Acceleration of primary and secondary particles in galaxy clusters by compressible MHD turbulence: from radio haloes to gamma-rays

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ABSTRACT

Radio observations discovered large-scale non-thermal sources in the central Mpc regions of dynamically disturbed galaxy clusters (radio haloes). The morphological and spectral properties of these sources suggest that the emitting electrons are accelerated by spatially distributed and gentle mechanisms, providing some indirect evidence for turbulent acceleration in the intergalactic medium (IGM).

Only deep upper limits to the energy associated with relativistic protons in the IGM have been recently obtained through gamma and radio observations. Yet these protons should be (theoretically) the main non-thermal particle component in the IGM implying the unavoidable production, at some level, of secondary particles that may have a deep impact on the gamma-ray and radio properties of galaxy clusters.

Following Brunetti & Lazarian, in this paper we consider the advances in the theory of magnetohydrodynamics (MHD) turbulence to develop a comprehensive picture of turbulence in the IGM and extend our previous calculations of particle acceleration by compressible MHD turbulence by considering self-consistently the re-acceleration of both primary and secondary particles. Under these conditions we expect that radio to gamma-ray emission is generated from galaxy clusters with a complex spectrum that depends on the dynamics of the thermal gas and dark matter. The non-thermal emission results in very good agreement with radio observations and with present constraints from hard X-ray and gamma-ray observations. In our model giant radio haloes are generated in merging (turbulent) clusters only. However, in case secondaries dominate the electron component in the IGM, we expect that the level of the Mpc-scale synchrotron emission in more relaxed clusters is already close to that of the radio upper limits derived by present observations of clusters without radio haloes. Important constraints on cluster physics from future observations with present and future telescopes are also discussed.

Key words: acceleration of particles – radiation mechanisms: non-thermal – turbulence – galaxies: clusters: general – radio continuum: general – X-rays: general.

1 INTRODUCTION

Radio observations of galaxy clusters prove the presence of non-thermal components, magnetic fields and relativistic particles, mixed with the hot intergalactic medium (IGM; e.g. Ferrari et al., 2008).

Potentially, cluster mergers can be responsible for the origin of the non-thermal components in IGM. During these events a fraction of the gravitational binding energy of dark matter haloes that is converted into internal energy of the baryonic matter can be channelled into the amplification of the magnetic fields (e.g. Dolag, Bartelmann & Lesch 2002; Subramanian, Shukurov & Haugen 2006; Ryu et al. 2008) and into the acceleration of particles via shocks and turbulence (e.g. Ensslin et al. 1998; Sarazin 1999; Blasi 2001; Brunetti et al. 2001, 2004; Miniati et al. 2001; Petrosian 2001; Berrington & Dermer 2003; Fujita, Takizawa & Sarazin 2003; Gabici & Blasi 2003; Ryu et al. 2003; Cassano & Brunetti 2005; Brunetti & Lazarian 2007; Hoeft & Brueggen 2007; Pfrommer 2008).

Theoretically relativistic protons are expected to be the dominant non-thermal particles component since they have long lifetimes and remain confined within galaxy clusters for a Hubble time (Völk, Aharonian & Breitschwerdt 1996; Berezinsky, Blasi & Ptsukin 1997; Ensslin et al. 1998). Proton–proton (p–p) collisions in the IGM inject secondary particles, including neutral pions that decay...
into gamma-rays and relativistic electrons that produce synchrotron and inverse Compton (IC) emission. So far only upper limits to the gamma-ray emission from galaxy clusters have been obtained by Fermi and Cherenkov telescopes (e.g. Aharonian et al. 2009a,b; Ackermann et al. 2010; Aleksic et al. 2010). These upper limits, together with constraints from complementary approaches based on radio observations (e.g. Reimer et al. 2004; Brunetti et al. 2007), suggest that relativistic protons contribute to less than a few per cent of the energy of the IGM, at least in the central Mpc-sized regions.

Relativistic electrons in the IGM are nowadays studied by radio observations of diffuse synchrotron radiation from galaxy clusters. Radio haloes are the most spectacular examples of cluster-scale radio sources, they are diffuse radio sources that extend on Mpc-scales in the cluster central regions and are found in about 1/3 of massive galaxy clusters (e.g. Feretti 2002; Ferrari et al. 2008; Cassano 2009).

The origin of relativistic electrons in radio haloes is still debated. In the context of the hadronic model (Dennison 1980; Blasi & Colafrancesco 1999; Pfrommer & Ensslin 2004) radio haloes are due to synchrotron emission from secondary electrons generated by p–p collisions, in which case clusters are (unavoidably) gamma-ray emitters due to the decay of the π⁰ produced by the same collisions. The very recent non-detections of nearby galaxy clusters at GeV energies by Fermi significantly constrain the role of secondary electrons in the non-thermal emission (Ackermann et al. 2010). Similarly, previous works pointed out that the spectral and morphological properties of a number of radio haloes appear inconsistent with a simple hadronic origin of the emitting particles (e.g. Brunetti et al. 2008; Brunetti 2009; Donnert et al. 2010a,b; Macario et al. 2010).

A second scenario proposed for the origin of radio haloes is based on turbulent re-acceleration of relativistic particles in connection with cluster-mergers events (e.g. Brunetti et al. 2001; Petrosian 2001; Fujita et al. 2003; Cassano & Brunetti 2005). The acceleration of thermal electrons to relativistic energies by magnetohydrodynamics (MHD) turbulence in the IGM faces serious drawbacks based on energy arguments (e.g. Petrosian & East 2008), consequently in these models it must be assumed a pre-existing population of relativistic electrons in the cluster volume that provides the seed particles to re-accelerate during mergers. These seeds may be primary electrons injected by supernova (SN), active galactic nucleus (AGN), galaxies and shocks in the cluster volume that can be accumulated for a few Gyr at energies of a few hundred MeV (e.g. Sarazin 1999; Brunetti et al. 2001).

MHD turbulence theory seriously advanced in the last decades also affecting our view of particle acceleration in astrophysical plasmas (e.g. Chandran 2000; Yan & Lazarian 2002; Lazarian 2006a and references therein). For this reason in Brunetti & Lazarian (2007) we considered the advances in the theory of MHD turbulence to develop a comprehensive picture of turbulence in the IGM and to study the re-acceleration of relativistic particles. In this respect, our main conclusions were that the compressible MHD turbulence (essentially fact modes), generated in connection with energetic cluster mergers, is the most important source of stochastic particle acceleration and evolution, in Section 4 we first discuss the connection between mergers and particle re-acceleration by turbulence and then show our results and in Section 5 we give our conclusions and discuss model simplifications, future extensions of the work and a comparison with previous works.

2 TURBULENCE IN THE IGM

Numerical simulations of galaxy clusters suggest that turbulent motions may store an appreciable fraction, 5–30 per cent, of the thermal energy of the IGM (e.g. Roettiger, Burns & Loken 1996; Roettiger, Loken & Burns 1997; Ricker & Sarazin 2001; Sunyaev, Norman & Bryan 2003; Dolag et al. 2005; Vazza et al. 2006; Vazza, Brunetti & Gheller 2009a; Paul et al. 2010). The largest turbulent eddies should decay into a turbulent velocity field on smaller scales, possibly developing a turbulent cascade.

It is known that the mean free path of thermal protons arising from Coulomb collisions in the hot IGM may be very large, 10 to 100 kpc. Fluids in such a collisionless regime can be very different  

¹ Assuming the same cluster magnetic field and for a given synchrotron luminosity.
from their collisional counterparts (Schekochihin et al. 2005, 2010). The parallel to magnetic field viscosity of IGM can be very large and the collisionless plasmas are subject to various instabilities. Instabilities however may change the effective collisionality of the fluid, justifying the application of MHD to describe the IGM at least on its large scales (see Lazarian et al. 2010). Indeed, particles in plasmas can interact through the mediation of the perturbed magnetic fields, and effective collisionality of plasmas may differ dramatically from the textbook estimates. The difference stems from the various instabilities that are present in the IGM plasmas (firehose, mirror, gyroresonance etc.). These instabilities are expected to transfer the energy from the turbulent compressions on the scales less or equal to the particle mean free path to the perturbations at the particle gyroscale. This has been described in Lazarian & Beresnyak (2006), as a result of the scattering, the mean free path of particles decreases, which makes the fluid essentially collisional over a wide range of scales, with the critical scale for which the fluid gets effectively collisional that is expected to decrease with the increase of turbulent driving rate. While some parts of the aforementioned paper dealing with the interaction of turbulence and cosmic rays remain controversial, a similar approach should be reliably applicable to thermal plasma particles. Therefore we believe that on scales much larger that the thermal particle gyroradius the turbulence can be treated in the MHD approximation and shall apply below the theory of MHD turbulence to the IGM.

Turbulence generated during cluster mergers is expected to be injected at large scales, $L_o \sim 100$–400 kpc, with typical velocity of the turbulent eddies at the injection scale around $V_o \sim 300$–700 km s$^{-1}$ (e.g. Subramanian et al. 2006). This makes turbulence subsonic, with $M_o = V_o/c_s \approx 0.25$–0.6, but strongly super-Alfvénic, with $M_A = V_A/v_A \approx 5$–10. Turbulent motions at large scales are thus essentially hydrodynamics and the cascading of compressive (magnetosonic) modes may couple with that of solenoidal motions (Kolmogorov eddies).

In Brunetti & Lazarian (2007) we discussed that the important consequence of the turbulence in magnetized plasma is that both solenoidal and compressive modes in hot galaxy clusters would not be strongly affected by viscosity at large scales and an inertial range is established, provided that the velocity of the eddies at large scales exceeds $\approx 300$ km s$^{-1}$.

In the Kolmogorov cascade the turbulent velocity $V_i$ scales as $V_i(l/L_o)^{1/3}$, and at scales less than $L_A \sim L_o M_A^{-1.3}$ the turbulence gets sub-Alfvénic and we enter into the MHD regime (see discussion in Lazarian 2006b). For the parameters above the scale $L_A \sim 0.1$–1 kpc, but the actual number of $M_A$ is not certain and therefore our estimate of $L_A$ should be treated with caution. Fortunately, the above uncertainty does not change the results of our paper appreciably.

Compressible MHD turbulence is a subject where a number of important insights have been obtained much before these ideas can be tested; the pioneering works in the area include Montgomery & Turner (1981), Shebalin, Matthaus & Montgomery (1983) and Higdon (1984). The key idea of critical balance by Goldreich & Sridhar (1995) has influenced in a profound way our further thinking of the MHD cascade.

In the MHD regime, at smaller scales where $V_i \leq v_A$, three types of modes should exist in a compressible magnetized plasma: Alfvén, slow and fast modes. Slow and fast modes may be roughly thought as the MHD counterpart of the compressible modes, while Alfvén modes may be thought as the MHD counterpart of solenoidal Kolmogorov eddies (a more extended discussion can be found in Cho, Lazarian & Vishniac 2002, and references therein).

Turbulence in the IGM is most likely a complex mixture of several turbulent modes. We shall assume that a sizeable part of turbulence at large scales (namely at scales where the magnetic tension does not affect the turbulent motions) is in the form of compressible motions. This is reasonable as these modes are easily generated in high $\beta$ medium (e.g. Brunetti & Lazarian 2007 and references therein).

Situation may be radically different at smaller scales where the magnetic field tension affects turbulent motions, i.e. in the MHD regime, $l \leq l_A$. In this case, MHD numerical simulations have shown that a solenoidal turbulent forcing gets the ratio between the amplitude of fast and Alfvén modes in the form (Cho & Lazarian 2003)

$$\frac{\langle \delta V \rangle^2}{\langle \delta V \rangle^2} \sim \frac{\langle \delta V \rangle v_A}{c_s^2 + v_A^2}$$

This essentially means that coupling between these two modes may be important only at $l \approx l_A$ (in the MHD regime it should be $\delta V \ll v_A$) since the drain of energy from Alfvénic cascade is marginal when the amplitudes of perturbations become weaker. Most importantly in galaxy clusters it is $c_s^2 \geq v_A^2$ and thus the ratio between the amplitude of fast and Alfvén modes at scales $l < l_A$ is expected to be small, $(\delta V)^2/(\delta V)^2 \leq (v_A/c_s)^2 \sim 10^{-2}$ (this for solenoidal forcing at $l \approx l_A$).

A more recent work by Kowal & Lazarian (2010) decomposed turbulent motions into slow, fast and Alfvén modes using wavelets. This approach is better justified than the decomposition in Fourier space employed in Cho & Lazarian (2002, 2003). Indeed, Alfvén and slow modes are defined in the local system of coordinates (see Lazarian & Vishniac 1999, hereafter LV99; Cho & Vishniac 2000; Maron & Goldreich 2001) and therefore the procedure of Fourier decomposition by Cho & Lazarian (2002, 2003, see 2005 for a review) can only be statistically true. The wavelet decomposition procedure is more localized in space and therefore is potentially more precise. Nevertheless, the results on mode decomposition in Kowal & Lazarian (2010) agree well with those in Cho & Lazarian (2003), which provides us with more confidence about the properties of MHD turbulence that we employ in the paper to describe turbulence interaction with energetic particles.

Our treatment of turbulence assumes that the turbulence is balanced, i.e. the energy flux of waves$^2$ moving in one direction is equal to the energy flux in the opposite direction. While the theory of imbalanced turbulence is being intensively developed (see Lithwick & Goldreich 2001; Beresnyak & Lazarian 2008, 2009, 2010; Chandran 2008, Perez & Boldyrev 2009) we do not expect that the effects of imbalance would dominate in the cluster environments. First of all, the turbulence driving is not expected to be strongly localized and then the effects of compressibility should decrease the local turbulence imbalance.

3 PARTICLE ACCELERATION, ENERGY LOSSES AND SECONDARY PARTICLES

In this paper we provide an extension of our previous calculations. We assume the picture of MHD turbulence in the IGM, as derived in Brunetti & Lazarian (2007), to calculate the re-acceleration of relativistic particles by compressible MHD turbulence by taking into

$^2$ Strong MHD turbulence presents a case of dualism of waves and eddies (see LV99, Cho, Lazarian & Vishniac 2003). Strong non-linear damping of oppositely moving Alfvén wave packets makes them act like eddies.
account self-consistently also the generation and re-acceleration of secondary particles.

The aim of this section is to present the formalism and the main assumptions used in our calculations.

3.1 Basic formalism

We model the re-acceleration of relativistic particles by MHD turbulence in the most simple situation in which only relativistic protons are initially present in a turbulent IGM. These protons generate secondary electrons via p–p collisions and in turns secondaries (as well as protons) are re-accelerated by MHD turbulence.

We model the time evolution of the spectral energy distribution (SED) of electrons, $N^e_\gamma$, and positrons, $N^e_\gamma$, with an isotropic Fokker–Planck equation

$$\frac{\partial N^e_\gamma(p, t)}{\partial t} = \frac{\partial}{\partial p} \left[ N^e_\gamma(p, t) \left( \frac{dp}{dt} \right) - \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D^e_{pp}(p, t) \right) \right] + \frac{\partial^2}{\partial p^2} \left[ D^e_{pp} N^e_\gamma(p, t) \right] + \mathcal{Q}^e_\gamma[p, t; N^p_\gamma(p, t)],$$

where $|dp/dt|$ marks radiative (r) and Coulomb (i) losses (Section 3.3), $D^e_{pp}$ is the electron/positron diffusion coefficient in the momentum space due to the coupling with magnetosonic modes (Section 3.2) and the term $\mathcal{Q}^e_\gamma$ accounts for the injection rate of secondary electrons and positrons due to p–p collisions in the IGM (Section 3.4).

The time evolution of the SED of protons, $N^p_\gamma$, is given by

$$\frac{\partial N^p_\gamma(p, t)}{\partial t} = \frac{\partial}{\partial p} \left[ N^p_\gamma(p, t) \left( \frac{dp}{dt} \right) - \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D^p_{pp}(p, t) \right) \right] + \frac{\partial^2}{\partial p^2} \left[ D^p_{pp} N^p_\gamma(p, t) \right] - \frac{N^p_\gamma(p, t)}{\tau_{pp}(p)},$$

where $|dp/dt|$ marks Coulomb losses (Section 3.3), $D^p_{pp}$ is the diffusion coefficient in the momentum space of protons due to the coupling with magnetosonic modes (Section 3.2) and $\tau_{pp}$ is the proton lifetime due to p–p collisions in the IGM (Section 3.3; see also Ensslin et al. 2007).

For isotropic turbulence (Section 3.2) the diffusion equation in the $k$-space is given by

$$\frac{\partial \mathcal{W}(k, t)}{\partial t} = \frac{\partial}{\partial k} \left[ k^2 D_{kk} \frac{\partial}{\partial k} \left( \frac{\mathcal{W}(k, t)}{k^2} \right) \right] + i(k, t) - \sum_i \Gamma_i(k, t) \mathcal{W}(k, t),$$

where $D_{kk}$ is the diffusion coefficient in the $k$-space, $\Gamma_i(k, t)$ are the different damping terms and $i(k, t) = I \delta(k - k_o)$ is the turbulence injection term, i.e. we consider the most simple situation where turbulence is injected at a single scale, with wavenumber $k_o$. The wave–wave diffusion coefficient of magnetosonic modes (Kraichnan treatment) is given by (Brunetti & Lazarian 2007 and references therein)

$$D_{kk} \approx \frac{V_{pk} k^4}{(V_{ph})^2} \frac{\mathcal{W}(k, t)}{\rho (V_{ph})^2},$$

where $V_{ph}$ is a representative, averaged (with respect to $\theta$), phase velocity.

We shall assume isotropic MHD turbulence to calculate the particle acceleration rate at any time. This assumption is appropriate for super-Alfvénic turbulence and fast modes (e.g. Cho & Lazarian 2003), provided that collisionless dampings are not efficient. At smaller scales, collisionless dampings with thermal particles in the IGM become severe and modify the spectrum of turbulent modes, and a cut-off in the turbulent spectrum is generated at $k = k_c$, where the damping time-scale becomes shorter than the cascading time.

The most important damping with thermal (and relativistic) particles is the transit-time damping (TTD) that is highly anisotropic (e.g. Schlickeiser & Miller 1998; Brunetti & Lazarian 2007; Yan, Lazarian & Petrosian 2008) being stronger for $\theta \sim \pi/2$ and causing the spectrum of the turbulent modes to become anisotropic at scales $k \sim k_c(\theta)$. On the other hand, in Brunetti & Lazarian (2007) we have shown that, under physical conditions typical of the IGM, hydromotions bend the magnetic-field lines in a time-scale, $\tau_{th} \approx I_a/v_L$, that is comparable to the damping time-scale of turbulence, in which case there is chance that approximate isotropization of the turbulent spectrum is maintained even at scales where dampings are severe.

Thus following Brunetti & Lazarian (2007) we shall assume a simplified turbulent (isotropic) spectrum in the form

$$\mathcal{W}(k) \approx (I_o \rho (V_{ph})^{1/2} k^{3/2})^{-2},$$

for $k_o < k < k_c$, with the cut-off wavenumber estimated from the condition that the damping time-scale becomes smaller than the cascading time-scale:

$$\tau_d \approx 1/\left( \sum \Gamma_i(k, \theta) \right)^2 \approx k^2/D_{kk},$$

that is

$$k_c \approx c I_o \rho (V_{ph}) \left( \sum \Gamma_i(k, \theta) \right)^{-2} k,$$

where quantities $\langle \cdots \rangle$ are averaged with respect to $\theta$ and $\sum \Gamma_i$ is the total (thermal and non-thermal) damping term due to TTD resonance and the constant $C \sim$ a few (Brunetti & Lazarian 2007; see also Matthaeus & Zhou 1989, for details on Kraichnan constants).

3.2 Momentum diffusion coefficient due to compressible MHD turbulence

Compressible turbulence can affect particle motion through the action of the mode electric field via gyroresonant interaction (e.g. Melrose 1968), the condition for which is

$$\omega - k_v v_i - n^\gamma = 0,$$

where $n = \pm 1, \pm 2, \ldots$ gives the first (fundamental), second, . . . harmonics of the resonance, while $v_i = \mu v$ and $k_v = \eta k$ are the parallel (projected along the magnetic field) speed of the particles and the wavenumber, respectively.

Following Brunetti & Lazarian (2007) we assume particle–mode coupling through the TTD, $n = 0$, resonance (e.g. Fisk 1976; Eilek 1979; Miller, La Rosa & Moore 1996; Schlickeiser & Miller 1998). In principle, this resonance changes only the component of the particle momentum parallel to the seed magnetic field and this would cause an increasing degree of anisotropy of the particle distribution leading to a less and less efficient process with time. Thus an important aspect in this working picture is the need of isotropization of particle momenta during acceleration (e.g. Schlickeiser & Miller 1998). In this paper we shall assume continuous isotropization of particle momenta. Isotropy may be provided by several processes.
discussed in the literature. These include electron firehose instability (Plipp & Völk 1971; Paesold & Benz 1999), and gyrosion to the Alfvén (and slow) modes at small scales, provided that these modes are not too much anisotropic (cf. Yan & Lazarian 2004). Gyrosion may also occur with electrostatic lower hybrid modes generated by anomalous Doppler resonance instability due to pitch angle anisotropies (e.g. Liu & Mok 1977; Moghadam-Taaheri et al. 1985) and, possibly, with whistlers (e.g. Steinacker & Miller 1992). In addition, Lazarian & Beresnyak (2006) proposed isotropization of cosmic rays due to gyrosion instability that arises as the distribution of cosmic rays gets anisotropic in phase space.

We adopt the momentum-diffusion coefficient of particles, \( D_{pp} \), as derived from detailed balancing argument, i.e. relating the diffusion coefficient of a \( \alpha \)-species to the damping rate of the modes themselves with the same particles (e.g. Eilek 1979; Brunetti & Lazarian 2007):

\[
D_{pp}(p) = \frac{\pi^2}{2e} p^2 \frac{1}{B_0^2} \int_0^{\pi} \sin(\theta) \left( 1 - \frac{V_{th}}{c} \right) \left( 1 - \frac{V_{th}}{c} \right)^2 \frac{dW}{dk} \left( \frac{V_{th}}{c} \right) \right)^2 \int dk W(k),
\]

(10)

where \( B_0 \) is the unperturbed magnetic field, \( k_c \) is given in equation (8) and

\[
W(p) = \frac{1}{B_0^2} \left( \frac{B_0}{16nW(k)} \right) W(k),
\]

(11)

where \( B_0^2/W(k) \) is the ratio between magnetic field fluctuations and total energy in the mode (the quantity \( \langle \cdots \rangle \) in equation 11 indicates average with respect to \( \theta \) and is of order unity, see Brunetti & Lazarian 2007 for further details).

3.3 Energy losses for electrons and protons

The energy losses of relativistic electrons in the IGM are dominated by ionization and Coulomb scattering, at low energies, and by synchrotron and IC losses, at higher energies (e.g. Sarazin 1999). The rate of losses due to the combination of ionization and Coulomb scattering is (in cgs units)

\[
\left( \frac{dp}{dt} \right)_{rad} = -3.3 \times 10^{-29} n_h \left[ 1 + \frac{\ln(\gamma / n_h)}{75} \right].
\]

(12)

where \( n_h \) is the number density of the thermal plasma. The rate of synchrotron and IC losses is (in cgs units)

\[
\left( \frac{dp}{dt} \right)_{rad} = -4.8 \times 10^{-4} p^2 \left[ \frac{B_{\mu0}}{3.2} \right]^2 \frac{\sin^2 \theta}{2/3} + (1 + z)^4.
\]

(13)

where \( B_{\mu0} \) is the magnetic field strength in units of \( \mu \)G and \( \theta \) is the pitch angle of the emitting leptons; in case of efficient isotropization of the electron momenta, the \( \sin \theta \) is averaged to 2/3.

For relativistic protons, the main channel of energy losses in the IGM is produced by inelastic p–p collisions. The lifetime of protons due to p–p collisions is given by

\[
\tau_{pp}(p) = \frac{1}{c n_h} \sum \sigma_{i = \pm} \cdot \theta.
\]

(14)

\[\text{In this paper we use the inclusive cross-section, } \sigma_{\pm}(p_p), \text{ given by the fitting formulae in Dermer (1986a) which allow to describe separately the rates of generation of neutral and charged pions.}
\]

For trans-relativistic and mildly relativistic protons, energy losses are dominated by ionization and Coulomb scattering. Protons more energetics than the thermal electrons, namely with \( \beta_p \gg \beta_e \equiv (3/2m_e/m_p)^{1/2} \beta_e \) (\( \beta_e \approx 0.18(\sqrt{T/(10^8 K)})^{1/2} \) is the velocity of the thermal electrons), are affected by Coulomb interactions. Defining \( x_m = \sqrt{3/\beta_e^2} \), one has (Schlickeiser, 2002)

\[
\left( \frac{dp}{dt} \right) \approx -1.7 \times 10^{-29} \left( \frac{n_h}{10^{-3}} \right) \left( \frac{B_p}{3 \text{ GeV}} \right) \left( \frac{\beta_p}{x_m^2 + \beta_p^2} \right) \text{ (cgs).}
\]

(15)

3.4 Injection of secondary electrons

The decay chain that we consider for the injection of secondary particles in the IGM due to p–p collisions is (Blasi & Colafrancesco 1999)

\[p + p \rightarrow \pi^0 + \pi^+ + \pi^- + \text{anything},\]

\[\pi^0 \rightarrow \gamma \gamma,\]

\[\pi^\pm \rightarrow \mu^\pm + v_\mu \rightarrow e^\pm v_\mu v_e,\]

that is a threshold reaction that requires protons with kinetic energy larger than \( T_p \approx 300 \text{ MeV}.\)

A practical and useful approach to describe the pion spectrum both in the high energy \( (E_\pi > 10 \text{ GeV}) \) and low energy regimes was proposed in Dermer (1986b) and reviewed by Moskalenko & Strong (1998) and Brunetti & Blasi (2005), and is based on the combination of the isobaric model (Stecker 1970) and scaling model (Badhwar, Golden & Stephens 1977; Stephens & Badhwar 1981).

The injection rate of pions is given by

\[
Q_{\pi^+}(E, t) = n_0 e^{-F_p(E, t)} \frac{F_{\pi}(E, t) \sigma_{\pi^0}(p_p)}{1 + (m_e/c_p)^2},
\]

(16)

where we adopted \( F_p \) as given in Brunetti & Blasi (2005) that use the isobaric model for \( E_\pi < 3 \text{ GeV} \), a scaling model for \( E_\pi > 10 \text{ GeV} \) and a linear combination of the two models for intermediate energies, and where \( p_\mu \) is the threshold momentum of protons for the process to occur.

The injection rate of relativistic electrons/positrons is given by

\[
Q_{e^\pm}(E, t) = \int_{E_\pi}^\infty Q_{\pi^\pm}(E_\pi, t) dE_\pi \int dE_p \times F_p(E_\pi, \mu, E_\pi) F_{\pi}(E_p, E_\pi),
\]

(17)

where \( F_{\pi}(E_\pi, \mu, E_\pi) \) is the spectrum of electrons and positrons from the decay of a muon of energy \( E_\mu \) produced in the decay of a pion with energy \( E_\pi \) and \( F_p(E_p, E_\pi) \) is the muon spectrum generated by the decay of a pion of energy \( E_\pi \) that is

\[
F_{\pi}(E_\mu, E_\pi) = \frac{m_\pi^2}{m_\pi^2 + m_e^2} \frac{1}{E_\pi^2 - m_\pi^2}
\]

(18)

between a kinematic minimum and maximum muon energy given by

\[
m_\mu \gamma_\nu (1 - \beta_\pi \beta_\nu) \leq E_\mu \leq m_\mu \gamma_\nu (1 + \beta_\pi \beta_\nu),
\]

(19)

where \( \gamma_\nu \) is the Lorentz factor of the muon in the pion frame, \( \beta_\nu \approx 0.2714 \) (Moskalenko & Strong 1998), and from kinematics:

\[
m_\mu \gamma_\nu (1 + \beta_\pi \beta_\nu) = \frac{m_\mu^2 - m_e^2}{2m_\pi^2}.
\]

(20)
In order to simplify calculations, following Brunetti & Blasi (2005), we assume that the spectrum of muons is a delta-function:

$$F(\mu, E_x) = \delta (\mu_x - E_x),$$  

where

$$E_\mu = \frac{m_\mu^2 - m_\nu^2}{m_\mu^2} E_x.$$  

We use the spectrum of electrons and positrons from the muon decay, $F(\mu_x, E_x, E_\mu, E_\nu)$, as given by Blasi & Colafrancesco (1999), and combining their results with equations (16), (17) and (21), we obtain the rate of production of secondary electrons/positrons:

$$Q_{ee}(p, t) = \frac{8\beta_\mu^2 m_\mu^2 n_0 c}{m_\mu^2 - m_\nu^2} \int_\epsilon_{min} \int_{E_\mu} dE_\mu \frac{dE_\nu}{E_\nu} \beta_\mu N(E_\nu)$$

$$\times \left[ 1 - \left( \frac{m_\mu^2 c^2}{E_\nu} \right)^2 \right] \sigma^{\delta}(E_\nu) F(E_x, E_\nu) F,$$

where $E_{min} = 2E_\mu\beta_\mu^2 m_\mu^2/(m_\mu^2 - m_\nu^2)$, and

$$F = \frac{5}{12} - \frac{3}{4} \lambda^2 + \frac{1}{3} \lambda^3$$

$$- \frac{1}{2} \frac{\beta_\mu}{\beta_\nu} \left[ 1 - \frac{1}{6} \left( \beta_\mu + \frac{1}{2} \right) \lambda^2 + \left( \frac{1}{3} \beta_\mu \right) \lambda^3 \right]$$

for $\frac{1}{1 - \beta_\nu} \leq \lambda \leq 1$ and

$$= \frac{\lambda^3 \beta_\mu}{1 - \beta_\nu} \left[ 3 - \frac{2}{3} \lambda \left( \frac{3 + \beta_\mu}{1 - \beta_\mu} \right) - \frac{1}{1 - \beta_\mu} \left( \lambda^2(1 + \beta_\mu) \right) \right]$$

$$- \frac{2}{1 - \beta_\mu} \left[ \frac{1}{2} + \lambda(1 + \beta_\mu) + \frac{2}{3} \lambda \left( \frac{3 + \beta_\mu}{1 - \beta_\mu} \right)^2 \right]$$

for $0 \leq \lambda \leq \frac{1 - \beta_\mu}{1 + \beta_\mu}$,

and where we use the following definitions:

$$\bar{\lambda} = \frac{E_x}{E_\nu} = \frac{E_x}{E_\nu} \left( \frac{2\beta_\mu^2 m_\mu^2}{m_\mu^2 - m_\nu^2} \right),$$

$$\tilde{\beta}_\mu = \left( 1 - \frac{m_\nu^2}{E_\nu} \right)^{1/2}$$

and

$$\tilde{\beta}_\mu = \frac{1}{\beta_\mu} \left( \frac{m_\mu^4}{m_\mu^2 - m_\nu^2} \right)^2 \left[ 4\beta_\mu^2 - 1 + \left( \frac{m_\mu}{m_\pi} \right)^4 \right].$$

4 RESULTS

In this section we calculate particle acceleration and non-thermal emission in galaxy clusters by assuming that MHD turbulence is generated during mergers between clusters and re-accelerates primary and secondary particles.

Following previous works (e.g. Brunetti & Blasi 2005) we adopt a simplified situation where we do not consider primary electrons in the IGM (see Section 5 for discussion), and where the two main ingredients are (i) relativistic protons, that are believed to be the most important non-thermal particle components in the IGM (e.g. Blasi, Gabici & Brunetti 2007 for review), and (ii) the MHD turbulence.

On one hand, relativistic protons inject secondary particles via p–p collisions in the IGM that produce radiation from the radio to the gamma-ray band, at the same time MHD turbulence may re-accelerate relativistic protons and secondary electrons in the IGM, generating radio haloes and leaving an imprint in the general non-thermal properties of galaxy clusters.

At variance with the aforementioned paper that focus on the Alfvénic case, following the model of MHD turbulence in galaxy clusters by Brunetti & Lazarian (2007), we assume that MHD turbulence in the IGM is in the form of compressible modes whose cascading from large to small scales results in an isotropic turbulent spectrum. This allows us to readily connect the injection of turbulence at large scales with the particle acceleration process and to study the theoretical framework of the connection between cluster mergers and turbulent re-acceleration of relativistic particles in the IGM. As a matter of fact calculations reported in this section provide an extension of those in Brunetti & Lazarian (2007) that consider re-acceleration of (only) primary particles by compressible MHD modes.

As already stressed, one of the main motivations for these new calculations comes from the recent gamma-ray and radio observations that put severe constraints on the energy density of relativistic protons in galaxy clusters (Brunetti et al. 2007; Aharonian et al. 2009a,b; Ackermann et al. 2010; Alekseev et al. 2010), allowing for including secondary particles in turbulent-acceleration models with substantially less degree of freedom than in the past.

4.1 Turbulent re-acceleration, time-scales and connection with mergers

In the framework adopted in our paper the idea is that radio haloes are generated by the re-acceleration of relativistic electrons (secondaries in our specific case) by turbulence generated during cluster mergers.

The leading processes in the context of our scenario are the generation, cascading and dissipation of turbulence in the IGM, and the acceleration and cooling of relativistic particles. These processes, and the cluster–cluster collisions themselves, have their own time-scale and the general picture breaks out from the interplay of all these time-scales. Fig. 1 reports the time-scales of the most relevant processes as calculated in different, relevant, regions of the cluster volume.

The generation and dissipation of turbulent motions in the IGM is not studied in great details, however, present numerical simulations suggest that these motions can be generated in galaxy clusters for a substantial fraction of the period of cluster–cluster interaction, a few Gyr, possibly driven by shock waves that cross the cluster volume and by the sloshing (and stripping) of cluster cores (e.g. Dolag et al. 2005; Vazza et al. 2009a; Paul et al. 2010; ZuHone 2010). Under our working picture, the compressible turbulence, after being injected at larger scales, decays at smaller scales where it dissipates through dampings with thermal and non-thermal particles in the IGM (Section 3.1). The turbulence decay requires about one eddy turnover time, that in the case of fast mode is $t_{k\lambda} \sim (V_{ph}/V_k)^2 (L/k)^{1/2}$ (e.g. Yan & Lazarian 2004), implying a unavoidable delay between the first generation of large-scale turbulence in a given region and the beginning of the phase of particle re-acceleration (that is mainly due to the non-linear interaction of particles with turbulent modes at smaller scales) in the same region. We believe however that this delay does not break the temporal connection between mergers and particle acceleration, since the eddy turnover time of compressible turbulence in massive (hot) clusters is $t_{k\lambda} \approx 0.2$–1 Gyr (by
assumed typical injection scales $L \approx 200–300\kpc$ and $(V_L/c)_1^2 \approx 0.1–0.3$, that is smaller than the typical duration of cluster–cluster interaction.

Under our simplified working picture, where turbulence is (only) generated by energetic cluster mergers, compressible turbulence dissipates completely in a few eddy turnover times, as soon as galaxy clusters becomes more relaxed. The decrease of the efficiency of turbulent-particle acceleration and the suppression of non-thermal cluster-scale emission, i.e. the ‘dissipation’ of radio haloes, are even faster due to the fact that (i) the acceleration efficiency scales non-linearly with the turbulent spectrum and (ii) the cooling time of the radio-emitting electrons is short, $\approx 0.1\Gyr$ (Fig. 1). The consequence is a tight connection between radio haloes and cluster mergers, although the picture may be more complex as the secondary particles, continuously injected in the IGM, should generate synchrotron emission at some level also in relaxed clusters (see Section 4.3.2).

Radio observations of statistical samples of galaxy clusters show that (i) giant radio haloes form (only) in merging clusters, (ii) that their lifetime in merging clusters is of the order of $1\Gyr$ and (iii) suggest that halo emission should dissipate in relaxed clusters in a short, $\lesssim \Gyr$, time-scale (e.g. Hwang 2004; Brunetti et al. 2007, 2009b; Venturi et al. 2008); these observational points are consistent with the working picture described in this section.

### 4.2 Spectral evolution of re-accelerated protons and electrons

In this section we report on some relevant results on the evolution of the particles spectrum (protons and secondary electrons/positrons) subject to TTD resonance with compressible MHD turbulence; the approach followed in this section is than used in next section to calculate non-thermal emission from galaxy clusters.

For example, $r_{\text{acc}} \propto D_{\text{pp}} \propto W_T^2$, combining equations (6), (8) and (10), see also Brunetti & Lazarian (2007).

![Figure 1](https://example.com/figure1.png)  
**Figure 1.** Left-hand panel: the lifetime of relativistic electrons in the IGM at $z = 0.2$ as a function of their Lorentz factor. Thick (blue) lines are for cluster cores ($B = 3\muG$, $n_{th} = 2 \times 10^{-3}\cm^{-3}$) and thin (red) lines are for cluster periphery ($B = 0.5\muG$, $n_{th} = 10^{-4}\cm^{-3}$). We report the total lifetime (solid lines) and the lifetimes due to single processes: Coulomb losses (dashed lines), synchrotron and IC losses (dotted lines) and bremsstrahlung losses (long dashed lines). The yellow region marks the turbulence eddy turnover time (assuming $L \sim 200–300\kpc$, $(V_L/c)_1^2 \approx 0.1–0.3$, $T \approx 10^8\K$), while the blue region marks the range of Lorentz factors of the relativistic electrons emitting at frequencies $\approx 300–1400\MHz$. Right-hand panel: the lifetime of cosmic ray protons in the IGM as a function of the particle momentum. Thick (blue) lines are for cluster cores and thin (red) lines are for cluster periphery. We report the total lifetime (solid lines) and the lifetimes due to single processes: Coulomb losses (dotted lines) and $p$–$p$ collisions (dashed lines). The yellow region marks the turbulence eddy turnover time, while the blue region marks the range of momentum of relativistic protons that mostly contribute to the injection of secondary electrons emitting at frequencies $\approx 300–1400\MHz$.

![Figure 2](https://example.com/figure2.png)  
**Figure 2.** A scheme of the processes taken into account in our calculations and the coupling between them. The starting points are the properties of the magnetized IGM ($n_{th}$, $T$, $B_0$), the initial spectrum of relativistic protons ($N_p(p,0)$) and the injection rate of turbulence ($I(k)$).

We adopt a simplified situation: Fig. 2 shows the chain of physical processes that we consider and their interplay. We assume that the thermal IGM is magnetized and, at time $t = 0$, consider (only) relativistic protons, with initial spectrum $N_p(p) = K_p p^{-2.6}$. The presence of relativistic and thermal protons determines the initial efficiency of injection of secondary particles in the IGM (Section 3.4), whose initial spectrum is calculated assuming stationary conditions (e.g. Dolag & Ensslin 2000; i.e. by taking $D_{\text{pp}} = 0$ and $\delta N/\delta t = 0$ in equation 2).

Following Brunetti & Lazarian (2007), we assume that compressible turbulence is injected at large scales and develops a quasi-stationary spectrum (equation 6) due to the interplay between non-linear wave–wave interaction and collisionless dampings at smaller...
scales (Section 3.1). Following Brunetti & Lazarian (2007) we consider a time-independent damping, $\sum \Gamma_i \simeq \Gamma_{\text{th}}$, that is obtained by assuming the (initial) physical properties of the IGM. This is motivated by the fact that in our model TTD dampings with relativistic particles are subdominant with respect to those with thermal IGM (see Cassano & Brunetti 2005; Brunetti & Lazarian 2007) and by the fact that the thermal properties of the IGM are not greatly modified by turbulence.

In our calculations we follow self-consistently the re-acceleration of relativistic protons due to TTD with the compressible turbulent modes, the generation (and its evolution with time due to the evolution of the spectrum of protons) of secondary electrons and positrons through collisions between these protons and the IGM and the re-acceleration of the secondaries (see Fig. 2).

Theoretically we expect that magnetic field can be amplified by turbulence in the IGM (e.g. Subramanian et al. 2006), although as a necessary simplification we do not include that amplification process in our calculations. The main reason is that the spectrum of the re-accelerated relativistic electrons is expected to evolve more rapidly than the magnetic field in the IGM (e.g. Cassano 2010). We also stress that present data do not show a clear connection between the magnetic field properties and cluster dynamics (e.g. Clarke, Kronberg & Böhringer 2001; Govoni 2006) that leaves the process of magnetic field amplification still poorly constrained.

Turbulent acceleration can be thought as the combination of a systematic effect, that causes the boosting of the spectrum of particles at higher energies, and a stochastic effect, that causes a broadening of the spectrum without net acceleration (e.g. Melrose 1968; Petsanis 2001). The time-scale of the systematic acceleration is

$$\tau_{\text{acc}} = \frac{p^3}{\partial p^2 D_{\text{pp}}/\partial p} = \frac{p^2}{4D_{\text{pp}}}$$

that does not depend on the particle energy in the case of TTD acceleration (Section 3.2), provided that the spectrum of compressible turbulence is isotropic (Section 3.1).

By considering a reference value of the acceleration time due to TTD resonance in the IGM, $\tau_{\text{acc}} \approx 10^9$ yr (e.g. Cassano & Brunetti 2005; Brunetti & Lazarian 2007), Coulomb\(^7\) and radiative losses in the IGM prevent the re-acceleration of electrons with energies $E < 10$ MeV and $E > 10$ GeV, respectively (Fig. 1).

On the other hand, TTD resonance in the IGM may re-accelerate suprathermal protons up to high energies, and consequently in our model the energy density of relativistic protons increases with mergers, similarly to the energy of the thermal IGM. By considering a toy scenario where these protons are simply injected in the cluster volume, with initial energy density $\epsilon_{\text{RC}}(0)$, and then re-accelerated by MHD turbulence during major cluster mergers, the rate of increase of their energy is

$$\frac{d\epsilon_{\text{CR}}}{dt} \sim \int dk W(k, t) \Gamma_{\text{CR}}(k, t),$$

where the TTD damping of compressible MHD turbulence by relativistic protons in case $\beta_{\text{pp}} \gg 1$ (from equation 31 in Brunetti & Lazarian 2007) is

$$\Gamma_{\text{CR}}/\omega_{\text{cr}} \simeq \frac{\pi^2}{4} c \epsilon_{\text{CR}} \sin^2 \beta \left( \frac{\partial^2 \hat{f}}{\partial p \partial f} \right),$$

where $\{\ldots\} = s + 2$ assuming a power-law energy distribution of relativistic protons $N(p) \propto p^{-s}$. By assuming a ‘sonic’ turbulent forcing, $L_0 \tau_{\text{cl}} \approx \epsilon_{\text{th}}$, where $\tau_{\text{cl}} \sim 3–6$ Gyr is the lifetime of massive clusters, and a typical merging history of massive clusters (e.g. Cassano & Brunetti 2005 and references therein), we expect $\epsilon_{\text{CR}} \lesssim$ few per cent of the thermal energy density, provided that relativistic protons are injected in the IGM with an ‘initial’ energy density $\epsilon_{\text{CR}}(0)/\epsilon_{\text{th}} \sim 0.001–0.01$.\(^8\)

In Fig. 3 we report the time evolution of relativistic electrons and protons in a hot, $T \sim 10^8$ K, IGM by assuming $(V_L/c_s)^2 = 0.22$, in which case the TTD acceleration time is $\tau_{\text{acc}} \sim 10^8$ yr. Radiative losses prevent the acceleration of relativistic electrons

\(^7\)In the external regions of galaxy clusters Coulomb losses are less severe and lower energy electrons can be re-accelerated (Fig. 1).

\(^8\)This is obtained by using the approximate scalings $[kW(k)/\tau_{\text{cl}}] \sim L_0$ in equation (28).
above a maximum energy, \( \gamma_{\text{max}} \approx 10^4 \), producing a bump in the spectrum of electrons (Fig. 3a). The number density of high-energy electrons increases with time also for \( \gamma > \gamma_{\text{max}} \) (Fig. 3a), that is because the injection rate of secondary electrons is enhanced with time as protons are accelerated (Fig. 3b). Overall the mechanism is very efficient: on one hand the acceleration of relativistic protons enhances the injection rate of secondary electrons, at the same time an increasing number of secondary electrons accumulates at energies \( \approx \gamma_{\text{max}} \) where cooling is balanced by acceleration. The combination of these two effects boosts the spectrum of electrons at energies \( \gamma \approx \gamma_{\text{max}} \).

In Fig. 4 we report the evolution of the spectrum of electrons with time assuming physical conditions that span the volume occupied by radio haloes, from cluster core (solid lines) to 1 Mpc distance from the cluster centre (dashed lines); \( (V_L/c)_L = 0.22 \) is assumed in both cases. Fig. 4 highlights the effect of decreasing radiative and Coulomb losses on the spectrum of the re-accelerated electrons: the boosting of the particle spectrum increases in the external (Mpc distance) regions because Coulomb losses are less severe (which allows the re-acceleration of the bulk of secondary electrons) and because synchrotron losses become subdominant with respect to inverse Compton losses due to the scattering of the CMB photons.

### 4.3 The non-thermal spectrum of turbulent galaxy clusters

In this section we calculate the non-thermal spectrum from galaxy clusters assuming re-acceleration of primary protons and of their secondary products by compressible MHD turbulence in the IGM.

#### 4.3.1 A toy model for the Coma cluster

The Coma cluster hosts the best studied, prototype, giant radio halo (Willson 1970; Giovannini et al. 1993). Constraints on the high-energy emission from the Coma cluster are presently available from hard X-ray (BeppoSAX, Fusco-Femiano et al. 1999, 2004; Rossetti & Molendi 2004; RXTE, Rephaeli, Gruber & Blanco 1999; INTEGRAL, Eckert et al. 2007; Lutovinov et al. 2008; Suzaku, Wik et al. 2009; Swift-BAT, Ajello et al. 2009) and gamma-ray (The Energetic Gamma Ray Experiment Telescope (EGRET), Reimer et al. 2003; Fermi, Ackermann et al. 2010; High Energy Stereoscopic System (HESS), Aharonian et al. 2009b; Very Energetic Radiation Imaging Telescope Array System (VERITAS), Perkins 2008) observations. It is thus a natural first step to compare our model expectations with the SED of the non-thermal emission from the Coma cluster.

We assume a simplified model for the thermal gas distribution in the Coma cluster that is anchored to the observed \( \beta \)-model profile (e.g. Briel, Henry & Böhringer 1992, considering \( \Lambda \)CDM cosmology). The gas temperature, \( k_B T \approx 8 \text{ keV} \) (David et al. 1993), is assumed to be constant on Mpc-scale.

The magnetic field in the Coma cluster, and its spatial distribution, is a crucial ingredient in our modelling of the synchrotron (radio halo) properties. We adopt the recent results by Bonafede et al. (2010) that carried out a detailed analysis of the rotation measures of a sample of extended cluster radio galaxies in the Coma cluster. The best fit to their data implies a magnetic field \( B(r) \propto \sqrt{\rho_m} \) with a central value \( B(0) \approx 5 \mu \text{G} \).

Once the thermal gas and magnetic field properties are anchored to present data, the free parameters in our calculations are (i) the energy density (and spatial distribution) of relativistic protons (at time \( t = 0 \)),\(^9\) and (ii) the injection rate (and scale) of compressible turbulence in the IGM, \( I_{\text{inj}} \).

In our reference toy model we assume (i) a flat spatial distribution of relativistic protons on the halo-scale, \( r \sim 5 \text{r}_c \), and a constant ratio between relativistic and thermal particles energy densities at larger distances, and (ii) a specific injection rate of turbulence, \( I_{\text{inj}}/\rho \), constant on the halo-scale. Assumption (i) is motivated by the fact that, similarly to other giant haloes, the Coma radio halo has a very broad synchrotron-brightness distribution (Govoni et al. 2001; see also discussion in Cassano et al. 2007; Donnert et al. 2010b) implying a very broad spatial distribution of relativistic protons on the halo-scale. Assumption (ii) is motivated by hydrodynamical and MHD cosmological simulations that found very extended turbulent regions in simulated galaxy clusters (Sunyaev et al. 2003; Dolag et al. 2005; Vazza et al. 2009a; Paul et al. 2010).

In Fig. 5 (upper panels) we show the expected SED emitted by the Coma cluster (from \( r \lesssim 3-4.5 \text{r}_c \)) assuming our reference model (see caption). We find that the radio spectrum of the Coma radio halo can be well reproduced by assuming a total energy content of relativistic protons, on the halo-scale, \( E_{\text{CR}} \sim 3.5 \text{ per cent of the thermal gas} \) (see caption for details). The presence of a break in the spectrum of the Coma radio halo at higher frequencies has been interpreted as a signature of turbulent acceleration,\(^10\) since it implies a corresponding break in the spectrum of the emitting electrons at the energy where turbulent acceleration is balanced by radiative losses (e.g. Schlickeiser et al. 1987; Brunetti et al. 2001; Petro시안 2001). Fig. 5 demonstrates that also models considering the re-acceleration of secondary particles can explain the steepening of the Coma radio halo, although a tail at higher frequencies due to freshly injected

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\(^9\) We assume an initial spectrum of protons \( N_p(\rho) = K_p \rho^{−2.6} \).

\(^10\) More recently Donnert et al. (2010a) have shown that the observed steepening cannot be due to the SZ decrement due to the hot gas in the central Mpc region of the cluster.
secondary electrons shows up in the (emitted) spectrum. For a consistency check, in Fig. 6 we also show that, although very simplified, our reference model allows us to (roughly) reproduce the observed radio brightness profile of the Coma halo (although the predicted profile is still slightly steeper than the observed one). Such expected broad synchrotron profile is due to the combination of the flat spatial distribution of relativistic protons with the increasing efficiency of re-acceleration of secondary electrons at larger distances from the cluster centre (see Fig. 4).

The gamma-ray emission from the cluster (from $r \leq 3-4.5r_c$, see caption) is dominated by the decay process of $\pi^0$ (Fig. 5) and is expected at about 10 per cent level of the present gamma-ray upper limits ($\text{Fermi}$, Ackermann et al. 2010). In our model the energy content of relativistic protons that is necessary to reproduce the

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**Figure 5.** The radio (left) and high-energy (right) emitted spectra from the Coma cluster (note that the Sunyaev–Zeldovich (SZ) decrement at higher radio frequencies is not included, see e.g. Donnert et al. 2010a). Upper panels show our reference model (see text), lower panels refer to the case of a cluster magnetic field 2.5 times smaller than that in our reference model (for a better comparison red dot–dashed lines in the right-hand panel show the IC and gamma-ray emission expected in our reference model). Solid lines show the emitted spectra obtained after $6.5 \times 10^8$ yr of re-acceleration (upper and lower lines in the synchrotron and gamma-ray spectra refer to the emission within 4.5 and 3 core radii, respectively, while X-rays are calculated within 4.5 core radii), dashed lines show the emitted spectra obtained by assuming that MHD turbulence is dissipated in the cluster. Dotted lines show the emitted spectra at intermediate stages of re-acceleration: after 2 and $4 \times 10^8$ yr, from bottom to top, respectively. Radio data points are taken from Thierbach, Klein & Wielebinski (2003), the hard X-ray upper limit is from Suzaku observations (Wik et al. 2009) and the EGRET and Fermi upper limits are taken from Reimer et al. (2003) and Ackermann et al. (2010), respectively. Calculations assume $(V_L/c_s)^2 \sim 0.18$ for our reference model and 0.2 for the lower panels ($L_o = 300$ kpc is assumed in all calculations). For reference, the energy of the population of relativistic protons ‘measured’ after 0.65 Gyr of re-acceleration (solid lines) is 0.035 and 0.25 times that of the thermal cluster in our reference model and lower panels, respectively.

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11 A similar conclusion comes from calculations of re-acceleration of secondary particles in the Alfvenic case (Brunetti & Blasi 2005).
observed luminosity and brightness profile of the Coma radio halo is much smaller than that from calculations based on pure hadronic models that assume a similar magnetic field in the Coma cluster (Donnert et al. 2010a), implying also a much smaller gamma-ray luminosity.

For seek of completeness, in Fig. 5 (lower panels) we show the expected SED of the Coma cluster by assuming a magnetic field in the cluster 2.5 times smaller than that inferred from Bonafede et al. (2010) (see caption). In this case the properties of the Coma radio halo can be reproduced by assuming an energy content of relativistic protons \( E_{\text{CR}} \sim 25 \) per cent of the thermal energy, i.e. seven–eight times larger than that of our reference model, and the expected gamma-ray emission increases significantly. Interestingly, in this case there would be a chance to detect the Coma cluster in the next years with the Fermi telescope.

Remarkably, on the other way round Fig. 5 (lower panels) demonstrates the importance of present gamma-ray upper limits: in the turbulent re-acceleration picture, the magnetic field in the central regions of the Coma cluster cannot be significantly smaller than about 2 \( \mu G \), provided that hadronic collisions are the main source of the (seed) electrons in the cluster volume.

In the case of our reference model, the inverse Compton emission in the hard X-ray band is expected at a \( \sim \) per cent level than the present Suzaku upper limit and a detection of the Coma cluster will be challenging even with the future hard X-ray experiments [e.g. ASTRO-H, Nuclear Spectroscopic Telescope Array (NuSTAR)]. On the other hand, by assuming a magnetic field in the cluster 2.5 times smaller than that inferred from Bonafede et al. (2010) the expected inverse Compton emission increases significantly and there would be a chance to detect the Coma cluster with future hard X-ray telescopes. However, by assuming a magnetic field substantially smaller than that of our reference model, we find that the predicted radio profile is much steeper than the observed one (Fig. 6).

\[ \delta P/P \lesssim (E Q^3(E)/E_0)/(E Q^0(E))_{E_0}, \]

where \( E_1 \) and \( E_0 \) are about 100 MeV and 1 GeV, respectively, that implies \( \delta P/P \sim 10–15 \) for typical values of the slope of the proton spectrum, \( s \approx 2.4–2.8 \).

The predicted level of amplification and suppression of radio emission in galaxy clusters and its connection with cluster dynamics can be constrained by radio and X-ray observations. The recent radio follow up of a complete X-ray sample of galaxy clusters, the ‘GMRT Radio Halo Survey’ (Venturi et al. 2007, 2008), leads to the discovery of a bi-modal behaviour of the clusters radio properties

4.3.2 Transient and long-living spectral components in galaxy clusters

In the context of our model the non-thermal emission from galaxy clusters is a mixture of two main spectral components: a long-living one that is emitted by the chain of secondary particles continuously generated by collisions between thermal and long-living (several Gyr) relativistic protons, and a transient amplification of the SED that appears when relativistic particles are re-accelerated by the MHD turbulence generated (and then dissipated) in connection with cluster mergers (see also Brunetti et al. 2009a); in this scenario the idea is that the transient synchrotron component generates the observed radio haloes.

The non-thermal spectrum of dynamically relaxed galaxy clusters should be mainly due to the long-living spectral component. An example of this spectral component is shown in Fig. 5 (dashed lines) where we report the synchrotron, inverse Compton and \( \pi^0 \) emission calculated by assuming that MHD turbulence in the Coma\footnote{Also the radial synchrotron profile in non-turbulent clusters is steeper than that in turbulent clusters (Fig. 6).} halo is dissipated.

The main effect of turbulent re-acceleration is to produce a amplification of the synchrotron (radio), inverse Compton (hard X) and \( \pi^0 \) (gamma) emission in merging clusters (Fig. 5), while the effect of the dissipation of MHD turbulence in those clusters that become more relaxed is to suppress (only) the radio and the hard X-ray emission. Consequently in our model we expect a tight correlation between clusters dynamics and the radio (radio haloes) and hard X-ray properties of galaxy clusters, but do not expect a tight correlation between gamma-rays and cluster dynamics.

An important issue is the evolution of the radio haloes in connection with cluster mergers and the difference between the radio properties of merging and relaxed clusters.

In the ‘classical’ scenario, where turbulence re-accelerates primary electrons, the boosting of the synchrotron and hard X-ray emission during cluster mergers can be extremely large (e.g. fig. 5 in Brunetti et al. 2009b; fig. 6 in Cassano 2010). Contrary to that, in our model the presence of relativistic protons in the IGM and the chain of secondary-particles decay induced by p–p collisions put stringent constraints to the radio evolution of galaxy clusters, since long-living spectral components are generated for about a Hubble time. Our calculations exploit this point and unveil the interplay between transient and long-living spectral components. In our model the boosting of the radio and hard X-ray luminosities due to re-acceleration is fairly constrained, since it is essentially due to the re-acceleration of secondary electrons from the energies where the injection spectrum of secondary electrons peaks (or from the energy where re-acceleration is stronger than Coulomb losses) to the energies necessary to electrons to radiate synchrotron emission in the radio band (provided re-acceleration is sufficient to accelerate particles up to these energies). This is \( \delta P/P \approx \frac{(E Q^3(E)/E_0)}{(E Q^0(E))_{E_0}}, \) where \( E_1 \) and \( E_0 \) are about 100 MeV and 1 GeV, respectively, that implies \( \delta P/P \sim 10–15 \) for typical values of the slope of the proton spectrum, \( s \approx 2.4–2.8 \).

The predicted level of amplification and suppression of radio emission in galaxy clusters and its connection with cluster dynamics can be constrained by radio and X-ray observations. The recent radio follow up of a complete X-ray sample of galaxy clusters, the ‘GMRT Radio Halo Survey’ (Venturi et al. 2007, 2008), leads to the discovery of a bi-modal behaviour of the clusters radio properties...
We have shown that a population of relativistic protons consistent with the above limits may be sufficient to generate the observed radio haloes in merging clusters (including their observed brightness profiles and spectra) via the re-acceleration of their secondary electrons by compressible MHD turbulence, provided that the spatial distribution of the relativistic protons is relatively flat and that the magnetic field in galaxy clusters is at the level derived from rotation measurements.

The main consequence of this theoretical scenario is that the non-thermal SED of galaxy clusters is given by the interplay of a transient component, generated by the re-acceleration of secondary particles by MHD turbulence during cluster mergers (that generates radio haloes), and a long-living component, that is generated by the secondary particles that are continuously injected by p–p collisions in the IGM. In this case we expect a tight connection between radio haloes and cluster mergers, as well as an amplification of the level of hard X-ray emission in merging clusters, while we expect no tight correlation between gamma-rays and cluster dynamics.

At the same time, we also expect that diffuse radio emission, due to secondary particles, must be common in galaxy clusters at a level that is about one order of magnitude below that of nowadays observed radio haloes, i.e. at the same level of the upper limits derived by present radio observations of ‘radio quiet’ clusters. This expectation can be tested by future (deeper) radio observations of ‘radio quiet’ clusters.

The level of gamma-ray emission from nearby, massive, galaxy clusters is expected at about 10 per cent of the level of present upper limits, assuming a magnetic field strength in these clusters in line with present studies of rotation measures. This implies that only future telescopes [e.g. Cherenkov Telescope Array (CTA)] may lead to the detection of galaxy clusters in the gamma-ray band. On the other hand, if the magnetic field in galaxy clusters is smaller, detection of galaxy clusters with Fermi could be possible in next years. In this respect, in the context of our model, present Fermi upper limits for the Coma cluster already provide a limit to the central value of the magnetic field B(0) ≥ 1–2 µG.

5.1 Model simplifications and future steps

In our paper we focus on the role played by compressible MHD turbulence, essentially the fast modes. Our educated guess, motivated in Section 2 and in Brunetti & Lazarian (2007), is that these modes are the most relevant for the re-acceleration of relativistic particles in galaxy clusters. These modes were also identified as major scattering agent for Galactic cosmic rays (Yan & Lazarian 2002, 2004). The Alfven modes and slow modes are not efficient for scattering if the turbulent energy is being injected at large scales (Chandran 2000; Yan & Lazarian 2002). Their inefficiency stems from the both the spectra being steep in terms of parallel perturbations as well as fluctuations being very anisotropic (Goldreich & Sridhar 1995; LV99; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002). If Alfven modes are injected by instabilities they may have radically different properties from the modes of the large-scale cascade. For instance, Alfven modes arising from particle streaming have slab structure. Such waves efficiently interact with energetic particles. Other instabilities, e.g. gyroresonance one, can produce slab waves (see Gary 1993). The instabilities producing slab Alfven modes may be induced by large-scale compressible turbulence (see Lazarian & Bersenyak 2006). Indeed it is worth mentioning that the mode composition at smaller scales, l ≪ lA, in the IGM could becomes rather complex (e.g. Kato 1968; Eilek & Henriksen 1984).
To what extend our calculations are accurate depends on our understanding of properties of IGM turbulence. We assumed that the damping of fast modes with thermal particles can be obtained considering a collisionless plasma. At the same time, it can be argued (see e.g. Lazarian et al. 2010) that the degree of collisionality of astrophysical plasmas can be underestimated if only Coulomb collisions are taken into account, as particles in plasmas can interact through the mediation of the perturbed magnetic fields. The inevitable conclusion is that the collisionless formulae describing damping of these modes should be only applied to scales less than the mean free path, which is much shorter than the Coulomb mean free path. As a result, fast modes should be substantially less damped and be present on the scales which are much shorter than the earlier estimates, including those in this paper. The consequence of this is that a more appreciable portion of energy gets available for the acceleration of cosmic rays. Therefore, our present calculations may underestimate the efficiency of cosmic ray acceleration by turbulence. We plan to address the self-consistent problem elsewhere.

Following Brunetti & Lazarian (2007) in our paper we adopt quasi-linear theory (QLT) to calculate particle acceleration by fast modes. In Brunetti & Lazarian (2007) we indeed have shown that for fast modes in the IGM it is \( \langle \omega \rangle \gg \langle \Gamma \rangle \), where \( \langle \cdots \rangle \) indicates angle-averaged quantities, that provides some justification to the use of QLT. More recent studies in Yan et al. (2008) presented an approach to particle acceleration that generalizes the QLT and allows to take into account the effect of large-scale variations of magnetic field. As numerical simulations which use the data from the actual MHD turbulence simulations (Berensyak, Yan & Lazarian 2010) support the theory, we believe that, in future, going beyond the standard QLT approach may provide a more accurate description of the particle acceleration process, although we do not expect a large change (see Yan et al. 2008).

Finally, an unavoidable simplification in our semi-analytical calculations is that turbulence is homogeneous, in space and time, on the radio halo volume, during cluster mergers. On the other hand, cosmological numerical simulations of galaxy clusters show a more complex situation where intermittent and patchy large-scale turbulent motions are generated during multiple collisions between galaxy clusters (e.g. Vazza et al. 2009a; Paul et al. 2010). We believe that to be more realistic one may need to vary the intensity of turbulence driving to describe different parts of galaxy clusters. Our idealized calculations of particle acceleration and evolution of compressible turbulence in the IGM provide a first step, implementing our formalism in detailed cluster simulations is necessary to obtain a more reliable description of the morphology and spectral distribution of the non-thermal emission and of the connection between radio haloes and cluster mergers.

5.2 A comparison with the Alfvenic approach

Earlier calculations of turbulent acceleration of primary protons and secondary electrons in galaxy clusters focus on the re-acceleration by Alfven modes (Brunetti & Blasi 2005). These studies first provided a description of the expected transient and long-living spectral components in galaxy clusters (e.g. Brunetti et al. 2009a).

It is well known that the damping of these modes is mainly due to the interaction (gyroresonance) with relativistic particles, that provides the main motivation to explore this possibility. On the other hand, as already mentioned, a complication of this approach is the anisotropy of Alfven modes that develops when turbulence cascades from larger to smaller scales. Consequently in Alfvenic models the injection of Alfven modes ‘directly’ at small scales, i.e. comparable with the gyroradius of high-energy particles, must be postulated, in which case it is also difficult to derive a overall picture connecting clusters mergers and the generation of these modes at such small scales.

A second issue in the modelling of Alfvenic acceleration of relativistic protons and of their secondary products is that the primary and secondary particles interact with Alfven modes with different scales. This is because secondary electrons of energy \( E_e \) are mostly generated by collisions between higher energy protons, \( E_p \sim 10–50 \times E_e \), and thermal targets that implies that these secondaries interact with modes on scales 10–50 times smaller than those of the parent primary protons (by consider the gyroresonant conditions \( k \sim eB/(c p \cos \theta) \)). Consequently the ratio between the transient and long-living components in these models depends also on the spectrum of Alfven modes. This implies a larger degree of freedom in Alfvenic models with respect to the more straightforward case treated in our paper where compressible MHD turbulence interact with relativistic particles.

5.3 Sources of primary electrons in a turbulent IGM

In general, if relativistic protons and electrons are present in the IGM, we expect that the MHD turbulence, generated during cluster mergers, would re-accelerate both these particles. If we assume an efficient confinement of cosmic rays in galaxy clusters, the unavoidable consequence of this scenario is that also the energy density of secondary products, due to p-p collisions, should increase and consequently their contribution to the non-thermal clusters spectrum.

In our paper we have addressed this problem under the most extreme (and simplified) condition where only protons and their secondaries are present in the IGM; we note that under these conditions the ratio of the energy densities of relativistic protons and of the emitting electrons in the IGM is maximized. In this context, we have shown that, assuming turbulent re-acceleration at the level necessary to explain radio haloes, the radio emission generated by secondary particles when turbulence is dissipated is consistent with present upper limits to the diffuse radio luminosity of clusters without radio haloes. Also we have shown that the gamma-ray emission from the decay of secondary neutral pions, that should be common in galaxy clusters and not tightly connected with their dynamical status, is expected at a level much smaller than that constrained from present upper limits with Fermi.

At the same time, however, under these conditions our calculations suggest that the difference between the cluster-scale radio emission of ‘turbulent’ (merging) and ‘non-turbulent’ (relaxed) clusters cannot be larger than about a factor of 10, and that we expect to detect diffuse radio emission potentially in ‘all’ massive clusters as soon as much deeper observations of clusters that are presently defined ‘radio quiet’ will become available.

On the other hand, primary relativistic electrons should be present, at some level, in the IGM. It is well known that active radio galaxies may fill large volumes in the IGM with relativistic plasma, relativistic electrons age rapidly but they can be accumulated at 100 MeV energies for longer times, especially in the external regions (Fig. 1). Other sources of relativistic primary electrons that are usually considered in the literature are Galactic winds and starburst galaxies (e.g. Völk & Atoyan 1999; Atoyan & Völk 2000) and shock waves (e.g. Ensslin et al. 1998; Sarazin 1999; Ryu et al. 2003; Pfommer et al. 2006; Skillman et al. 2008; Vazza et al. 2009b), evidence for the latter process come from the observations of radio relics that indeed suggest a connection between shocks and electron...
acceleration (or re-acceleration) in the IGM (e.g. Markevitch et al. 2005; Giacintucci et al. 2008 for observations of a shocks–relics connection in clusters).

In addition to these processes, another possibility that requires more attention is that the primary electrons can be ‘created’ in the IGM. Magnetic reconnection presents a natural way for doing this, as electrons bounce back and forth between converging magnetic fluxes can gain energy through the first-order Fermi acceleration (de Gouveia dal Pino & Lazarian 2003). Although it is generally believed that reconnection is a slow process, potentially turbulence can significantly enhance the reconnection rate (LV99). Exploring this mechanism and its interplay with particle re-acceleration by MHD turbulence in the IGM may open new perspective in our understanding of non-thermal cluster-scale emission and of its connection with cluster mergers. We aim to discuss in detail this point in a future paper, yet for completeness here we dedicate an appendix to turbulent reconnection.

The presence of primary electrons in the IGM may significantly affect the SED of galaxy clusters and their evolution. In particular if the number density of primary electrons is comparable to (or larger than) that of secondary particles, a smaller number of relativistic protons is required by our model to match the observed spectrum of radio halos. This has an impact on the expected gamma-ray emission from galaxy clusters that would be consequently smaller than that expected from the calculations presented in Section 4.3. A second important point is that if a population of re-accelerated primary electrons significantly contributes to the observed radio halo emission, the expected ratio between transient and long-living components in the radio and hard X-ray bands increases with respect to that derived in Section 4.3. As a matter of fact, assuming that primary electrons are dominant with respect to secondaries our picture evolves into the ‘classical’ re-acceleration model (e.g. Brunetti et al. 2001, 2004; Petrosian 2001; Cassano & Brunetti 2005) where indeed the luminosity of giant radio halos is suppressed by several orders of magnitude when galaxy clusters evolve into relaxed systems (e.g. Brunetti et al. 2009b; Cassano 2010).

Consequently, we believe that future gamma-ray and radio observations will be crucial also to constrain the ratio of primary and secondary electrons in the IGM.

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is the size of the magnetic regions and \( \equiv \) magnetic diffusivity. As Lundquist number can be for the IGM and higher, it is clear that the Sweet–Parker reconnection can (410, 399).

**APPENDIX A: MAGNETIC FIELD RECONNECTION AS SOURCE OF PRIMARY ELECTRONS IN THE IGM**

Magnetic reconnection is a ubiquitous process in magnetized flows, expected to happen when magnetic fields of non-parallel direction get into contact. However, the textbook processes of magnetic reconnection seem to fall short of providing the desired solution. The Sweet–Parker reconnection (Parker 1957, Sweet 1958) is too slow. The speed of the reconnection scales as Alfvén velocity \( V_A \) times the inverse value of the square root of the Lundquist number \( S^{1/2} = (L_B/\eta)^{1/2} \), where \( L \) is the size of the magnetic regions and \( \eta \) is magnetic diffusivity. As Lundquist number can be for the IGM \( 10^{15} \) and higher, it is clear that the Sweet–Parker reconnection can...
handle only a negligible fraction of magnetic flux in the Hubble time. Petscheck proposed a solution where magnetic fields get into contact at a sharp angle (Petscheck 1964).

In the Petscheck model no efficient acceleration of particles is expected at reconnection sites. Indeed the traditional processes of acceleration that rely on the electric field in the reconnection region are inefficient, as the reconnection region is too small and releases only a small portion of magnetic energy. Slow shocks predicted in the Petscheck model are likely to be inefficient for the particle acceleration (see discussion in Beresnyak, Jones & Lazarian 2009). In addition, observational data do not support the X-point reconnection predicted in the Petscheck model either (see Ciaravella & Raymond 2008).

A shortcoming of many discussions of magnetic reconnection is that the traditional set-up does not include ubiquitous pre-existing astrophysical turbulence. As turbulence radically changes many astrophysical processes, the influence of turbulence on reconnection has attracted the attention of researchers for a long time (see Speiser 1970; Matthaeus & Lamkin 1985, 1986; Strauss 1988).

A new approach to the effects of turbulence was adopted in LV99. This model predicts reconnection speeds close to the turbulent velocity in the fluid. More precisely, assuming isotropically driven turbulence characterized by an injection scale, $l$, smaller than the current sheet length $L$, LV99 obtained

$$V_{\text{rec}} \approx v_A (l/L)^{1/2} (V_l/v_A)^2,$$  \hspace{3cm} (A1)

where the turbulent injection velocity $V_l$ is assumed to be less than $v_A$. If $L < l$, the first factor in equation (A1) should be changed to $(l/l)^{1/2}$ (LV99). If turbulent injection velocity is larger than $v_A$, the reconnection happens at the Alfvén speed for $L > l_A$. For $L < l_A$, a factor $(l/l_A)^{1/2}$ should substitute the factor of $(l/L)^{1/2}$ in equation (A1). Physically this reflects the fact that at sufficiently small scales magnetic field energy dominates the kinetic energy and the magnetic field lines get only weakly perturbed by turbulent motions. Fig. A1 provides the simplest realization of the acceleration of particles within the reconnection region expected within LV99 model. As a particle bounces back and forth between converging magnetic fluxes, it gains energy through the first-order Fermi acceleration (de Gouveia dal Pino & Lazarian 2003, 2005; see also Lazarian 2005). The first-order acceleration of particles entrained on contracting magnetic loop can be understood from the Liouville theorem, i.e. the preservation of the phase volume which includes the spatial and momentum coordinates. As in the process of reconnection the magnetic tubes are contracting and the configuration space presented by magnetic field shrinks, the regular increase of the particle’s energies is expected. The requirement for the process to proceed efficiently is to keep the accelerated particles within the contracting magnetic loop. This introduces limitations on the particle diffusivities perpendicular to magnetic field direction. Thus high perpendicular diffusion of particles may decouple them from the magnetic field. Indeed, it is easy to see that while the particles within a magnetic flux rope depicted in Fig. A1 bounce back and forth between the converging mirrors and get accelerated, if these particles leave the flux rope fast, they may start bouncing between the magnetic fields of different flux ropes which may sometimes decrease their energy. Thus it is important that the particle diffusion parallel and perpendicular magnetic field stays different. Particle anisotropy that arises from particle preferentially getting acceleration in terms of the parallel momentum may also be important. The energy spectrum was derived in GL03:

$$N(E) \, dE = C \, E^{-5/2} \, dE.$$  \hspace{3cm} (A2)

In Drake et al. (2006) this idea was enriched by taking into account the back reaction of particles.

We believe that such an acceleration can be present in IGM. Two-dimensional numerical simulations of acceleration can be found in Drake et al. (2006, 2010) and first three-dimensional simulations were presented in Lazarian et al. (2010). In typical IGM the small-scale reconnection of turbulent field should be collisionless and during this collisionless with electrons and ions decoupled. This should enable the acceleration of electrons. The idea of particle acceleration in reconnection regions was recently applied to describe anomalous cosmic ray acceleration in heliosphere (Lazarian & Opher 2009, Drake et al. 2010) and acceleration of cosmic rays in heliotail (Lazarian & Desiati 2010). In IGM magnetic turbulence creates magnetic reversals and therefore we expect to see additional sources of energetic electrons. In terms of the model that we discuss the electron acceleration in magnetic reconnection sites should increase the energy density of electrons and modify their spectrum. We shall discuss these effects elsewhere.

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