The Effect of Turbulent Flow of Fluid Containing Nano-particles of Al₂O₃ on Free Vibrations of Pipe for Elliptical Cross Section at Different Boundary Conditions

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Abstract
In this research the Rayleigh method was used to estimate the natural frequency of elliptical cross-sectional pipe where the major axis (2a) and the minor axis (2b) conveyed water containing nanoparticles of Al₂O₃ at the ratio of volume concentration (Φ% = 0, 0.1, 0.3, 0.5). The study examined different boundary conditions (clamped – clamped, clamped – pin, clamped – free and pin – pin). The ratios were firstly b/a=1, 2, 3, 4, 5 and then a/b=1, 2, 3, 4, 5 for different thickness "t" (1mm, 2mm), where the half major axis "a" (1cm, 2cm) and the half minor axis "b" (1cm, 2cm) at the length of pipe "L" (1m), and using various pipe materials (PVC, aluminum and steel). This study predicted the natural frequency (ωₙ) and critical velocity (Vₖ) which in the first case was larger than that at the second case for any construction of pipe. The natural frequency decreased slightly with the increase in the ratio (Φ% ). Finally, the natural frequency and the critical velocity were changed directly with increase of the ratio (b/a) while the inverse changes occurred with the increase of the ratio ( a/b) for PVC pipe but increased little for aluminum and steel pipes, due to different physical properties. Results have been compared with published theoretical and experimental results and found to conform.

Key words: Axes ratio, Elliptical section, Internal flow, Nano-particles, Turbulent flow.

List of Symbols

| Symbol | Description |
|--------|-------------|
| a      | Semi length of major axes, (cm). |
| b      | Semi length of minor axes, (cm). |
| Aₘₜ   | Cross section area of water, (cm²). |
| Aₙₕ   | Cross section area of nano-fluid, (cm²). |
| Aₚ    | Cross section area of pipe, (cm²). |
| Eₚ     | Modulus of elasticity of pipe (N/m²). |
| L      | Length of the pipe (m). |
| Iₚ     | Second moment of area of pipe, (m⁴). |
| Dₜ     | Hydraulic diameter for pipe, (cm). |
| mₙₕ    | Mass of Nano-fluid per unit length, (kg/m). |
| mₜ      | Mass of water per unit length, (kg/m). |
| Re     | Reynolds Number. |
| Vₙₕ    | Velocity of nano-fluid (m/sec). |
| Vₜ     | Critical velocity of nano-fluid flows in the pipe (m/sec). |
| t      | Thickness of pipe, (mm). |

Greek symbols

| Symbol | Description |
|--------|-------------|
| ρₚ     | Mass density of pipe material, (kg/m³). |
| ρₚₙₕ   | Mass density of Nano-particles, (kg/m³). |
| Pₙₕ    | Mass density of Nano-fluid in the pipe, (kg/m³). |
| Φ      | Ratio of Nano-particles. |
\[ \omega \] Natural frequency of pipe at velocity of flow \( V_{nf} \) (rad/sec).

\[ \omega_{0} \] Fundamental natural frequency of pipe in absence of flow, (rad/sec).

1. Introduction

Fluid mixed with nano-particles produces nano-fluid, which has a wide range of uses in the fields of industrial production, nuclear and medical treatment and many other applications such as photonics, transportation, electronics, thermal therapy for cancer treatment, micro manufacturing, metallurgical chemical sectors, cooling, heating, air conditioning and energy supply industries. Research has been carried out to determine how best to optimize the use of nano-fluids in terms of heat transfer and vibration. The base fluids may be oil, water or ethylene glycol, while nano-particles are made of oxides, carbides or metal. The design of pipes that convey such fluids does not only comprise the calculations of strength according to the specific symbols, but also their behavior, which is influenced by the boundary conditions exerted by those pipes.

Vibration is of great importance when designing, and it is important to know the relevant natural frequency to avoid structural failure. Matthew T., (2004) conducted a practical study explaining the effect of the ratio of flow volume on pipe vibration. Where there is an increase in the acceleration of the pipe, with an increase in the volume of flow of fluid for any material of the pipe and at any diameter, acceleration occurs with decreasing diameter of the pipe while deceleration occurs with increasing density of pipe material. Kameswara R., (2008) analytically investigated the problem of pipe-related vibration using different fixations such as pinned – pinned, clamped – clamped and clamped – clamped, founded on the Pasternak-Winkler model using Fourier series and Galerkin’s technique.

The effect of foundation on the stability of the pipe during vibration was observed by Andrew (2009), who conducted a practical study to test the natural vibration of a pipe conveying fluid at turbulent flow and to explain the effect of flow velocity, diameter and thickness of pipe on the vibration of the pipe at clamped – pinned boundary conditions. The study explored pipes of various materials: PVC, aluminum and steel. The vibration of pipe increased with increasing dynamic pressure flow, and the pipe wall accelerated when the flow rate of the fluid increased.

Mohsen J., (2011) conducted a practical study using two models of circular pipes conveying water at boundary conditions clamped – clamped, clamped – pinned and pinned – pinned for the ventilate of critical velocity at natural frequency of the pipe, indicating that the critical velocity of the pipe at the clamped fixation is stabilized only at high flow. B. Mediano, (2014) examined the concept of increasing the appropriate quality of engineering materials versus the reduction of cost, in the context of stabilizing the vibration of the fluid-conveying pipes in clamped – pinned conditions, using the Hamilton method. The materials used for the pipes were PVC, PE, concrete, steel and aluminum). The study concluded that stability of the fluid conveyor pipe depends not only on the installation, but on the materials of the pipe and the pressure.

N. Chavda, (2014) studied a double pipe heat exchanger that conveyed nano-fluid containing water and Al_{2}O_{3} particles, at the parallel and counter- flow arrangement, from 0.001% to 0.01%. Numerical analysis in a finite volume method was performed on a channel that transmitted a nano-fluid with a disturbed flow, containing nanoparticles of CuO and TiO_{2}. Water was used as a base fluid with a 1–3% weight ratio. As the concentration increased, shear stress increased on the wall. As turbulent flow conditions the Reynolds number increased and the friction coefficient decreased.
M. Abdulbaqqi, (2015) performed numerical analysis with a finite volume method on a straight channel that transmitted nano-fluid, with a turbulent flow, containing nano-particles of CuO and TiO$_2$. Water was a base fluid with a 1 – 3% weight ratio. As the concentration increased, shear stress increased on the wall. When the turbulent flow increases, the Reynolds number increases and coefficient increases. Hameed K., (2016) carried out a practical study to evaluate the natural frequency of a carbon steel pipe with a circular section conveying a nano-fluid composed of water and alumina particles at turbulent flow. In A. Hosseinian, (2017) the research was carried out on a double pipe heat exchanger with a multi wall carbon nano tube, (MWCNT) and nano-fluid to study differences in heat transfer due to wall vibration at different rates of mass fracture. The study showed that when the vibration increased, the heat transfer coefficient increased, while the deposition of the nanoparticles decreased.

Ali J., (2017) conducted a study was in which the effect of adding polymeric (poly isobutylene) suspended in kerosene to reduce the drag in the flow of turbulence, and show the increase in concentration, increased volumetric flow of the fluid with an increase in speed and reduction of the pressure difference and pressure gradient. Etim S., (2018) made a practical and theoretical study using a plastic pipe conveying fluid at fixed clamping conditions and simply supported to estimate the natural frequency and deflection of pipe. That study also studied the stress and critical velocity of the structure, and showed that the natural frequency decreased as the critical velocity increased and the velocity increased with the thickness of the pipe.

This study used Rayleigh’s method to investigate the natural frequency of pipe in different boundary conditions, which were clamped – clamped, clamped – free, pin – pin, clamped – pin, at an elliptical cross section conveying nano-fluid. That nano fluid contained water as the base fluid and Al$_2$O$_3$ as nano-particles. The study used various constructions of pipe with different ratios between semi minor axes and semi major axes, different values of thickness and different materials such as PVC, aluminum and steel.

2. Theoretical Analysis

Figure 1 shows the pipe in elliptical cross section and clamped – clamped boundary conditions, h thickness and the length L can be derived:

![Flow of Nano-fluid](image)

*Figure 1. Pipe at clamped – clamped boundary conditions*

Figure 2 shows the cross-sectional area of pipe in the two cases as follows:

![Cross-sectional Area](image)
In elliptical cross section of pipe at any length of pipe, the cross-section area is
\[ A_p = \pi (a_1 \times b_1 - a \times b) \]  
(1)

and second moment of area is
\[ I_p = \frac{\pi}{4} (a_1 \times b_1^3 - a \times b^3) \]  
(2)

where: \( a_1 = a + t \) and \( b_1 = b + t \), therefore the mass of pipe per unit length can be obtained as follow:
\[ m_p = \rho_p \times A_p \]  
(3)

Here the density of pipe (\( \rho_p \)) is represented as the density of pipe materials.

Now the calculation the mass of nanofluid per unit length, such as,
\[ m_{nf} = \rho_{nf} \times A_{pi} \]  
(4)

Where,
\[ \rho_{nf} = (1 - \varnothing) \rho_{bf} + \varnothing \rho_{pa}, \text{ (Pak, 1998)} \]  
(5)

\( \rho_{bf} \) the density of base fluid in this study is the density of water, while \( \rho_{pa} \) the density of nano-particles is represented AL₂O₃ where is equal (3970 Kg/m³), and \( A_{pi} = \pi \times a \times b \)

Therefore, the Reynolds number of nano-fluid can be calculated from the following equation:-
\[ Re_{nf} = \frac{\rho_{nf} U_{nf} \times D, \text{ (Holmen, 2010)}}{\mu_{nf}} \]  
(6)

Where \( U_{nf} \), velocity of nano-fluid and \( D \) the hydraulic diameter of pipe,
\[ D = 4A_c/p, \text{ (Pijush, 2011)} \]  
(7)

where \( A_c \) is the cross-sectional area of the pipe and \( p \) is its wetted perimeter.

However, in the elliptical cross-sectional area of pipe the hydraulic diameter can be written as follow:
\[ D = 4 \pi \times a \times b / 2 \pi (\frac{a^2 + b^2}{2}) \]  
(8)

The viscosity of nano-fluid (\( \mu_{nf} \)) can be calculated from the following equation:-
\[ \mu_{nf} = \mu_{bf} (1 + 2.5 \varnothing), \text{ (Raja, 2012)} \]  
(9)

Where \( \mu_{bf} \) is the dynamic viscosity for water \( \mu = 1*10^{-3} \text{ kg/m} \cdot \text{sec} \).

Now it is possible to derive the natural frequency for transverse motion of pipe by applying the Rayleigh method where the term approximate for different boundary condition is as follows:-
\[
\begin{align*}
\text{Clamped – free} & \quad Y_1(x) = a \left[ 1 - \cos \left( \frac{2\pi x}{L} \right) \right], \text{ (Benoraya, 1998)} \quad (10) \\
\text{Clamped – clamped} & \quad Y_1(x) = c_1 \left[ 1 - \cos \left( \frac{2\pi x}{L} \right) \right], \text{ (Desai, 1971)} \quad (11) \\
\text{Pinned – pinned} & \quad Y_1(x) = Y_0 \left[ 1 - \cos \left( \frac{2\pi x}{L} \right) \right], \text{ (Livesley, 1983)} \quad (12) \\
\text{Clamped – pinned} & \quad Y_1(x) = Y_0 \left[ \frac{dY}{dx} \right], \text{ (Livesley, 1983)} \quad (13)
\end{align*}
\]

In order to find the approximate of the natural frequency can equate the maximum potential energy to the maximum kinetic energy. The potential energy is given below:
\[
V_{\text{max}}(x) = \frac{1}{2} \int_0^L E I(x) \left( \frac{d^2Y}{dx^2} \right)^2 dx, \text{ (Benoraya,1998)} \]  
(14)
And the maximum kinetic energy by:-

\[ T_{\text{max}}(x) = \frac{\omega^2}{2} \int_0^L m(x)Y^2(x)dx \]  

(Benoraya, 1998) (15)

By the equation \( V_{\text{max}} \) to \( T_{\text{max}} \), can be obtained:-

\[ \omega_n = \frac{\int_0^L E I(x) \left( \frac{d^2 Y}{dx^2} \right) dx}{\int_0^L m(x)Y^2(x)dx} \]  

(16)

Substitute the expressions of boundary conditions for \( Y_1(x) \) and the second differentiation in equation (16) and after integration the approximate of natural frequency can be yielded:-

\[
\begin{align*}
\omega_n &= \frac{1}{1.915} \left( \frac{E_p I_p}{m_{\text{tot}}} \right)^{1/2}, \ (\text{Clamped – free}) \ (17) \\
\omega_n &= \frac{3.14}{L} \left( \frac{E_p I_p}{m_{\text{tot}}} \right)^{1/2}, \ (\text{Pinned – pinned}) \ (18) \\
\omega_n &= \frac{3.927}{L} \left( \frac{E_p I_p}{m_{\text{tot}}} \right)^{1/2}, \ (\text{Clamped – pinned}) \ (19) \\
\omega_n &= \frac{4.73}{L} \left( \frac{E_p I_p}{m_{\text{tot}}} \right)^{1/2}, \ (\text{Clamped – clamped}) \ (20)
\end{align*}
\]

Where, \( m_{\text{tot}} = m_{nf} + m_p \) (21)

These equations can be used to evaluate the natural frequency \( \omega_1 \) for the pipe carrying fluid which not moved. In order to estimation the natural frequency of pipe when the fluid moved at any velocity, firstly should be determined the critical velocity of fluid from the flowing equations.

\[
\begin{align*}
V_c &= \frac{1.915}{L} \sqrt{\frac{E_p I_p}{m_{nf}}}, \ (\text{Clamped – free}), \ (Ivan, 2011) \ (22) \\
V_c &= \frac{3.14}{L} \sqrt{\frac{E_p I_p}{m_{nf}}}, \ (\text{Pinned – pinned}), \ (Mohsen J., 2011) \ (23) \\
V_c &= \frac{3.927}{L} \sqrt{\frac{E_p I_p}{0.747 m_{nf}}}, \ (\text{Clamped – pinned}), \ (Mohsen J., 2011) \ (24) \\
V_c &= \frac{4.73}{L} \sqrt{\frac{E_p I_p}{0.555 m_{nf}}}, \ (\text{Clamped – clamped}), \ (Mohsen J., 2011) \ (25)
\end{align*}
\]

In this study it is possible to use the mass of nano-fluid \( (m_{nf}) \) instead of \( (m_f) \) in the above equations. Thus, the natural frequency of pipe at any velocity of nano-fluid can be determined from the following equation:-

\[ \frac{\omega}{\omega_1} = \left( 1 - \left( \frac{V_f}{V_c} \right) \right)^{1/2}, \ [Blivens, 1993] \ (26) \]

It is also possible to use velocity of nano-fluid \( (V_{nf}) \) as a replacement for \( (V_f) \) in the above equation.

3. Results and Discussion

When the velocity of nano-fluids increases for enhancement of system performance, the potentiality of the flow induced vibrations is raised, where the flow of fluid through the pipe forms the interactive system in a dynamic form. Table 1a shows the specification of the three test models used in this study, while Table 1b shows the dimensions of the different pipes and Table 1c shows the properties
of nano-fluid at different concentrations of Al₂O₃ nano-particles. Table 2a shows comparison of the natural frequency of the transverse free vibration of plastic pipe for clamped ends and Table 2b presents the properties of material. Table 3a shows comparison of the natural frequency of the transverse free vibration of aluminum pipe with simple support ends and the Table 3b gives the properties of material those at different values of flow velocity.

Where comparisons are between the Rayleigh method in the present work and other methods, these are taken from the literature specified.

Table 1a. Specifications of the test pipe models (Ashrae, 2000 & E.J. Hearn, 1977)

| Model No. | Material   | Density (kg/m³) | Modulus of Elasticity (Gpa) |
|-----------|------------|-----------------|----------------------------|
| 1         | P.V.C.     | 1000            | 7.12                       |
| 2         | Aluminum   | 2710            | 70                         |
| 3         | Steel      | 7860            | 200                        |

Table 1b. Dimensions of pipe in the present work

| Thickness, t (mm) | Major axis, a (cm) | Ratio, b/a | Minor axis, b (cm) | Ratio, a/b | Length, L (m) |
|-------------------|--------------------|------------|--------------------|------------|---------------|
| 1 or 2            | 0.5 or 1           | (1-5)      | 0.5 or 1           | (1-5)      | 1             |

Table 1c. Properties of water with different concentrations of Al₂O₃

| Φ % | ρnf (kg/m³) | μnf *10⁻³ (kg/m.sec) |
|-----|-------------|----------------------|
| 0   | 1000        | 1                    |
| 0.1 | 1002.97     | 1.0025               |
| 0.3 | 1008.97     | 1.0075               |
| 0.5 | 1014.85     | 1.0125               |

Table 2a. Natural frequency (rad/sec) of plastic pipe conveying fluid with clamped ends

| Vf (m/sec) | ω(rad/sec) Exp. Zhang etal, 1999 | ω(rad/sec) Zhang etal, 1999 | ω(rad/sec) Geometric Analysis Etim, 2018 | ω(rad/sec) Present work | Difference δ% |
|------------|---------------------------------|----------------------------|-----------------------------------------|-------------------------|--------------|
| 0          | 14.5                            | 14.37                      | 14.751                                  | 16.5                    | 11%          |
| 1.54       | 14.26                           | 14.27                      | 14.621                                  | 16.3                    | 11%          |
| 3.07       | 14.13                           | 13.95                      | 14.228                                  | 16.1                    | 13%          |
| 4.34       | 13.66                           | 13.51                      | 13.685                                  | 15.5                    | 13%          |
| 5.42       | 13.27                           | 13.03                      | 13.051                                  | 15.1                    | 15%          |
| 6.43       | 12.92                           | 12.48                      | 12.291                                  | 14.2                    | 15%          |
| 7.19       | 12.53                           | 12.2                       | 11.594                                  | 13.3                    | 14%          |

δ = (Rayleigh method – Geometric Analysis method / Geometric Analysis method) × 100%

Table 2b. Constant properties for plastic conveying fluid pipe [Etim S., 2018]

| Da (m) | Dl (m) | L (m) | E (Pa) | Pipe density (Kg/m³) | Fluid density (Kg/m³) |
|-------|--------|-------|--------|----------------------|-----------------------|
| 0.0097| 0.006  | 0.362 | 2.09E+6| 1128.56              | 1000                  |
Table 3a. Natural frequency (rad/sec) of aluminum pipe conveying fluid with (S – S) ends.

| \(V_f\) (m/sec) | \(\omega\) (rad/sec) Exp. Dodds et al, 1965 | \(\omega\) (rad/sec) Geometric Analysis Etim, 2018 | \(\omega\) (rad/sec) Present work | Difference \(\delta\%\) |
|-----------------|-----------------------------------|---------------------------------|----------------------------|------------------|
| 6.59            | 29.905                            | 30.29                          | 29.98                      | -1.023%          |
| 13.97           | 27.207                            | 28.93                          | 28.93                      | 0                |
| 21.43           | 26.099                            | 26.21                          | 26.205                     | -0.019%          |
| 27.19           | 19.255                            | 22.96                          | 22.96                      | 0                |
| 29.68           | 20.98                             | 21.14                          | 21.13                      | -0.047%          |

\[ \delta = \left[ \frac{\text{Raighly method} - \text{Geometric Analysis method}}{\text{Geometric Analysis method}} \right] \times 100\% \]

Table 3b. Constant properties for aluminum pipe conveying fluid [Etim S., 2018]

| \(D_o\) (m) | \(D_i\) (m) | \(L\) (m) | \(E\) (Pa) | Pipe density \((\text{Kg/m}^3)\) | Fluid density \((\text{Kg/m}^3)\) |
|-------------|-------------|-----------|------------|-------------------------------|-------------------------------|
| 0.0254      | 0.0220      | 3.048     | 6.89E+10   | 2698.791                      | 1000                          |

![Figure 3a](image1)

![Figure 3b](image2)
Figure 3c

Figure 3. Natural frequency for 1st mode as a function of axes ratio (b/a) in different values of thickness (t), different length of major semi axis (a), different boundary conditions and Φ=0.1% at one-meter length for PVC pipe.

Figures 3a to 5d show the first mode of natural frequency for pipe at elliptical cross section in absence flow (V_{eff} = 0) as a function of the ratio (b/a) obtained for the approximate Rayleigh method for different values of thickness (t), semi major (a) and the different physical properties of pipe materials, which have different of boundary conditions. All of these use a one-meter length of pipe. It is clear that the frequency increased with the increase in the length of semi major axis (a), the ratio (b/a) and the thickness (t) and this comportment is due to an increase of the second moment of inertia, which has led to an increase in the strain energy of pipe that causes increase in the stiffness of structure. At the same time, the amount of water increased with increase in the ratio (b/a), that caused an increase in the mass of the system and increase in the kinetic energy of the structure, but at value less than the stiffness of the system.

At the same figures, the frequency increased at different values of semi major axis, with higher values at constant thickness. It is possible to clarify the effect of boundary conditions on frequency of pipe where the frequency at cantilever pipe is less than that for the three others, but the clamped – clamped pipe had the highest frequency value, because the flexibility of cantilever pipe is very high compared with other pipes for different boundary conditions. This reduced the stiffness of the pipe and consequently decreased the natural frequency of the structure. However, the clamped – clamped pipe was strongly supported, more so than the other three types, but the slope at the ends of pipe caused an increase the stiffness. That increase in the stiffness of the pipe led to an increase in the natural frequency of the pipe.

In terms of the effect of pipe material on frequency, he steel pipe had a higher frequency than the aluminum pipe, which in turn had a higher frequency than the PVC pipe for all boundary conditions.
This is attributed to the differences between them in stiffness of the pipes due to their different physical properties.
Figure 4. Natural frequency for 1st mode as a function of axes ratio (b/a) in different values of thickness (t), different length of major semi axis (a), different boundary conditions and Φ=0.1% at one-meter length for aluminum pipe.

Figure 5a

Figure 5b

Figure 5c

Figure 5d
Figure 5. Natural frequency for 1st mode as a function of axes ratio (b/a) in different values of thickness (t), different length of major semi axis (a), different boundary conditions and Φ=0.1% at one-meter length for steel pipe.

**Figure 6a**

**Figure 6b**

**Figure 6c**

**Figure 6d**
Figure 6. Natural frequency for 1st mode as a function of axes ratio (a/b) in different values of thickness (t), different length of minor semi axis (b), different boundary conditions and Φ=0.1% at one-meter length for PVC pipe.

Figures 6a to 8d show that the first mode of natural frequency for pipe for the second case at a function of the different ratios of (a/b) for different values of thickness (t) and the semi minor axis (b) which had different boundary conditions, using a one-meter length of pipe. The frequency of PVC pipe decreased with increase in the ratio (a/b) which was contrary to the behaviors of another pipes (aluminum and steel) due to their different physical properties. While increase in the ratio (a/b) led to increase in the strain energy and thus caused increase in the stiffness of structure. on the other hand there was an increase in the amount of mass flowing through the pipes and then increase in the kinetic energy. The behavior of the pipes can be explained by the differences in stiffness, therefore when the strain energy was greater than the kinetic energy this caused increase the natural frequency against the increase for the ratio (a/b).
Figure 7. Natural frequency for 1st mode as a function of axes ratio \(a/b\) in different values of thickness \((t)\), different length of minor semi axis \((b)\), different boundary conditions and \(\Phi=0.1\%\) at one-meter length for aluminum pipe.
Figure 8. Natural frequency for 1st mode as a function of axes ratio (a/b) in different values of thickness (t), different length of minor semi axis (b), different boundary conditions and Φ=0.1% at one-meter length for steel pipe.
Figure 9a
Figure 9. Natural frequency for 1st mode as a function of volume concentration ($\Phi$) for different values of ($b/a$) at thickness ($t$) =0.1mm, length of major semi axis ($a$)=0.5cm for different boundary conditions at one-meter length for PVC pipe.
Figure 10. Natural frequency for 1st mode as a function of volume concentration ($\Phi$) for different values of ($b/a$) at thickness ($t$) =0.1mm, length of major semi axis ($a$)=0.5cm for different boundary conditions at one-meter length for aluminum pipe.
Figure 11. Natural frequency for 1st mode as a function of volume concentration (Φ) for different values of (b/a) at thickness (t) =0.1mm, length of major semi axis (a)=0.5cm for different boundary conditions at one-meter length for steel pipe.

Figures 9a to 14b show that adding nano-particles to the water caused the cracking of the boundary layers, and thus increased the flow rate then increased the velocity of the nano-fluid and caused an increase in the movement of fluid towards the pipe walls, which increased the system damping. That led to a decrease in the natural frequency. The additional of nano-particles to the fluid led to a slight increase in the mass of the system, thus causing an increase in the kinetic energy of the pipe, after which the natural frequency of the structure decreased slightly.
Figure 12. Natural frequency for 1st mode as a function of volume concentration ($\Phi$) for different values of ($a/b$) at thickness ($t$) =0.1mm, length of minor semi axis ($b$)=0.5cm for different boundary conditions at one meter length for PVC pipe.

Figure 13. Natural frequency for 1st mode as a function of volume concentration ($\Phi$) for different values of ($a/b$) at thickness ($t$) =0.1mm, length of minor semi axis ($b$)=0.5cm for different boundary conditions at one-meter length for aluminum pipe.
Figure 14a

Figure 14. Natural frequency for 1st mode as a function of volume concentration (Φ) for different values of (a/b) at thickness (t) =1mm, length of minor semi axis (b)=0.5cm for different boundary conditions at one-meter length for steel pipe.

Figure 14b

Figure 15a

Figure 15. Natural frequency for 1st mode as a function of velocity of flow Vnf in different ratio of length of axis, thickness 1mm & Φ=0.3% for PVC pipe.

\[
\begin{align*}
&V_c = 32.62\text{m/s} \\
&V_c = 27.14 \\
&V_c = 21.7 \\
&V_c = 13.23
\end{align*}
\]

\[
\begin{align*}
&V_c = 63.47\text{m/s} \\
&V_c = 52.7 \\
&V_c = 12.45 \\
&V_c = 25.88
\end{align*}
\]

\[
\begin{align*}
&V_c = 28.8\text{m/s} \\
&V_c = 24.3 \\
&V_c = 19.1 \\
&V_c = 11.7
\end{align*}
\]
Figures 15a to 17c show the relationships between the frequency of pipe and the different values for velocity flow of the nano-fluid through pipes having different material and boundary conditions. For any construction of pipe, the flow of fluid through the pipe generates a pressure force on the wall of the pipe that leads to deformation of structure and results in the aggravation of system vibration. High velocity of nano-fluid causes buckling or failure of the wall and increases the velocity which leads to the state of resonance and fatigue failure, where it can be observed that the frequency decreases as the velocity of the flow towards to the critical speed. The pipe with clamped ends was more stable than other pipes and operated at a wide range of velocity values, and was most able to ensure safety and failure prevention.

Figure 16a

Figure 16b

Figure 16c

Figure 16. Natural frequency for 1st mode as a function of velocity of flow Vnf in different ratio of length of axis, thickness 1mm & Φ=0.3% for aluminum pipe.
4. Conclusion
In this study, the flow of nano-fluids induced vibrations of elliptical cross sections of pipe having different boundary conditions and different materials. The approximate Rayleigh method was appropriate for use in this study. The pipe had different constructions for different values of wall thickness and the dimensions of the pipe cross section at one-meter length. The nano-fluids (Al₂O₃ dispersed in water) were utilized at the ratio Φ = 0, 0.1, 0.3, 0.5. The natural frequency and the critical velocity decreased with increasing ratio (Φ) for all pipes, and increased with increasing ratios of (b/a) and (a/b). However, the frequency decreased with increase in the ratio (a/b) for PVC pipe only. In all types of pipe the frequency and critical velocity increased with increasing thickness of pipe. The results have been compared with theoretical and experimental data, obtainable in the literature, for validation.

5. References
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