To Phenomenological Theory of Superfluidity.
Superfluidity - Viscousless Flow in Viscous Medium

Yu.L.Klimontovich
1179899 Russian Moscow
Moscow M.V.Lomonosov State University
Physical Department

Despite of remarkable successes the theory of superfluidity, until now we have no a physical explanation of an opportunity of existence of viscousless flow of liquid helium in viscous medium. The existence of superfluid flow becomes possible due to occurrence of flicker noise and appropriate residual temporary correlations of the superfluid velocity fluctuations.

I. INTRODUCTION

In 1932 Keesom and Klausius (see Keesom, 1949) have found out in the region temperatures $T_C = 2.19^0$ a anomalous temperature dependence of the heat capacity. That to emphasize distinction of states of the helium higher and below of the of the phase transition point, the names were entered: "helium I" for temperatures $T > T_C$ and "helium II" for temperatures $T < T_C$.

In 1938 (Kapitza, 1938, 1941, 1944) has found out, that the helium has superfluidity - the ability of viscousless of flow through thin cracks and capillaries. He has found out also presence in capillaries the opposite flow of viscous "normal" and viscousless "superfluid" component liquid helium.

In the Kapitza experiments the viscous flow run out from the small vessel is filled by liquid helium. The flow is caused by the warming up helium in the vessel and it is discovered by a deviation of a target. It is essential, that thus the level of helium in he vessel remain constant. This indicates on the presence of a opposite flow - the flow of the superfluid helium.

To explain this phenomenon he accepts "two-liquid model" of helium II, offered in work (Tisza, 1938) and essential developed by (Landau, 1941), but in (Kapitza, 1944) he writes:

"If this theoretical supposition was not so full supported by the experimental proofs, it would sound as idea, which it is very difficult to recognize reasonable."

Thus, two-liquid model, even for the physicist of so high level, it is not represented rather clear.

Use of two-liquid model is justified by that on its basis it is possible to describe observable phenomena. In particulars, it gives an explanation the existence of the second sound (Landau, 1944), explains results of the Andronikashvily experiences with rotating helium (Andronokashvily, 1940).

Despite of these doubtless successes, the physical picture of the phenomenon superfluidity remains nevertheless not quite clear. Two basic ques-
The basic purpose of the present work - to discuss the possible answers on these questions on the following suppositions:

1). Superfluidity the macroscopic phenomenon in a dissipative continuous medium is. The description of superfluidity will be carried out on the basis of the corresponding equations of the mechanics of continuous medium.

2). Helium II is the example of a quantum liquid, as length of de Broglie wave is order of the average distance between atoms. However, in the approximation of continuous medium the de Broglie length is much less of size of a “point”, in which many particles contain. Due to this condition the description of superfluidity on a basis of the phenomenological kinetic equations for classical distribution function it is possible.

3). The superfluidity by two phenomena is caused.

First, by the phase transition of the second order, as a result of which arise not only fast, but also slow (coherence) relaxation processes. The last provide a spatial coherence on scales cracks or capillaries.

Secondly, there is a reorganization of structure of viscous friction. It is caused by occurrence of flicker noise ("1/f-noise) of hydrodynamic velocity fluctuations. The appropriate distribution on wave numbers represents an analogue of the "bose-condensation", as the dispersion of wave numbers become proportional to frequency for $\omega \rightarrow 0$.

II. PHASE TRANSITION HELIUM I - HELIUM II

A level with the phenomenological theory, the microscopic theory of superfluidity is developed also. The basic work in this direction the Bogolubov paper "To the theory of superfluidity" is (Bogolubov, 1947). By the object of research in the Bogolubov paper a weak nonideal Bose-gas was served.

A. The condensation of a weakly nonideal gas

In the classic Einstein (1924, 1925) papers is shown, that the continuous Bose-Einstein distribution it is valid only at temperatures $T > 3.14^0K$. In the appropriate critical point the distribution of particles on momenta breaks up to two parts: continuous distribution with zero value of the chemical potential and the "condensate" with the distribution $\delta(p)$. On a measure downturn of temperature the number of particles with a zero momentum grows and at zero temperature coincides with full number of particles. At the critical temperature the number of particles in a condensate is equal to zero.
Attempts (Tisza, 1938 and London 1954) to explain superfluidity of helium on the basis of the Bose-condensation phenomenon of ideal gas were not successful.

B. Weak nonideal Bose-gas. Bogolubov theory

In the ideal gas the condensate does not form the connected collective and therefore has not property of superfluidity.

In a weak nonideal gas because of interaction of particles, as Bogolubov has shown, the condensate forms collective and at it the motion as the whole only at the birth of collective elementary excitation is possible. At small momenta the energy of excitations is defined by classical expression:

\[ \varepsilon(p) = v_S p, \quad v_S = \sqrt{n \frac{\nu(|p = 0|)}{m}} p = \sqrt{\frac{4\pi \hbar^2 a}{m^2 n}}. \]  

which answers to a phonon part of the Landau spectrum. The speed of sound \( v_S \) via the amplitude \( a \) of the Born scattering is expressed

The Bogolubov spectra is valid that the pushing away of atoms prevails above their attraction ones. For the liquid helium the quantity \( a \) is significant less of the mean distance \( r_{av} \) between atoms, therefore in the theory of superfluidity it is possible introduce the corresponding small density parameter \( \varepsilon_S = na^3 \).

At zero temperature the number of particles of nonideal gas in the condensate is less than the complete number of particles. The difference is defined by the small parameter \( \varepsilon_S \). Thus for the weak nonideal gas the basic part of atoms of gas has a zero momentum.

The next years the essential development of the microscopic theory of weakly nonideal Bose-gas at \( T = 0 \) has undergone. Among many it is necessary to allocate the followings.

In the paper (Bogolubov, Zubarev, 1955) for Bose-gas the method of collective variables was used. In the paper (Belyaev, 1958) the perturbation theory on small density parameter \( \varepsilon_S = na^3 \) was developed. In the series of papers (Kirpatrick and Dorfman, 1985) the kinetic description of collective behavior of Bose-gas was carried out.

At last, in the papers (Tserkovnikov, 1992, 1995) the most detailed description of weakly nonideal Bose-gas at \( T = 0 \) is given and the comparative analysis of various method also is spent.

Bogolubov has noticed that the generalization on his theory for a liquid helium it is impossible, therefore the use of a phenomenological equations of continuous medium it is necessary. This remark and to the subsequent works concerns. The essential step in the theory of liquid helium by Feinman was made.
C. Feinman formula - the connection of the elementary excitations energy and the formfactor

Feinman (see in (Feinman, 1974) has established at $T = 0$ the general connection of the elementary excitation spectrum in liquid helium with the static formfactor $S(p)$

$$\varepsilon(p) = \frac{p^2}{2mS(p)}; \quad p = \hbar k.$$  \hspace{1cm} (2)

The static formfactor is defined via the spatial Fourier component of a two-point correlation function or, that is equivalent, through the spatial density of the number particles density fluctuations. For a liquid helium it can be established on a basis of experiments on the scattering of x-ray or neutrons.

In the paper (Bogolubov, Zubarev, 1955) the Feinman relation on the example of weakly nonideal Bose gas by the collective variable method was derivated. Thus it is possible simultaneously to receive expressions both for a elementary exitation spectra, and for the formfactor.

D. Kinetic derivation of the Feinman relation for the nonideal Bose gas

Being based on the kinetic equation for the Vigner function $f(r, p, t)$ with the Langevin source the account of the particles number density fluctuations it is possible to carry out At zero temperature ($T = 0$) is received the following expression for the required spectral density (Klimontovich and Silin, 1952; Pins and Nozier, 1966; Nozier, Pines, 1980):

$$\langle \delta n \delta n \rangle_{\omega, k} = \frac{\hbar}{\nu(k)} \frac{Im\chi(\omega, k)}{|\chi(\omega, k)|^2}.$$  \hspace{1cm} (3)

Here $\chi(\omega, k)$ is the dynamic susceptibility. By integration on frequency in the region of a transparency we find the spatial spectral density, which is connected with the static formfactor - the Feinman relation for a weak nonideal Bose gas

$$\frac{\langle \delta n \delta n \rangle_k}{n} \equiv S(k) = \frac{\hbar^2 k^2}{2m\hbar \omega(k)}.$$  \hspace{1cm} (4)

The Feinman relation was established under condition of $T = 0$ and for this reason it is non sufficient for the description of phase transition in superfluid state, which occurs at non zero temperatures.

E. Feinman relation in a classical limit

Let’s address to the Feinman relation. Let’s rewrite it as

$$\omega(k) = \frac{\hbar^2}{2m} \frac{\langle \delta n \delta n \rangle_k}{n}.$$  \hspace{1cm} (5)
The transition in the right part to the classical limit is impossible. It is connected to that in the formula (3) only a zero oscillations were taken into account. In the classical limit, instead of (3), we have the following expression (Klimontovich, Silin, 1960; Pines, Nozier, 1966):

\[(\delta n\delta n)_{\omega,k} = \frac{2}{\omega
u(k)} \frac{Im\chi(\omega,k)}{|\chi(\omega,k)|^2} k_BT \]  

(6)

By integration on \(\omega\), we shall receive expression for the connection of the spectral density of fluctuations and the component Fourier for potential of atoms interaction:

\[\frac{(\delta n\delta n)_k}{n} = \frac{1}{1 + \frac{\nu(|k|)}{k_BT}}.\]  

(7)

Using the definition of the isothermal compressity coefficient \(\beta_T\) we shall obtain at \(k = 0\) the following relations:

\[\frac{(\delta n\delta n)_{k=0}}{n} \equiv nk_BT/\beta_T = \frac{1}{1 + \frac{\nu(0)}{k_BT}}.\]  

(8)

Let’s compare this expression with the appropriate the Ornstein-Zernike expression, containing a component Fourier of the ”direct correlation function \(C(k)\) (Klimontovich, 1982; Martynov, 1992)

\[\frac{(\delta n\delta n)_{k=0}}{n} \equiv nk_BT/\beta_T = \frac{1}{1 - C(k = 0)} \equiv \frac{1}{1 + \frac{\nu_{eff}(0)}{k_BT}}.\]  

(9)

Here, a level with a direct correlation function \(C(k = 0)\), the definition for the effective potential \(\nu_{eff}(0)\) of interaction of atoms in a liquid helium is given.

Last equalities for dense gases and liquids are used. For this purpose it is necessary to have the additional equations for definition of a direct correlation function \(C(k = 0)\), or the corresponding effective potential \(\nu_{eff}(0)\) (Klimontovich, 1982; Martynov, 1992).

For the critical region at the phase transitions the effective potential with the critical temperature will be connected by the equality: \(n|\nu_{eff}(0)| = kB_TC\).

From the condition of positivity of the function \((\delta n\delta n)_{k=0}, nk_BT/\beta_T\) follows the restrictions

\[C(k = 0) < 1, \quad \frac{n|\nu_{eff}(0)|}{k_BT} < 1.\]  

(10)

At presence of the phase transition (it physical sense will be explained below) at the approach to the critical point from the side of high temperature the isothermal compressity \(\beta_T\) on the Cure law grows. At the critical region the functions \(1 - C(k = 0)\), \(1 + \nu_{eff}(0)/kB_T\) are proportional to the difference of temperature \(T - T_C\). From this follow that for temperature \(T < T_C\) the instability appears.
As well as in the chapters of the second volume, to the second order phase transitions are devoted, the restriction of instability occurs due to nonlinearity. This is provided by the replacement:

$$\frac{T - T_C}{T_C} \rightarrow \frac{T - T_C}{T_C} + \frac{|\psi|^2}{n}.$$  \hspace{1cm} (11)

The unknown function $|\psi|^2$ will be found below by the solution of the corresponding kinetic equations.

In result for the critical region are received the following expressions for correlator $(\delta n\delta n)_{k=0}$ and the isothermal compressibility coefficient $nk_BT\beta_T$:

$$\frac{(\delta n\delta n)_{k=0}}{n} \equiv nk_BT\beta_T = \frac{1}{\frac{T - T_C}{T_C} + \frac{|\psi|^2}{n}}, \quad |\psi|^2 = n_S.$$  \hspace{1cm} (12)

From stated follows that the prevailing role plays the attractive between molecules.

We can remark, that on the existence of the second order phase transition in a liquid helium shows an anomalous temperature dependence of heat capacity - "\(\lambda\)-curve" (Keesom, 1949).

F. Physical definition of constants in the Ginsburg - Landau (GL) equation

Follow to Ginsburg-Landau (GL) theory, we introduce the complex local effective wave function:

$$\psi(R,t) = |\psi|e^{i\varphi}, \quad |\psi|^2 = \frac{N_S}{V} = n_S.$$  \hspace{1cm} (13)

\(n_S\) - the density of number of particles in a condensate. In the stationary state the effective wave function to the GL equation satisfies (GL, 1950; Ginsburg, Sobyanin, 1976).

One of opportunities of the description of temporary evolution in the GL theory - the transition to the reversible Hartree equation. To reveal the physical sense of coefficients in the GL equation we can compare two ways of account of the susceptibility: on the GL equation, and on the quantum kinetic equation in the self-consistent approximation for a nonideal Bose gas (Klimontovich and Silin, 1952). In result is received relations:

$$bn = \alpha_L = n|\nu_{eff}(0)| = k_BT_C.$$  \hspace{1cm} (14)

It specifies analogy of the phase transition in a liquid helium and the phase transition in Van der Waals system (Klimontovich, 1998).

At transition in a superfluid state varies not only thermodynamics, but and hydrodynamics - there are "two-liquid flows" normal and superfluid components of helium. It puts a question on a physical nature of the occurrence
"two-liquid" state. For the answer it is necessary to use the evolutionary
equations with the account of dissipation. For the answer on it the dissipative
kinetic equations will be used.

The temporary dependence can be entered in the stationary GL equation
by two extreme ways.

G. Nonlinear Schroedinger equation

With $\alpha_L = k_B T_C$ we have the following nonlinear Schroedinger equation -
the Hartree equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial R^2} + k_B T_C \left[ \frac{T - T_C}{T_C} \frac{|\psi|^2}{n} \right] \psi.$$  \hspace{1cm} (15)

H. Relaxation G-L equation (RGLE)

Let’s carry out in the Hartree equation formal replacement $t$ by imaginary
time $it$. In result we come to the relaxation GL equation (RGLE):

$$\frac{\partial \psi}{\partial t} = -\gamma \left( \frac{T - T_C}{T_C} \frac{|\psi|^2}{n} \right) \psi + D \frac{\partial^2 \psi}{\partial R^2}.$$ \hspace{1cm} (16)

This is an example of the reaction diffusion equation. The coefficients $D$ and
$\gamma$ are determined by the formulas:

$$D = \frac{\hbar}{2m}, \hspace{0.5cm} \gamma = \frac{\alpha_L}{\hbar}, \hspace{0.5cm} \alpha_L = n |\nu_{eff}(0)| = k_B T_C.$$ \hspace{1cm} (17)

Let’s consider an opportunity of other derscription of temporary evolution on
a basis of the kinetic equation for the kinetic equation for the local distribution
function $f(n_S, R, t)$:

$$\frac{\partial f}{\partial t} = 2 \frac{\partial}{\partial n_S} \left[ \gamma D_{n_S} n_S \frac{\partial f}{\partial n_S} \right] + \frac{\partial}{\partial n_S} \left[ 2\gamma \left( \frac{T - T_C}{T_C} \frac{n_S}{n} n_S f \right) \frac{\partial^2 f}{\partial R^2} \right] + D \frac{\partial^2 f}{\partial R^2}.$$ \hspace{1cm} (18)

The diffusion coefficient $D_{n_S} = n/N_{ph}$ is defined by number of particles in a
point of continuous medium. In an equilibrium state:

$$f_0(n_S) = C \exp \left( -\frac{h_{eff}}{D_{n_S}} \right), \hspace{0.5cm} h_{eff} = \gamma \left[ \frac{T - T_C}{T_C} n_S + \frac{n_S^2}{2n} \right].$$ \hspace{1cm} (19)

Here the designation for the effective Hamilton function in account on one
particle is entered. In the first moment approximation the distribution function

$$f(n_S, t) = \delta \left( n_S - \langle n_S \rangle_{R,t} \right).$$ \hspace{1cm} (20)
and we have the following equation for the first moment:

\[
\frac{d \langle nS \rangle}{dt} = 2 \left[ \gamma D_{ns} - \gamma \left( \frac{T - T_C}{T_C} + \frac{\langle nS \rangle}{n} \right) \langle nS \rangle \right] + D \frac{\partial^2 \langle nS \rangle_{R,t}}{\partial R^2}. \tag{21}
\]

It is another example of the reaction diffusion equation. For stationary and homogeneous state it is reduced to the algebraic equation for \( \langle nS \rangle \):

\[
D_{ns} - \left( \frac{T - T_C}{T_C} + \frac{\langle nS \rangle}{n} \right) \langle nS \rangle = 0, \quad D_{ns} = \frac{n}{N_{ph}}, \quad \gamma = \frac{k_B T_C}{\hbar}. \tag{22}
\]

Its solution allows to find average value of number of superfluid atoms \( \langle nS \rangle \) at all value of temperature. In particular, in the critical point:

\[
\langle nS \rangle_{st}^{(2)} = \sqrt{nD_{ns}} = n\sqrt{\frac{1}{N_{ph}}} \ll n. \tag{23}
\]

In the thermodynamic limit the average density of superfluid atoms is equal to zero. Below the critical point:

\[
\langle nS \rangle_{st}^{(3)} = n \frac{T_C - T}{T_C}. \tag{24}
\]

I. Distribution function of amplitudes and phases

Instead the effective wave function we shall enter two real functions \( X(R,t), Y(R,t) \):

\[
\psi(R,t) = X(R,t) + iY(R,t), \quad nS(R,t) = X^2(R,t) + Y^2(R,t). \tag{25}
\]

The appropriate kinetic equation for the distribution function \( f(X,Y,R,t) \) with is taking into account the spatial diffusion, has the following form (see Klimontovich, 1995):

\[
\frac{\partial f}{\partial t} = \left\{ \frac{\partial}{\partial X} \left[ \frac{1}{2} \gamma D_{ns} \frac{\partial f}{\partial X} \right] + \frac{\partial}{\partial Y} \left[ \frac{1}{2} \gamma D_{ns} \frac{\partial f}{\partial Y} \right] \right\} +
\]

\[
\left\{ \frac{\partial}{\partial X} \left[ \gamma \left( \frac{T - T_C}{T_C} + \frac{X^2 + Y^2}{n} \right) Xf \right] + \frac{\partial}{\partial Y} \left[ \gamma \left( \frac{T - T_C}{T_C} + \frac{X^2 + Y^2}{n} \right) Yf \right] \right\} +
\]

\[
D \frac{\partial^2 f}{\partial R^2}, \quad D_{ns} = \frac{n}{N_{ph}}, \quad \gamma = \frac{k_B T_C}{\hbar}. \tag{26}
\]

We can find now the equation for the distribution function of values \( nS \):
\[ f(n_S, R, t) = \int \delta(n_S - (X^2 + Y^2)) f(X, Y, R, t) dX dY \]  

(27)

In the self-consistent approximation for \( n_S \) follows equation which coincides with the equation (18).

The relaxation processes below the critical point it is possible to divide into two parts: on fast - for density of number of particles or amplitude, and slow - for a phase or appropriate combination variable \( X, Y \).

For the description of the slow relaxation in the equation (26) the following replacement is introduced (see in Klimontovich, 1999):

\[
\frac{T - T_C}{T_C} \left( \frac{X^2 + Y^2}{n} \right) \rightarrow \frac{T - T_C}{T_C} + \frac{n_S}{n}.
\]  

(28)

In result is received the kinetic equation:

\[
\frac{\partial f}{\partial t} = \left\{ \frac{\partial}{\partial X} \left[ \frac{1}{2} \gamma D_{n_S} \frac{\partial f}{\partial X} \right] + \frac{\partial}{\partial Y} \left[ \frac{1}{2} D_{n_S} \gamma \frac{\partial f}{\partial Y} \right] \right\} + \left\{ \frac{\partial}{\partial X} \left( \frac{D_{n_S}}{\langle n_S \rangle_{st}} X f \right) + \frac{\partial}{\partial Y} \left( \frac{D_{n_S}}{\langle n_S \rangle_{st}} Y f \right) \right\} + D \frac{\partial^2 f}{\partial R^2}, \quad D_{n_S} = \frac{n}{N_{ph}}, \quad \gamma = \frac{k_B T_C}{\hbar}.
\]  

(29)

The equilibrium decision of this equation - the Gauss distribution on two variables:

\[
f_0(X, Y) = C \exp \left( -\frac{X^2 + Y^2}{\langle n_S \rangle_{st}} \right), \quad \int f_0(X, Y) dX dY = 1.
\]  

(30)

The average values \( X, Y \) are equal to zero. The information on phase transition contains in the expression for the dispersion \( \langle n_S \rangle_{st} \). It at all values of temperature is defined by the solution of the equations (22).

### III. FAST AND SLOW FLUCTUATIONS AT PHASE TRANSITIONS

At phenomenological description of phase transitions the linear part of the friction coefficient changes its sing.

In this respect there is an analogy with transition through the critical point in Van der Waals system and with a second order phase transition in segnetoelectrics We shall show that at transition via the critical point changes radically and the character of relaxation and fluctuating processes. The states at temperature \( T > T_C \) will be characterized by fast processes. At phase transition to states with temperature \( T_C > T \), on a level with fast fluctuations appears and slow ones. We shall show that slow fluctuations provide the coherence flow of helium in the Kapitza experiments.
A. Relaxation of fast processes at phase transitions

Let’s return to the evolutionary equation (21) and consider a small deviation from the stationary and the spatial homogeneous solution $\langle n_S \rangle_{st}$. The appropriate time relaxation and half-width of a spectral line are defined by expressions:

$$\frac{1}{\tau_{n_S}}(k) = \Delta_{n_S} = 2\gamma \left( \frac{T - T_C}{T_C} + 2 \frac{\langle n_S \rangle}{n} \right) + Dk^2, \quad \gamma = \frac{n |\nu_{eff}(0)|}{\hbar}. \quad (31)$$

At zero value of wave number the relaxation time grows at approach to the critical point under the Curi law. In the critical point it has the maximal finite value $\sqrt{N_{ph}/2\gamma}$.

The complex response $\langle n_S \rangle^{(1)}$ on the external action:

$$\chi(n_S)(\omega, k) = \frac{1}{-i\omega + \Delta_{n_S}(k)}, \quad \Delta_{n_S}(k) = 2\frac{k_B T_C}{\hbar} \left( \frac{T - T_C}{T_C} + 2 \frac{\langle n_S \rangle_{st}}{n} \right) + Dk^2. \quad (32)$$

We shall use connection of function $\chi(n_S)(0, 0)$ and the isothermal compressibility:

$$\chi(n_S)(0, 0) = \frac{1}{\Delta_{n_S}(0)} = 2\frac{k_B T_C}{\hbar} nk_B T_C \beta_T, \quad nk_B T_C \beta_T = \frac{1}{T - T_C + 2 \langle n_S \rangle_{st}}. \quad (33)$$

For the Landau region at approach to the critical point $\beta_T$ and the relaxation time $\tau_{n_S}$ grow under the Curi law. In the critical point $k = 0$ the compressibility is finite

$$nk_B T_C \beta_T = \sqrt{N_{ph}}. \quad (34)$$

B. Relaxation of slow processes

From the kinetic equation (29) we find the equations for average values $\langle X \rangle_{R,t}, \langle Y \rangle_{R,t}$ and with their help - the expressions for correlation time and the width of spectral lines:

$$\frac{1}{\tau_{(X,Y)}}(k) = \Delta_{(X,Y)}(k) = \gamma \left( \frac{D_{n_S}}{\langle n_S \rangle_{st}} \right) + Dk^2, \quad D_{n_S} = \frac{n}{N_{ph}}. \quad (35)$$

The $\langle n_S \rangle_{st}$ - the solution of the equation (22) at all temperatures. The expression for a complex susceptibility has the form:

$$\chi_X(\omega, k) = \chi_Y(\omega, k) = \frac{1}{-i\omega + \gamma \frac{D_{n_S}}{\langle n_S \rangle_{st}} \left( 1 + r_C^2 k^2 \right)^2}, \quad r_C^2 = \frac{D_{n_S}}{\gamma D_{n_S}}. \quad (36)$$
The designation for a square of the correlation radius here is entered. The isothermal compressibility is defined now by expression:

\[
 nk_B T_C \beta_T = \frac{\langle n_S \rangle_{st}}{D_{n_S}} 
\]  

(37)

For the region of applicability of the Landau theory at approach to the critical point from the side of high temperatures the isothermal compressibility grows under the Curi law. In the critical point it is finite also and is defined by expression:

\[
 nk_B T_C \beta_T = \frac{k_B T}{n |\nu_{eff}(0)|} \sqrt{N_{ph}}, \quad T = T_C. 
\]  

(38)

However, for the region of temperatures, below the critical one

\[
 nk_B T_C \beta_T = N_{ph} \frac{T_C - T}{T_C}, \quad T < T_C, 
\]  

(39)

the isothermal compressibility continues grow at a measure of downturn of temperature.

From received formulas follows that at temperature enough below critical one the ration of fast and slow correlation times is proportional to $1/N_{ph}$.

C. Fast and slow fluctuations

The task of an account of fluctuations at phase transition in a superfluid state is similar, solved in Ch.24 (Klimontovich, 1999) ” Kinetic fluctuations in the critical point ”.

1. Fast fluctuations

The account is similar, carried out in the section (19.6) (Klimontovich, 1999).

We enter in the equation (21) for the function $\langle n_S \rangle_{R,t}$ the Langevin source, which reflects the atomic structure of a liquid helium. The relaxation time of the parameter of order coincides with expression (31).

The complex response to a random source is defined above mentioned, expressions at all values of temperature. The quantity $\langle n_S \rangle$ is defined by the solution of the equations (22). At sufficient distance from the critical point the response $\chi(n_S)(\omega = 0, k = 0)$ varies under the Curi law. In the critical point the isothermal compressibility has finite value (34). A square of correlation radius at approach to the critical point in the region field of the Landau theory varies under the Curi law, but it is finite in the critical point.
2. Slow fluctuations

Let’s show, that the existence of superfluidity is possible due to slow fluctuations.

For this let’s address to the kinetic equation (29). Let’s enter into it the Langevin source, which intensity is determined by two dissipative characteristics (“by integrals of collisions”). In self-consistent approximation the equation for the first moments - functions $X(R,t), Y(R,t)$ have the following form:

$$\frac{\partial X}{\partial t} + \gamma \frac{D_{ns}}{\langle n_S \rangle_{st}} X = D \frac{\partial^2 X}{\partial R^2} + y(X)(R,T), \quad \frac{\partial Y}{\partial t} + \gamma \frac{D_{ns}}{\langle n_S \rangle_{st}} Y = D \frac{\partial^2 Y}{\partial R^2} + y(Y)(R,T).$$

(40)

The width of spectral lines and the dynamic susceptibility are defined by former expressions (35) - (36).

For the region of the Landau theory at approach to the critical point from the side of high temperatures the square of correlation radius grows on the Curi law:

$$r_C^2 = \frac{D}{\gamma} \frac{T_C}{T - T_C}, \quad T > T_C. \quad (41)$$

In the critical point the square of radius of correlation is finite:

$$r_C^2 = \sqrt{N_{ph} \frac{D}{\gamma}}, \quad T = T_C. \quad (42)$$

At downturn of temperature from critical

$$r_C^2 = N_{ph} \frac{D}{\gamma} \frac{T_C - T}{T_C} \sim N_{ph} \lambda_S^2 \frac{T_C - T}{T_C}, \quad T < T_C. \quad (43)$$

The following estimation of parameter $D/\gamma$ here is used:

$$\frac{D}{\gamma} \sim \frac{\hbar^2}{mk_BT_C} \sim \lambda_S^2. \quad (44)$$

$v_S$ - speed of the sound, $\lambda_S$ is the appropriate length of de Broglie.

Thus, value of the isothermal compressibility and the square of correlation radius for slow fluctuations at temperatures lower of the critical temperature ($T < T_C$) are defined by number of atoms in a point of continuous medium $N_{ph}$. Thus $r_C^2$ at $T < T_C$ is macroscopic characteristic.

Now we can give an estimation of $N_{ph}$ for the hydrodynamic description.

D. Physically infinitesimal scales

As the description of superfluidity is carried out on hydrodynamical level, it is necessary to use and appropriate definition of physically infinitesimal
scales (Klimontovich, 1982, 1995, 1999). Thus the required scales depend from external parameter of length \( L \) - one of the characteristic sizes of a vessel with a liquid helium.

According (Klimontovich, 1995) the formula for \( r^2 \) at low temperatures it is possible to rewrite as:

\[
\frac{r^2}{C} = \left( nL^3 \right)^{2/5} \frac{T_C - T}{T_C} \sim \left( nL^3 \right)^{2/5} \lambda_S^2, \quad T < T_C. \tag{45}
\]

To ensure the spatial coherence of the superfluid flow on capillary of a diameter \( d \), it is necessary to limit size \( d \) to a condition:

\[
d < r_C = \left( nL^3 \right)^{1/5} \lambda_S. \tag{46}
\]

This inequality is carried out for the Kapitza experiments

**E. Spectral density of slow fluctuations**

The account is similar carried out in Chs.19, 24 (Klimontovich, 1999), devoted the kinetic theory of fluctuations at phase transitions in segnetoelectrics and in the Van der Waals system. The structure of Langevin sources in the equations for the first moments:

\[
\left( y(X, Y) y(X, Y) \right)_{\omega, k} = 2\Delta(X, Y) (k) \frac{1}{2} \frac{\langle n_s \rangle_{st}}{n}. \tag{47}
\]

The appropriate expressions for spectral density of slow fluctuations look like:

\[
\langle \delta X \delta X \rangle_{\omega, k} = \frac{2\Delta(X) (k)}{\omega^2 + \Delta^2(X) (k)} \frac{\langle (\delta X)^2 \rangle}{n}, \quad \langle (\delta X)^2 \rangle = \frac{1}{2} \langle n_s \rangle_{st}. \tag{48}
\]

\[
\langle \delta Y \delta Y \rangle_{\omega, k} = \frac{2\Delta(Y) (k)}{\omega^2 + \Delta^2(Y) (k)} \frac{\langle (\delta Y)^2 \rangle}{n}, \quad \langle (\delta Y)^2 \rangle = \frac{1}{2} \langle n_s \rangle_{st}. \tag{49}
\]

The spatial spectral density does not depend from wave numbers - "spatial white noise"", therefore the spatial correlators:

\[
\langle \delta X \delta X \rangle_{R - R'} = \langle \delta Y \delta Y \rangle_{R - R'} = \frac{1}{2} \frac{\langle n_s \rangle_{st}}{n} \delta(R - R'). \tag{50}
\]

Thus, the spatial correlations are different from zero only in limits of a point of continuous medium (in physically infinitesimal volume \( V_{ph} \)), as function \( \delta(R - R') \big|_{R = R'} = V_{ph}^{-1} \). In result for the one-dot correlator slow (large-scale) fluctuation \( \langle \delta X \delta X \rangle_{R = R'} \) we have the following expression:

\[
\langle \delta X \delta X \rangle_{R = R'} = \langle \delta Y \delta Y \rangle_{R = R'} = \frac{1}{2} \frac{\langle n_s \rangle}{N_{ph}}, \quad N_{ph} = nV_{ph}. \tag{51}
\]

The dispersion of fluctuations, smoothed on volume of point of continuous medium, in \( N_{ph} \) of times there is less fluctuations of one-dot distribution - the Boltzmann distribution.
F. Intermediate conclusion

Was shown, that at temperatures below critical in a liquid helium are available two fluctuation process - "fast" and "slow". Behavior of fast fluctuations is similar what is offered by the theory of phase transitions, advanced by Landau. The thermodynamic and the fluctuation characteristic of fast processes demonstrate anomalous dependences on temperature - the Curi law for the isothermal compressibility, the static susceptibility and the square of radius of correlation.

The basic difference in behavior of fast fluctuations from the prediction of the Landau theory and the more general "the fluctuation theory of phase transitions" (Patashinskii, Pokrovskii, 1982) consists in absence of "the problems of infinity". Just in difference from the traditional theory here the isothermal compressibility, the static susceptibility and the square of radius of correlation have finite value in the critical point!

Moreover, in the Landau theory at removal from the critical point (in the region of high temperatures and in the region of low temperatures) the values of these characteristics decrease under the Curi up to their "normal" values. In a "normal" state the correlation radius of is the order of the average distance between atoms. On this reason fast correlations can not ensure the spatial and temporary coherence of more ordered asymmetrical state, which exists at temperatures $T < T_C$.

On the contrary, behavior of slow fluctuations at temperatures $T < T_C$ is essentially other. In process of removal downwards from the critical temperature the value of correlation radius continues to increase and aspires to macroscopic value. In a thermodynamic limit this value, formally, is equal to infinity. However, such limiting transition contradict to the model of continuous medium. It is enough, that radius of correlation was macroscopic, in particular, would exceed a diameter of a capillary in the Kapitza installations.

The basic features of the phenomenon of superfluidity are shown not in the thermodynamics only, but also in hydrodynamics. It is emphasized also by term, which has entered by Kapitza - "superfluidity". So, the basic task of the theory is reduced here to two questions:

1. Why the viscousless flow can exists a dissipative medium?
2. What physical distinction of superfluid and normal component in hydrodynamics of helium?

Let's try to give the answer to these basic questions.

IV. THE VISCOUSLESS FLOW IN THE VISCOUS MEDIUM

A. Dynamic-reaction-diffusion equation in the theory of Superfluidity

Above it was supposed, that the distribution of helium is the spatially homogeneous. However, a liquid helium is viscous medium, therefore at the spatially non-uniform state the viscous enters in the game. It is necessary
to show, that the carried out (at \( k = 0 \)) the division on ” normal ” and ”
superfluid ” components at the account of the viscous is valid.

In liquid helium the wave length of de Broglie \( \lambda_B^{(T)} \) is of the order of the
average distance between atoms \( r_{av} \). However, in continuous medium both
these sizes it is a lot of less of physically infinitesimal scale \( l_{ph} \), as in a point
of continuous medium there are many atoms. On this reason the contributions
determined by the Planck constant, in many cases are not essential.

The appropriate dissipative kinetic equation for local distribution functions
\( f(X, Y, R, v, t) \) of values of real and imaginary parts of the effective wave function
has the following structure:

\[
\frac{\partial f}{\partial t} + V \frac{\partial f}{\partial R} + F(R, t) \frac{\partial f}{\partial v} = I(v) + \left\{ \frac{\partial}{\partial X} \left[ \frac{1}{2} \gamma D_{ns} \frac{\partial f}{\partial X} \right] + \frac{\partial}{\partial Y} \left[ \frac{1}{2} D_{ns} \gamma \frac{\partial f}{\partial Y} \right] \right\} + \\
\left\{ \frac{\partial}{\partial X} \left( \gamma D_{ns} \frac{\langle n_S \rangle_{st}}{X f} \right) + \frac{\partial}{\partial Y} \left( \gamma D_{ns} \frac{\langle n_S \rangle_{st}}{Y f} \right) \right\} + D \frac{\partial^2 f}{\partial R^2} \tag{51}
\]

The diffusion and friction coefficient are defined by former expressions:

\[
D = \frac{\hbar}{2m}, \quad D_{ns} = \frac{n}{N_{ph}}, \quad \gamma = k_B T. \tag{52}
\]

**B. Transition to the hydrodynamical equations**

The equation of a continuity now has the following form:

\[
\frac{\partial}{\partial t} \langle n_S \rangle_{R,t} + \frac{\partial}{\partial R} j_S(R, t) = 2 \left[ D_{ns} - \gamma \left( \frac{T - T_C}{T_C} + \frac{\langle n_S \rangle_{R,t}}{n} \right) \langle n_S \rangle_{R,t} \right]. \tag{53}
\]

The designation for a flow of matter with the account as the convective, so and spatial diffusion flows, here is used:

\[
j_S(R, t) = \langle n_S \rangle_{R,t} u_S(R, t) - D \frac{\partial \langle n_S \rangle_{R,t}}{\partial R}. \tag{54}
\]

The right part of the equation of a continuity describes ” the chemical reaction ” - a birth and a destruction of average density of the superfluid component.

In the stationary state the equation of a continuity can be reduced to the two
equations. The first :

\[
\frac{\partial u_S(R, t)}{\partial R} = 0. \tag{55}
\]

allows to express the quantity of a velocity in capillary through difference of helium density on the ends of a capillary.
Let’s take into account, that the relaxation of the function $\langle n_S \rangle_{R,t}$ is fast. In the stationary state $\langle n_S \rangle_t$ for all values of temperature by the solution of the equation (22) can be defined.

In result the Navier-Stokes equation for the velocity of the superfluid component is received:

$$\frac{\partial u_S}{\partial t} + \left( u_S \frac{\partial}{\partial R} \right) u_S = \frac{F}{m} + \nu \frac{\partial^2 u_S}{\partial R^2}, \quad j_S(R, t) = \langle n_S \rangle_t u_S(R, t). \quad (56)$$

The equation of motion is not include the dependence on temperature. The flow of superfluid helium through density $\langle n_S \rangle_t$ depends on temperature. Here $D \to \nu$.

At values of speed $u_S$ much smaller of the critical one $u_C$, it is possible to neglect by the nonlinear terms. In result at $F = 0$ we come for the velocity $u_S$ to the diffusion equation:

$$\frac{\partial u_S}{\partial t} = \nu \frac{\partial^2 u_S}{\partial R^2}, \quad j_S(R, t) = \langle n_S \rangle_{R,t} u_S(R, t). \quad (57)$$

Thus the question arises: Why the viscousless flow can exists a viscous medium?

V. FLICKER NOISE AND VISCOSLESS FLOW IN VISCOUS MEDIUM

A. Flicker noise

Let’s result only minimum of the necessary information from the theory Flicker noise (Klimontovich, 1982; Kogan, 1985).

Flicker noise from the side of high frequencies is limited by diffusion time $\tau_\nu = L^2/\nu$, $L$— a minimal characteristic scale of a sample. Here it will be a diameter of a capillary $d$. The region of the existence of the flicker noise on frequencies is defined by inequalities:

$$\frac{1}{\tau_{life}} \ll \omega \ll \frac{1}{\tau_\nu} = \frac{\nu}{d^2}. \quad (58)$$

$\tau_{life}$ - time of life of installation. In the region of flicker noise there is a new scale and appropriate volume:

$$L_\omega = \sqrt{\frac{\nu}{\omega}} \gg L, \quad V_\omega = L_\omega^3 \gg V. \quad (59)$$

Let’s result the appropriate chain of inequalities for volumes:

$$V \ll V_\omega \ll V_{life}. \quad (60)$$

The equilibrium (natural) flicker noise arises at the diffusion in limited volume. Here it the diffusion of velocity $u_S$. 

16
The expression for the spatial - temporary spectrum fluctuations of velocity \( U_S \) follows from the appropriate Langevin equation and has the form (Klimontovich, 1982, 1990):

\[
(\delta u_S \delta u_S)_{\omega,k} = \frac{(yy)_{\omega,k}}{\omega^2 + (\nu k^2)^2}, \quad (yy)_{\omega,k} = 2\nu k^2 AV \omega \langle \delta u_S \delta u_S \rangle_V \exp \left( -\frac{\nu k^2}{2\omega} \right).
\]

(61)

Here \( \langle \delta u_S \delta u_S \rangle_V \) - the correlator fluctuations, average on volume of a sample. The \( A \) will be determined below from the normalization condition. From the last formula follows, that has a place strong dependence on frequency and on wave number. Thus dispersion of wave numbers is proportional to frequency \( \omega \):

\[
\left\langle \left( \delta k \right)^2 \right\rangle \sim \frac{1}{L_\omega^2} = \frac{\omega}{\nu}.
\]

(62)

Thus in the region of flicker noise has a place " the original Bose condensation ". This shows the presence of the spatial coherence.

In expression for spatially temporary spectral density

\[
(\delta u_S \delta u_S)_{\omega,k} = \frac{2\nu k^2}{\omega^2 + (\nu k^2)^2} AV \omega \langle \delta u_S \delta u_S \rangle_V \exp \left( -\frac{\nu k^2}{2\omega} \right).
\]

(63)

it is possible to execute the integration on \( k \). In result is received expression for the appropriate temporary spectral density:

\[
(\delta u_S \delta u_S)_{\omega} = \frac{\pi \langle \delta u_S \delta u_S \rangle_V 1}{\ln \left( \frac{\tau_{life}}{\tau_\nu} \right)} \omega, \quad \frac{1}{\tau_{life}} \ll \omega \ll \frac{1}{\tau_\nu}.
\]

(64)

\( A \) is determined from the normalization condition:

\[
\int_{1/\tau_\nu}^{1/\tau_{life}} (\delta u_S \delta u_S)_{\omega} \frac{d\omega}{\pi} = \langle \delta u_S \delta u_S \rangle_V.
\]

(65)

It is supposed, thus, that the basic contribution to the correlator \( \langle \delta u_S \delta u_S \rangle_V \) has to the region of flicker noise.

### B. Temporary correlation

The temporary correlation is connected to temporary spectral density by relation:

\[
\langle \delta u_S \delta u_S \rangle_\tau = \int_{1/\tau_{life}}^{1/\tau_\nu} (\delta u_S \delta u_S)_{\omega} \frac{d\omega}{\pi}.
\]

(66)

From here follows that
\[
\langle \delta u_S \delta u_S \rangle_\tau = \left( C - \frac{\ln (\tau / \tau_\nu)}{\ln (\tau_{life} / \tau_\nu)} \right) \langle \delta u_S \delta u_S \rangle_{\nu}, \quad \tau_\nu \ll \tau \ll \tau_{life}, \quad (67)
\]

\[
C = 1 - \frac{\gamma}{\ln (\tau_{life} / \tau_\nu)}, \quad \gamma = 0.577.
\]

Here are used the Euler constant.

In the region of flicker noise the dependence from \( \tau \) is logarithmic at the large value of argument. It gives the basis to speak about presence of the residual correlations.

The characteristic time of the correlations is defined by expression:

\[
\tau_{cor} = \int_{\tau_D}^{\tau_{life}} \frac{\langle \delta u_S \delta u_S \rangle_\tau \, d\tau}{\langle \delta u_S \delta u_S \rangle_\tau}. \quad (68)
\]

In result we find, that

\[
\tau_{cor} \sim \frac{\tau_{life}}{\ln (\tau_{life} / \tau_\nu)}. \quad (69)
\]

The time of correlation at the unlimited time \( \tau_{life} \) tends to infinity.

The stated in the present section shows, that in the region of flicker noise has a place as the spatial, and also the temporary coherence. It also gives the basis for to establish connection of two coherent phenomena: flicker noise and superfluidity.

### C. Flicker noise and superfluidity

Let’s return to the equation (57) for velocity of the speed superfluid helium. The account of fluctuations of velocity in the region of the flicker noise leads to the formula (54) for the temporary spectrum fluctuations of the superfluid components of velocity of liquid helium.

For the dissipative term we shall use the simplest approximation of kind ”\( 1/\tau_{rel} \)”. In result for the velocity of superfluid component is received the following relaxation equation:

\[
\frac{\partial u_S}{\partial t} = - \frac{1}{\tau_{rel}} u_S. \quad (70)
\]

In zero approximation on dimensionless parameter

\[
\frac{\tau_{obs}}{\tau_{life}}, \quad \tau_{life} \gg \tau_{obs} \gg \tau_D \quad (71)
\]

in the equation (70) it is possible to neglect by dissipation. Thus we come, to the equation
\[ \frac{\partial u_S}{\partial t} = 0, \quad u_S = \text{const}. \] (72)

The value of constant velocity is defined by boundary conditions on the ends of capillary.

Let’s estimate the diffusion time \( \tau_\nu \). It defines the high boundary of the flicker noise region: \( \omega_{\text{max}} \sim 1/\tau_\nu \). The diameter of a capillary in the Kapitza experiences of the order \( 10^{-4} - 10^{-5} \text{cm} \). The diffusion coefficient can be estimated on one of two formulas: \( D \approx \bar{h}/2m^* \nu_T l \). (i.e. effective length of free paths.) Thus the list value of the time observation of superfluidity

\[ (\tau_{\text{obs}})_{\min} \geq \tau_\nu = d^2/\nu \sim 10^{-5} - 10^{-6} \text{sec}. \] (73)

D. Superfluidity is viscousless flow in viscous medium

Let’s address to definition of concept ”superfluidity” - viscousless flow in viscous medium.

One of the definitions of this concept is ”the measuring”. It is connected with the observation time \( \tau_{\text{obs}} \).

However, as in process of increase of the observation time it fails to find out the reduction of velocity, it is natural to assume, that the constancy of velocity has a place within the limits of greatest temporary interval \( \tau_{\text{life}} \), i.e. ” of the time life of installation

VI. CONCLUSION

The results can be presented as two parts:

1. Thermodynamics of helium-II.

On the basis of the corresponding kinetic equations was shown, that for the spatially homogeneous state \( (k = 0) \), when the diffusion processes drop out, at temperatures \( T < T_C \) there are two kinds relaxation and fluctuation processes: the fast and the slow.

For the fast processes the radius of correlation and the isothermal compressibility at approach to a critical point grow under by the Curi law, but in the critical point they have finite values! At removal from the critical point ”downwards” the radius of correlation again becomes about average distances between atoms. For this reason the fast fluctuations can not to ensure the coherence of an asymmetrical phase on macroscopic scales.

On the contrary, the slow fluctuations at \( T < T_C \) become macroscopic. The appropriate radius of correlation are more than a diameter of capillary. Is provided, thus, spatial coherence of superfluid components in the Kapitza experiences.

2. Hydrodynamics of helium -II.
Was shown, that the existence of superfluidity in viscous medium is possible due to occurrence of the natural flicker noise. Thus there is ”the Bose-condensation” in the space of wave vectors. The dispersion of distribution on wave numbers is proportional to frequency. In results in the equations for the Fourier component of velocity it is possible the following replacement of hydrodynamical friction: $\nu k^2 \rightarrow \omega$. Accordingly to this arise ”residual” temporary correlations are limited on duration by time life of installation $\tau_{life}$ only. This time defines and the relaxation time for the velocity of superfluid component.

The new opportunity of physical treatment of two-liquid model of Tisza and Landau is opened. The independent existense of two motions it is possible only in linear approximation. The superfluid flow is broken, when it begins the change vortical motion of normal component. The appropriate critical velocity: $u_C \sim h/\md$. It corresponds to known estimation of critical speed (see in Lifshitz and Pitaevskii, 1978).

The physical treatment, submitted in these chapters, is possible to hope, allows better to understand the essence of the phenomenon of superfluidity, opened by Kapitza and detailed investigated by him in a number of remarkable papers.

VII. REFERENCES

1. Andronikashvily E.L. (JETP, 18 (1940) 424).
2. Belyaev S.T. (JETP 34 (1958) 417).
3. Bogolubov N.N. To theory of superfluidity. (Izv. AN, FIZ. v.11 (1047) 77).
4. Bogolubov N.N., Zubarev D.N. (JETP 28 (1955) 129).
5. Ginsburg V.L., Landau L.D. (JETF 20 (1950) 106).
6. Ginsburg V.L., Sobyanin A.A. (Uspehi Fiz. Nauk 120 (1976) 153).
7. Einstein A. Quantentheorie der einatomigen idealen Gases. (Berl. Ber. 22 (1924) 261; 23 (1925) 3, 18).
8. Kapitza P.L. (DAN U.S.S.R. 18 (1938) 21; Nature 141 (1938) 74).
9. Kapitza P.L. (JETP 11 (1941) 1).
10. Kapitza P.L. (JETP 11 (1941) 581).
11. Kapitza P.L. (JETF 20 (1944) 2).
12. Keesom V. Helium (”International Literature”, Moscow, 1949).
13. Kirpatrick T.R. and Dorfman J.R. (J. Low Temp. Phys. 58 (1955) 301; 58. (1955) 399; 59 (1955) 1).
14 Klimontovich Yu.L., Silin V.P. (JETP 23 (1952) 151; Uspehi Fiz. Nauk 70 (1960) 247).
15. Klimontovich Yu.L. Statistical Physics (”Nauka” , Moscow, 1982; Harwood, New York, 1986).
16. Klimontovich Y.L. (Pis’ma v JTP 9 (1983) 406; Sov. Techn. Phys. Let. 9 (1983) 174).
17. Klimontovich Y.L. (Pis’ma v JETP 51 (1990) 43; Physica A 167 (1990) 782).
18. Klimontovich Yu.L. Turbulent Motion and the Structure of chaos. ("Nauka", Moscow, 1990; Kluwer, Dordrecht, 1991).
19. Klimontovich Yu.L. (TMP 115 (1998) 437).
20. Klimontovich Yu.L. Statistical Theory of Open Systems. (V.I "Yanus" Moscow, 1995; Kluwer, Dordrecht, 1995); V.II "Yanus", Moscow, 1999; V.III, 2001).
21. Kogan Sh.M. (Uspehi Fiz. Nauk 145 (1985) 286).
22. Landau L.D. (JETP 11 (1941), 592).
23. Landau L.D. (JETP 14 (1944) 112).
24. Lifshitz E.M., Pitaevskii L.P. Statistical Physics. Part 2 ("Nauka" Moscow, 1978).
25. London F. Superfluids, Vol.I Superconductivity. (Wiley New York, 1954).
26. Martynov G.A. Fundamental Theory of Liquids. (Adam Hilger, Bristol, New York, 1992).
27. Nozieres Ph., Pines D. Theory of quantum liquids. V.II. Superfluid Bose Liquids (Addison New York, 1980).
28. Patashinskii A.Z., Pokrovskii V.L. Fluctuation Theory of Phase Transitions. ("Nauka" Moscow, 1982).
29. Pines D. and Nosier P. The Theory of Quantum Liquids (Benjamin, New York, 1966).
30. Tserkovnikov Y.A. (TMP 93 (1992) 412; TMP 105 (1955) 77).