Cluster-shell competition in systems with a few alpha particles and valence neutrons

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Abstract. We construct nuclear wave functions from alpha clusters and some additional valence nucleons, and allow the inter-cluster distance to change and one alpha to dissolve from the $(0s)^4$ structure as a result of the nuclear interaction. The change of the inter-cluster distance and the dissolution of the alpha particle can be interpreted as resulting from the competition of the ‘shell model’ and ‘cluster model’. We demonstrate this competition through a few parameters.

1. Introduction: cluster-shell competition

The structure of nuclei has a variety due to the particular forms in which nuclei with different proton and neutron numbers accommodate themselves to the nature of the nucleon-nucleon interaction. The structural variety consists in particular forms of the motion, localization, density etc. of the nucleons. Two of the typical structures are the mean-field and the cluster structure. The standard framework for describing the mean-field-like nature of nuclei is provided by the shell model. The shell model reproduces the nuclear magic numbers by treating valence nucleons as particles moving in the potential induced by a core, with the spin-orbit interaction between the core and valence nucleons playing an important role. The clustering aspect is embodied in the cluster model, in which the nucleons form groups called ‘clusters’ due to the strong correlation induced by the nucleon-nucleon interaction, and each cluster is spatially localized. These two aspects are realized not only in the two approaches of theoretical description but also in two pictures for the interpretation of nuclear phenomena.

We proposed a simplified parameterization for describing the structural change of nuclei from the shell model to the cluster model and vice versa. In this parameterization, the key parameters are the distance of the alpha particles inside the nucleus and a degree of the dissolution of the alpha particle(s). We call this parameterization the ‘cluster-shell competition’ [1, 2, 3, 4]. The dissolution of the alpha particle is caused by an explicit inclusion of the spin-orbit interaction. We also take into account the tensor correlation in the cluster model in a similar manner [2].

In this work, we briefly introduce the concept of the cluster-shell competition and show its applications to carbon isotopes. We investigate the role of valence neutrons on the structure of $^{12}$C, which is assumed to be a $3\alpha$ structure including one dissolved $\alpha$ particle [3]. Then we extend the approach to other carbon isotopes.
2. Formalism for the inclusion of the spin-orbit interaction
We briefly introduce the framework of the cluster-shell competition as a simplified modelling of the spin-orbit interaction (SMSO) [1]. We use an effective nucleon-nucleon interaction, which includes central, spin-orbit and Coulomb terms. We choose the Volkov No. 2 interaction [5] for the central part of $v_{ij}$,

$$V(r) = (W - MP^aP^r + BP^a - HP^r)[V_1 e^{-\rho_1 r^2} + V_2 e^{-\rho_2 r^2}],$$

where the exchange parameters are taken to be $W = 1 - M$, $M = 0.61$, and $B = H = 0.125$. For the spin-orbit term, we use the G3RS interaction [6],

$$V_{ls}(r) = V_0^{ls}[e^{-d_1 r^2} + e^{-d_2 r^2}]P^{(3)O} L \cdot S,$$

where $d_1 = 5.0$ fm$^{-2}$, $d_2 = 2.778$ fm$^{-2}$, $V_0 = 2000$ MeV, and $P^{(3)O}$ is a projection operator onto the triplet odd state. The parameters of the interaction have been determined from $\alpha + n$ and $\alpha + \alpha$ scattering phase shifts and the binding energy of the deuteron [7].

The total wave function is fully antisymmetrized and the spatial part of the ith nucleon is described by a Gaussian wave packet centered at $\vec{z}_i/\sqrt{\nu}$. Here $\nu$ is the width parameter of the wave packet. Its value is $\nu = 1/2b^2$, $b = 1.46$ fm. For the calculation of the carbon isotopes, we assume that the $^{12}$C core is described as three alpha particles forming an equilateral triangle. A length parameter, $R_1$, is introduced for the distance between two alpha particles in the triangle. The limit of $R_1 \to 0$ results in the wave function of the lowest state of the SU(3) shell-model.

To introduce the concept of the SMSO, we loosen up one of the alpha clusters by replacing each Gaussian centre $\vec{z}_i/\sqrt{\nu}$ by an expression containing a ‘dissolution parameter’ $\Lambda$:

$$\vec{z}_i/\sqrt{\nu} = \vec{R} + i\Lambda(\vec{e}_{spin}^i \times \vec{R}),$$

where $\vec{e}_{spin}^i$ is the unit vector along the spin-direction and the vector $\vec{R}$ is real. The $\Lambda = 0$ limit corresponds to the $(0s)^4$ configuration in the $\alpha$ particle, while the $\Lambda = 1$ case corresponds to four nucleons in one of the alpha clusters being elevated to the $p_{3/2}$-orbit. In this case, the spin-orbit interaction acts attractively for the $j_{z} = l + 1/2$ component of the wave function [1].

In order to describe the motion of the valence neutrons around the $^{12}$C core, we use a stochastic variational procedure [8, 9, 10].

3. Results
3.1. Application of SMSO to systems with several alpha particles
First, we briefly review the application of SMSO to systems with several alpha particles. This is based on the previous works [1, 2, 3, 4].

As a first step for incorporating the spin-orbit interaction within the SMSO framework, we studied $^9$Be ($2\alpha + n$), $^{10}$Be ($2\alpha + 2n$), $^{12}$C ($3\alpha$) and $^{20}$Ne ($4\alpha$) [1]. We simplified the cluster-shell competition by using just two parameters: the alpha-alpha distance and a single dissolution parameter of one alpha cluster. By optimizing these two parameters, we found that their optimum values depend on the nuclei, and, as a general tendency, the alpha-alpha distance fixed to a smaller value favours a larger dissolution parameter and vice versa. Furthermore, as the dissolution parameter becomes larger, the expectation value of the kinetic energy becomes large due to the increase of the higher orbital components. On the other hand, the spin-orbit interaction causes a larger gain in binding energy for a larger value of the dissolution parameter.

Next, we used our SMSO formalism to investigate the efficiency of taking into account the tensor interaction [2]. We introduced a similar parameterization to allow for the effect of the tensor force on the wave function. Unlike for the SMSO, for the tensor case we used a real
number for the geometrical shift of one spin-up proton and one spin-down neutron in opposite directions. This shift enhances the $S = 1$, $T = 0$ component, in other words, a deuteron-like component. We call this parameterization SMT (simple method to include the tensor force). By combining SMSO and SMT, we calculated the energy of some $N\alpha$ systems, $^4\text{He}$, $^8\text{Be}$, and $^{12}\text{C}$. Since the tensor interaction in this calculation is not the bare interaction, the energy gain from the tensor part is not as large as that which can be obtained from the tensor force in an ab initio calculation with a realistic nucleon-nucleon interaction.

A comparison of the results of SMSO-SMT calculations for $^8\text{Be}$ and $^{12}\text{C}$ reveals a competition between the spin-orbit and tensor forces. For $^8\text{Be}$, the optimum alpha-alpha distance is large and the dissolution is small. The energy gain from the tensor force is larger than that obtained in the $^4\text{He}$ case. On the other hand, for $^{12}\text{C}$, the optimum value of the alpha-alpha distance is not so large as in $^8\text{Be}$, and the dissolution parameter becomes large so as to allow the spin-orbit interaction to have a larger effect. The energy gained from the tensor interaction becomes small in this case.

The cause of such a difference between $^8\text{Be}$ and $^{12}\text{C}$ can be attributed to the following mechanism. A small alpha-alpha distance like that in $^{12}\text{C}$ hinders the particle-hole-like excitations, which would be involved in the dissolution of the alpha particle, due to the Pauli principle. A large alpha-alpha distance, in contrast, does not suppress the particle-hole excitations. Hence, in the $^8\text{Be}$ case, the energy gain of the tensor interaction becomes large.

### 3.2. Application to the carbon isotopes

The SMSO formalism was applied to investigating the carbon isotopes, especially $^{14}\text{C}$ and $^{16}\text{C}$ [3]. For the nucleons outside the $^{12}\text{C}$ core, we assume that they are described by randomly located wave packets distributed according to the weight function $w(r) = \exp(-r/R_2)$, where $r$ is the radial coordinate of the wave packet. Here, the range parameter of the distribution $R_2$ was taken to be 1.5 fm. The optimum values of $R_1$ and $\Lambda$ are specific to each nucleus. When the valence neutrons are added to $^{12}\text{C}$, the optimum value of $R_1$ becomes smaller, and simultaneously the optimum $\Lambda$ becomes larger. Therefore, it can be concluded that the $3\alpha$ structure of $^{12}\text{C}$ is dissolved by adding valence neutrons. The results are shown in Table 1. From the values of $R_1$ and $\Lambda$ it is seen that in $^{16}\text{C}$ the structure of the $^{12}\text{C}$ core is completely different from that of the ‘free’ $^{12}\text{C}$.

| Isotopes     | $(R_1, \Lambda)$ | $B(E2; 2^+_1 \rightarrow 0^+_1)$ | $B(E2)_n$ |
|--------------|-------------------|----------------------------------|-----------|
| $^{12}\text{C} (3\alpha)$ | (1.5, 0.4)         | 3.51                             | 8.2 ± 0.1 [12] |
| $^{14}\text{C} (3\alpha + 2n)$ | (0.5, 0.8)         | 2.99                             | 3.74 ± 0.5 [12] |
| $^{16}\text{C} (3\alpha + 4n)$ | (0.5, 0.8)         | 0.01                             | 2.6 ± 0.2 ± 0.7 [13] |

We sought clustering in the excited states of $^{16}\text{C}$. To this end, we mixed wave functions with different $R_1$ and $\Lambda$ values. The two terms to be mixed, (a) $R_1 = 0.5$ fm, $\Lambda = 0.8$ and (b) $R_1 = 2.5$ fm, $\Lambda = 0.0$, are shell-model-like and cluster-model-like components, respectively. First, we performed the calculation with the basis set (a). When we reached energy convergence, we appended the basis with set (b). We found that the energy of the $0^+_3$ state comes down significantly, whereas the energies of the ground and second $0^+$ states do not change drastically. Therefore, it is justifiable to say that the $3\alpha$ cluster structure with a geometric shape appears in the $E_x = 10$ MeV region of $^{16}\text{C}$, in analogy with the crystallization of the clusters in $^{14}\text{C}$ [11].
The observed $B(E2, 2^+_1 \rightarrow 0^+_1)$ value, $2.6 \pm 0.2 \pm 0.7 \text{ e}^2\text{fm}^4$ [13], is smaller than that of $^{12}\text{C}$, although the two nuclei have the same proton number. Here, we calculated $B(E2; 2^+_1 \rightarrow 0^+_1)$ with the optimum values of $R_1$ and $\Lambda$. The $B(E2)$ value obtained is very small, $0.01 \text{ e}^2\text{fm}^4$, much smaller than the experimental value shown in Table 1. We also calculated the ‘neutron $B(E2)$’ value ($B(E2)_n$ in Table 1) by exchanging the charges of neutrons and protons as $e^p = 0$ and $e^n = e$. The obtained value is relatively large; $B(E2; 2^+_1 \rightarrow 0^+_1)_n = 4.11 \text{ e}^2\text{fm}^4$. From this we conclude that the neutron part of $^{16}\text{C}$ is deformed, and the proton part is spherical.

3.3. Clustering and role of the tensor force

According to the level order of the shell model, the ground state of $^{15}\text{C}$ should be $5/2^+$ ($0d_{5/2}$-orbit). However, the experimental ground state of $^{15}\text{C}$ is $1/2^+$. We conjectured that this level inversion may be accounted for in terms of the SMSO and the first-order tensor force contribution [14]. Therefore, we investigated the effect of the tensor correlation on the evolution of the clustering in the carbon isotopes, $^{15}\text{C}$ and $^{17}\text{C}$. It was found that, for larger alpha-alpha distances, the repulsive effect of the first-order tensor correlation acts on the $5/2^+$-state strongly, and the $1/2^+$-state comes lower than the $5/2^+$-state in the $^{15}\text{C}$, with the $R_1 \geq 1.5 \text{ fm}$. However, such a large $R_1$ value does not yield the energy minimum. This indicates that other mechanisms, for example, the inclusion of the bare tensor interaction, are necessary.

4. Summary and discussion

We studied the cluster-shell competition by introducing a simple parameterization for the alpha-alpha distance in the core nucleus and for the dissolution of an alpha particle. In this way the characteristics of nuclei consisting of a few alpha particles and valence neutrons can be classified. The alpha particle is dissolved by the spin-orbit force, and the shell-model-like and cluster-model-like behaviour can be related to a dissolution parameter.

In this study, only one alpha particle was allowed to dissolve. We think that is not enough to describe the system realistically, and hence, in a future work, we proceed to breaking further alpha particles in the calculations.

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