Dimensional Synthesis of Watt-I Metal Mechanism based on Motion Generation

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Abstract. In industrial automation, there is always increasing demand of generating desired motion with accuracy and precision. It is possible when the mechanism generates motion for large number of precision points. Therefore, in present work, the dimensional synthesis of a 6-bar, Watt-I mechanism has been carried for twenty coupler positions. The single degree of freedom mechanism has revolute pairs at each of its joint and is able to generate required motion. All the links of the mechanism are made of metal. The dyad-triad design equations are formed in terms of complex number algebra. To solve these equations, a MATLAB code is generated and utilized to yield the dimensional parameters i.e. lengths and angular orientations of all links of the mechanism. The complete synthesis methodology is modeled and demonstrated on a numerical problem. To verify the outcomes, SAM 7.0 application software has been incorporated and applied on the given example.

1. Introduction
Inventing useful machine has always been keen interest of mankind. A machine consists of single mechanism or a group of mechanisms that work together to accomplish some useful work. Dimensional synthesis of such mechanisms is required to develop new advanced machines for modern applications. It includes assurance of correct link length along with its orientation to satisfy the required kinematic task. The number of links and their configuration varies depending upon the intricacy of the kinematic task to be performed. The configuration of the mechanism may be of 4-bar, 6-bar, 8-bar, slider crank mechanism or any other configuration that fulfills the requirements of kinematic task viz., path generation, motion generation, function generation to be performed. Dimensional synthesis for any such kinematic task involves finding out approximate/exact length of all the links of the mechanism that constitute and configure the mechanism. Also, it involves determination of orientations of all the links of the mechanism at its home position.

Dimensional synthesis can be carried out using graphical or analytical methods. But, graphical methods are restricted due to drawing accuracy. On the other hand, the analytical methods are in general simple and easy to apply. When they are used in combination with high-end programming softwares, they yield dimensions of all design parameters of the mechanism with improved accuracy. Sometimes, any optimization technique is also incorporated to determine the optimal design parameters of the mechanism to fulfill the sophisticated requirement of the task to be performed with minimum error. E.g., the optimal synthesis of a 6-bar linkage having rotational constraints was carried out in which coupler point generates the desired path [1]. The aim of synthesis is to get the required path as close to the specified path as possible. The author(s) considered that the specified path as a conjunction of a circular
are along with rectilinear segment. Many researchers have evaluated dimensional synthesis of various mechanisms for different kinematic tasks as discussed under subsequent section of related work.

2. Related work
Till starting of 20th century, only graphical methods were more prevalent to carry out dimensional synthesis of mechanisms. Rose [2] carried out synthesis of 5-bar mechanism using loop equations. Subsequently, Rawat [3] performed synthesis of variable topology mechanisms. Sun et al. [4] proposed a synthesis methodology of spherical 4-bar mechanism for motion generation. The author(s) performed harmonic analysis and observed that the output parameters are dependent on the rotation angle operator of the rigid body. Farhang et al. [5] discussed the motion generation based synthesis of those mechanisms which have input cranks with relatively small length. The author concluded that the coupler tracing point generate only elliptical curve for such mechanisms. Soh and Ying [6] projected design procedure for synthesis of 6-bar mechanism and 8-bar slider mechanisms to generate complex motion. The author(s) demonstrated the design process on wheelchair that can renovate its configuration as walking support for rehabilitation purposes. Shen et al. [7] formulated a computer code to generate prescribed coupler positions of fixed and moving pivot loci for 4-bar mechanism based on maximum driver static torques and coupler load. Schreiber et al. [8] combined circle point search method with homotopy method to dimensionally synthesis the planar Stephenson mechanisms for generating given motion. Martin et al. [9] suggested a MathCAD computer code and demonstrated the synthesis of a planar 4-bar crank rocker mechanism. The author(s) also considered feasible transmission angles while synthesizing the mechanisms. Balli and Chand [10-11] carried out the dimensional synthesis of variable topology 5-bar mechanism for motion between extreme positions. Their work was confined to single coupler tracing point. So, Nafees and Mohammad [12-13] extended the dimensional synthesis work of 5-bar mechanism for double offset tracing points. Balli and Chand [14-15], carried out synthesis of planar 7-link mechanisms that transfer motion between two extreme positions. Gadad et al. [16-17] synthesized variable topology 7-bar mechanisms for transmitting motion between extreme positions. Furthermore, Daivagna and Balli [18-20] performed synthesis of 5-bar and 7-bar variable topology mechanism to transfer motion between two dead-centre positions. Recently, Qaiyum and Mohammad [21] performed PSO based optimal synthesis of Stephenson-III mechanism.

Researchers have carried out synthesis, but they lack in performing synthesis for large number of precision points. To fulfill the requirements of high accuracy and precision in industrial automation, it is necessary that mechanisms should generate motion for greater number of precision points. Therefore, in present work, dimensional synthesis of a 6-bar (Watt-I) mechanism has been performed with 20 precision points. The design equations have been derived and represented in terms of complex number mathematics. Finally, a numerical problem has been demonstrated and verified to elaborate the synthesis methodology for given mechanism.

3. Arrangement of planar Watt-I linkage
The Watt-I is a planar kinematic six-bar linkage comprising of binary and ternary links with revolute pair at each of its joints. There are 3 binary, 2 ternary and 1 binary offset links (Refer Figure 1).

The connection of various links of the Watt-I linkage is shown in Fig. 1. The binary link 1 is fixed at pivots O₁ and O₂. The crank link 2 rotates about fixed point O₁ and the input motion is imparted to the linkage through this link. The ternary link 3 connects with crank link 2, binary offset link 4 and ternary link 6 at points, A, B and F respectively. The binary link 4 has an offset at point C which is called as coupler tracing point. The dimensional synthesis of the whole linkage is based on precision points and orientation of the binary offset link 4. Another ternary link 6 connects with fixed link 1, binary link 5 and ternary link 3 at points O₂, E and F respectively. The ternary link 6 is also called oscillating link as it oscillates about fixed point O₂. The binary link 5 makes revolute pair with link 4 and link 6 at points D and E respectively.
4. Arrangement of planar Watt-I linkage for prime position
Consider the home position of the linkage as shown in Figure 2. The initial position of each revolute joint is expressed by $O_1 A_0 B_0 C_0 D_0 E_0 F_0$. With the rotation of crank link from $O_1A_0$ to $O_1A_j$ by an angle $\theta$, the position of each revolute joint of the linkage is also shifted. The linkage acquires a prime position which is expressed by $O_1A_j B_j C_j D_j E_j F_j O_2$ (Refer Figure 2). All the remaining links i.e., ternary link ABF, ternary link $O_2EF$, binary link DE and binary offset link BCD have rotated by orientation angles of $\alpha_j$, $\beta_j$, $\phi_j$ and $\gamma_j$ respectively. The coupler tracing point C is also shifted from position $C_0$ to $C_j$ thereby the displacement of this point is expressed by $\delta_j$. For motion generation, there is requirement of 20 prescribed precision points ($C_j$) along with 20 orientations of binary offset links, which yields 20 prime position of the given linkage.

Figure 2 Displacement of Planar Watt-I Linkage from Initial to Prime Position By $\delta_j$

5. Formation of design equations
By considering the various independent vector loops, the equations are derived as explained below:

Consider the independent vector loop $O_1A_jB_jC_jB_0A_0O_1$ (Refer Figure 2), the design equation \([22]\) is formed as

\[
Z_t e^{i\theta} + Z_3 e^{i\alpha} + Z_{10} e^{i\beta} - \delta_j - Z_{10} - Z_3 - Z_t = 0
\]

\[
Z_t(e^{i\theta} - 1) + Z_3(e^{i\alpha} - 1) + Z_{10}(e^{i\beta} - 1) = \delta_j
\]

(1)

Consider the independent vector loop $O_2E_jD_jC_jD_0E_0O_2$ (Refer Figure 2), the design equation \([22]\) is formed as
\[ Z_e e^{j\beta} + Z_e e^{j\phi} + Z_{i1} e^{j\gamma} - \delta_j - Z_{s} - Z_{j} = 0 \]
\[ Z_s (e^{j\beta} - 1) + Z_s (e^{j\phi} - 1) + Z_{i1} (e^{j\gamma} - 1) = \delta_j \]  \hspace{1cm} (2)

Consider the independent vector loop \( E_0F_0B_0D_0E_0 \) (Refer Figure 2), the design equation [22] is formed as
\[ Z_6 + Z_7 + Z_0 - Z_8 = 0 \]  \hspace{1cm} (3a)

Consider the independent vector loop \( E_jF_jB_jD_jE_j \) (Refer Figure 2), the design equation [22] is formed as
\[ Z_6 e^{j\beta} + Z_6 e^{j\phi} + Z_{i0} e^{j\gamma} - Z_8 e^{j\phi} = 0 \]  \hspace{1cm} (3b)

On subtracting equation (3a) from (3b), we get
\[ Z_s (e^{j\beta} - 1) + Z_s (e^{j\phi} - 1) + Z_{i0} (e^{j\gamma} - 1) - Z_8 (e^{j\phi} - 1) = 0 \]  \hspace{1cm} (3)

For 20 precision points, \( j \) varies from 1 to 20 in design equations (1) to (3).

Consider closed loop \( F_0A_0B_0F_0 \), the unknown vector \( Z_2 \) is determined as (Refer Figure 2)
\[ Z_2 = Z_4 - Z_1 \]  \hspace{1cm} (4)

Consider closed loop \( O_2F_0E_0O_2 \), the unknown vector \( Z_3 \) is determined as (Refer Figure 2)
\[ Z_3 = Z_6 + Z_7 \]  \hspace{1cm} (5)

Consider closed loop \( O_1O_2F_0A_0O_1 \), the unknown vector \( Z_{12} \) is determined as (Refer Figure 2)
\[ Z_{12} = Z_1 - Z_3 - Z_5 \]  \hspace{1cm} (6)

6. Algorithm to obtain solution of design equations

As the number of unknown parameters is less than number of equations, the solution of equations (1) to (6) cannot be obtained manually. Therefore, a MATLAB computer code is developed to determine the solution of these equations. The algorithm for this program involves the following steps:

Step1. Input the value of prescribed parameters that comprises of coupler point orientation of binary offset link BCD (\( \gamma \)) and coordinates of coupler point C (P) for 20 precision points.

Step2. Determine displacement vector (\( \delta_j \)) for each coupler point C by subtracting coordinates of its prime position from initial position based on all precision points.

Step3. Input the values of assumed parameters and freely chosen parameters.

Step4. For \( j = 1, 2, \ldots, 20 \), calculate values \( e^{j\beta}, e^{j\phi}, e^{j\gamma}, \) and \( e^{j\phi} \).

Step5. Determine value of lengths \( Z_1, Z_3 \) and \( Z_{10} \) in vector form using equation (1).

Step6. Determine value of lengths \( Z_5, Z_6 \) and \( Z_{11} \) in vector form using equation (2).

Step7. Determine value of lengths \( Z_8, Z_4 \) and \( Z_9 \) in vector form using equation (3).

Step8. Determine value of length \( Z_2 \) in vector form using equation (4).

Step9. Determine value of length \( Z_5 \) in vector form using equation (5).

Step10. Determine value of length \( Z_{12} \) in vector form using equation (6).

7. Numerical problem

7.1 Problem Statement

It is required to carry out to dimensionally synthesize a planar kinematic six-bar Watt-I linkage for motion generation. The mechanism is prescribed with 20 motion generation positions corresponding to coupler orientations as given in Table 1 (Refer Figure 3).

7.2 Prescribed Parameters

The prescribed parameters include coupler point motion of binary offset link BCD (\( \gamma \)) and displacement vector (\( \delta_j \)). The displacement vector (\( \delta_j \)) is obtained by subtracting coordinates of the prime position from initial position at equal intervals (i.e. \( \theta_j \in \{0, 2\pi\} \), where \( j = 1, 2, \ldots, 20 \)).
Table 1 Prescribed Parameters

| Prescribed Point \( (P_j) \) | Abscissa \( (X_j) \) | Ordinate \( (Y_j) \) | Prescribed Coupler Orientation \( (\gamma_j) \) rad. |
|-----------------------------|------------------|------------------|---------------------------------|
|   \( P_1 \)                |  0.950           |  0.941           |  5.500                          |
|   \( P_2 \)                |  0.886           |  0.935           |  8.995                          |
|   \( P_3 \)                |  0.838           |  0.925           | 10.428                          |
|   \( P_4 \)                |  0.809           |  0.915           |  9.740                          |
|   \( P_5 \)                |  0.801           |  0.906           |  6.875                          |
|   \( P_6 \)                |  0.816           |  0.897           |  1.776                          |
|   \( P_7 \)                |  0.857           |  0.888           | -5.615                          |
|   \( P_8 \)                |  0.927           |  0.875           | -15.470                         |
|   \( P_9 \)                |  1.027           |  0.851           | -27.674                         |
|   \( P_{10} \)             |  1.152           |  0.805           | -41.081                         |
|   \( P_{11} \)             |  1.280           |  0.732           | -52.540                         |
|   \( P_{12} \)             |  1.378           |  0.655           | -58.098                         |
|   \( P_{13} \)             |  1.423           |  0.621           | -57.067                         |
|   \( P_{14} \)             |  1.426           |  0.645           | -51.566                         |
|   \( P_{15} \)             |  1.400           |  0.703           | -43.602                         |
|   \( P_{16} \)             |  1.351           |  0.771           | -34.435                         |
|   \( P_{17} \)             |  1.283           |  0.836           | -24.866                         |
|   \( P_{18} \)             |  1.202           |  0.887           | -15.584                         |
|   \( P_{19} \)             |  1.114           |  0.921           |  -7.162                         |
|   \( P_{20} \)             |  1.028           |  0.938           |  0.000                          |

7.3. Assumed Parameters
The assumed parameters include orientation of crank \( O_1A (\theta_j) \), ternary link ABF \( (\alpha_j) \), ternary link \( O_2EF (\beta_j) \) and binary link DE \( (\phi_j) \) and the corresponding range of these parameters are \{-2\pi/45, 5\pi/36\}, \{-5\pi/12, \pi/18\} and \{-\pi/18, \pi/6\} respectively [23].

7.4. Prescribed Parameters
The design parameters include dimensional lengths of fixed link \( O_1O_2 \), binary links \( (O_1A \) and DE), ternary links \( (ABF \) and \( O_2EF \) and binary offset link BCD. Each side of these links is considered as design parameter and represented in vector notation as \( Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9, Z_{10}, Z_{11} \) and \( Z_{12} \) (Refer Figure 2).
8. Result and discussion

The dimensional length and angular positions of each link is determined by solving equations (1) to (6) with the help of MATLAB coding are:

\[
Z_1 = 0.20 \angle 157^\circ; \quad Z_3 = 0.74 \angle 15^\circ; \quad Z_5 = 0.77 \angle 37^\circ; \quad Z_4 = 0.29 \angle 111^\circ; \\
Z_2 = 0.31 \angle 119^\circ; \quad Z_6 = 0.31 \angle 12^\circ; \quad Z_7 = 0.40 \angle 246^\circ; \quad Z_8 = 0.25 \angle 95^\circ; \\
Z_9 = 0.39 \angle 7^\circ; \quad Z_{10} = 0.29 \angle 46^\circ; \quad Z_{11} = 0.25 \angle 138^\circ; \quad Z_{12} = 0.70 \angle 0.27^\circ
\]

These final dimensional length with angular orientations of each link of given Watt-I mechanism for motion generations based on 20 precision points are graphically shown in Figure 4.

![Figure 4](image)

**Figure 4** Final dimensions of each link of Watt-I mechanism based on twenty precision points motion generation

9. Conclusions

In this paper, the state-of-art is summarised for dimensional synthesis of a 6-bar, Watt-I mechanism that generates required motion for 20 precision points. For this purpose, various design equations are derived and formed by considering various independent vector loops. The parameters have been identified as prescribed parameters, assumed parameters and design parameters. The SAM 7.0 software has been used to verify the final dimensional parameters determined using MATLAB coding. The advantage of present work is that it will assist in the design of mechanisms that will fulfil the accuracy and precision requirement of automation industry. These useful mechanisms will be effective in transforming intricate motions for a greater number of precision positions. The existing methodologies have low accuracy and poor precision due to limited number of precision points. Therefore, comparing with other methods, this work overcomes the limitation of existing methodologies and provides benefit of improved accuracy and precision.

**Nomenclature**

\( i \): A subscript used for representing link length parameters and varies from 1 to 12.
\( j \): A subscript used for representing precision point parameters and varies from 1 to 20.
\( Z \): Vector symbol representing side of each link.
\( \delta_j \): Displacement of coupler point from initial position \( C_0 \) to prime position \( C_j \)
\( \theta_j \): Angular movement of binary crank link \( O_1A \) from initial position \( O_1A_0 \) to prime position \( O_1A_j \)
\( \alpha_j \): Angular movement of ternary link \( ABF \) from initial position \( A_0B_0F_0 \) to prime position \( A_jB_jF_j \)
\( \beta_j \): Angular movement of ternary link \( O_2EF \) from initial position \( O_2E_0F_0 \) to prime position \( O_2E_jF_j \)
\( \gamma_j \): Angular movement of binary offset link \( BCD \) from initial position \( B_0C_0D_0 \) to prime position \( B_jC_jD_j \)
\( \phi_j \): Angular movement of binary link \( DE \) from initial position \( D_0E_0 \) to prime position \( D_jE_j \)
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