Probing pairing gap in Fermi atoms by light scattering

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We study stimulated scattering of polarized light in a two-component Fermi gas of atoms at zero temperature. Within the framework of Nambu-Gorkov formalism, we calculate the response function of superfluid gas taking into account the final state interactions. The dynamic structure factor deduced from the response function provides information about the pairing gap and the momentum distributions of atoms. Model calculations using local density approximation indicates that the pairing gap of trapped Fermi gas may be detectable by Bragg spectroscopy due to stimulated scattering.

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I. INTRODUCTION

Since the first realization of quantum degeneracy in an atomic Fermi gas by Jin’s group [1] in 1999, cold Fermi atoms have been in focus of research interest in current physics. In a series of remarkable experiments, several groups [2, 3, 4, 5, 6, 7] have demonstrated many new aspects of cold degenerate atomic Fermi gases. The ability to change interatomic interaction ranging from strong attraction to strong repulsion makes Fermi atoms the most favorable laboratory system for testing theoretical models in diverse fields. In particular, research with Fermi atoms has relevance in the field of superconductivity [9, 10]. The basic mechanism behind superconductivity and Fermi superfluidity is particle-particle pairing with an energy gap. Recently, two groups-Innsbruck [11] and JILA [12] have independently reported the measurement of pairing gap in Fermi atoms. Furthermore, two groups-Duke and Innsbruck [13, 14] have measured collective oscillations which indicate the occurrence of fermionic superfluidity [15] in atomic Fermi gas. The superfluid pairing is believed to occur near the crossover [16, 17, 18] between the predicted BCS state of atoms and the Bose-Einstein condensation of molecules formed from Fermi atoms. Several groups have produced Bose-Einstein condensates (BEC) [19] of molecules formed from degenerate Fermi atoms. Several other recent experimental [20] and theoretical investigations [21] have revealed many intriguing aspects of cold Fermi gases.

Several theoretical proposals [22, 23] have been made for probing pairing gap. A method has been suggested to use resonant light [22] to make an interface between normal and superfluid atoms. This has been recently implemented [11, 24], albeit in the radio frequency domain. A number of authors [25, 26, 27] have theoretically investigated Bogoliubov-Anderson (BA) mode [28, 29, 30] in fermionic atoms as a signature of superfluidity. BA mode constitutes a distinctive feature of superfluidity in neutral Fermi systems since it is associated with long wave Cooper-pair density fluctuations.

Our purpose here is to study Bragg spectroscopy with off-resonant polarized lasers as a method for detecting the pairing gap. Bragg spectroscopy has been used by Ketterle’s group for measuring structure factor of an atomic BEC [31]. Bragg scattering in superfluid Fermi atoms has an analogy with Raman scattering in electronic superconductors [32]. In the next section we describe polarization-selective light scattering. We then discuss briefly the method of calculation of response function of superfluid Fermi gas using Green function techniques [33, 34]. We present the main results which suggest the possibility of detecting pairing gap by scattering of circularly polarized light.

II. POLARIZATION-SELECTIVE LIGHT SCATTERING

When two off-resonant laser beams with a small frequency difference are impinged on atoms, the scattering of one laser photon is stimulated by the other photon. In this process, one laser photon is annihilated and reappeared as a scattered photon propagating along the other laser beam. The magnitude of momentum transfer is \( q \approx 2k_\perp \sin(\theta/2) \), where \( \theta \) is the angle between the two beams and \( k_\perp \) is the momentum of a laser photon. To illustrate the main idea, we specifically consider trapped \(^6\)Li Fermi atoms in their two lowest hyperfine spin states \( | g_1 \rangle = | 2S_{1/2}, F = 1/2, m_F = 1/2 \rangle \) and \( | g_2 \rangle = | 2S_{1/2}, F = 1/2, m_F = -1/2 \rangle \). For simplicity, we consider that the number of atoms in each spin component is the same. An applied magnetic field tuned near the Feshbach resonance (\( \sim 850 \) Gauss) results in splitting between the two spin states by \( \sim 75 \) MHz [35], while the corresponding splitting between the excited states \( | e_1 \rangle = | 2P_{3/2}, F = 3/2, m_F = -1/2 \rangle \) and \( | e_2 \rangle = | 2P_{3/2}, F = 3/2, m_F = -3/2 \rangle \) is \( \sim 994 \) MHz [4]. Let both the laser beams be \( \sigma \) polarized and tuned near the transition \( | g_2 \rangle \rightarrow | e_2 \rangle \). Then the transition between the states \( | g_1 \rangle \) and \( | e_2 \rangle \) would be forbidden while the transition \( | g_1 \rangle \rightarrow | e_1 \rangle \) will be suppressed due to the large detuning...
where the Bragg-scattered atoms remain in the same initial internal state $|e_2\rangle$. Similarly, atoms in state $|g_1\rangle$ only would undergo Bragg scattering when two $\sigma_+$ polarized lasers are tuned near the transition $|g_1\rangle \rightarrow 2P_{3/2}, F = 3/2, m_F = 3/2)$. Thus, we infer that in the presence of a high magnetic field, it is possible to scatter atoms selectively of either spin components only by using circularly polarized Bragg lasers. We assume that both the laser beams are $\sigma_-$ polarized and tuned near the transition $|g_2\rangle \rightarrow |e_2\rangle$. Under such conditions, considering a uniform gas of atoms, the effective laser-atom interaction Hamiltonian in electric-dipole approximation can be written as

$$H_I = \hbar \Omega \sum_{k,\sigma = 1,2} \gamma_\sigma \hat{c}_\sigma^\dagger(k + q)\hat{c}_\sigma(k) + \text{H.c.}$$

(1)

where $\hat{c}_\sigma(k)$ represents annihilation operator of an atom with momentum $k$ in the internal state $\sigma$. The subscript $1(2)$ refer to the state $|g_1\rangle (|g_2\rangle)$, $\Omega = (\Omega_1 + \Omega_2)/2$, and $\gamma_i = \Omega_i/\Omega$. Here $\Omega_i$ denotes the two-photon Rabi frequency for the transitions $|g_i\rangle \rightarrow |e_i\rangle \rightarrow |g_i\rangle$. For both the laser beams having $\sigma_-$ polarization tuned near $|g_2\rangle \rightarrow |e_2\rangle$, we have $\Omega_2 \gg \Omega_1$. One can identify the operator $\hat{\rho}_q^{(0)}(q) = \sum_k \hat{c}_\sigma^\dagger(k + q)\hat{c}_\sigma(k)$ as the Fourier transform of the density operator.

### III. THE FORMALISM

The scattering probability is given by the susceptibility

$$\chi(q, \tau - \tau') = -\langle T_\tau [\rho_q^{(\gamma)}(\tau)\rho_{-q}^{(\gamma)}(\tau')] \rangle.$$  

(2)

where $\rho_q^{(\gamma)} = \sum_{k,\sigma} \gamma_\sigma a_{q+k,\sigma}^\dagger a_{k,\sigma}$. $T_\tau$ is the complex time ordering operator and $\langle \cdots \rangle$ means thermal averaging. The dynamic structure factor is related to $\chi$ by $\chi(q, \omega_n)$ as

$$S(q, \omega) = -\frac{1}{\pi} \text{Im} \chi(q, \omega + i0^+).$$

(3)

This follows from generalized fluctuation-dissipation theorem. In order to treat collective excitations, it is essential to go beyond Hartree approximation and apply either a kinetic equation or a time-dependent Hartree-Fock equation or a random phase approximation [24]. The essential idea is to take into account the residual terms which are neglected in the BCS approximation and thereby treat the off-diagonal matrix elements (vertex functions) of single-particle operators in a more accurate way [25, 26].

To study light scattering in Cooper-paired fermionic atoms, we apply Nambu-Gor’kov formalism [33, 34] of superconductivity [22]. Using the familiar Pauli matrices, the susceptibility can be expressed as

$$\chi(q, \omega) = \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\hat{\gamma}_k \hat{G}(k_+)\Gamma(k_+, k_-) \hat{G}(k_-)]$$

(4)

where the Green function has a matrix form as

$$\hat{G}(k) = \begin{pmatrix} k_0 \tau_0 + \xi_k \tau_3 + \Delta_k \tau_1 & i\delta \\ -i\delta & k_0 - E_k^2 + i\delta \end{pmatrix},$$

(5)

where $E_k = \sqrt{k_+^2 + \Delta_k^2}$ and $\xi_k = \epsilon_k - \mu$ with $\epsilon_k = \hbar^2 k^2/(2m)$. Here $\tau_0$ is a $2 \times 2$ unit matrix. The vertex equation is

$$\Gamma(k_+, k_-) = \hat{\gamma} i + \int \frac{d^4k'}{(2\pi)^4} \tau_3 \hat{G}(k'_+)\Gamma(k'_+, k'_-)\hat{G}(k'_-) \tau_3 V(k, k'),$$

(6)

where $k_\pm = k \pm q/2$ and $k = (k, 0)$ is the energy-momentum 4-vector whose components are $k_\pm = \xi_k$ and $k_4 = ik_0$. The bare vertex is a diagonal matrix: $\hat{\gamma} = \text{Diag}(|\gamma_1, -\gamma_2|$. Using Pauli matrices $\tau_0$ and $\tau_3$, this can be rewritten as $\hat{\gamma} = \gamma_0(\gamma_1 - \gamma_2)/2$. For unpolarized light in the absence of magnetic field, $\gamma_1 = \gamma_2$. However, if the incident light is polarized, $\gamma_1 \neq \gamma_2$. In the specific case of $\sigma_-$ polarization as discussed above, we have $\gamma_3 = -\gamma_0 \approx -\gamma_2/2$. If the potential $V(k, k')$ is separable in $k$ and $k'$, then Eq. (6) is analytically solvable. We replace $V(k, k')$ by the potential $V = 4\pi\hbar^2 a_s/(2m)$, ($m = m/2$ being the reduced mass) expressed in terms of s-wave scattering length $a_s$. The strong-coupling limit may be accessed by first renormalizing the BCS mean-filed interaction and then taking the limit $a_s \rightarrow \pm \infty$ as we will discuss later. The four-dimensional integrals of Eq. (6) can be performed following the established method of relativistic quantum electrodynamics as applied for...
studying collective excitations in a superconductor \cite{37}. The detailed method of solution is discussed elsewhere \cite{38}. We here present the final result

\[
\chi(q, \omega) = 2N(0)\gamma_0^2(B) + 2N(0) \left[ A + \frac{\omega^2(f)^2}{4\Delta^2(\beta^2f)} \right] \gamma_3^2
\]

where \(N(0)\) is the density of states at the Fermi surface and

\[
A = \frac{(v_k \cdot P_q)^2 - \omega^2f}{\omega^2 - (v_k \cdot P_q)^2}, \quad B = \frac{(v_k \cdot P_q)^2(1 - f)}{\omega^2 - (v_k \cdot P_q)^2}
\]

Here \(P_q = \hbar q\) and \(v_k\) is the velocity of the atoms with momentum \(k\) and

\[
f(q) = \frac{\arcsin \beta}{\beta \sqrt{1 - \beta^2}}, \quad \beta^2 = \frac{\omega^2 - (v_k \cdot P_q)^2}{4\Delta^2}.
\]

The symbol \(\langle X \rangle\) implies averaging of a function \(X\) over the chemical potential surface: \(\langle X \rangle = [N(0)]^{-1} \int d^3k \delta(\epsilon_k) X\).

As \(\omega \to \omega + i0^+, \beta \to \beta + i0^+,\) we have the following analytic property of \(f(\beta)\):

\[
f(\beta) = -\frac{\arcsinh \sqrt{\beta^2 - 1}}{\beta \sqrt{\beta^2 - 1}} + \frac{i\pi/2}{\beta \sqrt{\beta^2 - 1}}, \quad \beta > 1
\]

**IV. STRONG-COUPLING LIMIT**

To access the strong-coupling limit, the chemical potential \(\mu\) and the gap \(\Delta_k\) should be obtained by solving the gap equation

\[
\frac{m}{4\pi\hbar^2a_s} = \frac{1}{V} \sum_k \left( \frac{1}{2\epsilon_k} - \frac{1}{2E_k} \right)
\]

along with the equation

\[
n = \frac{1}{6\pi^2}k_F^3 = \frac{1}{V} \sum_k \left( 1 - \frac{\xi_k}{E_k} \right).
\]

of the density of single component. Note that the eq. (11) is obtained by regularizing the zero-temperature BCS gap equation with a mean-field parameterized by two-body scattering length as done in Ref. \cite{17}. This approach fails to account for pairing fluctuation effects which are particularly significant near \(T_c\) in strong-coupling regime. However, far below \(T_c\), the correction due to the pairing fluctuation is very small \cite{17}. Based on this regularized mean-field approach and local density approximation (LDA), the zero-temperature density profiles \cite{39}, momentum distribution \cite{40} and the finite temperature effects \cite{41} of superfluid trapped Fermi atoms have been recently studied. The two coupled eqs. (11) and (12) admit analytical solutions \cite{42}. In the unitarity limit the solutions yield \(\Delta \approx 1.16\mu,\ \mu = (1 + \beta)e_F,\) where \(\beta = -0.41\) \cite{40,42} is a constant. Many other recent theoretical \cite{43,44} and experimental \cite{3,45} studies have established the universality of Fermi gas in the unitarity limit.

**V. LEADING APPROXIMATIONS**

With the use of Eqs. (11,10) in Eq. (9), the dynamic structure factor in the leading approximation in terms of \(\beta^{-1}\) can be written as

\[
S(q, \omega) = N(0) \frac{1}{4\Delta^2} \left[ \gamma_3^2 \left( \frac{\omega^2}{\beta^3 \sqrt{\beta^2 - 1}} \right) + \gamma_0^2 \left( \frac{(P_q \cdot v_k)^2}{\beta^3 \sqrt{\beta^2 - 1}} \right) \right], \quad \beta > 1
\]

This is also obtainable from the BCS- Bogoliubov mean-fielded treatment as shown in \cite{38}. Although this leading approximation takes into account excitations in the particle-hole continuum, it fails to account for the in-gap collective
FIG. 1: Dimensionless dynamic structure factor $S(\delta, q)/N(0)$ of a uniform superfluid Fermi gas is plotted as a function of dimensionless energy transfer $\omega/\varepsilon_0$ ($\varepsilon_0$ is the Fermi energy) for different values of the scattering length $a_s = 2.76k_F^{-1}$ (solid), $a_s = 3.89k_F^{-1}$ (dotted), $a_s = 5.47k_F^{-1}$ (dashed) for a fixed momentum transfer $q = 0.8k_F$. The dash-dotted curve is plotted for $a_s = 2.76k_F^{-1}$ and $q = 0.4k_F$. The inset shows the variation of the gap $\Delta$ and the chemical potential $\mu$ as a function of $a_s$.

FIG. 2: Same as in Fig. 1 but for a trapped superfluid Fermi gas for a fixed momentum transfer $q = 0.8k_F$. Here $\varepsilon_0$ is the Fermi energy at the trap center.

modes. It can be verified that in the regime of large momentum and energy transfer ($\beta \gg 1$), the dynamic structure factor of Eq. (13) approximately satisfies the f-sum rule

$$\int \omega S(q, \omega)d\omega \simeq \frac{Nq^2}{2m}, \quad \xi q \gg 1$$

where $N$ is the total number of particles.

Now let us consider the case $0 \leq \beta \ll 1$, that is $v_F p_q \leq \omega \ll 2\Delta$. In this case, the second term in the coefficient of $\gamma_3^2$ in Eq. (7) dominates over all other terms. This term leads to BA mode appearing as a pole of $\chi$. BA mode restores the continuous symmetry that is broken by BCS ground state. In the limit $q \to 0$ and $\omega \to 0$, $f \simeq 1$ and hence the pole is

$$\omega_{\text{BA}} = \frac{1}{\sqrt{3}}v_F p_q.$$ 

In the low momentum and low energy limit ($0 \leq \beta \ll 1$) the dynamic structure factor can be obtained by linearizing
the denominator of the second term in Eq. (7) around the BA mode. By approximating \( f \approx 1 \), we then obtain

\[
S(q, \omega) = N(0) \gamma_3^2 \frac{\omega^2 \delta(\omega - \omega_{BA})}{2\omega_{BA}}.
\] (16)

With \( \gamma_3 \rightarrow 1 \), this satisfies the \( f \)-sum rule. BA mode is well defined in the low momentum regime, i.e., for \( \xi q = v_F p_q/(2\Delta) \ll 1 \). For large momentum, it becomes ill defined due to Landau damping. Ohashi and Griffin \[26\] have provided a detailed theoretical treatment of this mode in the BCS-BEC crossover in Fermi atoms. Minguzzi \textit{et. al.} \[27\] have found that this mode appears as a prominent asymmetric peak in the spectrum of density fluctuation at a very low momentum and energy.

VI. RESULTS AND DISCUSSIONS

Figure 1 and 2 show \( S(\omega, q) \) as a function of \( \omega \) for a uniform and trapped gas, respectively, for different values of \( a_s \). In the case of trapped gas, we use LDA with local chemical potential \( \mu(r) \) determined from equation of state of interacting Fermi atoms in a harmonic trap. When \( a_s \) is large, the behavior of \( S(\delta, q) \) is quite different from that of normal as well as weak-coupling BCS superfluid. This can be attributed to the occurrence of large gap for large \( a_s \). In contrast to the case of a uniform superfluid \[32\], \( S(\delta, q) \) for a superfluid trapped Fermi gas has a structure below \( 2\Delta(0) \), where \( \Delta(0) \) is the gap at the trap center. As the energy transfer decreases below \( 2\Delta(0) \), the slope of \( S(\delta, q) \) gradually reduces. Particularly distinguishing feature of \( S(\delta, q) \) of a superfluid compared to normal fluid is gradual shift of the peak as \( a_s \) increases. The quasiparticle excitations occur only when \( 2\Delta(x) < \omega \). This implies that, when \( \omega \) is less than \( 2\Delta(0) \), the atoms at the central region of the trap cannot contribute to quasiparticle response.

VII. CONCLUSIONS

Order of magnitude analysis of Ref. \[46\] suggests that, with large momentum transfer, it may be possible to distinguish the scattered atoms in time of flight images. A comparison of images with and without Bragg pulses may reveal information about the momentum and density distribution of the scattered atoms. Furthermore, the polarization-selective Bragg spectroscopy may lead to better precision in time-of-flight spin-selective measurements \[8, 19\] since they will be in the same spin component. It is possible to select counter propagating scattered atoms using three or four beam scattering configurations \[46\]. One can then explore the possibility of measuring the correlation of two scattered atoms with opposite momentum by the technique as used in recent studies \[47, 48\].

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