The static friction force in commensurate structures with an asymmetric deformable substrate potential

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Abstract. It is used that a series of interacting atoms subjected in an asymmetric deformable substrate potential. With the increase of external driving force, the system moves from locked state to sliding state. This critical force is also known as depinning force or the maximum static friction force. It has been analyzed for the shape parameter.

1. Introduction

The Frenkel–Kontorova(FK) model represents a series of harmonically interacting particles chain subjected to a sinusoidal substrate potential. It is one of the most suitable model for studying many condensed matter systems\cite{1-3}. Especially in solid friction, under-damped FK model has attracted more and more attention. It can make people understand the complex field of nano-tribology more deeply\cite{4-8}.

When two workpieces are in contact with each other, the minimum force required to achieve sliding is referred to as the maximum static friction. Especially, some researchers\cite{9} have improved the one-dimensional FK model to more complex models to explain more complex physical phenomena. It is studied that the effect of system parameters on the maximum static friction force. A series of interacting atoms confined between two periodic sinusoidal substrate potentials is studied by O. M. Braun et al.\cite{10}. When the upper layer was driven by a spring running at a constant speed, they found an interesting phenomenon. The golden mean incommensurability reveals a very regular time-periodic dynamics that has relatively high kinetic friction values compared with the spiral mean case. However, the influence of an asymmetric deformable substrate potential on the static frictional force in a confined system is seldom studied.

In the present paper, we have built a one-dimensional system of an asymmetric deformable substrate potential and a chain of interacting particles. We have examined the influence of the shape parameter $r$ on the chain of interacting particles is driven by the force $F_{ext}$.

2. Model

We introduced a series of particles with simple harmonic interactions (upper layer). They are subject to the bottom of an ASDP\cite{11}(lower layer). Each particle in the upper layer is subjected to an external driving force $F_{ext}$.

The asymmetric deformable substrate potential (ASDP)\cite{11} is given as follows:
\[ V(u) = \frac{K}{(2\pi)^2} \frac{(1-r^2)^2[1-\cos(2\pi u)]}{[1+r^2+2r\cos(\pi u)]^2}, \quad (1) \]

Where \( r \) is the shape parameter \((-1 < r < 1)\) and \( K \) is the pinning strength. In Fig. 1, we plotted the asymmetric deformable substrate potential for different values of \( r \).

![Graph showing the dependence of the substrate potential on different values of \( r \)](image)

FIG. 1. Dependence substrate potential \( V(u) \) on different values of the shape parameter \( r \) for \( K = 4 \).

The particles are driven by the external force \( F_{ext} \). The equations of motion that this system satisfies is as follows:

\[ m \ddot{u}_i = -\gamma u_i + g(u_{i+1} + u_{i-1} - 2u_i) + \frac{dV}{du_i} + F_{ext}, \quad (2) \]

Where \( u_i (i=1,2,3,\ldots,N) \) stands for position coordinates of \( N \) particles. The damping \( \gamma \) terms in Eqs. (2) describe the dissipative forces. They are proportional to the relative velocities of the particles with substrate. In our system, we choose \( \gamma = 0.1 \), so that it is a underdamped model. Simulations have provided indirect evidence that such phenomenological viscosity terms serve well this purpose. We have used dimensionless units with substrate period \( a = 1.0 \) and chain atom mass \( m = 1 \). The fourth terms in Eqs. (2) represents the interaction between the particle and the bottom potential. Particle interaction [third term in Eqs. (2)] is harmonic with equilibrium spacing \( b \) and strength \( g \). It is used that the fourth-order Runge-Kutta to solve Eqs. (2). In the simulation, the time step we used is \( 0.02 \tau \). The time interval for the system to reach equilibrium is \( 100\tau \). The force is varied with the step of \( 10^{-3} \).

3. Results and discussion

In the field of tribology research, maximum static friction is one of the main problems, which describes the minimum force that causes relative motion. In Fig. 2, it is shown that the variation of the average velocities of the particles as a function of the driving force \( F_{ext} \) with different system parameters.
FIG. 2. Average velocity $\bar{v}$ as a function of the driving force $F_{ext}$ for $K = 4$, and different values of
the shape parameter $r$: (a) $r = 0.0$, $g = 1.0$; (b) $r = 0.5$, $g = 1.0$.

As far as locked to the sliding transition is concerned, for different intensities $g$, there is a critical
disengagement force below which the system will be locked and the system will slide over it. We have
determined the static friction in the simulation as the system's first external drive with non-zero
velocity. It can be seen that the parameter $r$ has a great influence on the static friction.

The static friction force $F_s$ as a function of the shape parameter $r$ for $K = 4$ is presented in Fig.
4. We can see that when $r$ increases, the maximum static friction increases to a certain value first,
and then presents a chaotic form.

FIG. 3. The static friction force $F_s$ as a function of the shape parameter $r$ for $K = 4$.

4. Conclusions
In this paper, we study the interaction of a simple harmonic interaction atomic chain subjected to an
asymmetric deformable substrate potential. The harmonic interaction atomic chain is driven by
external force. When the external driving force increases, the average velocity of the upper atom
changes from zero to non-zero. We define this force as the maximum static friction force. The maximum static force as a function of the shape parameter is presented. We have the conclusion that when \( r \) increases, the maximum static friction increases to a certain value first, and then presents a chaotic form.

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**References**

[1] Kokubo, N., Besseling, R., Vinokur, V. M. and Kes, P. H.(2002)Mode Locking of Vortex Matter Driven Through Mesoscopic IC Channels. Phys. Rev.Lett.,88:247004.

[2] Lee, S. J. and Halsey, T. C. (1993)Staircase dynamics of Josephson-junction arrays. Phys. Rev. B, 47:5133.

[3] Thorne, R. E., Hubacek, J. S., Lyons, W. G., Lyding, J. W. and Tucker, J. R.(1988) ac-dc interference, complete mode locking, and origin of coherent oscillations in sliding charge-density-wave systems. Phys. Rev. B, 37:10055.

[4] Granato, E., Baldan, M. R., Ying, S. C. (1995) Sliding Friction in the Frenkel-Kontorova Model. Appl. Mater. Interfaces.,10: 96.

[5] Granato, E., Ying, S. C.(1999) Dynamical transitions and sliding friction in the two-dimensional Frenkel-Kontorova model. Phys. Rev. B, 57: 7.

[6] Hentschel, H., Family, F., Braiman, Y.(1999) Friction Selection in Nonlinear Particle Arrays. Phys. Rev. Lett., 83: 104.

[7] Liu, Z., Li,B.(2007) Heat conduction in simple networks: the effect of interchain coupling. Phys. Rev. E,76:051118

[8] Granato, E., Ying, S. C. (1999) Dynamical transitions and sliding friction in the two-dimensional Frenkel-Kontorova model. Phys. Rev. B, 57: 7.

[9] Wang, C. L., Duan, W. S., Chen, J. M. and Hong, X. R.(2008)Investigation of superlubricity in a two dimensional Frenkel–Kontorova model with square lattice symmetry. Appl. Phys. Lett.,93:153116.

[10] Braun, O. M., Vanossi, A. and Tosatti, E. (2005)Incommensurability of a Confined System under Shear. Phys. Rev. Lett., 95: 026102.

[11] Hu, B. and Tekić, J.(2005)Dynamical mode locking in commensurate structures with an asymmetric deformable substrate potential. Phys. Rev. E, 72:056602.