Abstract

In the present work, we search static charged black hole solutions to Hořava-Lifshitz gravity with or without projectability condition. We consider the most general form of action which electromagnetic field couples with Hořava-Lifshitz gravity. With the projectability condition, we find (A)dS-Reissner-Nordstrom black hole solution in Painlevé-Gullstrand type coordinates in the IR region and a de-Sitter space-time solution in the UV region. Without the projectability condition, in the IR region, we find an especial static charged black hole solution.

PACS numbers: 98.80.Cq
I. INTRODUCTION

The Hořava-Lifshitz gravity which was introduced in [1, 2] is intended to be a power-counting renormalizable gravity theory. The basic idea behind Hořava’s theory is that time and space may have different dynamical scaling in UV limit. This was inspired by the development in quantum critical phenomena in condensed matter physics, with the typical model being Lifshitz scalar field theory [3, 4]. In this Hořava-Lifshitz theory, time and space will take different scaling behavior as

\[ x \to b x, \quad t \to b^z t, \]

(1)

where \( z \) is the dynamical critical exponent characterizing the anisotropy between space and time. Due to the anisotropy, instead of diffeomorphism, we have the so-called foliation-preserving diffeomorphism. The transformation is now just

\[ t \to \tilde{t}(t), \quad x^i \to \tilde{x}^i(x^j, t). \]

(2)

As a result, there is one more dynamical degree of freedom in Hořava-Lifshitz-like gravity than in the usual general relativity. Such a degree of freedom could play important role in UV physics, especially in early cosmology [5, 6]. At IR, due to the emergence of new gauge symmetry, this degree of freedom is not dynamical any more such that the kinetic part of the theory recovers the one of the general relativity.

Since time direction plays a privileged role in the whole construction, it is more convenient to work with ADM metric

\[ ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]

(3)

in which \( N \) and \( N_i \) are called “lapse” and “shift” variables respectively.

Taking Hořava-Lifshitz gravity as a new gravitational theory, it is an important issue to study its black hole solutions. In papers [8–15], it was assumed that the metric of the black solutions had the following Schwarzschild coordinates form

\[ ds^2 = -N(r)^2 dt_S^2 + \left( \frac{dr}{g(r)} \right)^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \]

(4)

From this metric ansatz, it was found that there were new black hole solutions, even at IR. For example, in [10], based on a modified Hořava-Lifshitz-type action, an asymptotically flat solution with

\[ g = N^2 = 1 + \omega r^2 - \sqrt{r(\omega^2 r^3 + 4\omega M)} \]

(5)

was found. And paper [14] find a new static charged black hole solution in the IR region, paper [15] had studied the extremal rotating no-charged black hole solutions. However, in the above ansatz (4) the “lapse function” \( N(r) \) obviously breaks the “projectability condition” which means that \( N \) only is the function of \( t \). This is introduced in [1, 2]. For the metric of the form (4), we can work in the Painlevé-Gullstrand coordinates by making a transformation

\[ dt_S = dt_{PG} - \frac{\sqrt{1 - N^2}}{N^2} dr. \]

(6)

Then the ansatz (4) becomes

\[ ds^2 = -dt_{PG}^2 + (dr + \sqrt{1 - N^2 dt_{PG}})^2 + \left( \frac{1}{g} - \frac{1}{N^2} \right)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \]

(7)
Comparing with the ADM metric, we find that \( N(t_{PG}) = 1 \) which is in accord with the projectability condition and if \( g = N^2 \) we reach (3). For instance, a Reissner-Nordstrom black hole after the transform (6) has the form

\[
ds^2 = -dt_{PG}^2 + \left( dr \pm \sqrt{\frac{2GM}{r} - \frac{Q^2}{r^3}dt_{PG}} \right)^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right). \tag{8}
\]

So paper [25] search non-charge black hole solutions to modified Hořava-Lifshitz gravity which was introduced by [17] with the metric ansatz as

\[
ds^2 = -N^2 dt^2 + \frac{1}{f(r)}(dr + N'r dt)(dr + N'r dt) + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \tag{9}
\]

where \( N \) only is a function of \( t \). Paper [25] found maximally symmetric space solution with curvature \( \Lambda_W \) and in the IR region, and (A)dS-Schwarzschild black hole solution in Painlevé-Gullstrand type coordinates was found. In the UV region, it found a de-Sitter space-time solution.

Then papers [26, 27] search black hole solutions with the same metric ansatz as (9), but \( N \) only is the function of \( r \). They seem get some new black hole solutions. Actually the solutions of paper [27] in the IR region is very similar as paper [12]. Furthermore, paper [28] had studied the plane symmetric solutions in Hořava-Lifshitz theory.

In the present work, we search static charged black hole solutions to Hořava-Lifshitz gravity coupling with electromagnetic field. We work on the same metric ansatz as (9). Whether the metric ansatz respects the projectability condition is determined on whether \( N \) only is the function of \( r \). With the projectability condition, we find (A)dS-Reissner-Nordstrom black hole solution in Painlevé-Gullstrand type coordinates in the IR region and a de-Sitter space-time solution in the UV region. Without the projectability condition, in the IR region, we find a static charged black hole solution which is similar as the results of [14]. In the UV region, we find the same solution as [8].

II. THE HOŘAVA-LIFSHITZ GRAVITY

In this section, we give a brief review of Hořava-Lifshitz gravity. Using the ADM formalism, the action of this Hořava-Lifshitz gravitational theory is given by [1, 2]

\[
S_{HL} = \int dt d^3x (L_K + L_V),
\]

\[
L_K = \sqrt{\hbar N} \left\{ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) \right\},
\]

\[
L_V = \sqrt{\hbar N} \left\{ \kappa^2 \mu^2 \left( \Lambda_W R - 3 \Lambda_W^2 \right) \right\} \frac{6(1 - 3\lambda)}{32(1 - 3\lambda)} R^2
\]

\[
- \frac{\kappa^2}{2\omega^4} \left( C_{ij} - \frac{\mu \omega^2}{2} R_{ij} \right) \left( C_{ij} - \frac{\mu \omega^2}{2} R_{ij} \right), \tag{10}
\]

where \( L_K \) is the kinetic term and \( L_V \) is the potential term. If we consider the term which represents a “soft” violation of the “detailed balance” condition in [2] and add the term in the action. The
The kinetic term $L_K$ is the same, and the potential term $L_V$ becomes

$$L_V = \sqrt{\hbar N} \left\{ \frac{k^2 \mu^2 (\Lambda_W R - 3 \Lambda_W^2)}{8(1 - 3 \lambda)} + \frac{k^2 \mu^2 (1 - 4.1)}{32(1 - 4 \lambda)} R^2 \right. $$

$$- \frac{k^2}{2 \omega^4} \left[ C_{ij} - \frac{\mu \omega^2}{2} R_{ij} \right] \left[ C_{ij} - \frac{\mu \omega^2}{2} R_{ij} \right] \right\}. \quad (11)$$

The last term has been introduced in [2, 9, 10]. In the action, $\lambda, \kappa, \mu, \omega, \Lambda_W,$ and $\Omega$ are the coupling parameters, and $C_{ij}$ is the Cotton tensor defined by

$$C_{ij} = \epsilon^{ikl} \nabla_k \left( R_{jl} - \frac{1}{4} R \delta_{ij} \right). \quad (12)$$

The study of the perturbations around the Minkowski vacuum shows that there is ghost excitation when $\frac{1}{3} < \lambda < 1$. This indicates that the theory is only well-defined in the region $\lambda \leq \frac{1}{3}$ and $\lambda \geq 1$. Since the theory should be RG flow to IR with $\lambda = 1$, we expect that at UV, $\lambda > 1$ to have a well-defined RG flow. At IR, $\lambda = 1$, the kinetic term recovers the one of standard general relativity. Comparing (10) to the action of the general relativity in the ADM formalism, the speed of light, the Newton’s constant and the cosmological constant emerge as

$$c = \frac{k^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3 \lambda}}, \quad G = \frac{k^2}{32 \pi c}, \quad \Lambda = \frac{3}{2} \Lambda_W. \quad (13)$$

It follows from (13) that for $\lambda > 1/3$, the cosmological constant $\Lambda_W$ has to be negative. It was noticed in [8] that if we make an analytic continuation of the parameters

$$\mu \rightarrow i \mu, \quad \omega^2 \rightarrow -i \omega^2, \quad (14)$$

the four-dimensional action (10) remains real as

$$L_K = \sqrt{\hbar N} \left\{ \frac{2}{k^2} (K_{ij} K^{ij} - \lambda K^2) \right\}, $$

$$L_V = \sqrt{\hbar N} \left\{ \frac{k^2 \mu^2 (\Lambda_W R - 3 \Lambda_W^2)}{8(3 \lambda - 1)} + \frac{k^2 \mu^2 (1 - 4 \lambda)}{32(3 \lambda - 1)} R^2 \right. $$

$$+ \frac{k^2}{2 \omega^4} \left[ C_{ij} - \frac{\mu \omega^2}{2} R_{ij} \right] \left[ C_{ij} - \frac{\mu \omega^2}{2} R_{ij} \right] \right\}. \quad (15)$$

In this case, the emergent speed of light becomes

$$c = \frac{k^2 \mu}{4} \sqrt{\frac{\Lambda_W}{3 \lambda - 1}}. \quad (16)$$

The requirement that this speed be real implies that $\Lambda_W$ must be positive for $\lambda > \frac{1}{3}$.

### III. ELECTROMAGNETIC FIELD IN HOŘAVA-LIFSHITZ STATIC CURVED SPACETIME

The electrodynamics in curved spacetime has been discussed many times in the old days. Ellis(1973) first wrote down the Maxwell’s equations in 3 + 1 congruence language. Later the
Maxwell’s equations in $3+1$ form were fully discussed by Thorne, Price, and Macdonald (1986).

The Maxwell’s equations in a curved spacetime could be written as

\begin{align}
\nabla \nu F_{\mu \nu} &= 0, \\
\partial_\mu F_{\nu \rho} + \partial_\nu F_{\rho \mu} + \partial_\rho F_{\mu \nu} &= 0,
\end{align}

where $F_{\mu \nu}$ is the antisymmetric electromagnetic tensor and $F^\mu{}_{\nu} = F_{\alpha \beta} g^{\alpha \mu} g^{\beta \nu}$. The electric field $E_i$ and magnetic field $H_i$ are related to $F_{\mu \nu}$ as

\begin{align}
E_i &= F_{i \mu}, \quad H_i = -\frac{\epsilon_{ijk}}{2} \sqrt{-g} F^{jk},
\end{align}

where $E_i$ and $H_i$ are spatial vectors. The electromagnetic action in the curved spacetime could be written in $3+1$ form

\begin{align}
S_{em} &= \int dt d^3x \sqrt{h} N g_{em} \left[ F_{\mu \nu} F^{\mu \nu} + F_{\mu \rho} F^{\mu \rho} + F_{\mu \nu} F^{\mu \nu} \right],
\end{align}

where $g_{em}$ is a constant. But the action at the Lifshitz point may be very different. This has been discussed by Hořava [23], Chen and Huang [24]. In the paper [24], Chen and Huang showed the action of Yang-Mills gauge field at the Lifshitz point in the flat spacetime as

\begin{align}
S_{YM} &= \frac{1}{2} \int dt d^d x \left[ \frac{1}{g_E^2} Tr(E_i E_i) - \sum_{J \geq 2} O_J \star F^J \right],
\end{align}

where

\begin{align}
O_J &= \frac{1}{g_E^2} \sum_{n=0}^{n_J} (-1)^n \frac{\lambda_{J,n}}{M^{2n+\frac{d}{2}}} \frac{1}{J-d-1} D^{2n}.
\end{align}

Here $F$ and $D$ are the abbreviated denotation for $F_{ij}$ and $D_k$ respectively, and $\lambda_{J,n}$ are the the coupling with zero energy dimension. Similarly $D^{2n} \star F^J$ also contains all possible independent combinations of $D_k$ and $F_{ij}$. The action of Yang-Mills theory at Lifshitz point in curved spacetime should be similar with action show above. To the static charged black hole, $F_{tr}$ component of $F_{\mu \nu}$ is not zero and it must be functions of $r$. If there are independent magnetic charges in the world and the static black hole absorbs some magnetic charges, the magnetic field component $H_r$ should not be zero. From (19), $F^{\theta \phi}$ should not be zero. So the action of electromagnetic field to static charged black hole will be reduced to

\begin{align}
S_{em} &= \int dt d^3 x L_{em} = \int dt d^3 x \sqrt{h} N 2 g_{em} \left[ F_{tr} F^{tr} + F_{\theta \phi} F^{\theta \phi} \right].
\end{align}

From the equations (17) and (18) we get four independent equations

\begin{align}
\partial_r \left( \sqrt{h} NF^{tr} \right) &= 0, \\
\partial_\theta \left( \sqrt{h} NF^{\theta \phi} \right) &= 0, \quad \partial_\phi \left( \sqrt{h} NF^{\theta \phi} \right) = 0, \\
\partial_r F_{\theta \phi} + \partial_\theta F_{r \phi} + \partial_\phi F_{r \theta} &= 0.
\end{align}
IV. STATIC CHARGED BLACK HOLE SOLUTIONS

As the discussion in the first paragraph, we now seek the static charged black hole solutions with the metric ansatz

\[ ds^2 = -N(t, r)^2 dt^2 + \frac{1}{f(r)} (dr + N' dt)(dr + N' dt) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \] (27)

With this metric ansatz, from (24), \( F^{tr} \) should satisfy the equation

\[ \partial_r \left( \frac{N}{\sqrt{f}} r^2 F^{tr} \right) = 0. \] (28)

The solution of this equation is

\[ F^{tr} = \frac{Q_e \sqrt{f}}{N r^2}, \] (29)

where \( Q_e \) is an integration constant. From (25) and (26), \( F^{\theta\phi} \) and \( F_{\theta\phi} \) should satisfy

\[ \partial_\theta \left( \frac{N}{\sqrt{f}} r^2 \sin \theta F^{\theta\phi} \right) = 0, \quad \partial_\phi \left( \frac{N}{\sqrt{f}} r^2 \sin \theta F^{\theta\phi} \right) = 0, \quad \partial_r F_{\theta\phi} = 0. \] (30)

The solution of the three equations are

\[ F^{\theta\phi} = \frac{Q_m}{r^4 \sin \theta}, \quad F_{\theta\phi} = Q_m \sin \theta, \] (31)

where \( Q_m \) is an integration constant. So from the action (23), the Lagrangian of electromagnetic field is

\[ \mathcal{L}_{em} = -2 g_{em} \frac{N}{\sqrt{f}} \frac{Q_e^2 - Q_m^2}{r^2}. \] (32)

The whole action of electromagnetic field couples with Hořava-Lifshitz gravity is \( S = S_{HL} + S_{em} \). Substituting the metric ansatz (27) into the Lagrangians (10) and (23), up to an overall scaling constant, we get

\[ \mathcal{L}_K = \frac{1}{N \sqrt{f}} \left\{ (1 - \lambda) r^2 f^2 \left( N' + N \frac{f'}{2f} \right)^2 \right\} + 2(1 - 2\lambda) f^2 N_r^2 \]

\[ - 4\lambda r^2 f^2 N_r \left( N' + N \frac{f'}{2f} \right), \] (33)

\[ \mathcal{L}_V = \frac{N}{\sqrt{f}} \left\{ 2 - 3 \Lambda_W r^2 - 2 f - 2 r f' + \frac{\lambda - 1}{2 \Lambda_W} f'^2 \right\} 

\[ - \frac{2(2\lambda - 1)(f - 1)^2}{\Lambda_W r^2} \right\}, \] (34)

\[ \mathcal{L}_{em} = -2 g_{em} \frac{N}{\sqrt{f}} \frac{Q_e^2 - Q_m^2}{r^2}. \] (35)

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Their solution is \( c = 1 \) and \( g_{em} = k^2 g_{em}/2 \). The full Lagrangian is \( L = L_K + L_V + L_{em} \). By varying the action with respect to the functions \( N_r, f \) and \( N(t) \), we obtain three equations of motions,

\[
0 = 2(1 - \lambda) r^2 f^2 \frac{1}{N} \left\{ N'' + \frac{f''}{2f} N_r + \frac{3f'}{2f} N'_r + 2 N_r \frac{1}{r} + \frac{2 \lambda f'}{f} N_r - 2 N_r \frac{r}{r^2} \right\} - \frac{N'}{N} \left( N_r + N_r \frac{f'}{2f} \frac{2 \lambda N_r}{1 - \lambda r} \right),
\]

\( (36) \)

For the projectability condition, lapse function \( N \) only is a function of \( t \), \( N' = 0 \) and we can set \( N(t) = 1 \) with the time rescaling. Obviously for all \( \lambda, N_r = 0 \) is a solution of \( (36) \). In this case, we assume ansatz \( f(r) = 1 + yr^2 \), just when \( Q = 0 \), from \( (37), (38) \), we get quadratic equations of \( y \),

\[
y^2 - 2\Lambda Wy - 3\Lambda_w^2 = 0, \quad y^2 + 2\Lambda Wy + \Lambda_w^2 = 0.
\]

\( (39), (40) \)

Their solution is \( y = -\Lambda_w \). This solutions corresponds to a maximally symmetric space with curvature \( \Lambda_w \). This result has been shown in paper \( [25] \).

In the IR region where \( \lambda = 1 \), the equation \( (36) \) is reduced to

\[
\frac{f'}{f} N_r = 0.
\]

\( (41) \)

Its \( N_r = 0 \) solution has been discussed above. It has another solution as \( f = constant \). when \( f \) is a constant, we could get the same equation from \( (37), (38) \) as

\[
(N_r')^2 + \frac{N_r^2}{r} + \left\{ -3\Lambda_wr + \frac{2(1 - f)}{r} + \frac{(1 - f)^2}{\Lambda_wr^3} - 2 \frac{Q_e^2 - Q_m^2}{r^2} \right\} = 0.
\]

\( (42) \)

The solutions of this equation are

\[
N_r = \pm \frac{1}{f} \sqrt{\frac{M}{r} + \frac{\Lambda_w}{2} r^2 + (f - 1) + \frac{(1 - f)^2}{2\Lambda_wr^2} - \frac{Q_e^2 - Q_m^2}{r^2}}.
\]

\( (43) \)
Especially when $f = 1$, the solution reduces to

$$N_r = \pm \sqrt{\frac{M}{r} + \frac{\Lambda_W r^2 - \tilde{g}_{em} Q_e^2 - Q_m^2}{r^2}}. \quad (44)$$

It is a (A)dS-Reissner-Nordstrom black hole written in Painlevé-Gullstrand type coordinates. Especially when $Q_e = Q_m = 0$, it is (A)dS-Schwarzschild black hole which has been shown in [25].

The electric field of the black hole (44) is

$$E_r = F_{tr} = \left(\frac{-Q_e}{r^2}\right).$$

Its charge is

$$Q_{BH} = \frac{1}{4\pi} \int_S \vec{E} \cdot d\vec{\sigma} = \frac{1}{4\pi} E_r \cdot 4\pi r^2 = -Q_e, \quad (45)$$

where $S$ is a closed surface everywhere with the same $r$ and $d\vec{\sigma}$ is surface integral element. We should choose $\tilde{g}_{em} = 1$ when the solution (44) is a charged black hole with charge $\pm Q_e$.

In the UV region where $\lambda \neq 1$, when $f$ is a constant, from (36), (37) and (38), we get three equations,

$$0 = N_r'' + 2 \frac{N_r'}{r} - 2 \frac{N_r}{r^2}, \quad (46)$$

$$0 = (1 - \lambda) r^2 N_r'^2 - 4 \lambda r N_r N_r' + 2(1 - 2\lambda) N_r^2 - \frac{4(1 - \lambda)(f - 1)}{\Lambda_W r^2 f} - \frac{4\tilde{g}_{em} Q_e^2 - Q_m^2}{f^2 r^2} + 2 \frac{2(1 - f) - 3\Lambda_W r^2}{\Lambda_2 r^2} + \frac{(2\lambda - 1)(f - 1)^2}{\Lambda_2 r^2}. \quad (47)$$

$$0 = (1 - \lambda) r^2 N_r'^2 - 4 \lambda r N_r N_r' + 2(1 - 2\lambda) N_r^2 - \frac{4\tilde{g}_{em} Q_e^2 - Q_m^2}{f^2 r^2} + 2 \frac{2(1 - f) - 3\Lambda_W r^2}{\Lambda_2 r^2} + \frac{(2\lambda - 1)(f - 1)^2}{\Lambda_2 r^2}. \quad (48)$$

Just when $f = 1$ and $Q_e^2 - Q_m^2 = 0$, they have a solution as

$$N_r = \pm \sqrt{\frac{\Lambda_W}{3\lambda - 1}}r. \quad (49)$$

This solution actually describes the same de-Sitter space-time. One easy way to see this point is to change inversely into the Schwarzschild coordinates. This result is the same as paper [25].

**B. Solutions without Projectability Condition**

Without projectability condition, $N$ is function of $r$, $N'$ isn’t zero. In the IR region where $\lambda = 1$, the equation (36) is reduced to

$$\left(\frac{2N'}{N} - \frac{f'}{f}\right) \frac{N_r}{r} = 0. \quad (50)$$

Its solutions are $N_r = 0$ or $N^2 = f$. In the case $N_r = 0$, with the same discussion above, we find a solution

$$f = 1 - \Lambda_W r^2, \quad (51)$$

and the function $N(r)$ is unconstrained. This result is the same as paper [8].
When $\lambda = 1$, in the case $N^2 = f$, from (37), (38), we get one same equation,

$$0 = 2\left[rfN_r^2\right] + \left\{2(1 - f) - 3\Lambda_wr^2 - 2rf' + \frac{2(1 - f)}{\Lambda_wr^2} f' + \frac{(f - 1)^2}{\Lambda_wr^2}\right\} - 2\tilde{g}_{em}\frac{Q_e^2 - Q_m^2}{r^2}. \quad (52)$$

This equation has a solution as

$$N_r^2 = \frac{\beta}{rf}, \quad f = 1 - \Lambda_wr^2 - \sqrt{cr + 2\tilde{g}_{em}\Lambda_w (Q_e^2 - Q_m^2)}, \quad (53)$$

Where $\beta, c$ are integration constants. If we reconsider the action (11) which contains the “soft” violation term, we could get a equation similar as (52) as

$$0 = 2\left[rfN_r^2\right] + \left\{2\Omega - \frac{\Omega - \Lambda_w}{\Lambda_w}(1 - f - rf') - 3\Lambda_wr^2 + \frac{2(1 - f)}{\Lambda_wr^2} f' + \frac{(f - 1)^2}{\Lambda_wr^2}\right\} - 2\tilde{g}_{em}\frac{Q_e^2 - Q_m^2}{r^2} \quad (54)$$

Its solution is

$$N_r^2 = \frac{\beta}{rf}, \quad f = 1 + \left(\Omega - \Lambda_w\right)r^2 - \sqrt{\Omega(\Omega - 2\Lambda_w)r^4 + cr + 2\tilde{g}_{em}\Lambda_w (Q_e^2 - Q_m^2)}, \quad (55)$$

Where $\beta, c$ are integration constants. When $Q_m = 0$, this solution is similar as paper [14] and When $Q_e^2 - Q_m^2 = 0$ it is the similar as [12]. The solutions (53), (55) are two especial static charged black hole solutions.

In the UV region where $\lambda \neq 1$, when $N_r = 0$, from (37), (38), we get two equations,

$$0 = 2 - 3\Lambda_wr^2 - 2f - 2rf' + \frac{\lambda - 1}{2\Lambda_w} f'' - \frac{2\lambda(f - 1)}{\Lambda_wr^2} f' + \frac{(2\lambda - 1)(f - 1)}{\Lambda_wr^2} - 2\tilde{g}_{em}\frac{Q_e^2 - Q_m^2}{r^2} \quad (56)$$

$$0 = \left(N' - \frac{f'}{2f}\right)\left\{-2r + \frac{\lambda - 1}{\Lambda_w} f' - \frac{2\lambda(f - 1)}{\Lambda_wr^2}\right\} + N\left\{\frac{\lambda - 1}{\Lambda_w} f'' + \frac{2(1 - \lambda)(f - 1)}{\Lambda_wr^2}\right\} \quad (57)$$

They have two new solutions as

$$Q_e^2 - Q_m^2 = 0, \quad f = 1 - \Lambda_wr^2 - ar\frac{2\lambda \sqrt{1-w^2}}{\lambda + 1}, \quad N = ar\frac{1 + \lambda w}{\lambda + 1} \sqrt{f}, \quad (58)$$

where $\alpha, \alpha$ are constants. The solutions has been got by paper [8].

Acknowledgments

The work was partially supported by NSFC Grant No.10535060, 10775002, 10975005 and RFDP. I would like to thank Bin Chen for drawing my attention to Horava-Lifshitz gravity and giving me many useful suggestions.

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