Dynamical bag in a chiral quark model

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Abstract

A type of bag function is proposed to make the MIT bag surface
of baryon dynamical. It is illustrated through renormalization of the
quark field that the softening of chiral bag gives rise to a model of chi-
ral quark with effectively-generated mass of quark, in which confined
quark moves in the background of nonlinear pion. A prediction of bag
constant $B \simeq 2f_{\pi}^2 m_{\pi}^2$ is made. With two free parameters, the self-
coupling $e$ of pion and the confining scale $a$, the computed mass, the
charge root-mean-square radius and magnetic moment of the proton
are in good agreement with the experimental values.

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Key Words Bag, chiral quark, nucleon properties

1 Introduction

In principle, the fundamental theory of strong interaction, the Quantum
Chromodynamics (QCD), provides the whole information of the hadrons
from the underlying degrees of freedom, the quark and gluon. The com-
plexity of QCD vacuum, however, makes the direct calculation via QCD,
e.g., the simulation via QCD on lattice, quite nontrivial for the light sector
of quark[1](below $\sim 1$GeV).
On the other hand, the effective models of hadrons, such as the Skyrme model, the MIT bag models, the cloudy bag model, the quark models in the nonrelativistic form (NQM) and the relativistic form, etc., also enable us to explore the strong interaction at low-energy. While the NQM is successful, the MIT bag model provides a conceptually simple picture of hadrons. Though it incorporates both confinement and asymptotic freedom of QCD, the MIT bag model, in its original form, did not possess chiral symmetry, another main feature of QCD at low energy. In the hybrid chiral bag model, Chodos and Thorn and Brown and Rho restored chiral symmetry by introducing pions into the bag Lagrangian. Thus a baryon can be thought to be three-quarks inside a bag, surrounded by a cloud of pions, which makes the nucleon-nucleon interaction description via the exchange of pions accessible.

In the nonlinear-pion treatment the HCB model (see for a review) the confined quarks inside bag interact with the pion through the chirally-invariant coupling at the bag surface by interpolating the Skyrme model in the limit of the small-bag (the bag radius $R \to 0$) and the MIT bag model in the large-bag limits ($R \to \infty$). To do so, a bag function $\theta_V$ (unit in the bag volume $V$ and zero outside) and its delta function $\delta_S (= \infty$ on the surface $S$ of $V$ and zero elsewhere) were used to imitate the permanent confinement of quarks in hadrons and ensure the continuity of the axial-vector current. The HCB Lagrangian reads

$$L^{HCB} = (\bar{q}i\gamma\partial q - B)\theta_V - \frac{1}{2}\bar{q}U_5 q\delta_V + L^\pi \theta_V,$$

(1)

where $L^\pi$ is the Lagrangian of pion $\pi$, $U_5 = \exp(i\tau \cdot \pi \gamma_5/f_\pi)$, $\theta_V = 1 - \theta_V$. In the case that $L^\pi$ is the Skyrme Lagrangian the model describes a quark bag as a defect in the Skyrme field, and realizes the chiral symmetry in the Wigner mode inside the bag and in the Nambu-Goldstone mode outside, with the baryon charge ($B$) conserved topologically. Due to the sharp surface of the bag which allows the creation of virtual pions with arbitrarily high momenta, however, the self-energy of nucleon remains to be infinite and the origin of the bag to be unknown in the bag models including the HCB model.

One way to avoid self-energy divergence of nucleon is to include the exterior fermions with the finite self-energy of nucleon arising from Casimir effect. Another approach is to replace the unphysical bag boundary by a finite surface thickness of the quark core within the framework of the relativistic quark model with static potential. Also, an attempt was made to regularize the self-energy by introducing a soft (fuzzy) bag surface.

In this paper, we propose a type of soft bag function formed by nonlinear
pion to make the bag surface of baryon dynamical. It is illustrated that the softening of chiral bag gives rise to a chiral quark model with an effectively generated mass of quark, in which confined valence quark interacts with a chiral cloudy of the pion. The finite mass of nucleon including pion dressing is obtained through renormalization of the quark field in terms of pion, and a prediction \( B \simeq 2f_\pi^2 m_\pi^2 \) for the bag constant is made. Using two free parameters of the model, the self coupling \( e \) of self-pion and the confining scale \( a \), some properties (the mass, the charge root-mean-square radius and magnetic moment) of the proton are computed, in good agreement with the experimental values. The bag radius and the size of quark core are estimated to be about 0.91 fm and 0.33 fm, respectively. This is done by using a trial profile for the hedgehog Skyrmion \[18\] in the process of solving model.

## 2 The quarks in a cloudy pion

To incorporate the surface dynamics into the HCB model in which the bag functions (\( \theta_V, \delta_S \)) are singular, we propose the following continuous bag functions constructed in terms of the nonlinear chiral field \( U(r) \) of pion, to replace the singular bag functions in the HCB model,

\[
\begin{align*}
\theta_V \to \theta_U &= \frac{1}{8} \text{tr}[2 - (U + U^\dagger)], \\
\delta_S \to \delta_U &= -\frac{1}{8} \frac{d}{dn} \text{tr}[2 - (U + U^\dagger)], \\
\theta_{\bar{V}} \to \theta_{\bar{U}} &= \frac{1}{8} \text{tr}[2 + (U + U^\dagger)]
\end{align*}
\]

where \( U = \exp(i \tau \cdot \pi / f_\pi) \) and \( d/dn \) stands for the gradual derivatives along the direction normal to the bag surface \( (S) \). Here, \( U \) is the nonlinear representation of the Goldstone bosons (\( \pi \) mesons) under the constraint \( U^\dagger U = 1 \).

To see how \( \theta_U \) and \( \delta_U \) behave as the soft bag functions, we utilize as a first step a simple Skyrmion profile in the Skyrme model for the chiral angle

\[
F(r) \simeq 4 \arctan[\exp(-c_0 r)],
\]

with \( r \) the radial coordinate measured in the unit of \( (e f_\pi)^{-1} \) (the parameters in the Skyrme Lagrangian, see \[3\] below). It is shown numerically \[19\] \[18\] that a Skyrmion in finite region can be approximated by \[3\] \( c_0 \approx 1.0 \). With the hedgehog ansatz for \( U = \exp[i F(r) \hat{r} \cdot \tau] \in SU(2) \) in \[2\], one finds

\[
\begin{align*}
\theta_U &= \frac{1}{2} (1 - \cos(F)) = \sin^2(F/2) \to \sec h^2[c_0 r], \\
\theta_{\bar{U}} &= \frac{1}{2} (1 + \cos(F)) = \cos^2(F/2) \to \tanh^2[c_0 r], \\
\delta_U &= -\frac{1}{2} \sin(F) \frac{dF}{dr} \to 2c_0 \sec h^2[c_0 r] \tanh(c_0 r).
\end{align*}
\]

Here, the symbol arrow \( \rightarrow \) refers the further reduction upon using \[3\], which are plotted explicitly in FIG.1.
Imposing the (chiral) boundary condition $F(0) = \pi, F(\infty) = 0$, corresponding to the physical vacua $U = \pm 1$, respectively, one can check from (2) alone that the relations $\theta_U = 1 - \theta_V$ and $\int_0^\infty dr \delta_U = 1$ hold, just as $\theta_V$ and $\delta_S$ do in the HCB model. We note that these relations hold simply as a pure result of the boundary condition on $F$, with nothing to do with the explicit profile (3) for the Skyrmion. Given the success of the Skyrmion model in description of the light baryons below 1 GeV, it is natural to extend the solitonic profile (4) into a bag-like functions constructed by the dynamical degree of freedom, $U$.

We consider the following reformulation of the HCB model (1) with the replacement (2) used and a scalar-like confining potential $S(x)$ added,

$$\mathcal{L}^{DCB} = \left[\bar{q}(i\gamma^\mu \partial_\mu - S(x)U_5)q - B\right]\theta_U - \frac{1}{2} \bar{q}U_5q\delta_U + \mathcal{L}^{SK}(U)\theta_U,$$

in which $q$ is the current quark of light flavor, $\mathcal{L}^{SK}(U)$ describes the chiral dynamics of the pion in terms of nonlinear chiral field $U$ (or equivalently, of $F$). Upon using (2) in (5), one finds

$$\mathcal{L}^{DCB} = \frac{1}{2}\left[\bar{q}(i\gamma^\mu \partial_\mu - SU_5)q\right] - \frac{1}{2} tr(U + U^\dagger)\left[\bar{q}(i\gamma^\mu \partial_\mu - SU_5)q\right] - \frac{1}{16} \bar{q}U_5q \left(n^\mu tr(\partial_\mu U + \partial_\mu U^\dagger)\right) + \mathcal{L}^{\pi}$$

FIG. 1: The three functions of soft bag.
in which \( n^\mu = (0, \vec{r}) \). Assuming the pion Lagrangian \( \mathcal{L}^{SK}(U) \) to be, for instance, the twice of Skyrme model \( 2\mathcal{L}^{Skyrme} \) (this corresponds to redefinition of the model parameters) one has explicitly for the pion dynamics in (6),

\[
\mathcal{L}_\pi = \frac{f^2_\pi}{4} tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{8} B tr[U + U^\dagger - 2] + \frac{f^2_\pi}{16} tr[U + U^\dagger] tr[\partial_\mu U \partial^\mu U] + \frac{1}{4 e^2} tr[U + U^\dagger] tr[(\partial_\mu U \partial_\nu U)^2 - (\partial^\mu U^\dagger \partial^\nu U)^2] + \frac{1}{64} e^2 tr[U + U^\dagger] tr[(\partial_\mu U \partial_\nu U)^2 - (\partial^\mu U^\dagger \partial^\nu U)^2].
\]

Here, the Skyrme Lagrangian is

\[
\mathcal{L}^{SK} = \frac{f^2_\pi}{4} tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32 e^2} tr[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2,
\]

with \( f_\pi \) the pion decay constant, and \( e \) the self-coupling of the pion.

The model (6) describes an interacted theory of valence quark and pion, which breaks chiral symmetry through the coupling the quark bilinear operators to the terms \( tr(U + U^\dagger) \) and \( tr(\partial_\mu U + \partial_\mu U^\dagger) \). We note here that the pion dynamics (7) is closed upon the replacement (2) in the sense that it simply picks up some of the higher terms (up to the order-six terms of the momentum expansion) from the chiral perturbative theory (ChPT) without extra parameters added. By comparing (7) with the ChPT, one finds \( B \simeq 2f^2_\pi m^2_\pi \), which results in the bag constant \( B \simeq (134 MeV)^4 \) (with the pion mass \( m_\pi \simeq 137 MeV \)), in agreement with \( B^{1/4} = 146 MeV \) in the MIT bag model.

The difference of the bag constant estimated here with that of the MIT bag model and the value \( B^{1/4} \simeq 150 MeV \) in chiral bag model may be due to the pion self-coupling near the bag region. In fact, the estimate \( B \simeq 2f^2_\pi m^2_\pi \) is made through the chiral (weak-\( \pi \)) expansion of \( U \): \( U \approx 1 + i\pi/f_\pi - (\pi/f_\pi)^2/2 \). In fact, the chiral (weak-\( \pi \)) expansion

\[
\theta_U \approx (\pi/2 f_\pi)^2 - (\pi/2 f_\pi)^4/3
\]

does not apply near the bag region (where \( \theta_U \sim 1 \), see FIG.1), as it ignores the strong self-interaction of the nonlinear pion there.

### 3 The renormalization of quark by pion

We see that the model (6), as it stands, does not assume the standard form of relativistic field theory. To see the role of pion clearly, it is very useful to reformulate (6) into the form of relativistic field theory by changing the quark field \( q \) into a new variable of effective quark, that is normalized.
We note first that the MIT wavefunction $q_{MIT}$, when normalized as
\[
\int q_{MIT}^\dagger q_{MIT} d^3x = 1,
\]
can be written in whole space as
\[
q_{MIT} = \psi_{\kappa m}(x) \theta(R - r)^{1/2}
\] (9)
where $\psi_{\kappa m}$ is the solution to the MIT bag model. The form (9) is also valid outside the bag ($r > R$) since $\theta(R - r)$ vanishes there. The wavefunction (9) can also be obtained by solving an equivalent problem\[17\] $L_{MIT} = q[i\partial - m(r)]q - B$ in an infinite vallum potential $m(r)$, namely, the Dirac equation for the quark having the effective vanishing mass $m(r) = 0$ (for $r \leq R$) and the infinite mass ($m(r) = \infty$, for $r > R$).

The similar picture of the effective mass occurs for our model, if one redefines a renormalized quark as\[14\]
\[
Q = \theta^{1/2} q = \left\{ \frac{1}{8} tr[2 - (U + U^\dagger)] \right\}^{1/2} q
\] (10)
and requires $Q$ to fulfill the reorganization $\int Q^\dagger Q dV = 1$ as $q_{MIT}$ do in the MIT bag model. Upon substitution of (10) into the (5), one finds
\[
L^{CQ} = \bar{Q} [i\gamma^\mu (\partial_\mu - \frac{1}{2} \partial_\mu \ln\theta_U) - SU_5 + \frac{1}{2} U_5 n^\mu \partial_\mu \ln\theta_U] Q + L^\pi
\] (11)
where the pion dynamics $L^\pi$ is given by (7).

The static Hamilton associated with (11) is
\[
H^{CQ} = \int d^3x \bar{Q} \left\{ -i\alpha \cdot (\nabla - \hat{r} \cot(F/2) F_r/2) \right. + \gamma^0 (M(r) + S(r)) \cos F \left. \right\} Q + B \int d^3x \sin^2(F/2) + H^\pi,
\] (12)
with
\[
M(r) \equiv \cot(F/2)(-F_r/2),
\] (13)
and $F_r = dF/dr$. Here, in this work, the Hamilton $H^\pi$ for the pion dynamics is truncated, for simplicity, to be that of the Skyrme model(three foregoing terms) in (7).

\[
H^\pi = \int d^3x \left\{ \frac{\alpha^2}{4} tr(\partial^i U \partial^j U^\dagger) + \frac{1}{4} B tr[1 - U] \right. \\
+ \left. \frac{1}{16\pi^2} tr[(\partial_i U^\dagger \partial_i U)^2 - (\partial_i U^\dagger \partial_j U)^2] \right\}
\] (14)

Taking the wave function for the (valence) quark $Q$ to be
\[
\psi_Q = \frac{N}{r} \left( \begin{array}{cc} G(r) \\ -iH(r) \sigma \cdot \hat{r} \end{array} \right) y_{jm}(\theta \varphi) \chi_f
\] (15)
with $N^2 \equiv 1/[\int dr (G^2 + H^2)]$ the normalization factor, $y_{ljm}$ the Pauli spinor and $\chi_f$ the flavor wavefunction, one finds for (12)

$$H^{CQ} = N^2 \int dr \left\{ H \frac{dG}{dr} - G \frac{dH}{dr} + \frac{2\kappa}{r} GH + (M + S) \cos F (G^2 - H^2) \right\} + 4\pi B \int dr r^2 \sin^2 (F/2) + H^\pi,$$

with $-\kappa$ the eigenvalue of the operator $K = \gamma^0 [\Sigma \cdot (r \times p)] + 1$ corresponding to the eigen-state $y_{ljm}$, and it, upon the re-scaling $r = Lz (L = 1/efe_z)$, becomes

$$H^{CQ} = N^2 \int dz \left\{ H \frac{dG}{dz} - G \frac{dH}{dz} + \frac{2\kappa}{z} GH + (LM + LS) \cos F (G^2 - H^2) \right\} + 4\pi L^4 B \frac{z^2}{z^2} (1 - \cos F) + E^\pi,$$

with $N^2 L = 1/\int dz [G^2 + H^2]$, and $E^\pi$ given by

$$E^\pi = \frac{2\pi f_w}{e} \int dz \left\{ z^2 F_z^2 + 2 \sin^2 (F) (1 + F_z^2) + \frac{\sin^4 F}{2} + e^2 L^4 B z^2 (1 - \cos F) \right\}$$

where $F_z \equiv dF/dz$. The equation of motion for (16) with (17) is

$$\frac{dG}{dz} + \frac{2}{z} G = \left[ \varepsilon + L (M + S) \cos F \right] H$$

$$- \frac{dH}{dz} + \frac{2}{z} H = \left[ \varepsilon - L (M + S) \cos F \right] G$$

$$\left( 1 + \frac{2 \sin^2 F}{z^2} \right) F_{zz} + \frac{2 \sin (2F)}{z} \left( F_z^2 - 1 - \frac{\sin^2 (F)}{z^2} \right)$$

$$= e^2 \left[ L^4 B - \frac{LN^2}{4\pi} (LM + LS) (G^2 - F^2) \right] \sin F$$

where $\varepsilon = E_q L$ is the eigen-energy of $Q$, and

$$LM(z) = \cot (F/2) (F_z^2/2),$$

$$LS(z) = L (r/a^2) = L a_z,$$

with $L_a = L/a = 1/(efe_z a)$. Here we have taken the scalar potential $S$ to be of the linear form $S = r/a^2$, with $a$ the confining scale.

It follows from the asymptotic form of (18) that

$$G(z \to \infty) \sim e^{-L_a z^2/2} \sim -H(z \to \infty),$$

$$F(z \to \infty) \sim e^{-2\sqrt{BL^2}z/2},$$

and

$$G(z \approx 0) \approx \sqrt{z} Y_{\kappa+1/2} (\varepsilon z) \sim \sqrt{\frac{2\kappa}{\pi}} z,$$

$$H(z \approx 0) \sim -z J_{\kappa+3/2} (\varepsilon z) \sim -\sqrt{\frac{2\kappa}{\pi}} z^{3/2},$$

$$F(z \approx 0) \approx \pi - Az.$$
One sees from (12) that an effective($r$-dependent) mass appears, as a result of the pion dressing in the form of renormalization (10). This is similar in the spirit to the equivalent description of the MIT bag where quark has a $r$-dependent effective mass $m(r)(\simeq 0$ inside bag and $\simeq +\infty$ outside), or equivalently, moves in a vallum-like scalar potential[17]. This is in consistent with the mechanism of mass generation[20] of quark through chiral symmetry breaking of QCD at low energy. The effective mass (13) is plotted in FIG.2(a) against $z = ef_\pi r$ for the improved configuration (21) given below in section 4.

4 The proton as a 3-quark state

According to the n"aive quark model, a nucleon is a three-quark state of light quark u, d and s, constructed based on SU(6) flavor symmetry. This picture is valid approximately when the strange(s quark) component in the nucleon ignored, as is assumed commonly. We check our model in the $N_f = 2$ flavor case which is relevant to the low-lying state of nucleons. We first solve the coupled equations (18) for the quark-pion system and then take the nucleon(proton, here) state as a three-quark state with the SU(6)$_F$ symmetry of spin-flavor wavefunction. At last, some of static properties of proton are computed.

To solve the coupled equations (18), the third of which is nonlinear, we firstly choose the following profile for the Skyrmion[18] as a trial function solution to the third equation in (18)

$$F(x) = 4w \arctan[e^{-cz}] + \pi(1 - w) \times \left[1 - \left(\frac{\sinh^2(dz)}{a^2 + \sinh^2(dz)}\right)^{-1/2}\right]$$

and then solve the equation (18) self-consistently. $F$ is obtained by averaging $v = (LM + LS)(G^2 - F^2)/(4\pi z^2)$ over the interval $[0, x_N]$ and applying the optimizing (Neilder-Mead) algorithm used in [18]. For $x_N = a/L = 7.8011$, the optimal result is $\langle v \rangle = 0.0097$, and that for (21) is

$$(a_t, c, d, w) = (0.620, 0.904, 1.283, 1.174).$$

The model parameters are set to be

$$e = 2.80, a = 1/0.6676GeV^{-1},$$

$$f_\pi = 93MeV, B^{1/4} = 0.130GeV.$$  

only two ($e, a$) of which are free since $B$ is fixed by the proton mass $E_p = 938.27MeV$(input). The boundary condition are fixed by (19) and (20). We
FIG. 2: The three functions of soft bag.

plotted the resulting solution \( k = -1 \) for the quark wavefunction \((G, H)\) in FIG. 2, corresponding to the energy eigenvalues \( E_q = \epsilon f_\pi \)

\[
\epsilon = 0.9315, \quad E_q = 242.557 \text{MeV},
\]

and the chiral angle \( F \) as well as \( v \) in FIG. 3. The corresponding bag functions and the effective mass of quark are shown in FIG. 4.

The bag radius is calculated using the definition \( R = F^{-1} (\pi/2) \), and listed in the Table I in which the size of the quark core \( R_a/R = a \) is fixed by (19).

\[
R = 0.9092 \text{fm}, \quad a = 0.2955 \text{fm},
\]

Following the naïve quark model, we write the nucleon state for the proton to be that of three valence quarks

\[
|p \uparrow\rangle = \frac{1}{\sqrt{3}} [ |u \uparrow u \downarrow d \uparrow \rangle_S - 2 |u \uparrow u \uparrow d \downarrow \rangle_S ],
\]

in which the suffix \( S \) (here) stands for the symmetrization of the quark indices \((1, 2, 3)\), namely, the position in the ket representation \( |q(x_1)q(x_2)q(x_3)\rangle \).
\[ (LM + LS)(G^2 - H^2)/(4\pi z^2) \]

The chiral angle \( F = 2.80 \), \( f_{\pi} = 93 \text{MeV}; \)

\[ a = 1/0.6676 \text{GeV}; \]

\[ B = (0.130 \text{GeV})^4 \]

FIG. 3: The three functions of soft bag.

\[ \sin(F/2)^2 \]
\[ \cos(F/2)^2 \]
\[ \sin(F)(-F')/2 \]

Effective mass \( M_e = 2.80 \), \( f_{\pi} = 93 \text{MeV}; \)

\[ a = 1/0.6676 \text{GeV}; \]

\[ B = (0.130 \text{GeV})^4 \]

FIG. 4: The three functions of soft bag.
Using (25) and assuming the flavor $SU(3)_F$ symmetry, it can be shown that

$$
\langle r^2 \rangle_{ch} = \langle p \uparrow | \sum_{i=1}^{3} e_i r_i^2 | p \uparrow \rangle
= \int d\Omega_1 dr_1 r_1^4 \Psi_u^\dagger(x_1) \Psi_u^\dagger(x_1)
= L^2 \int_0^\infty dz \frac{dz}{dz[G^2 + H^2]},
$$

(26)

where $\Psi_u(x_1)$ is the space part of the wavefunction (15) of the quark (taken to be up quark here, for instance) at $x_1$, $e_i (i = 1, 2, 3)$ stands for the quark charge ($e_u = +2/3$, $e_d = -1/3$) in the unit of electric charge $Q_e$. Numerically, the result is $\langle r^2 \rangle_{ch} = 1.326 L^2 \simeq (0.872 fm)^2$.

Similarly, the magnetic moment can be shown to be (using the Dirac matrices $\hat{\alpha} = \gamma^0 \gamma$)

$$
\mu_p = \langle p \uparrow | \sum_{i=1}^{3} \frac{e_i}{2} (r \times \hat{\alpha}_i) | p \uparrow \rangle
= 3 \langle p \uparrow | \hat{\alpha}_1 (r \times \hat{\alpha}_1) | p \uparrow \rangle
= \frac{Q_e}{2} \int d\Omega_1 dr_1 r_1^2 \psi_u^\dagger(x_1) \left( \begin{array}{cc} 0 & \hat{r} \times \hat{\sigma} \\ \hat{r} \times \hat{\sigma} & 0 \end{array} \right) \psi_u^\dagger(x_1),
$$

(27)

which, upon using (15), becomes,

$$
\mu_p = \frac{Q_e N^2}{2} \int_0^\infty dr \frac{G(r) H(r)}{r} \int_0^1 \sin^2 \theta d\cos \theta,
= \frac{2Q_e N^2 L^2}{3} \int_0^\infty dzzG(z) H(z)
= \frac{2Q_e N^2 L^2}{3} \int_0^\infty dzzG(z) H(z)
\int_0^\infty dzz[G^2 + H^2].
$$

(28)

The numerical result for $\mu_p$ (in the unit of the nuclear magneton $\mu_N = Q_e / 2m_p$) yields

$$
\frac{\mu_p}{\mu_N} = \frac{4m_p L}{3} \int_0^\infty dzzG(z) H(z) \int_0^\infty dzz[G^2 + H^2] \simeq 2.704.
$$

In Table I we present our report for the calculations, being compared with the experimental data (Exp.) as well as the calculations by other models, including the MIT bag model, the Skyrme model [3], the HCB model [12], and the (perturbative) chiral quark model (PCQM) [8]. A nice agreement with the data is obtained.

We note that though (11) is motivated by the chiral bag model, it is in the range of the chiral quark models [8] where the confinement is put in phenomenologically, and chiral symmetry is implemented by construction. The running of the valence quark mass agrees with asymptotic freedom of
QCD in principle. The our model (11) imitates the Skyrme model when the confining scale(size of the quark core) $a \to 0$, while it tends to the chiral bag model when $S(x)$ becomes a vallum-like potential.

## Table I

| Quantities | MIT $^5$ | Skyrme $^3$ | HCB $^{12}$ | PCQM $^8$ | This Work | Exp. |
|------------|----------|-------------|-------------|-----------|-----------|------|
| $R[fm]$   | 1.0      | 1.0         | 0.6         | 0.55 $\sim$ 0.65 | 0.909     |      |
| $e$        | $[\alpha_c = 2.2]$ | 5.45 | 4.5         | 2.80      | 93        | 93   |
| $f_\pi[fm]$ | $[Z = 1.84]$ | 93 | 93          | 88        | 93        | 93   |
| $a[GeV^{-1}]$ |          | 0.296      | [ ]         |           |          |      |
| $B^{1/4}[MeV]$ | 146      | 150         | $[B_0 = 1400]$ | 134      | 938.27    | 938.27 |
| $m_p[MeV]$ | 938      | 939         | 1425        | 938.3     | 938.27    | 938.27 |
| $\langle r^2 \rangle_{ch}[fm]$ | 0.73     | 0.59        | 0.48        | 0.85      | 0.8723    | 0.877 |
| $|\mu_p|/\mu_N$ | 1.93     | 1.87        | 2.19        | 2.6       | 2.704     | 2.793 |

The main features of model (11), which differs from the chiral quark model, and the bag models as well as the Skyrme models mentioned above, lies in

(i) It identifies the dynamical role of the term $\sim tr[2 \pm (U + U^\dagger)]$ as a bag-like function, which breaks explicitly chiral symmetry of the model, as it does in ChPT $^{16}$.

(ii) The generation of the valence quark mass via the pion dressing is made explicit, in a way that is consistent with the chiral symmetry and breaking of QCD, and that the valence quark mass vanishes at short distance.

(iii) The bag radius($\sim 0.9 fm$) is much bigger than the quark-core size ($\sim 0.3 fm$), which emphasizes the role of pion, but agrees with the two scales of QCD: $\Lambda_{QCD}^{-1} = 0.6 \sim 1 fm$ and $\Lambda_\chi^{-1} \sim 0.2 fm$.

## 5 Summary

Motivated by the hybrid chiral bag model, we propose a bag-like function of nonlinear pion to make the bag surface of baryon dynamical. Assuming quark confinement, we illustrated that the softening of the bag surface via nonlinear pion gives rise to a chiral quark model with an effectively-generated mass of quark, in which the confined quark moves in a background of nonlinear pion in a chiral invariant way. The running effective mass of quark is obtained through renormalization of the quark field in terms of pion, and a relation for bag constant $B \simeq 2f_\pi^2 m_\pi^2$ is obtained. Some static properties(mass, charge radius, magnetic moment) of proton are computed by solving model with a trial Skyrmion profile, in good agreement with the experimental data. The bag radius are estimated to be about $0.91 fm$ and the size of quark core to be about $0.3 fm$. 

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References

[1] P. Hagler, Hadron structure from lattice quantum chromodynamics, Phys. Rept. 490(2010) 49-175.

[2] T.H.R. Skyrme, Nucl. Phys. 31(1961)556; I. Zahed and G. Brown, Phys. Rept.142(1986)1, for a review.

[3] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. B228(1983)552.

[4] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D9(1974)3471; A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D10(1979)2594.

[5] T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D12(1975)2060.

[6] N. Isgur and G. Karl, Phys. Lett. 72B (1977) 109; Phys. Rev. D 18 (1978) 4187; Phys. Rev. D 20 (1979) 1191.

[7] S. Théberge, A.W. Thomas, G.A. Miller, Phys. Rev. D 22 (1980) 2838; A.W. Thomas, Adv. Nucl. Phys. 13(1984)1.

[8] E. Oset, R. Tegen, W. Weise, Nucl. Phys. A. 426(1984)456; R. Tegen, Ann. Phys. 197(1990)439; S.A. Chin, Nucl. Phys. A 382(1982)355; V.E. Lyubovitskij, T. Gutsche, A. Faessler, Phys. Rev. C 64(2001)065203.

[9] A. Faessler, T. Gutsche and V.E. Lyubovitskij, Prog. Part. Nucl. Phys. 55(2005)1–11, for a review.

[10] A. Chodos and C. B. Thorn, Phys. Rev. D 12(1975)2733.

[11] G. E. Brown and M. Rho, Phys. Lett. 82B(1979)177; G. E. Brown, M. Rho, and V. Vento, ibid. 84B(1979)383.

[12] A.D. Johnson and M. Rho, Phys. Rev. Lett. 51(1983)751; A.D. Johnson, D.E. Kahana, et al., Nucl. Phys. A462(1987)661.
[13] A. Hosaka and H. Toki, Phys. Rept. 277(1996)65.

[14] Y. Nogami, A. Suzuki, Prog. Theor. Phys. 69(1983)1184; C. A. Z. Vasconcellos, H.T. Coelho, et al., Eur. Phys. J. C 4(1998)115-127

[15] L. Vepstas, A.D. Jackson, Phys. Rept. 187(1990)109-143.

[16] J. Gasser, H. Leutwyler, Ann. Phys. (NY)158(1984)142; Phys. Rep. 87 (1982)77.

[17] U. Mosel, *Fields, symmetries and quark*, Springer-Verlag (Berlin,Heidelberg) 1999.

[18] Duojie Jia, Xiao-Wei Wang, Mod. Phys. Lett. A26(2011)557-565; Duojie Jia, Xiao-Wei Wang, et al., Chin. Phys. Lett. 27(2010)121201.

[19] P. M. Sutcliffe, Phys. Lett. B292(1992)104

[20] Y. Numbu, Phys. Rev. Lett. 4(1960)380; Y. Numbu, G. Jona-Lasino, Phys. Rev. 122(1961)345