High frequency sources of gravitational waves

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Abstract. Sources of high frequency gravitational waves are reviewed. Gravitational collapse, rotational instabilities and oscillations of the remnant compact objects are potentially important sources of gravitational waves. Significant and unique information for the various stages of the collapse, the evolution of protoneutron stars and the details of the equations of state of such objects can be drawn from careful study of the gravitational wave signal.

1. Introduction

The new generation of gravitational wave (GW) detectors is already collecting data by improving the sensitivity by at least one order of magnitude compared to the operating resonant detectors. Broadband GW detectors are sensitive to frequencies between 50 and a few hundred Hz. The next generation will broaden the bandwidth but it will still not be sensitive enough to frequencies over 500-600Hz, unless they are operated as narrow-band detectors [1, 2]. There are also suggestions for wide band resonant detectors in the kHz band [3].

In this short review we will discuss some of the sources that are in the frequency band (≈ 500 – 600Hz), where the interferometers are sensitive enough only if they are narrowbanded. Since there is a variety of GW sources with very interesting physics associated to them, this high-frequency window deserves special attention. If either resonant or narrow-band interferometers achieve the needed sensitivity, there is a plethora of unique information that can be collected.

2. Gravitational collapse

One of the most spectacular events in the Universe is the supernova (SN) collapse to create a neutron star (NS) or a black hole (BH). Core collapse is a very complicated event and a proper study of the event demands a deeper understanding of neutrino emission, amplification of the magnetic fields, angular momentum distribution, pulsar kicks, etc. There are many viable explanations for each of the above issues but it is still not possible to combine all of them together into a consistent theory. Gravitational waves emanating from the very first moments of the core collapse might shed light on all the above problems and help us understand the details of this dramatic event.
Gravitational collapse compresses matter to nuclear densities, and is responsible for the core bounce and the shock generation. The event proceeds extremely fast, lasting only less than a second, and the dense fluid undergoes motions with relativistic speeds ($v/c \sim 0.2 - 0.4$). Even small deviations from spherical symmetry during this phase can generate copious amounts of GWs. However, the size of these asymmetries is not known. From observations in the electromagnetic spectrum we know that stars more massive than $\sim 8M_\odot$ end in core collapse and that $\sim 90\%$ of them are stars with masses $\sim 8 - 20M_\odot$. During the collapse most of the material is ejected and if the progenitor star has a mass $M \lesssim 20M_\odot$ it leaves behind a neutron star. If $M \gtrsim 20M_\odot$ more than 10% falls back and pushes the proto-neutron-star (PNS) above the maximum NS mass leading to the formation of black holes (type II collapsars). Finally, if the progenitor star has a mass $M \gtrsim 40M_\odot$ no supernova is launched and the star collapses to form a BH (type I collapsars).

A significant amount of the ejected material can fall back, subsequently spinning up and reheating the nascent NS. Instabilities can be excited again during such a process. If a BH formed its quasi-normal modes (QNM) can be excited for as long as the process lasts. “Collapsars” accrete material during the very first few seconds, at rates $\sim 1 - 2M_\odot$/sec. Later the accretion rate is reduced by an order of magnitude but still material is accreted for a few tenths of seconds. Typical frequencies of the emitted GWs are at the range 1-3kHz for $\sim 3 - 10M_\odot$ BHs. If the disk around the central object has a mass $\sim 1M_\odot$ self-gravity becomes important and gravitational instabilities (spiral arms, bars) might develop and radiate GWs. There is also the possibility that the collapsed material might fragment into clumps, which orbit for some cycles like a binary system (fragmentation instability).

The supernova event rate is 1-2 per century per galaxy [4] and about 5-40% of them produce BHs through the fall back material [5]. Conservation of angular momentum suggests that the final objects should rotate close to the mass shedding limit, but this is still an open question, since there is limited knowledge of the initial rotation rate of the final compact object. Pulsar statistics suggest that the initial periods are probably considerably shorter than 20ms. This strong increase of rotation during the collapse has been observed in many numerical simulations (see e.g. [6, 7]).

Core collapse as a potential source of GWs has been studied for more than three decades (some of the most recent calculations can be found in [8, 9, 10, 7, 11, 12]). All these numerical calculations show that signals from Galactic supernova ($d \sim 10$ kpc) are detectable even with the initial LIGO/Virgo sensitivity at frequencies $\lesssim 1$ kHz. Advanced LIGO can detect signals from distances of 1Mpc but it will be difficult with the designed broadband sensitivity to resolve signals from the Virgo cluster ($\sim 15$ Mpc). The typical GW amplitude from the 2D numerical simulations [7, 11] for an observer located on the equatorial plane of the source is

$$h \approx 9 \times 10^{-21} \varepsilon \left(\frac{10\text{kpc}}{d}\right)$$

where $\varepsilon \sim 1$ is the normalized GW amplitude. The total energy radiated in GWs during
the collapse is $\lesssim 10^{-6} - 10^{-8} M_\odot c^2$. These numerical estimates are not yet conclusive, important aspects such as 3D hydrodynamics combined with proper spacetime evolution have been neglected. The influence of the magnetic fields have been ignored in most calculations. The proper treatment of these issues might not change the above estimations by orders of magnitude but it will provide a conclusive answer. There are also issues that need to be understood such as the pulsar kicks (velocities even higher than 1000 km/s) which suggest that in a fraction of newly-born NSs (and BHs) the process may be strongly asymmetric, or the polarization of the light spectra in supernovae which also indicate significant asymmetries [13]. Better treatment of the microphysics and construction of accurate progenitor models for the angular momentum distributions are needed. All these issues are under investigation by many groups.

3. Rotational instabilities

Newly born neutron stars are expected to rotate rapidly, being subject to rotation induced instabilities. These arise from non-axisymmetric perturbations having angular dependence $e^{i m \phi}$. Early Newtonian estimates have shown that a dynamical bar-mode ($m = 2$) instability is excited if the ratio $\beta = T/W$ of the rotational kinetic energy $T$ to the gravitational binding energy $W$ is larger than $\beta_{\text{dyn}} = 0.27$. The instability develops on a dynamical time scale (the time that a sound wave needs to travel across the star) which is about one rotation period and may last from 1 to 100 rotations depending on the degree of differential rotation in the PNS. Another class of instabilities are those driven by dissipative effects such as fluid viscosity or gravitational radiation. Their growth time is much longer (many rotational periods) but they can be excited for significantly lower rotational rates, $\beta \gtrsim 0.14$.

3.1. Bar-mode instability

The bar-mode instability can be excited in a hot PNS, a few milliseconds after the core-bounce, given a sufficiently large $\beta$. It might also be excited a few tenths of seconds later, when the NS cools enough due to neutrino emission and contracts still further ($\beta \sim 1/R$). The amplitude of the emitted gravitational waves can be estimated as $h \sim M R^2 \Omega^2 / d$, where $M$ is the mass of the body, $R$ its size, $\Omega$ the rotational rate and $d$ the distance from Earth. This leads to an estimation of the GW amplitude

$$h \approx 9 \times 10^{-23} \left( \frac{\epsilon}{0.2} \right) \left( \frac{f}{3 \text{kHz}} \right)^2 \left( \frac{15 \text{Mpc}}{d} \right) M_{1.4} R_{10}^2.$$  \hspace{1cm} (2)

where $\epsilon$ measures the ellipticity of the bar. Note that the GW frequency $f$ is twice the rotational frequency $\Omega$. Such a signal is detectable only from sources in our galaxy or the nearby ones (our Local Group). If the sensitivity of the detectors is improved in the kHz region, then signals from the Virgo cluster will be detectable. If the bar persists for many ($\sim 10-100$) rotation periods, then even signals from distances considerably larger than the Virgo cluster will be detectable. The event rate is of the same order as the
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SN rate (a few events per century per galaxy): this means that given the appropriate sensitivity at frequencies between 1-3kHz we might be able to observe a few events per year. Bars can be also created during the merging of NS-NS, BH-NS, BH-WD and even type II collapsars (see discussion in [14]).

The above estimates in general, rely on Newtonian hydrodynamics calculations; GR enhances the onset of the instability, $\beta_{\text{dyn}} \sim 0.24$ [15] and $\beta_{\text{dyn}}$ may be even lower for large values of the compactness (larger $M/R$). The bar-mode instability may be excited for significantly smaller $\beta$ if centrifugal forces produce a peak in the density off the sources rotational center[16]. Rotating stars with a high degree of differential rotation are also dynamically unstable for significantly lower $\beta_{\text{dyn}} \gtrsim 0.01$ [17]. According to this scenario the unstable neutron star settles down to a non-axisymmetric quasi-stationary state which is a strong emitter of quasi-periodic gravitational waves

$$h_{\text{eff}} \approx 3 \times 10^{-22} \left(\frac{R_{\text{eq}}}{30\text{km}}\right) \left(\frac{f}{800\text{Hz}}\right)^{1/2} \left(\frac{100\text{Mpc}}{d}\right) M_{1.4}^{1/2}.$$  

The bar-mode instability of differentially rotating neutron stars is an excellent source of gravitational waves but it is based on the assumption that the dissipation of non-axisymmetric perturbations by viscosity and magnetic fields is negligible. Magnetic fields might actually enforce the uniform rotation of the star on a dynamical timescale and the persistent non-axisymmetric structure might not have time to develop at all.

Numerical simulations have shown that the $m=1$ one-armed spiral mode might become dynamically unstable for considerably lower rotational rates [16, 18]. The $m=1$ instability depends critically on the softness of the equation of state (EoS) and the degree of differential rotation.

3.2. CFS instability, $f$ and $r$-modes

After the initial bounce, neutron stars may maintain a considerable amount of deformation. They settle down to an axisymmetric configuration mainly due to emission of GWs, viscosity and magnetic fields. During this phase QNMs are excited. Technically speaking, an oscillating non-rotating star has equal values $\pm|\sigma|$ (the frequency of a mode) for the forward and backward propagating modes (corresponding to $m = \pm|m|$). Rotation changes the mode frequency by an amount $\delta\sigma \sim m\Omega$ and both the prograde and retrograde modes will be dragged forward by the stellar rotation. If the star spins sufficiently fast, this mode will appear moving forwards in the inertial frame (an observer at infinity), but still backwards in the rotating frame (an observer rotating with the star). Thus, an inertial observer sees GWs with positive angular momentum emitted by the retrograde mode, but since the perturbed fluid rotates slower than it would in absence of the perturbation, the angular momentum of the mode itself is negative. The emission of GWs consequently makes the angular momentum of the mode increasingly negative thus leading to an instability. From the above, one can easily conclude that a mode will be unstable if it is retrograde in the rotating frame and prograde for a distant observer measuring a mode frequency $\sigma - m\Omega$ i.e. the criterion will be $\sigma(\sigma - m\Omega) < 0$. 


This class of frame-dragging instabilities is usually referred to as Chandrasekhar-Friedman-Schutz (CFS) instabilities. For the high frequency (f and p) modes this is possible only for large values of Ω or for quite large m. In general, for every mode there will always be a specific value of m for which the mode will become unstable, although only modes with |m| < 5 have an astrophysically significant growth time. The CFS mechanism is not only active for fluid modes but also for the spacetime or the so called w-modes\[19\]. It is easy to see that the CFS mechanism is not unique to gravitational radiation: any radiative mechanism will have the same effect.

In GR, the f-mode (l = m = 2) becomes unstable for $\beta \approx 0.06 - 0.08$ \[20\]. If the star has significant differential rotation the instability is excited for somewhat higher values of $\beta$ (see e.g. \[21, 22\]). The f-mode instability is an excellent source of GWs. After the brief dynamical phase, the PNS becomes unstable and the instability deforms the star into a non-axisymmetric configuration via the $l = 2$ bar mode. Since the star loses angular momentum, it spins-down, and the GW frequency sweeps from 1kHz down to about 100Hz. Such a signal if properly modelled can be detected from a distance of 100Mpc (if the mode grows to a large nonlinear amplitude).

Rotation does not only shifts the spectra of these modes; it also gives rise to a new type of restoring force, the Coriolis force, with an associated new family of rotational or inertial modes. Inertial modes are primarily velocity perturbations and of special interest is the quadrupole inertial mode (r-mode) with $l = m = 2$. The frequency of the r-mode in the rotating frame of reference is $\sigma = 2\Omega/3$. Using the CFS criterion for stability we can easily show that the r-mode is unstable for any rotation rate of the star. For temperatures between $10^7 - 10^9$K and rotation rates larger than 5-10% of the Kepler limit, the growth time of the unstable mode is smaller than the damping times of the bulk and shear viscosity. The mode grows until it saturates due to non-linear effects. The amplitude of the emitted GWs depends on $\alpha$. Mode coupling might not allow the growth of the instability to high amplitudes $\alpha \approx 10^{-2} - 10^{-3}$ \[23\]. The existence of a crust or of hyperons in the core \[24\] and strong magnetic fields, affect the efficiency of the instability (for an extended review see \[25\]). For newly-born neutron stars the amplitude of GWs might not be large enough and the signals will be detectable only from the local group of galaxies ($d < 1$Mpc)

$$h(t) \approx 10^{-21} \alpha \left( \frac{\Omega}{1kHz} \right) \left( \frac{100\text{kpc}}{d} \right)$$ (4)

If the compact object is a strange star, then the instability will not reach high amplitudes ($\alpha \sim 10^{-3} - 10^{-4}$) but it will persist for a few hundred years and in this case there might be up to ten unstable stars per galaxy at any time \[26\]. Integrating data for a few weeks can lead to an effective amplitude $h_{\text{eff}} \sim 10^{-21}$ for galactic signals at frequencies $\sim 700 - 1000$Hz. The frequency of the signal changes only slightly on a timescale of a few months, so the radiation is practically monochromatic.

Old accreting neutron stars, radiating GWs due to the r-mode instability, at frequencies 400-700Hz, are probably a better source \[27, 28, 29, 30\]. Still, the efficiency and the actual duration of the process depends on the saturation amplitude $\alpha$. If the
accreting compact object is a strange star then it might be a persistent source which radiates GWs for as long as accretion lasts\cite{26}.

4. Oscillations of black holes and neutron stars

\textbf{Black-hole ringing.} If the collapse produces a black hole (collapsar type I or II) the black hole will ring until it settles down to the stationary Kerr state. Although the ringing phase does not last too long (a few tenths of msecs), the ringing due to the excitation by the fallback material might last for secs. The frequency and the damping time of the oscillations for the \( l = m = 2 \) mode can be estimated via the relations \cite{31}

\[
\begin{align*}
\sigma & \approx 3.2 \text{kHz} \; M_{10}^{-1} \left[ 1 - 0.63 \left( 1 - a/M \right)^{3/10} \right] \\
Q &= \pi \sigma \tau \approx 2 \left( 1 - a \right)^{-9/20}
\end{align*}
\]

These relations together with similar ones either for the 2nd QNM or the \( l = 2, m = 0 \) can uniquely determine the mass \( M \) and angular momentum \( a \) of the BH if the frequency and the damping time of the signal have been accurately extracted \cite{32,33}. The amplitude of the ring-down waves depends on the BH’s initial distortion. If the excitation of the BH is due to falling material then the energy is roughly \( \Delta E \gtrsim \epsilon \mu c^2 (\mu/M) \) where \( \epsilon \gtrsim 0.01 \). This leads to an effective GW amplitude

\[
h_{\text{eff}} \approx 2 \times 10^{-21} \left( \frac{\epsilon}{0.01} \right) \left( \frac{10 \text{Mpc}}{d} \right) \left( \frac{\mu}{M_\odot} \right)
\]

\textbf{Neutron star ringing.} If the collapse leaves behind a compact star, various types of oscillation modes might be excited which can help us estimate parameters of the star such as radius, mass, rotation rate and EoS\cite{34,35,36}. This gravitational wave asteroseismology is a unique way to find the radius and the EoS of compact stars. One can derive approximate formulas in order to connect the observable frequencies and damping times of the various stellar modes to the stellar parameters. For example, for the fundamental oscillation (\( l = 2 \)) mode (\( f \)-mode) of non-rotating stars we get \cite{35}

\[
\begin{align*}
\sigma(\text{kHz}) & \approx 0.8 + 1.6 M_{1.4}^{1/2} R_{10}^{-3/2} + \delta_1 m \bar{\Omega} \\
\tau^{-1}(\text{secs}^{-1}) & \approx M_{1.4}^3 R_{10}^{-4} \left( 22.9 - 14.7 M_{1.4} R_{10}^{-1} \right) + \delta_2 m \bar{\Omega}
\end{align*}
\]

where \( \bar{\Omega} \) is the normalized rotation frequency of the star, and \( \delta_1 \) and \( \delta_2 \) are constants estimated by sampling data from various EoS. The typical frequencies of the NS modes are larger than 1kHz. On the other hand, 2D simulations of rotating core-collapse have shown that if a rapidly rotating NS is created, then the dominant mode is the quasi-radial mode ("\( l = 0 \)"), radiating through its \( l = 2 \) piece at frequencies \( \sim 800\text{Hz-}1\text{kHz} \). Since each type of mode is sensitive to the physical conditions where the amplitude of the mode is greatest, the more information we get from the various types of modes the better we understand the details of the star.

Concluding, we should mention that the tidal disruption of a NS by a BH \cite{38} or the merging of two NSs \cite{39} may give valuable information for the radius and the EoS if we can recover the signal at frequencies higher than 1 kHz.
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