Spin Torque and its Relation to Spin Filtering

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(Dated: March 22, 2022)

The spin torque exerted on a magnetic moment is a reaction to spin filtering when spin-polarized electrons interact with a thin ferromagnetic film. We show that, for certain conditions, a spin transmission resonance (STR) gives rise to a failure of spin filtering. As a consequence, no spin is transferred to the ferromagnet. The condition for STR depends on the incoming energy of electrons and the thickness of the film. For a simple model we find that when the STR condition is satisfied, the ferromagnetic film is transparent to the incoming electrons.

PACS numbers: 75.70.Ak, 72.25.-b, 85.75.-d

Since the spin torque problem was conceptualized\cite{1,2} and observed experimentally\cite{3,4,5,6}, enormous attention has been paid to the spin torque of a magnetic moment, driven by a spin-polarized current, both theoretically\cite{7,8,9,10,11,12} and experimentally\cite{13,14,15,16,17}. These discoveries also open a door to new magnetic random access memory\cite{14}, spin-wave amplification by stimulated emission of radiation\cite{13}, and nanoscale microwave sources\cite{17}. However, it is still necessary to understand and explore the physics underlying this phenomenon. In particular, the significance of the thickness of the ferromagnetic film has not been fully studied at present.

The key interaction associated with the spin torque is an interaction between the incoming spins (\(s\)) and the magnetic moment (\(M\)): 

\[
-2J_H s \cdot M,
\]

where \(J_H\) is the coupling strength. Thus, the Hamiltonian for this problem has two ingredients: one is the kinetic energy of the incoming electrons and the other the interaction energy with the moment in the film. The magnetic moment is assumed to originate from the local spins in the ferromagnet and its magnitude \(M_0\) is a constant.

As we illustrate in what follows, the spin torque arises as a reaction to spin filtering. Suppose a spin polarized electron beam enters normally to the ferromagnetic film. After spin filtering, incoming electrons lose their spin components perpendicular to the magnetic moment. Because of conservation laws, this spin is transferred as a torque exerting on the moment. Interestingly, such a mechanism for ‘spin transfer’ implies that no spin torque will take place if the incoming spin is not filtered. In this paper we scrutinize a simple model to understand what conditions are necessary for zero spin transfer and concomitant absence of spin filtering.

No spin will be transferred from the incoming spins to the magnetic moment when the condition for a spin transmission resonance (STR) is satisfied. STR is a purely quantum mechanical phenomenon and is similar to the well known particle transmission resonance (PTR) \cite{18} for a potential barrier. In our case the spin state of incoming electrons remains unaltered even after interacting with the magnetic moment in the ferromagnet and, thus, no spin filtering occurs. Since the spin torque is a reaction to spin filtering, the magnetic moment does not experience a spin torque. For a given energy of incoming electrons, this phenomenon is periodic with a thickness depending only on the interaction energy and fundamental constants like the electron mass. STR is one of the unique characteristics of spin transfer from a current to a moment, and can distinguish this mechanism unambiguously from that obtained through an applied magnetic field (in this case the field would be induced by the applied current).

When a spin torque is exerted on the magnetic moment, it will align parallel to the direction of incoming spins after a relaxation time \(\tau_0\). Thus, a signature of zero spin transfer is that \(\tau_0 \rightarrow \infty\). Using the adiabatic approximation, we derive the equations of motion for the magnetic moment and obtain an analytic form of \(\tau_0\). The adiabatic approximation is valid if \(\tau_0\) is much longer than a time scale incoming electrons spend on interacting with the magnetic moment. Obviously, this approximation is always applicable near STR. Plots of \(\tau_0\) as a function of the thickness of the ferromagnetic film and the energy of incoming electrons illustrate the physics associated with STR. In the derivation of the equations of motion for the magnetic moment, it is necessary to obtain the electron wave function inside the film to evaluate the expectation value of the spin operator. For simplicity, we consider a single-domain ferromagnet as in Refs.\cite{1,2,3,4,5,6}.

Depicted in Fig. 1 is the quantum mechanical problem we consider to obtain the electron wave function. Region II represents the ferromagnetic film with the thickness \(L\) while regions I and III are non-ferromagnetic. Since the film is parallel to the YZ plane, the Schrödinger equation relevant to our problem is one-dimensional. We suppose incoming electrons with momentum \(k\) along the X axis. The direction of the polarized spins is chosen to be Z axis. Then the incoming wave function \(|\psi_{in}\rangle = |+\rangle e^{ikx}\), where \(|+\rangle\) is the spin-up state in the lab frame. The incoming energy \(\epsilon_{in} = (\hbar k)^2/2m\), where \(m\) is the electron mass. We will utilize a normalization constant \(C\) to obtain dimensionless equations of motion for the magnetic
a given direction of the magnetic moment, we solve the Schrödinger equation to obtain the electron wave function.

In region I, the wave function consists of the incoming \( |\psi_{in}\rangle \) and reflected \( |\psi_{re}\rangle \) wave function. In region III, there is only the transmitted \( |\psi_{tr}\rangle \) wave function. The reflected and transmitted wave functions are expressed in terms of eigenstates \( |\chi_\sigma\rangle \) of the interaction \( 2J_H \mathbf{M} \cdot \mathbf{s} \) such that \( 2J_H \mathbf{M} \cdot \mathbf{s}|\chi_\sigma\rangle = \pm J_H M_0 |\chi_\sigma\rangle \), where \( \sigma = \uparrow \) or \( \downarrow \), and \( +(-) \) is for \( \sigma = \uparrow (\downarrow) \). We assume that the ferromagnetic film is sufficiently clean that the mean free path of electrons within the film is much longer than the film thickness. Therefore the convergence factor introduced in Refs. \[2, 12\] is not necessary, and we can investigate the significance of the thickness explicitly. Because of the interaction, the momentum is spin split; \( k_\sigma = \sqrt{k^2 + 2mJ_H M_0} \), depending on the eigenstates \( |\chi_\sigma\rangle \). If \( \epsilon_{in} \) is less than \( J_H M_0 \), \( k_\downarrow = i\kappa_\downarrow \) becomes pure imaginary where \( \kappa_\downarrow = \sqrt{2mJ_H M_0 - k} \) and the corresponding wave function decays exponentially. In this case, however, STR will not occur, as one might expect based on the usual PTR conditions. Now the wave functions in regions I, II, and III can be written as follows:

\[
|\psi_I\rangle = |+\rangle e^{ikx} + \left[R_\uparrow |\chi_\uparrow\rangle \langle\chi_\uparrow| + R_\downarrow |\chi_\downarrow\rangle \langle\chi_\downarrow|\right] e^{-ikx} \\
|\psi_{II}\rangle = (A_\uparrow e^{ik_\uparrow x} + B_\uparrow e^{-ik_\uparrow x}) |\chi_\uparrow\rangle \langle\chi_\uparrow| + (A_\downarrow e^{ik_\downarrow x} + B_\downarrow e^{-ik_\downarrow x}) |\chi_\downarrow\rangle \langle\chi_\downarrow| \\
|\psi_{III}\rangle = \left[T_\uparrow |\chi_\uparrow\rangle \langle\chi_\uparrow| + T_\downarrow |\chi_\downarrow\rangle \langle\chi_\downarrow|\right] e^{ikx}.
\]

The coefficients \( R_\sigma, A_\sigma, B_\sigma, \) and \( T_\sigma \) are determined by the boundary conditions of wave functions at \( x = 0 \) and \( x = L \); namely, \( \langle \pm |\psi_I(0)\rangle = \langle \pm |\psi_{II}(L)\rangle = |\pm |\psi_{III}(L)\rangle \), and similar relations for their derivatives. Some straightforward algebra yields

\[
R_\sigma = \frac{(k^2 - k_\sigma^2) \left(1 - e^{2ik_\sigma L}\right)}{(k + k_\sigma)^2 - (k - k_\sigma)^2 e^{2ik_\sigma L}} \\
A_\sigma = \frac{2k(k_\sigma + k)}{(k + k_\sigma)^2 - (k - k_\sigma)^2 e^{2ik_\sigma L}} \\
B_\sigma = \frac{2k(k_\sigma - k)e^{2ik_\sigma L}}{(k + k_\sigma)^2 - (k - k_\sigma)^2 e^{2ik_\sigma L}} \\
T_\sigma = \frac{4k k_\sigma e^{i(k_\sigma - k)L}}{(k + k_\sigma)^2 - (k - k_\sigma)^2 e^{2ik_\sigma L}}.
\]

It is worthwhile noting the similarity between our case and the one-dimensional potential barrier problem. At a glance, one can see the above coefficients are exactly the same as those for the potential barrier except for their two-fold nature due to the spin index. In fact, the two-fold nature can be mapped onto a potential \textit{well} for \( \sigma = \uparrow \) and a potential \textit{barrier} for \( \sigma = \downarrow \). Note that PTR takes place in a potential barrier as well as in a potential well. Therefore, conditions for STR are effectively those for PTR for the corresponding barrier and well at the same time.

For PTR, zero reflectance guarantees a transmission resonance; this is not the case for STR. If \( k_\sigma L = n_\sigma \pi \) \((n_\sigma = 1, 2, 3, \cdots)\), then \( R_\sigma = 0 \). However, STR does not occur yet because \( T_\sigma = e^{i(k_\sigma - k)L} \); in other words, it is not guaranteed that \( \langle - |\psi_{III}\rangle = 0 \), which means the spin state in region III cannot be represented only by \( |+\rangle \). Since the incoming wave function has only \(|+\rangle\), non-zero \( \langle - |\psi_{III}\rangle \) indicates that spin is transferred to the magnetic moment. The condition for STR is \( k_\sigma L = (2n_\sigma - 1)\pi \) or \( 2n_\sigma \pi \) with \( n_\perp < n_\parallel \) due to \( k_\perp < k_\parallel \). If the above condition is satisfied, we obtain \( \langle + |\psi_{III}\rangle = e^{i\kappa(x - L)} \) and \( \langle - |\psi_{III}\rangle = 0 \). This means that the transmitted wave function remains unaltered even after interacting with the magnetic moment in the ferromagnetic thin film except for an additional phase \( e^{-ikL} \) depending only on the thickness. Consequently, spin filtering fails completely.
under this condition.

Let us examine the condition for STR in more detail. Suppose \(2n_\sigma \pi\) and the incoming energy is larger than the interaction energy \(J_HM_0\) by a factor of \(\epsilon\); \(\epsilon n_\sigma = \eta J_HM_0\), and \(k_{\perp} = \sqrt{\eta - 1}k_0L\) and \(k_{\parallel} = \sqrt{\eta - 1}k_0L\), where \(k_0 = \sqrt{2\eta J_HM_0}\). We now obtain constraints for \(\eta\) and \(L\) under the STR condition: \(\eta = \frac{1 + (n_\sigma/n_{\perp})^2}{1 - (n_\sigma/n_{\perp})^2}\) and \(L = n_\sigma \sqrt{2[1 - (n_\sigma/n_{\perp})^2]}L_0\), where \(L_0 = \pi/k_0\). For \(k_\sigma L = (2n_\sigma - 1)\pi\), a similar constraint can be obtained by replacing \(n_\sigma\) with \(n_{\perp} - 1/2\). This analysis tells us that the STR condition can be satisfied by controlling \(\eta\) and \(L\); for example, if \(\eta = 5/4\) and \(L = 2L_0\), then STR takes place. Another example is for \(\eta = 5/3\) and \(L = \sqrt{6}L_0\). A more interesting result of the analysis is that for a given value of \(\eta\), the thickness \(L\) is periodic. This arises because \(\eta\) is determined by the ratio \(n_\sigma/n_{\perp}\) while \(L\) is proportional to \(n_{\perp}\). Therefore, for a given value of \(\epsilon\), say, \(\eta = 5/3\), STR will take place when \(L = \sqrt{6}L_0\). 2\(\sqrt{6}L_0\), 3\(\sqrt{6}L_0\), \(\cdots\). This is inherently a quantum mechanical property and completely different from the effects of a current-induced magnetic field on the magnetic moment.

Using the wave function in region II, \(\langle \psi_I \rangle\), we evaluate the expectation value of the spin operators to derive equations of motion for the magnetic moment for an arbitrary case. If the STR condition is met, we will be able to see its signature in the equations. Since the spin expectation is evaluated by \(\langle \sigma_i \rangle = (1/2)\langle \psi_I \mid \sigma_i \mid \psi_I \rangle\), where \(\sigma_i\) \((i = x, y, z)\) are Pauli matrices, we obtain

\[
\langle s_x \rangle = \alpha m_x - \beta m_y - \gamma m_z m_x \\
\langle s_y \rangle = \alpha m_y + \beta m_x - \gamma m_z m_y \\
\langle s_z \rangle = \alpha m_z + \gamma (1 - m_z^2)
\]

where

\[
\alpha = \frac{1}{4} \int_0^L dx \left[ |C_x|^2 (1 + m_z) - |C_{\parallel}|^2 (1 - m_z) \right] \\
\beta = \frac{1}{2} \int_0^L dx \text{Im} \left[ C_x C_{\parallel}^* \right] \\
\gamma = \frac{1}{2} \int_0^L dx \text{Re} \left[ C_x C_{\parallel}^* \right]
\]

with \(C_x = A_x e^{ik_xx} + B_x e^{-ik_xx}\). Note that we also take an average over region II to take into account a net effect of the incoming spins on the magnetic moment as done in Refs. [2][11][12].

The equation of motion for the magnetic moment corresponding to the interaction is \(dM/dt = 2\gamma_0 J_H \mathbf{M} \times \langle s \rangle\), where \(\gamma_0\) is the gyromagnetic ratio. In order to obtain dimensionless equations we need to restore the normalization constant \(C\), with which the intensity of incoming beam can be controlled because \(|C|^2\) represents the number \((N_e)\) of incoming electrons per unit length. We also introduce an effective local spin \(S_{local}\) to semiclassically describe \(M_0 = \gamma_0 S_{local}\). A dimensionless time \(\tau\) can be defined as \(\tau = j_0 t\), where \(j_0 = (\pi N_e/S_{local})k_0/m\) is the one-dimensional current density with \((\pi N_e/S_{local})\) electrons per unit length. Then, we obtain \(d\mathbf{m}/d\tau = \mathbf{m} \times \langle s \rangle\):

\[
\frac{d\mathbf{m}}{d\tau} = -\beta m_x m_x + \gamma m_y \\
\frac{d\mathbf{m}}{d\tau} = -\beta m_y m_y - \gamma m_x \\
\frac{d\mathbf{m}}{d\tau} = \beta (1 - m_z^2).
\]

Note that the equation for \(m_z\) does not depend on other components of \(\mathbf{m}\). This equation can be solved analytically and we obtain

\[
m_z(\tau) = \tanh \left[ \beta \tau + \frac{1}{2} \ln \left( \frac{1 + m_0}{1 - m_0} \right) \right]
\]

where \(m_0\) is the initial value of \(m_z\) at \(\tau = 0\) and \(|m_0| < 1\). We found for \(\eta \leq 1\), \(\beta\) is positive definite. However, for \(\eta > 1\), \(\beta \geq 0\) depending on \(\eta\) and the thickness \(L\). When STR occurs, it can be shown analytically that \(\beta = 0\). Then one may assume that the magnetic moment will precess with a frequency \(\gamma\) based on Eq. (3): however, one can show that \(\gamma\) also vanishes under the condition for STR. Consequently, the magnetic moment remains a constant in this case.

If \(\beta > 0\), we obtain an asymptotic expression for \(m_z(\tau)\):

\[
m_z(\tau) \approx 1 - \frac{1 - m_0}{1 + m_0} e^{-2\beta \tau}
\]

It is natural to define the relaxation time \(\tau_0\) as \(1/2\beta\) based on the above equation. In Fig. 2 we plot \(\tau_0\) as a function of \(L\) for given values of \(\eta\). When the condition for STR is satisfied, \(\tau_0 \rightarrow \infty\) because \(\beta \rightarrow 0\). The singular behavior is periodic as we analyzed earlier. The period depends on \(\eta\). For \(\eta = 5/4\), it is \(2L_0\) while for \(\eta = 5/3\) it is \(\sqrt{6}L_0\). For comparison, \(\eta = 1\) (solid) and \(\eta = 3/2\) (dashed curve) are also plotted. The singularities illustrated in Fig. 2 are strong; for example, \(\tau_0 \approx (L - 2L_0)^{-2}\) for \(\eta = 5/4\). Therefore, it should be measurable as long as the ferromagnetic film is reasonably smooth. We also plot \(\tau_0\) as a function for given values of \(L\) in Fig. 3. For \(L = 2L_0\) and \(\sqrt{6}L_0\), \(\tau_0\) increases without bound when \(\eta = 5/4\) and \(5/3\), respectively. For any other value of \(L\), \(\tau_0\) is finite as shown by a solid curve \((L = L_0)\) and by a dashed curve \((L = 2.2L_0)\). This plot can be used for experiments to test our predictions because it does not require a set of ferromagnetic films with varying width, as required for Fig. 2. In our estimation, \(L_0 \approx 5\text{nm}\) assuming \(J_H M_0\) is of order \(10^{-2}\text{eV}\) (~the s-d exchange energy[13]). A typical relaxation time can be also estimated to be of order \(10^{-9}\) second if \((\pi N_e/S_{local}) \approx 10^4\text{cm}^{-1}\), which corresponds to an electron beam with intensity of order \(10^{19}/\text{sec/cm}^2\). Incoming electrons with \(\epsilon_{\text{in}} \approx 10^{-2}\text{eV}\) spend only \(10^{-14}\) second per nanometer within the ferromagnetic film. Under the
adiabatic approximation, the beam intensity may not be too large because if so, $\tau_0$ could be comparable with a typical time incoming electrons spend on interacting with the magnetic moment. However, the adiabatic approximation is always applicable near STR regardless of the beam intensity since $\tau_0 \to \infty$ under the STR conditions. For an experimental test one could use the experimental setup in Refs. [21, 22] or the ferromagnet-normal-metal-ferromagnet (F1NF2) junction. If the STR condition is satisfied in F1, then F1 is transparent to the incoming spins. Therefore, the spin torque will appear only in F2.

In summary, we investigated the conditions for spin transmission resonance (STR), and their consequences. We found that STR will occur for a variety of widths which are multiples of a fundamental thickness (for given electron energy). This is a quantum mechanical property and indicates the significance of the thickness of the ferromagnetic film. When the STR condition is satisfied, the magnetic moment in the ferromagnetic film remains unaltered since no spin transfer occurs.

[1] J.C. Slonczewski, J. Magn. Magn. 159, L1 (1996); 195, L261 (1999)
FIG. 2: Relaxation time $\tau_0$ as a function of $L$ for given values of $\eta$. When the condition for STR is satisfied, $\tau_0 \to \infty$ because $\beta \to 0$. The singular behavior is periodic. The period depends on $\eta$. For $\eta = 5/4$, it is $2L_0$ while for $\eta = 5/3$ it is $\sqrt{6}L_0$. For comparison, we also plot $\tau_0$ for $\eta = 1$ (solid) and $3/2$ (dashed curve). Note that for the estimates of parameters given in the text, a value of $\tau_0 \sim 10$ on this plot corresponds to a real relaxation time $\sim 10^{-9}$ second.

FIG. 3: Relaxation time $\tau_0$ as a function $\eta$ for given $L$. For $L = 2L_0$ and $\sqrt{6}L_0$, $\tau_0 \to \infty$ when $\eta = 5/4$ and $5/3$, respectively. For any other value of $L$, $\tau_0$ is finite as shown by the solid curve ($L = L_0$) and the dashed curve ($L = 2.2L_0$).