Characterization of Associative PU-algebras by the Notion of Derivations

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Abstract: In this manuscript we insert the concept of derivations in associative PU-algebras and discuss some of its important results such that we prove that for a mapping being a (Left, Right) or (Right, Left)-derivation of an associative PU-algebra then such a mapping is one-one. If a mapping is regular then it is identity. If any element of an associative PU-algebra satisfying the criteria of identity function then such a map is identity. We also prove some useful properties for a mapping being (Left, Right)-regular derivation of an associative PU-algebra and (Right, Left)-regular derivation of an associative PU-algebra. Moreover we prove that if a mapping is regular (Left, Right)-derivation of an associative PU-algebra then its Kernel is a subalgebra. We have no doubt that the research along this line can be kept up, and indeed, some results in this manuscript have already made up a foundation for further exploration concerning the further progression of PU-algebras. These definitions and main results can be similarly extended to some other algebraic systems such as BCH-algebras, Hilbert algebras, BF-algebras, J-algebras, WS-algebras, CI-algebras, SU-algebras, BCL-algebras, BP-algebras and BO-algebras, Z-algebras and so forth. The main purpose of our future work is to investigate the fuzzy derivations ideals in PU-algebras, which may have a lot of applications in different branches of theoretical physics and computer science.

Keywords: PU-Algebras, (Left, Right)-derivations of PU-algebras, (Right, Left)-derivations of PU-algebras, Regular Derivations of PU-algebras.

1. Introduction

In 1966, Y. Imai and K. Isèki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [1-3]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Neggers et al. [4] introduced a notions, called Q-algebras, which is a generalization of BCH / BCI / BCK-algebras and generalized some theorems discussed in BCI-algebras. Megalai and Tamilarasi [5] laid down the foundation of a notion, called TM-algebra. Moreover, Mostafa et al. [6] introduced a new algebraic structure called PU-algebra, which is a dual for TM-algebra and investigated severed basic properties. Moreover they derived new view of several ideals on PU-algebra and studied some properties of them. Derivation is a very interesting and important area of research in the theory of algebraic structures in mathematics. Several authors [7-11] have studied derivations in rings and near rings. Jun and Xin [12] applied the notions of derivations in ring and near-ring theory to BCI-algebras, and they also introduced a new concept called a regular Derivation in BCI-algebra. They investigated some of its properties, defined a d -derivations ideal and gave conditions for an ideal to be d-derivations. Later, Abujabal and Al-Shehri [13], defined a left derivations in BCI-algebras and investigated a regular left derivations. Zhan and Liu [14] studied f-derivations in BCI-algebras and proved some results. Muhiddin and Al-roqi [15, 16] introduced the notions of (α, β)-derivations in a BCI-algebras and investigated related properties. They provided a condition for a (α, β) - derivations to be regular. They also introduced the concepts of a (d (α, β) - invariant (α, β) - derivations and α-ideal, and then they investigated their relations. Furthermore, they obtained some results on regular (α, β) - derivations. Moreover, they studied the notions of t-derivations on BCI-algebras [17] and obtain some of its related properties. Further, they characterized the notions of p-semisimple BCI-algebras X by using the notions of t-derivations. Abujabal and Shehri in their pioneer paper [18], defined the derivations as, for a self-map, d, for any algebra X, d is a left-right derivation (briefly (l, r)-derivation) of X if it satisfies the identity d(a ∗ b) = (d(a) ∗ b) ∧ (a ∗ d(b)). For all a, b ∈ X. If d satisfies the identity d(a ∗ b) = (a ∗ d(b)) ∧ (d(a) ∗ b) for all a, b ∈ X,
then \( d \) is a right-left derivation (briefly \((r,l)\)-derivation) of \( X \). If \( d \) is both \((l, r)\)-derivation and \((r, l)\)-derivation, then \( d \) is a derivation of \( X \). The aim of the paper is to complete the studies on PU-algebra; in particular, we aim to apply the notion of derivation on associative PU-algebra and obtain some related properties. We start with definitions and propositions on PU-algebra taken from [6].

Then, we redefine the notion of derivation in associative PU-algebra and prove that for \( \phi \) being a \((Left,Right)\) or \((Right,Left)\)-derivation of an associative PU-algebra \( Z \) then \( \phi \) is one-one map. If \( \phi \) is a regular map then it is identity. If there exists an element \( a \in Z \) such that \( \phi(a)=a \) then the map \( \phi \) is identity. We prove that if \( \phi \) is \((Left, Right)\) -regular derivation of \( Z \) then \( \phi(a) = a \land \phi(a) \) also if \( \phi \) is \((Right,Left)\)-regular derivation of \( Z \) then \( \phi(a) = \phi(a) \wedge a \), \( \forall a \in Z \). We prove that if \( \phi \) is a self-map of an associative PU-Algebra \( Z \) then \( (a \ast (a \ast \phi(a))) = a \ast (\phi(a) \ast (\phi(a) \ast a)) = a \). We also prove that if \( \phi \) is a regular map \((Right, Left)\)-derivation of an associative PU-algebra \( Z \) then \( \text{Ker}(\phi)=\{a \in Z | \phi(a) = 0\} \) is a subalgebra of \( Z \).

2. Preliminaries

This section consists of some preliminary definitions and basic facts about PU-algebra which are useful in the proofs of our results. Throughout this research work we denote the PU-algebra always by \( Z \).

Definition 2.1: [6] PU-algebra \((Z, *, 0)\) is a class of the type \((2, 0)\) algebras which satisfies the \((P_1)\) and \((P_2)\) conditions for all \( p, a, r \in Z \) where
\[
(P_1) 0 \ast a = a \quad (P_2) (a \ast c) \ast (b \ast c) = b \ast a
\]

While the binary relation ‘\(\leq\)’ on \( Z \) is defined as \( a \leq b \iff b \ast a = 0 \).

Proposition 2.2: [6] In PU-algebra \((Z, *, 0)\) the following results are true for all \( a, b, c \in Z \):
\[
(P_3) a \ast a = 0 \quad (P_4) (a \ast c) \ast c = a \quad (P_5) a \ast (b \ast c) = b \ast (a \ast c)
\]
\[
(P_6) a \ast (b \ast a) = b \ast a \quad (P_7) \text{The following three results are similar in } (Z, *, 0)
\]
\[
(1): b \ast c = (2): b \ast a = a \ast c \quad (3): a \ast b = a \ast c
\]

\( (P_8) \) Both (left and right) cancellation properties hold in \((Z, *, 0)\).

Definition 2.3: [6] PU-algebra \((Z, *, 0)\) is said to be associative if it satisfies the condition \( a \ast (b \ast c) = (a \ast b) \ast c \) for all \( a, b, c \in Z \).

3. Main Results

Definition 3.1:- Let \((Z, *, 0)\) is an associative PU-algebra and \( \phi : Z \rightarrow Z \) is a self-map then \( \phi \) called \((Left,Right)\)-derivation on \( Z \) if \( \phi(a \ast b) = (\phi(a) \ast b) \wedge (a \ast \phi(b)) \).

Definition 3.2:- Let \((Z, *, 0)\) is an associative PU-algebra and \( \phi : Z \rightarrow Z \) is a self-map then \( \phi \) is called \((Right, Left)\)-derivation on \( Z \) if \( \phi(a \ast b) = (a \ast \phi(b)) \ast (\phi(a) \ast b) \).

Definition 3.3: If \( \phi \) is both \((Left,Right)\)-derivation and \((Right, Left)\)-derivation on \( Z \) then \( \phi \) is called derivation on \( Z \).

Definition 3.4: A self-map \( \phi : Z \rightarrow Z \) on associative PU-algebra \( Z \) is called regular if \( \phi(0) = 0 \).

Example 3.5: Let the set \( Z = \{0, a, b, c\} \) defined by the following table.

\[
\begin{array}{cccc}
0 & a & b & c \\
0 & 0 & a & b \\
a & a & 0 & c \\
b & b & c & 0 \\
c & c & b & a \\
\end{array}
\]

Is an associative PU-algebra and a map, \( \phi : Z \rightarrow Z \) defined by
\[
\phi(0)=c, \phi(a)=b, \phi(b)=a \quad \text{and} \quad \phi(c)=0
\]

Proposition 3.6: Let \( \phi \) be a \((Left,Right)\)-derivation of an associative PU-algebra \( Z \) then
\[
(P_9): \phi(0)=\phi(a) \ast a, \forall a \in Z \quad (P_{10}): \phi \text{ is one-one map.}
\]

Proposition 3.7: If \( \phi \) is a regular map then it is identity.

Proposition 3.8: If \( \phi \) is a right-left derivation on \( Z \) then \( \text{Ker}(\phi)=\{a \in Z | \phi(a) = 0\} \) is a subalgebra of \( Z \).

Proof (P_9):
\[
\phi(0)=\phi(a) \ast a, \quad \forall a \in Z
\]

From (P_9) we have
\[
\phi(0)=\phi(a) \ast a, \forall a \in Z
\]

Also from (P_9) we have
\[
\phi(0)=\phi(b) \ast b, \forall b \in Z
\]

From (2) and (3) we get
\[
\phi(a) \ast a = \phi(b) \ast b
\]

Using the result of equation (1) in equation (4) we get
\[
\phi(a) \ast a = \phi(b) \ast b
\]

By (P_9) left cancellation law holds in \( Z \) therefore from (5) we get \( a = b \). Hence \( \phi \) is one to one.

Proof (P_{11}):- Let \( \phi \) is a regular then we have
\[
\phi(0)=0
\]
From (P₃) we have
\[ \phi(0)=\phi(a) * a \forall a \in Z \] (7)
From (6) and (7) we get
\[ \phi(a) * a=0 \forall a \in Z \] (8)
Now by using (P₃) in the right hand side of equation (8) then (8) becomes
\[ \phi(a) * a = a * a \forall a \in Z, \therefore a * a=0 \] (9)
By (P₃) right cancellation law holds in \( Z \) therefore (9) becomes \( \phi(a) = a \forall a \in Z \).
Hence \( \phi \) is the identity map.
Proof (P₁₂): Let
\[ \phi(a)=a \] (10)
Now by proposition (P₁) equation (10) is equivalent to
\[ \phi(a) * a = a * a \Rightarrow \phi(a) * a=0, \therefore \text{by (P₃)} \] (11)
From (P₃) we have
\[ \phi(0)=\phi(a) * a \forall a \in Z \] (12)
So now using the result of equation (12) in the left hand side of equation (11) we get
\[ \phi(0)=0 \Rightarrow \phi \text{ is regular which by (P₁₁) } \phi \text{ is the identity map.} \]
Proof (P₁₁): Let
\[ \phi(b)*a=0 \] (13)
by proposition (P₁) equation (13) becomes
\[ \phi(b)*a = a * a \] (14)
by (P₃) right cancellation law holds in \( Z \) therefore (14) becomes \( \phi(b)=a \).
Similarly for
\[ a * \phi(b)=0 \] (15)
by (P₃) equation (15) becomes
\[ a * \phi(b)=a * a \] (16)
by (P₃) left cancellation law holds in \( Z \) therefore (16) becomes \( \phi(b)=a \).

Proposition 3.7:- Let \( \phi \) be a (Right,Left)-derivation of an associative PU-algebra \( Z \) then
(P₁₄): \( \phi(0)=a * \phi(a), \forall a \in Z \)
(P₁₅): \( \phi \) is one-one map.
(P₁₆): If \( \phi \) is a regular map then it is identity.
(P₁₇): If there exists an element \( a \in Z \) such that \( \phi(a)=a \) then the map \( \phi \) is identity.
(P₁₈): If \( \phi(b)*a=0 \text{ or } a * \phi(b)=0 \text{ then } \phi(b)=a, \forall a, b \in Z \text{ i.e. } \phi \text{ is constant.} \)
Proof (P₁₄):
\[ \phi(0)=\phi(a * a), \therefore \text{by (P₃)} \]
\[ =a * \phi(a) \wedge \phi(a) * a \]
\[ =\phi(a) * a * (a * \phi(a)) \]
\[ =[(\phi(a) * a) * (\phi(a) * a)] * (a * \phi(a)), \therefore \text{ } Z \text{ is associative} \]
\[ =0 * (a * \phi(a)), \therefore \text{by (P₃)} \]
\[ =a * \phi(a), \therefore \text{by (P₃)} \]
Proof (P₁₅): Let \( a, b \in Z \) such that
\[ \phi(a)=\phi(b) \] (17)
From (P₁₄) we have
\[ \phi(a) * a = \phi(a) * b \forall a \in Z \] (18)
Also from (P₁₄) we have
\[ \phi(0)=b * \phi(a) \forall b \in Z \] (19)
From (17) and (18) we get
\[ a * \phi(a)=b * \phi(a) \] (20)
Using (17) in the right hand side of equation (20) we get
\[ a * \phi(a)=a * \phi(a) \] (21)
By (P₃) right cancellation law holds in \( Z \) therefore from (21) we get \( a=b \) that is \( \phi \) is one to one.
Proof (P₁₆): Let \( \phi \) is regular then we have
\[ \phi(0)=0 \] (22)
From (P₁₄) we have
\[ \phi(0)=a * \phi(a) \forall a \in Z \] (23)
From (22) and (23) we get
\[ a * \phi(a)=0 \forall a \in Z \] (24)
Now by using (P₃) in the right hand side of equation (24) then (24) becomes
\[ a * \phi(a)=a * a \forall a \in Z, \therefore a * a=0 \] (25)
By (P₃) left cancellation law holds in \( Z \) therefore (iv) becomes \( \phi(a)=a, \forall a \in Z \).
Hence \( \phi \) is the identity map.
Proof (P₁₇): Let
\[ \phi(a)=a \] (26)
Now by proposition (P₁) equation (26) is equivalent to
\[ a * \phi(a)=a * a \Rightarrow a * \phi(a)=0, \therefore \text{by (P₃)} \] (27)
From (P₁₄) we have
\[ \phi(0)=a * \phi(a) \forall a \in Z \] (28)
so now using (28) in (27) we get \( \phi(0)=0 \Rightarrow \phi \) is regular which by (P₁₆) \( \phi \) is the identity map.
Proof (P₁₈):
Let $\phi(b) * a = 0$ (29)

by proposition (P3) the right hand side of equation (29) becomes

$\phi(b) * a = a * a$ (30)

by (P3) right cancellation law holds in $\mathbb{Z}$ therefore (30) becomes $\phi(b) = a$.

Similarly

$a * \phi(b) = 0$ (31)

by (P3) equation (31) becomes,

$a * \phi(b) = a * a$ (32)

by (P3) left cancellation law holds in $\mathbb{Z}$ therefore (32) becomes $\phi(b) = a$.

Theorem 3.8: Let $\mathbb{Z}$ is an associative PU-algebra.

(P3): If $\phi$ is a (Left, Right) -derivation of $\mathbb{Z}$ then $\phi(a) = a$ $\forall a \in \mathbb{Z}$.

(P3): If $\phi$ is a (Right, Left) -derivation of $\mathbb{Z}$ then $\phi(a) = a$ $\forall a \in \mathbb{Z}$.

Proof (P3): Since $\phi$ is regular therefore we have

$\phi(0) = 0$ (33)

Now consider for some $a \in \mathbb{Z}$ we have

$\phi(a) = \phi(0 * a), \because$ by (P3)

$= (\phi(0) * a) \land (0 * \phi(a)), \because$ by definition 3.1

$= (0 * a) \land (0 * \phi(a)), \because$ by using (33)

$= a \land \phi(a), \because$ by (P3)

Proof (P3): Since $\phi$ is regular therefore we have

$\phi(0) = 0$ (34)

Now consider for some $a \in \mathbb{Z}$, $\phi(a) = \phi(0 * a), \because$ by (P3)

$= (0 * \phi(a)) \land (0 * a), \because$ by definition 3.1

$= (0 * \phi(a)) \land (0 * a), \because$ by using (34)

$= \phi(a) \land a, \because$ by (P3)

Theorem 3.9: Let $\phi$ is a self-map of an associative PU-Algebra $\mathbb{Z}$ then $(a * (a * \phi(a))) * a = \phi(a) * (\phi(a) * a) * a$.

Proof: Since by (theorem 3.8 (P3)) we have,

$\phi(a) = \phi(a) \land a = a * (a * \phi(a))$ (35)

By (P3) equation (35) is equivalent to

$\phi(a) * a = (a * (a * \phi(a))) * a$ (36)

on the other hand from (theorem 3.8 (P3)) we have

$\phi(a) = a \land \phi(a) = \phi(a) * (\phi(a) * a)$ (37)

Similarly by (P3) equation (37) is equivalent to

$\phi(a) * a = (\phi(a) * (\phi(a) * a)) * a$ (38)

from (36) and (38) we get

$(a * (a * \phi(a))) * a = \phi(a) * (\phi(a) * a) * a$.

Theorem 3.10: If $\phi$ is a derivation on an associative PU-algebra $\mathbb{Z}$ then $\forall a \in \mathbb{Z}$

$(P_2): \phi(a * \phi(a)) = 0$

$(P_2): \phi(\phi(a) * a) = 0$

Proof (P2): Let $\phi$ is a (Left, Right) -derivation on $\mathbb{Z}$ then

$\phi(a * \phi(a)) = (\phi(a) * \phi(a)) \land (a * \phi(\phi(a))) = 0 \land (a * \phi(\phi(a))) \lor \because$ by using (P3)

$\Rightarrow \phi(a * \phi(a)) = (a * \phi(\phi(a))) \lor [(a * \phi(\phi(a))) * 0]$ (39)

As $\mathbb{Z}$ is an associative PU-algebra therefore we can write equation (39) as

$\phi(a * \phi(a)) = [a * \phi(\phi(a))] \lor (a * \phi(\phi(a))) = 0 \lor (a * \phi(\phi(a)))$. 

As $\mathbb{Z}$ is an associative PU-algebra therefore (39) can be written as

$\phi(\phi(a) * a) = [(\phi(\phi(a)) * a) \lor (\phi(\phi(a)) * a)] = 0 \lor (\phi(\phi(a)) * a). \because$ by using (P3)

Theorem 3.11: Let $\phi$ is a regular (Left, Right) -derivation of an associative PU-algebra $\mathbb{Z}$ then the following results hold in $\mathbb{Z}$.

$(P_2): \phi(a) = a$

$(P_2): \phi(a) * b = a * \phi(b) \forall a, b \in \mathbb{Z}$

$(P_2): \phi(b * a) = \phi(a) * b = a * \phi(b) = a * \phi(b)$

$(P_2): \text{Ker}(\phi) = \{a \in \mathbb{Z}; \phi(a) = 0\}$ is a subalgebra of $\mathbb{Z}$

Proof (P2): $\phi(a) = \phi(0 * a)$.

$\because$ by using (P1)

$= (\phi(0) * a) \land (0 * \phi(a))$

$= (0 * a) \land (0 * \phi(a))$, $\because$ $\phi$ is regular

$= a \land \phi(a), \because$ by using (P1)

$= \phi(a) * (\phi(a) * a)$

$= (\phi(a) * (\phi(a)) * a), \because \mathbb{Z}$ is associative

$= 0 * a = a \land \because$ by using (P1)

Hence $\phi(a) = a$.

Proof (P2): As $\phi$ is a regular (Left, Right) -derivation of an
associative PU-algebra \( Z \), then by (P₃) we have
\[
\phi(a) = a, \forall a \in Z
\]  
(41)

And
\[
\phi(b) = b, \forall b \in Z
\]  
(42)

By using the results of equations (41) and (42) we get\\( \phi(a) \ast b = a \ast b = a \ast \phi(b) \).

Proof (P₆): As \( \phi \) is a regular (Left, Right)-derivation of an associative PU-algebra \( Z \), then by (P₃) we have
\[
\phi(a) = a, \forall a \in Z
\]  
(43)

Therefore for any \( a, b \in Z \),

We have \( \phi(a \ast b) = a \ast b \) by using (P₅)
\[
\Rightarrow \phi(a \ast b) = \phi(a) \ast \phi(b), \forall a, b \in Z
\]  
(44)

By equation (43)
\[
\Rightarrow \phi(a \ast b) = \phi(a) \ast \phi(b)
\]  
(45)

And \( \phi(a \ast b) = a \ast \phi(b) \) by equation (43)
\[
\Rightarrow \phi(a \ast b) = a \ast \phi(b)
\]  
(46)

The equations (45), (46) and (47) imply that
\[
\phi(b \ast a) = \phi(a) \ast b = \phi(a) \ast \phi(b)
\]  
(48)

Proof (P₆): Let \( a, b \in \text{Ker}(\phi) \) then \( \phi(a) = 0 \)
\[
\Rightarrow \phi(b) = 0
\]  
(49)

As \( \phi \) is a regular derivation therefore from (P₆) we have,
\[
\phi(a \ast b) = \phi(a) \ast \phi(b)
\]  
(50)

Using (48) and (49) in the right hand side of equation (50) we get
\[
\Rightarrow a \ast b \in \text{Ker}(\phi) \Rightarrow \text{Ker}(\phi) \text{ is a subalgebra of } Z
\]

4. Conclusion

We see that derivations with special properties play a central role in the investigation of the structure of an algebraic system.

The forthcoming study of derivations in PU-algebras may be the following topics are worth to be taken into account.

To describe left derivations in PU-algebras and investigate a regular left derivations by using this concept.

To introduce the concept of \( f \)-derivations, \( t \)-derivations, \( t \)-bi-derivations and \( (\alpha, \beta) \)-derivations in PU-algebras.

To refer this concept to some other algebraic structures.

To consider the results of this concept to some possible applications in information systems and computer sciences.

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