COUPLING THE NON-GRAVITATIONAL FORCES AND MODIFIED NEWTON DYNAMICS FOR COMETARY ORBITS

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ABSTRACT. The sublimation of ices of a cometary nucleus affects its gravitational orbit. This sublimation triggers a so-called non-gravitational forces. Moreover, some comets with large orbit ($a > 15$ AU) are in low-acceleration regime at their aphelion. We investigate the effect of MOND and non-gravitational perturbations for three comets with various orbital elements (2P/Encke, 1P/Halley and 153P/Ikeya-Zhang). We used six different forms of MOND functions and compute the secular variations of the orbital elements due to both perturbations. We show that the MONDian effects are not negligible for comets with high semi-major axis compared to the non-gravitational perturbations and this effect are currently close to the detection threshold.

1. Introduction

According to several observations, the visible mass of the galaxies and clusters of galaxies determined by the Tully-Fisher law is insufficient to explain their dynamics with the Newton’s law.

One of the solution to explain this discrepancy was proposed by Jan Oort and Fritz Zwicky. It consists on considering the existence of non-baryonic Cold Dark Matter (CDM) and uses

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the framework of the standard gravitational physics.

Oppositely, the MOdified Newtonian Dynamics (MOND) [12] [13] [14] lies on a modification of the Newton’s law. The modification consists in the introduction of a function $\mu(y)$ in the universal gravitational law to explain the kinematics of astrophysical systems (galaxies and clusters of galaxies for example). The modified gravitational law can be written as:

$$ F = m \mu \left( \frac{a}{a_0} \right) a $$

where $a = \|a\|$ is the norm of the acceleration of an object with a mass $m$ submitted to the force $F$ and $a_0 = 1.2 \times 10^{-10} \text{m/s}^2$ is the MOND acceleration constant. The MOND function $\mu(y)$ with $y = \frac{a}{a_0}$ tends to 1 for $y \gg 1$ in a Newtonian strong-field regime, and tends to $y$ for $y \ll 1$ in a weak gravitational field regime.

According to [2], the object with a large semi-major axis are more sensitive to the effects of perturbations induced by MOND. In this way, we decided to study some comets with large semi-major axis to determine the magnitude of the effects of MOND theory. Indeed, the comets are good candidates because they have the particularity to go far from the sun on very excentric orbit and come back close to the earth to be observed accurately.

Nevertheless, it has to be noted that the gravitational orbits of a comet are affected by the sublimation of ices on the nucleus surface. The outgassing triggers a non-gravitational forces that significantly modifies the orbit of the comet close to the Sun (under 3 AU). This non-gravitational forces has been modeled for the first time in [10] and then improved in [11]. In this last model, the cometary nucleus is assumed to be spherical and the whole illuminated surface is isotropically outgassing. This semi-empirical model is commonly used to describe the non-gravitational acceleration to produce the orbits of comets. Some other approach more physical of the non-gravitational forces has been developed in [17], [16], [1] and [9] that takes into account only the outgassing of some area on the nucleus which describes more accurately the observations made by space probes. In this paper, we only consider the Marsden et al. (1973) model to compute the non-gravitational forces as it is easily implemented to only compute orbits contrary to the more sophisticated model.

The plan of the paper is as follows:
In section 2, we present a brief reminder about the Gauss equation of the perturbed two-body problem and the implementation of the non-gravitational and MOND perturbation. Section 3 shows the consequence in term of secular variation of the orbital elements due to the non-gravitational and the MOND perturbation of three comets. We conclude in section 4 and give some prospects.

2. Perturbed Sun-Comet System

2.1. Reminder about perturbed two-body problem. In this Section, we remind classical definitions and results about the perturbed two-body problem. We refer in particular to [3] for more details and proofs.

The unperturbed two-body problem of a comet $C$ with mass $M_C$ around the Sun $S$ with mass $M_S$ is described by six orbital elements. We adopt the most classical ones which are the semi-major axis $a$, the eccentricity $e$, the orbital plane inclination $i$, the argument of the perihelion $\omega$, the ascending node longitude $\Omega$ and the mean anomaly $M$ defined by $M = n(t - \tau)$ where $n = \frac{2\pi}{P}$ is the mean motion, $P$ the orbital period of the comet and $\tau$ the time passage at the perihelion (Fig. 1).

Let $(S, \mathbf{x}, \mathbf{y}, \mathbf{z})$ be the fixed reference frame attached to the Sun, typically the fixed heliocentric frame and let $(S, \mathbf{e}_R, \mathbf{e}_T, \mathbf{e}_N)$ the frame associated to the heliocentric motion of the comet in the orbital plane where $\mathbf{e}_R$ is the radial unit vector, $\mathbf{e}_T$ the tangential unit vector
and \( \mathbf{e}_N \) the normal unit vector. In \((S, \mathbf{e}_R, \mathbf{e}_T, \mathbf{e}_N)\), the position vector of the comet is written as \( \mathbf{r} = r \mathbf{e}_R \). We remind that the change of basis from \((S, \mathbf{e}_R, \mathbf{e}_T, \mathbf{e}_N)\) to \((S, \mathbf{x}, \mathbf{y}, \mathbf{z})\) is obtained by performing as usual three successive frame rotations with angles \( \Omega, i, f + \omega \) where \( f \) is the true anomaly.

Let \( \mathbf{a} \) be a perturbing acceleration of the comet with components \((R, T, N)\) in \((S, \mathbf{e}_R, \mathbf{e}_T, \mathbf{e}_N)\). Using the classical Gauss equations associate to the perturbed two-body problem Sun-Comet, we obtain the time variation of orbital elements as follow:

\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} \left[ e \sin f R + (1 + e \cos f)T \right], \\
\frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na} \left[ \sin f R + \left( \cos f + \frac{e + \cos f}{1 + e \cos f} \right) T \right], \\
\frac{di}{dt} &= \frac{\sqrt{1-e^2} \cos(f + \omega)}{an(1 + e \cos f)} N, \\
\frac{d\Omega}{dt} &= \frac{\sqrt{1-e^2} \sin(f + \omega)}{an \sin i(1 + e \cos f)} N, \\
\frac{d\omega}{dt} &= \frac{\sqrt{(1-e^2)}}{nae} \left[ -\cos f R + \left( \frac{2 + e \cos f}{1 + e \cos f} \right) \sin f T \right] - \cos i \frac{d\Omega}{dt}, \\
\frac{dM}{dt} &= n(t) - \frac{2(1-e^2)}{n a^2(1 + e \cos f)} R - \sqrt{(1-e^2)} \left( \frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} \right) .
\end{align*}
\]

If \( \mathbf{a} \) is determined by a perturbing function \( U \) as \( \mathbf{a} = \nabla U \) where \( \nabla \) denoting the gradient, then we have the classical relation between the perturbing force and the components \((R, T, N)\) as follow:

\[
\begin{align*}
R &= \frac{a}{r} \frac{\partial U}{\partial a}, \\
T &= \frac{1}{r} \frac{\partial U}{\partial \omega}, \\
N &= \frac{1}{r \sin(f + \omega)} \frac{\partial U}{\partial i}.
\end{align*}
\]

Denoting by \( \{c_i\}_{i=1,\ldots,6} = \{a, e, i, \Omega, \omega, M\} \), the secular variation of the orbital elements are obtained for all \( i = 1, \ldots, 6 \) as follow:

\[
\langle \frac{dc_i}{dt} \rangle = \frac{1}{P} \int_0^P \frac{dc_i}{dt} dt = \frac{1}{2\pi} \int_0^{2\pi} \frac{dc_i}{dt} (1 - e^2)^{3/2} df.
\]

2.2. **Non-gravitational and MOND perturbations.** In [11], the authors developed a semi-empirical model of the non-gravitational forces applied to comet. The illuminated surface of a spherical nucleus is assumed to be isotropically outgassing. Marsden et al. (1973) [11] introduced the dimensionless function \( g(r) \). This function represents the variation in the sublimation rate as a function of the heliocentric distance of the comet and is based on the observation of water sublimation rate curve. The non-gravitational perturbing acceleration is
given in \((S, e_R, e_T, e_N)\) by its components

\[ R_{NG} = A_1 g(r), \quad T_{NG} = A_2 g(r), \quad N_{NG} = A_3 g(r) \]

where

\[ g(r) = 0.111262 \left( \frac{r}{2.808} \right)^{-2.15} \left( 1 + \left( \frac{r}{2.808} \right)^{5.093} \right)^{-4.6142} \]

and \(A_1, A_2, A_3\) are constants obtained by fitting the astrometrical positions of the considered comet together with the orbital elements.

In [2], the authors formulated the MOND theory in order to test it in the Solar System. This formulation is done in a way that we can see this modification of the Newtonian gravity close to the Solar System (for domain with Solar distance \(r \lesssim r_0 \approx 7100\) AU) as a perturbation of the classical two-body problem. The perturbing acceleration \(a_{MOND}\) caused by the MOND theory in their formulation has two main perturbing parts \(a_{MOND,Q_2}\) and \(a_{MOND,Q_3}\). We are only considering \(a_{MOND,Q_2}\) as a first model. The MOND perturbation is determined by a perturbing function given by ([2, Eq. 40])

\[ U_{MOND,Q_2} = \frac{1}{2} r^2 Q_2(r) \left( (e \cdot e_R)^2 - \frac{1}{3} \right) \]

where \(Q_2\) is a function of \(r\) and \(e\) is the direction of the galactic center. In fact, as we consider only comets with maximum distance \(r \approx 100\)AU, \(Q_2\) is observed to be constant for \(r \leq 100\) AU. Indeed, according to Fig. 4 and 5 of [2], we can see that between 0 and 1000 AU, \(Q_2\) is varying from \(3.83 \times 10^{-26}\) to \(3.80 \times 10^{-26}\) s\(^{-2}\).

The values of \(Q_2\) depends on the MOND function \(\mu\) chosen. In [2], the authors deal with multiple MOND function such as

\[ \mu_n(y) = \frac{y}{(1 + y^n)^{1/n}}, \text{ for any integer } n \geq 1, \]

\[ \mu_{\text{exp}}(y) = 1 - e^{-y}, \quad \mu_{\text{TeVeS}}(y) = \frac{\sqrt{1 + 4y} - 1}{\sqrt{1 + 4y} + 1}. \]

For these MOND functions, the values of \(Q_2\) is given in the following:

| MOND function | \(\mu_1(y)\) | \(\mu_2(y)\) | \(\mu_5(y)\) | \(\mu_{20}(y)\) | \(\mu_{\text{exp}}(y)\) | \(\mu_{\text{TeVeS}}(y)\) |
|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(Q_2\) [s\(^{-2}\)] | \(3.8 \times 10^{-26}\) | \(2.2 \times 10^{-26}\) | \(7.4 \times 10^{-27}\) | \(2.1 \times 10^{-27}\) | \(3.0 \times 10^{-26}\) | \(4.1 \times 10^{-26}\) |
In what follow, we denote respectively by $\beta$ and $\lambda$ the latitude and longitude of the galactic center in the heliocentric coordinate system, then we obtain the expression of $U_{MOND,Q_2}$ as

$$U_{MOND,Q_2} = \frac{Q_2}{6} \left(3(x \cos \beta \cos \lambda + y \cos \beta \sin \lambda + z \sin \beta)^2 - (x^2 + y^2 + z^2)\right),$$

where $x, y$ and $z$ are the coordinates of the comet in the frame $(S, \mathbf{x}, \mathbf{y}, \mathbf{z})$. In order to obtain the expression of the perturbing accelerations $a_{MOND,Q_2}$ in $(S, \mathbf{e}_R, \mathbf{e}_T, \mathbf{e}_N)$, we express $x, y$ and $z$ in function of the orbital elements and then we use formulas [3] for each perturbing functions. Straightforward computations leads to the expressions of $R_{MOND,Q_2}$, $T_{MOND,Q_2}$ and $N_{MOND,Q_2}$ written as

$$R_{MOND,Q_2} = \frac{Q_2 (1 - e^2)}{3(1 + e \cos f)} \times \left(3 \left(\cos \beta (\sin \lambda \cos i \cos \Omega \sin (f + \omega) + \sin \Omega \cos (f + \omega)) + \cos \lambda (\cos \Omega \cos (f + \omega) - \cos i \sin \Omega \sin (f + \omega))\right) + \sin \beta \sin i \sin (f + \omega)\right)^2 - 1,$$

$$T_{MOND,Q_2} = -\frac{Q_2 a(1 - e^2)}{1 + e \cos f} \times \left(\cos(\beta) \sin(f + \omega) \cos(\lambda - \Omega)ight.$$

$$- \cos(f + \omega) (\cos(\beta) \cos(i) \sin(\lambda - \Omega) + \sin(\beta) \sin(i))\Bigg) \times \left(\cos(\beta) \cos(i) \sin(f + \omega) \sin(\lambda - \Omega) + \cos(f + \omega) \cos(\lambda - \Omega)) + \sin(\beta) \sin(i) \sin(f + \omega)\right),$$

$$N_{MOND,Q_2} = \frac{Q_2 a(1 - e^2)}{1 + e \cos f} \left(\sin(\beta) \cos(i) - \cos(\beta) \sin(i) \sin(\lambda - \Omega))\right) \times \left(\cos(\beta) \cos(i) \sin(f + \omega) \sin(\lambda - \Omega) + \cos(f + \omega) \cos(\lambda - \Omega)) + \sin(\beta) \sin(i) \sin(f + \omega)\right).$$

We now have all the expressions of the perturbing force so we can deduce the secular variations of the orbital elements of the comet which are given as follow :

$$\left\langle \frac{dc_i}{dt} \right\rangle = \left\langle \frac{dc_i}{dt} \right\rangle_{NG} + \left\langle \frac{dc_i}{dt} \right\rangle_{MOND,Q_2}. $$
for all $i = 2, ..., 6$. For the secular variation of the semi-major axis we only have
\begin{equation}
\langle \frac{da}{dt} \rangle = \langle \frac{da}{dt} \rangle_{NG}
\end{equation}

because the secular variation caused by MOND on the semi-major axis is zero.

3. Application to three comets

We now compute the effects of the non-gravitational and MOND perturbations on three comets. We computed analytically the MONDian part and numerically the non-gravitational part due to the expression of the equations.

The three comets were chosen in function of their orbital parameters. As we know that the MONDian effects are more bigger for objects far from the sun we choose 1P/Halley and 153P/Ikeya-Zhang because they have one of the largest semi-major axis known for periodic comets and are relatively well known (8154 and 1954 astrometrical observations respectively). Oppositely, we choose 2P/Encke for its small semi-major axis to be able to make a comparison of the two types of comet.

The orbital elements and non-gravitational parameters used for the computation are provided in the table 2. This orbital elements are given by the database of JPL Small-Bodies Browser\[1\]. The table only gives approximate values. For non-truncated values and uncertainties, we refer to the JPL Small-Bodies Browser website. It can be noted that, for most of the comet, the non-gravitational parameter $A_3$ is considered as zero. Indeed, the non-gravitational perturbation in this direction is very weak and can not be solved by the fit (see \[8\] and \[11\]).

Using the values of the latitude $\beta$ and longitude $\lambda$ of the galactic center in the fixed heliocentric reference frame which are $\beta = -5.5^\circ$ and $\lambda = -93.2^\circ$ (see for example \[1\]), the results of the computation are given in tables 3, 4 and 5. The secular variations of the angles induced by MOND are in range of a few milli-arc-seconds per century. We can note that for 2P/Encke (short orbit), the MONDian effects are very small and may be negligible in front of the non-gravitational perturbation. Oppositely, for 1P/Halley and 153P/Ikeya-Zhang (large orbit), the MONDian effects are much bigger and can not be neglected in front of non-gravitational perturbations. As noticed in \[2\], the effects of the MOND perturbations decreases by a factor $\approx 10$ for the MOND function $\mu_n$ between $n = 2$ and $n = 20$. But the effects are in the same range for $\mu_1$, $\mu_{exp}$ and $\mu_{T_eV_eS}$.

\[1\]https://ssd.jpl.nasa.gov/sbdb.cgi
According to table 3, 4 and 5, the cometary orbits are precessing under non-gravitational perturbations but also because of the modified dynamic. As the non gravitational parameter $A_3$ is zero, there is no secular variation of the inclination of the orbit and the ascending node longitude due to the outgassing the comet. The variation of these elements are due to the modified dynamic. We also computed the effects of MOND on the eccentricity but it was not significant ($\langle \frac{de}{dt} \rangle$ takes value around $10^{-10}$- $10^{-11}$ \( \text{cy}^{-1} \)).

For the well-known comet 1P/Halley\textsuperscript{2}, the MONDian effects are in the same range than the precision of the orbital element determination ($\sigma_i = 24.4$ mas, $\sigma_\omega = 42.2$ mas and $\sigma_\Omega = 32.6$ mas, see JPL Small Bodies browser website for more details). As the dynamical model of comet are continually improving, it will be soon possible to detect and quantify these effects in cometary orbits.

We also computed the perturbation induced by $a_{MOND,Q3}$. To give an idea of the effect, it is about $10^{-11}$ to $10^{-12}$ mas/cy for 153P/Ikeya-Zhang angles and for the two other comets it is much more less than these values. Under the precision of the observations, it is completely negligible.

**Table 2.** Orbital elements and non-gravitational parameters of the comets used in the computation from JPL

|                  | 2P/Encke | 1P/Halley | 153P/Ikeya-Zang |
|------------------|----------|-----------|-----------------|
| $P$ [yr]         | 3.30     | 75.31     | 366.51          |
| $a$ [AU]         | 2.215    | 17.834    | 51.214          |
| $e$              | 0.848    | 0.967     | 0.990           |
| $i$ [deg]        | 11.8     | 162.3     | 28.1            |
| $\omega$ [deg]   | 186.5    | 111.3     | 34.7            |
| $\Omega$ [deg]   | 334.6    | 58.4      | 93.4            |
| $n$ [deg.day\(^{-1}\)] | 0.299    | 0.013     | 0.003           |
| $q$ [AU]         | 0.336    | 0.586     | 0.507           |
| $A_1$ [AU.day\(^{-2}\)] | $1.58 \times 10^{-10}$ | $2.70 \times 10^{-10}$ | $3.33 \times 10^{-9}$ |
| $A_2$ [AU.day\(^{-2}\)] | $-5.05 \times 10^{-11}$ | $1.56 \times 10^{-10}$ | $-3.51 \times 10^{-10}$ |
| $A_3$ [AU.day\(^{-2}\)] | 0.0      | 0.0       | 0.0             |

\textsuperscript{2}1P/Halley as been seen by the ESA spacecraft Giotto in 1986
Table 3. Results for the secular variations of 2P/Encke due to the MOND and non-gravitational perturbations. $\langle \frac{da}{dt} \rangle$ is given in astronomical (AU/cy), $\langle \frac{de}{dt} \rangle$ is given in cy$^{-1}$ and all results for the angles are given in $\text{milli}$-arc-seconds per century (mas/cy).

| MOND function | $\langle \frac{da}{dt} \rangle$ | $\langle \frac{de}{dt} \rangle$ | $\langle \frac{di}{dt} \rangle$ | $\langle \frac{d\Omega}{dt} \rangle$ | $\langle \frac{d\omega}{dt} \rangle$ |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\mu_1(y)$    | -               | -               | 0.125           | -0.062          | -0.23           |
| $\mu_2(y)$    | -               | -               | 0.072           | -0.036          | -0.13           |
| $\mu_5(y)$    | -               | -               | 0.024           | -0.012          | -0.04           |
| $\mu_{20}(y)$ | -               | -               | 0.007           | -0.003          | -0.01           |
| $\mu_{\exp}(y)$ | -           | -               | 0.099           | -0.049          | -0.18           |
| $\mu_{\text{TeVeS}}(y)$ | -           | -               | 0.135           | -0.067          | -0.25           |

Non-gravitational $-6.76 \times 10^{-5}$ $-3.87 \times 10^{-6}$ - - -3477.51

Table 4. Results for the secular variations of 1P/Halley due to the MOND and non-gravitational perturbations. $\langle \frac{da}{dt} \rangle$ is given in astronomical (AU/cy), $\langle \frac{de}{dt} \rangle$ is given in cy$^{-1}$ and all results for the angles are given in $\text{milli}$-arc-seconds per century (mas/cy).

| MOND function | $\langle \frac{da}{dt} \rangle$ | $\langle \frac{de}{dt} \rangle$ | $\langle \frac{di}{dt} \rangle$ | $\langle \frac{d\Omega}{dt} \rangle$ | $\langle \frac{d\omega}{dt} \rangle$ |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\mu_1(y)$    | -               | -               | 3.06            | -33.95          | -19.07          |
| $\mu_2(y)$    | -               | -               | 1.77            | -19.66          | -11.04          |
| $\mu_5(y)$    | -               | -               | 0.60            | -6.61           | -3.71           |
| $\mu_{20}(y)$ | -               | -               | 0.17            | -1.88           | -1.05           |
| $\mu_{\exp}(y)$ | -           | -               | 2.41            | -26.80          | -15.05          |
| $\mu_{\text{TeVeS}}(y)$ | -           | -               | 3.30            | -36.63          | -20.572         |

Non-gravitational $2.62 \times 10^{-3}$ $4.33 \times 10^{-6}$ - - -508.72

4. Conclusion

This work shows that the effects of MOND theory are not negligible in front of other small perturbations like non-gravitational perturbations on the cometary orbits. In agree with the study of [2], the MONDian effects are stronger for large orbit than for short orbits.

If the MOND theory is validated, it is really important to take into account its effects on the secular variation of the cometary orbital elements, especially in the case of Halley family
Table 5. Results for the secular variations of 153P/Ikeya-Zang due to the MOND and non-gravitational perturbations. \( \langle \frac{da}{dt} \rangle \) is given in astronomical (AU/cy), \( \langle \frac{de}{dt} \rangle \) is given in cy\(^{-1} \) and all results for the angles are given in milli-arc-seconds per century (mas/cy).

| MOND function | \( \langle \frac{da}{dt} \rangle \) | \( \langle \frac{de}{dt} \rangle \) | \( \langle \frac{di}{dt} \rangle \) | \( \langle \frac{d\Omega}{dt} \rangle \) | \( \langle \frac{d\omega}{dt} \rangle \) |
|--------------|----------------|----------------|----------------|----------------|----------------|
| \( \mu_1(y) \) | -              | -              | -10.57         | -45.30         | -43.41         |
| \( \mu_2(y) \) | -              | -              | -6.12          | -26.22         | -25.13         |
| \( \mu_5(y) \) | -              | -              | -2.06          | -8.82          | -8.45          |
| \( \mu_{20}(y) \) | -              | -              | -0.58          | -2.50          | -2.40          |
| \( \mu_{\exp}(y) \) | -              | -              | -8.34          | -35.76         | -34.27         |
| \( \mu_{\text{TeVeS}}(y) \) | -              | -              | -11.40         | -48.87         | -46.83         |
| Non-gravitational | 0.17           | 3.23 \times 10^{-5} | -              | -              | -5168.5        |

comet that have large orbit and are more affected. Some long-term studies of cometary orbits has already been managed for example to constrain the Oort cloud density (for example [6]). Generally, this kind of studies take into account the galactic tide, the stars encounters and the non-gravitational effects of the new comets introduced in the inner solar system. According to this work and [7], it would be interesting to include MONDian perturbations to this kind of study to show the effects on the Oort cloud density and on the injection of new comets in the solar system.

Finally, this work show that the MONDian effects are close to the detection limit. Thanks to the space mission like Rosetta (see [15]), our understanding of cometary physic will be improved in a near future. The cometary dynamical model will be more accurate. Moreover thanks to the space mission Gaia (see [5]), we will have access to better astrometrical measurements. The orbital elements will be determined more and more accurately and probably the MONDian effects will be quantifiable and more constrained.

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