Formation of topological vortices during superfluid transition in a rotating vessel

SHREYANSH S. DAVE(a) and AJIT M. SRIVASTAVA(b)

Institute of Physics - Bhubaneswar 751005, India and
Homi Bhabha National Institute, Training School Complex - Anushakti Nagar, Mumbai 400085, India

received 18 February 2019; accepted in final form 29 April 2019
published online 31 May 2019

PACS 11.27.+d – Extended classical solutions; cosmic strings, domain walls, texture
PACS 67.25.dk – Vortices and turbulence
PACS 67.25.dg – Transport, hydrodynamics, and superflow

Abstract – Formation of topological defects during symmetry breaking phase transitions via the Kibble mechanism is extensively used in systems ranging from condensed-matter physics to the early stages of the universe. The Kibble mechanism uses topological arguments and predicts equal probabilities for the formation of defects and antidefects. Certain situations, however, require a net bias in the production of defects (or antidefects) during the transition, for example, superfluid transition in a rotating vessel, or flux tubes formation in a superconducting transition in the presence of external magnetic field. In this paper we present a modified Kibble mechanism for a specific system, ⁴He superfluid transition in a rotating vessel, which can produce the required bias of vortices over antivortices. Our results make distinctive predictions which can be tested in superfluid ⁴He experiments. These results also have important implications for superfluid phase transitions in rotating neutron stars and also for any superfluid phases of QCD arising in the non-central low-energy heavy-ion collision experiment due to an overall rotation.

Copyright © EPLA, 2019

Introduction. – Topological defects arise in a wide range of systems ranging from condensed-matter physics to the early stages of the universe. The first detailed theory of formation of topological defects via domain formation in a phase transition was proposed by Kibble [1] in the context of the early universe. Zurek proposed that certain aspects of the Kibble mechanism can be tested in superfluid helium systems [2]. It is now well recognized that the Kibble mechanism applies equally well to any symmetry breaking transition [3,4], indeed tests of the Kibble mechanism have been carried out in various condensed-matter systems, e.g., superfluid helium, superconductors, liquid crystals, etc., see refs. [5–11]. Critical slowing-down in continuous transitions requires special considerations (as pointed out by Zurek [2,3]) which are incorporated in the Kibble-Zurek (KZ) mechanism. In general, topological calculations of the Kibble mechanism give an equal probability for the formation of defects and antidefects on the average (though there can be an excess of defects or antidefects in a given event of phase transition). There are many physical situations which require a net average excess of defects or antidefects in a phase transition due to the external conditions, such as formation of flux tubes in type-II superconductors in external magnetic field, and a superfluid transition of ⁴He in a rotating vessel. Normally, the net defect formation (e.g., superfluid vortex formation in a rotating vessel) is studied using arguments of energetics [12,13]. But vortex formation in a rotating vessel during the superfluid transition can deviate from the vortex model prediction due to the contribution from the non-equilibrium defect production process via the Kibble mechanism. A deviation from the vortex model prediction was indeed observed by Hess and Fairbank in their experiment [14], which possibly may be due to such Kibble vortices. It is important here to mention that for the case of superconductors, the issue of external bias is of crucial importance for testing spontaneous flux creation even in the absence of any applied field as it is hard to ensure that there are no external stray magnetic fields [7,8]. In this context, the work presented in ref. [8] is important where spontaneous flux production in annular superconductors is investigated to study the Kibble-Zurek scenario, and the effects of external fields are examined. Using 1D and 3D simulations, it is found that the properties of

(a)E-mail: shreyansh@iopb.res.in
(b)E-mail: ajit@iopb.res.in
superconducting rings are represented by analytic Gaussian approximations, indirectly encoding KZ scales, and the findings are corroborated by experimental results.

Two most important ingredients of Kibble mechanism are the existence of correlation domains inside which the order parameter is taken to be uniform, while the order parameter varies randomly from one domain to another, and the geodesic rule which says that the order parameter between two domains traces the shortest path in the order parameter space. (We mention here that the geodesic rule becomes ambiguous for the case of superconductors due to gauge dependence of phase differences as discussed in [15].) This makes our considerations of the present paper non-trivial for superconductors. We plan to extend our study in this paper for the superconductor case. It will be interesting to analyze, e.g., the results in [8] by adapting our results in the present paper incorporating the considerations of gauge invariance as discussed in [15].) In this work we study vortex formation during superfluid transition of the $^4$He system and show that in the presence of external influence, specifically, rotation of the vessel, both of these aspects of the Kibble mechanism need to be modified.

**Description of system.** – The superfluid component in $^4$He system is characterized by a multi-particle condensate wave function, $\Psi = \Psi_0 e^{i\theta}$, where $\Psi_0$ gives the number density of the superfluid component. The superfluid velocity is given by $\vec{v}_s = \frac{\hbar}{m} \nabla \theta$, where $m$ is the mass of the $^4$He atom. We use the expression for the free energy of the superfluid system in the presence of rotation [12,16] as $F' = F - L \cdot \vec{\Omega}$, where $F$ is the free energy for superfluid without rotation and $L = \rho_s \int (\vec{r} \times \vec{v}_s) d^2x$ is the angular momentum of the superfluid (in the plane perpendicular to the axis of rotation) just after the phase transition generated due to external rotation ($\rho_s = n_0 \Psi_0^2$ is the mass density), $\vec{\Omega}$ being the angular velocity of the vessel containing $^4$He. Here we are assuming that part of normal component which undergoes superfluid condensation carries the same angular momentum as before the transition. (However, it may be possible that only a fraction of the momentum of the normal fluid part which is condensing is carried over to the superfluid momentum. Effects of this possibility on our analysis require a further study. One can determine the value of this fraction experimentally using a rotating annulus of the kind suggested in ref. [2].) In two spatial dimensions, the free-energy density is given by

$$f' = f - \rho_s (\vec{r} \times \vec{v}_s) \cdot \vec{\Omega},$$

where $f$ is the free-energy density of the superfluid without any rotation. We thus get [3],

$$f' = \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 + \frac{h^2}{2mn} \Psi_0^2 |\nabla \theta|^2 - \Omega \rho_s \frac{\hbar}{m} |\nabla \theta|,$$

where $\alpha$ and $\beta$ are phenomenological coefficients. For temperatures less than the superfluid transition temperature $\alpha < 0$ and we determine the local value of the condensate density $\Psi_0$ by minimizing the free energy neglecting the rotation. (One can discuss the effect of rotation on $\Psi_0$, even far away from vortices, especially in the presence of boundaries. We keep the analysis of this issue for future discussions.) With constant superfluid density $\Psi_0$, we minimize this free-energy density with respect to $|\nabla \theta|$ and get

$$|\nabla \theta|_{bias} = \frac{m \Omega \rho_s}{\hbar}.$$  

This shows that the equilibrium configuration of $\Psi$ requires a non-zero value of $|\nabla \theta|$ in the presence of rotation. (Note, for the non-rotating case, we get $\theta = const$, as is assumed inside a domain in the conventional Kibble mechanism.) Note that $|\nabla \theta|_{bias}$ is proportional to the distance from the origin, this will play an important role for the biasing in the production of vortices over antivortices as we will see below.

**Domain structure.** – One of the main ingredients of the Kibble mechanism is the randomness of the condensate phase $\theta$ from one correlation domain to another. As we have discussed, for a superfluid phase transition in the presence of rotation, the order parameter $\theta$ cannot be uniform inside any domain, it must vary systematically inside each domain. In this modified domain picture we still use the fact that all domains are independent of each other and have a completely random $\theta$ value at the center of the domain. (This type of picture was invoked in an earlier work by some of us where biased skyrmion production due to non-zero baryon chemical potential was studied via a modified Kibble mechanism for a toy model in 1 + 1 dimensions [17].) Further, the order parameter variation inside the domain has to be such that it preserves the curl-free motion of the superfluid. As we have mentioned, here we are assuming that part of the normal components which undergoes superfluid condensation carries the same angular momentum as before the transition, and we know that normal components follow rigid-body rotation with velocity given by $\vec{v}_n = \Omega \vec{r}$ which has non-zero curl. With transition to the superfluid phase, we model the domain structure in the presence of an initial rotation such that the curl-free property of the superfluid does not get violated inside a domain. We assume that only on the circular arc within a given domain, drawn using the center of the vessel and passing through the center of that domain it has superfluid velocity as that of the normal component before the transition. This will give the gradient of $\theta$ on that arc to be the same as given by eq. (3). We can see this by relating the velocity of the superfluid components with normal components on the circular arc, i.e., $v_s = v_n$, which gives $|\nabla \theta|_{bias} = \frac{m \Omega \rho_s}{\hbar}$, which is the same as what was obtained earlier by minimizing the free-energy density. It means that larger-$r$ domains will have more variation in $\theta$ than the domains with smaller $r$. As we will see, this is precisely the feature that will cause the biasing in the formation of vortices over antivortices.

Now as there is no initial radial flow, we do not expect any radial superfluid inside a domain, too. This means...
that \( \theta \) will be uniform in the radial direction inside each domain. With these considerations, we obtain well-defined values of \( \theta \) at every point of a domain. We note that inside a given domain, the gradient of \( \theta \) decreases with the increase in \( r \), this domain structure provides curl-free motion of the superfluid. So, with this, for the rotation of the initial normal component whose velocity increases with \( r \), after becoming superfluid, the velocity becomes \( 1/r \) dependent inside a given domain. This can be viewed as the effect of the superfluid transition on the velocity profile inside a given correlation domain. Since with all this outer domains have a stronger variation of \( \theta \) (see eq. (3)), for the anticlockwise rotation of vessel, we should get a higher number of vortices than antivortices. This bias will depend upon \( \Omega \), the system size (\( r \)-dependence) and also the correlation length \( \xi \) (large values of \( \xi \) will give higher \( \Delta \theta \) inside a domain). Below we will see that biasing will also depend on the inter-domain separation due to the modified geodesic rule.

**Geodesic rule.** – We now consider the effect of the rotation on the geodesic rule, the way phase \( \theta \) modified geodesic rule. This bias will also depend on the inter-domain separation due to the free-energy density. Thus, we have

\[ \xi \] the correlation length \( \Delta \theta \)-dependence) and also \( \theta \) variation is considered along a different direction, \( \theta \) and \( \nu \), the free-energy density \( f' \) given above will be for a clockwise path. For an anticlockwise path free-energy density will be \( f' = a(\theta_2 - \theta_1 + 2\pi)^2/d^2 - b(\theta_2 - \theta_1 - 2\pi)/d \). Out of these two paths, one of the paths will have lower free-energy density. The clockwise path will be preferable if condition, \( f'_2 - f'_1 < 0 \) gets satisfied, which gives, \( \theta_2 - \theta_1 > bd/(2a\pi) \). Using the values of \( a \) and \( b \), we get

\[
(\theta_2 - \theta_1) > d|\nabla\theta|_{bias} + \pi, \tag{5}
\]

which is a more restrictive condition to have a clockwise path on the order parameter space than the case in which there is no rotation. Now, if \( \theta_2 < \theta_1 \), the free-energy density \( f'_2 \) given above will be for a clockwise path. For an anticlockwise path free-energy density will be \( f'_2 = a(\theta_2 - \theta_1 - 2\pi)^2/d^2 - b(\theta_2 - \theta_1 + 2\pi)/d \). Now, in this case, condition \( f'_2 - f'_1 < 0 \) will be for anticlockwise variation on the order parameter space, which gives

\[
\theta_2 - \theta_1 < d|\nabla\theta|_{bias} - \pi, \tag{6}
\]

which is a more supportive condition to have an anticlockwise variation of \( \theta \) than without any rotation. Thus, in both cases, the rotation of a vessel supports the anticlockwise variation of \( \theta \) on the order parameter space over the clockwise variation even though the path is longer. This shows that rotation generates biasing in the geodesic rule, too. These modified geodesic rules (eq. (5) and eq. (6)) will also contribute to the biasing of vortices formation over antivortices, along with modified domain structure. Note that for eq. (5) and eq. (6), we have considered that the variation of \( \theta \) is along the direction of the initial flow. If the \( \theta \) variation is considered along a different direction, then a suitable projection of \( |\nabla\theta|_{bias} \) should be taken.

**System parameters and simulation details.** – We consider a cylindrical vessel of radius \( R = 40 \mu \text{m} \), and study the formation of vortices in an essentially two-dimensions system. We have taken such a small vessel because of computational limitations. Note that effective two dimensions require that the height of the cylinder should be small (i.e., not too large compared to the correlation length). This will avoid string bending and formation of string loops which has to be handled in a full three-dimensional simulation. Certainly, it will be very interesting to see the effects of a rotating cylinder in the formation of strings (including string loops) in a full three-dimensional simulation and we plan to investigate it in the future. We have taken the temperature of the system below the Ginzburg regime. The critical temperature \( T_G \) and the Ginzburg temperature \( T_{G0} \) for the \(^4\)He system is 2.17 K and 2.16 K, respectively [16]. The correlation length for this system is given by

\[
\xi = \xi_0 e^{-\nu}, \tag{7}
\]

where \( \xi_0 = 4 \text{Å}, \epsilon = (T_c - T)/T_c, \nu = 2/3 \). With this expression, the Ginzburg correlation length \( \xi_G \) (correlation length at \( T = T_G \)) for this system can be
calculated to be 144 Å. As ordered domain structure only can form a temperature below \( T_G \); therefore, we have taken the correlation length \( \xi \) of the system to be 140 Å corresponding to a lower temperature, which is smaller than \( T_G \). We have taken the inter-domain distance \( d = 10 \text{ Å} \) (as a sample value, we will discuss the effect varying \( d \) on our results). We have considered the antclockwise rotation of the vessel with angular velocity \( \Omega \). The critical angular velocity for this system, for production of a single vortex by energetics argument, will be \( \Omega_{cr} = \frac{h}{mR} \log(R/\xi) \cong 78 \text{ rad s}^{-1} \) (note that the radius of the vessel is very small giving very large \( \Omega_{cr} \)).

For our two-dimensional simulation, we take a square lattice with the correlation domains centered at the lattice points. Domains are assumed to be square with side \( \xi \) so that the lattice constant is \( (\xi + d) \) with \( d \) being the inter-domain separation as mentioned above. We have performed the simulation only in the first quadrant of the vessel. So the numbers we get should be multiplied by 4 to get the total number of vortices for the whole vessel. Our focus will be on the probability of vortices per domain. (Note that even for the whole system, the center of the vessel is within a domain so it cannot accommodate a vortex at that point.) We take the lattice to start from non-zero coordinates (excluding the \( x \) and \( y \) axes). For winding number calculations (to locate vortices) we have excluded plaquettes where the correlation domains touch the boundary of the vessel.

The essential physics of the Kibble mechanism is implemented by taking a random \( \theta \) value at each lattice point (i.e., at the center of the domains). From eq. (3) we know the gradient of \( \theta \) at the circular arc within a given domain, drawn using the center of the vessel and passing through that center of the domain. By knowing the value of \( \theta \) at the center of the domain, and the gradient of \( \theta \) on this arc, we can determine \( \theta \) at each point on the arc. With this, by using the fact that there is no flow in the radial direction, so \( \theta \) is uniform in this direction, we obtain the phase value at the domain boundaries which lie on the side of the lattice. We also use the modified geodesic rule, eq. (5) and eq. (6), for the variation of \( \theta \) in the inter-domain region. To implement this rule, as we mentioned, assume that at the center point of the inter-domain region (which is the middle point of a link) the superfluid has the same velocity as that of the normal components before the transition (given by eq. (3)). We project this velocity along the direction of the lattice side to get \( \nabla \theta \) along the lattice side. With this, and knowing the values of \( \theta \) at the domain boundaries, we implement the modified geodesic rule, eq. (5) and eq. (6), to know the \( \theta \) variation in that region. With all this, we can calculate the winding in each plaquette. Depending upon the winding, at the center of a plaquette we obtain vortex or antivortex.

**Results of simulation.** Now we present the results of our simulation. We consider different values of the angular velocity \( \Omega \), and for each \( \Omega \) we generate 5000 events for defect formation to get good statistics of vortex-antivortex production. Figure 1 shows the distribution of the net defect number \( \Delta n \) (=defect number – antidefect number) for 5000 events. The dashed (black) plot shows the distribution without any rotation of the vessel \((\Omega = 0)\). For this we get a standard distribution as predicted by the Kibble mechanism. This distribution follows the Gaussian distribution \( f(\Delta n) = e^{-\frac{(\Delta n - \bar{\Delta n})^2}{2\sigma^2}} \). By fitting the distribution, we obtain the parameters of this Gaussian as \( a = 656.40, \bar{\Delta n} = 0, \sigma = 30.46 \) (we have taken the bin width 10 with error bars on the plot taken as \( \pm \sigma \) for each bin value). An important point to note is that the center of the Gaussian \( \bar{\Delta n} \) has zero value which is the standard prediction of the Kibble mechanism; no biasing in the formation of vortices and antivortices (on the average). We obtain the average total number of defects from the simulation to be \( N = 1857948 \). The Kibble mechanism makes an important prediction of the relation between \( \sigma \) and \( N \), \( \sigma = CN^\nu \), where the value of \( C \) for square domains is 0.71 [11]. The exponent \( \nu \) is universal and its theoretical value is \( \nu = 1/4 \) for the present case. From the obtained value of \( \sigma \) and \( N \) with the simulation, we derive the value of \( \nu = 0.2604 \), which is quite close to the theoretical value 0.25 and matches well with the experimental value of \( \nu = 0.26 \pm 0.11 \) obtained for the liquid-crystal case, see ref. [11].

The dotted (red) plot in fig. 1 gives the distribution of \( \Delta n \) for the case of vortex formation during superfluid transition in a rotating vessel with angular velocity \( 10^3 \text{ rad s}^{-1} \). We see that in this case also we get a Gaussian distribution but shifted with the mean value \( \Delta n = 25 \), which clearly shows that there is a biasing in the formation of vortices over antivortices. For the whole cylinder, we thus expect to get on an average greater than 100 vortices over antivortices in the vessel. This bias in the net value of \( \Delta n \) occurs here because of the modification in the domain structure and geodesic rule in the presence of rotation. Thus, our proposed modification of the Kibble mechanism, with modified domain structure along with the modified geodesic rule, is able to accommodate the expected bias in the net value of \( \Delta n \) due to the rotation of the vessel.

Table 1 summarizes the results of the simulations at different \( \Omega \) values (note that for each \( \Omega \) we have generated 5000 events and performed simulations). The values of \( \nu \) are obtained from the relation \( \sigma = CN^\nu \), \( C = 0.71 \). From table 1 we find that \( \Delta n \) has linear dependence on \( \Omega \) with the best-fit line \( \Delta n = 0.024\Omega + 1.0 \). The slope will be about 0.1 (4 times higher) for the full cylindrical vessel. As shown in table 1, for \( \Omega = 0 \) we find \( \Delta n = 0.00 \) as expected from the usual Kibble mechanism. However, the straight-line fit gives \( \Delta n \approx 1 \) for \( \Omega = 0 \). For a full vessel this would mean \( \Delta n \approx 4 \) at \( \Omega = 0 \). This is clearly due to fluctuations in the simulation results for a finite number of runs. The fitted line gives \( \Delta n \approx 12 \) at \( \Omega_{cr} \approx 78 \text{ rad s}^{-1} \). Note that when the number of vortices is calculated using only energetics arguments in the vortex model, we expect...
Formation of topological vortices during superfluid transition in a rotating vessel

Table 1: Effect of rotation on the formation of vortices.

| $\Omega$  | $\Delta n$ | $\sigma$ | $N$   | $\nu$  |
|----------|-----------|---------|-------|--------|
| $0$      | 0.0       | 30.46   | 1857948 | 0.2604 |
| $10^3$   | 25.29 $\pm$ 0.44 | 30.41 $\pm$ 0.44 | 1858005 | 0.26029 |
| $10^4$   | 250.20 $\pm$ 0.26 | 30.90 $\pm$ 0.26 | 1858003 | 0.2614 |
| $10^5$   | 2492.88 $\pm$ 0.30 | 31.42 $\pm$ 0.30 | 1858010 | 0.26255 |
| $5 \times 10^5$ | 12466.9 $\pm$ 0.36 | 31.77 $\pm$ 0.35 | 1858031 | 0.26332 |
| $10^6$   | 24932.8 $\pm$ 0.34 | 35.81 $\pm$ 0.32 | 1858136 | 0.27161 |
| $10^7$   | 240603. $\pm$ 4.96 | 43.47 $\pm$ 0.35 | 2745682 | 0.27753 |

As mentioned above, the best fit line for results in table 1 gives $\Delta n = 0.12$ (ignoring the intercept, hence for large $\Omega$). This matches very well with the vortex model prediction which gives $n \approx 2 \pi R^2 n \Omega / \hbar \approx 0.12$ (ref. [16]). This is expected as, for very large $\Omega$, the number of vortices should be dominated by the effects of rotation. We again mention that our results depend on various parameters, such as $\xi$, $d$, etc. Thus, one needs to study whether this agreement with the vortex model prediction (for large $\Omega$) is valid in general.

We emphasize that the free energy of individual defects plays no role in the Kibble mechanism (even with the modifications we propose). Still, with our incorporation of the initial rotation of the normal fluid (and its fraction getting transferred to the superfluid flow after the transition) at least some part, if not all, of the “rotation induced vortices” have been included in this proposed modified Kibble mechanism. This point will be particularly important for small rotations where very few vortices are expected from energetics arguments. This modified Kibble mechanism gives defect density right after the transition which will evolve in time, and approach the density expected using equilibrium free-energy arguments. Thus, if the (modified) Kibble mechanism gives a different number of net produced vortices, then, with time, the number of vortices will change appropriately so that ultimately, in the equilibrium, the system will have $n$ number of vortices as predicted by the vortex model using energetics arguments. It is also interesting to study the distribution of vortices and antivortices as a function of distance from the center in our model. The equilibrium distribution is uniform but, as mentioned above, the distribution right after the transition may be different due to non-equilibrium contributions from the (modified) Kibble mechanism. A non-uniform initial distribution will have very important implications for the case of neutron stars where migration of vortices to achieve uniform (equilibrium) distribution will lead to a change in moment of inertia of the neutron star (as in the model discussed in [18]). This requires large statistics and this study is underway. It is important to make sure that no unwanted long-range correlations are established in our model due to the lattice implementation of the effects of
rotation on the phase distribution. We are making those checks and the results will be reported in a future work.

Table 1 shows that the width of the Gaussian σ also increases with Ω (slowly initially but strongly for large values of Ω). σ represents randomness in the formation of vortices and antivortices. If the formation of vortices and antivortices is completely uncorrelated, then value of σ behaves like \( N^{1/2} \), i.e., the width of the binomial distribution. But there is a correlation between the production of defects and that of antidefects in the Kibble mechanism (ref. [11]) causing the suppression in randomness and hence \( σ \sim N^{1/4} \). By writing \( σ \sim Nν \) we see from the table 1 that ν increases with Ω showing that the correlation between production of vortices and that of antivortices is getting suppressed with Ω. We also fit the dependence of σ on Ω. A reasonable fit for σ as a function of Ω is obtained by σ = aΩp + b, where the fitted values of the parameters are found to be \( a = 0.004 \pm 0.006, \quad p = 0.51 \pm 0.10, \quad b = 30.30 \pm 0.65 \). Even though the value of a is entirely dominated by errors, this fit does suggest a systematic variation of σ with Ω with exponent \( p \simeq 0.5 \). We plan to carry out a systematic study of this result and the increase of ν with Ω in the future. (We mention that defect distribution and hence defect-antidefect correlation may have a non-trivial dependence on the radial distance in the presence of rotation.)

Figure 2 presents the results for a single event for the probability of the defect formation (number of defects per domain) for defects with winding ±1 as a function of Ω. Both probabilities increase with Ω, with winding +1 defect probability increasing faster than the probability for winding −1 (antidefects), reflecting biasing in the formation of defects over antidefects. We also find an increase in the formation of the winding number two defects and antidefects as a function of Ω (we have not included those numbers here). We find non-zero probabilities for both the cases at \( Ω > 2 \times 10^6 \) rad s\(^{-1}\) and the probabilities change differently with Ω, again reflecting biasing in the formation of defects over antidefects. The winding number two defects are known to be unstable in superfluid systems and split into two single winding defects eventually enhancing single winding defects formation probabilities.

We note that while the increase of the vortex formation probability is expected as a function of increasing angular velocity, it may appear puzzling why antidefect probability also increases with the rotation. The explanation for this may lie in the correlation of defects and antidefects which is an important and non-trivial prediction of the Kibble mechanism. As we see from table 1, the defect-antidefect correlation exponent ν, while increasing slightly with angular velocity to a value of about 0.28, still remains far below the value of 0.5 for the uncorrelated case. Thus, while the vortex probability increases naturally with the rotation, the underlying domain structure forces a larger probability of formation of the antivortices close to vortices for the winding number 1 as well as for the winding number 2 case (basically from the fact that positive winding across two domains appears as antiwinding for the neighboring region.)

We have also checked the effects of varying the inter-domain separation d on our results. For \( Ω = 10^6 \), the increase of d from \( d = 2 \) A to \( d = 40 \) A increases the probabilities for winding one defect as well as antidefect by about 15%. The change in winding two defect probabilities is very small and dominated by fluctuations. For \( Ω \leq 10^5 \) the change in probabilities is very small and dominated by fluctuations. The effect of d on various probabilities is a complex issue and we plan to study it systematically in the future.

**Discussion.** – Our assumption that part of the normal component which undergoes superfluid condensation carries the same angular momentum as it had before the transition (along an arc at the center of the domain) just reflects the local conservation of linear momentum during the superfluid transition on that arc. However, even if there was no initial motion of the fluid, still during phase transition, spontaneous generation of flow of the superfluid will arise simply from the spatial variation of the condensate phase. Indeed, it is this (random) phase variation from one domain to another which leads to the formation of a vortex network and hence spontaneous generation of superflow. Then a question arises: what happens to the local linear momentum conservation? Basically, some fraction of \(^4\)He atoms form the superfluid condensate during the transition and develop momentum due to the non-zero gradient of the phase of the condensate. The only possibility is that the remaining fraction of atoms (which form the normal component of the fluid in the two-fluid picture) develop opposite linear momentum so that the momentum is locally conserved (this point was emphasized in our earlier work [19]). This means that there is no net momentum flow anywhere right after the transition. For superfluid transition in a rotating vessel, the same consideration will apply to the normal component in a domain in regions away from the central arc as in those regions the superflow will in general not
match with the initial flow due to the rotation implying generation of an extra counterbalancing normal flow component. These considerations must be incorporated for any experimental test of the Kibble mechanism (either the conventional one or the modified one presented here). It is possible that a due consideration of this spontaneously generated counterbalancing flow of the normal fluid may improve agreement of the results of various superfluid helium experiments with the Kibble mechanism.

Conclusions. – We have proposed a modification of the conventional Kibble mechanism for the situation of the production of topological defects when the physical situation requires an excess of windings of one sign over the opposite ones. We have considered the case of formation of vortices for the superfluid $^4$He system when the transition is carried out in a rotating vessel. As our results show, this biased formation of defects can strongly affect the estimates of net defect density. Also, these studies may be crucial in discussing the predictions relating to defect-antidefect correlations. The modified Kibble mechanism we presented here has very specific predictions about the net defect number which shows a clear pattern of larger fluctuations (about the mean value governed by the net rotation) compared to the conventional Kibble prediction. This can be easily tested in experiments. Further, even the average net defect number deviates from the number obtained from energetics considerations, especially for low values of $\Omega$. This implies that exactly at the time of the transition, a different net defect number will be formed on the average, which will slowly evolve to a value obtained from energetics considerations. These considerations can be extended for the case of flux tube formation in superconductors (with appropriate modifications for the gauged case), and we hope to present it in a future work. Such a modified Kibble mechanism is also needed to study the formation of baryons at finite chemical potential in the framework of the chiral sigma model where baryons appear as skyrmions which are topological solitons (extending our earlier work on $(1+1)$-dimension skyrmion formation to $3+1$ dimensions [17]). Our results will have implications for superfluid transition in rotating neutron stars (where phase-transition-induced density fluctuations could be detected by observing pulsar signal changes, as proposed by some of us [18]).

In an earlier work [19], we considered the possibility of superfluid phases of QCD, e.g., neutron superfluid and color-flavor–locked phase, in low-energy heavy-ion collisions and showed that this will lead to the production of few vortices via the (conventional) Kibble mechanism which can strongly affect the hydrodynamical evolution of the system and can be detected by measuring flow fluctuations. For low-energy non-central collisions, the superfluid phase transition is likely to happen in the presence of an overall rotation of the plasma region. The resulting vortex production for such a case must be studied by a modified Kibble mechanism, as we have proposed here.

**REFERENCES**

[1] Kibble T. W. B., J. Phys. A, 9 (1976) 1387; Phys. Rep., 67 (1980) 183.
[2] Zurek W. H., Nature, 317 (1985) 505.
[3] Zurek W. H., Phys. Rep., 276 (1996) 177.
[4] Rajantie A., Int. J. Mod. Phys. A, 17 (2002) 1.
[5] Hendry P. C. et al., Nature (London), 368 (1994) 315; Ruutu V. M. H. et al., Nature (London), 382 (1996) 334; Dodd M. E. et al., Phys. Rev. Lett., 81 (1998) 3703; Carmi R. et al., Phys. Rev. Lett., 84 (2000) 4966; see also Volovik G. E., Czech. J. Phys., 46 (1996) 3048.
[6] Carmi R., Polturak E. and Koren G., Phys. Rev. Lett., 84 (2000) 4966; Maniv A., Polturak E. and Koren G., Phys. Rev. Lett., 91 (2003) 197001 (cond-mat/0304359); Rivers R. J. and Swarup A., cond-mat/0312082; Kavoussanaki E., Monaco R. and Rivers R. J., Phys. Rev. Lett., 85 (2000) 3452; Rudaz S., Srivastava A. M. and Varma S., Int. J. Mod. Phys. A, 14 (1999) 1591.
[7] Monaco R., Aaroe M., Mygind J., Rivers R. J. and Koshelets V. P., Phys. Rev. B, 77 (2008) 054509.
[8] Weir D. J., Monaco R., Koshelets V. P., Mygind J. and Rivers R. J., J. Phys.: Condens. Matter, 25 (2013) 404207.
[9] Chuang I., Durrer R., Turok N. and Yurke B., Science, 251 (1991) 1336; Snyder R. et al., Phys. Rev. A, 45 (1992) R2169; Chuang I. et al., Phys. Rev. E, 47 (1993) 3343.
[10] Bowick M. J., Chandar L., Schiff E. A. and Srivastava A. M., Science, 263 (1994) 943.
[11] Digal S., Ray R. and Srivastava A. M., Phys. Rev. Lett., 83 (1999) 5030; Ray R. and Srivastava A. M., Phys. Rev. D, 69 (2004) 103525 (hep-ph/0110165).
[12] Landau L. D. and Lifshitz E. F., Course on Theoretical Physics, Vol. 9, Statistical Physics Part 2 (Pergamon Press Ltd.) 1980.
[13] Hess G. B., Phys. Rev., 161 (1967) 189.
[14] Hess G. B. and Fairbank W. M., Phys. Rev. Lett., 19 (1967) 216.
[15] Rudaz S. and Srivastava A. M., Mod. Phys. Lett. A, 8 (1993) 1443.
[16] Tilley D. R. and Tilley J., Superfluidity and Superconductivity, third edition (Overseas Press) 2005.
[17] Kumar V. S., Layek B., Srivastava A. M., Sanval S. and Tiwari V. K., Int. J. Mod. Phys. A, 21 (2006) 1199.
[18] Bagchi P., Das A., Layek B. and Srivastava A. M., Phys. Lett. B, 747 (2015) 120.
[19] Das A., Dave S. S., De S. and Srivastava A. M., Mod. Phys. Lett. A, 32 (2017) 1750170.