Bi-Maximal Neutrino Mixing in SO(10)_{GUT}

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Abstract

We find a grand unified SO(10) model which accommodates the bi-maximal neutrino mixing for vacuum-oscillation solutions to the atmospheric and the solar neutrino problems. This model maintains the original SO(10) mass relation between neutrino and up-type quark masses, \( m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2 \).
The recent data on the atmospheric neutrino from the SuperKamiokande (SuperK) Collaboration [1] have presented convincing evidence for neutrino oscillation with mass-squared difference $\delta m_{\text{atm}}^2 \simeq 5 \times 10^{-3}$ eV$^2$. It is now understood that the long-standing puzzle of the atmospheric muon neutrino ($\nu_\mu$) deficit in underground detectors [2] is indeed due to the neutrino oscillation. As for the solar neutrino problem, there are still two allowed solutions: one is matter enhanced neutrino oscillation (i.e. the MSW solution [3]) and the other is long-distance vacuum neutrino oscillation called as “just-so” oscillation [4, 5].

It is known [7] that the small angle MSW solution and the maximal mixing between the atmospheric $\nu_\mu$ and $\nu_\tau$ are quite naturally explained in a large class of see-saw models [8]. However, the electron energy spectrum recently reported by the SuperK Collaboration [9] seems to favor the “just-so” vacuum oscillation with $\delta m_{\text{sun}}^2 \simeq 10^{-10}$ eV$^2$ and the maximal mixing. If this vacuum oscillation of the solar neutrino is confirmed in future solar-neutrino experiments [9, 10], we are led to a quite surprising situation that two independent mixing angles in the lepton sector are very large in contrast with the quark sector in which all observed mixing angles among different families are small. This may point to that a rule governs the lepton mass matrices is significantly different from the one for the quark sector, which seems to be a contradiction to the idea of complete unification of quarks and leptons [6].

On the other hand, as noted recently by Barger et.al. [12] the required neutrino mass ratio $m_{\nu_2}/m_{\nu_3} \simeq 10^{-4}$ (provided $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$) is approximately equal to $(m_c/m_t)^2$ as predicted by a see-saw model in the SO(10) grand unified theory (GUT) [13].

The purpose of this paper is to construct a simple SO(10)$_{\text{GUT}}$ model which naturally accommodates the bi-maximal neutrino mixing for the atmospheric and the solar neutrino vacuum oscillations, keeping the interesting SO(10)$_{\text{GUT}}$ mass relation $m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2$. We assume supersymmetry throughout this paper.

Let us first discuss the minimal SO(10)$_{\text{GUT}}$ model which contains three families of

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1. The MSW solution has two distinct regions: the small and the large angle ones [3].
2. The so-called democratic mass matrices for quarks and leptons can generate large mixing in the neutrino sector [11].
3. Recent analyses on phenomenological consequences of the bi-maximal neutrino mixing are given in Ref. [12, 14].
quarks and leptons $\psi_i(16)$ ($i = 1, \cdots, 3$) belonging to 16 of the $SO(10)_{GUT}$ and one Higgs field $H(10)$. We will consider Higgs multiplets responsible for the breaking of $SO(10)_{GUT}$ down to the standard-model gauge group later. This minimal model is known to yield a mass degeneracy of up-type and down-type quarks and vanishing CKM mixing \cite{15}.

The simplest extension of the minimal model avoiding this unwanted mass degeneracy is to introduce another Higgs field $H'(10)$. This two-Higgs field 10 model, in fact, gives less stringent relations among quark and lepton mass matrices as

$$M_{\nu_D} = M_u, \quad M_l = M_d,$$

where $M_{\nu_D}$ is 3×3 Dirac mass matrix for neutrinos. It is well known \cite{16} that we need a large hierarchy in the Majorana mass matrix for right-handed neutrinos $N_i$ ($i = 1, \cdots, 3$) to obtain large neutrino mixing. However, if one assumes that the large hierarchy in the Majorana mass matrix for $N_i$ one loses the original $SO(10)_{GUT}$ relation, $m_{\nu_2}/m_{\nu_3} \sim \left(m_c/m_t\right)^2$, discussed in the introduction.

We, therefore, consider a different extension of the minimal $SO(10)_{GUT}$ model in this paper. Instead of adding an extra Higgs field $H'(10)$, we introduce one extra matter multiplet $\psi(10)$ belonging to 10 of the $SO(10)_{GUT}$ \cite{17}. Thus, the matter multiplets in our model are three families of $\psi_i(16)$ and one $\psi(10)$.

We now assume that the $SO(10)_{GUT}$ is broken down to SU(5) by condensation of Higgs fields \langle $\chi(16)$ $\rangle = \langle \bar{\chi}(16^*) \rangle = V$ with $V$ being $\sim 10^{16}$ GeV.\cite{18} This GUT breaking also induces a mass term for the matter multiplets through the following superpotential,

$$W = \sum_{i=1}^{3} f_i \psi_i(16) \psi(10) \langle \chi(16) \rangle.$$  \hspace{1cm} (2)

Namely, a linear combination $5^*_{\psi} \equiv \sum_{i=1}^{3} f_i 5^*_i$ in $\psi_i(16)$ receives a GUT scale mass together with $5_{\psi}$ in $\psi(10)$. We choose $f_2 = f_3 = 0$, here. The reason for this will be clearly understood later on.

\footnote{4 We introduce one extra matter multiplet $\psi(10)$ in this paper. We may, however, introduce three families of $\psi_i(10)$ ($i = 1, \cdots, 3$). In this case, the Froggatt-Nielsen mechanism \cite{18} may be used to account for observed quark and lepton mass matrices, assuming different charges for $\psi_i(16)$ and $\psi_i(10)$ in each families.}

\footnote{5 We need other Higgs multiplets such as 45 to complete the breaking of the $SO(10)_{GUT}$ down to the standard-model gauge group. We do not consider them in this paper, since they are irrelevant to our present analysis.}
After the spontaneous breakdown of SO(10) GUT to SU(5), massless matter multiplets are given by

\[
\begin{align*}
\text{10}_3 & + \text{5}^{*}_3 + N_3 = \psi_3(16), \\
\text{10}_2 & + \text{5}^{*}_2 + N_2 = \psi_2(16), \\
\text{10}_1 & + N_1 \subset \psi_1(16), \\
\text{5}^{*}_\psi & \subset \psi(10).
\end{align*}
\]

It should be clear that \text{10}_i (i = 1, \cdots, 3) in Eq. (3) are not \text{10} of the SO(10) GUT, but \text{10} of the SU(5).

We take a basis where the original Yukawa coupling matrix of the Higgs field \(H(10)\) to the matter \(\psi_i(16)\) is diagonal:

\[
W = h_i \psi_i(16) \psi_i(16) H(10).
\]

This leads to a diagonal mass matrix for the up-type quarks as

\[
M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix},
\]

where \(M_u\) is defined as

\[
(M_u)_{ii} = h_i \langle 5_H \rangle.
\]

Here, \(5_H\) is a SU(5)-5 component of \(H(10)\).

The down-type quark mass matrix is, however, incomplete, since the SU(5)-5* of \(\psi(10)\) (i.e. \(5^{*}_\psi\)), does not have any Yukawa coupling to \(H(10)\). To solve this problem we introduce a pair of Higgs fields \(H(16)\) and \(H(16^*)\) and consider a superpotential

\[
W = k H(10) \tilde{H}(16^*) \tilde{\chi}(16^*) + M H(16) \tilde{H}(16^*).\]

\(U(1)\ R\)-symmetry may be useful to have this form of superpotential. The \(U(1)_R\) charges are given in Table \(\text{II}\). The GUT condensation \(\langle \tilde{\chi}(16^*) \rangle \neq 0\) induces a mass mixing between 5*’s of \(H(10)\) and \(H(16)\) (i.e. \(5^{*}_{H(10)}\) and \(5^{*}_{H(16)}\)). Then, a linear combination

\[
\tilde{5}^{*}_H = \cos \theta 5^{*}_{H(10)} + \sin \theta 5^{*}_{H(16)};
\]

\[
\tan \theta = - \frac{k \langle \tilde{\chi}(16^*) \rangle}{M}.
\]

\(^6\) Precisely speaking, we assume \(f_2 = f_3 = 0\) in Eq. (2) in this basis. We discard small deviations from our assumption in the present analysis.
remains as a massless Higgs field $H(5^*)$ in the standard SU(5)$_{\text{GUT}}$ and contributes to the quark and lepton mass matrix. Then, $\tilde{5}_H^*$ can couple to $5_\psi^*$ as

$$W_{\text{eff}} = \sin \theta \sum_{i=1}^{3} g_i \mathbf{10}_i \tilde{5}_H^* 5_\psi^*,$$

(10)

where the coupling constants $g_i$ are defined as

$$W = \sum_{i=1}^{3} g_i \psi_i(16) \psi(10) H(16).$$

(11)

Now, the Yukawa coupling of $\tilde{5}_H^*$ is given by

$$W_{\text{eff}} = \cos \theta \left( \begin{array}{ccc} 0 & g_1 \tan \theta & 0 \\ h_2 & g_2 \tan \theta & 0 \\ 0 & g_3 \tan \theta & h_3 \end{array} \right) \left( \begin{array}{c} \tilde{5}_H^* \\ 5_\psi^* \\ 5_3^* \end{array} \right),$$

(12)

which yields the down-type quark and the charged lepton mass matrix,

$$M_{d/l} = m_t \begin{pmatrix} 0 & x & 0 \\ m_c/m_t & y & 0 \\ 0 & z & 1 \end{pmatrix} \times \frac{\cos \theta}{\tan \beta}. $$

(13)

Here, $\tan \beta \equiv \langle 5_H^* \rangle/\langle \tilde{5}_H^* \rangle$ and

$$x = \frac{g_1}{h_3} \tan \theta, \quad y = \frac{g_2}{h_3} \tan \theta, \quad z = \frac{g_3}{h_3} \tan \theta.$$

(14)

We see that a choice of $x \sim m_c/m_t$, $y \sim \sqrt{m_c/m_t}$ and $z \sim 1$ produces a nice fit of the observed quark and lepton mass ratios and the CKM matrix. Thus, we take $x \sim m_c/m_t$, $y \sim \sqrt{m_c/m_t}$ and $z \sim 1$. The $\tan \beta$ may be very large unless $\cos \theta$ is very small ($\tan \beta \simeq \sqrt{2}(m_t/m_b) \cos \theta$). It is now clear that in contrast with the CKM mixing we have a large

\[\text{Table 1: } U(1)_R \text{ charges.}\]

|   | $H(10)$ | $H(16)$ | $H(16^*)$ | $\chi(16)$ | $\bar{\chi}(16^*)$ | $\psi_i(16)$ | $\psi(10)$ |
|---|--------|--------|-----------|----------|----------------|-----------|--------|
| $R$ | 0      | 0      | 2         | 0        | 0              | 1         | 1      |

\footnote{To explain quark and lepton masses more precisely one must introduce SU(5) breaking effects, otherwise we have wrong SU(5)$_{\text{GUT}}$ relations, $m_\mu = m_e$ and $m_\tau = m_d$. A detailed analysis including these effects will be given in Ref. [19].}
mixing closed to the maximal between $5_3^*$ and $5_\psi^*$ which corresponds to a mixing between charged leptons of the third and second families.

Let us turn to the Dirac mass term for neutrinos which is given by the superpotential \[ W = \sum_{i=1}^{3} k_i \psi_i(16) \psi(10) \bar{H}(10) \frac{\chi(16^*)}{M_G}, \] where $M_G$ is the gravitational scale $M_G \simeq 2 \times 10^{18}$ GeV. Together with the original coupling in Eq. (4), the nonrenormalizable interaction (15) yields

$$M_{\nu D} = m_t \begin{pmatrix} 0 & \delta_1 & 0 \\ m_c/m_t & \delta_2 & 0 \\ 0 & \delta_3 & 1 \end{pmatrix}.$$ (16)

Here, $M_{\nu D}$ is defined as

$$W_{\text{eff}} = \left( \begin{array}{ccc} N_1, & N_2, & N_3 \end{array} \right) M_{\nu D} \left( \begin{array}{c} 5_2^* \\ 5_\psi^* \\ 5_3^* \end{array} \right),$$ (17)

and $\delta_i = (k_i/h_3)(\langle \chi(16) \rangle / M_G)$. Notice that $\delta_i \simeq O(10^{-2})$ as long as $k_i/h_3 \sim O(1)$. It is extremely interesting that when $\delta_2 \sim m_c/m_t$ we have a large mixing between $5_\psi^*$ and $5_2^*$ which produces a large mixing between left-handed neutrinos of the second and first families. This observation is crucial for our purpose, since this tells us that when we maintain the SO(10)GUT relation, $m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2$ (i.e. $\delta_2 \simeq m_c/m_t$), we necessarily obtain a large mixing closed to the maximal between $\nu_e$ and $\nu_\mu$ (provided that the Majorana mass matrix for $N_i$ does not have hierarchy). We take, for simplicity, $\delta_1 \simeq \delta_3 \simeq 0$ and $\delta_2 \simeq m_c/m_t^8$

The Majorana masses for the right-handed neutrino $N_i$ are given by the following nonrenormalizable superpotential,\[ W = j j^* \frac{1}{M_G} \psi_i(16) \psi_j(16) \bar{\chi}(16^*) \bar{\chi}(16^*). \] (18)

\(^8\) If $\delta_2 \simeq 1/5$, the small angle MSW solution can be accommodated instead of the “just-so” solution. In this case some SO(10)-singlet fields are required at the GUT scale.
After the SO(10)$_{\text{GUT}}$ breaking we obtain the Majorana mass matrix,

$$(M_N)_{ij} = \frac{\langle \bar{\chi}(16^*) \rangle^2}{M_G} (j_{ij}). \quad (19)$$

Simply assuming $\langle \bar{\chi}(16^*) \rangle = V \sim 10^{16}$ GeV and $j_{ij} \sim \delta_{ij}$ we get

$$M_N \sim (10^{14} \text{ GeV}) \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (20)$$

From the see-saw mechanism the light neutrino masses are given by

$$M_\nu \sim \frac{(m_t)^2}{M_N} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (m_c/m_t)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (21)$$

with the MNS neutrino mixing matrix [27] defined in the basis where the charged lepton mass matrix is diagonal,

$$U_{\text{MNS}} \sim \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \epsilon \\ 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \end{pmatrix}, \quad (22)$$

$$\epsilon = O(\sqrt{m_c/m_t}). \quad (23)$$

This neutrino mass matrix given by Eqs. (21, 22) is nothing but the one used for explaining the atmospheric neutrino oscillation [1] and the “just-so” oscillation of the solar neutrino [12, 14].

In this paper we have found a simple SO(10)$_{\text{GUT}}$ model which naturally generates the bi-maximal neutrino mixing suggested from the atmospheric $\nu_\mu$ deficit and the “just-so” oscillation solution to the solar neutrino problem. This model maintains the original SO(10)$_{\text{GUT}}$ mass relation, $m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2$, which is required for the “just-so” scenario [12]. However, one may think that the present model is already too complicated, and in this sense the “just-so” oscillation seems very unlikely as stressed by P. Ramond and one of the authors (T.Y.) [7].

Nevertheless, if it turns out to be the case, we will be forced to consider drastic changes of underlying physics governs the Yukawa couplings for quarks and leptons. We think that

\[ \text{From Eq.(21), we obtain } m_{\nu_3} \simeq \sqrt{\delta m_{\text{atm}}^2} \simeq O(0.1) \text{ eV.} \]
our modified SO(10)\textsubscript{GUT} model presented in this paper will be a rather mild change among them.

**Note added**

In the text we have restricted our discussion to a specific case of \( f_2 = f_3 = 0 \) in Eq. (2).

For a general case \((f_1, f_2, f_3 \neq 0)\) we have the following mass matrix for down-type quarks and charged leptons in the limit \( m_u = 0 \):

\[
M_{d/l} = m_t \begin{pmatrix} 0 & x & 0 \\ (\cos \alpha_1)m_c/m_t & y & (\sin \alpha_1 \sin \alpha_2)m_c/m_t \\ 0 & z & \cos \alpha_2 \end{pmatrix} \times \frac{\cos \theta}{\tan \beta},
\]

which may yield a better fit to the observations.

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References

[1] SuperKamiokande Collaboration, Talk by T. Kajita at Neutrino-98, Takayama, Japan, June 1998.

[2] Kamiokande Collaboration, K.S. Hirata et al., Phys. Lett. B280 (1992) 146; IMB Collaboration, R. Becker-szendy et al., Phys. Rev. D46 (1992) 3720; SOUDAN-2 Collaboration, W.W.M. Allison et al., Phys. Lett. B391 (1997) 491.

[3] L. Wolfenstein, Phys. Rev. D17 (1978) 2369; S.P. Mikheyev and A. Smirnov, Yad. Fiz. 42 (1985) 1441; Nuovo Cim. 9C (1986) 17.

[4] See, for example, N. Hata and P. Langacker, Phys. Rev. D50 (1994) 632.

[5] V. Barger, R.J.N. Phillips, and K. Whisnant, Phys. Rev. D24 (1981) 538.

[6] S.L. Glashow and L.M. Krauss, Phys. Lett. B190 (1987) 199.

[7] T. Yanagida, Talk at Neutrino-98, Takayama, Japan, June 1998; P. Ramond, Talk at Neutrino-98, Takayama, Japan, June 1998.

[8] J.A. Harvey, P. Ramond, and D.B. Reiss, Nucl. Phys. B199 (1982) 223; J. Sato and T. Yanagida, hep-ph/9710510; M. Bando, T. Kugo, and K. Yoshioka, Phys. Rev. Lett. 80 (1998) 3004; C.H. Albright, K.S. Babu, and S.M. Barr, hep-ph/9805266; J.C. Pati, Talk at Neutrino-98, Takayama, Japan, June 1998; J.K. Elwood, N. Irges, and P. Ramond, hep-ph/9807228.

[9] SuperKamiokande Collaboration, Talk by Y. Suzuki at Neutrino-98, Takayama, Japan, June 1998.

[10] SNO Collaboration, Talk by A. McDonald at Neutrino-98, Takayama, Japan, June 1998.

[11] M. Fukugita, M. Tanimoto, and T. Yanagida, Phys. Rev. D57 (1998) 4429; H. Fritzsch and Z.Z. Xing, Phys. Lett. B413 (1997) 396; See also, R. Barbieri, L.J. Hall, D. Smith, A. Strumia, and N. Weiner, hep-ph/9807235.

[12] V. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, hep-ph/9806387.
[13] T. Yanagida, in Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK report 79-18, 1979), p. 95; M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity, ed. P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979), p. 315.

[14] S.M. Bilenky and C. Giunti, hep-ph/9802201; A.J. Baltz, A.S. Goldhaber, and M. Goldhaber, hep-ph/9806540.

[15] P. Langacker, Phys. Rep. 72 (1981) 185.

[16] T. Yanagida and M. Yoshimura, Phys. Lett. B97 (1980) 99.

[17] J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. D49 (1994) 4966.

[18] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147 (1979) 277.

[19] Y. Nomura, T. Sugimoto, and T. Yanagida, in preparation.

[20] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.