Dear editor,

For many protocols, quantum strategies have advantages compared with their classical counterparts, and these advantages have attracted many interests and applications. One of the famous examples is the Clauser-Horne-Shimony-Holt (CHSH) game [1], which recasts Bell’s theorem [2] into the framework of a game. In the CHSH game, two space-like separated players, Alice and Bob are each assigned a classical bit $a$ and $b$ respectively. Then they return bits $x$ and $y$ according to some pre-agreed strategies. They will win the game when $x \oplus y = a \cdot b$. In the game, if the players use the classical strategies, the optimal success probability $w_{\text{CHSH}} = 0.75$. However, if they add some quantum resources, the success probability will increase and up to maximal value $\cos^2(\pi/8)$, which is known as the Tsirelson’s bound [3]. Moreover, Popescu and Rohrlich noted that the perfect success probability 1 can also be achieved in a more general theory without violating the no-signaling assumption [4].

Recently, a variant of the CHSH game, a so-called CHSH* game is proposed and theoretically investigated [5]. The CHSH* game involves only one player. As the Fig. 1(a) shown, the player applies the controlled transformation $A_a$ and $B_b$ on the input system in sequence and then performs measurement, obtaining an outcome $c$. The player wins if $c = a \cdot b \mod 2$ and the optimal winning probability:

$$w_{\text{CHSH}^*} = \max_{\text{all strategies}} \frac{1}{4} \sum_{a,b \in \mathbb{Z}_2} p(c = a \cdot b | a, b).$$

(1)

CHSH* game with a two dimensional quantum system. For a $d = 2$ quantum system in the unitary setting, without loss of generality, we set the initial state as $|+\rangle$ and the measurement as Pauli observable $X$. The optimal strategy can be obtained by optimizing the gates $A_a$ and $B_b$, where $a = 0,1$ and $b = 0,1$. As Ref. [5], there exists one to one correspondence between the strategy for a CHSH game and a CHSH* game, and the Tsirelson’s bound upper-bounds both of them by $A_0 = I, A_1 = R_z(\frac{\pi}{2}), B_0 = R_z(-\frac{\pi}{2})$ and $B_1 = B_0^\dagger$, where $R_z(\theta)$ represents the rotation around the $z$ axis. When varying $\theta$ in $B_b$ from 0 to $\pi/2$, $P_{\text{suc}}$ changes with $\theta$ and $P_{\text{suc}} = 1/2 + \sin\theta/4 + \cos\theta/4$. When $\theta = 0, \pi/2$, $P_{\text{suc}}$ has the maximum probability for the Clifford setting, where $A_a$ and $B_b$ are confined to the Clifford gates.

At last, by including the irreversible transfor-
In the two-dimensional photonic system, which is spanned by two orthogonal polarized directions of the photon. As Fig. 1(b) shown, the single photon source is prepared by the spontaneous parametric down-conversion (SPDC) process. All the components in the setup are designed for the Gaussian beam light at 808 nm wavelength. To improve the precision of the operation, the spectrum of the down-converter photons is filtered using a 3 nm bandwidth interference filter (IF). A single mode fiber is used to direct the photon to the CHSH* gate. For each run of the game, the control bit assigned to monitor the whole process of the CHSH* game. For the unitary gate, the winning probability increasing with $\theta$, in the setting of the CHSH* game reaches $0.8536\pm0.0018$, approaching the Tsirelson’s bound $w(\text{CHSH}^*) = w(\text{CHSH}) = \cos^2(\frac{\pi}{8})$. Furthermore, after removing the limit of irreversible transformations, we even win the game with an absolute probability 1, which is $0.9984\pm0.0018$ for our experimental realization.

In conclusion, we investigate the CHSH* game with a single photonic qubit. For the reversible case, all the probabilities for $\theta \in (0, \pi)$ are higher than the classical upper bound 0.75, moreover, the optimal quantum strategy($\theta = \pi/2$) can achieve the Tsirelson’s bound. It is known that there exists a temporal version of the CHSH scenario, which probes the correlations of measurements happening at different times and can also yield the Tsirelson’s bound with the single qubit system [6]. The quantum advantage of the temporal CHSH scenario comes from the violation of the assumption of ‘macroscopic realism’ and ‘non-invasiveness’. However, in contrast to the spacial and temporal CHSH scenario, the CHSH* game doesn’t involve any nonlocality property. One work related to the quantum advantages in CHSH* game is Ref. [7]. In the paper, the authors introduce a new notion of contextuality for transform-
The scheme of CHSH* game. (a) A 40 mW, 404 nm wavelength continuous-wave laser pumps a type-I phase matching beta-barium-borate (BBO) crystal, and pairs of photons with 808 nm wavelength are generated in SPDC process. One of the photons is detected by the single photon detector as the trigger signal, and the other photon is sent to the CHSH* game modular through a single mode fiber. In the game, the photons are measured with the measurement $M$ after undergoing the controlled operations $A_a$ and $B_b$, then collected by the fiber couplers $C_0$ and $C_1$ and detected by the single-photon detectors $D_0$ and $D_1$. The measurement outcome $c$ is returned to check the winning condition. The details of the ERASE gate is given in the black box. Abbreviations: PBS, polarization beam splitter; Q, quarter wave plate; H, half wave plate; IF, interference filter; Mirr, mirror. (c) Result for the CHSH* game. The optimal success probabilities for the different settings are given in the Table. The dots represent the practical winning probabilities in our experiment and the blue line gives a theoretical prediction. The error bars which belong to 0.0017-0.0019, are smaller than the dot size.

In our work, we also detect the winning probability for the irreversible gate. These results help us to understand how the Tsirelson’s bound arises in the strict physics condition and how the reversibility play a role in the advantages of quantum protocols. This work sheds light on the development of strategies in quantum information and computation.

In the future, it would be interesting to implement the CHSH* game with higher dimensions. As the proposition 6 in Ref. [5] points out, in the reversible setting with $d \geq 3$, we can always achieve an optimal winning probability $w(CHSH^*) = 1$. While $w(CHSH^*) = \cos^2(\frac{\pi}{d})$ for the case of $d = 2$, the CHSH* game can serve as a dimensional witness. The implementation will involve the generalized Pauli X, which can be realized with the method in Ref. [8, 9] for photonic qudit. It is also possible to use the experimental setup of the CHSH* game to probe macrorealistic physical theories, such as testing Leggett-Garg inequalities in various quantum systems.

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