ω - φ mixing at finite temperature

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Abstract

We compute the mass shifts and mixing of the ω and φ mesons at finite temperature due to scattering from thermal pions. The ρ and b₁ mesons are important intermediate states. Up to a temperature of 140 MeV the ω mass increases by 12 MeV and the φ mass decreases by 0.6 MeV. The change in mixing angles is negligible.

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The emission of lepton pairs from high temperature matter formed in high energy nucleus–nucleus collisions is of great theoretical and experimental interest. It can signal a change in the properties of hadrons (more precisely, correlation functions) in hot hadronic matter as it approaches a chiral symmetry or a quark deconfinement phase transition or rapid crossover. Lepton pair emission has been measured by the DLS collaboration \cite{1} at the (disassembled) Bevalac at LBNL, by the CERES \cite{2} and HELIOS \cite{3} collaborations at the SPS at CERN, and will be measured in the PHENIX experiment \cite{4} at RHIC at BNL.

Vector mesons are prominent in these studies because of their coupling to the electromagnetic current. Most studies have focussed on the lighter ones, $\rho$, $\omega$, and $\phi$, and on the heavier $J/\psi$. The best signals are those that have a characteristic shape or structure. The $\rho$ meson is very broad in vacuum and undoubtedly gets even broader at finite temperature. The $J/\psi$ is narrow; it has a whole literature of its own within the field. The $\omega$ and $\phi$ mesons are rather narrow, but not so narrow that they will all decay after the hot matter has blown apart in a high energy collision. This makes them good candidates to study.

In vacuum the $\phi$ meson is almost entirely $\bar{s}s$ in its valence quark content while the $\omega$ meson is almost entirely nonstrange. There is a small mixing as evidenced by the observed decay mode $\phi \to \pi \rho$ and by studies using effective hadronic Lagrangians. In this paper we will study the change in masses and mixing angles of the $\omega$ and $\phi$ at moderate temperatures using only conventional ideas. We consider our study a simple extrapolation of known physics which may help in deciphering future experiments. Specifically, we study the scattering of these mesons from thermal pions which are the most abundant mesons at temperatures below 100 MeV or so. We use effective Lagrangian techniques to model the relatively soft interactions coupling pions, $\omega$, and $\phi$ mesons, fitting the parameters to known physical quantities. The resulting scattering amplitudes are used in a virial expansion to compute the vector meson properties at finite temperature.

A survey of the literature on effective hadronic Lagrangians and the Review of Particle Physics \cite{5} suggests that the interactions of relevance involve the 3–point vertices $\phi \rho \pi$, $\omega \rho \pi$, $\phi b_1 \pi$, and $\omega b_1 \pi$. The vector self–energies are obtained by computing one loop diagrams at finite temperature. We only keep the contribution from thermal pions in the loop. This corresponds to a virial expansion where the self–energy is obtained from the forward scattering amplitude of the vector meson from a pion with a Bose–Einstein momentum distribution. We do not consider thermal scattering from kaons or $\eta$ mesons \cite{6}. Those contributions would be important at higher temperatures and are the same order in the Boltzmann factor as the scattering from two thermal pions consecutively. The latter would require the computation of two loop self–energy diagrams at finite temperature which is notoriously difficult.
The interaction involving the $\rho$ meson is described by the Wess–Zumino term \[7\].

\[ L_{(\omega,\phi)\rho\pi} = g e^{\alpha\beta\mu\nu} \partial_{\alpha} \rho_{\beta} \cdot \pi \left( \partial_{\mu} \omega_{\nu}^{\delta} + \sqrt{2} \partial_{\mu} \omega_{\nu}^{s} \right) / \sqrt{3} \]  
(1)

The octet and singlet fields, $\omega_{8}$ and $\omega_{s}$, are expressed in terms of the (vacuum) physical fields with a mixing angle $\theta_{V}$.

\[ \begin{align*}
\omega_{8} &= \phi \cos \theta_{V} + \omega \sin \theta_{V} \\
\omega_{s} &= \omega \cos \theta_{V} - \phi \sin \theta_{V}
\end{align*} \]  
(2)

Ideal mixing is defined such that the physical $\phi$ meson would not couple to nonstrange hadrons. This corresponds to $\theta_{V} = \theta_{\text{ideal}} = \tan^{-1}(1/\sqrt{2}) \approx 35.3^{\circ}$. The real world is not far from that. Durso \[8\], for example, fits $39.2^{\circ}$, while the Review of Particle Physics quotes $39^{\circ}$ based on the Gell–Mann Okubo mass formula. The coupling constant $g$ is related to the coupling $g_{VV\rho\pi}$ used by Gomm, Kaymakcalan, and Schechter \[9\] by

\[ g = -\sqrt{2} g_{VV\rho\pi}. \]  

Gauging the Wess–Zumino term yields the prediction \[10\]

\[ g = \frac{3g_{\rho\pi\pi}^{2}}{8\pi^{2}f_{\pi}^{2}}. \]  
(3)

This is consistent with phenomenological studies. For example, Durso fits $g^{2} = 1.62 \pm 0.19 \times 10^{-4}$ MeV$^{-2}$ with a pseudoscalar mixing angle $\theta_{P} = -9.7^{\circ}$. We will insure that the parameters chosen reproduce the measured decay rate \[4\] using

\[ \Gamma_{\phi \rightarrow \rho \pi} = \frac{(\cos \theta_{V} - \sqrt{2} \sin \theta_{V})^{2} g_{\rho\pi\pi}^{2}}{288\pi} \left[ (m_{\phi}^{2} + m_{\pi}^{2} - m_{\rho}^{2})^{2} - 4m_{\pi}^{2}m_{\phi}^{2} \right]^{3/2}. \]  
(4)

Accepting Durso’s value of $g$ we fit $\theta_{V} = 40.1^{\circ}$.

The $b_{1}(1235)$ has a branching ratio of more than 50% into $\omega \pi$ and less than 1.5% into $\phi \pi$. Thus, the $b_{1}$ meson is important for the mass shift of the $\omega$ meson at finite temperature, as noticed by Shuryak \[11\]. The interactions of this resonant meson is not much studied, hence we are obliged to do so here. The interactions are assumed to be $\text{SU}(3)$ symmetric with $\text{SU}(3)$ broken only by mass terms. Suppressing Lorentz indices the interaction is of the form $b \cdot \pi (\omega_{8} + \sqrt{2} \omega_{s}) / \sqrt{3}$. Suppressing isospin indices there are five possibilities: $b_{\mu} \omega^{\mu} \pi$, $b_{\mu} \omega^{\mu} \rho_{5} \pi$, $b_{\mu} \omega^{\mu} \partial_{\nu} \pi$, $b_{\mu} \omega^{\mu} \partial_{\nu} \pi$, $b_{\mu} \omega^{\mu} \partial_{\nu} \pi$. The last of these forms is not independent of the second one if it is integrated by parts and the field conditions $\partial \cdot b = \partial \cdot \omega = 0$ are used. The third and fourth forms are not independent either in the weak field limit. For example, the form $b_{\mu} \omega^{\mu} \partial_{\nu} \pi$ becomes $-\partial_{\nu} b_{\mu} \omega^{\mu} \pi - b_{\mu} \partial_{\nu} \omega^{\mu} \pi$ upon integration by parts. The equation of motion is $\partial_{\mu} \omega^{\mu} = m_{\omega}^{2} \omega^{\nu} + \text{nonlinear field terms}$. The term linear in the $\omega$ field does not give rise to a new interaction term, and the
term which is nonlinear in the fields is not of relevance here. Therefore the interaction Lagrangian is

\[
\mathcal{L}_{(\omega,\phi)b_{1}\pi} = g_{b_{1}} \pi \cdot b^{\mu} \left( \frac{\omega_{\mu}^{8} + \sqrt{2} \omega_{\mu}^{s}}{\sqrt{3}} \right) + h_{b_{1}} \pi \cdot b^{\mu \nu} \left( \frac{\omega_{\mu \nu}^{8} + \sqrt{2} \omega_{\mu \nu}^{s}}{\sqrt{3}} \right). \tag{5}
\]

The two coupling constants can be inferred from the decay rate \(b_{1} \rightarrow \omega \pi\) and from the ratio of the D wave content of the decay amplitude to its S wave content. The spin-averaged squared matrix element for the decay rate is

\[
|\mathcal{M}_{b_{1} \rightarrow \omega \pi}|^{2} = (\sin \theta_{V} + \sqrt{2} \cos \theta_{V})^{2} \left\{ g_{b_{1}}^{2} \left[ 2 + \frac{(k \cdot q)^{2}}{m_{b_{1}}^{2} m_{\omega}^{2}} \right] + 12g_{b_{1}}h_{b_{1}}(k \cdot q) + 4h_{b_{1}}^{2} \left[ m_{b_{1}}^{2} m_{\omega}^{2} + 2(k \cdot q)^{2} \right] \right\}, \tag{6}
\]

where \(k \cdot q\) is the scalar product of the \(b_{1}\) and \(\omega\) four-momenta. By expanding the decay amplitude in spherical harmonics and comparing this expansion with an expression of the total decay amplitude containing the actual helicities, we relate the individual S and D amplitudes to the coupling constants as

\[
\begin{align*}
 f^{S} &= \sqrt{\frac{4\pi}{3}} \left\{ g_{b_{1}} + 2h_{b_{1}}(k \cdot q) \right\} \frac{2m_{\omega} + E_{\omega}}{m_{\omega}} - 2h_{b_{1}}|q| \frac{2m_{b_{1}}}{m_{\omega}} \\
 f^{D} &= \sqrt{\frac{8\pi}{3}} \left\{ g_{b_{1}} + 2h_{b_{1}}(k \cdot q) \right\} \frac{m_{\omega} - E_{\omega}}{m_{\omega}} + 2h_{b_{1}}|q| \frac{2m_{b_{1}}}{m_{\omega}}. \tag{7}
\end{align*}
\]

Here \(E_{\omega}\) and \(q\) are the energy and momentum of the \(\omega\) in the \(b_{1}\) rest frame. The Review of Particle Physics gives the width \(142 \pm 8\) MeV and the measured D/S ratio \(0.26 \pm 0.04\). Fitting to central values determines the most likely values of the coupling constants to be \(g_{b_{1}} = -9.471\) GeV and \(h_{b_{1}} = 6.642\) GeV\(^{-1}\).

The location of the poles and the mixing angle at finite temperature are obtained by finding the zeros of the inverse propagator in the (vacuum) physical \(\omega-\phi\) basis.

\[
\mathcal{D}^{-1}(k_{0}, k) = k_{0}^{2} - k^{2} - M^{2} - \Sigma(k_{0}, k) \tag{8}
\]

\(M^{2}\) is the 2×2 mass matrix at zero temperature. It is diagonal with components \(M_{11}^{2} = m_{\omega}^{2}\) and \(M_{22}^{2} = m_{\phi}^{2}\). The self–energy has contributions from the \(\rho\) meson and from the \(b_{1}\) meson. The first was calculated by Haglin and Gale [12]. The second is readily calculated by the usual finite temperature rules [13]. We restrict our attention to the finite temperature contribution. We consider only \(\omega\) and \(\phi\) mesons at rest in the many–body system \((k = 0)\) since that is where temperature will have its maximum impact. As \(|k|\) increases, many–body effects will decrease, and in the limit \(|k| \gg T\) they will
Figure 1: Mass shifts of the $\omega$ and $\phi$ mesons as functions of temperature due to scattering from thermal pions. Multiple pion scattering and scattering from other thermal mesons should become important above 100 MeV temperature.

disappear altogether. Finally, the imaginary part of the self–energy is small compared to the real part (inclusive of $M^2$) and so we do not include it in our present calculation. We find

$$\Sigma(k_0, |k| = 0) = \left( \frac{\cos^2 \delta_V}{\frac{\sin^2 \delta_V}{2}} \right) \left( \Sigma_{\rho} + \Sigma_{b_1} \right), \quad (9)$$

where $\delta_V = \theta_V - \theta_{\text{ideal}}$ measures the deviation from ideal mixing. The scalar functions are

$$\Sigma_{\rho} = -\frac{g^2 k_0^2 (k_0^2 + m_\pi^2 - m_\rho^2) \pi^2}{2} \int_0^\infty \frac{dp}{p^4} \frac{1}{\omega_\pi} \exp(\omega_\pi/T - 1) \cdot \frac{1}{(k_0^2 + m_\pi^2 - m_\rho^2)^2 - 4k_0^2 \omega_\pi^2} \quad (10)$$

and

$$\Sigma_{b_1} = -\frac{1}{2\pi^2 m_{b_1}} \int_0^\infty \frac{dp}{p^4} \frac{1}{\omega_\pi} \exp(\omega_\pi/T - 1) \cdot \frac{1}{(k_0^2 + m_\pi^2 - m_{b_1}^2)^2 - 4k_0^2 \omega_\pi^2} \times \left\{ (k_0^2 + m_\pi^2 - m_{b_1}^2) \left( 3m_{b_1}^2 \left[ g_{b_1}^2 + 4h_{b_1} k_0^2 (k_0^2 - m_\pi^2) + 4g_{b_1} h_{b_1} k_0^2 \right] + g_{b_1}^2 p^2 \right) \\
+ 8h_{b_1} m_{b_1} k_0^2 \left[ 3m_\pi^2 \left( m_\pi^2 - k_0^2 - m_{b_1}^2 \right) + p^2 \left( m_\pi^2 - m_{b_1}^2 - 5k_0^2 \right) \right] \\
- 24g_{b_1} h_{b_1} k_0^2 m_{b_1}^2 \omega_\pi^2 \right\}, \quad (11)$$

where $\omega_\pi = \sqrt{p^2 + m_\pi^2}$. Even in the limit of zero pion mass these integrals cannot be evaluated exactly in terms of elementary functions and so we resort to numerical integration.
It should be remarked that both integrals can have a pole within the limits of integration depending on the value of $k_0$. This is handled by the principal value prescription; the imaginary parts are not displayed separately. Inclusion of a finite width for both the intermediate $\rho$ and $b_1$ mesons or, better yet, use of dressed vacuum propagators, would lead to finite integrals. Only if we find a significant many-body effect should it be necessary to include this next level of sophistication.

After diagonalization one finds the mass shifts as functions of the temperature. They are displayed in Fig. 1. Although the results are plotted up to a temperature of 150 MeV to see the effect, other scattering processes will come into play above 100 MeV, as mentioned earlier. The $\omega$ mass goes up and a shift of roughly 12 MeV is reached at a temperature of 140 MeV. The $\phi$ mass decreases monotonically to about 0.6 MeV below its vacuum value at a temperature of 140 MeV.

Although a rigorous exploration of the available parameter space is not the point of this work, one can test the effect of simple variations. We have thus chosen a combination that enhances the mass shift of the $\omega$. The values $g_{b_1} = -11.401$ GeV and $h_{b_1} = 7.647$ GeV$^{-1}$ correspond to $D/S = 0.3$ and $\Gamma_{b_1 \to \omega\pi} = 150$ MeV. We are therefore still inside the measured error bars of the $b_1$'s hadronic properties. This choice of couplings represents our “maximum effect” set. The result of this exercise is to increase the mass shift of the $\omega$ and the $\phi$ by about 30% and 25%, respectively. This illustrates the sensitivity of our effective Lagrangian approach to empirical hadronic properties.

The mixing angles at finite temperature are shown in Fig. 2. The mixing angles remain uninterestingly small at all temperatures.
In conclusion, we have computed the change in the masses of the $\omega$ and $\phi$ mesons and their mixing angle up to temperatures of 100 MeV (pushing 150 MeV with the above caveat). The calculation is based on conventional physical input. The change in the $\phi$ mass and mixing angles are totally negligible. The change in the $\omega$ mass is large enough to be potentially detectable in future heavy ion experiments.

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