses for these systems are likely to be extremely uncertain (see below). Thus we adopt \( \log Z' = -1.0 \) to 0.5 as a realistic range of metallicities in the data.

The THINGS sample is observed at a resolution of \( \sim 750 \, \text{pc} \), much larger than a single atomic-molecular complex, so we expect \( c > 1 \). The true value of \( c \) cannot be determined directly in external galaxies without higher resolution observations. A lower limit comes from the fact that the observations mix together spiral arm and inter-arm regions, and the arm-interarm density contrast is \( \sim 2 - 4 \) in galaxies observed at higher resolution ( Nakanishi & Sofue 2002, Schuster et al. 2007). The complexes themselves represent density peaks on top of the already-enhanced density within the arm, and in fully molecular regions clouds are observed to have surface densities higher than the mean by a factor of \( \sim 2 \) ( Rosolowsky & Blitz 2003). We therefore adopt \( c \approx 5 \), and thus we expect the data to be characterized by \( \log cZ' \approx -0.3 \) to 1.2, with the four low-metallicity
Star formation rate surface density $\dot{\Sigma}_*$ as a function of total gas surface density $\Sigma_g$. Lines and contours are the same as in Figure 1. Other points are a compilation of literature data from B08. We show individual apertures in M51 (black dots, Kennicutt et al. 2007), azimuthal averages (blue circles) in NGC4736 and NGC5055 (Wong & Blitz 2002), NGC6946 (Crosthwaite & Turner 2007), and M51 (Schuster et al. 2007), and global averages for starbursts (open green triangles, Kennicutt 1998), normal spirals (filled green triangles, Kennicutt 1998), and low surface brightness galaxies (yellow diamonds, Wyder et al. 2009). The gray arrows and labels indicate schematically the dominant physical process responsible for setting the slope in each region.

dwarfs lying at somewhat lower $\log cZ'$. Our simple model recovers a number of salient features in the observations. Figure 1 shows that we recover the observational result that the H$_i$ surface density reaches a maximum value, which is $\sim 10 M_\odot$ pc$^{-2}$ at Solar metallicity, that the star formation rate does not correlate well with $\Sigma_{\text{HI}}$ in resolved observations of galaxies. The RMS noise in the star formation rate surface density in the survey is $\sim 10^{-4} M_\odot$ kpc$^{-2}$ (Bigiel et al. 2008), so the apparent flattening of the contours below this value is an observational artifact. Figure 1 indicates that we recover a good approximation to the correct, nearly constant star formation rate in molecular gas at surface densities from roughly $5 - 100 M_\odot$ pc$^{-2}$. Combined, these two effects produce a star formation rate that increases superlinearly with total gas content below the H$_i$ saturation threshold and only linearly above it (Figure 1). Third, we recover the return to a superlinear increase of star formation rate above $\sim 100 M_\odot$ pc$^{-2}$, produced by the increase in molecular cloud density in high-pressure environments (Figure 2).

Uncertainties in the star formation rates come from a combination of uncertainties in dust corrections and in the stellar initial mass function (IMF). Comparing star formation rates in the THINGS sample based on FUV plus 24 µm emission to those based on H$_\alpha$, or H$_\alpha$ plus 24 µm emission, suggests uncertainties below the factor of $\sim 2$ level. Comparison to the star formation rate in the Milky Way inferred from radio catalogs of H$_\text{ii}$ regions suggests a slightly larger uncer-
tainty: McKee & Williams (1997) infer a star formation timescale in the molecular gas of \( \frac{\text{SFR}_{\text{ff}}}{\text{ff}} \sim 100 \text{Myr} \) from this technique, compared to 2 Gyr for the average of the THINGS sample. The origin of this discrepancy is unclear, but it suggests that significant caution is warranted in interpreting the star formation rates inferred from observations.

4. SUMMARY

We have shown that the observed relationship between the star formation rate and the atomic and molecular content of galaxies can be explained by a simple model, whose elements are summarized by the regions labelled in Figure 2. First, self-shielding of hydrogen determines the amount of gas in molecular form. This imposes a characteristic gas surface density of \( \sim 10/cZ' \, M_\odot \, \text{pc}^{-2} \) for the transition from atomic to molecular, where \( c \) is the factor by which the gas surface density is increased due to clumping unresolved by the observations and \( Z' \) is the metallicity relative to solar. Second, once molecules do form, molecular clouds reach a surface density of roughly \( 85 \, M_\odot \, \text{pc}^{-2} \) independent of galactic environment. This behavior can be understood as arising from the fact that molecular clouds are overpressured relative to their surroundings, so they must be regulated by internal processes, most likely HH regions (Matzner 2002, Krumholz et al. 2006), that do not depend on metallicity or other large-scale galaxy properties. The constant surface density imposes a roughly constant volume density and free-fall time on all molecular gas. The exception to this is galaxies where the mean galactic surface density is \( \gtrsim 100 \, M_\odot \, \text{pc}^{-2} \), in which the ambient pressure is high enough to force GMC densities to rise along with galactic surface density in order to keep the clouds in pressure balance. Third, once formed molecular clouds convert themselves into stars at a nearly universal rate of \( \sim 1\% \) of the mass per free-fall time as a result of turbulent regulation. Together these effects produce a total gas star formation law that is superlinear at low galactic column density (due to increasing molecular fraction), linear or slightly sub-linear at intermediate column (due to the invariance of molecular cloud surface densities and the weak dependence of GMC masses on galactic properties), and superlinear again at high column (due to the breakdown of this invariance at high galactic pressures).

It is worth noting that our model does not make any explicit reference to galactic-scale processes such as spiral shocks, gravitational instability, supernova feedback, or cloud-cloud collisions. In a sense all of this physics is “upstream” of our theory: processes such as these are almost certainly responsible for determining the distribution of gas surface density within a galaxy. Our model addresses the next step of how, once large-scale processes assemble the gas, some fraction of it forms molecular clouds and then turns into stars. We have therefore separated the problem of star formation into two parts, and provided a tentative solution for one of them: galactic-scale processes determine \( \Sigma_g \), but the physics responsible for determining the star formation rate thereafter is purely local, and can be understood without reference to galactic-scale behavior. Our model shows that a much of the recent observational work on star formation can be understood in terms of a simple model for that local process.

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