$K \to \pi\nu\bar{\nu}$ and new physics in the neutrinos

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Abstract. We discuss generic effects of new physics on the rare decay modes $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ from the point of view of the neutrino sector. We pay particular attention to the cases of right handed neutrino couplings and neutrino lepton flavour violating interactions. The first of these examples requires the existence of a new light (sterile) right-handed neutrino and its contribution to both rates is always additive to the standard model. It is motivated as a possible solution to the so called charged B anomalies. The case of neutrino lepton flavour violating couplings produces interesting constraints on new physics which are competitive with those from standard searches for charged lepton flavour violating in kaons and in some cases better than those from tau decay.

1. Introduction
The standard model predictions for the modes $K \to \pi\nu\bar{\nu}$ are unusually clean so that precise comparisons to experiment are eventually expected. The charged mode was measured by E787/949 which found $B(K^+ \to \pi^+\nu\bar{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$[1]. This $1\sigma$ range of this result covers the standard model prediction, but also allows for new physics at a rate just over three times higher than the standard model. The NA62 experiment is expected to increase the precision of this measurement and at this conference a new upper limit from the observation of 3 events was presented, $B(K^+ \to \pi^+\nu\bar{\nu}) \leq 1.85 \times 10^{-10}$ at 95%CL [2]. At the same time the KOTO experiment has published a 90% CL upper limit of $B(K_L \to \pi^0\nu\bar{\nu}) \leq 3.0 \times 10^{-9}$[3], still far from the expected standard model rate. These results leave an ample window for probing new physics and many models have been constructed with predictions that fall in this window.

In this talk we will concentrate on extensions of the standard model that always increase the two rates because they rely on the existence of new neutrino final states [4, 5], and therefore do not interfere with the standard model, as is schematically shown in Figure 1. The first case is the possible existence of a new light sterile neutrino [6], motivated as a potential explanation for the charged B anomalies in $R(D)$ [7, 8, 9] and $R(D^*)$ [7, 8, 9, 10, 11, 12]. These quantities are defined as $R(D^{(*)}) = B(B \to D^{(*)}\ell^-\nu)/B(B \to D^{(*)}\ell^-\nu)$, $\ell^- = \mu^-, e^-$ and are sensitive to deviations from lepton universality in the tau couplings. The second possibility consists of final states with neutrinos of different flavour. This is motivated by the renewed interest in searches for charged lepton flavour violation, because the gauge symmetry of the standard model connects the two types of processes.

2. Light sterile neutrino
In this case there is a new light neutrino that couples to quarks through a $Z'$ gauge boson, for example. The coupling to quarks is model dependent and can proceed at both tree and one loop
Figure 1. Schematic showing additional possibilities for unobserved final state neutrino pairs: light sterile neutrinos, or pairs with different flavour.

levels but is constrained by $B_s - \bar{B}_s$ mixing measurements [13]. In general we can parametrise the new effective interaction as

$$H_{eff} = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi s_W} V_{ts}^* V_{td} \frac{1}{2} \bar{s} \gamma_\mu d \left( X_t \sum_\ell \bar{\nu}_\ell \gamma_\mu P_L \nu_\ell + \bar{X} \bar{\nu}_R \gamma_\mu P_R \nu_R \right).$$

The first term is the top-quark contribution of the standard model and serves to calibrate the strength and phase on the new term. Since the interaction with the sterile neutrino is right-handed (and we assume it is very light), there is no interference with the standard model and it results in an increase over the standard model branching ratios given by

$$B_{K^+}(\nu_{RH}) = B_{K^+}(SM) + \frac{\kappa_+}{3} \left| \lambda_t \bar{X} \right|^2,$$

$$B_{K_L}(\nu_{RH}) = B_{K_L}(SM) + \frac{\kappa_L}{3} \left( \text{Im} \lambda_t \bar{X} \right)^2.$$ (2)

Unlike other extensions of the standard model that are commonly discussed in the context of these modes, models with this additional neutrino are typically not constrained by $\epsilon'$ [14]. This is because the couplings of the $Z'$ are non-universal.

The range of rates that can be covered by this scenario is illustrated in the left panel of Figure 2. The green, blue, and red lines correspond respectively to $\bar{X}$ being real (so the new physics has the same phase as the standard model), having a phase opposite to that of $\lambda_t$ (so the new physics contributes only to the charged mode), and having a phase complementary to that of $\lambda_t$ which maximises the neutral rate. The shaded pink area is accessible in this scenario with $|\bar{X}| \leq 5.5$. For reference the green shaded region marks the BNL E787/E949 result, the dotted line marks the 90\%CL upper bound reported by NA62 in this conference and the shaded grey area is the Grossman-Nir [15] excluded region. Finally, the purple ellipse is the 1\% standard model prediction obtained with the latest input in CKMfitter [16].

3. Lepton flavour violation

The effective interaction for kaon decays into neutrino pairs of different flavours can be conveniently written as

$$H_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi s_W} \bar{s} \gamma_\mu d \left( \sum_\ell V_{ts}^* V_{td} X_t \bar{\nu}_\ell \gamma_\mu P_L \nu_\ell + \lambda^5 \sum_{i \neq j} W_{ij} \bar{\nu}_i \gamma_\mu P_L \nu_j \right).$$

The first term is again the standard model top-quark contribution (to same flavour neutrino pair) and is included for comparison. The second term is normalised so that $W \sim O(1)$.
Figure 2. Left panel: in pink the rate window that can occur from an interaction as in Eq. (1), see the main text for details. Right panel: a specific model that can produce this interaction can explain the $R(D)$-$R(D^*)$ measurements with a $W'$ of mass near a TeV.

is approximately the same size as the top standard model contribution. Unlike the sterile neutrino case however, we have not included the standard model phase in the normalisation. It is important to point out that when this new physics is generated by say lepto-quark exchange, it will also be accompanied by a corresponding lepton flavour conserving term. Here we ignore that term as it can be subsumed in previous studies of new physics with left-handed neutrinos.

Under these assumptions, the new physics in this scenario does not interfere with the standard model so the range of rates that can be reached is the same as it was for the sterile neutrino case, as can be seen in the left panel of Figure 3. The different shaded regions and dashed line have the same meaning as in Figure 2. The results for the rates

$$B_{K^+}(\nu_{RH}) = B_{K^+}(SM) + \frac{\bar{\kappa}_+}{3} \sum_{i \neq j} |W_{ij}|^2,$$

$$B_{K_L}(\nu_{RH}) = B_{K_L}(SM) + \frac{\kappa}{3} \sum_{i \neq j} \left| \frac{(W_{ij} - W_{ji})}{2} \right|^2,$$

(4)

show that the conditions under which there is no contribution to the neutral mode are slightly different than usual. Unlike other cases where they reduce to there being no CP violation, in this case they are $W_{ij} = W_{ji}^*$. Again we observe that it is not possible to go over the Grossman-Nir bound with this scenario.

As mentioned above, this scenario is related to searches for charged lepton flavour violation if the new physics respects the gauge symmetries of the standard model. In this case this just means that the effective theory at the electroweak scale is written in terms of left-handed lepton doublets, and this connects the two types of processes, e.g.,

$$l = \left( \nu_{\tilde{e}}^+ \right), \quad \mathcal{L} \supset c_4 \ \bar{s}_{\gamma}{\mu}P_{\text{RL}} \ \bar{t}_1 \ \gamma_{\mu}P_{\text{L}}l_2 + \text{h.c.},$$

(5)

is then responsible for decays such as $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\nu_{\mu}\bar{\nu}_{\mu}$ and $\Omega^- \to \Xi^- \mu^+\mu^-\bar{\nu}_{\mu}$.

The complete effective Lagrangian at dimension six that changes strangeness by one unit and violates lepton flavour has six possible structures (with different lepton flavour permutations)

$$\mathcal{L}_{NP} = \frac{1}{\Lambda_{NP}^2} \left( \sum_{k=1}^6 c_k^{ijxy} Q_k^{ijxy} + (c_6^{ijxy} Q_6^{ijxy} + \text{h.c.}) \right),$$

(6)
where
\[
Q_{ijxy}^{1} = \bar{\nu}_{i} \gamma \eta q_{j} \bar{\nu}_{x} \gamma \eta l_{y}, \quad Q_{ijxy}^{2} = \bar{\nu}_{i} \gamma \eta q_{j} \bar{\nu}_{x} \gamma \eta l_{y}, \quad Q_{ijxy}^{3} = \bar{\nu}_{i} \gamma \eta q_{j} \bar{\nu}_{x} \gamma \eta l_{y}, \quad Q_{ijxy}^{4} = \bar{\nu}_{i} \gamma \eta q_{j} \bar{\nu}_{x} \gamma \eta l_{y}, \quad Q_{ijxy}^{5} = \bar{\nu}_{i} \gamma \eta q_{j} \bar{\nu}_{x} \gamma \eta l_{y}, \quad Q_{ijxy}^{6} = \bar{\nu}_{i} \gamma \eta q_{j} \bar{\nu}_{x} \gamma \eta l_{y}. \quad (7)
\]

The details can be found in Ref. [17] but it is easy to see there are several possibilities. For example,
\[
Q_{1,4} \sim (\bar{\nu} \Gamma \nu + l \Gamma l), \quad Q_{2} \sim (\bar{\nu} \Gamma \nu - l \Gamma l), \quad W_{\ell \ell'} \equiv 9727 \left( \frac{1 \text{ TeV}}{\Lambda_{NP}} \right)^{2} \left( c_{1}^{\ell \ell'} - c_{2}^{\ell \ell'} + c_{4}^{\ell \ell'} \right). \quad (8)
\]

whereas \( Q_{3,5,6} \) do not produce neutrino pairs. What all this means is that the different processes are all complementary in the sense of probing different combinations of the possible couplings.

Of course, some processes are more sensitive than others and this is illustrated in the central panel of Figure 3. This figure illustrates a constraint on \( c_{1}^{\ell \ell} - c_{2}^{\ell \ell} \) plane with \( c_{4}^{\mu e} = c_{4}^{e \mu} \) and \( c_{4} = 0 \). The constraint from the E787-E949 result is shown in green. The black region in the figure shows the improvement on this constraint that follows from the NA62 result we heard yesterday [2]. Finally, the red band is from muon conversion in gold at SINDRUM-II [18, 19]. The blue area constrains an orthogonal combination of couplings and is due to the BNL-865 result for \( \mathcal{B}(K^{+} \to \pi^{+} e^{-} \nu^{+}) \) [20].

An interesting observation is that one of the lepton generations in Eq. (6) can be the third generation. In this case the rare kaon decays, through their sensitivity to the tau-neutrino, compete with LFV tau-decay. This is discussed in Ref. [21] and illustrated in a particular scenario in the right panel of Figure 3. In this illustration the constraint from the E767-E949 result \( \mathcal{B}(K^{+} \to \pi^{+} \nu \bar{\nu}) \) (in red) is seen to be much stronger (but orthogonal) to that from \( \mathcal{B}(\tau^{-} \to \mu^{-} K_{s}) \) from Belle (in blue) [22] with \( c_{1}^{\mu e} = c_{1}^{e \mu} \) and \( c_{2} = -c_{1} \).

**Figure 3.** Left panel: rates accessible to LFV models. In pink we take \( |W_{e\mu}| \leq |W_{\mu e}| \) with arbitrary phases. This case covers all the possible rates. The blue line is for \( |W_{e\mu}| = |W_{\mu e}| \) and \( \phi_{\mu e} = -\phi_{e\mu} \) for which the neutral mode is not modified. The green line with \( |W_{e\mu}| = |W_{\mu e}| \) and \( \phi_{\mu e} = \pi - \phi_{e\mu} \) maximises the neutral rate for a given charged rate. Central panel: comparison of bounds with \( c_{1}^{\mu e} = c_{1}^{e \mu} \) and \( c_{4} = 0 \) from E787-E949 (green), NA62 (black), muon conversion in gold at SINDRUM-II (red) and the BNL-865 \( \mathcal{B}(K^{+} \to \pi^{+} e^{-} \mu^{+}) \) (blue). Right panel: comparison of bounds with \( c_{1}^{\mu e} = c_{1}^{e \mu} \) and \( c_{2} = -c_{1} \) from E767-E949 (red) and \( \mathcal{B}(\tau^{-} \to \mu^{-} K_{s}) \) from Belle in blue.
4. Conclusions
We studied the effects of new physics affecting the neutrinos on the rare decay modes $K \to \pi \nu \bar{\nu}$. We find that these modes can provide significant constraints on new light sterile neutrinos that avoid LEP counts and big bang nucleosynthesis bounds. This is especially interesting in models that address the charged B anomalies in $R(D)$ and $R(D^*)$ through a light sterile neutrino.

We found that these modes also provide constraints on lepton flavour violating new physics, and that these constraints are complementary to bounds from charged lepton flavour violation experiments and in some cases stronger. This discussion extends to charged lepton flavour violation involving tau leptons because the kaon modes are also sensitive to the presence of tau-neutrinos.

Both scenarios discussed in this talk can only increase the $K \to \pi \nu \bar{\nu}$ rates over their standard model values as they do not interfere with the later. They can both populate the same region of $K \to \pi \nu \bar{\nu}$ rates (of course, with different interpretation), and this region is currently limited by the new NA62 result we heard at this meeting [2] and by a stronger than usual form of the Grossman-Nir bound.

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