Analysis on physical properties of micropolar nanofluid past a constantly moving porous plate

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Abstract. The computational analysis is presented for boundary layer heat and mass transfer flow of hydro magnetic micropolar nanofluid flow. In the flow model, viscosity of the fluid is taken as temperature-dependent and varies linearly and the other physical properties such as radiative heat flux, the magnetic field, the viscous dissipation, chemical reaction are additionally assumed in the energy equation and spices concentration equation respectively. The PDEs representing the fluid flow have been changed into a framework of dimensionless ODEs and explained mathematically through the 4th order R-K and NS shooting technique. Temperature distribution, velocity distribution, micro rotation, and concentration distribution are explored graphically for a series of solid volume fraction (0<ϕ<2) of nano-solid particles. All the findings for various flow parameters agreed perfectly with physical situation of the flow. It is observed that for increasing value of magnetic parameter, the concentration and temperature of the micropolar nanofluid near the boundary layer declines and increasing value of the volume fraction of nano-solid particle ϕ leads to decrease in velocity and micro rotation of the fluid within the boundary layer decreases.

Keywords. MHD Nanofluid, Boundary layer, Micropolar, heat transfer, and mass transfer

1. Introduction

In present technology, there is an important development in the utilization of colloids mass-production by nano-sized particles scattered in a base fluid. Heat controlling has been accepted as important tools in the recent techno-industrial world and it should be used effectively. Prior research shows that thermophysical properties can be enhanced by nanofluids than base fluids such as oil or water. Das, S. K et al. (2006), Liu M.S et al. (2005), and Choi et al. (2004). Yu W, France et al. (2008) provided an excellent review on the recent state of nanofluid innovation of heat transfer applications and area in which the data are presently questionable or disagreeing.

Due to the increasing production of metal and polymer sheets in engineering industries, the fluid dynamics of continuously moving plate plays an important role. The theory of continuously moving surfaces was introduced by Sakiadis (1961). Due to their cherished influences in various industrial and engineering processes such as plastic extrusion and polymer, the process of crystal development makes a micropolar fluid flow is a significant space of exploration. The concept of micropolar fluid was initiated by Eringen (1964). Later, many researchers have put forward their work on micropolar fluid past a constantly stretching
plate and analyze the effect of energy transport on it [V.M. Soundalgekar, H.S. Takhar, (1983), A. Rapits, (1998)].

Fluid viscosity is another important property which quantify the flow but in most of the previous studies on heat transfer in fluid flow it is supposed to be constant, however, if the temperature variation is large this assumption is not valid. So, in the present work viscosity is assumed as variable. Moreover, due to engineering applications, radiation energy plays a significant role in the energy transport flow of moving fluid. The problem of heat and mass transfer flow of micropolar fluid in the presence of radiation is analyzed by Loganathan and Golden Stepha (2012). Chemical reaction effects of the above problem were studied by Loganathan and Golden stepha (2012).

Abdul Rehman and Nadeem (2012) studied the mixed convective heat transfer effect of nanofluid over a cylinder with micropolar fluid is a base and the solution is obtained by HAM. Bourantas and Loukopoulos (2014) studied the natural convection of micropolar model in a square cavity and the analysis are performed for different nano particles. Kazimierz Rup and Konrad Nering (2014) studied unsteady natural convection of nanofluids using explicit finite difference scheme.

In literature, several investigations done on the MHD influences over the stretching sheet in numerous heat transfer-related flows such as MHD power producers, solar energy devices, and in biomedical field for hyperthermia cancer cure, brain tumor treatment. Due to their cherished influences in various industrial and engineering processes such as plastic sheets extrusion and polymer, the process of crystal growth makes a flow of the micropolar fluid in the boundary layer on a constantly moving surface as an important area of research. Heat transfer effect of Magnetohydrodynamic micropolar nanofluid past a permeable stretching/shrinking sheet with Newtonian heating is examined by Gangadhar et.al (2017). Recently Abdullah Dawar et al. (2020) studied Chemically reactive MHD micropolar nanofluid flow with velocity slips and variable heat source/sink and they solved the system of equation both analytically and numerically. The mhd micropolar-nanofluid flow in natural convection heat transfer over a radiative truncated cone was studied by Waqar A. Khan et al. (2020) and this study reveals an enhancement in the applicable parameters progresses the heat transmission rate.

Khuram Rafique et al. (2020) deliberated hydromagnetic flow of micropolar nanofluid and for investigation Buongiorno mathematical model of hydromagnetic micropolar nanofluid over a permeable inclined stretching sheet is considered and the system is solved by Keller- box scheme.

The contemporary research aims to examine theoretically various physical properties of the micropolar model for Nano fluidic suspensions.

2. Problem Formulation

A Problem of two-dimensional, incompressible, steady, micropolar fluid flow on a flat absorbent plate which moves constantly with a continuous velocity in a water-based nanofluid containing different nanoparticles such as CuO, Al2O3, and TiO2, medium at rest is formulated mathematically. The rectangular coordinate system is taken to describe the problem and whose origin is located at the place where the plate is brought into the fluid medium as displayed in Figure 1.
Figure 1. Flow Model

The surface is kept along the x-axis and the y-axis is vertical to it. The temperature and concentration on the surface are maintained at uniform value $T_\infty$ and $C_\infty$ respectively. The fluid is seen to be gray, radiating, and absorbing. Heat fluctuation in the y-direction is significant while comparing with the flux in the x-direction. The fluid viscosity is assumed as variable and it varies with respect to temperature linearly. The governing equations of the above flow model are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}} \left( \frac{\partial}{\partial y} (\mu_{nf} + k) \frac{\partial u}{\partial y} + k \frac{\partial \sigma}{\partial y} \frac{\alpha B^2 u}{\rho_{nf}} \right)
\]  
(2)

\[
\rho_{nf} \left( u \frac{\partial \sigma}{\partial x} + v \frac{\partial \sigma}{\partial y} \right) = \gamma_{nf} \frac{\partial^2 \sigma}{\partial y^2} - k \left( \frac{\partial u}{\partial y} + \frac{\partial \sigma}{\partial y} \right)
\]  
(3)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{\gamma_{nf}}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{1}{(\rho c_p)_{nf}} \left( \frac{\partial q_{r}}{\partial y} \right) + \frac{\alpha B^2 u^2}{(\rho c_p)_{nf}}
\]  
(4)

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - R_C (C - C_\infty)
\]  
(5)

exposed to the following boundary condition

\[
At \ y = 0, \ u = U_0, v = V_0, \sigma = -n \frac{\partial u}{\partial y}, T = T_w, C = C_w
\]  
(6)

\[
At \ y = \infty, u = 0, v = 0, \sigma = 0, T = T_\infty, C = C_\infty
\]

Where,

$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s$

$\phi$ - volume fraction of nano-solid particle

$\rho_{nf}$ - density of nanofluid

$\rho_s$ - the density of the solid particle

$\rho_f$ - density of the base fluid
\( \left( \rho c_p \right)_n = (1 - \phi) \left( \rho c_p \right)_f + \phi \left( \rho c_p \right)_s \)

\( \mu_n = \frac{\mu_f}{(1 - \phi)^{2.5}} \)

Now consider the following dimensionless similarity variables

\[
\eta = y \sqrt{\frac{U_o}{2g_fx}}, \quad \psi = \sqrt{\frac{U_0^3}{2g_fx}} f(\eta), \quad \sigma = \sqrt{\frac{U_0^3}{2g_fx}} g(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \lambda = \frac{C - C_\infty}{C_w - C_\infty}
\]

(7)

Where, \( f(\eta) \) and \( g(\eta) \) are non-dimensional stream functions, with respect to the equation (7), equations (2)-(5) are changed to the system of ODE given below

\[
\left\{ \frac{\mu_n}{\mu_f} + k \right\} \frac{f'''}{f''} + \frac{\gamma_r}{(1 - \phi) + \phi \frac{\rho_s}{\rho_f}} \frac{f''}{f' - M (f')^2} = 0
\]

(8)

\[
\left\{ \frac{\mu_n}{\mu_f} + k \right\} \frac{g''}{g'} + \frac{2k}{(1 - \phi) + \phi \frac{\rho_s}{\rho_f}} (2g + f') = 0
\]

(9)

\[
(3N + 4) \theta'' + \frac{3N Pr Ec}{(1 - \phi)^{2.5} (1 - \phi) + \phi \frac{\rho_s}{\rho_f}} (f')^2 + 3N Pr f \theta' + 3M Ec Pr N (f')^2 = 0
\]

(10)

\[
\left( \frac{k'k_n}{k_n} \right) \frac{\lambda''}{\lambda - Sc} f' + Sc R \lambda = 0
\]

(11)

Where,

\[
Pr = \frac{\gamma_r \rho c_p}{k_n}, \quad N = \frac{k'k_n}{4\sigma T_\infty^3}, \quad Ec = \frac{U_0^2}{c_p(T_w - T_\infty)},
\]

\[
F_w = -V_w \sqrt{\frac{2x}{\gamma_r U_0}}, \quad R = -R_c \left( \frac{2x}{U_0} \right), \quad M = \frac{\alpha B^2}{\rho \rho_0 U_0}
\]

The initial and boundary conditions corresponding to dimensionless quantities are given by

\[
f(0) = F_w, \quad f'(0) = 1, \quad g(0) = -\eta f', \quad \theta(0) = 1, \quad \lambda(0) = 1 \quad \text{as} \quad \eta = 0
\]

\[
f'(\infty) = 0, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \lambda(\infty) = 0 \quad \text{as} \quad \eta = \infty
\]
3. Numerical Solution

The set of non-linear PDEs (2) - (5) with the equation (6) are converted into ODEs by similarity transformation technic. The system of non-linear ODEs (7) - (10) with the equation (11) are solved by using the 4th order R-K method along with NS procedure (Adams and Rogers (1973)) for the proposed parameter $\phi$, $F_w$, $N$, $Pr$, $K$, $N$, $M$ and $Sc$. The system of ODE (7) – (10) which is subjected to the condition (11) solved by implementing a computer program. A step size of $\Delta \eta = 0.01$ was designated to fulfill the convergence criterion of 10-4 in all cases.

4. Results and Discussion

The mathematical solution of the above system is given in terms of graphs. Figure 2 expresses the influence of hydro magnetic parameter on fluid velocity near the plate. Figure 2 displays that the velocity of the fluid decrease as the hydro magnetic parameter increases. Figure 3 displays the microrotation of the micropolar nanofluid close to the boundary layer for different values of hydro magnetic parameter. It observes that the angular velocity of the micropolar nanofluid decreases as $M$ increases.

![Figure 2 Impact of hydro magnetic parameter on velocity of the fluid.](image1)

![Figure 3. Impact of hydro magnetic parameter on Micro rotation of the fluid](image2)
Figure 4 exhibits the concentration distribution of the micropolar nanofluid and it is observed that fluid concentration close to the plate decrease as the hydro magnetic parameter increases. Figure 5 exhibits the temperature distribution of the micropolar nanofluid and it states a slight variation in the temperature of the fluid when hydro magnetic parameter changes.

![Figure 4 Influence of hydro magnetic parameter on Concentration.](image1)

![Figure 5 Impact of hydro magnetic parameter on temperature.](image2)
Figure 6 exhibits the influence of coupling constant on Velocity of the fluid with in the boundary layer and it is found that for increasing $K$ velocity of the fluid with in the boundary layer increases.

Figure 7 exhibits the Influence of coupling constant on angular Velocity of the fluid with in the boundary layer and it reveals that for increasing $K$ angular velocity of the fluid with in the boundary layer decreases.

Effect of coupling constant on concentration of the fluid with in the boundary layer of the flow is shown in figure 8 and it reveals that for increasing $K$ concentration of the fluid increases with in the boundary layer.

![Figure 6. Influence of different $K$ on velocity of the fluid.](image)

![Figure 7. Impact of different $K$ on angular Velocity of the fluid.](image)
Figure 8. Impact of different $K$ on Concentration of the fluid.

Figure 9 exposes the velocity of the micropolar nanofluid in the boundary layer of the flow for various values of volume fraction of nano-solid particles. Figure 9 shows that the velocity of the fluid decrease as the volume fraction of nano-solid particles increases.

Figure 10 shows the micro rotation of the micropolar nanofluid near the boundary layer for different values of volume fraction of nano-solid particles. It observes that the angular velocity of the micropolar nanofluid decreases as $\phi$ rises.

Figure 9. Velocity profile for different $\phi$. 
Figure 10. Micro rotation profile for different $\phi$.

Figure 11 exhibits the temperature distribution of the micropolar nanofluid and it states a slight variation in the temperature of the fluid when the volume fraction of nano-solid particle changes. Figure 12 exhibits the concentration distribution of the micropolar nanofluid and it shows that the concentration of the fluid falls when the volume fraction of nano-solid particle increases changes.
Figure (13)-(15) indicate the temperature, concentration, velocity, for various values of Pr. It shows that the velocity and temperature of the fluid close to the boundary layer decreases as Pr increases, but concentration of the fluid inside the boundary layer declines as Pr decreases.

Figure 12. Concentration profile for different φ.

Figure 13. Influence of Prandtl number on fluid Velocity.
Figure 14. Influence of Prandtl number on fluid temperature.

Figure 15. Influence of Prandtl number on fluid concentration.
5. Conclusion
The mathematical exploration has been performed on the influence of microrotation, velocity, concentration and temperature of the fluid flow inside the boundary layer. Execution is done for the parameters such as hydro magnetic parameter $M$, volume fraction of nano-solid particle $\phi$, coupling constant $K$, and Prandtl number. Moreover, the effect of the above-mentioned parameters on the fluid properties in the boundary layer are presented graphically.

Final conclusion are as follows;

- The concentration and temperature of the fluid in the boundary layer declines when magnetic parameter increase.
- The velocity and microrotation of the fluid close to the boundary layer drops when magnetic parameter increases.
- For increasing $K$, the fluid velocity close to the boundary layer increases, but angular velocity of the fluid decreases near the boundary layer.
- For increasing value of the volume fraction of nano-solid particle $\phi$, the velocity and microrotation of the fluid within the boundary layer decreases.
- An upsurge in $\phi$ reduces the concentration and temperature of the fluid flow significantly while it raises the velocity of the fluid flow.
- The temperature and velocity declines due to an rise in Prandtl number and concentration declines as the Prandtl number decreases.

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