Neutrino masses through a type II seesaw mechanism at TeV scale

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Abstract

In this work we show that we can generate neutrino masses through the type II seesaw mechanism working at TeV scale in the context of a 331 model.
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The explanation of the smallness of the neutrino masses and the profile of their mixing as required by recent experiments have being taken as a great puzzle in particle physics. This is so true that in the past three years a great amount of papers have been devoted to its solution. Despite the volume of papers, we still dispose of few basic ideas to explore the puzzle \[1\]. In the context of the electroweak \(SU(2)_L \otimes U(1)_Y\) model a very attractive idea is centered on a very heavy Higgs-triplet \(\Delta\) \[2\].

With this scalar triplet, \(\Delta\), it is possible to implement the spontaneously breakdown of the total lepton number and generate neutrino majorana masses \[3\]. Its main consequence was the existence of a Goldstone-boson named the majoron-triplet. This Goldstone boson has many implications in collider, astro-particle, and cosmo-particle physics, so that the model received great attention until it was ruled out by LEP data \[4\].

In order to save the idea a term that violates explicitly the lepton number,

\[
M' \phi^T \Delta^\dagger \phi,
\]

was considered in the scalar potential. If we decouple the Higgs-triplet of the electro-weak scale taking it as a very heavy triplet, the majoron gains a mass getting safe from LEP data, and the vacuum expectation value (VEV) of \(\Delta\) develops a tiny value. To see this, consider below the potential with the term that violates explicitly the lepton number:

\[
V(\phi, \Delta) = -M^2 \Delta^\dagger \Delta - \mu^2 \phi^\dagger \phi + \lambda_\phi (\phi^\dagger \phi)^2 + \lambda_{\Delta} (\Delta^\dagger \Delta)^2 + \lambda_{\Delta \phi} \Delta^\dagger \Delta \phi^\dagger \phi + M' \phi^T \Delta^\dagger \phi.
\]

From the condition that the neutral component of the Higgs-triplet develops a VEV, we find the following relation among the vacua of the model:

\[
v_\Delta \sim \frac{v_\phi^2}{M} \ll v_\phi.
\]

To find the relation above the condition \(M \sim M' \gg v_\phi\) was used. Choosing \(v_\phi = 10^2\) GeV and \(M = 10^{14}\) GeV, we get \(v_\Delta = 0.1\) eV. This mechanism was labeled type II seesaw and when used in conjunction with some additional global symmetries, in order to generate the
wanted entries in the neutrino mass matrices, is the main ingredient of various interesting extensions of the standard model [5].

In Refs. [6] it was shown that the Higgs-triplet appears in the minimal version of the 331 models [7] embedded in a scalar sextet $S$. To recognize the triplet we must know that when the 331 symmetry breaks to the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y(321)$ symmetry, the sextet $S$ decomposes under 321 as follows: $S \rightarrow \Delta_{(1,3,-2)} + \Phi_{3(1,2,1)} + H_{\chi_{(1,1,4)}}^+$. As in the triplet majoron scheme, when the neutral component of the triplet $\Delta$ develops a VEV, we have the spontaneous breaking of the total lepton number, and therefore, the model develops a majoron-triplet too [6]. However, in the present model the majoron-triplet can be safe under LEP data [8]. In view of this a natural step further in the development of the 331 model is to add to its scalar potential a term that is equivalent to that one that gave rise the type II seesaw mechanism in the standard electroweak model with the triplet $\Delta$.

The scalar sector of the minimal 331 model is composed by three triplet and a sextet of scalars:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^+ \\ \eta^\dagger_1 \\ \eta^\dagger_2 \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix}, \quad S = \begin{pmatrix} \sigma^0_1 & h^-_1 & h^+_1 \\ h^+_1 & \sigma^0_2 & h^-_2 \\ h^-_2 & h^+_2 & H^+ \end{pmatrix}. \quad (4)$$

After the breaking of the 331 symmetry to the standard 321 symmetry, the sextet above will decompose under $3 - 2 - 1$ in the following triplet, doublet and singlet of scalars:

$$\Delta = \begin{pmatrix} \sigma^0_1 & h^-_1 \\ h^+_1 & H^1 \end{pmatrix}, \quad \Phi_3 = 1 \sqrt{2} \begin{pmatrix} h^+_1 \\ \sigma_2 \end{pmatrix}, \quad H^+_2. \quad (5)$$

With all those scalar multiplets in (4) we have the following potential which is invariant under the 331 gauge symmetry [7,8]:

$$V(\eta, \rho, \chi, S) = \mu_\eta^2 \eta^\dagger \eta + \mu_\rho^2 \rho^\dagger \rho + \mu_\chi^2 \chi^\dagger \chi + \mu_S^2 Tr(S^\dagger S) + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 + (\eta^\dagger \eta) \left( \lambda_4 (\rho^\dagger \rho) + \lambda_5 (\chi^\dagger \chi) \right) + \lambda_6 (\rho^\dagger \rho) (\chi^\dagger \chi) + \lambda_7 (\rho^\dagger \eta) (\eta^\dagger \rho) + \lambda_8 (\chi^\dagger \eta) (\eta^\dagger \chi) + \lambda_9 (\rho^\dagger \chi) (\chi^\dagger \rho) + \lambda_{10} Tr(S^\dagger S)^2 + \lambda_{11} Tr(S^\dagger S)^2 + \left( \lambda_{12} (\eta^\dagger \eta) + \lambda_{13} (\rho^\dagger \rho) \right) Tr(S^\dagger S)$$
\[ +\lambda_14(\chi^\dagger\chi)Tr(S^\dagger S) + \left(\lambda_{15}\epsilon^{ijk}(\chi^\dagger S)_i\chi_j\eta_k + h.c.\right) + \left(\lambda_{16}\epsilon^{ijk}(\rho^\dagger S)_i\rho_j\eta_k + h.c.\right) \\
+ \left(\lambda_{17}\epsilon^{ijk}\epsilon^{lmn}\eta_n\eta_k S_{lmj} + h.c.\right) + \lambda_{18}\chi^\dagger S\chi + \lambda_{19}\eta^\dagger S\eta + \lambda_{20}\rho^\dagger S\rho. \tag{6} \]

In this work, for the sake of simplicity, we impose to the scalar potential the symmetry \( \chi \rightarrow -\chi \) in order to avoid other trilinear terms besides one that will generate the seesaw mechanism.

The scalar potential above is not the total potential permitted by the 331 gauge symmetry. It permits more four terms which violate explicitly the lepton number, but for what concern us here we just will consider one of them:

\[ M'\eta^T S^\dagger \eta, \tag{7} \]

since it contains, after the decomposition (5), the term (II) which generates the seesaw mechanism in the Gelmini-Roncadelli scheme. The other terms will not change the results found here [10].

Adding the term (II) to the potential in (6), we find the following minimum condition to that the scalar field \( \sigma_1^0 \) develops a VEV:

\[ v_{\sigma_1} \left( \mu_S^2 + \lambda_{10}v_{\sigma_2}^2 + \frac{\lambda_{12}}{2}v_\eta^2 + \frac{\lambda_{13}}{2}v_\rho^2 + \frac{\lambda_{14}}{2}v_\chi^2 + \frac{\lambda_{19}}{2}v_\eta^2 \right) + M'v_\eta^2 + \frac{\lambda_{11}}{2}v_{\sigma_1}^3 + \lambda_{19}v_{\sigma_1}^3 = 0. \tag{8} \]

Considering that \( v_\chi \) is dominant over the other vacua, which is a plausible consideration since this VEV is the only responsible by the breaking of the 331 symmetry, and taking also natural values for the parameters \( \lambda \)'s, i.e., \( \lambda \)'s \( \sim \mathcal{O}(1) \), we find the following expression to the VEV of the field \( \sigma_1^0 \)

\[ v_{\sigma_1} \sim M'\frac{v_\eta^2}{v_\chi^2}. \tag{9} \]

From the minimum condition to the scalar fields \( \sigma_2^0 \) and \( \eta_0 \) develop a VEV we have more two constraints over the vacua of the model:

\[ v_\eta \left( \mu_\eta^2 + \frac{\lambda_4}{2}v_\rho^2 + \frac{\lambda_5}{2}v_\chi^2 + \lambda_{12}\left(\frac{v_{\sigma_2}^2}{2} + \frac{v_\eta^2}{2}\right) - \lambda_{17}v_{\sigma_2}^2 + \frac{\lambda_{19}}{2}v_{\sigma_1}^2 \right) \\
+ \frac{v_{\sigma_2}^2}{2\sqrt{2}}(\lambda_{15}v_\chi^2 - \lambda_{16}v_\rho^2) + \lambda_1v_\eta^3 = 0, \]
\[
v_{\sigma_2} \left( \mu_2^2 + \lambda_{10} v_{\sigma_1}^2 + \frac{\lambda_{12}}{2} v_\eta^2 + \frac{\lambda_{13}}{2} v_\rho^2 + \frac{\lambda_{14}}{2} v_\chi^2 - \lambda_{17} v_\eta^2 + \frac{\lambda_{18}}{4} v_\chi^2 + \frac{\lambda_{20}}{4} v_\rho^2 \right) \\
+ \frac{\lambda_{15}}{2\sqrt{2}} v_\eta v_\chi^2 - \frac{\lambda_{16}}{2\sqrt{2}} v_\eta v_\rho^2 + \frac{\lambda_{11}}{2} v_{\sigma_2}^2 + \lambda_{10} v_{\sigma_2}^2 = 0,
\]
(10)

which gives, by using the same approximations used to obtain (9), the following relation among \( v_{\sigma_2} \) and \( v_\eta \):

\[
v_\eta \sim v_{\sigma_2}.
\]
(11)

The result above is interesting because, together with Eq. (9), provides a relation among the VEVs of the two neutral components of the sextet:

\[
v_{\sigma_1} \sim M' \frac{v_{\sigma_2}^2}{v_\chi^2}.
\]
(12)

As the two vacua \( v_{\sigma_1} \) and \( v_{\sigma_2} \) have the same origin, the sextet, we could expect that they have the same order of magnitude. But we know that \( v_{\sigma_1} \) should be of the order of eV to explain the neutrino mass. However if we take \( v_{\sigma_2} \) of the order of eV we can not explain the charged lepton masses. As the field \( \sigma_0^2 \) only contributes to the charged lepton masses it should develop a VEV around the scale of GeV. We can wonder if the scalar potential, with the VEVs above and the required \( \lambda \)'s, is bounded from below. Although we have not done a detailed analysis we note that this condition can be assured by the \( \lambda_3 \chi^\dagger \chi \) term in (6) with \( \lambda_3 > 0 \).

Next we are going to discuss the values of the parameters \( M' \), \( v_{\sigma_2} \) and \( v_\chi \) which could better explain the neutrino and charged lepton masses. In the minimal 331 model the neutrinos and the leptons obtain their masses from the following Yukawa interactions [11]

\[
\mathcal{L}_Y^l = \frac{1}{2} (\Psi_{aL})^c G_{ab} \Psi_{bL} S + \epsilon^{ijk} (\Psi_{iAL})^c F_{ab} \Psi_{jbl} \eta^*_k,
\]
(13)

where \( \Psi_{aL} = (\nu_a, l_a, l^c_a)^T ; a = e, \mu, \tau \); and we have omitted \( SU(3) \) indices. After the scalar fields \( \sigma_0^1, \sigma_0^2 \) and \( \eta_0 \) develop their VEVs the interactions above generate the following mass terms to the neutrinos and charged leptons

\[
\mathcal{L}_Y^l = \frac{v_{\sigma_1}}{2\sqrt{2}} (\nu_{aL})^c G_{ab} \nu_{bL} + \frac{v_{\sigma_2}}{4} G_{ab} + \frac{v_\eta}{\sqrt{2}} F_{ab} l_{bR},
\]
(14)
with the matrix $F_{ab}$ being anti-symmetric [11].

Using the relations (11) and (12) in (14), we find the following expressions to the masses of the neutrinos and charged leptons

$$m_{ab}^{\nu} = \frac{G_{ab}M'v_{\sigma_2}^2}{2\sqrt{2}v_{\chi}^2}, \quad m_{ab}^l = \left(\frac{G_{ab}}{4} + \frac{F_{ab}}{\sqrt{2}}\right)v_{\sigma_2}. \quad (15)$$

The best choice for the set of parameters $M', v_{\sigma_2}$ and $v_{\chi}$, in order to explain the smallness of the neutrinos masses and also the charged lepton masses, is : $M' = v_{\sigma_2} = 1$ GeV and $v_{\chi} = 10$ TeV. With these values we have the following mass matrices to both sectors:

$$m^{\nu} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{12} & G_{22} & G_{23} \\ G_{13} & G_{23} & G_{33} \end{pmatrix} eV,$$

$$m^l = \begin{pmatrix} \frac{G_{11}}{12} & \frac{G_{12}}{12} + \frac{F_{12}}{\sqrt{2}} & \frac{G_{13}}{12} + \frac{F_{13}}{\sqrt{2}} \\ \frac{G_{12}}{12} - \frac{F_{12}}{\sqrt{2}} & \frac{G_{22}}{12} & \frac{G_{23}}{12} + \frac{F_{23}}{\sqrt{2}} \\ \frac{G_{13}}{12} - \frac{F_{13}}{\sqrt{2}} & \frac{G_{23}}{12} - \frac{F_{23}}{\sqrt{2}} & \frac{G_{33}}{12} \end{pmatrix} GeV. \quad (16)$$

The texture of the neutrino mass matrices is a question of try to put extra global symmetries in order to generate the wanted entries [12]. That is not the intention in this work. Nevertheless, we can conclude from the matrices above that the minimal 331 model prefers textures where the charged lepton matrix is not diagonal, unless we find some symmetry to justify the fine-tuning $G_{ab} = -G_{ba} = 12 \frac{F_{ab}}{\sqrt{2}}, \; a \neq b$.

Now let us briefly analyze the scenario where the sextet is very heavy, i.e., considering $\mu_S \sim M' \gg v_{\chi}$, as in the conventional type II seesaw mechanism. In this scenario the minimum condition in (8) give us the following expression to the vacuum of the field $\sigma_1^0$:

$$v_{\sigma_1} \sim \frac{v_{\eta}^2}{\mu_S}. \quad (17)$$

Choosing $\mu_S = 10^{14}$ GeV and $v_{\eta} = 10^2$ GeV we have $v_{\sigma_1} \sim 0.1$ eV, which is completely similar to the conventional case. However, from the minimum condition to the field $\sigma_2^0$ in (11) we find the following expression to its VEV
Choosing $v_\chi = 10$ TeV and $v_\eta = 10^2$ GeV we have $v_{s2} = 10^{-20}$ GeV. With this value to $v_{s2}$ only $\eta$ is responsible by the charged lepton masses. However we already know that $\eta$ alone is not sufficient to generate the correct charged lepton masses \[14\]. Then to have a type II seesaw mechanism with a very heavy sextet we should extent the model in order to generate the correct charged lepton masses. In this case a minimal extension, for example, is one where two fermions transforming like singlet under the 331 symmetry, $E_L \sim (1, 1, 1)$ and $E_R \sim (1, 1, 1)$, are added to the model, as suggested by Duong and Ma \[13\] and developed in Ref. \[14\].

In conclusion, in this work we analyzed the type II seesaw mechanism for generating neutrino masses in 331 models. The main result found here is that in the minimal version of the models the mechanism works in a situation where the higher scale of energy involved is the scale of the breaking of the symmetry 331, which is of the order of few TeV’s. This is a very interesting result because only few models are able to explain the neutrino puzzle at the tree level without resort to very high scale of energy.

After this work was almost concluded we found that a similar idea was pointed out in Ref. \[15\].

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