Quantum Zeno effect as a topological phase transition in full counting statistics and spin noise spectroscopy

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received 13 October 2013; accepted in final form 7 January 2014
published online 5 February 2014

PACS 72.70.+m – Noise processes and phenomena
PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion
PACS 72.25.Rb – Spin relaxation and scattering

Abstract – When the interaction of a quantum system with a detector changes from weak to strong-coupling limits, the system experiences a transition from the regime with quantum-mechanical coherent oscillations to the regime with a frozen dynamics. In addition to this quantum Zeno transition, we show that the full counting statistics of detector signal events experiences a topological phase transition at the boundary between two phases at intermediate coupling of a quantum system to the detector. We demonstrate that this transition belongs to the class of topological phase transitions that can be classified by elements of the braid group. We predict that this transition can be explored experimentally by means of the optical spin noise spectroscopy.

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Introduction. – Frequently repeated measurements applied to a quantum system may suppress quantum coherent oscillations by forcing this system to remain in an eigenstate of the measurement operator, which is the essence of the quantum Zeno effect [1]. This effect has been successfully observed experimentally, e.g. in the escape rate of trapped ultracold sodium atoms from the trapping potential [2] or in the decay of an excitation of a Bose-Einstein condensate embedded in an optical lattice [3].

The weak measurement framework allows one to treat the coupling of a system to a detector as a continuous parameter, which can be varied between the limits of the weak and strong coupling [4–8]. One can expect that in the low coupling limit the system is only weakly perturbed and generally exhibits coherent quantum effects such as oscillations of the measurable characteristics with time [8]. Respectively, in the limit of a strong coupling, coherent quantum oscillations are suppressed and transitions between states of the system become possible only due to the additional coupling of the system to its environment, which generally leads to incoherent stochastic behaviors.

It has been argued previously that such a transition between the two regimes is usually marked by a critical boundary that separates phases with and without coherent oscillations [9]. For example, suppression of coherent spin precessions due to a continuous measurement has been recently observed in a solid-state qubit in the diamond [10]. In this paper we explore such a phase transition from the point of view of the full counting statistics of detector signal events [11,12]. We assume that in a weak measurement process, the detector output is a series of discrete “clicks” separated by random intervals of time. Clicks correspond to successful measurements of the system [4]. Statistics of such events can be described by a generating function, which is akin to the one that describes statistics of observed photons in quantum optics [13]. The main finding of this letter is that, when the system’s interaction with a detector changes between weak- and strong-coupling limits, the full counting statistics undergoes a topological phase transition of the type that has been introduced recently to classify band structures of non-Hermitian Hamiltonians [14].

We will prove that the cumulant generating function and the time correlator of the detector output signal show damped oscillations in one phase and a monotonous decay with time in the other phase. The frequency of damped oscillations is finite in one phase but approaches the zero value near the critical point. At higher than critical couplings, oscillations disappear, which makes the oscillation frequency a natural order parameter whose
value distinguishes the quantum coherent phase from the quantum Zeno phase. The observation that different phases are topologically distinct is important, in particular, to conclude that the characteristics of different phases are topologically protected, i.e. they should be conserved upon finite changes of parameters and adding small changes to the measurement model.

**Setup.** – We consider a weak measurement model that was discussed in detail in [4]. It consists of a single-electron spin, such as in a quantum dot or a spin-(1/2) atom, which is continuously measured by means of the optical spin noise spectroscopy [15–22]. The bare Hamiltonian (without dissipation sources) describes spin precession in an in-plane magnetic field $B_y$:

$$\hat{H} = \frac{1}{2} g B_y \hat{S}_y,$$

and the density matrix of a spin-(1/2) can be written in the form

$$\hat{\rho} = \frac{1}{2} \hat{\rho} \cdot \hat{\sigma} = \frac{1}{2} \left( \rho_0 \hat{1} + \rho_x \hat{S}_x + \rho_y \hat{S}_y + \rho_z \hat{S}_z \right),$$

with $\rho_0 = 1$. The spin rotates around the $y$-axis with Larmor frequency $\omega_L = g B_y$, so that we can have $\rho_y(t) = 0$, as only incoherent relaxation occurs along the $y$-axis. We assume that spin relaxations along all axes occur with the same relaxation time $T$. As such, dynamics of the spin with time $t$ can be described by the evolution operator $\hat{U}[\hat{\rho}]$ through $\hat{\rho}(t) = \hat{U}[\hat{\rho}(0)]$, reading explicitly

$$\hat{\rho}(t) = \frac{1}{2} \left( \rho_0 \hat{1} + e^{-i t \omega_L \hat{S}_x} \left\{ \rho_x(0) \cos \omega_L t + \rho_z(0) \sin \omega_L t \hat{S}_z \right\} + \rho_z(0) \cos \omega_L t - \rho_x(0) \sin \omega_L t \hat{S}_x \right),$$

where $r = 1/T$ denotes the relaxation rate.

We further assume that measurements are performed in small discrete steps $\tau \ll T, 1/\omega_L$. Hence, it is possible to develop a continuous limit of our weak measurement scheme. The term “weak” means here that there is only a small probability per measurement for the detector to collapse the state vector of the spin to a measurement operator eigenstate. Let our measurement axis be the $z$-axis. The spin detection occurs due to the Faraday rotation experienced by the linearly polarized beam [23–26]. Following [4], we will assume that the detector is tuned to be insensitive to the beam passing through the spin state $|+\rangle$, but if the spin is in the state $|-\rangle$, the beam experiences additional rotation of its polarization, which has a finite but small probability per one measurement interval $\tau$ to produce an elementary distinguishable “click” of the detector.

Consider now the spin in a superposition

$$|\psi\rangle = a_+ |+\rangle + a_- |-\rangle,$$

with coefficients $a_+$ and $a_-$. After one weak measurement, with a probability $p_D|a_-|^2$, the detector shows a “click”, i.e. its output signal (e.g. voltage) intensity $I(t)$ shows a pulse denoted as 1, and the density matrix of the spin collapses to the pure state $|-\rangle\langle-|$. Here, the probability $p_D \ll 1$ characterizes the capability of the detector to induce the collapse of the wave function per one measurement. Respectively, with the probability $(1 - p_D)|a_-|^2$, the detector will not respond, i.e. its output voltage is zero at such time intervals. Thus, the output of such an ideal detector, $I(t)$, is a sequence, e.g.: 

$$I(t) \sim \ldots 001000000010000001000\ldots$$

(5)

The information content of a detector signal $I(t)$ can be determined from the signal statistical characteristics, which are in the focus of this article.

**Counting statistics of detector events.** – Following [4], we use POVM formalism [5] to obtain the probability of observing the sequence, such as (5), by introducing the Kraus operators:

i) The result “1” and projection of the density matrix on the $|-\rangle$ state are described by the Kraus operator

$$\hat{M}_1 = \sqrt{p_D} |\langle -| \rangle_.$$

(6)

ii) The result “0” does not correspond to a collapse of the state vector. It is described by the Kraus operator

$$\hat{M}_0 = \sqrt{1 - p_D} |\langle -| \rangle_{-} + |+\rangle\langle+|_{-}.$$

(7)

It will be convenient to introduce the parameter $\lambda_D$ such that $p_D = 4 \lambda_D \tau$ and consider the limit $\tau \to 0$. Since $p_D$ in this limit is proportional to the beam intensity, so is the parameter $\lambda_D$. Another physical meaning of this parameter (as we will show later) is half of the mean value of the inverse time between successive detector clicks separated by zeros. This means that the value of $\lambda_D$ is a natural characteristic of the strength of the coupling to the detector. At large values of $\lambda_D$, the system should show a pronounced Zeno effect and at small $\lambda_D$ coherent oscillations should be observed in characteristics of the detector output. We will explore what happens at intermediate values of $\lambda_D$.

The probability of a sequence $X \equiv [x_1 x_2 \ldots x_n]$ as a string of binary numbers is given by $P_X = \text{Tr} \left( \hat{M}_X[\hat{\rho}] \right)$, where

$$\hat{M}_X = M_{x_n} \hat{U}_T M_{x_{n-1}} \ldots M_{x_2} \hat{U}_T M_{x_1},$$

(8)

with $\hat{M}_{x_n} [\hat{\rho}] \equiv \hat{M}_{x_n} \hat{\rho} \hat{M}_{x_n}$ and $\hat{U}_T[\hat{\rho}]$ defined above eq. (3). Let $P_i(n, t)$ be the contribution to the $i$-th component of the spin density matrix at time $t$ produced by trajectories that make exactly $n$ detector clicks during the measurement time $t$. It is obtained by summing over all $M_X(t)$ in which the operator $\hat{M}_0[\hat{\rho}]$ encounters exactly $n$ times. The component $p_0 \rho_0$ remains zero during the evolution. Using eq. (8) and considering the continuous limit $\tau \to 0$, we obtain the equation of motion for the vector $P(n, t) \equiv \{ P_0(n, t), P_1(n, t), P_2(n, t) \}$:

$$\frac{\partial}{\partial t} P(n, t) = (\hat{K}_0 - \hat{V}) P(n, t) + \hat{V} P(n - 1, t),$$

(9)
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with

\[ \hat{K}_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -r & -\omega_L \\ 0 & \omega_L & -(2\lambda_D + r) \end{pmatrix}, \tag{10} \]

and

\[ \hat{\mathbf{V}} = \begin{pmatrix} 2\lambda_D & -2\lambda_D & 0 \\ -2\lambda_D & 2\lambda_D & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{11} \]

The full accessible information about the system and the measurement sequence is contained in the vector of generating functions:

\[ Z(\chi, t) = \sum_{n=0}^{\infty} P(n, t)e^{i\chi n}, \tag{12} \]

where \( \chi \) is the counting field, conjugated to the number \( n \). When \( \chi = 0 \), \( Z \) reduces to the vector \( \mathbf{P} \):

\[ Z(\chi = 0, t) = \sum_n P(n, t) = \mathbf{P}(t) = \{ P_0(t), P_1(t), P_2(t) \}. \]

The evolution equation for \( Z(\chi, t) \equiv \{ Z_0(\chi, t), Z_1(\chi, t), Z_2(\chi, t) \} \) can be obtained by multiplying (9) by \( e^{i\chi n} \) and summing over \( n \):

\[ \frac{\partial}{\partial t} Z(\chi, t) = \hat{K}(\chi)Z(\chi, t), \tag{13} \]

with

\[ \hat{K} = \hat{K}_0 + (e^{i\chi} - 1)\hat{\mathbf{V}} = \begin{pmatrix} 2(e^{i\chi} - 1)\lambda_D & -2(e^{i\chi} - 1)\lambda_D & 0 \\ -2(e^{i\chi} - 1)\lambda_D & 2(e^{i\chi} - 1)\lambda_D - r & -\omega_L \\ 0 & \omega_L & -(2\lambda_D + r) \end{pmatrix}. \tag{14} \]

We will concentrate on the generating function \( Z_0(\chi, t) = \sum_n P_0(n, t)e^{i\chi n} \) for probabilities \( P_0(n, t) \) to observe \( n \) detector clicks during time \( t \).

Usually the behavior of \( Z_0(\chi, t) \) is of particular theoretical interest at large total measurement time \( t \). For Markovian stochastic processes, \( Z_0(\chi, t) \) generally has a universal form in this limit, as

\[ Z_0(\chi, t) \sim e^{\chi f(\chi)t}, \tag{15} \]

with a function \( f(\chi) \) having the meaning of the cumulant generating function for the number of detector clicks. This universality follows from the fact that at large \( t \) the evolution of the generating function is dominated by the largest eigenvalue of some effective Hamiltonian. Two of the present authors showed in [14] that counting statistics of classical Markovian systems can undergo topological phase transitions, with distinct phases classified by the elements of the braid group. As a result, the function \( f(\chi) \) becomes nonanalytic at some values of \( \chi \) in topologically nontrivial phases. Below we show that the analysis of the counting statistics of the quantum-mechanical system with the evolution equation (13) can be performed along essentially the same steps, revealing a braid phase transition with physical manifestations in the short-time behavior of high-order cumulants of the click probability distribution.

Quantum Zeno effect and braid phase transition. — First we observe the analogy between eq. (13) and the Schrödinger equation with a non-Hermitian Hamiltonian \( \hat{K}(\chi) \). Note that by definition \( \hat{K}(\chi) = \hat{K}(\chi + 2\pi) \). One can think of eigenvalues of \( \hat{K}(\chi) \) as a braid structure with \( \chi \) playing the role of the Bloch vector. For a non-Hermitian Hamiltonian, the eigenvalues are generally complex. The consequence is that the periodicity of the Hamiltonian implies only the periodicity of the whole nondegenerate complex eigenspectrum but not the periodicity of each energy band as a function of \( \chi \).

We explored the band structure of the operator \( \hat{K}(\chi) \) numerically for different values of the parameter \( \lambda_D \). We observed that, by increasing \( \lambda_D \) from small to large values, the band structure of \( \hat{K}(\chi) \) passes through states with eigenvalue crossings at several different values of the parameter \( \chi \). The most interesting crossing point appears at \( \chi = 0 \) because the vicinity of the point \( \chi = 0 \) describes the most accessible lowest cumulants of the statistics of detector clicks. Such a crossing point appears in our system when \( \lambda_D = \omega_L \).

In fig. 1 we illustrate two bands of \( \hat{K}(\chi) \) for \( \lambda_D < \omega_L \) and \( \lambda_D > \omega_L \) and the corresponding schematic representations of twisting patterns of eigenvalues in terms of the braid diagrams (for clarity, we omit one band that has trivial behavior). For the case of fig. 1(a), it can be seen that each single band does not maintain the periodicity as does \( \hat{K}(\chi) \). Instead, the two displayed eigenvalues \( \lambda_{1,2}(\chi) \) of \( \hat{K}(\chi) \) satisfy the conditions

\[ \lambda_1(\chi) = \lambda_2(\chi + 2\pi), \quad \lambda_2(\chi) = \lambda_1(\chi + 2\pi), \tag{16} \]

i.e. the two complex bands twist with each other forming an element of the braid group. When we adjust the system...
parameters to make \( \lambda_D > \omega_L \), the periodicity is restored for each single band, and the original two complex bands twist twice with each other forming a different element of the braid group. The band structures for the two cases, as shown in fig. 1, are topologically inequivalent. Therefore, we encounter a topological phase transition.

Braid transitions correspond to emergence or disappearance of certain oscillating modes in the full counting statistics [14]. Indeed, let \( \epsilon_i \) be the eigenvalues of \( K(\chi) \). The evolution of the \( Z_0(\chi, t) \) would be \( Z_0(\chi, t) = \sum_i \epsilon_i(\chi) A_i^t \) with some coefficients \( A_i \) that depend on the initial state of the system. When \( \lambda_D < \omega_L \), there are two complex conjugate eigenvalues that determine behaviors of the generating function near \( \chi = 0 \) (explicitly, \( \epsilon_{1,2} = -\lambda_D \pm i\sqrt{\omega_L^2 - \lambda_D^2} \) at this point). On the other hand, when the coupling to the detector is stronger than the phase transition value, \( \lambda_D > \omega_L \), both these eigenvalues of the operator \( K(\chi) \) are real and negative near \( \chi = 0 \), which corresponds to the monotonous decay of their contributions to the generating function.

To obtain a better intuition about the physical consequences, consider the cumulants of the distribution of the number \( n \) of detected clicks:

\[
\begin{align*}
\epsilon_1 & \equiv \langle n \rangle = \frac{\partial Z_0(\chi)}{\partial(\chi^2)}|_{\chi=0}, \\
\epsilon_2 & \equiv \langle n^2 \rangle - \langle n \rangle^2 = \frac{\partial^2 Z_0(\chi)}{\partial(\chi^2)}|_{\chi=0} - \epsilon_1^2.
\end{align*}
\]  

One can obtain evolution equations for \( \langle n \rangle \) and \( \langle n^2 \rangle \) by differentiating eq. (13) over \( \chi \) once and twice and setting \( \chi \) to zero. Integrating them with equilibrium initial conditions \( Z(\chi = 0, t = 0) = (1, 0, 0) \), \( \epsilon_1(t = 0) = 0 \) and \( \epsilon_2(t = 0) = 0 \), we find

\[
\epsilon_1(t) = 2\lambda_D t,
\]

\( \text{i.e.} \) the average number of clicks just linearly increases with time, and critical behavior is not observed near \( \chi = \omega_L \) on the level of the mean number of detector clicks. However, for \( \epsilon_2(t) \), \( \text{i.e.} \) at the fluctuation level, we find an exponentially decaying correction in addition to a linearly growing contribution. A particularly simple expression for this component of \( \epsilon_2(t) \) appears after we take the 2nd derivative of \( \epsilon_2(t) \) over time:

\[
\frac{\partial^2 \epsilon_2(t)}{\partial t^2} = 8\lambda_D^2 \left( e^{\hat{Q}_0 t} \right)_{11},
\]

where

\[
\hat{Q}_0 = \begin{pmatrix}
    -\lambda_D & -\omega_L \\
    \omega_L & -2\lambda_D + r
  \end{pmatrix}
\]

is the nonzero \( 2 \times 2 \) sub-matrix of the matrix \( \hat{K}_0 \). For \( \lambda_D > \omega_L \), \( \text{i.e.} \) for a strong coupling to the detector, \( \hat{Q}_0 \) has two real eigenvalues \( \epsilon_{1,2} \), which correspond to the monotonous decay of (20). In contrast, for \( \lambda_D < \omega_L \), i.e. for a weak coupling, \( \hat{Q}_0 \) has two complex conjugated eigenvalues which correspond to damped oscillating behavior of (20). Similar oscillating behavior is found in all higher-order cumulants of the click distribution. Oscillation frequency is given by the imaginary part of such an eigenvalue: \( \omega = \sqrt{\omega_L^2 - \lambda_D^2} \). It gradually decreases with increasing \( \lambda_D \) and becomes zero at the phase transition point. Hence \( \omega \) is the natural order parameter that distinguishes the two phases. We note that high-order cumulants can also be obtained, and show similar behaviors as \( \epsilon_2(t) \).

Finally, we discuss the possibility to explore this phase transition experimentally. Measurements of individual physical detector events can be a very hard task. Instead, we suggest to use the recently developed method of the optical spin noise spectroscopy, which allows one to measure the intensity correlator of the detector output signal at equilibrium:

\[
\epsilon_2(t) = \langle I(t)I(0) \rangle - \langle I(t) \rangle \langle I(0) \rangle,
\]

where the signal \( I(t) \) is given by a sequence of physical detector clicks, such as (5). Importantly, spin noise spectroscopy can effectively extract the physical correlator (22) from a signal with a considerable background noise even when it is impossible to resolve individual useful detector clicks [16–20,22].

If we know the spin density matrix at time \( t \) after the system was successfully measured to be at the state \( |\psi\rangle \), then the intensity correlator can be expressed as

\[
\epsilon_2(t) = -4\lambda_D^2 \rho_{z}(t).
\]

Evolution equation for \( \rho(t) \) can be found by noticing that \( \rho(t) = Z(\chi = 0, t) \). Hence the operator \( \hat{K}(\chi = 0, t) \) describes the evolution of the components \( \rho_i \), defined in (2). Only its \( 2 \times 2 \) sub-matrix \( \hat{Q}_0 \) is nonzero in this case so that \( \rho_{z}(t) = |\exp[\hat{Q}_0 t]|_{11}, \text{i.e.} \) we obtain the relation \( 2\epsilon_2(t) = \partial^2 \epsilon_2(t)/\partial t^2 \). Consequently, the braid phase transitions are also directly responsible for the qualitative change of the behavior of the intensity correlator \( \epsilon_2(t) \) measured at the steady conditions. At \( \lambda_D < \omega_L \), the correlator \( \epsilon_2(t) \) shows damped oscillations that continue for arbitrary time, while at \( \lambda_D > \omega_L \) the correlator \( \epsilon_2(t) \) monotonously decays with time, as we illustrate in fig. 2(a). In fig. 2(b), for convenience, we also show the behavior of this correlator in the frequency domain.

Spin noise correlators at equilibrium are particularly convenient to study in atomic gases by measuring the spin noise power spectrum [27]. Experimentally, the parameter \( \lambda_D \) can be varied by changing the intensity of the measurement beam. To achieve this, the Larmor frequency should be sufficiently small, \( \text{i.e.} \) \( \omega_L \sim 1/T \). We predict that varying the intensity of the beam one can observe a transition from damped oscillations of the spin-spin correlator in real time to its monotonous decay. The major challenge is to achieve the Zeno effect regime at strong intensities of the beam that do not substantially heat the system and
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Fig. 2: (Colour on-line) (a) The correlator $C_2(t)$ as a function of time for different values of the parameter $\lambda_D$. (b) The Fourier transform of $C_2(t)$ (the noise power spectrum) for the same values of parameters as in (a). Black, blue, purple, green and orange curves correspond to, respectively, $\lambda_D = 0.94, 0.1, 0.3, 1.0, 2.0$. The Larmor frequency is $\omega_L = 1$ and the relaxation rate is $r = 0.05$. We normalized $C_2(0) = 1$ in (a) for all curves.

vary relaxation rates. For this reason, one should choose systems with large relaxation times $T$, so that the values of $\lambda_D$ and $\omega_L$ are relatively small. A possible candidate system is, e.g., a $^{85}$Rb atomic gas, in which the spin noise power has been studied at mG magnetic field values with the relaxation rate $1/T$ of only several kHz at $112 \, ^{\circ}\text{C}$ [27]. It is possible that by increasing the measurement beam intensity, the transition to the Zeno regime will be achieved in this system without affecting values of basic system parameters.

Conclusions. – We have shown that the path between the Zeno effect and quantum coherent dynamics in the weak measurement framework is marked by a topological phase transition at an intermediate value of a system coupling to the detector. Different phases correspond to different topologically nontrivial braid group elements, which classify the band structure of a non-Hermitian Hamiltonian that governs the evolution of the moment generating function of the detector event counting statistics. Oscillations of cumulants of detector clicks at low couplings is the signature of the quantum coherent regime, while the lack of such oscillations in the phase with a strong coupling can be interpreted as the on-set of the quantum Zeno effect. The oscillation frequency is the order parameter distinguishing between those two phases. The above-discussed phase transition could be observed in atomic gases by means of the optical spin noise spectroscopy.

The braid group finds implications in non-Hermitian quantum phase transitions [28]. The phase transitions at the fluctuation level are critical phenomena that can be observed only in higher than the first order cumulants of the event counting statistics. These phase transitions have attracted considerable attention recently [11,14,29] but their experimental studies in condensed-matter systems have been complicated because most of such phenomena can be observed only on the level of extremely rare unusual events. Our results prove, in particular, that some of such critical phenomena appear already on the level of the easily accessible time-dependent spin-spin correlator measurements. Moreover, the existing experimental results on the detection of the Zeno effect can, in fact, be reinterpreted in terms of such phase transitions.

We thank Yan Li and S. A. Crooker for useful discussions. Work at LANL was carried out under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396.

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