Abstract:

An analysis of MHD wave propagating in a gravitating and rotating medium permeated by non-uniform magnetic field has been done. It has been found that the Gradient of Magnetic Field when coupled with Rotation becomes capable to generate few instabilities (Temporal or Spatial) leading to the damping or amplification of MHD waves. The Jean’s criterion is not sufficient for stability always. Rather, the waves will suffer instability unless their wave length (frequency) is less (greater) than certain critical values. Otherwise, those will smoothly propagate outward. Out of different scenarios depending on the direction of the magnetic field, its gradient, rotation and wave propagation three important Special Cases have been discussed and different stability criteria have been derived.

Finally, using the above theory we have obtained the stability/instability criteria for the waves moving parallel and perpendicular to the galactic plane in the Core and Periphery of the Central Region of Galaxy (C.R.G.) due to the coupled action of Rotation and Non-Uniform Magnetic field. The possibility of heating or occurring diffused condition inside the central region by MHD waves or smooth propagation of these waves (under some restrictions) through the C.R.G. has been briefly discussed. The numerical values of the parameters of those waves for instabilities or smooth propagation have been estimated roughly. One may find some clues for the formation of Halo and Spiral Arms.

Keywords: MHD Instability, Magnetic field gradient and rotation, Central region of galaxy.

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1. INTRODUCTION

Any disturbance propagating through a non-uniform MHD fluid or plasma may be stable or undergoes instability. The damping of MHD waves is one of the causes of heating of such medium (Sturrock, 1966; Barnes, 1968; Bondyopadhyaya et. al. 1972, 1974). In general, if streaming flow is present it may help instability in this medium (Sturrock, 1960; Gold, 1965; Sudan, 1965). The strong rotation perpendicular to gravity, however, may help the stability of magnetized fluid (Gilman, 1970). The rotation can influence in the damping or heating of MHD medium like stellar interior (Bondyopadhyaya 1972, 1974, 1978). The MHD instability due to temperature gradient and possibility of mass-outflow from the central region of galaxy has been investigated by Chakraborty and Bondyopadhyaya (1998), Chakraborty (1998, Thesis). Sarkar and Bondyopadhyaya (2007) have discussed the MHD instability due to non-uniform magnetic field and its effect on the propagation of waves of wave length greater than certain critical value. They also discussed the role of MHD instability in heating due to damping of waves having length less than that value. In reality, MHD flow seems to be connected with different physical processes of AGN (See, e.g. Blandford, 1990) or MHD Jets from AGN (See, e.g. Rosen and Hardee, 2000; Wiita, 2002).

In this paper we have considered a gravitating and rotating MHD medium permeated by non-uniform magnetic field. We shall use the perturbation technique to linearise the guiding equations. Assuming the wave structure of the perturbation, General Dispersion Relation will be derived. Next investigation will be made about the wave propagation in different situations characterised by the Magnetic Field Gradient and Rotation. A number of stability/instability criteria will be derived which, specially, are produced by the coupled action of Non-Uniform Magnetic Field and Rotation. Finally, these results will be used to have better understanding of instabilities (heating or diffused condition or other phenomena) or stabilities (smooth wave propagation) occurring in Galactic Central Region.

2. BASIC EQUATIONS

Let us consider a gravitating, rotating MHD fluid medium having finite conductivity. The basic equations of this fluid flow are the following :

\[ \frac{\partial \vec{u}}{\partial t} = \frac{1}{(4\pi \mu \rho)} (\nabla \times \vec{B}) \times \vec{B} - (\nabla \times \vec{u}) \times \vec{B} - \rho^{-1} \nabla p - \nabla (\frac{\vec{u}^2}{2} + \phi) - 2(\vec{w} \times \vec{u}) \quad \text{[Equation of Motion]} \]  

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B} \quad \text{[MHD Field Equation]} \]  

\[ \frac{\partial \rho}{\partial t} = -\nabla (\rho \vec{u}) \quad \text{[Continuity Equation]} \]  

\[ \nabla^2 \phi = 4\pi G \rho \quad \text{[Poisson’s Equation]} \]  

\[ p = \left(\frac{C_s^2}{\gamma}\right) \rho \quad \text{[Equation of State]} \]

where,

- \( \vec{u} \) = Fluid velocity
- \( \vec{w} \) = Angular velocity
- \( \vec{B} \) = Magnetic field
- \( \rho \) = Gas density
- \( \phi \) = Gravitational potential
- \( \mu \) = Permeability
- \( \sigma \) = Electrical Conductivity
- \( \gamma \) = Ratio of specific heats
- \( C_s \) = Sound speed
- \( G \) = Gravitational constant
- \( \eta = c/(4\pi \mu \sigma) \) = Electrical Resistivity
- \( c \) = Velocity of light in vacuum

All other symbols have there usual meaning.
3. PERTURBED LINEARISED EQUATION

We shall now consider the situation when initially all the variables was at equilibrium but subsequently made perturbed as $\psi = \psi_0 + \psi'$ where $\psi_0$ and $\psi'$ are respectively unchanged value and small perturbation value of the variable $\psi$. First all the variables of equations (1) to (5) are perturbed, then the equilibrium conditions (obtained by putting equilibrium value in (1) to (5)) are used and finally the square or higher power of perturbation variables are neglected so that we get the following linearised equations (assuming, however, there is no initial streamflow i.e. $u_0 = 0$ and $\gamma = 1$) (See, Sarkar and Bondyopadhaya, 2007)

$$\frac{\partial \bar{u}}{\partial t} = \alpha \left[ (\nabla \times \vec{B}) \times \vec{B}_0 + (\vec{\nabla} \times \vec{B}_0) \times \vec{B} \right] - (\rho_0^{-1}) \nabla p - \nabla \phi - 2(\bar{u}_0 \times \bar{u}) \quad (6)$$

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{u} \times \vec{B}_0) + \eta \nabla^2 \vec{B} \quad (7)$$

$$\frac{\partial \rho}{\partial t} = -\rho_0 \nabla \cdot \bar{u} \quad (8)$$

$$\nabla^2 \phi = 4\pi G \rho \quad (9)$$

$$p = C_s^2 \rho \quad (10)$$

where

$C_s^2 = (RT_0)$, $T_0$=Initial Temperature, $\alpha = 1/(4\pi \mu r_0)$

$R$= Gas constant(obtained by dividing Universal Gas constant by mean mol. wt. of the gas). Here the primes have been dropped and suffixes zero denote the equilibrium values.

4. PERTURBATION STRUCTURE AND GUIDING EQUATION

Let us assume that all the perturbation parameters are proportional to $\text{Exp } i(kx - \omega t)$ i.e. they posses wave like structure , having the direction of propagation as x-axis, k and $\omega$ being the wave number and wave frequency respectively. Then using the equations (8),(9) and (10) we can write the equations (6) and (7) componentwise as follows :

From Equation of Motion :

$$u_x(\omega^2 - k^2 C_s^2 + u_y^2)/\omega + u_y(-2iw_0z) + u_z(2iw_0y) +$$

$$B_y[\alpha(-kB_{0y} + i(\partial B_{0y}/\partial x - \partial B_{0z}/\partial y))] + B_z[\alpha(-kB_{0z} - i(\partial B_{0z}/\partial y - \partial B_{0y}/\partial z))] = 0 \quad (11)$$

$$u_x(2iw_0z) + u_y(\omega) + u_z(-2iw_0x) +$$

$$B_x[-i\alpha(\partial B_{0y}/\partial x - \partial B_{0z}/\partial y)] + B_y(k\alpha B_{0x}) + B_z[i\alpha(\partial B_{0z}/\partial y - \partial B_{0y}/\partial z)] = 0 \quad (12)$$

$$u_x(-2iw_0y) + u_y(2iw_0x) + u_z(\omega) +$$

$$B_x[i\alpha(\partial B_{0z}/\partial z - \partial B_{0x}/\partial y)] + B_y[-i\alpha(\partial B_{0z}/\partial y - \partial B_{0y}/\partial z)] + B_z(k\alpha B_{0x}) = 0 \quad (13)$$

From Field Equation :

$$u_x[-i(\partial B_{0y}/\partial y + \partial B_{0z}/\partial z)] + u_y(i\partial B_{0x}/\partial y) + u_z(i\partial B_{0x}/\partial z) + B_x(\omega + i\eta k^2) = 0 \quad (14)$$

$$u_x(-kB_{0y} + i\partial B_{0y}/\partial x) + u_y[kB_{0x} - i(\partial B_{0x}/\partial x + \partial B_{0z}/\partial y)] + u_z(i\partial B_{0y}/\partial y) +$$

$$B_y(\omega + i\eta k^2) = 0 \quad (15)$$

$$u_x(-kB_{0z} + i\partial B_{0z}/\partial x) + u_y(i\partial B_{0z}/\partial y) + u_z[kB_{0x} - i(\partial B_{0z}/\partial x + \partial B_{0y}/\partial y)] +$$

$$B_z(\omega + i\eta k^2) = 0 \quad (16)$$

The equations can be put in matrix form as,
where, \( X = (u_x, u_y, u_z, B_x, B_y, B_z)^T \) is a column matrix and \( A = (a_{ij}) \) is a square matrix of order 6, \( a_{ij} \) denoting the coefficient of \( j \)-th variable in the \( i \)-th equation, \( i,j=1 \) to 6, has the following values:

\[
\begin{align*}
a_{11} &= (\omega^2 - k^2 c_s^2 + w_g^2)/\omega, \\
a_{12} &= -2iw_{0z}, \\
a_{13} &= 2iw_{0y}, \\
a_{14} &= 0, \\
a_{15} &= \alpha[-kB_{0y} + i(\partial B_{0y}/\partial x - \partial B_{0x}/\partial y)], \\
a_{16} &= \alpha[-kB_{0z} - i(\partial B_{0z}/\partial z - \partial B_{0x}/\partial x)] \\
a_{21} &= 2iw_{0z}, \\
a_{22} &= \omega, \\
a_{23} &= -2iw_{0x}, \\
a_{24} &= -i\alpha(\partial B_{0y}/\partial x - \partial B_{0x}/\partial y), \\
a_{25} &= k\alpha B_{0x}, \\
a_{26} &= i\alpha(\partial B_{0z}/\partial y - \partial B_{0y}/\partial z) \\
a_{31} &= -2iw_{0y}, \\
a_{32} &= 2iw_{0x}, \\
a_{33} &= \omega, \\
a_{34} &= i\alpha(\partial B_{0x}/\partial z - \partial B_{0z}/\partial x), \\
a_{35} &= -i\alpha(\partial B_{0z}/\partial y - \partial B_{0y}/\partial z), \\
a_{36} &= k\alpha B_{0x} \\
a_{41} &= i\alpha(\partial B_{0y}/\partial y + \partial B_{0z}/\partial z), \\
a_{42} &= i\partial B_{0x}/\partial y, \\
a_{43} &= i\partial B_{0x}/\partial z, \\
a_{44} &= \omega + \eta k^2, \\
a_{45} &= 0, \\
a_{46} &= 0 \\
a_{51} &= -kB_{0y} + i\partial B_{0y}/\partial x, \\
a_{52} &= kB_{0z} - i(\partial B_{0z}/\partial x + \partial B_{0y}/\partial z), \\
a_{53} &= i\partial B_{0y}/\partial z, \\
a_{54} &= 0, \\
a_{55} &= a_{44}, \\
a_{56} &= 0 \\
a_{61} &= -kB_{0z} + i\partial B_{0z}/\partial x, \\
a_{62} &= i\partial B_{0z}/\partial y, \\
a_{63} &= kB_{0x} - i(\partial B_{0x}/\partial x + \partial B_{0y}/\partial y), \\
a_{64} &= 0, \\
a_{65} &= 0, \\
a_{66} &= a_{55} = a_{44}.
\end{align*}
\]

where, \( w_g^2 = 4\pi G\rho_0 \) = Gravitational potential.

5. GENERAL DISPERSION RELATION (G.D.R.)

Eliminating the variables from the equation (17) we get the General Dispersion Relation (G.D.R.) as,

\[
\det(A) = 0 \tag{18}
\]

This is a relation of degree six representing six modes of wave propagation.

6. CONDITIONS FOR INSTABILITIES

We shall now analyse the wave instabilities under different conditions depending on direction of magnetic field.

6.1 The magnetic field in the direction of wave propagation i.e. \( \vec{B}_0 = (B_{0x}, 0, 0) \).

The dispersion relation (18) reduces to,

\[
J(A^2 - 4\omega^2 w_g^2) - 4\omega \dot{\omega} A(w_{0y}^2 + w_{0z}^2) + JAV_{Ax}^2 (L_y^{-2} + L_z^{-2}) + 2\omega k \dot{k}^2 (V_{Ax})^2 (w_{0y}L_z^{-1} - w_{0z}L_y^{-1}) + 2\omega \ddot{\omega} k^2 V_{Ax}^2 [L_y^{-1}(\omega w_{0z} - 2iw_{0x}w_{0y}) - L_z^{-1}(\omega w_{0y} + 2iw_{0x}w_{0z})] - 4\omega \dot{\omega} V_{Ax}^2 (w_{0y}L_y^{-1} + w_{0z}L_z^{-1})^2 = 0 \tag{19}
\]

where \( \dot{k} = k - iL_x, J = \omega^2 - k^2 c_s^2 + w_g^2, V_{Ax} = B_{0x}/(4\pi \mu \rho_0)^{1/2} \) = Alven velocity, \( A = \omega \dot{\omega} - k\ddot{k} V_{Ax}^2, L_s \) being the characteristic length of variation of magnetic field along \( s \) i.e. \( x, y \) and \( z \) direction for \( s=x,y,z \). It would be interesting to note that 2nd term shows that the waves will be affected by the joint action of non-uniformity of magnetic field along the wave propagation and rotation component perpendicular to wave propagation.
Further, the 4th, 5th and 6th term indicate that the waves which propagate along the magnetic field but perpendicular to non-uniformity of magnetic field (i.e. $L_y, L_z = \text{finite}$) as well as rotation components (i.e $w_0y, w_0z \neq 0$), must be affected by joint action of non-uniformity and rotation. Thus the joint action on the wave propagation along the magnetic field requires rotation component must be perpendicular to the wave propagation but non-uniformity may be parallel or perpendicular to wave propagation.

**Special case 1:** We assume that the waves are propagating along the gradient of magnetic field (i.e. $L_x = \text{finite}$ but $L_y, L_z \to \infty$) and rotation component is in the direction of wave propagation only i.e. $\vec{w}_0 \equiv (w_0x, 0, 0)$ (See Fig. 1) then from relation (19),

\[ J = 0 \]  \hspace{1cm} (19.1)
\[ \dot{\omega}(\omega \mp 2w_0x)/k^2 = V_{Ax}^2[1-i/(kL_x)] \]  \hspace{1cm} (19.2)

Now relation (19.1) and (19.2) yield [taking $\eta \to 0$ i.e $\dot{\omega} \to \omega$] the following,

\[ \omega^2 = k^2C_s^2 - w_0^2 \]  \hspace{1cm} (19.3)
\[ k = \pm[m-n^2]^{1/2} + in \]  \hspace{1cm} (19.4)

where

\[ m = \omega(\omega \mp 2w_0x)/V_{Ax}^2, \quad n = L_x^{-1}/2 \]

From (19.3) one can observe that the temporal stable mode of waves propagating along the common direction of magnetic field, its gradient and rotation requires:

Wave length $, \lambda < \frac{2\pi C_s}{w_0}$ = $\lambda_f$ (Jean’s wave length)

phase velocity, $(\frac{\omega}{k}) = C_s[1 - \frac{w_0^2}{kC_s^2}]^{1/2} < C_s$ = Sound Speed. \hspace{1cm} (I a)

The second relation (19.4) shows that the waves can avoid instability provided its wave length $\lambda < \frac{4\pi L_x}{2}$

Otherwise, such waves will undergo damping in the direction of increasing magnetic field ($n > 0$). This instability will be further enhanced at least for one mode whose wave frequency ($\omega$) < 2 rotational frequency ($w_0z$). In this situation the Instability Factor is given by–

\[ \beta = \left[ \frac{\omega(\pm 2w_0\omega - \omega)}{V_{Ax}^2} \right] + \left( \frac{L_z^{-1}}{2} \right)^2 \left( \frac{L_z^{-1}}{2} \right)^{1/2} + \frac{L_z^{-1}}{4} \] \hspace{1cm} (I b)

All these exhibit that the presence of magnetic field and rotation in the direction of wave propagation imposes restrictions on the stable and unstable mode’s frequency and wave length.

**Special case 2:** In this case we assume the magnetic field acts along the direction of wave propagation, but its gradient and rotation act perpendicular to the wave propagation direction i.e. $\vec{B}_0 \equiv (B_0x, 0, 0), L_y, L_x \to \infty, w_0 \equiv (0, 0, w_0z)$ (See, Fig. 2) then relation (19) reduces to,

\[ \omega \dot{\omega} + V_{Az}^2(L_z^{-2} - k^2) = 0 \] \hspace{1cm} (20.1)
\[ J(\omega \dot{\omega} - k^2V_{Az}^2) - 4\omega \dot{w}_0z^2 = 0 \] \hspace{1cm} (20.2)
We may note that the 1st relation involves non-uniformity of magnetic field but is independent of rotation while the 2nd relation involves rotation only but is independent of non-uniformity of magnetic field. Now, the relation (20.1) yields (taking \( \eta \rightarrow 0 \)) i.e. \( \dot{\omega} \rightarrow \omega \),

\[
\omega^2 = V_A^2_{A_x}(k^2 - L_z^2)
\]

Or,

\[
k^2 = \left(\omega^2 + V_A^2_{A_x}L_z^{-2}\right)/V_A^2_{A_x} > 0
\]  

(20.3)

These indicate that one mode will be temporally stable for \( \lambda < 2\pi L_z \).

The other mode is governed by (20.2) which, in turn, can be written as,

\[
\omega^4 + \omega^2[-k^2C_s^2 + w_g^2 - 4w_0^2z - k^2V_A^2_{A_x}] + k^2V_A^2_{A_x}[k^2C_s^2 - w_g^2] = 0
\]

Or,

\[
k^4(C_s^2V_A^2_{A_x}) - k^2[\omega^2C_s^2 + V_A^2_{A_x}(\omega^2 + w_g^2)] + \omega^2(\omega^2 + w_g^2 - 4w_0^2z) = 0
\]  

(20.4)

If \( \omega_1^2 \) and \( \omega_2^2 \) are two roots of (20.4) then,

\[
\omega_1^2 + \omega_2^2 = k^2C_s^2 - w_g^2 + 4w_0^2z + k^2V_A^2_{A_x}
\]

\[
\omega_1^2\omega_2^2 = k^2V_A^2_{A_x}[k^2C_s^2 - w_g^2]
\]

Evidently, two roots \( \omega_1^2 \) and \( \omega_2^2 \) are positive i.e the mode will be temporal stable if

\[
\omega_1^2C_s^2 > w_g^2 \quad \text{i.e} \quad \lambda < \lambda_J = \text{Jean’s wave length (See, Ferraro and Plumpton, 1966)}.
\]

If \( k_1^2 \) and \( k_2^2 \) are two roots of (20.4) then,

\[
k_1^2 + k_2^2 = [\omega^2C_s^2 + V_A^2_{A_x}(\omega^2 + w_g^2)]/(C_s^2V_A^2_{A_x})
\]

\[
k_1^2k_2^2 = \omega^2(\omega^2 + w_g^2 - 4w_0^2z)/(C_s^2V_A^2_{A_x})
\]

Clearly the two roots \( k_1^2 \) and \( k_2^2 \) are positive i.e the mode will be spatially stable provided

\[
\omega^2 + w_g^2 > 4w_0^2z
\]

Thus the stable mode must have

\[
\lambda < 2\pi L_z, \quad \lambda < \lambda_J \quad \text{or} \quad \omega^2 + w_g^2 > 4w_0^2z \quad \text{(II)}
\]

Note that the second condition is Jean’s condition. Thus the stable waves will be of higher frequency and of lesser wave length. Here both rotation and non-uniform magnetic field has been found to constrain the frequency and wave length but separately.

### 6.2 Magnetic field perpendicular to wave propagation

Here we will investigate the waves moving in the direction of the magnetic field but field gradient and rotation act perpendicular to the direction of wave propagation i.e. \( \vec{B}_0 \equiv (0, B_{0y}, 0) \) or \( \vec{B}_0 \equiv (0, 0, B_{0z}) \).

#### 6.2.1 For the first situation the dispersion relation (18) reduces to,

\[
J[\dot{\omega}^2(\omega^2 - 4w_0^2z) + \omega V_A^2_{A_y}L_z^{-1}(\omega L_z^{-1} - 2iw_0zL_y^{-1})] - 4\dot{\omega}^2\omega^2(w_0^2y + w_0^2z) + 2\omega kL_y^{-1}L_z^{-1}(V_A^2_{A_y}2(w_0zL_z^{-1} + ikw_0x) - 4\omega \dot{\omega} w_0zL_z^{-1}V_A^2_{A_y}(w_0yL_y^{-1} + w_0zL_z^{-1}) - \omega \dot{\omega} V_A^2_{A_y}(\omega^2 - 4w_0^2z)k^2 + 2i\omega \dot{\omega} L_z^{-1}L_y^{-1}(\omega w_0z + 2iw_0xw_0y) - 8i\omega \dot{\omega} w_0z w_0z L_z^{-1}V_A^2_{A_y}k = 0
\]

(21)

where \( V_A = B_{0y}/(4\pi \rho_0)^{1/2}, L_z \) being the characteristic length of variation of mag-
netic field along s i.e. x, y and z direction for s=x,y,z.
It is interesting to note that there are product terms representing combined effect of rotation and magnetic field gradients (for example, 2nd part of 1st and 5th term, the 3rd, 4th, 6th and last term).
If we analyse the relation (21) we can arrive at the conclusion that for gradient of magnetic field along the direction of wave propagation, the combined effect is peroduced provided \( w_{0z} \) is present. However, the gradient of magnetic field along the direction of magnetic field can not produce combined effect. Again if the gradient of magnetic field is in the z-direction, then the combined effect is present for \( w_{0z} \) only.

**Special case 3 :** Here we investigate the waves propagating (x-direction) perpendicular to magnetic field \( \vec{B}_0 \equiv (0, B_{0y}, 0) \) but its gradient and rotation both act perpendicular to both magnetic field and wave propagation (i.e. z-direction). Therefore, \( L_x, L_y \rightarrow \infty \) , \( \vec{w}_0 \equiv (0, 0, w_{0z}) \) (See, Fig. 3) then the relation (21) reduces to,

\[
(J - 4w_{0z}^2)(\omega \dot{\omega} + V_{Ay}^2L_{z}^{-2}) = \omega^2k^2V_{Ay}^2 \tag{21.1}
\]

This relation reveals that the combined effect could be produced by magnetic field gradient and rotation provided both act perpendicular to magnetic field as well as wave propagaton.

Now, relation (21.1) yields (taking \( \eta \rightarrow 0 \)),

\[
w^4 + w^2[V_{Ay}^2(L_{z}^{-2} - k^2) - (k^2C_s^2 - w_g^2 + 4w_{0z}^2)] + V_{Ay}^2L_{z}^{-2}[-k^2C_s^2 + w_g^2 - 4w_{0z}^2] = 0
\]

Or \( k^2 = (\omega^2 + V_{Ay}^2L_{z}^{-2})(\omega^2 + w_g^2 - 4w_{0z}^2)/[\omega^2V_{Ay}^2 + C_s^2(\omega^2 + V_{Ay}^2L_{z}^{-2})] \) \tag{21.2}

If \( \omega_1^2 \) and \( \omega_2^2 \) are two roots of the first relation in (21.2) then,

\[
\omega_1^2 + \omega_2^2 = V_{Ay}^2(k^2 - L_{z}^{-2}) + k^2C_s^2 - w_g^2 + 4w_{0z}^2
\]

\[
\omega_1^2\omega_2^2 = -V_{Ay}^2L_{z}^{-2}(k^2C_s^2 - w_g^2 + 4w_{0z}^2)
\]

It is to be noted that if \( k^2C_s^2 > w_g^2 \) i.e. \( \lambda < \lambda_f \), then both the roots can not be positive. This Jean’s criteria can not ensure stable mode so long as magnetic field, its gradient and wave propagation are mutually perpendicular to each other (See, Fig. 3). However, if the non-uniformity of magnetic field disappears i.e. \( L_z \rightarrow \infty \), then the first relation of (21.2) yields,

\[
\omega^2 = k^2(V_{Ay}^2 + C_s^2) - w_g^2 + 4w_{0z}^2
\]

This shows that for temporal stability Jean’s criterion is sufficient but for spatial stability the requirement is \( \omega^2 + w_g^2 > 4w_{0z}^2 \)

Clearly, the temporal stability of the waves require that both the roots \( (\omega_1^2 \) and \( \omega_2^2 \) must be positive. This means

\[
k^2(C_s^2 + V_{Ay}^2) > (V_{Ay}^2L_{z}^{-2} + w_g^2 - 4w_{0z}^2) \quad \text{i.e.} \quad \lambda < 2\pi \left[ \frac{C_s^2 + V_{Ay}^2}{V_{Ay}^2L_{z}^{-2} + w_g^2 - 4w_{0z}^2} \right]^{1/2}
\]

and
\[ k^2 C_s^2 - w_y^2 + 4w_{0z}^2 < 0 \quad \text{i.e.} \quad \lambda > 2\pi \frac{C_s}{|w_y^2 - 4w_{0z}^2|}^{1/2} \]

These give a range of wave length for the temporal stability of waves. Namely,

\[ 2\pi \left[ \frac{C_s^2 + V_{A_x}^2}{V_{A_y}^2 L_z^{-2} + w_y^2 - 4w_{0z}^2} \right]^{1/2} > \lambda > 2\pi \frac{C_s}{|w_y^2 - 4w_{0z}^2|}^{1/2} \quad \text{(III)} \]

Evidently, the rotation as well as magnetic field gradient both are capable to impose restrictions on the upper bound of the wave length of temporally stable modes while only the rotation is capable to restrict the lower bound of the temporally stable mode.

### 6.2.2 For the second situation, the dispersion relation (18) reduces to,

\[
J[\omega^2(\omega^2 - 4w_{0z}^2) + \omega V_{A_z}^2 L_y^{-1}(\omega L_y^{-1} + 2i w_{0x} L_z^{-1})] - 4\omega^2 \omega^2(w_{0y}^2 + w_{0z}^2) - 2\omega k L_y^{-1} L_z^{-1}(V_{A_z}^2)^2(w_{0y} L_y^{-1} + i k w_{0x}) - 4\omega \omega w_{0y} L_y^{-1} V_{A_z}^2(w_{0y} L_y^{-1} + w_{0z} L_z^{-1}) - \omega \omega V_{A_z}^2(\omega^2 - 4w_{0z}^2)k^2 - 2i\omega \omega L_x^{-1} L_z^{-1}(\omega w_{0y} - 2i w_{0x} w_{0z}) - 8i\omega \omega w_{0x} w_{0y} L_y^{-1} V_{A_z}^2 k = 0 \quad \text{(21.3)}
\]

where \( V_{A_z} = B_{0z}/(4\pi\mu_0)^{1/2}, L_s \) being the characteristic length of variation of magnetic field along \( s \) i.e. \( x, y \) and \( z \) direction for \( s=x,y,z \).

The conclusions which can be drawn from the relation (21.3) are similar to that from (21).

**SUMMARY :** We have discussed a few instabilities of the many cases which could be covered by the dispersion relation (18). For such cases pertaining to different situations we can study the instabilities, the role of combined effect of rotation and non-uniformity of magnetic field. But these will be done elsewhere. At present we can summarise the above study as follows.

When the wave propagates along \( x \)-direction we have considered three cases results of which are the following.

| Special Cases | \( B_0 \) | \( \nabla B_0 \) | \( w_0 \) | Dispersion Relation | Stability Condition | Figures |
|---------------|-----------|-------------|--------|-------------------|-------------------|--------|
| 1             | \( B_{0x} \) | \( \nabla_x B_{0x} \) | \( w_{0x} \) | \( J = \omega(\omega + 2w_{0x})/k^2 = V_{A_z}^2 [1 - i/(kL_x)] \) | \( \omega < \lambda_J \) or \( \lambda << 4\pi L_x \) | Fig. 1 |
| 2             | \( B_{0x} \) | \( \nabla_z B_{0x} \) | \( w_{0z} \) | \( J(\omega - k^2 V_{A_z}^2) - 4\omega w_{0z}^2 = 0 \) | \( \omega^2 + w_{0z}^2 > 4w_{0z}^2 \) or \( \lambda < 2\pi L_z \) or \( \lambda < \lambda_J \) | Fig. 2 |
| 3             | \( B_{0y} \) | \( \nabla_z B_{0y} \) | \( w_{0z} \) | \( (J - 4w_{0z}^2)(\omega + V_{A_y}^2 L_z^{-2}) = \omega^2 k^2 V_{A_y}^2 \) | \( \omega^2 + w_{0y}^2 > 4w_{0y}^2 \) or \( 2\pi[(V_{A_z}^2 L_z^{-2} + w_y^2 - 4w_{0z}^2)]^{1/2} > \lambda > 2\pi \frac{C_s}{|w_y^2 - 4w_{0z}^2|}^{1/2} \) | Fig. 3 |

It is found that Jean’s criterion is necessary but not sufficient for the temporal stability of
waves in Special Cases 1 and 2. However, the waves with shorter length and higher frequency are generally stable. For the Spatial stability the condition for Cases 2 and 3 are same. The Temporal stability for case 3 is too restricted to exist so long as non-uniformity of magnetic field exits.

7. CENTRAL REGION OF GALAXIES

The theory which has been discussed so far may be useful to explain different astrophysical phenomena like heating of the medium and mass outflow from it, provided, the medium could be treated essentially as MHD or Plasma.

Let us first note down the characteristic features of the medium embedded in the central region of galaxies (See, e.g. Chakraborty and Bondyopadhaya, 1998).

1. Fluid characteristics:

We know that the ionised gas medium could be treated as fluid if the mean free path of the charged particles is less than the linear dimension of the medium i.e. $\lambda_m < L$ (See, e.g. Woltjer, 1965). Now observations have revealed that huge material in different galactic central regions exists which may satisfy this criterion (See, e.g. Krotik et al., 1981; Perola et al., 1986; Uberoi, 1988). We shall study the dynamics of this fluid in the central regions by the help of hydromagnetic theory developed in the text (See also, Chakraborty and Bondyopadhaya, 1998).

2. Magnetic field:

Woltjer (1965, 1971, 1990) discussed the important role of magnetic field in the activities of Galactic Nuclei, Spiral Arms, Seyfert Galaxies and Radio jets (See also, Zeilik and Niley, 1997). This view was supported by many others (e.g. Osterbrock and Mathews, 1986). In fact, the existence of $\sim 1$ gauss magnetic field in AGN has been reported by many authors (See, e.g. Rees, 1987, Wielebinski, 1990, Morris, 1990). There is, however, indication that the field is generally parallel to the plane in denser region (core) and perpendicular to the plane in low density region (periphery) (See, Chuss et al., 2003). The magnetic field parallel or perpendicular to the galactic plane is likely to be non-uniform.

3. Rotation:

Generally rotation axis is either parallel or slightly inclined to the normal of the galactic plane. Therefore, for convenience we take the dominant component of rotation as perpendicular to the plane.

4. Division of Central Region:

There are two zones in the central region of the galaxy.

i) Core Region: Obviously the core of the central region is a denser medium. Following Chuss et al., 2003, let us assume that this region has magnetic field parallel to galactic plane. As per discussion above the rotation can be taken as perpendicular to this galactic plane. Further, the magnetic field gradient can be taken parallel and perpendicular to the galactic plane.

ii) Peripheral Region: Similarly, the peripheral region of the centre is a lighter medium. As per suggestion Chuss et al., 2003, the magnetic field can be taken as perpendicular to the galactic plane. The magnetic field gradient and rotation can also be taken as before.
8. STABILITY OR INSTABILITY OF THE CENTRAL REGION AND SIMILAR MEDIA

Now, we shall study the stability or instability of the wave propagation in the Core and Periphery of central region of the Galaxy.

i) The Core region: It is of higher density where the magnetic field may be taken as parallel but its gradient may be taken as perpendicular to the galactic plane. The MHD wave propagation in such system has been discussed in Special Case 3. The combined effect of rotation and magnetic field gradient has been found on the waves propagating parallel to the galactic plane. In this case, it is observed that the waves will be spatially stable for
\[ 4w_0^2 < \omega^2 + w_g^2 \]
And, the wave length of temporally stable wave requires the constrain (III) to be satisfied. We may note that if L.H.S.is greater than R.H.S. then
\[ w_g^2 > 4w_0^2 + C_s^2/L_z^2 \]
i.e. the density \( \rho_0 > \frac{\left[C_s^2/L_z^2+4w_0^2z\right]^{1/2}}{4\pi G} \)
The condition (III) is true when the galactic plane flow (x-direction) is perpendicular to magnetic field (y-direction). However, the flow along the magnetic field could be described by the theory outlined in Special Case 2 (Fig. 2). For stability such wave requires
\[ \omega^2 + w_g^2 > 4w_0^2 \] (as before) and \( \lambda < \lambda_J, \lambda < 2\pi L_z \)

When all these stable waves parallel to the galactic plane, and both along and perpendicular to the magnetic field continue to propagate for million of years those may feed the Spiral Arms or even may lead to the formation of Spiral Arms or other outer structures of the galaxies including our Galaxy.

ii) Peripheral Region: This region is of low density where the magnetic field, its gradient and rotation all act perpendicular to the galactic plane (as shown in Special Case 1, Fig. 1). Here, the waves propagating perpendicular to the galactic plane undergo instability due to the joint action of rotation and magnetic field gradient. One temporal stable mode can propagate whose phase velocity is less than sound velocity and whose wavelength \( \lambda < \lambda_J \). The other mode may be stable provided \( \lambda << \frac{4\pi L_z}{\lambda} \). If the wavelength does not satisfy this condition the waves will be amplified in the direction of decreasing magnetic field. In fact, the magnetic field decreases away from the galactic plane. In addition the waves with frequency \( \omega < 2w_{0x} \) will undergo instability with instability factor (I b) (as shown in Special Case 1), otherwise, I.F. =1/(2L_x) for the waves with period \( \tau > \frac{\pi}{w_{0x}} \) these unstable waves can create a diffused condition of material similar to Halo. While the stable waves can propagate in upward direction. However, it is interesting to note that if there is any flow towards the galactic plane in the direction of increasing magnetic field, the waves will be damped leading to the heating of the medium. In the long run these may help to create hot Halo like Solar Corona.

Now it is reported that from the galactic central region most of the energy come out in the form of infrared emission. In general, active galaxies have not only radio, u-v and x-ray but also infrared emission. In fact, later emission may be from heated dust or hot ionised gases (See, e.g. Zeilik, 1997). Heated dust around the nucleus of galaxies may absorb high energy radiation emitted from energised Core and re-emits it at larger infrared wave length (See, e.g. Moche, 2004).
Infact, any instability may initiate heating of the medium via damping of MHD waves. Of course, not all waves are allowed to do it. Generally, waves with shorter wave lengths can pass through and longer waves are damped or amplified (See e.g. Sarkar and Bondyopadhaya, 2007). Again, the system rotating with very high velocity may cause the instability of the MHD waves (See, Bondyopadhaya, 1972). Therefore, both rotation and non-uniform magnetic field may be held responsible for the heating of the Central Region. But both in the Core and Periphery of the central region the waves with shorter wavelength or high frequency can move smoothly. For such waves moving parallel or perpendicular to the galactic plane, when continues for long time may be responsible for the formation of the Spiral Arms or Halo region respectively.

We have seen in the text that the non-uniform magnetic field when coupled with rotation is capable to produce instability. This type of instability could not be produced by them separately (See, e.g. Barnes, 1967; Bondyopadhaya, 1972; Paul and Bondyopadhaya, 1973; Yusef-Zadeh et al., 1986; Chakraborty and Bondyopadhaya, 1998). That is to say, this instability is a product function of non-uniform magnetic field and rotation. In reality, many astrophysical media posses both rotation and non-uniform magnetic field (the rotation axis, however, may not coincide with magnetic axis always). Therefore, the theory like above may have good relevance for such media like AGN, Galactic Central Regions, Seyfert Galaxies and Infrared Galaxies. Hence, the joint effort of rotation and non-uniform magnetic field for initiating instability can be found there. In other words, the events occuring there may be at least partially due to joint action of non-uniform magnetic field and rotation.

9. NUMERICAL ESTIMATION

In the previous section we have discussed the instability criteria in the central region of galaxy (C.R.G.). Now, we will make a few numerical estimations of those criteria. We have considered the central region of our galaxy as a hot plasma bed described by the MHD equations (1 to 5) of Sec. 2 (See e.g., Chakraborty and Bondyopadhaya, 1993, 1998; Sarkar and Bondyopadhaya, 2007). Let us first consider some physical parameters in the central region of our Galaxy.

The radius of the central region may be taken as 0.5 to 100 pc because well defined circumnuclear gas disks have been found upto that region in a number of galaxies (Morris, 1998).

Further, the central region may be considered as a medium of uniform density, which is given by \( \rho_0 \approx 3.5 \times 10^{-20} g \text{ cm}^{-3} \) i.e. \( 10^{36} g \text{ pc}^{-3} \) (See Balick and Sanders, 1974; Genzel et al., 1985; Lo, 1986). Consequently, gravitational frequency becomes \( w_g = (4\pi G \rho_0)^{1/2} \approx 1.7 \times 10^{-13} s^{-1} \approx 5.36 \text{ per mil. yr.} \)

The adiabatic sound speed is given by \( C_s \approx 6.5 \text{pc per mil yr} \) (for \( T_0 = 10^4 K \) at 10 pc from the centre, vide Yusef-Zadeh et al., 1984).

The strength of general magnetic field is 4-6 \( \mu G \) with a cell size of 10-100 pc (See, Ohno and Shibata, 1993). Let us take magnetic field strength as 5\( \mu G \) and the variation of magnetic field as 2\( \mu G \) over 50 pc range. Thus the characteristic length of variation \( L_x \) or \( L_z \approx 25 \text{pc} \). Hence, Alfven velocity \( V_A \approx 1.4 \times 10^{-18} \text{pc s}^{-1} \approx 4.41 \times 10^{-5} \text{ pc per mil. yr.} \).
The central region of the Galaxy rotates like a rigid body. Beyond this region the velocity first decreases and then increases. At nearly 8 kpc from the centre the velocity becomes maximum. The Sun is situated at 8.5 kpc from the centre having linear velocity $220 \text{ km s}^{-1}$. Then Sun’s angular velocity about galactic centre $\approx 84.6 \times 10^{-17} \text{ s}^{-1}$. (See, Karttunen, 2007). Let us take this value as same as the angular velocity of central region i.e. $w_{0z} \approx 84.6 \times 10^{-17} \text{ s}^{-1} \approx 2.7 \times 10^{-2}$ per mil. yr.

Therefore, we obtain a critical wave length (i.e. Jeans wave length) $\lambda_J = \frac{2\pi C_s}{w_g} \approx 7.62\text{ pc}$.

Next, let us find what are the waves which can propagate or which will suffer instability inside the two areas of the central region of our Galaxy.

i) Peripheral Region : (Special Case 1, Fig. 1)

Here the magnetic field, its gradient and rotation all act perpendicular to the galactic plane. One modes of the waves moving perpendicular to this plane will be temporal stable provided

$\lambda < \lambda_J \approx 7.6 \text{ pc}$ and the other mode require

$\lambda << 4\pi L_x \approx 315 \text{ pc}$. Conversely if the waves have length greater than 7.6 pc those will suffer instability. In addition, if $\lambda$ is not much less than 315 pc some more modes will be unstable. Moreover, the waves having frequency less than $2w_{0x} \approx 17 \times 10^{-16} \text{ s}^{-1}$ (i.e. $\tau > 10^8 \text{ yr}$) the spatial instability will be enhanced. The actual instability factor is given by $\beta$ in Special case 1.

Such waves will be amplified leading to the defused condition if it moves along the decreasing direction of magnetic field i.e. away from the Galactic plane towards Halo. If, however, there is inflow of MHD waves from outside towards the Galactic plane i.e. in the direction of increasing magnetic field then the waves will be damped leading to the heating of the Halo.

ii) Core Region : (Special case 2,3, Fig 2,3)

The spatial stability of waves in this region require $w_g > 2w_{0z}$. As we have already discussed here $w_g \approx 1.7 \times 10^{-13} \text{ s}^{-1}$ and $w_{0z} \approx 8.5 \times 10^{-16} \text{ s}^{-1}$. Thus the inequality is satisfied. Hence, waves of all frequencies will spatially stable.

Now, the waves will propagate, in the direction perpendicular to the magnetic field, smoothly (i.e. temporally stable) must have the wave length restricted by the following inequality

$$2\pi \left[ \frac{C_s^2 + V_A^2}{4w_g L_z + w_g^2 - 4w_{0z}^2} \right]^{1/2} > \lambda > 2\pi \left[ \frac{C_s}{w_g^2 - 4w_{0z}^2} \right]^{1/2}$$

Incidentally, $w_g(\approx 1.7 \times 10^{-13}) >> 2w_{0z}(\approx 1.7 \times 10^{-15})$ and $C_s >> V_A$, $V_A/L_z \approx 3 \times 10^{-20} \text{ s}^{-1}$. These values will make R.H.S and L.H.S of the above inequality almost same. Therefore, no temporally stable wave can exist. However, those waves moving along the magnetic field will be temporally stable whose wave length is given by

$\beta$ We make a note of the fact that the source of energy of the solar corona is still not known but it is thought to be some how associated with magnetic reconnection and, therefore must be understood in terms of MHD process (See, Cravens, 1997).
As we have seen that in the central region of our Galaxy \( w_g >> 2w_{0z} \) and \( C_s >> V_Ay \) but the situation might be different in the long past in the life of our Galaxy itself. When both magnetic field and rotation was much higher. Similar situation may be present in some other galaxies also. For example, in AGN magnetic field is \( 10^6 \) times (See, Sec. 7) the field that we have taken, so that the contribution due to magnetic field or its gradient is much more significant. The derived condition could be applicable in those galaxies particularly at the time of formation.

10. CONCLUSION

In this paper the General Dispersion Relation (G.D.R.) for the unidirectional (x-direction) wave propagation has been derived in the rotating medium where magnetic field is non-uniform. Then the guiding dispersion relations have also been obtained for magnetic field either along or perpendicular to wave propagation. There could be many different situations depending on the magnetic field, its gradient and the direction of rotation. But out of these only three Special Cases have been discussed.

A number of temporal/spatial instability/stability conditions have been derived. Comparing the Special Cases it is observed that the combined effect of rotation and magnetic field gradient is present in the product form provided the wave propagates along the magnetic field. The coupled action of rotation and magnetic field’s non-uniformity have also been noticed when the wave propagates perpendicular to the magnetic field and its gradient. In general the instability conditions reveal that the waves having shorter frequency or longer wave length become unstable.

The instability conditions, thus derived, could be used to study different phenomena occurring in the the medium similar to the central region of our Galaxy. It is suggested that such stabilities or instabilities are more significant in the early stage of evolution of many galaxies when both rotation and magnetic field are much higher. Those waves which can propagate from the central region may help in the formation of Spiral Arms and Halo of the galaxy in long run.

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i) Mathematical analysis have been given more stress in this paper.
ii) A number of sub cases have also been analysed but all those are not included in this paper.
iii) The theory of the formation of hot Halo and Spiral Arms from the Galactic central material appears to be consistence with the underlined proposition of this paper.
iv) It is well known that the electrical resistivity causes the wave unstable. If \( \eta \) does not tend to 0, the dispersion relations obtained in the text could be easily derived. This will, however, be considered in separate communication.
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