Holographic perspectives on models of moduli stabilization in M-theory

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ABSTRACT: Recent holographic analyses on IIA and IIB models of moduli stabilization have led to many interesting results. Here we extend this approach to M-Theory. We consider both flux-stabilized models and non-perturbative stabilization methods. We perform a holographic analysis to determine the spectrum of the assumed dual CFT\(_3\) to see its AdS/CFT implication. For the flux stabilization, which relies on a large complex Chern-Simons invariant, moduli have integer dimensions similar to the DGKT flux-stabilized model in type IIA. For the non-perturbative stabilization, the results are similar to race-track models in type IIB.

KEYWORDS: AdS-CFT Correspondence, Conformal Field Models in String Theory, M-Theory, String and Brane Phenomenology

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1 Introduction

The properties of the vacuum in string theory are important because if string theory is true, then the vacuum will be able to describe our universe. The approach to connect the original formulation of string theory and the real world is string compactification (for reviews see [1–4]), which reduces the full 10d string vacuum to a (3+1)-d spacetime, while other dimensions are compactified. However, in the process of string compactification, new massless scalar fields will emerge. The vacuum expectation values of them, called moduli, do not at first appear in the 4d effective potential, so they are not constrained.

If moduli can take arbitrary values (for example, time dependent), then this will conflict with observations. Therefore, it is essential to stabilize the moduli. The ingredients of moduli stabilization are fluxes and non-perturbative effects (for example, world sheet instantons [5] and gaugino condensation) to make new terms appear in the 4d effective potential to minimize the moduli. Fluxes can be seen as a higher dimensional generalization of Dirac quantization, so they take integer values. Therefore, the corresponding vacua are also labeled by discrete integer fluxes. The set of all vacua generated by these fluxes form the so-called “landscape”.

Examples of scenarios of moduli stabilization include the type IIA string compactifications like DGKT [6] and type IIB string compactifications like the Large Volume Scenario(LVS) [7–10], KKLT [11] and Racetrack [12]. In DGKT all geometric moduli are stabilized at SUSY AdS vacua in the large flux limit. In the Large Volume Scenario the moduli are stabilized at non-SUSY AdS vacua with an exponentially large volume. KKLT stabilizes the moduli with both fluxes and non-perturbative effects at SUSY vacua in the limit of small $W_0$. The Racetrack model uses two different non-perturbative effects to stabilize the moduli at a SUSY vacuum without fluxes.
M-Theory is another limit of string theory. Moduli stabilization is also interesting for M-Theory compactification. This has been studied in [13–15] by both flux and non-perturbative stabilization. In the flux-stabilized vacua, all the saxions and a linear combination of axions are stabilized at SUSY AdS vacua under certain topological conditions. The non-perturbative stabilized vacuum is the multiple moduli version of the racetrack model.

The above method goes from top to down. However, in recent years an alternative approach has risen: the swampland program [16–18]. This aims at using basic principles in quantum gravity to exclude many effective field theories and work out properties that low energy effective field theory must satisfy. It is beneficial to understand the swampland program from holographic perspectives: for references see [19–28].

A novel approach from holography was proposed [29–34], which provides a new perspective to the problem. The motivation is to determine the consistency of these 4d vacua from the CFT side. So far, what has been studied are the LVS, the fibred LVS and DGKT. In particular DGKT gives an interesting spectrum leading to the integer conformal dimensions.

In this paper we extend the holographic swampland story to M-Theory, both models with fluxes and without fluxes. The paper is organised as follows. In section 2 we describe general aspects of M-Theory moduli stabilization on a $G_2$-holonomy manifold and the mass matrix elements. In section 3 we study the holographic dual of M-Theory vacuum with flux and compare it with the previous results of DGKT [32, 33]. In section 4 we study the holographic dual of M-Theory vacuum with zero flux background and compare it with the previous results of KKLT and racetrack [31]. In section 5 we give our conclusions. In the appendix the detailed calculations are presented.

## 2 General aspects of M-theory moduli stabilization

In this section we give a brief description of M-Theory moduli stabilization on a $G_2$ holonomy manifold [14]. The 11d low energy supergravity description of M-theory is given by the following action [35]:

$$S = \frac{1}{2k_{11}^2} \left[ \int d^{11}x \sqrt{-g} R - \int \left( \frac{1}{2} G_4 \wedge * G_4 - \frac{1}{6} C_3 \wedge G_4 \wedge G_4 \right) \right],$$

(2.1)

where the 11d metric $g$ and a 3-form $C_3$ consists of the bosonic components. $G_4 = dC_3$ is the field strength. By compactifying this theory on a 7d compact manifold $X$ with $G_2$ holonomy group, one obtains the 4d $\mathcal{N} = 1$ supergravity theory. A covariantly constant 3-form $\phi$ always accompanies a manifold $X$ with $G_2$ holonomy. Following the argument in [14], the moduli space of $X$ has the same dimension as $H^3(X, R)$, therefore the 4d $\mathcal{N} = 1$ supergravity theory has $b_3(X)$ moduli. The corresponding chiral fields read:

$$z_i = t_i + is_i, \quad i = 1, \ldots N,$$

(2.2)

$s_i$ are volume of 3-cycles and $t_i$ are the corresponding axions. In the case that is consistent with $G_2$ holonomy [15], the Kähler potential is expressed by:

$$K = -3 \text{Log}(\mathcal{V}).$$

(2.3)
We assume, following [15], the volume of the 7d-manifold can be written as \( V = \prod_{i=1}^{N} s_i^{a_i} \).

The parameters have the following constraints:

\[ \sum_{i=1}^{N} a_i = \frac{7}{3}. \]  

(2.4)

The Kähler potential satisfies the following no scale relationships:

\[ K^{ij} K_j = -s^i, \quad K^{ij} K_i K_j = 7. \]  

(2.5)

We now derive general expressions for the Hessians of both volume moduli and axions, assuming that we have a SUSY vacuum. Details are presented in appendix A. We start from the standard \( \mathcal{N} = 1 \) supergravity potential form (taking \( M_P = 1 \)):

\[ V = e^K (K^{ij} D_i W D_j \overline{W} - 3|W|^2). \]  

(2.6)

The SUSY condition is:

\[ \partial_a W = \partial^a W + K_a W = 0. \]  

(2.7)

For the volume moduli, the corresponding Hessian is expressed:

\[ \partial_b \partial_a V = K^{ab} V - 3 e^K K_a \partial_a |W|^2 - 3 e^K \partial_b \partial_a |W|^2 + e^K K^{ij} (\partial_a D_i W)(\partial_b D_j \overline{W}) + e^K K^{ij} (\partial_b D_i W)(\partial_a D_j \overline{W}). \]  

(2.8)

Following appendix A (also see [33]), putting everything together:

\[ H_{ab}^V = \frac{V_{ab}}{e^K |W|^2} = -K_{ab} + 3K_a K_b + 2 \frac{W_{ab}}{W} + 8 K^{ij} \frac{W_{ia} W_{jb}}{|W|^2} + 2 s_i K_b \frac{W_{ia}}{W} + 2 s_j K_a \frac{W_{jb}}{W}, \]  

(2.9)

where \( K, W \) are the Kähler potential and superpotential evaluated at the SUSY vacuum.

The Hessian for the axions can be obtained similarly from eq. (2.8), with all derivatives of the Kähler potential vanishing:

\[ \partial_b \partial_a V = -3 e^K \partial_b \partial_a |W|^2 + e^K K^{ij} (\partial_a D_i W)(\partial_b D_j \overline{W}) + e^K K^{ij} (\partial_b D_i W)(\partial_a D_j \overline{W}). \]  

(2.10)

where \( \partial_a, \partial_b \) means \( \partial_{s_a}, \partial_{s_b} \). Again, following appendix A (also see [33]), we can get the axion Hessians similarly:

\[ H_{ab}^A = \frac{\partial_b \partial_a V}{e^K |W|^2} = 2 K_a K_b + 6 \frac{W_{ab}}{W} + 8 K^{ij} \frac{W_{ia} W_{jb}}{|W|^2} + 2 s_i K_b \frac{W_{ia}}{W} + 2 s_j K_a \frac{W_{jb}}{W}, \]  

(2.11)

where \( \partial_a, \partial_b \) means derivatives to \( \partial_{s_a}, \partial_{s_b} \).

We now apply this formalism to two models of moduli stabilization. The first is flux-stabilization and the second is non-perturbative stabilization. In practice the superpotential will be either eq. (3.3) or eq. (4.2).
3 Flux-stabilized M-theory vacuum

This section investigates the moduli stabilization method of [14, 36], which aims at stabilizing the moduli with fluxes. This is analogous to the DGKT model in type IIA, which also uses fluxes. We give an introduction to M-Theory compactifications on $G_2$ manifolds with fluxes turned on.

In order to stabilize the moduli, certain topological conditions have to be imposed: the $G_2$ manifold $X$ needs to have an ADE singularity along a 3-manifold $Q$ [14]. Following [14, 37], the superpotential induced by turning on a background flux $G$ and background fields at the singularities is:

$$W = \frac{1}{8\pi^2} \int \left( \frac{C_3}{2} + i\phi \right) \wedge G + c_1 + ic_2.$$  \hspace{1cm} (3.1)

Expanding the 4-form flux $G$ into a harmonic basis:

$$G = N_i \rho_i,$$  \hspace{1cm} (3.2)

where $N_i$ are fluxes and $\rho_i \in H^4(X, \mathbb{Z})$, results in the following superpotential:

$$W(z) = N_i z_i + c_1 + ic_2.$$  \hspace{1cm} (3.3)

Following [14], $c_1 + ic_2$ is a complex Chern-Simons invariant. This term is essential for this mechanism because without this term the moduli could not be stabilized and the cosmological constant would be zero. Without loss of generality we can take $c_2$ to be positive.

The SUSY condition implies:

$$-\frac{3a_i}{2i s_i} (N^j z_j + c_1 + ic_2) + N_i = 0.$$  \hspace{1cm} (3.4)

Separating the real and imaginary part of this equation, we have:

$$N^i t_i + c_1 = 0,$$

$$\frac{3a_i}{2s_i} (N^j s_j + c_2) = N^i,$$  \hspace{1cm} (3.5)

which results in:

$$\frac{3a_i}{2s_i} (N^j s_j + c_2) = \frac{7}{2} (N^j s_j + c_2) = N^i s_i.$$  \hspace{1cm} (3.6)

This leads to the following solution:

$$c_2 = -\frac{5}{7} N^i s_i, \hspace{1cm} s_i = -\frac{3a_i}{5N_i} c_2,$$  \hspace{1cm} (3.7)

which implies a vacuum expectation value $W = -i\frac{2c_2}{5}$. Before we proceed, we need to check the validity of the effective field theory description. In order to realize scale separation, we need the Kaluza-Klein radius to be much smaller than the AdS radius:

$$R_{AdS}^2 = \frac{1}{e^K |W|^2} = \frac{\mathcal{V}^3}{|W|^2} \approx \frac{c_2^3}{c_2^2} \sim c_2^3,$$

$$R_{KK}^2 \sim \mathcal{V}^2 \sim c_2^2.$$  \hspace{1cm} (3.8)
so $R_{KK} \ll R_{\text{AdS}}$ if $c_2 \gg 1$. Following [14], we assume that large values of $c_2$ can be obtained for some special $G_2$-manifolds.

This superpotential implies that the second order derivatives of the superpotential all vanish. Following eq. (2.9), the Hessian corresponding to the volume moduli is:

$$M_{ij} = -K_{ij} + 3K_iK_j. \quad (3.9)$$

The physical masses can be extracted as the eigenvalues of $m = 2K^{-1}M$, where the matrix elements read:

$$m_{ij} = 2K^{ia}M_{aj}$$
$$= 2K^{ia}(-K_{aj} + 3K_aK_j)$$
$$= -2\delta_{ij} - 6s_iK_j,$$

where we use the no scale relationship eq. (2.5).

The eigenvalues are:

$$\lambda_1 = \ldots = \lambda_{N-1} = -2,$$
$$\lambda_N = -2 - 6s_iK_i$$
$$= -2 - 6s_i\frac{\partial V}{\partial s_i} \frac{1}{V}$$
$$= -2 + 18 \times \frac{7V}{V}$$
$$= 40, \quad (3.11)$$

where we use the property that $V$ is a homogeneous function of $s_i$ of degree $\frac{7}{3}$.

Using the standard relationship $\Delta(\Delta - 3) = m^2R_{\text{AdS}}^2$, the corresponding conformal dimensions are:

$$\Delta_1 = \ldots = \Delta_{N-1} = 2, \quad (3.12)$$
$$\Delta_N = 8. \quad (3.13)$$

For the axions, the corresponding Hessian is:

$$M_{aiaj} = 2K_iK_j. \quad (3.14)$$

Similarly, the matrix elements of $m = 2K^{-1}M$ are expressed as:

$$m_{ij} = 2K^{ia}M_{aj}$$
$$= 4K^{ia}K_aK_j$$
$$= -4s_iK_j.$$

The corresponding conformal dimensions for the axions are:

$$\Delta_1 = \ldots = \Delta_{N-1} = 3, \quad (3.16)$$
$$\Delta_N = 7. \quad (3.17)$$
This is rather interesting because all the conformal dimensions are integers. This is reminiscent of DGKT, where similar results were found for general Calabi Yau manifolds [32, 33]. It was argued in [38] that for a special class of M-theory compactifications on 7d manifolds $X$ with $G_2$ holonomy, special loci in their moduli space are well described by type IIA orientifolds. Therefore, it is natural to compare with DGKT. There may be some connections between the integers of M-Theory and the integers in DGKT, but their values are different. We give a summary of the results for general DGKT.

Following [33], for the Kähler moduli and axion-dilaton sector:

\[
\Delta_1 = 10, \quad \Delta_{2, h^{1,1}+1} = 6,
\]

(3.18)

for the saxions and

\[
\Delta_1 = 11, \quad \Delta_{2, h^{1,1}+1} = 5.
\]

(3.19)

for the corresponding axions. For the complex structure moduli sector:

\[
\Delta_{u_a} = 2, \quad \Delta_{a_a} = 3, \quad a = 1, \ldots h^{2,1}.
\]

(3.20)

In DGKT, the Kähler potential and superpotential can be expressed as [6]:

\[
K = K^K + K^Q = -\log(V) + 4D, \quad W = W^K + W^Q,
\]

(3.21)

(3.22)

where

\[
W^K = c_0 + c_a z^a + \frac{1}{2} k_{abc} m_a z_b z_c - \frac{m_0}{6} k_{abc} z_a z_b z_c,
\]

(3.23)

and

\[
W^Q = -2p_k N_k - iq_\lambda T_\lambda.
\]

(3.24)

Scale separation and geometric limit rely on large $c_2 \gg 1$. Unlike DGKT, no flux number can be dial up. Note that in DGKT fluxes can be chosen arbitrary large for any Calabi Yau 3-folds. However, in the M-Theory flux-stabilization, the value of $c_2$ depends on the topological conditions of the 7d $G_2$ manifold. An example of large $c_2$ is given in [14], where $Q = H^3/\Gamma$.

Note that the complex structure moduli sector in DGKT is similar to M-theory flux vacuum. This is because $W_Q$ is also a linear combination of complex structure moduli, which is alone the same line as eq. (3.3). The resemblance has been predicted in [6]. However, the volume moduli and axion-dilaton sector seems to be different because there is no quadratic and cubic terms in eq. (3.3).

4 Stabilization by non-perturbative effects

Up to now we have focused on scenarios arising from M-theory flux vacua. In this section, we study a different model based on M-theory. The authors took a further step in [13, 15]: they considered the M-theory vacuum with zero flux background. We will revisit this model and focus on the detailed properties of its holographic dual in this section.
We give a quick summary to the model. The Kähler potential remains the same in this case:
\[ K = -3 \log(\mathcal{V}), \] 
where \( \mathcal{V} = \prod_{i=1}^{N} s_i^a \) with \( \Sigma_i^N a_i = \frac{7}{3} \). The superpotential is generated by the non-perturbative effects [13, 15]:
\[ W = A_1 e^{ib_1 \Sigma_i^N N_1^1 z_i} + A_2 e^{ib_2 \Sigma_i^N N_2^2 z_i}, \]
where \( A_k \) are numerical constants. \( b_1 = \frac{2\pi}{7} \) and \( b_2 = \frac{2\pi}{7} \), with \( P, Q \) being the rank of the gauge group for gauge condensation. The sets of \( N_1^1, N_2^2 \) are all integers. Therefore, the M-theory vacuum is fully determined by the constants \((a_i, b_1, b_2, N_1^1, N_2^2, A_1, A_2)\). In this paper, without loss of generality, we take positive \( A_1, A_2 \).

The SUSY condition is:
\[ D_i W = \partial_i W + K_i W = 0. \] (4.3)
The vacuum solution to eq. (4.3) is:
\[ \frac{A_1}{A_2} = -\cos[(b_1 \Sigma_i^N N_1^1 s_i - b_2 \Sigma_i^N N_2^2 s_i)] \frac{2b_2 N_2^2 s_i + 3a_i e^{(b_1 \Sigma_i^N N_1^1 s_i - b_2 \Sigma_i^N N_2^2 s_i)}}{2b_1 N_1^1 s_i + 3a_i}, \] (4.4)
\[ \sin[(b_1 \Sigma_i^N N_1^1 s_i - b_2 \Sigma_i^N N_2^2 s_i)] = 0. \] (4.5)
To prove them, we can start by taking the expression of \( W \) and \( K \) into SUSY condition:
\[ i(A_1 b_1 N_1^1 e^{ib_1 \Sigma_i^N N_1^1 z_i} + A_2 b_2 N_2^2 e^{ib_2 \Sigma_i^N N_2^2 z_i}) - \frac{3a_i}{2is_i} (A_1 e^{ib_1 \Sigma_i^N N_1^1 z_i} + A_2 e^{ib_2 \Sigma_i^N N_2^2 z_i}) = 0. \] (4.6)
which implies:
\[ \frac{A_1}{A_2} = -\frac{2b_2 N_2^2 s_i + 3a_i e^{ib_2 \Sigma_i^N N_2^2 s_i - ib_1 \Sigma_i^N N_1^1 s_i}}{2b_1 N_1^1 s_i + 3a_i}, \] (4.7)
\[ (\cos[-b_2 \Sigma_i^N N_2^2 t_i + b_1 \Sigma_i^N N_1^1 t_i] - i\sin[-b_2 \Sigma_i^N N_2^2 t_i + b_1 \Sigma_i^N N_1^1 t_i]). \]
So we prove the SUSY condition.

The overall phase of \( W, e^{ib_1 \Sigma_i^N N_1^1 t_i} \), does not have physical meaning, since it is only relative phase factors that matter. If we have \( A_1, A_2 \) real and positive, the SUSY condition implies:
\[ \cos([-b_2 \Sigma_i^N N_2^2 t_i + b_1 \Sigma_i^N N_1^1 t_i]) = -1, \] (4.8)
which also gives:
\[ \frac{A_1}{A_2} = \frac{2b_2 N_2^2 s_i + 3a_i e^{-b_2 \Sigma_i^N N_2^2 s_i + b_1 \Sigma_i^N N_1^1 s_i}}{2b_1 N_1^1 s_i + 3a_i}, \] (4.9)
which results in the following equations:
\[ \frac{A_2}{A_1} = \frac{1}{\alpha} e^{-b_1 \sum_{i=1}^{N} N_1^1 s_i + b_2 \sum_{i=1}^{N} N_2^2 s_i}, \] (4.10)
\[ s_i = -\frac{3a_i(\alpha - 1)}{2(b_1 N_1^1 \alpha - b_2 N_2^2)}, \] (4.11)
where one can solve \( \alpha \) and \( s_i \) numerically.
The Hessians of the volume moduli and the axions in AdS units read:

\[
H_{ab}^V = R^2_{\text{AdS}} V_{ab} = \frac{V_{ab}}{e^K |W|^2} = -K_{ab} + 3K_a K_b + 2 \frac{W_{ab}}{W} + 8 K^{ij} W_{ia} W_{jb} \frac{W_{ia} W_{jb}}{|W|^2} + 2 s_i K_b \frac{W_{ia}}{W} + 2 s_j K_a \frac{W_{jb}}{W}, \tag{4.12}
\]

and

\[
H_{ab}^A = R^2_{\text{AdS}} V_{ab} = \frac{V_{ab}}{e^K |W|^2} = 2K_a K_b + 6 \frac{W_{ab}}{W} + 8 K^{ij} W_{ia} W_{jb} \frac{W_{ia} W_{jb}}{|W|^2} + 2 s_i K_b \frac{W_{ia}}{W} + 2 s_j K_a \frac{W_{jb}}{W}. \tag{4.13}
\]

The physical masses can be acquired as the eigenvalues of \( M = 2K^{-1} H \). Using the explicit expressions for the Hessians in the appendix, we have:

\[
M_{ab}^V = 2K^{ai} H_{ib}^V = -2 \delta_{ab} + \alpha \frac{s_a^2}{3a} \left( b_1 N_a^1 + 3a_s \right) \left( b_1 N_b^1 + 3a_b \right) \left( \sum_{i=1}^N \frac{8}{3a_i} \left( b_1 N_i^1 s_i + \frac{3a_i}{2} \right)^2 - 2 \right), \tag{4.14}
\]

\[
M_{ab}^A = 2K^{ai} H_{ib}^A = \alpha \frac{s_a^2}{3a} \left( b_1 N_a^1 + 3a_s \right) \left( b_1 N_b^1 + 3a_b \right) \left( \sum_{i=1}^N \frac{8}{3a_i} \left( b_1 N_i^1 s_i + \frac{3a_i}{2} \right)^2 - 6 \right). \tag{4.15}
\]

The eigenvalues are:

\[
\lambda_i^V = -2, \quad i = 1, \ldots, N - 1, \tag{4.16}
\]

\[
\lambda_N^V = -2 + \alpha \sum_{a=1}^N \frac{s_a^2}{3a} \left( b_1 N_a^1 + 3a_s \right) \left( \sum_{i=1}^N \frac{8}{3a_i} \left( b_1 N_i^1 s_i + \frac{3a_i}{2} \right)^2 - 2 \right), \tag{4.17}
\]

\[
\lambda_i^A = 0, \quad i = 1, \ldots, N - 1, \tag{4.18}
\]

\[
\lambda_N^A = \alpha \sum_{a=1}^N \frac{s_a^2}{3a} \left( b_1 N_a^1 + 3a_s \right) \left( \sum_{i=1}^N \frac{8}{3a_i} \left( b_1 N_i^1 s_i + \frac{3a_i}{2} \right)^2 - 6 \right), \tag{4.19}
\]

respectively.

Using the standard relationship \( \Delta(\Delta - 3) = m^2 R^2_{\text{AdS}} \), the corresponding conformal dimensions are:

\[
\Delta_i^V = 2, \quad i = 1, \ldots, N - 1, \tag{4.20}
\]

\[
\Delta_N^V = \frac{3 + \sqrt{1 + 4 \alpha \sum_{a=1}^N \frac{1}{3a} \left( b_1 N_a^1 s_a + \frac{3a_s}{2} \right)^2 \left( \sum_{i=1}^N \frac{8}{3a_i} \left( b_1 N_i^1 s_i + \frac{3a_i}{2} \right)^2 - 2 \right)}}{2}, \tag{4.21}
\]
and
\[
\Delta_i^A = 3, \quad i = 1, \ldots N - 1, \quad (4.22)
\]
\[
\Delta_N^A = \frac{3 + \sqrt{9 + 4\alpha \Sigma_{a=1}^N \frac{1}{3a_a} \left( b_1 N_i^1 s_i + \frac{3a_a}{2} \right)^2 \left( \Sigma_{i=1}^N \frac{8}{3a_a} \alpha \left( b_1 N_i^1 s_i + \frac{3a_a}{2} \right)^2 - 6 \right)}}{2}. \quad (4.23)
\]

Note that the option \( \Delta_i^V = 1 \) is excluded by \( N = 1 \) supersymmetry, as the volume moduli and the axions are in the same 3d \( N = 1 \) supermultiplet \[39\].

Following \[15\], we work in the following two branches in which the supergravity description is meaningful:

\[
a) \frac{A_2}{A_1} > 1, \min_i \frac{b_2 N_i^2}{b_1 N_i^1}; i = 1 \ldots N > \alpha > \max_i \frac{b_2 N_i^2}{b_1 N_i^1 + \frac{3a_a}{2}}; i = 1 \ldots N
\]
\[
b) \frac{A_2}{A_1} < 1, \max_i \frac{b_2 N_i^2}{b_1 N_i^1}; i = 1 \ldots N < \alpha < \min_i \frac{b_2 N_i^2}{b_1 N_i^1 + \frac{3a_a}{2}}; i = 1 \ldots N
\]
\[
(4.24)
\]

It remains to show how the conformal dimension \( (4.21) \) grows with the \( R_{\text{AdS}} \) in the scale separation limit where \( R_{\text{AdS}} \) goes to infinity. Since we have a supersymmetric vacuum, it follows that \( V = -3e^K|W|^2 \), therefore \( \frac{1}{R^2_{\text{AdS}}} = e^K|W|^2 \), which implies:

\[
\log R_{\text{AdS}} = -\frac{1}{2} K - \log W. \quad (4.25)
\]

Following the definition of Kähler potential, superpotential, eq. \( (4.10) \), eq. \( (B.1) \) and combining them, we have:

\[
\log R_{\text{AdS}} = \Sigma_{i=1}^N \frac{3}{2} a_i \log s_i + b_1 \Sigma N_i^1 s_i + \log \left( A_1 \left( 1 - \frac{1}{\alpha} \right) \right). \quad (4.26)
\]

In the scale separation limit, the leading order contribution of the conformal dimension and the \( \log R_{\text{AdS}} \) is:

\[
\Delta_N \propto \alpha \Sigma_{a=1}^N \left( b_1 N_i^1 s_i + \frac{3a_a}{2} \right)^2 \propto b_1 (\Sigma_{i=1}^N N_i^1 s_i)^2, \quad (4.27)
\]
\[
\log^2 R_{\text{AdS}} \propto b_1 (\Sigma_{i=1}^N N_i^1 s_i)^2, \quad (4.28)
\]

Therefore, in the scale separation limit:

\[
\Delta_{\text{Heavy}} \propto \log^2 R_{\text{AdS}}. \quad (4.29)
\]

We end this section by comparing the results with other scenarios like KKLT \[11\] and Racetrack \[12\], as they are both non-perturbative stabilized. KKLT is a type IIB flux compactification model in which both fluxes and non-perturbative effects are used to stabilize the moduli supersymmetrically. The AdS minimum is found after moduli stabilization then lifted to dS. The 4d effective field theory is described by:

\[
K = -3 \log(-i(\rho - \bar{\rho})),
\]
\[
W = W_0 + Ae^{i\alpha \rho}. \quad (4.30)
\]

In the limit when \( |W_0| \ll 1 \), one obtains \( \Delta \propto \log R_{\text{AdS}} \) \[29, 40\].

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Racetrack still occurs in the context of both type IIB and heterotic string compactification but with only non-perturbative effects. The Kähler potential is the same as KKLT while the superpotential are dominated by two different non-perturbative effects:

\[ W = A e^{i\alpha \rho} - B e^{i\beta \rho}. \]  \hspace{1cm} (4.31)

Following [31], one can show \( \Delta \propto \log^2 R_{\text{AdS}} \) in the scale separation limit.

It is natural that M-theory gives similar results to IIB racetrack because racetrack model corresponds to the one modulus case of M-theory with zero flux background.

5 Conclusion

We finish this paper by summarizing the results and proposing open questions. In this paper, we study two different models of M-theory moduli stabilization.

First, for the flux-stabilized M-Theory vacuum, we have shown that the spectrum of the dual CFT3 is characterized by a set of integer conformal dimensions. The presence of integer conformal dimensions dual to the moduli and axions in M-Theory flux vacuum is quite intriguing. Similar results also appear in DGKT type IIA string compactification scenarios [32, 33]. Therefore we compared them and found the flux-stabilized M-Theory vacuum resembles the complex structure sector of DGKT, which is not surprising because their superpotential has similar forms, as predicted in [6]. The validity of the results heavily rely on the existence of large \( c_2 \).

Second, for non-perturbatively effects stabilized M-Theory vacuum, we prove that there is one heavy operator with \( \Delta \propto \log R_{\text{AdS}}^2 \), the rest are the same as flux-stabilized M-Theory vacuum. Then we compare the results with other scenarios like KKLT [11] and racetrack [12]. It is different with KKLT because there are two different non-perturbative effects in the superpotential, while KKLT only has one non-perturbative term. Racetrack is quite similar because it is the one modulus version of the non-perturbative effects stabilized M-Theory vacuum.

An extremely interesting question is: what is the origin of these integer conformal dimensions? Currently we have no explanation for this, but we believe there must be some deeper structures behind this. Another interesting open question is: can we prove the existence of \( G_2 \) manifold with large \( c_2 \)? It would certainly be of interest to explore these questions in the future.

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A General expressions for mass matrices

In this section we derive eq. (2.9) and eq. (2.11). The general expressions of moduli and axions Hessians are:

\[ H_{ab}^{V} = \partial_{b} \partial_{a} V = K_{ab} V - 3e^{K} K_{a} \partial_{a} |W|^{2} - 3e^{K} \partial_{b} \partial_{a} |W|^{2} \]
\[ + e^{K} K^{ij} (\partial_{a} D_{i} W) (\partial_{b} D_{j} W) + e^{K} K^{ij} (\partial_{a} D_{i} W) (\partial_{b} D_{j} W), \]  

(A.1)

where \( \partial_{a}, \partial_{b} \) means \( \partial_{a_{a}}, \partial_{b_{a}} \) and \( \partial_{i}, \partial_{j} \) means \( \partial_{a_{i}}, \partial_{a_{j}} \).

\[ H_{ab}^{A} = \partial_{b} \partial_{a} V = -3e^{K} \partial_{b} \partial_{a} |W|^{2} + e^{K} K^{ij} (\partial_{a} D_{i} W) (\partial_{b} D_{j} W) \]
\[ + e^{K} K^{ij} (\partial_{a} D_{i} W) (\partial_{b} D_{j} W), \]

(A.2)

where \( \partial_{a}, \partial_{b} \) means \( \partial_{a_{a}}, \partial_{b_{a}} \) and \( \partial_{i}, \partial_{j} \) means \( \partial_{a_{i}}, \partial_{a_{j}} \).

For the first line of (A.1), the SUSY condition implies that:

\[ \partial_{a} |W|^{2} = 2 \partial_{a} W \bar{W} = -K_{a} |W|^{2}, \]  

(A.3)

and

\[ \partial_{b} \partial_{a} |W|^{2} = 2 \partial_{b} \partial_{a} W \bar{W} + 2 \partial_{a} W \partial_{b} \bar{W} \]
\[ = \left( 2 \frac{W_{ab}}{W} + \frac{1}{2} K_{a} K_{b} \right) |W|^{2}. \]  

(A.4)

Following [33], for the second line of (A.1), we have:

\[ e^{K} K^{ij} (\partial_{a} D_{i} W) (\partial_{b} D_{j} W) = e^{K}(K^{ij}W_{ia}W_{jb} + 4W_{ab}W - \frac{1}{2} K^{ij}W_{ia}K_{j}K_{b}W \]
\[ - \frac{1}{2} K^{ij} W_{ia} K_{j}K_{b} \bar{W} + \left( K_{ab} + \frac{3}{4} K_{a} K_{b} \right) |W|^{2}). \]  

(A.5)

The fact that \( K = K(z_{i} + \bar{z}_{i}) \) and the superpotential is holomorphic implies:

\[ \partial_{z_{i}} K = \frac{\partial_{a} K}{2i}, \partial_{\bar{z}_{i}} K = -\frac{\partial_{a} K}{2i}, \]  

(A.6)

and

\[ \partial_{z_{i}} W = \frac{1}{i} \partial_{a} W; \partial_{\bar{z}_{i}} W = -\frac{1}{i} \partial_{a} W, \]  

(A.7)

which results in:

\[ e^{K} K^{ij} (\partial_{a} D_{i} W) (\partial_{b} D_{j} W) = e^{K}(4K^{ij}W_{ia}W_{jb} + 4W_{ab}W + s_{i} W_{ia} K_{b} W \]
\[ + s_{j} W_{jb} K_{a} W + \left( K_{ab} + \frac{3}{4} K_{a} K_{b} \right) |W|^{2}). \]  

(A.8)

Combining (A.3), (A.4) and (A.8) leads to:

\[ M_{ab}^{V} = \frac{V_{ab}}{e^{K}|W|^{2}} = -K_{ab} + 3K_{a} K_{b} + 2 \frac{W_{ab}}{W} + 8K^{ij} \frac{W_{ia} W_{jb}}{|W|^{2}} \]
\[ + 2s_{i} K_{b} \frac{W_{ia}}{W} + 2s_{j} K_{a} \frac{W_{jb}}{W}, \]  

(A.9)
The axion Hessians (A.2) can be obtained similarly, note that:

\[
\frac{\partial W}{\partial a} = \frac{\partial W}{\partial z_a} = \frac{1}{i} \frac{\partial W}{\partial s_a},
\]

and

\[
\partial_a D_i W = \partial_a (\partial_{z_i} W + K_i W) = \partial_{z_i} \partial_{z_a} W + K_i \partial_{z_a} W = W_{z_i z_a} - K_i K_a W (A.10)
\]

which results in:

\[
\partial_b \partial_a |W|^2 = 2 \partial_b \partial_a W \overline{W} + 2 \partial_a W \partial_b \overline{W} = -2W_{ab} \overline{W} + \frac{1}{2} K_a K_b |W|^2.
\]

For the second term of (A.2), using (A.10), (A.11) we have:

\[
e^K K^{-ij} (\partial_a D_i W) (\partial_b D_j \overline{W}) = e^K 4K^{ij} \left( -W_{ia} + \frac{1}{4} K_i K_a W \right) \left( -\overline{W}_{jb} + \frac{1}{4} K_j K_b \overline{W} \right) = e^K \left( 4K^{ij} W_{ia} \overline{W}_{jb} + s_i W_{ia} K_b W + s_j W_{jb} K_a W + \frac{7}{4} K_a K_b |W|^2 \right).
\]

Putting (A.12), (A.13) together into (A.2):

\[
M_{ab} = \frac{V_{ab}}{e^K |W|^2} = 2K_a K_b + 6 \frac{W_{ab}}{W} + 8K^{ij} \frac{W_{ia} W_{jb}}{|W|^2} + 2s_i K_b W_{ia} \frac{W}{W} + 2s_j K_a W_{jb} \frac{W}{W}.
\]

B Hessians of zero fluxes background

In this appendix we give the explicit expressions for the Hessians of the moduli and axions.

We can substitute the vacuum expectation value of axions eqs. (4.8) into the superpotential because the Hessians for moduli and axions factorise:

\[
W = A_1 e^{-b_1 \sum_{i=1}^N N_i^1 s_i} - A_2 e^{-b_2 \sum_{i=1}^N N_i^2 s_i}.
\]

The first and second derivatives for W are:

\[
W_a = -b_1 N_a^1 A_1 e^{-b_1 \sum_{i=1}^N N_i^1 s_i} + b_2 N_a^2 A_2 e^{-b_2 \sum_{i=1}^N N_i^2 s_i} (B.2)
\]

\[
W_{ab} = b_1^2 N_a^1 N_b^1 A_1 e^{-b_1 \sum_{i=1}^N N_i^1 s_i} - b_2^2 N_a^2 N_b^2 A_2 e^{-b_2 \sum_{i=1}^N N_i^2 s_i} (B.3)
\]

We substitute the SUSY condition (4.9) into (B.3):

\[
\frac{W_{ab}}{W} = \frac{\alpha b_1^2 N_a^1 N_b^1 - b_2^2 N_a^2 N_b^2}{\alpha - 1}.
\]

– 12 –
Eqs. (4.11) implies:
\begin{equation}
N_a^2 = \frac{b_1 N_a^1}{b_2} \alpha + \frac{3a_a(\alpha - 1)}{2s_a b_2}.
\end{equation}

Putting them together, the second order derivatives of the superpotential are expressed as:
\begin{equation}
W_{ab} \frac{W}{W} = -\alpha b_1^2 N_a^1 N_b^1 - \frac{3}{2} b_1 \alpha \left( \frac{a_b}{s_b} N_a^1 + \frac{a_a}{s_a} N_b^1 \right) + \frac{9a_a a_b}{4s_a s_b} (1 - \alpha)
\end{equation}
\begin{equation}
= -\alpha \left( b_1 N_a^1 + \frac{3a_a}{2s_a} \right) \left( b_1 N_b^1 + \frac{3a_b}{2s_b} \right) + \frac{9a_a a_b}{4s_a s_b}.
\end{equation}

The Hessians of the moduli and axions can be expressed in AdS units:
\begin{equation}
H_{ab}^V = R_{\text{AdS}}^2 V_{ab} = \frac{V_{ab}}{e^K |W|^2} = -K_{ab} + 3K_a K_b + 2W_{ab} W W W_{ab} + 8K_{ij} W_{ia} W_{jb} \frac{W}{W} |W|^2
\end{equation}
\begin{equation}
= 2s_i K_{ab} W_{ia} W_{jb} - 2s_j K_a W_{jb} W_{ab}.
\end{equation}

The derivatives of Kähler potential and superpotential are given by:
\begin{equation}
K_a K_b = \frac{9a_a a_b}{s_a s_b},
\end{equation}
\begin{equation}
2W_{ab} W = -2\alpha \left( b_1 N_a^1 + \frac{3a_a}{2s_a} \right) \left( b_1 N_b^1 + \frac{3a_b}{2s_b} \right) + \frac{9a_a a_b}{4s_a s_b}.
\end{equation}

These are used to evaluate the last term in the first line in (B.7):
\begin{equation}
8K_{ij} W_{ia} W_{jb} \frac{W}{W} |W|^2
\end{equation}
\begin{equation}
= 8 \sum_{i=1}^N \frac{s_i^2}{3a_i} \left( -\alpha \left( b_1 N_a^1 + \frac{3a_a}{2s_a} \right) + \frac{9a_a a_b}{4s_i s_a} \right) \left( b_1 N_b^1 + \frac{3a_b}{2s_b} \right) + \frac{9a_a a_b}{4s_i s_b}
\end{equation}
\begin{equation}
= \frac{8}{3a_i} \alpha^2 \left( b_1 N_a^1 s_i + \frac{3a_i}{2} \right)^2 \left( b_1 N_a^1 + \frac{3a_a}{2s_a} \right) \left( b_1 N_b^1 + \frac{3a_b}{2s_b} \right)
\end{equation}
\begin{equation}
- \frac{8s_i^2}{3a_i} \alpha \left( b_1 N_a^1 + \frac{3a_i}{2s_i} \right) \frac{9a_i}{4s_i} \left( b_1 N_a^1 + \frac{3a_a}{2s_a} \right) \frac{a_b}{s_b} \left( b_1 N_b^1 + \frac{3a_b}{2s_b} \right) + \frac{8s_i^2}{3a_i} \frac{81a_i^2 a_a a_b}{16s_i^2 s_a s_b}
\end{equation}
\begin{equation}
= \frac{8}{3a_i} \alpha^2 \left( b_1 N_a^1 s_i + \frac{3a_i}{2} \right)^2 \left( b_1 N_a^1 + \frac{3a_a}{2s_a} \right) \left( b_1 N_b^1 + \frac{3a_b}{2s_b} \right)
\end{equation}
\begin{equation}
- 6s_i \alpha \left( b_1 N_a^1 + \frac{3a_i}{2s_i} \right) \left( b_1 N_a^1 \frac{a_a}{s_a} + b_1 N_b^1 \frac{a_a}{s_a} + \frac{3a_a a_b}{2s_a s_b} \right) + \frac{63 a_a a_b}{2 s_a s_b}.
\end{equation}
The second line of (B.7) can be obtained similarly:

\[
2 s_j K_b \frac{W_{ia}}{W} = 2 s_i - 3 a_b W_{ia} - \frac{s_b}{s_a} W
\]

\[
= - \frac{6 a_b}{s_b} \alpha_{i=1} \left( 3 a_i + 2 a_i \right) \left( b_1 N_i^1 s_i + 3 a_i \right) \left( b_1 N_a^1 + 3 a_a \right) - 9 a_a a_i \right) (B.13)
\]

\[
2 s_i K_a \frac{W_{db}}{W} = 2 s_i - 3 a_a W_{ib} - \frac{s_a}{s_b} W
\]

\[
= - \frac{6 a_a}{s_a} \alpha_{i=1} \left( 3 a_i + 2 a_i \right) \left( b_1 N_i^1 s_i + 3 a_i \right) \left( b_1 N_b^1 + 3 a_b \right) - 63 a_a a_b \right) (B.14)
\]

Combining these results in:

\[
H_{ab}^V = - K_{ab} + 3 K_a K_b + 2 \frac{W_{ab}}{W} + 8 K^{ij} \frac{W_{ia} W_{jb}}{|W|^2}
\]

\[
+ 2 s_j K_b \frac{W_{ia}}{W} + 2 s_j K_a \frac{W_{ib}}{W}.
\]

\[
= - \frac{s_a^2}{3 a_a} \delta_{ab} + \frac{8}{3 a_i} \alpha \left( b_1 N_i^1 s_i + 3 a_i \right) \left( b_1 N_a^1 + 3 a_a \right) \left( b_1 N_b^1 + 3 a_b \right) - 2 \right) (B.15)
\]

and

\[
H_{ab}^A = 2 K_a K_b + 6 \frac{W_{ab}}{W} + 8 K^{ij} \frac{W_{ia} W_{jb}}{|W|^2}
\]

\[
+ 2 s_j K_b \frac{W_{ia}}{W} + 2 s_j K_a \frac{W_{ib}}{W}.
\]

\[
= \frac{8}{3 a_i} \alpha \left( b_1 N_i^1 s_i + 3 a_i \right) \left( b_1 N_a^1 + 3 a_a \right) \left( b_1 N_b^1 + 3 a_b \right) - 6 \right) (B.16)
\]

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