Numerically-analytical solution of problem gaming confrontation hardware-redundant dynamic system with the enemy operating in conditions of incomplete information about the behavior of participants in the game

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Numerically-analytical solution of problem gaming confrontation hardware-redundant dynamic system with the enemy operating in conditions of incomplete information about the behavior of participants in the game

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Abstract. The mathematical model of a game task in which conflicting parties act and make decisions in conditions of incomplete information about the behavior of participants in the game. The attacked side has a hardware-redundant dynamic system, which is not only a resource of protection from enemy attacks, but also the resources of the active influence on the probability of finding an attacker in the appropriate States of the attack. It is believed that the behavior of warring parties approximarely Markov process. The solution of the considered differential game is reduced to a multi-step matrix game and its sequential solution at intervals of discretization with constant average probabilities of finding the attacker on these intervals. When solving a problem use numerical and analytical methods.

1. Introduction

Statement of the problem and the solution of game problems of confrontation of different nature systems in conflict situations, many papers [1-3], the closest of which to the contents of this article [10], devoted to the issues of formulating and solving the game problem of a technical system of confrontation with the enemy attacking in force under conditions of incomplete information in the course of the game.

In [10] has set and resolved the task's antagonism attacked, defending against attacks from its own resources, hardware and excessive dynamic system \( S_A(n, m, \bar{s}) \), with the attacking enemy force in the conditions of incomplete information about the behavior of the enemy attacked during the conflict. The attacking side in the game aims at the expense of resources of the attacks increase the failure rat \( \lambda_0(t), \lambda_1(t), \ldots, \lambda_q(t) \) primary and backup units attacked \( S_A(n, m, \bar{s}) \) - System up to its complete rejection.

The attacked party (player \( A \)) by appropriate strategy \( W^A = \{ \bar{r}, s(\bar{r}) \} \) redistribution \( m(m = s_1 + s_2 + \cdots + s_q) \), broken spare blocks on \( q \) corresponding groups between the number of the failed main functional \( n(n = n_1 + n_2 + \cdots + n_q) \), blocks hardware redundant system at appropriate times \( \tau \) during the playing time \([0, t_f]\), aims to maximize its probability of failure-free operation \( P(t_f) = \max P(t_f, \bar{s}(\tau_k)) \) by the end of the game (confrontation) with the attacking enemy.
The above notation corresponding notation in [10], have the following meanings: \( \mathbf{s} = (s_1 + s_2 + \ldots + s_q) \) - integral vector redundancy; \( \lambda_0(t) \) - the failure rate of reserve units; \( \lambda_1(t), \lambda_2(t), \ldots, \lambda_q(t) \) - the failure rate corresponding basic unit; \( \mathbf{r} = \{\tau_0, \tau_1, \ldots, \tau_L\} \) - adjustment vector (redistribution spare blocks between the failed basic) attacked \( S_A(n, m, s) \) - systems whose elements correspond torque redistributing spare blocks, wherein \( \tau_0 = 0, \tau_L < t_f \), where \( t_f \) - the end of the game; \( \mathbf{s}(\tau_k) \) - vector distribution of reserve units at a time \( \tau_k (0 \leq k \leq L) \).

In [10] It is also believed that the attacker system \( S_A(n, m, s) \), opponent (player \( B \)). During the game it can be located in one of the states \( B_1, B_2, \ldots, B_N \), characterized by the corresponding result of the attack resource attacks on hardware-redundant dynamic system as a vector of the system failure rates \( \mathbf{\lambda}(t) = \{\mathbf{\lambda}(t)\} \), each element of which represents the totality of intensities in failure \( q \) groups major functional blocks and not included in job spare blocks targeted system \( \mathbf{\lambda}^i(t) = \{\lambda_0^i(t), \lambda_1^i(t), \ldots, \lambda_q^i(t)\} \), \( i = 1, 2, \ldots, N \). In this case, if for each state \( B_i \) attacking enemy given the probability of its presence in these states \( Q(t) = \{Q_i(t)\} \), the set of policies attacking the opponent (Player \( B \)) is defined as \( W^B = (Q(t), \mathbf{\lambda}(t)) \). Thus, the player's actions in the process of the game is a random selection of \( N \) states, which correspond to intensity non-stationary poison flows bounce primary and backup units \( S_A(n, m, s) \), system in order to minimize the probability of failure-free operation of the targeted system.

For the numerical solution of the problem of confrontation game players \( A \) and \( B \) [10] are two algorithms. First - for calculating the vector \( \mathbf{s}(\tau_k) \) maximizing the probability of failure-free operation \( P(t_f, \mathbf{s}(\tau_k)) \) targeted system \( S_A(n, m, s) \), by the end of the game, and the second - the actual game problem solving using a first algorithm for finding the vectors \( \mathbf{r} \) and \( \mathbf{s} \) unknown player \( A \) strategy.

2. Formulation of the problem
In this paper, the development of the above stated game problem is put more complicated game problem, which proposes the development and expansion of the game model discussed above under conditions of incomplete information about the behavior of its members, taking into account the fact that the attacked party \( A \) is not only passively protected against enemy attacks, (side \( B \) ) due to its resource protection - hardware redundancy, but is actively using the resource and the impact on the attacking side in that as a result of exposure unlike [10] changes the probabilities be \( Q_i(t) \) in an appropriate state finding \( B_i \) among the plurality of states \( i = 0, 1, 2, \ldots, N + 1 \) not abruptly in a predetermined fixed times and continuously for the whole time interval \([0, t_f]\) games.

The solution to this problem the game with the payoff function \( P(t_f) = \max P(t_f, \mathbf{s}(\tau_k)) \) based on the methods and numerical algorithms developed by the author of this article [10].

3. Experimental results
The solution to this problem the game with the payoff function \( P(t_f) = \max P(t_f, \mathbf{s}(\tau_k)) \) based on the methods and numerical algorithms developed by the author of this article [10].

We assume that in this game \( G_2 \) under conditions of incomplete information from the participants in the game on the behavior of the opposing side are two players \( A \) and \( B \).

Player \( A \) has a system \( S_A^*(n, m, s) \) structurally and functionally identical hardware and excessive dynamic system, discussed in [10], and differs from it in that it has not only a "passive" protection against attacks of enemy resources (player \( B \)), but also the resources active influence on the probability of being in the respective states \( B_i(i = 0, 1, \ldots, N + 1) \) offensive player \( B \). As a probabilistic model of changes in the state of the attacking side (of the enemy) will use homogeneous Markov processes with discrete states and continuous time. We assume that the enemy of the state transitions \( B_i(i = 0, 1, \ldots, N + 1) \) with one resource attack \( \mathbf{\lambda}^i(t) = \{\lambda_0^i(t), \lambda_1^i(t), \ldots, \lambda_q^i(t)\} \), \( i = 1, 2, \ldots, N \) he player \( A \) to the state from other resources occurs under the active influence on the part of player \( A \) in the form of non-stationary poison flow of events with intensities \( y_j(t) \), the transitions from state \( B_0 \) in state \( B_j(j = 1, 2, \ldots, N) \) and with intensities \( \delta_j(t) \) in transitions from \( B_j \) to \( B_{N+1} \). It
is easy to understand that in the state $B_{N+1}$. The player has exhausted all its resources to attack the player

In the course of the game $G_2$ in the time interval $[0, t_f]$ player $A$ optimally using its resource protection in the form of $m$ standby units, their redistribution in the game at the appropriate times $\tau_k (0 \leq k \leq L)$ between the basic, refuse from the enemy attacks (player $B$), functional $n$ blocks hardware excessive dynamic system $S_A^*(n, m, \bar{s})$ to maximize the probability of its uptime $P(t_f)$ by the end of the game.

Given the above assumptions the probability $Q_i(t)$ Spent attacking side in the states $B_i (i = 0, 1, ..., N + 1)$ describes a system of differential equations with varying coefficients according to the known procedure [11]

$$\frac{dQ_0(t)}{dt} = - \sum_{j=1}^{N} y_j(t)Q_0(t).$$

$$\frac{dQ_i(t)}{dt} = y_i(t)Q_0(t) - \delta_i(t)Q_i(t). \quad (1)$$

$$i = 1, 2, ..., N$$

$$\frac{dQ_{N+1}(t)}{dt} = - \sum_{j=1}^{N} \delta_j(t)Q_j(t).$$

with the initial conditions at the beginning of the game when the attacking side is in a state $B_0$, equal

$$Q_0(0) = 1, \quad Q_1(0) = Q_2(0) = \cdots = Q_{N+1}(0) = 0.$$

The analytical solution of the equations (1) with variable coefficients is not possible, therefore, for an approximate calculation of probabilities $Q_i(t)$. We will use numerical-analytical method of solving the problem.

We use for this sampling technique [7], the essence of which is as follows.

We compute the minimum positive integer $r$ satisfying the following conditions:

$$\max_{1 \leq i \leq N} \max_{1 \leq v \leq r} |y_i(t) - y_{iv}| \leq \varepsilon,$$

$$\max_{1 \leq i \leq N} \max_{1 \leq v \leq r} |\delta_i(t) - \delta_{iv}| \leq \varepsilon,$$

Where

$$\Delta_v = [t_{v-1}, t_v], \quad t_v = v \cdot \Delta t, \quad \Delta t = T/r,$$

$$y_{iv} = \frac{1}{2} [y_i(t_{v-1}) + y_i(t_v)],$$

$$\beta_{iv} = \frac{1}{2} [\delta_i(t_{v-1}) + \delta_i(t_v)].$$
\( \varepsilon \) - the maximum allowable deviation functions \( y_i(t) \), and \( \delta_i(t) \) from the respective constants \( y_{iv} \), \( \delta_{iv} \) at sampling intervals \( \Delta_t \) for all \( v(1 \leq v \leq r) \), \( T = t_r \).

Then the system of equations (1) splits into \( r \) systems with constant coefficients:

\[
\frac{dQ_v^0(t)}{dt} = - \sum_{j=1}^{N} y_j^v Q_v^0(t),
\]

\[
\frac{dQ_v^i(t)}{dt} = y_i^v Q_v^0(t) - \delta_i^v Q_v^i(t), \quad (2)
\]

\( i = 1, 2, ..., N \)

\[
\frac{dQ_v^{N+1}(t)}{dt} = \sum_{i=1}^{N} \delta_i^v Q_v^i(t),
\]

with the initial conditions

\[
Q_v^i(t_{v-1}) = \begin{cases} 
Q_i(0), & \text{если } v = 1, \\
Q_i^{v-1}(t_{v-1}), & \text{если } 2 \leq v \leq r.
\end{cases}
\]

Integrating the system (2) by means of Laplace transform we obtain the expression for the aggressor state probabilities for \( v \)-th sampling interval:

\[
Q_v^0(t) = Q_v^{v-1}(t_{v-1}) \exp \left( - \sum_{j=1}^{N} y_j^t \right),
\]

\[
Q_v^i(t) = Q_v^{v-1}(t_{v-1}) y_i \frac{\exp\left(\sum_{j=1}^{N} y_j^t\right) - \exp(-\delta_i^t)}{\delta_i^t - \sum_{j=1}^{N} y_j} + Q_i^{v-1}(t_{v-1}) \exp(-\delta_i^t),
\]

\( i = 1, 2, ..., N \)

\[
Q_v^{N+1}(t) = \sum_{i=0}^{N+1} Q_i^{v-1}(t_{v-1}) - \sum_{i=0}^{N} Q_i^v(t).
\]

In the interval \([\tau_k, \tau_{k+1}]\) determine the probability of finding the mean values of the attacking side in the states \( B_i \):

\[
\bar{Q}_i = \frac{1}{\tau_{k+1} - \tau_k} \int_{\tau_k}^{\tau_{k+1}} Q_i(t) \, dt = \frac{\varphi_i(\tau_{k+1}) - \varphi_i(\tau_k)}{\tau_{k+1} - \tau_k}, \quad (3)
\]

Where
Based on the above arguments, if \( T = \tau \) we can write the approximate equality

\[
\varphi_1(\tau) \approx \sum_{v=1}^{\tau} \int_{\tau_{v-1}}^{\tau_v} Q_1^v(t) \, dt \approx \sum_{v=1}^{\tau} \psi_1^v. \tag{4}
\]

expression for \( \psi_1^v \) are as follows:

\[
\psi_0^v = \frac{Q_0^{v-1}(t_{v-1})}{c} [\exp(-ct_{v-1}) - \exp(-ct_v)], \tag{5}
\]

\[
\psi_i^v = \frac{Q_0^{v-1}(t_{v-1})}{c} \left[ \frac{\exp(-ct_{v-1}) - \exp(-ct_v)}{(\delta_i - c)} + \frac{\exp(-\delta_i t_{v-1})}{\delta_i (c - \delta_i)} \right] + \frac{Q_i^{v-1}(t_{v-1})}{\delta_i} [\exp(-\delta_i t_{v-1}) - \exp(-\delta_i t_v)] \tag{6}
\]

\[
\psi_{N+1}^v = (t_v - t_{v-1}) \sum_{i=0}^{N} Q_i^{v-1} (t_{v-1}) - \sum_{i=0}^{N} \psi_i^v. \tag{7}
\]

Where

\[
c = - \sum_{j=1}^{N} y_i.
\]

The solution of the game \( G_2 \) multiple-matrix reduces to the game and the successive solution in accordance with the procedure and numerical algorithms presented in \cite{10}, \((L + 1)\) games slots \( [\tau_k, \tau_{k+1}] \) \((0 \leq k < L; \tau_{L+1} = t_f)\) with constant average probability of finding the attacker in the states \( B_i (i = 0, 1, ..., N + 1)\). Calculated by the formula (3-7). set of solutions \((L + 1)\) games, for which the probability of failure-free operation \( P(t_f) \) the targeted hardware redundant dynamical system has the highest value, yields the desired settings vector \( \bar{\tau} = \{\tau_0, \tau_1, \tau_2, ..., \tau_L\} \), \( \tau_0 = 0, \tau_L < t_f \) system \( S_2(n, m, \bar{s}) \), whose elements correspond to the moments of reserve elements redistribution attacked system between the failed basic in \( q \)-groups and corresponding to these settings backup vectors \( s(\tau_k) = \{s_1(\tau_k), s_2(\tau_k), ..., s_q(\tau_k)\} \) dynamic system, which are the solution \( G_2 \) games.

4. Conclusion

In conclusion, it should be noted that the considered numerical and analytical model of the game problem of confrontation of two dynamical systems under incomplete information from players of the behavior of participants in the game in the process of conflict is applicable not only to study and optimize the behavior of conflicting technical systems but also for systems of different physical nature with appropriate identification.
It will be appreciated that the analytic solution of the problem is possible only in the simplest cases. In other cases it is necessary to use numerical methods and modeling with the help of personal computers.

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