d-Wave Order Parameter in Bi2212 from a Phenomenological Model of High $T_c$ Cuprates

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Abstract

A phenomenological lattice model of high $T_c$ cuprates including order parameter phase fluctuations is considered within the BCS approximation, to interpret the experimental data from ARPES measurements on Bi2212 samples. A Kosterlitz-Thouless (KT) transition temperature $T_{cKT}$ is estimated below the mean field transition $T_{cMF}$, phase boundaries between competing order parameters of different symmetries are obtained and best model parameters, fitting the ARPES gap of $d_{x^2-y^2}$ symmetry, are determined. Variation of $T_{cKT}$, as a function of the dopant concentration $\delta$, is in qualitative agreement with experiments.

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The symmetry of the superconducting order parameter (OP) in the high $T_c$ cuprate materials has been a widely debated issue in the last few years. In the recent past, several experiments done on the copper oxide materials [1-7], including those of phase sensitive experiments [2], interference measurements [3] and c-axis Josephson tunneling [4], gave contrasting or inconclusive results regarding the OP symmetry. While a number of experiments [1, 2, 5] noted signatures of $d_{x^2-y^2}$ OP symmetry, others argued in favour of the anisotropic $S$-wave or more exotic $S+i\text{d}$ wave OP symmetry [3, 4, 6, 7]. However, recently there has been considerable progress in the angle resolved photoemission spectroscopy (ARPES) measurements [8, 9], and a consensus seems to be emerging about the OP symmetry [10] in the high $T_c$ cuprates.

ARPES can give quantitative estimate of the momentum dependence of the superconducting gap on the Fermi surface (FS) in terms of the spectral function representation of data and a detailed study of the FS is also possible [3, 9]. Due to its high angular as well as energy resolution [3, 9], it can provide detailed and reliable knowledge about the nodes of the gap on the FS. Shen and coworkers, by ARPES measurements on Bi2212 compounds [4], found nodes of the superconducting gap on the FS along the $45^\circ$ ($\pi$, $\pi$) direction, suggestive of a d-wave OP symmetry. Ding et. al., by similar measurements [6] on high quality Bi2212 single crystal, showed that the gap on the FS vanishes at two points (per quadrant) symmetrically displaced about the $45^\circ$ direction, consistent with an anisotropic S-wave ($S_{xy}$) symmetry. However, reanalysis of their data revealed that the two node gap was an artifact of the superstructure producing “ghost” bands [6]. In this light, the ARPES measurements on several Bi2212 samples were redone by Ding et. al. using dense sampling of the Brillouin zone (BZ) in the vicinity of the FS [9]. The results are now consistent with a $|\cos(k_x) - \cos(k_y)|$ type gap function implying a $d_{x^2-y^2}$ OP symmetry.

In order to interpret the two node gap data [6], a phenomenological BCS like lattice model in two dimension was introduced [12] and mean field (MF) analysis of the instabilities in the spin singlet Cooper channel was done. The model is of interacting electrons on a square lattice, with an on-site repulsive ($V_0$) and attractive nearest neighbour ($V_1$) as well as next nearest neighbour ($V_2$) interactions. The model Hamiltonian $H = H_0 + H_1$
is

\[ H_0 = \sum_{k,\sigma} (\epsilon_k - \epsilon_F)c_{k,\sigma}^\dagger c_{k,\sigma} \]  

(1a)

and

\[ H_1 = V_0 \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + V_1 \sum_{i,\sigma,\sigma'} \sum_{\delta_{nn}} \hat{n}_{i,\sigma} \hat{n}_{i+\delta_{nn},\sigma'} + V_2 \sum_{i,\sigma,\sigma'} \sum_{\delta_{nnn}} \hat{n}_{i,\sigma} \hat{n}_{i+\delta_{nnn},\sigma'} \]  

(1b)

Where, \( c_{k,\sigma}^\dagger \) (\( c_{k,\sigma} \)) is the quasiparticle creation (annihilation) operator of momentum \( \vec{k} \) and spin \( \sigma \), \( \hat{n} \) is the number operator and \( \delta_{nn}, \delta_{nnn} \) are nearest neighbour and next nearest neighbour lattice vectors. The band dispersion \( \epsilon_k \) was obtained by a six parameter tight binding fit to the normal state ARPES data on Bi2212 single crystal \[12\] where the parameters are \([t_0, \ldots, t_5] = [0.131, -0.149, 0.041, -0.013, -0.014, 0.013]\) (in eV). Here \( t_0 \) is the orbital energy, \( t_1 \) nearest neighbour (nn), \( t_2 \) next nearest neighbour (nnn) etc. hopping matrix elements. Quasiparticle dispersion \( \epsilon_k \) incorporates flat dispersion around \((0, \pi)\) and \((\pi, 0)\) points which results van Hove singularity (vHS) in the single particle density of states (DOS).

The mean field analysis of the model \[12\] found strong instabilities for the \( d_{x^2-y^2} \) and \( S_{xy} \) OPs which can best exploit the large single particle DOS just below the FS. Ratio of the interaction parameters \( V_1/V_2 \) determines the relative stability of these two states. Further extension of the mean field work, to include strong fluctuations present in the quasi two dimensional cuprate materials, was carried out and results were reported in a previous communication \[13\].

In this paper, we reanalyse the phenomenological model, including order parameter phase fluctuations, in view of the conclusive finding of a \( d_{x^2-y^2} \) OP symmetry by the ARPES measurements on Bi2212 \[3,4\]. We first calculate a Kosterlitz-Thouless (KT) transition temperature \( T_c^{KT} \), using the helicity modulus or superfluid phase stiffness \( (\rho_s) \) expression of the present model together with the KT relation \( \rho_s(T_c^{KT}) = \frac{2\pi k_B T_c^{KT}}{\pi} \), within each of the irreducible representation \( B_1 \) (\( d_{x^2-y^2} \)), \( B_2 \) (\( d_{xy} \)) and \( A_1 \) (\( S, S^* \) and \( S_{xy} \)).

We then find out the phase diagrams showing regions of relative stability of \( B_1 \), \( B_2 \) and \( A_1 \) states in the interaction parameter planes and determine the best model parameters corresponding to the \( d_{x^2-y^2} \) gap with \( T_c^{KT} \sim 100 \text{ K} \). We also study the momentum
dependence of the gap function on the FS and dopant concentration ($\delta$) dependence of $T_c^{KT}$. Our main results are summarized below:

(a) Best model parameters at the optimal doping level $\delta = 0.17$, with a $T_c^{KT} \sim 100 \text{ K}$ and for the stable $B_1 (d_{x^2-y^2})$ state, are $V_0 \geq 400 \text{ meV}$, $V_1 \approx -48 \text{ meV}$ and $V_2 \sim -85 \text{ meV}$.

(b) Zero temperature $d_{x^2-y^2}$ gap on the FS, with $V_1 = -48 \text{ meV}$, match well the ARPES data [9] in the vicinity of 45° direction, but deviates from it as one moves beyond about 7 degrees on either sides.

(c) With varying dopant concentration ($\delta$), the gap magnitude at 0° angular direction changes considerably, although the node position is same for all $\delta$ by virtue of the momentum dependent part of the $d_{x^2-y^2}$ gap function.

(d) $T_c^{KT}$ for $B_1 (d_{x^2-y^2})$ state as a function of $\delta$ shows correct qualitative behaviour as in the high $T_c$ cuprate materials.

Within the standard BCS approximation, the Hamiltonian of Eq.(1) yields the gap equation

$$\Delta_{\vec{k}} = \frac{1}{N} \sum_{\vec{k}'} V(\vec{k} - \vec{k}') \frac{\Delta_{\vec{k}'}}{2E_{\vec{k}'}} \tanh \left( \frac{\beta E_{\vec{k}'}}{2} \right)$$

(2)

where the quasiparticle energy is $E_{\vec{k}'} = \sqrt{(\epsilon_{\vec{k}} - \epsilon_{F})^2 + |\Delta_{\vec{k}}|^2}$ and $\Delta_{\vec{k}}$ is the BCS gap function. The pairing interaction, $V(q) = V_0 + 4V_1 (\cos q_x + \cos q_y) + 8V_2 \cos q_x \cos q_y$, is separable as $V(\vec{k} - \vec{k}') = \sum_{i=0}^{4} V_i \eta_i(\vec{k}) \eta_i(\vec{k}')$. An expansion of the order parameter $\Delta_{\vec{k}} = \sum_{i=0}^{4} \eta_i(\vec{k}) \Delta_i$ gives the linearized gap equation

$$\Delta_i = -\frac{\bar{V}_i}{2N} \sum_{\vec{k}} \frac{\eta_i(\vec{k})}{E_{\vec{k}}} \tanh \left( \frac{\beta E_{\vec{k}}}{2} \right) \sum_{j \in R} \Delta_j \eta_j(\vec{k})$$

(3)

where $\eta_0(\vec{k}) = 1$, $\eta_1(\vec{k}) = \frac{1}{2} (\cos k_x + \cos k_y)$, $\eta_2(\vec{k}) = \frac{i}{2} (\cos k_x - \cos k_y)$, $\eta_3(\vec{k}) = \cos k_x \cos k_y$, $\eta_4(\vec{k}) = \sin k_x \sin k_y$ are the basis functions corresponding to $[S, S^*, d_{x^2-y^2}, S_{xy}, d_{xy}]$ symmetries, and $(\bar{V}_0...\bar{V}_4) = (V_0, 8V_1, 8V_1, 8V_2, 8V_2)$. We suppress writing the terms corresponding to triplet pairing and ignore them in our analysis.
\( R \equiv (A_1, B_1, B_2) \) are different irreducible representations of the \( C_{4v} \) group and the gap equation factorizes to independent \( \Delta_2 \) (\( B_1 \) representation), \( \Delta_4 \) (\( B_2 \) representations) and three coupled linear equations \( \Delta_0, \Delta_1, \Delta_3 \) (\( A_1 \) representation). Since \( \Delta_3 \) is the predominant component within the \( A_1 \) representation, it is identified as a state of \( S_{xy} \) OP symmetry.

To calculate the helicity modulus \( \rho_s \) (and hence \( T^c_{KT} \) thereafter), a transverse vector potential with the gauge \( A_y = 0 \) is considered. This introduces an extra phase which the carriers acquire while moving between the lattice sites. Hence the hopping matrix elements in \( H_0 \) (Eq.(1a)) should be changed through Peierls substitution \( t_{ij} \rightarrow t_{ij} \exp \left[ \frac{i e}{\hbar c} \int_{R_i}^{R_j} \vec{A} \cdot d\vec{l} \right] \). We work here with the units \( \hbar = c = 1 \), but explicitly write them whenever necessary.

The electron current operator \( \hat{j}_x(\vec{R}_i) \) consists of the usual paramagnetic and diamagnetic terms \([14]\). To linear order in \( A_x \), \( \hat{j}_x(\vec{R}_i) \) is obtained by differentiating \( H_0 \) with respect to \( A_x(\vec{R}_i) \)

\[
\hat{j}_x(\vec{R}_i) = -c \frac{\partial H_0}{\partial A_x(\vec{R}_i)} = \hat{j}^\text{para}_x(\vec{R}_i) + \hat{j}^\text{dia}_x(\vec{R}_i)
\]

In Eq.(4), the paramagnetic term does not involve \( A_x \) and is the electron velocity operator. The diamagnetic term is linear in \( A_x \) and stems from the Meissner screening of the condensate. Average value of the diamagnetic current density is obtained as

\[
j_x^\text{dia}(\vec{q}) = -\frac{e^2}{\hbar c N} \sum_{k,\sigma} \left\langle c_{k,\sigma}^\dagger c_{k,\sigma} \right\rangle \frac{\partial^2 E_k}{\partial k_x^2} A_x(\vec{q})
\]

where the \( \langle \rangle \) represents an average in the mean field superconducting state. In a London like relation \( j_x(\vec{q}) \propto -\rho_s A_x(\vec{q}) \), the diamagnetic contribution to the phase stiffness \( (\rho_s^{\text{dia}}) \) is proportional to \( \frac{1}{N} \sum_{k,\sigma} \left( c_{k,\sigma}^\dagger c_{k,\sigma} \right) \frac{\partial^2 E_k}{\partial k_x^2} \), the mean electronic kinetic energy along the \( x \)-direction \([13]\). The average in the lattice model turns out to be \( \left\langle c_{k,\sigma}^\dagger c_{k,\sigma} \right\rangle = \frac{1}{2} \left[ 1 - \frac{\epsilon_x - \epsilon_F}{E_k} \tanh \left( \frac{\beta E_k}{2} \right) \right] \), unlike the continuum case where \( \rho_s^{\text{dia}} \propto -\sum_{k,\sigma} \left( \frac{\partial f(\epsilon_k)}{\partial \epsilon_k} \right)^2 \frac{\partial f(\epsilon_k)}{\partial \epsilon_k} \) \( (f \) is the Fermi function).

Contribution of the paramagnetic part is evaluated using linear response theory. In the long wavelength limit, the paramagnetic current is found to be

\[
j_x^\text{para}(\vec{q}) = -\frac{1}{c} \left[ \lim_{|\vec{q}| \to 0} \lim_{\omega \to 0} K^{xx}(\vec{q}, \omega) \right] A_x(\vec{q})
\]
where \( K^{xx}(q, \omega) = -i \int dt \theta(t) e^{i\omega t} \langle [j_{xx}(q, t), j_{xx}(-q, 0)] \rangle \). The correlation function in Eq.(6) is evaluated to be \( K^{xx}(q \to 0, \omega \to 0) = e^{2} \bar{\hbar}^{2} \sum_{\vec{k}} \left( \frac{\partial \varepsilon_{\vec{k}}}{\partial k_{x}} \right)^{2} \frac{\partial f(E_{\vec{k}})}{\partial E_{\vec{k}}} \). Taking the contributions from diamagnetic and paramagnetic parts, from Eqs.(5) and (6), we obtain the expression for superfluid phase stiffness

\[
\rho_{s}(T) = \frac{1}{2N} \sum_{\vec{k}} \left[ \left( \frac{\partial \varepsilon_{\vec{k}}}{\partial k_{x}} \right)^{2} \frac{\partial f(E_{\vec{k}})}{\partial E_{\vec{k}}} + \frac{1}{2} \frac{\partial^{2} \varepsilon_{\vec{k}}}{\partial k_{x}^{2}} \left\{ 1 - \frac{\varepsilon_{\vec{k}} - \varepsilon_{F}}{E_{\vec{k}}} \tanh \left( \frac{\beta E_{\vec{k}}}{2} \right) \right\} \right].
\]  

(7)

It should be mentioned here that, we work in a transverse gauge and vertex corrections required to get a gauge invariant current [14] have not been included.

Above expression for \( \rho_{s} \) involves linearized BCS gaps. In the inset of Fig.1 we plot the superconducting gaps for different order parameters corresponding to \( B_{1}, B_{2} \) and \( A_{1} \) states, as a function of temperature. The point where gaps become nonzero mark the mean field transition \( T_{c}^{MF} \) for a state. These gaps, as shown in the inset of Fig.1, are used to calculate \( \rho_{s}(T) \) for different states. The KT transition temperature for each state is found by comparing the \( \rho_{s}(T) \) curve for each state, with the KT relation \( \rho_{s}(T_{c}^{KT}) = \frac{2}{\pi} T_{c}^{KT} \). In Fig.1 the intersecting point of a \( \rho_{s} \) curve with the KT straight line (emerging from the origin) gives \( T_{c}^{KT} < T_{c}^{MF} \). Similar technique was applied previously by Danteneer and coworkers [16] for the two dimensional attractive Hubbard model and correct behaviour of \( T_{c}^{KT} \), including its inverse coupling dependence in the strong coupling limit, was found. We too find a similar strong coupling dependence of \( T_{c}^{KT} \) in the present model. Thus, order parameter phase fluctuation degrades the mean field transition temperature. However, \( T_{c}^{KT} \) in our case is an upper bound of the actual KT transition temperature, that could be calculated only by considering superconducting gap renormalization due to the presence of vortex like fluctuations.

Next, we consider the phase boundaries, calculated by comparing the \( T_{c}^{KT} \) values of the competing states. In Fig.2, we plot phase boundaries separating \( d_{x^{2}-y^{2}} \) and \( A_{1} \) states in the \((-V_{1}, -V_{2})\) plane, for various values of \( V_{0} \). It clearly shows an widening of the the \( d_{x^{2}-y^{2}} \) stable region with increasing on site repulsion for small values of \(-V_{1} \). Rate of this widening is faster for small \( V_{0} \). For a fixed \(-V_{1} \), \( d_{x^{2}-y^{2}} \) solution is preferred over \( A_{1} \) up to a maximum \( V_{2} \). As for example, with \( V_{0} = 100 \) meV and \( V_{1} = -48 \) meV, \( d_{x^{2}-y^{2}} \) solution is stable for \(-V_{2} \leq 80 \) meV [17]. A comparison of the KT phase diagrams with
those from mean field calculations, can be found in Ref.[13].

To determine the optimal model parameters, one must also consider the phase boundary between $A_1$ and $d_{xy}$ states and find out the region where $A_1$ solution is not stable. In Fig.3, we plot such phase boundaries in the $(-V_2, V_0)$ plane for different values of $V_1$. To get the KT transition temperature $T_{cKT} \sim 100K$ for $d_{x^2-y^2}$ OP, one must have $V_1 \approx -48\text{meV}$. In Fig.3, if we set $V_1 = -48\text{meV}$ and $V_2 = -85\text{meV}$, then to rule out the stability of $A_1$ state, one must have $V_0 \geq 400\text{meV}$. Thus, we fix the optimal model parameters to be $V_1 = -48\text{meV}$, $V_2 = -85\text{meV}$ and $V_0 \geq 400\text{meV}$.

With these optimal parameters, we study the momentum dependence of the zero temperature $d_{x^2-y^2}$ gap on the FS at $\delta = 0.17$. A plot of $|\Delta_k(T=0)| = |\Delta_2(T=0)|\times\frac{1}{2}(|\cos(k_x)|-|\cos(k_y)|)$ is presented in Fig.4 for two different values of $V_1$. Solid circles are ARPES data [9] of $T_c \sim 87\text{K}$ Bi2212 single crystal sample in the Y quadrant [18]. We find that, our $V_1 = -48\text{meV}$ curve matches well the experimental data till about 7° away from node position 45° on either sides, and falls below ARPES data beyond this. A much better match could be obtained with $V_1 = -58\text{meV}$. But, this makes $T_{cKT} \sim 135\text{K}$ which is well above the sample $T_c \sim 87\text{K}$. One probable argument in favour of this could be that, our $T_{cKT}$ is an upper bound of the KT transition temperature. In actual case, gap renormalization due to the vortex fluctuations might reduce $T_{cKT}$. If one assumes a 25% reduction, a new set of best model parameters can be obtained as $V_1 = -58\text{meV}$, $V_2 = -90\text{meV}$ and $V_0 \geq 400\text{meV}$. In the inset of Fig.4, we plot the $\delta$ dependence of the zero temperature gap which provides a feel for the $d_{x^2-y^2}$ gap magnitude at the FS angle $\phi = 0$ degree.

Variation of $T_{cKT}$ for $d_{x^2-y^2}$ with dopant concentration $\delta$ is presented in Fig.5. This shows a correct qualitative bell shaped behaviour as in the high $T_c$ experiments. As a comparison, we also include $T_{cKT}$ for $A_1$ state for the optimal parameters, which is below the $d_{x^2-y^2}$ $T_{cKT}$ curve for all $\delta$. The $T_{cKT}$ curve here (and also $\Delta_2$ vs $\delta$ curve in the inset of Fig.4) peaks at around $\delta = 0.25$, that is consistent with the peak in the pairing density of states around $\epsilon_{vHS} \approx -20\text{meV}$ which corresponds to $\delta \approx 0.25$.

To summarize, we studied a phenomenological BCS model of high $T_c$ cuprate superconductors including fluctuations, calculated phase boundaries separating order pa-
ters of different symmetries within the model and determined optimal model parameters to fit the $d_{x^2-y^2}$ superconducting gap as in the experiment. We also studied the dopant concentration dependence of the transition temperature which is in qualitative agreement with the experiments.

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References

[1] D. H. Wu et. al., Phys. Rev. Lett. 70, 85 (1993); W. N. Hardy et. al., Phys. Rev. Lett. 70, 3999 (1993).

[2] D. A. Wollman et. al., Phys. Rev. Lett 71, 2134 (1993); ibid 74, 797 (1995); D. A. Brawner and H. R. Ott, Phys. Rev. B 50, 6530 (1994); C. C. Tsuei et. al., Phys. Rev. Lett. 73, 593 (1994); J. R. Kirtley et. al., Nature (London) 373, 225 (1995).

[3] P. Chaudhari and S. -Y. Lin, Phys. Rev. Lett 72, 1084 (1994).

[4] A. G. Sun et. al. Phys. Rev. Lett. 72, 2267 (1994); R. Kleiner et. al. Phys. Rev. Lett. 76, 2161 (1996).

[5] Z. -X. Shen et. al., Phys. Rev. Lett. 70, 1553 (1993).

[6] H. Ding et. al., Phys. Rev. Lett. 74, 2784 (1995).

[7] S. Chakravarty, A. Sudbø and P. W. Anderson, Science 261, 3999 (1993).

[8] M. Randeria et. al. Phys. Rev. Lett. 74, 4951 (1995).

[9] H. Ding et. al., Phys. Rev. Lett. 76, 1533 (1996); Preprint, cond-mat/9603044 (1996).

[10] D. J. Scalapino, Physics Reports 250, 392 (1995); M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991); J. Annet et. al., Preprint, cond-mat/9601060 (1996).

[11] M. R. Norman et. al., Phys. Rev. B 52, 15107 (1995).

[12] R. Fehrenbacher and M. R. Norman, Phys. Rev. Lett. 74, 3884 (1995); M. R. Norman et. al., Phys. Rev. B 52, 615 (1995).

[13] B. Chattopadhyay, D. Gaitonde and A. Taraphder, Europhys. Lett. (in press).

[14] J. R. Schrieffer, Theory of Superconductivity, Addison-Wesley Publishing Company, New York (1964).

[15] D. J. Scalapino, S. R. White and S. C. Zhang, Phys. Rev. Lett. 68, 2830 (1992).
[16] P. J. H. Denteneer et. al., Europhys. Lett. 16, 5 (1991).

[17] Below some values of $-V_1$ and $-V_2$, KT transition temperature $T^{KT}_c < 1 K$ and it's not possible to determine the phase boundaries beyond these parameter values.

[18] In Y quadrant, the main CuO FS is widely separated from the two ghost FS and gap data are more reliable. Hence, Y quadrant data from Ref.[9] is chosen for comparison.
FIGURE CAPTIONS

Fig.1. Helicity modulus $\rho_s$ is plotted as a function of temperature for different irreducible representations, at optimal doping $\delta = 0.17$ and with best model parameters $V_0 = 400 \text{ meV}$, $V_1 = -48 \text{ meV}$ and $V_2 = -85 \text{ meV}$. The solid straight line, originating from $(0,0)$ point, is the $\rho_s(T_{cKT}) = \frac{2}{\pi}T_{cKT}$ line. [Inset: Mean field order parameters of different symmetries corresponding to $A_1$, $B_1$ and $B_2$ representations.]

Fig.2. Phase boundaries, separating the regions of stability of the $A_1$ and $d_{x^2-y^2}$ states, in the $(-V_1, -V_2)$ plane, for $\delta = 0.17$ and various $V_0$ values as shown in the figure. [Inset: Phase diagram for $V_0 = 0.4 \text{ eV}$ is plotted on larger scales.]

Fig.3. Phase boundaries indicating the regions of stability of the $A_1$ and $d_{xy}$ states in the $(-V_2, V_0)$ plane at $\delta = 0.17$ for various $V_1$ values shown. Inset shows the phase diagram for $V_1 = -48 \text{ meV}$ on an expanded scale.

Fig.4. Momentum dependence of the gap function $|\Delta_k(T = 0)|$ on the Fermi surface is plotted against the Fermi surface angle $\phi$, for $\delta = 0.17$ and for $V_1$ given in the figure. The angle $\phi$ is measured with respect to the line joining $(\pi, \pi)$ and $(0, \pi)$ points. Solid circles are ARPES data from Ref.[9]. [Inset: Zero temperature $d_{x^2-y^2}$ gap as a function of $\delta$.]

Fig.5. Kosterlitz-Thouless transition temperature $T_{cKT}$ for $d_{x^2-y^2}$ and $A_1$ states as a function of dopant concentration $\delta$, for optimal model parameters.