Emergence and oscillation of cosmic space by joining M1-branes

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Recently, it has been proposed by Padmanabhan that the difference between the number of degrees of freedom on the boundary surface and the number of degrees of freedom in a bulk region leads to the expansion of the universe. Now, a natural question arises; how this model could explain the oscillation of universe between contraction and expansion branches? We try to address this issue in the framework of BIonic system. In this model, M0-branes join to each other and give rise to a pair of M1-anti-M1-branes. The fields which live on these branes play the roles of massive gravitons that cause the emergence of a wormhole between them and formation of a BIon system. This wormhole dissolves into M1-branes and causes a divergence between the number of degrees of freedom on the boundary surface of M1 and the bulk leading to an expansion of M1-branes. When M1-branes become close to each other, the square energy of their system becomes negative and some tachyonic states emerge. To removes these states, M1-branes compact, the sign of compacted gravity changes, causing the arising of anti-gravity: in this case, branes get away from each other. By articulating M1-Blons, an M3-brane and an anti-M3-brane are created and connected by three wormholes forming an M3-BIon. This new system behaves like the initial system and by closing branes to each other, they compact and, by getting away from each other, they open. Our universe is located on one of these M3-branes and, by compacting M3-brane, it contracts and, by opening it, it expands.

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I. INTRODUCTION

The origin of the universe expansion has been described recently by Padmanabhan [1]. It has been proposed that the expansion of the universe happens as a result of a deviation between the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom [1]. To date, several papers investigated this interesting proposal and its implications for cosmology [2][8]. For example, the Padmanabhan proposal has been used to deduce the
In previous studies, it has been shown that by joining $M_0$-branes, a pair of $M_1$-anti-$M_1$-branes could be constructed. At that stage, two types of fields are produced and interact with branes. One type plays the role of scalar field in transverse dimensions and another one appears as graviton fields on the $M_1$-branes. These gravitons lead to the emergence of a wormhole between branes and hence the formation of a BIonic system. The evolution of BIonic leads to the difference between the number of degrees of freedom on the boundary surface of $M_1$-branes and the bulk and this difference is the main cause of the expansion of $M_1$-branes in the Padmanabhan picture. When $M_1$-branes approach to each other, the square energy of branes system becomes negatives and the system transits to tachyon phase. To remove these tachyon states, $M_1$-branes become compact and gravity turns to be anti-gravity. In that conditions, branes get away from each other and begin to be opened. These BIons glue each other and form a bigger BIonic which includes $M_3$-brane and anti-$M_3$-brane in addition to three wormholes connecting them. The $M_3$-branes oscillate between compact and open branches as due to the oscillation of initial $M_1$-branes. Our universe is placed on one of these $M_3$-branes. By compacting the $M_3$-branes, it contracts and by opening $M_3$-branes, it expands.

The outline of the paper is the following. In section II, we consider the formation and the expansion of $M_1$-branes. We also study the process of formation of $M_3$ from $M_1$-branes and obtain the difference between the number of degrees of freedom of the universe in terms of BIonic evolution. In section III, we discuss how, by compacting branes, gravity turns to anti-gravity and the contraction of the branches begins. The last section is devoted to discussion and conclusions.

II. COSMIC EXPANSION IN PADMANABHAN MODEL

In previous studies, it has been shown that by joining $M_0$-branes to each other, a pair of $M_1$-anti-$M_1$-branes can be formed. The fields on these branes play the role of graviton and cause the formation of a wormhole between these branes. These graviton fields are the main cause of difference between the number of degrees of freedom of brane and bulk and hence causing an expansion. By closing $M_1$-branes, they bend, compact and gravity turns to be anti-gravity. By gluing $M_1$-branes, $M_3$-branes are formed which our universe is located on one of them. These branes expand and compact like the initial $M_1$-branes and this fact leads to an oscillation of our universe between expansion and contraction branches. The action of $M_1$ can be written as:

\[ S = -T_{M1} \int d^2 \sigma \text{Str} \sqrt{-\det(P_{abc}[E_{mnl} + E_{mi}j(Q^{-1} - \delta)^{ij}E_{klm}] + \lambda F_{abc})\det(Q_{j,k})} \]

where

\begin{align}
E_{mnl}^{\alpha\beta\gamma} &= C_{mnl}^{\alpha\beta\gamma} + B_{mnl}^{\alpha\beta\gamma}, \\
Q_{j,k}^{i} &= \delta_{j,k}^{i} + i\lambda[X_{a}^{j}T_{\alpha}^{\gamma}, X_{b}^{k}T_{\beta}^{\gamma}, X_{c}^{l}T_{\gamma}^{\gamma}]E_{j,k}^{\alpha\beta\gamma},
\end{align}

(1)
Here $X^M = X^M_\alpha\tau$, $A_{ab}$ is 2-form gauge field,

$$F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab}. \quad (2)$$

where $\lambda = 2\pi l_s^2$, $G_{mn} = g_{mn} \delta^\eta_{l\eta} + \partial_m X^i \partial_n X^j \sum_i (X^i)^2 \delta^\eta_{l\eta} + \partial_m \partial_n X^i \partial_l \partial_{\eta} X^i \delta^\eta_{l\eta}$ and $X^i$ are scalar fields of mass dimension. Here $a, b, c = 0, 1, ..., p$ are the world-volume indices of the $Mp$-branes, $i, j, k = p + 1, ..., 9$ are indices of the transverse space, and $m, n$ are the eleven-dimensional spacetime indices. Also, $T_{M0} = 1/(g_s(2\pi)^{p+1})$ is the tension of $Mp$-brane, $l_s$ is the string length and $g_s$ is the string coupling. In previous studies, it has been shown that this action can be obtained by summing over the actions of $pM0$-branes which is given by [12]:

$$S_{M0} = T_{M0} \int dt \text{Tr} \left( \Sigma^{10}_{M, N, L = 0} \langle [X^M, X^N, X^L], [X^M, X^N, X^L] \rangle \right) \quad (4)$$

To obtain the action (1) from the action of M0, we should use of following mappings [12, 15–24]:

$$\langle [X^a, X^i, X^j], [X^a, X^i, X^j] \rangle = \frac{1}{2} \varepsilon^{abc} \varepsilon^{ab'} \varepsilon^{cd}(\partial_c X^a)(\partial_c X^a)(\partial_d X^b)(\partial_d X^b) \sum_j (X^j)^2 = \frac{1}{2} (\partial_a X^i, \partial_a X^i) \sum_j (X^j)^2 \quad (5)$$

To obtain a similarity between branes and our real world, we assume that two form fields play the role of gravitons and obtain following results:

$$A^{ab} = g^{ab} = h^{ab} + \eta^{ab} \quad \text{and} \quad a, b, c = \mu, \nu, \lambda \Rightarrow$$

$$F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab} = 2(\partial_{\mu} g_{\nu\lambda} + \partial_{\nu} g_{\mu\lambda} - \partial_{\lambda} g_{\mu\nu}) = 2\Gamma_{\mu\nu\lambda}$$

$$\langle F^\rho_{\sigma\lambda}, F^\lambda_{\mu\nu} \rangle = \langle [X^\rho, X^\lambda, X^\mu], [X^\lambda, X^\mu, X^\nu] \rangle =$$

$$[X^\rho, [X^\rho, X^\lambda, X^\mu]] - [X^\mu, [X^\rho, X^\lambda, X^\nu]] + [X^\rho, X^\lambda, X^\nu][X^\lambda, X^\mu, X^\nu] - [X^\rho, X^\lambda, X^\mu][X^\lambda, X^\nu, X^\rho] =$$

$$\partial_{\rho} \Gamma^\rho_{\sigma\nu} - \partial_{\nu} \Gamma^\rho_{\sigma\rho} + \partial_{\rho} \Gamma^\rho_{\lambda\nu} \Gamma^\lambda_{\sigma\rho} - \partial_{\lambda} \Gamma^\rho_{\mu\nu} \Gamma^\lambda_{\rho\sigma} = R^\rho_{\sigma\mu\nu} \quad (6)$$

and

$$\kappa^{\rho}_{\mu\nu} = \delta^{\rho}_{\mu\nu} - \sqrt{\delta^\rho_{\mu\nu} - H^\rho_{\mu\nu}}$$

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \partial_{\mu} X^m \partial_{\nu} X^m$$

$$H_{\mu\nu} = \eta_{\mu\nu} + 2\Pi_{\mu\nu} - \eta_{\alpha\beta} \Pi_{\mu\alpha} \Pi_{\nu\beta}$$

$$X^m = x^m - \eta_{\mu\nu} \partial_{\mu} x^n$$

$$\Pi_{\mu\nu} = \partial_{\mu} \partial_{\nu} x^n$$

where $\pi$ is the scalar mode and $h^{ab}$ is the tensor mode of graviton. As can be seen from above equations, non-commutative relations between two form fields produce the exact form of curvature tensor. Also, when scalars are
attached to branes, their index changes from \(i, j \rightarrow \mu \nu\) and they transit to graviton mode. Previously, it has been shown that there are direct relations between \(\kappa\) and curvature scalars \((R)\) \cite{25, 27}:

\[
\delta_{\mu \nu}^\alpha \kappa_{\rho \sigma}^\gamma = R \tag{7}
\]

Thus, gravity can be easily obtained from the non-commutative relations in \(M\)-theory. At this stage, we can derive the explicit form of the relevant action of \(M1\) in equation (1) in terms of gravity terms. We can write:

\[
\det(Z) = \delta_{a_1 b_1 \ldots a_n b_n}^{a_1 b_1 \ldots a_n b_n} \ z_r^{a_1} \ldots z_r^{b_n} \ a, b, c = \mu, \nu, \lambda
\]

\[
Z_{abc} = P_{abc} [E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm}] + \lambda F_{abc}
\]

\[
\det(Z) = \det \left( P_{abc} [E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm}] \right) + \lambda^2 \det(F) \tag{8}
\]

This equation helps us to derive the relevant terms of determinant in action (1) separately. Applying relations in equation (7) in determinant(s), we obtain

\[
\det(F) = \delta_{\rho \sigma}^{\nu \alpha} (F^{\rho \sigma} \chi, F^{\lambda \mu}) = \delta_{\rho \sigma}^{\nu \alpha} R^{\rho \sigma}_{\mu \nu}
\]

\[
\delta_{\rho \sigma}^{\nu \alpha} [g_{\rho \sigma}^{\nu \alpha} + g_{\rho \sigma}^{\nu \alpha} \ (\partial^\mu X^\nu, \partial_{\nu} X^\alpha) \sum(X^i)^2 + (\partial^\rho \partial^\nu X^i, \partial_{\nu} \partial_{\rho} X^i) + \ldots] - \frac{1}{[(\lambda)^2 \det([X_1^k T^\alpha, X_1^k T^\beta, X_1^k T^\gamma])]}
\]

\[
\delta_{\rho \sigma}^{\nu \alpha} [\kappa_{\rho \sigma}^{\nu \alpha} \sum(X^i)^2 + (\partial_\lambda \kappa_{\rho \sigma}^{\nu \alpha} \partial_\lambda \kappa_{\rho \sigma}^{\nu \alpha})] \left(1 - \frac{1}{[(\lambda)^2 \det([X_1^k T^\alpha, X_1^k T^\beta, X_1^k T^\gamma])]} \right)
\]

\[
\delta_{\rho \sigma}^{\nu \alpha} [k_{\rho \sigma}^{\nu \alpha} \sum(X^i)^2 + (\partial_\lambda k_{\rho \sigma}^{\nu \alpha} \partial_\lambda k_{\rho \sigma}^{\nu \alpha})] \left(1 - \frac{1}{m_g^2} \right) \tag{10}
\]

where \(m_g^2 = [(\lambda)^2 \det([X_1^k T^\alpha, X_1^k T^\beta, X_1^k T^\gamma])]\) is the square of graviton mass. It is clear that the graviton mass depends on the scalars which interact with branes. This is because when scalars collide with branes, their index changes and they transit to graviton. With this definition, we can calculate another term of the determinant:

\[
\det(Q) \sim (i)^2 (\lambda)^2 \det([X_1^k T^\alpha, X_1^k T^\beta, X_1^k T^\gamma]) \det(E) \sim -[(\lambda)^2 \det([X_1^k T^\alpha, X_1^k T^\beta, X_1^k T^\gamma])] \det(g) = m_g^2 \det(g) \tag{11}
\]

By inserting equations (7), (9), (10) and (11) into the action (1) and replacing \(\sum(X^i)^2 \rightarrow F(X)\), we get:

\[
S_{M1} = -T_{M1} \int d^2 \sigma \left[ \sqrt{-g} \left( \delta_{\rho \sigma}^{\nu \alpha} [k_{\rho \sigma}^{\nu \alpha} \sum(X^i)^2 + (\partial_\lambda k_{\rho \sigma}^{\nu \alpha} \partial_\lambda k_{\rho \sigma}^{\nu \alpha})] - m_g^2 \delta_{\rho \sigma}^{\nu \alpha} \left( R^{\rho \sigma}_{\mu \nu} + [k_{\rho \sigma}^{\nu \alpha} \sum(X^i)^2 + (\partial_\lambda k_{\rho \sigma}^{\nu \alpha} \partial_\lambda k_{\rho \sigma}^{\nu \alpha})] \right) \right) \right]
\]

\[
= -T_{M1} \int d^2 \sigma \left[ \sqrt{-g} \left( F(X) R - m_g^2 \delta_{\rho \sigma}^{\nu \alpha} R^{\rho \sigma}_{\mu \nu} - m_g^2 F(X) R + \delta_{\rho \sigma}^{\nu \alpha} (1 - m_g^2) (\partial_\lambda k_{\rho \sigma}^{\nu \alpha} \partial_\lambda k_{\rho \sigma}^{\nu \alpha}) \right) \right] \tag{12}
\]

Obviously, first order terms in nonlinear theories, like Lovelock and massive gravity, are present in this action. This means that there is a direct relation between \(M\)-theory and effective theories of gravity. According to these calculations, there are two types of modes for gravitons. Scalar modes which are produced by attaching scalars to branes and tensor modes which are produced in the process of formation of \(M1\) from \(M0\)-branes (see also \cite{28, 29}).

Using the equation (1) and assuming the separation distance between two \(M1\) be \(l_4\) and the length of each \(M1\) be \(l_1\), we can obtain the relevant action for the interaction of an \(M1\) with an anti-\(M1\)-brane:
\[ A^{ab} \rightarrow l_1 \quad X^2 \rightarrow l_d \quad X^0 = t \quad X^i = 0, i \neq 0, 2 \]

\[ S = -T_{M1} \int d^2 \sigma \sqrt{\dot{l}_d^2 + \dot{l}_d^2} \left[ \left( l_d^2 + (l_d')^2 + (l_d'')^2(1 + \dot{l}_d^2)^{-1} \right) \left( 1 - \frac{1}{\dot{l}_d^2} \right) - (l_d')^2(l_d''^2) \right] = \]

\[-T_{M1} \int d^2 \sigma V(l_d) \sqrt{D_{l_d,t_1}} \]

\[ V(l_d) = \sqrt{\dot{l}_d^2 + l_d^5} \]

\[ D_{l_d,t_1} = \left[ (\dot{l}_d^2 + (l_d')^2 + (l_d'')^2(1 + \dot{l}_d^2)^{-1}) \left( 1 - \frac{1}{\dot{l}_d^2} \right) - (l_d')^2(l_d''^2) \right] \]

where the prime denotes the derivative respect to time. The equations of motion obtained from this action are:

\[
\left( \frac{(l_d')}{{D_{l_d,t_1}}} \right)' = \frac{1}{{D_{l_d,t_1}}} \left( 2l_d^{-4}[(l_d')^2 + (l_d')^2(1 + l_d^2)^{-1}](l_d') - 2(l_d')^2(l_d')(l_d'') + \frac{V'}{V} D_{l_d,t_1} - (l_d') \left[ 1 - \frac{1}{l_d^2} \right] \right) 
\]

\[
\left( \frac{2(l_d')(l_d'')}{D_{l_d,t_1}} \right)' = \frac{1}{{D_{l_d,t_1}}} (l_1 \left[ 1 - \frac{1}{l_d^2} \right] - \frac{V'}{V} [l_d']) 
\]

(14)

Solving these equations simultaneously, we obtain the approximate form of \( l_d \) and \( l_1 \) in terms of time:

\[
l_1 \sim \frac{l_{s,0}}{t_s^2} (t_1 - t) e^{-l_0(t_1 - t)} \\
l_d \sim \frac{l_{d,0}}{t_s^2} (t_1 - t)^{1/2} e^{-l_s t} 
\]

(15)

where \( t_s \) is time of collision between \( M1 \)-branes and \( l_{d,0} \) is the maximum distance between two \( M1 \)-branes. To be sure that these solutions are true, we must verify near the point that branes collide to each other, we consider their correctness when branes are closed to each other \( (l_d \sim 0) \). In this case, the size of two branes is very big \( (l_1 \sim \infty) \) and the velocity of their motion and rate of their growth is large \( (l_d' \sim l_1' \sim \infty) \). For this state, equations in (14) reduce to following equations:

\[
l_d \sim 0 \quad l_d' \sim \infty \]

\[
\frac{V'}{V} D_{l_d,t_1} \sim \frac{l_d'(2l_d + 5l_d')}{l_d^2 + l_d''^2} \left[ (l_d^2 + (l_d')^2 + (l_d'')^2(1 + l_d^2)^{-1}) \left[ 1 - \frac{1}{l_d^2} \right] - (l_d')^2(l_d''^2) \right] \sim \\
2l_d^{-4}[(l_d')^2 + (l_d')^2(1 + l_d^2)^{-1}](l_d') - 2(l_d')^2(l_d)(l_d'') 
\]

(13)

\[
\Rightarrow \left( \frac{(l_d')}{{D_{l_d,t_1}}} \right)' \sim \frac{1}{{D_{l_d,t_1}}} \left( \frac{(l_d')^2}{l_d^2 + l_d''^2} \left[ 1 - \frac{1}{l_d^2} \right] \right) 
\]

\[
\Rightarrow \frac{1}{X} = \frac{1}{{D_{l_d,t_1}}} \left( \frac{(l_d')^2}{l_d^2 + l_d''^2} \left[ 1 - \frac{1}{l_d^2} \right] \right) \rightarrow \frac{dX}{dt} = \lambda X \rightarrow X = e^{\lambda t} 
\]

\[
Y = \frac{1}{X} \sim \frac{(l_d'')^2(1 + l_d^2)^{-1}}{(l_d')^2 \left[ 1 - \frac{1}{l_d^2} \right]^2} 
\]

\[
\Rightarrow \frac{d}{dt} l_d \sim (B - t)^{1/2} e^{-\lambda t} 
\]

\[
l_d(t = t_s) = 0 \rightarrow B = t_s \quad \lambda = \frac{2}{t_s} 
\]
Applying these relations in equation (1), we obtain:

\[
\begin{align*}
\left(\frac{l_d}{\sqrt{D_{ia,i_1}}}\right)^2 & \simeq \frac{1}{\sqrt{D_{ia,i_1}}} \left(\frac{l_d}{l_i^2}\right) \\
\int & \sim (t_s - t)^{\frac{3}{2}} e^{-\frac{t}{l_s}} \Rightarrow D_{ia,i_1} \sim (t - t_s)^{-3} \\
\Rightarrow & \left(\frac{l_d}{(t_s - t)}\right)^2 \simeq \left(\frac{l_d}{(t_s - t)^2}\right) \\
\Rightarrow & l_1 \sim \frac{1}{(t_s - t)} (t_s - t) = 1 - e^{-\lambda(t_s - t)}
\end{align*}
\]

This equation shows that by passing time, two M1-branes move towards each other and \( l_d \) decreases while the size of \( M1 \) increases. We can show that a wormhole is formed between these \( M1 \)-branes that, by dissolving in them, causes to their growth. Before discussing this subject, we will construct \( M_p \)-branes from gluing \( M1 \)-branes. To this end, by equation (9), we use the following replacements in the action of the branes.

\[
i, j = a, b \Rightarrow \langle [X^i, X^j], [X^i, X^j]\rangle \Rightarrow \langle [X^a, X^b], [X^a, X^b]\rangle = \frac{1}{2} \langle \partial_a X^i, \partial_b X^i \rangle \sum X_j^2
\]

\[
i, j = a, b \Rightarrow \langle [X^i, X^j], [X^i, X^j]\rangle \Rightarrow \langle [X^a, X^b], [X^a, X^b]\rangle = \frac{1}{2} \langle \partial_a X^i, \partial_b X^i \rangle \\
i, j, k = a, b, c \Rightarrow \langle [X^i, X^j, X^k], [X^i, X^j, X^k]\rangle \Rightarrow \langle [X^a, X^b, X^c], [X^a, X^b, X^c]\rangle = \langle F^{abc}, F^{abc}\rangle
\]

\[
Q_{i,j} = \delta_{i,j} + i \int X^j T^a, X^k T^b \Rightarrow E_{i,j} = \langle F^{abc}, F^{abc}\rangle
\]

\[
T_{M1} \int d^2\sigma \Rightarrow T_{M_p} \int d^p\sigma
\]

Applying these relations in equation (11), we obtain:

\[
S = -T_{M_p} \int d^p\sigma \sqrt{-\det(O + 2\pi l_s^2 G(F))}
\]

\[
G = \sum_{n=0}^{p} \frac{1}{n!} \left(\frac{F}{\beta^2}\right)^n
\]

\[
O = \frac{1}{p} \sum_{n} \frac{(p-n)!}{n!} Y_n^p
\]

\[
F = \langle F^{abc}, F^{abc}\rangle \quad Y = \langle \partial_a X^i, \partial^a X^i \rangle \sum (X_j)^2 + \langle \partial_a \partial_b X^i, \partial^a \partial^b X^i \rangle \quad \beta = \frac{1}{2\pi l_s^2}
\]

(18)

where the nonlinear field \((G)\) has been introduced in [30–32]. Now, we can show that this action can be reproduced by multiplying the terms of relevant actions of \( p \) M1's. For simplicity, we choose \( X^1 = \sigma \) and \( X^4 = z, \sum(X_i)^2 \rightarrow F(z) \) where \( z \) is the transverse direction between branes. Using the action in (18), the Lagrangian for \( M_p \)-brane can be written as

\[
L = -T_{M_p} \int d\sigma \sqrt{(1 + z'^2 + z''^2)^p + (2\pi l_s^2)^2 G(F)}
\]

(19)

where \( (') \) denotes the derivative respect to \( \sigma \) and \( z' \) and \( z'' \) are the velocity and acceleration of branes in transverse dimension. To derive the Hamiltonian, we must obtain the canonical momentum density for graviton. For simplicity, we will use the method in [33] and [34] and assume that \( F_{001} \neq 0 \) and other components of \( F \) are zero. We get:

\[
\Pi = \frac{\delta L}{\delta \partial_{i}A^{0i}} = \frac{\sum_{n=0}^{p} \frac{n}{n!} (\frac{F_{001}}{2\pi l_s^2})^{n-1} F_{001}}{(1 + z'^2 + z''^2)^p + (2\pi l_s^2)^2 G(F)}
\]

(20)
Thus the Hamiltonian can be written as:

\[ H = T_{Mp} \int d\sigma \sigma^p \Pi \partial_t A^{01} - L = 4\pi \int d\sigma [\sigma^p \Pi (\partial_t A^{01} - \partial_\sigma A^{00}) - \partial_\sigma (\sigma^2 \Pi) A_{00}] - L \]  

(21)

where we use the integration by parts and applied the term proportional to \( \partial_\sigma A^{01} \). Using the constraint \( (\partial_\sigma (\sigma^p \Pi) = 0) \), we obtain:

\[ \Pi = \frac{k}{4\pi \sigma^p} \]  

(22)

where \( k \) is a constant. Replacing \( \Pi \) from the above equation into equation (21) gives the following Hamiltonian:

\[ H_1 = T_{Mp} \int d\sigma \sigma^p \left[ \left(1 + z'^2 F(z) + z'^2 r^2\right)^p + \left(2\pi l_s^2\right)^2 \sum_{n=0}^{p} \frac{n}{n!} \left(-\frac{F}{\beta^2}\right)^n F_1 \right] \]

(23)

For obtaining the explicit form of the wormhole between branes, we need a Hamiltonian which can be expressed in terms of separation distance between branes. To this end, the Lagrangian can be redefined as:

\[ L = -T_{Mp} \int d\sigma \sigma^p F_1 \Pi \partial_t A^{01} - L = \int d\sigma [\sigma^p F_1 \Pi (\partial_t A^{01} - \partial_\sigma A^{00}) - \partial_\sigma (F_1 \sigma^p \Pi) A_{00}] - L \]

(24)

With the help of this Lagrangian, we repeat our previous calculations. We can obtain:

\[ \Pi = \frac{\delta L}{\delta \partial_t A^{01}} = \frac{\sum_{n}^{n(n-1)} \frac{n(n-1)}{n!} (-\frac{F}{\pi})^{n-1} F_001}{\sqrt{(1 + z'^2 F(z) + z'^2 r^2)^p + \left(2\pi l_s^2\right)^2 \sum_{n=0}^{p} \frac{n}{n!} \left(-\frac{F}{\pi}\right)^n}} \]

(25)

Therefore the new Hamiltonian can be constructed as:

\[ H_2 = T_{Mp} \int d\sigma \sigma^p F_1 \Pi \partial_t A^{01} - L = \int d\sigma [\sigma^p F_1 \Pi (\partial_t A^{01} - \partial_\sigma A^{00}) - \partial_\sigma (F_1 \sigma^p \Pi) A_{00}] - L \]

(26)

where like the previous step, we have used in the second step an integration in parts for the term proportional to \( \partial_\sigma A^{01} \) like the method in [33]. Imposing the constraint \( (\partial_\sigma (\sigma^p \Pi) = 0) \), we obtain:

\[ \Pi = \frac{k}{4F_1 \pi \sigma^p} \]

(27)

By replacing the momentum in equation (27) into equation (26) we derive the following Hamiltonian:

\[ H_p = T_{Mp} \int d\sigma \sigma^p \left[ \left(1 + z'^2 F(z) + z'^2 r^2\right)^p + \left(2\pi l_s^2\right)^2 \sum_{n=0}^{p} \frac{n(n-1)}{n!} \left(-\frac{F}{\beta^2}\right)^n F_p \right] \]

\[ F_2 = F_1 \sqrt{1 + \frac{k^2}{F_1^2 \sigma^2 p}} \]

(28)

and by repeating these calculations for \( p \) times, we obtain:

\[ H_p = T_{Mp} \int d\sigma \sigma^p \left[ \sqrt{1 + z'^2 F(z) + z'^2 r^2} F_{tot} \right] \]

\[ F_{tot} = \sqrt{1 + \frac{k^2}{F_{p-1}^2 \sigma^2 p}} \sqrt{1 + \frac{k^2}{F_{p-2}^2 \sigma^2 p}} \cdots \sqrt{1 + \frac{k^2}{F_1^2 \sigma^2 p}} \sqrt{1 + \frac{k^2}{\sigma^2 p}} \]

\[ F_n = F_{n-1} \sqrt{1 + \frac{k^2}{F_{n-1}^2 \sigma^2 p}} \]

(29)
By growing branes \((\sigma \to \infty)\), the canonical density \((\Pi)\) in equation \((27)\) becomes small. This is because that this momentum relates to effect of one graviton on total size of one brane and consequently, by increasing the length of one brane, this effect decreases. However, by joining M1-branes to each other, the number of gravitons on the branes increases and total momentum density of gravity which is the sum over the momentum densities of gravitons enhances. Thus, this total momentum becomes large and plays the main role in evolution of universe branes. At this stage, for the case of \(\frac{k}{\sigma^2} \ll 1\), we can reproduce the Hamiltonian of \(Mp\)-brane by multiplying the Hamiltonians of M1's:

\[
H_p = T_{Mp} \int d\sigma \sigma^p (\sqrt{1 + z'^2 F(z)} + z''^2)^p F_{tot}
\]

\[
k = \frac{k}{p} \to F_{tot} = \sqrt{1 + \frac{k^2}{F_{p-1}^2} + \frac{k^2}{F_{p-2}^2} + \ldots + \frac{k^2}{F_{2}^2} + \frac{k^2}{\sigma^2}} \Rightarrow
\]

\[
H_p = T_{Mp} \int d\sigma \sigma^p (\sqrt{1 + z'^2 F(z)} + z''^2)^p (1 + \frac{k^2}{\sigma^2}) = T_{Mp} \int d^p \sigma \sigma^p (\sqrt{1 + z'^2})^p (1 + \frac{k^2}{\sigma^2}) \Rightarrow
\]

\[
H_p = \left( T_{M1} \int d\sigma \sigma \sqrt{1 + z'^2 F(z)} + z''^2 \right) (1 + \frac{k^2}{\sigma^2}) = H_1^p
\]

\[
H_1 = T_{M1} \int d\sigma \sigma \sqrt{1 + z'^2 F(z)} + z''^2 (1 + \frac{k^2}{\sigma^2}) = H_1
\]

\((30)\)

where we have used of this assumption that \(T_{Mp} = (T_{M1})^p\). As can be seen from the above equation, each \(Mp\)-brane can be constructed of \(pM1\)-brane. Also, we can show that each \(M1\)-brane produces a wormhole. For this end, using the above Hamiltonian and assuming that the acceleration of branes be smaller than the velocity of branes in transverse dimension \((z'' \ll z')\) and \(F(z) \sim z^2\), we derive the following equation of motion \(z\) for any M1:

\[
-z'^{M1} = (\frac{V_1(\sigma)^2}{V_1(\sigma_0)^2})^{-1/2} z
\]

\[
V_1 = \sigma F_1 = \sigma \sqrt{1 + \frac{k^2}{\sigma^2}}
\]

\((31)\)

The solution of this equation is:

\[
z_{M1} = e^{\int_{\sigma_0}^{\sigma} d\sigma' (\frac{V_1(\sigma')^2}{V_1(\sigma_0)^2})^{-1/2}}
\]

\((32)\)

Thus, the separation distance between two branes is:

\[
\Delta_{M1} = 2z_{M1} = 2e^{\int_{\sigma_0}^{\sigma} d\sigma' (\frac{V_1(\sigma')^2}{V_1(\sigma_0)^2})^{-1/2}}
\]

\((33)\)

where \(\sigma_0\) is the throat of wormhole between two \(M1\)-branes of two different branes. Thus, each \(Mp\)-brane is constructed from \(pM1\)-branes which each of them produces a wormhole and connects with the \(M1\) of other branes.

Now, we can derive the relevant action for \(Mp\)-branes by multiplying the action of \(p M1\)-branes by using the antisymmetric form \(\delta\):

\[
S_{Mp} = -T_{Mp} \int dt L_{Mp}
\]

\[
L_{Mp} = \text{det}(M) \quad L_{M1,i} = L_{a_i} = \text{det}(M_i) \sim M_i \quad \text{where,} \quad \text{det}(M) = \delta_{b_1, b_2, \ldots, b_n} M_{a_1}^{b_1} \ldots M_{a_n}^{b_n} \Rightarrow
\]

\[
L_{Mp} = \text{det}(M) = \delta_{b_1, b_2, \ldots, b_n} L_{a_1}^{b_1} \ldots L_{a_n}^{b_n} \quad \text{where,} \quad \delta_{b_1, b_2, \ldots, b_n} \delta_{a_1} \sigma_1 \ldots \delta_{a_n} \sigma_n = \delta_{b_1} \sigma_1 \ldots \delta_{b_n} \sigma_n
\]

\[
\sqrt{-g} = \sqrt{-\text{det}(g)} = \sqrt{-\text{det}(g_1 g_2 \ldots g_p)} = \sqrt{-\text{det}(g_1) \text{det}(g_2) \ldots \text{det}(g_p)}
\]

\((34)\)
so we have

\( S_{MP} = -(T_{M})^{p} \int dt \delta_{c_{1}c_{2}...c_{n}}^{a_{1}a_{2}...a_{n}} f_{a_{1}}...f_{a_{p}} = \)

\(- (T_{M})^{p} \int dt \int d^{p} \sigma \delta_{c_{1}c_{2}...c_{n}}^{a_{1}a_{2}...a_{n}} [ \sqrt{-g} ( \delta_{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} \sum (X^{i})^{2} + (\partial_{\lambda} K^{\mu_{1}}_{\nu_{1}} \partial_{\lambda} K^{\mu_{2}}_{\nu_{2}}) - m^{2}_{g} \delta^{\mu_{1}\nu_{1}}_{\nu_{1}\nu_{2}} (R_{\mu_{1}...\mu_{n}\nu_{n}}^{\nu_{1}} + [K^{\mu_{1}}_{\nu_{1}} \sum (X^{i})^{2} + (\partial_{\lambda} K^{\mu_{1}}_{\nu_{1}} \partial_{\lambda} K^{\mu_{2}}_{\nu_{2}})]) ) ]_{b_{1}}^{b_{p}} \times \)

\[ \left[ \sqrt{-g} ( \delta_{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} \sum (X^{i})^{2} + (\partial_{\lambda} K^{\mu_{1}}_{\nu_{1}} \partial_{\lambda} K^{\mu_{2}}_{\nu_{2}}) - m^{2}_{g} \delta^{\mu_{1}\nu_{1}}_{\nu_{1}\nu_{2}} (R_{\mu_{1}...\mu_{n}\nu_{n}}^{\nu_{1}} + [K^{\mu_{1}}_{\nu_{1}} \sum (X^{i})^{2} + (\partial_{\lambda} K^{\mu_{1}}_{\nu_{1}} \partial_{\lambda} K^{\mu_{2}}_{\nu_{2}})]) ) \right]_{a_{p}} \]

\(- (T_{M})^{p} \int dt \int d^{p} \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} F(X)^{\mu_{1}\nu_{1}} K^{\mu_{1}}_{\nu_{1}} K^{\mu_{2}}_{\nu_{2}} - \sum_{n=1}^{p} m^{2}_{g} \delta^{\mu_{1}\nu_{1}}_{\nu_{1}\nu_{2}} R_{\mu_{1}...\mu_{n}\nu_{n}}^{\nu_{1}} \right) \right]

\(- \sum_{n=1}^{p} m^{2}_{g} F(X)^{\mu_{1}\nu_{1}} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} K^{\mu_{1}}_{\nu_{1}} K^{\mu_{2}}_{\nu_{2}} + \sum_{n=1}^{p} (1 - m^{2}_{g}) \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} \partial_{\lambda} K^{\mu_{1}}_{\nu_{1}} \partial_{\lambda} K^{\mu_{2}}_{\nu_{2}} + \ldots ) \right) \]

(35)

This action includes all terms in nonlinear gravity theories like Lovelock [33, 36] and massive gravity [25, 27]. In addition, some extra terms are predicted only in this model. Now, we calculate the number of degrees freedom on the universe brane and in a bulk. Previously, we showed that two form gauge fields are the main cause of appearance of wormhole between M-branes. Thus, difference between number of degrees freedom on the brane and bulk is due to these fields. Using equations (9) and (15) and assuming \( A^{22} \sim g^{22} \sim l_{1} \) and \( A^{ii} = g_{ij} = 0 \), we get:

\[ A^{00} = g^{00} = -1 \quad A^{22} \sim g^{22} \sim l_{1} \quad A^{ij} = 0, i,j \neq 0,2 \]

\[ N_{sur} - N_{bulk} = 2(T_{M})^{p} \int dt \int d^{p} \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} \{ F^{\mu_{1}}_{\lambda \nu_{1}}, F^{\lambda} \}_{\nu_{1}...\nu_{n}} \right) \right] \]

\[ 2(T_{M})^{p} \int dt \int d^{p} \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} R_{\mu_{1}...\mu_{n}\nu_{n}}^{\nu_{1}} \right) \right] \approx \]

\[ (T_{M})^{p} V \sum_{n} \left[ \frac{n^{0}_{\gamma}}{2n(2n-1)(t_{s} - t)^{2n-1}} \left( 1 - \frac{t_{s}}{\sqrt{t_{s} - t}} \right)^{2n} - \frac{l_{2n}^{0}}{l_{2n}^{0}} (t_{s} - t)^{n} e^{-\frac{2n}{t_{s}}} \right] \]

(36)

where \( V \) is the volume of brane, \( p = 3 \) is related to our universe and the number 2 is related to exchanging graviton between two branes which produce two sections of a wormhole. We also have used of this fact that \( m_{g}^{2} = ((\lambda)^{2} det([X_{\alpha}^{\alpha}, X_{\beta}^{\beta}, X_{\gamma}^{\gamma}]) \sim 1 + \frac{3}{d_{t}} \). This equation shows that by approaching branes, difference between number of degrees freedom increases and this causes to the growth of branes and universe expansion. We also have:

\[ A^{00} = g^{00} = -1 \quad A^{22} \sim g^{22} \sim l_{1} \quad A^{ij} = 0, i,j \neq 0,2 \quad X^{2} = l_{1} \quad X^{i} = 0 \neq 2 \]

\[ N_{sur} - N_{bulk} = E_{M}p + E_{anti-M}p = \]

\[ -2(T_{M})^{p} \int dt \int d^{p} \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} F(X)^{\mu_{1}\nu_{1}} K^{\mu_{1}}_{\nu_{1}} K^{\mu_{2}}_{\nu_{2}} - \sum_{n=1}^{p} m^{2}_{g} \delta^{\mu_{1}\nu_{1}}_{\nu_{1}\nu_{2}} R_{\mu_{1}...\mu_{n}\nu_{n}}^{\nu_{1}} - \right) \right] \]

\[ \sum_{n=1}^{p} m^{2}_{g} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} R_{\mu_{1}...\mu_{n}\nu_{n}}^{\nu_{1}} - \]

\[ \sum_{n=1}^{p} m^{2}_{g} F(X)^{\mu_{1}\nu_{1}} \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} K^{\mu_{1}}_{\nu_{1}} K^{\mu_{2}}_{\nu_{2}} + \]

\[ \sum_{n=1}^{p} (1 - m^{2}_{g}) \delta^{\mu_{1}...\mu_{n}\sigma}^{\nu_{1}...\nu_{n}} \partial_{\lambda} K^{\mu_{1}}_{\nu_{1}} \partial_{\lambda} K^{\mu_{2}}_{\nu_{2}} + \ldots ) \]


\[
(T_{M_{p}}) V \sum_{n} \left[ \frac{6^{n} l_{1,n}^{2n} + l_{1,0}^{2n}}{(t_{s} - t)^{2n}} \right] e^{-2n} = \sum_{n} \left[ \frac{l_{d,0}^{2n} (t_{s} - t)^{n}}{t_{s}^{2n}} \right] e^{-2n t_{s}/t_{s}}
\] (37)

Solving equations (36) and (37), we can obtain the explicit form of degrees of freedom of bulk and brane:

\[
N_{\text{sur}} = (T_{M_{p}}) V \sum_{n} \left[ \frac{6^{n} l_{1,n}^{2n} + l_{1,0}^{2n}}{(t_{s} - t)^{2n}} \right] e^{-2n} = \sum_{n} \left[ \frac{l_{d,0}^{2n} (t_{s} - t)^{n}}{t_{s}^{2n}} \right] e^{-2n t_{s}/t_{s}}
\] (38)

\[
N_{\text{bulk}} = 2(T_{M_{p}}) V \sum_{n} \left[ \frac{6^{n} l_{1,n}^{2n} + l_{1,0}^{2n}}{(t_{s} - t)^{2n}} \right] e^{-2n} = \sum_{n} \left[ \frac{l_{d,0}^{2n} (t_{s} - t)^{n}}{t_{s}^{2n}} \right] e^{-2n t_{s}/t_{s}}
\] (39)

Clearly, at the beginning \((t = 0)\), the number of degrees of freedom on the surface of the brane is zero; while, by evolving the time and approaching the branes towards each other, the number of degrees of freedom on the brane increases and tends to infinity (see Figure 1. Left). On the other hand, the number of degrees of freedom in the bulk decreases with time and shrinks to zero at colliding point \((t = t_{s})\) (see Figure 1. Right).

FIG. 1: (Left) \(N_{\text{sur}}\) is increasing from zero at \(t = 0\) to infinity at \(t = t_{s}\). (Right) \(N_{\text{bulk}}\) is decreasing from certain value \(att = 0\) to zero \(t = t_{s}\). We have assumed \(p = 3\) for 3+1 dimensional M3 which our universe is located on it and time of collision between branes \(t_{s} = 33\text{Gyr}\).

III. CONTRACTION BRANCH OF COSMIC SPACE IN PADMANABHAN MODEL

Until now, we have shown that by approaching branes, their size grows causing the expansion of the universe. Now, we will show that near the collision point, branes compact, universe contracts and gravity changes to anti-gravity. This causes that branes get away from each other. To this end, let us to consider equations (13) and (15) near the colliding point:

\[
t \to t_{s} \Rightarrow l_{d} \sim \frac{l_{d,0} t_{s}}{t_{s}} (t_{s} - t)^{3/2} \to 0 \Rightarrow
\]

\[
l_{1} \sim \frac{l_{1,0} (t_{s} - t)^{2}}{l_{d} (t_{s} - t)} e^{-l_{0}(t_{s} - t)} \to l_{1}^{'(t_{s} - t)} \sim \frac{1}{t_{s} - t} \to \infty \Rightarrow
\]
This equation shows that by closing M1-branes to each other, $D_{a_1 b_1} \ll 0$ and thus the expression under $\sqrt{}$ in the action in equation (13) becomes negative. This means that the square energy of system becomes negative and some tachyonic states are produced. To solve this problem, M1-branes compact and the sign of gravity changes. To show this, we use of the method in [12, 24] and define $X_i^{\alpha} \rightarrow X_i^{\alpha} - \frac{R}{l_p^{3/2}}$ where $l_p$ is the Planck length. We can write:

$$\Sigma_{a,b,c=0}^{10} (F_{abc}, F_{abc}) = \Sigma_{a,b,c=0}^{10} \langle [X^a, X^b, X^c], [X_a, X_b, X_c] \rangle$$

$$- \Sigma_{a,b,c,a',b'=0}^{10} \varepsilon_{abc} \delta_{a'b'} X^a X^b X^c X^d X^e X^f X^g X^h X^i X^j$$

$$- \Sigma_{a,b,c,a',b'=0}^{10} \varepsilon_{abc} \delta_{a'b'} X^a X^b X^c X^d X^e X^f X^g X^h X^i X^j$$

$$- \Sigma_{a,b,c,a',b'=0}^{10} \varepsilon_{abc} \delta_{a'b'} X^a X^b X^c X^d X^e X^f X^g X^h X^i X^j$$

$$- \Sigma_{a,b,c,a',b'=0}^{10} \varepsilon_{abc} \delta_{a'b'} X^a X^b X^c X^d X^e X^f X^g X^h X^i X^j$$

This equation shows that two form fields in eleven dimensional space-time transit to one form field as due to compaction and the sign of self energy changes. Using equation (41), we can replace all two-form terms in gravity theories by one-form terms:

$$A_b = e_b, \quad F_{ab} = \partial_a e_b - \partial_b e_a, \quad \kappa_{ab} = \partial^c e_b$$

$$\Sigma_{\rho,\sigma,\mu,\nu=0} R^\rho_{\mu\nu} = \Sigma_{\rho,\sigma,\mu,\nu,\lambda=0} (F^\rho_{\sigma\lambda}, F^\lambda_{\mu\nu}) = \Sigma_{\rho,\sigma,\mu,\nu,\lambda=0} \langle [X^\rho, X^\sigma, X^\lambda], [X^\lambda, X^\mu, X^\nu] \rangle$$

$$- 6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,\nu=0} \langle [X^\rho, X^\sigma], [X^\mu, X^\nu] \rangle = - 6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,\nu=0} F^\rho_{\sigma\mu} F^\mu_{\nu}$$

$$- 6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,\nu=0} \delta^\rho_{\rho'} \delta^\mu_{\mu'} \delta^\nu_{\nu'}$$

$$\Sigma_{\rho,\sigma,\mu,\nu=0} R = - 6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,\nu=0} \delta^\rho_{\rho'} \delta^\mu_{\mu'} \delta^\nu_{\nu'}$$

With the help of these relations, we can show that the sign of Lovelock gravity changes:

$$\sum_{n=1}^p m_{g}^{2n} \delta_{\mu_1 \mu_2 \cdots \mu_n} \kappa_{\sigma_1 \sigma_2 \cdots \sigma_n} R_{\mu_1 \mu_2 \cdots \mu_n} \kappa_{\sigma_1 \sigma_2 \cdots \sigma_n} = - \sum_{n=1}^p m_{g}^{2n} \left( \frac{R^2}{l_p^3} \right) \delta_{\mu_1 \mu_2 \cdots \mu_n} \kappa_{\sigma_1 \sigma_2 \cdots \sigma_n}$$

This equation shows that by compacting Mp-brane, nonlinear theories like Lovelock gravity converts to other type of non-linear gravity theories with opposite sign. This means that by compacting branes, gravity changes to anti-gravity. We can study other effects of compactifications of Mp-brane by extending the relations in (43):
When scalars attached to branes give the index of branes, they play the role of graviton. In these conditions, using equation (44), we can obtain the following relations.

\[ -6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{i,j=0}^9 [X^a, X^j][X_a, X_j] = -6 \left( \frac{R^2}{l_p^2} \right) \partial_a X^i \partial_a X^i \Rightarrow F(X) = \sum (X^i)^2 = 1 \]

\[ \partial_a \partial_b X^i \partial_a \partial_b X^i = \Sigma_{a,b,i=0}^{10} ([X^a, X^b, X^i], [X_a, X_b, X_i]) \Rightarrow \]

\[ -6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{i,b=0}^9 [X^b, X^i][X_b, X_i] = -6 \left( \frac{R^2}{l_p^2} \right) \partial_a X^i \partial_a X^i \] (44)

When scalars attached to branes give the index of branes, they play the role of graviton. In these conditions, using equation (44), we can obtain the following relations.

\[ X^i \rightarrow e^c \Rightarrow \partial_a \partial_b X^i \rightarrow \partial_a \kappa^c_b \]
\[ \partial_a \kappa^c_b \partial_a \kappa^c_b \rightarrow \partial_a \partial_b X^i \partial_a \partial_b X^i = \sum_{a,b,i=0}^{10} ([X^a, X^b, X^i], [X_a, X_b, X_i]) \Rightarrow \]

\[ -6 \left( \frac{R^2}{l_p^2} \right) \Sigma_{i,b=0}^9 [X^b, X^i][X_b, X_i] = -6 \left( \frac{R^2}{l_p^2} \right) \partial_a X^i \partial_a X^i \rightarrow -6 \left( \frac{R^2}{l_p^2} \right) \kappa^c_b \] (45)

Substituting equations (43) and (45) into action (37), we observe that four non-linear terms of five will be removed by each other and only one term remains:

\[ S_{M_p} = -(T_{M_p}) \int dt \int d^p \sigma \sqrt{-g} \left( \sum_{n=1}^p m_g^{2n} \left( 1 + 6^n \left( \frac{R^2}{l_p^2} \right) \right) \delta^{\pi_1 \pi_2 \ldots \pi_n}_{\mu_1 \nu_1 \ldots \mu_n \nu_n} \partial^{\lambda}_{\pi_1} \partial^{\mu}_{\pi_2} \ldots \partial^{\nu_{n-1}}_{\pi_n} \partial^{\nu_n}_{\pi_n} \right) \] (46)

where \( m_g^2 = [(\lambda)^2 \det([X^i, X^k])] \) is the square of graviton mass. This equation show that compacting Mp-branes gives rise to anti-gravity. For example, it is

\[ \sqrt{-g} R \rightarrow -m_g^2 \sqrt{-g} R \] (47)

for general relativity. In fact, by approaching branes, they compact, universe contracts and gravity changes to anti gravity. In these conditions, branes are getting away from each other and contraction branch ends. To show this, similar to the previous section, we consider the action of M1-branes and then extend it to higher dimensional compact branes. We can rewrite the action of compacted M1-brane as [12] [15] [24]:

\[ S = -T_{M1} \int d^2 \sigma \sqrt{g} R \sqrt{-\det(P_{ab}[E_{mn} + E_{mi}(Q^{-1} - \delta^{ij} E_{jk})] + \lambda F_{ab} \det(Q^i_j))} \] (48)

where

\[ E_{mn} = G_{mn} + B_{mn}, \quad Q^j_j = \delta^j_j + i \lambda [X^j, X^k] E_{kj}, \quad F_{ab} = \partial_a A_b - \partial_b A_a \] (49)

where \( \lambda = 2\pi s^2, G_{mn} = g_{mn} + \partial_m X^i \partial_n X^i \) and \( X^i \) are scalar fields with mass dimension. Using the above action and assuming that, as in previous section, the separation distance between two M1 be \( l_d \) and the length of each M1 be \( l_1 \), we can derive the relevant action for the interaction of an M1 with an anti-M1-brane:
where the prime denotes the derivative respect to time. The equations of motion extracted from the above action are:

\[
\left(\frac{(l_d'[1 - \frac{1}{l_d'2}])}{\sqrt{D_{l_d,l_1}}}\right)' = \frac{1}{\sqrt{D_{l_d,l_1}}} \left(2l_d^{-3}[l_d'(l_d'^2 + l_1'^2)]' + \frac{V'}{V}[D_{l_d,l_1} - (l_d'[1 - \frac{1}{l_d'^2}])]\right)
\]

\[
\left(\frac{2(l_1')}{\sqrt{D_{l_d,l_1}}}\right)' = \frac{1}{\sqrt{D_{l_d,l_1}}} \left(l_1 \left[1 - \frac{1}{l_d'^2}\right] - \frac{V'}{V}[l_1'^2]l_d'\right)
\]

The approximate solutions of the above equations are:

\[
l_1 \sim \frac{l_1(1 - \frac{t}{t_s})^2}{(t - t_s)^2} \left[1 + \frac{t}{(t - t_s)}\right] e^{\frac{t}{(t - t_s)}}
\]

\[
l_d \sim \frac{l_d(1 - \frac{t}{t_s})}{t_s - (t - t_s)} \left[1 + 2 \ln \left[1 + \frac{(t - t_s)}{t_s}\right]\right]
\]

(52)

It is clear that at \(t = t_s\), the separation of distance between branes \((l_d = 0)\) is zero and the length of \(M1\) is approximately infinite; while, by passing time, the distance between \(M1\)-branes increases and the length of branes decreases. We can examine the correctness of these solutions near the colliding point that branes are very closed where the prime denotes the derivative respect to time. The above action are:

\[
\left(\frac{(l_d'[1 - \frac{1}{l_d'2}])}{\sqrt{D_{l_d,l_1}}}\right)' = \frac{1}{\sqrt{D_{l_d,l_1}}} \left(2l_d^{-3}[l_d'(l_d'^2 + l_1'^2)]' + \frac{V'}{V}[D_{l_d,l_1} - (l_d'[1 - \frac{1}{l_d'^2}])]\right)
\]

\[
\left(\frac{2(l_1')}{\sqrt{D_{l_d,l_1}}}\right)' = \frac{1}{\sqrt{D_{l_d,l_1}}} \left(l_1 \left[1 - \frac{1}{l_d'^2}\right] - \frac{V'}{V}[l_1'^2]l_d'\right)
\]

The approximate solutions of the above equations are:

\[
l_1 \sim \frac{l_1(1 - \frac{t}{t_s})^2}{(t - t_s)^2} \left[1 + \frac{t}{(t - t_s)}\right] e^{\frac{t}{(t - t_s)}}
\]

\[
l_d \sim \frac{l_d(1 - \frac{t}{t_s})}{t_s - (t - t_s)} \left[1 + 2 \ln \left[1 + \frac{(t - t_s)}{t_s}\right]\right]
\]

(52)

It is clear that at \(t = t_s\), the separation of distance between branes \((l_d = 0)\) is zero and the length of \(M1\) is approximately infinite; while, by passing time, the distance between \(M1\)-branes increases and the length of branes decreases. We can examine the correctness of these solutions near the colliding point that branes are very closed to each other. In this case, the size of branes before and after collision are approximately equal \((l_1, before(t = t_s) = l_1, after(t = t_s))\). Using this assumption, equations in (52) reduce to following equations:

\[
l_d \sim 0 \quad l_1' \sim 0
\]

\[
\Rightarrow \frac{V'}{V}D_{l_d,l_1} \sim \frac{l_d l_1'}{l_d'^2 + 1} \left[l_1^2 + (l_1')^2\right] \left[1 - \frac{1}{l_d'^2}\right] + (l_1')^2 \sim
\]

\[-2l_d^{-3}(l_1')^2 + l_1'^2 l_d' + \frac{l_1'^2}{l_d}\]

And

\[
l_1, before(t \rightarrow t_s) = l_1, after(t \rightarrow t_s) \approx \frac{t_s}{(t - t_s)^2}
\]

\[
\Rightarrow l_1^2 + (l_1')^2 = (l_1)^2 \left[1 + \frac{(t - t_s)^2}{t_s^2}\right] = (l_1)^2 \left[1 + \frac{(t - t_s)^2}{t_s^2}\right] - 2\frac{(t - t_s)^2}{t_s}
\]

\[
\Rightarrow \left(\frac{(l_d'[1 - \frac{1}{l_d'^2}])}{\sqrt{D_{l_d,l_1}}}\right)' = \frac{2}{1 + \frac{(t - t_s)}{t_s}}
\]

\[
\Rightarrow l_d \sim \frac{l_d(1 - \frac{t}{t_s})}{t_s - (t - t_s)} \left[1 + 2 \ln \left[1 + \frac{(t - t_s)}{t_s}\right]\right]
\]

\[
\left(\frac{2(l_1')}{\sqrt{D_{l_d,l_1}}}\right)' \sim \frac{2(l_1')}{\sqrt{D_{l_d,l_1}}} \Rightarrow
\]

\[
l_1 \sim \frac{l_1(1 - \frac{t}{t_s})^2}{(t - t_s)^2} \left[1 + \frac{t}{(t - t_s)}\right] e^{\frac{t}{(t - t_s)}}
\]

(53)

These results can be extended to higher dimensional branes. We have constructed the action of \((56)\) from compaction terms in action \((35)\). On the other hand, in previous section, we have proved that each \(Mp\)-branes can be built from \(pM1\)-branes.

\[
S_{M_p} = -T_{M_p} \int dt\delta^{a_1, a_2, \ldots, a_n} L_{a_1}^{b_1} \cdots L_{a_p}^{b_p} \quad H \sim H_1^p
\]

(54)

This means that results of equation \((52)\) can be generalized to \(Mp\)-branes and we can choose the same lengths for all dimensions of brane :
\[ l_2 = \ldots l_p = l_1 \sim \frac{l_1 \cdot - (t - t_s)^2}{(t - t_s)^2} \left( 1 + \frac{t_s}{t - t_s} \right) e^{l_0(t_s - t)} \] (55)

At this stage, we can write the relations between the number of degrees of freedom on the brane and in the bulk and the energy of the system. Until now, we have shown that one-form gauge fields produce anti-gravity which are the main cause of inequality between number of degrees of freedom on the brane and in a bulk. Substituting equations (43), (45) and (46) in equations (30) and (37), we obtain:

\[ N_{\text{sur}} - N_{\text{bulk}} = 2(T_{M_p}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n} F_{\rho_1 \rho_2 \ldots \rho_n} F_{\sigma_1 \sigma_2 \ldots \sigma_n} \right) \right] = 2(T_{M_p}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} m_g^n (1 + 6^n \left( \frac{R^{2n}}{T^3} \right)) \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n} \right) \right] \] (56)

\[ N_{\text{sur}} + N_{\text{bulk}} = E_{\text{compact} \rightarrow M3} + E_{\text{compact} \rightarrow M3} = 2(T_{M_p}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} m_g^n (1 + 6^n \left( \frac{R^{2n}}{T^3} \right)) \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n} \right) \right] \] (57)

Solving the above equations, using equation (52) and assuming \( m_g^2 = [\lambda]^2 \text{det}([X^j, X^k]) = 1 \) + \( l_d^2 \) and \( R = \frac{l_d^2}{6} \), we obtain the surface degrees of freedom and the one of bulk as follows:

\[ A^a \rightarrow l_1 \quad X^2 \rightarrow l_d \quad X^0 = t \quad X^i = 0, i \neq 0, 2 \]

\[ N_{\text{sur}} = (T_{M_p}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \left( \frac{l_d^{2n}}{l_s^{2n}} \right) \frac{1}{(t - t_s)^{2n-1}} + \frac{t^2}{t_s} (t - t_s) \ln \left[ 1 + \frac{(t - t_s)}{t_s} \right] + 2 \ln \left[ 1 + \frac{(t - t_s)}{t_s} \right] \right)^{2n-1} \right] \] (58)

\[ N_{\text{bulk}} = 2(T_{M_p}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^{p} \left( \frac{l_d^{2n}}{l_s^{2n}} \right) \delta^{\rho_1 \sigma_1 \ldots \rho_n \sigma_n} \right) \right] \] (59)

These equations show that at the colliding point \( (t = t_s) \), \( m_g^2 = 1 \), the number of degrees of freedom in the bulk is zero \( (N_{\text{bulk}} = 0) \) and the number of degrees of freedom on the brane surface becomes infinite \( (N_{\text{sur}} = \infty) \). However by passing time, degrees of freedom on the brane surface decrease and shrinks to zero while, degrees of freedom in the bulk increases (see figure 2).

IV. SUMMARY AND DISCUSSION

In this paper, we have investigated the Padmanabhan proposal in a system of oscillating branes. In this model, first, a pair of \( M1 \)-anti-\( M1 \)-branes are constructed from joining \( M0 \)-branes. During the processes of formation of these branes, two types of fields emerge. The first type is a scalar field which moves in transverse direction and when glues to branes, plays the role of a graviton scalar mode. The second type lives on the brane, plays the role of graviton.
FIG. 2: (Left) $N_{surf}$ is decreasing from large value at $t = t_s$ to zero at large time. (Right) $N_{bulk}$ is increasing from zero at $t = t_s$ to large value for large time. We have assumed $p = 3$ for 3+1 dimensional M3 which our universe is located on it and time of collision between branes $t_s = 33\text{Gyr}$.

tensor modes and causes the formation of a wormhole between the branes. By closing two branes towards each other, the wormhole dissolves in them and leads to an inequality between the number of degrees of freedom on the surface of the branes and in the bulk. Near the colliding point, the square of energy of system becomes negative and for solving this problem, the $M1$-branes compact, two-form gauge fields convert to one-form gravity with opposite sign and anti-gravity comes out. In these conditions, branes get away from each other and their size decreases. By joining $M1$-branes, higher dimensional branes like $M3$-branes are produced which compact and open like the initial $M1$'s. Our universe is located on one of these $M3$-branes and by compacting them, contracts and by opening, expands. By expanding universe, the number of degrees of freedom on the surface increases, while the one in the bulk decreases. However, by contracting universe, the number of degrees of freedom on the surface decreases and the one in bulk enhances. In a forthcoming paper, possible observational signatures of this dynamics will be discussed.

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