On Some Types of Fuzzy $\delta$-Connected Spaces in Fuzzy Topological Space on Fuzzy Set

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Abstract: In this paper we introduced and study some types of fuzzy connected spaces like ($\Omega$-connected space, $\alpha-\Omega$-connected space, feebly -connected space, $\alpha$-connected space, $\beta$-connected space, $Sp$-connected space, $a$-connected space) and the relationships between them and fuzzy $\delta$-connected space in fuzzy topological space on fuzzy set. And We give counter examples if they are invalid And introduce Some theorems are included about this object.

Introduction:
The recent concept is introduced by Zadeh in (1965) [1], In (1968) Chang [2] introduced the definition of fuzzy topological spaces and extended in a straightforward manner some concepts of crisp topological spaces to fuzzy topological spaces. In (1973) wrong given The definition of fuzzy point such away that an ordinary point was not special case of fuzzy point.

In (1974) While Wong [3] discussed and generalized some properties of fuzzy topological spaces. In (1980) Ming, p.p. and Ming, L.Y. [4] used fuzzy topology to define the neighborhood structure of fuzzy point.

In (1982) Hdeib [7,13] introduced the concept of fuzzy $\Omega$-open set in topological space, In (1982) Maheshwari S.N. and Jain P.G. [12] defined the notion of fuzzy feebly open and fuzzy feebly closed set in fuzzy topological space and studied their properties.

In (1986) Mashhour A.S. and others [9] introduced the notion of $\alpha$-open sets in topological space. In (1987) Mashhour A.S. and others [8,15] introduced
the concept fuzzy $\beta$-open set in general topology, In (1995) A.M.Zahran [10] introduced the notion of fuzzy $\delta$-open set in fuzzy topological spaces, In (1996) Dontchev and Przemski have introduced the concept of Sp-open set in general topology [11].
In (1998) Bai Shi – Zhong and Wang Wan – Liang [5] have introduced The notion of fuzzy topology on fuzzy set and they defined the quasi-coincident in fuzzy topological space on fuzzy set. In (2003) Mahmoud, fath-Alla and Abd.Ellah [6] defined fuzzy interior and fuzzy closure in fuzzy topological space on fuzzy set and investigate their properties, In (2016) otchana and others introduced the concept of $\alpha$-$\Omega$ open set in topological space [13].

**Definition 1.1 [2]**
Let $X$ be a nonempty set, a fuzzy set $\tilde{A}$ in $X$ is characterized by a function

$$
\mu_{\tilde{A}} : X \rightarrow [0,1],
$$
where $I = [0,1]$ which is written as

$$
\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\},
$$
the collection of all fuzzy sets in $X$ will be denoted by $I^X$, that is

$$
I^X = \{ \tilde{A} : \tilde{A} \text{ is a fuzzy sets in } X \} \text{ where } \mu_{\tilde{A}} \text{ is called the membership function}
$$

**Proposition 1.2 [13]**
Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets in $X$ with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively, then for all $x \in X$:

1. $\tilde{A} \subseteq \tilde{B} \iff \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$.
2. $\tilde{A} = \tilde{B} \iff \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.
3. $\tilde{C} = \tilde{A} \cap \tilde{B} \iff C(x) = \min\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.
4. $\tilde{D} = \tilde{A} \cup \tilde{B} \iff D(x) = \max\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.
5. $\tilde{B}^c$ the complement of $\tilde{B}$ with membership function

$$
\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x).
$$
Definition 1.3 [2]:
A fuzzy point \( x_\alpha \) is a fuzzy set such that :
\[
\mu_{x_\alpha}(y) = \begin{cases} 
  r > 0 & \text{if } x = y, \forall y \in X \quad \text{and} \\
  0 & \text{if } x \neq y, \forall y \in X
\end{cases}
\]
The family of all fuzzy points of \( \tilde{A} \) will be denoted by \( \text{FP}(\tilde{A}) \).

Definition 1.4 [2]:
A collection \( \tilde{T} \) of a fuzzy subsets of \( \tilde{A} \), such that \( \tilde{T} \subseteq \mathcal{P}(\tilde{A}) \) is said to be fuzzy topology on \( \tilde{A} \) if it satisfied the following conditions
1. \( \tilde{A}, \emptyset \in \tilde{T} \)
2. If \( \tilde{B}, \tilde{C} \in \tilde{T} \), then \( \tilde{B} \cap \tilde{C} \in \tilde{T} \)
3. If \( \tilde{B}_\alpha \in \tilde{T} \), then \( \bigcup_{\alpha} \tilde{B}_\alpha \in \tilde{T} \), \( \alpha \in \Lambda \)
(\( \tilde{A}, \tilde{T} \)) is said to be Fuzzy topological space and every member of \( \tilde{T} \) is called fuzzy open set in \( \tilde{A} \) and its complement is a fuzzy closed set.

Definition 1.5 [14]:
A fuzzy set \( B \) in a fuzzy topological space \( (\tilde{A}, \tilde{T}) \) is said to be fuzzy delta set if,
\[
\mu_{\text{Int(Cl}(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x) \leq \mu_{\text{Cl(Int}(\tilde{B}))}(x)
\]
Such that,
- Fuzzy \( \delta \)-open set if \( \mu_{\text{Int(Cl}(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x) \).
- Fuzzy \( \delta \)-closed set if \( \mu_{\tilde{B}}(x) \leq \mu_{\text{Cl(Int}(\tilde{B}))}(x) \).
- Fuzzy \( \delta \)-closed set if \( A=\delta\text{cl}(A) \), where
\[
\mu_{\delta\text{cl}(\tilde{B})}(x) = \min\{ \mu_{\tilde{F}}(x) : \tilde{F} \text{ is a fuzzy } \delta \text{– closed set in } \tilde{A} \},
\]
\[
\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)
\]
The complement of fuzzy \( \delta \)-closed set is fuzzy \( \delta \)-open set.
Some Types of Fuzzy Open Sets 1.6:

In this section we study the properties and relations of various types of fuzzy open set in fuzzy topological spaces on fuzzy set which will be needed later on

**Definition 1.7:**
A fuzzy set $\tilde{B}$ of a fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be:

1) **Fuzzy $\Omega$-open (Fuzzy $\Omega$-closed)** set if
   \[ \mu_{\text{Cl}(\tilde{B})}(x) \leq \mu_{\text{Cl}(\text{Int} (\tilde{B}))}(x), \quad (\mu_{\text{IntCl}(\tilde{B})}(x) \leq \mu_{\text{Cl}(\tilde{B})}(x)), \quad \forall \ x \in X. \]
   The family of all fuzzy $\Omega$-open (fuzzy $\Omega$-closed) sets in $\tilde{A}$ will be denoted by $F\Omega O(\tilde{A})$ ( $F\Omega C(\tilde{A})$).

2) **Fuzzy $\alpha-\Omega$ open (Fuzzy $\alpha-\Omega$ closed)** set if
   \[ \mu_{\tilde{B}}(x) \leq \mu_{\text{Int}(\text{Cl}(\text{Int} \tilde{B}))}(x), \quad (\mu_{\text{Cl}(\text{Int}(\text{Cl} \tilde{B})))} \leq \mu_{\tilde{B}}(x)), \quad .\]
   $\tilde{B}$ is called (Fuzzy $\alpha-\Omega$ closed) set if its complement is Fuzzy $\alpha-\Omega$ open sets.
   The family of all Fuzzy $\alpha-\Omega$ open (Fuzzy $\alpha-\Omega$ closed) sets in $\tilde{A}$ will be denoted by $F\alpha-O(\tilde{A})$ ( $F\alpha-C(\tilde{A})$).

3) **Fuzzy feebly – open (feebly – closed)** set if
   \[ \mu_{\tilde{B}}(x) \leq \mu_{\alpha}(\text{Cl}(\text{Int} \tilde{B}))(x), \quad (\mu_{\alpha}(\text{Int}(\text{Cl} \tilde{B}))) \leq \mu_{\tilde{B}}(x)), \quad \forall \ x \in X \]
   The family of all fuzzy feebly – open (fuzzy feebly – closed) sets in $\tilde{A}$ will be denoted by $FfeeblyO(\tilde{A})$ ( $FfeeblyC(\tilde{A})$).

4) **Fuzzy $\alpha$-open (fuzzy $\alpha$-closed)** set if
   \[ \mu_{\tilde{B}}(x) \leq \mu_{\text{Int}(\text{Cl}(\text{Int} \tilde{B})))} (x), \quad (\mu_{\text{Cl}(\text{Int}(\text{Cl} \tilde{B})))} \leq \mu_{\tilde{B}}(x)). \]
The family of all fuzzy $\alpha$-open (fuzzy $\alpha$-closed) sets in $\tilde{A}$ will be denoted by $F_{\alpha}O(\tilde{A})$ ($F_{\alpha}C(\tilde{A})$).

5) **Fuzzy $\beta$-open (fuzzy $\beta$-closed)** set if

$$\mu_B(x) \leq \mu_{int(Cl(\tilde{B}))}(x), \quad (\mu_{cl(Int(\tilde{B}))} \leq \mu_B(x)) \quad \forall \ x \in X$$

The family of all fuzzy $\beta$-open (fuzzy $\beta$-closed) sets in $\tilde{A}$ will be denoted by $F_f\beta O(\tilde{A})$ ($F_f\beta C(\tilde{A})$).

6) **Fuzzy Sp-open (fuzzy Sp-closed)** set if,

$$\mu_B(x) \leq \max\{\mu_{int(Cl(\tilde{B}))}(x), \mu_{cl(Int(\tilde{B}))}(x)\}$$

$$\mu_B(x) \geq \min\{\mu_{int(Cl(\tilde{B}))}(x), \mu_{cl(Int(\tilde{B}))}(x)\} \quad \forall \ x \in X$$

The family of all fuzzy Sp-open (fuzzy Sp-closed) sets in $\tilde{A}$ will be denoted by $FSpO(\tilde{A})$ ($FSpC(\tilde{A})$).

7) **Fuzzy a-open (fuzzy a-closed)** set if,

$$\mu_B(x) \leq \mu_{int(Cl(\tilde{B}))}(x), \quad (\mu_{cl(Int(\tilde{B})))} \leq \mu_B(x))$$

The family of all fuzzy a-open (fuzzy a-closed) sets in $\tilde{A}$ will be denoted by $FaO(\tilde{A})$ ($FaC(\tilde{A})$).

**Definition 1.8:**
Let $B$ is a fuzzy set in a fuzzy topological space $(\tilde{A}, \tilde{T})$ then:

- **The $\Omega$–closure** of $B$ is denoted by $(\Omega cl(\tilde{B}))$ and defined by
  $$\mu_{\Omega cl(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F} \text{ is a fuzzy } \Omega \text{– closed set in } \tilde{A}, \ \mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$$

- **The $\alpha$–closure** of $B$ is denoted by $(\alpha – \Omega cl(\tilde{B}))$ and defined by
  $$\mu_{\alpha – \Omega cl(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F} \text{ is a fuzzy } \alpha\text{ – }\Omega \text{ closed set in } \tilde{A}, \ \mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$$

- **The feeably–closure** of $B$ is denoted by $(feeably cl(\tilde{B}))$ and defined by
\[ \mu_{\text{feebly cl}}(B)(x) = \min \{ \mu_F(x) : \tilde{F} \text{ is a fuzzy feebly-closed set in } \tilde{A}, \mu_B(x) \leq \mu_F(x) \} \]

- **The \( \alpha \)-closure** of \( \tilde{B} \) is denoted by \( (\alpha \text{cl}(\tilde{B})) \) and defined by
\[ \mu_{\alpha \text{cl}}(B)(x) = \min \{ \mu_{\text{cl}}(\tilde{F})(x) : \tilde{F} \text{ is a fuzzy open set in } \tilde{A}, \mu_B(x) \leq \mu_{\text{cl}}(\tilde{F})(x) \} \]

**Proposition 1.9:**
Let \((\tilde{A}, \tilde{T})\) be a fuzzy topological space then:
1) The complement of fuzzy \( \Omega \)-open set is fuzzy \( \Omega \)-closed set.
2) The complement of fuzzy \( \alpha \)-\( \Omega \)-open set is fuzzy \( \alpha \)-\( \Omega \)-closed set
3) The complement of fuzzy feebly-open set is fuzzy feebly-closed set
4) The complement of fuzzy \( \alpha \)-open set is fuzzy \( \alpha \)-closed set
5) The complement of fuzzy \( \beta \)-open set is fuzzy \( \beta \)-closed set
6) The complement of fuzzy Sp-open set is fuzzy Sp-closed set
7) The complement of fuzzy a-open set is fuzzy a-closed set

**Proof:** Obvious.

**Proposition 1.10:**
Let \((\tilde{A}, \tilde{T})\) be a fuzzy topological space then:
1) Every fuzzy \( \delta \)-open set is fuzzy open set (fuzzy \( \Omega \)-open set, fuzzy feebly-open set, fuzzy a-open set)
2) Every fuzzy open set is fuzzy \( \Omega \)-open set (fuzzy feebly open set, fuzzy \( \alpha \)-open set)
3) Every fuzzy \( \Omega \)-open set is fuzzy \( \alpha \)-\( \Omega \) open set (fuzzy a-open set)
4) Every fuzzy \( \alpha \)-open set is fuzzy \( \alpha \)-\( \Omega \) open set (fuzzy \( \beta \)-open set, fuzzy Sp-open set)
5) Every fuzzy \( \beta \)-open set is fuzzy Sp-open set.
6) Every fuzzy a-open set is fuzzy \( \alpha \)-open set.
Remark 1.3.7:

Figure - 1 – illustrates the relation between fuzzy δ-open set and some types of fuzzy open sets.

Fuzzy δ-Connected Spaces 2.0:
In this section we present fuzzy δ-connected and fuzzy δ-disconnected spaces where some of their theorems are proved.

Definition 2.1:
A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be fuzzy $\delta$-connected if there is no proper non-empty maximal fuzzy $\delta$-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall \ x \in X
\]
If $(\tilde{A}, \tilde{T})$ is not fuzzy $\delta$-connected then is said to be fuzzy $\delta$-disconnected spaces.

**Theorem 2.2:**
A fuzzy topological space $(\tilde{A}, \tilde{T})$ is fuzzy $\delta$-connected if and only if there exist no non-empty fuzzy $\delta$-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\emptyset}(x)
\]
**Proof:** Obvious

**Corollary 2.3:**
A fuzzy topological space $(\tilde{A}, \tilde{T})$ is fuzzy $\delta$-connected if and only if there exist no non-empty fuzzy $\delta$-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$ such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\emptyset}(x)
\]
**Proof:** Obvious

**Some Types Of Fuzzy Connected Spaces 3.0:**
In this section we introduce another types of fuzzy connected spaces ($\Omega$-connected space, $\alpha - \Omega$-connected space, feebly-connected space, $\alpha$-connected space, $\beta$-connected space, $Sp$-connected space and $a$-connected space), and the relationship between of them and fuzzy $\delta$-connected space and some theorems are included throughout this work.

**Definition 3.1:**
A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be:
1. fuzzy $\Omega$-connected if there is no proper non-empty maximal fuzzy $\Omega$-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that,
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall \ x \in X
\]
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(\Omega\)-connected then is said to be fuzzy \(\Omega\)-disconnected spaces.

2. Fuzzy \(\alpha - \Omega\)-connected if there is no proper non-empty maximal fuzzy \(\alpha - \Omega\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall x \in X
\]
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(\alpha - \Omega\)-connected then is said to be fuzzy \(\alpha - \Omega\)-disconnected spaces.

3. Fuzzy \(f\)-\(\alpha\)-connected if there is no proper non-empty maximal fuzzy \(f\)-\(\alpha\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall x \in X
\]
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(f\)-\(\alpha\)-connected then is said to be fuzzy \(f\)-\(\alpha\)-disconnected spaces.

4. Fuzzy \(\tilde{f}\)-\(\alpha\)-connected if there is no proper non-empty maximal fuzzy \(\tilde{f}\)-\(\alpha\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall x \in X
\]
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(\tilde{f}\)-\(\alpha\)-connected then is said to be fuzzy \(\tilde{f}\)-\(\alpha\)-disconnected spaces.

5. Fuzzy \(\beta\)-\(\alpha\)-connected if there is no proper non-empty maximal fuzzy \(\beta\)-\(\alpha\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall x \in X
\]
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(\beta\)-\(\alpha\)-connected then is said to be fuzzy \(\beta\)-\(\alpha\)-disconnected spaces.

6. Fuzzy \(Sp\)-\(\alpha\)-connected if there is no proper non-empty maximal fuzzy \(Sp\)-\(\alpha\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall x \in X
\]
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(Sp\)-\(\alpha\)-connected then is said to be fuzzy \(Sp\)-\(\alpha\)-disconnected spaces.

7. Fuzzy \(\sigma\)-\(\alpha\)-connected if there is no proper non-empty maximal fuzzy \(\sigma\)-\(\alpha\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \} \quad \forall x \in X
If \((\tilde{A}, \tilde{T})\) is not fuzzy \(\alpha\)-connected then is said to be fuzzy \(\alpha\)-disconnected spaces.

**Theorem 3.2:**
A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(Sp\)-connected if and only if there exist no non-empty fuzzy \(Sp\)-closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\emptyset}(x)
\]

**Proof:**
\((\Rightarrow)\) suppose that \((\tilde{A}, \tilde{T})\) is fuzzy \(Sp\)-connected space
Suppose that there exist non empty fuzzy \(Sp\)-closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\emptyset}(x)
\]
Since \(\tilde{E}\) and \(\tilde{F}\) are fuzzy \(Sp\)-closed sets in \(\tilde{A}\) and,
\[
\min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\emptyset}(x)
\]
then \(\tilde{E}\) and \(\tilde{F}\) are fuzzy \(Sp\)-separated sets in \(\tilde{A}\)
Since \(\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \}\) then \((\tilde{A}, \tilde{T})\) is fuzzy \(Sp\)-disconnected, which is a contradiction.

\((\Leftarrow)\) Suppose that there exist no non-empty fuzzy \(Sp\)-closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\emptyset}(x)
\]
Suppose that \((\tilde{A}, \tilde{T})\) is fuzzy \(Sp\)-disconnected space, then this implies that there exist non-empty maximal fuzzy \(Sp\)-separated sets \(\tilde{B}\) and \(\tilde{C}\) in \(\tilde{A}\) such that
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \}
\]
since \(\tilde{B}\) and \(\tilde{C}\) are fuzzy \(Sp\)-separated in \(\tilde{A}\) then
\[
\min \{ \mu_{Spcl(\tilde{B})}(x), \mu_{\tilde{C}}(x) \} = \mu_{\emptyset}(x) \quad \text{and}
\]
\[
\min \{ \mu_{Spcl(\tilde{C})}(x), \mu_{\tilde{B}}(x) \} = \mu_{\emptyset}(x)
\]
implies that \(\mu_{\tilde{C}}(x) \leq [\mu_{Spcl(\tilde{B})}(x)]^{\mathsf{c}}\) and \(\mu_{\tilde{B}}(x) \leq [\mu_{Spcl(\tilde{C})}(x)]^{\mathsf{c}}\) since
\[ \mu_{\bar{A}}(x) = \max \{ \mu_{B}(x), \mu_{C}(x) \} \leq \max \{ [\mu_{\text{spcl}(B)}(x)]^c, [\mu_{\text{spcl}(C)}(x)]^c \} \]

Then
\[ \mu_{\bar{A}}(x) = \max \{ [\mu_{\text{spcl}(B)}(x)]^c, [\mu_{\text{spcl}(C)}(x)]^c \} \]

Then
\[ \mu_{\bar{A}}(x) = \min \{ [\mu_{\text{spcl}(B)}(x), \mu_{\text{spcl}(C)}(x)]^c \} \]

\[ [\mu_{\bar{A}}(x)]^c = \min \{ [\mu_{\text{spcl}(B)}(x), \mu_{\text{spcl}(C)}(x)] \} \]

\[ \mu_{\bar{0}}(x) = \min \{ [\mu_{\text{spcl}(B)}(x), \mu_{\text{spcl}(C)}(x)] \} \]

Let \( \mu_{\text{spcl}(C)}(x) = \mu_{\bar{E}}(x) \) and \( \mu_{\text{spcl}(B)}(x) = \mu_{\bar{F}}(x) \),
then\
\[ \min \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \mu_{\bar{0}}(x) \]

And \( \max \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \max \{ \mu_{\text{spcl}(C)}(x), \mu_{\text{spcl}(B)}(x) \} \)

\[ \max \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \mu_{\text{spcl}(\max \{ \mu_{C}(x), \mu_{B}(x) \})}(x) \]

\[ \max \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \mu_{\text{spcl}(\bar{A})}(x) \]

\[ \max \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \mu_{\bar{A}}(x) \]

which is a contradiction

Hence \((\bar{A}, \bar{T})\) is fuzzy \(Sp\)-connected space. □

**Theorem 3.3:**

1) A fuzzy topological space \((\bar{A}, \bar{T})\) is fuzzy \(\Omega\)-connected if and only if there exist no non-empty fuzzy \(\Omega\)-closed sets \(\bar{E}\) and \(\bar{F}\) in \(\bar{A}\) such that
\[ \mu_{\bar{A}}(x) = \max \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \mu_{\bar{0}}(x) \]

2) A fuzzy topological space \((\bar{A}, \bar{T})\) is fuzzy \(\alpha - \Omega\)-connected if and only if there exist no non-empty fuzzy \(\alpha - \Omega\)-closed sets \(\bar{E}\) and \(\bar{F}\) in \(\bar{A}\) such that
\[ \mu_{\bar{A}}(x) = \max \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\bar{E}}(x), \mu_{\bar{F}}(x) \} = \mu_{\bar{0}}(x) \]
3) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \textit{fuzzy freely-connected} if and only if there exist no non-empty fuzzy \textit{fuzzy freely-connected} closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)
\]

4) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\alpha\)-connected if and only if there exist no non-empty fuzzy \(\alpha\)-closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)
\]

5) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\beta\)-connected if and only if there exist no non-empty fuzzy \(\beta\)-closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)
\]

6) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\gamma\)-connected if and only if there exist no non-empty fuzzy \(\gamma\)-closed sets \(\tilde{E}\) and \(\tilde{F}\) in \(\tilde{A}\) such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)
\]

**Proof:** \((1)\) \((2)\) \((3)\) \((4)\) \((5)\) and \((6)\) similar proof theorem \((3.2)\)

**Corollary 3.4:**
A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\alpha\)-connected if and only if there exist no non-empty fuzzy \(\alpha\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\emptyset}}(x)
\]

**Proof:**
\((\Rightarrow)\) suppose that \((\tilde{A}, \tilde{T})\) is fuzzy \(\alpha\)-connected space

Suppose that there exist non empty fuzzy \(\alpha\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that 
\[
\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \quad \text{and} \quad \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\emptyset}}(x)
\]
Implies that \([\mu_{\tilde{A}}(x)]^c = \min \{ [\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c \}\)
and
\[
\mu_{\tilde{\emptyset}}(x) = \min \{ [\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c \}\]
and 
\[
\max \{ [\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c \} = [\mu_{\tilde{\emptyset}}(x)]^c
\]

Let \([\mu_{\tilde{G}}(x)]^c = \mu_{\tilde{E}}(x)\) and \([\mu_{\tilde{H}}(x)]^c = \mu_{\tilde{F}}(x)\)
Implies that that there exist no non-empty fuzzy $\alpha$-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that

$$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x) , \mu_{\tilde{F}}(x) \} \text{ and } \min \{ \mu_{\tilde{E}}(x) , \mu_{\tilde{F}}(x) \} = \mu_{\tilde{O}}(x)$$

Then $(\tilde{A} ,\tilde{T})$ is fuzzy $\alpha$-disconnected space which is a contradiction $(\Leftarrow)$

Suppose that there exist no non-empty fuzzy $\alpha$-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$ such that

$$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x) , \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x) , \mu_{\tilde{H}}(x) \} = \mu_{\tilde{O}}(x)$$

Suppose that $(\tilde{A} ,\tilde{T})$ is fuzzy $\alpha$-disconnected space, then this implies that there exist non-empty maximal fuzzy $\alpha$-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x) , \mu_{\tilde{C}}(x) \}$, since $\tilde{B}$ and $\tilde{C}$ are fuzzy $\alpha$-separated in $\tilde{A}$, then

$$\min \{ \mu_{\text{acl}(\tilde{B})}(x) , \mu_{\tilde{C}}(x) \} = \mu_{\tilde{O}}(x) \text{ and }$$

$$\min \{ \mu_{\text{acl}(\tilde{C})}(x) , \mu_{\tilde{B}}(x) \} = \mu_{\tilde{O}}(x)$$

implies that $\mu_{\tilde{C}}(x) \leq \lceil \mu_{\text{acl}(\tilde{B})}(x) \rceil$ and $\mu_{\tilde{B}}(x) \leq \lceil \mu_{\text{acl}(\tilde{C})}(x) \rceil$ since

$$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x) , \mu_{\tilde{C}}(x) \} \leq \max \{ \lceil \mu_{\text{acl}(\tilde{B})}(x) \rceil , \lceil \mu_{\text{acl}(\tilde{C})}(x) \rceil \}$$

Then $\mu_{\tilde{A}}(x) = \max \{ \lceil \mu_{\text{acl}(\tilde{B})}(x) \rceil , \lceil \mu_{\text{acl}(\tilde{C})}(x) \rceil \}$

Let $\lceil \mu_{\text{acl}(\tilde{B})}(x) \rceil = \mu_{\tilde{G}}(x)$ and $\lceil \mu_{\text{acl}(\tilde{C})}(x) \rceil = \mu_{\tilde{H}}(x)$

Then $\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x) , \mu_{\tilde{H}}(x) \}$

$$\min \{ \mu_{\tilde{G}}(x) , \mu_{\tilde{H}}(x) \} = \min \{ \lceil \mu_{\text{acl}(\tilde{B})}(x) \rceil , \lceil \mu_{\text{acl}(\tilde{C})}(x) \rceil \}$$

$$\min \{ \mu_{\tilde{G}}(x) , \mu_{\tilde{H}}(x) \} = \max \{ \mu_{\text{acl}(\tilde{B})}(x) , \mu_{\text{acl}(\tilde{C})}(x) \}$$

$$\min \{ \mu_{\tilde{G}}(x) , \mu_{\tilde{H}}(x) \} = \lfloor \mu_{\text{acl}(\tilde{A})}(x) \rfloor = \lceil \mu_{\tilde{A}}(x) \rceil = \mu_{\tilde{O}}(x)$$

which is a contradiction
Hence \((\tilde{A}, \tilde{T})\) is fuzzy \(a\)-connected space. ■

**Corollary 3.5:**
1) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\Omega\)-connected if and only if there exist no non-empty fuzzy \(\Omega\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that
   \[\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\theta}}(x)\]
2) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\alpha - \Omega\)-connected if and only if there exist no non-empty fuzzy \(\alpha - \Omega\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that
   \[\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\theta}}(x)\]
3) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\text{feebly-}\alpha\)-connected if and only if there exist no non-empty fuzzy \(\text{feebly-}\alpha\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that
   \[\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\theta}}(x)\]
4) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\alpha\)-connected if and only if there exist no non-empty fuzzy \(\alpha\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that
   \[\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\theta}}(x)\]
5) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(\beta\)-connected if and only if there exist no non-empty fuzzy \(\beta\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that
   \[\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\theta}}(x)\]
6) A fuzzy topological space \((\tilde{A}, \tilde{T})\) is fuzzy \(Sp\)-connected if and only if there exist no non-empty fuzzy \(Sp\)-open sets \(\tilde{G}\) and \(\tilde{H}\) in \(\tilde{A}\) such that
   \[\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} \text{ and } \min \{ \mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x) \} = \mu_{\tilde{\theta}}(x)\]

**Proof:** (1) (2) (3) (4) (5) and (6) similar proof corollary (3.4)

**Theorem 3.6:**
Let \((\tilde{A}, \tilde{T})\) be a fuzzy topological space then:-
1. Every fuzzy connected space is fuzzy $\delta$-connected space
2. Every fuzzy $\Omega$-connected space is fuzzy $\delta$-connected space (fuzzy connected space)
3. Every fuzzy $a$-open set is fuzzy $\Omega$-connected space (fuzzy $\delta$-connected space)
4. Every fuzzy $a$-connected space is fuzzy-connected space (fuzzy $a$-connected space)
5. Every fuzzy $a$-$\Omega$-connected space is fuzzy $\Omega$-connected space (fuzzy $a$-connected space).
6. Every fuzzy $\beta$-connected space is fuzzy $\alpha$-connected space
7. Every fuzzy $Sp$-connected space is fuzzy feebly-connected space (fuzzy $\alpha$-connected space, fuzzy $\beta$-connected space)

**Proof:** (1)

Let $(\tilde{A}, \tilde{T})$ be a fuzzy connected space

And suppose that $(\tilde{A}, \tilde{T})$ is fuzzy $\delta$-disconnected space

Then this implies that there exist non-empty maximal fuzzy $\delta$-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that

$$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \}$$

Then by theorem (2.2.8) there exist non-empty maximal fuzzy $\delta$-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that

$$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \}$$

Implies that $(\tilde{A}, \tilde{T})$ is fuzzy $\delta$-disconnected which is a contradiction

Hence $(\tilde{A}, \tilde{T})$ is fuzzy $\delta$-connected space. ■

**Proof (2) (3) (4) (5) (6) (7) and (8) similarly (1)**

**Remark 3.7:**
The converse of theorem (2.4.6) is not true in general as following examples show:-

**Examples 3.8:**
Let $X = \{ a, b \}$ and $\widetilde{H}_1, \widetilde{H}_2, \widetilde{H}_3, \widetilde{H}_4, \widetilde{H}_5, \widetilde{H}_6, \widetilde{H}_7, \widetilde{H}_8, \widetilde{H}_9, \widetilde{H}_{10}, \widetilde{H}_{11}, \widetilde{H}_{12}, \widetilde{H}_{13}, \widetilde{H}_{14}, \widetilde{H}_{15}, \widetilde{H}_{16}, \widetilde{H}_{17}$ are fuzzy subset in $\widetilde{A}$ where

$\widetilde{A} = \{ (a, 0.6), (b, 0.7) \}$

$\widetilde{H}_1 = \{ (a, 0.4), (b, 0.0) \}$, $\widetilde{H}_2 = \{ (a, 0.5), (b, 0.1) \}$

$\widetilde{H}_3 = \{ (a, 0.1), (b, 0.7) \}$, $\widetilde{H}_4 = \{ (a, 0.0), (b, 0.6) \}$

$\widetilde{H}_5 = \{ (a, 0.1), (b, 0.0) \}$, $\widetilde{H}_6 = \{ (a, 0.4), (b, 0.6) \}$

$\widetilde{H}_7 = \{ (a, 0.4), (b, 0.6) \}$, $\widetilde{H}_8 = \{ (a, 0.1), (b, 0.1) \}$

$\widetilde{H}_9 = \{ (a, 0.0), (b, 0.1) \}$, $\widetilde{H}_{10} = \{ (a, 0.4), (b, 0.1) \}$

$\widetilde{H}_{11} = \{ (a, 0.5), (b, 0.7) \}$, $\widetilde{H}_{12} = \{ (a, 0.5), (b, 0.6) \}$

$\widetilde{H}_{13} = \{ (a, 0.1), (b, 0.6) \}$, $\widetilde{H}_{14} = \{ (a, 0.6), (b, 0.1) \}$

$\widetilde{H}_{15} = \{ (a, 0.6), (b, 0.6) \}$, $\widetilde{H}_{16} = \{ (a, 0.6), (b, 0.0) \}$

$\widetilde{H}_{17} = \{ (a, 0.0), (b, 0.7) \}$

The fuzzy topology defined on $\widetilde{A}$ is

$\widetilde{T} = \{ \emptyset, \widetilde{A}, \widetilde{H}_1, \widetilde{H}_2, \widetilde{H}_3, \widetilde{H}_4, \widetilde{H}_5, \widetilde{H}_6, \widetilde{H}_7, \widetilde{H}_8, \widetilde{H}_9, \widetilde{H}_{10}, \widetilde{H}_{11}, \widetilde{H}_{12}, \widetilde{H}_{13}, \widetilde{H}_{14}, \widetilde{H}_{15}, \widetilde{H}_{16}, \widetilde{H}_{17} \}$.

By theorem (2.4.3),

- the fuzzy topological space $(\widetilde{A}, \widetilde{T})$ is fuzzy $\alpha$-disconnected space but it is fuzzy $\alpha$-connected space
- the fuzzy topological space $(\widetilde{A}, \widetilde{T})$ is fuzzy $\Omega$-disconnected space but it is fuzzy $\alpha$-$\Omega$-connected space
- the fuzzy topological space $(\widetilde{A}, \widetilde{T})$ is fuzzy $\alpha$-disconnected space but it is fuzzy $\beta$-connected space

Then, $\min \{ \mu_{\widetilde{H}_{16}}(x), \mu_{\widetilde{H}_{17}}(x) \} = \mu_{\emptyset}(x)$

Also, $\mu_{\widetilde{A}}(x) = \max \{ \mu_{\widetilde{H}_{16}}(x), \mu_{\widetilde{H}_{17}}(x) \}$

So,
\[
\min\{ \mu_{\tilde{H}_{16}}(x), \mu_{\tilde{H}_{17}}(x) \} \neq \mu_{\bar{\emptyset}}(x) \quad \text{and},
\]
\[
\mu_{\tilde{A}}(x) \neq \max \{ \mu_{\tilde{H}_{16}}(x), \mu_{\tilde{H}_{17}}(x) \}
\]

Hence \( \tilde{H}_{16}, \tilde{H}_{17} \) are fuzzy \( \alpha \)-connected space, fuzzy \( \alpha - \Omega \)-connected space, fuzzy \( \beta \)-connected space.

**Example 3.9:**

Let \( X = \{ a, b \} \) and \( \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F} \) are fuzzy subset in \( \tilde{A} \) where
\[
\tilde{A} = \{ (a, 0.7), (b, 0.7) \}, \quad \tilde{B} = \{ (a, 0.4), (b, 0.0) \}
\]
\[
\tilde{C} = \{ (a, 0.0), (b, 0.7) \}, \quad \tilde{D} = \{ (a, 0.4), (b, 0.7) \}
\]
\[
\tilde{E} = \{ (a, 0.1), (b, 0.7) \}, \quad \tilde{F} = \{ (a, 0.7), (b, 0.0) \}
\]

By corollary (2.4.5), the fuzzy topological space \( (\tilde{A}, \tilde{T}) \) is fuzzy \( \delta \)-disconnected space but it is fuzzy \( \alpha \)-connected space \( (\Omega \)-connected space, connected space, feebly connected space).

Then, \( \min\{ \mu_{\tilde{C}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\bar{\emptyset}}(x) \)

Also, \( \mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{C}}(x), \mu_{\tilde{F}}(x) \} \)

So,
\[
\min\{ \mu_{\tilde{C}}(x), \mu_{\tilde{F}}(x) \} \neq \mu_{\bar{\emptyset}}(x) \quad \text{and}
\]
\[
\mu_{\tilde{A}}(x) \neq \max \{ \mu_{\tilde{C}}(x), \mu_{\tilde{F}}(x) \}
\]

Hence \( \tilde{C}, \tilde{F} \) are fuzzy \( \delta \)-disconnected space.

**Remark 2.4.9:**

Figure - 2 – illustrates the relation between fuzzy \( \delta \)-connected space and some types of fuzzy connected space.
Fuzzy δ-connected space

Fuzzy Ω-connected space

Fuzzy α-connected space

Fuzzy α-Ω-connected space

Fuzzy α-connected space

Fuzzy δ-connected space

Fuzzy Feebly-connected space

Figure -2-

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