A High-Precision Magnetic Induction Through-the-Earth Positioning Scheme

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ABSTRACT Through-the-earth positioning technology plays a vital role in mine rescue. To achieve through-the-earth positioning in the complex earth that contains geomagnetic noise, water, and rocks, we design a magnetic induction through-the-earth positioning scheme that includes positioning methods and a noise reduction method. The positioning scheme achieves three-dimensional through-the-earth positioning with high precision and long distance. Using the directional characteristics of the magnetic field vector distributed by the transmitter coil horizontally placed underground, we design two positioning methods to determine the horizontal two-dimensional position and vertical dimensional position of the transmitter, respectively. Taking advantage of the extremely narrow bandwidth of the sinusoidal signal in the spectrum, we design a noise reduction method to reduce the influence of noise on positioning accuracy. The simulation results show that the positioning error of the positioning method based on the signal direction is less than that of the positioning methods based on the equations, path loss, and the optimization algorithm, and the noise reduction performance of the frequency-point amplitude acquisition method is better than that of the bandpass filter. The simulation results also show that the positioning scheme proposed in this paper can penetrate 1000 meters of the earth to achieve high-precision through-the-earth positioning with errors of less than 2 meters.

INDEX TERMS High-precision, magnetic induction, noise reduction, positioning, through-the-earth.

I. INTRODUCTION
When the mine disaster occurs, traditional communication methods such as wired and cellular wireless communications will be interrupted. Rescuers on the ground cannot obtain the exact location of the trapped miners underground in time. This is very detrimental to the rescue. The trapped miners face dangers such as tunnel collapse, water leakage, lack of oxygen, and gas explosion at any time. Their lives are severely threatened.

The through-the-earth positioning technology can play a vital role in mine rescue [1], [2]. This technology can penetrate the earth to convey the location information of the trapped miners to rescuers. Based on this information, rescuers can rescue the trapped miners by drilling holes and dredging roadways.

However, in the complex earth media containing water and rocks, it is a great challenge to penetrate several hundred meters of earth media to achieve high-precision three-dimensional (3D) through-the-earth positioning.

The electromagnetic wave method suffers from severe path loss, multipath effect, and large antenna size [3]–[10], which seriously affect the signal transmission distance, positioning accuracy, and underground feasibility. Therefore, the electromagnetic wave method is not suitable for through-the-earth positioning. Unlike the electromagnetic wave method, the magnetic induction method uses low frequency quasi-static sinusoidal alternating magnetic field to transmit the positioning signal, which has the advantages of no multipath effect, lower path loss, and smaller antenna size [11], [12]. Since the earth media usually do not contain ferromagnetic materials such as iron, cobalt, and nickel, the magnetic signal will not encounter refraction. Therefore, this method has a broad application prospect in through-the-earth positioning.

In the design of the magnetic induction positioning method, several hardware schemes can be selected. To achieve long-distance high-precision 3D through-the-earth
positioning, we should choose an appropriate hardware scheme among them.

When selecting single-axis or three-axis transceivers, we should consider the requirements of the application scenario of through-the-earth positioning. The single-axis transceivers usually use single-axis multi-turn coils as the transceiver antennas [13], while the 3-axis transceivers use three mutually orthogonal coils as the transceiver antennas [14]. We first analyze the transmitter. The signals of the single-axis transmitter antenna are mainly concentrated on the axis of the transmitter coil. In contrast, the signals of the three-axis transmitter antenna are approximately evenly distributed around the transmitter. In the high conductivity earth media, the signals of the single-axis transmitter can transmit a longer distance. Therefore, the single-axis transmitter is suitable for through-the-earth positioning. We then analyze the receiver. The 3-axis receiver can receive positioning signals in three directions once [15], while the single-axis receiver needs to rotate the receiver coil repeatedly to receive positioning signals in three directions [16]. The use of the 3-axis receiver can significantly reduce the complexity of the positioning operations. Therefore, the 3-axis receiver is suitable for the application scenario of through-the-earth positioning.

When deciding whether to add cores to the transceiver coils, we should consider the feasibility and efficiency. In the aspect of the transmitter, according to the Biot-Savart law, adding a high-permeability magnetic core to the transmitter coil can increase the radiation intensity of the transmitter. However, the increase of signal strength is affected by the demagnetization coefficient of the core [17], which is proportional to the slenderness ratio of the core. Since the limited space underground cannot accommodate an excessively long core, and a short core has a limited increase in the positioning signal, it is not feasible to add a magnetic core to the coil transmitter. In the aspect of the receiver, according to the Faraday law of electromagnetic induction, adding magnetic cores in the receiver coils can significantly improve the sensitivity of the receiver [18]. A high-sensitivity receiver can receive weaker signals at a longer distance, which increases the receiving efficiency. Therefore, in through-the-earth positioning, we should choose the air coil transmitter and the magnetic core receiver.

After determining the hardware scheme, we need to select the appropriate positioning method.

The 2D positioning methods determine the vertical projection position of the underground transmitter on the horizontal ground. Wait [19] have achieved 2D positioning by using the feature that the horizontal component of the positioning signal at the vertical projection position is 0. Powell [16] have achieved 2D positioning by using the feature that the horizontal component of the positioning signals on the ground point to the projection position.

The 3D positioning methods further determine the 3-dimensional position of the transmitter.

Tsai and Yang, [20], Yao et al. [21] have calculated the position of the receiver by using the corresponding relationship between the positioning signal vectors and the coordinates of the receiver. In the three-dimensional rectangular coordinate system with the cylindrical permanent magnet transmitter as the origin, Tsai et al. first established three equations of the magnetic field intensity at the receiver and the coordinates of the receiver. Then Tsai et al. solved the three equations to obtain the coordinates of the receiver.

Arumugam et al. [22] have estimated the position of the transmitter by using the path loss of positioning signals. Arumugam et al. first use multiple receivers placed at known locations to receive the signal strengths corresponding to these locations. Then, according to the path loss rule of the magnetic induction signals, Arumugam et al. obtain five pass loss equations of positioning signals with five variables of transmitter position and pose. Finally, Arumugam et al. estimate the position of the transmitter by using a numerical nonlinear least-squares optimization algorithm.

Song et al. [23] have proposed a method to achieve 3D positioning by using positioning signal strength. Song et al. first rotate the single-axis coil of the receiver in the horizontal and vertical dimensions. At this time, the signal strength of the three-axis transmitter will change. When the signal strength reaches the peak, the axis of the receiver points to the transmitter. Then, Song et al. calculate the distance between transceivers by using the path loss rule of the positioning signal. Finally, Song et al. calculate the position of the transmitter by using the distance between the transceivers and the rotation angle of the receiver.

A large number of field tests have confirmed the feasibility of magnetic induction through-the-earth positioning. Liu et al. [24] have tested the through-the-earth transmission system at a distance of 469m between the ground and the underground testing point. In [25], the Lockheed Martin MS2 magnetic communication system has been tested at a depth of about 457.2m below the surface. The maximum range achieved for through-the-mine testing is 1494m [13]. It is also shown in [13] that the beacon mode signal can transmit a longer distance. The beacon mode is another name for through-the-earth positioning. These tests have shown that it is possible to penetrate 1000m of the earth to achieve through-the-earth positioning.

Although the above positioning methods have achieved the expected positioning targets, these methods still have several obvious problems when applied to the mine rescue scenario requiring long-distance high-precision through-the-earth positioning.

1. The positioning methods of equations ignore the positioning error caused by the skin effect [20], [21]. The electrical conductivity of the earth media is much higher than that of the air. When propagating in the earth, the alternating magnetic field will attenuate due to the skin effect. Ignoring the skin effect will increase the error of the through-the-earth positioning.
2. There are errors in the positioning methods of using the path loss rule of the positioning signals [22], [23]. In the stratified earth media with different electrical conductivity, the attenuation rates of magnetic induction positioning signals are different. The difference will lead to errors.

3. The estimation methods of using the optimization algorithms are complex and inaccurate [26], [27]. The estimation methods must gradually adjust the variables in the path loss equations. Hundreds of complex iterative calculations are required in this process. At the same time, the jump of variables required by iterative calculations will also cause errors. Since the variables are discrete, there is an interval between the adjacent calculation results. The interval causes an error.

Aiming at the above problems in the existing positioning methods, we propose a high-precision magnetic induction through-the-earth positioning scheme achieving 3D positioning in this paper. The transmitter of the positioning scheme is a single-axis annular energized air coil placed horizontally underground. The receiver on the ground consists of three mutually orthogonal inductive core coils. On this hardware scheme, we design a horizontal positioning method and a vertical positioning method based on the directional characteristics of the positioning signal vectors, which determine the orientation of the underground transmitter in the horizontal two-dimension and vertical dimension, respectively. To reduce the positioning error caused by geomagnetic noise, we design a noise reduction method called the frequency-point amplitude acquisition method. This method achieves noise reduction by collecting the amplitude of the positioning signal in the frequency domain. The noise reduction method proposed in this paper provides a new idea for noise reduction, which has less computation and a better noise reduction effect than the traditional bandpass filter. The positioning methods and the noise reduction method proposed in this paper jointly realize the through-the-earth positioning with an error of fewer than 2m at a depth of 1000m.

Specifically, to ensure positioning accuracy, we add the exponential expression of skin effect to the formula. However, this change makes the calculation formula a transcendental function, and it is impossible to solve the position of the transmitter directly by using the relationship equations in [20] and [21]. For this reason, we perform vector decomposition and simplification calculations on the positioning signal vectors and solve the positioning of the transmitter by using the directions of the positioning signals. Since the directions of the vectors are not affected by the conductivity of the earth media, the positioning methods in this paper eliminate the positioning errors caused by the path loss. When determining the position of the transmitter, we replace the estimation methods with an accurate calculation, which eliminates the error caused by the interval of iterative calculations and reduces the computational complexity.

The remainder of this paper is organized as follows. Section 2 introduces the magnetic induction through-the-earth positioning scheme. Sections 3 and 4 achieve horizontal positioning and vertical positioning, respectively. Section 5 introduces the frequency-point amplitude acquisition noise reduction method. Section 6 evaluates the positioning error of the scheme. Section 7 concludes the paper.

II. POSITI ONING SCHEME INTRODUCTION

In this section, we first introduce the composition and workflow of the magnetic induction through-the-earth positioning scheme. Then we analyze the principles and implementation ideas of the through-the-earth positioning in spherical coordinates.

The composition and workflow of the magnetic induction through-the-earth positioning scheme are as follows. Fig. 1 shows the high-precision magnetic induction through-the-earth positioning scheme proposed in this paper. The transmitter placed horizontally underground is a single-axis multi-turn loop energized coil. The two receivers placed at different positions on the horizontal ground are three-axis inductive core coils. The axes of the three coils are orthogonal to each other. One of the axes is vertical, and two of the axes are horizontal. Between the transceivers are the multiple layers of electrically conductive earth media. A low-frequency (such as 10 Hz) sinusoidal current flows through the multi-turn coil of the transmitter. The current in the coil induces a sinusoidal quasi-static magnetic field in the surrounding earth media. The quasi-static magnetic field passes through the earth media to the receivers on the ground and generates induced voltage signals at both ends of the three inductive core coils. By using the signals, we can determine the directions and amplitudes of the positioning signal vectors.

Since the positioning signal vector at each receiver changes with the distance and angle between the transceivers, the receivers can determine the position of the transmitter using the direction of the positioning signal vectors.

To achieve through-the-earth positioning by using the directions of the positioning signal vectors, we need to determine the distribution rule of the positioning signal vectors in the coordinate system.
We first determine the distribution rule of the positioning signal under the ideal condition. As shown in Fig. 2, the spherical coordinate system is established with the center of the transmitter coil as the origin and the vertically upward direction as the z-axis. The ideal value of the positioning signal vector received by the receiver at point Q1 is [28]

$$\overrightarrow{B_{Q1}} = \frac{n\mu_0 R^2}{4r_1^3} \cdot \left( e_{r_1} \cdot 2 \cos \theta_1 + e_{\theta_1} \sin \theta_1 \right)$$

(1)

where $e_{r_1}$ and $e_{\theta_1}$ are the unit distance vector and unit elevation angle vector of $\overrightarrow{B_{Q1}}$ in the spherical coordinate system, respectively. $r_1$ is the distance between the transmitter coil center O and the receiver Q1. $\theta_1$ is the angle of $\angle COQ_1$, which represents the angle between the transceiver connection Q1O and the axis OC of the transmitter coil, and point C is the intersection of the transmitter coil axis and the ground. R, n, and I represent the radius, number of turns, and current of the transmitter coil, respectively. $\mu_0$ represents the permeability of the earth.

We then determine the distribution rule of the positioning signal with the earth media and noise. We assume that the conductivity of the earth media is $\sigma$, the geomagnetic noise in the spectrum is $n_0(t)$, and the current of the transmitter coil is $I = I_0 \cdot \sin(2\pi ft)$, where $f$ is the frequency, $t$ is the time, and $I_0$ is the peak value of the sinusoidal current. Ignoring the influence of current waveform distortion and coil shape irregularity on the positioning signal, the value of the positioning signal vector received by the three-axis receiver at point Q1 is

$$\overrightarrow{B_{Q1}} = \frac{n\mu_0 R^2 I_0 \sin(2\pi ft)}{4r_1^3} \cdot \left( e_{r_1} \cdot 2 \cos \theta_1 + e_{\theta_1} \sin \theta_1 \right) \cdot e^{-\sqrt{\sigma ft} \mu_0 \sigma r'_1} + n_0(t)$$

(2)

By replacing Q1 ($r'_1$, $\theta_1$, $\phi_1$) with Q2 ($r'_2$, $\theta_2$, $\phi_2$) in equation (2), we can also obtain the positioning signal vector $\overrightarrow{B_{Q2}}$ at point Q2.

Using the directions of $\overrightarrow{B_{Q1}}$ and $\overrightarrow{B_{Q2}}$, we can achieve the through-the-earth positioning. Since the magnetic induction lines in Fig. 1 are arc-shaped curves, $\overrightarrow{B_{Q1}}$ and $\overrightarrow{B_{Q2}}$ do not point to the transmitter coil along the straight line, we cannot locate the transmitter coil directly by the direction of the positioning signals at the receivers. However, we can locate the transmitter coil indirectly through two steps of horizontal positioning and vertical positioning.

The horizontal positioning method locates the transmitter in two horizontal dimensions. Point C has the same horizontal dimensions as the transmitter in Fig. 2. The position of point C can be determined by intersecting the horizontal components of the positioning signal vectors at the two receivers. According to (2), the positioning signal vector $\overrightarrow{B_{Q1}}$ at point Q1 contains only the component in the direction of $e_{r_1}$ and the component in the direction of $e_{\theta_1}$, the component in the direction of $e_{\phi_1}$ is 0. Therefore, the straight line where the horizontal component $B_{H1}$ of $\overrightarrow{B_{Q1}}$ lies passes through point C. Point C is the intersection point between the axis of the transmitter coil and the ground in Fig. 2. Similarly, the straight line where the horizontal component $B_{H2}$ of $\overrightarrow{B_{Q2}}$ lies also passes through point C. In the three-dimensional rectangular coordinate system with the origin of point Q1, we can determine the coordinates of point C by intersecting the lines where $B_{H1}$ and $B_{H2}$ are located.

The vertical positioning method determines the depth of the transmitter coil from the ground, that is, the length of OC in Fig. 2. We determine the transmitter depth by using the direction of the positioning signal vector $\overrightarrow{B_{Q1}}$ at point Q1 in Fig. 2. Take point Q1 as an example. In the right triangle composed of the receiver at point Q1, the transmitter at point O, and point C, we can calculate the length of OC by performing a trigonometric function on $Q_1C$ and $\angle Q_1OC$. The length of Q1C has been obtained in the horizontal positioning method, and we need to determine the angle $\theta_1$ of $\angle Q_1OC$ by the direction of $\overrightarrow{B_{Q1}}$. However, since (2) is a transcendental function, we cannot calculate the transmitter position directly by a polynomial, so we need to decompose $\overrightarrow{B_{Q1}}$. According to analytic geometry theory, the direction of $\overrightarrow{B_{Q1}}$ in Fig. 2 determines the ratio $\| \overrightarrow{B_{Z1}}/\| B_{H1} \|$ of its vertical component $\| \overrightarrow{B_{Z1}} \|$ to its horizontal component $\| B_{H1} \|$. By vector decompostion of $\overrightarrow{B_{Q1}}$ in equation (2), we get the expressions of $\| \overrightarrow{B_{Z1}} \|$ and $\| B_{H1} \|$. Then by dividing the expressions of $\| \overrightarrow{B_{Z1}} \|$ and $\| B_{H1} \|$, we get the expression containing only $\| \overrightarrow{B_{Z1}}/\| B_{H1} \|$ and $\theta_1$. We can calculate the value of $\theta_1$ by this expression. So far, the length of Q1C and the angle $\theta_1$ of $\angle Q_1OC$ have been obtained, and we can use the trigonometric function to calculate the depth OC of the transmitter coil from the ground.

The flow of the proposed positioning methods are shown in Fig. 2. Next, we will introduce the realization process of through-the-earth positioning.

**III. THE HORIZONTAL POSITIONING METHOD**

This section achieves the horizontal positioning of the underground transmitter. As shown in Fig. 2, there is a three-dimensional rectangular coordinate system with point Q1 as the origin. In the coordinate system Q1, the horizontal
positioning method acquires the horizontal two-dimensional coordinates of the transmitter. Point Q₁ can also be replaced by point Q₂. Point C is on the same vertical line as point O where the transmitter is located, so the horizontal coordinates of point C are the horizontal coordinates of point O. We first figure out the horizontal coordinates of point C and then assign them to point O, thereby achieving horizontal positioning.

To obtain the coordinates of point C, we express the transceivers and positioning signal vectors in the three-dimensional rectangular coordinate systems. As shown in Fig. 4, Point O is the transmitter, Q₁ and Q₂ are receivers, where point O is underground, Q₁ and Q₂ are on the ground. Unlike the ideal horizontal ground in Fig. 2, the ground in Fig. 4 has fluctuations, so points Q₁ and Q₂ may not be on the same horizontal plane. Taking points O, Q₁, and Q₂ as origins, we establish the rectangular coordinate system O, the coordinate system Q₁, and the coordinate system Q₂. The Z axis, ZQ₁ axis, and ZQ₂ axis are all perpendicular to the ground. The Z axis coincides with the axis of the transmitter coil. The three coordinate axes of coordinate systems Q₁ and Q₂ correspond to the axes of the three magnetic core coils of the receivers Q₁ and Q₂, respectively. To reduce the errors caused by the transformation of the positioning signal vectors among different coordinate systems, we rotate the two horizontal axes of the receivers Q₁ and Q₂ gradually until plane XQ₁Q₂ZQ₁ and plane XQ₂Q₁ZQ₂ coincide. This operation can be realized precisely with the aid of the total station. For the convenience of calculation, we make Q₁XQ₁ and Q₂XQ₂ in the same direction. At this time, the three coordinate axes of the coordinate system Q₁ and the coordinate system Q₂ are parallel to each other.

According to the positional relationship between the coordinate systems Q₁ and Q₂, we calculate the coordinates and positioning signal vector of point Q₂ in coordinate system Q₁. Through measurement, we know that Q₁Q₂ = d and Q₁XQ₂ = α in Fig. 4. So, the horizontal distance dH and vertical distance dV between points Q₁ and Q₂ are respectively

\[
\begin{align*}
    d_H &= d \cdot \cos \alpha \\
    d_V &= d \cdot \sin \alpha.
\end{align*}
\]

So, the coordinates of point Q₂ in coordinate system Q₁ are (dH, 0, dV).

According to the positional relationship between the coordinate systems Q₁ and Q₂, we can also calculate the positioning signal vector of point Q₂ in coordinate system Q₁. It can be seen from Fig. 4 that the horizontal component of the positioning signal vector B₀Q₂ is at the point Q₂ is B_H2. The coordinates of B_H2 in the coordinate system Q₂ are (x₂, y₂, 0). Since the three coordinate axes of coordinate systems Q₁ and Q₂ are parallel to each other, the coordinates of B_H2 in coordinate system Q₁ are also (x₂, y₂, 0).

We can use the direction of B_H1 and B_H2 to determine the position of Point C. As shown in (2), B₀Q₂ contains only the component in the direction of ê₁ and the component in the direction of ê₂, and the component in the direction of ê₃ is 0. So, the line where B₀Q₂ lies passes through the Z axis, and the line where its horizontal component B_H1 (x₁, y₁, 0) lies passes through point C on the Z axis. Since point Q₂ and point Q₁ are not on the same horizontal plane, the straight line where the horizontal component B_H2 of B₀Q₂ lies passes through point C’ on the Z axis. To determine the position of point C, we change the coordinate −dV of point Q₂ in coordinate system Q₁ to 0 and get the coordinate (−dH, 0, 0) of point Q₂’ in coordinate system Q₁. Then we translate B_H2 (x₂, y₂, 0) to point Q’ to get B’_H2 (x₂, y₂, 0), the line where B’_H2 (x₂, y₂, 0) lies passes through point C. By intersecting the lines where B’_H2 and B’_H2 lie, we can get the coordinates of point C in the coordinate system Q₁.

According to analytic geometry theory, the equation of a line in a rectangular coordinate system can be expressed by the coordinates of a point on the line and the coordinates of a vector parallel to the line. So, the equations of straight lines Q₁C and Q₂C in coordinate system Q₁ are respectively

\[
\begin{align*}
    \frac{x}{x_{Q_1}} &= \frac{y}{y_{Q_1}},
\end{align*}
\]
and

\[ x + d_H \frac{x_{Q2}}{y_{Q2}} = y \frac{y_{Q2}}{y_{Q2}}. \]  

(5)

By combining the two equations in (5), we can obtain the horizontal coordinates of the intersection point C in the coordinate system Q1. So, the horizontal coordinates of the transmitter in coordinate system Q1 are respectively

\[
\begin{align*}
x_1 &= \frac{x_{Q1}y_{Q2}d_H}{x_{Q2}y_{Q2} - x_{Q1}y_{Q2}} \text{ and } \\nonumber \\nonumber 
y_1 &= \frac{y_{Q1}y_{Q2}d_H}{x_{Q2}y_{Q2} - x_{Q1}y_{Q2}}.
\end{align*}
\]  

(6)

IV. THE VERTICAL POSITIONING METHOD

This section will achieve vertical positioning of the transmitter by using the direction of the positioning signal. The vertical positioning method based on the signal direction is realized by precise calculation, which has not been mentioned in previous research. After that, we will compare the positioning accuracy of this method and other existing methods.

A. THE VERTICAL POSITIONING METHOD BASED ON THE SIGNAL DIRECTION

The vertical positioning method acquires the length of OC in Fig. 4, which means transmitter depth at point O from the ground. Under the condition that we have obtained the XQ1 coordinate xo and YQ1 coordinate yo of point O in coordinate system Q1, we will continue to calculate the ZQ1 coordinate z0 of point O, \( z_0 = OC \).

Since OC in Fig. 2 and Fig. 4 are the same, to get the value of \( z_0 \) in Fig. 4, we need to determine the value of \( z \) in Fig. 2. As shown in Fig. 2, in the right triangle composed of points Q1, O, and C, we can acquire the value of \( z \) by performing trigonometric function calculation on Q1C and \( \angle Q1OC \), that is

\[ z = \frac{r_1}{\tan \theta_1}, \]  

(7)

where the length \( r_1 \) of Q1C in (7) is \( \sqrt{x_0^2 + y_0^2} \), while the angle \( \theta_1 \) of \( \angle Q1OC \) has not been determined.

We find that the direction of \( \vec{B}_{Q1} \) can be used to obtain the value of \( \theta_1 \), thus determining the value of \( z \). As shown in (2), there is a functional relationship between \( \vec{B}_{Q1} \) and \( \theta_1 \). However, because (2) is a transcendental function, we cannot solve \( \theta_1 \) directly through \( \vec{B}_{Q1} \).

To get the value of \( \theta_1 \), we need to perform a series of mathematical calculations on (2). Since the receiver coordinates cannot be obtained directly by using transcendental functions, We first decompose \( \vec{B}_{Q1} \) into vertical component \( |\vec{B}_{Z1}| \) and horizontal component \( |\vec{B}_{H1}| \) step by step. Then, we establish the functional relationship between the ratio of \( |\vec{B}_{Z1}|/|\vec{B}_{H1}| \) and \( \theta_1 \). After assigning \( |\vec{B}_{Z1}|/|\vec{B}_{H1}| \), we calculate the value of \( \theta_1 \). To avoid multiple positioning results, the vector \( \vec{B}_{H1} \)

should be in the same direction as the vector \( \vec{CQ}_2 \). The process of vector decomposition and calculation is as follows.

We first perform vector decomposition on \( \vec{B}_{Q1} \). According to (2), if we ignore the effect of geomagnetic noise, the peak value \( \vec{B}_{Q1,p} \) of the through-the-earth positioning signal strength \( B_{Q1} \) at point Q1 is

\[
\begin{align*}
\vec{B}_{Q1,p} &= \frac{n_{\mu_0}I_0 R^2}{4r_1^3} \left( \frac{\vec{e}_b}{2} \cos \theta_1 + \vec{e}_d \sin \theta_1 \right) e^{-\sqrt{\frac{\mu_0 \sigma}{\sigma}} \cdot r_1},
\end{align*}
\]  

(8)

where \( \vec{B}_{Q1,p} \) can be decomposed into components in the \( \vec{e}_b \) direction and in the \( \vec{e}_d \) direction. The strengths of the positioning signals used in the through-the-earth positioning process are usually at the peak. Unless otherwise specified, the following signal strengths represent the peak values of the signals.

To conveniently describe the vector decomposition process of \( \vec{B}_{Q1,p} \), we establish a rectangular coordinate system in the plane of points Q1, O, and C in Fig. 2. The origin of the coordinate system is point O, the Z axis coincides with the straight line OC, the upper direction is the positive direction of the Z axis, and the H1 is the horizontal axis of the coordinate system. As shown in Fig. 5, in the newly established rectangular coordinate system, Q1L corresponds to \( B_{Q1,L} \), Q1B and Q1F correspond to the \( \vec{e}_b \) component \( \vec{B}_{Q1,B} \) and \( \vec{e}_d \) component \( \vec{B}_{Q1,F} \) of \( \vec{B}_{Q1,F} \), respectively. Therefore, \( \vec{B}_{Q1,B} \) and \( \vec{B}_{Q1,F} \) are respectively

\[
\begin{align*}
\vec{B}_{Q1,B} &= \left( n_{\mu_0}I_0 R^2 \cos \theta_1 / 2r_1^3 \right) e^{-\sqrt{\frac{\mu_0 \sigma}{\sigma}} \cdot r_1} \cdot \vec{e}_b, \\
\vec{B}_{Q1,F} &= \left( n_{\mu_0}I_0 R^2 \sin \theta_1 / 2r_1^3 \right) e^{-\sqrt{\frac{\mu_0 \sigma}{\sigma}} \cdot r_1} \cdot \vec{e}_d.
\end{align*}
\]  

(9)

To obtain \( \vec{B}_{Z1} \) and \( \vec{B}_{H1} \), we continue to decompose \( \vec{B}_{Q1,B} \) and \( \vec{B}_{Q1,F} \) into horizontal and vertical vectors. Since \( \angle AQ1B = \angle EQ1F = \theta_1 \) in Fig. 5, the values of the horizontal component \( \vec{B}_{Q1,H} \) and the vertical component \( \vec{B}_{Q1,V} \) of \( \vec{B}_{Q1,B} \) are

\[
\begin{align*}
\vec{B}_{Q1,H} &= \vec{B}_{Q1,B} \cdot \sin \theta_1, \\
\vec{B}_{Q1,V} &= \vec{B}_{Q1,B} \cdot \cos \theta_1.
\end{align*}
\]  

(10)

The values of the horizontal component \( \vec{B}_{Q1,E} \) and the vertical component \( \vec{B}_{Q1,G} \) of \( \vec{B}_{Q1,F} \) are

\[
\begin{align*}
\vec{B}_{Q1,E} &= \vec{B}_{Q1,F} \cdot \cos \theta_1, \\
\vec{B}_{Q1,G} &= \vec{B}_{Q1,F} \cdot \sin \theta_1.
\end{align*}
\]  

(11)

As shown in Fig. 5, the directions of \( \vec{B}_{Q1,A} \) and \( \vec{B}_{Q1,G} \) are opposite, and the directions of \( \vec{B}_{Q1,E} \) and \( \vec{B}_{Q1,D} \) are the same. Therefore, the values of \( \vec{B}_{Z1} \) and \( \vec{B}_{H1} \) are

\[
\begin{align*}
\vec{B}_{Z1} &= \left| \vec{B}_{Q1,A} - \vec{B}_{Q1,G} \right| = \frac{n_{\mu_0}I_0 R^2}{4r_1^3} \cdot (2 \cos^2 \theta_1 - \sin^2 \theta_1) e^{-\sqrt{\frac{\mu_0 \sigma}{\sigma}} \cdot r_1}, \\
\vec{B}_{H1} &= \left| \vec{B}_{Q1,B} + \vec{B}_{Q1,E} \right| = \frac{n_{\mu_0}I_0 R^2}{4r_1^3} \cdot 3 \cos \theta_1 \sin \theta_1 e^{-\sqrt{\frac{\mu_0 \sigma}{\sigma}} \cdot r_1}.
\end{align*}
\]  

(12)
Dividing the two equations in (12), we have
\[
\frac{|B_{Z1}|}{|B_{H1}|} = \frac{2 \cos^2 \theta_1 - \sin^2 \theta_1}{3 \cos \theta_1 \sin \theta_1}.
\] (13)

In (12) and (13), both \( |B_{Z1}| \) and \( |B_{H1}| \) are the positioning signal vectors at point \( Q_1 \). In (12), \(\mu_0 \) and \( \sigma \) in the two equations are the same. So, the \( e^{-\sqrt{\mu_0 \sigma}} \) in the two equations can be canceled out. In this way, (13) holds in non-uniform earth media. Similarly, (6) also holds.

From (13), we can see that there is a functional relationship between \( \theta_1 \) and \( \frac{|B_{Z1}|}{|B_{H1}|} \). Since \( |B_{Z1}| \) corresponds to the output of the vertical axis of the receiver \( Q_1 \) and \( |B_{H1}| \) corresponds to the sum vector of the output of the horizontal two axes, \( \frac{|B_{Z1}|}{|B_{H1}|} \) in (13) is a known quantity. So, we can determine the value of \( \theta_1 \) by (13). Let \( \frac{|B_{Z1}|}{|B_{H1}|} = s \), we have
\[
\tan \theta_1 = \frac{-3s \pm \sqrt{9s^2 + 8}}{2}.
\] (14)

As shown in Fig. 5, the value range of \( \theta_1 \) is \( [0, \pi / 2] \), so the “±” sign in (14) should be the “+” sign. At this point, the value of \( \theta_1 \) is
\[
\theta_1 = \arctan \left( \frac{-3s + \sqrt{9s^2 + 8}}{2} \right).
\] (15)

By substituting the length \( r_1 \) of \( \angle Q_1 OC \) and the angle \( \theta_1 \) of \( \angle Q_1 OC \) into (7), we can obtain the value \( z \) of the depth OC of the transmitter coil from the ground as
\[
z = \frac{r_1}{-3s + \sqrt{9s^2 + 8}}.
\] (16)

Therefore, the three-dimensional coordinates of the transmitter in the coordinate system \( Q_1 \) are \( \left( \frac{x_{Q1}y_{Q1}z_{Q1}}{x_{Q2}y_{Q2}-x_{Q1}y_{Q1}}, \frac{y_{Q1}z_{Q1}-y_{Q2}z_{Q2}}{x_{Q2}y_{Q2}-x_{Q1}y_{Q1}}, -z \right) \).

B. POSITIONING ERROR COMPARISON

The positioning errors of the vertical positioning methods based on the equations, the signal direction, path loss, and the optimization algorithm will be compared here. To make it easy to express, we simplify the names of the four positioning methods into the equation method (EM), the direction method (DM), the path method (PM), and the optimization method (OM).

In [20] and [21], the corresponding relationship between the magnetic field intensity and the receiver coordinate is established. The positioning results of the equation method are determined based on the relationship. However, as shown in (1), the equation method ignores the influence of skin effect on positioning accuracy. After penetrating the earth, the positioning signal strength at the receiver will be smaller than the theoretical strength calculated by the equation. In this way, the calculated transceiver distance will be greater than the actual distance. We should evaluate the positioning error caused by ignoring the skin effect.

As described in [22] and [23], the positioning results of the path method are determined by the signal strength. However, as shown in (2), the signal strength at the receiver is affected by the earth conductivity \( \sigma \), the transmitter coil radius \( R \), and the transmitter coil current \( I \). When the locations of the transceivers are changed, \( \sigma \) will change accordingly. Similarly, when the transmitter coil is made incorrectly, \( R \) will change accordingly, and \( I \) will decrease as the battery decays. The changes of \( \sigma \), \( R \), and \( I \) lead to changes in signal strength at the receiver, which may cause positioning errors. Since these changes often occur in the application scenario of through-the-earth positioning, we should evaluate the errors of the path method above in these changes.

As described in [26], and [27], the positioning results of the optimization method are determined by the iterative calculations. In the application scenario of through-the-earth positioning in this paper, the optimization algorithm needs to gradually increase the value of \( \theta_1 \) in (12) until \( \frac{|B_{Z1}|}{|B_{H1}|} \) of (12) is closest to the output of the receiver. The change of \( \theta_1 \) in each iterative calculation is set as 0.01°.

The error evaluation results are shown in Fig. 6. The real depth of the transmitter is 1000m. As shown in Fig. 6, with the increase of \( \sigma \), \( R \), and \( I \), the errors of the equation method, the path method, and the optimization method increase accordingly, while the errors of the direction method are always 0. Besides, since the signal attenuation caused by the skin effect is ignored, the initial error of the equation method is greater than that of the path method and the optimization method. Therefore, the direction method has stronger environmental adaptability and fault tolerance than the equation method, the path method, and the optimization method, which can effectively reduce the positioning error in practical application.

V. NOISE PROCESSING

The above positioning process does not consider the influence of geomagnetic noise on positioning accuracy. To improve...
reason, the frequency-point amplitude acquisition method
achieves noise reduction by collecting the amplitude of the
frequency domain amplitude of the positioning signal on
the spectrum. After the receiver on the ground collects the
through-the-earth positioning signal doped with geomagnetic
noise for a certain period (such as 10 minutes), this method
first converts the positioning signal from the time domain to
the frequency domain by Discrete Fourier Transform. Then,
by using the characteristic of the extremely narrow bandwidth
of the sinusoidal positioning signal in the frequency domain,
this method acquires the amplitude of the frequency point
where the positioning signal is located from the spectrum.
Finally, by applying the amplitude correspondence between
the frequency domain and time domain of the positioning
signal derived in this section, this method converts the
acquired frequency domain amplitude of the positioning
signal into the time domain amplitude. In the process of
noise reduction, only one of the many frequency points
in the spectrum is involved in the conversion, and only
the geomagnetic noise at this frequency point affects the
accuracy of the positioning signal converted to the time
domain. Therefore, this method eliminates the influence of
the geomagnetic noise at other frequency points on the time
domain positioning signal. Compared with the traditional
filter, the frequency-point amplitude acquisition method has
less computation and a better noise reduction effect.

The frequency-point amplitude acquisition method is
implemented as follows.

A. THE FREQUENCY-POINT AMPLITUDE ACQUISITION
METHOD
The frequency-point amplitude acquisition method consists
of three steps: Discrete Fourier Transform, frequency-point
amplitude acquisition, and frequency domain amplitude-time
domain amplitude conversion. The first two steps can be
easily achieved by using digital signal processing technology,
and will not be repeated here. In the third step, we need to
convert the frequency domain amplitude of the positioning
signal to the time domain amplitude.

The conversion process consists of two steps. First,
we establish and simplify the Discrete Fourier Transform of
the positioning signal under certain conditions, and establish
the corresponding relationship between the frequency domain
amplitude and the time domain amplitude of the positioning
signal. Then, we solve the time domain amplitude of the posi-
tioning signal according to the corresponding relationship.
The conversion process is as follows.

We first establish the Discrete Fourier Transform of
the positioning signal. According to (2), the time-domain
amplitude of the positioning signal at point \( Q_1 \) is
\[
A_1 = \frac{n_0 \mu_0 R^2 \cdot \left( e^{\frac{1}{2} - \frac{1}{2} \cos \theta_1 + e^{\frac{1}{2} \sin \theta_1} \right) \cdot e^{-\sqrt{\pi f} \cdot \mu_0 \sigma \cdot \frac{1}{4} r_1^3}}
\]
the frequency of the positioning signal is \( f \). Assuming that
the initial phase of the positioning signal is \( \phi_0 \), the sampling
frequency of the receiver is \( f_s \), and the total number of
sampling points is \( N \), then the positioning signal received by
the receiver at point $Q_1$ is

$$x(m) = A_f \cos \left( \frac{2\pi f_m}{f_s} + \phi_0 \right),$$

where $m$ is any one of the $N$ sampling points. The Discrete Fourier Transform of $x(m)$ at $f$ Hz is

$$X \left( e^{j2\pi f} \right) = \sum_{m=1}^{N} \left[ A_f \cos \left( \frac{2\pi f_m}{f_s} + \phi_0 \right) e^{-j\frac{2\pi f m}{f_s}} \right]$$

$$= \sum_{m=1}^{N} \left[ A_f / 2 \cdot \cos \phi_0 + \frac{A_f}{2} \cdot \cos \left( \frac{4\pi f m}{f_s} \right) \cdot \cos \phi_0 \right.$$  

$$- A_f / 2 \cdot \sin \left( \frac{4\pi f m}{f_s} \right) \cdot \sin \phi_0 - j \cdot \frac{A_f}{2} \cdot \sin \left( \frac{4\pi f m}{f_s} \right) \cdot \cos \phi_0$$  

$$+ j \cdot A_f / 2 \cdot \sin \phi_0 - j \cdot \frac{A_f}{2} \cdot \cos \left( \frac{4\pi f m}{f_s} \right) \cdot \sin \phi_0 \right]. \quad (18)$$

The discrete Fourier transform of (18) can be simplified under certain conditions. In (18), the four terms $(A_f / 2) \cos(4\pi f m / f_s) \cos \phi_0$, $-(A_f / 2) \sin(4\pi f m / f_s) \sin \phi_0$, $-j(A_f / 2) \sin(4\pi f m / f_s) \cos \phi_0$, and $-j(A_f / 2) \cos(4\pi f m / f_s) \sin \phi_0$ change periodically with $m$, and the sampling values of each term are opposite numbers. Therefore, when $f_s$ is even, $f_s \gg f$ and $N$ is an integer multiple of $f_s$, the sum of the consecutive $f_s$ samples of each item is 0. So, (18) can be simplified as

$$X \left( e^{j2\pi f} \right) = \frac{A_f N}{2} \cdot \cos \phi_0 + j \cdot \frac{A_f N}{2} \cdot \sin \phi_0. \quad (19)$$

We then establish the corresponding relationship between the frequency domain amplitude and the time domain amplitude of the positioning signal. We perform the modular operation on (19) and get the corresponding relationship between $|X \left( e^{j2\pi f} \right)|$ and $A_f$ as

$$|X \left( e^{j2\pi f} \right)| = \sqrt{\left( \frac{A_f N}{2} \cdot \cos \phi_0 \right)^2 + \left( \frac{A_f N}{2} \cdot \sin \phi_0 \right)^2}$$

$$= \frac{A_f N}{2}. \quad (20)$$

Therefore, the time domain amplitude of the positioning signal at point $Q_1$ is

$$A_f = \frac{2 |X \left( e^{j2\pi f} \right)|}{N}. \quad (21)$$

According to (21), the time domain amplitude $A_f$ of the through-the-earth positioning signal is $2 \sqrt{N}$ of the frequency domain amplitude $|X \left( e^{j2\pi f} \right)|$. By using (21), we convert the frequency-domain amplitude of the frequency point to the time-domain amplitude. However, the conversion in (21) does not consider the influence of geomagnetic noise.

To demonstrate the real noise reduction capability of the frequency-point amplitude acquisition method, we need to determine the output signal of the frequency-point amplitude acquisition method with geomagnetic noise.

We superimpose the noise $n_0(t)$ in the frequency range of 0.1 – 32Hz obtained from the China Geophysical Science Data Center and the China Earthquake Administration with the magnetic induction through-the-earth positioning signal of (17) [29], [30]. The superimposed positioning signal is

$$x_n(m) = A \cos \left( \frac{2\pi f m}{f_s} + \phi_0 \right) + n_0(t). \quad (22)$$

According to the steps of the frequency-point amplitude acquisition method, we collect the amplitude of the positioning signal at $f$ Hz. Assuming that the noise amplitude at $f$ Hz in the spectrum is $n_f$, the positioning signal strength at $f$ Hz is $|X \left( e^{j2\pi f} \right)| + n_f$. Then we convert the frequency domain amplitude of the collected positioning signal into time-domain amplitude. According to (21), the time-domain amplitude $A_f$ of the positioning signal output by the frequency-point amplitude acquisition method is

$$A_f = \frac{2 |X \left( e^{j2\pi f} \right)| + 2n_f}{N}. \quad (23)$$

where the amplitude of the positioning signal in $A_f$ is $2 \sqrt{N}$ of the frequency domain amplitude $|X \left( e^{j2\pi f} \right)|$, and the amplitude of the noise is $2n_f / N$. Since the amplitude of noise in (23) is inversely proportional to $N$, and the growth rate of the noise that is approximately randomly distributed on the spectrum is much smaller than $N$, the longer the signal acquisition time is, the smaller the amplitude of the noise will be. In the simulation, a 1-minute signal can achieve a good noise reduction effect. However, to provide more accurate location information for mine rescue, it is worth to take 10 minutes to achieve a more accurate through-the-earth positioning.

To verify the noise reduction performance of the frequency-point amplitude acquisition method proposed in this section, we will compare the noise reduction performance of this method with that of the narrow bandpass filter.

### B. NOISE REDUCTION PERFORMANCE COMPARISON

This section compares the noise reduction performance of the frequency-point amplitude acquisition method and the Kaiser window bandpass filter. The comparison of noise reduction performance is achieved by the mean value, deviation ratio, and variance of the signals output by the two noise reduction methods. The mean value is the average value of the signal amplitudes output by either of the two noise reduction methods several times. The deviation ratio is the ratio of the mean value deviating from the ideal amplitude of the signal, and the variance represents the degree of fluctuation of the signal amplitudes output by the two noise reduction methods several times.

In this section, we first configure the parameters of the Kaiser window bandpass filter, and then we define the calculations of mean value, deviation ratio, and variance by using (24) - (26). Finally, after using the above two noise reduction methods several times to obtain the noise-reduced positioning signals, we substitute these signals into (24) - (26) to compare the noise reduction performance.
We first configure the Kaiser window bandpass filter [31], [32]. We set the passband width of the Kaiser window bandpass filter to 1Hz, the transition band to 0.4 Hz, the passband ripple to 1 dB, the stopband minimum attenuation to 40 dB, and the center frequency of the filter to be set at the frequency of 1Hz to 15 Hz.

We then define the calculations of mean value $E(A_f)$, deviation ratio $P(f)$, and variance $D(A_f)$. The definitions of $E(A_f)$, $P(f)$, and $D(A_f)$ are respectively

$$E(A_f) = \frac{\sum_{i=1}^{M} A_{f_i}}{M}, \quad (24)$$

$$P(f) = \frac{E(A_f) - A_{Tf}}{A_{Tf}}, \quad (25)$$

and

$$D(A_f) = \frac{\sum_{i=1}^{M} (A_{f_i} - E(A_f))^2}{M}, \quad (26)$$

where $M = 11$ is the number of sets of positioning signals doped with geomagnetic noise for 10 minutes, the $A_{f_i}$ is the positioning signal output by each of the two noise reduction methods, $f$ is the frequency of the positioning signals, the value range of $f$ is $1-15$ Hz, and $i$ is the number of the current positioning signal in the $M$ groups of positioning signals, the value range of $i$ is 1-11.

We compare the noise reduction performance of the two noise reduction methods, finally. As shown in Fig. 8, the contents of the comparison are the mean value $E(A_f)$, the deviation ratio $P(f)$, and the variance $D(A_f)$ of the output $A_{f_i}$ of the two noise reduction methods. Figs. 8 (a) -6 (c) correspond to $E(A_f)$, $P(f)$, and $D(A_f)$, respectively. Fig. 8 (a) shows that, in the range of $1-15$ Hz, the mean values of the positioning signals output by the frequency-point amplitude acquisition method almost coincide with the ideal values of positioning signals. In contrast, there are several obvious deviations from the ideal values of positioning signals in the output results of the Kaiser window bandpass filter. Fig. 8 (b) further shows that the deviation ratio of the frequency-point amplitude acquisition method is generally smaller than the Kaiser window bandpass filter. In the frequency range of $5-15$ Hz, the mean deviation ratio of the two is 0.50% and 7.11%, respectively, and the former is 14 times smaller than the latter. Fig. 8 (c) shows that the variance of the positioning signal output by the Kaiser window bandpass filter is generally larger than the frequency-point amplitude acquisition method, the output result of the frequency-point amplitude acquisition method is more stable. So, we can conclude that the frequency-point amplitude acquisition method has better noise reduction performance than the Kaiser window bandpass filter.

VI. POSITIONING ERROR ASSESSMENT

We have proposed and analyzed a variety of positioning methods and noise reduction methods above. However, the above contents do not evaluate the positioning accuracy of the complete positioning scheme with the positioning methods and the noise reduction method.

To achieve high precision through-the-earth positioning, we will evaluate the positioning error of the positioning scheme.

Since this paper has proved that the vertical positioning method based on the signal direction has a smaller positioning error than the vertical positioning methods based on path loss and the optimization algorithm, we will not compare the latter two. This section will evaluate the positioning errors of the horizontal positioning method, the vertical positioning method based on the signal direction, the Kaiser window band-pass filter, and the frequency-point amplitude acquisition method.
The parameters of the positioning scheme evaluated in this section are as follows. Due to the limited underground space, the size of the transmitter antenna should not be too large. The maximum radius \( R \) of the transmitter coil should not exceed 2m. The number of turns \( n \) of the transmitter coil is set to 3300, and the effective value of the transmitter coil current \( I \) is 4A. Since the earth media usually do not contain ferromagnetic substances such as iron, cobalt, and nickel, the earth permeability is taken as the vacuum permeability \( \mu_0 = 4\pi \times 10^{-7} \text{H/m} \). The conductivity \( \sigma \) of the earth media is set to \( 5 \times 10^{-3} \text{S/m} \). The signal frequency \( f \) is set to 10 Hz.

**FIGURE 9.** Positioning errors of the through-the-earth positioning scheme.

Using the parameters provided above, we simulate the positioning errors of the positioning schemes in Fig. 9. The horizontal axis of Fig. 9 represents the distance \( Q_1 \) between the receiver \( Q_1 \) and the point \( C \) in Fig. 2. The vertical axis represents the positioning errors of the through-the-earth positioning schemes. The actual depth of the transmitter from the ground is 1000m. As shown in Fig. 9, the positioning errors of the positioning schemes increase as \( Q_1 \) increases, the positioning error of the Kaiser window bandpass filter is always greater than the frequency-point amplitude acquisition method. Therefore, reducing the distance between the transceivers and using the frequency-point amplitude acquisition method for noise reduction can improve the precision of the through-the-earth positioning. When the vertical distance between the transceivers is 1000m, the horizontal distance is less than 500m, and the frequency-point amplitude acquisition method is used for noise reduction, the errors of horizontal positioning and vertical positioning are both less than 2m.

**VII. CONCLUSION**

This paper proposes a high-precision magnetic induction through-the-earth positioning scheme that can achieve horizontal positioning and vertical positioning. The three-axis receivers used in the scheme can determine the direction and amplitude of the positioning signal at one time, which reduces the complexity of horizontal positioning. The proposed positioning methods based on the direction of the positioning signal vector reduces the influence of variable path loss of the earth media on positioning accuracy. The frequency-point amplitude acquisition method greatly reduces the influence of geomagnetic noise on positioning accuracy. Compared with the Kaiser window bandpass filter, the frequency-point amplitude acquisition method can achieve higher-precision through-the-earth positioning. The simulation results show that: when the vertical distance between the transceivers is 1000m, the horizontal distance is less than 500m, and the frequency-point amplitude acquisition method is used for noise reduction, the errors of horizontal positioning and vertical positioning are both less than 2m.

In addition, due to the relatively long transmission distance of the positioning signal [13], the scheme can also be used to find the signal source of the through-the-earth communication system, which provides conditions for further text and voice through-the-earth communication.

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