Energy Absorption of Reinforced Concrete Constructions with Nonlinear Dynamic Deformation

Mikhail Berlino\textsuperscript{v}1, Marina Berlino\textsuperscript{v}1
\textsuperscript{1}Moscow State University of Civil Engineering, National Research University, 26, Yaroslavskoye Shosse, Moscow, Russia
berlinov2010@mail.ru

Abstract. A method is proposed for taking into account the specific features of the energy absorption of reinforced concrete constructions under dynamic loads under non-linear deformation conditions based on the application of the finite element method. Based on the consideration of the triaxial problem, the inconstancy of the energy absorption coefficient throughout the entire volume of the deformable body was revealed. It is shown that the hysteresis energy absorption depends on the level of the stress state and is determined by the deformation properties of materials. The solution of the problem is proposed to be obtained using an iterative process based on the method of integral estimates. Considering that the amount of energy absorbed by the structure depends on the level of the stress state, it becomes possible to control the process of energy absorption by changing the stress-strain state possible by varying the design, fixing conditions, changing reinforcement or any other constructive measures.

1. Introduction
All real building structures experience variable load times. Even with static deformation, that part of the existing force effects, which refers to the so-called temporary loading components (long-acting and especially short-term) cannot act continuously, decreasing to their minimum values and reaching maximum during any time periods, causing the loading conditions to be non-constant\textsuperscript{[1-4, 8]}. The variability of the cycles “loading - unloading” causes the formation of a closed hysteresis loop, which contributes to the absorption of energy per unit volume of the body. It depends on the fixed coordinate position of the area under consideration, and is determined by the features of the static scheme, the geometry of the sections or the shape of the structure and the level of the stress state determined from the results of the calculation \textsuperscript{[5]}.

This feature of the work becomes especially evident in the case of dynamic effects and requires special consideration. At the same time, in the course of dynamic calculation, the deformation features characteristic of this vibration process, the nature of the cycle asymmetry in each considered area and other characteristics necessary for determining the hysteresis energy absorption in the structures of buildings and constructions should be determined \textsuperscript{[6, 7]}.

2. Results section
The problem of the method of calculating the constructions of buildings and structures under dynamic effects can be solved using the discrete method of calculation, namely, the finite element method, which is widely used \textsuperscript{[9]}. The basic resolving equations of motion of the finite element method for dynamic deformation, in the absence of external dampers, are well known and look like:
\[ M \ddot{Z} + \beta R \ddot{Z} + R \dot{Z} = \ddot{P}(t), \]  

(1)

where: \( M \ddot{Z} \) - is the vector of inertia forces; \( \beta R \ddot{Z} \) - vector of damping forces; \( R \dot{Z} \) - vector of internal forces; \( \ddot{P}(t) \) - vector of external dynamic forces; \( \beta \) is the diagonal matrix of attenuation coefficients (in the general case it may depend on the level of the stress state); \( R \) - is the common stiffness matrix of the entire set of finite elements of the system under consideration.

Constructions under external loads are deformed, which is accompanied by energy expenditure that occurs due to the work of internal forces on the corresponding displacements. Considering the energy balance under uniaxial deformation, we can write down:

\[ \Delta W = W - W_0, \]  

(2)

where \( \Delta W \) - irreversible energy loss corresponding to the non-recoverable part of the deformations; \( W \) is the total energy consumption for deformation; \( W_0 \) is the part of the potential energy corresponding to the work to be restored in case of recovering deformations.

The amount of energy absorbed by a unit volume of a body in one cycle of oscillations or “loading - unloading” is estimated by the coefficient of energy absorption, which, as we know, is equal to taking into account equation (2):

\[ \psi = \frac{\Delta W}{W} = 1 - \frac{W_0}{W} \]  

(3)

\[ W = \frac{1}{2} \left( \sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{yz} \gamma_{yz} + \tau_{xz} \gamma_{xz} \right). \]  

(4)

This equation, often called the Klyperon formula, can be interpreted differently as the sum of the six independent components of the energy components accumulated by the unit volume of the body along the same three coordinate axes, namely:

\[ W = W_x + W_y + W_z + W_{xy} + W_{yz} + W_{xz}. \]  

(5)

The energy expended on the work of internal forces in a unit volume of a three-dimensional body at the corresponding displacements can be obtained by considering the uniaxial stress state. When determining these losses under conditions of triaxial deformation, it is necessary to take into account the six components of stresses, in contrast to one, which will somewhat complicate the calculation. For this it is necessary to use the fundamental principles established by the nonlinear mathematical theory of elasticity, which are widely used to solve a number of specific problems of mechanics of continuous deformable media.

In order to obtain the calculated dependencies, this theory provides for operating not with individual components of the stress tensor, but with certain averaged values, which are called strain and stress intensities, such an approach, without introducing significant errors, makes it possible to somewhat simplify the calculated dependences. The postulates of the nonlinear theory of elasticity can be used in this case and because the creep processes occurring during the time of one period of oscillations, characteristic of the work of modern machines (usually a fraction of a second), can be ignored, since
they simply do not have time to appear. In this theory, all the main dependencies of the deformation of a solid body retain their strength, and therefore the mathematical relationship between the intensities of stresses and strains has the following form:

$$\sigma_{\text{int}} = E_{\text{variable}} \varepsilon_{\text{int}},$$

(6)

here: $E_{\text{variable}}$ - variable (integral) strain modulus, depending on the level of the stress state; $\varepsilon_{\text{int}}$ - is the strain rate, determined from the following expression:

$$\varepsilon_{\text{int}} = \frac{\sqrt{2}}{3} \left( (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2} (\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2) \right),$$

(7)

$\sigma_{\text{int}}$ - is the stress intensity, calculated on the basis of the following relationship:

$$\sigma_{\text{int}} = \frac{1}{\sqrt{2}} \sqrt{ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6 (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) },$$

(8)

To determine the energy losses of deforming the body, it is necessary to use the initial postulate that the functional relationship between stress and strain intensities for each point in a complex stress state uniquely depends on the functional relationship between stresses and strains in simple compression and tension. Those. Any complex stress state can be reduced to the analysis of uniaxial compression-tension, which in principle can be described by a nonlinear deformation function of a general form.

Considering the above, the hypothesis of hysteresis energy losses during uniaxial deformation can be extended to the case of a volume stress-deformed state, moreover, the following reasoning can be used to substantiate this assumption. Each component of the stresses within the elementary volume of the body during the dynamic operation of the material will form a closed hysteresis loop in the corresponding coordinate plane.

But in this case, if we assume that the intensity of the stresses is a certain averaged value, then we can assume that the stresses that these stresses will also form a kind of hypothetical loop, which can reflect the total energy loss in an elementary three-dimensional body volume.

The absorbed energy of the construction corresponds to the area OBCD (figure 1). Based on this assumption, for the case of a uniaxial stress-strain state, one can obtain the value for the irreversible part of the hysteresis absorbed energy.
Figure 1. Hysteresis loop with triaxial deformation.

Then, the energy loses can be interpreted as the following expression:

\[ W = (\sigma_{\text{int,max}} - \sigma_{\text{int,min}}) \varepsilon_{\text{int}} - \int_{\sigma_{\text{int,min}}}^{\sigma_{\text{int,max}}} \varepsilon_{\text{int}} d\sigma_{\text{int}}. \]  

(9)

Energy corresponding to the recoverable part of the work expended during unloading:

\[ W_0 = \frac{1}{2} (\varepsilon_{\text{int,ob}}) \sigma_{\text{int,ob}}. \]

(10)

here: \( \varepsilon_{\text{int,ob}} \) – is the reversible part of the strain intensity, which is determined using the strain reversibility coefficient.

Using some simplifications, it is similarly possible for practical calculations to take the following form of the equation for the equation of the mechanical state of the material:

\[ \varepsilon_{\text{int,ob}} = \frac{\sigma_{\text{int}}}{E_{\text{variable}}} \].

(11)

From a joint consideration of equations (1), (2), (9) and (10), it follows that the unit of volume recovered by the unit volume of the three-dimensional body relative to each of the coordinate axes will be equal to:

\[ \Delta W = (\sigma_{\text{int,max}} - \sigma_{\text{int,min}}) \left( \varepsilon_{\text{int}} - \frac{1}{2} \varepsilon_{\text{int,ob}} \right) \int_{\sigma_{\text{int,min}}}^{\sigma_{\text{int,max}}} \varepsilon \, d\sigma. \]

(12)

For the case of uniaxial deformation, when we consider equations (2) and (3) together, we can get the value:

\[ \psi = 1 - \frac{1}{C}. \]

(13)

where:

\[ C = 1 + 2V \left( 1 - \frac{1}{2 + m} \right) \left( \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{R} \right)^m. \]

(14)

where: \( V \) and \( m \) – are the nonlinearity parameters of the deformation, determined from experimental data on the basis of the dependence:

\[ S = 1 + V \left( \frac{\sigma}{R} \right)^m. \]

(15)
By resorting to similar considerations, summing up the energy loses for the case of a triaxial stress state, one can obtain a value for the energy absorption coefficient taking into account the action of all stress components along all three coordinate axes in the following form:

$$\psi = 1 - \left[ 1 + \frac{2m}{1 + m} \left( \frac{\sigma_{\text{intmax}} - \sigma_{\text{intmin}}}{R} \right)^m \right]^4 \quad \text{(16)}$$

### 3. Results and discussions

From dependencies (16), it can be concluded that the value of the energy absorption coefficient is not constant and depends on the level of normal and tangential stresses acting on each of the three axes of the material of the deformable body.

Analysing all the above calculations and reasoning, we can conclude that even the averaged, not to mention constant, formulation of the problem of hysteresis energy absorption is not completely correct. Thus, the hysteresis absorption of energy depends on the level of the stress state and is determined by the deformation properties of the materials.

All building constructions have different levels of stress-strain state at different points, varying both in the coordinates of three-dimensional space and time, from negative to zero and further to positive values. In structural elements consisting of two materials, in particular steel and concrete, this process is accompanied by asynchronous manifestation of creep properties in different directions for different types of stress state and is further complicated by the process of redistributing stresses in time characteristic of heterogeneous systems.

### 4. Conclusions

Consequently, the amount of energy absorbed in the process of deformation varies, both in one cycle of oscillations and during the whole process of deformation, according to the coordinates of space and time.

Attention should be paid to some features of using the finite element method for solving nonlinear and non-equilibrium problems. The main difference is at each step of integrating equation (1), when building the global stiffness matrix of the system is developed, the values of the elements of this matrix should be refined depending on the level of the stress-strain state of each finite element. This is possible according to the values of reduction coefficients that improve the convergence of the iterative process stress values of the previous approximation. We note that at the first iterative step of such an integral method, the problem is solved in an elastic-linear formulation, and later the values of the deformation moduli are refined.

A similar operation will have to be carried out when calculating the damping matrix, since the values of energy absorption coefficients in each cell in this case also become dependent on the level of the stress-strain state and all elements of this matrix require sequential refinement at each step of the iterative process.

Considering that the amount of energy absorbed by the structure depends on the level of the stress state, it becomes possible to control the process of energy absorption by changing the stress-strain state possible by varying the design scheme, conditions for fixing the change in reinforcement or any other constructive measures.

It should be borne in mind that taking into account the energy absorption coefficient is very important when considering the resonant or near-vibration mode. In this case, it is possible to identify the pattern of variable energy absorption during oscillations, depending on the change in the level of the stress state over the volume of a deformable body.

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