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Analysis, Control and Circuit Design of a Novel Chaotic System with Line Equilibrium

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Abstract. In this paper, we describe a new 3-D chaotic system with line equilibrium. A detailed analysis of the proposed chaotic system is provided with MATLAB phase portraits, equilibrium points, symmetry, invariance, Lyapunov exponents, Kaplan-Yorke dimension, bifurcation diagram and a Poincaré map. For specific values of the parameters, the proposed system displays chaotic behaviour. We apply adaptive control method for the global chaos synchronization of the new chaotic systems with unknown system parameters. In addition, a new circuit implementation of the new chaotic system is reported and examined in MultiSIM. A good qualitative agreement is shown between the simulations and the MultiSIM results.

1. Introduction

The first experimental model of a chaotic system was obtained by Lorenz [1], when he was modelling the atmospheric convection with a 3-D model in 1963. Lorenz system [1] is a classical chaotic system with a two-scroll chaotic attractor. Since this monumental work by Lorenz, chaos has been studied extensively and many chaotic systems have been developed in the literature. Both theoretical and experimental chaotic systems have been reported and their applications have been discussed as well.

In [2], Rössler constructed a seven-term chaotic system with just one quadratic nonlinearity. In [3-4], Rikitake discovered a new chaotic system to explain the irregular polarity switching of the earth’s geomagnetic field. A model to explain the irregular variability in the luminosity of stars was presented in 1976 [5] and this system is known as the Moore-Spiegel system. In [6], Rucklidge discovered a double convection chaotic model in a horizontal layer of Boussinesq fluid with lateral constraints.

In 1994, Sprott described 19 algebraically simple chaotic systems [7]. In 2000, Malasoma proposed the simplest dissipative jerk equation that is parity invariant [8]. In 2009, Sun and Sprott constructed a piecewise exponential jerk system [9]. In the last few years, several new chaotic systems with an infinite number of equilibrium points have been proposed [10-14]. Specifically, the works [10-14] propose chaotic systems with a line or curve of equilibrium points.

Chaos has been widely applied to many scientific disciplines such as physics [15], biology [16], ecology [17], economy [18], random bit generators [19], psychology [20], lasers [21], astronomy [22],...
chemical reactions [23], memristors [24], neural networks [25], robotics [26], encryption [27], secure communication systems [28-31], DC motor [32], chaotic masking [33], etc.

Synchronization of chaotic systems [34] deals with a pair of chaotic systems called the master and slave systems and the design goal is to synchronize the respective state trajectories of the two systems asymptotically with the help of a controller attached to the slave system.

The synchronization of chaotic systems is a challenging research problem as the state trajectories of chaotic systems are very sensitive to small changes in initial conditions [35]. Several methods have been proposed in the literature for the synchronization of chaotic systems [36-38].

In this paper, we describe a new 3-D chaotic system with line equilibrium. Our novel chaotic system has six terms with four quadratic nonlinearities. In Section 2, we first provide the dynamics and phase portraits of the new chaotic system. We also present a qualitative analysis of the new chaotic system with properties such as dissipativity, equilibrium points, symmetry, invariance, Lyapunov exponents, Kaplan-Yorke dimension, bifurcation diagram and a Poincarè map. In Section 3, we apply adaptive control method to derive new results for the global chaos synchronization of the new chaotic systems with unknown system parameters. In Section 4, we, a new circuit implementation of the new chaotic system is reported and examined in MultiSIM. A good qualitative agreement is shown between the simulations and the MultiSIM results. In Section 5, we give final conclusions of this work.

2. A new 3-D chaotic system

In this work, inspired by the method and structure proposed in [39-40], we announce a new 3-D chaotic system described by the nonlinear dynamics

\[
\begin{align*}
x &= y - xz - yz \\
y &= axz \\
z &= y^2 - bz
\end{align*}
\]  

(1)

where \( x, y, z \) are state variables and \( a, b \) are positive system parameters.

We shall show that the system (1) is chaotic when the system parameters take the values

\[
a = 2.8, \quad b = 0.6
\]  

(2)

For numerical simulations, we take the initial conditions as

\[
x(0) = 0.1, \quad y(0) = 0.1, \quad z(0) = 0.1
\]  

(3)

With the parameter values as in (2) and the initial conditions as in (3), the Lyapunov exponents of the new system (1) are calculated using Wolf algorithm [41] as

\[
L_1 = 0.1292, \quad L_2 = 0, \quad L_3 = -1.6192
\]  

(4)

This shows that the new 3-D system (1) is chaotic.

Also, the sum of the Lyapunov exponents of the system (1) is found as

\[
L_1 + L_2 + L_3 = -1.49 < 0
\]  

(5)

Hence, the system (1) is a dissipative chaotic system with a strange chaotic attractor.

The phase portraits of the new chaotic system (1) are shown in Figure 1. The Lyapunov exponents of the new chaotic system (1) are shown in Figure 2.

The Kaplan-Yorke dimension of the new chaotic system (1) is calculated as

\[
D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0798
\]  

(6)

Thus, the new chaotic system (1) has fractional dimension of \( D_{KY} \).
Figure 1. Phase portraits of the new chaotic system

Figure 2. Lyapunov exponents of the new chaotic system
The equilibrium points of the new chaotic system (1) are obtained by solving the following system

\[
\begin{align*}
    y - xz - yz &= 0 \\
    axz &= 0 \\
    y^2 - bz &= 0 
\end{align*}
\]

(7)

We take the parameter values as in the chaotic case (2).

A simple calculation shows that the new chaotic system (1) has a line of equilibrium points

\[ S = \{(x, y, z) \in \mathbb{R}^3 : y = 0, z = 0\} \]

(8)

which is the \( x \)-axis in \( \mathbb{R}^3 \).

It is easy to see that the new chaotic system (1) is invariant under the change of coordinates

\[ (x, y, z) \rightarrow (-x, -y, z) \]

(9)

This shows that the system (1) has rotation symmetry about the \( z \)-axis.

Also, we find that \( z \)-axis is invariant under the flow of the new chaotic system (1). This invariant motion is characterized by the 1-D dynamics

\[ z = -bz \quad (b > 0) \]

(10)

which is exponentially stable.

Next, to have a detailed view of the new chaotic system (1), the behaviour of the system with respect to the bifurcation parameter \( b \) is studied. The new chaotic system (1) displays the expected chaotic behavior for \( 0.5 \leq b \leq 0.84 \) and for \( 0.85 \leq b \leq 0.86 \) periodic behavior is noted in the system. Also, the new chaotic system has chaotic behavior for \( b > 0.86 \) as presented in Figure 3. Poincaré map of the system (1) is also illustrated in Figure 4, which exhibits that the new chaotic system (1) has complex dynamics.

![Figure 3. Bifurcation diagram of the new chaotic system (1) with \( b \) as varying parameter](image.png)
3. Adaptive synchronization of the new chaotic system

In this section, we apply adaptive control method [42-45] for the global chaos synchronization of identical new chaotic systems with unknown system parameters.

As the master system, we consider the new chaotic system dynamics

\begin{align}
    x_1 &= y_1 - x_1 z_1 - y_1 z_1 \\
    y_1 &= a x_1 z_1 \\
    z_1 &= y_1^2 - b z_1
\end{align}

(11)

where \( x_1, y_1, z_1 \) are the states and \( a, b \) are unknown system parameters.

As the slave system, we consider the controlled new chaotic system dynamics

\begin{align}
    x_2 &= y_2 - x_2 z_2 - y_2 z_2 + u_x \\
    y_2 &= a x_2 z_2 + u_y \\
    z_2 &= y_2^2 - b z_2 + u_z
\end{align}

(12)

where \( x_2, y_2, z_2 \) are the states and \( u_x, u_y, u_z \) are the adaptive controls to be designed.

We define the synchronization error as follows:

\begin{align}
    e_x &= x_2 - x_1 \\
    e_y &= y_2 - y_1 \\
    e_z &= z_2 - z_1
\end{align}

(13)
Then the error dynamics is obtained as
\begin{align}
  e_x &= e_y - x_2 z_2 - y_2 z_2 + x_1 z_1 + y_1 z_1 + u_x \\
  e_y &= a (x_2 z_2 - x_1 z_1) + u_y \\
  e_z &= -be_z + y_2^2 - y_1^2 + u_z \\
\end{align}  \tag{14}

We consider the adaptive control defined by
\begin{align}
  u_x &= -e_x + x_2 z_2 + y_2 z_2 - x_1 z_1 - y_1 z_1 - k_x e_x \\
  u_y &= -\hat{a}(t) (x_2 z_2 - x_1 z_1) - k_y e_y \\
  u_z &= \hat{b}(t) e_z - y_2^2 + y_1^2 - k_z e_z \\
\end{align}  \tag{15}

where $k_x, k_y, k_z$ are positive constants.

By substitution of (15) into (14), the following closed-loop system is obtained:
\begin{align}
  e_x &= -k_x e_x \\
  e_y &= [a - \hat{a}(t)] (x_2 z_2 - x_1 z_1) - k_y e_y \\
  e_z &= -[b - \hat{b}(t)] e_z - k_z e_z \\
\end{align}  \tag{16}

To simplify the notations, we define the parameter estimation errors as
\begin{align}
  e_a &= a - \hat{a}(t) \\
  e_b &= b - \hat{b}(t) \\
\end{align}  \tag{17}

Substituting (17) into (16), we obtain the closed-loop system as
\begin{align}
  e_x &= -k_x e_x \\
  e_y &= e_a (x_2 z_2 - x_1 z_1) - k_y e_y \\
  e_z &= -e_b e_z - k_z e_z \\
\end{align}  \tag{18}

Time-differentiation of (17) yields the differential equations
\begin{align}
  \dot{e}_a &= -\dot{a}(t) \\
  \dot{e}_b &= -\dot{b}(t) \\
\end{align}  \tag{19}

Next, we define a candidate Lyapunov function given by
\begin{align}
  V(e_x, e_y, e_z, e_a, e_b) &= \frac{1}{2} (e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2) \\
\end{align}  \tag{20}

Differentiating $V$ along the trajectories of (18) and (19), we obtain
\begin{align}
  \dot{V} &= -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a \left[ e_y (x_2 z_2 - x_1 z_1) - \dot{a} \right] + e_b \left[ -e_z^2 - \dot{b} \right] \\
\end{align}  \tag{21}

In view of (21), we consider the following update law for parameter estimates:
\begin{align}
  \dot{a} &= e_y (x_2 z_2 - x_1 z_1) \\
  \dot{b} &= -e_z^2 \\
\end{align}  \tag{22}

Next, we state and prove the main result of this section.

**Theorem 1.** The identical new chaotic systems (11) and (12) are globally and exponentially synchronized by the adaptive control law (15) and the parameter update law (22) for all initial conditions, where $k_x, k_y, k_z$ are positive constants.

**Proof.** We establish this result by using Lyapunov stability theory [46].
First, we note that the quadratic Lyapunov function $V$ defined by (20) is positive definite in $\mathbb{R}^5$. After substituting the parameter update law (22) into (21), we obtain the time-derivative of $V$ as

$$V = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2$$

which is negative semi-definite in $\mathbb{R}^5$.

Hence, by Barbafat's lemma [46], we conclude that the closed-loop synchronization error dynamics (18) is globally exponentially stable.

This completes the proof.

For numerical simulations, we take the parameters as in the chaotic case (2), i.e. $a = 2.8$ and $b = 0.6$. We take the initial values of the parameter estimates as $\hat{a}(0) = 6.4$ and $\hat{b}(0) = 9.5$.

Also, we take the gain constants as $k_x = k_y = k_z = 10$.

As the initial state of the master system (11), we take

$$x_1(0) = 8.4, \quad y_1(0) = 3.6, \quad z_1(0) = 12.4$$

As the initial state of the slave system (12), we take

$$x_2(0) = 17.5, \quad y_2(0) = 6.3, \quad z_2(0) = 4.2$$

Figures 5-7 show the complete synchronization of the new chaotic systems (10) and (11), while Figure 8 shows the time-history of the synchronization errors $e_x, e_y, e_z$.

![Figure 5](image-url)  

**Figure 5.** Complete synchronization of the states $x_1$ and $x_2$ of the new chaotic systems
Figure 6. Complete synchronization of the states $y_1$ and $y_2$ of the new chaotic systems

Figure 7. Complete synchronization of the states $z_1$ and $z_2$ of the new chaotic systems
4. Circuit realization of the new chaotic system

In this section, an electronic circuit which emulates the new chaotic system (1) is described to show its feasibility. The circuit design in Figure 9 has been described following an approach based on operational amplifiers [47-52], where the state variables $x, y, z$ of the new system (1) are associated with the voltages across the capacitors $C_1, C_2$ and $C_3$ respectively.

By applying Kirchhoff’s circuit laws, the corresponding circuit equations of the designed circuit can be written as

\[
\begin{align*}
\dot{x} &= \frac{1}{C_1 R_1} y - \frac{1}{C_1 R_2} y z - \frac{1}{10C_1 R_3} x z \\
\dot{y} &= \frac{1}{10C_2 R_4} x z \\
\dot{z} &= \frac{1}{10C_3 R_5} y^2 - \frac{1}{10C_3 R_6} z
\end{align*}
\]

We choose the values of the circuit elements as

\[
\begin{align*}
R_4 &= 14.285K\Omega, & R_5 &= 666.66K\Omega, & R_2 &= R_3 = R_6 &= 40K\Omega \\
R_1 &= R_7 = R_8 = R_9 = R_{10} &= 400K\Omega \\
C_1 &= C_2 = C_3 &= 1nF
\end{align*}
\]

The circuit has three integrators by using Op-amp TL082CD in a feedback loop and four multipliers IC AD633. The supplies of all active devices are ±15 volt. With MultiSIM 10.0, we obtain the experimental observations of new system (1) as shown in Figures 10-12. The agreement between the experimental phase portraits with MultiSIM simulation (Figs. 6-8) and the numerical simulation with MATLAB (Fig. 1) confirms the feasibility of our new chaotic system (1).
Figure 9 Schematic of the proposed new chaotic system by using MultiSIM

Figure 10 2-D projection of the new chaotic system on the $(x, y)$ plane
5. Conclusions
In this paper, we derived a new chaotic system with a line of equilibrium points. The dynamical behaviors of the modified Rucklidge chaotic system are analyzed, both analytically and numerically, including some basic dynamical properties, symmetry, invariance, phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, bifurcation diagram and Poincaré map. The designed circuit has been implemented and examined using the MultiSIM and numerical results using MATLAB. Comparison of the MultiSIM result and MATLAB simulations showed good qualitative agreement between the chaotic system and its circuitry realization.
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