1. Introduction

While the use of computers and digital equipment more and more rules the daily live especially of scientists, the usage of computers for PALS is more or less limited to the evaluation of the final spectra. The advent of (ultra-)fast digitizers and fast enough computers opens up new possibilities of connecting the photomultipliers directly to the computer. Now all the evaluation of the anode-pulse can be done in software, thus replacing the almost 50 years old analog electronic chain.

As this change of the setup introduces new sources for errors, it is vital to know which part of the processing introduces which errors. Several Monte-Carlo-Simulations were done to simulate the different parts of the combination of photomultiplier, pre-amplifier and ADC.

Investigations on their influence on the final result have been made both independent of each other and in combination. The results show that the vertical resolution has a very big influence on the timing resolution partly because of the limited number of bits (usually 8-10 bits) available in this range of needed sampling rates of 2-8 GS/s and partly because of the noise introduced by pre-amp and ADC reducing the bit-depth to an effective value as low as 6 bits. Apart from the uncertainties from scintillator and photomultiplier this introduces a limit on the timing resolution which might come as a surprise.

Additionally noise reduction via lowpass-filtering is applied to the raw pulses to test whether this improves the timing resolution.

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2. Simulated pulses

The theory of signal processing says that the discrete event of the gamma-quantum is folded with the resolution functions of scintillator, photo-multiplier and used analog electronics. These are widely accepted to following a Gaussian-function:

\[ y \sim \exp \left( -\frac{(x-x_0)^2}{2\sigma^2} \right) \]

As the \( \delta \)-peak of a discrete event folded with the gaussian function results in the gaussian function itself, at least the rising slope of the photomultipliers signal can be estimated very well by this function. Note that the works of [2, 3] show that the gaussian function can also be used for interpolation on the real data. However this is slow compared to other methods because of the addtional step of fitting the Gaussian to the data.

The upper row of figure 1 shows four pulses as generated for these simulations. The shape and risetime are similar to what we measured with LSO-scintillators on Hamamatsu H3378-50 photomultipliers digitized with a 4 GS/s digitizer. To investigate whether the noise and the quantization from the vertical direction have an influence on the horizontal time resolution is the aim of this paper.

2.1. Parameters for the simulations

For the pulses several parameters can be set in the EPOS-Software [4]:

(i) The amplitude (and distribution) of each channel individually, used to simulate a \(^{22}\)Na energy spectrum. The values were choosen so that the stop signal as a bit less than half the amplitude of the start signal. With the energy distribution the absolute levels were choosen to fit in the vertical full-scale without distortions. These were not changed through the simulations of this work to obtain comparable data.

(ii) The distribution of the time-spread between the pulses. While one might think that not shifting the pulses in time gives the best values, this would in fact give unrealistic values. In real life (even when measuring the time resolution with \(^{60}\)Co) there is a slight deviation (a) in the time-derivation from the center of the pulse to the digitizer clock (this is implemented...
in the simulations as a box-distributed random number of \( x \in [0, 1) \) samples) and (b) in the distance between the two signals because of geometrical, scintillator and photomultiplier uncertainties. The last one is simulated by using a Gaussian distribution with \( \sigma = 0.001 \) (\( \equiv \) FWHM of 0.00235482 samples).

(iii) **The bit-depth of the vertical resolution.** This allows to simulate the analog-digital-conversion and its effects on the signal. Possible values range from 1 to 32 and more bits while native double resolution is also possible.

(iv) **The level of white noise added to simulate the analog electronics.** The noise added is white noise from a random number generator evenly distributed in the \([-\text{max}, \text{max}]\)-range, the difference between the fullscale and the noise level is called the signal-to-noise-ratio (SNR) and calculated as follows:

\[
SNR \ [\text{dB}] = 20 \log \frac{A_{\text{noise}}}{A_{\text{fullscale}}}
\]

In the following simulations the bit-depth and the noise level are changed according to the setup, the “energy spectrum” and the time distribution of the simulated pulses are left the same to create comparable results. This is an advantage of the simulation over reality where \(^{60}\text{Co}\) is used for reference timing measurements with an energy spectrum different from \(^{22}\text{Na}\).

### 2.2. The true-constant-fraction algorithm

To extract the timing information the following algorithm is applied to the data (our pulses are negative, so the rising slope is actually falling):

(i) Find and interpolate the minimum of the pulses by simple 3rd order polynom interpolation (using four points around the minimum sample).

(ii) Find and average the baseline before the pulses.

(iii) Find the (true) constant fraction point-in-time on the falling slope between the baseline and the minimum on the pulses. This is again done by 3rd order polynomial interpolation. The fraction used was 20%, which proved to be stable in various simulations and real digital measurements and is also used in the analog measurements at least here in the Halle positron lab.

(iv) Subtract one cf-time from the other to get the “lifetime” between the two pulses.

As this algorithm has different details but the same results as the analog constant-fraction-modules, the name true constant fraction was proposed [5].

### 3. Simulated 4 GS/s digitizer

The time unit used for the following simulations is natively a [sample], which makes the results independent of the real sampling rate used in the digitizer. However, as the shape of the pulses is targeted at rebuilding real signals digitized with 4 GS/s, the results simulated here do not apply to sampling rates much higher or lower than this. Multiplying the resulting numbers with 250 ps gives the timing values to expect from real measurements.

Still the results will be shown with the unit [sample] as the transition to real [ps] depends on the parameters of the simulation, specifically the shape of the rising slope of the pulses. Here the values were chosen to closely resemble pulses from Hamamatsu photomultipliers with BaF\(_2\)-scintillators digitized with 4GS digitizers simulations for other sampling rates, other photomultipliers and other scintillators.
3.1. Reducing the bit-depth
The first error-source to investigate is the vertical quantization, the bit-depth. The green circles in figure 3 show the determined timing resolutions for various bit-depths. The vertical range of signals fitted well into the vertical full scale. Given was a timing distribution with 0.00235 samples FWHM. It is clearly shown that higher bit-depths give better timing resolutions (with the same sampling rate). Calculating this to a 4 GS/s digitizer gives ∼50 ps FWHM with 8-bit and ∼5.2 ps FWHM with 10-bit resolution.

3.2. Effective bits-depth: noise reducing the accuracy
As the datasheets for (at least) Acqiris digitizers state only an effective bit-depth of 1.5 bits less than the real bit-depth due to noise, the next simulations were done a) with vertical double resolution and noise of the right level and b) with reduced bit-depths and the according noise levels combined. The blue triangles in figure 3 show the results of just the added noise, while the red squares are noise and bit-depth. These two show no real difference which makes the noise added from the digitizers analog electronic (pre-amps and converters) the main source for the limited timing resolution. Computed to real 4 GS sampling rate this gives 152 ps FWHM with 8-bit (6.5 effective bits) and 41 ps for 10-bit (8.5 effective bits).

4. Less noise with lowpass filter
To reduce the noise, one of the best options is to apply a lowpass filter on the raw data before the normal time-extracting algorithm is applied. A Butterworth filter from literature [6] was used for these tests. Given an input of white noise, figure 4 shows the frequency response of the filter for various cutoff frequencies and filter orders. The frequency is used relative to the sampling rate which makes the implementation of the filter module independent of the digitizer used. Figure 1 shows in the lower row the effect of the filtering on the pulses compared to the original raw pulses above.

Figure 4 shows the timing resolutions determined at different cutoff frequencies (f) and filter orders (N). All the simulations were done with double resolution and added noise of 6.5 effective bits. Clearly, the timing resolution is better than without the filtering. The best resolution with N = 1 and f = 0.05 evaluates to 75 ps for an 8bit-4GS/s-digitizer. Compared to without the filter, this is an improvement of ∼2 in the timing resolution.
On first thought it seems strange that the lowest filtering order with the weakest suppression of higher frequencies (see figure 4) yields the best results. The explanation is that the Gaussian shaped slope of the pulses contains a lot of higher frequencies (an ideal $\delta$-peak contains all frequencies with same phase). So while the higher orders are good at suppressing the high frequencies of the noise, they also suppress the high frequencies present in the rising slope which results in a more shallow slope and longer risetime. Which in turn gives a worse timing resolution.

5. Comparison of the results
Table 1 shows the three best resulting timing resolutions compared to each other for clarity. It shows that applying a lowpass filter almost removes the effect of the noise. Calculated to 4 GS/s digitizers the timing resolution of 153 ps is comparable to traditional analog positron lifetime setups [5] and to the first digital positron lifetime setups with basic constant fraction methods. However, somehow there seems to be a “magic border” at $\sim$100 ps FWHM timing resolution.
Table 1. Comparing the results. The resulting (best) timing resolutions from the simulations are listed here. It is clear to see that lowpass filtering can almost remove the effect of the noise added from the analog electronics.

| Method                        | Relative Timing FWHM [samples] | 4GS/s “real” FWHM [ps] |
|-------------------------------|-------------------------------|------------------------|
| Vertical quantization only (8-bit) | 0.202 samples                   | 50 ps                  |
| Noise of effective 6.5 bit    | 0.612 samples                   | 153 ps                 |
| Butterworth-Lowpass $f = 0.05 \ N = 1$ | 0.314 samples                   | 78.5 ps                |

Following our results this border can be crossed by applying optimal noise filtering.

6. Conclusion
One of the weak points of this simulation is definitely the self-choosen restriction to using [sample] as time unit. This means that the results given in [samples] are directly comparable, the transition to real life [ps] on the other hand remains to be checked.

We have shown that the main source for the limited timing resolution seems to be the noise reducing the bit-depth, especially in the 8-bit digitizers. It is pretty clear, removing the noise from the sampled data improves the timing resolution. Different strategies can be applied for this, we found that applying lowpass-filters from the dsp-world should give good results.

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