Large-scale cosmological perturbations on the brane

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In brane-world cosmologies of Randall-Sundrum type, we show that evolution of large-scale curvature perturbations may be determined on the brane, without solving the bulk perturbation equations. The influence of the bulk gravitational field on the brane is felt through a projected Weyl tensor which behaves effectively like an imperfect radiation fluid with anisotropic stress. We define curvature perturbations on uniform density surfaces for both the matter and Weyl fluids, and show that their evolution on large scales follows directly from the energy conservation equations for each fluid. The total curvature perturbation is not necessarily constant for adiabatic matter perturbations, but can change due to the Weyl entropy perturbation. To relate this curvature perturbation to the longitudinal gauge metric potentials requires knowledge of the Weyl anisotropic stress which is not determined by the equations on the brane. We discuss the implications for large-angle anisotropies on the cosmic microwave background sky.

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I. INTRODUCTION

Recently, a lot of attention has been devoted to the cosmology of a brane-universe embedded in a higher dimensional spacetime, stimulated by suggestions from string theory that there may exist extra dimensions which are large but inaccessible to ordinary matter \cite{1}. In this paper we will investigate the simplest such model with a single extra dimension, obeying the five-dimensional Einstein equations in the bulk, with matter fields confined to a single brane located at a $Z_2$-symmetric fixed point.

The standard Friedmann cosmology is not recovered in such a model \cite{4}, unless one assumes the existence of a constant tension in the brane \cite{3,4} (in addition to ordinary matter) and a suitably adjusted negative cosmological constant in the bulk, as in the (second) Randall-Sundrum model \cite{5}. The evolution is then indistinguishable from the standard one in the low-energy regime where the matter density in the universe is much smaller than the brane tension. Therefore the background brane cosmology reproduces the properties of the standard Friedmann background cosmology at the present day if one requires standard evolution since at least nucleosynthesis. To be able to discriminate between a brane cosmology and standard four-dimensional cosmology, it is necessary to go one step further and study perturbations about the background models. Present and future data on large-scale structure and cosmic microwave background (CMB) anisotropies provide extensive information on the spectrum and evolution of cosmological perturbations.

Our purpose here is to present the evolution equations for perturbations in brane cosmology as close as possible to the standard four-dimensional approach in order to discuss the possible imprint of the fifth dimension on cosmological observations, and in particular CMB anisotropies. The influence of the bulk gravitational field on the brane is felt through a projected Weyl tensor which behaves effectively like an imperfect radiation fluid with anisotropic stress. The present work results from the combination of two approaches to brane perturbations: a covariant approach \cite{6,7} based on the effective four-dimensional Einstein equations on the brane \cite{8}, and a metric-based approach treating the bulk metric perturbations in a Gaussian normal coordinate system \cite{9,10} (see also \cite{11,12} for other metric-based approaches, and \cite{13,14} for a covariant Green’s function approach). We will also follow the approach adopted in Refs. \cite{15,16} in using the energy conservation equations to compute the evolution of large-scale curvature perturbations and to identify the effect of non-adiabatic modes.

We focus our attention on the evolution of large-scale perturbations, i.e., perturbations on scales larger than the Hubble radius. The reason is that the scales of cosmological interest (e.g., for large-angle CMB anisotropies) have spent most of their time far outside the Hubble radius and have re-entered only relatively recently in the history of the Universe. Large-scale perturbations generated from quantum fluctuations during de Sitter inflation on the brane have been calculated \cite{15,17,18}. The spectrum of tensor perturbations contains a massless zero mode and massive modes...
that remain in the vacuum state \cite{11,18}. The amplitude of the zero mode is enhanced at high energies compared with the usual four-dimensional result, and is constant on large scales \cite{18}. In \cite{19}, the vector metric perturbations were shown to have no normalizable zero mode, while the normalizable massive modes remain in the vacuum state during inflation (see also \cite{14}). It follows that large-scale vector and tensor perturbations from brane inflation are expected to have the same qualitative properties as in general relativity (apart from an enhanced tensor amplitude), and therefore the same qualitative impact on CMB anisotropies.

Scalar perturbations were computed in \cite{15}, and it was shown that the amplitude of the curvature perturbation on uniform density hypersurfaces is enhanced at high energies relative to the standard four-dimensional result (see also \cite{20,10}). In \cite{15}, the effects of the bulk Weyl tensor on the brane were neglected. Large-scale scalar perturbations, incorporating the full bulk Weyl effects, have been investigated via a comoving covariant approach in \cite{6}, where it was shown that, even when bulk Weyl effects are included, the covariant density perturbation equations contain a closed system on the brane without solving the bulk perturbation equations. In \cite{7}, it was then shown that the covariant analog of the longitudinal gauge metric potential due to matter perturbations is non-constant on large scales in the early universe.

In this paper, we define the curvature perturbation on uniform density surfaces for matter and an entropy perturbation due to the ‘Weyl’ fluid. Their evolution on large scales follows directly from the energy conservation equations for each fluid. The total curvature perturbation is not necessarily constant for adiabatic matter perturbations, but can change due to the Weyl entropy perturbation. We go further to show that, while our approach is sufficient to determine the curvature perturbation at late times due to matter and Weyl effective density perturbations, it cannot determine the anisotropic stress exerted on the brane by the projected Weyl tensor, and hence the contribution of the scalar shear to CMB anisotropies. Thus the effect of brane-world scalar perturbations on large-angle CMB anisotropies cannot in general be determined in the same simple way used in general relativity. Further investigation is required to solve the bulk perturbation equations and determine the behavior of the Weyl anisotropic stress.

II. FIELD EQUATIONS

A. Five-dimensional equations

We assume that the gravitational field in the bulk obeys the five-dimensional Einstein equations

\[
(5)G_{AB} + \Lambda_5 (5)g_{AB} = \kappa_5^2 (5)T_{AB},
\]

where \(\kappa_5^2\) is the five-dimensional gravitational constant and \(\Lambda_5\) the cosmological constant in the bulk. We further assume that the spacetime is vacuum except at the brane. The gravitational field is also subject to appropriate boundary conditions at the brane. The energy-momentum tensor for matter on the brane, \(T_{\mu\nu}\), and the brane tension, \(\lambda\), cause a discontinuity in the extrinsic curvature, \(K_{\mu\nu}\), given by the junction conditions \cite{2,8}

\[
[K_{\mu\nu}] = \kappa_5^2 \left\{ \frac{1}{3} (\lambda - T) g_{\mu\nu} + T_{\mu\nu} \right\},
\]

where \(T = g^{\mu\nu} T_{\mu\nu}\), \(K_{\mu\nu} = g^{(5)}_{\mu\nu} (5)\nabla A n_B\), \(n^A\) is the spacelike unit normal to the brane, and the projected metric on the brane is given by

\[
g_{AB} = (5)g_{AB} - n_A n_B.
\]

We note that the division of the energy-momentum tensor into \(T_{\mu\nu}\) and \(\lambda g_{\mu\nu}\) is rather arbitrary; we choose \(\lambda\) in such a way that the original Randall-Sundrum brane is recovered when \(T_{\mu\nu} = 0\). If we assume that the brane is located at a \(Z_2\)-symmetric orbifold fixed point, then the matter energy-momentum tensor and the brane tension determine the extrinsic curvature close to the brane:

\[
K_{\mu\nu} = -\frac{\kappa_5^2}{2} \left\{ \frac{1}{3} (\lambda - T) g_{\mu\nu} + T_{\mu\nu} \right\}.
\]
B. The view from the brane

The effective 4-dimensional Einstein equations on the brane can be obtained \[8\] by projecting the 5-dimensional quantities. The Gauss equation leads to the 4-dimensional effective equations:

\[
G_{\mu\nu} = -\frac{\Lambda_5}{2}g_{\mu\nu} + KK_{\mu\nu} - K_\mu^\sigma K_\nu^\sigma - \frac{1}{2}g_{\mu\nu} (K^2 - K^\alpha_\beta K_{\alpha\beta}) - \mathcal{E}_{\mu\nu},
\]

where \(K = K^\mu_\mu\) and the effect of the non-local bulk gravitational field is described by the projected 5-dimensional Weyl tensor

\[
\mathcal{E}_{\mu\nu} = \langle 5 \rangle C^E_{AFB} n_E n^B g_{\mu A} g_{\nu B}. \tag{2.6}
\]

Using the junction conditions given in Eq. \(2.4\), we can give the extrinsic curvature in terms of the energy-momentum tensor on the brane so that

\[
G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \kappa_4^2 T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{2.7}
\]

where

\[
\Lambda_4 = \frac{\Lambda_5}{2} + \frac{\kappa_4^4}{12} \lambda^2, \tag{2.8}
\]

\[
\kappa_4^2 = 8 \pi G_N = \frac{\kappa_5^4}{6} \lambda, \tag{2.9}
\]

\[
\Pi_{\mu\nu} = -\frac{1}{4}T_{\mu\alpha} T^\alpha_\nu + \frac{1}{12}TT_{\mu\nu} + \frac{1}{24} \left(3T_{\alpha\beta} T^{\alpha\beta} - T^2\right) g_{\mu\nu}. \tag{2.10}
\]

Using the arbitrariness in the choice of \(\lambda\) as noted before, we set \(\Lambda_4 = 0\). The usual conservation laws for matter, \(\nabla_\mu T^\mu_\nu = 0\), still apply (they are obtained by substituting Eq. \(2.4\) into the Codazzi equations \[8\]).

The power of this approach is that the above form of the 4-dimensional effective equations of motion is independent of the evolution of the bulk spacetime, being given entirely in terms of quantities defined on the brane. Thus these equations apply to brane-world scenarios with infinite or finite bulk, stabilised or evolving.

Near the brane it is always possible to use Gaussian normal coordinates \(x^A = (x^\mu, y)\) in which the 5-dimensional line-element takes the form

\[
^{(5)}ds^2 = g_{\mu\nu}(x^\alpha, y)dx^\mu dx^\nu + dy^2, \tag{2.11}
\]

where the brane is located at \(y = 0\).

III. COSMOLOGICAL PERTURBATIONS ON THE BRANE

The most general linear scalar metric perturbation about a Friedmann-Robertson-Walker (FRW) brane is \[21\]

\[
g_{\mu\nu} = \begin{bmatrix}
-\left(1 + 2A\right) & aB_{ij} \\
 aB_{ij} & a^2 \left\{ (1 + 2R)\gamma_{ij} + 2E_{ij} \right\}
\end{bmatrix}, \tag{3.1}
\]

where \(a(t)\) is the scale factor, \(\gamma_{ij}\) is the metric for a maximally symmetric 3-space with comoving curvature \(K = 0, \pm 1\), and a vertical bar denotes the covariant derivative of \(\gamma_{ij}\).

The perturbed energy-momentum tensor for matter on the brane, with background energy density \(\rho\) and pressure \(P\), can be given as

\[
T^\mu_\nu = \begin{bmatrix}
-(\rho + \delta\rho) & a(\rho + P)(v + B)_{ij} \\
-a^{-1}(\rho + P)v^{ij} & (P + \delta P)\delta^i_j + \delta\pi^i_j
\end{bmatrix}, \tag{3.2}
\]

where \(\delta\pi^i_j = \delta\pi^{ij}_{|j} - \frac{1}{3}\delta^i_j\delta\pi^{kl}_{|k} |_k\) is the tracefree anisotropic stress perturbation. The perturbed quadratic energy-momentum tensor is (compare \[1\])
\[ \Pi^\mu_\nu = \frac{\rho}{12} \left[ -(\rho + 2\delta\rho) \quad 2a(\rho + P)(v + B) \right] - 2a^{-1}(\rho + P) v^i \left[ \right] \{ 2P + \rho + 2(1 + P/\rho)\delta\rho + 2\delta P \} \delta^i_j - (1 + 3P/\rho)\delta\pi^i_j \].

(3.3)

The remaining contribution of metric perturbations in the bulk to the modified Einstein equations on the brane is given by the projected Weyl tensor \( \xi^\mu_\nu \). Although this is due to the effect of bulk metric perturbations not defined on the brane, we can nonetheless parametrize this as an effective energy-momentum tensor [6, 10].

In the background FRW cosmology, Eq. (2.7) yields the modified Friedmann equation

\[ 3H^2 + \frac{3K}{a^2} = \kappa_4^2 \rho \left( 1 + \frac{\rho}{2\lambda} \right) + \xi^0_0, \]

(3.5)

where \( H = \dot{a}/a \) is the Hubble expansion rate. For matter on the brane, one can define an effective gravitational energy density and pressure

\[ \rho_{\text{eff}} = \rho \left( 1 + \frac{\rho}{2\lambda} \right), \]

(3.6)

\[ P_{\text{eff}} = P + \frac{\rho}{2\lambda} (2P + \rho), \]

(3.7)

which obey the standard Friedmann equation if \( \xi^0_0 = \kappa_4^2 \rho \) vanishes. The adiabatic sound speed for matter is given by \( c_s^2 = \frac{P}{\rho} \), and the effective adiabatic sound speed for the effective matter fluid is given by

\[ c_s^2_{\text{eff}} = \frac{P_{\text{eff}}}{\rho_{\text{eff}}}, \]

(3.8)

In the low energy regime, \( \rho \ll \lambda \), the effective density and pressure tend to the real quantities, and standard cosmology is recovered, up to the Weyl term, as is clear from Eq. (3.3). Given the agreement between the abundance of light elements and the nucleosynthesis predictions, there is not much freedom for non-standard evolution of the scale factor from the time of nucleosynthesis. The Universe is thus in a low-energy regime since at least nucleosynthesis, i.e. \( \lambda > \rho_{\text{nuel}} \sim (1 \text{ MeV})^4 \). This implies a lower bound on the 5-dimensional mass \( M_5 \) (defined by \( \kappa_4^2 = 8\pi/M_5^3 \)) of \( M_5 \gtrsim 10^5 \text{ TeV} \). In fact, it turns out there is a more stringent constraint coming from small-scale gravity experiments; the absence of deviations from Newton’s law on the millimeter scale (see, e.g. [22]) imposes \( M_5 \gtrsim 10^5 \text{ TeV} \), corresponding to

\[ \lambda^{1/4} \gtrsim 100 \text{ GeV}. \]

(3.9)

At times much earlier than nucleosynthesis, there is no a priori argument against a non-standard evolution of the Universe. In a very high-energy regime, \( \rho \gg \lambda \), the contribution from the matter in Eq. (3.3) becomes quadratic in the energy density. Thus a barotropic fluid with \( P/\rho = c_s^2 \) constant, leads to an effective sound speed given by \( c_{s,\text{eff}}^2 = 2c_s^2 + 1 \) at high energies (\( \rho \gg \lambda \)). In particular, ordinary radiation with \( c_s^2 = \frac{1}{3} \) yields an effective sound speed given by \( c_{s,\text{eff}}^2 = \frac{4}{3} \) at high energies in the early brane-world universe.

There is an additional contribution to the modified Friedmann equation (3.3) from the projected Weyl tensor, equivalent to an additional energy density, \( \rho_{\xi} = \xi^0_0/\kappa_4^2 \). The tracefree property of \( \xi^\mu_\nu \) implies that the pressure obeys \( P_{\xi} = \frac{1}{3} \rho_{\xi} \) and the effective sound speed is given by \( c_{\xi}^2 = \frac{1}{3} \).

We define a total effective energy density and pressure on the brane:

\[ \rho_{\text{tot}} = \rho \left( 1 + \frac{\rho}{2\lambda} \right) + \rho_{\xi}, \]

(3.10)

\[ P_{\text{tot}} = P + \frac{\rho}{2\lambda} (2P + \rho) + \frac{1}{3} \rho_{\xi}. \]

(3.11)

There are again constraints on the contribution of \( \rho_{\xi} \) to the total energy density in the Universe [4] from nucleosynthesis; since the effective number of light neutrino species must be less than 3.2 [23] we have

\[ \frac{\rho_{\xi}}{\rho} \lesssim 0.03 \]

(3.12)

at the time of nucleosynthesis. This implies, since \( \rho_{\xi} \propto 1/a^4 \), that the Weyl contribution would be extremely small today.

4
Conservation equations

Together with the junction conditions at the brane, the 4-dimensional modified Einstein equations (2.7) are equivalent to the 4-dimensional part of the 5-dimensional Einstein equations. Two of the remaining 5-dimensional Einstein equations are equivalent to the conservation of the matter energy and momentum on the brane:

\[
\dot{\delta \rho} + 3H(\delta \rho + \delta P) + 3(\rho + P)\dot{\mathcal{R}} + a^{-1}\left[\nabla^2 \delta q + (\rho + P)\nabla^2 \left(a\dot{E} - B\right)\right] = 0, \tag{3.13}
\]

\[
\dot{\delta q} + 4H\delta q + a^{-1}\left[(\rho + P)A + \delta P + \frac{2}{3}(\nabla^2 + 3K)\delta \pi\right] = 0, \tag{3.14}
\]

where the momentum perturbation is \(\delta q = (\rho + P)(v + B)\).

The final 5-dimensional Einstein equation yields an equation of state for the Weyl fluid [10], which in the 4-dimensional equations follows from the symmetry properties of the projected Weyl tensor, requiring \(P_E = \frac{1}{3}\rho_E\) in the background and \(\delta P_E = \frac{1}{3}\delta \rho_E\) at first order. We still require equations of motion for the effective energy and momentum of the projected Weyl tensor, and these are provided by the 4-dimensional contracted Bianchi identities. Note that these equations are \textit{intrinsically four-dimensional}, only being defined on the brane, and are not part of the five-dimensional field equations. The contracted Bianchi identities (\(\nabla_\mu A_{\nu} = 0\)) and energy-momentum conservation for matter on the brane (\(\nabla_\mu T_{\mu\nu} = 0\)) ensure, using Eq. (2.7), that

\[
\nabla_\mu E^\mu_{\nu} = \kappa_5^4 \nabla_\mu \Pi^\mu_{\nu}. \tag{3.15}
\]

In the background we have

\[
\dot{\rho}_E + 4H\rho_E = 0, \tag{3.16}
\]

and for the first-order perturbations we have

\[
\dot{\delta \rho}_E + 4H\delta \rho_E + 4\delta \rho_E \dot{R} + a^{-1}\left[\nabla^2 \delta \rho_E + \frac{4}{3}\rho_E \nabla^2 \left(a\dot{E} - B\right)\right] = 0 \tag{3.17}
\]

(which can be compared with the covariant form given in [1]). The key result here is that the effective energy of the projected Weyl tensor is conserved independently of the quadratic energy-momentum tensor. The only interaction is a momentum transfer [8,6], as shown by the perturbed momentum conservation equation

\[
\dot{\delta \rho}_E + 4H\delta \rho_E + a^{-1}\left[\frac{4}{3}\delta \rho_E A + \frac{1}{3}\delta \rho_E + \frac{2}{3}(\nabla^2 + 3K)\delta \pi\right] = \frac{(\rho + P)}{a\lambda} \left[\delta \rho - 3H\alpha \dot{\delta q} - (\nabla^2 + 3K)\delta \pi\right], \tag{3.18}
\]

where the terms on the right hand side represent the momentum transfer from the quadratic energy-momentum tensor. Note that the combination \(\delta \rho - 3H\alpha \dot{\delta q}\) that appears on the right hand side is gauge-invariant and is equal to the density perturbation on comoving hypersurfaces. Note also that, in the low-energy regime, the right hand side becomes extremely small and one gets a \textit{quasi-conservation} for the Weyl momentum, in addition to the exact conservation of the Weyl energy density.

**IV. CURVATURE PERTURBATIONS**

For the matter fluid we can define a gauge-invariant variable corresponding to the curvature perturbation on hypersurfaces of uniform density [24]

\[
\zeta \equiv \mathcal{R} + \frac{\delta \rho}{3(\rho + P)}. \tag{4.1}
\]

The energy conservation equation (3.13) can then be written as [16]

\[
\dot{\zeta} = -H \left(\frac{\delta P_{\text{nad}}}{\rho + P}\right) - \frac{1}{3} \nabla^2 \left(\frac{\rho}{a} + \dot{E}\right), \tag{4.2}
\]

where the non-adiabatic pressure perturbation is
\[ \delta P_{\text{nad}} \equiv \delta P - c_s^2 \delta \rho. \quad (4.3) \]

For adiabatic perturbations, \( \zeta \) becomes a constant on large scales, i.e., where gradient terms can be neglected\(^1\). This follows directly from the energy-conservation equation, independently of the Einstein equations for the gravitational field \( [16] \). In fact it is possible to define a curvature perturbation, \( \zeta_I \), on hypersurfaces of uniform density for any matter component separately, such as dust or radiation. This will also be constant for adiabatic perturbations in that component on sufficiently large scales if energy conservation holds for that component separately \( [16] \), as shown in the Appendix.

We will also define a gauge-invariant curvature perturbation on hypersurfaces of uniform total effective energy density on the brane,

\[ \zeta_{\text{tot}} \equiv R + \frac{\delta \rho_{\text{tot}}}{3(\rho_{\text{tot}} + P_{\text{tot}})} = R + \frac{\delta \rho(1 + \rho/\lambda) + \delta \rho_{\varepsilon}}{3(\rho + P)(1 + \rho/\lambda) + 4\rho_{\varepsilon}}. \quad (4.4) \]

A. Schwarzschild-Anti-de-Sitter background bulk

If the projected Weyl tensor is non-vanishing in the background, \( \rho_{\varepsilon} \neq 0 \), then we can define a gauge-invariant curvature perturbation on the brane with respect to the projected Weyl tensor, entirely analogous to that defined in Eq. (4.1) with respect to ordinary matter,

\[ \zeta_{\varepsilon} = R + \frac{\delta \rho_{\varepsilon}}{4\rho_{\varepsilon}}. \quad (4.5) \]

Because the effective energy-momentum tensor of the projected Weyl tensor has a definite equation of state and its effective energy is conserved, the energy-conservation equation \( (3.17) \) gives

\[ \dot{\zeta}_{\varepsilon} = 0, \quad (4.6) \]

on large scales (i.e., neglecting gradient terms).

The total curvature perturbation is then a weighted sum of \( \zeta \) for matter and \( \zeta_{\varepsilon} \) for the projected Weyl tensor:

\[ \zeta_{\text{tot}} = W \zeta + (1 - W) \zeta_{\varepsilon}, \quad (4.7) \]

where

\[ W = \frac{3(\rho + P)(1 + \rho/\lambda)}{3(\rho + P)(1 + \rho/\lambda) + 4\rho_{\varepsilon}}, \quad (4.8) \]

It is instructive to study the behaviour of \( W \). During the low-energy radiation era, \( W \) is a constant and \( 1 - W \lesssim \rho_{\varepsilon}/\rho \) is very small. In the subsequent matter era, \( 1 - W \) will decrease like \( a^{-1} \). By contrast, in a very high-energy radiation era (\( \rho \gg \lambda \)) before the standard, i.e. low-energy, radiation era, \( 1 - W \) would increase like \( a^4 \). From this analysis, it is clear that \( 1 - W \) reaches its maximum value during the low-energy radiation era, where it is constrained by nucleosynthesis to be rather small \( \lesssim 0.03 \) from Eqs. \( (3.12) \) and \( (4.8) \). During all other eras, it will be still smaller.

If we define a (gauge-invariant) Weyl entropy perturbation,

\[ S_{\varepsilon} = \zeta_{\varepsilon} - \zeta = \frac{\delta \rho_{\varepsilon}}{4\rho_{\varepsilon}} - \frac{\delta \rho}{3(\rho + P)}, \quad (4.9) \]

then we have

\(^1\)We assume that the universe looks locally like a FRW universe on sufficiently large scales. Thus the local momentum, shear and anisotropic stresses (and any other quantities derived from spatial gradients of scalars) must become negligible on large enough scales with respect to density, pressure or curvature perturbations.
\[ \zeta_{\text{tot}} = \zeta + (1 - W)S_{\zeta}. \] (4.10)

On large scales,

\[ \dot{\zeta}_{\text{tot}} = W\dot{\zeta} + 3HW(1 - W) \left( c_{\text{eff}}^2 - \frac{1}{3} \right) S_{\zeta}. \] (4.11)

Note that non-zero \( \dot{\zeta} \) arises if there is a non-adiabatic matter perturbation, whereas \( \dot{\zeta}_{\text{E}} = 0 \) always. Thus for adiabatic matter perturbations, both \( \zeta \) and \( \zeta_{\text{E}} \) are constant, and the only change in \( \zeta_{\text{tot}} \) is then due to the change in \( W \) when \( S_{\zeta} \neq 0 \) and \( c_{\text{eff}}^2 \neq \frac{1}{3} \).

From Eqs. (4.2) and (4.6), as long as we may neglect gradient terms, we may express \( \zeta_{\text{tot}} \) as

\[ \zeta_{\text{tot}}(t) = \zeta_* - W(t) \int_{t_*}^{t} dt' H \left( \frac{\delta P_{\text{nad}}}{\rho + P} \right) + [1 - W(t)] S_{\zeta_*}, \] (4.12)

where \( t_* \) is some early epoch and \( \zeta_* = \zeta(t_*) \), etc. In particular, when the matter consists of radiation and dust, we have

\[ \zeta = \zeta_* + \left( \frac{\rho_d/\rho_r}{4 + 3\rho_d/\rho_r} \right) S_{\text{dr}}, \] (4.13)

where \( \rho_r \) and \( \rho_d \) are the radiation and dust energy densities, respectively, and

\[ S_{\text{dr}} = 3 (\zeta_d - \zeta_r) = \frac{\delta \rho_d}{\rho_d} - \frac{3}{4} \frac{\delta \rho_r}{\rho_r}, \] (4.14)

is the entropy perturbation between the radiation and dust, which remains constant on superhorizon scales since \( \zeta_d \) and \( \zeta_r \) are separately conserved on large scales. The total curvature perturbation is then given by

\[ \zeta_{\text{tot}} = \zeta_* + W \left( \frac{\rho_d/\rho_r}{4 + 3\rho_d/\rho_r} \right) S_{\text{dr}} + (1 - W)S_{\zeta_*}. \] (4.15)

The general form for \( \zeta \) and \( \zeta_{\text{tot}} \) in a multi-component matter system is given in the Appendix.

**B. Anti-de-Sitter background bulk**

If there is no projected Weyl tensor in the background, \( \rho_{\text{E}} = 0 \), then any contribution from \( \delta \rho_{\text{E}} \) is non-adiabatic (and automatically gauge-invariant). The total curvature perturbation is then, on using Eq. (4.4) in Eq. (4.4),

\[ \zeta_{\text{tot}} = \zeta + \frac{\delta \rho_{\text{E}}}{3(\rho + P)(1 + \rho/\lambda)}. \] (4.16)

The continuity equation (3.17) becomes \( \delta \rho_{\text{E}} + 4H \delta \rho_{\text{E}} = 0 \) on large scales, and hence

\[ \delta \rho_{\text{E}} \propto \frac{1}{a^4}. \] (4.17)

We find that

\[ \dot{\zeta}_{\text{tot}} = \dot{\zeta} + H \left( c_{\text{eff}}^2 - \frac{1}{3} \right) \frac{\delta \rho_{\text{E}}}{(\rho + P)(1 + \rho/\lambda)}. \] (4.18)

The second term on the right may be compared with the expression in [7] for the total non-adiabatic pressure perturbation. Note that in the very high-energy radiation era, this term is a nonzero constant, whereas it is zero in the low-energy radiation era.

Similar to Eq. (4.13), we may express \( \zeta_{\text{tot}} \) on superhorizon scales in the present case as

\[ \zeta_{\text{tot}} = \zeta_* - \int_{t_*}^{t} dt' H \left( \frac{\delta P_{\text{nad}}}{\rho + P} \right) + \frac{\delta \rho_{\text{E}}(a_*/a)^4}{3(\rho + P)(1 + \rho/\lambda)}. \] (4.19)
Then for a universe with radiation and dust, we have

\[ \zeta_{\text{tot}} = \zeta_s + \left( \frac{\rho_\Lambda/\rho_r}{4 + 3\rho_\Lambda/\rho_r} \right) S_{d\tau} + \frac{1}{(4 + 3\rho_\Lambda/\rho_r)(1 + \rho/\lambda)} \frac{\delta\rho\nu}{\rho_{\nu}}. \]  

(4.20)

Before concluding this section, it is worthwhile to note the following fact. If we introduce the initial entropy perturbation \( S_E \) equivalent to \( S_{E*} \) by

\[ S_E = \frac{\rho_E}{\rho_r} S_{E*}, \]  

(4.21)

we can treat both \( \rho_E \neq 0 \) and \( \rho_E = 0 \) cases of the radiation-dust universe in a unified manner. Namely, Eqs. (4.15) and (4.20) are expressed in the single form,

\[ \zeta_{\text{tot}} = \zeta_s + \left[ \frac{\rho_\Lambda/(\rho_r + \tilde{\rho}_E)}{4 + 3\rho_\Lambda/(\rho_r + \tilde{\rho}_E)} \right] S_{d\tau} + \frac{4}{(4 + 3\rho_\Lambda/\rho_r)(1 + \rho/\lambda)} S_E, \]  

(4.22)

where \( \tilde{\rho}_E = \rho_E (1 + \rho/\lambda)^{-1} \), and \( \zeta_s, S_{d\tau}, \) and \( S_E \) are constants to be determined by the initial condition.

V. LONGITUDINAL GAUGE METRIC PERTURBATIONS

The curvature perturbation on hypersurfaces of uniform total effective energy density, \( \zeta_{\text{tot}} \), can be directly related, using the 4-dimensional (modified) Einstein equations, to the gauge-invariant metric perturbations \( \Phi \) and \( \Psi \) in the longitudinal (or zero-shear or conformal Newtonian) gauge:

\[ \zeta_{\text{tot}} = \Phi - \frac{3H \dot{\Phi} - 3H^2 \Psi - a^{-2}(\nabla^2 + 3K)\Phi}{3(\dot{H} - Ka^{-2})}, \]

\[ = \frac{(5 + 3w_{\text{tot}})}{3(1 + w_{\text{tot}})} \Phi + \frac{2H(\dot{\Phi} - (2/3a^2)(\nabla^2 + 6K)\Phi)}{3(\dot{H}^2 + K a^{-2})(1 + w_{\text{tot}})} + \frac{2a^2 H^2}{\rho_{\text{tot}}(1 + w_{\text{tot}})} \delta\pi_{\text{tot}}, \]  

(5.1)

where \( w_{\text{tot}} = P_{\text{tot}}/\rho_{\text{tot}} \) and

\[ \Psi = A + \left[ a(B - a\dot{E}) \right], \]  

(5.2)

\[ \Phi = R + a(B - a\dot{E}), \]  

(5.3)

\[ \delta\pi_{\text{tot}} = \left( 1 - \frac{\rho + 3P}{2\lambda} \right) \delta\pi + \delta\pi_E. \]  

(5.4)

Equation (5.1) is obtained by using the background (modified) Friedmann equations, i.e., the standard ones with ‘total’ fluid as matter, and the traceless part of the spatial perturbed (modified) Einstein equations, which yields

\[ \Phi + \Psi = -\kappa^2 a^2 \delta\pi_{\text{tot}}, \]  

(5.5)

as in general relativity [21]. Note that in the absence of anistropic stresses (\( \delta\pi_{\text{tot}} = 0 \)) there is essentially only one gauge-invariant scalar metric perturbation, \( \Phi = -\Psi \), which is determined directly on large scales from the primordial \( \zeta_{\text{tot}} \). But in the presence of anisotropic stresses it will no longer be possible to determine the metric perturbations from \( \zeta_{\text{tot}} \) alone. Even if there are no (or negligible) matter anisotropic stresses, \( \delta\pi_{\text{tot}} = \delta\pi_E \) may be non-zero so that \( \Phi + \Psi \neq 0 \).

In this section, our goal will be to relate the curvature perturbations to the metric perturbations, \( \Phi \) and \( \Psi \), using the results obtained in the previous section for the curvature perturbations. This will be useful in the next section where we will compute the large-scale anisotropies. This is also useful if one wishes to make the connection between the primordial curvature fluctuations and the late-time metric fluctuations. For simplicity, we will assume here that the universe is spatially flat. We will distinguish the three following cases of cosmological interest: a (low-energy) dust dominated era, a (low-energy) radiation dominated era, and finally a very high-energy radiation era.
A. Dust dominated era

In a dust-dominated universe, the curvature perturbation $\zeta_{\text{tot}}$ given by Eq. (4.22) reduces to

$$\zeta_{\text{tot}} = \zeta_{\ast} + \frac{4}{3} S_{\text{dr}} + \frac{2}{3} \rho_{\text{d}} S_{\text{E}}.$$  \hspace{1cm} (5.6)

The origin of each term on the right hand side is apparent. The first describes the adiabatic perturbation, the second the primordially isocurvature perturbation, and the third the Weyl entropy perturbation. On the other hand, Eq. (5.1), for a spatially flat ($K = 0$) FRW cosmology on large scales, reduces to

$$\zeta_{\text{tot}} = \frac{5}{3} \Phi + \frac{2}{3} \frac{d}{da} \Phi + \frac{2}{3} \rho_{\text{d}} a^{2} \delta \pi_{\text{tot}}.$$  \hspace{1cm} (5.7)

Hence we find that the parts of $\Phi$ corresponding to each term in Eq. (5.6) at the dust-dominated stage are given by

$$\Phi_{\text{ad}} = \frac{3}{5} \zeta_{\ast}, \quad \Phi_{\text{iso}} = \frac{1}{5} S_{\text{dr}}, \quad \Phi_{\text{E}} = \left( \frac{4 \rho_{\text{r}}}{3 \rho_{\text{d}}} \right) E_{\text{E}}, \quad \Phi_{\pi} = - \frac{\kappa_{4}}{a^{3/2}} \int \delta \pi_{\text{tot}} a^{7/2} da.$$  \hspace{1cm} (5.8)

B. Low-energy radiation era

We repeat here the same analysis as before for the (low-energy) radiation era with $\lambda \gg \rho_{\text{r}} \gg \rho_{\text{d}}$. In this case the curvature perturbation $\zeta_{\text{tot}}$ given by Eq. (4.22) simply reduces to

$$\zeta_{\text{tot}} = \zeta_{\ast} + \frac{\rho_{\text{d}}}{4 (\rho_{\text{r}} + \rho_{\text{E}})} S_{\text{dr}} + S_{\text{E}}.$$  \hspace{1cm} (5.10)

Here $w_{\text{tot}} = \frac{1}{3}$, and the relation between the total curvature perturbation and the metric fluctuation, given by Eq. (5.1), reduces to

$$\zeta_{\text{tot}} = \frac{3}{2} \Phi + \frac{1}{2} \frac{d}{da} \Phi + \frac{\kappa_{2}^{2}}{2} a^{2} \delta \pi_{\text{tot}}.$$  \hspace{1cm} (5.11)

One thus gets

$$\Phi_{\text{ad}} = \frac{3}{5} \zeta_{\ast}, \quad \Phi_{\text{iso}} = \frac{1}{5} S_{\text{dr}}, \quad \Phi_{\text{E}} = \left( \frac{4 \rho_{\text{r}}}{3 \rho_{\text{d}}} \right) E_{\text{E}}, \quad \Phi_{\pi} = - \frac{\kappa_{2}^{2}}{a^{3}} \int \delta \pi_{\text{tot}} a^{7/2} da.$$  \hspace{1cm} (5.12)

C. Very high-energy radiation era

We finally consider the case of a radiation era where the background evolution is highly non-standard, with $\rho_{\text{r}} \gg \lambda$, as well as $\rho_{\text{r}} \gg \rho_{\text{d}}$. Equation (4.22) gives

$$\zeta_{\text{tot}} = \zeta_{\ast} + \frac{\rho_{\text{d}}}{4 \rho_{\text{r}}} S_{\text{dr}} + \frac{\lambda}{\rho} S_{\text{E}}.$$  \hspace{1cm} (5.13)

and, as in the dust-dominated case, the contribution from the Weyl component in $\zeta_{\text{tot}}$ is time-dependent. Since $w_{\text{tot}} = \frac{5}{3}$, Eq. (5.1) now yields,

$$\zeta_{\text{tot}} = \frac{5}{4} \Phi + \frac{1}{4} \frac{d}{da} \Phi + \frac{\kappa_{2}^{2}}{2} a^{2} \delta \pi_{\text{tot}}.$$  \hspace{1cm} (5.14)

One then gets

$$\Phi_{\text{ad}} = \frac{4}{5} \zeta_{\ast}, \quad \Phi_{\text{iso}} = \frac{1}{6} \left( \frac{\rho_{\text{d}}}{\rho_{\text{r}}} \right) S_{\text{dr}}, \quad \Phi_{\text{E}} = \frac{4}{9} \left( \frac{\lambda}{\rho} \right) S_{\text{E}}, \quad \Phi_{\pi} = - \frac{\kappa_{2}^{2}}{a^{3}} \int \delta \pi_{\text{tot}} a^{6} da.$$  \hspace{1cm} (5.15)
VI. LARGE-ANGLE CMB ANISOTROPY

Let us consider the large-angle CMB anisotropy in our scenario. Since the energy density $\rho$ is much smaller than the tension $\lambda$ at and after the decoupling of photons and baryons, we may safely neglect the $\rho/\lambda$ corrections in all the equations.

Assuming a spatially flat universe, the (generalized) Sachs-Wolfe effect is described as

$$
\left( \frac{\delta T}{T} \right)_{SW} (\vec{\gamma}, \eta_0) = \left( \frac{1}{4} \Delta_{s,r} + \Psi \right) (\eta_{\text{dec}}, \vec{x}(\eta_{\text{dec}})) + \int_{\eta_{\text{dec}}}^{\eta_0} d\eta \partial_\eta (\Psi - \Phi) (\eta, \vec{x}(\eta)),
$$

where $\vec{x}(\eta) = \vec{\gamma}(\eta_0 - \eta)$, $\eta$ is the conformal time ($d\eta = dt/a(t)$), and $\Delta_{s,r} = \delta \rho_c/\rho_r + 4Ha(\dot{E} - B)$ is the photon density perturbation on the shear-free hypersurfaces. The last integral along photon null geodesics is called the integrated Sachs-Wolfe effect. In contrast with it, for convenience, let us call the first two terms in the parentheses the ‘direct’ Sachs-Wolfe effect.

By evaluating the right hand side of Eq. (4.1) in the shear-free gauge, we find the curvature perturbation on hypersurfaces of uniform photon density, $\zeta_r$, is expressed in terms of $\Delta_{s,r}$ and $\Phi$ as

$$
\zeta_r = \Phi + \frac{1}{4} \Delta_{s,r}.
$$

Thus the Sachs-Wolfe formula (6.1) may be expressed as

$$
\left( \frac{\delta T}{T} \right)_{SW} (\vec{\gamma}, \eta_0) = (\zeta_r + \Psi - \Phi) (\eta_{\text{dec}}, \vec{x}(\eta_{\text{dec}})) + \int_{\eta_{\text{dec}}}^{\eta_0} d\eta \partial_\eta (\Psi - \Phi) (\eta, \vec{x}(\eta)).
$$

To evaluate the quantities appearing in the Sachs-Wolfe formula, let us further assume the universe is dust-dominated at decoupling. One can thus apply the results obtained in the previous section in the case of a dust-dominated universe. Moreover, $\zeta_r$ is related to $\zeta$ as

$$
\zeta_r = \zeta - \left( \frac{\rho_d}{3\rho_d + 4\rho_r} \right) S_{dr}.
$$

Comparing this with Eq. (4.13), we find

$$
\zeta_r = \zeta^*.\n$$

Thus the curvature perturbation on hypersurfaces of uniform photon density exactly represents the adiabatic curvature perturbation.

Gathering all the terms given in Eqs. (5.8), (5.9), and (6.5) together, the terms contributing to the direct Sachs-Wolfe effect become

$$
\zeta_r + \Psi - \Phi = -\frac{1}{5} \zeta^* - \frac{2}{5} S_{dr} - \frac{8}{3} \left( \frac{\rho_r}{\rho_d} \right) S_E - \kappa_2^2 a^2 \delta \pi_{\text{tot}} + \frac{2\kappa_2^2}{a^{5/2}} \int \delta \pi_{\text{tot}} a^{7/2} da.
$$

The first and second terms on the right hand side describe the conventional adiabatic and isocurvature Sachs-Wolfe effects, which may be expressed in terms of $\Psi$ as $\frac{1}{5} \Psi_{\text{ad}}$ and $2\Psi_{\text{iso}}$, respectively. The third term due to the Weyl entropy perturbation may be expressed as $2\Psi_E$. One may be tempted to regard it as a kind of isocurvature perturbation. However, if we recall Eq. (4.22), we see it gives a time-independent contribution to $\zeta_{\text{tot}}$ during the radiation-dominated stage. The magnitude of $S_E$ depends very much on the early history of the universe. If the universe undergoes inflation at an early stage, $S_E$ will be totally negligible after inflation. Observationally the strongest constraint on $S_E$ comes from the COBE CMB anisotropies [20]:

$$
S_E \lesssim 10^{-4},
$$

since $(\rho_r/\rho_d) \sim 0.1$ at decoupling.

\footnote{For a spatially curved universe, the only change in the Sachs-Wolfe formula is the expression for $\vec{x}(\eta)$, which can only be obtained by integrating the null geodesic equations.}
Note that one can also relate the perturbations at last scattering to the perturbations in the early universe where these perturbations might have been generated. Consider for instance the adiabatic part of the metric perturbations. \( \Psi_{ad} \) at last scattering, i.e. during dust domination, is related to the corresponding primordial perturbation in a low-energy radiation era by

\[
\Psi_{ad} = \frac{9}{10} \Psi_{ad,r(low)},
\]

and to the primordial perturbation in a very high-energy radiation era by

\[
\Psi_{ad} = \frac{3}{4} \Psi_{ad,r(high)},
\]

The last two terms in Eq. (6.6) due to anisotropic stress are generally negligible except for possibly the Weyl contribution, \( \delta \pi_E \), which cannot be theoretically constrained within the present approach. Even inflation does not seem to be necessarily effective for reducing the amplitude of \( \delta \pi_E \). The magnitude of anisotropic stress on a comoving scale \( k^{-1} \) is given by

\[
\frac{|\delta \pi_{ij}|}{\rho + P} \sim \frac{k^2}{a^2 H^2} \left( \frac{a^2 H^2 \delta \pi}{\rho + P} \right),
\]

where \( k \) is the comoving wavenumber. Assuming all the perturbations behave regularly in the limit \( t \to 0 \) and in the large-scale limit, the only restriction is that \( H^2 a^2 \delta \pi_E / (\rho + P) \) be regular in both of the limits. From the amplitude of the COBE CMB anisotropies [26] and Eq. (6.6), we obtain the observational bound

\[
\kappa_4^2 a^2 \delta \pi_E \lesssim 10^{-5}.
\]

Finally, we consider the integrated Sachs-Wolfe effect. In addition to the conventional contributions discussed in the literature [27], there are contributions specific to our scenario. From Eqs. (5.8) and (5.9), we have

\[
\partial_\eta (\Psi - \Phi) = -\partial_\eta \left( \frac{8 \rho}{3 \rho_d} S_E + \kappa_4^2 a^2 \delta \pi_{tot} - \frac{2 \kappa_4^2}{a^{3/2}} \int \delta \pi_{tot} a^{7/2} da \right).
\]

The first term due to \( S_E \) gives the same effect as the one due to insufficient dust-dominance in conventional 4-dimensional cosmological models [27]. It is effective only in the vicinity of the last scattering surface, and its effect is expected to be of the same order of magnitude as the corresponding contribution in the direct Sachs-Wolfe effect. On the other hand, we are unable to constrain the magnitude of the last two terms due to \( \delta \pi_{tot} \), because of the presence of the Weyl anisotropic stress, \( \delta \pi_E \), whose behavior is undetermined within our approach.

**VII. DISCUSSION**

In this paper we have shown that it is possible to extend some results for the evolution of scalar perturbations about four-dimensional FRW cosmological solutions to a five-dimensional brane-world scenario by working solely with the induced four-dimensional Einstein equations on the brane. In particular, the curvature perturbation on uniform matter density hypersurfaces, \( \zeta \), remains constant for adiabatic matter perturbations on sufficiently large scales, where gradient terms become negligible. This remains applicable in a wide variety of higher-dimensional models so long as local conservation of energy holds for some or all matter fields on the four-dimensional brane-world.

We have focused on the case of five-dimensional Einstein gravity with a cosmological constant in the bulk, which ensures energy-conservation for matter on the brane. In addition to ordinary cosmological matter, a new component appears in the induced four-dimensional Einstein equations on the brane, which is the manifestation of the five-dimensional bulk gravitons. This component, which we call the Weyl component because it corresponds to the projected five-dimensional Weyl tensor, can be described effectively as a fluid from the brane point of view, but is constrained to remain small with respect to the ordinary radiation component.

The effective energy of the Weyl component is locally conserved independently of ordinary matter for linear perturbations, even though there may be momentum transfer at high energies. We are therefore able to define another perturbation, \( \zeta_E \), when \( \rho_E \neq 0 \), corresponding to the curvature perturbation on hypersurfaces of uniform effective Weyl density, which remains constant on large scales. If \( \rho_E = 0 \), then \( \delta \rho_E \) is a (gauge-invariant) non-adiabatic perturbation whose evolution is determined by the energy conservation equation. We are then able to model the evolution.
of the total effective curvature perturbation for matter plus Weyl fluid, $\zeta_{\text{tot}}$, constructed from the matter $\zeta$ and the Weyl fluid $\zeta_E$ or $\delta\rho_E$. This in turn can be related to the longitudinal-gauge metric perturbation, $\Phi$, either in the early radiation-dominated era (where non-conventional background evolution can change the usual relation between $\Phi$ and $\zeta_{\text{tot}}$ at very high energies), or later during dust-domination.

We have also studied the possible impact upon cosmic microwave background anisotropies. The presence of the Weyl component has essentially two possible effects. On the one hand, there is an additional contribution from the Weyl entropy perturbation $S_E$ that is similar to an extra isocurvature contribution. On the other hand, the anisotropic stress of the Weyl component, $\delta\pi_E$, also contributes to the CMB anisotropies. In the absence of anisotropic stresses, the curvature perturbation $\zeta_{\text{tot}}$ is sufficient to determine the metric perturbation $\Phi$ and hence the large-angle CMB anisotropies. However bulk gravitons can also generate anisotropic stresses which, although they do not affect the large-scale curvature perturbation $\zeta_{\text{tot}}$, can affect the relation between $\zeta_{\text{tot}}$ and $\Phi$ and hence the CMB anisotropies on large angles. There is no intrinsic brane equation determining the evolution of $\delta\pi$ and thus allowing us to estimate it during and after inflation. On intuitive grounds, the part of $\delta\pi_E$ generated by density inhomogeneity on the brane is expected to be no greater than the matter anisotropic stress $\delta\pi$, which may be neglected for calculating large-angle CMB anisotropies. In the absence of anisotropic stresses, the curvature perturbation $\zeta_{\text{tot}}$ is sufficient to determine the metric perturbation $\Phi$ and hence the large-angle CMB anisotropies. However bulk gravitons can also generate anisotropic stresses which, although they do not affect the large-scale curvature perturbation $\zeta_{\text{tot}}$, can affect the relation between $\zeta_{\text{tot}}$ and $\Phi$ and hence the CMB anisotropies on large angles. There is no intrinsic brane equation determining the evolution of $\delta\pi$ and thus allowing us to estimate it during and after inflation. On intuitive grounds, the part of $\delta\pi_E$ generated by density inhomogeneity on the brane is expected to be no greater than the matter anisotropic stress $\delta\pi$, which may be neglected for calculating large-angle CMB anisotropies. As for the other part due to quantum fluctuations of scalar gravitons during inflation, a dimensional analysis suggests $\kappa^2a^2\delta\pi$ is at most of the order of $\kappa^2H^2$. However, this remains to be proved.

Therefore, while the present approach based on the study of the perturbation equations solely on the brane has led us to significant results on large scales, it has also clearly shown us its limits. There is still a need to determine the evolution of the metric perturbations in the bulk in order to determine (i) the amplitude of the Weyl anisotropic stress $\delta\pi_E$, and (ii) the evolution of metric perturbations on the brane at sub-Hubble wavelengths. In this respect, it is important to devise a specific model for the description of the bulk metric perturbations. One would then have to relate the bulk perturbations with their fluid description on the brane in terms of an energy density, a momentum density and anisotropic stress [10,13]. Some such modelling of the evolution of perturbations will be required to make predictions for the shape of the CMB power spectrum over a range of angular scales, to compare with existing observational data.

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**APPENDIX A: EXTENSION TO MULTI-COMPONENT MATTER**

Here we present an extension of our approach to a multi-component matter system. The curvature perturbation on hypersurfaces of uniform $I$-th matter density is defined by

$$\zeta_I = R + \frac{\delta\rho_I}{3(\rho_I + P_I)}. \quad (A1)$$

For simplicity, let us assume $\rho_E \neq 0$. Then defining the weight $W_I$ for the $I$-th component by

$$W_I = \frac{3(\rho_I + P_I)}{3(\rho + P) + 4\tilde{\rho}_E} \quad \text{for} \quad I \neq E, \quad W_E = \frac{4\tilde{\rho}_E}{3(\rho + P) + 4\tilde{\rho}_E}, \quad (A2)$$

where $\tilde{\rho}_E = \rho_E(1 + \rho/\lambda)^{-1}$, we have

$$\zeta_{\text{tot}} = \sum_I W_I \zeta_I. \quad (A3)$$

If $\rho_E = 0$, the only modification is to replace the Weyl contribution as

$$W_E\zeta_E \rightarrow \frac{\delta\rho_E}{3(\rho + P)(1 + \rho/\lambda) + 4\rho_E}. \quad (A4)$$
If we assume all the components are non-interacting with each other, then the energy-momentum conservation equations (3.13) and (3.14) hold for each component separately. From the energy conservation (3.13), the equation of motion for $\zeta_I$ is given by

$$\dot{\zeta}_I = -H \left( \frac{\delta P_{I,\text{nad}}}{\rho_I + P_I} \right) - \frac{1}{3} \nabla^2 \left( \frac{v_I}{a} + \dot{E} \right),$$  

(A5)

where $v_I$ is the velocity potential of the $I$-th matter component. Thus on sufficiently large scales,

$$\dot{\zeta}_I = -H \left( \frac{\delta P_{I,\text{nad}}}{\rho_I + P_I} \right),$$  

(A6)

and $\zeta_I$ will remain constant on large scales for adiabatic perturbations of the $I$-th matter component, such that $\delta P_{I,\text{nad}} = 0$.

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