Local Bursts Model of CMB Temperature Fluctuations: Scattering in Resonance Lines of Primordial Hydrogen and Helium

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Within the framework of a flat cosmological model a propagation of an instantaneous burst of isotropic radiation is considered from the moment of its beginning at some initial redshift $z_0$ to the moment of its registration now (at $z = 0$). We take into account Thomson scattering by free electrons and scattering in $L_\alpha$ and $L_\beta$ lines of primordial hydrogen and in lines $1s2 - 1s2p$, $1s3p$ ($^1S - ^1P^*$) of HeI. It is shown that relative amplitude of spectrum distortions caused by scattering in these lines may be from $10^3$ to $10^4$ times greater than maximum possible amplitude due to scattering in subordinate lines considered in our previous paper (Dubrovich, Grachev, 2015). In a linear approximation on the optical thickness in the lines the profiles of distortions in resonance lines turn out to be purely in absorption and do not depend on both direction and distance to the burst center in contradistinction to the profiles in subordinate lines. The profiles contain the jumps on the frequencies corresponding to appearance of a source (a burst) at a given redshift $z = z_0$. For $z_0 = 5000$ the jumps in hydrogen $L_\alpha$ and $L_\beta$ lines are on the frequencies 493 and 584 GHz respectively and the jumps in the two HeI lines are on 855 and 930 GHz for $z_0 = 6000$. Relative magnitude of jumps amounts to $10^{-4} - 3 \cdot 10^{-3}$.

\textit{Key words:} cosmology, early Universe, cosmological recombination, radiative transfer, Thomson scattering, line scattering

\textbf{Introduction}

New results of the PLANCK mission (see Adam et al., 2015a) greatly improve exactness of our knowledge about the power spectrum of primordial spatial fluctuations of the matter density in the early Universe, about peculiarities of primordial matter recombination dynamics, about some global fundamental parameters of the Universe and about CMB polarization. Essential progress is achieved also in the CMB spectroscopy. However all these important successes do not cancel further more detailed and deep investigations of CMB properties.

In particular the medium resolution spectroscopy of separate objects on the CMB brightness map seems to be very important. Novelty here is in turning from investigation of statistical CMB properties to searching and learning local phenomena and “protoobjects”. The last ones may be somewhat rare events having practically no effect on the average statistical CMB parameters. But they can carry some information about completely new physical laws. So for example one can expect an existence of primordial black holes of great mass and new local forms of matter and fields (see, for example, Dubrovich (2003), Grachev and Dubrovich (2011), Dubrovich and Glazirin (2012)).

Besides of more or less probable but still hypothetical objects there evidently exists the whole class of local sources in the early Universe which can be learned separately. These are the same standard primordial CMB temperature fluctuations thoroughly learned now statistically. In fact we deal with some spatial domains where temperature increase or decrease takes
place for some reasons or other. It is very important that besides spatial apartness of these regions their temperature deviation is also nonstationary. Depending on a mechanism of a given inhomogeneity formation a typical time of its development and damping can be different. With regard to such events one can speak about local-bursts model of fluctuations.

The present work is a continuation of our previous paper (Dubrovich and Grachev, 2015) devoted to calculations of radiation field evolution due to Thomson scattering and scattering in subordinate lines H$_{\alpha}$, H$_{\beta}$, P$_{\alpha}$ and P$_{\beta}$ of primordial hydrogen in a homogeneous expanding and recombining Universe in assumption that radiation arises instantaneously at some redshift $z_0$. Now we calculate spectral distortions arising from scattering in HI and HeI lines of Lyman series. Several distances from the burst center and several initial redshifts $z_0$ are considered. The moments $z_0$ are selected in such a way that optical thicknesses of the Universe in these lines were less than unity. Polarization of radiation is neglected. Scattering both Thomson and in lines is assumed to be isotropic. The absorption coefficient profile is approximated by delta-function in the comoving frame of reference i.e. we neglect own line broadening in comparison with the broadening due to space expansion. It is assumed also that the burst radiation do not affect on electron concentration and on occupation numbers of atom levels which are calculated beforehand by using Boltzmann and Saha equations because conditions in considered range of redshifts ($z = 3500 - 6000$) are close to the local thermodynamic equilibrium (LTE).

The main equations and relations

We are interested in distortions in a source (burst) spectrum which may arise as a result of scattering in HI and HeII resonance lines in the epoch when optical thickness of the Universe in these lines was sufficiently small ($< 1$). In Sobolev approximation (a narrow line approximation) an optical thickness in a line arising in a transition $1 \leftrightarrow k$, is given by (see e.g. Dubrovich and Grachev, 2004)

$$
\tau_{1k}(z) = \frac{hc}{4\pi} n_1(z) B_{1k} \left[ 1 - \frac{n_k(z) g_1}{n_1(z) g_k} \right],
$$

(1)

where $h$ is the Planck constant, $c$ is the speed of light, $n_1$ an $n_k$ are occupation numbers of the levels, $g_1$ and $g_k$ are statistical weights of the levels, $B_{1k}$ is the Einstein coefficient for radiation absorption in the transition $1 \rightarrow k$, $H(z)$ is the Hubble factor.

Let us find concentrations of atoms and ions by using Saha formulae. Let $N_H$ and $N_{He}$ are total concentrations of atoms and ions for hydrogen and helium correspondingly, $f_{He} \equiv N_{He}/N_H$ is the helium abundance in number of atoms. Further let $N_{He^0}$ and $N_{He^+}$ of hydrogen neutral atoms and ions respectively, $N_{He^0}$, $N_{He^+}$ and $N_{He^{++}}$ are concentrations of neutral, singly ionized and doubly ionized helium atoms. We will measure all concentrations in units $N_H$: $x_{He^0} = N_{He^0}/N_H$, $x_{He^+} = N_{He^+}/N_H$, $x_{He^{++}} = N_{He^{++}}/N_H$, $x_{He^{++}} = N_{He^{++}}/N_H$, $x_{He^{++}} = N_{He^{++}}/N_H$, $x_{He^{++}} = N_{He^{++}}/N_H$, $x_{He^{++}} = N_{He^{++}}/N_H$, $x_{Elect} = n_e/N_H$, where $n_e$ is the electron concentration. Then Saha equations and charge conservation equation are written as

$$
x_{He^+} \frac{x_{H^+}}{1 - x_{H^+}} = \frac{2U(H^+)}{U(H^0)} \frac{F(T_e)}{N_H} \exp(-\chi_H/kT_e),
$$

(2)

$$
x_{He^+} \frac{x_{He^+}}{f_{He} - x_{He^+} - x_{He^{++}}} = \frac{2U(He^+)}{U(He^0)} \frac{F(T_e)}{N_H} \exp(-\chi_{He}/kT_e),
$$

(3)
When calculating albedo we take into account spontaneous and induced transitions. The optical thicknesses are given by:

$$x_e x_{He^{++}} = \frac{2U(He^{++}) F(T_e)}{U(He^+)} \frac{N_H}{N_{He^{++}}} \exp(-\chi_{He^+/kT_e}),$$

$$x_e = x_{H^+} + x_{He^+} + 2x_{He^{++}},$$

where $F(T_e) = (2\pi m_e kT_e)^{3/2} h^{-3}$, $U$ are partition functions ($U(H^0) = 2$, $U(H^+) = 1$, $U(He^0) = 1$, $U(He^+) = 2$, $U(He^{++}) = 1$), $\chi$ is ionization potentials, $k$ is the Boltzmann constant, $m_e$ is the electron mass, $T_e$ is the electron temperature. For the considered range of redshifts one may adopt that the electron temperature coincides with the temperature of background blackbody radiation: $T_e(z) = T(z) = T_0(1 + z)$, where $T_0$ is the present-day CMB temperature.

By using Boltzmann equation for occupation numbers and taking into account that the overwhelming part of atoms is in the ground state, i.e. $n_{1,H^0} = N_{H^0} N_{H}$, $n_{1,He^0} = x_{He^0} N_{H}$, one can rewrite eq. (1) for HI and HeI lines in the form

$$\tau_{1k}(z) = \frac{c^3}{8\pi \nu_{1k}^3} \frac{x_0(z) N_H(z) g_k}{H(z)} g_{l1} A_{k1} \left[ 1 - e^{-h\nu_{1k}/kT(z)} \right],$$

where $h\nu_{1k}$ is the transition energy, $A_{k1}$ is the Einstein coefficient of spontaneous transitions, $x_0 = x_{H^0}$ or $x_{He^0}$ for HI or HeI respectively.

As for the other parameters (except for $\theta$) appearing in the problem they enter in particular in the Hubble factor

$$H(z) = H_0 \sqrt{\Omega_{\Lambda} + (1 - \Omega)(1 + z)^2 + \Omega_M(1 + z)^3 + \Omega_{rel}(1 + z)^4},$$

where $H_0 = 2.4306 \cdot 10^{-18} h_0^{-1}$, $h_0$ is the Hubble constant in the units 75 km/(s-Mpc); $\Omega_M$, $\Omega_{\Lambda}$ and $\Omega_{rel}$ are the ratios of matter, dark energy and relativistic particles (radiation, massless neutrino) densities to the critical density now; $\Omega = \Omega_M + \Omega_{\Lambda} + \Omega_{rel}$, $\Omega_{rel} = \rho_R^0 (1 + f_n)/\rho_c$, $\rho_R^0 = a_R T_0^4/c^2$ is the radiation mass density now, $f_n$ is the part of relativistic (massless) neutrino (usually $f_n = 0.68$). For the flat model of the Universe $\Omega = 1$, and then $\Omega_M = 1 - \Omega_{\Lambda} - \Omega_{rel}$.

Moreover the total concentration of hydrogen atoms and ions

$$N_H(z) = N_{H^0}(1 + z)^3, \quad N_{H^0} = 0.63144 \cdot 10^{-5} X_{O\beta} h_0^2 \cdot 3,$$

enter the equations. Here $\Omega_{\beta}$ is the ratio of baryon density to the critical density now, $X$ is the hydrogen abundance (by mass). As the base we use the following values of parameters: $\Omega = 1$, $\Omega_{\Lambda} = 0.7$, $\Omega_B = 0.04$, $T_0 = 2.728$ K, $X = 0.76$, $\Omega_{rel} = 0.85 \cdot 10^{-4}$, the Hubble constant $H_0 = 70$ km/(s-Mpc).

Figure 1 shows optical thicknesses in hydrogen $L_\alpha$ and $L_\beta$ lines and in HeI lines 1s2-1s2p ($\text{1S} - \text{1P}^*$) and 1s2-1s3p ($\text{1S} - \text{1P}^*$). We choose such a range of redshifts in order that the optical thicknesses ($\tau_{1k}$) of the Universe in these lines were less than unity. For hydrogen $\tau_{12} < 1$ for $z > 3060$ and $\tau_{13} < 1$ for $z > 2750$ which corresponds to the frequency ranges $\nu < 805$ and $\nu < 1062$ GHz, where $\nu = \nu_{1k}/(1 + z)$, $\nu_{1k}$ is the laboratory frequency of transition. For the two considered HeI lines the corresponding ranges of frequencies are $\nu < 1120$ and $\nu < 1310$ GHz.

Besides optical thicknesses one must know albedo of a single scattering $\lambda_{1k}(z)$ in the considered lines. When calculating albedo we take into account spontaneous and induced transitions due to blackbody background radiation with the temperature $T(z)$:

$$\lambda_{1k}(z) = R_{k1}(z)/\left[ \sum_{i'=1, i' \neq k}^{\infty} R_{k'i'}(z) + R_{kc}(z) \right],$$
where
\[
R_{ki}(z) = \frac{g_i}{g_k} \frac{A_{ik}}{\exp[h\nu_{ki}/kT(z)] - 1}, \quad k < i,
\]
(10)

are the coefficients of transitions upwards and
\[
R_{ki}(z) = \frac{A_{ki}}{1 - \exp[-h\nu_{ki}/kT(z)]}, \quad k > i,
\]
(11)

are the coefficients of transitions downwards, \(R_{kc}\) are the coefficients of transitions due to ionization by blackbody radiation, \(A_{ki}\) are Einstein coefficients of spontaneous transitions. For hydrogen we calculate albedo by using 60-levels atom model (see Grachev and Dubrovich, 1991) and for HeI we take into account transitions 1s2p, 1s3p – 1sk, 1skd (1P* – 1S), 1skd (1P* – 1D) for \(k \leq 10\) (the data from the base NIST: Ralchenko et al., 2011). For HeI photoionization crosssections from the states kP* for \(k = 2\) and 3 we use approximate formula
\[
\sigma_{kc}(\nu) = \sigma_0 \left( \frac{\nu_{kc}}{\nu} \right)^3
\]
where \(\sigma_0\) and \(\nu_{kc}\) are the threshold values of the crosssection and frequency of ionization. According to Hummer and Storey (1998) \(\sigma_0 = 1.35 \cdot 10^{-17}\) and \(2.70 \cdot 10^{-17}\) cm\(^2\) for \(k = 2\) and 3 respectively. Figure 2 shows albedos of a single scattering in considered lines.

To solve the problem of radiation transfer taking into account both Thomson scattering and scattering in resonance lines we use the same method as in our previous paper (Dubrovich and Grachev, 2015) devoted to subordinate hydrogen lines. The only additional condition is a limitation on the value of the optical thickness in resonance lines. The thing is that in contradiction to subordinate lines which are always optically thin resonance lines can become optically very thick starting with some moment of the Universe evolution. In this case the calculations must be fulfilled with a proper account of multiple scatterings which is much more difficult. So we consider the period of evolution corresponding to sufficiently large redshifts when the optical thickness of the Universe in HI and HeI resonance lines was less than unity. To the first order in line optical thickness we have obtained (Dubrovich and Grachev, 2015, eq. (38)) the following relative energy distribution in a burst spectrum in the present-day epoch \((z = 0, u = u_0, \eta = \eta_0)\):
\[
\frac{n - n_0}{n_0} = -\tau_l \left\{ 1 - \frac{\lambda_l}{n_0} \left[ e^{u_l - u_0} s_0^0(r_l', u_l) + \int_{u_l}^{u_0} s_0^0(r', u') e^{u' - u_0} du' \right] \right\},
\]
(12)

where \(n\) is the radiation intensity (in average photon occupation numbers), \(n_0\) and \(s_0^0\) are radiation intensity and source function without account of line scattering (but with a proper account of multiple scattering on free electrons), \(\tau_l = \tau_{1k}(z_l), u_l = u(z_l), \lambda_l = \lambda_{1k}(z_l), r_l' = r'|_{\eta' = \eta_l}.\) Here
\[
z_l = \frac{\nu_{1k}}{\nu} - 1, \quad \frac{\nu_{1k}}{1 + z_0} \leq \nu \leq \nu_{1k},
\]
(13)

where \(\nu\) is a radiation frequency in the present-day epoch. By its physical sense \(z_l\) is a resonance redshift at which photons with the frequency \(\nu\) were scattered and \(\tau_l\) is the Sobolev optical thickness for this redshift. These quantities appear in the theory of primordial hydrogen recombination lines formation (see e.g., Dubrovich and Grachev, 2004). Further, in eq. (12) there is
\[
r' = \sqrt{r^2 - 2r\mu(\eta_0 - \eta') + (\eta_0 - \eta')^2},
\]
(14)
where \( r \) is a distance from the burst center, \( \arccos \mu \) is an angular distance from the direction on the burst center, \( \eta \) is a conformal time:

\[
\eta = c \int_0^t dt'/a(t') = c \int_z^{z_0} dz'/H(z'),
\]

where \( a(t) \) is a scale factor, \( t \) is a time, \( a = 1/(1+z) \) is a dimensionless time

\[
u = c \sigma_e \int_0^t n_e(t') dt' = c \sigma_e \int_z^{z_0} \frac{n_e(z')}{(1+z')H(z')} dz'
\]

which has a sense of an optical distance (for Thomson scattering) between the moments \( z \) and \( z_0 \), \( \sigma_e = 6.65 \cdot 10^{-25} \text{ cm}^2 \) is the Thomson scattering crossection. Here redshift \( z_0 \) corresponds to initial moment of time (a burst moment): \( u = \eta = t = 0 \) for \( z = z_0 \).

As to \( n^0 \) we assume that at the initial moment of time \( t = 0 \) \( (\eta = 0, z = z_0) \) the burst radiation is isotropical and spherically symmetrical:

\[
n^0(r, \mu, 0) = n_0(r),
\]

where \( n_0(r) \) is a given function which we take in the form

\[
n_0(r) = \pi^{-3/2}r_*^{-3}\exp[-(r/r_*)^2] \rightarrow \delta(r)/4\pi r^2 \text{ for } r_* \rightarrow 0,
\]

where \( r_* \) is a parameter defining characteristic size of the burst.

**Main results**

By using the code developed in previous our paper (Dubrovich and Grachev, 2015) we calculate summary profiles of hydrogen \( L_\alpha \) and \( L_\beta \) lines and HeI lines \( 1s^2-1s2p \) and \( 1s^2-1s3p \) for three initial redshifts \( z_0 \): 4000 and 5000 for hydrogen and 6000 for helium.

For the width of initial intensity distribution as a function of \( r \) (see eq. (15)) we use \( r_* = 50 \) Mpc in a distance scale at \( z = 0 \). But in a distance scale corresponding to the burst moment (at \( z = z_0 \)) the width of initial distribution (for \( z_0 \gg 1 \)) turns out to be significantly lower:

\[
a(z_0)r_* = r_*/(1+z_0).
\]

Figure 3 shows spatial distributions of a mean radiation intensity and Figure 4 shows angular distributions of intensity as a result of multiple scatterings on free electrons. Computations of these distributions were fulfilled using the code developed by us earlier (Grachev and Dubrovich, 2011). Summary line profiles are shown in Figures 5 – 7. They are in frequency regions for which the Universe is only weakly opaque in the corresponding lines (see Fig. 1 and also above in the text). It turned out that the profiles in resonance lines do not depend both on the distance from the burst center and on direction in contradistinction to the profiles in subordinate lines considered by us earlier (Dubrovich, Grachev, 2015). The jumps in the profiles are on the frequencies \( \nu = \nu_{1k}/(1+z_0) \) corresponding to the moment of appearance of the burst radiation absorption in the lines with laboratory frequencies \( \nu_{1k} \). For hydrogen lines \( L_\alpha \) and \( L_\beta \) \( \nu = 616 \) and 730 GHz for \( z_0 = 4000 \) and \( \nu = 493 \) and 584 GHz for \( z_0 = 5000 \). For corresponding HeI lines \( \nu = 855 \) and 930 GHz for \( z_0 = 6000 \). Relative magnitude of the jumps amounts to \( 10^{-4} \) — \( 3 \cdot 10^{-3} \).
It should be marked essential distinction of spectrum distortions profiles for resonance and subordinate lines. The first ones are purely in absorption and do not depend both on distance and direction. The second ones may contain also appreciable emission components and relative contribution of emission and absorption components noticeably depends both on distance from the source and on direction (see Dubrovich and Grachev, 2015). The thing is that optical thickness of the Universe in subordinate lines has quite narrow maximum over $z$ with subsequent fast decrease but optical thickness in resonance lines grows nearly exponentially with $z$ decrease (see Fig. 1). Moreover, optical thickness in all subordinate lines is quite small ($< 10^{-3}$) and one may use linear approximation over optical thickness. But an optical thickness in resonance lines can become very large so that spectrum calculations becomes very complicated. However it is clear that in the case of very large optical thickness a relative amplitude of spectrum distortions may be of the order unity.
Conclusion

The local bursts model of primordial plasma and radiation fluctuations in the early Universe is considered. These fluctuations may be represented as the local fastly variable sources of initial blackbody radiation with the temperature which is slightly different from the average CMB temperature. More generally, any really detached object (e.g. primordial accreting black hole) may come out as a source. In this work we calculate transition function from the radiation intensity of these sources to the observed intensity with the account of photons scattering on free electrons and in resonance lines of hydrogen and helium. In the first approximation this function has a physical sense of an optical thickness for scattering in lines.

As a model we consider scattering of continuous radiation of a source (instantaneous burst of radiation at a given $z_0$) on free electrons and on atoms of primordial hydrogen in $L_\alpha$ and $L_\beta$ lines and on atoms of primordial helium (HeI) in lines $1s2-1s2p$ ($^1S-^1P^*$) and $1s2-1s3p$ ($^1S-^1P^*$) at the epoch before hydrogen and He I recombination. For example, we consider three values of $z_0$: 4000, 5000 and 6000, which are in a region where optical thickness in the mentioned lines is less than unity. It is shown that thus arising lines in the source spectrum are in absorption with the jumps of the value from $10^{-4}$ to $3 \cdot 10^{-5}$ and their shape do not depend both on a distance from a source and on angular distance from direction to the source center. It should be noted that in a redshifts region where optical thickness in considered resonance lines is greater than unity a relative depth of absorption can reach a value of the order unity. Real observations give maximum amplitude of temperature deviations in spots about 300 $\mu$K (Adam et al., 2015b). So that the relative magnitude of temperature fluctuations is in the limits $\delta T/T \approx 10^{-4} - 10^{-5}$ which is more than 1000 times greater than the value predicted by subordinate lines.

Obtained results are very important for estimation of the prospects to reveal primordial protoobjects on redshifts up to 5000. One can search such objects by using the same methods as in searching quasars with great redshifts i.e. by searching Lyman jumps. So it is necessary to obtain a map of sufficiently large part of sky with a medium spectral resolution ($\Delta \lambda/\lambda \approx 1-2\%$) in submillimeter range of wavelengths.

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Figure 1: Optical thicknesses in hydrogen $L_\alpha$ and $L_\beta$ lines and in HeI lines $1s2-1skp (^1S - ^1P^*)$ ($k=2$ and 3).
Figure 2: Albedo of a single scattering in the same lines as in the preceding figure.
Figure 3: Mean intensity of scattered radiation for bursts at different $z_0$ (in Mpc). Crosses mark the points in which line profiles were calculated.
Figure 4: Angular distributions of radiation on different distances \( r \) (in Mpc) from the burst center at the present-day epoch. Here \( \theta \) is the angular distance from the direction on the burst center. In direction on the burst center \( n(0) = 4.3 \cdot 10^{-8}, 8.8 \cdot 10^{-7} \) and \( 1.0 \cdot 10^{-7} \) for \( r = 13500, \ 13600 \) and \( 13700 \) Mpc respectively.
Figure 5: Summary profile of hydrogen lines $L_\alpha$ and $L_\beta$ for the burst at $z_0 = 4000$. 
Figure 6: The same as in the preceding figure but for $z_0 = 5000$. 

\[ \frac{(n-n^0)/n^0}{10^{-3}} \]
Figure 7: Summary profile of HeI lines $1s2-1s2p \left(^1S - ^1P^*\right)$ and $1s2-1s3p \left(^1S - ^1P^*\right)$ for the burst at $z_0 = 6000$. 