Quantum integrable multi atom matter-radiation models with and without rotating wave approximation

Anjan Kundu
Saha Institute of Nuclear Physics, Theory Group
1/AF Bidhan Nagar, Calcutta 700 064, India.
email: anjan@tnp.saha.ernet.in *

July 18, 2018

Abstract

New integrable multi-atom matter-radiation models with and without rotating wave approximation (RWA) are constructed and exactly solved through algebraic Bethe ansatz. The models with RWA are generated through ancestor model approach in an unified way. The rational case yields the standard type of matter-radiation models, while the trigonometric case corresponds to their q-deformations. The models without RWA are obtained from the elliptic case at the Gaudin and high spin limit.

42.50 Pq, 03.65 Fd, 32.80 -t

1 Introduction

It is amazing to note that real systems, like those in quantum optics induced by resonance interaction between atom and a quantized laser field, in cavity QED both in microwave and optical domain [1, 2], in trapped ion interacting with its center of mass motion irradiated by a laser beam [3, 4] etc., all involving complex matter radiation (MR) interactions can be described so successfully by simple models, like Jaynes-Cummings (JC) [5] and Buck Sukumar (BS) [6] models. Many theoretical predictions based on these models, like vacuum Rabi splitting, Rabi oscillation and its quantum collapse and revival etc. have been verified in maser and laser experiments.

In such simplest interacting MR models the quantized radiation field is taken in single bosonic mode: $h = b e^{-i\omega_f t} + b^\dagger e^{i\omega_f t}$, while the atom is considered to be a two-level spin system with polarization vector $S = \sigma^- e^{-i\omega_a t} + \sigma^+ e^{i\omega_a t}$. The interaction ($h \cdot S$) therefore contains in general both fast (with frequency $w_f + w_a$) and slow oscillating (with frequency $w_f - w_a$) components. However it is customary to neglect the fast oscillating part by considering approximation $|w_f - w_a| << w_f$, which corresponds to the rotating wave approximation (RWA) and make the model solvable. This yields with the addition of free field and atomic excitation terms the well known JC model

$$H_{JC} = \omega_f b^\dagger b + \omega_a \sigma^z + \alpha (b^\dagger \sigma^- + b \sigma^+).$$

(1)

*Talk presented at NEEDS04, Gallipoli, Italy, July 2004
However this approximation, which is justified only near the resonance point \( \omega_f \approx \omega_a \), should be avoided in the general case [11], when additional counter rotating wave (CRW) terms must appear:

\[
H_{\text{crw}} = \beta (b^\dagger \sigma^+ + b \sigma^-)
\]

Moreover, for describing physical situations more accurately, one has to look for generalizations of the basic models, i.e. has to consider their q-deformations [7, 8], introduce higher nonlinearities [4, 9], involve multiple atoms [2, 9, 10] etc. However, while the exact solutions for the JC and the BS models together with their simple multi-atom extensions are known [12, 13, 14], the same appears to be no longer true for most of the above generalizations. More precisely, integrable models without RWA as well as those with explicit inter-atomic couplings are not known for most of the MR models, except for a few recent attempts [15, 16]. Moreover, though q-deformation, signifying introduction of anisotropy together with specific nonlinearity into the system, was considered earlier for a few models [7, 8], their multi-atom and integrable variants are not known. Therefore, it is indeed a challenge to find a scheme for generating integrable MR models having the desired properties mentioned above.

2 Major aims and strategy of our construction

To meet this challenge we construct new classes of integrable MR models with and without RWA. The result concerning our models with RWA has been reported recently [17], where we propose a general integrable system based on our ancestor Lax operator approach [18]. Through various reductions we generate from it a series of integrable MR models with explicit inter-atomic interactions. This includes such new multi-atom generalizations of JC, BS and trapped ion (TI) models as well as their integrable q-deformations involving quantum group. Moreover, since the construction of our models is based on a general Yang-Baxter (YB) algebra, we can also solve them in a unified way through the Bethe ansatz (BA). Our aim here is to review briefly this result and then present our new result obtained without RWA.

Our strategy for constructing models with RWA is to start with a combination of Lax operators:

\[
T(\lambda) = L^s(\lambda) \prod_j N_a \ L^S_j(\lambda),
\]

where \( L^s(\lambda) \) involving bosons represents the radiation or the oscillatory mode and \( L^S_j(\lambda) \) involving spin operators represents \( N_a \)-number of atoms. By construction it must satisfy the quantum YB equation (QYBE)

\[
R(\lambda - \mu) T(\lambda) \otimes T(\mu) = (I \otimes T(\mu))(T(\lambda) \otimes I) R(\lambda - \mu),
\]

yielding the integrability condition \([\tau(\lambda), \tau(\mu)] = 0\), with mutually commuting set of conserved operators, obtained through expansion \( \tau(\lambda) = tr T(\lambda) = \sum_a C_a \lambda^a \) [19]. For constructing the bosonic Lax operator we implement our ancestor model approach [18] and from a generalized Lax operator linked with a generalized quadratic algebra or its quantum deformation generate different \( L^s(\lambda) \) through various bosonic realizations. Our standard MR models are linked to the Lie algebra and the rational \( R \)-matrix of the \( xxx \) spin chain [19], while their \( q \)-deformations are related to the quantum algebra and the trigonometric \( R \)-matrix of the \( xxz \) chain [20].

For constructing integrable MR models without RWA, however, as we explain below, the above approach fails due to the difficulty in bosonizing the Sklyanin algebra. Therefore for such models, which are linked to the elliptic \( R \)-matrix of the \( xyz \) spin chain, we have to change our strategy and switch over to the Gaudin limit together with a high spin limit.
3 Integrable MR models with RWA

We concentrate first on standard MR models and recall that in the rational case the $2 \times 2$ ancestor Lax operator may be given as [18]

$$L^s(\lambda) = \begin{pmatrix} c_1^0(\lambda + s^3) + c_1^1 & s^- \\ s^+ & c_2^0(\lambda - s^3) - c_2^1 \end{pmatrix}$$

with operators $s$ satisfying a quadratic algebra

$$[s^+, s^-] = 2m^+s^3 + m^-, \quad [s^3, s^\pm] = \pm s^\pm, \quad [m^\pm, \cdot] = 0.$$  \hspace{1cm} (4)

The central elements $m^\pm$ are expressed through arbitrary parameters appearing in (3) as $m^+ = c_1^0c_2^0$, $m^- = c_1^1c_2^1 + c_1^0c_2^1$ and as it is easy to see, their different choices reduce (4) to different simple algebras:

i) $su(u)$, at $m^+ = 1, m^- = 0$,

ii) $su(1, 1)$, at $m^+ = -1, m^- = 0$,

iii) bosonic, at $m^+ = 0, m^- = -1$,

iv) canonical, at $m^+ = m^- = 0$  \hspace{1cm} (5)

and the corresponding limits yield from (3) the respective Lax operators. In case i), choosing $c_1^0 = 1$ and $c_1^1 = -c_2^1 = c_j$, (3) we get the spin Lax operator $L_j^s(\lambda), j \in [1, N_a]$ describing $N_a$ atoms with inhomogeneity parameters $c_j$, while the other cases can reduce to different types of bosonic Lax operators linked to the radiation mode.

Remarkably, such reductions yield in a unified way new integrable multi-atom BS, JC and TI models, at the limits ii), iii) and iv) of (5). Thus case ii) with choice $c_1^0 = -c_2^0 = 1, c_1^1 = c_2^1 \equiv c$, yields

$$H_{ba} = \omega_f s^3 + \sum_j \left( \omega_{aj} S_j^z + \alpha(s^+ S_j^- + s^- S_j^+) \right) + \alpha \sum_{i<j}(S_i^- S_j^+ - S_i^+ S_j^-),$$

which with a bosonic realization of $su(1, 1)$: $s^+ = \sqrt{N}b^+, s^- = b\sqrt{N}, s^3 = N + \frac{1}{2}$ and taking the spin-s operator as $\vec{S} = \frac{1}{2} \sum_k \vec{\sigma}_k$, represents a new integrable multi-atom BS model with inter-atomic interactions and nondegenerate atomic frequencies. At $N_a = 1$ the matter-matter interactions obviously vanish, recovering the known model [14]. The radiation frequency $\omega_f$ and the atomic frequencies $\omega_{aj}$ in our models are defined in general through the inhomogeneity parameters as

$$\omega_f = \sum_j w_j, \quad w_j = \alpha(c_1^0 - c_2^0)c_j, \quad \omega_{aj} = \omega_f - w_j + \alpha(c_1^1 + c_2^1)$$

Similarly, under reduction iii), choosing $c_1^0 = \alpha, c_2^0 = 0, c_1^1 \equiv c, c_2^1 = -\alpha^{-1}$ and taking direct bosonic realization $s^- = b, s^+ = b^+, s^3 = b^+b$, we obtain from the same (3) an integrable multi-atom JC model with matter-matter coupling.

From reduction iv), by fixing the parameter values as $c_1^0 = -1, c_1^1 \equiv c, c_2^0 = c_2^1 = 0$ and considering a consistent realization through canonical variables: $s^+ = e^{+ix}, \quad s^3 = p + x$, we generate an integrable multi-atom TI model with full exponential nonlinearity: $\alpha(e^{-ix}S^+ + e^{ix}S^-)$. Such models however are difficult to solve by standard BA, due to the absence of pseudovacuum like in the Toda chain.
For constructing integrable q-deformed MR models the strategy is the same; only one has to start from the trigonometric type ancestor Lax operator involving generalized q-deformed operators [18]. This yields an integrable system with Hamiltonian in the form

\[ H_{qMR} = H_d + (s^+_q S^-_q + s^-_q S^+_q) \sin \alpha, \]

\[ H_d = -ic_0 \cos(\alpha X) + c \sin(\alpha X), \quad X = (s^3_q - S^z_q + \omega), \]

which represent a new class of MR models with \( S_q \) belonging to the quantum group \( U_q(su(2)) \) and \( s_q \) to a more general quantum algebra [18].

Integrable q-deformed BS model is obtained from (8) by realizing \( s_q \) through q-oscillator: \( s^+_q = \sqrt{[N]_q} b^+_q, \ s^-_q = b_q \sqrt{[N]_q}, \ s^3_q = N + \frac{1}{2}, \) and using quantum spin operator \( S_q \) as its co-product [20]:

\[ S^\pm_q = \sum_s q^{-s} \sum_{k<j} \sigma^k_q \sigma^\pm_j q^{\sum_{i>j} \sigma^z_i}, \ S^z = \sum_j \sigma^z_j. \]

At \( s = 1 \), one gets an integrable version of an earlier model [8].

Similarly realizing \( s^+_q = b^+_q, s^-_q = b_q, s^3 = N \) yield a new integrable q-deformation of the JC model, and realization through canonical operators results an integrable q-deformed TI model.

We emphasize again that all integrable MR models we proposed, similar to their unified construction, allow exact BA solutions also in a unified way. More details on the above models and their exact solutions can be found in [17].

4 Integrable MR models without RWA

The basic idea for constructing integrable matter-radiation models with RWA, as we have explained above, is to consider first the xxx or the xxz spin chain with arbitrary spins and then replace one of the spins by its bosonic realization. The first case belongs to the rational class, where the spins satisfy the Lie type algebra, while the second case falls in the trigonometric class, where the higher spin operators satisfy the quantum algebra. Fortunately both these algebras allow bosonic realization, which yields the required radiation mode.

For constructing more general integrable models with additional CRW terms (like (2) and its multiatom generalizations), one may expect to apply the same idea, but for the xyz spin chain. However in this anisotropic case, linked to the elliptic R-matrix, the higher spin operators satisfy the Sklyanin algebra [21], for which unfortunately no bosonic realization is known. In search for a way out, one observes that the Sklyanin algebra fortunately reduces again to the usual \( su(2) \) algebra at the elliptic Gaudin limit [22], allowing the required bosonization. However, a direct bosonic realization of a single spin operator in Gaudin models fails again, since it results either nonintegrable or unphysical models. The situation is saved finally by a further limit of \( s \to \infty \), for the bosonic mode, which yields the desired integrable multi-atom JC type model with nontrivial CRW terms. We should mention that similar high spin limit has been used recently for constructing integrable multi-atom JC model, but with RWA [16].

4.1 Elliptic Gaudin model

Since the first step in our construction is the elliptic Gaudin model (EGM), we review briefly the related results from [22]. EGM is obtained at the \( \alpha \to 0 \) limit from the inhomogeneous xyz spin model and therefore all relevant objects like the Lax operator, R-matrix, conserved quantities, as
well as important formulas in the Bethe ansatz method, like the eigenfunctions and the eigenvalues together with the Bethe equations etc. concerning this model can be derived directly from those of the $xyz$ model \cite{23} at the said limit. The Lax operator with inhomogeneity parameters $z_n$ and the $r$-matrix of the EGM can be expressed through elliptic functions in the form \cite{22}

$$L_n(\lambda) = \sum_{a=1}^{3} w_a(\lambda - z_n) S_n^a \otimes \sigma^a, \quad r(\lambda) = \sum_{a=1}^{3} w_a(\lambda) \sigma^a \otimes \sigma^a, \quad n = 0, 1, \ldots, N_n$$

where

$$w_1(\lambda) = \frac{cn}{sn}(\lambda), \quad w_2(\lambda) = \frac{dn}{sn}(\lambda), \quad w_3(\lambda) = \frac{1}{sn}(\lambda). \quad (9)$$

Operators (9) satisfy a semiclassical YBE: \[ [L_n(\lambda) \otimes L_n(\mu)] = [r(\lambda - \mu), (L_n(\lambda) \otimes I + I \otimes L_n(\mu))] \]

obtained at $\alpha \to 0$ limit from the QYBE. Mutually commuting conserved quantities $H_n$ of the EGM are obtained from the expansion coefficient of the $xyz$ transfer matrix

$$\tau_{xyz}(\lambda, \alpha) = I + \alpha^2 \tau^{(2)}(\lambda) + O(\alpha^3) \quad (10)$$

as

$$\tau^{(2)}(\lambda \to z_n) = H_n = \sum_{a,m \neq n} w_n(z_n - z_m) S_n^a S_m^a \quad (11)$$

with $\sum_n H_n = 0$.

It is most crucial for our purpose to note that, higher spin operators $S_n^a$ appearing in the Gaudin model (9), are reduced from the generators of the Sklyanin algebra to simply those of

$$[S_n^+, S_m^-] = 2 \rho \delta_{nm} S_n^3, \quad [S_n^3, S_m^\pm] = \pm \delta_{nm} S_n^\pm, \quad (12)$$

with $\rho = \pm$ corresponding to $su(2)$ (or $su(1,1)$). Recall that in constructing standard MR models with RWA we have taken the spin operators for $N_a$ number of atoms as generators of $su(2)$, while for the radiation field we have considered bosonic realizations of different algebraic structures in (5). For EMG (9) however apart from $su(2)$, only other possible algebra is $su(1,1)$, as given in (12). And even this can not be used, since different algebras can not be mixed in integrable Gaudin model. On the other hand, if we choose $su(2)$ algebra for all operators and use the Holstein-Primakoff transformation (HPT) $S_0^+ = b^\dagger \sqrt{\frac{2}{\epsilon}} - N, \quad S_0^- = \sqrt{\frac{2}{\epsilon}} - Nb, \quad S_0^3 = N - \frac{\rho_0}{2}, \quad N = b^\dagger b$ for bosonizing the radiative mode, though we can retain the integrability, but get unphysical Hamiltonian for $|N| > \frac{\rho_0}{2}$, with $S_0^+$ becoming nonhermitian!

### 4.2 High spin limit

Fortunately, the above difficulties can be bypassed again by considering further a high spin limit $s_0 \to \infty$ for the radiation field, which reduces the HPT to

$$S_0^+ = \frac{1}{\sqrt{2\epsilon}} b^\dagger, \quad S_0^- = \frac{1}{\sqrt{2\epsilon}} b, \quad S_0^3 = -\frac{1}{4\epsilon^2}, \quad \epsilon = \frac{1}{\sqrt{s_0}} \to 0, \quad (13)$$

by retaining terms up to order $O(\frac{1}{\sqrt{\epsilon}})$.

For deriving now the integrable matter-radiation models without RWA, we replace spin operators at $n = 0$ by bosons for the radiative mode in (11) by using (13) and choose the arbitrary parameters at $n = 0, j$ as

$$z_0 = K + \epsilon x_0, \quad z_j = \epsilon x_j, \quad j = 1, \ldots N_a, \quad (14)$$
where $K$ is the elliptic integral [23] related to a period of the elliptic functions. At the limit $\epsilon \to 0$ therefore the coupling parameters $w_n(z_n - z_m)$ can be expanded as

$$w_\pm(z_0 - z_k) = -w_\pm(z_k - z_0) \to \pm \frac{k'}{2} + O(\epsilon), \quad w_+(z_0 - z_k) = -w_3(z_k - z_0) \to 1 + O(\epsilon^2),$$

Denoting $w_\pm = \frac{1}{2}(w_1 \pm w_2)$, which reduce the elliptic Gaudin Hamiltonian to $H_n = \frac{1}{\epsilon}H_n^{(2)} + \frac{1}{\epsilon^2}H_n^{(1)} + O(\epsilon^0)$ with $H_n^{(a)} = -\sum_j H_j^{(a)}, a = 1, 2$. Therefore, $H_j^{(a)}$ give mutually commuting independent conserved quantities: $[H_j^{(a)}, H_k^{(b)}] = 0$, with

$$H_j^{(2)} = S_j^3, \quad H_j^{(1)} = H_j^{(bs)} + H_j^{(SS)}, \quad j = 1, \ldots, N_a,$$

where

$$H_j^{(bs)} = \Omega \left((bS_j^+ + b^\dagger S_j^-) + (b^\dagger S_j^+ + bS_j^-)\right), \quad \Omega = \frac{k'}{2\sqrt{2}},$$

with elliptic modulus $k = \sqrt{1 - k'^2}$, describes matter-radiation interaction having explicit CRW terms, where by an allowed phase transformation $e^{i\tau}$ in both $b, S_j^\pm$, we have changed the sign of the RW term. In (16) the part

$$H_j^{(ss)} = \sum_{k \neq j} \frac{1}{x_j - x_k}(S_j^+ S_k^- + S_j^- S_k^+ + S_j^3 S_k^3),$$

accounts for interatomic interactions with inhomogenous coupling constants expressed through $x_j$. These arbitrary parameters may be adjusted to make the interaction strengths diminishing with increasing distance between the atoms. Various combinations of Hamiltonians (16) with arbitrary coupling constants: $H_j = \omega_j S_j^3 + J_j(H_j^{(bs)} + H_j^{(SS)})$ can yield different integrable multi-atom matter-radiation models without RWA. For example, by taking simply $\sum_j H_j$ with $\omega_j = \omega, J_j = 1$, we get an integrable multi-atom JC-type model with nontrivial CRW terms:

$$H_0 = \sum_j \left(\omega S_j^3 + \Omega(b + b^\dagger)(S_j^+ + S_j^-)\right)$$

which in contrast to the popular belief [11] is exactly solvable through Bethe ansatz, as we see below. To make the model more physical we should add a field term $\omega_f b^\dagger b$ to the Hamiltonian (19). Considering the field frequency $\omega_f$ to be small we can treat this additional term perturbatively over the Bethe solvable original integrable model. It is curious to note that, essentially at this small field frequency limit the RWA becomes a bad approximation and the CRW terms, which we have considered, become significant.

5 Exact Bethe ansatz solutions

In the basic algebraic Bethe ansatz (ABA) method the trace of the monodromy matrix $\tau(\lambda) = trT(\lambda)$ produces conserved operators, while the off-diagonal elements $T_{21}(\lambda) \equiv B(\lambda)$ and $T_{12}(\lambda) \equiv C(\lambda)$ act
like creation and annihilation operators of pseudoparticles, with the M-particle state given as \( |M >_B = B(\lambda_1) \cdots B(\lambda_M)|0 > \), with the pseudovacuum \( |0 > \) defined through \( C(\lambda)|0 >= 0 \). The major aim of ABA \([19]\) is to find the eigenvalue solution: \( \tau(\lambda)|M >_B = \Lambda(\lambda, \{\lambda_a\})|M >_B \).

All our models with RWA, constructed in a unified way through the ancestor model scheme, can be solved in a unified manner following the ABA method in its standard formulation \([19]\). Omitting here the details \([17]\), we focus on the ABA solution of our integrable models without RWA \((16-19)\). For deriving the ABA relations of these models we have to repeat the steps we have followed in obtaining their Hamiltonians. That is we have to start from the ABA relations for the \( x y z \) spin chain with arbitrary spins, where a gauge transformed version of operators and states has to be used to define a modified pseudovacuum \([23, 24]\). Considering carefully the limit \( \alpha \to 0 \) the corresponding ABA relations for the elliptic Gaudin model is obtained \([22]\). Taking the high spin limit for the radiative mode in these relations we derive finally the eigenvalues and the Bethe equations for our integrable models without RWA.

We start therefore from the elliptic Gaudin model, and using the key ABA relations from \([22]\), we derive the energy spectrum as

\[
E^{(egaud)}_n = 2 s_n \theta'_1(0)(i \pi \nu + \sum_{k=1}^M \frac{\theta'_{11}(z_n - \lambda_b)}{\theta_{11}(z_n - \lambda_b)} + \sum_{m \neq n} s_m \frac{\theta'_{11}(z_n - z_m)}{\theta_{11}(z_n - z_m)})
\]  

(20)

with arbitrary spin values \( s_n \) of operators at the \( n \)-th site and involving elliptic function \( \theta_{11} \). The corresponding Bethe equations can be found in \([22]\) as

\[
\sum_{n=1}^N s_n \frac{\theta'_{11}(\lambda_a - z_n)}{\theta_{11}(\lambda_a - z_n)} = -i \pi \nu + \sum_{b \neq a} \frac{\theta'_{11}(\lambda_a - \lambda_b)}{\theta_{11}(\lambda_a - \lambda_b)}
\]  

(21)

Choosing the inhomogeneity parameters at \( n = 0, j \) as in \((14)\), introducing a scaling also for the Bethe parameters: \( \lambda_a = \epsilon l_a \) and using some properties of the \( \theta_{11} \) function, we can derive now from \((20)\) for \( n = j \) the exact eigenvalue for our matter-radiation model \((16)\) without RWA, at the high spin limit (or equivalently, at \( \epsilon \to 0 \)) for the radiation field as

\[
E^{(crw)}_j = 2 s_j \theta'_1 \left( \sum_{b=1}^M \frac{1}{(x_j - l_b)} + \sum_{k \neq j} \frac{s_k}{(x_j - x_k)} + \frac{\theta''_{10}}{\theta_{10}} (x_j - x_0) \right)
\]  

(22)

where we have used short-hand notations \( \theta^{(l)}_1(0) = \theta^{(l)}_{11} \), for \( l = 0, 1, 2 \). Similarly expanding \((21)\) we extract the corresponding Bethe equation

\[
\frac{\theta''_{10}}{\theta_{10}} (x_0 - l_a) + \sum_{k=1}^N s_k \frac{1}{(x_k - l_a)} = \sum_{b \neq a} \frac{1}{(l_b - l_a)}, \quad a = 1, \ldots, M,
\]  

(23)

the solutions of which should determine the Bethe momentums \( l_b, b = 1, \ldots, M \) for a given arbitrary nondegenerate set of inhomogeneity parameters \( x_j, j = 1, \ldots, N_a \). Note that \( x_0 \) can be absorbed by shifting the parameters \( x_j \) and \( l_a \).

Either expanding \((20)\) for \( n = 0 \) and combining terms of different orders, or by taking the sum of \((22)\) with an additional term with \( \omega \) and using the Bethe equation \((23)\), we can obtain the exact eigenvalues for our multi-atom model \((19)\), with explicit CRW terms as

\[
E^{(crw)}_0 = \frac{\theta'_{11}}{\theta_{10}} \left( \sum_{b=1}^M (x_0 - l_b) + \sum_{k=1}^N s_k (x_0 - x_k) \right) + 2i \omega \pi \theta'_{11} \nu
\]  

(24)
6 Concluding remarks

In real matter-radiation models involving many atoms interatomic interactions must appear. Likewise the customary rotating wave approximation in general must be avoided, at least when away from the resonance point. However these important cases could not be included without spoiling the integrability in most of the solvable models proposed earlier. We have proposed new integrable models, where both these cases can be incorporated retaining the integrability of the system and extract exact solutions through the Bethe ansatz. For models with rotating wave approximation we find a series of integrable multi-atom models with inter-atomic interactions, which gives generalizations of Jaynes-Cummings, Buck-Sukumar and trapped ion models together with their q-deformations. These models belong to the rational or the trigonometric class and can be solved exactly in a unified way.

We also construct for the first time exactly solvable multi-atom matter-radiation models without rotating wave approximation (RWA), which are close to the physical systems with the required counter rotating wave (CRW) terms. We achieve this by taking the high spin limit for the radiation field in the elliptic Gaudin model. However in contrast to our models with RWA, where we could construct different types of matter-radiation models, our success with models without RWA is restricted, where we could obtain only JC type models with CRW terms. Moreover, the free radiation term does not naturally appear in such integrable models and has to be included perturbatively over the exact BA result.

Identifying the models in real systems and experimental verification of the related results presented here, especially in many-atom microlasers [25] or in trapped ions away from resonance point [11] would be an important application.

References

[1] G. Rempe, H. Walther and N. Klein , Phys. Rev. Lett. 58, 353 (1987); G. Rempe, F. Schmidt-kaler and H. Walther, Phys. Rev. Lett. 64, 2783 (1990)

[2] M. Raizen et al , Phys. Rev. Lett. 63, 240 (1989)

[3] C. A. Blockley, D. F. Walls and H. Risken, Europhys. Lett. 17, 509 (1992) ;

[4] W. Vogel and R. de Mitos Filho, Phys. Rev. A 52, 4214 (1995)

[5] E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963)

[6] B. Buck and C. V. Sukumar, Phys. Lett. 81 A, 132 (1981)

[7] V. Buzek, J. Mod. Opt. 39, 949 (1992)

[8] M. Chaichian, D. Ellinas and P. Kulish , Phys. Rev. Lett. 65, 980 (1990)

[9] H. S. Zheng, L. M. Kuang, K. L. Gao , Jaynes-Cummings model dynamics in two trapped ions, arXiv:quant-ph/0106020 (2001)

[10] G. S. Agarwal, Phys. Rev. Lett. 53, 1732 (1984)
[11] R. Angelo et al, Phys. Rev. A 64, 043801 (2001)
[12] M. Tavis and F. W. Cummings, Phys. Rev. 170, 379 (1968); W. R. Mallory, Phys. Rev. 188, 1976 (1969)
[13] N. Bogolubov et al, J. Phys. A 29, 6305 (1996)
[14] A. Rybin et al, J. Phys. A 31, 4705 (1998)
[15] L. Amico and K. Hikami, cond-mat/0309680 (2003)
[16] J. Dukelsky et al, Phys. Rev. Lett. 93, 050403 (2004)
[17] A. Kundu, J. Phys. 37, L281 (2004)
[18] A. Kundu, Phys. Rev. Lett. 82, 3936 (1999)
[19] L. Takhtajan, *Exactly solvable problems in condensed matter and relativistic field theory*, (Springer Verlag, 1985), 175
[20] V. Pasquier and H. Saleur, Nucl Phys. B 330, 523 (1990)
[21] E. K. Sklyanin, Func. Anal. Appl. 16, 263 (1983); 17, 273 (1984)
[22] E. K. Sklyanin and T. Takebe, Phys. Lett. A 219, 217 (1996)
[23] L. Takhtajan and L. Faddeev, Russ. Math. Surveys, 34, 11 (1979)
[24] T. Takebe, J. Phys. A 25, 1071 (1992); 28, 6675 (1995)
[25] K. An et al, Phys. Rev. Lett. 73, 3375 (1994); K. An, J. Phys. Soc. Jpn. 72, 811 (2003)