Instabilities in nonminimally coupled theories

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In this work we explore the viability of nonminimally coupled $f(R)$ theories, namely the conditions required for the absence of tachyon instabilities and ghost degrees of freedom. We contrast our finds with recent claims of a pathological behaviour of this class of models, which resorted to, in our view, an incorrect analogy with $k$-essence.

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I. INTRODUCTION

Despite its great experimental success (see e.g. Refs. [1,2]), it is well known that General Relativity (GR) does not exhibit the most general form to couple matter with curvature. Indeed, these can be coupled, for instance, in a nonminimal way [3] (see also Refs. [4–6] for early proposals in cosmology), a fact that can have a bearing on the dark matter [7,8] and dark energy [9,10] problems, as well as inflation [11,12] and structure formation [13]. This putative nonminimal coupling (NMC) modifies the well-known energy conditions [14] and can give rise to several implications, from Solar System [15] and stellar dynamics [16–19] to close time-like curves [20] and wormholes [21].

Following the argument that $f(R)$ theories should be derived from a more complete theory as low-energy phenomenological models, one also finds strong fundamental motivation for the presence of a NMC, as it arises from one-loop vacuum-polarization effects in the formulation of Quantum Electrodynamics in a curved spacetime [22], as well as in the context of multi-scalar-tensor theories, when considering matter scalar fields [23] (as explicitly shown in Ref. [24]). Furthermore, a NMC was put forward in an earlier proposal [25], developed in the context of Riemann-Cartan geometry, with another study showing that it clearly affects the features of the ground state [26].

Thus, one considers that the Einstein-Hilbert action is extended by the action functional $\mathcal{S}$,

$$ S = \int \left[ \kappa f_1(R) + f_2(R) \mathcal{L} \right] \sqrt{-g} \, d^4x , \quad (1) $$

where $f_i((R))$ ($i = 1,2$) are arbitrary functions of the scalar curvature, $R$, $g$ is the determinant of the metric and $\kappa = \epsilon^2/16\pi G$. The above encompasses the well-known $f(R)$ theories, which are widely used to study the effect of modifications of gravity in a plethora of scenarios, e.g. the Starobinsky inflationary model $f(R) = R + \alpha R^2$ [12], the accelerated expansion of the Universe [27], Solar System tests [28], amongst many other studies (see Ref. [29] for a review).

Variation with respect to the metric yields the modified field equations,

$$ (\kappa F_1 + F_2 \mathcal{L}) G_{\mu\nu} = \frac{1}{2} f_2 T_{\mu\nu} + \frac{1}{2} F_{\mu\nu} \left[ \kappa (f_1 - F_1 R) - F_2 R \mathcal{L} \right] , \quad (2) $$

with $F_i \equiv df_i/dR$ and $\Delta_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} \Box$. As expected, GR is recovered by setting $f_1(R) = R$ and $f_2(R) = 1$.

The trace of Eq. (2) reads

$$ (\kappa F_1 + F_2 \mathcal{L}) R = \frac{1}{2} f_2 T - 3 \Box (\kappa F_1 + F_2 \mathcal{L}) + 2\kappa f_1 . \quad (3) $$

Resorting to the Bianchi identities, one concludes that the energy-momentum tensor of matter may not be (covariantly) conserved, since

$$ \nabla_\mu T^{\mu\nu} = \frac{F_2}{f_2} (g^{\mu\nu} \mathcal{L} - T^{\mu\nu}) \nabla_\mu R , \quad (4) $$

can be non-vanishing (see Refs. [30,31] for thorough discussions).

II. TACHYONIC INSTABILITIES

When considering any novel extension of GR, one should not only show that it can be applied to a variety of problems, but also that it does not suffer from pathological behaviours that can lead to unphysical results. In the present case of a model exhibiting a NMC between curvature and matter, it has been shown that cosmological perturbations are compatible with the observed cosmological constraints [13,32], that it yields the correct weak-field limit at Newtonian and post-Newtonian levels [13], obeys the energy conditions [14] and is free from Dolgov-Kawasaki instabilities [14,33,34]. The latter arise if the Ricci curvature scalar acquires a negative mass-squared, which signals a tachyonic instability with associated exponential growth of perturbations [35]; in $f(R)$ theories, this is avoided if the condition $f''(R) \geq 0$.
is obeyed. In NMC models, this is generalised to
\[ f''(R) + f''(R)\mathcal{L} \geq 0. \]

In astrophysical and cosmological contexts, it was shown that a negative power-law
\[ f_2(R) = 1 + \left( \frac{R}{R_0} \right)^n, \quad n < 0, \quad (5) \]
is required to account for the dark matter or dark energy components, respectively (with a linear \( f_1(R) \) function). Thus, one has
\[ f''_1(R) + f''_2(R)\mathcal{L} = -n(n-1) \left( \frac{R}{R_0} \right)^n \frac{\rho}{R^2} < 0, \quad (6) \]
so that the viability condition discussed above for the absence of Dolgov-Kawasaki instabilities does not hold for the choice of Lagrangian density \( \mathcal{L} = -\rho \). However, one finds that the typical length or timescale of the curvature instabilities is much smaller than the scale over which the background curvature may be considered spatially or temporally constant (see Ref. \[17\] for a discussion), so that the derivations leading to the above constraint do not hold.

If, instead of assuming a perturbative expansion around a flat spacetime, a Friedmann-Robertson-Walker (FRW) metric is taken with background cosmological curvature \( R_0(t) \), and local curvature perturbations \( R_1(r) \ll R_0(t) \) are then considered, one finds that the equation of motion for \( R_1(t) \) may be written as
\[ \nabla^2 U - m^2 U = 0, \quad (7) \]
with \( U = [f''_1(R_0) + f''_2(R_0)\mathcal{L}_0]R_1 \), and a mass-squared defined as
\[ m^2 = \frac{1}{3} \left[ \frac{f_1(R_0) - f_2(R_0)\mathcal{L}_0}{f''_1(R_0) + f''_2(R_0)\mathcal{L}_0} + \right. \]
\[ \left. - \frac{6\mathcal{L}_0[f''_2(R_0) - f''_2(R_0)\mathcal{L}_0]}{f''_1(R_0) + f''_2(R_0)\mathcal{L}_0} - R_0 \right], \quad (8) \]
where \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \), \( \mathcal{L}_0 \) is the Lagrangian density of the cosmological background matter (which, if assumed as a pressureless dust, is given by \( \mathcal{L}_0 = -\rho_0(t) \)) and \( \mathcal{L}_1 \) is the corresponding quantity for the central body that imposes spherical symmetry (so that \( \mathcal{L}_1 = 0 \) away from it).

One finds that considering a FRW spacetime instead of a Minkowski background changes the previously discussed result, i.e. the avoidance of a negative mass-squared for curvature perturbations requires a more convoluted expression. Indeed, following Ref. \[15\], one finds that the choice for the functions
\[ f_1(R) = R, \quad f_2(R) = 1 + \left( \frac{R}{R_0} \right)^n, \quad (9) \]
yields \( m^2 > 0 \) for \( n \ll -10^{-25} \) — a result which basically has no practical consequences when attempting to describe, for instance, dark energy via a NMC model \[3\].

### III. Ghosts

Another issue faced by any extension of GR is the presence of ghost fields, that is, degrees of freedom with a kinetic term that has the wrong sign, so that the Hamiltonian can be unbounded from below and an explosive production of pairs particles out of the vacuum occurs, preventing stable atoms and a physical Universe. When attempting to quantize the theory, this manifests itself as states of negative norm, which leads to loss of unitarity and conservation of probability.

When addressing the possibility of ghosts in extensions of GR, two competing views arise: some argue that modifications of GR should be regarded as effective theories, valid in the weak-field limit of some fundamental quantum theory of gravity (such as string theory), and thus should not be probed beyond the classical level in which they are valid; conversely, others consider that extensions of GR should be quantized in order to achieve the yet unknown quantum theory of gravity. Naturally, the first interpretation relaxes the need to properly quantize the model under scrutiny, so that the treatment on instabilities should be carried out only classically.

Several studies approach the issue of ghost instabilities by expanding the action up to second order in perturbation to a chosen background metric (usually Minkowski or Friedmann-Robertson-Walker) to inspect the resulting kinetic terms and then searching for cross terms between the canonical variables and their momenta, \( P_iQ_i \) (which, since either quantity can attain positive or negative values, are unbounded).

Alternatively, one computes the propagators from the linear expansion of the modified field equations around a chosen background metric and then identifies the sign of the propagators of additional degrees of freedom relative to the usual propagator of GR or the presence of additional poles: this approach in a flat or constant curvature background confirms that \( f(R) \) theories introduce an additional degree of freedom with mass \( m^2 \sim f''(R = 0) \), although no treatment is available including a NMC Eq. \[11\] — which must abandon the former case of a Minkowski background metric, as this would imply to zeroth-order that no matter is present, \( \mathcal{L} = 0 \).

A key result, due to its generality and broadness, is the Ostrogradski theorem. It states that any non-degenerate theory with second or higher derivatives appearing in the Lagrangian leads to linear instabilities, i.e. the aforementioned unbounded cross-terms between canonical variables and their momenta (this can be avoided by the inclusion of constraints that restrict the phase space).

The requirement of non-degeneracy is essential to this result: indeed, GR and \( f(R) \) theories both express second derivatives in the Lagrangian density (via the Ricci curvature scalar), but are degenerate theories, as one cannot invert the definition of the canonical variables and their momenta to write the higher derivative term as a func-
The extension posited in Eq. (11) presents an extension of $f(R)$ theories, while preserving the overall structure of the field Eqs. (2), i.e. the NMC provides additional terms, but does not modify those stemming from a non-linear $f(R)$: thus, one concludes that the issue of inverting the relation between canonical variables/ momenta and the higher derivative term is worsened, and thus the model here considered is also degenerate — implying that the Ostrogradski theorem does not apply and cannot be used to assess the existence of ghosts in the theory.

Notwithstanding, the simplest cure to Ostrogradski ghosts in non-degenerate theories is to require that the equations of motion are second order in all fields, as stated above. The most general Lagrangian density including one additional scalar field satisfying this is the so-called Horndeski Lagrangian [46], given by

$$\mathcal{L} = R - 2\Lambda + G_2(X, \chi) + G_3(X, \chi)[\chi^2] + G_4(X, \chi)\chi^2 + G_5(X, \chi)\chi^{4\mu\nu} - \frac{G_6}{6}(\chi^3 - 3[\chi][\chi^2] + 2[\chi^3]),$$  

where $X \equiv -(1/2)g^{\mu\nu}\chi_{,\mu}\chi_{,\nu}$ and $[\chi^n]$ is the trace of the quantity

$$\chi_{\mu\nu} \equiv \chi_{\alpha\beta}\chi^{\alpha\beta\cdots\lambda\mu\nu\cdots}^{-1},$$  

e.g. $[\chi] \equiv \Box \chi$ and $[\chi^2] \equiv \chi_{\mu\nu}\chi^{\mu\nu}$. The various contributions act as “counterterms” that suitably cancel out any pathological terms on the ensuing field equations arising from the second derivatives of $\chi$ (see Ref. [47] for a discussion).

Inspection shows that the $\mathcal{L}_4$ is the one relevant to this study, as it displays a linear coupling with the Ricci curvature. It can be viewed as a modification of k-essence models [48], which consider a scalar field $\chi$ with a Lagrangian density given by a generic function of the scalar field and its kinetic term, $\mathcal{L} = p(\chi, X)$, but linearly coupled to the scalar curvature (so that $G_4 = p(\chi, X)$ and all other functions $G_i$ vanish):

$$\mathcal{L}_4 = p(\chi, X)R + p,X \left(\Box \chi^2 - \chi_{;\mu\nu}\chi^{;\mu\nu}\right).$$

Thus, a linear coupling requires the addition of suitable “counterterms” so that the ensuing modified field equations remain second order.

However, it is again pointed out that, since Eq. (11) is a degenerate theory, failure to include these does not immediately lead to Ostrogradski linear instabilities. Conversely, Lagrangian densities including only first derivatives are not automatically free from ghosts: popular phantom-type models for inflation [49] or dark energy [50] are afflicted by this problem as they present negative kinetic energy terms.

Indeed, the requirement for second-order field equations should instead be viewed as arising from considerations of naturalness: furthermore, even higher-order theories such as $f(R)$ can be recast as second-order theories, via the well-known equivalence between the Jordan frame formulation where the curvature appears non-linearly in the action, and the Einstein frame where one instead resorts to a scalar field nonminimally coupled to the scalar curvature [51-53].

By the same token, the structure of Eq. (2) hints that the four initial conditions required for a well-posed Cauchy problem do not have to be written in terms of the metric and its derivatives up to third order, but one can instead consider only the value and first derivative of both the metric and the Ricci curvature. This treatment is reminiscent of Ostrogradski’s Hamiltonian formulation of $f(R)$ theories, where the curvature is promoted to a canonical variable [54-56], and recalls the equivalence between the model posited in Eq. (11) and a two scalar field model: indeed, in Ref. [57] it was shown that Eq. (11) is equivalent to the action (in the Einstein frame)

$$S = \int \sqrt{-g}d^4x \times \left(2\kappa \left[R - 2g^{\mu\nu}\sigma_{ij}\varphi_i\varphi_j - 4U(\varphi^1, \varphi^2)\right] + \mathcal{L}^\ast\right),$$

where $\varphi^1$ and $\varphi^2$ are scalar fields, $\sigma_{ij}$ is the field-metric

$$\sigma_{ij} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix},$$

the potential is given by

$$U(\varphi^1, \varphi^2) = \frac{1}{4} \exp \left(-\frac{2\sqrt{3}}{3}\varphi^1\right) \left[\varphi^2 - f_1(\varphi^2) \exp \left(-\frac{2\sqrt{3}}{3}\varphi^1\right)\right],$$

and a Lagrangian density coupling matter with these two scalar fields also arises,

$$\mathcal{L}^\ast = \exp\left[-(4\sqrt{3}/3)\varphi^1\right]f_2(\varphi^2)\mathcal{L}.$$  

The two scalar fields are related with the scalar curvature and the non-trivial $f_1(R)$ and $f_2(R)$ functions through

$$\varphi^1 = \frac{\sqrt{3}}{2} \log \left[\frac{F_1(R) + F_2(R)\mathcal{L}}{2\kappa}\right], \quad \varphi^2 = R.$$  

Clearly, Eq. (13) gives rise to field equations which are second-order in the derivatives of $\varphi^i$, as stated before. While $\varphi^2$ has no kinetic term, and can be considered as an auxiliary field, the kinetic term of $\varphi^1$ has the usual sign, as is then positive-defined: no ghosts arise as long as condition $F_1(R) + F_2(R)\mathcal{L}/2\kappa > 0$ is obeyed and the above definition of fields is valid — a natural result following the condition $f'(R) > 0$ for ghost-free $f(R)$ theories [29].

Inserting a linear $f_1(R) = R$ function and a power-law NMC, Eq. (5), used in astrophysical [7, 8] and cosmological contexts [9-11, 13] to account for dark matter and dark energy, respectively, the above condition becomes

$$1 - n\left(\frac{R}{R_0}\right)^n \rho \frac{\rho}{2\kappa R} > 0,$$  


IV. ANALOGY WITH $\kappa$-ESSENCE

In a recent study \cite{58}, it has been claimed that a NMC model is not viable, due to the appearance of ghost degrees of freedom. Notwithstanding the dedicated discussion in the previous section, we now directly address the issues raised in Ref. \cite{58}, and show that they are based on improper assumptions and arguments.

1. Ref. \cite{58} starts by integrating away the auxiliary scalar field $\varphi^2$, thus rewriting (in the present notation) the action, Eq. (13), as

$$S = \int \sqrt{-g} d^4x \left[ 2\kappa \left( R - 2g^{\mu\nu} \varphi^1,_{\mu} \varphi^1,_{\nu} \right) - P(\mathcal{L}, \varphi^1) \right] ,$$

where $P(\mathcal{L}, \varphi^1)$ is a function obtained from the substitution of the solution $\varphi^2 = \varphi^{2\ast}$ of the field equation for this scalar field.

2. The authors then argue that a viable NMC model should allow the coupling to any Lagrangian density for matter $\mathcal{L}$, and choose $\mathcal{L} = -g^{\mu\nu} \chi,_{\mu} \chi,_{\nu} \equiv 2X$, i.e. a matter scalar field with no potential. One remarks that, although this freedom of choice of $\mathcal{L}$ should be a desirable feature, it can be argued that only certain forms for $\mathcal{L}$ may be nonminimally coupled to curvature, while others that give rise to pathological behaviours are deemed unphysical, and thus not allowed.

3. The above action with the choice $\mathcal{L} = X$ is then compared with k-essence models \cite{15}, given by the action

$$S = \int \sqrt{-g} d^4x \left[ 2\kappa R + p(X, \chi) \right]$$

This is done with little regard for the differences between k-essence and a NMC model, as no in depth justification is offered: instead, it is simply stated that such comparison is warranted because the kinetic term of $\varphi^1$ is canonical and separated from the function $P(\phi, \mathcal{L} = X)$.

Clearly, this is insufficient, as the dynamical effect of the scalar field $\varphi^1$ is left completely out of the ensuing discussion: this additional degree of freedom should have some impact on what otherwise bears a resemblance with k-essence models, specially since it is not an independent matter field, but is dynamically related to both the curvature and the Lagrangian density of matter through Eq. (17) — which goes against the claim that $\varphi^1$ can basically be disregarded.

4. Indeed, the above insufficiencies manifest themselves in the incorrect use of the conditions for positivity of the energy density $\epsilon(\chi)$ of the matter scalar field $\chi$ and the speed of sound $c_s(\chi)$ at which its perturbations propagate: the authors resort to the results obtained in Refs. \cite{48, 59, 61}, namely that

$$\epsilon(\chi) = 2Xp,_{X} - p > 0 ,$$

$$c_s^2(\chi) = \frac{p,_{X}}{2Xp,_{XX} + p,_{X}} > 0 .$$

The above are paramount in the overall argument posed by Ref. \cite{58}. However, closer examination of the computations behind the expression for $c_s(\chi)$ (namely Ref. \cite{59}) shows that its treatment of cosmological perturbations relies upon the (covariant) conservation of energy density,

$$\dot{\epsilon} = -3H(\epsilon + p) ,$$

and the Ansatz for the perturbed metric

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)^2(t)g_{ij}dx^i dx^j .$$

As it turns out, none of these hold in NMC models (although they are valid in k-essence, of course): perhaps the most striking and fundamental consequence of the latter (see Refs. \cite{2, 61} for a discussion) is that the energy-momentum tensor of matter is no longer covariantly conserved, as seen in Eq. (24) — which, for $\mathcal{L} = X$ and the ensuing

$$T_{\mu\nu} = (\epsilon + p) \frac{\chi,_{\mu} \chi,_{\nu}}{2X} - pg_{\mu\nu} ,$$

yields (for $\nu = 0$),

$$\dot{\epsilon} + 3H(\epsilon + p) = \frac{f_2}{f_2} (p - \epsilon) \dot{R} ,$$

which in general is non-vanishing (e.g. if $p \sim X^n$, then $p = \epsilon/(2n - 1)$).

The identification of $c_s$ as the speed of sound of perturbations of $\chi$ builds upon the incorrect use of Eq. (22) by assuming that these affect the metric as expressed in Eq. (23). This assumption is also wrong in NMC models: as Ref. \cite{15} has shown, the metric should instead include two potentials,

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)^2(t)g_{ij}dx^i dx^j .$$

related through

$$\Phi - \Psi = -\delta(F_1 + F_2) = -\delta \left[ \exp \left( \frac{-2\sqrt{3}}{3} \varphi^1 \right) \right] ,$$

with $\delta$ signalling a perturbation of the quantity in parenthesis.

This highlights the fact that, as stated before, one cannot freely disregard the contribution of the scalar field $\varphi^1$ and summarily proceed with a comparison with k-essence: as it turns, out that the treatment of cosmological perturbations in the latter is fundamentally incompatible with the results arising from the direct computations found in Ref. \cite{15}. As such, the objections posited in Ref. \cite{58} are ill-defined.
V. DISCUSSION AND OUTLOOK

In this work we have discussed the viability of non-minimally coupled models, focusing on the issue of tachyonic instabilities and ghost degrees of freedom; we have also addressed the claim, stemming from an incorrect analogy with k-essence models, that models with a NMC between matter and curvature exhibit a pathological behaviour.

As stated in Ref. [14], in a flat background no negative mass-squared arises if the condition $f''_0 + f'_0 \mathcal{L} > 0$ is obeyed; this is replaced by the more evolved expression Eq. (8) in a FRW background, as found in Ref. [15].

The absence of ghost degrees is obtained via a discussion of the equivalence with of the model under scrutiny with a multi-scalar-tensor theory; the non-applicability of Ostrogradski’s theorem is discussed in terms of the degeneracy of a NMC model, which is in itself an extension of degenerate $f(R)$ theories. Notwithstanding, we also discuss the issue of how the modified field equations, which are of fourth order, can be rewritten as second-order differential equations for both the metric and the scalar curvature — in line with the aforementioned equivalence and the related introduction of two appropriate scalar fields to express the dynamical effect of non-trivial $f_i(R)$ functions.

Finally, the claims of lack of viability put forward in Ref. [58] are discussed, focusing on the improper comparison between NMC models and k-essence: we argue that the contribution of the scalar field $\phi$ cannot be disregarded, as shown by its crucial role in distinguishing the treatment of cosmological perturbations between these two types of proposals — as is the conservation of energy-momentum, that holds in k-essence, but is broken in NMC models.

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[1] C. M. Will, Living Rev. Rel. 9, 3 (2006).
[2] O. Bertolami and J. Páramos, “The experimental status of Special and General Relativity”, Handbook of Spacetime, Springer, Berlin (2014); arXiv:1212.2177 [gr-qc].
[3] O. Bertolami, C. G. Böhmer, T. Harko and F. S. N. Lobo, Phys. Rev. D 75, 104016 (2007).
[4] L. Amendola and D. Tocchini-Valentini, Phys. Rev. D 64, 043509 (2001).
[5] S. ’i. Nojiri and S. D. Odintsov, PoS WC 2004, 024 (2004).
[6] G. Allemandi, A. Borowiec, M. Francaviglia and S. D. Odintsov, Phys. Rev. D 72, 063505 (2005).
[7] O. Bertolami and J. Páramos, JCAP 03, 009 (2010).
[8] O. Bertolami, P. Frazão and J. Páramos, Phys. Rev. D 86, 044034 (2012).
[9] O. Bertolami, P. Frazão and J. Páramos, Phys. Rev. D 81, 104046 (2010).
[10] O. Bertolami and J. Páramos, Phys. Rev. D 84, 064022 (2011).
[11] O. Bertolami, P. Frazão and J. Páramos, Phys. Rev. D 83, 044010 (2011).
[12] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).
[13] O. Bertolami, P. Frazão and J. Páramos, JCAP 1305, 029 (2013).
[14] O. Bertolami and M. C. Sequeira, Phys. Rev. D 79, 104010 (2009).
[15] O. Bertolami, R. March and J. Páramos, Phys. Rev. D 88, 064019 (2013).
[16] O. Bertolami and J. Páramos, Phys. Rev. D 77, 084018 (2008).
[17] O. Bertolami and A Martins, Phys. Rev. D 85, 024012 (2012).
[18] J. Páramos and C. Bastos, Phys. Rev. D86, 103007 (2012).
[19] O. Bertolami and J. Páramos, arXiv:1306.1177 [gr-qc].
[20] O. Bertolami and R. Z. Ferreira, Phys. Rev. D 85, 104050 (2012).
[21] N. Montelongo Garcia and F. S. N. Lobo, Class. Quantum Gravity 28, 085018 (2011).
[22] I. T. Drummond and S. J. Hathrell, Phys. Rev. D 22, 343 (1980).
[23] T. Damour and G. Esposito-Farèse, Class. Quantum Gravity 9, 2093 (1992).
[24] O. Bertolami and J. Páramos, Class. Quantum Gravity 25, 245017 (2008).
[25] H. F. M. Goenner, Found. Phys. 14, 9 (1984).
[26] O. Bertolami, Phys. Lett. B 186, 161 (1987).
[27] S. Capozziello, Int. J. Mod. Phys. D 11, 483 (2002); S. Capozziello, V. F. Cardone, S. Carloni and A. Troisi, Int. J. Mod. Phys. D 12, 1969 (2003).
[28] S. Capozziello, V. Cardone, S. Carloni and A. Troisi, Phys. Lett. A 326, 292 (2004); J. Mbelek, Astron. and Astrophys. 424, 761 (2004); S. Capozziello, V. Cardone and A. Troisi, Mon. Not. R. Ast. Soc. 375, 1423 (2007); S. Capozziello, A. Stabile and A. Troisi, Phys. Rev. D 76, 104019 (2007); T. Chiha, T. L. Smith and A. L. Erickcek, Phys. Rev. D 75, 124014 (2007).
[29] A. D. Felice and S. Tsujikawa, Living Reviews in Relativity 13 (2010) 3.
[30] T. Koivisto, Class. Quantum Gravity 23, 4289 (2006).
[31] T. P. Sotiriou and V. Faraoni, Class. Quantum Gravity 25, 5002 (2008).
[32] S. Thakur and A. A. Sen, Phys. Rev. D88, 044043 (2013).
[33] A.D. Dolgov and M. Kawasaki, Phys. Lett. B 573, 1 (2003).
[34] S. Nojiri and S.D. Odintsov, Phys. Lett. B 68, 123512 (2003); Gen. Relativity and Gravitation 36, 1765 (2004).
[35] V. Faraoni, Phys. Rev. D 74, 104017 (2006).
[36] O. Bertolami and J. Páramos, Phys. Rev. D 89, 044012 (2014).
[37] O. Bertolami, F. S. N. Lobo and J. Parámos, *Phys. Rev.* D 78, 064036 (2008).
[38] S. W. Hawking and T. Hertog, *Phys. Rev.* D 65, 103515 (2002).
[39] A. De Felice, M. Hindmarsh and M. Trodden, *JCAP* 0608, 005 (2006).
[40] N. Deruelle, Y. Sendouda and A. Youssef, *Phys. Rev.* D 80, 084032 (2009).
[41] M. Chaichian, M. Oksanen and A. Tureanu, *Phys. Lett.* B 693, 404 (2010) [Erratum-ibid. B 713, 514 (2012)].
[42] M. Chaichian, S. 'i. Nojiri, S. D. Odintsov, M. Oksanen and A. Tureanu, *Class. Quantum Gravity* 27, 185021 (2010) [Erratum-ibid. B 713, 514 (2012)].
[43] A. Nunez and S. Solganik, hep-th/0403159; *Phys. Lett.* B 608, 189 (2005).
[44] T. S. Koivisto and N. Tamanini, *Phys. Rev.* D 87, 104030 (2013).
[45] T. -j. Chen, M. Fasiello, E. A. Lim and A. J. Tolley, *JCAP* 1302, 042 (2013).
[46] G. W. Horndeski, *Int. J. Theor. Phys.* 10, 363 (1974).
[47] M. Zumalacáregui and J. García-Bellido, arXiv:1308.4685 [gr-qc].
[48] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, *Phys. Rev.* D 63, 103510 (2001).
[49] N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, *JCAP* 0404, 001 (2004).
[50] F. R. Urban and A. R. Zhitnitsky, *Phys. Lett.* B 688, 9 (2010).
[51] P. Teyssandier and P. Tourranc, *J. Math. Phys.* 24, 2793 (1983).
[52] H. Schmidt, *Class. Quantum Gravity* 7, 1023 (1990).
[53] D. Wands, *Class. Quantum Gravity* 11, 269 (1994).
[54] P. P. Fiziev, *Phys. Rev.* D 87, 044053 (2013).
[55] M. Ostrogradski, Mem. Acad. St. Petersbourg SeriesVI 4, 385 (1850).
[56] D. A. Eliezer and R. P. Woodard, *Nucl. Phys.* B 325, 389 (1989).
[57] O. Bertolami and J. Páramos, *Class. Quantum Gravity* 25, 045017 (2008).
[58] N. Tamanini and T. S. Koivisto, *Phys. Rev.* D 88, 064052 (2013).
[59] C. Armendariz-Picon, T.Damour and V.F. Mukhanov, *Phys. Lett.* B 458, 205 (1999).
[60] T. Chiba, T. Okabe and M. Yamaguchi, *Phys. Rev.* D 62, 023511 (2000).
[61] J. Garriga and V.F. Mukhanov, *Phys. Lett.* B 458, 219 (1999).