Influence of Atmospheric Parameters on Aircraft Characteristics

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Abstract. In this paper, a comparison of the standard atmosphere from ISO 4401-81 and several empirical models of the atmosphere obtained from sounding data is carried out. The main purpose of the work is to establish the influence of parameters of the atmosphere on the aerodynamic characteristics of an aircraft.

1. Introduction

Many developers use the standard atmosphere (SA) ISO 4401-81 model in the design of aircraft. According to SA at the increase of altitude $H$, air temperature $t_a$ first linearly decreases from 15 $^\circ$C to -56.5 $^\circ$C (at $H=0...11$ km), then has a constant value -56.5 $^\circ$C (at $H=11...20$ km) and then increases from -56.5 $^\circ$C to -2.5 $^\circ$C (at $H=20...47.4$ km). The other parameters of the atmosphere in ISO 4401-81 are determined using equations from a mandatory application. So, the parameters of the atmosphere are functions of altitude.

However, the parameters in real atmosphere also depend on weather conditions established at the moment in an area. It means that SA gives only an average estimate of atmosphere parameters. This can lead to errors in the calculation of aerodynamic and flight characteristics of an aircraft for weather conditions different from the conditions of SA.

More accurate data on parameters of the atmosphere corresponding to a specified time and specified place, may be obtained with sounding of atmosphere, which are the mart of meteorological observations. The use of such data makes it possible to improve the accuracy of calculations in problems that require the assessment of aerodynamic and flight characteristics of an aircraft, as well as in aircraft parameter identification [1], aircraft control [2], onboard measurement errors estimation [3].

2. Comparison of parameters of standard atmosphere and empirical models of the atmosphere

We will carry out a comparative analysis of SA and several empirical models of the atmosphere (EMA) in the altitude range from ground level to $H=30$ km. To do this, we take the atmosphere model from ISO 4401-81 and four EMA defined on a basis of $t_d(H)$. The dependences $t_d(H)$ are taken from [4] and correspond to the data of serological atmospheric sounding carried out in January and July 2017 at various meteorological observation stations. Information about these stations, as well as the dates and times of the sounding, is presented in table 1.
Table 1. Information of meteorological observation stations.

| Number of meteorological observation station | Nearest town    | State        | Date and time of sounding                  |
|---------------------------------------------|-----------------|--------------|-------------------------------------------|
| 24266                                       | Verkhoyansk     | Russia       | 30 January 2017, 12.00 UTC                |
| 34882                                       | Astrakhan       | Russia       | 30 January 2017, 12.00 UTC                |
| 34882                                       | Astrakhan       | Russia       | 24 July 2017, 12.00 UTC                   |
| 40417                                       | Dammam          | Saudi Arabia | 24 July 2017, 12.00 UTC                   |

First, let’s compare the dependence of the air temperature \( t_a \) on the altitude. Figure 1 shows that all considered EMA have significant differences from the SA in nature of changes of air temperature in height. Indeed, both the rate of air temperature change in an troposphere and stratosphere and the location of the boundaries between these atmospheric layers are different from the SA.

![Figure 1. The air temperature in SA and EMA](image)

If \( t_a(H) \) is known, we can determine a dependence of the sonic speed and a dynamic air viscosity coefficient \( \mu_a \) as a function of height

\[
\alpha = 20.0468 \cdot \sqrt{t_a + 273.2} \\
\mu_a = \frac{1.458 \cdot 10^{-6}}{t_a + 383.6} \cdot (t_a + 273.2)^{\frac{3}{2}}
\]

Formulas (1) and (2) are taken from the mandatory Appendix to ISO 4401-81. These formulae for considered height ranges are valid not only for SA, but also for any other model of the Earth’s atmosphere. This is due to the fact that the proportional ratio of gases in the air is constant throughout the near-earth space up to \( H=90-95 \) km. The dependence \( a(H) \) determines the Mach number at a known velocity of the incoming air flow, and \( \mu_a(H) \) determines the Reynolds number. The values of the Mach and Reynolds numbers are used in determining aerodynamic characteristics of aircraft.

Let us determine atmospheric pressure \( p_a \) and the density of air \( \rho_a \) as functions of height \( H \) for SA and the four models of EMA described above. To do this, we solve a system of two equations: the equations of state of an ideal gas (3) and the equations of atmospheric statics (4).

\[
p_a = 287,053 \cdot \rho_a \cdot (t_a + 273.2) \\
\frac{dp_a}{dH} = -\rho_a \cdot g, H=H_0 \Rightarrow p_a=p_{a0}
\]
where $H_0$ – height of ground level relative to sea level; $p_{a0}$ – atmospheric pressure at ground level, $g$ – acceleration of gravity, defined by the formula (5)

$$g = 9.7803(1 + 5.302 \cdot 10^{-3} \cdot \sin^2(\Psi_E)) – 3.086 \cdot 10^{-3} \cdot H,$$

where $\Psi_E$ – geographic latitude, rad, $H$ – height, km.

Equations (3) and (4) are also taken from the mandatory Appendix to ISO 4401-81. Similar to equations (1) and (2), these equations are applicable in the considered altitude range not only for SA, but also for any other model of the earth’s atmosphere, including the considered EMA.

The values of $\Psi_E, H_0$ and $p_{a0}$ are presented in table 2.

**Table 2. Initial conditions for solving the system of equations (3), (4)**

| Atmospheric model | Town       | Month    | $\Psi_E$, degree | $H_0$, km | $p_{a0}$, Pa |
|-------------------|------------|----------|------------------|-----------|--------------|
| SA                | ---        | ---      | 45.3             | 0         | 101325       |
| EMA               | Verkhoyansk| January  | 67.6             | 0.138     | 98800        |
| EMA               | Astrakhan  | January  | 46.3             | -0.022    | 103100       |
| EMA               | Astrakhan  | July     | 46.3             | -0.022    | 101200       |
| EMA               | Dammam     | July     | 26.4             | 0.012     | 99400        |

Having solved the system of equations (3) and (4), we compare the differences in $p_a(H)$ and $\rho_a(H)$ between SA and the considered EMA. To do this, imagine the values of $p_a$ and $\rho_a$ in EMA as a percentage deviation with respect to $p_a$ and $\rho_a$ in SA. These deviations $\delta p_a(H)$ and $\delta \rho_a(H)$ are shown in figure 2 and 3.

**Figure 2. Atmospheric pressure in EMA relative to atmospheric pressure in SA**

As shown in figure 2, atmospheric pressure in the EMA is markedly different from $p_a$ in the SA. In this case, the highest value $\left| \delta p_a \right|$ are obtained at medium and high altitudes. The decrease in $p_a$ relative to SA is typical for January EMA, and the increase – for July. Function $p_a(H)$ is used in the calculation of the characteristics of the aerial vehicle as a value that determines the altitude correction to the thrust of the engine.
Figure 3. Density of air in the EMA relative to the density of air in the SA

For Figure 3 are also visible differences between the density of the air \( \rho_a \) in SA and EMA. At low altitudes, the highest \( \rho_a \) values are reached in the EMA for Verkhoyansk, and the lowest in the EMA for the city of Dammam. At large \( H \) the most \( \rho_a \) is in the city of Dammam and the city of Astrakhan in July and the lowest over the city of Verkhoyansk. The \( \rho_a(H) \) dependence is used to determine the dynamic pressure and Reynolds number.

Based on the comparative analysis of SA and EMA, we conclude that the considered models of the atmosphere have significant differences. As it turned out, for a range of heights from ground level up to \( H=30 \) km, the difference in speed of sound between the SA and EMA is in the range -11...7%, in dynamic coefficient of viscosity is in the range of -17...11%, atmospheric static pressure – in the range -17...14% and the air density – in the range of -22...26%. The greatest deviations from the standard \( \rho_a \) values are observed in EMA, appropriate to the city of Verkhoyansk and the city of Dammam.

3. The algorithm for calculating the aerodynamic forces acting on the aircraft in flight.

When considering the flight of an aircraft in the speed coordinate system vector of the total aerodynamic force can be divided into two components: the drag force of the aircraft \( Q_a \) (directed along the velocity vector of the aircraft) and the lift of the aircraft \( Y_a \) (directed perpendicular to the velocity vector of the aircraft). The values of \( Q_a \) and \( Y_a \) are determined according to the formulas (6) and (7). [5]

\[
Q_a = \left( c_{xp}(M) + c_{xf}(M, Re) + c_{xibal}(\alpha, M) \right) \cdot q \cdot S_m, \tag{6}
\]

\[
Y_a = c_{yibal}(\alpha, M) \cdot q \cdot S_m, \tag{7}
\]

where \( c_{xp} \) – resistance pressure coefficient of aircraft, \( c_{xf} \) – resistance friction coefficient of an aircraft, \( c_{xibal} \) – inductive resistance coefficient of an aircraft, \( M \) – Mach number (determined by the formula (8)), \( Re \) – Reynolds number (determined by the formula (9)), \( \alpha \) – angle of attack, \( q \) – dynamic pressure (determined by the formula (10)), \( S_m \) – characteristic surface of aircraft, \( c_{yibal} \) – lift coefficient of the aircraft.

\[
M = \frac{V}{a}, \tag{8}
\]

\[
Re = \frac{\mu_s}{\rho_s a}, \tag{9}
\]

\[
q = \frac{\rho_s a^2}{2}, \tag{10}
\]

where \( V \) – air speed of the aircraft.

From formulas (6) - (10) it may be seen that at the same values of the parameters \( V, H \) and \( \alpha \) in different models of the atmosphere aerodynamic forces of the aircraft are be different. These differences
are determined by differences in the values of the aerodynamic coefficients and the dynamic pressure $q$, which depends on the air density $\rho_a$. The coefficients of $c_{x f}$ vary between different models of the atmosphere due to differences in the numbers of $M$ and $Re$ and the coefficients $c_{x p}$, $c_{x r}$, $c_{xahal}$ and $c_{xbal}$ due to differences in $M$. Because of the difference in aerodynamic forces of aircraft for different models of the atmosphere, it follows that flight characteristics of aircraft will also depend on the model of the atmosphere. As shown by the study of the flight characteristics hypothetical aircraft, conducted for the SA and four previously considered EMA, the values of the aircraft maximum range $D_{max}$ between the SA and the EMA are different. Thus, when considering the flight of a hypothetical aircraft in January EMA the values of $D_{max}$ at low altitude $H=0.5$ km are smaller than in SA up to -6%, and at $H=11$ km – greater more than to 14%.

When considering the flight of a hypothetical aircraft in July EMA, the opposite situation is observed: at low altitudes $D_{max}$ is greater than in SA up to 6%, and at high altitudes – smaller up to -9% at $H=15$ km.

Therefore, when solving problems that require the precision in calculating the aircraft aerodynamic and flight characteristics, the use of EMA is more preferable than the SA of ISO 4401-81.

4. Conclusion
As a result of comparison of SA and four EMA it is established that the difference between parameters of the atmosphere in SA and considered EMA is in the range from -22% to 26%. Considering the model of aerodynamic forces it was found that in different models of the atmosphere aerodynamic characteristics of an aircraft are different from each other. For a hypothetical aircraft, the values of the maximum ranges $D_{max}$ for SA and four EMA show the difference in the range from -9 to 14%.

References
[1] Korsun O N, Stulovskii A V, Ovcharenko V N and Kanyshev A V 2018 Journal of computer and systems sciences international 57(3) 374-389
[2] Zybin E Y and Kos'yanchuk V V 2016 Journal of Computer and Systems Sciences International 55(4) 546–557
[3] Pushkov S G, Lovitskii L L and Korsun O N 2018 Measurement Techniques 57(3) 140-147
[4] Atmospheric soundings (Univ. of Wyoming) http://weather.uwyo.edu/upperair/sounding.html
[5] Lebedev A, Chernobrovkin L 1962 Dynamics of flight, pp 121-128 (In Russian)