Assembly force modelling method on fixtures of automotive body compliant sheet metal parts

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Abstract. Some assembly forces will be generated on fixtures because of some existing variation sources during the welding process of automotive body compliant metal parts. This article proposes an assembly force modelling method on fixtures during the welding process of compliant sheet metal parts to decrease the assembly forces. The assembly force models on fixtures are established to show the influence relationship between assembly forces and all variation sources, and the finite element method is used to obtain all influence coefficients of assembly forces. The obtained assembly force models on fixtures describe the propagating principle of assembly forces in all welding subprocesses. Cases of different fixture layouts show that the fixture layout can greatly influence assembly forces on fixtures. So assembly forces on fixtures can be decreased through the optimization of variation sources and the fixture layout. This method can also be referred for other assembly processes, such as adhesive connection and fastener connection of compliant sheet metal parts.

1. Introduction

Some assembly forces will be generated on fixtures because of some existing variation sources during these assembly processes. These forces will cause some assembly variation and some residual stresses for the assembly after fixtures are released. So decreasing these assembly forces will greatly decrease the residual stresses risk, and improve the assembly quality. This research will focus on the generating principles of assembly forces on fixtures during the welding process of automotive body compliant sheet metal parts.

It is very important to analyze the generating principles of assembly forces on fixtures to decrease assembly forces of automotive body compliant metal parts. Some relevant research has been done in recent years. Dong and Kang developed an approach based on the response surface methodology to analyze the assembly variation and residual stress of compliant structures [1]. The finite element method and regression models were used to obtain the virtual experiment data and fit them. The illustrating result using a composite-metal assembly showed that the method was a practical and reliable solution to the analysis of compliant structures. Yi et al. used a 3D thermo-mechanical finite element analysis method to predict the welding distortion and residual stress of cylinder-shaped multi-pass layer weldments. A continuous welding procedure was adopted to investigate the influence of deposition sequence and welding heat input on the welding distortion and residual stress [2]. Zia and Qiao studied the influence of the clearances in mating parts on different stresses and deformation.
adopting a structure from automotive spot welding equipments. The effect of these clearances was determined using finite element analysis simulations for the process model of the product [3]. Faraz et al. used Monte Carlo simulations to investigate the expected size and location of the working regions of parts in a sheet metal family subjected to random resistance spot welding forces [4]. Astarita et al. finished the study and the experimental characterization of the influence of the process parameters on forces acting on the tool during the friction-stir welding process, and studied the correlation between the forces and the grain size [5].

To investigate the variation propagating principle of automotive compliant sheet metal parts during the welding process, some finite element modeling method had bee proposed to effectively model the assembly variation of compliant sheet metal parts. Liu and Hu firstly proposed the Method of Influence Coefficients (MIC) using the Finite Element Method (FEM) to set variation relationship between assembly variation and variation sources when deformable sheet metal parts were welded [6]. Camelio et al. proposed a multi-station assembly variation model of compliant parts considering part geometric variation, fixture variation and welding gun variation [7]. Liao and Wang applied wavelets analysis and the FEM in the variation modelling of compliant assembly [8]. Yu and Yang proposed a system assembly variation modelling method of multi assembly subprocesses using the FEM [9].

In above research, though some modelling methods of assembly forces or residual stresses were given, the generating principle of assembly forces or residual stresses and how to decrease the assembly forces or residual stresses by optimizing the assembly process were not clear. So in this research the assembly variation finite element modeling method of compliant parts is referred to set the generating principle of assembly forces on fixtures during the welding process of compliant sheet metal parts. And assembly force modelling methods in every assembly subprocess on fixtures and the influencing relationship between assembly forces and variation sources and the fixture locating layout will be given.

2. Assembly process and assembly force propagating process of compliant sheet metal parts

The assembly process is shown in Figure 1. To analyze the assembly process it can be divided into four subprocesses, i.e. the fixture locating subprocess, the welding gun clamping subprocess, the welding operation subprocess and the releasing subprocess. Assembly forces on all fixtures of parts must be generated during these assembly subprocesses because of the variation sources, i.e. manufacturing variation of parts and fixture locating variation.

![Figure 1. Assembly process of compliant sheet metal parts.](image)

Figure 2 demonstrates the assembly force propagating process during these assembly subprocesses caused by variation sources. An over located fixture layout is usually adopted for compliant sheet metal parts because of the compliant characteristic of parts. Usually there are some manufacturing variation in the additional locating points and welding points of parts and fixture locating variation. These variation sources will cause assembly forces on fixtures to overcome themselves during the assembly process.

In Liu and Hu [6], to set up the assembly spring-back model after the weld, some forces relationship caused during the welding process as shown in Figure 1 were described. In this research the referred assembly forces are focused on fixtures during the assembly process because the assembly...
forces on fixtures greatly influence the assembly variation and the residual stresses in the next assembly process. In some assembly subprocesses of Figure 2, the existing variation sources will cause new assembly forces on fixtures. So the assembly forces on fixtures are generated and propagated during the whole assembly process. The detail assembly force models in every assembly subprocess will be described in the next section.

**Figure 2.** Assembly force propagation during assembly subprocesses.

### 3. Assembly force modelling method of compliant sheet metal parts

To introduce the assembly force modelling method, Figure 3 gives an assembly of compliant sheet metal parts. It shows that the part 1 and part 2 adopted a 5-2-1 and a 4-2-1 fixture layouts, respectively. All fixture locating points on parts are divided into determinate locating points and additional locating points. The determinate locating points are consisted of six locating points composing a 3-2-1 locating layout to constrain six motion freedoms of a part, and the additional locating points are added to constrain more compliant deformations. In Figure 3 the locating points on B14, B15 and B24 are all additional locating points.

In the part 1, the locating pins P11 and P12 constrain the movements in the x and y directions and the rotating motion by the z direction, and the locating blocks B11~B15 constrain the z direction movement and rotating motions by the x and y directions. Similarly, in the part 2 the locating pins P21 and P22 constrain the movements in the x and y directions and the rotating motion by the z direction, and the locating blocks B21~B24 constrain the z direction movement and rotating motions by the x and y directions. W11~W13 and W21~W23 are the welding points of these parts.

**Figure 3.** An assembly demonstration of compliant sheet metal parts.
Only assembly forces on locating blocks are considered in this research, so the fixture variation in locating pins are ignored. Then the existing variation sources are given as vector denotations.

The additional point variation of the part 1 is \( \{ \ldots \} = \{ \ldots \} \).

The additional point variation of the part 2 is \( \{ \ldots \} = \{ \ldots \} \).

The welding point variation of the part 1 is \( \{ \ldots \} = \{ \ldots \} \).

The welding point variation of the part 2 is \( \{ \ldots \} = \{ \ldots \} \).

The fixture locating variation of the part 1 is \( \{ \ldots \} = \{ \ldots \} \).

The fixture locating variation of the part 2 is \( \{ \ldots \} = \{ \ldots \} \).

The superscript \( (0) \) denotes the initial status before the additional location of parts.

3.1. Assembly force models on fixtures in the fixture locating subprocess

In this subprocess, two types of variation sources can cause assembly forces on fixtures of one part, i.e. the additional locating point variation and fixture locating variation.

(1) Assembly force modelling caused by the additional locating point variation

If there exists some manufacturing variation on the additional locating points of one part, some forces will be produced to overcome these variation when these additional locating points are clamped to the nominal. Then these additional locating point forces will cause some assembly reaction forces on the other locating points, i.e. the determinate locating points.

Assume that the additional locating forces on B_{14} and B_{15} are \( f_{14}^{(1)} \) and \( f_{15}^{(1)} \), and that on B_{24} is \( f_{24}^{(1)} \).

The solving process using the finite element method is shown as follows.

The finite element model of the part 1 is created in which the boundary conditions on the locating points B_{11}, B_{12}, B_{13}, P_{11} and P_{12} are shown in Figure 3. The movements on these locating points are all constrained to be zero. Apply one unit force on B_{14} and B_{15} which is in accordance with the locating direction on B_{14} and B_{15}, respectively, and the displacements on B_{14} and B_{15} can be obtained. They are denoted as vectors \( \{ \ldots \} \) and \( \{ \ldots \} \). Then the relationship between the caused displacements and the applied forces on the additional locating points is

\[
\{ V \} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} f_{14}^{(1)} \\ f_{15}^{(1)} \end{bmatrix} = \begin{bmatrix} [c] \end{bmatrix} \begin{bmatrix} f_{14}^{(1)} \\ f_{15}^{(1)} \end{bmatrix}
\]

(1)

So the caused forces on the additional locating points of the part 1 to overcome the additional locating point variation are

\[
\{ F_{1(N-3)}^{(1)} \} = \begin{bmatrix} f_{14}^{(1)} \\ f_{15}^{(1)} \end{bmatrix} = [c]^{-1} \{ V \} = [K_1] \begin{bmatrix} -v_{14}^{(0)} \\ -v_{15}^{(0)} \end{bmatrix}
\]

(2)

where the force vector \( \{ F_{1(N-3)}^{(1)} \} \) is the additional locating point forces of the part 1, \( [K_1] \) is the rigidness matrix of the part 1 on the additional locating points B_{14} and B_{15}, which is the inversion of the matrix \( [c] \).

Also, the additional locating force of the part 2 on B_{24} can be obtained as follows.

\[
\{ F_{2(N-3)}^{(1)} \} = \begin{bmatrix} f_{24}^{(1)} \end{bmatrix} = [K_2] \begin{bmatrix} -v_{14}^{(0)} \end{bmatrix} = [K_2] \begin{bmatrix} -f_{14}^{(0)} \end{bmatrix}
\]

(3)

where the force vector \( \{ F_{2(N-3)}^{(1)} \} \) is the additional locating point force of the part 2, \( [K_2] \) is the rigidness matrix of the part 2 on the additional locating point B_{24}. 

4
After the caused forces on the additional locating points are obtained, assembly forces on the determinate locating points can be solved through the propagating relationship analysis between the additional locating points and the determinate locating points using the finite element method. Then all locating point force vector containing the additional locating points and the determinate locating points is

\[
\begin{bmatrix}
F_{1(1)} \\
F_{2(1)}
\end{bmatrix} = 
\begin{bmatrix}
G_{1B} & G_{1B(1)} \\
G_{1B(21)} & G_{1B(22)}
\end{bmatrix}
\begin{bmatrix}
F_{1(N-3)} \\
F_{2(N-3)}
\end{bmatrix}
\]  

(4)

where \( \{F_{1(1)}\} \) and \( \{F_{2(1)}\} \) are the assembly force vectors of the part 1 and part 2, respectively, \( \left[ G_{1B} \right] \) and \( \left[ G_{2B} \right] \) are the reaction coefficients between all locating points and the additional locating points of the part 1 and part 2, respectively.

The expressions of \( \left[ G_{1B} \right] \) and \( \left[ G_{2B} \right] \) are

\[
\left[ G_{1B} \right] =
\begin{bmatrix}
g_{1B(11)} & g_{1B(12)} \\
g_{1B(21)} & g_{1B(22)}
g_{1B(31)} & g_{1B(32)}
\end{bmatrix}
\]  

(6)

\[
\left[ G_{2B} \right] =
\begin{bmatrix}
g_{2B(11)} \\
g_{2B(21)} \\
g_{2B(31)} & 1
\end{bmatrix}
\]  

(7)

where the parameters \( \{g_{1B(11)} \ g_{1B(21)} \ g_{1B(31)}\} \) and \( \{g_{1B(12)} \ g_{1B(22)} \ g_{1B(32)}\} \) are the caused forces on \( B_{11}, B_{12} \) and \( B_{13} \) when one unit force is applied on \( B_{14} \) and \( B_{15} \), respectively.

Similarly, after the additional locating points \( B_{14} \) and \( B_{15} \) are located, the welding point variation of the part 1 will be changed to

\[
\{V_{w(1)}\} = \left[ S_{w_{B_1}} \right] \{V_{w_{B}}\}_{B} + \{V_{w_{1(1)}}\}
\]  

(8)

where the matrix \( \left[ S_{w_{B_1}} \right] \) is the sensitivity matrix of the welding point variation of the part 1 to its additional locating variation. The following gives the solving method using the finite element method.

Apply one unit displacement separately on the additional locating points and compute the displacement variation of all welding points of the part 1. The displacement variation vectors composed of all the welding point variation constitute the matrix \( \left[ S_{w_{B_1}} \right] \).

Similarly, the welding point variation of the part 2 is also obtained as follows.

\[
\{V_{w(2)}\} = \left[ S_{w_{B_2}} \right] \{V_{w_{B}}\}_{B} + \{V_{w_{2(1)}}\}
\]  

(9)

where the matrix \( \left[ S_{w_{B_2}} \right] \) is the sensitivity matrix of the welding point variation of the part 2 to its additional locating variation.

(2) Assembly force modelling caused by fixture locating variation

When the fixtures of one part are located and clamped, some new assembly forces will be added on these fixtures and some new assembly variation will be added on welding points if there exists some fixture locating variation.

Assume that the fixture locating variation for the part 1 is \( \{V_{f_1}\} \), then the welding point variation of the part 1 is changed to

\[
\{V_{w_{1(1)}}\} = \left[ S_{w_{B_1}} \right] \{V_{f_1}\} + \{V_{w_{1(1)}}\}
\]  

(10)
where the matrix \([S_{W1}]\) denotes the sensitivity matrix of the part 1’s welding point variation to its fixture locating variation. Its solving process using the finite element method is as follows.

Separately apply one unit displacement on all fixtures of the part 1, and compute the variation vectors of its all welding points. All variation vectors compose the matrix \([S_{W1}]\).

Similarly, the welding point variation of the part 2 is obtained as follows.

\[
\{V_{W2}\} = [S_{W2}]\{V_{F2}\} + \{V_{F2}\}^\top
\]

where the matrix \([S_{W2}]\) denotes the sensitivity matrix of the part 2’s welding point variation to its fixture locating variation.

For the fixtures on the part 1 and part 2, assembly forces are changed to

\[
\{F_{W1}\} = \left[ G_{1f} \right] \{V_{F1}\} + \{F_{1}\}
\]

\[
\{F_{W2}\} = \left[ G_{2f} \right] \{V_{F2}\} + \{F_{2}\}
\]

where the matrices \([G_{1f}]\) and \([G_{2f}]\) are the influence coefficients of assembly forces on fixture locating points of the part 1 and part 2 to their own fixture locating variation, respectively. The following gives the solving process using the finite element.

Apply one unit displacement on the j-th fixture, and compute the force vector \([G_{i,f}]\) on all fixtures of the part. After one unit displacement is separately applied on all fixtures, all obtained force vectors compose the coefficient matrix \([G_{i,f}]\). Similarly, the matrix \([G_{2,j}]\) can also be obtained.

### 3.2. Assembly force models on fixtures in the welding gun clamping subprocess

In this subprocess, welding areas of parts will be clamped by welding guns to the nominal. So when there exists some variation on these welding points, some forces \([F_{W}]\) are needed to be applied on these welding points to overcome these variation. These forces will cause new reaction forces on fixtures.

The solving process of \([F_{W}]\) is shown as follows.

Apply one unit force on the j-th welding point of one part, and the displacements of all welding points of this part \([C_{Wj}]\) \((j=1, \ldots, N)\) can be obtained. \(N\) is the welding point number of this part. After one unit force is separately applied on all welding points of this part, the obtained displacement vectors on all welding points compose one displacement matrix. The relationship between the displacements and forces of all welding points is

\[
\{V_{W}\} = \left[ \begin{array}{cccc}
C_{W11} & C_{W12} & \cdots & C_{W1N} \\
C_{W21} & C_{W22} & \cdots & C_{W2N} \\
\vdots & \vdots & \ddots & \vdots \\
C_{WN1} & C_{WN2} & \cdots & C_{WNN}
\end{array} \right] \left[ \begin{array}{c}
f_{W1} \\
f_{W2} \\
\vdots \\
f_{WN}
\end{array} \right] = [C_{W}]\{F_{W}\}
\]

Then the needed forces applied on all welding points of the part 1 and the part 2 can be obtained as follows, respectively.

\[
\{F_{W1}\} = [C_{W1}]^{-1}\{-V_{W1}^{(2)}\}
\]

\[
\{F_{W2}\} = [C_{W2}]^{-1}\{-V_{W2}^{(2)}\}
\]

The caused reaction forces on fixtures of the part 1 and the part 2 can be obtained as follows, respectively.

\[
\{F_{1}^{(3)}\} = [G_{1W}]\{F_{W1}\} + \{F_{1}^{(2)}\}
\]

\[
\{F_{2}^{(3)}\} = [G_{2W}]\{F_{W2}\} + \{F_{2}^{(2)}\}
\]
where the matrices $[G_{1w}]$ and $[G_{2w}]$ are the influence coefficients between the fixture forces and the welding point forces of the part 1 and the part 2, respectively. The solving process is shown as follows.

Separately apply one unit force on all welding points of one part, and compute the force vector on all fixtures of this part. All force vectors caused by all the welding point forces of this part compose the matrices $[G_{1w}]$ or $[G_{2w}]$.

3.3. Assembly force models on fixtures in the welding gun releasing subprocess

After the welding operation is finished between parts, welding guns will be removed. Because there exist the welding point forces $\{F_{W_1}\}$ and $\{F_{W_2}\}$, the assembly will cause the spring-back after the welding guns are released. The spring-back forces are equal to the forces $\{F_{W_1}\}$ and $\{F_{W_2}\}$, but their directions are reverse.

Solving the influence relationship between the spring-back forces and the reaction forces on all fixtures of the part 1 and the part 2, then assembly forces on all fixtures of the part 1 and the part 2 are obtained after the spring-back as follows.

$$\{F\} = \begin{cases} F_{1}^{(4)} \\ F_{2}^{(4)} \end{cases} = \begin{bmatrix} G_{S_1} & G_{S_2} \\ G_{S_3} & G_{S_4} \end{bmatrix} \begin{cases} -F_{W_1} \\ -F_{W_2} \end{cases} + \begin{cases} F_{1}^{(3)} \\ F_{2}^{(3)} \end{cases}$$

where the matrix $[G_{S}]$ is the influence coefficient matrix between all fixture forces and all welding point forces of the assembly.

The solving process using the finite element method is shown as follows.

Apply one unit force on every welding point of the assembly, and compute the reaction forces on all fixtures of the assembly which compose the matrix $[G_{S}]$.

In the given case in Figure 3, there are nine fixtures in which five is on the part 1 and four is on the part 2, and six welding points in which the part 1 and the part 2 all have three. So the matrix $[G_{S}]$ is a 9×6 matrix, it can be changed to

$$[G_{S}] = \begin{bmatrix} G_{S_{11}} & G_{S_{12}} \\ G_{S_{21}} & G_{S_{22}} \end{bmatrix}$$

where the matrix $[G_{S_{11}}]$ is a 5×3 matrix which denotes the influence relationship between the five fixture forces of the part 1 and its three welding point forces under the assembly status, the matrix $[G_{S_{12}}]$ is also a 5×3 matrix which denotes the influence relationship between the five fixture forces of the part 1 and three welding point forces of the part 2 under the assembly status, the matrix $[G_{S_{21}}]$ is a 4×3 matrix which denotes the influence relationship between the four fixture forces of the part 2 and three welding point forces of the part 1 under the assembly status, and the matrix $[G_{S_{22}}]$ is a 4×3 matrix which denotes the influence relationship between the four fixture forces of the part 2 and its three welding point forces under the assembly status.

Then the equation (19) is changed to

$$\begin{cases} F_{1}^{(4)} = -G_{S_{11}}F_{W_1} - G_{S_{12}}F_{W_2} + F_{1}^{(3)} \\ F_{2}^{(4)} = -G_{S_{21}}F_{W_1} - G_{S_{22}}F_{W_2} + F_{2}^{(3)} \end{cases}$$

3.4. Combined assembly force models

Combine the above assembly force models of fixtures in every assembly subprocess, and the final combined assembly force models of fixtures are shown as follows.
\[ F_{1}^{(4)} = \begin{bmatrix} S_{B11} \end{bmatrix} [V_{B1}^{(0)}] + \begin{bmatrix} S_{B12} \end{bmatrix} [V_{B2}^{(0)}] + \begin{bmatrix} S_{W11} \end{bmatrix} [V_{W1}^{(0)}] + \begin{bmatrix} S_{W12} \end{bmatrix} [V_{W2}^{(0)}] + \begin{bmatrix} S_{F11} \end{bmatrix} [V_{F1}^{(0)}] + \begin{bmatrix} S_{F12} \end{bmatrix} [V_{F2}^{(0)}] \] (22)

\[ F_{2}^{(4)} = \begin{bmatrix} S_{B21} \end{bmatrix} [V_{B1}^{(0)}] + \begin{bmatrix} S_{B22} \end{bmatrix} [V_{B2}^{(0)}] + \begin{bmatrix} S_{W21} \end{bmatrix} [V_{W1}^{(0)}] + \begin{bmatrix} S_{W22} \end{bmatrix} [V_{W2}^{(0)}] + \begin{bmatrix} S_{F21} \end{bmatrix} [V_{F1}^{(0)}] + \begin{bmatrix} S_{F22} \end{bmatrix} [V_{F2}^{(0)}] \] (23)

where \( F_{1}^{(4)} \) and \( F_{2}^{(4)} \) are the assembly force vectors caused on all fixtures of the part 1 and the part 2, respectively, \( [S_{B11}] \) and \( [S_{B21}] \) are the sensitivity matrices of the fixture forces of the part 1 and the part 2 to the additional locating point variation of the part 1, respectively, \( [S_{B12}] \) and \( [S_{B22}] \) are the sensitivity matrices of the fixture forces of the part 1 and the part 2 to the additional locating point variation of the part 2, respectively, \( [S_{W11}] \) and \( [S_{W21}] \) are the sensitivity matrices of the fixture forces of the part 1 and the part 2 to the welding point variation of the part 1, \( [S_{W12}] \) and \( [S_{W22}] \) are the sensitivity matrices of the fixture forces of the part 1 and the part 2 to the welding point variation of the part 2, \( [S_{F11}] \) and \( [S_{F21}] \) are the sensitivity matrices of the fixture forces of the part 1 and the part 2 to the fixture locating point variation of the part 1, \( [S_{F12}] \) and \( [S_{F22}] \) are the sensitivity matrices of the fixture forces of the part 1 and the part 2 to the fixture locating point variation of the part 2.

These sensitivity matrix expressions are listed as follows.

\[ \begin{align*}
[S_{B11}] &= -G_{wb} C_{w}^{-1} S_{wb} - G_{b1} C_{1}^{-1} - G_{s1} C_{1}^{-1} S_{wb} \\
[S_{B12}] &= -G_{s1} C_{1}^{-1} S_{wb} \\
[S_{W11}] &= G_{s1} C_{1}^{-1} S_{wb} - G_{wb} C_{w}^{-1} \\
[S_{W12}] &= G_{s1} C_{1}^{-1} S_{wb} \\
[S_{F11}] &= G_{f} - G_{wb} C_{w}^{-1} S_{wb} + G_{s1} C_{1}^{-1} S_{wb} \\
[S_{F12}] &= G_{s1} C_{1}^{-1} S_{wb} \\
[S_{B21}] &= -G_{s2} C_{2}^{-1} S_{wb} \\
[S_{B22}] &= G_{wb} C_{w}^{-1} S_{wb} - G_{s2} C_{2}^{-1} - G_{s1} C_{1}^{-1} S_{wb} \\
[S_{W21}] &= G_{s2} C_{2}^{-1} \\
[S_{W22}] &= G_{s2} C_{2}^{-1} S_{wb} - G_{wb} C_{w}^{-1} - G_{s2} C_{2}^{-1} S_{wb} \\
[S_{F21}] &= G_{s2} C_{2}^{-1} S_{wb} \\
[S_{F22}] &= G_{s2} C_{2}^{-1} S_{wb} + G_{s2} C_{2}^{-1} S_{wb} \\
\end{align*} \] (25)

Using these equations, assembly forces on fixtures after the weld can be predicted according to variation sources, and the sensitivity analysis of assembly forces to variation sources can be made using these obtained assembly force models to optimize variation source design to decrease assembly forces.

The above proposed modeling method of assembly forces on fixtures can also be referred for other assembly processes to help the optimization of assembly processes, such as adhesive connection and fastener connection of compliant sheet metal parts.

4. Assembly case analysis
4.1. Case description
The assembly case in Figure 3 is adopted to demonstrate the proposed assembly force modelling method. The parameters of the part 1 are $160 \times 240 \times 1 \text{mm}^3$ and those of the part 2 are $200 \times 200 \times 1 \text{mm}^3$. The modulus of elasticity and poisson’s ratio are adopted as $207,000 \text{N/mm}^2$ and 0.3, respectively. The software of ABAQUS is used to analyze the assembly process. The finite element models are shown in Figure 4.

In Figure 4 the symbol ‘Δ’ denotes the fixture constraint position corresponding to one node of the finite element model below which the letter means the constrained direction of the part. The symbol ‘●’ denotes the welding point position between these two parts. In this analysis the tie constraint is adopted for welding points of the two parts during the welding operation. The information of fixtures and welding points are listed in Table 1.

![Figure 4. Finite element models.](image)

| Fixtures | Coordinates | Welding points |
|----------|-------------|----------------|
| B₁₁      | (-30,50,0)  | W₁₁            |
| B₁₂      | (-30,-50,0) | W₁₂            |
| B₁₃      | (-190,0,0)  | W₁₃            |
| B₁₄      | (-120,50,0) |                |
| B₁₅      | (-120,-50,0)|                |
| P₁₁      | (-150,0,0)  |                |
| P₁₂      | (-60,0,0)   |                |
| B₂₁      | (50,70,-1)  | W₂₁            |
| B₂₂      | (50,-70,-1) | W₂₂            |
| B₂₃      | (170,70,-1) | W₂₃            |
| B₂₄      | (170,-70,-1)|                |
| P₂₁      | (100,50,-1) |                |
| P₂₂      | (100,-50,-1)|                |

4.2. Assembly force computation
According to the solving method of process parameters in former sections, the sensitivity matrices in Equations (24) and (25) can be obtained using the finite element models in Figure 4 as follows. They are
The variation measurement data are usually followed by the normal distribution. Then assume that manufacturing variation data of the part 1 and the part 2 follow $N(0, 0.33332)$ and $N(0, 0.52)$, and fixture locating variation data of the part 1 and the part 2 follow all $N(0, 0.12)$. The assembly variation results are obtained by the Monte Carlo Simulation Method. Assembly force results are listed in Table 2.

**Table 2.** Assembly force statistical results.

| Fixtures | Assembly force ($N$) | $\mu$ | $6 \sigma$ |
|----------|----------------------|-------|-----------|
| B_{11}   | 2.1316               | 647.46|
| B_{12}   | -2.1283              | 647.46|
| B_{13}   | 0.0003               | 0.3589|
| B_{14}   | -2.045               | 647.22|
| B_{15}   | 2.0435               | 647.22|
| B_{21}   | -0.1606              | 366.62|
| B_{22}   | 0.1577               | 366.77|
| B_{23}   | 0.0981               | 364.75|
| B_{24}   | -0.0974              | 364.79|
From the results of the part 1, the assembly forces on all fixtures are greatly unequal. The assembly force on $B_{13}$ is much less than those on others. The forces on fixtures of the part 2 are nearly equal.

To compare the different fixture layouts, another two fixture layouts are given for the part 1 which are shown in Figure 5. Comparing to the first fixture layout in Figure 4, the second fixture layout in Figure 5(a) removes the fixture $B_{13}$, and the third fixture layout in Figure 5(b) removes the fixtures $B_{14}$ and $B_{15}$. Assembly forces on fixtures are computed in these two fixture layouts and the results are shown in Table 3.

![Figure 5. Another two fixture layouts.](image)

**Table 3.** Assembly force statistical results.

| Fixtures | Second Layout | Third Layout |
|----------|---------------|--------------|
|          | $\mu$ | $6\sigma$ | $\mu$ | $6\sigma$ |
| $B_{11}$ | 0.13 | 532.81 | 0 | 35.3 |
| $B_{12}$ | -0.13 | 532.9 | 0 | 35.29 |
| $B_{13}$ | 0 | 0.18 | 0 | 0.18 |
| $B_{14}$ | 0.18 | 491.47 | 0.19 | 491.49 |
| $B_{15}$ | -0.19 | 491.49 | -0.19 | 491.49 |
| $B_{21}$ | 0.73 | 391.76 | 0.27 | 367.83 |
| $B_{22}$ | -0.72 | 391.72 | -0.72 | 367.9 |
| $B_{23}$ | -0.94 | 366.79 | -0.27 | 365.92 |
| $B_{24}$ | 0.94 | 366.8 | 0.27 | 365.94 |

Combing the results in Table 2 and 3, two conclusions can be made.

On the one hand, it can be seen that the fixture layout will greatly influence assembly forces on fixtures. The assembly forces on fixtures of the part 1 in the second fixture layout are less than those in the first fixture layout, and those in the third fixture layout are much less than those in the other two layouts while they are unequal on all fixtures of the part 1. Generally, the third fixture layout is best in these three fixture locating layouts. And the most optimization fixture locating layout can be obtained to minimize the assembly forces on fixtures.

On the other hand, in these three fixture layouts the assembly forces on fixtures of the part 2 are nearly equal. It shows that the fixture layout of one part only influences the assembly forces on its own fixtures. So the fixture locating layouts of all assembled parts can be separately optimized.
5. Conclusion
This article proposes an assembly force modelling method on fixtures during the welding process of compliant sheet metal parts to control the assembly force. This method can also be referred for other assembly processes, such as adhesive connection and fastener connection.

In this method the welding process is divided into four subprocesses. The assembly force propagating model in every subprocess is given. And the combined assembly force model is built after the weld is finished. The assembly force models on fixtures are established to show the influence relationship between assembly forces and all variation sources, and the FEM is used to obtain all influence coefficients of assembly forces. These variation sources contain the additional point variation, the welding point variation and the fixture locating variation. Using this modelling method, assembly forces on fixtures after the weld can be predicted according to variation sources, and the sensitivity analysis of assembly forces to variation sources can be made using these obtained assembly force models to optimize variation source design to decrease assembly forces.

A compliant assembly is adopted to analyze assembly forces on fixtures considering different fixture layouts. The result shows that the fixture layout can also greatly influence assembly forces on fixtures. So assembly forces on fixtures can be decreased through the separate fixture locating layout optimization of all assembled parts.

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