What gravity mediated entanglement can really tell us about quantum gravity

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We revisit the Bose-Marletto-Vedral (BMV) tabletop experimental proposal—which aims to witness quantum gravity using gravity mediated entanglement—analyzing the role of locality in the experiment. We first carry out a fully quantum modelling of the interaction of matter and gravity and then show in what way gravity mediated entanglement in the BMV experiment could be accounted for without appealing to quantum degrees of freedom of the gravitational field. We discuss what assumptions are needed in order to interpret the current BMV experiment proposals as a proof of quantum gravity, and also identify the modifications that a BMV-like experiment could have in order to serve as proof of quantum gravity without having to assume the existence of a local mediators in the gravitational field.

Introduction. A full formulation of quantum gravity is among the most coveted theories in physics. However, we lack experimental testbeds to probe gravity in the quantum regime. Recently, exciting proposals for tabletop experiments combining quantum mechanics and gravity have been put forward by Bose et. al [1] and Marletto and Vedral [2] (the so-called BMV experiment). These proposals have captured the eye of the community for their potential to provide insights on the interaction of gravity and quantum matter.

The experiment consists of two particles, each prepared in a superposition of two different trajectories. The two particles interact only gravitationally. The rough idea is that the different superposed paths generate different gravitational fields, which can entangle the particles. Despite the inherent experimental challenges to isolate the particles while shielding them from decoherence, we are close to having technology that will allow the experiment in the near future [1][8]. Although it is exciting to have an experiment that can explore aspects of gravity when quantum theory is relevant, some possible objections to the fact that the BMV experiment can reveal the quantum nature of gravity exist in the literature [9][14]. This indicates that the conclusions that can be drawn from the experiment require careful analysis. For example, it has been claimed that if the gravitational field is capable of entangling the masses, then either we must abandon the principle of locality, or the gravitational field must be quantum [15][16]. In this paper we present a series of arguments which qualify this statement. We discuss that while the experiment, as proposed, can prove that gravity can establish a quantum channel between the particles, it cannot decide whether gravity has quantum degrees of freedom. Furthermore, we propose a modification to the experiment so that it can demonstrate the fundamental quantum behaviour of gravity.

It has been argued that when one tries to describe the coupling of classical and quantum systems, one is faced with theoretical inconsistencies (see, e.g., [17][18]). From this perspective, theories where matter is quantum and gravity is classical cannot be fundamental. However, the theoretical argument alone cannot be used to claim that gravity is quantum. Instead, for an experimental claim of quantuness, one requires observing specific markers of quantum behaviour. While different authors may disagree on what conditions are sufficient to prove that a system is quantum, there are some uncontroversial properties of a system that, if observed, would prove its quantuness. For example, observing Wigner negativity [19][20], or violations of Bell inequalities [21][22] in the system itself.

In this light, our goal is twofold. First, we will discuss that while the current proposals of the BMV experiment can explore whether gravity can mediate entanglement between two quantum systems, it cannot directly witness quantum degrees of freedom of gravity, unless one also assumes the existence of local gravitational mediators. That is, we will argue that the mere observation that two masses (in a quantum superposition of trajectories) get entangled through the gravitational interaction is not sufficient to prove the existence of gravitational quantum degrees of freedom. Second, we will also identify under which regimes a BMV-like experiment could reveal more about the quantum behaviour of gravity with fewer assumptions.

A fully quantum description of the experiment. We first employ a quantum field theoretic description of gravity in the BMV experiment, where the masses are coupled to linearized quantum gravity: a weak field limit of the gravitational field which can be quantized [22][23]. This weak-field limit description should be a prediction of any theory of quantum gravity. Consider the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu},$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and $h_{\mu\nu}$ is a metric perturbation with units of energy. We choose these conventions so that the field propagators do not pick up factors of $G$. We quantize the gravitational perturbations as the quan-
The predictions of the theory are entirely determined by the field’s Wightman and Feynman propagator distributions,
\[
W_{\mu\alpha'\beta'}(x,x') = \langle \hat{h}_{\mu\nu}(x)\hat{h}_{\alpha'\beta'}(x') \rangle = \frac{i}{2}E_{\mu\alpha'\beta'}(x,x') + \frac{1}{2}H_{\mu\alpha'\beta'}(x,x'),
\]
\[
G_{\mu\alpha'\beta'}(x,x') = \langle T\hat{h}_{\mu\nu}(x)\hat{h}_{\alpha'\beta'}(x') \rangle = -\frac{i}{2}\Delta_{\mu\alpha'\beta'}(x,x') + \frac{1}{2}H_{\mu\alpha'\beta'}(x,x'),
\]
where \( T \) denotes the time ordering operation, \( E_{\mu\alpha'\beta'}(x,x') \) is the causal propagator (the advanced minus retarded Green’s functions), \( \Delta_{\mu\alpha'\beta'}(x,x') \) is the radiation Green’s function (the retarded plus advanced Green’s function), and \( H_{\mu\alpha'\beta'}(x,x') \) is the Hadamard distribution. It is important to distinguish which of these functions are a consequence of the field quantization, and which ones are also present for a classical field. The (state independent) propagator

\[
\exp\{ \frac{i}{\hbar}S(x) \}
\]

includes the fundamental quantum contributions from the vacuum and the presence of entanglement in the field (see [24]).

The specific BMV setup considers both particles to be in a superposition of the two paths, so that the initial state of the system is

\[
|\psi_0\rangle = \frac{1}{\sqrt{2}} (|L_1\rangle + |R_1\rangle) \otimes \frac{1}{\sqrt{2}} (|L_2\rangle + |R_2\rangle),
\]
corresponding to the density operator

\[
\hat{\rho}_0 = |\psi_0\rangle\langle \psi_0 | = \frac{1}{4} \sum_{p_1 \in \{L_1, R_1\}} \sum_{p_2 \in \{L_2, R_2\}} |p_1 p_2\rangle\langle q_1 q_2 |.
\]

To obtain the final state of the particles we assume that the field is in the vacuum state. After the interaction, one must trace-out the gravitational degrees of freedom. This results in a mixed state for the two masses, because the particles become entangled with the gravitational field itself. To simplify the result, we assume that all trajectories are related by rotations and translations in space, so that the local vacuum effect in each trajectory is the same. Under this assumption, the leading order negativity is computed in Appendix [3]

\[
N_0 = \pi \frac{G}{6} \left( |G_{L_1L_2} + G_{R_1R_2} - G_{L_1R_2} - G_{R_1L_2} | - \mathcal{L} \right) + O(G^2),
\]

where \( \mathcal{L} \) is a local noise term associated to the particle’s interaction with the gravitational field vacuum—which are given by integrals of \( W_{\mu\alpha'\beta'}(x,x') \) and

\[
G_{p_1 p_2} = \int dV dV' T_{p_1}(x) G_{\alpha'\beta'}(x,x') T_{p_2}(x'),
\]

where \( dV \) denotes the invariant spacetime volume element. As per the discussion after Eq. (2), the role played by the quantum degrees of freedom of the gravitational
field in the entanglement mediation is then encoded in the
dependence of $N_3$ on real part of $G_{\mu\nu\alpha'\beta'}(x,x')$ and
the local noise terms $L$.

**Quantum-controlled classical fields can entangle.**— We now present a description of the
BMV experiment which does not rely on any quantum
property of gravity and is compatible with relativity. We
will show that this description is able to precisely predict
the expected results of the experiment in the regimes
previously proposed [1]. In this model, we consider
linearized perturbations as in Eq. (1) and write the
classical solution of the linearized Einstein’s equations
for a source $T_{\mu\nu}$ as

$$h^{\mu\nu}(x) = \sqrt{4\pi G} \int dV' G_{\mu\nu}^{R_{\alpha'\beta'}}(x,x')T^{\alpha'\beta'}(x'),$$

where $G^{R_{\alpha'\beta'}}_{\mu\nu}(x,x')$ denotes a retarded Green’s function
for the linearized Einstein's equations. Each possible
path combination $|R_1R_2|, |R_1L_2|, |L_1R_2|, |L_1L_2|$ will be
associated with a different classical gravitational interaction
between the masses. In this sense, this description is
a quantum-classical model of gravity, where gravity has no quantum degrees of
freedom. The interaction Hamiltonian can be written as

$$\hat{H}_I(t) = \sum_{p_1 \in \{L_1, R_1\}} \Phi_{p_1p_2}(t) |p_1p_2\rangle \langle p_1p_2|,$$

where $\Phi_{p_1p_2}(t)$ denotes the retarded propagation of the
classical gravitational field between the two particles under-
going trajectories $z_{p_1}(t)$ and $z_{p_2}(t)$, i.e.,

$$\Phi_{p_1p_2}(t) = -\sqrt{4\pi G} \int d^3 x \left( T^{\mu\nu}_{p_1}(x) h^{\mu\nu}_{p_2}(x) + T^{\mu\nu}_{p_2}(x) h^{\mu\nu}_{p_1}(x) \right),$$

where

$$h^{\mu\nu}_{p_i}(x) = \sqrt{4\pi G} \int dV' G^{R_{\alpha'\beta'}}_{\mu\nu}(x,x')T^{\alpha'\beta'}(x').$$

The interaction [11] defines a quantum channel be-
tween the particles that respects relativity. However, estab-
lishing a quantum channel cannot be taken as proof
that gravity has quantum degrees of freedom: it is merely
a consequence of having quantum sources and not of any
assumptions about the field (see [21]). Indeed, this model
does not assume that the gravitational field has degrees
of freedom of its own that carry information between the
particles (i.e. gravity is not an active mediator). We use
the term quantum-controlled classical to refer to [11] be-
cause it associates to each state of the particles the clas-
sical field sourced by each particle undergoing each path.

Since the gravitational interaction is implemented by a
classical field, $\hat{H}_I(t)$ commutes with itself at different
times. The time-evolution operator is

$$\hat{U}_I = \exp \left(-i \int dt \hat{H}_I(t) \right) = \sum_{p_1 \in \{L_1, R_1\}} \sum_{p_2 \in \{L_2, R_2\}} \rho_{c p_1p_2} |p_1p_2\rangle \langle p_1p_2|,$$

where $\Delta_{p_1p_2}$ is a double integral in spacetime of the
retarded plus advanced propagator contracted with the
stress-energy tensor of the sources corresponding to each path:

$$\Delta_{p_1p_2} := \int dV dV' T^{\mu\nu}_{p_1}(x) \Delta_{\mu\nu\alpha'\beta'}(x,x')T^{\alpha'\beta'}(x').$$

Using the initial state for the particles given in Eq. (8),
we obtain the following final density operator after the
interaction

$$\hat{\rho}_c = \frac{1}{N} \sum_{p_1 \in \{L_1, R_1\}} \sum_{p_2 \in \{L_2, R_2\}} e^{2\pi G \Delta_{p_1p_2}} |p_1p_2\rangle \langle p_1p_2|,$$

The entanglement between the two particles can be eval-
uated through the negativity [25] of the state $\hat{\rho}_c$, which
reads

$$N_c = \frac{1}{2} \sin \left( \pi G |\Delta L_{L_1}\Delta_{L_1L_2} - \Delta_{R_1R_2} - \Delta_{L_1R_2}| \right).$$

With the typical choice of paths for the BMV experiment,
the above quantity is non-zero. Therefore, it is possible
to model entanglement creation with a relativistic inter-
action that does not involve local quantum degrees of
freedom of the gravitational field. Moreover, Eq. (18)
is exactly the result obtained when one sets the funda-
mental quantum parts of the two-point functions (i.e.,
$E_{\mu\nu\alpha'\beta'}(x,x')$ and $H_{\mu\nu\alpha'\beta'}(x,x')$) to 0 in Eq. (5). That is,
this model is the limit of the quantum field theoretical
description when one gets rid of the quantum degrees of
freedom of the gravitational perturbations. Overall, this
causal description does not involve local quantum degrees.
of freedom for gravity and yet it is able to predict the expected gravity-mediated entanglement generation in the BMV experiment.

**Comparing the models.**—Comparing Eqs. (2) and (8) with (18), we see that, apart from the vacuum noise $\mathcal{L}$ that appears in the quantum case, the entanglement acquired in the quantum field description is larger than the one obtained in the quantum-controlled classical description. In the regime of long interaction times, the noise term $\mathcal{L}$ is insignificant compared to the effect of the propagators, so that it can be neglected [24]. The contribution to the negativity due to the imaginary part of the Feynman propagator can be associated with entanglement mediated by communication via the field $\bar{\Phi}'$, while the real part of the propagator is associated with the entanglement extracted from the vacuum state of the gravitational field (in relativistic quantum information this phenomenon is known as entanglement harvesting [29,31]). In this sense, the real part of the propagator $(H_{\mu\nu\alpha}(x,x'))$ is the one that captures the entanglement associated to the quantum degrees of freedom of the gravitational field. Violating Bell inequalities with this entanglement would then be proof of the quantum behaviour of gravity. In most setups where the paths are causally connected, the contribution of the real part of the propagator is negligible compared to its imaginary counterpart $(\Delta_{\mu\nu\alpha}(x,x'))$. In particular, the proposed implementations of the BMV experiment use masses with smallest separation of the order of $L \sim 10^{-4}$m and interaction times of the order of $T \sim 1s$. For these parameters we find that the imaginary part contribution of the propagator is $10^{14}$ times larger than its real part, making the BMV experiment agnostic to the existence of local quantum degrees of freedom for the gravitational field.

In order to discuss the interpretation of the experiment, it is also important to distinguish between two fundamentally different notions of locality. The first notion comes from the description of spacetime and is deeply linked with causality. It states operations happen at events in spacetime, and do not affect other events which are causally disconnected from them. We will call this notion **event locality**. The second notion of locality comes from quantum mechanics, and states that operations that independently affect two quantum systems must be separable. We call this notion **system locality** [32]. The notion of system locality alone is agnostic about causal structure or any underlying notion of spacetime. Although these notions of locality are different there are particular frameworks in which we link the two. E.g., in quantum field theory (QFT) the postulate of microcausality prevents operations in local systems from violating the notion of event locality prescribed by relativity.

The distinction between the two notions of locality is particularly important when talking about local operations and classical communication (LOCC). While the notion of system locality is operationally captured by the ‘L’ and the ‘O’, we often define what we mean by classical communication (the ‘CC’) in terms of event locality. After all, what is classical communication if not sending information from one event to another?

Many works in the literature which argue that the BMV experiment can be used to probe the quantum nature of gravity use an argument based on LOCC. In essence, the argument goes as follows: LOCC does not increase the entanglement between quantum systems, thus, if the masses interact only gravitationally and get entangled, the gravitational field which mediates the interaction is going beyond ‘CC’ [33], hence the field cannot be classical [1,2,15,16]. However, these arguments assume a relationship between event and system locality in order to reach their conclusion: they assume that, in order to get the masses entangled, a mediator is required in order not to violate system locality, ruling out a direct interaction between the masses. This assumption can be reworded as follows: mass $A$ couples to the field and then the field carries quantum information to mass $B$, or otherwise we would have action-at-a-distance. However, the assumption that the gravitational interaction is implemented through system-local operations is not based on first principles. It is reasonable to demand that gravity must satisfy event locality to prevent action-at-a-distance, but it is not unthinkable to consider that it may not necessarily be system-local since this notion is operational rather than fundamental.

We know that if we use QFT to describe gravity, the interaction will be both event-local and system-local. However, assuming this relationship between the two notions of locality to interpret the results of the experiment implicitly assumes a-priori that the system is described by a framework like QFT. This is not satisfactory if our objective is to prove the quantum nature of a relativistic theory. For instance, the fact that a classical Coulomb potential can entangle two charges is not why we believe that the electromagnetic field is quantum: $\mathcal{A}$ is clearly not a quantum field. The electromagnetic field has only been proved to be quantum when QFT was required to model experiments which no classical theory could account for, not when the quantum description for the hydrogen atom was verified.

Importantly, the time evolution generated by the quantum-controlled classical field interaction is not local in the operational quantum sense (that is, $U_1 \neq U_1 \otimes U_2$ is not system-local), although the interaction is intrinsically local in the relativistic sense and thus satisfies event locality. This means that time evolution implemented by $U_1$ can create entanglement, even though the gravitational field does not have quantum degrees of freedom and the interaction is event-local in this description. The notion of event locality is the one that comes from first principles and captures no-action-at-a-distance: the field only interacts with the particles locally at each instant of time, due to the causal retarded propagation. This means that seeing the BMV experiment establish a quantum channel between the masses and assuming Lorentz invariance is not enough to infer the existence of local quantum degrees of freedom of gravity: one would also
need to assume that the interaction is implemented by local mediators.

A proposal for witnessing quantum gravity with the BMV experiment—Overall, the BMV experiment can be described either by treating gravity as a quantum field, or by considering it to be a quantum-controlled field devoid of quantum degrees of freedom. Although these descriptions yield different predictions (therefore the experiment does have the potential to acknowledge quantum gravitational degrees of freedom), they can only be experimentally distinguished in specific regimes.

It is then possible to adapt the experiment to directly witness the quantum degrees of freedom of the gravitational field. If it could be implemented for times of the order of the light-crossing time of the separation between paths, then the Hadamard term of the Feynman propagator would be larger than its imaginary part, and entanglement present in the gravitational field would be meaningful in the experiment. If the experiment can be adapted to implement these short interaction times (or can be sensitive to changes in negativity of the order of $10^{-14}$ with the current proposed parameters [1]) one can claim to witness quantum degrees of freedom of the gravitational field with no extra assumptions.

On the other hand, it would be unfair to say that the BMV experiment—even in the regimes considered in [1, 2]—gives no information about the interplay of gravity and quantum mechanics. If the gravitational field proves to be able to entangle the particles, we experimentally confirm that gravity can mediate quantum channels between masses. In fact, if the BMV experiment confirms that gravity can mediate entanglement, it would rule out many classical descriptions for gravity coupled to quantum matter (such as some of the examples in [34]). However, as we discussed, this is different from proving that gravity itself has quantum degrees of freedom.

Conclusions—We discussed the implications of the BMV experiment for quantum gravity. If the experiment is implemented for long times, classical gravity mediates entanglement between the masses. This can give us information about how gravity couple to quantum systems, but may not be argued as proof of existence of local quantum degrees of freedom of the gravitational field. If on the other hand the experiment can be performed in times of the order of the light-crossing time between the paths, then the entanglement between the particles would be significantly affected by the entanglement previously present in the gravitational field. This could be used to experimentally assess whether gravity admits a quantum field theoretic description or not.

In order to adapt the previously proposed experiment so that the entanglement of the masses is due to the quantum nature of the field, either the spatial separation between the trajectories would have to be increased, or the interaction time would have to be decreased to guarantee spacelike separation. Alternatively, one could also see quantum behaviour with enough sensitivity in the experiment to distinguish between the quantum-controlled (Eq. (18)) and the quantum field result (Eq. (3)). It is even possible to find regimes where the quantum prediction yields less entanglement than the classical model predicts. This is due to the fact that for short interaction times, the local vacuum noise term $\mathcal{L}$ can dominate the sum in Eq. (3). An effect that is well known within entanglement harvesting [29, 31, 35].

While the physical relevance of the BMV experiment is unquestionable, we argued that the experiment as prescribed is not enough to assert (without further assumptions) the existence of local quantum degrees of freedom in the gravitational field, unless taken to regimes where a relativistically local quantum-controlled model cannot predict the entanglement acquired by the masses. While we do not want to subtract from the many merits of the BMV proposal, it is important to qualify the range of implications of its experimental realization and the assumptions needed in order to consider it a full experimental proof of quantum gravity. While the assumption that gravity is mediated by some kind of ‘graviton’ degree of freedom may seem easy to accept for most, we believe it is important to emphasize that BMV experiment (in the regimes that it was originally proposed) cannot directly witness these ‘gravitons’. We also discussed that it is not enough to assume relativistic causality/Lorentz invariance in order to bypass the assumption of local mediators. We have shown, however, that the experiment can be adapted to not have to rely on this assumption, although at the price of making it more experimentally challenging.

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Appendix A: The retarded propagator of the gravitational field

The retarded propagator \( G_{R}^{\mu\nu\alpha\beta}(x,x') \) can be written as

\[
G_{R}^{\mu\nu\alpha\beta}(x,x') = \frac{1}{2\pi} \theta(t-t') \delta \left((t-t')^2 - |x-x'|^2\right) \mathcal{P}^{\mu\nu\alpha\beta} = \frac{1}{4\pi|x-x'|} \delta(t-t' - |x-x'|) \mathcal{P}^{\mu\nu\alpha\beta}, \tag{A1}
\]

where \( \mathcal{P} \) is a bitensor. We then have the linearized metric given by \( g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} \), where

\[
h^{\mu\nu}(x) = \sqrt{4\pi G} \int dV' G_{R}^{\mu\nu\alpha\beta}(x,x') T^{\alpha\beta'}(x'), \tag{A2}
\]

where \( T_{\alpha\beta} \) denotes the stress-energy tensor of the source. For the case of a pointlike particle undergoing a trajectory \( z_1(t) \) with four-velocity \( u_1^\mu(t) \), it reads

\[
T_{1}^{\mu\nu}(x) = m_1 u_1^\mu(t) u_1^\nu(t) \frac{\delta^{(3)}(x-z_1(t))}{u_1^0(t) \sqrt{-g}}, \tag{A3}
\]

so that we obtain

\[
\int dV' G_{R}^{\mu\nu\alpha\beta}(x,x') T_1^{\alpha\beta'}(x') = m_1 \int d\tau' d^3x' \frac{1}{u_1^0(t')} \delta^{(3)}(x' - z_1(t')) \delta(t-t' - |x-x'|) \mathcal{P}^{\mu\nu\alpha\beta} u_1^\alpha(t') u_1^\beta(t') \approx \frac{\delta(t' - t)}{1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_r)}. \tag{A4}
\]

Now, let \( t_r \) be the retarded time such that \( t - t_r - |x-z(t_r)| = 0 \), so that

\[
\delta(t-t' - |x-z_1(t')|) = \frac{\delta(t' - t)}{1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_r)}, \tag{A5}
\]

where \( \hat{r}_1(t') = (x-z_1(t'))/|x-z_1(t')| \) and we obtain

\[
\int dV' G_{R}^{\mu\nu\alpha\beta}(x,x') T_1^{\alpha\beta'}(x') = \frac{\mathcal{P}^{\mu\nu\alpha\beta} u_1^\alpha(t') u_1^\beta(t)}{4\pi u_1^0(t_r)(1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_r)) |x-z_1(t_r)|}. \tag{A6}
\]

The interaction Hamiltonian of a particle labelled 2 with the gravitational potential sourced by particle 1 will then be

\[
H_{12}(t) = -\frac{1}{2} \sqrt{16\pi G} \int d^3x h_{\mu\nu}(x) T_2^{\mu\nu}(x) = - \frac{G m_1 m_2}{|z_2(t) - z_1(t)|} \frac{\mathcal{P}^{\mu\nu\alpha\beta} u_2^\mu(t) u_2^\nu(t) u_1^\alpha(t_r) u_1^\beta(t_r)}{|z_2(t) - z_1(t_r)| (1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_{t_r}))}, \tag{A7}
\]

\[
= - \frac{G m_1 m_2}{|z_2(t) - z_1(t)|} \frac{2(\eta_{\mu\nu} u_2^\mu(t) u_2^\nu(t_{t_r}))^2 - 1}{|z_2(t) - z_1(t_{t_r})| (1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_{t_r}))}, \tag{A8}
\]

where \( t_{t_r} \) is the solution to \( t - t_{t_r} = |z_2(t) - z_1(t_{t_r})| \) and \( \hat{r}_{t_r} = (z_2(t) - z_1(t_{t_r}))/|z_2(t) - z_1(t_{t_r})| \), and we used \( \mathcal{P}^{\mu\nu\alpha\beta} = \eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\alpha} \eta_{\nu\beta} \). The total interaction between the two particles is then given by

\[
H_1(t) = \frac{1}{2} (H_{12}(t) + H_{21}(t)) = - \frac{G m_1 m_2}{|z_2(t) - z_1(t_{t_r})|} \frac{(\eta_{\mu\nu} u_2^\mu(t) u_2^\nu(t_{t_r}))^2 - 1}{|z_2(t) - z_1(t_{t_r})| (1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_{t_r}))}, \tag{A9}
\]

\[
= - \frac{G m_1 m_2}{|z_1(t) - z_2(t_{t_r})|} \frac{(\eta_{\mu\nu} u_2^\mu(t) u_2^\nu(t_{t_r}))^2 - 1}{|z_1(t) - z_2(t_{t_r})| (1 - \hat{r}_1(t_r) \cdot \hat{z}_1(t_{t_r}))}, \tag{A10}
\]

\( t_{t_1} \) is the solution to \( t - t_{t_1} = |z_2(t) - z_1(t_{t_1})| \) and \( \hat{r}_{t_1} = (z_1(t) - z_2(t_{t_1}))/|z_1(t) - z_2(t_{t_1})| \), and we used \( \mathcal{P}^{\mu\nu\alpha\beta} = \eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\alpha} \eta_{\nu\beta} \). Notice that in the non-relativistic limit, we have \( t_{t_r} \approx t_{t_1} \approx t, u_1^0(t) \approx u_2^0(t) \approx 1 \) and \( \eta_{\mu\nu} u_1^\mu(t) u_2^\nu(t) \approx 1 \), allowing one to recover the non-local Newtonian interaction between the particles:

\[
H_1(t) \approx - \frac{G m_1 m_2}{|z_1(t) - z_2(t)|}. \tag{A11}
\]
Also notice that
\[ \int dt H_I(t) = 2\pi G \int dVdV' \left( T^1_{\mu\nu}(x)G^R_{\mu\nu'\alpha'\beta'}(x,x')T^2_{\mu\nu}(x)G^R_{\mu'\nu'\alpha'\beta'}(x,x') \right) \]
\[ = 2\pi G \int dVdV' \left( T^1_{\mu\nu}(x)G^R_{\mu\nu'\alpha'\beta'}(x,x')T^2_{\mu\nu}(x)G^R_{\mu'\nu'\alpha'\beta'}(x,x') \right) \]
\[ = 2\pi G \int dVdV'T^1_{\mu\nu}(x)\Delta^{\mu\nu\alpha'\beta'}(x,x')T^2_{\mu\nu}(x), \]
where \( \Delta^{\mu\nu\alpha'\beta'}(x,x') = G^R_{\mu\nu\alpha'\beta'}(x,x') + G^A_{\mu\nu\alpha'\beta'}(x,x') \) and we used \( G^R_{\mu'\nu'\alpha'\beta'}(x,x') = G^A_{\alpha'\beta'}(x,x') \).

Appendix B: Perturbation theory in the quantum case

In this appendix we perform the calculations of the time evolution operator for the quantum treatment of the BMV experiment. For convenience, we write the interaction Hamiltonian that considers the particles coupled to linearized quantum gravity as
\[ \hat{H}_I(t) = -\sqrt{4\pi G} \sum_{p=L,R}^2 |p_i\rangle\langle p_i| \hat{\phi}_{p_i}(z_{p_i}(t)), \]
where we defined
\[ \hat{\phi}_{p_i}(z_{p_i}(t)) = \int d^3x \sum_{\mu\nu} \frac{\partial^{\mu\nu}Z(\nu)}{\partial \nu_p(t)} \hat{h}_{\mu\nu}(z_{p_i}(t)). \]
The time evolution operator can be written as
\[ \hat{U}_I = T\exp \left( -i \int \hat{H}_I(t) \right), \]
and the Dyson expansion gives
\[ \hat{U}_I = \mathbb{1} + \hat{U}_I^{(1)} + \hat{U}_I^{(2)} + O(3), \]
where \( O(3) \) denotes terms of third order in the interaction Hamiltonian. Explicitly,
\[ \hat{U}_I^{(1)} = -i \int dt \hat{H}_I(t), \quad \hat{U}_I^{(2)} = - \int dt dt' \hat{H}_I(t)\hat{H}_I(t')\theta(t-t'), \]
where \( \theta(t-t') \) is the Heaviside step function and implements time ordering. Plugging Eq. [B1] for the Hamiltonian, we obtain
\[ \hat{U}_I^{(1)} = i\sqrt{4\pi G} \sum_{p=L,R} |p_1\rangle\langle p_1| \int dt \hat{\phi}_{p_1}(z_{p_1}(t)) + i\sqrt{4\pi G} |p_2\rangle\langle p_2| \int dt \hat{\phi}_{p_2}(z_{p_2}(t)), \]
\[ \hat{U}_I^{(2)} = -4\pi G \sum_{p=L,R} |p_1\rangle\langle p_1| \int dt dt' \hat{\phi}_{p_1}(z_{p_1}(t))\hat{\phi}_{p_1}(z_{p_1}(t'))\theta(t-t') + |p_2\rangle\langle p_2| \int dt dt' \hat{\phi}_{p_2}(z_{p_2}(t))\hat{\phi}_{p_2}(z_{p_2}(t'))\theta(t-t'), \]
\[ - 4\pi G \sum_{p=L,R} |p_1\rangle\langle p_2| \int dt dt' \hat{\phi}_{p_1}(z_{p_1}(t))\hat{\phi}_{p_2}(z_{p_2}(t'))\theta(t-t') + \hat{\phi}_{p_1}(z_{p_1}(t'))\hat{\phi}_{p_2}(z_{p_2}(t))\theta(t-t'), \]
where we implemented the change of variables \( t \leftrightarrow t' \) in the double integrals of \( \hat{U}_I^{(2)} \).
Appendix C: Final states of the particles

The initial state of the two particles in matrix form in the basis \( \{|L_1L_2\rangle, |R_1L_2\rangle, |L_1R_2\rangle, |R_1R_2\rangle\} \) reads

\[
\hat{\rho} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.
\] (C1)

To leading order in the gravitational field, the updated state of the particles can be written as \( \hat{\rho} + \delta \hat{\rho}_c \), where

\[
\delta \hat{\rho}_c = -\frac{i\pi G}{2} \begin{pmatrix} 0 & \Delta R_{1L_2} - \Delta L_{1L_2} & \Delta L_{1R_2} - \Delta L_{1L_2} & \Delta R_{1R_2} - \Delta L_{1L_2} \\ \Delta L_{1L_2} - \Delta L_{1R_2} & 0 & \Delta R_{1R_2} - \Delta R_{1L_2} & \Delta R_{1R_2} - \Delta R_{1L_2} \\ \Delta L_{1L_2} - \Delta R_{1R_2} & \Delta L_{1R_2} - \Delta R_{1L_2} & 0 & \Delta R_{1R_2} - \Delta R_{1L_2} \\ \Delta L_{1L_2} - \Delta R_{1L_2} & \Delta L_{1R_2} - \Delta R_{1L_2} & \Delta L_{1R_2} - \Delta R_{1L_2} & 0 \end{pmatrix},
\] (C2)

In the quantum case, the final state of the particles to leading order can be written as \( \hat{\rho} + (\delta \hat{\rho}_c + \delta \hat{\rho}_g + \delta \hat{\rho}_l) \), where

\[
\delta \hat{\rho}_l = \pi G (\mathcal{L}_V - \mathcal{L}_I) \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix},
\] (C3)

and

\[
\delta \hat{\rho}_q = \frac{\pi G}{2} (H_{L_1R_2} + H_{R_1L_2} - H_{L_1L_2} - H_{R_1R_2}) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},
\] (C4)

where \( H_{p_1p_2} \) denotes the integrated Hadamard function along each pair of paths for particles 1 and 2,

\[
H_{p_1p_2} = \int \text{d}V \text{d}V' \text{d}X \text{d}X' T^\mu_1^\nu \mu_{\alpha'\beta'}(X,X') T^\alpha'_\beta'_R(X').
\] (C5)

while \( \mathcal{L}_V \) and \( \mathcal{L}_I \) are noise terms due to the interaction of each particle with the vacuum of the field. These are explicitly given by

\[
\mathcal{L}_V = \int \text{d}V \text{d}V' \text{d}X \text{d}X' \langle \hat{h}_{\mu\nu}(X) \hat{h}_{\alpha'\beta'}(X') \rangle_0 T^\alpha'_\beta'_R(X'),
\] (C6)

\[
\mathcal{L}_I = \int \text{d}V \text{d}V' \text{d}X \text{d}X' \langle \hat{h}_{\mu\nu}(X) \hat{h}_{\alpha'\beta'}(X') \rangle_0 T^\alpha'_\beta'_R(X'),
\] (C7)

where, due to choice of paths, the indices \( i \) can be 1 or 2, and still yield the same result due to the fact that the paths are related to translations and rotations, which are symmetries of the quantum field theory. Then, the \( \mathcal{L}_V \) term is a local noise associated to each path, while the \( \mathcal{L}_I \) term represents an interference term associated with each particle undergoing the superposition of paths. Also notice that due to the fact that the propagator decreases with distance, we have \( \mathcal{L}_I \leq \mathcal{L}_V \), with equality holding only if the paths 1 and 2 are identical. Moreover, these vacuum noise terms decay fast with the interaction time, so that they are negligible for long interaction times. Then we can interpret \( \delta \hat{\rho}_l \) as a local vacuum contribution and \( \delta \hat{\rho}_q \) can be seen as the additional correlation contribution due to the quantum nature of the field.

[1] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. A. Geraci, P. F. Barker, M. S. Kim, and G. Milburn, Spin entanglement witness for quantum gravity, Phys. Rev. Lett. 119, 240401 (2017)
[2] C. Marletto and V. Vedral, Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity, Phys. Rev. Lett. 119, 240402 (2017)
[3] M. Christodoulou and C. Rovelli, On the possibility of laboratory evidence for quantum superposition of geometries, Phys. Lett. B 792, 64 (2019)
Generalizations of this argument have been presented in the literature beyond quantum mechanics and LOCC [37], but T. R. Perche and E. Martín-Martínez, The role of quantum degrees of freedom of relativistic fields in quantum information [38].

T. D. Galley, F. Giacomini, and J. H. Selby, A no-go theorem on the nature of the gravitational field beyond quantum theory, Quantum 6, 779 (2022).

We use negativity instead of entanglement entropy to compare this classical calculation with the quantum description. Notice that in the nomenclature of quantum information an interaction between two systems of the form (11) is called quantum because it establishes a quantum channel. However in this manuscript we argue that establishing a quantum channel is not a valid proof that the gravitational interaction has a Hilbert space structure of any kind or any local quantum because it establishes a quantum channel. However in this manuscript we argue that establishing a quantum channel is not a valid proof that the gravitational interaction has a Hilbert space structure of any kind or any local quantum degrees of freedom. A proof that gravity is quantum requires one to see an experimental phenomenon that cannot be explained unless there are local degrees of freedom for the gravitational field, as we argue below.

We use negativity instead of entanglement entropy to compare this classical calculation with the quantum description for the gravitational field below. When the field is quantum there will be particle-field entanglement and the state of the particles will not be pure. Entanglement entropy is therefore not a valid entanglement measure. Negativity is always a faithful entanglement quantifier for bipartite two-level systems [36].
assumption of a relationship between system and event locality.

[34] S. Donadi and A. Bassi, Seven nonstandard models coupling quantum matter and gravity, AVS Quantum Sci. 4, 025601 (2022)

[35] B. Reznik, Entanglement from the vacuum, Foundations of Phys. 33, 167 (2003)

[36] G. Vidal and R. F. Werner, Computable measure of entanglement, Phys. Rev. A 65, 032314 (2002)

[37] C. Marletto and V. Vedral, Witnessing nonclassicality beyond quantum theory, Phys. Rev. D 102, 086012 (2020)