The processes of nonequilibrium exchange in rotating plasma flows

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Abstract. The mechanisms of energy/momentum exchange in rotating and compressing plasma flows have been discussed. It has been shown that such flows are capable of transforming the energy of different degrees of freedom into the energy of one degree owing to the interaction of the coupled nonlinear radial, axial and azimuthal electron-ion oscillations. These processes may lead to the additional acceleration of the flow in azimuthal or axial direction so they might be instrumental for the creation of space thrusters employing pulse transformations for propulsion.

1. The problem to be analyzed

In the present piece, we shall consider the processes of energy/momentum exchange in rotating two-dimensional, cold plasmas which may come about when the coupled nonlinear electron-ion oscillations are excited in the plasma. Our starting point is a natural statement to the effect that under some conditions, the strong nonequilibrium systems are capable of absorbing and accumulating energy/momentum in one macroscopic degree of freedom from other degrees of freedom (here we call such process as a intermode exchange).

There is some evidence that this assumption stands to reason. For example, everyone knows the phenomenon of energy localization of initial excitations in the chain of one-dimensional nonlinear linked oscillators, so called Fermi-Pasta-Ulam recurrence effect [1]. It was found that under some initial conditions, such a simple dynamic system, instead of manifesting equipartition of energy on all degrees of freedom, shows an unbalanced distribution of energy. Proceeding from this point, one can expect similar behavior for the multidimensional, time-dependent system too [2 - 6]. Owing to different kinds of interactions which may exist in these systems and due to a variety of their initial states, different modes of collective motion are possible. Therefore, unlike one-dimensional systems, time-dependent nonequilibrium states for these systems arising, for example, in the nonlinear stage of a few instabilities will be more versatile in character and then one can expect more favorable conditions

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for the energy/momentum exchange. As result, the energy/momentum from different scales may be transformed into one macroscopic degree of freedom [7, 8, 2].

2. The coupled nonlinear electron-ion oscillations

Such intermode exchange was established under studying the interaction of radial, axial and azimuthal nonlinear waves in the rotating cold plasmas. We have studied a plasma flow rotating around the z-axis and propagating along this axis. It is assumed that at the initial moment of time the plasma fills the cylinder of some radius and unlimited in z-direction. The displacement current is assumed to be fully compensated by the conduction current, so that the temporal dependence of the magnetic field can be represented by an azimuthal electric field. The dynamics of such system is investigated analytically by solving the cold-fluid equations of the electrons and ions together with the Maxwell’s equations.

Depending on the initial state, these flows are capable of transforming the energy of different degrees of freedom into the energy of one degree of freedom. In particular, one can obtain such effect for the coupled azimuthal and radial oscillations when the radial velocity \( v_r \) and the azimuthal velocity \( v_\phi \) for electron and ion fluids are defined by the relations [9, 10]:

\[
v_r = A(t)r\sin(2\phi) + B(t)\cos(2\phi) + u_r(t), \quad v_\phi = B(t)r\sin(2\phi) - A(t)\cos(2\phi) + u_\phi(t)
\]

Here the electron and ion fluids are marked by the indices \( e \) and \( i \), respectively. The corresponding electric field has the form

\[
\vec{E} = [C(t)\cos(2\phi) + D(t)\sin(2\phi) + re_\varepsilon(t)/2]\vec{e}_r + [D(t)\cos(2\phi) - C(t)\sin(2\phi) + re_\varepsilon(t)/2]e_\phi
\]

In this case one can get the transfer of energy to the rotation mode of the flow. The similar nonlinear plasma waves were studied in [11, 12].

Under exciting the coupled radial, axial and azimuthal oscillations when the velocity field is specified by the dependence

\[
v_r = A_j(t)r, \quad v_\phi = C_j(t)r, \quad v_z = B_j(t)z
\]

and the wave electric field is

\[
\vec{E} = re_\varepsilon_j(t)\vec{e}_r + re_\varepsilon_j(t)\vec{e}_\phi + ze_\varepsilon_z(t)\vec{e}_z
\]

we also observed the intermode exchange [13, 14]. It should be noted that the time-dependent amplitudes in (1)-(4) are associated with the radial, azimuthal and axial components of velocity and electric fields.

Figure 1 and figure 2 present the typical behavior for these cases [13]. For the convenience of presentation, here and in all further calculations, all values are presented in dimensionless form: we take the initial electron density \( n_0 \), the radius of plasma flow \( R \) and the inverse electron frequency \( \omega_{pe} \) as the scale of all densities, lengths and times. So all velocities are normalized by \( R\omega_{pe} \) and electric field is normalized by \( 4\pi eRe_0n_0 \). Also in all computations we shall use \( \mu = m_e/m_i = 10^3 \). Comparing figure 1 and figure 2, we see that a small difference in the initial flow leads to very different oscillation patterns. In particular, it is possible to have oscillations occurring only in the azimuthal flow component. We note that the initial disturbance of the basis radial flow has a strong effect on the azimuthal and axial flows. However, initial disturbance of the basis azimuthal flow, even when strong, does not affect the radial and axial flow components. That is, the energy in the azimuthal flow oscillations cannot be converted into the other degrees of freedom, but the energy in the latter can be converted into the azimuthal component.
Figure 1. The evolution of $A_j$, $B_j$ and $C_j$ given by (3) where $j = e, i$ for $A_{e0} = 10^{-3}$.

Figure 2. The evolution of $A_j$, $B_j$ and $C_j$ given by (3) where $j = e, i$ for $A_{e0} = 0$. 
Such behavior is caused by some modulation between the velocity of electron and ion fluids and the electric field (see equations (1) and (2)). Moreover, it was found that the flow asymmetry tend to enhance the axial acceleration of the plasma fluid, and the oscillation energy tend to remain in the azimuthal and axial directions. Such situation is depicted in figure 3 for $V_r(0) = -10^{-2}$, $V_\phi(0) = V_z(0) = 10^{-2}$, $V_{ze}(0) = 1.1 \times 10^{-2}$ [14].

In fact, the oscillation energy in the radial mode tends to be transferred to the other degrees of freedom.

3. Discussion

The present results allow us to speak about the redistribution of energy/momentum between the macroscopic degrees of freedom of the cold plasma flow. They show that the strong nonlinear cylindrical waves can perform nonzero work on the closed curve, as result, the plasma particles will acquire kinetic energy from the waves [15, 9, 10, 13, 14]. In particular, for axisymmetric flows the energy in the radial and axial flow components can be transferred to the rotating component but not vice versa [9, 13]. If we pay no regard to the cylindrical flow symmetry, then there is a connection between radial and axial flows which in the case of the radial compression of the flow leads to the axial acceleration in the asymmetric flow [14, 16]. The symmetrical flows are an exception, often there are asymmetrical flows. So it would be of interest to study how the violation of cylindrical flow symmetry affects the energy transfer among the different flow directions taking into account the plasma inhomogeneity on the nonlinear coupling among the electron and ion flow components and oscillations [17, 18].

Besides, the effect of intermode exchange has been established in the hydrodynamic approach for a cold, neutral flow model of the cylindrical plasma. However, these results suggest that the processes of energy/momentum transfer in the real systems may come about on the same footing. So we may venture a guess that there are some conditions under which the thermal mode might serve as a tank of "internal" energy which can be transformed in one degree of freedom. For example, as a result of this process the energy of chaotic degrees of freedom will be transferred into the rotation mode of the flow. We assume that this peculiarity is the basic property of rotating, strong non-equilibrium plasma flows which differs them from other hydrodynamic system. In order to show this we have to expand our pattern to include the finite electron and ion pressure. In fact, this process is directly opposite to the
Kolmogorov cascade related to the energy transfer from large to small scales where it is dissipated [5]. In this case, unlike the Carnot thermal machine being an evident technical embodiment of the local balance principle, here the mechanism beyond local equilibrium physical paradigm, energy production stems from redistributing the energy in space and time.

In this regard, the connection of present researches with kinetic problem of particle-wave interaction should be noted. The two-dimensional hydrodynamic solutions obtained in [13, 14] are very close in the form to the moments of the distribution function obtained in [19, 13, 20]. Such similarity suggests that these hydrodynamic solutions may also be obtained in the framework of the kinetic formalism extended on the cylindrical geometry to describe the intermode exchange in the rotating plasma flows [19, 18, 20]. This opens up fundamentally new opportunities since more complex problems can be considered further on. In the one-dimensional geometry there is no possibility to get energy and momentum redistribution, one can obtain only phenomena like the Landau damping. However, such opportunities appear in the two-dimensional rotating plasmas. So we should try to explore this partial model and the resulting physical effects in more detail for multi-dimensional cases.

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