Some FRW viscous models with $G$, $c$ and $\Lambda$ variables

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Abstract
We consider several viscous models with metrics FRW ($k = 0$) but with variable $G$, $c$ and $\Lambda$. We find trivially a set of solutions through Dimensional Analysis.

1 Introduction.

Recently have been studied several models with metrics FRW where the "constants" $G$ and $\Lambda$ ([3]) are considered as dependent functions on time $t$ ([1]). Most recently this type of models have been generalized by Arbab ([2]) who considers a viscous fluid. Other authors ([3]) study models with $c$ and $G$ variables for perfect fluids. In this paper we want to calculate as vary these "constants" $G$, $c$ and $\Lambda$ within the models FRW with a viscous fluid. We want to emphasize as the use of the Dimensional Analysis (D.A.) permits us to find in a trivial way a set of solutions to this type of models (but with

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$k = 0$), taking into account the conservation principle ($\text{div} T_{ij} = 0$), since the type of differential equations that go emerging can be (are) cumbersome. We have not wanted to make a meticulous study of the solutions obtained for each $(\omega, n)$ (we let this paragraph for a subsequent paper) but to show the obtained results.

The paper is organized as follows: In the second paragraph are made some small considerations on the followed dimensional method (address to reader to the classic literature on the topic ([5])). In the third paragraph we make use of the Dimensional Analysis (Pi theorem) to obtain a solution to the principal quantities that appear in the model and finally in the fourth paragraph we end with a short exposition of the obtained cases.

2 The model.

The modified field equations are:

$$R_{ij} - \frac{1}{2}g_{ij}R - \Lambda(t)g_{ij} = \frac{8\pi G(t)}{c^2(t)}T_{ij}$$

and we impose that

$$\text{div}(T_{ij}) = 0$$

where $\Lambda(t)$ represents the "cosmological constant". The basic ingredients of the model are:

The line element is defined by:

$$ds^2 = -c^2 dt^2 + f^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]$$

here only we will consider the case $k = 0$. The energy-momentum tensor is defined by:

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad p = \omega \rho$$

The developed equations of the field are:

$$2\frac{f''}{f} + \left( \frac{f'}{f} \right)^2 = \frac{8\pi G(t)}{c^2(t)}p + c^2(t)\Lambda(t)$$

$$\frac{(f')^2}{f^2} = \frac{8\pi G(t)}{3c^2(t)}\rho + c^2(t)\Lambda(t)$$
\[ \text{div}(T_{ij}) = 0 \iff \rho' + 3(\omega + 1)\rho \frac{f'}{f} = 0 \quad (4) \]

Integrating the equation (4) we obtain the following equation.

\[ \rho = A_\omega f^{-3(\omega+1)} \quad (5) \]

where \( f \) represents the scale factor that appears in the metrics and \( A_\omega \) is the constant of integration that depends on the equation of state that is imposed.

The effect of the viscosity in the equations is shown replacing \( p \) by \( p - 3\eta H \) \((H = f'/f)\) \((2)\) and \((4)\) for details). where:

\[ \eta = \eta_0 \rho^n \quad (6) \]

This last equation (of state) in our opinion does not verify the dimensional homogeneity principle, by this reason we have the changed it by:

\[ \eta = k_n \rho^n \quad (7) \]

where the constant \( k_n \) causes that the above equation will be dimensionally homogeneous for any value of \( n \). The dimensional analysis that we apply needs to make the following distinctions. We need to know beforehand the set of fundamental quantities, in this case it is solely the cosmic time \( t \) as is deduced of the homogeneity and isotropy of the model and to distinguish the set of constants, universal and unavoidable or characteristic, in this case there are no universal constants since are all functions on \( t \) \((G(t), c(t), \Lambda(t))\) \((G(t), c(t))\), that is to say we do not consider them, and the two only constants that appear are respectively the constant of integration \( A_\omega \) that depending on the equation of state that is imposed will have different dimensions and physical meaning and the constant \( k_n \) that it will depend on the equation of state that we impose for \( \eta \).

In a previous paper \((2)\) was calculated the dimensional base of this type of models, being this \( B = \{L, M, T, \theta\} \) where \( \theta \) represents the dimension of the temperature. The corresponding dimensions of each magnitude (with respect to this base) are:

\[ [t] = T \quad [A_\omega] = L^{2+3\omega} MT^{-2} \quad [k_n] = L^{n-1} M^{1-n} T^{2n-1} \]

All the magnitude that we go to calculate we will make it exclusively in function of the cosmic time \( t \) and of the constant unavoidable \( k_n \) and \( A_\omega \) with respect to a dimensional base \( B = \{L, M, T, \theta\} \).
3 Solutions through A.D.

We go to calculate through dimensional analysis D.A. the variation of $G(t)$ in function of $t$, the speed of the light $c(t)$, energy density $\rho(t)$, the radius of the universe $f(t)$, the temperature $\theta(t)$, and finally $\Lambda(t)@$.

3.1 Calculation of $G(t)$

As have indicated above, we go to accomplish the calculation applying the Pi theorem. The magnitude that we consider are: $G = G(t, k_n, A_\omega), B = \{L, M, T, \theta\}$. We know that $[G] = L^3M^{-1}T^{-2}$

$$
\begin{pmatrix}
G & t & k_n & A_\omega \\
L & 3 & 0 & n-1 & 2 + 3\omega \\
M & -1 & 0 & 1-n & 1 \\
T & -2 & 1 & 2n-1 & -2
\end{pmatrix}
$$

$$
G \propto A_\omega^{-1 + \frac{3\omega+5}{4(\omega+1)}}k_n^{\frac{3\omega+5}{4(\omega+1)(n-1)}}t^{-4 - \frac{3\omega+5}{4(\omega+1)(n-1)}}
$$

(8)

3.2 Calculation of $c(t)$

$c(t) = c(t, k_n, A_\omega) \implies$

$$
c(t) \propto A_\omega^{\frac{1}{3(\omega+1)}}k_n^{\frac{1}{4(\omega+1)(n-1)}}t^{-1 - \frac{1}{3(\omega+1)(n-1)}}
$$

(9)

3.3 Calculation of energy density $\rho(t)$

$\rho = \rho(t, k_n, A_\omega)$ with respect to the dimensional base $B$

$$
\rho \propto k_n^{-\frac{1}{1-n}}t^{-\frac{1}{\frac{1}{3(\omega+1)}}}
$$

(9)

3.4 Calculation of the radius of the universe $f(t)$.

$f = f(t, k_n, A_\omega) \implies$

$$
f \propto A_\omega^{\frac{1}{3(\omega+1)}}k_n^{\frac{1}{4(\omega+1)(n-1)}}t^{\frac{1}{3(\omega+1)(n-1)}}
$$

(10)
We can observe that:

\[q = -\frac{f''f}{(f')^2} = - [3(\omega + 1)(n - 1) + 1]\]

\[H = \frac{f'}{f} = - \left(\frac{1}{3(\omega + 1)(n - 1)}\right) \frac{1}{t}\]

\[dH = ct \lim_{t_0 \to 0} \int_{t_0}^{t} \frac{dt'}{f(t')} = \infty\]

Thus, the model has no horizon because \(dH\) diverges for \(t_0 \to 0\). (depending of the model obviously)

3.5 Calculation of the temperature \(\theta(t)\).

\[\theta = \theta(t, k_n, A_\omega, k_B) \text{ where } k_B \text{ is the Bolztmann constant} \implies\]

\[k_B\theta \propto A^{\frac{1}{3(\omega + 1)}} k_n^{\frac{\omega}{3(\omega + 1)}} t^{-2 + \frac{\omega}{3(\omega + 1)(n - 1)}}\]  \hspace{1cm} (11)

3.6 Calculation of the cosmological constant: \(\Lambda(t)\).

\[\Lambda = \Lambda(t, k_n, A_\omega) \implies\]

\[\Lambda \propto A^{\frac{2}{3(\omega + 1)}} k_n^{\frac{2}{3(\omega + 1)}} t^{-2 + \frac{2}{3(\omega + 1)(n - 1)}}\]  \hspace{1cm} (12)

4 Different cases.

All the following cases have been studied by Arbab \cite{2} and can be without difficulty calculated the rest of the cases.

4.1 \(n = 0 \text{ y } \omega = 0\)

\[G \propto A_{\omega}^{2/3} k_n^{-5/3} t^{-7/3}\]

\[c \propto A_{\omega}^{1/3} k_n^{-1/3} t^{-2/3}\]

\[\Lambda \propto A_{\omega}^{-2/3} k_n^{2/3} t^{-2/3}\]

\[\rho \propto k_n t^{-1}\]

\[k_B\theta \propto A_{\omega}^{1/3} k_n^{-2/3} t^{-2}\]

\[f \propto A_{\omega}^{1/3} k_n^{-1/3} t^{1/3}\]

\[q = 2\]
4.2 \( n = 1/2 \) y \( \omega = 1/3 \)

\[
\begin{align*}
G &\propto A^{1/2}_\omega k_n^{-3} t^{-1} & c &\propto A^{1/4}_\omega k_n^{-1/2} t^{-1/2} & \Lambda &\propto A^{-1/2}_\omega k_n t^{-1} \\
\rho &\propto k_n^2 t^{-2} & k_B \theta &\propto A^{1/4}_\omega k_n^{-1/2} t^{-3/2} & f &\propto A^{1/4}_\omega k_n^{-1/2} t^{1/2} & q = 1 \\
\end{align*}
\]

4.3 \( n = 3/4 \) y \( \omega = 1/3 \)

\[
\begin{align*}
G &\propto A^{1/2}_\omega k_n^{-6} t^{2} & c &\propto A^{1/4}_\omega k_n^{-1} t^{0} = const. & \Lambda &\propto A^{-1/2}_\omega k_n^2 t^{-2} \\
\rho &\propto k_n^4 t^{-4} & k_B \theta &\propto A^{1/4}_\omega k_n t^{-1} & f &\propto A^{1/4}_\omega k_n^{-1} t & q = 0 \\
\end{align*}
\]

4.4 \( n = 2/3 \) y \( \omega = 0 \)

\[
\begin{align*}
G &\propto A^{2/3}_\omega k_n^{-5} t & c &\propto A^{1/3}_\omega k_n^{-1} t^{0} = const. & \Lambda &\propto A^{-2/3}_\omega k_n^2 t^{-2} \\
\rho &\propto k_n^3 t^{-3} & k_B \theta &\propto A^{1/3}_\omega k_n^{0} t^{-2} & f &\propto A^{1/3}_\omega k_n^{-1} t & q = 0 \\
\end{align*}
\]

5 Conclusions

We have solved through Dimensional Analysis a viscous FRW model with \( k = 0 \) and taking into account the energy-momentum tensor and with \( G, \Lambda \) and \( c \) variables. We observe that the solutions that we have obtained coincide with the already obtained by Arbab unless that in this case our model envisages the variation on \( c \), generalizing thus the solutions. We let the conclusions and study of each case for a subsequent paper.

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