Study of $b \to c$ induced $\bar{B}^* \to V\ell\bar{\nu}_\ell$ decays

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Abstract

In this paper, we investigate the tree-dominated $\bar{B}^*_{u,d,s,c} \to V\ell^-\bar{\nu}_\ell$ ($V = D^*_{u,d}, D^*_s, J/\psi$ and $\ell = e, \mu, \tau$) decays in the Standard Model with the relevant form factors obtained in the light-front quark model. These decays involve much more helicity states relative to the corresponding $\bar{B}^* \to P\ell^-\bar{\nu}_\ell$ and $\bar{B} \to V\ell^-\bar{\nu}_\ell$ decays, and moreover, the contribution of longitudinal polarization mode ($V$ meson) is relatively small, $\sim 30\%$, compared with the corresponding $B$ meson decays. We have also computed the branching fraction, lepton spin asymmetry, forward-backward asymmetry and ratio $R_{V}^{(L)} \equiv \frac{B(\bar{B}^* \to V\ell^-\bar{\nu}_\ell)}{B(\bar{B}^* \to V\ell'^-\bar{\nu}_{\ell'})}$ ($\ell' = e, \mu$). Numerically, the branching fractions of $\bar{B}^* \to V\ell'^-\bar{\nu}_{\ell'}$ decays are at the level of $\mathcal{O}(10^{-7})$, and are hopeful to be observed by LHC and Belle-II experiments. The ratios $R_{D^*, D^*_s, J/\psi}^{(L)}$ have relatively small theoretical uncertainties and are close to each other, $R_{D^*}^{(L)} \simeq R_{D^*_s}^{(L)} \simeq R_{J/\psi}^{(L)} \simeq [0.26, 0.27]$ ($[0.27, 0.29]$), which are a bit different from the predictions in some previous works. The future measurements are expected to make tests on these predictions.

1
1 Introduction

In the past years, a large amount of $B\bar{B}$ events have been accumulated by Babar, Belle, Tevatron and LHCb experiments, and most of $B$-meson decays having branching fractions $\gtrsim \mathcal{O}(10^{-7})$ have been measured [1]. Moreover, some deviations between the standard model (SM) predictions and the experimental data have been observed, for instance, the angular observable $P_5'$ of $B \to K^* \mu^+ \mu^-$ decay with $2.6\sigma$ discrepancy [2–6], the differential branching fraction of $B_s \to \phi \mu^+ \mu^-$ decay with $3.3\sigma$ discrepancy [7, 8], the well-known “$\pi K$ CP puzzle” [9, 10], and so on. Besides the flavor-changing-neutral-current processes mentioned above, the $B$-meson semileptonic decays induced by $b \to c\ell \bar{\nu}_\ell$ transition also play an important role in testing the SM and probing the hints of possible new physics (NP). For instance, the well-known “$R_{D^*}$ anomaly” reported by BaBar [11, 12], Belle [13–15] and LHCb [16, 17] collaborations exhibits a significant deviation between the SM prediction and experimental data [1, 18, 19]. Many studies have been done within the model-independent frameworks [20–27], as well as in some specific NP models, for instance Refs. [28–47]. One can refer to Refs. [48, 49] for recent reviews.

The spin-triplet vector $B^*_q$ meson with quantum number of $n^2s^1L_J = 1^3S_1$ and $J^P = 1^-$ [50–53] has the same flavor components as the spin-singlet pseudoscalar $B_q$ ($q = u, d, s$ and $c$) meson, and can also decay through the $b \to c\ell \bar{\nu}_\ell$ transition at quark-level, therefore its $b \to c$ induced semileptonic decays can play a similar role as $B$ meson decays for testing the SM and probing possible hints of NP.

The $B^*_q$ meson is unstable particle, it cannot decay via strong interaction due to that $m_{B^*_q} - m_{B_q} \lesssim 50$ MeV $\ll m_{B_q}$ [54]; $B^*_q$ meson decay is dominated by the radiative process [54], $B^*_q \to B_q \gamma$; the weak decay modes via the bottom-changing transition (for instance, the $b \to c$ induced semileptonic $B^*_q$ decays considered in this work) are generally very rare, and their branching fractions are expected to be very small within the SM. Until now, there is no experimental information and few theoretical works concentrating on the $B^*_q$ weak decays. Fortunately, thanks to the high luminosity and large production cross section at the running LHC and SuperKEKB/Belle-II experiments, a huge amount of the $B^*_q$ meson data samples would be accumulated. At Belle-II experiment, the $B^*$ and $B^*_s$ mesons are produced mainly via $\Upsilon(5S)$ decays. With the target annual integrated luminosity, $\sim 13$ ab$^{-1}$ [55], and the cross section of $\Upsilon(5S)$ production in $e^+e^-$ collisions, $\sigma(e^+e^- \to \Upsilon(5S)) = (0.301 \pm 0.002 \pm 0.039)$ nb [56], it is
expected that about $4 \times 10^9 \Upsilon(5S)$ samples could be produced per year by Belle-II. Further considering that $\Upsilon(5S)$ meson mainly decays to final states with a pair of $B^{(*)}_s$ mesons and using the branching fractions of $\Upsilon(5S)$ decays given by PDG [54], it can be estimated that about $N(B^* + \bar{B}^*)/\text{year} \sim 4 \times 10^9$ and $N(B^*_s + \bar{B}^*_s)/\text{year} \sim 2 \times 10^9$ samples can be accumulated by Belle-II per year. Unfortunately, the $B^*_c$ meson and its decays are out of the scope of Belle-II experiment. In addition, a lot of $B^*_q$ samples can also be produced via $pp$ collision and be accumulated in the future by LHC with high collision energy, high luminosity and rather large production cross section [57–59], and some $B^*_q$ weak decays are hopeful to be observed, such as the leptonic $B^*_s \rightarrow \ell^+\ell^-$ decay with branching fraction $\sim \mathcal{O}(10^{-11})$ [60].

Encouraged by the abundant $B^*_q$ data samples at future heavy-flavor experiments, some interesting theoretical studies for the $B^*_q$ weak decays have been made within the SM, for instance, the pure leptonic $\bar{B}^*_s \rightarrow \ell^+\ell^-$ and $\bar{B}^*_u,c \rightarrow \ell^+\bar{\nu}_\ell$ decays [60], the impact of $\bar{B}_{s,d} \rightarrow \mu^+\mu^-$ decays [61], the studies of the semileptonic $B^*_c$ decays within the QCD sum rules [62–64], the semileptonic $B^*_{u,d,c,s} \rightarrow (P,V)\ell^-\bar{\nu}_\ell$ with $P = D, D_s, \eta_c, V = D^*, D^*_s, J/\psi$ decays within the Bethe-Salpeter (BS) method [65] and a approach under the assumption of heavy quark symmetry (HQS) [66], $B^* \rightarrow P\ell^-\bar{\nu}_\ell$ with $P = D, D_s, \pi, K$ [67] and the nonleptonic $\bar{B}^*_{d,s} \rightarrow D^+_{d,s}M^- (M = \pi, K, \rho$ and $K^*)$ [68, 69], $\bar{B}^*_{d,s} \rightarrow D_{d,s}V$ [70], $B^*_c \rightarrow B_{u,d,s}V, B_{u,d,s}P$ [71], $B^*_c \rightarrow \eta_cV$ [72], $B^* \rightarrow \bar{D}D$ [73] and $B^*_c \rightarrow \psi(1S, 2S)P, \eta_c(1S, 2S)P$ [74] decays. Moreover, the NP effects on the semileptonic $\bar{B}^* \rightarrow P\ell^-\bar{\nu}_\ell$ with $P = D, D_s, \pi, K$ decays have been investigated in a model-independent scheme [75] and the vector leptoquark model [76]. In this paper, we pay our attention to the CKM-favored and tree-dominated semileptonic $\bar{B}^*_{u,d,s,c} \rightarrow V\ell\bar{\nu}_\ell (V = D^*_{u,d}, D^*_s, J/\psi)$ weak decays, which are generally much more complicated than the corresponding $B$ decay modes because they involve much more allowed helicity states.

Our paper is organized as follows. In section 2, the helicity amplitudes and observables of $\bar{B}^* \rightarrow V\ell\bar{\nu}_\ell$ decays are calculated. Section 3 is devoted to the numerical results and discussions, and the $\bar{B}^* \rightarrow V$ transition form factors obtained within the covariant light-front quark mode are used in the computation. Finally, we give our summary in section 4.
2 Theoretical framework and results

2.1 Effective Lagrangian and amplitude

In the SM, $\bar{B}^*_{u,d,s,c} \rightarrow V \ell \bar{\nu}_\ell \ (V = D^*_{u,d}, D^*_s, J/\psi)$ decays are induced by $b \rightarrow c \ell \bar{\nu}_\ell$ transition at quark level via $W$-exchange, and can be described by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -2\sqrt{2} G_F V_{cb} \bar{c} \gamma^\mu b \bar{L} \gamma^\nu L + \text{h.c.}, \quad (1)$$

at low energy scale $\mu = \mathcal{O}(m_b)$, where $G_F$ is the Fermi coupling constant and $V_{cb}$ denotes the CKM matrix element. Using Eq. (1), the amplitude of $\bar{B}^* \rightarrow V \ell \bar{\nu}_\ell$ decay can be written as the product of hadronic matrix element and leptonic current. Then, in terms of leptonic $(L_{\mu\nu})$ and hadronic $(H^{\mu\nu})$ tensors built from the respective products of the leptonic and hadronic currents, the square amplitude can be expressed as

$$|\mathcal{M}(\bar{B}^* \rightarrow V \ell \bar{\nu}_\ell)|^2 = |\langle V \ell \bar{\nu}_\ell | \mathcal{L}_{\text{eff}} | \bar{B}^* \rangle|^2 = \frac{G_F^2 |V_{cb}|^2}{2} L_{\mu\nu} H^{\mu\nu}. \quad (2)$$

Inserting the completeness relation of the polarization vector of virtual $W^*$ boson,

$$\sum_{m,n} \bar{\epsilon}_{\mu}(m) \epsilon^*_\nu(n) g_{mn} = g_{\mu\nu}, \quad (3)$$

the product of $L_{\mu\nu}$ and $H^{\mu\nu}$ can be rewritten as

$$L_{\mu\nu} H^{\mu\nu} = \sum_{m,m',n,n'} L(m,n) H(m',n') g_{mm'} g_{nn'}, \quad (4)$$

where $L(m,n) \equiv L^{\mu\nu} \bar{\epsilon}_{\mu}(m) \epsilon^*_\nu(n)$ and $H(m,n) \equiv H^{\mu\nu} \bar{\epsilon}_{\mu}(m) \epsilon^*_\nu(n)$ are Lorentz invariant and therefore can be evaluated in different reference frames. In our following evaluation, $H(m,n)$ and $L(m,n)$ will be calculated in the $B^*$-meson rest frame and the $\ell - \bar{\nu}_\ell$ center-of-mass frame, respectively.

2.2 Kinematics

In the rest frame of $B^*$ meson, assuming the final state $V$-meson moving along with positive $z$-direction, the momenta of $B^*$, $V$ and $W^*$ could be written as

$$p_{B^*}^\mu = (m_{B^*}, 0, 0, 0), \quad p_{V}^\mu = (E_V, 0, 0, |\vec{p}|), \quad q^\mu = (q^0, 0, 0, -|\vec{p}|), \quad (5)$$
respectively, where \( q^0 = (m_B^2 - m_V^2 + q^2)/2m_B \) and \( |\vec{p}| = \lambda^{1/2}(m_B^2, m_V^2, q^2)/2m_B \), with 
\[
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)
\]
and \( q^2 = (p_B - p_V)^2 \) being the momentum transfer squared, are the energy and momentum of virtual \( W^* \). The polarization vectors of the initial \( B^* \)-meson and daughter \( V \)-meson, \( \epsilon^\mu_1(0, \pm) \) and \( \epsilon^\mu_2(0, \pm) \), can be written as

\[
\epsilon^\mu_1(0) = (0, 0, 0, 1), \quad \epsilon^\mu_1(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0); \quad (6)
\]
\[
\epsilon^\mu_2(0) = \frac{1}{m_V}(|\vec{p}|, 0, 0, E_V), \quad \epsilon^\mu_2(\pm) = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad (7)
\]
respectively. For the four polarization vectors of virtual \( W^* \), \( \bar{\epsilon}^\mu(t, 0, \pm) \), one can conveniently choose [77][78]

\[
\bar{\epsilon}^\mu(t) = \frac{1}{\sqrt{q^2}}(q^0, 0, 0, -|\vec{p}|), \quad \bar{\epsilon}^\mu(0) = \frac{1}{\sqrt{q^2}}(|\vec{p}|, 0, 0, -q^0), \quad \bar{\epsilon}^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0), \quad (8)
\]
in which, \( \lambda_{W^*} = t \) has to be understood as \( \lambda_{W^*} = 0 \) and \( J = 0 \).

Turning to the \( \ell - \bar{\nu}_\ell \) center-of-mass frame, the four-momenta of lepton and antineutrino are given as

\[
p^\mu_\ell = (E_\ell, |\vec{p}_\ell| \sin \theta, 0, |\vec{p}_\ell| \cos \theta), \quad p^\mu_{\nu_\ell} = (|\vec{p}_\ell|, -|\vec{p}_\ell| \sin \theta, 0, -|\vec{p}_\ell| \cos \theta), \quad (9)
\]
where \( E_\ell = (q^2 + m_\ell^2)/2\sqrt{q^2} \), \( |\vec{p}_\ell| = (q^2 - m_\ell^2)/2\sqrt{q^2} \), and \( \theta \) is the angle between \( V \) and \( \ell \) three-momenta. In this frame, the polarization vectors \( \bar{\epsilon}^\mu(\lambda_{W^*}) \) have the form

\[
\bar{\epsilon}^\mu(t) = (1, 0, 0, 0), \quad \bar{\epsilon}^\mu(0) = (0, 0, 0, 1), \quad \bar{\epsilon}^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \quad (10)
\]

### 2.3 Hadronic helicity amplitudes

For hadronic part, one has to calculate the hadronic helicity amplitudes \( H_{\lambda_{W^*}, \lambda_{B^*}, \lambda_V} \) of \( B^* \rightarrow V \ell^+ \bar{\nu}_\ell \) decay defined by

\[
H_{\lambda_{W^*}, \lambda_{B^*}, \lambda_V}(q^2) = \langle V(p_V, \lambda_V)|\bar{\epsilon}^\mu(1 - \gamma_5)b|B^*(p_{B^*}, \lambda_{B^*})\rangle\bar{\epsilon}^\ast\mu(\lambda_{W^*}), \quad (11)
\]
which describes the decay of three helicity states of \( B^* \) meson into the three helicity states of daughter \( V \) meson and the four helicity states of virtual \( W^* \). For the \( B^* \rightarrow V \) transition, the matrix elements \( \langle V(p_V, \lambda_V)|\bar{\epsilon}^\mu(1 - \gamma_5)b|B^*(p_{B^*}, \lambda_{B^*})\rangle \) can be factorized in terms of ten form factors \( V_{1,2,3,4,5,6}(q^2) \) and \( A_{1,2,3,4}(q^2) \) as [79][80]

\[
\langle V(\epsilon_2, p_V)|\bar{\epsilon}^\mu(1 - \gamma_5)b|B^*(\epsilon_1, p_{B^*})\rangle = (\epsilon_1 \cdot \epsilon_2^\ast) [-P_\mu V_1(q^2) + q_\mu V_2(q^2)]
\]
with the sign convention \(\epsilon_{0123} = -1\).

Then, by contracting these hadronic matrix elements with the polarization vector of virtual \(W^*\) boson, we can finally obtain the non-vanishing hadronic helicity amplitudes, \(H_{\lambda_{W^*} \lambda_B \lambda_V}\), given as

\[
H_{0++}(q^2) = -\frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} A_1(q^2) + \sqrt{q^2} A_2(q^2) + \frac{2m_{B^*}|\vec{p}|}{\sqrt{q^2}} V_1(q^2),
\]

\[
H_{t++}(q^2) = -\frac{2m_{B^*}|\vec{p}|}{\sqrt{q^2}} A_1(q^2) + \frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} V_1(q^2) - \sqrt{q^2} V_2(q^2),
\]

\[
H_{-0+}(q^2) = -\frac{m_{B^*}^2 + 3m_V^2 - q^2}{2m_V} A_1(q^2) + \frac{(m_{B^*}^2 - m_V^2 - q^2)}{2m_V} A_2(q^2)
\]

\[
- \frac{2m_{B^*}|\vec{p}|}{m_V(m_{B^*}^2 - m_V^2)} A_3(q^2) - \frac{m_{B^*}|\vec{p}|}{m_V} V_6(q^2),
\]

\[
H_{0--}(q^2) = \frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} A_1(q^2) - \sqrt{q^2} A_2(q^2) + \frac{2m_{B^*}|\vec{p}|}{\sqrt{q^2}} V_1(q^2),
\]

\[
H_{t--}(q^2) = \frac{2m_{B^*}|\vec{p}|}{\sqrt{q^2}} A_1(q^2) + \frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} V_1(q^2) - \sqrt{q^2} V_2(q^2),
\]

\[
H_{+-0}(q^2) = \frac{m_{B^*}^2 + 3m_V^2 - q^2}{2m_V} A_1(q^2) - \frac{(m_{B^*}^2 - m_V^2 - q^2)}{2m_V} A_2(q^2)
\]

\[
+ \frac{2m_{B^*}|\vec{p}|}{m_V(m_{B^*}^2 - m_V^2)} A_3(q^2) - \frac{m_{B^*}|\vec{p}|}{m_V} V_6(q^2),
\]

\[
H_{++0}(q^2) = 3\frac{m_{B^*}^2 + m_V^2 - q^2}{2m_{B^*}} A_1(q^2) - \frac{(m_{B^*}^2 - m_V^2 + q^2)}{2m_{B^*}} A_2(q^2)
\]

\[
+ \frac{2m_{B^*}|\vec{p}|}{m_{B^*}^2 - m_V^2} A_4(q^2) - |\vec{p}| V_5(q^2),
\]

\[
H_{--0}(q^2) = -\frac{3m_{B^*}^2 + m_V^2 - q^2}{2m_{B^*}} A_1(q^2) + \frac{(m_{B^*}^2 - m_V^2 + q^2)}{2m_{B^*}} A_2(q^2)
\]

\[
- \frac{2m_{B^*}|\vec{p}|}{m_{B^*}^2 - m_V^2} A_4(q^2) - |\vec{p}| V_5(q^2),
\]

\[
H_{000}(q^2) = \frac{|\vec{p}|(m_{B^*}^2 + m_V^2 - q^2)}{\sqrt{q^2} m_V} V_1(q^2) + \frac{2m_{B^*}|\vec{p}|^3}{\sqrt{q^2 m_V(m_{B^*}^2 - m_V^2)}} V_3(q^2)
\]
Taking the exact forms of spinors and where $J$ expression of $h$

Obviously, only the amplitudes with $\lambda_{B^*} = \lambda_V - \lambda_W^*$ survive due to the helicity conservation.

### 2.4 Helicity amplitudes and observables

For the leptonic part, the leptonic tensor could be expanded in terms of a complete set of Wigner’s $d^J$-functions, which has been widely used in the study of hadron semileptonic [77,81,82]. As a result, $L_{\mu\nu}H^{\mu\nu}$ can be reduced to a very compact form

$$L_{\mu\nu}H^{\mu\nu} = \frac{1}{8} \sum_{\lambda_\rho,\lambda_\rho, J^J, J} (-1)^{J+J^J} [h_{\lambda_\rho,\lambda_\rho} |^2 \delta_{\lambda_B^*, \lambda_V - \lambda_W^*} \delta_{\lambda_B^*, \lambda_V - \lambda_W^*}$$

$$\times d_J^{\lambda_B^*, \lambda_V - \frac{1}{2}, J^J} d_J^{\lambda_W^*, \lambda_V - \frac{1}{2}, J^J} H_{\lambda_W^*, \lambda_B^*, \lambda_V} H_{\lambda_B^*, \lambda_W^*, \lambda_V}$$  

(24)

where $J$ and $J^J$ run over 1 and 0, $\lambda_W^{(l)}$ and $\lambda_\ell$ run over their components. For the standard expression of $d^J$ function, we take their value from PDG [54]. The leptonic helicity amplitude $h_{\lambda_\rho,\lambda_\rho}$ in Eq. (24) defined as

$$h_{\lambda_\rho,\lambda_\rho} = \bar{u}_\ell(\lambda_\ell)\gamma^\mu(1 - \gamma_5)\nu_\rho\left(\frac{1}{2}\right)\bar{\epsilon}_\mu(\lambda_W^*)$$

(25)

Taking the exact forms of spinors and $W^*$ polarization vectors given in Eq. (10), we obtain

$$|h_{-\frac{1}{2}, \frac{1}{2}}|^2 = 8(q_2^2 - m_\ell^2)$$

$$|h_{\frac{1}{2}, \frac{1}{2}}|^2 = \frac{8m_\ell^2}{2q_2^2}(q_2^2 - m_\ell^2)$$

(26)

which are the same as the results obtained in semileptonic $B$ and hyperon decays [81,82].

Using the amplitudes obtained above, we can then further evaluate the observables of $\bar{B}^* \rightarrow V\ell^-\bar{\nu}_\ell$ decays. The double differential decay rate is written as

$$\frac{d^2\Gamma}{dq_2^2d\cos\theta} = \frac{G_F^2|V_{cb}|^2}{(2\pi)^3} \frac{|p|}{8m_{B^*}^2} \frac{1}{3}(1 - \frac{m_\ell^2}{q_2^2})L_{\mu\nu}H^{\mu\nu},$$

(27)
where the factor 1/3 is caused by averaging over the spins of initial $\bar{B}^*$ meson. The double differential decay rate with a given helicity state of lepton ($\lambda_\ell = \pm \frac{1}{2}$) is written as

$$
\frac{d^2\Gamma}{dq^2 d \cos \theta} = \frac{G_F^2 |V_{cb}|^2 |\rho|}{256\pi^3 m_{B^*}^2} \frac{1}{3} q^2 (1 - \frac{m_{\ell}^2}{q^2})^2 \times \left[ (1 - \cos \theta)^2 (H_{0+}^2 + H_{-0}^2) + (1 + \cos \theta)^2 (H_{-0}^2 + H_{0+}^2) \right] \\
+ 2\sin^2 \theta (H_{0+}^2 + H_{0-}^2 + H_{-0}^2 + H_{0+}^2) + 2(H_{0+} - \cos \theta H_{0-})^2 \\
+ 2(H_{0-} - \cos \theta H_{0+})^2 + 2(H_{00} - \cos \theta H_{00})^2 \right].
$$

Integrating over $\cos \theta$ and summing over the lepton helicity, we can obtain the differential decay rate written as

$$
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 |\rho|}{96\pi^3 m_{B^*}^2} \frac{1}{3} q^2 (1 - \frac{m_{\ell}^2}{q^2})^2 \times \left[ \frac{3m_{\ell}^2}{2q^2} (H_{0+}^2 + H_{0-}^2 + H_{00}^2) \right] \\
+ (H_{0+}^2 + H_{0-}^2 + H_{-0}^2 + H_{0+}^2 + H_{00}^2 + H_{-0}^2 + H_{0+}^2)(1 + \frac{m_{\ell}^2}{2q^2})],
$$

in which, the three non-diagonal interference terms in Eq. (29) vanish. In addition, paying attention to the polarization states of $V$ meson, one can obtain the longitudinal differential decay width $\frac{d\Gamma^L}{dq^2}$ by picking out $H_{00}^2$, $H_{0-}^2$, $H_{0+}^2$ and $H_{00}^2$ terms in Eq. (30).

Using Eqs. (28) and (29) given above, we can also construct some useful observables as follows. The $q^2$ dependent ratios is defined as

$$
R_V^{(L)}(q^2) \equiv \frac{d\Gamma^{(L)}(\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau)/dq^2}{d\Gamma^{(L)}(\bar{B}^* \rightarrow V \nu^- \bar{\nu}_\nu)/dq^2},
$$

where $\ell'$ denotes the light leptons $\mu$ and $e$ (in the following calculations, we take $m_{e,\mu} = 0$).

The lepton spin asymmetry and forward-backward asymmetry are defined as

$$
A_{\lambda}^{V}(q^2) = \frac{d\Gamma[\lambda_\ell = -1/2]/dq^2 - d\Gamma[\lambda_\ell = 1/2]/dq^2}{d\Gamma[\lambda_\ell = -1/2]/dq^2 + d\Gamma[\lambda_\ell = 1/2]/dq^2},
$$

and

$$
A_{\theta}^{V}(q^2) = \frac{\int_{-1}^{0} d\cos \theta \left( d^2\Gamma/dq^2 d\cos \theta \right) - \int_{0}^{1} d\cos \theta \left( d^2\Gamma/dq^2 d\cos \theta \right)}{d\Gamma/dq^2},
$$

respectively. These observables are independent of the CKM matrix elements, and the hadronic uncertainties canceled to a large extent, therefore, they can be predicted with a rather high accuracy.
3 Numerical results and discussions

In our numerical calculation, for the well-known Fermi coupling constant $G_F$ and the masses of mesons and $\tau$, we take their central values given by PDG \[54\]. For the CKM element, we take $|V_{cb}| = 41.80^{+0.28}_{-0.60} \times 10^{-3}$ given by CKMFitter Group \[83\]. In order to evaluate the branching fractions, the total decay widths (or lifetimes), $\Gamma_{\text{tot}}(B^{*}_{u,d,s,c})$, are also essential inputs. However, there is no available experimental or theoretical information until now. While, due to the fact that the electromagnetic processes $B^{*} \to B\gamma$ dominates $B^{*}$ decays, we can take the approximation $\Gamma_{\text{tot}}(B^{*}) \approx \Gamma(B^{*} \to B\gamma)$. In the light-front quark model (LFQM), the decay width of $B^{*} \to B\gamma$ decay is given by \[84\]

$$\Gamma(B^{*} \to B\gamma) = \frac{\alpha}{3} \left[ \epsilon_1 I(m_1, m_2, 0) + \epsilon_2 I(m_2, m_1, 0) \right]^2 \kappa^3, \quad (34)$$

$$I(m_1, m_2, q^2) = \int_0^1 \frac{dx}{8\pi^3} \int d^2k \frac{\psi(x, k'_{\perp}) \psi(x, k_{\perp})}{xM_0M_0'} \times \left\{ A + \frac{2}{M_0} \left[ k_{\perp} - \frac{(k_{\perp} \cdot q_{\perp})^2}{q_{\perp}^2} \right] \right\}, \quad (35)$$

where $A = \bar{x}m_1 + x m_2$ with $\bar{x} = 1 - x$, $M_0 = M_0 + m_1 + m_2$ with $M_0$ being the invariant mass of bound-state, $\alpha$ is the fine-structure constant, $\kappa = (m_{B^*}^2 - m_{B}^2)^2/2m_{B^*}$ is the kinematically allowed energy of the outgoing photon. The radial wavefunction (WF) $\psi(x, k_{\perp})$ of bound-state is responsible for describing the momentum distribution of the constituent quarks. In this paper, we shall use the Gaussian-type WF

$$\psi(x, k_{\perp}) = 4\pi^{\frac{3}{2}} \sqrt{\frac{\partial k_z}{\partial x}} \exp \left[ -\frac{k_z^2 + k_{\perp}^2}{2\beta^2} \right], \quad (36)$$

where $k_z$ is the relative momentum in $z$-direction and has the form $k_z = (x - \frac{1}{2})M_0 + \frac{m_2^2 - m_1^2}{2M_0}$. One can refer to Ref. \[84\] for more details. Using the constituent quark masses and the Gaussian parameter $\beta$ given in Table 1, we obtain the numerical results for $\Gamma(B^{*} \to B\gamma)$ as follows,

$$\Gamma_{\text{tot}}(B^{*+}) \approx \Gamma(B^{*+} \to B^{+}\gamma) = (349 \pm 18) \text{ eV}, \quad (37)$$

$$\Gamma_{\text{tot}}(B^{*0}) \approx \Gamma(B^{*0} \to B^0\gamma) = (116 \pm 6) \text{ eV}, \quad (38)$$

$$\Gamma_{\text{tot}}(B^{*}_s) \approx \Gamma(B^{*0}_s \to B^0_s\gamma) = (84^{+11}_{-9}) \text{ eV}, \quad (39)$$

$$\Gamma_{\text{tot}}(B^{*+}_c) \approx \Gamma(B^{*+}_c \to B^0_c\gamma) = (49^{+28}_{-21}) \text{ eV}. \quad (40)$$

These theoretical predictions are generally in agreement with the ones obtained in the previous work based on different theoretical models \[84,87,92\].
Table 1: The values of constituent quark masses and Gaussian parameters (in units of MeV) obtained by fitting to the data of decay constants [85,86], where $q = u, d$.

| Masses | Parameters |
|--------|------------|
| $m_q = 250$, $m_s = 450$, $m_c = 1400$, $m_b = 4640$; | $eta_{bq} = 540.7 \pm 9.6$, $\beta_{bs} = 601.9 \pm 7.4$, $\beta_{bc} = 933.9 \pm 11.1$, for P-meson |
| $\beta_{cq} = 413.0 \pm 12.0$, $\beta_{cs} = 514.1 \pm 18.5$, $\beta_{cc} = 684.4 \pm 6.7$; | $\beta_{bq} = 504.4 \pm 14.2$, $\beta_{bs} = 556.4 \pm 10.1$, $\beta_{bc} = 863.4 \pm 32.8$, for V-meson |

Besides the inputs given above, the $B^* \to V$ transition form factors are also crucial inputs for evaluating observables, especially for the branching fraction. In this work, we adopt the covariant light-front quark model (CLFQM) [93–95] to evaluate their values. The theoretical formulas for the form factors of $V' \to V''$ have been given in our previous work (see Eqs. (39-48) in the appendix of Ref. [96]). These theoretical results are obtained within Drell-Yan-West frame, $q^+ = 0$, which implies that the form factors are known only for space-like momentum transfer, $q^2 = -q_{\perp}^2 \leq 0$, and the ones in the physical time-like region need an additional $q^2$ extrapolation. Following the strategy employed in Refs. [80, 93–95], one can parameterize the form factors as functions of $q^2$ by using dipole model in the space-like region and then extend the them to the whole physical region $0 \leq q^2 \leq (m_{B^*} - m_V)^2$. The form factors in the dipole model have the form

$$F(q^2) = \frac{F(0)}{1 - a \frac{q^2}{m_{B^*}^2} + b \left( \frac{q^2}{m_{B^*}^2} \right)^2},$$

(41)

where $F$ denotes $A_{1-4}$ and $V_{1-6}$. Using the inputs given in Table 1, we then present our theoretical prediction for the form factors of $\bar{B}^* \to D^*$, $\bar{B}_s^* \to D_s^*$ and $\bar{B}_c^* \to J/\psi$ transitions in Table 2. Their $q^2$-dependences are shown in Fig. 1.

Using the formulas given in the last section and inputs given above, we then present our numerical results for the $q^2$-integrated observables of $\bar{B}^* \to V \ell^+ \bar{\nu}_\ell$ decays in Tables 3 and 4. For the branching fractions, the three errors in Table 3 are caused by the uncertainties of form factors, $V_{cb}$ and $\Gamma_{tot}(B^*)$, respectively. For the other observables listed in Table 4, the theoretical uncertainties are caused only by the form factors. Besides, the $q^2$-dependence of differential decay rates $d\Gamma^{(L)}/q^2$ and $A_{V,\ell}^{(V)}$, $R_{V}^{(L)}$ are shown in Figs. 2 and 3. The following are
Table 2: The numerical results of form factors for $\bar{B}^* \to D^*$, $\bar{B}^*_s \to D^*_s$ and $\bar{B}^*_c \to J/\psi$ transitions within the CLFQM. The uncertainties are caused by the Gaussian parameters listed in Table I.

| $B^* \to D^*$ | $\bar{B}^*_s \to D^*_s$ | $\bar{B}^*_c \to J/\psi$ |
|----------------|----------------|----------------|
| $F(0)$ | $0.66^{+0.01}_{-0.01}$ | $0.36^{+0.00}_{-0.00}$ | $0.07^{+0.00}_{-0.00}$ | $0.08^{+0.00}_{-0.00}$ | $0.67^{+0.01}_{-0.01}$ | $0.36^{+0.00}_{-0.00}$ | $0.13^{+0.00}_{-0.00}$ | $0.00^{+0.00}_{-0.00}$ | $1.17^{+0.01}_{-0.01}$ | $0.48^{+0.01}_{-0.01}$ |
| $F(1)$ | $1.31^{+0.02}_{-0.02}$ | $1.32^{+0.02}_{-0.02}$ | $1.79^{+0.02}_{-0.02}$ | $1.81^{+0.02}_{-0.02}$ | $1.30^{+0.02}_{-0.02}$ | $1.32^{+0.02}_{-0.02}$ | $1.72^{+0.02}_{-0.02}$ | $-0.09^{+0.45}_{-0.40}$ | $1.30^{+0.02}_{-0.02}$ | $1.29^{+0.02}_{-0.02}$ |
| $F(2)$ | $0.42^{+0.02}_{-0.02}$ | $0.42^{+0.02}_{-0.02}$ | $1.10^{+0.03}_{-0.03}$ | $1.15^{+0.04}_{-0.04}$ | $0.43^{+0.02}_{-0.02}$ | $0.42^{+0.02}_{-0.02}$ | $1.01^{+0.04}_{-0.04}$ | $1.27^{+0.38}_{-0.28}$ | $0.41^{+0.02}_{-0.02}$ | $0.40^{+0.02}_{-0.02}$ |

Table 3: The SM predictions for the branching fractions of $B^* \to V\ell^{-}\bar{\nu}_\ell$ decays.

| Decay mode | This Work | BS method [65] | HQS [66] |
|------------|-----------|----------------|---------|
| $\bar{B}^*\to D^{*0}\ell^-\bar{\nu}_\ell$ | $8.42^{+0.23+0.11+0.45}_{-0.24-0.24-0.42} \times 10^{-8}$ | $1.26 \times 10^{-7}$ | $6.41 \times 10^{-8}$ |
| $\bar{B}^*\to D^{*0}\tau^-\bar{\nu}_\tau$ | $2.26^{+0.08+0.03+0.12}_{-0.08-0.08-0.11} \times 10^{-8}$ | $2.74 \times 10^{-8}$ | $1.29 \times 10^{-8}$ |
| $\bar{B}^{*0}\to D^{*+}\ell^-\bar{\nu}_\ell$ | $2.51^{+0.08+0.03+0.13}_{-0.07-0.07-0.12} \times 10^{-7}$ | $1.92 \times 10^{-7}$ | $1.92 \times 10^{-7}$ |
| $\bar{B}^{*0}\to D^{*+}\tau^-\bar{\nu}_\tau$ | $6.73^{+0.24+0.09+0.35}_{-0.25-0.19-0.32} \times 10^{-8}$ | $3.88 \times 10^{-8}$ | $3.88 \times 10^{-8}$ |
| $\bar{B}^{*0}_s\to D^{*+}\ell^-\bar{\nu}_\ell$ | $3.46^{+0.17+0.05+0.41}_{-0.17-0.10-0.41} \times 10^{-7}$ | $4.63 \times 10^{-7}$ | $2.53 \times 10^{-7}$ |
| $\bar{B}^{*0}_s\to D^{*+}\tau^-\bar{\nu}_\tau$ | $9.10^{+0.60+0.12+1.07}_{-0.59-0.26-1.07} \times 10^{-8}$ | $1.05 \times 10^{-7}$ | $5.05 \times 10^{-8}$ |
| $\bar{B}^{*0}_c\to J/\psi\ell^-\bar{\nu}_\ell$ | $5.44^{+0.33+0.07+4.06}_{-0.33-0.16-2.00} \times 10^{-7}$ | $5.37 \times 10^{-7}$ | $2.91 \times 10^{-7}$ |
| $\bar{B}^{*0}_c\to J/\psi\tau^-\bar{\nu}_\tau$ | $1.43^{+0.13+0.02+1.07}_{-0.12-0.04-0.52} \times 10^{-7}$ | $1.49 \times 10^{-7}$ | $5.65 \times 10^{-8}$ |

some analyses and discussions:

(1) From Table 3 one can find a clear relation $B(\bar{B}^{*-} \to D^{*0}\ell^-\bar{\nu}_\ell) : B(\bar{B}^{*0} \to D^{*+}\ell^-\bar{\nu}_\ell) :$
Figure 1: The $q^2$-dependences of form factors for $\bar{B}^* \to D^*$, $\bar{B}_s^* \to D_s^*$ and $\bar{B}_c^* \to J/\psi$ transitions.

\[ B(\bar{B}_s^{*0} \to D_s^{*+} \ell^- \bar{\nu}_\ell); B(\bar{B}_c^{*-} \to J/\psi \ell^- \bar{\nu}_\ell) \approx 1 : 3 : 4 : 6, \] which is caused mainly by their total decay widths $\Gamma_{\text{tot}}(B^*)$ illustrated by Eqs. (37), (38), (39) and (40).

In Table 3, the previous predictions based on the Bethe-Salpeter (BS) method [65] and the assumption of heavy quark symmetry (HQS) [66] are also listed for comparison. It can be found that the results based on the BS method and the assumption of HQS are a little bit smaller and larger respectively than our results, but they are also in agreement.
Table 4: Predictions for $q^2$-integrated observables $A^{V}_{\lambda\theta}$ ($\ell = \tau$), $R^{(L)}_{V}$ and $F^{*V}_{L}$.

| Obs. | Prediction | Obs. | Prediction | Obs. | Prediction |
|------|------------|------|------------|------|------------|
| $A^{D*}_{\lambda}$ | $0.237^{+0.017}_{-0.016}$ | $A^{D*}_{\lambda}$ | $0.231^{+0.029}_{-0.030}$ | $A^{J/\psi}_{\lambda}$ | $0.214^{+0.043}_{-0.040}$ |
| $A^{*D*}_{\theta}$ | $0.070^{+0.007}_{-0.006}$ | $A^{*D*}_{\theta}$ | $0.071^{+0.011}_{-0.011}$ | $A^{*J/\psi}_{\theta}$ | $0.078^{+0.014}_{-0.013}$ |
| $R^{*D*}_{D}$ | $0.269^{+0.003}_{-0.003}$ | $R^{*D*}_{D}$ | $0.263^{+0.005}_{-0.005}$ | $R^{*J/\psi}_{D}$ | $0.262^{+0.009}_{-0.007}$ |
| $R^{*L}_{D}$ | $0.285^{+0.004}_{-0.003}$ | $R^{*L}_{D}$ | $0.277^{+0.006}_{-0.007}$ | $R^{*J/\psi}_{L}$ | $0.278^{+0.009}_{-0.009}$ |
| $F^{*D*}_{L}$ | $0.304^{+0.003}_{-0.004}$ | $F^{*D*}_{L}$ | $0.306^{+0.005}_{-0.006}$ | $F^{*J/\psi}_{L}$ | $0.303^{+0.006}_{-0.007}$ |

Figure 2: The $q^2$ dependences of differential decay rates $d\Gamma/dq^2$ (solid lines) and $d\Gamma^L/dq^2$ (dashed lines).

in the order of magnitude. These $b \to c\ell^- \bar{\nu}_\ell$ induced $B^*$ weak decays have the branching fractions of the order $\mathcal{O}(10^{-8} - 10^{-7}) > 10^{-9}$, and therefore are in the scope of Belle-II or LHC experiments. In addition, due to the fact that $V_{ub}/V_{cb} \approx 0.088$, the $b \to u\ell^- \bar{\nu}_\ell$ induced $B^*$ weak decays should have much smaller branching fractions, which are at the level of $\mathcal{O}(10^{-10} - 10^{-9})$, and thus are hard to be observed in the near future.

(2) Deviations from the SM predictions in $B \to D^{(*)}\ell\bar{\nu}_\ell$ decay modes have been observed by the BaBar [11,12], Belle [13,15] and LHCb [16,17] collaborations in the ratios $R_{D^{(*)}} \equiv \frac{B(B \to D^{(*)}\ell^- \bar{\nu}_\ell)}{B(B \to D^{(*)}\ell^- \bar{\nu}_\mu)}$ ($\ell' = \mu, e$). The combination of these measurements performed by the Heavy Flavour Averaging Group (HFLAV) [1] reads

$$R^{HFLAV}_{D} = 0.407 \pm 0.039 \pm 0.024, \quad R^{HFLAV}_{D^*} = 0.306 \pm 0.013 \pm 0.007$$

(42)
Figure 3: The $q^2$-dependences of $R^{(L)}_V(q^2)$, $A^V_\theta(q^2)$ and $A^\ast_V(q^2)$.

which show tensions of about 2 and 4σ, respectively, with the SM predictions [97]. Very recent measurement of $R_{D^*}$ by Belle [98] results in values more compatible with the SM and yield a downward shift in the average. However, even though such measurement is included in the global average, the deviation is still larger than 3σ [97]. If this "$R_{D^*}$ anomaly" is the truth, it possibly exists also in the $b \to c$ induced $\bar{B}^* \to V\ell\bar{\nu}_\ell$ decays, which therefore can provide another useful test on the lepton flavor universality and the various method based on the SM and NP for resolving "$R_{D^*}$ anomaly". Our numerical results for $R_V^{(L)}$ are summarized in Table 4 and the $q^2$-spectra of $R_V^{(L)}$ are shown in Fig. 3. It can be found that

$$R_{D^*}^{(L)} \simeq R_{D_s^*}^{(L)} \simeq R_{J/\psi}^{(L)}$$  (43)

within theoretical uncertainties. Moreover, their $q^2$-spectra almost overlap with each other as shown in Figs. 3 (a) and (b). Using the results summarized in Table 3, we can
also obtain the predictions based on the BS method and HQS,

\begin{align*}
R_{D^*}^s &= 0.217, \quad R_{D_s^*}^s = 0.227, \quad R_{J/\psi}^s = 0.277, \quad \text{BS method} \quad (44) \\
R_{D^*}^s &= 0.202, \quad R_{D_s^*}^s = 0.200, \quad R_{J/\psi}^s = 0.194, \quad \text{HQS} \quad (45)
\end{align*}

It can be found that these results are different from our predictions more or less because different models and parameterizations are used for evaluating form factors, which has been observed in the case of \( R_{D^*} \) \cite{18}. Future measurement will make a judgement on these results.

(3) Besides, the lepton spin asymmetry and the forward-backward asymmetry are also important observables for testing the SM and NP scenarios, for instance, two-Higgs-doublet models, R-parity violating supersymmetry models and so on \cite{42,47}, because their theoretical uncertainties can be well controlled and the zero-crossing points of their \( q^2 \)-spectra are sensitive to the NP effects \cite{42}. Our numerical results for \( q^2 \)-integrated \( A^{sV}_\lambda \) and \( A^{sV}_\theta \) are collected in Table 4, and the \( q^2 \) dependences of \( A^{sV}_\lambda (q^2) \) and \( A^{sV}_\theta \) are shown by Figs. 3 (c) and (d). One can easily find that \( A^{sV}_\lambda, \theta \simeq A^{sV}_{D^*}, s \simeq A^{sV}_{J/\psi} \). Moreover, the \( q^2 \)-spectra of \( A^{sV}_\lambda \) are almost overlap with each other as shown in Fig. 3 (c), and the case of \( A^{sV}_\theta \) is similar except that the \( q^2 \)-spectrum of \( A^{sV}_{J/\psi} \) deviates from the ones of \( A^{sV}_{D^*}, s \) at large \( q^2 \). In addition, \( A^{sV}_\lambda \) crosses the zero point at \( q^2 \approx 5 \text{GeV}^2 \), however \( A^{sV}_\theta \) does not have the zero point in all \( q^2 \) region.

(4) The \( D^* \) longitudinal polarization fraction in semileptonic \( B^0 \to D^{*+} \tau^+ \nu_\tau \) decay, defined as \( F_L^{D^*} = \Gamma_{\lambda_{D^*}} \gamma_0(B^0 \to D^{*+} \tau^+ \nu_\tau) / \Gamma(B^0 \to D^{*+} \tau^+ \nu_\tau) \), has been measured by Belle experiment with \( F_L^{D^*} = 0.60 \pm 0.08 \text{(stat.)} \pm 0.04 \text{(syst.)} \) \cite{99}, which deviates from the SM prediction \( (F_L^{D^*})_{\text{SM}} = 0.457 \pm 0.010 \) \cite{100} by 1.6\( \sigma \). Similarly, we can define the longitudinal polarization fraction

\[
F_L^{sV} = \frac{\Gamma_{\lambda_{D^*}} \gamma_0(\bar{B}^* \to V \tau^- \bar{\nu}_\tau)}{\Gamma(B^* \to V \tau^- \bar{\nu}_\tau)} \quad (46)
\]

for \( \bar{B}^* \to V \tau^- \bar{\nu}_\tau \) decay modes. From the numerical results given in the last row of Table 4, one can easily find that

\[
F_L^{sV} \simeq F_L^{sD^*} \simeq F_L^{sJ/\psi} \simeq 30\%, \quad (47)
\]
which implies that $\bar{B}^* \to V\tau^-\bar{\nu}_\tau$ decay is dominated by the transverse polarization. It is obviously different from the corresponding $\bar{B} \to V\tau^-\bar{\nu}_\tau$ decay mode, which is dominated by the longitudinal polarization state.

4 Summary

In this paper, motivated by abundant $B^*$ data samples at high-luminosity heavy-flavor experiments in the future, we have studied the $b \to c$ induced $\bar{B}_{u,d,s,c}^* \to V\ell^-\bar{\nu}_\ell$ ($V = D_{u,d}^*, D_s^*, J/\psi$ and $\ell = e, \mu, \tau$) decays within the SM. The helicity amplitudes are investigated in detail, and the form factors of $\bar{B}^* \to V$ transitions are computed within the covariant light-front quark model. After that we present our predictions for the observables including branching fraction (decay width), leptonic spin asymmetry, forward-backward asymmetry, ratio $R_V^{(L)}$ and longitudinal polarization fraction in Tables 3, 4 and Figs. 2, 3. It is found that all these semileptonic $B^*$ decays have relatively large branching fractions of $O(10^{-8}) \sim O(10^{-7})$, in which $\mathcal{B}(\bar{B}_c^* \to J/\psi\ell^-\bar{\nu}_\ell) \sim 5 \times 10^{-7}$ is the largest one, and are hopeful to be observed at running LHC and SuperKEKB/Belle-II experiments; in addition, for the $\bar{B}^* \to V\tau^-\bar{\nu}_\tau$ decay, the longitudinal polarization state of $V$ meson presents only about 30% contribution to the integrated decay width, which is obviously different from the corresponding $\bar{B} \to V\tau^-\bar{\nu}_\tau$ decay. All of results and findings in this paper are waiting for the experimental test in the future.

Data Availability Statement

This manuscript has no associated data or the data will not be deposited. This is a theoretical research work, no additional data are associated with this work.

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