Collision Avoidance Based on Robust Lexicographical Task Assignment

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Abstract—Traditional task assignment approaches for multi-agent motion control do not take the possibility of collisions into account. This can lead to challenging requirements for the path planning and guidance of the individual agents. To address this issue, we derive an assignment method that not only minimises the largest distance between an agent and its assigned destination but also provides local constraints for guaranteed collision avoidance. To this end, we introduce a sequential bottleneck assignment problem and define a notion of robustness of an optimal assignment. Conditioned on a sufficient level of robustness in relation to the size of the agents, we construct local time-varying position bounds for each individual agent. These local constraints are a direct byproduct of the assignment procedure and only depend on the initial agent positions, the locations that are to be visited, and a timing parameter. We show that no agent that is assigned to move to one of the target destinations collides with any other agent if all agents satisfy their local position constraints. We demonstrate the applicability of the method in a numerical case study.

I. INTRODUCTION

For autonomous systems with multiple agents and multiple tasks the first decision to be made is how to allocate the tasks to the agents. This type of decision is referred to as an assignment problem and may involve any of a variety of objectives. In [1], [2] the sum of individual costs incurred for assigning robotic agents to tasks is minimised in a so-called Linear Assignment Problem (LAP). An overview of assignment problems is given in [3]–[5].

We focus here on a specific type of assignment problem where the largest incurred individual agent-to-task assignment cost is minimised. This objective is referred to as the Bottleneck Assignment Problem (BAP). It commonly applies to multi-agent problems where tasks are completed in parallel and the overall completion time is of interest, as in [6] for instance. A Lexicographical Assignment Problem (LexAP) describes a subclass of the BAP where not only the largest assignment cost but also all other assignment costs are minimised with a sequence of decreasing hierarchy [7].

A review of the state-of-the-art methods to solve the BAP is provided in [5]. In [8] an algorithm to solve the BAP with distributed computation is introduced. The sensitivity of the bottleneck optimising assignment with respect to changes in the assignment costs is investigated in [9].

In applications where the tasks involve agents moving towards desired locations, collisions between agents may occur. Motion control with inter-agent collision avoidance is a heavily researched problem and many different strategies have been developed, e.g., reciprocal collision avoidance [10], [11], barrier certificates [12], buffered Voronoi cells [13], and multi-agent rapidly-exploring random trees [14]. In [15] collision avoidance awareness criteria are included in the decision of allocating tasks. In particular, a quadratic assignment cost function is designed that incentivises collision-free paths.

Here, we investigate intrinsic properties of assignment problems that provide conditions for which collisions are avoided. This is similar to [16], where collision avoidance properties are shown as an artefact of solving an LAP, with assignment costs consisting of Euclidean distances between initial agent positions and target destinations. Specifically, given the optimal LAP solution, straight lines connecting agents to their assigned tasks do not intersect. We study properties resulting from sequentially applying the BAP given an arbitrary distance metric to quantify the assignment costs. The non-intersection properties of the LAP solution do not hold for the BAP, however.

The contribution of this paper is the derivation of local position constraints for every agent. Rather than determining trajectories for all agents, the method proposed here provides time-varying sets of positions that guarantee collision avoidance but leave some degree of freedom for low-level path planning and motion control. We propose a sequential bottleneck task assignment approach and relate the physical size of the agents to bounds on the sensitivity of the optimal assignment. We show that if the assignment is robust at every step of the assignment sequence, it is the unique optimal solution of the LexAP. By accounting for the robustness of the optimal assignment, we construct the local constraints such that they can be applied to prevent collisions. The constraints for an individual agent do not explicitly depend on the positions of the other agents. Information on other agents only gets accounted for via the assignment costs. We prove that it is sufficient for every agent to satisfy its resulting local constraints in order to guarantee that no agent that is assigned to one of the tasks will collide with any other agent.

The paper is structured as follows. We first formulate the assignment and collision avoidance objectives in Section II. In Section III we address the assignment problem and investigate properties of a sequential bottleneck assignment procedure. Based on these tools we derive sufficient conditions for collision avoidance in Section IV. We illustrate the derived agent position constraints in a case study in Section V before concluding in Section VI.
II. PROBLEM FORMULATION

We consider $m$ agents, $\mathcal{A} := \{\alpha_i\}_{i=1}^m$, with initial positions, $p_i(0) \in \mathbb{R}^n_p, n_p \in \mathbb{N}$, $i \in \mathcal{A}$, and $n$ target destinations, $g_j \in \mathbb{R}^n_p, j \in \mathcal{T}$, where $\mathcal{T} := \{\tau\}_{\tau=1}^n$ and $\tau$ represents the task of going destination $g_\tau$. Without loss of generality we assume there are at least as many agents as tasks, $m \geq n$, and consider only non-trivial cases where there are multiple agents, $m > 1$, and at least one task, $n \geq 1$.

We define a set of binary variables, $\Pi := \{\pi_{i,j} \in \{0, 1\} | (i, j) \in \mathcal{A} \times \mathcal{T}\}$, that indicate the allocation of task $j \in \mathcal{T}$ to agent $i \in \mathcal{A}$ if and only if $\pi_{i,j} = 1$. We call such a set an assignment and denote the set of all assignments by $\mathbb{B}_{\mathcal{A}, \mathcal{T}}$. We assume that a cost is incurred when an agent proceeds to an assigned location.

Assumption 1: The cost of assigning an agent, $i \in \mathcal{A}$, to complete a task, $j \in \mathcal{T}$, is given by the distance of the initial agent position to the corresponding destination,

$$w_{i,j} := d(p_i(0), g_j),$$

where $d : \mathbb{R}^n_p \times \mathbb{R}^n_p \to [0, \infty)$ is an arbitrary distance function that satisfies the triangle inequality,

$$d(p, p') \leq d(p, p'') + d(p'', p'),$$

for all $p, p', p'' \in \mathbb{R}^n_p$.

At any given time, $t$, the centroid location of agent $i \in \mathcal{A}$ is given by a point, $p_i(t)$. However, the body of the agent occupies a finite volume in $\mathbb{R}^n_p$. If two agents lie in close proximity, they may collide.

Definition 1 (Inter-agent safety distance): Agents $i, i' \in \mathcal{A}, i \neq i'$, do not collide with each other at time $t$ if

$$d(p_i(t), p_{i'}(t)) > s_{i,i'},$$

where $s_{i,i'} = s_{i',i} \geq 0$ is known and named the safety distance between $i$ and $i'$.

We note that the collision avoidance condition in (3) couples the motion control problem of agents $i$ and $i'$. Satisfying this condition introduces a non-convex constraint with respect to the positions of the agents, $p_i(t)$ and $p_{i'}(t)$, and is therefore challenging. The objective in this paper is to derive local policies that sufficiently guarantee avoidance of collisions of agents that are assigned to tasks such that the maximum agent-to-destination distance is minimised.

Problem 1: Given the set of assignment costs, $\{w_{i,j} \geq 0 | (i, j) \in \mathcal{A} \times \mathcal{T}\}$, and suppose that one agent is assigned to every target destination such that the largest assigned cost is minimised,

$$\min_{\Pi \in \mathbb{B}_{\mathcal{A}, \mathcal{T}}} \max_{(i,j) \in \mathcal{A} \times \mathcal{T}} \pi_{i,j} w_{i,j}$$

subject to

$$\sum_{i \in \mathcal{A}} \pi_{i,j} = 1 \quad \forall j \in \mathcal{T},$$

$$\sum_{j \in \mathcal{T}} \pi_{i,j} \leq 1 \quad \forall i \in \mathcal{A}. $$

Find time-varying position constraints, $\mathcal{L}_i(t) \subset \mathbb{R}^n_p$, for every agent, $i \in \mathcal{A}$, such that no agent assigned to a task collides with any other agent over a time interval, $t \in [0, T]$, i.e., if $p_i(t) \in \mathcal{L}_i(t)$ for all $i \in \mathcal{A}, t \in [0, T]$, then

$$d(p_i(t), p'_{i'}(t)) > s_{i,i'},$$

for all $i^* \in \{i \in \mathcal{A} | \sum_{j \in \mathcal{T}} \pi_{i,j} = 1\}, i' \in \mathcal{A}$, and $t \in [0, T]$.

III. TASK ASSIGNMENT

In this section we introduce task assignment criteria and tools that will be used to derive collision avoidance conditions in Section IV. We illustrate details of the assignment process with an example assignment in Section III-C.

We consider $\mathcal{A}$, and the set of tasks, $\mathcal{T}$, where $|\mathcal{A}| = m > 1$ and $m \geq |\mathcal{T}| = n > 1$, we define the complete bipartite assignment graph $\mathcal{G} := (\mathcal{A}, \mathcal{T}, \mathcal{W})$, with vertex set, $\mathcal{V} := \mathcal{A} \cup \mathcal{T}$, and edge set, $\mathcal{E} := \mathcal{A} \times \mathcal{T}$. We also define the set of assignment weights, $\mathcal{W} := \{w_{i,j} \geq 0 | (i, j) \in \mathcal{E}\}$. An assignment, $\Pi \in \mathbb{B}_{\mathcal{A}, \mathcal{T}}$, is an admissible allocation of a subset of tasks, $\mathcal{T} \subseteq \mathcal{T}$, to a subset of agents, $\mathcal{A} \subseteq \mathcal{A}$, with respect to edge subset, $\mathcal{E} \subseteq \mathcal{E} := \mathcal{A} \times \mathcal{T}$, if all considered tasks in $\mathcal{T}$ are assigned to one agent in the considered subset of agents $\mathcal{A}$ and all these agents are assigned to at most one task. The set of such admissible assignments for subgraph $\mathcal{G} := (\mathcal{A}, \mathcal{T}, \mathcal{E})$ is given by,

$$\mathcal{P}_{\mathcal{A}, \mathcal{T}}(\mathcal{E}) := \{\Pi \in \mathbb{B}_{\mathcal{A}, \mathcal{T}} | \forall j \in \mathcal{T} \sum_{i \in \mathcal{A}} \pi_{i,j} = 1, \forall i \in \mathcal{A} \sum_{j \in \mathcal{T}} \pi_{i,j} \leq 1\}.$$

A. Bottleneck Assignment

The criteria given in (4) specify the assignments among the set admissible assignments for the graph, $\mathcal{G}$, for which the largest assignment weight is minimal. This defines the BAP. The BAP can be solved efficiently in polynomial time, see [5]. Independent of how the BAP is solved, we define operators associated to it in the following.

Definition 2 (Bottleneck assignment): We consider a subgraph of the assignment graph, $\mathcal{G} := (\mathcal{A}, \mathcal{T}, \mathcal{E})$, the assignment weights, $\mathcal{W}$, and an assignment, $\Pi \in \mathbb{B}_{\mathcal{A}, \mathcal{T}}$. We define a function that returns the largest value among the weights corresponding to assigned edges in $\mathcal{E}$,

$$b(\Pi, \mathcal{E}, \mathcal{W}) := \max_{(i,j) \in \mathcal{E}} \pi_{i,j} w_{i,j}.$$ (5)

We also define a function that returns the so-called bottleneck weight of subgraph $\mathcal{G}$,

$$B_{\mathcal{A}, \mathcal{T}}(\mathcal{E}, \mathcal{W}) := \min_{\Pi \in \mathcal{P}_{\mathcal{A}, \mathcal{T}}(\mathcal{E})} b(\Pi, \mathcal{E}, \mathcal{W}),$$ (6)

and a map that returns the set of minimising assignments

$$B_{\mathcal{A}, \mathcal{T}}(\mathcal{E}, \mathcal{W}) := \arg \min_{\Pi \in \mathcal{P}_{\mathcal{A}, \mathcal{T}}(\mathcal{E})} b(\Pi, \mathcal{E}, \mathcal{W}).$$

The set of edges with weight equal to the bottleneck is

$$E_{\mathcal{A}, \mathcal{T}}(\mathcal{E}, \mathcal{W}) := \{(i, j) \in \mathcal{E} | w_{i,j} = B_{\mathcal{A}, \mathcal{T}}(\mathcal{E}, \mathcal{W})\}.$$ (7)

We introduce a robustness margin, related to the concept of critical edge weights in [9], that characterises the next best assignments when a specific edge is discarded.
Definition 3 (Robustness margin): Given assignment weights, $W$, we consider the complete bipartite graph formed from a subset of agents, $A$, and a subset of tasks, $T$, i.e. subgraph $\tilde{G} := (\tilde{A}, \tilde{T}, \tilde{E})$, with edge set $\tilde{E} = \tilde{A} \times \tilde{T}$. For $|\tilde{E}| > 1$, we define the set of so-called maximum-margin bottleneck edges as,

$$e_{\tilde{A}, \tilde{T}}(W) := \arg \max_{(i,j) \in E_{\tilde{A}, \tilde{T}}(\tilde{E}, W)} B_{\tilde{A}, \tilde{T}}(\tilde{E} \setminus \{(i,j)\}, W),$$

and the corresponding robustness margin,

$$r_{\tilde{A}, \tilde{T}}(W) := \max_{(i,j) \in E_{\tilde{A}, \tilde{T}}(\tilde{E}, W)} B_{\tilde{A}, \tilde{T}}(\tilde{E} \setminus \{(i,j)\}, W) - w_{i,j}.$$ 

For $|\tilde{A}| = |\tilde{T}| = |\tilde{E}| = 1$, the maximum-margin bottleneck edge is set to be the singleton edge $e_{\tilde{A}, \tilde{T}}(W) = \tilde{E}$ and the robustness margin is assumed to be infinity, $r_{\tilde{A}, \tilde{T}}(W) = \infty$.

The following proposition links the robustness margin to uniqueness properties of optimal bottleneck assignments. While a strictly positive robustness margin does not imply that there is a unique optimising bottleneck assignment, it does ensure that all optimising assignments have in common that they assign all agent-task pairs in the set of maximum margin bottleneck edges. Based on this property, uniqueness criteria for a particular sequence of assignment problems will be derived in Section III.B.

Proposition 1 (Proof in Appendix I): Given assignment weights, $W$, and the complete bipartite graph, $\tilde{G} = (\tilde{A}, \tilde{T}, \tilde{E})$, formed from a subset of agents, $\tilde{A} \subseteq A$, and a subset of tasks, $\tilde{T} \subseteq T$, with edge set $\tilde{E} = \tilde{A} \times \tilde{T}$, where $|\tilde{E}| > 1$. If the robustness margin is strictly positive, $r_{\tilde{A}, \tilde{T}}(W) > 0$, then all bottleneck optimising assignments for subgraph $\tilde{G}$ are maximum-margin bottleneck edges, i.e., $\pi_{*, *'} = 1, (i^{*}, j^{*}') \in e_{\tilde{A}, \tilde{T}}(W), \forall \Pi = \{\pi_{i,j} \in \{0,1\} | (i,j) \in \tilde{E} \} \in B_{\tilde{A}, \tilde{T}}(\tilde{E}, W)$.

B. Sequential Bottleneck Assignment

The BAP does not fully determine all agent-task pairings. Given the primary assignment criterion in (4), we define a sequence of criteria of decreasing hierarchy.

Definition 4 (Sequential Bottleneck assignment): Given agents, $A$, tasks, $T$, and assignment weights, $W$. An assignment, $\Pi^*$, is sequential bottleneck optimising if it is bottleneck minimising for the assignment graph, $\tilde{G} = (A, T, E)$, and the sequence of subgraphs $\tilde{G}_1, \tilde{G}_2, \ldots, \tilde{G}_n$, where $\tilde{G}_k = (\tilde{A}_k, \tilde{T}_k, \tilde{E}_k)$ is the complete bipartite graph of the subset of agents, $\tilde{A}_k \subseteq A$, and the subset of tasks, $\tilde{T}_k \subseteq T$, of $A \times T$, obtained by removing a maximum-margin bottleneck agent and task from $\tilde{G}_{k-1}$, i.e., $\Pi^* \in S_{A,T}(W)$,

$$S_{A,T}(W) := \{ \Pi \in B_{A,T} \mid \forall k \in \{1, \ldots, n\} \Pi \in B_{\tilde{A}_k, \tilde{T}_k}(\tilde{E}_k, W) \},$$

where $\tilde{E}_k = \tilde{A}_k \times \tilde{T}_k, \tilde{A}_1 = A, \tilde{T}_1 = T, \tilde{A}_k = A \setminus \{i^{*}_1, \ldots, i^{*}_{k-1}\}, \tilde{T}_k = T \setminus \{j^{*}_1, \ldots, j^{*}_{k-1}\},$ (8a) (8b)

with so-called $k$-th order bottleneck edge,

$$(i^{*}_k, j^{*}_k) \in e_{\tilde{A}_k, \tilde{T}_k}(W),$$

and $k$-th order robustness margin,

$$\mu_k = \mu_{\tilde{A}_k, \tilde{T}_k}(W).$$

Given a sequential bottleneck assignment, $\Pi^* \in S_{A,T}(W)$, the set of all assigned agents is defined by the set of bottleneck agents of all orders, $\{i^{*}_1, \ldots, i^{*}_n\} \subseteq A$. For the case where there are more agents than tasks, $m > n$, the set of unassigned agents is given by $A \setminus \{i^{*}_1, \ldots, i^{*}_n\}$. We use the following properties of the robustness margin to derive sufficient conditions for collision avoidance in Section IV.

Proposition 2 (Proof in Appendix II): Given the set of agents, $A$ with $|A| = m > 1$, the set of tasks, $T$ with $|T| = n > 1$, the assignment weights, $W = \{w_{i,j} \geq 0 | (i,j) \in A \times T\}$, and a sequential bottleneck assignment, $\Pi^* \in S_{A,T}(W)$, with $k$-th order bottleneck edges, $(i^{*}_k, j^{*}_k)$ defined in (9), and corresponding robustness margins, $\mu_k$ defined in (10), for $k \in \{1, \ldots, n\}$. We have

$$w_{i^{*}_a, j^{*}_b} + \mu_a \leq w_{i^{*}_a, j^{*}_b} + \mu_a,$$

for all $a \in \{1, \ldots, n-1\}$ and $b \in \{a+1, \ldots, n\}$.

Proposition 3 (Proof in Appendix III): Given the set of agents, $A$ with $|A| = m > 1$, the strictly smaller set of tasks, $\bar{T}$ with $|\bar{T}| = |T| - 1 > 1$, the assignment weights, $\bar{W} = \{w_{i,j} \geq 0 | (i,j) \in A \times \bar{T}\}$, and a sequential bottleneck assignment, $\Pi^* \in S_{A,T}(\bar{W})$, with $k$-th order bottleneck edges, $(i^{*}_k, j^{*}_k)$ defined in (9), and corresponding robustness margins, $\mu_k$ defined in (10), for $k \in \{1, \ldots, n\}$. We have

$$w_{i^{*}_a, j^{*}_b} + \mu_a \leq w_{i^{*}_a, j^{*}_b} + \mu_a,$$

for all $a \in \{1, \ldots, n\}$ and $b \in \{1, \ldots, n\}$.

We note that the $k$-th order bottleneck edge defined in (9) may not be unique. However, if the robustness margin is strictly positive for all orders, it follows from Proposition 1 that all possible $k$-th order bottleneck edges are associated with the same unique set of binary variables for the remaining higher order bottleneck assignments.

Corollary 1: For a sequential bottleneck optimising assignment, $\Pi^* \in S_{A,T}(W)$, if the bottleneck margin is strictly positive, $r_{\tilde{A}_k, \tilde{T}_k}(W) > 0$, for all orders $k \in \{1, \ldots, n\}$, then the sequence of bottleneck weight values, $(w_{i^{*}_1, j^{*}_1}, \ldots, w_{i^{*}_n, j^{*}_n})$, is unique and lexicographically optimal, i.e., for all other assignments, $\Pi' \in P_{A,T}(\tilde{E}) \setminus \{\Pi^*\}$, either $w_{i^{*}_l, j^{*}_l} \leq w_{i^{*}_l, j^{*}_l}$ or there exists an order, $l \in \{2, \ldots, n\}$, for which $w_{i^{*}_l, j^{*}_l} \neq w_{i^{*}_l, j^{*}_l}$, where $(w_{i^{*}_1, j^{*}_1}, \ldots, w_{i^{*}_n, j^{*}_n})$ is a sequence of weights associated to assigned edges according to $\Pi'$ that are ordered with non-increasing value.

We call a sequential bottleneck optimising assignment with strictly positive robustness margins for all orders a robust lexicographical assignment. A robust lexicographical assignment is a special case solution of a LexAP, where the hierarchically nested optimality criteria have a unique optimiser.

Finding the $k$-th order bottleneck and the corresponding robustness margin requires solving two BAPs that can each be computed with a complexity of $O(|\tilde{E}_k||\tilde{T}_k|)$, see [8]. From (8) it follows that $|\tilde{E}_k| = (m - k + 1)(n - k + 1)$ and
Finding the bottleneck edges and robustness margins for all orders \( k \in \{1, \ldots, n\} \) can therefore be achieved with an overall complexity of \( O(mn^3) \).

**Remark 1:** The algorithm introduced in [8] solves the BAP without centralised decision making. Agents only have knowledge of weights associated to them, i.e., agent \( i \in A \) has access to the subset of weights \( W_i := \{w_{i,j} \mid j \in T\} \), and communicates local estimates of maximal and minimal edge weights to other agents. Because solving the sequential bottleneck assignment consist of solving \( 2n \) nested BAPs, it follows that a sequential bottleneck optimising assignment and the corresponding robustness margins can also be obtained with distributed computation.

### C. Assignment Example

Consider the sequential bottleneck assignment of \( n = 3 \) tasks, \( T = \{\tau_i\}_{i=1}^{n} \), to \( m = 4 \) agents, \( A = \{\alpha_i\}_{i=1}^{n} \), with assignment weights, \( W \), illustrated in Fig. 1. The sequence of BAPs is initialised with the full assignment graph, \( G_1 = G = (A, T, E) \), with \( E = A \times T \), as shown in Fig. 1a. The bottleneck weight is \( B_{A,T}(E, W) = 4 \). There are two edges that have weight equal to the bottleneck, \( (i_1^*, j_1^*) = (\alpha_2, \tau_2) = e_{A,T}(W) \). That is, \( B_{A,T}(E \setminus \{(\alpha_1, \tau_1)\}, W) = 4, B_{A,T}(E \setminus \{(\alpha_2, \tau_2)\}, W) = 7 \), and the associated first order robustness margin is therefore \( \mu_1 = r_{A,T}(W) = 3 \). Since the first order bottleneck robustness margin is greater than zero, it follows that all bottleneck optimal assignments allocate \( \alpha_2 \) to \( \tau_2 \). We note that there exist multiple valid assignments with maximum weight equal to the bottleneck weight. Assignment \( \Pi \), corresponding to the task-agent pairings \( \{(\alpha_1, \tau_1), (\alpha_2, \tau_2), (\alpha_4, \tau_3)\} \), and assignment \( \Pi' \), corresponding to the task-agent pairings \( \{(\alpha_1, \tau_3), (\alpha_2, \tau_2), (\alpha_4, \tau_1)\} \), both result in a bottleneck cost of 4, i.e., \( \Pi, \Pi' \in B_{A,T}(E, W) \). These bottleneck optimal assignments differ in the higher order criteria of the sequential bottleneck assignment however.

By removing the first order bottleneck agent and task, the bottleneck edge, and all edges adjacent to them, we obtain the second order subgraph, \( G_2 = (A_{2}, \tilde{T}_2, \tilde{E}_2) \), with \( A_{2} = A \setminus \{\alpha_2\}, \tilde{T}_2 = T \setminus \{\tau_2\}, \tilde{E}_2 = A_{2} \times \tilde{T}_2 \), shown in Fig. 1b. The second order bottleneck edge is \( B_{A_{2}, \tilde{T}_2}(\tilde{E}_2, W) = 2 \). There are two edges with weight equal to the bottleneck, \( E_{A_{2}, \tilde{T}_2}(\tilde{E}_2, W) = \{(\alpha_1, \tau_1), (\alpha_4, \tau_3)\} \), and both are maximum-margin bottlenecks, \( e_{A_{2}, \tilde{T}_2}(W) = E_{A_{2}, \tilde{T}_2}(\tilde{E}_2, W) \), with \( B_{A_{2}, \tilde{T}_2}(\tilde{E}_2 \setminus \{(\alpha_1, \tau_1)\}, W) = B_{A_{2}, \tilde{T}_2}(\tilde{E}_2 \setminus \{(\alpha_4, \tau_3)\}, W) = 4 \). From Proposition 1 it follows that all sequential bottleneck optimal assignments involve the agent-task pairings corresponding to both edges in \( e_{A_{2}, \tilde{T}_2}(W) \). From these two edges, we arbitrarily select the second order bottleneck edge, \( (i_2^*, j_2^*) = (\alpha_4, \tau_3) \), with corresponding robustness margin, \( \mu_2 = r_{A_{2}, \tilde{T}_2}(W) = 2 \). In the final step we consider the subgraph, \( G_3 = (A_3, \tilde{T}_3, \tilde{E}_3) \), with \( A_3 = A_{2} \setminus \{\alpha_4\}, \tilde{T}_3 = \tilde{T}_2 \setminus \{\tau_1\}, \tilde{E}_3 = A_3 \times \tilde{T}_3 \), shown in Fig. 1c. The third order bottleneck weight is, \( B_{A_{3}, \tilde{T}_3}(\tilde{E}_3, W) = 2 \), with unique bottleneck edge, \( (i_3^*, j_3^*) = E_{A_{3}, \tilde{T}_3}(\tilde{E}_3, W) = e_{A_{3}, \tilde{T}_3}(W) = (\alpha_1, \tau_3) \), and robustness margin, \( \mu_3 = r_{A_{3}, \tilde{T}_3}(W) = 6 \).

The resulting assignment \( \Pi^* = \Pi' \in S_{A_{3}, \tilde{T}_3}(W) \) is the unique optimal sequential bottleneck assignment and a robust lexicographic assignment because the robustness margins are strictly positive for all orders, i.e., every other admissible assignment, \( \Pi \in \mathcal{P}_{A,T}(E \setminus \{\Pi'\}) \), has a lexicographical larger weight sequence than \( \{w_{i_1^*, j_1^*}, w_{i_2^*, j_2^*}, w_{i_3^*, j_3^*}\} = (4, 2, 2) \). Furthermore, we observe that \( w_{i_1^*, j_1^*} + \mu_1 < \max\{w_{i_2^*, j_2^*}, w_{i_3^*, j_3^*}\} = 8, w_{i_2^*, j_2^*} + \mu_1 < \max\{w_{i_1^*, j_1^*}, w_{i_3^*, j_3^*}\} = 9, \) and \( w_{i_3^*, j_3^*} + \mu_2 = \max\{w_{i_2^*, j_2^*}, w_{i_3^*, j_3^*}\} = 4 \) in accordance with Proposition 2.

We also see that \( w_{i_1^*, j_1^*} + \mu_1 < w_{i_2^*, j_2^*} + \mu_2 < w_{i_3^*, j_3^*} + \mu_3 = w_{i_3^*, j_3^*} = 8 \) in agreement with Proposition 3.

### IV. Collision Avoidance

In this section we address inter-agent collision avoidance. We first investigate sufficient conditions related to the sequential bottleneck assignment in Section IV-A. Then, in Section IV-B we introduce time-dependent position constraints for the individual agents.

#### A. Sufficient Conditions for Collision Avoidance

We consider the problem setup introduced in Section II. Assumption 2: Given assignment weights, \( W = \{w_{i,j}\} \mid (i,j) \in A \times T \), based on distances between target destinations and initial agent positions, defined in (I). Agents, \( A \), are allocated to targets, \( T \), with a sequential bottleneck optimising assignment, \( \Pi^* \in \mathcal{S}_{A,T}(W) \), as in Definition 2 with \( k \)-th order bottleneck agent-task pair, \( (i_k^*, j_k^*) \), and the \( k \)-th order robustness margin, \( \mu_k \), for all \( k \in \{1, \ldots, n\} \).
Using the safety distances from Definition 1, we provide a first condition which guarantees that an assigned agent does not collide with any other agent at a particular time.

Lemma 1: Given Assumptions 1 and 2, The k-th order bottleneck agent, \( i^*_k \), \( k \in \{1, \ldots, n\} \), does not collide with any other agent, \( i' \in A \setminus \{i^*_k\} \), at time \( t \) if

\[
d(p_i(0), p_i(t)) + d(p_{i^*_k}(t), g_{i^*_k}) < w_{i^*_k} + \mu_k - s_{i^*_k}; \tag{11}
\]

Proof: Considering the triangle inequality in (1), the distance assigned to agent \( i^*_k \) is bounded by

\[
d(p_i(0), p_i(t)) + d(p_{i^*_k}(t), g_{i^*_k}) \leq d(p_i(0), p_{i^*_k}(t)) + d(p_{i^*_k}(t), g_{i^*_k}) \leq d(p_i(0), g_{i^*_k}) + d(p_{i^*_k}(t), g_{i^*_k}).
\]

By applying (11), with \( w_{i^*_k} = d(p_i(0), g_{i^*_k}) \), we obtain \( d(p_i(0), p_{i^*_k}(t)) > s_{i^*_k} \) as required from Definition 1.

For guaranteed collision avoidance among assigned agents we combine the concept of robustness margin with the condition given in Lemma 1.

Proposition 4: Given Assumptions 1 and 2, The k-th order bottleneck agent, \( i^*_k \), \( k \in \{1, \ldots, n-1\} \), does not collide with a higher order bottleneck agent, \( i^*_l \), \( l \in \{k+1, \ldots, n\} \), at time \( t \) if both following conditions are satisfied,

\[
d(p_i(0), p_i(t)) + d(p_{i^*_l}(t), g_{i^*_l}) < w_{i^*_l} + \mu_k - s_{i^*_l}; \tag{12a}
\]

\[
d(p_{i^*_k}(0), p_{i^*_k}(t)) + d(p_i(t), g_{i^*_k}) < w_{i^*_k} + \mu_k - s_{i^*_k}. \tag{12b}
\]

Proof: For an arbitrary order, \( k \in \{1, \ldots, n-1\} \), let \( l \in \{k+1, \ldots, n\} \) be an arbitrary higher order. If \( w_{i^*_l} - d(p_i(0), p_i(t)) + d(p_{i^*_l}(t), g_{i^*_l}) + s_{i^*_l} \) agents \( i^*_k \) and \( i^*_l \) do not collide at time \( t \) as shown in Lemma 1 with agent \( i^*_l \) playing the role of the ‘other agent’.

It remains to consider the case where \( w_{i^*_l} - d(p_i(0), p_i(t)) > d(p_{i^*_l}(t), g_{i^*_l}) + s_{i^*_l} \). Assume for the sake of contradiction that the condition in (11) also does not hold from perspective of agent \( i^*_l \), where agent \( i^*_l \) is the ‘other agent’, i.e., \( w_{i^*_l} - d(p_i(0), p_i(t)) + d(p_{i^*_l}(t), g_{i^*_l}) + s_{i^*_l} \). Then, from (12a) we have \( w_{i^*_l} - d(p_i(0), p_i(t)) + d(p_{i^*_l}(t), g_{i^*_l}) + s_{i^*_l} \) and from (12b) we have \( w_{i^*_l} - d(p_i(0), p_i(t)) + d(p_{i^*_l}(t), g_{i^*_l}) + s_{i^*_l} \). This however contradicts Proposition 2. It follows that \( w_{i^*_l} - d(p_i(0), p_i(t)) + d(p_{i^*_l}(t), g_{i^*_l}) + s_{i^*_l} \). Thus, according to Lemma 1, agents \( i^*_k \) and \( i^*_l \) do not collide at time \( t \).

B. Local Constraints for Guaranteed Collision Avoidance

We now derive individual position constraints for every agent such that if all agents satisfy their associated constraints, collisions involving assigned agents are avoided. The constraints rely on the robustness of the sequential bottleneck assignment as formalised in the following assumption.

Assumption 3: There exists an upper bound on the safety distance, \( s \geq s_i \), between all agents, \( i, i' \in A \), \( i \neq i' \), that is smaller than the k-th order bottleneck robustness margin for all \( k \in \{1, \ldots, n\} \), i.e.,

\[
s < \mu := \min_{k \in \{1, \ldots, n\}} \mu_k. \tag{13}
\]

The agent position constraints are composed of up to two components. The first component is a bound on the distance of an agent from its initial position. This bound is limited by the same time-varying parameter, \( a(t) \), for all agents up to a saturation value, \( A_k \), that is agent dependent.

Assumption 4: The distance of an agent \( i \), to its initial position is bounded by \( a_i(t) \), i.e.,

\[
d(p_i(0), p_i(t)) < a_i(t),
\]

for all \( i \in A \) and \( t \in [0, T] \), with

\[
a_i(t) = \begin{cases} a(t) & \text{if } a(t) \leq A_k, \\ A_k & \text{otherwise}, \end{cases}
\]

for all assigned agents \( i^*_k \), \( k \in \{1, \ldots, n\} \), and \( a_i(t) = a_i^*(t) \), for all unassigned, \( i' \in A \setminus \{i^*_1, \ldots, i^*_n\} \), where

\[
A_k := \min_{l \in \{1, \ldots, k\}} \left( w_{l} + \mu_l + \frac{1}{2}(\mu + s) + 1 \right) \tag{14}
\]

and \( a(t) \geq \frac{1}{2}(\mu - s) \).

The second component of the position constraints applies only to assigned agents. It consists of a bound on the distance of an agent to its target destination that decreases when the bound on the distance from the initial position increases.

Assumption 5: The distance of an assigned agents, \( i^*_k \), to its destination is bounded by \( b_{i^*_k}(t) \), i.e.,

\[
d(p_{i^*_k}(t), g_{i^*_k}) < b_{i^*_k}(t),
\]

for all assigned agents \( i^*_k \), \( k \in \{1, \ldots, n\} \), and \( t \in [0, T] \), with

\[
b_{i^*_k}(t) = A_k - a_{i^*_k}(t) + \frac{1}{2}(\mu - s).
\]

Satisfaction of these local bounds provides a sufficient condition for collision avoidance as shown in the following.

Theorem 1: Given that \( n \) target destinations are allocated to \( m \) agents with a sequential bottleneck optimising assignment, as specified in Assumption 2 with robustness margins satisfying Assumption 3 based on distance weights introduced in Assumption 1.

No assigned agent, \( i \in \{i^*_1, \ldots, i^*_n\} \), collides with any other agent, \( i' \in A \), at any time, \( t \in [0, T] \), if all agents satisfy the position bounds of Assumption 4 and all assigned agents additionally satisfy the position bounds of Assumption 5.

Proof: By construction the bounds in Assumptions 4 and 5 satisfy \( a_i(t) \geq a_i^*(t) \), \( b_i^*(t) \geq b_i(t) \), and \( a_i(t) + b_i(t) \geq \mu_k - s \) for all \( k \in \{1, \ldots, n\} \), \( l \in \{1, \ldots, n\} \), and \( t \in [0, T] \). With Assumption 3 it follows that the conditions in (12) hold for all \( k \in \{1, \ldots, n-1\} \) and \( t \in [0, T] \). Thus, no assigned agents collide with each other. Because of Proposition 3, Lemma 1 is also satisfied for all \( k \in \{1, \ldots, n\} \) and \( t \in [0, T] \). It follows that assigned agents also do not collide with any unassigned agents.
balls intersect for all $t \in [0, T]$ if $\mu > s$ because the sum of the radii is larger than the distance between the centres,

$$a_i^*(t) + b_{j^*}(t) \geq w_{i^*, j^*} + \mu - s > w_{i^*, j^*} = d(p_i^*(0), g_{j^*})$$

It follows that the constraint sets constructed from the bounds in Assumptions 4 and 5 i.e.,

$$\mathcal{L}_{i^*}^k(t) = \{ x \in \mathbb{R}^{n_p} \mid d(p_{i^*}^k(0), x) < a_{i^*}^k(t), \quad \mathcal{L}_{i^*}^k(t) = \{ x \in \mathbb{R}^{n_p} \mid d(x, g_{j^*}) < b_{j^*}(t) \}$$

for all assigned agents $i^*_k$, $k \in \{1, \ldots, n\}$, and

$$\mathcal{L}_i(t) = \{ x \in \mathbb{R}^{n_p} \mid d(p_i^k(0), x) < a_i^k(t) \},$$

for unassigned agents, $i' \in \mathcal{A} \setminus \{i_1^*, \ldots, i_n^*\}$, are all non-empty if Assumption 3 holds.

For any agent, $i \in \mathcal{A}$, the constraint set, $\mathcal{L}_i(t)$, depends on the timing parameter, $a(t)$, the bottleneck weights and robustness margins of the sequential bottleneck assignment, $w_{i^*, j^*}$ and $\mu_k$, respectively, with $k \in \{1, \ldots, n\}$, the agents own initial position, $p_i(0)$, and potentially its assigned target destination, $g_j$, if $\pi_{i^*, j^*} = 1$. The optimising assignment, $\Pi^* \in \mathcal{S}_{A,T}^{\mathcal{A}}(W)$, is determined based on information on distances between tasks and agents. Further information on other agents is not required to compute the individual constraints. These local position constraints can be applied to address the problem statement outlined in Section II.

**Corollary 2:** The agent position constraints given in (15) can be obtained with a complexity $O(mn^3)$ via a sequential bottleneck assignment. The resulting local constraints are a solution for Problem 1 if Assumption 3 holds.

**Remark 2:** The proposed collision avoidance constraints given in (15) can be obtained with distributed computation. Agents only need to coordinate through the shared scheduling variable, $a(t)$, and by exchanging estimates of all the bottleneck distances and robustness margins during the assignment as suggested in Remark 1.

The local agent position constraints proposed in (15) can be incorporated in many different motion control or trajectory planning applications. For instance, because the position bounds provide collision avoidance guarantees without fully specifying position trajectories, they can be included in predictive optimisation based approaches with objective functions that do not consider the coordination among the agents. The resulting optimisation problems may include other additional constraints such as avoidance of collisions with other objects. The bounds in (15) are convex as they are constructed from distance functions. This allows to bypass an extra procedure for convex approximation of a safe region which is typically required in model predictive motion control, see [17] for instance. For specific choices of the applied distance functions, e.g., the 1-norm or the infinity-norm distances, the constraints are linear in the position variables and can be efficiently encoded.

The time-varying position bounds can also be used to verify that motion control strategies derived from simplified assumptions do not result in collisions in practice or in higher fidelity simulations. We investigate a numerical example in the following Section.

### V. Case Study

We consider $m = 8$ mobile robots, $\mathcal{A} = \{\alpha_i\}_{i=1}^8$, operating on a plane. The location of each agent, $i \in \mathcal{A}$, at time $t$ is represented by a point position, $p_i(t) = (x_i(t), y_i(t))$. We assume there are $n = 6$ tasks, $T = \{\tau_i\}_{i=1}^6$, corresponding to the actions of visiting target destinations, $\{g_j \in \mathbb{R}^2, j \in T\}$, that are each to be completed by one agent given the initial positions $\{p_i(0) \in \mathbb{R}^2 | i \in \mathcal{A}\}$. Fig. 2a illustrates the configuration at time $t = 0$. The tasks are allocated to the agents according to a sequential bottleneck assignment, as proposed in Assumption 2 with the distance function in Assumption 1 specified to be the Euclidean distance.

$$d(p, p') = ||p' - p||_2$$

for $p, p' \in \mathbb{R}^2$. The resulting optimising assignment, $\Pi^* \in \mathcal{S}_{A,T}^{\mathcal{A}}(W)$, is described in Table II with the bottleneck edges, defined in (9), and the robustness margins, defined in (10), for all orders $k \in \{1, \ldots, 6\}$. We note that for every order the set of maximum-margin bottleneck edges is a singleton, all robustness margins are strictly positive, and $\Pi^*$ is therefore a robust lexicographical assignment.

**TABLE I:** Sequential bottleneck assignment of target destinations to agents with initial positions and colour coding shown in Fig. 2a.

| Order | Bottleneck edge | Bottleneck weight | Robustness margin | Bound limit | Assignment |
|-------|-----------------|-------------------|-------------------|-------------|------------|
| 1     | $\alpha_5, \alpha_1$ | 87.95m           | 10.78m            | 92.72m      | A1         |
| 2     | $\alpha_4, \alpha_7$ | 78.67m           | 9.99m             | 82.64m      | A2         |
| 3     | $\alpha_1, \alpha_3$ | 73.53m           | 9.02m             | 76.54m      | A3         |
| 4     | $\alpha_2, \alpha_4$ | 64.56m           | 27.82m            | 76.54m      | A4         |
| 5     | $\alpha_3, \alpha_5$ | 60.53m           | 21.30m            | 75.83m      | A5         |
| 6     | $\alpha_7, \alpha_6$ | 59.08m           | 23.38m            | 75.83m      | A6         |

The agents are of varying size but we assume that no agent occupies any space outside of a radius of 1.5m from its position point, i.e., at time $t$ the body of agent, $i \in \mathcal{A}$, lies within a safety circle with diameter $s = 3$m centred at $p_i(t)$. The smallest bottleneck robustness margin of the task assignment, defined in (13), is $\mu = 9.02m$. We note that because $s < \mu$, Assumption 3 is satisfied and the constraints, $\mathcal{L}_i(t) \subset \mathbb{R}^2$ in (15) are non-empty for all agents $i \in \mathcal{A}$. Figs. 2b to 2c shows the time-varying areas that satisfy the position constraints for every agent given the time-dependent coordination parameter,

$$a(t) = v^\text{ref}t + \frac{1}{2}(\mu - s)$$

where $v^\text{ref} = 10$m/s. We observe that for any assigned agent, $\pi_{i^*, k}^\text{ref}$, $k \in \{1, \ldots, 6\}$, there is always at least a distance of 3m between any point in position constraint set, $\mathcal{L}_{i^*}(t)$, and any point in the constraint set of any other agent, $\mathcal{L}_i(t)$, $i' \in \mathcal{A} \setminus \{i^*_k\}$. The only constraint sets that ever intersect are those of the unassigned agents. For the assigned agents, $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_7\}$, the constraints consist of the intersection of both bounds introduced in
Fig. 2: Illustration of time-varying bounds for guaranteed collision avoidance. Based on the sequential bottleneck assignment shown in (a), the positions satisfying the individual constraints, $L_i(t)$, are shown as shaded areas, where different colours are used for every agent $i \in A$, in (b)-(e), with initial positions, $p_i(0)$ [blue dots], positions at the time $t$, $p_i(t)$ [small black dots], surrounded by safety circle with diameter $s$ [red circles], past trajectories, $p_i(t')$, $t' \in [0, t]$ [dashed lines], and with target destinations, $g_j$ [green dots], for all tasks $j \in T$.

Assumptions [4] and [5]. The two bounds overlap by a different amount for every bottleneck agent, $i^*_{k}$, $k \in \{1, \ldots, 6\}$, depending on the difference of the corresponding saturation value, $A_k$ defined in [14], and the bottleneck weight, $w_{i^*_{k}, j}$. We observe from Table II that the saturation values of agents $\alpha_2$ and $\alpha_7$, corresponding to bottleneck orders $k = 4$ and $k = 6$ respectively, are not given by their own associated robustness margins but rather by the value of a lower order robustness margin. That is, the sequence of saturation values, $(A_1, \ldots, A_6)$, is set to be non-increasing in agreement with [14]. The constraints for the unassigned agents, $(\alpha_6, \alpha_8)$, only consist of limits on the distances of the agents from their initial positions that are bound by $A_6 = 75.83$m. If the positions of all agents remain inside the individual constraints at all considered times, it is guaranteed from Theorem I that none of the six assigned agents collide with any agent, including the two unassigned agents.

We consider a scenario where the motion of the robot agents is governed by nonlinear unicycle models. The agents are guided from their initial positions to their assigned destinations with decentralised feedback controllers. The controllers are derived by an optimal control scheme in which the models are linearised around reference straight-line trajectories, with constant reference velocity, $v(t) = v_{\text{ref}}$, and where input constraints are neglected. Figs. 2b to 2e show a sample simulation in which the control scheme is applied to the nonlinear models with input constraints, steering rate disturbances, and where the initial heading angles are not aligned with the targets. The resulting agent position trajectories clearly deviate from the straight-line references. However, because the positions of all agents never leave the local constraint sets, collisions involving the assigned agents are guaranteed not to occur. It is enough to tune or design the controllers for every agent individually such that the deviations from the straight-line paths do not violate the conditions that sufficiently guarantee collision avoidance. This demonstrates how the motion control of the agents can be decoupled while still guaranteeing collision avoidance based on the satisfaction of the local constraints.

VI. CONCLUSIONS

We derived local time-varying position constraints for agents that are assigned to move to different target destinations. We specified conditions for which the constraints guarantee that agents do not collide with each other. The targets are allocated to agents such that the largest distance between an initial position of an agent and the target destination is minimised. There may be agents that are not assigned to any task but all agents are constrained such that they do not collide with any assigned agent. Collisions among unassigned agent are not taken into account in this work. The collision avoidance conditions rely on a sequential bottleneck task assignment and depend only on the distances between agent initial positions and target destinations. The existence of the collision avoidance guarantees is conditioned on a level of robustness in the assignment. We defined a specific notion of a robustness margin and derived a method to quantify it.

The constraints can be constructed with distributed computation with agents accessing only limited information on other agents. The procedure consists of solving multiple coupled BAPs and involves structure which may be exploited for improved computation speed in future work. The proposed constraints and associated existence conditions are sufficient but not necessary for collision avoidance. That is, if there is a large enough robustness margin in the assignment,
the proposed local position constraints are feasible; if the constraints are satisfied for all agents, sufficient collision avoidance conditions are satisfied. In cases where not all conditions within this chain of arguments are satisfied, other strategies may exist to avoid collisions. Incorporating other strategies and investigating alternative local agent constraints, that ensure the collision avoidance conditions derived from the properties of the assignment, motivates further research on this topic. Extending the approach to other assignment objectives, the LAP for instance, provides another interesting future research direction.

APPENDIX I

PROOF OF PROPOSITION 1

From (7) we know that \( w_{i'j'} = B_{A,W}(\tilde{\mathcal{E}}, W) \) for any \( (i', j') \in e_{A,W}(W) \). Given \( r_{A,W}(W) > 0 \), assume for the sake of contradiction that there exists a \( \Pi \in B_{A,W}(\tilde{\mathcal{E}}, W) \) and a \( (i', j') \in e_{A,W}(W) \) with \( \pi_{i',j'} = 0 \). From this assumption and (6) it follows that \( b(\Pi, \tilde{\mathcal{E}}, W) = B_{A,W}(\tilde{\mathcal{E}}, W) \). Because \( \pi_{i',j'} = 0 \) in (5), we have \( b(\Pi, \tilde{\mathcal{E}}, W) = b(\Pi, \tilde{\mathcal{E}} \setminus \{(i', j')\}, W) \). From (6) it follows that \( b(\Pi, \tilde{\mathcal{E}} \setminus \{(i', j')\}, W) \geq B_{A,W}(\tilde{\mathcal{E}} \setminus \{(i', j')\}, W) \) and thus \( B_{A,W}(\tilde{\mathcal{E}} \setminus \{(i', j')\}, W) \leq B_{A,W}(\tilde{\mathcal{E}}, W) \). On the other hand we know that \( B_{A,W}(\tilde{\mathcal{E}} \setminus \{(i', j')\}, W) \geq B_{A,W}(\tilde{\mathcal{E}}, W) \) from (6) because \( \Pi \in P_{A,W}(\tilde{\mathcal{E}} \setminus \{(i', j')\}) \) if \( \pi_{i',j'} = 0 \) \( \in \) \( P_{A,W}(\tilde{\mathcal{E}}, W) \). It follows therefore that \( B_{A,W}(\tilde{\mathcal{E}} \setminus \{(i', j')\}, W) = B_{A,W}(\tilde{\mathcal{E}}, W) \) which contradicts \( r_{A,W}(W) = B_{A,W}(\tilde{\mathcal{E}} \setminus \{(i', j')\}, W) > 0 \).

APPENDIX II

PROOF OF PROPOSITION 2

Proof: For an arbitrary order, \( a \in \{1, \ldots, n-1\} \), and an arbitrary higher order, \( b \in \{a+1, \ldots, n\} \), consider the auxiliary assignment, \( \Pi = \{\pi_{i,j} \in \{0,1\} | (i,j) \in \mathcal{A} \times \mathcal{T} \} \), obtained from \( \Pi^* = \{\pi_{i,j}^* \in \{0,1\} | (i,j) \in \mathcal{A} \times \mathcal{T} \} \) by switching the agent-task pairings of \((i_a, j_a)\) and \((i_b, j_b)\) to \((i_a', j_a')\) and \((i_b', j_b')\), i.e.,

\[
\hat{\pi}_{i,j} = \begin{cases} 
0 & \text{if } (i,j) = (i_a', j_a') \text{ or } (i,j) = (i_b', j_b') \\
1 & \text{if } (i,j) = (i_a, j_a) \text{ or } (i,j) = (i_b, j_b) \\
\pi_{i,j}^* & \text{otherwise}
\end{cases}
\]

We note that \((i_a', j_a'), (i_b', j_b') \in \tilde{\mathcal{E}}_a\) by definition in (8). Because \((i_a', j_a'), (i_b', j_b') \notin \Pi^*\) and \((i_a', j_a'), (i_b', j_b') \notin \Pi^*\) we choose the auxiliary assignment, \( \Pi' = \{\pi_{i,j}^* \in \{0,1\} | (i,j) \in \mathcal{A} \times \mathcal{T} \} \), obtained from \( \Pi' = \{\pi_{i,j}^* \in \{0,1\} | (i,j) \in \mathcal{A} \times \mathcal{T} \} \) by switching the agent assigned to task \( j_a' \) from \( i_a \) to \( i', \) i.e.,

\[
\pi_{i,j}^* = \begin{cases} 
0 & \text{if } (i,j) = (i_a', j_a') \\
1 & \text{if } (i,j) = (i', j_a') \\
\pi_{i,j}^* & \text{otherwise}
\end{cases}
\]

We note that \((i', j_a') \in \tilde{\mathcal{E}}_a\) by definition in (8). Because \((i', j_a') \notin \Pi^*\) and \((i', j_a') \notin \Pi^*\) we choose the auxiliary assignment, \( \Pi' = \{\pi_{i,j}^* \in \{0,1\} | (i,j) \in \mathcal{A} \times \mathcal{T} \} \), obtained from \( \Pi' = \{\pi_{i,j}^* \in \{0,1\} | (i,j) \in \mathcal{A} \times \mathcal{T} \} \) by switching the agent assigned to task \( j_a' \) from \( i_a \) to \( i', \)

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