Heavy flavour mass corrections to the longitudinal and transverse cross sections in e⁺ e⁻-collisions

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Abstract

We present the heavy flavour mass corrections to the order $\alpha_s$ corrected longitudinal ($\sigma_L$) and transverse ($\sigma_T$) cross sections in $e^+ e^-$-collisions. Its effect on the value of the running coupling constant extracted from the longitudinal cross section is investigated. Furthermore we make a comparison between the size of these mass corrections and the magnitude of the order $\alpha_s^2$ contribution to $\sigma_T$ and $\sigma_L$ which has been recently calculated for massless quarks. Also studied will be the changes in the above quantities when the fixed pole mass scheme is replaced by the running mass approach.

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Experiments carried out at electron positron colliders like LEP and LSD have provided us with a wealth of information about the constants appearing in the standard model of the electroweak and strong interactions. One of them is the strong coupling constant $\alpha_s$ which can be extracted from various quantities. Examples are the hadronic width of the Z-boson, the total hadronic cross section, event shapes of jet distributions and jet rates (for a review on this subject see [2]). Another quantity from which $\alpha_s$ can be extracted is the longitudinal cross section $\sigma_L$ which is measured in the semi inclusive reaction

$$e^- + e^+ \rightarrow V \rightarrow P + \text{“}X\text{”}.$$  

(1)

Here $V$ represents the intermediate vector bosons $Z$ and $\gamma$ and “$X$” denotes any inclusive final state. Furthermore $P$ stands for a hadron which is produced via fragmentation by a quark or a gluon. As we will see below the perturbation series in QCD for $\sigma_L$ starts in order $\alpha_s$ provided the quark which couples to the vector boson is massless. Therefore this quantity will become very sensitive to the value of the strong coupling constant. Some time ago the perturbation series was only known up to order $\alpha_s$ (see [5], [6]). However it turned out that the result for the longitudinal cross section was much smaller than its experimental value measured at LEP [3, 4] which indicated that the higher order corrections may become rather large. The latter was confirmed by the second order corrections, computed very recently in [7], which amount to about 35% with respect to the lowest order contribution to $\sigma_L$. If the second order corrections are included the longitudinal cross section agrees now very well with experiment so that it can be used to extract $\alpha_s$ (see [8]). Until now $\sigma_L$ was only evaluated for massless quarks. This will be a correct approximation for the light quarks like $u, d, s$ and probably also for $c$ but it will certainly fail for the $b$-quark, leave alone for the top quark. Therefore one has to compute the mass corrections to this quantity at least up to first order in $\alpha_s$. This contribution will be presented in this paper and we discuss its effect on the value of $\alpha_s$ when extracted from experiment via $\sigma_L$.

The cross section corresponding to process (1) where the hadron $P$ emerges directly, or indirectly via the gluon, from the heavy quark $H$ is given by

$$\frac{d^2\sigma^{H,P}(x,Q^2)}{dx \, d \cos \theta} = \frac{3}{8} \left(1 + \cos^2 \theta \right) \frac{d\sigma_T^{H,P}(x,Q^2)}{dx} + \frac{3}{4} \sin^2 \theta \frac{d\sigma_L^{H,P}(x,Q^2)}{dx} + \frac{3}{4} \cos \theta \frac{d\sigma_A^{H,P}(x,Q^2)}{dx}.$$ 

(2)

From the equation above one can extract the transverse ($\sigma_T$), the longitudinal ($\sigma_L$) and the asymmetric ($\sigma_A$) cross sections. Further $\theta$ denotes the angle between the outgoing hadron $P$ and the incoming electron. The Bjørken scaling variable is defined by

$$x = \frac{2pq}{Q^2}, \quad q^2 = Q^2 > 0, \quad 0 < x \leq 1,$$ 

(3)

where the momenta $p$ and $q$ correspond with the outgoing hadron $P$ and the virtual vector boson ($Z, \gamma$) respectively. To obtain the total transverse and longitudinal cross sections one has to multiply Eq. (2) with $x$. After integration over $x$ and summation over all hadron species $P$ emerging from the heavy flavour $H$ one obtains

$$\sigma_k^H(Q^2) = \frac{1}{2} \sum_P \int_0^1 dx \, x \frac{d\sigma_k^{H,P}}{dx}(x,Q^2) \quad k = T, L.$$ 

(4)
The result above can be written as follows
\[
\sigma_k^H(Q^2) = \sigma_{VV}(Q^2)h_k^V(\rho) + \sigma_{AA}(Q^2)h_k^A(\rho) \quad \text{with} \quad \rho = \frac{4m^2}{Q^2},
\] (5)
where \(m\) denotes the heavy quark mass. The above definitions for the total cross sections should not be confused with the ones which are obtained by integrating \(d\sigma_k/dx\) without multiplication by \(x\). The heavy flavour contributions to the latter quantities were computed for the first time in [9] (see also [10]-[12]). The difference between these two definitions will be pointed out below. Finally the total cross section for heavy flavour production is given by
\[
\sigma_{\text{tot}}^H(Q^2) = \sigma_{T}^H(Q^2) + \sigma_{L}^H(Q^2).
\] (6)
In Eq. (3) the pointlike cross sections are defined by
\[
\sigma_{VV}(Q^2) = \frac{4\pi\alpha^2}{3Q^2} N \left[ e_\ell e_q^2 + \frac{2Q^2(Q^2 - M_Z^2)}{|Z(Q^2)|^2} e_\ell e_q C_{\ell} C_{V,q} \right] + \frac{(Q^2)^2}{|Z(Q^2)|^2} \left( C_{\ell}^2 + C_{A,\ell}^2 \right) C_{V,q}^2,
\] (7)
\[
\sigma_{AA}(Q^2) = \frac{4\pi\alpha^2}{3Q^2} N \left[ \frac{(Q^2)^2}{|Z(Q^2)|^2} \left( C_{\ell}^2 + C_{A,\ell}^2 \right) C_{A,q}^2 \right],
\] (8)
where \(N\) denotes the number of colours corresponding to the gauge group \(SU(N)\) (in the case of QCD one has \(N = 3\)). Furthermore we adopt for the Z-propagator the energy independent width approximation
\[
Z(Q^2) = Q^2 - M_Z^2 + iM_Z \Gamma_Z.
\] (9)
In Eqs. (5), (6) the charges of the lepton and the quark are denoted by \(e_\ell\) and \(e_q\) respectively and the electroweak constants are given by
\[
C_{A,\ell} = \frac{1}{2\sin 2\theta_W}, \quad C_{V,\ell} = -C_{A,\ell} (1 - 4\sin^2 \theta_W),
\]
\[
C_{A,u} = -C_{A,d} = -C_{A,\ell},
\]
\[
C_{V,u} = C_{A,\ell} (1 - \frac{8}{3}\sin^2 \theta_W), \quad C_{V,d} = -C_{A,\ell} (1 - \frac{4}{3}\sin^2 \theta_W),
\] (10)
The functions \(h_k^l\) \((k = T, L, l = v, a)\) in Eq. (3) can be obtained order by order in perturbative QCD from the singlet quark and the gluon coefficient functions in the following way
\[
h_k^l(\rho) = \int_{\sqrt{\rho}}^1 dx x C_{k,q}^{l,s}(x, \rho, Q^2/\mu^2) + \frac{1}{2} \int_0^{1-\rho} dx x C_{k,q}^{l,a}(x, \rho, Q^2/\mu^2).
\] (11)
Here \(\mu\) stands for the factorization as well as the renormalization scale. The result in Eq. (11) has been derived from the differential cross section \(d\sigma_k^H/dx\) (see e.g. [3]). The latter can be written as a convolution of the fragmentation densities \(D_i^P\) and the coefficient functions \(C_{k,i} (i = q, g)\) (see Eq. (2.4) in [14]). Because of the momentum sum rule satisfied by the fragmentation densities (see Eq. (2.9) in [3]) expression (11) follows immediately. Notice that the functions \(h_k^l\) differ from the functions \(f_k^l\) computed in [12] (see also the functions \(H_i (i = 2, 6)\) in [14]). The latter were derived from the first moment of the non-singlet quark coefficient function and they differ

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from the former which are given by the second moment of the singlet quark and gluon coefficient functions presented in Eq. (11). However because of Eq. (3) the sum of the transverse and longitudinal components of both functions have to lead to the same perturbation series of the total cross section so that one has the relation $h_T^l + h_L^l = f_T^l + f_L^l$ ($l = v, a$). The functions $h_k^l$ can be expanded in the strong coupling constant $\alpha_s$ as follows

$$h_k^l(\rho) = \sum_{n=0}^{\infty} \left( \frac{\alpha_s(\mu)}{4\pi} \right)^n h_k^{l,(n)}(\rho).$$  \hspace{1cm} (12)

The lowest order contributions corresponding to the Born reaction

$$V \to H + \bar{H},$$  \hspace{1cm} (13)

with $V = \gamma, Z$ are given by

$$h_T^{v,(0)}(\rho) = \sqrt{1 - \rho} \quad h_L^{v,(0)}(\rho) = \frac{\rho}{2}\sqrt{1 - \rho}$$

$$h_T^{a,(0)}(\rho) = (1 - \rho)^{3/2} \quad h_L^{a,(0)}(\rho) = 0,$$  \hspace{1cm} (14)

where $\rho$ is defined in Eq. (3). The next-to-leading order (NLO) contributions originate from the one-loop virtual corrections to reaction (13) and the gluon bremsstrahlungs process

$$V \to H + \bar{H} + g.$$  \hspace{1cm} (15)

We have computed the order $\alpha_s$ contributions to the quark $c_{k,q}^{t,S}$ and gluon $c_{k,g}^{t,S}$ coefficients functions for the case $m \neq 0$ in Eq. (11) and confirmed the result obtained in appendix A of [3]. Notice that the quark coefficient functions are derived from the process where the hadron $P$ emerges from the quark $H$. The gluon coefficient function corresponds with the process where the hadron is emitted by the gluon. After performing the integral in Eq. (11) we obtain the following results for $k = T, L$

$$h_T^{v,(1)}(\rho) = C_F \left[ \frac{1}{2} \rho(1 - 3\rho)F_1(t) + \rho^{3/2}(1 + \rho)F_2(t) + (32 - \frac{39}{2}\rho + \frac{21}{2}\rho^2)\text{Li}_2(t) 
+ (16 - 10\rho + 6\rho^2)F_3(t) + 2\sqrt{1 - \rho}F_4(t) + (8 - 6\rho + 6\rho^2)\ln(t)\ln(1 + t) 
+ (-12 + 3\rho - \frac{3}{2}\rho^2)\ln(t) - 5\rho\sqrt{1 - \rho} \right],$$  \hspace{1cm} (16)

$$h_L^{v,(1)}(\rho) = C_F \left[ -\frac{1}{2} \rho(1 - 3\rho)F_1(t) - \rho^{3/2}(1 + \rho)F_2(t) + \left(\frac{39}{2}\rho - \frac{37}{2}\rho^2\right)\text{Li}_2(t) 
+ 10\rho(1 - \rho)F_3(t) + \rho\sqrt{1 - \rho}F_4(t) + (6\rho - 8\rho^2)\ln(t)\ln(1 + t) 
+ (-\rho + \frac{13}{4}\rho^2)\ln(t) + (3 + \frac{19}{2}\rho)\sqrt{1 - \rho} \right],$$  \hspace{1cm} (17)

$$h_T^{a,(1)}(\rho) = C_F \left[ \frac{1}{2} \rho(1 - 4\rho)F_1(t) + \rho^{3/2}(1 + 2\rho)F_2(t) + (32 - \frac{103}{2}\rho + 30\rho^2)\text{Li}_2(t) 
+ (16 - 26\rho + 16\rho^2)F_3(t) + 2(1 - \rho)^{3/2}F_4(t) + (8 - 14\rho + 12\rho^2)\ln(t)\ln(1 + t) 
+ (-12 + 9\rho - \frac{9}{2}\rho^2 + \frac{3}{4}\rho^3)\ln(t) - (12\rho + \frac{3}{2}\rho^2)\sqrt{1 - \rho} \right],$$  \hspace{1cm} (18)

$$h_L^{a,(1)}(\rho) = C_F \left[ -\frac{1}{2} \rho(1 - 4\rho)(F_1(t) - 4F_3(t) - 7\text{Li}_2(t) - 4\ln(t)\ln(1 + t)) - \rho^{3/2}(1 + 2\rho)F_2(t) 
+ (2\rho + \frac{13}{4}\rho^2 - \frac{9}{8}\rho^3)\ln(t) + (3 + 3\rho + \frac{9}{4}\rho^2)\sqrt{1 - \rho} \right],$$  \hspace{1cm} (19)
with
\[ t = \frac{1 - \sqrt{1 - \rho}}{1 + \sqrt{1 - \rho}} \quad (20) \]
and the colour factor \( C_F \) is given by \( C_F = (N^2 - 1)/2N \). The functions \( F_i(t) \) appearing above are defined by
\[
F_1(t) = \text{Li}_2(t^3) + 4\zeta(2) + \frac{1}{2} \ln^2(t) + 3 \ln(t) \ln(1 + t + t^2) \\
F_2(t) = \text{Li}_2(-t^{3/2}) - \text{Li}_2(t^{3/2}) + \text{Li}_2(-t^{1/2}) - \text{Li}_2(t^{1/2}) + 3\zeta(2) + 2 \ln(t) \ln(1 + \sqrt{t}) \\
-2 \ln(t) \ln(1 - \sqrt{t}) + \frac{3}{2} \ln(t) \ln(1 + t - \sqrt{t}) - \frac{3}{2} \ln(t) \ln(1 + t + \sqrt{t}) \\
F_3(t) = \text{Li}_2(-t) + \ln(t) \ln(1 - t) \\
F_4(t) = 6 \ln(t) - 8 \ln(1 - t) - 4 \ln(1 + t), 
\]
where \( \zeta(n) \), which occurs in the formulae of this paper for \( n = 2, 3 \), represents the Riemann \( \zeta \)-function and \( \text{Li}_2(x) \) denotes the dilogarithm. Using Eqs. (16) - (19) one can check that the order \( \alpha_s \) contribution to the total cross section in (6) is in agreement with the literature [13]. The next-to-next-to-leading order (NNLO) contributions come from the following processes. First one has to calculate the two-loop vertex corrections to the Born process (13) and the one-loop corrections to (15). Second one has to add the radiative corrections due to the following reactions
\[
V \to H + \overline{H} + g + g \quad (25) \\
V \to H + \overline{H} + H + \overline{H} \quad (26) \\
V \to H + \overline{H} + q + \overline{q} \quad (27)
\]
where \( q \) denotes the light quark. The results for \( h_k^{(2)} \) presented below are only computed for those contributions containing Feynman graphs where the vector boson \( V \) is always coupled to the heavy quark \( H \) so that the light quarks can be only produced via fermion pair production emerging from a gluon splitting. Because we are only interested in the ratios
\[
R_k(Q^2) = \frac{\sigma_k^{H}(Q^2)}{\sigma_{\text{tot}}^{H}(Q^2)} \quad \text{for} \quad k = T, L 
\]
we do not consider other contributions which drop out in the expression above provided we put the mass of \( H \) equal to zero in the second order correction. The contributions which can be omitted are given by one- and two-loop vertex corrections which contain the triangular quark-loop graphs [13], [14]. They only show up if the quarks are massive and are coupled to the Z-boson via the axial-vector vertex. Notice that one has to sum over all members of one quark family in order to cancel the anomaly. Further we exclude all contributions from reaction (27) which involve Feynman graphs where the light quarks \( q \) are coupled to the vector boson \( V \). Notice that interference terms of the latter with diagrams where heavy quark are attached to the vector boson vanish if the heavy quark is taken to be massless provided one sums over all members in one family.

The order \( \alpha_s^2 \) contributions to the quark and gluon coefficient functions have been calculated in [7], [13]. Because of the complexity of the calculation of these functions the heavy quark mass
was taken to be zero. This approximation is good for the charm and bottom quark but not for the top quark as we will see below. Substituting these coefficient functions in the integrand of Eq. (11) (here \( \rho = 0 \)) we obtain

\[
\begin{align*}
\tilde{h}^{v(2)}_T &= h^{a(2)}_T = C_F^2 \left\{ 6 \right\} + C_AC_F \left\{ \frac{89}{15} - \frac{196}{5} \zeta(3) \right\} + n_f C_FT_f \left\{ \frac{8}{3} + 16\zeta(3) \right\} \\
\tilde{h}^{v(2)}_L &= h^{a(2)}_L = C_F^2 \left\{ -\frac{15}{2} \right\} + C_AC_F \left\{ -11 \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{2023}{30} - \frac{24}{5} \zeta(3) \right\} \\
&\quad + n_f C_FT_f \left\{ 4 \ln \left( \frac{Q^2}{\mu^2} \right) - \frac{74}{3} \right\}
\end{align*}
\]  

(29)

(30)

Where the colour factors are given by \( C_A = N \) and \( T_f = 1/2 \) (for \( C_F \) see below Eq. (20)). Further \( n_f \) denotes the number of light flavours which originate from process \( \tilde{f} \). Finally \( \mu \) appearing in the strong coupling constant \( \alpha_s \) and the logarithms in Eq. (30) represents the renormalization scale. Notice that the coefficient of the logarithm is proportional to the lowest order coefficient of the \( \beta \)-function. The logarithm does not appear in \( h^{l(2)}_T \) because \( h^{l(1)}_T = 0 \) for \( m = 0 \) and \( l = v, a \). Since \( m = 0 \) there is no distinction between \( h^{v(2)}_k \) and \( h^{a(2)}_k \) for \( k = T, L \) anymore unlike in the case of the first order corrections in Eqs. (16)-(19) where the heavy quark was taken to be massive. Furthermore one can check that substitution of Eqs. (29), (30) into Eq. (3) provides us with the order \( \alpha_s^2 \) contribution to the total cross section which is in agreement with the results obtained in [17]. When the quark is massless we get from Eqs. (4), (8) the following perturbation series for the ratio in Eq. (28) up to order \( \alpha_s^2 \)

\[
\begin{align*}
R^H_L(Q^2) &= \frac{\alpha_s(\mu)}{4\pi} C_F \left\{ 3 \right\} + \left( \frac{\alpha_s(\mu)}{4\pi} \right)^2 \left[ C_F \left\{ -\frac{33}{2} \right\} + C_AC_F \left\{ -11 \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{123}{2} - 44\zeta(3) \right\} \\
&\quad + n_f C_FT_f \left\{ 4 \ln \left( \frac{Q^2}{\mu^2} \right) - \frac{74}{3} \right\} \right], \\
R^H_T(Q^2) &= 1 - R^H_L(Q^2)
\end{align*}
\]  

(31)

(32)

We will now show how the results for the above ratios will be modified when the Born and the first order contribution are computed for massive quarks. Further we discuss the consequences of our findings for the extraction of \( \alpha_s \). In order to make the comparison between the massless and massive approach to Eq. (28) we have chosen the following parameters (see [18]). The electroweak constants are: \( M_Z = 91.187 \text{ GeV/c}^2, \Gamma_Z = 2.490 \text{ GeV/c}^2 \) and \( \sin^2 \theta_W = 0.23116 \). For the strong parameters we choose : \( \Lambda^{\overline{MS}}_5 = 237 \text{ MeV/c} \) \( n_f = 5 \) which implies \( \alpha_s(M_Z) = 0.119 \) (two-loop corrected running coupling constant). Further we take for the renormalization scale \( \mu = Q \) unless mentioned otherwise. Notice that we study \( R^H_k \) for \( H = c, b \) at the CM energy \( Q = M_Z \). For the heavy flavour masses the following values are adopted : \( m_c = 1.5 \text{ GeV/c}^2, m_b = 4.50 \text{ GeV/c}^2 \) and \( m_t = 173.8 \text{ GeV/c}^2 \). The results for the bottom quark can be found in table 1. A comparison between the first and second column reveals that on the Born level the difference between the massive and massless approach to \( R^b_T \) is very small (about two promille). For \( R^b_L \) it is more conspicuous but the correction due to mass effects, which equals 0.0014 for \( R^{(1)}_L \), is still much smaller than the order \( \alpha_s^2 \) contribution which amounts to 0.0121. The corrections due to mass effects are also smaller than the changes caused by a different choice of the renormalization scale. If we choose \( \mu = Q/2 \) or \( \mu = 2 Q \) one gets \( R^{(2)}_L = 0.0509 \) and \( R^{(2)}_L = 0.0461 \) respectively which differ by 0.0024 from the central value \( R^{(2)}_L = 0.0485 \). We also studied the effect of the running
The ratio $R_k = \sigma_k / \sigma_{\text{tot}}$ ($k = T, L$) for $b\bar{b}$-production.

Table 1: The ratio $R_k = \sigma_k / \sigma_{\text{tot}}$ ($k = T, L$) for $b\bar{b}$-production.

| $m_b = 0.0$ GeV/$c^2$ | $m_b = 4.50$ GeV/$c^2$ | $m_b(M_Z) = 2.80$ GeV/$c^2$ |
|------------------------|------------------------|------------------------|
| $R_T^{(0)}$            | 1.0                    | 0.999                  |
| $R_T^{(1)}$            | 0.962                  | 0.964                  |
| $R_T^{(2)}$            | 0.950                  | 0.952                  |
| $R_L^{(0)}$            | 0.0                    | 0.0016                 |
| $R_L^{(1)}$            | 0.0378                 | 0.0364                 |
| $R_L^{(2)}$            | 0.0499                 | 0.0485                 |

where $\mu$ stands for the mass renormalization scale for which we choose $\mu = Q$. Further we adopt the two-loop corrected running mass with the initial condition $m(\mu_0) = \mu_0$. Using the relation between the $\overline{MS}$-mass and the fixed pole mass, as is indicated by the first factor on the righthand side in Eq. (33), we have taken for bottom production $\mu_0 = 4.10$ GeV/$c^2$ which corresponds with a pole mass $m_b = 4.50$ GeV/$c^2$. This choice leads to $m_b(M_Z) = 2.80$ GeV/$c^2$ which is 5 % above the experimental value 2.67 GeV/$c^2$ measured at LEP [19]. The results are presented in the third column of table 1. Comparing the second with the third column we observe that the mass corrections decrease because the running mass is smaller than the fixed pole mass. It is now interesting to see how the mass terms contributing to $R_L$ affect the extraction of the running coupling constant at $\mu = Q = M_Z$. To that order we equate the mass corrected formula for $R_L$ with the massless result presented in Eq. (31). The latter yields $R_L^{(2)} = 0.0499$ for $\alpha_s = 0.119$ (see first column, third row of table 1). Using the same number for the massive expression of $R_L^{(2)}$ we obtain $\alpha_s = 0.122$ which amounts to an enhancement of 2.5 % for $m_b = 4.50$ GeV/$c^2$. In the case of a running mass i.e. $m_b(M_Z) = 2.80$ GeV/$c^2$ we get $\alpha_s = 0.120$ so that the enhancement becomes 1.0 %. However as we have said before these mass effects are smaller than those caused by a variation in the renormalization scale. Following the same procedure we equate $R_L$ at different values of $\mu$. Choosing $\mu = Q$ one infers from table 1 that $R_L^{(2)} = 0.0499$ for $\alpha_s = 0.119$. This value of $R_L^{(2)}$ can also be obtained at $\mu = Q/2$ but then one has to take $\alpha_s = 0.127$. If we choose $\mu = 2Q$ the result is $\alpha_s = 0.112$. Therefore the uncertainty is about 6% which is larger than the mass correction. The latter becomes even smaller when we also add the part coming from the bottom quark to the longitudinal cross section consisting of the contributions of the light quarks and the charm quark. Concerning the latter quark we want to remark that its mass is so small that the massive approach will become indistinguishable from the massless results. Since the mass effects up to the order $\alpha_s$ level are rather small even for the bottom quark we
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & $m_t = 0 \text{ GeV/c}^2$ & $m_t = 173.8 \text{ GeV/c}^2$ & $m_t(Q) = 153.5 \text{ GeV/c}^2$ \\
\hline
$R_T^{(0)}$ & 1.0 & 0.826 & 0.862 \\
$R_T^{(1)}$ & 0.970 & 0.828 & 0.834 \\
$R_T^{(2)}$ & 0.962 & 0.821 & 0.826 \\
\hline
$R_L^{(0)}$ & 0.0 & 0.175 & 0.138 \\
$R_L^{(1)}$ & 0.030 & 0.172 & 0.167 \\
$R_L^{(2)}$ & 0.038 & 0.180 & 0.174 \\
\hline
\end{tabular}
\caption{The ratio $R_k = \sigma_k/\sigma_{tot}$ ($k = T, L$) for $\bar{t}t$-production at $Q = 500 \text{ GeV/c}$.}
\end{table}

can assume that the second order corrections, derived for $m = 0$, also apply for $m \neq 0$ at least as long as $Q \gg m$. This will be correct at large collider energies for all quarks except for the top quark as we will see below.

For $\bar{t}t$ production we will study the mass effects up to order $\alpha_s$ at a CM energy $Q = 500 \text{ GeV/c}$. Here we have chosen $\mu_0 = 166.1 \text{ GeV/c}^2$ which corresponds with a fixed pole mass of $m_t = 173.8 \text{ GeV/c}^2$. This choice leads to $m_t(Q) = 153.5 \text{ GeV/c}^2$. From table 2 we infer that for this process the mass corrections are huge and they are much larger than the first and second order corrections. Therefore the numbers in the last row, presented for $R_T$ and $R_L$, are unreliable because they only hold if the mass corrections in the order $\alpha_s^2$ contributions can be neglected as was done in Eqs. (29), (30). This implies that for the top quark $R_T^{(2)}$ and $R_L^{(2)}$ have to be computed for $m \neq 0$ which will be an enormous enterprise. We also studied the effect of the running mass presented in the third column of table 2. Here the difference between the fixed pole mass and the running mass approach is larger than the one observed for the bottom quark in table 1. Another feature is that the order $\alpha_s$ correction increases when the running mass scheme is used and its sign is reversed with respect to the correction obtained for the fixed pole mass approach. Notice that a study of the change in $R_k$ ($k = T, L$) under variation of the renormalization scale makes no sense because of the missing exact result in second order for $m \neq 0$.

Summarizing our findings we have computed the mass corrections up order $\alpha_s$ for the transverse and longitudinal cross sections which are due to heavy flavours. In the case of charm there is no observable difference between the massless and massive approach. For the bottom quark we see a difference which leads to an enhancement of the extracted value of $\alpha_s$ which is of the order of 2.5 % (fixed pole mass $m_b = 4.50 \text{ GeV/c}^2$). or 1.0 % (running mass $m_b(M_Z) = 2.80 \text{ GeV/c}^2$). These numbers become much smaller when the light flavour contributions are added to the cross sections. A variation in the renormalization scale introduces larger effects on the value for $\alpha_s$ which are about 6.0 % (see e.g also [4]). In the case of top-quark production the mass terms cannot be neglected anymore since they are much larger than the second order corrections computed for massless quarks.

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