Quantum transport through a double Aharonov-Bohm-interferometer in the presence of Andreev reflection

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Quantum transport through a double Aharonov-Bohm-interferometer in the presence of Andreev reflection is investigated in terms of the nonequilibrium Green function method with which the reflection current is obtained. Tunable Andreev reflection probabilities depending on the interdot coupling strength and magnetic flux as well are analysed in detail. It is found that the oscillation period of the reflection probability with respect to the magnetic flux for the double interferometer depends linearly on the ratio of two parts magnetic fluxes \( n \), i.e. \( 2(n + 1)\pi \), while that of a single interferometer is \( 2\pi \). The coupling strength not only affects the height and the linewidth of Andreev reflection current peaks vs gate voltage but also shifts the peak positions. It is furthermore demonstrated that the Andreev reflection current peaks can be tuned by the magnetic fluxes.

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I. INTRODUCTION

Quantum dots (QDs) containing discrete energy levels which can be tuned easily in experiment have been attracting considerable attention continuously, and the study of quantum transport properties through QDs has become an interesting field which serves as a testing ground for the physical phenomena, such as Kondo \[1,2\] and Fano effects \[3\]. Recent developments on the study include Coulomb on-site repulsion \[4\], spin-flip effect caused by the magnetic leads \[5\] or rotating magnetic field \[6\], and phonon-assisted tunneling in the molecular QD \[7\] with time-dependent field or ac gate voltage \[8,9\].

Electron transports coherently through an open QD can be investigated in the standard procedure by means of nonequilibrium Green function (NGF) \[8\], the rate equation \[10\] and some other methods \[11\]. The current characteristic properties change dramatically if the QD is connected to additional devices, so various kinds of hybrid QD systems have been studied in the literature. Two aspects mainly involved in those hybrid systems are the ferromagnetic and superconductor properties which evoke the spin degree of freedom of electrons. The F-QD-F system, i.e., QD coupled with two ferromagnetic leads, has been studied as spin-valve device \[5,12\]. In the superconductor case, the Andreev reflection, conserving the momentum of the reflected particle but not the charge, plays a dominant role at the interface between a normal metal and a superconductor. Using multichannel S-matrix method, Beenakker demonstrated the resonant Andreev tunneling in the normal metal-normal QD-superconductor (N-NQD-S) hybrid system in the zero-bias limit \[13\]; later Claughton et al. extended this theory to the finite-bias case \[14\]. Sun et al. explored the resonant Andreev reflection in this system with multienergy levels in normal QD and found that some extra peaks originating from the Andreev reflection superimposed on the conventional current plateaus \[15\]. And they also investigated the photon-assisted Andreev and normal tunnels in this system. With an applied time-varying external field, it was observed that a series of photon-assisted Andreev tunneling peaks appeared in the curve of the average current \( \langle I \rangle \) vs the gate voltage with the negative peaks on the left-hand side and positive peaks on the right-hand side of the original resonant peak in the absence of the external field \[16\]. Another interesting hybrid system consists of ferromagnet and superconductor leads coupled with a normal QD (F-QD-S). In Ref. \[17\], Zhu et al. provided an efficient method for introducing a spin degree of freedom in this system, and also derived the general current formula of the Andreev reflection in a quantum dot connected with two ferromagnetic and one superconductor leads (F\(_1\), F\(_2\)-QD-S) \[18\]. Considering both the Coulomb interaction and the multienergy levels in QD, Feng and Xiong studied the transport properties in the F-QD-S system \[19\]. Very recently, Cao et al. considered the spin-flip effect in QD of a F-QD-S system and found that the single-resonance peak of the Andreev reflection conductance versus the gate voltage would spill into a double-resonance peak with the increasing of the spin-flip scattering strength \[20\]. Thus far, most of studies relating to Andreev reflection have been focused on a single-QD system.

On the other hand, the electron transport through a double-quantum-dot (DQD), especially the parallel-coupled double-quantum-dot (PDQD), is an interesting subject which may involve the quantum phase interference. An open PDQD threaded by a magnetic flux is regarded as an Aharonov-Bohm (AB) interferometer in which the coherence of the electron remains partly even in the presence of Coulomb repulsion \[1,21\]. Tunable coupling strength between two parallel dots makes the transport properties more complex and interesting \[22,23\]. The tunable Andreev reflection in an AB interferometer has been investigated recently by Peng et al. \[25\].
In this paper we shall study the electron transport through a N-PDQD-S structure, i.e., a parallel-coupled double-quantum dot that is connected with two leads, one is a normal metal and the other is a superconductor. The device threaded by two magnetic fluxes (see Fig.1) with the interdot coupling can be regarded as a double AB interferometer. By using the NGF method, the current and the probability of the Andreev reflection are derived.

The paper is organized as follows: In Sec. II, we present the model Hamiltonian and derive the general formula of the current and the probability of the Andreev reflection with NGF method. In Sec. III, the properties of the current due to Andreev reflection depending on bias voltage, magnetic flux, gate voltage, and interdot coupling strength are studied in detail, and several different effects are predicted. Finally, a brief summary is given in Sec. IV.

II. MODEL HAMILTONIAN AND FORMULATION

The Hamiltonian of the system schematically shown in Fig. 1 can be decomposed into four parts of the following form:

\[ H = H_L + H_R + H_D + H_T, \]

where

\[ H_L = \sum_{k\sigma}(\varepsilon_k - eV)a_{k\sigma}^\dagger a_{k\sigma}, \]

\[ H_R = \sum_{p\sigma}\varepsilon_p b_{p\sigma}^\dagger b_{p\sigma} + \sum_p(\Delta b_{p\sigma} b_{-p\sigma}^\dagger + h.c.), \]

\[ H_D = \sum_{\sigma,i=1,2}\varepsilon_i d_{i\sigma}^\dagger d_{i\sigma} + \sum_\sigma(\Omega e^{i\Phi_{12}}d_{2\sigma}^\dagger d_{1\sigma} + h.c.), \]

\[ H_T = \sum_{k\sigma,i=1,2}(L_i e^{i\Phi_{1L}}a_{k\sigma}^\dagger d_{i\sigma} + h.c.) + \sum_{p\sigma,i=1,2}(R_i e^{i\Phi_{1R}}b_{p\sigma}^\dagger d_{i\sigma} + h.c.). \]

\( H_L \) describes the left normal metal lead, \( a_{k\sigma} \) (\( a_{k\sigma}^\dagger \)) is the corresponding annihilation (creation) operator, and \( V \) is the bias voltage. We assume the chemical potential of right lead \( \mu_R = 0 \). \( H_R \) describes the right superconductor lead with the energy gap \( \Delta \). \( H_D \) models the coupled double quantum dots, \( \Omega \) is the interdot coupling strength, and \( \Phi_{12} \) is the corresponding phase shift induced by the magnetic flux. Here we consider the single energy level in each quantum dot. \( H_T \) is the tunneling part of Hamiltonian, \( L_i \) (\( R_i \)), independent of momentum \( k \) (\( p \)), is the hopping strength between \( i \)th quantum dot [i.e. the quantum dot with energy level \( \varepsilon_i \) (\( i = 1, 2 \))] and left (right) lead, and \( \Phi_{1L} \) is the phase shift of the electron tunneling from the left lead to first quantum dot, where

\[ \Phi_{1L} = \Phi_{1R} = \frac{\Phi}{4}, \]

where \( \Phi = 2\pi(\alpha_1 + \alpha_2) \) with \( \Phi_i = \alpha_i \Phi_0 \) (\( i = 1, 2 \)) being the magnetic flux in the quantum unit \( \Phi_0 = \hbar e / c \) and \( \Phi_{12} = -\Phi_{21} = 2\pi(\alpha_1 - \alpha_2) \).

The current from the left lead to a quantum dot can be written as [4,8]

\[ J_L(t) = -\frac{2e}{h} \text{Im} \int \text{d}\varepsilon \sum_{i,j=1,2} \sum_{k\sigma} L_i L_j^* e^{i(\Phi_{1L} + \Phi_{1J})} \]

\[ \times \delta(\varepsilon - \varepsilon_k) \int_{-\infty}^{\infty} dt_1 G_{ij\sigma}(t, t_1) f_L(\varepsilon + eV) \]

\[ + \theta(t' - t_1) G_{ij\sigma}^<(t, t_1)) e^{-i\varepsilon(t_1 - t')}, \]  

where we define \( G_{ij\sigma}(t, t') = -i\theta(t - t') \langle \{ d_{i\sigma}(t), d_{j\sigma}(t') \} \rangle \) and \( G_{ij\sigma}^<(t, t') = i \langle d_{j\sigma}(t')d_{i\sigma}(t) \rangle \). Because of the right superconductor lead, it is convenient to introduce 4×4 matrix representation in which \( \mathbf{G}^>(t, t') \) and \( \mathbf{G}^<(t, t') \) have the forms,

\[ \mathbf{G}^>(t, t') = -i\theta(t - t') \langle \{ \Psi(t), \Psi^\dagger(t') \} \rangle, \]
\[ G^<(t,t') \equiv i \langle \Psi(t') \Psi(t) \rangle, \]  

where \( \Psi^i = (d_{1\uparrow}^i, d_{1\downarrow}^i, d_{2\uparrow}^i, d_{2\downarrow}^i) \). So Eq. (3) can be rewritten as

\[ J_{L^\uparrow}(t) = -\frac{2e}{h} \text{Im} \int \frac{d\varepsilon}{2\pi} \int_{-\infty}^{t} dt' e^{-i(t-t')} \{ \Gamma^L [G^r(t,t')f(t + eV) + G^< (t,t')] \}^{11+33}, \tag{6} \]

where

\[ \Gamma^L = 2\pi \sum_k \delta(\varepsilon - \varepsilon_k) \begin{pmatrix} L_1 L_1^* & 0 & L_2 L_1^* e^{i\Phi_k} & 0 \\ L_1 L_2^* e^{-i\Phi_k} & 0 & L_2 L_2^* e^{-i\Phi_k} & 0 \\ 0 & L_2 L_1^* e^{i\Phi_k} & 0 & L_2 L_2^* e^{i\Phi_k} \\ 0 & 0 & L_2 L_1^* e^{-i\Phi_k} & 0 \end{pmatrix}, \]  

\[ = \begin{pmatrix} \Gamma^L_1 & 0 & \sqrt{\Gamma^L_1 \Gamma^L_2} e^{i\Phi_k} & 0 \\ 0 & \Gamma^L_1 & 0 & \sqrt{\Gamma^L_1 \Gamma^L_2} e^{-i\Phi_k} \\ \sqrt{\Gamma^L_1 \Gamma^L_2} e^{-i\Phi_k} & 0 & \Gamma^L_2 & 0 \\ 0 & \sqrt{\Gamma^L_1 \Gamma^L_2} e^{i\Phi_k} & 0 & \Gamma^L_2 \end{pmatrix}. \tag{7} \]

with \( \Gamma^L_k = 2\pi \sum_k \delta(\varepsilon - \varepsilon_k)L_1 L_1^*, \quad \Gamma^R_k = 2\pi \sum_k \delta(\varepsilon - \varepsilon_k)L_2 L_2^* \), here we consider only a spin-up current \( J_{L^\uparrow}(t) \) [ \( J_{L^\downarrow}(t) \) is easily obtained from \( J_{L^\uparrow}(t) \) by exchanging the spin index].

To calculate \( G^r(t,t') \), we start from the Dyson equation,

\[ G^r(t,t') = g^r(t,t') + \int dt_1 dt_2 G^r(t,t_1) \Sigma^r(t_1,t_2) g^r(t_2,t'), \tag{8} \]

in which \( g^r(t,t') \) is the retarded Green function for the isolated quantum dots, and can be easily obtained as,

\[ g^r(t,t') = -i\theta(t - t') \begin{pmatrix} e^{-i\varepsilon_1(t-t')} & 0 & 0 & 0 \\ 0 & e^{i\varepsilon_1(t-t')} & 0 & 0 \\ 0 & 0 & e^{-i\varepsilon_2(t-t')} & 0 \\ 0 & 0 & 0 & e^{i\varepsilon_2(t-t')} \end{pmatrix}. \tag{9} \]

Self-energy \( \Sigma^r(t_1,t_2) \) consists of three parts, such that

\[ \Sigma^r(t_1,t_2) = \Sigma_{12} + \Sigma^r_L(t_1,t_2) + \Sigma^r_R(t_1,t_2), \tag{10} \]

where \( \Sigma^r_L(t_1,t_2) \) results from the tunneling coupling between QD and the normal metal lead, \( \Sigma^r_R(t_1,t_2) \) between QD and superconductor lead, and \( \Sigma_{12} \) is from interdot coupling contribution. Under the wide-bandwidth approximation [8,16,20], the self-energy becomes

\[ \Sigma_{12} = \begin{pmatrix} 0 & 0 & \Omega e^{-i\Phi_{12}} & 0 \\ 0 & 0 & 0 & -\Omega e^{i\Phi_{12}} \\ \Omega e^{i\Phi_{12}} & 0 & 0 & 0 \\ 0 & -\Omega e^{-i\Phi_{12}} & 0 & 0 \end{pmatrix}, \tag{11} \]

and \( \Sigma^r_L(t_1,t_2) \) is written as

\[ \Sigma_L^r(t_1,t_2) = \sum_k L_k^* g^r_k(t_1,t_2) L_k \]

\[ = -\frac{i}{2} \delta(t_1 - t_2) \Gamma^L, \]

where

\[ L = \begin{pmatrix} L_1 e^{-i\Phi_k} & 0 & 0 & 0 \\ 0 & -L_1^* e^{i\Phi_k} & 0 & 0 \\ 0 & 0 & L_2 e^{i\Phi_k} & 0 \\ 0 & 0 & 0 & -L_2^* e^{-i\Phi_k} \end{pmatrix}. \tag{13} \]
The self-energy from the coupling between QD and right superconductor lead is given by [16],

\[
\Sigma_R(t_1, t_2) = \sum_p R^* g^p(t_1, t_2) R
\]

\[
= -i \theta(t_1 - t_2) \times \int \frac{d\varepsilon}{2\pi} e^{-iE(t-t')} \left| \varepsilon \right| \left( \begin{array}{cccc}
\Gamma^R_1 & -\Gamma^R e^{-i\frac{\pi}{4}} \Delta & -\sqrt{\Gamma^R_1 \Gamma^R_2} e^{-i\frac{\pi}{4}} & -\Gamma^R e^{-i\frac{\pi}{4}} \Delta \\
-\Gamma^R e^{-i\frac{\pi}{4}} \Delta & \Gamma^R_1 & -\sqrt{\Gamma^R_1 \Gamma^R_2} e^{-i\frac{\pi}{4}} & -\Gamma^R e^{-i\frac{\pi}{4}} \Delta \\
-\sqrt{\Gamma^R_1 \Gamma^R_2} e^{i\frac{\pi}{4}} \Delta & -\sqrt{\Gamma^R_1 \Gamma^R_2} e^{i\frac{\pi}{4}} \Delta & \Gamma^R_2 & -\Gamma^R e^{-i\frac{\pi}{4}} \Delta \\
-\Gamma^R e^{-i\frac{\pi}{4}} \Delta & -\Gamma^R e^{-i\frac{\pi}{4}} \Delta & -\Gamma^R e^{-i\frac{\pi}{4}} \Delta & \Gamma^R_2
\end{array} \right),
\]

(14)

where

\[
R = \left( \begin{array}{cccc}
R_1 e^{i\frac{\pi}{4}} & 0 & 0 & 0 \\
0 & -R_1^* e^{-i\frac{\pi}{4}} & 0 & 0 \\
0 & 0 & R_2 e^{-i\frac{\pi}{4}} & 0 \\
0 & 0 & 0 & -R_2^* e^{i\frac{\pi}{4}}
\end{array} \right),
\]

(15)

and the retarded Green function of the isolated superconductor lead \( g_R^p(t, t') \) is read as

\[
g_R^p(t, t') = -i \theta(t - t') \int d\varepsilon \rho^N R e^{-iE(t-t')} \left| \varepsilon \right| \left( \begin{array}{cccc}
1 & \frac{\Delta}{|\varepsilon|} & \frac{\Delta}{|\varepsilon|} & \frac{\Delta}{|\varepsilon|} \\
\frac{\Delta}{|\varepsilon|} & 1 & \frac{\Delta}{|\varepsilon|} & \frac{\Delta}{|\varepsilon|} \\
\frac{\Delta}{|\varepsilon|} & \frac{\Delta}{|\varepsilon|} & 1 & \frac{\Delta}{|\varepsilon|} \\
\frac{\Delta}{|\varepsilon|} & \frac{\Delta}{|\varepsilon|} & \frac{\Delta}{|\varepsilon|} & 1
\end{array} \right).
\]

(16)

with \( \rho^N_R \) being the density states of the right lead in a normal metal state. We define \( \Gamma^R = 2\pi |R_1|^2 \rho^N_R \). By substituting Eqs. (9)–(13) into Eq. (8), the Green function \( G^R(t, t') \) can be derived. It is, however, convenient to transform Eq. (8) into the energy representation by making use of the Fourier transformation,

\[
G^R(E, E') = \int dt dt' G^R(t, t') e^{iEt} e^{-iE't}.
\]

(17)

Then solving Eq. (8) in the energy representation, we get (see Appendix A)

\[
G_{ij}^R(E, E') = 2\pi \tilde{G}_{ij}^R(E') \delta(E - E'),
\]

(18)

here, \( i, j = 1, 2, 3, 4 \). \( G_{ij}^R(E, E') \) are the elements of the matrix \( G^R(E, E') \), and the detailed calculation of \( \tilde{G}_{ij}^R(E') \) is given in Appendix A. The Green function \( G^R(t, t') \) is obtained after making an inverse Fourier transformation as

\[
G^R(t, t') = \frac{1}{(2\pi)^2} \int dE dE' G^R(E, E') e^{-iEt} e^{iE't}.
\]

(19)

In the following, we derive the smaller Green function using the Keldysh equation

\[
G^<(t, t') = \int dt_1 dt_2 G^>(t_1, t_2) \Sigma^<(t_1, t_2) G^a_>(t_2, t'),
\]

(20)

where

\[
\Sigma^<(t_1, t_2) = \Sigma^<_L(t_1, t_2) + \Sigma^<_R(t_1, t_2),
\]

(21)

with
\[
\Sigma^<_{L}(t_1, t_2) = \sum_k L^* g^<_{k} (t_1, t_2) L
\]

\[
= i \int \frac{d\varepsilon}{2\pi} e^{-i\varepsilon(t_1-t_2)} \begin{pmatrix}
  f_L(\varepsilon + eV) & 0 & 0 & 0 \\
  0 & f_L(\varepsilon - eV) & 0 & 0 \\
  0 & 0 & f_L(\varepsilon + eV) & 0 \\
  0 & 0 & 0 & f_L(\varepsilon - eV)
\end{pmatrix} \Gamma^L,
\]

(22)

and

\[
\Sigma^<_{R}(t_1, t_2) = \sum_p R^* g^<_{p} (t_1, t_2) R
\]

\[
= i \int \frac{d\varepsilon}{2\pi} e^{-i\varepsilon(t_1-t_2)} f_R(\varepsilon) \tilde{\rho}_R(\varepsilon)
\]

\[
\times \begin{pmatrix}
  \Gamma^R_1 & -\Gamma^R_2 e^{-i\phi} & \sqrt{\Gamma^R_1 \Gamma^2_2} e^{-i\phi} & -\sqrt{\Gamma^R_1 \Gamma^2_2} e^{i\phi} \\
  -\Gamma^R_2 e^{i\phi} & \Gamma^R_1 & -\sqrt{\Gamma^R_1 \Gamma^2_2} e^{i\phi} & \sqrt{\Gamma^R_1 \Gamma^2_2} e^{-i\phi} \\
  \sqrt{\Gamma^R_1 \Gamma^2_2} e^{-i\phi} & -\sqrt{\Gamma^R_1 \Gamma^2_2} e^{i\phi} & \Gamma^R_2 & -\Gamma^R_2 e^{-i\phi} \\
  -\sqrt{\Gamma^R_1 \Gamma^2_2} e^{i\phi} & \sqrt{\Gamma^R_1 \Gamma^2_2} e^{-i\phi} & -\Gamma^R_2 e^{i\phi} & \Gamma^R_2
\end{pmatrix},
\]

(23)

where

\[
\tilde{\rho}_R(\varepsilon) = \frac{|\varepsilon| \theta(|\varepsilon| - \Delta)}{\sqrt{\varepsilon^2 - \Delta^2}}.
\]

(24)

\(\tilde{\rho}_R(\varepsilon)\) is the ratio of superconducting density of states to the normal density of states [15]. According to Eq. (20), \(G^<_{\ast}(t, t)\) can be obtained by substituting Eqs. (21)-(23) and the Green functions \(G^r(t_1, t_2)\) into Eq. (20) with the results of matrix elements \(G^r_1(q, t), G^r_2(q, t), G^r_3(q, t), G^r_4(q, t),\) and \(G^r_{34}(q, t)\) shown in Appendix B.

The total current, i.e., the sum of both spin-up and spin-down parts, is found by substituting the retarded Green function \(G^r_{ij}(t, t')\) and the smaller Green function \(G^r_{34}(t, t)\) shown in Appendix B into Eq. (6), and the result is

\[
J_L = J_A + J_T
\]

(1)

where

\[
J_A = \frac{2e}{\hbar} \int \frac{d\varepsilon}{2\pi} [f_L(\varepsilon + eV) - f_L(\varepsilon - eV)] T_{AR},
\]

and

\[
J_T = \frac{2e}{\hbar} \int \frac{d\varepsilon}{2\pi} [f_L(\varepsilon + eV) - f_R(\varepsilon)] T_{LR}.
\]

\(J_A\) is seen to be the current due to the Andreev reflection contribution and the probability of the Andreev reflection \(T_{AR}\) is given by

\[
T_{AR} = (\Gamma^L_1)^2 \left| G^r_{12}(\varepsilon) \right|^2 + (\Gamma^L_2)^2 \left| G^r_{34}(\varepsilon) \right|^2 + \Gamma^L_1 \Gamma^L_2 \left| G^r_{14}(\varepsilon) \right|^2 + \left| G^r_{32}(\varepsilon) \right|^2
\]

\[
+ 2\sqrt{\Gamma^L_1 \Gamma^L_2} \text{Re} \Gamma^L_1 e^{-i\phi} G^r_{12}(\varepsilon) G^r_{14}(\varepsilon)
\]

\[
+ \Gamma^L_2 e^{-i\phi} \left| G^r_{32}(\varepsilon) \right|^2 + \Gamma^L_2 e^{i\phi} G^r_{12}(\varepsilon) G^r_{14}(\varepsilon)
\]

\[
+ \Gamma^L_2 e^{-i\phi} G^r_{12}(\varepsilon) G^r_{32}(\varepsilon) + \Gamma^L_1 e^{-i\phi} G^r_{14}(\varepsilon) G^r_{32}(\varepsilon)
\]

\[
+ \Gamma^L_2 e^{-i\phi} G^r_{14}(\varepsilon) G^r_{34}(\varepsilon).
\]

(26)

As explained in Refs. [15,24], the term \((\Gamma^L_1)^2 \left| G^r_{12}(\varepsilon) \right|^2 \left[ (\Gamma^L_2)^2 \left| G^r_{34}(\varepsilon) \right|^2 \right] \) describes an electron with spin-up tunnelling from the left lead through quantum dot 1 (2) to the superconductor lead and reflecting a hole back to the left lead.
through the quantum dot 1 (2). While the term $\gamma_1 \Gamma_1^R \left| \tilde{G}_{14}(\epsilon) \right|^2 \left[ \Gamma_1^R \Gamma_2^L \left| \tilde{G}_{52}(\epsilon) \right|^2 \right]$ is for the spin-up electron tunneling from left lead through dot 1 (2) to the superconductor lead and a hole reflecting back to the left lead through the dot 2 (1). There exist other channels in which the tunnel electron and reflecting hole go through both two dots. The remaining part in Eq. (26) describes the interference effect of all possible multichannels of the device. The transmission probability from left to right leads reads

$$T_{LR} = \Gamma_1^R \Gamma_1^R \left| \tilde{G}_{14}(\epsilon) \right|^2 + \Gamma_1^R \left| \tilde{G}_{12}(\epsilon) \right|^2$$

$$+ \Gamma_2^R \left| \tilde{G}_{13}(\epsilon) \right|^2 + \Gamma_2^R \left| \tilde{G}_{14}(\epsilon) \right|^2$$

$$+ \Gamma_2^R \left| \tilde{G}_{51}(\epsilon) \right|^2 + \Gamma_1^R \left| \tilde{G}_{52}(\epsilon) \right|^2$$

$$+ \Gamma_2^R \left| \tilde{G}_{53}(\epsilon) \right|^2 + \Gamma_2^R \left| \tilde{G}_{54}(\epsilon) \right|^2$$

$$+ 2 \text{Re} \left[ - \Gamma_1^R \Gamma_1^R e^{-i\phi} \frac{\Delta}{\epsilon} \tilde{G}_{11}(\epsilon) \tilde{G}_{12}(\epsilon) + \cdots \right].$$

containing many terms which are understood. We are mainly interested in the physics of the Andreev reflections. As a matter of fact only the Andreev reflection process contributes to the current (i.e., $J_R = 0$) at zero temperature, if $|eV| < \Delta$. In the following section, we analyze time evolution of the probability of the Andreev reflection $T_{AR}$ and the Andreev reflection current $J_A$ as well depending on dc bias voltage, gate voltage, magnetic flux, and interdot coupling strength.

### III. NUMERICAL RESULTS

First of all we transform the coupled-QD system into an effective decoupled one with bonding and antibonding dressed states of the QD molecule using the following transformation [23],

$$\left( \begin{array}{c} f_+ \\ f_- \end{array} \right) = \left( \begin{array}{cc} -\cos \beta e^{\Phi_{12}} & -\sin \beta \\ -\sin \beta & \cos \beta e^{-\Phi_{12}} \end{array} \right) \left( \begin{array}{c} d_1 \\ d_2 \end{array} \right),$$

where, $f_-$ and $f_+$ are the annihilation operators of the bonding and antibonding dressed states for the QD molecule. And $\beta$ is defined as $1/2 \tan^{-1}[2\Omega/(\epsilon_1 - \epsilon_2)]$. Thus the Hamiltonian for coupled double quantum dot $H_D$ is diagonalized as

$$\tilde{H}_D = E_+ f_+^\dagger f_+ + E_- f_-^\dagger f_-,$$

where

$$E_{\pm} = \frac{1}{2} |\epsilon_1 + \epsilon_2 \pm \sqrt{(\epsilon_1 - \epsilon_2)^2 + 4\Omega^2}|.$$ 

Considering the tunneling Hamiltonian between double QD and the left lead, we obtain the linewidth $\Gamma_+^L$ and $\Gamma_-^L$ corresponding to the antibonding and bonding states coupled to the left lead, respectively, as

$$\Gamma_+^L = \Gamma_1^L \cos^2 \beta + \Gamma_2^L \sin^2 \beta + \sqrt{\Gamma_1^L \Gamma_2^L \sin(2\beta) \cos(\frac{\Phi}{2}) + \Phi_{12}},$$

$$\Gamma_-^L = \Gamma_1^L \sin^2 \beta + \Gamma_2^L \cos^2 \beta - \sqrt{\Gamma_1^L \Gamma_2^L \sin(2\beta) \cos(\frac{\Phi}{2}) + \Phi_{12}}.$$ 

In the numerical analysis we set $\Delta = 1$ as the energy unit. The energy dependence of reflection probability $T_{AR}$ on the interdot coupling strength is shown in Fig. 2 for the symmetry [with $\Gamma_1^L = \Gamma_1^R = \Gamma_2^L = \Gamma_2^R = 0.02$ , Fig.2(a)] and asymmetry [with $\Gamma_1^L = \Gamma_2^L = 0.08$, $\Gamma_1^R = \Gamma_2^R = 0.02$ , Fig.2(b) and $\Gamma_1^L = \Gamma_1^R = 0.08$, $\Gamma_2^L = \Gamma_2^R = 0.02$ , Fig.2(c)] cases, respectively. Other parameters are chosen as $\epsilon_1 = \epsilon_2 = 0$, and $\Phi = \Phi_{12} = 0$. It is seen in Fig. 2(a) that if the two QDs are completely decoupled (i.e., $\Omega = 0$) only a single peak appears (solid line), and when $\Omega \neq 0$, two peaks emerges at $\epsilon = \pm \Omega$ due to the existence of two energy levels. The maximum of peaks is getting lower when the coupling constant $\Omega$ is increasing. For the asymmetry case Fig. 2(b) shows that there are no significant two peaks. Fig. 2(c) exhibits the complex spectra due to the possible overlap of the broad levels similar to the interpretation of
Fano effect in a parallel coupled double QD connected to two normal metal leads system [23]. And when $\Omega$ is large enough such that the broad two energy levels are completely separated, the two single peaks remain at $\varepsilon = E_+ $ and $E_- $.

It is interesting to see the effect of the magnetic flux on the Andreev reflection and Fig. 3 presents the dependence of the Andreev reflection probability on the magnetic flux $\Phi$ with $\alpha_1 = \alpha_2$ and therefore $\Phi_{12} = 0$. The height of peaks at $\varepsilon = \pm \Omega$ can reach 2 when the magnetic flux is nonzero, and magnetic flux also has an influence on the width of the peaks, i. e., on the linewidth function $\Gamma$, here we choose $\Phi = \pi/3$ for a solid line, and $2\pi/3$ for a dotted line, the height of peaks are not changed. Electron phase coherence is shown in Fig. 4. When $\Omega = 0$, i. e., there is no the interdot coupling, the oscillation period of the Andreev reflection probability vs magnetic flux is $2\pi \Phi_0$ (solid line), while $\Omega \neq 0$, the period changes to $4\pi \Phi_0$; when $\alpha_1 = \alpha_2$ [see Fig. 4(a)]. Here, we choose the different energy eigenvalues such that $\varepsilon = 0.11$, since at those values the probability of the Andreev reflection reaches maximum. To see the reason of the period changing, we give a general result of the oscillation period when $\alpha_1/\alpha_2 = n$. The channel paths contain two parts due to the Andreev reflection in the hybrid system. One is the incident electron from the left lead to the superconductor in Fig. 5a, the other is the reflecting hole from the superconductor to the left lead in Fig. 5b. The phase shift of electron in the case (3) in Fig. 5a is $(\Phi_1 - \Phi_2)/2$, and the corresponding phase shift of hole in the case (1) in Fig. 5b is $(\Phi_1 + \Phi_2)/2$, so the total shift of the channel (3) for the electron and (1) for the hole is $\Phi_1$. The interference effect corresponds to all the channel paths reads [25]

$$T = \begin{vmatrix}
A_0 + A_1 e^{i\Phi_1} + A_2 e^{-i\Phi_1} + A_3 e^{i\Phi_2} + A_4 e^{-i\Phi_2} + A_5 e^{i(\Phi_1 + \Phi_2)} \\
+ A_6 e^{-i(\Phi_1 + \Phi_2)} + A_7 e^{i(\Phi_1 - \Phi_2)} + A_8 e^{-i(\Phi_1 - \Phi_2)}
\end{vmatrix}^2$$

$$= B_0 + B_1 \cos \Phi_1 + B_2 \cos \Phi_2 + B_3 \cos(\Phi_1 + \Phi_2)$$
$$+ B_4 \cos(\Phi_1 - \Phi_2) + B_5 \cos 2\Phi_1 + B_6 \cos 2\Phi_2$$
$$+ B_7 \cos 2(\Phi_1 + \Phi_2) + B_8 \cos(2\Phi_1 + 2\Phi_2) + B_9 \cos(\Phi_1 + 2\Phi_2)$$
$$+ B_{10} \cos(2\Phi_1 - 2\Phi_2) + B_{11} \cos(2\Phi_1 + 2\Phi_2) + B_{12} \cos(\Phi_1 - 2\Phi_2)$$

(33)

where parameters $A$ are the amplitudes of channel paths. With $\Phi_1 = \Phi n/(n+1), \Phi_2 = \Phi/(n+1)$, Eq. (33) shows the oscillation periods might have $\pi, 2\pi, 2\pi(n+1)/n, 2\pi(n+1), 2\pi(n+1)/2n, 2\pi(n+1)/2, 2\pi(n+1)/(n-1)$, $2\pi(n+1)/(n-2), 2\pi(n+1)/(n+2), 2\pi(n+1)/(2n+1)$, and $2\pi(n+1)/(2n-1)$, suggesting that the oscillation period of reflect probability should be $2\pi(1+1)\pi$. The linear dependence of the ratio $n$ is further confirmed in Fig. 4(b), it shows that the oscillation period is $6\pi$ for $n = 2$ (solid line), $8\pi$ for $n = 3$ (dotted line), and $10\pi$ for $n = 4$ (dashed line). The linear manner of the oscillation period is the same as the one in Ref. 22, however, the mechanism is different: there the transmitted current is analyzed.

Fig. 6 shows current-gate-voltage curve ($J_A$ vs $V_g$) for the case that $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.3$, $\Phi = \Phi_{12} = 0$, and the bias voltage $V = 0.4$. We assume that the energy levels in both dots are connected to the same voltage through the capacitive coupling. When $\Omega = 0$ (solid line), there exist two smaller peaks at $V_g = 0.1$, and 0.3 located symmetrically beside the larger peak at $V_g = 0.2$ with the height that is two times higher than the height of the smaller peaks. The interpretation of the phenomena is similar to that in Ref. [15] such that when $V_g = 0.1$ the energy level $\varepsilon_1$ matches with chemical potential $u_R = 0$ in the right superconductor lead and the Andreev reflection takes place at this level. When $V_g = 0.2$ two energy levels have a symmetric localization about $u_R$ ( $\varepsilon_2 = 0.1$ above $u_R$ while $\varepsilon_1 = 0.1$ below $u_R$ ) and both $\varepsilon_1$ and $\varepsilon_2$ are below the bias voltage, so the Andreev reflection that involves those two levels leads to the height of the peaks that are two times higher than that containing only one energy level. The interpretation is the same for the gate voltage $V_g = 0.3$ where the energy level $\varepsilon_2$ matches with $u_R$. Some new properties are observed for the coupled case $\Omega \neq 0$: (1) the location of the middle peak does not change, while the positions of left and right peaks shift towards the left and right sides, respectively, with the increasing coupling strength $\Omega$. (2) the heights of the left and the middle peaks are suppressed significantly; meantime, the height of the right peak is enhanced. (3) the widths of the left and middle peaks decrease while the width of the right peak widened. These interesting phenomena can be explained with Eq. (30)-(32). The energy level of dressed state $E_+$ gets larger and $E_-$ becomes smaller with increasing coupling strength $\Omega$, and at the end $E_+$ is higher than energy level of bare state $\varepsilon_2$, when $E_-$ is smaller than $\varepsilon_1$. So when the gate voltage takes action on the dressed states, the left peak runs towards the left while the right peak goes opposite, however, the locations of the two peaks remain symmetric about the middle peak. We assume $\varepsilon_2 = \varepsilon_1 - a = 0.1 - a$, $E_+ = \varepsilon_2 + a = 0.3 + a$, where $a$ is introduced due to the coupling strength $\Omega$. So when the gate voltage becomes $V_g = 0.1 - a$, the left peak emerges and with increasing $V_g$ (up to 0.2), both two dressed states levels locate symmetrically about $u_R$. Thus, the location of middle peak does not change at $V_g = 0.2$, no matter how the parameter $a$ changes. The linewidth of dressed states, $\Gamma_{\pm}$, increases with the increasing of $\Omega$, while $\Gamma_b$ decreases under the parameters considered here, and hence, the right peak gets wider and left peak becomes narrower with the increasing of $\Omega$. The linewidths $\Gamma_{\pm}$ are directly connected with the coupling strengths between
QD and the leads. The widening of the right-side peak means the increasing of coupling strength between dressed state $E_+$ and the leads. Therefore, the height of the right-side peak is enhanced, and opposite to this, the height of the left-side peak is suppressed. The suppression of the middle-peak height suggests that the Andreev resonance involving two energy levels gets maximum when two energy levels are symmetric.

Figures. 7 – 9 show a tunable Andreev reflection current depending on the magnetic flux when $\Omega \neq 0$. In Fig. 7, we fix $\Phi = \pi/2$, and tune the value of $\Phi_{12}$ for different curves. One can see that the height of the right peak is enhanced and the height of the left one is suppressed while the height of the middle peak does not change with the increase of $\Phi_{12}$. It is interesting to notice that the left peaks has essentially no changes when $\Phi = 0$ (solid line) and $\pi/2$ (dotted line), while the right peak changes conspicuously with respect to the case without the magnetic flux. Figure 8 has the opposite effect compared with Fig. 7, namely, the height of the left peak is suppressed while the height of the middle peak does not change with the increase of $\Phi_{12}$. We fix $\Phi_{12} = \pi/5$, and vary values of magnetic flux $\Phi$ in Fig. 9, it is shown that the height of three peaks can be tuned simultaneously.

IV. CONCLUSIONS

We have studied the tunable Andreev reflection in a double AB interferometer in terms of the nonequilibrium Green function method and observed several features of the reflection current. It is found that the oscillation period of the Andreev reflection probability with the magnetic flux is $2\pi$ when interdot coupling vanishes ($\Omega = 0$, in this case the system reduces to a single interferometer), while it is $2(n + 1)\pi$ for our double interferometer ($\Omega \neq 0$) where $n$ is the ratio of two parts magnetic fluxes, i.e. $n = \Phi_1/\Phi_2$. The Andreev reflection current peaks cannot only be tuned by coupling strength $\Omega$, but also by magnetic flux.

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VI. APPENDIX A

In this appendix, we present the derivation of the Green function of Eq. (18) in detail. From Eqs. (8)-(13) and (17) we have the following formulas for the matrix elements of Green function $G^r(t,t')$ in the energy representation:

\[
g_{11}^{-1}(E')G_{11}(E,E') = 2\pi\delta(E-E') - A_1G_{11}^r(E,E') + B\Gamma_1^R e^{i\phi_2} G_{12}^r(E,E') + x_1G_{13}^r(E,E') + B\sqrt{\Gamma_1^R \Gamma_2^R} G_{14}^r(E,E'), \quad (A1)
\]

\[
g_{22}^{-1}(E')G_{12}(E,E') = B\Gamma_2^R e^{-i\phi_2} G_{11}^r(E,E') - A_2G_{12}^r(E,E') + B\sqrt{\Gamma_1^R \Gamma_2^R} G_{13}^r(E,E') + x_2G_{14}^r(E,E'), \quad (A2)
\]

\[
g_{33}^{-1}(E')G_{13}(E,E') = x_3G_{11}^r(E,E') + B\sqrt{\Gamma_1^R \Gamma_2^R} G_{12}^r(E,E') - A_2G_{13}^r(E,E') + B\Gamma_2^R e^{-i\phi_2} G_{14}^r(E,E'), \quad (A3)
\]

\[
g_{44}^{-1}(E')G_{14}(E,E') = B\sqrt{\Gamma_1^R \Gamma_2^R} G_{11}^r(E,E') + x_1G_{12}^r(E,E') + B\Gamma_2^R e^{i\phi_2} G_{13}^r(E,E') - A_2G_{14}^r(E,E'), \quad (A4)
\]

where
\[ A_1 = \frac{i}{2} (\Gamma_L^L + \Gamma_R^L) \frac{|E'|}{\sqrt{(E')^2 - \Delta^2}} \],
\[ A_2 = \frac{i}{2} (\Gamma_L^R + \Gamma_R^R) \frac{|E'|}{\sqrt{(E')^2 - \Delta^2}} \],
\[ B = \frac{i}{2} \sqrt{(E')^2 - \Delta^2} \],

and
\[ x_1 = \Omega e^{i\Phi_{12}} - \frac{i}{2} \sqrt{\Gamma_1^L \Gamma_2^L} e^{-i\frac{\Delta}{2}} - \frac{i}{2} \sqrt{(E')^2 - \Delta^2} \sqrt{\Gamma_1^R \Gamma_2^R} e^{i\frac{\Delta}{2}}, \]
\[ x_2 = -\Omega^* e^{-i\Phi_{12}} - \frac{i}{2} \sqrt{\Gamma_1^L \Gamma_2^L} e^{i\frac{\Delta}{2}} - \frac{i}{2} \sqrt{(E')^2 - \Delta^2} \sqrt{\Gamma_1^R \Gamma_2^R} e^{-i\frac{\Delta}{2}}, \]
\[ x_3 = \Omega^* e^{-i\Phi_{12}} - \frac{i}{2} \sqrt{\Gamma_1^L \Gamma_2^L} e^{i\frac{\Delta}{2}} - \frac{i}{2} \sqrt{(E')^2 - \Delta^2} \sqrt{\Gamma_1^R \Gamma_2^R} e^{-i\frac{\Delta}{2}}, \]
\[ x_4 = -\Omega e^{i\Phi_{12}} - \frac{i}{2} \sqrt{\Gamma_1^L \Gamma_2^L} e^{-i\frac{\Delta}{2}} - \frac{i}{2} \sqrt{(E')^2 - \Delta^2} \sqrt{\Gamma_1^R \Gamma_2^R} e^{i\frac{\Delta}{2}}. \]

Solving those coupled equations, the results are obtained as
\[ G_{11}'(E, E') = 2\pi \tilde{G}_{11}(E') \delta(E - E'), \]
\[ G_{12}'(E, E') = 2\pi \tilde{G}_{12}(E') \delta(E - E'), \]
\[ G_{13}'(E, E') = 2\pi \tilde{G}_{13}(E') \delta(E - E'), \]
\[ G_{14}'(E, E') = 2\pi \tilde{G}_{14}(E') \delta(E - E'), \]

where
\[ \tilde{G}_{11}(E') = \frac{1}{M(E')} \begin{vmatrix} g_{22}^{-1}(E') + A_1 & -x_2 & -B \sqrt{\Gamma_1^R \Gamma_2^R} \\ -B \sqrt{\Gamma_1^R \Gamma_2^R} & g_{33}^{-1}(E') + A_2 & -x_2 \\ -x_4 & -B \Gamma_2^R e^{i\frac{\Delta}{2}} & g_{44}^{-1}(E') + A_2 \end{vmatrix}, \]
\[ \tilde{G}_{12}(E') = -\frac{1}{M(E')} \begin{vmatrix} -B \Gamma_1^R e^{-i\frac{\Delta}{2}} & -B \sqrt{\Gamma_1^R \Gamma_2^R} & -x_2 \\ -x_3 & g_{33}^{-1}(E') + A_2 & -B \Gamma_2^R e^{-i\frac{\Delta}{2}} \\ -x_4 & -B \Gamma_2^R e^{i\frac{\Delta}{2}} & g_{44}^{-1}(E') + A_2 \end{vmatrix}, \]
\[ \tilde{G}_{13}(E') = \frac{1}{M(E')} \begin{vmatrix} -B \Gamma_1^R e^{-i\frac{\Delta}{2}} & g_{22}^{-1}(E') + A_1 & -x_2 \\ -x_3 & -B \sqrt{\Gamma_1^R \Gamma_2^R} & -B \Gamma_2^R e^{-i\frac{\Delta}{2}} \\ -x_4 & g_{44}^{-1}(E') + A_2 \end{vmatrix}, \]
\[ \tilde{G}_{14}(E') = -\frac{1}{M(E')} \begin{vmatrix} -B \Gamma_1^R e^{-i\frac{\Delta}{2}} & g_{22}^{-1}(E') + A_1 & -x_2 \\ -x_3 & -B \sqrt{\Gamma_1^R \Gamma_2^R} & g_{33}^{-1}(E') + A_2 \\ -x_4 & -B \Gamma_2^R e^{i\frac{\Delta}{2}} \end{vmatrix}, \]

here,
\[ M(E') = \begin{vmatrix} g_{11}^{-1}(E') + A_1 & -x_1 & -B \sqrt{\Gamma_1^R \Gamma_2^R} \\ -B \Gamma_1^R e^{-i\frac{\Delta}{2}} & g_{22}^{-1}(E') + A_1 & -x_2 \\ -x_3 & -B \sqrt{\Gamma_1^R \Gamma_2^R} & g_{33}^{-1}(E') + A_2 \\ -x_4 & -B \Gamma_2^R e^{i\frac{\Delta}{2}} \end{vmatrix}. \]

With the same procedure we have
\[ g_{11}^{-1}(E')G_{31}(E, E') = -A_1 G_{31}(E, E') + B \Gamma e^{-i \phi} G_{32}(E, E') \]
\[ x_1 G_{33}(E, E') + B \sqrt{\Gamma^{31}} \Gamma^{32} G_{34}(E, E'), \]  
(A13)

\[ g_{22}^{-1}(E')G_{32}(E, E') = B \Gamma^{21} e^{-i \phi} G_{31}(E, E') - A_1 G_{32}(E, E') \]
\[ + B \sqrt{\Gamma^{31}} \Gamma^{32} G_{33}(E, E') + x_2 G_{34}(E, E'), \]  
(A14)

\[ g_{33}^{-1}(E')G_{33}(E, E') = 2\pi \delta(E - E') + x_3 G_{31}(E, E') \]
\[ + B \sqrt{\Gamma^{31}} \Gamma^{32} G_{32}(E, E') - A_2 G_{33}(E, E') \]
\[ + B \Gamma e^{-i \phi} G_{34}(E, E'), \]  
(A15)

\[ g_{44}^{-1}(E')G_{34}(E, E') = B \sqrt{\Gamma^{31}} \Gamma^{32} G_{31}(E, E') + x_4 G_{32}(E, E') \]
\[ + B \Gamma^{21} e^{-i \phi} G_{33}(E, E') - A_2 G_{34}(E, E'). \]  
(A16)

So the final results are

\[ G_{31}(E, E') = 2\pi G_{31}(E') \delta(E - E') \]
\[ G_{32}(E, E') = 2\pi G_{32}(E') \delta(E - E') \]
\[ G_{33}(E, E') = 2\pi G_{33}(E') \delta(E - E') \]
\[ G_{34}(E, E') = 2\pi G_{34}(E') \delta(E - E') \]  
(A17)

where

\[ \tilde{G}_{31}^{-1}(E') = \frac{1}{M(E')} \begin{vmatrix} -B \Gamma^{13} e^{i \phi} & -x_1 & -B \sqrt{\Gamma^{13} \Gamma^{12}} & -x_2 \\ g_{22}^{-1}(E') + A_1 & -B \sqrt{\Gamma^{31} \Gamma^{32}} & -x_2 & g_{44}^{-1}(E') + A_2 \\ -x_4 & -B \Gamma^{32} e^{i \phi} & \end{vmatrix} \]  
(A18)

\[ \tilde{G}_{32}^{-1}(E') = -\frac{1}{M(E')} \begin{vmatrix} g_{11}^{-1}(E') + A_1 & -x_1 & -B \sqrt{\Gamma^{13} \Gamma^{12}} & -x_2 \\ -B \Gamma^{31} e^{-i \phi} & -B \sqrt{\Gamma^{31} \Gamma^{32}} & -x_2 & g_{44}^{-1}(E') + A_2 \\ -B \sqrt{\Gamma^{31} \Gamma^{32}} & -B \Gamma^{32} e^{i \phi} & \end{vmatrix} \]  
(A19)

\[ \tilde{G}_{33}^{-1}(E') = \frac{1}{M(E')} \begin{vmatrix} g_{11}^{-1}(E') + A_1 & -B \Gamma^{13} e^{i \phi} & -B \sqrt{\Gamma^{13} \Gamma^{12}} & -x_2 \\ -B \Gamma^{31} e^{-i \phi} & g_{22}^{-1}(E') + A_1 & -x_2 & g_{44}^{-1}(E') + A_2 \\ -B \sqrt{\Gamma^{31} \Gamma^{32}} & -x_4 & \end{vmatrix} \]  
(A20)

\[ \tilde{G}_{34}^{-1}(E') = -\frac{1}{M(E')} \begin{vmatrix} g_{11}^{-1}(E') + A_1 & -B \Gamma^{13} e^{i \phi} & -x_1 & -B \sqrt{\Gamma^{13} \Gamma^{12}} \\ -B \Gamma^{31} e^{-i \phi} & g_{22}^{-1}(E') + A_1 & -B \sqrt{\Gamma^{31} \Gamma^{32}} & -x_2 \\ -B \sqrt{\Gamma^{31} \Gamma^{32}} & -x_4 & \end{vmatrix} \]  
(A21)

VII. APPENDIX B

The complete expression of matrix elements of the Green function \( G^<(t, t) \) is
\[ G_{11}^r(t, t) = i \int \frac{d\epsilon}{2\pi} \left[ f_L(\epsilon + eV)\Gamma_t^L \left| \bar{G}_{11}^r(\epsilon) \right|^2 + f_L(\epsilon - eV)\Gamma_t^L \left| \bar{G}_{12}^r(\epsilon) \right|^2 \right] 
\]
\[ + f_L(\epsilon + eV)\Gamma_t^L \left| \bar{G}_{13}^r(\epsilon) \right|^2 + f_L(\epsilon - eV)\Gamma_t^L \left| \bar{G}_{14}^r(\epsilon) \right|^2 \]
\[ + 2\sqrt{\Gamma_t^L T_2^L} \text{Re}[f_L(\epsilon + eV)e^{i\Phi} \bar{G}_{11}^r(\epsilon) \bar{G}_{13}^r(\epsilon)] 
\]
\[ + f_L(\epsilon - eV)e^{-i\Phi} \bar{G}_{12}^r(\epsilon) \bar{G}_{14}^r(\epsilon)] \}
\[ + i \int \frac{d\epsilon}{2\pi} f_R(\epsilon)\bar{\rho}_R(\epsilon)\left\{ \Gamma_t^R \left| \bar{G}_{11}^r(\epsilon) \right|^2 + \Gamma_t^R \left| \bar{G}_{12}^r(\epsilon) \right|^2 \right\} 
\]
\[ + \Gamma_t^R \left| \bar{G}_{13}^r(\epsilon) \right|^2 + \Gamma_t^R \left| \bar{G}_{14}^r(\epsilon) \right|^2 \] 
\[ + 2\text{Re}[-\Gamma_t^Re^{-i\Phi}\Delta_{\bar{G}}^r(\epsilon)\bar{G}_{11}^r(\epsilon)\bar{G}_{13}^r(\epsilon)] 
\]
\[ + \sqrt{\Gamma_t^R T_2^R} e^{-i\Phi} \bar{G}_{11}^r(\epsilon) \bar{G}_{13}^r(\epsilon) + \sqrt{\Gamma_t^R T_2^R} e^{i\Phi} \bar{G}_{12}^r(\epsilon) \bar{G}_{14}^r(\epsilon) 
\]
\[ - \Gamma_t^R e^{i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{13}^r(\epsilon) - \Gamma_t^R e^{-i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{14}^r(\epsilon) \] 
\[ - \sqrt{\Gamma_t^R T_2^R} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{12}^r(\epsilon) \bar{G}_{13}^r(\epsilon) \],

\[ G_{13}^r(t, t) = i \int \frac{d\epsilon}{2\pi} \left[ f_L(\epsilon + eV)\Gamma_t^L \left| \bar{G}_{11}^a(\epsilon) \right| \bar{G}_{13}^a(\epsilon) + f_L(\epsilon + eV)\sqrt{\Gamma_t^L T_2^L} e^{-i\Phi} \bar{G}_{13}^r(\epsilon) \bar{G}_{13}^a(\epsilon) \right] 
\]
\[ + f_L(\epsilon - eV)\Gamma_t^L \left| \bar{G}_{23}^a(\epsilon) \right| \bar{G}_{13}^a(\epsilon) + f_L(\epsilon - eV)\sqrt{\Gamma_t^L T_2^L} e^{i\Phi} \bar{G}_{13}^r(\epsilon) \bar{G}_{23}^a(\epsilon) \]
\[ + f_L(\epsilon + eV)\sqrt{\Gamma_t^L T_2^L} e^{i\Phi} \bar{G}_{11}^a(\epsilon) \bar{G}_{23}^a(\epsilon) + f_L(\epsilon + eV)\Gamma_t^L \left| \bar{G}_{23}^a(\epsilon) \right| \bar{G}_{13}^a(\epsilon) \]
\[ + f_L(\epsilon - eV)\sqrt{\Gamma_t^L T_2^L} e^{-i\Phi} \bar{G}_{12}^a(\epsilon) \bar{G}_{23}^a(\epsilon) + f_L(\epsilon - eV)\Gamma_t^L \left| \bar{G}_{23}^a(\epsilon) \right| \bar{G}_{13}^a(\epsilon) \]
\[ + i \int \frac{d\epsilon}{2\pi} f_R(\epsilon)\bar{\rho}_R(\epsilon)\left\{ \Gamma_t^R \left| \bar{G}_{11}^r(\epsilon) \right|^2 - \Gamma_t^R e^{-i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{11}^r(\epsilon) + \Gamma_t^R \bar{G}_{12}^a(\epsilon) \right\} 
\]
\[ - \sqrt{\Gamma_t^R T_2^R} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{12}^a(\epsilon) \bar{G}_{13}^a(\epsilon) + \sqrt{\Gamma_t^R T_2^R} e^{i\Phi} \bar{G}_{13}^r(\epsilon) \bar{G}_{23}^a(\epsilon) \]
\[ + \sqrt{\Gamma_t^R T_2^R} e^{-i\Phi} \bar{G}_{11}^a(\epsilon) - \Gamma_t^R \left| \bar{G}_{12}^a(\epsilon) \right| \bar{G}_{13}^a(\epsilon) + \Gamma_t^R \left| \bar{G}_{13}^a(\epsilon) \right| \bar{G}_{23}^a(\epsilon) \]
\[ - \Gamma_t^R e^{i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{13}^a(\epsilon) - \Gamma_t^R e^{-i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{23}^a(\epsilon) \] 
\[ - \sqrt{\Gamma_t^R T_2^R} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{11}^a(\epsilon) \bar{G}_{13}^a(\epsilon) \] 
\[ - \sqrt{\Gamma_t^R T_2^R} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{12}^a(\epsilon) \bar{G}_{13}^a(\epsilon) \]

\[ G_{31}^r(t, t) = i \int \frac{d\epsilon}{2\pi} \left[ f_L(\epsilon + eV)\Gamma_t^L \left| \bar{G}_{31}^r(\epsilon) \right|^2 + f_L(\epsilon + eV)\sqrt{\Gamma_t^L T_2^L} e^{-i\Phi} \bar{G}_{35}^r(\epsilon) \bar{G}_{11}^a(\epsilon) \right] 
\]
\[ + f_L(\epsilon - eV)\Gamma_t^L \left| \bar{G}_{52}^a(\epsilon) \right| \bar{G}_{31}^a(\epsilon) + f_L(\epsilon - eV)\sqrt{\Gamma_t^L T_2^L} e^{i\Phi} \bar{G}_{34}^r(\epsilon) \bar{G}_{21}^a(\epsilon) \]
\[ + f_L(\epsilon + eV)\sqrt{\Gamma_t^L T_2^L} e^{i\Phi} \bar{G}_{31}^a(\epsilon) \bar{G}_{33}^a(\epsilon) + f_L(\epsilon + eV)\Gamma_t^L \left| \bar{G}_{33}^a(\epsilon) \right| \bar{G}_{31}^a(\epsilon) \]
\[ + f_L(\epsilon - eV)\sqrt{\Gamma_t^L T_2^L} e^{-i\Phi} \bar{G}_{32}^a(\epsilon) \bar{G}_{34}^a(\epsilon) + f_L(\epsilon - eV)\Gamma_t^L \left| \bar{G}_{34}^a(\epsilon) \right| \bar{G}_{31}^a(\epsilon) \]
\[ + i \int \frac{d\epsilon}{2\pi} f_R(\epsilon)\bar{\rho}_R(\epsilon)\left\{ \Gamma_t^R \left| \bar{G}_{31}^a(\epsilon) \right|^2 - \Gamma_t^R e^{-i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{31}^a(\epsilon) + \Gamma_t^R \bar{G}_{32}^r(\epsilon) \right\} 
\]
\[ - \sqrt{\Gamma_t^R T_2^R} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{31}^a(\epsilon) \bar{G}_{33}^a(\epsilon) + \sqrt{\Gamma_t^R T_2^R} e^{i\Phi} \bar{G}_{34}^r(\epsilon) \bar{G}_{33}^a(\epsilon) \]
\[ - \sqrt{\Gamma_t^R T_2^R} e^{-i\Phi} \bar{G}_{31}^a(\epsilon) - \Gamma_t^R \left| \bar{G}_{32}^r(\epsilon) \right| \bar{G}_{33}^a(\epsilon) + \Gamma_t^R \left| \bar{G}_{33}^a(\epsilon) \right| \bar{G}_{34}^a(\epsilon) \] 
\[ - \Gamma_t^R e^{i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{33}^a(\epsilon) - \Gamma_t^R e^{-i\Phi} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{34}^a(epsilon) \] 
\[ - \sqrt{\Gamma_t^R T_2^R} \Delta_{\bar{G}}^r(\epsilon) \bar{G}_{31}^a(\epsilon) \bar{G}_{33}^a(\epsilon) \]

(B1)
\[
-G^{<}_{33}(t, t) = i \int \frac{d\varepsilon}{2\pi} \left[ f_L(\varepsilon + eV) \Gamma^R_L \left| \tilde{G}^r_{33}(\varepsilon) \right|^2 + f_L(\varepsilon - eV) \Gamma^L_R \left| \tilde{G}^a_{32}(\varepsilon) \right|^2 + f_L(\varepsilon + eV) e^{i\phi} \Gamma^R_L \left| \tilde{G}^r_{33}(\varepsilon) \tilde{G}^a_{32}(\varepsilon) \right|^2 + f_L(\varepsilon - eV) e^{-i\phi} \Gamma^R_L \left| \tilde{G}^a_{32}(\varepsilon) \tilde{G}^r_{33}(\varepsilon) \right|^2 
\right. 
\left. + f_R(\varepsilon) \tilde{\rho}_R(\varepsilon) \left\{ \Gamma^R_L \left| \tilde{G}^r_{33}(\varepsilon) \right|^2 + \Gamma^L_R \left| \tilde{G}^a_{32}(\varepsilon) \right|^2 + \Gamma^R_R \left| \tilde{G}^a_{33}(\varepsilon) \right|^2 \right\} \right] 
\]

The results here show that the above matrix elements of $G^<(t, t)$ do not depend on time $t$, because for the N-QD-S hybrid system, the current should be independent of time for the dc bias [15].

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FIGURE CAPTION
FIG. 1: Schematic diagram for the double AB interferometer connected with normal metal and the superconductor leads, respectively.

FIG. 2: The probability of the Andreev reflection $T_{AR}$ vs $\varepsilon$ in the unit of the energy gap $\Delta$ for (a) the symmetry case ($\Gamma_L^1 = \Gamma_R^1 = \Gamma_L^2 = \Gamma_R^2 = 0.02$) and asymmetry case (b) $\Gamma_L^1 = \Gamma_R^1 = 0.08, \Gamma_L^2 = \Gamma_R^2 = 0.02$, (c) $\Gamma_L^1 = \Gamma_R^1 = 0.08, \Gamma_L^2 = \Gamma_R^2 = 0.02$. The solid, dotted, dot-dashed, and dashed lines correspond to $\Omega = 0, 0.02, 0.04, \text{and } 0.08$, respectively.

FIG. 3: The magnetic flux $\Phi$ dependence of the Andreev reflection probability for $\Gamma_L^1 = \Gamma_R^1 = \Gamma_L^2 = \Gamma_R^2 = 0.02$, $\varepsilon_1 = \varepsilon_2 = 0$, $\Omega = 0.2$, and $\Phi_{12} = 0$, solid line: $\Phi = \pi/3$, dotted line: $\Phi = 2\pi/3$.

FIG. 4: The periodic oscillation of the Andreev reflection probability $T_{AR}$ with the magnetic flux $\Phi$ ($\Gamma_L^1 = \Gamma_R^1 = \Gamma_L^2 = \Gamma_R^2 = 0.02$, $\varepsilon_1 = -0.1$, $\varepsilon_2 = 0.1$), (a) $\alpha_1/\alpha_2 = 1$. Solid line: $\Omega = 0$, $\varepsilon = 0.1$, dotted line: $\Omega = 0.05$, $\varepsilon = 0.11$, and dashed line: $\Omega = 0.1$, $\varepsilon = 0.14$. (b) $\Omega = 0.05$, $\varepsilon = 0.11$. Solid line: $\alpha_1/\alpha_2 = 2$, dotted line: $\alpha_1/\alpha_2 = 3$, and 4 for dashed line.

FIG. 5: Paths and corresponding phase shift. (a) for the incident electron from left lead to superconductor (b) for the reflecting hole from superconductor to left lead.

FIG. 6: Andreev reflection current $J_A$ vs gate voltage $V_g$ ($\varepsilon_1 = 0.1, \varepsilon_2 = 0.3, \Gamma_L^1 = \Gamma_R^1 = \Gamma_L^2 = \Gamma_R^2 = 0.02$, and $\Phi = \Phi_{12} = 0$) for bias voltage $V = 0.4$, $\Omega = 0$ (solid line), $\Omega = 0.05$ (dotted line), and $\Omega = 0.1$ (dashed line).

FIG. 7: Andreev reflection current vs gate voltage with the fixed total magnetic flux that $\Phi = \pi/2$ for $\Omega = 0.05$. $\Phi_{12} = 0$ (solid line), $\pi/2$ (dotted line), $3\pi/4$ (dot-dashed line), $\pi$ (dashed line).

FIG. 8: Andreev reflection current vs gate voltage for $\Phi = 3\pi/2$. $\Phi_{12} = 0$ (solid line), $\pi/4$ (short-dashed line), $\pi/2$ (dotted line), $\pi$ (long-dashed line).

FIG. 9: Andreev reflection current vs gate voltage with $\Phi_{12} = \pi/5$ for $\Phi = 0$ (solid line), $2\pi/3$ (dotted line), $5\pi/3$ (dashed line).
\[
\begin{align*}
\Omega e^{i\phi_2}
\end{align*}
\]
