Decay Rate Asymmetry of the Top Quark through Light Squarks

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Abstract

A decay rate asymmetry of the top quark is discussed within the framework of the supersymmetric standard model. Although new sources of $CP$ violation in this model are severely constrained from the electric dipole moments of the neutron and the electron, there is a possibility that a top squark has a mass of around 100 GeV and $CP$ violation in its interaction with a top quark is unsuppressed. Then partial widths could be different between the decays $t \rightarrow bW^+$ and $\bar{t} \rightarrow \bar{b}W^-$. The magnitude of this $CP$ asymmetry can be of order $10^{-3}$, which may be detectable in the near future.

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1 Introduction

In quantum field theory, $CPT$ invariance holds under only fundamental assumptions, while $CP$ violation can be implemented by various ways. Phenomenologically, $CP$ violation is observed in $K^0-\bar{K}^0$ mixing, and possibly in the universe as baryon asymmetry. Although the former is described well by the Kobayashi-Maskawa mechanism of the standard model (SM), it is difficult to produce observed baryon asymmetry through this mechanism. It seems necessary to extend the SM to have new sources of $CP$ violation.

One plausible extension of the SM is the supersymmetric standard model (SSM) based on $N=1$ supergravity coupled to grand unified theories (GUTs). This model contains new sources of $CP$ violation, which could generate baryon asymmetry sufficiently [1]. The electric dipole moments (EDMs) of the neutron and the electron are also predicted to have large magnitudes [2]. However, the new sources of $CP$ violation do not affect much the $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ systems. The SSM can only contribute to $CP$ asymmetries in $B$-meson decays indirectly through $B^0-\bar{B}^0$ mixing [3]. Other observables have to be invoked for examining $CP$ violation by the SSM in collider experiments.

In this paper, I report a recent study by Aoki and Oshimo [4] on $CP$ violation induced by the SSM in the production and decay of the top quark. If one top squark is light enough, a top quark can decay into a top squark and a neutralino, which subsequently become a bottom quark and a $W$ boson through final state interactions. The interaction of a top squark with a top quark could violate $CP$ invariance maximally. Then, $CP$ violation may be observed as a difference of partial decay rates between the decays of a top quark and an anti-top quark, which is measured by the decay rate asymmetry

$$A_{CP} = \frac{\Gamma(t \to bW^+) - \Gamma(\bar{t} \to \bar{b}W^-)}{\Gamma(t \to bW^+) + \Gamma(\bar{t} \to \bar{b}W^-)}.$$  \hfill (1)

Taking into account the constraints from the EDMs of the neutron and the electron, we discuss $A_{CP}$ and explore parameter region where a sizable asymmetry is expected.

2 The model

The SSM has several complex parameters. In the supersymmetric part of the interaction Lagrangian, complex are a mass parameter $m_H$ in the linear coupling of Higgs superfields and coefficients of the trilinear Yukawa couplings. The soft-breaking part
contains gaugino mass parameters \( \tilde{m}_3, \tilde{m}_2, \) and \( \tilde{m}_1 \) for the SU(3), SU(2), and U(1) gauge groups, respectively, dimensionless coupling constants \( A_f \)'s and \( B \) for the trilinear and linear terms of scalar fields, respectively. We assume unification of the gaugino masses at the GUT scale, giving the relation \( \tilde{m}_3/\alpha_3 = \tilde{m}_2/\alpha_2 = 3\tilde{m}_1/5\alpha_1 \) at the electroweak scale. Since \( A_f \)'s are considered to have the same value of order unity at the GUT scale, their differences at the electroweak scale are neglected and thus we put \( A_f = A \). Under these assumptions, by redefining particle fields, we can take \( m_H \) and \( A \) as physical complex parameters without loss of generality. We express these parameter as \( m_H = |m_H| \exp(i\theta) \) and \( A = |A| \exp(i\alpha) \). The phase \( \theta \) is contained in the chargino and neutralino mass matrices and the squark and slepton mass-squared matrices. The phase \( \alpha \) is contained only in the latter.

The mass-squared matrix \( M^2_q \) for the squarks corresponding to a quark \( q \) with mass \( m_q \), electric charge \( Q_q \), and third component of the weak isospin \( T^3_q \) is given by

\[
M^2_q = \begin{pmatrix}
    m^2_q + \cos 2\beta (T^3_q - Q_q \sin^2 \theta W) M^2_Z + \tilde{M}^2_{qL} & m_q(R_q m_H + A^* m_{3/2}) \\
    m_q(R_q m_H' + A m_{3/2}) & m^2_q + Q_q \cos 2\beta \sin^2 \theta W M^2_Z + \tilde{M}^2_{qR}
\end{pmatrix},
\]

\[
R_q = \frac{1}{\tan \beta} \left( \begin{array}{c}
    T_{3q} = \frac{1}{2} \\
    T_{3q} = -\frac{1}{2}
\end{array} \right), \quad \tan \beta = \frac{v_2}{v_1}, \quad (2)
\]

where \( \tilde{M}^2_{qL} \) and \( \tilde{M}^2_{qR} \) denote the mass-squared parameters for the left-handed squark and the right-handed squark, respectively, and \( m_{3/2} \) is the gravitino mass. Generation mixings are not necessary to induce \( CP \) violation, so that we neglect them. The mass eigenstates \( \tilde{q}_1 \) and \( \tilde{q}_2 \) are obtained by diagonalizing the mass-squared matrix as

\[
S^\dagger_q \tilde{M}^2_q S_q = \mathrm{diag}(\tilde{M}^2_{q1}, \tilde{M}^2_{q2}) \quad (\tilde{M}^2_{q1} < \tilde{M}^2_{q2}), \quad (3)
\]

where \( S_q \) is a unitary matrix. The slepton mass-squared matrices are obtained by appropriately changing \( M^2_q \) in Eq. (2).

At the GUT scale, we consider the squark and slepton masses to have a common value of the gravitino mass. Then, at the electroweak scale, the values of the mass-squared parameters for the squarks of the first two generations and all the sleptons are approximately the same,

\[
\tilde{M}^2_{qL} \simeq \tilde{M}^2_{qR} \simeq \tilde{M}^2_{lL} \simeq \tilde{M}^2_{lR} \equiv \tilde{M}^2. \quad (4)
\]
On the other hand, those for the squarks of the third generation receive large quantum corrections through Yukawa interactions proportional to the top quark mass $m_t$, expressed as

$$
\tilde{M}_{L}^2 = \tilde{M}^2 - cm_t^2, \quad \tilde{M}_{R}^2 = \tilde{M}^2 - 2cm_t^2,
$$

$$
\tilde{M}_{bL}^2 = \tilde{M}^2 - cm_t^2, \quad \tilde{M}_{bR}^2 = \tilde{M}^2,
$$

with $c = 0.1 - 1$. Under this scheme, if $\tilde{M}$ is around $m_t$, the quantum corrections and the large values of the off-diagonal elements of the top squark mass-squared matrix make one top squark rather light.

In our scheme, the SSM parameters which determine the interactions at the electroweak scale are $\tan \beta$, $A$, $m_H$, $\tilde{m}_2$, $\tilde{M}$, $m_{3/2}$, and $c$. Although these parameters are not all independent each other, they can have various sets of values depending on assumptions for underlying GUTs and parameter values. Therefore, for simplicity, we take those parameters independent.

## 3 Constraints from EDMs

The complex mass matrices for the $R$-odd particles lead to $CP$-violating interactions, which give rise to the EDMs of the neutron and the electron at the one-loop level. The exchanged particles in the loop diagrams are charginos, neutralinos, and gluinos with squarks or sleptons. The present experimental upper bounds on the neutron and the electron EDMs are approximately $10^{-25}e.cm$ and $10^{-26}e.cm$, respectively. For an unsuppressed value of the $CP$-violating phase $\theta$, both the neutron and the electron EDMs generally receive dominant contributions from the chargino-loop diagrams, which are approximately proportional to $\sin \theta$. The experimental bounds on the EDMs impose the constraints that the squarks and the sleptons should be heavier than 1 TeV \cite{2}. The chargino contributions become small as the magnitude of $\theta$ decreases, irrespective of the value of another $CP$-violating phase $\alpha$. For a sufficiently small value of $\theta$, the constraints on the squark and the slepton masses are relaxed.

Assuming $\theta \ll 1$ and $\alpha \sim 1$, the neutron and the electron EDMs receive dominant contributions from the gluino- and the neutralino-loop diagrams, respectively. In Table 1, the absolute values of the EDMs of the neutron $|d_n|$ and the electron $|d_e|$ are shown for several values of the SU(2) gaugino mass $\tilde{m}_2$, taking $\tilde{M} = 200$ GeV, $\theta = 0$, and $\alpha = \pi/4$. The other parameters are taken as $\tan \beta = 2$, $|m_H| = 100$ GeV,
Table 1: The absolute values of the EDMs of the neutron $|d_n|$ and the electron $|d_e|$ for several different values of $\tilde{m}_2$. The $CP$-violating phases are taken as $\alpha = \pi/4$ and $\theta = 0$. We have chosen $\tan \beta = 2$, $|m_H| = 100$ GeV, and $|A|m_{3/2} = \tilde{M} = 200$ GeV.

| $\tilde{m}_2$ (GeV) | $|d_n|$ | $|d_e|$ |
|------------------|-------|-------|
| 300 3.4 x 10^{-25} | 1.3 x 10^{-26} |
| 400 1.8 x 10^{-25} | 1.1 x 10^{-26} |
| 500 1.1 x 10^{-25} | 9.5 x 10^{-27} |
| 600 7.5 x 10^{-26} | 8.1 x 10^{-27} |
| 700 5.2 x 10^{-26} | 6.9 x 10^{-27} |
| 800 3.8 x 10^{-26} | 5.8 x 10^{-27} |
| 900 2.9 x 10^{-26} | 5.0 x 10^{-27} |

and $|A|m_{3/2} = \tilde{M}$. If $\tilde{m}_2$ is around or greater than 500 GeV, the neutron EDM is consistent with its experimental bound. In all the range of $\tilde{m}_2$, the predicted value of the electron EDM lies within the experimental bound. This numerical analysis shows that even if the phase $\alpha$ is of order unity, squarks and sleptons are allowed to have masses of order 100 GeV, without causing inconsistency for the EDMs of the neutron and the electron.

4 Numerical results of decay rate asymmetry

The constraints from the EDMs do not exclude a possibility that the lighter top squark is lighter than the top quark. There is a parameter region where a top quark can decay into a top squark and a neutralino. The produced top squark and neutralino can yield a bottom quark and a $W$ boson by exchanging charginos or bottom squarks as shown in Fig. 1. If $CP$ invariance is violated in these reactions, the interference of the decay amplitudes at the tree level and the one-loop level makes the partial decay rates different between the decays $t \rightarrow bW^+$ and $\bar{t} \rightarrow \bar{b}W^-$. The decay rate asymmetry $A_{CP}$ is obtained as

$$A_{CP} = \frac{\alpha_2}{2} \left[ \left\lbrace m_t^2 + m_b^2 - 2M_W^2 + \frac{(m_t^2 - m_b^2)^2}{M_W^2} \right\rbrace \sqrt{\lambda(m_t^2, M_W^2, m_b^2)} \right]^{-1} T, \quad (6)$$

where $T$ consists of $CP$-odd terms coming from the contributions of the diagrams (a) and (b) in Fig. 1. We refer its analytical formula to Ref. [4].
Figure 1: The one-loop diagrams for the top quark decay producing a bottom quark and a $W$ boson, where $\tilde{t}$, $\tilde{b}$, $\omega$, and $\chi$ denote respectively the top squark, bottom squark, chargino, and neutralino.

We give numerical results of the absolute value of the $CP$ asymmetry $|A_{CP}|$ in Table 2 for $\theta = 0$, $\alpha = \pi/4$ and several different values of $\tilde{m}_2$, taking three sets of values for $\tilde{M}$ and $c$: (i)$\tilde{M} = 180$ GeV, $c = 0.3$, (ii)$\tilde{M} = 200$ GeV, $c = 0.4$, (iii)$\tilde{M} = 220$ GeV, $c = 0.5$. The other parameters are taken as $\tan \beta = 2$, $|m_H| = 100$ GeV, and $|A|m_{3/2} = \tilde{M}$. In these parameter regions the experimental constraints on the squark and chargino masses are satisfied. The top quark mass is taken to be 180 GeV. Resultant values of the lighter top squark mass are given by (i) 97 GeV, (ii) 92 GeV, (iii) 91 GeV. For larger values of $\tilde{m}_2$ in Table 2 for which no value of $A_{CP}$ is presented, the decay $t\rightarrow \tilde{t}\chi$ is not allowed kinematically.

The numerical analysis shows that, for $\theta \ll 1$, $\alpha \sim 1$, $\tilde{M} \sim 200$ GeV, and $m_H \sim 100$ GeV, the absolute value of the $CP$ asymmetry $A_{CP}$ is of order $10^{-3}$ and maximally $2 \times 10^{-3}$. This amount of asymmetry can be detected if the decays $t \rightarrow bW^+$ and $\bar{t} \rightarrow \bar{b}W^-$ are found by an amount of order $10^6$. At hadron colliders, it is not so easy to find these decays by purely hadronic final states, so that an isolated lepton coming from the $W$ boson decay may be used as a signal of the top quark decay. In the parameter space where the asymmetry $A_{CP}$ has a sizable value, the branching ratio for $t \rightarrow bW^+$ is around 0.8. Since the branching ratio for the leptonic decay of a $W$ boson is around 0.2, the production of $t\bar{t}$ pairs of order $10^7$ will enable the detection of the decay rate asymmetry. At the CERN Large Hadron Collider (LHC), the top quark pairs are expected to be produced at a rate of order $10^7$. The asymmetry may be detectable in near-future experiments at LHC.
Table 2: The decay rate asymmetry $A_{CP}$ is shown for several different values of $\tilde{m}_2$. We take the $CP$-violating phases for $\alpha = \pi/4$ and $\theta = 0$. The other parameters are taken as $\tan \beta = 2$, $|m_H| = 100$ GeV, and $|A|m_{3/2} = \tilde{M}$. We have chosen three sets of values for $\tilde{M}$ and $c$: (i) $\tilde{M} = 180$ GeV, $c = 0.3$, (ii) $\tilde{M} = 200$ GeV, $c = 0.4$, (iii) $\tilde{M} = 220$ GeV, $c = 0.5$.

| $\tilde{m}_2$ (GeV) | $(i)$ | $(ii)$ | $(iii)$ |
|---------------------|-------|--------|--------|
| 300                 | $1.7 \times 10^{-3}$ | $1.9 \times 10^{-3}$ | $1.9 \times 10^{-3}$ |
| 400                 | $1.7 \times 10^{-3}$ | $2.0 \times 10^{-3}$ | $2.1 \times 10^{-3}$ |
| 500                 | $1.4 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $2.0 \times 10^{-3}$ |
| 600                 | $7.7 \times 10^{-4}$ | $1.6 \times 10^{-3}$ | $1.7 \times 10^{-3}$ |
| 700                 | $1.2 \times 10^{-3}$ | $1.4 \times 10^{-3}$ | $1.4 \times 10^{-3}$ |
| 800                 | $8.0 \times 10^{-4}$ | $1.0 \times 10^{-3}$ | $7.0 \times 10^{-4}$ |
| 900                 |       |        |        |

The interactions which induce the rate asymmetry between the standard decays $t \to bW^+$ and $\bar{t} \to \bar{b}W^-$ also yield a rate asymmetry between the non-standard decays $t \to \tilde{t}\chi$ and $\bar{t} \to \tilde{t}^*\chi$. This difference, however, compensates the difference for the standard decays, satisfying the relation

$$\Gamma(t \to bW^+) - \Gamma(\bar{t} \to \bar{b}W^-) = - \left\{ \Gamma(t \to \tilde{t}\chi) - \Gamma(\bar{t} \to \tilde{t}^*\chi) \right\}.$$  

(7)

As a result, the total width of the top quark is the same as that of the anti-top quark, as required by $CPT$ invariance. On the other hand, this relation makes the detection of the decay rate asymmetry more involved. In most of the parameter region where the asymmetry is sizable, the top squark decays through $\tilde{t}_1 \to b\omega^+ \to bW^*\chi$, where $W^*$ denotes the virtual state of the $W$ boson. Therefore, the final states of the non-standard decay and the standard decay have the same particles except invisible neutralinos. In order to measure the asymmetry, therefore, detailed analysis of final states is needed to distinguish between the decays $t \to bW^+$ and $t \to \tilde{t}\chi$. In addition, top squark pairs are directly produced at a rate larger than the top quark pairs. Since the decays $t \to bW^+$ and $\bar{t} \to b\omega^+$ yield the same visible particles, we also need to distinguish between these decays. Such distinctions can be made by examining energy spectra of the particles in the final states.

We have seen a possibility that $CP$ violation is observed at the electroweak energy scale if the decay $t \to \tilde{t}\chi$ is allowed kinematically. Such a non-standard
decay, however, may be detected at Tevatron, if it has a large branching ratio. The branching ratio of $t \to \tilde{t}_1 \chi_1$ is approximately 0.2. Since the final states of the top quark decay and top squark decay have the same visible particles, as mentioned above, energy spectra of these particles have to be examined in detail to find the non-standard decay mode. At present, it is still not ruled out that the non-standard decay may have escaped detection, even if its branching ratio is comparable to that of the standard decay.

Through our discussions, we have assumed the masses of the squarks and the sleptons to be degenerate at the GUT scale. By some unknown reasons, however, it may happen that the squarks of the third generation are of order 100 GeV and not related to the mass of the other squarks and sleptons. In this case, if the squarks and sleptons of the first generation are heavier than 1 TeV, the EDMs of the neutron and the electron do not impose constraints on the $CP$-violating phases. We have also computed the $CP$ asymmetry taking both $\theta$ and $\alpha$ of order unity. The asymmetry is at most approximately $3 \times 10^{-3}$ and not significantly large compared to the asymmetry obtained under the assumption of unification for the squark and slepton masses.

5 Summary

We have discussed a partial decay rate asymmetry between the two decays $t \to bW^+$ and $\bar{t} \to \bar{b}W^-$ within the framework of the SSM. As long as one $CP$-violating phase $\theta$ is sufficiently small, even if another phase $\alpha$ is of order unity, squarks, sleptons, one chargino, and some of neutralinos are allowed to have masses of order 100 GeV without inconsistency with the constraints from the EDMs of the neutron and the electron. In this case there is a parameter space where the non-standard decay $t \to \tilde{t}_1 \chi$ is allowed kinematically. In a wide region of this parameter space, the asymmetry is of order $10^{-3}$ and maximally $2 \times 10^{-3}$. Since $10^7$ pairs of top quarks are expected to be produced at LHC, such an amount of asymmetry may be detectable in the near future.

The lighter top squark may be heavier than the top quark. In this case, the decay rate asymmetry $A_{CP}$ in Eq. (1) cannot be used to measure $CP$ violation. However, if the top squark is sufficiently heavy, another rate asymmetry between the decays $\tilde{t} \to t \chi$ and $\tilde{t}^* \to \bar{t} \chi$ is induced by the same sources of $CP$ violation in the SSM \[5\]. The parameter space where this asymmetry has a non-vanishing value is wide, compared to that for $A_{CP}$ which is restricted because of the kinematical
constraint \( m_t > \tilde{M}_1 + \tilde{m}_{\chi_1} \). The investigation of the decay rate asymmetry for the top squark and anti-top squark is another window for \( CP \) violation in the SSM.

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