Experimental demonstration of a high speed quantum random number generation scheme based on measuring phase noise of a single mode laser

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We present a high speed random number generation scheme based on measuring the quantum phase noise of a single mode diode laser operating at a low intensity level near the lasing threshold. A delayed self-heterodyning system has been developed to measure the random phase fluctuation. By actively stabilizing the phase of the fiber interferometer, a random number generation rate of 500Mbit/s has been demonstrated and the generated random numbers have passed all the DIEHARD tests.

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I. INTRODUCTION

Random numbers have been widely used in many branches of science and technology, such as statistical analysis, computer simulation [1], cryptography [2], etc. One recent example is quantum key distribution (QKD) [3], where truly random numbers are required for both quantum state preparation and quantum state detection. Most recently, truly random numbers have also been employed in testing fundamental principles of physics [4, 5].

In practice, it is not easy to obtain high quality random numbers with proven randomness [6]. In a cryptographic system, the application of a weak random number generator (RNG) could be catastrophic, as evidenced by Goldberg and Wagner’s attack on the Netscape SSL implementation [7].

A pseudorandom generator generates a long train of “random” bits from a short random seed by employing deterministic algorithms. The generated long bit string could meet a number of statistical measures, which allows it to pass all existing randomness tests. However, the entropy of the long bit string is ultimately determined by the length of the random seed. In principle, random numbers generated by deterministic algorithms are not truly random. John von Neumann once famously said “Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.”[8].

A physical RNG, on the other hand, generates random numbers from unpredictable physical processes, such as thermal noise [9], radioactive decay [10], air turbulence [11], etc. For example, the Intel 80802 Firmware Hub chip included a hardware random number generator [12]. It is important to distinguish two different types of physical random number generation processes, based on the source of randomness: the chaotic behavior of classical deterministic systems, which we shall call type-one randomness henceforth; or the truly probabilistic nature of fundamental quantum processes [13], which we call type-two randomness. In this paper, we use the term “classical noise” to represent the unpredictability of a deterministic chaotic system and the term “quantum noise” to describe the fundamental uncertainty in a quantum process.

For example, a RNG based on atmospheric conditions can be treated as a type-one RNG, since the randomness mainly originates from the absence of enough information about the weather system. In other words, the observed fluctuation can be treated as classical noise. Obviously, the more knowledge we have, the less random the system appears. The unpredictable weather change appeared to a layman might well be predictable to an expert equipped with supercomputers. This raises a serious question: how much can we trust a RNG? The weakness of a RNG could be fatal if the generated random numbers have been used as secure keys in a cryptographic system where the security relies on the true randomness of the key.

We remark that ultrahigh speed RNGs based on chaotic semiconductor lasers have been proposed and random number generation rates above Gbit/s have been demonstrated [14]. In [14], random numbers are generated from the amplitude noise of chaotic semiconductor lasers. However, as we discussed above, it is still arguable whether random numbers generated from a deterministic chaotic system are suitable for cryptographic applications.

Fortunately, a type-two RNG, or a quantum RNG (QRNG), can provide us with true random numbers with proven randomness. One of the simplest QRNG is constructed by a single photon source, a symmetric beam splitter and two single photon detectors [13]. Each photon has the same probability to be either transmitted or reflected by the beam splitter and thus “which detector clicks” is completely unpredictable. This unpredictability is not due to the absence of information about the quantum state of the photon or the measurement device, it is due to the probabilistic nature of a projection measurement. The randomness of the result is guaranteed by the fundamental laws of quantum mechanics.

One natural source of quantum randomness is the thermal noise in an electrical amplifier [9]. However, a practical high gain, broadband electrical amplifier also exhibits classical noises which typically dominate over the quantum noise. So far, the reported random number genera-
tion rate based on this scheme is only a few Mbit/s [9].

To date, most QRNGs are based on performing single photon detections [13, 15] and the highest random number generation rate achieved is 16 Mbit/s [16]. This random number generation rate is too low for certain applications, such as high speed QKD systems operated at GHz clock rates [17, 18]. Indeed, current high-speed QKD set-ups often use either a) deterministic random number generation algorithms or b) repeatedly a fixed pattern and, therefore, are not really unconditionally secure. Although there is still some room for improvement, the ultimate speed of these devices is limited by the performance of the single photon detector (SPD), such as its dead time, efficiency, afterpulsing probability, etc. For example, a typical silicon APD-SPD has a dead time of tens of ns [19], which suggests that the ultimate random number generation rate based on this type of SPD is in the order of tens of Mbit/s.

Another promising quantum random number generation scheme is based on measuring the random field fluctuation of vacuum with a homodyne detector [20]. However, the fabrication of a high speed, shot noise limited homodyne detector is also a challenging task and a high speed QRNG (> tens of Mbit/s) based on this scheme has not been demonstrated.

In this paper, we present a QRNG scheme based on measuring the quantum phase noise of a single mode semiconductor laser operating at a low intensity level near the lasing threshold [21]. This ensures that the main contribution to the phase noise is from spontaneous emission (SE) [22], rather than from chaotic evolution of the macroscopic field [14]. One significant advantage of this scheme is the potential high random number generation rate. In this paper, we achieve a 500Mbit/s random number generation rate with commercial off-the-shelf components.

We remark that any practical devices present both quantum noise and classical noise, and true random numbers can only originate from the former one. How can we distinguish these two different noises in practice? We will address this question briefly in Section V.

II. THEORETICAL MODEL

A. Quantum phase noise and linewidth of semiconductor lasers

The linewidth of a single mode semiconductor laser can be viewed as due to the random phase fluctuation of the optical field [22]. Experimental studies have shown that the linewidth of a single mode injection diode laser varies linearly with reciprocal laser output power [23, 24]. To explain these experimental discoveries, Henry developed a theoretical model which attributes the fundamental laser linewidth to the phase fluctuations arise from spontaneous emission [22]. Furthermore, it has been shown that the phase noise of an InGaAsP DFB laser can be well described by this model [25].

In [22], the dependence of the linewidth of a single mode semiconductor laser on its output laser power is described by

$$\Delta f = \frac{\nu_0^2 h \gamma \eta_p \alpha_m (1 + \alpha^2)}{8 \pi F_0}$$

(1)

Here, $\nu_0$ is the group velocity, $h\nu$ is the energy of photon, $g$ is the gain of laser medium, $\eta_p$ is the spontaneous emission factor, $F_0$ is the output power per facet. $\alpha_m$ is the facet loss which is defined as $\alpha_m \equiv g - \alpha_L$, where $\alpha_L$ is the waveguide loss of the laser. $\alpha \equiv \frac{\Delta n}{\Delta n''}$, where $\Delta n'$ and $\Delta n''$ are the deviations of the real part and imaginary part of the refractive index from their steady-state values.

An intuitive physical picture is as follows [22]: each spontaneous emitted photon has a random phase, which in turn contributes a random phase fluctuation to the total electric field and results in a linewidth broadening. This is represented by the term “1” in the parentheses on the right hand side of (1). On the other hand, the same spontaneous emitted photon also alters the amplitude of the laser field, which results in a change of the carrier density. The change of carrier density further triggers a change of $n''$, which is the imaginary part of the refractive index of the laser medium. Finally, the change in $n''$ has an associated change of the real part of the refractive index $n'$, which contributes to an additional phase shift of the laser field and linewidth broadening. This additional linewidth broadening is described by the term $\alpha^2$ in the parentheses on the right hand side of (1).

Note that the spontaneous emission is a quantum mechanical effect and the corresponding phase noise can be treated as quantum noise. However, a practical laser source also presents additional classical noises, such as occupation fluctuation [26] and 1/f noise [27]. Fortunately, these classical noises are laser power independent [26, 27]. By operating a semiconductor laser under certain power level, the noise properties are mainly determined by quantum effects [26]. In the following Sections we will show how to harness this quantum noise to generate true random numbers.

B. Phase measurement with a delayed self-heterodyning scheme

The optical phase of a laser field can be measured by performing an interferometric experiment. A delayed self-heterodyning scheme has been employed to measure the linewidth of a semiconductor laser [24]. Fig.1 shows its basic structure.

The electric field of a laser beam can be described by

$$E(t) = E_0 \exp[i(\omega t + \theta(t))]$$

(2)

where $\theta(t)$ represents the random phase fluctuation of the laser source.
The interference signal detected by the photo detector (PD) can be described by

\[ S(t) \propto |E_0 \exp[i(\omega_0 t + \theta(t + T_d))] + E_0 \exp[i(\omega_0 t + \omega_0 T_d + \theta(t))]|^2 \]  

Here \( T_d \) is the time delay difference between the two arms of the Mach-Zehnder interferometer (MZI), which can be determined by \( T_d = n\Delta L/C \). \( \Delta L \) is the path length imbalance, \( n \) is the refractive index of fiber and \( C \) is the speed of light in vacuum.

After removing a DC background, equation (3) can be simplified as

\[ S(t) \propto \cos[\omega_0 T_d + \Delta \theta(t, T_d)] \]  

where \( \Delta \theta(t, T_d) \equiv \theta(t) - \theta(t + T_d) \).

In (4), the term \( \Delta \theta(t, T_d) \) represents the quantum phase noise of the laser source, while the term \( \omega_0 T_d \) represents the phase delay introduced by the path length difference. In practice, for an interferometer without phase stabilization, the term \( \omega_0 T_d \) will not be a constant due to ambient temperature fluctuations, for example. This in turn will contribute additional classical phase noise. Intuitively, if the time delay difference \( T_d \) is much larger than the coherence time of the laser, then \( \Delta \theta(t, T_d) \) will present an uniform distribution in the range of \([−\pi, \pi]\). Under this condition, the total phase \( \omega_0 T_d + \Delta \theta(t, T_d) \) also uniformly distributes in the range of \([−\pi, \pi]\) regardless of the actual value of \( \omega_0 T_d \). Thus we can generate binary random numbers by simply measuring \( S(t) \) using a fast detector, sampling the output at the fixed intervals \( (T_S) \) to generate a series of \( S(t_i) \), and taking the sign of the individual \( S(t_i) \) in the series.

A more rigorous discussion is as follows: the net contribution of a large number of SE photons can be characterized by a random walk process, and the phase fluctuation \( \Delta \theta(t, T_d) \) can be treated as Gaussian white noise with a variance of [28]:

\[ \langle (\Delta \theta(t, T_d))^2 \rangle = \frac{2T_d}{\tau_c} \]  

Here \( \tau_c \) is the coherence time of the laser, which is related to its linewidth \( \Delta f \) as \( \tau_c \sim \frac{1}{\pi \Delta f} \) [28].

Equation (5) shows that as long as \( T_d \gg \tau_c \), the resulting Gaussian distribution can be treated as a uniform distribution in the range of \([−\pi, \pi]\) in practice.

It is useful to define two other time constants here. The response time of the photo-detection system \( T_R \) is defined as the reciprocal of the detection system’s bandwidth. The sampling period \( T_S \) is defined as the reciprocal of the sampling rate.

The necessary condition for random number generation without phase stabilization is summarized as

\[ T_d \gg \tau_c; T_S - T_d \gg \tau_c + T_R \]  

From (6), the maximum sampling rate (or the random number generation rate) is determined by the coherence time of the laser. The coherence time of the laser used in our experiment can be set to a few ns by tuning its driving current. To generate high quality random numbers, the sample period \( T_S \) should be larger than 10ns, which corresponds to a maximum sample rate of 100MHz.

One way to go beyond the limitation imposed by the coherence time of the laser source is to employ phase stabilization technique. This can be seen from (4). By stabilizing the phase of the MZI, the term \( \omega_0 T_d \) in the cosine function can be treated as a constant. Furthermore, if we can set \( \omega_0 T_d = 2m\pi + \pi/2 \) (where \( m \) is an integral), equation (4) can be further simplified as

\[ S(t) \propto \sin[\Delta \theta(t, T_d)] \]  

Note in the derivation of (7), we have ignored the error of the phase feedback control system, which will contribute to additional classical noise. It will be interesting to quantify how the performance of the proposed RNG depends on the phase control error.

From equation (7), the discrete time series samples of \( S(t) \), which is labeled as \( S(t_i) \), has a symmetric distribution around zero. Again, we can generate binary random numbers by simply taking the sign of \( S(t_i) \). Here, we don’t need to assume that \( \Delta \theta(t, T_d) \) is uniformly distributed in the range of \([−\pi, \pi]\). In principle, the sampling rate is mainly limited by the bandwidth of the detection system but not the coherence time (or linewidth) of the laser.

To minimize the correlation between adjacent samples, the time delay imbalance \( T_d \) should be smaller than the sampling period \( T_S \). This is illustrated in Fig.2: suppose that the first sampling result \( S(t_1) \) is determined by phase noise from the SE photons emitted in the time period of \( (t_1 - T_d, t_1) \), while the second sampling result \( S(t_2) \) is determined by phase noise from the SE photons emitted in the time period of \( (t_2 - T_d, t_2) \). By choosing \( T_S = t_2 - t_1 > T_d \), \( S(t_1) \) and \( S(t_2) \) are contributed by SE photons emitted at different time windows, thus there is no correlation between them.

In practice there are other factors to be considered on determining the optimal coherence time. On one hand, the coherence time \( \tau_c \) should be larger than the response time \( T_R \) of the detection system. Otherwise, the interference signal will be averaged out due to its random fluctuation within the time period for acquiring one sample.
On the other hand, from (5), $\tau_c$ cannot be too large, otherwise, the variance of phase noise $\Delta \theta(t, T_d)$ could be too small to be resolved.

The necessary condition for random number generation with phase stabilization is summarized as

$$T_S - T_d > T_R$$

III. EXPERIMENTAL SETUP

A 1.5 $\mu$m single mode cw DFB diode laser (ILX Lightwave) is employed as the source of quantum phase noise. From (1), the linewidth of the laser diode varies linearly with reciprocal laser output power which can be conveniently controlled by adjusting the driving current. By operating the laser diode at a power level where its linewidth is much larger than the linewidth at the high power limit, the majority of the phase noise can be attributed to quantum noise.

A delayed self-heterodyning system has been developed to measure the random phase fluctuation of the single mode DFB diode laser. The experimental setup is shown in Fig.3. Two symmetric fiber couplers are used to construct a fiber MZI with a length imbalance of $\Delta L$. The interference signals from the two output ports of the second fiber coupler are fed into two detection channels: the first channel, including a 5GHz bandwidth InGaAs photo-detector ($PD_1$ in Fig.3) and a 3GHz bandwidth real time oscilloscope, is used to generate random numbers; the second channel ($PD_2$ in Fig.3), which has a bandwidth of 1MHz, is used to monitor the relatively slow phase drift of the MZI due to ambient fluctuations. Due to its small bandwidth, $PD_2$ can only sense the slow phase drift of MZI, while the high frequency phase fluctuation due to SE will be averaged out. The output from $PD_2$ is sampled by a computer together with a DAQ card (NI PCI6115), which in turn provides a feedback control signal to a phase modulator inside the MZI. Two polarization controllers are used in this setup: $PC_1$ is used to make sure the polarization state of the input light aligned with the axis of the phase modulator, while $PC_2$ is employed to achieve high interference visibility.

IV. EXPERIMENTAL RESULTS

As we have discussed in Section II, by stabilizing the phase of MZI to satisfy the condition of $\omega_0 T_d = 2m\pi + \pi/2$ (where $m$ is an integral), the sampling rate is not limited by the coherence time of the laser, so we can achieve a very high random number generation rate.

During this experiment, the driving current of the DFB laser has been set to $I = 12mA$. Note, if the driving current is too small, the laser power will be too weak and the detected signal will be dominated by the noise of the detection system rather than the quantum phase noise of the laser; on the other hand, if the driving current is too large, the quantum phase fluctuation could be too small to be resolved. By using the technique described in [24], the coherence time $\tau_c$ of the laser has been determined to be about 10ns (or a linewidth of 30MHz) under the condition of $I = 12mA$ and 320ns (or a linewidth of 1MHz) at a high driving current ($I = 50mA$). From (5), for a fixed $T_d$, the phase noise variance is proportional to $1/\tau_c$. Thus the phase noise variance at $I = 12mA$ is about 32 times larger than that at $I = 50mA$. Since we attribute the phase noise variance at the high power limit (or the high driving current) to classical noise and assume that it is laser power independent, we conclude that under our experimental conditions, the phase noise of the laser is dominated by quantum noise.

For an ideal RNG, there is no correlation between its outputs at different times. From the correlation theorem in Fourier transformation, the spectrum of an ideal RNG is expected to be flat [29].

We have measured the noise spectrum of the setup shown in Fig.3. In this experiment, a spectrum analyzer (HP8564E) has been employed to replace the oscilloscope. Measurements have been performed with two different imbalanced MZIs, $T_{d1} = 650 \pm 100$ps and $T_{d2} = 250 \pm 100$ps. The measurement results are shown in Fig.4. The electrical noise of the detection system has been measured by blocking the laser output.

There are several remarkable features in Fig.4. First of all, the electrical noise, which looks quite random in time domain, presents a few dominant spectral lines.
These spikes could be due to the environmental EM noises picked up by our detection system. This highlights the challenge in random number generation from the thermal noise of a broadband electrical amplifier: the residual classical noises could dominate over the quantum noise. On the other hand, the spectra of phase noise are broadband as expected. Secondly, the noise spectrum measured with a $T_d$ of $650 \pm 100$ ps presents a clear low-pass character, while the noise spectrum measured with a smaller $T_d$ ($250 \pm 100$ ps) shows a flatter frequency response. As we have shown in Fig.2, the measured phase noise is contributed by the SE photons emitted in a time interval of $T_d$. This introduces an equivalent integration time in the order of $T_d$ and thus reduces the bandwidth of the whole system to $1/T_d$. Third, at low frequency region, the noise power measured at $T_d = 250 \pm 100$ ps is about 7dB lower than that at $T_d = 650 \pm 100$ ps. This is consistent with (5), which suggests a linear relation between phase variance and the time delay $T_d$.

To generate random numbers, $T_d$ was set to be $650 \pm 100$ ps and the output of PD1 was sampled by the 3GHz oscilloscope at a sampling rate of 1G samples per second (corresponding to $T_S = 1$ ns). The sampling results were saved onto the hard drive of the oscilloscope in frames: the oscilloscope continuously samples 1M data, transfers the data into the hard drive and then starts to sample another frame of data. To generate binary random numbers from the sampling results $S(t_i)$, we simply compare them with the mean value $S_0$: the $i$th bit is assigned as either “1” if $S(t_i) > S_0$ or “0” if $S(t_i) < S_0$.

Two 100Mbits binary random number trains have been generated, which are named as Bin1 and Bin2. By performing a bitwise XOR operation between Bin1 and Bin2, the randomness can be further improved. This XOR operation has been commonly used on improving randomness of a RNG [6, 30]. The random number train generated through this XOR process is named as Bin3. Since Bin1 and Bin2 have been generated at 1Gbit/s, the equivalent generation rate of Bin3 is 500Mbit/s.

To evaluate the qualities of these random numbers, we first calculated the autocorrelation of each random train. The results are shown in Fig.5. From Fig.5, we can see that the residual correlation of Bin3 is significantly lower than that of Bin1.

We further test the randomness of Bin3 with the DIEHARD test suite [31]. As shown in Table 1, Bin3 passed all the tests.

![FIG. 4: Spectral power density of electrical and phase noise. The solid-line, dot-line and circle-line represent the spectral power densities of the detection system, the phase noise with a long delay ($T_d = 650 \pm 100$ ps), and the phase noise with a short delay ($T_d = 250 \pm 100$ ps), correspondingly.](image1)

![FIG. 5: Autocorrelations of the random number trains acquired at 1Gbit/s. Note for Bin3, the equivalent random number generation rate is 500Mbit/s.](image2)

| Table 1: Diehard Test Results (500Mbits/s) |
|-----------------------------------------|
| Statistical Test                      | P-Value | Result |
| BIRTHDAY SPACINGS                     | 0.856068 (KS) | Success |
| OVERLAPPING 5- PERMUTATION             | 0.45120 | Success |
| BINARY RANK TEST for 31x31 matrices   | 0.642062 | Success |
| BINARY RANK TEST for 32x32 matrices   | 0.461172 | Success |
| BINARY RANK TEST for 64x64 matrices   | 0.607734 (KS) | Success |
| BITSTREAM                              | 0.93957 | Success |
| Overlapping-Pairs-Sparse-Occupancy     | 0.2575 | Success |
| Overlapping-Quadruples-Sparse-Occupancy| 0.1860 | Success |
| DNA                                    | 0.149 | Success |
| COUNT-TH-1’s TEST                     | 0.910419 | Success |
| COUNT-TH-1’s TEST for specific bytes   | 0.751492 | Success |
| PARKING LOT                            | 0.199468 (KS) | Success |
| MINIMUM DISTANCE                       | 0.721785 (KS) | Success |
| 3D SPHERES                             | 0.405863 (KS) | Success |
| SOE-ZS                                 | 0.306944 | Success |
| OVERLAPPING SUMS                       | 0.246453 (KS) | Success |
| RUNS                                   | 0.829651 (KS) | Success |
| CRAPS                                  | 0.838686 | Success |

KS: Kolmogorov-Smirnov TEST
V. DISCUSSION AND CONCLUSION

In this paper, we have presented a high speed random number generation scheme based on measuring the quantum phase noise of a DFB laser diode. The whole system is constructed with off-the-shelf components and a random number generation rate of 500Mbit/s has been demonstrated. Currently, the random number generation rate is mainly limited by the speeds of the oscilloscope (3GHz bandwidth) and the photo detector (5GHz bandwidth). By employing a detection/sampling system with a larger bandwidth and a higher sensitivity, we believe higher random number generation rate is achievable.

Comparing with the random number generation scheme based on measuring vacuum noise [20], the scheme we proposed here is realized by interfering two relatively strong laser beams. The photo-detector does not have to be shot noise limited. So it is easier to implement.

There are also similarities and differences between our scheme and the one reported in [14]. The SE contributes to both amplitude fluctuation and phase fluctuation. However, for a laser operated under normal conditions, it is very difficult to resolve the small amplitude fluctuation due to SE. In [14], the authors operated the lasers under chaotic conditions by introducing strong external feedbacks. The observed noise is mainly due to the chaotic behavior of the lasers rather than the quantum noise from SE. In contrast, the quantum phase fluctuation can readily be measured with a conventional interferometric setup, as we have shown in this paper.

There is a lot of room to further improve the QRNG presented here. The sensitivity of the detection system can be further improved by replacing the photo detector with a balanced detector followed by an electrical subtraction circuit; the real time oscilloscope can be replaced by either a high speed comparator or a high speed analogue to digital convertor; the DFB laser used in the current system could be replaced by a combination of a broadband light source and a narrowband optical filter. In this case, the coherence time (or linewidth) is determined by the bandwidth of the filter.

We would like to end this paper with some general comments on RNGs for secure communications.

Typically, a true RNG consists of two components: a high entropy source and a randomness extractor [6]. The high entropy source could be a physical device whose output is more or less unpredictable, while the randomness extractor could be an algorithm which generates nearly perfect random numbers from the output of the high entropy source. For example, in the QRNG described in this paper, the combination of the laser source and the delayed self-heterodyning system can be treated as a high entropy source, while the bitwise XOR operation, which has been adopted to improve the randomness, can be treated as a randomness extractor. Note the randomness extractor normally generates a short, nearly perfect random number train from a long, imperfect one. Obviously, to design an appropriate randomness extractor, we need to know the entropy of the source in advance.

The entropy of a practical device originates from both quantum noise and classical noise. As we have discussed in Section I, true random numbers with proven randomness can only be generated from irreducible quantum randomness. Thus it is important to quantify with respect to the observed entropy, how much is contributed by the quantum noise. In [32], the authors proposed a scheme to quantify the “min-entropy” (or the irreducible quantum entropy) of a two dimensional quantum system by employing quantum state tomography. This is an interesting idea. However, it does not take into account the classical noise contributed by the detection system. Moreover, it is not clear how to apply this idea to continuous variables.

Another interesting topic is how to deal with the finite response time $T_R$ of a practical detection system. Normally, we assume that the resulting correlation between adjacent samples drops exponentially when we increase the sampling period $T_S$. In practice, this correlation can be neglected by using a $T_S$ much larger than $T_R$. However, this will reduce the random number generation rate. To achieve the highest random number generation rate, it might be more efficient to tolerate a finite correlation at the sampling stage and let the randomness extractor to remove the residual correlation later on.

Finally, in the special case of QKD, the randomness extractor might be integrated into the privacy amplification process. In QKD, after the quantum transmission stage, the two users need to perform error correction and privacy amplification (to correct errors and remove the eavesdropper’s information) on the raw key to generate the final secure key. Since the data size of the raw key is much less than that of the random numbers used in the QKD experiment, it might be much efficient to treat the imperfection of the QRNG as partial information leaked to the eavesdropper, which can be removed during privacy amplification.

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Notes Added: after the completion of a preliminary version of this paper, we notice that a preprint [33] has recently been posted on quant-ph.

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The testing of Bell inequalities also requires random numbers. Hidden correlations between the supposedly local random numbers can lead to a loophole in the testing of Bell inequalities. In fact, people have proposed generating the random numbers for testing Bell inequalities by observing lights from distant stars in different sectors of the sky. However, the inflation theory shows that even different sectors of the sky might previously been in causal contacts with each other and, therefore, their star lights might still have hidden correlations.

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