We calculate average magnetopolarizability of an isolated metallic sample at frequency \( \omega \) comparable to the mean level spacing \( \Delta \). The frequency dependence of the magnetopolarizability is described by a universal function of \( \omega/\Delta \).

Mesoscopic effects in electric polarizability of small metallic particles have been discussed in the literature starting from the seminal work by Gor’kov and Eliashberg (GE) \[1\]. The role of screening effects, which were not taken into account in the original paper \[1\], has been subsequently emphasized \[2\]. Whereas GE predicted a giant effect of level correlations on the polarizability, screening actually reduces this quantum effect to a relatively small correction to the classical value. Like other mesoscopic effects related to quantum interference, this correction is affected by magnetic field. It can thus be observed in magnetopolarizability of a grain, similarly to the weak localization correction to resistivity \[3–5\].

An experimental study of quantum corrections in electrical properties of wavefunctions, obtained with the supersymmetric sigma-model can be employed to calculate the average magnetopolarizability of an isolated metallic sample at frequency \( \omega \) of the magnetopolarizability on the frequency \( \omega \) samples has been reported recently \[6,7\].

The (complex-valued) polarizability is defined as \( \alpha(\omega) = \alpha(\omega)E(\omega) \), where \( E \) and \( d \) are the external electric field and the induced dipole moment, respectively. The expression for the ensemble-averaged magnetopolarizability \( \alpha_B(\omega) = \alpha(\omega, B) - \alpha(\omega, 0) \), where \( B \) is an applied magnetic field (strong enough to break the time-reversal symmetry) has the form:

\[
\alpha_B(\omega) = \frac{2e^2}{E^2} \int dr_1dr_2 \Phi(r_1)\delta(\Pi(r_1, r_2; \omega)\Phi(r_2),
\]

where \( \Phi(r) \) is the local potential in the sample (resulting from screening of the external electric field), the symbol \( \delta \) denotes the difference between quantities with and without magnetic field, and \( \Pi \) is the polarization operator.

Here \( m \) and \( n \) label exact single-particle states, the angular brackets denote the ensemble averaging, and it is assumed that \( \omega \) has an (infinitesimally small) positive imaginary part, \( \omega \equiv \omega + i0 \). We will consider the low-temperature limit, \( T \ll \Delta \), thus setting \( T = 0 \) in the sequel. Substitution of \[3\] into \[1\] yields

\[
\alpha_B(\omega) = \frac{2e^2}{E^2} \delta \left\{ \sum_{\epsilon_m<\epsilon_F<\epsilon_n} |\Phi_m| \right\}^2 
\]

\[
\times \left( \frac{1}{\epsilon_m - \epsilon_n - \omega} + \frac{1}{\epsilon_m - \epsilon_n + \omega} \right),
\]

where \( \Phi_{mn} = \int d\psi^*_m(r)\Phi(r)\psi_n(r) \).

The canonical ensemble is realized by pinning the Fermi-level to one of the single-particle levels \( \epsilon_k \): \( \epsilon_F = \epsilon_k + 0 \). Splitting the sum in \[3\] into two contributions with \( n = k \) and \( n \neq k \), we find

\[
\alpha_B = \frac{2e^2}{E^2\Delta^2} \delta \left\{ \int_{\epsilon_k}^{\infty} d\epsilon \frac{1}{\epsilon - \omega + 1/\epsilon + \omega} \right\} 
\]

\[
\times \left[ R_2(\epsilon)\Delta + \int_{\epsilon_k}^{\infty} d\epsilon_1 R_3(\epsilon, \epsilon_1) \langle |\Phi|^2 \rangle \right],
\]

where \( R_2(\epsilon) \) and \( R_3(\epsilon, \epsilon_1) \) are the two-level and the three-level correlation functions (normalized to unity at \( \epsilon, \epsilon_1 \gg \Delta \)).
\[ R_2(\epsilon) = \Delta^2 \left( \sum_{ij} \delta(E - E_i)\delta(E + \epsilon - E_j) \right); \]
\[ R_3(\epsilon, \epsilon_1) = \Delta^3 \left( \sum_{ijkj} \delta(E - E_i)\delta(E + \epsilon - E_j) \right) \times \delta(E + \epsilon_1 - E_k), \]
\[ \langle \Phi^2 \rangle_\epsilon \text{ is the average squared matrix element,} \]
\[ \langle \Phi^2 \rangle_\epsilon = R_2^{-1}(\epsilon)\Delta^2 \left( \sum_{ij} |\Phi_{ij}|^2 \right) \times \delta(E - E_i)\delta(E + \epsilon - E_j) \].

When writing Eq. (4), we decoupled the wave function correlations \( R_3(\epsilon, \epsilon_1) \) from the three-level correlation function \( R_3(\epsilon, \epsilon_1) \). Indeed, we know from the supersymmetry calculations of two-level correlation functions that (i) the level correlation function has the RMT form, up to 1\(/g^2 \) corrections; (ii) the wavefunction correlations have no dependence on \( \omega/\Delta \) in the order 1\(/g \). Here \( g \sim E_c/\Delta \gg 1 \) is the dimensionless conductance of the grain, \( E_c \) is the Thouless energy, and we consider the frequency range \( \omega \ll E_c \). We make thus an (extremely plausible) assumption that these properties hold also for higher order correlation functions, which allows us to proceed in the case of the canonical ensemble.

We begin the evaluation of Eq. (4) by considering the term which contains the three-level correlator. We write \( R_3(\epsilon, \epsilon_1) = R_2(\epsilon) + \bar{R}_3(\epsilon, \epsilon_1) \) and denote the corresponding contributions as \( \alpha_B^{(1)} \) and \( \alpha_B^{(2)} \). In the leading order in \( 1/g \) the \( \epsilon \)-integral in \( \alpha_B^{(1)} \) is a sum of contributions from the regions \( \epsilon \sim E_c \) and \( \epsilon \sim \Delta \) (to be denoted as \( \alpha_B^{(1a)} \) and \( \alpha_B^{(1b)} \) respectively). In the former we can neglect \( \omega \), which yields after taking into account the orthogonality and completeness of the eigenfunctions \( \langle \Phi | \Phi \rangle \equiv 1 \).

\[ \alpha_B^{(1a)} = \frac{4e^2}{E^2\Delta^2} \delta \int_{+0}^\infty d\epsilon R_2(\epsilon) \langle |\Phi|^2 \rangle_\epsilon \]
\[ = -\frac{2e^2}{E^2\Delta} \langle |\Phi|^2 \rangle_\epsilon = \frac{2e^2}{E^2\Delta} \langle |\Phi|^2 \rangle_\epsilon, \]
\[ \langle |\Phi|^2 \rangle_\epsilon = \langle |\Phi|^2 \rangle_{\epsilon \sim E_c} \]
\[ = \frac{1}{V^2} \int dr_1 dr_2 \Phi(r_1)\Phi(r_2)\Pi_D(r_1, r_2), \]
\[ \Pi_D \text{ is the diffusion propagator satisfying} \]
\[ -D\nabla^2 \Pi_D(r_1, r_2) = (\pi V)^{-1} \left[ \delta(r_1 - r_2) - V^{-1} \right], \]

with the boundary condition \( \nabla \Pi_D = 0 \). In the contribution \( \alpha_B^{(1b)} \) we can neglect the \( \epsilon \)-dependence of the matrix element, thus replacing it by \( \langle |\Phi|^2 \rangle_\epsilon \).

\[ \alpha_B^{(1b)} = \frac{2e^2}{E^2\Delta} \langle |\Phi|^2 \rangle_\epsilon \]
\[ \times \int_{+0}^\infty d\epsilon \left( \frac{1}{\epsilon - \omega} + \frac{1}{\epsilon + \omega} \right) \epsilon \delta R_2(\epsilon) . \]

The same is valid for the term \( \alpha_B^{(2)} \), as well as for the remaining contribution of the first term in square brackets in Eq. (4). Collecting everything, we finally get

\[ \alpha_B = \frac{2e^2}{E^2\Delta} \langle |\Phi|^2 \rangle_\epsilon F(\omega) , \]

where

\[ F(\omega) = 1 + \int_{+0}^\infty d\epsilon \left( \frac{1}{\epsilon - \omega} + \frac{1}{\epsilon + \omega} \right) \]
\[ \times \left[ \epsilon \delta R_2(\epsilon) + \Delta \delta R_3(\epsilon, \epsilon_1) + \int_{+0}^\epsilon d\epsilon_1 \delta \bar{R}_3(\epsilon, \epsilon_1) \right]. \]

Since the integrals in (12) are determined by the range \( \epsilon, \epsilon_1 \sim \Delta \), we can use the RMT results [12] for the level correlation functions \( R_2 \) and \( R_3 \) entering this formula. Therefore, \( F(\omega) \) is in fact a universal function of the dimensionless parameter \( s = \omega/\Delta \). Real and imaginary parts of this function describe the influence of the magnetic field on polarizability and absorption of grains, respectively, and are plotted in Fig. 1.

![FIG. 1. Real and imaginary parts of the function F [Eq. (13)] representing the frequency dependence of the magnetopolarizability \( \alpha_B(\omega) \).](image-url)

Note that \( F(s) = 0 \) at \( s = 0 \), which is an identity relating the two-level and the three-level correlation functions entering Eq. (12). This identity can be derived by using the invariance of the level correlations with respect to a perturbation [13].

As is seen from Fig. 1, our results predict a positive magnetopolarizability for all values of \( \omega \) and a negative magnetooptical absorption for almost all \( \omega \). This is in qualitative agreement with the findings of Ref. [1], where the
measurements have been performed at a frequency corresponding to \( s \approx 0.21 \). It would be very interesting to have the experimental data for several frequencies in the range \( \omega \sim \Delta \) in order to check our prediction for the universal scaling function \( F(s) \) describing the frequency dependence of the magnetopolarizability induced by the level and eigenfunction statistics in the grains.

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[13] In our paper a non-zero value of \( \alpha_B(0) \) was given, which was a result of an unfortunate arithmetic error in evaluation of the \( \omega = 0 \) limit of the integral (4). We are grateful to H. Bouchiat for giving a qualitative argument implying \( \alpha_B(0) = 0 \), which helped us to find this error.