Proton Decay and Fermion Masses in Supersymmetric SO(10) Model with Unified Higgs Sector

Yunfei Wu and Da-Xin Zhang

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

We make a detailed analysis on the proton decay in a supersymmetric SO(10) model proposed by K.Babu, I.Gogoladze, P.Nath, and R. Syed. We introduce quark mixing, and find that this model can generate fermion masses without breaking the experimental bound on proton decay. We also predict large CKM unitarity violations.

I. INTRODUCTION

In grand unification models quarks and leptons are usually contained in the same multiplets. Consequently, baryon and lepton numbers are not conserved in general. If the models are supersymmetric, dimension-five operators mediated by the color-triplet Higgs superfields are dominant in these baryon and lepton number non-conservation processes. These dimension-five operators are also related to the fermion masses, thus are highly predictable in most of the supersymmetric unification models.

To build up unification models that generate correct fermion masses and fulfill the stability of baryons, one usually needs to add into more Higgs multiplets and/or fermion masses and fulfill the stability of baryons, one cation models. If the models are supersymmetric, dimension-five operators are also related to the fermion masses, thus are highly predictable in most of the supersymmetric unification models.

In our work, we analyze the problems of fermion masses and proton decay in this new model. By introducing quarks mixing, we obtain the CKM unitarity breaking effects and the diagonal quark masses. The general form of the dimension-five operators for baryon decay is given. We pick out the most important proton decay mode $p \rightarrow K^+ \nu \mu$. We fit the quartic and cubic Yukawa couplings of the second and third generation at unification scale. By using these values, we get the couplings of dimension-five operators to analyze the proton lifetime.

This paper is organized in the following way: A short review of the model is presented in section II. In section III, we give the quark mixing and mass generation. The CKM unitarity breaking is analyzed. In section IV, we present the general dimension-five operators and low energy Lagrangian for proton decay. In section V, the numerical results are discussed. We found the model can survive in some parameter space. Finally, we summarize our results.

II. REVIEW OF THE MODEL

The model of Ref. uses the following superpotential

$$W = M(\bar{144}_H \times 144_H)$$

$$+ \frac{\lambda_{145}}{M'}(\bar{144}_H \times 144_H)_{451}(\bar{144}_H \times 144_H)_{451}$$

$$+ \frac{\lambda_{142}}{M'}(\bar{144}_H \times 144_H)_{452}(\bar{144}_H \times 144_H)_{452}$$

$$+ \frac{\lambda_{210}}{M'}(\bar{144}_H \times 144_H)_{210}(\bar{144}_H \times 144_H)_{210},$$

where $M'$ is supposed to be at the Planck scale and $\lambda$'s are the couplings after integrating out the corresponding 45 or 210 dimension component fields. $M$ is the mass of 144 Higgs. Eq. gives the one step breaking of SO(10) to the supersymmetric stand model and the doublet-triplet splitting.

The terms responsible for the symmetry breaking is

$$W_{SB} = MQ^i_j P^i_j + \frac{1}{M'} \left[ -\lambda_{145} + \frac{1}{6} \lambda_{210} \right] Q^i_j P^i_j Q^i_k P^i_k$$

$$+ \frac{1}{M'} \left[ -4\lambda_{451} - \frac{1}{2} \lambda_{452} - \lambda_{210} \right] Q^i_j P^i_j Q^i_k P^i_k,$$

where the all the fields are chiral supermultiplets and indices $i, j$ take 1 to 5. $P$'s and $Q$'s are 24 dimension Higgs of SU(5) coming from 144 and 210 respectively. To get the following vacuum expectation values and minimization of $W_{SB}$

$$\langle Q^i_j \rangle = q \text{ diag}(2, 2, 2, -3, -3),$$

$$\langle P^i_j \rangle = p \text{ diag}(2, 2, 2, -3, -3),$$

we need

$$\frac{MM'}{qp} = 116\lambda_{451} + 7\lambda_{452} + 4\lambda_{210}.$$
The $D$-flat condition needs $q = p$. The vacuum expectation values in Eq. (3) break the SO(10) down to the Standard Model gauge group.

The further electroweak symmetry breaking requires two of the Higgs doublets light and all the Higgs triplets heavy. The superpotential governs the doublet-triplet splitting is given in Ref. [5]. We only give out masses splitting results. The Higgs doublet pairs and triplet pairs before splitting are

$$D_1 : (Q^a, P_a), \quad T_1 : (Q^a, P_a),$$
$$D_2 : (Q^a, P^a), \quad T_2 : (Q^a, P^a),$$
$$D_3 : (Q^a, P^a), \quad T_3 : (Q^a, P^a),$$
$$T_4 : (\tilde{Q}^a, \tilde{P}_a),$$

where $\alpha, \beta$ take 1, 2 and 3 while $a, b$ take 4 and 5. The diagonalization of the doublets’ and triplets’ mass matrices use the rotations

$$\begin{bmatrix}
Q_a' P'^a \\
\tilde{Q}_a' \tilde{P}'^a
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_D & \sin \theta_D \\
-\sin \theta_D & \cos \theta_D
\end{bmatrix}
\begin{bmatrix}
Q_a P^a \\
\tilde{Q}_a \tilde{P}^a
\end{bmatrix},$$

$$\begin{bmatrix}
Q'_a P'^a \\
\tilde{Q}'_a \tilde{P}'^a
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_T & \sin \theta_T \\
-\sin \theta_T & \cos \theta_T
\end{bmatrix}
\begin{bmatrix}
Q_a P^a \\
\tilde{Q}_a \tilde{P}^a
\end{bmatrix},$$

where

$$\tan \theta_D = \frac{1}{d_3} \left( d_2 + \sqrt{d_2^2 + d_3^2} \right),$$
$$\tan \theta_T = \frac{1}{t_3} \left( t_2 + \sqrt{t_2^2 + t_3^2} \right).$$

Here

$$d_1 = -\frac{2}{5} M + \frac{q \mu}{M'} \left( \frac{296}{5} \lambda_{451} - 16 \lambda_{452} - \frac{392}{15} \lambda_{210} \right),$$
$$d_2 = -\frac{8}{5} M + \frac{q \mu}{M'} \left( -\frac{1036}{5} \lambda_{451} + \frac{1}{2} \lambda_{452} + \frac{427}{15} \lambda_{210} \right),$$
$$d_3 = 2 \sqrt{\frac{3}{5} \frac{q \mu}{M'}} \left( 10 \lambda_{451} + \frac{5}{4} \lambda_{452} - \frac{5}{6} \lambda_{210} \right).$$

The mass eigenvalues are found to be

$$M_{D_1} = M + \frac{q \mu}{M'} (180 \lambda_{451} + 9 \lambda_{452} - 10 \lambda_{210}),$$
$$M_{D_2, D_3} = \frac{1}{2} \left( d_1 \pm \sqrt{d_2^2 + d_3^2} \right).$$

We will set $M_{D_3}$ to be small, and get

$$H_u = \tilde{P}'_a, \quad H_d = \tilde{Q}'_a.$$

Moreover, the triplet eigenstates’ masses are

$$M_{T_1} = M + \frac{q \mu}{M'} (180 \lambda_{451} + 4 \lambda_{452} - 10 \lambda_{210}),$$
$$M_{T_2} = -M + \frac{q \mu}{M'} (-84 \lambda_{451} - 4 \lambda_{452} + 2 \lambda_{210}),$$
$$M_{T_3} = \frac{1}{2} \left( t_1 \pm \sqrt{t_2^2 + t_3^2} \right),$$

where

$$t_1 = -\frac{2}{5} M + \frac{q \mu}{M'} \left( \frac{576}{5} \lambda_{451} - 11 \lambda_{452} - \frac{302}{15} \lambda_{210} \right),$$
$$t_2 = -\frac{8}{5} M + \frac{q \mu}{M'} \left( -\frac{816}{5} \lambda_{451} + 2 \lambda_{452} + \frac{421}{15} \lambda_{210} \right),$$
$$t_3 = \sqrt{\frac{q \mu}{M'}} \left( 3 \lambda_{451} + 2 \lambda_{452} \right).$$

The authors of [5] introduce two kinds of couplings to gain fermion masses. All the three generations achieve masses from the quartic couplings [5, 8]

$$\begin{bmatrix}
\xi^{(10)}_{ij} \\
\xi^{(10)}_{ij}
\end{bmatrix} (16_i \times 16_j),$$
$$\begin{bmatrix}
\xi^{(126)}_{ij} \\
\xi^{(126)}_{ij}
\end{bmatrix} (16_i \times 16_j),$$
$$\begin{bmatrix}
\lambda^{(45)}_{ij} \\
\lambda^{(45)}_{ij}
\end{bmatrix} (16_i \times 16_j),$$
$$\begin{bmatrix}
\lambda^{(120)}_{ij} \\
\lambda^{(120)}_{ij}
\end{bmatrix} (16_i \times 16_j),$$
$$\begin{bmatrix}
\lambda^{(10)}_{ij} \\
\lambda^{(10)}_{ij}
\end{bmatrix} (16_i \times 144_j),$$
$$\begin{bmatrix}
\lambda^{(10)}_{ij} \\
\lambda^{(10)}_{ij}
\end{bmatrix} (16_i \times 144_j),$$

where $i, j$ are the generation indices. The extra 10-plet and 45-plet extra heavy matters couple only to the third generation fermions through following superpotential [6]

$$W^{16 \times 144 \times 10} = \frac{1}{21} \bar{h}^{(45)} < \hat{\Psi}^*_+ | B | \tilde{T}_+ > \tilde{F}_{\mu \nu}^{(45)},$$
$$W^{(45)}_{mass} = m_F^{(45)} \tilde{F}_{\mu \nu}^{(45)},$$

and

$$W^{16 \times 144 \times 10} = h^{(10)} < \hat{\Psi}^*_+ | B | \tilde{T}_- > \tilde{F}_\mu^{(10)},$$
$$W^{(10)}_{mass} = m_F^{(10)} \tilde{F}_\mu^{(10)},$$

where $\hat{\Psi}$ represents the 16 dimension fermion and $\tilde{T}$ represents the 144 dimension Higgs. We follow the authors of [5] defining

$$f^{(i)} = ih^{(i)},$$

to get the real couplings. From the above coupling forms, we can get all the Yukawa couplings contributing to the fermion masses.

### III. Quark Mixing and Mass Generation

The model of Ref. [5] provides a mechanism of generating fermion masses. From Eqs. (13), (14) and (15), we
We assume the quartic parts of mass matrix for up type quarks are already diagonalized to reduce complexity. The main difference from Ref. 4 is the authors of 4 neglect the fact that the quartic coupling matrices might induce mixing between light quarks and the extra 45- and 10-plots. In this new scenario the down quark mass matrix can be written as follows

\[
M_d = \begin{pmatrix}
\begin{array}{cccc}
16b & 45b & 10b \\
16b & 45b & 10b \\
31 & 32 & 33 & m'_{10}
\end{array}
\end{pmatrix}
\]

where

\[
m'_{10} = \frac{1}{2} f(10) \left( \frac{Q_5}{\sqrt{10}} + \frac{Q_5}{\sqrt{10}} \right),
\]

\[
m''_{45} = -2\sqrt{2} f(45)(P_5),
\]

\[
m_D^{(10)} = -2\sqrt{2} f(10) p, m_D^{(10)} = 2\sqrt{f(10) q},
\]

and all the m’s in the upper-left block are from the quartic couplings.

Noting that the quartic components of the upper-left 3 \times 3 part of \( M_D \) is extremely small, the cubic components will receive little effect from the quartic couplings when diagonalizing. In this sense, we can diagonalize the cubic part first by taking \( m_{33} \sim 0 \) as in 4, and then consider how the quartic couplings take effects. The matrices \( U_{t,b} \) and \( V_{t,b} \) can be found in Appendix A, which is slightly different from Ref. 8. After diagonalizing the cubic part, we get

\[
U_b M_d V_b^T = \begin{pmatrix}
\begin{array}{cccc}
11 & 12 & 13 & 0 \\
21 & 22 & 23 & 0 \\
31 & 32 & 33 & 0 \\
0 & 0 & 0 & 0
\end{array}
\end{pmatrix}
\]

where \( \lambda_2 \) and \( \lambda_3 \) are the eigenmasses of rotated extra heavy fermions.

For \( \lambda_2 \) and \( \lambda_3 \) are extremely large, we can diagonalize the light down-type quark mass matrix

\[
m_d^{ij} = \begin{pmatrix}
\begin{array}{cccc}
11 & 12 & 13 \\
21 & 22 & 23 \\
31 & 32 & 33
\end{array}
\end{pmatrix}
\]

If we denote the Yukawa couplings \( m_d^{ij} = m_d^{ij} V' \delta_i \), in the up quark diagonalized basis \( V' \) is analogous to the CKM matrix.

The matrices diagonalizing \( M_d \) are

\[
V_d^T = V_b^T \times V_{ij} + \mathcal{O}(\frac{\lambda_1}{\lambda_2}, \frac{\lambda_1}{\lambda_3}) + \cdots
\]

\[
\begin{pmatrix}
V_{ud} & V'_{us} & V'_{ub} \\
V'_{cd} & V'_{cs} & V'_{cb} \\
0 & 0 & 0
\end{pmatrix}
\]

\[
= \begin{pmatrix}
V_{td} \cos \theta_{V_t} & V'_{ts} \cos \theta_{V_t} & V'_{tb} \cos \theta_{V_t} \\
V'_{td} \sin \theta_{V_t} & -V'_{ts} \sin \theta_{V_t} & -V'_{tb} \sin \theta_{V_t} \\
0 & 0 & 0
\end{pmatrix}
\]

\[
+ \mathcal{O}(\frac{\lambda_1}{\lambda_2}, \frac{\lambda_1}{\lambda_3}) + \cdots
\]

and

\[
U_d = U_b + \mathcal{O}(\frac{\lambda_1}{\lambda_2}, \frac{\lambda_1}{\lambda_3}) + \cdots
\]

The upper-left 3 \times 3 part of \( V_d^T \) is just the transpose of the CKM matrix. When taking the quartic coupling for the up quarks to be diagonalized, the up quark mass matrix can be diagonalized easily by

\[
V'_u = V_t^T \quad \text{and} \quad U_u = U_t.
\]

The mass matrices for the charged leptons have the same structure as the down quarks for they share the same Yukawa couplings.

Before mass matrix diagonalization, the CKM unitarity violation can also derived from Eq. (1) easily.

\[
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 - |V_{td}|^2 \tan^2 \theta_{V_t},
\]

\[
|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 - |V_{ts}|^2 \tan^2 \theta_{V_t},
\]

\[
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 - |V_{tb}|^2 \tan^2 \theta_{V_t},
\]

\[
|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 - \sin^2 \theta_{V_t}.
\]

For \( V_{td} \) and \( V_{ts} \) are very small in the right-hand side of first two equations, the last two equations give the most important unitarity violation. We denote

\[
\delta_b = |V_{ub}|^2 \tan^2 \theta_{V_t},
\]

\[
\delta_t = \sin^2 \theta_{V_t}.
\]

We need \( |\tan \theta_{V_t}| \ll 1 \) by examining the unitarity bound on the CKM matrix. This is different from Ref. 8, where the authors took \( |\tan \theta_{V_t}| \gg 1 \). Under this new condition, the \( b \) - \( \tau \) unification \( f_b = f_\tau \) gives

\[
\tan \theta_D = \frac{5\sqrt{30}}{83}.
\]

From section V we will see that \( \delta_b \) and \( \delta_\tau \) are given at the percent level by fixing the mass Yukawa couplings.

The other CKM violations are

\[
V_{ud}V_{us} + V_{cd}V_{cs} + V_{td}V_{ts} = -V_{td}V_{ts} \tan^2 \theta_{V_t},
\]

\[
V_{us}V_{ub} + V_{cs}V_{cb} + V_{ts}V_{tb} = -V_{ts}V_{tb} \tan^2 \theta_{V_t},
\]

\[
V_{ud}V_{ub} + V_{cd}V_{cb} + V_{td}V_{tb} = -V_{td}V_{tb} \tan^2 \theta_{V_t}.
\]
IV. DIMENSION-FIVE OPERATORS AND DECAY RATES

In supersymmetric unification models, the dominant mechanism of inducing proton decay is through the color-triplet Higgsino mediation. The resulting dimension-five operators are of the type of LLLL and RRRR. We will focus on LLLL-type only to simplify our discussion although the RRRR-types can also be important.

The Yukawa coupling of the Higgs to the matter multiplets are as follows

\[ W_Y = h^u_i u_i^c Q_i H_u - V^e_i e^c_i Q_i H_d - f^f_i f_i^c L_i H_d \]

\[ + Y^{ij}_L Q_i Q_j H_{cf} + Y^{ij}_L Q_i L_j H_{cf} \]

\[ + Y^{ij}_e e^c_i Q_j H_{cf}, \tag{28} \]

where \( h^u_i \)'s, \( f^f_i \)'s and \( f^e_i \)'s are the Yukawa couplings giving masses and \( Y \)'s are the Yukawa coupling with the color triplet Higgs. \( f \) denotes different color triplets.

The dimension-five operators that cause the nucleon decay can be written explicitly as

\[ W_5 = \frac{1}{M_{T1}} Y^{ij}_L q^c_i Y^{ij}_L (Q_i Q_j) (Q_k L_l) \]

\[ + \frac{1}{M_{T2}} Y^{ij}_e e^c_i Y^{ij}_e (u^c_k e^c_l) (u^c_k d^c_l). \tag{29} \]

The total anti-symmetry in color index requires \( i \neq k \), which implies the dominant mode is \( p \to K \bar{\nu} \).

Dressing of wino to dimension-five operators gives the triangle diagram factor

\[ f(u, d) = \frac{M_2}{m_5^2 - m_3^2} \left( \frac{m_5^2}{M_2^2} \ln \frac{m_5^2}{M_2^2} - \frac{m_3^2}{M_2^2} \ln \frac{m_3^2}{M_2^2} \right), \tag{30} \]

where \( M_2 \) is the wino masses. The resulting four-fermion operators can be written as

\[ \mathcal{L} = Y(ijk)A_S(i, j, k)A_L \epsilon_{\alpha \beta \gamma} \left[ (u^c_i d^c_j) (d^c_j u_k) (f(u_j, e_k) + f(u_i, d_i^c)) + (d^c_i u^c_j) (u^c_j e_k) (f(u_j, d_i^c) + f(d_j, u_k)) \right. \]

\[ + (d^c_i u^c_j) (d^c_j u_k) (f(u_j, e_k) + f(u_i, d_i^c)) + (u^c_i d^c_j) (u^c_j e_k) (f(d_j, u_k) + f(d_i^c, u_k)) \left. \right], \tag{31} \]

where the coupling \( Y(ijk) \) defined as follows

\[ Y(ijk) = \left\{ \frac{1}{M_{T1}} Y^{ij}_L Y^{jk}_L \cos^2 \theta_T + Y^{ij}_L Y^{jk}_L \sin^2 \theta_T + (Y^{ij}_L Y^{jk}_L + Y^{ij}_L Y^{jk}_L) \cos \theta_T \sin \theta_T \right\} \frac{\alpha_2}{2 \pi}. \tag{32} \]

In Eq. (31), the function \( A_S \) refers to the short range renormalization effect between the unification and the supersymmetry breaking scale and \( A_L \) the long range renormalization effect between the supersymmetry scale and 1 GeV. All of these have been investigated thoroughly in [10, 11].

The relevant terms for \( p \to K^+ + \bar{\nu}_\mu \) from Eq. (31) are

\[ \mathcal{L} = A_L \epsilon_{\alpha \beta \gamma} \left( (d^c_i u^c_j) (s^c_j u_k) + (s^c_i u^c_j) (d^c_j u_k) \right) \]

\[ + A_S(c, u, s) Y(2, 1, 2) W_{cs} V_{cd} (f(c, \mu) + f(c, d^c)) \]

\[ + A_S(t, u, s) Y(3, 1, 2) W_{ts} V_{td} (f(t, \mu) + f(t, d^c)). \tag{33} \]

We neglect the \( \nu_\mu \) mode for the smallness of the first generation Yukawa couplings The direct coupling to \( \nu_\tau \) is suppressed by CKM matrix element. Although the coupling to \( ^{10} \nu \) is order 1, \( ^{10} \nu \) is heavy and its contribution to rotated \( \nu_\tau \) is highly suppressed.

Noting that \( Y^{ij}_L \) has only diagonal elements, it will have no contribution. So \( H_{t1} \) or the first term in \( Y(i, j, k) \) does not contribute to Eq. (33).

We can use the chiral Lagrangian technique [13, 14] to obtain hadronic level matrix elements

\[ \langle K^+(u, d)_L S_L | p \rangle = \frac{\beta}{f} \left( 1 + \left( \frac{D}{3} + F \right) \frac{m_N}{m_B} \right), \tag{34} \]

\[ \langle K^+(u, s)_L d_L | p \rangle = \frac{\beta}{f} \frac{2D m_N}{3 m_B}, \tag{35} \]

in the limit \( m_{u, d, s} \ll m_{N, B} \). All the parameters can be found in [10, 11].

V. NUMERICAL RESULTS AND DISCUSSION

In this section, we present some numerical results. The recent Super-Kamiokande bound on proton decay is

\[ \tau_{p \to K^+ + \bar{\nu}} > 1.6 \times 10^{33} \text{yrs.} \tag{36} \]
TABLE I: Proton life and CKM unitarity violation. $\delta_b$ is got with $|V_{ub}| = 0.77$.2

| $\tan \beta$ | 2 | 3 | 6 | 10 | 20 |
|--------------|---|---|---|----|----|
| $\zeta^{(10)}/10^{-20}\text{GeV}^{-1}$ | -0.10 | -0.20 | -0.30 | -0.40 | -0.80 |
| $\zeta^{(120)}/10^{-20}\text{GeV}^{-1}$ | 1.5 | 2.5 | 4.5 | 7.2 | 14.5 |
| $\zeta^{(10)}/10^{-20}\text{GeV}^{-1}$ | -0.40 | -0.10 | -0.10 | -0.70 | -0.90 |
| $\zeta^{(120)}/10^{-20}\text{GeV}^{-1}$ | 5.0 | 2.8 | 2.3 | 6.2 | 7.2 |
| $\lambda_{22}^{(45)}/10^{-19}\text{GeV}^{-1}$ | -8.48 | -1.58 | -1.35 | -7.82 | -6.47 |
| $\lambda_{11}^{(54)}/10^{-19}\text{GeV}^{-1}$ | 22.0 | 4.10 | 3.52 | 20.3 | 16.8 |
| $f^{(45)}$ | 1.24 | 1.04 | 0.96 | 0.95 | 0.95 |
| $f^{(10)}$ | 0.960 | 1.14 | 1.59 | 2.05 | 2.85 |

$\tau_p/(10^{17}\text{yrs})$ 1.4 $\times 10^7$ 84 38 4.3 1.9
$\delta_b/\%$ 2.4 1.7 1.4 1.3 1.3
$\delta_t/\%$ 3.0 2.2 2.0 1.8 1.8

In the present model, the doublet and triplet masses connect to the 144 Higgs mass. We keep a pair Higgs doublets light.

When we setting $M'$ in Eq.9 to be Planck scale $10^{19}\text{GeV}$, we can get the relation between light Higgs masses and 144 Higgs mass. This is a fine tuning problem relating to the doublet-triplet splitting. We choose one of the three doublets to be light while leaving others heavy. Then we automatically get the heavy triplet Higgs masses.

From Eq.13, the condition $\tan \theta_{U_h} \ll 1$ and $\tan \theta_{V_b} \ll 1$ give

$$m_{F}^{(10)} \gg p, \quad m_{F}^{(10)} \gg q,$$

if we take $f^{(10)}$ and $f^{(45)}$ to be of order 1.

The unification scale can be

$$M_{\text{GUT}} \sim 2 \times 10^{16}\text{GeV}.$$ (38)

The supersymmetry breaks at about 1 TeV. All the sfermion masses in Eq.20 are taken to be 1 TeV. The wino mass is taken as $M_2 = 300\text{GeV}$.

We take $p = q = 10^{16}\text{GeV}$ and $m_{F}^{(10)} = m_{F}^{(45)} = 10^{17}\text{GeV}$. After fine tuning $M_{D_2}$ to the order of $10^{16}\text{GeV}$, the other Higgs doublet and triplets are fixed at the order of $10^{16}\text{GeV}$. Because the number of Yukawa couplings in this model are redundant, we can just choose some of them to generate the light fermion masses. Here we take $\lambda_{ij}^{(10)} = g_{ij}^{(126)} = 0$. At the unification scale we fit $\zeta^{(10)}$, $\zeta^{(120)}$, $\zeta^{(120)}$, $f^{(10)}$ and $f^{(45)}$ to get the correct fermion masses. Besides fermion masses, this model can have long enough proton life time even without cancellation introduced in Ref.8, which implies vanishing down-type fermion masses of the second generation. We get the longest proton life times without affecting the fermion masses by fine tuning $\lambda^{(45)}$ and taking $\lambda^{(54)} = -7/27\lambda^{(45)}$. The results are given in table II while we take all the parameters in Eqs.33,35 as 12.

$\beta = 0.0118\text{GeV}^3$, $D = 0.8$, $F = 0.47$, $f = 0.131\text{GeV}$, $m_N = 0.94\text{GeV}$, $m_B = 1.15\text{GeV}$. (39)

From table II, the longest possible proton life time decreases when $\tan \beta$ increases. The present model dilute the relation between fermion masses and dimension-five operators. We can choose some parameters to suppress the dimension-five operators without affecting the fermion masses. Even for very large $\tan \beta$, this model could have long enough proton life time.

The unitarity breaking of the CKM matrix can be obtained directly from Eq.24. Unitarity is good for the first and second columns of CKM up to $O(\lambda_{22}, \lambda_{23})$, because $\lambda_2$ and $\lambda_3$ are extra fermion masses at about $10^{16}\text{GeV}$. For the first two equations of Eqs.24, unitarity violations are about $10^{-6}$ due to the smallness of $V_{td}$ and $V_{ts}$, within experimental constraints[7]. This model predicts relatively large CKM breaking for $\delta_b$ and $\delta_t$. Their values are at percentage level.

VI. SUMMARY

In this work we analyze the supersymmetric SO(10) model with a unified 144+144 Higgs. We introduce the fermion mixing, and find large unitarity violations of the CKM matrix. We find proton life time is in agreement with the experiment bound for a rather large value of $\tan \beta$.

This work was supported in part by the National Natural Science Foundation of China (NSFC) under the grant No. 10435040.

APPENDIX A: MASS MATRICES AND YUKAWA COUPLINGS

In this section we list the detailed results of the mass matrices and the diagonalization matrices. From Eqs.13,14 and 15 we can get all the couplings for the component fields. We put all of them in tables II and III. The Baryon-Lepton number violating terms can also be found in Ref.8.

From tables II and III we can easily write out the mass matrix. The matrices that used to diagonalize the lower right 3 $\times$ 3 part of mass matrix Eq.18 are given by

$$U_{b(t,\tau)} =$$

$$= \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & \cos \theta_{U_{b(t,\tau)}} - \sin \theta_{U_{b(t,\tau)}} & 0 & 0 \\
0 & 0 & \sin \theta_{U_{b(t,\tau)}} \cos \theta_{U_{b(t,\tau)}} & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix},$$

and

$$V_{b(t,\tau)} =$$

$$= \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & \cos \theta_{V_{b(t,\tau)}} - \sin \theta_{V_{b(t,\tau)}} & 0 & 0 \\
0 & 0 & \sin \theta_{V_{b(t,\tau)}} \cos \theta_{V_{b(t,\tau)}} & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}.$$ (A2)
TABLE II: All the quartic couplings that contribute to masses and Baryon-Lepton number violations. Generation indices are neglected. All the couplings are matrices of the three generations.

| Coupling constants | Mass terms | Baryon number violating |
|--------------------|------------|-------------------------|
| \( J_1 \) \( M^{ij}M_jP_i^aP_k^b \) | \( 2 \left( -\lambda^{(45)} - \lambda^{(54)} + 8\xi^{(10)} + \frac{8}{3} \epsilon^{(120)} \right) \) | \(-3p q^{a\alpha} d_c^a + e^{a\beta} e^Lb_\alpha P_a \) \( 2p q^{a\alpha} d_c^a - 3p q^{a\beta} e^Lb_\alpha P_a \) |
| \( J_2 \) \( \epsilon_{ijklm} M^{ijkl}P_i^aP_j^bP_k^cP_l^d \) | \( \frac{1}{\sqrt{5}} \left( -\lambda^{(45)} + \frac{4}{3} \xi^{(120)} \right) \) | \(-24p u_c^a q^{b\alpha} e^Lb_\alpha P_a \) \( 16p \left( u_c^a e^+ - \epsilon_{a\alpha\gamma} Q^{b\gamma} Q^{c\alpha} \right) P^a \) |
| \( J_3 \) \( \epsilon_{ijklm} M^{ijkl}P_i^aP_j^b \) | \( \frac{1}{\sqrt{5}} \left( -\lambda^{(45)} - \frac{4}{3} \lambda^{(54)} + 4\xi^{(120)} \right) \) | \(-20p u_c^a q^{b\alpha} e^Lb_\alpha P_a \) \( 20p \left( u_c^a e^+ - \epsilon_{a\alpha\gamma} Q^{b\gamma} Q^{c\alpha} \right) P^a \) |
| \( K_1 \) \( M^{ij}M_jQ_i^aQ_k^b \) | \( 2 \left( \frac{2}{15} \theta^{(120)} + \frac{2}{3} \chi^{(120)} \right) \) | \(-3q q^{a\alpha} d_c^a + e^{a\beta} e^Lb_\alpha Q^a \) \( 2q \left( -\epsilon_{a\beta} u_j^a d_c^a - Q^{b\alpha} Lb_\alpha \right) Q_a \) |
| \( K_2 \) \( M^{ij}M_jQ_i^aQ_k^b \) | \( \frac{1}{\sqrt{5}} \left( \theta^{(120)} + 8\xi^{(10)} + 4\xi^{(120)} \right) \) | \(-3q q^{a\alpha} d_c^a + 3q e^{a\beta} e^Lb_\alpha Q^a \) \( 2q \left( -\epsilon_{a\beta} u_j^a d_c^a - Q^{b\alpha} Lb_\alpha \right) Q_a \) |
| \( K_3 \) \( M^{ij}M_jQ_i^aQ_k^b \) | \( \frac{1}{\sqrt{5}} \left( -\theta^{(120)} - \frac{2}{3} \xi^{(120)} \right) \) | \( 4q q^{a\alpha} d_c^a + e^{a\beta} e^Lb_\alpha Q^a \) \( -5q \left( Q^{b\alpha} Lb_\alpha - \epsilon_{a\beta} u_j^a d_c^a \right) Q_a \) |

They are just the matrices used in Ref. [9], when eliminating the upper-left 2 x 2 identity matrices.

The matrices used for \( t \) and \( \tau \) have the same form. All the matrices elements are given by [6]

\[
\tan \theta_{U_3} = \frac{-f^{(10)}q}{\sqrt{2}m_f^{(10)}}, \quad \tan \theta_{U_5} = \frac{\sqrt{2}f^{(45)}_p}{m_f^{(45)}}, \quad (A3)
\]

\[
\tan \theta_{U_3} = \frac{6\sqrt{2}f^{(45)}_p}{m_f^{(45)}}, \quad \tan \theta_{U_5} = \frac{3\sqrt{2}f^{(10)}q}{4m_f^{(10)}}, \quad (A4)
\]

and

\[
\tan \theta_{U_3} = \frac{4\sqrt{2}f^{(45)}_q}{m_f^{(45)}}, \quad \tan \theta_{U_5} = \frac{\sqrt{2}f^{(45)}_p}{m_f^{(45)}}. \quad (A5)
\]

When taking the quartic couplings for up type quarks to be diagonalized, we can easily get the Yukawa coupling constants for masses.

\[
h_u = \left[ \frac{24p \sqrt{3} \xi^{(10)} + 6p \sqrt{3} \left( -\lambda^{(45)} + \frac{4}{3} \xi^{(120)} \right)}{\sqrt{3} \xi^{(10)} + \frac{7}{12} \lambda^{(45)} - \frac{9}{4} \xi^{(54)} + \frac{20}{9} \xi^{(120)} \right] \sin \theta_D, \quad (A6)
\]

\[
f_{d} = \frac{q \cos \theta_D}{\sqrt{3}} \left( \frac{11}{15} \theta^{(120)} + 8\xi^{(10)} - \lambda^{(10)} + \frac{56}{3} \xi^{(120)} \right) + 2\sqrt{6}q \left( \frac{1}{3} \theta^{(120)} - 8\xi^{(10)} - \frac{4}{3} \xi^{(120)} \right) \sin \theta_D, \quad (A7)
\]

\[
f_{e} = \frac{3q \cos \theta_D}{\sqrt{5}} \left( -\frac{3}{15} \theta^{(120)} + 8\xi^{(10)} - \lambda^{(10)} - \frac{8}{3} \xi^{(120)} \right) + 2\sqrt{6}q \left( \frac{1}{3} \theta^{(120)} - 8\xi^{(10)} - \frac{4}{3} \xi^{(120)} \right) \sin \theta_D, \quad (A8)
\]

\[
h_{t} = 3 \sin \theta_{U_3} \cos \theta_{U_5} f^{(45)} \left( \frac{\cos \theta_D}{\sqrt{10}} + \frac{\sin \theta_D}{2\sqrt{3}} \right), \quad (A9)
\]

\[
f_{b} = \frac{1}{2} \sin \theta_{U_3} \cos \theta_{U_5} f^{(10)} \left( \frac{\cos \theta_D}{\sqrt{10}} + \frac{\sin \theta_D}{2\sqrt{3}} \right), \quad (A10)
\]
and
\[ f_\tau = -\frac{1}{2} \sin \theta_{U\tau} \cos \theta_{V\tau} f^{(10)} \left( -\frac{\cos \theta_D}{\sqrt{10}} + \frac{\sqrt{3} \sin \theta_D}{2} \right). \]

(A11)

We have neglected the generation indices in \( h_u, f_d \) and \( f_\tau \), which should be \( 3 \times 3 \) matrices. \( h_u, f_d \) and \( f_\tau \) are the Yukawa couplings after diagonalization of cubic coupling matrices.

The couplings for Baryon-Lepton number violating terms \( QQ' \)'s and \( QL \)'s used in section IV are given by
\[ Y_{Q1} = -32q \left( \frac{2}{15} \tilde{q}^{(126)} + \zeta^{(10)} + \frac{2}{3} \xi^{(120)} \right), \]
\[ Y_{Q2} = \frac{16p}{\sqrt{5}} \zeta^{(10)} + \frac{4p}{\sqrt{5}} \left( -\lambda^{(45)} + \frac{4}{3} \xi^{(120)} \right), \]
\[ Y_{Q3} = 5\sqrt{2}p \left( 3\lambda^{(54)} + \lambda^{(45)} - \frac{16}{3} \xi^{(120)} - 4\xi^{(10)} \right), \]
\[ Y_{L1} = 2p(-\lambda^{(45)} + 5\lambda^{(54)} - 16\xi^{(10)} - \frac{16}{3} \xi^{(120)}), \]
\[ Y_{L2} = \frac{p}{\sqrt{5}} \left( -\frac{14}{15} \tilde{q}^{(126)} + 3\lambda^{(10)} - 16\xi^{(10)} - \frac{64}{3} \xi^{(120)} \right), \]
\[ Y_{L3} = \sqrt{2}q \left( 40\xi^{(10)} - \frac{1}{15} \tilde{q}^{(126)} - \frac{16}{3} \xi^{(120)} \right), \]

(A12) \hspace{1cm} (A13) \hspace{1cm} (A14) \hspace{1cm} (A15) \hspace{1cm} (A16) \hspace{1cm} (A17)

APPENDIX B: FIELDS NORMALIZATION

In deducing table II and III we have used
\[ \partial \widehat{P}_{k}^{\alpha} \partial \widehat{P}_{\alpha}^{(k)} = \partial \widehat{P}_{k}^{\alpha} \partial \widehat{P}_{\alpha}^{(k)} + \partial \widehat{P}_{k}^{\alpha} \partial \widehat{P}_{\alpha}^{(k)} + \cdots. \]

(B1)

So the triplets and doublets are normalized according to
\[ (\partial \widehat{Q}_{\alpha}, \partial \widehat{P}_{\alpha}^{(k)}) \rightarrow \frac{\sqrt{3}}{2\sqrt{2}} (\partial \widehat{Q}_{\alpha}, \partial \widehat{P}_{\alpha}^{(k)}), \]
\[ (\partial \widehat{Q}_{\alpha}, \partial \widehat{P}_{\alpha}^{(k)}) \rightarrow \frac{\sqrt{7}}{2} (\partial \widehat{Q}_{\alpha}, \partial \widehat{P}_{\alpha}^{(k)}). \]

(B2)

The other fields’ normalizations can be found in [5, 6].

[1] H. Georgi, in Particles and Fields 1974, proceedings of the Williamsburg Meeting of the Division of Particles and Fields of the American Physical Society, edited by C. E. Carlson p. 575 (AIP, New York, 1975).
[2] H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975).
[3] N. Sakai and T. Yanagida, Nucl. Phys. B 197, 533 (1982).
[4] S. Weinberg, Phys. Rev. D 26, 287 (1982).
[5] K. S. Babu, I. Gogoladze, P. Nath, and R. M. Syed, Phys. Rev. D 72, 095011 (2005).
[6] K. Babu, I. Gogoladze, P. Nath, and R. Syed, Phys. Rev. D 74, 075004 (2006).
[7] C. Amsler et al. (Particle Data Group), Phys. Lett. B667, 1 (2008).
[8] P. Nath and R. M. Syed, Phys. Rev. D 77, 015015 (2008).
[9] T. Goto and T. Nihei, Phys. Rev. D 59, 115009 (1999).
[10] J. Hisano, H. Murayama, and T. Yanagida, Nucl. Phys. B 402, 46 (1993).
[11] P. Nath and R. Arnowitt, Phys. Rev. D 38, 1479 (1988).
[12] J. Ellis, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B 202, 43 (1982).
[13] M. Claudson, M. B. Wise, and L. J. Hall, Nucl. Phys. B 195, 297 (1982).
[14] S. Chadha and M. Daniels, Nucl. Phys. B 229, 105 (1983).
[15] Y. Aoki, C. Dawson, J. Noaki, and A. Soni, Phys. Rev. D 75, 014507 (2007).