On non-Fermi liquid quantum critical points in heavy fermion metals

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Abstract

Heavy electron metals on the verge of a quantum phase transition to magnetism show a number of unusual non-fermi liquid properties which are poorly understood. This article discusses in a general way various theoretical aspects of this phase transition with an eye toward understanding the non-fermi liquid phenomena. We suggest that the non-Fermi liquid quantum critical state may have a sharp Fermi surface with power law quasiparticles but with a volume not set by the usual Luttinger rule. We also discuss the possibility that the electronic structure change associated with the possible Fermi surface reconstruction may diverge at a different time/length scale from that associated with magnetic phenomena.

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1 Introduction

In recent years, a number of experiments have studied the quantum phase transition between a non-magnetic Fermi liquid metal and an antiferromagnetically ordered metal in the heavy fermion materials[1]. In the best studied cases, a host of strange phenomena have been discovered in the vicinity of the transition. Most remarkably the transition has been found to be accompanied by a rather severe breakdown of Fermi liquid theory. A proper theoretical description of this phase transition and the observed non-Fermi liquid physics is currently lacking and is the subject of much theoretical discussion[2].

Microscopically the heavy fermion materials may be modelled as a lattice of localized magnetic moments that are coupled to an itinerant band of conduction electrons through spin exchange. A crucial question is the ultimate fate
of the localized moments in the quantum ground state of the coupled system. In the fermi liquid state the local moments are absorbed into the Fermi sea by a lattice analog of the Kondo effect. The resulting Fermi surface has a volume consistent with Luttingers theorem only if the local moments are included in a count of the total electron density. This is known as the ‘large’ Fermi surface. Further the quasiparticles near the Fermi surface typically have large effective masses - this is the origin of the term ‘heavy fermi liquid’.

In the magnetically ordered states on the other hand the local moments are frozen in time in specific directions. An important though often overlooked point is that there may actually be two distinct kinds of magnetic metals. Crudely speaking in one kind the magnetism may be viewed as arising from imperfectly Kondo screened local moments, essentially as a spin density wave formed out of the parent heavy Fermi liquid. This state will be dubbed SDW. The other kind of magnetic state arises when the moments order directly due to RKKY exchange interactions and are not part of the fermi surface of the metal. As in Ref. [3] we will refer to this state as the local moment magnetic metal (LMM). The distinction between the two magnetic metals can be sharp in the sense that the two are not connected smoothly without any intervening phase transitions.

The purpose of the present paper is to discuss in a general way several aspects of the quantum phase transition associated with the onset of magnetism in the heavy Fermi liquid with a focus on understanding the non-Fermi liquid phenomena observed in experiments. We begin by elaborating on the distinction between the two kinds of magnetic metals mentioned above. We then discuss the possibility that the strong breakdown of Fermi liquid theory observed in experiments is associated with the transition to the LMM state. In contrast the transition to the SDW state is expected to be described by the well-developed Moriya-Hertz-Millis theory[4] which provides for only weak deviations from Fermi liquid theory.

A theory of the quantum phase transition between the heavy Fermi liquid and LMM states does not exist at present. Here we will first examine in a rather general manner some properties that such a phase transition will have if it is second order. We will then examine some possible ways in which such a second order transition might actually occur. We will suggest various experiments that could potentially pave the way for a theoretical understanding of the non-Fermi liquid physics.
The Kondo lattice model provides a useful framework to think about general issues in heavy fermion physics:

\[
H = H_t + H_K + H_J
\]  
\[
H_t = \sum_k \epsilon_k c_k^\dagger c_k \tag{2}
\]  
\[
H_K = \frac{J_K}{2} \sum_r \vec{S}_r \cdot c_r^\dagger \vec{\sigma} c_r \tag{3}
\]  
\[
H_J = J \sum_{<rr'>} \vec{S}_r \cdot \vec{S}_{r'} \tag{4}
\]

The first term represents the kinetic energy of conduction electrons \(c_k\) on some lattice. The second term is a Kondo spin exchange between conduction electrons and localized spin-1/2 moments \(\vec{S}_r\) on the sites \(r\) of the lattice. It is conceptually convenient to allow for a Heisenberg exchange between the local moments (the last term in the Hamiltonian).

Let us first describe the LMM metal. Imagine that the Heisenberg exchange dominates over the Kondo exchange in the Hamiltonian. In that situation, the local moments will predominantly talk to each other rather than to the conduction electrons. With unfrustrated Heisenberg interactions, the local moments will then order magnetically. If we further assume that the conduction band does not interact the magnetic Brillouin zone, then the \(c\)-electrons will essentially be unaffected by the magnetic ordering. Thus the local moments and \(c\)-electrons will more or less stay decoupled in this state. The Fermi surface of this metal will basically be determined by the \(c\)-electron band structure (see Fig. 1).

Now consider the other kind of magnetic metal - the SDW state. To understand it we first need to understand its parent state, the heavy Fermi liquid. To that end let us first ignore the Heisenberg exchange term. The Kondo process responsible for the occurrence of the Fermi liquid is conveniently described in a mean field theory which represents the local moments in terms of fermionic degrees of freedom \(f_{r\alpha}\) (\(\alpha = 1, 2\)):

\[
\vec{S}_r = \frac{1}{2} f_{r}^\dagger \vec{\sigma} f_r \tag{5}
\]

together with the constraint \(f_{r}^\dagger f_r = 1\) at each site. The mean field approximation consists of choosing an appropriate self-consistently determined quadratic Hamiltonian in terms of the \(c\) and \(f\) operators. The Kondo process is described
in terms of a 'hybridization’ between the $c$ and $f$ bands. In the heavy Fermi liquid, the hybridization amplitude is non-zero, and leads to quasiparticles whose band structure is modified from that of the bare conduction electrons. The Fermi surface is large in the sense that its volume counts both the $c$ and $f$ particles (see Fig. 1).

Now consider the effect of increasing $J_H$ on this heavy Fermi liquid. It is conceivable that at some point it becomes favorable to form a spin density wave state out of the renormalized quasiparticles of the heavy Fermi liquid (while the Kondo hybridization stays non-zero). Is this state smoothly connected to the LMM described above? To answer this let us examine the Fermi surface of this SDW state in the magnetic Brillouin zone (assuming the same magnetic ordering pattern as in the LMM). This is shown in Fig 1. Comparing with the Fermi surface of the LMM state we see that the two Fermi surfaces have different orientation. Thus at least in this model these two kinds of magnetic metals cannot be smoothly connected to each other. A Lifshitz transition associated with reconstruction of the Fermi surface must separate the two phases. Note though that both states satisfy Luttinger’s theorem which relates the total Fermi surface volume modulo the Brillouin zone volume to the total density of electrons.

Though the preceding discussion focused on the situation where the large Fermi surface did not intersect the magnetic Brillouin zone a similar distinction will in general hold in that situation as well. In complex real systems with many partially filled bands and (possibly) incommensurate magnetic order it is harder to clearly demonstrate a distinction between the two phases based on the orientation/topology of the Fermi surface. However it seems rather likely
Fig. 2. Schematic phase diagram showing the heavy fermi liquid (HF), spin density wave (SDW) and Local Moment Magnetic metal (LMM) phases. The transition between HF and SDW is expected to have a description within the Moriya-Hertz-Millis description unlike that between the HF and LMM phases.

that the distinction will survive. Finding a useful criterion to distinguish these two magnetic phases in such a general setting is an important open theoretical question.

3 Magnetic phase transition: generalities

What may we say about the quantum phase transition associated with the onset of magnetic order from the Fermi liquid? A priori we should expect that the universality class of the transition depends on which of the two distinct kinds of magnetic phases the transition is to (see Fig. 2. Considerable effort has gone into examining the transition to the spin density wave metal from the Fermi liquid[4]. This theory (known as the Moriya-Hertz-Millis theory) focuses on the long wavelength low energy fluctuations of the natural magnetic order parameter in a metallic environment and makes specific predictions for a number of physical properties near the quantum phase transition. However these predictions seem inconsistent with the strong non-Fermi liquid physics seen in some heavy fermion metals (such as $CeCu_{6-x}Au_x$ or $YbRh_2Si_2$) near the magnetic ordering transition. Thus at least in these materials it is worthwhile to explore the possibility that the observed transition is to the LMM phase. As the Fermi surface of the LMM is rather different from that of the heavy Fermi liquid, such a transition is associated with a substantial reconstruction of the Fermi surface. This is over and above the onset of magnetic order.

Empirically the hypothesis that non-Fermi liquid physics is associated with a transition to the LMM may in principle be checked in some ways. First the Hall coefficient is expected to jump on passing through the transition[5]. Sup-
porting evidence has appeared in recent measurements[6] on $YbRh_2Si_2$. It is important to point out though that as a matter of principle confirming/ruling out this hypothesis does not require detailed measurements of the Fermi surface structure near the quantum critical point. Rather one can go far away from the transition into either the magnetic or Fermi liquid phases and obtain information about their respective Fermi surfaces. Now upon tuning toward the quantum critical point if there are no other phase transitions then the Fermi surface data can be directly compared to see if major reconstruction has occured.

The properties of a phase transition between the Fermi liquid and the LMM state are rather poorly understood. Can it even be second order? Two rather distinct phenomena (magnetic ordering and Fermi surface reconstruction) are required to occur at the same point as we tune some parameter across the transition. Thus naively it might appear that a direct second order transition might occur only if more than one parameter is fine-tuned (as at a multicritical point). Interestingly recent theoretical work on simpler problems has raised the possibility that this naive expectation might be incorrect[7]. If such a second order transition were to exist, it must clearly have rather unusual properties. How do we describe such a transition? Will it reproduce the observed non-Fermi liquid physics in experiments? We do not currently know the answers to these questions. In the rest of the paper we will discuss some ideas on how the answers might eventually work out with a view to motivating future experimental and theoretical work.

4 Critical non-fermi liquid: “superlarge” Fermi surface?

The reconstruction of the Fermi surface necessary to go from the heavy Fermi liquid to the LMM is quite drastic. Even without considering the added complication of magnetic ordering how can the transition be second order? One possible way out is the following. On approaching the quantum critical point from the Fermi liquid side suppose the quasiparticle residue $Z$ vanishes everywhere on the large Fermi surface. Then the quasiparticle on this Fermi surface dies on approaching the quantum critical point. Moving further out into the LMM side one can then imagine that there is no quasiparticle-like peak in the electron spectral function at the location of the large Fermi surface. If the $Z$ vanishes continuously this would be a mechanism for the large Fermi surface to disappear continuously on going from the Fermi liquid side to the LMM. Exactly the same process can also be associated with the continuous disappearance of the Fermi surface of the LMM metal. Specifically suppose the quasiparticle residue $Z$ vanishes continuously everywhere on this Fermi surface when approaching the critical point.
Within this line of thought what is the nature of the electronic excitations right at the quantum critical point? Consider first the fate of the quasiparticle peak of the large Fermi surface. For a second order transition where $Z$ vanishes continuously it seems reasonable that the $\delta$-function quasiparticle peak is replaced by a singular power law peak. Thus one might expect that right at the quantum critical point the large Fermi surface survives but with a singular power law quasiparticle. Repeating this reasoning from the LMM side, we are lead to suppose that in addition a small Fermi surface also survives with its own power law quasiparticle.

These considerations provide a rather interesting picture of the electronic excitations of the critical ground state. It has two identifiable “Fermi surfaces” - one large and the other small - but with power law quasiparticle peaks (rather than $\delta$-function) in the electron spectral function. Clearly this is a strongly non-Fermi liquid state. The presence of a sharply defined Fermi surface (through the power law singularity) in this non-Fermi liquid suggests that it may be viewed as a higher dimensional analog of the familiar Luttinger liquid state of one dimensional systems. In two dimensions such a Luttinger liquid picture was advocated by Anderson[8] as a means to describe the optimally doped cuprates. However in striking contrast to one dimensional Luttinger liquids or their two dimensional avatars envisaged by Anderson, the quantum critical metal discussed in this section does not satisfy the Luttinger rule on the total volume of the Fermi surface. Indeed if the number of conduction electrons per unit cell is $n_c$, the presence of both the small and large Fermi surfaces implies a total Fermi volume corresponding to $(1 + n_c) + n_c = 1 + 2n_c$ electrons per unit cell. We may call this a 'super-large' Fermi surface.

One can also conceive of an alternate (tamer) process through which the Fermi surface reconstruction might take place in a second order transition. This is motivated by the calculations in Ref. [9] for a specific model. Suppose that on the Fermi liquid side the large Fermi surface actually consists of more than one sheet. For concreteness consider the case with two sheets - one of these sheets (denoted the cold sheet) simply evolves into the Fermi surface of the LMM on going through the transition. Its quasiparticle peak is retained throughout the transition including right at the critical point. Upon approaching the transition from the Fermi liquid the volume of this cold sheet changes continuously to match on to that set by the conduction electron density alone at the critical point. The other sheet - denoted the hot sheet - is where all the important action associated with the critical phenomena happen. This hot sheet disappears continuously on approaching the critical point from the Fermi liquid side through a vanishing of its quasiparticle residue. Further the volume of the hot sheet evolves continuously so that right at the critical point it only accomodates the local moments. Again right at the critical point we might expect that the hot Fermi surface survives but with a power law singularity in the electron spectral function. The nature of the excitations of
the quantum critical ground state will then be somewhat simple. There is a
cold Fermi surface with volume set by conduction electron density and with a
well-defined quasiparticle peak. In addition there is a hot Fermi surface with
volume set by the local moment density and with a power law quasiparticle
singularity. The total Fermi surface volume is the same as that of the large
Fermi surface and thus satisfies Luttinger’s theorem. This would thus be a
non-Fermi liquid ground state with a sharp Fermi surface satisfying Luttinger
theorem. However the breakdown of the Fermi liquid occurs in only one sheet
and not in all sheets of the Fermi surface.

5 Two diverging scales

As mentioned above at the Fermi liquid-LMM transition two rather differ-
ent things seem to happen simultaneously. The magnetic order appears con-
comitantly with the electronic structure change associated with Fermi surface
reconstruction. Here we will distinguish two different ways in which such a
second order transition might work out. We will call these the single scale and
two-scale hypotheses.

Let the characteristic time scale of the magnetism be denoted \( t_m \) and for the
electronic structure by \( t_{el} \). For instance \( t_m \) may be taken as the correlation time
of the fluctuations of the magnetic order parameter and can be measured by
neutron scattering experiments. Similarly \( t_{el} \) may be defined from the electron
spectral function measurable ((at least in principle) through photoemission
experiments. Specifically the electron spectral function of the Fermi liquid
very near the Fermi surface will crossover from the critical power law form to
the quasiparticle peak at some characteristic energy scale. The inverse of this
energy scale may be taken to define \( t_{el} \). If the transition were second order we
might expect that these characteristic time scales \( t_m \) and \( t_{el} \) both diverge.

Now typically near many familiar critical points there is only a single diverg-
ing length or time scale. Thus one possibility is that \( t_m, t_{el} \) diverge identically
with their ratio remaining constant. We will call this the single scale hypothesis.
However in view of the rather different physical phenomena that they charac-
terize it may be surprising to find the same divergence for both time scales.
It is therefore very interesting to contemplate the possibility that \( t_m \) and \( t_{el} \)
diverge with different exponents so that their ratio also has singular behavior
at the quantum critical point. We will call this the two-scale hypothesis. This
was first formulated in Ref. [3]. To be precise let

\[
t_m \sim |g - g_c|^{-\phi_m}
\]

where \( g \) is the control parameter used to drive the transition (pressure, mag-
netic field, etc) and \( g_c \) is the critical point. Similarly

\[
t_{el} \sim |g - g_c|^{-\phi_{el}} \tag{7}
\]

The single scale hypothesis corresponds to \( \phi_m = \phi_l \) while the two-scale hypothesis corresponds to \( \phi_m \neq \phi_{el} \). In the second case we should further distinguish the two cases \( \phi_m > \phi_l \) and \( \phi_m < \phi_l \). For concreteness let us consider the possibility that \( \phi_m > \phi_{el} \). In this case the magnetic time scale diverges faster than the electronic one. The electronic structure changes first and is followed later by the development of magnetic order. As the ratio \( t_m/t_{el} \) diverges at the quantum critical point the magnetic phenomena and the electronic structure change are well separated dynamically, occurring on vastly differing time scales.

The two-scale hypothesis is theoretically appealing for a few different reasons. First it provides a possible resolution of one of the key difficulties associated with the transition between the heavy Fermi liquid and the LMM state. Namely how can a transition that involves the vanishing of two distinct competing orders (magnetic in the LMM and ‘Kondo order’ in the Fermi liquid) be generically second order? Shouldn’t the two competing orders be separated as a function of tuning parameter? The two-scale hypothesis proposes that the two competing orders are indeed separated but not as a function of tuning parameter. Rather the separation is dynamical as a function of scale (time or length scale).

The two diverging time scales will manifest themselves as two distinct vanishing energy scales as the quantum critical point is approached. In turn these will set two distinct vanishing temperature scales \( T_m \) and \( T_{el} \) corresponding to magnetic and Kondo phenomena respectively.

If correct the two-scale hypothesis also has practical benefits for developing a theory of the quantum transition. The separation as a function of scale suggests that it may be possible to treat the two crossovers at \( t_{el}, t_m \) separately. Either crossover then involves only one fluctuating order and is therefore easier to think about. Let us now consider in a bit more detail the two separate possibilities with \( \phi_m \) greater (less) than \( \phi_{el} \).

5.1 Kondo-driven: \( \phi_m > \phi_{el} \)

If \( \phi_m > \phi_{el} \), then the underlying primary transition is associated with the Fermi surface reconstruction. The magnetism is a secondary low energy complication that happens once the local moments drop out of the Fermi surface. The transition may then be regarded as being driven by the loss of Kondo
screening.

What is the nature of the state that obtains between the two diverging time scales $t_{el}$ and $t_m$? In this intermediate state the local moments are quenched neither by Kondo screening nor by static antiferromagnetic or any other form of true long range order. A reasonable assumption then is that the local moments have developed singlet bond correlations without settling down into any particular long range ordered state. Such a state may be thought of as a resonating valence bond spin liquid formed out of the local moments. In the absence of Kondo screening the conduction electrons are essentially decoupled from the local moments, and fill a small Fermi surface. Thus this intermediate time scale state is rather exotic. Such a spin liquid state coexisting with a small Fermi surface of conduction electrons was dubbed a ‘fractionalized Fermi liquid’ (FL*) in Ref. [10]. Different kinds of FL* phases may be distinguished based on the different kinds of possible spin liquid structures.

In thinking about the possible quantum phase transition with $\phi_m > \phi_{el}$ theoretically the Fermi surface reconstruction phenomena may be treated first without reference to the magnetic ordering. The theoretical problem thus reduces to two subproblems which are hopefully simpler. First we need to describe the transition between the heavy Fermi liquid and an appropriate FL* phase. Next we need to show how magnetic long range order appears as a low energy instability of this FL* phase. Note that as the ratio of the two time scales $t_m/t_{el}$ also diverges at the transition, the unstable FL* phase obtains in a wide window asymptotically close to the transition.

5.2 Magnetism driven: $\phi_{el} > \phi_m$

Next we consider the situation where $\phi_{el} > \phi_m$ so that $t_{el}$ diverges faster than $t_m$. In this case the primary transition is the loss of magnetic order. The Kondo screening develops as a secondary phenomenon once the magnetic long range order is destroyed. In this situation at intermediate times in the paramagnetic side, a state with no magnetic order and a small Fermi surface of conduction electrons is obtained. Presumably the local moments again have short ranged antiferromagnetic correlations and may be thought of as forming a spin liquid. This too is an FL* state albeit a different one from the one discussed above. This state must then eventually be unstable (at scales $t_{el}$) towards the development of the large Fermi surface heavy Fermi liquid for this scenario to work.
6 Towards a theory

It has thus far not been possible to develop the thinking outlined in previous sections into a serious theory of the non-fermi liquid heavy electron quantum critical points. However substantial progress has been achieved in understanding various theoretical issues that may pave the way for future work in this direction. In the following sections we will quickly review these results.

7 The two-scale hypothesis

From a renormalization group standpoint the two-scale hypothesis would obtain most naturally where the ordering in one of the two phases is due to a perturbation that is dangerously irrelevant at the critical fixed point. Below we discuss the theoretical evidence for this phenomenon at quantum critical points.

7.1 Deconfined quantum criticality

The naive intuition against a direct second order transition between the HF and LMM states is based on the order parameter based theory of phase transitions pioneered by Landau. Specifically for a classical finite temperature phase transition between two distinct ordered phases with different broken symmetry characterized by two distinct order parameters, Landau’s theory forbids it from being generically second order. In applying this intuition to the present phase transition we must keep in mind two distinctions with the situation traditionally considered in Landau theory. First the transition is a quantum phase transition occurring at zero temperature. Second the Kondo order in the heavy Fermi liquid phase is not captured by any local order parameter.

Recent theoretical work[7] has shown that magnetic quantum phase transitions in two dimensional insulators can violate this natural expectation from Landau theory. Specifically a direct second order quantum phase transition between a Neel state (which breaks spin rotation symmetry) and a spin-Peierls or valence bond solid state (which breaks lattice rotation symmetry) was described for spin-1/2 magnets on a square lattice. The resulting critical theory is rather non-trivial, and is most naturally described in terms of fractional spin (spinon) variables that interact with a fluctuating emergent gauge field. Neither the spinons nor the gauge field have any legitimacy in the excitation structure of the two phases far away from the critical point. They become useful degrees of freedom only right in the vicinity of the quantum critical
point. The analogy between these results and the heavy fermi liquid critical points has been well explained in the literature\cite{7,3}. Two important features are worth emphasizing. First in these insulating magnets two distinct diverging length/time scales do appear for the two fluctuating orders. Second the deconfined quantum critical point is the moral equivalent of a non-Fermi liquid in the context of insulating magnets. The magnon spectral function measured in neutron scattering experiments will be anomalously broad right at the quantum critical point. This strong breakdown of the magnon as a quasiparticle can be roughly understood as being due to decay into two spinons at the critical point.

Since the original proposal of deconfined criticality a number of numerical model calculations have sought to confirm the predicted phenomena. Unfortunately the majority of such calculations seem to find first order transitions or indications for unusual second order transitions not predicted by the theory\cite{11}. A notable exception may be the very recent calculations of Sandvik\cite{12} on a fully \textit{SU(2)} symmetric spin-1/2 model on a square lattice where results consistent with the theory of Ref. \cite{7} is obtained. Finding clear numerical support for the theory of Ref. \cite{7} is clearly an important open problem.

As far as lessons for the heavy fermion critical points go, it is sufficient to consider the theoretical example of the same transition in general \textit{SU(N)} symmetric spin models\cite{13} and study it in a systematic large-$N$ expansion. This provides an analytically controlled example\cite{7} of such a deconfined quantum critical point and therefore gives some legitimacy to exploring such phenomena in the heavy fermion context.

7.2 ‘Local’ quantum criticality

A popular approach to the non-Fermi liquid heavy electron critical points is through the notion of ‘local’ quantum criticality advocated by Si and coworkers\cite{14} In this subsection we make several comments on this approach. Most importantly we point out that even within this framework the two-scale hypothesis emerges as a natural possibility.

The first comment is that the term ‘local’ criticality does not imply that there is no diverging length scale. Rather the hypothesis is that the spatial correlation length for spin fluctuations near the ordering wavevector shows a simple mean field divergence with no anomalous exponent. However it is also hypothesized that there is a diverging correlation time that is strongly not mean field like, and has a non-trivial anomalous exponent. Equivalently right
at criticality the spin susceptibility is assumed to take the form
\[ \chi(\vec{q}, \omega) \sim \frac{1}{|\vec{q} - \vec{Q}|^2 + A(-i\omega)^\alpha} \] (8)
for \( \vec{q} \) close to the ordering wavevector \( \vec{Q} \); \( A \) is a non-universal constant and \( \alpha \) a universal exponent. This form is motivated by fits to neutron scattering data\[15\] on \( \text{CeCu}_6-\text{xAu}_x \). Existing theoretical justifications of this assumed form are based on Extended Dynamical Mean Field Theory (EDMFT) treatments of Kondo-Heisenberg models. It should be emphasized however that this kind of ‘locality’ is to some extent built into EDMFT as it does not allow for \( q \) dependence in the self energies.

The second comment is that it is important to distinguish the hypothesis of ‘local’ criticality from the hypothesis that the non-fermi liquid behavior is associated with the transition from the heavy fermi liquid to the LMM state. The former is a specific statement about the structure of the singularities in, say the spin susceptibility at the critical point. The latter is a statement on the nature of the two phases involved in the transition. It is this latter hypothesis that requires large reconstruction of the Fermi surface and other related phenomena at the transition. It is by no means obvious that such a phase transition necessarily has the specific structure of critical singularities assumed in the ‘local’ criticality scenario. Thus experiments such as that in Ref. [6] showing a jump in the Hall coefficient must be taken as evidence for the latter hypothesis rather than directly for the ‘local’ criticality scenario.

Finally we comment on the two-scale hypothesis within the framework of the EDMFT approach to the Kondo-Heisenberg lattice. In the presence of full \( SU(2) \) spin symmetry, the EDMFT calculations describe a transition between a Fermi liquid phase and a paramagnetic non-fermi liquid phase. The paramagnetic non-fermi liquid fixed point occurs at zero Kondo coupling. It thus seems correct to interpret this paramagnetic fixed point as a non-trivial gapless spin liquid formed from the local moments that is decoupled from conduction electrons. It is therefore to be seen as some version of an FL\(^*\) phase. To argue that this transition describes the true transition from the heavy Fermi liquid to the LMM it is necessary to make two assumptions: first that the paramagnetic non-fermi liquid fixed point has a relevant perturbation that drives an instability toward a magnetically ordered phase (to be interpreted as LMM) and second that this perturbation is also irrelevant at the critical fixed point associated with the transition to the fermi liquid. In other words the magnetic ordering is driven by a perturbation that is dangerously irrelevant at the critical fixed point. It then follows that this transition will also be characterized by two diverging time scales. The time scale for magnetic ordering will diverge much faster than the time scale associated with loss of Kondo screening exactly as proposed in Refs. [3,9].
If the two scale hypothesis is correct near the non-fermi liquid heavy fermion critical point then it should be possible to consider the Kondo and magnetic ordering phenomena as separate crossovers. Consider first the Kondo-driven case of Section 5.1. In this case the primary transition involves loss of Kondo screening and the subsequent jump in the Fermi volume. A theory for such a transition is conveniently developed\[9\] using a fermionic representation of the local moment. (For prior calculations on an analogous transition in a random Kondo lattice see Ref. [16]). At a mean field level the Kondo process is described in terms of a hybridization amplitude between the $c$ and $f$ bands. The Heisenberg exchange leads to the presence of some dispersion in the $f$-band. As a function of the ratio $J_K/J_H$ two distinct states are found. For large Kondo coupling the Kondo hybridization is non-zero and a heavy Fermi liquid state obtains. For small Kondo coupling on the other hand the hybridization is zero. The $c$-electrons are then decoupled from the local moments which have settled into a spin liquid state. This is an example of an FL* phase. The transition between the two phases is second order in mean field theory, and is accompanied by a jump in the Fermi volume. The critical point describes a non-Fermi liquid state.

This calculation provides an explicit realization of the scenario (see the end of Section 4) in which the Fermi surface of the heavy Fermi liquid consists of two sheets near the transition. All the important action is associated with just one sheet (the hot sheet) while the other cold sheet remains more or less a benign spectator. In particular the quasiparticle residue vanishes all over the hot sheet but stays finite on the cold sheet.

Fluctuation effects beyond the mean field theory were also examined in Ref. [9], and shown to lead to singular non-Fermi liquid temperature dependences for a number of physical quantities (for example specific heat and resistivity). Very recently Coleman et al[17] have studied the evolution of the zero temperature transport properties across the transition, and argued that both the longitudinal and Hall resistivities will jump at the critical point.

It remains to be seen whether this kind of transition (to a suitable unstable FL* phase) can provide a platform for describing a Kondo-driven transition between the heavy Fermi liquid and the LMM state.

It is natural to expect that the magnetism-driven scenario of Section 5.2 is better accessed by employing a description of the spins in terms of Schwinger bosons advocated in Ref. [18]. It again remains to be seen whether such an approach can eventually lead to a theory of the LMM- heavy fermi liquid transition.
8 Conclusion

In this paper we have discussed some ideas on the non-fermi liquid behavior found near the magnetic quantum critical point in some heavy fermion materials. Clearly there is much work left (and more ideas needed) to understand the basic physics of this non-fermi liquid. Our goal was to outline in a general way some of the theoretical issues and to motivate experiments. Here we reiterate some of the most important questions that future experiments will hopefully clarify.

(i) It is important to establish a clear connection (if any) between the nature of the magnetic phase to which the transition happens (LMM or SDW) and the occurrence of non-fermi liquid criticality. If as in the simple situation discussed in Section 2 the two magnetic metals are sharply distinct then it may be possible to establish the nature of the magnetic phase through measurements far from the critical point.

(ii) What is the fate of the Fermi surface right at the quantum critical ground state? As discussed in Section 4 it seems reasonable to assume that a sharp Fermi surface albeit with power law quasiparticles survives. If so will it be super-large or will it have several sheets with some sheets with power law quasiparticles and the others with ordinary (delta function) quasiparticles?

(iii) Are the fermi surface reconstruction and magnetic ordering phenomena characterized by two distinct diverging length/time scales? As we have discussed such a possibility is theoretically appealing and finds support in other analogous simpler theoretical problems.

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