Measurement of W Polarisation at LEP

The L3 Collaboration

Abstract

The three different helicity states of W bosons produced in the reaction $e^+e^- \rightarrow W^+W^- \rightarrow ℓνqq'$ at LEP are studied using leptonic and hadronic W decays. Data at centre-of-mass energies $\sqrt{s} = 183 - 209$ GeV are used to measure the polarisation of W bosons, and its dependence on the W boson production angle. The fraction of longitudinally polarised W bosons is measured to be $0.218 \pm 0.027 \pm 0.016$ where the first uncertainty is statistical and the second systematic, in agreement with the Standard Model expectation.

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Introduction

The existence of all three W boson helicity states, +1, −1 and 0, is a consequence of the non-vanishing mass of the W boson, that, in the Standard Model [1], is generated by the Higgs mechanism of electroweak symmetry breaking. The measurement of the fractions of longitudinally and transversely polarised W bosons constitutes a test of the Standard Model predictions for the triple gauge boson couplings γWW and ZWW.

To determine the W helicity fractions, events of the type $e^+e^- \rightarrow W^+W^- \rightarrow \ell\nu qq'$ are used, with $\ell$ denoting either an electron or a muon. These events are essentially background free and allow a measurement, with good accuracy, of the W momentum vector, the W charge and the polar decay angles. The W helicity states are accessible in a model independent way through the shape of the distributions of the polar decay angle, $\theta^*_\ell$, between the charged lepton and the W direction in the W rest frame. Transversely polarised W bosons have angular distributions $(1 \mp \cos \theta^*_\ell)^2$ for a $W^-$ with helicity ±1, and $(1 \pm \cos \theta^*_\ell)^2$ for a $W^+$ with helicity ±1. For longitudinally polarised W bosons, a $\sin^2 \theta^*_\ell$ dependence is expected. For simplicity, we refer in the following only to the fractions $f_-$, $f_+$ and $f_0$ of the helicity states −1, +1 and 0 of the $W^-$ boson, respectively. Assuming CP invariance these equal the fractions of the corresponding helicity states +1, −1 and 0 of the $W^+$ boson.

The differential distribution of leptonic $W^-$ decays at Born level is:

$$\frac{1}{N} \frac{dN}{d \cos \theta^*_\ell} = f_- \frac{3}{8} (1 + \cos \theta^*_\ell)^2 + f_+ \frac{3}{8} (1 - \cos \theta^*_\ell)^2 + f_0 \frac{3}{4} \sin^2 \theta^*_\ell. \quad (1)$$

For hadronic W decays, the quark charge is difficult to reconstruct experimentally and only the absolute value of the cosine of the decay angle, $|\cos \theta^*_q|$, is used:

$$\frac{1}{N} \frac{dN}{d |\cos \theta^*_q|} = f_+ \frac{3}{4} (1 + |\cos \theta^*_q|^2) + f_0 \frac{3}{2} (1 - |\cos \theta^*_q|^2), \quad (2)$$

with $f_+ = f_+ + f_-$. After correcting the data for selection efficiencies and background, the different fractions of W helicity states are obtained from a fit to these distributions. The fractions $f_-$, $f_+$ and $f_0$ are also determined as a function of the $W^-$ production angle $\Theta_{W^-}$ in the laboratory frame. The helicity composition of the W bosons depends strongly on the centre-of-mass energy, $\sqrt{s}$.

Data and Monte Carlo

The analysis presented in this Letter is based on the whole data set collected with the L3 detector [2] and supersedes our previous results [3] based on about one third of the data. An integrated luminosity of 684.8 pb$^{-1}$, collected at different centre-of-mass energies between 183 GeV and 209 GeV, as shown in Table 1, is analysed.

The $e^+e^- \rightarrow W^+W^- \rightarrow e\nu qq'$, $\mu\nu qq'$ Monte Carlo events are generated using KORALW [4]. The Standard Model predictions for $f_-$, $f_+$ and $f_0$ are obtained from these samples by fitting the generated decay angular distributions for each value of $\sqrt{s}$. As an example, the expected fraction of longitudinally polarised W bosons changes from 0.271 at $\sqrt{s} = 183$ GeV to 0.223 at $\sqrt{s} = 206$ GeV. The luminosity averaged Standard Model expectations for $f_-$, $f_+$ and $f_0$ are 0.590, 0.169 and 0.241, respectively.

Background processes are generated using KORALW for W pair production decaying to other final states, and PYTHIA [5] and KK2F [6] for $e^+e^- \rightarrow q\bar{q}(\gamma)$. For studies of systematic
effects, signal events are also generated using EEWW \[7\] and EXCALIBUR \[8\]. The L3 detector response is simulated with the GEANT \[9\] and GEISHA \[10\] packages. Detector inefficiencies, as monitored during the data taking period, are included.

A large sample of signal events is generated using the EEWW Monte Carlo program. This program assigns, differently from KORALW, W helicities on an event-by-event basis but uses the zero-width approximation for the W boson and does not include higher order radiative corrections and interference terms. The W$^-$ helicity fractions obtained from a fit to the generated decay angle distributions agree with the input values. A comparison of the fractions obtained from EEWW and YFSWW \[11\], which includes improved $O(\alpha)$ corrections, with those obtained from KORALW also shows good agreement. Therefore the Born level formulæ (1) and (2) are applicable after radiative corrections.

**Selection of $W^+W^- \to e\nu\bar{q}q'$, $\mu\nu\bar{q}q'$ events**

Only events which contain exactly one electron or one muon candidate are accepted \[3\]. Electrons are identified as isolated energy depositions in the electromagnetic calorimeter with an electromagnetic shower shape. A match in azimuthal angle with a track reconstructed in the central tracking chamber is required. Muons are identified and measured as tracks reconstructed in the muon chambers which point back to the interaction vertex. All other energy depositions in the calorimeters are assumed to originate from the hadronically decaying W. The neutrino momentum vector is assumed to be the missing momentum vector of the event. The following additional criteria are applied:

- The reconstructed momentum must be greater than 20 GeV for electrons and 15 GeV for muons.
- The neutrino momentum must be greater than 10 GeV and its polar angle, $\theta_\nu$, has to satisfy $|\cos \theta_\nu| < 0.95$.
- The invariant mass of the lepton-neutrino system has to be greater than 60 GeV.
- The invariant mass of the hadronic system has to be between 50 and 110 GeV.

Figure 1 shows some distributions of those variables for data and Monte Carlo.

The number of events selected by these criteria are listed in Table 1. In total, 2010 events are selected with an efficiency of 65.7% and a purity of 96.3%. The contamination from $W^+W^-\to\tau\nu\bar{q}q'$ and $e^+e^-\to q\bar{q}(\gamma)$ is 2.4% and 1.3%, respectively, independent of $\sqrt{s}$ and the W production angle.

**Analysis of the W helicity states**

For the selected events, the rest frames of the W bosons are calculated from the lepton and neutrino momenta, and the decay angles $\theta^*_\ell$ and $\theta^*_q$ of the lepton and the quarks are determined. The angle $\theta^*_q$ is approximated by the polar angle of the thrust axis with respect to the W direction in the rest frame of the hadronically decaying W.

The fractions of the W helicity states are obtained from the event distributions, $dN/d\cos \theta^*_\ell$ and $dN/d|\cos \theta^*_q|$. For each energy point, the background, as obtained from Monte Carlo simulations, is subtracted from the data, and the resulting distributions are corrected for selection
efficiencies as obtained from large samples of KORALW Monte Carlo events. The corrected
decay angle distributions at the different centre-of-mass energies are combined into single
distributions for leptonic and hadronic decays, which are then fitted to the functions \((1)\) and \((2)\),
respectively. A binned fit is performed on the normalised distributions, shown in Figure 2
using \(f_-\) and \(f_0\) as the fit parameters. The fraction \(f_+\) is obtained by constraining the sum of
all three parameters to unity.

Detector resolution introduces migration effects that bias the fitted parameters. For example,
purely longitudinally polarised leptonically decaying W bosons at \(\sqrt{s} = 206\) GeV would
be measured to have a helicity composition: \(f_0 = 0.945, f_- = 0.043\) and \(f_+ = 0.012\). The
magnitude of these effects depends on the helicity fractions and on \(\sqrt{s}\). Corrections for this
bias as a function of the helicity fractions are determined from EEWW Monte Carlo samples.
If the ratio of two helicity fractions is constant the bias correction function of the third fraction
is linear to a good approximation. For the correction of \(f_0\) in the hadronic W decay, the ratio
\(f_-/f_+\) is taken from the measurement in the leptonic W decay, as only the sum of \(f_+\) and \(f_-\) is
known from hadronic decays. Bias correction functions are determined for the analysis of the
complete data sample, separately for the W\(^+\) and W\(^-\) events and in bins of the W\(^-\) production
angle.

Results

The results of the fits to the decay angle distributions for leptonic and hadronic W decays are
shown in Figure 2. The data are well described only if all three W helicity states are used. Fits
omitting the helicity 0 state fail to describe the data. For leptonic W decays, the \(\chi^2\) increases
from 12.7 for eight degrees of freedom if all helicity states are included to 56.2 for nine degrees
of freedom if only the helicities +1 and −1 are used in the fit. For hadronic W decays, the \(\chi^2\)
increases from 6.6 for four degrees of freedom if all helicity states are included to 59.1 for four
degrees of freedom if only the helicities ±1 are used.

The measured fractions of the W helicity states in data, at an average centre-of-mass energy
\(\sqrt{s} = 196.7\) GeV, are presented together with the Standard Model expectation in Tables 2, 3
and 4. The parameters \(f_-\) and \(f_0\) derived from the fit are about 90% anti-correlated. These
results include a bias correction of 0.005 on \(f_0\) for leptonic decays and 0.044 for hadronic
decays. The bias correction adds 0.003 to the statistical uncertainty on \(f_0\) for leptonic decays
and 0.007 to the one for hadronic decays. The measured W helicity fractions agree with
the Standard Model expectations for the leptonic and hadronic decays, as well as for the
combined sample. Longitudinal W polarisation is observed with a significance of seven standard
deviations, including systematic uncertainties.

A number of systematic uncertainties are considered. These include selection criteria, bin-
ning effects, bias corrections, the contamination due to non double resonant four fermion pro-
cesses, background levels, and efficiencies. Selection cuts are varied over a range of one standard
deviation of the corresponding reconstruction accuracy. Fits are repeated with one bin more or
one bin less in the decay angle distributions. Uncertainties on the bias and efficiency corrections
are determined with large Monte Carlo samples, the latter being negligible. The contamina-
tion due to non double resonant four fermion processes is studied by using the EXCALIBUR
Monte Carlo. Background levels are varied according to Monte Carlo statistics for both the
\(e^+e^-\rightarrow W^+W^-\rightarrow \tau\nu qq'\) and \(e^+e^-\rightarrow q\bar{q}(\gamma)\) processes. The largest uncertainties arise from selec-
tion criteria and binning effects. As an example, Table 5 summarises those effects on \(f_0\).

Within the Standard Model, CP symmetry is conserved in the reaction \(e^+e^-\rightarrow W^+W^-\) and
the helicity fractions $f_+, f_-$ and $f_0$ for the $W^+$ are expected to be identical to the fractions $f_-, f_+$ and $f_0$, for the $W^-$, respectively. CP invariance is tested by measuring the helicity fractions for $W^+$ and $W^-$ separately. The charge of the $W$ bosons is obtained from the charge of the lepton. We select 1020 $W^+ \rightarrow \ell^+ \nu$, and 990 $W^- \rightarrow \ell^- \bar{\nu}$ events. Results of separate fits for the $W^-$ helicity fractions are given in Tables 2, 3 and 4 for leptonic, hadronic and combined fits. Good agreement is found, consistent with CP invariance.

To test the variation of the helicity fractions with the $W^-$ production angle, $\Theta_{W^-}$, the data are grouped in four bins of $\cos \Theta_{W^-}$. The ranges have been chosen such that large and statistically significant variations of the different helicity fractions are expected. Figure 3 shows the four decay angle distributions for the leptonic $W$ decays. The corrected distributions are fitted for leptonic and hadronic $W$ decays separately in each bin of $\cos \Theta_{W^-}$. The fit results, combining leptonic and hadronic $W$ decays, are shown in Table 4 and Figure 4 together with the Standard Model expectations from the KORALW Monte Carlo. The results agree with the Standard Model expectation and demonstrate a strong variation of the $W$ helicity fractions with the $W^-$ production angle.

In conclusion, all three helicity states of the $W$ boson are required in order to describe the data. Their fractions and their variations as a function of $\cos \Theta_{W^-}$ are in agreement with the Standard Model expectation. The fraction of longitudinally polarised $W$ bosons at $\sqrt{s} = 183 - 209$ GeV is measured as $0.218 \pm 0.027 \pm 0.016$. Separate analyses of the $W^+$ and $W^-$ events are consistent with CP conservation.

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Table 1: Average centre-of-mass energies, integrated luminosities and numbers of selected events.

|       | 182.7 | 188.6 | 191.6 | 195.5 | 199.5 | 201.8 | 205.9 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| √s [GeV] | 55.5  | 176.8 | 29.8  | 84.1  | 83.3  | 37.2  | 218.1 |
| Integrated luminosity [pb⁻¹] | 18.5  | 5.8   | 13.3  | 11.0  | 5.9   | 3.5   | 35.5  |
| Selected eνq̄q′ events | 82    | 293   | 59    | 133   | 110   | 56    | 355   |
| Selected μνq̄q′ events | 67    | 255   | 43    | 110   | 99    | 59    | 289   |

Table 2: W⁻ helicity fractions for the leptonic decays for the combined data sample. All the helicities are converted to W⁻ parameters using CP invariance. The first uncertainty is statistical, the second systematic. The corresponding helicity fractions in the Standard Model as implemented in the KORALW Monte Carlo program are also given with their statistical uncertainties.

| Sample       | f⁻     | f⁺     | f₀      |
|--------------|--------|--------|---------|
| W⁻→ℓ⁻ν Data | 0.559  ± 0.038 ± 0.016 | 0.201 ± 0.026 ± 0.015 | 0.240 ± 0.051 ± 0.017 |
| W⁺→ℓ⁺ν Data | 0.625  ± 0.037 ± 0.016 | 0.179 ± 0.023 ± 0.015 | 0.196 ± 0.050 ± 0.017 |
| W±→ℓ±ν Data | 0.589  ± 0.027 ± 0.016 | 0.189 ± 0.017 ± 0.015 | 0.221 ± 0.036 ± 0.017 |
| Monte Carlo  | 0.592  ± 0.003 | 0.170 ± 0.002 | 0.238 ± 0.004 |

Table 3: W⁻ helicity fractions for the hadronic decays for the combined data sample. All the helicities are converted to W⁻ parameters using CP invariance. The first uncertainty is statistical, the second systematic. The corresponding helicity fractions in the Standard Model as implemented in the KORALW Monte Carlo program are also given with their statistical uncertainties.

| Sample       | f⁻       | f⁺       | f₀        |
|--------------|----------|----------|-----------|
| W⁻→hadrons Data | 0.750 ± 0.056 ± 0.039 | 0.250 ± 0.056 ± 0.039 |
| W⁺→hadrons Data | 0.833 ± 0.062 ± 0.039 | 0.167 ± 0.062 ± 0.039 |
| W±→hadrons Data | 0.785 ± 0.042 ± 0.039 | 0.215 ± 0.042 ± 0.039 |
| Monte Carlo  | 0.757 ± 0.004 | 0.243 ± 0.004 |
Table 4: W⁻ helicity fractions, measured combining leptonic and hadronic decays. All the helicities are converted to W⁻ parameters using CP invariance. The first uncertainty is statistical, the second systematic. The corresponding helicity fractions in the Standard Model as implemented in the KORALW Monte Carlo program are also given with their statistical uncertainties.

|         | f⁻         | f⁺         | f₀         |
|---------|------------|------------|------------|
| W⁻ Data| 0.555 ± 0.037 ± 0.016 | 0.200 ± 0.026 ± 0.015 | 0.245 ± 0.038 ± 0.016 |
| W⁺ Data| 0.634 ± 0.038 ± 0.016 | 0.181 ± 0.024 ± 0.015 | 0.185 ± 0.039 ± 0.016 |
| W⁻ Data| 0.592 ± 0.027 ± 0.016 | 0.190 ± 0.017 ± 0.015 | 0.218 ± 0.027 ± 0.016 |
| Monte Carlo| 0.590 ± 0.003 | 0.169 ± 0.002 | 0.241 ± 0.003 |

Table 5: Systematic uncertainties on the measurement of f₀ for leptonic and hadronic W decays.

| Selection          | W→ℓν | W→hadrons |
|--------------------|------|-----------|
| Binning effects    | 0.013| 0.024     |
| Bias correction    | 0.007| 0.029     |
| Four fermion contamination | 0.006| 0.011     |
| Background corrections | 0.004| 0.001     |
| Total              | 0.017| 0.039     |

Table 6: The W⁻ helicity fractions measured as a function of cos Θ_{W⁻} combining leptonic and hadronic W decays. The first uncertainty is statistical, the second systematic. The KORALW Monte Carlo expectations are also given with their statistical uncertainties.

$$
\begin{array}{cccc}
\text{cos } \Theta_{W^-} & \text{Fraction} & \text{W± Data} & \text{Monte Carlo} \\
\hline
[-1.0, -0.3] & f_- & 0.173 ± 0.041 ± 0.033 & 0.156 ± 0.006 \\
 & f_+ & 0.418 ± 0.060 ± 0.043 & 0.431 ± 0.008 \\
 & f_0 & 0.409 ± 0.082 ± 0.051 & 0.413 ± 0.008 \\
[-0.3, 0.3] & f_- & 0.509 ± 0.055 ± 0.029 & 0.446 ± 0.006 \\
 & f_+ & 0.303 ± 0.040 ± 0.032 & 0.282 ± 0.005 \\
 & f_0 & 0.188 ± 0.060 ± 0.043 & 0.272 ± 0.006 \\
[0.3, 0.9] & f_- & 0.683 ± 0.042 ± 0.026 & 0.723 ± 0.004 \\
 & f_+ & 0.135 ± 0.027 ± 0.030 & 0.119 ± 0.003 \\
 & f_0 & 0.182 ± 0.039 ± 0.027 & 0.158 ± 0.004 \\
[0.9, 1.0] & f_- & 0.708 ± 0.093 ± 0.056 & 0.647 ± 0.007 \\
 & f_+ & -0.010 ± 0.055 ± 0.028 & 0.029 ± 0.004 \\
 & f_0 & 0.302 ± 0.082 ± 0.059 & 0.324 ± 0.007 \\
\end{array}
$$
Figure 1: Distributions used for the event selections: (a) momentum of the electrons, (b) momentum of the muons, (c) absolute value of the cosine of the polar angle of the neutrino, (d) mass of the lepton-neutrino system. In each plot, all other selection criteria are applied. The arrows indicate cut positions.
Figure 2: Corrected decay angle distributions for (a) leptonic W decays and (b) for hadronic W decays at $\sqrt{s} = 183 - 209$ GeV. Fit results for the different W helicity hypotheses are also shown.
Figure 3: Corrected decay angle distributions for leptonic W decays separated into four different $W^-$ production angle ranges, together with the KORALW expectation and fit results. Assuming CP invariance, $W^+$ decays are included with $\cos \Theta_{W^-} = -\cos \Theta_{W^+}$. 
Figure 4: W helicity fractions $f_-, f_+$ and $f_0$ and their statistical uncertainties for four different bins of $\cos \Theta_W$ in the combined data sample and in the KORALW Monte Carlo for $\sqrt{s} = 183 - 209$ GeV.