On spin 2 electromagnetic interactions

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Abstract

In this paper we (re)consider the problem of electromagnetic interactions for massless spin 2 particles and show that in (A)dS spaces with non-zero cosmological constant it is indeed possible (at least in linear approximation) to switch on minimal electromagnetic interactions supplemented by third derivative non-minimal ones which are necessary to restore gauge invariance.

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Introduction

It was known for a long time that it is not possible to construct standard gravitational interaction for massless high spin $s \geq 5/2$ particles in flat Minkowski space \cite{1,2,3} (see also recent discussion in \cite{4}). At the same time, it has been shown \cite{5,6} that this task indeed has a solution in $(A)dS$ space with non-zero cosmological term. The reason is that gauge invariance, that turn out to be broken when one replace ordinary partial derivatives by the gravitational covariant ones, could be restored with the introduction of higher derivative corrections containing gauge invariant Riemann tensor. These corrections have coefficients proportional to inverse powers of cosmological constant so that such theories do not have naive flat limit. But it is perfectly possible to have a limit when both cosmological term and gravitational coupling constant simultaneously go to zero in such a way that only interactions with highest number of derivatives survive. So the crucial point is the existence of cubic higher derivative spin $s-s-2$ vertex containing (linearized) Riemann tensor and two massless spin $s$ particles in flat Minkowski space. For spin $s = 3$ case an appropriate candidate has been constructed recently in \cite{7} (see also \cite{8} where this vertex was reconsidered and an appropriate one for $s = 4$ case has been constructed). And in our recent paper \cite{9} we have shown that deformation of this vertex into $(A)dS$ space indeed can reproduce (at least in linear approximation) standard minimal gravitational interaction.

Besides gravitational interaction one more classical and important test for any high spin theory is electromagnetic interaction. The problem of switching on of such interaction for massless high spin particles looks very similar to the problem with gravitational interactions. Namely, if one replace ordinary partial derivatives by the gauge covariant ones the resulting Lagrangian looses its gauge invariance and this non-invariance (arising due to non-commutativity of covariant derivatives) is proportional to field strength of vector field. In this, for the massless field with $s \geq 3/2$ in flat Minkowski space there is no possibility to restore gauge invariance by adding non-minimal terms to Lagrangian and/or modifying gauge transformations. But such restoration becomes possible if one goes to $(A)dS$ space with non-zero cosmological constant. By the same reason as in the gravitational case, such theories do not have naive flat limit, but it is possible to consider a limit where both cosmological constant and electric charge simultaneously go to zero so that only highest derivative non-minimal terms survive. Again it is crucial to have some higher derivative cubic $s-s-1$ vertex containing $e/m$ field strength.

In this paper we give an example of such restoration for the case of massless spin $2$ particles. In Section 1, motivated by results of \cite{8}, we (re)consider cubic third derivative 2-2-1 vertex and show that interaction Lagrangian indeed could be written in terms of field strength of vector field (so it is trivially invariant under vector field gauge transformations) while spin $2$ fields gauge transformations have exactly the same form as in \cite{8}. The structure of interaction Lagrangian and gauge transformations suggests transition to so called frame-like formulation where one uses general non-symmetric second rank tensors for the description of spin $2$ particles and in Section 2 we reconstruct the same vertex in such formalism. At last, in Section 3 we consider deformation of this vertex into $(A)dS$ space and show that it is indeed possible to produce minimal $e/m$ interaction at least in linear approximation.
1 Cubic vertex 2-2-1

In this Section we (re)consider cubic three derivative 2-2-1 vertex motivated by results of [8]. In particular, from the results of this paper it follows that one needs at least two different spin 2 fields in order such vertex can be constructed. So we consider two symmetric second rank tensors $h_{\mu \nu}^i$, $i = 1, 2$ and vector field $A_\mu$. We take sum of the standard kinetic terms for all three fields as our free Lagrangian:

$$L_0 = \frac{1}{2} \partial_\mu h_{\alpha \beta} \partial_\mu h_{\alpha \beta} - (\partial h)_\mu^i (\partial h)_\mu^i + (\partial h)_\mu^i \partial_\mu h^i - \frac{1}{2} \partial_\mu h^i \partial_\mu h^i - \frac{1}{4} A_{\mu \nu}^2$$

(1)

This Lagrangian is invariant under usual gauge transformations:

$$\delta h_{\mu \nu}^i = \partial_\mu \xi_\nu^i + \partial_\nu \xi_\mu^i, \quad \delta A_\mu = \partial_\mu \lambda$$

(2)

The most general (up to total divergence) cubic vertex with three derivatives could be written (schematically):

$$L_1 \sim \partial^3 h_1 h_2 A \oplus \partial^2 h_1 \partial h_2 A \oplus \partial h_1 \partial^2 h_2 A \oplus h_1 \partial^3 h_2 A$$

Correspondingly, we choose the following ansatz for modification of gauge transformations with vector parameters:

$$\delta_1 A \sim \partial h_1 \partial \xi_2 \oplus \partial h_2 \partial \xi_1, \quad \delta_1 h_1 \sim \partial A \partial \xi_2, \quad \delta_1 h_2 \sim \partial A \partial \xi_1$$

As for the corrections for $\lambda$-transformations, all of them (up to possible change of variables) are trivial (i.e. leave sum of kinetic terms invariant). Indeed, it is easy to check that transformations

$$\delta h_{\mu \nu}^i = \varepsilon^{ij} [c_1 R_{\mu \nu}^j + c_2 g_{\mu \nu} R^j] \lambda$$

where $R_{\mu \nu}^i$ — linearized Ricci tensor, leave free Lagrangian invariant and do not correspond to any interaction vertex. It contradicts with [8], but it is exactly what one needs for possible application to electromagnetic interactions, because in this case the Lagrangian being trivially invariant under $\lambda$-transformations can be written in terms of gauge invariant field strength for the vector field. Then, the requirement that the Lagrangian be invariant under corrected $\xi_\mu^i$-transformations leads to the following cubic vertex:

$$M^2 L_1 = \varepsilon_{ij} A^\mu_{\alpha \beta} [\partial_\mu h_{\alpha \beta}^i \partial_\alpha h_{\beta \nu}^j + \frac{1}{2} \partial_\alpha h_{\beta \nu}^i \partial_\alpha h_{\beta \nu}^j - \partial_\alpha h_{\beta \nu}^i \partial_\beta h_{\alpha \nu}^j - \frac{1}{2} \partial_\mu h_{\alpha \beta}^i \partial_\nu h_{\alpha \beta}^j + \partial_\mu h_{\nu \alpha}^i (\partial h)_\alpha^j + \frac{1}{2} (\partial h)_\mu^i (\partial h)_\nu^j - (\partial h)_\mu^i \partial_\nu h^j - \partial_\mu h_{\nu \alpha}^i \partial_\alpha h^j + \frac{1}{2} \partial_\mu h^i \partial_\nu h^j]$$

(3)

Here, using the fact that cubic bosonic three derivative vertex must have coupling constant with dimension $1/m^2$, we introduce appropriate mass scale $M$. Corrections to gauge transformations look as follows:

$$M^2 \delta h_{\mu \nu}^i = -\varepsilon^{ij} \left[ \frac{1}{2} (A_{\mu \alpha} \partial_\alpha \xi_\nu^j + A_{\nu \alpha} \partial_\alpha \xi_\mu^j) + \frac{1}{d-2} g_{\mu \nu} A_{\alpha \beta} \partial_\alpha \xi_\beta^j \right]$$

$$M^2 \delta A_\mu = \varepsilon_{ij} \partial_\alpha h_{\beta \mu}^i \partial_\alpha \xi_\beta^j$$

(4)

Note, that they have exactly the same form as in [8].

The structure of interaction Lagrangian and gauge transformations suggests transition to so called ”frame-like” formulation where one uses general (non-symmetric) second rank tensor for description of spin 2 particle. So, before turning on to possible electromagnetic interactions, we reconstruct the same cubic vertex in such formulation.
2 Frame-like second order formulation

As is well known, for symmetric second rank tensor it is impossible to construct gauge invariant object out of first derivatives of this field. But if our tensors $h_{\mu\nu}^i$ are non-symmetric, we can choose gauge transformations to be:

$$\delta h_{\mu\nu}^i = \partial_\mu \xi_\nu^i$$

As a result, we can easily construct gauge invariant tensors ("Lorentz connections"):

$$\Omega_{\mu,\alpha\beta}^i = \frac{1}{2} \left[ \partial_\mu (h_{\alpha\beta}^i - h_{\beta\alpha}^i) - \partial_\alpha (h_{\mu\beta}^i + h_{\beta\mu}^i) + \partial_\beta (h_{\mu\alpha}^i + h_{\alpha\mu}^i) \right]$$ (5)

which are antisymmetric on last two indices $\Omega_{\mu,\alpha\beta}^i = -\Omega_{\mu,\beta\alpha}^i$. But making transition from symmetric tensors to general ones, we introduce additional degrees of freedom (namely, antisymmetric parts of our tensors). To compensate this difference, we introduce additional local transformations:

$$\delta h_{\mu\nu}^i = \eta_{\mu\nu}^i, \quad \eta_{\mu\nu}^i = -\eta_{\nu\mu}^i$$

In this, $\delta \Omega_{\mu,\alpha\beta}^i = \partial_\mu \eta_{\alpha\beta}^i$. Now the free Lagrangian could be written as:

$$L_0 = \frac{1}{2} (\Omega_{\mu,\alpha\beta}^i \Omega_{\alpha,\mu\beta}^i - \Omega_{\beta}^i \Omega_{\beta}^i) - \frac{1}{4} A_{\mu\nu} A_{\mu\nu}$$ (6)

where $\Omega_\beta^i = g^{\mu\alpha} \Omega_{\mu,\alpha\beta}^i$.

In terms of these variables the interaction Lagrangian takes the form:

$$M^2 L_1 = \varepsilon_{ij} A^{\mu\nu} \left[ \frac{1}{2} \Omega_{\mu,\alpha\beta}^i \Omega_{\nu,\alpha\beta}^i - \Omega_{\alpha,\mu\beta}^i \Omega_{\beta,\nu\alpha}^j + \Omega_{\alpha,\mu\nu}^i \Omega_{\alpha,\nu\beta}^j + \Omega_{\mu,\alpha\nu}^i \Omega_{\nu,\beta\alpha}^j \right]$$ (7)

It is trivially (by construction) invariant under the $\xi_\mu^i$-transformations, while invariance under $\eta_{\mu\nu}^i$-transformations requires non-trivial deformations:

$$M^2 \delta_1 h_{\mu\nu}^i = -\varepsilon_{ij} [A_{\alpha\mu} \eta_{\alpha\nu}^j + A_{\alpha\nu} \eta_{\alpha\mu}^j + \frac{1}{d-2} g_{\mu\nu} A^{\alpha\beta} \eta_{\alpha\beta}^j]$$

$$M^2 \delta_1 A_{\mu} = \varepsilon_{ij} \Omega_{\mu,\alpha\beta}^i \eta_{\alpha\beta}^j$$ (8)

In order to go back to symmetric second rank tensors, one has supplement $\xi_\mu^i$ transformations with $\eta_{\mu\nu}^i$-ones where $\eta_{\mu\nu}^i = -\frac{1}{2} (\partial_\mu \xi_\nu^j - \partial_\nu \xi_\mu^j)$ reproducing results of previous Section.

Note also, that if we formally separate indices into "local" and "world" ones then the Lagrangian could be rewritten in a rather suggestive form:

$$M^2 L_1 = \frac{1}{4} (\delta_a^\mu \delta_b^\nu - \delta_a^\nu \delta_b^\mu) \varepsilon_{ij} \left[ 4 \Omega_{\mu,\alpha\beta}^{ac,i} \Omega_{\nu,\delta\beta}^{bd,j} A^{cd} + \Omega_{\mu}^{cd,i} \Omega_{\nu}^{cd,j} A^{ab} + \Omega_{\mu}^{ab,i} \Omega_{\nu}^{cd,j} A^{cd} \right]$$ (9)

3 Deformation to (A)dS

In this Section we consider transition from flat Minkowski space to constant curvature (A)dS ones and show that the three derivative cubic vertex considered above is exactly what one
needs to compensate the non-invariance that arises when one introduce minimal electromagnetic interaction for massless spin 2 field. We will use the following convention on covariant derivatives:

\[
[D_\mu, D_\nu]v_\alpha = R_{\mu \nu, \alpha \beta} v^\beta = -\kappa (g_{\mu \alpha} v_\nu - g_{\nu \alpha} v_\mu), \quad \kappa = \frac{2\Lambda}{(d-1)(d-2)} \tag{10}
\]

Now, due to non-commutativity of covariant derivatives, \( \Omega^i_j \) are not invariant under gauge transformations with vector parameters:

\[
\delta \Omega_{\mu, \alpha \beta}^i = \kappa (g_{\mu \alpha} \xi^\beta_i - g_{\mu \beta} \xi^\alpha_i)
\]

Resulting non-invariance of free Lagrangian:

\[
\delta L_0 = -\kappa (d-2) \Omega^{\alpha i} \xi^\alpha_i
\]

could be compensated by adding:

\[
\Delta L_0 = \frac{\kappa (d-2)}{2} [h^{\alpha \beta} i h^{\beta \alpha} - h^i h^i] \tag{11}
\]

In this, total Lagrangian is invariant under transformations with tensor parameters as well.

Let us introduce minimal e/m interaction by replacing \((A)dS\) covariant derivatives by fully covariant ones, e.g.:

\[
\nabla_\mu \xi^\alpha_i = D_\mu \xi^\alpha_i - e_0 \varepsilon^{ij} A_\mu \xi^\alpha_j, \quad [\nabla_\mu, \nabla_\nu] \xi^\alpha_i = [D_\mu, D_\nu] \xi^\alpha_i - e_0 \varepsilon^{ij} A_{\mu \nu} \xi^\alpha_j \tag{12}
\]

As a result, additional non-invariance under the transformations with vector parameters appear:

\[
\delta \Omega_{\mu, \alpha \beta}^i = -\frac{e_0}{2} \varepsilon^{ij} [A_{\mu \alpha} \xi^\beta_j - A_{\mu \beta} \xi^\alpha_j - A_{\alpha \beta} \xi^\mu_j]
\]

This leads to non-invariance of free Lagrangian:

\[
\delta \xi L_0 = \frac{e_0}{2} \varepsilon^{ij} A_{\mu \nu} [\Omega_{\alpha, \mu \nu}^i \xi^\alpha_j + 2 \Omega_{\mu}^i \xi^\nu_j]
\]

At the same time, invariance of free Lagrangian under the transformations with tensor parameters is also broken:

\[
\delta \eta L_0 = \frac{e_0}{2} \varepsilon^{ij} A_{\mu \nu} [2 h_{\alpha \mu}^i \eta_{\nu \alpha}^j + h^i \eta_{\mu \nu}^j]
\]

Now we add the third derivative cubic vertex, obtained in the previous Section, where all derivatives in the Lagrangian and gauge transformations are replaced by the covariant ones. Non-invariance of this vertex due to non-commutativity of covariant derivatives has the form (up to the terms quadratic in \(A_{\mu \nu}\)):

\[
M^2 \delta \xi L_1 = -\kappa (d-3) \varepsilon^{ij} A_{\mu \nu} [\Omega_{\alpha, \mu \nu}^i \xi^\alpha_j + 2 \Omega_{\mu}^i \xi^\nu_j] - 2 \kappa \varepsilon^{ij} A_{\mu \nu} \Omega_{\nu, \alpha \nu}^i \xi^\alpha_j
\tag{13}
\]

For \(e_0 = 2\kappa (d-3)/M^2\) first term compensate non-invariance of free Lagrangian, while to compensate the second term we add finally:

\[
\Delta L_1 = a_0 \varepsilon^{ij} A_{\mu \nu} h_{\mu \alpha}^i h_{\nu \alpha}^j, \quad \delta A_\mu = b_0 \varepsilon^{ij} h_{\mu \alpha}^i \xi^\alpha_j \tag{14}
\]

When for \(a_0 = 2\kappa, b_0 = 2\kappa\) all variations cancel. Moreover, resulting Lagrangian, as we have explicitly checked, turns out to be invariant under transformations with tensor parameters as well.
Conclusion

Thus we have shown that for massless spin 2 particles in ($A)dS$ space it is indeed possible to switch on minimal electromagnetic interactions supplemented by third derivative non-minimal interactions. But similar possibility exists for the massive particles in flat Minkowski space [10, 11, 12, 13]. By the same reasoning it is natural to suggest that there must exist a limit where both mass and electric charge go to zero so that some non-minimal higher derivative terms survive. Non-minimal terms similar to the ones considered here appeared in [10], while in our investigation [11], based on gauge invariant description of massive spin 2 particles, it turned out that to restore gauge invariance after switching on minimal e/m interactions it is enough to introduce corrections with two derivatives only. So it appears that the relation between massless particles in ($A)dS$ space and massive ones in flat Minkowski space is not so simple and straightforward as it seemed. Anyway, this question certainly deserves further study.

References

[1] C. Aragone, S. Deser "Consistency problem of hypergravity", Phys. Lett. B86 (1979) 161.

[2] D. de Wit, D. Z. Freedman "Systematics of higher-spin gauge fields", Phys. Rev. D21 (1980) 358.

[3] F. A. Berends, G. J. H. Bugrers, H. van Dam "On the theoretical problems in constructing interactions involving higher-spin massless particles", Nucl. Phys. B260 (1985) 295.

[4] M. Porrati "Universal Limits on Massless High-Spin Particles", arXiv:0804.4672.

[5] E. S. Fradkin, M. A. Vasiliev "On the gravitational interaction of massless higher-spin fields", Phys. Lett. B189 (1987) 89.

[6] E. S. Fradkin, M. A. Vasiliev "Cubic interaction in extended theories of massless higher-spin fields", Nucl. Phys. B291 (1987) 141.

[7] N. Boulanger, S. Leclercq "Consistent couplings between spin-2 and spin-3 massless fields", JHEP 0611 (2006) 034, arXiv:hep-th/0609221.

[8] N. Boulanger, S. Leclercq, P. Sundell "On The Uniqueness of Minimal Coupling in Higher-Spin Gauge Theory", arXiv:0805.2764.

[9] Yu. M. Zinoviev "On spin 3 interacting with gravity", arXiv:0805.2226.

[10] S. Ferrara, M. Porrati, V. L. Telegdi "$g = 2$ as the natural value of the tree-level gyromagnetic ratio of elementary particles", Phys. Rev. D46 (1992) 3529.

[11] S. M. Klishevich, Yu. M. Zinoviev "On electromagnetic interaction of massive spin-2 particle", Phys. Atom. Nucl. 61 (1998) 1527, arXiv:hep-th/9708150.
[12] S. Deser, A. Waldron  "Inconsistencies of massive charged gravitating higher spins", Nucl. Phys. B631 (2002) 369, arXiv:hep-th/0112182.

[13] M. Porrati, R. Rahman  "Intrinsic Cutoff and Acausality for Massive Spin 2 Fields Coupled to Electromagnetism", arXiv:0801.2581