Cancellation Mechanism of FCNCs in $S_4 \times Z_2$ Flavor Symmetric Extra U(1) Model

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Abstract

We propose a $E_6$ inspired supersymmetric model with a non-Abelian discrete flavor symmetry: $SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)_X \times S_4 \times Z_2$. In our scenario, the additional abelian gauge symmetry; $U(1)_X$, not only solves the $\mu$-problem in the minimal Supersymmetric Standard Model (MSSM), but also requires new exotic fields which play an important role in solving flavor puzzles. If our exotic quarks can be embedded into a $S_4$ triplet, which corresponds to the number of the generation, one finds that dangerous proton decay can be well-suppressed. Hence, it might be expected that the generation structure for lepton and quark in the SM (Standard Model) can be understood as a new system in order to stabilize the proton in a supersymmetric standard model (SUSY). Moreover, due to the nature of the discrete non-Abelian symmetry itself, Yukawa coupling constants of our model are drastically reduced.

In our previous work, we actually have found much success. However, we also have to solve Higgs mediated FCNC at tree level, as is often the case with such extended Higgs models. In this paper, we propose a promising mechanism which could make cancellation between Higgs and SUSY contributions.

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1 Introduction

It is well-known that the standard model based on $G_{SM} = SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge symmetry is a quite promising theory to describe interactions of the particles. However, there are unsolved or non-verifiable points enough, in particular, the followings are underlying to be clarified:

1. The electroweak symmetry breaking scale $M_W \sim 10^2$ GeV is unnaturally small in comparison with the fundamental energy scale such as Planck scale $M_P \sim 10^{18}$ GeV.

2. The number of Yukawa coupling constants is too many to give predictions of the quark and lepton mass matrices.

3. There is no understanding about the meaning of generations.

It is believed that the first point is solved by introducing SUSY [1], but there is still naturalness problem in the MSSM. The superpotential of MSSM has $\mu$-term:

$$\mu H^U H^D.$$ (1)

The parameter $\mu$ has to be fine-tuned to $O(1\,\text{TeV})$ in order to give appropriate electroweak breaking scale, but it is unnatural. This problem is elegantly solved by introducing an additional $U(1)$ gauge symmetry. This extra $U(1)$ model is proposed in the context of superstring-inspired $E_6$ model [2]. In this model, the bare $\mu$-term is forbidden by the new $U(1)_X$ symmetry, but the trilinear term including $G_{SM}$ singlet superfield $S$ is allowed:

$$\lambda S H^U H^D.$$ (2)

When this singlet field $S$ develops a vacuum expectation value (VEV), the $U(1)_X$ gauge symmetry is spontaneously broken and an effective $\mu$-term; $\mu_{\text{eff}} H^U H^D$, is generated from this term, where $\mu_{\text{eff}} = \lambda \langle S \rangle$ [3].

A promising solution for the second point is a flavor symmetry

1. In fact, the flavor symmetry strongly reduces the Yukawa coupling constants. Here, we introduce a non-Abelian discrete flavor symmetry involved in triplet representations, expecting that the number of the generations for lepton and quark is three. The triplet representations are contained in several non-Abelian discrete symmetry groups [5], for examples, $S_4$ [6], $A_4$ [7], $T'$ [8], $\Delta(27)$ [9] and $\Delta(54)$ [10]. In our work, we consider $S_4 \times Z_2$.

A promising solution for the third point can be arose by the cooperation with the flavor symmetry and supersymmetry. In the MSSM, the R-parity conserving operators such as $QQQL, E^c U^c U^c D^c$ induce the proton decay at unacceptable level. But, in the extra $U(1)$ model, these operators are forbidden by the additional gauge symmetry. However, since the extra $U(1)$ model has additional exotic fields, the Yukawa interactions for the exotic quarks and leptons and quarks reduce proton life time to unacceptable level, again. With the $S_4$ flavor symmetry, such a dangerous proton decay is sufficiently suppressed. Hence, it might be expected that the generation structure can be understood as a new system in order to stabilize the proton [11].

Considering the Higgs sector of our model, there is a serious problem of flavor changing neutral current (FCNC). Multiple Higgs interactions with leptons and quarks induce too large FCNC, if the mass scale of Higgs bosons is at $O(TeV)$ region [12]. In this paper, we show Higgs contributions to FCNC may be cancelled by SUSY FCNC contributions. This cancellation solution softens the FCNC constraint on Higgs mass. Because the mass bound of Higgs mass is at $O(TeV)$ region, our model is testable at LHC or future colliders.

The paper is organized as follows. In section 2, we explain the basic structure of $S_4$ flavor symmetric extra $U(1)$ model. We give the superpotential of quark and lepton sector in section 3, and of Higgs sector in section 4. In section 5, we discuss the Higgs and SUSY contributions to FCNC. Finally, we make a brief summary in section 6. Experimental values of mixing matrices and masses of quarks and leptons are given in appendix, which are used to test our models.

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1 The $E_6$ inspired supersymmetric extension of SM with discrete flavor symmetry has been considered by authors [4].
2 The Extra U(1) Model with $S_4$ Flavor Symmetry

2.1 The Extra U(1) Model

The basic structure of the extra U(1) model is given as follows. At high energy scale, the gauge symmetry of model has two extra U(1)s, which consists maximal subgroup of $E_6$ as $G_2 = G_{SM} \times U(1)_X \times U(1)_Z \subset E_6$. MSSM superfields and additional superfields are embedded in three 27 multiplets of $E_6$ to cancel anomalies, which is illustrated in Table 1. The 27 multiplets are decomposed as $27 = \{Q, U^c, E^c, D^c, L, N^c, H^D, g^c, H^U, g, S\}$, where $N^c$ are right-handed neutrinos (RHN), $g$ and $g^c$ are exotic quarks, and $S$ are $G_{SM}$ singlets. We introduce $G_{SM} \times U(1)_X$ singlets $\Phi$ and $\Phi^c$ to break $U(1)_Z$ which prevents the RHNs from having Majorana mass terms. If the $G_{SM} \times U(1)_X$ singlets develop the intermediate scale VEVs along the D-flat direction of $\langle \Phi \rangle = \langle \Phi^c \rangle$, then the $U(1)_Z$ is broken and the RHNs obtain the mass terms through the trilinear terms $Y^M \Phi N^c N^c$ in the superpotential. After the symmetry is broken, as the R-parity symmetry

$$R = \exp \left[ \frac{i \pi}{20} (3x - 8y + 15z) \right]$$

remains unbroken, $G_1 = G_{SM} \times U(1)_X \times R$ survives at low energy. This is the symmetry of the low energy extra U(1) model.

Within the renormalizable operators, the full $G_2$ symmetric superpotential is given as follows:

$$W_1 = W_0 + W_S + W_B,$$

$$W_0 = Y^U H^U Q^c + Y^D H^D Q^c + Y^E H^D L^c + Y^N H^U L N^c + Y^M \Phi N^c N^c,$$

$$W_S = k S g^c + \lambda S H^U H^D,$$

$$W_B = \lambda_1 Q_{gg} + \lambda_2 g^c D^c + \lambda_3 g^c U^c + \lambda_4 g^c L^c + \lambda_5 g D^c N^c.$$  

For simplicity, we drop gauge and generation indices. Where $W_0$ is the same as the superpotential of the MSSM with the RHNs besides the absence of $\mu$-term, and $W_S$ and $W_B$ are the new interactions. In $W_S$, $k S g^c$ drives the soft SUSY breaking scalar squared mass of $S$ to negative through the renormalization group equations (RGEs) and then breaks $U(1)_X$ and generates mass terms of exotic quarks, and $\lambda S H^U H^D$ is source of the effective $\mu$-term. Therefore, $W_0$ and $W_S$ are phenomenologically necessary. In contrast, $W_B$ breaks baryon number and leads to very rapid proton decay, which are phenomenologically unacceptable, so this must be forbidden.

| $SU(3)_c$ | $Q$ | $U^c$ | $E^c$ | $D^c$ | $L$ | $N^c$ | $H^D$ | $g^c$ | $H^U$ | $g$ | $S$ | $\Phi$ | $\Phi^c$ |
|-----------|-----|-------|-------|-------|-----|-------|-------|-------|-------|-----|-----|-------|-------|
| 3         | 3   | 3     | 8     | 1     | 1   | 1     | 3     | 1     | 1     | 3   | 1   | 1     | 1     |
| $SU(2)_W$ | 2   | 1     | 1     | 1     | 2   | 1     | 1     | 2     | 1     | 1   | 1   | 1     | 1     |
| $y = 6Y$  | 1   | -4    | 6     | 2     | -3  | 0     | -3    | 2     | 3     | -2  | 0   | 0     | 0     |
| $x$       | 1   | 1     | 1     | 2     | 2   | 0     | -3    | -3    | -2    | -2  | 5   | 0     | 0     |
| $z$       | -1  | -1    | -1    | 2     | 2   | -4    | -1    | -1    | -1    | 2   | -1  | 8     | -8    |
| $R$       | -   | -     | -     | -     | -   | -     | -     | -     | -     | -   | -   | +     | +     |

Table 1: $G_2$ assignment of fields. Where the $x$, $y$ and $z$ are charges of $U(1)_X$, $U(1)_Y$ and $U(1)_Z$, and $Y$ is hypercharge.

2.2 $S_4$ Flavor Symmetry

We show how the $S_4$ flavor symmetry forbids the baryon number violating superpotential $W_B$. Non-Abelian group $S_4$ has two singlet representations $1$, $1'$, one doublet representation $2$ and two triplet representations $3$, $3'$, where $1$ is the trivial representation. As the generation number of quarks and leptons is three, at least one superfield of $\{Q, U^c, E^c, D^c, L, N^c, H^D, g^c, H^U, g, S\}$ must be assigned to triplet of $S_4$ in order to solve flavor puzzle. As we assume that full $E_6$ symmetry does not realize at Planck scale, there is no need to assign all superfields to the same $S_4$ representations. The multiplication rules of these representations are

\[ SU(3)_c \times SU(2)_W \times U(1)_X \times U(1)_Y \times U(1)_Z \]
as follows:

\[
\begin{align*}
3 \times 3 &= 1 + 2 + 3 + 3', \\
3 \times 3' &= 1' + 2 + 3 + 3', \\
2 \times 3' &= 3 + 3', \\
1' \times 3 &= 3', \\
1' \times 2 &= 2, \\
1' \times 1' &= 1.
\end{align*}
\] (8)

With these rules, it is easily shown that all the \( S_4 \) invariants consist of two or three non-trivial representations are given by

\[
\begin{align*}
1' \cdot 1', & \quad 2 \cdot 2, \quad 3 \cdot 3, \quad 3' \cdot 3', \quad 1' \cdot 2 \cdot 2, \quad 1' \cdot 3 \cdot 3', \quad 2 \cdot 2 \cdot 2, \quad 2 \cdot 3 \cdot 3, \\
2 \cdot 3 \cdot 3', & \quad 2 \cdot 3' \cdot 3', \quad 3 \cdot 3 \cdot 3, \quad 3 \cdot 3' \cdot 3', \quad 3' \cdot 3' \cdot 3'.
\end{align*}
\] (9)

From these, one can see that there is no invariant including only one triplet. Therefore, if \( g \) and \( g^c \) are assigned to triplets and the others are assigned to singlets or doublets, then \( W_B \) is forbidden. This provides a solution to the proton life time problem.

### 2.3 Exotic Quark Decay and Proton Decay Suppression

The absence of \( W_B \) makes exotic quarks and proton stable, but the existence of exotic quarks which have life time longer than 0.1 second spoils the success of Big Ban nucleosynthesis. In order to evade this problem, the \( S_4 \) symmetry must be broken. Therefore, it is assumed that the \( S_4 \) breaking terms are induced from non-renormalizable terms. We introduce \( G_2 \) singlet \( T \) as triplet of \( S_4 \) and add the quartic terms:

\[
W_{NRB} = \frac{1}{M_P} T (\bar{Q}Qg + g^c U^c D^c + g^c E^c L^c + g^c LQ + gD^c N^c).
\] (10)

Where the order one coefficients in front of each terms are omitted for simplicity. When \( T \) develops VEV with

\[
\frac{\langle T \rangle}{M_P} \sim 10^{-12},
\] (11)

the phenomenological constraints on the life times of proton and exotic quarks are satisfied at the same time [4]. The violation of \( S_4 \) symmetry gives \( S_4 \) breaking corrections to effective superpotential through the non-renormalizable terms which are expressed in the same manner as Eq.(10):

\[
W_{NRFV} = \frac{1}{M_P} T^2 (H^U QU^c + H^D QD^c + H^D LE^c + H^U LN^c + M' N^c N^c + SH^U H^D) + \frac{1}{M_P} TS gg^c.
\] (12)

Since the above corrections are negligibly small, the \( S_4 \) flavor symmetry approximately holds in low energy effective theory. One finds that the most economical flavon sector is the one which is exchanged \( T \) into superfield-product; \( \Phi \Phi^c / M_P \), by embedding \( \Phi^c \) to a \( S_4 \) triplet (Hereafter, we call \( \Phi \) and \( \Phi^c \) as flavon which is the trigger of flavor violation.). In this case, the condition of Eq. (11) correspond to the following relation:

\[
\frac{\langle \Phi \rangle \langle \Phi^c \rangle}{M_P^2} \sim 10^{-12},
\] (13)

and then the right-handed neutrino mass scale can be predicted as follows:

\[
M_R \sim \langle \Phi \rangle \sim 10^{-6} M_P \sim 10^{12} \text{ GeV}.
\] (14)

Hence, by applying the above relation to the measurement of proton and exotic quarks (In our model, we call exotic quarks as \( g \)-quark.) life time, it is expected that one can determine the right-handed neutrino mass scale.

### 3 Quark and Lepton Sector

At first, we define \( W_0 \) that contributes mass matrices of quarks and leptons. Although the \( S_4 \) symmetry reduces the Yukawa coupling constants, there is still an overabundance of parameters. In order to reduce
chiral superfields are assigned to the representations of the Yukawa coupling constants further, we extend the flavor symmetry to $S_4 \times Z_2$ [13]. In our model, all chiral superfields are assigned to the representations of $S_4 \times Z_2$ as Table 2.

The superpotential $W_0$ which is consistent with $G_2$ and the symmetries of Table 2 is given by

$$W_0 = Y_1^U H_3^U (Q_1 U^c_1 + Q_2 U^c_2) + Y_3^U H_3^U Q_3 U^c_3$$

$$+ Y_1^D H_3^D (Q_1 D^c_1 + Q_2 D^c_2) + Y_3^D H_3^D D^c_3$$

$$+ Y_1^H Q_3 H_3^U (H_3^L Q_1 + H_3^L Q_2) + Y_2^H (H_3^D Q_1 + H_3^D Q_2) D^c_3$$

$$+ Y_2^N [H_3^L (L_1 N_1^c + L_2 N_2^c) + H_3^D (L_1 N_1^c - L_2 N_2^c)]$$

$$+ Y_3^N H_3^L L_3 N_3^c + Y_4^L N_3 (H_3^L N_1^c + H_3^D N_2^c)$$

$$+ Y_d^E E_5^c (H_2^D L_1 + H_2^D L_2) + Y_2^E E_3^c H_2^D L_3 + Y_4^E E_3^c (H_1^D L_2 - H_2^D L_1)$$

$$+ \frac{1}{2} Y_M^M \Phi(N_1^c N_1^c + N_2^c N_2^c) + \frac{1}{2} Y_M^M \Phi(N_5^c N_5^c).$$

(15)

There are sixteen complex Yukawa coupling constants in this superpotential. The twelve phases of these can be absorbed by redefinition of the five of six quark superfields $\{Q_i, Q_3, U^c_i, U^c_3, D^c_i, D^c_3\}$ and seven lepton superfields $\{L_1, L_3, E_1^c, E_2, E_3^c, N_1^c, N_3^c\}$. Without loss of generality, we can define $Y_3^E, Y_4^D, Y_2^N, Y_4^E, Y_1^M$ to be real. We define the phases of complex Yukawa couplings as follows:

$$Y_1^U = e^{i\alpha} |Y_1^U|, \quad Y_1^D = e^{i\beta} |Y_1^D|, \quad Y_3^D = e^{i\gamma} |Y_3^D|, \quad Y_3^N = e^{i\delta} |Y_3^N|.$$

(16)

We write the VEV of the flavon as

$$\langle \Phi \rangle = V,$$

(17)

and the VEVs of the $SU(2)_W$ doublet Higgses as

$$\langle H_1^U \rangle = v_u \cos \theta_u, \quad \langle H_2^U \rangle = v_u \sin \theta_u, \quad \langle H_3^U \rangle = v'_u,$$

$$\langle H_1^D \rangle = v_d \cos \theta_d, \quad \langle H_2^D \rangle = v_d \sin \theta_d, \quad \langle H_3^D \rangle = v'_d,$$

(18)

where we assume these VEVs are real and the parameters $V, v_u, v'_u, v_d$ are non-negative and the relation

$$\sqrt{v_u^2 + v_d^2 + v'_u^2 + v'_d^2} = 174 \text{ GeV}$$

(19)

is satisfied. If we define the non-negative mass parameters as follows:

$$M_1 = Y_M^M V, \quad M_2 = Y_M^M V,$$

$$m_u^2 = |Y_1^U|v_u, \quad m_d^2 = |Y_1^D|v_d,$$

$$m_u^2 = |Y_2^N|v_u, \quad m_d^2 = |Y_2^N|v_d,$$

$$m_u^2 = |Y_3^N|v_u, \quad m_d^2 = |Y_3^N|v_d,$$

(20)

$^2T'$ does not have this property but $A_4, \Delta(27)$ and $\Delta(54)$ have.
then the mass matrices of up-type quarks \((M_u)\), down-type quarks \((M_d)\), charged leptons \((M_l)\), Dirac neutrinos \((M_D)\) and Majorana neutrinos \((M_R)\) are given by

\[
M_u = \begin{pmatrix}
& e^{i\alpha}m_1^u & 0 & m_2^l \\
& 0 & e^{i\alpha}m_1^u & m_3^l \\
m_1^u \sin \theta_u & m_2^l \cos \theta_u & m_3^l & 0
\end{pmatrix},
\quad M_d = \begin{pmatrix}
& e^{i\beta}m_1^d & 0 & m_2^d \sin \theta_d \\
& 0 & e^{i\beta}m_1^d & m_3^d \sin \theta_d \\
m_1^d \sin \theta_d & m_2^d \cos \theta_d & m_3^d \sin \theta_d & 0
\end{pmatrix},
\quad M_l = \begin{pmatrix}
& 0 & m_1^l \cos \theta_d & 0 \\
& m_1^l \sin \theta_d & 0 & m_3^l \cos \theta_d \\
& 0 & m_2^l & 0
\end{pmatrix},
\quad M_D = \begin{pmatrix}
& 0 & m_2^d \sin \theta_d \\
& m_2^d \cos \theta_d & m_3^d \sin \theta_d & 0
\end{pmatrix},
\quad M_R = \begin{pmatrix}
& M_1 & 0 & 0 \\
& 0 & M_1 & 0 \\
& 0 & 0 & M_3
\end{pmatrix}.
\]

After the seesaw mechanism, the light neutrino mass matrix is given by

\[
M_\nu = M_D M_R^{-1} M_D^T = \begin{pmatrix}
\rho_2^2 & 0 & \rho_2 \rho_4 \sin 2\theta_u \\
0 & \rho_2^2 & \rho_2 \rho_4 \cos 2\theta_u \\
\rho_2 \rho_4 \sin 2\theta_u & \rho_2 \rho_4 \cos 2\theta_u & \rho_2^2 + e^{i\beta} \rho_3^2
\end{pmatrix},
\]

where

\[
\rho_2 = \frac{m_2^l}{\sqrt{M_1}}, \quad \rho_4 = \frac{m_4^l}{\sqrt{M_1}}, \quad \rho_3 = \frac{m_3^l}{\sqrt{M_3}}.
\]

In the lepton sector, the mass eigenvalues and diagonalization matrix of charged leptons are given by

\[
V_{1R}^\dagger M_l V_{LL} = \text{diag}(m_1^l, m_2^l, m_3^l) = (m_1^l, m_2^l, m_3^l),
\]

\[
V_{LL} = \begin{pmatrix}
0 & -\sin \theta_d & \cos \theta_d \\
0 & \cos \theta_d & \sin \theta_d \\
-1 & 0 & 0
\end{pmatrix},
\]

\[
V_{IR} = \begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix},
\]

and those of the light neutrinos are given by

\[
V_\nu^T M_\nu V_\nu = \text{diag}(e^{i\phi_1+\phi}, m_{\nu_1}, e^{i\phi_2+\phi}, m_{\nu_2}, e^{i\phi_3+\phi}, m_{\nu_3}),
\]

\[
V_\nu = \begin{pmatrix}
\sin 2\theta_\nu & -\cos 2\theta_\nu & 0 \\
\cos 2\theta_\nu & \sin 2\theta_\nu & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
-\sin \theta_\nu & e^{i\phi} \cos \theta_\nu & 0 \\
-\cos \theta_\nu \sin \theta_\nu & e^{i\phi} \cos \theta_\nu \sin \theta_\nu & \sin \bar{\theta} \\
-\sin \theta_\nu \cos \theta_\nu & e^{i\phi} \sin \theta_\nu \cos \theta_\nu & -\cos \bar{\theta}
\end{pmatrix} P_\nu,
\]

\[
V_{MNS}^\dagger = V_{1L}^\dagger V_{\nu} P_\nu = \begin{pmatrix}
-e^{-i\phi} \cos \theta_\nu & -\sin \theta_\nu & 0 \\
-\cos \bar{\theta} \sin \theta_\nu & e^{i\phi} \cos \theta_\nu \cos \bar{\theta} & \sin \bar{\theta} \\
-\sin \bar{\theta} \sin \theta_\nu & e^{i\phi} \sin \theta_\nu \cos \bar{\theta} & -\cos \bar{\theta}
\end{pmatrix} P_\nu,
\]

where

\[
\bar{\theta} = \theta_d + 2\theta_u,
\]

\[
P_\nu = \text{diag}(e^{-i(\phi_1+\phi)/2}, e^{-i(\phi_2+\phi)/2}, 1).
\]

Following ref. [13], we get

\[
\tan^2 \theta_\nu = \frac{m_{\nu_3}^2 |\sin^2 \phi - m_{\nu_3}| \cos \phi}{m_{\nu_1}^2 - m_{\nu_3}^2 \sin^2 \phi + m_{\nu_3}^2 \cos \phi},
\]

\[
\sin(\phi_1 - \phi_2) = \frac{m_{\nu_2} \sin \phi}{m_{\nu_1} m_{\nu_2}} \left[ \sqrt{m_{\nu_2}^2 - m_{\nu_3}^2 \sin^2 \phi} + \sqrt{m_{\nu_1}^2 - m_{\nu_3}^2 \sin^2 \phi} \right],
\]

\[
\sin(\phi_1 - \phi) = \frac{\sin \phi}{m_{\nu_1}} \left[ m_{\nu_3} \sqrt{1 - \sin^2 \phi} + \sqrt{m_{\nu_1}^2 - m_{\nu_3}^2 \sin^2 \phi} \right].
\]
After the redefinition of the fields, the MNS matrix is transformed to the standard form in Eq.(106) where the parameters are given by

$$\theta_{13} = 0, \quad \theta_{12} = \theta_\nu, \quad \theta_{23} = \bar{\theta}, \quad \alpha' = \frac{\phi_1 - \phi_2}{2}, \quad \beta' = \frac{\phi_1 - \phi_3}{2}. \quad (35)$$

If the neutrino masses have been measured, the two Majorana phases $\alpha'$ and $\beta'$ would be predicted by Eqs.(32), (33), (34) and (35). In addition, $\theta_{13} = 0$ is predicted, so totally three predictions are given in the lepton sector.

In the quark sector, the mass eigenvalues and diagonalization matrices of quarks are given as follows:

$$V_{uR}^\dagger M_u^u V_{uL} = \text{diag}(m_u, m_c, m_t), \quad (36)$$

$$V_{uL} = V_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi_{uL}} \end{pmatrix} \begin{pmatrix} \cos \theta_{uL} & 0 & \sin \theta_{uL} \\ 0 & 1 & 0 \\ -\sin \theta_{uL} & 0 & \cos \theta_{uL} \end{pmatrix} S_{12}, \quad (37)$$

$$V_{uR} = V_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi_{uR}} \end{pmatrix} \begin{pmatrix} \cos \theta_{uR} & 0 & \sin \theta_{uR} \\ 0 & 1 & 0 \\ -\sin \theta_{uR} & 0 & \cos \theta_{uR} \end{pmatrix} S_{12}, \quad (38)$$

$$V_u = \begin{pmatrix} \cos \theta_u & -\sin \theta_u & 0 \\ \sin \theta_u & \cos \theta_u & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (39)$$

$$m_u^2 = (m_u^d)^2, \quad (40)$$

$$m_c^2 = \frac{1}{2} \left[ (m_1^d)^2 + (m_3^d)^2 + (m_4^d)^2 + (m_5^d)^2 - \mu_u^2 \right], \quad (41)$$

$$m_t^2 = \frac{1}{2} \left[ (m_1^d)^2 + (m_3^d)^2 + (m_4^d)^2 + (m_5^d)^2 + \mu_u^2 \right], \quad (42)$$

$$\mu_u^2 = \sqrt{(m_3^u)^2 + (m_4^u)^2 - (m_1^u)^2 - (m_5^u)^2}^2 + 4L_u^2, \quad (43)$$

$$L_u = \sqrt{(m_4^u)^2 + (m_5^u)^2 + (m_1^u)^2 + (m_3^u)^2} \cos \alpha + m_4^u m_5^u \sin \alpha)^2, \quad (44)$$

$$R_u = \sqrt{(m_4^u)^2 + (m_5^u)^2 + (m_1^u)^2 + (m_3^u)^2} \cos \alpha + m_4^u m_5^u \sin \alpha)^2, \quad (45)$$

$$\tan 2\theta_{uL} = \frac{2L_u}{(m_3^u)^2 + (m_4^u)^2 - (m_1^u)^2 - (m_5^u)^2}, \quad (46)$$

$$\tan \phi_{uL} = \frac{m_4^u m_5^u \sin \alpha}{m_4^u m_5^u \cos \alpha + m_3^u m_5^u}, \quad (47)$$

$$\tan 2\theta_{uR} = \frac{2R_u}{(m_3^u)^2 + (m_4^u)^2 - (m_1^u)^2 - (m_5^u)^2}, \quad (48)$$

$$\tan \phi_{uR} = \frac{-m_4^u m_5^u \sin \alpha}{m_4^u m_5^u \cos \alpha + m_3^u m_5^u}, \quad (49)$$

$$V_{dR}^\dagger M_d^d V_{dL} = \text{diag}(m_d, m_s, m_b), \quad (50)$$

$$V_{dL} = V_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi_{dL}} \end{pmatrix} \begin{pmatrix} \cos \theta_{dL} & 0 & \sin \theta_{dL} \\ 0 & 1 & 0 \\ -\sin \theta_{dL} & 0 & \cos \theta_{dL} \end{pmatrix} S_{12}, \quad (51)$$

$$V_{dR} = V_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi_{dR}} \end{pmatrix} \begin{pmatrix} \cos \theta_{dR} & 0 & \sin \theta_{dR} \\ 0 & 1 & 0 \\ -\sin \theta_{dR} & 0 & \cos \theta_{dR} \end{pmatrix} S_{12}, \quad (52)$$

$$V_d = \begin{pmatrix} \cos \theta_d & -\sin \theta_d & 0 \\ \sin \theta_d & \cos \theta_d & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (53)$$

$$m_d^2 = (m_1^d)^2, \quad (54)$$

$$m_s^2 = \frac{1}{2} \left[ (m_1^d)^2 + (m_3^d)^2 + (m_4^d)^2 + (m_5^d)^2 - \mu_d^2 \right], \quad (55)$$

$$m_b^2 = \frac{1}{2} \left[ (m_1^d)^2 + (m_3^d)^2 + (m_4^d)^2 + (m_5^d)^2 + \mu_d^2 \right], \quad (56)$$
\[ \mu_d^2 = \sqrt{(m_4^d)^2 + (m_1^d)^2 - (m_1^d)^2 + 4L_d^2}, \]  
\[ L_d = \sqrt{(m_4^d m_1^d \cos \beta + m_2^d m_3^d \cos \gamma)^2 + (m_4^d m_1^d \sin \beta - m_2^d m_3^d \sin \gamma)^2}, \]  
\[ R_d = \sqrt{(m_4^d m_5^d \cos \beta + m_2^d m_3^d \cos \gamma)^2 + (m_4^d m_5^d \sin \beta - m_2^d m_3^d \sin \gamma)^2}, \]  
\[ \tan 2\theta_{dL} = \frac{2L_d}{(m_4^d)^2 + (m_1^d)^2 - (m_1^d)^2 - (m_2^d)^2}, \]  
\[ \tan \phi_{dL} = \frac{m_4^d m_1^d \sin \beta - m_2^d m_3^d \sin \gamma}{m_4^d m_1^d \cos \beta + m_2^d m_3^d \cos \gamma}, \]  
\[ \tan 2\theta_{dR} = \frac{2R_d}{(m_4^d)^2 + (m_1^d)^2 - (m_1^d)^2 - (m_2^d)^2}, \]  
\[ \tan \phi_{dR} = \frac{-m_4^d m_5^d \sin \beta + m_3^d m_4^d \sin \gamma}{m_4^d m_5^d \cos \beta + m_3^d m_4^d \cos \gamma}, \]  
\[ S_{12} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

from which the Cabibbo-Kobayashi-Maskawa (CKM) matrix is given by

\[ V_{CKM} = V_{ud}^\dagger V_{dl} = \begin{pmatrix} \cos \tilde{\theta} & -\sin \tilde{\theta} \cos \theta_{dL} & -\sin \tilde{\theta} \sin \theta_{dL} \\ \sin \tilde{\theta} \cos \theta_{dL} & \cos \tilde{\theta} \cos \theta_{dL} + e^{i\phi} \sin \theta_{dL} \sin \theta_{dL} & \cos \tilde{\theta} \sin \theta_{dL} \sin \theta_{dL} - e^{i\phi} \sin \theta_{dL} \cos \theta_{dL} \\ \sin \tilde{\theta} \sin \theta_{dL} & -\sin \tilde{\theta} \cos \theta_{dL} & \cos \tilde{\theta} \sin \theta_{dL} \sin \theta_{dL} + e^{i\phi} \sin \theta_{dL} \cos \theta_{dL} \end{pmatrix}, \]

where

\[ \tilde{\theta} = \theta_d - \theta_u, \quad \phi = \phi_{dL} - \phi_{uL}. \]

The experimental values of the matrix elements and Jarlskog invariant in Eq.(172) are reproduced by putting

\[ \tilde{\theta} = 13.3^\circ, \quad \theta_{uL} = 2.05^\circ, \quad \theta_{dL} = 0.99^\circ, \quad \phi = -83.9^\circ. \]

In ref. [13], it is assumed that the VEVs of Higgs \( S_3 \) doublets are fixed in the direction of \( \theta_u = \theta_d = \frac{\pi}{4} \), which enforces \( \tilde{\theta} = 0 \) (and predicts the atmospheric neutrino mixing angle is maximal). This means the Cabibbo angle is zero. In contrast, there is no such a condition of vacuum directions in this model.

Due to an overabundance of free parameters, there is no prediction in quark sector. But we can show that there exist consistent parameter sets. For example, if we put

\[ \alpha = 0.00^\circ, \quad \beta = -83.9^\circ, \quad \gamma = 83.9^\circ, \]

\[ m_1^u = 1.28 \text{ MeV}, \quad m_2^u = 172 \text{ GeV}, \quad m_3^u = 17.2 \text{ GeV}, \quad m_4^u = 6.23 \text{ GeV}, \]

\[ m_1^d = 2.91 \text{ MeV}, \quad m_2^d = 1.94 \text{ GeV}, \quad m_3^d = 2.14 \text{ GeV}, \quad m_4^d = 74.2 \text{ MeV}, \]

\[ m_1^l = 1.75 \text{ GeV}, \quad m_2^l = 487 \text{ KeV}, \quad m_3^l = 103 \text{ MeV}, \]

then the quark masses in Eq.(171) and the parameters of CKM matrix in Eq.(67) are reproduced. In this case, unknown mixing angles \( \theta_{uR}, \theta_{dR} \) and phases \( \phi_{uR}, \phi_{dR} \) are given by

\[ \theta_{uR} = 5.70^\circ, \quad \theta_{dR} = 47.8^\circ, \quad \phi_{uR} = \phi_{uL} = 0.00^\circ, \quad \phi_{dR} = -\phi_{dL} = 83.9^\circ. \]

These parameters can be expressed by the perturbative Yukawa coupling constants and the VEVs of Higgs fields through Eq.(20), for example as follows:

\[ v_u = 41.4 \text{ GeV}, \quad v_u' = 150 \text{ GeV}, \quad v_d = 60.0 \text{ GeV}, \quad v_d' = 49.5 \text{ GeV}, \]

\[ |Y_1^U| = 8.53 \times 10^{-6}, \quad |Y_3^U| = 1.15, \quad |Y_4^U| = 0.415, \quad |Y_5^U| = 0.150, \]

\[ |Y_1^D| = 5.87 \times 10^{-5}, \quad |Y_3^D| = 0.0392, \quad |Y_4^D| = 0.0357, \quad |Y_5^D| = 1.23 \times 10^{-3}, \]

\[ |Y_1^E| = 0.0292, \quad |Y_2^E| = 9.84 \times 10^{-6}, \quad |Y_3^E| = 1.72 \times 10^{-3}. \]

As all the coupling constants of the model are perturbative, it is consistent that the fundamental energy scale is much larger than the electroweak scale, which is the base of naturalness problem.
4 Higgs sector

Next, we define Higgs potential and solve its minimum condition approximately. With gauge symmetry in table 1 and flavor symmetry in table 2, superpotential of Higgs sector is given by

\[ W_H = \lambda_1 S_3 (H_U^2 H_D^2 + H_U^2 H_D^2) + \lambda_3 S_3 H_U^4 H_1^2 + \lambda_4 H_1^2 (S_1 H_U^2 + S_2 H_D^2) + \lambda_5 H_1^2 (S_1 H_U^2 + S_2 H_D^2) \subset W_S, \]  

(71)

where one can take, without any loss of the generalities, \( \lambda_{1,3,4,5} \) as real, by redefining of four arbitrary fields of \( \{ S, S, H_U^2, H_U^2, H_D^2, H_D^2 \} \). However, this superpotential could have would-be goldstone bosons when all of the Higgs fields acquire VEVs, because of an accidental \( O(2) \) symmetry induced by the common rotation of the \( S_4 \) doublet. In order to avoid the problem, we assume that the flavor symmetry should be explicitly broken in the soft scalar mass terms, which can play role in giving the controllable parameters for the direction of the \( SU(2) \) doublet Higgs VEVs.

As the Higgs potential has too many unknown parameters, we make several assumptions. In the superpotential, we assume the parameters \( \lambda_i \) are hierarchical for examples, as follows:

\[ \lambda_5 \ll \lambda_1 = 0.03 \ll \lambda_3 = \lambda_4 = 0.3. \]  

(72)

Then, we can neglect the first and fourth term in \( W_H \). Note that, too small \( \lambda_1 \) is not consistent with chargino mass bound \( M_{\text{chargino}} > 94 \text{GeV} \), and too large \( \lambda_{3,4} \) make \( Y_3^U \) reach Landau pole below \( M_P \). With this assumption, F-term and D-term contribution to Higgs potential is given by

\[ V_{\text{SUSY}} = |\lambda_3 H_U^2 H_D^2|^2 + |\lambda_4 H_1^2 H_D^2|^2 + |\lambda_5 H_1^2 H_2^2|^2 \]

\[ + |\lambda_3 S_3 H_D^3 + \lambda_4 (S_1 H_D^1 + S_2 H_D^2)|^2 \]

\[ + |\lambda_3 S_3 H_D^3|^2 + |\lambda_5 H_1^2 S_1|^2 + |\lambda_5 H_2^2 S_2|^2 \]

\[ + \frac{1}{8} g_2^2 \sum_{a=1}^3 \left( |H_a^U|^2 \sigma_A H_a^U + (H_a^U)^\dagger \sigma_A H_a^D \right)^2 + \frac{1}{8} g_Y^2 \left[ |H_a^D|^2 - |H_a^D|^2 \right]^2 \]

\[ + \frac{1}{2} g_2^2 \left[ x_{H_D} |H_a^U|^2 + x_{H_D} |H_a^D|^2 + x_{S_4} |S_4|^2 \right], \]  

(73)

where index \( a \) runs \( a = 1, 2, 3 \), and flavor symmetric SUSY breaking terms are given by

\[ V_{\text{SB}} = m^2_{H_U} (|H_1^U|^2 + |H_2^U|^2) - m^2_{H_D} (|H_3^D|^2 + |H_2^D|^2) + m_{H_D}^2 |H_3^D|^2 \]

\[ + m_{S_3}^2 (|S_2|^2 + |S_3|^2) - m_{S_3}^2 |S_3|^2 \]

\[ - \left\{ \lambda_3 A_3 S_3 H_D^3, S_3 H_1^2, S_3 H_2^2 \right\} \], \]  

(74)

where all parameters in \( V_{\text{SB}} \) can be real in some SUSY breaking scenario, for example, in the case that A-parameters are induced by gaugino mass through RGEs, A-parameters become real. In order to avoid goldstone bosons, we assume flavor violation in soft scalar mass terms, and add flavor violating terms as follows:

\[ V_{\text{SBFB}} = -m^2_{BH_U} (H_1^U) (H_1^U c H_U + H_2^U s H_U) - m^2_{BS} (S_3) (S_1 c S + S_2 s S) + h.c., \]  

(75)

where we assume flavor violation is induced by VEV of \( S_4 \) doublet \( Z_2 \) odd auxiliary field in hidden sector.

In this paper, we do not consider hidden sector which is beyond our paper. With this assumption, the term \( m_{H_D}^2 (H_3^D)^\dagger (H_1^U c H_D + H_2^U s H_D) \) should be included in \( V_{\text{SBFB}} \). Here we assume this term is approximately negligible. In this approximation, all parameters of potential \( V = V_{\text{SUSY}} + V_{\text{SB}} + V_{\text{SBFB}} \) are real, because we can remove the phases of \( m_{BH_U}^2 \) and \( m_{BS}^2 \) by field redefinition. After the redefinition, three phases of \( m_{BH_U}^2, BH_D^0, BS \) are transformed into \( \lambda_{1,5}, m_{BH_D}^2 \) which are assumed to be small and negligible.

From the defined potential above, the potential minimum conditions are given by

\[ 0 = \frac{\partial V}{\partial H_U^2} / (v_u c_u) = m_{BH_U}^2 - m_{BH_U}^2 c H_U (v'_u / v_u c_u) + \frac{1}{2} g_Y^2 x_{H_D} x_{S} (v'_u + (v'_u)^2) \]

\[ + \left\{ \frac{1}{4} (g_Y^2 + g_2^2) (v'_u + (v'_u)^2 - v'_d - (v'_d)^2) \right\} \]
\[0 = \frac{\partial V}{\partial H_2^f} / (v_u s_u) = m_{H_2}^2 v_u - m_{BH}^2 s_H v_u / v_u s_u + g_s^2 x_H v_x (v_u^2 + (v'_u)^2) \]

\[0 = \frac{\partial V}{\partial H_2^d} / v'_u = -m_{H_2}^2 - \lambda_3 A_3 v'_u (v'_d / v'_u) - \lambda_4 A_4 \{ v_u c_a (v'_d / v'_u) + v_s s_d (v'_d / v'_u) \}
+ m_{BH}^2 c_H v_u / v'_u + s_H / v_u s_u / v'_u \]
+ \lambda_3^2 (v'_u)^2 + \lambda_3^2 (v'_d)^2 + \frac{1}{4} (g'_s + g_s^2) (v'_u)^2 + (v'_d)^2 - (v'_u)^2)
+ \frac{g_s^2 x_H}{x_H} (v_u^2 + (v'_u)^2) + x_H (v_d^2 + (v'_d)^2) \}

\[0 = \frac{\partial V}{\partial H_2^d} / (v_d s_d) = m_{H_2}^2 - \lambda_3 A_3 v'_u (v'_d / v'_u) + \lambda_4 (v_u c_a / v_d s_d) [\lambda_3 v'_u v'_d + \lambda_4 (v_u c_a v_d c_d + v_s s_d v_d s_d)]
+ \frac{g_s^2 x_H}{x_H} (v_u^2 + (v'_u)^2)
+ \left\{ \lambda_3^2 (v'_u)^2 - \frac{1}{4} (g'_s + g_s^2) (v'_u)^2 + (v'_d)^2 - (v'_u)^2 \right\}
+ \frac{g_s^2 x_H}{x_H} (v_u^2 + (v'_u)^2) + x_H (v_d^2 + (v'_d)^2) \}

\[0 = \frac{\partial V}{\partial H_2^d} / (v' u) = m_{H_2}^2 - \lambda_3 A_3 v'_u (v'_d / v'_u) + \lambda_4 (v_u c_a / v_d s_d) [\lambda_3 v'_u v'_d + \lambda_4 (v_u c_a v_d c_d + v_s s_d v_d s_d)]
+ \frac{g_s^2 x_H}{x_H} (v_u^2 + (v'_u)^2)
+ \left\{ \lambda_3^2 (v'_u)^2 - \frac{1}{4} (g'_s + g_s^2) (v'_u)^2 + (v'_d)^2 - (v'_u)^2 \right\}
+ \frac{g_s^2 x_H}{x_H} (v_u^2 + (v'_u)^2) + x_H (v_d^2 + (v'_d)^2) \}

\[0 = \frac{\partial V}{\partial S_1} / (v_u c_a) = m_{S_2}^2 - m_{BS}^2 (v'_d / v_u c_a) c_S + \frac{g_s^2 x_S}{x_S} (v_u^2 + (v'_u)^2)
+ \left\{ -\lambda_4 A_1 v'_u (v_d c_a / v_u c_a) + \lambda_4^2 (v'_u)^2 + \lambda_4 (v_d c_a / v_u c_a) [\lambda_3 v'_u v'_d + \lambda_4 (v_u c_a v_d c_d + v_s s_d v_d s_d)]
+ \frac{g_s^2 x_S}{x_S} [x_H (v_u^2 + (v'_u)^2) + x_H (v_d^2 + (v'_d)^2)] \}

\[0 = \frac{\partial V}{\partial S_2} / (v_s s_d) = m_{S_2}^2 - m_{BS}^2 (v'_d / v_s s_d) s_S + \frac{g_s^2 x_S}{x_S} (v_u^2 + (v'_u)^2)
+ \left\{ -\lambda_4 A_1 v'_u (v_d s_d / v_s s_d) + \lambda_4^2 (v'_u)^2 + \lambda_4 (v_d s_d / v_s s_d) [\lambda_3 v'_u v'_d + \lambda_4 (v_u c_a v_d c_d + v_s s_d v_d s_d)]
+ \frac{g_s^2 x_S}{x_S} [x_H (v_u^2 + (v'_u)^2) + x_H (v_d^2 + (v'_d)^2)] \}

\[0 = \frac{\partial V}{\partial S_3} / (v'_u) = -m_{S_2}^2 - m_{BS}^2 [(v_u c_a / v'_u) c_S + (v_s s_d / v'_u) s_S] + \frac{g_s^2 x_S}{x_S} (v_u^2 + (v'_u)^2)
+ \left\{ -\lambda_4 A_1 v'_u (v_d c_a / v'_u) + \lambda_4^2 (v'_u)^2 + \lambda_4 (v_d c_a / v'_u) [\lambda_3 v'_u v'_d + \lambda_4 (v_u c_a v_d c_d + v_s s_d v_d s_d)]
+ \frac{g_s^2 x_S}{x_S} [x_H (v_u^2 + (v'_u)^2) + x_H (v_d^2 + (v'_d)^2)] \}

\[\langle S_1 \rangle = v_u \cos \theta_s, \quad \langle S_2 \rangle = v_u \sin \theta_s, \quad \langle S_3 \rangle = v'_u, \]

and hereafter we neglect the terms in bracket \{ \}, because those are very small (Note that $v_{u,d}, v'_{u,d} \ll$...
Solving the Eqs.(76)-(84), we get

\[ m_{BS}^2(v_s/v_s) \left( \frac{c_s}{c_d} - \frac{s_s}{s_d} \right) = 0 \quad (\because \quad \theta_s = \theta_S) \tag{86} \]
\[ m_{H}^2 - m_{BS}^2(v'_s/v_s) + g_x^2 x_S^2 (v_s^2 + (v_s')^2) = 0 \tag{87} \]
\[ -m_{S}^2 - m_{BS}^2(v_s/v_s') + g_x^2 x_S^2 (v_s^2 + (v_s')^2) = 0 \tag{88} \]
\[ \lambda_d \left[ \lambda_4 v_d' + \lambda_4 v_d (c_d c_d + s_s s_d) - A_4 v_u' \left( \frac{c_s}{c_d} - \frac{s_s}{s_d} \right) \right] v_s = 0 \quad (\because \quad \theta_d = \theta_s) \tag{89} \]
\[ m_{H}^2 - \lambda_4 A_4 v_s (v'_s/v_d) + \lambda_3 \lambda_4 v_s (v'_s/v_d) + \lambda_4^2 v_s^2 + g_x^2 x_H^2 (v_s^2 + (v_s')^2) = 0 \tag{90} \]
\[ m_{H}^2 - \lambda_3 \lambda_4 v_s (v'_s/v_d') + \lambda_3^2 (v_s')^2 + \lambda_3 \lambda_4 v_s (v'_s/v_d') + g_x^2 x_H^2 (v'_s^2 + (v'_s')^2) = 0 \tag{91} \]
\[ m_{B_{HV}}^2 (v'_s/v_u) \left( \frac{c_{H_{HV}}}{c_u} - \frac{s_{H_{HV}}}{s_u} \right) = 0 \quad (\because \quad \theta_u = \theta_{H_{HV}}) \tag{92} \]
\[ m_{B_{HV}}^2 - m_{B_{HV}}^2 (v'_s/v_u) + g_x^2 x_{H_{HV}} (v_s^2 + (v_s')^2) = 0 \tag{93} \]
\[ m_{B_{HV}}^2 - \lambda_4 A_4 v_s (v'_s/v_d') - \lambda_4 A_4 v_s (v'_s/v_d') - m_{B_{HV}}^2 (v'_s/v_u) \tag{94} \]

Using Eqs.(86)-(94), mass matrices of neutral CP-even (\(\phi\)) and CP-odd (\(\rho\)) Higgs bosons are given by

\[ M_\phi^2 = \begin{pmatrix}
M_{uu}^2 & M_{ud}^2 & 0 \\
M_{du}^2 & M_{dd}^2 & 0 \\
0 & 0 & M_{ss}^2
\end{pmatrix}, \tag{95} \]
\[ M_\rho^2 = \begin{pmatrix}
M_{uu}^2 & -M_{ud}^2 & 0 \\
-M_{du}^2 & M_{dd}^2 & 0 \\
0 & 0 & M_{ss}^2
\end{pmatrix}, \tag{96} \]
\[ M_{ss}^2 = \begin{pmatrix}
m_{BS}^2 (v'_s/v_u) + g_x^2 x_S^2 (v_s c_s) \\
2g_x^2 x_S^2 v_s c_s \\
-m_{BS}^2 c_s + 2g_x^2 x_S^2 v_s c_s
\end{pmatrix}, \tag{97} \]
\[ M_{uu}^2 = \begin{pmatrix}
m_{BS}^2 (v'_s/v_u) \\
0 \\
-m_{BS}^2 c_s
\end{pmatrix}, \tag{98} \]
\[ M_{dd}^2 = \begin{pmatrix}
m_{BS}^2 (v'_s/v_u) + g_x^2 x_H^2 (v_s s_s) \\
-\lambda_3^2 (v_s s_s)^2 - \lambda_3 \lambda_4 v_s (v'_s/v_d) \\
0
\end{pmatrix}, \tag{99} \]
\[ M_{dd}^2 = \begin{pmatrix}
\lambda_3 \lambda_4 v_s (v'_s/v_d) \\
0 \\
-\lambda_3 \lambda_4 v_s (v'_s/v_d)
\end{pmatrix} \tag{100} \]
\[ M_{uu}^2 = \begin{pmatrix}
m_{BS}^2 (v'_s/v_u) + g_x^2 x_H^2 (v_s s_s) \\
0 \\
-\lambda_3 \lambda_4 v_s (v'_s/v_d)
\end{pmatrix} \tag{101} \]

where \(v_u, v'_u, v_d, v'_d \ll v_s, v'_s\) is assumed and defined as

\[ (H_i^U)^0 = \frac{\phi_{s,i} + i \rho_{s,i}}{\sqrt{2}}, \quad (H_i^D)^0 = \frac{\phi_{d,i} + i \rho_{d,i}}{\sqrt{2}}, \quad S_i = \frac{\phi_{s,i} + i \rho_{s,i}}{\sqrt{2}} (i = 1, 2, 3). \tag{102} \]

Hereafter we do not consider \(\phi, \rho,\) because these fields do not mix with \(\phi_{s,d} \rho_{s,d}\) and never contribute FCNC. Because the mass matrices are partially diagonalized as follows

\[ (M_{uu}^2)^{\prime} = V_u^\dagger M_{uu}^2 V_u \]
\[
\begin{align*}
\mathcal{L} &= \left( m^2_{BHv} (v'_u/v_u) \right) 0 - m^2_{BH\nu} (v'_u/v_u) 0 \left( m^2_{BHv} (v'_u/v_u) + m^2_{BH\nu} (v'_{d}/v_d) + \lambda_3 A_3 v'_u (v'_d/v_d) + \lambda_4 A_4 v_s (v_d/v_u) \right), \\
(M^2_{dd})' &= V_d^t M^2_{dd} V_d = \left( \begin{array}{ccc} \lambda_4 v_s [A_4 (v'_u/v_d) - \lambda_3 v'_s (v'_d/v_d)] & 0 & \lambda_3 \lambda_4 v_s v'_s \\ 0 & M^2_{\phi_{d,2}} & 0 \\ \lambda_3 \lambda_4 v_s v'_s & 0 & \lambda_3 v'_s [A_3 (v'_u/v_d) - \lambda_4 v_s (v_d/v'_u)] \end{array} \right),
\end{align*}
\]

where \( V_u \) and \( V_d \) are defined in Eq. (39) and Eq. (53), respectively, one can see that the mixed states

\[
\begin{align*}
\phi^'_{u,1} &= -\phi_{u,1} s_u + \phi_{u,2} c_u, & \rho^'_{u,2} &= -\rho_{u,1} s_u + \rho_{u,2} c_u \\
\phi^'_{d,2} &= -\phi_{d,1} s_d + \phi_{d,2} c_d, & \rho^'_{d,2} &= -\rho_{d,1} s_d + \rho_{d,2} c_d
\end{align*}
\]

are mass eigenstates. Note that CP-even Higgs bosons \( \phi^'_{u,2}, \phi^'_{d,2} \) and CP-odd Higgs bosons \( \rho^'_{u,2}, \rho^'_{d,2} \) have the same mass eigenvalues in this approximation, respectively.

## 5 Cancellation of Higgs and SUSY-FCNC Contributions

Finally, we evaluate the Higgs and SUSY contributions to FCNC. Here we calculate \( K^0 - \bar{K}^0 \), \( B^0 - \bar{B}^0 \) and \( D^0 - \bar{D}^0 \) mass differences.

### 5.1 Higgs contributions

First, we explain how Higgs bosons mediate FCNCs. Yukawa coupling interactions of quarks and charged lepton masses between neutral Higgs bosons are given by

\[
\mathcal{L}_Y = (\bar{u}_1, \bar{u}_2, \bar{u}_3)_R \left( \begin{array}{ccc} Y^U_1 (H^U_3)^0 & 0 & Y^U_4 (H^U_3)^0 \\ 0 & Y^U_1 (H^U_3)^0 & Y^U_4 (H^U_3)^0 \\ Y^U_5 (H^U_3)^0 & Y^U_5 (H^U_3)^0 & Y^U_5 (H^U_3)^0 \end{array} \right) \left( \begin{array}{c} u_1 \\ u_2 \\ u_3 \end{array} \right)_L + (\bar{d}_1, \bar{d}_2, \bar{d}_3)_R \left( \begin{array}{ccc} Y^D_1 (H^D_3)^0 & 0 & Y^D_4 (H^D_3)^0 \\ 0 & Y^D_1 (H^D_3)^0 & Y^D_4 (H^D_3)^0 \\ Y^D_5 (H^D_3)^0 & Y^D_5 (H^D_3)^0 & Y^D_5 (H^D_3)^0 \end{array} \right) \left( \begin{array}{c} d_1 \\ d_2 \\ d_3 \end{array} \right)_L + (\bar{l}_1, \bar{l}_2, \bar{l}_3)_R \left( \begin{array}{ccc} Y^E_1 (H^E_3)^0 & Y^E_4 (H^E_3)^0 & 0 \\ 0 & Y^E_3 (H^E_3)^0 & 0 \\ -Y^E_3 (H^E_3)^0 & Y^E_3 (H^E_3)^0 & 0 \end{array} \right) \left( \begin{array}{c} l_1 \\ l_2 \\ l_3 \end{array} \right)_L + h.c.. \right)
\]

With the basis that quark and lepton mass matrices are diagonal, these terms are rewritten by

\[
\begin{align*}
\mathcal{L}_Y &= \frac{1}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t})_R \left( \begin{array}{ccc} Y^U_1 (\phi_{u,3} + i \rho_{u,3}) & Y^U_4 s_{uL} (\phi^'_{u,2} + i \rho^'_{u,2}) & -Y^U_4 c_{uL} (\phi^'_{u,2} + i \rho^'_{u,2}) \\ Y^U_5 s_{uR} (\phi^'_{u,2} + i \rho^'_{u,2}) & H^U_{22} & H^U_{23} \\ -Y^U_5 c_{uR} (\phi^'_{u,2} + i \rho^'_{u,2}) & H^U_{32} & H^U_{33} \end{array} \right) \left( \begin{array}{c} u \\ c \\ t \end{array} \right)_L + \frac{1}{\sqrt{2}} (\bar{d}, \bar{s}, \bar{b})_R \left( \begin{array}{ccc} Y^D_1 (\phi_{d,3} + i \rho_{d,3}) & \eta Y^D_4 s_{dL} (\phi^'_{d,2} + i \rho^'_{d,2}) & -\eta Y^D_4 c_{dL} (\phi^'_{d,2} + i \rho^'_{d,2}) \\ \eta Y^D_5 s_{dR} (\phi^'_{d,2} + i \rho^'_{d,2}) & H^D_{22} & H^D_{23} \\ -\eta Y^D_5 c_{dR} (\phi^'_{d,2} + i \rho^'_{d,2}) & H^D_{32} & H^D_{33} \end{array} \right) \left( \begin{array}{c} d \\ s \\ b \end{array} \right)_L + \frac{1}{\sqrt{2}} (\bar{l}, \bar{\mu}, \bar{\tau})_R \left( \begin{array}{ccc} -Y^E_2 (\phi_{d,3} + i \rho_{d,3}) & 0 & 0 \\ 0 & Y^E_3 (\phi^'_{d,2} + i \rho^'_{d,2}) & 0 \\ 0 & Y^E_3 (\phi^'_{d,2} + i \rho^'_{d,2}) & Y^E_3 (\phi^'_{d,2} + i \rho^'_{d,2}) \end{array} \right) \left( \begin{array}{c} e \\ \mu \\ \tau \end{array} \right)_L + h.c.,
\end{align*}
\]

\[
H^U_{22} = [Y^U_1 c_{uR} (\phi_{u,3} + i \rho_{u,3}) - Y^U_5 s_{uR} (\phi^'_{u,2} + i \rho^'_{u,2})] s_{uL},
\]

\[
H^U_{23} = [Y^U_1 c_{uR} (\phi_{u,3} + i \rho_{u,3}) - Y^U_5 s_{uR} (\phi^'_{u,2} + i \rho^'_{u,2})] c_{uL}.
\]
where

\[ \phi'_{a,1} = \phi_{a,1}c_u + \phi_{a,2}s_u, \quad \rho'_{a,1} = \rho_{a,1}c_u + \rho_{a,2}s_u, \quad \phi'_{d,1} = \phi_{d,1}c_d + \phi_{d,2}s_d, \quad \rho'_{d,1} = \rho_{d,1}c_d + \rho_{d,2}s_d. \]

From these interactions, we can evaluate FCNC processes. For example, one can see that \( \phi'_{d,2} \) and \( \rho'_{d,2} \) mediate flavor changing operator such as \( (d_Rd_L)(\bar{d}_Ls_R) \), which contributes \( K^0 - \bar{K}^0 \) mass difference \( \Delta m_K \). Note that the terms \( (d_Rs_L)(\bar{d}_Ls_R) \) and \( (d_Ls_R)(\bar{d}_Ls_R) \) are not induced because contributions to them from \( \phi'_{d,2} \) and \( \rho'_{d,2} \) are cancelled due to degeneration of masses. However, lepton flavor changing processes such as \( \mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma \) and \( \tau \rightarrow e\gamma \) are not induced.

Flavor violating effective interactions are given by

\[
\mathcal{L}_{\text{Higgs–FCNC}} = \frac{Y^U_{11}}{4 m^2_{\phi'_{a,2}}} Y^U_{sLs_R} (\bar{u}_{R,\alpha} c_{L}^\alpha) (\bar{u}_{L,\beta} s_{R}^\beta) + \frac{1}{4} \frac{Y^D_{11}}{m^2_{\phi'_{d,2}}} (\bar{d}_{R,\alpha} s_{L}^\alpha) (\bar{d}_{L,\beta} s_{R}^\beta)
\]

where \( \alpha \) and \( \beta \) are color indices. From this Lagrangian, we can evaluate the Higgs contributions to \( \Delta m_K \), \( \Delta m_B \) and \( \Delta m_D \) as follows:

\[
\langle \Delta m_K \rangle_{\text{Higgs}} = 2 \text{Re} \left( K^0 | (\mathcal{L}_{\text{Higgs–FCNC}}) | \bar{K}^0 \rangle \right) = -2 \frac{Y^D_{11} Y^D_{sLs_R}}{m^2_{\phi'_{d,2}}} \left( K^0 \left| (\bar{d}_{R,\alpha} s_{L}^\alpha) (\bar{d}_{L,\beta} s_{R}^\beta) \right| \bar{K}^0 \right),
\]

(122)

\[
\langle \Delta m_B \rangle_{\text{Higgs}} = 2 \text{Re} \left( B^0 | (\mathcal{L}_{\text{Higgs–FCNC}}) | \bar{B}^0 \rangle \right) = -2 \frac{Y^D_{11} Y^D_{cLcR}}{m^2_{\phi'_{d,2}}} \left( B^0 \left| (\bar{d}_{R,\alpha} b_{L}^\alpha) (\bar{d}_{L,\beta} b_{R}^\beta) \right| \bar{B}^0 \right),
\]

(123)

\[
\langle \Delta m_D \rangle_{\text{Higgs}} = 2 \text{Re} \left( D^0 | (\mathcal{L}_{\text{Higgs–FCNC}}) | \bar{D}^0 \rangle \right) = -2 \frac{Y^U_{11} Y^U_{sLs_R}}{m^2_{\phi'_{a,2}}} \left( D^0 \left| (\bar{u}_{R,\alpha} c_{L}^\alpha) (\bar{u}_{L,\beta} c_{R}^\beta) \right| \bar{D}^0 \right),
\]

(124)

where

\[
\langle K^0 | (\bar{d}_{R,\alpha} s_{L}^\alpha) (\bar{d}_{L,\beta} s_{R}^\beta) | \bar{K}^0 \rangle = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{m_{K^0}}{m_s(2\text{GeV}) + m_d(2\text{GeV})} \right)^2 \right] m_{K^0} f_K^2
\]
which are evaluated by using the parameters given in appendix. Requiring $K, B, D$, we get

$$\langle B^0 \left| (\bar{d}_{R, \alpha} b_{L}^\alpha)(\bar{d}_{L, \beta} b_{R}^\beta) \right| B^0 \rangle = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{m_{B^0}}{m_b(m_b) + m_d(m_d)} \right)^2 \right] m_{B^0} f_B^2 = 9.21 \times 10^7 \text{MeV}^3,$$

$$\langle D^0 \left| (\bar{u}_{R, \alpha} c_{L}^\alpha)(\bar{u}_{L, \beta} c_{R}^\beta) \right| D^0 \rangle = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{m_{D^0}}{m_c(m_c) + m_u(m_u)} \right)^2 \right] m_{D^0} f_D^2 = 4.99 \times 10^7 \text{MeV}^3,$$

These constraints are too strong. In our model, SUSY contributions to FCNC may be used to cancel these Higgs contributions. However, in order to suppress $\Delta m_{K,B,D}$, we must give three cancellation conditions:

$$\left| (\Delta m_M)_{Higgs} + (\Delta m_M)_{SUSY} \right| < \left| (\Delta m_M)_{Higgs} \right| \quad (M = K, B, D),$$

which is unnatural. In the next subsection, we show the number of cancellation conditions are reduced to two from three.

### 5.2 Squark and gluino contributions

Now we evaluate SUSY-FCNC contributions. As we assume $\Delta m_D$ is suppressed by cancellation:

$$\left| (\Delta m_D)_{Higgs} + (\Delta m_D)_{SUSY} \right| < \left| (\Delta m_D)_{Higgs} \right|,$$

we consider only $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mass differences induced by squark and gluino box diagrams. These contributions depend only on down type squark mass matrices. Considering the following squark Lagrangian

$$\mathcal{L}_{s quark} = m_2^2 (|Q|^2 + |Q^2|^2) + m_2^2 (|Q_s|^2 + |Q_s|^2) + m_2^2 (|D| + |D|^2) + m_2^2 (|D_s|^2 + |D_s|^2) + \{ e^{-i\phi_Q} m_{BQ}^2 |Q^2|^2 (c_Q Q_1 + s_Q Q_2) + e^{i\phi_D} m_{BD}^2 |D_s|^2 (c_D D_1 + s_D D_2) + h.c. \} + (D - \text{terms}),$$

one can see that down type squark mass matrix is given by

$$\mathcal{L}_{down - squark} = \left( D^\dagger, D^\dagger \right) \left( \begin{array}{cc} M^2_{LL} & 0 \\ 0 & M^2_{RR} \end{array} \right) \left( \begin{array}{c} D^\dagger \\ (D^\dagger) \end{array} \right).$$

$$M^2_{LL} = \begin{pmatrix} m_2^2 & 0 & e^{i\phi_Q} m_{BQ}^2 |c_Q| & e^{i\phi_Q} m_{BQ}^2 |s_Q| \\ 0 & m_2^2 & e^{-i\phi_Q} m_{BQ}^2 |c_Q| & e^{-i\phi_Q} m_{BQ}^2 |s_Q| \\ e^{-i\phi_Q} m_{BQ}^2 |c_Q| & e^{-i\phi_Q} m_{BQ}^2 |s_Q| & m_2^2 & 0 \\ e^{i\phi_D} m_{BD}^2 |c_D| & e^{i\phi_D} m_{BD}^2 |s_D| & 0 & m_2^2 \end{pmatrix},$$

$$M^2_{RR} = \begin{pmatrix} m_2^2 & 0 & e^{i\phi_D} m_{BD}^2 |c_D| & e^{i\phi_D} m_{BD}^2 |s_D| \\ 0 & m_2^2 & e^{-i\phi_D} m_{BD}^2 |c_D| & e^{-i\phi_D} m_{BD}^2 |s_D| \\ e^{-i\phi_D} m_{BD}^2 |c_D| & e^{-i\phi_D} m_{BD}^2 |s_D| & m_2^2 & 0 \\ e^{i\phi_Q} m_{BQ}^2 |c_Q| & e^{i\phi_Q} m_{BQ}^2 |s_Q| & e^{-i\phi_Q} m_{BQ}^2 |c_Q| & e^{-i\phi_Q} m_{BQ}^2 |s_Q| \end{pmatrix}.\$$

Where D-term contributions are absorbed into $m^2_{Q,Q_s,D,D_s}$. In super-CKM basis, squark mass matrices are given by

$$(M^2_{LL})' = V_{dL}^* M^2_{LL} V_{dL},$$

$$(M^2_{LL})' = \begin{pmatrix} m_2^2 & \eta_Q m_{BQ}^2 |s_Q| d_L |d_L| & -\eta_Q m_{BQ}^2 |s_Q| d_L |d_L| \\ \eta_Q m_{BQ}^2 |s_Q| d_L |d_L| & (M^2_{LL})_{22} & (M^2_{LL})_{23} \\ -\eta_Q m_{BQ}^2 |s_Q| d_L |d_L| & (M^2_{LL})_{23} & (M^2_{LL})_{33} \end{pmatrix},$$

$$(M^2_{LL})_{22} = \begin{array}{c} m_2^2 (c_L d_L)^2 + m_2^2 (s_L d_L)^2 - m_2^2 c_L c_L |d_L|^2 \eta_d + \eta_d^2, \\ (M^2_{LL})_{23} = (m_2^2 - m_2^2 c_L c_L) c_L d_L |d_L|^2 + m_2^2 c_L c_L |d_L|^2 - \eta_d (s_L d_L)^2. \end{array}$$
\[
\begin{align*}
(M_{Ld})_{33} &= m_Q^2(s_{dL})^2 + m_Q^2(c_{dL})^2 + m_{BQ}q_{sL}c_{dL}(\eta_Q + \eta^*_Q), \\
s_{Qd} &= \sin(\theta_Q - \theta_d), \\
\eta_Q &= \eta e^{i\phi_Q}, \\
(M_{RR}^2)' &= V_{dR} M_{RR} V_{dR} \\
&= \left( \begin{array}{ccc}
\eta_D m_{BD}^2 |s_Dd| s_{dR} & -\eta_D m_{BD}^2 |s_Dd| s_{dR} & \eta_D m_{BD}^2 |s_Dd| s_{dR}
\eta_D m_{BD}^2 |s_Dd| s_{dR} & m_{BD}^2 (s_{dR})^2 - |m_{BD}|^2 c_{dR}(\eta_D + \eta^*_D) & -\eta_D m_{BD}^2 |s_Dd| s_{dR}
\eta_D m_{BD}^2 |s_Dd| s_{dR} & -\eta_D m_{BD}^2 |s_Dd| s_{dR} & m_{BD}^2 (s_{dR})^2 - |m_{BD}|^2 c_{dR}(\eta_D + \eta^*_D)
\end{array} \right),
\end{align*}
\]

Here we assume degenerate mass squared parameters as

\[
m_Q^2 = m_Q^2, \quad m_D^2 = m_D^2,
\]

which are essential assumptions to realize cancellation between Higgs and SUSY-FCNC contributions. These relations are realized if gaugino mass contributions dominate in RGEs. With this assumption, diagonal elements of mass squared matrix are also degenerated approximately as follows:

\[
(M_{Ld})_{22} \simeq (M_{Ld})_{33} \simeq m_Q^2, \\
(M_{RR}^2)_{22} \simeq (M_{RR}^2)_{33} \simeq m_D^2.
\]

Where we assume that the contributions from \(m_{BD,BQ}^2\) are negligible. Furthermore, we assume \(\eta_Q = \eta_D = 1\) to suppress

\[
\text{Im} \langle K^0 | L_{SUSY-FCNC} | K^0 \rangle.
\]

Here flavor changing effective interactions induced by squark and gluino box diagrams are calculated in mass insertion approximation [14] as follows:

\[
\begin{align*}
L_{SUSY-FCNC} &= \frac{\alpha_3^2}{216 M_Q^2 K} \{ (\delta_{12})^2_{LL} [24 x f_1(x) + 66 f_2(x)] O_1 + (\delta_{12})^2_{LR} [24 x f_1(x) + 66 f_2(x)] O_2 \\
&+ (\delta_{12})^2_{LL} (\delta_{12})^2_{RR} \{ [504 x f_1(x) - 72 f_2(x)] O_3 + [24 x f_1(x) + 120 f_2(x)] O_4 \} \}
&+ \frac{\alpha_3^2}{216 M_Q^2 B} \{ (\delta_{13})^2_{LL} [24 y f_1(y) + 66 f_2(y)] P_1 + (\delta_{13})^2_{RR} [24 y f_1(y) + 66 f_2(y)] P_2 \\
&+ (\delta_{13})^2_{LL} (\delta_{13})^2_{RR} \{ [504 y f_1(y) - 72 f_2(y)] P_3 + [24 y f_1(y) + 120 f_2(y)] P_4 \} \},
\end{align*}
\]

where \(\alpha_3\) is \(SU(3)_c\) gauge coupling, \(M_{Q,K}\) and \(M_{Q,B}\) are averaged squark mass, and the other parameters are defined as

\[
\begin{align*}
f_1(x) &= \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}, \\
f_2(x) &= \frac{6x(1 + x) \ln x - x^3 + 9x^2 + 9x + 1}{3(x - 1)^5}, \\
O_1 &= (\bar{d}_{L,R} \gamma^\mu \gamma^\nu b_L^\alpha)(\bar{d}_{L,R} \gamma^\nu s_L^\beta), \quad P_1 = (\bar{d}_{L,R} \gamma^\mu b_L^\alpha)(\bar{d}_{L,R} \gamma^\nu b_L^\beta), \\
O_2 &= (\bar{d}_{R,R} \gamma^\mu s_R^\beta)(\bar{d}_{R,R} \gamma^\nu s_R^\beta), \quad P_2 = (\bar{d}_{R,R} \gamma^\mu b_R^\beta)(\bar{d}_{R,R} \gamma^\nu b_R^\beta), \\
O_3 &= (\bar{d}_{R,R} s_R^\alpha)(\bar{d}_{L,L} \gamma^\nu s_L^\beta), \quad P_3 = (\bar{d}_{L,R} b_R^\beta)(\bar{d}_{L,R} \gamma^\nu b_R^\beta), \\
O_4 &= (\bar{d}_{R,R} s_R^\alpha)(\bar{d}_{L,L} \gamma^\nu s_L^\beta), \quad P_4 = (\bar{d}_{R,R} b_R^\beta)(\bar{d}_{L,L} \gamma^\nu b_R^\beta), \\
(\delta_{12})_{LL} &= \frac{m_{BQ}^2 s_{Q4dL}}{M^2_{Q,K}}, \quad (\delta_{13})_{LL} = -\frac{m_{BQ}^2 s_{Q4dL}}{M^2_{Q,B}},
\end{align*}
\]
Note that the dominant contributions to $\Delta m_K$ follows:

One finds that these constraints are weaker than Eqs. (128) and (129). Therefore three cancellation condition:

where $M_3$ is gluino mass. With the assumptions of Eqs. (150) and (151), one can assume $M_{Q,K}^2 = M_{Q,B}^2$. Note that the dominant contributions to $\Delta m_K$ and $\Delta m_B$ come from $O_3$ and $P_3$ in Eq. (153) due to the large coefficients. Total contributions to $O_3$ and $P_3$ from Higgs (Eq. (121)) and SUSY (Eq. (153)) are written as follows:

If accidental cancellation occurs between the terms in bracket $[\ ]$, new physics contributions to $\Delta m_K$ and $\Delta m_B$ are well-suppressed at same time. Assuming $x = 1$ and $|m_{BQ}| s_{Qd} = -2 |m_{BD}| s_{Dd}$, we get

and cancellation condition:

One finds that Eq. (166) is satisfied, for example, if we put

Then the sub-dominant contributions from Eq. (153) are evaluated as follows:

where

are used. Requiring $|\langle \Delta m_M \rangle_{SUSY}| < \Delta m_M (M = K, B)$, we get

One finds that these constraints are weaker than Eqs. (128) and (129). Therefore three cancellation conditions Eq. (131) are reduced to two conditions Eq. (132) and Eq. (166).
6 Summary

In this paper, we have considered the Higgs-FCNC problem in $S_4 \times Z_2$ flavor symmetric extra U(1) model, and have shown that the Higgs mass bounds from FCNCs are weakened by cancellation between Higgs and SUSY contributions. As the result, SUSY breaking scale may be around $O(\text{TeV})$ region. It might be expected that the new gauge symmetry and the flavor symmetry are tested in LHC or future colliders.

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Appendix

A Experimental Values

Running masses of quarks and charged leptons [15]:

\[
\begin{align*}
 m_u(m_Z) &= 1.28^{+0.30}_{-0.29}(\text{MeV}), \\
 m_c(m_Z) &= 624 \pm 83 \text{ (MeV)}, \\
 m_t(m_Z) &= 172.5 \pm 3.0 \text{ (GeV)}, \\
 m_d(m_Z) &= 2.91^{+0.29}_{-0.28} \text{ (MeV)}, \\
 m_s(m_Z) &= 55^{+16}_{-15} \text{ (MeV)}, \\
 m_b(m_Z) &= 2.89 \pm 0.09 \text{ (GeV)}, \\
 m_e(m_Z) &= 0.48657 \text{ (MeV)}, \\
 m_{\mu}(m_Z) &= 102.72 \text{ (MeV)}, \\
 m_{\tau}(m_Z) &= 1746 \text{ (MeV)}.
\end{align*}
\]

CKM matrix elements and Jarlskog invariant [16]:

\[
\begin{align*}
 |V_{ud}| &= 0.97418 \pm 0.00027, \\
 |V_{us}| &= 0.2255 \pm 0.0019, \\
 |V_{ub}| &= (3.93 \pm 0.36) \times 10^{-3}, \\
 |V_{cd}| &= 0.230 \pm 0.011, \\
 |V_{cs}| &= 1.04 \pm 0.06, \\
 |V_{cb}| &= (41.2 \pm 1.1) \times 10^{-3}, \\
 |V_{td}| &= (8.1 \pm 0.6) \times 10^{-3}, \\
 |V_{ts}| &= (38.7 \pm 2.3) \times 10^{-3}, \\
 |V_{tb}| &> 0.74,
\end{align*}
\]

\[
J_{CP} = \text{Im}(V_{ud}V_{ub}^*V_{cd}V_{cb}^*) = (3.05^{+0.19}_{-0.20}) \times 10^{-5}.
\]

Neutrino mass-squared differences and the parameters of MNS matrix [16]:

\[
\Delta m_{21} = m_2^2 - m_1^2 = (8.0 \pm 0.3) \times 10^{-5} \text{ (eV}^2),
\]

\[
\Delta m_{32} = |m_{32}^2 - m_{22}^2| = (1.9 - 3.0) \times 10^{-3} \text{ (eV}^2),
\]

\[
V_{MNS} = \begin{pmatrix}
   c_{12}c_{13} & c_{12}s_{13}e^{i\delta'} & s_{13}e^{-i\delta'} \\
   -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta'} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta'} & s_{23}c_{13} \\
   s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta'} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta'} & c_{23}c_{13}
\end{pmatrix}
\begin{pmatrix}
   1 & 0 & 0 \\
   0 & e^{i\alpha'} & 0 \\
   0 & 0 & e^{i\beta'}
\end{pmatrix},
\]

\[
\theta_{12} = 34.0^{+1.3}_{-1.5}, \quad 45.0^o > \theta_{23} > 36.8^o, \quad 12.9^o > \theta_{13} > 0.0^o.
\]

Meson masses [16]:

\[
 m_{K^0} = 497.614 \pm 0.024 \text{MeV}, \quad m_{B^0} = 5279.50 \pm 0.30 \text{MeV}, \quad m_{D^0} = 1864.84 \pm 0.17 \text{MeV}.
\]

Meson mass differences [16]:

\[
\Delta m_K = (3.483 \pm 0.006) \times 10^{-12} \text{MeV},
\]

\[
\Delta m_B = (3.337 \pm 0.033) \times 10^{-10} \text{MeV},
\]

\[
\Delta m_D = (1.56 \pm 0.43) \times 10^{-11} \text{MeV}.
\]

Meson decay constants [17]:

\[
 f_K = 159.8 \pm 1.4 \pm 0.44 \text{MeV}, \quad f_B = 200 \pm 20 \text{MeV}, \quad f_D = 212 \pm 14 \text{MeV}.
\]

Running quark mass [15]:

\[
 m_d(2\text{GeV}) = 5.04^{+0.95}_{-1.54} \text{MeV}, \quad m_s(2\text{GeV}) = 105^{+25}_{-35} \text{MeV},
\]

\[
 m_d(m_u) = 4.23^{+1.74}_{-1.71} \text{MeV}, \quad m_b(m_b) = 4.20 \pm 0.07 \text{GeV},
\]

\[
 m_u(m_c) = 2.57^{+0.99}_{-0.84} \text{MeV}, \quad m_c(m_c) = 1.25 \pm 0.09 \text{GeV}.
\]
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