Research Article

Degree-Based Topological Properties of Molecular Polymeric Networks Composed by Sierpinski Networks

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1. Introduction

Graph theory is concerned with network of points connected by lines. The beginning of graph theory deals with recreational math problems, but now it has grown into significant research area. Recently, it acquires sufficient attention due to its use in computer science, circuit boards, interrelated systems, living systems, sociology, and so on. Chemical graph theory (CGT) plays a very vital role in the composition and arrangement of substances. Chemical compounds are usually supposed to be the chemical networks, consequently whose compound’s atoms represent vertices, and bonds represent edges of a graph. Scientists are very attentive to know about the topology of chemical networks through some mathematical parameters gained from the network graphs of chemical molecules. A large scale of topological indices has been examined in biochemistry through QSPECTS analysis to analyze chemical networks topology related to pharmacy, medical engineering, and experimental science, which are important fields of CGT [1]. A quick study of QSPECTS is one of the main reasons to apply graph theory in chemistry [2, 3].

The required technique for the study of some graphical structures is to break down the structures into substructures containing significant properties and also some associated structures that are supposed to have similar properties. We need to know how the network-related substructures are similar to each other in these circumstances. For example, to make polymeric networks, Sierpinski graphs are used. A class of polymeric materials which are directly or indirectly linked together as polymeric chains are polymeric networks. In mathematics, fractal theory and other areas of science, the Sierpinski, and these types of different graphs are used. In the recent decade, fractal nature-based graphs such as Sierpinski and related graphs were studied with applications in topology and computer science. Nowadays, the modern communication systems like radio and mobile radiations as far as pattern-based log-periodic behaviour have been studied by fractal nature of Sierpinski networks. The family of Sierpinski graphs has some unique properties and was discussed in the literature [4, 5].

Topological indices are numerical numbers associated with the networks that can be helpful to predict some of its
properties. Topological indices are mostly used for QSAR and QSPR studies. QSAR/QSPR studies have a lot of use in biochemistry and modern chemistry. It is important to select an appropriate descriptor to get a good correlation. There are many descriptors that can predict the simple properties of a molecular structure and give an insight into the properties of the structure under consideration [6]. Hence, we need to employ such molecular descriptors that can help to characterize the molecular properties and influence its activities [4, 7–9]. Harold Wiener introduces the concept of path number of a chemical graph and later calls it Wiener index. After that many topological indices were introduced. This theory became the central topic of research. The most studied topological indices among these are the degree-based having many applications in chemical graph theory.

Let $\mathcal{H}$ be a simple connected graph with its edge set and vertex set denoted by $E$ and $V$, respectively. The order of $\mathcal{H}$ is the cardinality of set $V$. Similarly, the size of $\mathcal{H}$ is the cardinality of set $E$. Let $u_1 \in V$, then $N(u_1)$ denotes the set containing neighbors of $u_1$ and is called open neighborhood of $u_1$. The closed neighborhood of a vertex $u_1$ is denoted by $N[u_1]$ and defined as $N[u_1] = \{u_1\} \cup N(u_1)$. We denote the degree of a vertex $u_1$ by $\Lambda(u_1)$ and count of elements in $N(u_1)$. For basic concepts related to graph theory, we refer the readers to the book by West [10].

First degree-based topological index was proposed by Randić [5] in 1975 and named as “branching index.” It was later called as Randic connectivity index $R_{(-1/2)}(\mathcal{H})$. It was found to be useful to determine the degree of branching of saturated hydrocarbons in their carbon-atom skeleton. This index was later generalized by Bollobás and Erdős [11] in 1988 and they called it general Randić index. It was observed by Randić that a good correlation exists between various physical/chemical properties of alkanes and the Randic index $R_e$. The mathematical formulas for Randić index and general Randić index are

$$R_{(-1/2)}(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} \frac{1}{\Lambda(u_1)\Lambda(v_1)}.$$  \hfill (1)

$$R_e(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} (\Lambda(u_1)\Lambda(v_1))^e.$$  \hfill (2)

Gutman introduces the first and second Zagreb indices in [12]. These indices were first applied to the branching problem [13]. The mathematical formulas for Zagreb indices are

$$M_1(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} \Lambda(u_1) + \Lambda(v_1),$$

$$M_2(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} (\Lambda(u_1) \times \Lambda(v_1)).$$  \hfill (3)

The sum-connectivity index, denoted by SCI, was proposed by Zhou et al. [14] and has mathematical formula as

$$SCI(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} \frac{1}{\Lambda(u_1) + \Lambda(v_1)}.$$  \hfill (4)

Zhong [15] introduced the harmonic index as follows:

$$H(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} \left( \frac{2}{\Lambda(u_1) + \Lambda(v_1)} \right).$$  \hfill (5)

Estrada et al. [16] introduced the atom-bond connectivity (ABC) index which is formulated as

$$ABC(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} \sqrt{\frac{\Lambda(u_1) + \Lambda(v_1) - 2}{\Lambda(u_1)\Lambda(v_1)}}.$$  \hfill (6)

This index gives a good stability model for cycloalkanes and linear and branched alkanes strain energy [16, 17].

Geometric-arithmetic index (GA) was put forward by Vukičević et al. in [18] and is defined as

$$GA(\mathcal{H}) = \sum_{u_1, v_1 \in E(\mathcal{H})} \frac{2\Lambda(u_1)\Lambda(v_1)}{\Lambda(u_1) + \Lambda(v_1)}.$$  \hfill (7)

Using octane isomers, it was proved that GA index shows good correlation with different physical/chemical properties.

Recently, two new degree definitions, namely, ve-degree and ev-degree, were proposed by Chellali et al. [19]. Their mathematical properties were discussed in more detail later by Horoldagva et al. [20]. We denote the ev-degree of an edge $u_1v_1 = e_1 \in E(\mathcal{H})$ by $\Lambda_{ev}(e_1)$ and is defined as the cardinality of the set $N[u_1] \cup N[v_1]$. Similarly, for any vertex $v_1$, its ve-degree is denoted by $\Lambda_{ve}(v_1)$ and is equal to the total number of edges that are incident to the vertices in the set $N[v_1]$. The classical degree-based topological indices definitions were converted into ev- and ve-degree. The Zagreb ($M^v$) index and Randic ($R^v$) index based on ev-degree for any edge $e_1 = u_1v_1 \in E(\mathcal{H})$ are defined as

$$M^v(\mathcal{H}) = \sum_{e_1 \in E(\mathcal{H})} \Lambda_{ev}(e_1)^2,$$

$$R^v(\mathcal{H}) = \sum_{e_1 \in E(\mathcal{H})} \Lambda_{ev}(e_1)^{-(1/2)}.$$  \hfill (8)

The first Zagreb alpha ($M^v_{1}$) index of a vertex $v_1 \in V(\mathcal{H})$ based on ve-degree is defined as

$$M^v_{1}(\mathcal{H}) = \sum_{v_1 \in V(\mathcal{H})} \Lambda_{ve}(v_1)^2.$$  \hfill (9)

Similarly, the ve-degree type first Zagreb beta ($M^v_{2}$) index, geometric-arithmetic ($G^v$) index, atom-bond connectivity ($ABC^v$) index, second Zagreb ($M^v_{2}$) index, sum-connectivity ($\chi^v$) index, Randic ($R^v$) index, and harmonic ($H^v$) index for each edge $u_1v_1 \in E(\mathcal{H})$ are defined as
A lot of research is going on these topological indices. The results related to these topological indices can be found in [21–25]. The objective of this paper is to calculate exact values of topological indices for Sierpinski network \( S(n,m) \) based on ve- and ev-degree.

2. Methodology

For the calculation and computation of topological indexes such as \( M^v \), \( R^v \), \( M^e \), \( M_1^e \), \( ABC^e \), \( GA^e \), \( H^v \), \( \chi^v \), and \( R^v \) indexes for polymeric molecular network composed by Sierpinski networks \( S(n,m) \), we compute the ev- and ve-degree of each edge and vertex, respectively. Moreover, for mathematical estimation, calculations, and confirmation, MATLAB software was used. To plot these mathematical results, we used the software maple.

3. Main Results

This section deals with the computation of ve- and ev-degree-based topological indices for Sierpinski network \( S(n,m) \). Let \( m,n \geq 1 \), then the Sierpinski network denoted by \( S(n,m) \) is a graph on vertex set \( V(S(n,m)) = \{1, 2, \ldots, m\}^n \). Any vertex \( u \in V(S(n,m)) \) has the representation \( u = (u_1, u_2, \ldots, u_n) \), where \( u_i \in \{1, 2, \ldots, m\} \) and \( i = 1, 2, \ldots, n \). Two vertices \( u, v \in V(S(n,m)) \) are connected by an edge if and only if there exist and \( p \in 1, 2, \ldots, n \) such that

(i) \( u_q = v_q \) for \( q = 1, 2, \ldots, p - 1 \).
(ii) \( u_p = v_p \), and
(iii) \( u_q = v_p \) and \( u_p = v_q \) for \( p = q + 1, \ldots, n \).

Sierpinski networks \( S(n,m) \) definition comes from the study of Lipscomb’s space. For \( n = 3 \) in the definition of \( S(n,m) \), we get the tower of Hanoi graph. The graphs of \( S(2,5) \) and \( S(3,5) \) are depicted in Figure 1. It is easy to compute that the number of vertices and number of edges of \( S(n,m) \) are \( m^n \) and \( m^{n+1} - m/2 \), respectively.

**Theorem 1.** Let \( \mathcal{H} \) be a graph of Sierpinski network \( S(n,m) \), then

\[
R^v(\mathcal{H}) = \frac{1}{2} \left(7m^{n+2} - 4m^n - 7m^3 - 2m^2 + 6m\right),
\]

\[
R^e(\mathcal{H}) = \frac{m^2 - m}{\sqrt{m + 1}} + \frac{m^n - m^{n+1} - m^n - 2m^2 + 2m}{2\sqrt{m + 2}}.
\]

**Proof.** Let \( E_{(p,q)} \) denote the set of edges of \( \mathcal{H} \) with end vertices of degree \( p \) and \( q \), respectively. Then, \( E(\mathcal{H}) \) can be partitioned into two sets \( E_{(m-1,m)}(\mathcal{H}) \) and \( E_{(m,m)}(\mathcal{H}) \). The ev-degree of each edge in the partition sets is presented in Table 1.

From Table 1, the ev-degree-based Randic and Zagreb indices can be computed as...
\[M^v(\mathcal{H}) = \sum_{e \in E(\mathcal{H})} \Lambda_{ev}(e_1)^2,\]

\[R^v(\mathcal{H}) = \sum_{e \in E(\mathcal{H})} \Lambda_{ev}(e_1)^{-1/2},\]

\[M^v(\mathcal{H}) = (m + 1)^2|E_{(m,m-1)}| + (2m)^2|E_{(m,m)}| + (m + 2)^2|E_{(m,m)}|\]
\[= (m + 1)^2(m^2 - m) + 2(m)^2 \left( \frac{m^n - m}{2} \right) + (m + 2)^2 \left( \frac{m^{n+1} - m^{n} - 2m^2 + 2m}{2} \right)\]
\[= \frac{1}{2} \left( 7m^{n+2} - 4m^n - 7m^3 - 2m^2 + 6m \right),\]

\[R^v(\mathcal{H}) = (m + 1)^{-1/2}|E_{(m,m-1)}| + (2m)^{-1/2}|E_{(m,m)}| + (m + 2)^{-1/2}|E_{(m,m)}|\]
\[= (m + 1)^{-1/2} \left( m^2 - m \right) + \frac{(2m)^{-1/2} \left( m^n - m \right)}{2}\]
\[+ \frac{(m + 2)^{-1/2} \left( m^{n+1} - m^n - 2m^2 + 2m \right)}{2}\]
\[= \frac{m^2 - m}{\sqrt{m + 1}} + \frac{m^n - m}{2\sqrt{2m}} + \frac{m^{n+1} - m^n - 2m^2 + 2m}{2\sqrt{m + 2}}.\]

**Theorem 2.** Let \( \mathcal{H} \) be a graph of Sierpinski network \( S(n, m) \), then

\[M^{\text{ev}}(\mathcal{H}) = \frac{1}{4} (m^{n+4} + 6m^{n+3} + 5m^{n+2} - 12m^{n+1} + 4m^n - 8m^4 - 16m^3 + 32m^2 - 12m).\]  

**Proof.** Let \( v_1 \in V(\mathcal{H}) \), then the degree of \( v_1 \) is either \( m - 1 \) or \( m \). The ve-degree of a vertex is either \( m^2 + m - 2/2 \) or \( m^2 + 3m - 4/2 \). These data are presented in Table 2. Using Table 2, the value of \( M^{\text{ev}}(\mathcal{H}) \) can be calculated as \( \square \).
Theorem 3. Let $\mathcal{H}$ be a molecular graph of Sierpinski network $S(n,m)$, then

\[ H_{1}^{\text{ve}}(\mathcal{H}) = \frac{1}{4} \left( 2m^{n+3} + 6m^{n+2} - 4m^{n+1} - 10m^{3} + 2m^{2} + 4m \right), \]

\[ H_{2}^{\text{ve}}(\mathcal{H}) = \frac{1}{8} \left( m^{n+5} + 6m^{n+4} + 5m^{n+3} - 12m^{n+2} + 4m^{n+1} - 9m^{5} - 22m^{4} + 47m^{3} - 24m^{2} + 4m \right), \]

\[ \text{ABC}^{\text{ve}}(\mathcal{H}) = 2(m^{2} - m) \left( \sqrt{\frac{m^{2} + 2m - 5}{m^{4} + 4m^{3} - 3m^{2} - 10m + 8}} + \sqrt{\frac{m^{2} + 3m - 5}{m^{4} + 6m^{3} + 3m^{2} - 18m}} \right) \]

\[ + \left( m^{3} - 3m^{2} + 2m \right) \sqrt{\frac{m^{2} + 3m - 6}{m^{4} + 6m^{3} + m^{2} - 24m + 16} + \frac{1}{m^{4} + 5m^{3} + 6m^{2} - 12m + 4}} \]

\[ \text{GA}^{\text{ve}}(\mathcal{H}) = \left( m^{2} - m \right) \sqrt{\frac{m^{4} + 4m^{3} - 3m^{2} - 10m + 8}{m^{2} + 2m - 3} + \frac{m^{4} + 6m^{3} + 3m^{2} - 18m}{m^{2} + 3m - 3} \right) \]

\[ + \frac{1}{2} \left( m^{3} - 3m^{2} + 2m \right) \sqrt{\frac{m^{4} + 4m^{3} - 3m^{2} - 10m + 8}{m^{2} + 2m - 3} + \frac{m^{4} + 6m^{3} + 3m^{2} - 18m}{m^{2} + 3m - 3} \right) \]

\[ H^{\text{ve}}(\mathcal{H}) = \frac{2m}{m + 3} + \frac{2(m^{2} - m)}{m^{2} + 3m - 3} + \frac{m^{2} - 2m}{m + 4} + \frac{m(m^{n} - m^{2} - m + 1)}{m^{2} + 3m - 2}, \]

\[ \chi^{\text{ve}}(\mathcal{H}) = \left( m^{2} - m \right) \left( \sqrt{\frac{1}{m^{2} + 2m - 3} + \frac{1}{m^{2} + 3m - 3}} \right) + \frac{1}{2} \left( m^{3} - 3m^{2} + 2m \right) \sqrt{\frac{m^{4} + 4m^{3} - 3m^{2} - 10m + 8}{m^{2} + 3m - 3} + \frac{m^{4} + 6m^{3} + 3m^{2} - 18m + 8}{m^{2} + 3m - 3} \right). \]

\[ R^{\text{ve}}(\mathcal{H}) = 2(m^{2} - m) \sqrt{\frac{1}{m^{4} + 4m^{3} - 3m^{2} - 10m + 8} + \frac{1}{m^{4} + 6m^{3} + 3m^{2} - 18m + 8}} \]

\[ + \frac{m^{3} - 3m^{2} + 2m}{\sqrt{m^{4} + 6m^{3} + m^{2} - 24m + 16} + \frac{m^{n+1} - m^{n} - m^{2} + m}{\sqrt{m^{4} + 5m^{3} + 6m^{2} - 12m + 4}}}, \]

(14)

Proof. Based on the degree of end vertices of each edge, we can partition $E(\mathcal{H})$ into two sets $E_{(m-1,m)}$ and $E_{(m,m)}$. Now, depending on the ve-degree of end vertex of each edge, we can further partition the sets $E_{(m-1,m)}$ and $E_{(m,m)}$ into four sets. Table 3 shows the details of this partition. \qed

| $\lambda(u_{i})$ | $\lambda_{w}(u_{i})$ | Frequency |
|-----------------|---------------------|----------|
| $m-1$           | $m^{2} + m - 2/2$   | $m$      |
| $m$             | $m^{2} + 3m - 4/2$  | $m^{2} - m$ |
| $m$             | $m^{2} + 3m - 2/2$  | $m^{n} - m^{2}$ |

Table 2: ve-degree of each vertex of $\mathcal{H}$. |
Table 3: ev-degree of each edge of $\mathcal{H}$.

| Edge         | $(\Lambda_{ve}(u_i), \Lambda_{ve}(v_1))$ | Frequency |
|--------------|------------------------------------------|-----------|
| $E^*_1(m-1,m)$ | $(m^2 + m - 2, m^2 + 3m - 4/2)$          | $m^2 - m$ |
| $E^*_2(m,m)$  | $((m^2 + 3m - 4)/2, (m^2 + 3m - 2)/2)$  | $m^2 - m$ |
| $E^*_3(m,m)$  | $((m^2 + 3m - 4)/2, (m^2 + 3m - 4)/2)$  | $1/2(m^4 - 3m^2 + 2m)$ |
| $E^*_4(m,m)$  | $((m^2 + 3m - 2)/2, (m^2 + 3m - 2)/2)$  | $1/2(m^2 + 3m^2 + m)$ |

Using Table 3, we can compute the indices as

$$M^{\text{free}}_1(\mathcal{H}) = \sum_{u_i, v \in E(\mathcal{H})} (\Lambda_{ve}(u_i) + \Lambda_{ve}(v_1)),$$

$$M^{\text{free}}_1(\mathcal{H}) = \left( \frac{m^2 + m - 2}{2} + \frac{m^2 + 3m - 4}{2} \right) E^*_1(m-1,m) + \left( \frac{m^2 + 3m - 4}{2} + \frac{m^2 + 3m - 2}{2} \right) E^*_2(m-1,m)$$

$$+ \left( \frac{m^2 + 3m - 2}{2} + \frac{m^2 + 3m - 2}{2} \right) E^*_4(m-1,m)$$

$$= (m^2 + 2m - 3)(m^2 - m) + (m^2 + 3m - 3)(m^2 - m) + \frac{1}{2}(m^2 + 3m - 4)(m^2 - 3m^2 + 2m)$$

$$+ \frac{1}{2}(m^2 + 3m - 2)(m^4 + m^3 - m^2 + m)$$

$$= \frac{1}{4}(2m^{n+3} + 6m^{n+2} - 4m^{n+1} - 10m^3 + 2m^2 + 4m).$$

$$M^{\text{free}}_2(\mathcal{H}) = \sum_{u_i, v \in E(\mathcal{H})} (\Lambda_{ve}(u_1) \Lambda_{ve}(v_1)),$$

$$M^{\text{free}}_2(\mathcal{H}) = \left( \frac{m^2 + m - 2}{2} \right) \left( \frac{m^2 + 3m - 4}{2} \right) E^*_1(m-1,m) + \left( \frac{m^2 + 3m - 4}{2} \right) \left( \frac{m^2 + 3m - 2}{2} \right) E^*_2(m-1,m)$$

$$+ \left( \frac{m^2 + 3m - 2}{2} \right) \left( \frac{m^2 + 3m - 2}{2} \right) E^*_4(m-1,m)$$

$$= \frac{1}{4}(m^4 + 4m^3 - 3m^2 - 10m + 8)(m^2 - m) + \frac{1}{4}(m^2 - m)(m^4 + 6m^3 + 3m^2 - 18m + 8)$$

$$+ \frac{1}{8}(m^4 + 6m^3 + m^2 - 24m + 16)(m^3 - 3m^2 + 2m)$$

$$+ \frac{1}{8}(m^4 + 6m^3 + 5m^2 - 12m + 4)(m^{n+1} - m^3 - m^2 + m)$$

$$= \frac{1}{8}(m^{n+5} + 6m^{n+4} + 5m^{n+3} - 12m^{n+2} + 4m^{n+1} - 9m^n - 22m^4 + 47m^3 - 24m^2 + 4m).$$

### 4. Numerical Discussions and Results

In this portion, we have represented the numerical outcomes of the above computed topological descriptors related to the ev-degrees and ve-degrees for molecular polymeric networks composed by Sierpinski network. We have computed numerical tables/results by using different values of $n$ and $k$ (see Tables 4, 5, and 6). Moreover, we have drawn the graphical results (see Figures 2–7) to test and review the behaviour of topological descriptors computed above. It is clear from Tables 4, 5, and 6 and Figures 2–Figure 7 that, as there is increase in $n$ and $k$, the computed topological descriptors are increasing.
Table 4: Numerical table related to ev-degree-based indices.

| [n,k]  | [1,1] | [2,2] | [3,3] | [4,4] | [5,5] | [6,6] |
|--------|-------|-------|-------|-------|-------|-------|
| M^{ev} | 0     | 22    | 702   | 13596 | 2.6674e+05 | 5.7846e+06 |
| R^{ev} | 0     | 1.6547 | 17.29 | 201.78 | 2856.2 | 14282 |

Table 5: Numerical table related to ve-degree index.

| [n,m]  | [1,1] | [2,2] | [3,3] | [4,4] | [5,5] | [6,6] | [7,7] |
|--------|-------|-------|-------|-------|-------|-------|-------|
| M_{i}^{ve} | 0 | 26 | 1521 | 42612 | 1126560 | 31536270 | 952009905 |

Table 6: Numerical table related to end vertices ve-degrees indices.

| [n,m]  | [1,1] | [2,2] | [3,3] | [4,4] | [5,5] | [6,6] |
|--------|-------|-------|-------|-------|-------|-------|
| M_{i}^{ve} | 0 | 16 | 588 | 13164 | 296580 | 7277820 |
| M_{v}^{ve} | 0 | 20 | 2229 | 85002 | 2.8157e+06 | 9.4607e+07 |
| ABC^{ve} | 0 | 0.99654 | 5.4946 | 39.799 | 513.58 | 8978.6 |
| G^{ve} | 1403774.74 | 1403778.72 | 1403786.68 | 1403798.63 | 1403814.55 | 1403834.46 |
| H^{ve} | 0.3 | 1.1214 | 5.2286 | 39.564 | 411.37 | 5383.6 |
| \chi^{ve} | 14917 | 14917 | 14928 | 14976 | 15097 | 15342 |
| R^{ve} | 0 | 1.1438 | 5.2445 | 39.577 | 411.38 | 5383.6 |

Figure 2: (a) The graph M^{ev}(\mathcal{G}). (b) The graph of R^{ev}(\mathcal{G}).
ABC(v) = \sum_{u_i, v_i \in E(\mathcal{G})} \sqrt{\frac{\Lambda_{ve}(u_i) + \Lambda_{ve}(v_i) - 2}{\Lambda_{ve}(u_i)\Lambda_{ve}(v_i)}}

\begin{align*}
\text{ABC}^\text{ve} (\mathcal{G}) &= 2 \left( \frac{m^2 + 2m - 5}{m^2 + 4m^3 - 3m^2 - 10m + 8} |E^*_{(m-1,m)}| \right) \\
&+ 2 \left( \frac{m^2 + 3m - 5}{m^4 + 6m^2 + 3m^2 - 18m} |E^*_{(m-1,m)}| \right) \\
&+ 2 \left( \frac{m^2 + 3m - 6}{m^4 + 6m^2 + m^2 - 24m + 16} |E^*_{(m-1,m)}| \right)
\end{align*}

Figure 3: The graph of \( M^\text{ve}_1 (\mathcal{G}) \).

Figure 4: (a) The graph of \( M^\text{ve}_1 (\mathcal{G}) \). (b) The graph of \( M^\text{ve}_2 (\mathcal{G}) \).
\[\begin{align*}
&+ 2 \left( m^2 - m \right) \sqrt{\frac{m^2 + 3m - 4}{m^4 + 5m^2 + 6m^2 - 12m + 4}} E_{(m-1,m)}^{(m)} \\
&= 2(m^2 - m) \frac{m^2 + 2m - 5}{m^4 + 4m^2 - 3m^2 - 10m + 8} + 2(m^2 - m) \frac{m^2 + 3m - 5}{m^4 + 6m^2 + 3m^2 - 18m} \\
&+ (m^3 - 3m^2 + 2m) \frac{m^2 + 3m - 6}{m^4 + 6m^2 + m^2 - 24m + 16} \\
&+ \frac{1}{4} \left( m^{n+1} - m^3 + m^2 \right) \frac{m^2 + 3m - 4}{m^4 + 5m^2 + 6m^2 - 12m + 4} \\
&= 2(m^2 - m) \left( \frac{m^2 + 2m - 5}{m^4 + 4m^2 - 3m^2 - 10m + 8} + \frac{m^2 + 3m - 5}{m^4 + 6m^2 + 3m^2 - 18m} \right) \\
&+ (m^3 - 3m^2 + 2m) \frac{m^2 + 3m - 6}{m^4 + 6m^2 + m^2 - 24m + 16} \\
&+ \frac{1}{4} \left( m^{n+1} - m^3 + m^2 \right) \frac{m^2 + 3m - 4}{m^4 + 5m^2 + 6m^2 - 12m + 4} \\
&\text{GA}^\text{v} (\mathcal{H}) = \sum_{u_i, v_i \in \mathcal{E} (\mathcal{H})} 2 \sqrt{\Lambda_v (u_i) \times \Lambda_v (v_i)} \\
&\quad \left( \frac{\sqrt{m^4 + 4m^3 - 3m^2 - 10m + 8}}{m^2 + 2m - 3} \frac{E_{(m-1,m)}^*}{E_{(m-1,m)}^{(m)}} \right) \\
&= \frac{\sqrt{m^4 + 4m^3 - 3m^2 - 10m + 8}}{m^2 + 2m - 3} + \frac{\sqrt{m^4 + 6m^3 + 3m^2 - 18m}}{m^2 + 3m - 3} \\
&+ \frac{\sqrt{m^4 + 6m^3 + 5m^2 - 12m + 4}}{2(m^2 + 3m - 2)} \\
&= \frac{\sqrt{m^4 + 4m^3 - 3m^2 - 10m + 8}}{m^2 + 2m - 3} + \frac{\sqrt{m^4 + 6m^3 + 3m^2 - 18m}}{m^2 + 3m - 3} \\
&+ \frac{\sqrt{m^4 + 6m^3 + 5m^2 - 12m + 4}}{2(m^2 + 3m - 2)}.
\end{align*}\]
\[ H^e(\mathcal{R}) = \sum_{u_iv_i\in E(\mathcal{R})} \frac{2}{\Lambda_{ve}(u_i) + \Lambda_{ve}(v_i)} |E^*_{(m-n,m)}| + \left( \frac{2}{m^2} \right) |E^*_{(m,m)}| + \left( \frac{1}{m^2 + 3m - 2} \right) |E^*_{(m,m)}| \]

\[ = \left( \frac{2}{m^2 + 2m - 3} \right) (m^2 - m) + \left( \frac{1}{m^2 + 3m - 3} \right) (m^2 - m) + \left( \frac{1}{2m^2 + 3m - 4} \right) (m^3 - 3m^2 + 2m) + \left( \frac{1}{m^2 + 3m - 2} \right) (m^{n+1} - m^3 - m^2 + m) \]

\[ = \frac{2m + 2m(m-1) + m(m-2) + m(n^m - m^2 - m + 1)}{m^2 + 3m - 3 + m + 4 + m^2 + 3m - 2} \]

\[ \chi^e(\mathcal{R}) = \sum_{u_iv_i\in E(\mathcal{R})} (\Lambda_{ve}(u_i) + \Lambda_{ve}(v_i))^{-(1/2)} \]

\[ = \chi^e(\mathcal{R}) = \left( \frac{m^2 + 2m - 3}{m^2 + 3m - 2} \right)^{-(1/2)} |E^*_{(m-n,m)}| + \left( \frac{m^2 + 3m - 3}{m^2 + 3m - 2} \right)^{-(1/2)} |E^*_{(m,m)}| + \left( \frac{m^2 + 3m - 4}{m^2 + 3m - 2} \right)^{-(1/2)} |E^*_{(m,1,m)}| \]

\[ = \left( m^2 + 2m - 3 \right)^{-(1/2)} (m^2 - m) + \left( m^2 + 3m - 3 \right)^{-(1/2)} (m^2 - m) + \left( m^2 + 3m - 4 \right)^{-(1/2)} (m^2 - m) \]

\[ + \frac{1}{2} \left( \frac{m^3 - 3m^2 + 2m}{\sqrt{m^2 + 3m - 4}} \right)^{-(1/2)} (m^{n+1} - m^3 - m^2 + m) \]

\[ + \frac{1}{2} \left( \frac{m^3 - 3m^2 + 2m}{\sqrt{m^2 + 3m - 4}} \right)^{-(1/2)} (m^{n+1} - m^3 - m^2 + m) \]
\[
R^{\nu} (\mathcal{H}) = \sum_{u_i, v_j \in E(\mathcal{H})} (\Lambda_{\nu} (u_i) \Lambda_{\nu} (v_j))^{-1/2},
\]

\[
R^{\nu} (\mathcal{H}) = \left[ \frac{1}{4} \left( m^4 + 4m^3 - 3m^2 - 10m + 8 \right) \right]^{-(1/2)} \left| E^*_{(m-1,m)} \right| \\
+ \left[ \frac{1}{4} \left( m^4 + 6m^3 + 3m^2 - 18m + 8 \right) \right]^{-(1/2)} \left| E^*_{(m-1,m)} \right| \\
+ \left[ \frac{1}{4} \left( m^4 + 6m^3 + m^2 - 24m + 16 \right) \right]^{-(1/2)} \left| E^*_{(m-1,m)} \right| \\
+ \left[ \frac{1}{4} \left( m^4 + 6m^3 + 5m^2 - 12m + 4 \right) \right]^{-(1/2)} \left| E^*_{(m-1,m)} \right| \\
= \left[ \frac{1}{4} \left( m^4 + 4m^3 - 3m^2 - 10m + 8 \right) \right]^{-(1/2)} \left( m^2 - m \right) \\
+ \left[ \frac{1}{4} \left( m^4 + 6m^3 + 3m^2 - 18m + 8 \right) \right]^{-(1/2)} \left( m^2 - m \right) \\
+ \frac{1}{2} \left[ \frac{1}{4} \left( m^4 + 6m^3 + m^2 - 24m + 16 \right) \right]^{-(1/2)} \left( m^3 - 3m^2 + 2m \right) \\
+ \frac{1}{2} \left[ \frac{1}{4} \left( m^4 + 6m^3 + 5m^2 - 12m + 4 \right) \right]^{-(1/2)} \left( m^{n+1} - m^3 - m^2 + m \right) \\
= \frac{2(m^2 - m)}{\sqrt{m^4 + 4m^3 - 3m^2 - 10m + 8}} + \frac{2(m^2 - m)}{\sqrt{m^4 + 6m^3 + 3m^2 - 18m + 8}} \\
+ \frac{m^3 - 3m^2 + 2m}{\sqrt{m^4 + 6m^3 + m^2 - 24m + 16}} + \frac{m^{n+1} - m^3 - m^2 + m}{\sqrt{m^4 + 6m^3 + 5m^2 - 12m + 4}}
\]

Figure 6: (a) The graph of $H^{\nu} (\mathcal{H})$. (b) The graph of $\chi^{\nu} (\mathcal{H})$. 
5. Conclusion

To understand the basic topology, the analysis of chemical graphs and networks by topological descriptors is required. Such research has a broad variety of uses in the fields of bioinformatics, cheminformatics, biostatistics, and drug delivery, where a variety of daunting schemes are tackled by diagnostics based on different graph endeavours. Graphs are essential methods for estimating and forecasting the characteristics of chemical and biological compounds in the study of QSPR and QSAR.

In this paper, we have presented the results for ev- and ve-degree-based topological indices for the polymeric molecular networks modeled by Sierpinski networks that can be utilized for QSPR and QSAR study.

Data Availability

No data are required to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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