Based on experimental mass hierarchy, a set of flavor–democratic (FD) quark mass matrices at low energies is discussed. The model predicts CP violation parameters $J_{CP} = (0.3 \pm 0.2) \times 10^{-4}$ and $\epsilon'/\epsilon = (0.6 \pm 0.5) \times 10^{-3}$. However, this simple FD model also predicts a physical top quark mass not much higher than 100 GeV. As a next step, we assume that the Standard Model (SM) breaks down around some high energy $\Lambda$, and is replaced by a new FD flavor gauge theory (FGT). This possibility can be investigated by studying renormalization group equations for the Yukawa couplings of SM with two Higgs doublets for various $m_t$ and $v_U/v_D$. With appropriate flavor–democratic boundary conditions at $\Lambda_{FGT}$, bounds on masses of top quark and tau-neutrino are derived, which are compatible with experimental bounds.

1. Flavor Democracy at Low Energy

In the standard electroweak theory, the hierarchical pattern of the quark masses and their mixing remains an outstanding issue. While a gauge interaction is characterized by its universal coupling constant, the Yukawa interactions have as many coupling constants as there are fields coupled to the Higgs boson. There is no apparent underlying principle which governs the hierarchy of the various Yukawa couplings, and as a result, the Standard Model of strong and electroweak interactions can predict neither the quark (or lepton) masses nor their mixing. This situation can be improved by assuming a universal Yukawa interaction – the resulting spectrum consists then of one massive and two massless quarks in each (up and down) sector in the three generation Standard Model. Flavor–democratic (FD) quark mass matrices, and a perturbed form of such FD matrices, were introduced already in 1978 by Harari, Haut and Weyers$^1$ in a left-right symmetric framework. Flavor democracy

$^1$Talk given by C.S. Kim at the Workshop on Masses and Mixings of Quarks and Leptons, Shizuoka, Japan, March 19-21, 1997. Proceedings will be published.
has recently been suggested by Koide, Fritzsch and Plank\(^2\), as well as Nambu\(^3\) and many other authors\(^3\) as an analogy with the BCS theory of superconductivity. In this Section we will discuss how this flavor symmetry can be broken by a slight perturbation at low energies, in order to reproduce the quark masses and the CKM matrix\(^4\).

As a result, predictions for the top quark mass and for the CP violation parameter \(J_{CP}\) are obtained. This Section is based on a work by Cuypers and Kim\(^12\).

Considering only quark fields, the gauge invariant Yukawa Lagrangian is

\[ \mathcal{L}_Y = - \sum_{i,j} (\bar{Q}'_i L \Gamma^D_{ij} d'_j R \phi + \bar{Q}'_i L \Gamma^U_{ij} u'_j R \tilde{\phi} + \text{h.c.}) . \]  

(1)

Here, the primed quark fields are in a flavor \([SU(2)]\) basis of the \(SU(2) \times U(1)\) electroweak gauge group – the left-handed quarks form doublets under the \(SU(2)\) transformation, \(Q'_L = (\bar{u}'_L, \bar{d}'_L)\), and the right-handed quarks are singlets. The indices \(i\) and \(j\) run over the number of fermion generations. The Yukawa coupling matrices \(\Gamma^U, \Gamma^D\) are arbitrary and not necessarily diagonal. After spontaneous symmetry breaking, the Higgs field \(\phi\) acquires a nonvanishing vacuum expectation value (VEV) \(v\) which yields quark mass terms in the original Lagrangian

\[ \mathcal{L}_{\text{mass}} = - \sum_{i,j} (\bar{d}'_i L M^D_{ij} d'_j R + \bar{u}'_i L M^U_{ij} u'_j R + \text{h.c.}) , \]  

(2)

and the quark mass matrices are defined as

\[ M^U, D_{ij} \equiv \frac{v}{\sqrt{2}} \Gamma^U, D_{ij} . \]  

(3)

Mass matrices \(M^U, D\) are diagonalized by biunitary transformations involving unitary matrices \(U^U, D_L\) and \(U^U, D_R\), and the flavor eigenstates are tranformed to physical mass eigenstates by the same unitary transformations,

\[ U^U, D_L M^U, D (U^U, D_R)^\dagger = M^U, D_{\text{diag}} \quad \text{and} \quad U^U, D_L u'_L = u_{L,R}, \quad U^U, D_R d'_R = d_{L,R} . \]  

(4)

Using the recent CDF data\(^5\) of the physical top mass \(m_t^{phys} \approx 175\) GeV, the diagonalized mass matrices \(M^U, D_{\text{diag}}\) at a mass scale of 1 GeV are

\[ M^U_{\text{diag}} \approx m_t \begin{bmatrix} 2.5 \times 10^{-5} & 0.006 & \cr & 1 & \cr \end{bmatrix} \quad \text{and} \quad M^D_{\text{diag}} \approx m_b \begin{bmatrix} 1.7 \times 10^{-3} & 0.03 & \cr & 1 & \cr \end{bmatrix} . \]  

(5)

The first two eigenvalues in both matrices are almost zero (almost degenerate) when compared to the eigenvalue of the third generation. In order to account for this large mass gap, one can use mass matrices which have in a flavor basis the flavor–democratic (FD) form

\[ M^U_0 = \frac{m_t}{3} \begin{bmatrix} 1 & 1 & 1 & \cr 1 & 1 & 1 & \cr 1 & 1 & 1 & \cr \end{bmatrix} \quad \text{and} \quad M^D_0 = \frac{m_b}{3} \begin{bmatrix} 1 & 1 & 1 & \cr 1 & 1 & 1 & \cr 1 & 1 & 1 & \cr \end{bmatrix} . \]  

(6)
Diagonalization leads to a pattern similar to the experimental spectrum (5)

\[
M_{\text{diag}}^U = m_t \begin{bmatrix} 0 & 0 & 1 \\ \end{bmatrix} \quad \text{and} \quad M_{\text{diag}}^D = m_b \begin{bmatrix} 0 & 0 & 1 \\ \end{bmatrix}.
\]  

Arbitrariness in the choice of the Yukawa Lagrangian has been substantially reduced with this symmetric choice. Each (up or down) quark sector is determined in this pure FD approximation by a single universal Yukawa coupling.

To induce nonzero masses for the lighter quarks and to reproduce the experimental CKM matrix, small perturbations have to be added to the universal Yukawa interactions. One possibility is to analyze effects of the following two kinds of independent perturbation matrices

\[
P_1 = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & 0 \end{bmatrix},
\]  

\(\alpha, \beta, a \) and \(b\) being real parameters to be determined from the quark masses. For simplicity, these perturbations can be applied separately. Quark mass matrices (in a flavor basis) are then sums of the dominant universal FD matrices (6) plus one kind of the perturbation matrices (8). One then has to solve the eigenvalue problem

\[
\det |M_{\text{U,D}}^U - \lambda| = 0, \text{ where } M_{\text{U,D}}^U = M_0^{U,D} + P_i \text{ and } \lambda = m_1, -m_2, m_3, \quad \text{(9)}
\]

and \(m_1 = m_d \) or \(m_u\), \(m_2 = m_s \) or \(m_c\) and \(m_3 = m_b \) or \(m_t\). The six parameters of the perturbed matrices \(M_{\text{U,D}}^U\) (e.g., \(m_t, \alpha^{(u)}, \beta^{(u)}; m_b, \alpha^{(d)}, \beta^{(d)}\)) are uniquely determined from the experimental input of the five light (current) quark masses and the choice of a particular mass for the top quark. CKM matrix is then constructed as

\[
V = U_L^U \begin{bmatrix} 1 & e^{i\sigma} & e^{i\tau} \end{bmatrix} U_L^{D\dagger},
\]  

where phase angles \(\sigma\) and \(\tau\) are introduced phenomenologically to generate possible CP violation in the framework of the three generation standard CKM model. The CKM matrix is then uniquely determined by the arbitrary input of the two angles \(\sigma\) and \(\tau\) in (10).

To determine these eight perturbation parameters, a \(\chi^2\) analysis was used. For the first five quarks, the masses obtained by Gasser and Leutwyler\(^6\) can be used. No constraints on the top quark mass were imposed. Additional constraints were used – for four degrees of freedom of the CKM matrix coming from two sources. Information on the quark mixing angles comes from the measurements of the three absolute values\(^7\): 

\[
|V_{us}| = \sin \theta_C = 0.221 \pm 0.002, \quad |V_{cb}| = 0.040 \pm 0.004, \quad |V_{ub}/V_{cb}| = 0.08 \pm 0.02. \quad \text{(11)}
\]
Information on the CP violating phase was taken from the experimental value of $\varepsilon$ parameter of K decay

$$\varepsilon = (2.26 \pm 0.02) \times 10^{-3} = B_K \cdot f(m_c, m_t, V), \quad (12)$$

where $f$ is a complicated function of the charmed and top quark masses and of CKM matrix elements, and $B_K$ is the parameter connecting a free quark estimate to the actual value of $\Delta S = 2$ matrix element describing $K - \bar{K}$ mixing. Following Ref. 8, we used the value of $B_K \approx 2/3 \pm 1/3$.

Analysis showed that only the combination of perturbations $P_U = P_1$ and $P_D = P_2$ resulted in an acceptable value of $\chi^2/d.o.f. \approx 0.6/1$. The best fit was obtained for

$$m_s = 183 \text{ MeV}, \quad m_t = 100 \text{ GeV}, \quad \sigma = 0.6^\circ, \quad \text{and} \quad \tau = 5.7^\circ, \quad (13)$$

the other quark masses being close to their central values. The three other combinations gave much larger values $\chi^2 > 4$. It appears thus that the prediction for the top quark mass from the low energy FD mass matrices cannot satisfy the TEVATRON value of $m_t^{\text{phys}} \approx 175$ GeV. This model’s prediction for

$$J_{CP} = \text{Im}(V_{ub}V_{td}^*V_{ub}^*V_{tb}^*) , \quad (14)$$

as a function of $m_t$ can also be obtained – the approximate value $J_{CP} = (0.3\pm0.2) \times 10^{-4}$ is predicted, which corresponds to $\sin \delta_{13} \approx (0.56\pm0.37)$. This result is used to predict

$$\varepsilon'/\varepsilon = (290) \cdot J_{CP} \cdot H(m_t), \quad (15)$$

where $H(m_t)$ is a decreasing function of the top quark mass. The predicted value in the model is $\varepsilon'/\varepsilon = (0.6 \pm 0.5) \times 10^{-3}$, with a weak dependence on the top quark mass. This prediction seems to favor the data from E731 over the data from NA31.

To conclude this Section, we described a new set of quark mass matrices based on a perturbation of a universal (FD) Yukawa interaction at low energy. The model contains eight parameters, which have been fitted to reproduce the five known quark masses (except $m_t$), moduli of three known elements of the CKM matrix, and the $K$-physics parameter $\varepsilon$. As a result, the physical top quark mass is predicted to be not much heavier than $\approx 100$ GeV, and the direct CP violation parameters are predicted to be $J_{CP} = (0.3 \pm 0.2) \times 10^{-4}$ and $\varepsilon'/\varepsilon = (0.6 \pm 0.5) \times 10^{-3}$. The analysis will be improved substantially with a better theoretical knowledge of $B_K$, a more precise determination of the light quark masses as well as by taking into account the more accurate measurement of $|V_{ub}|$ and the ratio $|V_{ub}/V_{cb}|$. This low energy model, based on a simple perturbation of a universal FD Yukawa interaction at low energies, has been invalidated by the discovery of the top quark much heavier than 100 GeV.

2. Flavor Democracy at High Energies

Many attempts to unify the gauge interactions of the Standard Model (SM) have been made in the past – within the framework of the Grand Unified Theories (GUT’s).
These theories give a unification energy $E_{\text{GUT}} \gtrsim 10^{16}$ GeV, i.e., the energy where the SM gauge couplings would coincide: $5\alpha_1/3 = \alpha_2 = \alpha_3$. Here, $\alpha_j = g_j^2/4\pi$ (j = 1, 2, 3) are the gauge couplings of $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, respectively. For the unification condition to be satisfied at a single point $\mu (= E_{\text{GUT}})$ exactly, supersymmetric theories (SUSY) were used, replacing the SM above the energies $\mu \approx M_{\text{SUSY}} \approx 1$ TeV. This changed the slopes of $\alpha_j = \alpha_j(\mu)$ at $\mu \geq M_{\text{SUSY}}$, and for certain values of parameters of SUSY the three lines met at a single point.

There are several deficiencies in such an approach. The unification energy is exceedingly large ($E_{\text{GUT}} \gtrsim 10^{16}$ GeV) since the proton decay time is large ($\tau_{\text{proton}} \geq 5.5 \times 10^{32}$ yr). This implies a large desert between $M_{\text{SUSY}}$ and $E_{\text{GUT}}$. While eliminating several of the previously free parameters of the SM, SUSY introduces several new parameters and new elementary particles which haven’t been observed.

It is our belief that it is more reasonable to attempt first to reduce the number of degrees of freedom (d.o.f.’s) in the Yukawa sector, since this sector seems to be at least as problematic as the gauge boson sector. Any such attempt should be required to lead to an overall reduction of the seemingly independent d.o.f.’s, unlike the GUT–SUSY approach. The symmetry responsible for this reduction of the number of parameters can be “flavor democracy” (FD), valid possibly in certain separate sectors of fermions (e.g., up-type sector, down-type sector). This symmetry could be realized in a flavor gauge theory (FGT)\textsuperscript{14} – this is a theory blind to fermionic flavors at high energies $E > \Lambda_{\text{FGT}}$ and leading at “lower” energies $E \sim \Lambda_{\text{FGT}}$ to flavor–democratic (FD) Yukawa interactions. Requirement of reduction of as many d.o.f.’s as possible would make it natural for FGT’s to be without elementary Higgs. The scalars of the SM are then tightly bound states of fermion pairs $\bar{f}f$, with $\bar{f}f$ condensation taking place at energies $\Lambda$: $E_{\text{ew}} \ll \Lambda \lesssim \Lambda_{\text{FGT}}$. The idea of FD, and deviations from the exact FD, at low energies ($E \sim 1 - 10^2$ GeV) have been investigated by several authors\textsuperscript{2,3,12}. On the other hand, in this Section we discuss FD and deviations from it at higher energies $E \gg E_{\text{ew}}$, and possible connection with FGT’s. This discussion is motivated and partly based on works of Ref.\textsuperscript{14}.

Let us illustrate first these concepts with a simple scheme of an FGT. Assume that at energies $E \gtrsim \Lambda_{\text{FGT}}$ we have no SM scalars, but new gauge bosons $B_\mu$, i.e., the symmetry group of the gauge theory is extended to a group $G_{\text{SM}} \times G_{\text{FGT}}$. Furthermore, we assume that the new gauge bosons obtain a heavy mass $M_B \sim \Lambda_{\text{FGT}}$ by an unspecified mechanism (e.g., dynamically, or via a mechanism mediated by an elementary Higgs). At thus high energies, the SM–part $G_{\text{SM}} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ is without Higgses, and hence with (as yet) massless gauge bosons and fermions. The FGT–part of Lagrangian in the fermionic sector is written schematically as

\begin{equation}
L_{\text{g.b.–f}}^{\text{FGT}} = -g^f_\Psi \gamma^\mu B_\mu \Psi \quad (\text{for} \ E \gtrsim \Lambda_{\text{FGT}}),
\end{equation}

where $\Psi$ is the column of all fermions and $B_\mu = B^j_\mu T_j$. $T_j$’s are the generator matrices of the new symmetry group $G_{\text{FGT}}$. Furthermore, we assume that the $T_j$’s correspond-
ing to the electrically neutral $B_j^\mu$’s do not mix flavors (i.e., no FCNC’s at tree level) and are proportional to identity matrices in the flavor space (“flavor blindness”). We will argue in the following lines that the FGT Lagrangian (16) can imply creation of composite Higgs particles through condensation of fermion pairs, and can subsequently lead at lower energies to Yukawa couplings with a flavor democracy.

The effective current–current interaction, corresponding to exchanges of neutral gauge bosons $B$ at “low” cutoff energies $E$ ($E \sim \Lambda_{\text{FGT}} \sim M_B$), is

$$L^\text{FGT}_{4f} \approx -\frac{g^2}{2M_B^2} \sum_{i,j}(\bar{f}_i \gamma^\mu f_i)(\bar{f}_j \gamma_\mu f_j) \quad (\text{for} \quad E \sim \Lambda_{\text{FGT}} \sim M_B).$$

(17)

Since we are interested in the possibility of Yukawa interactions of SM originating from (17), and since such interactions connect left–handed to right–handed fermions, we have to deal only with the left–to–right (and right–to–left) part of (17). Applying a Fierz transformation\(^\text{15}\) to this part, we obtain four-fermion interactions without $\gamma^\mu$’s

$$L^\text{FGT}_{4f} \approx \frac{2g^2}{M_B} \sum_{i,j}(\bar{f}_{iL} f_{jR})(\bar{f}_{jR} f_{iL}) \quad (\text{for} \quad E \sim \Lambda_{\text{FGT}} \sim M_B).$$

(18)

These interactions can be rewritten in a formally equivalent (Yukawa) form with auxiliary (i.e., as yet nondynamical) scalar fields. One possibility is to introduce only one $SU(2)$ doublet auxiliary scalar $H$ with (as yet arbitrary) bare mass $M_H$, by employing a familiar mathematical trick\(^\text{16}\)

$$L_{Y}^\text{E} \approx -M_H \sqrt{2}g \frac{3}{M_B} \sum_{i,j=1} \left\{ \left[ (\bar{\psi}^q_{iL} \tilde{H}^R + \bar{\psi}^d_{iL} \tilde{H}^R) u^q_{jR} + \bar{\psi}^d_{iL} \tilde{H}^R u^d_{jR} + \text{h.c.} \right] 
+ \left[ (\bar{\psi}^q_{iL} H^L) d^q_{jR} + (\bar{\psi}^d_{iL} H^L) d^d_{jR} + \text{h.c.} \right] \right\} - M_H^2 H^\dagger H,$$

(19)

where $M_H$ is an unspecified bare mass of the auxiliary $H$, and we use the notations

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad \tilde{H} = i\tau_2 H^* \quad \psi^q_i = \begin{pmatrix} u^q_i \\ d^q_i \end{pmatrix}, \quad \psi^d_i = \begin{pmatrix} u^d_i \\ d^d_i \end{pmatrix},$$

where $u^q_1 = u, \quad u^d_1 = \nu_e, \quad u^q_2 = c$, etc. Another possibility is to introduce two auxiliary scalar isodoublets $H^{(U)}, \quad H^{(D)}$, with (as yet) arbitrary bare masses $M_H^{(U)}, \quad M_H^{(D)}$, and express (18) in the two-Higgs ‘Yukawa’ form

$$L_{Y}^{(E)} \approx -M_H^{(U)} \sqrt{2}g \frac{3}{M_B} \sum_{i,j=1} \left\{ \left[ (\bar{\psi}^q_{iL} \tilde{H}^{(U)}^R + \bar{\psi}^d_{iL} \tilde{H}^{(U)}^R) u^q_{jR} + \bar{\psi}^d_{iL} \tilde{H}^{(U)}^R u^d_{jR} + \text{h.c.} \right] 
- M_H^{(U)} \left( H^{(U)} \right)^\dagger H^{(U)} \right\}$$

$-M_H^{(D)} \sqrt{2}g \frac{3}{M_B} \sum_{i,j=1} \left\{ \left[ (\bar{\psi}^q_{iL} \tilde{H}^{(D)}^R + \bar{\psi}^d_{iL} \tilde{H}^{(D)}^R) d^q_{jR} + \bar{\psi}^d_{iL} \tilde{H}^{(D)}^R d^d_{jR} + \text{h.c.} \right] 
- M_H^{(D)} \left( H^{(D)} \right)^\dagger H^{(D)} \right\}.$

(20)
The cutoff superscript $E \sim \Lambda_{\text{FGT}}$ at the “bare” parameters and fields in (19) and (20) is suppressed for simplicity of notation. Yukawa terms there involve nondynamical scalar fields and are formally equivalent to (18). Equations of motion show that the (yet) nondynamical scalars $H, H^{(U)}, H^{(D)}$ are proportional to condensates involving fermions and antifermions – i.e., they are composite. When further decreasing the energy cutoff $E$ in the sense of the renormalization group, the composite scalars in (19) and (20) obtain kinetic energy terms and vacuum expectation values (VEV’s) through quantum effects if the FGT gauge coupling $g$ is strong enough – i.e., they become dynamical in an effective SM (or: two-Higgs-doublet SM) framework and they induce dynamically electroweak symmetry breaking (DEWSB). The neutral physical components of these composite Higgs doublets are scalar condensates of fermion pairs $H^0 \sim \bar{f}f$. The low energy effective theory is the minimal SM (MSM) in the case (19) and the SM with two Higgs doublets – type II [2HDM(II)] in the case (20). Hence, although (19) and (20) are formally equivalent to four-fermion interactions (18), they lead to two physically different low energy theories. The condensation scenario with the smaller vacuum energy density would physically materialize. We emphasize that the central ingredient distinguishing the described scheme from most of the other scenarios of DEWSB is the flavor democracy in the Yukawa sector near the transition energies, as expressed in (19) and (20).

We note that (19) implies that the MSM, if it is to be replaced by an FGT at high energies, should show up a trend of the Yukawa coupling matrix (or equivalently: of the mass matrix) in a flavor basis toward a complete flavor democracy for all fermions, with a common overall factor, as the cutoff energy is increased within the effective MSM toward a transition energy $E_0(\sim \Lambda_{\text{FGT}})$

$$M^{(U)} \text{ and } M^{(D)} \rightarrow \frac{1}{3} m_t^0 \left( \begin{array}{cc} \mathcal{N}_{FD}^q & 0 \\ 0 & \mathcal{N}_{FD}^l \end{array} \right) \text{ as } E \uparrow E_0 ,$$

where $m_t^0 = m_t(\mu = E_0)$ and $N_{FD}$ is the $3 \times 3$ flavor–democratic matrix

$$N_{FD}^f = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} ,$$

with the superscript $f = q$ for the quark sector and $f = l$ for the leptonic sector. On the other hand, if the SM with two Higgses (type II) is to experience such a transition, then (20) implies separate trends toward FD for the up–type and down–type fermions

$$M^{(U)} \left( M^{(D)} \right) \rightarrow \frac{1}{3} m_t^0 \left( m_b^0 \right) \left[ \begin{array}{cc} \mathcal{N}_{FD}^q & 0 \\ 0 & \mathcal{N}_{FD}^l \end{array} \right] \text{ as } E \uparrow E_0 ,$$

where $m_t^0$ and $m_b^0$ can in general be different. Note that $N_{FD}$, when written in the
diagonal form in the mass basis, has the form

\[
N_{FD}^{\text{mass basis}} = 3 \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\] (24)

Hence, FD (and FGT) implies in the mass basis as \(E\) increases to \(E_0 \sim \Lambda_{\text{FGT}}\):

\[
\frac{m_u}{m_t}, \frac{m_c}{m_t}, \frac{m_{\nu_e}}{m_{\nu_e}}, \frac{m_{\nu_{\mu}}}{m_{\nu_{\mu}}} \to 0;
\]

\[
\frac{m_d}{m_b}, \frac{m_s}{m_b}, \frac{m_e}{m_{\tau}}, \frac{m_{\mu}}{m_{\tau}} \to 0;
\]

\[
\frac{m_{\nu_{\tau}}}{m_{\tau}}, \frac{m_{\nu_{\mu}}}{m_{\tau}} \to 1,
\] (25)

and in the case of the minimal SM \textbf{in addition}

\[
\frac{m_b}{m_t}, \frac{m_{\tau}}{m_{\nu_{\tau}}} \to 1.
\] (26)

In our previous papers\(^{14}\) we showed, by considering the quark sector, that the minimal SM does not have the required trend toward FD, but that SM with two Higgs doublets (type II) does. We also checked that these conclusions remain true when we include the leptonic sector. When including also leptons (Ref. \(^{14}\), first entry), we can neglect for simplicity masses of the first two families of fermions, i.e., only \((t, b)\) and \((\nu_{\tau}, \tau)\) are dealt with (here \(\nu_{\tau}\) is the Dirac tau–neutrino), and then investigate evolution of their Yukawa coupling parameters (or: their masses) with energy. In the case of the effective 2HDM(II) with only the third fermion family, the FD conditions read as (25) (last line).

The one–loop renormalization group equations (RGE’s) for the Yukawa coupling parameters \(g_t, g_b, g_{\nu_{\tau}}, g_{\tau}\) of the third family fermions in any fixed flavor basis for various Standard Models with two Higgs doublets are available, for example, in Ref.\(^{18}\). The running masses (at evolution, or cutoff, energies \(E\)), are proportional to these parameters and to the (running) VEV’s of the two Higgs doublets:

\[
\begin{bmatrix}
  m_t(E) \\
  m_{\nu_{\tau}}(E)
\end{bmatrix} = \frac{v_u(E)}{\sqrt{2}} \begin{bmatrix}
  g_t(E) \\
  g_{\nu_{\tau}}(E)
\end{bmatrix},
\]

\[
\begin{bmatrix}
m_b(E) \\
m_{\tau}(E)
\end{bmatrix} = \frac{v_d(E)}{\sqrt{2}} \begin{bmatrix}
g_b(E) \\
g_{\tau}(E)
\end{bmatrix},
\] (27)

where

\[
\langle H^{(U)}(E) \rangle_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v_u(E) \end{bmatrix}, \quad \langle H^{(D)(E)} \rangle_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v_d(E) \end{bmatrix}
\]

and

\[
v_u^2(E) + v_d^2(E) = v^2(E) \approx 246^2 \text{ GeV}^2 \text{ for } E \approx E_{\text{ew}}.
\] (28)

We recall that the transition energy \(E_0\), appearing in FD conditions (25) and (26), is the energy above which SM starts being replaced by an FGT and the composite
scalors start “de-condensing.” In Ref.\textsuperscript{14}, we argued that this $E_0$ lies near the pole of the running fermion masses ($E_0 \approx \Lambda_{\text{pole}}$). We then simply approximate: $E_0 = \Lambda_{\text{FGT}} = \Lambda_{\text{pole}}$. Hence, the high energy boundary conditions (25) are then

$$\frac{g_{\nu_T}}{g_t} = 1, \frac{g_{\tau}}{g_b} = 1 \quad \text{at} \quad E \approx \Lambda_{\text{pole}}.$$  \hspace{1cm} (29)

These conditions are taken into account in numerical calculations, together with the low energy boundary conditions

$$m_{\tau} = 1.78 \text{ GeV}, \quad m_b(\mu = 1 \text{ GeV}) = 5.3 \text{ GeV},$$

$$m_t(\mu = m_t) \approx 167 \text{ GeV},$$  \hspace{1cm} (30)

where $m_{\tau}$ and $m_b$ are based on the available data of the measured masses\textsuperscript{19,20}. The above value of mass $m_t(\mu) \approx m_t^{\text{phys}}.(1 + 4\alpha_3(m_t)/(3\pi))^{-1} \approx m_t^{\text{phys}}./1.047$ is based on the experimental value of $m_t^{\text{phys}}. \approx 175 \text{ GeV}$ measured at Tevatron\textsuperscript{5}. For chosen values of VEV’s ratio $v_U/v_D$, we found the masses of Dirac tau–neutrino $m_{\nu_{\tau}},$ which satisfy the above boundary conditions (29,30), by using numerical integration of RGE’s from $\mu = 1 \text{ GeV}$ to $\Lambda_{\text{pole}}$. The calculated Dirac neutrino masses are too large to be compatible with results of the available experimental predictions\textsuperscript{22}. Therefore, we invoke the usual “see–saw mechanism”\textsuperscript{23} of the mixing of the Dirac neutrino masses and the much larger right–handed Majorana neutrino masses $M_R$, in order to obtain a small physical neutrino mass

$$m_{\nu_{\tau}}^{\text{phys.}} \approx \frac{m_{\nu_{\tau}}^{\text{Dirac}}}{4 M_R}.$$  \hspace{1cm} (31)

Majorana mass term breaks the lepton number conservation. Therefore, Majorana masses $M_R$ are expected to be of the order of some new (unification) scale $\Lambda \gg E_{\text{ew}}$. We assume: $M_R \approx \Lambda$. Within our context, the simplest choice of this new unification scale would be the energy $\Lambda_{\text{FGT}} = \Lambda_{\text{pole}}$ where SM is replaced by FGT.

$$m_{\nu_{\tau}}^{\text{phys.}} \approx \frac{m_{\nu_{\tau}}^{\text{Dirac}}}{4 \Lambda_{\text{FGT}}}.$$  \hspace{1cm} (32)

The physical tau–neutrino masses $m_{\nu_{\tau}}^{\text{phys.}}$ predicted in this way are very small for the most cases of chosen values of $v_U/v_D$ and $m_t^{\text{phys.}}$, i.e., in most cases they are acceptable since being below the experimentally predicted upper bounds\textsuperscript{22}.

The see–saw scenario leading to our predictions of $m_{\nu_{\tau}}^{\text{phys.}}$ implicitly assumes that: (a) FGT contains in addition Majorana neutrinos, and its energy range of validity also provides the scale for the heavy Majorana masses [i.e., $M_R \sim \Lambda_{\text{FGT}}.$] (b) At low (SM) energies, Majorana neutrinos remain decoupled from (or very weakly coupled to) the Dirac neutrinos, which is a very plausible assumption in view of assumption (a). In general, it could be assumed $M_R \sim \Lambda_{\text{new–scale}} \geq \Lambda_{\text{FGT}}$, leading thus to even smaller $m_{\nu_{\tau}}^{\text{phys.}}$ than those in (32).
When increasing $m_t^{\text{phys}}$ at a fixed $v_U/v_D$, $m_{\nu_\tau}^{\text{Dirac}}$ increases and $\Lambda_{\text{FGT}}$ decreases, and hence $m_{\nu_\tau}^{\text{phys}}$ increases. This provides us, at a given ratio $v_U/v_D$, with: (a) upper bounds on $m_t^{\text{phys}}$ for (various) specific upper bounds imposed on $m_{\nu_\tau}^{\text{phys}}$ (e.g., $\leq 31$ MeV$^{22}$, or $\leq 1$ MeV, or $\leq 17$ KeV$^{24}$); (b) lower bounds on $m_t^{\text{phys}}$ for (various) specific upper bounds imposed on $\Lambda_{\text{FGT}}$ (e.g., $\leq \Lambda_{\text{Planck}}$, or $\leq 10^{10}$ GeV, or $\leq 10^5$ GeV). Even with the largest possible upper bounds on $m_{\nu_\tau}^{\text{phys}} \leq 31$ MeV and $\Lambda_{\text{FGT}} \leq \Lambda_{\text{Planck}}$, we can still get rather narrow bands on the values of $m_t^{\text{phys}}$ at any given $v_U/v_D$. E.g., if $v_U/v_D = 1$, then $155$ GeV $\leq m_t^{\text{phys}} \leq 225$ GeV. Inversely, if $m_t^{\text{phys}} = 175$ GeV $[m_t(m_t) = 167$ GeV$, m_{\nu_\tau}^{\text{phys}} \leq 31$ MeV and $\Lambda_{\text{FGT}} \leq \Lambda_{\text{Planck}}$, then we obtain rather stringent bounds on the VEV ratio: $0.64 \leq v_U/v_D \leq 1.35$.

To conclude this Section, we stress that we can estimate the masses of top and tau–neutrino within SM with two Higgs doublets, assuming solely that the complete flavor democracy should set in at energies where SM starts breaking down. The gauge theories (FGT’s) which presumably replace SM at such energies remain to be further investigated. For related detailed information, see Ref.$^{14}$.

3. Discussions and Conclusion

We discussed on the one hand flavor–democratic (FD) mass matrices at low energies, and on the other hand conditions under which mass matrices show a trend to flavor–democratic forms at high energies (in a flavor basis) – a behavior possibly related to flavor gauge theories (FGT’s) at high energies. However, we found that the model based on our simple perturbation of a universal FD Yukawa interaction at low energies has been invalidated, because of the discovery of a top quark much heavier than 100 GeV. On the contrary, at high energies, assuming solely that the complete flavor democracy should set in at energies where an effective perturbative two-Higgs-doublet SM (type II) starts breaking down, we can estimate the masses of top and tau–neutrino, which are compatible with the present experimental results. Therefore, the gauge theories (FGT’s) which presumably replace SM at such energies remain to be further investigated.

In our forthcoming work$^{25}$, we would like to investigate further the simple FD mass matrices ansatz which had been applied earlier$^{12}$ at low energies and had given experimentally unacceptable $m_t$. We would like to apply this ansatz at a high energy scale $E \sim \Lambda_{\text{pole}}$, employing RGE evolution within a two-Higgs-doublet SM model (type II). Furthermore, the compositeness nature of the scalars in this framework should be further investigated, particularly in view of the fact that, for cases when VEV ratio is $v_U/v_D \sim 1$, the usual RGE compositeness conditions at $\Lambda_{\text{pole}}$ suggest
that only $H^{(U)}$ can be fully composite, but not $H^{(D)}$ (cf. Ref. 26).

4. Acknowledgements

CSK would like to thank Prof. Y. Koide for his kind invitation to the Workshop of MMQL97. The work of CSK was supported in part by the CTP, Seoul National University, in part by Yonsei University Faculty Research Fund of 1997, in part by the BSRI Program, Ministry of Education, Project No. BSRI-97-2425, and in part by the KOSEF-DFG large collaboration project, Project No. 96-0702-01-01-2. The work of GC was supported in part by the German Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Project No. 057DO93P(7).

5. References

1. H. Harari, H. Haut and J. Weyers, Phys. Lett. B78 (1978) 459.
2. Y. Koide, Phys. Rev. D39 (1989) 3500; H. Fritzsch and J. Plankl, Phys. Lett B237 (1990) 451.
3. Y. Nambu, Proceedings of XI Warsaw symmetry on High Energy Physics (1988); P. Kaus and S. Meshkov, Phys. Rev. D42 (1990) 1863; F. Cuypers and C.S. Kim, Phys. Lett B254 (1991) 462; H. Fusaoka and Y. Koide, Mod. Phys. Lett. A10 (1995) 289.
4. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
5. CDF Collaboration: F. Abe et al., Phys. Rev. Lett. 74 (1995) 2626, Phys. Rev. Lett. 77 (1996) 438; D0 Collaboration: S. Adachi et al., Phys. Rev. Lett. 74 (1995) 2632.
6. J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77.
7. Particle Data Book: Phys. Rev. D54 (1996) 1; A. Ali and D. London, Nucl. Phys. Proc. Suppl. 54A (1997) 297.
8. C.S. Kim, J.L. Rosner and C.-P. Yuan, Phys. Rev. D42 (1990) 96.
9. G. Buchalla, A.J. Buras and M.K. Harlander, Nucl. Phys. B337 (1990) 313.
10. E731 Collaboration: E.J. Ramberg et al., Phys. Rev. Lett. 70 (1993) 2529.
11. NA31 Collaboration: G.D. Barr et al., Phys. Lett. B317 (1993) 233.
12. F. Cuypers and C.S. Kim, in Ref. 3.
13. U. Amaldi, W. de Boer and H. Furstennau, Phys. Lett. B260 (1991) 447; P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817.
14. G. Cvetič and C.S. Kim, Mod. Phys. Lett. A9 (1994) 289; Int. J. Mod. Phys. A9 (1994) 1495; Nucl. Phys. B407 (1993) 290; Phys. Rev. D51 (1995) 201.
15. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B120 (1977) 316.
16. T. Kugo, Prog. Theor. Phys. 55 (1976), 2032; K. Kikkawa, Prog. Theor.
Phys. 56 (1976) 947; T. Eguchi, Phys. Rev. D14 (1976) 2755; see also Ref. 14 (third entry), App. A.
17. V.G. Vaks and A.I. Larkin, Zh. Exp. Teor. Fiz. 40, No. 1 (1961) [Sov. Phys. JETP 13 (1961) 192]; Y. Nambu and G. Jona–Lasinio, Phys. Rev 122 (1961) 345; 124 (1961) 246.
18. C.T. Hill, C.N. Leung and S. Rao, Nucl. Phys. B262 (1985) 517; G. Cvetič, S.S. Hwang and C.S. Kim, hep-ph/9706323 (June 1997).
19. BES Collaboration: Jing-Zhi Bai et al., Phys. Rev. Lett. 69 (1992) 3021.
20. J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77; S. Narison, Phys. Lett. B197 (1987) 405.
21. ALEPH Collaboration: D. Decamp et al., Z. Phys. C53 (1992) 1; L3 Collaboration: B. Adeva et al., Z. Phys. C51 (1991) 179.
22. ARGUS Collaboration: H. Albrecht et al., Phys. Lett B202 (1988) 149; B292 (1992) 221.
23. M. Gell–Mann, P. Ramond and R. Slansky, in Supergravity, edited by P. Van Nieuwenhuizen and D.Z. Freedman (North–Holland, Amsterdam, 1979); T. Yanagida, Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe (KEK, Japan, 1979).
24. A. Hime, R.J.N. Phillips, G.G. Ross and S. Sankar, Phys. Lett. B260 (1991) 381.
25. G. Cvetič, S.S. Hwang and C.S. Kim, work in progress (1997).
26. G. Cvetič, “Top quark condensation – a review,” (Subsec. VI.A.3), hep-ph/9702381, to appear in Rev. Mod. Phys.