On Modifications of the Sp(2) Covariant Superfield Quantization

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Abstract

We propose a modification of the $Sp(2)$ covariant superfield quantization to realize a superalgebra of generating operators isomorphic to the massless limit of the corresponding superalgebra of the $osp(1,2)$ covariant formalism. The modified scheme ensures the compatibility of the superalgebra of generating operators with extended BRST symmetry without imposing restrictions eliminating superfield components from the quantum action. The formalism coincides with the $Sp(2)$ covariant superfield scheme and with the massless limit of the $osp(1,2)$ covariant quantization in particular cases of gauge-fixing and solutions of the quantum master equations.

1 Introduction

The covariant quantization of gauge theories is based on the concept of quantum master equations, realized in terms of the corresponding generating operators and antibrackets (see, e.g., [1, 2, 3]). Quantum master equations encode the presence of BRST symmetry, being a global supersymmetry of the integrand in the vacuum functional. This symmetry was discovered in Yang–Mills theories and then generalized to extended BRST symmetry, which combines BRST [4] and antiBRST [5] transformations. Extended BRST symmetry permitted to find a superspace description [6] of quantum Yang–Mills theories, where this symmetry was realized in terms of supertranslations along additional anticommuting coordinates.

In general gauge theories (with an arbitrary gauge algebra and stage of reducibility), extended BRST symmetry is realized within the $Sp(2)$ covariant quantization scheme [1] and its different modifications, e.g., [2, 3], including the $osp(1,2)$ covariant formalism [3]. The quantization scheme [1] describes the structure of the complete configuration space of a theory in terms of irreducible representations of the group $Sp(2)$. The scheme [3] modifies the formalism [1] in a way which ensures the $Sp(2)$ invariance of a theory by imposing on the quantum action a set of master equations and analogous subsidiary conditions with the corresponding generating operators subject to a superalgebra isomorphic

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to \(osp(1,2)\). The superalgebra of the generating operators \[3\] depends on a mass parameter, inherited by the quantum action. The incorporated mass is intended to serve as a regularization parameter of an \(Sp(2)\) invariant renormalization of the quantum theory \[3\]. The superfield versions \[7, 8, 9\] of the \(Sp(2)\) and \(osp(1,2)\) covariant schemes realize superspace formulations of extended BRST symmetry in general gauge theories.

The superfield formalism \[7\] combines the variables used in the \(Sp(2)\) covariant scheme \[1\] into a set of superfields and supersources defined in a superspace with two anticommuting coordinates. The quantum action is given by a functional of superfields and supersources, which makes it possible to present extended BRST symmetry in terms of supertranslations and transformations generated by superfield antibrackets \[7\]. A possible arbitrariness in the form of superfield antibrackets compatible with the superspace interpretation of extended BRST symmetry was also examined in \[7\].

There are two alternative superfield formulations \[8, 9\] of the \(osp(1,2)\) covariant scheme \[3\]. Both formulations, constructed along the lines of \[7\], are not free from difficulties. Thus, in \[8\] there exists an inconsistency (see \[9\] for a detailed discussion) between the form of superfield antibrackets and the extended BRST symmetry realized in terms of supertranslations. In \[9\], this problem is solved at the cost of eliminating some superfield components from the quantum action, which implies that the extended BRST symmetry in \[9\] is not entirely controlled by the quantum master equations.

In \[9\], it was also remarked that a consistent superspace formulation of the \(osp(1,2)\) covariant approach should contain the \(Sp(2)\) covariant superfield scheme \[7\] in the massless limit. On the one hand, this can be explained by the fact that the massless limit of the \(osp(1,2)\) covariant scheme contains the original \(Sp(2)\) covariant formalism, regarded in a special case of gauge-fixing and solutions of the master equations. On the other hand, the superfield description of the \(Sp(2)\) covariant scheme \[7\] realizes the only form of superfield antibrackets which respects the superspace interpretation of extended BRST symmetry without additional restrictions on the quantum action. A non-trivial problem facing the proposal \[9\] is to ensure a compatibility of the superfield antibrackets \[7\] with the \(osp(1,2)\) superalgebra of generating operators (see also \[8, 9\]).

To advance in the solution of this problem, we demonstrate the existence of a superfield scheme which can be identified with a massless limit suggested in \[9\]. To this end, we propose a superfield scheme based on a set of generating operators which form a superalgebra isomorphic to the massless limit of the superalgebra realized in the \(osp(1,2)\) covariant scheme \[3\]. The choice of generating operators is consistent with the superfield antibrackets \[7\] and the superspace form of extended BRST symmetry, without imposing restrictions eliminating superfield components. The formalism contains the \(Sp(2)\) covariant superfield scheme \[7\] and the massless limit of the \(osp(1,2)\) covariant approach \[3\]. Massive extensions of the proposed formalism may provide a key to constructing a superfield \(osp(1,2)\) covariant scheme free from the problems that remain in \[8, 9\].

The paper is organized as follows. In Section 2, we introduce the main definitions. In Section 3, we formulate the quantization rules. In Section 4, we discuss the relation of the proposed formalism to the quantization schemes \[3, 7\]. In Section 5, we summarize the results and make concluding remarks.

We use the notation adopted in \[3, 7\]. Derivatives with respect to (super)sources and antifields are taken from the left, and those with respect to (super)fields, from the right. Left derivatives with respect to (super)fields are labeled by the subscript “\(l\)”. Integration over superfields and supersources is understood as integration over their components.
2 Main Definitions

Let us consider a superspace \((x^\mu, \theta^a)\), where \(x^\mu\) are space-time coordinates, and \(\theta^a\) is an \(Sp(2)\) doublet of anticommuting coordinates. Notice that any function \(f(\theta)\) has a component representation,

\[
f(\theta) = f_0 + \theta^a f_a + \theta^2 f_3, \quad \theta^2 \equiv \frac{1}{2} \theta_a \theta^a,
\]

and an integral representation,

\[
f(\theta) = \int d^2 \theta' \delta(\theta' - \theta) f(\theta'), \quad \delta(\theta' - \theta) = (\theta' - \theta)^2,
\]

where raising and lowering the \(Sp(2)\) indices is performed by the rule \(\theta^a = \varepsilon^{ab} \theta_b\), \(\theta_a = \varepsilon_{ab} \theta^b\), with \(\varepsilon^{ab}\) being a constant antisymmetric tensor, \(\varepsilon^{12} = 1\), and integration over \(\theta^a\) is given by

\[
\int d^2 \theta = 0, \quad \int d^2 \theta \theta^a = 0, \quad \int d^2 \theta \theta^a \theta^b = \varepsilon^{ab}.
\]

In particular, for any function \(f(\theta)\) we have

\[
\int d^2 \theta \frac{\partial f(\theta)}{\partial \theta^a} = 0,
\]

which implies the property of integration by parts

\[
\int d^2 \theta \frac{\partial f(\theta)}{\partial \theta^a} g(\theta) = - \int d^2 \theta (-1)^{\varepsilon(f)} f(\theta) \frac{\partial g(\theta)}{\partial \theta^a}, \quad (1)
\]

where derivatives with respect to \(\theta^a\) are taken from the left.

We now introduce a set of superfields \(\Phi^A(\theta)\), \(\varepsilon(\Phi^A) = \varepsilon_A\), with the boundary condition

\[
\Phi^A(\theta)\big|_{\theta = 0} = \phi^A,
\]

and a set of supersources \(\bar{\Phi}_A(\theta)\) of the same Grassmann parity, \(\varepsilon(\bar{\Phi}_A) = \varepsilon_A\). The structure [1] of the complete configuration space \(\phi^A\) of a general gauge theory of \(L\)-stage reducibility is given by

\[
\phi^A = (A^i, B^{a_s|a_1\cdots a_s}, C^{a_s|a_0\cdots a_s}), \quad s = 0, \ldots, L, \quad (2)
\]

where \(A^i\) are the initial classical fields, while \(B^{a_s|a_1\cdots a_s}, C^{a_s|a_0\cdots a_s}\) are the pyramids of auxiliary and (anti)ghost fields, being completely symmetric \(Sp(2)\) tensors of rank \(s\) and \(s + 1\), respectively.

For arbitrary functionals \(F = F(\Phi, \bar{\Phi})\), \(G = G(\Phi, \bar{\Phi})\), we define the superbracket operations \((,)^a\) and \(\{ , \}_a\)

\[
(F, G)^a = \int d^2 \theta \left\{ \frac{\delta F}{\delta \Phi^A(\theta)} \frac{\partial}{\partial \theta^a} \frac{\delta G}{\delta \bar{\Phi}_A(\theta)} (-1)^{\varepsilon_A + 1} - (-1)^{\varepsilon(F) + 1} \varepsilon(G) (F \leftrightarrow G) \right\},
\]

\[
\{F, G\}_a = - \int d^2 \theta \left\{ (\sigma_a)_B^A \left[ \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta F}{\delta \Phi^A(\theta)} \right) \theta^2 + \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta F}{\delta \Phi^A(\theta)} \theta^2 \right) \right] \frac{\delta G}{\delta \bar{\Phi}_B(\theta)} \right. \\
+ \left. \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta F}{\delta \Phi^A(\theta)} \right) \theta_a (\sigma_a)_B^A \theta^a \frac{\delta G}{\delta \bar{\Phi}_A(\theta)} + (\delta \theta^2 \delta \bar{\Phi}_B(\theta)) \varepsilon(F) \varepsilon(G) (F \leftrightarrow G) \right\}, \quad (3)
\]
where
\[
\frac{\partial^2}{\partial \theta^2} \equiv \frac{1}{2} \varepsilon^{ab} \frac{\partial}{\partial \theta^b} \frac{\partial}{\partial \theta^a}.
\]

Notice the properties of derivatives
\[
\frac{\delta_i \Phi^A(\theta)}{\delta \Phi^B(\theta')} = \frac{\delta \Phi^A(\theta)}{\delta \Phi^B(\theta')} = \delta(\theta' - \theta) \delta^A_B, \quad \frac{\delta \Phi^A(\theta)}{\delta \Phi^B(\theta')} = \delta(\theta - \theta') \delta^A_B.
\]

In (3), the matrices \((\sigma^a)_A^B \equiv -(\sigma^a)_A^B\), with the indices (2), are given by
\[
(\sigma^a)_A^B = (\sigma^a)_A^b (P_\pm)^B_a.
\]

Here, \((\sigma^a)_a^b\), with \(a = (0, +, -)\), stands for a set of matrices which possess the properties
\[
(\sigma^a)_a^b = -(\sigma^a)_a^b, \quad (\sigma^a)^b = \varepsilon^{ac}(\sigma^c)_a^b = (\sigma^a)^c \varepsilon^{cb} = \varepsilon^{ac}(\sigma^a)_c^d \varepsilon^{db}, \quad (\sigma^a)^b = (\sigma^a)^b,
\]

\[
(\sigma^a)_a^a = (\sigma^a)_a^a = 0, \quad \varepsilon^{ab} \delta^c_b + \varepsilon^{ba} \delta^c_a = -(\sigma^a)^b (\sigma^a)_c^d
\]

and form the algebra \(sl(2)\)
\[
\sigma_{\alpha \beta} = g_{\alpha \beta} + \frac{1}{2} \epsilon_{\alpha \beta \gamma} \sigma^\gamma, \quad \sigma^\alpha = g^{\alpha \beta} \sigma_{\beta}, \quad \text{Tr}(\sigma_{\alpha \beta}) = 2g_{\alpha \beta},
\]

\[
g^{\alpha \beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}, \quad g^{\alpha \gamma} g_{\gamma \beta} = \delta^\alpha_\beta,
\]

with \(\epsilon_{\alpha \beta \gamma}\) being an antisymmetric tensor, \(\epsilon_{0+ -} = 1\).

In (4), the matrices \((P_\pm)^B_a\) are given by
\[
(P_+)^B_a = (P_-)^B_a = (P_\pm)^B_A \delta_A^a + \delta_B^a g_{a}^b, \quad (P_\pm)^B_A = \delta_A^b (P_\pm)^B_a,
\]

where
\[
(P_+)^B_a = \left\{ \begin{array}{ll}
\delta^i_j \delta^a_b & A = i, B = j, \\
\delta^a_s (s + 1) S^b_1 \cdots S^a_1 & A = \alpha, a_1 \cdots a_s, B = \beta, b_1 \cdots b_s, \\
\delta^a_s (s + 2) S^b_0 \cdots S^a_0 & A = \alpha, a_0 \cdots a_s, B = \beta, b_0 \cdots b_s, \\
0 & \text{otherwise.} \\
\end{array} \right.
\]

Here, \(S^b_0 \cdots S^a_0\) is a symmetrizer (\(X^a\) being independent bosonic variables)
\[
S^b_0 \cdots S^a_0 = \frac{1}{(s + 2)!} \frac{\partial}{\partial X^a_0} \cdots \frac{\partial}{\partial X^b_s} X^a X^b \cdots X^b,
\]

with the properties
\[
S^b_0 \cdots S^a_0 = \frac{1}{s + 2} \left( \sum_{r=0}^s \delta_{b_r}^b \delta_{a_r}^a S^b_0 \cdots b_{r+1} \cdots b_s + \frac{1}{s + 1} \sum_{r=0}^s \delta_{b_r}^b \delta_{a_r}^a S^b_r \cdots b_{r+1} \cdots b_s \right),
\]

\[
S^b_0 \cdots S^a_0 = \frac{1}{s + 1} \sum_{r=0}^s \delta_{b_r}^b \delta_{a_r}^a S^b_0 \cdots b_{r+1} \cdots b_s.
\]

From the above definitions follow the properties [3]
\[
(P_\pm)^A_B (P_\pm)^C_d = 0, \quad \varepsilon^{ad} (P_\pm)^B_d + \varepsilon^{bd} (P_\pm)^B_a = -(\sigma^a)^b (\sigma^a)_A^B.
\]
\[ \varepsilon^{ad}(P_\pm)_{Bb} + \varepsilon^{bd}(P_\pm)_{Ba} - (\sigma^a)_{\alpha}(\sigma^e)_{c}(P_\pm)_{Ac} = -(\sigma^a)_{\alpha}(\sigma^d)_{c} \delta_{B}^{F} + \delta_{e}^{c}(\sigma^a)_{B} \).

We now introduce a set of first-order operators \( V^a \), \( U^a \) (odd) and \( V^{\alpha} \), \( U^{\alpha} \) (even),

\[ V^a = \int d^2\theta \frac{\partial}{\partial \theta^a} \frac{\delta}{\delta \Phi_{A}(\theta)}, \]
\[ U^a = \int d^2\theta \frac{\partial}{\partial \theta^a} \frac{\delta}{\delta \Phi_{A}(\theta)}, \]
\[ V^{\alpha} = \int d^2\theta \left( \Phi_{B}(\sigma^a)_{A} \delta_{\Phi_{B}(\sigma^a)_{A}} - \frac{\partial^2}{\partial \theta^2} \left( \Phi_{A}(\theta)\theta_{b} \right) (\sigma^a)_{\alpha} \delta_{\Phi_{A}(\theta)} \right), \]
\[ U^{\alpha} = \int d^2\theta \left( \Phi^{A}(\sigma^a)_{A} \frac{\delta_{\Phi_{B}(\sigma^a)_{A}}}{\delta \Phi_{B}(\theta)} \right) + \frac{\partial^2}{\partial \theta^2} \left( \Phi^{A}(\theta)\theta_{a} \right) (\sigma^a)_{\alpha} \theta_{b} \frac{\delta_{l}}{\delta \Phi_{A}(\theta)}. \] (6)

These operators obey a superalgebra with the following non-trivial (anti)commutation relations:

\[ [V^{\alpha}, V^{\beta}] = \epsilon_{\alpha\beta}^{\gamma} V^{\gamma}, \quad [V^{\alpha}, U^{a}] = V^{b} (\sigma^{a})_{b}^{\alpha}, \quad \{V^{\alpha}, V^{b}\} = 0, \]

\[ [U^{\alpha}, U^{\beta}] = -\epsilon_{\alpha\beta}^{\gamma} V^{\gamma}, \quad [U^{\alpha}, U^{a}] = -U^{b} (\sigma^{a})_{b}^{\alpha}, \quad \{U^{\alpha}, U^{b}\} = 0. \] (7)

We also introduce a set of second-order operators \( \Delta^{a} \) (odd) and \( \Delta^{\alpha} \) (even)

\[ \Delta^{a} = -\int d^2\theta \frac{\delta l}{\delta \Phi_{A}(\theta)} \frac{\partial}{\partial \theta^a} \frac{\delta}{\delta \Phi_{A}(\theta)}, \]
\[ \Delta^{\alpha} = (-1)^{\varepsilon_{A}+1} \int d^2\theta \left\{ (\sigma^a)_{A}^{\alpha} \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta l}{\delta \Phi_{A}(\theta)} \right) \theta^2 - \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta l}{\delta \Phi_{A}(\theta)} \right)^2 \right\} + \frac{\partial^2}{\partial \theta^2} \left( \frac{\delta l}{\delta \Phi_{B}(\theta)} \right) \theta_{b} (\sigma^a)_{b}^{\alpha} + \frac{\delta l}{\delta \Phi_{A}(\theta)} \theta_{a} (\sigma^a)_{B}^{\alpha} \theta^a \frac{\delta}{\delta \Phi_{B}(\theta)}. \] (8)

These operators possess the algebraic properties

\[ [\Delta^{\alpha}, \Delta^{\beta}] = 0, \quad \{\Delta^{a}, \Delta^{b}\} = 0, \quad [\Delta^{\alpha}, \Delta^{a}] = 0, \] (9)

\[ [\Delta^{\alpha}, V^{b}] + [V^{\alpha}, \Delta^{b}] = \epsilon_{\alpha\beta}^{\gamma} \Delta^{\gamma}, \]
\[ \{\Delta^{a}, V^{b}\} + \{V^{a}, \Delta^{b}\} = 0, \]
\[ [\Delta^{\alpha}, V^{a}] + [V^{\alpha}, \Delta^{a}] = \Delta^{b} (\sigma^a)_{b}^{\alpha}. \] (10)

From (8) it follows that the action of the operators \( \Delta^{a} \) and \( \Delta^{\alpha} \) on the product of two functionals defines the superbracket operations (3), namely,

\[ \Delta^{a}(FG) = (\Delta^{a}F)G + F(\Delta^{a}G) + \{F, G\}_{a}, \]
\[ \Delta^{\alpha}(FG) = (\Delta^{\alpha}F)G + F(\Delta^{\alpha}G)(-1)^{\varepsilon(F)} + (F, G)^{a}(-1)^{\varepsilon(F)}. \] (11)

Using the relations (9), (10), (11), one can establish the properties of the superbrackets (3) at the algebraic level [3].

Finally, we introduce the operators

\[ \bar{\Delta}^{a} \equiv \Delta^{a} + \frac{i}{\hbar} V^{a}, \quad \bar{\Delta}^{\alpha} \equiv \Delta^{\alpha} + \frac{i}{\hbar} V^{\alpha}. \]
From (7), (9), (10) it follows that these operators obey the superalgebra

\[
[\bar{\Delta}_\alpha, \bar{\Delta}_\beta] = \left(\frac{i}{\hbar}\right) \epsilon^{\gamma}_{\alpha\beta} \bar{\Delta}_\gamma,
\]

\[
[\bar{\Delta}_\alpha, \bar{\Delta}^a] = \left(\frac{i}{\hbar}\right) \bar{\Delta}^b (\sigma_a)_b^a,
\]

\[
\{\Delta^a, \bar{\Delta}^b\} = 0,
\]

isomorphic to the massless limit of the superalgebra of generating operators used in the \textit{osp}(1,2)-covariant quantization scheme [3].

3 Quantization Rules

Let us define the vacuum functional \(Z\) as the following path integral:

\[
Z = \int d\Phi \, d\bar{\Phi} \, \exp \left[ \frac{i}{\hbar} \left( W(\Phi, \bar{\Phi}) - \frac{1}{2} \epsilon_{ab} U^a U^b F(\Phi) + \bar{\Phi} \Phi \right) \right].
\]  

(12)

Here, \(W = W(\Phi, \bar{\Phi})\) is the quantum action, satisfying the boundary condition

\[W|_{\Phi=\bar{\Phi}=0} = S,\]

where \(S = S(A)\) is the action of the original gauge theory. The quantum action \(W\) is subject to the master equations

\[
\bar{\Delta}^a \exp \left( \frac{i}{\hbar} W \right) = 0,
\]

(13)

and the subsidiary conditions

\[
\bar{\Delta}_\alpha \exp \left( \frac{i}{\hbar} W \right) = 0,
\]

(14)

with \(\bar{\Delta}^a\) and \(\bar{\Delta}_\alpha\) given by (8). Equations (13) and (14) are equivalent to

\[
\frac{1}{2} \{W, W\}^a + V^a W = i\hbar \Delta^a W,
\]

(15)

\[
\frac{1}{2} \{W, W\}_\alpha + V_\alpha W = i\hbar \Delta_\alpha W,
\]

(16)

where the superbrackets \(\{,\}_{\alpha}\), \(\{,\}_a\) and the operators \(V^a, V_\alpha, \Delta^a, \Delta_\alpha\) are defined by (3), (6), (8). The quantum action \(W\) is also assumed to be an admissible solution of (15) and (16). Namely, it is subject to the restriction

\[
\int d^2 \theta \, \theta^2 \left( \frac{\delta W}{\delta \Phi^A(\theta)} + \Phi^A(\theta) \right) = 0.
\]

(17)

In (12), \(\bar{\Phi} \Phi\) is a functional of the form

\[
\bar{\Phi} \Phi = \int d^2 \theta \, \bar{\Phi}^A(\theta) \Phi^A(\theta),
\]

(18)

while \(F(\Phi)\) is a gauge-fixing Boson restricted by the conditions

\[U_\alpha F(\Phi) = 0,\]

(19)
where \( U_a \) are the operators (6).

An important property of the integrand in (12) is its invariance under the following transformations:

\[
\begin{align*}
\delta \Phi^A(\theta) &= \mu_a U^a \Phi^A(\theta), \quad \delta \bar{\Phi}_A(\theta) = \mu_a V^a \bar{\Phi}_A(\theta) + \mu_a (W, \bar{\Phi}_A(\theta))^a, \\
\delta \bar{\Phi}^A(\theta) &= \mu^A U_a \bar{\Phi}^A(\theta), \quad \delta \bar{\Phi}_A(\theta) = \mu^A V_a \bar{\Phi}_A(\theta) + \mu^A \{W, \bar{\Phi}_A(\theta)\}_a,
\end{align*}
\]

where \( U^a \) are operators given by (6), while \( \mu_a \) and \( \mu^A \) are constant (anti)commuting parameters, \( \varepsilon(\mu_a) = 1, \varepsilon(\mu^A) = 0 \). The validity of the symmetry transformations (20), (21) follows from the master equations (15), (16) and the conditions (19) for the gauge-fixing Boson, with allowance for integration by parts (1) and the algebraic properties (7).

The transformations (20) realize the extended BRST symmetry, while the transformations (21) express the symmetry related to the \( Sp(2) \) invariance of the quantum action. This interpretation is explained in the following section, by the relation of the present formalism to the \( Sp(2) \) covariant superfield scheme [7] and the \( osp(1,2) \) covariant approach [3]. Note that the admissibility condition (17) is not required for the proof of invariance. As will be shown in the following section, this condition establishes the relation between the proposed formalism and the quantization schemes [3, 7].

The transformations of extended BRST symmetry (20) permit establishing the independence of the vacuum functional (12) from a choice of the gauge Boson \( F(\Phi) \). Indeed, any infinitesimal change \( F \to F + \delta F \) can be compensated by a change of variables (20) with the parameters \( \mu_a = -(i/2\hbar)\varepsilon_{ab} U_b \delta F \), and therefore \( Z_{F+\delta F} = Z_F \), which implies the independence of the \( S \)-matrix from the choice of gauge within the proposed formalism.

### 4 Component Analysis

Let us consider the component representation of the formalism proposed in the previous section in order to establish its relation with the \( osp(1,2) \) covariant approach [3] and the \( Sp(2) \) covariant superfield scheme [7].

The component form of superfields \( \Phi^A(\theta) \) and supersources \( \bar{\Phi}_A(\theta) \) reads

\[
\begin{align*}
\Phi^A(\theta) &= \phi^A + \pi^{Aa} \theta_a + \lambda^A \theta^2, \\
\bar{\Phi}_A(\theta) &= \bar{\phi}_A - \theta^a \phi^*_{Aa} - \theta^2 \eta_A.
\end{align*}
\]

Here, the components \( (\phi^A, \pi^{Aa}, \lambda^A, \bar{\phi}_A, \phi^*_{Aa}, \eta_A) \) are identical with the set of variables used for the construction of the vacuum functional in the quantization schemes [3, 7].

By virtue of the manifest structure of \( \Phi^A(\theta), \bar{\Phi}_A(\theta) \), the component representation of the integration measure in (12) is given by

\[
d\Phi \, d\bar{\Phi} = d\phi \, d\pi \, d\lambda \, d\bar{\phi} \, d\phi^* \, d\eta,
\]

and the functional \( \Phi \bar{\Phi} \) in (18) has the form

\[
\Phi \bar{\Phi} = \bar{\phi}_A \lambda^A + \phi^*_{Aa} \pi^{Aa} - \eta_A \phi^A.
\]
Let us denote \( F(\Phi, \bar{\Phi}) \equiv \tilde{F}(\phi, \pi, \lambda, \bar{\phi}, \phi^*, \eta) \). Then the \( \{ \), \( \} \) and \( \{ \), \( \} \) of (3) acquire the following component structure:

\[
\langle F, G \rangle^a = \frac{\delta \tilde{F}}{\delta \phi^A} \frac{\delta \tilde{G}}{\delta \phi^*_A} + \varepsilon^{ab} \frac{\delta \tilde{F}}{\delta \pi_{Ab}} \frac{\delta \tilde{G}}{\delta \phi^*_A} - (\tilde{F} \leftrightarrow \tilde{G}) (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)},
\]

\[
\{ F, G \}_\alpha = (\sigma_\alpha)_B^A \left( \frac{\delta \tilde{F}}{\delta \phi^A} \frac{\delta \tilde{G}}{\delta \phi^*_A} + \frac{\delta \tilde{F}}{\delta \lambda^A} \frac{\delta \tilde{G}}{\delta \phi^*_B} \right) + \left( \frac{\delta \tilde{F}}{\delta \pi_{Ab}} (\sigma_\alpha)_b^a + \frac{\delta \tilde{F}}{\delta \pi_{Ba}} (\sigma_\alpha)_b^A \right) \frac{\delta \tilde{G}}{\delta \phi^*_A} + (\tilde{F} \leftrightarrow \tilde{G})(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)},
\]

while the second-order operators \( \Delta^a \) and \( \Delta_\alpha \) in (8) take the form

\[
\Delta^a = (-1)^{\varepsilon_A} \frac{\delta l}{\delta \phi^A} \frac{\delta}{\delta \phi^*_A} + (-1)^{\varepsilon_A+1} \varepsilon^{ab} \frac{\delta l}{\delta \pi_{Ab}} \frac{\delta}{\delta \phi^*_A},
\]

\[
\Delta_\alpha = (-1)^{\varepsilon_A} (\sigma_\alpha)_B^A \left( \frac{\delta l}{\delta \phi^A} \frac{\delta}{\delta \phi^*_A} + \frac{\delta l}{\delta \lambda^A} \frac{\delta}{\delta \phi^*_B} \right) \frac{\delta}{\delta \phi^*_A} + (-1)^{\varepsilon_A+1} \left( \frac{\delta l}{\delta \pi_{Ab}} (\sigma_\alpha)_b^a + \frac{\delta l}{\delta \pi_{Ba}} (\sigma_\alpha)_b^A \right) \frac{\delta}{\delta \phi^*_A}.
\]

In (6), the first-order operators \( V^a \) and \( V_\alpha \) have the component representation

\[
V^a = \varepsilon^{ab} \phi^*_A \frac{\delta}{\delta \phi^A} - \eta_A \frac{\delta}{\delta \phi^*_A},
\]

\[
V_\alpha = \bar{\phi}_B (\sigma_\alpha)_B^A \frac{\delta}{\delta \phi^A} \left( \phi^*_A (\sigma_\alpha)_b^a + \phi^*_B (\sigma_\alpha)_b^A \right) \frac{\delta}{\delta \phi^*_A} + \eta_B (\sigma_\alpha)_B^A \frac{\delta}{\delta \eta_A},
\]

while the first-order operators \( U^a \) and \( U_\alpha \) are given by

\[
U^a = (-1)^{\varepsilon_A} \varepsilon^{ab} \lambda^A \frac{\delta l}{\delta \pi_{Ab}} - (-1)^{\varepsilon_A} \pi^{Aa} \frac{\delta l}{\delta \phi^A},
\]

\[
U_\alpha = \phi^B (\sigma_\alpha)_B^A \frac{\delta l}{\delta \phi^A} \left( \lambda^A (\sigma_\alpha)_b^a + \pi^{Ba} (\sigma_\alpha)_b^A \right) \frac{\delta}{\delta \eta_A} + \lambda^B (\sigma_\alpha)_B^A \frac{\delta l}{\delta \lambda^A}.
\]

Finally, the component form of the admissibility condition (17)

\[
\frac{\delta \tilde{W}}{\delta \eta_A} = \phi^A
\]

implies a simplification of the quantum action:

\[
\tilde{W} = W(\phi, \pi, \lambda, \bar{\phi}, \phi^*) + \eta_A \phi^A.
\]

To establish the relation between the proposed superfield scheme and the \( osp(1,2) \) covariant formalism [3], we note, first of all, that the operators \( U_\alpha \) and \( V_\alpha \) in (26), (27) coincide with the generators of \( Sp(2) \) invariance [3]. In particular, equation (19) is the condition of \( Sp(2) \) invariance for the gauge Boson \( \tilde{F}(\phi, \pi, \lambda) \).

Let us subject the quantum action \( \tilde{W} \) to the restrictions

\[
\frac{\delta \tilde{W}}{\delta \lambda^A} = \frac{\delta \tilde{W}}{\delta \pi^{Aa}} = 0,
\]
reducing the variables of $\tilde{W}$ to the set $(\phi^A, \bar{\phi}_A, \phi^*_A, \eta_A)$, parameterizing the quantum action in the $osp(1,2)$ covariant scheme. By virtue of (30) and the component representations (24)–(27), the set of equations (15), (16) becomes identical to the massless limit of the master equations in the $osp(1,2)$ covariant formalism.

Using the conditions (28), (30), with allowance for the properties of the matrices $\sigma_\alpha$ in (4), (5), one can transform the subsidiary master equations (16) into the condition of $Sp(2)$ invariance for the quantum action \[ (\sigma_\alpha)^A_B \frac{\delta \tilde{W}}{\delta \phi^A} \phi^B + V_\alpha \tilde{W} = 0, \]

which thus establishes the interpretation of the symmetry transformations (21) related to equations (16).

Let us also restrict the gauge-fixing Boson to the class of gauges used in the $osp(1,2)$ covariant scheme: $\tilde{F} = \tilde{F}(\phi)$. Then, with allowance for the component form (27) of the operators $U_\alpha$, the condition of $Sp(2)$ invariance (19) reduces to \[ (\sigma_\alpha)^A_B \frac{\delta \tilde{F}}{\delta \phi^A} \phi^B = 0, \] which, in view of the admissibility condition (28), can be rewritten as \[ (\sigma_\alpha)^A_B \frac{\delta \tilde{F}}{\delta \phi^A} \frac{\delta \tilde{W}}{\delta \eta_B} = 0. \] Equations (31) and (32) reproduce the whole set of subsidiary conditions used in the $osp(1,2)$ covariant scheme to provide an $Sp(2)$ invariant gauge-fixing [3].

Let us establish the relation of the vacuum functional (12), given in terms of $\tilde{W} = \tilde{W}(\phi, \bar{\phi}, \phi^*, \eta)$ and $\tilde{F} = \tilde{F}(\phi)$, to the vacuum functional of the $osp(1,2)$ covariant scheme [3]. Using the component form (27) of the operators $U^\alpha$, and integrating out the variables $\eta_A$, with allowance for (22), (23), (29), we can represent the vacuum functional (12) in the form \[ Z = \int d\phi d\phi^* d\pi d\bar{\phi} d\lambda \exp \left[ \frac{i}{\hbar} \left( W + \lambda^A + \phi^*_A \pi^A + \phi^*_A \pi^A \right) \right], \]

where the functional $W = W + \eta_A \phi^A$ satisfies (15), (16), (28), and the gauge-fixing term $\lambda^A$ is given by

$$
\lambda^A = -\frac{\delta \tilde{F}}{\delta \phi^A} \phi^A - \frac{1}{2} \varepsilon_{ab} \pi^A \frac{\delta^2 \tilde{F}}{\delta \phi^A \delta \phi^B} \pi^B,
$$

with $\tilde{F}$ subject to (31). On the other hand, the vacuum functional in the massless limit of the $osp(1,2)$ covariant formalism [3] can be represented as \[ Z = \int d\phi \exp \left( \frac{i}{\hbar} S_{\text{eff}} \right), \]

$$
S_{\text{eff}}(\phi) = S_{\text{ext}}(\phi, \bar{\phi}, \phi^*, \eta)\big|_{\bar{\phi} = \phi^* = \eta = 0}, \quad \exp \left[ \left( \frac{i}{\hbar} S_{\text{ext}} \right) \right] = \hat{U}(Y) \exp \left[ \frac{i}{\hbar} S \right].
$$

Here, $S = S(\phi, \bar{\phi}, \phi^*, \eta)$ is the quantum action subject to the system of master equations and subsidiary conditions (15), (16), (28) satisfied by $\tilde{W} = \tilde{W}(\phi, \bar{\phi}, \phi^*, \eta)$, while $\hat{U}(Y)$ is an operator of the form

$$
\hat{U}(Y) = \exp \left( \frac{\delta Y}{\delta \phi^A} \frac{\delta}{\delta \phi^A} + \frac{i\hbar}{2} \varepsilon_{ab} \frac{\delta}{\delta \phi^A} \frac{\delta Y}{\delta \phi^B} \frac{\delta}{\delta \phi^B} \right),
$$
where \( Y = Y(\phi) \) is a gauge-fixing Boson restricted by the same condition of \( Sp(2) \) invariance (31) which is imposed on \( \tilde{F} = \tilde{F}(\phi) \). To establish the identity between the vacuum functionals (33) and (34), it is sufficient to set \( S = \tilde{W} \) and \( Y = \tilde{F} \).

Let us finally establish the relation of the proposed superfield scheme to the original \( Sp(2) \) covariant superfield formalism [7]. First, note that the operators \( U^a, V^a \) (6), which also appear in the symmetry transformations (20), are naturally interpreted [7] as generators of transformations induced by supertranslations, \( \theta^a \rightarrow \theta^a + \mu^a \). Next, the form of \( (,)^a \) and \( \Delta^a \) in (3), (8) implies that equations (15) are identical with the master equations of the approach [7]. Then, the admissibility condition (28) and the related dependence (29) of \( \tilde{W} \) on the variables \( \eta^A \), with allowance for (22), (23), permit us to rewrite the vacuum functional (12) in the form

\[
Z = \int d\Phi d\bar{\Phi} \rho(\bar{\Phi}) \exp \left[ \frac{i}{\hbar} \left( W(\Phi, \bar{\Phi}) - \frac{1}{2} \varepsilon_{ab} U^a U^b F(\Phi) + \bar{\Phi} \Phi \right) \right],
\]

(35)

where \( \rho(\bar{\Phi}) \) is an integration weight, given by

\[
\rho(\bar{\Phi}) = \delta \left( \int d^2 \theta \bar{\Phi}(\theta) \right) = \delta (\eta).
\]

The integral (35) is identical with the vacuum functional of the \( Sp(2) \) covariant superfield scheme [7], where the corresponding objects \( W(\Phi, \bar{\Phi}) \) and \( F(\Phi) \) are subject to additional restrictions, (16), (17), (19), which ensure the \( Sp(2) \) invariance of the quantum theory. Note that the symmetry transformations (20) related to the master equations (15) coincide with the superfield form of extended BRST symmetry [7] in terms of supertranslations.

5 Summary

The present work is motivated by the problem of a consistent superspace formulation of extended BRST symmetry on the basis of the \( osp(1, 2) \) covariant quantization scheme [3] for general gauge theories. Here, by a consistent superspace formulation we understand a superfield quantization scheme in which extended BRST symmetry, realized in terms of supertranslations, is completely controlled by the quantum master equations (see, e.g., [7]). This consistency condition requires [9] that a superfield \( osp(1, 2) \) covariant scheme should contain the \( Sp(2) \) covariant superfield formalism [7] in the limit of a vanishing mass (a parameter introduced to provide an \( Sp(2) \) invariant renormalization [3]), which arises in the superalgebra [3] of generating operators of quantum master equations. The fulfillment of the above requirement turns out to be a non-trivial problem (see, e.g., [9]), related to a realization of the \( osp(1, 2) \) superalgebra of generating operators in a form compatible with the superfield antibrackets used in [7]. To approach this problem, we propose a superfield scheme which can be regarded as the massless limit of a consistent superspace formulation of the \( osp(1, 2) \) covariant formalism. Namely, we propose a modification of the \( Sp(2) \) covariant superfield scheme [7] on the basis of a superalgebra of generating operators isomorphic to the massless limit of the corresponding superalgebra of \( osp(1, 2) \) covariant quantization [3]. The realization of generating operators is consistent with the superfield antibrackets [7]. As a result, the superspace form of extended BRST symmetry is encoded by the quantum master equations without imposing restrictions eliminating superfield components (cf. [9]). An additional admissibility condition reduces the formalism to the original \( Sp(2) \) covariant superfield scheme and to the massless limit of the
$osp(1,2)$ covariant scheme in particular cases of gauge-fixing and solutions of the master equations. Analysis of massive extensions of the proposed scheme (as well as the study of its possible arbitrariness) may provide a constructive way of finding a consistent superspace formulation of the $osp(1,2)$ covariant approach. It appears interesting to extend the consideration of the present work to the superfield scheme [8], where a superspace description of $osp(1,2)$ covariant quantization is proposed by considering the $osp(1,2)$ superalgebra as a subalgebra of the $sl(1,2)$ superalgebra [10], which can be regarded as the algebra of conformal generators in a superspace with two anticommuting coordinates. The approach [8] suggests an intriguing possibility to realize extended BRST symmetry in terms of conformal transformations in superspace, whereby supertranslations [7] are included as a particular case.

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