Certain quantum key distribution achieved by using Bell states

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(Dated: June 27, 2018)

A new protocol for quantum key distribution based on entanglement swapping is presented. In this protocol, both certain key and random key can be generated without any loss of security. It is this property differs our protocol from the previous ones. The rate of generated key bits per particle is improved which can approach six bits (4 random bits and 2 certain bits) per four particles.

PACS numbers: 03.67.-a, 42.50.Dv, 85.25.Dq

In cryptography, a message is uneavesdropped to any eavesdroppers. To achieve this goal, the message is combined with a key to produce a cryptogram. For a cipher to be secure, it should be impossible to unlock the cryptogram without the key. In this sense, the security of the cipher depends on the security of the key, which is difficult to be assured by classical means. Quantum cryptography, also called quantum key distribution (QKD), is defined as a procedure, in which two legitimate communicators can establish a sequence of key bits secretly and any unauthorized user can be detected. Since the BB84 protocol[1], the first quantum key distribution scheme, was proposed, various quantum encryption schemes have been proposed such as B92 protocol[2], the EPR protocol[3] and other protocols[4-11]. All these protocols for QKD produce random key bits, the security of the protocols is assured by this way. Here we propose a new QKD scheme in which not only random key bits, but also certain key bits can be generated without any loss of security.

In this paper, we present a new protocol for quantum key distribution based on entanglement swapping. In our proposed protocol, Alice only transports 2 particles to Bob, she can share 6 bits, which include 2 certain bits and 4 random bits, with Bob secretly. This protocol brings some merits over the previous protocols, for example, the use efficiency can approach 100%.

It has been proposed by Zukowski et al.[13] for two pairs of entangled particles with each pair in one of the Bell states. Entanglement swapping works as follows. Consider two pairs of entangled particles 1, 2, 3 and 4, prepared in Bell states respectively $|\phi^+\rangle_{12}$ and $|\psi^-\rangle_{14}$. If a Bell operator measurement is performed on particles 2 and 3, then we have

$$|\Psi\rangle_{1234} = |\phi^+\rangle_{12} \otimes |\psi^-\rangle_{14}$$

$$= \frac{1}{2}(|\phi^+\rangle_{23}|\psi^-\rangle_{14} + |\phi^+\rangle_{23}|\psi^-\rangle_{14}$$

$$- |\phi^+\rangle_{23}|\psi^-\rangle_{14} - |\psi^-\rangle_{23}|\phi^+\rangle_{14}$$

(1)

As is obvious, from above equation, the four possible results $|\phi^+\rangle_{23}, |\phi^-\rangle_{23}, |\psi^+\rangle_{23}$ and $|\psi^-\rangle_{23}$ occur with same probability and the outcome of each measurement is purely random. Suppose that the result $|\phi^+\rangle_{23}$ is obtained, the state of the pair 1 and 4 after the measurement is consequently $|\psi^-\rangle_{14}$. Which means particles 1 and 4 become entangled although they have never interacted.

Recently, the entanglement swapping between two pairs of qubits has been used to realize schemes for quantum key distribution[9]. Which do not need the legitimate users to choose between possible measurements to generate the key and assure the security. But only random key bits can be produced in these schemes.

Our scheme, illustrated in Fig. 1, can be described as follows.

Alice has a EPR source. She can apply a local operation $X$, where $X \in \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$ on her particles. Alice and Bob agree beforehand as following encoding

$$|\phi^+\rangle \rightarrow 00, |\psi^+\rangle \rightarrow 01, |\psi^-\rangle \rightarrow 10, |\phi^-\rangle \rightarrow 11$$

(2)

where $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ are Bell states.

$$\sigma_0 \rightarrow 00, \sigma_1 \rightarrow 01, \sigma_2 \rightarrow 10, \sigma_3 \rightarrow 11$$

(3)

They can perform series of operations as following,

1. Alice creates EPR pairs and sends some of her particles to Bob.
Alice creates two EPR pairs $|\phi^+\rangle_{12}$ and $|\phi^+\rangle_{34}$, she sent particles 2 and 4 to

Bob, and tells Bob the state in which the particles are.

2. Alice applies a local operation on particle 1.

Let the initial state of particles 1 and 2 be $|\phi^+\rangle_{AB}$, and the state of particles 3 and 4 be $|\phi^+\rangle_{AB}$. Alice performs a local operation $\sigma_1$ on particle 1 and thus, the state $|\phi^+\rangle_{AB}^1$ is turned into $|\psi^+\rangle_{AB}^1$.

3. Alice makes a Bell operator measurement on particles 1 and 3. We assume that the result of Alice’s measurement as $|\psi^-\rangle_{AA}^1$, then she can infer the state of particles 2 and 4 as $|\phi^-\rangle_{BB}^{24}$ by the following equation,

$$|\Phi^{1234}_{ABAB}\rangle = |\psi^+\rangle_{AB}^1 \otimes |\phi^+\rangle_{AB}^{34}$$

$$= \frac{1}{2} (|\phi^+\rangle_{AA}^{13} |\psi^+\rangle_{BB}^{24} - |\phi^-\rangle_{AA}^{13} |\psi^-\rangle_{BB}^{24} + |\psi^+\rangle_{AA}^{13} |\phi^-\rangle_{BB}^{24} - |\psi^-\rangle_{AA}^{13} |\phi^+\rangle_{BB}^{24})$$

4. Alice calculates which state of particles 1 and 3 should be without local operation applied on particle 1 when the EPR state of particles 2 and 4 is $|\phi^-\rangle_{BB}^{24}$.

$$|\Phi^{1234}_{ABAB}\rangle = |\phi^+\rangle_{AB}^{12} \otimes |\phi^+\rangle_{AB}^{34}$$

$$= \frac{1}{2} (|\phi^+\rangle_{AA}^{13} |\psi^+\rangle_{BB}^{24} + |\phi^-\rangle_{AA}^{13} |\psi^-\rangle_{BB}^{24} + |\psi^+\rangle_{AA}^{13} |\phi^-\rangle_{BB}^{24} + |\psi^-\rangle_{AA}^{13} |\phi^+\rangle_{BB}^{24})$$

From Eq.(5), Alice get the state of particles 1 and 3 as $|\phi^-\rangle_{AA}^{13}$, compared this result with expression(3), she decodes the bits as 11.

5. Alice tells Bob she has made a Bell operator measurement on her particles 1 and 3, but without mention the result of her measurement through classical channel.

6. Bob performs a Bell operator measurement on particles 2 and 4 and infers the outcomes of Alice’s measurement.

From the calculation of entanglement swapping, we know, after Alice has performed a Bell operator on particles 1 and 3, Bob’s measurement result of his particles should be $|\phi^-\rangle_{BB}$. Note that the result of Alice’s measurement inferred by Bob is the corresponding state determined using Eq.(5). Because Bob does not know the exact state of entangled pair 1 and 2 after Alice subjects her particle 1 to a local operation. What he can do is to use known information: entangled pairs $|\phi^+\rangle_{AB}^{12}$ and $|\phi^+\rangle_{AB}^{34}$, to calculate the state particles 1 and 3 is in. In this case, it is $|\phi^-\rangle_{AA}^{13}$, which can be decoded as 11.

7. Bob asks Alice’s corresponding outcome of her Bell operator measurement on particles 1 and 3 by classical channel. He knows the Bell operator measurement of particles 1 and 3 is $|\psi^-\rangle_{AA}^{13}$.

8. Bob compares the measurement result of particles 1 and 3 announced by Alice with his calculation result, and he will find what the Alice’s local operation on particle 1 is $|\psi^-\rangle_{AA}^{13} \sigma_1 \leftrightarrow |\phi^-\rangle_{AA}^{13}$. Bob gets the local operation is $\sigma_1$. Then he knows the certain bits is 01. Obviously, both Alice and Bob know the state $|\phi^-\rangle_{BB}^{24}$ and the imaginary state of particles 1 and 3 as $|\psi^-\rangle_{AA}^{13}$. Thus they establish the random key bits 11 at the same time.

Bob makes another ES calculation as eq.(5), he obtain the imaginary states of particles 1 and 3 as $|\psi^-\rangle_{AA}^{13}$. He decodes the random bits as 10.

They perform above procedures on each group of particles repeat.

9. Bob told Alice some his qubits, Alice will be sure if there is any eavesdropper.

Then Alice and Bob securely share the certain key bit and the random key.

I. SECURITY

Security is an important issue of QKD. One protocol for QKD is said to be secure if it can either generate the key secretly or stops the protocol when any eavesdroppers appear. In this section, we proof that our proposed protocol is secure, even if the shared quantum channels are public.

First, we consider the security of the random key which is encoded in the outcomes of Bob’s measurement and the calculation result of “Alice’s measurement” assuming that she does not apply any local operations on her particle. Where the symbol “” implies that Alice does not really apply this measurement and get the outcome, it is just an imaginary measurement. After Alice has performed a local operation on particle 1 and a Bell operator measurement, no eavesdroppers can gain any information, although Alice announces publicly her Bell measurement result on particles 1 and 3. The reason is that no one except Alice knows exactly the entangled states of particles 1 and 2 once Alice performs a local operation on her particle. There is only a probability of $\frac{1}{3}$ to guess the correct EPR state in which particles 1 and 2 are in. If Eve eavesdrops 4n key bits, she tends to succeed with a probability of $(\frac{1}{3})^n$ which approaches 0 as n is sufficiently large. Further, if Eve is so clever that she shares the quantum channels with Alice and Bob, or replaces the particles in transit to Alice and Bob by the particles prepared by her, she can succeed in eavesdropping with probability $\frac{1}{3}$ or $\frac{1}{2}$ respectively. This has been certified by us in[14]. Then Bob can send some results to Alice to detect the action of Eve and thus, Eve cannot gain any information of the random key.

Now turns to the problem of the certain key which is encoded in the local operations performed by Alice on her particle. It is available to Bob by comparing the state of particles 1 and 3 gotten by Bob with the result announced by Alice. Thus the security of the certain key
depends on whether Bob can secretly infer the correct corresponding state of particles 1 and 3 using the entanglement swapping between two initial shared entangled pairs. In our previous work[14], we showed that it is impossible for any eavesdroppers to know the Bell operator measurement results by inner-spring the transmission. More detail, Eve cannot determine the outcomes of the sender and receiver’s measurements in which the key is encoded, no matter what methods Eve may choose. There is a probability of 2/3 to gain the correct information on the key when Eve tries to share the quantum communication channels with Alice and Bob, and a success probability of 1/3 by replacing the particles transported to legitimate users by the particles accessible to her. This is a useful conclusion even for this different protocol. That is to say, Eve can eavesdrop no information of the certain key.

This protocol is secure, even the shared entangled states are public (in another words, everyone know that the Alice and Bob shared states are $|\phi^+\rangle_{AB}$). If Alice and Bob do not publish their’s measured result, eavesdropper (Eve) cannot get the information from the known entangled channels. For example, both the quantum channels are the same Bell states $|\phi^+\rangle_{AB}$ and $|\phi^+\rangle_{AB}$. By the entanglement swapping calculation, we know the state of these entangled particles are

$$|\Phi\rangle_{AB} = \frac{1}{2} \{ |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} + |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} + |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} \} (6)$$

So the four Bell-operator measurement outcomes of Alice are equally likely, each occurring with probability $1/4$. Yet the eavesdropper can not get any information, although she owned many entangled pairs in state $|\phi^+\rangle_{EE}$.

If the Eve was smart enough, she made the state

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}} [ (|00\rangle_{AB} \alpha_E + |11\rangle_{AB} \beta_E) ]$$

instead of $|\phi^+\rangle_{AB}$, when Alice made a joint measurement on particle 1 and particle 3, the state become

$$|\Psi\rangle_{ABE} = \frac{1}{2\sqrt{2}} \{ |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} |\phi^+\rangle_{EE} + |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} |\phi^+\rangle_{EE} + |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} |\phi^+\rangle_{EE} \} (9)$$

Compare eq.(3) with eq.(4), we found if there is an eavesdropper, there is only $1/2$ probability that the two result are same. Bob can send some result random to Alice, then Alice can know there is eavesdropper with different result, they give up this key. Namely, Eve can not gain any information about the key.

If Eve shared the entangled pairs $|\phi^+\rangle_{AE}$ with Alice instead of Bob, Eve shared entangled pairs $|\phi^+\rangle_{EB}$ with Bob, before the key distribution, both Alice and Bob do not know this. Then the process of key distribution become

$$|\Phi\rangle_{AB} = \frac{1}{2} \{ |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} + |\phi^+\rangle_{AB} |\phi^+\rangle_{BB} \} (8)$$

and

$$|\Phi\rangle_{BB} = \frac{1}{2} \{ |\phi^+\rangle_{BB} |\phi^+\rangle_{BB} + |\phi^+\rangle_{BB} |\phi^+\rangle_{BB} \} (9)$$

From Eq.(5) and Eq.(6), we know that, the probability, which Bell operator measurement of particle 1 and 3 is same as particle 1' and 3', is only $1/4$, when Bob send some his measurement results to Alice, Alice can find if there is eavesdropper.

So this protocol is secret and secure.

Acknowledgments

This work was partially supported by the CNSF (grant No.90203018) the Knowledgeable Innovation Program (KIP) of the Chinese Academy of Sciences, the National Fundamental Research Program of China with No.001GB309310, K. C. Wong Education Foundation, HongKong, and China Postdoctoral Science Foundation.

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