A problem facing world commercial aviation is a provision of the flight range and an increase in the fuel efficiency of transport category airplanes using fuel trim transfer application, which allows for decreasing airplane trim drag at cruise flight. In the existing mathematical models, center-of-gravity position is usually assumed fixed, but with fuel usage, center-of-gravity shifts within the definite range of center-of-gravity positions. Until the fuel trim transfer was not used in airplanes, the center-of-gravity shift range was rather short, that allowed to use the specified assumption without any considerable mistakes. In case of fuel trim transfer use, center-of-gravity shifts can reach 15…20% of mean aerodynamic chord, that requires considering the center-of-gravity actual position during the flight range calculation. Early made estimated calculations showed the necessity of following mathematical model improvement using accounting the real engine throttling characteristics. The goal of this publication is to develop a method of flight range calculation taking transport category airplane into account actual center-of-gravity position with fuel using and variation in engine-specific fuel consumption according to their throttling characteristics. On the basis of real data from engine maintenance manuals, formulas are obtained for approximation throttling characteristics of turbofan engines in the form of dimensionless specific fuel consumption (related to the specific fuel consumption at full thrust) dependence on the engine throttling coefficient. A mathematical model (algorithm and its program implementation using C language in Power Unit 11.7 R03 system) has been developed to calculate the airplane flight range accounting its actual center-of-gravity position shift with fuel usage and variation in specific fuel consumption according to engine throttling characteristics. Using comparison with known payload-range diagram, adequacy of developed mathematical model is shown. Recommendations to improve the mathematical model are also given.

**Keywords:** center-of-gravity; flight range; fuel system; aerodynamic performance; required thrust; specific fuel consumption.

**Introduction**

The main problem facing world commercial aviation and practically determining the competitiveness of transport category airplanes is operating costs decreasing, the main part of which is fuel expenses.

One of the ways to increase transport category airplanes’ fuel efficiency is fuel trim transfer application, which allows decreasing airplane trim drag at cruise flight [1, 2].

In the past during calculation of the airplane flight range, center-of-gravity (CG) position was usually assumed fixed, but with fuel usage, CG shifts within the definite range of CG positions. Until the fuel trim transfer was not used in airplanes, the CG shift range was rather short, which allowed using the specified assumption without any considerable mistakes.

In case of fuel trim transfer utilization, CG shift can reach 15…20% of mean aerodynamic chord (MAC), that requires to take into account the CG actual position during flight range calculation.

Early made estimated calculations showed the necessity of following mathematical model improvement using accounting the real engine throttling characteristics.

**The goal** of this publication is to develop the method of flight range calculation of transport category airplanes taking into account actual CG position with fuel use and variation in engine specific fuel consumption according to their throttling characteristics.

**1. Approximation of Turbofan Throttling Characteristics**

Turbofan throttling characteristics are given in engine maintenance manuals in various forms:

a) In the form of specific fuel consumption dependence on high-pressure compressor rpm (Д-30 [3, p. 34], ПС-90 [4, p. 113]);

b) In the form of hourly fuel consumption dependence on engine throttle settings (Д-436 [5, p. 66]);

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c) In the form of dimensionless hourly fuel consumption (ratio to hourly fuel consumption at full thrust) dependence on high-pressure compressor rpm (Д-18Т [6, p. 221]).

Approximation is performed as follows. Initially, coefficients $A_i$ and $B_i$ of the second order polynomials, which approximate dependencies from engine maintenance manuals, have been obtained:

$$P(p) = A_1 p^2 + A_2 p + A_3 = P_0 \cdot \xi_{th}^{*};$$  \hspace{1cm} (1)

$$C_p(p) = B_1 p^2 + B_2 p + B_3 = C_{p0} \cdot \overline{C}_p(\xi_{th}^{*}).$$  \hspace{1cm} (2)

where $p$ is the parameter, engine characteristics are shown in engine maintenance manuals as its function; $P(p)$ is the engine thrust as a function of the parameter $p$; $C_p(p)$ is the specific fuel consumption as a function of the parameter $p$; $P_0$ is the engine full thrust at $M = 0$, $H = 0$; $\xi_{th}^{*}$ is the engine throttling coefficient; $C_{p0}$ is the specific fuel consumption at full thrust and $M = 0$, $H = 0$; $\overline{C}_p(\xi_{th}^{*})$ is the dimensionless specific fuel consumption (ratio to $C_{p0}$) as a function of the engine throttling coefficient $\xi_{th}^{*}$.

Fig. 1. Engine throttling characteristics
Next, by solving the quadratic equation (1), the parameter \( p \) was expressed as a function of \( \xi_{th} \):

\[
p_{1,2}(\xi_{th}) = \frac{1}{2A_1} \left[ -A_2 \pm \sqrt{A_2^2 - 4A_1(A_3 - P_0\xi_{th})} \right]. (3)
\]

As when throttling coefficient \( \xi_{th} \) increases, the parameter \( p \), and therefore the thrust \( P \) should increase, the first root \( p_1(\xi_{th}) \), which corresponds to sign «plus», has been selected.

Finally, the obtained expression (3) for the parameter was substituted in the formula for specific fuel consumption (2). It allows to plot engine throttling characteristics in the form of dimensionless specific fuel consumption (2). It allows to plot engine throttling

characteristics in the form of dimensionless specific fuel consumption (related to the specific fuel consumption at full thrust) dependence on the engine throttling coefficient (Fig. 1):

\[
\bar{C}_p(\xi_{th}) = \frac{C_p[p_1(\xi_{th})]}{C_{p0}} = \frac{B_1[p_1(\xi_{th})]^2 + B_2[p_1(\xi_{th})] + B_3}{C_{p0}}. (4)
\]

The most reasonable dependence takes place in the ПС-90A engine, thus its parameters have been accepted for the following calculations.

2. Estimation of Engine Throttling Characteristics Influence on Airplane Flight Range

Fig. 2 shows dependence of the engine throttling coefficient, required for an airplane steady level flight, calculated as the required thrust \( (P_{req}) \) of all engines (that is an airplane aerodynamic drag \( -X_a \)) ratio to their available thrust \( (P_{av}) \) at the same mode:

\[
\xi_{th} = \frac{P_{req}(M, x_{CG}, m_{flight})}{P_{av}(M, H)} = \frac{X_a(M, x_{CG}, m_{flight})}{P_{av}(M, H)}. (5)
\]

Fig. 2 also shows the dimensionless specific fuel consumption \( \bar{C}_p \), which corresponds to obtained engine throttling coefficient, and a mean (within a flight) dimensionless specific fuel consumption \( \bar{C}_{p mean} \).

It can be seen in Fig. 2, that as engines have excessive thrust, values of their throttling coefficient are intolerably low, which leads to an increase of the dimensionless specific fuel consumption at their thrust decrease. One of the possible reasons for the effect is, that the engine's full thrust and its specific fuel consumption at the designing flight mode \( (M, H) \) were determined by old statistical formulas \[8\]. In case the real engine thrust drops strongly with Mach number increase, then the required throttling coefficient increases, but the dimensionless specific fuel consumption decreases.

Considering the variation of the specific fuel consumption (depending on the engine throttling concerning the specific fuel consumption at the same flight mode but with full thrust) results in flight range increase (in the example below) approximately by 6 \%.

![Fig. 2. Engine throttling coefficient and dimensionless specific fuel consumption vs. current flight mass](image-url)
3. Calculation of Airplane Center-of-Gravity, Lift-to-Drag Ratio, and Flight Range in PU System

To make the calculations, Power Unit (PU) system was improved. The improvements include:
1) possibility of each tank incomplete refueling;
2) accounting of the specific fuel consumption variation depending on the engine throttling coefficient.

As it is known [7], an airplane flight range in steady level flight is expressed by the formula

\[ L = 3.6 \int_{m_1}^{m_2} V \, dm \, C_p(\frac{\alpha}{S_{th}}), \]  

where V is the speed, m/s, \( P_{req} \) is the total required thrust of all engines installed in an airplane, N; \( C_p(\frac{\alpha}{S_{th}}) \) is the engine specific fuel consumption depending on the engine throttling coefficient, kg/N-hr, \( m_1 \) and \( m_2 \) are the masses of airplane at the beginning and at the end of the cruise flight.

In the first approximation, flight speed and altitude of airplane at cruise flight can be assumed constant and known. So except for specific fuel consumption, only the required thrust, which is equal to the airplane drag at the steady level flight, is unknown in formula (3).

The expression for the required engine thrust taking into account actual CG position was found in the publication [8]:

\[ P_{req} = 0.7 p_H M^2 S C_{x0} + \frac{(mg)^2}{0.7 p_H M^2 S} \times \]

\[ \times \left[ A_{WHT} \left( \frac{\Delta L + x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right)^2 \right] + \]

\[ + A_{HT} \frac{S}{S_{HT}} \left( \frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right)^2, \]  

where \( p_H \) is the atmospheric pressure at the flight altitude; M is the flight Mach number; \( S \) and \( S_{HT} \) are the areas of wing and horizontal tail (HT), correspondingly; \( C_{x0} \) is the airplane drag coefficient at zero lift; \( m \) is the airplane current mass; \( A_{WHT} \) and \( A_{HT} \) are the drag-to-lift factors of airplane without HT and separate HT; \( x_{CG} \) is the airplane CG position relatively wing MAC leading edge; \( x_{pWHT} \) is the center of pressure (CP) position of the airplane without HT relatively wing MAC leading edge; \( x_{pHT} \) is the CP position of separate HT relatively HT MAC leading edge; \( \Delta L \) is the distance between MAC leading edges of wing and HT.

CG coordinate is a known function of the current mass and pitch angle \( \upsilon \) [9] (which is equal to the airplane angle-of-attack \( \alpha \) at cruise flight)

\[ x_{CG} = f(m, \upsilon). \]  

Airplane flight performance \( C_{x0}, A_{WHT} \) and \( A_{HT} \) are determined by the airplane shape and its flight mode (altitude and Mach number), and therefore they do not depend on airplane CG variation.

CP position of the airplane without HT and separate HT are determined by the airplane shape, flight mode and the angle-of-attack:

\[ x_{pWHT} = b_a \left[ x_{FaWHT} + \frac{m_{z0WHT}}{C_{yaWHT}(\alpha - \alpha_{0WHT})} \right], \]  

\[ x_{pHT} = b_{aHT} \left[ x_{FaHT} + \frac{m_{z0HT}}{C_{yaHT}(\alpha - \alpha_{0HT})} \right], \]  

where \( b_a \), \( b_{aHT} \) are the MAC of wing and HT correspondingly, \( x_{FaWHT} \), \( x_{FaHT} \) are the coordinates of aerodynamic centers of the airplane without HT and separate HT correspondingly, \( m_{z0WHT} \), \( m_{z0HT} \) are the coefficients of pitching moment at zero lift of the airplane without HT and separate HT correspondingly, \( C_{yaWHT} \), \( C_{yaHT} \) are the derivatives of lift coefficient of the airplane without HT and separate HT correspondingly, \( \alpha_{0WHT}, \alpha_{0HT} \) are the zero lift angles-of-attack of the airplane without HT and separate HT correspondingly.

Thus, to calculate the flight range, it is necessary to find the airplane angle-of-attack under the current flight mass and CG position. Formulas for the required lift coefficient of the airplane without HT and separate HT correspondingly were found in publication [8]

\[ C_{yaWHT} = \frac{mg}{0.7 p_H M^2 S} \left[ \frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right], \]

\[ C_{yaHT} = \frac{mg}{0.7 p_H M^2 S_{HT}} \left[ \frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right]. \]

At the same time, the lift coefficient is related with the angle-of-attack by known formula

\[ C_{yaWHT} = C_{yaWHT}^\alpha (\alpha - \alpha_{0WHT}), \]

whence the cruise flight angle-of-attack can be determined

\[ \alpha = \alpha_{0WHT} + \frac{mg}{0.7 p_H M^2 S_{yaWHT}} \left[ \frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}} \right]. \]

To check the calculation correctness, the flight range can also be computed by known integral formula
\[ L_{\text{simp}} = 3.6 \frac{V}{C_P \text{mean}} \ln \left( \frac{m_1}{m_2} \right) \]  \hspace{1cm} (14)

where \( K_{\text{mean}} \) is the airplane mean lift-to-drag ratio, 
\( C_P \text{mean} \) is the mean specific fuel consumption, \( g \) is the gravity acceleration.

The algorithm of airplane flight range calculation accounting actual CG position and variation in specific fuel consumption according to engine throttling characteristics can be presented in the following form (Fig. 3).

4. Calculation Results

The considered algorithm is implemented by Ruslan U. Tsukanov in the calculation module of the Power Unit 11.7 R03 software. Flight range calculations of the An-188 airplane were made as an example.

To calculate the airplane flight performance, the methodology of Prof. V. I. Holiavko [10] was used as a basic one, which was updated to calculate wing with kinks at leading and trailing edges, and also with airfoil thickness ratio and its setting angle linearly varying spanwise.

So, the following aerodynamic performance was assumed to calculate the flight range of the considered airplane (Table 1). Information about the flight range and payload of the considered airplane is shown in Table 2.

Point 1 corresponds to a flight with maximum fuel load and without any payload. Point 2 matches to flight with maximum fuel load and maximum takeoff mass.

![Flight range calculation algorithm](image-url)
Table 1

| Parameter        | Value  | Parameter        | Value  |
|------------------|--------|------------------|--------|
| $C_{ya\ WHT}$, l/rad | 4.2828 | $C_{ya\ HT}$, l/rad | 3.9077 |
| $\alpha_{0\ WHT}$, rad | -0.0824 | $\alpha_{0\ HT}$, rad | -0.0098 |
| $A_{WHT}$ | 0.0386 | $A_{HT}$ | 0.0790 |
| $C_{x0}$ | 0.0432 | $\bar{x}_{Fa}$ | 0.9875 |
| $m_{z0\ WHT}$ | -0.1826 | $m_{z0\ HT}$ | -0.0589 |

Results of lift-to-drag ratio and flight range calculation

| Parameter                        | Point 1 | Point 2 | Point 3 |
|----------------------------------|---------|---------|---------|
| $m_{pl}$, kg                     | 20000   | 35000   | 47000   |
| $m_{r}$, kg                      | 42530   | 36600   | 24600   |
| $m_{0}$, kg                      | 135930  | 145000  | 145000  |
| ZFW, kg                          | 93400   | 108400  | 120400  |
| $K(ZFCG = 0.20)$                 | 17.77   | 17.94   | 17.97   |
| $K(ZFCG = 0.35)$                 | 17.90   | 18.12   | 18.18   |
| $\Delta K = K(0.35) - K(0.20)$  | 0.124   | 0.175   | 0.211   |
| $\Delta K = 100 \Delta K/K(0.20)$, % | 0.697   | 0.975   | 1.174   |
| $L_{188}$, km                    | 6300    | 5000    | 3000    |
| $L(ZFCG = 0.20)$, km             | 6716    | 5307    | 3411    |
| $L(ZFCG = 0.35)$, km             | 6758    | 5355    | 3448    |
| $L_{simp}(ZFCG = 0.20)$, km      | 6553    | 5174    | 3329    |
| $\Delta L_{simp} = 100(L(0.2) - L_{simp})/L(0.2)$, % | 2.26    | 2.31    | 2.23    |
| $\Delta L_{188} = L(0.20) - L_{188}$, km | 416.5   | 307.2   | 411.4   |
| $\Delta L_{188} = 100(L(0.2) - L_{188})/L_{188}$, % | 6.61    | 6.14    | 13.71   |
| $\Delta L = L(0.35) - L(0.20)$, km | 41.3    | 47.4    | 37.0    |
| $\Delta L = 100 \Delta L/L(0.20)$, % | 0.82    | 1.11    | 1.26    |

Point 3 corresponds to a flight with maximum payload and maximum takeoff mass. Results of calculation by the Power Unit 11.7 R03 software are shown in the bottom part of the table.

Center-of-gravity diagrams calculated in the Power Unit 11.7 R03 system for three various Zero Fuel Weight (ZFW) of the airplane for the extreme aft airplane Zero Fuel Center-of-Gravity (ZFCG = 0.35) are shown in Figs 4-6.

Table 2 allows to make the following conclusions.

1. Calculation increase of lift-to-drag ratio, caused by CG shift backwards from $x_{CG} = 0.20$ till $x_{CG} = 0.35$, makes 0.12 to 0.21 units or 0.69 to 1.17%.

2. Range values $L$ obtained by formula (6) differ from the values $L_{simp}$ obtained by simplified formula (14) less than by 2.5%, that indicates adequacy of formulas used in the algorithm.

3. Calculation flight range $L$ exceeds the real range $L_{188}$ by 307 to 416 km or 6.14 to 13.71% in accordance with the fact, that all the fuel is used in the calculation for the cruise flight (that is, fuel storage for take-off and landing is not accounted).

4. Calculation increase of the flight range, caused by CG shift backwards from $x_{CG} = 0.20$ till $x_{CG} = 0.35$, makes 37.0 to 47.4 km or 0.82 to 1.26%. Thus, taking into account the variation of specific fuel consumption, according to throttling characteristics, resulted in rise of relative flight range.
increase in comparison with the relative lift-to-drag ratio increase (as it takes place in reality).

Real and calculating payload-range diagrams for the considered airplane are shown in Fig. 7.

Fig. 4. Airplane CG diagram for $Z_{FW} = 93400$ kg: 1 – at minimal pitch angle without anti-surge ribs; 2 – at minimal pitch angle with anti-surge ribs; 3 – at cruise pitch angle, determined by formula (13); 4 – at mean cruise pitch angle ($2^\circ$); 5 – at maximum pitch angle

Fig. 5. Airplane CG diagram at $Z_{FW} = 108400$ kg
Fig. 6. Airplane CG diagram at $ZFW = 120400$ kg

Fig. 7. Range-payload diagrams of considered airplane:
1 – real; 2 – calculated
Conclusion

1. Mathematical model (algorithm and its program implementation using C language in Power Unit 11.7 R03 system) has been developed to calculate airplane flight range accounting its actual center-of-gravity position shift with fuel usage and variation of specific fuel consumption following throttling characteristics of engines installed in the airplane.

2. Adequacy of the developed mathematical model is shown using a comparison with the known payload-range diagram of the An-188 airplane.

3. For more effective utilization of fuel trim transfer process as for An-188, during the development of its modifications, it is necessary to change geometrical wrapping in plane view taking into account ellipticity factor, adjust its polar diagram, and change the fuel usage schedule in the new layout of wing fuel tanks. This solution implements to the full the fuel trim transfer process influence on the flight range increase, especially with full commercial payload. A mathematical model (algorithm and its program implementation using C language in Power Unit 11.7 system) has been developed for CG position numerical simulation of an airplane with a swept-back wing, which keeps fuel both in wings, and in fuselage tanks, in the process of fuel utilization at specified pitch angles, taking into account specified number and arrangement of ribs with baffle check valves, and also specified fuel burn schedule.

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Одною з проблем, що стоїть перед світовою цивільною авіацією, є забезпечення дальність та підвищення паливої ефективності літаків транспортної категорії шляхом використання балансувального перекачування палива, яке дає змогу знизити балансувальний опір літака на крейсерській ділянці польоту. В існуючих моделях положення центру мас зазвичай приймається фіксованим, однак у міру вироблення палива центр мас змінюється в деякому діапазоні поточних центрувань. Доти, доки на літаках не використовувалося балансуване перекачування палива, діапазон зміщення центру мас був досить вузьким, що давало змогу використовувати вказані припущення без істотних похибок. У разі використання балансувального перекачування палива зміщення центру мас може сягати 15…20 % середньої аеродинамічної хорди, що потребує врахування дійсного положення центру мас під час розрахунку дальності польоту. Раніше проведені оцінювальні розрахунки показали, необхідність подальшого уточнення математичної моделі шляхом врахування реальної дросельної характеристики двигунів. Цілю цієї роботи є розробка метода розрахунку дальності польоту літака транспортної категорії з урахуванням дійсного положення центру мас у міру вироблення палива та змінення питомої витрати палива двигунами згідно їх дросельної характеристики. На основі реальних даних з посібників з технічної експлуатації двигунів отримано формули для апроксимації дросельної характеристики двоконтурних двигунів у вигляді залежностей безрозмірної питомої витрати палива (віднесеної до питомої витрати палива при повній тязі) від коефіцієнта дроселювання двигуна. Розроблено математичну модель (алгоритм та його програмна реалізація на мові C у системі Power Unit 11.7 R03) для розрахунку дальності польоту літака з урахуванням дійсного положення його центру мас у міру вироблення палива і змінення питомої витрати палива згідно дросельної характеристики двигунів. Шляхом порівняння із відомої діаграмою «вантаж-дальність» показано адекватність розробленої математичної моделі. Також надано рекомендації щодо подальшого уточнення математичної моделі.

Ключові слова: центр мас, дальність польоту, паливна система, аеродинамічні характеристики, потрібна тяга; питома витрата палива.

РОСЧЁТ ДАЛЬНОСТИ ПОЛЁТА САМОЛЁТА ТРАНСПОРТНОЙ КАТЕГОРИИ
C УЧЁТОМ ИЗМЕНЕНИЯ ПОЛОЖЕНИЯ ЦЕНТРА МАСС
И ДРОСЕЛЬНЫХ ХАРАКТЕРИСТИК ДВИГАТЕЛЯ

Одной из проблем, стоящей перед мировой гражданской авиацией, является обеспечение дальности и повышение топливной эффективности самолётов транспортной категории путём применения балансировочной перекачки топлива, которая позволяет снизить балансировочное сопротивление самолёта на крейсерском участке полёта. В существующих моделях положение центра масс обычно принимается фиксированным, однако по мере выработки топлива центр масс смещается в некотором диапазоне полётных
центровок. До тех пор, пока на самолётах не применялась балансировочная перекачка топлива, диапазон смещения центра масс был достаточно узок, что позволяло пользоваться указанным допущением без существенных погрешностей. В случае применения балансировочной перекачки топлива смещение центра масс может достигать 15...20 % средней аэродинамической хорды, что требует учёта действительного положения центра масс при расчёте дальности полёта. Ранее проведённые оценочные расчёты показали, необходимость дальнейшего уточнения математической модели путём учёта реальной дроссельной характеристики двигателей. Целью данной работы является разработка метода расчёта дальности полёта самолётов транспортной категории с учётом действительного положения центра масс по мере выработки топлива и изменения удельного расхода топлива двигателями согласно их дроссельной характеристике. На основе реальных данных из руководств по технической эксплуатации двигателей получены формулы для аппроксимации дроссельной характеристики двухконтурных двигателей в виде зависимости безразмерного удельного расхода топлива (отнёсённого к удельному расходу топлива при полной тяге) от коэффициента дросселирования двигателя. Разработана математическая модель (алгоритм и его программная реализация на языке С в системе Power Unit 11.7 R03) для расчёта дальности полёта самолёта с учётом действительного положения его центра масс по мере выработки топлива и изменения удельного расхода топлива согласно дроссельной характеристике двигателей. Путём сравнения с известной диаграммой «груз-дальность» показана адекватность разработанной математической модели. Также даны рекомендации по дальнейшему уточнению математической модели.

**Ключевые слова:** центр масс, дальность полёта, топливная система, аэродинамические характеристики, потребная тяга; удельный расход топлива.

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