Combining Modified Weibull Distribution Models for Power System Reliability Forecast

Ming Dong, Senior Member, IEEE and Alexandre B. Nassif, Senior Member, IEEE

Abstract—In recent years, under deregulated environment, electric utility companies have been encouraged to ensure maximum system reliability through the employment of cost-effective long-term asset management strategies. To help achieve this goal, this research proposes a novel statistical approach to forecast power system asset population reliability. It uniquely combines a few modified Weibull distribution models to build a robust joint forecast model. At first, the classic age based Weibull distribution model is reviewed. In comparison, this paper proposes a few modified Weibull distribution models to incorporate special considerations for power system applications. Furthermore, this paper proposes a novel method to effectively measure the forecast accuracy and evaluate different Weibull distribution models. As a result, for a specific asset population, the suitable model(s) can be selected. More importantly, if more than one suitable model exists, these models can be mathematically combined as a joint forecast model to forecast future asset reliability. Finally, the proposed methods were applied to a Canadian utility company for the reliability forecast of electromechanical relays and the results are discussed in detail to demonstrate the practicality and usefulness of this research.

Index Terms—Weibull Distribution, Power System Reliability, Asset Management

I. INTRODUCTION

Nowadays, under deregulated environment, electric utility companies are encouraged to reduce overall cost while maintaining system reliability risk at an acceptable level. To achieve this goal, understanding and forecasting the reliability trends of different asset populations is the key. Sophisticated and optimal asset management measures can only be established based on the accurate forecasting of asset reliability change in the future.

Previously, the standard age based Weibull distribution has been widely used in reliability engineering as a statistical tool to model equipment aging failures [1-4]. However, this classic model cannot effectively incorporate additional information such as asset health condition data, asset warranty, energization delay, asset infant mortality period and minimum spare requirements which many electric utility companies often have to consider. To resolve this problem, this paper proposes a novel statistical approach to forecast power system asset population reliability. The main contributions of this paper include:

- it proposes four modified Weibull distribution models in response to additional information and considerations a power system may have;
- it proposes a unique method to measure the forecast accuracy of different Weibull distribution models. Based on this method, suitable Weibull distribution model(s) can be identified for a specific asset population;
- it proposes a unique method to combine different Weibull distribution models as a joint forecast model.

Fig.1. Flowchart of proposed approach for power system asset reliability forecast

The flowchart of the proposed methods is shown in Fig.1. In the beginning, the operation status data of a specific asset population is analyzed, converted to the cumulative failure probability table. This table is then split into training records and testing records. Training records are used to model the asset failure progression using different Weibull distributions.
models. Testing records are used to evaluate the forecast accuracy of the distribution models. The suitable Weibull distribution model(s) can be identified based on the test results. When there is more than one suitable Weibull distribution models, the models can be mathematically combined as a joint model. In the next stage, the combined joint model is applied to the broader asset population data and the future reliability change of this population can be forecasted.

This paper firstly describes the required asset operation status dataset as the foundation before applying the proposed methods. Then it reviewed the standard Weibull distribution for aging analysis. Based on this, a few modified Weibull distributions including health index based Weibull distribution, X-shifting Weibull distribution, Y-shifting Weibull distribution and XY-shifting Weibull distribution are explained with reference to practical scenarios in power systems. A few methods to determine Weibull distribution parameters such as Maximum Likelihood Estimation (MLE), Least Squares Estimation (LSE) are reviewed. After this, the proposed methods of evaluating and combining Weibull distribution models are presented. Then the process of forecasting asset reliability using the produced joint forecast model is explained. Finally, this paper provides a real example of applying proposed methods to forecast future electromechanical relay failures and spare requirements for a Canadian utility company.

II. ASSET OPERATION STATUS DATASET

Nowadays, in light of the significant value embedded in data, many electric utility companies have employed sophisticated Computerized Maintenance Management Systems (CMMS) to track and store various asset data [5-6]. For the purpose of asset reliability forecast, the asset operation status data is used. It is organized by asset type and asset population. Asset operation status data must include age and operating statuses (i.e. working or failed). The data is continuously recorded as new asset gets installed and old asset retires due to failures. Just like any data mining tasks, low quality input data will undoubtedly lead to inaccurate data observations and analysis [7]. Understanding the data requirements of asset operation status dataset and preparing qualified asset operation status dataset is a key step towards the successful implementation of the methods proposed in this paper. It should be noted that:

- One type of asset often contains many sub types due to different technology adoptions and manufacturing standards. The sub types should be manually identified by utility asset engineers based on equipment domain knowledge and separated into different datasets. For example, underground cables normally have paper insulated cables, cross-linked Polyethylene (XLPE), Ethylene Propylene Rubber (EPR) [8] and these cables are often in service at the same time in the same utility company’s power system because they were installed at different times in history adopting different technologies. These cables demonstrate different failure characteristics and should be viewed as different asset types and analyzed separately; another example is mineral oil immersed power transformers. Power transformers manufactured before 1970 are rated at 55°C temperature rise. Power transformers manufactured in 1970s are rated 55/65°C and became 65°C after 1970s till today [9]. Since the failure characteristics of transformers are closely related to winding and top-oil temperature rises, power transformers should be separated into different datasets by temperature rating or approximately by manufacturing time.

- The same type of asset can be installed in different ways. For example, underground cables can be direct buried in soil or installed in PVC conduit. In general, the direct buried cables are more prone to fail due to soil corrosion and moisture ingress [10]; distribution and transmission power poles are constructed as tangent structures (carrying straight conductors), angle structures (having deflection angles), dead-end structures (at the end of the line) or transformer poles (carrying mounted transformers). These structures incur different mechanical forces which lead to different failure characteristics. Similarly to dealing with the sub types, asset with different installations should be separated into different populations and form different datasets.

- Utility companies may also perform inspection or testing for certain asset types and populations. In cases like this, the latest inspection/testing data can be linked with the age and status data. This is because the modified Weibull distribution models this paper proposes can incorporate asset condition data and derive health index based models.

- Asset operation status dataset used by the proposed methods does not have to be the entire population for a specific type of asset. Oftentimes, a randomly sampled population fraction with hundreds of records will be good enough to establish a confident Weibull distribution model. Data completeness is another consideration. Sometimes only limited members in a population have inspection or testing data and these members should be utilized for establishing the model. The established model can still be applied to the entire population for a later stage.

As an example, a 138 kV in-duct XLPE transmission underground cable operation status dataset is shown in Table I which follows the above discussed data requirements.

| TABLE I | ASSET OPERATION STATUS DATASET FOR 138KVF IN-DUCT XLPE TRANSMISSION UNDERGROUND CABLE |
| _______ | ___________ | ___________ | ___________ | ___________ | ___________ |
| Cable ID | Age | Operating Status | Partial Discharge Result | Neutral Condition | Splice Condition |
| _______ | _______ | ___________ | ___________ | ___________ | ___________ |
| 1 | 10 | Working | Good | Good | Good |
| 2 | 11 | Working | Good | Poor | Good |
| 3 | 17 | Working | Medium | Good | Medium |
| 4 | 37 | Failed | Medium | Good | Good |
| 5 | 45 | Working | Good | Poor | Medium |
| 6 | 43 | Working | Good | Good | Medium |
| 7 | 52 | Failed | Poor | Poor | Poor |
| 8 | 25 | Failed | Good | Good | Poor |
| 9 | 35 | Working | Good | Poor | Good |
| 10 | 40 | Working | Good | Good | Good |
| ... | ... | ... | ... | ... | ... |

III. STANDARD AGE BASED WEIBULL DISTRIBUTION

The Weibull distribution is a continuous probability distribution widely used in the field of reliability
engineering[1-4]. The standard probability density function of equipment age is

$$f(A) = \left( \frac{\beta}{\alpha} \right) \left( \frac{A}{\alpha} \right)^{\beta-1} e^{-\left( \frac{A}{\alpha} \right)^{\beta}}, A \geq 0$$  \hspace{1cm} (1)

where $\alpha > 0$ is the scale parameter; $\beta > 0$ is the shape parameter; $A$ in this application is the equipment age; $f(A)$ is the probability of failure. The corresponding cumulative Weibull distribution function is given as:

$$F(A) = 1 - e^{-\left( \frac{A}{\alpha} \right)^{\beta}}, A \geq 0$$  \hspace{1cm} (2)

where $F(A)$ is the cumulative probability of failure.

Fig. 2. Standard age based Weibull Distribution

Fig.2. shows an example of cumulative Weibull distribution function for a certain type of equipment. As age progresses, the cumulative probability of failure increases until it reaches 100% at a later age. This is a statistical description of an asset aging failure process and can be used for equipment survival analysis [11].

This model can be derived from the asset operation status dataset discussed in Section II. For a specific age $A$, the observed cumulative failure probability is

$$\hat{F}(A) = \frac{N_{fa}}{N_{fa} + N_{wa}} \times 100\%$$  \hspace{1cm} (3)

where $N_{fa}$ is the total number of failed asset in the dataset with an age less than or equal to $A$; $N_{wa}$ is the total number of working asset in the dataset with an age greater than age $A$. Based on this calculation, a new table can be produced to depict the relationship of Age $A$ and the observed corresponding cumulative failure probability $\hat{F}(A)$.

Table II shows an example of cumulative failure probability vs. age. $\hat{F}(A)$ increases to 100% at the age of 35. It should be noted sometimes due to data availability, observed $\hat{F}(A)$ may not immediately change between consecutive years such as when $A=3$ and 4 in this case. This does not impact the modelling as the observed $\hat{F}(A)$ is based on frequency and does not always comply with $F(A)$. Table II will be split into training and testing records. The training records will be fed into the Weibull distribution modeling module as shown in Fig.1, following the methods explained in Section V to estimate the parameters $\alpha$ and $\beta$.

| A | $\hat{F}(A)$ |
|---|---|
| 0 | 0.0% |
| 1 | 0.0% |
| 2 | 0.8% |
| 3 | 1.5% |
| 4 | 1.5% |
| 5 | 2.2% |
| 6 | 3.5% |
| 7 | 5.7% |
| ... | ... |
| 32 | 98.2% |
| 33 | 98.2% |
| 34 | 99.1% |
| 35 | 100% |

IV. MODIFIED WEIBULL DISTRIBUTIONS

This paper proposes a few modified Weibull distribution models. The physical meanings of these distributions and their applications are discussed in this section.

A. Two-Parameter Health index based Weibull Distribution

Equation (2) only considers age as the sole factor in asset’s failure progression modeling. This could be accurate for assets that are used in a near homogenous environment where operating conditions such as loading level and temperatures are about the same. However, for a large power system where operating conditions vary significantly, it is not uncommon to find a young asset with higher failure probability and an old asset with a lower failure probability. In cases like this, the classic age based Weibull distribution modeling will not be able to provide accurate forecast. However, as mentioned in Section II, if additional asset condition information is available, the independent variable $A$ can be converted to health index $H$ as below.

$$F(H) = 1 - e^{-\left( \frac{H}{\alpha} \right)^{\beta}}, H \geq 0$$ \hspace{1cm} (4)

where $A$ is normalized asset age; $W_a$ is the weighting factor for asset age; $W_i$ is the weighting factor for $i$th condition attribute; $C_i$ is $i$th normalized condition attribute. The above equation uses different weighting factors to combine age and other condition data to generate an index between 0 and 100 [12]. Similar to using age alone, health index $H$ progresses from 0(brand new, all healthy) to 100 (most unhealthy observation in the system). The higher $H$ is, the higher the corresponding failure probability $F(H)$ is. As can be seen, the standard age based Weibull distribution (2) is just a special case of (4) when other condition weighting factors $W_i = 0$ and $W_a = 1$. Fig.3. shows an example of health index based Weibull distribution.

This model can be derived from the asset operation status dataset discussed in Section II. For a specific health index $H$, the observed cumulative failure probability

$$\hat{F}(H) = \frac{N_{fH}}{N_{fH} + N_{wh}} \times 100\%$$  \hspace{1cm} (5)

where $N_{fH}$ is the total number of failed asset in the dataset with a health index greater than or equal to $H$; $N_{wh}$ is the total
number of working asset in the dataset with a health index greater than $H$. Based on this calculation, Table III can be produced to depict the relationship between health index $H$ and cumulative failure probability $F(H)$. Table III shows an example of cumulative failure probability vs. health index.

**Table III**

| $H$ | $F(H)$ |
|-----|--------|
| 0   | 0.0%   |
| 1   | 0.3%   |
| 2   | 2.6%   |
| 3   | 2.9%   |
| 4   | 6.5%   |
| 5   | 9.3%   |
| 6   | 9.5%   |
| ... | ...    |
| 97  | 98.7%  |
| 98  | 99.0%  |
| 99  | 99.3%  |
| 100 | 100%   |

**B. X-Shift Weibull Distribution**

X-Shift Weibull distribution has one additional parameter $\gamma$ ($\gamma \geq 0$). This parameter $\gamma$ shifts the Weibull distribution curve along $X$ axis. Fig.4. shows an example of X-Shift Weibull distribution. Mathematically, it is given as below:

$$F(H) = 1 - e^{-(H^{-\gamma})^\beta}, H \geq 0$$

$$H = W_a A + \sum_{i=1}^{r} W_i C_i, W_a + \sum_{i=1}^{r} W_i = 1 \quad (6)$$

The physical meanings of parameter $\gamma$ in the power system context can mean the following:

- A failure-free period: some assets naturally have an extremely low failure probability during a certain initial period. This is more true for assets in protected, controlled and sometimes enclosed environment, for example a GIS switchgear almost never fails in the first few years.

- Asset warranty and insurance: some assets are warranted by manufacturers. This implies assuming free of failure during the warranted period of time. Even if there is a failure, the manufacturer could cover the cost and loss of the failure and from the utility perspective, it is like “failure-free”.

- Energization delay: in power systems, after the installation of an asset, there could be an energization delay before this asset is energized and put into actual use. For example, an underground feeder system is often constructed by stages. The cable is often installed at an early stage and will stay in place for a few months or even years before the other parts of system get constructed. In the end, the entire system will be energized at the same time. This kind of construction is very common for the development of green field projects. In this case, cable will not fail before energization time.

**Fig.3. Health index based Weibull Distribution**

**C. Y-Shift Weibull Distribution**

Y-Shift Weibull distribution has one additional parameter $\delta$. This parameter $\delta$ shifts the Weibull Distribution curve along $Y$ axis. Fig.5. shows an example of Y-Shift Weibull distribution. Mathematically, it is given as below:

$$F(H) = 1 + \delta - e^{-(H^{1/\alpha})^\beta}, H \geq 0$$

$$H = W_a A + \sum_{i=1}^{r} W_i C_i, W_a + \sum_{i=1}^{r} W_i = 1 \quad (7)$$

When $\delta < 0$, this distribution curve is shifted downward and the physical meaning becomes very similar to the X-shift Weibull distribution where a starting period of time has no
positive failure probability (negative failure probability does not have a statistical meaning and can be capped to zero).

When $\delta > 0$, this distribution curve is shifted upward and the physical meaning is to capture the initial failure probability associated with an infant mortality period [13-14]. In reliability engineering, some assets could easily fail at the initial period and the failure probability will reduce to stable stage after this initial period, this period is called the infant mortality period. Reflected by the cumulative distribution function, this initial failure can be described as a greater than zero cumulative failure probability $\delta$ when $H$ is zero. Another occasion for applying a positive $\delta$ is when making spare forecast for critical asset. A minimum spare level is required even in the early time to ensure maximum system reliability.

D. XY-Shift Weibull Distribution

XY-Shift Weibull distribution has two additional parameter $\gamma$ and $\delta$ to allow the Weibull distribution curve to shift along both $X$ axis and $Y$ axis. Fig.6. shows an example of Y-Shift Weibull distribution. Mathematically, it is given as below.

\[
\begin{align*}
F(H) &= 1 + \delta - e^{-(H-\gamma)^{\beta}} + \delta, H \geq 0 \\
H &= W_a A + \sum_{i=1}^{t} W_i c_i, \quad W_a + \sum_{i=1}^{t} W_t = 1
\end{align*}
\]

(8)

![Fig.6. XY-Shift Weibull Distribution](image)

XY-Shift Weibull distribution combines the characteristics of X-Shift Weibull distribution and Y-Shift Weibull distribution. For example, it can be used when an asset type has an energization delay as well as a failure probability in its infant mortality period.

V. ESTIMATION OF MODEL PARAMETERS

Table II and Table III contain the cumulative failure probability records that can be used to estimate Weibull distribution parameters and establish the models. However, not all the data should be used at the stage of model establishment. This is because this paper uses a unique method to evaluate and combine multiple Weibull distribution models. Similar to a typical supervised learning process [15], Table II and Table III should be split into two parts:

- Training records: the records that are used to estimate Weibull distribution parameters. A certain percentage, for example 80% of the data, can be randomly picked and grouped as training records. When picking the records, a uniform distribution can be applied to sample the entire range of $H$ and $F(H)$. This is to ensure the entire failure progression can be modeled with enough supporting data.
- Testing records: the remaining records that will be used later to measure the accuracy of a model.

There are many existing ways of estimating two-parameter Weibull distribution parameters $\alpha$ and $\beta$. A traditional method is the Maximum Likelihood Estimation (MLE) method [16]. Considering the Weibull probability density function given in (1), the likelihood function is given as:

\[
L(x_1, x_2, ..., x_n; \beta, \alpha) = \prod_{i=1}^{n} \left( \frac{\beta}{\alpha} \frac{x_i^{\beta-1}}{\Gamma(\beta)} \exp \left[ -\left( \frac{x_i}{\alpha} \right)^{\beta} \right] \right)
\]

Taking the natural logarithm of both sides and partially differentiating $\ln L$ with respect to $\alpha$ and $\beta$, we get:

\[
\begin{align*}
\frac{\partial}{\partial \beta} \ln L &= \frac{n}{\beta} + \frac{1}{\alpha^2} \sum_{t=1}^{n} x_t^{\beta-1} \ln x_t = 0 \\
\frac{\partial}{\partial \alpha} \ln L &= -\frac{n}{\alpha} + \frac{1}{\alpha^2} \sum_{t=1}^{n} x_t^{\beta} = 0
\end{align*}
\]

(10)

Solving the above equations, we get:

\[
\frac{1}{\beta} + \frac{1}{n} \sum_{t=1}^{n} x_t - \frac{\sum_{t=1}^{n} x_t^{\beta} \ln x_t}{\sum_{t=1}^{n} x_t^{\beta}} = 0
\]

(11)

$\beta$ can be solved using Newton-Raphson method or any other numerical method. When $\hat{\beta}$ is obtained, the value of $\hat{\alpha}$ can be obtained by using:

\[
\hat{\alpha} = \frac{1}{n} \sum_{t=1}^{n} x_t^{\hat{\beta}}
\]

(12)

Least squares estimation (LSE) is another common method used for Weibull distribution parameter estimation [16]. The cumulative Weibull distribution function can be linearized by taking logarithmic transformation to the form of:

\[
y_t = p + qx_t
\]

(13)

Proper $p$ and $q$ should be chosen to minimize the sum of squared errors:

\[
e = \sum_{t=1}^{n} (y_t - p + qx_t)^2
\]

(14)

The then unbiased estimators can be calculated as:

\[
\begin{align*}
\hat{q} &= \frac{n \sum x y - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \\
c &= \bar{y} - \hat{q} \bar{x}
\end{align*}
\]

(15)
In addition to MLE and LSE, there are also some modern optimization algorithms for estimating Weibull distribution parameters such as using simulated annealing algorithm [17] and genetic algorithm [18].

As discussed in Section IV., X-Shift, Y-Shift and XY-Shift Weibull distributions have additional shifting parameters. However, these parameters are often not random parameters and have physical meanings. Therefore based on human expert’s domain knowledge, these parameters can be initialized. A practical way is to initialize these parameters to discrete values in a numerical range. For example, if we know that the typical energization delay for a transmission cable population is 1-2 years, γ can be set to 1.0, 1.25, 1.5, 1.75 and 2.0. For any of these five γ values, the corresponding X-Shift Weibull distribution will become equivalent to a two-parameter Weibull Distribution model and can therefore estimate the remaining parameters α and β using the methods discussed above. Instead of having to choose optimal γ or δ at this stage, we can keep all established models with discrete γ or δ values. They can be all evaluated, selected and combined as one joint model, which will be discussed in Section VI.

VI. EVALUATING AND COMBINING WEIBULL DISTRIBUTION MODELS

As discussed in Section V., the asset operation status dataset is split to two parts: training records and testing records. The testing records are now used to evaluate the accuracy of the established models.

A. Evaluating Weibull distribution models

Suppose S is the set of established Weibull distribution models with different forms and parameters. We have:

\[ S = \{ F_1, F_2, F_3, \ldots \} \]  \hspace{1cm} (16)

For each model \( F_i \), we can calculate its Mean Squared Error (MSE) for the testing records, which is given by:

\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} \left[ \hat{F}(x_i) - F(x_i) \right]^2 \]  \hspace{1cm} (17)

Only testing records are used for evaluation because testing records were not used previously for model parameter estimation and they are unknown points to this established model. Traditionally, Weibull distribution models were not evaluated based on pre-selected testing data and therefore cannot effectively describe the model’s forecasting capability when cope with unknowns. This paper proposes this new evaluation method for Weibull distribution and it has two advantages:

- A good MSE on testing records indicate the true forecast accuracy of this model and can be generalized to describe the forecasting capability for future unknown data using this model. This is especially important for health index based models because a calculated health index using (4) can be any real value in [0,100] and the original asset operating status dataset used for Weibull distribution modelling may not contain these values.
- Furthermore, as explained in Section V., the training records are intentionally selected following a uniform distribution so that the corresponding testing records will also be able to cover the entire range of available \( H \) and \( F(H) \) data. The MSE therefore can indicate the forecast capability of the entire failure progression.

Similar ideas of using training and testing sets have been applied very successfully in many supervised learning applications [15].

Suitable models are defined as a subset \( S' \subseteq S \) where for an \( F_i^{s} \in S' \), its MSE for testing records is less than a pre-defined maximum threshold. The other way to choose \( F_i^{s} \) is to rank all MSEs for \( F_i \in S \) and only choose the top models with smallest MSEs, for example the top 3 models. Only \( S' \) is taken into the next step to establish the joint forecast model.

B. Combining Weibull Distribution Models as one joint model

On the basis of evaluating Weibull distribution models, this paper further proposes a powerful method to combine different Weibull distribution models as one joint forecasting model. This was never studied before when traditionally only single Weibull distribution model is applied for a forecasting task. One challenge utility asset engineers face is that they cannot pre-determine which Weibull distribution model will be able to yield the most accurate forecast. They also cannot pre-determine the most optimal shifting parameters although they know a probable range these parameters should fall under. A joint model \( F_i \) can keep all the suitable models \( F_i^{s} \in S' \) and leverage the strength of each model towards the final forecast output. Similar ideas can be found in some ensemble learning methods which attempt to build an accurate joint classifier using multiple weaker classifiers [15]. The joint model \( F^c \) can be created as:

\[ F^c(H) = \sum_{i=1}^{k} F_i(H) \left( \frac{1}{\text{MSE}_i} \right) \]  \hspace{1cm} (18)

In ( ), given a new health index input \( H \), a weak model’s output \( F_i(H) \) with a higher MSE will be assigned with a smaller weighting factor while a strong model’s output \( F_i(H) \) with a lower MSE will be assigned with a bigger weighting factor. All suitable models’ outputs \( F_i^s \) will be averaged by the weighting factors to produce a combined output \( F^c(H) \). This \( F^c(H) \) is the final forecasted cumulative failure probability for input \( H \). In many cases, the joint model can have a smaller MSE than any single model. If \( k = 1 \), this method is equivalent to selecting the best model.

VII. ASSET RELIABILITY FORECAST USING THE MODELS

The joint forecasting model \( F^c(H) \) can be used to forecast asset reliability in the future [19].

For a specific future time span \([T_1, T_2]\), a single asset’s forecasted failure probability during this time span is:

\[
\begin{align*}
P(H_1, H_2) &= F^c(H_2) - F^c(H_1) \\
H_1 &= h(\Delta T_1, h) \approx H \times \left( \frac{A + \Delta T_1}{A} \right) \\
H_2 &= h(\Delta T_2, h) \approx H \times \left( \frac{A + \Delta T_2}{A} \right)
\end{align*}
\]  \hspace{1cm} (19)

where \( \Delta T_1 \) and \( \Delta T_2 \) are the time intervals in years from today till \( T_1 \) and \( T_2 \); \( H \) is today’s asset health index; \( H_1 \) and \( H_2 \) are the future health indexes at \( T_1 \) and \( T_2 \); \( h(\Delta T, H) \) is a health index.
estimator and are dependent on the initial health index \( H \) and time interval \( \Delta T \). This estimator can be derived based on statistically checking historical asset health index values that are close to \( H \) and their average change after a time interval close to \( \Delta T \). It can also be approximated based on the current \( H \), \( A \) and \( \Delta T \) using the shown equations. This approximation is more accurate when \( W_0 \) is much higher than \( W_t \) or when most condition attributes are strongly correlated with age, which in many cases are true.

If only age is used in Weibull distribution modeling, the above equations can be simplified as:

\[
\begin{align*}
  P(A_1, A_2) &= F^c(A_2) - F^c(A_1) \\
  A_1 &= A + \Delta T_1 \\
  A_2 &= A + \Delta T_2
\end{align*}
\] (20)

For an asset population with \( N \) members, the forecast total number of failures during time span \( [T_1, T_2] \) is

\[
N_f = \sum_{i=1}^{N} P^i(H^i_1, H^i_2)
\] (21)

where \( P^i(H^i_1, H^i_2) \) is the forecasted failure probability using (20) for asset member \( i \), assuming there are in total \( N \) members in this asset population.

\( N_f \) can be further converted to consequence function \( C \) given the average loss factor \( L_{avg} \) per failure.

\[
C = N_f \times L_{avg}
\] (22)

If \( L_{avg} \) is unknown or the loss significantly differs between different population members, Monte Carlo simulation can be employed to forecast future system reliability consequences for a given time span [20]-[21].

VIII. CASE STUDY

The proposed method was recently applied to a utility company in West Canada for the reliability forecast of electromechanical relay population. The process and results are discussed as below.

A. Electromechanical relay operation status dataset

The electromechanical relay operation dataset contain age data, operating status data and three condition attributes: enclosure condition, mechanical condition and electrical condition. Utility substation crews annually inspect these electromechanical relays for these three conditions. Based on the inspection results, they assign health ratings for each condition attributes. For a limited population, they also perform a low-voltage simulation test and check if the relay can correctly react to the testing signals and control the trip circuits [23]. If the relay fails to make the correct actions, it will be labeled as a failed relay and will get replaced otherwise labeled as a working relay. Health index \( H \) is determined by the normalized relay age, enclosure condition, mechanical condition and electrical condition ratings. Weighting factors are pre-determined by asset engineers based on domain knowledge and experience. They are listed in Table IV.

| Item                     | Weighting factor |
|--------------------------|------------------|
| Age \( W_a \)            | 0.7              |
| Enclosure Condition \( W_e \) | 0.1              |
| Mechanical condition \( W_m \) | 0.1              |
| Electrical Condition \( W_e \) | 0.1              |

The dataset contain 560 relay records with operating statuses obtained from the historical low-voltage simulation tests. Their ages and the latest condition ratings are also included. For each relay, the health index is calculated using (4). Then the dataset can be converted to a cumulative failure probability vs. health index table in the format of Table III. This table is further split into 80 records for training and 20 records for testing.

B. Modified Weibull Distribution Models

In this utility company, the electromechanical relays typically have an approximate 2-year energization delay due to substation, transmission and distribution line construction. By checking the inspection records, it is found all relays less than 2-years old have a health index less than 5. Therefore \( \gamma < 5 \) and for modeling, it is discretized as \( \gamma \in \{2.5, 5\} \). In addition, the asset engineer is also certain that these electromechanical relays’ infant mortality rate is less than 10%, which is discretized as \( \delta \in \{5\% , 10\% \} \). Based on these priori knowledge and assumptions, 12 Weibull distribution models are established in the form of (3), (4), (6), (7) and (8). After evaluating each model using the 20 testing records, their \( MSEs \) are calculated and ranked in Table V.

| ID | Model Description | MSE    | Ranking |
|----|-------------------|--------|---------|
| 1  | Two-parameter     | 0.0015 | 3       |
| 2  | X-Shift(\( \gamma = 2.5 \)) | 0.0015 | 4       |
| 3  | X-Shift(\( \gamma = 5 \)) | 0.0016 | 6       |
| 4  | Y-shift(\( \delta = 0.05 \)) | 0.0015 | 5       |
| 5  | Y-shift(\( \delta = 0.10 \)) | 0.0040 | 9       |
| 6  | XY-shift(\( \gamma = 2.5, \delta = 0.05 \)) | 0.0013 | 2       |
| 7  | XY-shift(\( \gamma = 2.5, \delta = 0.10 \)) | 0.0035 | 8       |
| 8  | XY-Shift(\( \delta = 5, \delta = 0.05 \)) | 0.0013 | 1       |
|    | XY-Shift(\( \delta = 5, \delta = 0.10 \)) | 0.0034 | 7       |

C. Joint Weibull Distribution Model

From Table IV, top 3 models are selected as suitable models and are combined as \( F^c(H) \) using (18). Graphically, these three suitable models and \( F^c(H) \) are shown in Fig.7. The joint model \( F^c \) is also evaluated using the testing records and its \( MSE \) is 0.0012. It is better than any single Weibull distribution model listed in Table V.

![Comparison of models](image)


D. Reliability forecast for the next 5 years

This joint model is then applied to the entire 2334 electromechanical relay population. The health index composition is shown in Fig.8. From 2019 to 2023, the failure probability for each relay is calculated using (19) and the total number of failures is calculated using (21). The results show that there could be 277 relay failures from 2019 to 2023. Accordingly, the utility company could consider preparing at least 277 new relays and replace the failed relays proactively from 2019 to 2023. Some electromechanical relays may have to be replaced with digital relays. This finding is important for supply and construction resource planning for the utility company.

Health Index Percentage in Relay Population

![Fig.8. Health index percentage in the electromechanical relay population](image)

IX. CONCLUSIONS

This paper presents a novel approach for creating and combining a certain number of modified Weibull distribution models to forecast power system asset reliability. Compared to previous works, the proposed methods have the following advantages:

- It is not reliant on a single Weibull distribution model. Instead, it uses a few modified Weibull distribution models to incorporate additional information and considerations;
- It uses a unique method to measure the forecast accuracy of different Weibull distribution models which can better describe the model’s forecasting capability;
- It can effectively combine different Weibull distribution models as a joint forecast model which could have a better performance than individual model.

The proposed method was applied to a utility company in West Canada to study electromechanical relay reliability and demonstrated success. The proposed methods can also be used for other power asset populations.

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Ming Dong (S’08, M’13, SM’18) received his doctoral degree from Department of Electrical and Computer Engineering, University of Alberta, Canada in 2013. Since graduation, he has been working in various roles in two major electric utility companies in West Canada as a Professional Engineer (P.Eng.) and Senior Engineer for more than 5 years. In 2017, he received the Certificate of Data Science and Big Data Analytics from Massachusetts Institute of Technology. His research interests include applications of artificial intelligence and big data technologies in power system planning and operation, power quality data analytics, power equipment testing and system grounding.

Alexandre Nassif (S’05, M’09, SM’13) is a specialist engineer in ATCO Electric. He published more than 50 technical papers in international journals and conferences in the areas of power quality, DER, microgrids and power system protection and stability. Before joining ATCO, he simultaneously worked for Hydro One as a protection planning engineer and Ryerson University as a post-doctoral research fellow. He holds a doctoral degree from the University of Alberta and is a Professional Engineer in Alberta.