Unusual spin dynamics in the low-temperature magnetically ordered state of Ag₃LiIr₂O₆

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Recently, there have been contrary claims of Kitaev spin-liquid behaviour and ordered behavior in the honeycomb compound Ag₃LiIr₂O₆ based on various experimental signatures. Our investigations on this system reveal a low-temperature ordered state with persistent dynamics down to the lowest temperatures. Magnetic order is confirmed by clear oscillations in the muon spin relaxation (μSR) time spectrum below 9 K till 52 mK. Coincidentally in ⁷Li nuclear magnetic resonance, a wipe-out of the signal is observed below ~ 10 K which again strongly indicates magnetic order in the low temperature regime. This is supported by our density functional theory calculations which show an appreciable Heisenberg exchange term in the spin Hamiltonian that favors magnetic ordering. The ⁷Li shift and spin-lattice relaxation rate also show anomalies at ~ 50 K. They are likely related to the onset of dynamic magnetic correlations, but their origin is not completely clear. Detailed analysis of our μSR data is consistent with a co-existence of incommensurate Néel and striped environments. A significant and undiminished dynamical relaxation rate (~ 5 MHz) as seen in μSR deep into the ordered phase indicates enhanced quantum fluctuations in the ordered state.

I. INTRODUCTION

Kitaev’s seminal proposal of bond-dependent magnetic interactions stabilizing a novel Z₂ spin-liquid ground state with Majorana excitations, followed by the important material-specific advance of Jackeli and Khaliullin [1] has triggered significant experimental effort to synthesize such materials. They advocated honeycomb lattice structures of 4d/5d element based oxides with edge-sharing oxygen octahedra and strong spin-orbit coupling as having the necessary ingredients to host the Kitaev model [2]. Several promising candidates with such layered honeycomb structure have since been investigated: Na₂IrO₄ [3–5], α-Li₂IrO₃ (as also its three dimensional polymorphs) [6, 7], and α-RuCl₃ [8, 9]. However, it has been revealed that these materials order magnetically [9–15] due to the presence of Heisenberg and other non-Kitaev terms. In the Kitaev interactions may only be realised either at higher temperatures or under application of a magnetic field.

In this family of materials a new addition has been H₃LiIr₂O₆ [16], where all of the interlayer Li⁺ ions of α-Li₂IrO₃ (LIO) are replaced by H⁺ ions, retaining the LiIr₂O₆ planes. Various measurements have confirmed the absence of magnetic ordering down to 0.05 K in H₃LiIr₂O₆ [17] which has been argued to be a spin-orbit entangled quantum spin-liquid [17]. To complicate matters further, x-ray diffraction (XRD) has revealed the presence of stacking faults between the honeycomb planes [16], and the low temperature behavior was attributed to local moments induced by these defects. Theoretical ab-initio calculations have also shown for these systems that although the bond-dependent Kitaev interactions are significant, the Heisenberg and other non-Kitaev terms are not negligible. It has been suggested that these systems lie close to the tricritical point between ferromagnetic, zigzag and incommensurate spiral order resulting in the absence of magnetic order [18]. Calculations further reveal that the interlayer O-H-O geometry as well as lack of hydrogen order also strongly influence the Kitaev and other exchange interactions having strong impact on its magnetic properties [18, 19].

Very recently, the compound Ag₃LiIr₂O₆ (ALIO) has been synthesized [20], where proximate Kitaev spin-liquid physics has been claimed based on scaling behaviour of various thermodynamic quantities in the presence of quenched disorder and a two step release of magnetic entropy. Replacement of the lighter H⁺ ions by the heavier Ag⁺ ions leads to an increase in the inter-layer separation which can significantly influence the various magnetic exchange interactions. Estimates of the magnetic interactions using ab-initio study are currently lacking for this system.

We have been working on the hexagonal Ag₃LiM₂O₆ (M = Mn, Ru, Ir) system with the intention of devel-

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oping a comprehensive understanding of this honeycomb system. The Mn-based (3d³ or \(S = 3/2\)) system exhibits long-range order below about 50 K. There is also evidence of Berezinskii-Kosterlitz-Thouless behavior from an analysis of the electron spin resonance line broadening data near the transition temperature \([21]\). The Ru-based system (4d⁴ which might be \(S = 1\) or \(J = 0\)) was expected to be a possible candidate for excitonic magnetism \([22]\). Our investigations however revealed that spin-orbit coupling might not be significant in this case. But the expected long-range ordered state was not seen in spite of the apparently unfrustrated geometry of the honeycomb lattice. Rather, the local moments were found to be on the boron sublattice, apparently unfrustrated geometry of the honeycomb lattice.

Continuing our investigations in this series of systems, we focus in this paper on the Ir analog containing Ir⁴⁺ ions \((J_{\text{eff}} = 1/2)\) with the purpose of looking for possible Kitaev spin-liquid physics due to the enhanced spin-orbit coupling of the Ir moments. Our experimental and theoretical results as itemized below are however quite far from such expectations:

- We observe clear oscillations in \(\mu\)SR relaxation data below \(\sim 9\)K providing strong evidence for magnetic order below this temperature. Analysis of the muon data in the ordered state, complemented with density function theory (DFT) simulations of the muon stopping site points towards the co-existence of incommensurate Néel and stripe ordered magnetic domains.

- We also provide theoretical estimates for the various magnetic interactions in ALIO via DFT-based computations. We find the ratio of the nearest neighbor Kitaev exchange \((\mathcal{K}_1)\) to the Heisenberg exchange \((\mathcal{J}_1)\) to be in the range \(\frac{\mathcal{K}_1}{\mathcal{J}_1} \sim 2.0-3.5\), placing the ALIO system far from the pure Kitaev limit, and closer to the phase boundaries between stripe, Néel and 120° order of the phase diagram \([24]\).

- We observe two anomalies in the \(^7\)Li nuclear magnetic resonance (NMR) shift variation with temperature. The first one that is present at \(T \sim 10\) K clearly signifies the onset of magnetic long-range order as seen from the so-called wipe-out of the NMR signal, which agrees with our conclusions from muon data.

- The second anomaly is a broad maximum at about 50 K in the temperature variation of the \(^7\)Li NMR line shift (which tracks the intrinsic spin susceptibility) and is reminiscent of such features in quasi-one and quasi-two dimensional Heisenberg antiferromagnetic systems due to short-range magnetic correlations. The \(^7\)Li NMR 1/T₁ also has a maximum at \(T \sim 50\) K. Together with the observed progressive loss of the NMR intensity at this temperature, this could be signifying dynamic short-range magnetic correlations similar to those seen in CeCu₂Si₂ \([25, 26]\).

- Finally, we also find a large value of the muon relaxation rate \((\sim 5\) MHz\)) that remains essentially flat and undiminished deep into the ordered phase, i.e. down to 52 mK which is about 1/200⁰ of the ordering temperature of \(\sim 10\) K. This is quite striking and noteworthy, rather reminiscent of spin-liquid behavior \([27]\) in spite of the unambiguous evidence for magnetic order mentioned above (i.e., clear oscillations in \(\mu\)SR relaxation data.) We interpret this as a signature of persistent spatio-temporal fluctuations of the Néel and stripe ordered domains, possibly driven by quantum effects given our theoretical estimates of the various magnetic exchange energy scales in ALIO.

Our results are in sharp contrast with a recent report on the same compound \([28]\). This report was based on the low-temperature scaling behaviors as stated before, and in particular, on an apparent two step release of magnetic entropy suggesting ALIO may be a proximate Kitaev spin-liquid. We argue that the (extracted) magnetic specific heat in the high-temperature regime is very uncertain. This is simply due to the overwhelming contribution of the lattice to the total specific heat, especially in the high-temperature region \((T \gtrsim 30\) K\)). Consequently, the inference of the high-temperature peak (position and magnitude) and that of a two-step entropy release is rather insecure, let alone ascribing it to Majorana excitations. We note here that an even more recent report \([29]\) has found evidence for magnetic ordering in a cleaner batch of samples in line with our observations.

The remainder of the paper is organised as follows: we start by giving the details on the structure of ALIO and relevant technical details on measurements and theoretical methods in Sec. II. We next present our main pieces of experimental evidence that establish a low-temperature magnetic ordered state coming from \(\mu\)SR and NMR data in Sec. III. This is followed by a presentation of our theoretical estimates for the various magnetic exchange couplings based on DFT in Sec. IV. Sec. V is devoted to a detailed discussion of our observations using bulk probes (heat capacity and susceptibility) and the high temperature anomaly in NMR vis-a-vis long-range magnetic order below 10 K as inferred from our observations and computation, versus Kitaev spin-liquid scenario as in Ref. \([28]\). Concluding remarks are given in Sec. VI.

II. STRUCTURE AND TECHNICAL DETAILS

ALIO crystallizes in base centered monoclinic symmetry having space group C2/m. The crystal structure is
FIG. 1. (a) shows the unit cell of Ag₃LiIr₂O₆. The edge shared IrO₆ honeycomb network is shown (b). The x, y, z local axes point toward the transition metal (Ir) to ligand (O) direction. X (red dotted line), Y (green dotted line) and Z (blue dotted line) type Ir-Ir nearest-neighbor bonds are perpendicular to the chosen x, y, z local axes respectively.

shown in Fig. 1(a). The magnetic building block consists of Ir in an octahedral environment with nearest neighbor (nn) oxygen ligands. The IrO₆ octahedra form an edge shared honeycomb geometry in the a-b plane containing Li ions at the center (see Fig. 1(b)). The honeycomb layers in ALIO are identical to its parent compound, α-LIO but the chemical bonds between the layers are modified. The interlayer Li atoms in α-LIO are octahedrally coordinated with six oxygens in the two adjacent O₆ honeycomb layers, whereas the Ag atoms in ALIO are linearly connected to two oxygens in neighboring layers making a 180° O-Ag-O bond-angle.

Polycrystalline samples of ALIO were prepared by a two-step process as described in the supplementary information (SM) [30]. The resulting product was Ag₃LiIr₂O₆ as verified by lab x-ray diffraction measurements using a PANalytical X’Pert PRO diffractometer using Cu-Kα radiation (λ = 1.54182 Å). Small amounts of residual Ag and α-Li₂IrO₃ were detected in the x-ray diffraction pattern.

The magnetization measurements have been performed in a Quantum Design SQUID Vibrating Sample Magnetometer in the temperature range 2-400 K and in applied fields ranging from 0 Oe to 70 kOe. The heat capacity measurements have been done in a Quantum Design PPMS in the temperature range 2-250 K, in various field values in the range 0-90 kOe. μSR measurements were carried out using the MUSR spectrometer at the ISIS Neutron and Muon Source at the STFC Rutherford Appleton Laboratory in the UK. The powder sample was loaded on a silver sample holder to minimize the background signal. The holder was then mounted on a dilution refrigerator insert and a standard cryostat stick for measuring temperatures ranging from 50 mK up to 150 K. ⁷Li NMR measurements have been performed in a fixed field of 93.954 kOe, using a Tecmag spectrometer in a continuous flow cryostat in the temperature range of 4-300 K. Measurements have also been performed in a swept field magnet down to 1.5 K at various frequencies (and therefore, fields). From our measurements, we obtained ⁷Li NMR spectra, spin-lattice (1/T₁) and spin-spin (1/T₂) relaxation rates as a function of temperature in various fields.

The first-principles electronic structure calculations in the framework of density functional theory (DFT) are carried out within the generalized gradient approximation (GGA) for the exchange-correlation functional following the Perdew-Burke-Ernzerhof prescription. We have employed the plane-wave basis as implemented within the Vienna Ab initio Simulation Package (VASP) [31, 32] with projector augmented wave potentials [33, 34] as well as in the Nᵗʰ-order muffin-tin orbital (NMO) and linear muffin-tin orbital (LMTO) basis sets as implemented in the STUTTGART code [35]. The consistency between the two sets of calculations in two choices of basis sets is cross-checked in terms of band structure, density of states etc. The VASP calculations are done with usual values of Coulomb correlation U [36] and Hund’s coupling (J_H) chosen for Ir with U_eff (≡ U - J_H) = 1.5 eV in Dudarev scheme [37]. The details of the VASP, LMTO and NMTO calculations are described in the SM [30].

III. THE LOW TEMPERATURE ORDERED STATE OF ALIO

A. μSR

The depolarisation of the muons as a function of temperature is shown in Fig. 2. At high temperatures, a slow (Gaussian-like) decay of the muon polarisation is seen whereas below about 20 K, a faster (exponential-like) decay is discernible which gets even faster with a decrease in temperature. Finally, below about 9 K, clear oscillations in the muon asymmetry as a function of time.
are seen. Fits of the time decay of the muon asymmetry at various temperatures have then been carried out to obtain the variation of the local moment dynamics with temperature.

We find that at temperatures above 15 K, the data are well fit to a product of a static Kubo-Toyabe function with an exponential in addition to a constant background $A_0$, i.e., $A(t) = A_2 G_{KT}(\Delta, t)e^{-\lambda_2(T)t} + A_0$. Here, $G_{KT}(\Delta, t)$ is the Kubo-Toyabe function which models the relaxation of muons in a Gaussian distribution of magnetic fields from nuclear moments. From these fits we obtain the field distribution $\Delta$ to be about 1.6 Oe. This value is typical of nuclear dipolar fields at the muon site, in the present case arising from $^{107,109}$Ag, $^{6,7}$Li, and $^{191,193}$Ir nuclei. The exponential term $e^{-\lambda_2(T)t}$ arises from the relaxation due to fluctuations of the electronic local moments. This relaxation rate $\lambda_2(T)$ is small at high temperatures and gradually increases as the local moment fluctuation rate gets smaller (see Fig. 4). We notice a sharper increase of $\lambda_2(T)$ below about 20 K.

The intermediate region of 13 K to 9 K shows a sharply falling muon asymmetry with $\lambda_2(T)$ showing a sharp increase with decreasing temperature as an approach to long-range order. Going further down in temperature, we find that below 9 K, there are clear oscillations in the muon asymmetry as a function of time. This is a classic signature of the presence of long-range magnetic order. The temporal decay of the muon asymmetry is nearly unchanged from 7 K down to 52 mK. The data in the range 52 mK-7 K were well fit (see Fig. 3) by the following equation:

$$A(t) = A_0 + A_1 e^{-\lambda_1(T)t} J_0(\gamma H_1 t) + A_2 e^{-\lambda_2(T)t} J_0(\gamma H_2 t) + A_3 e^{-\lambda_3(T)t}$$  \hspace{1cm} (1)$$

where $\gamma$ is the muon gyromagnetic ratio ($\gamma = 2\pi \times 135.539$ MHz/Tesla).

The $A_3 e^{-\lambda_3(T)t}$ term is ascribed to muons which are initially parallel to the internal field components and hence do not precess. As for the other significant terms, $J_0(\gamma H_1 t)$ and $J_0(\gamma H_2 t)$ are zeroth order Bessel functions. In case of ordering that is commensurate with the lattice, one expects an exponentially damped sinusoidal variation of $A(t)$ in the ordered state. The Bessel function variation observed here is indicative of magnetic order incommensurate with the lattice [38] (such as for a spin density wave), where the muon experiences fields up to a maximum of $H_1$ or $H_2$, in the present case. We found that the fit was better with two Bessel functions rather

FIG. 2. Variation of the muon asymmetry with time is shown at selected temperatures. Clear oscillations are seen below about 9 K indicative of long-range magnetic order. Fits at some representative temperatures are shown as explained in the text.

FIG. 3. Variation of the muon asymmetry with time is shown at 52 mK together with a fit to Eq. 1. The fit parameters are nearly unchanged up to 7 K.

FIG. 4. Variation of the muon relaxation rate (main figure) and the local field at the muon site (inset) as a function of temperature for ALIO is shown.
than one (see SM [30] for a comparison), suggesting the presence of two types of magnetic environments for the muons. These could arise either from the presence of two kinds of regions with different spin order, or possibly from crystallographically inequivalent muon stopping sites.

From our DFT calculations (see Sec. IV for details), we conclude that while the Néel type order has the lowest energy, the stripy phase is not much higher in energy, which lends credence to the first possibility. Thus assuming this scenario of a single muon stopping site, the site was determined from calculations of the electrostatic energy and was found to be about 1 Å from the oxygen ion (see SM [30] for details). This is similar to that found in cuprates and other oxide materials [39]. We then calculated the dipolar magnetic fields at the muon stopping site in Néel and stripy environments respectively. The calculated field values of 139 Oe for the Néel phase and 266 Oe for the stripy phase (assuming a moment of 0.5 $\mu_B$/Ir which is typical for Ir$^{4+}$) are in reasonable register with the values of 129 Oe and 232 Oe obtained as the averages of fit parameters $H_1$ and $H_2$ respectively between 52 mK to 1.4 K (see inset of Fig. 4).

Finally, we look at the variation of the muon relaxation rates $\lambda_1$ and $\lambda_2$ vs $T$. It shows a peak at about 9 K, but does not fall to low values even at 52 mK as is expected to happen for progressively slower dynamics as we go deeper into the ordered state (Fig. 4). It rather stays almost flat and undiminished at a value of about 5 MHz in the low temperature side as seen in Fig. 4. We speculate that persistent spatio-temporal fluctuations of the stripy and Néel regions are responsible for this. In such a scenario, this would present an interesting example where these fluctuations persist even at temperatures more than two orders of magnitude lower than the transition temperature ($\sim$ 9 K) till at least a thermal energy scale of $\sim$ 50 mK (4.3 $\mu$eV). Could these then be quantum mechanical in origin?

From a quantitative point of view, the rather large value of 5 MHz for the muon depolarisation rate is quite remarkable, comparable to (or even larger than) those seen in a variety of spin liquids (for example, see the review Ref. [27] and the references therein; also see recent Refs. [40, 41] not covered in this review). Even in $\alpha$-RuCl$_3$ which has been established to be magnetically ordered, the muon relaxation rate is appreciable ($\sim$ 0.5 MHz in a single crystal [42] and $\sim$ 2 MHz/$\sim$ 4 MHz in a polycrystalline sample [43]) though smaller compared to ALIO. However, these observations on $\alpha$-RuCl$_3$ are limited respectively to roughly $1/2$ the ordering temperature in the single crystal work [42], and about $1/5^{th}$ of the ordering temperature in the polycrystalline sample study [43]. Whereas our observations on ALIO go down to $1/200^{th}$ of the ordering temperature and can be considered well-representative of the ground state physics. This suggests that the persistent dynamics are really a feature of the magnetically ordered many-body ground state.

![Figure 5](image1.png)

**FIG. 5.** The $^7$Li NMR shift increases with decreasing temperature and then shows a broad plateau below 50 K.

### B. NMR

Having established the presence of magnetic order in ALIO from our zero field $\mu$SR data at low temperatures, we now move over to the local probe technique of NMR to examine the variation of the intrinsic spin susceptibility in the paramagnetic state, as well as to look for complementary evidence of ordering. The bulk susceptibility on the other hand can have a low-$T$ upturn arising from extrinsic contributions or orphan spins which may not be reflective of the intrinsic properties. We have therefore performed $^7$Li local probe NMR measurements to determine the shift of the $^7$Li resonance ($^7$K) with respect to a diamagnetic reference as a function of temperature. The results are shown in Fig. 5 where it is seen that the intrinsic susceptibility (in the form of a $^7$K line shift) increases

![Figure 6](image2.png)

**FIG. 6.** The variation of the $^7$Li NMR spin-lattice relaxation rate is shown with $T$. This also shows a peak at about 50 K where there is a susceptibility anomaly.
with decreasing temperature and then exhibits a broad plateau region below about 50 K.

Furthermore, as NMR is a good probe of low-energy excitations, we have performed $^7$Li NMR spin-lattice relaxation rate measurements as a function of temperature. The recovery of the longitudinal nuclear magnetisation after a saturating pulse sequence was well fitted with a single exponential. As shown in Fig. 6, we find that $1/T_1$ increases with decreasing temperature and has a peak around 50 K similar to systems which show ordering. The $^7$K and $1/T_1$ results are found independent of the applied field between 18-94 kOe.

We now contrast our observations with those reported in a very recent Ref. [44] performed on two different batches of samples (A and B) of ALIO. Wang et al. found that the cleaner sample A showed a single peak in the NMR spectrum as opposed to two peaks for sample B. The longitudinal nuclear magnetisation recovery for sample A was found to be single exponential in contrast to a stretched exponential variation for sample B. Also, the values of the relaxation rate were higher for sample A compared to sample B. The near absence of a second peak in the NMR lineshape of our sample of ALIO, together with a single exponential recovery in $T_1$ and with the absolute value of the $^7$Li NMR relaxation rate $1/T_1$ on the higher side (similar to sample A of Ref. [44]) suggests that our sample is of high quality.

We also monitored the total NMR spectral intensity as a function of temperature which naturally involves the measurement of the spin-spin relaxation rate ($1/T_2$). The total spectral intensity should normally vary as a Curie law due to a Curie variation of the nuclear magnetisation. Hence, the product of the nuclear magnetisation $M_0$ and $T$ should remain constant with temperature in case the same number of nuclei contribute to the signal at all temperatures. Our results for $1/T_2$ and $M_0T$ are shown in Fig. 7. We observe an onset of a decrease in intensity already around 50 K and a near complete wipe-out below 10 K. The observed wipe-out is a classic signature of the onset of long-range magnetic ordering. An anomaly is also seen in the $1/T_2$ data around 10 K. We thus conclude that there is an onset of short-range correlations around 50 K and eventually long-range magnetic ordering sets in at about 10 K.

IV. FIRST PRINCIPLES ELECTRONIC STRUCTURE CALCULATIONS

In order to gain a microscopic understanding of the electronic and magnetic behavior, we first optimised the crystal structure parameters using first-principles density functional theory calculations. The crystal structure of ALIO has been shown in Fig. 1, and we recall that the various exchange parameters, in particular the Kitaev exchange interaction are strongly dependent on the nearest-neighbor ($nn$) bond-length and bond-angles. To cross-check the experimental refinement of the position of the light atoms (such as O) based on X-ray diffraction, we have independently determined the structural parameters by carrying out an ionic relaxation simulation for ALIO while maintaining the crystal symmetries of space group C2/m (see SM [30] for details). A comparison of the structural data for the experimental and the relaxed structures are presented in Table I along with the structural data of $H_3LiIr_2O_6$ (HLIO) and the parent compound $\alpha$-LIO with similar stoichiometries. We find for the relaxed structure of ALIO, the Ag-O bond lengths along the O-Ag-O are nearly equal and substantially larger in comparison to its H counterpart while the Ir-O-Ir angle for the Z-bond are nearly identical for both the systems. All the subsequent calculations are performed with this relaxed structure of ALIO.

We begin with an investigation of the electronic structure of ALIO without magnetic order. The results of our calculations are summarized in the top panel of Fig. 8, where we have plotted the total as well as the Ir projected density of states (DOS). We find that the octahedral environment of Ir splits its $d$ states into $t_{2g}$ and $e_g$ states.

### Table I: Bond lengths are given in Å and bond-angles are in degrees (°).

| Parameters                  | $r$-Li$\text{Ir}_2$O$_6$ (exp) [45] | $r$-Li$\text{Ir}_2$O$_6$ (exp) [18] | $Ag_3$Li$\text{Ir}_2$O$_6$ (exp) [20] | $Ag_3$Li$\text{Ir}_2$O$_6$ (relax) |
|-----------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Ir-Ir distance (X/Y-bond)   | 2.98                              | 3.10                              | 3.06                              | 3.04                              |
| Ir-Ir distance (Z-bond)     | 2.98                              | 3.05                              | 3.04                              | 3.08                              |
| Ir-O-Ir angle (X/Y-bond)    | 94.74                             | 99.77                             | 98.55                             | 97.76                             |
| Ir-O-Ir angle (Z-bond)      | 95.42                             | 99.03                             | 92.43                             | 100.08                            |
| (Li,H,Ag)-O distance        | 1.88, 2.13, 1.23, 1.27             | 1.94, 2.09                        | 2.08, 2.10                        | 2.08, 2.10                        |
by a large crystal field splitting ($\Delta CF \sim 4.3$ eV) characteristic of iridates. The $t_{2g}$ states are further split due to monoclinic distortion and host the Fermi level. The oxygen states are completely occupied, while the Ir-$e_g$, Li-$s$ and Ag-$d$ states are completely empty consistent with the nominal ionic formula $\text{Ag}_3^{1+}\text{Li}^{+}\text{Ir}^{4+}\text{O}_6^{-2}$.

In view of the large $\Delta CF$, the entire physics of these systems is essentially governed by the $t_{2g}$ states. We have therefore constructed a low energy tight-binding model retaining only the $t_{2g}$ states in a local basis (where the local $x$, $y$, and $z$ axes point towards the ligands) and downfolded all other higher degrees of freedom using the NMTO downfolding method [35]. The C2/m group splits the $t_{2g}$ states retaining two-fold symmetry at each metal site. The various hopping interactions between the Ir atoms reveal that the hopping corresponding to the first $nn$ is stronger compared to the other interactions. The C2/m space group provides two types of symmetry inequivalent nearest-neighbor bonds: (i) Z bonds, parallel to the crystallographic b-axis, are of local C2h symmetry; (ii) X or Y bonds with lower symmetry. The nearest neighbor $d-d$ hopping integrals for the Z-bond expected for the ideal structure are extracted from NMTO downfolding calculations by suitable averaging and the values are $t_1 \equiv t_{xx,xx} = t_{yy,yy} = -49.1$ meV, $t_2 \equiv t_{zz,zz} = t_{yz,zy} = 204.0$ meV, $t_3 \equiv t_{xy,xy} = -1.4$ meV, $t_4 \equiv t_{xz,xz} = t_{yz,yz} = t_{xy,yx} = 46.2$ meV. While $t_2$ is strongest as expected, $t_3$ is however strongly suppressed. This will have a profound impact on the exchange interactions to be discussed later in the section.

We now present the effect of spin-orbit coupling (SOC) on the electronic structure of ALIO. The total and the Ir projected DOS are displayed in Fig. 8. We find that the SOC further splits the $t_{2g}$ states into low-lying, four-fold degenerate $J_{\text{eff}} = 3/2$ and high-lying two-fold degenerate $J_{\text{eff}} = 1/2$ states. The five electrons of nominal Ir$^{4+}$ completely fill the $J_{\text{eff}} = 3/2$ states, while the $J_{\text{eff}} = 1/2$ state is half-filled which upon inclusion of a moderate Hubbard interaction $U$ makes the system insulating. In the limit $U \gg t$, the holes occupying the $J_{\text{eff}} = 1/2$ states are nearly localized on the metal sites and the low energy degrees of freedom are pseudo-spin-1/2 variables $S_i$ connected to the $J_{\text{eff}} = 1/2$ states.

This spin-orbit entangled pseudo-spin state of Ir atom on a honeycomb lattice in the strong coupling limit hosts bond dependent anisotropic Kitaev exchanges in addition to the usual isotropic Heisenberg exchange terms. The nearest neighbor spin Hamiltonian may be written as $H_{\text{spin}} = \sum_{\langle i,j \rangle} S_i \cdot J_{ij} \cdot S_j$, where $J_{ij}$ is a $3 \times 3$ symmetric matrix due to the presence of local inversion symmetry and is given by:

$$
\begin{bmatrix}
J_1 & \Gamma_1 & \Gamma'_1 \\
\Gamma_1 & J_1 & \Gamma'_1 \\
\Gamma'_1 & \Gamma'_1 & J_1 + K_1
\end{bmatrix}
$$

(2)

The various parameters of the spin Hamiltonian (Eq. 2) are calculated following Refs. [15, 24] using the hopping integrals obtained from the NMTO downfolding method mentioned earlier and neglecting the crystal field terms. For $U = 1.7$ eV, $J_H = 0.3$ eV and $\lambda = 0.4$ eV suitable for ALIO [15], we obtain for the spin Hamiltonian ($J_1, K_1, \Gamma_1, \Gamma'_1$) in Eq. 2 as ($+3.19$, $-11.4$, $-1.3$, $-2.99$) meV and ($2.86$, $-5.85$, $-6.7$, $-1.54$) meV by using the strong coupling expressions from Ref. [15] and Ref. [24] respectively. The reported values of ($J_1, K_1, \Gamma_1, \Gamma'_1$) from two different studies on similarly stoichiometric HLIO structure that has been suggested to be a Kitaev quantum spin-liquid [17] are ($1.8$ meV, $-12.0$ meV, $-0.2$ meV, $-3.2$ meV) [19] and ($-1.3$ meV, $-15.4$ meV, $+1.5$ meV, $-5.1$ meV) [18] respectively. While the ferromagnetic nature of $K_2$ obtained for ALIO is similar to that of HLIO, the magnitude of $|\frac{K_2}{J_1}|$ is 2-3.5. This is much smaller in comparison to HLIO (8.5-12.5). It is therefore likely that ALIO will order like the parent compound $\alpha$-LIO [7], for which the (two) reported estimates of ($J_1, K_1, \Gamma_1, \Gamma'_1$) are ($-4.6$ meV, $-4.2$ meV, $+11.6$ meV, $-4.3$ meV) and ($-3.1$ meV, $-6.3$ meV, $+9.4$ meV, $-0.1$ meV) [15].

Perturbative analysis [24] estimates that $J \sim \frac{t_{dd}^4}{\lambda^2}$ and $K \sim \frac{t_{dd}^4}{\lambda^2} \frac{J_H}{U^{1/2}}$ with the Heisenberg term predominantly governed by direct exchange, while the Kitaev interaction is mostly due to superexchange processes along the Ir-O-Ir paths. Here, $t_{dd}$ and $t_{pd}$ stand for the hopping amplitudes between $d$ orbitals of neighboring Ir ions and between Ir-$d$ and O-$p$ states respectively. $\Delta_{pd}$ is the charge-transfer energy. Unlike Li-$s$ in honeycomb $\alpha$-LIO,
the Ag-d orbitals form strong covalent bonds with ligand O-p orbitals (see from the crystal orbital Hamiltonian population (COHP) plot in SM [30]) resulting in strong d-p mixing. This modifies the Ir-O-Ir superexchange interaction and enhances the Kitaev term ($K$). In comparison to $\alpha$-LIO ($\Gamma_1 = 11.6$ meV) [15], the increased Ir-O-Ir bond angle in ALIO suppresses $t_3$ and leads to the reduced $\Gamma_1$ similar to HLIO. We have also calculated the second and third neighbor interaction strengths and these are found to be much weaker, ($J_{2nd} < 6\%$ of $J_{nn}$; $J_{3rd} < 1\%$ of $J_{nn}$). Our calculated parameters place ALIO in the vicinity of the striped phase and the Néel (AFM) phase in the phase diagram reported in Ref. [24]. The $120^\circ$ magnetic phase is, as well, in close proximity for these parameters.

To determine the likely ground state of ALIO, we have considered several magnetic configurations whose spin and orbital moments are consistent with the $J_{\text{eff}} = 1/2$ state of Ir, and calculated their energies within GGA+SOC+U scheme (see Fig. 9). Apart from the zigzag AFM order that has been observed in the parent compound $\alpha$-LIO [7], we have also examined three other representative magnetic orders: ferromagnet (FM), Néel AFM, and stripe AFM which have been observed in honeycomb materials for the $q = 0$ magnetic structure [4, 9]. Whereas the Néel AFM order has the lowest energy among the chosen configurations, the stripy phase is not much higher in energy. The $q \neq 0$ solutions and the incommensurate phases are not taken into consideration which may also dictate the ground state magnetism, especially the $120^\circ$ ordered phase.

\section{DISCUSSION ON BULK PROBES AND HIGH-TEMPERATURE ANOMALY}

In this section, we discuss bulk probes — heat capacity (Fig. 10) and magnetic susceptibility (Fig. 11) —, and the high temperature anomaly in the NMR data in light of the low-temperature ordered state established in the previous sections. We start with heat capacity data. As is well-known, the measured heat capacity contains a contribution from the crystal lattice degrees of freedom in addition to magnetic contributions. At higher temperatures, it is the lattice contribution which generally dominates the heat capacity. To determine the lattice part, one can make use of structurally analogous nonmagnetic variants of the given compound if available. In such cases, corrections are necessary in inferring the lattice contribution of the magnetic compound from that of the non-magnetic analogue to account for the differences in their effective Debye temperatures of the two compounds due to the different ionic masses and unit cell volumes. Following such a correction, the heat capacities of the two compounds must coincide in a high-temperature region where the magnetic contribution is negligible. In Ref. [28, 29], the authors have made use of $\text{Ag}_3\text{LiS}_3\text{O}_6$ (ALSO) as the nonmagnetic analog. However, it appears that no correction has been applied before subtracting these data from the measured heat capacity of ALIO. This can be seen in Fig. 3(a) of Ref. [28] and Fig. 2(a) of Ref. [29] where the data for of ALIO and ALSO do not appear to overlap at high-temperatures.

The magnetic specific heat of our sample, using the data for the structurally analogous nonmagnetic $\text{Ag}_3\text{LiTi}_2\text{O}_6$ (ALTO) for the lattice contribution (see SM [30] for details), is given in Fig. 10. Anomalies are

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{Comparison of different magnetic configurations within GGA+SOC+U calculation}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{The magnetic heat capacity ($C_m$) and the entropy change ($\Delta S_m$) are plotted in units of Rln2 as a function of temperature. Representative error bars are given at a few temperatures. Anomalies are observed at $T_H \simeq 50$ K and $T_L \simeq 13$ K.}
\end{figure}
seen in our data at about 13 K and 50 K. Based on our NMR and μSR data described in Sec. III, the lower-temperature anomaly is from long-range order. The higher-temperature 50 K anomaly could be from short-range correlations (recall the broad plateau seen in the 7Li NMR shift with temperature, Fig. 5). However, it should be noted that at high temperatures, the magnetic heat capacity is quite small compared to the lattice contribution, and hence a small error in the lattice heat capacity gives rise to a large error in the magnetic heat capacity. For instance, at about 50 K, the inferred magnetic specific heat is only about 13% of the total specific heat. Therefore, a 5% error in the lattice heat capacity will result in nearly 50% error in magnetic heat capacity at 50 K. Consequently, the magnetic entropy change as well suffers from high uncertainty, especially in the high-temperature region. As a result, the inference of a two-stage entropy release is suspect in our opinion which calls into question an interpretation in terms of localised and itinerant Majorana excitations.

The DC magnetic susceptibility $\chi(T) = M/H$ of Ag$_3$LiIr$_2$O$_6$ is shown in Fig. 11. It varies in a Curie-Weiss manner at high-temperature but has an anomaly at about 50 K. In particular, as shown in the inset of Fig. 11, in a low field of 100 Oe, there is a clear bifurcation between the zero-field-cooled ZFC and field-cooled FC curves a little below 100 K. Such a measurement is currently not available in the literature for comparison.

Given the above bulk probe data, we revisit the high temperature anomaly seen in NMR data which one might want to optimistically interpret as having to do with Kitaev physics. Broad anomalies, such as the one in the 7Li NMR line shift of ALIO at about 50 K (Fig. 5), are expected in low-dimensional quantum magnets with antiferromagnetic couplings at temperatures on the scale of the dominant exchange coupling. In such a case, an anomaly is also expected in the temperature dependence of $1/T_1$. It should be noted that 7Li nuclei are located symmetrically with respect to the Ir moments on the lattice, which will filter out Neél-type antiferromagnetic fluctuations. However, given the evidence from μSR in Sec. III A of the eventual incommensuration below 9 K, as also the possibility of the (incommensurate) stripey phase as a competing phase for which the fluctuations are not cancelled at the 7Li site, one expects now incomplete filtering at the 7Li site. The appearance of a peak at 50 K (Fig. 6) then likely reflects a build-up of short-range dynamic magnetic correlations at the dominant exchange scale as concluded in Sec. III B. This 50 K scale is indeed in consistent with our DFT estimates (Sec. IV) for the various exchange couplings (also, see SM [30]). On the other hand, as discussed in the NMR section (Sec. III B), the absence of a sharp divergence in $1/T_1$ at the ordering temperature ($\sim$ 10 K) is likely due to a wipe-out effect where a large fraction of the 7Li nuclei (nearly 90% as seen from Fig. 7) are already out of the window of observation by 10 K.

We now return to the bifurcation seen in the low-field ZFC/FC susceptibility data at about 100 K suggesting a freezing of magnetic moments (inset of Fig. 11). In higher fields, this anomaly moves to about 50 K. If this arose from intrinsic regions, it would result in a sharp loss of the NMR spectral intensity at this temperature. On the other hand, the observed loss of NMR intensity is rather gradual as the temperature decreases from 100 K (Fig. 7). Further, one should have seen a reflection of the freezing of moments in the μSR relaxation rate $\lambda_2$ which is a zero-field measurement. However, $\lambda_2$ shows a rather gentle variation with temperature with a sharper increase only below 20 K (Fig. 4). This then suggests that the ZFC/FC bifurcation is extrinsic in origin, possibly related to moments localized at stacking faults that naturally occur these systems, and one has to look for a different cause for the decrease in the NMR intensity.

We note that such a decrease of the NMR intensity has been seen in the heavy Fermion superconductor CeCu$_2$Ge$_2$ above the superconducting transition temperature as well as in its non-superconducting variants [25, 26]. In these cases, it was ascribed to dynamical magnetic correlations arising from the itinerant physics of the “heavy-fermion band magnetism”. In the present case, our DFT estimates of the Heisenberg and Kitaev terms tell us that they are comparable ($|J_\text{Heisenberg}|$ is about 2-3.5) and in the range of the energy scale $\sim$ 50 K. As discussed earlier in this section, these competing interactions can be driving the development of short-range dynamical magnetic correlations in ALIO. Whether there is some form of itinerancy — by which, we have in mind some form of a quasi-particle excitation continuum [46] — in the ALIO system driven by the Kitaev terms leading to the above decrease of the NMR intensity is an open question.
VI. CONCLUSIONS

In summary, ALIO is shown to exhibit magnetic long-range order below 9 K. This observation is consistent with our DFT calculations which find comparable Heisenberg and Kitaev exchange couplings. Though 5d Ir$^{4+}$ possesses strong spin-orbit coupling that lie at the origin of substantial bond-dependent anisotropic Kitaev exchange terms, however they are tamed by the Heisenberg exchange term in this system which leads to magnetic long-range order. More specifically, the introduction of the heavy Ag atom in ALIO in place of Li in $\alpha$-Li$_2$IrO$_3$ or H in HLIO strongly affects the local structure and enhances the interlayer Ag-d and O-p hybridization along linear O-Ag-O bonds. The increased bond angle originating due to Ag-O electronic repulsion essentially contributes to the nearest-neighbor Kitaev exchange. The nearest-neighbor Kitaev and Heisenberg exchanges are found to be ferromagnetic and antiferromagnetic respectively. Although a comparison of the energy among various magnetic configurations within the unit cell finds the Néel state to be the lowest in energy, the stripy ordered state is not very far in energy (Fig. 9). Both $J$ and $K$ parameters strongly depend on the local geometry, an interesting aspect that can be explored in future studies by the tuning of bond-length and bond-angles driven by pressure or the application of a magnetic field. This can open up the possibility of suppressing $J$ to negligible values so that the Kitaev term $K$ dominates the physics of the ground state which might then be a Kitaev spin-liquid.

We end with the most unusual finding from our study: the continued presence of a high muon relaxation rate (∼5 MHz) deep into the ordered state as exemplified by Fig. 4. Our observations go down to temperatures as low as 52 mK which is 1/200$^{th}$ of the ordering temperature as highlighted earlier in the introduction and the text surrounding Fig. 4. This points to considerable dynamics of the ordered moments even in the ground state. The $\alpha$-RuCl$_3$ system also shows a significant muon depolarisation rate on the ordered side [42, 43], however as mentioned in Sec. III A, these datasets are limited to about 1/5$^{th}$ of the ordering temperature. A somewhat similar conclusion of persistent dynamics has been drawn [44, 47] based on an analysis of the $^7$Li NMR $1/T_1$ in the ordered state of ALIO. The natural question is what is the source of this dynamics. Our data taken together with our computations of the muon stopping site and their dipolar fields are consistent with the co-existence of Néel and stripe ordered domains. We conjecture that the persistent dynamics are due to spatio-temporal fluctuations of these two kinds of ordered domains that are likely driven by quantum effects in presence of non-negligible Kitaev terms. This is a well-motivated question for theory, i.e. what is the effect of quantum fluctuations coming from the spin-flip Kitaev terms in the ordered regions of the phase diagram [24]. Ultimately, an explanation for the persistent dynamics whether driven by Kitaev terms or not is needed.

VII. ACKNOWLEDGMENTS

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Supplemental Material: Unusual spin dynamics in the low-temperature magnetically ordered state of Ag₃LiIr₂O₆

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I. SAMPLE PREPARATION

The sample preparation of Ag₃LiIr₂O₆ (ALIO) has been done in two steps. The first step was to prepare the precursor α-Li₂IrO₃ by a solid state reaction [1]. After preparing phase pure α-Li₂IrO₃, we mixed AgNO₃ in it (in a 1:3 molar ratio) and ground the mixture. This was followed by heating at 200°C for 6 hrs. In the final step, we washed the product with deionized water several times to remove the excess nitrates and silver. The absence of nitrates was checked with a solution of KCl.

The resulting product was Ag₃LiIr₂O₆ (ALIO) as verified by lab x-ray diffraction measurements using a PANalytical X’Pert PRO diffractometer using Cu-Kα radiation (λ = 1.54182Å). Small amounts of residual Ag and Li₂IrO₃ were detected in the x-ray diffraction pattern. Ag₃LiTi₂O₆ (ALTO) was prepared in a similar manner.

II. RIETVELD REFINEMENT

The X-ray Diffraction data were taken at room temperature on a powder specimen. Table I summarizes the unit cell parameters and quality factors for the Rietveld refinement of Ag₃LiIr₂O₆. A 2D ordering peak, also known as the Warren peak is observed in the range 19° to 24°, shown in inset of Fig.1 with a fit [2] as given below,

\[ I_w(2\theta) = Ae^{-g(2\theta)^2} + B/(C + (2\theta)^2), \]

where A,B and C are constants, \( g \) is the exponent of the Gaussian term and it measures the percentage of the stacking faults known as the \( g \)-factor:

\[ g = \delta^2/d^2, \delta^2 = <d^2> - <d>^2 \]

where \( d \) is the interlayer spacing. A good fit with exponent, \( g = 0.06(6) \) in the inset of Fig.1 corresponds to at least 6% volume fraction of stacking disorder. After excluding the Warren peak, Rietveld refinement is performed with faultless model (stacking faults are ignored) to extract different quality factors, atomic coordinates, site occupancies, and the isotropic Debye-Waller factors (\( B_{iso} = 8\pi^2U_{iso} \)) of Ag₃LiIr₂O₆, which are tabulated in Table I and Table II respectively.

III. MUON SPIN RELAXATION (µSR)

An introduction to the µSR technique can be found in Refs. 3 and 4. As mentioned in the main text, clear oscillations were found in the zero field muon time spectrum below about 9 K. Attempts to fit the data to one or two

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TABLE I. Unit cell parameters and quality factors are reported for the Rietveld refinement of Ag$_3$LiIr$_2$O$_6$ at room temperature.

| Unit Cell Parameters for space group C2/m | Quality Factors |
|------------------------------------------|-----------------|
| $a$(Å) 5.277(7)                           | $R_{Bragg}$ (%) 5.61 |
| $b$(Å) 9.136(8)                           | $R_F$ (%) 3.82   |
| $c$(Å) 6.493(8)                           | $R_{exp}$ (%) 5.8 |
| $\alpha=\gamma$ (°) 90                   | $R_p$ (%) 23.1   |
| $\beta$ (°) 106.01(7)                     | $R_{wp}$ (%) 19.9|
| Z 2                                        | $\chi^2$ 11.7   |
| $V$ (Å$^3$) 300.98(0)                     |                 |

FIG. 1. Rietveld refinement of the xrd pattern of Ag$_3$LiIr$_2$O$_6$ by using the ideal, faultless monoclinic structures (Space group: C2/m). *,* indicates extrinsic peaks from residual $\alpha$-Li$_2$IrO$_3$ and Ag, respectively.

sinusoidal functions modulated by exponentials did not lead to satisfactory fits. We then tried to fit the data using a Bessel function multiplied by an exponential (in addition, another exponential function was used to account for muons which have their magnetic moment parallel to the internal field). Finally, we found that two Bessel functions were essential to obtain a satisfactory fit. A comparison of the fits to single and double Bessel functions is shown in Fig. 2.
TABLE II. Atomic coordinates, Normalized site occupancies, and the isotropic Debye-Waller factors ($B_{iso} = 8\pi^2U_{iso}$) are reported for the Rietveld refinement of $\text{Ag}_3\text{LiIr}_2\text{O}_6$.

| Atom | Wyckoff position | Site | x   | y   | z   | Norm. Site Occ. | $B_{iso}$(Å$^2$) |
|------|------------------|------|-----|-----|-----|-----------------|-----------------|
| Ir(1)| 4g               | 2    | 0   | 0.334 | 0   | 1               | 0.4             |
| Li(1)| 2a               | 2/m | 0   | 0   | 0   | 1               | 0.4             |
| O(1) | 4i               | m   | 0.408 | 0   | 0.222 | 1               | 0.5             |
| O(2) | 8j               | 1   | 0.394 | 0.332 | 0.183 | 1               | 0.5             |
| Ag(1)| 4h               | 2   | 0   | 0.163 | 1/2 | 1               | 0.5             |
| Ag(2)| 2d               | 2/m | 0   | 1/2 | 1/2 | 1               | 0.5             |

FIG. 2. Variation of the muon asymmetry with time is shown at 52 mK together with a fit to both single and double Bessel functions.

IV. HEAT CAPACITY

The heat capacity was measured for ALTO in the temperature range 2 K to 163 K. The data for ALIO and ALTO are shown in Fig. 3. As there is a molar mass difference and a unit cell volume difference between ALIO and ALTO, the lattice contribution will be different for the two systems. A Bouvier scaling procedure [5] is often used to obtain
a correction factor to scale the Debye temperature of the nonmagnetic compound. Correcting the temperature axis of the nonmagnetic compound by this factor then enables one to subtract the heat capacity of the nonmagnetic compound from the magnetic one. In the present case, Bouvier scaling gives the scaling factor \( r = \frac{\theta_{Ir}}{\theta_{Ti}} = 0.78(6) \) where \( \theta_{Ir} \) and \( \theta_{Ti} \) are the Debye temperatures of ALIO and ALTO, respectively. However, this did not lead to a complete cancellation of the heat capacity in the high-\( T \) range (where the magnetic contribution is expected to be zero) when the data of the nonmagnetic compound ALTO (after temperature scaling) was subtracted from the magnetic compound ALIO. Finally, we used \( r = \frac{\theta_{Ir}}{\theta_{Ti}} = 0.88(7) \) to correct the temperature axis of ALTO in Fig. 3 which gives a better cancellation in the high temperature region. These data were subtracted from the ALIO data to obtain the magnetic heat capacity of ALIO.

**FIG. 3.** Heat capacity is plotted as a function of temperature in \( \text{Ag}_3\text{LiIr}_2\text{O}_6 \) (red open circles) and its nonmagnetic analog, \( \text{Ag}_3\text{LiTi}_2\text{O}_6 \) (blue solid line).
V. MAGNETIZATION

We checked our $M(H, T)$ data for data collapse when $T^{\alpha-1}M$ was plotted as a function of $H/T$. As shown in Fig. 4 our data exhibit a data collapse for $\alpha = 0.1$. The corresponding exponent $\alpha$ was found to be 0.17 in Ref. [2].

We also performed neutron diffraction on ALIO at 300 K and 3 K where no additional Bragg peaks were seen at low-$T$. Given the large absorption cross section that Ir has for neutrons together with the incommensurate and two types of order as also persistent fluctuations inferred from our $\mu$SR data analysis, the absence of additional Bragg peaks is not surprising.

VI. COMPUTATIONAL DETAILS

In order to achieve the convergence of energy eigenvalues, the kinetic energy cut off of the plane wave basis of Vienna Ab initio Simulation Package (VASP) code [6, 7] was chosen to be 600 eV. The Brillonin-Zone integrations are performed with $8 \times 8 \times 6$ grid of kpoints. As mentioned in main text the GGA+U [8] calculations are done in Dudarev scheme [9] with $U_{eff} = 1.5$ eV. The symmetry protected ionic relaxation of the crystal structure has been carried out within VASP calculation using the conjugate-gradient algorithm until the Hellman-Feynman forces on each atom were less than the tolerance value of 0.01 eV/Å. In order to ascertain the accuracy of our VASP calculations we have also performed the electronic structure calculation using Stuttgart TB-LMTO-47 code [10] with K-mesh $8 \times 8 \times 6$, based on tight binding linearized muffin-tin orbital (TB-LMTO) method in the atomic sphere approximation (ASA). The
space filling in the ASA is obtained by inserting appropriate empty spheres in the interstitial regions. The hopping parameters as well as onsite energies of the low-energy tight-binding model retaining only the Ir-t_{2g} states in the basis are obtained from the muffin-tin orbital (MTO) based Nth order MTO (NMTO) downfolding method[11].

A. COHP calculation

The primary structural difference of ALIO with its parent is the enhanced interlayer distance due to the insertion of relatively large Ag atoms at the place of Li. In this regard the overlap between Ag-d and O-p increases than Li-s O-p hybridization in α−LIO. To get the quantitative estimation of hybridization we have plotted the an energy resolved visualization of the chemical bonding between the interlayer atoms with O which can be obtained from the crystal orbital Hamiltonian population (COHP) plot as shown in Fig.5. We have analyzed the chemical bonding by computing the crystal orbital Hamiltonian population (COHP) as implemented in the Stuttgart tight-binding linear muffin-tin orbital (TB-LMTO) code [10]. The COHP provides the information about the specific pairs of atoms participating in the bonding, while the integrated COHP (ICOHP) provides the strength of the interactions. The black and red solid (dotted) lines show the calculated COHP (ICOHP) of Ag-O and Li-O covalency for ALIO and α-LIO respectively. At E_F the quantitative value of ICOHP of Ag-O is higher than that for Li-O bond in α−LIO which indicates that interlayer Ag-d is more hybridized with O that connects the superexchange path between nn Ir atoms within a-b plane. The electronic repulsion exerted on ligand O along the Ag-O linear bonds due to extended Ag-d orbitals increases the Ir-O-Ir bond angles which essentially enhances the Kitaev interaction between nn Ir atoms.

B. Crystal Field and Hopping Integrals

In order to find the crystal-field splitting of Ir-d orbitals in the distorted octahedral environment and hopping interactions between various Ir atoms, the NMTO downfolding method [11] is employed. To calculate crystal-field splitting, only Ir-t_{2g} orbitals are retained in the basis and the rest are downfolded. Due to rotation and distortions in the IrO₆ octahedral network t_{2g} levels are primarily contributed by d_{xy}, d_{3z^2−1} and d_{xz} orbitals of Ir⁴⁺ ions. In order to extract the low energy Hamiltonian within local frame of reference pointed along the Ir-O bond lengths (as shown in Fig.1(b) of main text) we have rotated the basis (Euler angles: α = 178.55⁰, β = −70.92⁰, γ = −46.03⁰). The monoclinic distortion of the octahedra completely lifts the degeneracy of the t_{2g} levels with gap of ∼ 0.27 eV and ∼ 0.29 meV w.r.t the lowest energy level.

FIG. 5. COHP (black) and ICOHP (red) is plotted for bonds of Ag/Li atoms with O atoms for ALIO (solid line) and LIO (dotted line).
C. Spin-polarized calculation

As a representative spin polarized calculation for ALIO we have considered here GGA+U calculation with ferromagnetic (FM) arrangement of Ir spins. A plot of the spin-polarized density of states (DOS) for the FM configuration is shown in the left panel of Fig.6. Calculations reveal that in the majority spin channel the Ir-d $t_{2g}$ states are completely filled while as expected the minority spin channel is partially occupied. The magnetic moment per Ir site is calculated to be 0.63$\mu_B$, and the rest of the moment are found to be at the ligand sites (0.10$\mu_B$/O) due to hybridization. Next we have introduced SOC in our calculation (see Fig.6). The spin and the orbital moments in the GGA+U+SOC scheme at the Ir site is calculated to be 0.19$\mu_B$ and 0.41$\mu_B$ respectively. This large value of orbital moment suggests that spin orbit coupling induces the pseudo-spin $J_{eff} = 1/2$ ground state (right panel of see Fig.6) for this system. Further incorporation of U in presence of SOC splits the two fold degenerate $J_{eff} = 1/2$ state creating a gap of value $\sim 66$ meV, which leads to the insulating nature of the system.

![DOS plot](image)

**FIG. 6.** Ferromagnetic total (black line) and partial Ir-d (blue shaded region) DOS plot within GGA+U (left panel) and GGA+SO+U (right panel) calculations. The zoomed in view of the GGA+SO+U DOS is shown in the inset of right panel to show the insulating nature.

D. Details of calculation of the muon stopping site

In order to locate the muon stopping site, we have implanted a muon within the crystallographic unit cell in the scope of our DFT calculation by inserting H$^+$ ion. In oxides muons are generally found to stop approximately 1 Å away from an oxygen atom, similar to hydrogen in a hydroxyl bond [12, 13]. Starting from a grid of interstitial positions near oxygen, the effect of muon embedding is inspected. Hence we consider a sphere around an oxygen of type-1 then for oxygen type-2. The muon implanted unit cell geometry and lattice parameters, for each set of starting guess, are allowed to optimize within VASP calculation using the conjugate-gradient algorithm until the Hellman-Feynman forces on each atom are less than the tolerance value of 0.01 eV/Å. The muon implanted near type-2 oxygen gives lower energies than that near type-1 oxygens. The optimal location of the muon that gives rise to the lowest energy is $r_\mu = (0.0043, 0.0658, 0.1918)$, in fractional co-ordinate and at a distance 0.996 Å from the nearest type-2 oxygen. The presence of the muon distorts the cell geometry. After obtaining the muon stopping site from DFT calculations, the dipole field can be calculated for a given spin order of the magnetic atoms with the formula [14, 15],

$$B^p(r) = \frac{\mu_0}{4\pi} \sum_{i,q} \mu_{eff,i}m^q_i (\frac{3R_i^p R_q^q}{R_i^3} - \delta^{pq}), \tag{3}$$

where $p, q$ run over $x, y, z$ directions, $r_i$ is the position of the $i^{th}$ magnetic ion, within the Lorentz sphere centered at muon site, $r_\mu$. Here $R_i = r_\mu - r_i$ with $R_i = |R_i|$. The effective magnetic moment of $i^{th}$ magnetic Ir ion is mentioned as $\mu_{eff,i}$ and $m^q_i$ is the direction cosine of that moment along direction $q$. Magnetic fields at the muon site are calculated.
with dipole sums performed in real space with a Lorentz sphere radius of 120 Å. The chosen radius produced results at a muon stopping site which converged within 0.2 Oe compared to those using spheres of radii 150 Å and 200 Å. The components of the dipolar field calculated with $\mu_{\text{eff},i} = 0.5 \, \mu_B / \text{Ir}$ (as obtained within the GGA+SOC+U calculations) are $B_x = -126.93$ Oe, $B_y = 9.90$ and $B_z = 55.05$. The magnitude of the obtained field was found to be $|B| \sim 139$ Oe for the Néel phase. The value of effective magnetic moment obtained from our theoretical calculations agrees well with that of experimental mapping except slight variation may be due to presence of finite moments at neighboring ligand sites or choice of $U$ values for Ir atoms in our DFT calculations.

From the DFT calculations, we find Néel and stripy phases are close in energy (energy difference $\sim 7$ meV/f.u.), hence we have calculated the magnitude of the dipolar field for the stripy phase as well with $\mu_{\text{eff},i} = 0.5 \, \mu_B / \text{Ir}$ and obtained a field strength 266 Oe which is close to the experimentally observed second component around 230 Oe.

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