Breathing Dissipative Soliton Molecule Switching in a Bidirectional Mode-Locked Fiber Laser

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Breathing optical solitons propagating in a dissipative nonlinear system can interact and bind stably, forming an optical soliton molecule that presents striking molecule-like dynamics. To date, the breathing soliton pair has been mainly observed in the microcavity platform, and the peculiar dynamic evolution of breathing soliton molecules (BSMs) remains largely unexplored in mode-locked fiber lasers. Herein, the transient switching dynamics of BSMs in a bidirectional ultrafast fiber laser is revealed, specifically triggered at different parameter spaces of saturable absorption with manipulation polarizations that maintain constant pulse separation or undergo strong repulsion and kink after switching. All-optical switching of breather molecules and significantly increasing multiple soliton molecules’ switching of two or three solitons between states with different binding separations by applying a strong stimulus with periodic pump modulation is demonstrated. The instantaneous pulse breakup can be induced by the collision of bidirectional breathing solitons in each switching, which is characteristic of the breathing solitons in a bidirectional fiber laser and further corroborated by numerical simulation. The study unveils new perspectives into the ultrafast transient process of BSMs in various dissipative systems.

1. Introduction

Dissipative solitons are self-localized structures, resulting from the balance between nonlinearity and dispersion, for which the existence and stability highly depend on the interplay between gain and loss. The dissipative solitons have been extensively observed in a variety of fields, including nonlinear optics, hydrodynamics, plasma physics, atomic physics, chemistry, and biology.

Besides the parameter-invariant stationary solitons, the breathing solitons are also supported in many nonlinear systems, which undergo periodic oscillation in their amplitude and duration. Breathers were first experimentally demonstrated in passive fiber cavity and recently extensively investigated in optical microresonators. Meanwhile, the breathing solitons that experience large periodic variations operated in laser oscillators have been predicted theoretically by the cubic–quintic complex Ginzburg–Landau equation. Similar to the Kuznetsov–Ma and Akhmediev breathers in conservative systems, breathing dissipative solitons are strongly linked to the Fermi–Pasta–Ulam recurrence, which is a paradoxical evolution of nonlinearly coupled oscillators that periodically return to the initial state. Exploring breathing dissipative solitons dynamics is not only of fundamental interest in nonlinear science, but also important for the application, for example, increase in the resolution in a dual-comb source.

In various nonlinear optical systems, the mode-locked fiber laser, as an ideal dissipative system, provides an excellent testbed for exploring complex nonlinear dynamics, such as soliton buildup, rogue waves, soliton explosion, soliton molecules, and breathing solitons. In particular, the soliton molecules that exhibit fascinating particle-like interactions, for example, vibration, synthesis, and switch, constitute the fundamental problems in soliton physics. In terms of application, soliton molecules may increase the capacity of telecommunication, which is also attractive in all-optical information storage. In addition to stationary soliton molecules, there are also dynamic soliton molecules, among which the time interval or phase difference between individual solitons changes over time, and breathing soliton molecules (BSMs), that experience periodic spectra and temporal evolutions.

Despite the stable breathing dynamics of soliton molecules that have been reported previously in unidirectional lasers, the most exotic dynamic evolution of BSMs is still largely unexplored, particularly in the normal dispersion bidirectional mode-locked fiber laser, which is an excellent platform for dissipative solitons involving rare dynamics. In addition, the bidirectional mode-locked fiber laser is regarded as a novel light source...
that simultaneously generates two sets of pulse trains from counter-propagating directions. It is interesting to study the interplay of the breathing soliton molecules in bidirectional lasers that the counter-propagating solitons collision in each roundtrip (RT). The counterpropagating pulses could interact in different ways, for example, direct pulse interaction by colliding and indirect interactions due to time-dependent parameters of laser cavity components (such as saturable absorber [SA] or gain media). These interactions and underlying physical processes significantly stand out from interactions of copropagating pulses in a unidirectional fiber laser. Continuous interest in bidirectional mode locking also reinforced their practical applications, such as ultrafast laser ring gyroscopes\(^{[38]}\) and dual-comb spectroscopy.\(^{[39]}\) However, the lack of underlying research on soliton dynamics poses significant limitations to the development of bidirectional ultrafast laser applications. Moreover, the probe and trigger of the dynamical BSM switching in the normal dispersion dissipative systems may contribute to control and generation of desired pulse patterns in an optical oscillator, further extending the “molecular” analogy and potential applications.

In this work, we reveal the transient switching dynamics of breathing dissipative soliton molecules via the real-time time-stretch dispersive Fourier transform (TS-DFT) technique in a carbon nanotube (CNT) mode-locked bidirectional ultrafast fiber laser operating in the net-normal-dispersion regime. The switching of BSMs could be triggered at different parameter spaces of saturable absorption with manipulation polarization that maintains constant pulse separation or undergoes strong repulsion and kink after switching. The transient pulse breakup in soliton molecules could be induced in each switching that characterizes the breathing soliton in the bidirectional fiber laser in contrast to the unidirectional laser. Applying a strong stimulus with periodic pump modulation, the all-optical switching of BSMs of two or three solitons with different binding separation was resolved. Meanwhile, the soliton molecules from the clockwise (CW) and counterclockwise (CCW) directions possess high behavior similarity during switching owing to common gain/loss modulation, further supported by numerical simulations. We anticipate that these results shed light on the further understanding of the complex nonlinear dynamics of BSMs in different dissipative systems.

2. Results

2.1. Stable Bidirectional BSM

The switching dynamics of bidirectional BSM was investigated in a passively mode-locked Er-doped fiber laser based on the CNT as the SA (Figure 1a). The laser cavity incorporated an 11.5 m erbium-doped fiber (EDF) (M12-980-125) with a group velocity dispersion (GVD) of about 18.5 ps\(^2\) km\(^{-1}\) and 3.85 m SM28-e fiber with a GVD of \(\approx -22\) ps\(^2\) km\(^{-1}\), operated in a normal dispersion regime with a net cavity dispersion of \(\approx 0.13\) ps\(^2\). The length of the whole cavity was around 15.35 m, corresponding to a fundamental repetition rate of 13.01 MHz. Polynvinyl alcohol (PVA)-based CNT SA (CNT SA) is used to realize the modelocking operation. The polarization controller (PC) was inserted into the cavity to optimize the polarization state. A coupler with a 10:90 splitting ratio was used to extract 10% power of pulses in each direction for measurement. A 50/50 coupler combined counterpropagating pulses for consistent and synchronized analysis. A proper delay was introduced at the input ports of the 50/50 coupler to avoid overlapping of spectral profiles during stretching in the DFT branch. The temporal information (undispersed) was detected by a 20 GHz photodiode (Agilent 83440C) and digitized by a 20 GHz real-time oscilloscope (Lecroy SDA 820Zi-B), while the spectra were recorded by an optical spectrum analyzer (OSA, YOKOGAWA AQ6370D) and TS-DFT technique simultaneously. The DFT branch was composed of a spool of dispersion-compensating fiber (DCF) with \(-577\) ps nm\(^{-1}\) dispersion and was detected by a 12 GHz photodiode (New Focus 1544-B). The temporal and spectra resolution were 50 ps and 0.17 nm,\(^{[40]}\) respectively.

The stable BSMs can be directly generated at a pump power of 23 mW with a careful maneuver of the intracavity polarization. Figure 1d,e shows the spectral evolution of counterpropagating soliton molecules with clear breathing behavior corresponding to the same period of 70 RTs. The soliton spectra for both CCW and CW directions suggest the same center wavelength of 1600 nm (Figure 1b) but with distinguished bandwidths (22 nm for CCW pulses and 19.8 nm for CW pulses). The CW pulses possess higher energy and amplified spontaneous emission (ASE) noise that is attributed to the asymmetric pump structure of the EDF. The CW solitons separate from CCW solitons with a constant temporal distance of 35.56 ns, and only one fundamental frequency in the radio frequency (RF) spectrum demonstrates the synchronization and identical repetition rate in both directions (see Figure S1, Supporting Information). The apparent oscillation in the energy evolution curves (integration of spectral density) further demonstrates the breathing behavior of bidirectional soliton molecules (indicated by the white line in Figure 1d,e). The bidirectional soliton pair with a larger breathing ratio and temporal separation could also be observed (Figure 1c) by varying the pump power and polarization setting. It should be noted that the BSMs in opposite propagating directions collide in each cavity RT. The collision point in the cavity is inferred as the location of the CNT that leads to good temporal synchronization between counterpropagating pulse trains. The CNT produces equivalent loss modulation for the counterpropagating soliton molecules as the common transmission path through the CNT simultaneously. Therefore, the bidirectional BSMs have similar performance in this laser system, such as the same breathing period, spectral center wavelength, and repetition rate.

2.2. BSM Single Switching

Experimentally, different switching processes of BSMs could be triggered at different parameter spaces of saturable absorption by manipulation polarization. Initially, we investigated a unique single switching process and explored its transition dynamics. Figure 2a,c shows the spectrum evolution of counterpropagating BSM. The measured DFT spectra evolved dramatically during the switching process. The Fourier transforms of the single-shot spectra (dashed rectangle of Figure 2a,c) provided the field autocorrelation traces (Figure 2b1,d1) that show a detailed switching process of counterpropagating BSM. The breathing behavior of
The bidirectional soliton molecules in this single switching process is further demonstrated by the periodic spectral evolution at RT 3000–4000 corresponding to Figure 2a,c (see Figure S2, Supporting Information).

Over the switching process, the number of spikes in the field autocorrelation traces significantly increases in both directions. Two pulses of soliton molecules in both directions split into multiple pulses and then transit to a new BSM with larger temporal separation. The peak number and separation in autocorrelation trace at the six typical RTs varied in the switching process (Figure 2b2,d2). Note that if the pulse bunch contains $n$ pulses with equal pulse separation, its field autocorrelation trace features $2n−1$ peaks. The unequally spaced molecules possess relatively complicated autocorrelation traces owing to the complex multipulse structures. Although the temporal pulse separation of soliton molecules is less than the temporal resolution of 50 ps that impedes direct observation of the pulse distribution in the time domain, the soliton temporal distribution in the switching process can be determined by analytical fit according to the autocorrelation traces.[41] The actual pulse distribution also needs to obey the rule that the multipulses are usually dense in the front and sparse at the end. In the analytical fit, each soliton in the soliton molecule was a sech pulse profile with a temporal width of 500 fs, and the intensity and separation of the pulses were inferred from the autocorrelation traces (Figure 2b2,d2). The soliton temporal distribution experiences a similar evolution in both directions (Figure 2b3,d3). Two solitons in the BSM were first split into four solitons with almost equal temporal separation during transition. The intensity of each soliton varies from RT label 2 to 4 that suggests the existence of energy transfer among solitons over the cavity RTs. Then, the four solitons transit to three solitons at RT label 5, and the second pulse has a higher temporal intensity than each side pulse in both directions. The three solitons are bound with different pulse separations to form the so-called $(2+1)$-type molecule, which can also be interpreted as a molecule assembly of a soliton pair and a soliton singlet. Thus, the stronger soliton interaction force and the weaker soliton interaction force both play crucial roles in the formation of these complex unequally spaced soliton molecules. Finally, the three solitons in the soliton molecule transit to two solitons with a larger pulse separation (compare label 6 and label 1). This switching of both directions of BSMs shows high behavior similarity, including close temporal pulse separation and similar transient pulse evolution in the transition stage. The calculated autocorrelation traces according to analytical fit temporal pulse distribution (Figure 2b3,d3) agree well with the experiment (Figure 2b2,d2) that further corroborates our hypothesis (see Figure S3, Supporting Information).
Furthermore, for the internal dynamics of bidirectional BSMs, the relative pulse temporal separation $\tau$ and phase $\phi$ constitute the relevant internal degrees of freedom for the soliton molecules\cite{28,36} that can be retrieved from fringes of the recorded DFT spectrum (see Figure S4 and S5, Supporting Information). For the internal dynamics before [RTs from 1 to 4000] and after (RTs from 5000 to 10 000) the switching process (black dashed lines in Figure 2a,c), the retrieved temporal separation and relative phase suggest the same evolution trend of the bidirectional BSMs. The minor difference of bidirectional soliton molecules in this switching is that the temporal separation and relative phase oscillation amplitude of CCW is larger than that in the CW direction. The 3D interaction space of the BSMs before and after switching is introduced to visualize the internal motion (Figure 2e,f corresponding to RTs from 1 to 4000 and RTs from 5000 to 10 000 in Figure 2a,c). Considering the breathing dynamics, the trajectory traces spatial circles with varying turning points, while the overall trajectory envelope completes one circle representing a phase variation of around $-\pi$ within one period, and monotonically slides over the cavity RTs. The larger numbers of turning points and circles in the CCW interaction space that is attributed to the CCW soliton molecules show larger phase oscillating amplitude than the CW direction. The more phase unwrapping of CCW soliton molecules significantly increases

**Figure 2.** BSM single switching maneuvered by polarization. a) The real-time spectral evolution measured via DFT in CW direction. b1) The field autocorrelation traces corresponding to the dashed box in (a), b2) the field autocorrelation traces, and b3) the temporal pulse distribution of several typical RTs during the BSM transition process corresponding to the black dashed lines in (b1). c) The real-time spectral evolution measured via DFT in CCW direction. d1) The field autocorrelation traces corresponding to the dashed box in (c), d2) The field autocorrelation traces and d3) the temporal pulse distribution of several typical RTs corresponding to the black dashed lines in (d1), black dashed lines that label 1 to 6 in (b1), and (d1) corresponding to RTs 4000, 4200, 4335, 4415, 4540, and 4900. Trajectories of the internal degrees of freedom in the interaction space in e) CW and f) CCW directions, namely, the interpulse separations $\tau$ and relative phases $\phi$; the RT number is displayed in color scale.
the number of turning points and circles. The spectral behaviors of the bidirectional BSM (dashed rectangle of Figure 2a,c) were also compared through cross correlation (see Figure S6, Supporting Information) between the single-shot spectra in opposite directions. The high spectrum similarity can be observed both in the stable breathing state and in the dramatic soliton molecule switching process.

2.3. BSM Multiple Switching

Furthermore, the peculiar multiple switching processes of BSMs with intense repulsive interaction and a transient kink were triggered by changing the polarization setting at the fixed pump power (Figure 3). Moreover, we also observed another type of BSM multiple switching that maintains a fixed temporal separation after each switching with different polarization settings at the same pump power. Figure 3a,d shows the spectrum evolution of counterpropagating BSM; the spectra experience a dramatic change in each switching process. The field autocorrelation traces (Figure 3b,e) suggest that bidirectional BSM experiences several different stages during switching and maintains a similar evolution trend. First, the bidirectional BSM maintains stable periodic oscillation in stage 1 with a pulse separation of 28.8 ps (28.9 ps) in the CW (CCW) direction. Then, the BSM enters a chaotic evolution stage with a significantly enhanced oscillation amplitude in pulse temporal separation and energy evolution (stage 2). In the switching process (stage 3), repulsion dominates the interaction with the BSMs. Finally, the pulses of

![Figure 3. BSM multiple switching with intensive repulsive interaction. a) The real-time spectral evolution measured via DFT and b) the field autocorrelation traces calculated via the Fourier transform of each single-shot spectrum in CW direction. c1,c2,c3) Zoom-in plot of three switching processes corresponding to (b). d) The real-time spectral evolution measured via DFT and e) the corresponding field autocorrelation traces in CCW direction. f1,f2,f3) Zoom-in plot of three switching processes corresponding to (e). g) Spectra cross-correlation map corresponds to RTs 15 000 to 16 500, RTs 19 500 to 20 700, and RTs 24 300 to 25 500, respectively. h) Energy evolution.](image-url)
BSM maintain a fixed temporal separation and stable breathing behavior (stage 4). The breathing dynamics of bidirectional soliton molecules in this multiple switching process are further demonstrated by the obvious energy oscillation and periodic spectral evolution at RT 1–1000 corresponding to Figure 3a,d (see Figure S7, Supporting Information).

The close-up view (Figure 3c1–c3,f1–f3) of Figure 3b,e shows the details of the three switching processes in CW and CCW directions. In those switching processes, the two pulses in soliton molecules both split into multiple pulses (three and four pulses) and then recover to two pulses. In the first and second switching process, the pulses with separation of 31.7 ps (31.9 ps) at RT 15 000 swiftly increase to 41.6 ps (41.9 ps) at RT 16 500 and 52.2 ps (52.6 ps) at RT 19 500 swiftly increase to 59.2 ps (59.4 ps) at RT 20 700 in CW (CCW) direction, respectively. The two pulses of soliton molecules experience a strongly repulsive interaction and the pulse separation further monotonically increases after switching. During the third switching process, the pulse separation began to swiftly decrease from 70.2 ps (70.6 ps) at RT 24 300 to 67.1 ps (67.3 ps) at RT 25 500 in the CW (CCW) direction. Then two pulses of the soliton molecule maintained fixed temporal separation and periodic oscillation after the transition (stage 4). The spectral behaviors of the BSMs with multiple switching (Figure 3a,d) were further compared through the cross correlation (Figure 3g) between single-shot spectra from opposite directions. Typically, for every RT with index N propagating in the CW direction, the spectrum cross correlation with RTs from N-50 to N+50 in the CCW direction is calculated. In Figure 3g, the cross-correlation magnitude reveals the similarity in the two single-shot spectra from opposite directions. Notably, the spectra show an insignificant change in the quasistable breathing state. For each RT from the CW direction, many RTs from the CCW direction have high spectral similarity with it. The high spectral similarity can also be found in the most variant part of the soliton molecule switching process. In contrast to the behavior similarity in the stationary soliton, the behavior similarity also exists in the bidirectional BSM switching process. The energy evolution follows the same trend for counterpropagating BSMs (Figure 3h), further suggesting high behavior similarity during the multiple switching. The slight difference in the energy evolution is that the soliton molecule exhibits a larger energy oscillation amplitude in CW compared with that in the CCW direction during chaotic evolution (stage 2) from RT 6450 to 15 000 (dashed rectangle of Figure 3a,d). This is attributed to the soliton molecule in the CW direction acquiring higher gain than in CCW that leads to stronger nonlinearity for the CW soliton molecule in this chaotic process, accompanying larger and dramatic energy oscillation. Meanwhile, the intensive repulsion and transient kink of the two pulses in the BSMs arises from the gain depletion and recovery during the three switching processes (see Figure S8, Supporting Information).

2.4. BSM All-Optical Switching

By applying a strong stimulus with periodic pump modulation that abruptly decreases the pump power by ≈70% in each period (see Figure S9, Supporting Information), the events of BSM switching can be significantly increased. The pump power was modulated by a fiber-coupled intensity modulator (FIM) with a period of 25 μs and 10% duty ratio with a power drop of ≈1.7 and 0.4 μs rising and falling edges. Figure 4a,c shows the spectral evolution of the BSM consecutive switching of two solitons under pump power modulation combined with an adjustment of polarization. The spectra experience a dramatic change in each switching process. The behavior similarity during pulse evolution was corroborated by the field autocorrelation traces (Figure 4b,d) for the bidirectional BSM. The soliton molecules with 36 switching in 70 000 RTs correspond to ≈150 μs for each switching that is ≈6 times related to the modulation period (25 μs). It should be noted that this switching time is longer than the modulation period compared with that reported in a Ti: sapphire laser. Compare to the reported only 110 fs pulse separation with strong direct interaction in the soliton molecule, in our case, the pulse separation of tens of picoseconds relative to the femtosecond pulse width indicates a weak long-range interaction between pulses in the soliton molecules. Therefore, the pump power needs a higher modulation depth of 70% in our experiment compared with the 10% drop in the report, and the larger rising and falling time of 0.4 μs also leads to the slow gain response of the soliton molecule that leads to the longer switching period (≈150 μs) compared with the applied control signal period (25 μs). For the multiple switching process of the bidirectional soliton molecule, the temporal pulse separation will change in each switching process from the initial separation of 36.1 ps (35.3 ps) at RT 1000 to the separation of 76.5 ps (75.7 ps) at RT 70 000 in CW (CCW) direction (RT 1000 and 70 000 corresponding to label 1 and label 3 in Figure 4b,d, respectively). The two solitons in soliton molecule split into multiple solitons (three or four) and then recover to two solitons after each switching process. The most peculiar part in multiple switching corresponds to RT from 41 500 to 43 500 (label 2 in Figure 4b,d). The longer duration of ≈900 RT of this switching process is attributed to higher gain for each soliton in this stage that significantly increases the lifetime of the solitons. In this switching, the two solitons split into three solitons with interpulse separation of 13.9 ps (13.2 ps) and 54.3 ps (53.5 ps) in the CW (CCW) direction and then recover to two solitons.

Another exotic consecutive switching process was obtained (Figure 4e–h) by altering the polarization at the same pump power modulation. Figure 4e,f shows the spectral evolution of the bidirectional BSMs multiple switching, including two and three solitons in BSM, and the corresponding temporal intensity and energy evolution can be found in Figure S10, Supporting Information. The spectra experienced a dramatic change in the multiple switching process. The field autocorrelation traces (Figure 4g1,h1) suggest the same evolution behavior of the BSM in both directions. First, the two solitons of soliton molecule experienced 6 switching in 6400 RTs corresponding to ≈82 μs for each switching, that is, ≈3.3 times related to the modulation period (25 μs). This shorter switching time compared with 150 μs in Figure 4a–d suggests higher gain and nonlinearity that decrease the gain response time of soliton molecules at this polarization setting. The two solitons split into multiple solitons (three or four) and then recovered to two solitons in each switching process. The temporal pulse separation from 32.8 ps (32.1 ps) at RT 300 transits to 39.9 ps (39.3 ps) at RT 5850 in CW (CCW) direction.
The three solitons of the soliton molecule experience 10 switching in the remaining 44 600 RTs corresponding to \( \mu s \) for each switching that is obviously larger than two solitons' switching time, which means that the increase in the number of pulses in the soliton molecule will greatly lengthen the gain response time of the soliton molecule. The three solitons split into multiple solitons (four or five) and then recovered to three solitons in each switching process. The temporal pulse distribution of several typical RT corresponding to (g1) and (h1), indicating that the pulse separation changed in each switching process. Here, the intensity of the pulses was

Figure 4. Consecutive switching of BSM with pump power modulation. a–d) Consecutive BSM switching of two solitons. a) The real-time spectral evolution measured via DFT and b) the field autocorrelation traces calculated via the Fourier transform of each single-shot spectrum in CW direction. c) The real-time spectral evolution and d) the corresponding field autocorrelation traces in CCW direction. e–h) Consecutive switching of BSM with two and three solitons. e) The real-time spectral evolution and g1) the field autocorrelation traces in CW direction. g2) The temporal pulse distribution of several typical RTs corresponding to (g1), and black lines that label 1 to 10 in (g1) and (h1) corresponding to RTs 8400, 12 360, 17 610, 24 060, 28 540, 32 900, 37 660, 42 130, 47 350, and 50 400.
neglected in the calculation, and the pulse separation was inferred from the autocorrelation traces (Figure 4g1,h1). The interpulse separation for three pulses gradually increased during consecutive switching from 18.5 ps (17.6 ps) and 20.6 ps (19.7 ps) at RT 8400 (label 1) to 29.1 ps (28.5 ps) and 32.4 ps (31.5 ps) at RT 50 400 (label 10) in CW (CCW) direction.

2.5. Numerical Simulations

To provide insight into BSM dynamics in bidirectional mode-locked fiber lasers, numerical simulation was executed in the lumped propagation model, and each component of the cavity is modeled by a separate equation. Pulse propagation within the fiber is modeled with a modified nonlinear Schrödinger equation (NLSE) for a slowly varying pulse envelope. The experimental results indicate that manipulation polarization and gain are critical to trigger the switching of BSMs. In the NLSE, the gain saturation energy $E_s$ is determined by pump power. $q_0$ is the modulation depth, and $P_{sat}$ is the saturation power of the SA, which is modeled by a nonlinear transfer function. The parameters in the nonlinear transfer function could be tuned by changing the polarization, thus modifying the interactions among pulses in soliton molecules.

A scalar iterative map was calculated to identify the regimes for soliton molecule switching (see Figure S11, Supporting Information). The simulation results of two solitons’ transition to three solitons and three solitons switching in the BSMs match well with the experiment in Figure 4g1,h1. Figure 5a–c shows the simulation results of BSM transition from two solitons to three solitons with $E_s = 3.571$ pJ, $q_0 = 0.71$, and $P_{sat} = 210$ W. The initial optical fields of two propagating directions are a weak sech-shape pulse with 5 ps pulse width combined with random noise. In the transition process, a new soliton was formed in the middle of the original two solitons (Figure 5a1,b1). The corresponding field traces evolution (Figure 5a2,b2) agree well with the experiment results (Figure 4g1,h1) with minor differences in temporal separation among the three solitons in the BSM. The three solitons switching with different temporal separations in BSMs is shown in Figure 5d–f with $E_s = 3.571$ pJ, $q_0 = 0.68$, and $P_{sat} = 195$ W. The temporal intensity evolution of the

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Simulation of BSMs. a–c) Transition from two solitons to three solitons in BSMs. a1) Temporal intensity evolution and a2) the corresponding field autocorrelation traces in CW direction. b1) Temporal intensity evolution and b2) the corresponding field autocorrelation traces in CCW direction. c) Energy evolution. d–f) Three solitons switching in BSMs. d1) Temporal intensity evolution and d2) the corresponding field autocorrelation traces in CW direction. e1) Temporal intensity evolution and e2) the corresponding field autocorrelation traces in CCW direction. f) Energy evolution.
A bidirectional BSM is shown in Figure 5d, e. The corresponding field autocorrelation trace evolutions (Figure 5d, e) indicate that the physical process involved is similar to experimental observations depicted in Figure 4g, h (see Figure S12, Supporting Information). The pulse temporal separation and intensity both change in the switching process adjacent to energy transfer between solitons. The three solitons split into multiple solitons and then revert to three solitons in the transition stage. Further, the simulated energy evolution follows the same trend for counterpropagating BSM in this switching process, suggesting the high behavior similarity of bidirectional BSMs. The simulation also confirms that the most critical influence factor of the behavior similarity in soliton molecules is the common modulation by SA and not sensitive to the initial condition: slightly change the initial seed intensity difference of the counterpropagation pulse envelope, the simulation will still converge to a state where bidirectional pulses possess high behavior similarity under certain parameters.

In this bidirectional mode-locked fiber laser, the counterpropagating BSMs share the same dispersion and optical path but in the opposite direction and pass through the CNT together with the same cavity loss and similar gain profile. Therefore, the bidirectional breathers naturally synchronize in time and possess high behavior similarity during dynamic evolution. Different from the unidirectional laser where the periodical transition occurs between the vibrational and static soliton molecules in an anomalous dispersion nonlinear polarization rotation (NPR) mode-locked fiber laser, or consecutive switching between two states of stationary soliton molecules in a Ti: sapphire laser, our work unveils completely different characteristics in a normal dispersion bidirectional ultrafast fiber laser. The unidirectional mode-locked laser pulse does not split and features a smaller change in temporal pulse separation (≈100 fs) during switching of two pulses in soliton molecules. In contrast, the switching dynamics of bidirectional BSM show a more dramatic transition that the two or three solitons split to multiple pulses, then revert to the number of pulses before switching, and show a significant change of temporal pulse separation of approximately picoseconds in each switching. The collision of counterpropagating BSM plays an important role in the formation of multipulse structures in each switching process; meanwhile, the presence of multisoliton structures in switching could be referred to as the inelastic collision of counterpropagating breathers. The collision of BSM combined with polarization or pump power variation leads to the system parameter experiencing a dramatic change, further affecting the energy evolution and nonlinearity in the cavity for each soliton. Then, the breathing solitons experienced instability and split. Finally, owing to the self-stabilization effect, the split solitons recovered to the number of pulses before switching.

As we know, the breathing ratio of dissipative solitons, defined as the widest to the narrowest spectrum bandwidth within a breathing period, depends on cavity dispersion and nonlinear gain. In this experiment, the BSM switching could be triggered at different parameter spaces of saturable absorption with manipulation polarization or perturbation through periodic pump power modulation. The bidirectional soliton molecules during the switching process have relatively weaker breathing behavior compared with the stable BSM that corresponds to the different system parameters, and the stable breathing dissipative soliton with a strong breathing ratio is less sensitive to external disturbances and the nonequilibrium dynamics is less observed. Moreover, the recently reported soliton molecule switching with direct electronic control identified a universal bound-state formation mechanism different from broadly considered models, wherein the soliton coupling mechanism is based on linear reflections. However, the linear reflections will lead to the soliton molecule binding with only several specific pulse temporal separations that may not be applicable to the all-optical switching of bidirectional BSM in our experiment. The consecutive 36 switching of doublet and 10 switching of triplet BSMs (Figure 4) with different temporal separation are dominated by gain relaxation through gain depletion and recovery mechanism rather than the linear reflections.

3. Conclusion

In conclusion, we have revealed the instantaneous switching of BSMs in a bidirectional ultrafast fiber laser. The diversified switching dynamics in soliton molecules could be triggered at different parameter spaces of saturable absorption with manipulation polarization that maintains constant pulse separation or undergoes strong repulsion and kink after switching. The switching of breather molecules is more prone to be triggered in the parameter interval close to the boundary of the doublet and triplet soliton molecules that are further corroborated in numerical simulation. Meanwhile, the time-varying separations between breathers and transient pulses’ splitting during switching are closely related to the collision of counterpropagating breather molecules and the modulation of CNT. The intensive repulsion interaction and a transient twist of the two pulses in the breather molecules arise from the gain depletion and recovery. Further, the gain/loss dynamics also play an important role in the switching of soliton molecules. The switching period of three solitons is longer than that of two solitons owing to the slower gain response. Moreover, ultrafast fiber lasers are well known for their versatile design and compatible with a variety of mode-locking mechanisms. It is significantly interesting to explore the versatile dynamic evolution of BSMs such as build-up or switching dynamics in fiber laser cavities with different characteristics and operating conditions. We anticipate that our results unveil new perspectives into the BSM transient dynamics of mode-locked lasers and will provide useful insights into laser design and applications.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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