Experimental Reconstruction Images of Tissue Phantom by Diffuse Optical Tomography

Young Sik Jun and Woon Sik Baek
Department of Electronics and Radio Engineering, Kyung Hee University, South Korea.
E-mail: wsbaek@khu.ac.kr

Abstract. As one of the new modality to imaging in vivo, Diffuse Optical Tomography(DOT) is using near-infrared(NIR) light sources and detectors by which the photon propagation is dominated by scattering rather than absorption in human tissue. In this paper, we reconstructed images of tissue phantom from measured data that is transmitted through highly scattering turbid media, using DOT experimental setup. In imaging procedure, we used FEM based forward and inverse solver and it is very difficult to obtain a good inverse matrix, since the Jacobian matrix which is composed of differential components of optical parameters(scattering coefficient and diffusion coefficient) is ill-conditioned by limited number of detectors. Therefore, we have to update these parameters repeatedly to obtain a reasonable optical parameters. The optimization of this updating process can be achieved by regularization by which the weighted diagonal matrix are added to the Jacobian matrix. And we present the preliminary experimental reconstruction images of optical parameters for the liquid phantom of human breast tissue.

1. Introduction
In Diffuse Optical Tomography light is considered as particles with energy and velocity, that is photons. These photons are scattered or absorbed in turbid media such as biological tissues and are reflected at boundaries according to the Fresnel’s law[1]. The operation between photons and turbid media can be described by the probability per unit length of a photon being scattered or absorbed. And these parameters denoted as scattering coefficient $\mu_s$ and absorption coefficient $\mu_a$ respectively.

In this paper, we used the diffusion equation that is approximated Boltzmann transport equation which is well describing the light transport in biological tissues. And the FEM forward solver was developed using finite-element method(FEM) to solve the diffusion equation numerically. And calculated images of tissue phantom was reconstructed. Preliminary experimental results with tissue phantom would be presented.

2. FEM forward solver for frequency-domain DOT
2.1. Frequency-domain diffusion equation
Photon transport in highly scattering media such as biological tissue is well modeled by the Boltzmann transport equation. However, it has many variables for consideration and it is very difficult and time consuming to obtain the solution of Boltzmann transport equation. For the diffuse optical applications
in which the light transport in medium is dominated by scattering rather than absorption, it can be simplified with diffusion approximation. Using P₁-approximation for the radiance, diffusion equation can be obtained[1],

$$\nabla \cdot (D(r)\nabla \Phi(r,t)) - \mu_a(r)\Phi(r,t) - \frac{1}{v}\frac{\partial}{\partial t}\Phi(r,t) = -S(r,t)$$

(1)

where $v$ is the speed of light in the medium, $r$ is the position vector and $t$ is time. $\Phi(r,t)$ is the photon fluence rate [Watt/cm²] which is defined as integral of the radiance over all solid angle, $D(r)$ is the diffusion coefficient and $S(r,t)$ is the isotropic light source [Watt/cm³].

In the frequency-domain DOT, fluence rate can be assumed $\Phi(r,t) = \Phi(r)e^{-i\omega t}$ and the $e^{-i\omega t}$ terms have factored out[2]. Therefore frequency-domain diffusion equation can be obtained,

$$\nabla \cdot (D(r)\nabla \Phi(r)) - \left(\mu_a(r) - \frac{i\omega}{v}\right)\Phi(r) = -S(r)$$

(2)

where $\omega$ is the light source modulation angular frequency.

2.2. Finite element method

To solve the diffusion equation we adopted the finite element method. If the medium is homogeneous, the optical properties $\mu_a$ and $D$ has uniform distribution over all region and it only needs the discretized photon fluence rate $\Phi$ and photon flux $F = -D\nabla \Phi$. If the medium is inhomogeneous, the optical properties has local heterogeneities and also needs to be discretized. Therefore we can obtain the discretized diffusion equation,

$$\sum_{j=1}^{N} \Phi_j \left[ \left( - \sum_{k=1}^{K} D_{ij} \nabla \phi_j \cdot \nabla \phi_i \right) - \left( \sum_{l=1}^{I} \mu_a \phi_l - \frac{i\omega}{c} \phi_l \phi_l \right) \right] = -\langle S \phi \rangle + \sum_{j=1}^{M} F_j \hat{\phi}_j \phi_j ds$$

(3)

where $\phi$, $\psi$, and $\varphi$ are the Lagrangian basis functions, $\langle \rangle$ indicates integration over all element and $\hat{\int}$ represents integration over the boundary surface. This complicated equation by the FE approximation can be written in the matrix form,

$$\begin{bmatrix} A_{bb} & A_{bi} \\ A_{bi} & A_{ii} \end{bmatrix} \begin{bmatrix} \Phi_b \\ \Phi_i \end{bmatrix} = \begin{bmatrix} B_{bb} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_b \\ 0 \end{bmatrix} + \begin{bmatrix} C_b \\ C_i \end{bmatrix}$$

(4)

where the subscripts $i$ and $b$ denote interior and boundary nodes of the medium respectively.

3. Inverse solver

Generally, human tissues are inhomogeneous and if the rate of change of the $D$ and $\mu_a$ can be found, we can obtain the image in vivo using these parameters[3]. The numerical solutions(i.e., photon fluence rate $\Phi$ and photon flux $F$) of the diffusion equation which is approximated by FE can be Taylor expanded in terms of absorption and diffusion coefficient as follows,

$$\Phi(D,\mu_a) = \Phi(D,\mu_a) + \frac{\partial \Phi}{\partial D} \Delta D + \frac{\partial \Phi}{\partial \mu_a} \Delta \mu_a + \cdots$$

(5)
\[ F(\mathbf{T}, \mathbf{\mu}_a) = F(D, \mu_a) + \frac{\partial F}{\partial D} \Delta D + \frac{\partial F}{\partial \mu_a} \Delta \mu_a + \cdots . \]  

(6)

If we define the vector \( \Delta \chi \) which including \( \Delta D \) and \( \Delta \mu_a \), then we can derive eq. (8) with the Jacobian matrix of eq. (7),

\[
J = \begin{bmatrix}
\frac{\partial \Phi_1}{\partial D_1} & \frac{\partial \Phi_1}{\partial D_2} & \ldots & \frac{\partial \Phi_1}{\partial D_k} & \frac{\partial \Phi_1}{\partial \mu_1} & \frac{\partial \Phi_1}{\partial \mu_2} & \ldots & \frac{\partial \Phi_1}{\partial \mu_L} \\
\frac{\partial \Phi_2}{\partial D_1} & \frac{\partial \Phi_2}{\partial D_2} & \ldots & \frac{\partial \Phi_2}{\partial D_k} & \frac{\partial \Phi_2}{\partial \mu_1} & \frac{\partial \Phi_2}{\partial \mu_2} & \ldots & \frac{\partial \Phi_2}{\partial \mu_L} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial \Phi_M}{\partial D_1} & \frac{\partial \Phi_M}{\partial D_2} & \ldots & \frac{\partial \Phi_M}{\partial D_k} & \frac{\partial \Phi_M}{\partial \mu_1} & \frac{\partial \Phi_M}{\partial \mu_2} & \ldots & \frac{\partial \Phi_M}{\partial \mu_L}
\end{bmatrix},
\]

(7)

\[ J \Delta \chi = \Phi^o - \Phi^e, \]

(8)

where \( \Phi^o \) and \( \Phi^e \) denote observed and calculated photon fluence rates respectively. Since the detectors are located only at boundaries, the number of measurements are limited. Consequently, the Jacobian matrix is ill-conditioned. Therefore, we have to update these parameters repeatedly to acquire reasonable \( \Delta \chi \). The optimization of this updating process can be achieved by regularization in which the weighted diagonal matrix are added to the Jacobian as below,

\[ (J^T J + \lambda I) \Delta \chi = J^T (\Phi^o - \Phi^e). \]

(9)

And we utilized the Levenberg-Marquardt algorithm that minimize least-square errors between observed and calculated values by updating \( \lambda \).

### Figure 1.

Diagram of the single channel frequency-domain DOT setup.

### 4. Experimental setup

Fig. 1 shows the diagram of the single channel frequency-domain DOT setup we developed. We used 830nm, 30mW diode laser(Sanyo, DL-5032-001) as a light source which was directly modulated with 70MHz sinusoidal radio-frequency by Bias-Tee(Mini-Circuits, ZFBT-4R2G) and the modulation depth was greater than 90 percent. The modulated light was delivered to an optical fiber by LD-to-Fiber coupler(OzOptics, LDPC-01-830) and the coupling efficiency was greater than 75 percent. We adopt maximally-flat RF band-pass filter to ensure long-term stability.

We could obtain amplitude and phase of propagated light by detecting the photons that scattered out around the boundaries of medium with APD(Hamamatsu, C5331-03) and Lock-in amplifier(SRS,
SR830 DSP Lock-in Amplifier). Consequently, we can obtain unknown optical properties by substitute these data into inverse solver.

5. Results

Fig. 2 shows the photon fluence rate distribution in the medium and it has the homogeneous background properties of $\mu_a = 0.5 \text{[cm}^{-1}]$, $\mu_s = 140 \text{[cm}^{-1}]$, $\gamma = 0.9$ and the index of refraction $n = 1.33$.

First, we assumed that the anomaly has $\mu_a = 14 \text{[cm}^{-1}]$ (smaller than background) and the other parameters are equal to the background. Second, we assumed that the anomaly has $\mu_a = 1400 \text{[cm}^{-1}]$ (larger than background). Fig. 2 (a) and (b) shows the fluence rate distribution by above assumptions respectively. It was observed that the smaller scattering coefficient in the anomaly made the light propagate further.

![Figure 2. Fluence rate distribution](image)

6. Conclusions

We developed the FEM forward solver and showed the distribution of the photon fluence rate in the medium according to the anomaly’s scattering coefficient changes. And we also developed the inverse solver for image reconstruction using Jacobian matrix. By updating the calculated data from the FEM forward solver, we showed that the image reconstruction could be achieved with the experimental setup of frequency-domain DOT we developed. And we could obtain an electrical drift characteristic under 0.3 [degree/hour] by adopting the maximally-flat band-pass filter and that was reasonable for stable phase detection.

Acknowledgements

This work was partly supported by a grant from the Kyung Hee University in 2004 (KHU-20040079) and was partly supported by the Korea Science and Engineering Foundation(KOSEF) grant funded by the Korea government(MEST) (R11-2002-103-07002-0).

References

[1] A.J. Welch, M. J. C. van Gemert, *Optical-Thermal Response of Laser-Irradiated Tissue*, Plenum Press, 1995.
[2] R. Choe, *Diffuse Optical tomography and spectroscopy of breast cancer and fetal brain*, Ph.D thesis, University of Pennsylvania, 2005.
[3] K. D. Paulsen, H. Jiang, *Spatially varying optical property reconstruction using a finite element diffusion equation approximation*, Med. Phys., vol. 22, pp. 691-701, 1995.