The $b \to s\gamma$ decay in SUSY models with non–universal $A$–terms

E. Gabrielli$^1$, S. Khalil$^{1,2}$, and E. Torrente-Lujan$^1$

$^1$ Dept. de Física Teórica C-XI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.

$^2$ Ain Shams University, Faculty of Science, Cairo 11566, Egypt.

Abstract

We analyse the predictions for the inclusive branching ratio for $B \to X_s\gamma$ in a class of string-inspired SUSY models with non–universal soft-breaking $A$–terms. These models are particularly interesting since the non–universality of the $A$–terms plays an important role in providing new significant contributions to CP violation effects in kaon physics while respecting the severe bounds on electric dipole moments. We show that $b \to s\gamma$ do not severely constrain the non–universality of these models. In particular, at low $\tan \beta$ ($\tan \beta \simeq 2$), our predictions are close to the universal case. For large $\tan \beta$ ($\tan \beta \gtrsim 15$), the effect of non–universality is enhanced and stronger constraints hold. We find that the parameter regions which are important for generating sizeable contribution to $\varepsilon'/\varepsilon$, of order $2 \times 10^{-3}$, are not excluded.
1 Introduction

Recently there has been a growing interest concerning supersymmetric models with non-universal soft-breaking terms [1–5]. The main theoretical reason is that some string-inspired models naturally favour SUSY models with non-universality in the soft-breaking sector [6–9]. Within this class of string-inspired models particularly interesting are those with non-universal $A$-terms [3, 4]. This is mainly due to the relevant role they can play in solving the SUSY CP problem, especially in the light of the recent experimental results on the direct CP violation parameter $\varepsilon'/\varepsilon$.

The new measurements of $\varepsilon'/\varepsilon$ at KTeV [10] and NA48 [11] lead to a world average of $\text{Re}\,\varepsilon'/\varepsilon = (21.4 \pm 4.0) \times 10^{-4}$ [12]. This result is higher than the Standard Model (SM) predictions [13], opening the way to the interpretation that it may be a signal of new physics beyond the SM. Clearly it is still premature to claim that this is a genuine new physics effect, mainly due to the large theoretical uncertainties in the non-perturbative hadronic sector which affect the SM predictions for $\varepsilon'/\varepsilon$ [14]. However if one accepts the point of view that new CP sources are needed in order to obtain large values for $\varepsilon'/\varepsilon$, one may wonder if the minimal supersymmetric extension of the SM (MSSM) can help in solving this problem. The answer is no, mainly due to the assumption of universal boundary conditions of the soft-breaking terms [3, 15–17]. Moreover we stress that without new flavor structure beyond the usual Yukawa couplings, general SUSY models with phases of the soft terms of order $\mathcal{O}(1)$ (but with a vanishing CKM phase $\delta_{\text{CKM}} = 0$) can not give a sizeable contribution to the CP violating processes [2, 3, 17, 18].

The impact of the new flavor structure in the non-universality of the $A$–terms have been studied in Refs. [1–5]. In these works it was emphasized that the non-degenerate $A$–terms can generate the experimentally observed CP violation $\varepsilon$ and $\varepsilon'/\varepsilon$ even with a vanishing $\delta_{\text{CKM}}$, i.e., fully supersymmetric CP violation in the kaon system is possible in a class of models with non-universal $A$–terms. This effect can be simply understood by making use of the mass-insertion approximation [19]: the non-degenerate $A$–terms enhance the gluino contributions to $\varepsilon$ and $\varepsilon'/\varepsilon$ through enhancing the imaginary parts of the L-R mass insertions $\text{Im}(\delta_{LR}^d)_{12}$ and $\text{Im}(\delta_{RL}^d)_{12}$ [3].

It is well known that the experimental $b \to s\gamma$ constraints cause a dramatic re-
duction of the allowed parameter space in case of universal soft terms \[20, 21\]. Hence one may worry if these constraints would be even more severe in the case of the non-degenerate $A$–terms. However a complete analysis of the $b \to s\gamma$ constraints has not been considered in Refs. [3–5]. In particular in these works it was roughly checked, by using the results of Ref. [19], that the $b \to s\gamma$ constraints are satisfied, namely \((\delta^d_{LR})_{23} \leq 1.6 \times 10^{-2}\) and \((\delta^d_{LL})_{23} \leq 8.2\). Note that these constraints are obtained by assuming that the gluino amplitude is the dominant contribution to $b \to s\gamma$ (dominant also with respect to the SM).

Although the gluino contribution to $b \to s\gamma$ decay is usually very small in the universal case, being proportional to the mass insertions $(\delta^d_{LR})_{23}$, we could expect that this contribution may be enhanced by the non–universality of the $A$–terms. However the non–universal $A$–terms could also give large contributions to the chargino amplitude through the $(\delta^u_{LR})_{23}$. Even though the non-degenerate $A$–terms enhance the gluino contribution, one can not a priori expect that it will be the dominant effect. Therefore a careful analysis of the $b \to s\gamma$ predictions, including the full SUSY contributions, is necessary in this scenario.

The purpose of this work is to perform a complete analysis of the $b \to s\gamma$ constraints for the SUSY models with non–universal $A$–terms studied in Refs. [3–5]. Indeed we will show that our results are different from the naive expectations based only on the gluino-dominance approximation.

The paper is organized as follows. In section 2 we present two models with non–universal $A$–terms that have recently been considered for solving the SUSY CP problem. These two models are based on weakly coupled heterotic string and type I string theories respectively. In section 3 we present formulas for the total branching ratio taking into account the QCD corrections and we discuss the different SUSY contributions to the $b \to s\gamma$ decay in these models. Our numerical results for the predictions of the $b \to s\gamma$ branching ratio are presented in section 4. The last section is devoted to conclusions. Finally, various formulas are summarized in the appendix.
2 String inspired models with non-degenerate $A$–terms

In this work we consider the class of string inspired model which has been recently studied in Refs. [3–5]. In this class of models, the trilinear $A$–terms of the soft SUSY breaking are non-universal. It was shown that this non-universality among the $A$–terms plays an important role on CP violating processes. In particular, it has been shown that non-degenerate $A$-parameters can generate the experimentally observed CP violation $\varepsilon$ and $\varepsilon'/\varepsilon$ even with a vanishing $\delta_{\text{CKM}}$.

Here we consider two models for non-degenerate $A$–terms. The first model (model A) is based on weakly coupled heterotic strings, where the dilaton and the moduli fields contribute to SUSY breaking [6]. The second model (model B) is based on type I string theory where the gauge group $SU(3) \times U(1)_Y$ is originated from the 9 brane and the gauge group $SU(2)$ is originated from one of the 5 branes [7].

In order to fix the conventions, the following Lagrangian $L_{SB}$ for the soft-breaking terms is assumed

$$-L_{SB} = \frac{1}{6} h_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} (\mu B)^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)^i \phi^* i \phi_j + \frac{1}{2} M_a \lambda_a \lambda_a + h.c.$$  \hspace{1cm} (1)

where the $\phi_i$ are the scalar parts of the chiral superfields $\Phi_i$ and $\lambda_a$ are the gaugino fields. In the notation for the trilinear couplings, the $A$–terms are defined as $h_{ijk} = Y_{ijk} A_{ijk}$ (indices not summed) where $Y_{ijk}$ are the corresponding Yukawa couplings.

2.1 Model A

We start with the weakly coupled string-inspired supergravity theory. In this class of models, it is assumed that the superpotential of the dilaton ($S$) and moduli ($T$) fields is generated by some non-perturbative mechanism and the $F$-terms of $S$ and $T$ contribute to the SUSY breaking. Then one can parametrize the $F$-terms as [6]

$$F^S = \sqrt{3} m_{3/2} (S + S^*) \sin \theta, \quad F^T = m_{3/2} (T + T^*) \cos \theta.$$  \hspace{1cm} (2)

Here $m_{3/2}$ is the gravitino mass, $n_i$ is the modular weight and $\tan \theta$ corresponds to the ratio between the $F$-terms of $S$ and $T$. In this framework, the soft scalar masses $m_i$
and the gaugino masses $M_a$ are given by \[8\]

\[ m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta), \quad M_a = \sqrt{3} m_{3/2} \sin \theta. \quad (3) \]

The $A_{u,d}$-terms are written as

\[ (A_{u,d})_{ij} = -\sqrt{3} m_{3/2} \sin \theta - m_{3/2} \cos \theta (3 + n_i + n_j + n_{H_{u,d}}), \quad (5) \]

where $n_{i,j,k}$ are the modular weights of the fields that are coupled by this $A$–term. As shown in Eqs.(3-5), the values of the soft SUSY breaking parameters depend on the modular weight of the matter fields. These modular weights $n_i$ are negative integers, and their 'natural' values (in case of $Z_N$ orbifolds) are $-1, -2, \text{ and } -3 \ [6]$. If we assign $n_i = -1$ for the third family and $n_i = -2$ for the first and second families (we also assume that $n_{H_1} = -1$ and $n_{H_2} = -2$). Note that with this choice of modular weights we have $m_{H_2}^2 < m_{H_1}^2$ which is favored for the electroweak breaking (EW) and all the squark mass matrices are equal. Also we find the following texture for the $A$-parameter matrix at the string scale

\[ A_{u,d} = \begin{pmatrix} x_{u,d} & x_{u,d} & y_{u,d} \\ x_{u,d} & x_{u,d} & y_{u,d} \\ y_{u,d} & y_{u,d} & z_{u,d} \end{pmatrix}, \quad (6) \]

where

\[ x_u = m_{3/2} (-\sqrt{3} \sin \theta + 3 \cos \theta), \quad y_u = m_{3/2} (-\sqrt{3} \sin \theta + 2 \cos \theta), \quad y_d = z_u = m_{3/2} (-\sqrt{3} \sin \theta + \cos \theta), \quad (7) \]

\[ z_d = -\sqrt{3} m_{3/2} \sin \theta. \quad (10) \]

By fixing the value of $\tan \beta$ we can determine the values of $\mu$ and $B$ from the radiatively EW breaking conditions. Then all the SUSY particle spectrum is completely determined in terms of $m_{3/2}$ and $\theta$. The non–universality of this model is parameterized by the angle $\theta$ and the value $\theta = \pi/2$ corresponds to the universal limit for the soft terms. In order to avoid negative mass squared in the scalar masses we restrict ourselves
to the case with $\cos^2 \theta < 1/2$. Such restriction on $\theta$ makes the non-universality in the whole soft SUSY breaking terms very limited. However, as shown in [3, 4], this small range of variation for the non-universality is enough to generate sizeable SUSY CP violations in $K$ system. We emphasize that choosing modular weights different from those assigned above, the allowed range of the soft SUSY breaking terms do not essentially change. For instance, if we assign $n_i = -3$ for the first family instead of $-2$, it may appear that the non-universality among the entries of the $A$-terms is enhanced. However in this case the angle $\theta$ is more constrained than before ($\cos^2 \theta < 1/3$). We have checked that different choices for $n_i$ do not significantly affect our results for the $b \to s \gamma$ branching ratio.

### 2.2 Model B

As mentioned in the introduction, this model is based on type I string theory. Like model A, this is a good candidate for generating sizeable SUSY CP violations. Recently, there has been considerable interest in studying the phenomenological implications of this class of models [22]. In type I string theory, non universality in the scalar masses, $A$-terms and gaugino masses can be naturally obtained [9]. Type I models contain either 9 branes and three types of $5_i$($i = 1, 2, 3$) branes or $7_i$ branes and 3 branes. From the phenomenological point of view there is no difference between these two scenarios. Here we consider the same model used in Ref. [5], where the gauge group $SU(3)_C \times U(1)_Y$ is associated with 9 brane while $SU(2)_L$ is associated with $5_1$ brane.

If SUSY breaking is analysed, as in model A, in terms of the vevs of the dilaton and moduli fields [9]

$$F^S = \sqrt{3}m_{3/2}(S + S^*) \sin \theta, \quad F^{T_i} = m_{3/2}(T_i + T_i^*)\Theta_i \cos \theta,$$

(11)

where the angle $\theta$ and the parameter $\Theta_i$ with $\sum_i |\Theta_i|^2 = 1$, just parametrize the direction of the goldstino in the $S$ and $T_i$ fields space. Within this framework, the gaugino masses are [9]

$$M_1 = M_3 = \sqrt{3}m_{3/2} \sin \theta,$$

(12)

$$M_2 = \sqrt{3}m_{3/2}\Theta_1 \cos \theta.$$  

(13)
In this case the quark doublets and the Higgs fields are assigned to the open string which spans between the $5_1$ and $9$ branes. While the quark singlets correspond to the open string which starts and ends on the $9$ brane, such open string includes three sectors which correspond to the three complex compact dimensions. If we assign the quark singlets to different sectors we obtain non–universal $A$–terms. It turns out that in this model the trilinear couplings $A^u$ and $A^d$ are given by

$$A^u = A^d = \begin{pmatrix} x & y & z \\ x & y & z \\ x & y & z \end{pmatrix},$$ \hspace{1cm} (14)$$

where

$$x = -\sqrt{3} m_{3/2} \left( \sin \theta + (\Theta_1 - \Theta_3) \cos \theta \right),$$ \hspace{1cm} (15)$$
$$y = -\sqrt{3} m_{3/2} \left( \sin \theta + (\Theta_1 - \Theta_2) \cos \theta \right),$$ \hspace{1cm} (16)$$
$$z = -\sqrt{3} m_{3/2} \sin \theta.$$ \hspace{1cm} (17)

The soft scalar masses for quark-doublets and Higgs fields ($m^2_L$), and the quark-singlets ($m^2_{R_i}$) are given by

$$m^2_L = m^2_{3/2} \left( 1 - \frac{3}{2} (1 - \Theta^2_i) \cos^2 \theta \right),$$ \hspace{1cm} (18)$$
$$m^2_{R_i} = m^2_{3/2} \left( 1 - 3 \Theta^2_i \cos^2 \theta \right),$$ \hspace{1cm} (19)$$

where $i$ refers to the three families. For $\Theta_i = 1/\sqrt{3}$ the $A$–terms and the scalar masses are universal while the gaugino masses could be non–universal. The universal gaugino masses are obtained at $\theta = \pi/6$.

It is worth mentioning that in these models (A and B) the gaugino masses, the $A$–terms, and the $\mu$–term are in general complex. However, by using $R$-rotation we can make the gaugino masses real and we end up, in addition to the phase of $\mu$, with the phases of the $A$–terms. The phase of $\mu$ is severely constrained by the electric dipole moment (EDM) of the electron and the neutron \cite{18}, while the phases of the $A$–terms are essentially unconstrained. Thus one can set the phase of $\mu$ to zero, as done in Ref. \cite{3–5}. Moreover it has been shown that the phases of the $A$–terms can
lead to sizeable supersymmetric contribution to CP observables, in particular on the
direct CP violation $\varepsilon'/\varepsilon$. However, since the total branching ratio $b \to s\gamma$ decay is a
CP conserving observable, this should not be very effective in constraining the phases
of the SUSY soft-breaking terms. For this reason we have made in our analysis the
simplifying assumption to set to zero all the phases.

2.3 Yukawa textures

As emphasized in Refs. [3–5], in models with non-degenerate $A$–terms we have to fix
the Yukawa matrices to completely specify the model. In fact, with universal $A$–terms
the textures of the Yukawa matrices at GUT scale affect the physics at EW scale only
through the quark masses and usual CKM matrix, since the extra parameters contained
in the Yukawa matrices can be eliminated by unitary fields transformations. This is
no longer true with non-degenerate $A$–terms since in the scalar potential the $A$–terms
enter through the tensorial product $(Y_q^A)_{ij} = (Y_q)_{ij}(A_q)_{ij}$. Thus the diagonalization
of $Y_q$ can not be done simultaneously with $Y_q^A$ (unlike the universal case). Thus in
the models with non–universal $A$–terms, some extra degrees of freedom (in addition
to the quark masses and CKM matrix) contained in the Yukawa matrices become
observable. Hence, the analysis of the non-degenerate $A$–terms could shed some light
on the favoured Yukawa textures. For instance in Ref. [4], using a symmetric Yukawa
texture with a symmetric $A$–terms, an accidental cancellation between the different
SUSY contributions to $\varepsilon'/\varepsilon$ was found, cancellation which leads to a very small value
for $\varepsilon'/\varepsilon$. On the contrary, by using asymmetric Yukawa matrices with symmetric $A$–
terms, this cancellation does not occur and it is found that $\varepsilon'/\varepsilon$ can be easily of the
order of the KTeV result.

Here we show two realistic examples of Yukawa matrix textures that have already
been used in Refs. [3–5]. In the first one we have the following symmetric Yukawa
matrices

\[ Y^d = y^b \begin{pmatrix} 0 & \frac{m_s}{m_b} & V_{12} \\ \frac{m_s}{m_b} & \frac{m_s}{m_b} & V_{13} \\ V_{12} & V_{13} & V_{23} \end{pmatrix}, \quad Y^u = y^t \begin{pmatrix} 0 & 0 & V_{13} \\ 0 & \frac{m_c}{m_t} & 0 \\ V_{13} & 0 & 1 \end{pmatrix}, \quad (20) \]

where $y^{b,t}$ are the Yukawa couplings of the bottom and top respectively, and $V$ is the
CKM matrix. The second example is based on the assumption that the CKM mixing
matrix originates from the down Yukawa couplings and that the Yukawa matrices are
hermitian.

\[ Y^u = \frac{1}{v \cos \beta} \text{diag} (m_u, m_c, m_t), \quad Y^d = \frac{1}{v \sin \beta} V^\dagger \cdot \text{diag} (m_d, m_s, m_b) \cdot V \quad (21) \]

Although the analysis of the CP violation is quite sensitive to the specific Yukawa
matrix, we found that the branching ratio of \( b \to s \gamma \) does not essentially depend on
it. We checked this property by using different Yukawa textures (the two examples
presented here and others). We will only present through all the paper the results
concerning the second example just as a representative case.

Now we present the general expressions for the squark mass matrices in the non-
universal case in the SCKM basis, in this basis the unitary matrices \( S^{U,R,L} \) and \( S^{D,R,L} \)
(obtained by a superfield rotation) are chosen to diagonalize the up– and down– Yukawa
couplings \( Y^{u,d} \)

\[ m_U = \frac{v \sin \beta}{\sqrt{2}} S_{UR} (Y^{u,d})^T S_{UL}^\dagger, \quad m_D = \frac{v \cos \beta}{\sqrt{2}} S_{DR} (Y^{u,d})^T S_{DL}^\dagger \quad (22) \]

where \( T \) stands for the transpose and \( m_{U,D} \) are the diagonal up– and down–quark
mass matrices respectively. In this basis the up and down squark mass matrices at low
energy (respectively \( M^2_{\tilde{u}} \) and \( M^2_{\tilde{d}} \)) are given by

\[ M^2_{\tilde{u},d} = \begin{pmatrix}
\left(M^2_{\tilde{u},d}\right)_{LL} & \left(M^2_{\tilde{u},d}\right)_{LR} \\
\left(M^2_{\tilde{u},d}\right)_{RL} & \left(M^2_{\tilde{u},d}\right)_{RR}
\end{pmatrix}, \quad (23) \]

where for the up-sector

\[ \begin{align*}
\left(M^2_{\tilde{u}}\right)_{LL} &= S_{UL} M^2_Q S_{UL}^\dagger + m_U^2 + \frac{m_Z^2}{6} (3 - 4 \sin^2 \theta_W) \cos 2\beta, \\
\left(M^2_{\tilde{u}}\right)_{RR} &= S_{UR} (M^2_{\tilde{u}})^T S_{UL}^\dagger + m_U^2 + \frac{2 m_Z^2}{3} \sin^2 \theta_W \cos 2\beta, \\
\left(M^2_{\tilde{u}}\right)_{LR} &= \left(M^2_{\tilde{u}}\right)^\dagger_{RL} = \mu m_U \cot \beta + \frac{v \sin \beta}{\sqrt{2}} S_{UL} Y^A_{u} S_{UR}^\dagger,
\end{align*} \]

and for the down-sector

\[ \begin{align*}
\left(M^2_{\tilde{d}}\right)_{LL} &= S_{DL} M^2_Q S_{DL}^\dagger + m_D^2 - \frac{m_Z^2}{6} (3 - 2 \sin^2 \theta_W) \cos 2\beta, \\
\left(M^2_{\tilde{d}}\right)_{RR} &= S_{DR} (M^2_{\tilde{d}})^T S_{DL}^\dagger + m_D^2 + \frac{2 m_Z^2}{3} \sin^2 \theta_W \cos 2\beta, \\
\left(M^2_{\tilde{d}}\right)_{LR} &= \left(M^2_{\tilde{d}}\right)^\dagger_{RL} = \mu m_D \tan \beta + \frac{v \cos \beta}{\sqrt{2}} S_{DL} Y^A_{d} S_{DR}^\dagger.
\end{align*} \]

\[ \text{(25)} \]
where $M_2^2$ and $M_{2c,dc}$ are the soft-breaking $(3 \times 3)$ mass matrices for the squark doublet and singlets respectively. Our convention for the sign of the $\mu$–term is opposite to the same one of Ref. [23]. Note that the matrices $S_{U,D}$, unlike in the universal case, can not be re–absorbed in the definition of diagonal Yukawa couplings.

3 The $b \to s\gamma$ decay in SUSY models

In this section we analyse the $b \to s\gamma$ decay in SUSY models with non–universal $A$–terms. As pointed out previously, these models are particularly interesting because they can give sizeable contributions to the CP violating processes through their large contributions to the mass insertions $(\delta_{u,d}^{LR})_{ij}$ or $(\delta_{u,d}^{RL})_{ij}$. For this reason one may expect that large effects can be also induced in the processes mediated by the dipole-magnetic operators, such as the rare decay $b \to s\gamma$. Indeed, if the $\delta_{u,d}^{LR}$ are large enough, then the SUSY contributions to these operators are enhanced since the typical chiral suppression is removed by the insertion of the internal gaugino mass, thus allowing for a competition with the chiral-suppressed SM amplitude.

Let us start with the experimental results. The most recent result reported by CLEO collaboration for the total (inclusive) B meson branching ratio $B \to X_s\gamma$ is [24]

\[
\text{BR}(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}
\]  

where the first error is statistical, the second systematic, and the third one accounts for model dependence. From this result the following bounds (each of them at 95\% C.L.) are obtained

\[
2.0 \times 10^{-4} < \text{BR}(B \to X_s\gamma) < 4.5 \times 10^{-4}. 
\]  

In addition the ALEPH collaboration at LEP reported a compatible measurement of the corresponding branching ratio for $b$ hadrons at the $Z$ resonance [25].

The starting point for the theoretical study of $b \to s\gamma$ decay is given by the effective Hamiltonian

\[
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{32}^* V_{33} \sum_{i=1}^{8} C_i(\mu_b) Q_i(\mu_b)
\]  

(28)
where the complete basis of operators in the SM can be found in Ref. [26]. Recently the main theoretical uncertainties present in the previous leading order (LO) SM calculations have been reduced by including the NLO corrections to the $b \to s\gamma$ decay, through the calculation of the three-loop anomalous dimension matrix of the effective theory [26]. The relevant SUSY contributions to the effective Hamiltonian in Eq.(28) affect only the $Q_7$ and $Q_8$ operators, the expression for these operators are given (in the usual notation) by

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu},$$

$$Q_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a.$$  \hspace{1cm} (29)

The Wilson coefficients $C_i(\mu)$ are evaluated at the renormalization scale $\mu_b \simeq O(m_b)$ by including the NLO corrections [26]. They can be formally decomposed as follows

$$C_i(\mu) = C^{(0)}_i(\mu) + \frac{\alpha_s(\mu_b)}{4\pi} C^{(1)}_i(\mu) + O(\alpha_s^2).$$  \hspace{1cm} (30)

where $C^{(0)}_i$ and $C^{(1)}_i$ stand for the LO and NLO order respectively. Finally the branching ratio $BR(B \to X_s\gamma)$, conventionally normalized to the semileptonic branching ratio

$$BR^{exp}(B \to X_c e\nu) = (10.4 \pm 0.4\%) [27],$$

is given by [26]

$$BR^{NLO}(B \to X_s\gamma) = BR^{exp}(B \to X_c e\nu) \frac{|V_{32}^* V_{33}|^2}{|V_{23}|^2} \frac{6\alpha_e}{\pi g(z) k(z)} \left(1 - \frac{8}{3} \frac{\alpha_s(m_b)}{\pi}\right)$$

$$\times \left(|D| + A \right)(1 + \delta_{np}),$$  \hspace{1cm} (31)

with

$$D = \frac{\alpha_s(\mu_b)}{4\pi} \left(C^{(1)}(\mu_b) + \sum_{i=1}^8 C^{(0)}_i(\mu_b) \left[r_i(z) + \gamma_r^{(0)} \log \frac{m_b}{\mu_b}\right]\right),$$

$$A = \left(e^{-\alpha_s(\mu_b) \log \delta(7+2\log \delta)/3\pi - 1}\right) |C^{(0)}(\mu_b)|^2 + \frac{\alpha_s(\mu_b)}{\pi} \sum_{i\leq j=1}^8 C^{(0)}_i(\mu_b) C^{(0)}_j(\mu_b) f_{ij}(\delta),$$

where $z = m^2_c/m^2_b$. The expressions for $C_i^{(0)}$, $C_i^{(1)}$, and the anomalous dimension matrix $\gamma$, together with the functions $g(z)$, $k(z)$, $r_i(z)$ and $f_{ij}(\delta)$, can be found in Ref. [26]. The term $\delta_{np}$ (of order a few percent) includes the non-perturbative $1/m_b$ [28] and $1/m_c$ [29] corrections. From the formula above we obtain the theoretical result for $BR(B \to X_s\gamma)$ in the SM which is given by

$$BR^{NLO}(B \to X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4}$$  \hspace{1cm} (32)
where the main theoretical uncertainty comes from uncertainties in the SM input parameters, namely \( m_t \), \( \alpha_s(M_Z) \), \( \alpha_{em} \), \( m_c/m_b \), \( m_b \), \( V_{ij} \), and the small residual scale dependence.\(^6\) The central value in Eq.(32) corresponds to the following central values for the SM parameters

\[
\begin{align*}
\text{pole} t &\approx m_{\text{MS}}(m_Z) \approx 174 \text{ GeV}, \\
\text{pole} b &\approx 4.8 \text{ GeV}, \\
m_c/m_b & \approx 1.3 \text{ GeV}, \\
\mu & = m_b, \\
\alpha_s(m_Z) & = 0.118, \\
\alpha_e^{-1}(m_Z) & = 128, \\
\sin^2 \theta_W & = 0.23 \\
\text{and a photon energy resolution corresponding to } \delta = 0.9 \text{ is assumed. Note that in Eq.(31) the (small) } 1/m_c \text{ corrections have not been included.}
\end{align*}
\]

The SUSY contributions to the Wilson coefficients \( C_{7,8}^{(0,1)} \) are obtained by calculating the \( b \to s\gamma \) and \( b \to sg \) amplitudes at EW scale respectively. The LO contributions to these amplitudes are given by the 1-loop magnetic-dipole and chromomagnetic dipole penguin diagrams respectively, mediated by charged Higgs boson, chargino, gluino, and neutralino exchanges. The corresponding results for these amplitudes can be found in Ref.\(^{[21]}\). It is known that the charged Higgs contribution always interferes with the SM contribution \(^{[20, 21]}\). The chargino contribution could give rise to a substantial destructive interference with SM and charged Higgs amplitudes, depending on the sign of \( \mu \), the value of \( \tan \beta \), and the mass difference between the stop masses \(^{[20, 21]}\).

We point out that the SUSY models with non–universal \( A \)–terms may induce non-negligible contributions to the dipole operators \( \tilde{Q}_{7,8} \) which have opposite chirality with respect to \( Q_{7,8} \). It is worth mentioning that these operators are also induced in the SM and in the MSSM with supergravity scenario, but their contributions are negligible being suppressed by terms of order \( \mathcal{O}(m_s/m_b) \). In particular in MSSM, due to the universality of the \( A \)–terms, the gluino and chargino contributions to \( \tilde{Q}_{7,8} \) turn out to be of order \( \mathcal{O}(m_s/m_b) \). This argument does not hold in the models with non–universal \( A \)–terms and in particular in our case. It can be simply understood by using the mass insertion method \(^{[19]}\). For instance, the gluino contributions to \( Q_7 \) and \( \tilde{Q}_7 \) operators are proportional to \( (\delta_{LR})^d_{23} \approx (S_{DL}Y_d A^* S^d_{DR})_{23}/m_\tilde{q}^2 \) and \( (\delta_{RL})^d_{23} \approx (S_{DR} Y_d A S^d_{DL})_{23}/m_\tilde{q}^2 \) respectively. Since the \( A^D \) matrix is symmetric in model A and \( A^D_{ij} \approx A^D_{ji} \) in model B, then \( (\delta_{LR})^d_{23} \approx (\delta_{RL})^d_{23} \). Then in our case we should consistently take into account the

\(^6\) Recently in Ref.\(^{[30]}\) the current method of extracting the inclusive rate for \( b \to s\gamma \) from the currently published CLEO data has been criticized arguing that the theoretical uncertainties have so far been underestimated and only a precise measurement of the photon spectrum would be helpful in reducing these uncertainties.
SUSY contributions to \( \tilde{Q}_7 \) in \( b \to s\gamma \). Analogous considerations hold for the operator \( \tilde{Q}_8 \).

By taking into account the above considerations regarding the operators \( \tilde{Q}_7, \tilde{Q}_8 \), the new physics effects in \( b \to s\gamma \) can be parametrized in a model independent way by introducing the so called \( R_{7,8} \) and \( \tilde{R}_{7,8} \) parameters defined at EW scale as

\[
R_{7,8} = \frac{(C_{7,8}^{(0)} - C_{7,8}^{(0)\text{SM}})}{C_{7,8}^{(0)\text{SM}}}, \quad \tilde{R}_{7,8} = \frac{\tilde{C}_{7,8}^{(0)}}{C_{7,8}^{(0)\text{SM}}},
\]

where \( C_{7,8} \) include the total contribution while \( C_{7,8}^{\text{SM}} \) contains only the SM ones. Note that in \( \tilde{C}_{7,8} \), which are the corresponding Wilson coefficients for \( \tilde{Q}_7, \tilde{Q}_8 \) respectively, we have set to zero the SM contribution. In Ref. [21] only the expressions for the \( R_{7,8} \) are given, for completeness we report the corresponding expressions for \( \tilde{R}_{7,8} \) in the appendix.

Inserting these definitions into the BR(\( B \to X_s\gamma \)) formula in Eq.(31) yields a general parametrization of the branching ratio in terms of the new physics contributions

\[
\text{BR}(B \to X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4} \left( 1 + 0.622 R_7 + 0.090 (R_7^2 + \tilde{R}_7^2) + 0.066 R_8 + 0.019 (R_7 R_8 + \tilde{R}_7 \tilde{R}_8) + 0.002 (R_8^2 + \tilde{R}_8^2) \right),
\]

where the overall SM uncertainty has been factorized outside. We have checked explicitly that the result in Eq.(34) is in agreement with the corresponding one used in Ref. [32]. Recently the leading EW corrections in the SM have been included in the \( C_{7,8} \) coefficients [33]. However, we have not included them since they should affect the results in a few percent which is within the theoretical uncertainties present in the SUSY sector. In particular, in order to be consistent with the NLO calculations, one should also include the corresponding two-loop QCD corrections to the SUSY amplitudes, namely \( C_{7,8}^{(1)} \). In this respect recent works [34] have been done in this direction. They calculated the two-loop \( \alpha_s \) corrections to the chargino and gluino amplitudes. However these corrections can be consistently applied only in a restricted SUSY scenario with low tan \( \beta \) and large gluino masses, since on the contrary other two-loop diagrams with

\footnote{Note that the SM central value for BR(\( B \to X_s\gamma \)) in Ref. [31] slightly differs from the result in Eq.(32), since in Ref. [31] the non-perturbative \( \Lambda/m_c \) corrections have been included.}
no QCD interactions at all could become relevant. In our work we are going to explore SUSY scenarios in the full range of \( \tan \beta \) \((2 \leq \tan \beta \leq m_t/m_b)\) and gluino mass, so we do not include these corrections since they would not change our main conclusions.

4 Numerical results and discussions

In this section we present our results for the total branching ratio \( BR(B \to X_s\gamma) \) as a function of the fundamental parameters of the soft-breaking sector of models A and B. We start our discussion by analysing the constraints on these models set by the condition of vacuum stability and the experimental bounds on the SUSY particle spectrum.

We calculate the low energy SUSY spectrum by running the soft-breaking terms and the other SUSY parameters (by means of the renormalization group equations (RGE) of MSSM generalized to the non–universal soft-breaking terms\footnote{\textsuperscript{21}}) from the GUT scale \((\simeq 2 \times 10^{16} \text{ GeV})\) to the EW scale \((\simeq M_Z)\), by using the boundary conditions given in section 2. For fixed \( \tan \beta \) and sign of \( \mu \), we restrict the parameter space by imposing the present experimental bounds on the SUSY spectra\footnote{\textsuperscript{27}}. Since the spectrum of these models is given in terms of few parameters \((m_{3/2}, \theta \text{ and/or } \Theta_i)\) we find that by requiring the lightest chargino mass to be \( m_{\chi^\pm} \geq 90 \text{ GeV} \) implies that all the other SUSY particle and the lightest Higgs masses are above their experimental bounds.

In order to avoid vacuum instabilities and color-charge breaking, we require that all the square scalar masses in Eqs.\footnote{\textsuperscript{3}} and \footnote{\textsuperscript{19}} should be positive. All these constraints set strong restrictions on the parameter space of both models. As pointed out in section 2, in model A this leads to \( \sin \theta > 1/\sqrt{2} \simeq 0.7 \). The bounds on \( m_{3/2} \) from the lightest chargino mass are as follows: for low \( \tan \beta \) \((\tan \beta = 2)\) \( m_{3/2} \geq 80 \) (60) GeV for \( \sin \theta \simeq 1/\sqrt{2} \) \footnote{\textsuperscript{1}}. For large \( \tan \beta \) these bounds are increased by 20 GeV in both cases.

In model B the non–universality is parameterized by the angle \( \theta \) and the \( \Theta_i \)'s. The values \( \Theta_i = 1/\sqrt{3} \) give universal \( A \) terms and scalar masses, the gaugino masses are universal only at \( \theta = \pi/6 \). To avoid negative scalar masses in this model one needs to impose the constraints

\[
\cos^2 \theta \Theta_i < 1/3, \quad (i = 1, 2, 3),
\]
\[(1 - \Theta_1^2) \cos^2 \theta < \frac{2}{3}. \] (35)

In the limit of universal $A$ terms these two constraints are satisfied for any value of $\theta$.

Further constraints on the parameter space of model B are obtained from the gaugino sector. As shown in Eq.\(\text{(13)}\), for a very large $m_{3/2}$, the limit of $\theta \simeq \pi/2$ cannot be reached since $M_2$ would approach zero and the mass of the lightest chargino would be too small. Moreover, the lower bound on the gluino mass and the condition of having the EW breaking at the correct scale, lead also to a lower bound on the angle $\theta$. In the case of universal $A$–terms we find that $\theta > 0.1$, while in the non–universal case Eqs.\(\text{(35)}\) set more severe constraints on $\theta$. For example, for $\Theta_1 \simeq 1$ we obtain that $0.9 \leq \theta < \pi/2$ and for $\theta$ close to $\pi/2$ the gravitino mass $m_{3/2}$ should be quite heavy (of order TeV) to make the lightest chargino higher than the experimental bound. We find that these constraints together strongly reduce the allowed parameter space of model B.

We have checked that, in both models, the $B - \bar{B}$ mixing measurements do not set further constraints on the allowed ranges of the parameter space. This mixing, being a $\Delta B = 2$ process, is proportional to $(\delta_{AB})_{13} \times (\delta_{CD})_{13}$ where $A, B, C, D = (L, R)$. We have found that, in our case, the values of these mass insertions satisfy the constraints given in Ref.\[\text{[19]}\].

Our results for the partial SUSY amplitude contributions are presented in Figs.\[\text{[1-3]}\]. The total branching ratio $BR(B \to X_s \gamma)$ is shown in Figs.\[\text{[4-5]}\] and \[\text{[6]}\] for model A and B respectively. In Figs.\[\text{[1-3]}\] we show, for model A, the individual SUSY contributions to the $R_7$ variable, (see Eq.\(\text{(13)}\) for its definition) versus $\sin \theta$. We see that the chargino and charged Higgs contributions give the dominant effect in all the range of $\theta$, while the gluino is sub–dominant, such as in the universal case. For model B, as expected from its tightly constrained parameter space, we have found that the results of the separate amplitude contributions do not differ from the corresponding ones in the universal scenario, and will not be presented here.

We can understand this behavior by using the mass insertion method. The gluino amplitude gets two leading contributions: one is proportional to the single–mass–insertion $(\delta_{LR}^d)_{23}$ and the another one to the double–mass–insertion, namely $(\delta_{LR}^d)_{22} (\delta_{LL}^d)_{23}$. In the low and intermediate $\tan \beta$ regions these two mass insertions are comparable, so
a possible destructive or constructive interference between them may appear depending on the sign of $\mu$. In the large $\tan \beta$ region the double mass insertion becomes dominant, since $(\delta^d_{LR})_{22}$ is proportional to $\mu \tan \beta$ and the amplitude (normalized to the SM one) is

$$
\frac{A_{\tilde{g}}}{A_{SM}} \propto \frac{\alpha_s}{\alpha_W V_{32}} \tan \beta \left( m_W^2 M_{\tilde{g}} \mu \right) \frac{M_{bL\tilde{s}L}^2}{m_{\tilde{g}}^2},
$$

(36)

where $m_{\tilde{g}}$ is the average squark mass in the down sector and $M_{bL\tilde{s}L}^2$ is the off diagonal element of the down-squark mass matrix defined in Eqs.(23,24). In this way, the sensitivity of the gluino amplitude to the sign of $\mu$ for large $\tan \beta$ can be understood.

The chargino amplitude, like in the gluino case, is enhanced by $\tan \beta$ and the dominant contributions to it are the (Higgsino) $A_{\tilde{h}^-}$ and (Wino-Higgsino) $A_{\tilde{W}h^-}$ amplitudes, given by

$$
\frac{A_{\tilde{h}^-}}{A_{SM}} \propto \tan \beta \left( \mu m_t \right) \frac{M_{tL\tilde{t}R}^2}{m_{\tilde{t}_L}^2 m_{\tilde{t}_R}^2},
$$

(37)

$$
\frac{A_{\tilde{W}h^-}}{A_{SM}} \propto \frac{1}{V_{32}} \tan \beta \left( \mu M_2 \right) \frac{M_{tL\tilde{t}L}^2}{m_{\tilde{t}_L}^2 m_{\tilde{t}_R}^2}.
$$

(38)

We see that these contributions are large with respect to the gluino one, such as in the universal case of MSSM, mainly because the mass insertions get a large enhancement of the light-stop mass $m_{\tilde{t}_L}^2$ in the denominator, unlike in the gluino case where the $1/m_{\tilde{g}}^6$ suppression is effective.

Regarding the contribution of the $\tilde{Q}_{7,8}$ operators to the total branching ratio, we checked that their effect is negligible, almost an order of magnitude smaller than the total contribution to $Q_{7,8}$. This can be explained by observing that the dominant effect to these operators comes from the gluino amplitude which is much smaller than the chargino or charged Higgs one.

In Figs. [4,5] we plot the results for the branching ratio $\text{BR}(B \to X_s\gamma)$, in model A, versus $\sin \theta$ for different values of $\tan \beta$, the sign of $\mu$ and for two representative values of gravitino mass $m_{3/2}$, namely $m_{3/2} = 150, 300$ GeV. The main message arising from these results is that the sensitivity of $\text{BR}(B \to X_s\gamma)$ respect to $\sin \theta$ increases with $\tan \beta$. In particular for the low $\tan \beta$ region the $b \to s\gamma$ result does not differ significantly from the universal case. In the large $\tan \beta$ region, $\tan \beta = 15 - 40$, the CLEO measurement of $b \to s\gamma$ set severe constraints on the angle $\theta$ for low gravitino
masses. For $\mu < 0$ almost the whole range of parameter space is excluded as shown in Fig. 4,5.

Comparing Top and Bottom plots in Figs.4,5, we see that that for positive (negative) sign of $\mu$ the branching ratio $BR(B \rightarrow X_s \gamma)$ decreases (increase) when the departure from the universality increases ($\sin \theta \rightarrow 1/\sqrt{2}$). Clearly, due to decoupling effects, the deviations from universality tend to be reduced for large gravitino masses, as can be seen by comparing the plots at $m_{3/2} = 150$ GeV with the corresponding ones at $m_{3/2} = 300$ GeV.

Now we discuss results in model B. In Fig.6 we plot the branching ratio $BR(B \rightarrow X_s \gamma)$ versus $\tan \beta$ for three different values of $\Theta_1, \Theta_2$ (see the figure caption) which are representative examples for universal and highly non–universal cases. From these figures it is clear that $BR(B \rightarrow X_s \gamma)$ is not very sensitive to the values of $\Theta_i$’s parameters, even at very large $\tan \beta$, unlike model A. The constraints from CLEO measurement are almost the same in the universal and non–universal cases. For $\mu > 0$ the branching ratio is constrained from the lower bound of CLEO only at very large $\tan \beta$, while for $\mu < 0$ the branching ratio is almost excluded except at low $\tan \beta$.

5 Conclusions

The recent CP violation measurements of $\varepsilon'/\varepsilon$ indicate a large deviation from the SM predictions which might be interpreted like a signal of new CP violation sources beyond the SM. It is unlikely that the minimal supersymmetric standard model with universal boundary conditions at GUT scale can explain these large enhancements in the direct CP violations. On the contrary, non-minimal SUSY scenarios with non–universal $A$–terms, derived from some string inspired models, have been found effective in explaining large values for $\varepsilon'/\varepsilon$ while keeping the electric dipole moments below the experimental bounds.

In this paper we have considered two models based on weakly coupled heterotic string (model A) and type I string theories (model B). In this framework we carefully analysed the constraints set by the $b \rightarrow s\gamma$ decay on these two models by taking into account the relevant set of SUSY diagrams. In the calculation of the total branching
ratio we take into account the NLO QCD corrections to the SM. We found that in the model based on the weakly coupled heterotic string, the $b \to s\gamma$ branching ratio is more sensitive to the non-universality at large $\tan \beta$ and that the dominant SUSY contribution comes from the chargino amplitude for any value of $\tan \beta$. For type I string-derived model, we found that the sensitivity of the $b \to s\gamma$ branching ratio to the non-universality parameters $\theta$ and $\Theta_i$ is quite weak. The main reason for this weakness is because in this model the allowed ranges for these parameters are strongly constrained by the vacuum stability bounds and the experimental limits on the lightest chargino mass.

We conclude that the recent CLEO measurements on the total inclusive B meson branching ratio $\text{BR}(B \to X_s\gamma)$ do not set severe constraints on the non-universality of these models. Moreover the constraints set on $\tan \beta$ and gravitino mass are almost the same as in the universal case. In this respect we have found that the parameter regions which are important for generating sizeable contributions to $\varepsilon'/\varepsilon$ [3-5], in particular the low $\tan \beta$ regions, are not excluded by $b \to s\gamma$ decay.

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**Appendix**

Here we give the expressions for the dominant SUSY contributions to $\tilde{R}_{7,8}$ defined in Eq.(33), namely the chargino and gluino ones, in the approximation $\mathcal{O}(m_s/m_b) = 0$

\[
\begin{align*}
\tilde{R}_{7,8} &= \tilde{R}_{7,8}^\chi + \tilde{R}_{7,8}^\tilde{g} \\
\tilde{R}_{7}^\chi &= -\frac{2}{3V_{32}^2 V_{33} x_{tW} F_7(x_{tW})}
\end{align*}
\]
\[ X^L_i \cdot X^R_i \cdot \left( X^L_i \cdot X^R_i \right)^* \cdot \left( x_{\chi I k_1} - \frac{m_{\chi I}}{m_b} \cdot \left( F_3(x_{\chi I k_1}) + \frac{2}{3} F_4(x_{\chi I k_1}) \right) \right) \]

\[ \tilde{R}^X_{8} = \frac{2}{3V_{32}^* V_{33} x_{tW} \cdot F_1(x_{tW})} \sum_{i=1}^{6} x_{W \bar{u}_k} \left( X^L_i \cdot X^R_i \right)^* \cdot \left( x_{\chi I k_1} - \frac{m_{\chi I}}{m_b} \cdot \left( F_3(x_{\chi I k_1}) + \frac{2}{3} F_4(x_{\chi I k_1}) \right) \right) \]

\[ \tilde{R}^\tilde{g}_{7} = \frac{16\alpha_s}{27\alpha W V_{32}^* V_{33} x_{tW} \cdot F_7(x_{tW})} \sum_{k=1}^{6} x_{W \tilde{d}_k} \left( \Gamma^D_L \cdot \Gamma^D_R \right)^* \cdot \frac{m_{\tilde{g}}}{m_b} \left( 9 F_3(x_{\tilde{g}_d k}) + F_4(x_{\tilde{g}_d k}) \right), \] (39)

with

\[ \left( X^L_i \right)_{ki} = -V_{11} \left( \Gamma^U_L \cdot V \right)_{ki} + \frac{1}{\sqrt{2 m_W \sin \beta}} V_{12} \left( \Gamma^D_L \cdot V \cdot M_U \cdot V \right)_{ki} \]

\[ \left( X^R_i \right)_{ki} = \frac{1}{\sqrt{2 m_W \cos \beta}} U_{12} \left( \Gamma^U_L \cdot V \cdot M_D \right)_{ki} \] (40)

where \( x_{ij} \equiv m_i^2/m_j^2 \), \( F_i(x) = \frac{2}{3} F_1(x) + F_2(x) \), \( U \) and \( V \) are the \( 2 \times 2 \) diagonalization matrices for the chargino mass matrix defined as in Ref. [21], and the \( 6 \times 6 \) matrices \( \Gamma^{(U,D)} \equiv \left[ \Gamma^{(D_L, U_L)}_{6 \times 3}, \Gamma^{(D_R, U_R)}_{6 \times 3} \right] \) respectively diagonalize the Up- and Down-squark mass matrices in Eqs. (23,25). The expressions for the functions \( F_i(x) \) can be found in Ref. [21].

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Figure 1: (Top) $R_7$ for chargino contribution versus $\sin \theta$ in model A, for $\mu > 0$, $m_{3/2} = 150$ GeV, and for $\tan \beta = 2, 15, 40$. (Bottom) The same as before, but for $\mu < 0$. 


Figure 2: (Top) $R_7^g$ for the gluino contribution versus $\sin \theta$ in model A, for $\mu > 0$, $m_{3/2} = 150$ GeV, and for $\tan \beta = 2, 15, 40$. (Bottom) The same as before, but for $\mu < 0$. 
Figure 3: $R_7$ for charged Higgs contribution versus $\sin \theta$ in model A, $m_{3/2} = 150$ GeV, and for $\tan \beta = 2, 15, 40$. 
Figure 4: (Top) The BR($B \rightarrow X_s \gamma$) versus sin θ in model A, for $\mu > 0$, $m_{3/2} = 150$ GeV and tan $\beta = 2, 15, 40$. (Bottom) The same as before, but for $\mu < 0$. 
Figure 5: (Top) The BR($B \rightarrow X_s \gamma$) versus sin $\theta$ in model A, for $\mu > 0$, $m_{3/2} = 300$ GeV and tan $\beta = 2, 15, 40$. (Bottom) The same as before, but for $\mu < 0$. 
Figure 6: (Top) The branching ratio \( \text{BR}(B \to X_s \gamma) \) versus \( \tan \beta \) in model B, for \( \mu > 0 \), \( m_{3/2} = 150 \text{ GeV} \), and for some values of \((\Theta_1, \Theta_2) = (1/\sqrt{3}, 1/\sqrt{3}), (0.9, 0.2), (0.6, 0.2)\), corresponding to the continuous, dashed, and dot-dashed lines respectively. (Bottom) The same as before, but for \( \mu < 0 \).