The detection of subsolar mass dark matter halos

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Abstract. Dark matter halos of subsolar mass are the first bound objects to form in cold dark matter theories. In this paper, I discuss the present understanding of ‘microhalos’, their role in structure formation and the implications of their potential presence, in the interpretation of dark matter experiments.

Contents

1. Introduction 2
2. Kinetic decoupling and microhalo scales 2
3. Microhalos in numerical simulations 3
   3.1. Formation of microhalos 3
   3.2. Challenges in numerical simulations 4
4. Survival of microhalos in the Milky Way halo 5
5. Characterizing the physical properties of microhalos 6
6. Detection of microhalos using dark matter annihilation products 8
   6.1. Microhalos as γ-ray sources 9
7. Microhalos and the local Milky Way dark matter distribution 11
   7.1. Microhalos and the mean dark matter density 12
8. Conclusion 14
Acknowledgments 14
References 15
1. Introduction

The term ‘microhalo’ loosely refers to a generic dark matter, gravitationally bound object, whose mass is typically less than that of the Sun. Objects of subsolar mass scales are predicted to be the first objects formed in the high-redshift Universe in theories where the dark matter particle is ‘cold’ (such as ΛCDM). In this short paper, I summarize the present understanding of microhalos. In section 2, I review the theoretical motivation behind the presence of microhalos; in section 3, I present the results of numerical simulations that studied their formation. Section 4 is an overview of the present state of understanding the survival of these objects, whereas in section 5, I discuss the procedure of how to characterize their physical properties. I present their connection to the direct and indirect detection experiments in sections 6 and 7, and I conclude in section 8. Throughout the paper, the assumed cosmology is flat ΛCDM, with Ω_m h^2 = 0.1358, Ω_b h^2 = 0.02267, h = 0.705, σ_8 = 0.812 and a spectral index of n_s = 0.96 [1].

2. Kinetic decoupling and microhalo scales

In the standard cold dark matter cosmological model, the energy density in the Universe is balanced by approximately 4% baryonic matter, 23% cold dark matter and 73% dark energy. Cold dark matter refers to gravitationally interacting matter that is hypothesized to be in the form of a yet-to-be-discovered particle. From the particle physics aspect, new particles arise in essentially all extensions to the standard model of particle physics, and usually the lowest mass particle is stable due to a new symmetry. Dark matter particle candidates include (among many others), neutralinos, the lightest Kaluza–Klein particle, axions and sterile neutrinos. Even though there is no a priori physical reason as to one candidate being more favorable than another, neutralinos are attractive and well studied because they are experimentally accessible at present. Neutralinos arise in supersymmetric extensions to the standard model of particle physics [2, 3], and they are part of a generic class of dark matter candidates, called WIMPs (weakly interacting massive particles).

In the early Universe, conditions are such that a WIMP is in chemical, thermal and kinetic equilibrium. As the rates of these interactions are diluted due to the expansion of the Universe, the WIMP falls out of equilibrium and decouples. The decoupling temperature is determined by the scattering cross section that is responsible for the equilibrium condition the particle decouples from, whereas the velocity distribution function of the particle is set by its mass. The temperature at which a WIMP is kinetically decoupled, i.e. when momentum-changing interactions cease to be effective, is called the kinetic decoupling temperature. For supersymmetric dark matter, such as the neutralino, the kinetic decoupling temperature is T_d \sim 10–1000 MeV [4]–[6], [68]. The free-streaming of particles after kinematic decoupling tends to smooth out fluctuations on small scales [4], [7]–[9], [68]. This physical effect leads to a cutoff in the dark matter power spectrum.

The cutoff of the power spectrum includes an imprint of the acoustic oscillations of the cosmic radiation fluid [10]. In general, the transfer function of the dark matter density perturbation amplitude is obtained by solving the Boltzman equation, but under certain approximations, it is possible to be derived analytically [8, 9]. Figure 1 shows an example of the analytic calculation of the cutoff of the power spectrum, derived from [8, 9]. What is shown is the damping factor D(k) that arises for a supersymmetric WIMP dark matter particle with a decoupling scale of k_d \sim 40 {\text{pc}}^{-1}, and a free-streaming scale of k_{fs} \sim 0.94 {\text{pc}}^{-1} (corresponding

New Journal of Physics 11 (2009) 105012 (http://www.njp.org/)
to a kinetic decoupling temperature of \( T_d \sim 20 \text{ MeV} \). In general, the cutoff scale is related to the kinetic decoupling temperature as
\[
M_{\text{cut}} \approx 10^{-4} M_{\odot} (T_d / 10 \text{ MeV}^{-1})^{-3}
\] [10] (for a recent review, see [69]). Recently, Martinez et al [11] presented the posterior of the cutoff scale from a Bayesian analysis of the presently allowed supersymmetric parameter space.

3. Microhalos in numerical simulations

3.1. Formation of microhalos

The first attempt to numerically simulate the formation of microhalos was done in 2005 by the group at the University of Zürich [12]. In that work, a small region of the Universe was simulated using a multiscale technique [13] in order to achieve the required spatial resolution needed to resolve objects below the cutoff scale of the power spectrum. The input transfer function was taken from [8]; corresponding to a supersymmetric dark matter candidate of mass \( \sim 100 \text{ GeV} \). This particular choice leads to an exponential cutoff at \( \sim 10^{-6} M_{\odot} \). The simulation was evolved from a redshift of \( z = 350 \) to a redshift of \( z = 26 \), when the simulated region itself approached nonlinearity. In the small, high-resolution volume of roughly average density, the first nonlinear structures with mass near the cutoff scale of \( \sim 10^{-6} M_{\odot} \) are observed to form by redshift \( z \approx 60 \). Their physical size is few \( \times 10^{-2} \) pc, and they are described by a single power-law density profile. Even though halos of mass in the range of \( 10^{-6} - 10^{0} M_{\odot} \) are formed roughly at the same time, they all share the same concentrations at \( z = 26 \). As expected, these objects contained no substructure, and no objects with smaller mass were formed, confirming the input shape of the power spectrum. Even though the statistics in this first simulation were low, the deduced mass function suggested a power law and normalization consistent with the halo mass function derived from numerous orders of magnitude larger scales, namely, \( dN/d \ln M \sim M^{-1} \). This finding is an important piece of information when it comes to studies aimed at the
long-term survival of microhalos in the potential well of a present-day galaxy, such as the Milky Way.

In a subsequent paper [14], the numerical simulation of the formation and evolution of microhalos was studied by using 64 million dark matter particles within a comoving volume box of 3 kpc on the side (implying a particle mass of $9.8 \times 10^{-10} M_\odot$). The initial redshift was $z = 456$, and the initial conditions were similar to the simulation discussed in [12] (see the previous paragraph). By the end redshift of the simulation, $z \approx 75$, the volume contained almost 2000 virialized dark matter subsolar mass halos. The most massive halo is $\sim 10^{-2} M_\odot$, and corresponds to a $3.5 \sigma$ fluctuation in the density field. For a comparison of the evolution and substructure content of this simulation, the authors performed a separate simulation, of similar dynamic range, of a cluster-size halo at $z = 0$. Comparison between the substructure mass functions in the two simulations showed confirmation of the earlier result found in [12], namely a power-law behavior that is self-similar over more than ten orders of magnitude in subhalo mass.

It should be emphasized that the simulation presented in [12] was performed in a region of average density, thus probing the formation of ‘field’ subsolar mass dark matter halos. On the other hand, the high-resolution simulation presented in [14] was centered around the highest density peak of the initial 3 kpc volume. This allowed the study of the assembly and structure of an object with mass higher than the cutoff scale of the power spectrum, whereas resolving scales down to the cutoff scale. In addition, it provided the first estimate of the survival rate of dark matter microhalos, a very important result, which relies on the nonlinear evolution of structure formation.

### 3.2. Challenges in numerical simulations

The numerical simulation of the formation and survival of microhalos is a very difficult task. The difficulties arise from two main sources. Firstly, by the fact that the power spectrum $P(k)$ of subsolar mass scales asymptotically approaches the $k^{-3}$ behavior, thus leading to the formation of multiple scales roughly at similar epochs, and secondly, by uncertainties in the actual implementation of numerical techniques in numerical simulations [15].

The cutoff in the power spectrum that enters the initial conditions in the numerical simulations does not fully suppress the formation of structure at scales smaller than the cutoff scale. Even though the fraction of substructure in objects below the cutoff scale is small, their mere presence deserves further study in order to determine whether this effect is an artifact of the simulation techniques employed, or whether it is a physical effect. It should be noted that such effects have also been found in simulations of warm dark matter (WDM), where the dark matter particle possesses a nonzero thermal velocity [16]–[19]. A possible numerical source that leads to this artificial effect is the implementation of grid initial conditions [20]. If not implemented at an early enough starting redshift, the initial Zel’dovich displacement can be a large fraction of the inter-particle separation [21]. This can lead to artificial fragmentation along filaments; however, the effect has also been observed when the use of glass initial conditions [22] is implemented. The simulations of [14] also show that the formation of sub-cutoff scale structure is not limited to a preferred separation along a grid axis, suggesting that it may perhaps be a real effect.

Another source that can introduce bias in the measurement of a mass function is the starting redshift of simulations. As shown in [21], the requirement that the Fourier modes within the
simulation box are all linear is very naive. The mass function cannot be robustly determined at 
redshifts, which are close to the redshift of first crossing (the redshift where halos form). It has 
also been pointed out that even perhaps the choice of a halo finder in the simulation can also 
lead to an uncertainty in the measurement of the mass function [14, 15], due to the fact that the 
density contrast in subsolar mass halos is low due to similar formation times. Therefore, caution 
must be taken in the interpretation of these results, and a thorough convergence study on the 
mass function of subsolar mass halos is of paramount importance, and urgently needed.

4. Survival of microhalos in the Milky Way halo

The survival (and thus existence) of microhalos in the present-day Milky Way halo has been a 
subject of debate. The approach typically taken in these studies has progressed in two fronts: 
simple analytical estimates, and through numerical simulations. In principle the two should 
agree; however, given the vast orders of magnitude in dynamic range and highly complex 
nonlinear effects, it has proven to be a difficult task.

The question of surviving microhalos was first addressed analytically in [23]. A detailed 
analytical study of the effects of tidal destruction on microhalos due to rapid, early-time 
hierarchical merger phase, showed that only 0.1–0.5% of formed microhalos would survive 
to the present epoch [23]. The initial claim that a large number of microhalos could be present 
in the solar neighborhood [12] was challenged only days later [24]. The argument presented was 
that strong impulses by individual stars in the disc can tidally disrupt microhalos, leading to a 
large number of tidal streams. Within days, a response [25] showed that the application of the 
impulse approximation requires microhalos to experience multiple stellar interactions and as a 
result the survival timescale is many Hubble times.

One of the key advantages of numerical simulations in the study of the formation and 
abundance of microhalos is the ability to trace the history of each bound structure. In doing so, 
the work of [14] found that substructure formed at early times is highly prone to tidal disruption 
due to the high central density of the host halo. In addition, most of the remaining microhalos 
that survived to the end-redshift of the simulation run are significantly tidally stripped. As 
expected, the amount of mass loss experienced due to tidal effects depends on the history of 
the subhalo and in particular, whether it experienced passages near the center of the host halo. 
However, most importantly, the study of [14] demonstrated that even though the density contrast 
of subsolar mass scales is small, microhalos of mass $10^{-6} \, M_\odot$ are able to survive in a host at 
least $10^4$ times more massive, thus addressing numerically the question of survival in the initial 
merging process.

A separate study [26] looked at the energy input to microhalos due to stellar interactions by 
using analytic calculations, and then tested their results in a numerical simulation [27, 28]. The 
main result is that at large impact parameters, the energy input to microhalos is independent of 
mass. They show that the survival timescale of microhalos in the Milky Way is of the order of 
the age of the Milky Way, but decreases significantly as the microhalo mass increases. 
In an independent test of the impulse approximation [29], it was shown that multiple stellar 
encounters can remove most of the mass from microhalos, however, the very inner cores can 
survive to the present time. Nevertheless, the normalization and shape of the density distribution 
of dark matter in microhalos will be decreased, with significant implications to direct and 
indirect detection experiments (see the following sections). It is important to realize that it is 
currently not feasible to fully address the issue of survival, mass and spatial distribution of

New Journal of Physics 11 (2009) 105012 (http://www.njp.org/)
microhalos, due to the enormous dynamic range required and the limited computing power that technologically exists. Studies such as the ones mentioned here are invaluable in shedding light on what might happen in ideal situations, and therefore should be further explored.

5. Characterizing the physical properties of microhalos

In order to assess the detectability of microhalos in any scheme (e.g. direct or indirect detection experiments), it is first necessary to determine the distribution of dark matter within the virialized region that comprises the microhalo. Typically, this can be done by assuming a profile and a normalization of that profile.

The numerical work of [12, 14] finds that microhalos at the redshift at which they were studied in the simulation are described by an NFW profile [30]. The form of this profile is \( \rho(r) = \rho_s/(1 + r/r_s)^2 \), where \( \rho_s \) is a density normalization, called the characteristic density, \( r_s \) is the scale radius, and \( r = r/r_s \). The two parameters needed to specify the profile are the characteristic density and the scale radius. The numerical simulation findings suggest that \( r_s \) is approximately equal to the virial radius \( R_{\text{vir}} \) of the microhalo, suggesting an NFW concentration parameter \( (c_v = R_{\text{vir}}/r_s) \) at that redshift which is of order one (more specifically \( c_v \approx 2-4.5 \) [31]). For field microhalos, the concentration parameter grows as \( (1 + z)^{-1} \), indicating that ‘field’ subhalos have concentrations in the range of 50–100 for a formation redshift around \( z \approx 20 \) (see below).

Obviously, the profile of a present-day microhalo in the Milky Way has most likely evolved from this initial distribution of dark matter, to a distribution which is the result of tidal interactions throughout the course of structure formation. As I alluded to in sections 3 and 4, these processes are very difficult to accurately model. Nevertheless, it is a reasonable assumption to consider the initial profile at the redshift of formation as a representation of a maximal distribution, and keep in mind that any experimental ramifications are going to actually be upper limits and not absolute values.

In order to normalize the dark matter density profile of microhalos we need to first determine their formation time, which can be obtained by knowledge of the CDM power spectrum, and the cosmological growth factor.

The normalization of a profile of a dark matter halo can be obtained by assuming that mean density of the halo is \( \Delta_{\text{vir}} \) times the mean matter density of Universe \( \rho_{\text{M}} \), with \( \Delta_{\text{vir}} \) being the virial overdensity (see e.g. [32]). The mass of the halo is then \( M = 4\pi \rho_{\text{M}} \Delta_{\text{vir}}(z) R_{\text{vir}}^3/3 \). By using this definition of the mass of a dark matter halo, the normalization of the NFW profile is then simply obtained via \( \rho_s = \rho_{\text{M}} \Delta_{\text{vir}} c_v^3/3 f(c_v) \), where \( f(x) = \ln(1 + x) - x/(1 + x) \). The rarity of a particular mass scale at a particular redshift can be obtained from the power spectrum of fluctuations. In general, if we define the power in each logarithmic interval in \( k \) as \( \Delta^2(k) \sim k^3 P(k) \), where \( P(k) \) is the CDM power spectrum, then the mean square fluctuations on a scale \( M \) is given by \( \sigma^2(M) = \int \Delta^2(k)|W(k; M)|^2 d\ln k \). Here, \( W(k; M) \) is the Fourier transform of a real-space spherical top-hat window that contains mass \( M \). The left panel of figure 2 shows the root-mean-square fluctuations on microhalo scales, derived by assuming the WMAP5 cosmological parameters [1]. Note that for illustration purposes, the primordial power spectrum in this example does not contain the cutoff shown in figure 1. Knowledge of the underlying cosmological model (i.e. the values of the cosmological parameters), allows us to compute the growth function of fluctuations \( D(z) \). Then, the redshift of formation of a mass scale \( M \) is defined as the redshift \( z \) where \( \sigma(M) D(z) = \delta_c \), where \( \delta_c = 1.686 \) is the value of the characteristic density for collapse (derived from linear theory). The rarity of a particular peak
Figure 2. Left: the rms fluctuation on subsolar mass scales for the WMAP5 cosmology. Right: the concentration parameter of ‘field’ subsolar mass halos at $z = 0$ based on the model of [33]. This is obtained from an extrapolation of the $c_v(M)$ relationship found on galactic scales, all the way down to microhalo scales. It does not represent the $c_v(M)$ relationship of Milky Way progenitors that were of microhalo scale at the time of their formation.

is typically defined to be in units of the standard deviation of the smoothed density distribution, so that $\nu = \delta_c/\sigma(M)D(z)$.

Note that the rarity of a particular fluctuation in the density field is directly related to the abundance of that scale in the present Milky Way halo—a key quantity in any dark matter experimental effort. For example, a smaller fraction of the total Milky Way mass originates from $3\sigma$ peaks in the density field as those are rarer than e.g. $1\sigma$ peaks. However, the $3\sigma$ peaks are formed earlier, thus they will be denser. As such, they are not as susceptible to tidal disruption as the $1\sigma$ peaks. The abundance of microhalos in the present epoch will be determined by a balance of these competing effects and a full description of the microhalo population of the Milky Way must include all of these factors. A first step in that direction was the study of [34], where it was found that high-$\nu$ progenitors are found predominantly near the centers of dark matter halos today. For example, according to [34], the median progenitor at the solar neighborhood of the Milky Way corresponds to a $2\sigma$ peak.

An additional piece of information that is of importance when it comes to dark matter detection experiments, is the tidal radii of survived microhalos in the Milky Way. A reasonable approach would be to assume that the truncation radius at the present epoch can be estimated by finding the radius of the microhalo where the density is equal to the density at the solar radius [35]. This assumption results in the truncation radius being equal to the virial radius for the microhalos considered here, reflecting the dense state of the Universe at the time they were formed. Clearly, the assumption here is that the halo profile normalization is intact from the time it was formed until the present epoch. Again, this points at the lack of knowledge of the
mass function and history (thus properties) of microhalos in the solar neighborhood. But given the high initial densities reflected in microhalos and the steep power law of the density profile, it seems likely that the tidal radius will not be very different from their virial radius set at the time of collapse.

A more general note regarding the physical description of the dark matter profiles of microhalos is with regard to the concentration parameter. It is dangerous to assign concentration parameters that would correspond to present-epoch ‘field halos’ (i.e. a microhalo formed at high redshift and evolved uninterruptedly to the present day). Field halos grow hierarchically by the accretion of smaller halos and by smooth accretion of dark matter. It is not clear exactly how this process occurs at very high redshifts on microhalo scales, as the flatness of the power spectrum implies the simultaneous collapse of many different scales above the cutoff in the power spectrum. The maximum value of the concentration parameter for a field microhalo can be obtained by extrapolating the concentration–mass relationship deduced from numerical simulations [33, 36] (see figure 2). Even though this is a daring extrapolation over numerous orders of magnitude, it can be considered as an upper limit for field microhalos, as the concentration–mass relationship stems from the shape of the power spectrum. The power spectrum flattens out at large $k$, and so is $c_v$ with respect to mass [33] (see figure 2). The danger from assuming concentration parameters of field halos in microhalo studies near the solar neighborhood arises due to the fact that the density profile normalization is rather sensitive to the value of the concentration parameter $\rho_s \sim c_v^3$, therefore assuming concentration parameters of the order of 100, can result in a very large overestimate of the normalization of the profile.

To summarize, given the uncertainty in the evolution of the profile of microhalos, the only reasonable approach is to determine their physical properties at the time they were formed, and consider those as upper limits to what they could be at the present epoch. Figures 3 and 4 show the formation redshift, characteristic density and virial radius of subsolar mass dark matter halos. It is of vital importance to emphasize that any treatment of annihilation signals from microhalos (see e.g. $\gamma$-rays in the next section) must be consistent with a description of the physical properties of microhalos, which are set by the underlying structure formation paradigm. Arbitrary choices of concentration parameters (which affect the normalization of the profile), or inconsistent choices of formation redshift will only result in an estimate for a particular toy microhalo model. Such an approach cannot assess the structure formation relevance, and/or distribution function of expected signals and must be treated with caution.

6. Detection of microhalos using dark matter annihilation products

It might be possible to detect the presence of microhalos in the Milky Way halo by searching for the annihilation products of the WIMP dark matter particle, and specifically for $\gamma$-rays. Under the supersymmetric WIMP scenario, the dark matter particle couples to quarks and leptons at the tree level, and to photons and the $Z^0$ gauge boson via one-loop diagrams. The search of $\gamma$-rays from the annihilation of the dark matter particle is attractive for two reasons: (i) a swath of $\gamma$-ray experiments are operational (e.g. Fermi/GLAST [37], VERITAS [38], HESS [39], MAGIC [40], CANGAROO [41]), and (ii) the shape of the emitted spectrum can be estimated (given a supersymmetric dark matter candidate).

In general, the luminosity of a subhalo as a function of energy is given by $L(E) = S \int P(E) \, dE$, where $S = \int \rho^2(r) \, d^3r$ (see figure 4), and $P(E) = (dN/dE)\langle \sigma v \rangle / M^2_\chi$. Here, $dN/dE$ is the energy spectrum of the annihilation products under study (e.g. photons, positrons,
neutrinos, etc), \(\langle \sigma v \rangle\) is the annihilation cross section, and \(M_f\) is the mass of the dark matter particle. Maximal values of \(\int P(E) \, dE\) in supersymmetric theories with a valid dark matter candidate (e.g. the neutralino) yield a value of \(\leq 10^{-28} \text{cm}^3 \text{GeV}^{-2}\) for a threshold energy of \(\sim 1 \text{ GeV}\).

### 6.1. Microhalos as \(\gamma\)-ray sources

In this subsection, I discuss the possible detection of microhalos with \(\gamma\)-rays, though the formalism can be well extended to other annihilation products, such as positrons and neutrinos. In order for a subsolar mass halo at a distance \(d\) and flux on Earth \(\Phi = L/4\pi d^2\) to be detected above a detector threshold \(\Phi_0\), it must be located at a distance \(d < \sqrt{\Phi/\Phi_0}\). Such an example is shown in figure 5, for a supersymmetric WIMP dark matter candidate with \(P \leq 10^{-28} \text{cm}^3 \text{GeV}^{-2}\), and a detector threshold of \(\Phi_0 = 10^{-9} \text{cm}^{-2} \text{s}^{-1}\). It should be noted that for subhalo mass functions that scale as \(dN/d\ln M dV \sim M^{-1}\), the typical visibility distance between microhalos of mass \(M\) scales as \(\langle r \rangle \sim M^{1/3}\). However, as the luminosity of a halo goes as \(L \sim \rho^2 r^3 \sim M\), the ‘visibility’ distance of microhalos is proportional to \(d_{\text{max}} \sim M^{-1/2}\), i.e. it is ‘easier’ to detect more massive halos at larger distance [42, 43], than small halos nearby. This scalings will be altered if the dark matter particle’s annihilation cross section is Sommerfeld enhanced [44]–[48]. The increased annihilation rate due to low microhalo internal velocities leads to the possibility that small nearby microhalos will be more luminous than the canonical case where the annihilation cross section is independent of velocity.

*New Journal of Physics 11* (2009) 105012 (http://www.njp.org/)
Figure 4. Left: the virial radius of microhalos at the redshift of collapse. Right: the annihilation luminosity of a subsolar mass dark matter halo as a function of its mass. If the halo has lost 99% of its mass due to tidal interactions with other halos and/or baryons (stars, disc, clouds, etc), then the luminosity should be decreased by a factor of 30% from the values shown. Line types and colors as in figure 2.

One approach to investigate the observability of a population of microhalos in the Milky Way is to Monte Carlo the distribution and properties of microhalos as followed in [49]. The key unknown in this approach (or any approach for that matter) is the abundance of microhalos as a function of their mass, as well as their central densities (as there is a distribution of mass scales that collapse in each redshift interval—see section 5). The results of [49] show that for a fixed concentration microhalos (i.e. isolated density peaks), it is highly unlikely that there should be any high signal-to-noise single-source detection of microhalos with the all-sky Fermi gamma-ray space telescope (FGST, formerly GLAST).

Another approach was followed in [50]. In that study, the uncertainty that stems from the lack of knowledge of the survival rate of microhalos was parameterized as a fixed contribution to the local dark matter density, so that a particular fluctuation peak (i.e. collapsed microhalo at a particular redshift) results in a certain abundance. Again, the main caveat here is the unknown survival rate. The result of [50] suggested that a large number of microhalos could potentially be visible, albeit the normalization of the number was directly deduced from the unknown density parametrization.

One unique feature of the potential presence of survived microhalos in the Milky Way is the probability that they can be present in very short distances (subparsec) from the Solar system. Thus, it may be possible to observe a proper motion of the $\gamma$-ray signal [25, 50, 51]. The two crucial conditions for the observation of the proper motion of a subhalo is that it is bright enough to be detected at some high signal-to-noise level, and it must be close enough so that the proper motion exhibited over the life-time of an experiment is above the threshold angular resolution of the detector. The details of this potential observation were studied in [50] under the assumption...
that a certain fraction of the Milky Way mass at the solar radius is in microhalos, which were all formed in a particular epoch.

Any potential detection of proper motion implies the presence of a large number of ‘unresolved’ microhalos at distances far beyond the flux-limited distance set by the sensitivity of a particular detector. The presence of a large number of microhalos below individual detection threshold would lead to a background radiation that would correlate with the angular and radial distribution of microhalos in the Milky Way. This was the subject of study in [52], where it was shown that the measured value of the $\gamma$-ray background measured by EGRET [53] is already placing stringent limits on the probability of proper motion detection (due to the implied low-number density of microhalos in the Milky Way). Addressing the same issue, [54] showed that regardless of the assumptions of the inner density profile of microhalos, the EGRET measurement of the $\gamma$-ray flux from the Galactic center [55] (see [56], but also [57]) is strongly suggesting that the probability of detecting microhalos with measurable proper motion is negligible. Nevertheless, given the simplicity of this measurement, a search for the proper motion of $\gamma$-ray-only sources should be performed due to the valuable information contained in such a potential detection.

7. Microhalos and the local Milky Way dark matter distribution

The presence of substructure in dark matter halos leads to interesting consequences when it comes to direct detection experiments. In general, the rate of events in a direct detection experiment is proportional to $\Gamma \sim \rho_\odot$, where $\rho_\odot$ is the dark matter density in the solar...
neighborhood. Typically, the value of $\rho_\odot$ is obtained from dynamical measurements of the structure of the Milky Way (e.g. [35]), with a canonical value of $\rho_\odot = 0.4 \text{ GeV cm}^{-3}$. This value is obtained from averaging regions, which are of the order of $\sim \text{kpc}$, much larger than the regions probed in the duration of a direct detection experiment (subparsec scales), and it is susceptible to uncertainties that are introduced due to the unknown shape of the Milky Way halo (see e.g. [58, 59]). If there are fluctuations on subparsec scales (as what one might expect from the presence of microhalos), then direct detection experiments are prone to structure formation uncertainties. In this section, I give an overview of the experimental implications of fluctuations in the density field at the solar radius.

7.1. Microhalos and the mean dark matter density

It is expected that if a certain amount of mass in a dark matter halo is distributed in high-density regions, then the density of the smoothly distributed dark matter will be lower than the case where all matter is distributed smoothly. The presence of subsolar mass microhalos in the Milky Way raises one important question: what is the value of the smooth component if the substructure mass function extends down to microhalo scales, and what is the probability of the solar system being in an overdense region at any given epoch? The answer to both of these questions is directly related to the local flux enhancement in the annihilation signal (the so-called ‘boost factor’), as the probability of being in a density-enhanced region is inversely proportional to the boost factor.

Addressing this question was the study presented in [60]. Under the assumption of hierarchical structure formation, it was shown that the derived local density probability distribution function (PDF) is positively skewed due to the presence of substructure, and that the peak of the distribution is always less than the canonical mean value of the local dark matter density in the solar neighborhood (see figure 6). This conclusion is derived by a simple analytic model based on the scale-invariant nature of the hierarchical structure formation, as well as by investigating the implications of the distribution and profiles of dark matter halos as seen in numerical simulations. Both approaches showed similar results. These results were recently confirmed in numerical simulations by analyzing a suite of halos taken from the Aquarius Project [61, 62] and independently, by an analysis of the Via Lactea II Milky Way simulation [59, 63].

An interesting outcome of the approach of [60] is a derivation of the local annihilation rate due to the granularity of the local dark matter halo. The local boost factor should not be confused with the global boost factor that is commonly referred in the literature (see e.g. [11, 64]). The least granular cases studied in [60] yield a boost factor, which is roughly consistent with estimates based on the survival rate of microhalos [23] as well as direct simulation results, as shown in [63]. The local boost factor is $\sim 1–5$, with a weak dependence on the cutoff scale of the subhalo mass function.

The introduced variance in the local dark matter density has significant implications to the interpretation of combined results from indirect and direct detection experiments, as the expected signal is proportional to different powers of the local particle density. The volume probed during a 3-year direct detection experiment is very small ($\sim \text{few} \times 10^{-4} \text{ pc}$), while dynamical estimates of the local dark matter halo provide averages over much larger scales. The uncertainties implied by fluctuations in the local dark matter density due to the presence of substructure are manifested as uncertainties in the predicted rates in a dark matter detector [60].
Figure 6. The local dark matter density PDF [60]. The peak of the distribution defines the dark matter density of the smooth distribution of dark matter, while the power-law behavior at higher densities arises from the presence of dark matter subhalos. This function is derived under the assumption that the substructure mass fraction in the Milky Way of all substructure in the mass range of $10^{-10} M_\odot - M_{MW}$ is $\sim 20\%$.

If substructure in the form of microhalos is abundant in the solar neighborhood, then the local boost factor will be large (favoring indirect detection searches); however, the probability of being in an overdense region will be low (disfavoring direct detection experiments). Fortuitously, present simulation results suggest that these effects (and similarly perhaps the uncertainties due to structure in phase space) are small, and a smooth dark matter halo is a safe assumption [59], [61]–[63], though the consequences for indirect detection searches may be more pronounced (e.g. the explanation of the positron excess in the PAMELA [65] and ATIC [66] data by the presence of a nearby subhalo [67]).

An interesting question would be to ask whether it is possible that a long-duration direct detection experiment will be able to map the local distribution of dark matter. If we assume the velocity of the Sun to be $220 \text{ km s}^{-1}$, then the distance swept over $\sim 20$ years, is $x \sim \text{few } \times 10^{-3} \text{ pc}$. An extrapolation of the subhalo mass function to microhalo masses (normalized so that 10% of the Milky Way halo is in objects with mass greater than $10^{-5} M_{MW}$) and assuming an NFW profile for the radial distribution of subhalos, results in a mean distance between microhalos, which depends on mass as $\langle r \rangle \approx 52 M_\odot^{0.3} \text{ pc}$. For microhalos of mass $10^{-6} M_\odot$, this is $\langle r \rangle \approx 0.8 \text{ pc}$. As $x \ll \langle r \rangle$, it seems unlikely that a direct detection experiment will measure the transition from a smooth component to an overdensity of dark matter. Coupled with the very small likelihood of being inside a microhalo at the present time (see [59]–[61]) it implies that the rate in direct detection experiments should be time independent (note that this statement does not include the annual modulation that should be present due to the relative motions of the Sun and the Earth in the Milky Way dark matter halo). Figure 7 shows the expected time dependence.
Figure 7. The time-dependent flux enhancement expected from a toy-example of a passage of the solar system through a $10^{-6}M_\odot$ microhalo. The left panel shows the effects of the impact parameter on the flux enhancement (normalized to the maximum flux at closest approach), whereas the right panel shows the same quantity but at a fixed impact parameter, for different mass microhalos.

of the signal in a long-duration direct detection experiment in the hypothetical scenario where the solar system passes through a microhalo. It is important to emphasize that these effects are not convolved with the likelihood of such an occurrence, they simply demonstrate the magnitude of any effect for each particular case shown.

8. Conclusion

Subsolar mass dark matter halos are interesting, not only because they are linked to the nature of dark matter, but also because a detection of their presence would provide insight into structure formation at extremely early times. The detection of microhalos would first and foremost show that the dark matter particle is cold. In addition, it will place constraints on the value of the kinetic decoupling temperature and mass of the dark matter particle. From the cosmological perspective, any detection of subsolar mass halos would provide insights into halo merging and growth at extremely high redshifts, a task unattainable by any other observational method. Their mere presence would imply that at least a certain fraction of them survived the rapid merger phase. Regardless of the difficulties presented by these highly nonlinear structures, subsolar mass dark matter halos are extremely interesting objects and certainly warrant further investigations.

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References

[1] Komatsu E et al 2009 Five-year Wilkinson Microwave Anisotropy Probe observations: cosmological interpretation Astrophys. J. Suppl. 180 330–76
[2] Jungman G, Kamionkowski M and Griest K 1996 Supersymmetric dark matter Phys. Rep. 267 195–373
[3] Bertone G et al 2005 Particle dark matter: evidence, candidates and constraints Phys. Rep. 405 279–390
[4] Hofmann S et al 2001 Damping scales of neutralino cold dark matter Phys. Rev. D 64 083507
[5] Chen X, Kamionkowski M and Zhang X 2001 Kinetic decoupling of neutralino dark matter Phys. Rev. D 64 021302
[6] Profumo S, Sigurdson K and Kamionkowski M 2006 What mass are the smallest protohalos? Phys. Rev. Lett. 97 031301
[7] Schmid C et al 1999 Amplification of cosmological inhomogeneities from the QCD transition Phys. Rev. D 59 043517
[8] Green A M, Hofmann S and Schwarz D J 2004 The power spectrum of SUSY–CDM on sub-galactic scales Mon. Not. R. Astron. Soc. 353 L23
[9] Green A M et al 2005 The first wimpy halos J. Cosmol. Astropart. Phys. JCAP08(2005)003
[10] Loeb A and Zaldarriaga M 2005 The small-scale power spectrum of cold dark matter Phys. Rev. D 71 103520
[11] Martinez G D, Bullock J S, Kaplinghat M, Strigari L E and Trotta R 2009 Indirect dark matter detection from dwarf satellites: joint expectations from astrophysics and supersymmetry, arXiv:0902.4715
[12] Diemand J et al 2005 Earth-mass dark-matter halos as the first structures in the early universe Nature 433 389–91
[13] Bertschinger E 2001 Multiscale Gaussian random fields for cosmological simulations Astrophys. J. Suppl. 137 1
[14] Diemand J, Kuhlen M and Madau P 2006 Early supersymmetric cold dark matter substructure Astrophys. J. 649 1–13
[15] Elahi P J, Thacker R J, Widrow L M and Scannapieco E 2008 Subhalos in scale-free cosmologies, arXiv:0811.0206
[16] Bode P, Ostriker J P and Turok N 2001 Halo formation in warm dark matter models Astrophys. J. 556 93–107
[17] Knebe A, Devriendt J E G, Gibson B K and Silk J 2003 Top-down fragmentation of a warm dark matter filament Mon. Not. R. Astron. Soc. 345 1285
[18] Yoshida N, Sokasian A, Hernquist L and Springel V 2003 Early structure formation and reionization in a warm dark matter cosmology Astrophys. J. 591 L1–4
[19] Colin P, Avila-Reese V and Valenzuela O 2000 Substructure and halo density profiles in a warm dark matter cosmology Astrophys. J. 542 622–30
[20] Goetz M and Sommer-Larsen J 2003 Galaxy formation: warm dark matter, missing satellites, and the angular momentum problem Astrophys. Space Sci. 284 341–4
[21] Heitmann K et al 2006 Capturing halos at high redshifts Astrophys. J. 642 L85–8
[22] White S D M 1996 Formation and evolution of galaxies Cosmology and Large Scale Structure ed R Schaeffer, J Silk, M Spiro and J Zinn-Justin p 349
[23] Berezinsky V et al 2003 Small-scale clumps in the galactic halo and dark matter annihilation Phys. Rev. D 68 103003
[24] Zhao H S et al 2005 Earth-mass dark matter halos are torn into dark mini-streams by stars, arXiv:astro-ph/0502049
[25] Moore B et al 2005 On the survival and disruption of earth mass CDM micro-halos, arXiv: astro-ph/0502213
[26] Green A M and Goodwin S P 2007 Mini-halo disruption due to encounters with stars Mon. Not. R. Astron. Soc. 375 1111–20
[27] Goodwin S P, Whitworth A P and Ward-Thompson D 2004 Simulating star formation in molecular cloud cores I. The influence of low levels of turbulence on fragmentation and multiplicity Astron. Astrophys. 414 633–50

New Journal of Physics 11 (2009) 105012 (http://www.njp.org/)
[28] Goodwin S P, Whitworth A P and Ward-Thompson D 2004 Simulating star formation in molecular cores II. The effects of different levels of turbulence Astron. Astrophys. 423 169–82
[29] Goerdt T et al 2007 The survival and disruption of CDM micro-halos: implications for direct and indirect detection experiments Mon. Not. R. Astron. Soc. 375 191–8
[30] Navarro J F et al 1996 The structure of cold dark matter halos Astrophys. J. 462 563–75
[31] Diemand J 2009 private communication
[32] Bryan G L and Norman M L 1998 Statistical properties of x-ray clusters: analytic and numerical comparisons Astrophys. J. 495 80
[33] Bullock J S et al 2001 Profiles of dark halos: evolution, scatter and environment Mon. Not. R. Astron. Soc. 321 559–75
[34] Diemand J, Madau P and Moore B 2005 The distribution and kinematics of early high-sigma peaks in present-day halos: implications for rare objects and old stellar populations Mon. Not. R. Astron. Soc. 364 367–83
[35] Klypin A, Zhao H S and Somerville R S 2002 LCDM-based models for the Milky Way and M31 I: dynamical models Astrophys. J. 573 597–613
[36] Eke V R, Navarro J F and Steinmetz M 2001 The power spectrum dependence of dark matter halo concentrations Astrophys. J. 554 114–25
[37] Ritz S, Grindlay J, Meegan C, Michelson P F and GLAST Mission Team 2005 The gamma-ray large area space telescope (GLAST) mission Bull. Am. Astron. Soc. p 1198
[38] Weekes T C et al 2002 VERITAS: The very energetic radiation imaging telescope array system Astropart. Phys. 17 221–43
[39] Hofmann W (HESS Collaboration) 2003 Status of the HESS Project Int. Cosmic Ray Conference (Jul. 2003, Tsukuba, Japan) p 2811
[40] Martinez M (MAGIC Collaboration) 2003 Status of the MAGIC Telescope Int. Cosmic Ray Conference (Jul. 2003, Tsukuba, Japan) p 2815
[41] Yoshikoshi T et al 1999 Present status of the 7-m to 10-m telescope of CANGAROO II. Astropart. Phys. 11 267–9
[42] Koussiappas S et al 2004 Observability of gamma rays from neutralino annihilations in the Milky Way substructure Phys. Rev. 69 043501
[43] Kuhlen M, Diemand J and Madau P 2008 The dark matter annihilation signal from galactic substructure: predictions for GLAST, arXiv: 0805.4416
[44] Sommerfeld A 1931 Ann. Phys., LPZ 403 257
[45] Hisano J, Matsumoto S and Nojiri M M 2004 Explosive dark matter annihilation Phys. Rev. Lett. 92 031303
[46] Profumo S 2005 TeV gamma-rays and the largest masses and annihilation cross sections of neutralino dark matter Phys. Rev. D 72 103521
[47] Arkani-Hamed N, Finkbeiner D P, Slatyer T R and Weiner N 2009 A theory of dark matter Phys. Rev. D 79 015014
[48] March-Russell J D and West S M 2008 WIMPonium and boost factors for indirect dark matter detection, arXiv: 0812.0559
[49] Pieri L et al 2005 Difficulty of detecting minihalos via gamma rays from dark matter annihilation Phys. Rev. Lett. 95 211301
[50] Koussiappas S M 2006 Proper motion of gamma-rays from microhalo sources Phys. Rev. Lett. 97 191301
[51] Koussiappas S M 2007 Detecting the dark matter via the proper motion of gamma-rays from microhalos AIP Conf. Proc. 921 142–6
[52] Pieri L, Bertone G and Branchini E 2008 Dark matter annihilation in substructures revised Mon. Not. R. Astron. Soc. 384 1627
[53] Sreekumar P et al 1998 EGRET observations of the extragalactic gamma ray emission Astrophys. J. 494 523–34
[54] Ando S, Kamionkowski M, Lee S K and Koussiappas S M 2008 Can proper motions of dark-matter subhalos be detected? Phys. Rev. D 78 101301

New Journal of Physics 11 (2009) 105012 (http://www.njp.org/)
[55] http://heasarc.gsfc.nasa.gov/docs/cgro/egret

[56] Pullen A R, Chary R-R and Kamionkowski M 2007 Search with EGRET for a gamma ray line from the galactic center Phys. Rev. D 76 063006

[57] Mack G D, Jacques T D, Beacom J F, Bell N F and Yuksel H 2008 Conservative constraints on dark matter annihilation into gamma rays Phys. Rev. D 78 063542

[58] Kamionkowski M and Kinkhabwala A 1998 Galactic halo models and particle dark matter detection Phys. Rev. D 57 3256–63

[59] Zemp M et al 2008 The graininess of dark matter halos, arXiv: 0812.2033

[60] Kamionkowski M and Koushiappas S M 2008 Galactic substructure and direct detection of dark matter Phys. Rev. D 77 103509

[61] Vogelsberger M et al 2008 Phase-space structure in the local dark matter distribution and its signature in direct detection experiments, arXiv: 0812.0362

[62] Springel V et al 2008 The Aquarius Project: the subhalos of galactic halos, arXiv: 0809.0898

[63] Diemand J et al 2008 Clumps and streams in the local dark matter distribution Nature 454 735–8

[64] Strigari L E, Koushiappas S M, Bullock J S and Kaplinghat M 2007 Precise constraints on the dark matter content of Milky Way dwarf galaxies for gamma-ray experiments Phys. Rev. D 75 083526

[65] Adriani O et al 2009 An anomalous positron abundance in cosmic rays with energies 1.5–100 GeV Nature 458 607–9

[66] Chang J et al 2008 An excess of cosmic ray electrons at energies of 300–800 GeV Nature 456 362–5

[67] Hooper D, Stebbins A and Zurek K M 2008 The PAMELA and ATIC excesses from a nearby clump of neutralino dark matter, arXiv: 0812.3202

[68] Bringmann T and Hofmann S 2007 Thermal decoupling of WIMPs from first principles J. Cosmol. Astropart. Phys. JCAP04(2007)016B

[69] Bringmann T 2009 Particle models and the small-scale structure of dark matter New J. Phys. 11 105027 arXiv:0906.0189B