Numerical Simulation of the Development of a Powerful Convective Cloud Based on a Three-dimensional Model with Explicit Microphysics and Taking into Account Electrical Coagulation

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Abstract. This paper describes the calculation of electrical parameters in a three-dimensional mathematical model of a convective cloud with detailed consideration of thermodynamic, microphysical and electrical processes. The formation of electric charges occurs due to freezing of droplets and accretion processes of liquid and solid particles. The calculated values of volume charges are used to determine the potential of the electric field. A three-dimensional Poisson equation is solved for this purpose - at each step of numerical integration of the model's system of equations. The values of the electric field strength in discrete cloud cells at each time step are also taken into account when adjusting the coefficients of gravitational coagulation of cloud particles using appropriate multipliers. Some results of calculations of microstructural parameters of the cloud at different stages of development are presented with and without taking into account the electrical coagulation of cloud particles.

1. Introduction
An important problem is the protection of people and infrastructure from powerful convective processes (showers, thunderstorms, hail, tornadoes). Mathematical modeling allows us to investigate many complex questions of cloud physics, for example, to assess how much cloud electrification affects microphysical processes. Numerical models of convective clouds with electrical processes taken into account are developing successfully. Currently, three-dimensional models of convective clouds with both parameterized (bulk) and explicit (bin, spectral) microphysics have been developed. Models with explicit microphysics are more difficult to implement, so there are fewer such works. Let's briefly discuss the issues of modeling electrical processes.

Article [1] describes one of the latest implementations of lightning physics in the framework of weather research in the WRF forecasting model. To provide a scientifically based assessment of the occurrence of lightning, an explicit charge/discharge model was used (with an explicit elliptical solution of the three-dimensional component of the surrounding electric field). An algorithm which is in the parameterized microphysics and based on a two-stage scheme was developed for this purpose. Hydrometeor charging consists of five different non-inductive parameterizations, cloud water polarization and charge exchange during contact mass transfer. Later, the physics of lightning discharge...
in this model was updated, and a well-documented hybrid three-dimensional scheme was added to the source code to predict the flash type and polarity [2].

In article [3], a non-stationary numerical model is used to simulate the growth of the electric field in thunderclouds by the polarization mechanism, including both due to the growth of hydrometeors and the growth of the electric charge centers. The results show a direct relation between the growth of hydrometeors and the electric field. Various types of clouds are analyzed with taking into account their electrical properties. It was found that clouds containing large ice particles and small supercooled water droplets and fully crystallized clouds create sufficient electric fields for lightning to occur. Electric forces in clouds slow down the rate of precipitation particles falling and reduce the rate of their interaction. Thus, the resulting effect is to slow down the growth of hydrometeors and the rate of increase of the electric field.

Without dwelling in detail on the advantages and disadvantages of existing models taking into account electrical processes, we note that there is still no consensus on the issue of accounting for electrification and the impact of electrical processes on microphysical processes. The results obtained using existing models should be considered as an initial stage of cloud electricity research.

This paper describes the calculation of electrical parameters in a three-dimensional mathematical model of a convective cloud developed at the High-Mountain Geophysical Institute (HMGI) with detailed consideration of thermodynamic, microphysical and electrical processes [4]. The formation of electric charges in the model cloud occurs due to freezing of droplets and accretion of liquid and solid particles. Some results of calculations of microstructural and electrical parameters of the cloud at different stages of development are given with and without taking into account the electrical coagulation of cloud particles.

2. Brief description of the numerical model
The hydrothermodynamic block of the model developed at HMGI includes equations of motion describing wet convection in the Boussinesq approximation (the rationale for the approach is given in [5]). The microphysical block describes the processes of nucleation, condensation, coagulation of droplets with droplets, sublimation, accretion, freezing of droplets, deposition of cloud particles in the field of gravity, their transfer by air flows, as well as the interaction of cloud particles under the influence of the electric field of the cloud. A system of kinetic equations is used for the mass distribution functions of droplets $f_1(r, m, t)$, ice particles $f_2(r, m, t)$, and fragments of freezing droplets $f_3(r, m, t)$ [4]. The model calculates the density of bulk charges in the cloud, the potential and intensity of the electric field, and examines in detail the influence of the electric field of the cloud on the microphysical processes of interaction of cloud particles (electric coagulation).

The accumulation of negative charge on the ice particles occurs due to the freezing of droplets and accretion in the cloud. At the same time, a positive charge is formed, consisting of charges of individual particles-micro fragments formed when the droplets freeze. For freezing droplets with a diameter greater than 200 microns, the electrification process is described with sufficient accuracy by the expression [6]:

$$ q(m) = a \cdot m, $$

where $m$ - is the mass of the frozen drop; $a$ - is the proportionality coefficient, the value of which varies depending on the content of impurities in the drop and its freezing temperature ($a \approx 3.5 \times 10^{-10}$ C/g at $T = -8 \ldots -16$ °C). In addition, large ice crystals, graupel and hailstone accumulate an electric charge by capturing supercooled drops. The acquired charge is proportional to the mass of the frozen water on ice particles. In this case, the proportionality coefficient depends on the temperature of the growing particle, as well as on the concentration and chemical composition of impurities in the cloud water and takes a value from $10^{-10}$ to $10^{-8}$ C/g.

The formation of microscopic fragments during freezing of droplets is taken into account as follows [7]:
The calculated electric field strength at each time step is determined according to the experimental dependences of micro particle emissions on the size of the freezing drop [6].

Microscopic fragments of freezing are carried by streams to the upper part of the cloud, where a predominantly positive volume charge $\rho_+ (\vec{r}, t)$ is formed. The area of concentration of negatively charged ice particles forms a zone of predominantly negative volume charge $\rho_- (\vec{r}, t)$. The calculated total values of volume charges in the nodes of the spatial grid are used to determine the potential $U(\vec{r}, t)$ of the electric field. To do this, a three-dimensional Poisson equation is solved at each step of numerical integration of the model's system of equations. Three components of the electric field strength vector $E(\vec{r}, t)$ are calculated as a potential gradient along each of the coordinate axes.

In addition, the values of the electric field strength are taken into account at each time step to adjust the coefficients of gravitational coagulation of cloud particles. The correction is performed using correction coefficients that depend on the electric field strength. Since the electric field strength differs in the different points of the cloud, the correction coefficients change accordingly from one spatial node to another [4]. Correction coefficients are calculated based on the results of experimental and theoretical studies of electrical coagulation in articles [8, 9].

3. Some results of numerical experiments

The cloud was modeled in the 50x50x16 km domain. The spatial distribution of microstructural parameters of convective clouds (water and ice content), positive, negative and total volume charges in the cloud, electrostatic field strength, radar reflectivity at wavelengths of 3.2 and 10 cm, and other parameters at various times is studied based on the results of modeling. Two versions of calculations were performed for each model cloud: one version took into account the electrical coagulation of particles, the second one did not take it into account. Figures 1 and 2 show the isolines of water and ice content in a powerful convective cloud at the 20th and 30th minutes of development for version calculation 1. The author's program for three-dimensional visualization of parameters is used [10]. Figure 3 shows the isosurfaces of the ascending flow of 15 m/s and the volume charge of $8 \times 10^{-10}$ C/m$^3$ and the isolines of the radar reflectance at the 30th minute, and figure 4 shows the isolines of the vertical component of the electric field strength. The color palette is shown on the left in the figures to analyze the isolines.
Figure 1. Isosurfaces of water content $3.0 \text{ g/m}^3$ (1) and of ice content $3.0 \text{ g/m}^3$ (2) against the background of isolines of ascending air flows in the vertical section of the cloud at 20th minutes.

Figure 2. Isosurfaces of water content $3.0 \text{ g/m}^3$ (1) and of ice content $3.0 \text{ g/m}^3$ (2) against the background of the isolines of the vertical component of the air velocity in the vertical section at 30th minutes.

Figure 3. Isosurfaces of ascending air flows of $15.0 \text{ m/s}$ (1) and a volume electric charge of $8.0 \cdot 10^{-10} \text{ C/m}^3$ (2) against the background of isolines of radar reflectivity in the vertical section of the cloud at 30th minutes.
Figure 4. Isolines of the vertical component of the electric field strength in the vertical section at 30th minutes.

The maximum parameters for the 20th, 30th, and 40th minutes of cloud development for these two cases are shown in the table 1. The left column shows the parameters obtained during calculations with electric coagulation, while the right column shows parameters without electric coagulation.

| t, min | water content, g m⁻¹ | ice content, gm⁻¹ | updraft and downdraft, ms⁻¹ | reflectivity (3.2 cm), dBZ | charge per unit volume, C/m³ | value of the potential, V | electric field strength, V/cm |
|--------|----------------------|-------------------|-----------------------------|-----------------------------|----------------------------|--------------------------|-----------------------------|
| 20     | 3.9                  | 4.4               | 4.7                         | 2.8                         | 21.3                       | 21.1                     | 39                          | 6                           | 2.3·10⁻¹⁰                   | 8.2·10⁻¹²                   | 1.5·10⁻⁷                   | 0.0                         | 50                          | 6                           |
|        | -2.1                 |                   | -2.0                        |                             |                            |                          |                             |                             |                             | -1.9·10⁻¹⁰                  | -1.6·10⁻¹¹                  | -1.1·10⁻⁷                  | -2.4·10⁶                  | -170                        | -5                          |
| 30     | 3.7                  | 4.2               | 5.1                         | 6.6                         | 21.4                       | 21.0                     | 59                          | 54                          | 1.2·10⁹                    | 8.8·10⁻¹⁰                  | 6.7·10⁶                    | 2.8·10⁸                  | 808                         | 379                         |
|        | -3.9                 |                   | -3.7                        |                             |                            |                          |                             |                             |                             | -6.8·10⁻¹⁰                 | -7.0·10⁻¹⁰                 | -5.1·10⁻⁷                 | -1.2·10⁸                 | -1470                       | -975                        |
| 40     | 4.0                  | 3.9               | 4.0                         | 5.9                         | 19.0                       | 18.7                     | 63                          | 55                          | 1.3·10⁹                    | 1.1·10⁻⁹                  | 1.6·10⁹                    | 1.1·10⁸                  | 1025                        | 813                         |
|        | -4.2                 |                   | -4.2                        |                             |                            |                          |                             |                             |                             | -5.3·10⁻¹⁰                 | -5.7·10⁻¹⁰                 | 0.0                       | -3.1·10⁷                 | -2641                       | -2155                       |

4. Discussion of results
In powerful convective clouds that exceed the height of the isotherm minus 22 °C, large volume electric charges occur. Usually, the upper part of the cloud is positively charged, while the middle part is negatively charged. In our case, the formation of electric charges and fields in the model cloud occurred because of the freezing of droplets and accretion (interaction of droplets and crystals). The spatial charge separation occurs due to different velocities of fall in air of micro-fragments (mainly positively charged) and the larger ice particles and hail grains (mostly negatively charged): the top of the cloud accumulate positive volume charge in the middle of the cloud – negative volume charges. The density of positive and negative volume charges reached ±10⁹ C/m³ (with taking into account the charge sign). According to the numerical simulations, at the 40th minute of cloud development, the electric field potential exceeds 10⁹ V. The field strength reaches a breakdown value of ≈ 2000 V/cm (table 1). As the cloud develops, the volume charge and, consequently, the electric potential increase (table 1). The obtained quantitative values of electrical parameters of powerful convective clouds are consistent with the calculated data of other authors and with experimental data [11-14].

Electrical coagulation significantly accelerates the growth of particles in the cloud, and at the same time increases the values of volume electric charges, potential, and field strength, as can be seen from the comparison of the left and right columns in table 1.
Along with other calculated parameters, the analysis of radar reflectivity [15] of simulated clouds at wavelengths of 3 and 10 cm was carried out. In the numerical experiments, reflectivity of up to 60 dBZ or more was observed, which corresponds to the presence of hail in the cloud. In addition, this parameter was used for the comparison with the radar reflectance of real clouds observed using weather radars of the Russian Federal hydrometeorological service. The radar structure of clouds according to numerical modeling data is in satisfactory agreement with the observational data.

5. Conclusions
The three-dimensional spatial structure and quantitative values of volumetric electric charges and field strength in and around the cloud at successive points in time during evolution are determined. It is found that the mechanisms of spontaneous crystallization of large supercooled droplets and the growth of cereals and hailstones due to accretion are sufficient to explain the electrification of powerful convective clouds at the stage of their growth and maximum development.

Electrical coagulation leads to the faster growth of particles in the clouds, while generating a greater amount of electric charge and the field strength reaches the breakdown values at which lightning occurs inside the cloud or on the ground. The model can be used to study electrical processes in convective clouds.

6. References
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