Gauge Coupling Unification
with Anomalous $U(1)_A$ Gauge Symmetry

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Abstract

Recently we proposed a natural scenario of grand unified theories with anomalous $U(1)_A$ gauge symmetry, in which doublet-triplet splitting is realized in $SO(10)$ unification using Dimopoulos-Wilczek mechanism and realistic quark and lepton mass matrices can be obtained in a simple way. The scenario has an additional remarkable feature that the symmetry breaking scale and the mass spectrum of super heavy particles are determined essentially by anomalous $U(1)_A$ charges. Therefore once all the anomalous $U(1)_A$ charges are fixed, the gauge coupling flows can be calculated. We examine several models in which the gauge coupling unification is realized. Examining the conditions for the coupling unification, we show that when all the fields except those of the minimal SUSY standard model become super-heavy, the unification scale generically becomes just below the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV and the cutoff scale becomes around $\Lambda_G$. Since the lower GUT scale leads to shorter life time of nucleon, the proton decay via dimension six operator $p \to e^+\pi^0$ can be seen in future experiment. On the other hand, the lower cutoff scale than the Planck scale may imply the existence of extra dimension in which only gravity modes can propagate.

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1 Introduction

There is strong evidence supporting grand unified theories (GUT)\cite{1}, in which the quarks and leptons are unified in several multiplets in a simple gauge group. They explain various matters that cannot be understood within the standard model: the miracle of anomaly cancellation between quarks and leptons, the hierarchy of gauge couplings, charge quantization, etc. The three gauge groups in the standard model are unified into a simple gauge group at a GUT scale, which is considered to be just below the Planck scale. On the other hand, the GUT scale destabilizes the weak scale. One of the most promising ways to avoid this problem is to introduce supersymmetry (SUSY). One of the most important successes of SUSY is regarded as the gauge coupling unification. In the minimal SUSY standard model (MSSM), three gauge couplings meet at a single scale $\Lambda_G \sim 2 \times 10^{16}$ GeV.

However, it is not easy to obtain a realistic SUSY GUT\cite{2}. First, it is difficult to obtain realistic fermion mass matrices in a simple way. In particular, unification of quarks and leptons puts strong constraints on the Yukawa couplings. But concerning the fermion masses, recent progress in neutrino experiments\cite{3} provides important information on family structure. There are several impressive works\cite{4, 5, 6, 7, 8, 9} in which the large neutrino mixing angle is realized within GUT framework. It is now natural to examine $SO(10)$ and higher gauge groups, because they allow for every quark and lepton, including the right-handed neutrino, to be unified in a single multiplet, which is important in addressing neutrino masses.

Second, one of the most difficult obstacles is the “doublet-triplet (DT) splitting problem”. Generally, a fine-tuning is required to obtain the light $SU(2)_L$ doublet Higgs multiplet of the weak scale while keeping the triplet Higgs sufficiently heavy to suppress the dangerous proton decay. There have been several attempts to solve this problem.\cite{10, 11} Among them, the Dimopoulos-Wilczek mechanism is a promising way to realize DT splitting in the $SO(10)$ SUSY GUT.\cite{11, 12, 13, 14}

Finally, there is a rather theoretical problem, which has not been emphasized so much in the literature. If we adopt an adjoint Higgs field $A$ to break the GUT gauge group, the superpotential is generically given by $W = \sum_n A^n$. In the vacua, the GUT gauge group is generically broken to $U(1)^r$, where $r$ is the rank of the GUT gauge group. It is unnatural to obtain the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ by this superpotential. At least for $SU(5)$ unification, we can impose renormalizability to avoid this problem. Then the superpotential becomes $W = A^2 + A^3$, which naturally gives the standard gauge group below the GUT scale. However, for $SO(10)$ or $E_6$ unification, it is not workable because $A^3$ is not allowed under the gauge symmetry. Moreover, in the context of Wilsonian renormalization group, renormalizability is not a principle to be imposed, but a resulting feature which low energy effective theories happen to obtain. When the cutoff scale is much higher than the typical scale of the
theory, higher dimensional operators become irrelevant, and only a few operators are important to determine the low energy physics. However, since the GUT scale is near the Planck scale, it is not natural to impose the renormalizability on the grand unified theories. Therefore we have to answer the question why the non-Abelian gauge group remains at the low energy scale, or, why the allowed interactions in the superpotential are restricted.

Recently we proposed a scenario of $SO(10)$ (also $E_6$) grand unified theory (GUT) with anomalous $U(1)_A$ gauge symmetry[15, 16], which has the following interesting features;

1. The interaction is generic, namely, all the interactions, which are allowed by the symmetry, are introduced. Therefore, once we fix the field contents with their quantum numbers (integer), all the interactions are determined except the coefficients of order one.

2. Even with generic interaction, non-Abelian gauge group at the low energy scale can be obtained. (Necessary restriction to the superpotential is realized by SUSY zero mechanism.)

3. The Dimopoulos-Wilczek mechanism, which realizes the doublet-triplet (DT) splitting, is naturally embedded.

4. The proton decay via the dimension-five operator is suppressed.

5. Realistic quark and lepton mass matrices can be obtained in a simple way. In particular, in the neutrino sector, bi-large neutrino mixing is realized.

6. The symmetry breaking scales are determined by the anomalous $U(1)_A$ charges.

7. The mass spectrum of the super heavy particles is fixed by the anomalous $U(1)_A$ charges.

As a consequence of the above features, the fact that the GUT scale is smaller than the Planck scale leads to modification of the undesired GUT relation between the Yukawa couplings $y_\mu = y_s$ (and also $y_e = y_d$) while preserving $y_\tau = y_\nu$.

The anomalous $U(1)_A$ gauge symmetry,[17] whose anomaly is cancelled by the Green-Schwarz mechanism,[18] plays an essential role in explaining the DT splitting mechanism at the unification scale and the restriction of the interactions in the superpotential as well as in reproducing Yukawa hierarchies[19, 20, 21]. Unfortunately to solve the DT splitting problem, several super-heavy particles become lighter than the GUT scale. Generically, the existence of these fields destroys the success of the gauge coupling unification. However, the spectrum of the super heavy particles are determined by the anomalous $U(1)_A$ charges, we can calculate the running gauge couplings and easily examine whether these
couplings meet at an unified scale or not. In other words, requirement of gauge coupling unification gives some constraints on the anomalous $U(1)_A$ charges. In this paper, we examine the constraints and try to find out models in which gauge coupling unification is realized. It is suggestive that when all the other fields but those of the MSSM become super-heavy, only a condition leads to the gauge coupling unification. It is interesting that the cutoff scale becomes the usual GUT scale $\Lambda_G$ and the unified scale becomes just below the scale $\Lambda_G$.

In section 2, we explain how the SUSY vacua are determined in the anomalous $U(1)_A$ framework. Using this argument, we recall the discussion of the DT splitting mechanism in section 3, and the resulting mass spectrum of super-heavy particles in section 4. In section 5, we review how to determine the anomalous $U(1)_A$ charges to realize Quark and Lepton mass matrices and bi-large neutrino mixing angles. These have been discussed in Ref. [15]. In section 6, we briefly explain how to solve the $\mu$ problem in our scenario, following the discussion in Ref. [22]. In section 7, we discuss the conditions for gauge coupling unification and in section 8, we examine several models in which these conditions are satisfied.

2 Vacuum determination

In this section, we explain how the vacua of the Higgs fields are determined by the anomalous $U(1)_A$ quantum numbers.

First, we show that none of the fields with positive anomalous $U(1)_A$ charge acquire non-zero VEV if the Froggatt-Nielsen (FN) mechanism [23] acts effectively in the vacuum. For simplicity, we here introduce just gauge singlet fields $Z^\pm_i$ ($i = 1, 2, \cdots n_{\pm}$) with charges $z^\pm_i$ ($z^+_i > 0$ and $z^-_i < 0$). From the $F$-flatness conditions of the superpotential, we get $n = n_+ + n_-$ equations plus one $D$-flatness condition,

$$\frac{\delta W}{\delta Z_i} = 0, \quad D_A = g_A \left(\sum_i z_i |Z_i|^2 + \xi^2\right) = 0, \quad (2.1)$$

where $\xi^2 = \frac{g^2_{\text{tr}} Q_4}{192 \pi^2} (\equiv \lambda^2 \Lambda^2)$ is the coefficient of Fayet-Iliopoulos $D$-term and $\Lambda$ is a cutoff scale of the theory. Throughout this paper we use a unit in which $\Lambda = 1$ and denote all the superfields with uppercase letters and their anomalous $U(1)_A$ charges with the corresponding lowercase letters. At first glance, these look to be over determined. However, the $F$-flatness conditions are not independent, because the gauge invariance of the superpotential $W$ leads to the relation

$$\frac{\delta W}{\delta Z_i} z_i Z_i = 0. \quad (2.2)$$

Therefore, generically a SUSY vacuum with $\langle Z_i \rangle \sim \Lambda$ exists (Vacuum a), because the coefficients of the above conditions are generically of order 1. However, if
\( n_+ \leq n_- \), we can choose another vacuum (Vacuum b) with \( \langle Z_i^+ \rangle = 0 \), which automatically satisfies the \( F \)-flatness conditions \( \frac{\delta W}{\delta Z_i^+} = 0 \). Then the \( \langle Z_i^- \rangle \) are determined by the \( F \)-flatness conditions \( \frac{\delta W}{\delta Z_i^-} = 0 \) with the constraint \( (2.2) \) and the \( D \)-flatness condition \( D_A = 0 \). Note that if \( \xi < 1 \) (In this paper, we take \( \xi = \lambda \sim 0.2 \)), the VEVs of \( Z_{-i} \) are less than the cutoff scale. This can lead to the Froggatt-Nielsen mechanism. If we fix the normalization of \( U(1)_A \) gauge symmetry so that the largest value \( z_{-i} \) in the negative charges \( z_{-i} \) equals -1, then the VEV of the field \( Z_{-1}^- \) is determined from \( D_A = 0 \) as \( \langle Z_{-1}^- \rangle \sim \lambda \), which breaks \( U(1)_A \) gauge symmetry. (We explain later why we take the field \( Z_{-1}^- \) with the largest charges \( z_{-1} \).) In the following, we denote the Froggatt-Nielsen field \( Z_{-1}^- \) as \( \Theta \). On the other hand, other VEVs are determined by the \( F \)-flatness conditions of \( Z_i^+ \) as \( \langle Z_i^- \rangle \sim \lambda^{-z_i} \), which is shown below. Since \( \langle Z_i^+ \rangle = 0 \), it is sufficient to examine the terms linear in \( Z_i^+ \) in the superpotential in order to determine \( \langle Z_i^- \rangle \). Therefore, in general the superpotential to determine the VEVs can be written

\[
W = \sum_{i} W_{Z_i^+},
\]

\[
W_{Z_i^+} = \lambda_{z_i}^{z_i} Z_i^+ \left( \sum_j \lambda_{z_j}^{z_j} Z_j^- + \sum_{j,k} \lambda_{z_j}^{z_j+} Z_j^- Z_k^- + \cdots \right) \quad (2.4)
\]

\[
= \sum_{i} \tilde{Z}_i^+ \left( \sum_j \tilde{Z}_j^- + \sum_{j,k} \tilde{Z}_j^- \tilde{Z}_k^- + \cdots \right),
\]

where \( \lambda = \langle \Theta \rangle \) and \( \tilde{Z}_i \equiv \lambda_{z_i}^{-z_i} Z_i \). The \( F \)-flatness conditions of the \( Z_i^+ \) fields require

\[
\lambda_{z_i}^{z_i} \left( 1 + \sum_j \tilde{Z}_j^- + \cdots \right) = 0, \quad (2.6)
\]

which generally lead to solutions \( \tilde{Z}_j^- \sim O(1) \) if these \( F \)-flatness conditions determine the VEVs. Thus the \( F \)-flatness condition requires

\[
\langle Z_j^- \rangle \sim O(\lambda^{-z_j^-}). \quad (2.7)
\]

Note that if \( n_+ = n_- \), generically all the VEVs of \( Z_{-i}^- \) are fixed, therefore there appears no flat direction in the potential. It means that there is no massless field. On the other hand, if \( n_+ < n_- \), generally the \( n_+ \) equations of \( F \)-flatness and \( D \)-flatness conditions do not determine all the VEVs of \( n_- \) fields \( Z_{-i}^- \). Therefore, there are flat directions in the potential, namely there must be some massless fields. Roughly speaking, if we would like to realize no massless mode in the
Higgs sector, \( n_+ = n_- \) must be imposed in Higgs sector. \(^1\)

Here we have examined the VEVs of singlets fields, but generally the gauge invariant operator \( O \) with negative charge \( o \) has non-vanishing VEV \( \langle O \rangle \sim \lambda^{-o} \) if the \( F \)-flatness conditions determine the VEV. For example, let us introduce spinors \( C(16) \) and \( \bar{C}(\overline{16}) \) of \( SO(10) \). The VEV of the gauge singlet operator \( \bar{C}C \) is estimated as \( \langle \bar{C}C \rangle \sim \lambda^{-\left(c + \bar{c}\right)} \). The \( D \)-flatness condition of \( SO(10) \) gauge theory requires

\[
\langle C \rangle = \langle \bar{C} \rangle \sim \lambda^{-\left(c + \bar{c}\right)/2}.
\]

(2.8)

Note that these VEVs are also determined by the anomalous \( U(1)_A \) charges but they are different from the naive expectation \( \langle C \rangle \sim \lambda^{-c} \). This is because the \( D \)-flatness condition plays an important role to fix the VEVs.

Note that if there is another field \( Z_i^- \) which has smaller charge than the FN field \( Z_{-1} \), then the VEV of \( Z_i^- \) becomes larger than the \( \xi \), which is inconsistent with \( D \)-flatness condition. Therefore we have to take the field with the largest negative charge as the FN field.

If Vacuum a is selected, the anomalous \( U(1)_A \) gauge symmetry is broken at the Planck scale, and the FN mechanism does not act. Therefore, we cannot know the existence of the \( U(1)_A \) gauge symmetry from the low energy physics. On the other hand, if Vacuum b is selected, the FN mechanism acts effectively and we can understand the signature of the \( U(1)_A \) gauge symmetry from the low energy physics. Therefore, it is natural to assume that Vacuum b is selected in our scenario, in which the \( U(1)_A \) gauge symmetry plays an important role for the FN mechanism. The VEVs of the fields \( Z_i^+ \) vanish, which guarantees that the SUSY zero mechanism\(^2\) acts effectively.

In summary,

1. Gauge singlet operators with positive total charge have vanishing VEVs, in order that the FN mechanism acts effectively. This guarantees that the SUSY zero mechanism works well.

2. The \( F \)-flatness conditions of fields with positive charges determine the VEVs of singlet operators \( O \) with negative total charges \( o \) as \( \langle O \rangle \sim \lambda^{-o} \), while the \( F \)-flatness conditions of fields with negative charges are automatically satisfied.

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\(^1\) This rough argument of number counting is based on an assumption that the Higgs sector has no other structure by which the freedom of \( F \)-flatness conditions is reduced as realized in this section by taking \( \langle Z \rangle = 0 \). Such a structure is easily realized by imposing some symmetry, for example, \( Z_2 \) parity or \( R \) parity. However, it is obvious that \( n_+ \geq n_- \) is required to make all the Higgs fields super-heavy.

\(^2\) Note that if total charge of an operator is negative, the \( U(1)_A \) invariance forbids the operator in the superpotential since the field \( \Theta \) with negative charge cannot compensate for the negative total charge of the operator (SUSY zero mechanism).
3. If the number of the fields with positive charges equals to that of the fields with negative charges, generically no massless field appears.

4. General superpotential to determine the VEVs is the following structure $W = \sum_i W^+_i$, where $W^+_i$ is linear in the field $Z^+_i$ with positive charges.

# 3 Doublet-triplet splitting mechanism

In this section, we review the mechanism which naturally realizes the doublet-triplet splitting in $SO(10)$ unified scenario \cite{15}.

The contents of the Higgs sector with $SO(10) \times U(1)_A$ gauge symmetry is given in Table I, where the symbols $\pm$ denote $Z_2$ parity quantum numbers.

| Negative charge | Positive charge |
|-----------------|----------------|
| $45$ $A(a = -2, -)$ | $A'(a' = 6, -)$ |
| $16$ $C(c = -4, +)$ | $C'(c' = 4, -)$ |
| $\bar{16}$ $\bar{C}(\bar{c} = -1, +)$ | $\bar{C}'(\bar{c}' = 7, -)$ |
| $10$ $H(h = -6, +)$ | $H'(h' = 8, -)$ |
| $1$ $Z(z = -3, -), Z(\bar{z} = -3)$ | $S(s = 5, +)$ |

The adjoint Higgs field $A$, whose VEV $\langle A(45) \rangle_{B-L} = i \tau_2 \times \text{diag}(v, v, v, 0, 0)$ breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This Dimopoulos-Wilczek form of the VEV plays an important role in solving the DT splitting problem. The spinor Higgs fields $C$ and $\bar{C}$, that break $SU(2)_R \times U(1)_{B-L}$ into $U(1)_Y$ by developing $\langle C \rangle = \langle \bar{C} \rangle = \lambda^{-(c+\bar{c})/2}$. The Higgs field $H$ contains usual $SU(2)_L$ doublet. All these Higgs fields must have negative anomalous $U(1)_A$ charges $a, c, \bar{c}$ and $h$ to obtain non-vanishing VEVs because only the fields with negative charge can get non-vanishing VEVs, as discussed in the previous section. On the other hand, in order to give masses to all the Higgs fields, we have to introduce the fields with positive charges, whose freedom must be the same as that of the fields with negative charges.\footnote{Strictly speaking, since some of the Higgs fields are eaten by Higgs mechanism, in principle, less number of positive fields can give superheavy masses to all the Higgs fields. Here we do not examine the possibilities.} Therefore we introduced $A', C', \bar{C}'$ and $H'$, which have positive anomalous $U(1)_A$ charges. Therefore, in a sense, we introduce the minimal Higgs contents here. It is surprising that the mechanism, in which DT splitting is realized, is naturally embedded in such minimal Higgs contents.

As discussed in the previous section, since the fields with non-vanishing VEVs have negative charges, only the $F$-flatness conditions of fields with positive charge must be taken into account for determination of their VEVs. (Generically $c$ or $\bar{c}$
The argument does not change significantly if \( c \) or \( \bar{c} \) is positive. This is because the terms \( C^4 \) or \( \bar{C}^4 \) does not include \( N_C^4 \) or \( N_{\bar{C}}^4 \), where \( N \) is a neutral component under the standard gauge group. We have only to take account of the terms in the superpotential which contain only one field with positive charge. Therefore, in general, the superpotential required by determination of the VEVs can be written as
\[
W = W_{H'} + W_{A'} + W_S + W_{C'} + W_{\bar{C}'}.
\]
Here \( W_X \) denotes the terms linear in the \( X \) field, which has positive anomalous \( U(1)_A \) charge. Note, however, that terms including two fields with positive charge like \( \lambda H' H' \) give contributions to the mass terms but not to the VEVs.

In the following argument, for simplicity, we neglect the terms like \( 16^1, \bar{16}^1, 10 \cdot 16^2, 10 \cdot \bar{16}^2 \) and \( 1 \cdot 10^2 \), even if these terms are allowed by the symmetry. This is because these interactions do not play a significant role in our argument since they do not include the products of only the neutral components under the standard gauge group. It is easy to include these terms in our analysis.

We now discuss the determination of the VEVs. If \( -3a \leq a' < -5a \), the superpotential \( W_{A'} \) is in general written as
\[
W_{A'} = \lambda^{a+a'} \alpha A'A + \lambda^{a' + 3a} (\beta(A'A)_{1}(A^2)_{1} + \gamma(A'A)_{54}(A^2)_{54}),
\]
where the suffixes \( 1 \) and \( 54 \) indicate the representation of the composite operators under the \( SO(10) \) gauge symmetry, and \( \alpha, \beta \) and \( \gamma \) are parameters of order 1. Here we assume \( a + a' + c + \bar{c} < 0 \) to forbid the term \( \bar{C}A'AC \), which destabilizes the DW form of the VEV \( \langle A \rangle \). If we take \( \langle A \rangle = i \tau_2 \times \text{diag}(x_1, x_2, x_3, x_4, x_5) \), the \( F \)-flatness of the \( A' \) field requires \( x_i(\alpha \lambda^{-2a} + 2(\beta - \gamma)(\sum_j x_j^2) + \gamma x_i^2) = 0 \), which gives only two solutions \( x_i^2 = 0, x_j^2 = \frac{\alpha}{(2N - 1)\tau_2 - 2N\beta} \lambda^{-2a} \). Here \( N = 1 - 5 \) is the number of \( x_i \neq 0 \) solutions. The DW form is obtained when \( N = 3 \). Note that the higher terms \( A'A^{2L+1} \) (\( L > 1 \)) are forbidden by the SUSY zero mechanism. If they are allowed, the number of possible VEVs other than the DW form becomes larger, and thus it becomes less natural to obtain the DW form. This is a critical point of this mechanism, and the anomalous \( U(1)_A \) gauge symmetry plays an essential role to forbid the undesired terms. It is also interesting that the scale of the VEV is automatically determined by the anomalous \( U(1)_A \) charge of \( A \), as noted in the previous section.

Next we discuss the \( F \)-flatness condition of \( S \), which determines the scale of the VEV \( \langle \bar{C}C \rangle \). \( W_S \), which is linear in the \( S \) field, is given by
\[
W_S = \lambda^{s+c+\bar{c}} S \left( (\bar{C}C) + \lambda^{-(c+\bar{c})} + \sum_k \lambda^{-(c+\bar{c})+2ka} A^{2k} \right)
\]
if \( s \geq -(c + \bar{c}) \). Then the \( F \)-flatness condition of \( S \) implies \( \langle \bar{C}C \rangle \sim \lambda^{-(c+\bar{c})} \), and the \( D \)-flatness condition requires \( |\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-(c+\bar{c})/2} \). The scale of the
VEV is determined only by the charges of $C$ and $\bar{C}$ again. If we take $c + \bar{c} = -5$, then we obtain the VEVs of the fields $C$ and $\bar{C}$ as $\lambda^{5/2}$, which differ from the expected values $\lambda^{-c}$ and $\lambda^{-\bar{c}}$ if $c \neq \bar{c}$. Note that a composite operator with positive anomalous $U(1)_A$ charge larger than $-(c + \bar{c}) - 1$ may play the same role as the singlet $S$ if such a composite operator exists. (In the above example, there is no such composite operator.)

Next, we discuss the $F$-flatness of $C'$ and $\bar{C}'$, which realizes the alignment of the VEVs $\langle C \rangle$ and $\langle \bar{C} \rangle$ and imparts masses on the PNG fields. This simple mechanism was proposed by Barr and Raby. [12] We can easily assign anomalous $U(1)_A$ charges which allow the following superpotential:

$$W_{C'} = \bar{C}(\lambda^{c+\alpha}A + \lambda^{\bar{c}+\delta}Z)C', \quad \text{(3.4)}$$
$$W_{\bar{C}'} = C'(\lambda^{c+\alpha}A + \lambda^{\bar{c}+\delta}Z)C. \quad \text{(3.5)}$$

The $F$-flatness conditions $F_{C'} = F_{\bar{C}'} = 0$ give $(\lambda^{a-z}A + Z)C = \bar{C}(\lambda^{a-z}A + \bar{Z}) = 0$. Recall that the VEV of $A$ is proportional to the $B - L$ generator $Q_{B-L}$ as $\langle A \rangle = \frac{3}{2}vQ_{B-L}$. Also $C$, 16, is decomposed into $(3, 2, 1)_{1/3}$, $(\bar{3}, 1, 2)_{-1/3}$, $(1, 2, 1)_{-1}$ and $(1, 1, 2)_1$ under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Since $\langle \bar{C}C \rangle \neq 0$, not all components in the spinor $C$ vanish. Then $Z$ is fixed to be $Z \sim -\frac{3}{2}\lambda vQ_{B-L}^0$, where $Q_{B-L}^0$ is the $B - L$ charge of the component field in $C$, which has non-vanishing VEV. It is interesting that no other component fields can have non-vanishing VEVs because of the $F$-flatness conditions. If the $(1, 1, 2)_1$ field obtains a non-zero VEV (therefore, $\langle Z \rangle \sim -\frac{3}{2}\lambda v$), then the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken to the standard gauge group. Once the direction of the VEV $\langle C \rangle$ is determined, the VEV $\langle \bar{C} \rangle$ must have the same direction because of the $D$-flatness condition. Therefore, $\langle \bar{Z} \rangle \sim -\frac{3}{2}\lambda v$. Thus, all VEVs have now been fixed.

Finally the $F$-flatness condition of $H'$ is examined. $W_{H'}$, which is linear in the $H'$ field, is written

$$W_{H'} = \lambda^{h+a+h'}H'AH. \quad \text{(3.6)}$$

The $F_{H'}$ leads to the vanishing VEV of the triplet Higgs $\langle H_T \rangle = 0$.

There are several terms which must be forbidden for the stability of the DW mechanism. For example, $H^2$, $HZH'$ and $HZH'$ induce a large mass of the doublet Higgs, and the term $\bar{C}A'AC$ would destabilize the DW form of $\langle A \rangle$. We can easily forbid these terms using the SUSY zero mechanism. For example, if we choose $h < 0$, then $H^2$ is forbidden, and if we choose $\bar{c} + c + a + a' < 0$, then $\bar{C}A'AC$ is forbidden. Once these dangerous terms are forbidden by the SUSY zero mechanism, higher-dimensional terms which also become dangerous; for example, $\bar{C}A'^3C$ and $\bar{C}A'CCAC$ are automatically forbidden, since only gauge invariant operators with negative charge can have non-vanishing VEVs. This is also an attractive point of our scenario.
In the end of this section, we would like to explain how to determine the symmetry and the quantum numbers in the Higgs sector to realize DT splitting. It is essential that the dangerous terms are forbidden by SUSY zero mechanism and the necessary terms must be allowed by the symmetry. The dangerous terms are

\[ H^2, HH', HZH', \bar{C}A'C, \bar{C}A'AC, \bar{C}A'ZA, A'A^4, A'A^5. \]  

(3.7)

On the other hand, the terms required to realize DT splitting well are

\[ A'A, A'A^3, HAH', \bar{C}'(A + Z)C, \bar{C}(A + Z)C', S\bar{C}C. \]  

(3.8)

Here we denote both \( Z \) and \( \bar{Z} \) as \( Z \). In order to forbid \( HH' \) while \( HAH' \) is allowed, we introduce \( Z_2 \) parity. We have some ambiguities to assign the \( Z_2 \) parity, but once the parity is fixed, the above requirements become just inequalities, which are easily satisfied as discussed in this section.

Of course, the above conditions are necessary but not sufficient. As in the next section, we have to write down the mass matrices of Higgs sector to know whether an assignment truly works well or not.

### 4 Mass spectrum of Higgs sector

In this section, we examine the mass spectrum of the super-heavy particles. Before going to the detail, we classify the fields by the quantum number of the standard gauge group. Using the definition of the fields \( Q(3, 2)_{\frac{1}{4}}, Uc(\bar{3}, 1)_{-\frac{2}{3}}, Dc(\bar{3}, 1)_{\frac{1}{3}}, L(1, 2)_{-\frac{1}{2}}, Ec(1, 1)_{1}, Nc(1, 1)_{0}, X(3, 2)_{-\frac{1}{6}} \) and their conjugate fields, and \( G(8, 1)_{0} \) and \( W(1, 3)_{0} \) with the standard gauge symmetry, under \( SO(10) \supset SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y \), the spinor 16, vector 10 and the adjoint 45 are

\[
\begin{align*}
16 & \rightarrow \underbrace{[Q + U^c + E^c]}_{10} + \underbrace{[D^c + L]}_{5} + \underbrace{N^c}_{1}, \\
10 & \rightarrow \underbrace{[D^c + L]}_{5} + \underbrace{[\bar{D}^c + \bar{L}]}_{5}, \\
45 & \rightarrow \underbrace{[G + W + X + \bar{X} + N^c]}_{24} + \underbrace{[Q + U^c + E^c]}_{10} + \underbrace{[\bar{Q} + \bar{U}^c + \bar{E}^c]}_{10} + \underbrace{N^c}_{1}.
\end{align*}
\]

(4.1)  

(4.2)  

(4.3)

In the followings, we study how mass matrices of the above fields are determined by anomalous \( U(1)_A \) charges. For the mass terms, we must take account of not only the terms in the previous section but also the terms that contain two fields with vanishing VEVs.

First we examine the mass spectrum of 5 and 5 of \( SU(5) \). Considering the additional terms \( \lambda^{2h'}H'H', \lambda^{e'+h'}C'C', \lambda^{e'+e+h'}CH', \lambda^{e'+e+h'}\bar{C}'CH' \) and
we write the mass matrices $M_I$, which are for the representations $I = D^c(H_T), L(H_D)$ and their conjugates:

$$M_I = \begin{pmatrix}
I_H & I_{H'} & I_C & I_{C'} \\
\bar I_H & \lambda^{h+h'+a} \langle A \rangle & 0 & 0 \\
\bar I_{H'} & \lambda^{h+h'+a} (\bar C) & \lambda^{h'+e'} (\bar C) & \lambda^{h'+e'} + \alpha_I \beta_1 v \\
\bar I_C & \lambda^{h+h'+a} (\bar C) & \lambda^{h'+e'} (\bar C) & \lambda^{h'+e'} + \alpha_I \beta_1 v \\
\bar I_{C'} & 0 & 0 & \lambda^{h+h'+a} \langle A \rangle
\end{pmatrix} \quad (4.4)
$$

The colored Higgs obtain their masses of order $\lambda^{h+h'+a} \langle A \rangle \sim \lambda^{h+h'}$. Since in general $\lambda^{h+h'} > \lambda^{2h'}$, the proton decay is naturally suppressed. The effective colored Higgs mass is estimated as $\langle (\lambda^{h+h'})^2 / \lambda^{2h'} \rangle = \lambda^2 h^2$, which is larger than the cutoff scale, because $h < 0$. One pair of the doublet Higgs is massless, while another pair of doublet Higgs acquires a mass of order $\lambda^{2h'}$. The DW mechanism works well, although we have to examine the effect of the rather light super-heavy particles. Since $\beta_{D^c} = -2$ and $\beta_L = -3$, the color triplets acquire masses $2\lambda^{e+e'}$ and $2\lambda^{c+e'}$, while the weak doublets acquire masses $3\lambda^{e+e'}$ and $3\lambda^{c+e'}$.

Note that if the term $\bar C' A \bar C H$, which is not allowed with the typical charge assignment in Table I, is allowed by the symmetry, the massless Higgs doublet becomes

$$\bar C' + \frac{1}{2} (\bar C - c) \bar C,$$

and the effect of the mixing must be taken into account in considering the quark and lepton mass matrices.

Next we examine the mass matrices for the representations $I = Q, U^c$ and $E^c$, which are contained in the 10 of SU(5). Like the superpotential previously discussed, the additional terms $\lambda^{2a} A' A', \lambda^{e+e'} \bar C' C', \lambda^{e+a+e} \bar C' A' C'$ and $\lambda^{e+a+e} C' A' C$ must be taken into account. The mass matrices are written as $4 \times 4$ matrices,

$$M_I = \begin{pmatrix}
I_A & I_{A'} & I_C & I_{C'} \\
\bar I_A & 0 & \lambda^{a+a'} \alpha_I & \lambda^{e+a+e'} (\bar C) \\
\bar I_{A'} & \lambda^{a+a'} \alpha_I & 0 & \lambda^{e+a+e'} (\bar C) \\
\bar I_C & \lambda^{e+a+e'} (\bar C) & \lambda^{e+a+e'} (\bar C) & \lambda^{e+a+e'} \beta_1 v \\
\bar I_{C'} & 0 & 0 & \lambda^{e+a+e'} \beta_1 v
\end{pmatrix} \quad (4.6)
$$

where $\alpha_I$ vanishes for $I = Q$ and $U^c$ because these are Nambu-Goldstone modes, but $\alpha_{E^c} \neq 0$. On the other hand, $\beta_1 = \frac{1}{2} ((B-L)_I - 1)$; that is, $\beta_Q = -1, \beta_{U^c} = -2$ and $\beta_{E^c} = 0$. Thus for each $I$, the $4 \times 4$ matrix has one vanishing eigenvalue, which corresponds to the Nambu-Goldstone mode eaten by the Higgs mechanism. The mass spectrum of the remaining three modes is $(\lambda^{e+a} v, \lambda^{e+e+a} v, \lambda^{2a'})$ for the color-triplet modes $Q$ and $U^c$, and $(\lambda^{e+a'}, \lambda^{a+a'}, \lambda^{e+e'})$ or $(\lambda^{e+a+e} (\bar C), \lambda^{e+e+e} (\bar C), \lambda^{2a'})$ for the color-singlet modes $E^c$. 


The adjoint fields \(A\) and \(A'\) contain two \(G\), two \(W\) and two pairs of \(X\) and \(\bar{X}\), whose mass matrices \(M_I(I = G, W, X)\) is given by

\[
M_I = \tilde{I}_A \begin{pmatrix}
I_A & I_{A'} \\
0 & \alpha_I \lambda^{a+a'}/\lambda^{2a'}
\end{pmatrix}.
\]

(4.7)

Two \(G\) and two \(W\) acquire masses \(\lambda^{a+a'}\). Since \(\alpha_X = 0\), one pair of \(X\) is massless, which is eaten by Higgs mechanism. However, the other pair has a rather light mass of \(\lambda^{2a'}\).

Once we determine the anomalous \(U(1)_A\) charges, the mass spectrum of all fields is determined, and hence we can examine whether the running couplings from the low energy scale meet at the unification scale or not. Before going to the discussion of the conditions for gauge coupling unification, in the next section, we will examine several models with this DT splitting mechanism and conditions with which realistic mass matrices of quarks and leptons can be obtained.

5 Quark and Lepton masses and Neutrino relation

In this section, we examine models to demonstrate how to determine everything from the anomalous \(U(1)_A\) charges.

In addition to the Higgs sector in Table I, we introduce only three 16 representations \(\Psi_i\) with anomalous \(U(1)_A\) charges (\(\psi_1 = n + 3, \psi_2 = n + 2, \psi_3 = n\)) and one 10 field \(T\) with charge \(t\) as the matter contents. These matter fields are assigned odd R-parity, while those of the Higgs sector are assigned even R-parity. Such an assignment of R-parity guarantees that the argument regarding VEVs in the previous section does not change if these matter fields have vanishing VEVs.

Then the mass term of \(5\) and \(\bar{5}\) of \(SU(5)\) is written

\[
5_T(\lambda^{t+\psi_1+c} \langle C \rangle, \lambda^{t+\psi_2+c} \langle C \rangle, \lambda^{t+\psi_3+c} \langle C \rangle, \lambda^{2t}) \begin{pmatrix}
\bar{5}_{\psi_1} \\
\bar{5}_{\psi_2} \\
\bar{5}_{\psi_3} \\
\bar{5}_T
\end{pmatrix} = 5_T(\lambda^{t+\psi_1+(c-\bar{c})/2}, \lambda^{t+\psi_2+(c-\bar{c})/2}, \lambda^{t+\psi_3+(c-\bar{c})/2}, \lambda^{2t}) \begin{pmatrix}
\bar{5}_{\psi_1} \\
\bar{5}_{\psi_2} \\
\bar{5}_{\psi_3} \\
\bar{5}_T
\end{pmatrix},
\]

(5.1)

(5.2)

where \(\langle \bar{C} \rangle = \langle C \rangle \sim \lambda^{-(c+\bar{c})/2}\). Since \(\psi_3 < \psi_2 < \psi_1\), the massive mode \(\bar{5}_M\), the partner of \(\bar{5}_T\), must be \(\bar{5}_{\psi_3}(\Delta = 2t - (t + \psi_3 + (c-\bar{c})/2) > 0)\) or \(\bar{5}_T(\Delta < 0)\). The former case is interesting and the massive mode is given by

\[
\bar{5}_M \sim \bar{5}_{\psi_3} + \lambda^2\bar{5}_T + \lambda^{\psi_2-\psi_3}\bar{5}_{\psi_2} + \lambda^{\psi_1-\psi_3}\bar{5}_{\psi_1}.
\]

(5.3)
Therefore the three massless modes ($\bar{5}_1$, $\bar{5}_2$, $\bar{5}_3$) are written ($\bar{5}_{\psi_1} + \lambda_{\psi_1-\psi_1} \bar{5}_{\psi_3}$, $\bar{5}_{\psi_2}$, $\bar{5}_{\psi_3} + \lambda_{\psi_2-\psi_3} \bar{5}_{\psi_3}$). The Dirac mass matrices for quarks and leptons can be obtained from the interaction

$$\lambda_{\psi_i+\psi_j} \Psi_i \Psi_j H. \quad (5.4)$$

The mass matrices for the up quark sector and the down quark sector are

$$M_u = \begin{pmatrix} \lambda_6 & \lambda_5 & \lambda_3 \\ \lambda_5 & \lambda_4 & \lambda_2 \\ \lambda_3 & \lambda_2 & 1 \end{pmatrix} \langle H_u \rangle, \quad M_d = \lambda^2 \begin{pmatrix} \lambda_4 & \lambda_{\Delta+1} & \lambda_3 \\ \lambda_3 & \lambda_{\Delta} & \lambda_2 \\ \lambda_1 & \lambda_{\Delta-2} & 1 \end{pmatrix} \langle H_d \rangle. \quad (5.5)$$

Here taking $1 \leq \Delta \leq 3$ leads reasonable value of the ratio $m_s/m_b$. Note that the Yukawa couplings for $\bar{5}_2 \sim \bar{5}_T + \lambda \bar{5}_{\psi_3}$ are obtained only through the Yukawa couplings for the component $\bar{5}_{\psi_3}$, because we have no Yukawa couplings for $T$ without the Higgs mixing ($4.3)$. With the Higgs mixing ($4.3$), the interaction $\lambda_{\psi_i+t} \Psi_i TC$ induces the correction to the mass matrix of down-type quarks. It is easily checked that the correction to the down-type Yukawa couplings are the same order as in Eq. ($5.5$).

We can estimate the Cabbibo-Kobayashi-Maskawa (CKM) matrix\(^4\) from these quark matrices\(^6\) as

$$U_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (5.6)$$

which is consistent with the experimental value if we choose $\lambda \sim 0.2$. Since the ratio of the Yukawa couplings of top and bottom quarks is $\lambda$, a small value of $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ is predicted by these mass matrices.

The Yukawa matrix for the charged lepton sector is the same as the transpose of $M_d$ at this stage, except for an overall factor $\eta$ induced by the renormalization group effect. The mass matrix for the Dirac mass of neutrinos is given by

$$M_{\nu_D} = \lambda^2 \begin{pmatrix} \lambda_4 & \lambda_3 & \lambda \\ \lambda_{\Delta+1} & \lambda_{\Delta} & \lambda_{\Delta-2} \\ \lambda_3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \eta. \quad (5.7)$$

The right-handed neutrino masses come from the interaction

$$\lambda_{\psi_i+\psi_j}^{\psi_i+\psi_j} \Psi_i \Psi_j \bar{C} \bar{C} \quad (5.8)$$

\(^4\) Strictly speaking, if the Yukawa coupling originated only from the interaction ($5.4$), the mixing concerning to the first generation becomes too small because of a cancellation. In order to get the expected value of CKM matrix as in Eq. ($5.6$), non-renormalizable terms, for example, $\Psi_i \Psi_j H \bar{C} \bar{C}$ must be taken into account. It is required that $c + \bar{c} \geq -5$. 

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as
\[ M_R = \lambda \psi_i + \psi_j + 2 \bar{c} \langle \bar{O} \rangle^2 = \lambda^{2n+c-\bar{c}} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \] (5.9)

Therefore we can estimate the neutrino mass matrix:
\[ M_\nu = M_{\nu_D}M_R^{-1}M_{\nu_D}^T = \lambda^{4-2n+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{\Delta-1} & \lambda \\ \lambda^{\Delta-1} & \lambda^{2\Delta-4} & \lambda^{\Delta-2} \\ \lambda & \lambda^{\Delta-2} & 1 \end{pmatrix} \langle H_u \rangle^2 \eta^2. \] (5.10)

Note that the overall factor \( \lambda^{4-2n+c-\bar{c}} \) can have negative power. From these mass matrices in the lepton sector the Maki-Nakagawa-Sakata (MNS) matrix is obtained as
\[ U_{\text{MNS}} = \begin{pmatrix} 1 & \lambda^{3-\Delta} & \lambda \\ \lambda^{3-\Delta} & 1 & \lambda^{\Delta-2} \\ \lambda & \lambda^{\Delta-2} & 1 \end{pmatrix} \] for \( 2 \leq \Delta \leq 3 \) and
\[ U_{\text{MNS}} = \begin{pmatrix} 1 & \lambda & \lambda^{3-\Delta} \\ \lambda & 1 & \lambda^{2-\Delta} \\ \lambda^{3-\Delta} & \lambda^{2-\Delta} & 1 \end{pmatrix} \] for \( 1 \leq \Delta \leq 2 \). If we take \( \Delta = 5/2 \), namely,
\[ t = n + \frac{1}{2}(c - \bar{c} + 5), \] (5.13)

bi-large neutrino mixing angle is obtained. We then obtain the prediction \( m_{\nu_\mu}/m_{\nu_\tau} \sim \lambda \), which is consistent with the experimental data: \( 1.6 \times 10^{-3} \text{eV}^2 \leq \Delta m^2_{\text{atm}} \leq 4 \times 10^{-3} \text{eV}^2 \) and \( 2 \times 10^{-5} \text{eV}^2 \leq \Delta m^2_{\text{sun}} \leq 1 \times 10^{-4} \text{eV}^2 \) (LMA). The relation \( V_{e3} \sim \lambda \) is also an interesting prediction from this matrix, though CHOOZ gives a restrictive upper limit \( V_{e3} \leq 0.15 \). Moreover, if we take \( 4-2n+c-\bar{c} = -(5+l) \), the parameter \( l \) is determined from
\[ \lambda^l = \lambda^{-5} \frac{H_u^2 \eta^2}{m_{\nu_\tau} \Lambda}, \] (5.14)
where \( m_{\nu_\tau} \) is tau neutrino mass. We are supposing that the cutoff scale \( \Lambda \) is in a range \( 10^{16} \text{(GeV)} < \Lambda < 10^{20} \text{(GeV)} \), which allows us to take \( -2 \leq l \leq 2 \). If we take \( l = 0 \), the neutrino masses are given by \( m_{\nu_\tau} \sim \lambda^{-5} \langle H(10, 5) \rangle^2 \eta^2 / \Lambda \sim m_{\nu_\mu} / \lambda \sim m_{\nu_e} / \lambda^2 \). If we take \( \eta \langle H(10, 5) \rangle = 100 \text{ GeV}, \Lambda \sim 10^{18} \text{ GeV} \) and \( \lambda = 0.2 \), then we get \( m_{\nu_\tau} \sim 3 \times 10^{-2} \text{ eV}, m_{\nu_\mu} \sim 6 \times 10^{-3} \text{ eV} \) and \( m_{\nu_e} \sim 1 \times 10^{-3} \text{ eV} \). From such a rough estimation, we can obtain almost desirable values for explaining the experimental data from the atmospheric neutrino and large mixing angle (LMA)
MSW solution for solar neutrino problem.\textsuperscript{[27]} This LMA solution for the solar neutrino problem gives the best fitting to the present experimental data.\textsuperscript{[28]}

In addition to Eq. (5.4), the interactions

\[ \lambda \psi_i + \psi_j + 2a + h \Psi_i A^2 \Psi_j H \]  

also contribute to the Yukawa couplings. Here \( A \) is squared because it has odd parity. Since \( A \) is proportional to the generator of \( B - L \), the contribution to the lepton Yukawa coupling is nine times larger than that to quark Yukawa coupling, which can change the unrealistic prediction \( m_\mu = m_s \) at the GUT scale. Since the prediction \( m_s/m_b \sim \lambda^{5/2} \) at the GUT scale is consistent with experiment, the enhancement factor \( 2 \sim 3 \) of \( m_\mu \) can improve the situation. Note that the additional terms contribute mainly in the lepton sector. If we set \( a = -2 \), the additional matrices are

\[
\begin{align*}
\Delta M_u \langle H_u \rangle &= \frac{v^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\Delta M_d \langle H_d \rangle &= \frac{v^2}{4} \begin{pmatrix} \lambda^2 & 0 & \lambda \\ \lambda & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \\
\Delta M_e \langle H_d \rangle &= \frac{9v^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ 0 & 0 & 0 \\ \lambda & 1 & 0 \end{pmatrix},
\end{align*}
\]  

(5.16)

(5.17)

It is interesting that this modification essentially changes the eigenvalues of only the first and second generation. Therefore it is natural to expect that a realistic mass pattern can be obtained by this modification. This is one of the largest motivations to choose \( a = -2 \). Note that this charge assignment also determines the scale \( \langle A \rangle \sim \lambda^2 \). It is suggestive that the fact that the GUT scale is slightly smaller than the Planck scale is correlated with the discrepancy between the naive prediction of the ratio \( m_\mu/m_s \) from the unification and the experimental value. It is also interesting that the SUSY zero mechanism plays an essential role again. When \( z, \bar{z} \geq -4 \), the terms \( \lambda \psi_i + \psi_j + a + z + h Z \Psi_i A \Psi_j H + \lambda \psi_i + \psi_j + 2z + h Z^2 \Psi_i \Psi_j H \) also contribute to the fermion mass matrices, though only to the first generation. It is useful to examine other charge assignment to \( a \). If \( a \leq -3 \), then the modification changes the eigenvalue of at the most first generation, which is inconsistent with the present experimental results. If \( a = -1 \), then the modification changes the eigenvalues of all generations. It is consistent with the present experimental values, though it does not explain the Yukawa coupling relation \( y_b = y_\tau \) at the GUT scale. Since the GUT relation \( y_b = y_\tau \) is still controvertible\textsuperscript{[30]}, this option \( a = -1 \) may be realistic.

Proton decay mediated by the colored Higgs is strongly suppressed in this model. As mentioned in the previous section, the effective mass of the colored

\footnote{If we take \( \Delta = 2 \), namely \( t = n + \frac{1}{2}(c - \bar{c} + 4) \), the MNS matrix becomes lopsided type. It has been argued that even in this case, the desirable values can be obtained, using the ambiguity of coefficients.\textsuperscript{[29]}}
Higgs is of order $\lambda^{2h} \sim \lambda^{-12}$, which is much larger than the cutoff scale. Proton decay is also induced by the non-renormalizable term
\[ \lambda^{\psi_i+\psi_j+\psi_k+\psi_l}\Psi_i\Psi_j\Psi_k\Psi_l, \] (5.18)
which has also the same suppression as via the colored Higgs mediation.

6 A natural solution for the $\mu$ problem

In our scenario, SUSY zero mechanism forbids the SUSY Higgs mass term $\mu HH$. However, once SUSY is broken, the Higgs mass $\mu$ must be induced. The induced mass must be proportional to the SUSY breaking scale.

We now examine a solution for the $\mu$ problem in a simple example [22]. The essential point of this mechanism is that the VEV shift of a heavy singlet field by SUSY breaking. We introduce the superpotential
\[ W = \lambda^{s'}S' + \lambda^{s+p}s'P, \]
where $S'$ and $P$ are singlet fields with positive anomalous $U(1)_A$ charge $s$ and with negative charge $p$, respectively ($s' + p \geq 0$). Note that the single term of $P$ is not allowed by SUSY zero mechanism, while usual symmetry cannot forbid this term. This is an essential point of this mechanism. The SUSY vacuum is at $\langle S'\rangle = 0$ and $\langle P\rangle = \lambda^{-p}$. After SUSY is broken, these VEVs are modified. To determine the VEV shift of $S'$, which we would like to know because the singlet $S'$ with positive charge can couple to the Higgs field with negative charge, the most important SUSY breaking term is the tadpole term of $S'$, namely $\lambda^{s'}M_p^2AS'$. Here $A$ is a SUSY breaking parameter of order of the weak scale. By this tadpole term, the VEV of $S'$ appears as $\langle S'\rangle = \lambda^{-s'-2p}A$. If we have $\lambda^{s'+2h}S'H^2$, the SUSY Higgs mass is obtained as $\mu = \lambda^{2h-2p}m_{SB}$, which is proportional to the SUSY breaking parameter $m_{SB}$ and the proportional coefficient can be of order 1 if $h \sim p$. Note that the $F$-term of $S'$ is calculated as $F_{S'} \sim \lambda^{-s'-2p}m_{SB}^2$. The Higgs mixing term $B\mu$ can be obtained from the SUSY term $\lambda^{s'+2h}S'H^2$ and the SUSY breaking term $\lambda^{s'+2h}A_{S'H^2}S'H^2$ as $\lambda^{s'+2h}F_{S'} \sim \lambda^{2h-2p}m_{SB}^2$ and $\lambda^{2h-2p}A^2 \sim \mu A$, respectively. Therefore the relation $B \sim m_{SB}$ is naturally obtained [6]. This is a solution for the $\mu$ problem. Note that the condition $h \sim p$ can be satisfied because both fields $H$ and $P$ have negative charges. Note that $S'$ or $P$ can be a composite operator, for example, a composite operator $\bar{C}C$ can play the same role as $P$ in the above mechanism. In this case, the condition becomes
\[ h \sim p = \frac{1}{2}(c + \bar{c}). \] (6.1)

We call this condition the economical condition for the $\mu$ problem.

6 If doublet-triplet splitting is realized by fine-tuning or some accidental cancellation, the Higgs mixing $B\mu$ can become intermediated scale $m_{SB}M_X$ as discussed in Ref. [31], where $M_X$ is the GUT scale. However, once the doublet-triplet splitting is naturally solved as in Ref. [15], such a problem disappears.
7 Conditions for gauge coupling unification

In order to stabilize the DW form of $\langle A \rangle$, the term $\tilde{C}A'AC$ must be forbidden by SUSY zero mechanism, namely, $\tilde{c} + c + a' + a < 0$. On the other hand, $a' + 3a \geq 0$ is required to obtain the term $A'A^3$. From these inequalities, we obtain $\frac{1}{2}(c + \tilde{c}) < a$, which leads to $\langle A \rangle \sim \lambda^{-a} > \lambda^{-(c+\tilde{c})/2} \sim \langle C \rangle$. Therefore at the scale $\Lambda_A \equiv \langle A \rangle \sim \lambda^{-a}$, $SO(10)$ gauge group is broken into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which is broken into the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ at the scale $\Lambda_C \equiv \langle C \rangle \sim \lambda^{-(c+\tilde{c})/2}$.

In this paper, we make an analysis based on the renormalization group equations up to one loop. The conditions of the gauge coupling unification are given by

$$\alpha_3(\Lambda_A) = \alpha_2(\Lambda_A) = \frac{3}{5} \alpha_Y(\Lambda_A) \equiv \alpha_1(\Lambda_A), \quad (7.1)$$

where $\alpha_1(\mu > \Lambda_C) \equiv \frac{3}{5} \alpha_1^{-1}(\mu > \Lambda_C) + \frac{2}{5} \alpha_{B-L}^{-1}(\mu > \Lambda_C)$. Here $\alpha_X = \frac{g_X^2}{4\pi}$ and the parameters $g_X(X = 3, 2, R, B - L, Y)$ are the gauge couplings of $SU(3)_C$, $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$ and $U(1)_Y$, respectively.

The gauge couplings at the scale $\Lambda_A$ are roughly described by

$$\alpha_1^{-1}(\Lambda_A) = \alpha_1^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_1 \ln \left( \frac{M_{SB}}{\Lambda_A} \right) + \Sigma_i \Delta b_i \ln \left( \frac{m_i}{\Lambda_A} \right) - \frac{12}{5} \ln \left( \frac{\Lambda_C}{\Lambda_A} \right) \right), \quad (7.2)$$

$$\alpha_2^{-1}(\Lambda_A) = \alpha_2^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_2 \ln \left( \frac{M_{SB}}{\Lambda_A} \right) + \Sigma_i \Delta b_i \ln \left( \frac{m_i}{\Lambda_A} \right) \right), \quad (7.3)$$

$$\alpha_3^{-1}(\Lambda_A) = \alpha_3^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_3 \ln \left( \frac{M_{SB}}{\Lambda_A} \right) + \Sigma_i \Delta b_i \ln \left( \frac{m_i}{\Lambda_U} \right) \right), \quad (7.4)$$

where $M_{SB}$ is a SUSY breaking scale, $(b_1, b_2, b_3) = (33/5, 1, -3)$ are the renormalization group coefficients for the minimal SUSY standard model(MSSM) and $\Delta b_{ai}(a = 1, 2, 3)$ are the correction to the coefficients from the massive fields with mass $m_i$. The last term in Eq. (7.2) is from the breaking $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ by the VEV $\langle C \rangle$. Since the gauge couplings at the SUSY breaking scale $M_{SB}$ are given by

$$\alpha_1^{-1}(M_{SB}) = \alpha_1^{-1}(\Lambda_G) + \frac{1}{2\pi} \left( b_1 \ln \left( \frac{\Lambda_G}{M_{SB}} \right) \right), \quad (7.5)$$

$$\alpha_2^{-1}(M_{SB}) = \alpha_2^{-1}(\Lambda_G) + \frac{1}{2\pi} \left( b_2 \ln \left( \frac{\Lambda_G}{M_{SB}} \right) \right), \quad (7.6)$$

$$\alpha_3^{-1}(M_{SB}) = \alpha_3^{-1}(\Lambda_G) + \frac{1}{2\pi} \left( b_3 \ln \left( \frac{\Lambda_G}{M_{SB}} \right) \right), \quad (7.7)$$

where $\alpha_1^{-1}(\Lambda_G) \sim 25$ and $\Lambda_G \sim 2 \times 10^{16}$ GeV, the above conditions for unification are rewritten as

$$b_1 \ln \left( \frac{\Lambda_A}{\Lambda_G} \right) + \Sigma_i \Delta b_{1i} \ln \left( \frac{\Lambda_G}{\det M_I} \right) - \frac{12}{5} \ln \left( \frac{\Lambda_A}{\Lambda_C} \right) \quad (7.8)$$
In our scenario, the unification scale \( \Lambda_A \sim \lambda^{-a} \), the symmetry breaking scale \( \Lambda_C \sim \lambda^{-\frac{1}{2}(c+e)} \) and the determinants of the reduced mass matrices are fixed by the anomalous \( U(1)_A \) charges;

\[
\begin{align*}
\det \bar{M}_Q &\sim \lambda^{2a'+e'+c+e'}, \\
\det \bar{M}_{E^c} &\sim \lambda^{2a'+2a'+e'+e'}, \\
\det \bar{M}_{D^c} &\sim \lambda^{2b'+2b'+c+e'+e'}, \\
\det \bar{M}_L &\sim \lambda^{2h'+e'+c'+e'}, \\
\det \bar{M}_G &\sim \lambda^{2a'+2a'}, \\
\det \bar{M}_X &\sim \lambda^{2a'}.
\end{align*}
\]  

The unification conditions \( \alpha_1(\Lambda_A) = \alpha_2(\Lambda_A) = \alpha_3(\Lambda_A) \) and \( \alpha_2(\Lambda_A) = \alpha_3(\Lambda_A) \) are rewritten as

\[
\begin{align*}
&\left( \frac{\Lambda_A}{\Lambda_G} \right)^{14} \left( \frac{\Lambda_C}{\Lambda_A} \right)^{6} \left( \frac{\det \bar{M}_L}{\det \bar{M}_D} \right) \left( \frac{\det \bar{M}_Q}{\det \bar{M}_U} \right) \left( \frac{\det \bar{M}_Q}{\det \bar{M}_E} \right) \left( \frac{\det \bar{M}_W}{\det \bar{M}_X} \right), \\
&\left( \frac{\Lambda_A}{\Lambda_G} \right)^{16} \left( \frac{\det \bar{M}_D}{\det \bar{M}_L} \right) \left( \frac{\det \bar{M}_Q}{\det \bar{M}_U} \right) \left( \frac{\det \bar{M}_Q}{\det \bar{M}_E} \right) \left( \frac{\det \bar{M}_G}{\det \bar{M}_X} \right), \\
&\left( \frac{\Lambda_A}{\Lambda_G} \right)^4 \left( \frac{\det \bar{M}_D}{\det \bar{M}_L} \right) \left( \frac{\det \bar{M}_U}{\det \bar{M}_Q} \right) \left( \frac{\det \bar{M}_G}{\det \bar{M}_W} \right) \left( \frac{\det \bar{M}_G}{\det \bar{M}_X} \right)
\end{align*}
\]
which lead to $\Lambda \sim \lambda^{\frac{1}{2}} \Lambda_G$, $\Lambda \sim \lambda^{-\frac{1}{2}} \Lambda_G$ and $\Lambda \sim \lambda^{-\frac{1}{2}} \Lambda_G$, respectively. So the unification condition becomes $h \sim 0$, and then the cutoff scale must be taken as $\Lambda \sim \Lambda_G$. Note that these relation are independent on the anomalous $U(1)_A$ charges except that of the doublet Higgs. It implies that this result can be applied to rather general cases. On the other hand, we should not take this relation $h \sim 0$ seriously because we have an ambiguity of order one coefficients and use only one loop renormalization group equations. However, in order to catch the tendency, the above analysis is fairly useful.

Before going to the discussion of model buildings, it is useful to examine the reason to obtain the above result. The essential point appears in estimating the ratio of determinants of mass matrices between the components in the same multiplet of $SU(5)$ gauge group. Note that in Eqs. (7.12)–(7.17), the powers are given by simple sums of the anomalous $U(1)_A$ charges. Therefore, the relation $\det \bar{M}_L/\det M_{Dc}$ can be easily estimated from the trivial relation $\lambda^{2h} \det \bar{M}_L/\det M_{Dc} = 1$, where $2h$ is the total charge of massless modes (a pair of doublet Higgs fields). The ratio $\det \bar{M}_Q/\det M_{Ec}$ is also determined by the relation $\lambda^{2a} \det \bar{M}_Q/(\lambda^{c+\bar{c}} \det M_{Ec}) = 1$, where $2a$ and $c + \bar{c}$ are the total charges of massless modes (Nambu-Goldstone (NG) modes which appear by breaking $SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$, respectively.) The relation $\lambda^{2a} \det \bar{M}_G/\det M_X = 1$, where $2a$ is the total charge of massless modes (NG modes which appear by breaking $SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, respectively.) It is interesting that all the effect of massless modes except Higgs doublet are cancelled out in deriving the conditions for the gauge coupling unification. It means that it is not accidental to realize the coupling unification in our scenario though the cutoff scale becomes around $\Lambda_G \sim 2 \times 10^{16}$ GeV and the unification scale becomes $\lambda^{-a} \Lambda_G$. (So we cannot take $a \leq -2$, because of proton stability.) Actually we have no solution to realize the coupling unification if DT splitting does not happen (i.e., $(b_1, b_2, b_3) = (6, 0, -3)$ and $\det \bar{M}_L/\det M_{Dc} = 1$). Therefore this result is non-trivial and it is stimulating that the proton decay via dimension six operator may be seen in future because the unification scale must be smaller than the usual unification scale $\Lambda_G \sim 2 \times 10^{16}$ GeV.

From these estimation, it is obvious that the charges of massless modes are essential to examine whether gauge couplings are unified at a scale or not. It is independent on the detail of the contents of Higgs sector. Therefore we can formally examine the possibility of gauge coupling unification without building models of Higgs sector explicitly. For example, to examine the case in which other massless modes than one pair of Higgs doublet appear, it is sufficient to take account of the anomalous $U(1)_A$ charges of the massless modes. Unfortunately we could not find natural example in which coupling unification of gauge couplings and DT splitting are realized. For example, if we introduce an additional adjoint field with negative charge, the additional massless modes $G$ and $W$ appear. The masses are controlled by the SUSY breaking terms, so are expected to be around

18
the SUSY breaking scale. (Strictly speaking, we can compute the mass scale, using the same mechanism for the $\mu$ problem as discussed in section 6.) Then we can calculate the running flow of the gauge couplings and examine whether coupling unification happens or not. Since the ratios of the mass determinants are given by

\[
\begin{align*}
\frac{\lambda_a \det \bar{M}_L}{\det \bar{M}_D} & \sim \lambda^{-2h-a}, \\
\frac{\det \bar{M}_Q}{\det \bar{M}_E} & \sim \lambda^{c+\bar{c}-2a}, \\
\frac{m_G \det \bar{M}_G}{\lambda^{-a} \det \bar{M}_X} & \sim \lambda^{3a-2\Delta+\omega},
\end{align*}
\]

where $m_G = \lambda^a \Lambda$ is the mass of the massless mode of $G$ and $\Delta$ is the charge of the massless fields $G$ and $W$, the above conditions for coupling unification become

\[
\begin{align*}
\alpha_2 & = \alpha_3 \rightarrow \Lambda \sim \lambda^{\frac{1}{4}(2h-2\Delta+\omega)} \Lambda_G, \\
\alpha_1 & = \alpha_2 \rightarrow \Lambda \sim \lambda^{\frac{1}{4}(-2h-10\Delta+5\omega)} \Lambda_G.
\end{align*}
\]

These equations lead to unrealistic relation $2\Delta - \omega = -6h$.

In the next section, we will find out several models in which the condition for the gauge coupling unification $h \sim 0$ is almost satisfied.

### 8 Some models

In this section, we examine the cases in which all the fields become massive except one pair of Higgs doublets. Then unification scale becomes $\lambda^{-a} \Lambda_G$ as discussed in the previous section. So we should take $a = -1$ to stabilize nucleon. Then $a' = 3$ or 4 because the term $A' A^5$ must be forbidden and the term $A' A^3$ is required. The unification condition is $h \sim 0$, but we have to take negative $h$ to forbid the Higgs mass term $H^2$. Therefore we would like to know how large negative charge $h = -2n$ can be adopted in our scenario.

Necessary conditions for realizing DT splitting and bi-large neutrino mixing ($\Delta = 5/2$) are

\[
\begin{align*}
c - \bar{c} & = 2n - 9 - l, \\
t & = n + \frac{1}{2}(c - \bar{c} + 5), \\
n + t + c & \geq 0, \\
t & \geq 0, \\
c + \bar{c} + a' + a & < 0.
\end{align*}
\]

The third and forth conditions are required because the terms $\Psi_3 T C$ and $T^2$ are needed in our scenario. Since the cutoff scale $\Lambda \sim \Lambda_G = 2 \times 10^{16}$ GeV, we must
adopt \( l = -1 \) or \(-2\) for correct size of neutrino masses. If we assume that all
the charges are integer, then we have to take \( l = -2 \) to realize integer \( t \). Under
this assumption, the minimum value of \( n \) is 2 (namely \((\psi_1, \psi_2, \psi_3) = (5, 4, 2), t = 3, h = -4\)), and we obtain essentially three solutions which satisfy the above
necessary conditions: \((a' = 3, c = -3, \bar{c} = 0)\), \((a' = 3, 4, c = -4, \bar{c} = -1)\) and
\((a' = 3, 4, c = -5, \bar{c} = -2)\). We have some freedom to choose the charges
\( z, \bar{z}, h', c', \bar{c}' \). Typical values are \( z = \bar{z} = -2, h' = 5, c' = 2 - \bar{c}, \bar{c}' = 2 - c, s = - (c + \bar{c}) \).

If we allow to take half integer charges, then the minimum value of \( n \) satisfying the above necessary conditions becomes \( 3/2 \) (namely \((\psi_1, \psi_2, \psi_3) = (9/2, 7/2, 3/2), t = 2, h = -3\)). We can get only a solution: \( a' = 3, c = -7/2, \bar{c} = 1/2 \). We have some freedom to choose the charges \( z, \bar{z}, h', c', \bar{c}', s \). Typical values are \( z = \bar{z} = -2, h' = 4, c' = 3/2, \bar{c}' = 11/2, s = 3 \).

When all the charges are determined, we can calculate the running flows of
gauge couplings (see Fig. [32]). Here we use the ambiguities of the coefficients
\( 0.5 \leq y \leq 2 \). It is shown that the three gauge couplings actually meet around
\( \lambda^{-a} \Lambda_G \sim 5 \times 10^{15} \text{ GeV} \). Even the cases \( n = 2 \), gauge coupling unification
is possible, though we have to use larger ambiguities of the coefficients.

In these cases, since the unification scale \( \Lambda_U \sim \lambda \Lambda_G \) becomes smaller than the
usual GUT scale \( \Lambda_G \sim 2 \times 10^{16} \text{ GeV} \), proton decay via dimension six operator
\( p \to e^+ \pi^0 \) may be seen in near future. If we roughly estimate the lifetime of proton
using the formula in Ref. [33] and the recent result of the lattice calculation for
the hadron matrix element parameter \( \alpha \)

\[
\tau_p \sim 4.4 \times 10^{34} \left( \frac{\Lambda_U}{10^{16} \text{ GeV}} \right)^4 \left( \frac{0.015}{\alpha} \right)^2 \text{ years}, \tag{8.7}
\]

the lifetime of the proton in these cases becomes

\[
\tau_p \sim 2.8 \times 10^{33} \text{ years} \tag{8.8}
\]

---

7 The last candidate is not so good because \( c + \bar{c} = -7 \) which may lead to smaller Cabbibo angle by a cancellation.

8 If we adopt lopsided type neutrino mass matrix, the second condition \((8.2)\) is replaced by

\[
t = n + \frac{1}{2} (c - \bar{c} + 4) \tag{8.6}
\]

The minimum value of \( n \) is also 2(namely, \((\psi_1, \psi_2, \psi_3) = (5, 4, 2), t = 2, h = -4\)), and we have
only one solution \( a' = 3, 4, c = -4, \bar{c} = 0 \), and typical values of charges \( z, \bar{z}, h', c', \bar{c}' \), \( s \) are
\( z = \bar{z} = -2, h' = 5, c' = 2 - \bar{c}, \bar{c}' = 2 - c, s = - (c + \bar{c}) \).

9 To adopt half integer charges with the FN field's charge \( \theta = -1 \) becomes essentially the same as to adopt only integer charges with \( \theta = -2 \). If we have no singlet field with charge \(-1\),
the model has naturally half integer charges with \( U(1)_A \) normalization \( \theta = -1 \).

10 This is not inconsistent with the discussion in ref. [32], though they concluded that the
coupling unification with a simple gauge group is generally impossible. The essential difference
is that we have not adopted their assumption \( \Lambda \sim 10^{18} \text{ GeV} \).
because the unification scale is around $5 \times 10^{15}$ GeV. It is interesting that this value of the lifetime is just around the present experimental limit \cite{34}

$$\tau_{\text{exp}}(p \to e^+\pi^0) > 2.9 \times 10^{33}\text{ year}. \quad (8.9)$$

Of course, since we have an ambiguity of order one coefficients and of the hadron matrix element parameter $\alpha$, and the lifetime of proton is strongly dependent on the GUT scale and the parameter, this prediction may not be so reliable. However, the above rough estimation gives us a strong motivation for experiments of proton decay search, because the lifetime of nucleon via dimension six operator must be less than that in the usual SUSY GUT scenario.

We have to comment on the proton decay via dimension five operators. The effective colored Higgs mass is given by $\lambda^2 \Lambda \sim 2 \times 10^{20}$ GeV even if we take $\Lambda = 2 \times 10^{16}$ GeV. Therefore the proton decay via dimension five operator is still suppressed.

In $E_6$ unification case, the above analysis is a bit changed as discussed in
Ref. [15]. We have to introduce the effective anomalous $U(1)_A$ charges, which are available only for the estimation of the mass determinants. Actually in the above analysis, the charge of Higgs field must be replaced by

$$h \rightarrow h + \frac{1}{4}(\phi - \bar{\phi}),$$

(8.10)

where $\phi$ and $\bar{\phi}$ are the anomalous $U(1)_A$ charges of $\Phi$ and $\bar{\Phi}$, whose VEVs $\langle \Phi \rangle = \langle \bar{\Phi} \rangle \sim \lambda^{-\frac{1}{2}}(\phi + \bar{\phi})$ break $E_6$ into $SO(10)$. This is because the mass of the fields $D_c$ and $L$ is determined not only by the charges but also by the VEV $\langle \Phi \rangle$.

It is easily checked that all the effective charges can be defined consistently, though the effective charges for the mass matrices of different representation are generically different even if they originate from the same multiplet of $E_6$. In principle, this modification can change the above situation of coupling unification. Unfortunately the situation is not so improved but even worse, since $\phi - \bar{\phi}$ must be negative for small $n$ in order to obtain the realistic quark and lepton mass matrices. As discussed in Ref. [16], the conditions for obtaining the realistic quark and lepton mass matrices are

$$c - \bar{c} = \phi - \bar{\phi} + 1 = 2n - 9 - l,$$

(8.11)

where we take $l = -1$ or $-2$ because $\Lambda \sim \Lambda_G$. In $E_6$ DT splitting mechanism [33], Higgs $H$ is naturally unified into the multiplet $\Phi$, namely $h = \phi = -2n$. In order to obtain the effective charge of Higgs $h + \frac{1}{4}(\phi - \bar{\phi}) = \frac{1}{4}(-6n - 10 - l) \sim 0$, the small $n$ is required. From the condition $\phi + \bar{\phi} = -6n + 10 + l \leq -1$, the smallest value of $n$ becomes $3/2$ for $l = -2$. Then $\phi = -3$ and $\bar{\phi} = 2$. In order to satisfy the economical condition for the $\mu$ problem

$$-1 \leq 2h - (c + \bar{c}) + \frac{1}{2}(\phi - \bar{\phi}) \leq 1,$$

(8.12)

we adopt $c + \bar{c} = -8$, then $c = -6$ and $\bar{c} = -2$. It is interesting that in this model we do not have to introduce R-parity because half integer anomalous $U(1)_A$ charges can play the same role. Unfortunately the effective Higgs mass becomes $h + \frac{1}{4}(\phi - \bar{\phi}) = \frac{17}{16}$, which is a bit larger than the minimum value in $SO(10)$ unification case, though the gauge coupling unification may be possible using larger ambiguities of order one coefficients. Of course, we have to examine whether such a charge assignment is consistent with the DT splitting mechanism in $E_6$ unification or not, that will be discussed in separate paper [35].

---

11 Strictly speaking, even in $SO(10)$ unification case, we have to introduce the effective charges for the mass matrices, because the mass term between (5, 16) and (5, 10) is dependent on the VEV $\langle C \rangle \sim \lambda^{-\frac{1}{2}}(c + \bar{c})$. However, in the calculation in this paper, these effects happen to be cancelled. If the Higgs doublet $H_d$ originates from (5, 16), then these effects must be taken into account.
9 Discussions and Summary

In this paper, we have examined the conditions for gauge coupling unification with the anomalous $U(1)_A$ gauge theory and discussed several models which satisfy the conditions. Since the unification scale and the spectrum of super-heavy particles are determined only by the anomalous $U(1)_A$ charges, the unification conditions are described by the charges. We obtained a remarkable result that if all the fields except the MSSM fields have super-heavy masses, only a condition $h \sim 0$ realizes the gauge coupling unification. The unification scale becomes $\lambda^{-a}A_G$ and the cutoff scale becomes around the usual GUT scale $A_G \sim 2 \times 10^{16}$ GeV. It is surprising that these results are independent on the details of the Higgs contents and their charge assignment. Therefore the predictions are rather rigid, though we have some ambiguities of order 1 coefficients.

It is interesting that the unification scale is smaller than the usual GUT scale $A_G$, since $a < 0$. Therefore, proton decay through dimension six operator can be seen in future experiment. Actually, if we adopt $a = -1$, the lifetime of nucleon becomes around the present experimental limit. Moreover, our scenario predicts smaller cutoff scale than the Planck scale. One way to explain this discrepancy is to introduce extra dimension in which only gravity modes can propagate. Such a structure has been examined in the context of strongly coupled Heterotic string theory. It is interesting that the structure may give a solution for the FCNC problem in SUSY breaking sector, if only gravity modes mediate the SUSY breaking effect from the hidden brane to our visible brane.

10 Acknowledgement

We would like to thank M. Bando, T. Kugo, M. Yamaguchi and T. Yamashita for useful comments.

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