Assessing the true role of coauthors in the 
\( h \)-index measure of an author scientific impact

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Abstract
A method based on the classical principal component analysis leads to demonstrate that the role of co-authors should give a \( h \)-index measure to a group leader higher than usually accepted. The method rather easily gives what is usually searched for, i.e. an estimate of the role (or "weight") of co-authors, as the additional value to an author papers’ popularity. The construction of the co-authorship popularity \( H \)-matrix is exemplified and the role of eigenvalues and the main eigenvector component are discussed. An example illustrates the points and serves as the basis for suggesting a generally practical application of the concept.

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1 Introduction
The \( h \)-index value of an author results from the counting of his/her quoted publications \cite{1}, ranked according to their popularity (the most quoted paper gets a rank \( r=1 \), etc.), and is obtained by the rank value \( (h=r) \) such that the papers above that rank \( (r \geq h) \) have less citations than \( h \). The \( h \)-index has been invented to quantify an author impact, though it is rather a measure of an author paper productivity and/or popularity \cite{2,3} - which maybe partially due to some paper content quality \cite{5} or to co-author fame \cite{6}.

It has been much discussed what publication is (or has to be) considered, when measuring \( h \). Sometimes book citations are not counted; sometimes, there is double counting, or sometimes two papers deposited on different websites are counted as different papers, or sometimes not; sometimes papers in proceedings are not (or sometimes are) counted as of equal value as those in classical

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peer-review journals, etc. Therefore, $h$ depends on the type of selection criteria and big data search engine: Google Scholar (Google), Web of Science (Thomson Reuters), Scopus (Elsevier) databases, etc. However, the following considerations apply to all search engine results.

Many variants have been proposed in order to remediate several so-called defects \cite{11-14} of the original $h$-measurement\footnote{see also http://sci2s.ugr.es/hindex/}. Sometimes self-citations have been scorned upon \cite{15}, sometimes not \cite{16}. Another, considered very important, criticism has been about the counting of coauthors and their role \cite{17-34}. It has been often argued that the number of quotations of a paper should be weighted according to the number of coauthors of this paper, thereby reducing the $h$-index of an author having many co-authors, as in the $profit(p) - index$ \cite{33}. It is true that sometimes team leaders are unaware of the papers they have published. Sometimes there is complaisance co-authorship as well \cite{36}.

The present paper has for main aims to propose a practical and basically sound (i.e., physics-prone) way of remeasuring the $h$-index (keeping the same "name" ($h$) and quasi-similar notations as in the "$h$-index" literature, for simplicity) of an author publishing with co-authors. It is argued that the original $h$-index, in fact, undervalues the role of co-authors. The following study and method, therefore, emphasize that the impact of a team leader, or more generally co-authorship, is underrepresented by the classical $h$-index.

The "theory arguments" seem to follow better from practical examples, in a deductive way rather than through an inductive presentation. The methodology idea is based on the principal component analysis (PCA) method which aims at reducing the dimensionality of a data set, consisting of a large number of interrelated variables, while retaining as much as possible of the variation present in the data set. This is achieved by transforming the raw data into a new set of measures, the principal components (PCs), which are uncorrelated, and which are ordered so that the first few retain most of the variations present in all of the original variables. Here, the data set is the $h$-index values for authors, but considering that they can have 1, 2, 3, ... co-authors, forming teams. For these teams, one can calculate also the corresponding $h$-index, in a usual way. This leads to write down a square matrix with dimensions equal to the number of considered co-authors. To calculate the eigenvalues and eigenvectors is next a classical matter. Then, the result leads the true measure of co-authorship popularity from the set of ranked papers of such co-authors. The example cases, illustrating the argument, are limited to a few co-authors, but could obviously be extended. Their finite size is wholly irrelevant; in fact, this allows a better comprehension of details. Other extensions are briefly discussed in the conclusion section.

2 Methodology

At the start, get the $h$-index ($h_{ij}$) for each authors ($i$), from some search engine, e.g., Google Scholar or Web of Science. The source is irrelevant, since it will
be seen that the method applies whatever the search engine. Of course, initial and final numbers will be different, but the discussion on whether some source is "better" than another is not part of the present development.

Next, reduce the publication list of the $i$ authors only to the joint papers by a couple of co-authors, e.g. $i$ and $j$, i.e. $N_{i,j}$. Thus, after ranking these papers, one easily obtains the equivalent of the $h$-index, i.e. an $h_{ij}$, for the couple of authors, from the list of $N_{i,j}$ papers. A warning: this list might include papers which have a "large" citation number, yet not large enough to have a rank lower than $h_{ii}$ for author $i$ (or $j$). Indeed, for the author $i$ a paper might not be often quoted, whence have a rank higher than $h_{ii}$, although such a number of citations might be important enough to have a rank lower than the $h$-index for the "couple of authors", i.e. $h_{ij}$.

There is no need to emphasize that the citation lists should be taken from the same data base, for coherence purpose. A practical point can be also mentioned. It is useful to cross-check the lists, i.e. repeating the procedure stating from $j$, and obtaining $h_{ji}$, thereby observing that one truly obtains $h_{ij} \equiv h_{ji}$.

Thereafter, define the co-authorship popularity $\mathcal{H}$-matrix, having $h_{ij}$ as its off-diagonal elements and has $h_{ii}$ on the diagonal. The order $i$ is irrelevant. However, for a discussion, it seems appropriate to rank the authors $i$ according to their $h_{ii}$ value. In so doing, $h_{ij}$ (or $h_{ji}$) $\leq h_{jj} \leq h_{ii}$. This matrix $\mathcal{H}$ differs from the co-occurrence matrices introduced in [37, 38] which only consider the frequency of partnerships.

Finally, calculate the eigenvalues and eigenvectors of $\mathcal{H}$. For emphasizing the partner weights, the lowest component of the eigenvector corresponding to the largest eigenvalue has always been imposed equal to 1.

3 A real case

Consider the following ($i = 1, 2, 3, 4$) co-authors: MAU, PCL, APE, and JPE, respectively, having worked in statistical mechanics independently, together, or with various co-authors. A few characteristics of their publication lists is given in Table 1. Next, take the whole publication list of each author, e.g. from Google Scholar, without any loss of generality for the argument.

For the present case, one obtains (six) 2x2 matrices; the same procedure is repeated for finding the matrix elements of the (four) 3x3 matrices, and for the unique 4x4 matrix. The number of joint papers is of course not increasing in this process. Recall that it seems convenient to order the authors ($i = 1, 2, 3, 4$) according to their $h$-index.

For space saving, the $\mathcal{H}$-matrices of this example are displayed below, with the same line, the relevant number of joint papers; the matrix eigenvalues, but only the (unnormalized) eigenvector components corresponding to the largest eigenvalue (designated by (1)) are also given here below. For immediately emphasizing the partner weights, the lowest component of this eigenvector is imposed to be equal to 1; also the index $i$ of the component refers to the author rather than to its usual order when writing a vector.
The 6 matrices emphasizing links between two authors, among the 4 considered, are

\[
h_{MAU, PCL} = \begin{pmatrix} 35 & 10 \\ 10 & 11 \end{pmatrix}; \quad N_{1,2} = 30;
\]
\[
=> \lambda_{1,2}^{(1)} = 38.620; \quad \lambda_{1,2}^{(2)} = 7.380; \quad x_1^{(1)} = 2.765 \quad x_2^{(1)} = 1
\]

\[
h_{MAU, APE} = \begin{pmatrix} 35 & 7 \\ 7 & 11 \end{pmatrix}; \quad N_{1,3} = 30;
\]
\[
=> \lambda_{1,3}^{(1)} = 36.827; \quad \lambda_{1,3}^{(2)} = 8.173; \quad x_1^{(1)} = 3.841 \quad x_2^{(1)} = 1
\]

\[
h_{MAU, JPE} = \begin{pmatrix} 35 & 7 \\ 7 & 10 \end{pmatrix}; \quad N_{1,4} = 30;
\]
\[
=> \lambda_{1,4}^{(1)} = 41.041; \quad \lambda_{1,4}^{(2)} = 7.380; \quad x_1^{(1)} = 2.765 \quad x_2^{(1)} = 1
\]

The 4 matrices emphasizing links between three authors, among the 4 considered, are

\[
h_{PCL, APE} = \begin{pmatrix} 11 & 6 \\ 6 & 10 \end{pmatrix}; \quad N_{2,3} = 8;
\]
\[
=> \lambda_{2,3}^{(1)} = 41.041; \quad \lambda_{2,3}^{(2)} = 7.380; \quad x_1^{(1)} = 2.765 \quad x_2^{(1)} = 1
\]

\[
h_{PCL, JPE} = \begin{pmatrix} 11 & 2 \\ 2 & 10 \end{pmatrix}; \quad N_{2,4} = 8;
\]
\[
=> \lambda_{2,4}^{(1)} = 41.041; \quad \lambda_{2,4}^{(2)} = 7.380; \quad x_1^{(1)} = 2.765 \quad x_2^{(1)} = 1
\]

\[
h_{APE, JPE} = \begin{pmatrix} 10 & 2 \\ 2 & 10 \end{pmatrix}; \quad N_{3,4} = 8;
\]
\[
=> \lambda_{3,4}^{(1)} = 41.041; \quad \lambda_{3,4}^{(2)} = 7.380; \quad x_1^{(1)} = 2.765 \quad x_2^{(1)} = 1
\]
\[
h_{1,2,4} = \begin{pmatrix}
35 & 10 & 2 \\
10 & 11 & 2 \\
2 & 2 & 2 \\
\end{pmatrix}; \quad N_{1,2,4} = 2; \\
\Rightarrow \lambda_{1,2,4}^{(1)} = 38.799; \quad \lambda_{1,2,4}^{(2)} = 7.626; \quad \lambda_{1,2,4}^{(3)} = 1.575; \\
x_1^{(1)} = 13.388; \quad x_2^{(1)} = 4.888; \quad x_4^{(1)} = 1
\]

\[
h_{1,3,4} = \begin{pmatrix}
35 & 7 & 2 \\
7 & 11 & 2 \\
2 & 2 & 2 \\
\end{pmatrix}; \quad N_{1,3,4} = 2; \\
\Rightarrow \lambda_{1,3,4}^{(1)} = 37.064; \quad \lambda_{1,3,4}^{(2)} = 9.369; \quad \lambda_{1,3,4}^{(3)} = 1.567; \\
x_1^{(1)} = 13.743; \quad x_3^{(1)} = 3.771; \quad x_4^{(1)} = 1
\]

\[
h_{2,3,4} = \begin{pmatrix}
11 & 6 & 2 \\
6 & 10 & 2 \\
2 & 2 & 2 \\
\end{pmatrix}; \quad N_{2,3,4} = 2; \\
\Rightarrow \lambda_{2,3,4}^{(1)} = 17.051; \quad \lambda_{2,3,4}^{(2)} = 4.848; \quad \lambda_{2,3,4}^{(3)} = 1.465; \\
x_2^{(1)} = 3.903; \quad x_3^{(1)} = 3.605; \quad x_4^{(1)} = 1
\]

The matrix emphasizing the links between the four authors is

\[
h_{1,2,3,4} = \begin{pmatrix}
35 & 10 & 7 & 2 \\
10 & 11 & 6 & 2 \\
7 & 6 & 10 & 2 \\
2 & 2 & 2 & 2 \\
\end{pmatrix}; \quad N_{1,2,3,4} = 2; \\
\Rightarrow \lambda_{1,2,3,4}^{(1)} = 41.277; \quad \lambda_{1,2,3,4}^{(2)} = 10.921; \quad \lambda_{1,2,3,4}^{(3)} = 4.339; \quad \lambda_{1,2,3,4}^{(4)} = 1.463; \\
x_1^{(1)} = 11.539; \quad x_2^{(1)} = 4.576; \quad x_3^{(1)} = 3.524; \quad x_4^{(1)} = 1.
\]

N.B. Those 4 authors have only 2 papers in common.

4 Case analysis and implications

It can be immediately observed that the (here called) "average h-index" for MAU, resulting from having co-authored papers at least with PCL or with APE or with JPE, leads to \( h >_{1}^{(1)} = (38.62 + 38.83 + 35.12) / 3 = 37.52 \), instead of \( h >_{1}^{(1)} (\equiv h_{11}) = 35 \).

In the same line of thought, consider the (average) h-index for MAU resulting from having co-authored papers at least with PCL and with APE or MAU with PCL and with JPE or MAU with APE and with JPE. It easily found that \( h >_{1}^{(1)} = (41.04 + 39.80 + 37.06) / 3 = 39.30 \).
Table 1: Productivity characteristics of the 4 co-authors considered in the text

| co-authors i: | MAU | PCL | APE | JPE |
|---------------|-----|-----|-----|-----|
| $h_{i}$       | 35  | 11  | 10  | 2   |
| N. citations of most cited paper | 152 | 127 | 37  | 7   |
| N. citations till $h$ | 1113 | 296 | 224 | 14  |
| N. coauthors  | 317 | 32  | 46  | 4   |
| N. papers with "best" coauthor | 155 | 30  | 21  | 2   |
| N. publications (<2012) | 571 | 34  | 111 | 2   |

The effective $h$-index value can be calculated, in a similar manner, for another author, e.g. APE, due to his partnership in this particular 4-member team. It is easily obtained that $< h >^{(2)}_3 = (17.05 + 10.62 + 7.626)/3 = 11.77$, instead of APE $h_{22} = 11$.

Thus, one can emphasize that $< h >^{(1)}_4 = \lambda^{(1)}_{1,2,3,4} = 41.277$ is not some "average", but is the truly effective value for MAU due to publishing (and being quoted) when participating in this group of 4 co-authors.

Moreover, the largest principal component is also giving some relevant information on the relative impact of a co-author. It is sufficient to normalize the vector components indeed and consider the absolute weights. For example, for MAU in the 4-member team, the largest PC is found equal to $11.539/\sqrt{(11.539^2 + 4.576^2 + 3.524^2 + 1)} \sim 89\%$. This results in the effective $h$ due to team partnership being equal to $(41.277 \times 0.89) \approx 36.80$; in contrast to the raw value 35 which is not taking into account various co-authorships.

The output due to the eigenvector components as indicators and measures of the respective weight gains and losses is postposed to Sect. 5.2 for better emphasis of the method interest.

Note that the argument on the proportionality factor can be applied to each level or participation, considering sub-groups of co-authors.

5 Two other cases

A reviewer of the initial version of this paper claimed that the conclusion is based on the use of a very small sample of 4 authors, which in addition is not arbitrary since it includes the author himself. So contrary to the author claim that he uses "an arbitrarily selected example, but without loss of generality", he is using a specific sample not at all arbitrarily (sic). At least a few more cases should be treated to provide a more solid basis to the idea.

Therefore, two other cases are outlined, though without going through the complete details as above.
5.1 Extended "real case"

Consider two other authors, \( (i = 5) \) JMK and \( (i = 6) \) DAH, having worked with the previous 4 authors, but not all of them. Therefore a few off-diagonal elements are necessarily vanishing. N.B. Those 6 authors have no paper in common: \( N_{1,.6} = 0 \). Moreover, for comparison with the above, the authors have not been ranked according to their \( h^- \) index when writing the matrix emphasizing the links between the six authors

\[
h_{1,...,6} = \begin{pmatrix}
35 & 10 & 7 & 2 & 3 & 2 \\
10 & 11 & 6 & 2 & 3 & 2 \\
7 & 6 & 10 & 2 & 3 & 0 \\
2 & 2 & 2 & 2 & 1 & 0 \\
3 & 3 & 3 & 1 & 9 & 2 \\
2 & 2 & 0 & 0 & 2 & 17
\end{pmatrix};
\]

\( \Rightarrow \lambda_{1,...,6}^{(1)} = 42.232; \lambda_{1,...,6}^{(2)} = 17.207; \lambda_{1,...,6}^{(3)} = 12.262; \lambda_{1,...,6}^{(4)} = 6.658; \lambda_{1,...,6}^{(5)} = 4.188; \lambda_{1,...,6}^{(6)} = 1.452, \)

with

\[x_1^{(1)} = 11.167; \quad x_2^{(1)} = 4.577; \quad x_3^{(1)} = 3.513; \quad x_4^{(1)} = 1.0; \quad x_5^{(1)} = 1.859; \quad x_6^{(1)} = 1.397.\]

It is noticed that the main author weight goes down from 11.539 to 11.167. This is due to the influence of the sixth author which has a \( h_{6,6} \), in contrast to the 2nd author who has a smaller \( h_{2,2} = 11 \). Therefore the method allows to test the loss (or gain, in the previous section) of some author's influence due to some co-author.

5.2 Shortened "real case"

Indications on the weight gains and losses are best described on a "shortened case", i.e. when the number of components of the eigenvectors, or the matrix rank, is small. In contrast to a 6 member team, consider the real case with three (for concision) authors like MD, SG and AP who all have a rather high \( h_{i,i} \) (as
of Oct. 2014) such that the popularity matrix reads

\[
h_{MD,SG,AP} = \begin{pmatrix}
35 & 3 & 8 \\
3 & 30 & 0 \\
8 & 0 & 12
\end{pmatrix};
\]

\[
=> \lambda^{(1)}_{MD,SG,AP} = 38.479; \quad \lambda^{(2)}_{MD,SG,AP} = 29.070; \quad \lambda^{(3)}_{MD,SG,AP} = 9.451;
\]

In this (real) case SG and AP have no paper in common. However, both other couples (MD, SG) and (MD,AP) have a few papers in common, although they are often cited, whence the relatively small \(h_{12}\) and \(h_{13}\). It is of interest to write the three (normalized) eigenvectors:

\[
x^{(1)} = (0.303; -0.907; 0.293);
\]

\[
x^{(2)} = (-0.044; -0.321; 0.946);
\]

\[
x^{(3)} = (-0.952; -0.274; 0.137).
\]

Two points can be made as a brief conclusion of this subsection. On one hand, as before, the "highest h-" author, i.e., \(i = 1\) gains from the other two, \(h_{11} = 35 \rightarrow 38.5\), but the second (or middle one, in this case) does not lose much \(h_{22} = 30 \rightarrow 29\) in having no publication, whence of course no citation, with the third partner. On the other hand, it is well seen that the eigenvectors indicate and measure the respective weight gains and losses.

6 Conclusions

In summary, the above indicates that the effect of co-authors on evaluating the popularity of an author through the \(h\)-index method can be investigated through a principal component analysis method. Through an arbitrarily selected example, but without loss of generality, it has been proved that the \(h\)-index undervalues the role of the team, in particular on the team "leader". Two other cases have served to indicate that the method can be applied in larger or smaller samples. It is found that an effective \(h\)-index can be calculated from the co-authorship popularity \(H\)-matrix eigenvalues, through the selection of team partners, but also up to the whole team size.

It has been remarked that the co-authorship popularity \(H\)-matrix is sensitive to the size of the team and to the own \(h\)-index of the various members, - as should be expected, but also on the joint \(h\)-index of co-author couples. The relative weight is therefore nicely measured when imagining team members influence, e.g. on a finished project. There would be no need for a posteriori (and previously rather non objectively) asking the weight of a co-author, as it is often done in some dossier evaluation (see also comments on such a consideration in the Appendix).

An interesting application would occur when the ranking of teams with a leader is necessary in fund raising processes, but also in hiring and promotion processes when the team partnership capacity of an author has to be quantified.
The above demonstration seems easily applied through any web search engine, if, e.g., evaluation committees wish to consider a specific search engine.

The approach seems much more realistic and valuable than the classical $h$-index and several variants, - the more so nowadays in presence of co-authorship inflation. The method, based on a standard physics approach, is fundamentally different from Schreiber’s $h_m$ [24] and the many variant (fractional [30], normalized [39], ...) estimates of the $h$-index (see Appendix).

In that respect, a final note about the construction of the co-authorship popularity $H$-matrix seems of interest. The usual co-authorship network considerations suggest that one could examine a not-necessarily symmetric matrix, but evaluate the $h_{ij}$ elements, taking into account the order of authors, i.e. $h_{ij} \neq h_{ji}$. This would not change much the eigenvalues, but would modify the eigenvectors and the relative weights. However, it is well known that the order of authors obeys different criteria, with different justifications, in different scientific fields [40]. In the main text, the position of the co-author in the list has not been a criterion. Nevertheless, this consideration could be easily implemented.

In conclusion, it can be claimed that the PCA method rather easily gives what is searched for, i.e. an estimate of the role of co-authors, as the additional weighted value to the measure of an author research paper’s popularity within the $h$–index scheme.

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Appendix

This Appendix follows a remark by a reviewer: In addition, the author claims that "The method, based on a standard physics approach, is fundamentally different from Schreiber’s $h_m$ [24] and the many variant (fractional [30], normalized [39], ...) estimates of the $h$-index." without proving it. Indeed, he should compare those different methods using the same sample.

The point is of interest, but the reviewer is misunderstanding the main point. There is no need for a long proof to show that something is obviously different: to any reader, the present method should appear to be (completely) different from these weighting the number of citations through the number of authors, in the cited references. The latter methods in fine lead to a decrease of the apparent $h$-index of the "team leader", while the present approach shows that its value should be considered to be increased instead, (because co-authors unduly take some part of the popularity value of the team leader)!
Nevertheless, discussion of the fractional aspect being much related to the present work, values of the fractional $h_{ii}$ index, according to [24] and [30] can be calculated as an illustration. However, the comparison with all the normalized indices introduced in [39] or others discussed by Schreiber [12] is not going to be made here in order to keep the size of the appendix at a reasonable length. On the other hand, it seems sufficient, for the present point, only to consider the data of the Sect. 5.2 case.

In summary, Schreiber [24] proposes to count each paper citation only fractionally according to the inverse of the number of authors. In doing so, the following results $h_{11} = 22$, $h_{22} = 25$, $h_{33} = 5$ are obtained for the Sect. 5.2 case. Note here that SG ($i = 2$) has rarely more than one co-author and is much publishing single author papers, whence his fractional $h$-index remains very high. Taking the off-diagonal element fractionalized as well into account, one reaches $\lambda_{MD,SG,AP}^{(1)} = 27.206$; $\lambda_{MD,SG,AP}^{(2)} = 21.185$; $\lambda_{MD,SG,AP}^{(3)} = 3.609$, all obviously smaller than in the non-fractionalized case, but also showing a similar evolution toward an increase of the "leader value" with respect to the other authors.

In a more complicated way, Galam’s tailor based allocation (TBA) [30] gives a set of weight possibilities, - although without providing an optimum one. The weight of an author is (in short) related to his/her position in the co-author list, but is (expectedly and supposedly) decided by the co-authors. In order to get a finite size appendix here, a specific constraint has to be selected for finding the fractionalized $h_{ij}$. In order to contrast with the uniform distribution in Schreiber’s approach, a practically admitted constraint can be implemented for this illustration. The choice of the weight given to an author at position $p$ of a given $q$ authors paper, i.e., $g(p,q)$, is hereby made similar to that used in evaluating rules at FNRS (Fonds National de la Recherche Scientifique) in Belgium. Let the value of a paper be $2q$. For a paper with two authors, each one gets the same weight ($1/q =50\%$). Otherwise, the first author gets 50\% of the weight; the last author gets 25\%, the rest being equally divided between the other authors. It is obvious that the weight of the "middle list" co-authors gets quickly small when their number increases. Implementing such a rule leads in the Sect. 5.2 case to $h_{11} = 20$, $h_{22} = 24$, $h_{33} = 6$. Taking the off-diagonal element, fractionalized as well, into account, one reaches $\lambda_{MD,SG,AP}^{(1)} = 24.339$; $\lambda_{MD,SG,AP}^{(2)} = 18.300$; $\lambda_{MD,SG,AP}^{(3)} = 5.362$. Again, observe the strong effect of publishing with only a few co-authors and taking the first or last place in the list.

It is worth as a conclusion to quote from [30]: the TBA rescaling disadvantages senior authors who usually sit last with several co-authors .... (while) ... a low citation paper does contribute mainly to first and last authors. It could be added that according to the here called FNRS rule or constraint implemented as in the TBA, it is not very interesting to be one among many co-authors, - the more so if the paper is not often cited, and if the position in the list is toward the middle. Fortunately, the present approach corrects a little bit the drastic rule. Nevertheless, this indicates that the a priori choice of the $g(p,q)$ weight also much influences the final values, as expected, - but the index evolution is
again the same: the "leader" gets a higher $h_{ii}$ value to the detriment of his/her colleagues.

Notice that as in [24], the present method does not need a rearrangement of the citation records in contrast to the fractional methods, and ... it is not sensitive to extreme values of the number of co-authors ... cannot decrease when the number of citations increases, and ... its construction does not push highly cited papers out of the core.

In conclusion, it seems obvious that these approaches ([24], [30], [39], ...) giving an a priori weight according to the position in a co-author list are different from the present one.

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