Multi-target Joint Detection, Tracking and Classification Based on Generalized Bayesian Risk using Radar and ESM sensors

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Abstract—In this paper, a novel approach is proposed for multi-target joint detection, tracking and classification based on the labeled random finite set and generalized Bayesian risk using Radar and ESM sensors. A new Bayesian risk is defined for the labeled random finite set variables involving the costs of multi-target cardinality estimation (detection), state estimation (tracking) and classification. The inter-dependence of detection, tracking and classification is then utilized with the minimum Bayesian risk. Furthermore, the conditional labeled multi-Bernoulli filter is developed to calculate the estimates and costs for different hypotheses and decisions of target classes using attribute and dynamical measurements. Moreover, the performance is analyzed. The effectiveness and superiority of the proposed approach are verified using numerical simulations.

Index Terms—Joint detection, tracking and classification, Labeled multi-Bernoulli, Bayesian risk

I. INTRODUCTION

Multi-target joint detection, tracking and classification (JDTC) using Radar and ESM sensors is a critical problem in airborne surveillance systems. In this problem, both kinematic measurements and attribute measurements are used to estimate the number of the targets, estimate their kinematic states, and determine their classes. Actually, these three subproblems are usually coupled: tracking may provide flight envelop and kinematic feature to distinguish the target type, according to the target class, appropriate dynamic models can be chosen for accurate tracking, and the change of the target number implies a modification of tracking and classification procedures [1]. Actually, multi-target JDTC is a joint decision and estimation (JDE) problem.

Most traditional multi-target JDTC algorithms can be classified into the following categories. 1) Estimation-Then-Decision (ETD): In this category, target tracking is usually performed using data from kinematic sensors, and the classification is then derived based on the flight envelopes and kinematic estimates [2]-[4]. The drawback of this two-step strategy is that, the classification is significantly dependent on the estimates. As shown in [5], the classification performance was deteriorated due to the inaccurate state estimates derived with the error data association. 2) Decision-Then-Estimation (DTE). In this category, the decision is made using data from identity or attribute sensors, and the estimates are then calculated based on the decisions made before [6]. The disadvantage of this strategy is that, the error of the decision is not considered. In [7], the state estimates were calculated with classification-aided data association, however, the classification was done without regarding the quality of the estimation it would lead to. 3) Based on the joint probability density: In this category, the target state and class are inferred by the joint state-class probability density function. In [8]-[9], the joint decision and estimation goal was not directly reached [10]. In [11], the class dependent multi-target density was calculated using the particle implementation of PHD/MeMBer filter [10][11] with corresponding motion model set, and the probability of target class could then be inferred by the weights of particles in the cluster. However, in these methods, the state and class of each target were not explicitly obtained. Furthermore, the overall performance may not be necessarily good because the final joint decision and estimation goal was not directly reached [12].

In [13], Li proposed a new approach for the problems involving inter-dependent decision and estimation based on a generalized Bayesian risk. In this method, the decision and estimation costs were converted to a unified measure using additional weight coefficients, and the optimal solution was derived to minimize the Bayesian risk. Because the inter-dependence between decision and estimation was considered, this method is inherently superior to the conventional approaches. In [14][15], the recursive JDE (RJDE) algorithm was developed to fit the dynamic system and solve target JTC problem. Moreover, a joint performance metric (JPM) was proposed for evaluating the overall performance. In [16], the conditional JDE (CJDE) algorithm was proposed based on a new Bayesian risk defined conditioned on data and used to solve the target JTC and JDT problems [17]. Because the estimates and costs were directly calculated using corresponding measurements once the decision is made, the computation of the algorithm is simplified greatly.

In this paper, a novel approach is developed for multi-target JDTC based on the generalized Bayesian risk using Radar and ESM sensors. A new Bayesian risk is defined based on the labeled RFS involving the costs of multi-target detection, tracking and classification, and the optimal solution is then derived to minimize this new risk. Given the class decision sets of multiple targets, the posterior state estimates and class probabilities are calculated using kinematic measurements and attribute measurements within the Bayes recursion. For the explicit expression of the multi-target posterior density involving the measurement-target-associations (MTA’s), the RFS based
estimation and classification costs are exact calculated and
the optimal IDE solution is directly derived. The Gaussian
mixture implementation of the proposed algorithm is also
developed, and the performance of the approach is analyzed.
Simulations show that the proposed approach performs better
than traditional methods.

This paper is organized as follows: An introduction of
LMB filter and CJDE approach is presented in Section 2.
The recursive multi-target JDTC algorithm is developed in
Section 3. The simulation results of the proposed algorithm
are provided in Section 4. Conclusions are summarized in Section 5.

II. BACKGROUND

A. Labeled multi-Bernoulli RFS and multi-target Bayes filter

In [10], a Bernoulli RFS was used to represents the uncertainty
about the existence of a single object. The probability
density function of a Bernoulli RFS \( X \) can be given by

\[
 f(X) = \begin{cases} 
 1 - p, & \text{if } X = \emptyset \\
 p \cdot f(x), & \text{if } X = \{x\}
\end{cases}
\]  

As expressed in the equation, the Bernoulli RFS can either be
empty with a probability of \( 1 - p \), or have one element \( x \) with
probability \( p \), and \( f(x) \) is the probability density function of
variable \( x \) over space \( \mathcal{X} \). In [19], Vo et al. introduced the notion of labeled RFS.
Assume that \( X \) denotes the RFS of target states, the multi-
target exponential of a real valued function \( h \) for all the state
vectors \( x \) is \( h^X_\delta \triangleq \prod_{x \in X} h(x) \), where \( h^b = 1 \). The Kronecker
delta function and the inclusion function are

\[
 \delta_Y(X) = \begin{cases} 1, & \text{if } X = Y \\
 0, & \text{otherwise} 
\end{cases}, \quad 1_Y(X) = \begin{cases} 1, & \text{if } X \subseteq Y \\
 0, & \text{otherwise} 
\end{cases}
\]

Suppose that the state vector \( x \) in the space \( \mathcal{X} \) is augmented
with a unique label \( \ell \in \mathbb{L} \), where \( \mathbb{L} \) is a discrete label space,
and \( \mathbf{X} \) represents the labeled RFS. Let \( \mathcal{L} : \mathbb{L} \times \mathcal{X} \rightarrow \mathbb{L} \) be
the projection \( \mathcal{L}(x, \ell) = \ell \), \( \mathcal{L}(X) \) is the label set of \( X \).
The distinct label indicator \( \Delta(X) = \delta_{\mathcal{L}}(\mathcal{L}(X)) \) ensures the
distinctness of the labels of \( X \). All finite subsets of \( \mathbb{L} \) are denoted by \( \mathcal{F}(\mathbb{L}) \).

Augment the state with an unique label, the labeled multi-
Bernoulli (LMB) RFS \( X \) in the state space \( \mathcal{X} \) and label space \( \mathbb{L} \) can then be represented by the parameter set \( \pi = \{r^{(\ell)}, p^{(\ell)}(x)\}_{\ell \in \mathbb{L}} \), and the density function is

\[
 \pi(X) = \Delta(X) \omega(\mathcal{L}(X)) p^X
\]

where the weight

\[
 \omega(L) = \prod_{\ell \in \mathbb{L}} (1 - r^{(i)}) \prod_{\ell \in \mathbb{L}} \frac{1_{L}(\ell)r^{(\ell)}}{1 - r^{(\ell)}}
\]

Based on the labeled multi-Bernoulli RFS, an approximation of the multi-target Bayes filter was proposed in [21], which
consists of the following two steps:

1. Prediction: Suppose that the multi-target prior density and
birth density are LMB RFSs. Then the predicted multi-target
density is also a LMB RFS with state space \( \mathcal{X} \) and label space
\( \mathcal{L}_+ = \mathbb{L} \cup \mathbb{B} (\mathbb{B} \cap \mathbb{L} = \emptyset) \), where \( \mathbb{L} \) and \( \mathbb{B} \) are the label spaces of
surviving and birth target. This predicted density can be
represented by the parameter set

\[
 \pi_+ = \{ (r^{(\ell)}_{+; L}, p^{(\ell)}_{+; L}) \}_{\ell \in \mathbb{L}} \cup \{ (r^{(\ell)}_{+; B}, p^{(\ell)}_{+; B}) \}_{\ell \in \mathbb{B}}
\]

where

\[
 r^{(\ell)}_{+; L} = \eta_S(\ell)r^{(\ell)} \\
 p^{(\ell)}_{+; L} = \langle p_s(\cdot, \ell), f(x|\cdot, \ell), p(\cdot, \ell) \rangle / \eta_S(\ell) \\
 \eta_S(\ell) = \langle p_s(\cdot, \ell), p(\cdot, \ell) \rangle
\]

Here, \( p_s \) is the state dependent survival probability, and
\( f(x|\cdot, \ell) \) is the state transition density. \( r^{(\ell)}_{+; B} \) and \( p^{(\ell)}_{+; B} \) are the prior birth probability and state density of a new birth target,
respectively.

2. Update: Suppose that the predicted multi-target LMB RFS is represented by the parameter set \( \pi_+ = \{ (r^{(\ell)}, p^{(\ell)}(x)) \}_{\ell \in \mathcal{L}_+} \) on \( \mathcal{X} \times \mathcal{L}_+ \). The multi-target predicted
density can then be given by

\[
 \pi_+(X) = \Delta(X) \sum_{\ell \in \mathcal{L}(X)} \omega^{(\ell)} \delta_{\ell, \mathcal{L}(\mathcal{L}(X))} [p_+]^X
\]

where

\[
 \omega^{(\ell)} = \prod_{\ell \in \mathbb{L}} (1 - r^{(i)}_{+}) \prod_{\ell \in \mathbb{L}} \frac{1_{L}(\ell)r^{(\ell)}_{+}}{1 - r^{(\ell)}_{+}}
\]

After receiving the measurements, the LMB RFS that matches
exactly the first moment of the multi-target posterior den-
sity can be denoted by the parameter set \( \pi(X|Z) = \{ (r^{(\ell)}, p^{(\ell)}(x)) \}_{\ell \in \mathcal{L}_+} \), in which, the updated existence probabilities \( r^{(\ell)} \) and spatial distributions \( p^{(\ell)}(x) \) of track \( \ell \) are

\[
 r^{(\ell)} = \sum_{\ell \in \mathcal{L_+}, \ell' \in \mathcal{L_+} \times \Theta} \omega^{(\ell, \theta)}(Z) 1_{\ell}(\ell) \\
 p^{(\ell)}(x) = \frac{1}{r^{(\ell)}} \sum_{\ell \in \mathcal{L_+}, \ell' \in \mathcal{L_+} \times \Theta} \omega^{(\ell, \theta)}(Z) 1_{\ell}(\ell) p^{(\ell)}(x) \]

where \( \Theta \) is the space of mappings \( \theta : \mathbb{L} \rightarrow \{0, 1, ..., |Z|\} \),
such that \( \theta(i) = \theta(i') > 0 \) implies \( i = i' \), and

\[
 \omega^{(\ell, \theta)}(Z) \propto \omega^{(\ell)}(Z) |\eta^{(\theta)}_{\mathcal{L}}(\ell)| \\
 p^{(\theta)}(x, \ell|Z) = \frac{p(x, \ell)p^{(\theta)}(x, \ell; \theta)}{\eta^{(\theta)}_{\mathcal{L}}(\ell)} \\
 \eta^{(\theta)}_{\mathcal{L}}(\ell) = \langle p_s(\cdot, \ell), \psi_{\mathcal{L}}(\cdot, \ell; \theta) \rangle \\
 \psi_{\mathcal{L}}(x, \ell; \theta) = \delta_{\theta}(\theta(q_{\mathcal{L}}(x, \ell)))
\]

Here, \( p_d(x, \ell) \) is the detection probability of the target,
\( q_d(x, \ell) = 1 - p_d(x, \ell) \) is the probability for missed detection,
\( g(z_{\theta(|\ell)}) x, \ell \rangle \) is the measurement likelihood, and \( \kappa(z_{\theta(|\ell)}) \) is
the intensity of the clutter process.
B. Conditional joint decision and estimation

The foundation of the CJDE method [12] is a novel Bayesian risk depending on the particular received measurement $z$. The decision and estimation costs are converted to a unified measurement by introducing additional weight coefficients $\{\alpha_{ij}, \beta_{ij}\}$, that is

$$
\tilde{R}(z) = \sum_{i} \sum_{j} (\alpha_{ij}c_{ij} + \beta_{ij}E[C(x, \hat{x})|D^i, H^j, z])P\{D^i, H^j|z\}
$$

(18)

where $P\{D^i, H^j|z\}$ is the joint probability of decision and hypothesis, $c_{ij}$ is the cost of decision $D^i$ while the true hypothesis is $H^j$, and the conditional expected estimation cost $E[C(x, \hat{x})|D^i, H^j] = mse(\hat{x}|D^i, H^j)$ is the mean square error. The optimal solution is derived to minimize this new Bayes risk, the optimal decision $D$ is

$$
D = D^i \quad \text{if} \quad C_C^i(z) \leq C_C^0(z) \quad \forall n
$$

(19)

where the posterior cost is

$$
C_C^i(z) = \sum_{j} (\alpha_{ij}c_{ij} + \beta_{ij}E[C(x, \hat{x})|D^i, H^j, z])P\{H^j|z\}
$$

(20)

To calculate $C_C^i(z)$ with $C(x, \hat{x}) = \hat{x}'\hat{x}$, the key is to obtain the estimation cost $\epsilon_{ij}$. Assuming the optimal target estimate is

$$
\hat{x}_n = \sum_j E(\hat{x}(j)|H^j, z)P\{H^j|z\}
$$

(21)

then the estimation cost is

$$
\epsilon_{ij}(z) = E[\hat{x}(j)|D^i, H^j, z] + E[(\hat{x}(j) - \hat{x}(j))(\hat{x}(j) - \hat{x}(j))'|D^i, H^j, z]
$$

$$
= mse(\hat{x}(j)|H^j, z) + (e_{ij} - \hat{x}(j)')(\hat{x}(j) - \hat{x}(j)), \forall z \in D
$$

(22)

The recursive CJDE algorithm is shown as follows:

1. Initialize the parameters: $\hat{x}^{(j)}_{k-1}$, $P\{H^j|Z^{k-1}\}$ and so on.
2. Predict the state based on dynamics of $x_k$. Update $\hat{x}^{(j)}_k$ and $P\{H^j|Z^k\}$ by $z_k$. Then compute $\hat{x}^{(i)}_k$ for decision $i$.
3. Compute $\epsilon_{ij}(Z^k)$ and get cost $C_C(Z^k)$. Then $D^i_k: C_C(Z^k) \leq C_C^0(Z^k), \forall n$.
4. Output the CJDE solution for time $k$. $D_k = D^i_k$ and $\hat{x}_k = \hat{x}^{(i)}_k$.

III. THE RECURSIVE MULTI-TARGET JDTC APPROACH

In this section, the mathematical formulation of the problem is firstly presented in 3.1. The multi-target JDTC algorithm and its Gaussian mixture (GM) implementation is then developed in 3.2 and 3.3, respectively. At last, the performance of the algorithm is analyzed in 3.4.

A. Problem formulation

Suppose that the class of a target is a time-invariant attribute, which can be distinguished according to the dynamic behavior. The target kinematic state at time $k$ for class $c_i$ can be modeled as

$$
x_k = F_{k-1,c}x_{k-1} + \Gamma_{k-1}w_{k-1,c}
$$

(23)

where $F_{k-1,c}$ is the class-dependent state transition matrix, $w_{k,c}$ is the Gaussian process noise, and $\Gamma_{k-1}$ is the gain matrix. The target can be observed by both Radar and ESM sensors, and the kinematic measurement of radar contains the range and angle measurements of the target, which can be given by

$$
z_k^r = H_k x_k + v_k
$$

(24)

where $H_k$ is the measurement matrix, and $v_k$ is the Gaussian noise with covariance $R_k$. The ESM sensors scan the frequency range to intercept emitted electromagnetic signals from the targets and identify the likely source emitters. The signal are processed and the angle of arrival can be obtained. The bearing measurement is

$$
z_k^e = H_k^e x_k + v_k^e
$$

(25)

where $H_k^e$ and $v_k$ are the measurement matrix and Gaussian, respectively. Furthermore, the identification of the source emitters can be derived by sorting the received signals according to the radio frequency, signal parameters like modulation format, pulse repetition frequency, and so on. To account for the measurement error, the confusion matrix $P$ can be defined. Assume there are $N$ types emitters, the matrix $P$ contains $m \times m$ elements, where $m = 2^N$, and the element $\pi_{ij}$ in the matrix is the probability that

$$
\pi_{ij} = Pr\{\text{declare } E^j|\text{true } E^i\} \quad i, j = 1, 2, \ldots, m
$$

(26)

Assume that, at time $k$, $X_k = \{x_{k,1}, \ldots, x_{k,n}\}$ is the set of multi-target states, $Z_k = \{z_{k,1}^r, \ldots, z_{k,m}^e\}$ is the set of noisy and cluttered measurements, where $\{x_{k,1}, \ldots, x_{k,m}\}$ is the measurement set generated from the targets and $\{c_1, \ldots, c_i\}$ is the set of clutter. Similarly, $Z_k = \{z_{k,1}^e, \ldots, z_{k,m}^e\}$ is measurement set of the ESM sensor, the measurement $z_{k,m} = [\beta_{k,m}, c_{k,m}]$ contains the angle of the target and the probability of the target type. The multi-target JDTC algorithm aims to estimate the target number and states, and determine their classes from a sequence of noisy and cluttered measurement sets.

B. The multi-target JDTC approach based on the generalized Bayesian risk

As multi-target JDTC is a dynamic problem and measurements are usually obtained sequentially, a new recursive Bayesian risk is firstly defined based on the labeled RFS. Suppose that $C = \{C_i\}_{j=1}^J$ is the set which contains $J$ possible target classes, $X$ is the multi-target state RFS, $H_m = \{H_{m}^{i}\}_{\ell \in L(X)}$ and $D_{k,m} = \{D_{k,m}^{i}\}_{\ell \in L(X)}$ are the class hypothesis and decision sets of all the targets, respectively, where $H_{m}^{i}$ and $D_{k,m}^{i}$ are the class hypothesis and decision for track $\ell$. The new Bayesian risk is then given by

$$
\tilde{R}_C(Z_k) = \sum_{m,n} (\alpha_{mn}c_{mn} + \beta_{mn}E[C(X, \hat{X})|D_{k,m}^{n}, H_{m}^{n}, Z_k])P\{D_{k,m}^{n}, H_{m}^{n}|Z_k\}
$$

(27)
where \( c_{mn} \) is the cost of deciding on \( D^n_k \) when the hypothesis \( H^m \) is true, \( C([X,X]|D^n_k, H^m, Z_k) \) is the conditional expected estimation cost of multi-target states, and \( E[|I_{mn} - \hat{I}|]|D^n_k, H^m, Z_k \) is the conditional expected multi-target cardinality estimation error, \( P(D^n_k, H^m|Z_k) \) is the posterior probability of decision and hypothesis set, \( \alpha_{mn}, \beta_{mn} \), and \( \gamma_{mn} \) are the nonnegative weights used to unify the costs.

To minimize \( \bar{R}_C(Z_k) \), the optimal decision \( D_k \) is

\[
D_k = D^n_k \text{ if } C_n(Z_k) \leq C(Z_k), \forall i
\]  

(28)

where the cost \( C_n(Z_k) \) for the decision \( n \) is given by

\[
C_n(Z_k) = \sum_m \left( \alpha_{mn}c_{mn} + \beta_{mn}E[C(X, \hat{X})|D^n_k, H^m, Z_k] + \gamma_{mn}E[|I_{mn} - \hat{I}|]|D^n_k, H^m, Z_k] \right) P_n(H^m|Z_k)
\]  

(29)

Similar to (*) , the decision conditioned estimation and costs are calculated using the measurements lie in the region of the decision region \( D^i \). Based on the Bayes decision method, for target \( t \), a set of Radar and ESM measurements \( Z \) lie in the region of the decision region \( D^i \) when

\[
C^k_i(Z|Z^{k-1}) \leq C^k_n(Z|Z^{k-1}), \forall n
\]  

(30)

where, \( C^k_i(Z|Z^{k-1}) \) is the intermediate cost of target state estimation and classification

\[
C^k_i(Z|Z^{k-1}) = \frac{1}{\rho} \sum_j \left( \alpha_{ij}c_{ij} + \beta_{ij}e_{ij} \right) L(Z|Z^{k-1}, H^j_i) P\{H^j_i|Z^{k-1}\}
\]  

(31)

where, \( \rho \) is the normalization factor. \( L(Z|Z^{k-1}, H^j_i) \) is the likelihood functions conditioned on target type of \( H^j_i \) of the kinematic and attribute measurements

\[
L(Z|Z^{k-1}, H^j_i) = f(z^r|z^{k-1}, H^j_i)f(z^a|z^{k-1}, H^j_i) \text{Pr}(z_c=j|H^j_i)
\]  

(32)

Especially, when the target belongs to two possible classes \( H^j_i, j=1,2 \)

\[
\frac{C_1}{C_2} \geq \frac{D^i_2}{D^i_1} L(Z|Z^{k-1}, H^j_i) P\{H^j_i|Z^{k-1}\}
\]  

(33)

where \( C_1 = \alpha_{12}c_{12} + \alpha_{11}c_{11} + \beta_{11}e_{11} + \beta_{12}e_{12} \). Actually, the class decisions of each target form a partition of the measurement space. Here, the inclusion function \( 1_{D^j_i}(z) \) is used in (*) to indicate whether the measurement \( z \) lies inside the region \( D^j_i \). If \( z \in D^j_i \), \( 1_{D^j_i}(z) = 1 \); if \( z \notin D^j_i \), \( 1_{D^j_i}(z) = 0 \). When the measurement is missing, according to the mapping \( \theta \), the corresponding likelihood function is equal to \( 1 - p_d \).

Assume at \( k - 1 \), the posterior density of target \( \ell \) can be given by

\[
p_{k-1}(x, \ell) = \sum_{j=1}^J f_{k-1}(x, \ell|H^j_i) P(H^j_i)
\]  

(34)

where \( P(H^j_i) \) is the probability of the class hypothesis, and \( f_{k-1}(x, \ell|H^j_i) \) is the class dependent target density. Then, the multi-target posterior density at \( k - 1 \) can be represented as

\[
\pi_{k-1} = \{(r_{k-1}^{(\ell)}, p_{k-1}^{(\ell)}(x|H^j_i) P(H^j_i)) \}\in \mathbb{E}_k
\]  

Suppose that the multi-target birth density is also LMB RFS with label set \( \mathbb{E} \), the posterior density conditioned on the decision set \( D^i_k = \{D^i_k, \ell \in L(X) \cup \mathbb{E} \} \) at time \( k \) can be given by

\[
\pi(X|D^n_k) = \frac{1}{\eta(X)} \sum_{I_{k-1} \in \mathbb{E}} \Theta(I_{k-1} \cup \mathbb{E}) \psi_k \bigg( \sum_j p_n^{(\ell)}(\cdot, |H^j_i, D^i_k, Z_k) P_n^{(\ell)}(H^j_i|D^i_k, Z_k) \bigg)^{x/2} \prod_{j=1}^J p_n^{(\ell)}(\cdot, |H^j_i, D^i_k, Z_k) P_n^{(\ell)}(H^j_i|D^i_k, Z_k)
\]

(35)

where \( \Theta \) is the space of mappings \( \theta \) between the targets and the measurements from Radar and ESM sensors, i.e., \( \Theta : L \rightarrow \{0,1,...,|Z_k|\} \times \{0,1,...,|Z_k|\} \), \( \eta = \sum_{I_{k-1} \cup \mathbb{E}} \prod_{j=1}^J p_n^{(\ell)}(\cdot, |H^j_i, D^i_k, Z_k) P_n^{(\ell)}(H^j_i|D^i_k, Z_k) \) is the normalization factor, and \( \psi_k \) is the inclusion function that indicates whether the measurement lies inside the region of corresponding decision \( D^i_k \) according to the mapping. The posterior density and class probability of each target can be calculated as

\[
p_n^{(\ell)}(x, \ell|H^j_i, D^i_k, Z_k) = \frac{1_{D^i_k}(x, \ell) \psi_k(x, \ell; \theta, f_{k-1}(x, \ell)p_{k-1}(x|H^j_i))}{\eta(Z_k^{\ell}(\ell)) P_{k-1}(H^j_i)}
\]

(36)

\[
P_n^{(\ell)}(H^j_i|D^i_k, Z_k) = \frac{\eta(Z_k^{\ell}(\ell)) P_{k-1}(H^j_i)}{\sum_{j=1}^J \eta(Z_k^{\ell}(\ell)) P_{k-1}(H^j_i)}
\]

(37)

\[
\Psi_k(x, \ell; \theta) = \psi_k(x, \ell; \theta) \psi_k^r(x, \ell; \theta)
\]

(38)

where \( f_{k-1}(x, \ell) \) is the state transition function, \( \psi_k^r(x, \ell; \theta) \) are the likelihood functions of the Radar and ESM measurements, respectively.

\[
\psi_k^r(x, \ell; \theta) = \begin{cases} 1 - p_d(x, \ell), & \text{z}_\theta(x, \ell) = \emptyset \\ p_d(x, \ell) g(\text{z}_\theta(x, \ell), \ell), & \text{other} \end{cases}
\]

(40)

\[
\psi_k^r(x, \ell; \theta) = \begin{cases} 1 - p(x, \ell), & \text{z}_\theta(x, \ell) = \emptyset \\ p(x, \ell) g(\text{z}_\theta(x, \ell), \ell) \text{Pr}(z_c=j|H^j_i), & \text{other} \end{cases}
\]

(41)

In (*) , the weights \( \omega_k^i \) is equal to

\[
\gamma_k^{Z_k^\ell}(\ell) = \begin{cases} 1 - p_k(x, \ell), & \forall \ell \in I_{k-1}, \ell \notin I_k \\ p_k(x, \ell) \eta_{k-1}^\ell(\ell), & \forall \ell \in I_{k-1}, \ell \notin I_k \end{cases}
\]

(42)

The LMB RFS that matches exactly the first moment of the multi-target posterior density can then be given by

\[
\pi_n^k(x|Z_k) = \{(r_n^{(\ell)}, p_n^{(\ell)}(x|H^j_i) P_n(H^j_i)) \}\in \mathbb{E}_k
\]

(43)
where
\[ p_n^{(e)}(x|H_k^l) = \frac{1}{y^{(e)}} \sum_{l_k,\theta} \omega_n^{(l_k,\theta)}(Z_k)1_{l_k}(\ell) \]

\[ P_n(H_k^l) = \frac{1}{y^{(e)}} \sum_{l_k,\theta} \omega_n^{(l_k,\theta)}(Z_k)1_{l_k}(\ell)P_0^{(\theta)}(H_k^l) \]

In the update step, the multi-target posterior density is computed conditioned on the decision. Additionally, multi-target distribution is approximated by preserving the spatial density of each track with exact match of the first moment.

To derive the optimal CJDE solution, the costs of multi-target detection, tracking and classification need to be calculated. For the exact calculation of the posterior density for each target involving the MTA's, the CJDE cost can be calculated as
\[ C_n(Z_k) = \sum_m \left( \sum_c \omega_n^c(\alpha_{mn}c_{mn} + \beta_{mn}\varepsilon_X) + \gamma_{mn}\varepsilon_I \right) P_n(H^m|Z_k) \]

where \( c \in \mathbb{C} \) represents \((I_k, \theta) \in \mathcal{F}(\mathbb{L}) \times \Theta \), and the hypothesis probability
\[ P_n(H^m|Z_k) = \prod_{\ell \in l_k} P_n(H_k^\ell) \]

The calculation of the CJDE cost can be divided into two parts. Firstly, the joint cost of target detection and classification can be calculated as
\[ C_n(Z_k) = \sum_m \sum_c \omega_n^c(\alpha_{mn}c_{mn} + \beta_{mn}\varepsilon_X) \prod_{\ell \in l_k} P_n(H_k^\ell) \]

where \( c \in \mathbb{C} \) represents \((I_k, \theta) \in \mathcal{F}(\mathbb{L}) \times \Theta \), and the hypothesis probability
\[ P_n(H^m|Z_k) = \prod_{\ell \in l_k} P_n(H_k^\ell) \]

The recursive CJDE-LMB Algorithm
1. Predict prior multi-target density using the class-dependent dynamic model according to the hypothesis.
2. Update \( \hat{x}_{k,\ell} \), \( P_n(H_k^\ell) \), and \( \omega_n^{(l_k,\theta)} \) for decision \( D^\ell_k \) using the conditional LMB filter.
3. Calculate the joint detection, tracking and classification cost \( C_n(Z_k) \) using (45)-(48), and the optimal decision is then \( D^\ell_k : C_n(Z_k) \leq C_i(Z_k), \forall i \), and the corresponding target state estimates are derived using the conditional LMB filter.
4. Output the CJDE solution for time k: the optimal decision \( D_k = D^\ell_k \), the target existence probability \( r_n^{(e)} \) and the state estimate \( \hat{x}_{k,\ell} \).

C. Gaussian mixture implementation
In this subsection, the Gaussian mixture implementation of the proposed recursive JDTC approach is developed.

1) Prediction: Suppose that at time \( k - 1 \), the multi-target density can be represented as \( \pi_{k-1}(X) = \{ (r_{k-1}^{(l)}, p_{k-1}^{(l)}(x|H_k^l)) \}_{\ell \in \mathbb{L}} \), where \( p_{k-1}^{(l)}(x|H_k^l) \) is the density of track \( \ell \) that can be typically modeled by a Gaussian mixture
\[ p_{k-1}^{(l)}(x|H_k^l) = \sum_{n=1}^{N_{k-1,\ell}} \omega_{n,k-1,j}(x)N(x, m_{n,k-1,j}, P_{n,k-1,j}) \]

where \( m_{n,k-1,j} \) and \( P_{n,k-1,j} \) are the mean value and covariance of the state vector, the predicted multi-target density can then be represented as (29). Suppose that the predicted multi-target density can be represented by the parameters \( \pi_{k|k-1}(X) = \{ (r_{k|k-1}^{(l)}, p_{k|k-1}^{(l)}(x|H_k^l)) \}_{\ell \in \mathbb{L}} \), where the density \( p_{k|k-1}^{(l)}(x|H_k^l) \) can be represented by a Gaussian mixture as
\[ p_{k|k-1}^{(l)}(x|H_k^l) = \sum_{n=1}^{N_{k|k-1,\ell}} N(x; m_{n,k|k-1,j}, P_{n,k|k-1,j}) \]
When the measurement set $Z_k$ is collected at time $k$, the posterior multi-target density conditioned on the decision $\{D_k^n\}$ is

$$\pi_k^n(X|Z_k) = \Delta(X) \sum_{(J,\theta)\in\mathcal{F}(k_+,n)} \omega_n^{(I+\theta)}(Z_k)\delta_{I_+}(\mathcal{L}(X)) \times \left[ \sum_j p_n^{(\theta)}(\cdot,\ell|H_k^j, D_k^j, Z_k) P_n^{(\theta)}(H_k^j|D_k^j, Z_k) \right]^{x}$$

where the weight

$$\omega_n^{(I+\theta)}(Z_k) \sim \omega_n^{(I+)} \left[ \eta_n^{(\theta)}(\ell|D_k^j, H_k^j) \right]$$

and

$$\eta_n^{(\theta)}(\ell|D_k^j, H_k^j) = 1_{D_k^j}(z_{\theta(\ell)}) \left(1 - p_d\right) + p_d \frac{1}{\lambda(\ell)} \sum_{n=1}^{N_k^{i,j-1}} \omega_n^{(n)}(x_{\ell}^{k-1}, j) N(z; H_k m_k^{(n)}(x_{\ell}^{k-1}), H_k P_k^{(n)}(x_{\ell}^{k-1}, j, H_k^j + R_k)) \right)$$

The posterior density of each target can be calculated using the measurement augmented optimal Kalman filtering method as follows

$$p_n^{(\theta)}(x, \ell|Z_k) = \sum_{n=1}^{N_k^{i,j-1}} \omega_n^{(n)}(x_{\ell}^{k-1}, j) N(z_{\theta(\ell)}; H_k m_k^{(n)}(x_{\ell}^{k-1}, j), P_k^{(n)}(x_{\ell}^{k-1}, j, H_k^j + R_k))$$

where

$$m_k^{(n)}(x_{\ell}^{k-1}, j) = F_k^{ij} m_k^{(n)}(x_{\ell}^{k-1}, j)$$

$$P_k^{(n)}(x_{\ell}^{k-1}, j) = F_k^{ij} P_k^{(n)}(x_{\ell}^{k-1}, j, F_k^{ij} + Q_k^{ij})$$

$$q_k^{(n)}(z_{\theta(\ell)}; H_k m_k^{(n)}(x_{\ell}^{k-1}, j), P_k^{(n)}(x_{\ell}^{k-1}, j, H_k^j + R_k))$$

$$m_k = \hat{x}_{k|k-1,\ell} + K_k(z_{\theta(\ell)} - z_+)$$

$$z_+ = H_k \hat{x}_{k|k-1,\ell} + b$$

$$K_k = P_k|k-1,H_k^j[H_k^j + R_k]^{-1}$$

$$P_k = (I - K_k H) P_k|k-1$$

$$S_k = H P_k|k-1 H^T + R$$

3) Calculate the risk: Compute the class dependent posterior estimate and associated covariance with respect to the distribution given in (60), that is

$$\hat{x}_{k,\ell} = \sum_{n=1}^{N_k^{i,j}} \omega_n^{(n)} m_k^{(n)}(x_{\ell}^{k-1}, j)$$

$$P_{k,\ell} = \sum_{n=1}^{N_k^{i,j}} \omega_n^{(n)} (P_{k,ij}^{(n)} + (m_k^{(n)} - \hat{x}_{k,\ell})(m_k^{(n)} - \hat{x}_{k,\ell})^T)$$

Then, the optimal estimate of track $\ell$ is

$$\hat{x}_{k,\ell} = \sum_{j=1}^{J} \hat{x}_{k,\ell} P_{k,\ell}(H_k^j)$$

For the explicit Gaussian mixture implementation of the conditioned LMB filter, the estimation cost $\varepsilon_X$ in (45) can be given by

$$\varepsilon_X = \sum_{i \in \mathcal{L}(X)} \left( (\hat{x}_{k,\ell} - \hat{x}_{k,\ell})^T (\hat{x}_{k,\ell} - \hat{x}_{k,\ell}) \right)$$

Finally, compute the CJDE cost for decision $D_k^n$ using (45)-(49), then the optimal solution can be derived.

D. Performance analysis

Because the detection of the target is the prerequisite of tracking and classification, if $\gamma_i$ is relative small, the CJDE cost $C_{m}(Z) \approx \sum_{i \in \mathcal{L}(X)} \frac{\epsilon_i}{C_m(m, \ell) + \beta_i^{\ell} \tau_i^{\ell}}$. Because the estimation and classification costs in the Bayes risk are nonnegative, in this case, the target tends to be judged as missed for less state estimation and classification costs, and an incorrect JDTC solution maybe derived. Assume that no measurements lie inside the region of $D_k^n$, the weight is nonnegative the existence probability of the target is

$$r_n^{(\ell)} = \sum_{I_k\ell} \omega_n^{(I+\theta)}(Z_k) 1_{I_+}(\ell)$$

$$= \sum_{I_k\ell \in \mathcal{L}(X)} \frac{1_{I_k\ell}(\ell)}{\sum_{I_k\ell \in \mathcal{L}(X)} 1_{I_k\ell}(\ell)}$$

Therefore, $\gamma_i$ can be chosen to make the maximum cost of the target detection approximate equal to the sum of the maximum costs of estimation and classification, i.e., $\gamma_i \approx (\alpha_{mn} \cdot \beta_{mn} \cdot \max(\varepsilon))/(1 - \bar{p})$, where $\bar{p}$ is the target existence probability estimate calculated with an empty set of measurements. In this case, the target detection cost will be predominant and the multi-target JDTC problem is solved with optimal estimate of the target number.

IV. SIMULATIONS

In this section, numerical examples are presented to illustrate the effectiveness and superiority of the proposed CJDE-LMB algorithm. In addition, the results derived with different parameters are also compared.
A. Example 1

Suppose that there are several targets with two possible classes move in a two-dimensional scenario. The classes differ from each other in terms of the dynamic behaviors, each class has a corresponding set of possible motion models. The \( i \)th model for class \( j \) is

\[
x_k = F_{k,i} x_{k-1} + w_{k,i}
\]

(75)

where \( F_{k,i} \) is the model-dependent state transition matrix, and \( w_{k,i} \) is Gaussian noise with covariance \( Q_{k,i} \). The target of class 1 only has the constant velocity (CV) model with the following parameters

\[
F_{k,1} = \text{diag} \left[ \begin{bmatrix} 1 & T \ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & T \ 0 & 1 \end{bmatrix} \right],
\]

\[
Q_{k,1} = \text{diag} \left[ \begin{bmatrix} T^2 & T \\ T & 1 \end{bmatrix}, \begin{bmatrix} T^2 & T \\ T & 1 \end{bmatrix} \right] \sigma_v^2
\]

where \( \sigma_v \) is the process noise with the covariance \( \sigma_v^2 = 1 \text{ m}^2/\text{s}^2 \).

The target of class 2 has two possible dynamic models, the CV model as before, and the constant accelerate (CA) model with parameters

\[
F_{k,2} = \text{diag} \left[ \begin{bmatrix} 1 & 0 & T^2 \\ 0 & 1 & T \end{bmatrix}, \begin{bmatrix} 1 & 0 & T^2 \\ 0 & 1 & T \end{bmatrix} \right],
\]

\[
Q_{k,2} = \text{diag} \left[ \begin{bmatrix} T^4 & T^3 & T^2 & T \\ T^3 & T^2 & T & 1 \end{bmatrix}, \begin{bmatrix} T^4 & T^3 & T^2 & T \\ T^3 & T^2 & T & 1 \end{bmatrix} \right] \sigma_v^2.
\]

(77)

where \( \sigma_v \) is the process noise with the covariance \( \sigma_v^2 = 10 \text{ m}^2/\text{s}^2 \). The model transition probability matrix is set as

\[
\pi = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}
\]

(78)

The kinematic measurement is \( z_k = [x_k, y_k]^T + w_k \), where \( [x_k, y_k] \) is the position of the target, and \( w_k \) is the Gaussian measurement noise with the covariance \( R_k = \text{diag}[\sigma_x^2, \sigma_y^2] \), \( \sigma_x = \sigma_y = 2 \text{ m} \). The target detection probability \( p_d = 0.98 \), and the intensity of the Poisson distributed clutter is \( 6 \times 10^{-5} \).

In the scenario, there are two non-maneuvering targets and one maneuvering target move within the two-dimensional scenario. Target 1 moves straight from the beginning to the end, with the initial location \([-200, 700]\) m and velocity \([50, 0]\) m/s. Target 2 appears at \( k = 5 \) with the initial location \([-200, 1000]\) m, and moves straight with constant velocity \([40,30]\) m/s until it disappears at \( k = 25 \). The maneuvering target 3 appears at the \( k = 3 \) and disappears at \( k = 27 \). It moves straight from location \([0, 1900]\) m with a constant acceleration of \([4, -3]\) m/s².

The multi-target detection, tracking, and classification performance of the CJDE-LMB algorithm is compared with the traditional methods in terms of the multi-target cardinality estimates, optimal subpattern assignment (OSPA) distance \([23]\), and the probability of correct classification, respectively. Moreover, the overall performance is evaluated by the joint performance metric (JPM), which is calculated with the costs of target detection, tracking, and classification.

The compared methods are the follows:

1) Estimation-Then-Decision: The target state is first estimated using the GNN approach, and the decision is then made based on the ratio of current measurement likelihoods of the predicted states conditioned on different hypotheses.

2) Decision-Then-Estimation: The target class is first determined, which minimizes the Bayes decision risk, and the target state is then estimated given the decided class.

3) Estimate the joint target-state-class probability density: As proposed in \([1]\), the class-dependent posterior density is firstly calculated using the particle implementation of the PHD filter with corresponding dynamic models. Then, the target state and class probabilities are obtained by clustering the particles. This method is referred to as YW-JDTC here.

In the simulation, the target survival probability is \( p_s = 0.98 \), and the target birth probability is \( p_b = 0.02 \). The density of the new birth target is \( b_k = N(x; m_b, Q_b) \), where the parameters \( m_{1,k} = [−200, 50, 0, 700, 0, 0]^T \), \( m_{2,k} = [−200, 40, 0, 1000, 30, 0]^T \), and \( m_{3,k} = [20, 4, 1900, −15, −3]^T \), while the state covariances are \( P_{1,k} = P_{2,k} = P_{3,k} = \text{diag}[100, 10, 1, 100, 10, 1] \).

The model transition probability matrix is set as

\[
\pi = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

(77)

Moreover, the proposed algorithm achieves the final goal because the interdependence between them are considered. The CJDE-LMB algorithm also performs best while the ETD method is the worst. The reason for this phenomenon is that the decision is only dependent on the current state estimation in the ETD method. In addition, although the superiority of the proposed CJDE-LMB algorithm over the YW-JDTC method is not very obvious, CJDE-LMB provides explicit decisions of the target classes, whereas YW-JDTC only computes the class probabilities. Summing up all the costs and the overall performance is evaluated in terms of the JPM. As depicted in Fig. 1(d), the performance of the CJDE-LMB algorithm is better than that of the other methods. This example shows that the performance of estimation and decision are improved because the interdependence between them are considered. Moreover, the proposed algorithm achieves the final goal

Figure 1(a) illustrates the estimate of the multi-target cardinality. The targets are correctly detected by the proposed CJDE-LMB approach. The reason is that, because the coefficient \( \gamma \) in the new CJDE risk is relatively large, the penalty of the target miss detection is severe. The tracking performance is shown in Fig. 1(b). As illustrated, the CJDE-LMB is the best in terms of the OSPA distance.
directly and the explicit estimation and classification result are derived.

B. Example 2

In order to illustrate the importance of the coefficients in the new Bayesian risk, the JDTC results are derived with different parameters in this example. Suppose that the coefficients are set to be $\alpha_{ij}^1 = 20, \beta_{ij}^1 = 1, \gamma_{ij}^1 = 100$, and $\alpha_{ij}^2 = 20, \beta_{ij}^2 = 1, \gamma_{ij}^2 = 10$, respectively. The values of $\alpha$ and $\beta$ make the costs of state estimation and classification balance. When $\gamma = 100$, the target detection plays a dual role as before, on contrary, when $\gamma = 10$, the cost of target miss detection contributes to $\bar{R}_C$ less significantly.

The performance of target detection, tracking and classification under different parameters is illustrated in Fig. 2. As shown in Fig. 2(a), when all the targets keep their motion modes, all the tracks are detected correctly. After the target 3 executes constant acceleration, all the tracks are maintained under $\gamma = 100$, whereas there exists target miss detection on some trials under $\gamma = 10$. The reason for this phenomenon is that after the target 3 performs maneuver, the optimal Bayesian decision converts to maneuvering, both the costs of estimation and decision increase due to the transition of dynamic model and the change of optimal Bayesian decision, respectively. In this case, all the targets can be correctly detected when $\gamma = 100$ because the penalization is heavier on target miss detection. On contrary, the decision with less state estimation and classification costs is chosen when $\gamma = 10$, in this case, the target is judged to be undetected. Due to the incorrect target detection results, the average tracking and classification performance given $\gamma = 10$ is worse than $\gamma = 100$ as illustrated in Fig. 2(b) and 2(c). As a result, the overall performance given $\gamma = 100$ is also better as shown in Fig. 2(d).

This example shows that, because target detection is the prerequisite for accurate tracking and correct classification in the multi-target JDTC problem, the penalization on target miss detection need to be heavier.

V. CONCLUSION

In this paper, a novel recursive approach was proposed to solve the multi-target joint detection, tracking, and classification problem. The optimal solution was derived based on a new generalized Bayesian risk involving the costs of target number estimation, state estimation and classification. Because the interdependence between the decision and estimation was considered, the performances of multi-target detection, tracking and classification were improved. Moreover, as the multi-target density was approximated by a sum of class dependent components, the computational complexity was largely reduced. The performance of the proposed approach was also analyzed, and the method of the coefficient selection was provided in order to derive reasonable results. As illustrated in the simulations, the targets can be detected correctly under appropriate cost coefficients, and the state estimation and classification performances of the proposed approach were
better than traditional methods.

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REFERENCES
[1] W. Yang, Y.W. Fu, J.Q. Long, X. Li, Joint Detection, Tracking and Classification of Multiple Targets in Clutter using the PHD Filter, IEEE Trans. Aerosp. Electron. Syst. 48(4) (2012) 3594-3609.
[2] B. Ristic, N. Gordon, A. Bessell, On target classification using kinematic data, Information Fusion 5 (2004) 15-21.
[3] B. Ristic, P. Smets, Target classification approach based on the belief function theory, IEEE Trans. Aerosp. Electron. Syst. 41(2) (2005) 574-583.
[4] G. Powell, D. Marshall, P. Smets, et al, Joint tracking and classification of airborne objects using particle filters and the continuous transferable belief model, in: 9th International Conference on Information Fusion, Florence, Italy, 2006, pp. 1-8.
[5] S. Hachour, F. Delmotte, D. Mercier, E. Lefvre, Object tracking and credal classification with kinematic data in a multi-target context, Information Fusion 20 (2014) 174-188.
[6] H. Lang, C. Shan, M.T. Pronobis, S. Scott, Wavelets feature aided tracking (WFAT) using GMTI/HRR data, Signal Process. 83(12) (2003) 2683-2690.
[7] Y. Bar-Shalom, T. Kirubarajan, C. Gokberk, Tracking with classification-aided multiframe data association, IEEE Trans. Aerosp. Electron. Syst. 41(5) (2005) 868-878.
[8] T. Zajic, B. Ravichandra, R. Mahler, R. Mehra, M. Noviskey, Joint tracking and identification with robustness against unmodeled targets, in: Signal Processing, Sensor Fusion and Target Recognition XII, in: Proc. SPIE, 2003, 5096.
[9] G. Lin, W. Sun, P. Wei, Extensions of the CBreMBer filter for joint detection, tracking, and classification of multiple maneuvering targets, Digital Signal Process., 56 (2016) 3542.
[10] R. Mahler, Statistical Multisource-Multitarget Information Fusion, Artech House, Norwood, MA, 2007.
[11] K. Punithakumar, T. Kirubarajan, A. Sinha, Multiple-model Probability Hypothesis Density Filter for Tracking Maneuvering Targets, IEEE Trans. Aerosp. Electron. Syst. 44(1) (2008) 87-98.
[12] W. Cao, J. Lan, X.R. Li, Conditional joint decision and estimation with application to joint tracking and classification, IEEE Trans. Syst. Man Cybern. 46(4) (2016) 459-471.
[13] X.R. Li, Optimal bayes joint decision and estimation, in: 10th International Conference on Information Fusion, Quebec City, Canada, 2007, pp. 1-8.
[14] Y. Liu, X.R. Li, Recursive Joint Decision and Estimation Based on Generalized Bayes Risk, in: 14th International Conference on Information Fusion, Chicago, USA, 2011, pp. 2066-2073.
[15] W. Cao, J. Lan, X.R. Li, Joint tracking and classification based on recursive joint decision and estimation using multi-sensor data, in: 14th International Conference on Information Fusion, Istanbul, Turkey, 2014, pp. 1-8.
[16] W. Cao, J. Lan, X.R. Li, Joint tracking and classification based on conditional joint decision and estimation, in: 18th International Conference on Information Fusion, WASHINGTON DC, USA, 2015, pp. 1764-1771.
[17] W. Cao, J. Lan, X.R. Li, Joint multi-target detection and tracking using conditional joint decision and estimation with OSPA-like cost, in: 18th International Conference on Information Fusion, Washington DC, USA, 2015, pp. 1740-1747.
[18] M.Z. Li, Z.L. Jing, P. Dong, H. Pan, Multi-target Joint Detection, Tracking and Classification Using Generalized Labeled Multi-Bernoulli Filter with Bayes Risk, in: 19th International Conference on Information Fusion, Heidelberg, Germany, 2016.
[19] B.-T. Vo, B.-N. Vo, Labeled random finite sets and multi-object conjugate priors, IEEE Trans. Signal Process., 61(13) (2013) 3460-3475.
[20] B.-N. Vo, B.-T. Vo, D. Phung, Labeled random finite sets and the Bayes multi-target tracking filter, IEEE Trans. Signal Process. 62(24) (2014) 6554-6567.
[21] S. Reuter, B.-T. Vo, B.-N. Vo, K. Dietmayer, The labeled multi-bernoulli filter, IEEE Trans. Signal Process. 62(12) (2014) 3246-3260.

[22] R. Mahler, Advances in Statistical Multisource-Multitarget Information Fusion, Artech House, Norwood, MA, 2014.

[23] B. Ristic, B.-N. Vo, D. Clark, B.-T. Vo, A metric for performance evaluation of multi-target tracking algorithms, IEEE Trans. Signal Process. 59(7) (2011) 3452-3457.