Elastic double diffractive production of axial-vector $\chi_c(1^{++})$ mesons and the Landau-Yang theorem

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Abstract

We discuss exclusive elastic double diffractive axial-vector $\chi_c(1^+)$ meson production in proton-antiproton collisions at the Tevatron. The amplitude for the process is derived within the $k_t$-factorisation approach with unintegrated gluon distribution functions (UGDFs). We show that the famous Landau-Yang theorem is not applicable in the case of off-shell gluons. Differential cross sections for different UGDFs are calculated. We compare exclusive production of $\chi_c(1^+)$ and $\chi_c(0^+)$. The contribution of $\chi_c(1^+)$ to the $J/\Psi + \gamma$ channel is smaller than that of the $\chi_c(0^+)$ decay, but not negligible and can be measured. The numerical value of the ratio of the both contributions is almost independent of UGDFs modeling.

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1. INTRODUCTION

The central exclusive production of mesons has been recently revived. This is essentially because of two reasons.

Firstly, the theoretical QCD inspired approach has been developed. This is because of interest in the double diffractive production of the Higgs boson firstly proposed by A. B. Kaidalov, V. A. Khoze, A. D. Martin and M. G. Ryskin [1, 2, 3] (KKMR) as an alternative to inclusive production for Higgs searches. In principle, very similar methods can be used for scalar [7], pseudoscalar [8], axial-vector and tensor mesons. The situation for vector meson production is somewhat different. Here the dominant mechanism is photon-pomeron (pomeron-photon) fusion [9] or pomeron-odderon (odderon-pomeron) fusion [10]. Recently a \( k_t \)-factorization approach has been used to calculate exclusive \( \Upsilon \) production [11] at Tevatron.

Secondly, some experimental efforts have been done to facilitate real measurements at Tevatron [12] and in the future at RHIC [13] and LHC [14]. Some preliminary results from Tevatron have been presented recently [12, 15].

Some kinematical variables are shown in addition.

In the present paper we are concentrated on the exclusive double-diffractive production of axial-vector \( \chi_c(1^+) \) mesons\(^1\). Here the dominant mechanism is the two-pomeron fusion, which in the QCD language is a “fusion” of two QCD ladders. The mechanism is shown schematically in Fig. 1. Compared to the Higgs production, where a hard scale is guaranteed by the large mass of the Higgs boson, here the natural scale (mass of the \( \chi_c(1^+) \) meson) is much lower and the method proposed by KKMR is a bit questionable as a big part of the strength may come from the region of relatively small gluon transverse momenta. A pragmatic solution is to use nonperturbative models of UGDFs instead of the pQCD inspired KKMR procedure (for more details see [7]).

The situation with the axial-vector production is new compared to both zero-spin case (scalar [7], pseudoscalar [8] mesons) as well as to the vector meson production where the

\[^1\] Some general aspects of this process were discussed previously in Refs. 3, 4, 5.
vector meson is dominantly transversely polarized \[^{9,11}\] , at least, for small transferred four-momenta in the nucleon lines. The axial-vector meson, as it will be discussed here, can be polarized both transversely and longitudinally. We shall calculate the cross section for different polarisation states of the $\chi_c(1^+)$ meson.

There is interesting theoretical aspect of the double diffra ctive production of the $\chi_c(1^+)$ meson. The coupling $g^*g^*\chi_c(1^+)$ (see Fig. 1) vanishes for on-shell gluons (so-called Landau-Yang theorem). According to the original Landau-Yang theorem \[^{16}\] the symmetries under space rotation and inversion forbid the decay of the spin-1 particle into two (on-shell) spin-1 particles (two photons, two gluons). The same is true for the fusion of two on-shell gluons. The symmetry arguments cannot be strictly applied for off-shell gluons. This fact has been already explored in inclusive production of $\chi_c(1^+)$ \[^{17,18,19,20}\] , in the production of spin-1 glueballs \[^{21}\] , and recently in the studies of decays of hypothetical $Z'$ bosons into pair of standard $Z$ bosons \[^{22}\] . One of the goals of our paper is to confirm explicitly that the Landau-Yang theorem is violated by virtual effects in diffractive production of $\chi_c(1^+)$ leading to very important observational consequences. In our approach the off-shell effects are treated explicitly. For comparison, in the standard KKMR approach the corresponding cross section would vanish due to their on-shell approximation. The measurement of the cross section can be therefore a good test of the off-shell effects and, consequently, UGDFs used in the calculation.

At the Tevatron the $\chi_c$ mesons are measured through the $\gamma + J/\Psi$ decay channel. The axial-vector $\chi_c(1^+)$ meson has a large branching fraction of the radiative decay $\chi_c(1^+) \rightarrow \gamma + J/\psi$ (BR = 0.36 \[^{23}\] ). This is much bigger than for the scalar $\chi_c(0^+)$ where it is only about 1 \% \[^{23}\] . Therefore, the discussed off-shell effects may be very important to understand the situation in the $\gamma + J/\Psi$ channel observed experimentally.

II. FORMALISM

The kinematics of the process was already discussed in our previous paper on $\chi_c(0^+) \[^{7}\]$. Here we discuss only details of the matrix element for exclusive $\chi_c(1^+)$ production. This is derived for the first time, at least in the generalized KKMR approach.

A. General $\chi_cJ$ production amplitude

In the following we employ the general Kaidalov-Khoze-Martin-Ryskin approach \[^{1,2,3}\] , and write the amplitude of the exclusive double diffractive color singlet production $pp \rightarrow pp\chi_cJ$ as

$$\mathcal{M}_{pp \rightarrow pp\chi_cJ}^{pp} = \frac{s}{2} \pi^2 \delta_{c1c2}^2 \frac{1}{N_c^2 - 1} \int d^2 q_0 \frac{1}{t_0} \left( f^{eff}_{g,1}(x_1, x_1', q_0^2, t_0, t_1) f^{eff}_{g,2}(x_2, x_2', q_0^2, t_0, t_1) \right) \frac{V^{c_1c_2}_{J,\lambda}(q_1, q_2, p_M)}{q_0^2 q_1^2 q_2^2}. \quad (2.1)$$

In this expression $f^{eff}_{g,i}(x_i, x_i', q_0^2, t_0, t_i)$ are off-diagonal unintegrated gluon distributions. In the general case we do not know the off-diagonal UGDFs very well. In Ref. \[^{7,8}\] we have proposed a prescription how to calculate the off-diagonal UGDFs with the help of their
diagonal counterparts:

\[ f_{g,1}^{\text{off}} = \sqrt{f_{g}^{(1)}(x_1, q_0, \mu_0^2) \cdot f_{g}^{(1)}(x_1, q_1, \mu_2^2) \cdot F_1(t_1)}, \]

\[ f_{g,2}^{\text{off}} = \sqrt{f_{g}^{(2)}(x_2, q_0, \mu_0^2) \cdot f_{g}^{(2)}(x_2, q_2, \mu_2^2) \cdot F_1(t_2)}, \]  

(2.2)

where \( F_1(t_1) \) and \( F_1(t_2) \) are the isoscalar nucleon form factors. In the present work we shall use a few sets of unintegrated gluon distribution functions (UGDFs), which aim at description of phenomena where small gluon transverse momenta are involved. Some details concerning these distributions can be found in Ref. [24]. We shall follow notations there.

Following our previous work [7], the vertex factor \( V_{j,\bar{c}c}^{\text{c1c2}}(q_1, q_2) \) describes the coupling of two virtual gluons to \( \chi_{cJ} \)-meson follows from

\[ V_{j,\bar{c}c}^{\text{c1c2}}(q_1, q_2) = \mathcal{P}(q\bar{q} \to \chi_{cJ}) \cdot \Psi_{ik}^{\text{c1c2}}(q_1, q_2), \]  

(2.3)

where \( \mathcal{P}(q\bar{q} \to \chi_{cJ}) \) is the operator that projects the \( q\bar{q} \) pair onto the charmonium bound state (see below), \( \Psi_{ik}^{\text{c1c2}}(q_1, q_2) \) is the production amplitude of a pair of massive quark \( q \) and antiquark \( \bar{q} \) with momenta \( k_1, k_2 \), respectively.

Within the quasi-multi-Regge-kinematics (QMRK) approach [25] we have

\[ \Psi(c_1, c_2; i, k; q_1, q_2) = -g^2 (t^e_{ij} t^e_{jk} b(k_1, k_2) - t^e_{kj} t^e_{ji} b(k_2, k_1)), \quad \alpha_s = \frac{g^2}{4\pi}, \]  

(2.4)

where \( t^e \) are the colour group generators in the fundamental representation, \( b, \bar{b} \) are the effective vertices arising from the Feynman rules in QMRK

\[ b(k_1, k_2) = \gamma^- \hat{q}_1 - \hat{k}_1 - m \frac{(q_1 - k_1)^2 - m^2}{(q_1 - k_1)^2 - m^2} \gamma^+, \quad \bar{b}(k_1, k_2) = \gamma^+ \hat{q}_1 - \hat{k}_1 + m \frac{(q_1 - k_1)^2 - m^2}{(q_1 - k_1)^2 - m^2} \gamma^- . \]  

(2.5)

While projecting on the color singlet the \( ggg \)-vertex contributions disappear from the resulting matrix element, so we did not write them explicitly in Eq. (2.5). Taking into account standard definitions of the light-cone vectors \( n^+ = p_2 / E_{\text{cms}}, \ n^- = p_1 / E_{\text{cms}} \) and momentum decompositions \( q_1 = x_1 p_1 + q_1, q_2 = x_2 p_2 + q_2 \), and using the gauge invariance property (Gribov’s trick) one gets the following projection

\[ q_1^0 V_{j,\mu\nu}^{\text{c1c2}} = q_2^\mu V_{j,\mu\nu}^{\text{c1c2}} = 0, \]

\[ V_{j,\bar{c}c}^{\text{c1c2}}(q_1, q_2) = n_\mu n_\nu V_{j,\mu\nu}^{\text{c1c2}}(q_1, q_2) = \frac{4 q_{1,\mu}^t q_{2,\nu}^t}{x_1 x_2} V_{j,\mu\nu}^{\text{c1c2}}(q_1, q_2). \]  

(2.6)

Since we adopt here the definition of the polarization vectors proportional to gluon transverse momenta \( q_{1,2} \), then we must take into account the longitudinal momenta in the numerators of vertices (2.5).

Projection of the hard amplitude onto the singlet charmonium bound state \( V_{j,\mu\nu}^{\text{c1c2}} \) is given by the 4-dimensional integral over relative momentum of quark and antiquark \( q = (k_1 - k_2)/2 \) [17, 26]:

\[ V_{j,\mu\nu}^{\text{c1c2}}(q_1, q_2) = \mathcal{P}(q\bar{q} \to \chi_{cJ}) \cdot \Psi_{ik,\mu\nu}^{\text{c1c2}}(q_1, q_2) = 2\pi \cdot \sum_{i,k} \sum_{L_z,S_z} \frac{1}{\sqrt{m}} \int \frac{d^4q}{(2\pi)^4} \delta \left( q_0 - \frac{q^2}{M} \right) \times \Phi_{L=1, L_z}(q) \cdot \langle L = 1, L_z; S = 1, S_z; J, J_z \rangle \langle 3i, \bar{3}k|1 \rangle \text{Tr} \left\{ \Psi_{ik,\mu\nu}^{\text{c1c2}} \mathcal{P}_{S=1,S_z} \right\}, \]  

(2.7)

\[ \Psi_{ik,\mu\nu}^{\text{c1c2}} = -g^2 \sum_j \left[ t^c_{ij} t^c_{jk} \chi_{\nu} \left( \gamma^\nu \hat{q}_{1,t} - \hat{k}_{1,t} - m \frac{(q_1 - k_1)^2 - m^2}{(q_1 - k_1)^2 - m^2} \gamma^\mu \right) - t^c_{kj} t^c_{ji} \chi_{\mu} \left( \gamma^\mu \hat{q}_{1,t} - \hat{k}_{2,t} + m \frac{(q_1 - k_2)^2 - m^2}{(q_1 - k_2)^2 - m^2} \gamma^\nu \right) \right]. \]
where the function $\Phi_{L=1,L_z}(q)$ is the momentum space wave function of charmonium, and for a small relative momentum $q$ the projection operator $\mathcal{P}_{S=1,S_z}$ has the form
\[
\mathcal{P}_{S=1,S_z} = \frac{1}{2m} (\hat{k}_2 - m) \frac{\epsilon(S_z)}{\sqrt{2}} (\hat{k}_1 + m).
\] (2.8)

Since $P$-wave function $\Phi_{L=1,L_z}$ vanishes at the origin, we may expand the trace in Eq. (2.7) in Taylor series around $q = 0$, and only the linear terms in $q^n$ in the trace survive. This yields an expression proportional to
\[
\int \frac{d^3q}{(2\pi)^3} q^n \Phi_{L=1,L_z}(q) = -i \sqrt{\frac{3}{4\pi}} \epsilon^n(L_z) \mathcal{R}'(0),
\] (2.9)
with the derivative of the $P$-wave radial wave function at the origin $\mathcal{R}'(0)$ whose numerical values can be found in Ref. [27]. The general $P$-wave result (2.7) may be further reduced by employing the Clebsch-Gordan identity which for the vector $\chi_{cJ=1}$ charmonium reads
\[
T_{J=1}^{\sigma \rho} = \sum_{L_z,S_z} (1, L_z; 1, S_z|1, J_z) \epsilon^n(L_z) \epsilon^n(S_z) = -i \sqrt{\frac{1}{2}} \epsilon^{\sigma \rho \alpha \beta} \frac{P^\alpha}{M} \epsilon^\beta(J_z). \] (2.10)

**B. $gg \rightarrow \chi_{c(1^+)}$-vertex function**

Summarizing all ingredients above in Eqns. (2.6), (2.7), (2.9) and (2.10), we get the vertex factor in the following covariant form
\[
V_{J=1}^{\sigma \rho} = 2g^2 \delta^{\sigma \rho} \sqrt{\frac{6}{M \pi N_c M^2(q_1q_2)}} \epsilon^{\sigma \rho \alpha \beta} \epsilon^\beta(J_z) \left[ q_1^\sigma q_2^\rho (q_1^2 + q_2^2)(q_1^2 + q_2^2) - \right.
\]
\[\left. - \frac{2}{s} p_1^\rho p_2^\sigma \left( q_1^\sigma (2q_2^2 q_1q_2) - (q_1, q_2, t)(q_1^2 + q_2^2) - q_2^\rho (2q_1^2 q_1q_2) - (q_1, q_2, t)(q_1^2 + q_2^2) + \right) \right]. \] (2.11)

The general vertex function (2.11) possesses the Bose symmetry under simultaneous permutation of gluon momenta $q_1 \leftrightarrow q_2$ and polarisation vectors $n^+ \leftrightarrow n^-$ defined in Eq. (2.3), or, equivalently, under simultaneous permutations of protons ($p_1 \leftrightarrow p_2$) and gluons (both transverse $q_{1,t} \leftrightarrow q_{2,t}$ and longitudinal $x_1 p_1 \leftrightarrow x_2 p_2$) momenta.

We write the decomposition of the polarisation vector of a heavy meson with a given helicity $\lambda = 0, \pm 1$ as
\[
\epsilon^\beta(P, \lambda) = (1 - |\lambda|)n_3^\beta - \frac{1}{\sqrt{2}} (\lambda n_1^\beta + i|\lambda| n_2^\beta), \quad n_0^\mu = \frac{P^\mu}{M}, \quad n_\alpha^\mu n_\beta^\mu g_{\mu\nu} = g_{\alpha\beta}, \quad \epsilon^\mu(\lambda) \epsilon^\nu(\lambda') = -\delta^{\lambda\lambda'}.
\]

In the c.m.s. frame we choose the basis with collinear $n_3$ and $P$ vectors (so, we have $P = (E, 0, 0, P), P_2 = |P| > 0$) as a simplest one
\[
n_1^\beta = (0, 1, 0, 0), \quad n_2^\beta = (0, 0, 1, 0), \quad n_3^\beta = \frac{1}{M} (|P|, 0, 0, E), \quad |P| = \sqrt{E^2 - M^2}. \] (2.12)

Note, that we choose $n_2$ to be transverse to the c.m.s beam axis (see Fig. 2), while $n_1, n_3$
FIG. 2: Coordinate basis in the center-of-mass system of incoming protons $p_1, p_2$.

are turned around by the polar angle $\psi = [0 \ldots \pi]$ between $P$ and the c.m.s. beam axis. In the considered basis $\{n_1, n_2, n_3\}$ we have the following coordinates of the incoming protons

$$p_1 = \frac{\sqrt{s}}{2} (1, -\sin \psi, 0, \cos \psi), \quad p_2 = \frac{\sqrt{s}}{2} (1, \sin \psi, 0, -\cos \psi).$$

(2.13)

The gluon transverse momenta with respect to the c.m.s. beam axis are

$$q_{1,t} = (0, Q_{1,t}^x \cos \psi, Q_{1,t}^y, Q_{1,t}^z \sin \psi), \quad q_{2,t} = (0, Q_{2,t}^x \cos \psi, -Q_{2,t}^y, Q_{2,t}^z \sin \psi),$$

where $Q_{1/2,t}^x, \pm Q_{1/2,t}^y$ are the components of the gluon transverse momenta in the basis with the $z$-axis collinear to the c.m.s. beam axis.

From definition (2.13) it follows that energy of the meson and polar angle $\psi$ are related to covariant scalar products in the considered coordinate system as

$$E = \frac{(p_1P) + (p_2P)}{\sqrt{s}}, \quad \cos \psi = \frac{(p_1P) - (p_2P)}{\sqrt{s}|P|}, \quad \sin \psi = \frac{(p_2n_1) - (p_1n_1)}{\sqrt{s}}.$$  (2.14)

Further, we also see that from $q_1 = x_1p_1 + q_{1,t}, \quad q_2 = x_2p_2 + q_{2,t}$ and $q_1 + q_2 = P$ we have

$$x_1 = \frac{E + |P| \cos \psi}{\sqrt{s}}, \quad x_2 = \frac{E - |P| \cos \psi}{\sqrt{s}}.$$  (2.15)

Relations (2.14) and (2.15) show that the interchange of proton momenta $p_1 \leftrightarrow p_2$ is equivalent to the interchange of the angle $\psi \leftrightarrow \psi \pm \pi$, i.e. $\sin \psi \leftrightarrow -\sin \psi$ and $\cos \psi \leftrightarrow -\cos \psi$, simultaneously. The last permutation also provides the interchange of the longitudinal components of gluons momenta $x_1 \leftrightarrow x_2$.

Conservation laws provide us with the following relations between components of gluon transverse momenta and covariant scalar products

$$Q_{1,t}^x = -\frac{q_{1,t}^2 + (q_{1,t}q_{2,t})}{|P| \sin \psi}, \quad Q_{2,t}^x = -\frac{q_{2,t}^2 + (q_{1,t}q_{2,t})}{|P| \sin \psi}, \quad Q_{t}^y = \frac{\sqrt{q_{1,t}^2q_{2,t}^2 - (q_{1,t}q_{2,t})^2}}{|P| |\text{sign}(Q_{t}^y)|},$$

$$P_{t}^2 = -|P_t|^2 = -|P|^2 \sin^2 \psi = q_{1,t}^2 + q_{2,t}^2 + 2(q_{1,t}q_{2,t}), \quad q_{1/2,t}^2 = -|q_{1/2,t}|^2,$$

where $|P_t| = |P||\sin \psi|$ is the meson transverse momentum with respect to $z$-axis. The appearance of the factor $\text{sign}(Q_{t}^y)$ guarantees the applicability of Eq. (2.16) for positive and
Let us consider firstly the limit of the "coherent" scattering processes. The sections are sensitive to larger values of gluon transverse momenta than, e.g., in the case of necessary to get a nonzero cross section. It also means that the amplitude and the cross section when \( q \) production amplitude.

The calculation leads to the following vertex function in these coordinates

\[
V_{j=1,\lambda}^{c_1c_2} = -8g^2\delta^{c_1c_2}\sqrt{\frac{6}{M\pi N_c}} \frac{\mathcal{R}'(0)}{|P_t|(M^2 - q_{1,t}^2 - q_{2,t}^2)^2} \left\{ \frac{1}{\sqrt{2}} i|\lambda|(q_{1,t}^2 - q_{2,t}^2)(q_{1,t}q_{2,t})\text{sign}(\sin \psi) + \lambda(q_{1,t}^2 + q_{2,t}^2) |[q_{1,t} \times q_{2,t}] \times n_1| \text{sign}(Q)^y \text{sign}(\cos \psi) \right\} + (1 - |\lambda|)(q_{1,t}^2 + q_{2,t}^2) |[q_{1,t} \times q_{2,t}] \times n_3| \text{sign}(Q)^y \text{sign}(\sin \psi) \right\} \tag{2.16}
\]

where

\[
|[q_{1,t} \times q_{2,t}] \times n_1| = \sqrt{q_{1,t}^2 q_{2,t}^2 - (q_{1,t}q_{2,t})^2} |\cos \psi|, \\
|[q_{1,t} \times q_{2,t}] \times n_3| = \frac{E}{M} \sqrt{q_{1,t}^2 q_{2,t}^2 - (q_{1,t}q_{2,t})^2} |\sin \psi|.
\]

The amplitude (2.16) explicitly obeys the Bose symmetry under the interchange of gluon momenta and polarisations due to resulting simultaneous permutations \( \cos \psi \leftrightarrow -\cos \psi, \sin \psi \leftrightarrow -\sin \psi \) and \( Q^y \leftrightarrow -Q^y \).

A short inspection of Eq. (2.16) shows that

\[
V_{j=1,\lambda}^{c_1c_2}(q_{1,t}, q_{2,t}) \to 0 \tag{2.17}
\]

when \( q_{1,t} \to 0 \) or \( q_{2,t} \to 0 \). It shows that gluon transverse momenta (gluon virtualities) are necessary to get a nonzero cross section. It also means that the amplitude and the cross section are sensitive to larger values of gluon transverse momenta than, e.g., in the case of \( \chi_c(0^+) \) production.

It follows from the conservation laws that

\[
q_{1t} + p'_{1t} = -q_{0t}, \quad q_{2t} + p'_{2t} = q_{0t}, \quad P_t = -(p'_{1t} + p'_{2t})
\]

Let us consider firstly the limit of the “coherent” scattering protons \( p'_{1t} = p'_{2t} \equiv p_t \), so

\[
q_{1t} = -(p_t + q_{0t}), \quad q_{2t} = -(p_t - q_{0t}), \quad P_t = -2p_t, \quad p_t^y = 0. \tag{2.18}
\]

The production vertex (2.16) in this limit and considered coordinates has a form

\[
V_{j=1,\lambda}^{c_1c_2}(q_{0t}^x, q_{0t}^y, p_t) = -16g^2\delta^{c_1c_2}\sqrt{\frac{3}{M\pi N_c}} \frac{\mathcal{R}'(0)}{(M^2 - 2(p_t^2 + q_{0t}^2))} \left\{ i|\lambda|(q_{0t}^2 - p_t^2)q_{0t}^x \text{sign}(\sin \psi) + (p_t^2 + q_{0t}^2)q_{0t}^y \left[ \lambda \cos \psi + \frac{\sqrt{2}E}{M} (1 - |\lambda|) \sin \psi \right] \right\} \tag{2.19}
\]

\footnote{We are thankful to M. G. Ryskin for enlightening correspondence on the issues of Bose symmetry of production amplitude.
This vertex is antisymmetric w.r.t. simultaneous interchanges \( q_{0t}^{x} \leftrightarrow -q_{0t}^{x} \) and \( q_{0t}^{y} \leftrightarrow -q_{0t}^{y} \):

\[
V_{J=1,\lambda}^{c_{1}c_{2}}(q_{0t}^{x},q_{0t}^{y},p_{t}) = -V_{J=1,\lambda}^{c_{1}c_{2}}(-q_{0t}^{x},-q_{0t}^{y},p_{t})
\]  \tag{2.20}

In the considered “coherent” limit \([2.18]\) the integrand of the diffractive amplitude

\[
V_{J=1,\lambda}^{c_{1}c_{2}}(q_{0t}^{x},q_{0t}^{y},p_{t}) \cdot f_{g,1}^{off}(x_{1},x_{1}',q_{0t}^{2},(p_{t} + q_{0t})^{2},t_{1}) f_{g,2}^{off}(x_{2},x_{2}',q_{0t}^{2},(p_{t} - q_{0t})^{2},t_{2})
\]

will be antisymmetric only if \( x_{1} = x_{2} = E/\sqrt{s} \equiv x \) (while \( x_{1}' \sim x_{2}' \ll x_{1,2} \)), i.e. in the case when \( y = 0 \), while the deviation from zero at \( y \neq 0 \) manifests the violation of Regge factorization which was used to examine this limit earlier \([3]\). So, the diffractive amplitude in this case

\[
\mathcal{M}_{y \rightarrow 0} \sim F_{1}(t_{1})F_{1}(t_{2}) \int dq_{0t}^{x}dq_{0t}^{y} V_{J=1}(q_{0t}^{x},q_{0t}^{y},p_{t}) \cdot f(x,q_{0t}^{2},t_{1})f(x,q_{0t}^{2},t_{2}) = 0.
\]

In the forward limit \( p_{t} \rightarrow 0 \) (which is the particular case of coherent one) the amplitude turns to zero at any \( y \). Indeed, we have \( P_{t} \rightarrow 0 \) and \( \sin \psi \rightarrow \pm 0 \) and the amplitude turns into

\[
V_{J=1,\lambda}^{c_{1}c_{2}}(q_{0t}^{x},q_{0t}^{y},p_{t} \rightarrow 0) = -16g^{2}2^{c_{1}c_{2}}R'(0)\sqrt{\frac{3}{M\pi N_{c}}} \times \frac{q_{0t}^{2}}{(M^{2} - 2q_{0t}^{2})^{2}} \left\{ i\lambda q_{0t}^{x} \text{sign}(\sin \psi)\big|_{\psi \rightarrow 0,\pi} + \lambda q_{0t}^{y} \text{sign}(\cos \psi)\big|_{\psi \rightarrow 0,\pi} \right\}
\]  \tag{2.21}

As in the previous case, it is obviously antisymmetric under interchanges \( q_{0t}^{x} \leftrightarrow -q_{0t}^{x} \) and \( q_{0t}^{y} \leftrightarrow -q_{0t}^{y} \). Since in this case \( q_{1t} = -q_{0t}, q_{2t} = q_{0t} \), then the diffractive amplitude has an antisymmetric integrand and turns to zero

\[
\mathcal{M}_{p_{t} \rightarrow 0} \sim F_{1}(t_{1})F_{1}(t_{2}) \int dq_{0t}^{x}dq_{0t}^{y} V_{J=1}(q_{0t}^{x},q_{0t}^{y},p_{t} \rightarrow 0) \cdot f(x_{1},q_{0t}^{2},t_{1})f(x_{2},q_{0t}^{2},t_{2}) = 0.
\]

This explicitly confirms the observation made in Refs. \([3, 5]\)^3.

It is also possible to express the results in terms of transverse 3-momenta of fusing off-shell gluons \(|\mathbf{q}_{1,t}|\) and \(|\mathbf{q}_{2,t}|\), and the angle between them \( \phi \) in the center-of-mass system of colliding nucleons with \( z \)-axis fixed along meson momentum \( \mathbf{P} \). In this case, summing the squared matrix elements over meson polarizations we get the expression

\[
\frac{|\mathbf{q}_{1,t}|^{2}|\mathbf{q}_{2,t}|^{2}}{((|\mathbf{q}_{1,t}|^{2} + |\mathbf{q}_{1,t}|^{2})^{2} \sin^{2}\phi + M^{2}(|\mathbf{q}_{1,t}|^{2} + |\mathbf{q}_{2,t}|^{2} - 2|\mathbf{q}_{1,t}||\mathbf{q}_{2,t}| \cos \phi))^{4}}
\]

equal (up to different normalisations of gluon polarization vectors) to the one derived in Ref. \([18]\),

\(^{3}\) We are grateful to V. A. Khoze for very interesting and helpful correspondence on this problem.
C. Three-body phase space

At high energies and small momentum transfers the phase space volume element can be written as

\[ d^3PS = \frac{1}{2^8 \pi^4} dt_1 dt_2 d\xi_1 d\xi_2 d\Phi \delta(s(1-\xi_1)(1-\xi_2) - M^2), \]  

(2.22)

where \( \xi_1, \xi_2 \) are longitudinal momentum fractions carried by outgoing protons with respect to their parent protons and the relative angle between outgoing protons \( \Phi \in (0, 2\pi) \). Changing variables \( (\xi_1, \xi_2) \rightarrow (x_F, M^2) \) one gets

\[ d^3PS = \frac{1}{2^8 \pi^4} dt_1 dt_2 \frac{dx_F}{s \sqrt{x_F^2 + 4(M^2 + |P_{M,t}|^2)/s}} d\Phi. \]  

(2.23)

III. RESULTS

Let us start from presenting the differential cross sections. In Fig. 3 we show distributions in rapidity \( y \) for different UGDFs from the literature. The results for different UGDFs significantly vary. The biggest cross section is obtained with BFKL UGDF and the smallest one with Gaussian UGDFs. The big spread of the results is due to quite different distributions of UGDFs in gluon transverse momenta \( q_1t, q_2t \), although when integrated over transverse momenta distributions in longitudinal momentum fractions \( x_1, x_2 \) are fairly similar.

![Fig. 3: Distributions in rapidity of \( \chi_c(1^+) \) meson (left panel) and \( \chi_c(0^+) \) meson (right panel) for different UGDFs. Dash-dotted line corresponds to BFKL UGDF, long-dashed line – GBW, short-dashed line – KL, and two solid lines – Gaussian UGDFs for \( \sigma_0 = 0.5 \text{ GeV}^2 \) (upper line) and \( \sigma_0 = 1.0 \text{ GeV}^2 \) (lower line).](image)

Comparing the left and right panels, the cross section for the axial-vector \( \chi_c(1^+) \) production is much smaller (more than an order of magnitude) than the cross section for the scalar
\(\chi_c(0^+)\) production. This is related to the Landau-Yang theorem, which “causes” vanishing of the cross section for on-shell gluons. For axial-vector quarkonia the effect is purely of off-shell nature and is due to the interplay of the off-shell matrix element and off-diagonal UGDFs. This interplay causes a huge sensitivity of differential distributions to UGDFs observed in Fig. 3.

In Fig. 4 we show corresponding distributions in \(t = t_1\) or \(t = t_2\) (identical) again for different UGDFs. Except of normalisation the shapes are rather similar. This is because of the \(t_1\) and \(t_2\) dependencies of form factors, describing the off-diagonal effect, taken the same for different UGDFs.

In Fig. 5 we show the correlation function in relative azimuthal angle between outgoing protons. The shapes of the distributions are almost independent of UGDFs. In the case when energy resolution is not enough to separate contributions form different states of \(\chi_c\) (\(\chi_c(0^+), \chi_c(1^+), \chi_c(2^+)\)) the distribution in relative azimuthal angle may, at least in principle, be helpful.

Summarizing differential distributions, the cross sections (especially their absolute normalisation) strongly depend on the model of UGDF. In spite of the huge uncertainty in predicting the absolute cross section it becomes obvious that the cross section for \(\chi_c(0^+)\) is much bigger than the cross section for the \(\chi_c(1^+)\) production. This result could be expected based on the Landau-Yang theorem. However, the size of the suppression cannot be predicted without actual calculations.

In Table 1 we have collected cross sections integrated in \(y, t_1, t_2, \phi\) over the full phase space for the Tevatron energy \(W = 1960\) GeV. More than an order of magnitude suppression of \(\chi_c(1^+)\) relative to \(\chi_c(0^+)\) can be seen by comparing numbers in appropriate columns\(^4\).

\(^4\) This ratio should be only weakly modified by absorption effects.
FIG. 5: Distribution in relative azimuthal angle $\Phi$ of $\chi_c(1^+)$ (left panel) and $\chi_c(0^+)$ (right panel) meson production for different UGDFs.

TABLE I: Integrated cross section $\sigma_{tot}$ (in nb) for exclusive $\chi_c(0^+)$ and $\chi_c(1^+)$ production for different UGDFs and the Tevatron energy $W = 1960$ GeV. Branching ratios of radiative decays were taken from [29]: $\text{BR}(\chi_c(0^+) \to J/\Psi \gamma) = 0.0128$ and $\text{BR}(\chi_c(1^+) \to J/\Psi \gamma) = 0.36$.

| UGDF   | $\chi_c(0^+)$ | $\chi_c(1^+)$ | ratio |
|--------|---------------|---------------|-------|
|        | $\sigma_{tot}$ | $\text{BR} \cdot \sigma_{tot}$ | $\sigma_{tot}$ | $\text{BR} \cdot \sigma_{tot}$ | $\frac{\text{BR} \cdot \sigma_{tot}(\chi_c(1^+))}{\text{BR} \cdot \sigma_{tot}(\chi_c(0^+))}$ |
| KL     | 55.2          | 0.7           | 0.5   | 0.2   | 0.3 | |
| GBW    | 160           | 2             | 4.2   | 1.5   | 0.8 | |
| BFKL   | 1200          | 15.4          | 14.2  | 5.1   | 0.3 | |
| Gauss, $\sigma_0 = 0.5$ GeV | 26 | 0.3 | 0.2 | 0.09 | 0.3 | |
| Gauss, $\sigma_0 = 1.0$ GeV | 2.2 | 0.03 | 0.02 | 0.006 | 0.2 | |

The best method to measure $\chi_c$ mesons at the Tevatron is via $\gamma + J/\Psi$ decay channel. All P-wave $\chi_c$-quarkonia decay into this channel. However, the branching fractions to this channel are very different [23]. While the branching fraction for $\chi_c(0^+)$ is very small (of the order of 1 %), the branching fraction for $\chi_c(1^+)$ is one and half order of magnitude larger. In the third and fifth columns we present the total cross sections multiplied by the appropriate branching fractions. After multiplying the cross section by the branching fraction for $\gamma + J/\Psi$ decay the situation somewhat changes, i.e. now $\chi_c(1^+)$ becomes closer to $\chi_c(0^+) - \text{BR} \cdot \sigma_{tot}(\chi_c(1^+))$ is about two times smaller than that for $\chi_c(0^+)$ for all UGDFs used in our calculation, so it seems to be almost model independent statement.
In a preliminary analysis the CDF collaboration [15] assumes that the observed strength comes dominantly from $\chi_c(0^+)$, thus conforming results of our investigation. In order to make comparison with the experimental results one would still need to include experimental cuts on lepton and photon rapidities and transverse momenta. Also including absorption effects may be important as slightly larger absorption can be expected for $\chi_c(1^+)$ (harder distributions in $t_1$ and $t_2$ -- see Fig. 4). These points need further studies.

Our calculation suggests that the inclusion of $\chi_c(1^+)$ in the experimental analysis is not negligible and may be necessary. The present energy resolution does not allow for separating different P-wave quarkonia. Perhaps, looking to other decay channels may help in disentangling the contributions from different states and allowing for extracting the cross sections separately for each of them.

Another interesting option to shed more light to the problem is to study the angular distributions of outgoing $J/\psi$ in the $\chi_c$ rest frame. Different states should have, in principle, different distributions. We leave the analysis of those distributions for a separate study.

| Table II: Integrated cross section $\sigma_{\text{tot}}$ (in nb) for exclusive $\chi_c(1^+)$ production at different energies. |
|---------------------------------------------------------------|
| UGDF  | RHIC  | Tevatron  | LHC  |
|-------|-------|-----------|------|
| KL    | 0.05  | 0.5       | 1.7  |
| GBW   | 0.04  | 4.2       | 73.1 |
| BFKL  | 0.07  | 14.2      | 1064 |
| Gauss, $\sigma_0 = 0.5$ GeV | 0.007 | 0.2       | 2.5  |
| Gauss, $\sigma_0 = 1.0$ GeV | 0.0005 | 0.02    | 0.2  |

Finally in Table 2 we present the total cross sections for $\chi_c(1^+)$ also for RHIC and LHC energies. The question of separation of different $\chi_c$ states should be similar, except that other decay channels should be available [13, 14].

IV. CONCLUSIONS AND DISCUSSION

Our results can be summarized as follows:

We have derived the QCD amplitude for exclusive elastic double diffractive production of axial-vector $\chi_c(1^+)$ meson. According to the Landau-Yang theorem the amplitude vanishes for the fusion of on-shell gluons. In the present analysis we have generalized the formalism proposed recently for diffractive production of the Higgs boson. We have derived corresponding $g^*g^* \to \chi_c(1^+)$ vertex function. Our effect is purely off-shell type, i.e. requires off-shell gluons, which demands nonvanishing transverse momenta of gluons in the high-energy regime.

We have calculated the corresponding differential cross sections. Different unintegrated gluon distributions from the literature have been used. The absolute cross section is very sensitive to the choice of UGDF in contrast to the shapes of distributions. The predicted total (integrated over phase space) cross section, obtained from the bare amplitude, is from a fraction to several nanobarns, depending on the model of UGDFs. This is one and a
half order of magnitude less than a similar cross section for $\chi_c(0^+)$ \[7\]. This is a direct consequence of the Landau-Yang theorem. However, because the branching fraction $BR(\chi_c(1^+) \to J/\psi + \gamma) \gg BR(\chi_c(0^+) \to J/\psi + \gamma)$, one may expect a different situation in the $J/\psi + \gamma$ channel. This has an analogy with the inclusive production of P-wave quarkonia, where the signal (in the $J/\psi + \gamma$ channel) of $\chi_c(1^+)$ is larger than that for $\chi_c(0^+)$. We have observed that $BR \cdot \sigma_{tot}(\chi_c(1^+))$ is (only) several times smaller than that for $\chi_c(0^+)$ for all UGDFs used in our calculation.

Moreover, we have also calculated \[30\] differential cross sections for different spin polarizations of $\chi_c(1^+)$. The integrated cross section for spin polarization $\lambda = \pm 1$ is approximately an order of magnitude greater than that for the $\lambda = 0$ polarization. Similar observation has already been made in Refs. \[3, 4\] and verified by the WA102 data for $f_1(1285), f_1(1420)$ production \[31\]. The ratio of the cross sections integrated over the phase space is only weekly dependent on UGDFs but strongly depends on $t_1$ and $t_2$.

In the present analysis we have neglected the absorption effects. The latter clearly go beyond the scope of the present analysis. At the Tevatron energies they lead, however, to a large damping of the cross section. In zeroth approximation they can be taken into account by multiplying the cross section by a so-called soft survival probability \[1, 2\]. At the Tevatron energies the soft survival probability is of the order of 0.1 \[1, 2\]. A better approximation is to convolute the bare amplitude with nucleon-nucleon elastic (re)scattering amplitude (see e.g. Refs. \[9, 11, 32\]). We leave the inclusion and discussion of the absorption/rescattering effects for a separate analysis, including all quarkonium states ($\chi_c(0^+), \chi_c(1^+), \chi_c(2^+)$).

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[1] V.A. Khoze, A.D. Martin and M.G. Ryskin, Phys. Lett. B 401, 330 (1997);
V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C 23, 311 (2002).
[2] A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C 33, 261 (2004).
[3] A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C 31, 387 (2003) [arXiv:hep-ph/0307064].
[4] F. E. Close, Phys. Lett. B 419, 387 (1998) [arXiv:hep-ph/9710450];
F. E. Close and G. A. Schuler, Phys. Lett. B 464, 279 (1999) [arXiv:hep-ph/9905305];
F. E. Close and G. A. Schuler, Phys. Lett. B 458, 127 (1999) [arXiv:hep-ph/9902243].
[5] F. Yuan, Phys. Lett. B 510, 155 (2001).
[6] V.A. Khoze, A.D. Martin, M.G. Ryskin and W.J. Stirling, Eur. Phys. J. C 35, 211 (2004).
[7] R. S. Pasechnik, A. Szczurek and O. V. Teryaev, Phys. Rev. D 78 (2008) 014007, arXiv:0709.0857 [hep-ph].
[8] A. Szczurek, R. S. Pasechnik and O. V. Teryaev, Phys. Rev. D 75, 054021 (2007) arXiv:hep-ph/0608302.
[9] W. Schäfer and A. Szczurek, Phys. Rev. D76 (2007) 094014.
[10] A. Bzdak, L. Motyka, L. Szymanowski and J.-R. Cudell, hep-ph/0702134, Phys. Rev. D75 (2007) 094023.
[11] A. Rybarska, W. Schäfer and A. Szczurek, Phys. Lett. B668 (2008) 126.
[12] J. Pinfold, a talk at the international conference PHOTON2007, Paris, Sorbonne, July 2007; J. Pinfold, a talk at the international conference MESON2008, Cracow, June 2008.
[13] W. Guryn, private communication.
[14] R. Schicker, private communication.
[15] M. Albrow, a talk at the international conference DIFFRACTION2008, La Londe Les Maures, France, September 2008.
[16] L.D. Landau, Dokl. Akad. Nauk. USSR 60 (1948) 207; C.N. Yang, Phys. Rev. 17 (1950) 242.
[17] P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. V. Teryaev, Phys. Rev. Lett. 86, 1446 (2001) arXiv:hep-ph/0004263.
[18] B. A. Kniehl, D. V. Vasin and V. A. Saleev, Phys. Rev. D 73, 074022 (2006) arXiv:hep-ph/0602179.
[19] A. K. Likhoded and A. V. Luchinsky, Phys. Atom. Nucl. 71, 294 (2008) arXiv:hep-ph/0703091.
[20] S. P. Baranov and A. Szczurek, Phys. Rev. D 77, 054016 (2008) arXiv:0710.1792 [hep-ph]; S. P. Baranov, Phys. Lett. B 594, 277 (2004).
[21] M. Melis, F. Murgia and J. Parisi, Phys. Rev. D 70, 034021 (2004) arXiv:hep-ph/0404070.
[22] W.-Y. Keung, I. Low and J. Shu, Phys. Rev. Lett. 101 (2008) 091802.
[23] W. M. Yao et al. (Particle Data Group), Jour. Phys. G33 1 (2006).
[24] M. Luszczak and A. Szczurek, Phys. Rev. D73, 054028 (2006).
[25] V. S. Fadin and L. N. Lipatov, Nucl. Phys. B 477, 767 (1996) arXiv:hep-ph/9602287; V. S. Fadin, R. Fiore, A. Flachi and M. I. Kotsky, Phys. Lett. B 422, 287 (1998) arXiv:hep-ph/9711427; V. Fadin, ”BFKL News”, Talk given at ”LISHEP98”, LAFEX school on high energy physics, February 14-21, Rio de Janeiro, Brazil, 1998, hep-ph/9807528.
[26] P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. Teryaev, Phys. Rev. D 62, 071502 (2000) arXiv:hep-ph/0002077; Ph. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. V. Teryaev, Phys. Rev. D 63, 077501 (2001) arXiv:hep-ph/0008316.
[27] E. J. Eichten and C. Quigg, Phys. Rev. D 52, 1726 (1995) arXiv:hep-ph/9503356.
[28] N.I. Kochelev, T. Morii and A.V. Vinnikov, Phys. Lett. B457 (1999) 202.
[29] C. Amsler et al. (Particle Data Group), Phys. Lett. B667 1 (2008).
[30] R. S. Pasechnik, A. Szczurek and O. V. Teryaev, work in progress.
[31] D. Barberis et al. [WA102 Collaboration], Phys. Lett. B 440, 225 (1998) arXiv:hep-ex/9810003.
[32] V.A. Petrov, R.A. Ryutin, A.E. Sobol and J.-P. Guillaud, arXiv:hep-ph/0409118.