The Dark Energy Regulated by Emergent Conformal Symmetry

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We have found a mechanism which regulates the dark energy in our universe. With an emergent conformal symmetry, the dark energy density is regulated to the order of a conformal anomaly parameter in the conformally coupled gravity. In the late time cosmological evolution, we have obtained a set of exact cosmological equations which deviate from the Friedmann equations significantly. Based on the recent observational cosmic expansion data, it is shown that the dark energy density is about $1/4$ of the matter density at present, which is quite smaller than determined by General Relativity. The jerk parameter at present is also determined as a definite value 0.47.

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I. INTRODUCTION

The cosmological constant problem has been a big mystery since the development of General Relativity. There have been many attempts to cancel or remove the cosmological constant without success. Recently, there have been new astronomical observations of the late time cosmic acceleration of our universe [1, 2] adding another mystery to the cosmological constant problem: the dark energy density in our present universe is not zero, but as small as $\rho_\Lambda^{\text{obs}} \sim (10^{-3}\text{eV})^4$, which is quite close to the matter density at present [3, 4].

Thus, on the top of the old puzzle for the dark energy: how the vacuum energy in matter Lagrangian could not contribute to the dark energy, we are now facing another formidable new puzzle: why the dark energy density is so close to the matter density at present in our universe. As yet, there has been no successful mechanism to relax the vacuum energy leaving such a small dark energy density which is so close to the matter density in the present universe.

Before the success of Weinberg-Salam model, the weak interactions were characterized by the dimensional Fermi’s coupling constant, $G_F \simeq (300\text{Gev})^{-2}$. But later the Fermi’s coupling constant is replaced with the Higgs scalar field which seems to be discovered recently [5]. Inspired by the success, it seems plausible to replace the dimensionful Newton’s constant with a gravity scalar field as Brans-Dicke gravity model [6, 7], or the induced gravity models [8–23]. Furthermore, there is no room to conceive any mechanism to resolve the dark energy puzzles in General Relativity. Thus, we consider a gravity model replacing the Newton’s constant with a gravity scalar field, adding a cosmic scalar potential.

It is known that, in any gravity models with a gravity scalar field which can give a flat spacetime solution with a constant gravity scalar field in vacuum, it is not possible to find an exact cancellation mechanism of the cosmological constant [24].

However, with an evolving gravity scalar field in the late time universe, we have found a regulation mechanism of dark energy in the conformally coupled gravity with a uniquely determined cosmic scalar potential, which might have an origin of a conformal anomaly generated by quantum effects in matter Lagrangian. Even though the conformally coupled gravity model does not give a flat spacetime for a constant gravity scalar field in vacuum, the non-flatness would be quite negligible in a non-cosmic scale gravity with the uniquely determined cosmic potential.

In the matter dominated late time universe, we have obtained a set of exact cosmological equations which deviate from the Friedmann equations significantly. However, in the radiation dominated early universe, we regain the Friedmann equations of General Relativity having a constant gravity scalar field. Thus, we can say that the conformally coupled gravity with the logarithmic cosmic potential is a minimal extension of General Relativity to implement the dark energy regulation mechanism, replacing the dimensionful Newton’s constant with the gravity scalar field.

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II. THE CONFORMALLY COUPLED GRAVITY

Replacing the Newton’s constant $G$ with a gravity scalar field $\Phi$ as $8\pi G = 1/\Phi^2$, we consider the following conformally coupled gravity action with a cosmic scalar potential $V(\Phi)$ which is to be determined in the next section,

$$S = -\int d^4x \sqrt{-g} \left[ \frac{1}{2} \Phi^2 R + 3(\partial \Phi)^2 \right] + \int d^4x \sqrt{-g} (\mathcal{L}_m - V(\Phi)), \tag{1}$$

where $\mathcal{L}_m$ is the matter Lagrangian. The first gravity part of the action (1) is invariant under the local conformal transformation, $g_{\mu\nu}(x) \rightarrow e^{2\sigma(x)} g_{\mu\nu}(x), \quad \Phi(x) \rightarrow e^{-\sigma(x)} \Phi(x)$, which is the reason why we call the action (1) as the conformally coupled gravity. However, the second matter part of the action (1) does not have the conformal symmetry, in general.

We have the equations of motion for the gravity scalar field $\Phi$ and the metric $g_{\mu\nu}$ as follows,

$$6\nabla^2 \Phi = \Phi R + \frac{\partial V(\Phi)}{\partial \Phi}, \tag{2}$$

$$\Phi^2 G_{\mu\nu} - 2\Phi \nabla_\mu \nabla_\nu \Phi + 2g_{\mu\nu} \Phi \nabla^2 \Phi + 4\nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} (\nabla \Phi)^2 - g_{\mu\nu} V(\Phi) = T^{(m)}_{\mu\nu}, \tag{3}$$

where $T^{(m)}_{\mu\nu}$ is the matter energy-momentum tensor.

Taking the trace of Eq.(3) gives an expression of $\Phi^2 R$. Plugging it into Eq.(2), we have a characteristic constraint equation of the conformally coupled gravity,

$$T^{(m)} + 4V(\Phi) - \Phi \frac{\partial V(\Phi)}{\partial \Phi} = 0, \tag{4}$$

where $T^{(m)}$ is the trace of the matter energy-momentum tensor. The origin of the constraint (4) is that the gravity scalar field $\Phi$ is not dynamical, but auxiliary in the conformally coupled gravity. Even though the total classical action (1) was not imposed to have a conformal symmetry in general, the consistency in equations of motion (2,3) requires the conformal symmetry equation (4) in the conformally coupled gravity [25]. Therefore, the conformally coupled gravity (1) describes a system of having an emergent conformal symmetry, but not having the conformal symmetry in the classical action [28].

Due to the presence of the cosmic scalar potential $V(\Phi)$, the equations (2,3) do not allow the exact flat spacetime solution even for a constant gravity scalar field in vacuum. However, in a non-cosmic scale, it is found that the deviation from the flat spacetime could be negligible provided that the cosmic potential satisfies a certain condition.

III. THE COSMOLOGICAL EVOLUTION

We investigate the cosmological evolution, assuming a homogeneous gravity scalar field $\Phi(t)$ depending only on time, and adopting the Robertson-Walker metric with a vanishing spatial curvature($k = 0$),

$$ds^2 = dt^2 - S^2(t)[dr^2 + r^2 d\Omega^2]. \tag{5}$$

Then, Eqs.(2,3) are reduced to a set of cosmological equations as shown below. Denoting the time derivative as an over-dot,

$$3\Phi^2 \dot{H}^2 + 6H \dot{\Phi}^2 + 3\ddot{\Phi}^2 = \rho_m + \rho_\Lambda(\Phi), \tag{6}$$

$$2a\dddot{\Phi}^2 + H^2 \dot{\Phi}^2 + 4H \dot{\Phi} \ddot{\Phi} + 2\dot{\Phi}^2 - \dot{\Phi}^2 = -p_m + \rho_\Lambda(\Phi), \tag{7}$$

$$\rho_m - 3p_m + 4\rho_{\text{vac}} + 4V(\Phi) = \dot{\Phi} \frac{\partial V(\Phi)}{\partial \Phi}, \tag{8}$$

where the Hubble parameter $H \equiv \dot{S}/S$, the acceleration parameter $a \equiv \ddot{S}/S$. With the matter pressure $p_m$, the matter density $\rho_m$, $T^{(m)0}_0 = \rho_m + \rho_{\text{vac}}$, $T^{(m)1}_1 = T^{(m)2}_2 = T^{(m)3}_3 = -p_m + \rho_{\text{vac}}$, where $\rho_{\text{vac}}$ is the vacuum energy which could be generated by quantum effects in the matter Lagrangian. We can see that the sum of the vacuum energy $\rho_{\text{vac}}$ in the matter Lagrangian and the cosmic potential $V(\Phi)$ plays the role of the total dark energy density, $\rho_\Lambda(\Phi) \equiv \rho_{\text{vac}} + V(\Phi)$. 
IV. THE COSMIC POTENTIAL TO REGULATE THE DARK ENERGY

We try to find a cosmic potential in the conformally coupled gravity which can regulate the vacuum energy in the matter Lagrangian. For a given vacuum energy $\rho_{\text{vac}}$ with the matter density $\rho_m$ and the matter pressure $p_m$ from the matter Lagrangian, a gravity scalar field $\Phi$ would satisfy Eq. (8). For a different vacuum energy $\rho_{\text{vac}}'$, the equation (8) would be satisfied with a different gravity scalar field $\Phi'$ instead of $\Phi$. For these two different vacuum energies $\rho_{\text{vac}}$ and $\rho_{\text{vac}}'$, the difference in the total dark energy density is found as

$$\Delta \rho_\Lambda = \rho_\Lambda(\Phi') - \rho_\Lambda(\Phi) = V(\Phi') - V(\Phi) + \rho_{\text{vac}}' - \rho_{\text{vac}} = \frac{1}{4} \left[ \Phi' \frac{\partial V(\Phi)}{\partial \Phi} |_{\Phi'} - \Phi \frac{\partial V(\Phi)}{\partial \Phi} |_{\Phi} \right].$$ (9)

Therefore, we can have the same total dark energy density $\rho_\Lambda$ in the conformally coupled gravity whatever the vacuum energy $\rho_{\text{vac}}$ is in matter Lagrangian provided that the cosmic scalar potential satisfies the equation,

$$\Phi \frac{\partial V(\Phi)}{\partial \Phi} |_{\Phi} = \alpha$$

for a constant $\alpha$. Thus we choose the cosmic scalar potential quite uniquely as:

$$V(\Phi) = \alpha \ln \frac{\Phi}{\mu}, \quad \rho_\Lambda(\Phi) = \rho_{\text{vac}} + \alpha \ln \frac{\Phi}{\mu}.$$ (10)

where $\alpha$ is a dimensionful constant which would have some other physical origin. Both the dimensionful constant $\mu$ and the vacuum energy $\rho_{\text{vac}}$ in matter Lagrangian hide into the total dark energy density $\rho_\Lambda(\Phi)$ and do not appear as independent physical quantities at all.

For the cosmic potential Eq. (10), the total dark energy density $\rho_\Lambda$ satisfies the constraint equation (8) with the matter density $\rho_m$ and the matter pressure $p_m$,

$$\rho_m - 3p_m + 4\rho_\Lambda(\Phi) = \alpha.$$ (11)

The left hand side of this equation is the trace of the total energy momentum tensor including all matter and total dark energy of our universe. If we interpret Eq. (11) as the trace anomaly cancellation equation which states that the total trace anomaly of our universe vanishes, then the constant $-\alpha$ in Eq. (11) would be the conformal anomaly which is generated in matter Lagrangian by quantum effects. Thus, we may speculate that the cosmic potential (10) is an effective potential to describe the conformal anomaly generated in matter Lagrangian by quantum effects. It is found that the cosmic potential (10) is the linear form of coupling $\phi \Theta^\mu_\mu$ between a scalar field $\phi$ and a conformal anomaly $\Theta^\mu_\mu$, which might have an origin in quantum chromodynamics, considered in [24, 26] with $\Phi \equiv e^{-\phi}/\kappa_0$. Thus, we can say $\alpha$ as a conformal anomaly parameter.

Taking a time derivative Eq. (10), and using Eqs. (7, 8, 10), we have the matter energy conservation equation,

$$d \frac{\rho_m + 3H(\rho_m + p_m)}{dt} = 0,$$ (12)

which is independent of the gravity scalar field $\Phi$.

V. THE LATE TIME COSMOLOGY OF OUR UNIVERSE

In the late time evolution of our universe dominated by cold matter with the matter pressure $p_m = 0$, the matter energy conservation equation (12) tells $\rho_m = A/S^3(t)$ with a constant $A$ [27]. Taking a time derivative of equation (11) with the help of the explicit form Eq. (10), we find that

$$\frac{d \Phi}{dt} = -\frac{\Phi}{4\alpha} \frac{d \rho_m}{dt} = \frac{3}{4\alpha} H\Phi \rho_m.$$ (13)

Thus, we can find the solution of the gravity scalar $\Phi(t)$ and the corresponding gravitational constant $8\pi G = 1/\Phi^2(t)$ as a function of the scale factor $S(t)$,

$$\Phi(S) = \Phi_\infty e^{-\frac{4\alpha}{3\kappa_0} S^3(t)}, \quad G(S) = G_\infty e^{\frac{4\alpha}{3\kappa_0} S^3(t)}.$$ (14)

where $\Phi_\infty$ and $G_\infty$ are the values of the gravity scalar field and the gravitational constant in the infinite future, respectively.
The relative variations of the gravity scalar field $\Phi$ and the gravitational constant $G$ in the matter dominated universe are determined by the matter density ratio $\rho_m/\alpha$ as

$$\ddot{\Phi}/H\dot{\Phi} = \frac{3}{4} \frac{\rho_m}{\alpha}, \quad \frac{\dot{G}}{HG} = -\frac{3}{2} \frac{\rho_m}{\alpha}. \quad (15)$$

However, $\Phi$ should be a constant in the radiation dominated early universe from Eq. (11) observing $\rho_{m,rad} = \rho_{m,rad}/3$ for relativistic matter. Therefore, this conformally coupled gravity model is reduced to General Relativity giving the Friedmann equations with a constant gravitational constant $G_{rad} = G_{\infty}$ and a constant dark energy density $\rho_{\Lambda,rad} = \alpha/4$ in the radiation dominated early universe.

Inserting Eq. (13) into the cosmological equations (16,17), we have a set of exact cosmological evolution equations in the matter dominated late time universe as follows,

$$3\Phi^2 H^2 = \alpha \frac{4(1 + 3x)}{(4 + 3x)^2},$$

$$6\Phi^2 a = \frac{3(1-x)(4 + 3x)^2 - (1 + 3x)(16 - 48x + 9x^2)}{(4 + 3x)^3}, \quad (17)$$

where we have defined $x \equiv \rho_m/\alpha$, and used $\rho_{\Lambda}(\Phi) = (1-x)\alpha/4$ from Eq. (11) with $\rho_m = 0$. The cosmological equations (16,17) approximate to the Friedmann equations only if $x$ is much smaller than 1, i.e. $x \ll 1$.

The jerk parameter which measures time variation of the cosmic acceleration parameter $a$ is obtained as

$$j \equiv \frac{\dddot{S}(t)}{S(t) dt} = \frac{\ddot{a}}{H^3} + a = 1 -\frac{9}{16} x \frac{64 - 32x + 3x^2}{4 + 3x} + \frac{9}{8} x \frac{4 + 3x}{1 + 3x} + \frac{9}{4} \frac{a^2}{H^2} \frac{8 - 3x}{4 + 3x}, \quad (18)$$

which depends only on $x$ because $a/H^2$ is a function of $x$ only from Eqs. (16,17).

**VI. IMPLICATIONS ON THE COSMOLOGY IN THE PRESENT UNIVERSE**

Because the matter in our universe is believed to be cold with $\rho_m = 0$ at present [27], the matter density $\rho_m$ and the current dark energy density $\rho_\Lambda$ would satisfy

$$\rho_m + 4\rho_\Lambda = \alpha \equiv 4\rho_{\Lambda,\infty}. \quad (19)$$

As our universe expands, the matter density will eventually die out to zero. Thus, the total dark energy density $\rho_{\Lambda,\infty} = \alpha/4$ will prevail our universe in the infinite future. $\rho_{\Lambda,\infty}$ is the very dark energy density $\rho_{\Lambda,rad}$ in the radiation dominated early universe with $\rho_{m,rad} = \rho_{m,rad}/3$.

Recently, the ratio of the cosmic acceleration parameter $a$ to the Hubble parameter $H^2$ was determined through the help of Type-I supernovae as standard candles [1, 2, 27],

$$\left(\frac{a}{H^2}\right)_{\text{exp}} \approx 0.55. \quad (20)$$

From the ratio of Eq. (17) to Eq. (10), $\frac{a}{H^2}$, which is a monotonically decreasing function of $x$, we can find a single solution $x_{\text{exp}}$ which satisfies the experimental ratio $(a/H^2)_{\text{exp}}$ numerically. If we use the experimental value (20), then we find the corresponding solution $x_{\text{exp}} = 0.476$. For this $x_{\text{exp}}$, the cosmological equations (16,17) do not approximate to the Friedmann equations because the higher $x$ terms are not negligible. This $x_{\text{exp}} = 0.476$ determines the matter density $\rho_m$, the dark energy density $\rho_\Lambda$, the jerk parameter $j$, the critical density $\rho_c$ at present, respectively, as

$$\rho_m = 0.476\alpha, \quad \rho_\Lambda = 0.131\alpha, \quad j = 0.47, \quad \rho_c = 3\Phi^2 H^2 = \frac{3}{8\pi G} H^2 = 0.33\alpha. \quad (21)$$

Thus, the ratio of the dark energy density to the matter density in the present universe is $\rho_\Lambda/\rho_m \cong 1/4$, which is quite smaller than the value $\rho_\Lambda/\rho_m \cong 7/3$ determined by the Friedmann equations based on the same data (20). The jerk parameter $j = 0.47$ predicted by this model is quite definite as General Relativity predicts $j = 1$.

Because only about 4% of the critical density $\rho_c = (\frac{3}{8\pi G}) H^2_{\text{exp}}$ is the ordinary baryonic matter [27], the baryonic matter density is estimated as $\rho_{\text{baryon}} \approx 0.013\alpha$. Thus, the dark matter density is $\rho_{DM} \approx 0.463\alpha$, which dominates all matter density $\rho_m$ as $\rho_{DM}/\rho_m \approx 0.97$ in this model.

From the difference between the infinite future and the current total dark energy densities, we can estimate the increase of gravity scalar field $\Phi^2 \approx 1.27\Phi^2_{\text{now}}$ and the corresponding decrease of gravitational constant $G_{N,\infty} \approx 0.79G_{N,\text{now}}$ in the infinite future, restoring to the gravitational constant in the radiation dominated early universe.
VII. CONCLUSIONS

We have proposed the conformally coupled gravity with the uniquely determined cosmic potential, which describes a system of having the emergent conformal symmetry. We have found that the dark energy density is regulated to the order of the conformal anomaly parameter. The logarithmic cosmic potential might be an effective way to describe the conformal anomaly which is generated by quantum effects in the matter Lagrangian. The conformal anomaly parameter $\alpha$ introduced in the cosmic potential may provide a new fundamental length scale of gravity instead of the Newton’s constant as $\alpha \sim (10^{-8} \text{eV})^4 \sim (0.1 \text{mm})^4$, which is an order of the matter density in our present universe. The smallness of the conformal parameter $\alpha$ would ensure that the local gravity in a solar system scale is not affected by the presence of the cosmic potential.

In the matter dominated late time universe, we have a set of exact cosmological equations which deviate from the Friedmann equations significantly. However, in the radiation dominated early universe, we regain the Friedmann equations of General Relativity having a constant gravity scalar field. Thus, the conformally coupled gravity with the logarithmic cosmic potential is a minimal extension of General Relativity to implement the dark energy regulation mechanism, replacing the dimensionful Newton’s constant with the gravity scalar field.

Based on the recent observational cosmic expansion data for $(a/H_0)^2_{\text{exp}}$, we have found that the dark energy density is about $1/4$ of the matter density in the present universe, which is quite smaller than the value $7/3$ determined by the Friedmann equations. As a characteristic prediction of this model, the jerk parameter at present has a quite definite value $j = 0.47$ as General Relativity predicts $j = 1$.

Since the regulation mechanism of the dark energy, through the emergent conformal symmetry in the conformally coupled gravity with the unique cosmic potential, may resolve the puzzles for the dark energy, we hope that the jerk parameter $j$ will be measured precisely to test this model in near future.

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