Dual proximity effect near superconductor-insulator transition

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We show that quantum vortex-loop proliferation may be one possible explanation of the super-long-range proximity effect observed in an insulating underdoped cuprate, using the dual theory of quantum vortices. As a test of this scenario, we propose that a dual proximity effect experiment can confirm the superfluid motion of the quantum vortices in the vortex-proliferated insulator and can measure the divergent correlation lengths near the superconductor-insulator transition.

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Cuprate superconducting materials exhibit a rich structure of phases depending on the charge-carrier doping and the temperature. Most notably, they exhibit superconducting, antiferromagnetic (AF) Mott-insulating, and \( T > T_c \) strange metallic phases. To this date, however, the nature of the intervening phases between the superconducting state and the AF Mott-insulating state is not understood [1]. In attempts to understand this region of the phase diagram, there have been a number of theoretical proposals on the mechanism of destruction of superconductivity such as the superconductor-insulator transition into the quantum-disordered insulator with vortex-loop proliferation [2–5], superconductor into AF insulator transition with the SO(5)-symmetric theory [6], transition into novel ground states which are possible in two-dimensional (2D) doped Mott insulators [7,8], or striped phases [9]. Therefore, it would be desirable to devise experiments which can shed light on the nature of the underdoped insulating state.

In an experiment performed by Decca et al. [2], Josephson junctions of superconductor-insulator-superconductor (SIS) type were made by generating photo-induced superconducting electrodes on insulating underdoped thin films of YBa\(_2\)Cu\(_3\)O\(_{6+x}\) family. Surprisingly, they observed the Josephson effect with an invariant product of the Josephson critical current and the normal states junction resistance for separations as large as 100 nm between superconducting electrodes, much larger than the superconducting coherence length \( \xi_0 \sim 1 \) nm. This finding indicates enhancement of the effective correlation length \( \xi_{\text{eff}} \) of the superconducting order parameter in the insulator, which can be explained by the proximity to a continuous quantum phase transition. For instance, the SO(5) theory is one possible scenario which predicts such an effect [10]. In this paper, as an alternative scenario, we will demonstrate that the vortex proliferation does provide a long correlation length scale near SIT, which may explain the super-long-range proximity effect in the underdoped insulator. In fact, experiments on the optical conductivity [11] and the thermal Nernst effects [12] provided evidences for strong thermal fluctuations of unbound vortices over a wide range of temperatures above \( T_c \) in the underdoped pseudogap region, which suggest a Kosterlitz-Thouless like transition with a broad phase fluctuation regime [13]. In addition, an experiment on current-voltage characteristic of superconducting Bi2212 shows indications of a large density of quantum vortex pairs well below \( T_c \) and far away from SIT [14]. Therefore, given the indications of strong quantum/thermal phase fluctuations in cuprates, vortex-proliferation is a prospective candidate scenario of SIT.

We also propose an experiment which can detect a qualitatively distinct effect of Josephson tunneling of superfluid vortices in an insulator-superconductor-insulator (ISI) junction. If the SIT is induced by the vortex proliferation, we expect that the divergence in correlation lengths is symmetric between the superconducting and insulating side of the transition due to the electromagnetic duality [15]. More precisely, the vortex proliferated insulator can be considered as the bose-condensed state of vortices, and consequently superfluid motion of vortices is possible. Therefore, by choosing a superconducting junction sufficiently close to SIT, it will be possible to detect the super-long-range proximity effect of the vortex-condensate superfluid in the dual Josephson (ISI) junction experiment [16]. In Fig. 1 we show an ISI junction which can be prepared by the same experimental technique used in Ref. [2]. We consider applying an external magnetic field gradient across the junction and subsequently letting it relax, which will impose a time-dependent phase difference \( \Delta \phi \) in the vortex-condensate order parameter \( \Phi \) across the junction. The

FIG. 1. A dual Josephson junction experiment of insulator-superconductor-insulator type.
phase difference induces a dual Josephson current of magnetic vortices ($\vec{J}$) across the superconducting junction. The vortex current leads to a transverse electric voltage difference ($\Delta V_{\text{tr}}$) across the superconducting junction which can be subsequently measured. As in the superconducting proximity effect experiment, we can imagine varying the length ($\delta$) of the junction to probe the correlation length of the magnetic vortex field $\Phi_S$ in the superconducting junction. If the superconductor is in proximity to the SIT, we expect a large vortex correlation length ($\xi_v$), analogous to the super-long-range proximity effect in the insulator observed in Ref. [9]. We show that a simple oscillatory solution to the equation of dual Josephson vortices is clear from the form of Eqs. (1) and (2) which obtained from the energy of a pancake vortex and given by $m_v v_c^2 \approx \rho_s \pi \kappa$. We have assumed a vortex core size of $\xi_0$ which provides the momentum (short-time) cutoff at $\Lambda_c = \pi/\xi_0$ and the frequency (short-time) cutoff at $\nu_f \Lambda_c$. Here we take $v_c \approx v_F$ on physical grounds [3]. From Eq. (3), the vortex number density ($n_v$) and the vortex current can be expressed by $\Phi$ as

$$n_v = J_0/2\pi = \text{Im } 2\Phi^* \partial_0 \Phi + 4\pi G_0|\Phi|^2$$

$$\vec{J}/2\pi = v_c^2 (\text{Im } 2\Phi^* \nabla \Phi + 4\pi G|\Phi|^2).$$

In addition to Eq. (2), there is a term that couples the vortex currents to the background superfluid density which acts as a dual magnetic field for vortices, but we assume that this does not affect the superfluidity of the quantum vortices in the insulating state where the Cooper pairs are localized.

We now discuss the superconducting proximity effects in the insulating state. The action in Eq. (2), if $t$ is analytically continued to the imaginary time, can be viewed as the effective 3D Ginzburg-Landau functional of $\Phi$. Then SIT occurs upon Bose-condensation of the $\Phi$ fields, when $m_v^2 < 0$ so that $|\Phi| \neq 0$ and the correlation function in the mean-field approximation is $\langle \Phi(x) \Phi^*(y) \rangle \approx |\Phi_0|^2 > 0$, independent of $|x - y|$. The correlation function $\langle \Phi(x) \Phi^*(y) \rangle$ can be considered as the expectation value of the number of magnetic vortex paths that connect $x$ and $y$, and therefore $\langle \Phi(x) \Phi^*(y) \rangle \approx |\Phi_0|^2 > 0$ implies a finite probability of arbitrarily long 2+1D vortex loops. Here we demonstrate that the Bose-condensed $\Phi$ field introduces a new length scale related to the observed super-long correlation length $\xi_v$.

We first derive an effective Ginzburg-Landau functional of the superconducting order parameter ($\Psi$) in the vortex-proliferated insulating state. The derivation is straightforward once we identify the trajectories of Cooper pairs with dual (electric) vortex lines in the dual theory [17]. The symmetry between the Cooper pairs and vortices is clear from the form of Eqs. (1) and (2) which have similar functional forms in $\rho_s e^{i\phi}$ and $\Phi$ respectively, except for the topological term $\partial_\mu \phi n_s/2$ in Eq. (1) which is absent in Eq. (2) and action of the gauge fields. Besides, just as a Cooper pair picks up a phase factor of $2\pi$ upon encircling a vortex, a vortex picks up the same phase factor upon encircling a Cooper pair. Therefore, in the superfluid state of vortices (insulating state), a Cooper pair is the topological defect, playing the role of a vortex. Thus, we can perform a dual transformation on Eq. (3) to obtain an effective theory of superconducting order parameter, in the same way as the dual theory of vortices in Eqs. (2) and (3) is derived from the original
superconducting theory in Eq. (1). The result is following:

\[ S[\Psi, \Psi^*, A_{\mu}] = \int d^2 x \, dt \left[ \left( \partial_\mu + 2iA_\mu \right) \Psi^* \frac{\Psi}{m_c v_c^2} - \left( \nabla + 2iA \right) \frac{\Psi^2}{m_c} - m_c v_c^2 \frac{\Psi^2}{2} \right] + S[A_{\mu}], \quad (6) \]

\[ S[A_{\mu}] = \sum_k \frac{1}{2(2\pi)^2 |\Phi_0|^2 v_c^2} \left\{ [k^2 + 2|\Phi_0|^2(2\pi)^2 \mathcal{K}_0(k)] A_0^2 - [v_c^2 k^2 - \omega^2 + v_c^2 |\Phi_0|^2(2\pi)^2 \mathcal{K}_T(k)] A^2 \right\}. \quad (7) \]

Here we estimate that \( m_c v_c^2 = 4\pi |\Phi_0|^2 v_c^2 \ln(L/\xi_0) \) and take \( v_c \approx v_0 \) on physical grounds, where \( L \) is the linear dimension of the sample. Therefore, we may take the correlation length of the superconducting order parameter from \( \xi^{-1}_c \approx m_c v_c \approx 4\pi |\Phi_0|^2 v_c \ln(L/\xi_0) \), and so \( \xi_c \) can be a very large number near SIT where \( |\Phi_0|^2 \) is small. This can explain the anomalous proximity effect observed in Ref. [3], assuming that the insulating compound is in the vortex-proliferated state. The vortex-proliferated insulating state has a physically distinct property of vortex superfluidity as shown below.

Next we consider the dual proximity effect near SIT. We consider an ISI junction surrounded by a superconducting region so that magnetic flux can be trapped inside. We assume that the vortex condensate density in \( I_1 \) and \( I_2 \) is \( |\Phi_0|^2 \). We imagine applying a magnetic flux density difference \( \Delta B \) between \( I_1 \) and \( I_2 \) regions of the ISI junction and relaxing it at a certain time \( t = 0 \) as shown in Fig. [4], and consider the time evolution of the magnetic field difference and the vortex supercurrent across the S region. Here the difference in the total number of magnetic vortices is \( \int d^2 x (\Delta J_0)/2\pi = \Delta B L^2/\phi_0 \) where \( \phi_0 = hc/2e \) is a quantum of magnetic flux in a superconductor and \( L \) is the linear size of the \( I_1 \) and \( I_2 \) regions. This condition can be implemented by adding a Lagrange multiplier \( \int dt \mu_v(t) \int d^2 x [\Delta J_0(x, t) - 2\pi \Delta B/\phi_0] \) to Eq. (3).

The chemical potential difference \( \mu_v \) can be absorbed into the dual electro-chemical potential difference \( G_0 \) which is the sum of the chemical potential and the dual scalar potential generated by the presence of extra vortices introduced by the external magnetic field. Ignoring the spatial fluctuations in \( G_0 \), we can absorb \( G_0 \) into the phase difference of the field \( \Phi \) by the transformation \( \Phi \rightarrow \Phi e^{i\Delta \varphi} \) where

\[ \partial_t \Delta \varphi(t) = -2\pi \tilde{G}_0(t). \quad (8) \]

The difference in the vortex number density can be related to \( G_0 \) from Eq. (4) as follows:

\[ \Delta n_v = \langle \Delta J_0/2\pi \rangle \approx 4\pi \tilde{G}_0(t)|\Phi|^2, \quad (9) \]

neglecting the time derivatives of the condensate \( \Phi \). The value of \( \tilde{G}_0 \) is determined by the condition \( \Delta n_v = \Delta B/\phi_0 \approx 4\pi \tilde{G}_0|\Phi|^2 \), which gives \( \tilde{G}_0 \approx \Delta B/(4\pi |\Phi_0|^2 \phi_0) \).

Inside the superconducting junction \( S \) of length \( \delta \) and width \( L \), the \( \Phi_S \) field is phase-incoherent and does not have non-zero vacuum expectation value. From Eq. (3), ignoring non-linear effects, the semi-classical \( \Phi_S \) field minimizes the following action:

\[ S = \int d^2 x dt \left| \partial_t \Phi \right|^2 - v_c^2 |\nabla \Phi|^2 + v_c^2 |\Phi/\xi|^2, \quad (10) \]

with a correlation length of \( \xi_v \approx 1/m_c v_c \). If the field \( \Phi_S(x, t) \) is solved in the \( S \) region, we can obtain the vortex number current \( |J_v| = L J_S(x, t)/2\pi \) between \( I_1 \) and \( I_2 \) (in the \( x \) direction) flowing through the \( S \) region. Then the rate of change of the vortex number density difference can be related to the vortex current as follows:

\[ -\partial_t \Delta n_v = J_S(x, t)/2\pi L. \quad (11) \]

Below we show that if we assume a nearly one-dimensional geometry of the junction, a simple solution to Eqs. (8), (9), (10), and (11) can be obtained under following conditions: (a) The magnetic field difference \( \Delta B \) is small so that \( |\Phi|^2 \) in the insulators can be approximately taken as a constant as a function of \( \Delta n_v \). (b) Inside the superconducting junction region, the following conditions hold: \( \Phi_S/\xi_v \ll |\partial_x \Phi_S|^2 \), which is valid when \( \delta \ll \xi_v \), and \( |\partial_x \Phi_S/v_F|^2 \ll |\partial_x \Phi_S|^2 \). In this case, from Eq. (10), the equation of motion for \( \Phi_S(x, t) \) is simply reduced to a static Laplace equation:

\[ \partial_t^2 \Phi_S = 0. \]

(c) The phase difference remains small \( (|\Delta \varphi| \ll 1) \) due to the small magnitude of the magnetic field difference \( \Delta B \). From condition (b), along with the boundary conditions \( \Phi_S(0) = \Phi_0 e^{i\Delta \varphi} \) and \( \Phi_S(\delta) = \Phi_0 \), we can obtain the solution of the Laplace equation in the region \( S \):

\[ \Phi_S \approx \Phi_0 [e^{i\Delta \varphi}(1 - x/\delta) + (x/\delta)]. \quad (12) \]

Then one can can show that there is a magnetic vortex supercurrent through the region \( S \) as follows:

\[ J_v \approx 2\pi v_c^2 n_v (\Phi_0 \partial_x \Phi_0^* - \Phi_0^* \partial_x \Phi_0) \approx -\hat{x} J_v \sin \Delta \varphi. \quad (13) \]

where \( J_v = 4\pi v_c^2 |\Phi_0|^2 /\delta \). If \( \delta \) exceeds the correlation length \( \xi_v \), \( J_v \) will be exponentially suppressed in \( \delta/\xi_v \) [14]. From Eqs. (8) and (12), and substituting the vortex current obtained from Eq. (12) into Eq. (11), we obtain the equation

\[ \partial_t^2 \Delta \varphi = -\omega_v^2 \sin \Delta \varphi, \]

which reduces to \( \partial_t^2 \Delta \varphi \approx -\omega_v^2 \Delta \varphi \) from condition (c) that \( |\Delta \varphi| \ll 1 \), where \( \omega_v = \sqrt{J_v/4\pi L |\Phi_0|^2} = v_c/\sqrt{\epsilon L} \delta \) is the natural frequency of the given ISI junction. The solution in this case is

\[ \Delta \varphi(t) \approx \varphi_M \sin(\omega_v t + \alpha). \quad (14) \]

with \( |\varphi_M| \ll 1 \). In the presence of such vortex condensate phase oscillations, we may detect the magnetic vortex supercurrents by measuring the transverse voltage difference across the region \( S \) (see Fig. [4]) which is induced by the vortex motion as follows:
As the junction thickness $\delta$ is increased above $\xi_\nu$, the Laplace equation of $\varphi$ does not hold anymore in the region $S$, and the relation $J_\nu \propto 1/\delta$ breaks down. Therefore, by varying the thickness $\delta$ and studying $J_\nu(\delta)$, the correlation length $\xi_\nu$ can be determined as the length scale of $\delta$ where $J_\nu(\delta)$ deviates from $J_\nu \propto 1/\delta$. As we further underdope the charge-carriers of the region $S$, we expect that $\xi_\nu$ will grow, resulting in a super-long-ranged dual proximity effect.

We are now able to make estimates of various quantities related to the (dual) proximity effects in the SIS (ISI) junctions, to see if we can find a regime of physical parameters within experimental access. The information on one particular set of parameters is provided by the experimental results of Decca et al. [9] on the SIS junction. The insulating junction (with thickness $\delta_c$) described by Eq. (6) leads to a Josephson current $I_J = I_s \sin \Delta \varphi$ where $I_c = e n_s L/2 m \delta_c$, if $\delta_c < \xi_\nu$. In Ref. [9], $I_s \approx 2.6 \mu A$ with $\delta_c \approx 45 \text{ nm}$ and $L \approx 200 \text{ nm}$, which leads to $\rho_s = n_s/4 m \approx 1.2 \text{ meV}$. From this, using the 1D Josephson weak link model [15] one can estimate the magnetic vortex rest energy $m_\nu \nu_s^2 \approx \rho_s \ln \kappa \approx 2.6 \times 10^{-2} \text{ eV}$. Note that the above small magnitude of the superfluid phase stiffness $\rho_s$ already indicates that the material is under strong vortex fluctuations and the vortex mass $m_\nu$ may have been significantly renormalized since $\rho_s < 2 \text{ meV}$ [16], although we will keep the bare mass for order of magnitude estimates.

Now we consider the ISI junction made of the same superconducting (S) and insulating (I$_1$ and I$_2$) regions as were used in Ref. [9]. Then the correlation length of the vortex fields in the region $S$ is expected to be $\xi_\nu \approx (m_\nu \nu_s)^{-1} \approx 5.1 \text{ nm}$, which is a few times larger than the superconducting coherence length. We can enhance the correlation length $\xi_\nu$ by further undoping the charge-carrier density. Here we assume a junction of length $\delta \approx 5.1 \text{ nm}$ and explore the possibility of the simple solution in Eq. (14). The magnitude of the fluctuations of $\Phi$ in time [condition (a)] cannot be checked within the phenomenological theory presented here. Condition (b) ($\delta < \xi_\nu$ and $|\partial \Phi_S/v_F| < |\partial \varphi_s|/\varepsilon_F$) can be checked by comparing $\nu_s |\partial \varphi_s/\varepsilon_F| \sim v_s/\delta \approx 10 \text{ THz}$ with the natural frequency $|\partial \Phi/|/\varepsilon_s \sim \nu_s = \sqrt{\varepsilon_s^2/L^2}$ which is about 1 THz; we confirm that condition (b) can be satisfied with the given parameters. Condition (c) ($|\Delta \varphi| \sim \Delta B/(2\varepsilon_0 v_s |\Phi_0|^2) \ll 1$) gives the upper limit of the magnetic field difference that allows the solution in Eq. (14). From $\xi_s \approx 100 \text{ nm}$ and $\xi_s^{-1} \approx 4 m_\nu |\Phi_0|^2 v_s \ln(L/\xi_0)$, we find that the upper limit of $\Delta B$ is $2.2 \times 10^2 \text{ Gauss}$. The upper limit of the transverse voltage in this case is about $|\Delta V_t| \sim L \nu_s \varphi_M/2e \approx 11 \text{ mV}$, obtained from Eq. (15) and the relation $(J_c \xi_\nu)^{-1} = \delta \ln(L/\xi_0)/\nu_s$. Therefore, based on the information obtained by Decca et al. [9], the proposed experiment of dual Josephson junction and the observation of the solution in Eq. (14) may be within the reach of experimental access.

We have shown that the anomalous proximity effect in an underdoped insulating cuprate [9] can be interpreted in terms of vortex proliferation. We proposed that the dual counterpart of the proximity effect can be observed in an ISI junction experiment. We anticipate that a similar super-long-range proximity effect will be observed in the dual Josephson junction experiment as long as the junctions are prepared in close proximity to the SIT.

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