Triply resonant cavity electro-optomechanics at X-band

Xu Han\textsuperscript{1}, Chi Xiong\textsuperscript{1}, King Y Fong, Xufeng Zhang and Hong X Tang

Department of Electrical Engineering, Yale University, 15 Prospect St, New Haven, CT 06511 USA
E-mail: hong.tang@yale.edu

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Abstract
Optomechanical microcavities with high-frequency mechanical resonances facilitate experimental access to mechanical states with low phonon occupation and also hold promise for practical device applications including compact microwave sources. However, the weak radiation pressure force poses practical limits on achievable amplitudes at super-high frequencies. Here, we demonstrate a piezoelectric force-enhanced microcavity system that simultaneously supports microwave, optical and mechanical resonant modes. The combination of the highly sensitive optical readout and resonantly enhanced strong piezoelectric actuation enables us to build a microwave oscillator with excellent phase noise performance, which pushes the micromechanical signal source into the microwave X-band.

Keywords: optomechanics, piezo-electricity, oscillator, AlN

1. Introduction

The cavity optomechanical system, which exploits the enhanced light–matter interactions in optical resonators [1, 2], is a platform of fundamental importance. Intriguing physical phenomena such as the optical cooling of a mechanical resonator [3–7] and the optomechanical squeezing of light [8–10] have been experimentally demonstrated in cavity optomechanical...
systems. At the same time, the combination of a high-finesse optical microcavity and high-quality-factor ($Q$-factor) mechanical resonator also finds promising applications such as the ultra-sensitive detection of mechanical motions [11] and forces [12], photonic switches and amplifiers [13–15], non-volatile mechanical memory [16] and self-sustained oscillators [17–20]. Achieving optomechanical resonators vibrating at a high-frequency regime is important. From a quantum physics perspective, higher mechanical frequency translates to higher phonon energy quantum and hence leads to a less stringent temperature requirement for reaching the mechanical ground state. From an application standpoint, high-frequency optomechanical oscillators create opportunities for high-performance microwave sources, which are in demand for modern communications, networking and radio-over-fiber applications [21–23].

Since the first radiation pressure-induced regenerative oscillation was observed in microtoroid cavities with mechanical modes of up to 100 MHz [17], significant efforts have been made in developing optomechanical oscillators of a higher frequency regime [19, 20]. However, for oscillator applications which require amplification of mechanical motion, radiation pressure backaction becomes inefficient in driving stiff gigahertz resonators, which have a spring constant as large as $10^6 \text{N m}^{-1}$ [24, 25]. Brillouin laser-based microwave oscillators are able to operate at X-band frequencies (8∼12 GHz) [26–28] and even up to K-band (18∼27 GHz) by photomixing cascaded Brillouin lines [28]. Nevertheless, Brillouin oscillators require microcavities of ultra-high optical $Q$-factors ($10^8 \sim 10^9$) and delicate control of device geometries, both of which are challenging tasks for scalable implementations.

Here we demonstrate a triple-resonance cavity electro-optomechanical system at X-band frequency, in which mechanical and optical modes supported in an AlN microwheel cavity are coupled to a microwave resonance supported in a half-lambda microstrip resonator. Such design combines strong piezoelectric actuation [29] and ultrasensitive displacement readout [30–32] of an optomechanical cavity, which greatly improve the overall electro-optomechanical transduction. Using this triply-resonant electro-optomechanical system as the frequency selection component, we are able to demonstrate self-sustaining microwave oscillations at 8.256 GHz with phase noise as low as $-88 \, \text{dBc Hz}^{-1}$ at a 10 kHz offset from the carrier. This high-performance chip-based oscillator is expected to meet the demand for stable high-frequency sources in communications and radar systems. Moreover, it provides a hybrid information conversion system between microwave and optical domains, creating opportunities for building integrated information networks with high complexity and efficiency [33].

2. Electro-optomechanical triple-resonance coupling

An electro-optomechanical resonator is essentially a hybrid cavity in which a mechanical mode couples to two electromagnetic modes, one at microwave frequency and the other at optical frequency. Such integration of microwave and optical resonances in one single device is a challenging task since metal electrodes of the microwave components absorb light. One design is to use a metal-coated silicon nitride membrane to act as both the electrode of an LC resonator and the reflective mirror of a Fabry–Perot cavity [33–35]. However, in such a design the mechanical frequency is generally in the megahertz range or even lower and is hard to scale to microwave frequency. Another possible approach is to fabricate the microwave
resonator off-chip and then wire-bond it to the integrated electrodes in the device, as demonstrated in a nanomechanical system [36]. Recently, a microwave-to-optical interface has also been achieved using a piezoelectric optomechanical crystal [37]. In this work, we achieve electro-optomechanical triple resonance at X-band by using an AlN microwheel resonator with a microstrip resonator aligned in close proximity. A schematic of the triple-resonance system is shown in figure 1(a). The microwheel structure serves as both the high-finesse optical cavity and the high-$Q$ mechanical resonator, which are coupled to each other through radiation pressure. Meanwhile, the microwheel’s mechanical motions are piezoelectrically coupled to the microstrip resonator, whose resonant frequency is carefully designed to match that of the mechanical mode so that the actuation electric field can be resonantly enhanced. The microstrip resonator is fabricated on a separate substrate and is flipped over and aligned to the top of the microwheel. The distance between the two chips is precisely adjusted to be $\sim 50 \mu m$ so that the evanescent field of the microstrip resonator reaches the microwheel while the optical absorption due to the metallic part is negligible. Figure 1(b) illustrates the frequency scales of the three modes used in our triple-resonance system: the optical whispering gallery mode at around 196 THz, the high-frequency mechanical mode at around 8 GHz and the fundamental mode of the half-lambda microstrip resonator with a frequency aligned to that of the mechanical mode.

An optical micrograph of the microwheel device is shown in figure 1(d). The microwheel resonator is fabricated in a 650 nm-thick $c$-axis-oriented AlN thin film on silicon substrate. Fabrication details are described in [30]. The microwheel consists of a ring with inner radius

![Figure 1](image_url)
$R_i = 32.6 \, \mu m$ and outer radius $R_o = 37.6 \, \mu m$, and four $0.5 \, \mu m$-wide spokes which are connected to a central disk anchored to the underlying oxide. The dimensions are carefully chosen so that the clamping loss is minimized and at the same time the bending loss of the optical cavity is low enough to achieve high optical $Q$-factors. A waveguide wrapping around the microwheel structure is fabricated for efficient coupling to the cavity and a pair of grating couplers is used to couple free-space light into and out of the waveguide. The device is studied using the measurement configuration shown in figure 1(c). A driving signal is sent from the network analyzer to the microstrip resonator to drive the mechanical motion of the wheel resonator, which is then optically transduced via a laser light from a tunable diode laser. The light coming out from the device is amplified by an erbium-doped fiber amplifier (EDFA) and detected by a high-speed InGaAs photoreceiver, and the signal is fed back to the network analyzer. For operation of the device in self-oscillation mode a closed-loop configuration is used (dashed line) and −16 dB of the in-loop signal is coupled out for characterization.

3. Cavity optomechanics in a high-frequency regime

Figure 2(a) shows the normalized optical transmission spectrum of the device. Optical whispering gallery resonances with various mode numbers are observed. In particular, two optical resonances of interest are highlighted: one with a $Q$-factor of more than 120,000 and one with a lower $Q$ of $\sim 23,000$. Normally, a high-$Q$ cavity is used to improve the optomechanical readout efficiency. However, for high-frequency oscillation, finite cavity
lifetime has to be taken into account. Especially in oscillator operation, large amplitude and high-frequency mechanical motion translate to fast speed sweeping of optical resonance with respect to the laser input, which significantly impacts the dynamics of photon storage and release in the cavity [16, 38, 39]. A trade-off has to be made in choosing the optimal optical $Q$ in our device in order to achieve efficient transduction for high-frequency oscillators. For a mechanical resonator oscillating at frequency $\Omega$ with amplitude $x_0$, it can be shown that the detected radio frequency (RF) power at frequency $\Omega$ is given by [40]

$$P_{RF,\Omega} = \eta P_{in}^2 \frac{g_{om}^2 x_0^2}{\Omega^2} C_{MOT},$$

where $P_{in}$ is the input optical power, $g_{om}$ is the optomechanical coupling rate, $\eta$ takes into account the detection efficiency, amplifier gain and insertion loss, and $C_{MOT}$ is the optical transduction factor, defined as

$$C_{MOT} \equiv 4 \times \frac{\kappa_e^2 \Delta^2}{\left[(\kappa'/2)^2 + \Delta^2\right]^2},$$

where $\kappa_e$, $\kappa_i$, $\kappa' = \kappa_i + \kappa_e$ and $\Delta'$ are respectively the external waveguide coupling rate, intrinsic cavity dissipation rate, total cavity dissipation rate and optical detuning, all normalized with respect to the oscillation frequency $\Omega$. The expression is derived with the assumption $g_{om} x_0 / \Omega \ll 1$. Note that $C_{MOT}$ depends only on the intrinsic properties of the optical cavity. To visualize the dependence of $C_{MOT}$ on the detuning and dissipation rate, we assume the critical coupling condition ($\kappa = \kappa_e = \kappa_i$) and plot $C_{MOT}$ against $\kappa'/2$ and $\Delta$ in a contour chart, as shown in the left panel in figure 2(b). The white dashed lines indicate the optimal detuning for any given $\kappa'/2$. In the resolved sideband limit ($\kappa'/2 \ll 1$), $C_{MOT}$ has two optimal detuning points at $|\Delta'| = \kappa'/2$ and $|\Delta'| = 1$ with equal transduction factor $C_{MOT} \approx 1$ (0 dB), while in the unresolved sideband limit ($\kappa'/2 \gg 1$) the optimal detuning is at $|\Delta'| = \kappa'/2 \sqrt{3}$, which corresponds to the maximum slope of the Lorentzian shaped optical transmission. For comparison, detuning for optimal optomechanical dynamical backaction in the unresolved sideband limit is given by $|\Delta'| = \kappa'/2 \sqrt{5}$ [41]. The value of $C_{MOT}$ at optimal detuning is plotted against $\kappa'/2$ in the right panel of figure 2(b). It is notable that $C_{MOT}$ has a global maximum at $\kappa'/2 \approx 0.39$, $|\Delta'| \approx 0.61$ (intersection of the white dashed lines), around which the value of $C_{MOT}$ varies slowly within a wide detuning range, indicating minimum susceptibility against detuning fluctuation. This stability is important in applications such as optomechanical oscillators where thermal fluctuations and detuning drifting in the system need to be suppressed.

The above analysis indicates that the optimal optical $Q$-factor one should use to obtain the best transduction efficiency is $Q_{O, opt} = (\Omega_O / \Omega_M) / (2 \times 0.39)$, where $\Omega_O$ and $\Omega_M$ are the optical and mechanical resonant frequencies, respectively. For our system $\Omega_O \approx 2 \pi \times 196$ THz and $\Omega_M \approx 2 \pi \times 8.256$ GHz, and so the optimal $Q$ is $Q_{O, opt} \approx 30,000$. Therefore, in our experiment, the optical resonance with a relatively lower $Q$-factor of 23,000 is used. Experimentally, we also confirm that a higher microwave transmission is obtained compared with that using the optical resonance with a higher $Q$-factor.
4. Half-lambda microstrip resonators

Another challenge, in addition to optimization of the readout scheme, is improving the actuation efficiency of mechanical motion at high frequency. The bandwidth of commonly used techniques such as electrostatic actuation is usually limited by parasitic resistance and capacitance. Actuation using optical force can overcome the bandwidth issue [42] but is relatively weak in driving the stiff high-frequency resonator. To overcome the bandwidth limit and further enhance the actuation force, we utilize the evanescent field of a microwave frequency half-lambda microstrip resonator to drive the AlN mechanical resonator piezoelectrically. We design the resonance frequency of the microstrip resonator to match to that of the mechanical resonator so that the actuation electric field can be resonantly enhanced.

4.1. Design and fabrication

Figure 3(a) shows an optical micrograph of the microstrip resonator. A straight microstrip feedline is used for excitation. The darker region on the right side of the resonator is the area where the bottom gold ground layer is removed to provide optical access to the microwheel. (b) A simulated resonance profile of the perpendicular electric field ($E_z$). A zoomed-in chart along the central line in the tip region shows an enhancement factor of 5 at the tip. (c) Physical alignment between the flipped-over microstrip resonator and the microwheel resonator. The blue dashed line indicates the tip region of the microstrip resonator behind the gold ground layer. (d) Microwave reflection spectra of microstrip resonators with different resonator lengths. The upper panel shows the simulated data, which match the experimental results shown in the lower panel very well. The background ripples in the experimental data arise from the imperfect impedance matching between the feedline and the SMA connector.
thin (127 μm), transparent, high-index sapphire substrate. A 10 nm chromium adhesion layer and a 40 nm gold seed layer are first deposited on both sides of the substrate by thermal evaporation. Gold plating is then performed to get a 3.8 μm-thick gold layer. Four-point probe measurement shows that the resistivity of the gold layer is 3.4 × 10⁻⁸ Ω m, which is close to that of the bulk gold (2.2 × 10⁻⁸ Ω m), indicating good plating quality. The patterns of the microstrip lines are defined using photolithography on the front-side gold layer and are etched by potassium iodine/iodine solution. The back-side gold serves as the ground. The sapphire chip is secured on a mounting plate fitted with SMA edge connectors. The device utilizes a half-lambda resonator design with two ends open where the electric field is maximized. A microstrip feedline is fabricated next to the resonator for side-coupling and a window is opened on the ground gold layer (the dark region in figure 3(a)) to allow optical access to the device. The microstrip resonator is flipped over with the resonator tip aligned on top of the AlN microwheel as shown in figure 3(c).

4.2. Numerical simulation and experimental characterization

We perform numerical simulation to determine the dimension of the device. The feedline has a width of 117 μm to achieve 50 Ω impedance while the resonator has a width of 500 μm to minimize the Ohmic loss. The coupling gap g is set to 200 μm to achieve critical coupling. To further enhance the electric field, a tiny tip with a dimension of 200 μm × 80 μm is added at the end of the microstrip resonator to act as an electric field concentrator. The simulated electric field profile of the fundamental mode of the half-lambda resonator is shown in figure 3(b). When the resonator is excited on-resonance, electromagnetic standing waves are formed and both open ends of the resonator are voltage anti-node, which gives a voltage enhancement of √Q_e, where Q_e is the microstrip resonator quality factor. This field is further enhanced by the tip concentrator. A plot of the perpendicular electric field (E_z) along the central line near the tip region is shown in the zoomed-in chart of figure 3(b). An extra five-fold enhancement is obtained compared to the resonator without the concentrator.

Figure 3(d) shows both the simulated and measured reflection spectra of three microstrip resonators with different lengths. It can be seen that the experimental measurements match the simulation results very well. The background ripples in the experimental data are attributed to imperfect impedance matching at the connection between the SMA connector and the microstrip feedline, as confirmed by the measured reflection of the feedline structure alone without a resonator (plotted in square scatters in figure 3(d)). When the AlN chip approaches the microstrip resonator, the resonant frequency of the microstrip resonator decreases due to the increase of the surrounding dielectric constant. Therefore, a microstrip resonator with higher resonant frequency (8.7 GHz) in the air surrounding is used to match the 8.25 GHz mechanical resonance of the AlN wheel resonator. The Lorentzian fitting shows a loaded Q-factor of 87 and an intrinsic Q-factor of 125.

5. Performance characterization of an 8.25 GHz microwave oscillator

With the optimized optical transduction and resonantly enhanced piezoelectric actuation, our triple-resonator gives an unprecedentedly high transduction efficiency and excellent signal-to-noise ratio for super-high-frequency mechanical oscillation. To demonstrate the
power of this triple-resonance enhanced transduction, we utilize our system to construct a low-phase noise X-band oscillator.

5.1. Open-loop response

The open-loop characterization of the mechanical resonances is measured using the configuration in figure 1(c). As shown in figure 4(a), a mechanical resonance at 8.256 GHz with a $Q$-factor of $Q_m = 1400$ is obtained. The two minor peaks come from mode splitting due to the presence of the supporting spokes. For high-frequency resonance, the $Q$-factor of the mechanical mode is limited by the $f \cdot Q$ product [43]. In our case, the X-band mechanical mode has an $f \cdot Q$ product of $1.15 \times 10^{13}$, which is close to the limit of AlN reported in [43]. The polar chart around this resonance in figure 4(b) shows the expected circular trajectory.

5.2. Closed-loop oscillation and phase noise performance

The closed-loop oscillator is formed by feeding back the transduced signal to the microstrip resonator (shown in figure 1(c)—dashed lines). Self-oscillation starts when the RF amplifiers and the EDFA provide sufficient gain to compensate the loop loss. The Barkhausen criterion for the phase is fulfilled by adjusting the phase shifter. For a high-performance oscillator, the phase noise (directly related to the signal-to-noise ratio) is of utmost importance. As we can see in equation (1), the power of the RF signal is quadratically dependent on the input optical power, whereas the laser shot noise has a linear dependence on it. Therefore the signal-to-noise ratio can be improved by using high optical power. We amplify the output power of the tunable diode laser by an EDFA to 15 dBm, which is still low enough to avoid undesired heating effects. It can be shown that this power is much smaller than the power threshold needed for radiation pressure-driven self-oscillation [19]. As a passive device, the microstrip resonator only

![Figure 4. X-band mechanical resonance. (a) The optomechanically transduced RF transmission spectrum showing the mechanical resonances at around 8 GHz. The mechanical resonance at 8.256 GHz has a fitted $Q$-factor of 1400. (b) The RF transmission $S_{21}$ plotted in a polar chart around the mechanical resonance at 8.256 GHz.](image-url)
contributes electrical thermal noise (Johnson noise), which can be shown to be negligible compared to laser shot noise. The signal from the photoreceiver is further amplified by two-stage RF amplifiers (low-noise/high-power) to optimize the phase noise performance. After RF amplification a small portion (−16 dB) of the in-loop RF signal is coupled out for signal analysis.

The power spectrum in the inset of figure 5 shows the oscillation signal at 8.256 GHz with a microwave carrier power of 6.8 dBm. The phase noise of the oscillator is characterized using a signal analyzer (Agilent E5052B). As shown in figure 5, the oscillator demonstrates an excellent phase noise of −88 dBc Hz\(^{-1}\) at 10 kHz offset. At large offset frequency, the phase noise reaches a noise floor of −131 dBc Hz\(^{-1}\). A 1/f\(^2\) dependence is observed in the range of 10 kHz ∼ 1 MHz. In the low offset frequency region (<10 kHz), the phase noise is dominated by flicker noise of the RF amplifiers, which appears as the 1/f\(^3\) dependence in the spectrum. Although lower phase noise can be achieved in other systems, such as tunable YIG sphere oscillators [44] and opto-electronic oscillators [45] (typically −105 dBc Hz\(^{-1}\) and −140 dBc Hz\(^{-1}\) at 10 kHz offset from a ∼10 GHz carrier, respectively), our electro-optomechanical oscillator system is amenable to monolithic integration and is compatible with planar semiconductor technology. Further reduction of phase noise in the electro-optomechanical oscillator can be achieved by using amplifiers with lower flicker noise and minimizing the optical insertion loss.

6. Conclusions

In this work, we demonstrate a hybrid electro-optomechanical cavity which provides highly efficient microwave-to-optical transduction. The electro-optomechanical system provides an
opportunity to electrically control the microcavity system and allows us to construct an integrated AlN electro-optomechanical oscillator at microwave X-band. The super-high-frequency mechanical resonator we show here is realized at room temperature. Operating a similar device at 300 mK in a $^3$He fridge would place both mechanical and microwave modes at ground state and enable a quantum interface between microwave and optical fields when the high-$Q$ microstrip resonator is fabricated from superconducting materials.

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References

[1] Kippenberg T J and Vahala K J 2008 Science 321 1172–6
[2] Aspelmeyer M, Kippenberg T J and Marquardt F 2013 arXiv:1303.0733
[3] Kleckner D and Bouwmeester D 2008 Nature 444 75–8
[4] Arcizet O, Cohadon P F, Barrant T, Pinard M and Heidmann A 2006 Nature 444 71–4
[5] Gigan S, Böhm H R, Paternostro M, Blaser F, Langer G, Hertzberg J B, Schwab K C, Bäuerle D, Aspelmeyer M and Zeilinger A 2006 Nature 444 67–70
[6] Chan J, Alegre T P M, Safavi-Naeini A H, Hill J T, Krause A, Gröblacher S, Aspelmeyer M and Painter O 2011 Nature 478 89–92
[7] Teufel J D, Donner T, Li D, Harlow J W, Allman M S, Cicak K, Sirois A J, Whittaker J D, Lehnert K W and Simmonds R W 2011 Nature 475 359–63
[8] Purdy T P, Yu P-L, Peterson R W, Kampel N S and Regal C A 2013 Phys. Rev. X 3 031012
[9] Safavi-Naeini A H, Gröblacher S, Hill J T, Chan J, Aspelmeyer M and Painter O 2013 Nature 500 185–9
[10] Brooks D W, Botter T, Schreppler S, Purdy T P, Brahms N and Stamper-Kurn D M 2012 Nature 488 476–80
[11] Teufel J D, Donner T, Castellanos-Beltran M A, Harlow J W and Lehnert K W 2009 Nature Nanotechnol. 4 820–3
[12] Krause A G, Winger M, Blasius T D, Lin Q and Painter O 2012 Nature Photon. 6 768–72
[13] Wiederhecker G S, Chen L, Gondarenko A and Lipson M 2009 Nature 462 633–6
[14] Rosenberg J, Lin Q and Painter O 2009 Nature Photon. 3 478–83
[15] Li H, Chen Y, Noh J, Tadese S and Li M 2012 Nat. Commun. 3 1091
[16] Bagheri M, Poot M, Li M, Pernice W P and Tang H X 2011 Nature Nanotechnol. 6 726–32
[17] Rokhsari H, Kippenberg T J, Carmon T and Vahala K J 2005 Opt. Express 13 5293–301
[18] Rocheleau T O, Grine A J, Gutter K E, Schneider R A, Quack N, Wu M C and Nguyen C T-C 2013 Proc. Micro Electro Mechanical Systems (MEMS), IEEE 26th International Conference on IEEE pp 118–21
[19] Jiang W C, Lu X, Zhang J and Lin Q 2012 Opt. Express 20 15991–6
[20] Tallur S and Bhave S A 2012 Proc. IEEE Int. Conf. on Solid-State Sensors, Actuators and Microsystems pp 1472–5
[21] Capmany J and Novak D 2007 Nature Photon. 1 319–30
[22] Khanna A 2009 Microwave J. 49 22–8
[23] Seeds A J and Williams K J 2006 *J. Lightw. Technol.* **24** 4628–41
[24] Sun X, Zhang X and Tang H X 2012 *Appl. Phys. Lett.* **100** 173116
[25] Ding L, Baker C, Senellart P, Lemaitre A, Ducci S, Leo G and Favero I 2011 *Appl. Phys. Lett.* **98** 113108–113108
[26] Tomes M and Carmon T 2009 *Phys. Rev. Lett.* **102** 113601
[27] Grudinin I S, Matsko A B and Maleki L 2009 *Phys. Rev. Lett.* **102** 043902
[28] Li J, Lee H and Vahala K J 2013 *Nat. Commun.* **4** 2097
[29] Xiong C, Fan L, Sun X and Tang H X 2013 *Appl. Phys. Lett.* **102** 021110
[30] Xiong C, Sun X, Fong K Y and Tang H X 2012 *Appl. Phys. Lett.* **100** 171111
[31] McRae T G, Lee K H, Harris G I, Knittel J and Bowen W P 2010 *Phys. Rev. A* **82** 023825
[32] Lee K H, McRae T G, Harris G I, Knittel J and Bowen W P 2010 *Phys. Rev. Lett.* **104** 123604
[33] Andrews R W, Peterson R W, Purdy T P, Cicak K, Simmonds R W, Regal C A and Lehnert K W 2013 arXiv:1310.5276
[34] Bagci T, Simonsen A, Schmid S, Villanueva L G, Zeuthen E, Appel J, Taylor J M, Sorensen A, Usami K, Schliesser A and Polziket E S 2013 *Nature* **507** 81–85
[35] Regal C A and Lehnert K W 2011 *J. Phys. Conf. Ser.* **264** 012025
[36] Faust T, Krenn P, Manus S, Kotthaus J P and Weig E M 2012 *Nat. Commun.* **3** 728
[37] Bochmann J, Vainsencher A, Awschalom D D and Cleland A N 2013 *Nature Phys.* **9** 712–6
[38] Poot M, Fong K Y, Bagheri M, Pernice W H P and Tang H X 2012 *Phys. Rev. A* **86** 053826
[39] Baghri M, Poot M, Fan L, Marquardt F and Tang H X 2013 *Phys. Rev. Lett.* **111** 213902
[40] Gorodetksy M L, Schliesser A, Anetsberger G, Deleglise S and Kippenberg T J 2010 *Opt. Express* **18** 23236–46
[41] Lin Q, Rosenberg J, Jiang X, Vahala K J and Painter O 2009 *Phys. Rev. Lett.* **103** 103601
[42] Li M, Pernice W H P, Xiong C, Baehr-Jones T, Hochberg M and Tang H X 2008 *Nature* **456** 480–4
[43] Chandorkar S A, Agarwal M, Melamud R, Candler R N, Goodson K E and Kenny T W 2008 *Proc. IEEE Int. Conf. MEMS* pp 74–7
[44] Tanbakuchi H, Nicholson D, Kunz B and Ishak W 1989 *IEEE Trans. Magn.* **25** 3248–53
[45] Yao X S and Maleki L 1996 *JOSA B* **13** 1725–35