Can black holes exist in an accelerating universe?

Pedro F. González-Díaz.
Instituto de Matemáticas y Física Fundamental
Consejo Superior de Investigaciones Científicas
Serrano 121, 28006 Madrid, SPAIN

May 15, 2002

Abstract

This letter deals with an analysis of the space-time static metric that corresponds to a quintessential state equation with constant characteristic parameter. Following a procedure parallel to as it is used in the case of de Sitter space, we have tried to generalize the metric components that correspond to the quintessential case to also embrace black-hole terms and shown that this is not possible. We argue therefore that, in the absence of a cosmological constant, black holes seem to be prevented in a cosmological space-time which is asymptotically accelerating when the acceleration is driven by a quintessence-like field.

Report-no: IMAFF-RCA-02-05
Some time ago it was not infrequent to witness scientific debates, both at meetings and private desks, on whether black holes may actually exist. Many physicists were then reluctant to accept black holes to indeed be among the existing physical objects. Even Einstein showed a very critical attitude against the existence of black holes and advanced some proofs for their nonexistence [1]. Since then, the situation has dramatically shifted to favor the opinion that black holes must exist and be formed by gravitational collapse in many places of the universe, including star binaries and the center of galaxies. Although nobody has actually found a completely convincing case in which the black-hole occurrence is out of any doubt [2], black holes are currently being also used as the engine to deliver the high energies observed in many astrophysical processes, and often claims that a black hole has been observed in binaries or at galactic centers are being released by the press.

Basing on the gravitational physics induced by a cosmic quintessential field [3], we argue in this letter that the present accelerating expansion of the universe [4,5] may not be compatible with the occurrence of large black holes, provided that the accelerating expansion is driven by a dynamic vacuum scalar field with negative pressure which did not imply a future cosmic event horizon [6,7] and, therefore, that the fundamental string and M theories be mathematically consistent [8]. Even in this case, room is left however for the existence of primordial black holes [9] during the whole cosmic period from big bang to the onset of the accelerating regime. If the recent results in supernova type Ia [4,5] are nevertheless explained by means of a positive cosmological constant, any kinds of black holes are allowed, but then no string or M theory could be consistently defined. Our result becomes then a kind of cosmic dilemma by which one must somehow choose between black holes and fundamental string or M theories.

The Schwarzschild solution is usually interpreted as describing a black hole of given
mass $M$ in an asymptotically flat space. There is also the straightforward generalization to the case of nonzero positive cosmological constant $\Lambda$, which is provided by the so-called Schwarzschild-de Sitter static metric [10,11], which represents a black hole in asymptotically de Sitter space [12]. This solution reads

$$ds^2 = - \left( 1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} \right) dt^2 + \left( 1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} \right)^{-1} dr^2 + r^2 d\Omega_2^2,$$

(1)

where $d\Omega_2^2$ is the metric on the unit three-sphere. This generalization of Schwarzschild space can actually be also viewed as a generalization of de Sitter space in static coordinates because of the following argument. If we assume for the considered static metric a general spherically-symmetric ansatz

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega_2^2,$$

(2)

in the case of a de Sitter space the metric components $A(r) = B(r)^{-1}$ satisfies a differential equation which is the particular case for $\omega = -1$ of the most general differential equation that corresponds to a static, spherically-symmetric space for a quintessence-like vacuum scalar field with equation of state $p = \omega \rho$ [3,13]

$$\left( \frac{A'A^{-1(1+\omega)/(2\omega)}}{r^{(1+2\omega)/\omega}} \right)' + \left( \frac{A^{(\omega-1)/(2\omega)}}{r^{(1+3\omega)/\omega}} \right)' + \frac{\alpha (A'r^2)'}{8\pi G\omega r^{(1+6\omega)/\omega}} = 0,$$

(3)

where the prime $'$ denotes differentiation with respect to the radial coordinate $r$, $\alpha$ is an integration constant and the state equation parameter $\omega$ runs in the interval $-1/3 \geq \omega \geq -1$, and either [13]

$$B = 1 + \frac{\left[ (A')^{-2\omega/(1+\omega)} A'r^2 \right]'}{A(A')^{-(3\omega+1)/(\omega+1)} \left( A'r + \frac{2(3\omega+1)A}{\omega+1} \right)},$$

(4)

for $-1/3 > \omega > -1$, or $B = A^{-1}$ for $\omega = -1$. Note that Eqs (3) and (4) admit a particular solution for any $\omega$ other than $\omega = -1$ which has the form $A \propto r^{-1}$, $B = 1 - r^{-2}$, and is singular at $r = 0$ but does not show any event horizon.
At the extreme de Sitter case where \( \omega = -1 \) for which the positive cosmological constant is given by \( \Lambda = 8\pi G/\alpha \), Eq. (3) reduces to

\[
\left( \frac{A'}{r} \right)' + \left( \frac{A}{r^2} \right)' = \frac{1}{\Lambda r^5} \left( A'r^2 \right)'.
\] (5)

Of course, a solution to Eq. (4) is the well-known static de Sitter metric \( A = B^{-1} = 1 - \Lambda r^2/3. \) In addition, there is the generalization of this solution to a de Sitter space-time containing a black hole when we make the transformation \( A \rightarrow \tilde{A} = A(r) + \xi(r) \), where \( \xi(r) \) is also a function of the radial coordinate \( r \) that satisfies the following two conditions separately

\[
\left( \frac{\xi'}{r} \right)' + \left( \frac{\xi}{r^2} \right)' = 0, \quad \xi' r^2 = \text{const}.
\] (6)

Eq. (5) is then invariant under \( A \rightarrow \tilde{A} \) if \( \xi = \text{const.}/r \), so that we finally obtain the Schwarzschild-de Sitter solution \( \tilde{A} = \tilde{B}^{-1} = 1 - \Lambda r^2 - 2GM/r \), after fixing \( \text{const.} = -2GM \) in the limit \( \alpha \rightarrow \infty \).

In order to investigate whether there could exist black holes in asymptotically accelerating cosmological spaces corresponding to a dynamic vacuum with a quintessence-like equation of state characterized by a constant parameter \(-1/3 > \omega > -1\), rather than \( \omega = -1 \) of de Sitter space, let us try to follow a procedure parallel to that we have used for the case that we have just a positive cosmological constant, starting also with Eq. (3). We first notice that in all cases where \( \omega \neq -1, -1/3 \), this general differential equation can be cast in a simpler form if we (i) re-define \( A(r) \) such that

\[
D(r) = A(r)^{(\omega-1)/(2\omega)},
\] (7)

and (ii) use the general solution to Eq. (3) for \(-1/3 > \omega > -1\), i.e. [13]

\[
D(r) = kr^{2(\omega-1)/(\omega+1)},
\] (8)
where
\[ \kappa \equiv \kappa(\omega, \alpha) = \left( \frac{2\pi G (\omega^2 + 6\omega + 1)}{\omega \alpha} \right)^{(\omega-1)/(\omega+1)}. \] (9)

In terms of the new function \( D(r) \), the differential equation (3) can then be re-written as
\[ \left( \frac{D'}{r^{(2\omega+1)/\omega}} \right)' + \left( \frac{\omega - 1}{2\omega} \right) \left( \frac{D}{r^{(3\omega+1)/\omega}} \right)' + \frac{\alpha \kappa^{(\omega+1)/(\omega-1)}}{8\pi G \omega r^{(6\omega+1)/\omega}} (D' r^4)' = 0. \] (10)

This equation is obviously satisfied by solution (8). What we shall try to see now is whether it will be also satisfied by the generalized function \( \tilde{D} \), defined by
\[ D \rightarrow \tilde{D} = D(r) + \psi(r), \] (11)
in such a way that, analogous to the function \( \xi(r) \) in de Sitter space, the function \( \psi(r) \) will satisfy the following two conditions:
\[ \left( \frac{\psi'}{r^{(2\omega+1)/\omega}} \right)' + \left( \frac{\omega - 1}{2\omega} \right) \left( \frac{\psi}{r^{(3\omega+1)/\omega}} \right)' = 0, \quad \psi' r^4 = \text{const.}, \] (12)
so that Eq. (11) is satisfied by the solution \( \psi(r) \) too. It can however be straightforwardly checked that for the two conditions (12) to be simultaneously satisfied by \( \psi(r) \) it is necessary that \( (\omega - 1)/(2\omega) = 3 \), which can only hold for \( \omega = -1/5 \), i.e. outside the range that corresponds to the accelerating solutions \( -1/3 > \omega > -1 \). This shows that, even though for \( \omega = -1 \) the de Sitter (or Schwarzschild) static solution can be consistently generalized to the Schwarzschild-de Sitter metric, so allowing black holes to exist in asymptotically de Sitter space, Schwarzschild black holes are not allowed to occur in universes which asymptotically tend to a spherically symmetric space that corresponds to the accelerating cosmological spaces whose expansion is driven by a quintessence-like field with an equation of state characterized by a constant parameter \( -1/3 > \omega > -1 \). It is worth noticing that even for the case \( \omega = -1/5 \) we had no Schwarzschild event horizon at all, as for in that case we obtain \( \psi \propto D \propto r^{-3} \).
The question now arises: if the present accelerated expansion of the universe is driven by a quintessence field, is then the above result a definitive proof for the nonexistence of Schwarzschild black holes (or actually any of their generalizations with charge or angular momentum) in the accelerating universe we live in?. Even though the considered values of $\omega, -1/3 > \omega > -1$, strictly make the usual sense only for homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetimes, if we keep an equation of state $p = \omega \rho$ also in the case of spacetimes with static, spherically symmetric coordinates, one can as well obtain the static metrics which correspond to the above $\omega$-values [13] for a given fixed relation between the energy density and the metric components, much in the same way as the static metric for de Sitter or Schwarzschild-de Sitter spaces can be derived from the Einstein equations for static, spherically coordinates and an equation of state $p = \omega \rho$, whenever we set $\omega = -1$. That static metric for de Sitter space can, in fact, be directly related with the corresponding FRW de Sitter metric obtained from the Friedmann equations also for $\omega = -1$ by means of an embedding in a common five-dimensional hyperboloid. Thus, to the extent at which Schwarzschild-de Sitter metric be interpreted as representing a black hole in asymptotically de Sitter space, our result should in fact imply a positive answer to the above question. Schwarzschild, Kerr and Reissner-Nordström black holes could not then exist in an accelerating universe. Moreover, since black holes with sizes larger than that of the atomic scale would completely evaporate off by the Hawking process only in a time which is much longer than the present age of the universe, it appears that for quintessence-driven accelerated expansion none of the known black holes with masses larger than $\sim 10^{15}$ g could exist at any time along the entire cosmological evolution, from big bang to now.

However, even though neither Schwarzschild event horizon nor any of its rotating
or charged generalizations can exist in an accelerating universe, what such a universe might still have is a new kind of generalized horizons which would in turn be forbidden for the case $\omega = -1$. These possibilities would correspond to those remaining generalizations which, together with $A \rightarrow \bar{A}$, $\bar{B} = \bar{A}^{-1}$, will exhaust all possible generalizations for the type of spacetimes we are considering. Let us introduce then the additional transformation

$$D \rightarrow \bar{D} = D(r) \exp[\chi(r)]. \quad (13)$$

Insertion of this transformation into the differential equation (10) and further separate application of this equation to $D(r)$ alone leads then to:

$$\exp[\chi(r)] = C_0 + C_1 \left( \frac{\omega + 1}{2(\omega - 1)} \right) \int x^\frac{m(\omega + 1)}{2(\omega - 1)} \left( \frac{\omega - 3}{\omega + 1} \right) e^{-x} dx, \quad (14)$$

where $C_0$ and $C_1$ are arbitrary integration constants, and we have introduced the definitions

$$x = r^{2(\omega - 1)/(\omega + 1)}, \quad m = \frac{10\omega^2 + 30\omega + 4}{3\omega^2 - 6\omega + 1}.$$

It can be checked that (i) for $\omega = -1$ Eq. (14) would imply $\chi = \text{const.}$, meaning that no solution other than de Sitter or Schwarzschild-de Sitter are allowed for $\omega = -1$, and (ii) for $-1/3 > \omega > -1$ the function $\chi$ does actually depend on $r$ and therefore may, in principle, eventually show new kinds of event horizons. Eq. (14) admits an integration in terms of the incomplete gamma function $\gamma(\alpha, x)$ [14], such that

$$\exp(\chi) = C_0 + C_1 \left( \frac{\omega + 1}{2(\omega - 1)} \right) \gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)}, x \right]$$

$$= C_0 + C_1 \left( \frac{\omega + 1}{2(\omega - 1)} \right) \gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)}, r^{2(\omega - 1)/(\omega + 1)} \right]. \quad (15)$$

Hence, we can obtain an expression for the metric component $A(r)$ which reads

$$A(r) = \left[ \bar{D}(r) \right]^\frac{2}{\omega - 1}.$$
\[
\kappa \frac{\omega}{r^{\omega+1}} \left\{ C_0 + C_1 \left( \frac{\omega + 1}{2(\omega - 1)} \right) \gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)} \right. \right. \right. \\
\left. \left. \left. + r^{2(\omega-1)} \right] \right\} \right\}^{\frac{\omega}{\omega-1}}. \tag{16}
\]

Inspection of Eqs. (2), (4) and (8) allows us to deduce that all possible singularities of the metric must arise from the following two conditions: (i) \( A = 0 \), and (ii) \( A' r + 2(3\omega + 1)A/\omega + 1 = 0 \), but not from \( A' = 0 \) alone, in such a way that either \( A = 0 \) and \( B = \infty \) simultaneously (i.e. a coordinate singularity at an event horizon) or \( A \to B \to \infty \) (i.e. a curvature singularity which was already present before making generalization (13)). Let us first analyze condition (ii), noting that the \( r \)-derivative of \( A(r) \) can be written as [15]

\[
A(r)' = \kappa \frac{\omega}{r^{\omega+1}} \left\{ \frac{4\omega}{\omega + 1} - C_1 \exp \left[ - \left( \chi + r^{\frac{2(\omega-1)}{\omega + 1}} \right) \right] \right\} \exp \left( \frac{2\omega \chi}{\omega - 1} \right). \tag{17}
\]

Using then expression (17) in condition (ii) we get for the possible singularities at \( r = r_h \) and \( r = r_s \)

\[
\{ P(r_h) \} \times \{ Q(r_s) \} = \left\{ \frac{4\omega}{\omega + 1} \exp \left( \frac{2\omega \chi(r_h)}{\omega - 1} \right) \right\} \times

\left\{ \frac{2(5\omega + 1)}{\omega + 1} - C_1 \exp \left[ - \left( \chi(r_s) + r_s^{\frac{2(\omega-1)}{\omega + 1}} \right) \right] \right\} = 0. \tag{18}
\]

We have then two different possible singularities coming from condition (ii). If we set \( \{ P(r_h) \} = 0 \), then the use of definitions (15) leads to \( r_h = \infty \), so that, since \( r \geq 0 \) and in the considered \( \omega \)-interval \( 2(\omega - 1)/(\omega + 1) \) is definite negative and \( m(\omega + 1)/[2(\omega - 1)] \) is definite positive, we also have \( \gamma(r_h) = 0 \) and \( A(r_h) = 0 \), which makes conditions (i) and (ii) to hold simultaneously. Such a singularity would then describe a black hole or cosmological event horizon at \( r_h = \infty \), and hence the resulting static spacetime would correspond to an accelerating universe either within an asymptotic black hole space with infinite radius, or with a cosmological event horizon at \( r = \infty \). It follows that the curvature singularity at \( r_s = 0 \), at which both metric components \( A \) and \( B \)
simultaneously tend to infinity, must correspond to the condition \( \{Q(r_s)\} = 0 \). This would in turn imply

\[
2\frac{5\omega + 1}{\omega + 1} C_0 = C_1 \left[ (\omega + 1) \exp \left(-\frac{2\omega + 1}{r_s^{\omega + 1}}\right) - \frac{5\omega + 1}{\omega - 1} \gamma(r_s) \right],
\]

where [15]

\[
\lim_{r \to 0} \gamma(r) = \Gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)} \right],
\]

with \( \Gamma \) being the complete gamma function. One can now set the relative value of the integration constants \( C_0 \) and \( C_1 \) to be

\[
C_0 = \frac{(\omega + 1)\Gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)} \right] C_1}{2(1 - \omega)},
\]

with which we finally obtain for the metric components

\[
A(r) = \kappa \frac{2\omega}{\omega + 1} \frac{4\omega}{\omega + 1} \left[ \frac{(\omega + 1)C_1}{2(1 - \omega)} \right]^{2\omega + 1} \left\{ \Gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)} \right] - \gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)} + \frac{2(\omega - 1)}{r_s^{\omega + 1}} \right] \right\}^{2\omega + 1} - \gamma \left[ \frac{m(\omega + 1)}{2(\omega - 1)} + \frac{2(\omega - 1)}{r_s^{\omega + 1}} \right].
\]

\[
B(r) = \frac{F(r, \omega)}{A(r)},
\]

where \( F(r, \omega) \) is a generally finite function of \( r \) and \( \omega \) which diverges only as \( r \to 0 \) and as \( r \to \infty \).

We have in this way obtained that the generalization (13) implies the existence of a spacetime which can be interpreted as representing a space that corresponds to an accelerating universe which evolves either within an infinite black hole, or toward an asymptotic cosmological event horizon at infinity. In both of these interpretations fundamental string and M theories would preserve mathematical consistency as such interpretations do not allow for causally disconnected regions anywhere up to infinite future. This result encompasses all \( \omega \)-values within the interval \(-1/3 > \omega > -1\) and confirms therefore our above conclusion that neither cosmological nor black hole event horizons can exist at finite distances for state equations \( p = \omega \rho \) with \(-1/3 > \omega > -1\).
Thus, the only black holes which are allowed to occur within the universe before
the onset of the accelerating regime in the above scenario are those having a mass
scale $M \leq 10^{15}$ g. These tiny black holes are currently known as primordial black
holes [9] and were formed quite before galaxies existed. They could have evaporated
off completely before or just at the coincidence time [16], when dark energy started
to dominate (see Fig. 1). In fact, the black hole evaporation loss rate is given by the
approximate expression [17]

$$\frac{dM}{dt} \simeq -T_{BH}^4 \times A_{BH} \sim \frac{\nu}{M^2},$$

where $T_{BH} = 1/(8\pi GM)$ is the Hawking temperature of the black hole and $A_{BH} =
16\pi G^2 M^2$ is the black hole surface area. The constant $\nu$ is expected to be a number of
order unity which reflects the number of degrees of freedom accounting for the species
of particles created by the black hole gravitational field. Thus, an estimate of the time
lasted by the largest possible black hole in completely evaporating, $t \sim 64G^2 M^3$, should
then roughly equal the coincidence time. This would bound the mass of the largest
possible black hole to be of the order $10^{15}$ g and moreover imply that short gamma
ray bursts which were assumed by Cline, Motthely and Otwinowski [18] to be produced
by nearby primordial black holes had necessarily stopped to occur at the onset of the
accelerating regime. It would then appear that the onset of dark energy dominance
and the end of black hole occurrence are strongly correlated to each other. At present,
we had no explanation for that correlation.

If we want to preserve consistency in the fundamental theories for gravity and par-
ticle physics, such as predictive string theories or M theory, one cannot describe our
present accelerating universe using a gravitational framework with a positive cosmo-
Figure 1: Pictorial representation of the evolution from big bang to now of a universe which contains a quintessence field with constant state equation responsible for the present accelerating expansion. No black hole at all may exist in such a universe after the onset of the present accelerating expansion. Before the coincidence time at which dark energy started dominating, that universe only allows the existence of primordial black holes which exhausted all their energy by Hawking evaporation before or at the onset of the present accelerating expansion.

logical constant, for in that case the above theories become mathematically ambiguous because they all rely on the existence of an $S$ matrix at infinite distances which is incompatible with the existence of a cosmological event horizon in the future [6-8]. The accelerating universe that corresponds to a constant quintessential field has nevertheless no event horizons at finite distances [13] and therefore does not pose this kind of challenge to the fundamental theories. If we accept the result obtained in this paper, then one is thus brought to the following dilemma: either one allows for the existence
of black holes of any size at any time during cosmological evolution at the cost of having to renounce to any mathematically consistent description of particle physics and quantum gravity based on the existence of an S matrix at infinite distances [6-8], or, if the mathematical consistency of such theories is taken to be prioritary, then one has to renounce to having black holes at our disposal to make the nuclei of galaxies, or for igniting the engines of so many high-energy processes that are currently thought to occur in astrophysics and cosmology.

Although the present status of string and M theories, on the one hand, and that of accelerating cosmology, on the other hand, does not allow one to succeed in finding any solid, definitive argument in favor of any of these two possibilities, one might be more akin to preserving the predictive character and mathematical consistency of fundamental particle-physics and quantum-gravity (if any) theories, and then reluctantly shift the black-hole concept to just the quantum realm where it could be implemented either (i) as the kind of primordial black holes we considered before, or, together with other space-time constructs such as wormholes, ringholes or warp drives, as just another more component of the so-called quantum space-time foam [19]. What this attitude would definitely prevent is any large black hole formed by the gravitational collapse of stars.

**Acknowledgements** The author thanks C.L. Sigüenza for many useful discussions. This work was supported by DGICYT under Research Project No. PB98-0520.
References

1 A. Einstein, Ann. Math. 40 (1939) 992.

2 R. Narayan and J.S. Heyl, *Evidence for the Black Hole Event Horizon*, to appear in Proceedings of the *Coral Gable Conference on High Energy Physics and Cosmology*, edited by B. Kursunoglu and A. Perlmutter (American Institute of Physics, USA).

3 C. Wetterich, Nucl. Phys. B302 (1988) 668; J.C. Jackson and M. Dodgson, Mon. Not. R. Astron. Soc. 297 (1998) 923; J.C. Jackson, Mon. Not. R. Astron. Soc. 296 (1998) 619; R.R. Caldwell, R. Dave and P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582; L. Wang and P.J. Steinhardt, Astrophys. J. 508 (1998) 483; R.R. Caldwell and P.J. Steinhardt, Phys. Rev. D57 (1998) 6057; G. Huey, L. Wang, R. Dave, R.R. Caldwell and P.J. Steinhardt, Phys. Rev. D59 (1999) 063005; P.F. González-Díaz, Phys. Rev. D62 (2000) 023513.

4 S. Perlmutter et al., Astrophys. J. 483 (1997) 565; S. Perlmutter et al. (The Supernova Cosmology Project), Nature 391 (1998) 51; P.M. Garnavich et al. Astrophys. J. Lett. 493 (1998) L53; B.P. Schmidt, Astrophys. J. 507 (1998) 46; A.G. Riess et al. Astrophys. J. 116 (1998) 1009.

5 A. Riess et al., Astrophys. J. 560 (2001) 49.

6 W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, JHEP 0107 (2001) 003; S. Hellerman, N. Kaloper and L. Susskind, JHEP 0106 (2001) 003.

7 T. Banks, Int. J. Mod. Phys. A46 (2001) 910; E. Witten, *Quantum gravity in DeSitter space*, hep-th/0106109; T. Banks and W. Fischler, *M-theory observables for cosmological space-times*, hep-th/0102077.
8  Xiao-Gang He, *Accelerating universe and event horizon*, astro-ph/0105005; S.M. Carroll, Phys. Rev. Lett. 81 (1998) 3067.

9  Ya. B. Zeldovich and I.D. Novikov, Sov. Astron. A.J. 10 (1967) 602; S.W. Hawking, Mont. Not. R. Astron. Soc. 152 (1971) 75.

10 F. Kottler, Ann. Phys. (Leipzig) 56 (1918) 410.

11 D. Kramer, H. Stephani, M.A.H. MacCallum, and H. Herlt, Exact Solutions of the Einstein’s Field Equations (Cambridge University Press, Cambridge, UK, 1980).

12 G.W. Gibbons and S.W. Hawking, Phys. Rev. D15 (1977) 2738.

13 P.F. González-Díaz, Phys. Lett. B522 (2001) 211; Phys. Rev. D65, 104035 (2002).

14 I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, Inc., Orlando, USA, 1980).

15 M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, USA, 1964).

16 P.J. Steinhardt, in *Critical Problems in Physics*, edited by V.L. Fitch and D.R. Marlow (Princeton University Press, Princeton, N.J., USA, 1997).

17 D.N. Page and S.W. Hawking, Astrophys. J. 206 (1976) 1.

16 D.B. Cline, C. Motthey and S. Otwinowski, Astrophys. J. 527 (1999) 827.

19 P.F. González-Díaz, Phys. Rev. D54 (1996) 6122; L.J. Garay and P.F. González-Díaz, Gen. Rel. Grav. 33 (2001) 353.