CONSTRAINTS ON COMPACT DARK MATTER FROM GRAVITATIONAL WAVE MICROLENSING

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ABSTRACT

If a significant fraction of dark matter is in the form of compact objects, they will cause microlensing effects in the gravitational wave (GW) signals observable by LIGO and Virgo. From the non-observation of microlensing signatures in the binary black hole events from the first two observing runs and the first half of the third observing run, we constrain the fraction of compact dark matter in the mass range \(10^{-7} - 10^3 M_\odot\) to be less than \(\sim 50-80\%\) (details depend on the assumed source population properties and the Bayesian priors). These modest constraints will be significantly improved in the next few years with the expected detection of thousands of binary black hole events, providing a new avenue to probe the nature of dark matter.

1. INTRODUCTION

Astronomical observations have firmly established that a significant fraction of the mass energy in the Universe is in the form of dark matter, which interacts only through gravity (Bertone & Hooper 2018). Fundamental particles that are beyond the Standard Model of particle physics are the most popular candidate for dark matter. However, such particles have so far evaded a confident detection through direct or indirect methods (Roszkowski et al. 2018). Massive astrophysical compact halo objects (MACHOs), in particular primordial black holes (PBHs), are also potential candidates of dark matter (Carr & Kuhnel 2020).

PBHs could be formed via the collapse of large overdensities in the early Universe. Their abundance is heavily constrained by the non-observation of their signatures in a variety of astronomical probes. These include the effects of Hawking evaporation, microlensing of stars and supernovae, gravitational wave (GW) observations, accretion effects from X-ray binaries, distortions of cosmic microwave background, dynamical effects such as the stability of certain wide binaries and stellar clusters, formation of large-scale structures, etc. (Carr & Kuhnel 2020; Carr et al. 2020). Nevertheless, the possibility of them contributing to the dark matter cannot be ruled out in several mass windows. Recent observations of GWs from massive black hole binaries (Abbott et al. 2016; Abbott et al. 2016a, 2019a, 2021a) have resulted in renewed interest in PBHs (Sasaki et al. 2018; Carr & Kuhnel 2020).

Here we present constraints on the abundance of MACHOs through the non-observation of gravitational microlensing signatures in the GW signals detected by LIGO and Virgo. If a significant fraction of the dark matter is in the form of MACHOs in the mass range \(\sim 10^{-7} - 10^3 M_\odot\), microlensing will introduce characteristic deformations on the GW signals produced by binary black hole mergers (Takahashi & Nakamura 2003; Jung & Shin 2017; Urrutia & Väskonen 2021). Our search included the binary black hole events detected by LIGO and Virgo during their first (O1) (Abbott et al. 2016a) and second (O2) (Abbott et al. 2019a) observing runs as well as the first half of the third (O3a) observing run (Abbott et al. 2021a). We use the non-observation of such signatures to constrain the fraction of dark matter in the form of MACHOs to be less than \(\sim 50-80\%\). The precise constraints depend on the assumed source population properties and the Bayesian priors. While these constraints are indeed modest, this method provides a new way to probe the abundance of MACHOs in the high mass window. These constraints will significantly improve in the next few years with the detection of thousands of binary black hole events.

2. SEARCH FOR MICROLENSING SIGNATURES IN LIGO-VIRGO BINARY BLACK HOLE EVENTS

![Figure 1. Cumulative distribution of ln Bℓ from LIGO-Virgo events from O1, O2 and O3a. (number of events with ln Bℓ < less than the value shown in the horizontal axis). The largest value is ln Bℓ = 1.15, which is not large enough to provide strong evidence for lensing.](image)

Although the prime candidates of MACHOs are PBHs, the microlensing effects are practically the same for any compact object. Hence we keep the generic name MACHOs throughout this article.
the radiation is comparable to the gravitational radius of the lens \(\lambda \sim GM/c^2\). Hence wave diffraction effects will be apparent and the lensing has to be treated in the wave optics regime (Takahashi & Nakamura 2003).

Lensing effects on the GW signal \(h|f; \lambda\) (in Fourier domain) due to a point mass lens can be modeled in terms of a frequency dependent, complex magnification \(F(f)\), so that the resulting lensed waveform is

\[
h_l(f; \lambda, M^*_l, y) = h(f; \lambda)F(f; M^*_l, y),
\]

where \(M^*_l \equiv M_l(1+z_l)\) is the redshifted mass\(^2\) of the lens (\(M_l\) being its actual mass and \(z_l\) the cosmological redshift) and \(y\) is the dimensionless source position defined with respect to the optical axis (Takahashi & Nakamura 2003). Also, \(\lambda\) is the set of parameters that describe the (unlensed) GW signal in the detector, such as the redshifted masses \((m_1^*, m_2^*)\), the dimensionless spin vectors \((\chi_1, \chi_2)\), sky location of the binary \((\alpha, \delta)\), luminosity distance \((d_L)\), inclination and polarisation angles \((\iota, \psi)\), and the time and phase of coalescence \((t_0, \phi_0)\).

Given the data \(d\) containing a GW signal, and models of lensed and unlensed waveforms \((H_\ell \text{ and } H_\nu)\), we can compute the Bayesian likelihood ratio between the “lensed” hypothesis \(H_\ell\) and “unlensed” hypothesis \(H_\nu\):

\[
B^\nu = \frac{P(d|H_\ell)}{P(d|H_\nu)} = \frac{\int P(\lambda|H_\ell)P(d|\lambda, H_\ell)d\lambda}{\int P(\lambda|H_\nu)P(d|\lambda, H_\nu)d\lambda}, \tag{2}
\]

Where \(\lambda^\ell \equiv \{\lambda, M^*_l, y\}\) denotes the set of parameters describing the lensed waveform model.

We searched for evidence of microlensing effects in the 10 binary black hole events reported by the LIGO-Virgo Collaboration from the first two observing runs (Abbott et al. 2019b). Our search is similar to what is reported in Hannuksela et al. (2019). However, we also include in our analysis, 8 additional events reported by Zackay et al. (2019b); Venumadhav et al. (2020); Zackay et al. (2019a). The unlensed BBH waveforms \(h(f)\) were generated using the IMPHENOMv2 waveform approximant (Hannam et al. 2014; Husa et al. 2016; Khan et al. 2016) coded in the LALSUITE software package (LIGO Scientific Collaboration 2018). We use the Dynamic Nested Sampling (Speagle 2020) implementation (DYNESTY) in BILBY package (Ashton et al. 2019) to compute the posteriors of the signal parameters and the marginal likelihoods of \(H_\ell\) and \(H_\nu\). In addition to this, we also make use of the results (that is, the \(B^\nu\) values) of the microlensing search on the 36 binary black hole events from the first half of the third observing run (O3a) reported by Abbott et al. (2021b).

For the Bayesian parameter estimation, we use uniform priors in the detector frame chirp mass \(M^* \in [3, 60] M_\odot\) and the mass ratio \(q \equiv m_2/m_1 \in [0.125, 1]\), along with the constraint on the component masses \(m_1^*, m_2^* \in [5, 80] M_\odot\). We also use isotropic sky location (uniform in \(\hat{\alpha}, \hat{\delta}\)) and orientation (uniform in \(\cos \iota, \phi_0\)) uniform in polarisation angle \(\psi\), and a volumetric prior \(\propto d^2 L\) on luminosity distance. Additionally, we use a uniform prior in \(\log_{10}(M_0^*/M_*^\ell) \in [0, 5]\) and \(\rho(y) \propto y\) with a cutoff \(y \in [0, 1.3]\). In addition, we restrict the parameter space of lens parameters \((M^*_l, y)\) such that the time delay \(\Delta t_{\text{lens}}(M^*_l, y)\) due to lensing is always less than the duration \(\tau_{\text{signal}}(\lambda)\) of the corresponding signal.

\(^2\) The frequency of GWs will be redshifted due to the cosmological expansion. This effect is equivalent to redefining the masses involved in the process \(M \rightarrow M^* \equiv M(1+z)\) (both in the GW generation and lensing).

\(^3\) If the lensing time delay is larger than the duration of the waveforms, the resulting waveform will appear as two separate GW events in the LIGO-Virgo data. The non-observation of multiple images can also be used to put constraints on \(f_{\text{DM}}\) at higher lens masses. This is being explored in an ongoing work.
lenses. We consider three different cosmological redshift distributions of binary black holes — uniform distribution in comoving volume as well as that the ones predicted by population synthesis models presented in Dominik et al. (2013) and Belczynski et al. (2016a,b). We use a power-law mass distribution model, \( P(m_1) = m_1^{-2.35} \), on the mass of the heavier black hole while the mass ratio \( m_2 / m_1 \) is distributed uniformly in the interval \([1, 1/18]\) with the total mass lying in the interval \([5−200]M_\odot\). (Abbott et al. 2016b). We consider spinning black holes with component spin magnitudes distributed uniformly between 0 and 0.99 with spins aligned/antialigned with the orbital angular momentum. The binaries are distributed uniformly in the sky with isotropic orientations.

In our simulations, MACHOs are approximated by point mass lenses and distributed uniformly in comoving volume, and the microlensing optical depth depends on the \( f_{\text{DM}} \). Lensing effects on the GW signal are computed using Eq. (1). Binaries producing a network signal-to-noise (SNR) of 8 or above in the LIGO-Virgo detectors were detectable. Since the calculation of the lensing likelihood radio \( B_\ell^t \) using Nested Sampling from all the simulated signals is computationally expensive, we use an approximation that is expected to be accurate in the high SNR regime (Cornish et al. 2011; Vallisneri 2012). We then compute the fraction of detected events that produce a \( B_\ell^t \) that is larger than the highest \( B_\ell^c \) obtained from real LIGO-Virgo events. This lensing fraction is shown as a function of the \( f_{\text{DM}} \) in Fig. 3. This data can be used to compute the Jacobian \( \frac{du}{df_{\text{DM}}} \).

4. RESULTS AND DISCUSSION

The largest value of the microlensing likelihood ratio obtained from O1, O2, O3a events is \( \text{ln} B_\ell^c = 1.15 \). The fraction of simulated events with \( \text{ln} B_\ell^c \geq 1.15 \) is shown as a function of the \( f_{\text{DM}} \) in Fig. 3, for different lens masses. This allows us to compute the Jacobian \( \frac{du}{df_{\text{DM}}} \) and thus the posterior on \( f_{\text{DM}} \) as described by Eq. (8). As commonly done in the literature, we assume monochromatic spectra for MACHOs (Carr & Kuhnel 2020). Figure 4 shows the posterior of \( f_{\text{DM}} \), with masses given in the legends\(^4\). The 90% upper limits are shown as filled circles in each plot. The upper limits depend on the

\(^4\) For lens mass \( \lesssim 100M_\odot \), lensing effects on the waveform are typically too weak to be identified (Fig. 11). If we estimate the \( f_{\text{DM}} \) posteriors for these lens masses, we will be practically recovering the priors. Also, for lens mass \( \gtrsim 10^3M_\odot \), lensing time delays are typically large enough to produce multiple resolvable copies of the signals (geometric optics regime). This is assumed redshift distribution of binary black holes as well as the Bayesian priors used in the analysis. Nevertheless, we are able to place upper bounds on \( f_{\text{DM}} \) of the order of 50–80%. The 90% upper limits are shown as a function of the lens mass in Fig. 5.

We point out some limitations of our study: We assume that the GW signals are (possibly) lensed by only one microlens. However, if \( f_{\text{DM}} \approx 1 \), a small number of sources at high redshifts (\( z \gtrsim 1.5 \)) could be potentially lensed by more than one lens. Even then, we expect the dominant lensing effect on the waveform will be due to one single lens. The loss of sensitivity of our search due to neglecting the contributions of additional lenses is expected to be negligible. Also, in order to estimate the sensitivity of our search (or, the Jacobian of the lensing fraction and dark matter fraction), we use an approximation of the Bayesian likelihood ratio that is expected to be valid in high SNRs. While we expect this approximation to be reasonable for the SNRs that we consider, the quantitative effect of this needs to be verified by extensive simulations.

We approximate MACHOs as isolated point masses. Since these micro lenses are embedded in the lensing potential of the galaxy, the macro lens can cause additional effects when the micro lenses are within the Einstein radius of the macro lens (Cheung et al. 2021, e.g.). This is especially important when the micro lenses are very close to the image locations of the macro lens, which is expected to happen only for a small fraction of MACHOs. We also neglect any additional effect of lensing by sub-structures in dark matter halos (Dai et al. 2018, e.g.). The clustering of MACHOs, which we neglect, is unlikely to change our results significantly (Zackrisson & Riehm 2007).

The bounds that we obtain are weaker than some of the existing constraints (Carr & Kuhnel 2020; Carr et al. 2020). However, the GW lensing bounds will get significantly better in the next few years as the sensitivity of GW detectors improve. The sensitivity improvement will bring about two effects: Firstly, the increased number of total detections will allow us to estimate the lensing fraction \( u \) better (see, e.g. Fig. 2). Secondly, the increased horizon distance of the detectors will increase the lensing optical depth and hence the fraction of lensed events (see, e.g., Fig. 3). The expected \( f_{\text{DM}} \) upper limits from future detections (Abbott et al. 2018) are shown in Fig. 6 (for lens mass \( 10^3M_\odot \)) as a function of the
Figure 3. The fraction of simulated events with \( \ln B^\text{fDM}_i \) greater than the threshold value 1.15 shown as a function of the \( f_{\text{DM}} \). The left, middle and right plots correspond to different assumed redshift distribution of mergers. In each plot, different colors correspond to different lensing fraction estimated using the noise power spectral densities of LIGO-Virgo detectors from O3a, O3 and O1 observing runs, respectively. The error bars indicate the counting errors due to the finite number of samples of simulated binaries and the curves show quadratic polynomial fits.

Figure 4. Posteriors on \( f_{\text{DM}} \) obtained by the non-observation of microlensing signatures in the 54 binary black hole events detected in O1, O2 and O3a. Posteriors shown by solid (dotted) lines are obtained by assuming flat (Jeffreys) prior in \( \Lambda_{\text{C}} \) and \( \Lambda_{\ell} \). The left, middle and right plots correspond to different assumed redshift distribution models of binary black holes. In each sub-plot, different curves correspond to different assumed lens masses (shown in legends). The 90\% credible upper limits are shown by dots.

number of detected binary black hole mergers, assuming that none of them show signatures of lensing. The upcoming third generation of GW detectors that will detect hundreds of thousands of binary black hole mergers every year by probing the high-redshift Universe \( (z \simeq 15) \), the constraints will improve by orders of magnitude. It is fair to say that microlensing of GWs is opening a powerful probe of the nature of dark matter.

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used the redshift distribution of binary mergers given by Belczynski et al. with increased sensitivity anticipated in upcoming observing runs. We have assumed flat priors for $\Lambda$ and $\Lambda_\ell$; that is,

$$p(\Lambda) = \frac{1}{\Lambda_{\text{max}}} \Theta(\Lambda - \Lambda_{\text{max}}), \quad (A1)$$

$$p(\Lambda_\ell) = \frac{1}{\Lambda_{\ell_{\text{max}}}^\Lambda} \Theta(\Lambda_\ell - \Lambda_{\ell_{\text{max}}}^\Lambda) = \frac{1}{\Lambda u_{\text{max}}^\Lambda} \Theta(u - u_{\text{max}}^\Lambda),$$

where $\Lambda_{\text{max}}, \Lambda_{\ell_{\text{max}}}^\Lambda$ ans $u_{\text{max}}^\Lambda$ are the maximum possible values of $\Lambda, \Lambda_\ell$ and $u$, respectively ($u_{\text{max}}^\Lambda$ corresponds to $f_{DM} = 1$). This results in the explicit expression

$$p(u \mid \{N_\ell = 0, N\}) \propto \Theta(u_{\text{max}}^\Lambda - u) \int_0^{\Lambda_{\text{max}}^\Lambda} \frac{\Lambda^{N+1} e^{-\Lambda(u+1)}}{1 - e^{-u_{\text{max}}^\Lambda}} d\Lambda. \quad (A2)$$

On the other hand, if we assume Jeffreys prior for $\Lambda$ and $\Lambda_\ell$; that is,

$$p(\Lambda) = \frac{1}{\sqrt{\Lambda \Lambda_{\text{max}}}^\Lambda} \Theta(\Lambda - \Lambda_{\text{max}}), \quad (A3)$$

$$p(\Lambda_\ell) = \frac{1}{\sqrt{\Lambda_\ell \Lambda_{\ell_{\text{max}}}^\Lambda}} \Theta(\Lambda_\ell - \Lambda_{\ell_{\text{max}}}^\Lambda) = \frac{1}{\Lambda} \frac{1}{\sqrt{4u_{\text{max}}^\Lambda}} \Theta(u - u_{\text{max}}^\Lambda),$$

this results in the explicit expression

$$p(u \mid \{N_\ell = 0, N\}) \propto \Theta(u_{\text{max}}^\Lambda - u) \int_0^{\Lambda_{\text{max}}^\Lambda} \frac{\Lambda^N e^{-\Lambda(u+1)}}{\text{erf}(\sqrt{u_{\text{max}}^\Lambda})} d\Lambda. \quad (A4)$$

We use these expressions to compute the posteriors shown in Fig. 2. The normalisation is fixed by $\int_0^{u_{\text{max}}^\Lambda} p(u \mid \{N_\ell = 0, N\}) \; du = 1$.

### APPENDIX

#### POSTERIORS OF LENSING FRACTION ASSUMING DIFFERENT PRIORS

Here we present the explicit expressions of the posteriors of the lensing fraction $u \equiv \Lambda_\ell / \Lambda$ using different priors. Here $\Lambda$ and $\Lambda_\ell$ are the Poisson means of the number of binary black hole detections and the lensing detections, respectively. If we assume flat priors for $\Lambda$ and $\Lambda_\ell$; that is,

$$p(\Lambda) = \frac{1}{\Lambda_{\text{max}}} \Theta(\Lambda - \Lambda_{\text{max}}), \quad (A1)$$

$$p(\Lambda_\ell) = \frac{1}{\Lambda_{\ell_{\text{max}}}^\Lambda} \Theta(\Lambda_\ell - \Lambda_{\ell_{\text{max}}}^\Lambda) = \frac{1}{\Lambda u_{\text{max}}^\Lambda} \Theta(u - u_{\text{max}}^\Lambda),$$

where $\Lambda_{\text{max}}, \Lambda_{\ell_{\text{max}}}^\Lambda$ ans $u_{\text{max}}^\Lambda$ are the maximum possible values of $\Lambda, \Lambda_\ell$ and $u$, respectively ($u_{\text{max}}^\Lambda$ corresponds to $f_{DM} = 1$). This results in the explicit expression

$$p(u \mid \{N_\ell = 0, N\}) \propto \Theta(u_{\text{max}}^\Lambda - u) \int_0^{\Lambda_{\text{max}}^\Lambda} \frac{\Lambda^{N+1} e^{-\Lambda(u+1)}}{1 - e^{-u_{\text{max}}^\Lambda}} d\Lambda. \quad (A2)$$

On the other hand, if we assume Jeffreys prior for $\Lambda$ and $\Lambda_\ell$; that is,

$$p(\Lambda) = \frac{1}{\sqrt{\Lambda \Lambda_{\text{max}}}^\Lambda} \Theta(\Lambda - \Lambda_{\text{max}}), \quad (A3)$$

$$p(\Lambda_\ell) = \frac{1}{\sqrt{\Lambda_\ell \Lambda_{\ell_{\text{max}}}^\Lambda}} \Theta(\Lambda_\ell - \Lambda_{\ell_{\text{max}}}^\Lambda) = \frac{1}{\Lambda} \frac{1}{\sqrt{4u_{\text{max}}^\Lambda}} \Theta(u - u_{\text{max}}^\Lambda),$$

this results in the explicit expression

$$p(u \mid \{N_\ell = 0, N\}) \propto \Theta(u_{\text{max}}^\Lambda - u) \int_0^{\Lambda_{\text{max}}^\Lambda} \frac{\Lambda^N e^{-\Lambda(u+1)}}{\text{erf}(\sqrt{u_{\text{max}}^\Lambda})} d\Lambda. \quad (A4)$$

We use these expressions to compute the posteriors shown in Fig. 2. The normalisation is fixed by $\int_0^{u_{\text{max}}^\Lambda} p(u \mid \{N_\ell = 0, N\}) \; du = 1$.

### ASTROPHYSICAL SIMULATIONS OF LENSED MERGERS

Here we describe the astrophysical simulations used to evaluate the efficiency of our Bayesian model selection method in distinguishing lensed merger events from unlensed events, thus estimating the Jacobian between the compact dark matter fraction $f_{DM}$ and the fraction of lensed events $u$. Here are the steps involved:

1. **Generate a population of mergers:** The source redshifts $z_s$ are drawn from three redshift distributions (Fig. 7) — uniformly in comoving volume as well as from the population synthesis models given by Dominik et al. (2013)
and Belczynski et al. (2016a,b). We use a power-law mass distribution model, $P(m_1) = m_1^{-2.35}$, on the mass of the heavier black hole while the mass ratio $m_2/m_1$ is distributed uniformly in the interval $[1, 1/18]$ with the total mass lying in the interval $[5 - 200] M_\odot$. We consider spinning black holes with component spin magnitudes distributed uniformly between 0 and 0.99 with spins aligned/antialigned with the orbital angular momentum. The binaries are distributed uniformly in the sky with isotropic orientations (Fig. 8).

2. Identify the detectable events: Compute the optimal signal-to-noise ratio (SNR) of the binaries observed by the LIGO-Virgo detectors, using appropriate noise power spectral density (PSD) and antenna pattern functions. Binaries producing a network SNR of 8 or above are considered detectable (Fig. 8). A given choice of redshift and mass distribution of the mergers yields $N$ number of detectable events.

For O1, we used the representative PSDs given in LIGO-Virgo Collaboration (2015a,b). For O2, the representative PSDs given in LIGO-Virgo Collaboration (2018) were used, while, for O3, O4 and O5 scenarios, the representative/anticipated PSDs given in LIGO-Virgo Collaboration (2020) were used.

3. Identify the lensed events: Assuming that the MA-CHOs are distributed uniformly in comoving volume, the probability that GWs from a binary located at a redshift of $z_S$ is lensed is given by $P(z_S) = 1 - e^{-\tau(z_S)}$, where $\tau$ is the lensing optical depth (Fig. 9)

$$\tau(z_s, f_{DM}, y_0) = \int_0^{z_s} \frac{dz}{dz} d\tau,$$

with the differential optical depth given by Jung & Shin (2017)

$$\frac{d\tau}{dz} = f_{DM} \frac{3}{2} y_0^2 \Omega_{DM} \frac{H_0^2}{c} \frac{(1+z_s)^2 D_s D_l}{D_c}.$$

Above, $y_0$ is a fiducial dimensionless radius of influence of the lens, or the maximum impact parameter (in units of the Einstein angle $\theta_E$) within which the lens can potentially produce desired lensing effect.

We identify a binary as lensed when the lensing probability $P(z_S)$ of that binary is larger than a random number uniformly distributed between 0 and 1. This ensures that $P(z_S)$ fraction of binaries located at a redshift $z_S$ is counted as lensed.

4. Assign lens properties: When a merger located at a redshift $z_S$ is identified as lensed, the lens redshift $z_L$ is randomly drawn from a probability distribution given by the differential optical depth [Eq. (B2)]. For a lens mass $M_L$, the redshifted lens mass is computed as $M_L' = M_L(1+z_L)$. The impact parameter $y$ is drawn from the distribution $P(y) \propto y$, with $y \in [0, 0.01, y_0]$. We choose $y_0 = 5$, since signals with $y > 5$ are unlikely to contain identifiable lensing signatures (Fig. 11).

5. Identify events with wave optics effects: Wave optics effects in the waveform are observed when the time delay caused by lensing is smaller than the duration of the signal. (Otherwise, lensing will produce multiple signals separated in time). The time delay produced by a point mass lens is given by (Takahashi & Nakamura 2003)

$$\Delta T^L = 4M^2_L \left[ \frac{y}{2} \sqrt{y^2 + 4} + \ln \left( \frac{y^2 + 4 + y}{y^2 + 4 - y} \right) \right],$$

We approximate the duration of a GW signal by the Newtonian chirp time (Sathyaprakash 1994), with some extra time to adjust for the presence of the merger and ringdown part.

$$\tau_{\text{signal}} = \frac{5}{256} M^2_S \left( \frac{\pi f_{\text{low}}}{\sqrt{2}} \right)^{-8/3} + 10^4 M^3_S,$$

where $M_S$ and $M'$ are the redshifted chirp mass and total mass of the binary, respectively, while $f_{\text{low}}$ is the low-frequency cutoff of the detector. We consider those lensed binaries with $\Delta T^L < \tau_{\text{signal}}$ as the ones potentially containing wave optics effects (Fig. 10).

6. Generate lensed waveforms: Generate gravitational waveforms corresponding to the source parameters. Apply the wave optics lensing effects using Eq. (1).

7. Compute the approximate Bayes factor for the microlensed events at each detector: In the high SNR limit, the Bayes factor $B^\ell_0$ between the lensed and unlensed hypotheses [Eq. (2)] can be approximated as (Cornish et al. 2011; Vallisneri 2012)

$$\ln B^\ell_0 \approx (1 - \text{FF}) \rho^2,$$

where $\rho \equiv \sqrt{\langle h_l | h_0 \rangle} \approx \sqrt{\langle h_l | h_l \rangle}$ is the optimal SNR of the signal while FF is the fitting factor of the unlensed signal.

As long as $y_0$ is chosen sufficiently large and the actual impact parameters are distributed up to $y_0$ (step 4), the precise choice of $y_0$ does not affect our estimation of the lensing fraction.

Since the optical depth is also scaled with the same value of $y_0$, this will not change the fraction of identifiable lensed events.

In the wave optics regime that we consider, the amplification of the signals is not substantial unlike in the geometric optics regime. Hence the approximation $(h_l | h_0) \approx \langle h_l | h_l \rangle$ is a good one. The bias in $\rho^2$ incurred by using this approximation is less than 10% for over 90% of the lensed signals.

Figure 7. Redshift distribution of binary black hole mergers assumed predicted by different models — uniform distribution in comoving volume, population synthesis models predicted by Belczynski et al. (2016a,b) and Dominik et al. (2013). We also show, in thin grey lines, several models of primordial black hole mergers given in Mandic et al. (2016). Since most of them are “bracketed” by the three models that we consider, we do not use them explicitly in the computation of upper limits.
waveform family \( h(\Theta) \) with the lensed waveform \( h_\ell \)

\[
\text{FF} = \max_{\Theta} (h_\ell | h(\Theta)) .
\]  

(B6)

Here the brackets denote the following noise weighted inner product

\[
(a, b) = 4 \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{a(f) h^*(f)}{S_\delta(f)} df
\]

(B7)

where \( S_\delta(f) \) is the one sided power spectral density of the detector noise. In Eq. (B6), \( \Theta \) comprises the intrinsic source parameters \( \{ \mathcal{M}_s, \eta_\ell, \chi_\ell \} \) of the unlensed template. It is not necessary to maximize the match explicitly over the extrinsic parameters, as this is performed semi-analytically by the match calculation (for non-precessing signals containing only the dominant mode of the gravitational radiation) (Fig. 11).

8. Combine Bayes factors from multiple detectors: Assuming that the noise of different detectors are statistically independent, the Bayes factors \( B_{U}^{(D)} \) obtained from the individual detector \( D \) can be combined as

\[
B_{U} = \prod_{D} B_{U}^{(D)}. 
\]  

(B8)

9. Compute the fraction of detectable events that have a \( B_{U}^{(D)} \) greater than a threshold: If an event has a \( B_{U}^{(D)} \) greater than a threshold, it is deemed as an event that is identifiable as lensed. If there are \( N_\ell \) such identifiable lensed events in the simulation, the lensing fraction \( u \) is computed as \( u \equiv \Lambda_\ell / \Lambda \simeq N_\ell / \hat{N} \), where \( \hat{N} \) the number of detectable events from the simulation. This lensing fraction \( u \) as a function of \( f_{\text{DM}} \) can be used to compute the Jacobian \( du / d f_{\text{DM}} \) for a given choice of source population and lens mass (Fig. 3).

Since the PSD of the detector noise is different between observing runs, this affects the fraction of lensed events for a given \( f_{\text{DM}} \). We combine the lensing fraction \( u(f_{\text{DM}}) \) computed from simulations using different PSDs, with the number of detected events from that observing run as the weight. That is,

\[
u(f_{\text{DM}}) = \frac{1}{\hat{N}} \sum_{R} N_R u_R(f_{\text{DM}}),
\]  

(B9)

where \( N_R \) is the number of events detected (or, expected to be detected, in the case of future observing runs) in an observing run \( R \) and \( u_R(f_{\text{DM}}) \) is the lensing fraction estimated from simulations using the PSD of that observing run. \( \hat{N} \) is the total number of detected events considered.

BAYESIAN MODEL SELECTION AND INJECTION STUDIES

Here we provide some additional details on the Bayesian model selection performed to compute the likelihood ratio \( B_{U}^{(D)} \), which, in turn, is used to determine whether a GW event contains signatures of microlensing or not. Figure 12 (left panel) shows the posterior distributions of redshifted lens mass \( M_{\ell} \) (marginalised over all other parameters) and the Bayesian likelihood ratio between lensed and unlensed hypotheses obtained from the binary black hole signals observed during O1 and O2. None of the likelihood ratios are significant enough to favour the lensing hypothesis.

In order to check the accuracy of our Bayesian model selection, we perform a simulation study where an unlensed GW signal with redshifted masses \( m_1^e = 35.2 M_\odot, m_2^e = 31.7 M_\odot \) (broadly consistent with the GW150914 event) and \( \text{SNR} = 16.1 \) was added to Gaussian noise with the noise PSD from O3a. We then perform the Bayesian model selection us-
ing both the lensed and unlensed GW signal models. The true parameters of the simulated signal are well recovered within 90% confidence interval of the posterior distribution of the parameters. The Bayesian likelihood ratio between lensed and unlensed hypotheses computed from this simulated event is $\ln B^i_e = -0.2$, showing no evidence of lensing, as expected (Fig. 12 right panel). The recovered posterior on $M_i^L$ is consistent with zero, as seen in the case of real events. We also simulate a signal with the same source parameters that is lensed by a compact object with redshifted mass $M_i^L = 10^{13} M_\odot$ and impact parameter $y = 0.47$ and repeat the same analysis on it. Here we find that the lensing hypothesis is significantly preferred ($\ln B^i_c = 26.7$), as expected. The recovered posterior on $M_i^L$ is also consistent with simulated lens mass (Fig. 12 right panel).

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Figure 12. The left panel shows the posterior distributions of redshifted lens mass $M_{\ell}z$ (violin plots) and the Bayesian likelihood ratio between lensed and unlensed hypotheses (top horizontal axis) obtained from the binary black hole signals observed during O1 and O2. None of the likelihood ratios are significant enough to favor the lensing hypothesis. The right panel shows the same estimated from a simulated lensed/unlensed binary black hole event with redshifted masses $m_{z1} = 35.2M_\odot$, $m_{z2} = 31.7M_\odot$ (broadly consistent with the GW150914 event) and SNR = 16.1. For the simulated lensed event, the lensing hypothesis is significantly preferred ($\ln B_{\ell U} = 26.7$) and the posterior on lens mass is consistent with the injected value $M_{\ell}z = 10^{3.4}M_\odot$. For the simulated unlensed event, $\ln B_{\ell U} = -0.2$ is consistent with the values derived from real events shown in the left panel (within noise induced fluctuations).

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