The Orbit of the Close Companion of Polaris: Hubble Space Telescope Imaging, 2007 to 2014*

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Abstract

As part of a program to determine the dynamical masses of Cepheids, we have imaged the nearest and brightest Cepheid, Polaris, with the Hubble Space Telescope (HST) Wide Field Planetary Camera 2 and Wide Field Camera 3. Observations were obtained at three epochs between 2007 and 2014. In these images, as in HST frames obtained in 2005 and 2006, which we discussed in a 2008 paper, we resolve the close companion Polaris Ab from the Cepheid Polaris Aa. Because of the small separation and large magnitude difference between Polaris Aa and Ab, we used point-spread function deconvolution techniques to carry out astrometry of the binary. Based on these new measurements, we have updated the elements for the 29.59 year orbit. Adopting the distance to the system from the recent Gaia Data Release 2, we find a dynamical mass of 3.45 ± 0.75 M⊙ for the Cepheid, although this is preliminary and will be improved by CHARA measurements covering periastron. As is the case for the recently determined dynamical mass for the Cepheid V1334 Cyg, the mass of Polaris is significantly lower than the “evolutionary mass” predicted by fitting to evolutionary tracks in the Hertzsprung–Russell diagram. We discuss several questions and implications raised by these measurements, including the pulsation mode, which instability-strip crossing the stars are in, and possible complications such as rotation, mass loss, and binary mergers. The distant third star in the system, Polaris B, appears to be older than the Cepheid, based on isochrone fitting. This may indicate that the Cepheid Polaris is relatively old and is the result of a binary merger, rather than being a young single star.

Key words: stars: binaries: spectroscopic – stars: massive – stars: variables: Cepheids

1. Introduction

The link between Cepheids and the extragalactic distance scale illustrates the importance of a thorough understanding of Cepheid physics. In addition to their significance as standard candles, they are benchmarks for confronting stellar-evolution theory. They ultimately become white dwarfs, which means a subset will be compact objects in binary/multiple systems, a part of a population from which exotic objects are formed (novae, supernovae, X-ray binaries). The long-standing “Cepheid mass problem” (the mass predictions from evolutionary tracks are larger than predictions from pulsation calculations) has been reduced through revision of interior opacities (e.g., Bono et al. 2001), but still persists. Measured dynamical masses for Cepheids, coupled with a well-determined luminosity, are needed to confront these questions as well as to provide predictions about the origin of late-stage exotic objects.

Polaris (α Ursae Minoris) is the nearest and brightest Cepheid. It is a member of a triple system, with its resolved eighth-magnitude F3 V physical companion, Polaris B, lying at a separation of 18″. The Cepheid itself has been known for many years to be a single-lined spectroscopic binary with a period of about 30 years (Roemer 1965; Kamper 1996, hereafter K96, and references therein), the components of which we denote Polaris Aa and Ab. Thus, Polaris offers an opportunity for the direct determination of a Cepheid’s dynamical mass. To date, V1334 Cyg is the only other Cepheid with a measured purely dynamical mass (Gallenne et al. 2013, 2018) in the Milky Way; however, interferometric studies that resolve orbits will increase the number over the next few years (Gallenne et al. 2015). Cepheids in eclipsing binaries in the Large Magellanic Cloud summarized by Pilecki et al. (2018) provide an important comparison.

Recent papers have disagreed about the distance of the Polaris system. The Hipparcos mission (van Leeuwen 2007, hereafter vl07) measured an absolute parallax of 7.54 ± 0.11 mas (d = 132.6 ± 1.9 pc). However, Turner et al. (2013, hereafter T13, and references therein) gave astrophysical arguments that the parallax of Polaris must be considerably larger, 10.10 ± 0.20 mas (d = 99 ± 2 pc), but this was disputed by van Leeuwen (2013). More recently, Bond et al. (2018, hereafter B18) determined a parallax of only 6.26 ± 0.24 mas (d = 160 ± 6 pc), using the Fine Guidance Sensors (FGS) on the Hubble Space Telescope (HST). Since Polaris itself is too bright for FGS measurements, B18 determined the parallax of the wide physical companion, Polaris B. The large distance based upon the FGS parallax implies a relatively high luminosity, indicating that Polaris pulsates in the second overtone rather than in the fundamental mode. For the Hipparcos parallax, the pulsation is likely to be at the first overtone (e.g., Feast & Catchpole 1997; Bono et al. 2001; van Leeuwen et al. 2007; Neilson 2014). For the
In the recent Data Release 2 (DR2), the Gaia mission derived a parallax of 7.292 ± 0.028 mas for Polaris B (Gaia Collaboration et al. 2016, 2018a, 2018b). Polaris itself is too bright to have been measured for DR2. This result corresponds to a distance of 137.2 pc with a range of 136.6–137.67 pc. Applying the correction found by Lindegren et al. (2018b) of +0.03 mas to the parallax reduces the distance to 136.6 pc. Polaris B has a magnitude $V = 8.65$, well below the brightness limit for Gaia. The discussion on Cepheids by Riess et al. (2018a) finds a slightly larger zero-point offset (−0.046 mas) for the Cepheids they discuss, although Polaris B is somewhat hotter (F3 V) and the zero point may have a color dependence. Further discussion is given in Section 3.

Polaris is very unusual among Cepheids, in that its pulsation amplitude decreased over much of the 20th century. However, in recent years, the amplitude has begun to increase again (e.g., Bruntt et al. 2008). This may indicate a very long-period pulsation/interference cycle.

Our team obtained near-ultraviolet images of Polaris with the High-Resolution Channel (HRC) of the Advanced Camera for Surveys (ACS) on board HST in 2005 and 2006. These images succeeded in resolving the Polaris Aa and Ab pair for the first time, at a magnitude difference of 5.4 mag and a separation of about 0′′.17 (Evans et al. 2008, hereafter E08). This allowed us to estimate the first purely dynamical mass for any Cepheid (albeit with a large uncertainty). Since then, we have used HST cameras to make three more observations, which now cover a larger portion of the orbit. However, the new measurements were considerably more difficult and inexact, because the separation between Polaris and its close companion has been decreasing, and because the HRC was no longer available. In this paper, we report these new measurements and update the dynamical mass estimates.

### 2. Observations, Image Processing, and Deconvolution

Table 1 lists information about our three new HST observations, along with the previous 2005 and 2006 measurements given by E08. The new HST observations reported here were made in 2007, 2009, and 2014. In this section, we give details of our observing strategy, the creation of combined images at each epoch, and finally the deconvolution of the images and astrometry of the binary.

### 2.1. Observing Strategies

Two different HST cameras were used for our new observations, as follows:

1. **Wide Field Planetary Camera 2 (WFPC2) 2007.** Following the successful detection by E08 of the close companion of Polaris with ACS/HRC in 2005 and 2006, we had planned to obtain an additional HRC observation in 2007. Unfortunately, the ACS suffered a failure in 2007 January, making it unavailable. We used instead the WFPC2, and the observations were made on 2007 July 17. We chose the near-ultraviolet broadband F218W filter, both because Polaris AB is slightly hotter than the Cepheid and because of the smaller point-spread function (PSF) at shorter wavelengths. We used a similar observing strategy as for the HRC observations, placing the target in the Planetary Camera (PC) CCD. The PC plate scale is 0′′.046 pixel$^{-1}$, as compared with the ACS/HRC scale of 0′′.026 pixel$^{-1}$. We first obtained a series of eight short (0.8 s) exposures spread over a three-point dither pattern. The telescope orientation was specified such that Polaris A and B would lie parallel to the edge of the PC field and symmetrically placed relative to the corners, and we positioned these targets close to the edge so as to minimize charge-transfer-inefficiency effects. This orientation also placed the location of Ab away from the diffraction spikes of Aa, for its anticipated position angle (P.A.). We then moved the telescope to put the wide companion Polaris B at the same position used for A and obtained six longer exposures (50 s) in the same dither pattern. This provided a PSF standard at the same field position as Polaris A, exposed to about the same count level.

2. **WFPC3 2009.** The WFPC2 camera was removed from HST during the 2009 May Servicing Mission 4 and replaced with the Wide Field Camera 3 (WFC3). We observed Polaris with the WFC3 UVIS channel on 2009 November 18. The UVIS plate scale is 0′′.400 pixel$^{-1}$. Because of the increased sensitivity of WFC3 relative to WFPC2, we used the narrowband near-ultraviolet filter FQ232N in order to avoid saturating the image of Polaris Aa. We again chose a telescope orientation to place Ab between the diffraction spikes of Aa. We obtained a total of nine dithered exposures of 0.5 and 0.7 s, with Polaris A placed near the center of a 2048 × 2048 pixel subarray (8′′ × 8′′). Then, we placed Polaris B near the center of the field and obtained five dithered exposures of 45 s each to use as a PSF reference.
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(3) WFC3 2014. By the time of our final HST observations of Polaris, it had become known that short exposures with WFC3 are affected by vibrations in the camera due to the shutter mechanism (“shutter jitter”; see Sahu et al. 2014 for a technical discussion). Exposures with the shutter in the “A” blade position are less affected than those in the “B” position, so we used the newly available BLADE=A option for all of our exposures. Instead of using longer exposures on Polaris B as a PSF reference star, which would be much less affected by shutter jitter, we observed γ Cygni—a star with nearly the same magnitude, color, and spectral type as the Cepheid—immediately after the Polaris observation. We obtained 14 dithered FQ222N exposures on Polaris of 1.5 s each (which the 2009 exposures indicated would not be saturated) and chose a telescope orientation that placed Polaris Ab at an optimum location in the PSF to avoid diffraction spikes and filter ghosts, based on our analysis of the 2009 images. This was followed by a set of 12 essentially identical dithered exposures on γ Cyg. The observations were otherwise very similar to those made in 2009.

Our first attempt at these observations (2014 March 17) suffered a loss of guide-star lock partway through the γ Cyg visit, so that we did not have useful PSF images. These data were not included in our data analysis. Both sets of observations were repeated successfully on 2014 June 26.

2.2. Combining the Dithered Images

As described above, we obtained a set of dithered exposures of Polaris A and B at two epochs, and of Polaris A and γ Cyg at a single epoch. In order to combine these images into optimal single images at each epoch, we used the AstroDrizzle package developed at the Space Telescope Science Institute (STScI), running under PyRAF. The resulting combined images have a pixel scale of 0.02. For the WFC3 data in 2009, the problem of “shutter jitter” was a concern. We experimented with the images and found that the “even” images in the dither pattern were better quality, and were able to test the effect on the location of the companion.

2.3. Deconvolution and Astrometry of Polaris Ab

The observed stellar images are blurred by the PSF of the telescope and camera optics, and by the finite size of the detector pixels. To remove the effects of this blurring, we have in principle several options, including a subtraction of the PSF (based on images of a reference point source), fitting of a model for the source plus PSF to the observed source, or a full deconvolution of the PSF.

Because of the small separation, the large magnitude difference between the Cepheid Polaris Aa and its companion Polaris Ab, and the larger plate scales compared to the ACS/HRC used in our earlier observations, we could not use direct PSF model fitting for any of the more recent observations. Direct PSF subtraction was also unable to detect the companion. Therefore, we applied deconvolution/restoration techniques using the observed single point-like reference stars as models of the PSF.

We applied two image-restoration methods. One of them is the well-known Richardson–Lucy (R-L) deconvolution technique (Richardson 1972; Lucy 1974), and the other is a statistical deconvolution (restoration) technique called EMC2 (Expectation through Markov Chain Monte Carlo). The EMC2 technique uses a wavelet-like multiscale representation of the true image in order to achieve smoothing at all scales of resolution simultaneously (see Esch et al. 2004; Karovska et al. 2005, 2010).

As described above, for the 2007 and 2009 observations, we used Polaris B as a PSF reference star, which we had placed at the same field location as Polaris A. For these two epochs, the R-L technique successfully resolved the companion. Figures 1 and 2 depict the results of the deconvolutions, with the locations of the companion Polaris Ab marked with circles, connected by arrows to the center of Polaris Aa. The last two columns in Table 1 give the derived P.A. and separation of Polaris Ab. The quoted errors estimated by eye correspond to one pixel in the drizzled image (0.02). This is conservative for the 2009 WFC3 observation, where they may be smaller.

By the time of the 2014 observation, Polaris Ab had moved much closer to Polaris Aa, making it considerably more difficult to separate the two components. We did, however, have images of the PSF reference star γ Cyg, obtained with the same exposure times as Polaris and thus affected by the same amount of shutter jitter. For this observation, we applied the EMC2 deconvolution technique, which is better suited for detecting faint companions.

Given the small separation and large magnitude difference, we had to search for Polaris Ab amidst the PSF structure and noisy remnants of the deconvolution. However, our search was guided by a predicted location of the companion, based on an existing orbital fit to the previous measurements. This facilitated the search for the companion, and we believe we have detected it.

In Figure 3, we show the deconvolved 2014 image. A circle marks the region where we believe we may have detected the companion. Also included are two smaller circles indicating the predicted locations. The first uses the orbit from the spectroscopic orbit, our HST observations from 2005 to 2007, and the astrometric measurements from Wielen et al. (2000). The second uses a slightly modified orbit with a 1σ change in the inclination. The resulting P.A. and separation are listed in the bottom row in Table 1. The quoted error corresponds to the drizzled pixel size of 0.02. Although the 2014 detection is not as clear as in the earlier observations, it may provide an additional constraint on the orbit. Unfortunately, for the remaining lifetime of HST, the separation will be too close to be resolved.

3. Visual Orbit for Polaris Aa–Ab

Our additional HST measurements of Polaris Aa–Ab substantially improve the orbital coverage, compared with the results presented by E08. We have fitted new orbits to the five astrometric data points presented in Table 1, and also to a subset of only the first four points, using a Newton–Raphson method to minimize χ² by calculating a first-order Taylor expansion for the equations of orbital motion. Since we have only four or five astrometric observations, covering only a small portion of the orbit, we fixed the values of the spectroscopic parameters of the orbital period P, eccentricity e, and longitude of periastron passage ω to those determined by K96. The date of periastron passage T₀ reported by K96 was discussed and updated by Wielen et al. (2000), and we adopt their value. We then solved for the remaining parameters of the
angular semimajor axis $a$, orbital inclination $i$, and the position angle of the line of nodes $\Omega$. We obtained formal errors for each of the free parameters from the diagonal elements of the covariance matrix. To account for the uncertainties in the spectroscopic orbital parameters, we computed a series of 10,000 orbital solutions where we randomly varied $P$, $T_0$, $e$, and $\omega$ within their $3\sigma$ uncertainties and solved for the best-fit values of $a$, $i$, and $\Omega$ for each iteration. We computed the standard deviation for each parameter’s distribution and added these uncertainties in quadrature to the formal errors computed from the best-fit solution when fixing the spectroscopic parameters.

We present two orbital solutions. One uses all of the $HST$ data, and the other omits the 2014 measurement, which is the most uncertain. The final orbital parameters and uncertainties are presented in Table 2 for both solutions.

In Figure 4, we show the $HST$ measurements compared with the best-fit orbit for all data. The shared gray area shows the range of orbital solutions that fit the data. This region was computed using two different methods. As described above, in the first method, we randomly varied the spectroscopic parameters within their 1$\sigma$ uncertainties and rederived the best-fitting orbit for each iteration. In the second method, we fixed the spectroscopic parameters and randomly varied $a$, $i$, and $\Omega$ in order to search for orbital solutions within $\Delta \chi^2 = 3.53$ of the minimum, corresponding to a 68.3% confidence interval, for the three free parameters. The shaded region in the figure represents the maximum deviations in orbital solutions obtained from these two samples. The residuals in separation and P.A. compared with the best-fit orbit are plotted in Figure 5.

By assuming a parallax $\pi$ for the system, we can then compute the total mass of Polaris Aa and Ab through Kepler’s Third Law: 
\[ M_{\text{tot}} = M_{\text{Aa}} + M_{\text{Ab}} = a^3/(\pi^2 P^2), \]
where the masses are in solar units, $a$ and $\pi$ in seconds of arc, and $P$ in years. Using the radial-velocity amplitude of Polaris Aa ($K_{\text{Aa}} = 3.72 \pm 0.03 \text{ km s}^{-1}$) from the spectroscopic orbit of K96, we can derive the individual masses of Aa and Ab.

We show the resulting dynamical masses using both the $Gaia$ parallax and the $Hipparcos$ parallax in Table 3. We note that the errors for the distance from $Gaia$ are small, but the mass itself depends on the orbital solution, which will become better defined during the periastron passage. The comparison in Table 2 of the orbit omitting the least accurate point illustrates the possible differences in the solutions.

The mass derived depends heavily on the parallax. The parallax from $Gaia$ DR2 for Polaris B has a reasonable “excess noise” of 0.0 and a reduced $\chi^2$ of 2.7. The “goodness of fit” is 12.2, which is higher than the desirable 9, but within a reasonable range. The use of a zero-point correction to $Gaia$ parallaxes adds to the uncertainty. The mass of the Cepheid without the correction becomes $3.50 \pm 0.76 M_\odot$. We anticipate improvement in the mass determination from subsequent $Gaia$ releases such as DR3, as well as more complete observations of the orbit.

We note two further points about the adopted parallax for the Polaris system. (1) Using the $T13$ parallax would result in a mass for the Cepheid of only $1.08 M_\odot$, far smaller than either the observational or theoretical predictions for a Cepheid. We believe the large parallax advocated by $T13$ is now definitively ruled out. (2) We recomputed the FGS parallax reported by B18 by adopting as input values the parallaxes and proper motions for the background reference stars as given in the $Gaia$
DR2. This actually makes the derived parallax of Polaris B even smaller than given by B18. At this point, given the good agreement between the Hipparcos parallax for Polaris A and the DR2 parallax for Polaris B, we no longer advocate for the FGS parallax. We are examining reasons why it might be incorrect (such as the possibility that Polaris B could be a marginally resolved close binary).

Table 3 also lists, in the bottom line, the mean visual absolute magnitude of the Cepheid Polaris Aa, calculated by assuming a mean apparent magnitude of $\langle V \rangle = 1.982$ and a reddening of $E(B - V) = 0.01 \pm 0.01$ (see B18 and references therein), and adopting the two different parallaxes.

### 4. Discussion

#### 4.1. Properties

Based on the updated results given in this paper (which still covers only a fraction of the entire 29.95 year orbit), we now compare the masses in Table 3 with expectations from current Cepheid calibrations. For the Cepheid Polaris Aa, the mass prediction is compared below with the measured mass of the Cepheid V1334 Cyg (Gallenne et al. 2018) as well as with evolutionary tracks. There is also information about the astrometric companion, Polaris Ab, which was resolved by the HST HRC of the ACS at an epoch when the separation was wider. In E08, its mass was estimated based on photometric measurements to be $1.3 M_\odot$. This is in reasonable agreement with the estimate from the orbit (Table 3), but more precise masses for the Cepheid and the companion await more complete coverage of the orbit. The properties of both the Ab and B components of the system are discussed further compared with an isochrone in the discussion of mergers below.
Dynamical Masses for Polaris Aa and Ab for Different Adopted Parallaxes

| Parameter          | Parallax Adopted from: | vL07       | G18       |
|--------------------|------------------------|------------|-----------|
| ξ [mas]            |                        | 7.54 ± 0.11| 7.3218 ± 0.0281 |
| d [pc]             |                        | 132.6 ± 1.9 | 136.58 ± 0.52   |
| $M_{Aa}$ [M$_\odot$] |                     | 4.65 ± 0.71 | 5.07 ± 0.75    |
| $M_{Ab}$ [M$_\odot$] |                     | 3.11 ± 0.70 | 3.45 ± 0.75    |
| $M_{Aa} [M_{co}]$  |                        | 1.54 ± 0.46 | 1.63 ± 0.49    |
| $\langle M_{e, Aa} \rangle$ |                 | −3.66 ± 0.04 | −3.73 ± 0.03   |

Notes.

* Abbreviations for sources of the adopted parallaxes are vL07 = van Leeuwen (2007); G18 = Gaia including the Lindegren correction.
* Mean visual absolute magnitude of Polaris Aa, assuming $(V) = 1.982$, $E(B - V) = 0.01 ± 0.01$, and the adopted parallax.

### 4.2. Questions and Implications

As the nearest Cepheid, Polaris has measured properties available for only a few other Cepheids which provide a number of distinctive clues and raise questions that may be answered as the orbit becomes better defined. In this section, we discuss recent results in several areas relevant to the interpretation of a measurement of the mass (and luminosity) of Polaris. The most important comparison with the preliminary mass of Polaris is with the recently determined mass of V1334 Cyg and a comparison of these two stars with evolutionary tracks. Subsequently, we discuss the period change (often interpreted as a direct measure of evolution), instability-strip crossing, possible mass loss or a previous merger, and implications from abundances.

#### 4.2.1. Pulsation Mode

Figure 6 shows the location of Polaris in the Leavitt period–luminosity relation using the *Gaia* absolute magnitude $M_V$. The comparison Cepheids have FGS trigonometric parallaxes (Benedict et al. 2007), *HST* spatial-scan parallaxes (Riess et al. 2018b), and the light-echo distance of RS Pup (Kervella et al. 2014). $(V)$ and $E(B - V)$ have been taken from the Galactic Cepheid Database (Fernie et al. 1995). While *Gaia* results for a large Cepheid sample will undoubtedly contribute immensely to the Leavitt law, the brightest Cepheids are challenging for *Gaia*, so we use the previous *HST* results for this figure. The result from *Gaia* for Polaris B and hence for the Cepheid Polaris Aa is consistent with pulsation in the first-overtone mode (though the second overtone is still within the errors). We will treat the Cepheid as a first-overtone pulsator below.

#### 4.2.2. Comparison with V1334 Cyg

V1334 Cyg is another overtone pulsator with a similar (fundamentalized) period (4.74 days) and luminosity to Polaris. Its mass and distance have been measured by Gallenne et al. (2018) to unprecedented accuracy using interferometry combined with radial velocities from the visible and ultraviolet, based on the orbit of Evans (2000). They find a Cepheid mass of $4.288 ± 0.133 M_\odot$ and a distance of $720.35 ± 7.84$ pc, corresponding to an absolute magnitude $M_V = -3.37$ (including a correction for the companion), which probably provides the best estimate of the expectation for Polaris. Polaris has a somewhat longer period than V1334 Cyg and a slightly brighter $M_V = -3.73 ± 0.03$. The mass of the Cepheid Aa (*Gaia*; Table 3) is smaller ($3.45 ± 0.75 M_\odot$), but the mass will not be final until the orbit is more fully covered. The mass of...
Polaris Aa only differs by a little more than 1σ from that of V1334 Cyg, and the luminosities are similar. We discuss the rapid period change and variation in pulsation amplitude of Polaris below, but we draw attention to the fact that V1334 Cyg shows no indication of a period change, at least since 1968 (Berdnikov et al. 1997). In the previous discussion (B18), when Polaris appeared to be a second-overtone pulsator, it seemed likely that a different pulsation mode might be responsible for the difference in the envelope pulsation properties (period change and amplitude variation). However, it now seems most likely that both stars are pulsating in the same mode, illustrating a range of envelope pulsation properties.

4.2.3. Evolutionary Tracks

The combination of observed mass and luminosity is needed for comparison with evolutionary tracks. For a number of years, it has been clear that a moderate amount of core convective overshoot in intermediate-mass main-sequence stars increases the fuel available and ultimately the luminosity of the “blue loops,” the stage at which Cepheids are found. (See Neilson et al. 2012a for examples of this effect.) This has the unfortunate countereffect of decreasing the temperature range of the blue loops so that evolutionary tracks for stars less massive than $5M_\odot$ typically do not penetrate the instability strip (e.g., Figure 2 in Neilson et al. 2012b). Recently, an additional parameter has been explored in detail: rotational velocity. Anderson et al. (2014) provided evolutionary tracks for a range of initial rotations, which produce results similar to convective overshoot calculations: more luminous blue loops for a given mass. However, there is less suppression of loops for lower mass stars. Either of these possibilities or a combination may ultimately prove to be an accurate description, and the blue loops are notoriously sensitive to input parameters. However, at present, it is not clear whether either theoretical approach matches the masses available or those expected in the near future.

In Figure 7, we show the comparison between the evolutionary tracks with rotation discussed in Anderson et al. (2014), which are taken from Georgy et al. (2013). Polaris and V1334 Cyg have luminosities from Gaia (Table 3) and Gallenne et al. (2018), respectively. Temperatures are from $(B-V)_0$ (Evans 1988 and Evans et al. 2013, respectively) and the temperature color calibration as discussed in Evans & Teays (1996). However, masses inferred from the luminosities of the evolutionary tracks in Figure 7 are about $5M_\odot$, which is larger than the measured values, even for V1334 Cyg. This is true even when the stars start their main-sequence life with a large rotational velocity. The mass of Polaris estimated using the evolutionary tracks in Figure 7 (the “evolutionary mass”) is approximately $5.5M_\odot$ for the tracks incorporating maximum rotation, but $6.0M_\odot$ without rotation. These values are discrepant by $2.7\sigma$ and $3.4\sigma$ from the measured mass of $3.45 \pm 0.75 M_\odot$ (Table 3). However, only a small portion of the visual orbit is currently covered; a systematic offset of $3\sigma$ in the semimajor axis as the periastron passage is mapped in future observations would bring the mass of Polaris in agreement with the evolutionary masses.

In Figure 8, we compare several representative evolutionary tracks in the “blue loop” section of the Hertzsprung–Russell diagram. Specifically, $5M_\odot$ tracks are plotted from the Geneva series (Georgy et al. 2013), the MIST series (Choi et al. 2016), and the PARSEC set (from padova_tracks), based on the results from the Bressan et al. (2012) group.
We selected 5\(M_\odot\) tracks because they cover the temperature range of the instability strip and are available for a range of rotational velocities from several codes. For all three codes, tracks are shown with zero initial rotation. For Geneva and MIST, a track with substantial initial rotation is also shown (0.95 critical breakup speed for Geneva, 0.4 critical velocity for MIST). Figure 8 shows the resulting variation in luminosity from the different codes as well as differing predictions for the incorporation of main-sequence rotation.

V1334 Cyg is the first Cepheid with a dynamically determined mass with a small enough uncertainty (4.288 ± 0.133 \(M_\odot\)) to strongly constrain the tracks. The most important point in Figure 8 is that V1334 Cyg is significantly more luminous than the predictions for its mass from the calculations regardless of the code or rotation. An alternate description is that the Geneva tracks in Figure 7 indicate that its mass is approximately 5\(M_\odot\), which is 0.7\(M_\odot\) larger than the measured mass.

A further note about the mass of V1334 Cyg is that it strongly constrains the lower mass at which a star destined to become a Cepheid can enter the instability region after the red giant branch. Current tracks in Figure 7 show that a 4\(M_\odot\) star would not become hot enough. This is doubly true in that overtone pulsators are found among shorter period stars (even when “fundamentalized”) on the blue side of the instability strip. Thus, a track has to become hot enough to reach the blue region of the Cepheid instability strip.

Another parameter that is directly comparable with the predictions of evolutionary tracks is the radius, Mérand et al. (2006) measured an angular diameter of Polaris of 3.123 ± 0.008 mas, which corresponds to 46\(R_\odot\) using the Gaia distance. We can make an approximate comparison between this and the results of the evolutionary tracks (Anderson et al. 2016) using the luminosity and temperature tabulated in their Table A.4. In the comparison, we use the Milky Way metallicity, an initial rotation which is half of the critical breakup rotation, a mass of 5\(M_\odot\), and first-overtone pulsation. The 5\(M_\odot\) evolutionary track does not cover the full temperature width of the instability strip for the second and third crossings (discussed below), but rather enters the cool edge and then turns in the middle and returns to the cool edge, being the lowest mass of the computed tracks to enter the instability strip. However, the truncated second and third crossings cover the range of radii of 42–52\(R_\odot\) while the first crossing spans 26–33\(R_\odot\). Thus, in this approximate comparison, the third crossing (increasing period) is in reasonable agreement with the measured radius.

### 4.2.4. Period Change

Polaris undoubtedly has a changing period (Neilson et al. 2012a and references therein). A changing period is typically interpreted as resulting from evolution through the instability strip; however, there are a number of indications that more than this simple interpretation is needed. Overtone pulsators appear to have higher rates of period change than fundamental-mode pulsators of comparable period (Szabados 1983; Evans et al. 2002). This would, in fact, explain what is described as the anomaly of large period changes in short-period Cepheids by Turner et al. (2006). Studies using spectacular satellite observations (Kepler, CoRot, MOST, WIRE) are alerting us to hitherto unknown pulsation behavior: new modes and interactions. For instance, first-overtone Cepheids in the Small Magellanic Cloud have peculiarities that may be related to nonradial pulsation (Smolec & Sniegowska 2016). In addition, continuous sequences of high-quality photometry with the MOST satellite of RT Aur (fundamental mode) and SZ Tau (first overtone) confirm that overtone cycles are less stable than fundamental-mode cycles (Evans et al. 2015), though this has been disputed by Poretti et al. (2015).

Thus, while Polaris’s rapid period change has sometimes been interpreted as indicating a first rather than a third crossing of the instability strip (Turner et al. 2005), the working model here is that for overtone pulsators, period changes are not strictly due to evolution through the instability strip. Rather, period changes are at least partially due to some aspect of pulsation that we do not yet fully understand. Other characteristics of Polaris’s pulsation support this. While the period change \(O – C\) (observed minus computed) diagrams suggest a parabola, the actual form is more complex, with a “glitch” about 40 years ago (Turner et al. 2005; Neilson et al. 2012b). This requires two tightly constrained parabolas to fit the data. Furthermore, the pulsation amplitude of Polaris has a marked drop followed by a gradual increase over many decades, a phenomenon shared by at most one or two other Cepheids (Bruntt et al. 2008; Spreckley & Stevens 2008). A variable amplitude (decreasing and increasing) is much more likely to be due to a pulsation interaction than to an evolutionary effect, which would be expected to go in one direction only. In fact, the classical Cepheid with the most prominent amplitude variation is V473 Lyr, which is a second-overtone pulsator.

Neilson et al. (2012a) have made a population synthesis analysis of period changes in Cepheids and conclude that both main-sequence core convective overshoot and mass loss (see below) are needed to match the observations to the predictions. This is based both on the size of the period change and also
the proportion of second-crossing (period increase) to
third-crossing (period decrease) stars. However, the working
model above is that many of the short-period stars are overtone
pulsators, and hence the period changes are determined by a
pulsation component as well as an evolutionary component.
Neilson et al. draw attention to the fact that short-period stars
fit the predictions less well than long-period stars. If the short-
period stars are omitted for this reason, Figure 3 in Neilson
et al. shows that for the long-period stars, the distribution of
period changes is well matched by evolutionary predictions. It
is likely that this also changes the proportion of positive to
negative changes as well, but a firm statement requires the
detailed identification of overtone pulsators. Note further that
the period changes discussed are biased in the sense that they
include only stars with measurable period changes. A large
fraction of Cepheids in the Szabados catalogs (1977,
1980, 1981, 1989, 1991) show no period change, at least over
the 60 years for which we have photometric observations.
Presumably their periods also change as they evolve, but more
slowly. This means that Figures 2 and 4 in Neilson et al., where
the predictions are larger than the observed values, are even
more difficult to reconcile with the expected mean period
changes.

The period variation in Polaris challenges the current
models. In a number of stars, period change is not monotonic,
but rather a combination of increase and decrease
(e.g., Figure 3 in Berdnikov et al. 1997). The simplest
explanation for Polaris and other overtone pulsators would be
that the current epoch reflects, at least partly, short-term
unsteadiness rather than long-term evolution. Some possible
scenarios where pulsation might influence the perceived
period change in overtone pulsators are sketched in
Evans et al. (2002).

That said, Polaris does have an unusually large period
change.

4.2.5. Which Instability-strip Crossing?

Neilson (2014) provides a recent thorough discussion of the
observations of Polaris, together with evolutionary models,
mass loss, rotation, and abundance. A question related to the
period change is the crossing of the instability strip. T13 have
asserted that their distance is consistent with a first crossing.
This, however, has been refuted by van Leeuwen (2013) and
Neilson (2014) and is not consistent with our mass summary
above (Table 3). Following our “working model” above that
period changes in overtone pulsators are not entirely due to
simple monotonic evolution removes rapid period change as an
argument for Polaris as a star on the first crossing of the
instability strip.

Anderson (2018) has recently proposed a different inter-
pretation for Polaris, that it is in the first crossing of the
instability strip and pulsating in the first overtone. This is based
on the comparison of the luminosity in B18 with evolutionary
tracks from the Geneva group (Anderson et al. 2014), as well as
CNO abundance enhancements (discussed below). A major
implication of this interpretation is that the mass of the Cepheid
is 7 M⊙. However, the mass of Polaris based on the new Gaia
distance (3.45 ± 0.75 M⊙; Table 3) is inconsistent with this.

Thus, based on the period and luminosity, Polaris is on the
third crossing of the instability strip.

4.2.6. Mass Loss

Could the current mass of Polaris have been affected by
mass loss? The question of whether Cepheids lose mass, and in
particular whether pulsation fosters mass loss, is an important
one. This has been a suggestion to solve the “Cepheid mass
problem,” reconciling the larger evolutionary track masses with
smaller pulsation predictions. Neilson et al. (2012a) and
references therein discuss studies that indicate that pulsation
can drive mass loss. They conclude that mass loss is necessary
to explain period change predictions (but we argue above for
complexities in the interpretation of the period change).
Observationally, there have been a series of interferometric
observations indicating circumstellar envelopes (CSEs) in the
K-band (Kervella et al. 2006; Mérand et al. 2006, 2007). This
feature seems to be shared by all of the Cepheids observed, but
not the nonvariable yellow supergiant α Per. These shells are
particularly important to characterize and understand since they
are dependent both on wavelength and pulsation period. Thus,
they could affect the infrared (IR) Leavitt (period–luminosity)
law and may indicate mass loss. On the other hand, Spitzer
observations have found only occasional indications of extended IR emission (Barmby et al. 2011). Specifically,
10% have fairly clear emission, with a further 14% with
tentative emission. This agrees with the summary of mid-IR
excess by Schmidt (2015). He finds that only 9% of classical
Cepheids have IR excess (omitting three with anomalously
energy distributions) and only stars with periods 11 days or
longer. Thus, while there is evidence that a small fraction of
Cepheids have extended IR emission, it is by no means
universal. Uniquely in the case of δ Cep, there is also a more
extended feature in the Spitzer study that looks like a bow
shock (Marengo et al. 2010). This was followed up by
Matthews et al. (2012) with a H 1 21 cm observation consistent
with an outflow.

For Polaris itself, a CSE has been detected (Mérand
et al. 2006). However, Spitzer IRAC and MIPS observations
have found only a suggestion of IR excess in Polaris (Marengo
et al. 2010) and no extended IR emission (Barmby et al. 2011).

Recent X-ray observations have revealed another process
that may contribute to upper-atmosphere mass motions. The
passage of the pulsation wave at minimum radius through the
photosphere and chromosphere is marked by disturbances such
as ultraviolet emission lines, most clearly shown in δ Cep itself
(Engle et al. 2017). X-rays, on the other hand, remain low and
constant at minimum radius but exhibit a sharp rise and fall just
after maximum radius. One possible mechanism for this is
flares such as those frequently found on the Sun and other stars
with convective surfaces. Flares are often associated with
coronal mass ejections. The burst of X-rays in Cepheids
appears to be tied to the pulsation cycle, above the photosphere,
and triggered by the collapse of the atmosphere after maximum
radius. Thus, it is possible that the X-ray bursts may be part of a
chain that moves material upward in Cepheid atmospheres.

Polaris is a very low-amplitude Cepheid, and as yet an X-ray
burst has not been detected (Evans et al. 2018). However, the
phase range where it would be expected has not been fully
covered by X-ray observations.

4.2.7. Mergers

Figure 9 shows the positions of V1334 Cyg and Polaris
Aa and B in the color–magnitude diagram (CMD;
Also plotted are MIST isochrones for solar metallicity and zero rotation, for ages of 100 Myr and 2.25 Gyr. The nominal 100 Myr isochrone in Figure 9 provides a schematic description of the evolution of a Cepheid, which is clearly a reasonably massive evolved star with an age of order 100 Myr. The apparent discordance between the ages of Polaris and its wide companion was discussed by B18. Setting aside the remote possibility that B is not actually a physical companion of the Cepheid, B18 considered two scenarios: (1) The Polaris system is young, and the B component is abnormally luminous. This could conceivably be due to B being an unresolved binary, even at HST resolution, although there is no direct evidence for this. We are currently planning an observation with the CHARA array to test whether B can be resolved. This might, of course, affect the Gaia parallax. It is also possible that B is temporarily overluminous due to a recent stellar merger. (2) Alternatively, the system may in fact be old, and it is Polaris itself that appears anomalously young. For example, reddening-corrected absolute V magnitude versus B − V color). B18 pointed out that Polaris B lies above the main sequence in the CMD, if the relatively small parallax that they found is assumed. This remains true even if we use the larger Gaia parallax, as shown in Figure 9. Also plotted are MIST isochrones for solar metallicity and zero rotation, for ages of 100 Myr and 2.25 Gyr. The nominal 100 Myr isochrone fits the locations of the two Cepheids reasonably well. The 2.25 Gyr isochrone was chosen because it fits the position of Polaris B. We have argued in Section 4.2.3 that evolutionary tracks for the "blue loop" phase require some adjustment, specifically that the observed masses of the two Cepheids are significantly smaller than predicted by the luminosity of the isochrones. However, the 100 Myr isochrone in Figure 9 provides a schematic description of the evolution of a Cepheid, which is clearly a reasonably massive evolved star with an age of order 100 Myr.

Polaris might be a merger product. It certainly seems likely that at least some Cepheids are descendants of merged binaries, given the high frequency of close binaries among their B-type progenitors. In fact, Neilson et al. (2015) have pointed out an age discrepancy for the binary eclipsing the Large Magellanic Cloud Cepheid OGLE-LMC-CEP1812 and presented arguments that this Cepheid is a post-merger product. Based on the MIST 2.25 Gyr isochrone, the mass of Polaris B is $\sim 1.4 M_\odot$, and stars of $\sim 1.6 M_\odot$ have essentially completed their evolution. Since we find a dynamical mass for Polaris Aa of $3.45 \pm 0.75 M_\odot$, a merger scenario based on two stars that are both near the main-sequence turnoff at $\sim 1.5 M_\odot$ appears possible.

We investigated whether Polaris Ab provides any useful constraint on these two scenarios. For example, if Ab also appeared to be an old star, that would greatly strengthen the merger scenario for the Cepheid progenitor. Unfortunately, other than the dynamical mass of 1.63 ± 0.49 $M_\odot$, the only information we have on the nature of Polaris Ab is its absolute magnitude in the F220W (2200 Å) bandpass, which we measure in our ACS/HRC frames to be about +4.25 (Vega scale, Gaia distance). We have no direct information on its color or spectral type. The MIST isochrones provide absolute magnitudes in the F220W bandpass, but such an absolute magnitude can be attained by a star of this mass with either the young or old isochrones plotted in Figure 9.

### 4.2.8. Abundances

Abundances provide clues to the history of an evolved star. Specifically, an increase in N and a decrease in C compared with main-sequence values indicates processed material. The abundances of Polaris have been measured several times, summarized by Usenko et al. (2005), including by Luck & Bond (1986). Similarly, abundances for V1334 Cyg have been measured by Takeda et al. (2013). Abundances from Usenko et al. and Takeda et al. are summarized in Table 4, showing very similar amounts of N enrichment and C decrease, suggesting comparable processing mechanisms and quantity. As discussed by Neilson (2014), the processing of material could come either through rotational mixing near the main sequence or dredge-up as a star becomes a red giant. He computes evolutionary tracks with a range of main-sequence rotational velocities. From these predictions and the relatively low observed rotational velocity, he concludes that rotational mixing is not responsible for the abundance distribution, hence the star must be beyond the first crossing of the instability strip. Anderson (2018) on the other hand comes to the opposite conclusion from the abundances, that they are consistent with a first crossing. We note that Figures 8, 10, and 11 in Anderson et al. (2014) show that the N and C abundances are roughly in agreement with either pre-dredge-up with a lot of initial rotation or post-dredge-up with little initial rotation. Takeda et al. surveyed 12 Cepheids at several phases in the pulsation cycle for each. The C and N abundances (in the mean) also

| Abundance | [C/H] | [N/H] | [O/H] |
|-----------|-------|-------|-------|
| Polaris Aa | -0.17 | +0.42 | -0.00 |
| V1334 Cyg | -0.24 ± 0.03 | +0.44 ± 0.03 | +0.03 ± 0.03 |
agree with those of V1334 Cyg and Polaris, providing no evidence for an evolutionary state other than the post-dredge-up state typical of Cepheids.

A further mechanism that would produce processed material is the merger of a close main-sequence binary (previous section).

4.3. Future Work

These results conclude the work possible with HST in resolving Polaris Aa and Ab since the separation between the two stars becomes progressively smaller during the following years. In this discussion, we are providing a summary of the understanding of the system at present. Clearly, continued study of the orbit over the next few years will provide an improved definition of the orbit and the mass. These observations are to be continued with interferometry (e.g., with the CHARA array). In fact, a tentative detection has been made, which is in reasonable agreement with the current orbit.

5. Summary

To summarize, Polaris now has a suite of directly measured parameters to identify its physical state: luminosity, mass, radius, period change, and abundance, although we stress that the mass presented here is preliminary, and Tables 2 and 3 illustrate the dependence on the data string and the distance. The mass is within approximately 1σ of the mass of V1334 Cyg, which is more precisely measured. However, for both Polaris and V1334 Cyg, the masses are smaller than predicted by the evolutionary tracks for the observed luminosities. There are several suggestions for the more complex history for Polaris in arriving at its present configuration. It has a CSE in the IR, making it likely that some mass has been lost. The interpretation of the rapid period change is likely to involve more than evolution through the instability strip alone, but it also may be characterized of overtone pulsation. In agreement with this is the fact that V1334 Cyg (also an overtone pulsator) has no measurable period change. The difference in age required for the isochrones for Polaris Aa and B suggests that the Cepheid might be a merger product. Probing these questions is the goal of further observations of the orbit.

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