Interference phenomenon and geometric phase for Dirac neutrino in $\pi^+$ decay

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We analyze the geometric phase in the neutrino oscillation phenomenon, which follows the pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$. Its value $\pi$ is consistent with the present-day global analysis of the Standard Model neutrino oscillation parameters, accounting for the nonzero value of $\theta_{13}$. The impact of the charge-parity (CP) violating phase $\delta$, the neutrino’s nature, and the new physics is discussed.

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I. Introduction. The aim of this brief paper is to discuss the idea that in measurement subtleties of the neutrino oscillation phenomenon, geometrical properties reflected in the geometric phase of the oscillating flavor neutrino are important. In Ref. [1], it was proposed that the production and detection of the neutrino shall be treated as the split-beam experiment in the energy space. In the present paper, we consider the muon neutrino which is produced in the decay of pion to muon and the Dirac neutrino, namely $\pi^+ \rightarrow \mu^+ + \nu_\mu$ [2]. The flavor neutrino state $|\nu_\mu\rangle$ is a superposition of the stationary states $|\nu_i\rangle_\lambda \equiv |p, \lambda, \lambda \rangle$ of the definite masses $m_i$, $i = 1, 2, 3$, helicities $\lambda = -1$ or $+1$, and four-momentum $p$. When the new physics (NP) interactions are included, this superposition composes the mixed state $|\bar{\nu}_\mu\rangle$. Thus, the flavor neutrino, here $\nu_\mu$, represents the beam of three massive states, which split at the moment of production of the $\alpha = \mu$-flavor superposition, propagate, and finally at the distance $L$, interfere in the detector in the $\beta$-flavor interference pattern. This interference experiment for the neutrino proposed in [1] and discussed in [1, 2] in two flavor neutrino cases, allows us to test the dependence of the type [3] of the Aharonov-Anandan geometric phase (GP) [2] on the particular field theory model to which this paper is devoted.

The global analysis of neutrino oscillation parameters [5] shows the discrepancy in the data for the atmospheric neutrino mixing angle $\theta_{23}$. For the normal neutrino mass ordering (and we will use this one), the profile of the $\Delta \chi^2$ test statistics has two almost equally deep minima—the “local minimum” (lm) for the solar plus reactor long-baseline and accelerator long-baseline neutrino experiments, with new data from the $\nu_\mu$ and $\bar{\nu}_\mu$ channels included, and the “global minimum” (gm), which includes data from atmospheric neutrinos, too. The profile is practically symmetric and the preference (if any, see [6]) of gm (with $\sin^2 \theta_{23} = 0.427$) over lm (with $\sin^2 \theta_{23} = 0.613$) is very weak as the difference of $\Delta \chi^2$ in these minima is equal to 0.02 [8]. The $2\sigma$ range (0.38, 0.66) covers both of them. The experimental reason is that $\theta_{23}$ strongly depends on the $CP$ violating phase $\delta$ [9], whose $1\sigma$ range is $(0, 2\pi)$ [8]. For further discussion of this problem, see [10, 11]. It will appear that the mean $\sin^2 \theta_{23} \approx 0.517$ is the robust one. The central values of the other oscillation parameters are $\sin^2 \theta_{12} = 0.320$, $\sin^2 \theta_{13} = 0.0246$, $\delta = 0.80\pi$, $\Delta m^2_{21} = 7.62 \times 10^{-5} eV^2$ and $\Delta m^2_{31} = 2.55 \times 10^{-3} eV^2$.

A. Muon neutrino density matrix: From the $\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay experiments [2], we know that the fraction of the right-handed $N_{\nu_{\mu R}}$ to the left-handed $N_{\nu_{\mu L}}$ neutrinos fulfills the constraint [12, 13]

$$N_{\nu_{\mu L}} / N_{\nu_{\mu R}} < 0.002.$$  (1)

Let us assume that the pion decays effectively both in the left ($L$) and right ($R$) chiral charge current (CC) interactions [4] via the exchange of the Standard Model ($\nu$SM) $W$ boson only. Then, at the $W$-boson energy scale, the $R$ and $L$ chiral pion decay constants [14] are equal [13]. Moreover, the pseudoscalar correction to the pion hadronic matrix element can be neglected due to its smallness [15]. Then the invariant amplitudes $A_i^{\mu \lambda \nu_\mu}(p)$ in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ are related as follows [3, 4],

$$|A_i^{\mu \lambda \nu_\mu}(p)|^2 = |A_i^{\mu \lambda \nu_\mu}(p)|^2 = \frac{|\varepsilon_R|^2 |U_{\mu \nu_\mu}|^2}{|U_{L \nu_\mu}|^2}.$$  (2)

Here, $U_{\alpha \nu_\mu}^L$ and $U_{\alpha \nu_\mu}^R$ are the $L$ and $R$ chiral neutrino mixing matrices, which enter into the $CP$ Lagrangian in the products with the coupling constants $\varepsilon_L$ and $\varepsilon_R$, respectively [4]. The NP values of $\varepsilon_L$ and $\varepsilon_R$ can deviate slightly from the $\nu$SM values 1 and 0, respectively. However, the Fermi constant constraint $\varepsilon_L^2 + \varepsilon_R^2 = 1$ should hold.

Under the above conditions, in the process of neutrino production (P) the nonzero neutrino density matrix elements in the mass-helicity basis $|\nu_i\rangle_\lambda$ and in the center-of-mass (CM) frame are as follows [3, 4]:

$$\vartheta_{-1; -1}^{P; i; i} = \frac{\varepsilon_L^2 |U_{\mu \nu_\mu}^L|^2}{|U_{\mu \nu_\mu}^L|^2 + |\varepsilon_L|^2}, \vartheta_{-1; -1}^{P; i; i} = \frac{|U_{\mu \nu_\mu}^R|^2 |U_{\mu \nu_\mu}^L|^2}{|U_{\mu \nu_\mu}^R|^2 + |\varepsilon_L|^2},$$  (3)

constituting the muon neutrino $6 \times 6$-dimensional block diagonal density matrix $\rho_P^{\mu \nu} = \text{diag}(\vartheta_{-1; -1}^{P; i; i} \vartheta_{-1; -1}^{P; i; i})$ with two $3 \times 3$ matrices given in [3]. Here we choose $U_{\alpha \nu_\mu}^L = U_{\alpha \nu_\mu}^R = U_{\alpha \nu_\mu}$, where $U$ is the Maki-Nakagawa-Sakata neutrino mixing matrix [17], as the full statistical analysis of this hypothesis is beyond the data accessible.
in the present-day experiments. Using (1) we obtain the bound on the ratio $|\varepsilon_R/\varepsilon_L| < 0.0447$. It constrains the density matrix $\rho^\nu_{i+1;+1}$ of the initial neutrino. Its evolution and the effective Hamiltonian during the neutrino propagation are described in the next section.

Next, for the neutrino energy $E_\nu > 100$ MeV, the neutrino is in practice the relativistic particle. Hence the effect of the helicity Wigner rotation is negligible \cite{3} and the result for the density matrix in the laboratory ($L$) frame is $\rho^L_{\nu}(p_L) = \rho^L(p)$. Finally, only the neutrino which is produced in the $L$ frame in the forward direction along the $z$ axis reaches the detector and we choose this axis as the quantization one.

II. Evolution of the density matrix. Under the requirement of the nondissipative homogeneous medium, the Liouville-von Neumann equation governs the density matrix evolution. Thus, in the ultrarelativistic case, where the distance and the propagation time approach the relation $z = t$, the evolution rule for the neutrino density matrix is as follows:

$$\rho^\nu(t = 0) \rightarrow \rho^\nu(t) = e^{-i \mathcal{H} t} \rho^\nu(t = 0) e^{i \mathcal{H} t}, \quad (4)$$

where $\rho^\nu$ is an initial density matrix \cite{3} and $\mathcal{H}$ is the effective Hamiltonian.

With three massive and two helicity neutrino states, the effective Hamiltonian $\mathcal{H}$ has the $6 \times 6$-dimensional representation. In the case of the axial-vector interactions only, the effective Hamiltonian $\mathcal{H}$ can be considered as block diagonal with two $3 \times 3$ matrices,

$$\mathcal{H} = \mathcal{M} + \text{diag}(\mathcal{H}_{--}, \mathcal{H}_{++}). \quad (5)$$

Here $\mathcal{M} = \text{diag}(E_0^1, E_0^2, E_0^3, E_1^0, E_2^0, E_3^0)$ with $E_i^0 = E_\nu + m_i^2/2E_\nu$ ($i = 1, 2, 3$) is the mass term, where $E_\nu$ is the energy for the massless neutrino \cite{2}. The interaction Hamiltonians for the coherent Dirac neutrino scattering inside unpolarized matter read \cite{2, 18}

$$(\mathcal{H}^{--})_{ij} = \sqrt{2} G_F N_e |\varepsilon_L|^2 U_{ei}^{L*} U_{ej}^{L} \quad (6)$$

$$(\mathcal{H}^{++})_{ij} = \sqrt{2} G_F \left\{N_e |\varepsilon_R|^2 U_{ei}^{R*} U_{ej}^{R} - \frac{\rho}{2} N_n \varepsilon_R^{\nu} \Omega_{ij}^{R} \right\},$$

where $N_e$ and $N_n$ stand for the number of background electrons ($e$) and neutrons ($n$) per unit volume, respectively and $g \approx 1$. Small NP deviations of the neutral coupling constant for background particles are also neglected. We choose the right chiral neutral current ($NC$) coupling constant equal to $|\varepsilon_R^{\nu}| = 1$ can be obtained from the analysis of the charge-parity-time reversal ($CPT$) symmetry violation in the neutrino oscillation survival events \cite{19}. In the analysis we assume that the relevant $\nu$SM and NP coupling constants are real.

III. Analysis of geometric phase. Various types of geometric phases have been studied for a long time in physical systems ranging from classical mechanics to high-energy physics \cite{20}. There are also examples of exploiting the notion of geometric phases in neutrino physics. Let us mention a few of them. In \cite{21}, it was shown that in the neutrino oscillations analysis, carried out under adiabatic conditions \cite{4}, the nonzero Berry phase \cite{22} appears in the $\nu$SM if a background consists of at least two varying densities. The case of the three-level neutrino systems was considered in \cite{23}. In \cite{1} it was shown that the Pancharatnam phase \cite{24}, which defines the relative phases between states in the Hilbert space, leads in two-flavor neutrino oscillation to the topological phase of the interference term, which is equal to zero or $\pi$ for the survival and appearance probability, respectively.

In the present case, the neutrino $\nu_\mu$ is produced in the $\pi^+$ decay and propagates in the ordinary matter of the crust (with the density $\rho = 2.6$ g/cm$^3$). It reaches the detector after one oscillation period, i.e. at the maximum of the survival transition rate $P(\nu_\mu \rightarrow \nu_\mu)$. If the detector lies at the distance $L = 800$ km which is the baseline for the NOvA–Low-Z Calorimeter experiment \cite{2}, it happens for $E_\nu = 0.803$ GeV (what matters is the ratio $L/E_\nu$). For the central value of the $CP$ violating phase $\delta = 0.80 \pi$ of the $U$-matrix, we obtain $P(\nu_\mu \rightarrow \nu_\mu) \approx 0.992$ (g.m) or $P(\nu_\mu \rightarrow \nu_\mu) \approx 0.991$ (l.m). For perfect cyclicity $P(\nu_\mu \rightarrow \nu_\mu) = 1$. Hence the evolution is not exactly cyclic. Another measure of the deviation from perfect cyclicity is the trace distance between initial state at $t = 0$ and the state at time $t = L$ \cite{23},

$$D = \frac{1}{2} ||\rho^\nu(t = 0) - \rho^\nu(t = L)||, \quad (7)$$

where the norm $||\rho|| = \text{Tr} \sqrt{\rho^\dagger \rho}$. For perfectly cyclic evolution, $D = 0$. The calculations show that depending on $\delta$ and at the central values of other parameters \cite{8}, the trace distance $D \in (0.012, 0.092)$ (g.m) with the minimum for $\delta = 0$ and maximum for $\delta = \pi$ (the cases when $CP$ is not violated). For $\delta = 0.80$ the minimal value $D = 0.089$ (g.m) is at $L = 800$ km, which is the period of the oscillation. The same is true for the “local minimum” \cite{8}. The deviation from the perfect cyclicity is due to the fact that the neutrino flavor state is a three-state system and is not an eigenvector of the effective Hamiltonian governing its propagation.

In this paper, we exploit the kinematic approach to the geometric phase \cite{8} which can be applied to arbitrary (also nonunitary and/or noncyclic) quantum evolution. It possesses the following fundamental features \cite{8}: it is gauge invariant, purification independent, and it reduces to well establish results in the limit of unitary evolution. This approach has already been utilized in \cite{8} for the two-flavor neutrino system both for nondissipative and dissipative cases.
In order to analyze the GP, it is convenient to present the density matrix \( \rho^a(t) \) in the spectral-decomposition form

\[
\rho^a(t) = \sum_{i=1}^{6} \lambda_i^a(t) |u_i^a(t)\rangle \langle u_i^a(t)|,
\]

where \( \lambda_i^a(t) \) and \( |u_i^a(t)\rangle \) are the eigenvalues and eigenvectors of the matrix \( \rho^a(t) \). Then the geometric phase \( \Phi^a(t) \) at time \( t \) associated with such an evolution is defined by the following relation \[\Box\]:

\[
\Phi^a(t) = \text{Arg} \left\{ \sum_{i=1}^{6} \lambda_i^a(0) |u_i^a(t)\rangle \langle u_i^a(0)| \right\}^{1/2} \langle u_i^a(0)|u_i^a(t)\rangle \times \exp(-\int_0^t \langle u_i^a(s)|\dot{u}_i^a(s)\rangle ds),
\]

where \( \text{Arg}[Z] \) denotes argument of the complex number \( Z \), \( \langle u_i^a(s)|\dot{u}_i^a(s)\rangle \) is a scalar product, and the dot indicates the derivative with respect to time \( s \). It is natural to analyze the GP at time \( t = L \), which corresponds to the period of neutrino oscillations. Below, we study the GP at this time and use the notation \( \Phi = \Phi^a(t = L) \).

In \[\Box\] it was assumed that neutrino oscillation realizes a kind of interference experiment, and under this assumption it was proven that in the two-flavor case, the topological phase of the interference term is reflected in the orthogonality of the mixing matrix. In the present paper, it is suggested that because this interference experiment reflects the orthogonality of the neutrino mixing matrix, the GP takes the topological value \( \pi \) (the correction from the CP violating phase \( \delta \) will appear very tiny). This value of GP influences self-consistently the parameters of the mixing matrix.

A. Geometric phase in \( \nu \)SM: Because of the mentioned discrepancy in the data, the analysis of the GP given by Eq. \( \Box \) is for \( \nu \)SM performed for \( bm \) and \( gm \) \[\Box\]. The results are presented in Fig. 1. The GP for the central values of \( bm \) and \( gm \) are equal to \( \Phi^{bm} = 1.1917 \pi \) and \( \Phi^{gm} = 0.8301 \pi \), respectively. The bottom line is plotted for \( \sin^2 \theta_{23} = 0.461 \) for \((+1\sigma \text{ bound } bm)\) and the upper one for \( \sin^2 \theta_{23} = 0.573 \) for \((-1\sigma \text{ bound } gm)\). We notice that (with other oscillation parameters fixed) \( \Phi \) changes linearly as the function of \( \sin^2 \theta_{13} \), where \( \theta_{13} \) is the third mixing angle of \( U \) \[\Box\]. Two examples of the GP solution with \( \Phi = \pi \) are pointed out, the first one for \( \sin^2 \theta_{23} = 0.517 \) \((sl)\) and the second one for \( \sin^2 \theta_{23} = 0.514 \) \((s2)\). The former value, \( \sin^2 \theta_{23} = 0.517 \), is the arithmetic mean of the \(+1\sigma \text{ bound } 0.461 \) for \( bm \) and \(-1\sigma \text{ bound } 0.573 \) for \( gm \). With this value, the change of \( \Phi/\pi \) is significantly weaker. That is, the change of \( \sin^2 \theta_{13} \) is approximately equal to \( 3 \times 10^{-4} \), \( 10^{-5} \), and \( 3 \times 10^{-6} \), respectively. The small dependence on the \( CP \)-violating phase \( \delta \) is presented in Fig. 2. Its impact is of the order of \( 10^{-4} \). The numerical calculations show that in \( \nu \)SM with the period \( L = 800 \text{ km} \) the GP takes the topological values \( \Phi^a = n \pi \text{ (mod } 2\pi) \), \( n \in N \) (up to the influence of the phase \( \delta \)).

![FIG. 1: The geometric phase \( \Phi \equiv \Phi^a(t = L) \) vs \( \sin^2 \theta_{13} \) plotted for the value of \( \sin^2 \theta_{23} = 0.461 \) \((+1\sigma \text{ bound } bm)\) and \( \sin^2 \theta_{23} = 0.573 \) \((-1\sigma \text{ bound } gm)\).](image)

![FIG. 2: The geometric phase difference \( \Delta \Phi = \Phi_{NP} - \Phi_{\nu \text{SM}} \) as a function of the NP coupling constant \( \varepsilon_R \). Each curve corresponds to one \( \delta \) and one NP value of \( \varepsilon_R \). As the reference level the \( \nu \)SM \( s2 \) solution (see Fig. 1) is taken for which \( \Phi_{NP} = \pi \) \((\text{up to the influence of } \delta = 0.80\pi \text{ equal to } -4.09 \times 10^{-4} \pi)\).](image)

Interestingly, for the old \( \nu \)SM global analysis \[\Box\] with the central value \( \sin^2 \theta_{13} = 0.010 \) (but when the non-zero value of \( \theta_{13} \) was still disputed), the GP analysis had suggested that the condition \( \Phi = \pi \) requires \( \sin^2 \theta_{13} \)
to be enlarged approximately to 0.0175, which value was then inside ±1σ limits, or alternatively that sin²θ_{23} shall be diminished from 0.51 to 0.506 [26].

**B. Geometric phase in NP:** The bounds on the CC and NC right-chiral coupling constants ε_{R} and ε_{R}^{Nν} are given in the Introduction. In Fig. 3, the difference ΔΦ = Φ_{NP} − Φ_{SM} between NP and SM values of Φ as the function of ε_{R} is depicted. Each curve corresponds to the different value of the phase δ. The upper impact of ε_{R} on Φ/π is of 10−6 order. Even weaker is the influence of ε_{R}^{Nν}. Yet, because it enters linearly into the Hamiltonian [13], it therefore depends on the ε_{R}^{Nν} sign, too.

Finally, let us comment on the Majorana neutrino case. In the case of νSM, there is no difference between GP for the Dirac and Majorana neutrinos [5]. In the case of NP, the difference Δ^{M−D}Φ = Φ^{M} − Φ^{D} of the geometric phases Φ^{M} and Φ^{D} for the Majorana neutrino and Dirac neutrino is depicted in Fig. 3 as a function of the NP couplings [18]. The impact of ε_{R}^{Nν} and ε_{R} on Φ/π is of order 10−4 and 10−5−10−6, respectively.

**IV. Conclusions.** With this brief paper we have shown that selected properties of the nonadiabatic noncyclic flavor neutrino oscillation can be analyzed in terms of the type of the Aharonov-Andanian GP introduced in [3]. At first, using the trace distance D, it has been checked that in one oscillation period, the muon neutrino state performs the evolution along the path in its Hilbert space, which shows some small departure from cyclicity. Hence the solid angle encircled in this space is close to 2π (similar to the spin particle moving in the mesoscopic ring [27]). This motivates the use of the kinematic approach to the geometric phase presented in [3] which attaches the geometric phase to the Pancharatnam relative one. As mentioned above, the described pattern of the interference in the energy space of the massive neutrino states is highly possible [1, 3]. This in [1] enables us to use the Pancharatnam relative phase for the explanation of the orthogonality of the two-flavor mixing matrix. In [3], the behavior of the GP attached to it was analyzed. In this paper it is pointed out that the present-day global analysis of the oscillation parameters [5] is consistent with the GP value equal to π, which is the reflection of both the unitarity of the mixing matrix and the values of its experimentally estimated parameters. The GP is sensitive to changes of sin²θ_{23} and sin²θ_{13} (see Fig. 4), currently the more disputable parameters [5], whereas the influence of the other νSM oscillation parameters is approximately of the relative order 10−6−10−4 in their 2σ ranges. The NP corrections connected with the right-chiral CC and NC currents are at most of the relative order of 10−6, being at present far beyond the experimental verification. Recent progress in entirely novel experimental techniques makes the verification of presented findings more realistic in the future. In the long term, our research may provide new tools for analysis of neutrino physics.

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