Relations of Pre Generalized Regular Weakly Locally Closed Sets in Topological Spaces

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Abstract: In this paper pgrw-locally closed set, pgrw-locally closed*-set and pgrw-locally closed**-set are introduced. A subset A of a topological space (X,τ) is called pgrw-locally closed (pgrw-lc) if A=GÇF where G is a pgrw-open set and F is a pgrw-closed set in (X,τ). A subset A of a topological space (X,τ) is a pgrw-lc* set if there exist a pgrw-open set G and a closed set F in X such that A=GÇF. A subset A of a topological space (X,τ) is a pgrw-lc**-set if there exists an open set G and a pgrw-closed set F such that A=GÇF.

The results regarding pgrw-locally closed sets, pgrw-locally closed* sets, pgrw-locally closed** sets, pgrw-lc-continuous maps and pgrw-lc-irresolute maps and some of the properties of these sets and their relation with other lc-sets are established.

Keywords: pgrw-lc, pgrw-lc*, pgrw-lc**-set, pgrw-sub-maximal space, pgrw-lc-continuous maps

1. Introduction

According to Bourbaki subset A of a topological space X is called locally closed in X if it is the intersection of an open set and a closed set in X. Gangster and Reilly used locally closed sets to define LC-Continuity and LC-irresoluteness. Balachandran, Sundaram and Maki introduced the concept of generalized locally closed sets in topological spaces and investigated some of their properties.

2. Preliminaries:

2.1 Definition: A subset A of a topological space (X, τ) is called

i. a semi-open set [4] if A ⊆ cl(int(A)) and semi-closed set if int(cl(A)) ⊆ A.

ii. a pre-open set [5] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.

iii. an α-open set [6] if A ⊆ int(cl(int(A))) and α-closed set if cl(int(cl(A))) ⊆ A.

iv. a semi-pre-open set (β-open) [7] if A⊆cl(int(cl(A))) and a semi-pre closed set (β-closed) if int(cl(int(A)))⊆A.

v. a regular open set [8] if A = int(cl(A)) and a regular closed set if A = cl(int(A)).

vi. δ-closed [9] if A = clδ(A), where clδ(A) = \{x ∈ cl(U) : U open in X and x ∈ U\}

vii. Regular semi open [10] set if there is a regular open set U such that U ⊆ A ⊆ cl(U).

viii. a regular generalized closed set(briefly rg-closed) [11] if cl(A)⊆U whenever A⊆U and U is regular open in X.

ix. a generalized semi-pre regular closed (gspr-closed) set [12] if spcl(A)⊆U whenever A⊆U and U is regular open in S.

x. a generalized semi-pre regular closed (gspr-closed) set [13] if spcl(A)⊆U whenever A⊆U and U is open in X.

xi. a pre generalized regular closed set [14] (pgpr-closed) if pcl(A)⊆U whenever A⊆U and U is open in X.

xii. a generalized pre closed (briefly gp-closed) set [3] if pcl(A)⊆U whenever A⊆U and U is open in X.

xiii. a regular w-closed set (rw-closed) [15] if cl(A)⊆U whenever A⊆U and U is regular semi-open in S.

xiv. a #regular generalized closed (briefly #rg-closed) set [16] if cl(A)⊆U whenever A⊆U and U is rw-open.
2.2 Definition: A subset A of a topological space \((X, \tau)\) is called a pre generalized regular weakly closed set if 
\[ pcl(A) \subseteq U \] whenever \( A \subseteq U \) and \( U \) is a rw-open set [17].

The complements of the abovementioned closed sets are their open sets respectively.

2.3 Definition: Let \((X, \tau)\) be a topological space and \(A \subseteq X\). The intersection of all closed (resp pre-closed, \(\alpha\)-closed and semi-pre-closed) subsets of space \(X\) containing \(A\) is called the closure (resp pre-closure, \(\alpha\)-closure and Semi-pre-closure) of \(A\) and denoted by \(cl(A)\) (resp \(pcl(A)\), \(acl(A)\), \(spcl(A)\)).

3. \(pgrw\)-locally closed sets

3.1 Definition: A subset \(A\) of a topological space \((X, \tau)\) is \(pgrw\)-locally closed (\(pgrw\)-lc) if 
\[ A = G \cap F \] where \(G\) is a \(pgrw\)-open set and \(F\) is a \(pgrw\)-closed set in \((X, \tau)\).

The set of all \(pgrw\)-locally closed subsets of \((X, \tau)\) is given by \(PGRWLC(X, \tau)\).

3.2 Example: \(X = \{1,2,3,4\}\) and \(\tau = \{X, \emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}\}\). 
\(rw\)-open sets are \(X, \emptyset, \{1\}, \{2\}, \{3\}, \{4\}\) \(pre\)-closed sets are \(X, \emptyset, \{3\}, \{4\}, \{3,4\}, \{2,3,4\}, \{1,3,4\}\). 
\(pgrw\)-closed sets are \(X, \emptyset, \{3\}, \{4\}, \{2,3\}, \{3,4\}, \{1,4\}, \{2,4\}, \{2,3,4\}, \{1,3,4\}, \{1,2,4\}\). 
The set \(\{2,3\} = \{1,2,3\} \cap \{2,3,4\}\) is a \(pgrw\)-lc set where \(\{1,2,3\}\) is \(pgrw\)-open and \(\{2,3,4\}\) is \(pgrw\)-closed.

3.3 Remark: In the space of 3.2 the set \(\{3\} = \{1,2,3\} \cap \{3,4\}\) is a \(pgrw\)-lc set where \(\{1,2,3\}\) is \(pgrw\)-open and \(\{3,4\}\) is \(pgrw\)-closed and also \(\{3\} = \{1,3\} \cap \{2,3,4\}\) where \(\{1,3\}\) is \(pgrw\)-open and \(\{2,3,4\}\) is \(pgrw\)-closed. Therefore \(G\) and \(F\) are not unique.

3.4 Theorem: subset \(A\) of \(X\) is \(pgrw\)-lc if and only if its complement \(A^c\) is the union of a \(pgrw\)-open set and a \(pgrw\)-closed set.

Proof: \(A\) is a \(pgrw\)-lc set in \((X, \tau)\). 
\[ A = G \cap F \] where \(G\) is a \(pgrw\)-open set and \(F\) is a \(pgrw\)-closed set.

\[ A^c = (G \cap F)^c = G^c \cup F^c \] where \(G^c\) is a \(pgrw\)-closed set and \(F^c\) is a \(pgrw\)-open set.

Conversely, \(A\) is a subset \((X, \tau)\) such that \(A^c = G \cup F\) where \(G\) is a \(pgrw\)-open set and \(F\) is a \(pgrw\)-closed set.

\[ (A^c)^c = (G \cup F)^c \]
\[ A = G^c \cap F^c = F^c \cap G^c \] where \(F^c\) is a \(pgrw\)-open set and \(G^c\) is a \(pgrw\)-closed set.

\(A\) is a \(pgrw\)-lc set.

3.5 Theorem:

i) Every \(pgrw\)-open set in \(X\) is \(pgrw\)-lc.

ii) Every \(pgrw\)-closed set in \(X\) is \(pgrw\)-lc

Proof: i) \(A\) is a \(pgrw\)-open set in \(X\).

\[ A = SA \] where \(A\) is \(pgrw\)-open and \(X\) is \(pgrw\)-closed.

\(A\) is \(pgrw\)-lc.

ii) \(A\) is a \(pgrw\)-closed subset of \(X\).

\[ A = X \cap A \] where \(X\) is \(pgrw\)-open and \(A\) is \(pgrw\)-closed.

\(A\) is \(pgrw\)-lc.

The converse statements are not true.

3.6 Example: In 3.2, the set \(\{2,4\} = X \cap \{2,4\}\) is \(pgrw\)-lc, but not \(pgrw\)-open. The set \(\{1,3\} = \{1,3\} \cap \{1,3,4\}\) is \(pgrw\)-lc, but not \(pgrw\)-closed.

3.7 Corollary: In \(X\)

Every open set is \(pgrw\)-lc.

i) every closed set is \(pgrw\)-lc.

Proof: i) \(A\) is open in \(X\).
⇒ Aispgrw-open in X.
⇒ A is pgrw-lc in X.

ii) A is closed in X.
⇒ A is pgrw-closed in X.
⇒ A is pgrw-lc in X.

The converse statements are not true.

3.8 Example: In 3.2, 2,4 is pgrw-lc, but not open and 1,3 is pgrw-lc, but not closed.

3.9 Theorem: Every locally closed set in X is pgrw-lc.

Proof: A is a locally closed subset of X.
⇒ A = G∩H, G is an open set and H is a closed set.
⇒ A = G∩H, G is pgrw-open and H is pgrw-closed.
⇒ A is pgrw-lc in X.

The converse statement is not true.

3.10 Example: In 3.2, the set 2, 4 is pgrw-lc, but not alc-set.

3.11 Theorem: In X
i) every locally-δ-closed set is pgrw-lc.
ii) every regular-locally closed set is pgrw-lc.
iii) every α-locally closed set is pgrw-lc.
iv) every #rg-locally closed set is pgrw-lc.
v) Every pgr-pr-locally-closed set is pgrw-lc.

Proof: i) A is a lδc-set in (X,τ).
⇒ A=G∩F, G is δ-open and F is δ-closed.
⇒ A=G∩F, G is pgrw-open and F is pgrw-closed in X.
⇒ A is a pgrw-lc set in (X,τ).

The other statements may be proved similarly.

The converse statements are not true.

3.12 Example: In 3.2, δ-closed sets in X are X, φ, [3,4],[2,3,4], {1,3,4}. The set {2,4} is pgrw-lc, but not lδc.

3.13 Example: In 3.2, regular-closed sets in X are X, φ, {2,3,4},{1,3,4}. The set {2,4} is pgrw-lc, but not regular-lc.

3.14 Example: In X = {1,2,3,4}, τ= {X,φ,[2,3],[1,2,3],[2,3,4]}, α-closed sets in X are X, φ, [1,4],[1],[4]. The set {1,3} = X∩{1,3} is pgrw-lc, but not α-lc.

3.15 Example: In 3.2 #rg-closed sets in X are X, φ, [4], {3,4},{[1,4],[2,4],[1,3],[2,3,4],[1,3,4]. The set {1,2,4} = X∩{1,2,4} is pgrw-lc, but not #rg-lc.

3.16 Example: In 3.2 pgr-pr-closed sets in X are X, φ, {3}, {4}, {3,4}, {1,3,4}, {2,3,4}. The set {1,2} = {1,2}∩X is pgrw-lc, but not pgr-pr-lc.

3.17 Theorem: In X every pgrw-locally closed set is
i) gp-lc ii) gpr-lc iii) gsp-lc iv) gspr-lc

Proof: i) A is a pgrw-lc set in X.
⇒ A=G∩H, G is pgrw-open and H is pgrw-closed.
⇒ A=G∩H, G is gp-open and H is gp-closed.
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3.18 Remark: The above results are shown in the following diagram

4. pgrw-locally closed*-sets

4.1 Definition: A subset \( A \) of a topological space \( (X, \tau) \) is a pgrw-\( \text{lc}^* \) set if there exist a pgrw-open set \( G \) and a closed set \( F \) in \( X \) such that \( A = G \cap F \).

The set of all pgrw-\( \text{lc}^* \) subsets of \( (X, \tau) \) is denoted by \( \text{PGRWLC}^*(X, \tau) \).

4.2 Example: Refer 3.2, \( \{2,3\} = \{1,2,3\} \cap \{b, c, d\} \) is pgrw-locally closed* set, because \( \{1,2,3\} \) is pgrw-open and \( \{2,3,4\} \) is closed.

4.3 Theorem: Every \( \text{lc} \)-set of \( X \) is a pgrw-\( \text{lc}^* \)-set .

Proof: \( A \) is alc-set in \( X \).

\[ \Rightarrow A = G \cap C, \text{ G is open and C is closed in X}. \]

\[ \Rightarrow A = G \cap C, \text{ G is pgrw-open and C is closed in X}. \]

\[ \Rightarrow A \text{ is a pgrw-\( \text{lc}^* \)-set in X}. \]

The converse statement is not true.

4.4 Example: \( X = \{1,2,3\}, \tau = \{X, \emptyset, \{1\}, \{2,3\}\} \).

pgrw-closed sets are all subsets of \( X \). The set \( \{1,2\} \) is pgrw-open and \( \{2,3\} \) is closed. Since \( \{2\} = \{1,2\} \cap \{2,3\} \) is a pgrw-\( \text{lc}^* \)-set, but not a \( \text{lc} \)-set.

4.5 Theorem: Every pgrw-\( \text{lc}^* \)-set of \( X \) is a pgrw-\( \text{lc} \) set.

Proof: \( A \) is a pgrw-\( \text{lc}^* \)-set in \( X \).

\[ \Rightarrow A = G \cap C \text{ where G is pgrw-open and C is closed in X}. \]

\[ \Rightarrow A = G \cap C \text{ where G is pgrw-open \& C is pgrw-closed in X}. \]

\[ \Rightarrow A \text{ is a pgrw-\( \text{lc} \)-set in X}. \]

4.6 Theorem: A subset \( A \) of \( X \) is pgrw-\( \text{lc}^* \) iff \( A = G \cap \text{cl}(A) \) for some pgrw-open set \( G \).

Proof: \( A \) is a pgrwlc*-set in \( X \).

\[ \Rightarrow A = G \cap F \text{ for a pgrw-open set G and a closed set F in X}. \]

\[ \Rightarrow A \subseteq G \text{ and A} \subseteq F, \text{ a closed set}. \]

\[ \Rightarrow A \subseteq G \cap \text{cl}(A) \text{ and cl}(A) \subseteq F \]

\[ \Rightarrow A \subseteq G \cap \text{cl}(A) \text{ and } G \cap \text{cl}(A) \subseteq G \cap F = A. \]

\[ \Rightarrow A = G \cap \text{cl}(A). \]
Conversely, \( A = G \cap \text{cl}(A) \) where \( G \) is a pgrw-open set.

\[ \Rightarrow A \text{ is the intersection of a pgrw-open set and a closed set.} \]

\[ \Rightarrow A \text{ is pgrw-le*}. \]

**4.7 Theorem:** If for a subset \( V \) of \( X \), \( VU(\text{cl}(V))^c \) is pgrw-open, then \( V \) is pgrw-le*.

**Proof:** \( \forall \) subset \( V \) of \( X \).

\[ V = VU\emptyset \]

\[ = VU((\text{cl}(V))^c \cap \text{cl}(V)) \]

\[ = (VU(\text{cl}(V))^c) \cap (VU\text{cl}(V)) \]

\[ = (VU(\text{cl}(V))^c) \cap \text{cl}(V), \text{ because } V \subseteq \text{cl}(V). \]

So if \( VU(\text{cl}(V))^c \) is pgrw-open, then \( V \) is the intersection of a pgrw-open set and a closed set. Therefore \( V \) is pgrw-le*.

**4.8 Corollary:** If for a subset \( V \) of \( X \) the set \( \text{cl}(V) - V \) is pgrw-closed, then \( A \) is pgrw-le*.

**Proof:** For any subset \( V \) of \( X \)

\[ \text{cl}(V) - V = \text{cl}(V) \cap V^c \]

Therefore \( \text{cl}(V) - V \) is pgrw-closed.

\[ \Rightarrow \text{cl}(V)^c \text{ is pgrw-open.} \]

\[ \Rightarrow V \text{ is pgrw-le*}. \]

**5. pgrw-locally closed**-sets

**5.1 Definition:** A subset \( A \) of \((X, \tau)\) is a pgrw-le**-set if there exists an open set \( G \) and a pgrw-closed set \( F \) such that \( A = G \cap F \).

The set of all pgrw-le**-sets of \((X, \tau)\) is denoted by \( \text{PGRWL}^{**}(X, \tau) \).

**5.2 Example:** Refer 3.2, \( \{1,2\} \cap \{2,3,4\} = \{2\} \) is pgrw-locally closed**-set, because \( \{1,2\} \) is open and \( \{2,3,4\} \) is pgrw-closed.

**5.3 Theorem:** Every lc-set of \( X \) is a pgrw-le**-set.

**Proof:** \( A \) is alc-set \( X \).

\[ \Rightarrow A = G \cap F \text{ where } G \text{ is open and } F \text{ is closed in } X. \]

\[ \Rightarrow A = G \cap F \text{ where } G \text{ is open and } F \text{ is pgrw-closed in } X. \]

\[ \Rightarrow A \text{ is a pgrw-le**-set in } X. \]

The converse statement is not true.

**5.4 Example:** \( X = \{1,2,3\}, \tau = \{X, \emptyset, \{1\}, \{2,3\}\} \).

pgrw-closed sets in \( X \) are all subsets of \( X \). The set \( \{3\} = \{2,3\} \cap \{3\} \) where \( \{2,3\} \) is open and \( \{3\} \) is pgrw-closed. So \( \{3\} \) is a pgrw-le**-set. But \( \{3\} \) is not alc-set.

**5.5 Theorem:** Every pgrw-le**-set in \( X \) is pgrw-le.

**Proof:** \( A \) is a pgrw-le**-set in \( X \).

\[ \Rightarrow A = G \cap F \text{ where } G \text{ is open and } F \text{ is pgrw-closed.} \]

\[ \Rightarrow A = G \cap F \text{ where } G \text{ is pgrw-open and } F \text{ is pgrw-closed.} \]

\[ \Rightarrow A \text{ is a pgrw-le-set.} \]

The converse statement is not true.

**5.6 Example:** \( X = \{1,2,3\}, \tau = \{X, \emptyset, \{1\}, \{1,3\}\} \)
pgrw-closed sets are $X, \phi, \{2\}, \{3\}, \{2,3\}, \{1,2\}$ is apgrw-lc set, but not pgrw-lc**.

5.7 Remark: The following diagram shows the relation between lc-set, pgrw-lc-set, pgrw-lc*-set and pgrw**-set.

5.8 Theorem:
i) If $A \in \text{PGRWL}^*(X, \tau)$ and $B$ is closed in $(X, \tau)$, then $A \cap B \in \text{PGRWL}^*(X, \tau)$.

Proof: i) $A \in \text{PGRWL}^*(X, \tau)$ and $B$ is closed in $X$.

$\Rightarrow A = P \cap F$ where $P$ is a pgrw-open set and $F$ is a closed set in $X$ and $B$ is closed.

$\Rightarrow A \cap B = (P \cap F) \cap B = P \cap (F \cap B)$, where $P$ is pgrw-open and $(F \cap B)$ is closed.

$\Rightarrow A \cap B \in \text{PGRWL}^*(X, \tau)$.

ii) $A \in \text{PGRWL}**(X, \tau)$ and $B$ is open in $X$.

$\Rightarrow A = P \cap F$ where $P$ is an open set and $F$ is a pgrw-closed set in $X$ and $B$ is open.

$\Rightarrow A \cap B = (P \cap F) \cap B = (P \cap B) \cap F$, where $(P \cap B)$ is open and $F$ is pgrw-closed.

$\Rightarrow A \cap B \in \text{PGRWL}**(X, \tau)$.

5.9 Theorem: If every pgrw-closed set is closed in $(X, \tau)$, then $\text{PGRWL} (X, \tau) = \text{LC} (X, \tau)$.

Proof: obvious.

6. pgrw-lc-continuous maps

6.1 Definition: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called pgrw-lc-continuous(pgrw-lc*-continuous, pgrw-lc**-continuous resp.) if $\forall V \in \sigma f^{-1}(V) \in \text{PGRWL}^*(X, \tau)$, $f^{-1}(V) \in \text{PGRWL}**(X, \tau)$ resp.)

6.2 Example: For $(X, \tau)$ refer 3.2, $Y = \{1, 2, 3, 4\}$ $\sigma = \{Y, \phi, \{1,2\}, \{3,4\}\}$. Define a map $f$ by $f(1) = 2$, $f(2) = 3$, $f(3) = 4$, $f(4) = 1$. Pre-images $X, \phi, \{1,4\}, \{2,3\}$ of $\sigma$-open sets belong to $\text{PGRWL}(X, \tau)$ ($\text{PGRWL}^*(X, \tau)$, $\text{PGRWL}**(X, \tau)$). So $f$ is a pgrw-lc continuous (pgrw-lc*-continuous, pgrw-lc**-continuous) map.

6.3 Theorem:
i) Every pgrw-lc*-continuous function is pgrw-lc-continuous.

ii) Every pgrw-lc**-continuous function is pgrw-lc-continuous.

Proof: i) A map $f$ is pgrw-lc*-continuous.

$\Rightarrow \forall V \in \sigma, f^{-1}(V) \in \text{PGRWL}^*(X, \tau)$.

$\Rightarrow \forall V \in \sigma, f^{-1}(V) \in \text{PGRWL} (X, \tau)$. 

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6.4 Theorem:
i) If \( f \) is alc-continuous function, then \( f \) is pgrw-lc-continuous (pgrw-lc*-continuous, pgrw-lc**-continuous).

ii) If \( f \) is lδc-continuous, then \( f \) is pgrw-lc-continuous.

iii) If \( f \) is regular-lc-continuous, then \( f \) is pgrw-lc-continuous.

iv) If \( f \) is #rg-lc-continuous, then \( f \) is pgrw-lc-continuous.

v) If \( f \) is \( \alpha \)-lc-continuous, then \( f \) is pgrw-lc-continuous.

Proof: i) A map \( f \) is lc-continuous.

\[ \forall V \in \sigma f^{-1}(V) \in L C(X, \tau) \]

\[ \Rightarrow \forall V \in \sigma f^{-1}(V) \in P G R W L C(X, \tau) \]

\[ \Rightarrow f \text{ is pgrw-lc-continuous.} \]

Similarly, the other statements may be proved.

The converse statements are not true.

6.5 Example: For \((X, \tau)\) refer 3.2, \(Y=\{1,2,3,4\}\), \(\sigma=\{X, \phi, \{1\}, \{3,4\}, \{1,3,4\}\} \). Define a map \( f \) by \( f(1)=2, f(2)=4, f(3)=1, f(4)=3 \). Pre-images \(X, \phi, \{3\}, \{2,4\}, \{2,3,4\}\) of \(\sigma\)-open sets are pgrw-lc in \(X\). So \( f \) is pgrw-lc continuous.

\(\delta\)-closed sets in \(X\) are \(X, \phi, \{3,4\}, \{2,3,4\}, \{1,3,4\}\).

Regular-closed sets in \(X\) are \(X, \phi, \{2,3,4\}, \{1,3,4\}\).

\(\alpha\)-closed sets in \(X\) are \(X, \phi, \{2\}, \{1,2\}, \{2,3,4\}\).

The set \(\{3,4\}\) is \(\sigma\)-open. \(f^{-1}(\{3,4\})=\{2,4\}\) is

i) not a lc-set. Therefore \( f \) is not lc-continuous.

ii) not a lδc-set. Therefore \( f \) is not lδc-continuous.

iii) not a regular-lc-set. Therefore \( f \) is not regular-lc-continuous.

iv) not a \(\alpha\)-lc-set. Therefore, \( f \) is not \(\alpha\)-lc-continuous.

6.6 Example: Consider the spaces in 6.5, #rg-closed sets in \(X\) are \(X, \phi, \{4\}, \{3,4\}, \{1,4\}, \{2,4\}, \{1,3,4\}\). Define a map \( f \) by \( f(1)=1, f(2)=3, f(3)=2, f(4)=4 \). Pre-images of \(\sigma\)-open sets are \(X, \phi, \{1\}, \{2,4\}, \{1,2,4\}\) which are pgrw-lc-sets. So \( f \) is pgrw-lc-continuous. But \(\{1,3,4\}\) is \(\sigma\)-open and \(f^{-1}(\{1,3,4\})=\{1,2,4\}\) is not #rg-lc set. Therefore \( f \) is not #rg-lc-continuous.

6.7 Theorem: If \( f \) is pgrw-lc-continuous, then it is

i) gp-lc-continuous. ii) gpr-lc-continuous. iii) gsp-lc-continuous iv) gspr-lc-continuous

Proof: i) A map \( f \) is pgrw-lc-continuous.

\[ \forall V \in \sigma f^{-1}(V) \in P G R W L C(X, \tau) \]

\[ \Rightarrow f \text{ is gp-lc-continuous.} \]

Similarly the other statements may be proved.

6.8 Theorem: If \( X \) is a door space, then every map \( i \) is

i. pgrw-lc-continuous.

ii. pgrw-lc*-continuous

iii. pgrw-lc**-continuous

Proof: i) \( X \) is a door space and \( f \) is a map.
\[ \forall A \in \sigma f^{-1}(A) \text{ is either open or closed in } X. \]
\[ \Rightarrow \forall A \in \sigma f^{-1}(A) \text{ is either pgrw-open or pgrw-closed in } X. \]
\[ \Rightarrow \forall A \in \sigma f^{-1}(A) = f^{-1}((A) \cap X \text{ where } f^{-1}(A) \text{ is pgrw-open and } X \text{ is pgrw-closed or } f^{-1}(A) = X \cap f^{-1}(A) \text{ where } X \text{ is pgrw-open and } f^{-1}(A) \text{ is pgrw-closed.} \]
\[ \Rightarrow \forall \text{Ac} f^{-1}(A) \text{ is a pgrw-lc set in } X. \]
\[ \Rightarrow f \text{ is pgrw-lc-continuous.} \]

Similarly the other statements may be proved.

6.9 Theorem: If X is pgrw-sub-maximal, then every function f is pgrw-lc*-continuous.

Proof: X is a pgrw-sub-maximal space.
\[ \Rightarrow \text{PGRWLC}^*(X, \tau) = P(X), \text{ the power set of } X. \]
\[ \Rightarrow \text{for any map } f^{-1}(V) \in \text{PGRWLC}^*(X, \tau) \forall V \subseteq Y. \]
\[ \Rightarrow f^{-1}(V) \in \text{PGRWLC}^*(X, \tau) \forall V \in \sigma. \]
\[ \Rightarrow f \text{ is pgrw-lc*-continuous.} \]

6.10 Corollary: If X is pgrw-sub-maximal, then every function f is pgrw-lc-continuous.

Proof: obvious.

6.11 Theorem: If f is a pgrw-lc-continuous (resp. pgrw-lc*-continuous, pgrw-lc**-continuous) map and g is a continuous map, then goi:(X,\tau) \rightarrow (Z,\eta) is pgrw-lc-continuous (resp. pgrw-lc*-continuous, pgrw-lc**-continuous).

Proof: g is continuous and f is pgrw-lc-continuous.
\[ \Rightarrow \forall \eta \text{-open set } U \subseteq Z \text{ such that } g^{-1}(U) \text{ is open in } (Y,\sigma) \text{ and } f^{-1}(g^{-1}(U)) \text{ is pgrw-lc in } X. \]
\[ \Rightarrow \forall \eta \text{-open set } U \subseteq Z (gof)^{-1}(U)) \text{ is pgrw-lc in } X. \]
\[ \Rightarrow \text{gof:(X,\tau) \rightarrow (Z,\eta) is pgrw-lc-continuous.} \]

Similarly the other statements may be proved.

6.12 Definition: A function g is sub-pgrw-lc*-continuous if there is a basis \( \beta \) for \((Y,\sigma)\) such that \( f^{-1}(U) \in \text{PGRWLC}^*(X,\tau) \forall U \in \beta. \)

6.13 Example: For \( (X,\tau) \) and pgrw-open sets in \( X \) refer 3.2.
\[ Y = \{1, 2, 3\}, \sigma = \{Y, \phi, \{1\}, \{2\}, \{1, 2\}\}; \beta = \{Y, \phi, \{1\}, \{2\}\} \text{ is a basis for } (Y, \sigma). \]
Define a function f by \( f(1) = 3, f(2) = 1, f(3) = 2, f(4) = 3. \)
Pre-images of elements of \( \beta \) are \( X, \phi, \{2\}, \{3\} \) and are pgrw-lc* sets. So f is sub-pgrw-lc*-continuous.

6.14 Theorem: If f is sub-lc-continuous, then it is sub-pgrw-lc*-continuous.

Proof: Follows from LC(X,\tau) \subseteq \text{PGRWLC}^*(X,\tau).

The converse statement is not true.

6.15 Example: For \( (X,\tau) \) refer 3.2, \( Y = \{1, 2, 3\}, \sigma = \{Y, \phi, \{1\}, \{2\}, \{1, 2\}\}; \beta = \{Y, \phi, \{1\}, \{2\}\} \text{ is a basis for } \sigma. \)
Define a function f:X \rightarrow Y by \( f(1) = 3, f(2) = 1, f(3) = 2, f(4) = 3. \)
Pre-images of elements of \( \beta \) are \( X, \phi, \{2\}, \{3\} \) and are pgrw-lc*-sets. So f is sub-pgrw-lc*-continuous. Then f is not sub-lc-continuous, because \( \{2\} \in \beta, f^{-1}(\{2\}) = \{3\} \) is not a lc-set in X.

6.16 Theorem: If f is pgrw-lc*-continuous, then it is sub-pgrw-lc*-continuous.

Proof: f is pgrw-lc*-continuous.
\[ \Rightarrow V \in \sigma f^{-1}(V) \in \text{PGRWLC}^*(X,\tau). \]
\[ \Rightarrow V \in \beta, \text{ a basis, } f^{-1}(V) \in \text{PGRWLC}^*(X,\tau), \text{ because } \beta \subseteq \sigma. \]
\[ \Rightarrow f \text{ is sub-pgrw-lc*-continuous.} \]
6.17 Theorem: If \( f \) is sub-pgrw-lc*-continuous, then there is a sub-basis \( S \) for \((Y,\sigma)\) such that \( f^{-1}(V) \in \text{PGRWL}C^*(X,\tau) \) \( \forall V \in S \).

Proof: If \( f \) is sub-pgrw-lc*-continuous, then there is a basis \( \beta \) for \((Y,\sigma)\) such that \( f^{-1}(U) \in \text{PGRWL}C^*(X,\tau) \) for each \( U \in \beta \). Since \( \beta \) is also a sub-basis for \((Y,\sigma)\) the proof is obvious.

6.18 Remark: The composition of a sub-pgrw-lc*-continuous function and a continuous function need not be a sub-pgrw-lc*-continuous.

Proof: Take a sub-pgrw-lc*-continuous function \( f \) which is not pgrw-lc*-continuous. Hence there is a set \( V \in \sigma \) such that \( f^{-1}(V) \notin \text{pgrw-lc}^*(X,\tau) \). Let \( \eta=[Y,\phi,V] \). Then \( \eta \) is a topology on \( Y \) and the identity function \( g \) is continuous. But the composition \( gof:(X,\tau) \rightarrow (Y,\eta) \) is not sub-pgrw-lc*-continuous.

7. pgrw-lc-irresolute maps

7.1 Definition: A map \( f:(X,\tau) \rightarrow (Y,\sigma) \) is called pgrw-lc irresolute if \( \forall \text{pgrw-lc-set} \ V \in Y. \ f^{-1}(V) \) is pgrw-lc in \( X \).

Similarly pgrw-lc*-irresolute and pgrw-lc*-irresolute functions are defined.

7.2 Example: \( X=\{1,2,3\}=Y, \tau=\{X,\phi,\{1\},\{1,3\}\}; \sigma=\{Y,\phi,\{1\},\{2,3\}\} \). pgrw-closed sets in \( X \) are \( X,\phi,\{2\},\{3\},\{2,3\} \). pgrw-closed sets in \( Y \) are all subsets of \( Y \). Define a map \( f:X \rightarrow Y \) by \( f(1)=2, f(2)=3, f(3)=1 \). \( f \) is pgrw-lc-irresolute.

7.3 Theorem: A map \( f \) is

i. pgrw-irresolute \( \Rightarrow \) \( f \) is pgrw-lc-irresolute.
ii. pgrw-lc-irresolute \( \Rightarrow \) \( f \) is pgrw-lc-continuous.
iii. pgrw-lc*-irresolute \( \Rightarrow \) \( f \) is pgrw-lc*-continuous.
iv. pgrw-lc*-irresolute \( \Rightarrow \) \( f \) is pgrw-lc*-continuous.

Proof: \( \forall \) map \( f \) and for sets \( U, F \in Y \),

\[ f^{-1}(U \cap F) = f^{-1}(U) \cap f^{-1}(F). \]

i) \( V \in \text{PGRW-LC}(Y,\sigma) \) and \( f \) is pgrw-irresolute.

\[ \Rightarrow V = U \cap F \] for a pgrw-open set \( U \) and a pgrw-closed set \( F \) and

\[ f^{-1}(V) = f^{-1}(U) \cap f^{-1}(F), f^{-1}(U) \] is pgrw-open and \( f^{-1}(U) \) is pgrw-closed in \( (X,\tau) \).

\[ \Rightarrow \forall V \in \text{PGRW-LC} (Y,\sigma), f^{-1}(V) \in \text{PGRW-LC}(X,\tau). \]

\( f \) is pgrw-lc-irresolute.

ii) \( V \in \sigma \) and \( f \) is pgrw-lc-irresolute.

\[ \Rightarrow V \in \text{PGRW-LC}(Y,\sigma) \]

\[ \Rightarrow f^{-1}(V) \in \text{PGRW-LC}(X,\tau). \]

Therefore \( f \) is pgrw-lc-continuous.

Similarly (iii) and (iv) follow.

7.4 Example: In 7.2, \( f \) is pgrw-lc-irresolute. As \( \{16\} \) is pgrw-closed in \( Y \) and \( f^{-1}(\{2\}) = \{1\} \) is not pgrw-closed in \( X \). So \( f \) is not pgrw-irresolute.

7.5 Theorem: If \( X \) is a door space, then every map \( f \) is pgrw-lc-irresolute.

Proof: \( X \) is a door space and \( f \) is a map.

\[ \Rightarrow f^{-1}(A) \] is either open or closed \( \forall A \in Y. \)

\[ \Rightarrow f^{-1}(A) \] is either pgrw-open or pgrw-closed \( \forall A \in Y. \)

\[ \Rightarrow f^{-1}(A) = f^{-1}(A) \cap X \] where \( f^{-1}(A) \) is pgrw-open and \( X \) is pgrw-closed or \( f^{-1}(A) = X \cap f^{-1}(A) \) where \( X \) is pgrw-open and \( f^{-1}(A) \) is pgrw-closed. Thus \( \forall A \in Y, f^{-1}(A) \) is pgrw-lc in \( (X,\tau) \) and so \( \forall V \in \text{PGRW-LC}(Y,\sigma), f^{-1}(V) \in \text{PGRW-LC}(X,\tau). \) \( \Rightarrow f \) is pgrw-lc-irresolute.

7.6 Theorem: \( f \) and \( g \) are two functions.

\( f \) and \( g \) are pgrw-lc-irresolute
⇒ gof is pgrw-lc-irresolute.

f is pgrw-lc-irresolute and g is pgrw-lc-continuous
⇒ gof: (X, τ) → (Z, η) is pgrw-lc-continuous.

**Proof:** i) The functions g and f are pgrw-lc-irresolute.

⇒ ∀V ∈ PGRW-LC(Z, η), g⁻¹(V) ∈ PGRW-LC(Y, σ) and

f⁻¹(g⁻¹(V)) ∈ PGRW-LC(X, τ).

⇒ ∀V ∈ PGRW-LC(Z, η), (gof)⁻¹(V) ∈ PGRW-LC(X, τ).

⇒ gof: (X, τ) → (Z, η) is pgrw-lc-irresolute.

ii) g is pgrw-lc-continuous and f is pgrw-lc-irresolute.

⇒ ∀V ∈ η, g⁻¹(V) ∈ PGRW-LC(Y, σ) and

f⁻¹((g⁻¹(V)) ∈ PGRW-LC(X, τ).

⇒ ∀V ∈ η, (gof)⁻¹(V) ∈ PGRW-LC(X, τ).

⇒ gof: (X, τ) → (Z, η) is pgrw-lc-continuous.

**7.7 Theorem:** f and g are two functions.

i) f and g are pgrw-lc*-irresolute
⇒ gof is pgrw-lc*-irresolute.

ii) f is pgrw-lc*-irresolute and g is pgrw-lc*-continuous
⇒ gof is pgrw-lc*-continuous.

**Proof:** Similar to 7.6.

**7.8 Theorem:** f and j are two functions.

i) f and g are pgrw-lc**-irresolute
⇒ gof is pgrw-lc**-irresolute.

ii) f is pgrw-lc**-irresolute and g is pgrw-lc**-continuous
⇒ gof is pgrw-lc**-continuous.

**Proof:** Similar to 7.6.

**References**

N. Bourkbaki, General Topology, Part I reading MA: Addison Wesley, 1996.

M. Ganster and I. L. Reilly, Locally closed sets and LC-continuous functions, Internal J. Math. and Math. Sci., 12, 417-424 1989.

Maki, I. H.J. Umehara and T. Noiri. Every topological space in Pre-T1/2, Mem. Fac. Sci, Kochi Univ. Ser. A. Maths. (17). 33-42. 1996.

Levine, N. Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70, 36-41, 1963.

Mashhour, A. S., M. E., Abd El-Monsef and El-Deeb S. N., On pre-continuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt, 1982; 53:47-53.

Njastad O., On some classes of nearly open sets, Pacific J. Math., 15(1965), 1961-1970.

Andrijevic, D., Semi-preopen sets, Mat. Vesnik., 38(1), 24-32, 1986.

Stone M., Application of the theory of Boolean rings to general topology, Trans. Amer. Math.Soc.1937; 41: 374-481.

J. K. Park, Locally δ closed sets and lδc-continuous functions, Indian J. Pure and Appl. Math.

Rajarubi P., Studies on regular semi-open sets intoptological spaces, Ph.D. Thesis, Annamalai University.

Palaniappan N and K. C. Rao, Regular generalized closed sets, Kyungpook Math. J. 33, 211-219, 1993.

GovindappaNavelga, A. S. Chandrashekharappa and S.V. Gurushantanavar, gspr-open functions in topological spaces. Int.J. of Mathematical Sciences & Applications, 2(1), 202, 231-237.

Sarsak M. Sand N. Rajesh, Generalized semi pre closed sets, Int. Math Forum 5(12), 573-578, 2010.

Anitha, M and P. Thangavelu, Locally And Weakly pgpr-Closed Sets, International Journal OfPure And Applied Mathematics,87(6), 2013, 757-762.
Wali R.S., Thesis: Some topics in general and fuzzy topological spaces, Karnataka University Dharwad.
Syed Ali Fathima and Mariasingam, On \#rg-closed sets in topological spaces, International journal of mathematical archive- 2(11), 2497 – 2502, 2011.
WaliR.S.andVijayakumari T. Chilakwad, On Pre Generalized Regular Weakly [pgrw]-Closed sets in a Topological Space, International Journal of Mathematical Archive, 76-85, 6, 2229 – 5046, 2015..