A Two-Ball Ellsberg Paradox

Experimental Evidence from a Representative US Sample

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Abstract

We conduct an incentivized experiment on a nationally representative US sample (N=708) to test how people prefer to avoid ambiguity even when the ambiguity improves the probability of receiving a fixed prize. We find that subjects prefer non-ambiguous acts to similar ambiguous acts, even when the ambiguous acts provide larger win probabilities. Furthermore, this preference for avoiding ambiguity is not entirely due to a lack of understanding, as subjects “correctly” select the act with a larger win probability when comparing two similar ambiguous acts. Traditional models of ambiguity aversion cannot explain such preferences.

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1 Introduction

Motivation. Since Ellsberg (1961)’s seminal paper, many mathematical models have been created for decision-making under ambiguity. Schmeidler (1989) axiomatized the theory of Choquet Expected Utility (CEU), wherein the decision-maker chooses an act to maximize the utility’s Choquet integral concerning a subjective “capacity” (non-additive measure). This theory depicts the decision-maker as behaving somewhat similarly to a subjective expected utility maximizer, and in particular, their preferences must satisfy the familiar “monotonicity” axiom of Anscombe and Aumann (1963). If, in each state of the world, act $f$ grants a weakly preferred lottery over act $g$, then the decision-maker must weakly prefer act $f$ to act $g$.

Still, suppose we play the following gamble proposed by Jabarian (2021). An urn contains only red and blue balls. Two balls are drawn randomly with replacement from the urn, and the player wins $3 if the two balls are the same color. Is it preferable to gamble when the urn has 50 red balls and 50 blue balls or when its contents are utterly unknown? Although the gamble with the 50–50 urn (call it $R$, as its contents are merely risky) may seem more attractive than the ambiguous urn (call it $A$), in actuality the gamble with $A$ can only have a larger win probability than $R$. The more unevenly distributed urn $A$’s contents, the higher the win probability. Urn $A$ having 50% red balls and 50% blue balls is the worst case for win probability, making it equally likely to win as gambling on urn $R$ is. We call this preference for the $R$-gamble over the $A$-gamble the 2-Ball Ellsberg Paradox, as two balls are drawn with replacement from the same urn.

Aims. This paper conducts an incentivized experiment on a nationally representative sample to accurately test the extent to which people show preferences, such as the 2-Ball Ellsberg Paradox, that traditional models of ambiguity aversion cannot explain. Our experiment aims to answer two main questions. First, do subjects sometimes avoid an ambiguous act by choosing an unambiguous lottery that is worse than any lottery the ambiguous act could produce? Second, how are subjects’ choices in this 2-Ball Ellsberg Paradox related to other modes of behavior, such as failing to reduce compound lotteries, that traditional models of ambiguity aversion cannot explain?

Preview of Results. This study found three main results. First, we find that subjects prefer non-ambiguous acts to similar ambiguous acts even when the ambiguous acts only have larger win probabilities. Second, these preferences are closely corre-
lated with, but not completely explained by, other modes of behavior that suggest an aversion to complexity. Third, lack of understanding does not entirely explain the preference for avoiding ambiguity; subjects “correctly” select the act with a larger win probability when comparing two similar acts that are both ambiguous.

Furthermore, the experiment design revealed several additional results. Subjects in the treatment were asked about comparisons between similar ambiguous acts and “learned” nothing from this experience; they still preferred the lower-win probability, unambiguous acts as subjects not in this treatment. This lack of “learning” suggests subjects’ preference for avoiding ambiguity, even when it can only improve their odds of receiving a prize, is deliberate and not entirely due to a lack of understanding.

**Related Literature.** Similar to Anscombe and Aumann (1963), Gilboa and Schmeidler (1989) axiomatized the theory of *Maxmin Expected Utility* (MMEU), in which a decision-maker has a subjective set of probability distributions over states that are possible for Nature to choose. The decision-maker chooses an act to maximize the minimum expected utility across all distributions in this set. Ghirardato et al. (2004) generalize the MMEU model to $\alpha$-*Maxmin Expected Utility*, wherein agents may care about some combination of the maximum and minimum payoffs that could follow from any action. Maccheroni et al. (2006) and Strzalecki (2011) also generalize the MMEU theory, axiomatizing *Variational Preferences* (VP) and *Multiplier Preferences*, wherein the decision-maker acts to maximize the worst-case payoff against an adversarial Nature; however, Nature may be forced to “pay a cost” for choosing certain (sufficiently unreasonable) probability distributions over states of the world. Finally, Klibanoff et al. (2005) axiomatize a “smooth” model of ambiguity preferences that separates individuals’ risk preferences, ambiguity preferences, and subjective measures of ambiguity.

The extant literature has produced many challenges to these models of decision-making under ambiguity. Machina (2009) presents thought experiments that demonstrate plausible violations of the CEU model. Baillon et al. (2011) employ variations on Machina (2009) to propose plausible violations of several of the decision-making models mentioned above. L’Haridon and Placido (2010) test one of these examples, the so-called “reflection example” of Machina (2009), in an experimental setting and reject the MEU and VP models and the smooth ambiguity model of Klibanoff et al. (2005). Blavatskyy (2013) proposes a variant of Machina’s reflection example, casting doubt on more recent decision-making models under ambiguity, such as Siniscalchi (2009)’s *Vector Expected Utility* (VEU).

These thought-experiment examples and the experimental tests by L’Haridon and...
Placido (2010) cast doubt on the models mentioned above; however, examples involve inconsistencies between *multiple* choices facing the decision-maker.\(^1\) Furthermore, a variety of somewhat contrived examples may be required to refute the various models of decision-making under ambiguity built on those above. A better approach may be to question more fundamental assumptions that underlie all such models.

All of the theories mentioned above imply that the monotonicity axiom of Anscombe and Aumann (1963) must hold. There is growing theoretical and experimental evidence that models involving this monotonicity axiom, or generally employing the framework of Anscombe and Aumann (1963), may fail to capture the decision maker’s behavior in the face of ambiguity.

Halevy (2007) demonstrates that individuals’ preferences may be inconsistent with the proper reduction of compound lotteries, an assumption that is implicitly a part of any model built on the framework of Anscombe and Aumann (1963). He further shows that ambiguity aversion in the classic experiment of Ellsberg (1961) is closely correlated with an adverse reaction to compound lotteries. Gillen et al. (2019) replicate Halevy (2007)’s experiment with a correction for measurement error, revealing that this correlation may be close to 1.

Schneider and Schonger (2019) examine whether subjects’ preferences satisfy a weak form of the monotonicity axiom (*weak separability*) and find nearly half of the subjects in violation. They find no evidence that complexity aversion or failure to reduce compound lotteries explains these violations. Furthermore, subjects violating the monotonicity axiom generally make choices consistent with first-order stochastic dominance, demonstrating that these violations are likely unrelated to a lack of understanding of their choices.

2 Experimental Design

2.1 Structure of the Experiment

This study examines subjects’ choices in the novel “2-Ball” Ellsberg gamble and determines how these relate to their choices in other scenarios, most notably the “classic” Ellsberg paradox. All subjects’ choices were elicited through incentivized MPLs as described in Section 2.2. Furthermore, all subjects were asked the same questions twice to correct for measurement error as described in Section 2.3. In addition to the 2-Ball and classic Ellsberg gambles, subjects were also asked a variety of other questions as

\(^1\)For example, they are of the form “If you prefer A to B, then you cannot also prefer C to D.”
described in Section 2.4.

The questions were split across four treatments, described in Section 2.5. Questions in this experiment are divided into blocks, with treatments consisting of a specific number of specific blocks randomized in a certain order as detailed in Section 2.5. A block contains either one or several similar questions. Before each block, subjects view the relevant instructions. Each question within a given block contains (1) a reiteration of the block’s instructions, (2) the new details of that particular question, highlighted in yellow, and (3) the MPL table for that question.\footnote{Certain questions require subjects to choose a color (red or blue) to place a bet. For these questions, subjects must select a color before the MPL appears on the screen.} Subjects must fill out the MPL table before they can move on to the next question screen. Questions are ordered uniformly at random and independently across subjects within a given block. To accommodate for online cognitive fatigue and prevent attention loss, each treatment could only contain 11 or 12 questions.

Below, we summarize each block’s contents that appear in at least one treatment. In each question, “winning” the gamble (i.e., an act or a lottery) means a payoff of 300 tokens (=\$3), and ”losing” means a payoff of 0 tokens. The notation “[x red, y blue]” means an urn that contains exactly x red balls, y blue balls, and no other balls. Similarly, “[Unknown red, Unknown blue]” means the urn contains an unknown number of red and blue balls and no other balls. For notational convenience, \( R = [50 \text{ red}, 50 \text{ blue}] \) and \( A = [\text{Unknown red}, \text{Unknown blue}] \).

Subjects were informed that the contents of urn \( A \) would vary from question to question (i.e., the contents of ambiguous urns are re-determined between questions). In practice, the contents of each urn \( A \) were determined by drawing an integer \( X \) uniformly at random between 0 and 100. A virtual urn containing \( X \) red balls and 100\( -X \) blue balls was created. Subjects were not informed of this procedure to determine the contents of ambiguous urns.

The following sections first describe this study’s measurement procedure, how it is incentivized, and how the experiment was designed to correct for measurement error.

2.2 Incentive Mechanism and Measurement Procedure

We elicit the subjects’ certainty equivalents using a multiple price list (MPL) to determine their preferences over various acts. Each question introduces a gamble as detailed above.

Subjects are then presented with a table containing 31 rows labeled “Fixed payment: x tokens.” The x values range from 0 to 300, in increments of 10.
In each row, subjects select either the left column ("Receive fixed payment") or the right column ("Play the gamble"). To make the process less time-consuming and enforce the consistency of choices, the subject's selection in each row is automatically completed based on a limited number of clicks. For example, if a subject clicks to indicate a preference for 150 tokens instead of the gamble, the software automatically completes rows 160 through 300 to indicate that the subject also prefers receiving tokens to the gamble. Similarly, if the subject prefers the gamble instead of receiving 140 tokens, the software automatically completes rows 0 through 130 to indicate a preference for playing the gamble over receiving tokens. Subjects can revise their choices (consistent with the autocompletion rules above) before moving on to the next question.

Each question contains, at most, one row in which the subject's preference switches from preferring the gamble to preferring a specific amount of tokens. The subject's certainty equivalent for the gamble must lie between the token amounts listed in this row and the previous row. We then record the subject's certainty equivalent as the \textit{midpoint} between the two rows, i.e., a number ending in 5.\footnote{If the subject prefers the gamble over 300 tokens or prefers 0 tokens to the gamble, then no such "switching" row exists. Nonetheless, if the subject prefers the gamble over a fixed payment of 300 tokens, their certainty equivalent may be considered 300 tokens, as the gamble cannot pay more than 300 tokens. Similarly, if the subject prefers 0 tokens to the gamble, their certainty equivalent is 0. In these cases, we record the subject's certainty equivalent as 300 or 0, respectively.}

Fourteen questions are selected uniformly at random for payment from among all the questions in a given treatment to make this mechanism incentive compatible.\footnote{Some experiments eliciting risk attitudes select only a single question for payment, avoiding the possibility of subjects using their choices in different questions to hedge their payoffs; however, doing so creates a significant variance in the monetary payments that different subjects receive, which was undesirable for this experiment.} If a question is selected for payment, then one row of that question's MPL table is selected at random, and the subject is given whatever their preference in that row.\footnote{For example, if row 120 was selected and the subject preferred the gamble to 120 tokens, then the gamble is simulated, and the subject wins the prize (usually 300 tokens) or receives 0 tokens if they lose. If the subject preferred 120 tokens to the gamble, they would receive 120 tokens.} To eliminate the possibility of wealth effects and ensure that subjects did not "learn" the distribution used to resolve ambiguity, the payoffs for each question (as well as which questions were selected for payment) were not determined until after the subject completed the entire experiment.\footnote{Subjects were invited to practice with the MPL mechanism (before the experiment) and see a summary of the results; they were informed that these practice questions would not be selected for payment. Furthermore, none of these questions involved ambiguity; hence, none presented an opportunity to learn how this experiment resolved ambiguity.} At the end of the experiment, subjects were presented with a table summarizing the questions selected for payment, the row selected in that
question’s MPL, the subject preference in that row, and (if they preferred the gamble) whether they won the gamble. At the end of the experiment, the subject’s total payment was $1 for every 100 tokens earned, plus a fixed payment of $10 for participation.

When agents do not make choices that correspond to the expected utility theory predictions, using the MPL mechanism may be problematic. For example, Karni and Safra (1987) showed that incentive-compatible mechanisms could not elicit certainty equivalents if the independence axiom does not hold.

Despite this concern, the MPL mechanism has been used extensively in experiments where agents face risk or ambiguity when making choices, many of which included the possibility of their choices over lotteries not satisfying the predictions of expected utility theory. This is perhaps because the MPL offers several advantages over other mechanisms. Andreoni and Kuhn (2019) argue that the MPL mechanism is very easy for subjects to understand and yields more consistent choices than other standard mechanisms for eliciting risk preferences. Furthermore, it grants externally valid predictions once adjusted for measurement error.

2.3 Double Elicitations, Measurement Error and Attention Screeners

With this in mind, we employ the MPL mechanism to elicit subjects’ preferences; however, as Gillen et al. (2019) demonstrate, laboratory experiments eliciting subjects’ certainty equivalents for gambles are often subject to significant measurement error. If not considered, this measurement error can create significant bias in estimated correlations and regression coefficients.

Methods to correct for such measurement error involve eliciting subjects’ certainty equivalents twice for each gamble of interest. Although many techniques can then be used to eliminate the bias in the estimation of coefficients and correlations; the Obviously Related Instrumental Variables (ORIV) approach proposed in Gillen et al. (2019) generally estimates these parameters with lower standard errors. Essentially, this estimation entails using multiple instrumentation strategies at the same time, then combining the results. Therefore, this experiment uses the ORIV approach to estimate correlations and coefficients. In particular, we double-elicit subjects’ CEs for all gambles of central importance to our analysis; however, due to time constraints and concerns that subjects may “zone out” and provide especially noisy answers if asked too many repeated similar questions, we could not double-elicit CEs for all of the less major gambles. See Table 1 for details.
Due to the complex nature of some of the questions, there is concern that some subjects may not comprehend the questions or may simply be answering largely at random to complete the experiment quickly. Although most of the financial reward comes from incentivized MPL questions, there is a small fixed reward for merely completing the experiment. To avoid this concern, subjects were screened based on three criteria:

(1) After receiving general instructions concerning the experiment, subjects were given a basic comprehension quiz with three questions regarding those instructions. Subjects unable to correctly answer the three questions were removed from the experiment.

(2) Between each of the experiment’s major sections, subjects were given a standard attention-screening question.

(3) If, in the course of our double elicitation of a subject’s preferences, two reported certainty equivalents for the same question differed by more than 100 tokens—that is, one third the size of the 300-token table—then the subject was deemed to be paying insufficient attention to the experiment.

Subjects failing criterion (1) were immediately removed from the experiment and received a minimum payment. Subjects failing at least one of the attention-screening questions in (2) were subsequently removed. Finally, subjects deemed to be paying insufficient attention according to (3) were removed. As a result, out of an initial 880 subjects, 172 were excluded from our data set.

2.4 Description of Elicitations

Each treatment is divided into blocks consisting of one or multiple gambles for which the subject must report their CE. The contents of each block are presented below.

See Section 2.5 for a description of which blocks are used in which treatments.

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7See Section 3.1 for details.
8They received a small fixed amount for their two-minute participation and were made aware of this scenario when they gave their consent.
9Other thresholds for exclusion, such as “differed by more than 150 tokens,” yield qualitatively similar results to those below.
10See section 3.1 for details.
11We will attach the symbol D to the name of an elicitation when we refer to a duplicate of this later.
2.4.1 Classical Ellsberg

Ellsberg  This block contains a replication of the classic Ellsberg experiment, in which subjects provide their certainty equivalents for two gambles. In each gamble, they choose a color (red or blue) and win if a single ball drawn at random matches their selection. The gambles are as follows:

(R2) The urn is [50 red, 50 blue].

(A2) The urn is [Unknown red, Unknown blue].

EllsbergD  This block is a duplicate of the Ellsberg block where all instances of the urn [50 red, 50 blue] are replaced with [25 red, 25 blue].

2.4.2 2-Ball Ellsberg

2Ball  This block contains this experiment’s central questions. Subjects provide their certainty equivalents for four gambles, each containing a ”1st urn” and a ”2nd urn,” where these two urns may be the same urn. A ball is drawn randomly from the 1st urn and returned to the same urn. Then a ball is drawn randomly from the 2nd urn. Subjects win if the two balls are the same color. The gambles are as follows:

(R2) 1st urn = [50 red, 50 blue], 2nd urn = 1st urn (i.e., the ”2nd urn” is the same urn as the 1st urn.)

(A2) 1st urn = [Unknown red, Unknown blue], 2nd urn = 1st urn.

(RA) 1st urn = [50 red, 50 blue], 2nd urn = [Unknown red, Unknown blue].

(AR) 1st urn = [Unknown red, Unknown blue], 2nd urn = [50 red, 50 blue].

2BallShort  This block contains questions R2 and A2 from the 2Ball block.\textsuperscript{12}

2BallD  The 2BallD block is a duplicate of 2Ball where all instances of the urn [50 red, 50 blue] are replaced by [40 red, 40 blue].

2BallShortD  This block is a duplicate of 2BallShort where all instances of the urn [50 red, 50 blue] are replaced by [40 red, 40 blue].

\textsuperscript{12}Due to time constraints, three of four treatments use 2BallShort instead of 2Ball; subjects’ CEs for R2 and A2 have more central importance than RA and AR.
2.4.3 Compound Lotteries

**Compound** This block contains questions about compound lotteries. Subjects provide their certainty equivalents for two gambles involving a new urn $N$. They are informed that urn $N = [X \text{ red}, 100 - X \text{ blue}]$ where $X$ is an integer drawn uniformly at random between 0 and 100. The gambles are as follows:

(C) Subjects pick a color (either red or blue), and a ball is randomly drawn from urn $N$. Subjects win if the ball matches the selected color.

(C2) A ball is drawn randomly from urn $N$, then put back. A second ball is then drawn at random from urn $N$. Subjects win if both balls are the same color.

2.4.4 Bounded 2-Ball Ellsberg

**Bounded** This block is similar to 2Ball, except balls are drawn from ambiguous urns with “bounds” on the number of balls of certain types contained. Subjects provide their certainty equivalents for three gambles. In each gamble, there is an urn $A = [\text{Unknown red}, \text{Unknown blue}]$ from which two balls are drawn with replacement. Subjects win if the two balls are the same color. The gambles are as follows:

$(B2^{40-60})$ Urn $A$ is known to contain between 40 and 60 red balls (but its contents are otherwise unknown).

$(B2^{60-100})$ Urn $A$ is known to contain between 60 and 100 red balls (but its contents are otherwise unknown).

$(B2^{95-100})$ Urn $A$ is known to contain between 95 and 100 red balls (but its contents are otherwise unknown).

2.4.5 Three-Ball Ellsberg

**3Ball** This block contains questions similar to 2Ball, except with three urns instead of two. Subjects provide their certainty equivalents for three gambles. In each gamble, there is a “1st urn,” “2nd urn,” and “3rd urn,” and any of these three urns may be the same urn. A ball is drawn randomly from the 1st urn and returned to the same urn. Another ball is drawn randomly from the 2nd urn and returned to the same urn. Finally, a ball is drawn at random from the 3rd urn. Subjects win if the three balls are all the same color. The gambles are as follows:

$(R3)$ 1st urn = [50 red, 50 blue], 2nd urn = 1st urn, 3rd urn = 1st urn.
(A3) 1st urn = [Unknown red, Unknown blue], 2nd urn = 1st urn, 3rd urn = 1st urn.
(RAA) 1st urn = [50 red, 50 blue], 2nd urn = [Unknown red, Unknown blue], 3rd urn = 2nd urn.

2.4.6 2-Ball Ellsberg With Independent Urns

Independent  This block contains a single question:

(Ia) There are two urns, A1 = [Unknown red, Unknown blue] and A2 = [Unknown red, Unknown blue], whose contents are determined independently. One ball is drawn at random from each of A1 and A2. Subjects win if the two drawn balls are the same color.

2.5 Treatments

The experiment consists of four treatments divided into blocks of questions. Most questions are asked twice, in two separate blocks (indicated with a D in the name of the block), to double-elicit the subject’s certainty equivalent.

Table 1: treatments

| Treatment | Contents of Treatment |
|-----------|----------------------|
| 1         | [(Ellsberg, 2Ball),  |
|           | (EllsbergD, 2BallD)] |
| 2         | [(Ellsberg, 2BallShort, Compound),  |
|           | (EllsbergD, 2BallShortD, CompoundD)] |
| 3         | (BoundedU,  |
|           | [(Ellsberg, 2BallShort), (EllsbergD, 2BallShortD)] ) |
| 4         | (Ellsberg, 2BallShort),  |
|           | (3Ball, Independent),  |
|           | (EllsbergD, 2BallShortD) ) |

Table 1 summarizes the structure of each treatment. Each item in bold is one of the blocks described in Section 2.4. Multiple items within parenthesis ( ) mean that
the order of these items is determined uniformly at random, independently for each subject. Items within brackets [ ] are not randomized; they always appear in the order listed within the brackets.

In each treatment, we double-elicit subjects’ certainty equivalents for the two classic Ellsberg gambles as well as the two 2-Ball gambles in the 2BallShort block (which also appear within its longer version 2Ball). Thus, using data from all four treatments, we can robustly determine if subjects prefer R2 (or R) over A2, even though the latter is more likely to win. Furthermore, by comparing a subject’s responses to these 2-Ball gambles with their responses to the classic Ellsberg gambles, we can determine the relationship between ambiguity aversion, risk aversion, and “falling for” the 2-Ball Ellsberg paradox.

Each of the four treatments also contains additional questions specific to that treatment, summarized below.

2.5.1 Treatment 1

Recall that the 2Ball block elicits subjects’ CEs for 2-Ball gambles involving urn combinations R2, A2, RA, and AR.13

Treatment 1 aims to determine the relationship between subjects’ CEs for these four gambles. In particular, if subjects report higher CEs for R2 than AR (despite each winning with 50% probability), this suggests that subjects are ”scared away” from gambles involving ambiguous urns. Additionally, if subjects report lower CEs for RA than AR, this perhaps suggests that subjects fear that the contents of urn A may only be determined after the first ball has been drawn from urn R, even though the instructions explain this not to be the case.

2.5.2 Treatment 2

Recall that the Compound block consists of two gambles: Halevy (2007)’s compound lottery C and a ”2-Ball Halevy” gamble C2. Gamble C2 is the same as the ambiguous gamble A2 from 2Ball, except its urns’ contents are determined by a known lottery rather than an unknown, ambiguous procedure.

The purpose of Treatment 2 is threefold. First, question C replicates Halevy (2007)’s experiment; it allows us to determine which subjects have certainty equivalents for gamble C that are not the same as those for a simple 50% lottery, such as gamble R

13The CEs for the R2 and A2 gambles are also elicited in all other treatments, in the shorter 2BallShort block.
from the Ellsberg block. Thus, we can re-test Halevy (2007) and Gillen et al. (2019)’s conclusions that ambiguity aversion in the classic Ellsberg paradox is closely linked to such failure to reduce compound lotteries. Second, Treatment 2 may allow us to extend these results by determining how these two modes of “irrational” behavior are linked to the “irrational” preference for R2 over A2 in block 2Ball.

Third, question C2 allows us to test if the “irrational” preference for R2 over A2 in block 2Ball is either

- due to a preference to avoid the ambiguity present in A2 or
- due to a preference to avoid the complexity in both A2 and C2 (or, perhaps, due to a lack of understanding of both of these gambles).

A subject preferring R2 over A2 and C2 to A2 would suggest that they try to avoid ambiguity more than they seek to avoid complexity. Similarly, a subject preferring R2 to both A2 and C2 would suggest that the ”irrational” choice of R2 is primarily explained by an aversion to complexity or a lack of understanding.

2.5.3 Treatment 3

Recall that BoundedA block elicits subjects’ CEs for three gambles. Each gamble is the same as A2 from 2Ball, except the ambiguous urn’s contents fall within a specific, bounded range in each gamble. Gamble B240−60 has “less ambiguity” than gamble B260−100 because the urn’s range of possible compositions is smaller in B240−60 than B260−100. Additionally, B260−100 is guaranteed to have a win probability at least as high as B240−60. Furthermore, B295−100 has a win probability of at least .905, which is likely higher than the win probability of gamble B260−100. Thus, a ”rational” subject should prefer B295−100 to B260−100 and B240−60.

The purpose of Treatment 3 is threefold. First, it allows us to determine the relationship between ”falling for” the 2-Ball Ellsberg paradox and preferring B240−60 to the other gambles in BoundedA; that is, making a choice that is even worse than the wrong choice in 2BallShort.

Second, by comparing the responses to 2BallShort across subjects who either (i) completed block BoundedA before 2BallShort or (ii) completed BoundedA after 2BallShort, we can determine if the process of answering the questions in BoundedA has a ”teaching” effect, i.e., if completing BoundedA before 2BallShort decreases the likelihood or strength of a subject falling for the 2-Ball Ellsberg paradox.14

14One might suspect that a subject completing BoundedA could learn to prefer more unevenly dis-
Third, Treatment 3 may allow us to rule out certain explanations for subjects falling for the 2-Ball Ellsberg paradox. Specifically, if a subject prefers $B_2^{40-60}$ to $B_2^{60-100}$, this suggests that either

(a) they prefer avoiding the ”larger amount of ambiguity” in $B_2^{60-100}$, even if this means sacrificing some probability of winning, or

(b) they prefer more ”evenly-distributed” (i.e., closer to 50-50) urns for this type of gamble, perhaps because they do not realize that a more unevenly distributed urn has a higher win probability.

If (b) is true but (a) is false, then the subject should prefer $B_2^{40-60}$ to $B_2^{60-100}$ and also prefer $B_2^{60-100}$ to $B_2^{95-100}$. If they do not, this suggests their preference for $B_2^{40-60}$ over $B_2^{60-100}$ is not from simply failing to calculate win probabilities correctly but instead is a deliberate sacrifice of win probability to avoid the increased ambiguity of $B_2^{60-100}$.

### 2.5.4 Treatment 4

Treatment 4 contains two blocks not appearing in other treatments: **3Ball** and **Independent**.

In **3Ball**, subjects report their CEs for the 3-Ball gambles $R_3$, $A_3$, and $RAA$. These questions are designed with two purposes in mind. First, the combination of questions $R_3$ and $A_3$ allows us to compare the ”ambiguity premium” (positive or negative) of a Three-Ball gamble to the ambiguity premium of a 2-Ball gamble determined in **2Ball**.

Second, question $RAA$ grants two useful comparisons. Comparing the $RAA$ and $A_3$ answers indicate how a subject’s ambiguity premium changes as the ”amount” of ambiguity increases. Similarly, comparing $RAA$ to $A_2$ from block **2Ball** allows us to determine if a subject’s certainty equivalent for $RAA$ is above, below, or equal to half the certainty equivalent of $A_2$, as $RAA$ has precisely half the win probability of $A_2$.

In **Independent**, subjects report their CEs for a 2-stage gamble, $IA$, where the two balls are drawn from separate ambiguous urns whose contents are determined independently. This block is designed for two reasons. First, comparing subjects’ CEs for distributed urns to more evenly distributed ones when considering a 2-Ball gamble, as (among other things) **BoundedA** asks the subject to consider how likely they are to win a 2-Ball gamble when the urn is at least 95% red balls.  

15The Three-Ball gamble where all balls are drawn from urn $R$ has only half the win probability of the analogous 2-Ball gamble; hence, comparisons of ambiguity premia must be adjusted accordingly. See Sections 3.4.1 and 3.4.2 for further details.
IA and A2 allows us to determine if they mistakenly interpret the two draws from the same ambiguous urn to be the same as the two draws from independent ambiguous urns. Second, since (as long as the subject believes the ambiguity is color-neutral) the gamble IA is equivalent to the “1-Ball” ambiguous gamble, A, from Ellsberg\textsuperscript{16}, comparing certainty equivalents for these two gambles allows us to test if subjects are indeed indifferent. We can then determine how a preference between these two gambles explains choosing R2 over A2 in 2Ball.

3 The Data

3.1 Data Sample Details and Summary Statistics

We used Prolific to run our experiment and collect our data\textsuperscript{17}. Prolific is an online survey platform that, due to its participant pool’s quality, is increasingly used in economics and other social sciences to run surveys and incentivized experiments.\textsuperscript{18} Our sample comprised 880 participants, selected to be nationally representative in age and gender. Of these initial 880 participants, 708 passed the basic attention-screening questions and criteria described in Section 2.3. All 708 received a $2 fixed participation payment, and they averaged an additional $3.50 in bonus payments from the incentivized parts of the experiment.

This experiment’s MPL table contains 31 rows corresponding to fixed prize values between 0 and 300 tokens, in increments of 10 tokens. There are 32 possible locations where a subject can place their “cutoff” (below which they prefer the gamble and after which they prefer the fixed prize). If a value $x \in \{0, 10, \ldots, 290\}$ exists such that the subject prefers the gamble to receiving $x$ tokens but prefers receiving $x + 10$ tokens to the gamble, then this was recorded numerically as “the subject’s certainty equivalent is $x + 5$.” If the subject preferred 0 tokens to the gamble, the certainty equivalent was 0. Finally, if the subject preferred the gamble to 300 tokens, the certainty equivalent was 300.

To correct for measurement error, we double-elicit subjects’ certainty equivalents

\textsuperscript{16}To see that these gambles are equivalent, fix the first ball drawn (from A1) in Independent. If the first ball is red, the subject wins if one ball drawn from an ambiguous urn (A2) is red. If the first ball is blue, the subject wins if one ball drawn from an ambiguous urn is blue. Thus, as long as the subject believes that the ambiguity is color-neutral, the gamble in Independent is the same as gamble (2) of Ellsberg, regardless of which color ball is drawn from A1.

\textsuperscript{17}Data was collected on Prolific in May 2020.

\textsuperscript{18}Please go to Appendix for more details

for most of the gambles. For notational convenience in the analysis below, superscripts denote the elicitation number for all double-elicited variables.

Table 2 summarizes the naming conventions for the various certainty equivalents. Variables with a superscript \(j\) were elicited twice; others were elicited only once. The variable \(R^j_i\) is the \(j\)th elicitation of subject \(i\)’s CE for a simple 50-50 gamble. For convenience, we will let

\[
U_i = \frac{U_i^1 + U_i^2}{2}
\]

denote the average certainty equivalent \(R\) reported by subject \(i\) across the two elicitation. Similarly, when we mention any variable that was double-elicited but exclude the superscript, we refer to the average value of that variable across the two elicitations.

| Name | Description |
|------|-------------|
| \(R^j\) | \(j\)th elicitation of CE for 50-50 urn of Ellsberg |
| \(A^j\) | \(j\)th elicitation of CE for ambiguous urn of Ellsberg |
| \(R_{2j}\) | \(j\)th elicitation of CE for 50-50 urn in 2Ball |
| \(A_{2j}\) | \(j\)th elicitation of CE for ambiguous urn in 2Ball |
| \(AR^j\) | \(j\)th elicitation of CE for "1st urn=A, 2nd=R" gamble of 2Ball |
| \(RA^j\) | \(j\)th elicitation of CE for "1st urn=R, 2nd=A" gamble of 2Ball |
| \(R3\) | CE for 3Ball with all three urns = R |
| \(A3\) | CE for 3Ball with all three urns = A |
| \(RAA\) | CE for 3Ball with 1st urn = R, latter two urns = A |
| \(IA\) | CE for Independent (2-Ball gamble with independent ambiguous urns) |
| \(C^j\) | \(j\)th elicitation of CE for single-urn gamble of Compound |
| \(C_{2j}\) | \(j\)th elicitation of CE for 2-Ball gamble of Compound |
| \(B_{240-60}\) | CE for BoundedA with ambiguous urn containing 40–60 red balls |
| \(B_{260-100}\) | CE for BoundedA with ambiguous urn containing 60–100 red balls |
| \(B_{295-100}\) | CE for BoundedA with ambiguous urn containing 95–100 red balls |

Table 2: raw variable names

Table 7 presents summary statistics of all of the variables that were double-elicited in the experiment: \(R, A, C, R2, A2, C2, RA,\) and \(AR\). The mean and 95% confidence interval for each of the two elicitation of that variable are presented below each vari-

\(^{19}\)For a few gambles, we only single-elicit values. Due to constraints on how long subjects were expected to spend on the experiment, we could not double-elicit all certainty equivalents. Hence, a few items of secondary interest are only single-elicited.
able. The correlation between the two elicitations (and its standard error) is presented below the confidence intervals. These correlations range between .855 and .883, suggesting that subjects are fairly consistent in their answers when asked the same question twice.\footnote{As mentioned in Section 2.3, answers for the same question that differed by more than 100 tokens were removed from the data set because the subjects seemed to be paying less attention. See Appendix ?? for analysis that includes these subjects.}

Table 8 contains similar summary statistics for the derived variables, such as $U - A$, that measure how much subjects prefer a simple 50-50 gamble over other gambles that are at least as good. From it, we see that the classic Ellsberg paradox is replicated in this experiment; subjects prefer the 50-50 gamble $R$ to the ambiguous gamble $A$. Averaging across elicitations, $R - A$ takes an average value of 12.31 cents ($t = 10.03$). Similarly, Halevy (2007)'s experiment is replicated; subjects prefer the simple 50-50 gamble $R$ to the compound 50-50 gamble $C$ by an average of 6.82 cents ($t = 3.26$).

### 3.2 2-Ball Ellsberg Paradox

The variable $R - A_2$ measures how subjects prefer the simple 50-50 gamble $R$ to the 2-Ball ambiguous gamble $A_2$. On average across subjects, $R - A_2$ is positive (mean = 16.41) and statistically significant ($t = 10.66$). This means that subjects are typically willing to pay about 16 cents more for $R$ than $A_2$, even though the latter is more likely to win.

This difference is not merely statistically significant but also of noticeable size. All of the variables listed in Table 7 have standard deviations between 55.00 (namely, $AR$) and 60.15 (namely, $A_2$). Therefore, a 16.41 difference represents a change of more than 0.27 standard deviations, or just over a 10 percentile change from the center of a Normal distribution. Although this change is not enormous, it is quite noticeable.

Figure 1 shows the complete distribution of $R - A_2$ for each elicitation.
Figure 1: Histogram of $R - A_2$, by elicitation

### Table 3: Relationships between CE differences

| Dependent Variable: $R - A_2$ | $R - A$ | $R - R_2$ | $R - C$ | $R - CC$ |
|-------------------------------|--------|---------|--------|--------|
| ORIV $\rho$                  | 0.892  | 0.952   | 0.954  | 0.917  |
|                               | (0.017)| (0.012) | (0.024)| (0.032)|
| $N$                           | 708    | 708     | 158    | 158    |

We now analyze the relationship between the preference for $R$ over $A_2$, (a) a general preference for "simplicity," and (b) ambiguity aversion in the classic Ellsberg paradox.

The variable $R - A$ measures subjects’ ambiguity aversion in the classic Ellsberg paradox, while $R - C$ measures their preference for a simple 50-50 gamble over a compound 50-50 gamble. $R - R_2$ measures subjects’ preference for a 1-Ball 50-50 gamble.
to a 2-Ball 50-50 gamble. The variables $R - C$ and $R - R2$ are not statistically different ($t = .614$), suggesting that they both measure subjects’ aversion to complexity; the first measures aversion to compound lotteries, while the second measures aversion to multi-draw gambles.\footnote{Although the 2-Ball lottery $R2$ can itself be thought of as a compound lottery, it has many fewer possibilities to consider than Halevy (2007)’s compound lottery $C$.}

$R - C2$ measures subjects’ preference for a simple 50-50 gamble over a gamble that is both compound and 2-Ball ($C2$). $R - C2$ is similar to $R - A2$, except the urn in $A2$ has ambiguous contents.

Table 3 shows that the ORIV-corrected correlations between $R - A2$ and each of $R - A$, $R - R2$, $R - C$, and $R - C2$ are quite high; however, $R - A2$ is larger than all of $R - A$, $R - R2$, and $R - C$ ($t > 4$ in all cases), but is only larger than $R - CC$ by a statistically insignificant amount ($t = 1.05$).

This insignificant difference suggests that subjects view $C2$ and $A2$ as somewhat similar, paying only a slightly larger premium to avoid $A2$ than its non-ambiguous analog $C2$; however, subjects’ preference for simplicity in choosing $R$ over the 2-Ball gamble $R2$ cannot explain the choice of $R$ over $A2$. Similarly, the choice of $R$ over the compound gamble $C$ and subjects’ ambiguity-averse choice of $R$ over $A$ are insufficient. The strength of subjects’ preference for $R$ over $A2$ can seemingly be explained by combining the compound nature, the 2-Ball feature, and possibly the ambiguity in $A2$.

One conceivable explanation for these aversions (compound nature, 2-Ball feature, ambiguity, or any combination thereof) is that subjects do not understand the presented gambles. Gamble $A2$ is one of the most complicated gambles in the experiment, so the preference for $R$ over $A2$ may ultimately be entirely due to a lack of understanding that $A2$ must have at least as high of a win probability as $R$. Perhaps subjects prefer simpler gambles or gambles involving urns whose contents are known to be 50-50 due to a lack of understanding. There are three possible hypotheses to explain the preference for $R$ over $A2$:

(A) Subjects do not understand that a 2-Ball gamble with an urn whose contents are not 50-50 is more likely to win than a simple 50-50.

(B) Subjects do not understand that in a 2-Ball gamble, a more “unevenly” distributed urn increases the win probability.

(C) Subjects at least partially understand both (A) and (B), but they prefer $R$ to $A2$, perhaps because $R$ is simpler and less ambiguous.
The **Bounded** and **Independent** blocks were designed to test these hypotheses. The data from **Bounded** suggest that (A) and (B) are implausible, making (C) the most plausible hypothesis. In contrast, the data from **Independent** questions whether subjects’ understanding of the gambles is complete and universal.

The **Bounded** block consists of gambles identical to the 2-Ball ambiguous gamble, $A_2$, except the subject is given a particular range for the ambiguous urn’s percentage of red balls. Gamble $B_2^{40-60}$’s ambiguous urn contains 40 to 60 red balls, $B_2^{60-100}$ has 60 to 100 red balls, and $B_2^{95-100}$ has 95 to 100 (in all cases, there are 100 balls total). Table 4 summarizes subjects’ certainty equivalents for these gambles.

|       | $B_2^{40-60}$ | $B_2^{60-100}$ | $B_2^{95-100}$ |
|-------|---------------|----------------|----------------|
| **Mean** | 98.603         | 132.235        | 207.654        |
| **SD**  | (52.113)       | (63.887)       | (90.784)       |
| **N**   | 179            | 179            | 179            |

Table 4: **Bounded-Ambiguity 2-Ball Gambles**

If hypothesis (B) is true, subjects should prefer $B_2^{40-60}$ to $B_2^{60-100}$, as the former has an ambiguous urn whose contents are closer to 50-50; however, the opposite is the case. Subjects prefer $B_2^{60-100}$ to $B_2^{40-60}$ by an average of 33.6 cents ($t > 9$). Similarly, they prefer $B_2^{95-100}$ to $B_2^{60-100}$ by an average of 75.4 cents ($t > 13$).

Indeed, even those 22 subjects who reported a larger certainty equivalent for $B_2^{40-60}$ than $B_2^{60-100}$ showed a preference for $B_2^{95-100}$ over $B_2^{60-100}$ (mean = 68.64, $t = 3.36$). This indicates that the former preference cannot be entirely explained by failing to comprehend the gambles or having a universal preference for urns closer to 50-50.

These significant differences suggest that subjects understand that having a more "unevenly" distributed urn is advantageous in 2-Ball gambles. This makes hypothesis (B) implausible. Similarly, hypothesis (A) seems implausible, as subjects strongly prefer $B_2^{60-100}$ or $B_2^{95-100}$ to the simple 50-50 gamble $R$ ($t > 3.9$ in both cases). This suggests that hypothesis (C) is the most plausible; subjects at least partially understand that $A_2$ is more likely to win than $R$, but they prefer to avoid its complexity and ambiguity.

Interestingly, subjects generally prefer $R$ to $B_2^{40-60}$ (mean = 16.47, $t = 5.62$) even though the latter has a larger win probability. Since a total lack of understanding cannot explain this difference, it seems more plausible that it comes from a preference to avoid the complexity and ambiguity in $B_2^{40-60}$. This preference to avoid com-
plexity/ambiguity is not sufficiently strong to cause a preference for \( R \) over \( B^{260-100} \) (which has a win probability somewhere between 52\% and 100\%); however, it is strong enough to cause a preference for \( R \) over \( B^{40-60} \) (which has a win probability only between 50\% and 52\%).

The **Independent** block consists of the single gamble \( IA \), which is identical to \( A_2 \) except that the two draws in gamble \( IA \) are from *separate ambiguous urns* whose contents are determined independently. For this gamble, the mean of subjects’ certainty equivalents is 107.839, and its standard deviation is 68.733.

Suppose subjects fully understand both gambles \( A_2 \) and \( IA \). In that case, they must realize that \( IA \) is (in terms of ultimate win probabilities) equivalent to the 1-Ball ambiguous gamble \( A). For any given realization of the contents of urn \( A \), \( A_2 \) has a win probability at least that of \( A \). If subjects only care about win probabilities and fully understand all the gambles, they should value \( IA \) and \( A \) identically and value \( A_2 \) the most.

In reality, among the 192 subjects in Treatment 4 (those asked about \( IA \)), \( A \) is slightly preferred to \( A_2 \); however, the figure is not statistically significant (mean = 2.63, \( t = 1.18 \)). Similarly, these subjects slightly prefer \( A_2 \) to \( IA \), but the preference is not significantly significant (mean = 1.73, \( t = .60 \)). Furthermore, when combining these two preferences, \( A \) is not significantly larger than \( IA \) (mean = 4.36, \( t = 1.36 \)).

Interestingly, among the entire pool of 708 subjects across all four treatments, subjects slightly prefer \( A \) to \( A_2 \) in a way that is statistically significant (mean = 4.79, \( t = 4.03 \)). This result suggests that an inadequate sample size may cause the lack of statistical significance between \( A \) and \( A_2 \) (perhaps also between \( A \) and \( IA \)) in Treatment 4. Altogether, this sample fails to disprove that subjects treat gamble \( IA \) differently from either \( A \) or \( A_2 \), questioning whether subjects fully understand all the gambles.

### 3.3 Distaste for the Presence of Ambiguity

Recall that \( RA \) is a 2-Ball gamble in which subjects win if matching balls are drawn first from a 50-50 urn and second from an ambiguous urn. \( AR \) is the same gamble, but the order of the urns is reversed; \( R_2 \) is the same gamble, except both draws are from a 50-50 urn. Both \( RA \) and \( AR \) have a 50\% win probability, so if subjects understand these gambles and care only about win probability, their certainty equivalents should be indistinguishable from those for the simple 50-50 gamble \( R \).

Subjects’ average certainty equivalents for the 2-Ball gambles \( RA \), \( AR \), and \( A_2 \) are

---

22Assuming the subject chooses red or blue at random in gamble \( A \)
not statistically different. The largest of these differences is between $AR$ and $RA$ and is not significant (mean = 1.80, $t = 1.39$); however, $R2$ is significantly larger than $RA$ (mean = 11.33, $t = 5.89$), and $A$ is also significantly larger than $RA$ (mean = 4.27, $t = 2.10$).\footnote{From Section 3.1, this implies that the average difference between $R$ and $RA$ is even larger than that between $A$ and $RA$, by about 12.3 cents.}

Notice that gambles $R2$ and $RA$ have a similar level of complexity, both being 2-Ball gambles; the only difference between them is that $RA$ involves a draw from an ambiguous urn. Furthermore, both gambles have the same 50% win probability. This means that subjects’ preference for $R2$ over $RA$ can only be explained as a distaste for ambiguity\footnote{We do not use the term “ambiguity aversion” since none of the classical ambiguity aversion models permit an agent to strictly prefer $X$ over $Y$, even though both yield a win probability of 50% in all states of the world.}, as the presence of an ambiguous urn is the only difference.

Subjects exhibit a statistically significant preference for $R2$ over $A2$ (which can only have a larger win probability); however, they do not exhibit a significant preference for $A2$ over $RA$. This result suggests that win probabilities have little effect on subjects’ preferences in these cases, at least relative to the other forces at play. Furthermore, from Section 3.2 we know that subjects do not entirely fail to understand that two draws from a single ambiguous urn have a higher chance of winning than two draws from a 50-50 urn. Thus, a distaste for the mere presence of ambiguity, or the complexity created by the mere presence of ambiguity, seems to be the only adequate explanation for the subjects’ preferences.

Our claim that subjects prefer to avoid ambiguity (even when it can only improve their odds of winning) is deliberate is reinforced because subjects did not seem to “learn” to choose $A2$ over $R$. This result held even when subjects were prompted with questions in the BoundedA block that demonstrated the principle that more “unevenly” distributed urns yield larger win probabilities in 2-Ball gambles.

Recall from Section 3.2 that subjects strongly preferred gamble $B2_{95−100}$ to $B2_{60−100}$ to $B2_{40−60}$. This preference shows that, by the time they had completed this block, subjects had at least some understanding that more unevenly distributed urns yield higher win probabilities in 2-Ball gambles. It stands to reason that if a subject did not understand this point before completing block BoundedA, they may have come to understand it during that block. Thus, if subjects exhibit a preference for $R$ over $A2$ because of a lack of understanding (such that, if subjects understood them, then they would prefer $A2$ to $R$), then we may expect that completing block BoundedA could have a “learning” effect. This effect might cause subjects to report a more nega-
tive certainty equivalent difference $R - A2$ than if they had not already completed the \textbf{BoundedA} block.

Only subjects in Treatment 3 completed the \textbf{BoundedA} block. Since subjects were randomly assigned to one of the four treatments, if such a learning effect exists, it should manifest as a statistically significant difference between the $R - A2$ values in Treatment 3 versus those in the other treatments.

With this in mind, if we let $I^{T3}$ be the indicator variable for Treatment 3 participation, then in a regression of $Z := R - A2$ on $I^{T3}$, the slope coefficient should represent the causal effect of being in Treatment 3 on how subjects prefer $R$ over $A2$ despite the latter being more likely to win. A statistically significant negative slope coefficient would indicate that Treatment 3 has a learning effect, causing subjects to manifest less preference for $R$ over $A2$.

|        | $Z^1$  | $Z^2$  | $Z^{avg}$ |
|--------|--------|--------|-----------|
| $I^{T3}$ | 2.615  | -0.418 | 1.098     |
|        | (3.659)| (3.917)| (3.366)   |
| Const. | 16.994*** | 16.786*** | 16.890*** |
|        | (1.840)| (1.969)| (1.692)   |

$N$ 708 708 708

\begin{center}
Table 5: Learning Effects
\end{center}

Table 5 shows the results of such a regression, first using individual elicitations and then the averages across elicitations. As shown, the slope coefficient is not statistically significant, and it is positive in the case using averages. Thus, we fail to reject the hypothesis that there is no learning effect ($p = .63$).

There are two possible explanations for the lack of a learning effect despite subjects successfully identifying that more unevenly distributed urns are more likely to win in the \textbf{BoundedA} block:

(I) Subjects cannot see the similarity between gamble $A2$ and the "bounded" versions of this gamble in \textbf{BoundedA}, or upon arriving at gamble $A2$, they forget what they might have learned from \textbf{BoundedA} and fail to realize that $A2$ is more likely to win than $R$.

(II) In \textbf{BoundedA} and in comparing gambles $R$ and $A2$, subjects understand that a more unevenly distributed urn is more likely to win. Their preference for $R$ over
A2 is deliberate and due to a distaste for the ambiguity or complexity in A2.

Although hypothesis (I) is conceivably correct, it seems implausible that it can entirely explain the lack of a learning effect. Hence, a distaste for ambiguity or complexity must partially explain that subjects generally prefer R to A2.

3.4 Additional Results

3.4.1 Three-Ball Gambles and Overweighting

Recall that the block 3Ball consists of 3-Ball gambles; the subject wins the gamble if all three balls drawn have the same color. Subjects were asked about three gambles, R3, A3, and RAA, and the summary statistics for their certainty equivalents appear in Table 6.

|       | R3     | A3     | RAA    |
|-------|--------|--------|--------|
| Mean  | 97.708 | 91.120 | 92.552 |
| SD    | (67.310)| (69.264)| (68.172)|
| N     | 192    | 192    | 192    |

Table 6: 3-Ball gambles

These reported certainty equivalents are too large for a classical risk-averse or risk-neutral agent who correctly calculates the probabilities of winning. Notice that R3 has a win probability of exactly \( \frac{1}{4} \). Therefore if subjects based their certainty equivalents only on correct calculations of win probabilities (or only on this plus a distaste for complexity), then any risk-neutral or risk-averse subject would report a certainty equivalent for R3 that is at most \( \frac{1}{4} \) times 300, i.e., at most 75; however, subjects report an average certainty equivalent of 97.71 for R3, significantly larger than 75 (\( t = 4.67 \)). Thus, subjects cannot be correctly calculating the win probability of the 3-Ball gamble R3; they overweight this win probability and choose a certainty equivalent that is larger than the actual win probabilities would suggest.

Still, subjects report smaller certainty equivalents for R3 than for R2 (mean = 17.92, \( t = 5.38 \)); however, the average certainty equivalent for R3 is massively larger than half the average CE for R2 (mean = 39.90, \( t = 11.14 \)), even though R3 has exactly half the win probability of R2. Similarly, subjects report an average certainty equivalent for RAA that is smaller than A2 (mean = 17.02, \( t = 5.52 \)) but massively larger than half.
that of $A_2$ (mean = 37.77, $t$ = 11.03), even though $RAA$ has exactly half the win probability of $A_2$. Similar findings apply to $A_3$ versus $A_2$. The relationship between the win probabilities of $A_3$ and $A_2$ is not obvious. For any model for resolving ambiguity, $A_3$ has at least half the win probability of $A_2$. Under any model other than ”the ambiguous urn always contains exactly 50% red balls,” $A_3$ has a win probability strictly larger than half that of $A_2$. The exact ratio of win probabilities depends on the model.; however, the statistical results mentioned for the other 3-Ball versus 2-Ball comparison remain qualitatively true and are of similar numerical magnitude.

Despite the general overweighting of win probabilities, comparisons between certainty equivalents for these 3-Ball gambles remain qualitatively similar to the comparisons between the certainty equivalents for 2-Ball gambles discussed in Section 3.3. Similar to how subjects were indifferent between $RA$ and $A_2$, we find no statistically significant difference between $RAA$ and $A_3$ (mean = 1.43, $t$ = .56). Likewise, just as subjects on average preferred $R_2$ to $A_2$, we find that subjects generally prefer $R_3$ to $A_3$ (mean = 6.59, $t$ = 2.16), even though $A_3$ can only have a higher win probability than $R_3$. As before, this suggests subjects’ distaste for ambiguity.

3.4.2 Does the ”amount” of ambiguity matter?

On average, subjects report a larger certainty equivalent for the 1-Ball ambiguous gamble $A$ than the 2-Ball ambiguous gamble $A_2$ (mean = 4.79, $t$ = 4.03), even though the latter has a higher win probability. One possible explanation for this difference is that $A_2$ is more complex than $A$, causing subjects to prefer $A$; however, the preference for $A$ over $A_2$ also suggests the subjects’ dislike for gambles with more ambiguity, as measured by the number or proportion of ambiguous draws.

Recall from Section 3.3 that the difference between subjects’ average certainty equivalents for gambles $RA$ and $A_2$ was statistically insignificant. Similarly, in Section 3.4.1 we found that $RAA$ and $A_3$ were not statistically different. Unfortunately, these results do not imply that an increased number/proportion of ambiguous draws affects certainty equivalents. For example, when comparing $A_2$ to $RA$, the certainty equivalent for $A_2$ is may fall below that of $RA$ due to $A_2$ having ”more” ambiguity. Simultaneously, the certainty equivalent for $A_2$ also grows larger than that of $RA$ because $A_2$ has a higher win probability than $RA$. These two effects could precisely cancel each other out on average.

Therefore, we compare $RAA$ to $A_2$ to control for win probability differences. $A_2$ has a larger proportion of ambiguous draws than $RAA$; however, $RAA$ has precisely half the win probability of $A_2$, making win probability comparisons easier.
As discussed in Section 3.4.1, subjects seem to overweight the win probabilities of all 3-Ball gambles relative to analogous 2-Ball gambles. For example, averaging across subjects (and across elicitations for $R_2$), we find that

$$\frac{R_2}{R_3} = 1.122$$

even though this ratio should be precisely two if subjects were risk-neutral and based their certainty equivalents only on win probabilities. It should be even more significant if subjects are also averse to the additional complexity of the 3-Ball gamble $R_3$ compared to the 2-Ball gamble $R_2$.

Thus, when comparing $R_{AA}$ to $A_2$, instead of using a correction factor of 2 for each subject’s win probability differences, we use a subject-specific correction factor of $R_2/R_3$ to account for the combination of perceived win probability differences and possible complexity differences between 2-Ball and 3-Ball gambles. Hence, we test whether

$$R_{AA} \cdot \frac{R_2}{R_3} - A_2$$

is statistically greater than 0. If so, this implies that subjects do prefer gambles with a smaller proportion of draws from ambiguous urns (after correcting for perceived win probability and complexity differences). Running this test, we find a $t$-statistic of 1.577, with a $p$-value of .058 for a 1-sided test.

At a significance level of .05, we (barely) fail to disprove the hypothesis that a larger proportion of ambiguous draws is associated with a lower certainty equivalent after correcting for perceived win probability differences and possible complexity differences. Thus, similar to the comparisons between $RA$ and $A_2$ or between $A$ and $A_2$, it is unclear whether subjects care about the amount of ambiguity. Further experiments may be necessary to resolve this question more precisely.

4 Discussion

The results in Section 3.3 show that subjects generally prefer $R_2$ to $RA$. Since the two gambles have equal win probability and are of similar complexity, this preference for $R_2$ over $RA$ must be explained in terms of a distaste for the presence of ambiguity.

Qualitatively, this preference for $R_2$ over $RA$ is somewhat similar to the preference for $R$ over $C$ in Halevy (2007)’s experiment. Substituting a more complex urn for a 50-50 urn in a way that does not affect the ultimate win probability causes subjects to report lower certainty equivalents. In $R_2$ versus $RA$, replacing one urn draw (pre-
viously from a known 50-50 urn) with a draw from an ambiguous urn is associated with a lower certainty equivalent for the gamble, even though the win probability is unchanged. Thus, one might argue that our supposed distaste for ambiguity is simply another instance of a distaste for complexity. Gambles involving ambiguous urns are less appealing than similar non-ambiguous gambles, but only because the ambiguity involved constitutes a type of complexity.

This interpretation seems perfectly plausible; however, our experiment has discovered new terrain, even if it is correct. We show that ambiguity can constitute a complexity sufficient to drive people away from otherwise-attractive gambles. That is, subjects may prefer a non-ambiguous gamble to an analogous ambiguous gamble in which the ambiguity can only make the subject better off than under the non-ambiguous gamble.

People harboring a distaste for ambiguity has potentially widespread implications for economic theory and policymaking. Regarding economic theory, subjects may prefer gamble $R$ to $A$ in the classic Ellsberg paradox, primarily because they dislike the presence of ambiguity, but not because, e.g., they hold concern for worst-case scenarios, as Gilboa and Schmeidler (1989) would suggest. This means that new models may be required to explain people’s behavior adequately.

If people exhibit a distaste for ambiguity, officials may wish to consider this in crafting policy. They could do so in two ways. (a) Realizing that people dislike exposure to ambiguity, officials could avoid creating ambiguous policies, sometimes even when it might be beneficial. (b) Realizing that people avoid action that results in ambiguous payoffs if left to their own devices, officials could deliberately introduce policies that create beneficial but ambiguous payoffs when individuals would have avoided these (better) payoffs simply due to a distaste for the potential ambiguity.

## 5 Conclusions

2-Ball gambles are a rich class of decision problems. Because they can involve ambiguity but guarantee a minimum win probability that is at least as large as that of some other gamble, they allow us to test whether subjects avoid ambiguity *per se* as opposed to avoiding ambiguity because it may yield a worse outcome.

The most striking case of preferring a gamble with lower win probability is that subjects preferred the 50-50 gamble, $R$, to the 2-Ball ambiguous gamble, $A_2$. This preference is closely correlated with the Ellsberg paradox preference for $R$ over a 1-Ball ambiguous gamble $A$, $R$ over the compound 50-50 $C$, and $R$ over the 2-Ball 50-50
gambles $R_2$. These close relationships suggest that much of the preference for $R$ over $A_2$ is from a distaste for ambiguity and complexity.

It is implausible that subjects prefer $R$ to $A_2$ simply due to a poor understanding of the gambles. In the BoundedA block, subjects correctly and strongly identified that more unevenly distributed urns are more likely to win. Moreover, the lack of a "learning" effect from being in the treatment containing BoundedA suggests that subjects’ preference for $R$ over $A_2$ is deliberate.

Although the presence of an ambiguous draw within a gamble is associated with a significantly lower certainty equivalent, it remains unclear whether having more ambiguity, as measured perhaps by the number or proportion of ambiguous draws present in a gamble, has an additional negative effect on the certainty equivalent. This presents an interesting question for further research.

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|        | R       | A       | C       | RA      |
|--------|---------|---------|---------|---------|
| Mean   | 118.55  | 105.42  | 108.67  | 95.67   |
| 95% Conf. Interval | [114.43, 122.66] | [101.10, 109.74] | [100.15, 117.19] | [87.64, 103.70] |
| ρ      | 0.875   | 0.871   | 0.878   | 0.855   |
|        | (0.018) | (0.018) | (0.038) | (0.039) |
| N      | 708     | 708     | 158     | 179     |
| R²     | 110.55  | 108.64  | 102.53  | 97.21   |
| 95% Conf. Interval | [106.25, 114.85] | [96.59, 105.30] | [93.51, 111.56] | [89.15, 105.26] |
| ρ      | 0.866   | 0.876   | 0.855   | 0.883   |
|        | (0.019) | (0.018) | (0.042) | (0.035) |
| N      | 708     | 708     | 158     | 179     |

Table 7: RAW VARIABLES: DECOMPOSED SUMMARY STATISTICS
Table 8: derived variables: decomposed summary statistics

|               | R − A      | R − C      | R − A2     | R − C2     |
|---------------|------------|------------|------------|------------|
| Mean          | 13.15      | 7.91       | 17.66      | 14.11      |
| 95% Conf. Interval | [10.46, 11.54] | [3.37, 12.45] | [14.53, 20.78] | [6.91, 21.32] |
| ρ             | 0.489      | 0.364      | 0.578      | 0.561      |
| N             | 708        | 158        | 708        | 158        |
|               | R − R2     | RA − AR    | RA − A2    | RA − R2    |
| Mean          | 8.02       | -1.51      | 0.84       | -11.68     |
| 95% Conf. Interval | [5.33, 9.08] | [-4.85, -2.07] | [-4.76, -2.85] | [-16.81, -11.01] |
| ρ             | 0.428      | 0.151      | 0.368      | 0.259      |
| N             | 708        | 179        | 179        | 179        |