USING ENERGY CONDITIONS TO DISTINGUISH BRANE MODELS AND STUDY BRANE MATTER

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Current universe (assumed here to be normal matter on the brane) is pressureless from observations. In this case the energy condition is $\rho_0 \geq 0$ and $p_0 = 0$. By using this condition, brane models can be distinguished. Then, assuming arbitrary component of matter in DGP model, we use four known energy conditions to study the matter on the brane. If there is nonnormal matter or energy (for example dark energy with $w < -1/3$) on the brane, the universe is accelerated.

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I. INTRODUCTION

In braneworld scenarios, our universe is a 3-brane embedded in a higher-dimensional bulk [1]-[3]. It is proposed that braneworld modification of gravity to explain the accelerating expansion of the universe by Dvali, Gabadadze and Porrati (DGP) [4]. (see [5] for a recent review). In this model our universe is a 3-brane embedded in an infinite-volume extra space. Meanwhile, a more general class of braneworld models is described by Sahni and Shtanov [6]. The cosmological constant in the bulk and the curvature term in the action for the brane with coefficient $m^2$ are included in this model. Then the exact global solutions of brane universes are given in [7]. It contains two arbitrary functions of time $t$. As we know, a number of recent observations indicate that the expansion rate of our universe is accelerating [8]. The cosmological solution of DGP theory exhibits self-acceleration on the brane [9]. This solution describes a universe that is accelerating beyond the crossover scale. Moreover, the braneworld models of dark energy are studied in [10]. Now the acceleration behavior in braneworld scenarios are widely studied in terms of current observational data [11].

The purpose of this paper is to use energy conditions to distinguish the different brane models and study the matter in DGP brane. As we know in the braneworld model, the standard matter particles and forces are confined on the 3-brane while gravity can freely propagate in all dimensions. From current observation, we can distinguish these brane models by energy conditions. Then, in DGP brane we assume there is normal matter or non-normal and apply energy conditions to study the matter on the brane.

II. USING ENERGY CONDITIONS TO DISTINGUISH THE BRANE UNIVERSES

In braneworld model, the standard matter particles and forces are confined on the 3-brane while gravity can freely propagate in all dimensions. The Hubble and deceleration parameters are respectively given on $y = 0$ brane by

$$H = \frac{\dot{a}}{a}, \quad q = -\frac{1}{H^2} \frac{\ddot{a}}{a},$$

(1)

The conservation equation for the energy-momentum tensor of the cosmic fluid is

$$\dot{\rho} + 3H(p + \rho) = 0.$$  

(2)

We note that observations show that current universe (assumed here to be normal matter, as opposed to dark energy) is pressureless. In this case the energy condition is $\rho_0 \geq 0$ and $p_0 = 0$, where the subindex 0 means the current quantity. For different brane models, we can obtain different constraints with this energy condition. The analysis is given as follows.

A. DGP brane

For the DGP brane model, the Friedmann equation is written as [3]

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{\rho}{3M_{pl}^2} + 1/4r_c^2 + \frac{1}{2r_c^2}}\right)^2,$$

(3)

where $\rho$ is the total cosmic fluid energy density, and $\epsilon = \pm 1$. $r_c = M_{pl}^2/2M_5^2$, $M_{pl}$ and $M_5$ are independent parameters (in general there could be an relation between the two quantity). For the self-inflationary solution, it was adopted $\epsilon = 1$. So we set $\epsilon = 1$ firstly. For $k = 0$, Eq. (3) is rewritten as

$$\frac{\rho}{3M_{pl}^2} = H^2 - \frac{H}{r_c}.$$  

(4)
with $H \geq 1/2r_c$. Then from $\rho_0 \geq 0$, the inequality is obtained as $H_0 \geq 1/r_c$. This inequality gives a relation between $H_0$ and $r_c$. Since $H$ is observable quantity and $r_c$ is a quantity from the theory model, this inequality is beneficial to choose $r_c$. From the conservation equation (2) and the Friedmann equation (4), the pressure is expressed as

$$\frac{p}{M^2_{pl}} = H^2(2q - 1) - \frac{H}{r_c}(q - 2).$$

(5)

By using $p_0 = 0$, we get the relation

$$\frac{1}{r_c} = \frac{H_0(2q_0 - 1)}{q_0 - 2}.$$  (6)

From the inequality $H_0 \geq 1/r_c > 0$ and the Eq. (6), we have $-1 < q_0 < 1/2$. According current observation value $q_0 = -0.81 \pm 0.14$ [21], this inequality is valid.

Then, if we choose $\epsilon = -1$, with $H \geq -1/2r_c$ the density and pressure are

$$\frac{\rho}{3M^2_{pl}} = H^2 + \frac{H}{r_c},$$

(7)

$$\frac{p}{M^2_{pl}} = H^2(2q - 1) + \frac{H}{r_c}(q - 2).$$

(8)

Since the energy conditions $\rho_0 \geq 0$ and $p_0 = 0$, we have two relations as

$$H_0 + \frac{1}{r_c} \geq 0,$$

(9)

$$H_0(2q_0 - 1) + \frac{1}{r_c}(q_0 - 2) = 0.$$  (10)

So we can get $-1 < q_0 < 2$. Since $H_0 > 0$, from (10), it is obtained that $1/2 < q_0 < 2$. This deceleration parameter shows our universe should be deaccelerating expansion. However according to the present observation this is not true, and our universe is accelerating. Therefore, to satisfy the energy conditions, $\epsilon \neq -1$ in this model.

B. Brane models with brane tension and cosmological constant

For the braneworld models in [22], the Friedmann equation takes the form

$$H^2 + \frac{k}{a^2} = \frac{\rho + \sigma}{3m^2} + \frac{2}{l^2} \left[ 1 \pm \sqrt{1 + \frac{2(\rho + \sigma)}{3m^2} \frac{\Lambda}{6} - \frac{C}{a^4}} \right],$$

(11)

where $\sigma$ is the brane tension, $\Lambda$ is the bulk cosmological constant, $l = m^2/M^3$, $C$ is a constant, and the term $C/a^4$ plays the role of “dark radiation”. Next, we set $k = 0$, $\Lambda = 0$ and neglect the radiation density. Eq. (11) is rewritten as

$$H^2 = \frac{\rho + \sigma}{3m^2} + \frac{2}{l^2} \left[ 1 \pm \sqrt{1 + \frac{2(\rho + \sigma)}{3m^2}} \right].$$

(12)

In this model, the Friedmann equation contains $\pm$ sign, so this model is separated two types, i.e. BRANE1 and BRANE2 with “−” and “+” respectively. Next, we will discuss them as follows:

For BRANE1, choosing the “−” sign, the Friedmann equation is

$$H^2 = \frac{\rho + \sigma}{3m^2} + \frac{2}{l^2} \left[ 1 - \sqrt{1 + \frac{2(\rho + \sigma)}{3m^2}} \right].$$

(13)

When the condition is

$$1 - (H^2 - \frac{\rho + \sigma}{3m^2})^2 \geq 0,$$

(14)

the density is

$$\rho = 3m^2(H^2 + \frac{2}{l}H - \sigma),$$

(15)

where, if $H^2 - (\rho + \sigma)/3m^2 \geq 0$, adopt “−”; while if $H^2 - (\rho + \sigma)/3m^2 \leq 0$, adopt “+”. These can ensure mathematical reasonability of our calculation.

Firstly, choosing “−” sign, the density and pressure are

$$\rho = 3m^2(H^2 - \frac{2}{l}H - \sigma),$$

(16)

$$p = m^2[H^2(2q_0 - 1) - \frac{2}{l}H(q_0 - 2)] + \sigma.$$  (17)

Because the inequality $\rho_0 \geq 0$ and the equality $p_0 = 0$, we get

$$3m^2(H_0^2 - \frac{2}{l}H_0) - \sigma \geq 0,$$

(18)

$$m^2[H_0^2(2q_0 - 1) - \frac{2}{l}H_0(q_0 - 2)] + \sigma = 0.$$  (19)

Substituted Eq. (19) into the inequality (18), one new inequality can be obtained as

$$(H_0 - \frac{1}{l})(q_0 + 1) \geq 0.$$  (20)

Therefore, we get two solutions of this inequality: one is $H_0 \geq 1/l$ and $q_0 \geq -1$, the other is $H_0 \leq 1/l$ and $q_0 \leq -1$. In terms of current observation, $q_0 = -0.81 \pm 0.14$ therefor we find $H_0 \geq 1/l$ and $q_0 \geq -1$. However, the inequality (14) and equation (16) imply $H \leq 1/l$. So, only the critical situation $H_0 = 1/l$ is suitable, but this is a critical condition.

Secondly, choosing “+” sign, we obtain the density and pressure as

$$\rho = 3m^2(H^2 + \frac{2}{l}H - \sigma),$$

(21)

$$p = m^2[H^2(2q_0 - 1) + \frac{2}{l}H(q_0 - 2)] + \sigma.$$  (22)

Form the energy conditions $\rho_0 \geq 0$ and $p_0 = 0$, they are obtained as

$$3m^2(H_0^2 + \frac{2}{l}H_0) - \sigma \geq 0,$$

(23)

$$m^2[H_0^2(2q_0 - 1) + \frac{2}{l}H_0(q_0 - 2)] + \sigma = 0.$$  (24)
Substituting Eq. (24) into the inequality (23), we have

\[ (H_0 + \frac{1}{l})(q_0 + 1) \geq 0. \tag{25} \]

Since \( H_0 > 0 \) and \( l > 0 \), we get the solution of this inequality as \( q_0 \geq -1 \). In terms of current observation, \( q_0 = -0.81 \pm 0.14 \), therefore \( q_0 \geq -1 \) is valid. At the same time, from (24), we can find \( \sigma \geq 0 \). This is a positive brane tension.

For BRANE2, taking “+” sign, the Friedmann equation is

\[ H^2 = \frac{\rho + \sigma}{3m^2} + \frac{2}{l^2} \left[ 1 + \sqrt{1 + \frac{4l^2 \rho + \sigma}{3m^2}} \right]. \tag{26} \]

When the condition is

\[ (H^2 - \frac{\rho + \sigma}{3m^2})^2/2 - 1 \geq 0, \tag{27} \]

the density is written as

\[ \rho = 3m^2(H^2 \pm \frac{2H}{l}) - \sigma. \tag{28} \]

where, if \( H^2 - (\rho + \sigma)/3m^2 \geq 0 \), we will take “-”; while if \( H^2 - (\rho + \sigma)/3m^2 \leq 0 \), we will take “+” for being reasonable in mathematics.

Firstly, taking “-” sign, we have

\[ \rho = 3m^2(H^2 - \frac{2}{l}H) - \sigma. \tag{29} \]

\[ p = m^2[H^2(2q - 1) - \frac{2}{l}H(q - 2)] + \sigma. \tag{30} \]

Eq.(29) and Eq.(24) imply the condition \( Hl - 1 \geq 0 \). Under the energy conditions \( \rho_0 \geq 0 \) and \( p_0 = 0 \), they are obtained as

\[ 3m^2(H_0^2 - \frac{2}{l}H_0) - \sigma \geq 0. \tag{31} \]

\[ m^2[H_0^2(2q_0 - 1) - \frac{2}{l}H_0(q_0 - 2)] + \sigma = 0. \tag{32} \]

Substituting Eq. (32) into the inequality (31), we have

\[ (H_0 - \frac{1}{l})(q_0 + 1) \geq 0. \tag{33} \]

Then, we get two solutions: one is \( H_0 \geq 1/l \) when \( q_0 \geq -1 \); the other is \( H_0 \leq 1/l \) when \( q_0 \leq -1 \). From the current value \( q_0 = -0.81 \pm 0.14 \), the latter solution is not satisfied. So, the suitable solution is \( H_0 \geq 1/l \) when \( q_0 \geq -1 \). However in this situation, from (24), since \( \sigma \) is determined by \( l \), we can not verify it is positive or negative.

Secondly, taking “+” sign, the density and the pressure are

\[ \rho = 3m^2(H^2 + \frac{2}{l}H) - \sigma, \tag{34} \]

\[ p = m^2[H^2(2q - 1) + \frac{2}{l}H(q - 2)] + \sigma. \tag{35} \]

Considering the energy conditions \( \rho_0 \geq 0 \) and \( p_0 = 0 \), we obtain

\[ 3m^2(H_0^2 + \frac{2}{l}H_0) - \sigma \geq 0. \tag{36} \]

\[ m^2[H_0^2(2q_0 - 1) + \frac{2}{l}H_0(q_0 - 2)] + \sigma = 0. \tag{37} \]

Substituting Eq. (37) into the inequality (36), the new inequality is

\[ (H_0 + \frac{1}{l})(q_0 + 1) \geq 0. \tag{38} \]

for \( H_0 > 0 \) and \( l > 0 \), the solution of this inequality is \( q_0 \geq -1 \). From the current value \( q_0 = -0.81 \pm 0.14 \), this satisfies the \( q_0 \geq -1 \). But from Eq. (34) and (24), it is obtained that \(-H_0(0) - 1 \geq 0 \). This is incompatible with \( H_0 > 0 \). Therefore there is no valid solution.

### III. STUDY MATTER ON DGP BRANE WITH ENERGY CONDITIONS

In this Section I, \( \epsilon = 1 \) DGP brane is considered. Now we assume arbitrary matter besides normal matter on the brane. From (4) and (5), the \( \rho \) and \( p \) with \( M_{pl}^2 = 1 \) are described as

\[ \rho = 3(H^2 - \frac{H}{r_c}) \tag{39} \]

\[ p = H^2(2q - 1) - \frac{H}{r_c}(q - 2). \tag{40} \]

The standard classical energy conditions are the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC), and dominant energy condition (DEC). Basic definitions of these energy conditions can be found in Ref. [12]. For the case in cosmology they are

\[ \text{NEC : } \rho + p \geq 0, \tag{41} \]

\[ \text{WEC : } \rho \geq 0 \text{ and } \rho + p \geq 0, \tag{42} \]

\[ \text{SEC : } \rho + 3p \geq 0 \text{ and } \rho + p \geq 0, \tag{43} \]

\[ \text{DEC : } \rho \geq 0 \text{ and } \rho \geq |p|. \tag{44} \]

They were used in deriving many theorems such as the singularity theorems [13], the censorship theorem [14] and so on. Other applications of the energy conditions to cosmology can be found in [15]-[18]. In the classical general relativity these energy conditions are satisfied, while if considering the quantum effect, energy conditions should be violated [19, 20].

We consider the universe is dominated by one fluid with \( \rho \) and \( p \). The equation of state is

\[ w = \frac{p}{\rho}, \tag{45} \]

where \( w \) may be arbitrary form on the brane. Substituting this equation in to the four energy conditions, we
Following previous usage (see, for example [1]) we call matter that satisfies all the four energy conditions “normal” and call matter that specifically violates the SEC “abnormal”. And we call matter that violates any one of the four energy conditions “non-normal”. In the following we will discuss the normal and nonnormal matter respectively.

Firstly, for normal matter all the four standard energy conditions should be satisfied. From (16)-(19) we have

Normal Matter : \( \rho \geq 0 \) and \(-1/3 \leq w \leq 1\). (50)

With the use of these constraints on the brane, we obtain from (39) and (40) that is

\[
H \geq \frac{1}{r_c}, \quad (51)
\]

\[
q \geq -\frac{3}{r_c} \frac{1}{2H - \frac{1}{r_c}}, \quad (52)
\]

\[
q \leq \frac{4H - \frac{5}{3}}{2H - \frac{1}{r_c}}. \quad (53)
\]

Therefore, when \(1/r_c \leq H \leq 5/(4r_c)\), the deceleration \(q\) satisfies \(-3 \leq q \leq 0\) and this show the universe is accelerating. Even when \(H \geq 5/(4r_c)\), from (52), we can not exclude that \(q\) is negative.

Secondly, we discuss “nonnormal” matter on the brane. The constraints are described as

\[
\rho \geq 0 \quad \text{and} \quad w \leq -\frac{1}{3}. \quad (54)
\]

Then, we have

\[
H \geq \frac{1}{r_c}, \quad (55)
\]

\[
q \leq -\frac{3}{r_c} \frac{1}{2H - \frac{1}{r_c}}. \quad (56)
\]

Since \(r_c \geq 0\) and \(2H - 1/r_c \geq 0\), the deceleration \(q\) is negative. If there are “abnormal” or “nonnormal” matter on the brane, the universe do be accelerating.

If normal matter in the standard FRW cosmology, all energy conditions are satisfied and the universe is decelerating. However, if normal matter on DGP brane, all energy conditions are also satisfied but the accelerating universe can be obtained. Moreover, the EOS of quintessence field is \(-1 \leq w \leq 1\). Therefore quintessence is a normal matter if \(-1/3 \leq w < 1\) and abnormal matter if \(-1 \leq w < -1/3\). If quintessence with \(-1 \leq w < -1/3\) on the brane, the universe is accelerated. Because the EOS of phantom is \(w < -1\), the universe is accelerated by this field on the brane.

**IV. CONCLUSION**

In this paper, energy conditions are used to distinguish different brane models and study the matter on DGP brane. We notice that observations show that the current of universe (assumed here to be normal matter, as opposed to dark energy) is pressureless. Therefore, the energy consititutions reduce to the inequality \(\rho_0 \geq 0\) and the equation \(p_0 = 0\). Then we use these two relations to analyze DGP model and the models with brane tension and cosmological constant. In the DGP model, we find when \(\epsilon = 1\), the energy conditions are satisfied, while they does not satisfy the \(\epsilon = -1\) case. For the models with brane tension and cosmological constant, there are two types, i.e. BRANE1 and BRANE2. To satisfy the energy conditions, in the BRANE1, we find the form of density is expressed as \(\rho = 3m^2(1 + 2H/l) - \sigma\) and the brane tension \(\sigma\) is positive; while for the BRANE2, the relation of density is described as \(\rho = 3m^2(1 - 2H/l) - \sigma\) but it is not known whether the brane tension is positive. Meanwhile, we get many inequalities and equations to limit these brane models, and these relations will be verified by measurement of cosmic fundamental constants and parameters in the future. At last, we assume arbitrary matter and use four known energy conditions to study the matter on the brane in DGP model. If only normal matter is on the brane, we obtain \(-3 \leq q \leq 0\) when \(1/r_c \leq H \leq 5/(4r_c)\) and can not exclude the accelerating universe. However, If there is nonnormal matter with \(w < -1/3\) on the brane, the universe is accelerated.

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