The Effects of Disorder on the $\nu = 1$ Quantum Hall State

Ganpathy Murthy

Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506

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A disorder-averaged Hartree-Fock treatment is used to compute the density of single particle states for quantum Hall systems at filling factor $\nu = 1$. It is found that transport and spin polarization experiments can be simultaneously explained by a model of mostly short-range effective disorder. The slope of the transport gap (due to quasiparticles) in parallel field emerges as a result of the interplay between disorder-induced broadening and exchange, and has implications for skyrmion localization.

One of the physical properties that points to a role for disorder is the small magnitude of the transport gap for $\nu = 1$ [3]. Even after assuming that skyrmions, which have smaller excitation energy than quasiparticles [4,5], contribute to reducing this excitation energy further, the predicted gap is nearly a factor of two above the data [6]. In this paper we will take a straightforward phenomenological approach to computing the physical properties of the $\nu = 1$ state in the presence of both disorder and interactions [7]. In this approach the single-particle Green’s function (sigma Green’s function) and occupations of various Landau levels (LLs) with energy $(n + \frac{1}{2})\omega_c$, where $\omega_c = eB/m$ is the cyclotron frequency. Each LL has a macroscopic degeneracy of $BA/\phi_0$, where $A$ is the area of the 2DEG, and $\phi_0 = h/e$ is the quantum unit of flux. A complete understanding of the $\nu = 1$ state should involve the effects of both interactions and disorder.

One of the physical properties that points to a role for disorder is the small magnitude of the transport gap for $\nu = 1$ [3]. Even after assuming that skyrmions, which have smaller excitation energy than quasiparticles [4,5], contribute to reducing this excitation energy further, the predicted gap is nearly a factor of two above the data [6]. In this paper we will take a straightforward phenomenological approach to computing the physical properties of the $\nu = 1$ state in the presence of both disorder and interactions [7]. In this approach the single-particle Green’s function in the Hartree-Fock approximation is averaged over disorder to obtain a Dyson equation

$$
(G)_{nn}^{-1} = (G_0)_{nn}^{-1} - \int \frac{d^2q}{(2\pi)^2} U(q) \sum_m |\rho_{nm}(q)|^2 G_{mm} 
$$

which $G_0$ is the Green’s function in the clean limit seen as a matrix in the the LL indices, $G$ is the disorder-averaged Green’s function, $\rho_{nm}(q)$ is the matrix element of the electron density operator $\rho(q)$ between the LLs $n$ and $m$, $E_C = e^2/\varepsilon l_0$, and $\phi_0 = h/e$ is the quantum unit of flux. This paper has recently been supported by scanning probe experiments which image the electron density of
the 2DEG. The images show that even within the incompressible strips there are short-range density fluctuations. Based on this picture, we will assume an effective disorder potential which is mostly short-ranged, but also has a small long-range component which is screened for \( q \leq W^{-1} \). In this case, due to the small-\( q \) properties of the density matrix elements \( \rho_{mn} \), the long-range part of the effective disorder adds a constant to all the diagonal elements \( \alpha_{nn} \) and makes a negligible contribution to the off-diagonal terms. We model the short-range part of the disorder with uncorrelated \( \delta \)-function impurities, with \( U(q) = \alpha_s E_{\text{C}}^2 q^2 \), where \( \alpha_s \) is the disorder strength. Our model disorder potential is characterized by the two dimensionless parameters \( \alpha_l \) and \( \alpha_s \) and has the simple form \( \alpha_{mn} = \alpha_s + \alpha_l \delta_{mn} \).

Consider the effect of disorder on the spin polarization and the transport gap. In the clean limit, the \( n = 0, \uparrow \) LL is completely occupied while the \( n = 0, \downarrow \) and all other LLs are empty. The transport gap is just the splitting between the \( n = 0, \uparrow \) and \( n = 0, \downarrow \) LLs. Under realistic conditions (magnetic fields of a few Tesla) the interaction energy \( E_C \) is usually larger than the cyclotron energy, and both the above scales are much larger than the Zeeman coupling \( E_Z \). At \( \nu = 1 \) the exchange energy dominates the gap. The splitting between the \( n = 0, \uparrow \) and \( n = 0, \downarrow \) levels (assuming that only these two are occupied) is

\[
\Delta = E_0(N_F(\uparrow, 0) - N_F(\downarrow, 0)) + E_Z \tag{5}
\]

where the exchange integral is \( E_0 = \int \frac{e^2 q}{2\pi\hbar^2} v(q) e^{-q^2 l_0^2/2} \).

Eq.(5) shows that disorder can be expected to reduce the gap. If the disorder broadening of the LLs is sufficient to make the single-particle states in \( n = 0, \uparrow \) and \( n = 0, \downarrow \) overlap, then the gap will be reduced when compared to the clean limit according to Eq.(5). However, the band overlap will simultaneously decrease the spin polarization. In order to see if this effect is operative in real samples, one needs to look at the spin polarization.

Aifer et al. made measurements of the absolute spin polarization near \( \nu = 1 \) using an optical absorption technique. Their data show a “flat top”, demonstrating that for \( 0.95 \leq \nu \leq 1.05 \) the sample is only 65% spin polarized even at the lowest temperatures, with an estimated error of 10% [21]. A similar feature has recently been observed in optically pumped nuclear magnetic resonance (OPNMR) measurements [22]. Outside this range of \( \nu \) the data show unambiguous evidence for skyrmion-induced depolarization [18,22]. Figure 1 shows our predictions for the spin polarization near \( \nu = 1 \) for short range + small amounts of long-range disorder. A Fang-Howard form [23] for the interaction \( v(q) \) with width \( b^{-1} = 0.5l_0 \) was used to compute exchange integrals, the \( n = 0, 1, 2 \) Landau levels were kept, and other parameters appropriate to Sample A of Aifer et al [6] were used \((B_{\nu=1} = 6.2T)\).

**FIG. 1.** Spin polarizations in a small range around \( \nu = 1 \) for different combinations of short and long range disorder. The legend shows the values \((\alpha_s, \alpha_l)\). The polarization is seen to be robust.

The lack of full polarization [7] shows that the disorder-broadened \( n = 0, \uparrow \) and \( n = 0, \downarrow \) LLs do overlap, so by Equation (5) we can expect reductions of the transport gap [4]. For definiteness, we will focus on the data of Schmeller et al. [8], who measured the transport gap \( \Delta \) at \( \nu = 1 \) as a function of the Zeeman coupling \( E_Z \). There were two noteworthy features in their data. Firstly, the largest gap they obtained (in the SI1 sample) for \( \nu = 1 \) (at \( E_Z = 0.01E_C \)) was approximately 0.25\( E_C \), almost a factor of two smaller than the smallest theoretical estimate for the transport gap [4, 13]. Secondly, they observed a high slope of \( \Delta \) at small \( E_Z \), and interpreted the result as showing evidence for large skyrmions. If skyrmions with \( s \) reversed spins are the charge carriers, then the transport gap should behave as

\[
\Delta(E_Z) = \Delta_0 + sE_Z \tag{6}
\]

Thus interpreted, the data suggest \( s \approx 7 \) skyrmions.

Figure 2 shows the results of our own self-consistent calculations of the transport gap. Once again the Fang-Howard form for the interaction with width \( b^{-1} = 0.5l_0 \) was used, with the parameters appropriate to the SI1 sample \((B_{\perp} = 2.3T)\) of Schmeller et al [8]. Since the zero-field mobilities of the SI1 sample \((\approx 3.4 \times 10^6 \text{cm}^2/\text{Vs})\) and Sample A of Aifer et al \((\approx 3.2 \times 10^6 \text{cm}^2/\text{Vs})\) are similar, we can roughly expect the same disorder strengths in the two samples. Based on this expectation, some of the same disorder strengths as in Figure 1 have been used in Figure 2. We assume that the extended quasiparticle states of the \( n = 0, \uparrow \) and \( n = 0, \downarrow \) LLs lie at the respective band centers, defined as the energy where the density of states of a particular band is the maximum (this is exact when there is no LL mixing due to disorder [21]). To conform to convention, based on fitting to the form \( R_{xx} \approx e^{-\Delta/2T} \), the transport gap is computed as twice the difference between the chemical potential and the nearest extended state.
Recently there have been reports of very large skyrmions \( 25, 26 \) (s in the range 36 – 50) for systems with very small effective \( E_Z \). This can also be understood within the context of our model. It was shown by Fogler and Shklovskii \( 27 \) that as disorder increases beyond a critical value the exchange gap collapses (in the absence of Zeeman coupling). Near the critical value of disorder, even a small change in \( E_Z \) makes a tremendous difference in the transport gap. Figure 3 shows a plot of the transport gap as a function of \( E_Z \). If the initial slope is interpreted in terms of Equation (6) it would correspond to very large skyrmions (s = 50). The slope decreases with increasing \( E_Z \), as in the data \( 24, 25 \).

Despite the semi-quantitative agreement with many experiments at \( \nu = 1 \), there are several caveats that should be mentioned. We have ignored correlation effects beyond Hartree-Fock. In addition to important qualitative phenomena such as the coulomb gap \( 28 \), these effects can also decrease the transport gap \( 29 \). In the experimentally relevant range of sample densities these effects are estimated to be unimportant for \( \nu = 1 \), but are significant for \( \nu = 3 \) and higher \( 24 \). Secondly, the parallel field causes mixing between the different electronic subbands, leading to modifications of the interaction \( 30 \). These effects are also expected to be more significant for higher LLs. Finally, we have assumed that the extended states lie at the band maxima. Numerical work on noninteracting models with disorder shows that this is a good approximation if LL mixing due to disorder is weak, but fails for large LL mixing \( 24 \). This effect is also more significant at higher \( \nu \). In this context, the transport gaps for \( \nu = 3 \) have also been measured as a function of \( E_Z \). A straightforward application of the methods of this paper gives a high slope similar to \( \nu = 1 \), in agreement with earlier work (see Usher et al \( 3 \)) but disagrees with other data \( 8 \). It is possible that the disorder + exchange mechanism does occur, but is modified in a sample-dependent way by the other effects mentioned above \( 24, 31, 32 \), which are all expected to be substantially greater for \( \nu = 3 \) than for \( \nu = 1 \). Work is in progress to take these effects into account \( 31 \) with a numerical HF approach for concrete disorder realizations \( 32 \).

If one assumes that the disorder + exchange mechanism is the dominant one for \( \nu = 1 \), there are some interesting consequences. The slope of the transport gap as a function of \( E_Z \) is consistent with skyrmions being invisible in transport at \( \nu = 1 \). However, from polarization measurements \( 19, 22 \) we know that skyrmions depolarize the system away from \( \nu = 1 \). This means that skyrmions, if they do exist at \( \nu = 1 \), must be localized. This is plausible, since skyrmions are extended objects, and therefore their “hopping” requires many spin overlaps. These overlaps are presumably reduced by disorder, possibly leading to skyrmion localization. In fact, the most recent OPNMR measurements \( 32 \) are consistent with skyrmion localization in a small range around \( \nu = 1 \), though the analysis is complicated by wavefunc-
tion and polarization profiles perpendicular to the 2DEG.

In summary, the interplay between disorder and exchange effects, when treated in a self-consistent disorder-averaged Hartree-Fock approximation, can provide a consistent account of many of the experimental observations at $\nu = 1$. This approach offers an alternative interpretation of the transport data which does not include skyrmions, and suggests that skyrmions may be localized and hence invisible in transport.

In order to test the model rigorously, it would be very helpful to have measurements of transport gaps and spin polarization on the same sample at $\nu = 1$. More work is needed at higher $\nu$ to tease apart the confounding effects of Landau level mixing due to disorder, correlation effects, and effects due to the parallel field.

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