OBSERVABLE CIRCLES-IN-THE-SKY IN FLAT UNIVERSES

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An important, and potentially detectable, signature of a non-trivial topology for the universe is the presence of so called circles-in-the-sky in the cosmic microwave background (CMB). Recent searches, confined to antipodal and nearly antipodal circles, have however failed to detect any. This outcome, coupled with recent theoretical results concerning the detectability of very nearly flat universes, is sufficient to exclude a detectable non-trivial cosmic topology for most observers in the inflationary limit \(0 < |\Omega_{\text{tot}} - 1| \lesssim 10^{-5}\). In a recent paper we have studied the consequences of these searches for circles if the Universe turns out to be exactly flat \(\Omega_{\text{tot}} = 1\) as is often assumed. More specifically, we have derived the maximum angles of deviation possible from antipodicity of pairs of matching circles associated with the shortest closed geodesic for all multiply-connected flat orientable 3-manifolds. These upper bounds on the deviation from antipodicity demonstrate that in a flat universe for some classes of topology there remains a substantial fraction of observers for whom the deviation from antipodicity of the matching circles is considerably larger than zero, which implies that the searches for circles-in-the-sky undertaken so far are not enough to exclude the possibility of a detectable non-trivial flat topology. Here we briefly review these results and discuss their consequences in the search for circles-in-the-sky in a flat universes.

1. Introduction

The overall global shape of the Universe remains among the fundamental open questions in cosmology. The main theoretical difficulty is that, while general relativity (as well any other local geometrical theory of gravity) can be used to constrain the space-time geometry, it leaves its spatial topology undetermined (see, e.g., the reviews Refs. [1]). The spatial topology of the universe, however, can in principle be inferred by making systematic searches using the high resolution cosmic microwave background (CMB) data [2]. Moreover, the combination of CMB with other observational data also provides strong support for near or exact flatness of the universe.

In standard cosmology the Universe is modeled by a 4–manifold \(\mathcal{M} = \mathbb{R} \times M_3\) endowed with the spatially homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker metric. Furthermore, the spatial sections \(M_3\) are often assumed to be the simply-connected 3–manifolds: Euclidean \(\mathbb{E}^3\), spherical \(S^3\), or hyperbolic space \(\mathbb{H}^3\). However, these choices are not unique. In fact, depending upon the spatial curvature of the three section, there are an infinity of classes of topologically inequivalent multiply-connected 3–manifolds. An important observational consequence of a non-trivial (multiply-connected) observable spatial non-trivial topology of \(M_3\) is the existence of the circles-in-the-sky, i.e. pairs of matching circles with the same
distribution of temperature fluctuations, identified by $\Gamma$, the so-called holonomy group.

Recently searches for antipodal and nearly antipodal (with the deviation from antipodicity $|\theta| \leq 10^\circ$) circles have been undertaken, with negative results$^{[3,4]}$. In Ref. $^7$ we have shown that a detectable spatial topology of very nearly flat universe can be ‘locally’ approximated by either a slab space ($\mathbb{R}^2 \times S^1$) or chimney space ($\mathbb{R} \times T^2$) irrespective of its global topology. These results allow an upper bound to be placed on the deviation from antipodicity in the inflationary limit, which again turns out to be less than $10^\circ$ for majority of observers$^8$. The combination of these results would (if confirmed) be in principle sufficient to exclude detectable manifolds with non-trivial (non-flat) topology for the overwhelming majority of observers in a very nearly flat universe$^8$.

In a recent paper$^9$ we have studied the consequences of these searches for circles if the spatial section of the Universe turns out to be exactly flat with an orientable topology. This possibility is important not only because a flat universe is compatible with observations, but also because it is the assumption usually made in the so called concordance model of cosmology. Here we briefly summarize the main results of Ref. $^9$ and discuss their consequences in connection with the recent searches for circles-in-the-sky.

2. Results and Concluding Remarks

Recently we made a detailed study of the circles-in-the-sky and in particular the maximum permissible deviations from antipodicity in all orientable multiply-connected flat manifolds (see Ref. $^9$ for further details). We recall that in addition to the simply connected flat Euclidean space $\mathbb{R}^3$, there are a total of 17 multiply connected three-dimensional flat manifolds which are quotient spaces of the form of $\mathbb{R}^3/\Gamma$, where $\mathbb{R}^3$ is the covering space. Nine out of these 17 classes of manifolds are orientable. These consist of the six compact manifolds ($E_i, i = 1, \cdots, 6$): three-torus, half turn space, quarter turn space third turn space, sixth turn space and Hantzsche-Wendt space, plus three non-compact ones: the chimney space $E_{11}$, the chimney space with half turn $E_{12}$ and the slab space $E_{16}$. The manifolds, $E_1$, $E_{11}$ and $E_{16}$ are globally homogeneous and hence would only give rise to antipodal pairs of circles. The important question is then: What type of circles would the remaining 6 manifolds produce and, specifically, what would be the maximum value for $\theta$, the deviation from antipodicity for their most readily detectable circle pairs?

We have calculated the maximum values for $\theta$, $\theta_{\text{max}}$, for all orientable flat three-manifolds with a non-trivial topology. Our results are summarized in Table 1 (see Ref. $^9$ for details).

An important outcome of our results is that the upper bounds on the deviation from antipodicity of pairs of matching circles-in-the-sky associated with the shortest closed geodesics of these manifolds can far exceed the $\pm 10^\circ$ antipodicity interval used in searches so far undertaken$^{[3,4]}$. This has the immediate consequence that the searches so far do not rule out the possibility of a detectable non-trivial flat
Table 1. Multiply-connected flat orientable 3-manifolds and the maximum deviation from antipodicity angles, $\theta_{\text{max}}$, of pairs of matching circles-in-the-sky associated with the shortest closed geodesic.

| Orientable Flat Manifold          | Symbol | $\theta_{\text{max}}$ |
|-----------------------------------|--------|----------------------|
| three-torus                       | $E_1$  | $0^\circ$            |
| half turn space                   | $E_2$  | $120^\circ$          |
| quarter turn space                | $E_3$  | $86^\circ$           |
| third turn space                  | $E_4$  | $109^\circ$          |
| sixth turn space                  | $E_5$  | $59^\circ$           |
| Hantzsche-Wendt space             | $E_6$  | $120^\circ$          |
| chimney space                     | $E_{11}$ | $0^\circ$           |
| chimney space with half turn      | $E_{12}$ | $120^\circ$          |
| slab space                        | $E_{16}$ | $0^\circ$           |

topology.

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