Single-layer Compton detectors for measurement of polarization correlations of annihilation quanta

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Abstract

Measurement of gamma-ray polarization can provide valuable insight in different areas of physics research. One possible application is in Positron Emission Tomography, where the annihilation quanta with orthogonal polarizations are emitted. Since polarization can be measured via Compton scattering, the initial orthogonality of polarizations can be translated to correlation of azimuthal scattering angles, and this correlation may be exploited as an additional handle to identify the true coincidence events. In order to examine the concept of utilizing the polarization correlations in PET, we have used a system of two compact, position and energy-sensitive Compton scattering detectors in coincidence mode. Each consists of a single matrix of scintillation pixels, read-out by a matching array of Silicon photomultipliers on the back side. The Compton events in each module are clearly identified and the scattering angles are reconstructed from the energy deposition and event topology. We have extracted the polarimetric modulation factors from the distributions of the difference of the azimuthal angles of the two Compton-scattered gammas and studied their dependence on Compton scattering angles $\theta$ and on azimuthal resolution $\Delta\phi$.

For scattering angles around $\theta_{1,2} = 82^\circ$, where the maximum modulation is expected, the modulation factors from $\mu = 0.15 \pm 0.01$ to $\mu = 0.27 \pm 0.02$ have been measured, depending on the azimuthal resolution, which is governed by event topology in the detectors. Analogously, for scattering around $\theta_{1,2} = 70^\circ$, modulation factors from $\mu = 0.12 \pm 0.01$ to $\mu = 0.21 \pm 0.02$ have been obtained. The results show that the measurement of the polarization correlations of annihilation quanta are feasible with compact single-layer, single-side read-out detectors, which may be used to build cost-efficient systems for various applications where gamma-ray polarization information is of interest.

Keywords: Gamma-ray polarization, Positron Emission Tomography, Compton imaging

1 Introduction

Gamma ray polarization measurement relies on Compton scattering, where according to Klein-Nishina cross-section, the most probable azimuthal scattering angle of the gamma is perpendicular to the incident polarization vector. Although explored for astrophysics (e.g. 1 2 3 4 5 6 7), the polarization of gammas has not yet been implemented in biomedical imaging. One potential use can be Positron Emission Tomography (PET). Gammas emitted from $e^+e^-$ annihilation have initially orthogonal polarizations. If both gammas undergo Compton scattering, the orthogonality of their polarizations will with a high probability result in orthogonality of their azimuthal scattering angles. Since the polarization is independent of energy, this azimuthal (polarization) correlation offers another independent handle to identify the true coincident events. Preliminary studies have shown that exploiting this feature has a potential to contribute to the image quality of a PET system, especially with sources of high activity where the probability of a false positive coincidence is significant 8. A Monte-Carlo model of a PET system utilizing Compton scattering for polarization measurement has been developed to demonstrate the feasibility of the approach 9 10. To date, however, this has not been demonstrated experimentally.

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In this paper we present that measurement of polarization correlations of annihilation quanta is possible with modules encompassing a single array of scintillation pixels and silicon photomultipliers (SiPM). The single-layer concept makes the modules compact and cost-efficient in comparison with more common dual layer Compton detectors, and opens possibility to use them in various applications where the information about polarization of gamma-rays is of interest.

2 Measurement of Polarization Correlations

Two photons originating from $e^+e^-$ annihilation are emitted back-to-back with 511 keV energy and orthogonal polarizations. In case both of them undergo Compton scattering with scattering angles $\theta_{1,2}$ and azimuthal angles $\phi_{1,2}$, respectively, the differential cross-section is given by [11, 12]:

$$\frac{d^2\sigma}{d\Omega_1d\Omega_2} = \frac{r^4}{16} F(\theta_1)F(\theta_2) \left\{ 1 - \frac{G(\theta_1)G(\theta_2)}{F(\theta_1)F(\theta_2)} \cos[2(\phi_1 - \phi_2)] \right\}$$  \hspace{1cm} (1)

with

$$F(\theta_i) = \frac{2 + (1 - \cos \theta_i)^3}{(2 - \cos \theta_i)^3}, \quad G(\theta_i) = \frac{\sin^2 \theta_i}{(2 - \cos \theta_i)^2}$$  \hspace{1cm} (2)

where $i = 1, 2$. Since initially, the polarization vectors of both photons are orthogonal, the cross-section has the maximum for $|\phi_1 - \phi_2| = 90^\circ$ (for the fixed scattering angles $\theta_{1,2}$), and so the polarization correlation is preserved in the Compton scattering process. The sensitivity of the measurement to the initial polarizations is characterized by the polarimetric modulation factor, defined as:

$$\mu \equiv \frac{P(\phi_1 - \phi_2 = 90^\circ) - P(\phi_1 - \phi_2 = 0^\circ)}{P(\phi_1 - \phi_2 = 90^\circ) + P(\phi_1 - \phi_2 = 0^\circ)}$$  \hspace{1cm} (3)

where $P(\phi_1 - \phi_2 = 90^\circ)$ and $P(\phi_1 - \phi_2 = 0^\circ)$ are the probabilities to observe the two scattered gammas with perpendicular and parallel azimuthal angles, respectively.

The polarimetric modulation, $\mu$, depends on Compton scattering angles $\theta_{1,2}$ and it has been shown that it reaches the maximum $\mu_{max} = 0.48$ for $\theta_1 = \theta_2 \approx 82^\circ$ [12]. Although the correlation is the strongest at $\theta_1 = \theta_2 \approx 82^\circ$, one has to take into account the probabilities to have Compton scattering at those angles (i.e. the cross-section) in order to take the optimal advantage of the polarization correlation as a tool to recognize the true coincident events in PET. For the 511 keV gamma photons, the cross-section for single photon Compton scattering will peak at forward angles, $\theta \approx 34^\circ$. Therefore, the region around $\theta_{1,2} \approx 70^\circ$ has been suggested as optimal [10], since it provides a relatively high scattering probability and a relatively strong polarimetric modulation factor, $\mu = 0.40$. It has to be noted that the modulation factors are somewhat reduced in finite (realistic) detector geometries, since they are integrated over the angular acceptance of the detectors [12].

Experimentally, we can determine the polarimetric modulation factor by measuring the distribution of azimuthal angle differences, $N(\phi_1 - \phi_2)$, for given range of scattering angles $\theta_1$ and $\theta_2$. This observed distribution must be corrected for the non-uniformities in detector acceptance, as:

$$N_{cor}(\phi_1 - \phi_2) = \frac{N(\phi_1 - \phi_2)}{A_n(\phi_1 - \phi_2)}$$  \hspace{1cm} (4)

where the $A_n(\phi_1 - \phi_2)$ is the normalized acceptance for the particular azimuthal angle difference, which can be obtained from simulation or experimentally by measuring the distributions of randomly-polarized sources. We adopt the latter approach, where the acceptance correction $A_n(\phi_1 - \phi_2)$ is obtained by event-mixing technique. In this case the azimuthal difference $\phi_1 - \phi_2$(mixed) is reconstructed using $\phi_1$ and $\phi_2$ from different events. The orientation of the polarization of single annihilation quantum does not have a preferred direction, so different events will have gammas with randomly oriented polarizations. Therefore the distribution $A_n(\phi_1 - \phi_2$(mixed)) does not contain the polarization correlation of annihilation quanta, but keeps the information of the detector pair acceptance, and can be used as the acceptance correction.

It has been shown in [10] that $\mu$ equals the $\frac{G(\theta_1)G(\theta_2)}{F(\theta_1)F(\theta_2)}$ from Eqn. 1. Hence, according to equation 1 experimentally one expects:

$$N_{cor}(\phi_1 - \phi_2) = M[1 + \mu \cos(2\phi_1 - \phi_2)]$$  \hspace{1cm} (5)

where $M$ will correspond to the average amplitude of the distribution and $\mu$ to the modulation factor.
3 Experimental Setup

We have set up a system of two detector modules, depicted in Figure 1. Each module encompasses a 4x4 matrix of Lutetium Fine Silicate scintillation pixels, with dimension of 3.14 x 3.14 x 20 mm$^3$, with a 3.2 mm pitch. The scintillator matrix is read out by a SiPM array in a one-to-one match. All channels are read-out, amplified and then digitized using pulse digitizers with 1.6 GS/s sampling speed. The experimental setup is described in detail in [13]. Under typical operating conditions, the voltage of $U_b = U_{breakdown} + 1.6$ V and temperature $t = 20^\circ - 22^\circ$ C, the modules achieve an average energy resolution of $\Delta E = 12.2\% \pm 0.7\%$ (FWHM) at 511 keV. The coincidence time resolution between the modules is $\Delta t = 0.54 \pm 0.02$ ns (FWHM) for the detection of annihilation gammas. The detector performance is described in detail in [14].

To select the Compton scattering events in the modules, we require that exactly two pixels fire in each module and that their energy sum is equal to 511 keV within $\pm 3\sigma$, determined by the energy resolution. In those events, the scattering angles $\theta_1, \theta_2$, in the first and the second module, respectively, are reconstructed from the energy of fired pixels, via Compton scattering kinematics. The angular resolution is $\Delta \theta \approx 18.8^{\circ}$ (FWHM) throughout the acceptance. For scattering angles $\theta > 60^{\circ}$, there is a possible ambiguity in the determination of the pixel corresponding to the recoil electron and the pixel corresponding to the scattered gamma, since a scattering at a forward angle can result in the same energy responses of the pixels as a scattering at one backward angle. However, the forward scattering is always favored owing to the higher cross-section and the lower absorption probability of the scattered gamma [14]. The azimuthal angle, $\phi$, is reconstructed from the relative position of the two fired pixels in a module. The angular uncertainty is caused by the uncertainty of the interaction position within a pixel, and it depends on the distance, d, of the fired pixels in a module as [14]:

$$\sigma_\phi = \frac{1}{\sqrt{6}} \left| \frac{a}{d} \right|$$

(6)

where $a = 3.14$ mm is the pixel width. Hence the azimuthal resolution ranges from $\Delta \phi = 54^{\circ}$ (FWHM) for the closest neighbors to $\Delta \phi = 12.7^{\circ}$ (FWHM) for the most distant pixels.

The measurement of polarization correlations was conducted with a $^{22}$Na-source ($\approx 1\mu$Ci) enclosed in an aluminum case, placed on the central system axis, 4 cm from the front face of each module. The trigger was set up to acquire only the events where coincidence between the modules occurred. The presented analysis is based on 50 million recorded events, of which 19.4 million had full energy deposition in both modules and 1.05 million had passed the additional Compton event selection as described above.

4 Results and discussion

For the events where Compton scattering occurs in both detector modules, the scattering angles $\theta_{1,2}$ and azimuthal angles $\phi_{1,2}$ are reconstructed and the acceptance-corrected distribution of azimuthal angle difference, $N_{cor}(\phi_1 - \phi_2)$ is obtained for a selected range of $\theta_{1,2}$.

First, we selected the scattering angles $\theta_{1,2} = 72^{\circ} - 90^{\circ}$, centered around $\theta = 82^{\circ}$ where the maximum azimuthal correlation is expected. The reconstructed distribution for all event topologies, corresponding to pixel distances $d = 3.2 - 13.6$ mm, is shown in Figure 2. The error bars represent the contribution of the statistical and the systematic error. The latter is determined by examining the acceptance corrected yield at $-90^{\circ}, 0^{\circ}, 90^{\circ}, 180^{\circ}$, in dependence of the histogram bin width and it is estimated to be 2% of the yield. The distribution is fit with the function from Eqn. 5 from which the modulation factor $\mu = 0.15 \pm 0.01$.
is obtained. Further, we explored the dependence of the modulation factor on event topology, i.e. on the
distance of fired pixels, which determines the azimuthal resolution (Eqn. 6). The modulation factors obtained
when pixels with specific distances are selected, are shown in Table 1, Set 1-4. It clearly shows that the lowest
modulation factors are obtained for \( d = 3.2 \), when the fired pixels are the adjacent neighbors, in which case
the azimuthal uncertainty is the largest. If events with fired adjacent neighbors are not used, the measured
modulation is significantly higher (Table 1, Set 6,7), as shown for example in Figure 3.

![Figure 2: The acceptance corrected \( \phi_1 - \phi_2 \) distribution for \( \theta_0 = 72^\circ - 90^\circ \) and all possible pixel distances \( d = 3.2 - 13.6 \) mm. The line is a fit of Eqn. 5.](image)

![Figure 3: The acceptance corrected \( \phi_1 - \phi_2 \) distribution for \( \theta_0 = 72^\circ - 90^\circ \) and for pixel distances \( d = 4.5 - 13.6 \) mm. The line is a fit of Eqn. 5.](image)

The same analysis was repeated for scattering angles in range \( \theta_{1,2} = 60^\circ - 80^\circ \), centered around \( \theta = 70^\circ \),
which was suggested as the optimal range with sufficient azimuthal correlation and abundant statistics [10].
The reconstructed distribution for all event topologies is shown in Figure 4, and the distribution excluding
the adjacent pixels is shown in Figure 5. The results are summarized in Table 2.

The results show the modulation of the \( \phi_1 - \phi_2 \) distribution, as expected due to initial orthogonality of
polarizations of the annihilation quanta. The strength of the modulation depends on the scattering angles
\( \theta_{1,2} \) and we have indeed observed a stronger modulation for scattering around \( \theta_{1,2} = 82^\circ \), than for scattering
around \( \theta_{1,2} = 70^\circ \). The modulation also depends on the angular resolution and we have observed stronger
modulation for smaller \( \Delta \phi_{1,2} \). Such behaviour is expected, as stated in [12], since the finite geometries reduce
the effective modulation strength with respect to the one that would be obtained for an infinite precision in
(\( \theta, \phi \)).

Standard PET devices are dominantly exploiting events, where the energy of each annihilation gamma is
deposited in a single pixel. The observed ratio of all Compton events to single-pixel events is \( R_{CE} = 1.6\% \) for
Table 1: Modulation factor $\mu$ for $\theta_{1,2} = 72^\circ - 90^\circ$, for different pixel distances $d$ and the corresponding mean azimuthal resolutions $< \Delta \phi_{1,2} >$ (FWHM).

| Set | $d$ [mm] | $< \Delta \phi_{1,2} >$ | $\mu$     |
|-----|---------|----------------|---------|
| 1   | 3.2     | 54.0$^\circ$   | 0.08 ± 0.01 |
| 2   | 4.5     | 38.1$^\circ$   | 0.23 ± 0.02 |
| 3   | 6.4     | 27.0$^\circ$   | 0.27 ± 0.03 |
| 4   | 7.2     | 24.1$^\circ$   | 0.29 ± 0.04 |
| 5   | 3.2 - 13.6 | 44.1$^\circ$   | 0.15 ± 0.01 |
| 6   | 4.5 - 13.6 | 31.0$^\circ$   | 0.25 ± 0.01 |
| 7   | 6.4 - 13.6 | 23.7$^\circ$   | 0.27 ± 0.02 |

Table 2: Modulation factor $\mu$ for $\theta_{1,2} = 60^\circ - 80^\circ$, for different pixel distances $d$ and the corresponding mean azimuthal resolutions $< \Delta \phi_{1,2} >$ (FWHM).

| Set | $d$ [mm] | $< \Delta \phi_{1,2} >$ | $\mu$     |
|-----|---------|----------------|---------|
| 1   | 3.2     | 54.0$^\circ$   | 0.05 ± 0.01 |
| 2   | 4.5     | 38.1$^\circ$   | 0.14 ± 0.03 |
| 3   | 6.4     | 27.0$^\circ$   | 0.26 ± 0.03 |
| 4   | 7.2     | 24.1$^\circ$   | 0.22 ± 0.04 |
| 5   | 3.2 - 13.6 | 44.1$^\circ$   | 0.12 ± 0.01 |
| 6   | 4.5 - 13.6 | 31.0$^\circ$   | 0.17 ± 0.01 |
| 7   | 6.4 - 13.6 | 23.7$^\circ$   | 0.21 ± 0.02 |

$\theta_{1,2} = 72^\circ - 90^\circ$, and $R_{CE} = 2.6\%$ for $\theta_{1,2} = 60^\circ - 80^\circ$. These ratios are modest, but this should not come as a surprise, since the LFS material is optimized for high photo-electric cross section and a high stopping power.

In order to increase the polarimetric sensitivity using the same detector concept, a better angular resolution should be provided. The improvement in $\Delta \theta$ may be achieved by improving the energy resolution, while the improvement in $\Delta \phi$ could be achieved either by finer segmentation or by using a detector material with lower stopping power that would allow more Compton events with more distant pixels fired. A promising candidate is GAGG:Ce, which offers a superior energy resolution, as well as lower density and lower effective atomic number than LFS or LYSO, which should also result in a larger $R_{CE}$, desirable in this concept.

![Figure 4: The acceptance corrected $\phi_1 - \phi_2$ distribution for $\theta_0 = 60^\circ - 80^\circ$ and all possible pixel distances $d = 3.2 - 13.6$ mm. The line is a fit of Eqn. 5.](image)
5 Conclusions

We have used a system of two compact, position and energy-sensitive, single-layer Compton detectors to investigate the feasibility of measuring the polarization correlations of annihilation quanta. The coincidence data from positron annihilations has been collected and events with Compton scattering in both modules are selected. The polarimetric modulation has been extracted from the difference in the azimuthal scattering angles of the two gammas, demonstrating the feasibility of the approach. Although a moderate polarimetric sensitivity has been observed, it may be improved by optimizing detector material and geometry to provide better angular resolutions. Such detectors might be exploited in PET or other experiments where measurement of gamma polarization is of interest. Importantly, the detectors based on the single-layer concept would significantly improve the cost-efficiency compared to typical two layer systems used for Compton scattering detection.

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