The Schwarz-Hora effect: present-day situation

Yu. N. Morokov
Institute of Computational Technologies, Siberian Branch of
the Russian Academy of Sciences, Novosibirsk 630090, Russia

The Schwarz-Hora effect, observed under attempts to modulate an electron beam by laser light, was discussed extensively in the literature in the early 1970s. The analysis of the literature shows there are the unresolved up to now contradictions between the theory and the Schwarz experiments. The new model for the interpretation of the Schwarz-Hora effect is proposed. The problem of the description of the long-wavelength spatial beating in the Schwarz-Hora radiation is essential for the model and is considered in detail.

Key words: electron-photon interaction, the Schwarz-Hora effect

I. INTRODUCTION

Thirty years ago, H. Schwarz attempted to modulate an electron beam with optical frequency. In his experiments, a new effect has been discovered. When a 50-keV electron beam inside an ultra-high-vacuum system crossed a thin crystalline dielectric film illuminated with laser light, electrons produced the electron-diffraction pattern not only at a fluorescent target but also at a nonfluorescent target (the Schwarz-Hora effect). In the latter case the pattern was roughly of the same color as the laser light. The effect was absent if the electrical vector of the polarized laser light was parallel to the film surface. When changing the distance between the thin crystalline film and the target, a periodic change in the light intensity was observed with spatial period of the order of centimeters. Since 1972 no reports on the results of further attempts to repeat those experiments in other groups have appeared, while the failures of the initial such attempts have been explained by Schwarz in Ref. [7]. The latest review can be found in Ref. [7].

The reported quantitative results were obtained for the films of about 1000 Å thickness. The main material used in the experiments was SiO₂. The films were illuminated by a 10²-W/cm² argon ion laser irradiation (λ₀= 4880 Å) perpendicular to the electron beam of about 0.4 µA current. These values will be used below for estimates.

There are several essential contradictions between the theory and the Schwarz experiments:

1. The radiation intensity. The relatively high intensity of the Schwarz-Hora radiation (at least of the order of 10⁻¹⁰ W) was observed in the Schwarz experiments. The calculated power of the coherent emission of light at the laser frequency turns out to be at least 10³ times smaller.

2. The dependence on the electron current. The presented in Refs. experimental photographs of the diffraction pattern allow to affirm that the intensity of the Schwarz-Hora radiation is linear on an electron beam current. The quantum-mechanical treatment shows that a sharp peak in the intensity at the laser frequency can be accounted for only the collective processes of light emission. It leads to the quadratic dependence of the radiation on the electron current.

3. The polarization angle dependence. The observed intensity of the Schwarz-Hora radiation falls off exponentially with the angle between the electric vector of the laser light and the electron beam direction. The exponential slope appears to be linear in the distance between the film surface and the target. A theoretical explanation of such dependence is absent.

4. An initial phase of the spatial beating. The Schwarz experiments indicate that there must be the maximum of the Schwarz-Hora radiation intensity at the dielectric film surface. The quantum-mechanical models predict the minimum.

5. The spatial beating period. There is the large discrepancy of more than 10% between the experimental and theoretical results for the period of the spatial beating of the Schwarz-Hora radiation.

These contradictions allow to conclude that we have not the adequate theory for the interpretation of the Schwarz-Hora effect. It seems natural in this situation to try to connect the known facts in some phenomenological scheme. Below we consider such phenomenological model. The quantitative aspects of the model are essentially connected with the interpretation of the long-wavelength spatial modulation of the Schwarz-Hora radiation. The quantum interpretation of such modulation was discussed in Ref. [21].

II. QUANTUM INTERPRETATION OF THE LONG-WAVELENGTH SPATIAL BEATING

Let the z axis be directed along the incident electron beam. The laser beam is along the x axis. The electrical vector of the laser light is in the z direction. Electrons pass through the dielectric slab restricted by the planes z = -d and z = 0. We consider without loss of generality only the central outgoing electron beam (zeroth-order diffraction).

Usually the following assumptions are used: An electron interacts with the light wave only within the slab; it interacts within the slab only with the light wave; the
spin effects can be neglected. In the simplest case the light field within the slab and incident electrons are represented by plane waves.

Using these assumptions, consider the origin of the long-wavelength spatial modulation in the one-electron quantum theory. The solution of the Klein-Gordon equation to first order in the light field (see, for example, Refs. [8,20]) gives the following expression for the electron probability density for \( z > 0 \):

\[
\rho(x, z, t) = \rho_0 \left\{ 1 - \beta \sin \left[ \frac{z}{2\hbar} (2p_0 - p_{1z} - p_{-1z}) \right] \right. \\
\times \sin \left( \frac{\pi d}{2d_0} \cos \left[ kx - \omega t + \frac{z}{2\hbar} (p_{1z} - p_{-1z}) \right] \right) \).
\] (1)

Here \( \rho_0 \) is the probability density for the initial incident electron beam and \( \omega \) and \( k \) denote the circular frequency and the wave number of the light wave inside the slab. The parameter \( \beta \) is proportional to the amplitude of the laser field and \( d_0 \) is the smallest optimum value of the slab thickness. For the conditions of the Schwarz experiments, these parameters are \( \beta = 0.35 \) (for \( \alpha \)-quartz) and \( d_0 = 1007 \) \( \AA \). This implies that the probability for absorption (or stimulated emission) of a photon by an electron inside the slab is \( (\beta/\alpha)^2 = 0.008 \) for \( d = d_0 \). The value \( n_\alpha = 1.550 \) was used here as the \( \alpha \)-quartz refractive index. The \( z \) components of the momentum \( p_{nz} \) are determined for free electrons of energy \( E_n \) and momentum \( p_n \) from the relativistic relationship

\[
E_n^2 = m^2 c^4 + p_n^2 c^2, \\
E_n = E_0 + n\hbar\omega, \quad p_{nz} = n\hbar k, \quad n = 0, \pm 1.
\] (2)

Here \( m \) is the electron mass.

The probability that an electron absorbs or emits a photon inside the dielectric slab is a periodic function of the slab thickness. This is indicated by the second sine term in Eq. (1). We obtain the following expression [21]

\[
\lambda_b = \lambda_{b0} \frac{1}{1 - (\frac{\lambda_b}{\lambda_0})^2(1 - n^2 \cos^2 \alpha)}.
\] (3)

This formula gives a better value for the spatial beating wavelength, \( \lambda_b = 1.47 \) cm, for \( \alpha \)-quartz. However, the condition for total internal reflection, \( n \cos \alpha > 1 \), limits the possibility to improve the agreement between the theory and experiment by using the formula (3). This implies that \( \lambda_b = \lambda_{b0} = 1.515 \) cm is the upper limit, which cannot be exceeded by any formal optimization of the parameters \( n \) and \( d \).

Thus the considered above quantum models cannot resolve the discrepancy of more than 10% between theory and experiment for the quantity \( \lambda_b \). This statement remains valid even if we take into account some uncertainty of the published experimental data on the parameters \( n \) and \( d \). Below we consider the more general model situation, taking into account the possible electron beam divergence.

## III. INFLUENCE OF THE ELECTRON BEAM DIVERGENCE ON THE LONG-WAVELENGTH SPATIAL BEATING

Let the incident electron beam is represented by a fragment of the spherical wave (Fig. 1), propagating from a focal point \( F \). This point is placed at a distance \( r \) from the slab surface \( z = 0 \). We shall consider incident electrons moving at small angles \( \beta \) to a beam optical axis FA. Below the label 0 marks electrons passing through the film without energy change. The labels 1 and 2 mark electrons absorbing or emitting a photon inside the film.

The film thickness \( d \) gives the additional phase differences among the three outgoing electron beams. However, these phase differences are the same in the considered approximation for paraxial rays as for the plane
incident electron wave and do not influence the spatial beating phase. We shall neglect below the film thickness and take \(d = 0\).

Consider the interference among the electron waves at the point \(A\) at the target (on the optical axis of the incident electron beam). The phases of the three electron waves at a moment \(t\) can be written as

\[
\phi_0(A, t) = \frac{1}{\hbar} [p_0(z + r) - E_0 t],
\]

\[
\phi_1(A, t) = \frac{1}{\hbar} [\vec{p}_{1z} r + p_{1z} z - E_0 t - h \omega t],
\]

\[
\phi_2(A, t) = \frac{1}{\hbar} [\vec{p}_{2z} r + p_{2z} z - E_0 t + h \omega t],
\]

where \(\vec{p}_{1z}\) and \(\vec{p}_{2z}\) are the \(z\) components of the momenta of electrons for \(z < 0\); \(p_{1z}\) and \(p_{2z}\) are the same for \(z > 0\).

The spatial beating phase is given by the next relationship

\[
\chi = \frac{1}{2} (2\phi_0 - \phi_1 - \phi_2) = \frac{r}{2\hbar} (2p_0 - \vec{p}_{1z} - \vec{p}_{2z})
+ \frac{2\hbar}{z^2} (2p_0 - p_{1z} - p_{2z}).
\]

When \(\chi(z)\) is a linear function of the distance \(z\), the constant spatial beating wavelength \(\lambda_b\) is determined by relationship \(\chi = 2\pi z/\lambda_b\). In more general case, we define the spatial beating wavelength as \(\lambda_b(z) = 2\pi(d\chi/dz)^{-1}\).

In the simplest case, \(r(z) = \text{const}\), Eq. (8) gives

\[
\lambda_b(z) = \lambda_{0b} \left[ 1 - \left( \frac{\hbar \omega}{E_0} \right)^2 \left[ 1 - (n \cos \alpha)^2 \frac{r}{z + r} \right] \right].
\]

Taking into account the smallness of \(h\omega/E_0\), we obtain

\[
\chi = \frac{2\pi z}{\lambda_{0b}} \left[ 1 - \left( \frac{\hbar \omega}{E_0} \right)^2 \left[ 1 - (n \cos \alpha)^2 \frac{r}{z + r} \right] \right].
\]

There are two limit values of \(\lambda_b\), which do not depend on the parameters \(n\) and \(d\): \(\lambda_b = 1.826\) cm and \(\lambda_{0b} = 1.515\) cm. The first is an asymptotic value for \(z \to \infty\) and the second is the upper limit for \(z = 0\).

The presented in Ref. \[3\] experimental recording includes only three maxima and, therefore, does not allow to verify the dependence \(\lambda_b\) on \(z\). However, there are reported also the maxima at two other points: \(z = 15.3\) cm and \(z = 34.0\) cm. These data are in agreement with the constant value \(\lambda_b = 1.704\) cm and contradict the plots of \(\lambda_b(z)\) presented in Fig. 2. This discrepancy can be explained only if the distances \(r\) and \(z\) are varied synchronously keeping fixed a factor \(r/(z + r)\) in Eq. (8). In this case we must assume that Schwarz varied also the focus position \(r\) in adjusting the electron optics \[3\] for each \(z\) so that the ratio \(r/z\) was kept up fixed. This assumption allows to explain the constancy of \(\lambda_b = 1.704\) cm for \(\alpha\)-quartz taking \(r = 4.55 - 4.57\) cm at \(z = 10.2\) cm. However, in this case the problem of the initial phase of the spatial beating remains unresolved. The constant value of \(r/z\) could be slightly different in different experiments. This could be a cause of some uncertainty of the experimental data on \(\lambda_b\).

IV. PHENOMENOLOGICAL MODEL FOR INTERPRETATION OF THE SCHWARTZ-HORA EFFECT

Thirty-year history of the Schwarz-Hora effect clearly shows that the modern formalism of the quantum electrodynamics cannot explain the reported by Schwarz experimental facts. To resolve finally this long-standing problem the new control experiments are necessary. In the absence of ones it seems reasonable to try to connect the known facts in some sufficiently simple phenomenological scheme.

A starting idea for the proposed below phenomenological scheme may be considered as a literal reading of the expressions from the papers by Schwarz and Hora: "... electron beams are modulated at transmission of a laser-illuminated solid, transferring and generating photons at nonluminescent targets" \[22\]: "This means that the electrons really "carried" to the screen photons "picked up" in the interaction region within the dielectric slab" \[3\].

According to the existing quantum theory, an electron interacting with laser light in the presence of third body can absorb a photon. Such process we shall also call below "the total absorption of a photon".

Supposition 1. Suppose there can be another final result of capturing of a photon by an electron besides the total absorption. Suppose some electrons can capture a photon and form some metastable state in which the captured photon keeps its individuality.

Supposition 2. Suppose the total energy, momentum and mass of such electron metastable state are the same as for an electron which has absorbed a photon.
We use here the term "metastability" to mark the instability of the electron state relative to the interaction between the metastable electron and a third body. Such interaction leads with a high probability to a release of the captured photon. At the same time, the conservation of energy-momentum forbids the emission of the captured photon in the absence of third bodies. The high instability of such states accounts for the difficulty of their experimental detection. From this point of view, the Schwarz experiments have given a unique possibility to observe a formation of the electron metastable states at the dielectric film surface and their decay at the target. The ultra-high vacuum in the Schwarz experiments allowed to save the metastable state on the way to the target.

The simple estimate shows that the intensity of the Schwarz-Hora radiation of $10^{-10}$ W can be reached if about 0.1% of electrons in the beam take part in the transport of the captured photons to the target. So, the probability for the formation of the electron metastable state is comparable in magnitude with the probability 0.8% considered in Sect. I for the total absorption (or stimulated emission) of a photon by an electron inside the film.

For further development of the model, we consider the spatial beating of the Schwarz-Hora radiation.

In our scheme, the spatial oscillations of the radiation intensity is a consequence of the interference among light fields formed by the "released" photons. For that the captured photon must transfer to the target the information on a phase of the laser light field.

The spatial oscillations cannot be obtained if we consider only the electron metastable states with energy $E_0 + \hbar \omega$. To explain the experiment we must suppose an existing of the metastable electrons which have the different energy. Moreover, the amplitudes of those two electron waves must be comparable in magnitude to obtain the modulation depth (~ 85%) observed in the experiment.

**Supposition 3.** Suppose the second beam of electrons in the metastable states consists of electrons with energy $E_0$.

Consider a process of stimulated emission of a photon by an electron in the laser field. We may consider this process as going through three steps: (i) a laser photon is captured by the electron; (ii) this photon stimulates a formation of a second (stimulated) photon on the electron; (iii) both the photons are emitted. To interpret the appearance of the second electron beam with captured photons we may suppose as a work hypothesis that these electrons appear as a result of the emission of only one photon at the third step (iii) of the stimulated emission process. If so, then the dielectric film surface gives a unique possibility for some electrons to "freeze" one of two stimulated photons.

Consider the phases $\varphi_0$ and $\varphi_1$ of the light fields formed by the photons that were "thrown off" at the target by electrons of those two electron beams.

**Supposition 4.** Suppose the light field phase, transferred by the captured photon, does not change on the way from the dielectric film to the target.

Then we have for the target point A ($x = 0, z$) at a moment $t$ (Fig. 1).

$$
\varphi_0 = \varphi_0(0, z, t) = -\omega(t - t_0),
\varphi_1 = \varphi_1(0, z, t) = k_x x_1 - \omega(t - t_1).
$$

(10)

Here $t_0$ and $t_1$ are the corresponding times of flight and $(x_1, 0)$ is a point where the electrons with energy $E_0 + \hbar \omega$ started. The light field at the target can be written now as

$$
\psi = ae^{i\varphi_0} + be^{i\varphi_1},
$$

(11)

where the positive constants $a$ and $b$ are determined by the currents of the corresponding electron beams. The radiation intensity at the target is determined as for the usual light interference

$$
I = a^2 + b^2 + 2ab \cos (\varphi_0 - \varphi_1).
$$

(12)

Using the smallness of $\hbar \omega / E_0$, we obtain in the case of the collimated incident electron beam

$$
\Delta \varphi = \varphi_0 - \varphi_1 = \omega(t_0 - t_1) - k_x x_1 = \frac{4\pi z}{\lambda_0} \left[ 1 - \frac{\nu_0^2}{c^2} (1 - n^2 \cos^2 \alpha) \right].
$$

(13)

We may consider as in Sect. II the more general situation when the incident electron beam is focused at the distance $r$ before the dielectric film. In this case our model gives the following generalization of Eq. (13)

$$
\Delta \varphi = \frac{4\pi z}{\lambda_0} \left[ 1 - \left( \frac{\nu_0}{c} \right)^2 \left( 1 - (n \cos \alpha)^2 \frac{r}{z + r} \right) \right].
$$

(14)

This expression differs only by factor 2 from the quantum expression \ref{quantumexpression} for the spatial beating phase $\chi$. Both the expressions lead to the same spatial period $(\lambda_0 / 2)$ for the radiation intensity. We consider this coincidence as a serious argument for the acceptance of **Supposition 4** in our phenomenological scheme. The supposition looks natural enough because a photon in a plane light wave in vacuum also carries a constant phase.

However, the essential difference between our model and the quantum treatment is in the initial phase of the spatial oscillations of the radiation intensity. The expression \ref{quantumexpression} gives the maximum of the Schwarz-Hora radiation intensity at the film surface $z = 0$ unlike the results of the previous quantum considerations. Thus our model allows to resolve the problem, discussed in the end of Sect. II, on the constancy of $\lambda_0 = 1.70 \text{Å}$.

**V. CONCLUSIONS**

There are several essential contradictions unresolved up to now between the theory and the Schwarz experiments. In this work we consider in detail one of these
contradictions - the problem connected with the interpretation of the long-wavelength spatial modulation of the Schwarz-Hora radiation. It is shown that the experimental values for the long beating wavelength $\lambda_b$ can be explained only if the influence of the electron beam divergence is taken into account. The model is considered in which the incident electron beam is focused at the distance $r$ before the dielectric film. The problem of the wavelength $\lambda_b$ can be formally resolved if we fix the ratio $r/(z + r)$ in the expression (8). The additional information on the Schwarz experiments or new experiments on the Schwarz-Hora effect may clear up the problem of the spatial beating wavelength dependence on the film-target distance $z$.

The physical model, proposed in this work, allows to give the more coherent picture of the Schwarz-Hora effect than the existing quantum theories:

1. The model allows to explain the relatively high intensity of the Schwarz-Hora radiation.

2. The model gives the linear dependence of the Schwarz-Hora radiation on the electron beam current in agreement with the experiment.

3. The reported by Schwarz strong dependence of the radiation intensity on the laser light polarization is a manifestation of some spin properties of the electron metastable state. However, we do not consider those properties at a present stage of the model development.

4. The model gives the maximum of the radiation intensity at the dielectric film surface in agreement with the experiment.

5. The model allows to reproduce the magnitude and the constancy of the spatial beating period reported by Schwarz in Ref. [3] if two suppositions about an experimental setup are made: (i) the incident electron beam was not strictly collimated and was focused at the distance $r$ before the dielectric film; (ii) the ratio $r/z$ was kept up fixed in the experiments reported in Ref. [3].

Thus, the Schwarz-Hora effect presents, in our view, the unique possibility to observe the metastable states of electrons with captured photons and the more detailed experimental study of this effect is necessary.

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**Figure 1.** Interference scheme for the diverging incident electron beam.

**Figure 2.** The spatial beating wavelength as a function of the film-target distance $z$ for various parameters $m$.