Some Aspects of the de Sitter/CFT Correspondence

Dietmar Klemm

Dipartimento di Fisica dell’Università di Milano and INFN, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy.

Abstract

We discuss several aspects of the proposed correspondence between quantum gravity on de Sitter spaces and Euclidean conformal field theories. The central charge appearing in the asymptotic symmetry algebra of three-dimensional de Sitter space is derived both from the conformal anomaly and the transformation law of the CFT stress tensor when going from $dS_3$ in planar coordinates to $dS_3$ with cosmological horizon. The two-point correlator for CFT operators coupling to bulk scalars is obtained in static coordinates, corresponding to a CFT on a cylinder. Correlation functions are also computed for CFTs on two-dimensional hyperbolic space. We furthermore determine the energy momentum tensor and the Casimir energy of the conformal field theory dual to the Schwarzschild-de Sitter solution in five dimensions. Requiring the pressure to be positive yields an upper bound for the black hole mass, given by the mass of the Nariai solution. Beyond that bound, which is similar to the one found by Strominger requiring the conformal weights of CFT operators to be real, one encounters naked singularities.

*dietmar.klemm@mi.infn.it
1 Introduction

In the last months there has been an increasing interest in gravity on de Sitter (dS) spacetimes \[1, 2, 3, 4, 5, 6, 7, 8, 9\]. This is partially motivated by recent astrophysical data indicating a positive cosmological constant \[10\]. Apart from phenomenological aspects, it would also be desirable to understand the role of de Sitter spaces in string theory, and to clarify the microscopic origin of the entropy of dS space\[1\]. Whereas string theory on anti-de Sitter spaces is known to have a dual description in terms of certain superconformal field theories \[16\], no such explicit duality was known up to now for dS spacetimes. The first evidence for a dS/CFT correspondence was given by Hull \[17\], who considered so-called IIA* and IIB* string theories, which are obtained by T-duality on a timelike circle from the IIB and IIA theories respectively. The type IIB* theory admits E4-branes, which are the images of D4-branes under T-duality along the time coordinate of the brane. The E4-branes interpolate between Minkowski space at infinity and dS\[5\] \(\times H^5\) near the horizon, where \(H^5\) denotes hyperbolic space. The effective action describing E4-brane excitations is a Euclidean \(D = 4, \mathcal{N} = 4\) U(N) super Yang-Mills theory, which is obtained from SYM in ten dimensions by reduction on a six-torus with one timelike circle. This leads to a duality between type IIB* string theory on dS\[5\] \(\times H^5\) and the mentioned euclidean SYM theory \[17\]. Unfortunately this example is pathological, because both theories have ghosts\[2\].

Based on \[17\] and related ideas that appeared in \[21, 22, 2\], Strominger proposed recently a more general holographic duality relating quantum gravity on dS\[D\] to a conformal field theory residing on the past boundary \(\mathcal{I}^-\) of dS\[D\] \[6\]. He argued that in general this CFT may be non-unitary, with operators having complex conformal weights, if the dual bulk fields are sufficiently massive. The asymptotic symmetry algebra of three-dimensional de Sitter space was found to consist of two copies of Virasoro algebras with central charges \(c = \tilde{c} = 3l/2G\) \[4\], where \(l\) is the dS\[3\] curvature radius, and \(G\) denotes Newton’s constant. This generalizes the result of Brown and Henneaux \[23\] to the case of positive cosmological constant. In this paper, we derive this central charge by two independent alternate methods. The first uses the relation to the trace anomaly, whereas the second is based on the transformation law of the CFT stress tensor when going from the plane, corresponding to dS\[3\] in planar coordinates, to the cylinder, corresponding to dS\[3\] with a cosmological horizon. The shift of \(-c/24\) in the Virasoro generators is thereby identified with the negative mass of dS\[3\] in presence of a cosmological horizon.

The two-point correlator for CFT operators coupling to bulk scalars is then obtained in static coordinates. This correlation function agrees with the result that one would get by starting with the two-point function on the plane, and then using the scaling relations

\[1\] For microscopic derivations of dS entropy based on the Chern-Simons formulation of 2+1 dimensional dS gravity, or on other approaches that are not directly related to string theory, cf. \[11-15\].

\[2\] Note however that Euclidean super Yang-Mills theory can be twisted to obtain a well-defined topological field theory in which the physical states are the BRST cohomology classes \[18, 19, 20\]. According to \[17\], this should correspond to a twisting of the type IIB* string theory, with a topological gravity limit.
for CFT operators under the coordinate transformation from the plane to the cylinder. Thereby, the conformal weights of the operators are given in terms of the masses of the bulk fields. Several properties of the correlator in static coordinates are discussed. We also compute correlation functions for CFTs on two-dimensional hyperbolic space, dual to dS$_3$ in hyperbolic slicing. Finally we consider the Schwarzschild-dS black hole in arbitrary dimension, and derive a Smarr-like formula. For the five-dimensional case, we determine then the stress tensor and the Casimir energy of the dual CFT. Requiring the pressure to be positive yields an upper bound on the black hole mass, much like the bound obtained in [6] for bulk scalars, following from reality of the CFT conformal weights. Our bound is exactly the mass of the Nariai solution [24], for which the event horizon and the cosmological horizon coalesce.

2 The Stress Tensor for de Sitter Space

The gravitational action of (n + 1)-dimensional de Sitter gravity has the form

$$I_{\text{bulk}} + I_{\text{surf}} = \frac{1}{16\pi G} \int_{M} d^{n+1}x \sqrt{-g} \left( R - \frac{n(n-1)}{l^2} \right) + \frac{1}{8\pi G} \int_{\partial M} d^{n}x \sqrt{\gamma} K. \tag{2.1}$$

The first term is the Einstein-Hilbert action with positive cosmological constant $\Lambda = n(n-1)/2l^2$, whereas the second is the Gibbons-Hawking boundary term necessary to have a well-defined variational principle. $K$ is the trace of the extrinsic curvature $K_{\mu\nu} = -\nabla_{(\mu} n_{\nu)}$ of the spacetime boundary $\partial M$, with $n^\mu$ denoting the outward pointing unit normal. $\gamma$ is the induced metric on the boundary. In evaluating expressions like (2.1) one usually encounters divergences coming from integration over the infinite volume of spacetime. In the case of adS gravity, a regularization procedure was proposed in [25, 26, 27], that consists of adding counterterms constructed from local curvature invariants of the boundary. These counterterms, which are essentially unique, can be easily generalized to the case of positive cosmological constant, yielding

$$I_{\text{ct}} = \frac{1}{8\pi G} \int_{\partial M} d^{n}x \sqrt{\gamma} \left[ \frac{n-1}{l} - \frac{lR}{2(n-2)} \right], \tag{2.2}$$

where $R$ is the Ricci scalar of the boundary metric $\gamma$. Using $I = I_{\text{bulk}} + I_{\text{surf}} + I_{\text{ct}}$, one can then construct a conserved stress tensor [28] associated to the boundary $\partial M$,

$$T^{\mu\nu} = -\frac{2}{\sqrt{\gamma}} \frac{\delta I}{\delta \gamma^{\mu\nu}} = \frac{1}{8\pi G} \left[ K^{\mu\nu} - K \gamma^{\mu\nu} - \frac{(n-1)}{l} \gamma^{\mu\nu} - \frac{lG^{\mu\nu}}{(n-2)} \right], \tag{2.3}$$

where $G^{\mu\nu}$ is the Einstein tensor of $\gamma$. This generalizes the result of [1] for three-dimensional de Sitter gravity, where the last term in (2.2) and (2.3) has to be omitted.
3 dS$_3$ Central Charge from Conformal Anomaly

We start from dS$_3$ in spherical slicing,

$$ds^2 = -l^2 d\tau^2 + l^2 \cosh^2 \tau (d\theta^2 + \sin^2 \theta d\phi^2).$$

(3.1)

These coordinates cover the entire spacetime. The Carter-Penrose diagram of (3.1) can be found in [6]. Using (2.3), we can now compute the stress tensor of the dual Euclidean CFT, residing on the past boundary $I^-(\tau \to -\infty)$, i.e. on a two-sphere $S^2$. This yields

$$T_{\theta\theta} = -\frac{l}{16\pi G}, \quad T_{\phi\phi} = -\frac{l}{16\pi G} \sin^2 \theta.$$  

(3.2)

The CFT metric $h_{\mu\nu}$ can be obtained by setting $d\tau = 0$ in (3.1) and dropping the diverging conformal factor $\cosh^2 \tau$. This gives the trace anomaly

$$T = h_{\mu\nu} T_{\mu\nu} = -\frac{1}{8\pi G l}.$$  

(3.3)

Comparing this with $T = -cR/24\pi$, where $R = 2/l^2$, we learn that

$$c = \frac{3l}{2G},$$

(3.4)

confirming the result of [6].

Alternatively, one could consider dS$_3$ in the coordinates (cf. appendix)

$$ds^2 = -l^2 d\tau^2 + l^2 \sinh^2 \tau (d\theta^2 + \sinh^2 \theta d\phi^2),$$

(3.5)

corresponding to a conformal field theory on the hyperbolic space $H^2$. Then one gets

$$T_{\theta\theta} = \frac{l}{16\pi G}, \quad T_{\phi\phi} = \frac{l}{16\pi G} \sinh^2 \theta,$$

(3.6)

so that $T = 1/8\pi G l$. As the scalar curvature is now $R = -2/l^2$, this yields again (3.4).

We finally note that the slicing (3.3) allows furthermore to consider CFTs on compact Riemann surfaces of genus $g > 1$, by quotienting dS$_3$ by a suitable discrete subgroup $\Gamma$ of the isometry group $SO(2, 1)$ of $H^2$. 


There is still another way to obtain the central charge (3.4). Consider $dS^3$ in planar coordinates, with metric
\[
 ds^2 = -d\tau^2 + e^{-2\tau/l}dzd\bar{z}, \tag{4.1}
\]
where $-\infty < \tau < \infty$, and $z, \bar{z}$ range over an infinite plane. Using $n = -\partial_\tau$, it can be easily checked that the energy momentum tensor (2.3) vanishes for (4.1).

By means of the transformation
\[
 \begin{align*}
 \tau &= t - \frac{l}{2} \ln |V(r)|, \quad V(r) = 1 - \frac{r^2}{l^2}, \\
 z &= r|V(r)|^{-\frac{1}{2}}e^{-i\phi+tl}, \\
 \bar{z} &= r|V(r)|^{-\frac{1}{2}}e^{i\phi+tl}, \tag{4.2}
\end{align*}
\]
(4.1) can be cast into the form
\[
 ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2d\phi^2, \tag{4.3}
\]
where $\phi$ is identified modulo $2\pi$. The Carter-Penrose diagram of (4.3) is shown in figure (1). The past and future boundaries $I^\pm$ correspond to $r = \infty$.

The characteristic feature is the appearance of a cosmological event horizon [29] at $r = l$, with temperature $T = 1/2\pi l$ and Bekenstein-Hawking entropy $S = \pi l/2G$. For $r > l$, the Killing vector $\partial_t = l^{-1}(z\partial_z + \bar{z}\partial_{\bar{z}}) + \partial_\tau$ becomes spacelike, whereas $\partial_r$ becomes timelike. On the past boundary $I^-$, we have $r \to \infty$, and the transformation (4.2) becomes
\[
 \begin{align*}
 z &= le^{-i\phi+tl}, \\
 \bar{z} &= le^{i\phi+tl}. \tag{4.4}
\end{align*}
\]
or equivalently $z = l \exp(-iw/l)$, where we defined $w = l \phi + it$. This is precisely the transformation from the plane to the cylinder, and it is well-known that this induces a shift in the Virasoro generators of a two-dimensional conformal field theory. The Hamiltonian of time translation in the $w$ frame is

$$lH = L_0 + \tilde{L}_0 - \frac{c + \tilde{c}}{24}. \quad (4.5)$$

As the stress tensor (2.3) vanishes for the metric (4.1), corresponding to $z, \bar{z}$, we know that the conformal weights $L_0$ and $\tilde{L}_0$ are zero. On the other hand, we can compute the stress tensor for dS$_3$ in static coordinates (4.3) (corresponding to $w, \bar{w}$), and from this the Hamiltonian $H$. A straightforward calculation yields for $r \to \infty$

$$T_{tt} = -\frac{1}{16\pi G l}, \quad T_{\phi\phi} = \frac{l}{16\pi G}. \quad (4.6)$$

The conserved charge $M$ associated with the Killing field $k = \partial_t$ is given by [28, 25]

$$M = \int_0^{2\pi} T_{\mu\nu} u^\mu k^\nu \sqrt{\sigma} d\phi, \quad (4.7)$$

where $u = (r^2/l^2 - 1)^{-1/2} \partial_t$ is the unit normal to the surface $\Sigma_t$ of constant $t$ in $\partial M$, and $\sigma$ denotes the induced metric on $\Sigma_t$. One then gets

$$M = -\frac{1}{8G}. \quad (4.8)$$

Equating this with the Hamiltonian $H$, and using $c = \tilde{c}$, we finally obtain

$$c = \frac{3l}{2G}. \quad (4.9)$$

5 Two-point Correlators in Static Coordinates

Like in [3], we can now compute correlation functions of CFT operators that couple to bulk fields, in the spirit of the AdS/CFT correspondence. For simplicity, we will consider only massive scalar fields. In the static coordinates (4.3), the Klein-Gordon equation reads

$$m^2 \Phi = \nabla^2 \Phi = \frac{1}{r} \partial_r (r V(r) \partial_r \Phi) - \frac{1}{V} \partial_t^2 \Phi + \frac{1}{r^2} \partial_\phi^2 \Phi. \quad (5.1)$$
Near $\mathcal{I}^-$, the last two terms in (5.1) can be neglected, leading to the asymptotic behaviour

$$\Phi \sim r^{-h_\pm}$$

for $r \to \infty$, with $h_\pm$ given by

$$h_\pm = 1 \pm \sqrt{1 - m^2 l^2}.$$  

(5.3)

We will consider only the case $0 < m^2 l^2 < 1$, which implies real $h_\pm$. Similar to [6], we impose the boundary condition

$$\lim_{r \to \infty} \Phi(r, t, \phi) = r^{-h_-} \Phi_-(t, \phi).$$

(5.4)

The two-point function of the operator $O$ coupling to $\Phi$ can be obtained [6] from

$$\lim_{r \to \infty} \int_{\mathcal{I}^-} dt' d\phi' dt d\phi \left[ \frac{(rr')^2}{l^2} \left( \Phi(r, t, \phi) \Phi(r', t', \phi') \right) \right]_{r=r'},$$

(5.5)

where $G$ denotes the Hadamard two-point function given in (B.5), and $dr_* = (-V(r))^{-1/2} dr$. For $r \to \infty$, $G$ behaves like (cf. (B.9))

$$\lim_{r, r' \to \infty} G(r, t, \phi; r', t', \phi') = \gamma_+ (rr')^{-h_+} \left[ \cosh \frac{\Delta t}{l} - \cos \Delta \phi \right]^{-h_+} + \gamma_- (rr')^{-h_-} \left[ \cosh \frac{\Delta t}{l} - \cos \Delta \phi \right]^{-h_-},$$

(5.6)

where $\gamma_\pm$ are constants, and $\Delta t = t - t'$, $\Delta \phi = \phi - \phi'$. Using (5.4) and (5.6), the expression (5.5) reduces modulo a constant prefactor to

$$\int_{\mathcal{I}^-} dt d\phi dt' d\phi' \Phi_-(t, \phi) \left[ \frac{\text{const.}}{\cosh \frac{\Delta t}{l} - \cos \Delta \phi} \right]^{h_+} \Phi_-(t', \phi').$$

(5.7)

This yields the two-point correlator

$$\langle O(t, \phi) O(t', \phi') \rangle = \frac{\text{const.}}{\left[ \cosh \frac{\Delta t}{l} - \cos \Delta \phi \right]^{h_+}},$$

(5.8)

or in the coordinates $w, \bar{w}$. 

6
\[ \langle O(w, \bar{w})O(w', \bar{w}') \rangle = \text{const.} \frac{e^{-\pi T h_+ (\Delta w + \Delta \bar{w})}}{(1 - e^{-2\pi T \Delta w})^{h_+} (1 - e^{-2\pi T \Delta \bar{w}})^{h_+}}, \]  

(5.9)

where \( T = 1/2\pi l \) denotes the Hawking temperature of the cosmological horizon. The AdS analogue of (5.9) has been obtained in [30]. In that case, it corresponds to a BTZ black hole in the bulk. (5.9) agrees with the result that one would get by starting with the two-point correlator

\[ \langle O(z, \bar{z})O(z', \bar{z}') \rangle \sim \frac{1}{(\Delta z \Delta \bar{z})^{h_+}}, \]  

(5.10)

obtained in [4], and using the scaling relations for dimension \((h_+, h_+)\) operators under the coordinate transformation from the plane to the cylinder [31].

Note that the finite extent of the system is not in \( t \) direction, but along the coordinate \( \phi \). The infinite cylinder geometry that we have should thus correspond rather to a quantum chain at zero temperature, but with periodic boundary conditions, and not to an infinite chain at finite temperature. The system would thus have a correlation length \( \xi = l/h_+ \), and a mass gap \( \delta E = h_+/l \) between the ground state and the first excited state [32]. Alternatively, one could interpret (5.9) as a thermal correlator at imaginary temperature \( iT = i/2\pi l \),

\[ \langle O(w, \bar{w})O(w', \bar{w}') \rangle \sim [\sinh(i\pi T \Delta w) \sinh(i\pi T \Delta \bar{w})]^{-h_+}. \]  

(5.11)

This would mean that the coordinate transformation \( z = l \exp(-iw/l) \) induces a Bogoljubov transformation of the operators \( O(z, \bar{z}) \) to new operators \( O(w, \bar{w}) \) that see the Poincaré vacuum as a thermal bath of excitations (at imaginary temperature). Note also that (5.9) is invariant under the shift

\[ \Delta t \mapsto \Delta t + i\beta, \]  

(5.12)

where \( \beta = 1/T \). This means that the temperature is imaginary, because \( t \) is already a Euclidean time. The significance of an imaginary temperature in this context remains rather obscure, but it may be related to the fact that \( I^- \) lies actually behind the horizon, whereas the concept of a (real) Hawking temperature is well-defined only outside the horizon.

It is possible to rederive the central charge (3.4) by expanding (5.8) in powers of \( \Delta t \) and \( \Delta \phi \) which yields the leading term (5.10) plus finite size corrections. Inserting the correct prefactor for the two-point function of dimension \((h_+, h_+)\) operators on the cylinder [33], we have

\[ ^3 \text{I would like to thank Tassos Petkou for pointing out this to me.} \]
\[ \langle \mathcal{O}(t, \phi) \mathcal{O}(0,0) \rangle = l^{-2h_+} \left[ 2 \cosh \frac{t}{l} - 2 \cos \phi \right]^{-h_+}, \quad (5.13) \]

which yields for small \( t, \phi \)
\[ \langle \mathcal{O}(t, \phi) \mathcal{O}(0,0) \rangle = (t^2 + l^2 \phi^2)^{-h_+} \left[ 1 - \frac{h_+}{12l^2}(t^2 - l^2 \phi^2) + \ldots \right]. \quad (5.14) \]

One can now compare this with the operator product expansion
\[ \mathcal{O}(r) \mathcal{O}(0,0) \sim r^{-2h_+} + C_{\mu\nu}(r) T_{\mu\nu}(0) + \ldots, \quad (5.15) \]
where in our conventions
\[ C_{\mu\nu}(r) = \frac{2\pi h_+}{c} \left( r_\mu r_\nu - \frac{1}{2} r^2 h_{\mu\nu} \right) r^{-2h_+}. \quad (5.16) \]

Using the expectation value (4.6) of the energy momentum tensor, one obtains again \( c = 3l/2G \).

### 6 Two-point Correlators in Hyperbolic Coordinates

In [6], the two-point function of CFT operators that couple to massive Klein-Gordon bulk fields was computed for the case where the CFT lives on a sphere \( S^2 \). Starting from \( dS_3 \) in hyperbolic coordinates (3.5), we can do similar calculations for CFTs on \( H^2 \). The Klein-Gordon equation reads
\[ m^2 l^2 \Phi = l^2 \nabla^2 \Phi = -\partial_\tau^2 \Phi - 2 \coth \tau \partial_\tau \Phi + \frac{1}{\sinh^2 \tau \sinh \theta} \partial_\theta (\sinh \theta \partial_\theta \Phi) + \frac{1}{\sinh^2 \tau \sinh^2 \theta} \partial_\phi^2 \Phi. \quad (6.1) \]

For \( \tau \to -\infty \), the last two terms in (6.1) can be neglected, and thus
\[ \lim_{\tau \to -\infty} \Phi(\tau, \theta, \phi) = \Phi_+(\theta, \phi) e^{h_+ \tau} + \Phi_-(\theta, \phi) e^{h_- \tau}. \quad (6.2) \]

The asymptotic behaviour of the propagator is (cf. (B.12))
\[ \lim_{\tau, \tau' \to \infty} G(\tau, w; \bar{w}; \tau', v; \bar{v}) = \gamma_+ e^{h_+(\tau + \tau')} \frac{(1 - w\overline{w})^{h_+}(1 - v\overline{v})^{h_+}}{|w - v|^{2h_+}} + \gamma_- e^{h_-(\tau - \tau')} \frac{(1 - w\overline{w})^{h_+}(1 - v\overline{v})^{h_-}}{|w - v|^{2h_-}}, \quad (6.3) \]
where \( w = \tanh \frac{\theta}{2} e^{i \phi} \) is a complex coordinate on \( H^2 \). The hyperbolic analogue of (5.3) reads

\[
\lim_{\tau \to -\infty} \int_{I_-} d^2 w d^2 v \sqrt{h(w)h(v)} \left[ e^{-2(\tau+\tau')} \left( \Phi(\tau, w, \bar{w}) \partial_{\tau'} G(\tau, w, \bar{w}; \tau', v, \bar{v}) \partial_{\tau'} \Phi(\tau', v, \bar{v}) \right) \right]_{\tau = \tau'},
\]

(6.4)

\[
\sqrt{h(w)} = 2(1 - w \bar{w})^{-2}
\]

denoting the measure on \( H^2 \). Inserting (6.2) and (6.3) into (6.4), one obtains, up to a normalization constant,

\[
\int_{I_-} d^2 w d^2 v \sqrt{h(w)h(v)} (\gamma_+ \Phi_-(w, \bar{w}) \Delta_{h_+} \Phi_-(v, \bar{v}) + \gamma_- \Phi_+(w, \bar{w}) \Delta_{h_-} \Phi_+(v, \bar{v})),
\]

(6.5)

where \( \Delta_{h_\pm} \) is the two-point correlator for a conformal field of dimension \( h_\pm \) on the hyperbolic space \( H^2 \),

\[
\Delta_{h_\pm} = \left[ \frac{(1 - w \bar{w})(1 - v \bar{v})}{|w - v|^2} \right]^{h_\pm}.
\]

(6.6)

### 7 Schwarzschild-de Sitter Black Holes in Arbitrary Dimension

The metric for the Schwarzschild-de Sitter black hole in \( D \) dimensions reads\(^4\)

\[
ds^2 = -V(r) dt^2 + V(r)^{-1} dr^2 + r^2 d\Omega_{D-2}^2,
\]

(7.1)

where

\[
V(r) = 1 - \frac{\mu}{r^{D-3}} - \frac{r^2}{l^2},
\]

(7.2)

and \( d\Omega_{D-2}^2 \) denotes the standard metric on the unit \( S^{D-2} \). For \( D = 5 \), the product of (7.1) with hyperbolic space \( H^5 \) is a solution of IIB* supergravity \[^{17}\], whereas for \( D = 4 \), the de Sitter black hole (7.1) times adS7 solves the equations of motion \[^{21}\] of the low energy effective action of M* theory \[^{38}\], which is the strong coupling limit of the IIA* theory, and has signature \( 9 + 2 \).

For mass parameters \( \mu \) with \( 0 < \mu < \mu_N \), where

\[^{4}\text{For generalizations cf. }^{34} -^{37}.\]
\[ \mu_N = \frac{2l^{D-3}}{D-1} \left( \frac{D-3}{D-1} \right)^{\frac{D-3}{2}}, \]  

(7.3)

one has a black hole in de Sitter space with event horizon at \( r = r_H > 0 \) and cosmological horizon at \( r = r_C > r_H \), with \( V(r_H) = V(r_C) = 0 \). For \( \mu = \mu_N \), the event horizon and the cosmological horizon coalesce, and one gets the Nariai solution [24]. For \( \mu > \mu_N \), (7.1) describes a naked singularity in de Sitter space. We thus notice that the absence of naked singularities yields an upper bound for the black hole mass. Below we will see (for the case \( D = 5 \)) that the same bound results from the requirement that the pressure of the dual CFT must be positive. It is interesting that analogous bounds arise for bulk fields coupling to CFT operators, if one wants to have real conformal weights [6].

We start by deriving a Smarr-like formula for (7.1), following the lines of [29]. Consider the Killing identity

\[ \nabla_\mu \nabla_\nu k^\mu = R_{\nu\rho k}^\rho = \frac{D-1}{l^2} k_\nu, \]  

(7.4)

where \( k^\mu \) is a Killing vector, \( \nabla (\nu k_\mu) = 0 \), and we used the Einstein equations in the last step. Now integrate (7.4) on a spacelike hypersurface \( \Sigma_t \) from the black hole horizon \( r_H \) to the cosmological horizon \( r_C \). On using Gauss’ law, this gives

\[ \frac{1}{2} \int_{\partial\Sigma_t} \nabla_\mu k_\nu d\Sigma^{\mu\nu} = \frac{D-1}{l^2} \int_{\Sigma_t} k_\nu d\Sigma^{\nu}, \]  

(7.5)

where the boundary \( \partial\Sigma_t \) consists of the intersection of \( \Sigma_t \) with the black hole- and the cosmological horizon,

\[ \partial\Sigma_t = S^{D-2}(r_H) \cup S^{D-2}(r_C). \]  

(7.6)

Applying (7.3) to the Killing vector \( k = \partial_t \) yields

\[
\frac{D-2}{16\pi G(D-3)} \int_{S^{D-2}(r_H)} \nabla_\mu k_\nu d\Sigma^{\mu\nu} + \frac{D-2}{16\pi G(D-3)} \int_{S^{D-2}(r_C)} \nabla_\mu k_\nu d\Sigma^{\mu\nu} = \frac{(D-1)(D-2)}{8\pi G l^2(D-3)} \int_{\Sigma_t} k_\nu d\Sigma^{\nu},
\]  

(7.7)

where \( G = \frac{l^{D-2}}{l^2} \) denotes Newton’s constant. One can regard the right-hand side of Eq. (7.4) as representing the contribution of the cosmological constant to the mass within the cosmological horizon. As in [29], we therefore identify the second term on the left-hand side as the total mass \( M_C \) within the cosmological horizon, and the first term as the
negative of the black hole mass $M_{BH}$. The latter can be rewritten by using the definition of the surface gravity $\kappa_H$, which, by the zeroth law, is constant on the horizon. In this way, one obtains

$$M_{BH} = \frac{D - 2}{8\pi G(D - 3)} \kappa_H A_H ,$$

(7.8)

where $A_H$ denotes the area of the event horizon. One therefore gets the Smarr-type formula

$$M_C = \frac{D - 2}{8\pi G(D - 3)} \kappa_H A_H + \frac{(D - 1)(D - 2)}{8\pi G l^2 (D - 3)} \int_{\Sigma_t} k_\nu d\Sigma^\nu ,$$

(7.9)

from which a first law of black hole mechanics can be derived. Evaluating (7.9) yields

$$M_C = \frac{(D - 2)V_{D-2}}{16\pi G} \left[ \mu - \frac{2}{D - 3} \frac{r_{C}^{D-1}}{l^2} \right],$$

(7.10)

where $V_{D-2}$ is the volume of the unit $S^{D-2}$.

Alternatively, we can use the stress tensor (2.3) in order to associate a mass to the spacetime (7.1). We will consider only the case $D = 5$, and write the metric on the unit $S^3$ as

$$d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2.$$

(7.11)

Then the stress tensor (2.3) of the CFT dual to the Schwarzschild-de Sitter black hole in five dimensions reads

$$8\pi G T_{tt} = \frac{12\mu - 3l^2}{8r^2} + \mathcal{O}(r^{-4}) ,$$

$$8\pi G T_{\theta\theta} = \frac{-4\mu l + l^3}{8r^2} + \mathcal{O}(r^{-4}) ,$$

$$8\pi G T_{\phi\phi} = \frac{-4\mu l + l^3}{8r^2} \sin^2 \theta + \mathcal{O}(r^{-4}) ,$$

$$8\pi G T_{\psi\psi} = \frac{-4\mu l + l^3}{8r^2} \cos^2 \theta + \mathcal{O}(r^{-4}).$$

(7.12)
where $\gamma_{\mu\nu}$ denotes the induced metric on the boundary $\partial M$. The field theory’s stress tensor $\hat{T}^{\mu\nu}$ is related to the one in (2.3) by the rescaling

$$\sqrt{h} h_{\mu\rho} \hat{T}^{\rho\nu} = \lim_{r \to \infty} \sqrt{\gamma} \gamma_{\mu\rho} T^{\rho\nu},$$

which amounts to multiplying all expressions for $T_{\mu\nu}$ in (7.12) by $r^2/l^2$ before taking the limit $r \to \infty$. Defining the “pressure”

$$p = \frac{-4 \mu + l^2}{64 \pi G l^3},$$

and $v = (1, 0, 0, 0)$, we can write the energy-momentum tensor as

$$\hat{T}_{\mu\nu} = p(-4v_{\mu}v_{\nu} + h_{\mu\nu}),$$

which is conserved and traceless. Note that the pressure is positive precisely if $\mu < \mu_N$, i.e. if naked singularities are absent.

The conserved charge associated with the Killing vector $k = \partial_t$ is given by

$$M = \int T_{\mu\nu} u^\mu k^\nu \sqrt{\sigma} d\theta d\phi d\psi,$$

where, as in (4.7), $u = (-V(r))^{-1/2} \partial_t$ is the unit normal to the surface $\Sigma_t$ of constant $t$ in $\partial M$, and $\sigma$ denotes the induced metric on $\Sigma_t$. One then gets

$$M = \frac{3\pi}{8G} \left( \mu - \frac{l^2}{4} \right).$$

This is to be identified with the energy of the dual CFT. Note that $M$ is negative for $\mu < \mu_N$. The Casimir energy $E_C$ of the conformal field theory, which lives on $\mathbb{R} \times S^3$, is obtained by setting $\mu = 0$, i.e., for pure de Sitter space without a black hole. We have thus

$$E_C = \frac{-3\pi l^2}{32G}.$$
Note added

Some time after this paper was posted on the web, a related computation of the Schwarzschild-de Sitter mass (7.19) appeared in [10]. There the same result (7.19) was obtained, but with opposite overall sign. This sign difference comes from the definition

\[ T^{\mu\nu} = \frac{2}{\sqrt{\gamma}} \frac{\delta I}{\delta \gamma_{\mu\nu}} \] (7.21)

for the stress tensor of the dual CFT used in [10], whereas in (2.3) we defined \( T^{\mu\nu} = -\frac{2}{\sqrt{\gamma}} \frac{\delta I}{\delta \gamma_{\mu\nu}} \). The motivation for choosing the opposite sign in [40] was to get an energy that increases with the entropy. Indeed the entropy of the cosmological horizon,

\[ S_C = \frac{r_C^3 \pi^2}{2G}, \] (7.22)

increases with decreasing \( \mu \) (for \( \mu = 0 \), it reaches its maximum value \( S_C = l^3 \pi^2 / 2G \)). At the same time, the mass

\[ M = \frac{3\pi}{8G} \left( \frac{l^2}{4} - \mu \right), \] (7.23)

obtained in [10] rises if we lower \( \mu \). The temperature \( T_C \) of the cosmological horizon, derived from the thermodynamic fundamental relation as \( T_C = \partial M / \partial S \), is then positive. Things change however if we consider the entropy \( S_H \) of the black hole event horizon instead of the entropy of the cosmological horizon. Inserting \( \mu = r_H^2 - r_{H/C}^4 / l^2 \) and \( S_H/C = r_H^3 / 2G \) into (7.19), we obtain, up to the Casimir term (7.20), the fundamental relation

\[ M(S) = \frac{3\pi}{8l_p} \left[ \left( \frac{2S}{\pi^2} \right)^{2/3} - \frac{l_p^2}{l^2} \left( \frac{2S}{\pi^2} \right)^{4/3} \right], \] (7.24)

where \( S \) can be either \( S_H \) or \( S_C \). The temperature is then given by

\[ T_{H/C} = \frac{\partial M}{\partial S_{H/C}} = \frac{1}{2\pi} \left( \frac{1}{r_{H/C}} - \frac{2r_{H/C}}{l^2} \right). \] (7.25)

As we have \( r_H^2 \leq l^2 / 2 \) and \( r_C^2 \geq l^2 / 2 \), the black hole temperature is positive with our definition, whereas the temperature associated to the cosmological horizon is negative. If we use instead the mass (7.23) of [10], \( T_C \) is positive, whereas the Hawking temperature \( T_H \) of the black hole is negative, or, in other words, the mass decreases with increasing black hole entropy.

\[ ^5 \text{The definition (2.3) was also used in [1].} \]
Acknowledgements

This work was partially supported by INFN, MURST and by the European Commission RTN program HPRN-CT-2000-00131, in which D. K. is associated to the University of Torino. The author would like to thank S. Cacciatori and A. C. Petkou for useful discussions, and A. Strominger for clarifying correspondence.

A Coordinate Systems in de Sitter Space

A.1 Static Coordinates

Consider $\mathbb{R}^{D+1}_1$ with coordinates $X^A$, $A = 0, 1, \ldots, D$, and metric $\eta_{AB} = \text{diag}(-1, 1, \ldots, 1)$. $D$-dimensional de Sitter space $dS_D$ can then be defined as the hypersurface

$$\eta_{AB} X^A X^B = l^2. \quad (A.1)$$

Fix now

$$(X^0)^2 - (X^D)^2 = -l^2 \left(1 - \frac{r^2}{l^2}\right) = -l^2 V(r). \quad (A.2)$$

One has then

$$(X^1)^2 + \ldots + (X^{D-1})^2 = r^2, \quad (A.3)$$

so the coordinates $X^1, \ldots, X^{D-1}$ range over a $(D-2)$-sphere $S^{D-2}$ with radius $r$. If we parametrize the hyperbola $[A.2]$ of fixed $r$ by

$$X^0 = \sqrt{r^2 - l^2} \cosh \frac{t}{l}, \quad X^D = \sqrt{r^2 - l^2} \sinh \frac{t}{l}, \quad (A.4)$$

then the induced metric on the hypersurface $[A.1]$ is given by

$$ds^2 = -V(r) dt^2 + V(r)^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \quad (A.5)$$

where $d\Omega_{D-2}^2$ denotes the standard metric on the unit $S^{D-2}$. $[A.3]$ describes de Sitter space in static coordinates, with horizon at $r = l$.

Below, we shall compute the de Sitter invariant Hadamard two-point function. To this aim, we need the geodesic distance $d(X, X')$ between two points $X$ and $X'$ on $[A.1]$, which reads $[6]$. 

14
\[ d = l \arccos P, \quad (A.6) \]

where

\[ l^2 P(X, X') = X^A \eta_{AB} X'^B. \quad (A.7) \]

In the coordinates (A.5), one gets

\[ l^2 P(X, X') = -\sqrt{r^2 - l^2 \cosh \theta} + r \cos \theta, \quad (A.8) \]

where \( \Theta = \Theta(\Omega, \Omega') \) denotes the geodesic distance of two points with angular variables \( \Omega \) and \( \Omega' \) on the unit \( S^{D-2} \), e. g. \( \Theta = \phi - \phi' \) for \( D = 3 \).

Alternatively, we can fix

\[ (X^{D-1})^2 + (X^D)^2 = \tau^2 + l^2. \quad (A.9) \]

This yields

\[ -(X^0)^2 + (X^1)^2 + \ldots + (X^{D-2})^2 = -\tau^2, \quad (A.10) \]

so the coordinates \( X^0, X^1, \ldots, X^{D-2} \) range over a hyperbolic space \( H^{D-2} \) with curvature radius \( \tau \). Parametrize the circle (A.4) of fixed \( \tau \) by

\[ X^{D-1} = \sqrt{\tau^2 + l^2} \cos \frac{v}{l}, \quad X^D = \sqrt{\tau^2 + l^2} \sin \frac{v}{l}. \quad (A.11) \]

This leads to the induced metric

\[ ds^2 = -\left(1 + \frac{\tau^2}{l^2}\right)^{-1} d\tau^2 + \left(1 + \frac{\tau^2}{l^2}\right) dv^2 + \tau^2 d\Sigma^2_{D-2} \quad (A.12) \]

on the hypersurface (A.1). \( d\Sigma^2_{D-2} \) is now the standard line element on the unit hyperbolic space \( H^{D-2} \).
A.2 Hyperbolic Coordinates

If we set instead

\[ X^D = l \cosh \tau , \]  

(A.13)

then the coordinates \( X^0, X^1, \ldots, X^{D-1} \) range over \( H^{D-1} \) with curvature radius \( l \sinh \tau \).

The induced metric on (A.1) takes the form

\[ ds^2 = -l^2 d\tau^2 + l^2 \sinh^2 \tau d\Sigma_{D-1}^2 . \]  

(A.14)

Let us write the hyperbolic metric \( d\Sigma_{D-1}^2 \) as

\[ d\Sigma_{D-1}^2 = d\theta^2 + \sinh^2 \theta d\Omega_{D-2}^2 , \]  

(A.15)

where \( d\Omega_{D-2}^2 \) is the line element on the unit \( S^{D-2} \). This yields

\[ P(X, X') = \cosh \tau \cosh \tau' - \sinh \tau \sinh \tau' \cosh \theta \cosh \theta' \]
\[ + \sinh \tau \sinh \tau' \sinh \theta \sinh \theta' \cos \Theta , \]  

(A.16)

where again \( \Theta = \Theta(\Omega, \Omega') \) denotes the geodesic distance of two points with angular variables \( \Omega \) and \( \Omega' \) on the unit \( S^{D-2} \).

B Green’s Functions

The de Sitter invariant Hadamard two-point function

\[ G(X, X') = \text{const.} \langle 0 | \{ \Phi(X), \Phi(X') \} | 0 \rangle \]  

(B.1)

obeys

\[ (\nabla_x^2 - m^2)G(x, x') = 0 , \]  

(B.2)

and depends on \( x \) and \( x' \) only through \( P \), as de Sitter space is maximally symmetric. As shown in [41], one can write

\[ l^2(\nabla_x^2 - m^2)f(P) = (1 - P^2) \frac{d^2 f}{dP^2} - DP \frac{df}{dP} - m^2 l^2 f \]  

(B.3)

for an arbitrary function \( f \). This leads to the differential equation
\[(P^2 - 1)\frac{d^2 G}{dP^2} + DP\frac{dG}{dP} + m^2 l^2 G = 0 \quad (B.4)\]

for the Hadamard two-point function, with the solution

\[G(P) = \text{Re} F(h_+, h_-; \frac{D}{2}, \frac{1 + P}{2}) , \quad (B.5)\]

where \(F\) is a hypergeometric function, and

\[h_\pm = \frac{1}{2}(D - 1 \pm \sqrt{(D - 1)^2 - 4m^2 l^2}) . \quad (B.6)\]

(B.5) generalizes the result of \([6]\) to arbitrary dimension.

In the static coordinates \((A.5)\), one has the asymptotic behaviour near \(I^-\)

\[\lim_{r, r' \to \infty} P(r, t, \Omega; r', t', \Omega') = -\frac{rr'}{l^2} \left[ \cosh \frac{t - t'}{l} - \cos \Theta \right] , \quad (B.7)\]

which diverges. Like in \([6]\), we can then use the formula

\[F(h_+, h_-; \frac{D}{2}; z) = \frac{\Gamma(D/2)\Gamma(h_- - h_+)}{\Gamma(h_-)\Gamma(D/2 - h_+)} (-z)^{-h_+} F(h_+, h_+ - \frac{D}{2} + 1, h_+ - h_- + 1; \frac{1}{z}) + (h_+ \leftrightarrow h_-) , \quad (B.8)\]

together with \(F(\alpha, \beta, \gamma; 0) = 1\), to obtain

\[\lim_{r, r' \to \infty} G(r, t, \Omega; r', t', \Omega') = \frac{\Gamma(D/2)\Gamma(h_- - h_+)}{\Gamma(h_-)\Gamma(D/2 - h_+)} \left( \frac{rr'}{2l^2} \right)^{-h_+} \left[ \cosh \frac{t - t'}{l} - \cos \Theta \right]^{-h_+} + (h_+ \leftrightarrow h_-) . \quad (B.9)\]

The asymptotic scaling behaviour of the two-point function \(G\) was first observed in \([12]\).

In the hyperbolic coordinates \((A.14)\), one has the asymptotic behaviour near \(I^-\)

\[\lim_{\tau, \tau' \to -\infty} P(\tau, \theta, \Omega; \tau', \theta', \Omega') = \frac{1}{4} e^{-\tau - \tau'} \left[ 1 - \cosh \theta \cosh \theta' + \sinh \theta \sinh \theta' \cos \Theta \right] , \quad (B.10)\]

which again diverges. Using (B.8), we get
\[
\lim_{\tau, \tau' \to \infty} G(\tau, \theta, \Omega; \tau', \theta', \Omega') = \\
\frac{\Gamma\left(\frac{D}{2}\right) \Gamma(h_+ - h_-)}{\Gamma(h_-) \Gamma\left(\frac{D}{2} - h_+\right)} 4^{h_+} e^{h_+(\tau + \tau')} \left[ \cosh \theta \cosh \theta' - \sinh \theta \sinh \theta' \cos \Theta - 1 \right]^{-h_+} + (h_+ \leftrightarrow h_-). \tag{B.11}
\]

For \(D = 3\), using the complex coordinate \(w = \tanh \frac{\theta}{2} e^{i\phi}\), (B.11) reads

\[
\lim_{\tau, \tau' \to \infty} G(\tau, w, \bar{w}; \tau', v, \bar{v}) = \\
\frac{\Gamma\left(\frac{3}{2}\right) \Gamma(h_+ - h_-)}{\Gamma(h_-) \Gamma\left(\frac{3}{2} - h_+\right)} 4^{h_+} e^{h_+(\tau + \tau')} \left[ \frac{(1 - w\bar{w})(1 - v\bar{v})}{|w - v|^2} \right]^{-h_+} + (h_+ \leftrightarrow h_-). \tag{B.12}
\]
References

[1] T. Banks, “Cosmological breaking of supersymmetry,” \texttt{hep-th/0007146}; T. Banks and W. Fischler, “M-theory observables for cosmological space-times,” \texttt{hep-th/0102077}.

[2] V. Balasubramanian, P. Horava and D. Minic, “Deconstructing de Sitter,” JHEP 0105 (2001) 043 \texttt{hep-th/0103171}.

[3] E. Witten, “Quantum gravity in de Sitter space,” \texttt{hep-th/0106108}.

[4] A. Volovich, “Discreteness in de Sitter space and quantization of Kaehler manifolds,” \texttt{hep-th/0101176}.

[5] A. Chamblin and N. D. Lambert, “De Sitter space from M-theory,” Phys. Lett. B 508 (2001) 369 \texttt{hep-th/0102159}.

[6] A. Strominger, “The ds/CFT correspondence,” JHEP 0110 (2001) 034 \texttt{hep-th/0106113}.

[7] P. O. Mazur and E. Mottola, “Weyl cohomology and the effective action for conformal anomalies,” \texttt{hep-th/0106151}.

[8] M. Li, “Matrix model for de Sitter,” \texttt{hep-th/0106184}.

[9] S. Nojiri and S. D. Odintsov, “Conformal anomaly from dS/CFT correspondence,” \texttt{hep-th/0106191}.

[10] S. Perlmutter, “Supernovae, dark energy, and the accelerating universe: The status of the cosmological parameters,” in \textit{Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy LP99} ed. J.A. Jaros and M.E. Peskin, Int. J. Mod. Phys. A 15S1 (2000) 715 [eConfC 990809 (2000) 715].

[11] J. Maldacena and A. Strominger, “Statistical entropy of de Sitter space,” JHEP 9802 (1998) 014 \texttt{gr-qc/9801090}.

[12] M. Banados, T. Brotz and M. E. Ortiz, “Quantum three-dimensional de Sitter space,” Phys. Rev. D 59 (1999) 046002 \texttt{hep-th/9807216}.

[13] W. T. Kim, “Entropy of 2+1 dimensional de Sitter space in terms of brick wall method,” Phys. Rev. D 59 (1999) 047503 \texttt{hep-th/9810169}.

[14] F. Lin and Y. Wu, “Near-horizon Virasoro symmetry and the entropy of de Sitter space in any dimension,” Phys. Lett. B 453 (1999) 222 \texttt{hep-th/9901147}.

[15] S. Hawking, J. Maldacena and A. Strominger, “De Sitter entropy, quantum entanglement and AdS/CFT,” JHEP 0105 (2001) 001 \texttt{hep-th/0002143}.
[16] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323 (2000) 183 [hep-th/9905111].

[17] C. M. Hull, “Timelike T-duality, de Sitter space, large N gauge theories and topological field theory,” JHEP 9807 (1998) 021 [hep-th/9806146].

[18] B. S. Acharya, M. O’Loughlin and B. Spence, “Higher-dimensional analogues of Donaldson-Witten theory,” Nucl. Phys. B 503 (1997) 657 [hep-th/9705138].

[19] B. S. Acharya, J. M. Figueroa-O’Farrill, B. Spence and M. O’Loughlin, “Euclidean D-branes and higher-dimensional gauge theory,” Nucl. Phys. B 514 (1998) 583 [hep-th/9707111].

[20] M. Blau and G. Thompson, “Euclidean SYM theories by time reduction and special holonomy manifolds,” Phys. Lett. B 415 (1997) 242 [hep-th/9706225].

[21] C. M. Hull and R. R. Khuri, “Worldvolume theories, holography, duality and time,” Nucl. Phys. B 575 (2000) 231 [hep-th/9911082].

[22] R. Bousso, “Holography in general space-times,” JHEP 9906 (1999) 028 [hep-th/9906022].

[23] J. D. Brown and M. Henneaux, “Central charges in the canonical realization of asymptotic symmetries: An example from three-dimensional gravity,” Commun. Math. Phys. 104 (1986) 207.

[24] H. Nariai, “On a new cosmological solution of Einstein’s field equations of gravitation,” Sci. Rep. Tohoku Univ. Ser. I 35 (1951) 62.

[25] V. Balasubramanian and P. Kraus, “A stress tensor for anti-de Sitter gravity,” Commun. Math. Phys. 208 (1999) 413 [hep-th/9902121].

[26] R. B. Mann, “Misner string entropy,” Phys. Rev. D 60 (1999) 104047 [arXiv:hep-th/9903229].

[27] R. Emparan, C. V. Johnson and R. C. Myers, “Surface terms as counterterms in the AdS/CFT correspondence,” Phys. Rev. D 60 (1999) 104001 [hep-th/9903238].

[28] J. D. Brown and J. W. York, “Quasilocal energy and conserved charges derived from the gravitational action,” Phys. Rev. D 47 (1993) 1407.

[29] G. W. Gibbons and S. W. Hawking, “Cosmological event horizons, thermodynamics, and particle creation,” Phys. Rev. D 15 (1977) 2738.

[30] E. Keski-Vakkuri, “Bulk and boundary dynamics in BTZ black holes,” Phys. Rev. D 59 (1999) 104001 [hep-th/9808037].
[31] J. L. Cardy, “Conformal invariance and universality in finite-size scaling,” J. Phys. A 17 (1984) L385.

[32] P. Di Francesco, P. Mathieu and D. Sénéchal, Conformal field theory, Springer 1997.

[33] J. L. Cardy, “Anisotropic corrections to correlation functions in finite size systems,” Nucl. Phys. B 290 (1987) 355.

[34] D. Kastor and J. Traschen, “Cosmological multi-black hole solutions,” Phys. Rev. D 47 (1993) 5370 [hep-th/9212033].

[35] J. T. Liu and W. A. Sabra, “Multi-centered black holes in gauged d = 5 supergravity,” Phys. Lett. B 498 (2001) 123 [hep-th/0010025].

[36] D. Klemm and W. A. Sabra, “Charged rotating black holes in 5d Einstein-Maxwell-(A)dS gravity,” Phys. Lett. B 503 (2001) 147 [hep-th/0010200].

[37] D. Klemm and W. A. Sabra, “General (anti-)de Sitter black holes in five dimensions,” JHEP 0102 (2001) 031 [hep-th/0011010].

[38] C. M. Hull, “Duality and the signature of space-time,” JHEP 9811 (1998) 017 [hep-th/9807127].

[39] R. C. Myers, “Stress tensors and Casimir energies in the AdS/CFT correspondence,” Phys. Rev. D 60 (1999) 046002.

[40] V. Balasubramanian, J. de Boer and D. Minic, “Mass, entropy and holography in asymptotically de Sitter spaces,” [hep-th/0110108].

[41] P. Candelas and D. J. Raine, “General relativistic quantum field theory - an exactly soluble model,” Phys. Rev. D 12 (1975) 965.

[42] I. Antoniadis, P. O. Mazur and E. Mottola, “Comment on Nongaussian isocurvature perturbations from inflation,” [astro-ph/9705200].