ON THE OCCURRENCE OF MASS IN FIELD THEORY

Giampiero Esposito

INFN, Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia, Edificio N’, 80126 Napoli, Italy; e-mail: giampiero.esposito@na.infn.it

Dipartimento di Scienze Fisiche, Università di Napoli “Federico II”, Complesso Universitario di Monte S. Angelo, Via Cintia, Edificio N’, 80126 Napoli, Italy

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This paper proves that it is possible to build a Lagrangian for quantum electrodynamics which makes it explicit that the photon mass is eventually set to zero in the physical part on observational ground. Gauge independence is achieved upon considering the joint effect of gauge-averaging term and ghost fields. It remains possible to obtain a counterterm Lagrangian where the only non-gauge-invariant term is proportional to the squared divergence of the potential, while the photon propagator in momentum space falls off like $k^{-2}$ at large $k$ which indeed agrees with perturbative renormalizability. The resulting radiative corrections to the Coulomb potential in QED are also shown to be gauge-independent. The experience acquired with quantum electrodynamics is used to investigate properties and problems of the extension of such ideas to non-Abelian gauge theories.

Key words: quantum electrodynamics, path integrals, perturbative renormalization.
1. INTRODUCTION

A key task of theoretical physics has been always the description of a wide variety of natural phenomena within a unified conceptual framework, where they can all be derived from a few basic principles which have been carefully tested against observation. The development of local or non-local field theories, the investigation of perturbative and nonperturbative properties, and the construction of gauge theories of fundamental interactions provide good examples of how such a task can be accomplished. Moreover, when a commonly accepted model remains unproven for a long time, the theoretical physicist has to perform a careful assessment of the ideas leading to such a prediction, and he/she is expected to find either an independent way to confirm it, or an alternative way to understand the phenomenon.

Within this framework, it is the aim of our paper to reconsider a longstanding problem in particle physics and field theory, i.e. the generation of mass in gauge theories of fundamental interactions. Although the Higgs mechanism provides a well understood theoretical model for the generation of mass,\(^{(1)}\) the analysis of alternative models appears necessary for at least a fundamental reason: no conclusive evidence on the existence of the Higgs field is available as yet. At present one can only say that, from the precision measurements of the mass of the \(W\) boson and the effective leptonic weak mixing angle at the \(Z\)-boson resonance, one finds a 95 per cent confidence level upper bound on the Higgs-boson mass given by \(M_H < 188\) GeV.\(^{(2)}\) For example, in the Weinberg–Salam model,\(^{(3−5)}\) the Lagrangian density \(\mathcal{L}\) (hereafter we omit the word “density” for simplicity) contains five terms describing gauge bosons, the coupling of gauge bosons to scalars, the coupling of
gauge bosons to left-handed and right-handed fermions, and the gauge-invariant interaction among scalars and fermions, respectively. In particular, the coupling of gauge bosons to scalars is described by the term

\[ \mathcal{L}_{GB-S} = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi^\dagger \phi), \quad (1.1) \]

where \( \phi \) is a Higgs field and the gauge-covariant derivative reads

\[ D_\mu \equiv \partial_\mu + ig \sum_{k=1}^{3} W^k_\mu \tau_k + ig' W^0_\mu \tau_0. \quad (1.2) \]

With a standard notation, \( W^k_\mu \) are the \( SU(2) \) gauge fields with associated generators \( \tau_k \), while \( W^0_\mu \) is the \( U(1) \) gauge field with generator \( \tau_0 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \). In the unitary gauge, the Higgs field is expressed by the “column vector” \( \phi = \begin{pmatrix} 0 \\ \tilde{\rho} \end{pmatrix} \), and after writing the transformation (\( \theta_w \) being the Weinberg angle)

\[ \begin{pmatrix} W^3_\mu \\ W^0_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \quad (1.3) \]

the kinetic term in Eq. (1.1) reads eventually

\[ (D^\mu \phi)^\dagger D_\mu \phi = \frac{g^2}{4} \left( W^1_\mu W^1_\mu + W^2_\mu W^2_\mu \right) \tilde{\rho}^2 + \frac{g^2}{4} Z^\mu Z_\mu \tilde{\rho}^2, \quad (1.4) \]

Thus, the vector mesons \( W_+ \), \( W_- \) and \( Z \) are found to have square masses \( \frac{1}{2} g^2 \tilde{\rho}^2 \), \( \frac{1}{2} g^2 \tilde{\rho}^2 \) and \( \frac{1}{2 \cos^2 \theta_w} g^2 \tilde{\rho}^2 \), respectively. From the known experimental value of the Weinberg angle, one then finds at tree level masses \( m_W \) and \( m_Z \) of order 80 GeV and 90 GeV, respectively.
Nevertheless, since the Higgs field remains unobserved, we are led to ask ourselves whether the fundamental principles of quantum field theory make it possible to fit the experimental data without having to assume the existence of a Higgs field.

Motivated by this outstanding problem, Secs. 2 and 3 study a new class of gauge-averaging functionals in the path integral for bosonic gauge theories, and other original results are derived in Secs. 4–8, which are devoted to photon propagators in quantum electrodynamics; perturbative renormalization of a QED model where the mass of the photon is set to zero only on observational ground at a later stage; radiative corrections in QED; mass terms for vector mesons in non-Abelian gauge theory. Concluding remarks and open problems are presented in Sec. 9.

2. GAUGE-AVERAGING FUNCTIONALS AND GAUGE-FIELD OPERATORS

At this stage, the fundamental point in our investigation is the need to recall a well-known property of all gauge theories: since an invariance group is present, the operator obtained from second functional derivatives $S_{,ij}$ of the classical action $S$ is not invertible. To obtain an invertible operator on field disturbances one has to add to $S_{,ij}$ a term obtained from the generators of infinitesimal gauge transformations and their adjoints.$^{(6)}$ In the corresponding quantum theory, the counterpart of this construction is the addition of a gauge-averaging (also called, more frequently, gauge-breaking or gauge-fixing) term to the
original Lagrangian $\mathcal{L}$.\( ^{7} \) The resulting Lagrangian leads to well defined functional determinants in the one-loop semiclassical theory and is part of the path-integral prescription for gauge theories, aimed at avoiding a “summation” over gauge-equivalent field configurations for the out-in amplitude. In other words, the key idea which inspires our model is as follows: a gauge-invariant Lagrangian is very elegant, but what one really needs is instead a Lagrangian leading to an invertible operator on gauge fields,\( ^{6−8} \) with the associated gauge-breaking term and ghost fields.\( ^{6−12} \) This is invariant under BRST transformations, which express the most important symmetry of modern quantum field theory. [We will find that the resulting theory cannot truly “generate” mass, but acquires technical tools for describing its occurrence]

Bearing in mind these properties, let us consider for simplicity the Lagrangian for Euclidean Maxwell theory via path integrals (here we write only the part involving the potential):

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\alpha} [\Phi(A)]^2.$$  \hspace{1cm} (2.1)

With a standard notation, $F_{\mu\nu}$ is the electromagnetic field strength that contributes the non-invertible operator ($R_{\mu\nu}$ being the Ricci tensor of the background with metric $g$)

$$-g_{\mu\nu} \Box + \nabla_\mu \nabla_\nu + R_{\mu\nu}$$

acting on the potential (with $\Box \equiv \nabla^\mu \nabla_\mu = g^{\mu\nu} \nabla_\mu \nabla_\nu$, and $\nabla$ the Levi-Civita connection on space-time). Moreover, $\alpha$ is a dimensionless parameter, and $\Phi$ is the gauge-averaging functional

$$\Phi : A \in \{A_\mu dx^\mu\} \to \Phi(A) \in \mathbb{R}.$$
The potential $A$ is mapped into the real number $\Phi(A)$ via the action of $\Phi$ in a way here expressed in the form

$$\Phi(A) \equiv T^\mu A_\mu = g^{\mu\nu}T_\nu A_\mu.$$  \hfill (2.2)

In a local formulation, $T^\mu = \nabla^\mu$ leads to the Lorenz gauge,\(^{(13)}\) with the associated gauge-field operator

$$P_{\mu\nu} = -g_{\mu\nu} \Box + \left(1 - \frac{1}{\alpha}\right)\nabla_\mu \nabla_\nu + R_{\mu\nu},$$  \hfill (2.3)

which becomes of Laplace type (in a Euclidean framework) when $\alpha$ is set equal to 1 (this is the Feynman choice for $\alpha$).

3. ROLE OF $\gamma$-MATRICES

Independently of being able to find an alternative to the Higgs mechanism in non-Abelian theories, we would now like it to understand whether mass terms can be considered with the help of a suitable formulation of the process of gauge-averaging in the path integral, while making sure that such masses are unaffected by any particular choice of gauge parameters. In the simpler case of Maxwell theory, we should find how a term proportional to $A_\mu A^\mu$ can be obtained. Indeed, in four dimensions, one can exploit the identity

$$A_\mu A^\mu = g^{\mu\nu}A_\mu A_\nu = \frac{1}{4}\text{Tr}(\gamma^\mu \gamma^\nu)A_\mu A_\nu.$$
where $g^{\mu\nu}$ are the contravariant components of the metric tensor.

We have therefore looked, in a first moment, for a gauge-fixing condition combining the effect of Lorenz gauge and $\gamma$-matrices. However, one cannot simply add the derivatives of $A_\mu$ in the Lorenz gauge and $\gamma^\mu$ terms, since the latter are four-vectors with components given by $4 \times 4$ matrices. The only well defined operation on such objects is the one giving rise to the matrix (here $i,j$ are matrix indices ranging from 1 through 4)

$$
\Phi^j_i(A) \equiv \left( \delta^j_i \partial^\mu + \beta (\gamma^\mu)^j_i \right) A_\mu(x), \tag{3.1}
$$

where we use a notation that makes it explicit how to add correctly partial derivatives and $\gamma$-matrix contributions, and the parameter $\beta$ is now introduced to ensure that all terms in $\Phi^j_i$ have the same dimension (i.e. $\beta$ has dimension $[\text{length}]^{-1}$). There is only one coefficient, $\beta$, since only one potential $A_\mu$ is available for contraction with $\gamma^\mu$ in the Abelian case. The resulting gauge-averaging term in the path integral for quantum electrodynamics is taken to be (with $\alpha$ a real parameter)

$$
\frac{\Phi^2(A)}{2\alpha} = \frac{1}{2\alpha} \Phi^j_i(A) \Omega^k_j \Phi^i_k(A),
$$

where, having defined the symmetric matrix

$$
\Omega^k_j \equiv \frac{1}{4} \delta^k_j, \tag{3.2}
$$

one finds

$$
\Phi^j_i(A) \Omega^k_j \Phi^i_k(A) = \frac{1}{4} \left[ 4(\partial^\mu A_\mu)(\partial^\nu A_\nu) + \beta (\partial^\mu A_\mu)(\gamma^\nu)^i_j A_\nu + \beta (\gamma^\mu)^i_j A_\mu \partial^\nu A_\nu \\
+ \beta^2 (\gamma^\mu)^i_j (\gamma^\nu)^j_i A_\mu A_\nu \right] = (\partial^\mu A_\mu)(\partial^\nu A_\nu) + \beta^2 A_\mu A^\mu, \tag{3.3}
$$
since the $\gamma$-matrices are traceless, and $\text{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$. When $\beta$ is set to zero, this reduces to the familiar Lorenz gauge-averaging term, which is why the numerical factors have been chosen as in the definition (3.2).

It should be stressed that the matrix (3.1) is a tool to express in a concise and elegant form the gauge-averaging term $\frac{\Phi^2(A)}{2\alpha}$ in the full action, but our gauge-averaging functional for QED is not a matrix and is equal to

$$\Phi(A) \equiv \sqrt{\Phi^j_i(A)\Omega_j^k\Phi^i_k(A)}.$$ (3.4)

One cannot regard $\Phi^j_i(A)$ itself as a gauge-averaging functional, since otherwise one would get 16 supplementary conditions which are totally extraneous to the quantum (as well as classical) theory and make it over-constrained. To make sure that the equation $\Phi(A) = \zeta$ admits a solution for all real $\zeta$, we have to require that the right-hand side of (3.3) should remain $\geq 0$ for all real $\beta$. In Minkowski space-time, this is achieved with $A_\mu A^\mu > 0$ and metric given by $\text{diag}(1,-1,-1,-1)$, or with $A_\mu A^\mu < 0$ and metric given by $\text{diag}(-1,1,1,1)$. In the Euclidean regime, where the $\gamma$-matrices are anti-Hermitian so that $g^{\mu\nu} = -\frac{1}{4}\text{Tr}(\gamma^\mu \gamma^\nu)$, one has then to take $\beta$ pure imaginary.

Note also that, on denoting by $I$ the $4 \times 4$ identity matrix, the operator $D^\mu \equiv I\partial^\mu + \beta \gamma^\mu$ in Eq. (3.1) bears apparently some resemblance with the super-covariant derivative first considered by Townsend\textsuperscript{(14)} within the framework of supergravity in anti-de Sitter space. However, the commutator $[D^\mu, D^\nu]$ does not vanish in Minkowski space-time, and is there equal to

$$[D^\mu, D^\nu] = \beta^2(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu).$$ (3.5)
Thus, it is impossible to relate the commutator of these derivatives to the space-time curvature, since the Riemann tensor of Minkowski space-time vanishes. The gauge curvature does not help either, because, on defining $\nabla_{\mu} \equiv \partial_{\mu} + qA_{\mu}$, one finds

$$[\nabla_{\mu}, \nabla_{\nu}] = q(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}) = qF_{\mu\nu},$$

which does not yield (3.5) upon mapping $A$ into $\gamma$. This means that no formal analogy can be actually proposed between the operator occurring in (3.1) and the Townsend construction\(^{(14)}\) for supergravity in anti-de Sitter.

4. PHOTON PROPAGATOR

As a first step towards quantization of non-Abelian theories\(^{(15–17)}\) we now consider a simpler but instructive problem, i.e. the photon propagator in the Euclidean version of quantum electrodynamics with gauge-averaging functional (3.4). In modern language, the path integral tells us that the photon propagator is obtained by first evaluating the gauge-field operator $P_{\mu\nu}$ resulting from the particular choice of gauge-averaging functional, then taking its symbol $\sigma(P_{\mu\nu})$ and inverting such a symbol to find $\sigma^{-1}(P_{\mu\nu})$ for which $\sigma\sigma^{-1} = \sigma^{-1}\sigma = I$. The photon propagator reads eventually (cf. Ref. 18)

$$\Delta^{\mu\nu}(x, y) = (2\pi)^{-4} \int_{\zeta} d^4k \, \sigma^{-1}(P_{\mu\nu})e^{i\zeta(x-y)}$$

(4.1)
for some contour $\zeta$, where $\sigma^{-1}(P_{\mu\nu}) \equiv \tilde{\sigma}^{\mu\nu}$ should be thought of as carrying contravariant indices, in agreement with the left-hand side. In the light of (3.1) and (3.3), our gauge-field Lagrangian (2.1) turns out to be, by virtue of gauge averaging,

$$\mathcal{L} = \partial^\mu \rho_\mu + \frac{1}{2} A^\mu P_{\mu\nu} A^\nu,$$

(4.2)

where

$$\rho_\mu \equiv -\frac{1}{2} A_\nu \partial^\nu A_\mu + \frac{1}{2} A_\nu \partial_\mu A^\nu + \frac{1}{2\alpha} A_\mu \partial^\nu A_\nu,$$

(4.3)

and

$$P_{\mu\nu} \equiv g_{\mu\nu} \left[ -\Box + \frac{\beta^2}{\alpha} \right] + \left( 1 - \frac{1}{\alpha} \right) \partial_\mu \partial_\nu.$$

(4.4)

Of course, the term $\rho_\mu$ only contributes to a total divergence and hence does not affect the photon propagator, while the parameter $\alpha$ can be set equal to 1 (Feynman choice) so that calculations are simplified. Thus, we can eventually obtain the gauge-field operator

$$P_{\mu\nu}(\alpha = 1) = g_{\mu\nu} \left( -\Box + \beta^2 \right).$$

(4.5)

The symbol of (4.5), which results from Fourier analysis of our translation-invariant operator, reads

$$\sigma(P_{\mu\nu}(\alpha = 1)) = (k^2 + \beta^2) g_{\mu\nu},$$

(4.6)

and hence our Euclidean photon propagator reads

$$\triangle^\mu_\nu(x, y) = (2\pi)^{-4} \int \frac{d^4k}{(k^2 + \beta^2)} e^{ik \cdot (x - y)},$$

(4.7)
where the points \( x \) and \( y \) refer to the indices \( \mu \) and \( \nu \), respectively. Note that integration along the real axis for \( k_0, k_1, k_2, k_3 \) avoids poles of the integrand, which are located at the complex points for which \( k^2 = -\beta^2 \). The choice \( \alpha = 1 \) has led to Eq. (4.7) which has the advantage of being very simple, but the correct interpretation of the term \( \beta^2 \) will only be clear after reading the following section.

In general, the ghost operator is obtained by contraction of functional derivatives of the gauge-averaging functional with the infinitesimal generators of gauge transformations.\(^{(6,7,17)}\) This leads to the \( - \Box \) operator in the Lorenz gauge, but we are considering the non-linear gauge-averaging functional (3.4) which therefore yields a more involved ghost operator \( P \), whose action is given by

\[
P : \varepsilon \to \left[ -\frac{(\partial^\nu A_\nu)}{\Phi(A)} \partial^\mu \partial_\mu - \beta^2 A^\mu \Phi(A) \partial_\mu \right] \varepsilon. \tag{4.8}
\]

It reduces to the ghost operator in the Lorenz gauge after imposing that photons are massless on observational ground (see Sec. 5).

5. OUTLINE OF PERTURBATIVE RENORMALIZATION

A crucial question is now in order, i.e. how the perturbative renormalization programme can be initiated with our choice (3.4) for the gauge-averaging functional. For this purpose, we study the full Lagrangian density \( \mathcal{L} \) for spinor electrodynamics, including the
gauge-averaging term. Since we are aiming to split $\mathcal{L}$ into a sum of physical and counterterm parts, we consider bare fields denoted by the $B$ subscript and physical fields written without such a subscript. To begin, for the gauge potential and the spinor field we assume that renormalization can be performed by considering the following relations:

$$ (A_\mu)_B = \sqrt{z_A} A_\mu, \quad \Rightarrow F_B^{\mu\nu} = \partial^\mu A^\nu_B - \partial^\nu A^\mu_B, \quad (5.1) $$

$$ \psi_B = \sqrt{z_\psi} \psi, \quad (5.2) $$

and similarly for mass, charge and gauge parameter respectively, i.e.

$$ m_B = \frac{z_m}{z_\psi} m, \quad (5.3) $$

$$ e_B = \frac{z_e}{z_\psi \sqrt{z_A}} e, \quad (5.4) $$

$$ \alpha_B = \frac{z_A}{z_\alpha} \alpha. \quad (5.5) $$

Moreover, we also introduce, at the beginning of renormalization, the equation

$$ \beta_B = \rho \beta, \quad (5.6) $$

where $\rho$ can be fixed in due course (see below). It should be stressed that multiplicative renormalizability of QED (i.e. the renormalization relying upon the $z$-factors and $\rho$-factor) with our gauge (3.4) is a conjecture at this stage, because multiplicative renormalizability is not a universal property of gauge theories independently of the gauge condition, but it will be justified ‘a posteriori’, once that the bare photon propagator is obtained in (5.19), and bearing in mind what we said about the ghost propagator following (4.8).
Now we are ready to write the Lagrangian density in Minkowski space-time (the part \( \mathcal{L}_{gh} \) involving Faddeev–Popov ghost fields is not written explicitly, since it does not affect the following calculations):

\[
\mathcal{L} - \mathcal{L}_{gh} = -\frac{1}{4} F_{\mu\nu} F_{\nu|^\mu} + \overline{\psi}_{B} \left( i\gamma^{\mu} \partial_{\mu} - e_{B} \gamma_{\mu} A_{\mu B} \right) \psi_{B} \\
- m_{B} \overline{\psi}_{B} \psi_{B} - \frac{1}{2\alpha_{B}} [\Phi(A_{B})]^{2}, \tag{5.7}
\]

where this general formula is here considered for \( \Phi(A_{B}) \) given by (3.4), with \( \beta \) and the potential \( A_{\mu} \) replaced by their renormalized values therein. By virtue of (5.3)–(5.7) our Lagrangian density admits the split

\[
\mathcal{L} - \mathcal{L}_{gh} = \mathcal{L}_{ph} + \mathcal{L}_{ct}, \tag{5.8}
\]

where the physical part (also called basic) reads

\[
\mathcal{L}_{ph} = \frac{1}{4} F_{\mu\nu} F_{\nu|^\mu} + \overline{\psi} i\gamma^{\mu} \partial_{\mu} \psi - e \overline{\psi} \gamma^{\mu} A_{\mu} \psi - m \overline{\psi} \psi \\
- \frac{1}{2\alpha} (\partial^{\mu} A_{\mu})^{2} - \frac{\beta^{2}}{2\alpha} A_{\mu} A^{\mu}, \tag{5.9}
\]

while the part involving counterterms is given by (cf. Ref. 18)

\[
\mathcal{L}_{ct} = -\frac{1}{4} (z_{A} - 1) F_{\mu\nu} F_{\nu|^\mu} + (z_{\psi} - 1) \overline{\psi} i\gamma^{\mu} \partial_{\mu} \psi - (z_{e} - 1) e \overline{\psi} \gamma^{\mu} A_{\mu} \psi \\
- (z_{m} - 1) m \overline{\psi} \psi - \frac{1}{2\alpha} (z_{\alpha} - 1) (\partial^{\mu} A_{\mu})^{2} - \frac{\beta^{2}}{2\alpha} (\rho^{2} z_{\alpha} - 1) A_{\mu} A^{\mu}. \tag{5.10}
\]

In the equation for \( \mathcal{L}_{ph} \) the parameters \( e, m \) and

\[
m_{\gamma}^{2} \equiv \frac{\beta^{2}}{\alpha} \tag{5.11}
\]
should be fixed by experiment. This simple equation is one of the most fundamental in our paper, and it tells us that the physical mass parameter $m_\gamma$ is not $\beta$ but actually the ratio $\frac{\beta}{\sqrt{\alpha}}$. At non-perturbative level, $\alpha$ and $\beta$ are independent, but upon implementing perturbative renormalization we end up by having a freely specifiable gauge parameter $\alpha$ (as expected) and a physical parameter $m_\gamma$ which is fixed on observational ground, so that $\beta$ disappears eventually as an independent parameter, being equal to $m_\gamma \sqrt{\alpha}$. Note also that, if

$$\rho = \frac{1}{\sqrt{z_\alpha}}, \quad (5.12)$$

the counterterm Lagrangian reduces to the familiar form in the Lorenz gauge,\(^{(18)}\) and the renormalization of $\beta$ is not independent of the renormalization of $\alpha$, in agreement with the definition (5.11). We shall therefore assume that Eq. (5.12) holds from now on, so that there are no sources of lack of gauge invariance in $L_{ct}$ apart from the term arising from the gauge fixing. Our approach to QED shows that the counterterm $A_\mu A^\mu$, which is compatible with Lorentz and charge conjugation invariance,\(^{(18)}\) can indeed be obtained from the gauge-fixed Lagrangian, while gauge invariance forces us to weight it with vanishing coefficient (otherwise gauge invariance would be broken by the quantum dynamics of QED\(^{(18)}\)).

We also find that, for arbitrary values of $\alpha_B$ and $\beta_B$, the symbol of the gauge-field operator in QED reads (cf. Sec. 4)

$$\sigma(P_{\mu\nu}) = \left( k^2 + \frac{m_\gamma^2}{z_A} \right) g_{\mu\nu} + \left( \frac{1}{\alpha_B} - 1 \right) k_\mu k_\nu \equiv \sigma_{\mu\nu}(k), \quad (5.13)$$
since
\[
\frac{\beta_B^2}{\alpha_B} = z_\alpha \rho \frac{1}{z_A} \frac{\beta^2}{\alpha} = \frac{m^2_{\gamma}}{z_A}, \tag{5.14}
\]
by virtue of (5.11) and (5.12). Its inverse \( \tilde{\sigma} \) is a combination of \( g^{\mu\nu} \) and \( k^\mu k^\nu \) with coefficients \( \mathcal{A} \) and \( \mathcal{B} \), respectively, determined from the condition
\[
\sigma_{\mu\nu} \tilde{\sigma}^{\nu\lambda} = \delta^\lambda_\mu, \tag{5.15}
\]
which implies
\[
\mathcal{A} = (k^2 + \bar{m}^2_\gamma)^{-1}, \tag{5.16}
\]
\[
\mathcal{B} = \frac{(\alpha_B - 1)}{(k^2 + \alpha_B \bar{m}^2_\gamma) (k^2 + \bar{m}^2_\gamma)}, \tag{5.17}
\]
upon defining
\[
\bar{m}^2_\gamma \equiv \frac{m^2_{\gamma}}{z_A}. \tag{5.18}
\]
At this stage, the bare photon propagator reads
\[
\Delta^{\mu\nu}(x, y) = \int \frac{d^4k}{(2\pi)^4} \left[ \frac{g^{\mu\nu}}{(k^2 + \bar{m}^2_\gamma)} + \frac{(\alpha_B - 1)k^\mu k^\nu}{(k^2 + \alpha_B \bar{m}^2_\gamma) (k^2 + \bar{m}^2_\gamma)} \right] e^{ik \cdot (x-y)}. \tag{5.19}
\]
We might have expressed Eq. (5.19) through the bare parameters \( \alpha_B \) and \( \beta_B \) only, but the explicit occurrence of the physical parameter \( \bar{m}^2_\gamma \) will prove useful in the next section.

Equation (5.19) shows a very important property: the photon propagator in momentum space falls off like \( k^{-2} \) at large \( k \) (the same occurs in the Stueckelberg model\(^{19}\)), which is the behaviour necessary to ensure perturbative renormalizability.\(^{18}\) As far as we
can see, the deeper roots of our explicit result (5.19) lie in the use of the BRST method which is known to lead to perturbative renormalizability independently of the particular choice of gauge-fixing condition,\(^{(15,16)}\) and also in the preservation of the linear nature of the gauge-field operator acting on \(A_\mu\) in the quantum theory (so that the rescaling (5.1) proves as useful as in the Lorenz gauge). Any addition by hand of mass terms to the original gauge-invariant Lagrangian leads instead to a photon propagator with no regular massless limit,\(^{(18)}\) since the purely Maxwell part of the Lagrangian does not lead to an invertible operator on \(A_\mu\).

In the massive QED model known so far in the literature one deals instead with the field equations\(^{(19)}\)

\[
(i\gamma^\mu\partial_\mu - MI)\psi = eA^\mu\gamma_\mu\psi, \tag{5.20}
\]

\[
\partial^\mu F_{\mu\nu} + m^2 A_\nu = -e\bar{\psi}\gamma_\nu\psi, \tag{5.21}
\]

leading to the photon propagator\(^{(19)}\)

\[
\triangle^{\mu\nu}(x,y) = \int \frac{d^4 k}{(2\pi)^4} \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{m^2} \right) \frac{-i}{(k^2 - m^2 + i\varepsilon)} e^{ik \cdot (x-y)}. \tag{5.22}
\]

Here the integrand is constant at large \(k\), and this leads to a non-renormalizable theory, in which the divergence of a Feynman diagram increases with the number of internal photon lines. A way out is provided by the introduction of an auxiliary vector field with free propagator in momentum space given by

\[
-i \left[ \frac{g^{\mu\nu}}{(k^2 - m^2 + i\varepsilon)} - \frac{k^\mu k^\nu}{m^2} \left( \frac{1}{(k^2 - m^2 + i\varepsilon)} - \frac{1}{(k^2 - m_0^2 + i\varepsilon)} \right) \right].
\]
A renormalizable model is now achieved in perturbation theory, but at a price: the Green functions depend on $m_0^2$ and describe an indefinite-metric Hilbert space with ghost particles.\textsuperscript{(19)} Such unpleasant features, however, are not shared by our model, where mass is not added by hand to the original Lagrangian, and the photon mass parameter is the physical mass, which is set to zero at the end of all calculations on observational ground.

6. RADIATIVE CORRECTIONS IN QED

A crucial step in quantum electrodynamics is the analysis of radiative corrections. For our purposes, we focus on the renormalized photon propagator with the associated polarization tensor, since they can be used to evaluate functions of physical interest, leading in turn to measurable predictions. At a deeper level, we are aiming to provide direct evidence that physical observables are independent of the $\beta$-parameter, as will be shown below.

We have just seen that, in the bare theory, one deals with the gauge-field operator $P_{\mu\nu}$ and its symbol $\sigma_{\mu\nu}$ with inverse $\bar{\sigma}^{\mu\nu}$. Integration of the latter in momentum space yields the photon propagator according to Eq. (5.19). Upon taking into account radiative corrections, $\bar{\sigma}^{\mu\nu}(k)$ is replaced by $\bar{\Sigma}^{\mu\nu}(k)$ according to the equation\textsuperscript{(20)}

$$
\bar{\Sigma}^{\mu\nu}(k) = \bar{\sigma}^{\mu\nu}(k) + \bar{\sigma}^{\mu\lambda} \Pi_{\lambda\rho} \bar{\Sigma}^{\rho\nu}(k),
$$

\textsuperscript{(6.1)}
where $\Pi_{\lambda\rho}$ is a symmetric rank-two tensor called the polarization, which is a sum of all diagrams that cannot be disconnected by cutting only one photon line. The tensor $\tilde{\Sigma}^{\mu\nu}$ is the inverse of the symmetric tensor here denoted by $\Sigma_{\mu\nu}$. Thus, Eq. (6.1) yields

$$\tilde{\Sigma}^{\mu\nu}\Sigma_{\nu\sigma} = \delta^\mu_\sigma = \tilde{\sigma}^{\mu\nu}\Sigma_{\nu\sigma} + \tilde{\sigma}^{\mu\lambda}\Pi_{\lambda\sigma},$$

and eventually

$$\sigma_{\gamma\mu}\delta^\mu_\sigma = \sigma_{\gamma\mu}\tilde{\sigma}^{\mu\nu}\Sigma_{\nu\sigma} + \sigma_{\gamma\mu}\tilde{\sigma}^{\mu\lambda}\Pi_{\lambda\sigma},$$

i.e.

$$\Pi_{\mu\nu}(k) = \sigma_{\mu\nu}(k) - \Sigma_{\mu\nu}(k). \quad (6.2)$$

The as yet unknown tensors $\Sigma_{\mu\nu}$ and $\tilde{\Sigma}^{\mu\nu}$ have the general form

$$\Sigma_{\mu\nu}(k) = g_{\mu\nu}u_1(k^2) + k_\mu k_\nu u_2(k^2), \quad (6.3)$$

$$\tilde{\Sigma}^{\mu\nu}(k) = g^{\mu\nu}d_1(k^2) + k^\mu k^\nu d_2(k^2), \quad (6.4)$$

and the condition

$$\Sigma_{\mu\nu}\tilde{\Sigma}^{\nu\lambda} = \delta^\lambda_\mu \quad (6.5)$$

yields

$$d_1 = \frac{1}{u_1}, \quad (6.6)$$

$$d_2 = -\frac{u_2 d_1}{(u_1 + k^2 u_2)}. \quad (6.7)$$

Equation (5.13) for $\sigma_{\mu\nu}(k)$, jointly with (6.2) and (6.3), yields

$$\Pi_{\mu\nu}(k) = g_{\mu\nu}(k^2 + \tilde{m}_\gamma^2 - u_1) + k_\mu k_\nu \left( \frac{1}{\alpha_B} - 1 - u_2 \right) = g_{\mu\nu}a_1(k^2) + k_\mu k_\nu a_2(k^2). \quad (6.8)$$
Moreover, the condition from current conservation that $\Pi_{\mu\nu}$ should be transverse, i.e. $k^\mu \Pi_{\mu\nu} = 0$, yields $a_1 = -k^2 a_2$, which leads to

$$u_1 + k^2 u_2 = \frac{1}{\alpha_B} (k^2 + \alpha_B \tilde{m}_\gamma^2), \quad (6.9)$$

and hence

$$\Pi_{\mu\nu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) (k^2 + \tilde{m}_\gamma^2 - u_1). \quad (6.10)$$

To express that $\Pi_{\mu\nu}$ is gauge-independent we require that, for some function $f$ of $k^2$ which cannot grow faster than $k^2$ at large $k$,\(^{(20)}\) one has

$$u_1 = \tilde{m}_\gamma^2 + f(k^2), \quad (6.11)$$

which implies from (6.9) that

$$u_2 = \frac{1}{\alpha_B} - \frac{f(k^2)}{k^2}. \quad (6.12)$$

In the bare theory, $f(k^2)$ reduces to $k^2$, and we find the bare tensor $\sigma_{\mu\nu}$, here re-written in the convenient form (cf. Eq. (5.13))

$$\sigma_{\mu\nu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) (k^2 + \tilde{m}_\gamma^2) + \frac{k_\mu k_\nu}{k^2} \frac{1}{\alpha_B} (k^2 + \alpha_B \tilde{m}_\gamma^2), \quad (6.13)$$

and the radiatively corrected tensor

$$\Sigma_{\mu\nu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) (f(k^2) + \tilde{m}_\gamma^2) + \frac{k_\mu k_\nu}{k^2} \frac{1}{\alpha_B} (k^2 + \alpha_B \tilde{m}_\gamma^2). \quad (6.14)$$

Thus, the coefficient of the longitudinal part $\frac{k_\mu k_\nu}{k^2}$ is the same in the bare as well as in the full theory\(^{(20)}\) in agreement with the Ward identity,\(^{(18)}\) while the coefficients of
the transverse part $g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$ depend on $\alpha_B$ and $\beta_B$ in such a way that the difference $\sigma_{\mu\nu}(k) - \Sigma_{\mu\nu}(k)$ is indeed gauge-independent:

$$
\Pi_{\mu\nu}(k) = \sigma_{\mu\nu}(k) - \Sigma_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \left(k^2 - f(k^2)\right).
$$

(6.15)

Eventually, Eqs. (6.6), (6.7), (6.11) and (6.12) yield the renormalized photon propagator by integrating in momentum space the tensor

$$
\tilde{\Sigma}^{\mu\nu}(k) = \frac{g^{\mu\nu}}{f(k^2) + \tilde{m}_\gamma^2} + \frac{\left(\alpha_B \frac{f(k^2)}{k^2} - 1\right) k^\mu k^\nu}{(k^2 + \alpha_B \tilde{m}_\gamma^2) \left(f(k^2) + \tilde{m}_\gamma^2\right)}.
$$

(6.16)

A first application of these formulae is given by radiative corrections to Coulomb’s law, which result from the polarization of the vacuum around a point charge. Indeed, the “effective” potential associated with a charge $q$ takes, in momentum space, the form

$$
\mathcal{A}^0 = A^0 + \tilde{\Sigma}^{0\rho} \Pi_{\rho\lambda} A^\lambda,
$$

(6.17)

where $A^0$ is proportional to $\frac{q}{k^2}$, while the contraction $\tilde{\Sigma}^{0\rho} \Pi_{\rho\lambda}$ reads, from our previous formulae,

$$
\tilde{\Sigma}^{0\rho} \Pi_{\rho\lambda} = \left(\delta^0_\lambda - \frac{k^0 k_\lambda}{k^2}\right) \frac{(k^2 - f(k^2))}{f(k^2) + \tilde{m}_\gamma^2},
$$

(6.18)

since $k^0 k^\rho d_2(k^2) \Pi_{\rho\lambda} = 0$. The renormalized potential is therefore

$$
\mathcal{A}^0 = A^0 + \frac{(k^2 - f(k^2))}{f(k^2) + \tilde{m}_\gamma^2} \left(A^0 - \frac{k^0}{k^2} k_\lambda A^\lambda\right).
$$

(6.19)

Note that the classical long-range part $\frac{q}{k^2}$ resulting from $A^0$ is still present, and eventually our $\tilde{m}_\gamma$ is set to zero on observational ground.
By virtue of the transverse nature of the polarization tensor $\Pi_{\rho\lambda}$, the full potential $A^0$ depends on gauge parameters $\alpha, \beta$ not separately, which would have led to unavoidable gauge dependence (since $\beta = m_\gamma \sqrt{\alpha}$), but only through the combination $\frac{1}{\epsilon_A} \frac{\beta^2}{\alpha}$. The latter is proportional to the photon mass parameter $m_\gamma^2$ in the physical Lagrangian of perturbative renormalization. Thus, the resulting short-range potential only depends on a mass parameter in the physical Lagrangian and is therefore, with the above understanding, gauge independent (and so are the coefficients of the transverse parts in Eqs. (6.13) and (6.14)). An example of gauge dependence is instead provided by the coefficient of $k^\mu k^\nu$ in Eq. (6.16) for the renormalized photon propagator in momentum space, where the numerator is equal to $\alpha_B \frac{f(k^2)}{k^2} - 1$, and the denominator contains the term $k^2 + \beta_B^2 = k^2 + \alpha_B \tilde{m}_\gamma^2$. But such a coefficient does not affect the renormalized potential, because Eq. (6.18) holds by virtue of the transverse nature of the polarization tensor.

Note also that, in the light of previous remarks, the most general form of Eq. (6.11) is

$$u_1 = u_1 \left( k^2 ; \frac{\beta_B^2}{\alpha_B} \right) = u_1 \left( k^2 ; \tilde{m}_\gamma^2 \right),$$

leading to $d_1 = \frac{1}{u_1(k^2;m_\gamma^2)}$ and

$$d_2 = \frac{1}{k^2} \left[ \frac{\alpha_B}{(k^2 + \alpha_B \tilde{m}_\gamma^2)} - \frac{1}{u_1(k^2;\tilde{m}_\gamma^2)} \right]$$

in Eq. (6.4). Once more, only $d_2$ is gauge-dependent, since its first term depends on $\alpha_B$.

The renormalized potential (6.19) can be therefore expressed in the general form

$$A_0 = A^0 + \frac{(k^2 + \tilde{m}_\gamma^2 - u_1(k^2;\tilde{m}_\gamma^2))}{u_1(k^2;\tilde{m}_\gamma^2)} \left( A^0 - \frac{k^0}{k^2} k_\lambda A^\lambda \right).$$

(6.22)
7. MASS TERMS IN NON-ABELIAN THEORY

The aim of the previous analysis was not to find an alternative to the Higgs mechanism for Abelian theories, for which no such mechanism is needed, but rather to prepare the ground for studying gauge theories relying upon non-Abelian groups. We can now work out how the above ideas can be applied to a non-Abelian gauge theory without Higgs field with group \( SU(2) \times U(1) \); an intriguing theoretical structure will be found to emerge, eventually.

First, the matrix (3.1) is replaced by an equation representing 4 of them (here no summation over the index \( a \) is understood)

\[
\Phi^a \equiv (I \partial^\mu + \beta_a \gamma^\mu) W^a_\mu. \tag{7.1}
\]

Note that, after the experience acquired in Secs. 3 and 4, we do not write explicitly matrix indices for \( I \) and \( \gamma^\mu \). According to the Faddeev-Popov\(^{(21)}\) path-integral prescription in the non-Abelian case, the resulting gauge-averaging term in the Lagrangian is written with the help of \( \Phi^a \) and of an invertible symmetric matrix \( I_{4 \tau_{ab}} \) in the form (here summation over repeated indices \( a \) and \( b = 0, 1, 2, 3 \) is instead understood)

\[
\frac{1}{2} \Phi^a I_{4 \tau_{ab}} \Phi^b.
\]

From now on we focus on the resulting mass-like term, which is our main goal. This reads, summing over all values of \( \mu, \nu \) and \( a, b \), and taking the matrix traces as in (3.3),

\[
\frac{1}{2} g^{\mu\nu} \beta_a W^a_\mu \tau_{ab} \beta_b W^b_\nu = f_1(W_1, W_2) + f_2(Z, Z) + f_3(A, A) + f_4(Z, A) + f_5(W_1, W_2, Z, A). \tag{7.2}
\]
With our notation, where we denote by \(k, l\) the indices \(a, b\) when taking the values 1, 2, we find, upon choosing

\[
\tau_{kl} = \tau \delta_{kl}, \tag{7.3}
\]

the formula

\[
f_1(W_1, W_2) = \frac{1}{2} g^{\mu \nu} \beta_k W^k_\mu \tau_{kl} \beta_l W^l_\nu = \frac{1}{2} \tau \beta_k^2 W^k_\mu W^\mu_k. \tag{7.4}
\]

The assumption (7.3) has been made since in the electroweak theory with Higgs field the mass term associated with \(W\) bosons can indeed be cast in the form (7.4). Moreover, by exploiting the identity \(g^{\mu \nu} Z_\mu Z_\nu = Z_\mu Z^\mu\), and choosing

\[
\tau_{03} = \tau_{30}, \tag{7.5}
\]

we find, by virtue of (1.3),

\[
f_2(Z, Z) = \frac{1}{2} \left( \beta_0^2 \sin^2 \theta \tau_{00} + \beta_3^2 \cos^2 \theta \tau_{33} - 2 \beta_0 \beta_3 \sin \theta \cos \theta \tau_{03} \right) Z_\mu Z^\mu, \tag{7.6}
\]

where we have set \(\theta_w \equiv \theta\) for simplicity of notation. Similarly, we find

\[
f_3(A, A) = \frac{1}{2} \left( \beta_0^2 \cos^2 \theta \tau_{00} + 2 \beta_0 \beta_3 \sin \theta \cos \theta \tau_{03} + \beta_3^2 \sin^2 \theta \tau_{33} \right) A_\mu A^\mu. \tag{7.7}
\]

Furthermore, by virtue of (7.5) and of the identity \(g^{\mu \nu}(Z_\mu A_\nu + A_\mu Z_\nu) = Z_\mu A^\mu + A_\mu Z^\mu\), the mixed term \(f_4(Z, A)\) reads

\[
f_4(Z, A) = \frac{1}{2} \left( \beta_3^2 \tau_{33} \sin \theta \cos \theta - \beta_0^2 \tau_{00} \sin \theta \cos \theta + \beta_0 \beta_3 \tau_{03} (\cos^2 \theta - \sin^2 \theta) \right) \times \left( Z_\mu A^\mu + A_\mu Z^\mu \right). \tag{7.8}
\]

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Last, the mixed term $f_5(W_1, W_2, Z, A)$ takes the form

$$f_5(W_1, W_2, Z, A) = \frac{1}{2} g^\mu\nu \left[ \beta_0 (-\sin \theta Z_\mu + \cos \theta A_\mu) \tau_{0k} \beta_k W_\nu^k + \beta_k W_\mu^k \tau_{k0} \beta_0 (-\sin \theta Z_\nu + \cos \theta A_\nu) + \beta_k W_\mu^k \tau_{k3} \beta_3 (\cos \theta Z_\nu + \sin \theta A_\nu) + \beta_3 (\cos \theta Z_\mu + \sin \theta A_\mu) \tau_{3k} \beta_k W_\nu^k \right].$$

(7.9)

Now we point out that, since the mixed term $f_5$ is not observed in nature, we have to set

$$\tau_{0k} = \tau_{k0} = 0, \tau_{3k} = \tau_{k3} = 0, \forall k = 1, 2.$$  

(7.10)

As far as the photon mass $m_\gamma$ is concerned, we keep it alive for the time being because the following calculations will show that it plays a key role in ensuring internal consistency of our model. Moreover, we bear in mind that no mixed term $f_4(Z, A)$ has ever been found in experiments. By virtue of (7.7) and (7.8), these two requirements lead to the equations

$$m_\gamma^2 = \left( \beta_0^2 \cos^2 \theta \tau_{00} + 2 \beta_0 \beta_3 \sin \theta \cos \theta \tau_{03} + \beta_3^2 \sin^2 \theta \tau_{33} \right),$$

(7.11)

$$-\beta_0^2 \sin \theta \cos \theta \tau_{00} + \beta_0 \beta_3 (\cos^2 \theta - \sin^2 \theta) \tau_{03} + \beta_3^2 \sin \theta \cos \theta \tau_{33} = 0.$$  

(7.12)

Such formulae imply that

$$\tau_{03} = \left( -\frac{\beta_3}{\beta_0} \tau_{33} + \frac{m_\gamma^2}{\beta_0 \beta_3} \right) \tan \theta,$$  

(7.13)

$$\tau_{00} = \frac{\beta_3^2}{\beta_0^2} \tau_{33} \tan^2 \theta + \frac{m_\gamma^2}{\beta_0^2} (1 - \tan^2 \theta).$$  

(7.14)
So far, the symmetric matrix $\tau_{ab}$ has been cast in the form

$$
\tau_{ab} = \begin{pmatrix}
\tau_{00} & 0 & 0 & \tau_{03} \\
0 & \tau & 0 & 0 \\
0 & 0 & \tau & 0 \\
\tau_{03} & 0 & 0 & \tau_{33}
\end{pmatrix}.
$$

(7.15)

For consistency, we have now to require that $\tau_{ab}$ should be non-singular, i.e.

$$
\det \tau_{ab} = \tau^2(\tau_{00}\tau_{33} - \tau_{03}^2) \neq 0.
$$

(7.16)

Remarkably, the contributions not involving $m_\gamma$ cancel each other exactly in the determinant (7.16), and one finds

$$
\tau_{00}\tau_{33} - \tau_{03}^2 = \frac{m_\gamma^2}{\beta_0^2} \left( \frac{\tau_{33}}{\cos^2 \theta} - \frac{m_\gamma^2 \tan^2 \theta}{\beta_3^2} \right).
$$

(7.17)

At this stage, the photon mass can be therefore very small for all practical purposes but nevertheless non-vanishing, for our model to be concretely applicable. The value of $\tau \beta_k^2$ in Eq. (7.4) is then fixed by requiring agreement with the experimental value of $m_W^2$, i.e. (cf. comments after (5.10))

$$
m_W^2 = \tau \beta_1^2 = \tau \beta_2^2,
$$

(7.18)

and the insertion of (7.13) and (7.14) into Eq. (7.6) makes it possible to fix the value of $\tau_{33}\beta_3^2$ by requiring agreement with the observed value of $m_Z^2$, i.e.

$$
m_Z^2 = \frac{1}{\cos^2 \theta} \left( \tau_{33}\beta_3^2 - m_\gamma^2 \sin^2 \theta \right).
$$

(7.19)

Interestingly, the first term on the right-hand side of Eq. (7.19) depends on the Weinberg angle exactly as in Eq. (1.4), which relies instead on the Higgs boson, but we now have
a correction resulting from the photon mass, which should be non-vanishing to ensure invertibility of the matrix (7.15) as we have seen. The issue of the photon mass is indeed not entirely settled. After the investigations in the seventies,(22–24) more recently high mass photon pairs have been considered at LEP,(25) while in other branches of modern physics the concept of photon (effective) mass is intimately related to the possibility of accelerating photons by moving plasma perturbations.(26) We will see in the following section how the $m_\gamma \to 0$ limit can be taken.

Note also that, since we only fix by experiment the products $\tau \beta_2^2$ and $\tau_33 \beta_3^2$, there is a residual gauge freedom in choosing, for example, non-vanishing values of $\tau$ and $\tau_33$ (see (7.16)), which then determine the $\beta$-parameters as functions of $m_W, m_Z, m_\gamma, \theta, \tau$ and $\tau_33$. This finding is in agreement with the general path-integral prescription for quantized gauge theories, according to which the masses of vector mesons should be independent of the particular invertible matrix $\tau_{ab}$.(7,17)

8. THE $m_\gamma \to 0$ LIMIT AND ITS IMPLICATIONS

The $m_\gamma \to 0$ limit deserves now a careful analysis to make sure that our model is viable. For this purpose, we first revert to quantum electrodynamics, since our $4 \times 4$ matrix $\tau_{ab}$ of gauge parameters corresponds to the $1 \times 1$ matrix $\frac{1}{\alpha}$ in QED and hence the det $\tau \to 0$ limit corresponds to the $\alpha \to \infty$ singular limit in QED. If the latter is understood, we
understand the former as well. Indeed, if we first impose that the four-momentum should have vanishing contraction with the four-current:

\[ k^\mu j_\mu = 0 \] (8.1)

by virtue of current conservation, and then take the limit

\[ \alpha \to \infty, \text{ with } m_\gamma^2 \to 0 \] (8.2)

according to experiment, we recover the massless photon propagator \( \frac{\eta^{\mu\nu}}{f(k^2)} \) in momentum space (cf. (6.16)).

The above order in which the operations are performed is crucial: first impose Eq. (8.1), which shows that \( k^\mu k^\nu \) terms do not affect physics and can be eventually omitted from the integrand defining the photon propagator. Then take the \( \alpha \to \infty \) limit while making sure that \( m_\gamma^2 \) approaches zero to agree with experiment. With this understanding, we can eventually set to zero \( m_\gamma \) in the mass formulae for \( SU(2) \times U(1) \) gauge theory, hence finding

\[
\lim_{m_\gamma \to 0} \frac{m_2^2}{m_W^2} = \frac{1}{\cos^2 \theta} \frac{\tau_{33}}{\tau} \left( \frac{\beta_3}{\beta_1} \right)^2 ,
\] (8.3)

where there exist infinitely many ways of making sure that

\[
\frac{\tau_{33}}{\tau} \left( \frac{\beta_3}{\beta_1} \right)^2 = 1,
\] (8.4)

so that full agreement with the standard formula for \( \frac{m_2^2}{m_W^2} \) is eventually recovered.
On setting $m_\gamma = 0$, the gauge-field operator acting on $A_\mu$ in our version of non-Abelian
gauge theory receives contributions from

$$\frac{1}{4}\left(\partial^\mu W^{3\nu} - \partial^\nu W^{3\mu}\right) \left(\partial_\mu W^{3}_\nu - \partial_\nu W^{3}_\mu\right),$$

$$\frac{1}{4}\left(\partial^\mu W^{0\nu} - \partial^\nu W^{0\mu}\right) \left(\partial_\mu W^{0}_\nu - \partial_\nu W^{0}_\mu\right),$$

and from the gauge-averaging term $\frac{1}{2} \Phi^a \frac{L}{4} \tau_{ab} \Phi^b$. Adding together the three resulting con-
tributions one gets, by virtue of (1.3) and (7.1), the following photon contribution to the
Lagrangian density: $\frac{\varepsilon}{2} A^\nu \mathcal{P}_\nu A_\mu$, where

$$\mathcal{P}_\mu = -\delta_\mu^\nu \partial^\rho \partial_\rho + \left(1 - \tau_{33} \sin^2 \theta \left(\frac{\beta_3}{\beta_0} - 1\right)^2\right) \partial_\nu \partial^\mu,$$

and $\varepsilon = 1$ for Euclidean theory, while $\varepsilon = -1$ in Minkowski space-time. At this stage, the
gauge-field operator for photons can be reduced to the minimal form $-\delta_\rho^\nu \partial^\rho \partial_\rho$ by choosing
the ratio of gauge parameters in such a way that

$$\frac{\beta_3}{\beta_0} = 1 + \frac{1}{\sqrt{\tau_{33}}} \frac{1}{\sin \theta}.$$  \hspace{1cm} (8.6)

Similarly to Eqs. (5.5) and (5.12), gauge parameters can be renormalized by setting

$$(\tau_{ab})_B = f_{ab} \tau_{ab},$$  \hspace{1cm} (8.7)

$$(\beta_a)_B = \rho_a \beta_a,$$  \hspace{1cm} (8.8)
and requiring that the counterterm Lagrangian should contain no massive term for photons.

This leads to the equations

\[ \rho_0^2 f_{00} = 1, \quad (8.9) \]

\[ \rho_0 \rho_3 f_{03} = 1, \quad (8.10) \]

\[ \rho_3^2 f_{33} = 1, \quad (8.11) \]

which are solved by

\[ \rho_0 = \frac{1}{\sqrt{f_{00}}}, \quad (8.12) \]

\[ \rho_3 = \frac{\sqrt{f_{00}}}{f_{03}} = \frac{1}{\sqrt{f_{33}}}. \quad (8.13) \]

It should however be stressed that, since there is no need to perform a gauge averaging in the path integral for spin-\(\frac{1}{2}\) fields,\(^{(7,17)}\) we do not succeed in finding alternatives to the Higgs mechanism for the generation of fermionic masses.

9. CONCLUDING REMARKS

The current models of mass generation in field theory rely on the assumption that Higgs bosons exist, with the associated Higgs mechanism.\(^{(1)}\) However, if Higgs bosons were to remain elusive, the problem remains to understand to which extent the general principles of quantum field theory make it possible to account for the existence of massive vector bosons.
Indeed, when Higgs elaborated his model\(^{(1)}\) the emphasis was very much on gauge-invariant Lagrangians, whereas it is by now clear and well accepted that the starting point for quantization of gauge theories is a Lagrangian (still called classical in Ref. 16) consisting of three ingredients: a gauge-invariant part, a gauge-breaking term and contribution of ghost fields.\(^{(7)}\) If, for a moment, we no longer assume that Higgs fields exist, we still have to work within this broader framework, although it is by no means obvious that we are going to find a viable scheme even just for describing the occurrence of mass. As a matter of fact, we have not succeeded in this respect, but our stronger findings, of field-theoretical interest, are as follows.

(i) Supplementary condition in QED chosen in the non-linear form (3.4). This suggests that $\gamma$-matrices generate the matrix $\Phi^i_j$ in (3.1) which, in turn, acts as a ‘potential’ for the gauge-fixing functional through Eq. (3.4).

(ii) New photon propagators in quantum electrodynamics, with a possibly deeper perspective on the massless nature of photons in vacuum QED (Secs. 4 and 5).

(iii) Renormalization of the gauge parameter $\beta$ in such a way that the counterterm Lagrangian has vanishing coefficient of $A_\mu A^\mu$ (Sec. 5), as in the ordinary formulation of QED in linear covariant gauges.\(^{(18)}\)

(iv) Evaluation of the renormalized photon propagator in our gauges, and proof of gauge independence of the associated short-range potential, adding evidence in favour of our model being physically relevant.
We have also considered mass terms in $SU(2) \times U(1)$ gauge theory, generalizing the Maxwell construction with the help of the invertible matrix (7.15), with gauge-averaging term reading

$$\frac{1}{2} \langle \Phi^a \rangle^j_i (\Omega_{ab})^k_j (\Phi^b)^i_k,$$

where $(\Omega_{ab})^k_j$ is the $16 \times 16$ matrix having diagonal form

$$(\Omega_{ab})^k_j \equiv \frac{1}{4} \delta^k_j \tau_{ab} = \frac{1}{4} \begin{pmatrix} \tau_{ab} & 0 & 0 & 0 \\ 0 & \tau_{ab} & 0 & 0 \\ 0 & 0 & \tau_{ab} & 0 \\ 0 & 0 & 0 & \tau_{ab} \end{pmatrix}.$$  \hspace{1cm} (9.1)

Note that both $\tau_{00}$ and $\tau_{03}$ depend linearly on $\tau_{33}$, and hence there exist infinitely many choices of $\tau_{33}$ leading to the same values of $m_Z^2$ (see (7.19)), and similarly for $\tau$ and $m_W^2$ (see (7.18)). Moreover, Eqs. (7.18) and (7.19) make it possible to express $\beta_1, \beta_2$ and $\beta_3$ in terms of $m_\gamma, \tau, m_\gamma, m_Z, \tau_{33}$ and the Weinberg angle $\theta$. Thus, eventually, the gauge-averaging functional is only found up to infinitely many possible choices of $\tau$ and $\tau_{33}$, in agreement with the basic requirement that masses of vector mesons should be independent of both $\Phi^a$ and $\tau_{ab}$.$^{(7,17)}$ The associated massless limit for photons has been studied in detail in Sec. 8; this is a singular limit which can only be taken after the general path-integral formulae for the photon propagator have been worked out, as we have shown therein. If our prescription for taking the $m_\gamma \to 0$ limit in Sec. 8 is rejected, the content of our paper remains purely Abelian but still quite interesting: the photon mass parameter is independent of any particular choice of the gauge parameter $\alpha$ and is set to zero on observational ground. It remains true that the gauge-averaging procedure has removed redundant degrees of freedom while not affecting the physics.
A legitimate objection against our scheme might be that, in the Abelian case, the mass has physical relevance since it has a non-trivial cohomological content\(^{(15,16)}\) whereas we might be hiding this property by appealing to gauge averaging in the path integral. However, non-trivial cohomology is relevant for the Abelian Higgs–Kibble model which assumes the existence of fundamental scalar fields, whereas we have assumed neither fundamental scalar fields nor massive photons from the outset.

While our paper was in preparation, the LEP collaboration has announced data which can be accounted for by assuming a Higgs boson with mass of about 115 GeV\(^{(27–31)}\). New theoretical investigations have been therefore performed, including a probability density calculation of the Higgs boson mass\(^{(32)}\). However, there is not yet conclusive evidence in favour of the existence of Higgs bosons, and only the Large Hadron Collider (LHC) can rule out some of the existing models. In particular, our model reflects the desire to develop theoretical physics with the minimal amount of structures and making use of known fields only.

As we acknowledge in Sec. 8, no path-integral approach can succeed in generating mass terms for spin-\(\frac{1}{2}\) fields. Nevertheless, it appears relevant to have found a mechanism for dealing with \textit{(but not truly generating!)} massive terms in the quantization of gauge theories while preserving perturbative renormalizability and independence of particular values of gauge parameters, as we have done explicitly in the Abelian case and advocated in non-Abelian gauge theories.

At field-theoretical level, it now appears important to prove perturbative renormalizability and investigate the possible occurrence of Gribov ambiguities\(^{(33)}\) in our gauges for
non-Abelian theories (although our analysis is inspired by perturbation theory, a framework where Gribov copies are expected to decouple\textsuperscript{(18)}, as well as mass terms for ghost fields.\textsuperscript{(16)} Moreover, since we end up by putting the emphasis on the space of four-vectors with components given by $4 \times 4$ matrices, which is a natural structure for theories incorporating fermions, a last effort is in order before ruling out that our scheme might be relevant for a deeper understanding of the standard model in particle physics. If this effort fails, one might have to resort to the mechanism suggested by Gribov in Ref. 34, according to which, in the standard model without elementary Higgs the fact that the $U(1)$ coupling becomes of order 1 at the Landau scale leads to spontaneous symmetry breaking and generation of masses.

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