Intersubband population inversion and stimulated terahertz transitions between Landau levels in resonant tunneling multiple quantum well structures

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Abstract. It is shown that a population inversion can be achieved in the system of Landau levels in cascade quantum well structures in a magnetic field under a condition of sequential resonant tunneling. This mechanism allows a wide range tuning of the emitted terahertz frequency by the magnetic field strength variation.

1. Introduction

In the present report we show the possibility to achieve a population inversion in the system of Landau levels (LL) in cascade quantum well structures in a strong magnetic field under a condition of sequential resonant tunneling, i.e. in a strong transverse electric field. The scheme of transitions is shown in Fig.1.

It was shown that in quantum wells with spacing between 1st and 2nd subbands lower than an optical phonon energy (i.e. when optical phonon scattering is suppressed) the population of 0th LL in 2nd subband exceeds that of 1st LL in the 1st subband. Therefore the stimulated emission of terahertz radiation can be achieved on the radiative transitions between these LLs.

The effect is caused by a considerable difference between intersubband and intrasubband scattering times (upper LL (2-0) is depopulated by intersubband scattering only, while the lower LL (1-1) – by intrasubband scattering to 0th level of the same subband).

In the considered system the electron-electron scattering is the most important inter-LL scattering mechanism, since other inter-LL scattering processed are strongly suppressed. So the main attention was paid to developing a model of e-e scattering in a system of LLs in quantum wells with several subbands.

The proposed mechanism allows the continuous tuning of the emitted radiation frequency by the variation of the magnetic field strength, thus giving the possibility to achieve a tunable stimulated emission.

2. Theory

Let us consider the electron kinetics in sequential resonant tunneling GaAs / AlGaAs quantum well structures in quantizing magnetic field \( \mathbf{B} = \mathbf{B} e_z \) (axis Z is perpendicular to the layers). In Landau
gauge the single-electron spectrum is a set of confinement subbands each splitting into a series of Landau levels:

\[ E = E_n + \hbar \omega_c (n + 1/2) \]

with wavefunctions \( \psi(x) = \frac{\exp(ikx)}{\sqrt{L}} \cdot \Phi_n(y - k \ell^2) \phi_n(z) \)
determined by three quantum numbers – subband index \( n \), Landau level number \( \nu \) and x-axis momentum component \( k \).

Using the Fermi golden rule approximation, the electron flux to Landau state \( f \rightarrow (\nu, n) \) caused by electron-electron scattering transitions \( (i, j) \rightarrow (f, g) \) is given by the expression

\[ f_{(i,j)\rightarrow(f,g)}^{e-e} = \frac{N_i}{\tau_{(i,j)\rightarrow(f,g)}} \]

where

\[ \tau_{(i,j)\rightarrow(f,g)} = \frac{1}{L^2\alpha} \sum_{k_i} \frac{1}{\tau_{(i,j)\rightarrow(f,g)}(k_i)} \]

– total \( (i, j) \rightarrow (f, g) \) transition rate,

\[ \frac{1}{\tau_{(i,j)\rightarrow(f,g)}(k_j)} = \frac{2\pi}{\hbar} \sum_{k_f, k_g} V_{(i,j)(f,g)}(k_i, k_f, k_f, k_g) \cdot \delta(E_i + E_f - E_j - E_g) \]

– scattering rate from initial state containing one electron in LL \( i \) with wave vector \( k_i \) and one electron in LL \( j \) to the final state containing electrons in LLs \( f \) and \( g \), \( \alpha \) - LL degeneracy, \( N \) - 2D electron concentration on the corresponding Landau state.
\[ V_{(i,j)(f,g)}(k_i, k_f, k_f, k_f, k_f) = \int d\mathbf{r}_1 d\mathbf{r}_2 \psi_{j,k_f}^*(\mathbf{r}_1) \psi_{j,k_i}(\mathbf{r}_1) - \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \psi_{g,k_i}(\mathbf{r}_2) \psi_{j,k_f}(\mathbf{r}_2) \]  

(4)

- matrix elements of electron-electron interaction.

The finite width of the LLs is taken into account by replacing the \( \delta \)-function in (3) by a form-factor \( F_{(i,j)(f,g)}(E_i + E_f - E_f - E_g) \), which is approximated by Lorentzian

\[ F_{(i,j)(f,g)}(E) = \frac{1}{\pi} \frac{\Gamma}{E^2 + \Gamma^2} \]  

(5)

of halfwidth \( \Gamma = 2 \) meV, typical values for quantum well structures [1,2].

3. Results

The results of the numerical calculation of the e-e scattering times are shown in Fig. 2. The calculations were carried out for GaAs/AlGaAs quantum wells of 25 nm width, with intersubband spacing \( \Delta E_{12} = 20.4 \) meV. The lifetime of the lower (1,1) LL was found to depend monotonically and approximately linearly upon magnetic field. On the contrary the intersubband scattering lifetime of LL (2,0) revealed an oscillating dependence on magnetic field similar to that obtained in [1,2]. The minima of lifetime correspond to the situation when LL (2,0) coincides with certain LL of lower subband or is situated exactly between them.

It is seen from Fig.2 that the intrasubband scattering time is considerably shorter than intersubband one. Within the range 5 – 10 T of magnetic field strength the lifetime of the upper (2,0) LL is always higher than that of the lower (1,1) LL, and in the intervals \( B = 4.9 \pm 5.6, 6.2 \pm 7.6, 8.1 \pm 11.0 \) the difference achieves the values of 3 – 10 times. It opens the possibility to achieve a population inversion for considered inter-LL transition by the resonant tunneling pumping of the upper (2,0) level.

The emission frequency \( \hbar \omega = \Delta E_{21} - \hbar \omega_C \) (\( \omega_C \) – cyclotron frequency) will vary within interval \( \hbar \omega = 11.8 – 3.2 \) meV (\( \nu = 95 – 26 \) cm\(^{-1}\), \( \lambda = 105 – 390 \) \( \mu \)m, \( f = 2.86 – 0.77 \) THz) for magnetic field range \( B = 5 – 10 \) T.

The important point is that the specified (2,0)→(1,1) optical transition is forbidden by selection rules when magnetic field \( B \) is orthogonal to structure layers. The problem can be solved by placing the structure into a tilted magnetic field where the mentioned selection rule is violated due to the mixing of in-plane and out-of-plane carrier motion similarly to that observed in resonant tunneling transitions between LLs in tilted magnetic field [3-6]. If the cyclotron energy is several times lower than intersubband spacing the following expression for the dipole matrix element can be obtained:

\[ |D_{(2,0)\rightarrow(1,1)}|^2 = \delta_{\xi,\xi'} \left| \langle \phi_{2}(z) | \phi_{1}(z) \rangle \right|^2 \xi^2 / 2 \cdot \exp\left(-\xi^2 / 2\right) \]  

(6)

Here \( \xi = \left[ \langle \hat{z} \rangle_2 - \langle \hat{z} \rangle_1 \right] / \ell_{\perp} \) and \( \ell_{\perp} = \hbar c / eB_{\perp} \) and \( \ell_{\parallel} = \hbar c / eB_{\parallel} \) – magnetic lengths for perpendicular (\( B_{\perp} \)) and parallel (\( B_{\parallel} \) ) component of magnetic field with respect to layers, \( \langle \hat{z} \rangle_\gamma = \langle \phi_\gamma | \hat{z} | \phi_\gamma \rangle = \int dz |\phi_{\gamma}(z)|^2 z \) - the average z-coordinate for the states of \( \nu \)-th subband. Therefore to make the dipole element nonzero there should be a difference in average coordinates of upper and lower states \( \langle \hat{z} \rangle_1 \) and \( \langle \hat{z} \rangle_2 \). It can be achieved by introducing asymmetric potential along the growth axis of the structure, in particular, by application of transverse electric field. In Fig. 3 the calculated according to (6) matrix element dependence on electric field for different tilt angles of magnetic field is shown.

The asymmetry can also be introduced by asymmetric well construction, say, using as an active element an asymmetric system of two or more strongly coupled quantum wells.
Fig. 3. The calculated dependence of dipole matrix element $|D_{(2,0)\rightarrow(1,1)}|^2$ on the voltage drop per quantum well eFa in tilted magnetic field for different values of the parallel to layers component $B_{\parallel} = 1 \div 5$ T. The calculations were made for GaAs/Al$_{0.3}$Ga$_{0.7}$As quantum well of 25 nm width. The perpendicular to layers component of magnetic field $B_{\perp} = 5$ T.

Conclusion

It was shown that due to a difference between intersubband and intrasubband e-e scattering rates a population of the Landau levels of the upper subbands can considerably exceed that of the Landau levels of the lowest one, and wide-range tunable terahertz stimulated emission on such inter-Landau-level transition may be achieved.

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References

[1] Kempa K, Zhou Y, Engelbrecht J R, Bakshi P, Ha H I, Moser J, Naughton M J, Ulrich J, Strasser G, Gornik E and Unterrainer K 2002 Phys. Rev. Lett. 88 226803-1
[2] Kempa K, Zhou Y, Engelbrecht J R and Bakshi P 2003 Phys. Rev. B 68 085302
[3] Leadbeater M L, Sheard F W and Eaves L 1991 Semicond. Sci. Technol. 6 1021
[4] Lyo S K 1998 Phys. Rev. B 57 9114
[5] Telenkov M P and Mityagin Yu A 2006 JETP 130 (3) 491-499
[6] Telenkov M P and Mityagin Yu A 2007 Int. J. Mod. Phys. B 21(8/9) 1594