Modelling carbon radio recombination line observations towards the ultracompact H\(\pi\) region W48A

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Accepted 2013 June 12. Received 2013 June 12; in original form 2012 December 12

ABSTRACT

We model the carbon recombination line (CRL) emission from the photodissociation region (PDR) surrounding the ultracompact (UC) H\(\pi\) region W48A. Our modelling shows that the inner regions \((A_V \sim 1)\) of the C\(\pi\) layer in the PDR contribute significantly to the CRL emission. Using models spanning over a large range of parameters that are typical for the environments of UCH\(\pi\) regions, we explore the dependence of the line ratios of CRL emission on the density of the PDR and on the far-ultraviolet radiation incident in the region. We find that it is possible to constrain the density of the PDR by observing a suitable set of CRLs. If the neutral density in the PDR is high \((\gtrsim 10^7 \text{ cm}^{-3})\), then the CRL emission is bright at high frequencies \((\gtrsim 20 \text{ GHz})\), and the absorption lines from such regions can be detected at low frequencies \((\lesssim 10 \text{ GHz})\). By modelling CRL observations towards W48A, we can show that the UCH\(\pi\) region is embedded in a molecular cloud with a density of about \(4 \times 10^7 \text{ cm}^{-3}\).

Key words: atomic processes – ISM: clouds – H\(\pi\) regions – radio lines: ISM.

1 INTRODUCTION

H\(\pi\) regions that are less than about 0.1 pc in size are called ultracompact H\(\pi\) (UCH\(\pi\)) regions. Their radio continuum emission is optically thick at frequencies below a few GHz, which indicates emission measures \(\gtrsim 10^7 \text{ pc cm}^{-6}\) and electron densities \(\gtrsim 10^4 \text{ cm}^{-3}\) (see Wood & Churchwell 1989a; Kurtz, Churchwell & Wood 1994; Garay & Lizano 1999). Molecular line observations have revealed that UCH\(\pi\) regions are embedded in dense molecular clouds (e.g. Kim & Koo 2003). Line emission from many high-density tracer molecules, such as NH\(_3\) and CS, has been detected towards some UCH\(\pi\) regions (see Churchwell, Walmsley & Cesaroni 1990; Garay & Lizano 1999; Kim & Koo 2003). The UCH\(\pi\) regions are bright at far-infrared wavelengths, and models of this emission suggest that hot dust envelopes surround newly formed stars (Churchwell 1993; Kurtz et al. 1994). Some of these sources also show evidence of internal density and velocity gradients within the ionized region (Phillips 2007; Kato, Zhang & Kurtz 2008; Sewilo et al. 2008). These observations imply that UCH\(\pi\) regions are the early evolutionary stages of massive stars, embedded in dense massive molecular clouds with high optical extinctions (see Churchwell 2002).

The age of UCH\(\pi\) regions estimated using the observed number of such H\(\pi\) regions in the Galaxy, and the galactic star formation rate is \(\sim 10^3 \text{ yr}\). This age is found to be longer than the time-scale of the expansion of the ionized gas to a size of about 0.1 pc (dynamical age; a few times \(10^3 \text{ yr}\)) at its sound speed (see Wood & Churchwell 1989b). Many models have been proposed to resolve this inconsistency in age, which is also called the age paradox (see Garay & Lizano 1999; Franco et al. 2007, and references therein). The proposed models include the following: (i) the confinement of ionized gas as a result of the thermal or turbulent pressure of the surrounding material; (ii) the confinement by the ram pressure of infalling material or bow shocks; (iii) champagne flows; (iv) disc evaporation; (v) mass loaded stellar wind. Recently, Peters et al. (2010) have proposed that the ‘flickering’ of the size of the ionized regions around massive stars, resulting from shielding by dense filaments in accretion flow, makes the size of the H\(\pi\) region independent of the age of the star. Many of these models need a dense external medium surrounding the ionized gas (e.g. de Pree, Rodrigue & Goss 1995). Indeed, observations of molecular and carbon recombination lines (CRLs) have also shown the presence of high-density \((> 10^5 \text{ cm}^{-3})\) molecular material in the vicinity of UCH\(\pi\) regions (Roshi et al. 2005b).

The high-density molecular material surrounding UCH\(\pi\) regions can be heated by the far-ultraviolet (FUV) radiation from the embedded massive stars, establishing a photodissociation region (PDR;
The PDR in the vicinity of the H II region forms a thin layer, mostly consisting of ionized carbon (C II) and neutral hydrogen (H I). The physical conditions in this layer can be inferred by observing [C II], [O I] and ro-vibrational H2 emission lines (see Hollenbach & Tielens 1997), which are all in the infrared band. The recombination of an electron with a carbon ion at high quantum numbers, n > ~40, and the subsequent cascade produce HII regions also indicate that these H II regions might be pressure-confined (Roshi et al. 2005a).

The PDRs can attain steady state if the age of these regions is much larger than the time-scales of the cooling and molecular processes. The cooling time-scale is of the order of 4 × (T/1000)−7/2 yr, which is much smaller than the evolutionary time-scales (Hollenbach & Natta 1995) for a typical temperature of a few tens to ~1000 K in the PDR. Of the chemical processes, the slower hydrogen formation time-scale, tH, is about 5 × 108/n yr, where n is the hydrogen density in cm−3 (Goldsmith & Sternberg 1995). For densities ~104 cm−3, tH, is < 5000 yr, which is shorter than the expected age or the time-scale for pressure equilibrium. This suggests that the PDR surrounding UCH II regions can attain steady state in a relatively short time-scale for high ambient gas densities.

For an expanding UCH II region, the PDR properties evolve with time (Franco, Tenorio-Tagle & Bodenheimer 1990; Roger & Dewdney 1992). In this case, thermal equilibrium is achieved faster than the dynamical time-scales (Hollenbach & Natta 1995), but the effect of time-dependent chemistry might be important. In this paper, we consider the case where the PDR is in a steady state.

2.1 Model for carbon recombination lines

We consider a spherical UCH II region of radius, RUCH, placed inside a molecular medium of density n, for modelling. The ionizing star is assumed to be at the centre of the UCH II region and is stationary with respect to the molecular cloud. The molecular material is heated by the FUV radiation from the embedded star, establishing a PDR layer as shown schematically in Fig. 1. The FUV field incident on the surface of the molecular medium, G, is specified in units of the mean interstellar radiation field G0 (see Le Petit et al. 2006). For a given FUV field and a molecular density, the PDR models might be pressure-confined for a long time. CRL observations towards UCH II regions also indicate that these H II regions might be pressure-confined (Roshi et al. 2005a).

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Figure 1. A schematic diagram of the geometry used for the CRL intensity calculations (not to scale). The PDR layer, shown as the shaded region, surrounds the UCH II region. The FUV radiation from the central star in the H II region produces the PDR layer. The background continuum Tb(v, p) for each ray, with impact parameter p, is calculated along the dotted lines, and Tl(v, p) is the calculated line brightness temperature corresponding to this ray.

2 PHOTODISSOCIATION REGION SURROUNDING AN ULTRACOMPACT H II REGION

The PDR is created at the interface between an UCH II region and the associated molecular cloud. During the early phases of the evolution of H II regions, the existence of the PDR layer depends on the relative speed between the ionization and the dissociation fronts (Bertoldi & Draine 1996). However, UCH II regions can attain pressure equilibrium within the dense molecular cores (Kurtz et al. 2001), and the high-pressure cores can stop the expansion at a timescale of about 3 × 105 yr (Franco et al. 2000). Arthur et al. (2004) have noted that the dust in the ionized region can stall the expansion even earlier. This time-scale is a factor of about 10 smaller than the age inferred from the observed number of UCH II regions (Wood & Churchwell 1989b), which suggests that the H II regions might be pressure-confined for a long time. CRL observations to-
solve the energy balance and the chemical network simultaneously. This solution provides the equilibrium gas temperature, \( T_e \), and the density of ionized carbon, \( n_{C^+} \), as well as that of the electron, \( n_e \), as a function of depth into the molecular medium measured from the surface of incidence of FUV radiation (see Le Bourlot et al. 1993). For the present work, these results for the PDR are obtained using the MEUDON PDR code (version 13X103) developed by Le Bourlot et al. (1993), in which a one-dimensional, semi-infinite plane-parallel slab of gas is heated from one side by the FUV field. The standard values of all other parameters of this model, such as the heating and cooling processes, can be obtained from the above reference. We have used the standard chemical network available with this code (the file called Drcos.chi), which uses a total of 69 chemical species and 539 chemical reactions.

For the calculation of the level populations of C II, we approximate the PDR layer into \( N_{\text{slab}} \) slabs, each with a constant \( T_e \) and \( n_{C^+} \). The level populations of C II are calculated for each slab. The effects of deviations of the level populations from the local thermodynamic equilibrium (LTE) on the line emission are characterized by the factors \( b_\alpha \) and \( \beta_\alpha \) (see Dupree & Goldberg 1970). These factors are calculated using the code originally developed by Salem & Brocklehurst (1979), which was later modified by Walmsley & Watson (1982) and Payne et al. (1994). The excitations of carbon levels are affected by the background radio continuum radiation and local gas collisions. Because the free–free optical depth of the PDR layer for the frequencies of interest here is negligible, all the slabs receive the same background continuum intensity. The temperature of the background radiation incident at the surface of the PDR layer, at any given frequency, is determined using the electron temperature and emission measure of the ionized gas in the spherical UCH II region (Mezger & Henderson 1967).

In order to estimate the CRL intensities, we have substituted the radial dependence of the physical quantities of the spherical PDR shell by the one-dimensional results obtained as described above. The emergent intensity is calculated for many rays (\( \sim 200 \)). Each ray is identified with an impact parameter, \( p \) (see Fig. 1), which is measured from the centre of the sphere and has values in the range 0 to \( R_{\text{HII}} \). The background continuum for each ray is calculated through the chord, shown as dashed lines in Fig. 1.

Following Shaver (1975), the line radiation temperature for a ray with impact parameter, \( p \), is given by

\[
T_l(v, p) = T_{l+c}(v, p) - T_c(v, p),
\]

where

\[
T_{l+c}(v, p) = T_0(v, p) \exp\left(-\tau_l^\prime - \tau_c^\prime\right)
\]

\[
+ \int_0^{\tau_l^\prime + \tau_c^\prime} T_{ex}(v, \tau') \exp(-\tau') \, d\tau',
\]

and

\[
T_c(v, p) = T_0(v, p) \exp\left(-\tau_c^\prime\right)
\]

\[
+ \int_0^{\tau_c^\prime} T_{ex}(v, \tau') \exp(-\tau') \, d\tau'.
\]

In the above equations, \( v \) is the frequency of the transition, \( \tau' \) is the optical depth to a point in the PDR layer measured from the outer boundary and \( \tau_l^\prime \) is the total optical depth of the PDR layer. The subscripts ‘l’ and ‘c’ denote the line and continuum, respectively, and \( T_0 \) and \( T_{ex} \) are the background radio continuum radiation temperature and excitation temperature of the C II levels, respectively.

The surface averaged line radiation temperature is obtained as

\[
T_l(v) = \frac{\int_0^{R_{\text{HII}}} T_l(v, p) 2\pi p \, dp}{\int_0^{R_{\text{HII}}} 2\pi p \, dp}.
\]

3 FORMATION OF CARBON RECOMBINATION LINES IN A PHOTODISSOCIATION REGION

In order to understand the CRL formation in a PDR, we study the contribution to the total CRL emission of each thin shell within the PDR layer surrounding the UCH II region. The combined PDR and CRL model, as described in Section 2, is calculated for the observed parameters of W48A (see Section 4). Using these calculations it is possible to understand the various characteristics of CRL formation.

The CRL optical depth as a function of depth into the PDR is obtained by considering a molecular cloud of density \( 10^7 \) cm\(^{-3} \), heated by a FUV flux of \( 10^4 \) G0. The densities of C\(^+\) and electrons and the gas temperature provided by the PDR model for these parameters are plotted as a function of \( A_V \) in Fig. 2 (upper panel). Using these quantities, the LTE and non-LTE optical depths of C53\(\alpha\) (45.4764 GHz) and C76\(\alpha\) (14.6973 GHz) are computed, and these are also plotted as a function of \( A_V \) in Fig. 2 (lower panel). As seen in Fig. 2, both LTE and non-LTE optical depths have large (absolute) values near \( A_V \sim 1 \) for the PDR properties considered for modelling. The CRL optical depth is proportional to \( E/M/T_s^{-3/2} \) and hence will have large values at those \( A_V \) where this factor is maximum. The non-LTE effect depends on a number of factors, such as background radiation field and electron and ion densities. It is expected that the

![Figure 2](https://example.com/figure2.png)
non-LTE optical depth will become negative for a set of frequencies, resulting in partial masing of the recombination line (Shaver 1975). For example, the non-LTE optical depth calculated for the model parameters considered here indicates that the intensity of C76α is amplified by stimulated emission (see Fig. 2).

By modelling the spectral energy distributions (SEDs) of smaller H II regions (including hypercompact H II regions) over a wide range of frequencies, Keto et al. (2008) have inferred density gradients with power-law indices between $-1.5$ and $-2.5$. The SEDs of H II regions with such density gradients will be different from those with uniform density, spherical ionized gas, as considered in the CRL modelling. Typically, such SEDs have higher radio flux densities near frequencies ($>10$ GHz) where the uniform density models become optically thin (e.g. Keto et al. 2008). At frequencies less than about 15 GHz, stimulated emission of CRLs occurs because of background radiation, and hence higher line flux density is expected compared to the uniform density models. Similarly, if the lines at these frequencies are in absorption, once again large line flux densities are expected. Therefore, the model results presented here are applicable for UCH II regions that do not show evidence of density profiles in their SEDs.

Such density gradients can also be present in the molecular gas in the vicinity of H II regions. Because the thickness of the layer that contributes to the CRLs is very small, any variation of the physical parameters within this layer is not expected to contribute significantly for the above observed indices. Further, we compare the variation of the LTE optical depths inside a medium with a density gradient with a power-law index of $-2.0$ (for $n = 10^7$ cm$^{-3}$, $G = 10^5$ G0 and $n = 10^6$ cm$^{-3}$, $G = 10^7$ G0), with those of the appropriate constant density models, such that the mean density of the C II layers in both models are the same. The differences in the variation of the optical depth with distance are negligible between these models. Therefore, constant molecular gas density models are a good approximation for modelling the CRLs. However, the densities inferred using the CRL-emitting PDR layer would imply higher densities at the surface of the H II region for a power-law density medium compared to a constant density medium.

3.1 Dependence of carbon recombination line intensity on frequency

Fig. 3 (left panel) shows the variation of CRL flux density with the frequency of the line transition for different PDR densities ($10^5$, $10^6$, $10^7$ and $10^8$ cm$^{-3}$). An FUV field of $10^6$ G0 and UCH II region with parameters described above are used for the flux density calculations. Qualitatively, Fig. 3 (left panel) shows that, for a given FUV field, the maxima of line emission shifts to higher frequencies with increasing density.

Fig. 3 (right panel) shows the variation of CRL flux density for different line transitions with an incident FUV field ($10^5$, $10^6$ and $10^7$ G0). For these calculations, we use a neutral density of $10^7$ cm$^{-3}$ for the PDR and UCH II region with parameters as described above. This figure shows that the flux density generally decreases with an increasing FUV field. This decrease in flux density is because of the general increase in the PDR temperature with the FUV field. The CRL optical depth inversely depends on the PDR temperature $(T_{\text{p}})^{-5/2}$ and hence the flux density decreases with the FUV field.

An examination of the frequency dependence of the CRL flux density (Fig. 3) shows that for densities $>10^7$ cm$^{-3}$, it might be possible to detect the CRL in absorption at frequencies below $\sim 10$ GHz. The absorption at low frequencies occurs because the level population of quantum states corresponding to these frequencies approaches the LTE value at higher densities. The LTE excitation temperature for these transitions is smaller than the background radiation temperature of the UCH II region.

3.2 Carbon recombination line from an expanding ultracompact H II region

The modelling shows that, for densities typical for PDRs near UCH II regions, it is possible to detect line emission at smaller quantum numbers (i.e. higher frequencies: $\gtrsim 20$ GHz or so). This is evident from Fig. 3 for densities $>10^7$ cm$^{-3}$ where line emission is detectable at frequencies above $\sim 20$ GHz. The non-LTE optical depths of these lines are positive (see, for example, Fig. 2), indicating that the line emission is not dominated by stimulated emission. Therefore, for the geometry of the PDR shown in Fig. 1, the detection of line emission at frequencies $\gtrsim 20$ GHz provides a unique opportunity to constrain the expansion speed of the UCH II region. At these frequencies, any expansion will produce a double profile for the CRL emission and the separation between the two line components gives a direct measure of the expansion of the UCH II region. The application of this method to the UCH II regions will be discussed further in a forthcoming paper.

3.3 Dependence of carbon recombination line intensity on photodissociation region parameters

The ranges of PDR parameters over which we present the model results are chosen based on the observational studies of the environments of the H II regions. The densities of the molecular material near UCH II regions inferred from observations of the high-density tracer molecule, NH$_3$, are in the range of a few times $10^4$ to $10^5$ cm$^{-3}$ (see Garay & Lizano 1999). Because of the embedded O4 to B0 type stars, the expected FUV field at the surface of UCH II regions ranges from $10^4$ to greater than $10^6$ G0 (Wood & Churchwell 1989b). Thus, we estimate the CRL intensities for molecular cloud densities range from $10^4$ to $10^6$ cm$^{-3}$ and G ranging from $10^4$ to $10^7$ G0. For the parameters of the UCH II region, the observed values of W48A are used (see Section 4).

The CRL emission is calculated for a set of 63 PDR models for a grid in the G--$n$ plane. The results are further re-gridded into a fine grid. In Fig. 4, we plot the variation of the ratio of line intensities between the transitions C92α(8.3135 GHz), C76α(14.6973 GHz).
and C53α (45.4764 GHz), in the G–n plane. The parameters that have been assumed for the FWHM of the telescope beam and the distance to the cloud do not affect the line ratios. The contours of the line ratio, C92α/C76α, clearly distinguish the density of the PDR but are mostly independent of the FUV field. The line ratio, C76α/C53α, shows a similar dependence but two densities are possible for a given observed line ratio. These figures show that observations at a suitable set of frequencies will be able to constrain the density of the PDR.

4 PHOTODISSOCIATION REGION AROUND THE ULTRACOMPACT H II REGION W48A

The UCH II region W48A is located in the high-mass star-forming region G35.20–1.74 at a distance of about 3.27 kpc (Wood & Churchwell 1989a; Zhang et al. 2009). Using the Very Large Array (VLA), Roshi et al. (2005b) have observed W48A at many frequencies in both the continuum and the spectral line. The continuum emission towards W48A can be well fitted by a constant density spherical UCH II region model. Based on this fit, the estimated parameters of W48A are as follows: the radius of the UCH II region RUCHII is 0.059 pc; the EM of the background continuum is $6.4 \times 10^6$ pc cm$^{-6}$; the temperature of the ionized gas is $0.99 \times 10^4$ K (Roshi et al. 2005b). The CRL emission is detected at 14 GHz (C76α) and 45 GHz (C53α) and the upper limits have been provided at 8 GHz (C92α) and 4 GHz (C110α, C111α). At a distance of 3.27 kpc, the UCH II region subtends an angle of 3.7 arcsec, which is used to convert the model line temperature to line flux density. For further details on the observations, see Roshi et al. (2005b).

The observed values of C76α/C53α and C92α/C76α are 0.73 ± 0.28 and <0.3, respectively (after correcting for different beam sizes at different frequencies). Because the C92α line flux density is an upper limit, it might also represent an undetected absorption line. Modelling also shows that the C92α transition can be in emission or in absorption, depending on the PDR density and background FUV field (see Section 3.3). Therefore, the ratio C92α/C76α used is 0.0 ± 0.3 in order to represent both positive and negative upper limits. Using the model results, in Fig. 5 we plot the CRL line ratio contours for those values observed towards W48A. The χ$^2$ contours corresponding to 1σ (cyan), 2σ (green) and 3σ (magenta) confidence regions of the fit are plotted with decreasing levels of grey-shaded regions. The intersection of the contours of the observed CRL ratios gives a PDR density of about $4 \times 10^6$ cm$^{-3}$ and an FUV field of about $7 \times 10^6 G_0$. However, the 1σ confidence region suggests that the allowed density and G values are larger because of large errors in the line ratios. Sensitive CRL observations are needed to further constrain the model parameters.

The FUV field incident on the PDR, estimated from the parameters of the stellar type (O8–O7.5) obtained by Roshi et al. (2005b), is $<10^6 G_0$ without considering any dust extinction for FUV photons. This value for G can be used to further constrain the PDR density to a range between 2.5 and $7 \times 10^5$ cm$^{-3}$. The range of PDR densities obtained here is about 30 per cent smaller than the range obtained using a homogeneous slab model (Roshi et al. 2005b).

5 CONCLUSIONS

We have modelled the CRL emission from the PDR layers surrounding an UCH II region. The depth dependence of temperature and the ionized carbon and electron densities obtained from PDR models have been incorporated into this model. The non-LTE population of the carbon levels is calculated using these temperatures and densities. The CRL emission is presented over a range of...
Figure 5. Contours of the observed line ratios $C76/α/C53/α$ (solid) and $C92/α/C76/α$ (dashed) towards W48A, are plotted in the $G–n$ plane. The intersection of the contours of the observed CRL ratios gives a PDR density of about $4 \times 10^7$ cm$^{-3}$ and an FUV field of about $7 \times 10^4$ G$_0$. The $\chi^2$ contours corresponding to $1\sigma$ (cyan), $2\sigma$ (green) and $3\sigma$ (magenta) confidence regions of the fit are plotted with decreasing levels of grey-shaded regions.

PDR parameters. The results are shown in the $G–n$ plane, where $G$ ranges from $10^4$ to $10^7$ G$_0$ and the density ranges from $10^4$ to $10^8$ cm$^{-3}$. By modelling the observed CRL emission towards W48A, we obtain a density for the ambient medium of about $4 \times 10^7$ cm$^{-3}$.

ACKNOWLEDGEMENTS

We are grateful to the anonymous referee for critical comments and suggestions. SJ is grateful for the support offered by the Raman Research Institute on a short visit. We thank J. Le Bourlot for providing us with the PDR code. This work was partially supported by PROMEP/103-5/07/2462 and Conacyt CB-2009-01/130523 grants.

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