Quantum pump for spin and charge transport in a Luttinger liquid

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We study two different parametric pumps for interacting quantum wires, one for pumping spin currents, the other for charge currents. We find that the spin or charge pumped per cycle has a non-universal crossover, depending on pumping details, between two universal fixed point values of 0 and twice the electronic spin or charge quantum number. These universal values are independent of interactions, but the direction of flow between the two values depends on whether the interactions are repulsive or attractive.

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In recent years there has been a tremendous interest in electron transport via the mechanism of pumping, in which periodic perturbations of the system yield a dc current [1]. The idea of pumping charge by cyclic variation of external couplings was first introduced by Thouless [2], with an emphasis on the quantisation of charge transport. More recently, a generalisation of this picture has led to the observation of charge pumping in the case of open quantum dots [3]. The corresponding theory for non-interacting systems has been developed quite extensively [3-5]. The picture that emerges is that, for a phase coherent quantum system, the out of phase variations of any pair of independent parameters will give rise to a dc current. In the case of electron pumps that operate in the regime of Coulomb blockade, a quantised electronic charge is pumped per cycle in the adiabatic limit [5]. An interesting question is how bulk electronic interactions could affect this quantization, for example in the case of Luttinger liquids, such as quantum wires, metallic carbon nanotubes, or fractional quantum Hall edges.

Further motivation to study pumping in interacting systems is provided by the recent developments in coherent spin transport in low dimensional semiconductors [6-8]. The study of spin transport is important not only for constructing devices based on manipulation of spins, the area of spintronics [9], but also because it offers the possibility of addressing fundamental issues of spin-charge dynamics in low-dimensional strongly correlated systems. A mechanism to pump a spin current through a quantum wire would be an alternative approach to existing coherent spin transport methods relying on injection from ferromagnetic interfaces [10-11].

In this paper we introduce the idea of a spin pump, and analyze the pumping of charge and spin through an interacting quantum wire. As we shall show, interactions play a crucial role in determining the response to pumping. However, universal features, independent of the interaction strength, are found in the asymptotic limits of slow and fast pumping frequencies.

A clean 1D interacting electronic system is a realization of a Luttinger liquid [12], which is characterised by power-law decays of various correlation functions with exponents that depend on the interaction parameters. As a consequence, the transport properties of a Luttinger liquid are strikingly different from that of a Fermi liquid [13-14]. We show in this paper that there is also a qualitative difference in the behaviour of a quantum pump in a Luttinger liquid versus one in a non-interacting electron system. Since the response to pumping is an average transfer of charge \( Q_c = eN_c \) (or spin \( Q_s = \hbar N_s \)) in a cycle, we can define charge and spin pumping conductances as \( g_{c,s} = \frac{e^2}{2}\hbar N_{c,s} \). These quantities are defined so as to have the same units as the corresponding dc conductances \( G_c \) and \( G_s \) [14]. For repulsive interactions, the pumping conductance \( g_c \), as well as \( G_s \), in the case of spin pumping, are quantised at \( T = 0 \) in the limit of slow pumping. The average charge pumped per cycle is \( Q_c = 2e \) while the average spin pumped per cycle is \( Q_s = \hbar \), irrespective of the strength of interactions. In the limit of fast pumping both these quantities go to zero. The picture for attractive interactions is reversed. Thus, in the slow pumping limit \( g_{c,s} = 0 \), while in the fast pumping limit both \( G_c \) and \( G_s \) are quantised (\( Q_c = 2e \) and \( Q_s = \hbar \)) independent of the interaction strength. The non-interacting case is special in that the two conductances are not quantised but are independent of the pumping frequency. For reasons given below, the asymptotic behaviour of the pumping conductances \( G_{c,s} \) in the regimes of repulsive and attractive interactions is opposite to the behaviour shown by the dc conductances \( G_{c,s} \) in Ref. [14].

Fig. [1] depicts two different arrangements for operating a quantum pump in a Luttinger liquid. While the set-up of Fig. (a) allows for pumping charge alone, that of Fig. (b) can pump a pure spin current under appropriate conditions as described below. Henceforth, we shall refer to these set-ups as Q-pump and S-pump, respectively. In the presence of the externally tunable interactions, indicated in Fig. 1, the Hamiltonian gets an explicitly time-dependent term:

\[
\delta H(t) = \sum_{\sigma,\sigma'} \int dx V_{\sigma\sigma'}(x,t) \psi_\sigma^\dagger(x) \psi_{\sigma'}(x)
\]

(1)

For the Q-pump, \( V_{\sigma\sigma'}(x,t) = V_{0}^\dagger(x,t) \delta_{\sigma\sigma'} + V_{0}^\dagger(x,t) \delta_{\sigma\sigma'} \) is the sum of the two potentials arising from the gate voltages, with \( V_{0}^\dagger(x,t) \) being essentially zero outside the gate’s point of contact \((x = \pm a)\). The S-pump has \( V_{\sigma\sigma'} = V_{0}^\dagger(x,t) \delta_{\sigma\sigma'} + V_{s}^\dagger(x,t) \tau_z^{\sigma\sigma'} \), where \( \tau^{i} \) is the \( i \)-th Pauli spin matrix, and \( V_{s}^\dagger \) is the coupling of the local magnetic field (in the \( i \)-direction)
rotonized about the two Fermi points (bosonization scheme, wherein the fermion fields, in all the current arises from the backscattering term: \( \psi_p \).

In the case of a quantum pump, no source-drain voltage is applied, so there is no direct contribution (\( I_d = 0 \)). Therefore, all the current arises from the backscattering term: \( I_p = -I_b \).

To proceed we look at the Hamiltonian in the canonical bosonization scheme, wherein the fermion fields, linearized about the two Fermi points (\( \pm k_F \)), are written as \( \psi(x) = e^{ik_F x} \psi_{R,\sigma}(x) + e^{-ik_F x} \psi_{L,\sigma}(x) \). Here \( \sigma = \uparrow, \downarrow \), and \( \psi_{R,\sigma}, \psi_{L,\sigma} \) are the right and left moving chiral fields, which are represented as normal ordered exponentials of bosonic fields, \( \psi_{R,\sigma} = e^{i\sqrt{\tau} \phi_{R,\sigma}(x,t)}; \psi_{L,\sigma} = e^{-i\sqrt{\tau} \phi_{L,\sigma}(x,t)} \). The combinations \( \phi_{R,\uparrow} + \phi_{L,\downarrow} = (\Phi_c + \Phi_s)/2 \) and \( \phi_{R,\downarrow} + \phi_{L,\uparrow} = (\Phi_c - \Phi_s)/2 \), separate the bulk Hamiltonian \( \mathcal{H}_b \) into independent spin \((\chi)\) and charge \((\sigma)\) sectors:

\[
\mathcal{H}_b = \mathcal{H}_c + \mathcal{H}_s = \int dx \left\{ \frac{V_c}{2 g_c} \left[ (\partial_x \Phi_c)^2 + \frac{1}{v_c} (\partial_t \Phi_c)^2 \right] + \frac{V_s}{2 g_s} \left[ (\partial_x \Phi_s)^2 + \frac{1}{v_s} (\partial_t \Phi_s)^2 \right] \right\}.
\]

The spin isotropic point with a global \( U(1) \times SU(2) \) symmetry corresponds to \( g_s = 2 \), and the non-interacting fermion limit is recovered for \( g_s = g_c = 2 \). In the absence of the backscatters, the dc two-terminal conductance is \( G_c = g_c e^2/h \), while the spin-conductance is \( G_s = g_s e^2/h \).

The time-dependent Hamiltonian \( \delta \mathcal{H}(t) \) in Eq. (1) describes both the backscattering and forward scattering processes by the two contacts. The two contacts in the Q- and S-pumps can be reduced to an effective single contact as long as the pumping frequency \( \omega_p \ll v_F / a \) (where \( v_F \) is the Fermi velocity), as shown in the context of fractional quantum Hall edges in Ref. (17). This leads to an effective backscattering amplitude centered around \( x = 0 \) and with magnitude \( \int dx \langle V_{\sigma'}(x,t) e^{-i2k_F x} \rangle \). Also, since the pumping current is determined entirely by the periodic variation in backscattering processes, we can drop the forward scattering part of the interactions from the Hamiltonian. As a result, the time-dependent term in the Hamiltonian can be written in a matrix form with a unified notation for the Q- and S-pumps:

\[
\delta \mathcal{H}(t) = \Psi^\dagger(0) \mathcal{Y}(t) \Psi(0), \tag{3}
\]

where \( \Psi^\dagger = \left( \psi_{R,\uparrow}^\dagger \psi_{R,\downarrow}^\dagger \psi_{L,\uparrow}^\dagger \psi_{L,\downarrow}^\dagger \right) \), and

\[
\mathcal{Y}(t) = \left[ \begin{array}{cccc} 0 & \Gamma^x(t) & \tau^y & \tau^z \\ \Gamma^x(t) & 0 & 0 & 0 \\ 0 & 0 & -i \omega_p & 0 \\ 0 & 0 & 0 & -i \omega_p \end{array} \right].
\]

For the Q-pump, the only non-vanishing term is \( \Gamma^x(t) = e^{-i2k_F x} \tilde{V}_0^c(2k_F,t) + e^{i2k_F x} \tilde{V}_0^c(2k_F,t) \), while the S-pump has \( \Gamma^x(t) = e^{-i2k_F x} \tilde{V}_0^s(2k_F,t) + e^{i2k_F x} \tilde{V}_0^s(2k_F,t) \). The \( \tilde{V}_0^c(k,t) \) are the Fourier modes of the \( V_p^c(x,t) \) potentials. Let us denote the two parameters whose periodic variations operate these pumps as \( X_1(t) \) and \( X_2(t) \). These parameters are identified as: \( X_1(t) = e^{-i2k_F x} \tilde{V}_0^c(2k_F,t) \) for both pumps: \( X_2(t) = e^{i2k_F x} \tilde{V}_0^c(2k_F,t) \) for the Q-pump, while \( X_2(t) = e^{i2k_F x} \tilde{V}_0^s(2k_F,t) \) for the S-pump.

The response to this parametric variation in the charge sector is given by the charge backscattering current: \( \tilde{I}_b^c = i [\tilde{N}_{R,L}, \mathcal{H}_b] = -i [\tilde{N}_{R,L}, \delta \mathcal{H}] \), where \( \tilde{N}_{R,L} \) is the charge density of right (left) movers. This expression can be generalized to include spin currents and written in the following form:

\[
\tilde{I}_b^\lambda = -\frac{1}{2} i \Psi^\dagger(0) [M^\lambda, \mathcal{Y}] \Psi(0), \tag{4}
\]

where \( M^\lambda = \tau^\lambda \otimes \mu^3 \), the \( \mu^3 \) matrix being a Pauli matrix in the chiral space.

Consider first the effect of harmonic variation of the parameters \( X_1(t) \) and \( X_2(t) \) perturbatively, for weak barriers. The leading order contribution to the dc pumping current is \( I_{p,dc}^c \approx \int_0^\infty dt' (\mathcal{P}_f(t), \delta \mathcal{H}(t')) \mathcal{H}_0 \). Evaluating this at the spin isotropic point, we get:

\[
I_{p,dc}^c \approx \sum_{\mu, \nu} \text{Tr} \{ (\tau^\lambda, \tau^\mu), \tau^\nu \} \times \int dt' \text{Im} \{ \Gamma_\mu(t) \mathcal{P}_f(t') \} \text{Im} G^R(t-t'), \tag{5}
\]

where \( G^R(t-t') \) is the retarded Green’s function of the bosonised operator \( \tilde{\psi}_{R,\sigma}^\dagger(t) \tilde{\psi}_{L,\sigma}(t) \). For the Q-pump \( \mu = \nu = 0 \), so that the only non-zero component of the generalized current is the charge current \( I_{p,dc}^c \approx \frac{2}{\pi} \frac{1}{\omega_c} \text{Im} \omega_c / \omega_1^{(2k_F x - 2)} \omega_0 \), where \( \omega_1 \) is a cross-over energy scale set by the details of the path described by the amplitudes \( \Gamma_\mu(t) \). With \( X_1(t) = X_1 \cos(\omega_1 t - \varphi)/2 \) and \( X_2(t) = X_2 \cos(\omega_1 t + \varphi)/2 \), we have

\[
\mathcal{A} = \text{Im} [X_1 X_2^* \sin \varphi - \text{the area enclosed in a pumping cycle by the parameters } X_1(t) - X_2(t).\]

For the S-pump, with the magnetic field in the \( \hat{z} \) direction, we get only a spin-current
$I_p$ having the same expression as $I_p^0$ above. The reason is that terms giving a non-vanishing contribution to a dc current require $\mu \neq \nu$, in which case the trace term is non-zero only for $\lambda = 3$.

The perturbative expansion is meaningful for $g_c > 2$ only in the IR limit ($\omega_0 \ll \omega_T$), and for $g_c < 2$ only in the UV limit ($\omega_0 \gg \omega_T$). In both these limits $G_{c,s} = 0$. For non-interacting electrons ($g_c = g_s = 2$), we get charge pumping in the Q-pump with a frequency independent pumping conductance: $G_e = \frac{e^2}{h} I_p^0 = \frac{e^2}{h} \sin \varphi \Im \left[4X_1X_2^*\right]$, similar to Ref. [4]. Also, for non-interacting electrons, the S-pump operates as a pure spin pump, with a spin pumping conductance $G_s$ identical in form to $G_e$ above. Both these expressions display non-universal behaviour, being dependent on the form of the external perturbations.

Let us now turn to the non-perturbative case of repulsive interactions in the IR limit, and attractive interactions in the UV limit of pumping. To understand the behaviour of the Q-pump we need only consider the case of spinless electrons, where $g < 1$ for repulsive interactions. For the special case of $g = 1/2$ the problem can be mapped into that of a time dependent scattering problem involving free chiral fermions and an impurity state. We have solved this problem exactly [19], and the pumping current $I_p$ is given by:

$$I_p(t) = \frac{e}{2} \left[1 - \sum_\omega 4n_\omega \Re \left\{ \int_0^t dt' \frac{\Gamma_0(t_0)}{\Gamma_0(t)} \right. \right. \times e^{i\omega(t-t_0)} e^{\frac{e^2}{h} I_p^0 dt' |\Gamma_0(t')|^2} \left. \right\},$$

(6)

where $n_\omega$ is the equilibrium fermion occupation number. In the UV limit the charge-pumping conductance $G$ vanishes, as anticipated by the perturbative calculation. In the IR limit, at $T = 0$, the charge pumped in a cycle is:

$$Q_c = eN_c = \int_0^{\omega_0} dt I_p^0 = \frac{e}{2\pi} \int_0^{\omega_0} \frac{d\Gamma_0}{\Gamma_0} = e.$$

(7)

Thus, in the adiabatic limit and for $g = 1/2$, a quantum of charge is pumped in a cycle of the Q-pump, irrespective of the form of the pumping function $\Gamma_0(t)$. This universality allows us to define the charge-pumping conductance for spinless electrons:

$$G \equiv \frac{e^2}{h} N_c = e^2/h.$$

(8)

We can interpret these results along the lines of the renormalization group (RG) arguments by Kane and Fisher [4] for spinless electrons. In this picture, for single impurity interactions, there are two fixed points: (i) the perfectly transmitting limit, and (ii) the perfectly backscattering limit of the Luttinger liquid. For repulsive interactions the barrier is a relevant perturbation for fixed point (i) and is irrelevant for fixed point (ii). As a result, for $g < 1$, the dc conductance $G = 0$ in the IR limit. Thus, for a small applied dc voltage we get $I = I_d - I_0 = 0$ and all the current is backscattered. For the Q-pump, where only the backscattering currents matter, this picture implies maximal pumping response in the adiabatic (IR) limit. To calculate this we note (as suggested by the $g = 1/2$ case) that in the adiabatic limit the pumping current should be independent of the form of the pumping path. Therefore choosing $\Gamma_0(t) = X_+ e^{i\omega_0 t}$ should pump the same charge per cycle as would any other form of $\Gamma_0(t)$. For the purpose of calculating the pumping or backscattering current, this particular form of pumping corresponds to applying an effective source-drain voltage $V_{\text{eff}} = -\omega_0/g$. Consequently the backscattering current, which is also the (negative) pumping current, should be $I_b = I_d = gV_{\text{eff}}$. We then recover a quantised charge pumped in a cycle, $N_c = -\frac{2\pi}{g} I_b = 1$. Expression (8) defines $G$, in the repulsive regime, independent of interaction strength.

For attractive interactions ($g > 1$) the weak barrier perturbation is irrelevant for the fixed point (i) while it is relevant for the fixed point (ii). Consequently $I_b \to 0$ in the IR limit, upholding our earlier conclusion, based on perturbation theory, that $G = 0$ for attractive interactions in this limit. To access the behaviour in the non-perturbative UV limit of pumping, we note that at $T = 0$, and for an effective dc source-drain voltage, there exists an exact $g \to 1/g$ duality such that the backscattering current satisfies the relation: $I_b(V_{\text{eff}}, g) = \frac{e^2}{h} - g^2 I_b(V_{\text{eff}}, 1/g)$ [21]. At least in the particular case of $\Gamma_0(t) = X_+ e^{i\omega_0 t}$, this duality implies that the UV limit of pumping conductance for $g > 1$ should be equal to the IR limit for $g < 1$, which is given by (8). We thus get a complete picture of the universal behaviour of the charge pumping conductance which is summarized in Fig. 2, and compared with the dc source drain conductance $G$ in Table 1. We would like to point out that, in the particular case of pumping in a fractional quantum Hall bar, the charge pumped per cycle is always the electron charge $e$, irrespective of the filling fraction $\nu$ (= $g$). This follows for the particular geometry we studied; other pumping geometries, operating through anti-dots [21], can be designed so as to pump fractional charge per cycle.

As is deductible from our result (8) for $g = 1/2$, the cross-over regime as a function of pumping frequency is non-
Consequently, the behaviour of \( \Gamma \) pumping the duality relation is changed \([22]\). Also, the special form of the thermore, the approximate duality of Ref. \([14]\) seems to imply physics at the bulk spin-isotropic point \((g_s = 2)\). The only universal features seem to be in the asymptotic frequency limits, or fixed point pumping conductances.

Including spins in our description of the Q-pump, the physics at the bulk spin-isotropic point \((g_s = 2)\) is governed by the same fixed points as in the spinless case \([4]\). However, the duality relation is changed \([3]\). Also, the special form of pumping \( \Gamma_\text{p}(t) = X_c e^{i\omega_0 t} \), corresponds to \( V_{\text{eff}} = -2\omega_0/g_c \). Consequently, the behaviour of \( G_c \) is same as that of \( 2G \).

We now turn to the behaviour in the non-perturbative regimes for the S-pump. The external potential \( \delta \mathcal{H} \), can be written as:

\[
\delta \mathcal{H}(t) = |X_1| \cos \omega_0 t \cos \sqrt{\pi} \Phi_c(0) \cos \sqrt{\pi} \Phi_s(0) + |X_2| \cos(\omega_0 t + \varphi) \sin(\sqrt{\pi} \Phi_c(0) + \chi) \sin \sqrt{\pi} \Phi_s(0), \tag{9}
\]

where \( \chi \) is the constant phase difference between \( X_1 \) and \( X_2 \). From the RG analysis of Ref. \([4]\) we know that, for \( g_s = 2, g_c < 2 \), the most relevant perturbation due to a single barrier at \( x = 0 \) is: \( \nu_c \cos \sqrt{\pi} \Phi_s(0) \cos \sqrt{\pi} \Phi_c(0) \), and the system is a spin and charge insulator. Consequently, in the IR limit \( I_{\text{b}}^{0.3} = I_{\text{d}}^{0.3} \), which means that for the S-pump, the pumping current \( I_{\text{p}}^{0.3} = g_s V_{\text{eff}}/2 \pi \), where \( V_{\text{eff}} \) is the “voltage” that couples to the spin in the action. Such a “voltage” gives the barrier term a time-dependence: \( \nu_c \cos(\sqrt{\pi} \Phi_c) \cos(\sqrt{\pi} \Phi_s) \cos(g_s V_{\text{eff}} t/2) + \nu_s \cos(\sqrt{\pi} \Phi_s) \sin(\sqrt{\pi} \Phi_s) \sin(g_s V_{\text{eff}} t/2) \), and can only yield a spin current. This time-dependent barrier is the same as Eq. \([3]\) when \( \chi = \pi/2 = \varphi \), and \( |X_1| = |X_2| \), so that we can identify \( V_{\text{eff}} = -2\omega_0/g_c \). Thus, for this particular form of pumping, \( I_{\text{b}}^{0} = 0 \) and \( I_{\text{d}}^{0.3} = 2\omega_0/2\pi \). If in the IR limit of pumping the spin transferred per cycle is independent of the form of the perturbing parameters, as was argued earlier for the spinless charge pump, then the S-pump has an IR fixed point spin conductance \( G_c = 2e^2/h \), the same as the Q-pump’s \( G_c \). Furthermore, the approximate duality of Ref. \([4]\) seems to imply that the other non-perturbative regime for attractive interactions \((g_c > 2, g_s = 2)\) in the UV limit of pumping also has a fixed point value of \( G_c = 2e^2/h \).

In conclusion, we have proposed and analysed the behaviour of a charge and a spin pump through a Luttinger liquid wire, and found universal behaviour in the IR and UV limits of pumping. The frequency dependent cross-over between these values is non-universal, depending on the details of the path that the pumping amplitudes trace on the complex plane. The spin pump, in particular, could serve as an alternative way to coherently transport spin currents across a wire without ferromagnetic contacts.

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### REFERENCES

[1] B. L. Altshuler and L. I. Glazman, Science **283**, 1864 (1999).
[2] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
[3] M. Switkes, C. M. Marcus, K. Campman, A. C. Gossard, Science **283**, 1905 (1999).
[4] P. W. Brouwer, Phys. Rev. B **58**, 10135 (1998).
[5] B. Spivak, F. Zhou, and M. T. Beal Monod, Phys. Rev. B **51**, 13226 (1995).
[6] I. L. Aleiner and A. V. Andreev, Phys. Rev. Lett. **81**, 1286 (1998).
[7] M. Johnson and R. H. Silbey, Phys. Rev. Lett. **55**, 1790 (1985); M. Johnson, *ibid.* **70**, 2142 (1993).
[8] P. R. Hammar, B. R. Bennett, M. J. Yang and M. Johnson **83**, 203 (1999).
[9] G. A. Prinz, Science **282**, 1660 (1998).
[10] L. Balents and R. Egger, Phys. Rev. Lett. **85**, 3464 (2000).
[11] Q. Si, Phys. Rev. Lett. **81**, 3191 (1998).
[12] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[13] W. Apel and T. M. Rice, Phys. Rev. B **26**, 7063 (1982).
[14] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15232 (1992).
[15] P. A. Lee and D. S. Fisher, Phys. Rev. B **23**, 6851 (1981).
[16] J. Voit, Rep. Prog. Phys. **58**, 977 (1995).
[17] C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen, Phys. Rev. B **55**, 2331 (1997).
[18] C. Chamon, D. E. Freed, and X. G. Wen, Phys. Rev. B **51**, 2363 (1995).
[19] P. Sharma and C. Chamon, in preparation.
[20] P. Fendley, A. W. Ludwig, and H. Saleur, Phys. Rev. B **52**, 8934 (1995).
[21] S. H. Simon, Phys. Rev. B **61**, 16327 (2000).
[22] F. Lesage, H. Saleur, and P. Simonetti, Phys. Rev. B **56**, 7598 (1997).