The Maximum Accreted Mass of Recycled Pulsars

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Abstract

The maximum mass of neutron stars (NSs) is of great importance for constraining equations of state of NSs and understanding the mass gap between NSs and stellar-mass black holes. NSs in X-ray binaries increase in mass by accreting material from their companions (known as the recycling process), and the uncertainties in the accretion process make studying the NS mass at birth a challenge. In this work, we investigate the NS accreted mass while considering the effect of NS spin evolution and provide the maximum accreted mass for NSs in the recycling process. By exploring a series of binary evolution calculations, we obtain the final NS mass and the maximum accreted mass for a given birth mass of an NS and a mass transfer efficiency. Our results show that NSs can accrete relatively more material for binary systems with donor masses in the range of 1.8 < 2.0 M⊙, NSs accrete relatively more mass when the remnant WD mass is in the range of 0.25 ~ 0.30 M⊙, and the maximum accreted mass is positively correlated with the initial NS mass. For a 1.4 M⊙ NS at birth with a moderate mass transfer efficiency of 0.3, the maximum accreted mass could be 0.27 M⊙. The results can be used to estimate the minimum birth mass for systems with massive NSs in observations.

Unified Astronomy Thesaurus concepts: Binary pulsars (153); Pulsars (1306); Millisecond pulsars (1062)

1. Introduction

A neutron star (NS) is the remnant of a massive star. NSs are supposed to be produced from electron capture supernovae and core-collapse supernovae (Nomoto 1984, 1987; Burrows et al. 1995; Takahashi et al. 2013; Wang & Liu 2020; see Woosley et al. 2002 for a review). The mass of an NS at birth is strongly dependent on the supernova explosion processes (Timmes et al. 1996; Tauris et al. 2015, and references therein). Theoretically, according to the different density profiles of an NS, the maximum masses can range from ~1.5 to ~2.8 M⊙ (Rikovska Stone et al. 2007; Read et al. 2009; Goriely et al. 2010; Potekhin et al. 2013; Kojo et al. 2015; and Özel & Freire 2016 for a recent review). However, most NSs are observed with mass less than 2.0 M⊙. Recent pulsar radio timing and X-ray observations found several NSs with mass beyond ~2.0 M⊙, e.g., PSR J1600-3053 with NS mass of 2.3 ± 0.17 M⊙ (Arzoumanian et al. 2018), PSR J2215 +5135 with NS mass of 2.28 ± 0.10 M⊙ (Kandel & Romani 2020), PSR J1959+2048 with NS mass of 2.18 ± 0.09 M⊙ (Kandel & Romani 2020), PSR J0740+6620 (hereafter J0740) with NS mass of 2.07 ± 0.066 M⊙ (Cromartie et al. 2020; Riley et al. 2021), and PSR J0348+0432 (hereafter J0348) with NS mass of 2.01 ± 0.04 M⊙ (Antoniadis et al. 2013). The detection of the gravitational-wave event GW190814 also suggests that there is a possibility of the existence of an NS with a mass of around 2.6 M⊙ (Abbott et al. 2020). The massive NSs in observation are important for inferring the NS mass distribution (Valentin et al. 2011; Kiziltan et al. 2013; Alsing et al. 2018; Shao et al. 2020), and constraining the NS equation of state (Lim et al. 2021; Godzien et al. 2021).

Many NSs are in binary systems, including X-ray binaries (NSSs are in the accretion phase), double NSs, and NS+white dwarf binaries, NS+black hole binaries, etc. ( Özel & Freire 2016; Abbott et al. 2021). Most NSs can accrete material from the companions and have been spun-up (Alpar et al. 1982; Radhakrishnan & Srinivasan 1982; Bhattacharya & van den Heuvel 1991; see also Tauris et al. 2012). There are many uncertainties regarding the recycling phase, e.g., accretion efficiency (ratio of NS accreted mass to the transferred mass from the donor), accretion disk instability, propeller effect (van Paradijs 1996; Antoniadis et al. 2012, 2016; Romanova et al. 2018), etc. Therefore, it is unclear how much material can be accreted by an NS in that phase. In an extreme case, if we only consider the spin-up process, the spin period of an NS is inversely correlated to the NS accreted mass. For example, the NS can be spun up to 10 and 1 ns by accretion of ~0.01 M⊙ and ~0.22 M⊙, respectively (Tauris et al. 2012). However, due to the existence of the spin-down process during the mass transfer phase, an NS may accrete more mass for a given recycled spin period (Liu & Chen 2011). The exact amount of mass accreted by an NS during the recycling process is affected by the detailed treatment of the mass transfer, and is of great importance for constraining the birth mass of an NS (Tauris et al. 2011; Cognard et al. 2017).

In this work, we attempt to determine the maximum accreted mass of an NS during the recycling processes by modeling the binary evolution with NS companions. In many previous works, the NS mass accretion is only limited by the Eddington rate, e.g., Tauris & Savonije (1999), Podsiadlowski et al. (2002), Lin et al. (2011), and Van & Ivanova (2019). Such a treatment simplifies the accretion process, and likely overestimates the accreted masses of NSs. Here we consider the effect of the spin evolution of an NS during the accretion processes in addition to the limit of the Eddington rate, as was done in Tauris et al. (2011) and Liu & Chen (2011). In this case, the propeller effect may occur and prevent mass accretion.

The paper is structured as follows. We present the model inputs and methods in Section 2, and the results are given in Section 3. The main uncertainties in our simulations are discussed in Section 4. Finally, we provide a summary and a conclusion in Section 5.
2. Model Inputs and Methods

2.1. Binary Evolution Code

Since there are many uncertainties during supernovae and NS birth, we start our study from an NS with a zero-age main sequence star as a companion. The companion may overfill its Roche lobe and transfer material to the NS. If the mass transfer is dynamically unstable, the NS will be involved in the envelope of the companion and the binary will enter into common envelope evolution process. The common envelope evolution is complicated and whether the NS can accrete material during the common envelope phase is under debate (Ivanova et al. 2013; MacLeod & Ramirez-Ruiz 2015; Holgado et al. 2018). We therefore do not consider NS accretion in this case in our study. If the mass transfer is dynamically stable, an NS will accrete mass from the companion via stable mass transfer, and we focus on this case here.

Detailed binary evolution calculations are done with the state-of-the-art stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA version 9575; Paxton et al. 2011, 2013, 2015). For convenience, the NS is taken as a point mass. For the donor star, the initial element abundances of Population I stars, i.e., metallicity Z = 0.02, are adopted. The hydrogen mass fraction is given by X = 0.76 − 3Z (Pols et al. 1998). The mixing-length parameter is set to be αMLT = 1.9. The mass transfer rate is calculated with the Ritter scheme (Ritter 1988), that is,

$$\dot{M} \propto \frac{R_d^3}{M_d} \exp \left( \frac{R_d - R_{\text{Roche}}}{H_p} \right).$$  \hspace{1cm} (1)

where $R_d$ and $R_{\text{Roche}}$ are the donor radius and its Roche lobe radius, $M_d$ is the donor mass, and $H_p$ is the pressure scale height.

The initial NS, $M_{NS,i}$, ranges from 1.10–2.2 $M_\odot$, where the NS mass has a step of 0.2 $M_\odot$ from 1.4–2.2 $M_\odot$; the choice of 1.10 $M_\odot$ is intended to cover the minimum NS in the observations (i.e., 1.174 ± 0.004 $M_\odot$ for the companion of PSR J0453+1559; Martinez et al. 2015), and 1.25 $M_\odot$ is regarded as the mean mass of an NS from electron capture supernovae (Schwab et al. 2010). We assume that all transferred material from the donor flows to the NS, then some ($\beta_{\text{mt}}$) is lost from the binary, and some ($\delta_{\text{mt}}$) forms a circumbinary (CB) disk (see more details in Section 2.2). The remainder (1 − $\beta_{\text{mt}}$ − $\delta_{\text{mt}}$) is defined as mass transfer efficiency $\eta_{\text{mt}}$. The value of $\eta_{\text{mt}}$ is quite uncertain, and is set from 0.1–0.9 with a step of 0.1. Note that the accreted mass of an NS is also limited by the Eddington rate and an inefficient accretion stage (see more details in Section 2.3), and the real accretion efficiency is lower than $\eta_{\text{mt}}$. With a given $M_{NS,i}$ and $\eta_{\text{mt}}$, the initial donor masses range from 1.0–3.6 $M_\odot$ with a variable step size, i.e., the donor mass has a step of $\Delta M_d = 0.2 M_\odot$ for $M_d \leq 2.4 M_\odot$, and $\Delta M_d = 0.4 M_\odot$ for $2.4 < M_d \leq 3.6 M_\odot$. The initial orbital period ranges from 0.7–2.0 days with a step of 0.05 day, and from 2–20 days in a step of $\Delta \log_{10}(P_{\text{orb,i}}/$ days) = 0.025. For binary systems in wider orbits, the donors generally enter into the red giant branch at the onset of mass transfer. The mass transfer rate is significantly larger than the Eddington rate, and most of the envelope masses of the donors are lost from the system (see more details in Section 3.1). For a given $\eta_{\text{mt}}$ and $M_{NS,i}$, we will obtain the maximum increased mass of an NS. The evolution stops as the evolutionary age reaches 14 Gyr, but we mainly focus on the mass transfer stage, and the termination of mass transfer is defined as $M < 10^{-12} M_\odot$ yr$^{-1}$ (Chen et al. 2017).

In this work, we mainly consider the binaries that evolve into detached NS + WD systems. The case of accreting pulsars with very low-mass nondegenerate companions, e.g., redbacks and black widows (Chen et al. 2013; Roberts 2013), are not included in our simulations. With the loss of the orbital angular momentum due to the gravitational-wave radiation, the WD will fill its Roche lobe and transfer mass to the NS. The stability of mass transfer processes is still under debate (van Haften et al. 2012; Bobrick et al. 2017; Yu et al. 2021), therefore, we ignore the cases of NS accreting mass from the WDs.

2.2. Angular Momentum Loss

We consider three types of angular momentum loss mechanisms, which are gravitational-wave radiation, magnetic braking, and mass loss, respectively.

The orbital angular momentum carried away by the gravitational-wave radiation can be calculated as (Landau & Lifshitz 1975)

$$J_{\text{GW}} = -\frac{32G^{7/2}M_\odot^2 M_d^3 (M_{NS} + M_d)^{1/2}}{5c^5 a^{7/2}},$$  \hspace{1cm} (2)

where $M_{NS}$ and $M_d$ denote the NS and the donor star mass, $a$ is the semimajor axis of the orbit, $c$ is the speed of light in vacuum, and $G$ is the gravitational constant.

The angular momentum loss because of magnetic braking is calculated from the formula (Rappaport et al. 1983)

$$J_{\text{MB}} = -3.8 \times 10^{-39} M_d R_d^{3\omega_{\text{orb}}} \Omega^3 \text{ dyn cm},$$  \hspace{1cm} (3)

where $\gamma_{\text{MB}}$ is the magnetic braking index, and is set to be 4 according to the standard magnetic braking prescription (Chen et al. 2013; Van et al. 2019). $R_d$ is the radius of the donor, $\Omega$ is the spin angular velocity, which equals the orbital angular velocity $\omega_{\text{orb}}$ as tidal synchronization is assumed. The magnetic braking effect can be neglected if the convective envelope becomes too thin. In this work, we switch on magnetic braking when the convective envelope fraction is larger than 0.01, as is done in Chen et al. (2017).

During mass transfer, we also consider the angular momentum extracted by the CB disk. The angular momentum loss rate under this torque can be expressed as (Spruit & Taam 2001; see also

5 The definition of accretion efficiency here is a little different from that in Antoniadis et al. (2016). Antoniadis et al. (2016) suggest that the accretion efficiency should be less than 0.2 according to observations. However, it is noted that the accretion efficiency defined in their work is an average value, i.e., the fraction of NS accreted mass to the lost mass from the donor. While in this work, the accretion efficiency is defined in every time interval during the mass transfer phase. Due to the existence of the Eddington limit and inefficient accretion, even if a high mass transfer efficiency is adopted, the value of average accretion efficiency is comparable to that in Antoniadis et al. (2016).

6 The reason for the variable sizes of $M_d$ and $P_{\text{orb,i}}$ is for the sake of computational cost. We find the accreted mass is relatively small for binaries with massive donors and wide periods, as shown in Section 3.3. Therefore, the step size of the initial parameters for these binaries are properly widened.

7 The higher $\gamma_{\text{MB}}$ means a stronger angular momentum loss caused by magnetic braking. However, the orbital evolution during the mass transfer stage, which is the main stage we are concerned with, is mainly dominated by mass loss (Istrate et al. 2014). The varying of $\gamma_{\text{MB}}$ will have little impact on our results.
where $\dot{M}_d$ is the mass transfer rate, $\gamma^2$ is the scale factor, and given by $r_i/a$, where $r_i$ is the inner radius of the disk, $a$ is the binary separation, $t$ is the time since mass transfer began, $t_{\text{eq}}$ is the viscous timescale at the inner edge of the disk, which is defined by $t_{\text{eq}} = 2\gamma P_{\text{orb}}/(3\pi\alpha \beta^2)$, $\alpha$ is the viscosity parameter (Shakura & Sunyaev 1973), and $\beta$ is the ratio of the scale height to the radius of the disk. Based on the observed results, we set $\gamma^2 = 1.7$ (Muno & Mauerhan 2006), $\alpha = 0.01$, $\beta = 0.03$ (Belle et al. 2004), and $\delta_{\text{in}} = 3 \times 10^{-5}$ (Taam et al. 2003).

The extra material that leaves the systems is assumed to take away the specific angular momentum of the NS. Then, the angular momentum loss due to mass loss is

$$J_{\text{ML}} = -(1 - f_{\text{in}} - \delta_{\text{in}}) |\dot{M}_d| \left( \frac{M_{\text{NS}}}{M_{\text{NS}} + \dot{M}_d} \right)^2 \frac{2\pi a^2}{P_{\text{orb}}}.$$  

2.3. Mass Accumulation Process of the NS

First, the accretion of the NS is limited by the Eddington accretion limit, $M_{\text{Edd}}$.

$$M_{\text{Edd}} = 3.6 \times 10^{-8} \left( \frac{M_{\text{NS}}}{1.4 M_\odot} \right) \left( \frac{0.1}{GM_{\odot}/R_{\text{NS}} c^2} \right) \times \left( \frac{1.7}{1 + X} \right) M_\odot \text{yr}^{-1},$$  

where $R_{\text{NS}}$ is the NS radius, and can be approximately expressed as a simple nonrelativistic degenerate Fermi-gas polytrope: $R_{\text{NS}} = 15(M_{\text{NS}}/M_\odot)^{-1/3}$ (Tauris et al. 2012). Combining the limit of Eddington accretion rate, the accretion rate of the NS is

$$M_{\text{acc}} = \min(-f_{\text{in}} \dot{M}_d, M_{\text{Edd}}).$$  

Second, the spin evolution of the NS is considered in addition to the limit of Eddington rate, which leads to an inefficient accretion during the mass transfer stage, as described below.

We define the magnetosphere radius, $r_{\text{mag}}$ of the NS at which the ram pressure of the accreted material equals the magnetic pressure in the magnetosphere (Lamb et al. 1973; Ghosh & Lamb 1979a, 1979b; Liu & Chen 2011), that is,

$$r_{\text{mag}} = 1.8 \times 10^4 \left( \frac{B_s}{10^{12} \text{G}} \right)^{4/7} \left( \frac{M_{\text{acc}}}{10^{-8} M_\odot \text{yr}^{-1}} \right)^{-2/7} \text{cm},$$  

where $B_s$ is the surface magnetic field of the NS, and $\dot{M}_{\text{in}}$ is the mass inflow rate. The evolution of magnetic field during the accretion process is described as (Shibazaki et al. 1989; Wijers 1997)

$$B_s = \frac{B_i}{1 + \Delta M_{\text{acc}}/m_B},$$  

where $B_i$ is the initial magnetic field of the NS, and is set to be $10^{12} \text{G}$, $\Delta M_{\text{acc}}$ is the accreted mass of the NS, $m_B$ is the mass constant for the field decay, and is set to be $10^{-4} M_\odot$, according to observations (Shibazaki et al. 1989; Zhang & Kojima 2006; Wang et al. 2011). If the magnetosphere radius is less than the corotation radius, the infalling material can be accreted onto the NS surface. The corotation radius is defined as (Liu & Chen 2011; Romanova et al. 2018)

$$r_{\text{co}} = 1.5 \times 10^3 \left( \frac{M_{\text{NS}}}{M_\odot} \right)^{1/3} P_{\text{spin}}^{2/3} \text{cm},$$  

where $P_{\text{spin}}$ is the spin period of the NS in units of seconds. The spin-up torque during the accretion process is given by

$$J_{\text{acc}} = M_{\text{acc}} \sqrt{GM_{\text{NS}} R_{\text{NS}}},$$  

With the spin-up of the NS, $r_{\text{mag}}$ will be greater than $r_{\text{co}}$ at some point. In this situation, the centrifugal barrier at $r_{\text{mag}}$ prevents the infalling material from being accreted by the NS. This process is known as the propeller effect. The spin evolution during the propeller phase is approximately calculated by Alpar (2001) as

$$J_{\text{prop}} = M_{\text{acc}} r_{\text{mag}}^2 \left( \frac{1}{\Omega} - \Omega_k r_{\text{mag}} \right),$$  

Moreover, if $r_{\text{mag}}$ is larger than the light cylinder radius $r_{\text{lc}}$, where $r_{\text{lc}} = c/\Omega = 48 \text{km}(P_{\text{spin}}/1 \text{ms})$, the NS spins too fast to allow the infalling material to penetrate the light cylinder and the NS appears as a radio pulsar.

There is a maximum spin frequency of the NS, i.e., Keplerian frequency, $f_K(M_{\text{NS}})$. The accreted material is supposed to be ejected if the spin frequency of the NS equals $f_K(M_{\text{NS}})$. Here $f_K(M_{\text{NS}}) \simeq C \text{kHz} (M_{\text{NS}}/M_\odot)^{1/3} (R_{\text{NS}}/10 \text{km})^{-3/2}$, where $C$ is a fitted parameter and is set to be 1.15 (Haensel et al. 2009). Since the NS is an extremely compact object, the moment of inertia of the NS should be calculated with general relativity effects and the specific equation of state of the NS considered (Arnett & Bowers 1977). For convenience, we take the NS as a point mass, and adopt $I = 10^{45} \text{g cm}^2$ for all kinds of NSs. The influence of $I$ on the NS accreted mass will be discussed in Section 4.2.

From what has been introduced above, the accretion efficiency, $\epsilon_{\text{acc}}$, is given by

$$\epsilon_{\text{acc}} = \begin{cases} \min(f_{\text{in}}, |\dot{M}_{\text{Edd}}/\dot{M}_d|) & \text{accretion phase;} \\ 0 & \text{inefficient accretion;} \end{cases}$$  

where the inefficient accretion cases include that the propeller effects occur, the NS is in the radio phase, and the NS spins at the Keplerian frequency.

3. Binary Evolution Results

3.1. Evolutionary Examples

The increase in NS mass is connected with the mass transfer rate during binary evolution, as shown in the left panel of Figure 1, where the binary initially contains a $1.4 M_\odot$ NS and a $1.4 M_\odot$ donor. The mass accretion rate of an NS can be easily obtained by using Equation (7), and is not shown for clarity. At
the early stage of mass transfer, the mass transfer is on a thermal timescale. Therefore, the NS mass can increase rapidly. The propeller effect starts to work after the NS accretes a small part of masses. At this moment, the magnetosphere radius is larger than the corotation radius, as shown in the inset of the right panel of Figure 1 (where the cyan line is above the gray line). The matter is unable to be accreted onto the NS due to the centrifugal force exerted by the magnetosphere.\textsuperscript{9} Meanwhile, the centrifugal barrier exerts a propeller spin-down torque on the NS in that phase. As the spin period increases, the magnetosphere radius can be less than the corotation radius at some point (where the gray line is above the cyan line in the inset of the right panel of Figure 1), resulting in repeated accretion processes at the early mass transfer phase. And the NS can accrete about 0.15\,M\odot during that phase. After the initial thermal timescale mass transfer, the mass transfer rate decreases due to the radius expansion of a star driven by nuclear burning (Podsiadlowski et al. 2002), and the NS enters into a long-term propeller phase. The sudden decrease in mass transfer around 3.52 Gyr is due to the discontinuity of the composition gradient during the first dredge-up stage (Tauris & Savonije 1999; Istrate et al. 2016). There is still enough envelope material for burning after the dredge-up; the donor star expands again and a subsequent mass transfer occurs (Jia & Li 2014; Li et al. 2019). It is noted that an NS can only accrete very little material in that epoch and most transferred material is accreted during the thermal timescale mass transfer stage.

As shown in Figure 1, how much mass can be accreted by an NS during the recycling process strongly depends on the mass transfer process. For low-mass donors, e.g., \(M_{\text{d,i}} = 1.4\,M_{\odot}\) in the left panel of Figure 1, the mass transfer rate is always sub-Eddington. While for more massive donors, the mass transfer rate may exceed the Eddington limit, such as in the cases shown in Figure 2. We see that the the NS can accrete relatively more mass from intermediate-mass donors (\(1.8 \lesssim M_{\text{d,i}} \lesssim 2.4\,M_{\odot}\), e.g., the case shown in the left panel of Figure 2). The reason is that the main accretion process for an NS occurs during the thermal timescale mass transfer phase. While for massive donors (\(M_{\text{d,i}} \gtrsim 2.4\,M_{\odot}\), e.g., the case shown in the right panel of Figure 2), the thermal timescale mass transfer rate could be larger than the Eddington rate by several orders of magnitude. Therefore, a significant part of transferred mass will be ejected due to the Eddington limit (Podsiadlowski et al. 2002). As a result, an NS would not accrete too much material from a low-mass donor (\(M_{\text{d,i}} \lesssim 1.8\,M_{\odot}\) due to the low thermal timescale mass transfer rate, and also cannot accrete too much material from a massive donor since most of the transferred material is ejected on account of the Eddington limit.

\subsection{3.2. The Spin Evolution of an NS}

In Figure 3, we present the spin evolution of an NS with different initial parameters, as shown in the panels. Most material has been accreted by the NS during thermal timescale mass transfer, resulting in a rapid decrease in the spin period. The NS accreted masses from left to right are 0.13, 0.22, and 0.02\,M\odot, respectively, and the NS generally rotates fast with a large accreted mass. In the middle panel, the minimum spin period of the NS is less than 1 ms. However, no sub-millisecond pulsars have been discovered yet (Hessels et al. 2006; Papitto et al. 2014; Bassa et al. 2017; Patruno et al. 2017; Haskell et al. 2018). The cause may be that the timescale of an NS in the sub-millisecond stage is very short, about 10\,yr, as shown in the middle panel. As a comparison, the timescale of an NS with \(P_{\text{spin}} \lesssim 10\) ms is about 2 \times 10\,yr, which is 20 times larger than that of the sub-millisecond stage. At the end of mass transfer, the spin periods (as shown by blue circles) are several times larger than the minimum spin period, and the NS subsequently spins down due to the magnetic dipole radiation (Tauris et al. 2012).

The spin period of an NS versus the accreted mass is shown in Figure 4, where the open circles denote the spin period at the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Left panel: the mass transfer rate and NS mass vs. star age for \(M_{\text{d,i}} = 1.4\,M_{\odot}\), \(P_{\text{orb,i}} = 1.6\,d\), \(M_{\text{NS,i}} = 1.4\,M_{\odot}\), and \(f_{\text{acc}} = 0.9\). The mass transfer rate and NS mass are shown in black and green, respectively. The propeller effects are denoted by the red lines. The Eddington rate is denoted by the gray dotted line. Right panel: comparisons among \(r_{\text{mag}}\) (cyan line), \(r_{\text{co}}\) (gray line), and \(r_{\text{c}}\) (blue line) are shown. The repeated accretion processes from 2.96–3.02 Gyr are shown in the inset. More details are given in the text.
}
\end{figure}
end of mass transfer, and the solid circles denote the minimum spin period during the accretion process. In general, the NS rotates fast for a large $\Delta M_{\text{NS}}$, consistent with the theoretical result in Tauris et al. (2012). The reason for the dispersion around the theoretical curve is that we consider the specific mass transfer process during the accretion. We also found that some NSs may have a sub-millisecond spin period, but they will spin down due to the propeller effects and the spin periods for the simulated samples at the termination of mass transfer are larger than 1 ms. Besides, at the end of mass transfer, the accreted mass is larger than that calculated in Tauris et al. (2012) for a given recycled spin period (i.e., the open circles above the green line in Figure 4). The reason is that the NS may accrete more mass and obtain a relatively shorter spin period, and then spins down to the given recycled spin period (as shown in Figure 3).

### 3.3. The Mass Increase of an NS and the Remnant WD Mass

To find the maximum accreted mass of an NS for a given $f_{\text{mint}}$ and $M_{\text{NS,i}}$, we present the relation between NS accreted mass and the remnant WD mass, as shown in Figure 5, where $M_{\text{NS,i}} = 1.4 M_\odot$, $f_{\text{mint}} = 0.3$. The symbols represent the donors in the different mass ranges, as indicated in the panels. It is clear that the NSs accrete relatively more masses as $M_{\text{d,i}}$ ranges from 1.8–2.4 $M_\odot$. The reasons are that the mass loss caused by the Eddington limit is not too much, and the NS can accrete relatively more material during the thermal timescale mass transfer phase, as discussed in Section 3.1.

NSs can accrete relatively more masses for $M_{\text{WD}}$ around 0.25–0.30 $M_\odot$. As discussed above, the main accretion process occurs at the early stage of the mass transfer phase. A binary with a short orbital period generally leads to a low thermal timescale mass transfer rate and a small remnant WD mass.
Therefore, we can see that the NS accretes 0.1 \( M_e \) at most with \( M_{\text{WD}} \simeq 0.15 M_e \) in Figure 5, which is lower than that with \( M_{\text{WD}} \simeq 0.25-0.30 M_e \). For binaries with large orbital periods, most of the transferred material is lost due to the Eddington limit, and the NS accreted mass is less than 0.05 \( M_e \) for \( M_{\text{WD}} \) around 0.40 \( M_e \). The fluctuation of NS accreted mass is ascribed to the propeller effect, which depends not only on the mass transfer rate, but also on the surface magnetic field of the NS and the corotation radius (see Equations (8)-(10)).

Figure 5 shows the correlation between an NS accreted mass and the remnant WD mass. If we assume all NSs have similar birth masses in binary pulsars, there should be a correlation between the final NS masses and the WD masses. For example, the NSs could be more massive with \( M_{\text{WD}} \) in the range of 0.25–0.30 \( M_e \). However, the relation becomes uncertain when the birth masses of NSs distribute in a large range, as found in observations (Faulkner et al. 2005; Janssen et al. 2008; Rawls et al. 2011; Arzoumanian et al. 2018; Kandel & Romani 2020).

In a further work, we will explore the mass distribution of an NS and its companion by combining the simulation results and binary population synthesis method, and try to find the correlation between NS mass and companion mass.

### 3.4. The Maximum Accreted Mass of NSs

The NS accreted mass is strongly dependent on the initial binary parameters, and is difficult to determine. However, for a given \( f_{\text{mt}} \) and \( M_{\text{NS,i}} \), there is a maximum accreted mass in the simulations. For example, when \( f_{\text{mt}} = 0.3 \) and \( M_{\text{NS,i}} = 1.4 M_e \), the maximum accreted mass of an NS is about 0.27 \( M_e \), as shown in Figure 5. By changing the values of \( f_{\text{mt}} \) and \( M_{\text{NS,i}} \), we may get the corresponding maximum accreted mass in a similar way. In the upper panel of Figure 6, we present the maximum NS mass after the accretion with given mass transfer efficiencies and initial NS masses, where the final NS masses are shown in colors. For clarity, we plot several dotted lines to express a given final NS mass as noted in the figure. The curves are given by the linear interpolation between adjoining grids. The relations between NS maximum accreted mass and mass transfer efficiency for different initial NS masses are shown in the lower panel. We can see that the maximum accreted masses vary little for \( f_{\text{mt}} \) larger than \( \sim 0.5 \), due to the existence of the propeller effect. In general, massive NSs can accrete more material in comparison with low-mass NSs if other parameters are fixed. For example, with a binary with a 1.25 \( M_e \) NS, the NS can accrete about 0.39 \( M_e \) mass for \( f_{\text{mt}} = 0.9 \). However, for an initial NS mass of 2.2 \( M_e \), the maximum accreted mass could be \( \sim 0.66 M_e \). The reason is that the massive NS has a relatively large corotation radius, resulting in more material captured by the NS.

The two massive pulsars with He WD companions, i.e., PSR J0348 and J0740, are shown by solid lines in the upper panel of Figure 6. It is noted that the maximum accreted mass of an NS is different for each \( M_{\text{NS,i}} \). If a moderate mass transfer efficiency of 0.3 is assumed, the minimum birth mass of an NS can be obtained by interpolation between the adjoining grids, as shown in the upper panel of Figure 6. We find that the NS birth masses should be larger than 1.70 and 1.75 \( M_e \) for PSR J0348 and J0740, respectively. The results of minimum NS birth masses with different mass transfer efficiencies speculated for the two observed samples are presented in Table 1.

Black widows and redbacks are one particular class of recycled pulsars that have been suggested to have significant accretion during the recycled processes (Roberts 2013). In these pulsar binaries, the very low-mass nondegenerate companions are irradiated and ablated by the NSs (Chen et al. 2013). However, the NSs in such binaries are still in the accretion phase, which are not considered in our simulations. In recent observations, two of such binaries are found with massive NSs, i.e., an NS mass of 2.18 \( \pm 0.09 M_e \) for PSR J1959+2048 and an NS mass of 2.28 \( \pm 0.10 M_e \) for J2215+5135. Chen et al. (2013) studied the formation of this kind of pulsar binary, and found that low-mass companions can be produced from donors with mass around 1.0–1.2 \( M_e \) by considering the evaporation effects. In the formation scenario of an NS with a low-mass companion, a degenerate core is not be developed at the onset of mass transfer. Therefore, progenitor binaries are supposed to have a short orbital period, which leads to a relatively lower mass transfer rate. As discussed in Section 3.1,
than ∼1.40, 1.60, 1.80, 2.00, and 2.20 lines. Lower panel: the maximum accreted mass of NSs with different inertia. In this section, we discuss the assumptions of initial magnetic processes. The main uncertainties in the simulations are the calculated in this work. should not be larger than the maximum accreted mass mass transfer, and the accreted mass of an NS in such binaries an NS mainly increases its mass during the thermal timescale mass transfer, and the accreted mass of an NS in such binaries should not be larger than the maximum accreted mass calculated in this work.

4. Uncertainties in the Simulations

We consider the spin evolution of an NS during its recycling processes. The main uncertainties in the simulations are the assumptions of initial magnetic field and the NS moment of inertia. In this section, we discuss the influences of two parameters.

4.1. The Effect of the Initial NS Magnetic Field

Most NSs at birth may have a magnetic field higher than ∼10¹³ G (Haberl 2007). Before the onset of mass transfer, the magnetic field decays due to the ohmic decay of electric currents located in the NS crust or core. Then the NS may have a magnetic field weaker than 10¹² G at the onset of the mass transfer process (Aguilera et al. 2008; Gullón et al. 2014; Bransgrove et al. 2018). A weak pulsar magnetic field will lead to a small accretion radius (magnetosphere radius), and angular momentum can be effectively transferred to the NS, which results in a large spin-up rate (Longair 2011). In general, the propeller phase occurs (r_mag > r_co) slightly later for an NS with a weak magnetic field, and then more material could be accreted by the NS.

We calculated the evolution of binaries with M_NS,i = 1.4 M_☉, B_mag,i = 0.5 × 10¹² G. For f_m = 0.9, the NS accreted mass for models with small B_mag,i is higher than that for models with B_mag,i = 10¹² G by about 0.22 M_☉. With the decrease in mass transfer efficiency f_m, the influence of the initial magnetic field correspondingly decreases. For example, the difference in the maximum NS accreted mass between B_mag,i = 10¹² G and 10¹² G for f_m = 0.1 is about 0.025 M_☉. Therefore, the choice of the initial magnetic field has a limited effect for a small f_m, but has a significant effect for a large f_m.

4.2. The Influence of the NS Moment of Inertia

The NS moment of inertia is difficult to solve due to the uncertainties of the NS equation of state. In this work, we adopt a constant value of I (10⁴⁵ g cm²) in the calculations, which is almost the lower limit for an NS. The true value of the moment of inertia for a massive NS may be larger than 3 × 10⁴⁵ g cm² (Greif et al. 2020, and references therein). Here we discuss the effect of the moment of inertia on our results.

A larger I means the acceleration of the spin-up process during the accretion phase is small, which results in a long spin period during the accretion phase. According to Equation (10), the corotation radius is larger than that for an NS with a low-value I. As a result, the binary spends a relatively shorter time in the propeller phase for an NS with a larger I, and the NS can accrete more material during the accretion phase. To illustrate this issue, we additionally calculate the cases of 1.4 M_☉ with I = 2 × 10⁴⁵ g cm². For a large mass transfer efficiency of f_m = 0.9, the NS accreted mass is about 0.1 M_☉ greater than the case of I = 1 × 10⁴⁵ g cm². Similarly, the influence of I decreases when f_m becomes small.

5. Summary and Conclusion

In this work, we consider the spin evolution of NSs, and calculate the maximum accreted mass of an NS in a binary system. Our main conclusions are summarized as follows:

(1) The accreted masses are strongly dependent on the initial donor mass and the remnant WD mass. In general, an NS can accrete relatively more material for a donor mass in the
range of $1.8 M_\odot \sim 2.4 M_\odot$ than that of donors in the other mass range. An NS accretes relatively more mass when the remnant WD mass is in the range of $\sim 0.25-0.30 M_\odot$.

(2) The maximum accreted mass of an NS is positively correlated to the initial NS mass. In other words, massive NSs can accrete more material than low-mass NSs with other initial parameters fixed.

(3) With the consideration of the spin evolution of an NS, the maximum NS accreted masses change little for mass transfer efficiency $f_{\text{mt}} > 0.5$, because of the propeller effects.

(4) The maximum accreted masses of NSs with different $f_{\text{mt}}$ and $M_{\text{NS,i}}$ are given. For example, for an NS with a birth mass of $1.4 M_\odot$, if we assume a moderate mass transfer efficiency of 0.3, the NS can accrete $\sim 0.27 M_\odot$ at most. In the extreme case, if $f_{\text{mt}} = 0.9$, the maximum accreted mass of an NS is about $0.465 M_\odot$.

(5) We analyze two massive pulsars with WD companions, i.e., J0348 ($M_{\text{NS}} = 2.01 \pm 0.04 M_\odot$) and J0740 ($M_{\text{NS}} = 2.062^{+0.067}_{-0.066} M_\odot$). Both of them are supposed to experience a recycling process during the mass transfer phase. If a moderate mass transfer efficiency of 0.3 is adopted, the birth masses of NSs should be larger than $1.70$ and $1.75 M_\odot$ for PSR J0348 and J0740, respectively. In the extreme case of $f_{\text{mt}} = 0.9$, the birth masses of NSs should be larger than $1.51$ and $1.56 M_\odot$ for PSR J0348 and J0740, respectively.

The results addressed in this work can be used to estimate the minimum birth mass for the observed pulsars. It is difficult to provide a further constraint on the likely birth mass of NSs since the accreted mass is strongly dependent on the initial progenitor binary parameters. However, by comparison with the WD mass distribution for the NS+WD populations between observations and that of binary population synthesis could provide some clues on this. For example, according to the relation between the WD mass distribution and the accreted mass distribution, we may estimate the likely accreted mass for the NS. Besides, the mass transfer efficiency has a significant effect on the NS accreted mass, which can be limited by the NS mass distribution.

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