Mathematical model to calculate key parameters of forming irregular lateral cuttings on sheet articles to eliminate such defects as “under-forging”

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Abstract. The paper considers irregular lateral cutting beyond OST 1.52468-80. It shows that standard methods do not provide solutions to forming of either element of a workpiece. The study suggests methods to find parameters of forming and technical characteristics of a part containing the given element. It also proposes method allowing eliminating such defects as “under-forging”. The results are compared with finite element analysis of forming and natural experiment. The paper makes the conclusion on the possibility of applying the suggested formulas.

1. Introduction
The analysis of the main technological tasks of elastic, liquid or gas forging states that forming with these methods may lead to various schemes of stress-strain behavior: from the simplest (linear compression or stretching) to the most complex (including deformation with additional high stress of hydrostatic compression). In some cases one and the same process may indicate diverse strained condition of deformable workpiece.

2. Results
When forming an element similar to “cutting” by elastic environment the stress-strain behavior is quite complex. At present, key parameters of cuttings during forming (pressure demand and minimum thickness) are suggested to calculate according to models proposed by E.I. Isachenkov [1], which may be presented as follows:

$$S_{(min)} = \frac{a s_0 \sin \beta}{2K}$$

(1)

where l – cutting runout;
$$s_0$$ – workpiece material thickness;
h – height (depth) of cutting;
h / l = K – steepness of cutting;
a – flat inclined area on cutting;
$$\beta$$ – angle of flat inclined area on cutting;
r – approximate cutting radius while forming.
\[ q = \frac{s_0 \sqrt{2\sigma_0 + 4\theta} \left( \frac{2l \cos \beta - 2r}{2l \cos \beta} \left( \sqrt{1 + K^2} - 1 \right) \right)^{2r \beta}}{r \left( 2l \cos \beta \left( \sqrt{1 + K^2} - 1 \right) \right)^{2r \beta}} \]

where \( \Pi \) – hardening module;
\( \sigma_0 \) – yield limit.

Upon closer analysis these formulas (1) and (2) work only in a zone of the nomogram OST 1.52468-80 (Figure 1). Area of high-quality cuttings without corrugation C or with their fitting without under-forging H and gap P.

![Figure 1. Nomogram for “cuttings”](image1.png)

In case of going beyond the given nomogram these formulas show inadequate results. The model part from material D16AM is chosen as an example (Figure 2). Area of high-quality cuttings without corrugation C or with their fitting without under-forging H and gap P.

![Figure 2. Model piece](image2.png)

Using formulas (1) and (2) let us find thinning and pressure demand. We get \( s_{i(\text{min})} = 0.111 \text{ mm.} \) and \( q = 145.47 \text{ MPa.} \) Analyzing the obtained data we see that based on the calculated minimum thickness the thinning makes 94.45\%, and the pressure exceeds the current pressure of industrial presses. This is not correct since forming of this part may only lead to defect like “under-forging” and by thinning
we may state that the crack has appeared. This indicates the impossibility of using these formulas for calculations.

However, if we break the process into two transitions (at the second transition there is a need to use a mobile clip [2]) and change stress-strain behavior while forming, then we will get the following scenario. For parts going beyond the nomogram the calculation on the basis of deformation calculated according to forming area is suggested at the left. Supposing that the surface area built on the basis of four tops and a curve belonging to the cutting top is the area to obtain cutting before forming and that up to this point the metal is not made thinner. Then ratios of the cutting area and the area of designed surface is deformation characterizing thinning (Figure 3). According to Brahmagupta’s formula [3] through determinant it is possible to define the surface area of this surface using the Sarrus’ rule [4] to find the determinant (Formula 3).

$$S = \frac{1}{4} \left| \begin{array}{cccc}
\frac{a^4 - 2 \cdot a^2 \cdot b^2 - 2 \cdot a^2 \cdot c^2 - 2 \cdot a^2 \cdot d^2 - 2 \cdot b^2 \cdot c^2 - 2 \cdot b^2 \cdot d^2 + c^4 - 2 \cdot c^2 \cdot d^2 + d^4}{8 \cdot a \cdot b \cdot c \cdot d + b^4 - 2 \cdot b^2 \cdot c^2 - 2 \cdot b^2 \cdot d^2 + c^4 - 2 \cdot c^2 \cdot d^2 + d^4} \right| = 0.25 \cdot \sqrt{8 \cdot a \cdot b \cdot c \cdot d - b^4 + 2 \cdot b^2 \cdot c^2 + 2 \cdot b^2 \cdot d^2 - c^4 + 2 \cdot c^2 \cdot d^2 - d^4}$$

Figure 3. For thinning calculations

Due to allowance the minimum thickness after forming is equal

$$S_{i(min)} = S_0 - \frac{S_{i-1} - S_0}{100} = 2 \text{ mm} - \frac{16.11\% \times 2 \text{ mm}}{100} = 1.678 \text{ mm}$$

In a general view the formula for the minimum thickness looks as follows (Formula 5):

$$S_{i(min)} = S_0 \left( \frac{2 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 2 \cdot a^4 + 2 \cdot a^2 \cdot b^2 \cdot c^2 + 2 \cdot a^2 \cdot c^2 + 2 \cdot a^2 \cdot d^2 + 2 \cdot a^2 \cdot b^2 \cdot c^2 + 2 \cdot a^2 \cdot b^2 \cdot d^2 + c^4 - 2 \cdot c^2 \cdot d^2 + d^4}{2 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 2 \cdot a^4 + 2 \cdot a^2 \cdot b^2 \cdot c^2 + 2 \cdot a^2 \cdot c^2 + 2 \cdot a^2 \cdot d^2 + 2 \cdot a^2 \cdot b^2 \cdot c^2 + 2 \cdot a^2 \cdot b^2 \cdot d^2 + c^4 - 2 \cdot c^2 \cdot d^2 + d^4} \right)$$

To find the pressure demand let us define complete deformation in undercutting zone of a piece through the ratio of undercutting zone areas and the projected area of this piece.
Let us find the surface area of undercutting zone of a piece by Brahmagupta’s formula (Figure 5, Formula 6):

\[
S_3 = 0.25 \cdot \sqrt{2 \cdot a^2 \cdot b^2 - a^4 + 2 \cdot a^2 \cdot c^2 + 2 \cdot a^2 \cdot d^2 + 8 \cdot a \cdot b \cdot c \cdot d - b^4 + 2 \cdot b^2 \cdot c^2 + 2 \cdot b^2 \cdot d^2 - c^4 + 2 \cdot c^2 \cdot d^2 - d^4}
\]

\[
= 0.25 \cdot \sqrt{2 \cdot (32,653 \, mm)^2 \cdot (12,047 \, mm)^2 - (32,653 \, mm)^4 + 2 \cdot (32,653 \, mm)^2 \cdot (31,873 \, mm)^2 + 2 \cdot (32,653 \, mm)^2 \cdot (12,122 \, mm)^2 + 8 \cdot (32,653 \, mm) \cdot (12,047 \, mm) \cdot (31,873 \, mm) \times \times (12,122 \, mm) - (31,873 \, mm)^4 + 2 \cdot (12,047 \, mm)^2 \cdot (12,122 \, mm)^2 - (31,873 \, mm)^4}
\]

\[
= 0.25 \cdot 1583,689 \, mm^2 = 395,922 \, mm^2
\]

\( (6) \)

The projection surface area as the area of a rectangle (Figure 6, Formula 7):

\[
S_4 = a \cdot b = 9 \, mm \cdot 30,0073 \, mm = 270,0657 \, mm^2
\]

\( (7) \)
Hence, complete deformation is equal (Formula 8):

\[ \varepsilon_{pol} = \frac{S_3 - S_4}{S_3} \cdot 100\% = \frac{395.922 \text{mm}^2 - 270.0657 \text{mm}^2}{395.922 \text{mm}^2} \cdot 100\% = 31.8\% \]  

To find the pressure demand it is necessary to calculate stress in plastic part of a flow curve. To describe the behavior of a material in a plastic zone let us use the Krupkowsky law function [5]. It is the mathematical function (Formula 9) considering strain hardening and connecting equivalent stress with plastic deformation:

\[ \sigma = K(\varepsilon_0 + \varepsilon_p)^n \]  

where \( K \) – mathematical constant of this material; 
\( n \) – coefficient of strain hardening; 
\( \varepsilon_0 \) – deformation of counting the beginning of plastic deformations.

For D16AM \( K = 324.17 \text{ MPa}, n = 0.2183; \varepsilon_0=0.0003 \) and \( \varepsilon_p=\varepsilon_{pol}/100\% \)

In our case stresses are equal (Formula 10):

\[ \sigma_{pod} = 252,627 \text{ MPa} (0.0003 + 0.318)^{0.2183} = 252,627 \text{ MPa} \]  

For verification finite element modeling in PAM-STAMP system was carried out and the natural experiment was performed. The following results are obtained (Table 1).

As a result we see that:
- the difference between \( \sigma_{pod} = 252,627 \text{ MPa} \) calculated analytically and \( \sigma_{eq} = 247,426 \) makes 2.1\%, so the results can be considered identical;
- the difference between \( \varepsilon_{pod} = 31.8\% \) calculated analytically and \( \varepsilon = 31.3\% \) makes 2.4\%, so the results can be considered identical;
- the difference between \( s_i(\text{min}) = 1,678 \text{ mm} \) calculated analytically and \( s_{\text{min}} = 1,696 \text{ mm} \) makes 1.1\%, so the results can be considered identical;
- the difference between \( s_i(\text{min}) = 1,678 \text{ mm} \) calculated analytically by proposed technique differs from the natural experiment \( s_{\text{nat(\text{min})}} = 1,6862 \text{ mm} \) by 0.488\%, so the results can be considered identical.
Table 1. Results of modeling and natural experiment

\[ \sigma_{eq} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \]

Stress

\[ \varepsilon = \sqrt{\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2} \]

Deformation

3. Conclusion
Based on comparisons it is possible to conclude that the suggested formulas can be used to calculate the minimum thickness, deformation and stress while forming irregular cuttings going beyond the nomogram into a defect zone like “under-forging”.

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