Research Article
Rescaled Local Interaction Simulation Approach for Shear Wave Propagation Modelling in Magnetic Resonance Elastography

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Received 13 November 2015; Revised 16 December 2015; Accepted 17 December 2015

1. Introduction

Mechanical properties of tissues are one of the most significant indicators used for detection of various abnormalities in medical diagnosis. Tumors and other pathologies often exhibit values of elastic moduli that are significantly different from healthy tissues. It is well known that none of the classical medical approaches, such as Computed Tomography (CT), Magnetic Resonance Imaging (MRI), and Ultrasonography (US), are able to detect mechanical properties of tissues that are diagnosed by palpation [1, 2]. Elastography is used extensively in diagnostic applications (e.g., liver fibrosis or breast tumors detection [3–9]) due to flexibility and noninvasiveness. Since abnormal tissues are often stiffer than the normal ones, medical diagnosis can be achieved. Although the method was developed in the late 1980s [10–12] the major breakthrough came in the mid 1990s when a dynamic approach to elastography was proposed [13]. A Motion-Encoding Gradient (MEG) was introduced to a conventional MRI system leading to Magnetic Resonance Elastography (MRE) [13–17].

Modelling in elastography relies on direct and inverse problems. The former relates to measurements of tissue responses to applied stresses. The latter is related to estimation of unknown mechanical properties from measured mechanical responses. Both problems are formulated using physical laws, which provide equations that relate biomechanical properties, such as shear modulus, Poisson’s ratio, viscosity, nonlinearity, and poroelasticity, to measured mechanical responses. Accurate models are required to predict displacement responses to different mechanical excitations to solve the inverse problem. For simple setups the equations that describe the direct problem have been solved analytically [18]. A similar approach used for irregular domains of elastically heterogeneous tissues is not possible in practice. Consequently, numerical simulations are used to ease this task. Modelling is used in MRE applications in order to create forward models that capture complex mechanisms of wave propagation in soft tissues. Previous studies in this field include various finite difference (FD) [17–19] and Finite Element methods (FE) [15, 20–24]. FE modelling has been used in previous studies for visualization of ultrasonic wave propagation [25–31], elasticity reconstruction [21, 32], and shear wave propagation analysis in gelatin phantoms [33–39].

The paper aims to develop a full three-dimensional (3D) model of shear wave propagation in a gelatin phantom for MRE applications. Some primary investigation has been performed for the bulk wave propagation model based on the Local Interaction Simulation Approach (LISA) [40]. In contrast to the previous work, current investigation focuses...
on the guided wave propagation with rescaling procedure. The major novelty of the presented work relates to the application of the Local Interaction Simulation Approach (LISA) for guided wave propagation and a rescaling procedure for the LISA is proposed for shear wave propagation modelling. This major novelty is considered to tackle numerical problems.

Then the LISA model is developed to examine density, shear modulus, and shear wavelength in a gelatin phantom. This study proposes the rescaling solution method in order to avoid numerical problems, especially related to wave amplitude. Numerical simulation results are compared with FE simulation results and MRE experimental measurements from a soft tissue mimicking an agarose gelatin phantom.

2. Theoretical Background

Elastic wave propagation in an isotropic linear medium is governed by the momentum balance given as

$$\sigma_{ij, i} + b_i = \rho \ddot{W}_i,$$

where $\sigma_{ij,i}$ is the divergence of stress tensor, $b_i$ is an external volume force, and $\ddot{W}_i$ represents particle acceleration vector. The constitutive equation that relates stresses to strains in a linear elastic solid is given as

$$\sigma_{ij} = \lambda \Delta \delta_{ij} + 2\mu \epsilon_{ij},$$

where $\delta_{ij}$ is the Kronecker delta, $\Delta$ represents material dilatation given by $\Delta = \nabla \cdot W = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, and $\lambda, \mu$ represent the Lamé constants for the material. The strain ($\epsilon_{ij}$) is defined through the strain tensor using the following relationship

$$\epsilon_{ij} = \frac{1}{2} \left( W_{ij,j} + W_{ji,i} \right),$$

where $W_{ij}$ represents particle displacement components. Combining (1)-(3) the equation of equilibrium, that is, [41],

$$\rho \nabla^2 W + (\lambda + \mu) \nabla \cdot W = \rho \ddot{W}_i$$

governs wave propagation in an infinite elastic space and for practical problems must be amended by appropriate boundary and initial conditions describing the problem. Boundary conditions increase the complexity of the problem since they give rise to the so-called guided wave propagation problem, where global wave propagation patterns, that is, modes, travel at different, and possibly frequency-dependent, speeds, as explained in [41]. It is well known that the solution to (4) can be found only for simple canonical problems. Numerical simulations are used for more complex scenarios.

3. Numerical Models

This section describes numerical models used for shear wave propagation in soft tissues. Firstly FE model was developed as a reference. Then a LISA model is described. The major focus is on a rescaling procedure that is used to avoid numerical discrepancies.

3.1. Finite Element Model. The FE model used in the current investigations was developed using the Marc Mentat 2013 software package. Following the work presented in [36], a 3D cylindrical container, with a diameter of 200 mm and thickness of 20 mm, was modelled using gel phantom material properties. The bottom of the cylinder was fixed in the $y$ direction (see Figure 1). Altogether 36 000 elements of $2 \times 2$ mm radial and axial element size and 200 element along the circle were used. The phantom was modelled as a homogenous isotropic elastic solid with Poisson’s ratio $\nu = 0.495$. Harmonic sinusoidal motion of 150 Hz was applied to the center of the top cylinder surface as an excitation. Three different elastic moduli ($E$) were investigated, that is, 30, 60, and 120 kPa, to study the relationship between shear wavelengths and shear moduli. Similarly, numerical simulations were performed using three different material density ($\rho$) values, $0.5 \times 10^3, 1 \times 10^3,$ and $2 \times 10^3$ kg/m$^3$, for each Young’s modulus. Material damping was assumed to be zero.

The shear wavelength ($\lambda_f$) in the FE model was obtained by estimating the distances between wave peaks directly from response waveforms to make a direct comparisons with the results presented in [36].

3.2. Local Interaction Simulation Approach Model

3.2.1. Background of the LISA Model. The LISA, previously used for wave propagation in complex media [42–49], has been applied for MRE shear wave propagation modelling. The algorithm of the LISA model is based on an FD approximation of (4) which discretises any structure under investigation into a grid of cells. Similar discretization is also used in the time domain when modelling is performed. All material properties are assumed to be constant within each

![Figure 1: Elementary discretization scheme used for wave propagation modelling in the LISA 3D [42].](image-url)
cell but may differ between cells. The algorithm can be derived from the elastodynamic wave equation \[ (5) \]
\[
\nabla_x (KW_x W) = \rho W_{tt},
\]
where \( W = [u, v, w]^T \), \( W \) is the vector of particle displacement, \( K \) is the stiffness matrix, \( \nabla_x \) and \( \nabla_x \) are the differential operators matrices for stress and strain, respectively, and \( \rho \) is the density. A comma before the subscript in (5) denotes differentiation. The \( K \) matrix contains stiffness components \( K_{ij}^{(p)} \) that depend on Young's moduli and Poisson ratios. The structure is discretised into parallelepiped cells for the 3D LISA wave propagation simulation, as illustrated in Figure 1. The junction of the eight cells characterizes the nodal point \( P \). The second time derivatives across the eight cells are needed to converge towards a common value \( \Omega \) at the point \( P \). In order to calculate a spatial derivative in the eight surrounding cells to \( P \), the central difference scheme is utilized. Then to obtain the solution, stress continuity across adjacent cells is constrained.

The following iteration equations are acquired for each displacement component for a general orthotropic case [42, 49]
\[
\chi \left( u_{t+1} - 2u_t + u_{t-1} \right) = -2u_0 \sum_{(i,j)} \frac{1}{\Delta x_i^2} \sum_{p=p} (1,11), (2,66), (3,35) \] + \sum_{p=p} K_{ij}^{(p)} + \Delta x_2 \left( u_0 \sum_{p=SP} K_{13+55}^{(P)} + \frac{1}{\Delta x_i^2} \sum_{p=p} K_{ij}^{(p)} \right)
\]
\[
+ \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} + \Delta x_3 \left( u_0 \sum_{p=SP} K_{12+66}^{(P)} + \frac{1}{\Delta x_i^2} \sum_{p=p} K_{ij}^{(p)} \right)
\]
\[
+ \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} + \Delta x_4 \left( u_0 \sum_{p=SP} K_{13+55}^{(P)} + \frac{1}{\Delta x_i^2} \sum_{p=p} K_{ij}^{(p)} \right)
\]
\[
\chi \left( v_{t+1} - 2v_t + v_{t-1} \right) = -2v_0 \sum_{(i,j)} \frac{1}{\Delta x_i^2} \sum_{p=p} K_{ij}^{(p)} + \Delta x_1 \left( u_0 \sum_{p=SP} K_{23+44}^{(P)} + \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} \right)
\]
\[
+ \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} + \Delta x_3 \left( u_0 \sum_{p=SP} K_{12+66}^{(P)} + \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} \right)
\]
\[
+ \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} + \Delta x_4 \left( u_0 \sum_{p=SP} K_{13+55}^{(P)} + \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} \right)
\]
\[
\chi \left( w_{t+1} - 2w_t + w_{t-1} \right) = -2w_0 \sum_{(i,j)} \frac{1}{\Delta x_i^2} \sum_{p=p} K_{ij}^{(p)} + \Delta x_1 \left( u_0 \sum_{p=SP} K_{23+44}^{(P)} + \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} \right)
\]
\[
+ \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} + \Delta x_3 \left( u_0 \sum_{p=SP} K_{12+66}^{(P)} + \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} \right)
\]
\[
+ \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} + \Delta x_4 \left( u_0 \sum_{p=SP} K_{13+55}^{(P)} + \frac{2}{\Delta x_i^2} \sum_{p=p} \sum_{r} \sum_{p} \psi_{a(r)} \psi_{b(r)} K_{ij}^{(p)} \right)
\]
can be investigated in details through the iteration equations analysis. First, the severity of numerical damping can be analysed by considering roots of characteristic polynomial of a numerical scheme at hand, directly related to the Courant-Friedrichs-Lewy stability condition [50]. The latter is frequently invoked in the context of wave propagation modelling as the model parameters are required to meet certain restrictions for the analysis to be stable. This concept can be also used to quantify scheme’s accuracy, as will be shown next.

Soft tissues are highly demanding from computational point of view. From physical perspective it is well known that mainly transversally polarized waves propagate in these structures [41]. When numerical modelling is used both types of waves normally coexist. However, analyzing material properties characteristic to soft tissues (these properties are extraordinary when compared to solid media) the difference in longitudinal and shear wave velocities can be immediately found, reaching the ratio of 10. As a consequence, the shear wave component, which is of particular interest for MRE, propagates under conditions far from the stability limit. Namely, the roots associated with the characteristic polynomial drive the waves to decay.

This drawback can be resolved twofold: by reformulating constitutive relationships in order to eliminate the longitudinal wave component, or by manipulating model parameters to push the shear wave closer to the stability limit. In the following work the second approach was employed as this requires no intervention in the solver structure, maintaining the flexibility of the method to model wider class of materials (i.e., solid media and soft tissues).

The longitudinal and shear wave speeds can be expressed

\[
V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}},
\]

\[
V_T = \sqrt{\frac{\mu}{\rho}},
\]

These definitions show that the density is a parameter that uniformly influences both longitudinal and shear wave velocities. Hence, the approach presented in the paper aims at improving the model properties by rescaling wave speeds. Following the work presented in [36], scaled density is used in numerical simulations. This procedure can be explained using a 1D example of wave propagation. The major focus is on the stability and amplitude accuracy of LISA. The objective of this study is to obtain information about the effect of density on the LISA model. The 1D finite difference equation, involved in numerical simulations, can be expressed as

\[
u_i^{n+1} = -u_i^{n-1} + 2u_i^n + C^2 (u_{i+1}^n - 2u_i^n + u_{i-1}^n),\]

\[
C = c \frac{\Delta t}{\Delta x},\]

where \(u\) is the displacement, \(n\) is the time step index, \(i\) relates to the node position, \(C\) is known as the (dimensionless) Courant (or Courant-Friedrichs-Lewy (CFL)) number, \(c\) is the wave velocity, and \(\Delta t, \Delta x\) are related to the time step and the element size, respectively. The stability analysis by means of the Fourier transform is known as the von Neumann analysis [50]. This analysis allows for expression of a governing equation as a recurrence relation that is particularly useful for establishing stability conditions. The key idea is the analysis of the amplification polynomial of the scheme, which is obtained by applying the Fourier transform to the governing FD equation. Once the amplification polynomial is established, certain restrictions are put on its roots. Although the analysis presented is for a 1D case, the entire procedure can be easily extended to provide general stability conditions for higher dimensions.

When (10) is used and the stability condition is obtained for various parameters, stable and unstable conditions can be analysed for various values of density. It is important to note that it is beyond this paper to put all the equation and formula involved in this analysis. Potential readers are referred to [50] for further details. After obtaining the roots of amplification polynomial which are conjugate pairs of the same complex number, the magnitude is the same for both. The magnitude of the roots of amplification factor can be expressed as

\[
\|g\| = \sqrt{8C^4s^4 - 8C^2s^2 + 1},
\]
were selected and respective wave velocities calculated. The study investigates the effect of density scaling parameter on LISA approach were conducted and analysis was performed with (7) and (8) and the instability can be reached in 1D finite difference model.

It is clear that once wave propagation is simulated with scaled densities, an inverse spatial scaling procedure should be applied to the results to retrieve proper responses. This is accomplished by an inverse scaling procedure employed for the space sensor waveforms. Again (7) and (8) were employed and space (wavelength) signals were multiplied by the square root of the relevant scaling factors. The results, shown in Figure 3(b), illustrate that the wavelength of the original signal is recovered after the rescaling procedure.

To illustrate the approach, dispersion curves for respective rescaled models were calculated and used to recover the original waveforms. In the following analysis, the $A_0$ mode is considered, as it is the dominant mode in this frequency range. In Figure 4, dispersion curves for three different scaling parameters are given ($S$ in figures correspond by scaling factor). By applying the scaling factor to the original density, it affects the dispersion curve, respectively, which causes a certain change to the wave number of every dispersion curve. The rescaling parameter, square root of $S$ – based scaling factor, is also then obtained by analyzing dispersion curves plot between simulated original and scaled density by comparing the ratio of wave number of scaling density by wave number of original density and it proved the efficiency of propose method. To show an example, the waveform (density $\rho = 1 \times 10^3$ kg/m$^3$) and one rescaled waveform $(\rho = 3 \times 10^3$ kg/m$^3$) together with the
### Table 1: The effect of scaled density on wave propagation velocities and numerical stability.

| Density   | Courant number | Longitudinal wave velocity ($V_L$) and shear wave velocity ($V_T$) [m/s] | Scaling factor in Figures 3(a) and 3(b) | Comments |
|-----------|----------------|--------------------------------------------------------------------------|-----------------------------------------|----------|
| $\rho = 500 \text{ kg/m}^3$, $C = \frac{V_L}{V_{lim}} = 0.3$ | $V_L = 6.0045 \times 10^3$ $V_T = 1.4286 \times 10^3$ | Original density | Green line in Figure 4 relates to initial (original) original density and is used as reference; low amplitudes in numerical simulations |
| $\rho_1 = 1000 \text{ kg/m}^3$, $C_1 = \frac{V_{L1}}{V_{lim}} = 0.212$ | $V_{L1} = 4.2458 \times 10^3$ $V_{T1} = 1.0102 \times 10^3$ | $S_1 = 2$ | Wavelength was rescaled in Figure 3(b) with the square root of the scaling factor, that is, $\sqrt{2}$ |
| $\rho_2 = 1500 \text{ kg/m}^3$, $C_2 = \frac{V_{L2}}{V_{lim}} = 0.173$ | $V_{L2} = 3.4667 \times 10^3$ $V_{T2} = 824.8232$ | $S_2 = 3$ | Wavelength was rescaled in Figure 3(b) with the square root of the scaling factor, that is, $\sqrt{3}$ |
| $\rho_3 = 2000 \text{ kg/m}^3$, $C_3 = \frac{V_{L3}}{V_{lim}} = 0.150$ | $V_{L3} = 3.0022 \times 10^3$ $V_{T3} = 714.3179$ | $S_3 = 4$ | Wavelength was rescaled in Figure 3(b) with the square root of the scaling factor, that is, $\sqrt{4}$ |
| $\rho_4 = 2500 \text{ kg/m}^3$, $C_4 = \frac{V_{L4}}{V_{lim}} = 0.13$ | $V_{L4} = 2.6853 \times 10^3$ $V_{T4} = 638.9053$ | $S_4 = 5$ | Wavelength was rescaled in Figure 3(b) with the square root of the scaling factor, that is, $\sqrt{5}$ |
| $\rho_5 = 3000 \text{ kg/m}^3$, $C_5 = \frac{V_{L5}}{V_{lim}} = 0.12$ | $V_{L5} = 2.4513 \times 10^3$ $V_{T5} = 583.2381$ | $S_5 = 6$ | Wavelength was rescaled in Figure 3(b) with the square root of the scaling factor, that is, $\sqrt{6}$ |

**Figure 4:** The dispersion plot for the original density and three $S_1$, $S_2$, and $S_3$ scaled densities.

The corresponding dispersion curves are shown in Figure 5. The wavelength for the original waveform is equal to 21 mm whereas the wavelength for the rescaled waveform is equal to 12 mm. The original waveform (shown in Figure 5(a)) can be recovered from the rescaled waveform (Figure 5(c)) when the latter is multiplied (scaled back) by the square root of the scaling factor (i.e., square root of 3 in this case) and vice versa. The results are shown in Figure 6. The analysis of dispersion curves reinforces the condition that the guided not bulk wave theory should be used in the case investigated, as discussed further in Section 5.

In summary, two interesting observations can be made after the analysis performed in this section. Firstly, the wave amplitude increases when density is rescaled towards larger values. Secondly, the inverse rescaling of waveforms allows one to reproduce accurately the original wavelengths.

### 4. Magnetic Resonance Elastography: Experimental Data

The MRE data from the experiments reported in [36] were used as a reference in the current investigations. The phantom used in the experiment was a 3D cylinder filled with 2% agarose gel. The geometry of the cylinder was as follows: diameter 150 mm and height 20 mm. The MRE tests were conducted using the 1.5 T General Electric Signa CT scanner. The phantom was placed in a head coil and an electromechanical driver was placed on the top surface of the phantom in order to generate shear waves corresponding to the excitation frequency of 150 Hz. The experimental setup used is shown in Figure 7.

The propagation of elastic waves in the phantom was imaged with an MRE pulse sequence sensitive to motion in the horizontal direction. The shear wavelength was estimated manually by calculating the distances between the adjacent wave peaks. Also, the mean of shear wavelength was measured by averaging the wavelength over the four phase offsets. Subsequently, for isotropic elastic infinite solid, an estimate of the local shear modulus $G$ can be obtained from the local estimate of wavelength $\lambda$ as [13]

$$\lambda = \frac{1}{f} \sqrt{\frac{G}{\rho}},$$

(12)

The shear wavelength $\lambda$ for phantom estimated by MRE was $38.00 \pm 2.12$ mm at 150 Hz. This corresponds to the mean...
value of 28.5 kPa for the shear modulus $G$. The shear modulus $G$ was also estimated using a dynamic multifrequency shear test with the DMA 2980 machine for polymer testing to obtain the value of 30 kPa. The density was estimated experimentally as $\rho = 1.0 \times 10^3$ kg/m$^3$.

### 5. Numerical Simulation Results

Numerical simulations of shear wave propagation in the phantom described in Section 4 were performed using FE and LISA models. The results are presented in this section.
5.1. Shear Wave Propagation in Soft Tissue. Simulated FE, LISA, and experimental MRE shear wave propagation patterns are presented in Figures 8(a), 8(b), and 8(c), respectively. The simulated results were obtained for Young’s modulus $E = 90\,\text{kPa}$ and the density $\rho = 1.0 \times 10^3\,\text{kg/m}^3$. The density scaling was applied in LISA models to avoid numerical problems related to excessive wave attenuation. Subsequently, the rescaling procedure was used in postprocessing to recover proper waveforms. The results in Figure 8 show that the simulated and experimental wave patterns reveal the same wavelengths. Small differences between FE and LISA models can be attributed to different formulations of the FE and LISA equations used and differences in meshes.

Subsequently, the out-of-plane displacement component responses were acquired from the simulated (FE and LISA models before and after scaling) and experimental (MRE measurements) data. The results, presented in Figure 9, show good agreement between simulated and experimental displacements. It is also important to note that after scaling the amplitude of the LISA model is improved. Next, the shear wavelength was computed from the distance between two successive peaks (or valleys). The wavelengths were estimated as $\lambda_f = 37.5\,\text{mm}$ and $\lambda_f = 37\,\text{mm}$ for the simulated FE and LISA models, respectively. These results correspond quite well with the MRE-based experimental value of the wavelength $\lambda_m = 38\,\text{mm}$. However, the computational effort of 10 seconds the LISA model compares favorably if compared with the 2640 seconds for the FE model.

Following these investigations, simulated shear wavelengths, calculated for different values of elastic moduli and density, were compared with the relevant analytical values calculated from (7) and (8) for the bulk wave propagation problem. Four different elastic moduli, that is, 30, 60, 90, and 120 kPa, and three different densities, that is, $0.5 \times 10^3$, $1 \times 10^3$, and $2 \times 10^3\,\text{kg/m}^3$, were investigated. Figure 10 presents the results for the 150 Hz excitation frequency. Here, the three continuous solid, dashed, and dotted curves give the values of shear wavelengths calculated from (12) for infinite medium propagation model.

Although the results are quite consistent for lower values of elastic moduli, significant discrepancies between numerically (FE and LISA) and analytically (bulk wave propagation solution) estimated results can be observed for higher values of elastic moduli (corresponding to larger wavelengths), particularly for lower densities. These discrepancies are further discussed in the next section.

5.2. Guided Wave Propagation in Soft Tissues. Equations (12) provide the relationships between excitation frequency, wavelength, and elastic constants for an infinite elastic space. Thus any estimation of wavelengths, as discussed in the previous sections, and consequently estimation of elastic properties based on these wavelengths is accurate only for the infinite space assumption. This assumption can be approximately fulfilled for the following two conditions: (1) wavelength estimates are made sufficiently far from the object’s boundaries; (2) wavelengths are small when compared with distances from the boundaries. Both conditions can be achieved when excitation frequencies are selected to obtain sufficiently short wavelengths. However, near the boundaries wavelength estimations will not be accurate, unless the effect of the interfaces is taken into account.

Figure 11 shows the through-thickness cross-section of the wave propagation displacement field for the analysed phantom model. The results, obtained for the 150 Hz excitation frequency, show that the displacement varies across the thickness of the phantom, from a finite value (top) to zero (bottom). This nonuniform displacement distribution indicates that the wave field is strongly affected by the (top and bottom) boundaries; as a result the (global) wavelength is different from the wavelength for the assumed theoretical infinite space case.

These numerically estimated values were compared in Figure 10 with the theoretical values obtained for the assumed infinite space. The results show that the numerical and analytical results start to diverge for wavelengths larger than $20 \div 30\,\text{mm}$. This corresponds to the thickness of the phantom. Thus the analysed wave propagation field in the phantom corresponds to the guided wave field rather than to the bulk wave field (assumed in (12)). Guided wave propagation involves partial waves, that is, waves propagating in an infinite space that interact with the (top and bottom) boundaries. These partial waves undergo multiple reflections and mode conversions forming global displacement patterns, that is, wave modes. Therefore, wavelength estimation in the analysed model should involve the relevant dispersion equations for guided waves rather than (7) and (8) for bulk waves. This problem can be solved semianalytically or numerically, as illustrated in [48]. A semianalytical approach, based on the LISA iteration equations, was used in the current paper for wavelength estimation. The vertical (y) displacement component at the bottom surface of the phantom was constrained. The results for various Young’s moduli and densities are presented in Figure 12. This time, the wavelengths estimated from the guided wave propagation model are compared with the relevant wavelengths estimated from the FE infinite space model (i.e., from (12)). When the results are analysed two distinct wavelength ranges can be distinguished in Figure 12. The semianalytical solution for guided wave propagation compares very well with the bulk wave model for wavelengths shorter than $20 \div 30\,\text{mm}$.
In contrast, the results for the guided and unbounded media differ significantly for longer wavelengths. The results in Figures 11 and 12 indicate that the guided wave propagation model rather than the bulk wave propagation model (that was originally employed in [36]) should be used for wavelength estimation in the case investigated.
Figure 13: Five different model boundary condition, for both the Computational and Mathematical Methods in Medicine conditions, namely, fixed and free ends, were examined. Two different boundary conditions was investigated. Two different boundary with wave interactions with boundaries, the effect of boundary of interfacial conditions for complex wave propagation in problems related to numerical errors in soft tissues modelling. In contrast, the solutions based on guided wave propagation are more accurate for longer wavelengths. Also by the rescaling procedure which is presented in this paper, the wave amplitude problems related to numerical errors in soft tissues modelling can be avoided. This analysis can serve as an indicator of interfacial conditions for complex wave propagation in biological tissues.

Conflict of Interests

There is no conflict of interests involved.

Acknowledgments

The work presented in this paper was supported by the Foundation for Polish Science under the research WELCOME Project no. 2010-3/2 (Innovative Economy, National Cohesion Programme funded by EU). The fourth author would like to acknowledge research financial support from the Faculty of Mechanical Engineering and Robotics, AGH University of Science and Technology. The authors would also like to thank Professor Kai-Nan An from Mayo Clinic College of Medicine in Rochester, USA, and Dr. Frank Chen from Excelen in Minneapolis, USA, for experimental MRE data used in the paper.

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