Galaxies with fuzzy dark matter

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Abstract
This is a brief review on some properties of galaxies in the fuzzy dark matter model, where dark matter is an ultra-light scalar particle with mass \( m = \mathcal{O}(10^{-22}) \) eV. From quantum pressure, dark matter has a halo length scale which can solve the small scale issues of the cold dark matter model, such as the core-cusp problem, and explain many other observed mysteries of galaxies.

Keywords  Galaxy · Fuzzy dark matter · Bose–Einstein condensate

1 Introduction
Dark matter (DM) is one of the main ingredients of the universe providing a gravitational attraction to form cosmic structures [1]. The most popular DM model is the cold dark matter (CDM) model in which numerical simulations successfully reproduce the observed large-scale structures of the universe, such as clusters of galaxies. However, it encounters some difficulties in explaining small-scale structures at galactic scales, such as the core-cusp problem (predicting the cusped central halo density which is not observed) and the missing satellite problem (predicting many more small galaxies than observed) [2–5]. Therefore, we need a variant of the CDM that acts as CDM on a super-galactic scale while suppressing the formation of smaller structures. This requires a natural length scale with a small galaxy size on the order of \( kpc \).

Recently, interest in the fuzzy DM model as an alternative to the CDM has been revived. In this model, DM particles are ultra-light scalars with mass \( m = \mathcal{O}(10^{-22}) \) eV in a Bose–Einstein condensate (BEC) [6–13]. This tiny DM particle mass leads to a very high DM particle number density, which means the wave functions of the particles overlap. A huge, but finite, length scale related to the Compton wavelength \( \lambda_c = 1/m \sim 0.1pc \) of the particles naturally arises in this model. Unlike conventional CDM particles that move incoherently, fuzzy DM particles in the BEC state move collectively and form a coherent wave with the de Broglie wavelength \( \lambda_{db} = \mathcal{O}(kpc) > \lambda_c \). Despite the tiny mass, the fuzzy DM particles are non-relativistic because the particles in a BEC move as a heavy single entity. This model has many other names, such as BEC DM, scalar field DM, ultra-light axion (ULA), and wave DM.

The idea that galactic DMs are condensated ultra-light scalar particles has been repeatedly suggested [14–34]. In Refs. [35], self-gravitating bosons with the de Broglie wavelength of the typical galaxy size were considered. In Ref. [36], DM halos as a ground state of scalar fields are investigated [37]. In Ref. [38], Sin tried to explain the observed flat rotation curves (RCs) using excited states of the fuzzy DM and obtained the particle mass \( m \approx 3 \times 10^{-23} \) by fitting the observed RC of galaxy NGC2998. Lee and Koh [39] suggested that DM halos were giant boson stars and considered the effect of self-interaction. In this paper, we briefly review some properties of galaxies in the fuzzy DM model.

2 Fuzzy dark matter and the small scale crisis
We still lack an exact particle physics model of the fuzzy DM. The fuzzy DM field can be a scalar field \( \phi \) with an action

\[
S = \int \sqrt{-g} d^4x \left[ \frac{R}{16\pi G} - \frac{s}{2} \phi^{*\mu} \phi_{\mu} - U(\phi) \right]. \quad (1)
\]
where the potential is given by \( U(\phi) = \frac{m^2}{2} |\phi|^2 \). In the Newtonian limit, this action leads to the following Schrödinger Poisson equation (SPE), which the macroscopic wave function \( \psi \) satisfies:

\[
\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + mV\psi,\]

\[
\nabla^2 V = 4\pi G \rho,
\]

where the rescaled field \( \psi = \sqrt{m} \phi \), the DM mass density \( \rho = m|\psi|^2 = m^2|\phi|^2 \), and \( V(\psi) \) is the gravitation potential. We used the natural units \( \hbar = 1 = c \). Note that the SPE can be seen as a non-linear Schrödinger equation, which can have dispersion-less soliton solutions, unlike the ordinary Schrödinger equation. The ground state of the SPE is one of the solitons.

The SPE has a useful scaling property for numerical studies:

\[
\{t, r, \psi, \rho, V\} \to \{\lambda^{-2}t, \lambda^{-1}r, \lambda^2\psi, \lambda^4\rho, \lambda^2V\},
\]

where \( \lambda \) is a scaling parameter. This leads to the following scaling law of parameters:

\[
\{M, E, L\} \to \{\lambda M, \lambda^3E, \lambda L\},
\]

where \( M \) is the mass, \( E \) is the energy, and \( L \) is the angular momentum of a dark matter distribution.

For an understanding, the role of the fuzzy DM in the formation of cosmological structures, reducing the Schrödinger equation to a fluid equation using the Madelung relation [7, 10], \( \psi(r, t) = \sqrt{\rho(r, t)} e^{i \phi(r, t)} \), is useful. This gives an Euler-like equation

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla \mathbf{v} + \frac{V\rho}{m} - \nabla Q = 0,
\]

and a continuity equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.
\]

Here, the fluid velocity \( \mathbf{v} \equiv \nabla S/2m \) and the quantum potential \( Q \equiv \hbar^2 \Delta \sqrt{\rho}/(2 m \sqrt{\rho}) \). The pressure \( \rho \) can come from a self-interaction, if any self-interaction exists. (We have ignored the cosmic expansion for simplicity.)

The quantum pressure \( \nabla Q/m \) from the uncertainty principle is the key difference between the fuzzy DM model and the conventional CDM models. Below the galactic scale, the quantum pressure suppresses small structure formation, while at scales larger than galaxies, the quantum pressure becomes negligible, and the fuzzy DM behaves like CDM. This interesting property makes the fuzzy DM an ideal alternative to the CDM because it resolves the small-scale problems of the CDM model while sharing the merits of the CDM [40].

To find the length scale during structure formation perturbing the above equations around \( \rho = \bar{\rho}, \mathbf{v} = 0, \) and \( V = 0 \) is useful. One can get the following equation for the density perturbation \( \delta \rho \equiv \rho - \bar{\rho} \):

\[
\frac{\partial^2 \delta \rho}{\partial t^2} + \frac{\hbar^2}{4m^2} \nabla^2 (\nabla^2 \delta \rho) - c_s^2 \nabla^2 \delta \rho - 4\pi G \bar{\rho} \delta \rho = 0,
\]

where \( c_s \) is the sound velocity, and \( \bar{\rho} \) is the average density of background matter [41, 42].

An equation for the density contrast \( \delta \equiv \delta \rho/\bar{\rho} = \sum_k \delta_k e^{ik \cdot r} \) with a wave vector \( k \),

\[
\frac{d^2 \delta_k}{dt^2} + \left[ (c_s^2 + c_s^2)k^2 - 4\pi G \bar{\rho} \right] \delta_k = 0,
\]

can be derived from the Fourier-transformed version of Eq. (7), where \( c_s = \hbar k/2m \) is a quantum velocity. Because we can assume \( c_s \) to be almost independent of \( k \), we expect the \( c_s \)-dependent term from the quantum pressure to dominate only for large \( k \) (at a small scale) [19, 43–45]. This gives the quantum Jeans length scale [19, 46, 47] at a redshift \( z \):

\[
\lambda_J(z) = \frac{2\pi}{k} = \left( \frac{\pi^2 h^2}{Gm^2 \bar{\rho}(z)} \right)^{1/4} \approx 55.6 \left( \frac{\rho_b}{m_{22}^2 \Omega_m h^2 \bar{\rho}(z)} \right)^{1/4} \text{kpc},
\]

where \( m_{22} = m/10^{-22} \text{eV} \), the Hubble parameter \( h = 0.673 \), \( \rho_b \) is the current matter density, and the matter density parameter \( \Omega_m = 0.315 \) [48].

Interestingly, \( \lambda_J(z) \) determines the minimum length scale of galactic halos formed at \( z \) [49–51]. This fact might explain the observed size evolution of the early compact galaxies [49]. Any perturbation below \( \lambda_J(z) \) decays, and no DM structure below this scale can grow. This remarkable property resolves the small-scale issues of the CDM model by suppressing the formation of too many small structures [20, 23, 44, 52]. The average mass inside \( \lambda_J(z) \) is the quantum Jeans mass

\[
M_J(z) = \frac{4\pi}{3} \bar{\rho}(z) \lambda_J^3 = \frac{4}{3} \frac{\pi^{\frac{11}{2}}}{\Gamma(\frac{3}{2})} \left( \frac{h}{Gm} \right)^{\frac{3}{2}} \bar{\rho}(z)^{\frac{1}{2}},
\]

which is the minimum mass of DM structures forming at \( z \). Therefore, one can expect the minimum mass and size of a galaxy to have a quantum mechanical origin.

Understanding the core-cusp problem in the fuzzy DM model is easy. If no compact object exists at the center of a galaxy, a natural boundary condition there is a zero-derivative condition, i.e., \( \partial \mathbf{v}/\partial r = 0 \). This means the DM density is flat at the center, and this solves the core-cusp problem. If a supermassive black hole exists in the DM halo, the central boundary condition should be changed [53]. This property has been argued to explain the M-sigma relation of black
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3 Fuzzy dark matter and galaxies

Because dwarf galaxies are the smallest DM-dominated objects, they are ideal for studying the nature of DM. A density perturbation with a size larger than \( \lambda_0 \) can collapse to form a galactic DM halo. Therefore, we expect the galactic density perturbation with a size larger than objects, they are ideal for studying the nature of DM. A den-

\[ E(\xi) \approx \frac{\hbar^2}{2m^2} + \int_0^\xi d\xi' \frac{Gm}{r^2} \int_0^{r'} dr'' 4\pi r'' \rho(r''), \]

where \( \rho(r) \) is the DM density at \( r \). One can obtain the size of the ground state (the soliton),

\[ \xi = \frac{\hbar^2}{GMm^2}, \]

from the condition \( \frac{dE(\xi)}{d\xi} = 0 \) \([38, 55]\). Here, \( M \equiv \int_0^\xi d\xi' 4\pi r'^2 \rho(r') \) is the mass within \( \xi \). The size of the halo is inversely proportional to \( M \). A natural assumption is that the quantum Jeans mass is similar to the minimum value of \( M \). Therefore, the lightest galaxy formed at \( z \) has a typical size

\[ \xi(z) = \frac{\hbar^2}{GM_J(z)m^2} = \frac{3h^{1/2}}{4\pi^{3/4}(GM^2 \rho(z))^{1/4}}, \]

which has the same form of \( \lambda_0 \) (Eq. (9)) with a somewhat smaller constant. Note that according to the above equation the lightest (dwarf) galaxy has a maximum size among dwarf galaxies in this theory. Antlia II was shown to be close to this upper limit in size and the velocity of stars is consistent with the theory [56]. This model seems to explain the minimum length scale of galaxies [57] and the size evolution [49] of the most massive galaxies [58].

On the other hand, an approximate maximum mass of galaxies can also be obtained from the stability condition of boson star theory. The maximum mass of the ground state is \( 0.63m_p^2/m \). If we take this value for the maximum mass of galaxies (\( O(10^{12}M_\odot) \)), we can derive a constraint \( m \geq O(10^{-28}) \) eV. From the maximum stable central density, \( m \leq O(10^{-22}) \) eV [39].

The finite length scale also implies a finite acceleration scale in this model. Using an approximation \( \phi_s \sim 1/\xi \) in \( VQ/m \) in Eq. (5), one can obtain a typical acceleration scale for DM halos of

\[ g^3 \equiv \frac{h^2}{2m^2 \xi^3} = 2.2 \times 10^{-10} \left( \frac{10^{-22} \text{eV}}{m} \right)^2 \left( \frac{300 \text{pc}}{\xi} \right)^3 \text{m/s}^2, \]

which is absent in other DM models. This scale is relevant for the baryonic Tully–Fisher relation (BTFR) [59], which is an empirical relation between the total baryonic mass of a disk galaxy and its asymptotic rotation velocity. Interestingly, if we choose the core size of the dwarf galaxies (~ 300 pc [57]) for \( \xi \), we can reproduce the observed value \( g^3 = 1.2 \times 10^{-10} \text{m/s}^2 \) for Modified Newtonian dynamics (MOND), the radial acceleration relation (RAR), and BTFR [60]. Mond was proposed to explain the rotation curves without the DM [61]. According to MOND gravitational acceleration of baryonic matter, \( g_b \) should be replaced by

\[ g_{\text{obs}} = \sqrt{g_b^3}, \]

when \( g_b < g^3 \). MOND and RAR may just be effective phenomena of fuzzy DM.

Surprisingly, DM simulations using graphic processing units (GPUs) with an adaptive mesh refinement (AMR) scheme [62] revealed that a solitonic core exists in every halo surrounded by granules from DM interference (see Fig. 1). This configuration is different from the simple excited states of the fuzzy DM or CDM. An approximate numerical solution can be found in Ref. [62]:

\[ \rho(r) \approx \frac{\rho_0}{(1 + 0.091(r/r_c)^2)^8}, \]

FIG. 1 (Color online) The two-dimensional section of the three-dimensional DM density of a model galaxy from our numerical study, which shows a central soliton and surrounding granules.
where the central core density is \( \rho_0 = 1.9a^{-1}(m/10^{-22} eV)^2(kpc/r_c)^4M_\odot/pc^3 \) and \( r_c \) is the half-density radius. The outer profile of the halo is similar to the Navarro–Frenk–White (NFW) profile of the CDM (See Fig. 2.). The halo mass \( M_{\text{halo}} \) was also observed to be related to the soliton mass \( M \) by \( M \propto M_{\text{halo}}^{1/3} \).

Figure 1 shows the DM density in a halo in our DM-only numerical simulation using the spectral method. In this example, ten small halos with mass \( M = 10^9M_\odot \) collide with each other. From \( \nu \), one can predict the astrophysical properties of galaxies. For example, the rotation velocity at radius \( r \) is roughly given by \( \nu_{\text{rot}}(r) = \sqrt{GM(r)/r} \), where \( M(r) = 4\pi \int_0^r r^2 \rho(r) dr \) is the mass within \( r \). This equation can be used to investigate the RCs of galaxies [37, 63–65] in this model. The mass \( m \sim 10^{-22} eV \), which is consistent with other cosmological constraints, was obtained by fitting RCs.

4 Discussions

Numerical studies in this model so far are mainly DM-only simulations. For a more precise simulation of large galaxies, we need to understand the role of baryon matter such as stars or gas. For example, it was shown that the gravitational potentials of the fuzzy DM induce spiral arm patterns of stars in galaxies [33]. In Ref. [66] it was numerically shown that the flat RCs appear only when we include visible matter in large galaxies. In summary, the fuzzy DM with mass about \( 10^{-22} eV \) can explain many mysterious properties of galaxies. To find conclusive proofs, we need more precise fuzzy DM simulations with visible matter.

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