An alternative to dark matter: do braneworld effects hold the key?

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Abstract

An alternative way of explaining astrophysical observations without dark matter is proposed. In the braneworld scenario, where our universe is visualised as a 4-dimensional hypersurface embedded in a higher dimensional spacetime, the effective Einstein equation on the brane contains extra terms. We show that the astrophysical observations which are usually explained by particle dark matter, can be explained in this modified theory of gravity via those extra terms, without the need for dark matter. As specific example, we model X-ray profiles of clusters of galaxies and the galaxy rotation curves which are consistent with observations. Further, we investigate whether gravitational lensing can discriminate between the two possibilities. Significant differences between the values of the deflection angles are obtained, which can be tested in future observations.

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I. INTRODUCTION: A LOOK ON DARK MATTER

An important issue of present-day astrophysics is the understanding of dark matter. These are apparently invisible objects but their existence is inevitable in explaining several aspects of astrophysics and cosmology. There are a number of possible candidates for dark matter. To mention a few, the particle dark matter models propose ‘baryonic dark matter’ which include massive compact halo objects (MACHO). The second kind of these models are the ‘non-baryonic dark matter’ which constitute a pressureless medium. Examples include exotic supersymmetric particles, known as the weakly interacting massive particles, the WIMP [1, 2]. Models with substantial amount of pressure have also been proposed [3]. These are all material candidates for dark matter. There are alternative propositions, called the MOND [4, 5], that are based on the modifications of Newtonian dynamics. A relativistic version of the MOND is also available [6].

Astrophysicists need to introduce dark matter in two contexts [7, 8]. First, the observed X-ray emission from hot, intra-cluster gas can be used to measure its density and pressure. The mass estimated from these data using Einstein’s theory of gravitation (the so-called Newtonian analysis) is always found to be substantially in excess of what can be attributed to visible matter. So, to preserve standard Einstein’s theory, it is proposed that around \( \sim 80\% \) of the matter surrounding clusters is dark.

Secondly, the Doppler shift measured from the neutral hydrogen cloud rotating inside spiral galaxies can be used to estimate the rotational velocity. It has been observed that the rotational velocity decreases away from centre but remains constant after reaching a typical value of \( \sim 200 \) km/s. The velocity profile thus found, called the “rotation curve”, when applied to standard Einstein gravity, gives rise to an enhanced mass. Hence if Einstein’s theory has to be preserved, then one has to postulate that the galaxy is embedded in a halo made up of some kind of invisible matter, called dark matter [9, 10].

However, one should note that Einstein’s theory of gravitation has not been tested in these length scales. Further, the exact nature of dark matter is not well-understood till date. Hence one may start from the very beginning with a modified version of the standard Einstein theory of gravity by which the above observations might be explained without the need for dark matter. Here is one such theory, popularly known as the effective Einstein equation. The term “Effective Einstein Equation” means that if there is an \((N - 1)\)-dimensional
hypersurface embedded in an $N$-dimensional spacetime then the Einstein equation on the hypersurface takes into account the effects of the terms coming from the extra dimensions in the $N$-dimensional spacetime ($N > 4$). Hence, the geometry and physics on the hypersurface (our four dimensional world) is governed by this modified Einstein equation. We propose that it is possible to model spiral galaxies and clusters using this effective Einstein equation, that can successfully replace dark matter with the effects coming from extra dimensions.

II. EFFECTIVE EINSTEIN EQUATION : A REVIEW

Let us now switch over to a brief review of the effective Einstein equation. Brane world physics \[11, 12\] predicts that the observable universe might be a 4-dimensional hypersurface (3-brane) embedded in a 5-dimensional (or higher) spacetime (bulk). A new geometrical approach to the brane world physics, called the effective Einstein equation \[13\], is in vogue for the last few years. The basic notion is that if there is an extra dimension, then it has to have some effect on our 4D universe and one should take into account those effects in order to obtain correct physics. This formalism helps us take a different look at the physical phenomena that are not well-established till date. Some such aspects have been studied in \[14, 15, 16, 17, 18, 19, 20, 21\].

Without going into the technical details of how it is derived, let us straightaway try to find out the essence of the effective Einstein equation on the brane which reads \[13, 16\]

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa_4^2 T_{\mu\nu} + \kappa_5^4 S_{\mu\nu} - E_{\mu\nu}$$  \hspace{1cm} (2.1)

Before proceeding further, let us have a look on the above equation. The first two terms in the expression of $G_{\mu\nu}$ combine to give the standard 4D Einstein equation. The third term $S_{\mu\nu}$ is the quadratic contribution from the brane energy-momentum tensor $T_{\mu\nu}$ and the last term $E_{\mu\nu}$ is the projected bulk Weyl tensor on the brane, in absence of bulk matter \[13, 16\].

The 4D cosmological constant $\Lambda$ can be set to zero by proper fine-tuning \[11, 12\] and for weak field (which is relevant in the present study) the contribution from $S_{\mu\nu}$ is negligibly small \[13\]. Hence Eq (2.1) reduces to \[22, 23\]

$$G_{\mu\nu} = \kappa_4^2 T_{\mu\nu} - E_{\mu\nu}$$  \hspace{1cm} (2.2)
The net result is that the standard 4D Einstein equation is modified by a purely geometric term \( E_{\mu\nu} \) which is a traceless symmetric tensor. This is the property we are going to highlight in subsequent discussions.

III. THE SCHEME: FORMALISM WITHOUT DARK MATTER

We are now in a position to replace dark matter by our formalism. In particular, we look for modelling spiral galaxies and clusters consistent with observed rotation curves or X-ray profiles. Contrary to the usual concept, we assume that there is no such dark matter in form of halos surrounding the galaxies or the clusters but the visible matter is all that contributes. Since the gravitational fields inside the galaxies and clusters are weak, the metric can be completely specified by two unknown potentials: the Newtonian potential \( \Phi(r) \) and the relativistic one \( \Psi(r) \), in isotropic coordinates as \[ ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi + 2\Psi)(dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2) \] (3.1)

The Einstein tensor components are listed below.

\[
G^0_0 = -2\nabla^2(\Phi - \Psi), \quad G^r_r = 2\frac{\Psi'}{r}, \quad G^\theta_\theta = G^\phi_\phi = \Psi'' + \frac{\Psi'}{r}
\] (3.2)

In the usual Newtonian analysis based on the standard Einstein equation, and considering that \( T_{00} = -\rho c^2 \) is the only non-zero component, \( \Psi(r) \) comes out to be zero and we have

\[
\nabla^2 \Phi = \frac{4\pi G}{c^2} \rho
\] (3.3)

where \( \Phi \) is determined directly from observations of either rotation curves (for spiral galaxies) or X-ray profiles (for galaxy clusters). The mass estimated from Eq (3.3) with \( \Phi(r) \) only is always found to be substantially in excess of the visible matter. To solve the puzzle it was proposed that \( \sim 80\% \) of the matter surrounding the spiral galaxies and clusters in form of halos is dark matter.

Let us now look back to the essential information that emerges from the effective Einstein equation. Here we have an extra term \( E_{\mu\nu} \) which is traceless. We exploit this property by taking trace of Eq (2.2) to obtain \[ \nabla^2(\Phi - 2\Psi) = \frac{4\pi G}{c^2} \rho_v \] (3.4)
The message is now crystal clear. That we were probably by mistake attributing to the Newtonian potential $\Phi$ is, in reality, compensated by $\Psi$, the later being non-zero in the present scenario. One must have noticed that here there is no use of dark matter since the visible matter with density $\rho_v$ is all that contributes.

Solving Eq (3.4) we arrive at \[ \Psi = \frac{1}{2} \Phi - \frac{2\pi G}{c^2} (\nabla^2)^{-1} \rho_v \] (3.5)

The bottomline is that we have a solution for the spiral galaxies and clusters, consistent with observations, without the need for dark matter.

A. Modelling clusters with X-ray emission

Let us now apply the formalism to model galaxy clusters. The observed X-ray emission from hot intra-cluster gas allow us measure its density profile $\rho_g(r)$. The model that best fits this density profile is the isothermal $\beta$ model which reads \[ \rho_g(r) = \rho_0 [1 + (r/r_c)^2]^{-3/2} \] (3.6)

Use of the energy-momentum conservation leads to the following equation

\[ \Phi'(r) = -\frac{kT}{\mu m_p c^2} \frac{d \ln \rho_g}{dr} \] (3.7)

Assuming $\beta = 2/3$ and restricting our analysis to $r \gg r_c$ we obtain the Newtonian potential

\[ \Phi(r) = \frac{2kT}{\mu m_p c^2} \ln \frac{r}{r_c} \] (3.8)

Since there is no dark matter, ie, $\rho_v = \rho_g$ one can substitute the expressions for $\rho_g$ and $\Phi$ into Eq (3.4) to obtain

\[ \Psi(r) = \left[ \frac{kT}{\mu m_p c^2} - \frac{2\pi G \rho_0 r_c^2}{c^2} \right] \ln \frac{r}{r_c} \] (3.9)

that compensates for the extra gravitational acceleration.

B. Explaining rotation curves of spiral galaxies

Inside a spiral galaxy visible matter is mainly distributed in a disk the density profile of which is modelled as
\[ \rho_v = \rho_0 e^{-\gamma R^2} \delta(z) \]  
(3.10)
where \( z = z' \) is the position of the disk and \( \gamma = \frac{1}{r_c} \), the inverse scale length for the disk.

The circular velocity \( v_c(r) \), referred to as “rotation curve”, can be used to obtain the geodesic equation of the neutral hydrogen (HI) clouds rotating inside spiral galaxies.

\[ \Phi'(r) = \frac{v^2_c(r)}{c^2} \frac{1}{r} \]  
(3.11)

The Newtonian potential \( \Phi(r) \) can be readily obtained from the above equation [3]

\[ \Phi(r) = \frac{v^2_c}{c^2} \left[ \ln \left( \frac{r}{r_0} \right) - 1 \right] \]  
(3.12)

Note that the solution matches the exterior Schwarzschild metric at the boundary \( r = r_0 \).

Use of the density profile in Eq (3.4) gives the Green function solution for \( \Phi - 2\Psi \), which reads for large \( R \)

\[ \Phi - 2\Psi = \left[ \frac{8\pi^2 G \rho_0}{\gamma^2 c^2} \right] \frac{1}{\sqrt{R^2 + |z|^2}} \]  
(3.13)

wherefrom it is a trivial job to find out \( \Psi \) as

\[ \Psi(r) = \frac{v^2_c}{2c^2} \left[ \ln \left( \frac{r}{r_0} \right) - 1 \right] - \left[ \frac{4\pi^2 G \rho_0}{\gamma^2 c^2} \right] \frac{1}{r} \]  
(3.14)

Hence far from the origin, the potentials look spherically symmetric, which reveals that the disk appears as a point source to a very distant observer.

IV. GRAVITATIONAL LENSING: ALTERNATIVE PROBE

So far we have learned that the extra dimensional scenario can successfully replace dark matter by geometry. To be confirmed, one has to provide other observational techniques that can discriminate between the two scenarios. Here we suggest that gravitational lensing data can be an alternative probe that can possibly help us determine which scenario is correct.

In the dark matter concept, the deflection angle \( \hat{\alpha}_N \) of a photon passing through the halo is fixed by the Newtonian potential \( \Phi \) only.

\[ \hat{\alpha}_N = 2 \int_s^0 \nabla \perp \Phi \ dl \]  
(4.1)
On the other hand, in the effective Einstein equation scenario where $\Psi \neq 0$, the deflection angle is modified to

$$\hat{\alpha} = \int_s^o \hat{\nabla} \cdot (2\Phi - \Psi) \, dl$$  \hspace{1cm} (4.2)$$

Let us see by how much amount the modified deflection angle differs from its Newtonian value.

A. For clusters

For our choice of clusters, the Newtonian deflection angle is found to be

$$\hat{\alpha}_N = \frac{4\pi kT}{\mu m_p c^2}$$  \hspace{1cm} (4.3)$$

whereas the deflection angle from the proposed scenario turns out to be

$$\hat{\alpha} = \alpha_N \left[ 0.75 + \frac{\pi G \rho_0 r_0^2 \mu m_p}{2kT} \right]$$  \hspace{1cm} (4.4)$$

B. For spiral galaxies

In case of spiral galaxies, the Newtonian deflection angle is the usual one

$$\hat{\alpha}_N = \frac{2\pi v_c^2}{c^2}$$  \hspace{1cm} (4.5)$$

and the modified deflection angle is given by

$$\hat{\alpha} = \alpha_N \left[ 0.75 - \frac{4\pi G \rho_0}{\gamma^2 v_c^2 b} \right]$$  \hspace{1cm} (4.6)$$

The calculations above reveal that the deflection angle in the present scenario is around $\sim 75\%$ of the Newtonian value. Unfortunately, this difference falls within the present experimental error bars. We look forward for sufficiently accurate lensing data for a more conclusive remark.

V. SUMMARY AND OUTLOOK

Here we have proposed an alternative to dark matter in astrophysics. We note the standard Einstein theory of gravity has not been tested on galactic length scales, yet it has
been applied to estimate the mass of galaxies and clusters, that give rise to an inevitable prediction of dark matter. In stead of using the standard Einstein equation, we have applied its modified version, namely, the effective Einstein equation to explain the observed rotation curves of spiral galaxies and the X-ray emission from galaxy clusters. Thus, even in absence of dark matter, these astrophysical observations can possibly be realised as the effect of the embedding geometry alone.

We have also succeeded in providing an alternative observational technique that can discriminate between the two scenarios. This technique, namely, gravitational lensing calculation, reveals that the proposed scenario differs by a factor of $\sim 25\%$ from the dark matter analysis, that can be subjected to observational verification in future.

To conclude, our basic goal was to explain astrophysical observations without dark matter. But the context of dark matter also arises in cosmology to explain the expanding universe. Even models of galaxies with relativistic stresses are around \[25\]. It remains as further work to see if these aspects of gravity can be explained by the extra dimensional effects.

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