Quasi equilibrium state of expanding quantum fields and two-pion Bose-Einstein correlations in \( pp \) collisions at the LHC

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Abstract

We argue that the two-particle momentum correlation functions of high-multiplicity \( p + p \) collisions at the LHC provide a signal for a ground state structure of a quasi equilibrium state of the longitudinally boost-invariant expanding quantum field which lies in the future light cone of a collision. The physical picture is that pions are produced by the expanding quantum emitter with two different scales approximately attributed to the expanding ideal gas in local equilibrium state and ground-state condensate. Specifically, we show that the effect of suppressing the two-particle Bose-Einstein momentum correlation functions increases with increasing transverse momentum of a like-sign pion pair due to different momentum-dependence of the corresponding particle emission regions.
I. INTRODUCTION

It is firmly established now that collective phenomena in relativistic heavy ion collisions are associated with hydrodynamics [1–3]. Surprisingly, similar collective phenomena have been observed recently in high-multiplicity \( p + p \) collisions at the CERN Large Hadron Collider (LHC) [4]. It is not clear however whether such collective phenomena can be attributed to hydrodynamic evolution like in \( A + A \) collisions [5, 6]. Also, while there is some evidence that hydrodynamics can be successfully applied to describe flow-like features in high-multiplicity \( p + p \) collisions, see e.g. refs. [7–10] (for discussions whether hydrodynamics is applicable in small systems see e.g. refs. [11, 12]), till now there is no united description of one-particle momentum spectra and multi-particle momentum correlations in \( p + p \) collisions in any detail dynamical model.

Many-particle correlations play an important role in the understanding of multiparticle production mechanisms. In particular, the correlation femtoscopy method (commonly referred to as femtoscopy, or HBT interferometry) uses momentum correlations of two identical particles at low relative momenta to extract information about the space-time evolution and properties of the expanding matter in high energy nucleon and nuclear collisions, for reviews see e.g. refs. [13–19]. Because in such collisions most of produced particles are pions, the Bose-Einstein correlations of two identical pions are usually analyzed. The measured Bose-Einstein correlations of particles with some fixed momentum of a particle pair are typically described by a function with two sort of parameters: the effective radius parameters \( R_i \) (sometimes called “HBT radii”), which can be interpreted as mean (statistically averaged) distance between centers of particle emissions, and the “correlation strength” parameter \( \lambda \), the latter is also called the incoherence or chaoticity parameter. The interpretation of the latter is not unambiguous, because under the real experimental conditions \( \lambda \)-parameter can be affected by particle misidentifications, decays of long-lived resonances, and non-Gaussian features of the particle distributions. Practically, the values of \( \lambda \)-parameter obtained fitting experimental data are momentum-dependent and less than 1.

Results from femtoscopic studies of two-particle correlations from \( p + p \) collisions at the LHC have been presented in refs. [20–22]. It was found that the femtoscopic radii \(^1\) Loosely speaking, then typical interferometry radii of a detected pair are very large and cannot be resolved because the resolution requires very low difference of momenta of detected particles beyond the experimental limits, in a result, corresponding quantum statistical momentum correlations of particles are not seen.
measured in high-multiplicity $p+p$ collisions are smaller than the ones in relativistic heavy ion collisions, and decrease with increasing momentum of a pair. The latter in hydrodynamical approach can be interpreted as the decrease of “lengths of homogeneity” \cite{23,24} (sizes of the effective emission region). This is a direct consequence of generated by hydrodynamical flow $x-p$ correlations: Due to the collective flow pion pairs of higher momenta are effectively emitted from smaller regions of the source. Also, it was found that $\lambda$-parameter is essentially less than unity, and exhibits a decrease as the momentum of a like-sign pion pair increases \cite{21,22}. This seems to be at variance with the behavior of pionic $\lambda$ parameter in reliable hydrodynamical models of $A+A$ collisions, see e.g. ref. \cite{27}.

Hydrodynamic is, in essence, the local conservation equations of the expectation values of the stress-energy tensor and charge current operators. In particular, at the lowest order in the gradient expansion the mean value of the stress-energy tensor takes the ideal fluid form. It is worth to note that hydrodynamic equations can be derived using Zubarev’s formalism of non-equilibrium statistical operator \cite{28,32}, see also refs. \cite{33,36} for recent papers related to this method. The corresponding statistical operator is appropriate for reduced description of the system and can be used to calculate expectation values of relevant observables. Even if the statistical operator can be approximated by its quasi equilibrium form for non-interacting quantum fields, the corresponding expressions can demonstrate essential deviations from the ones at the global thermodynamic equilibrium with a temperature, a collective four-velocity, and a chemical potential equal to those in some space-time point, see refs. \cite{37,39}. In the present paper, using a simple and physically clear method of calculations we re-calculate particle momentum spectra and Bose-Einstein correlations first evaluated for a boost-invariant expanding free boson quantum field in refs. \cite{37,39}. It allows us, in particular, to reveal the underlying physical mechanism which is responsible for the non-trivial quasi equilibrium ground state of expanding non-interacting quantum fields. Then, unlike refs. \cite{37,39} where ground-state condensate contributions of quasi equilibrium statistical operator to particle momentum spectra were treated as physically meaningless and therefore subtracted from the particle spectra, in the present paper we argue its physical significance and possible relevance to the observed effect of suppression of the two-particle Bose-Einstein momentum correlation functions in high-multiplicity $p+p$ collisions.
II. QUASI EQUILIBRIUM STATE OF BOOST-IN Variant EXPANDING QUANTUM SCALAR FIELD

Following refs. [28–32], we define a quasi equilibrium statistical operator on Minkowski spacetime as

$$\rho = Z^{-1} \exp \left( -\int_{\sigma} d\sigma \nu(x) \beta(x) u_{\mu}(x) T^{\mu\nu}(x) \right),$$  \hspace{1cm} (1)

where $\beta(x) = 1/T(x)$ is the local inverse temperature, $u_{\mu}(x)$ is hydrodynamical four-velocity, $u_{\mu}(x)u^{\mu}(x) = 1$, $\sigma_\nu$ is a three-dimensional space-like hypersurface with a time-like normal vector $n_\nu(x)$, $T^{\mu\nu}(x)$ is the operator of an energy-momentum tensor, and $Z$ is the normalization factor making $Tr[\rho] = 1$. We assume for simplicity that a chemical potential $\mu = 0$. The expectation value of an operator $\hat{O}$ can be expressed as

$$\langle \hat{O} \rangle = Tr[\rho \hat{O}] .$$  \hspace{1cm} (2)

The quasi equilibrium statistical operator $\rho$ is determined from the maximization of the information entropy with specific constraints on the average local values of the energy-momentum density operator on the hypersurface $\sigma_\nu$: $n_\nu(x)\langle T^{\mu\nu}(x) \rangle$, see e.g. refs. [28–36]. Notice that because field operators commute for space-like separations, the corresponding operators are clearly local observables.

To apply this formalism to high-multiplicity $p+p$ scatterings at the LHC, we take into account that hydrodynamics with boost-invariance in the longitudinal direction [40] give reliable results for one-particle momentum spectra in $p+p$ collisions [9, 10]. Also, aiming to perform calculations analytically in some tractable approximations, we neglect finite-size effects in transverse directions (collective expansion, inhomogeneities, etc.). Then, assuming that matter produced in a high energy $p+p$ collision is locally restricted to the light cone with beginning at $t = z = 0$ plane of the Minkowski spacetime manifold, we utilize Bjorken coordinates $(\tau, \eta)$ instead of Cartesian ones, $(t, z)$. Namely, we define

$$t = \tau \cosh \eta ,$$  \hspace{1cm} (3)

$$z = \tau \sinh \eta .$$  \hspace{1cm} (4)

\footnote{For the Minkowskian metric tensor we use the convention $g^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$.}
The two other coordinates \( r_T = (r_x, r_y) \) are the Cartesian ones. The Minkowski line element restricted to the light cone is

\[
ds^2 = dt^2 - dr_T^2 - dz^2 = d\tau^2 - \tau^2 d\eta^2 - dr_T^2. \tag{5}
\]

Such a coordinate system is nothing but the Milne frame \([41]\) and is associated with the system of the (hypothetical) observers which move with different but constant longitudinal velocities in such a way that their world lines begin at \( z = t = 0 \). Then boost-invariant hydrodynamic four-velocity \( u_\mu \) reads

\[
u^\mu(x) = (\cosh \eta, 0, 0, \sinh \eta). \tag{6}
\]

Because small systems created in \( p+p \) collisions do not exhibit prolonged post-hydrodynamical kinetic stage of hadronic rescatterings, we assume that particle momentum spectra are frozen immediately after emission (so called sharp freeze-out assumption \([42]\)) at a hypersurface with constant energy density in the comoving coordinate system. Then \( \beta(x) \) is constant on the corresponding hypersurface. Such a hypersurface is defined by constant \( \tau = \sqrt{t^2 - z^2} \) \([40]\), that is proper time of the (hypothetical) inertial local observers comoving with a fluid element with a constant rapidity \( \eta \) and constant transverse coordinates \( r_T \). This implies that

\[
n^\mu(x) = u^\mu(x), \tag{7}
\]

and

\[
d\sigma = \tau d\eta dr_x dr_y. \tag{8}
\]

It is worth to note that \( \tau \) also controls a value of the collective velocity space-time gradients,

\[
\partial_\mu u^\mu = \frac{1}{\tau}. \tag{9}
\]

To proceed further, one needs to specify an energy-momentum tensor in eq. (1). Because main aim of this paper is to reveal a ground-state condensate associated with quasi equilibrium expansion of quantum fields, and then study its effects on the single-particle momentum spectra and the two-particle momentum correlation functions, we neglect here field self-interactions and use simple scalar quantum field model with the classical action

\[
S = \int dt d^3 r \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left( \frac{\partial \phi}{\partial r} \right)^2 - \frac{m^2}{2} \phi^2 \right] \equiv \int dt d^3 r L, \tag{10}
\]
where $L$ is the corresponding Lagrangian density. Then

$$T^{\mu\nu}(x) = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} L,$$

and, using eqs. (3), (4), (6), (7), (9), and (11) we get

$$n_\mu u_\nu T^{\mu\nu}(x) = u_\mu u_\nu T^{\mu\nu}(x) = \left( \frac{\partial \phi}{\partial \tau} \right)^2 - L,$$

where we took into account that $u_\mu \partial^\mu = \partial_\tau$. The Lagrangian density $L$ in the Bjorken coordinates (then $dtd^3r = \tau d\tau d\eta d^2r_T$) is

$$L = \frac{1}{2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 - \frac{1}{2} \frac{1}{\tau^2} \left( \frac{\partial \phi}{\partial \eta} \right)^2 - \frac{1}{2} \left( \frac{\partial \phi}{\partial r_T} \right)^2 - \frac{1}{2} m^2 \phi^2.$$

It provides

$$n_\mu u_\nu T^{\mu\nu}(x) = \frac{1}{2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} \frac{1}{\tau^2} \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial r_T} \right)^2 + \frac{1}{2} m^2 \phi^2.$$

It follows from eq. (13) that conjugate momentum with respect to $\tau$ is $\Pi^{[\tau]} = \frac{\partial \phi}{\partial \tau}$. Then one can notice that

$$\int d\sigma n_\nu(x) u_\mu(x) T^{\mu\nu}(x) = H^{[\tau]},$$

and as a result

$$\rho = Z^{-1} \exp \left( -\beta H^{[\tau]} \right),$$

where $H^{[\tau]}$ is the Hamiltonian that generates translation in the time-like direction with respect to $\tau$. One sees that such a Hamiltonian (and the corresponding metrics, see eq. (5)) is explicitly $\tau$-dependent. Despite the metrics under the coordinate transformation (3), (4) is reduced to the flat Minkowski metrics, it is not the case for $H^{[\tau]}$ which is not reduced to $H^{[t]}$.

It follows immediately from eq. (10) that $\phi(x)$ satisfies to the Klein-Gordon equation of motion

$$(\Box - m^2) \phi(x) = 0,$$

where $\Box = -\partial_\mu \partial^\mu$ is the d’Alembert operator associated with the Minkowski spacetime. It is well known that solution of eq. (17) can be written as

$$\phi(x) = \int \frac{d^3p}{\sqrt{2\omega_p (2\pi)^{3/2}}} \left( e^{-i\omega_p t - ip \vec{r}} a(\vec{p}) + e^{i\omega_p t - ip \vec{r}} a^\dagger(\vec{p}) \right),$$
where $\omega_p = \sqrt{p^2 + m^2}$. The conjugated field momentum at the hypersurface $t = \text{const}$ is $\Pi_t = \frac{\partial \phi}{\partial t}$. The quantization prescription at such a hypersurface,

$$[\phi(x), \Pi_t(x')] = i\delta^{(3)}(r - r'),$$

(19)

means that functions $a^\dagger(p)$ and $a(p)$ become creation and annihilation operators, respectively, which satisfy the following canonical commutation relations:

$$[a(p), a^\dagger(p')] = \delta^{(3)}(p - p'),$$

(20)

and $[a(p), a(p')] = [a^\dagger(p), a^\dagger(p')] = 0$.

It is worth to note that plane-wave mode representation used in eq. (18) has special meaning in our approach because corresponding particles are the observed ones, and therefore we are interested in expectation values for appropriate products of operators $a^\dagger(p)$ and $a(p)$. With this aim, it is convenient to find representation of the canonical commutation relations (with corresponding mode functions) that (at least, approximately) diagonalizes $H^{[r]}$. Because finally we are interested in expectation values for $a^\dagger(p)$ and $a(p)$, such a representation should be explicitly related with the plane mode representation (18). In a most simple way it can be done if we just rewrite eq. (18) for appropriate mode functions.

With this aim let us first introduce momentum rapidity $\theta$ and transverse mass $m_T$ instead of one-particle energy $\omega_p$ and longitudinal momentum $p_z$, then

$$\omega_p = m_T \cosh \theta,$$

(21)

$$p_z = m_T \sinh \theta,$$

(22)

$$m_T = \sqrt{p_T^2 + m^2},$$

(23)

where $p_T = (p_x, p_y)$ is the transverse momentum. Then we introduce new operators $\alpha(p_T, \theta)$, $\alpha^\dagger(p_T, \theta)$ as

$$\alpha(p_T, \theta) = (m_T \cosh \theta)^{1/2} a(p),$$

(24)

$$\alpha^\dagger(p_T, \theta) = (m_T \cosh \theta)^{1/2} a^\dagger(p),$$

(25)

with the commutation relation

$$[\alpha(p_T, \theta), \alpha^\dagger(p_T', \theta')] = \delta(\theta - \theta')\delta^{(2)}(p_T - p_T').$$

(26)
The solution \[18\] of the Klein-Gordon equation \[17\] can be expressed in terms of these new operators as follows
\[
\phi(x) = \int \frac{d\theta d^2p_T}{\sqrt{2(2\pi)^{3/2}}} \left[ \exp(-im_T \cosh \theta t + im_T \sinh \theta z + i\mathbf{p}_T \cdot \mathbf{r}_T)\alpha(p_T, \theta) + \right.
\]
\[
\left. \exp(im_T \cosh \theta t - im_T \sinh \theta z - i\mathbf{p}_T \cdot \mathbf{r}_T)\alpha^\dagger(p_T, \theta) \right].
\]
(27)

The next step is to introduce operators \(b(p_T, \mu), b^\dagger(p_T, \mu)\) that are related to operators \(\alpha(p_T, \theta), \alpha^\dagger(p_T, \theta)\) through the following formulas
\[
\alpha(p_T, \theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\mu \theta} b(p_T, \mu) d\mu,
\]
(28)
\[
\alpha^\dagger(p_T, \theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i\mu \theta} b^\dagger(p_T, \mu) d\mu.
\]
(29)

One can see that
\[
[b(p_T, \mu), b^\dagger(p_T', \mu')] = \delta(\mu - \mu')\delta^{(2)}(p_T - p'_T),
\]
(30)
with all other commutators vanishing. Using the definitions \[28\], \[29\] one can rewrite eq. \[27\] to the form
\[
\phi(x) = \int \frac{d\theta d^2p_T d\mu}{\sqrt{2(2\pi)^{3/2}}} \left[ \exp(-im_T \tau \cosh(\theta - \eta) + i\mathbf{p}_T \cdot \mathbf{r}_T + i\mu \theta) b(p_T, \mu) + \right.
\]
\[
\left. \exp(im_T \tau \cosh(\theta - \eta) - i\mathbf{p}_T \cdot \mathbf{r}_T - i\mu \theta) b^\dagger(p_T, \mu) \right].
\]
(31)

Performing change of integration variables in eq. \[31\], \(\theta = \vartheta + \eta\), we get
\[
\phi(x) = \int_{-\infty}^{+\infty} \frac{d^2p_T d\mu}{4\pi \sqrt{2}} \left[ -i \exp(\mu \pi/2 + i\mu \eta + i\mathbf{p}_T \cdot \mathbf{r}_T) H^{(2)}_{i\mu}(m_T \tau) b(p_T, \mu) + \right.
\]
\[
\left. i \exp(-\mu \pi/2 - i\mu \eta - i\mathbf{p}_T \cdot \mathbf{r}_T) H^{(1)}_{i\mu}(m_T \tau) b^\dagger(p_T, \mu) \right],
\]
(32)
where \(H^{(2)}_{i\mu}(m_T \tau)\) and \(H^{(1)}_{i\mu}(m_T \tau)\) are the Hankel functions \[43,\]
\[
H^{(2)}_{i\mu}(m_T \tau) = -\frac{1}{i\pi} \exp(-\mu \pi/2) \int_{-\infty}^{+\infty} d\vartheta \exp(-im_T \tau \cosh \vartheta + i\mu \vartheta),
\]
(33)
\[
H^{(1)}_{i\mu}(m_T \tau) = \frac{1}{i\pi} \exp(\mu \pi/2) \int_{-\infty}^{+\infty} d\vartheta \exp(im_T \tau \cosh \vartheta - i\mu \vartheta).
\]
(34)

Using eqs. \[30\], \[32\] and accounting for properties of the Hankel functions, one can see that such a representation realizes the quantization procedure on the hypersurface \(\tau = \text{const}\):
\[
[\phi(x), \tau \frac{\partial}{\partial \tau}(x')] = i\delta(\eta - \eta')\delta^{(2)}(r_T - r'_T).
\]
(35)
The mode representation (32) in the future light cone is well known, see e.g. refs. [41, 44–46], and the vacuum state with respect to operators $b$ and $b^\dagger$ is exactly the usual Minkowski vacuum.

Substituting (32) into (14) and performing integrations over space-time variables we bring $H^{[\tau]}$, see eqs. (8), (14), and (15), to a form

$$H^{[\tau]} = \frac{\tau \pi}{8} \int_{-\infty}^{+\infty} d^2 p_T d\mu [G_{(2,1)}(b^\dagger(p_T, \mu))b(p_T, \mu) + b(p_T, \mu)b^\dagger(p_T, \mu)] - e^{\mu \pi} G_{(2,2)}(p_T, \mu)b(-p_T, -\mu) - e^{-\mu \pi} G_{(1,1)}(p_T, \mu)b^\dagger(-p_T, -\mu),$$

(36)

where we introduced notations

$$G_{(l,n)} = [\partial_{\tau} H^{(l)}_{i\mu}(m_T \tau)] [\partial_{\tau} H^{(n)}_{i\mu}(m_T \tau)] + \left(\frac{\mu^2}{\tau^2} + m_T^2\right) H^{(l)}_{i\mu}(m_T \tau) H^{(n)}_{i\mu}(m_T \tau).$$

(37)

Equations (16), (36), and (37) provide the basis for a diagonalization procedure and particle momentum spectrum calculations in the next Section.

### III. GROUND-STATE CONDENSATE AND ITS EFFECTS ON PARTICLE MOMENTUM SPECTRA

We begin by noting that freeze-out of pion momentum spectra in high-multiplicity $p+p$ collisions happens when $\tau \gtrsim 1$ fm (and when $\beta \simeq 1/m$ where $m$ is pion mass). Then, to simplify matters we can utilize appropriate approximations to the Hankel functions in eq. (37) and then perform approximate diagonalization of $H^{[\tau]}$, see eq. (36). Namely, let us assume that $m_T \tau \gg 1$. Then, applying the saddle-point approximation to the Hankel functions we get

$$H^{(2)}_{i\mu}(m_T \tau) \approx \left(\frac{2}{\pi m_T \tau}\right)^{1/2} \left(1 + \frac{\mu^2}{m_T^2 \tau^2}\right)^{-1/4} \times \exp(i\pi/4 - \mu \pi/2 - im_T \tau \sqrt{1 + \mu^2/(m_T \tau)^2 + i\mu \vartheta}),$$

(38)

$$H^{(1)}_{i\mu}(m_T \tau) \approx \left(\frac{2}{\pi m_T \tau}\right)^{1/2} \left(1 + \frac{\mu^2}{m_T^2 \tau^2}\right)^{-1/4} \times \exp(-i\pi/4 + \mu \pi/2 + im_T \tau \sqrt{1 + \mu^2/(m_T \tau)^2 - i\mu \vartheta}).$$

(39)

where a value of the saddle-point $\vartheta$ is defined by the equation

$$\frac{d}{d\vartheta}[-im_T \tau \cosh \vartheta + i\mu \vartheta] = 0,$$

(40)
and is given by
\[
\sinh \vartheta_\sigma = \frac{\mu}{m_T \tau}.
\] (41)

Then, after some simple but length calculations we get in the leading order in \((m_T \tau)^{-1}\)
\[
H^{[\tau]} \approx \int_{-\infty}^{+\infty} d^2 p_T d\mu [(m_T \cosh \vartheta_\sigma) c^\dagger(p_T, \mu) c(p_T, \mu) +
\frac{1}{4\tau}(c(p_T, \mu) c(-p_T, -\mu) + c^\dagger(p_T, \mu) c^\dagger(-p_T, -\mu))],
\] (42)

where we omitted a constant term taking into account that such a term is canceled in the expression for the statistical operator \([16]\). Also, we included phase factors in the correspondingly re-defined annihilation and creation operators,
\[
c(p_T, \mu) = b(p_T, \mu) \exp \left( -\frac{i}{8\tau m_T \cosh \vartheta_\sigma} - \frac{i}{2\tau} m_T \cosh \vartheta_\sigma + \frac{i}{2} \mu \vartheta_\sigma \right),
\] (43)
\[
c^\dagger(p_T, \mu) = b^\dagger(p_T, \mu) \exp \left( \frac{i}{8\tau m_T \cosh \vartheta_\sigma} + \frac{i}{2\tau} m_T \cosh \vartheta_\sigma - \frac{i}{2} \mu \vartheta_\sigma \right).
\] (44)

One can see that \(H^{[\tau]}\) contains combinations of operators such as \(c^\dagger(p_T, \mu) c^\dagger(-p_T, -\mu)\) and \(c(p_T, \mu) c(-p_T, -\mu)\) which correspond to the creation and destruction of two particles with zero total momentum, respectively, due to the expansion. It is interesting to note similarity of the expression \((42)\) to the corresponding expressions which describe evolution of a scalar field in an expanding Universe, see e.g. refs. \([47, 48]\).

The Hamiltonian \((42)\) is rather easy to diagonalize by means of the canonical Bogolyubov transformations with real coefficients. The diagonalization is obtained by defining a new set of creation and destruction operators \(\xi^\dagger\) and \(\xi\):
\[
\xi(p_T, \mu) = \frac{c(p_T, \mu) - A(p_T, \mu) c^\dagger(-p_T, -\mu)}{\sqrt{1 - A^2(p_T, \mu)}},
\] (45)
\[
\xi^\dagger(p_T, \mu) = \frac{c^\dagger(p_T, \mu) - A(p_T, \mu) c(-p_T, -\mu)}{\sqrt{1 - A^2(p_T, \mu)}},
\] (46)

with canonical commutation relations
\[
[\xi(p_T, \mu), \xi^\dagger(p_T', \mu')] = \delta(\mu - \mu') \delta^{(2)}(p_T - p_T'),
\] (47)

and \([\xi^\dagger(p_T, \mu), \xi(p_T', \mu')] = [\xi(p_T, \mu), \xi(p_T', \mu')] = 0\). Then according to eqs. \([45], [46]\)
\[
c(p_T, \mu) = \frac{\xi(p_T, \mu) + A(p_T, \mu) \xi^\dagger(-p_T, -\mu)}{\sqrt{1 - A^2(p_T, \mu)}},
\] (48)
\[
c^\dagger(p_T, \mu) = \frac{\xi^\dagger(p_T, \mu) + A(p_T, \mu) \xi(-p_T, -\mu)}{\sqrt{1 - A^2(p_T, \mu)}}.
\] (49)
Substituting (48) and (49) into eq. (42) allows us to diagonalize $H^{[\tau]}$ in operators $\xi^\dagger$ and $\xi$. Such a diagonalization implies that $A(p_T, \mu)$ is a solution of the quadratic equation. Choosing the solution which tends to zero when $m_T \tau$ tends to infinity, we get

$$A(p_T, \mu) = -\frac{1}{4 \tau m_T \cosh \vartheta_\sigma}. \quad (50)$$

Under this transformation the Hamiltonian $H^{[\tau]}$ in the leading order in $(m_T \tau)^{-1}$ takes the form

$$H^{[\tau]} \approx \int_{-\infty}^{+\infty} d^2 p_T d\mu (m_T \cosh \vartheta_\sigma) \xi^\dagger(p_T, \mu) \xi(p_T, \mu). \quad (51)$$

A direct consequence of eqs. (48) and (49) is that the notion of a vacuum is not unique for "c" and "ξ" particles. Namely, the vacuum of "c" particles coincides with the ordinary Minkowski vacuum defined with respect to the plane-wave modes. On the other hand, vacuum with respect to "ξ" particles is well-known in the context of quantum optics two-mode squeezed state (see e.g. refs. 50, 51) of "c" particles. Therefore, ground state of the Hamiltonian $H^{[\tau]}$ is in leading order in $(m_T \tau)^{-1}$ a highly entangled state (condensate) of correlated pairs of $c^\dagger(p_T, \mu)$ and $c^\dagger(-p_T, -\mu)$ quanta with zero total momentum.

We now come back to our main objective: use the quasi equilibrium statistical operator $\rho$, see eq. (16), to calculate the two-boson Bose-Einstein correlation function. For such calculations it is convenient to use the method proposed in refs. 52, 53 (see also refs. 54, 55). First, let us introduce operator

$$\xi^\dagger(p_T, \mu, \beta) = e^{-\beta H^{[\tau]}} \xi^\dagger(p_T, \mu) e^{\beta H^{[\tau]}}. \quad (52)$$

It follows from

$$\frac{\partial \xi^\dagger(p_T, \mu, \beta)}{\partial \beta} = [\xi^\dagger(p_T, \mu, \beta), H^{[\tau]}] = -m_T \cosh \vartheta_\sigma \xi^\dagger(p_T, \mu, \beta) \quad (53)$$

that

$$\xi^\dagger(p_T, \mu, \beta) = e^{-\beta m_T \cosh \vartheta_\sigma} \xi^\dagger(p_T, \mu). \quad (54)$$

Using trace invariance under the cyclic permutation of an operator, we get

$$Tr[e^{-\beta H^{[\tau]}} \xi^\dagger(P_{T_1}, \mu_1) \xi(P_{T_2}, \mu_2)] = Tr[\xi^\dagger(P_{T_2}, \mu_2) e^{-\beta H^{[\tau]}} \xi^\dagger(P_{T_1}, \mu_1)] =$$

$$Tr[\xi^\dagger(P_{T_2}, \mu_2) e^{-\beta H^{[\tau]}} \xi^\dagger(P_{T_1}, \mu_1) e^{\beta H^{[\tau]}} e^{-\beta H^{[\tau]}}] =$$

$$Tr[e^{-\beta H^{[\tau]}} \xi^\dagger(P_{T_2}, \mu_2) e^{-\beta H^{[\tau]}} \xi^\dagger(P_{T_1}, \mu_1) e^{\beta H^{[\tau]}}]. \quad (55)$$

3 Such a state also appears in the context of the Bogolyubov’s microscopical theory of superfluidity, see e.g. ref. 49.
Using this equation together with eqs. (2), (16) and (52), one has

$$\langle \xi^\dagger(P_{T1}, \mu_1)\xi(P_{T2}, \mu_2) \rangle = \langle \xi(P_{T2}, \mu_2)\xi^\dagger(P_{T1}, \mu_1, \beta) \rangle. \quad (56)$$

Substituting (54) into (56) and using commutation relation (47) we get

$$\langle \xi^\dagger(P_{T1}, \mu_1)\xi(P_{T2}, \mu_2) \rangle = e^{-\beta m_{T1}} \cosh \vartheta_1 \times$$

$$\langle \xi^\dagger(P_{T1}, \mu_1)\xi(P_{T2}, \mu_2) \rangle + \delta(\mu_1 - \mu_1)\delta(2)(P_{T1} - P_{T2}). \quad (57)$$

It then follows from eq. (57) that

$$\langle \xi^\dagger(P_{T1}, \mu_1)\xi(P_{T2}, \mu_2) \rangle = \frac{\delta(\mu_1 - \mu_1)\delta(2)(P_{T1} - P_{T2})}{e^{-2\beta m_{T1}} \cosh \vartheta_1 - 1}. \quad (58)$$

Notice that \(\langle \xi^\dagger\xi^\dagger \rangle = \langle \xi\xi \rangle = \langle \xi^\dagger \rangle = \langle \xi \rangle = 0\). Other \(n\)-point operator expectation values can be calculated in a similar way.

Now, let us utilize eqs. (24), (25), (28), (29), (43), (44), (48), and (49) to relate operators \(a\) and \(a^\dagger\) with \(\xi\) and \(\xi^\dagger\). We obtain

$$a(P) = \frac{1}{\sqrt{2\pi m_T \cosh \theta}} \int_{-\infty}^{+\infty} d\mu \left(\frac{\xi(P,\mu) + A\xi^\dagger(-P,-\mu)}{\sqrt{1 - A^2}}\right) \times$$

$$\exp \left( i\mu\theta + \frac{i}{8\pi m_T \cosh \vartheta_0} + \frac{i}{2} \vartheta_0 \cosh \vartheta_0 - \frac{i}{2} \vartheta_0 \vartheta_0 \right), \quad (59)$$

and

$$a^\dagger(P) = \frac{1}{\sqrt{2\pi m_T \cosh \theta}} \int_{-\infty}^{+\infty} d\mu \left(\frac{\xi^\dagger(P,\mu) + A\xi(-P,-\mu)}{\sqrt{1 - A^2}}\right) \times$$

$$\exp \left( -i\mu\theta - \frac{i}{8\pi m_T \cosh \vartheta_0} - \frac{i}{2} \vartheta_0 \cosh \vartheta_0 + \frac{i}{2} \vartheta_0 \vartheta_0 \right). \quad (60)$$

Let us recall that the creation and annihilation operators for the pions are vectors in isotopic spin space, \(a^\dagger(p) = (a_1^\dagger(p), a_2^\dagger(p), a_3^\dagger(p))\), \(a(p) = (a_1(p), a_2(p), a_3(p))\), and \([a_i^\dagger, a_j] = 0\) if \(i \neq j\). Then the annihilation and creation operators for the charged (say, \(\pi^+\)) pions are

$$a_+(p) = \frac{1}{\sqrt{2}}(a_1(p) + ia_2(p)), \quad (61)$$

$$a_+^\dagger(p) = \frac{1}{\sqrt{2}}(a_1^\dagger(p) - ia_2^\dagger(p)), \quad (62)$$

where \(a_i, a_i^\dagger\) are related with \(\xi_i, \xi_i^\dagger\) by means of eqs. (59), (60), here \(i\) is 1 or 2 and \([\xi_i^\dagger, \xi_j] = 0\) if \(i \neq j\). Utilization of eqs. (61), (62) and isotopic symmetry yield \(\langle a_+^\dagger(p_1)a_+(p_2) \rangle = \).
Here we substitute \( \delta \) and \( \tau \) from the momentum spectrum, Also, after some basic but lengthy operator algebra one can see that the two-particle momentum spectrum at zero temperature. Indeed, one can rewrite eq. (63) to the form

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle = \langle a_{+}^{\dagger}(p_{1})a_{+}^{\dagger}(p_{2}) \rangle = 0. \tag{63}
\]

Then, using eqs. (58, 59, 60), we find at \( m_{T}\tau \gg 1 \)

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle = \frac{\delta^{(2)}(p_{T1} - p_{T2})}{2\pi \sqrt{\omega_{p1}\omega_{p2}}} \int_{-\infty}^{+\infty} \frac{e^{-i\mu(\theta_{1} - \theta_{2})}}{1 - A^{2} + \frac{1 + A^{2}}{e^{\beta m_{T1} \cosh \theta_{\sigma}} - 1}} d\mu. \tag{64}
\]

Also, after some basic but lengthy operator algebra one can see that the two-particle momentum spectrum, \( \langle a_{+}^{\dagger}(p_{1})a_{+}^{\dagger}(p_{2})a_{+}(p_{1})a_{+}(p_{2}) \rangle \), can be written as

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}^{\dagger}(p_{2})a_{+}(p_{1})a_{+}(p_{2}) \rangle = \langle a_{+}^{\dagger}(p_{1})a_{+}(p_{1}) \rangle \langle a_{+}^{\dagger}(p_{2})a_{+}(p_{2}) \rangle + \langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle \langle a_{+}^{\dagger}(p_{2})a_{+}(p_{1}) \rangle. \tag{65}
\]

The above expressions, (63) and (64), allows us to estimate the Bose-Einstein correlation function for identical charged bosons (e.g. \( \pi^{+} \)) which is defined as

\[
C(p, q) = \frac{\langle a_{+}^{\dagger}(p_{1})a_{+}^{\dagger}(p_{2})a_{+}(p_{1})a_{+}(p_{2}) \rangle}{\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{1}) \rangle \langle a_{+}^{\dagger}(p_{2})a_{+}(p_{2}) \rangle}, \tag{66}
\]

where \( p = (p_{1} + p_{2})/2, q = p_{2} - p_{1} \).

First, notice that our findings imply that one-particle momentum spectrum can be approximated by sum of local equilibrium ideal gas and ground-state condensate contributions, where the ground-state condensate formally corresponds to particle momentum spectrum at zero temperature. Indeed, one can rewrite eq. (63) to the form

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle = \langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle_{l.eq.} + \langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle_{cond}, \tag{67}
\]

where the last term does not vanish when temperature goes to zero. Then taking into account that \( A \ll 1 \) and retaining in each term only the leading power of \( (m_{T}\tau)^{-1} \), one has

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle_{l.eq.} = \frac{R_{T}^{2}}{2\pi^{3} \sqrt{\omega_{p1}\omega_{p2}}} \int_{-\infty}^{+\infty} d\mu e^{-i\mu(\theta_{1} - \theta_{2})} \frac{1}{e^{\beta m_{T1} \cosh \theta_{\sigma}} - 1}, \tag{68}
\]

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle_{cond} = \frac{R_{T}^{2}}{2\pi^{3} \sqrt{\omega_{p1}\omega_{p2}}} \int_{-\infty}^{+\infty} d\mu e^{-i\mu(\theta_{1} - \theta_{2})} A^{2}. \tag{69}
\]

Here we substitute \( \delta^{(2)}(p_{T1} - p_{T2}) \) by \( (2\pi)^{-2}R_{T}^{2} \) at \( p_{T1} = p_{T2} \). Then, taking into account eq. (64), one can change integration variable, \( \mu = (m_{T}\tau) \sinh(\eta - \theta) \), and get

\[
\langle a_{+}^{\dagger}(p_{1})a_{+}(p_{2}) \rangle_{l.eq.} = \frac{R_{T}^{2}}{2\pi^{3} \omega_{p}} \int_{-\infty}^{+\infty} d\eta m_{T}\tau \cosh(\eta - \theta) \frac{1}{e^{\beta m_{T} \cosh(\eta - \theta)} - 1} = \frac{1}{(2\pi)^{3} \omega_{p}} \int_{\sigma_{\mu}} d\sigma_{\mu} p_{\mu} e^{-i\mu p_{\mu}} - \frac{1}{(2\pi)^{3} \omega_{p}} \int_{\sigma_{\mu}} d\sigma_{\mu} p_{\mu} e^{\beta \omega_{p} p_{\mu}} - 1. \tag{69}
\]
where \( \frac{1}{(2\pi)^3} \frac{1}{\exp(\beta p) - 1} \) corresponds to the Bose-Einstein local equilibrium distribution function of the ideal gas.

It is instructive to write approximate expressions for one- and two-particle momentum spectra for \( \beta m_T \gg 1 \) and assuming \( p_T = p_T \) (then \( q_T = 0 \)). In such an approximation (67) is given by

\[
\langle a_+^\dagger(p_1) a_+(p_2) \rangle_{l.eq.} \approx n_{l.eq.}(p) \exp \left( -\frac{m_T^2 \tau^2}{2 \beta m_T} (\theta_1 - \theta_2)^2 \right),
\]

and calculation of (68) results in

\[
\langle a_+^\dagger(p_1) a_+(p_2) \rangle_{cond} \approx n_{cond}(p) \exp(-m_T \tau |\theta_1 - \theta_2|).
\]

Here the one-particle momentum spectra \( n_{l.eq.}(p) \) and \( n_{cond}(p) \) are approximately given by

\[
n_{l.eq.}(p) = \frac{R_{l.eq.}^2}{(2\pi)^3 \omega_p} \tau m_T \sqrt{\frac{2\pi}{\beta m_T}} \exp(-\beta m_T),
\]

\[
n_{cond}(p) = \frac{R_{cond}^2}{(2\pi)^3 \omega_p} \frac{\pi}{16 m_T \tau}.
\]

Taking into account that \( q_L = p_{2z} - p_{1z} = m_T (\sinh \theta_2 - \sinh \theta_1) \approx (m_T \cosh \theta)(\theta_2 - \theta_1) \), \( \theta = (\theta_1 + \theta_2)/2 \), one can rewrite eqs. (70) and (71) in terms of the more customary particle momentum difference \( q_L \),

\[
\langle a_+^\dagger(p_1) a_+(p_2) \rangle_{l.eq.} \approx n_{l.eq.}(p_T) \exp \left( -\frac{R_{l.eq.}^2}{2} q_L^2 \right),
\]

\[
\langle a_+^\dagger(p_1) a_+(p_2) \rangle_{cond} \approx n_{cond}(p_T) \exp \left( -\frac{R_{cond}^2}{2} |q_L| \right),
\]

where

\[
R_{l.eq.} = \frac{\tau}{\sqrt{\beta m_T} \cosh \theta}
\]

is well-known approximate expression for the longitudinal radius [56 –58] (see also refs. [23–26, 37–39]), and

\[
R_{cond} = \frac{2\tau}{\cosh \theta}
\]

is the longitudinal scale of the ground-state condensate.

The above results allows us to write the correlation function (65) as

\[
C(p, 0, 0, q_L) = 1 + \left( \sqrt{\lambda_{l.eq.}} \exp \left( -\frac{R_{l.eq.}^2}{2} q_L^2 \right) + \sqrt{\lambda_{cond}} \exp \left( -\frac{R_{cond}^2}{2} |q_L| \right) \right)^2,
\]
where
\[ \sqrt{\lambda_{\text{eq.}}} + \sqrt{\lambda_{\text{cond}}} = 1, \] (79)
and
\[ \lambda_{\text{eq.}} = \left( \frac{n_{\text{eq.}}}{n_{\text{eq.}} + n_{\text{cond}}} \right)^2, \] (80)
\[ \lambda_{\text{cond}} = \left( \frac{n_{\text{cond}}}{n_{\text{eq.}} + n_{\text{cond}}} \right)^2. \] (81)

It follows from eqs. (78) and (79) that intercept of the Bose-Einstein correlation function for identical charged pions at zero relative momentum is equal to 2, \( C(p, 0) = 2 \).

It is instructive to compare expression (78) with the fitted form of the correlation function of two identical charged pions, which in each \( p \)-bin looks like
\[ C_{\text{exp}}(p, q) = 1 + \lambda_{p} F_{p}(q), \] (82)
with the function \( F_{p}(q) \) depending on the shape of the boson source, \( F_{p}(0) = 1 \) and \( F_{p}(q) \to 0 \) for \( |q| \to \infty \), the latter condition follows from normalization of the correlation function that is applied by experimentalists: \( C_{\text{exp}}(p, q) \to 1 \) for \( |q| \to \infty \). One can see that \( C(p, q) \), see eqs. (63), (64), and (65), satisfies the proper normalization condition. The phenomenological chaoticity parameter \( 0 \leq \lambda_{p} \leq 1 \) describes the correlation strength. An estimate of the size of the emission region, in the form of the HBT radii, is then extracted from a fit of eq. (82) in wide enough \( q \)-interval to the measured in each \( p \)-bin two-particle correlations with, typically, Gaussian or exponential from of the function \( F_{p}(q) \).

Comparing eqs. (78) and (82), we see that, strictly speaking, we can not identify \( \lambda \)-parameters and radii in eq. (78) with the measured ones. Nevertheless, just to have an idea on the magnitude of the effect of the ground-state condensate on the two-particle correlation function, one can rewrite eq. (78) to the form
\[ C(p, 0, 0, q_{L}) = 1 + \Lambda(p_{T}, q_{L}) e^{-R_{l,\text{eq.}}^2 q_{L}^2}, \] (83)
where by definition
\[ \Lambda(p_{T}, q_{L}) = \left( 1 - \sqrt{\lambda_{\text{cond}}} \left( 1 - e^{-\frac{R_{\text{cond}}^2}{2}|q_{L}| + \frac{R_{l,\text{eq.}}^2}{2} q_{L}^2} \right) \right)^2. \] (84)

Identification of \( \Lambda(p_{T}, q_{L}) \) with phenomenological parameter \( \lambda_{p} \) can give us rough estimate of the latter in the region of the correlation peak: \( |q_{L}| \sim 1/R_{l,\text{eq.}}, |q_{T}| = 0 \). Namely, it
follows from eq. (84) that in such a region \( \Lambda(p_T, q_L) < 1 \) and decreases when transverse momentum of a pion pair increases, and that there is the correlation between the size of the emission region and \( \Lambda(p_T, q_L) \): for smaller radius, \( R_{l,eq} \), one can see that \( \Lambda(p_T, q_L) \) is also smaller. Namely, when transverse momentum increases, then \( \lambda_{cond} \) increases, see eqs. (72), (73) and (81). Also, \( R_{l,eq} < R_{cond} \), and \( R_{l,eq} \) decreases when transverse momentum increases while \( R_{cond} \) is constant, see eqs. (70) and (77), as a consequence \( \Lambda(p_T, q_L) \) decreases for \( |q_L| \sim 1/R_{l,eq} \) when \( p_T \) increases. Such a behavior seems to be at least qualitatively consistent with behavior of the phenomenological \( \lambda_p \)-parameter derived from experimental parametrization (82) [21, 22]. For very high transverse momenta the ground-state condensate contribution to one-particle momentum spectrum dominates, and \( \lambda_{l,eq} \rightarrow 0, \lambda_{cond} \rightarrow 1 \) in the two-particle correlation function \( (\ref{eq:78}) \) when \( p_T \rightarrow \infty \), see eqs. (72), (73), (80) and (81).

Finally, let us address the question how our results are changed if one uses next-to-leading order terms in \( (m_T \tau)^{-1} \). First at all, note that the ground-state condensate appears if the ground state (at zero temperature) of quasi equilibrium statistical operator \( H^{[\tau]} \) does not coincide with the ordinary vacuum state in Minkowski spacetime, in the model considered it is the case because the ground state with respect to \( H^{[\tau]} \) is not the same as the ground state (vacuum) with respect to the ordinary Hamiltonian. While we have calculated the momentum spectra of over-condensate excitations as well as condensate particles only in the leading order of perturbative expansion in \( (m_T \tau)^{-1} \), one can conclude that accounting for next orders in \( (m_T \tau)^{-1} \) does not change general qualitative features of our results. Namely, the two-component parametrization of the two-particle momentum correlation function, see eq. (78), remains unchanged, as well as decrease of \( R_{l,eq}/n_{cond} \) and \( R_{l,eq}/R_{cond} \) when transverse momentum increases. Then, in particular, one can conclude that estimation of \( \Lambda(p_T, q_L) \) behavior with \( p_T \) remains correct.

**IV. CONCLUSIONS**

Motivated by the recent experimental observations of the suppression of two-particle Bose-Einstein momentum correlations in the high-multiplicity \( p + p \) collisions at the LHC [20, 22], we relate these observations to the ground-state condensate contribution to particle momentum spectra. Our findings demonstrate that under certain circumstances quasi equilibrium expansion of quantum fields can lead to the condensate formation. We considered
a model for particles emission, based on the hypothesis of quasi equilibrium boost-invariant longitudinal expansion. In the present work, to make the problem tractable, we used the model of non-interacting scalar quantum field to describe the sudden particle momentum spectra freeze-out at the hypersurface $\tau = \text{const}$, and do not take into account transverse finite-size effects. In our opinion the proposed physical picture reflects in some degree the physics of the quasi equilibrium evolution and particles emission in the high-multiplicity $p + p$ collisions at the LHC. We demonstrated that the ground-state condensate can lead to suppression of the two-particle Bose-Einstein momentum correlations due to the two-scale mechanism of particles emission. This effect should be less important for relativistic heavy ion collisions which have prolonged post-hydrodynamical kinetic stage of hadronic rescatterings. To demonstrate reliability of the model proposed, we performed a simple analytical estimate of the apparent $\lambda$-parameter in the two-pion Bose-Einstein correlation function. The main point here is not a detail comparison with experimental data but a conclusion that the quasi equilibrium state of the longitudinally boost-invariant expanding quantum scalar field implies that hydrodynamics is accompanied by the ground-state condensate with the corresponding properties. This should be taken into account when hydrodynamical approach is applied to analyze high-energy $p + p$ collision experiments.

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4 Notice that two-scale mechanism of particles emission was also proposed in ref. 59 based on different underlying physical picture, and two-particle Bose-Einstein correlations in $p + p$ collisions at the LHC were fitted by the correspondingly parameterized correlation function 60.

5 To make it possible one needs to take into account that the particle-emitting sources produced in high-energy $p + p$ collisions are expanding also in the transverse direction, particle momentum spectra include feed-downs from the resonance decays, etc., but such an analysis goes beyond the scope of the present article.
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