Coulomb effects in high-energy $e^+e^-$ electroproduction by a heavy charged particles in an atomic field

P. A. Krachkov* and A. I. Milstein†

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia

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Abstract

The cross section of high-energy $e^+e^-$ pair production by a heavy charged particle in the atomic field is investigated in detail. We take into account the interaction with the atomic field of $e^+e^-$ pair and a heavy particle as well. The calculation is performed exactly in the parameters of the atomic field. It is shown that, in contrast to the commonly accepted point of view, the cross section differential with respect to the final momentum of a heavy particle is strongly affected by the interaction of a heavy particle with the atomic field. However, the cross section integrated over the final momentum of a heavy particle is independent of this interaction.

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*Electronic address: P.A.Krachkov@inp.nsk.su
†Electronic address: A.I.Milstein@inp.nsk.su
I. INTRODUCTION

Production of $e^+e^-$ pair by the ultra-relativistic heavy charged particle in the atomic field is very important because the cross section of this process is even larger than the cross section of bremsstrahlung of a heavy particle in the field. Thus, this process plays an important role in the energy losses of heavy particles in detectors. The cross section of the process under consideration in the leading Born approximation was derived many years ago. In this approximation, the cross section depends on the atomic charge number $Z$ and the charge number of a heavy particle $Z_p$ as $Z^2Z_p^2$. In papers the interaction of a heavy particle with the atomic field was not taken into account. Later, the Coulomb corrections with respect to the interaction of $e^+e^-$ pair with the atomic field were obtained in Refs. using the plane waves for the wave functions of a heavy particle and the Coulomb wave functions for electron and positron. Thus, the results in Refs. were exact in the parameter $Z\alpha$ but still proportional to $Z_p^2$, where $\alpha$ is the fine-structure constant. The authors of Refs. obtained the cross sections differential with respect to the final momentum of a heavy particle as well as that integrated over this momentum. For the latter case, the cross sections were obtained within another approach, see Refs. and reviews. In that approach, the cross sections were obtained for a fixed impact parameter $\rho$ of a heavy particle with respect to the atomic center. Therefore, the interaction of a heavy particle with the atomic field was not taken into account. Then, the results were integrated over the impact parameter. Thus, the final results correspond to the cross sections integrated over the final momentum of a heavy particle. Note that the energy $\omega$ of created $e^+e^-$ pair, which gives the main contribution to the cross section, is much smaller than the energy of a heavy particle. As a result, the cross sections of the process are independent of the spin and mass $m_p$ of a heavy particle but depend on the relativistic factor $\gamma = \varepsilon_p/m_p$, where $\varepsilon_p$ is the energy of a heavy particle, $\hbar = c = 1$. Thus, the formulas for the cross sections are the same for muons and light nuclei (with the corresponding substitutions of the charge numbers).

In the present paper, we investigate the differential cross section of high-energy $e^+e^-$ electroproduction by heavy charged particles in the atomic field taking into account the interaction of a heavy particle with the atomic field. Our consideration is based on the quasiclassical approximation, see review in Ref., developed in Ref. for the problem.
of high-energy $e^+e^-$ electroproduction by ultra-relativistic electron in the atomic field. Our results are exact in both parameters, $\eta = Z\alpha$ and $\eta_p = ZZ_p\alpha = Z_p\eta$. We show that the cross section differential over momentum of a heavy particle strongly depends on $\eta_p$, in contrast to the commonly accepted point of view. For light nuclei in the field of a heavy atom, this parameter can be large, $\eta_p \gtrsim 1$. However, the cross section integrated over the final momentum of a heavy particle is independent of the parameter $\eta_p$. It seems, the experimental investigation of a strong dependence of the differential cross section on $\eta_p$ for moderate values of the relativistic factor $\gamma$ is not a very difficult task.

II. GENERAL DISCUSSION

FIG. 1: Diagram for the amplitude of electroproduction by a heavy particle in the atomic field. Wavy line denotes the photon propagator, straight lines denote the wave functions in the atomic field.

The differential cross section of the process under consideration reads [11], see Fig. 1, where the corresponding Feynman diagrams in the Furry representation is shown:

$$d\sigma = \frac{(Z_p\alpha)^2}{(2\pi)^8} d\varepsilon_3 d\varepsilon_4 d\mathbf{p}_{2\perp} d\mathbf{p}_{3\perp} |T|^2,$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the initial and final momenta of a heavy particle, $\mathbf{p}_3$ and $\mathbf{p}_4$ are the momenta of electron and positron, $\varepsilon_1 = \varepsilon_2 + \omega$ is the energy of the incoming heavy particle, $\omega = \varepsilon_3 + \varepsilon_4$, $\varepsilon_{1,2} = \sqrt{p_{1,2}^2 + m_p^2}$, $\varepsilon_{3,4} = \sqrt{p_{3,4}^2 + m_e^2}$, $m_e$ is the electron mass and $m_p$ is the mass of a heavy particle. In Eq. (1) the notation $\mathbf{X}_\perp = \mathbf{X} - (\mathbf{X} \cdot \mathbf{\nu})\mathbf{\nu}$ for any vector $\mathbf{X}$ is used, $\mathbf{\nu} = \mathbf{p}_1/\varepsilon_1$. Below we assume that $\varepsilon_{1,2} \gg m_p$, $\varepsilon_{3,4} \gg m_e$, and $m_p \gg m_e$. The main contribution to the cross section is given by the energy region $\varepsilon_{3,4} \lesssim \gamma m_e$, where $\gamma = \varepsilon_1/m_p \gg 1$, so that $\omega/\varepsilon_1 \lesssim m_e/m_p \ll 1$.

The straightforward application of the method developed in our recent paper [10] results in the expression for $T$ exact in the parameters $\eta$ and $\eta_p$ for arbitrary atomic potential $V(r)$. 

\vspace{1cm}
We perform calculations of the matrix element $T$ for definite helicities $\mu_3$ and $\mu_4$ of electron and positron, respectively, $\mu_4 = \pm$ denotes a sign of the helicity.

We begin our consideration with the case of a pure Coulomb field and then discuss the effects of screening.

### A. Coulomb field

It is convenient to write the amplitude $T$ as a sum $T = T_\perp + T_\parallel$, where the corresponding contributions have the form (cf. Eq. (18) in Ref. [10]):

$$T_\perp = \frac{8i\eta}{\omega} |\Gamma(1 - i\eta)|^2 \int \frac{d\Delta_\perp A_{as}(\Delta_\perp + p_{2\perp})}{(Q^2 + \Delta_{0\parallel}^2) M^2 (\omega^2/\gamma^2 + \Delta_\perp^2)} (\xi_2/\xi_1)^{i\eta} \mathcal{M},$$

$$\mathcal{M} = -\frac{\delta_{\mu_3} \mu_4}{\omega} [\varepsilon_3 (s_{\mu_3} \cdot \Delta_\perp) (s_{\mu_3} \cdot I_1) - \varepsilon_4 (s_{\mu_4} \cdot \Delta_\perp) (s_{\mu_4} \cdot I_1)] + \mu_3 \delta_{\mu_3} \mu_4 \frac{m_e}{\sqrt{2}} (s_{\mu_3} \cdot \Delta_\perp) I_0,$$

$$T_\parallel = -\frac{8i\eta \varepsilon_3 \varepsilon_4}{\omega^3} |\Gamma(1 - i\eta)|^2 \int \frac{d\Delta_\perp A_{as}(\Delta_\perp + p_{2\parallel})}{(Q^2 + \Delta_{0\parallel}^2) M^2} (\xi_2/\xi_1)^{i\eta} I_0 \delta_{\mu_3} \mu_4,$$

where $\omega = \varepsilon_3 + \varepsilon_4$, $s_\lambda = (e_x + i \lambda e_y)/\sqrt{2}$, $e_x$ and $e_y$ are two orthogonal unit vectors perpendicular to $\mathbf{v} = p_1/p_1$, $\Gamma(x)$ is the Euler $\Gamma$ function, and the function $A_{as}(\Delta_\perp)$ reads

$$A_{as}(\Delta_\perp) = -\frac{4\pi \eta_p (L \Delta_\perp)^{2i\eta_p} \Gamma(1 - i\eta_p)}{\Delta_\perp^2 \Gamma(1 + i\eta_p)}.

A specific value of $L$ is irrelevant because the factor $L^{2i\eta_p}$ disappears in $|T|^2$. In Eq. (2) the following notations are used:

$$\Delta_{0\perp} = p_{2\perp} + p_{3\perp} + p_{4\perp}, \quad \Delta_{0\parallel} = -\frac{1}{2} \left[ \frac{\omega}{\gamma^2} + \frac{m_e^2 + \zeta^2}{\varepsilon_3 \varepsilon_4} + \frac{\delta^2}{\omega} + \frac{p_{2\perp}^2}{\varepsilon_1} \right],$$

$$M^2 = m_e^2 + \frac{\varepsilon_3 \varepsilon_4}{\gamma^2} + \frac{\varepsilon_3 \varepsilon_4 \Delta_\perp^2}{\omega^2}, \quad \zeta = \frac{\varepsilon_4}{\omega} p_{3\perp} - \frac{\varepsilon_3}{\omega} p_{4\perp}, \quad \delta = p_{3\perp} + p_{4\perp},$$

$$Q = \Delta_\perp - \delta, \quad q_1 = \frac{\varepsilon_3}{\omega} Q - \zeta, \quad q_2 = \frac{\varepsilon_4}{\omega} Q + \zeta,$$

$$I_0 = (\xi_1 - \xi_2) F(x) + (\xi_1 + \xi_2 - 1)(1 - x) F'(x)/i\eta,$$

$$I_1 = (\xi_1 q_1 + \xi_2 q_2) F(x) + (\xi_1 q_1 - \xi_2 q_2)(1 - x) F'(x)/i\eta,$$

$$\xi_1 = \frac{M^2}{M^2 + q_1^2}, \quad \xi_2 = \frac{M^2}{M^2 + q_2^2}, \quad x = 1 - \frac{Q^2 \xi_1 \xi_2}{M^2},$$

$$F(x) = F(i\eta, -i\eta, 1, x), \quad F'(x) = \frac{\partial}{\partial x} F(x),

where $F(a, b, c, x)$ is the hypergeometric function.
In terms of the variables $\zeta$, $\delta$, and $p_{2\perp}$, see Eq. (4), the main contribution to the total cross section is given by the region $\zeta$, $\delta$, $p_{2\perp} \lesssim m_e$ and $\omega \lesssim m_e \gamma$. In this region $\omega/\varepsilon_1 \lesssim m_e/m_p$ and $p_{2\perp}/\varepsilon_1 \ll p_{3\perp}/\varepsilon_3$, $p_{4\perp}/\varepsilon_4$, i.e., the angle between the momenta $p_2$ and $p_1$ is much smaller than the angles between $p_3$, $p_4$ and $p_1$.

Let us first consider the cross section integrated over $p_{2\perp}$, which is one of the most interesting quantity from the experimental point of view. We show that this cross section is independent of the parameter $\eta_p$, so that it is not affected by the interaction of a heavy particle with the Coulomb center. First of all we note that the last term $p_{2\perp}^2/\varepsilon_1$ in $\Delta_0|$ (see Eq. (4)) may be omitted because its contribution is small as compared with that of other terms (the relative contribution of this term to $\Delta_0|$ is $m_e/m_p$). After that the variable $p_{2\perp}$ presents in the amplitude $T$ solely in the function $A_{as}(\Delta_\perp + p_{2\perp})$, see Eq. (2). Let us consider the integral,

$$R = \int d\Delta_\perp A_{as}(\Delta_\perp + p_{2\perp})G(\Delta_\perp),$$

where $G(\Delta_\perp)$ is some function and $A_{as}(\Delta_\perp)$ is given by Eq. (3). The integral (4) is well-defined. However, to have a possibility to change the order of integration, we introduce the regularized function $A_{as}^{(r)}(\Delta_\perp)$,

$$A_{as}^{(r)}(\Delta) = -\frac{4\pi(\eta_p - i\epsilon)(L\Delta_\perp)^{2i\eta_p + 2\epsilon}\Gamma(1 - i\eta_p - \epsilon)}{\Delta^2 \Gamma(1 + i\eta_p + \epsilon)},$$

where $\epsilon$ is a small positive parameter of regularization. Then we have

$$R = \lim_{\epsilon \to 0} \int \int d\mathbf{x}d\mathbf{y} G(\mathbf{x})G(\mathbf{y}) \int d\mathbf{p}_{2\perp} A_{as}^{(r)}(\mathbf{x} + \mathbf{p}_{2\perp})A_{as}^{(r)*}(\mathbf{y} + \mathbf{p}_{2\perp}).$$

The integral over $\mathbf{p}_{2\perp}$ can be easily take by means of the Feynman parametrization. We have

$$R = \lim_{\epsilon \to 0} \int \int d\mathbf{x}d\mathbf{y} G(\mathbf{x})G(\mathbf{y}) \frac{32\pi^3 \epsilon}{|\mathbf{x} - \mathbf{y}|^{2-4\epsilon}} = (2\pi)^4 \int d\mathbf{x}|G(\mathbf{x})|^2.$$

Thus, the final result is independent of the parameter $\eta_p$, so that it can be evaluated in the limit $\eta_p \to 0$ using the relation

$$\lim_{\eta_p \to 0} \eta_p \int d\Delta_{\perp} |\Delta_{\perp} + \mathbf{p}_{2\perp}|^{2i\eta_p - 2}G(\Delta_\perp) = -i\pi G(-\mathbf{p}_{2\perp}).$$

The cross section $d\sigma_0$ in this limit is given by Eq. (1) with the replacement $T \to T = T_\perp + T_\parallel$ with

$$T_\perp = \frac{-32\pi^2 \eta |\Gamma(1 - i\eta)|^2 M_0}{\omega \Delta_0^2 M^2 (\omega^2/\gamma^2 + p_{2\perp}^2)} \left(\frac{\xi_2}{\xi_1}\right)^{\eta},$$

and

$$T_\parallel = \frac{\omega \Delta_0^2 M^2 (\omega^2/\gamma^2 + p_{\parallel}^2)}{\omega \Delta_0^2 M^2 (\omega^2/\gamma^2 + p_{2\perp}^2)} \left(\frac{\xi_2}{\xi_1}\right)^{\eta_2}. $$
\[ M_0 = \frac{\delta_{\mu_3 \mu_4}}{\omega} \left[ \varepsilon_3 (s_{\mu_3} \cdot p_{2\perp}) (s_{\mu_3} \cdot I_1) - \varepsilon_4 (s_{\mu_4} \cdot p_{2\perp}) (s_{\mu_4} \cdot I_1) \right], \]

\[ - \mu_3 \delta_{\mu_3 \mu_4} \frac{m_e}{\sqrt{2}} (s_{\mu_3} \cdot p_{2 \perp}) I_0, \]

\[ T_{\parallel} = \frac{32 \pi^2 \eta \varepsilon_3 \varepsilon_4 |\Gamma(1 - i\eta)|^2}{\omega^3 \Delta_0^4 M^2} \left( \frac{\xi_2}{\xi_1} \right)^{i\eta} I_0 \delta_{\mu_3 \mu_4}, \] (10)

where all notations are given in Eq. (4) with the replacement \( \Delta_{\perp} \rightarrow -p_{2\perp} \). The result (10) agrees with that obtained in Ref. [3].

Though the cross section integrated over \( p_{2\perp} \) is independent of \( \eta_p \), the cross section differential over \( p_{2\perp} \) strongly depends on this parameter. This statement is illustrated in Fig. 2 where the quantity \( \Sigma \),

\[ \Sigma = \frac{d\sigma}{Sp_{2\perp}d\varepsilon_3 d\varepsilon_4}, \quad S = \frac{(Z_p \alpha)^2}{\omega^2 m_3^3}, \] (11)

which is the differential cross section in units \( S \) integrated over \( p_{3\perp} \) and \( p_{4\perp} \), is shown as the function of \( p_{2\perp} \) for \( \omega = m_e \gamma / 4, \varepsilon_3 = \varepsilon_4 = \omega / 2, \gamma = 100, Z = 79 \) (gold), and a few values of \( Z_p \).

![Fig. 2: The dependence of \( \Sigma \), Eq. (11), on \( p_{2\perp}/m_e \) for \( \omega = m_e \gamma / 4, \varepsilon_3 = \varepsilon_4 = \omega / 2, \gamma = 100, Z = 79 \) (gold), and a few values of \( Z_p \); solid curve for \( Z_p = 3 \), dashed curve for \( Z_p = 2 \), dash-dotted curve for \( Z_p = 1 \), and dotted curve for \( Z_p \rightarrow 0 \) (without account for the interaction of a heavy particle with the Coulomb field).](image)

It is seen that impact of interaction of a heavy particle with the Coulomb field on the cross section differential over \( p_{2\perp} \) is significant. At small value of \( p_{2\perp}/m_e \), the cross section
exact in $\eta_p$ is essentially smaller than that obtained in the limit $\eta_p \to 0$. At large value of $p_{2\perp}/m_e$, the relation between these cross sections is opposite. As should be, $\int_0^\infty \Sigma dp_{2\perp}$ is independent of $\eta_p$.

FIG. 3: The dependence of $\Sigma$, Eq. (11), on $\omega/(m_e\gamma)$ for $p_{2\perp}/m_e = 2$ (left picture) and $p_{2\perp}/m_e = 0.5$ (right picture), $\varepsilon_3 = \varepsilon_4$, $\gamma = 100$, $Z = 79$ (gold), and a few values of $Z_p$; solid curve for $Z_p = 3$, dashed curve for $Z_p = 2$, dash-dotted curve for $Z_p = 1$, and dotted curve for $Z_p \to 0$ (without account for the interaction of a heavy particle with the Coulomb field).

In Fig. 3 the quantity $\Sigma$ is shown as the function of $\omega/(m_e\gamma)$ for $p_{2\perp}/m_e = 2$ (left picture) and $p_{2\perp}/m_e = 0.5$ (right picture), $\varepsilon_3 = \varepsilon_4$, $\gamma = 100$, $Z = 79$ (gold), and a few values of $Z_p$. It is seen that the dependence of the function $\Sigma$ on $Z_p$ is very strong for all $\omega/(m_e\gamma)$.

FIG. 4: The dependence of $\Sigma$, Eq. (11), on $x = \varepsilon_3/\omega$ for $\omega = m_e\gamma/4$, $\gamma = 100$, $Z = 79$ (gold), $p_{2\perp}/m_e = 2$ (left picture), $p_{2\perp}/m_e = 0.5$ (right picture), and a few values of $Z_p$; solid curve for $Z_p = 3$, dashed curve for $Z_p = 2$, dash-dotted curve for $Z_p = 1$, and dotted curve for $Z_p \to 0$ (without account for the interaction of a heavy particle with the Coulomb field).

In Fig. 4 we show the dependence of $\Sigma$ on $x = \varepsilon_3/\omega$ for $\omega = m_e\gamma/4$, $\gamma = 100$, $Z = 79$.
(gold), $p_{2\perp}/m_e = 2$ (left picture), $p_{2\perp}/m_e = 0.5$ (right picture), and a few values of $Z_p$. Again, it is seen that the account for the interaction of a heavy particle with the Coulomb field is very important for the differential cross section.

B. Effect of screening

An account for the effect of screening can be performed in the same way as it has been done in our paper [10]. In Eq. (2) one should replace $A_{as}(\Delta_\perp) \rightarrow A(\Delta_\perp)$,

$$A(\Delta_\perp) = i \int d\rho \exp[-i\Delta_\perp \cdot \rho - iZ_p\chi(\rho)]$$

$$\chi(\rho) = \int_{-\infty}^{\infty} dz V(\sqrt{z^2 + \rho^2}),$$

(12)

where $V(r)$ is the atomic potential, and multiply the integrand in Eq. (2) by the function $F_a(q^2)$ being the atomic form factor.

We show that the cross section integrated over $p_{2\perp}$ is independent of $\eta_p$ for any localized potential $V(r)$. Let us consider the function $R_1$,

$$R_1 = \int d\mathbf{p}_{2\perp} \left| \int d\Delta_\perp A(\Delta_\perp + \mathbf{p}_{2\perp}) G(\Delta_\perp) \right|^2.$$  

(13)

Substituting Eq. (12) to Eq. (13) we obtain

$$R_1 = \iint dx dy G(x)G^*(y) \int d\mathbf{p}_1 d\mathbf{p}_2 \exp\{iZ_p[\chi(\rho_2) - \chi(\rho_1)] + iy \cdot \mathbf{p}_2 - ix \cdot \mathbf{p}_1\}$$

$$\times \int d\mathbf{p}_{2\perp} \exp[i\mathbf{p}_{2\perp} \cdot (\mathbf{p}_2 - \mathbf{p}_1)].$$

(14)

Taking the integrals first over $\mathbf{p}_{2\perp}$ and then over $\mathbf{p}_1, \mathbf{p}_2,$ and $\mathbf{y}$, we find the following result

$$R_1 = (2\pi)^4 \int d\mathbf{p}_{2\perp} |G(\mathbf{p}_{2\perp})|^2,$$  

(15)

which is independent of $\eta_p$. Thus, the cross section integrated over $\mathbf{p}_{2\perp}$ can be evaluated by means of Eq. (11), where $T = F_a(\Delta_\perp^2)(\mathcal{T}_\perp + \mathcal{T}_\parallel)$, $\mathcal{T}_\perp$ and $\mathcal{T}_\parallel$ are given in Eq. (10).

Let us discuss the impact of screening on the differential cross section. Without account for the interaction of a heavy particle with the atomic field, this question was investigated in Ref. [12]. In this case the effect of screening is important for $\gamma \gg m_e r_{scr} \sim Z^{-1/3}/\alpha$, where $r_{scr}$ is the screening radius. Screening modifies the result of account for the interaction of a heavy particle with the atomic field in the cross section differential over $p_{2\perp}$ for $\gamma \gg (\omega/m_e)m_e r_{scr} \gg m_e r_{scr}$. Therefore, up to a very large $\gamma$ one can use $A_{as}(\Delta)$, Eq. (3), instead of $A(\Delta)$, Eq. (12), but take into account the atomic form factor $F_a$. 

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III. CONCLUSION

Using the quasiclassical approximation, we have derived the differential cross section of high-energy $e^+e^-$ electroproduction by heavy charged particles in the atomic field. The result is exact in the parameters $\eta$ and $\eta_p$. It is shown that the cross section differential in $p_{2\perp}$ strongly depends on the parameter $\eta_p$ while the cross section integrated over $p_{2\perp}$ is independent of this parameter. Though, at $\gamma \gtrsim m_e r_{scr} \gg 1$, screening is important for interaction of $e^+e^-$ pair with the atomic field, it is not necessary to take screening into account for interaction of a heavy particle with the atomic field up to a very large $\gamma$.

Situation with the dependence of the cross sections of electroproduction on $\eta_p$ reminds the situation with the Coulomb corrections to the cross sections of bremsstrahlung by high-energy muons in the atomic field, see Ref. [13]. The Coulomb corrections to the cross section of bremsstrahlung differential in both final muon and photon momenta modify essentially the result as compared with the Born result. However, the Coulomb corrections to the cross section integrated over the final muon momentum (or the photon momentum) vanish.

To observe experimentally a strong dependence of the cross section under discussion on the value of the parameter $\eta_p$, it is necessary to detect the final heavy particle. The angle between vectors $p_2$ and $p_1$ is essentially smaller than the angles between $p_3$, $p_4$, and $p_1$. However, this experiment for moderate values of the relativistic factor $\gamma$ seems to be not a very hard problem.

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