Propagating speed of primordial gravitational waves and inflation

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Abstract

We show that if the propagating speed of gravitational waves (GWs) gradually diminishes during inflation, the power spectrum of primordial GWs will be strongly blue, while that of the primordial scalar perturbation may be unaffected. We also illustrate that such a scenario is actually a disformal dual to the superinflation, but it does not have the ghost instability. The blue tilt obtained is $0 < n_T \lesssim 1$, which may significantly boost the stochastic GWs background at the frequency band of Advanced LIGO/Virgo, as well as the space-based detectors.

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I. INTRODUCTION

Recently, the LIGO Scientific Collaboration has observed a transient gravitational wave (GWs) signal with a significance in excess of 5.1σ, which is consistent with an event of the binary black hole coalescence. This discovery will be a scientific milestone for understanding our universe, if it is confirmed.

It is speculated that the stochastic GWs background contributed by the incoherent superposition of all merging binaries in the universe might be higher than expected previously, which is potentially measurable around 25Hz by the Advanced LIGO/Virgo detectors operating at their projected final sensitivity. However, some cosmological sources may also contribute a stochastic background of GWs at the corresponding frequency band, such as cosmic strings and cosmological phase transitions.

It is well known that the standard slow-roll inflation predicts a nearly flat spectrum of scalar perturbation, as well as primordial GWs. Recently, the BICEP2/Keck data, combined with the Planck data and the WMAP data, have put the constraint \( r < 0.09 \) (95% C.L.) on the amplitude of primordial GWs on large scale, or at ultra-low frequency, which corresponds to \( \Omega_{gw} \sim 10^{-15} \), but there is no strong limit for its tilt \( n_T \). Actually, as long as its spectrum is blue enough, the stochastic GWs background from primordial inflation is also not negligible at the frequency band of Advanced LIGO/Virgo.

The slow-roll inflation model with \( \epsilon = -\dot{H}/H^2 \ll 1 \) generally has \( n_T = -2\epsilon < 0 \). Thus \( n_T > 0 \) requires either the superinflation, which breaks the null energy condition (NEC), or an anisotropic stress source during inflation, e.g., the particle production. During the superinflation, the primordial GWs come from the amplification of vacuum tensor perturbations. However, since the almost scale-invariance of the scalar perturbation requires \(|\epsilon| \sim 0.01\), we generally have \(|n_T| \sim \mathcal{O}(0.01)\) for the superinflation. Obtaining a blue GWs spectrum \( n_T > 0.1 \) without the ghost instability while reserving a scale-invariant scalar spectrum with slightly red tilt is still a challenge for the inflation scenario, see e.g.,[20] for comments.

In Einstein gravity, the propagating speed \( c_T \) of GWs is the same as the speed of light, thus can naturally be set as unity. Nevertheless, it might be modified when dealing with

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1 It is found that in the pre-big bang scenario (obtained in the context of string cosmology) the primordial GWs spectrum is blue [17][18][19].
the extremely early universe, e.g., the low-energy effective string theory with higher-order corrections [21][22][23][24], see also [25][26]. Since the amplitude of the primordial GWs is determined by \(c_T\) and the Hubble radius \(\sim H^{-1}\), the running of \(c_T\) will inevitably affect the power spectrum of primordial GWs (see also [27] for the study from the point of view of the running of GWs’ refractive index \(n\)). It was found in [28][29] that the oscillation of \(c_T\) may leave some observable imprints in CMB B-mode polarization. The effect of the sound speed \(c_S\) of scalar perturbation on the scalar spectrum has been investigated in e.g.[30] [31].

Here, we show that if the propagating speed \(c_T\) of GWs gradually diminishes during inflation, the power spectrum of primordial GWs will be strongly blue, while the spectrum of scalar perturbation may be still that of slow-roll inflation. There is no the ghost instability. The blue tilt obtained is \(0 < n_T \lesssim 1\), which may significantly boost the stochastic GWs background within the window of Advanced LIGO, as well as those of the space-based detectors.

II. INFLATION AND \(c_T\)

A. The model

We follow the effective field theory of inflation [32], beginning with the Langrangian in unitary gauge

\[
S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left[ R - c_1(t) - c_2(t)g^{00} \right. \\
- \left. \left(1 - \frac{1}{c_T^2(t)}\right) \left( \delta K_{\mu\nu} \delta K^{\mu\nu} - \delta K^2 \right) \right],
\]

where \(M_p = 1/\sqrt{8\pi G}\), \(c_1(t) = 2(\dot{H} + 3H^2)\), \(c_2(t) = -2\dot{H}\), and a dot denotes the derivative with respect to cosmic time \(t\). We will work in the inflation background with \(0 < \epsilon \ll 1\), which may be set by requiring \(|\dot{H}| \ll H^2\) in (1). The scalar perturbation at quadratic order is not affected by \(\delta K_{\mu\nu} \delta K^{\mu\nu} - \delta K^2\), see Appendix A, and also [33], so its spectrum is determined by slow-roll parameters. However, the quadratic action of tensor perturbation

\[
S_T = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R_T - c_T^2(t)g^{00} \right. \\
- \left. \left(1 - \frac{1}{c_T^2(t)}\right) \left( \delta K_{\mu\nu} \delta K^{\mu\nu} - \delta K^2 \right) \right],
\]
is altered as
\[ S_{g}^{(2)} = \int d\tau d^{3}x \frac{M_{p}^{2} a^{2} c_{T}^{2}}{8} \left[ \left( \frac{d\gamma_{ij}}{d\tau} \right)^{2} - c_{T}^{2} (\vec{\nabla} \gamma_{ij})^{2} \right], \]
where \( \tau = \int dt/a \), and \( \gamma_{ij} \) satisfies \( \gamma_{ii} = 0 \) and \( \partial_{i} \gamma_{ij} = 0 \).

The Fourier series of \( \gamma_{ij} \) is
\[ \gamma_{ij}(\tau, x) = \int \frac{d^{3}k}{(2\pi)^{3}} e^{-ik \cdot x} \sum_{\lambda = \pm, \times} \hat{\gamma}_{\lambda}(\tau, k) \epsilon_{ij}^{(\lambda)}(k), \]
in which \( \hat{\gamma}_{\lambda}(\tau, k) = \gamma_{\lambda}(\tau, k) a_{\lambda}(k) + \gamma_{\lambda}^{*}(\tau, -k) a_{\lambda}^{\dagger}(-k) \), the polarization tensors \( \epsilon_{ij}^{(\lambda)}(k) \) satisfy \( k_{j} \epsilon_{ij}^{(\lambda)}(k) = 0 \), \( \epsilon_{ii}^{(\lambda)}(k) = 0 \), and \( \epsilon_{ij}^{(\lambda)}(k) \epsilon_{ij}^{*^{(\lambda')}}(k) = \delta_{\lambda \lambda'} \), \( \epsilon_{ij}^{(\lambda)}(k) = \epsilon_{ij}^{(\lambda)}(-k) \), the annihilation and creation operators \( a_{\lambda}(k) \) and \( a_{\lambda}^{\dagger}(k') \) satisfy \( [a_{\lambda}(k), a_{\lambda}^{\dagger}(k')] = \delta_{\lambda \lambda'} \delta(3k - k') \). The equation of motion for \( u(\tau, k) \) is
\[ \frac{d^{2}u}{d\tau^{2}} + \left( c_{T}^{2} k^{2} - \frac{d^{2}z_{T}/d\tau^{2}}{z_{T}} \right) u = 0, \]
where
\[ u(\tau, k) = \gamma_{\lambda}(\tau, k) z_{T}, \quad z_{T} = \frac{a M_{p} c_{T}^{-1}}{2}. \]
Initially, the perturbations are deep inside the sound horizon, i.e., \( c_{T}^{2} k^{2} \gg \frac{d^{2}z_{T}/d\tau^{2}}{z_{T}} \), the initial state is the Bunch-Davies vacuum, thus \( u \sim \frac{1}{\sqrt{2\pi c_{T} k}} e^{-ic_{T} k \tau} \). The power spectrum of primordial GWs is
\[ P_{T} = \frac{k^{3}}{2\pi^{2}} \sum_{\lambda = \pm, \times} |\gamma_{\lambda}|^{2} = \frac{4 k^{3}}{\pi^{2} M_{p}^{2}} \cdot \frac{c_{T}^{2}}{a^{2}} |u|^{2}, \quad a H/(c_{T} k) \gg 1. \]

The diminishment of \( c_{T} \) may be regarded as
\[ c_{T} = (-H_{in,f} \tau)^{p}, \]
in which \( p > 0 \), and \( H_{in,f} \) is the Hubble parameter during inflation, which is regarded as constant for simplicity. Additionally, Eq. (8) suggests \( \frac{\dot{c}_{T}}{H_{in,f} c_{T}} = -p \).

We set \( dy = c_{T} d\tau \), thus Eq. (5) is rewritten as
\[ u_{yy} + \left( k^{2} - \frac{z_{T,yy}}{z_{T}} \right) u = 0, \]

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\( ^{2} \)In [27], the author investigated the effect induced by the running of GWs’ refractive index \( n(\tau) \), which is similar to that of \( c_{T} \). But note that \( c_{T} \neq 1/n \), as can be seen from the difference between Eq.(3) here and the Eq. (2.8) in [27].
where \( u(y, k) = \gamma_\lambda(y, k)z_T \), \( z_T = \frac{aM_p c_T^{-1/2}}{2} \) and the subscript ‘, y’ denotes \( d/dy \). Note here \( u(y, k) \) and \( z_T \) are different from those in Eq. (5), but \( \gamma_\lambda \) is still the same. The solution of Eq. (9) is

\[
 u_k(y) = \frac{\sqrt{\pi}}{2\sqrt{k}} \sqrt{-ky} H^{(1)}_\nu(-ky),
\]

where

\[
 H^{(1)}_\nu(-ky) \approx -i \left( \frac{2}{-ky} \right)^\nu \frac{\Gamma(\nu)}{\pi},
\]

and \( \nu = 1 + \frac{1}{2(1+p)} \). Thus the spectrum (7) is

\[
 P_T = \frac{4k^3}{\pi^2 M_p^2} \frac{|u|^2}{a^2} = \frac{2\pi^p}{\pi} \Gamma^2 \left( \frac{1}{2(1+p)} \right) \frac{2H_{inf}^2}{\pi^2 M_p^2 c_T} (-ky)^{-\nu T},
\]

where \( y = \frac{c_T}{H_{inf}} = -\frac{c_T}{(1+p)aH_{inf}} \). Therefore,

\[
 n_T = \frac{p}{1 + p}
\]

is blue-tilt, which is \( n_T \approx p \) for \( p \ll 1 \) and \( n_T \approx 1 \) for \( p \gg 1 \). Here, the running of \( H_{inf} \) may contribute \( -2\epsilon \sim -0.01 \), which has been neglected.

Thus, we obtain a blue-tilt GWs spectrum with \( 0 < n_T \leq 1 \). Here, both the scalar perturbation and the background are unaffected by additional operator (2). The background is set by (1), which is the slow-roll inflation with \( 0 < \epsilon \ll 1 \), so the scalar spectrum is flat with a slightly red tilt, which is consistent with the observations. It is noticed that based on the effective field theory of inflation, the introducing of other operators may also result in the blue-tilt GWs spectrum [34][35], however, in [34] \( n_T > 0.1 \) requires that the graviton have a large mass \( m_{graviton} \approx H_{inf} \), while in [35] \( \left| \frac{\dot{c_T}}{c_T H_{inf}} \right| < 1 \) was implicitly assumed.

It is well known that the blue-tilt GWs spectrum is the hallmark of the superinflation. Here, the scenario proposed is actually a disformal dual to the superinflation. We will discuss this issue in detail in Sec. III.

B. The stochastic background of GWs

We will focus on the stochastic background of GWs from such a scenario of inflation. The present observations are still not able to put stringent constraints on \( c_T \) at present (see, e.g., [36][37], also [38] for the constraint on the phase velocity and [39] for the group velocity). Future observations may put more stringent constraints on \( c_T \) [40][41]. However, we will not
get involved in this issue too much and we will assume that $c_T(t)$ will return to $c_T = 1$ at certain time before the end of inflation. Conventionally, one define

$$\Omega_{gw}(k, \tau_0) = \frac{1}{\rho_c} \frac{d \rho_{gw}}{d \ln k} = \frac{k^2}{12 a_0^2 H_0^2} P_T T^2(k, \tau_0),$$

(14)

where $\rho_c = 3 H_0^2 / (8 \pi G)$, $\tau_0 = 1.41 \times 10^4$ Mpc, $a_0 = 1$, $H_0 = 67.8$ km s$^{-1}$ Mpc$^{-1}$, the reduced Hubble parameter $h = H / (100 \text{ km s}^{-1}\text{Mpc}^{-1})$, and $\rho_{gw}$ is the energy density of relic GWs at present, so $\Omega_{gw}(k, \tau_0)$ reflects the fraction of $\rho_{gw}$ per logarithmic frequency interval. The transfer function is [42][43][44]

$$T(k, \tau_0) = \frac{3 \Omega_m j_1(\tau_0)}{k \tau_0} \sqrt{1.0 + 1.36 \frac{k}{k_{eq}} + 2.50 \left( \frac{k}{k_{eq}} \right)^2},$$

(15)

where $k_{eq} = 0.073 \Omega_m h^2$ Mpc$^{-1}$ is that of the perturbation mode that entered the horizon at the equality of matter and radiation. We have neglected the effects of the neutrino free-streaming on $T(k, \tau_0)$ [45], which is actually negligible. The underlying assumption on the thermal history of the post-inflation universe is able to affect $T(k, \tau_0)$ significantly, see e.g.[46], but we will only focus on the simplest case described by Eq.(15).

One generally parameterizes $P_T$ as

$$P_T = A_T \left( \frac{k}{k_*} \right)^{n_T},$$

(16)

where $k_* = 0.01$ Mpc$^{-1}$ is the pivot scale. However, if $n_T > 0.4$, one will have $P_T > 1$ at high-frequency region ($f > 10^5$Hz). The GWs with $P_T \sim 1$ will induce the same-order scalar perturbation at nonlinear order, e.g.[20], which will result in the overproduction of primordial black holes at the corresponding scale, which is inconsistent with their abundance. The upper bound put by the production of primordial black holes is $P_T < 0.4$ [47]. In addition, the indirect upper bound given by the combination of CMB with lensing, BAO and BBN observations is $\Omega_{gw} < 3.8 \times 10^{-6}$ [48], which also puts a strong constraint on $n_T$, i.e., $n_T < 0.36$ at 95% C.L. for $r = 0.11$ [49], otherwise $\Omega_{gw}$ at higher frequency will exceed this bound.

However, in our scenario, $c_T(t)$ is assumed to return to unity at a certain time $t_c$ before the end of inflation, as has been mentioned. This means that the blue-tilt spectrum will acquire a cutoff around $k_c$, see Sec. IV for details, which may avoid the above constraints on $n_T$. We may parameterize the corresponding $P_T$ as

$$P_T = A_T \left[ 1 - e^{-\left( \frac{k}{k_c} \right)^{n_T}} \right] \left( \frac{k_c}{k_*} \right)^{n_T},$$

(17)
which is (16) for \( k \ll k_c \), and tends to a constant \( A_T \left( \frac{k}{k_c} \right)^{n_T} \) for \( k \gg k_c \). Though we will use (16) and (17) since we are mainly interested in the boosted blue-tilted spectrum, we should point out that \( P_T \) will decrease at \( k > k_c \) or \( k \gg k_c \) (which may be out of the range we are interested in), if we assume that \( c_T \) will increase back to unity. In such case, \( P_T \) may be parameterized as
\[
P_T = A_T \left( \frac{k}{k_*} \right)^{n_T} \frac{1}{1 + (k/k_c)^{n_{Tc}}},
\]
where \( n_{Tc} > n_T \), so that when \( k \gg k_c \), \( P_T = A_T (k_c/k_*)^{n_T} (k/k_c)^{n_T - n_{Tc}} \) has a red tilt. When \( n_{Tc} = n_T \), (18) is similar to (17).

We plot the stochastic background of our GWs in Fig.1. It is obvious that a blue-tilt primordial GWs with \( n_T \gtrsim 0.4 \) is able to contribute a large stochastic GWs background within the windows of Advanced LIGO/Virgo, which may be greater than the contribution from the incoherent superposition of all binary black hole coalescence. \( n_T \gtrsim 0.4 \) requires \( p \gtrsim 2/3 \) in (8), which suggests that the diminishment of \( c_T \) in units of Hubble time is not too fast. It is also interesting to notice that if such a GWs background could be detected by Advanced LIGO/Virgo in upcoming observing runs, it will also be able to be detected by the space-based interferometers at a lower frequency band, such as eLISA, and China’s Taiji program in space, see Fig.2, as well as the PTA, e.g.[49][50].

III. DISFORMAL DUAL TO SUPERINFLATION

The superinflation is the inflation with \( \epsilon = -\dot{H}/H^2 < 0 \), i.e. \( \dot{H} > 0 \), which breaks the NEC. The model we proposed in Sec. II A, i.e., inflation with a diminishing \( c_T = (-H_{inf}\tau)^p \), is actually disformally dual to superinflation. This can be inferred from the evolution of the GWs sound horizon.

The perturbation mode outside the comoving sound horizon \( 1/(aH_{Per}) \) of the perturbations\(^3\) (i.e., \( k \ll aH_{Per} \)) will freeze, while it will evolve inside \( 1/(aH_{Per}) \). In inflation scenario, the spectrum of GWs generally has similar shape to that of the scalar perturba-

\(^3\) Here, \( H_{Per} \) is defined as \( H_{Per} = \frac{(z_T/\tau_T)^{1/2}}{a_{cT}} \), where \( z_T \) is given by Eq.(6). For \( c_T \sim (-\tau)^p \) and \( a \sim (-\tau)^{-1} \), we have
\[
\frac{1}{aH_{Per}} = \sqrt{\frac{1}{2 + 3p + p^2}} \frac{c_T}{aH} \sim \frac{c_T}{aH},
\]

(19)
FIG. 1: The brown line is the stochastic GWs background from inflation with spectral index $n_T = 0.45$ and tensor-to-scalar ratio $r = 0.05$ at the CMB scale. O1, O2, and O5 curves, taken from [2], are the current Advanced LIGO/Virgo sensitivity, the observing run (2016-2017) and (2020-2022) sensitivities at 1σ C.L., respectively. The blue curve is the GWs background generated by all binary black hole coalescence without excluding potentially resolvable binaries.


dilation, since both GWs and scalar perturbations have a comoving sound horizon $1/(aH_{per})$ almost coincide with $1/(aH)$. Here, since the comoving sound horizon of GWs is $c_T/(aH)$, and its evolution is completely different from $1/(aH)$, the spectrum of GWs shows itself blue-tilt, see Fig.3. However, the tilt of $c_T/(aH)$-line in Fig.3 is the same as that of the superinflation with $c_T = 1$, see Fig.1 in [54]. This indicates the physical processes of horizon crossing of GWs modes are same in these two scenarios, thus will generate the same power spectra. In fact, these two scenarios can be connected by a disformal transformation. Below, we give the strict proof.

We make a disformal redefinition of the metric [33]

$$g_{\mu\nu} \rightarrow c_T^{-1} \left[ g_{\mu\nu} + (1 - c_T^2)n_\mu n_\nu \right].$$

with

$$\tilde{t} \equiv \int c_T^{1/2} dt, \quad \tilde{a}(\tilde{t}) \equiv c_T^{-1/2} a(t),$$

which makes (3) become

$$S^{(2)} = \int d\tilde{t} d^3x \frac{M_p^2 \tilde{a}^2}{8} \left[ \left( \frac{d\gamma_{ij}}{d\tilde{t}} \right)^2 - (\nabla^2 \gamma_{ij})^2 \right]$$

with $\tilde{c}_T = 1$. 

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FIG. 2: The green and the brown lines are the stochastic GWs backgrounds from inflation with \( n_T = 0.3 \) in (16) and \( n_T = 0.45 \) in (17), respectively. Both C1 and C4-lines are eLISA’s representative configurations given in [51]. The sensitivity curves of DECIGO and BBO are given in [52]. The red dashed curve is Taiji’s sensitivity curve, see, e.g., [53] for a preliminary report. Fig.1 is actually the amplification of image at the frequency band 10-400 Hz in this figure.

Here, with \( d\tilde{\tau} = d\tilde{t}/\tilde{a} \), which implies

\[
\tilde{\tau} = \int^{\tau} \left( -H_{inf} \right)^p d\tau = -\left( H_{inf} \right)^p \frac{(-\tau)^{p+1}}{p+1},
\]

we have

\[
\tilde{a} = c_T^{-1/2} a \sim (-\tilde{\tau})^{-\frac{2+p}{2(1+p)}}.
\]

\[
\tilde{H} = \frac{d\tilde{a}/d\tilde{t}}{\tilde{a}} = c_T^{-1/2} \left( H_{inf} - \frac{dc_T/dt}{2c_T} \right) \sim (-\tilde{\tau})^{-\frac{p}{2(1+p)}}.
\]

Thus the value of \( \tilde{H} \) is gradually increasing. This suggests that after the disformal transformation the background is actually the superinflation with \( \tilde{\epsilon} = -p/(2+p) \), which satisfies \( -1 \lesssim \tilde{\epsilon} < 0 \). The scenario with \( \tilde{\epsilon} \ll -1 \) is the slow expansion, which was implemented in [55].

The equation of motion for \( u(\tilde{\tau}, k) \) is

\[
\frac{d^2 u}{d\tilde{\tau}^2} + \left( k^2 - \frac{d^2 \tilde{z}_T/d\tilde{\tau}^2}{\tilde{z}_T} \right) u = 0,
\]

where \( u(\tilde{\tau}, k) = \gamma(\tilde{\tau}, k)\tilde{z}_T \) and \( \tilde{z}_T = \tilde{a}M_p/2 \). The initial state is still the Bunch-Davies vacuum \( u \sim \frac{1}{\sqrt{2k}} e^{-ik\tilde{\tau}} \). The solution is

\[
u_k(\tilde{\tau}) = \frac{\sqrt{\pi}}{2\sqrt{k}} \sqrt{-\tilde{\epsilon} \tilde{H}^{(1)}_{\nu}(-k\tilde{\tau})},
\]
FIG. 3: This sketch illustrates the evolutions of the primordial perturbations during inflation in our scenario. The brown line is $\sim 1/aH$. The blue line is $\sim c_T/aH$, which is the sound horizon of GWs. We assume that $c_T$ decrease to some value less than unit and begin to increase later, so that it could return to unity and both horizons coincides before or near the end of inflation.

where

$$H^{(1)}_\nu (-k\tilde{\tau}) \overset{-k\tilde{\tau}\to 0}{\approx} -i \left( \frac{2}{-k\tilde{\tau}} \right) \frac{\rho}{\pi} \cdot \frac{\Gamma (\tilde{\nu})}{\pi},$$

and $\tilde{\nu} = 1 + \frac{1}{2(1+p)}$. Thus the power spectrum is

$$P_T = \frac{k^3}{2\pi^2} \sum_{\lambda=+,-,\times} |\gamma_\lambda|^2$$

$$= \frac{4k^3}{\pi^2 M_p^2 a^2} \cdot \frac{\pi}{4k} (-k\tilde{\tau})^2 \frac{2^{2+\frac{1}{1+p}}}{(-k\tilde{\tau})^{2+\frac{1}{1+p}}} \cdot \frac{1}{4(1+p)^2} \cdot \frac{\Gamma^2 \left( \frac{1}{2(1+p)} \right)}{\pi^2}$$

$$= \frac{2\tilde{H}^2}{\pi^2 M_p^2} \cdot \frac{2^{1+\frac{1}{1+p}}}{\pi(2+p)^2} \Gamma^2 \left( \frac{1}{2(1+p)} \right) (-k\tilde{\tau})^{\frac{p}{1+p}}$$

$$= \frac{c_T k^2}{\pi^3 M_p^2} \cdot \frac{(-\tau)^2 H_{inf}^2 2^{1+\frac{1}{1+p}}}{(1+p)^2} \Gamma^2 \left( \frac{1}{2(1+p)} \right) \left( k \cdot \frac{(-\tau)^{1+p}}{1+p} \right) H_{inf}^p \frac{-\frac{p}{1+p}}{1+p}$$

$$= \frac{2H_{inf}^2}{\pi^2 M_p^2 c_T} \cdot \frac{2^{1+\frac{1}{1+p}}}{\pi} \Gamma^2 \left( \frac{1}{2(1+p)} \right) (-ky)^{\frac{p}{1+p}}.$$  (29)

This result is completely the same as Eq.(12).

When $p \ll 1$, we have

$$\frac{2^{1+\frac{1}{1+p}}}{\pi(2+p)^2} \Gamma^2 \left( \frac{1}{2(1+p)} \right) \approx 1 + 0.27p + \mathcal{O}(p^2)$$  (30)
in Eq.(29) and $\tilde{\epsilon} = -\frac{d\tilde{H}/dt}{\tilde{H}^2} \ll 1$. Thus with (29), we have

$$P_T = 2\tilde{H}^2/\pi^2 M_P^2,$$

(32)
i.e. Creminelli et.al’s result [33].

Actually, it is well known that the spectrum of GWs, as well as scalar perturbation, is independent of the disformal redefinition (20) of the metric [33][56]. An intuition argument for it is the comoving horizon of scalar perturbation

$$\frac{\tilde{c}_s}{\tilde{a}H} = \frac{1}{c_T\tilde{a}H} \sim (-\tilde{\tau})^{1/2} \sim \frac{1}{aH_{inf}}$$

(33)
i.e., the relation between the comoving wave number $k$ and the comoving sound horizon is not altered, where $\tilde{c}_s = 1/c_T$ [33].

Conventionally, the superinflation breaks the NEC. Implementing the superinflation without the ghost instability is still a significant issue, e.g.[20][57]. Here, we actually suggest such a superinflation scenario. It might be just a slow-roll inflation living in a disformal metric with $c_T$ gradually diminishing, however, if we see it with $c_T = 1$, what we will feel is the superinflation. The violation of NEC in modified gravity does not necessarily mean ghost instability. Because the quadratic actions (3) and (22) for the tensor (as well as those for scalar) are canonical, there is no ghost instability in both frames.

IV. CUTOFF OF BLUE SPECTRUM

To avoid $P_T \sim 1$ at high frequency, we have to require that the diminishment of $c_T$ stop at a certain time $\tau_c$. Additionally, we assume that $c_T(t)$ will return to unity before the end of inflation, as in Sec. II B.

We assume that

$$c_T = (-H_{inf}\tau)^p \quad \text{for} \quad \tau < \tau_c,$$

$$c_T = c_{Te} \quad \text{for} \quad \tau > \tau_c.$$  

(34)

We set $dy = c_Td\tau$. The solution of (9) is

$$u_2(y) = \sqrt{-ky} \left[ C_1(k)H_{3/2}^{(1)}(-ky) + C_2(k)H_{3/2}^{(2)}(-ky) \right]$$

(35)
for \( y > y_c \), and is \( u_1(y) \) for \( y < y_c \), which is actually (10), where \( \nu = 1 + \frac{1}{2(1+p)} \), \( y_c = c_T \tau_c \).

When \( -ky \ll 1 \),
\[
    u_2 \approx \frac{\sqrt{2}}{-ky\sqrt{\pi}} |C_1 - C_2|.
\]

Thus the spectrum of primordial GWs is
\[
P_T = \frac{4k^3}{\pi^2 M_p^2} \frac{c_T |u|^2}{a^2} = \frac{2H_{inf}^2}{\pi^2 M_p^2} f(p, y_c, k),
\]
where
\[
f(p, y_c, k) = \frac{4k}{\pi c_T} |C_1 - C_2|^2,
\]
and
\[
C_1 = -\frac{i\pi^{3/2}}{16\sqrt{k}} \left[ -2ky_c H^{(1)}_{\nu-1}(-ky_c) H^{(2)}_{3/2}(-ky_c) + H^{(1)}_{\nu}(-ky_c) \left( 2ky_c H^{(2)}_{1/2}(-ky_c) + (3 - 2\nu) H^{(2)}_{3/2}(-ky_c) \right) \right],
\]
\[
C_2 = \frac{i\pi^{3/2}}{16\sqrt{k}} \left[ 2ky_c H^{(1)}_{\nu}(-ky_c) H^{(1)}_{1/2}(-ky_c) + H^{(1)}_{3/2}(-ky_c) \left( -2ky_c H^{(1)}_{\nu-1}(-ky_c) + (3 - 2\nu) H^{(1)}_{\nu}(-ky_c) \right) \right]
\]
are set by the continuities of \( u(y) \) and \( du/dy \) at \( \tau_c \). We plot (37) in Fig.4, and see that, although \( P_T \) has a blue tilt, it is flat at a high frequency. We analytically calculate it as follows.

When \( -ky_c \ll 1 \),
\[
C_1 = -2^{-\frac{3+5p}{2(1+p)}} e^{iky_c} \frac{1}{\sqrt{k}} (-ky_c)^{-\frac{6+5p}{2(1+p)}} \frac{\Gamma\left(\frac{3+2p}{2(1+p)}\right)}{1 + p} \left[ ip + pky_c + 2ip(1 + p)(ky_c)^2 + 2(1 + p)^2(ky_c)^3 \right],
\]
\[
C_2 = 2^{-\frac{3+5p}{2(1+p)}} e^{-iky_c} \frac{1}{\sqrt{k}} (-ky_c)^{-\frac{6+5p}{2(1+p)}} \frac{\Gamma\left(\frac{3+2p}{2(1+p)}\right)}{1 + p} \left[ -ip + pky_c - 2ip(1 + p)(ky_c)^2 + 2(1 + p)^2(ky_c)^3 \right].
\]

We have
\[
f(p, y_c, k) = \frac{4k}{\pi c_T} |C_1 - C_2|^2 \approx \frac{2^{-\frac{3+5p}{2(1+p)}}}{9(1 + p)^2 \pi c_T} \Gamma^2 \left(\frac{3 + 2p}{2(1+p)}\right) (6 + 5p)^2(-ky_c)^{\frac{p}{1+p}}.
\]
FIG. 4: $P_T/P_T^{inf} = f(p, y_c, k)$. The parameters of the magenta dashed and brown solid curves are $c_T = 10^{-3}$ and $10^{-5}$, respectively, while we set $p = 0.7$.

Thus the tilt $n_T = p/(1+p)$, which is the same as (12).

When $-k y_c \gg 1$, 

$$C_1 = e^{\frac{i\pi}{4} - \frac{i\nu\pi}{2}} \frac{\sqrt{\pi}}{8k^{5/2}y_c^2} \left[ i(2\nu - 3) + (2\nu - 5)ky_c + 4i(ky_c)^2 \right],$$

(44)

$$C_2 = e^{\frac{i\pi}{4} - \frac{i(\nu+4ky_c)}{2}} \frac{\sqrt{\pi}}{8k^{5/2}y_c^2} \left[ i(2\nu - 3) + (1 - 2\nu)ky_c \right].$$

(45)

We have

$$f(p, y_c, k) \approx \frac{1}{c_T},$$

(46)

Thus the spectrum is flat.

From the above result, we can infer that if $c_T$ slowly diminishes to a value less than unity during inflation and then increases back to unity before the end of inflation, $\Omega_{gw}$ could be strongly boosted at the frequency band of Advanced LIGO/Virgo.

V. DISCUSSION

In the inflation scenario, obtaining a blue GWs spectrum ($n_T > 0.1$) without the ghost instability while reserving a scale-invariant scalar spectrum with a slightly red tilt is still a challenge. We find that if the propagating speed of GWs gradually diminishes during inflation, the power spectrum of primordial GWs will be strongly blue, while that of the scalar perturbation may be unaffected.
It is well known that the blue-tilt GWs is the hallmark of superinflation [9][10]. It may be implemented without ghost in G-inflation [58], but it is difficult, however, to simultaneously give it a slightly red-tilt scalar spectrum [20], see also [57]. Our scenario is actually a disformal dual to the superinflation, see Sec. III. In this duality, our background is actually a slow-roll inflation living in a disformal metric with $c_T$ gradually diminishing. However, if we see it with $c_T = 1$, what we will feel is the superinflation, but there is no ghost instability. Thus our work might offer a far-sighted perspective on superinflation.

The blue tilt obtained is $0 < n_T \lesssim 1$, which may significantly boost the stochastic GWs background at the frequency band of Advanced LIGO/Virgo, as well as the space-based detectors. This indicates that the primordial GWs recording the origin of the universe may be potentially measurable by the corresponding experiments.

To conclude, if a stochastic background of GWs is detected by Advanced LIGO/Virgo in the upcoming observing runs, it also possibly comes from the primordial inflation, and encodes the physics beyond GR at inflation scale.

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**Appendix A: Scalar perturbation**

We work with the ADM metric

$$g_{\mu\nu} = \begin{pmatrix} N_k N^k - N^2 N_j & N_j \\ N_j & h_{ij} \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} -N^{-2} & N^j \\ N^i & h^{ij} - N^i N^j \end{pmatrix},$$

where $h_{ij} = a^2 e^{2\zeta}(e^\gamma)_{ij}$, and $\gamma_{ii} = 0 = \partial_i \gamma_{ij}$. Generally, $N = 1 + \alpha$ and $N_i = \partial_i \beta$ are set for the scalar perturbations. It is convenient to define the normal vector of 3-dimensional hypersurface $n_\mu = n_0 dt/dx^\mu = (n_0, 0, 0, 0)$ and $n^\mu = g^{\mu\nu} n_\nu$. Using the normalization $n_\mu n^\mu = -1$, one has $n_0 = -N$, which suggests $n_\mu = (-N, 0, 0, 0), n^\mu = (\frac{1}{N}, \frac{N^i}{N^2})$, and the 3-dimensional induced metric, orthogonal to the normal vector, i.e., $H_{\mu\nu} n^\nu = 0$, can be defined to be
\[ H_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu, \]

\[ H_{\mu\nu} = \begin{pmatrix} N_k N^k & N_j \\ N_i & h_{ij} \end{pmatrix}, \quad H^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h^{ij} \end{pmatrix}. \] (A2)

The covariant derivative associated with \( H_{\mu\nu} \) is \( D_\mu \), which is applied to define the extrinsic curvature \( K_{\mu\nu} \):

\[ K_{\mu\nu} = \frac{1}{2N}(\dot{H}_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu). \] (A3)

We have

\[ \delta K_{\mu\nu} \delta K^{\mu\nu} - (\delta K)^2 = \frac{1}{(1 + \alpha)^2} \left\{ -6(\dot{\zeta} - \alpha H)^2 + 4a^{-2}e^{-2\zeta}(\dot{\zeta} - \alpha H)(\partial_i \partial_i \beta + \partial_i \beta \partial_i \zeta) \\
+ a^{-4}e^{-4\zeta} \left[ (\partial_i \partial_j \beta - \partial_i \beta \partial_j \zeta - \partial_j \beta \partial_i \zeta)^2 - 2(\partial_i \beta \partial_i \zeta)^2 - (\partial_i \partial_i \beta)^2 \right] \right\}, \] (A4)

where \( \delta K_{\mu\nu} = K_{\mu\nu} - H_{\mu\nu}H \).

Thus the quadratic action of scalar perturbation for (1) and (2) is

\[ S_\zeta^{(2)} = \int dx^4 M_p^2 \left\{ a^3 H^2 \alpha^2 \epsilon - 27a^3 H^2 \zeta^2 + 9a^3 H^2 \epsilon \zeta^2 - 18a^3 H \zeta^2 \\
+ a (\partial \zeta)^2 - 2a \alpha \partial_\zeta \partial_\zeta \zeta - \frac{1}{c_T^2}\left[ 3a^3 H^2 \alpha^2 - 6a^3 H \alpha \zeta + 3a^3 \zeta^2 \\
- 2a \partial_\zeta \partial_\zeta (\dot{\zeta} - H \alpha) \right] \right\}. \] (A5)

The constraints can be solved as

\[ \alpha = \frac{\dot{\zeta}}{H}, \] (A6)
\[ \partial_i \partial_i \beta = \frac{c_T^2}{H}(a^2 H \epsilon \dot{\zeta} - \partial_\zeta \partial_\zeta \zeta). \] (A7)

Inserting them into (A5),

\[ S_\zeta^{(2)} = \int dx^4 M_p^2 a^3 \epsilon \left[ \dot{\zeta}^2 - \frac{(\partial \zeta)^2}{a^2} \right] \] (A8)

is obtained. Therefore, the scalar perturbation is not affected by the operator \( \delta K_{\mu\nu} \delta K^{\mu\nu} - (\delta K)^2 \) at quadratic order.

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