GalRotpy\textsuperscript{†}: an educational tool to understand and parametrize the rotation curve and the gravitational potential of disk-like galaxies

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ABSTRACT

GalRotpy is an educational Python-based visual tool, which is useful to understand how is the contribution of each mass components to the gravitational potential of disk-like galaxies and in the corresponding rotation curve. The standard gravitational potential of disk-like galaxies can be built by the contribution of a Miyamoto-Nagai potential model for both bulge/core and thin disk together with an NFW (Navarro-Frenk-White) potential or Burkert (for cored density profile) potential for the Dark Matter halo. The GalRotpy tool can give a first approximation of the galaxy rotation curve using the following schemes: (i) a bulge model, (ii) a thin or thick disk (iii) and an exponential disk model for the stellar component and (iv) a Dark Matter halo. The axisymmetric gravitational potential calculations are based on galpy package. After a brief review of gravitation potential theory, we focus on the construction of the rotation curves of disk-like galaxies. We present two study cases, the rotation curves of NGC6361 and M33 galaxies. According to the results found, GalRotpy is useful to modeling rotation curves and estimate realistic gravitational parameters of galaxies. It can be used for the Milky Way galaxy and other extragalactic sources.

Key words: dark matter, galaxies – disk-like, galaxies – gravitational potential, galaxies – dynamics, galaxies

1 INTRODUCTION

In 1914 Vesto Slipher discovered that spiral galaxies rotate, by detecting inclined absorption lines in nuclear spectra from M31 and Sombrero galaxies (Slipher 1914). Later, Jan Oort in 1932 first found that there must be three times as much mass as is observed in visible light when he studied stellar motions above the galactic plane. This finding prompted him to include undetected components like interstellar medium to explain the missing mass. Similar observations for the external parts of NGC3115 galaxy showed that the mass to light ratio is about two orders of magnitude larger than in the solar neighborhood (Oort 1940) as evidence of no visible matter. The latter is known as the “missing mass” problem; it is, the mass contained in the bright objects of a defined region in space does not correspond to its dynamical mass brought to us by its gravitational interactions.

The mass to light ratio $\Upsilon = M/L$ is a quantity that describes how much the mass is a fraction of the light expressed in solar units ($\Upsilon_\odot = M_\odot/L_\odot$). It has been the main tool for investigating the “missing mass” problem in stellar systems like the Milky Way galaxy, external galaxies, and cluster of galaxies.

It has been raised some explanations concerning the “missing mass” problem. H. Babcock in 1939 found that the rotation curve is approximately flat on the periphery of M31 galaxy instead of the expected Keplerian decrease because of the diminishing in luminosity (predicted by the luminous profile) Babcock (1939). He concluded that the mass-to-light ratio must be not constant in the galactic radius, but it must increase. He suggested two explanations for this phenomenon: the light absorption must increase in external parts of the galaxy, or it is required a modification to the Newtonian dynamics (Sanders 2010).

The findings published by Fritz Zwicky (Zwicky 1933, 1937) suggested the existence of “unseen matter” or dark
matter in his results using the virial theorem applied to the velocities of galaxies in the Coma galaxy cluster. Zwicky measured the radial velocities of the galaxies in the cluster, and thus he estimated the cluster mass and therefore the average galaxy mass. Then comparing this value with the luminosity, he obtained the mass to light ratio for galaxies in the cluster \( \Upsilon = \frac{M}{L} = 500\Upsilon_\odot \) suggesting the major contribution is dark matter in the cluster. Later, in 1970 Vera Rubin first reveals an observational evidence of dark matter in M31 galaxy. She realized the flattened circular velocity in the external regions of the galaxy based on the galaxy rotation curve from 67 HII spectra between a range in the galactic radius of (3-24 kpc) (Rubin et al. 1970).

The rotation curve of disk-like galaxies is the main kinematic observable data that allow studying dynamical properties of its stars and interstellar gas, in addition to structure, evolutionary and formation processes of the galaxy (Sofue et al. 2001). The shape of rotation curves was related to the morphology of spiral galaxies (Rubin et al. 1980) looking for a universal rotation curve depending only on the galaxy luminosity (Persic et al. 1996), and not only in the luminosity but by a multi-parameter family such as morphological type, the shape of the light distribution and other optical properties (Noordermeer et al. 2007).

The mass distribution in a component of a galaxy can be estimated by the assumption that the mass-to-light ratio is constant. Given that the galaxy luminosity is one astrophysical observable, it can be obtained a light profile of a galaxy component and therefore infers its mass distribution. Then it is possible to find the rotation curve for each mass contribution and derive interesting quantities like bulge to disk or bulge to the dark matter halo mass ratios, or equally interesting the radial extension of each mass component. The modeling of a rotation curve using mass decomposition is widely used even in recent studies. It evidences the influence of all the mass contributions in each position, from the nucleus until several decades in galaxy radio.

Following the method of mass decomposition, GalRotpy is intended to visualize the building of a rotation curve of disk-like galaxies in its main mass constituents. A rotation curve can be fitted by changing in real time two or more gravitational potential parameters, to give a first glance of the mass distribution of a galaxy component and therefore infer its mass using GalRotpy code.

This paper was arranged in the following way. In section (2) we summarize the theory of gravitational potential as the foundations of GalRotpy, it is, the functional forms for the gravitational potentials for the galaxy’s mass components (2.2) and its circular velocities (2.3) of the discussed three models: Miyamoto-Nagai for the bulge and thin/thick disk, the exponential disk and the Navarro-Frenk-White or Burkert for the dark matter halo. Section (2.4) is dedicated to the estimation of the parameters of the gravitational potential following the three mass components

2 GRAVITATIONAL POTENTIAL OF DISK-LIKE GALAXIES

In this section we account the main results on the theory of gravitational potential related to different mass components in disk-like galaxies; we also show the equations of circular velocity and the parameters of these potentials to understand the basis to decompose the observed rotation curve of disk galaxies. The potential theory is the fundamental issue to extract, from the rotation curves, the kinematical features and deduce the dynamics of this systems.
2.1 Potential Theory

A galaxy is a system of stars, interstellar gas and dark matter that interact between them fundamentally following Newton’s theory of gravity. The mass distribution of a system may be associated with the different mass components, and it will give us the functional form for the potentials according to the Newton gravity law expressed in differential form by Poisson’s equation (Samurović 2007; Binney & Tremaine 2008):

\[ \nabla^2 \Phi(x) = 4\pi G \rho(x). \]  

(1)

The Poisson’s equation is an elliptical partial differential equation that allows linearity: if \( \Phi_1 \) generates potential \( \Phi_1 \), and \( \Phi_2 \) generates the sum \( \Phi_1 + \Phi_2 \). When we can set \( \Phi(x \to \infty) = 0 \) the solution to (1) is:

\[ \Phi(x) = -G \int \frac{\rho(x')}{|x-x'|} \mathrm{d}^3x'. \]  

(2)

where \( G \) is the gravitational constant, and the integral is taken over all the mass distribution.

As we see before, the equation (1) admits to overlap many mass components and to find the total potential for the galaxy which is necessary to compute the circular velocity. It is widely used to implement GalRotpy tool.

For a deep discussion on potential theory refer to Binney & Tremaine (2008).

2.2 Potentials of a disk-like galaxy

It is a tough task to resolve the Poisson equation for \( \Phi(x) \) given a mass density \( \rho(x) \). However, supported by symmetry considerations and observed luminosity profiles it is possible to simplify the problem and to find a functional form for the mass distribution of a spiral galaxy as in (Miyamoto & Nagai 1975).

As a first approximation to the real dynamics of disk-like galaxies, GalRotpy uses galpy calculations of four axisymmetric gravitational potentials. The mass distribution of a disk galaxy can be decomposed mainly in three masses: Bulge, Disk, and Dark Halo (Sofue 2016). Below we present in context the historical development of these components, the functional forms for its potentials and the importance in the understanding of the rotation curve of disk-like galaxies.

The Miyamoto - Nagai potential: expresses the mass components of a bulge and thin/thick disk of a galaxy. The Miyamoto - Nagai potential is a generalization of the Plummer and Kuzmin potentials.

In 1911, Plummer used an elementary solution of the Lane-Emden equation to finding the gravitational potential of a spherical system. The Lane-Emden equation relates the dimensionless radius \( x = r/d \) with \( \psi = \Psi/\Psi_0 \), a quantity involving the density \( \rho \) (Binney & Tremaine 2008):

\[ \frac{1}{x^2} \frac{d}{dx} \left( x^2 \frac{d \Psi}{dx} \right) = \left\{ \begin{array}{ll} -3 \Psi'' & \text{if } \psi > 0 \\
0 & \text{if } \psi \leq 0 \end{array} \right. \]

with \( d = (4/3\pi G \Psi_0^{-1} c_n)^{-1/2} \) and the boundary conditions \( \Psi(0) = 1, d\Psi/ds|_{s=0} = 0 \); of a polytropic gas spheres, where the pressure \( P \) is proportional to a power of the density \( \rho \) given by the polytropic index \( n \), \( P = K_\rho^{1+1/n} \).

A spherical self-consistent system known as Plummer model corresponds to an exact solution of the Lane-Emden equation, and its boundary conditions, with \( n = 5 \) polytrope according to Schuster’s results in 1883 (Miyamoto & Nagai 1975; Binney & Tremaine 2008):

\[ \Phi_p(R, z) = -\frac{GM}{\sqrt{R^2 + z^2 + b^2}} = -\frac{GM}{\sqrt{R^2 + b^2}} \]

with \( r^2 = R^2 + z^2 \). The Plummer model raised on the discussion about stars distribution in globular clusters. The mass density at different distances from the center of globular clusters is approximately the same and it is supposed this systems come from spherical distributions (Plummer 1911).

Moreover, for a flattened mass distribution of axisymmetric galaxies, Toomre in 1963 found the exact solutions for the Poisson’s equation:

\[ \nabla^2 \Phi(x) = 4\pi G \mu(x) \delta(z), \]

where \( \mu(R) \) and \( \delta(z) \) are the surface density and Dirac’s delta function. The solution is given by Miyamoto & Nagai (1975); Binney & Tremaine (2008):

\[ \Phi_K(R, z) = -\frac{GM}{\sqrt{R^2 + (a + |z|)^2}} \]  

named the Kuzmin model or Toomre’s model 1.

The Miyamoto - Nagai potential is an axisymmetric potential defined in cylindrical coordinates \((R, z)\) by:

\[ \Phi_{MN}(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}} \]  

(3)

where \( a, b, M \) are the length, height scales and mass enclosed at galacto-centric distance \( R \), and these can be used as free parameters to determinate the mass components. The Fig. 2 represents the changes in the rotation curve’s shape by three values of amplitude traced by the quantity \( GM \) (left) representing the mass in the Miyamoto-Nagai model and the height scale \( b \) (right) that determines the flatness of the mass density (better traced by the dimensionless scale ratio \( b/a \)). The model 3 is free of singularities and tends to the Newtonian point mass potential when \( R \) and \( z \) become large (Miyamoto & Nagai 1975).

This potential is a generalization of the Plummer and Kuzmin potentials \( \Phi_p(R, z) \) and \( \Phi_K(R, z) \) respectively. A spherically symmetric potential (i.e. a Plummer model) can be expressed using the scale parameters defined in the Miyamoto-Nagai potential (3) with \( a = 0 \) (Samurović 2007), and with \( b = 0 \) a Kuzmin model. For the latter case, the rotation curve is shown in Fig. 2 (right) getting smaller values of \( b \) such that the circular velocity have a lower rise in the central few kpc region.
The Miyamoto-Nagai potential describes continuously both a disk and a bulge geometries including Plumer and Kuzmin potentials without superposing them (Miyamoto & Nagai 1975). This potential is implemented in galpy and is used by GalRotpy to define the parameters $a, b, M$ in galaxies with a bulge and a thin or thick disk. The input parameters in GalRotpy can be fitted directly to the observed rotation curve.

**Razor thin exponential disk potential:** an axisymmetric thin disk can be considered a very flat spheroid. Then in the following we show the gravitational potential of a infinitely thin spheroid assuming some key physical and geometric conditions.

The volume of a oblate spheroid of equatorial and polar radius respectively $a, c$ is $V = 4\pi a^2 c/3$ and its density is $\rho$ then the total mass between the radius $a$ is $M(a) = \frac{4}{3}\pi pqa^3$ where the density is constant.

According to the Fig. (3) the distance along the polar axis allow us to express the enclosed mass in a radius $R$ for a radius respectively $a, R$.

Then, calculating the mass and surface density differentials by a variation in the distance $a$ it is obtained the respective flattened homocoid quantities:

$$\delta M(a) = 2\pi \Sigma_0 a \delta a; \quad \delta \Sigma(a, R) = \frac{\Sigma_0 \delta a}{\sqrt{a^2 - R^2}}$$

with the central density $\Sigma_0 = 2\rho a$ at fixed $a$. Now letting the galacto-centric radius $R$ as a parameter for the surface density $\Sigma_0(a)$ depending only of $a$, that satisfies (Binney & Tremaine 2008):

$$\Sigma(R) = \int_R^\infty da \frac{\Sigma_0(a)}{\sqrt{a^2 - R^2}}$$

with $\Sigma(R)$ an arbitrary observed surface density model.

The above equation have solution given by the Abel integral equation:

$$\Sigma_0(a) = -\frac{2}{\pi} \frac{d}{da} \int_a^\infty dR \frac{R\Sigma(R)}{\sqrt{R^2 - a^2}}$$

Now, following the method of separate the Laplace’s equation in cylindrical coordinates and use the Gauss’s Theorem to obtain a particular $\Sigma, \Phi$ pair as is detailed in (Cuddeback 1993):

$$\Sigma_k(R) = \frac{k}{2\pi} J_0(kR)$$

$$\Phi_k(R, z) = -G J_0(kR) \exp(-k|z|),$$

where $J_0$ is a cylindrical Bessel function; the resulting disk potential is a superposition of the above components:

$$\Phi(R, z) = -2\pi G \int_0^\infty \int_0^\infty \Sigma(R')J_0(kR')R'J_0(kR)\exp(-|z|)dR'dk.$$
it can be constructed a razor-thin disk potential like an infinitely flattened homocedoid:

\[ \delta \Phi = -2\pi G\Sigma_0 \delta \sin^{-1}\left(\frac{2a}{\sqrt{z^2 + (a + R)^2 + \sqrt{z^2 + (a - R)^2}}}ight). \]

then the potential for axisymmetric disks with surface density \( \Sigma(R) \), decomposed in homocedoids using the equations (4, 5) can be expressed by

\[ \Phi(R, z) = 4G \int_0^\infty d\alpha \sin^{-1}\left(\frac{2a}{\sqrt{\alpha^2 + z^2}}\right) \int_a^\infty \frac{dR'}{\sqrt{R^2 - \alpha^2}} \frac{R'S(R')}{\sqrt{R^2 - \alpha^2}} \]

Finally, integrating it in \( a \), the potential for an axisymmetric disk in the plane (when \( z \rightarrow 0 \)) is:

\[ \Phi(R, 0) = -4G \int_0^\infty \frac{da}{\sqrt{R^2 - a^2}} \int_a^\infty \frac{dR'}{\sqrt{R^2 - a^2}} \frac{R'S(R')}{\sqrt{R^2 - a^2}} \]  

According to Freeman (1970), it can be assumed that mass/luminosity ratio is approximately uniform, at least within the disk of a galaxy and then the disk surface density profile is given by the form:

\[ \Sigma_d(R) = \Sigma_{dc} \exp(-R/R_d) \]  

known as the exponential disk, where \( \Sigma_{dc} \) and \( R_d \) are the central surface mass density and radial scale respectively. The Fig. 4 shows the change in rotation curve shape by the changes in the potential parameters of equation 7. A change in surface density \( \Sigma_{dc} \) represents from a very flat circular velocity until a thick disk (left) and lower values in radius scale parameter \( h_t \) allow to reproduce a thin disk.

Replacing the surface density (7), in the second integral of the equation (6), it is obtained:

\[ \int_a^\infty dR' \frac{R'\Sigma_0 e^{-R'/R_d}}{\sqrt{R^2 - a^2}} = \Sigma_0 a K_1(a/R_d) \]

and the potential in the equatorial plane in terms of the modified Bessel functions \( K_0, K_1 \) and \( h, l \), takes the form:

\[ \Phi_{ED}(R, z) = -\pi G\Sigma_0 R (I_0(y)K_1(y) - I_1(y)K_0(y)) \]  

where \( y = R/2R_d \). For further discussion see Freeman (1970); Binney & Tremaine (2008); Cuddeford (1993).

For this model the total disk mass is given by Freeman (1970):

\[ M_d = 2\pi a^2 \Sigma_{dc}. \]  

that depends only on the two free parameters, the scale radius \( a \) and the central surface density \( \Sigma_{dc} \).

A complementary method is given by the definition of the galaxy luminosity by unit of area or “surface brightness” (Binney & Tremaine 2008):

\[ I = \int_0^\infty dr j(r) \]

with \( j(r) \) the distribution of luminosity density. Given the notation of Fig. 3 in cylindrical coordinates the projection of a spherical body where \( a = r \) and the LOS is along the \( z \) coordinate, results in the surface brightness:

\[ I(R) = 2 \int_0^\infty dz j(r) \]

and with the geometric constraint \( r^2 = R^2 + z^2 \) then \( r = \sqrt{R^2 - z^2} \) and therefore \( dz = rdr/z \). The above equation results in

\[ I(R) = 2 \int_0^R \frac{j(x)dx}{\sqrt{R^2 - x^2}} \]

that can be inverted in a direct way using the Abel integral identity:

\[ j(r) = \frac{1}{\pi} \int_r^\infty \frac{dl}{\sqrt{R^2 - l^2}} \frac{dR}{\sqrt{R^2 - r^2}} \]

Under the assumption of some mass to light ratio, the volumetric mass density \( \rho(r) \) is proportional to the brightness function \( j(r) \) by

\[ \rho(r) = \frac{M}{L} j(r). \]

Then, starting from the 2D surface density \( I(R) \) it can be found the 3D luminosity density \( j(r) \) and if the light traces the mass it can be derived the mass density of the system.

The Navarro-Frenk-White potential: collisionless N-body numerical simulations of the clustering of dark matter particles suggest that the mass density within a dark halo, has similar structure to a power density model and an universal scale behavior. It is interesting to see the similarity between the luminosity profile in elliptical galaxies (Binney...
The mass density is given by a two-power law:
\[ \rho(r) = \frac{\rho_0}{(r/a)^\alpha (1 + r/a)^\beta}. \] (10)

In particular \((\alpha, \beta) = (1, 3)\) is called Navarro-Frenk-White (NFW) (Navarro et al. 1997) model. This model have two free parameters: the scale radius \(a\) and the representative density \(\rho_0\). It has also two correlated parameters: the halo mass \(M_a(R)\) and its characteristic (dimensionless) density \(\delta_0\) (Navarro et al. 1997). Finally the NFW model density is:
\[ \rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}, \]
where \(\rho_c = 3H_0^2/8\pi G\) is the halo critical mass density.

And the potential is:
\[ \Phi_{NFW}(r) = -4\pi G \rho_0 a^2 \frac{\ln(1 + r/a)}{r/a}. \] (11)

The enclosed mass within radius \(R\) is (from Jimenez et al. (2003)):
\[ M_a(R) = 4\pi \rho_0 a^3 \left[ \ln \left(1 + \frac{R}{a}\right) - \frac{R/a}{1 + R/a} \right]. \] (12)

Alternatively the numerical simulations and the theory of gravitational collapse in the expanding universe showed the importance of defining the quantity called concentration dimensionless parameter defined as Bosch et al. (2010):
\[ X_{200} = r_{200}/a \] (13)
where:
\[ M_{200} = 200\rho_c \frac{4\pi}{3} r_{200}^3 \] (14)
is the critic mass of the dark matter halo, and \(r_{200}\) is the critic distance from the center of the halo, in which the mean density is 200 times the cosmological critical density \(\rho_c\).

Using the above set of equations it can be found the relation:
\[ \ln(1 + X_{200}) - X_{200} \frac{1}{1 + X_{200}} = 200 \rho_c \frac{r_{200}^2}{3\rho_0}, \] (15)
and solving (15) for \(X_{200}\) by successive approximation (Sofue 2016), the parameters \(M_{200}, r_{200}\) can be obtained from \(\rho_0\) and \(a\) by fitting the mass halo component on the rotation curve. As it can be seen in Fig. 5, an increment in the quantity \(GM\) results in greater amplitude and any change in scale parameter \(a\) gives a fast steep or larger coverage in the galactocentric distance of the galaxy. \texttt{GalRotpy} works directly over the scale \(a\) and the routines in \texttt{galpy} provides the concentration parameter. See section (5.2).

\textbf{The Burkert density profile:} some dwarf galaxies are completely dominated by DM (Dark Matter), and the density profiles are according to central density given by the modified isothermal law (16). It satisfies the outer rotation curve constant value because it falls proportional to \(r^{-2}\) (Burkert 1995):
\[ \rho(r) = \frac{\rho_0}{1 + r^2/r_c^2}, \] (16)
where \(r_c\) is the core radius and \(\rho_0\) is the central dark matter density.

Nevertheless cosmological simulations, the Cold Dark Matter (CDM) scenario predict halos with central density...
Figure 5. Rotation curve using a Navarro-Frenk-White potential with three values of the amplitude $GM$ (top) and the radius scale parameter $a$ (bottom).
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Rotation curve using a Burkert potential with three values of the central core density $\rho_c$ (top) and the radius scale parameter $r_c$ (bottom).

**NFW potential** (Bovy 2015):

$$v_{NFW}(R, z) = R \sqrt{\frac{4\pi G a^2 \rho_0}{r^2(r+a)} - 4\pi G a^3 \rho_0 \frac{\log(r/a) + 1}{r^3/2}}$$  \hspace{1cm} (23)

**Burkert potential** :

$$v_{BK} = \sqrt{\frac{4G M_0}{r} \left[ \ln \left( \frac{1 + r}{r_0} \right) - r g^{-1} \left( \frac{r}{r_0} \right) + \frac{1}{2} \ln \left( 1 + \left( \frac{r}{r_0} \right)^2 \right) \right]}$$  \hspace{1cm} (24)

where $M_0$ is the mass within the core.

**Table 1.** Mass components for The Milky Way galaxy with four Miyamoto-Nagai models.

| Mass component | $M \times 10^{11} M_\odot$ | $a$ (kpc) | $b$ (kpc) |
|----------------|-----------------------------|-----------|-----------|
| Core           | 0.05                        | 0.00      | 0.12      |
| Bulge          | 0.10                        | 0.00      | 0.75      |
| Disk           | 1.60                        | 6.00      | 0.50      |
| Halo           | 3.00                        | 15.00     | 15.00     |

2.4 Spiral galaxies potential parameters

In this section there are presented the mass decomposition using the three mass contributions to represent the observed rotation curve of disk-like galaxies. It could be used two Miyamoto-Nagai models (Sofue 1996) for Bulge and Disk, and a NFW model for a cuspy profile density of Dark Matter Halo.

In Sofue (1996) it was found the following values for The Milky Way galaxy using four Miyamoto-Nagai models (see Table 1):

A system with a massive, centrally concentrated thick disk is modeled by Pouliasis (2016) for the Milky Way galaxy potential, where the thick disk of a Miyamoto-Nagai potential have the parameters for two models, I and II, (see Table 2):

I : $\Phi_{tot}(R, z) = \Phi_{bulge}(R, z) + \Phi_{thin}(R, z) + \Phi_{thick}(R, z) + \Phi_{halo}(R)$

II : $\Phi_{tot}(R, z) = \Phi_{thin}(R, z) + \Phi_{thick}(R, z) + \Phi_{halo}(R)$

In Sofue (2016) it was fitted parameters of a de Vaucouleurs Bulge, an exponential Disk and a NFW dark halo, for more than one hundred of spiral galaxies according to Table 3.

To represent cored DM profile density is used the Burkert model. It has been found (Table 4) by Karukes et al. (2017) the best results for modeling Universal Rotation Curve mass components in dwarf galaxies using an exponential disk, a gaseous disk and a Burkert potential.

These potential parameters values are not in galpy but these give us a guess of the range in which it must be defined for a typical disk-like and dwarf galaxies.
Table 2. Parameters for the two models of the Milky Way galaxy (taken from Pouliasis (2016)). Masses are in units of \((2.32 \times 10^7 \text{M}_\odot)\).

| Component | Parameters | Model I | Model II |
|-----------|------------|---------|----------|
| Bulge     | M          | 460     | –        |
|           | b          | 0.3000  | –        |
| Thin Disk | M          | 1700.0  | 1600.0   |
|           | a          | 5.3000  | 4.8000   |
|           | b          | 0.25    | 0.25     |
| Thick Disk| M          | 1700.0  | 1700.0   |
|           | a          | 2.6     | 2.0      |
|           | b          | 0.8     | 0.8      |
| Halo      | M          | 6000.0  | 9000.0   |
|           | a          | 14.0    | 14.0     |

Table 3. Potential parameters for a de Vaucouleurs Bulge, an exponential Disk and a NFW dark halo. Masses are in units of \((\times 10^{11} \text{M}_\odot)\).

| Mass component | M\(^*\) (\text{M}_\odot) | a (kpc) | M\(_{200}\) (\text{M}_\odot) | R\(_{200}\) (kpc) |
|----------------|--------------------------|---------|-----------------------------|-----------------|
| Bulge          | 0.23                     | 1.5     | –                           | –               |
| Disk           | 0.57                     | 3.3     | –                           | –               |
| Dark Halo      | 2.23                     | 21.6    | 12.76                       | 193.7           |

Table 4. Mass parameters of dwarf galaxies universal rotation curve in Karukes et al. (2017)

| Mass component | \(\log(\rho)\) (\text{M}_\odot/ \text{kpc}^{-3}) | \(\eta_0\) (kpc) |
|----------------|--------------------------|-----------------|
| Dark Matter Halo | 7.55 \pm 0.04 | 2.30 \pm 0.13 |

3 GALPY PACKAGE

galpy\(^{1}\) is a python package that contains a set of tools for galactic dynamics including gravitational potentials and its derived quantities: mass density, circular velocity, total mass, etc. galpy performs numerical orbit integration with a variety of Runge–Kutta-type and symplectic integrators; and it supports the calculation of action-angle coordinates and orbital frequencies for spherical potentials. It includes a number of distribution functions (DF) also like two-dimensional axisymmetric and non-axisymmetric disk DFs, a three-dimensional disk DF, and a DF framework for tidal streams (Bovy 2015). Finally, galpy provides support for a consistent and convenient units system usage and a powerful graphical interface that allows a direct analysis for the physical problem.

3.1 Defining composed potential

The following is an example of using the MiyamotoNagaiPotential function for defining a Miyamoto-Nagai potential with galpy. A similar code is used for the disk potential and the result are shown in Figure 1:

```python
# Some general parameters
r_0 = 8*\text{units.kpc}
v_0 = 220*\text{units.km/units.s}

# Initial parameters for Bulge
a1=0.*\text{units.kpc}
b1=0.495*\text{units.kpc}
amp1=460*2.32e7*\text{units.Msun}

# The Bulge potential
MN_Bulge= MiyamotoNagaiPotential(amp=amp1, a=a1, b=b1, normalize=False, ro=r_0, vo=v_0)
```

3.2 Defining a rotation curve

The circular velocity function calcRotcurve is defined with a galactocentric radius \(R\), it is an array in \(\text{kpc}\). Then, making \text{phi=None} for an axisymmetric potential it is obtained the composed rotation curve in Figure 2:

```python
calcRotcurve([MN_Bulge, MN_Thin_Disk, NFW_Halo], R_array, phi=None)
```
4 GALROTOPY

GalRotpy requires an initial guess for the potential parameters of the galaxy in study by means a file named \texttt{input\_params.txt} that must contain a central value for the mass (in $M_\odot$), galactocentric distance and height scales (in kpc) and an associated threshold, for each mass component. The threshold must be dex of the mass estimate and \% for the size scales.

There are required the following three main \texttt{python} packages to run GalRotpy: \texttt{matplotlib} to generate interface and plots, \texttt{astropy} and \texttt{galpy}. For a detailed description about GalRotpy, see the repository\footnote{https://github.com/andresGranadosC/GalRotpy.git} page where is available the requirements and instructions for its use.

In the top of the left panel is shown a check list to select the potentials to include in the curve fitting. The mass contribution and scale parameters are the red indicators of the central guess value according to Figure (9).

The rotation curve must be a file named: \texttt{rot\_curve.txt} containing three columns with kpc units for the radial coordinate and $\text{kms}^{-1}$ for the velocity and its uncertainty.

The sliders in the left panel allows to calculate, in real time, the circular velocity in correspondence to the selected potentials shown in different colors and dashed lines. The black solid line is the resulting rotation curve. The following potential parameters are based in these used for the Milky Way galaxy and for dwarf galaxies. It can be taken by default.

4.1 Setting input potential parameters

In GalRotpy are defined a standard mass components for each potential as:

\begin{align}
0.01 < M < 0.1 & \ (\times 10^{11} M_\odot) \\
0.0 < a < 0.5 & \ \text{in kpc}
\end{align}

\begin{align}
0.1 < M < 0.5 & \ (\times 10^{11} M_\odot) \\
0.01 < a < 0.05 & \ \text{in kpc} \\
0.5 < b < 1.5 & \ \text{in kpc}
\end{align}
galaxy type (Sab edge-on) 2 each galaxy of the CALIFA sample (García et al. 2015). lines, stellar populations, and others physics features of kinematic properties from emission and from absorption and spectroscopy. The data cubes have information about

5.1 NGC6361 test case
To get the rotation curve of NGC6361, we first make a selection of some galaxies from CALIFA Survey, which provides data cubes of more than 600 galaxies of local universe with 0.005 < z < 0.03. CALIFA Survey uses Integral Field Spectroscopy (IFS) to integrate the properties of image and spectroscopy. The data cubes have information about kinematic properties from emission and from absorption lines, stellar populations, and others physics features of each galaxy of the CALIFA sample (García et al. 2015). We then selected the NGC6361 galaxy which is an spiral galaxy type (Sab edge-on) 2 and there is not presence of 

Figure 10. Gas velocity field (color bar indicates the velocity in km/s) of the NGC6361 galaxy (dataprocess was provided by Sánchez et al). X and Y axis indicates right ascencion and declination. White cross marks are taken as the circular velocity bar in it. After that, we obtained from CALIFA survey the dataprocess of NGC6361, which was derived from PIPE3D, a technique that was implemented by Sánchez et al. (2016). Based on the datacube of NGC6361 and velocity map for Hα emission line provided by Sánchez et al, we get the image of the Figure 10).

In the following example, a rotation curve of NGC6361 galaxy is shown. The points over the kinematic center and maximum velocities masked as white crosses are taken over the radial coordinate on the gas velocity field (see Figure 10). The observed velocity of a point in the galaxy plane as is seen projected in two dimensions is given by Beckman et al. (2004):

\[ V_{\text{obs}} = V_{\text{sys}} + V_\theta \sin(i) \cos(\theta) + V_\phi \sin(i) \cos(\theta) \]

where \( R_\theta \) are the polar coordinates of a point in the plane galaxy and \( i \) is the inclination of the galaxy relative to the observer. Then, with the assumptions that: it’s a disk galaxy, neglecting the radial and out the plane movements, and subtracting the systemic velocity of the galaxy \( V_{\text{sys}} \) (given by spectroscopic redshift), the above equation can be expressed as:

\[ V_r = V_{\text{obs}} - V_{\text{sys}} = V_\theta \sin(i) \]

tracking the circular velocity \( V_\theta \) (with \( \theta = 0 \)). Then a first approximation can be obtained by the marks on the velocity field, (see Figure 10) and correcting by the inclination \( i \) from photometric measurements. Then:

\[ V_\theta = \frac{V_r}{\sin(i)} \]  (32)

In this case we took the assumption that the gas velocity follows approximately the galaxy potential like the stars velocity field.

Figure 11 shows the main \texttt{GalRotpy} interface where a
5.2 M33 test case

In López Fune et al. (2017) is used a NFW model to fit the rotation curve of M33 nearby galaxy to derive the radial distribution of dark matter density in the halo. M33 is a spiral galaxy type (SA(s)cd, Face -on) and there is not presence of bar. It was found a best fitting values for the potential parameters for the stellar disk mass and scale radius $M_*$ and $h_\sigma$ respectively. And for the quantities $c$, $M_{NFW}$ of cosmological importance derived from the NFW model.

Based on the published rotation curve for M33 galaxy from Corbelli et al. (2014), we fitted it using GalRotpy and we found in a first approximation, the mass components and its gravitational potential parameters in NFW and Burkert scenarios for dark matter halo component.

NFW model: we found that it was required a NFW model for the dark halo and a exponential disk to reproduce the M33 rotation curve without a bulge or central core. Following, there are listed the potential parameters, i.e, the stellar mass given by equation (9) with the corresponding $\Sigma_\star$ and $h_\sigma$, the Halo virial mass $M_{NFW}$ and the concentration parameter $c$ for this galaxy:

$$\Sigma_\star = 3.2 \times 10^2 M_\odot \text{ pc}^{-2}$$
$$h_\sigma = 1.93 \text{ kpc}$$
$$M_\star = 7.5 \times 10^9 M_\odot$$
$$M_{NFW} = 5.2 \times 10^{11} M_\odot$$
$$c = 1.2 \times 10^1$$

The mass components for NFW model reported by López Fune et al. (2017) are the following:

$$M_\star = 4.9 \times 10^9 M_\odot$$
$$h_\sigma = 2.2 \text{ kpc}$$
$$M_{NFW} = (5.6 \pm 0.6) \times 10^{11} M_\odot$$
$$c = (9.5 \pm 0.7)$$

Burkert model:

$$\Sigma_\star = 4.1 \times 10^2 M_\odot \text{ pc}^{-2}$$
$$h_\sigma = 1.58 \text{ kpc}$$
$$M_\star = 6.6 \times 10^9 M_\odot$$
$$\rho_{BK} = 2.1 \times 10^7 M_\odot \text{ kpc}^{-3}$$
$$M_{BK}(22.7 \text{kpc}) = 7.2 \times 10^9 M_\odot$$
$$r_0 = 7.5 \text{ kpc}$$

Finally, the mass components for Burkert density profile reported by López Fune et al. (2017) are the following:

$$\rho_{BK} = (12.3 \pm 1.0) \times 10^8 M_\odot \text{ kpc}^{-3}$$
$$M_{BK}(23 \text{kpc}) = (6.7 \pm 1.2) \times 10^{10} M_\odot$$
$$r_0 = (9.6 \pm 0.5) \text{ kpc}$$

Our first approximation for the fitting of the observed rotation curve in Fig. 12 (top) are according to the qualitative characteristics of the composed rotation curve published by López Fune et al. (2017). It is, the proposed Dark Matter models dominates the far extreme beyond 5 kpc approximately while the exponential disk dominates the inner region ($R < 5 \text{ kpc}$).

The Dark Matter halo cored profile, represented by Burkert potential shows its most contribution in external parts of the rotation curve that the cusp profile using NFW potential. So, the exponential disk is more massive in the inner region using the Burkert profile than the NFW one in a similar way the rotation curves in López Fune et al. (2017).

6 DISCUSSION

GalRotpy is a tool for the real time composition of rotation curves of disk-like galaxies in a simple and powerful method. This method gives a approximation to the dynamics of stellar systems and its gravitational global features. Thus, GalRotpy allow to:

- check the presence of an assumed mass type component in a observed rotation curve, by including or removing quickly a mass model;
- determine quantitatively the main mass contribution in a galaxy by means of its mass ratios between pairs of mass components included in GalRotpy. Specially the bulge to disk and the disk to dark matter halo ratios are relevant;
- to bound the extension of each mass component given its radial and height scales. The gravitational potential parameters as result of the fitting of the rotation curve reveals the mass distribution in the galaxy;
- evidence the influence of the main components of the galaxy in whole of the gravitating stellar system.

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3 [http://simbad.u-strasbg.fr/simbad/sim-basicIdent=m33&submit=SIMBAD+search]
Figure 12. A rotation curve with potential parameters found for M33 galaxy, using a NFW (top) and a Burkert (bottom) models for dark matter halo. The rotation curve was taken from Corbelli et al. (2014).

We presented the results of use GalRotpy tool in determining the gravitational parameters of two disk like galaxies as a first stage to be continued with the study of the associated uncertainties of the circular velocity and how are the variations of its fitting parameters.

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