Phase transitions in the anisotropic XY ferromagnet with quenched nonmagnetic impurity

Olivia Mallick\textsuperscript{1} and Muktish Acharyya\textsuperscript{2,*}

Department of Physics, Presidency University,
86/1 College Street, Calcutta-700073, INDIA
\textsuperscript{1}E-mail: olivia.rs@presiuniv.ac.in
\textsuperscript{2}E-mail: muktish.physics@presiuniv.ac.in

Abstract: The equilibrium behaviours of the anisotropic XY ferromagnet, with nonmagnetic impurity, have been investigated in three dimensions by Monte Carlo simulation using Metropolis algorithm. Two different types of anisotropy, namely, the bilinear exchange type and single-site anisotropy are considered here. The thermodynamic behaviours of the components of the magnetisations ($M$), susceptibility ($\chi$) and the specific heat ($C$) have been studied systematically through extensive Monte Carlo simulations. The ferro-para phase transition has been observed to take place at a lower temperature for impure anisotropic XY ferromagnet. The pseudocritical temperature ($T^*_c$) has been found to decrease as the system gets more and more impure (impurity concentration $p$ increases). In the case of bilinear exchange type of anisotropy ($\lambda$), the pseudocritical temperature ($T^*_c$) increases linearly with $\lambda$ for any given concentration of nonmagnetic impurity ($p$). The slope of this linear function has been found to depend on the impurity concentration ($p$). The slope decreases linearly with the impurity concentration ($p$). In the case of the single site anisotropy ($D$), the pseudocritical temperature ($T^*_c$) has been found to decrease linearly with $p$ for fixed $D$. The critical temperature (for a fixed set of parameter values) has been estimated from the temperature variation of fourth order Binder cumulants ($U_L$) for different system sizes ($L$). The critical magnetisation ($M(T_c)$) and the maximum value of the susceptibility ($\chi_p$) are calculated for different system sizes ($L$). The critical exponents for the assumed scaling laws, $M(T_c) \sim L^{-\frac{\beta}{\nu}}$ and $\chi_p \sim L^{\frac{\gamma}{\nu}}$, are estimated through the finite size analysis. We have estimated, $\frac{\beta}{\nu}$, equals to $0.48 \pm 0.05$ and $0.37 \pm 0.04$ for bilinear exchange and single site anisotropy respectively. We have also estimated, $\frac{\gamma}{\nu}$ equals to $1.78 \pm 0.05$ and $1.81 \pm 0.05$ for bilinear exchange and single site anisotropy respectively.

Keywords: XY ferromagnet, Anisotropy, Monte Carlo simulation, Metropolis algorithm, Finite size analysis, Binder cumulant

* Corresponding author
I. Introduction

The planar ferromagnet (continuous SO(2) symmetric) in two dimensions, holds a distinctive significance within the realm of phase transitions. The existence of peculiar kind of ordered phase (vortex-antivortex pair) without any conventional long range ferromagnetic order has made it special [1–3]. The special kind of phase transitions has been observed in various thermodynamical systems [4] namely in two dimensional superconductors, superfluids, liquid-crystals etc..

The planar ferromagnet (classical XY ferromagnet) can give rise to usual phase transition (having long range ferromagnetic ordering) in higher dimensions ($d > 2$). The systematic investigations of the thermodynamic phase transitions are done in three dimensional XY ferromagnet [5, 6] by Monte Carlo simulations, mainly to estimate the critical temperature and the critical exponents. The XY universality class has also been specified.

How does the phase transition get affected by anisotropy? The XY ferromagnet with single site anisotropy has been investigated [7] quantum mechanically (bosonic field) and the critical temperature has been found to increase monotonically with the strength of single site anisotropy. The XY ferromagnet with bilinear exchange kind of anisotropy has been investigated recently [8] by Monte Carlo simulation. Here also, the critical temperature has been found to increase linearly with the strength of bilinear exchange anisotropy. Moreover, the phase transitions in three dimensional XY ferromagnet have been studied [8] with distributed anisotropy. The studies regarding the role of anisotropy are not limited in the thermodynamic phase transitions only. To study the vortex-loop scaling behaviour, the anisotropic (with different interplane/intraplane coupling ratios) XY ferromagnet was investigated [9] by using the technique of the renormalization group. The renormalization group technique has also been employed [10] to study the critical behaviours of coupled XY model. The Cantor spectra have been observed [11] in the one dimensional quasiperiodic anisotropic XY model.

The anisotropy may also be introduced by simply varying the type of interactions (ferromagnetic or antiferromagnetic) in different directions. The quantum critical behaviour of spin-1/2 anisotropic XY model has been investigated [12] with staggered Dzyaloshinskii-Moriya interaction. The XY model has recently been investigated for various kinds of interactions. Those are higher order exchange interactions [13], antinematic kind of interactions [14], geometrically frustrated interaction [15] and higher order antiferromagnetic interaction [16] on a triangular lattice. The phase transition was studied [17] in the three-dimensional XY layered antiferromagnet, where the intra-planar interaction is ferromagnetic with inter-planar antiferromagnetic interaction.

In the thermodynamic phase transition, it is quite natural to study the effects of disorder. Apart from the randomly distributed anisotropy [8], the quenched disorder may also be introduced in the system by random fields and randomly quenched nonmagnetic impurity. Recently, the role of random field in the aging and domain growth has been extensively investigated [18]. The random fields act as random disorder which usually reduces the critical temperature. On the other hand, the uniform anisotropy generally increases the critical temperature just by breaking the SO(2) symmetry.

A competitive behaviour of the random field and the bilinear exchange anisotropy has recently been investigated [19] by Monte Carlo simulation. Another kind of random disorder may be represented by the randomly quenched nonmagnetic impurity. The site diluted XY ferromagnet has been studied [20] by Monte Carlo simulation and found that the phase transition took place at lower temperatures for higher values of the nonmagnetic impurity. This feature is quite natural since the disorder usually reduces the transition temperature.

However, we have not yet observed any systematic and intensive study on the impure anisotropic XY ferromagnet. It may be a naive question, what will be the critical behaviour of the anisotropic XY ferromagnet in the presence of quenched random nonmagnetic impurity? We have addressed this question in this paper. We have systematically investigated, by Monte Carlo simulation, the effects of nonmagnetic impurity, both in the bilinear exchange kind of anisotropy as well as in the single site anisotropic XY ferromagnet. The manuscript is formatted as follows: the anisotropic XY model is introduced in section-II, the Monte Carlo methodology is mentioned in section-III, the simulational results are reported in section-IV and the paper ends with a summary accompanied by some concluding remarks given in section-V.

II. Anisotropic XY ferromagnetic model

The Hamiltonian of anisotropic (bilinear exchange type) XY ferromagnet is expressed as
\[
H = -J \sum_{<i,j>} (1 + \lambda)S_i^x S_j^x + (1 - \lambda)S_i^y S_j^y
\]

where \(S_i^x(= \cos \theta_i)\) and \(S_i^y(= \sin \theta_i)\) correspond to the components of the two dimensional vector (at the i-th lattice site) having unit length, \(|S|=1\). \(\theta\) is the angle (measured with respect to the positive X-axis) of the vector \(\vec{S}\). It may be considered as a two dimensional rotor at each lattice site, which can point in any direction (specified by \(\theta\)). The \(\lambda\) denotes the strength of bilinear exchange anisotropy. For \(\lambda = 0\), the system becomes conventional isotropic XY ferromagnet and the system represents ferromagnetic XX model for \(\lambda = 1\) (but \(S_i^x\) is continuous variable). This anisotropy usually breaks the SO(2) symmetry. The parameter \(J\) is the nearest neighbour (represented by \(<ij>\) in the summation) ferromagnetic \((J > 0)\) interaction strength. For simplicity, we have considered the uniform \((J\) constant\) ferromagnetic interaction, here.

One may also think of the anisotropic XY ferromagnet with the single site anisotropy \((D)\) represented by the following Hamiltonian:

\[
H = -J \sum_{<i,j>} S_i^x S_j^x + S_i^y S_j^y - D \sum (S_i^x)^2 - (S_i^y)^2 \right). \]

Here, \(D\) represents the strength of single site anisotropy.

The nonmagnetic impurity is implemented in the anisotropic model by considering simply a few site (randomly chosen with probability \(p\)) having \(\vec{S} = 0\). The concentration of such nonmagnetic impurity is represented by \(p\). This nonmagnetic impurity \((p)\) acts as quenched disorder in the system.

### III. Monte Carlo simulation methodology

In the present study, a three dimensional simple cubic lattice of size \(L\) (=20 here, apart from the finite size analysis) is considered here. The lattice dimensions of the system are three (simple cubic) and the dimensions of the spin vector are two (planar ferromagnet or XY model). We have applied the conventional periodic boundary conditions in all the three directions of the lattice.

The Monte Carlo simulation starts from a random initial configuration of the spin, corresponding to a very high temperature phase. This state corresponds to the usual paramagnetic phase (without long range ferromagnetic ordering) having zero magnetisation. At a finite temperature \(T\) (measured in the unit of \(J/k\), where \(k\) is Boltzmann constant), a lattice site (say \(x,y,z\)) is chosen randomly (at any instant of time \(t\)) having a random initial spin configuration (represented by an angle \(\theta_i(x,y,z,t)\)). A new configuration of the spin (at site \(x,y,z\)) is also chosen (represented by \(\theta_f(x,y,z,t)\)) randomly. The change in energy \((\Delta H(t))\) due to the change in configuration (angle) of spin (from \(\theta_i(x,y,z,t)\) to \(\theta_f(x,y,z,t)\)) is calculated from eqn. (1) and eqn. (1) for respective cases. The probability of accepting the new configuration is calculated from the Metropolis formula [21].

\[
P_f = \text{Min}[\exp(-\Delta H(t)/kT), 1].
\]

A uniformly distributed (between 0 and 1) random number \((r = [0,1])\) is chosen. The chosen site is assigned to the new spin configuration \(\theta_f(x,y,z,t')\) (for the next instant \(t'\)) if \(r \leq P_f\). In this way, \(L^3\) number of sites are updated randomly. \(L^3\) number of such random updates defines a unit time step and is called Monte Carlo step per site (MCSS). The time in this simulation is measured in the unit of MCSS. Throughout the study the system size \(L(= 20)\). The total length of simulation is \(2 \times 10^4\) MCSS, out of which the initial \(10^4\) MCSS times are discarded. Here, to make the system ergodic some initial MCSS are required. All statistical quantities are calculated by averaging over the rest \(10^4\) MCSS [21]. However, for finite size analysis the length of the simulation is much longer. Each thermodynamic quantities are further averaged over different random realizations (typically around 20) of fixed concentration of impurity \((p)\). But for smaller sizes of the systems \((L = 10, 15 \text{ etc.})\) the number of such random samples is quite large (around 200).

The instantaneous components of magnetisations are \(m_x(t) = 1/L^3 \sum s_x(x,y,z,t)\) and \(m_y(t) = 1/L^3 \sum s_y(x,y,z,t)\). Total instantaneous magnetisation is

\[
m = \sqrt{m_x^2 + m_y^2}.
\]
The equilibrium magnetisation is measured as \( M = < m > \). The susceptibility is determined by \( \chi = \frac{L^3}{kT} (< m^2 > - < m >^2) \). The specific heat has been calculated from \( C = \frac{L^3}{kT} (< H^2 > - < H >^2) \). The fourth order Binder cumulant (for system size \( L \)) has been calculated as \( U_L = 1 - \frac{<m^4>}{3<m^2>^2} \). The symbol \(< .. >\), represents the time averaging, which is approximately (within the length of simulation) equal to the ensemble averaging in the ergodic limit.

IV. Simulational Results

In this section, we report the simulational results of our study. The results for the bilinear exchange type of anisotropy (\( \lambda \)) is given in subsection (a) and that for single site anisotropy (\( D \)) are reported in subsection (b).

(a). Bilinear exchange type of Anisotropy (\( \lambda \))

Let us start by presenting the simulational outcomes, highlighting the findings obtained for bilinear exchange type of anisotropy (\( \lambda \)) We have calculated the magnetisation (\( M \)), susceptibility (\( \chi \)) and the specific heat (\( C \)) and studied these quantities as functions of the temperature (\( T \)). The temperature variations of all these quantities (for different values of bilinear exchange anisotropy \( \lambda \) and impurity concentration \( p \)) are shown in Fig. 1.

![Figure 1](image-url)

Figure 1: The temperature dependences of all thermodynamic quantities. In the left panel, for impurity concentration \( p = 0.2 \), (a) Magnetisation (\( M \)) for \( \lambda = 0.3 \) and \( \lambda = 0.7 \) (b) Susceptibility (\( \chi \)) for \( \lambda = 0.3 \) and \( \lambda = 0.7 \) (c) Specific heat (\( C \)) for \( \lambda = 0.3 \) and \( \lambda = 0.7 \). In the right panel, for impurity concentration \( p = 0.4 \), (d) Magnetisation (\( M \)) for \( \lambda = 0.3 \) and \( \lambda = 0.7 \) (e) Susceptibility (\( \chi \)) for \( \lambda = 0.3 \) and \( \lambda = 0.7 \) (f) Specific heat (\( C \)) for \( \lambda = 0.3 \) and \( \lambda = 0.7 \).

While cooling the system for a fixed set of values of the strength of bilinear exchange type of anisotropy (\( \gamma \)) and the concentration of impurity (\( p \)), it is observed that the order parameter or the magnetisation (\( M \)) transits...
from zero to a nonzero value continuously, indicating the continuous phase transition. The susceptibility ($\chi$) and specific heat ($C$) exhibit maxima at a particular temperature, signifying the occurrence of a phase transition. The pseudocritical temperature ($T_c^*$) is determined from the temperature at which the susceptibility ($\chi$) attains its maximum value. Moreover, the pseudocritical temperature has been observed to change depending on the value of $\lambda$ and $p$. For any fixed value of $\lambda$ the pseudocritical temperature ($T_c^*$) decreases as the concentration of impurity ($p$) increases. It is pertinent to note here that the decrease of the pseudocritical temperature ($T_c^*$) with the increase of the impurity concentration ($p$) has already been noticed \cite{20} in the three dimensional *isotropic* XY ferromagnet by Monte Carlo simulation. This was confirmed later \cite{22} and found the linear dependence of pseudocritical temperature $T_c^*$ on the impurity concentration $p$. On the other hand, the pseudocritical temperature ($T_c^*$) increases with $\lambda$ for the fixed value of the impurity concentration $p$. These are demonstrated in Fig.1.

Figure 2: The pseudocritical temperature ($T_c^*$) is shown as a function of the strengths of the bilinear exchange type of anisotropy ($\lambda$) for different impurity concentrations ($p$) for. The solid lines represent corresponding linear best fit $y = a + bx$. $p = 0.1$ (represented by Squares, $a = 2.05, b = 2.28$), $p = 0.2$ (represented by Bullets, $a = 1.81, b = 2.01$), $p = 0.3$ (represented by Triangles, $a = 1.54, b = 1.74$) and $p = 0.4$ (represented by Inverted triangles, $a = 1.29, b = 1.44$).

We have tried to study the dependence of $T_c^*$ on $\lambda$ and $p$, systematically. Fig.2 shows the variation of $T_c^*$ as function of $\lambda$ for four different values of $p$. $T_c^*$ is found to be linear in $\lambda$. The data show linear best fit $T_c^* = a + b\lambda$. However, the slope ($b$) and the intercept ($a$) depends on $p$. The scaled pseudocritical temperature ($T_c' = (T_c^* - a)/b$) has been plotted with $\lambda$. All data collapsed (Fig.3) on a straight line $T_c' = \lambda$.

Figure 3: The scaled pseudocritical temperature ($T_c' = (T_c^* - a)/b$) is shown as a function of the anisotropy ($\lambda$). The values of $a$ and $b$ are collected from Fig.2. The solid line is the fitted (linear best fit) function ($T_c' = A + B\lambda$) obtained from the best fit. Here, $A=0.00$ and $B=1.00$. 
The slope \((b)\) has been found to depend linearly \((b = A + Bp)\) on the impurity concentration \((p)\). We have estimated \(A = 2.56\) and \(B = -2.79\). This is shown in Fig.4. This has not been studied before.

![Figure 4: The slope of the fitted straight line \((b)\) is plotted against different impurity concentration \((p)\). The data are fitted to a straight line (solid line) \(b = 2.56 - 2.79p\).](image)

We have also done a systematic finite size analysis for \(L = 10, 15, 20, 25\) to substantiate the growth of critical correlation (along with divergence of susceptibility) at the phase transition temperature. The fourth order Binder cumulant has been studied as function of temperature with \(L\) as parameter, for fixed \(\lambda = 0.4\) and \(p = 0.3\). This is shown in Fig.5. The intersection of all such curves determines the true critical temperature \(T_c\) (i.e., \(T_c(L)\) for \(L \to \infty\)). Here, we have estimated \(T_c = 2.25\).

![Figure 5: The fourth order Binder cumulant \((U_L)\) is plotted against the temperature \((T)\) for different system sizes \((L)\) and fixed values of impurity concentration \((p = 0.3)\) and strength of bilinear exchange anisotropy \((\lambda = 0.4)\). The intersection point indicates the critical temperature \((T_c)\), shown by the vertical black straight line. Here, \(L = 10\) (Square), \(L = 15\) (Bullet), \(L = 20\) (Triangle) and \(L = 25\) (Inverted triangle).](image)

We have also studied the temperature dependence of magnetisation \((M)\) for different system sizes \((L = 10, 15, 20, 25)\) for fixed anisotropy \(\lambda = 0.4\) and impurity concentration \(p = 0.3\). This is depicted in Fig.6. The vertical line \((T_c = 2.25)\) intersects the curves at different points.

From these intersections, the critical magnetisations \((M(T_c))\), for various system sizes \((L = 10, 15, 20, 25)\), are determined. Assuming the scaling law \([23]\), \(M(T_c) \sim L^{-\frac{2}{\nu}}\), the \(\log((M(T_c)))\) is plotted against \(\log(L)\). This is shown in Fig.7.
Figure 6: The magnetisation ($M$) is plotted against the temperature ($T$) for different system sizes ($L$) and fixed values of impurity concentration ($p = 0.3$) and strength of the bilinear exchange anisotropy ($\lambda = 0.4$). The vertical black line indicates the critical temperature estimated from the intersection of Binder cumulant. The values of $M(T_c)$ are measured from the intersections of $M$ and the vertical line. Here, $L = 10$ (Square), $L = 15$ (Bullet), $L = 20$ (Triangle) and $L = 25$ (Inverted triangle).

$$\log(M(T_c)) = -0.22 - 0.48 \log(L)$$

This has been fitted with a straight line. The slope of the straight line results the scaling exponent $\frac{\beta}{\nu} = 0.48 \pm 0.05$.

The divergence of the susceptibility at the transition temperature is a crucial phenomenon in the equilibrium continuous phase transitions. This can be studied by checking the increase of peak height of the susceptibility with increase of the system size $L$. We have studied the temperature variation of $\chi$ for different system sizes ($L = 10, 15, 20, 25$) with fixed anisotropy $\lambda = 0.4$ and impurity concentration $p = 0.3$. Fig. 8 shows that peak height of the susceptibility ($\chi_p$) increases as the system size $L$ increases.

This indicates that the susceptibility would eventually diverge for $L \to \infty$. Here also, assuming the scaling law $\chi_p \sim L^{\gamma/\nu}$, we have plotted $\log(\chi_p)$ with $\log(L)$ for fixed $\lambda = 0.4$ and $p = 0.3$. This is shown in Fig. 9.

This has been fitted with a straight line. The slope of the straight line estimates the scaling exponent $\frac{\gamma}{\nu} = 1.78 \pm 0.05$. These finite size analysis confirms the transition as the true thermodynamic phase transition.
(b). Single site Anisotropy ($D$)

In this subsection, we report the simulational results of our study for the single site anisotropy ($D$). Here also, we have studied the temperature variation of the magnetisation ($M$), susceptibility ($\chi$) and the specific heat ($C$). All these quantities are shown in Fig-10 as functions of temperature for different values of single site anisotropy $D$ and impurity concentration $p$.

The results of pseudocritical temperature ($T^*_c$) as function of single site anisotropy ($D$) for pure ($p = 0$) system are shown in Fig-11.

This is first Monte Carlo results. This was initially done by a finite temperature quantum field theoretic calculation [7]. The Monte Carlo results show a linear variation $T^*_c = A + BD$ with $A = 2.26$ and $B = 0.32$ which is significantly different from that obtained in quatum calculations [7]. It may be noted here that the pseudocritical temperature ($T^*_c$) increases linearly [8] with the strength of anisotropy ($\lambda$) in the case of bilinear exchange anisotropy.

While cooling the system for fixed set of values (Fig-10) of the strength of the single site anisotropy ($D$) and the concentration of impurity ($p$), it is observed that the order parameter or the magnetisation ($M$) shows a continuous variation, from a zero to a non-zero value, indicative of the continuous phase transition. The susceptibility ($\chi$) and the specific heat ($C$) get peaked at any temperature indicating the phase transition. The pseudocritical temperature-
Figure 10: The temperature dependences of all thermodynamic quantities for single site anisotropy. In the left panel, for impurity concentration $p = 0.2$, (a) Magnetisation $(M)$ for $D = 1.0$ and $D = 2.5$ (b) Susceptibility $(\chi)$ for $D = 1.0$ and $D = 2.5$ (c) Specific heat $(C)$ for $D = 1.0$ and $D = 2.5$. In the right panel, for impurity concentration $p = 0.5$, (d) Magnetisation $(M)$ for $D = 1.0$ and $D = 2.5$ (e) Susceptibility $(\chi)$ for $D = 1.0$ and $D = 2.5$ (f) Specific heat $(C)$ for $D = 1.0$ and $D = 2.5$.

ture $(T^*_c)$ is calculated from the temperature which maximises the susceptibility $(\chi)$. Moreover, the pseudocritical temperature $(T^*_c)$ depends on the values of $D$ and $p$. For any fixed value of $D$ the pseudocritical temperature $(T^*_c)$ decreases as the concentration of impurity $(p)$ increases. Conversely, the pseudocritical temperature $(T^*_c)$ has been found to increase with the increase of $D$ for a fixed value of the impurity concentration $p$.

We have tried to study the dependence of $T^*_c$ on $D$ and $p$, systematically. Fig.12 shows the variation of $T^*_c$ as function of $p$ for four different values of anisotropy $D$. Here also, $T^*_c$ is found to be linear in $p$. The data show linear best fit $T^*_c = a - bp$. Unlike the case of bilinear exchange kind of anisotropy, we failed to make any conclusive remark regarding the dependence of the slope $b$ on the anisotropy $D$. However, the slope $(b)$ and the intercept $(a)$ depends on $D$. The scaled pseudocritical temperature $(T'^*_c = (T^*_c - a)/b)$ has been plotted with $p$. All data collapsed on a single straight line $T'^*_c = p$ and shown in Fig.13.
Figure 11: The pseudocritical temperature ($T_c^*$) is plotted against the strength of single-site anisotropy ($D$) for pure ($p = 0.0$) system. The inset shows the temperature ($T$) dependences of the susceptibility ($\chi$) for two different values of $D$.

We have also done a systematic finite size analysis for $L = 10, 15, 20, 25$ to check the growth of critical correlation (along with divergence of susceptibility) at the phase transition temperature. The fourth order Binder cumulant has been studied as function of temperature with $L$ as parameter, for fixed $D = 1.0$ and $p = 0.3$. This is shown in Fig.14.

The intersection of all such curves determines the true critical temperature $T_c$ (i.e., $T_c^*(L)$ for $L \to \infty$). Here, we have estimated $T_c = 1.78$. 
Figure 12: The pseudocritical temperature ($T^*_c$) is shown as a function of impurity concentration ($p$) for different strengths of anisotropy ($D$). The solid lines represent corresponding linear best fits $y = a - bx$. $D = 0.5$ (represented by Squares, $a = 2.44, b = 2.80$), $D = 1.0$ (represented by Bullets, $a = 2.61, b = 2.81$), $D = 2.0$ (represented by Triangles, $a = 2.96, b = 3.00$) and $D = 2.5$ (represented by Inverted triangles, $a = 3.15, b = 3.20$).

Figure 13: The scaled pseudocritical temperature ($T'^*_c = (T^*_c - a)/b$) is shown as a function of the concentration of impurity ($p$). The values of $a$ and $b$ are collected from Fig 12. The solid line is the fitted (linear best fit) function ($T'^*_c = A + Bp$) obtained from the best fit. Here, $A=0.00$ and $B=-1.00$.

We have also studied the temperature dependence of magnetisation ($M$) for different system sizes ($L = 10, 15, 20, 25$) for fixed anisotropy $D = 1.0$ and impurity concentration $p = 0.3$. This has been shown in Fig. 15. The vertical line ($T_c = 1.78$) intersects the curves at different points. From these intersections, the critical magnetisations ($M(T_c)$), for various system sizes ($L = 10, 15, 20, 25$), are determined. Assuming the scaling law $[23]$, $M(T_c) \sim L^{-\frac{\beta}{\nu}}$, the log($(M(T_c)$) is plotted against log($L$). This is shown in Fig. 16.
Figure 14: The fourth order Binder cumulant ($U_L$) is plotted against the temperature ($T$) for different system sizes ($L$) and fixed values of impurity concentration ($p = 0.3$) and strength of single site anisotropy ($D = 1.0$). The intersection point indicates the critical temperature ($T_c$), shown by the vertical black straight line. Here, $L = 10$ (Square), $L = 15$ (Bullet), $L = 20$ (Triangle) and $L = 25$ (Inverted triangle).

This has been fitted with a straight line. The slope of the straight line results the scaling exponent $\frac{\beta}{\nu} = 0.37 \pm 0.04$.

The divergence of the susceptibility at the transition temperature is a crucial phenomenon in the equilibrium continuous phase transitions. This can be examined by checking the increase of peak height of the susceptibility with increase of the system size $L$. We have studied the temperature variation of $\chi$ for different system sizes ($L = 10, 15, 20, 25$) with fixed anisotropy $D = 1.0$ and impurity concentration $p = 0.3$. Fig[17] shows that peak height of the susceptibility ($\chi_p$) increases as the system size $L$ increases.
Figure 15: The magnetisation ($M$) is plotted against the temperature ($T$) for different system sizes ($L$) and fixed values of impurity concentration ($p = 0.3$) and strength of single site anisotropy ($D = 1.0$). The vertical black line indicates the critical temperature estimated from the intersection of Binder cumulant. The values of $M(T_c)$ are measured from the intersections of $M$ and the vertical line. Here, $L = 10$ (Square), $L = 15$ (Bullet), $L = 20$ (Triangle) and $L = 25$ (Inverted triangle).

Figure 16: The logarithm of the values of critical magnetisation ($\log(M(T_c))$) is plotted for logarithm of different $L$. Here, $D = 1.0$ and $p = 0.3$. The solid line is the best fit. The exponent $\frac{D}{\nu} \sim 0.37 \pm 0.05$ for the scaling $M(T_c) \sim L^{-\frac{D}{\nu}}$.

This indicates that the susceptibility would eventually diverge for $L \to \infty$. Here also, assuming the scaling law $\chi_p \sim L^{\gamma}$, we have plotted $\log(\chi_p)$ with $\log(L)$ for fixed $D = 1.0$ and $p = 0.3$. This is shown in Fig. 18. This has been fitted with a straight line. The slope of the straight line estimates the scaling exponent $\frac{\gamma}{\nu} = 1.81 \pm 0.05$. These finite size analysis validates the transition as the true thermodynamic phase transition.
Figure 17: The susceptibility ($\chi_p$) is plotted against the temperature ($T$) for different $L$. Here, $D = 1.0$ and $p = 0.3$. The solid line is the best fit. Here, $L = 10$ (Square), $L = 15$ (Bullet), $L = 20$ (Triangle) and $L = 25$ (Inverted triangle).

Figure 18: The logarithm of the values of maximum susceptibility ($\log(\chi_p)$) is plotted for logarithm of different $L$. Here, $D = 1.0$ and $p = 0.3$. The solid line is the best fit. The exponent $\frac{\gamma}{\nu} \sim 1.81 \pm 0.05$ for the scaling $\chi_p \sim L^{\frac{7}{2}}$.

V. Summary

The role of nonmagnetic impurity on the critical behaviours of the anisotropic XY ferromagnet has not been studied before. We have tried to explore such role of nonmagnetic impurity on the anisotropic XY ferromagnet through a systematic investigation via Monte Carlo simulation in three dimensions with Metropolis algorithm. We have considered here two different kinds of anisotropy, namely, the bilinear exchange type and the single site anisotropy.

Our major findings are, in the case of bilinear exchange anisotropy, the pseudocritical temperature of the ferro-para phase transition increases linearly with the strength of anisotropy. However, the slope ($b$) of such linear increase, decreases linearly as the concentration ($p$) of the nonmagnetic impurity increases. The critical exponents for the assumed scaling laws $M(T_c) \sim L^{-\beta}$ and $\chi_p \sim L^{\frac{7}{2}}$ are estimated through the finite size analysis. We have estimated, $\frac{\beta}{\nu}$ equals to $0.48 \pm 0.05$ and $\frac{\gamma}{\nu}$ equals to $1.78 \pm 0.05$. It may be noted here that the exponent $\frac{\beta}{\nu}$ is very close to $\frac{1}{2}$.

On the other hand, for single site anisotropy, the pseudocritical temperature has been found to decrease linearly with the impurity concentration. The slope of this linear fall depends on the single site anisotropy ($D$). However, we failed to extract any systematic dependence of it. In the case of single site anisotropy, the critical exponents for the assumed scaling laws $M(T_c) \sim L^{-\beta}$ and $\chi_p \sim L^{\frac{7}{2}}$ are estimated through the systematic and usual finite size analysis. We have estimated, $\frac{\beta}{\nu}$ equals to $0.37 \pm 0.04$ and $\frac{\gamma}{\nu}$ equals to $1.81 \pm 0.05$. It may be noted here that
the exponent $\frac{\nu}{2}$ is very close to $\frac{1}{3}$.

The scaling exponents $\frac{\nu}{2}$, for both bilinear exchange anisotropy and single site anisotropy, provides almost same value. But the values of other exponent $\frac{\nu}{2}$, differ significantly for bilinear exchange and single site anisotropy. This prompted us to conclude that these two kinds of anisotropy belong to two different universality classes [23].

As far as the knowledge of these authors is concerned, this is the first study of the effects of randomly quenched disorder (of nonmagnetic impurity) in the phase transition of three dimensional anisotropic planar ferromagnet. The main conclusion of this study is that the two types of anisotropy (bilinear exchange type or single site type) belong to two different universality classes. The universality class generally depends on the dimensionality of the system and the symmetry of the order parameter in the case of equilibrium phase transitions. But in the present study, as the results show, that the types of anisotropy may lead to different universality classes. Recently, the breaking of the same universality class for the site percolation and bond percolation has been reported in the statistical physics of networks [24].

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