Violating traffic light behavior in the Biham-Middleton-Levine traffic flow model

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Abstract

Effects of violating traffic light behavior are studied in this paper based on the cellular automata (CA) model proposed by Biham-Middleton-Levine (BML). We define two kinds of drivers: one obeys the traffic light rule, the other ignores it. Simulation results show that the violator increases the average velocity of free flowing phase while decreases the critical car density. We predicted the average velocity in the free flowing phase by ignoring the correlation among cars. A phase separation phenomenon, where jams and freely flowing traffic coexist, has been found in intermediate car density range. The occurrence reason of phase separation has been explained.

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Keywords: Traffic flow; cellular automata; BML model; phase transition; traffic light

1. Introduction

In the past few decades, traffic problems have attracted considerable attention of many scientists. Various models have been proposed to study and solve the problem of traffic flow by using the methods and concepts of non-equilibrium statistical physics [1, 2, 3, 4]. Among these models, cellular automata (CA) has become a well-established method, because its evolution rules are simple, straightforward and efficient [5, 6, 7, 8, 9].
The two-dimensional CA traffic model was firstly proposed by Biham-Middleton-Levine in 1992 [10]. Although the model is extremely simple, the behaviours it displays are extraordinarily complex, e.g., phase transition and self-organization. Since then, many extensive researches have been done base on this model and it serves as a theoretical underpinning for physicists to modeling urban traffic [11, 12].

It is generally believed that the original BML model is either on freely flowing phase or jamming phase. However, D’Souza notes that the BML model does not necessarily exhibit a sharp phase transition from free flow to global jam, instead it has a range of intermediate states with free flow regions intersecting at jammed wave fronts [13, 14]. Recently, a phase separation phenomenon has been observed in the intermediate car density range when the traffic light period $T=2\tau$ in the BML model is extended to $\tau>1$[15].

Society has various rules to regulate the interaction among its members. These rules play an important role in controlling social order. Just like the social conventions, the urban traffic is controlled by the synchronous traffic light in the original BML model: all the eastbound cars move only in even time steps and the northbound cars move in odd time steps. The system is self-organized into maximal speed under the control of traffic light. However, in urban traffic, some drivers do not obey the traffic light rule, not stopping at the red light for saving time. Since this behaviour always has a bad influence, it is very important to study its effect on the whole urban traffic system.

The paper is organized as follows. In section 2, the model is presented in detail. In section 3, the numerical results were reported and the occurrence reason is explained. The conclusions are given in section 4.

2. Model

In the BML model, there are two species of cars, eastbound and northbound, populating on a two-dimensional periodic square lattice with same density. Each lattice site can be in one of three states: empty, occupied by an eastbound car, or occupied by a northbound car. The system dynamics is controlled by the traffic light which is divided into two phases: red and green. On even time steps, the traffic light is green for all the eastbound cars which synchronously attempt to advance one lattice site toward the east. If the site eastward is currently empty, it advances. Otherwise, it remains stationary even if the eastward site is to become empty during the same time step. On odd time steps, the northbound cars follow the analogous dynamics. The dynamics is fully deterministic; randomness enters only through the initial conditions. Furthermore, the dynamics conserves cars, and neither allow for an eastbound car to change its row, nor for a northbound car to change its column. So on an $L \times L$ lattice, there are $2L$ conservation laws.

The behavior of violating traffic light rule is modeled as follows. At each time step, some drivers are chosen as violators randomly. The ratio of violators to total cars is $p_v$. The detail of the rule is presented in Fig. 1(a). The violators (black arrow in situation A of Fig. 1(a)) drive to its empty front site every time step, even if the traffic light is red, while the normal drivers (light grey arrow in situation B of Fig. 1(a)) will obey the traffic light rule. When two cars (one eastbound, the other northbound) attempt to move to the same empty site at the same time step, there are four cases: the driver with green light has the priority over the normal driver with red light (situation C and D in Fig. 1(a)), while it gives way to the violator with red light (situation E and F in Fig. 1(a)), no matter the driver with green light is a normal driver or violator.

3. Simulation results and discussion

The simulation was implemented on the square lattice of size $256 \times 256$. We simulate 200 random
realizations for each density. All realizations are simulated until converged \((v = 1\) or \(0\)) or for times out to at least 106 time steps. The average velocity of a period is defined to be the number of successful move in a period divided by the number of total cars. In the original BML model, two time steps constitute a period of traffic light and the car only can move once in a period. Since the violators move forward every time step, it is possible for a car to move twice in a period, resulting in the average velocity larger than one.

Fig. 1. (a) The driving rules of the model. The symbols \(\rightarrow\), \(\uparrow\), represent the eastbound and northbound car respectively. The light grey arrow represents the normal driver while the black arrow presents the violator; (b) The average velocity \(v\) of each run versus the car density \(\rho\). The lattice size is 256 x 256.

Fig. 1(b) shows the average velocity \(v\) recorded for each random realization against the car density \(\rho\) as the violator's ratio \(p_v = 0, 0.2, 0.4, 0.6, 0.8, 1.0\). Similar as the BML model, the average velocity \(v > 0\) when the car density is low, indicating that the system is in the free flowing phase, while \(v\) becomes 0 at some point means the system comes into the jamming phase. There is a range of densities in which both the asymptotic states coexist. \(\rho_c\) is defined as the central of this region. The average velocity \(v\) at low car densities increases as \(p_v\) increases, reaching the value of \(v = 1 + p_v\) as \(p_v\rightarrow 0\). On the other hand, the value of \(\rho_c\) decreases compare to the original BML model. However, the \(\rho_c\) does not decrease monotonously with \(p_v\), e.g., the \(\rho_c\) of \(p_v = 1.0\) is bigger than that of \(p_v = 0.8\).

The non-monotonic decrease of \(\rho_c\) compare to the original BML model can be understood qualitatively as follows. When \(p_v = 0\), for low car densities, these cars are self-organized to form a moving pattern (see Fig. 2(a)) due to that they have the same, simple and regular moving rules. For \(p_v > 0\), the violators do not obey the regular rules, disturb other cars and disrupt the self-organized moving pattern (see Fig. 2(b)). Thus it is harder to form a moving pattern with the presence of violator, resulting in a lower \(\rho_c\). When \(p_v = 1.0\), since the moving rule is uniform for all the cars again, they are easier to keep a moving pattern without the disturbance of other kinds of cars. However, the interaction among the cars is still stronger and they need more room to form the moving pattern compare to the original BML model.

From Fig. 1(b), one can see that the velocity of free flowing phase varies with the car density and ratio of violator when \(0<p_v<1\). In the free flowing phase, the cars are distributed randomly and homogeneously (see Fig. 2(b)) instead of forming the ordered stripes of alternating east- and northbound cars (see Fig. 2(a)). Thus the velocity of free flowing phase could be obtained by ignoring the correlation. The probability that the front site is empty for a random car equals to \(1-p\). The number of attempted moves to total cars is \((1 + p_v)\) in a period. Thus the average velocity is

\[
v = (1-p)(1 + p_v).
\]

The dashed line in Fig. 1(b) is the results of equation (1). One can see that the equation (1) agrees with the simulation results well.
Except the freely flowing and jammed phase, a new phase whose velocity is neither 1 nor 0 could be observed in the intermediate car density range. The typical pattern of the new phase is shown in Fig. 2(c). It can be seen that the intermediate phase occurs where two free flow stripes (one eastbound and one northbound) intersect in front of the jammed region. This phase is different from the structured geometric patterns (SGP) found in [13], but similar to the phase separation phenomenon in [15]. It can be understood from the macroscopic view.

From the macroscopic view, one can see that there exists a bifurcation at the tail of the jamming strip. The bifurcation stops the spreading of the jam which always evolves to the global jam. Because the tail of each offset of the jamming strip needs one kind of car to expand itself. At the same time, the two freely flowing stripes intersecting at the head of jam prevent the dissipation of jammed wave front. Although the cars in the jammed wave front have chance to escape in the evolution process, the interface between free flow stripes and jamming area essentially remains dynamic stationary.

![Fig. 2.](image)

**Fig. 2.** (a) Typical configuration of free flowing phase of BML model where the system size is 256 and $\rho = 0.12$; (b) Typical configuration of free flowing phase where the system size is 256 and $\rho = 0.06$, $p_v = 0.2$; (c) Typical configurations of phase separation on 256 x 256 lattice when $p_v = 0.2$, $\rho = 0.14$. Blue and red denote east- and northbound cars respectively.

### 4. Conclusion

In summary, the effects of violating traffic light rule behaviour on urban traffic are studied in this paper. The simulation results show that the presence of violator increases the average velocity at low car density, while decreases the critical car density. The reason is the break of self-organization of moving pattern when the cars have different moving rules. We have developed a mean field analysis for the average velocity in the freely flowing phase, which is consistent with the simulation results well. The system exhibits a phase separation phenomenon, in which the system separates into coexistence of free flow and jam. We explained the occurrence reason from the macroscopic view.

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