PROBABILISTIC INFERENCE OF BASIC STELLAR PARAMETERS: APPLICATION TO FLICKERING STARS

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ABSTRACT

The relations between observable stellar parameters are usually assumed to be deterministic. That is, given an infinitely precise measurement of independent variable, “\(x\)”, and some model, the value of dependent variable, “\(y\)”, can be known exactly. In practice this assumption is rarely valid and intrinsic stochasticity means that two stars with exactly the same “\(x\)” will have slightly different “\(y\)”s. The relation between short-timescale brightness fluctuations (flicker) of stars and both surface gravity and stellar density are two such stochastic relations that have until now been treated as deterministic ones. We recalibrate these relations in a probabilistic framework, using hierarchical Bayesian modeling to constrain the intrinsic scatter in the relations. We find evidence for additional scatter in the relationship, that cannot be accounted for by the observational uncertainties alone. The scatter in surface gravity and stellar density does not depend on flicker, suggesting that using flicker as a proxy for log g and \(\rho_*\) is equally valid for dwarf and giant stars, despite the fact that the observational uncertainties tend to be larger for dwarfs.

Key words: methods: statistical – stars: fundamental parameters – techniques: photometric

1. INTRODUCTION

Accurate stellar characterization plays a vital role for many active research fields within astronomy. For example, the study of binary stars, asteroseismology, and exoplanet studies all rely on inferences of basic stellar parameters to varying degrees. Empirically derived and reliable estimates are of particular value, increasing our confidence in the end-product results built upon these inputs.

Basic stellar parameters, such as effective temperature and surface gravity, can be inferred using one (or more) of several types of observations, such as spectroscopy, photometry, interferometry, etc. This inference can be performed by invoking theoretical models or by building an empirical calibration library. For example, an observed stellar spectrum could be matched against a library of theoretical spectra generated using stellar atmosphere models, or against a library of observed spectra of “standard stars,” serving as calibrators. Regardless of the approach, be it theoretical or empirical, the methods used for the inference of stellar parameters are traditionally “deterministic.” In this context, a deterministic model can be loosely described as one in which a particular observational input always returns a single-valued output for a parameter of interest, i.e., nature itself has no variance and the underlying model is considered to be a perfect description of reality.

An alternative approach for inferring model parameters is to allow relationships between observables to be stochastic. In recent years, there has been a shift toward such methods in several areas of astronomy, particularly within the exoplanet community. For example, Wolfgang et al. (2015) considered that the mass–radius relationship of exoplanets is stochastic because a particular sized planet could be have a range of planet masses due to unmodeled variances in compositions, environment, and other complications. These recent demonstrations in exoplanetary science have prompted us to consider the need for treating the parent stars in the same probabilistic framework, with potential applications spanning many fields of astronomy.

The demand for probabilistic stellar parameters is not only motivated by the fact that probability distributions are far more representative of our “beliefs” about astrophysical parameters; it also has a practical purpose. When using data published in the astronomical literature to, for example, infer relationships between parameters that are themselves the product of an inference process (for example, exoplanet transit depth and period), inference can be performed as the final stage in a hierarchical treatment (see, e.g., Foreman-Mackey et al. 2014a). Studies such as these are benefited by posterior probability distribution function (PDF) samples, rather than point estimates of inferred properties.

One of the more recent tools developed to characterize stars is known as “flicker” (Bastien et al. 2013). Flicker is a proxy for the scatter on an 8 hr timescale (denoted as \(F_8\)) in a broad visible bandpass timeseries photometric light curve, such as that from \textit{Kepler} or the upcoming \textit{TESS} mission. A more detailed account of the procedure to calculate flicker is described in Bastien et al. (2013). As shown in Bastien et al. (2013), flicker displays a remarkable correlation to the asteroseismically determined parent star surface gravities (log g). Turning this around, the observation implies that flicker can be used to empirically infer surface gravities at the level of \(\sim 0.1\) dex, an attractive proposition given the wealth of photometric light curves available through the array of exoplanet transit missions flying and scheduled to launch.

Cranmer et al. (2014) demonstrated that models of stellar surface granulation indeed reproduce a flicker effect in close agreement with that observed by Bastien et al. (2013), providing a physically plausible explanation. Since surface gravity is highly correlated with mean stellar density (\(\rho_*\)) on evolutionary tracks, Kipping et al. (2014) showed that flicker can be also be used to infer \(\rho_*\), which is more useful for exoplanet transit analysis (Seager & Mallén-Ornelas 2003).
Whether one calibrates flicker to \( \log g \) or \( \rho_* \), there are several aspects of the problem that are attractive for our purposes of a simple demonstration of probabilistic inference of stellar parameters. First, in \( \log g \) space the relationship is very simple, appearing to be linear (Kipping et al. 2014). Second, there is a sufficiently large number of points in the sample (439 stars) to constrain a population-based model. Third, there is significant excess scatter around the best-fitting relation implying that a deterministic model is inadequate. This is not surprising, given that granulation is a complex and messy process for which one should not expect any parametric model to provide a perfect description. Finally, the physical processes that produce surface granulation, of which flicker is an observational tracer, may be more or less noisy for different types of stars. We will test whether flicker has greater predictive power in certain regions of parameter space; i.e., is flicker significantly more informative for subgiants than for dwarfs? For these reasons we identify the calibration of flicker to \( \log g \) and \( \rho_* \) as a well-posed problem to first demonstrate probabilistic inference in the arena of stellar characterization.

### 2. Probabilistic Calibration

#### 2.1. Calibration Data

For our calibration data we used a sample of Kepler stars with both asteroseismic and flicker measurements available. Chaplin et al. (2014) report asteroseismic \( \rho_* \) estimates (and the associated uncertainties) for 518 Kepler stars. The authors report three different sets of results, depending on the choice of \( T_{\text{eff}} \) and [Fe/H]. In this work we elected to use values reported in their Table 6 over Table 5, and Table 5 over Table 4. We additionally used the 71 additional planet hosting stars with asteroseismology reported in Huber et al. (2013) but not reported in Chaplin et al. (2014). Values for flicker and “range” were taken from Kipping et al. (2014), based upon the methods described in Bastien et al. (2013). In order to use the same data set as Kipping et al. (2014) and for reasons described therein, we only include targets in our calibration for which

1. Range (defined in Bastien et al. 2013) <1000 ppm
2. \( 4500 < T_{\text{eff}} < 6500 \) K
3. \( K_p < 14 \)
4. \( 1.2 < \log_{10} (F_8 \, [\text{ppm}]) < 2.2 \).

We use the same sample for our calibration of \( \log g \) except that we exclude the Huber et al. (2013) data, as those authors do not provide estimates of \( \log g \).

#### 2.2. Hierarchical Bayesian Model

We model the stochastic relationship between \( F_8, \log g \) and \( \rho_* \), accounting for the fact that there exists some intrinsic scatter in the dependent variable. There are two excellent reasons for modeling the relation stochastically. First, if the intrinsic scatter is ignored and the relation between variables is assumed to be deterministic, those data points with smaller measurement uncertainties may have an unrepresentative greater weighting during the fitting process (Kelly 2007; Hogg et al. 2010). Second, we are interested in producing probability distributions over stellar densities and surface gravities, as opposed to point estimates, and propagating these probability distributions through to subsequent analyses. Several recent studies have required posterior PDF samples to conduct their hierarchical analyses (e.g., Foreman-Mackey et al. 2014a; Angus et al. 2015; Rogers 2015).

The two models we use to describe the relationships between \( F_8, \log g \) and \( \rho_* \) are

\[
\text{log}_{10}(F_8) \sim \mathcal{N}(\alpha_{\rho} + \beta_{\rho} \log_{10}(\rho_*), \sigma^2_{\rho}), \quad (1)
\]

and

\[
\text{log}_{10}(F_8) \sim \mathcal{N}(\alpha_g + \beta_g \log_{10}(g), \sigma^2_g). \quad (2)
\]

The free parameters of the two models are \( \alpha_{\rho}, \beta_{\rho}, \sigma_{\rho}, \alpha_g, \beta_g \) and \( \sigma_g \). These relations are Gaussian distributions with means given by the equation of a straight line and standard deviations which describe the intrinsic scatter about the mean. We used the MCMC package, emcee (Foreman-Mackey et al. 2013) to explore the posterior PDFs of our model parameters.

We also tested a model in which the additional scatter depends on flicker itself, defined as

\[
\text{log}_{10}(F_8) \sim \mathcal{N}(\mu + \alpha_{\rho} \log_{10}(\rho_*), \sigma^2_{\rho} + \gamma_{\rho} \log_{10}(F_8)), \quad (3)
\]

for flicker versus \( \rho_* \) and similarly for \( \log g \). This model allowed us to determine whether there the magnitude of additional scatter varied as a function of flicker. In other words, whether flicker was a better proxy for \( \log g \) or \( \rho_* \), for either dwarf or giant stars. We found that the maximum a-posteriori values for the \( \gamma \) parameters were consistent with zero: \( \gamma_{\rho} = 0.006 \pm 0.02 \), \( \gamma_g = -0.01 \pm 0.01 \), and interpret this as evidence for a constant intrinsic scatter level across evolutionary stages.

We used a likelihood function which accounts for two-dimensional (2D) uncertainties but does not allow the intrinsic scatter to be a function of the dependent or independent variables. For the relation between flicker and \( \rho_* \), this likelihood function can be written as

\[
\ln \left[ \frac{p(F_8|\rho_*, \alpha, \beta, \sigma)}{\gamma_{\rho}} \right] \propto - \frac{1}{2} \sum_{n=1}^{N} \left[ \frac{(F_{8n} - (\alpha_{\rho} + \beta_{\rho} \rho_{*n}))^2}{\sigma^2_{\rho}} \right] + \ln(\sigma^2_{F_8,n}) + \ln(\sigma^2_{\rho})
\]

and similarly for \( \log g \) (Hogg 2010). We found that the posterior PDFs for the model parameters obtained using this likelihood function were consistent with those obtained using a model that only accounts for the uncertainties on the flicker measurements. The median values of the model parameters differed by around 0.05\( \sigma \) for the \( \alpha \) and \( \beta \) parameters, by 0.3\( \sigma \) for \( \sigma_{\rho} \), and by 0.8\( \sigma \) for \( \sigma_g \). Because they are so dependent on the observational uncertainties, the parameters that describe the intrinsic scatter in the relations are more sensitive to whether the uncertainties in the \( x \)-direction are included. Accounting for uncertainties on \( y \) and \( x \) is not essential in this case, but is still good practice and will, in general, produce more accurate model parameters and uncertainties.

We used the uninformative prior for the parameters of a straight line for data with unknown uncertainties, outlined in...
We also tested uniform, flat priors as defined below:
\[
\begin{align*}
\alpha, \beta & \sim U(-10 : 10) \\
\log(\sigma_{\text{flat}}), \log(\sigma_g) & \sim U(-10 : 10).
\end{align*}
\]

We found that the results were relatively insensitive to the choice of prior, with median parameter values differing by only around 0.05 \( \sigma \). MCMC chains were run until the Gelman & Rubin convergence criterion, \( \hat{R} \), reached a value of less than 1.002 and the number of autocorrelation times was greater than 35. Figures 1 and 2 show the data with the best-fit models. The shaded regions show the 1 and 2\( \sigma \) confidence interval which are representative of the intrinsic scatter in the relations. The marginal posterior PDFs of the model parameters for \( \rho \) are shown in Figure 3. The marginal posterior PDFs for \( \log g \) are similarly Gaussian and, as with \( \sigma_{\rho} \) and \( \sigma_{g} \), is clearly greater than zero. We checked the consistency between the two relations by calculating flicker values for the Sun, finding \( F_{\delta} = 1.24 \pm 0.07 \) and \( F_{\delta} = 1.21 \pm 0.1 \) from solar density and surface gravity measurements, respectively. All of the code used for this project and several Ipython notebooks explaining our analysis are available at https://github.com/RuthAngus/flicker.

### 3. DISCUSSION

We have recalibrated the relation between short-timescale brightness fluctuations in the Kepler light curves of stars (flicker) with both stellar density and surface gravity, while including parameters to describe the intrinsic scatter in these relationships presented in Table 1. The terms \( \sigma_{\rho} \) and \( \sigma_{g} \) are both non-zero, suggesting that there is an additional source of scatter in the relations, not accounted for by the observational uncertainties alone. This is either caused by intrinsic scatter in the physical relationship between flicker and density and

![Figure 1](image1.png)

**Figure 1.** Stellar density vs. flicker. This figure shows the model, conditioned on the data. The solid black line shows the model with the best-fitting parameter values quoted in the text. The solid pink lines show the 1\( \sigma \) region where the extra scatter is not included and the pink shaded regions show the 1 and 2\( \sigma \) regions with the additional scatter.

![Figure 2](image2.png)

**Figure 2.** \( \log(g) \) vs. flicker. As in Figure 1 this shows the model, conditioned on the data. The solid black line shows the model with the best-fitting parameter values quoted in the text. The solid blue lines show the 1\( \sigma \) region where the extra scatter is not included and the blue shaded regions show the 1 and 2\( \sigma \) regions with the additional scatter.

![Figure 3](image3.png)

**Figure 3.** Marginal posterior PDFs of the model parameters for \( \rho \). This figure was generated using the corner python package (Foreman-Mackey et al. 2014b).

| Parameter | Median Value |
|-----------|--------------|
| \( \alpha_{\rho} \) | \( 1.31 \pm 0.01 \) |
| \( \beta_{\rho} \) | \( -0.53 \pm 0.01 \) |
| \( \sigma_{\rho} \) | \( 0.060 \pm 0.003 \) |
| \( \alpha_g \) | \( 4.91 \pm 0.05 \) |
| \( \beta_g \) | \( -0.83 \pm 0.01 \) |
| \( \sigma_g \) | \( 0.060 \pm 0.003 \) |
log g, which is produced by some physical process that is not accounted for in the model or by an underestimation of the observational uncertainties. We also tested a model with both an additional variance term and a term that included flicker-dependent variance. We found that the need for additional flicker-dependent variance was not supported by the data, indicating that the intrinsic scatter in the relations between flicker, log g and $\rho_\star$ does not depend on evolutionary state.

This is a simple “fitting a line to data” exercise. However, it continues the discussion of probabilistic modeling that is an active topic within the fields of exoplanet and stellar astronomy. We used hierarchical Bayesian modeling to constrain the intrinsic scatter in the relationship between flicker, surface gravity, and density and included the effects of the non-negligible two-dimensional observational uncertainties. Relationships between astronomical parameters are almost always non-deterministic; an element of stochasticity effects the physical parameters of stars so one can never perfectly predict y given an observation of x. We advocate a probabilistic approach in both the “fitting the model to data” step, and when using an empirically calibrated model to predict parameter values. The fitting stage benefits because if the relationships between parameters are falsely assumed to be deterministic, they will be skewed by data points with uncertainties that only represent measurement errors and no additional scatter. The prediction stage benefits from the stochastic treatment both because a probability distribution is in many ways more representative of an observation than a point estimate, and because posterior PDF samples can be used in subsequent studies (provided the prior used during the fitting process is described).

We provide posterior PDF samples at https://zenodo.org/deposit/105051/. Whenever a prediction for the surface gravity or density of a star is required for a given estimate of flicker, we recommend using these posterior samples within the calculation of $\rho_\star$ or log g and its (Monte Carlo) uncertainty. These posterior samples will naturally fold in the covariances between parameters. Simple analytical uncertainty propagation is only valid when uncertainties are Gaussian and uncorrelated, which is rarely true and certainly not the case when the model is a straight line (the slope and intercept are always correlated). A flicker value with uncertainties (or even better, posterior PDF samples) input into our model will result in a probability distribution over stellar densities or surface gravities that reflects both the uncertainties on the flicker measurement, the uncertainties on the model parameters, and the intrinsic scatter in the flicker-$\rho_\star$-log g relations.

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