Conditional Inference and Activation of Knowledge Entities in ACT-R

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Abstract. Activation-based conditional inference applies conditional reasoning to ACT-R, a cognitive architecture developed to formalize human reasoning. The idea of activation-based conditional inference is to determine a reasonable subset of a conditional belief base in order to draw inductive inferences in time. Central to activation-based conditional inference is the activation function which assigns to the conditionals in the belief base a degree of activation mainly based on the conditional’s relevance for the current query and its usage history. Therewith, our approach integrates several aspects of human reasoning into expert systems such as focusing, forgetting, and remembering.

Keywords: Conditional Reasoning, Inductive Inference, Cognitive Model, ACT-R, Focusing, Forgetting, Remembering

1 Introduction

Expert systems \cite{718} are computer programs which infer implicit information from belief bases in order to solve complex reasoning tasks. Basically, expert systems consist of two components, a belief base and an inference engine. When the user inserts a query, the belief base and the query are transferred to the inference engine which decides whether the query can be drawn from the belief base or not. The aspiration of expert systems is to draw inferences of high quality from usually incomplete and uncertain beliefs.

The contribution of activation-based conditional inference to this inference process is a preselection of beliefs, yielding a reduced belief base that is transferred to the inference engine. The objective is both to reduce computational costs during the inference process, and to model human cognitive processes with expert systems more adequately. It is obvious and reasonable that human reasoners do not draw inferences based on all of their beliefs, in particular when they have to make snap decisions in time. Basically, there are two cognitive processes which affect the selection of beliefs: The long-term process of forgetting and
remembering and the short-term process of activating certain beliefs depending on the context. In ACT-R (Adaptive Control of Thought-Rational, [5,4]), a well-founded cognitive architecture established in cognitive science in order to formalize human reasoning, the long-term memory is represented by the base-level activation, while the context-dependent activation of beliefs is described by the spreading activation theory [3]. The core idea behind the spreading activation theory is that an initial priming caused by sensory stimuli triggers certain cognitive units [3] which again trigger related cognitive units and so on until the disposition for activation is too low to trigger further cognitive units. The triggered cognitive units settle the current focus in which reasoning takes place.

In this paper, we adapt the concept of (de)activation of knowledge entities from ACT-R and combine it with conditional inference formalisms from nonmonotonic reasoning. More precisely, we define a model for activation-based inference from conditional belief bases by adapting the activation function from ACT-R to conditionals of the form \((B|A)\) with the meaning “if \(A\) holds, then usually \(B\) holds, too.” Therewith, on the one hand, we generalize the concept of focused inference [16] and give it a profound cognitive meaning. And on the other hand, we equip ACT-R, which is typically realized as a production system [9,11], with a modern inference formalism of high quality.

The rest of the paper is organized as follows. First, we recall some basics on conditional logics, inductive inference formalisms, and the ACT-R architecture. Then, we give a brief outline of activation-based conditional inference which is based on the activation function for conditionals. This activation function is examined in more detail afterwards. Finally, we show how the concepts of forgetting and remembering can be integrated into our framework before we conclude with an outlook.

2 Preliminaries

2.1 Logical Foundations

We consider a propositional language \(\mathcal{L}\) which is defined over a finite set of propositional variables (or atoms) \(\Sigma\). Formulas in \(\mathcal{L}\) are built by the common connectives \(\neg\) (negation), \(\land\) (conjunction), and \(\lor\) (disjunction). The semantics of formulas in \(\mathcal{L}\) is given by interpretations \(I \in \mathcal{I}\) as usual. We further use the abbreviations \(AB = A \land B\), \(\neg A = \neg A\), \(A \Rightarrow B = A \lor B\), \(\top = A \lor \neg A\), and \(\bot = A \land \neg A\). An expression of the form \((B|A)\) with \(A, B \in \mathcal{L}\) is called conditional and has the intuitive meaning “if \(A\) holds, then usually \(B\) holds, too.” Formally, conditionals are interpreted by ranking functions over possible worlds [15]. Here, possible worlds are the interpretations in \(\mathcal{I}\) represented as complete conjunctions of literals, i.e. an atom or its negation. The set of all possible worlds is denoted by \(\Omega\). A ranking function \(\kappa : \Omega \to \mathbb{N}_0^\infty\) maps possible worlds to a degree of plausibility while satisfying the normalization condition \(\kappa^{-1}(0) \neq \emptyset\). Lower ranks indicate higher plausibility so that \(\kappa^{-1}(0)\) is the set of the most plausible worlds. The rank of a formula \(A\) is the minimal rank of its models, \(\kappa(A) = \min\{\kappa(\omega) \mid \omega \in \Omega, \ \omega \models A\}\), where the convention \(\min\emptyset = \infty\) applies.
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| Conditional | Meaning |
|-------------|---------|
| \( r_1 = (f|aw) \) | Winged animals usually fly. |
| \( r_2 = (\overline{f}|aw) \) | Wingless animals usually do not fly. |
| \( r_3 = (b \Rightarrow a|\top) \) | Birds are animals. |
| \( r_4 = (w|b) \) | Birds usually have wings. |
| \( r_5 = (d|b) \) | Birds usually drink water. |
| \( r_6 = (p \Rightarrow b|\top) \) | Penguins are birds. |
| \( r_7 = (\overline{f}|p) \) | Penguins usually do not fly. |
| \( r_8 = (c \Rightarrow b|\top) \) | Chickens are birds. |
| \( r_9 = (\overline{f}|c) \) | Chickens usually do not fly. |
| \( r_{10} = (f|cs) \) | Scared chickens usually fly. |
| \( r_{11} = (\overline{s}|c) \) | Chickens are usually not scared. |
| \( r_{12} = (i \Rightarrow a|\top) \) | Fish are animals. |
| \( r_{13} = (r \Rightarrow i|\top) \) | Freshwater fish are fish. |
| \( r_{14} = (l \Rightarrow i|\top) \) | Saltwater fish are fish. |
| \( r_{15} = (l \lor r|i) \) | Fish are usually saltwater fish or freshwater fish. |
| \( r_{16} = (\overline{d}|r) \) | Freshwater fish usually do not drink water. |
| \( r_{17} = (d|l) \) | Saltwater fish usually drink water. |
| \( r_{18} = (h \Rightarrow r|\top) \) | Hatchetfish are freshwater fish. |
| \( r_{19} = (\overline{f}|\overline{w}|h) \) | Hatchetfish usually fly but are wingless. |
| \( r_{20} = (k \Rightarrow m|\top) \) | Kangaroos are marsupials. |

Table 1. Belief base \( \Delta' = \{ r_1, \ldots, r_{20} \} \) (cf. Example 1).

\( \kappa \) accepts a conditional \( (B|A) \) iff \( \kappa(AB) < \kappa(\overline{A}B) \) or \( \kappa(A) = \infty \). A ranking function \( \kappa \) is a model of a belief base \( \Delta \), i.e. a finite set of conditionals, iff \( \kappa \) accepts all conditionals in \( \Delta \). A belief base is called consistent iff it has at least one model. The set of all belief bases over \( L \) is denoted by \( D \). If \( X \) is a formula, a conditional, or a belief base, then we denote the set of atoms mentioned in \( X \) with \( \Sigma(X) \), i.e., \( \Sigma(X) \) is the signature of \( X \).

Example 1. In Table 1 an example of a consistent belief base over the signature \( \Sigma' = \{ a, b, c, d, f, h, i, k, l, m, p, r, s, w \} \) is shown. For example, \( \Sigma(\Delta) = \Sigma' \) and \( \Sigma(r_6) = \{ b, p \} \).

2.2 Conditional Inference

We now consider the task of drawing inductive inferences from a belief base \( \Delta \) and recall the notion of focused inference from [16]. Roughly said, inductive inferences are conditionals which are plausible consequences from \( \Delta \).

Definition 1. (cf. [8]) An (inductive) inference operator \( \mathcal{I} : D \rightarrow L \times L \) is a mapping which assigns to each belief base \( \Delta \in D \) an inference relation \( \vdash^3_\Delta \subseteq L \times L \) such that:
- If \( (B|A) \in \Delta \), then \( A \vdash^3_\Delta B \). (Direct Inference)
If $\Delta = \emptyset$, then $A \vdash^\Delta B$ only if $A \models B$. (Trivial Vacuity)

$\mathcal{I}_\Delta = \{(B | A) \mid A \vdash^\Delta B\}$ denotes the set of inductive inferences from $\Delta$ wrt. $\mathcal{I}$. We further define a three-valued inference response to a query conditional $(B | A)$ by

$$[(B | A)]^3_\Delta = \begin{cases} 
\text{yes} & \text{iff } (B | A) \in \mathcal{I}_\Delta \\
\text{no} & \text{iff } (\overline{B} | A) \in \mathcal{I}_\Delta \\
\text{unknown} & \text{otherwise}
\end{cases}$$

An important representative of inference operators is the System $P$ inference operator $\mathcal{I}^P$, which is defined by $(B | A) \in \mathcal{I}^P$ iff every model of $\Delta$ accepts $(B | A)$.

$\mathcal{I}^P$ is characterized by a collection of inference rules which are well-established in nonmonotonic reasoning [1,10]. One also has $(B | A) \in \mathcal{I}^P_{\emptyset}$ iff $\Delta \cup \{(\overline{B} | A)\}$ is inconsistent [6].

Another well-founded inference operator is provided by System $Z$ [12]. Here, we are not interested in the inference operator $\mathcal{I}^Z$ itself but in the $Z$-partition of $\Delta$ which is an auxiliary structure for computing $\mathcal{I}^Z_\Delta$. $Z$-partitions are ordered partitions of belief bases based on the notion of tolerance. A belief base $\Delta$ tolerates a conditional $(B | A)$ iff there is a possible world $\omega$ in which $(B | A)$ is verified, i.e. $\omega \models AB$, and no conditional from $\Delta$ is falsified, i.e. $\omega \models (A' \Rightarrow B')$ for all $(B' | A') \in \Delta$. An ordered partition $(\Delta_0, \Delta_1, \ldots, \Delta_m)$ of $\Delta$ is a tolerance partition of $\Delta$ iff for $i = 0, \ldots, m$ every conditional in $\Delta_i$ is tolerated by $\bigcup_{j=i}^m \Delta_j$.

The $Z$-partition $Z(\Delta)$ is the unique tolerance partition of $\Delta$ which is obtained by iteratively determining $\Delta_i$, the maximal set of tolerated conditionals. If a conditional $r$ is in the $i$-th partition of $Z(\Delta)$, we say that $r$ has $Z$-rank $Z_\Delta(r) = i$.

System $Z$ satisfies the paradigm of maximum normality, i.e., the lower the $Z$-rank of a conditional is, the more normal the conditional is.

**Example 2.** The $Z$-ranks of the conditionals in $\Delta'$ (Table 1) are shown in Table 2. For example, the $Z$-rank of $\tau_1$ is $Z(\Delta')(\tau_1) = 0$ because $\tau_1$ is tolerated by $\Delta'$ (consider $\omega = abcd\overline{ef}\overline{ghi}\overline{mnpqrstu}$). Further, the $Z$-ranks $Z(\Delta)(\tau_1) = 2$ and $Z(\Delta)(\tau_0) = 1$ illustrate the concept of normality. While conditional $\tau_9$ is concerned about the flight ability of chickens in general, conditional $\tau_{10}$ makes a statement about the flight behavior of chicken when they are in a special mood. Hence, conditional $\tau_{10}$ applies to a more specific case than $\tau_9$ and accordingly has a higher $Z$-rank.

An inference operator $\mathcal{I}$ is semi-monotonic iff for every two belief bases $\Delta$ and $\Delta'$ it holds that $\Delta \subseteq \Delta'$ implies $\mathcal{I}_\Delta \subseteq \mathcal{I}_{\Delta'}$. While System $P$ inference is semi-monotonic (cf., e.g., [10]), System $Z$ inference is not. We give an example which illustrates the semi-monotony of System $P$.

**Example 3.** Consider $\Delta'' = \{\tau_9, \tau_{11}\} \subseteq \Delta'$ (Table 1). Since $\Delta'' \cup \{(f | c\overline{s})\}$ is inconsistent, $c\overline{s} \not\vdash_{\Delta''} f$ follows. That is, one can infer from $\Delta''$ wrt. System $P$ that chicken which are not scared usually do not fly. Due to the semi-monotony of System $P$, this inference can also be drawn from $\Delta'$ because of $\Delta'' \subseteq \Delta'$.

We now recall the concept of focused inference from [10]. The idea behind focused inference is to draw inferences from a reasonable (as small as possible)
subset of $\Delta$ in order to make snap but still well-founded decisions in time. In this context, the advantage of semi-monotonic inference operators like $I^P$ is that one does not risk to draw false inferences when focusing on a subset $\Delta' \subset \Delta$ because $\langle (B|A) \rangle^P_{\Delta'} = \text{yes (resp. no)}$ implies $\langle (B|A) \rangle^P_\Delta = \text{yes (resp. no)}$. In order to formalize focused inference, we consider mappings $\phi: \mathcal{D} \rightarrow \mathcal{D}$ with $\phi(\Delta) \subseteq \Delta$, i.e. mappings which return subsets of $\Delta$. We call such a mapping $\phi$ a focus.

**Definition 2.** Let $\Delta$ be a belief base, $(B|A)$ a conditional, $\mathcal{I}$ an inference operator, and $\phi$ a focus. Then, $(B|A)$ follows from $\Delta$ wrt. $\mathcal{I}$ in the focus $\phi$ iff $(B|A) \in \mathcal{I}_{\phi}(\Delta)$.

In [10], the focus $\phi$ is defined iteratively based on the query $q = (B|A)$: The conditionals in the direct focus $\phi_0^q(\Delta)$ are those conditionals which share at least one atom with $q$, i.e. $\phi_0^q(\Delta) = \{ r \in \Delta : \Sigma(r) \cap \Sigma(q) \neq \emptyset \}$. The conditionals in the $i$-th focus are determined by $\phi_i^q(\Delta) = \{ r \in \Delta : \exists r \in \phi_{i-1}(\Delta) : \Sigma(r) \cap \Sigma(q) \neq \emptyset \}$.

**Example 4.** The direct focus of $\Delta'$ (Table 1) wrt. $q = (f|c\sigma)$ is $\Delta_0 = \phi_0^q(\Delta')$ with $\Delta_0 = \{ t_1, t_2, t_7, t_8, t_9, t_{10}, t_{11}, t_{19} \}$. One has $[q]^P_{\Delta_0} = \text{no}$. According to Example 3 one already has $[q]^P_{\Delta''} = \text{no}$ where $\Delta'' = \{ t_9, t_{11} \} \subset \Delta_0$, though. Hence, the direct focus does not have to be the smallest possible focus in which an inference can be drawn. On the contrary, a focus can also be too small in order to decide a query. For instance, $[q]^P_{\Delta'} = \text{unknown}$ wrt. any $\Delta'' \subset \Delta'$ with $\{ t_9, t_{11} \} \not\subseteq \Delta''$.

Apart from the computational benefits of drawing inferences wrt. small foci, appropriate foci are also interesting from the knowledge representation and reasoning (KRR) perspective because they unveil the part of the belief base which is relevant for answering the query. Unfortunately, finding appropriate foci is challenging. In the following, we approach this problem from the cognitive science perspective and develop a framework for drawing focused inferences which are justified by cognitive principles.

### 2.3 ACT-R Architecture

*ACT-R* [54] is a production systems based cognitive architecture which formalizes human reasoning. In ACT-R a distinction is made between declarative and procedural memory. In the declarative memory, categorical knowledge about individuals or objects is stored in form of chunks (*knowing that*) while the procedural memory consists of production rules and describes how chunks are processed (*knowing how*, [14]). Reasoning in ACT-R starts with an initial priming, for example a stimulus from the environment, which causes an activation of chunks. The chunk with the highest activation is processed by production rules in order to compute a solution to the reasoning task. If this fails, the activation passes into an iterative process: The system obtains additional chunks from the declarative memory and tries to compute a solution again. The iteration stops when either the problem is solved or no further chunks are active. The retrieval of chunks is
a very refined process in ACT-R. Basically, it depends on an *activation function* which is calculated for each specific request and is based on a *usage history* of the chunks, associations between *cognitive units* and the *priming* [3]. There is no clear consensus about the kind of cognitive units despite of the perception that they form the basic building blocks of thinking [2].

How the activation of a chunk $A(c_i)$ is computed in detail depends on multiple parameters and the configurations of the ACT-R system, but is mainly given by the sum of the so-called *base-level activation* $B(c_i)$ and the *spreading activation* $S(c_i)$, which again is a sum of *degrees of associations* between chunks $S(c_i, c_j)$ weighted by some *weighting factors* $W(c_j)$:

$$A(c_i) = B(c_i) + \sum_j W(c_j) \cdot S(c_i, c_j).$$

(1)

The *base-level activation* of a chunk $B(c_i)$ reflects the *entrenchment* of $c_i$ in the reasoner’s memory and depends on the recency and frequency of its use. Typically, $B(c_i)$ is decreased over time (*fading out*) and is increased when the chunk is active. Further, $B(c_i)$ is independent of the priming.

In contrast, the *spreading activation* of a chunk $S(c_i)$ depends on the priming and exploits the well-known *spreading activation theory* [3] to formalize how the brain iterates through a network of associated ideas to retrieve information. In the spreading activation theory one breaks down the notion of ideas into *cognitive units*. Usually, the cognitive units are arranged as vertices in an undirected graph, the so-called *spreading activation network* $N(\Delta)$, and an initial *triggering* of some cognitive units caused by the priming is propagated through $N(\Delta)$. The spreading activation $S(c_i)$ can then be derived from the *triggering values* of the cognitive units of which $c_i$ makes use. The interrelation of cognitive units and of chunks is specified in more detail in the *degree of association* and the *weighting factor*.

The *degree of association* $S(c_i, c_j)$ reflects how strongly related $c_i$ and $c_j$ are. Chunks which deal with the same issue have a high degree of association while chunks which refer to different topics are only loosely or not related and, therefore, have a low degree of association. Technically, $S(c_i, c_j)$ is based on the cognitive units which $c_i$ and $c_j$ have in common. The degrees of association are weighted by the *weighting factors* $W(c_j)$. While the degree of association is independent of the priming, the weighting factors reflect the context-dependency of $A(c_i)$. Only if $c_i$ is associated to a chunk $c_j$ ($S(c_i, c_j) > 0$) which has positive weight ($W(c_j) > 0$), then the chunk $c_i$ has a positive spreading activation ($S(c_i) > 0$), too.

### 3 Activation-Based Conditional Inference

As common ACT-R implementations are production systems which process chunks that are represented as simple lists of attributes, the logical basis of ACT-R does not hold the pace with modern KRR formalisms in nonmonotonic reasoning. Thus, we propose a cognitively inspired model of inductive conditional
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| Conditional | $Z^\Delta(r)$ | $B^\Delta(r)$ | $W^\Delta_{q_1}(r) \cdot S^\Delta_{q_1}(r) \cdot A^\Delta_{q_1}(r)$ | $W^\Delta_{q_2}(r) \cdot S^\Delta_{q_2}(r) \cdot A^\Delta_{q_2}(r)$ |
|-------------|--------------|--------------|-------------------------------------------------|-------------------------------------------------|
| $r_1$       | 0            | 1            | $\frac{1}{3}$ | 1.36 | 2.36 | $\frac{1}{4}$ | 1.27 | 2.27 |
| $r_2$       | 0            | 1            | $\frac{1}{3}$ | 1.36 | 2.36 | $\frac{1}{4}$ | 1.27 | 2.27 |
| $r_3$       | 0            | 1            | $\frac{2}{3}$ | 1.41 | 2.41 | $\frac{1}{4}$ | 0.66 | 1.66 |
| $r_4$       | 0            | 1            | $\frac{1}{3}$ | 1.13 | 2.13 | $\frac{1}{4}$ | 0.70 | 1.70 |
| $r_5$       | 0            | 1            | $\frac{2}{15}$ | 0.82 | 1.82 | $\frac{1}{21}$ | 0.41 | 1.41 |
| $r_6$       | 0            | 1            | $\frac{2}{3}$ | 1.36 | 2.36 | $\frac{1}{4}$ | 0.60 | 1.60 |
| $r_7$       | 1            | $\frac{1}{2}$ | $\frac{2}{3}$ | 1.23 | 1.73 | $\frac{1}{4}$ | 1.10 | 1.60 |
| $r_8$       | 0            | 1            | $\frac{4}{15}$ | 1.03 | 2.03 | $\frac{1}{4}$ | 1.43 | 2.43 |
| $r_9$       | 1            | $\frac{1}{2}$ | $\frac{4}{15}$ | 0.93 | 1.43 | 1 | 2.35 | 2.85 |
| $r_{10}$    | 2            | $\frac{1}{3}$ | $\frac{7}{15}$ | 0.81 | 1.14 | 1 | 2.61 | 2.94 |
| $r_{11}$    | 1            | $\frac{1}{2}$ | $\frac{2}{15}$ | 0.40 | 0.90 | 1 | 2.08 | 2.58 |
| $r_{12}$    | 0            | 1            | $\frac{1}{3}$ | 0.78 | 1.78 | $\frac{1}{21}$ | 0.29 | 1.29 |
| $r_{13}$    | 0            | 1            | $\frac{7}{15}$ | 0.29 | 1.29 | $\frac{1}{21}$ | 0.12 | 1.12 |
| $r_{14}$    | 0            | 1            | $\frac{7}{15}$ | 0.27 | 1.27 | $\frac{4}{151}$ | 0.08 | 1.08 |
| $r_{15}$    | 0            | 1            | $\frac{7}{15}$ | 0.29 | 1.29 | $\frac{4}{151}$ | 0.12 | 1.12 |
| $r_{16}$    | 0            | 1            | $\frac{7}{15}$ | 0.19 | 1.19 | $\frac{1}{21}$ | 0.11 | 1.11 |
| $r_{17}$    | 0            | 1            | $\frac{7}{15}$ | 0.17 | 1.17 | $\frac{4}{151}$ | 0.07 | 1.07 |
| $r_{18}$    | 0            | 1            | $\frac{7}{15}$ | 0.18 | 1.18 | $\frac{1}{21}$ | 0.15 | 1.15 |
| $r_{19}$    | 1            | $\frac{1}{2}$ | $\frac{2}{3}$ | 0.89 | 1.39 | $\frac{1}{4}$ | 1.09 | 1.59 |
| $r_{20}$    | 0            | 1            | 0             | 0   | 1   | 0             | 0   | 1   |

Table 2. Z-ranks $Z^\Delta$, base-level activation $B^\Delta$, spreading activation $S^\Delta_{q_1}$ and activation function $A^\Delta_{q_1}$ wrt. $q_1 = (\phi \Rightarrow \psi)$ and $q_2 = (\overline{\theta} \land \overline{\psi})$ for the conditionals in $\Delta$. Selected conditionals are boxed (threshold $\theta = 2.3$).

reasoning by interpreting the concepts of ACT-R in terms of logic, conditionals, and inference. More precisely, we replace chunks by conditionals of a belief base $\Delta$ and derive a focus $\phi$ based on the activation function in \(1\) in order to draw focused inferences wrt. an inference operator $I_{\phi(\Delta)}$. Here, we rely on $I_{\phi(\Delta)}$ because of the semi-monotony of System P. In our formalism, atoms play the role of cognitive units, and the production rules are replaced by the inference operator. From the conditional logical perspective, the added value of this activation-based conditional inference approach are

- the cognitive justification of the focus,
- the possibility of a more fine-grained adjustment of the focus than in \(16\),
- and the option to integrate further cognitive concepts such as forgetting and remembering.

Formally, we calculate an activation value $A(r) > 0$ for every conditional $r$ in $\Delta$. If $A(r)$ is above a certain threshold $\theta$, $A(r) \geq \theta$, the conditional is selected for the focus on $\phi(\Delta)$. For this, we define a selection function $s^\theta_A : \Delta \rightarrow \{0, 1\}$ with $s^\theta_A(r) = 1$ iff $A(r) \geq \theta$ and $s^\theta_A(r) = 0$ otherwise. We denote the set of selected
Conditionals by
\[ \Delta^\theta_A = \{ r \in \Delta \mid s^\theta_A(r) = 1 \} \].

Note that \( \Delta^\theta_A \) will implicitly depend on a query \( q = \langle B|A \rangle \) since queries will serve as the initial priming and the spreading activation, which is part of \( A \), depends on the priming.

**Definition 3.** Let \( \Delta \) be a belief base, \( \langle B|A \rangle \) a conditional, \( \mathcal{I} \) an inference operator, \( A \) an activation function for \( \Delta \), and \( \theta \geq 0 \). Then, \( \langle B|A \rangle \) is activation-based inferred from \( \Delta \) wrt. \( \mathcal{I}, A, \) and \( \theta \) iff \( \langle B|A \rangle \in \mathcal{I} \Delta^\theta_A \).

If answering a query fails, i.e. \( \delta^{\theta}(\Delta) = unknown \), then the inference process can be repeated by iteratively choosing a lower threshold \( \theta_i < \theta \) which leads to a larger (or equal) set of selected conditionals. In the limit, when choosing \( \theta = 0 \), one has \( \Delta^\theta_A = \Delta \), thus \( \mathcal{I} \Delta^\theta_A = \mathcal{I} \Delta \). This iteration process is in analogy to the sequence \( \delta^\theta_i(\Delta) \) defined in [16] and can be used to approximate \( \mathcal{I} \Delta \) for any inductive inference operator \( \mathcal{I} \). In particular, the chance of successfully answering the query increases with each iteration step when \( \mathcal{I} \) is semi-monotonous.

### 4 Blueprint for Activation Based Conditional Inference

ACT-R does not formalize the activation function in [11] in more detail but describes its functionality informally. Hence, there is certain freedom in its configuration. We give a concrete instantiation of [11] in the conditional inference setting which can be seen as a blueprint for further investigations and empirical analyses. Note that we shift the dependence of the base-level activation on the usage history of conditionals to the next section.

Let \( \Delta \) be a belief base, \( t_i \in \Delta \), and \( q \) a conditional (the query resp. priming). Then, [11] becomes

\[ A^\Delta_q(t_i) = B^\Delta(t_i) + \sum_{t_j \in \Delta} W^\Delta_q(t_j) \cdot S(t_i, t_j). \]

We explain the single components of \( A^\Delta_q(t_i) \) in detail.

#### 4.1 Base-Level Activation

\( B^\Delta(t) \) reflects the entrenchment of \( t \) in the reasoner’s memory. Since epistemic entrenchment and ranking semantics are dual ratings, the normality of a conditional is a good estimator and we define

\[ B^\Delta(t) = \frac{1}{1 + Z^\Delta(t)}, \quad t \in \Delta, \]
where $Z^\Delta(r)$ is the Z-rank of $r$. Following this definition, $B^\Delta(r)$ is positive and normalized by 1. While the most normal conditionals have a base-level activation of $B^\Delta(r) = 1$, this value decreases with increasing specificity of $r$.

**Example 5.** Table 2 shows the base-level activations of the conditionals in $\Delta'$ (Table 1). For example, $B^\Delta'(r_9) = 1/2$ and $B^\Delta'(r_{10}) = 1/3$. Since $r_9$ is less specific than $r_{10}$ (cf. Example 2), its base-level activation is higher than $B^\Delta'(r_{10})$.

### 4.2 Degree of Association

$S(t_i, t_j)$ is a measure of connectedness between the conditionals in $\Delta$ and is defined by

$$S(t_i, t_j) = \frac{|\Sigma(t_i) \cap \Sigma(t_j)|}{|\Sigma(t_i) \cup \Sigma(t_j)|}, \quad t_i, t_j \in \Delta.$$  

Hence, it is the number of shared atoms relative to all atoms in $t_i$ or $t_j$ and, therefore, non-negative and normalized by 1. The degree of association of a conditional $t$ to itself is $S(t, t) = 1$ while the degree of association of conditionals which do not share any atoms is 0. The syntactically-driven definition of $S(t_i, t_j)$ is motivated by and extends the principle of relevance from nonmonotonic reasoning. This principle of relevance states that if the belief base $\Delta$ splits into two sub-belief bases $\Delta_1$ and $\Delta_2$ with $\Sigma(\Delta_1) \cap \Sigma(\Delta_2) = \emptyset$ and the query is defined over one of the signatures $\Sigma(\Delta_i)$, say $\Sigma(\Delta_1)$, only, then only the conditionals in $\Delta_1$ should be relevant for answering this query [8]. Not only the quantities

| Atom | $\tau^\Delta_{q_1}(a)$ | $\tau^\Delta_{q_2}(a)$ |
|------|------------------------|------------------------|
| $a$  | 1 (0)                  | 1/4 (1)                |
| $b$  | 2/3 (1)                | 1/4 (1)                |
| $c$  | 2/15 (2)               | 1 (0)                  |
| $d$  | 2/15 (2)               | 1/21 (2)               |
| $f$  | 2/3 (1)                | 1 (0)                  |
| $h$  | 1/5 (2)                | 1/4 (1)                |
| $i$  | 1/3 (1)                | 1/21 (2)               |
| $k$  | 0 (∞)                  | 0 (∞)                  |
| $l$  | 1/15 (2)               | 1/15 (3)               |
| $m$  | 0 (∞)                  | 0 (∞)                  |
| $p$  | 1 (0)                  | 1/4 (1)                |
| $r$  | 1/15 (2)               | 1/21 (2)               |
| $s$  | 2/15 (2)               | 1 (0)                  |
| $w$  | 1/3 (1)                | 1/4 (1)                |

Fig. 1. Unlabeled spreading activation network $N(\Delta')$ and labeling of $N(\Delta')$ wrt. the queries $q_1 = (p \Rightarrow a \top)$ and $q_2 = (f \mid c \& s)$. The numbers in the parentheses next to the labels (i.e., triggering values) are the iteration steps in which the atoms are labeled. 0 stands for the priming and $\infty$ for unreachable atoms.
$S(r_i, r_j)$ for $r_j \in \Delta$ themselves are essential for the spreading activation of a conditional $r_i$ but also how many conditionals $r_i$ is associated with. The more a conditional is cross-linked within $\Delta$, the more likely it is that this conditional has a high spreading activation and is selected by the $s$.

**Example 6.** The degrees of association between the conditionals in $\Delta'$ (Table 1) are shown in Table 3. For example,

$$S(r_9, r_{10}) = \frac{|\{c, f\} \cap \{c, f, s\}|}{|\{c, f\} \cup \{c, f, s\}|} = \frac{2}{3}.$$ 

### 4.3 Weighting Factor

$W_{\Delta q}(r)$ indicates how much the initial priming $q$ triggers the conditional $r$. We formalize the influence of the priming according to the spreading activation theory by a labeling of the spreading activation network $N(\Delta)$ between cognitive units. In our context, the cognitive units are the atoms $a \in \Sigma$ and the outcome of $N(\Delta)$ is a triggering value $\tau_{\Delta q}(a) \in [0, 1]$ which indicates how much $a$ is triggered by $q$. We follow the idea that a conditional $r$ is triggered not more than the atoms in $\Sigma(r)$ and define the weighting factor by

$$W_{\Delta q}(r) = \min\{\tau_{\Delta q}(a) \mid a \in \Sigma(r)\}.$$ 

**Example 7.** The weighting factors of the conditionals in $\Delta'$ (Table 1) wrt. queries $q_1$ and $q_2$ are shown in Table 2. The weighting factors depend on the labeling of the spreading activation network in Figure 1 which is explained in the next paragraph. For example, $\tau_{q_1}(c) = 4/15$ and $\tau_{q_1}(f) = 2/3$. Consequently, the weighting factor of $r_9$ wrt. $q_1$ is $W_{q_1}(r_9) = \min\{4/15, 2/3\} = 4/15$.

### 4.4 Spreading Activation Network

$N(\Delta) = (V, E)$ is an undirected graph with vertices $V = \Sigma$. Edges in $E$ represent associations between the atoms in $\Sigma$ along which the triggering of the atoms spreads. Two atoms are associated if they occur commonly in some conditionals in $\Delta$, i.e.

$$E = \{\{a, b\} \mid \exists r \in \Delta : \{a, b\} \subseteq \Sigma(r)\}.$$ 

The actual spreading of activation is modeled by iteratively labeling the vertices (atoms) in $N(\Delta)$ with their triggering value $\tau_{\Delta q}(a)$. The labeling algorithm is shown in Figure 2. It starts with labeling the atoms which are mentioned in the query $q$ with 1. In the subsequent step, the neighboring atoms are labeled and so on. The remaining atoms which are not reachable from the initially labeled atoms in $\Sigma(q)$ are labeled with 0. The labels of the atoms in between are the sum of the labels of the already labeled neighbors weighted by the sum of all labels so far plus 1. This guarantees that these labels are between 0 and 1 and decrease for increasing iteration steps. Therewith, the triggering value of an atom depends on both the triggering values of the associated (sooner triggered) atoms and their count.
Table 3. Degrees of association \( S(r_i, r_j) \) between the conditionals \( r_i, r_j \in \Delta' \). Since \( S(r_i, r_j) \) is symmetric in its arguments, only the entries in the upper right triangle of the table are shown. Also 0-entries are left out for a better readability.
Labeling Algorithm

Input: Spreading activation network $N(\Delta) = (V, E)$ (unlabeled); query $q = (B|A)$

Output: Labeling of $N(\Delta)$, i.e. triggering values $\tau^\Delta_q(a) = \text{label}(a)$ for $a \in \Sigma (= V)$

1. for $a \in V$ with $a \in \Sigma(q)$ do
   2. $\text{label}(a) = 1$
3. initialize
4. $\mathcal{L} = \{ a \in V \mid a \text{ is labeled} \}$,
5. $V' = \{ a \in V \mid \exists \{a, b\} \in E: a \in V \setminus \mathcal{L} \land b \in \mathcal{L} \}$
6. while $V' \neq \emptyset$ do
7.   for $a \in V'$ do
8.     $\text{label}(a) = \frac{\sum_{b \in \mathcal{L}, \{a, b\} \in E} \text{label}(b)}{1 + \sum_{b \in \mathcal{L}} \text{label}(b)}$
9.     update $\mathcal{L}$, $V'$
10. for $a \in V \setminus \mathcal{L}$ do
11.    $\text{label}(a) = 0$
12. return $\text{label}(a)$ for $a \in V$

Fig. 2. Labeling of a spreading activation network $N(\Delta)$ wrt. a query $q$.

Example 8. Figure 1 shows on the left-hand side the (unlabeled) spreading activation network of $\Delta'$ (Table 1). The labelings wrt. queries $q_1$ and $q_2$ are shown on the right-hand side. For example, $\Sigma(q_1) = \{a, p\}$ and consequently $\text{label}(a) = \text{label}(p) = 1$. Next, $b$, $f$, $i$, and $w$ are labeled as they are direct neighbors of at least one of the atoms $a$, $p$. For instance, $\{a, w\} \in E$ and, therefore,

$$\text{label}(w) = \frac{\text{label}(a)}{1 + \text{label}(a) + \text{label}(p)} = \frac{1}{3}.$$  

Atom $b$ is neighbor of $a$ and $p$ and is labeled with

$$\text{label}(b) = \frac{\text{label}(a) + \text{label}(p)}{1 + \text{label}(a) + \text{label}(p)} = \frac{2}{3}.$$  

Altogether, we are now able to compute $A^\Delta_q(r)$ (without usage history).

Example 9. Table 2 shows $A^\Delta_q$ (cf. also Table 1) wrt. the queries $q_1 = (p \Rightarrow a|\top)$ and $q_2 = (\overline{f}|c)$. If a threshold $\theta = 2.3$ is used, the conditionals which are selected for activation-based conditional inference are

$$\Delta'_1 = (\Delta')^\theta_{A_1} = \{r_1, r_2, r_3, r_6\},$$

where $A_1 = A^\Delta_{q_1}$, and

$$\Delta'_2 = (\Delta')^\theta_{A_2} = \{r_8, r_9, r_{10}, r_{11}\},$$

where $A_2 = A^\Delta_{q_2}$. One has $[q_1]_{\Delta'_1}^{\text{yes}} = \text{yes}$ and $[q_2]_{\Delta'_1}^{\text{yes}} = \text{no}$. That is, both queries can already be decided based on the reduced belief bases $\Delta'_1$ and $\Delta'_2$.
Table 4. Activation function $A_{\Delta q_1 q_2}^\Delta (\tau_i)$, where the base-level activation was updated by $\phi_{\delta,s}(\tau_i) = \begin{cases} 1 + \delta & \text{iff } s = 1 \\ 1 - \delta & \text{otherwise} \end{cases}$ beforehand. $A_{\Delta q_2}^\Delta$ is recalled for comparison. Selected conditionals are boxed (threshold $\theta = 2.3$).

with activation-based conditional inference. Note that $\Delta_1'$ and $\Delta_2'$ are smaller than the resp. direct foci according to (standard) focused inference (cf. Example 4 for $q_2$).

In the next section, we make the base-level activation dependent on the history of usage of conditionals and thereby integrate the concepts of forgetting and remembering into activation-based conditional inference.

5 Activation-Based Conditional Inference and Forgetting and Remembering

In ACT-R the base-level activation of a chunk is not constant but decreases over time and increases when the chunk is retrieved. In order to capture this dynamic view on the base-level activation, we introduce a forgetting factor wrt. a selection $s$ by

$$\phi_{\delta,s}(\tau) = \begin{cases} 1 + \delta & \text{iff } s = 1 \\ 1 - \delta & \text{otherwise} \end{cases}$$

(2)
with which we update the base-level activation $B^\Delta(r)$ after each inference request. By doing so, the base-level activation of a conditional is decreased when the conditional is not selected for answering the query, and it is increased otherwise. For the updated base-level activation we write $B^\Delta_{\delta,s}(r) = B^\Delta(r) \cdot \phi_{\delta,s}(r)$.

When applying this update of the base-level activation for every inference request, the usage history of the conditionals is implemented into $B^\Delta$ implicitly.

**Example 10.** We compare the activation function $A^\Delta_{q_2}$ wrt. query $q_2 = (f|c\neg s)$ with the activation function $A^\Delta_{q_1,q_2}$ which is obtained by querying $q_1$ first and by updating $B^{\Delta'}$ wtr. $s^{-1}(1) = (\Delta')^\theta_{A^\Delta_{q_1}}$ and querying $q_2$ afterwards (cf. Table 4, also for parameters, and Table 1). While in the first case the conditionals selected for activation-based conditional inference are \{r_8, r_9, r_{10}, r_{11}\} (cf. Example 9), in the second case \{r_1, r_2, r_9, r_{10}, r_{11}\} are selected. In particular, $r_8$ is forgotten because it did not play a role when answering $q_1$. In both cases, the query $q_2$ is answered with no.

The following example shows how remembering is realized within our approach.

**Example 11.** When querying $q_3 = (p \Rightarrow a | \top)$ from $\Delta'$ (Table 1) with threshold $\theta = 2.3$, the conditional $r_{10}$ is not selected (cf. Table 2) and consequently its base-level activation is decreased (cf. Table 3). Afterwards, it has the lowest base-level activation of all conditionals in $\Delta'$. However, it turns out that this conditional is selected and, hence, remembered when asking for $q_2 = (f|c\neg s)$ afterwards (cf. Example 10).

Although the base-level activation of a conditional may have been decreased by the forgetting factor over time to nearly zero, the conditional can still be selected by a selection $s$ if the spreading activation is high enough to compensate the low base-level activation.

### 6 Conclusions and Future Work

We applied conditional reasoning to ACT-R and developed a prototypical model for activation-based conditional inference. For this, we reformulated the activation function from ACT-R for conditionals and selected the conditionals with the highest degree of activation for focused inference. With activation-based conditional inference it is possible to implement several aspects of human reasoning into modern expert systems such as focusing, forgetting, and remembering.

The main challenge for future work is to find for a given query $q$ and a given inference operator $I$ a proper least subset $\Delta'$ of a belief base $\Delta$ such that the query is answered the same wtr. $\Delta'$ as to $\Delta$, i.e. $[q]_I^{\Delta'} = [q]_I^\Delta$, without having to draw the computationally expensive inference $[q]_I^\Delta$. 

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