Measuring galaxy mass

When we observe galaxies at high redshift, cosmological surface brightness dimming means that it is easiest to see very high luminosity, high surface brightness components such as starburst galaxies.

Most of the light from young galaxies comes from massive main sequence O and B stars.

Q: How does one measure mass for a low-redshift galaxy? How many of these techniques can be extended to high redshift?

→ Measure luminosity, assume M/L ratio, deduce mass

→ Use kinematic data with techniques such as fundamental plane, Tully-Fisher, to get a better handle on stellar mass.
Fig. 1.4. Luminosity and effective temperature during the main-sequence and later lives of stars with solar composition: the hatched region shows where the star burns hydrogen in its core. Only the main-sequence track is shown for the $0.8\, M_\odot$ star – Geneva Observatory tracks.

A better approximation is

$$\log(\tau_{\text{MS}}/10\, \text{Gyr}) = 1.015 - 3.49 \log(\mathcal{M}/M_\odot) + 0.83[\log(\mathcal{M}/M_\odot)]^2.$$  (1.9)

The most massive stars will burn out long before the Sun. None of the O stars shining today were born when dinosaurs walked the Earth 100 million years ago, and all those we now observe will burn out before the Sun has made another circuit of the Milky Way. But we have not included any stars with $\mathcal{M} < 0.8\, M_\odot$ in Figure 1.4, because none has left the main sequence since the Big Bang, $\sim 14$ Gyr.
1.1 The stars

Table 1.1 Stellar models with solar abundance, from Figure 1.4

| Mass (M⊙) | L_{ZAMS} (L⊙) | T_{eff} (K) | Spectral type | τ_{MS} (Myr) | τ_{red} (Myr) | f(L dτ)_{MS} (Gyr × L⊙) | f(L dτ)_{pMS} (Gyr × L⊙) |
|-----------|----------------|------------|---------------|--------------|--------------|--------------------------|--------------------------|
| 0.8       | 0.24           | 4860       | K2            | 25000        | 10           | 10                       | 10                       |
| 1.0       | 0.69           | 5640       | G5            | 9800         | 3200         | 10.8                     | 24                       |
| 1.25      | 2.1            | 6430       | F3            | 3900         | 1650         | 11.7                     | 38                       |
| 1.5       | 4.7            | 7110       | A2            | 2700         | 900          | 16.2                     | 13                       |
| 2         | 16             | 9080       | B7            | 1100         | 320          | 22.0                     | 18                       |
| 3         | 81             | 12250      | B4            | 350          | 86           | 38.5                     | 19                       |
| 5         | 550            | 17180      | B4            | 94           | 14           | 75.2                     | 23                       |
| 9         | 4100           | 25150      | B4            | 26           | 1.7          | 169                      | 40                       |
| 15        | 20000          | 31050      | B4            | 12           | 1.1          | 360                      | 67                       |
| 25        | 79000          | 37930      | B4            | 6.4          | 0.64         | 768                      | 145                      |
| 40        | 240000         | 43650      | B4            | 4.3          | 0.47         | 1500                     | 112                      |
| 60        | 530000         | 48190      | B4            | 3.4          | 0.43         | 2550                     | 9                        |
| 85        | 1000000        | 50700      | B4            | 2.8          |              | 3900                     |                          |
| 120       | 1800000        | 53330      | B4            | 2.6          |              | 5200                     |                          |

Note: L and T_{eff} are for the zero-age main sequence; spectral types are from Table 1.3; τ_{MS} is main-sequence life; τ_{red} is time spent later as a red star (T_{eff} ≤ 6000 K); integrals give energy output on the main sequence (MS), and in later stages (pMS).

As the particles slowly increases, and the core must become hotter to support the denser star against collapse. Nuclear reactions go faster at the higher temperature, and the star becomes brighter. The Sun is now about 4.5 Gyr old, and its luminosity is almost 50% higher than when it first reached the main sequence.

Problem 1.4 What mass of hydrogen must the Sun convert to helium each second in order to supply the luminosity that we observe? If it converted all of its initial hydrogen into helium, how long could it continue to burn at this rate? Since it can burn only the hydrogen in its core, and because it is gradually brightening, it will remain on the main sequence for only about 1/10 as long.

Problem 1.5 Use Equation 1.3 and data from Table 1.1 to show that, when the Sun arrived on the main sequence, its radius was about 0.87 R_{⊙}.

A star can continue on the main sequence until thermonuclear burning has consumed the hydrogen in its core, about 10% of the total. Table 1.1 lists the time τ_{MS} that stars of each mass spend there; it is most of the star’s life. So, at any given time, most of a galaxy’s stars will be on the main sequence. For an average value \(\alpha \approx 3.5\) in Equation 1.6, we have

\[
τ_{MS} = τ_{MS,⊙} \frac{M/L}{M_⊙/L_⊙} \approx 10 \text{Gyr} \left( \frac{M}{M_⊙} \right)^{-2.5} = 10 \text{Gyr} \left( \frac{L}{L_⊙} \right)^{-5/7}. \tag{1.8}
\]
somewhere between giants and supergiants. Supergiants are luminosity class I. We further divide supergiants into la and lb, with la being larger. When we look at the spectral lines from a star we can actually tell something about the size. Stars of different sizes will have different accelerations of gravity near their surface. The surface gravity affects the detailed appearance of certain spectral lines.

There are also stars that appear below the main sequence. These stars are typically 10 mag fainter than main sequence stars of the same temperature. They are clearly much smaller than main sequence stars. Since most of these are in the middle spectral types, and therefore appear white, we refer to them as white dwarfs. (Do not confuse dwarfs, which are main sequence stars, with white dwarfs, which are much smaller than ordinary dwarfs.)

Example 3.3 Size of white dwarfs
Suppose that some white dwarf has the same spectral type as the Sun, but has an absolute magnitude that is 10 mag fainter than the Sun. What is the ratio of the radius of the white dwarf, \( R_{wd} \), to that of the Sun, \( R_\odot \)?

**SOLUTION**
The luminosity is proportional to the square of the radius, so

\[
\frac{L_{wd}}{L_\odot} = \left( \frac{R_{wd}}{R_\odot} \right)^2.
\]

We use equation (2.2) to find the luminosity ratio for a 10 mag difference:

\[
\frac{L_{wd}}{L_\odot} = 10^{(M_\odot - M_{wd})/2.5}
\]

\[
= 10^{-4}
\]

Combining these two results to find the ratio of the radii yields

\[
\frac{R_{wd}}{R_\odot} = \left( \frac{L_{wd}}{L_\odot} \right)^{1/2}
\]

\[
= (10^{-4})^{1/2}
\]

\[
= 10^{-2}
\]

The radius of a white dwarf is 1% of the radius of the Sun!

For any cluster for which we plot an HR diagram, we only know the apparent magnitudes,
stars, the timescale to reach the MS is longer than the lifetime of the cluster, and so we observe them following their Hayashi tracks down to the MS (see Figure 5.10). Notice that the scattering of these stars in the CM diagram implies that they cannot all have formed simultaneously, as different stars with the same masses have evolved to different points in their pre-main-sequence evolution.

At the upper end of the MS, the stars have started to evolve away from the MS, giving the sequence more curvature than the ZAMS. Subsequently, massive stars evolve very rapidly across the CM diagram, have very brief lives as supergiant stars, and explode as supernovae. The short timescale on which these stars pass through the giant stages of their evolution means that the late-evolution sequences in the CM diagram are very sparsely populated with a pronounced Hertzsprung gap (§3.5).

To compare the CM diagrams of different clusters, we must place them on a common absolute-magnitude scale by MS fitting. The calibration of this scale is better established than for the globular clusters (§6.1.2) since there are a large number of nearby stars with measured trigonometric parallaxes that have similar metallicities to the open cluster members. The more extensive nature of the MS in open clusters also assists in the fitting
2.1 The solar neighborhood

Fig. 2.3. The histogram shows the luminosity function \(\Phi(M_V)\) for nearby stars: solid dots from stars of Figure 2.2, open circles from Reid et al. 2002 AJ 124, 2721. Lines with triangles show \(L_V \Phi(M_V)\), light from stars in each magnitude bin; the dotted curve is for main-sequence stars alone, the solid curve for the total. The dashed curve gives \(M \Phi_{MS}(M_V)\), the mass in main-sequence stars. Units are \(L_\odot\) or \(M_\odot\) per 10 pc cube; vertical bars show uncertainty, based on numbers of stars in each bin.

Problem 2.3 Show that the volume in Equation 2.3 is \(V_{\text{max}}(M) \approx 4\pi d_{\text{max}}^3/3\), where \(d_{\text{max}}\) is the smaller of 100 pc and 10 pc \(\times 10^{0.2(M-M_\odot)}\). Using Table 1.4 for \(M_V\), find \(d_{\text{max}}\) for an M4 dwarf. Why are you surprised to see such faint stars in Figure 2.2?

It is quite difficult to determine the faint end of the luminosity function, since dim stars are hard to find. The bright end of \(\Phi(M_V)\) also presents problems; because luminous stars are rare, we will not find enough of them unless we survey a volume larger than our 100 pc sphere. But stars are not spread out uniformly in space. For example, their density falls as we go further out of the Milky Way's disk in the direction of the Galactic poles. So, if we look far afield for luminous stars, the average density in our search region is lower than it is near the Sun's position. Finally, many stars are in binary systems so close that they are mistaken for a brighter single star. Despite these uncertainties, it is clear that dim stars are overwhelmingly more numerous than bright ones.

Figure 2.3 also shows how much of the V-band light is emitted by stars of each luminosity: stars in the range from \(M_V\) to \(M_V + \Delta M_V\) contribute an amount \(L_V \Phi(M_V) \Delta M_V\) of the total. Almost all the light comes from the brighter stars, mainly A and F main-sequence stars and K giants. Rare luminous stars such as main-sequence O and B stars, and bright supergiants, contribute more light than all the stars dimmer than the Sun; so the total luminosity of a galaxy
We have photometric observations, from red-frame UV to near IR, of a galaxy at z = 3.0. We want to get the best estimate we can of its stellar mass. How should we use our measurements, and what assumptions do we need to make?

→ color of galaxy will give clues as to age

→ population synthesis models:
  - star formation rate
  - metallicity
  - IMF
  - spectral libraries of stars for different metallicity, mass, evolutionary state
butions (SEDs), following the Calzetti et al. (1994) method of fitting to the 10 rest-frame UV bins defined by those authors.

As can be inferred from Table 1, the galaxies span a large range in all these properties. The "median galaxy" is red and faint in the observer’s optical, with \( R_{\text{mag}} = 25.9 \). We show the full distribution of the rest-frame \( U-V \) colors in Figure 2a. The bluest galaxies have \( U-V < -0.1 \) and are bluer than nearby irregular galaxies, whereas the colors of the reddest galaxies are similar to those of nearby elliptical galaxies (see, e.g., Fukugita et al. 1995). The median \( U-V \) = 0.6, which is similar to nearby spiral galaxies but also to nearby dust-enshrouded starburst galaxies (e.g., Armus et al. 1989).

The distribution of \( \beta \) is shown in Figure 2b. Remarkably, the distribution is rather flat and has no well-defined peak, in contrast to previous studies of optically selected samples (Adelberger & Steidel 2000). The median \( \beta = -0.39 \), indicating a relatively flat spectrum in \( F_{\lambda} \) (see also Papovich et al. 2005).

A potential worry is that individual values of \( \beta \) are uncertain, as many galaxies are very faint in the observer’s optical. We tested the robustness of the derived distribution of \( \beta \) by summing the observed optical fluxes of the galaxies in the lower and upper 25% quartiles, weighting by the inverse of the total optical flux. The power-law slopes of these summed SEDs are in very good agreement with the median \( \beta \)’s that we determined from the SED fits.

The large range of properties of massive galaxies at \( 2 < z < 3 \) is illustrated in Figure 3, which shows the full UBVRizJHK\(_{\text{S}}\) SEDs of three galaxies from MUSYC with different values of \( \beta \). The top galaxy has a very blue SED similar to those of UV-selected samples (see, e.g., Shapley et al. 2001), the middle object has an SED that resembles that of nearby spiral galaxies, and the bottom galaxy has a very red SED indicating strong extinction.

5. DISCUSSION

The main result of our analysis is that massive galaxies at \( z \sim 2.5 \) span a large range in rest-frame UV slopes, rest-frame optical colors, and rest-frame M/L\(_{\text{V}}\) ratios, indicating significant variation in dust content, star formation histories, or both. This result is not surprising in the light of the recent discoveries of DRGs, IEROs, and other populations. Here we have quantified the median colors and their range for a uniformly selected, large, mass-limited sample.

The large variation in the rest-frame color distributions of our mass-limited sample implies that “standard” color selection techniques produce biased samples. We consider two of the two most widely used selection techniques in this redshift range: the Lyman break technique of Steidel and collaborators and the \( J-K_s > 2.3 \) DRG selection of Franz et al. (2003). DRGs are identified in the following way. From the best-fitting Bruzual & Charlot (2003) SEDs (which include absorption due to the Ly\( \alpha \) forest), we calculated synthetic colors in Steidel’s \( U_GTR \) system. To qualify as an LBG, an object has to have \( R_{\text{AB}} < 25.5 \) and synthetic \( U_GTR \) colors that place it in the Lyman break, BX, or BM selection region (see Steidel et al. 2003, 2004). Combined, these criteria provide a continuous selection of galaxies over the redshift range considered here.

Figure 4 illustrates the LBG and DRG selection techniques, as applied to our sample. DRGs with \( J-K_s > 2.3 \) are indicated by red symbols, and LBGs by blue symbols. The DRG limit and the standard photometric LBG limit of \( R_{\text{AB}} = 25.5 \) are also indicated.

By number, DRGs make up 69% of the sample, and LBGs 20%. The DRG and LBG samples do not show much overlap: only 7% of objects fall in both categories. By rest-frame V-

12 An LBG in this definition is therefore an object that has \( R_{\text{AB}} < 25.5 \), \( 2 < z < 3 \), and is either a classical “V-dropout” or a BX/DM object.

13 We note that not all galaxies with \( J-K_s > 2.3 \) have redshifts in the range \( 2 < z < 3 \) to \( K_s = 21 \), we find that ~50% are in this redshift range, with the rest about equally split between \( z < 2 \) and \( z > 3 \) galaxies.
Stellar population synthesis at the resolution of 2003

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ABSTRACT

We present a new model for computing the spectral evolution of stellar populations at ages between $1 \times 10^5$ and $2 \times 10^{10}$ yr at a resolution of 3 Å across the whole wavelength range from 3200 to 9500 Å for a wide range of metallicities. These predictions are based on a newly available library of observed stellar spectra. We also compute the spectral evolution across a larger wavelength range, from 91 Å to 160 μm, at lower resolution. The model incorporates recent progress in stellar evolution theory and an observationally motivated prescription for thermally pulsing stars on the asymptotic giant branch. The latter is supported by observations of surface brightness fluctuations in nearby stellar populations. We show that this model reproduces well the observed optical and near-infrared colour–magnitude diagrams of Galactic star clusters of various ages and metallicities. Stochastic fluctuations in the numbers of stars in different evolutionary phases can account for the full range of observed integrated colours of star clusters in the Magellanic Clouds. The model reproduces in detail typical galaxy spectra from the Early Data Release (EDR) of the Sloan Digital Sky Survey (SDSS). We exemplify how this type of spectral fit can constrain physical parameters such as the star formation history, metallicity and dust content of galaxies. Our model is the first to enable accurate studies of absorption-line strengths in galaxies containing stars over the full range of ages. Using the highest-quality spectra of the SDSS EDR, we show that this model can reproduce simultaneously the observed strengths of those Lick indices that do not depend strongly on element abundance ratios. The interpretation of such indices with our model should be particularly useful for constraining the star formation histories and metallicities of galaxies.

Key words: stars: evolution – galaxies: evolution – galaxies: formation – galaxies: stellar content.

1 INTRODUCTION

The star formation history of galaxies is imprinted in their integrated light. The first attempts to interpret the light emitted from galaxies in terms of their stellar content relied on trial and error analyses (e.g. Spinrad & Taylor 1971; Faber 1972; O’Connell 1976; Turnrose 1976; Pritchet 1977; Pickles 1989). In this technique, one reproduces the integrated spectrum of a galaxy with a linear combination of individual stellar spectra of various types taken from a comprehensive library. The technique was abandoned in the early 1980s because the number of free parameters was too large to be constrained by typical galaxy spectra. More recent models are based on the evolutionary population synthesis technique (Tinsley 1978; Bruzual 1983; Arimoto & Yoshii 1987; Guiderdoni & Rocca-Volmerange 1987; Buzzoni 1989; Bruzual & Charlot 1993; Bressan, Chiosi & Fagotto 1994; Fritze-Von Alvensleben & Gerhard 1994; Worthey 1994; Leitherer & Heckman 1995; Fioc & Rocca-Volmerange 1997; Maraston 1998; Vazdekis 1999; Schulz et al. 2002). In this approach, the main adjustable parameters are the stellar initial mass function (IMF), the star formation rate (SFR) and, in some cases, the rate of chemical enrichment. Assumptions concerning the time evolution of these parameters allow one to compute the age-dependent distribution of stars in the Hertzsprung–Russell (HR) diagram, from which the integrated spectral evolution of the stellar population can be obtained. These models have become standard tools in the interpretation of galaxy colours and spectra.

Despite important progress over the last decade, modern population synthesis models still suffer from serious limitations. The largest intrinsic uncertainties of the models arise from the poor understanding of some advanced phases of stellar evolution, such as the supergiant phase and the asymptotic giant-branch (AGB) phase (see Charlot 1996; Charlot, Worthey & Bressan 1996; Yi 2003). Stars in these phases are very bright and have a strong influence

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Fig. 1. Evolution of the $B - V$ and $V - K$ colours and stellar mass-to-light ratio $M/L_V$ of simple stellar populations for different metallicities, $Z = 0.004$ (dotted line), $Z = Z_\odot = 0.02$ (solid line) and $Z = 0.05$ (dashed line), for the standard model of Section 3. All models have the Chabrier (2003b) IMF truncated at 0.1 and 100 $M_\odot$ (see equation 2).

Fig. 1 shows the evolution of the $B - V$ and $V - K$ colours and $M/L_V$ for three different metallicities, $Z = 0.004, 0.02$ and 0.05, for our standard SSP model. The irregularities in the photometric evolution arise both from the discrete sampling of initial stellar masses in the track library and from ‘phase’ transitions in stellar evolution. For example, the evolution of low-mass stars through the helium flash causes a characteristic feature in all properties in Fig. 1 at ages near $10^7$ yr. At fixed age, the main effect of increasing metallicity is to redden the colours and increase $M/L_V$. The reason for this is that, at fixed initial stellar mass, lowering metallicity causes stars to evolve at higher effective temperatures and higher luminosities (Schaller et al. 1992; Fagotto et al. 1994a; Girardi et al. 2000). Another noticeable effect of varying $Z$ is to change the relative numbers of red and blue supergiants. The evolution of the $B - V$ colour at early ages in Fig. 1 shows that the signature of red supergiants in the colour evolution of an SSP depends crucially on metallicity (see also Cerviño & Mas-Hesse 1994). We note that increasing metallicity at fixed age has a similar effect to increasing age at fixed metallicity, which leads to the well-known age–metallicity degeneracy.

In Fig. 2, we illustrate the influence of the stellar evolution prescription on the predicted photometric evolution of an SSP for fixed (solar) metallicity and fixed (STELIB/BaSeL 3.1) spectral calibration. We show models computed using the Padova 1994, the Geneva and the Padova 2000 track libraries (Section 2.1). The largest difference between the Padova 1994 and Geneva prescriptions arises at early ages and results from the larger number of evolved, blue massive (Wolf–Rayet) stars in the Padova models than in the Geneva models (see Fig. 2b of Charlot 1996). Also, since the minimum mass for quiet helium ignition is lower in the Geneva model than in the Padova 1994 model ($1.9$ $M_\odot$ versus $2.2$ $M_\odot$), the photometric signature of the helium flash occurs at slightly later ages in the Geneva model in Fig. 2. Differences between the Padova 1994 and Padova 2000 track libraries pertain only to stars less massive than $7$ $M_\odot$.

Fig. 2. Evolution of the $B - V$ and $V - K$ colours and stellar mass-to-light ratio $M/L_V$ of simple stellar populations of solar metallicity computed using the Geneva (dotted line), Padova 1994 (standard model; solid line) and Padova 2000 (dashed line) stellar evolution prescriptions and the STELIB/BaSeL 3.1 spectral calibration. All models have the Chabrier (2003b) IMF truncated at 0.1 and 100 $M_\odot$ (see equation 2).

with turn-off ages greater than about $5 \times 10^7$ yr (Section 2.1). In the Padova 2000 model, the finer resolution in initial stellar mass around $2.0$ $M_\odot$ makes the evolution through the helium flash much smoother than in the Padova 1994 model. At late ages, the $V - K$ colour is significantly bluer in the Padova 2000 model than in the Padova 1994 model. The reason for this is that the red giant branch is $50–200$ K warmer (from bottom to tip) in the Padova 2000 tracks than in the Padova 1994 tracks. As a result, the integrated $V - K$ colour of a solar-metallicity SSP in the Padova 2000 model reaches values typical of old elliptical galaxies ($V - K \sim 3.0-3.3$ along the colour–magnitude relation; Bower, Lucey & Ellis 1992) only at ages $15–20$ Gyr. Since this is older than the currently favoured estimates of the age of the Universe, and since the giant-branch temperature in the Padova 2000 tracks has not been tested against observational calibrations (e.g. Frogel, Persson & Cohen 1981), we have adopted here the Padova 1994 library rather than the Padova 2000 library in our standard model (see above).

It is intriguing that the Padova 2000 models, which include more recent input physics than the Padova 1994 models, tend to produce worse agreement with observed galaxy colours. The relatively high giant branch temperatures in the Padova 2000 models, though attributable to the adoption of new opacities, could be subject to significant coding uncertainties (L. Girardi, private communication). This is supported by the fact that the implementation of the same input physics as used in the Padova 2000 models into a different code produces giant branch temperatures to much better agreement with those of the Padova 1994 models (A. Weiss 2002, private communication). We regard the agreement between the Girardi et al. (2002) model and our standard model at late ages in Fig. 5 (see Section 3.2) as fortuitous, as the spectral calibration adopted by Girardi et al. (2002) relies on purely theoretical model atmospheres, which do not reproduce well the colour–temperature relations of cool stars (e.g. Lejeune et al. 1997).
We now consider the influence of the spectral calibration on the photometric evolution of an SSP for fixed (solar) metallicity and fixed (Padova 1994) stellar evolution prescription. In Fig. 3, we compare the results obtained with four different spectral libraries: the STELIB/BaSeL 3.1 library (standard model); the BaSeL 3.1 library; the STELIB/BaSeL 1.0 library; and the Pickles library (recall that, at solar metallicity, the BaSeL 3.1 library is identical to the BaSeL 2.2 library; Section 2.2.1). Fig. 3 shows that the differences between these spectral calibrations have only a weak influence on the predicted photometric evolution of an SSP. The good agreement between the STELIB/BaSeL 3.1, the BaSeL 3.1 and the Pickles calibrations follows in part from the consistent colour-temperature scale of the three libraries. Also the empirical corrections applied by Lejeune et al. (1997) and Westera et al. (2002) to the BaSeL 1.0 spectra, illustrated by the differences between the STELIB/BaSeL 3.1 and STELIB/BaSeL 1.0 models in Fig. 3, imply changes of at most a few hundredths of a magnitude in the evolution of the \( B - V \) and \( V - K \) colours. It is important to note that the spectral calibration has a stronger influence on observable quantities that are more sensitive than integrated colours to the details of the stellar luminosity function, such as colour-magnitude diagrams (Section 3.3) and surface brightness fluctuations (Liu et al. 2000). Fig. 8 of Liu et al. (2000) shows that, for example, the observed near-infrared surface brightness fluctuations of nearby galaxies clearly favour the BaSeL 2.2/3.1 spectral calibration over the BaSeL 1.0 one.

It is also of interest to examine the influence of the IMF on the photometric evolution of an SSP for fixed (solar) metallicity, fixed (Padova 1994) stellar evolution prescription and fixed (STELIB/BaSeL 3.1) spectral calibration. Fig. 4 shows the evolution of the \( B - V \) and \( V - K \) colours and \( M/L_v \) for four different IMFs: Chabrier (2003b, see equation 2 above), Kroupa (2001, universal IMF), Salpeter (1955) and Scalo (1998). In all cases, the IMF is truncated at 0.1 and 100 M\(_\odot\). The evolution of the \( B - V \) colour does not depend sensitively on the IMF, because the optical light is dominated at any age by stars near the turn-off. The \( V - K \) colour is slightly more sensitive to the relative weights of stars of different masses along the isochrone, especially at ages less than about 10\(^6\) yr, when the mass of the most evolved stars differs significantly from the turn-off mass. The \( M/L_v \) ratio is far more sensitive to the shape of the IMF, especially near the low-mass end that determines the fraction of the total mass of the stellar population locked into faint, slowly evolving stars. For reference, the fraction of mass returned to the ISM by evolved stars at the age of 10 Gyr is 31, 44, 46 and 48 per cent for the Salpeter, the Scalo, the Kroupa and the Chabrier IMFs, respectively.

### 3.2 Comparison with previous work

Most current population synthesis models rely on readily available computations of stellar evolutionary tracks and stellar atmospheres, such as those mentioned in Sections 2.1 and 2.2.1 above. In general, however, publically available stellar evolutionary tracks do not include the uncertain evolution of stars beyond the early-AGB phase. Also, the widely used model atmospheres of Kurucz (1992, and other releases) do not include spectra of stars outside the temperature range 3500 \( \leq T_{\text{eff}} \leq 50 000 \) K. We therefore expect differences between our model and previous work to originate mainly from our observationally motivated prescription for TP-AGB stars, the spectral calibration of very hot and very cool (giant) stars and the adoption of a new library of observed stellar spectra at various metallicities.

In Fig. 5, we compare the evolution of the \( B - V \) and \( V - K \) colours and the mass-to-visual light ratio \( M/L_v \) of a

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**Figure 3.** Evolution of the \( B - V \) and \( V - K \) colours and stellar mass-to-light ratio \( M/L_v \) of simple stellar populations of solar metallicity computed using the Padova 1994 stellar evolution prescription and the BaSeL 3.1 (dotted line), STELIB/BaSeL 1.0 (short-dashed line), STELIB/BaSeL 3.1 (standard model; solid line) and Pickles (long-dashed line) spectral calibrations. All models have the Chabrier (2003b) IMF truncated at 0.1 and 100 M\(_\odot\) (see equation 2).

**Figure 4.** Evolution of the \( B - V \) and \( V - K \) colours and stellar mass-to-light ratio \( M/L_v \) of simple stellar populations of solar metallicity computed using the Padova 1994 stellar evolution prescription and the STELIB/BaSeL 3.1 spectral calibration, for different IMFs: Chabrier (2003b, standard model; solid line; see equation 2), Kroupa (2001, dotted line), Salpeter (1955, short-dashed line) and Scalo (1998, long-dashed line). All IMFs are truncated at 0.1 and 100 M\(_\odot\).
For different values of evolving Kroupa IMP equation (3) relative to a standard Kroupa IMF, nor-
malized to unity at $z = 0$. The curves are computed from PEGASE.2 models for different values of $\gamma$, as described in Section 6. The data points are from fits to the observed $M_\ast$–SFR relation at $z = 0.45, 1, 2$ (described and shown in Fig. 2) at stellar masses of $10^{10.5} M_\odot$ (squares), $10^{10} M_\odot$ (triangles) and $10^{9.5} M_\odot$ (circles). The required evolution is reasonably well fit by $\gamma = 2$, i.e. $M_{\text{IMF}} = 0.5 (1 + z)^2 M_\odot$. Long and short dashed-dot models show results for the Larson (1998) scenario where the minimum temperature of molecular clouds is set by the CMB temperature, and $M_{\text{IMF}} \propto T_{\text{min}}^\beta$, where $\beta = 3.35$ (Larson 1985) and $\beta = 1.7$ (Jappsen et al. 2005), respectively. Because the CMB temperature does not exceed the local minimum temperature in molecular clouds of 8 K until $z \gtrsim 2$, this scenario does not yield substantial evolution at $z \lesssim 2$.

As a check, an independent method for determining $\dot{M}_{\text{IMF}}$ evolution was explored that is less comprehensive, but conceptually more straightforward. Here, the high-mass SFR is assumed to be unchanged, while observations of stellar mass are assumed to reflect stars near the main-sequence turn-off mass ($M_{\text{turn-off}}$), since red optical light is dominated by giant stars. For an evolving Kroupa IMF, to obtain a reduction of a factor of $x$ in the amount of stars produced at $M_{\text{turn-off}}$, it is easy to show that $\log \dot{M}_{\text{IMF}} = \log M_{\text{turn-off}} + \log x$. $M_{\text{turn-off}}$ may be estimated by noting that stellar lifetimes scale as $M^{-3}$, and that the Sun has a lifetime of 10 Gyr; this yields $M_{\text{turn-off}}^3 = 10/(\log x - 1)$, $(\log$ in Gyr), which is 1.24 and 1.58 $M_\odot$ at $z = 1, 2$, respectively. Inserting values of $x = 1.9, 3.3$ at $z = 1, 2$ yield $\dot{M}_{\text{IMF}} = 2.4$ and 5.2 $M_\odot$. These values are quite similar to those obtained from PEGASE modelling showing that the results are not critically dependent on details of PEGASE.

The same procedure can be applied to the scenario proposed by Larson (1998), where the CMB temperature is solely responsible for setting a floor to the ISM temperature. In that case, $T_{\text{min}} = \max(T_{\text{CMB}}, 8 \text{ K})$ and $\dot{M}_{\text{IMF}} \propto T_{\text{CMB}}^\beta$. Larson (1985) predicted $\beta = 3.35$, while Jappsen et al. (2005) found $\beta = 1.7$. The results of this scenario for these two values of $\beta$ are shown as the long and short dot-dashed lines, respectively, in Fig. 3. As is evident, this IMF evolution is not nearly sufficient to reconcile the observed and predicted $\alpha_d$ evolution out to $z \sim 2$, mainly because $T_{\text{CMB}}$ only exceeds 8 K at $z > 1.93$, and hence $\dot{M}_{\text{IMF}}$ does not change from $z \sim 0$–2.

Note that since CMB heating of the ISM is subdominant at all redshifts, the assumed form of IMF evolution (equation 3, scaling as $T_{\text{CMB}}^\beta$) is not physically well motivated. While the evolution out to $z \sim 2$ is well fit by such a form, it could be that the form changes at higher $z$, or that it should be parametrized as some other function of $z$. For instance, it is the virgouness of star formation activity that determines $\dot{M}_{\text{IMF}}$, perhaps IMF evolution actually reverses at very high redshifts. For lack of better constraints, the form for IMF evolution in equation (3) will be assumed at all $z$, but this should be taken as an illustrative example rather than a well-motivated model.

This analysis also makes predictions for $\alpha_d$ at $z > 2$ that would be inferred assuming a standard (non-evolving) IMF. For instance, Fig. 3 implies that at $10^{10} M_\odot$, $\alpha_d(z = 3) = 0.13$ and $\alpha_d(z = 4) = 0.12$. Notably, the evolution slows significantly at $z \gtrsim 3$, and actually reverses at $z \gtrsim 4$, though the simple SFH assumed in the PEGASE modelling may be insufficiently realistic to yield valid results at $z \gtrsim 4$. Any such extrapolation of this IMF evolution should be made with caution, as the $M_\ast$–SFR relation only constrains it out to $z \sim 2$. However, in the next section this will be compared to observations that suggest that such IMF evolution may be reasonable out to $z \sim 3$.

Fig. 4 illustrates the evolving IMF, in terms of stellar mass formed per unit logarithmic mass bin. A Salpeter IMF is shown for reference, and evolving Kroupa IMFs are shown at $z = 0, 1, 2$ with characteristic masses $M_{\text{IMF}} = 0.5 (1 + z)^2 M_\odot$, respectively. As expected, the evolution results in more high-mass stars compared to low-mass ones at higher redshifts. While such a dramatic increase in $\dot{M}_{\text{IMF}}$ may seem surprising, recall that studies of local highly active star-forming regions suggest a characteristic mass scale of a few $M_\odot$ (Rieke et al. 1993; Sirianni et al. 2000; Figer 2005). Note that an IMF with a steeper high-mass slope (e.g. Miller & Scalo 1979; Kroupa et al. 1993) would result in even stronger IMF evolution being required than in the Kroupa (2001) case.

In summary, an evolving Kroupa IMF whose characteristic mass increases with redshift as $M_{\text{IMF}} = 0.5 (1 + z)^2 M_\odot$ is able to alter the observed $\alpha_d$ evolution from $z \sim 2 \to 0$ into one with approximately no evolution, as predicted by models or inferred from...