Principal-component analysis of two-particle azimuthal correlations in PbPb and pPb collisions at CMS

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For the first time a principle-component analysis is used to separate out different orthogonal modes of the two-particle correlation matrix from heavy ion collisions. The analysis uses data from $\sqrt{s_{NN}} = 2.76$ TeV PbPb and $\sqrt{s_{NN}} = 5.02$ TeV pPb collisions collected by the CMS experiment at the CERN Large Hadron Collider. Two-particle azimuthal correlations have been extensively used to study hydrodynamic flow in heavy ion collisions. Recently it was shown that the expected factorization of two-particle results into a product of the constituent single-particle anisotropies is broken. The new information provided by these modes may shed light on the breakdown of flow factorization in heavy ion collisions. The first two modes ("leading" and "subleading") of two-particle correlations are presented for elliptical and triangular anisotropies in PbPb and pPb collisions as a function of $p_T$ over a wide range of event activity. The leading mode is found to be essentially equivalent to the anisotropy harmonic previously extracted from two-particle correlation methods. The subleading mode represents a new experimental observable and is shown to account for a large fraction of the factorization breaking recently observed at high transverse momentum. The principle-component analysis technique was also applied to multiplicity fluctuations. These also show a subleading mode. The connection of these new results to previous studies of factorization is discussed.

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1. INTRODUCTION

The primary goal of experiments with heavy ion collisions at ultrarelativistic energies is to study nuclear matter under extreme conditions. Quantum chromodynamics on the lattice predicts the formation of a quark-gluon plasma (QGP) at energy densities that are attainable in relativistic heavy ion collisions. Measurements carried out at the Relativistic Heavy Ion Collider (RHIC) indicate that a strongly interacting QGP is produced in heavy ion collisions [1–4]. The presence of azimuthal anisotropy in the emission of final state hadrons revealed a strong collective flow behavior of this strongly coupled hot and dense medium [5,6]. The significantly higher energies available at the CERN Large Hadron Collider (LHC) compared to RHIC have allowed the ALICE, ATLAS, and CMS experiments to make very detailed measurements of the QGP properties [7–15]. The collective expansion of the QGP can be described by hydrodynamic flow models [16–18]. In the context of these models, the azimuthal anisotropy of hadron emission is the response to the initial density profile of the overlap region of the colliding nuclei. Such anisotropic emission, for a given event, can be quantified through a Fourier decomposition of the single-particle distribution

$$\frac{dN}{dp} = \sum_{n=-\infty}^{\infty} V_n(p)e^{-in\phi}, \quad (1)$$

with $V_n(p) = v_n(p)e^{in\Psi_n(p)}$ and $dp = dp_T\,d\phi\,d\eta$, where $p$ is a shorthand notation for $p_T$ and $\eta$. This single-particle distribution is the invariant yield of emitted particles $N$ expressed in phase space $p_T$, $\eta$, and $\phi$, i.e., transverse momentum, pseudorapidity, and azimuthal angle. Here, $v_n$ corresponds to the real single-particle anisotropy and $\Psi_n(p)$ represents the $n$th order event plane angle. Also, because of the reflection symmetry of the overlap region, the relation $V_n^* = V_{-n}$ holds for the complex harmonics. Using this relation and integrating Eq. (1) over a given pseudorapidity and $p_T$ window yields

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2 \sum_{n=1}^{\infty} v_n(p) \cos[n(\phi - \Psi_n(p))]\right). \quad (2)$$

Note that the single-particle anisotropy coefficient $v_n$ is generally a function of $p_T$ and $\eta$, which is also the case for the event plane angle. The azimuthal correlation of $N_{\text{pairs}}$ emitted particle pairs (with particles labeled $a$ and $b$) as a function of their azimuthal separation $\Delta\phi^{ab} = \phi^a - \phi^b$ can be characterized by its own Fourier harmonics,

$$\frac{dN_{\text{pairs}}}{d\Delta\phi^{ab}} = \frac{N_{\text{pairs}}}{2\pi}\left(1 + 2 \sum_{n=1}^{\infty} V_{n,\Delta}(p^a, p^b) \cos(n\Delta\phi)\right), \quad (3)$$

where $V_{n,\Delta}$ is the two-particle harmonic. In a pure hydrodynamic picture, as a consequence of independent particle emission, the flow hypothesis connects the single- and two-particle spatial anisotropies from Eqs. (2) and (3) through factorization. In other words, particles carry information only about their orientation with respect to the whole system and the two-particle distribution can therefore be factorized based on

$$\frac{dN_{\text{pairs}}}{d\Delta\phi^{ab}} = \frac{dN}{d\phi^a}\frac{dN}{d\phi^b}.$$
with the bracket ⟨⟩ representing the average over all events of interest. This equality can be investigated by looking at the connection between the single- and two-particle harmonics:

\[
\langle V_n(\Delta(p^a, p^b)) \rangle = \langle V_n(p^a) V_n(p^b) \rangle = \langle \nu_n^a \nu_n^b \cos [n(\Psi_n^a - \Psi_n^b)] \rangle \leq \langle \nu_n^a \nu_n^b \rangle.
\]

From Eq. (5) we infer that factorization is preserved when the cosine value equals unity. This scenario is possible only when the event plane angle acts as a global phase, lacking any \( p_T \) or \( \eta \) dependence for a given event. Thus, measurements of the momentum space fluctuations (correlations) constrain the initial state and properties of QGP expansion dynamics. Previous measurements have shown a significant breakdown of factorization at high \( p_T \) in ultracentral (i.e., almost head-on) PbPb collisions [15]. A smaller effect was also seen in high-multiplicity PbPb collisions [19]. Furthermore, significant factorization breakdown effects as a function of \( \eta \) were observed in both PbPb and high-multiplicity PbPb collisions [19]. Several possible explanations for the observed factorization breaking have been proposed. One expected contribution arises from nonflow effects, i.e., short-range correlations mainly due to jet fragmentation and resonance decays. However, factorization breaking is also possible in hydrodynamic models, once the effects of event-by-event initial-state fluctuations are taken into account [20,21]. Such a nonuniform initial-state energy density can arise from fluctuations in the positions of nucleons within nuclei and/or the positions of quark and gluon constituents inside each nucleon, giving rise to variations in the collision points when the two nuclei collide. The resulting fluctuating initial energy density profile creates nonuniformities in pressure gradients which push particles in different regions of phase space in directions that vary randomly about a mean angle, thereby imprinting these fluctuations on the final particle distributions. Consequently, the event plane angles estimated from particles in different \( p_T \) and \( \eta \) ranges may vary with respect to each other. By introducing such a dependence, \( \Psi_n = \Psi_n(p_T, \eta) \), it is possible to describe the resulting final-state particle distributions using hydrodynamical models [20,21].

Principal-component analysis (PCA) is a multivariate technique that can separate out the different orthogonal contributions (also known as modes) to the fluctuations. Using the method introduced in Ref. [22], this paper presents the first experimental use of applying PCA to two-particle correlations in order to study factorization breaking as a function of \( p_T \). This allows the extraction of a new experimental observable, the subleading mode, which is directly connected to initial-state fluctuations and their effect on factorization breaking.

II. EXPERIMENTAL SETUP AND DATA SAMPLES

The Compact Muon Solenoid (CMS) is an axially symmetric detector with an onionlike structure, which consists of several subsystems concentrically placed around the interaction point. The CMS magnet is a superconducting solenoid providing a magnetic field of 3.8 T, which allows precise measurement of charged-particle momentum. The muon chambers are placed outside the solenoid. In this analysis the data used are extracted from the silicon tracker, which is the closest subdetector to the interaction point. This detector consists of 1440 silicon pixel and 15 148 silicon strip detector modules that detect hit locations, from which the charged-particle trajectories are reconstructed. The silicon tracker covers charged particles within the range \( |\eta| < 2.5 \) and provides an impact parameter resolution of \( \sim 15 \mu \text{m} \) and a \( p_T \) resolution better than 1.5% up to \( p_T \sim 100 \text{ GeV}/c \).

The other two subdetectors located inside the solenoid are the electromagnetic calorimeter (ECAL) and hadronic calorimeter (HCAL). The ECAL is constructed of 75 848 lead tungstate crystals which are arranged in a quasi-projective geometry and cover a pseudorapidity range of \( |\eta| < 1.48 \) units in the barrel and two endcaps that extend \( |\eta| \) up to 3.0. The HCAL barrel and endcaps are sampling calorimeters constructed from brass and scintillator plates, covering \( |\eta| \) up to 3.0. Additional extension in \( |\eta| \) from 2.9 up to 5.2 is achieved with the iron and quartz-fiber Čerenkov Hadron Forward (HF) calorimeters on either side of the interaction region. The HF calorimeters are segmented into towers, each of which is a two-dimensional cell with a granularity of \( 0.175 \times 0.175 \text{ rad}^2 (\Delta \eta \times \Delta \phi) \). The zero-degree calorimeters (ZDCs) are tungsten quartz Čerenkov calorimeters located \( \pm 140 \text{ mm} \) from the interaction point [23]. They are designed to measure the energy of photons and spectator neutrons emitted from heavy ion collisions. A set of scintillator tiles, the beam scintillator counters (BSCs), are mounted on the inner side of the HF calorimeters and are used for triggering and beam-halo rejection. The BSCs cover the range \( 3.23 < |\eta| < 4.65 \). A detailed description of the CMS detector can be found in Ref. [24].

This analysis is performed using data recorded by the CMS experiment during the LHC heavy ion runs in 2011 and 2013. The PbPb data set at a center-of-mass energy of \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) corresponds to an integrated luminosity of about \( 159 \mu \text{b}^{-1} \), while the pPb data set at \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \) corresponds to about \( 35 \text{ nb}^{-1} \). During the pPb run, the beam energies were 4 TeV for protons and 1.58 TeV per nucleon for lead nuclei.

III. SELECTION OF EVENTS AND TRACKS

Online triggers, track reconstruction, and offline event selections are the same as in Refs. [15,19,25] for PbPb and pPb data samples and are summarized in the following sections.

A. The PbPb data

Minimum bias PbPb events were collected using coincident trigger signals from both ends of the detector in either BSCs or the HF calorimeters. Events affected by cosmic rays, detector noise, out-of-time triggers, and beam backgrounds were suppressed by requiring a coincidence of the minimum bias trigger with bunches colliding in the interaction region. The efficiency of the trigger is more than 97% in the case of hadronic inelastic PbPb collisions. Because of hardware limits on the data acquisition rate, only a small fraction (2%) of all minimum bias events were recorded (i.e., the trigger is "prescaled"). To enhance the event sample for very central PbPb collisions, a dedicated online trigger was implemented by simultaneously requiring the HF transverse energy \( (E_T) \)
sum to be greater than 3260 GeV and the pixel cluster multiplicity to be greater than 51400 (which approximately corresponds to 9500 charged particles over 5 units of $\eta$). The selected events correspond to the 0–0.2% most central PbPb collisions. Other standard PbPb centrality classes presented in this paper were determined based on the total energy deposited in the HF calorimeters [13]. The inefficiencies of the minimum bias trigger and event selection for very peripheral events are taken into account.

To reduce further the background from single-beam interactions (e.g., beam gas and beam halo), cosmic muons, and ultraperipheral collisions leading to the electromagnetic breakup of one or both Pb nuclei [26], offline PbPb event selection criteria [13] were applied by requiring energy deposits in at least three towers in each of the HF calorimeters, with at least 3 GeV of energy in each tower, and the presence of a reconstructed primary vertex built of at least two tracks. The reconstructed primary vertex is required to be located within $\pm 15$ cm of the average interaction point along the beam axis and within a radius of 0.2 cm in the transverse plane. Following the procedure developed in Ref. [15], events with large signals of at least one additional interaction, or pileup event, and are thus rejected (about 0.1% of all events).

The reconstruction of the primary event vertex and of the trajectories of charged particles in PbPb collisions is based on signals in the silicon pixel and strip detectors and is described in detail in Ref. [13]. From studies based on PbPb events simulated using HYDJET version 1.8 [27], the combined geometrical acceptance and reconstruction efficiency of the primary tracks is about 70% at $p_T \sim 4$ GeV/$c$ and $|\eta| < 1.0$ for the most central (0–5%) PbPb events, but drops to about 50% for $p_T \sim 0.3$ GeV/$c$. The fraction of misidentified tracks is kept to <5% over most of the $p_T$ (>0.5 GeV/$c$) and $|\eta|$ (<1.6) ranges. It increases to about 20% for very low $p_T$ (<0.5 GeV/$c$) particles in the forward ($|\eta| > 2.0$) region.

### B. The PbPb data

Minimum bias PbPb events were triggered by requiring at least one track with $p_T > 0.4$ GeV/$c$ to be found in the pixel tracker in coincidence with an LHC PbPb bunch crossing. From all minimum bias triggered events, only a fraction ($\sim 10^{-3}$) was recorded. To select high-multiplicity PbPb collisions, a dedicated trigger was implemented using the CMS level 1 (L1) and high-level trigger (HLT) systems. At L1, the total transverse energy summed over the ECAL and HCAL is required to be greater than a given threshold (20 or 40 GeV). The online track reconstruction for the HLT is based on the three layers of pixel detectors and requires a track originated within a cylindrical region of length 30 cm along the beam and radius of 0.2 cm perpendicular to the beam. For each event, the vertex reconstructed with the highest number of pixel tracks is selected. The number of pixel tracks ($N_{\text{trk}}^{\text{offline}}$) with $|\eta| < 2.4$, $p_T > 0.4$ GeV/$c$, and having a distance of closest approach of 0.4 cm or less to this vertex is determined for each event.

In the offline analysis, hadronic PbPb collisions are selected by requiring a coincidence of at least one HF calorimeter tower with more than 3 GeV of total energy in each of the HF detectors. Events are also required to contain at least one reconstructed primary vertex within 15 cm of the nominal interaction point along the beam axis and within 0.15 cm transverse to the beam trajectory. At least two reconstructed tracks are required to be associated with the primary vertex. Beam-related background is suppressed by rejecting events for which fewer than 25% of all reconstructed tracks are of good quality (i.e., the tracks selected for physics analysis).

The instantaneous luminosity provided by the LHC in the 2013 PbPb run resulted in approximately 3% probability of at least one additional interaction occurring in the same bunch crossing, i.e., pileup events. Pileup was rejected using a procedure based on the number of tracks in a given vertex and the distance between that and an additional vertex (see Ref. [25]). The fraction of PbPb events selected by these criteria, which have at least one particle (proper lifetime $\tau > 10^{-18}$ s) with total energy $E > 3$ GeV in an $\eta$ range of $-5 < \eta < -3$ and at least one in the range $3 < \eta < 5$ (selection referred to as “double-sided”) has been found to be 97–98% by using the EPOS [28] and HIJING [29] event generators.

In this analysis, the CMS highPurity [30] tracks are used. Additionally, a reconstructed track is only considered as a primary-track candidate if the significance of the separation along the beam axis ($z$) between the track and the best vertex, $dz/\sigma(dz)$, and the significance of the impact parameter relative to the best vertex transverse to the beam, $d_{zT}/\sigma(d_{zT})$, are less than 3 in each case. The relative uncertainty of the $p_T$ measurement, $\sigma(p_T)/p_T$, is required to be less than 10%. To ensure high tracking efficiency and to reduce the rate of misidentified tracks, only tracks within $|\eta| < 2.4$ and with $p_T > 0.3$ GeV/$c$ are used in the analysis. The entire PbPb data set is divided into classes of reconstructed track multiplicity, $N_{\text{trk}}^{\text{offline}}$, where primary tracks with $|\eta| < 2.4$ and $p_T > 0.4$ GeV/$c$ are counted. The multiplicity classification in this analysis is identical to that used in Ref. [25], where more details are provided.

### IV. ANALYSIS TECHNIQUE

This analysis uses two-particle correlations and PCA as a new flow method that can make use of all the information contained in $V_n$ harmonics. Averaging Eq. (3) over all events of interest, within a given reference bin $p_T^{\text{ref}}$, and assuming factorization, one can write

$$\left(\frac{dN_{\text{pairs}}}{d\Delta\phi}\right) = \frac{(N_{\text{pairs}})}{2\pi} \left(1 + \sum_{n=1}^{\infty} \nu_n^2(2) \cos(n\Delta\phi)\right), \quad (6)$$

where $\nu_n(2)$ is the integrated reference flow calculated from the $V_n$ as

$$\nu_n(2) = \sqrt{\frac{V_n(\rho_{n}^{\text{ref}},\rho_{T}^{\text{ref}})}}{\sqrt{V_{0}(\rho_{n}^{\text{ref}},\rho_{T}^{\text{ref}})}}, \quad (7)$$

with

$$V_n(\rho_{n}^{\text{ref}},\rho_{T}^{\text{ref}}) \equiv \left(\sum_i \cos(n\Delta\phi_i)\right). \quad (8)$$
Here, the label \( V_{0\Delta} \) for \( N_{\text{pairs}} \) is used, since the sum over cosine counts the number of pairs for the \( n = 0 \) case. Calculating the differential flow one gets

\[
\varphi_n(p_T) \equiv \frac{V_{0\Delta}(p_T, p_T^{\text{ref}}) \sqrt{V_{0\Delta}(p_T, p_T^{\text{ref}})}}{V_{0\Delta}(p_T, p_T^{\text{ref}})} \]

(9)

or

\[
\varphi_n(p_T) = \frac{V_{0\Delta}(p_T, p_T^{\text{ref}})}{\sqrt{V_{0\Delta}(p_T, p_T^{\text{ref}})}} \frac{\sqrt{V_{0\Delta}(p_T, p_T^{\text{ref}})}}{V_{0\Delta}(p_T, p_T^{\text{ref}})}. \]

(10)

The single-particle anisotropy definition in Eq. (10) includes the \( V_{0\Delta} \) terms to compensate for the fact that the \( V_{n\Delta} \) Fourier harmonics are calculated per-event normalization by the number of pairs in the given bin [15,19]. This way of calculating the cosine term is essential for the PCA to work, since it gives a weight to a bin that is of the order of the number of particles in it [22].

In a realistic experiment, the \( V_{n\Delta} \) harmonics of Eq. (8) are affected by imperfections in the detector and take the following operational definition:

\[
V_{n\Delta}(p_T^a, p_T^b) = \langle \cos(n \Delta \phi) \rangle_s - \langle \cos(n \Delta \phi) \rangle_B,
\]

\[n = 1,2,3, \ldots.\]

(11)

Here, the first term on the right-hand side of Eq. (11), \( \langle \cos(n \Delta \phi) \rangle_s \), is the two-particle anisotropic signal where the correlated particles belong to the same event. The second term, \( \langle \cos(n \Delta \phi) \rangle_B \), is a background term that accounts for the nonuniform acceptance of the detector. This term is usually two orders of magnitude smaller than the corresponding signal. It is estimated by mixing particle tracks from two random events. These two events have the same 2-cm-wide range of the primary vertex position in the \( z \) direction and belong to the same centrality (track multiplicity) class. For both terms, in order to suppress nonflow correlations, a pseudorapidity difference requirement between the two tracks \( |\Delta \eta| > 2 \) is applied.

### A. Factorization breaking

The PCA is a multivariate analysis that orders the fluctuations in the data by size. The ordering is done through principal components that represent orthogonal eigenvectors of the corresponding covariance data matrix. In the context of flow fluctuations, the components should reveal any significant substructure caused by the fluctuating initial state geometry of colliding nuclei. Introducing PCA in terms of factorization breaking, one can write the Pearson correlation coefficient used for measurement of the effect as in Ref. [19]:

\[
r_n(p_T^a, p_T^b) \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)}} \approx \langle \cos n \Psi(p_T^a) - \Psi(p_T^b) \rangle.
\]

(12)

The ratio \( r_n \) is approximated by the cosine term, giving unity if the event plane angle is a global phase, as discussed previously. Expressing the ratio through the two-particle harmonic in complex form from Eq. (5), \( r_n \) can only be unity if the complex flow coefficient \( V_n(p_T) \) is generated from one initial geometry, for instance, where the initial geometry of the overlap region is defined by some complex eccentricity (\( \varepsilon_0 \)) and a fixed real function \( f(p_T) \), i.e., \( V_n(p_T) = f(p_T) r_n \). However, if events are described by multiple eccentricities then \( r_n \) may be less than unity and the flow pattern displays factorization breaking [31]. This last statement can be generalized by expanding the complex flow coefficient using the principal components \( V_n^{(1)}(p_T), V_n^{(2)}(p_T), \ldots \) as a basis built from a covariance data matrix of given size \( N_x \times N_y \).

\[
V_n(p_T) = \xi_n^{(1)}(p_T^1) + \xi_n^{(2)}(p_T^2) + \ldots + \xi_n^{(N_x N_y)}(p_T^{N_x N_y}),
\]

where \( \xi_n^{(i)} \) are complex uncorrelated variables with zero mean, i.e., \( \langle \xi_n^{(i)} \xi_n^{(j)} \rangle = \delta_{ij} \), \( \langle \xi_n^{(i)} \rangle = 0 \), and \( N_x \) represents the number of \( p_T \) differential bins. Therefore, the two-particle harmonics are the building elements of the covariance data matrix \( \langle \xi_n^{(i)} \xi_n^{(j)} \rangle \) are matrices of order \( N_x \times N_y \) elements.

A covariance matrix is symmetrical and positive semidefinite (i.e., with eigenvalues \( \lambda_i \geq 0 \)). For the flow matrix, the last trait is valid if there are no nonflow contributions and no strong statistical fluctuations [22]. Now, calculating the two-particle harmonic using the expansion from Eq. (13) one gets

\[
V_n(p_T^a, p_T^b) = \sum_{\alpha=1}^{N_x N_y} N_\alpha \langle p_T^{(a)} \rangle V_n^{(\alpha)}(p_T^{(b)}).
\]

(14)

Here, the principal components are referred to as modes [22,31,32]. To calculate the modes the spectral decomposition is rewritten as

\[
V_n(p_T^a, p_T^b) = \sum_{\alpha} \lambda^{(\alpha)} e^{(\alpha)}(p_T^a) e^{(\alpha)}(p_T^b),
\]

(15)

which gives

\[
V_n^{(\alpha)}(p_T) = \sqrt{\lambda^{(\alpha)}} e^{(\alpha)}(p_T).
\]

(16)

where \( e^{(\alpha)}(p_T) \) are \( (\alpha) \) index values of normalized eigenvectors and \( \lambda^{(\alpha)} \) eigenvalues that are sorted in a strict decreasing order \( \lambda^{(1)} > \lambda^{(2)} > \ldots > \lambda^{(n)} \). Equation (14) shows directly that factorization holds only in the case where just one mode is present. If multiple modes are present in the data, Eqs. (15) and (16) allow one to define a normalized orthogonal basis for the total \( \varphi_n \) given in Eq. (10). These basis vectors are defined by

\[
\varphi_n^{(\alpha)}(p_T) = V_n^{(\alpha)}(p_T) / V_0^{(\alpha)}.
\]

(17)

The normalization factor \( V_0^{(1)} \) is the first mode that would follow from Eq. (16) using the matrix of the number of pairs, i.e., the matrix of \( V_{0\Delta} \) terms. In practice, the mode \( V_0^{(1)} \) has a simple physical meaning: it is the average differential multiplicity \( \langle M(p_T) \rangle \). However, given the pseudorapidity requirement in the correlations, \( V_0^{(1)} \) is proportional to \( \langle N_{\text{pairs}}^{(\geq 2)}(p_T, p_T) \rangle \). To restore normalization by the average bin multiplicity \( \langle M(p_T) \rangle \) an intermediate step is made by multiplying the \( V_{n\Delta}(p_T^a, p_T^b) \) with

\[
\xi = \frac{\langle N_{\text{pairs}}^{(\geq 2)}(p_T^a, p_T^b) \rangle}{\langle N_{\text{pairs}}^{(\geq 2)}(p_T^a, p_T^b) \rangle},
\]

(18)
The leading (\(\alpha = 1\)) and the subleading (\(\alpha = 2\)) normalized modes (for simplicity, term modes are used) can be thought of as new experimental observables. Given that the eigenvalues \(\lambda^{(\alpha)}\) are strongly ordered, two components typically describe the variance in the harmonic flow to high accuracy. The leading mode is strongly correlated with the event plane and thus is essentially equivalent to the standard definition of the single-particle anisotropic flow, while the subleading mode is uncorrelated with the event plane and thus quantifies the magnitude of the factorization breaking caused by the initial-state fluctuations.

### B. Multiplicity fluctuations

The PCA can also be applied for investigating multiplicity fluctuations in heavy ion collisions. The multiplicity matrix that is used for extraction of the corresponding modes is built from the following matrix elements:

\[
\tilde{M}(p_T^a, p_T^b) = \left\langle V_{n\Delta}(p_T^a, p_T^b) \right\rangle = \left\langle M(p_T^a)\right\rangle \left\langle M(p_T^b)\right\rangle.
\]

The leading (\(\alpha = 1\)) and the subleading (\(\alpha = 2\)) normalized modes (for simplicity, term modes are used) can be thought of as new experimental observables. Given that the eigenvalues \(\lambda^{(\alpha)}\) are strongly ordered, two components typically describe the variance in the harmonic flow to high accuracy. The leading mode is strongly correlated with the event plane and thus is essentially equivalent to the standard definition of the single-particle anisotropic flow, while the subleading mode is uncorrelated with the event plane and thus quantifies the magnitude of the factorization breaking caused by the initial-state fluctuations.

#### V. SYSTEMATIC UNCERTAINTIES

Several sources of possible systematic uncertainties, such as the event selection, the dimension of the matrix, and the effect of the tracking efficiency, were investigated. Among these sources, only the effect of the tracking efficiency had a noticeable influence on the results. For all the considered cases

\[
\tilde{V}_{n\Delta}(p_T^a, p_T^b) = \tilde{V}_{n\Delta}(p_T^a, p_T^b).
\]

Equation (17) then becomes

\[
u_n^{(\alpha)}(p_T) = \frac{\tilde{V}_n^{(\alpha)}(p_T)}{\langle M(p_T)\rangle}.
\]

The leading (\(\alpha = 1\)) and the subleading (\(\alpha = 2\)) normalized modes (for simplicity, term modes are used) can be thought of as new experimental observables. Given that the eigenvalues \(\lambda^{(\alpha)}\) are strongly ordered, two components typically describe the variance in the harmonic flow to high accuracy. The leading mode is strongly correlated with the event plane and thus is essentially equivalent to the standard definition of the single-particle anisotropic flow, while the subleading mode is uncorrelated with the event plane and thus quantifies the magnitude of the factorization breaking caused by the initial-state fluctuations.

#### VI. RESULTS

Figure 1 shows leading and subleading modes for the elliptic case (\(n = 2\)) for eight centrality regions in PbPb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV as a function of \(p_T\). These centrality regions range from ultracentral (0–0.2%) to peripheral (50–60%). The data are binned into seven \(p_T\) bins covering the region 0.3 < \(p_T\) < 3.0 GeV/c. The number of differential \(p_T\) bins for constructing the covariance matrix is \(N_{\Delta} = 7\). In all the
Leading ($\alpha = 1$) and subleading ($\alpha = 2$) modes for $n = 2$ as a function of $p_T$, measured in a wide centrality range of PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The results for the leading mode ($\alpha = 1$) are compared to the standard elliptic flow magnitude measured by ALICE and CMS using the two-particle correlation method taken from Refs. [7,15], respectively. The error bars correspond to statistical uncertainties and boxes to systematic ones.

Leading ($\alpha = 1$) and subleading ($\alpha = 2$) modes for $n = 3$ as a function of $p_T$, measured in a wide centrality range of PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The results for the leading mode ($\alpha = 1$) are compared to the standard triangular flow magnitude measured by ALICE and CMS using the two-particle correlation method taken from Refs. [7,15], respectively. The error bars correspond to statistical uncertainties and boxes to systematic ones.
FIG. 3. Leading ($\alpha = 1$) and subleading ($\alpha = 2$) modes for $n = 2$ as a function of $p_T$, measured in high-multiplicity pPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, for four classes of reconstructed track multiplicity $N_{\text{trk}}^{\text{offline}}$. The results for the leading mode ($\alpha = 1$) are compared to the standard elliptic flow magnitude taken from Ref. [25]. The error bars correspond to statistical uncertainties and boxes to systematic ones.

FIG. 4. Leading ($\alpha = 1$) and subleading ($\alpha = 2$) modes for $n = 3$ as a function of $p_T$, measured in high-multiplicity pPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, for four classes of reconstructed track multiplicity $N_{\text{trk}}^{\text{offline}}$. The results for the leading mode ($\alpha = 1$) are compared to the standard triangular flow magnitude taken from Ref. [25]. The error bars correspond to statistical uncertainties and boxes to systematic ones.
FIG. 5. Comparison of the Pearson correlation coefficient $r_2$ reconstructed with harmonic decomposition, using the leading and subleading modes and $r_2$ values from Ref. [19], as a function of $p_T^a - p_T^b$ in bin of $p_T^a$ for six centrality classes in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The error bars correspond to statistical uncertainties and boxes to systematic ones.

FIG. 6. Comparison of the Pearson correlation coefficient $r_3$ reconstructed with harmonic decomposition, using the leading and subleading modes and $r_3$ values from Ref. [19], as a function of $p_T^a - p_T^b$ in bin of $p_T^a$ for six centrality classes in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The error bars correspond to statistical uncertainties and boxes to systematic ones.
is closer to unity for the PCA results than for the previous measurements. This is expected because the $V_{n\Delta}$ values are constructed from only two of the modes. Figure 6 shows the $n=3$ case, again using the comparison with $r_3$ from the previous two-particle correlation analysis [19]. Although the errors are large it is clear that the principle-component analysis tracks the previously measured divergence of $r_3$ from unity at high $p_T$.

The Pearson coefficient calculated from Eq. (12) can be expanded as a power series of ratios of modes. Figure 7 shows the ratio of the leading and subleading modes for both $p$Pb and PbPb collisions as a function of centrality (track multiplicity). The ratios are calculated for the highest $p_T$ bin used in the analysis. The top panel shows the elliptic case while the bottom panel shows the triangular case. For the elliptic case the ratio is clearly above zero, with $p$Pb high-multiplicity values being above the peripheral PbPb ones. For the triangular case half of the individual points are consistent with zero within the uncertainties. However, the ensemble of all the points suggests that the ratio is above zero.

Finally, Fig. 8 shows leading and subleading modes for the multiplicity case ($n=0$) for PbPb collisions as a function of $p_T$ for eight regions of centrality. For all centralities the leading mode depends only weakly on $p_T$, while the subleading mode increases rapidly with $p_T$ except for very central collisions. The observed increase of the subleading mode with $p_T$ for all centralities is a response to radial-flow fluctuations [22,33]. From a hydrodynamical point of view, the number of particles at high $p_T$ decreases exponentially as $\exp[\frac{p_T(u - u_0)}{T}]$. Here, $T$ is the temperature, $u$ is the maximum fluid velocity, and $u_0 = \sqrt{1 + u^2}$. A small variation in $u$ produces a relative yield that increases linearly with $p_T$. Such behavior is observed in the data for more peripheral collisions. At a given $p_T$
the subleading mode increases strongly from central to peripheral collisions. Since peripheral collisions correspond to smaller interaction volumes, it is expected that $p_T$ fluctuations are more important for peripheral than for central events.

VII. SUMMARY

For the first time the leading and subleading modes of elliptic and triangular flow have been measured for 5.02-TeV $p$Pb and 2.76-TeV PbPb collisions. For PbPb collisions the leading and subleading modes of multiplicity fluctuations were also measured. Since the principal-component analysis uses all the information encoded in the covariance matrix, it provides increased sensitivity to fluctuations. For a very wide range of $p_T$ and centrality, the leading modes of the elliptic and triangular flow are found to be essentially equal to the anisotropy coefficients measured using the standard two-particle correlation method. For both the elliptic and triangular cases the subleading modes are nonzero and increase with $p_T$. This behavior reflects a breakdown of flow factorization at high $p_T$ in both the $p$Pb and PbPb systems. For charged-particle multiplicity both the leading and subleading modes increase steadily from central to peripheral PbPb events. The leading mode depends only weakly upon $p_T$ while the subleading mode increases strongly with $p_T$. This centrality and $p_T$ dependence are suggestive of the presence of fluctuations in the radial flow.

In summary the subleading modes of the principal-component analysis capture new information from the spectra of flow and multiplicity fluctuations and provide an efficient method to quantify the breakdown of factorization in two-particle correlations.

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