Observational constraints for power-law spectra of density perturbations

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Abstract

We present an analysis of cosmological models with mixed dark matter (spectrum slope of density perturbations \(n_S \in [0.6, 1.6]\), hot dark matter abundance \(\Omega_\nu \in [0.0, 0.6]\)) in spatially flat Universe, based on Press-Schechter formalism (P&S). We found that the models with \(n_S > 1\) and \(\Omega_\nu < 0.5\) are preferable. Additionally, for all considered models we simulated \(\Delta T/T|_{10^0}\) and compared it with the COBE measured amplitude of CMB anisotropy. We found that the models favourable for galaxy cluster abundance need large amount of cosmological gravitational waves (\(T/S > 1\)).

1 Introduction

A reconstruction of density perturbation spectrum is a key problem of modern cosmology. It made a dramatic turn after detecting the primordial CMB anisotropy by DMR COBE (Smoot et al\(^[1]\), Bennet et al\(^[2]\)) as the signal found at \(10^0 \Delta T/T = 1.06 \times 10^{-5}\) appeared to be few times more than the expectable value of \(\Delta T/T\) in the most simple and developed cosmological model – standard CDM one (SCDM\(^[3]\)).

Currently there are a lot of experimental data (such as the spatial distributions of galaxies, clusters of galaxies and quasars, bulk velocities, CMB anisotropy, and others) which can be used to reconstruct the density perturbation spectrum. Characteristic scales of data are different and vary from \(\sim 10\) Mpc which is a scale of nonlinearity to the horizon scale. However, it seems now the most crucial tests are large-scale CMB anisotropy and the number of galaxy clusters in top-hat sphere with radius \(R = 8h^{-1}\) Mpc = 16 Mpc. The

\(^{1}\Omega_M = 1, \Omega_b = 0.06\) (Walker at al\(^[3]\)), \(\Omega_{CDM} = 0.94, h = 0.5\), no cosmological gravitational waves.
former can be easily related to the amplitude of density perturbations through the SW effect (Sachs & Wolfe): 

$$\frac{\Delta T}{T}(\vec{x}) = \frac{H_0^2}{2(2\pi)^{3/2}} \int_{-\infty}^{\infty} \frac{1}{k^2} \delta(k)e^{i\vec{k} \vec{x}}d^3k, \quad \vec{x} \approx 2\vec{e}_H.$$ 

where $\delta_k$ is a Fourier transform of density contrast $\delta(\vec{x}) \equiv \delta \rho/\rho$, $H_0$ is the Hubble constant, $\langle \delta_k \delta_{k'} \rangle = P(k)\delta(\vec{k}-\vec{k'})$, $P(k) = A_k n S T^2(\Omega_\nu, k)$ is a power spectrum of density perturbations, $A$ is the normalization constant, $T(\Omega_\nu, k)$ is a transfer function. The latter determines the value of biasing parameter $b^{-1} \equiv \sigma_R$ for a spatially flat Universe:

$$\sigma_R^2 = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} P(k)W^2(kR)d^3k,$$

where $W(kR) = \frac{3}{(kR)^3} \sin kR - kR \cos kR$ is the Fourier transform of hop-hat window function.

Obviously, both normalizations are model-dependent, the $\Delta T/T$ normalization depends on the amplitude of cosmological gravitational wave spectrum on large scale and, therefore, is related to the model of inflation, the $\sigma_R$ normalization does depend on the nature of dark matter. Here we prefer to fix $\sigma_{16}$ to consider the relative contribution of gravitational waves at COBE scale $T/S$ as an additional calculable parameter (instead of considering some inflationary model).

Below, we report results based on P&S formalism which deals with abundance of gravitationally bounded halos of dark matter.

## 2 P&S formalism

P&S formalism (Press & Schechter) is built up on two assumptions: firstly, the density contrast $\delta$ is a Gaussian random field, and, secondly, the mass distribution of virialized halos of dark matter is the same as the distribution of high density peaks.

Since the density contrast smoothed by a window function $\delta(\vec{x}, R)$ is the Gaussian random field too, we can easily calculate the fraction of space points at which the density exceeds any given value. Thus, the fraction of the points surrounded by a sphere of radius $R$ within which the density contrast exceeds $\delta_c$, is given by

$$P(\delta > \delta_c) = \frac{1}{\sqrt{2\pi}\sigma} \int_{\delta_c}^{\infty} e^{-\frac{\delta^2}{2\sigma^2}}d\delta.$$ 

According to P&S this fraction can be identified with the fraction of halos with mass exceeding $M_c = \frac{4}{3}\rho_0(1+\delta_c)R^3$, where $\delta_c = 1.69$ (Gunn & Gott).
Figure 1: Theoretical curves for 4 cosmological models: \( n_S = 1.4, \Omega_\nu = 0.1 \) (dot-dashed line); \( n_S = 1.0, \Omega_\nu = 0.1 \) (dotted line); \( n_S = 1.4, \Omega_\nu = 0.5 \) (3 dot-dashed line); \( n_S = 1.0, \Omega_\nu = 0.5 \) (dashed line). All models are normalized by \( \sigma_{16} = 0.57 \). Empty squares with error bars are observational points taken from Bachall & Cen [8].

The integral mass distribution of nonlinear lumps is the following:

\[
N(> M) = \int_{M}^{+\infty} \frac{dn}{dM} dM = \int_{M}^{+\infty} \frac{2}{\pi} \frac{\rho_0 \delta_c}{M \sigma^2} \left| \frac{d\sigma}{dM} \right| e^{-\frac{\delta_c^2}{2\sigma^2}} dM.
\]

We employed the P&S formalism to find theoretical mass distributions for all models described in the Introduction. Fig. 1 shows curves corresponding to 4 models with different parameters for \( \sigma_{16} = 0.57 \). To calculate \( \sigma_R \) we used the approximation for the transfer function proposed in Pogosyan & Starobinsky [7]. Observational data (7 points in total) were extracted from Bachall & Cen [8] (the error bars are about 30%). To find models satisfying the observations we used the following criteria: the averaged residues of observational point from P&S curve are inside 1\( \sigma \) for any model and for each point. Firstly, we found that for each model \( \sigma_{16} \in [0.42, 0.58] \) satisfies the observational data. If \( \sigma_{16} < 0.42 \) the P&S curve has too small amplitude relatively to observational data, if \( \sigma_{16} > 0.58 \) it lies to high. The result of the calculations for \( \sigma_{16} = 0.56, 0.57, 0.58 \) is presented at Fig. 2. The lines are the upper 1\( \sigma \) limits. The allowed region is under the lines. Notice, that the sum of squared residues is minimal for the model with \( n_S = 1.5, \Omega_\nu = 0.1 \) (\( \sigma_{16} = 0.56 \)); \( n_S = 1.6, \Omega_\nu = 0 \) (\( \sigma_{16} = 0.57 \)); \( n_S = 1.5, \Omega_\nu = 0 \) (\( \sigma_{16} = 0.58 \)). We used \( \sigma_{16} = 0.57 \) as a central point by the reasons.
Figure 2: Upper limits for models which are preferable by P&S method for three values of $\sigma_{16}$ parameter. Dotted line corresponds to $\sigma_{16} = 0.56$, the dashed line $\sigma_{16} = 0.57$, the dot-dashed line $\sigma_{16} = 0.58$.

proposed in White et al[9]. Statistically, error bars of data are too large to exclude any model even for the bottom line in Fig.2 but we argue that there is some tendency to large $n_S$ and small $\Omega_\nu$.

3 $\Delta T/T$

It is clear that a model should fit the COBE data $\Delta T/T = 1.06 \times 10^{-5}$, therefore, the simulated amplitude of CMB anisotropy should not exceed this value. We simulated $\Delta T/T|_{10^0} \equiv \Delta T/T|_{\text{simulated}}$ due to density perturbations and presented the result as slices of $\Im \equiv S/(S+T) = (\Delta T/T|_{\text{simulated}})/(\Delta T/T|_{\text{COBE}})$ (Fig.3).

We assume that $0.25 \leq \Im \leq 1$ is a reasonable range that does not contradict current theoretical models of inflation (Lukash & Mikheeva[10]). If $\Im > 1$ the CMB anisotropy is overproduced, and if $\Im < 0.25$ we have problems with constructing the model of inflation. We have obtained that the models with $0.75 \leq n_S \leq 1.25$, $0 \leq \Omega_\nu \leq 0.35$ ($\sigma_{16} = 0.57$) are available only.

4 Discussion and conclusions

Analyzing Fig.2,3 we can conclude (see Fig.4):
Figure 3: A relative contribution of density perturbations into $\Delta T/T|_{10^9} \Im$ as a function of model parameters $n_S$ and $\Omega_\nu$. $\sigma_{16} = 0.57$.

- P&S cluster mass function is strongly sensitive to the $\sigma_{16}$ value: $\sigma_{16} \in [0.42; 0.58]$;
- the advantaged models by P&S cluster mass function ($\sigma_{16} = 0.57 \pm 0.01$) are those with the “blue” spectra of density perturbations and an abundance of hot dark matter (see Fig.2);
- these “blue” models need a significant amount of cosmological gravitational waves ($T/S > 1$) to be consistent with the COBE data;
- Current observational data error bars are large, thus $1\sigma$ region covers about a half of the phase space of the models (see Fig.2);
- for the $\sigma_{16} = 0.57$, $1\sigma$ statistical criterion, and $0.25 < \Im < 1$ we obtain the following constrains for the model parameters: $0.75 < n_S < 1.25$, $\Omega_\nu < 0.35$ (see Fig.4, the allowable region is shaded);
- if we assume $\Im < 0.25$ then larger $n_S$ (till the boundary value 1.6) and $\Omega_\nu$ (till 0.5) are available (see Fig.2).

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Figure 4: Summarized constraints for power-law spectra of density perturbations. Labels "1" and "0.25" mark models with T/S = 1 and 0.25 consequently.

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