Is torsion needed in a theory of gravity? A reappraisal

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In Memory of Professor Roman S. Ingarden who was died July 12, 2011.

Abstract

It is known that General Relativity (GR) uses a Lorentzian Manifold \((M_4; g)\) as a geometrical model of the physical spacetime. The metric \(g\) is required to satisfy Einstein’s equations. Since the 1960s many authors have tried to generalize this geometrical model of the physical space–time by introducing torsion. In this paper we discuss the present status of torsion in a theory of gravity. Our conclusion is that the general–relativistic model of the physical spacetime is sufficient for the all physical applications and it seems to be the most satisfactory.

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1 Introduction

In past we were enthusiast of torsion, mainly under influence of excellent papers given by F.W. Hehl and A. Trautman. But studying Poincare’ field theories of gravity (PGT) one can easily see that torsion leads to serious complications, especially calculational.

About twenty years ago we have observed that the our idea of the superenergy and supermomentum tensors (very effective in general relativity) fails in a spacetime having torsion. So, our interest to torsion has diminished.

In the meantime we have read many papers by C.M. Will, G. Esposito-Farese, T. Damour, S. Kopeikin, S.G. Thuryshchev and others devoted recent experiments on gravity. As we understood all these experiments confirmed standard general relativity (GR) with a very high precision and excluded torsion, at least in vacuum.

Besides, during the last three decades there was given many interesting papers on universality of the GR equations. So, in consequence, we have decided to analyze status of torsion in gravitational physics. From this analysis the review has originated.

Of course, we do not prove that torsion is not admissible at all. Rather, we only give short information about recent gravitational experiments and collect problems which arise when one introduces torsion as a part of the geometrical structure of the physical spacetime. But, as you see, we will finish review with the conclusion (based mainly on Ockham razor):

1. Torsion in needn’t in a theory of gravity:

2. The Levi-Civita connection is sufficient for the all physical applications. This the most simple connection is exactly just what we need.

The paper is organized as follows. In Section II we remind a general definition of torsion and in Section III we consider motivations to introduce torsion into geometrical model of the physical spacetime. We will see that these motivations are not convincing. In Section IV we very shortly discuss experimental evidence for torsion and Section V we present arguments against torsion in a theory of gravity. We will conclude in Section VI (from the facts given in the two previous Sections) that torsion rather should not be introduced into a geometrical model of the physical spacetime.

2 Torsion of a linear connection $\omega^i_k$ on $L(M)$

We confine to the metric-compatible connection which satisfies $Dg_{ik} = dg_{ik} - \omega^p_i g_{pk} - \omega^p_k g_{ip} = 0$ because we do not see any reasons to consider more general connection. Here, and in the following, $D$ means exterior covariant derivative and $d$ is the ordinary exterior derivative.
One can give the following, general definition of torsion \( \Theta^i \) of a linear connection

\[
\Theta^i := D\theta^i = d\theta^i + \omega^i_k \wedge \theta^k := \frac{1}{2} Q^i_{kl} \theta^k \wedge \theta^l.
\]  

(1)

Here \( \theta^i \) are canonical 1-forms (or soldering 1-forms) on the principal bundle of the linear frames \( L[M, GL(n; R), \pi] \) (\( L(M) \) in short) over a manifold \( M \), and \( Q^i_{kl} \) denote components of the torsion tensor.

After pulling back by local section \( \sigma : U \rightarrow L(M); \ U \subset M \), one gets on the base \( M \)

\[
\bar{\Theta}^i = d\vartheta^i + \bar{\omega}^i_k \wedge \vartheta^k = \frac{1}{2} \bar{Q}^i_{kl} \vartheta^k \wedge \vartheta^l.
\]  

(2)

\( \bar{\omega}^i_k \) are pull-back of \( \omega^i_k \) and \( \vartheta^i \) are pull-back of \( \theta^i \). \( \vartheta^i := \vartheta^0, \vartheta^1, \vartheta^2, \vartheta^3 \) form a Lorentzian coframe on \( M \).

In a coordinate (= holonomic) frame \( \{ \partial_i \} \) and dual coframe \( \{ dx^i \} \) on \( M \) one has

\[
\bar{\omega}^i_k = \Gamma^i_{lk} dx^l, \quad \text{and} \quad \bar{Q}^i_{kl} = \Gamma^i_{kl} - \Gamma^i_{lk}.
\]

### 3 Motivation to introduce of torsion into gravity

In the 1960s–1970s some researchers introduced torsion into the theory of gravity\(^2\)[11][2][3][4][5][6]. The main motives (only theoretical) were the following:

1. Studies on geometric theory of dislocations (Theory of a generalized Cosserat continuum) led, following Günter, Hehl, Kondo, and Kröner, to heuristic arguments for a metric spacetime with torsion, i.e., to Riemann–Cartan spacetime.

2. Investigations of spinning matter in GR resulted in conclusion that the canonical energy–momentum tensor of matter \( cT^i_k \) can be source of curvature and the canonical intrinsic spin density tensor \( cS^{ikl} = (-)cS^{kli} \) can be source of torsion of the underlying spacetime. From this Einstein–Cartan–Sciama–Kibble (ECSK) theory and its generalizations originated.

3. Attempts to formulate gravity as a gauge theory for Lorentz group \( L \) or for Poincare’ group \( P \) by using Palatini’s formalism led to a space–time endowed with a metric–compatible connection which might have (but not necessarily) non–vanishing torsion, i.e., again one was led to Riemann–Cartan space–time\(^3\)[7][8][9][10][11].

Some remarks are in order concerning 3.

1. If we admit a metric–compatible connection with torsion when “gauging” groups \( L \) or \( P \) by using Palatini’s approach and Ehreshmann theory of connection, then

\(^2\)We omit here older attempts to introduce torsion because they have only historical meaning.

\(^3\)
we will end up with strange situation, different then in ordinary gauge fields: we get a “gauge theory” which has two gauge potentials

\[ \vartheta^i \quad = \quad \text{translational \ (= \ pseudoorthonormal coframe)}, \]

\[ \omega^i_k \quad = \quad \text{rotational \ (= \ metric \ – \ compatible linear connection)}, \]

and two gauge strengths

\[ \Theta^i = D\vartheta^i \quad = \quad \text{translational \ (torsion)}, \]

\[ \Omega^i_k = D\omega^i_k \quad = \quad \text{rotational \ (curvature)}. \]

Notice that \( \vartheta^i \) do not transform like gauge potentials and contribute to \( \omega^i_k = L_C \omega^i_k + K^i_k \); besides, the gauge strengths \( \Theta^i = K^i_k \wedge \vartheta^k \) contribute to \( \omega^i_k = L_C \omega^i_k + K^i_k \) and also to \( \Omega^i_k = d\omega^i_k + \omega^i_p \wedge \Omega^p_k \).

Here \( L_C \omega^i_k \) denotes the Levi-Civita Connection and \( K^i_k \) is the contortion.

So, the gauge potentials and gauge strengths are not independent in the case. This is not satisfactory and suggests other approach to “gauging” gravity.

2. Besides, the action integrals in these trials to gauge gravity didn’t have forms like an action integral for a gauge field, \( \int tr(F \wedge \star F) \), and led to very complicated field equations of 3rd order, different from GR equations. These field equations contain many arbitrary parameters (10 apart from \( \Lambda \) in the case of the so-called Poincare’ Gravity Theories, PGT). Here \( \star \) means Hodge duality operator.

There exist many serious problems connected with these field equations: tachyons, ghosts, instability of their solutions, ill–posedness Cauchy problem, etc., (see, e.g., [71]).

We would like to emphasize that there exists an old approach to “gauge” gravity proposed by Yang [69] which has action typical for a gauge field: \( \int \Omega^i_k \wedge \star \Omega^k_i \).

But, unfortunately, this approach leads to incorrect theory of gravity.

The above theoretical motives are not convincing. For example, the often used argument for torsion (following from study of spinning matter in GR) based on the (non-homogeneous) holonomy theorem \[ [10] [12] \] holds only if one uses Cartan displacement which displaces vectors and contactpoints \[ [12] \]. Ordinary parallel displacement (which displaces only vectors) gives only Lorentz rotations (= homogeneous holonomy group) even in a Riemann-Cartan space-time \[ [12] \]. Moreover, there are other geometrical interpretations of torsion, e.g., Bompiani \[ [13] \] connects torsion with rotations in tangent spaces, not with translations.

\[ ^3 \text{This theorem says that torsion gives translations, and curvature gives Lorentz rotations in tangent spaces of a Riemann-Cartan manifold induced by (Cartan) displacement along loops.} \]
We also needn’t to generalize GR in order to get a gauge theory with \( L \) or \( P \) as a gauge group [14, 48, 70]. The most convincing argument in this field is given by Cartan’s approach to connection and geometry [70].

Roughly speaking, in Cartan’s approach (for details, see [70]) one combines the linear Ehresmann connection form \( \omega \) and coframe field \( \theta \) into one connection \( A = \omega \oplus \theta \) valued in a larger Lie algebra \( g \) (In our case \( \omega \) is the Ehresmann connection on principal bundle of the orthonormal frames \( O[M, L, \pi] \) and \( \theta \) is the soldering form on this bundle. \( g \) is the algebra of the Poincare’ group \( P \) or de Sitter group).

In consequence, one has only one gauge potential \( A = \omega \oplus \theta \) and one gauge strength \( \hat{F} = \Omega - \frac{4}{3} \theta \wedge \theta \) (\( \Lambda \) is the cosmological constant) for gravity.

Using Cartan’s approach to connection one can write the ordinary GR action with \( \Lambda \), \( S_g = \int \sqrt{|g|} (R - \Lambda) d^4x \), in the form \( S_g = (-\frac{3}{20\Lambda}) \int tr(\hat{F} \wedge \star \hat{F}) \), i.e., exactly in the form of the action of a gauge field.

Thus, the Cartan’s (not Ehresmann) approach to connection and geometry suits to correct “gauging” of GR. The Ehresmann theory suits to ordinary gauge fields.

There exists also an other approach to GR as a gauge theory developed by A. Ashtekar, C. Rovelli, J. Lewandowski and coworkers (Ashtekar’s variables) [15, 16, 18, 74]. In this approach GR is also very akin to a Yang–Mills theory.

In resuming, one can say that we needn’t generalize or modify GR in order to obtain a gauge theory of gravity.

### 4 Experimental evidence for torsion

Up to now we have no experimental evidence for existence torsion in Nature (see, e.g., [75]). There exist only very stringent constraints on torsion components obtained in a speculative, purely theoretical, methods (see, eg., [75, 72]).

To the contrary, all gravitational experiments confirmed with a very high precision (\( \sim 10^{-14} \)) Einstein’s Equivalence Principle (EEP) and, with a smaller precision (up to 0,003\% in Solar System , i.e., in weak field, and up to 0,05\% in binary pulsars, i.e., in strong gravitational fields) the General Relativity (GR) equations [19, 21, 22, 23, 24, 25, 20]. Here by EEP we mean a formulation of this Principle given by C. W. Will [19]. In this formulation\(^4\) the EEP states:

1. The Weak Equivalence Principle (WEP) is valid. This means that the trajectory of a freely falling spherical test body (one not acted upon by non-gravitational forces and being too small to be affected by tidal forces) is independent of its internal structure and material composition.

2. Local Lorentz Invariance (LLI) is valid. This means that the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling and non-rotating reference frame in which it is performed.

\(^4\)This constructive formulation of the Principle can be experimentally tested.
3. Local Position Invariance (LPI) is valid. This means that the outcome of any local non-gravitational experiment performed in a freely-falling and non-rotating reference frame is independent of where and when in the Universe it is performed.

Following C.M. Will, the only theories of gravity that can embody EEP in the above constructive formulation of the Principle are those that satisfy the postulates of metric theories of gravity [19], which are:

1. Spacetime is endowed with a symmetric metric.
2. The trajectories of freely falling spherical test bodies are geodesics of that metric.
3. In local freely-falling and non-rotating reference frames the non-gravitational laws of physics are those written in the language of Special Relativity (SRT).

C.M. Will called the EEP “heart and soul of GR”.

The EEP implies a universal pure metric coupling between matter and gravity. It admits GR, of course, and, at most, some of the so-called scalar–tensor theories (these, which respect EEP) [19] [21] [22] without torsion.

So, torsion seems to be excluded in vacuum or at least very strongly constrained in vacuum by the latest gravitational experiments, i.e., at least propagating torsion is excluded or very strongly constrained by these experiments which have confirmed EEP with very high precision. As a consequence, at least freely propagating torsion still seems to be purely hypothetical.

We would like to emphasize that T. Damour already concluded in past [22]: “Einstein was right at least 99.9999999999% concerning EEP and 99.9% concerning Lagrangian and field equations”.

Thus, from the experimental point of view, up to now, torsion is needn’t in a theory of gravity.

5 Theoretical arguments against torsion

We begin this Section with the remark that if one utilizes the so-called “Ockham’s razor” then torsion is needn’t for him in a theory of gravity because the wonderful,

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5 Torsion is excluded or very strongly constrained at least in vacuum because if we neglect a cosmological background, then the all gravitational experiments were performed in vacuum. This means that ECSK theory can survive since this theory is identical in vacuum with GR. Of course, the same is true for other gravity theories which in vacuum reduce to GR. But the gravity theories of such a kind do not admit propagating free torsion.

6 By “Ockham’s razor” we mean a Philosophical Principle which states: “Entities are not to be multiplied without necessity”.

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the most simple and most symmetric Levi–Civita connection is sufficient for the all physical requirements.

The first our argument against torsion is given in the very important paper by J. Ehlers, F.A.E. Pirani, and A. Schild [73]. These authors have showed that requiring compatibility between conformal geometry $C$ defined by rays of light and the projective structure $P$ of spacetime determined by trajectories of freely-falling test particles leads to Weyl spacetime with a symmetric connection $\omega$. Then, admitting some, very natural axioms [73], we obtain Riemannian geometry.

So, studying the rays of light and freely-falling particles, leads us to Riemannian spacetime.

Now, let us pay our attention to the other, disadvantageous properties of torsion and metric-compatible spacetimes with torsion:

1. In a spacetime with torsion do not exist infinitesimal parallelograms [12, 29] because the operation of invariant geometric addition of infinitesimal coordinate segments is noncommutative. So, such spacetimes seem physically inadmissible as this result is in direct conflict with the operational and epistemological basis of our difference physics [30]. Besides, such spacetime cannot be approximated locally by a flat, Minkowskian spacetime already on classical level.

2. Torsion is topologically trivial. This means that the topological invariants of a real manifold $M$ and characteristic classes of vector bundles over $M$, as defined in [31, 32, 33] depend only on curvature and can be fully determined by the curvature $\text{LC}\Omega^i_k$ of the Levi–Civita connection. Roughly speaking, one can continuously deform any metric-compatible connection (or even general linear connection) into Levi-Civita connection without changing topological invariants and characteristic classes. So, torsion is not relevant for topological invariants and characteristic classes.

3. Torsion is not relevant from the dynamical point of view either. Namely, one can reformulate every metric theory of gravitation with a metric-compatible connection $\omega^i_k$ as a "Levi-Civita theory". Torsion is then treated as a matter field. Such reformulation preserves the all dynamical properties of the theory. An obvious example is given by ECSK theory in the so-called “combined formulation” [34, 35].

Some authors say that torsion which satisfies differential field equations might be topologically non-trivial. But this seems to be incorrect because one can still continuously deform the connection in the case into torsionless Levi-Civita connection without changing topological invariants and characteristic classes. The field equations will, of course, change during such deformation. So, it seems to us that one can say only that the torsion which satisfies differential field equations might be physically non-trivial. Of course, one cannot exclude that there exist other topological properties of spacetime which can substantially depend on torsion.

In this formulation ECSK theory is dynamically fully equivalent to the ordinary GR [35].
In general, one can prove [36] that any total Lagrangian of the type
\[ L_t = L_g(\vartheta^i, \omega^i_k) + L_m(\Psi, D\Psi) \] (7)
*admits an unique decomposition* into a pure geometric part \( \tilde{L}_g(\vartheta^i, \omega^i_k) \) containing no torsion plus a generalized matter Lagrangian \( \tilde{L}_m(\Psi, \nabla^\omega \Psi, K^i_k) \) which collects the pure matter terms and all the terms involving torsion
\[ L_t = L_g + L_m = \tilde{L}_g + \tilde{L}_m. \] (8)

Here \( \nabla \) means the exterior covariant derivative with respect to the Levi-Civita connection \( \omega^i_k \).

From the Lagrangian
\[ L_t = \tilde{L}_g + \tilde{L}_m \] (9)
there follow the *Levi-Civita equations associated with \( L_t \).*

So, torsion *can always be treated as a matter field.* This point of view is preferred e.g. in [37, 38] and it is supported by transformational properties of torsion: torsion transforms like a matter field i.e., it transforms as a tensor–valued form.

4. A gravitational theory with torsion *violates EEP*, which has so very good experimental evidence. It is because in a spacetime with torsion a tangent space \( T_p(M) \) *cannot be identified with Minkowskian spacetime*, i.e., there do not exist holonomic frames such that \( g_{ik}(P) = \eta_{ik}, \Gamma^i_{kl} = 0 \), and, in which geometry, in an infinitesimal vicinity of the point \( P \), is Minkowskian. \( P \) is here a preselected point. So, a gravitational theory with torsion *is not a covering theory for SRT* [54] and violates EEP (Strictly speaking, it violates LLI). A correct relativistic theory of gravity should be a covering theory for the both theories, SRT and Newton’s theory of gravity. Of course, GR satisfies this condition.

We also lose Fermi coordinates [12, 39, 40, 77] in a Riemann-Cartan space-time.\(^9\)

Some authors [41, 42, 44] formulate EEP in a weaker form than the constructive Will’s formulation, which we have adopted in this paper. Namely, in their formulation this Principle reads: there exists (anholonomic for a connection with torsion) *normal frame* \( \{ \vartheta^i \} \) such that in a preselected point \( P \) one has
\[ \Gamma^i_{kl}(P) = 0, \quad g_{ik}(P) = \eta_{ik}. \] (10)

But this *Equivalence Principle is a tautology* because, as it was showed in past [45], *every linear connection on a metric manifold* satisfies it.

Moreover, if the metric-compatible connection has torsion, then, the so-called *transposed connection* (see, e.g., [4]) \( \hat{\omega}^i_k(P) := \omega^i_k(P) + Q^i_{kl}(P)\vartheta^l \), torsion

\[^9\] Fermi coordinates realize in GR a local (freely-falling and non-rotating) inertial frame along a curve in which SRT is valid.
\( Q^i_{kl}(P) \) and the symmetric part \( \Gamma^i_{(kl)}(P) \) of the connection \( \omega^i_k = \Gamma^i_{lk} \vartheta^l \) do not vanish in \( P \) even, if in \( P, \omega^i_k(P) = \Gamma^i_{lk}(P) \vartheta^l = 0. \)

In consequence, even in a normal frame, the geometry of tangent space \( T_p(M) \) is not Minkowskian i.e., the constructive Will’s formulation of the EEP is violated.\(^\text{10}\)

The Equivalence Principle formulated in the form (10) needs holonomic frames in order to effectively work. Namely, in the set of the holonomic frames it chooses a symmetric, linear connection. Then, adding the most natural metricity postulate (or Hamiltonian Principle for trajectories of the test particles) univocally leads us to (pseudo)-Riemannian geometry i.e., to the Levi-Civita connection.

5. A connection having torsion can be determined neither by its own autoparallels (paths) nor by geodesics \(^\text{12}\). So, one cannot determine unequivocally a connection which has torsion by observation of the test particles (which could move along geodesics or autoparallels).

6. Study of the Einsteinian strength of the field equations of the proposed gravity theories favorize the purely metric theories of gravity (obtained with the help of Hilbert variational principle) which use Levi-Civita connection, \( \omega_{LC} \), in comparison with competitive Palatini’s theories of gravity (apart from ECSK theory) which use metric-compatible connection admitting torsion (see, e.g., \(^\text{47}\)).

Namely, the purely metric gravity theories have much more smaller strengths (48 in four dimensions) and numbers of dynamical degrees of freedom (16 in four dimensions) than the competitive Palatini’s PGT (120 and 40 in four dimensions respectively).

Following Einstein, from the two competitive gravity theories this one is better, which has smaller strength and smaller number dynamical degrees of freedom because such theory determines gravitational field more precisely. More precisely in the sense: it admits a smaller number of arbitrary initial data (putting in “by hand”) in the Cauchy problem, i.e., it admits smaller freedom in obtaining a solution to the field equations.

7. Reduction of the principal bundle of the linear frames \( L[M_n, \ GL(n; r), \ \pi] \) over \( M_n \) to subbundle of the (pseudo)orthonormal frames \( O[M_n, \ O(n; k) \ \pi] \) \(^\text{11}\) leads us univocally to the Levi-Civita connection. Namely, we have the Theorem \(^\text{76}\).

Theorem

Let \( [M_n, \ g] \) be a (pseudo)Riemannian manifold of an arbitrary signature, \( k, \).

Then, there exists one and only one linear connection \( \omega \) on \( L[M_n, \ GL(n; r), \ \pi] \)

\(^{10}\)As we have already emphasized, Will’s formulation of the EEP has very good experimental evidence.

\(^{11}\) For \( n = 4, \ k = 1 \) one has Lorentz group \( L \).
with null torsion $\Theta = D\theta = 0$ which can be reduced to the group $O(n; k)$, i.e., to the connection $\omega_R$ on the principal bundle $[M_n, O(n; k), \pi]$.

Interestingly, that $\omega$, and reduced connection $\omega_R$, are exactly the Levi-Civita connection $\omega_{LC}$ for the metric $g$.

So, the fibre bundle approach suggests choosing of the symmetric and metric Levi-Civita connection for the mathematical model $M_4(g, \Gamma)$ of the physical spacetime.

Torsion leads to ambiguities:

1. The Minimal Coupling Principle (MCP) differs from the Minimal Action Principle (MAP) in a spacetime with torsion [48].

The MCP can be formulated as follows. In SRT field equations obtained from the SRT Lagrangian density $L = L(\Psi, \partial_i \Psi)$ we replace $\partial_i \rightarrow \nabla_i$, $\eta_{ik} \rightarrow g_{ik}$ and get covariant field equations in $(M_4, g)$.

By the MAP we mean an application of the Minimal Action Principle (Hamiltonian Principle) to the covariant action integral $S = \int_{\Omega} L(\Psi, D\Psi) d^4\Omega$, where $L(\Psi, D\Psi)$ is a covariant Lagrangian density obtained from the SRT Lagrangian density $L(\Psi, \partial_i \Psi)$ by MCP.

It is natural to expect that the field equations in $(M_4, g)$ obtained by using MCP on SRT equations should coincide with the Euler-Lagrange equations obtained from $L(\Psi, D\Psi)$ by MAP. This holds in GR but not in the framework of the Riemann-Cartan geometry. So, we have there an ambiguity of the field equations [12].

2. In the framework of the ECSK theory of gravity we have four energy-momentum tensors for matter: Hilbert, canonical, combined, formal [34]. Which one is more important?

3. Let us consider now normal coordinates $NC(P)$ [12, 49, 50, 51] which are so very important in GR (see, eg., [49, 50, 51, 52]). In the framework of the Riemann-Cartan geometry we have two $NC(P)$: normal coordinates for the Levi-Civita part of the Riemann-Cartan connection $NC(LC\omega, P)$ and normal coordinates for the symmetric part of the full connection $NC(s\omega, P)$ [53]. Which one has a greater physical meaning?

The above ambiguity of the normal coordinates [13] leads us to ambiguities in superenergy and supermomentum tensors [53]. Moreover, the obtained expressions are too complicated for practical use. In fact, we lose here a possibility of

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12 Axial torsion removes this ambiguity. By $(M_4, g)$ we mean here a general metric manifold; not necessarily Riemannian.

13 Axial torsion removes this ambiguity.
effective use of the normal coordinates which give a very powerful tool in GR to extract physical content hidden in various non-covariant expressions.

Perhaps by use normal frames defined in [45, 78] instead of normal coordinates one could avoid these ambiguities and connected problems. This conjecture will be studied in future.

4. In the framework of Riemann-Cartan geometry [12] there holds

\[ R_{(ik)lm} = R_{ik(lm)} = 0, \]  
\[ \text{(11)} \]

but

\[ R_{iklm} \neq R_{lmik}. \]  
\[ \text{(12)} \]

The last asymmetry leads to an ambiguity in construction of the so-called “Maxwellian superenergy tensor” for the field \( R_{iklm} \) [53]. This tensor is uniquely constructed in GR owing to the symmetry \( R_{iklm} = R_{lmik} \) and it is proportional to the Bel-Robinson tensor [53]. In the framework of the Riemann-Cartan geometry the obtained result depends on which antisymmetric pair of the \( R_{iklm} \), the first or second, is used in the construction.

5. In a Riemann-Cartan spacetime we have geodesics and autoparallels (paths). Hamiltonian Principle demands geodesics as trajectories for the test particles [54]. Then, what about the physical meaning of the autoparallels?  

6. In a spacetime with torsion we have in fact three kinds of parallel displacement defined by

\[ dv^k = (-)\Gamma^k_{ij} v^j dx^i, \]  
\[ \text{(13)} \]

\[ dv^k = (-)\Gamma^k_{ij} v^i dx^j, \]  
\[ \text{(14)} \]

and

\[ dv^k = (-)\Gamma^k_{(ij)} v^i dx^j, \]  
\[ \text{(15)} \]

and three different curvatures. These results follow from that three kinds of covariant (and absolute) differentials

\[ \nabla^L_i v^k = \partial_i v^k + \Gamma^k_{il} v^l, \]  
\[ \text{(16)} \]

\[ \nabla^R_i v^k = \partial_i v^k + \Gamma^k_{li} v^l, \]  
\[ \text{(17)} \]

\[ \nabla^s_i v^k = \partial_i v^k + \Gamma^k_{(li)} v^l. \]  
\[ \text{(18)} \]

Authors usually use only one of the two first possibilities. What about the others?

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\[ 14 \text{Axial torsion removes this problem. One can also easily prove in the framework of the ECSK theory that spinless test particles move along geodesics.} \]
In a torsionless spacetime the above three possibilities coincide.

The ambiguities (13), (14)—(16), (17) arise from the two possibilities expanding of the local connection forms $\tilde{\omega}^i_k$ on the base space $M_n$ in coordinate frames:

$$\tilde{\omega}^i_k = \Gamma^i_{kl} dx^l,$$

$$\tilde{\omega}^i_k = \Gamma^i_{lk} dx^l.$$  \hfill (19)

In practice, one must consequently use one of the two above possibilities (or conventions) in order to avoid mistakes.

5.1 Symmetry of the energy–momentum tensor of matter.

In Special Relativity (SRT) the correct energy–momentum tensor for matter (electromagnetic field, continuous medium, dust, elastic body, solids) must be symmetric \[39, 55\].

One can always get such a tensor starting from the canonical pair $cT^i_k, cS^{ikl} = (-) cS^{kli}$, where $cT^i_k \neq c T^i_k$ is the canonical energy–momentum tensor and $cS^{ikl}$ — the canonical spin tensor. These two canonical tensors are connected by the equations

$$\partial_k cT^i_k = 0, \quad cT^i_k - c T^i_k = \partial_l cS^{ikl}.$$  \hfill (20)

By use of the Belinfante symmetrization procedure \[34, 48, 56, 57\] one can get the most simple new pair

$$sT^i_k = c T^i_k - \frac{1}{2} \partial_j (c S^{ijk} - c S^{ikj} + c S^{jki}),$$  \hfill (21)

$$S^{ijk} = c S^{ijk} - A^{jki} + A^{ikj} = 0.$$  \hfill (22)

Here

$$A^{ikj} = \frac{1}{2} (c S^{ikj} - c S^{jik} + c S^{jki}).$$  \hfill (23)

The obtained new "pair" ($sT^i_k$, 0) is the most simple and the most symmetric. Note that the symmetric tensor $sT^i_k = s T^i_k$ gives complete description of matter because the spin density tensor $cS^{ijk}$ is entirely absorbed into $sT^i_k$ by the symmetrization procedure.

Note also that the symmetric tensor $sT^i_k$ has 10 independent components and this number is exactly the same as the number of integral conserved quantities in an asymptotically flat closed system.

It is interesting that one can easily generalize the above symmetrization procedure onto a general metric manifold $(M_4, g)$ \[14, 34\] by using the Levi-Civita connection associated with the metric $g$. The generalized symmetrization procedure has the same form as above with the replacement $\eta^i_k \rightarrow g^i_k, \quad \partial_i \rightarrow \nabla_{\text{LC}}^i$.

So, one can always get on a metric manifold $(M_4, g)$ a symmetric energy–momentum tensor $sT^i_k = s T^i_k$ for matter (then, of course, corresponding $S^{ijk} = 0$). Observe that
the symmetric tensor \( sT^{ik} \), like as in SRT, consists of the canonical tensors \( cT^{ik} \) and \( cS^{ikl} \).

The symmetric energy–momentum tensor for matter is unique, i.e., it is uniquely determined by the matter equations of motion and reasonable boundary conditions \[58\]. This fact is essential for the uniqueness of the gravitational field equations. Moreover, the symmetric energy–momentum tensor is covariantly conserved (a canonical energy-momentum tensor is not conserved).

L. Rosenfeld has proved \[59\] that

\[
sT^{ik} = \frac{\delta L_m}{\delta g_{ik}}, \tag{24}
\]

where \( L_m = L_m(\Psi, \ _LC D\Psi) \) is a covariant Lagrangian density for matter. The tensor \( sT^{ik} \) given by (24) is the source in the Einstein equations

\[
G_{ik} = \chi sT_{ik}, \tag{25}
\]

where \( \chi = \frac{8\pi G}{c^4} \).

Note that these equations geometrize both the canonical quantities \( cT^{ik} \) and \( cS^{ikl} = (-)_{c}S^{kil} \) in some equivalent way because the tensor \( sT^{ik} \) is built from these two canonical tensors.

So, it is the most natural and most simple to postulate that, in general, the correct energy–momentum tensor for matter is the symmetric tensor \( sT^{ik} \). This leads us to a purely metric torsion-free theory of gravity with the field equations

\[
\frac{\delta L_a}{\delta g_{ik}} = \frac{\delta L_m}{\delta g_{ik}}. \tag{26}
\]

Then, if we take into account the dynamical universality of the Einstein equations \[38, 60, 61\], we will end up with General Relativity (possibly with \( \Lambda \neq 0 \)) which will have a sophisticated, symmetric energy-momentum tensor as a source.

### 5.2 Some remarks on the “teleparallel equivalent of general relativity”

After presenting the preliminary draft of the old our lectures in arXiv \[80\], we have got critical remarks from some persons which are working on the so-called teleparallel equivalent of general relativity (TEGR) in the framework of the Weitzenböck or teleparallel geometry \[12, 29, 62\]. Our reply was the following \[15\]. The Weitzenböck or teleparallel connection and geometric structure on spacetime is determined by a tetrad (or other anholonomic frame) field \( h^{(a)}_b(x) \) and can always be introduced independently of the geometric structure of the spacetime. Here \( (a), (b), \ldots \) are tetrad (= anholonomic) indices and \( a, b, c, \ldots \) mean holonomic (= world) indices.

\[15\] This reply was considerably extended and updated in the paper \[81\].
The fundamental formulas of the teleparallel geometry read

\[ g_{ik} := \eta_{(a)(b)} h^{(a)}_i h^{(b)}_k, \]  
\[ \Gamma^i_{kl} := h_{(a)}^i \partial_k h^{(a)}_l, \]  
\[ \nabla h^{(a)}_k = 0, \]  
\[ \Gamma^i_{kl} = \text{LC} \Gamma^i_{kl} + K^i_{kl}, \]  
\[ K^i_{kl} := 1/2(T^i_{kl} + T^i_{lk} - T^i_{kl}), \]  
\[ T^i_{kl} := \Gamma^i_{lk} - \Gamma^i_{kl}, \]

and

\[ R^i_{klm} = \text{LC} R^i_{klm} + Q^i_{klm} \equiv 0, \]  

where \( Q^i_{klm} \) is a tensor written in terms of the contortion \( K^i_{kl} \) and its covariant derivatives with respect to the Levi-Civita connection \( \text{LC} \Gamma^i_{kl} \) of the metric \( g_{ik} \).

Here \( \eta_{(a)(b)} \) means the interior metric (usually Minkowskian) of a tangent space and the duals \( h_{(a)}^i \) are defined by

\[ h_{(a)}^i h^{(a)}_k = \delta^i_k. \]

Those authors which work on \( \text{TEGR} \), by use the formulas (27), (30), and (33) of the teleparallel geometry rephrase, step-by-step, the all formalism of \( \text{GR} \) in terms of the Weitzenböck connection \( \Gamma^i_{kl} \) and its torsion \( T^i_{kl} \). Then, they call this formal reformulation of \( \text{GR} \) in terms of the Weitzenböck geometry the teleparallel equivalent of general relativity (TEGR) (What kind of “equivalence”?).

One can read in the papers \[62\] the following conclusion: ”Gravitational interaction, thus, can be described alternatively in terms of curvature, as is usually done in \( \text{GR} \), or in terms of torsion, in which case we have the so-called teleparallel gravity. Whether gravitation requires a curved or torsional spacetime, therefore, turns out to be a matter of convention”.

From the point of view of the \( \text{TEGR} \), therefore, teleparallel torsion has fundamental physical meaning and it has been already detected.

We cannot agree with such statements. In our opinion, the ”teleparallel equivalent of \( \text{GR} \)” is only formal and geometrically trivial rephrase of \( \text{GR} \) in terms of the Weitzenböck geometry. Such rephrase is, of course, always possible not only with \( \text{GR} \) but also with any other purely metric theory of gravity (see eg. \[63\]) but it has no profound physical motivation. It is because, as one can easily show, the teleparallel torsion is entirely expressed in terms of the Van Danzig and Schouten anholonomity object \( \Omega_{(a)(b)(c)} \) (see eg. \[12, 29\]). So, the torsion \( T^i_{kl} \) of a teleparallel connection describes only anholonomy of the used field of aholonomic frames \( h_{(a)}^i(x); \) not real
geometry of the spacetime.\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.} Contrary, Levi-Civita part of a Weitzenböck connection can have (and has) geometrical (and physical) meaning.

Resuming, it seems to us that TTEGR is rather a mathematical curiosity which gives, by no means, anything better than ordinary GR gives and one can doubt into its physical meaning.

Precise experimental confirmation of the EEP proved non-zero curvature of physical spacetime\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.} and supported ordinary GR. We think that this fact excludes a physical motivation for rephrasing GR into TTEGR.

One remark more is in order concerning TTEGR: TTEGR resulted in $f(T)$ theories where $T$ means the Lagrangian density\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.} of the TTEGR. In analogy to $f(R)$ extension of the Hilbert action of GR, the $f(T)$ theories are generalization of the action of TTEGR.

It seems that the only one positive property of these theories is the fact that they have 2-nd order field equations.

6 Concluding remarks

The GR model of the space-time has very good experimental confirmation in a weak-field approximation (Solar System) and in the strong fields (binary pulsars). On the other hand, torsion has no experimental evidence (at least in vacuum) and it is not needed in a theory of gravity. Moreover, the introduction of torsion into the geometric structure of space-time leads to many problems (apart from calculational, of course). Most of these problems are removed if only axial torsion $A_i = \frac{1}{2} \eta_{abc} Q^{abc}$, $Q^{[abc]} = Q^{abc}$ exists. So, it would be reasonable to confine themselves to the axial torsion only (If one still want to keep on torsion). This is also supported by the important fact that the matter fields (= Dirac’s particles) are coupled only to the axial part of torsion in the Riemann-Cartan space-time.

However, if we confine to the axial torsion, then (if we remember the dynamical triviality of torsion and the dynamical universality of the Einstein equations) we effectively will end up with GR + additional matter fields. In the most important case of the ECSK theory we will end up with GR + an additional pseudovector field $A_i$ (or with an additional pseudoscalar field $\varphi$ if the field $A_i$ is potential, i.e., if $A_i = \partial_i \varphi$)\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.}. But GR with an additional dynamical pseudovector field $A_i$ yields local gravitational physics which may have both location and velocity-dependent effects\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.} unobserved up to now. Besides, GR with an additional pseudoscalar field has a defect because there exist two distinguished frames, the Einstein frame and the Jordan frame, which are not equivalent physically\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.}.\footnote{Unless one can physically distinguish a tetrad field (or other anholonomic field of frames) and give it a fundamental geometrical and physical meaning. But we think that this could introduce a cristal–like structure on spacetime and, therefore, it would contradict local Lorentz invariance.}
Additionally, we would like to emphasize that there exist very strong experimental constraints on the components of the axial torsion: \( < 10^{(-15)} m^{(-1)} \) \[75\].

So, we will finish with the conclusion that the geometric model of the space-time given by ordinary GR and “wonderful” Levi-Civita connection seems to be the most satisfactory.

Interestingly that this model has a very strong support from the field-theoretic approach to gravity (see e.g., \[68\]).

It seems to us that the torsion was introduced into a theory of gravity in order to get some link between theory of gravity and quantum fundamental particles theory (It is commonly known that the role of the curvature in an atomic and smaller scale is neglegible). But these trials were not successful (see, e.g., \[75\]). It also seems that what we really need nowadays is a quantum model of the Riemannian geometry and a quantum gravity which is based on this model. The recent papers given by Ashtekhar \[16, 18, 17, 74\] and co–workers on this problem seems to be very promising.

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References

[1] R. Utiyama, *Phys. Rev.*, **101** (1956) 1597.

[2] T.W.B. Kibble, *Journ. Math. Phys.*, **2** (1961) 212.

[3] D.W. Sciama, “On the analogy between charge and spin in general relativity” in “Recent developments in general relativity”, PWN, Warsaw 1962.

[4] A. Trautman, “On the Structure of the Einstein–Cartan Equations”, *Istituto Nazionale di Alta Matematica, Symposia Mathematica*, **12** (1973) 139; A. Trautman, “The Einstein-Cartan Theory”, article in *Encyclopedia of Mathematical Physics*. Editors J.P. Francoise, G.L. Naber and S.T. Isou, Oxford, Elsevier, 2006, Vol.2,pp189-195 (gr-qc/0606062).

[5] F.W. Hehl, *Gen. Relativ. Grav.*, **4** (1973) 333; ibidem **5** (1974) 491.

[6] F.W. Hehl et al., *Rev. Mod. Phys.* **48** (1976) 393.
[7] F. W. Hehl et al., “Gravitation and the Poincare’ Gauge Field Theory with Quadratic Lagrangian” in “General Relativity and Gravitation” Vol.I. Ed. A. Held, Plenum Publishing Corporation 1980; F.W. Hehl et al., “Poincare’ gauge theory of gravity: Friedman cosmology with even and odd parity modes. Analytic part”, arXiv: 1009.5112 [gr-qc].

[8] A. Trautman, “Fiber Bundles, Gauge Fields and Gravitation” in “General Relativity and Gravitation” Vol.I. Ed. A. Held, Plenum Publishing Corporation 1980.

[9] K. Hayashi and T. Shirafuji, Prog. Theor. Phys., 64 (1980) 866; 64 (1980) 883; 64 (1980) 1435; 65 (1981) 2222; 65 (1981) 525; 66 (1981) 318; 66 (1981) 2258; 73 (1985) 54; 74 (1985) 852.

[10] J.Garecki, Gen.Relativ. Grav., 22 (1990) 111.

[11] A. Trautman, Class. Quantum Grav. 16 (1999) A157-A176.

[12] J.A. Schouten, “Ricci-Calculus”, Springer–Verlag, Berlin 1954.

[13] E. Bompiani, “Significato del tensore di torsione in une connesione affine”, Boll. Un. Mat. Ital. 6 (1951) 273.

[14] D. Grensing and G. Grensing, Phys. Rev., D 28 (1983) 286.

[15] D. Giulini, “Ashtekar Variables in Classical General Relativity” in “Canonical Gravity: From Classical to Quantum”, Eds. J. Ehlers and H. Friedrich, Springer–Verlag 1994.

[16] A. Ashtekar, “Quantum Mechanics of Geometry”, arXiv: gr-qc/9901023 A. Ashtekhar, Ann.Phys.(Leipzig), 9 (2000) 178. (ArXiv: gr-qc/99101101); “Quantum Geometry and Gravity: Recent Advances”, arXiv: gr-qc/0112038 “Gravity, Geometry and the Quantum”, arXiv:gr-qc/0605011; “Quantum space-times: beyond the continuum of Minkowski and Einstein”, arXiv:0810.0514[gr-qc].

[17] A. Ashtekar and J. Lewandowski, Class. Quant. Grav., 21 (2004) R 53 (arXiv:gr-qc/0404018).

[18] M.S. Iriondo et al., Adv.Theor.Math. Phys., 2 (1988) 1075.(ArXiv: gr-qc/9804019).

[19] Clifford M. Will, “Theory and experiment in gravitational physics”, Cambridge University Press, Cambridge 1993; Clifford M. Will, “The Confrontation Between General Relativity and Experiment: arXiv: gr-qc/0510072; Finally, results from Gravity Probe B”, in Matters of Gravity, No. 38, Fall 2011, arXiv:1109.3499 [gr-qc]pp. 6-9.
[20] Eric G. Adelberger et al., “Opportunities for Probing Fundamental Gravity with Solar System Experiments”. A Science White Paper submitted to the *Cosmology and Fundamental Physics*. Science Frontier Panel of Astro 2010. (arXiv:0902.3004[gr-qc]).

[21] G. Esposito-Farese, “Comparing Solar-System, Binary-Pulsars, and Gravitational-Wave Test of Gravity”, e-print gr-qc/9903058, “Motion in alternative theories of gravity”. Lecture given at the *School on Mass*, Orleans, France, 23-25 June 2008 (arXiv:0905.2575[gr-qc]).

[22] T. Damour, “Experimental Tests of Relativistic Gravity”, arXiv: gr-qc/9904057; “Black Hole and Neutron Star Binaries: Theoretical Challenges”, arXiv:0705.3109 [gr-qc]; “General Relativity Today”, arXiv: 0704.0754 [gr-qc].

[23] Slava G. Turyshev, “Experimental Tests of General Relativity: Recent Progress and Future Directions”, [arXiv:0809.3730][gr-qc]; Slava G. Turyshev et al., *Int. J. Mod. Phys. D* 16(12a) (2007) 1879 [arXiv:0711.0150][gr-qc]; S.G. Turyshev, “Tests of Relativistic Gravity from Space”, arXiv:0906.2520[gr-qc].

[24] S. Kopeikin, “Beyond the Standard IAU Framework”, [arXiv:0908.4108][gr-qc]; Reference Frames, Gauge Transformations and Gravitomagnetism in the Post-Newtonian Theory of the Lunar Motion”, arXiv:0905.2424[gr-qc]; S. Kopeikin et al., “Recent VLBA/VERA/IVS Tests of General Relativity”, arXiv:0912.3421[gr-qc].

[25] M. Kramer et al., “Testing GR with the Double Pulsars: Recent Results”, [arXiv:astro-ph/0503386]. J.M. Weisberg et al., “Timing Measurements of the Relativistic Pulsar PSR B1913+16”, [arXiv:1011.0718][astro-ph.GA]; Norbert Wex et.al., “Generic gravity teste with double pulsar”, arXiv:1001.4733[gr-qc]; Kosmos Lazaridis et al., “Precision timing of PSR J1012+5307 and strong-field GR tests”, arXiv:1001.4704[astro-ph.GA];

[26] F.W. Hehl, J.D. McCrea, E.W. Mielke and Y. Neéman, *Found.Phys.*, 19 (1989) 1075.

[27] F.W. Hehl et al., Phys. Rep. 258 (1995) 1–171.

[28] F. Gronwald, “Metric–Affine Gauge Theory of Gravity I. Fundamental Structure and Field Equations”, arXiv:gr-qc/9702034.

[29] S. Gołąb, “Tensor Calculus”, *PWN* (Warsaw 1966) (in Polish). [Exists English translation].

[30] D.G.B. Edelen, “The structure of field space”, Berkeley, University of California Press 1962.
[31] A. Trautman, “Differential Geometry for Physicists”, Stony Brook Lectures, Bibliopolis, Naples 1984.

[32] S. Kobayashi and K. Nomizu, “Foundations of Differential Geometry”, Interscience Publishers 1963 (Vol.I) and 1969 (Vol.II).

[33] K. Maurin, two articles in “Leksykon Matematyczny”. Eds. M. Skwarczyński, Wiedza Powszechna, Warsaw 1993 (in Polish).

[34] F. W. Hehl, Rep. Math. Phys., 9 (1976) 55.

[35] J. Nester, Phys. Rev. D 16 (1977) 239.

[36] M. A. Schweitzer, “Gauge Theory and Gravitation”, PhD Zürich 1980.

[37] M. Ferraris, J. Kijowski, Gen. Rel. Grav., 14 (1982) 37.

[38] A. Jakubiec, K. Kijowski, Phys. Rev., D 37 (1988) 1406; A. Jakubiec, J. Kijowski, J.Math.Phys. 30 (1989) 1073; A. Jakubiec, J. Kijowski, J. Math. Phys., 30 (1989) 2923.

[39] C. W. Misner, K. S. Thorne, J. A. Wheeler, “Gravitation”, W. H. Freeman and Company, San Francisco 1973.

[40] Wann-Quan Li and Wei-Tou Ni, J. Math. Phys., 20 (1979) 25; Wei-Tou Ni, “Testing relativistic gravity and detecting gravitational waves in space”, 8p. Work supported by the National Science Foundation of China (Grant Nos. 10778710 and 10875171).

[41] P. von der Heyde, Lett. Nuovo Cim., 14 (1975) 250.

[42] F.W. Hehl, “Four lectures on Poincare' Theory of Gravity”, Proceedings of the 6th Course of the International School of Cosmology and Gravitation on “Spin, Torsion, Rotation and Supergravity” held at Erice, Italy, May 1979. P.G. Bergmann and V. de Sabatta (editors), Plenum Press 1980.

[43] F.W. Hehl, “On the Gauge Field Theory of Gravitation”, 3 lectures given at the Dublin Institute for Advanced Studies from 18–21 Nov. 1981.

[44] M. Blagojević, “Gravitation and Gauge Symmetries”, Institute of Physics Publishing, Bristol 2002.

[45] D. Hartley, Class. Quantum Grav., 12 (1995) L103-L106 (gr-qc/9510013) B.Z Iliev, J. Phys. A: Math. Gen., 29 (1996) 6895; B.Z. Iliev, J. Geometry Phys., 45 (2003) 24.
[46] L.D. Landau, E.M. Lifschitz, “Field Theory”, Nauka Publishers, Moscow 1988 (In Russian) [English edition of this book: “The Classical Theory of Fields” Pergamon, Oxford 1975].

[47] J. Garecki, R. Schimming, Rep. Math. Phys, 51 (2003) 197 (arXiv:gr-qc/0201008).

[48] V. de Sabatta and M. Gasperini, “Introduction to Gravitation”, World Scientific, Singapore 1985.

[49] A.Z. Petrov, “New Methods in General Relativity”, Nauka, Moscow 1966 (In Russian). [English edition of Petrov’s book: “Einstein Spaces”, Pergamon, New York 1969].

[50] L. Brewin, Class. Quantum Grav., 25 (1998) 3085.

[51] A.I. Nesterov, Class. Quantum Grav., 16 (1999) 465.

[52] V. Müller et al., “A Closed Formula for the Riemann Normal Coordinate Expansion”, arXiv:gr-qc/9712092.

[53] J. Garecki, Acta Phys. Pol., B 9 (1978) 291; J. Garecki, Acta Phys. Pol., B 12 (1981) 739; J. Garecki, Class. Quantum Grav., 2 (1985) 403; J. Garecki, Acta Phys. Pol., B 12 (1981) 1017; J. Garecki, “Some Remarks on the Bel-Robinson Tensor”. An amended version of the Essay which received an “honorable mention” in the 1999 Essay Competition of the Gravity Research Foundation. Ann. Phys. (Leipzig) 10 (2001) 911 (arXiv:gr-qc/0003006).

[54] F. Rohrlich, “Classical Charge Particles. Foundations of their Theory”, Addison-Wesley Publishing Company, Inc. Reading, Massachusetts 1965.

[55] B. F. Schutz, “A First Course in General Relativity”, Cambridge University Press, Cambridge 1985.

[56] F. J. Belinfante, Physica, 6 (1939) 887.

[57] W. Kopczyński, A. Trautman, “Spacetime and Gravitation”, PWN, Warsaw 1984 (in Polish). [English edition of this book: “Space-Time and Gravitation”, J. Wiley, New York 1992].

[58] V. D. Fock, “The Theory of Space, Time and Gravitation”, Pergamon Press, London 1969.

[59] L. Rosenfeld, Mem. Acad. Roy. Belg., Cl.Sc., fasc. 6 (1940).

[60] G. Magnano et al., Gen Rel. Grav., 19 (1987) 465.
[61] A. Borowiec et al., *Gen Rel. Grav.*, **26** (1994) 637; A. Borowiec et al., *Class. Quantum Grav.*, **15** (1998) 43 (arXiv:gr-qc/9511067); A. Borowiec et al., “Alternative Lagrangians for Einstein Metrics”, arXiv: gr-qc/9806116.

[62] V.C. de Andrade et al., “Teleparallel gravity: an overview”, arXiv:gr-qc/0011087; V.C. de Andrade et al., ”Teleparallel gravity and the gravitational energy-momentum density”, arXiv:gr-qc/0011097; V.C. de Andrade et al., ”Teleparallel spin connection”, arXiv:gr-qc/0104102; R. Aldrovandi and J.G. Pereira, “An introduction to teleparallel gravity”, Instituto de Fisica Teorica, UNESP Sao Paulo, Brazil 2007.

[63] J.Garecki, R. Schimming, *Bulletin dela Societe’ des Sciences et des Lettres de Lódź, Vol. LII* (2002) 101 (ArXiv:gr-qc/0208084).

[64] M. Blagojević, “Three Lectures on Poincare’ Gauge Theory”, arXiv: gr-qc/0302040.

[65] R.T. Hammond, *Rep. Prog. Phys.*, **65** (2002) 599; F.W. Hehl and Y.N. Obukhov, “Elie Cartan torsion in geometry and field theory, an essay”, *Ann. Fond. L. de Broglie*, **32** (2007) N2-3; Yuyui Lam, “Totally asymmetric torsion on Riemann-Cartan manifold”, arXiv:gr-qc/0211009.

[66] A. Schild, “Lectures on General Relativity Theory” in *Relativity Theory and Astrophysics*. Ed. J. Ehlers, *AMS, Providence, Rhode Island, 1967*; “Gravitational Theories of Whitehead Type and the Principle of Equivalence” in *Evidence for Gravitational Theories*. Ed. C. Moller, *Academic Press, New York 1962*.

[67] A. Macias and A. Garcia, *Gen. Relativ. Grav.*, **33** (2001) 890.

[68] N Straumann, “Reflections on Gravity”. Concluding talk at the ESA-CERN WORKSHOP, CERN, 5-7 April 2000, arXiv:astro-ph/0006423.

[69] C.N. Yang, *Phys. Rev. Lett.*, **33** (1974) 445.

[70] Derek K. Wiese, *Class. Quantum Grav.*, **27** (2010) 155010; “Symmetric Space Cartan Connections and Gravity in Three and Four Dimensions”, arXiv:0904:1738[math.DG]. The last paper is a contribution to the Special Issue *Elie Cartan and Differential Geometry*: A.Randono, “Gauge Gravity: a forward looking introduction”, arXiv:1010.5822[gr-qc].

[71] N. Straumann, “Problems with Modified Theories of Gravity, as Alternative to Dark Energy”. Invited talk at the Conference *Beyond Einstein*, Mainz, 22-26 September 2008 (arXiv: 0809.5148[gr-qc]); Antonio De Felice et al., “Ghosts, Instabilities, and Superluminal Propagation in Modified Gravity Models”,
[72] R. March et al., “Constraining spacetime torsion with LAGEOS”, arXiv:1101.2791[gr-qc]; M. Tegmark et al., Phys. Rev., D 76 (2007) 1550.

[73] J. Ehlers et al., “The geometry of free-fall and light propagation”. An article in Synge Festschrift, Oxford University Press, Oxford 1972.

[74] C. Rovelli, “Quantum Gravity”, CUP, Cambridge 2004; “Loop quantum gravity: the first twenty five years”, arXiv:1012.4707[gr-qc]; T. Thiemann, “Modern canonical quantum General Relativity”, CUP, Cambridge 2007; H. Nicolai et al., Class. Quantum Grav., 22 (2005) R 193 (arXiv:hep-th/0501114); T. Thiemann, Lect. Notes Phys., 721 (2007) 185 (arXiv:hep-th/0608210); C. Kiefer, “Quantum Gravity: General Introduction and Recent Developments”, arXiv:gr-qc/0508120.

[75] I.L. Shapiro, Physics Reports, 357 (2002) 113; R.T. Hammond, Phys. Rev. D52 (1995) 6918; V. Alan Kostelecky et al., PRL, 100 (2008) 111102; Sean M. Carrol et al., Phys. Rev., D 50 (1994) 3867.

[76] R. Sulanke, P. Wintgen, “Differentialgeometrie und Faserbündel”, VEB Deutscher Verlag der Wissenschaften, Berlin 1972; Polish edition: “Geometria różniczkowa i teoria wiązek”, PWN, Warszawa 1977.

[77] Peter Collas and David Klein, “A simple criterion for nonrotating reference frames”, arXiv:0811.2474[gr-qc]; “General transformation formulas for Fermi-Walker coordinates for space-like curves”, arXiv: 0712.3838[gr-qc]; “Exact Fermi coordinates for a class of spacetimes”, arXiv:0912.2779[math-ph]; “Exact Fermi coordinates for anti-de Sitter and other spacetimes”, arXiv:0905.1367[gr-qc]; David Klein and Evan Randles, “Fermi coordinates, simultaneity, and expanding space in Robertsor-Walker cosmologies”, arXiv:1010.0588[math-ph]; P. Delva, “Extended Fermi coordinates”, arXiv:0901.4465[gr-qc]; Michael S. Underwood and Karl-Peter Marzlin, “Fermi-Frenet coordinates for space-like curves”, arXiv: 0706.3224[gr-qc]; J.W. Maluf anf F.F. Faria, “On the construction of Fermi-Walker transported frames”, arXiv:0804.2502[gr-qc]; Donato Bini, Andrea Geralico and Robert T. Jantzen, “Kerr metric, static observers and Fermi coordinates”, arXiv:gr-qc/0510023; Masaki Ishii, Masana Shibata, and Yasushi Mino, “Black hole tidal problem in the Fermi normal coordinates”, arXiv:gr-qc/0501084; Hrvoje Nikolic, “Fermi coordinates and relativistic effects in non-inertial frames”, arXiv:gr-qc/0009068.

[78] James M. Nester, Ann. Phys. (Berlin), 19 (2010) 45.
[79] J. Garecki, “On torsion in a theory of gravity”, an article in *Relativity, Gravitation, Cosmology*. Eds: V. Dvoeglazov and A. EspinozaGarrido, pp. 43-59. **2004** Nova Science Publishers, Inc.

[80] J. Garecki, “Is torsion needed in theory of gravity”? Lecture delivered on the XXXII Symposium on Mathematical Physics (Toruń 2000). [ArXiv:gr-qc/0103029]

[81] J. Garecki, “Teleparallel equivalent of general relativity: a critical review”. Lecture delivered at *Hypercomplex Seminar 2010*, 17-24 July 2010, Będlewo, Poland. [arXiv:1010.2654[gr-qc]]. Proceedings of the Seminar (in print).

[82] John D. Barrow et al., “$f(T)$ gravity and local Lorentz invariance”, [arXiv:1010.1041[gr-qc]]; John D. Barrow et al., “Large -scale Structure in $f(T)$ Gravity”, [arXiv:1103.2786[astro-ph.CO]]; John D. Barrow, “Generalization of teleparallel gravity and local Lorentz symmetry”, [arXiv:1012.4039[gr-qc]]; Cemsinan Deliduman et al., “Absence of Relativistic Stars in $f(T)$ Gravity”, [arXiv:1103.2225[gr-qc]]; Rafael Ferraro et al., “Non trivial frames for $f(T)$ theories of gravity and beyond”, [arXiv:1103.0824[gr-qc]]; Tower Wang, “Static Solutions with Spherical Symmetry in $f(T)$ Theories”, [arXiv:1102.4410[gr-qc]]; Surajit Chattopadhyay et al., “Emergent Universes in Chameleon $f(R)$ and $f(T)$ Gravity Theories”, [arXiv:1105.1091[gr-qc]].
Czy torsja jest potrzebna w teorii grawitacji? Nowe Spojrzenie

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Streszczenie

W pracy pokazano, że wprowadzenie skręcenia do modelu matematycznego fizycznej czasoprzestrzeni nie jest ani konieczne, ani wskazane.

Model matematyczny, który daje ogólna teoria względnosci jest wystarczający dla wszelkich potrzeb fizyki i, jak dotąd, jest bardzo dobrze potwierdzony przez eksperymenty.