Precritical Chiral Fluctuations in Nuclei$^a$

T. HATSUDA  
*Physics Department, Kyoto University, Kyoto 606-8502, Japan*  
E-mail: hatsuda@ruby.scphys.kyoto-u.ac.jp

T. KUNIHIRO  
*Faculty of Science and Technology, Ryukoku University, Seta, Otsu-city, 520-2194, Japan*  
E-mail: kuni@math.ryukoku.ac.jp

Spectral enhancement near $2m_\pi$ threshold in the $I=J=0$ channel in nuclei is shown to be a distinct signal of the partial restoration of chiral symmetry. The relevance of this phenomenon with possible detection of $2\pi^0$ and $2\gamma$ in hadron-nucleus and photo-nucleus reactions is discussed.

1 Introduction

The low density theorem in QCD and model calculations suggest that partial restoration of chiral symmetry takes place in nuclear medium. If the quark condensate decreases substantially in nuclear matter, fluctuation of the condensate becomes large in accordance with the general wisdom of statistical physics. This implies that there is a softening of a collective excitation in the scalar-isoscalar channel with the following physical consequences: (i) partial degeneracy of the scalar-isoscalar particle (traditionally called the $\sigma$-meson) with the pion, and (ii) decrease of the decay width of $\sigma$ due to the phase space suppression caused by (i) in the reaction $\sigma \rightarrow 2\pi$. Although $\sigma$ appears only as a broad resonance in the vacuum and is hard to be distinguished from the background, it was suggested that it may appear as a sharp resonance at finite temperature ($T$) because of (i) and (ii) discussed above.

In this talk, we demonstrate, using a toy model, that the spectral enhancement associated with the partial chiral restoration takes place also at finite baryon density close to $\rho_0 = 0.17\text{fm}^{-3}$. As possible experiments to detect this softening, we will discuss the production of the neutral-dipion ($2\pi^0$) and diphoton ($2\gamma$) in reactions with heavy nuclear targets. We also mention the relevance of the softening and the recent measurement of the near-threshold $\pi^+\pi^-$ production in $\pi^+$-nucleus reactions.

---

$^a$Talk given at the 1998 YITP-workshop on QCD and Hadron Physics (Oct. 14-18, 1998, Kyoto, Japan). Extended version of this talk is given in nucl/th-9810022 (T. Hatsuda, T. Kunihiro and H. Shimizu)
2 Basic idea

Let us first describe the general aspects of the spectral enhancement near the two-pion threshold. Consider the propagator of the $\sigma$-meson at rest in the medium:

$$D_{\sigma}^{-1}(\omega) = \frac{\omega^2 - m_{\sigma}^2 - \Sigma_{\sigma}(\omega; \rho)}{(\omega^2 - m_{\sigma}^2 - \Sigma_{\sigma}(\omega; \rho))^2 + (\text{Im}\Sigma_{\sigma})^2},$$

where $m_{\sigma}$ is the mass of $\sigma$ in the tree-level, and $\Sigma_{\sigma}(\omega; \rho)$ is the loop corrections in the vacuum as well as in the medium. The corresponding spectral function reads

$$\rho_{\sigma}(\omega) = -\frac{1}{\pi} \frac{\text{Im}\Sigma_{\sigma}}{(\omega^2 - m_{\sigma}^2 - \text{Re}\Sigma_{\sigma})^2 + (\text{Im}\Sigma_{\sigma})^2}.$$

Near the two-pion threshold, the phase space factor gives $\text{Im}\Sigma_{\sigma} \propto \theta(\omega - 2m_\pi) \left(1 - \frac{4m_\pi^2}{\omega^2}\right)^{1/2}$ in the one-loop order. On the other hand, partial restoration of chiral symmetry indicates that $m_{\sigma}^{\ast}$ ("effective mass" of $\sigma$ defined by $\text{Re}D_{\sigma}^{-1}(\omega = m_{\sigma}^{\ast}) = 0$) approaches to $m_\pi$. Therefore, there exists a density $\rho_c$ at which $\text{Re}D_{\sigma}^{-1}(\omega = 2m_\pi)$ vanishes even before the complete $\sigma$-$\pi$ degeneracy takes place. At $\rho = \rho_c$, the spectral function is solely dictated by the imaginary part of the self-energy:

$$\rho_{\sigma}(\omega \simeq 2m_\pi) = -\frac{\pi}{\omega} \frac{\text{Im}\Sigma_{\sigma}}{\text{Im}\Sigma_{\sigma}} \propto \theta(\omega - 2m_\pi) \left(1 - \frac{4m_\pi^2}{\omega^2}\right)^{-1/2}.$$

This implies that, even if there is no sharp resonance in the scalar channel in the vacuum, there arises a mild (integrable) singularity just above the threshold in the medium. This is a general phenomenon associated with the partial restoration of chiral symmetry.

3 Linear sigma model

Let us now evaluate $\rho_{\sigma}(\omega)$ in a toy model, namely the SU(2) linear $\sigma$-model:

$$L = \frac{1}{4} \text{tr}[\partial M\partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!} (M M^\dagger)^2 - h(M + M^\dagger)],$$

where tr is for the flavor index and $M = \sigma + i \vec{\tau} \cdot \vec{\pi}$. Although the model is not a precise low energy representation of QCD, it is known to describe the pion dynamics qualitatively well up to 1 GeV.

The coupling constants $\mu^2, \lambda$ and $h$, which are determined in the vacuum to reproduce $f_\pi = 93$ MeV, $m_\pi = 140$ MeV as well as the $s$-wave $\pi$-$\pi$ scattering phase shift in the one-loop order, are recapitulated for two characteristic cases in Table 1 in which $m_{\sigma}^{\text{peak}}$ is defined as a peak position of $\rho_{\sigma}(\omega)$.

Let us now consider the $\sigma$-meson embedded in nuclear matter. The interaction of $M$ with the nucleon $N$ in SU(2) chiral symmetry is modeled as

$$\mathcal{L}_I(N, M) = -g\chi \bar{N}U_5 N - m_0 \bar{N}U_5 N,$$
Table 1: Parameters for $m_{\sigma}^{\text{peak}} = 550, 750$ MeV taken from ref.6.

| (I) | $m_{\sigma}^{\text{peak}}$ (MeV) | $\sqrt{-\mu^2}$ (MeV) | $\lambda/4\pi$ | $h^{1/3}$ (MeV) |
|-----|----------------------------------|------------------------|----------------|----------------|
|     | 550                              | 284                    | 5.81           | 123            |
| (II)| 750                              | 375                    | 9.71           | 124            |

where we have used a polar representation $\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5 = \chi U_5$ for convenience. The first term in (I) with $g$ is a standard chiral invariant coupling in the linear $\sigma$ model. The second term with a new parameter $m_0$ is also chiral invariant and non-singular, but is not usually taken into account in the literatures.

After the dynamical breaking of chiral symmetry in the vacuum ($\langle \sigma \rangle_{\text{vac}} \equiv \sigma_0 \neq 0$), the coupling of $\tilde{\sigma} = \sigma - \sigma_0$ and $\pi$ with $N$ is dictated by $g_s \equiv g$ and $g_p \equiv g_s + m_0/\sigma_0$, respectively. Because of $m_0$, the standard constraint $g_s = g_p$ can be relaxed without conflicting with chiral symmetry. $g_p$ is constrained as $g_p = m_N/\sigma_0 \simeq m_N/f_\pi = g_{\pi N} = 13.5$, while $g_s$ is independent from $g_p$ and can be treated as a free parameter. This freedom partially avoid the well-known problem that $g_s = g_p$ combined with eq.(3) does not reproduce the known nuclear matter properties in the mean-field level.

4 One-loop estimate

Let us parametrize the chiral condensate in the medium as

$$\langle \sigma \rangle = \sigma_0 \Phi(\rho). \tag{5}$$

In the linear density approximation, $\Phi(\rho) = 1 - C\rho/\rho_0$ with $C = (g_s/\sigma_0 m_0^2)\rho_0$. The plausible value of $\Phi(\rho = \rho_0)$ is $0.7 \sim 0.9$.

The one-loop corrections to the self-energy for $\sigma$ can be read off from the diagrams in Fig.1: $\Sigma_\sigma(\omega; \rho) = \Sigma^A_{\text{vac}} + \Sigma^B_{\text{vac}} + \Sigma_{MF}(\rho) + \Sigma_{ph}(\rho)$. $\Sigma^A_{\text{vac}} (\Sigma^B_{\text{vac}})$ corresponds to Fig.1A and Fig.1B, respectively. The explicit form of $\Sigma^A_{\text{vac}} (\Sigma^B_{\text{vac}})$ is given in Appendix A of ref.6. $\Sigma_{MF}(\rho)$ corresponds to the mean field correction in the nuclear matter (Fig.1C). $\Sigma_{ph}(\rho)$ is a correction from the nuclear particle-hole excitation. We take only the density dependent part in these diagrams and neglect the problematic vacuum-loops of the nucleon.

The leading term in the mean-field part is easily estimated as

$$\Sigma_{MF}(\rho) = \lambda \sigma_0 (\langle \sigma \rangle - \sigma_0) = -\lambda \sigma_0^2 (1 - \Phi(\rho)), \tag{6}$$

Leading term in the particle-hole part (Fig.1D) starts from $O(\rho^{5/3})$ and is not more than a few % of $\Sigma_{MF}(\rho)$ at $\rho = \rho_0$. This is in contrast to the case of the pion, where both $\Sigma_{MF}(\rho)$ and $\Sigma_{ph}(\rho)$ are proportional to $\rho$ and cancel.
with each other due to chiral symmetry. Because of this cancellation, we can neglect the two-loop contribution related to the medium modification of the low-momentum pion near the two-pion threshold.

Up to the order we are considering, \( \text{Im} \Sigma_\sigma \) solely comes from \( \text{Im} \Sigma_{\text{vac}}^B \), since there is no Landau damping and scalar-vector mixing for the \( \sigma \)-meson at rest in nuclear matter. For \( 2m_\pi \leq \omega \leq 2m_\sigma \),

\[
\text{Im} \Sigma_\sigma (\omega; \rho) = \text{Im} \Sigma_{\text{vac}}^B = \frac{\lambda^2}{96\pi} \sigma_0^2 \sqrt{1 - \frac{4m_\pi^2}{\omega^2}}.
\]  

(7)

As we have already discussed, the threshold peak is expected to be prominent when \( \text{Re} D^{-1}_\sigma = \omega^2 - m_\sigma^2 - \text{Re} \Sigma_\sigma = 0 \). In terms of \( \Phi \), this condition is rewritten as \( \Phi(\rho_c) = 0.74 \) (for case (I) in table 1), and 0.76 (for case (II) in Table 1). The numbers in the right hand side are rather insensitive to the parameters in Table 1 as far as the physical quantities in the vacuum are fixed. In the linear density formula \( \Phi(\rho) \approx 1 - 0.2\rho/\rho_0 \), we obtain \( \rho_c \approx 1.25\rho_0 \). The spectral functions together with \( \text{Re} D^{-1}_\sigma (\omega) \) for two cases (I) and (II) are shown in Fig.2. The characteristic enhancements just above the \( 2m_\pi \) threshold are seen for \( \rho \approx \rho_c \).

We emphasize here that there are two reason behind the enhancement: partial restoration of chiral symmetry \((m_\sigma^* \rightarrow m_\pi)\), and the cusp structure of \( \text{Re} D^{-1}_\sigma (\omega = 2m_\pi) \). Both features can be seen in the lower panels of Fig.2. Although the cusp is not prominent at zero density, it eventually hits the real axis at \( \rho = \rho_c \) because \( \text{Re} D^{-1}_\sigma (\omega) \) increases associated with \( m_\sigma^* \rightarrow 2m_\pi \). This is a general phenomena for systems where the internal symmetry is partially restored in the medium. Another important observation is that, even at densities well below the point where \( m_\sigma^* \) and \( m_\pi \) are degenerate, one can expect a large enhancement of \( \rho_\sigma (\omega \approx 2m_\pi) \).

5 Experiments

To confirm the threshold enhancement associated with the partial chiral restoration, measuring \( 2\pi^0 \) and \( 2\gamma \) with hadron/photon beams off the heavy nuclear
targets should be most appropriate. By measuring $\sigma \rightarrow 2\pi^0 \rightarrow 4\gamma$, one can avoid the possible $I = J = 1$ background from the $\rho$ meson inherent in the $\pi^+\pi^-$ measurement. Measuring the electromagnetic decay $\sigma \rightarrow 2\gamma$ is also important because of the small final state interactions. There is also a possibility that one can detect dilepton through the scalar-vector mixing in matter: $\sigma \rightarrow \gamma^* \rightarrow e^+e^-$. To enhance the production cross section of the critical fluctuation in the $\sigma$ channel, $(d, ^3\text{He})$ reactions may be useful as the case of $\eta$ and $\omega$ mesic nuclei.

To cover the spectral function in the range $2m_\pi < \omega < 750$ MeV, the incident kinetic energies of $d$ in the laboratory system is estimated as $1 \text{GeV} < E < 10 \text{GeV}$.

Recently CHAOS collaboration measured the $\pi^+\pi^\pm$ invariant mass distribution $M^A_{\pi^+\pi^\pm}$ in the reaction $A(\pi^+, \pi^+\pi^\pm)X$ with the mass number $A$ ranging from 2 to 208: They observed that the yield for $M^A_{\pi^+\pi^-}$ near the $2m_\pi$ threshold is close to zero for $A = 2$, but increases dramatically with increasing $A$. They identified that the $\pi^+\pi^-$ pairs in this range of $M^A_{\pi^+\pi^-}$ is in the $I = J = 0$ state. Attempts so far in hadronic models without considering the partial chiral restoration failed to reproduce this enhancement. On the other hand, $A$ dependence of the the invariant mass distribution of the CHAOS data near $2m_\pi$ threshold has close resemblance to our model calculation in Fig.2, which suggests that this experiment may already provide a hint about how the
(partial) restoration of chiral symmetry manifests itself at finite density.

6 Summary

In summary, we have shown that the spectral function in the $I = J = 0$ channel has a large enhancement near the $2m_\pi$ threshold even at nuclear matter density due to the partial chiral restoration. Detection of the dipion and diphoton spectral distribution in the reactions of hadron/photon with heavy nucleus is suitable to confirm the idea of partial chiral restoration in nuclei.

References

1. T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 221 (1994). G. E. Brown and M. Rho, Phys. Rep. 269, 333 (1996).
2. P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977). S. Sachdev, cond-mat/9705268.
3. T. Hatsuda and T. Kunihiro, Phys. Lett. 145 (1984) 7; Phys. Rev. Lett. 55, 158 (1985); Phys. Lett. B185, 304 (1987).
4. Particle Data Group, Eur. Phys. J. C 363, 390 (1998).
5. H. A. Weldon, Phys. Lett. B274, 133 (1992). C. Song and V. Koch, Phys. Lett. B404, 1 (1997).
6. S. Chiku and T. Hatsuda, Phys. Rev. D58, 076001 (1998).
7. CHAOS Collaboration, Phys. Rev. Lett. 77, 603 (1996).
8. J. Gasser and H. Leutwyler, Ann. Phys. 158, 142 (1984).
9. L. H. Chan and R. W. Haymaker, Phys. Rev. D7, 402 (1973); ibid. D10, 4170 (1974).
10. B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E6, 515 (1997).
11. H. Shimizu, in Proceedings of XV RCNP Osaka International Symposium, Nuclear Physics Frontiers with Electro-weak Probes (FRONTIER 96), (Osaka, March 7-8, 1996), p.161.
12. T. Kunihiro, Prog. Theor. Phys. Supplement 120, 75 (1995).
13. R.S. Hayano, S. Hirenzaki and A. Gillitzer, nucl-th/9806012.
14. R. Rapp, J. W. Durso and J. Wambach, Nucl. Phys. A596, 436 (1996).
15. H. C. Chiang, E. Oset, and M. J. Vicente-Vacas, Nucl. Phys. A644, 77 (1998).