Heavy quark symmetry in $B \to D^{(*)}\ell\bar{\nu}$ spectra

Benjamín Grinstein$^1$ and Zoltan Ligeti$^2$

$^1$Physics Department, University of California at San Diego, La Jolla, CA 92093
$^2$Ernest Orlando Lawrence Berkeley National Laboratory
University of California, Berkeley, CA 94720

Abstract

We calculate heavy quark symmetry breaking in the slopes and curvatures of the $B \to D^{(*)}\ell\bar{\nu}$ spectra at zero recoil, including the order $\alpha_s^2 \beta_0$ corrections. We point out that the theoretical uncertainties in the differences between $B \to D$ and $B \to D^*$ slopes and curvatures are smaller than in the deviations of the slopes and curvatures themselves from their infinite mass limits. We find that the central values of the current experimental results for the difference of the slopes differ from our calculations when QCD sum rules are used to estimate subleading Isgur-Wise functions. A better understanding of the shapes of the $B \to D^{(*)}\ell\bar{\nu}$ spectra may also help to reduce the error of $|V_{cb}|$ extracted from the zero recoil limit of $B \to D^*\ell\bar{\nu}$. We argue that heavy quark symmetry requires that the same fitting procedure be used in the experimental determinations of the shape parameters and $|V_{cb}|$ from the $B \to D\ell\bar{\nu}$ and $B \to D^*\ell\bar{\nu}$ spectra.
I. INTRODUCTION

The determination of $|V_{cb}|$ from exclusive $B \to D^{(*)}\ell\bar{\nu}$ decays is based on the fact that heavy quark symmetry relates the form factors which occur in these decays to the Isgur-Wise function, whose value is known at zero recoil in the infinite mass limit. The symmetry breaking corrections can be organized in a simultaneous expansion in $\alpha_s$ and $\Lambda_{QCD}/m_Q$ ($Q = c, b$). The $B \to D^{(*)}\ell\bar{\nu}$ decay rates are given by

$$\frac{d\Gamma(B \to D^*\ell\bar{\nu})}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} r_*^3 (1 - r_s)^2 \sqrt{w^2 - 1} (w + 1)^2$$

$$\times \left[ 1 + \frac{4w}{1+w} \frac{1 - 2wr_* + r_*^2}{(1-r_s)^2} \right] |V_{cb}|^2 F_s^2(w),$$

$$\frac{d\Gamma(B \to D\ell\bar{\nu})}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} r^3 (1 + r)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 F^2(w),$$

where $w = v \cdot v' = (m_B^2 + m_D^{(*)^2} - q^2)/(2m_B m_D^{(*)})$ and $r_s = m_{D^{(*)}}/m_B$. Both $F(w)$ and $F_s(w)$ are equal to the Isgur-Wise function in the $m_Q \to \infty$ limit, and in particular $F(1) = F_s(1) = 1$, allowing for a model independent determination of $|V_{cb}|$.

The main theoretical uncertainties in such a determination of $|V_{cb}|$ come from the value of $F_s(1)$, and from the shape of $F_s(w)$ used to fit the data. Such a fit will continue to be important, since the number of $B \to D^*\ell\bar{\nu}$ events needed to measure $|V_{cb}|F_s(1)$ with a statistical error of order $(\Lambda_{QCD}/m_Q)^2$ scales parametrically as $(m_Q/\Lambda_{QCD})^7$. This can be seen from Eq. (1), making no assumption on the shape of $F_s(w)$, by considering a bin at zero recoil with width of order of the desired accuracy, that is, of order $(\Lambda_{QCD}/m_Q)^2$. Similarly, when unquenched lattice QCD calculations of $F_s(1)$ will be available with error $a$, the number of events needed to measure $|V_{cb}|F_s(1)$ with a comparable statistical error, without assumptions about the shapes of $F_s(w)$, will scale as $a^{-7/2}$ in $B \to D^*\ell\bar{\nu}$ and as $a^{-9/2}$ in $B \to D\ell\bar{\nu}$. Reliable unquenched lattice QCD results for $F_s(1)$ are likely to be available before comparable results for the functional form of $F_s(w)$. Constraining the shapes of $F_s(w)$ will remain important in the foreseeable future.

The zero recoil limit of $F_s(w)$, including symmetry breaking corrections, can be written schematically as

$$F_s(1) = 1 + c_A(\alpha_s) + \frac{0}{m_Q} + \frac{\ldots}{m_Q^2} + \ldots,$$

$$F(1) = 1 + c_V(\alpha_s) + \frac{\ldots}{m_Q} + \frac{\ldots}{m_Q^2} + \ldots$$

(2)
The perturbative corrections, $c_A = -0.04$ and $c_V = 0.02$, have been computed to order $\alpha_s^2$. The order $\Lambda_{\text{QCD}}/m_Q$ correction to $\mathcal{F}_s(1)$ vanishes due to Luke’s theorem. The terms indicated by (…) in Eqs. (2) are only known using phenomenological models or quenched lattice QCD at present. This is why the determination of $|V_{ub}|$ from $B \to D^* \ell \bar{\nu}$ is theoretically more reliable for now than that from $B \to D\ell \bar{\nu}$, although both QCD sum rules and quenched lattice QCD suggest that the order $\Lambda_{\text{QCD}}/m_Q$ correction to $\mathcal{F}(1)$ is small. Due to the extra $w^2 - 1$ helicity suppression near zero recoil, $B \to D\ell \bar{\nu}$ is also more difficult experimentally.

Some of the order $\Lambda_{\text{QCD}}/m_Q$ corrections which enter $\mathcal{F}_{\ast\ast}(w)$ also influence ratios of form factors measurable in $B \to D^* \ell \bar{\nu}$ decay. The exclusive semileptonic $B \to D^* \ell \bar{\nu}$ decay rate is parameterized by four form factors,

$$\langle D^*(v',\epsilon) | \bar{c} \gamma^\mu b | B(v) \rangle = \frac{i h_V \varepsilon^{\mu\nu\beta\gamma} \epsilon_A^* \epsilon_B^*}{\sqrt{m_D^* m_B}},$$

$$\langle D^*(v',\epsilon) | \bar{c} \gamma^\mu \gamma_5 b | B(v) \rangle = h_{A_1}(w + 1) \epsilon^{*\mu} - (h_{A_2} \epsilon^{*\mu} + h_{A_3} \epsilon^{*\mu}) (\epsilon^* \cdot v). \quad (3)$$

One linear combination is not measurable when the lepton masses are neglected. The form factor $h_{A_1}$ dominates the rate near zero recoil. It is conventional to define two measurable ratios of the form factors

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)}, \quad R_2(w) = \frac{h_{A_3}(w) + (m_D^* / m_B) h_{A_2}(w)}{h_{A_1}(w)}. \quad (4)$$

In the infinite mass limit $R_1(w) = R_2(w) = 1$, and deviations from this limit measure certain combinations of the subleading Isgur-Wise functions as it will be discussed it later.

Since there are several form factors in $B \to D^* \ell \bar{\nu}$, it is customary to fit the shape parameters of $h_{A_1}$, together with the form factor ratios $R_{1,2}$. For the shapes of $\mathcal{F}(w)$, $\mathcal{F}_s(w)$, and $h_{A_1}(w)$, analyticity imposes stringent constraints between the slopes and curvatures at zero recoil. It is convenient to write

$$\mathcal{F}(w) = \mathcal{F}(1) \left[ 1 - \rho_\mathcal{F}^2 (w - 1) + c_\mathcal{F} (w - 1)^2 + \cdots \right],$$

$$\mathcal{F}_s(w) = \mathcal{F}_s(1) \left[ 1 - \rho_\mathcal{F}_s^2 (w - 1) + c_\mathcal{F}_s (w - 1)^2 + \cdots \right],$$

$$h_{A_1}(w) = h_{A_1}(1) \left[ 1 - \rho_{A_1}^2 (w - 1) + c_{A_1} (w - 1)^2 + \cdots \right]. \quad (5)$$

A lower index $X$ will denote any of $\mathcal{F}$, $\mathcal{F}_s$ or $A_1$. In the $m_Q \to \infty$ limit, heavy quark symmetry predicts $\rho_X = \rho_0$ and $c_X = c_0$, the slope and curvature of the Isgur-Wise function,
respectively. These equalities are violated by corrections at order $\alpha_s$ and $\Lambda_{QCD}/m_Q$. In a linear fit to the data the expansions in (5) are terminated at linear order in $w - 1$, while in a quadratic (or “free curvature”) fit the expansions are terminated at order $(w - 1)^2$. Higher order terms are small but non-negligible, since the fitted range extends to $w = 1.59$ (1.50) in $B \to D^{(*)}$ decay. Unitarity constraints give a strong correlation between $\rho^2_{\mathcal{F}_*}$ and the coefficients of the higher order terms in $w - 1$ [7, 8], and also constrain the effect of the higher than quadratic terms in $w - 1$ [8]. For these reasons, an unconstrained quadratic fit to the data is not equivalent to a unitarity constrained fit.

In the next section we compute the violations to the symmetry relations between the slope and curvature parameters and find them to be small. Moreover, we observe that in the present data the slope parameters $\rho^2_{\mathcal{F}_*}$ and $\rho^2_{\mathcal{F}}$ differ significantly if the spectra are fit linearly or quadratically; see Table I and Fig. 1. However, when present data is fit to the functional form constrained by unitarity, the result is consistent with the symmetry relation $\rho^2_{\mathcal{F}_*} = \rho^2_{\mathcal{F}}$. Therefore, if in the future the data supports the necessity of a unitarity constrained form for $\mathcal{F}_*(w)$, then it will be equally necessary to use a unitarity constrained fit for $\mathcal{F}(w)$.

Furthermore, to the extent that we can reliably estimate the deviations from $\rho^2_{\mathcal{F}_*} = \rho^2_{\mathcal{F}}$,
TABLE I: The most recent available CLEO and BELLE measurements of the slope parameters, \( \rho^2_X \), in \( B \to D^{(*)} \ell \bar{\nu} \) decay. The difference between fitting to the unitarity constrained shapes of one or the other of Refs. \[7\] or \[8\] is very small.

| Fitted slope parameter | CLEO                  | BELLE                  |
|------------------------|-----------------------|------------------------|
| \( B \to D^* \ell \bar{\nu} \), unitarity constrained fit to \( \rho^2_{A_1} \) | \( 1.67 \pm 0.11 \pm 0.22 \) \([10]\) | \( 1.35 \pm 0.17 \pm 0.19 \) \([11]\) |
| \( B \to D^* \ell \bar{\nu} \), linear fit to \( \rho^2_{F_*} \) | \( 0.98 \pm 0.09 \pm 0.07 \) \([12]\) | \( 0.89 \pm 0.09 \pm 0.05 \) \([13]\) |
| \( B \to D \ell \bar{\nu} \), unitarity constrained fit to \( \rho^2_F \) | \( 1.30 \pm 0.27 \pm 0.14 \) \([14]\) | \( 1.16 \pm 0.25 \pm 0.15 \) \([15]\) |
| \( B \to D \ell \bar{\nu} \), linear fit to \( \rho^2_F \) | \( 0.76 \pm 0.16 \pm 0.08 \) \([14]\) | \( 0.69 \pm 0.14 \pm 0.09 \) \([15]\) |

The accuracy in the extraction of \( |V_{cb}| \) can be improved by a simultaneous fit to \( B \to D^* \) and \( B \to D \). While the value of \( |V_{cb}| \mathcal{F}(1) \) has larger uncertainty than \( |V_{cb}| \mathcal{F}_s(1) \), since \( B \to D \ell \bar{\nu} \) is helicity suppressed near zero recoil, the error on \( \rho^2_{\mathcal{F}_s(1)} \) is comparable as extracted from \( B \to D^* \) and \( B \to D \). Since there is a very strong correlation between \( \rho^2_{\mathcal{F}_s(1)} \) and \( |V_{cb}| \mathcal{F}_s(1) \), despite the fact that \( \mathcal{F}(1) \) is less well-known than \( \mathcal{F}_s(1) \), a better determined slope may have significant implications for the value of \( |V_{cb}| \) as it is extracted from the zero recoil limit of \( B \to D^* \ell \bar{\nu} \). When precise unquenched lattice calculations of \( \mathcal{F}_s(w) \) become available, consistency between the predictions and data for \( \rho^2_X \) may be an important cross-check of the \( |V_{cb}| \) determination.

II. ANALYTIC RESULTS

We next give the theoretical predictions for the heavy quark symmetry violation in \( \rho^2_X \), \( c_X \), and the form factor ratios \( R_{1,2} \) at order \( \Lambda_{QCD}/m_Q, \alpha_s \) and \( \alpha_s^2 \beta_0 \). The first two corrections are known in the literature, but the order \( \alpha_s^2 \beta_0 \) terms, which probably dominate the order \( \alpha_s^2 \) corrections (since \( \beta_0 = 11 - 2n_f/3 \) is large) are new. These are required to predict heavy quark symmetry breaking in the quantities under consideration at the \( \sim 5\% \) level, and may become important in testing lattice results. We can write the slope parameters \( \rho^2_X \) as

\[
\rho^2_X = \rho^2_0 + \delta^{(a)}_X + \frac{\bar{\Lambda}}{2m_c} \delta^{(1/m)}_X ,
\]

where \( X = \mathcal{F}, \mathcal{F}_s, A_1 \) denotes the functions under consideration. The perturbative corrections can be calculated model independently and are contained in \( \delta^{(a)}_X \), while \( \delta^{(1/m)}_X \) contains
the order \( \Lambda_{\text{QCD}}/m_Q \) corrections to \( \rho_X^2 \). The perturbative corrections can be computed including the order \( \alpha_s^2 \beta_0 \) corrections from expanding Eqs. (39) in Ref. \[17\]. We obtain

\[
\delta_{F}^{(\alpha_s)} - \delta_{F_*}^{(\alpha_s)} = \frac{\tilde{\alpha}_s}{\pi} \left[ \frac{8(1 - 3z + z^2)}{9(1 - z)^2} - \frac{4z(1 + z)}{9(1 - z)^3} \ln z \right] + \frac{\alpha_s^2}{\pi^2} \beta_0 \left[ \frac{27(1 - z)^2}{54(1 - z)^3} - \frac{(1 + z)(12 - 23z + 12z^2)}{108(1 - z)^3} \ln z \right],
\]

\[
\delta_{A_i}^{(\alpha_s)} - \delta_{F_*}^{(\alpha_s)} = \frac{\tilde{\alpha}_s}{\pi} \left[ \frac{4(1 - z + z^2)}{9(1 - z)^2} + \frac{2z(1 + z)}{9(1 - z)^3} \ln z \right] + \frac{\alpha_s^2}{\pi^2} \beta_0 \left[ \frac{54(1 - z)^2}{54(1 - z)^3} - \frac{(1 + z)(12 - 37z + 12z^2)}{108(1 - z)^3} \ln z \right],
\]

where \( z = m_c/m_b \), and \( \tilde{\alpha}_s \) denotes the strong coupling renormalized in the \( \overline{\text{MS}} \) scheme at the scale \( \mu = \sqrt{m_b m_c} \). The terms of order \( \alpha_s \) agree with Ref. \[17\]. Using \( z = 0.29, \tilde{\alpha}_s = 0.26, \) and \( \beta_0 = 25/3 \), we obtain

\[
\delta_{F}^{(\alpha_s)} - \delta_{F_*}^{(\alpha_s)} = 0.079 + 0.046, \quad \delta_{A_i}^{(\alpha_s)} - \delta_{F_*}^{(\alpha_s)} = 0.034 + 0.018,
\]

where the first and second terms come from the order \( \tilde{\alpha}_s \) and \( \tilde{\alpha}_s^2 \beta_0 \) corrections, respectively. The apparent bad convergence of the perturbation series in Eq. \[8\] may be due to the fact that they contain so-called renormalon ambiguities, and are only well-defined physical quantities when the nonperturbative corrections discussed next [see Eq. \[9\]] are included. This assertion is supported by the fact that we will find significantly better behavior when these series are expressed in terms of a short distance mass in Sec. III.

The corrections in \( \delta_{X}^{(1/m)} \) depend on the four subleading Isgur-Wise functions that parameterize first order deviations from the infinite mass limit. Using the notation of \[6\], one finds from Ref. \[17\]

\[
\delta_{F}^{(1/m)} - \delta_{F_*}^{(1/m)} = \frac{5(1 + z)}{6} + \frac{16}{3} \chi_2(1) - 16 \chi_3'(1) + \frac{1 - 2z + 5z^2}{3(1 - z)} \eta(1) + \frac{2(1 - z)^2}{1 + z} \eta'(1),
\]

\[
\delta_{A_1}^{(1/m)} - \delta_{F_*}^{(1/m)} = \frac{1 + z}{3} + \frac{4}{3} \chi_2(1) + \frac{1 + z + 2z^2}{3(1 - z)} \eta(1),
\]

where prime denotes \( d/dw \). \( \eta(w) \equiv \xi_3(w)/\xi(w) \) parameterizes the \( \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}) \) corrections to the \( b \to c \) current, and \( \chi_{2,3}(w) \) describe the matrix elements of the time ordered product of the chromomagnetic operator, \( (g_5/2) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v \), with the leading order current. An important point is that the poorly known function \( \chi_1(w) \) which parameterizes matrix elements involving the time ordered product of the kinetic energy operator, \( \bar{h}_v (iD)^2 h_v \), with the leading order current drops out from the differences in Eq. \[9\]. In general, \( \chi_1 \) does not
effect any quantity determined by heavy quark symmetry at leading order, such as the slope and curvature differences in Eqs. (9) and (13), and $R_{1,2}$ in Eqs. (14) and (15). The reason is that $\chi_1$ enters all form factors in the combination $\xi(w) + (\bar{\Lambda}/m_c + \bar{\Lambda}/m_b) \chi_1(w)$, where $\xi(w)$ is the Isgur-Wise function.

It is straightforward to compute the heavy quark symmetry breaking corrections between $c_F$, $c_{F^*}$, and $c_{A_1}$. Similar to Eq. (6), we define

$$c_X = c_0 + \frac{\bar{\Lambda}}{2m_c} \Delta_X^{(1/m)}.$$  (10)

For the differences of $\Delta^{(\alpha_s)}$ we obtain

$$\Delta_X^{(\alpha_s)} - \Delta_{F^*}^{(\alpha_s)} = \frac{2\bar{\alpha}_s}{135 \pi} \left[ \frac{47 - 148z + 282z^2 - 148z^3 + 47z^4}{(1 - z)^4} + \frac{5z(1 + z)(1 + 6z + z^2)}{(1 - z)^5} \ln z \right]$$

$$+ \frac{\bar{\alpha}_s^2}{405 \pi^2 \beta_0} \left[ \frac{509 - 1241z + 1084z^2 - 1241z^3 + 509z^4}{5(1 - z)^4} - \frac{(1 + z)(282 - 1277z + 2142z^2 - 1277z^3 + 282z^4)}{4(1 - z)^5} \ln z \right]$$

$$+ \rho_0^2 \left[ \delta_X^{(\alpha_s)} - \delta_{F^*}^{(\alpha_s)} \right],$$

$$\Delta_{A_1}^{(\alpha_s)} - \Delta_{F^*}^{(\alpha_s)} = \frac{2\bar{\alpha}_s}{27 \pi} \left[ \frac{7 - 23z + 12z^2 - 23z^3 + 7z^4}{(1 - z)^4} + \frac{z(1 + z)(1 - 12z + z^2)}{(1 - z)^5} \ln z \right]$$

$$+ \frac{\bar{\alpha}_s^2}{324 \pi^2 \beta_0} \left[ \frac{44 - 83z - 134z^2 - 83z^3 + 44z^4}{(1 - z)^4} - \frac{(1 + z)(42 - 187z + 396z^2 - 187z^3 + 42z^4)}{4(1 - z)^5} \ln z \right]$$

$$+ \rho_0^2 \left[ \delta_{A_1}^{(\alpha_s)} - \delta_{F^*}^{(\alpha_s)} \right].$$  (11)

The order $\alpha_s$ piece in the first of these equations agrees with Ref. [18]. To this order, $\rho_0^2$ can be taken as any of $\rho_F^2$, $\rho_{F^*}^2$, or $\rho_{A_1}^2$. With the previously used values of $z$ and $\bar{\alpha}_s$, we obtain

$$\Delta_X^{(\alpha_s)} - \Delta_{F^*}^{(\alpha_s)} = (0.074 + 0.043) + \rho_0^2 (0.079 + 0.046),$$

$$\Delta_{A_1}^{(\alpha_s)} - \Delta_{F^*}^{(\alpha_s)} = (0.058 + 0.031) + \rho_0^2 (0.034 + 0.018),$$  (12)

where the first and second terms in each parenthesis come from the order $\bar{\alpha}_s$ and $\bar{\alpha}_s^2 \beta_0$ corrections, respectively. The convergence of these series is again quite poor. The order
\( \Lambda_{\text{QCD}}/m_{c,b} \) heavy quark symmetry breaking in the curvature differences is given by
\[
\Delta_{F}^{(1/m)} - \Delta_{F^*}^{(1/m)} = \frac{(1 + z)(25 - 42z + 25z^2)}{36(1 - z)^2} + 4\chi_2(1) \frac{1 - 6z + z^2}{9(1 - z)^2} - \frac{16}{3} \chi_2'(1) + 8\chi_3''(1)
\]
\[
+ \eta(1) \frac{5 - 28z + 18z^2 - 52z^3 + 25z^4}{18(1 - z)^3} - \eta'(1) \frac{1 - 2z + 5z^2}{3(1 - z)} - \eta''(1) \frac{(1 - z)^2}{(1 + z)}
\]
\[
+ \rho_0^2 \left[ \delta_{F}^{(1/m)} - \delta_{F^*}^{(1/m)} - \frac{16}{3} \chi_2(1) + 16\chi_3(1) \right]
\]
\[
\Delta_{A_1}^{(1/m)} - \Delta_{F^*}^{(1/m)} = \frac{2(1 + z)(2 - 3z + 2z^2)}{9(1 - z)^2} + 4\chi_2(1) \frac{1 - 6z + z^2}{9(1 - z)^2} - \frac{4}{3} \chi_2'(1)
\]
\[
+ \eta(1) \frac{5 - 19z - 9z^2 - 25z^3 + 16z^4}{18(1 - z)^3} - \eta'(1) \frac{1 + z + 2z^2}{3(1 - z)}
\]
\[
+ \rho_0^2 \left[ \delta_{A_1}^{(1/m)} - \delta_{F^*}^{(1/m)} - \frac{4}{3} \chi_2(1) \right].
\] (13)

Note that the coefficients of \( \rho_0^2 \) are independent of \( \chi_i \). While not all parameters entering these formulae are known, it may be possible to get information on \( \chi_5(1) \) and \( \eta''(1) \) from precise measurements of \( R_2'(1) \) and \( R_1'(1) \), respectively, or from lattice QCD calculations.

Independent of model calculations, experimental data on the \( B \to D^* \ell \bar{\nu} \) form factor ratios defined in Eq. (4) will constrain some of the subleading Isgur-Wise functions entering Eqs. (9) and (13). Measurements of \( R_{1,2} \) can be used to constrain the quantities \( \eta(1) \), \( \eta'(1) \), and \( \chi_2(1) \) according to
\[
R_1(1) = 1 + \frac{4\alpha_s}{3\pi} + \frac{\alpha_s^2}{\pi^2} \beta_0 \left[ \frac{2}{9} - \frac{1 + z}{3(1 - z)} \ln z \right] + \frac{\Lambda}{2m_c} \left[ 1 + z - 2z \eta(1) \right] + \ldots,
\]
\[
R_1'(1) = -\frac{4\alpha_s}{9\pi} - \frac{\alpha_s^2}{\pi^2} \beta_0 \left[ \frac{2(1 + z^2)}{9(1 - z)^2} - \frac{(1 + z)(1 - 4z + z^2)}{9(1 - z)^3} \ln z \right]
\]
\[
- \frac{\Lambda}{2m_c} \left[ \frac{1 + z}{2} - z \eta(1) + 2z \eta'(1) \right] + \ldots. \] (14)

Yet again, one encounters badly behaved perturbation series, 0.110 + 0.055 for \( R_1(1) \) and
\(-0.037 - 0.025 \) for \( R_1'(1) \). For \( R_2 \) we obtain
\[
R_2(1) = 1 - \frac{2\alpha_s}{3\pi} \left[ \frac{2z}{1 - z} + \frac{z(1 + z)}{(1 - z)^2} \ln z \right] - \frac{13\alpha_s^2}{18\pi^2} \beta_0 \left[ \frac{z}{1 - z} + \frac{z(1 + z)}{2(1 - z)^2} \ln z \right]
\]
\[
- \frac{\Lambda}{2m_c} \left[ (1 + 3z) \eta(1) + 4(1 - z) \chi_2(1) \right] + \ldots,
\]
\[
R_2'(1) = -\frac{2\alpha_s}{3\pi} \left[ \frac{z(1 + 10z + z^2)}{3(1 - z)^3} + \frac{2z^2(1 + z)}{(1 - z)^4} \ln z \right]
\]
\[
+ \frac{\alpha_s^2}{18\pi^2} \beta_0 \left[ \frac{z(1 - 38z + z^2)}{2(1 - z)^3} + \frac{z(1 + z)(1 - 11z + z^2)}{(1 - z)^4} \ln z \right]
\]
\[
+ \frac{\Lambda}{2m_c} \left[ (1 + 3z) \left( \frac{\eta(1)}{2} - \eta'(1) \right) - 4(1 - z) \left( \chi_2'(1) + \chi_2(1) \rho_0^2 \right) \right] + \ldots. \] (15)
Note that the $O(\alpha_s)$ corrections to $R_2(1)$ and $R'_2(1)$ are very small, around $+0.0056$ and $-0.0011$, respectively, and the $O(\alpha_s^2\beta_0)$ corrections are even smaller, $+0.0021$ and $-0.0006$, respectively. This is not unexpected, since all the nonperturbative corrections to $R_2$ involve the subleading Isgur-Wise functions but not $\bar{A}$ by itself, so the perturbation series does not involve a leading renormalon.

III. DISCUSSION AND CONCLUSIONS

To evaluate the results of the previous section, they must be expressed in terms of short distance quark masses. We use the upsilon expansion [19], and express $m_c$ through $m_b - m_c = \bar{m}_B - \bar{m}_D + \lambda_1/2\bar{m}_B - \lambda_1/2\bar{m}_D$, where $\bar{m}_B = (m_B + 3m_{B^*})/4 = 5.313$ GeV and $\bar{m}_D = (m_D + 3m_{D^*})/4 = 1.973$ GeV. It is convenient to re-express $\bar{\alpha}_s$ in terms of $\alpha_s(m_b)$.

For the $1S b$ quark mass we use $m_{b^{1S}} = 4.75 \pm 0.07$ GeV, which follows from Ref. [20] using the CLEO measurement of $\langle E_\gamma \rangle = 2.346 \pm 0.034$ GeV in $B \to X_s\gamma$ corresponding to a photon energy cut $E_\gamma > 2$ GeV [21]. The uncertainty in $m_{b^{1S}}$, and the errors related to it which we quote below, will almost certainly be significantly reduced in the near future. In the upsilon expansion

$$\bar{A} = m_B - m_{b^{1S}} - 0.051 \epsilon - 0.091 \epsilon^2_{BLM} + \ldots,$$

where we used $\alpha_s(m_b) = 0.22$, $\epsilon \equiv 1$ is the parameter of the upsilon expansion, and $\epsilon^2_{BLM}$ denotes the part of the second order correction proportional to $\beta_0$. Since we include the $\alpha_s^2/\beta_0$ terms in all results, the residual scale dependence is very small.

The right-hand sides of Eqs. (9) and (13) can only be estimated at present using model predictions. The subleading Isgur-Wise functions have been computed including order $\alpha_s$ corrections in QCD sum rules, yielding [4]

$$\chi_2(1) \simeq -0.04, \quad \chi'_2(1) \simeq 0.02, \quad \eta(1) \simeq 0.6, \quad \eta'(1) \simeq 0.$$

Using the upsilon expansion to eliminate the quark masses, and these values for the numerical estimates, we obtain

$$\rho_F^2 - \rho_{F*}^2 = 0.203 + 0.053 \epsilon - 0.013 \epsilon^2_{BLM} + 0.075 \eta(1) + 0.14 \eta'(1)$$

$$+ 1.0 \chi_2(1) - 3.0 \chi'_2(1) - 0.018 \lambda_1/{\text{GeV}}^2 \simeq 0.19,$$

$$\rho_{A_1}^2 - \rho_{F*}^2 = 0.081 + 0.024 \epsilon - 0.006 \epsilon^2_{BLM} + 0.131 \eta(1) + 0.25 \chi_2(1) - 0.007 \lambda_1/{\text{GeV}}^2 \simeq 0.17,$$

where

$$\rho_F = (\rho_{F*} + \rho_F)/2$$

and

$$\rho_{F*} = (\rho_{F*} - \rho_F)/2.$$
We also used $\lambda_1 = -0.25 \text{GeV}^2$, although the results are hardly sensitive to the value of this parameter. The behavior of these perturbation series are clearly much better than those in Eq. (8) in terms of the quark pole masses. The uncertainty due to a $\pm 70 \text{MeV}$ change in $m_b^{1S}$ is $\mp 0.020$ and $\mp 0.022$ in these estimates of $\rho_F^2 - \rho_{F*}$ and $\rho_{A1}^2 - \rho_{F*}^2$, respectively.

Although these estimates are model dependent, they are less so than one might at first think, since the first terms on the right-hand sides of Eqs. (9), which are model independent, contribute a large part of the result. These results mostly depend on the value of $\eta(1)$ and on the smallness of the functions $\chi_{2,3}(w)$, which parameterize order $\Lambda_{\text{QCD}}/m_Q$ corrections due to the chromomagnetic operator. If $\chi_{2}(1)$ and $\chi_{3}(1)$ were order unity then these results could be dramatically different. However, $\chi_{2,3}(w)$ are expected to be small in most models (recall that $\chi_{3}(1) = 0$ due to Luke’s theorem [2]), and as we will see below, $\eta(1)$ can be constrained from $R_{1,2}(1)$.

For the curvature differences we obtain

$$
c_{F} - c_{F*} = 0.202 + 0.050 \epsilon - 0.012 \epsilon_{\text{BLM}}^2 - 0.087 \eta(1) - 0.08 \eta'(1) - 0.07 \eta''(1)
- 0.12 \chi_{2}(1) - 1.0 \chi_{2}'(1) + 1.5 \chi_{3}'(1) - 0.011 \lambda_1/\text{GeV}^2
+ \rho_0^2 [\rho_F^2 - \rho_{F*}^2 - 1.0 \chi_{2}(1) + 3.0 \chi_{3}(1)] \simeq 0.17 + 0.29 \rho_0^2,
$$

$$
c_{A1} - c_{F*} = 0.141 + 0.042 \epsilon - 0.007 \epsilon_{\text{BLM}}^2 - 0.059 \eta(1) - 0.13 \eta'(1)
- 0.12 \chi_{2}(1) - 0.25 \chi_{2}'(1) - 0.005 \lambda_1/\text{GeV}^2
+ \rho_0^2 [\rho_{A1}^2 - \rho_{F*}^2 - 0.25 \chi_{2}(1)] \simeq 0.14 + 0.18 \rho_0^2,
$$

(19)

where for the numerical estimates we also used $\chi_{2}'(1) \simeq 0.03$ [4], and $\chi_{3}'(1) = \eta''(1) = 0$.

The uncertainty due to a $\pm 70 \text{MeV}$ change in $m_b^{1S}$ is $\mp 0.021$ and $\mp 0.018$ in the 0.17 and 0.14 terms in these estimates of $c_F - c_{F*}$ and $c_{A1} - c_{F*}$, respectively. While there is a sizable uncertainty again due to the subleading Isgur-Wise functions, for their particular values predicted by QCD sum rules, the final result is dominated by terms which are model independent.

For the form factor ratios $R_1$ and $R_2$ we obtain

$$
R_1(1) = 1.243 + 0.079 \epsilon - 0.016 \epsilon_{\text{BLM}}^2 - 0.112 \eta(1) - 0.021 \lambda_1/\text{GeV}^2 \simeq 1.25,
$$

$$
R_1'(1) = -0.122 - 0.021 \epsilon + 0.010 \epsilon_{\text{BLM}}^2 + 0.056 \eta(1) - 0.112 \eta'(1) + 0.011 \lambda_1/\text{GeV}^2 \simeq -0.10,
$$

$$
R_2(1) = 1 + 0.006 \epsilon + 0.001 \epsilon_{\text{BLM}}^2 - 0.355 \eta(1) - 0.53 \chi_{2}(1) \simeq 0.81,
$$

$$
R_2'(1) = -0.001 \epsilon + 0.178 \eta(1) - 0.355 \eta'(1) - 0.53 \chi_{2}'(1) - 0.53 \chi_{2}(1) \rho_0^2 \simeq 0.09 + 0.02 \rho_0^2.
$$

(20)
The uncertainty due to a ±70 MeV change in \( m_{b}^{1S} \) is ±0.03, ±0.01, ±0.03, and ±0.01 in these estimates of \( R_{1}(1) \), \( R'_{1}(1) \), \( R_{2}(1) \), and \( R'_{2}(1) \), respectively. Unfortunately the sensitivity to the values of the subleading Isgur-Wise functions is not too large, since they enter with small coefficients. Still, useful constraints can be obtained, e.g., \( R_{2}(1) \) measures \( \eta(1) \) assuming that \( \chi_{2}(1) \) is small (or a linear combination of them otherwise), \( \eta'(1) \) can be measured using

\[
R_{1}(1) + 2R'_{1}(1) = 1 + 0.037 \epsilon + 0.004 \epsilon^{2}_{BLM} - 0.223 \eta'(1),
\]

and then \( R'_{2}(1) \) can be used to constrain a linear combination of \( \chi_{2}(1) \) and \( \chi'_{2}(1) \). The coefficient of \( \eta'(1) \) changes by ±0.032 under a ±70 MeV variation of \( m_{b}^{1S} \), while the other terms are essentially unaffected. Since \( R'_{1,2}(1) \) are expected to be even smaller than \( R'_{1,2}(1) \), it seems unlikely that they could give useful independent information. CLEO measured \( R_{1} = 1.18 \pm 0.30 \pm 0.12 \) and \( R_{2} = 0.71 \pm 0.22 \pm 0.07 \) \cite{22}, assuming that they are independent of \( w \) and that \( h_{A_{1}}(w) \) has a linear \( w \)-dependence. These results agree well with Eq. (20).

Comparing our results for heavy quark symmetry breaking in the slope parameters in Eq. (18) to the data in Table I, there are several points to be made:

(i) The result of the linear fits at both CLEO and BELLE indicate \( \rho^{2}_{F_{*}} - \rho^{2}_{F} \sim 0.2 \) (see Table I). This is opposite to what is expected based on the QCD sum rule predictions for the subleading Isgur-Wise function in Eq. (18). The simplest way to accommodate the central value of the data is if \( \chi'_{3}(1) \sim 0.15 \), which is several times larger than the QCD sum rule prediction. It should be straightforward to decide using lattice QCD whether this large value of \( \chi'_{3}(1) \) occurs.

(ii) The result of the unitarity constrained quadratic fits at both CLEO and BELLE indicate \( \rho^{2}_{A_{1}} - \rho^{2}_{F} \sim 0.3 \) (see Table I), whereas based on the QCD sum rule predictions for the subleading Isgur-Wise functions one would expect this difference to be close to zero,

\[
\rho^{2}_{A_{1}} - \rho^{2}_{F} = -0.14 + 0.06 \eta(1) - 0.14 \eta'(1) - 0.75 \chi_{2}(1) + 3.0 \chi'_{3}(1) + \ldots \simeq -0.02. \tag{22}
\]

The value \( \chi'_{3}(1) \sim 0.15 \) suggested above also accommodates this data well.

(iii) The value of \( \chi'_{3}(1) \) is hard to constrain from data, since its contribution to \( R'_{1,2}(1) \) is suppressed by \( \alpha_{s}/\pi \). A large value of \( \chi'_{3}(1) \) could explain a large heavy quark symmetry breaking in the slope parameters, so determining \( \chi'_{3}(1) \) from the lattice is very important. Of course, it would be desirable to compute all subleading Isgur-Wise functions from the lattice.

11
(iv) Since heavy quark symmetry breaking would be unacceptably large in the comparison of \( \rho_{A1}^2 \) obtained from the unitarity constrained quadratic fit with \( \rho_{F}^2 \) obtained from the linear fit, one must use the same fitting procedure to extract the slopes and compare them. Therefore, one must be careful not to draw wrong conclusions when comparing the two sides of Fig. 1.\(^1\) We expect the data for \( B \to D^{*}\ell\bar{\nu} \) will eventually favor the unitarity constrained fit, at which point the data for \( B \to D\ell\bar{\nu} \) will have to be fit with the same method lest one gives up the symmetry relations that follow from the heavy quark expansion or invokes surprisingly large values for subleading Isgur-Wise functions (making convergence of the expansion questionable).

(v) Measurements of heavy quark symmetry breaking in the slope and curvature parameters, \( \rho_X^2 \) and \( c_X \), together with measurements of the \( R_{1,2} \) form factor ratios will strongly constrain the order \( \Lambda_{QCD}/m_Q \) corrections. Eqs. (18), (19), and (20) can be used to test future lattice calculations, or model predictions, such as those from QCD sum rules used in this paper for some numerical estimates.

(vi) Most importantly, a better knowledge of the slope parameters will help to reduce the error of \( |V_{cb}| \), since there is a very strong correlation, as can be seen from Fig. 1.

In conclusion, we calculated heavy quark symmetry breaking in the slopes and curvatures of the \( B \to D^{(*)}\ell\bar{\nu} \) spectra at zero recoil, including the order \( \alpha_s^2 \beta_0 \) corrections. A combined fit to the shapes of the \( B \to D^{*}\ell\bar{\nu} \) and \( B \to D\ell\bar{\nu} \) spectra together with the form factor ratios \( R_1 \) and \( R_2 \) may lead to a better knowledge of the values of the subleading Isgur-Wise functions. This in turn may reduce the error of \( |V_{cb}| \), since the correlation between the \( \rho_{F,2}(\omega) \) and \( |V_{cb}| F(\omega) (1) \) is very large. Once \( \rho_{F}^2 - \rho_{F,2}^2 \) is better understood, it will also be interesting to see how well the \( c_F - c_{F,2} \) constraint is satisfied by the data fitted using the unitarity constraints. When \( F_{\omega}(w) \) is computed in unquenched lattice QCD, agreement of the predicted values of \( \rho_{F}^2 - \rho_{F,2}^2 \) and \( c_F - c_{F,2} \) with data would give additional confidence (beyond checking that \( |V_{cb}| \) extracted from \( B \to D \) and \( B \to D^{*} \) agree) that the errors are well understood. It will be especially reassuring if such a value of \( |V_{cb}| \) agrees with the inclusive determination at the few percent level.

\(^1\) As the present authors first did.
Acknowledgments

In the published article there is an error in the first relation in Eq. (13), involving the terms proportional to $\rho_0^2$, slightly affecting the numerical result in Eq. (19). We thank Matt Dorsten for communicating his results to us prior to publication [23]. Erratum to be published in Phys. Lett. B.

We thank Mike Luke and Mark Wise for useful discussions, and Tom Browder, Hyunki Jang, Jiwoo Nam and Karl Ecklund for correspondence about the BELLE and CLEO analyses. Z.L. thanks the Aspen Center for Physics for hospitality while part of this work was completed. B.G. was supported in part by the Department of Energy under contract No. DOE-FG03-97ER40546. Z.L. was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

[1] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113; Phys. Lett. B237 (1990) 527.
[2] A. Czarnecki, Phys. Rev. Lett. 76 (1996) 4124; A. Czarnecki and K. Melnikov, Nucl. Phys. B505 (1997) 65.
[3] M.E. Luke, Phys. Lett. B252 (1990) 447.
[4] M. Neubert, Z. Ligeti and Y. Nir, Phys. Lett. B301 (1993) 101; Phys. Rev. D47 (1993) 5060; Z. Ligeti, Y. Nir and M. Neubert, Phys. Rev. D49 (1994) 1302.
[5] S. Hashimoto et al., Phys. Rev. D61 (2000) 014502; S. Hashimoto et al., hep-ph/0110253.
[6] M. Neubert, Phys. Rept. 245 (1994) 259.
[7] C.G. Boyd, B. Grinstein, R.F. Lebed, Phys. Lett. B353 (1995) 306; Nucl. Phys. B461 (1996) 493; Phys. Rev. D56 (1997) 6895.
[8] I. Caprini, L. Lellouch, and M. Neubert, Nucl. Phys. B530 (1998) 153.
[9] E. Won, BELLE Collaboration, Talk given at the International Conference on High Energy Physics of the European Physical Society (12–18 July 2001, Budapest, Hungary); H. Tajima, BELLE Collaboration, Talk given at the XX International Symposium on Lepton and Photon Interactions at High Energies (23–28 July 2001, Rome, Italy); H. Kim, BELLE Collaboration, Talk given at the 9th International Symposium on Heavy
Flavor Physics (10–13 September, 2001, Pasadena, California).

[10] J.P. Alexander et al., CLEO Collaboration, hep-ex/0007052.
[11] K. Abe et al., BELLE Collaboration, hep-ex/0111060.
[12] Private communication, Karl Ecklund (CLEO Collaboration).
[13] Private communication, Hyunki Jang (BELLE Collaboration).
[14] J. Bartelt et al., CLEO Collaboration, Phys. Rev. Lett. 82 (1999) 3746.
[15] K. Abe et al., BELLE Collaboration, hep-ex/0111082.
    Private communication, Jiwoo Nam (BELLE Collaboration).
[16] M. Neubert and C.T. Sachrajda, Nucl. Phys. B438 (1995) 235.
[17] C.G. Boyd, Z. Ligeti, I.Z. Rothstein and M.B. Wise, Phys. Rev. D55 (1997) 3027.
[18] I. Caprini and M. Neubert, Phys. Lett. B380 (1996) 376.
[19] A.H. Hoang, Z. Ligeti, and A.V. Manohar, Phys. Rev. Lett. 82 (1999) 277; Phys. Rev. D59 (1999) 074017.
[20] Z. Ligeti, M.E. Luke, A.V. Manohar and M.B. Wise, Phys. Rev. D60 (1999) 034019.
[21] S. Chen et al., CLEO Collaboration, hep-ex/0108032.
[22] J.E. Duboscq et al., CLEO Collaboration, Phys. Rev. Lett. 76 (1996) 3898.
[23] M. P. Dorsten, hep-ph/0310025.