Update on the impact of the proton radius on the neutron star radius

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Abstract. We present the new result of our investigation on the extraction of proton radius and the impact of different proton radii on the radius of neutron star, after correcting the mistake in the previous calculation of the proton radius. The new value of the extracted proton radius is 0.864 fm. The effect of this correction on the calculated neutron star radius is trivial.

1. Introduction
It has been long believed that the standard and precise measurement of the proton radius would be obtained by using the elastic electron-proton scattering $e^- + p \rightarrow e^- + p$, for which the cross section reads (see Ref. [5] for further explanation)

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \frac{1}{1 + \tau} \left[1 + \frac{\tau}{\epsilon}\mu_p^2\right] G_{E,p}^2(Q^2, \Lambda).$$

(1)

Note that the radius is obtained from the relation $r_E \equiv \langle r_E^2 \rangle^{1/2} = \frac{1}{\epsilon} \frac{dG_{E,p}(Q^2)}{dQ^2}|_{Q^2=0}$.

The latest measurement at MAMI yields $r_E = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}}$ fm [1], in accordance with the standard CODATA value [2], $r_E = 0.8768(69)$ fm, obtained from measurements of the Lamb shift in hydrogen atom. However, a recent measurement of the Lamb shift in muonic hydrogen atom [3] leads to a problem, since this measurement yields a smaller proton charge radius, i.e. $r_E = 0.84184(67)$ fm. In the previous work we try to solve this problem by assuming that the radii of protons are not identical. As a consequence, the cross section given by Eq. (1) must be averaged over all proton sizes, i.e., averaged over the form factor cut-off $\Lambda$. Therefore, the proton form factor must be also averaged in the squared form,

$$\langle G_{E,p}^2(Q^2, \Lambda_1) \rangle = \frac{1}{2\Delta\Lambda} \int_{\Lambda_1-\Delta\Lambda}^{\Lambda_1+\Delta\Lambda} G_{E,p}^2(Q^2, \Lambda) d\Lambda,$$

(2)

where $2\Delta\Lambda$ represents the range of the cut-off variation near the middle value $\Lambda_1$.

2. Extraction of the proton radius
By using Eq.(2) and assuming a dipole for $G_{E,p}(Q^2, \Lambda)$ we fitted the lowest $Q^2$ form factor data given in Ref. [4]. The lowest value of $\chi^2/N$ was obtained for $\Delta\Lambda/\Lambda_1 = 21.5\%$ and $\Lambda_1 = 0.8203$ fm.
Figure 1. The value of $\chi^2/N$ as functions of the proton charge radius $r$ and $\Delta \Lambda/\Lambda_1$. The position of smallest $\chi^2/N$ is indicated by the intersection of the two dashed (blue) lines.

$\chi^2/N$ value as functions of the proton radius and the ratio of cut-off range is depicted in Fig. 1, where we can see that the lowest value of $\chi^2$ is obtained for $r_E = 0.864$ fm and $\Delta \Lambda/\Lambda_1 = 21.5\%$. Applying the relativistic correction explained in Ref. [11], the radius decreases to 0.844 fm [12], which is very close to the recent muonic hydrogen atom measurement [3].

3. Radius of the neutron star

As described in our previous work [5] the energy density of the neutron star matter depends on the densities of nucleon and scalar particles. Therefore, the effect of different nucleon radii enters the neutron star calculation via the nucleon and scalar densities

$$\rho_i = A \bar{\rho}_i \, , \quad \rho_{s,i} = A \bar{\rho}_{s,i} \, ,$$

(3)
Figure 2. The neutron star mass as a function of its radius obtained by assuming five different nucleon radii $r_N$. The result has been obtained by using $\beta = 0.01$ in Eq. (5). The gray horizontal band shows the mass of the SRJ164-2230 pulsar, which is believed to be the heaviest observed neutron star [10].

where $\bar{\rho}_i$ and $\bar{\rho}_{s,i}$ indicate the densities of the $i$th nucleon and scalar particles if they were point particles. In Eq. (3) the proportionality constant $A$ reads

$$A = \frac{1}{1 + V_p\bar{\rho}_p + V_n\bar{\rho}_n}, \quad (4)$$

where $V_p$ and $V_n$ are the proton and neutron volumes, respectively, and we have assumed that $V_p = V_n \equiv V_N = \frac{4}{3}\pi r_N^3$.

The dependency of the nucleon radius $r_N$ on the density of matter is parameterized by using

$$r_N(\rho) = r_N(0)\left\{1 + \beta \left(\frac{\rho}{\rho_0}\right)^2\right\}^{-2}, \quad (5)$$

where $\rho = \rho_p + \rho_n$ and $\rho_0$ is the value of $\rho$ at saturation point. The ratio between $r_N$ and $r_N(0)$ as a function of the matter density $\rho$ for different values of $\beta$ is shown in Fig. 6 of Ref. [5].

To obtain the neutron star mass as a function of its radius we solve the Tolman-Oppenheimer-Volkoff (TOV) equation with different particle densities in the neutron star core. The result is shown in Fig. 2, where we compare the effect of different nucleon radii on the mass–radius profile of the neutron star. Note that the new calculation is represented by $r_N = 0.86$ fm in Fig. 2, i.e., the solid black line.

Obviously, the calculated radius of the neutron star depends on the value of nucleon radius $r_N$, while the maximum mass of neutron star is determined by the dependence of the nucleon radius on the matter density ($\beta$). As shown in Ref. [5], the choice of $\beta$ is very decisive here, the use of $\beta = 0.0005$ could overshoot the experimental data. Figure 2 also shows that the corrected proton radius $r_N = 0.86$ fm only slightly shifts the mass-radius relation from our previous result, which used $r_N = 0.83$ fm. Nevertheless, the discrepancy between the present result and the point-particle approximation increases and becomes more apparent.
Figure 3. As in Fig. 2, but for the neutron star mass as a function of its center pressure. Solid square, star, triangle, circle, and diamond indicate the maximum masses with the corresponding center pressures.

Figure 3 displays the relation between the neutron star mass and its center pressure. Clearly, the effect of the correction on proton radius is quite subtle in this case. As a conclusion, we may say that the effect of the correction on proton radius is very mild in the calculation of the neutron star. Given that the real result would be obtained by averaging the uncertainty in the extracted proton radius, i.e., about 20%, it is safe to say that the effect of the correction is trivial in this case.

4. Conclusion
We have corrected our calculation of the proton radius extraction and found a new radius of 0.864 fm, instead of 0.833 fm. Without the relativistic correction this value is larger than the new measurement obtained from the muonic Hydrogen atom, but still smaller than the standard value. This correction does not, however, disturb our result of the neutron star calculation.

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