Gear loss factor using the load distribution model for varying contact ratio in spur gear drive for improved bending strength

Ravivarman R\(^1\), Palaniradja K\(^2\), Prabhu Sekar R\(^3\)

\(^1\)\(^2\) Department of Mechanical Engineering, Pondicherry Engineering College, India
\(^3\) Mechanical Engineering Department, Motilal Nehru National Institute of Technology, Allahabad, India
E-mail: varman92@gmail.com

Abstract. Gear loss factor is one of the imperative parameter in estimating the power loss in a meshing gear pair. This paper demonstrates the prediction of gear loss factor parameter in spur gear drive using the load distribution technique. The aptness of two distinct contact ratios is studied and compared in this paper: Normal Contact Ratio (NCR) and High Contact Ratio (HCR). The described model is studied for gearing strength which is improved through profile modification technique. The ultimate focus is to enhance the gearing strength of the pinion in a spur gear drive by discovering the optimal modification point in the tooth profile with at most balanced stresses in the gear pair considered for the study. Using contact analysis, the stresses developed over the contacting surfaces are predicted through finite element method and it is balanced between the gear and pinion. A gear loss factor is predicted along the path of contact for a complete meshing cycle for this balanced drive. It is uncovered that the methodology utilized encourages significant decrease in the gear loss factor of the HCR pair which is confined with the literature models. The values predicted using the load distribution model shows that the high contact ratio gear drive with increased addendum height has the gear loss factor minimum compared with a normal contact ratio gear drive.

1. Introduction

Gear drives are the most widely familiar transmission systems. Abundant researchers have focused on different highlights of the gears and gear trains. The main motive for the designers in spur gear transmission is to develop an efficient drive with improved life period. Design of gear drives with minimum power loss and enhanced life is a thrust for designers in gear design for developing more reliable and efficient mechanisms. In recent times more focus has been given to the HCR gear pair due to the increased load share with high strength and reduced weight. Several models and studies are available on the HCR spur gear pair [1, 2] in calculating the losses. The introduction of loss factor term in predicting the power loss gets its significance from the early stages of research in transmission drives which is majorly caused by the load acting in the gearing system. Buckingham [3] and Ohlendorf [4] was the first to introduce the loss factor \((H_v)\) parameter in predicting the power loss but the effect of the load acting on the tooth during meshing and the tooth profile changes were not considered in their model. The contact load plays a vital role in the prediction of the gear loss factor acting along the path of contact. Many researchers have discussed about the prediction of load share in the gear drive. Load share ratio is estimated along the contact path by Pedrero et al. [5] in his study. Li [6, 7] used contact analysis to predict the load distribution through Finite Element Methods (FEM). Thus the expansion of numerical simulation allows the designers to predict the load distribution accurately as if in the physical model using the Finite Element Methods. Here the load and stresses are estimated using FEM and the predicted stresses are balanced through profile modification.
technique (Figure 1) which has been elaborated in the previous studies made by the authors [8-11].
This paper presents a model for the calculation of gear loss factor by including the load distribution.
This load distribution is evaluated for an improved bending strength pinion by providing variation in
the tooth profile. With this equation, the gear loss factor, and accordingly the gear loss factor at any
point of the contact path has been evaluated, both for normal contact ratio and high contact ratio spur
gears.

![Figure 1. Variation in teeth height for varying contact ratio.](image)

2. Load Share Ratio and Gear Loss Factor for Balanced Gear Drive

In spur gear drives, the teeth come in contact at engagement points during meshing by sharing the
load for an instant of time. This sharing of tooth load take place in different pattern based on the type
of gear pair and contact ratio. Normally in spur gear drive the tooth load sharing is divided based on
the contact ratio (ε). For a normal contact ratio gear pair (ε<2) the tooth load is shared by single pair
and double pair regions which occurs for particular time period during meshing. Whereas for a high
contact ratio gear pair (ε>2) the tooth load is shared by double and triple pair regions. Hence it
become important to predict the way in which the load sharing takes place in a gearing pair for the
varying contact ratio. The load share ratio can be evaluated from the equation given below and the
contact forces are assessed based on the contact analysis technique.

\[
\text{Load Share Ratio, } LSR = \frac{F_i}{F_x}
\]

Where,
- \( F_i \) - Contact force at any instantaneous point
- \( F_x \) - Cumulative contact force

Gear loss factor (\( H_v \)) is an important parameter in predicting the efficiency of the drive. In this study,
the gear loss factor is estimated by including the load share ratio into account which has been
neglected in the previous methods of predicting the loss factor. The gear loss factor (\( H_v \)) of a spur gear
pair during meshing can be expressed from the load share ratio (\( LSR \)) as:

\[
(H_v) = \frac{1}{ep_p} (LSR) \left( \frac{v_i}{v_x} \right)_i
\]
Where, \( p_b \) is the base pitch, \( v_s \) and \( v_b \) is the sliding and base velocity.

The equation (2) is a generalized form of the Wimmer [12] approach in predicting the loss factor along the contact path for one complete meshing cycle.

3. Description of the finite element model

The finite element model used for this study is shown in the figure 2 which includes the gear and pinion with contact initiated. Here an APDL program is developed and used in ANSYS for computation. Generation of the tooth profile is initiated by the trochoidal portion followed by the involute portion. A Bi-dimensional FE study has been carried with element PLANE 82 to form the model. An equivalent torque is applied in the pinion hub which will induce a relative motion among the drive through tooth deflection. In terms of constraints, pinion can rotate around the axis whereas motion of the gear is prevented. CONTA172 and TARGE169 are the considered elements at the contact zones. The total number of element count is around 158,000 for NCR and 316,000 for HCR transmission drive studied. In which the critical areas at the contact zones are fine meshed with convergence study. Material considered for the study was steel with a modulus of rigidity of 210 GPa and Poisson’s ratio of 0.3. The parameters taken for the study is as follows: Module (m) =1, Number of teeth in pinion \((z_p)\) =50, Gear ratio \((i)\) =1.5, Pressure angle at pitch circle \((\alpha)\) =20°, Addendum height \((h_a)\) =1 (For NCR) and 1.2 (For HCR).

(a) NCR spur gear pair. (b) HCR spur gear pair.

Figure 2. FE model study for analysis.

4. Gear loss factor - a comparison with literature models

The gear loss factor is evaluated for the normal and high contact ratio spur gear pair using equation (2) by including the load share ratio as mentioned. Figure 3 shows the comparison of the gear loss factor \((H_v)\) with the literature models. As discussed in the introduction section Buckingham [3] and Ohlendorf [4] evaluated the gear loss factor using analytical equations. But these models failed to include the load sharing effect which is a vital parameter in estimating the gear loss factor \((H_v)\). From the figure 3, it can be assessed that the loss factor is achieved minimum for the present work for both the NCR and HCR gear pair.
Figure 3. Gear loss factor ($H_v$) vs. various literature models.

5. Results and discussions

For the considered parameters, $m=1$, $z_p=50$, $i=1.5$, $\alpha_o=20^\circ$ the balanced root stress is attained by varying the tooth thickness along the pitch circle of the gear pair. The tooth thickness factor achieved for NCR and HCR with balanced root stress is listed in Table 1. It can be noticed from the figure 4(a) that the tooth thickness factor is 0.518 for a balanced NCR gear pair that is primarily due to increased tooth thickness at the critical region which takes more load in it during balancing of stresses. Whereas for an HCR gear pair the stress is balanced at a thickness factor of 0.4854, here the effect of increased radius of curvature at the fillet region gets signified when compared to the effect of tooth thickness.

From the table 1, it can be noticed that the contact stress achieved is minimum for an HCR gear pair when compared with NCR gear pair drive. Figure 4(b) shows the load distribution along the contact path for the NCR and HCR gear drive. It is observed from the trend that in NCR gear pair, the load share is distributed at the A’B’ and C’D’ region as double pair region whereas the region B’C’ takes the full load as single pair region. For the case of HCR gear pair [10], it can be noted that there exist a triple-double-triple-double-triple pair regions for one mesh cycle. The obtained value of load share is maximum of 60% in HCR when compared with NCR of 100% which is a significant enhancement in load carrying ability of the drive.

Table 1. Comparison of NCR and HCR computed data.

| Type  | Contact Ratio ($\varepsilon$) | Tooth thickness factor ($k_p$) | Gear loss factor ($H_v$) | Maximum Balanced root stress ($\sigma_{b_{\text{max}}} = \sigma_{b_{\text{max}}} \text{MPa}$) | Maximum contact stress ($\sigma_{s_{\text{max}}} \text{MPa}$) |
|-------|-----------------------------|------------------------------|-------------------------|-------------------------------------------------|--------------------------|
| NCR   | 1.785936                    | 0.518                        | 0.088                   | 21.231                                          | 256.431                   |
| HCR   | 2.1074545                   | 0.4854                       | 0.0599                  | 15.269                                          | 199.724                   |
The gear loss factor ($H_v$) is estimated along the contact path for one complete mesh cycle and compared between the NCR and HCR gear pair as shown in Figure 4(c). The $H_v$ obtained for the HCR gear pair is less compared to the NCR gear pair throughout the mesh cycle due to reduced tooth load. The contact ratio increases significantly when the drive gets converted from NCR to HCR gear drive. It is also noticed that the maximum contact stress ($\sigma_s$)$_{\text{max}}$ decreases due to the decline in tooth load for the change in drive. The gear loss factor increases in the regions of increased load share at B'C' for NCR and at B and E for HCR spur gear pair which evidently proves that the gear loss factor depends on the load distribution.
6. Conclusions
The load share based estimation of gear loss factor has been proposed for improved root strength drive by varying tooth thickness factor. The contact load capacity is enhanced for the high contact ratio drive compared to normal contact ratio drive. The gear loss factor obtained using the proposed technique is validated with a comparative study carried among the earlier literature models. For the HCR spur gear pair, a noticeable reduction in loss factor (almost 50%) was achieved when compared with the Buckingham and Ohlendorf model.

7. References

[1] Yildirim N, Gasparini G and Sartori S 2008 An improvement on helicopter transmission performance through use of high contact ratio spur gears with suitable profile modification design P. I. Mech. Eng. Part G: J Aerospace Engineering 222(8) 1193-1210.

[2] Rameshkumar M, Sivakumar P, Sundaresh S and Gopinath K 2010 Load Sharing Analysis of High-Contact-Ratio Spur Gears in Military Tracked Vehicle Applications Gear Technology 1(3) 43-50.

[3] Buckingham E 1963 Efficiencies of gears. Analytical mechanics of gears New York, NY, USA: Dover Publications 395–425.

[4] Ohlendorf H, Verlustleistung und Erwärmung von Stirnrädern 1958 (Ph.D. thesis) Dissertation TU Munchen, Munchen.

[5] José I. Pedrero, Miguel Pleguezuelos, Mariano Artés, Juan A. Antona 2010 Load distribution model along the line of contact for involute external gears Mechanism and Machine Theory 45 (5) 780-794.

[6] Shuting Li 2007 Effects of machining errors, assembly errors and tooth modifications on load-carrying capacity, load-sharing rate and transmission error of a pair of spur gear, Mechanism and Machine Theory 42 698–726.

[7] Shuting Li 2015 Effects of misalignment error, tooth modifications and transmitted torque on tooth engagements of a pair of spur gears Mechanism and Machine Theory 83 125-136.

[8] Sekar RP and Muthuveerappan G. A 2015 Balanced Maximum Root stresses on Normal Contact Ratio Spur Gears to Improve the Load Carrying Capacity through Non-Standard Gears Mechanics based design of structure and machines, Taylor and Francis 43 150-163.

[9] Sekar RP and Sathishkumar R. 2017 Enhancement of Wear Resistance on Normal Contact Ratio Spur Gear Pairs through Non-Standard Gears Wear 380-381 228-239.

[10] Ravivarman R, Palaniradja K and Sekar RP. 2018 Evolution of balanced root stress and tribological properties in high contact ratio spur gear drive Mechanism and Machine Theory 126 491-513.

[11] Ravivarman R, Palaniradja K and Sekar RP. 2018 Influence of gear ratio on wear depth of nonstandard HCR spur gear drive with balanced fillet stress, Materials Today- Proceedings 5 17350–17359.

[12] Wimmer AJ. Lastverluste von Stirnradverzahnungen 2006 PhD thesis, Fakultät für Maschinenwesen der Technischen Universität München, München.

Acknowledgments
This research was supported by INSPIRE Fellowship scheme, Department of Science and Technology (DST) Government of India, for the corresponding author by providing financial assistance.