Stacking catalogue sources in WMAP data

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ABSTRACT
We stack Wilkinson Microwave Anisotropy Probe (WMAP) 7-yr temperature data around extragalactic point sources, showing that the profiles are consistent with WMAP’s beam models, in disagreement with the findings of Sawangwit and Shanks. These results require the point source catalogue’s selection to be free from biases due to cosmic microwave background (CMB) fluctuations. We compare profiles from sources in the standard WMAP catalogue, the WMAP catalogue selected from a CMB-free combination of data and the NRAO VLA Sky Survey catalogue, and quantify the agreement with fits to simple parametric beam models. We estimate the biases in source profiles due to alignments with positive CMB fluctuations, finding them roughly consistent with those biases found with the WMAP standard catalogue. Addressing those biases, we find source spectral indices significantly steeper than those found by WMAP, with strong evidence for spectral steepening above 61 GHz. We re-analyse parameters with a revised source correction, finding an $n_s$ increase by up to ~0.3σ. Finally, we discuss implications for current CMB experiments.

Key words: galaxies: active – quasars: general – cosmic background radiation – cosmology: observations.

1 INTRODUCTION
A telescope’s beam or point spread function, which dampens the instrumental response to fluctuations on small scales, is prominent among the systematic effects that must be well understood for cosmic microwave background (CMB) measurements. The act of observing convolves the sky with the telescope beam, so observations of bright objects that are point-like (compared to the beam size) provide an obvious check on the beam pattern. The Wilkinson Microwave Anisotropy Probe (WMAP) team used observations of Jupiter to measure the beams (Page et al. 2003; Jarosik et al. 2007; Hill et al. 2009; Jarosik et al. 2011), but in principle any suitable objects will work, including extragalactic point sources.

Sawangwit & Shanks (2010a) used the 5-yr WMAP data to construct stacked profiles around sources from the WMAP point source catalogue (Wright et al. 2009). Whitbourn, Shanks & Sawangwit (2011) repeated this analysis with the 7-yr data, using WMAP and external catalogues. Both papers report some intriguing discrepancies: for WMAP’s differencing assemblies (DAs) at 40–90 GHz, the authors found substantial offsets at large scales, and the profiles appear broader than the beam patterns from the Jupiter measurements. They address and discard several possible explanations for the effect: extended radio sources, source clustering and selection bias near the catalogue threshold, and favour a non-linearity in WMAP’s response to Jupiter, which has a peak temperature approximately three orders of magnitude higher than CMB fluctuations at these frequencies. They note that a cosmological analysis based on the window functions computed from their stacked profiles (instead of the Jupiter model) would significantly change WMAP’s basic cosmological results, for example by changing the height and location of the first acoustic peak in the power spectrum. In Sawangwit & Shanks (2010b), the authors present this finding as a challenge to the Λ-cold dark matter (ΛCDM) paradigm.

Despite these findings, several contrary lines of evidence indicate that the Jupiter-based beam models sufficiently represent the true beam patterns for the WMAP telescope. First, WMAP and several pre- and post-WMAP experiments (from the ground and from balloons) have over the past decade found similar CMB power spectra on the scales where they overlap. Telescopes for which the beam scale differs substantially from WMAP are particularly powerful probes of this consistency (see WMAP comparisons to QUaD: Brown et al. 2009; ACBAR: Reichardt et al. 2009; ACT: Hajian et al. 2011 and SPT: Keisler et al. 2011). Secondly, using WMAP data alone, comparison of the power spectra derived from the different DAs can be a useful test of errors in the beam models. A frequency-dependent beam systematic error of the size considered by Sawangwit & Shanks (2010a) would almost certainly show up in estimates for the unresolved point source contribution at high $l$. Nolta et al. (2009) and Huffenberger et al. (2008) extensively studied multifrequency combinations of Jupiter-beam-corrected power spectra from WMAP for this purpose, and found no such substantial beam anomaly. These methods constrain the relative beam window functions between different assemblies at roughly the per cent level. For example, a bump at $l < 200$ in the WMAP 3-yr data’s unresolved point source estimate disappeared with the beam revision

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of Hill et al. (2009) for the 5-yr data. This revision changed the measurement of the first acoustic peak’s height by 2–3 per cent, much smaller than the change in the beam proposed by Sawangwit & Shanks (2010a) or Whitbourn et al. (2011). (See also Giannantonio, Lewis & Crittenden 2010 for further discussion of the strengths of the $\Lambda$CDM model despite the misgivings of Sawangwit & Shanks 2010b.)

However, the issue with the stacked point sources remains: why should stacked extragalactic point sources present a profile so different from the telescope beam? This is a worthwhile question. Here, we explore it by stacking sources in the same way as Sawangwit & Shanks (2010a), and we are able to reproduce their basic results for the WMAP standard catalogue. However, by using alternative source catalogues, we by contrast find source profiles compatible with the WMAP beam models. We fit parametrized models to quantify the beam effects, and further explore systematic selection biases.

In Section 2, we present our data selection and analysis methods, while in Section 3, we discuss and interpret our results. Finally in Section 4, we draw our conclusions.

2 METHODS

2.1 Data selection

We base our analysis on the WMAP 7-yr maps and point source catalogues, available from the Legacy Archive for Microwave Background Data (LAMBDA) website. In turn we use maps for the individual DAs, with and without a foreground template removed. We focus on the $Q$ band (41 GHz), $V$ band (61 GHz) and $W$ band (94 GHz). All maps are at HEALPIX$^2$ resolution $N_{\text{side}} = 512$ (6.9 arcmin pixels).

We mask the sky to exclude pixels and sources near the Galactic plane or other extended structures (like the Large Magellanic Cloud), while retaining bright sources away from extended foregrounds. We begin with the WMAP 7-yr temperature analysis mask. Then we invert the WMAP point source mask to retain those pixels near sources excluded by temperature analysis mask. The negative side effect is that some pixels are now kept around sources in high-foreground regions. To eliminate these, we smooth with a 2$^\circ$ FWHM Gaussian and apply a threshold at 90 per cent of the smoothed maximum. This expands the mask slightly, eliminating the problem pixels. Our resultant mask excludes the highest foreground regions and leaves 76.8 per cent of the sky available for our analysis.

The standard WMAP catalogue contains 471 sources, but our mask excludes 38. Closely paired sources can spoil the profile, so we additionally require that the sources be isolated. Since we explore the source profiles out to $\theta_{\text{max}} = 1^\circ$, we exclude all catalogue sources which are separated by less than $2\theta_{\text{max}}$ from another source. (Both members of the pair are excluded.) This prevents overlaps in computing our stacked profiles and removes an additional 68 sources from our analysis (20 of these sources have another source even within 1$^\circ$). After the two cuts, we have 365 sources on which we base our stacking analysis. The positional uncertainty in the WMAP catalogue is 4 arcmin (Bennett et al. 2003; Chen & Wright 2008).

The WMAP team also provides a catalogue selected from CMB-subtracted maps using the multifrequency method of Chen & Wright (2008). The catalogue contains 417 sources. Our mask eliminates 52 sources; then we cull 34 members of close pairs, leaving 331. The positional uncertainty in the CMB-free catalogue is smaller, about 2 arcmin. Of these 331 sources, 260 (or 78.5 per cent) lie within 0.3 of a WMAP standard catalogue source.

Finally, we check our results by stacking on the source catalogue from the NRAO VLA Sky Survey (NVSS; Condon et al. 1998), which surveyed the sky at 1.4 GHz for $\delta > -40^\circ$. At the frequency and angular resolution of this survey, the CMB is not a significant component of the emission, and cannot affect the catalogue selection. We cut at 2 Jy (at 1.4 GHz) to take the brightest 762 sources. Masking eliminates 350 sources, and culling close pairs eliminates another 129 sources, leaving 283. Accounting for the reduced sky coverage, this gives a similar density of sources as the WMAP catalogues. The positional uncertainty of the bright NVSS sources is very small, <1 arcsec. Of these 283 NVSS sources, only 71 (or 25.1 per cent) lie within 0.2 of a WMAP standard source catalogue. Thus, the NVSS catalogue is probing a different set of sources than the WMAP standard and CMB-free catalogues.

2.2 Stacked profiles

We stack our sources simply by defining angular annuli around reported catalogue positions, and then averaging the pixels whose centres fall into those annuli.

The shape of the profile is subject to pixelization effects, both from the angular binning into annuli and from the map pixelization. WMAP’s beam models are tabulated at much higher angular resolution than the profiles, so must also be binned for direct comparison to the profile, or for the computation of $\chi^2$.

On an infinite resolution map, binning will simply average the beam model ($b$) over annuli (indexed by $i$) to get the binned model ($m$):

$$m_i = \frac{\int_{\theta_{\text{min},i}}^{\theta_{\text{max},i}} \theta \, d\theta \, b(\theta)}{\int_{\theta_{\text{min},i}}^{\theta_{\text{max},i}} d\theta}.$$  

The angular weighting causes the peak to be slightly suppressed.

For point sources in realistic maps, the map pixel size and shape also influence the profile, broadening and blunting the peak. This affects the smallest beams the most. To simulate this effect, we placed 1000 synthetic sources with the shape of each WMAP beam at random locations in a high-resolution HEALPIX map; we then reduced the resolution to $N_{\text{side}} = 512$ and computed a stacked profile around the source centre. (We used $N_{\text{side}} = 4096$ for the high-resolution map; using $N_{\text{side}} = 2048$ makes only a minor difference at the W band, and is immaterial for $V$ and larger beams.) This results in a profile which is notably blunted at the peak, and then larger than the input beam at $0.2 \lesssim \theta \lesssim 0.4$. At the peak, pixel effects suppress the $W$-band beams slightly more than 10 per cent, $V$-band by about 5 per cent, $Q$-band by 3 per cent, and the $K$- and $K_a$-band profiles by less than 2 per cent. Normalizing the pixelized, blunted profiles to the unbinned beam model at the peak boosts the binned profile’s tail, but this alone is insufficient to account for the broad profiles as seen by Sawangwit & Shanks (2010a). The source profiles in maps are well reproduced by convolving the beam with the map pixel window function and then binning. This is the strategy we use later for our parametric beam models. We illustrate these binning and pixel effects in Fig. 1 for the W3 DA. The $W$-band beams are WMAP’s narrowest in the main lobe and this particular beam has an interesting shape, with a shoulder from 0.2 to 0.4. We model the uncertainty in source position by convolving the profile with a Gaussian of appropriate size ($\sigma = 4$ arcmin for the standard WMAP
catalogue, \(\sigma = 2\) arcmin for the CMB-free catalogue and \(\sigma = 0\) arcmin for the NVSS catalogue).

We consider two contributions to the covariance matrix for these profiles, one from detector noise and the other from background CMB fluctuations. Our final covariance matrix is the sum of these components. Below we use these covariance matrices when minimizing \(\chi^2\) for model fitting. We do not make any correction for the finite number of sources stacked in the profile, which will slightly modify the errors because of positional uncertainty (deviating from the Gaussian convolution kernel appropriate for an infinite number of sources).

For non-overlapping source profiles, under the assumption of white noise, the detectors contribute a diagonal component to the covariance. We compute this term analytically from the noise variance per pixel provided by the WMAP team for each DA’s map (Appendix A).

Under the assumption that source positions are uncorrelated with CMB fluctuations, we can construct the covariance due to the background CMB. For the WMAP standard catalogue, we will later see that this assumption is unsound. The covariance can be computed analytically, but is more practically computed with a Monte Carlo method, as follows.

For the CMB power spectrum, we use the WMAP best-fitting \(\Lambda\)CDM model. We combine stacked profiles of CMB-only simulations (including the beam and pixel window functions) to produce our covariance estimate. We use 1600 Monte Carlo realizations, although using half that number changes our typical estimate of \(\chi^2\) by just one half of 1 per cent. Unlike the noise, the covariance due to CMB fluctuations is strongly correlated between profile bins (Fig. 2).

The covariance matrices for the standard WMAP catalogue and the WMAP CMB-free catalogue are very similar because they share a large number of sources. The covariance for NVSS sources is somewhat larger for two main reasons. First, the number of NVSS sources is smaller. Secondly, NVSS did not survey the South ecliptic pole \((\delta \approx -66.6)\). WMAP noise is notably suppressed at the ecliptic poles due to a higher number of observations, so the NVSS profiles are taken from higher noise portions of the sky. This increases the errors accordingly below.

### 2.3 Minimizing \(\chi^2\) for model fitting

Below we explore parametric models for the profile, typically applying an amplitude factor and an offset. To fit a model to our stacked source profile, we minimize

\[
\chi^2(\alpha) = \sum_{ij} \left[ P_i - m_j(\alpha) \right] C_{ij}^{-1} \left[ P_j - m_j(\alpha) \right].
\]

Here \(P\) is the stacked profile and \(m\) is a binned model for the profile, which in turn is based on the WMAP beam, the HEALpix pixel window function and the positional uncertainty appropriate for the catalogue. The indices \(i, j\) run over the angular bins. This model additionally depends on a set of parameters \(\alpha\) (like an amplitude, etc.). When the model depends on the parameters linearly, we can solve for the best-fitting parameters and their covariance matrix algebraically. Otherwise we use an implementation of Powell’s direction sets method to minimize \(\chi^2\) for non-linear models. We compute parameter covariances with

\[
(Cov(\alpha_p, \alpha_q))^{-1} = \sum_{ij} \frac{\partial m_i}{\partial \alpha_p} C_{ij}^{-1} \frac{\partial m_j}{\partial \alpha_q}.
\]

although this is strictly only applicable when the fit is good.

### 3 RESULTS

#### 3.1 Stacked source profiles

For WMAP catalogue sources, we show our stacked profiles from the DA maps in Fig. 3. All plots are in thermodynamic temperature and we leave off the error bars to avoid crowding the plot, but show them below. Consistent with the spectral energy distribution typical of these sources, the average profile is brighter in the \(Q\) band than in the \(V\) band, and brighter in \(V\) than in \(W\). Below
we quantify that the profiles are significantly discrepant from the Jupiter-modelled beams. The excess above the model is similar in \( V \) and \( W \) at \( \theta > 0.4 \), as if the beam profiles are sitting atop a common fluctuation.

We see large-scale offsets out to 1\(^\circ\), the same as Sawangwit & Shanks (2010a). The offset is 30–40\( \mu \)K in the \( Q \) band and 10–20\( \mu \)K in \( V \) and \( W \). Sawangwit & Shanks (2010a) attribute this offset to large-scale CMB fluctuations, and fit it at large angular scales. We disagree with this interpretation. CMB fluctuations would not cause the substantially larger offset in the \( Q \) band. Modelled as a constant (as we detail below) the offset is 33.1\( \mu \)K in \( Q \), 13.2\( \mu \)K in \( V \) and 16.6\( \mu \)K in \( W \), each with an error of about \( \pm 4\mu \)K. Since the same sources are stacked in each band, the underlying CMB fluctuations should be the same (in thermodynamic units), up to the effects of beam smoothing, suggesting that another foreground signal is responsible. Indeed, the profiles computed on the foreground-reduced map set do not show the large-scale offsets. Here, the \( Q \)-band profiles join \( V \) and \( W \) at \( \theta > 0.5 \) atop a common fluctuation.

Although they mitigate Galactic emission, the foreground-reduced maps are inappropriate for probing the stacked profiles in detail. The templates for foreground removal also subtract away a portion of each source’s flux. One foreground template is constructed from the \( K \) and \( Ka \) bands, and when the template is subtracted it creates a depression around each source which is the size of the larger beam scale in those two channels. (None of this should affect \( WMAP \)'s power spectrum analysis, because all these regions near sources are masked.)

The \( WMAP \) CMB-free catalogue also selects sources that are bright in the \( WMAP \) bands; nearly 80 per cent of the sources are the same. These stacks (Fig. 4) have similar peak temperatures to the \( WMAP \) sources in the \( Q \) band, and are slightly lower in \( V \) and \( W \). Furthermore, the tails of these profiles are much closer to the model than for sources from the standard catalogue.

The stacked profiles around NVSS catalogue sources show that on average the NVSS sources are dimmer than the \( WMAP \) catalogue sources, and the signal-to-noise ratio is lower. This is consistent with their selection at 1.4 GHz: a flat spectrum source that is bright in NVSS will be bright in \( WMAP \), and included in the catalogue. Since only \( \sim 25 \) per cent of the sources overlap between the catalogues, most of the NVSS sources must have falling spectra, so that they drop out of the \( WMAP \) catalogue. The same pattern of source peak temperature dropping from \( Q \) to \( V \) to \( W \) holds.

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Stacking catalogue sources in \( WMAP \) data

Figure 3. Stacked profiles around sources from the \( WMAP \) point source catalogue for all DAs, where the horizontal bars show the angular bin width. The source amplitude decreases from the \( Q \) band (red circles) to \( V \) band (green triangles) to \( W \) band (blue squares). The best-fitting models for \( Q1 \), \( V1 \) and \( W1 \) are shown as dotted lines, and are poor fits to the profiles. Left: using the raw \( WMAP \) maps for each DA. Right: the same, but using foreground-reduced maps.

Figure 4. Similar to the left-hand panel of Fig. 3, but stacking on the alternative source catalogues that are selected from maps without a significant CMB contribution. The beam models fit much better here. Left: \( WMAP \) CMB-free catalogue. Right: NVSS catalogue.
3.2 Parametric beam fits

We fit a two-parameter model to the stacked profiles:

\[
m(\theta) = Sb(\theta) + m_0,
\]

where \(S\) is the amplitude, \(b(\theta)\) is the Jupiter-modelled \(WMAP\) beam, normalized and smoothed with the pixel window function and Gaussian positional uncertainty, and \(m_0\) is an offset to represent large-scale foreground or CMB contamination. Comparisons to the data use the model after binning. We enforce model beam normalizations so that the 2D integrals under beams are unity,

\[
2\pi \int d\theta \, b(\theta) = 1. \quad (5)
\]

Thus, \(b(\theta)\) has the units of inverse solid angle. Therefore, the amplitude \(S\) has units of temperature times solid angle, or equivalently flux density. For each DA, conversions to flux densities (\(dB/dT\)) are computed at the effective frequencies given in Page et al. (2003). We show values using the effective frequencies for an \(\alpha = -0.1\) source, roughly the same as the mean source spectral index from Wright et al. (2009), \(\alpha \sim -0.09\). Below we find evidence that the source spectral indices are actually somewhat steeper than this, \(\alpha \sim -0.3\) to \(-0.5\). Even if the index is made as steep as \(\alpha = -0.7\), the change in effective frequency is small, and our source amplitudes change by less than 2 per cent.

We first examine the best-fitting models for the \(W3\) DA; the behaviour is similar for the other DAs in the \(V\) and \(W\) bands. Fig. 5 shows the best-fitting model for this DA and the model’s residuals for each of the three catalogues.

The stack on the \(WMAP\) standard catalogue shows large positive residuals compared to the fit. One seeming peculiarity of this fit is that the fitted model is below the data at most scales, and at least the model’s peak could be brought closer to the data by increasing the amplitude. Such a change nonetheless worsens the \(\chi^2\) of the fit, and the strong off-diagonal components in the covariance matrix caused by CMB fluctuations (Fig. 2) are responsible. Due to the shape of the model \(WMAP\) beam, the fit will fall below the data at 0.5 for any reasonable amplitude. Different bins are so strongly correlated that this in turn causes the \(\chi^2\) fit to prefer that all bins be below the stacked profile, including those at \(\theta < 0.2\). Boosting the amplitude to raise the model near the peak, which naively would appear to improve the fit, actually worsens \(\chi^2\). Because of the bin-to-bin correlations, in this case the \(\chi^2\) fit prefers, in order: (1) all bins low, (2) only some bins low and (3) some bins high and some bins low. The other DAs show this behaviour stacked on the \(WMAP\) standard catalogue, except for the \(Q\) band fitted to the stack from the foreground-reduced maps, where the residuals are negative and caused by the foreground template oversubtraction (and the same reasoning regarding correlated bins applies).

By contrast, the model fits the profiles from the CMB-free and NVSS catalogues much better. Even the shoulder in the \(W3\) beam appears to be recovered in the CMB-free profile. The NVSS source profiles have lower signal-to-noise ratio, but are similar. The other DAs show the same for the CMB-free and NVSS catalogues.

Thus, sources selected from maps that contain CMB produce stacked profiles with a bias, while sources selected from maps without a significant CMB contribution do not. These results differ from what Sawangwit & Shanks (2010a) and Whitbourn et al. (2011) report for their CMB-free and NVSS stacks. Below we find that this bias is roughly consistent with the expectations of source selection bias due to CMB fluctuations.

This bias for the \(WMAP\) standard catalogue causes the \(\chi^2\) of the fit to be very poor in every DA, seen in Table 1, which also shows the fitted parameters and the probability to exceed \(\chi^2\) by chance for 18 degrees of freedom (20 bins with two parameters).

The CMB-free catalogue probabilities are much more reasonable, except at the \(Q\) band, where they are low. The probabilities for the NVSS catalogue are everywhere reasonable. Despite some minor complications which we discuss below, we find no compelling evidence from this modelling that the Jupiter-based \(WMAP\) beams are radically insufficient to explain the source profiles, and we conclude that \(WMAP\)’s beams for \(V\) and \(W\) bands, which are used for the cosmological analysis, are sound at this level. Furthermore, the discrepancy with the profiles from the standard catalogue bears the mark of source selection bias due to CMB fluctuations. We explore this below.

3.3 CMB-free catalogue residuals

The CMB-free catalogue profiles have some features that warrant further discussion. For example, for every CMB-free profile, the first bin is low compared to the models for each DA (Fig. 4, left-hand panel).

Fig. 6 depicts the profiles and fits for Q1, which is the poorest fit for the CMB-free catalogue. Here the residuals show some
interesting patterns. In both $Q$-band DAs, the model for the CMB-free catalogue exceeds the data slightly at $0.3 < \theta < 0.5$, leading to negative residuals. One possibility for this relates to the positional uncertainty. Brighter sources will have more accurate measured positions, so stacked profiles, weighted towards the bright sources, should have less effective positional uncertainty than the catalogue overall (quoted as 2 arcmin for this case). In addition, the mix of bright sources in each stack changes from band to band based on the frequency dependence of each source, so even considering a single catalogue, the effective positional uncertainty can be different between the bands.

When we allow the positional uncertainty to float in the $Q$-band fit, the $\chi^2$ minimum has positional uncertainty less than 2 arcmin, but it is not consistent between $Q1$, where the fit prefers zero positional uncertainty, and $Q2$, which prefers 1.9 arcmin, and the probability to exceed $\chi^2$ is still low, never exceeding a few per cent (now for 17 degrees of freedom). Changing the positional uncertainty from 0.5 to 2.5 arcmin leads to a 5 per cent increase in the inferred source amplitude. Further letting the model adjust the positional uncertainty in a flux-dependent way (as a power law) makes the model so flexible that it is difficult to draw any conclusions.

The other peculiar feature in Fig. 6 for the CMB-free $Q1$ residuals is the alternating low–high pattern in the first several bins, and the first bin is quite low. This is visible in the first five bins in each DA except for $W1$ and $W4$, where only the first three and two bins (respectively) follow the pattern. This cannot be ringing in the maps due to the source signal, because the pattern is absent in the NVSS profiles. Positional uncertainty or systematic mis-centring of the sources does not cause an alternating pattern.

CMB fluctuations, which are common to all the DA maps (up to beam smoothing), are another candidate. The covariance matrix (Fig. 2) shows that bins 1–3–5 are more strongly correlated to each other (30–35 $\mu$K$^2$) than they are to bins 2–4 (20–25 $\mu$K$^2$), so CMB fluctuations could conceivably have an effect shaped like this (with the offset removing the fluctuation common to all bins) and be common across the bands. However, such an effect should only appear at the couple of $\mu$K level, smaller than what is seen here, and would not be particularly stronger in $Q$.

Slight underestimates of the covariance due to positional uncertainty and the finite number of sources, which would be stronger in $Q$, are another possibility, but we have not explored it in detail.

### 3.4 Flux density

The source amplitudes are similar in the $Q$ band for the WMAP standard and CMB-free catalogues in Table 1, but the amplitudes in $V$ and $W$ are substantially less in the CMB-free catalogue. The selection bias causes the sources in $V$ and $W$ to appear much too

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**Table 1.** Parameter values and the goodness of fit for a simple amplitude-offset model based on the WMAP beam, for stacked profiles from the three different catalogues. The probability to exceed $\chi^2$ indicates that the model fits the WMAP catalogue profiles poorly, the CMB-free catalogue profiles much better (although $Q$ band’s probability is low) and the NVSS profiles well.

| DA     | Amplitude (Jy) | Offset ($\mu$K) | $\chi^2_{r=18}$ | $P(>\chi^2)$ |
|--------|----------------|-----------------|-----------------|---------------|
| WMAP catalogue: Q1 | 1.51 ± 0.01 | 32.80 ± 4.07 | 57.08 | $6.01 \times 10^{-6}$ |
| Q2 | 1.51 ± 0.01 | 33.45 ± 4.17 | 44.24 | $5.34 \times 10^{-4}$ |
| V1 | 1.47 ± 0.02 | 12.49 ± 4.27 | 49.45 | $9.12 \times 10^{-5}$ |
| V2 | 1.43 ± 0.02 | 13.97 ± 4.14 | 67.23 | $1.32 \times 10^{-7}$ |
| W1 | 1.38 ± 0.04 | 16.57 ± 4.33 | 56.09 | $8.63 \times 10^{-6}$ |
| W2 | 1.44 ± 0.05 | 16.92 ± 4.27 | 69.83 | $4.83 \times 10^{-6}$ |
| W3 | 1.41 ± 0.05 | 16.39 ± 4.19 | 58.66 | $3.36 \times 10^{-6}$ |
| W4 | 1.39 ± 0.04 | 16.54 ± 4.21 | 47.35 | $1.88 \times 10^{-4}$ |

| WMAP CMB-free catalogue: Q1 | 1.51 ± 0.01 | 36.97 ± 4.28 | 35.41 | $8.39 \times 10^{-3}$ |
| Q2 | 1.49 ± 0.01 | 36.58 ± 4.27 | 30.34 | $0.034$ |
| V1 | 1.32 ± 0.02 | 15.97 ± 4.26 | 25.52 | $0.111$ |
| V2 | 1.28 ± 0.02 | 15.56 ± 4.39 | 19.22 | $0.378$ |
| W1 | 1.05 ± 0.03 | 17.32 ± 4.27 | 16.86 | $0.533$ |
| W2 | 1.05 ± 0.04 | 16.85 ± 4.40 | 24.60 | $0.136$ |
| W3 | 1.03 ± 0.04 | 16.25 ± 4.35 | 17.16 | $0.512$ |
| W4 | 1.05 ± 0.04 | 17.23 ± 4.32 | 8.89 | $0.962$ |

| NVSS catalogue: Q1 | 0.77 ± 0.01 | 31.04 ± 6.72 | 17.05 | $0.520$ |
| Q2 | 0.74 ± 0.01 | 28.86 ± 6.72 | 24.59 | $0.137$ |
| V1 | 0.63 ± 0.02 | 10.27 ± 6.57 | 14.40 | $0.703$ |
| V2 | 0.66 ± 0.02 | 12.14 ± 6.73 | 12.43 | $0.825$ |
| W1 | 0.59 ± 0.03 | 13.17 ± 6.70 | 12.02 | $0.846$ |
| W2 | 0.54 ± 0.04 | 12.81 ± 6.57 | 15.92 | $0.598$ |
| W3 | 0.59 ± 0.04 | 14.04 ± 6.77 | 24.41 | $0.142$ |
| W4 | 0.55 ± 0.04 | 13.81 ± 6.71 | 9.25 | $0.954$ |
bright, and this has a significant effect on the measured frequency dependence of sources.

From the best-fitted source amplitudes, we can examine the scaling for the mean source flux density, writing

\[ S/V = (V/V)_{\text{pow}} \tag{6} \]

and so forth. These are displayed in Table 2. For the WMAP standard catalogue, where we know the flux densities to be biased, we find a mean spectral index \( \alpha \sim -0.09 \), the same as Wright et al. (2009) found on a source-by-source basis. However, the spectral index from the CMB-free catalogue is much steeper and shows steepening above 61 GHz, from \( \alpha = -0.36 \pm 0.03 \) to \( \alpha = -0.50 \pm 0.05 \), a difference significant at the \( \sim 5\sigma \) level. The NVSS sources have similarly steeper indices than the WMAP standard catalogue sources, but there is no significant change in the spectral index with frequency.

### 3.5 Primordial spectral index

These frequency scalings are employed in the estimation of the power spectrum of unresolved point sources, which is necessary to correct the CMB power spectrum. The source correction effectively changes the tilt of the measured power spectrum (see Komatsu et al. 2008), we repeat the free catalogue scaling, and causes us to expect little change in the spectral index with the (bias-free) CMB-free catalogue values. The CMB power spectrum results, which are the basis of the cosmological measurement, employ a combination of V- and W-band data only.

The point source correction uses multifrequency estimators that scale the unresolved sources from Q to the V and W bands. If the power in Q is held constant, steepening the spectral index according to the CMB-free catalogue values reduces the required correction in V by about 20 percent and in W by about 40 percent. However, according to Larson et al. (2011), the source amplitude for the power spectrum correction is scaled by \( v^2 T^2 \) in thermodynamic temperature, which ignores the differing conversions for antenna temperature at these frequencies. This effectively steepens the sources in the correction by about the same amount as we see comparing the (biased) standard catalogue to the (bias-free) CMB-free catalogue scaling, and causes us to expect little change in the source correction.

As in Huffenberger, Eriksen & Hansen (2006) and Huffenberger et al. (2008), we repeat the \( n_s \) analysis with modified source corrections, here using the WMAP likelihood code (version 4.1) and \textsc{cosmomc} (2011 October version,3 Lewis & Bridle 2002). One difficulty was that the WMAP team does not release the specific weights of the cross spectra contributions to the final power spectrum (from the 15 VV, VW and WW combinations). The weights are necessary to determine a new source correction based on a different source model. WMAP does publish the source correction, but this alone is insufficient to determine the weights. We note that the WMAP published source correction appears not to approach our estimate of the W-band correction near \( l \sim 1200 \), which is surprising because the spectrum should come almost completely from W-band data at that multipole.

To build new source corrections, we took two strategies to estimate the cross-spectrum weights. First, we (incorrectly) assume that the power spectrum is a combination of VV and WW cross spectra only. Under this assumption, we can determine the weights from the published source correction, and then determine a new source correction from the weights. This is almost the same as the WMAP correction, causing no significant shift in the cosmological parameters.

Secondly, we make a simplified estimate for the inverse noise weights that WMAP uses for the spectrum at \( l > 600 \). We treat each DA as having the same noise and consider only the diagonal of the covariance of the cross spectra. Using the Knox (1997) formula, this model of noise and weights predicts power spectrum errors within \( 20 \) percent of the WMAP published power spectrum errors for \( 600 < l < 1000 \). The source correction generated by this weighting has a similar shape to the published one but is lower by \( \sim 50 \) percent. It causes an \( n_s \) increase of 0.003–0.004, which is an increase of about \( 0.3\sigma \). We emphasize that this shift in the spectral index is caused by the weighting scheme for the cross spectra, not by the frequency scaling from the sources.

### 3.6 Selection bias from CMB anisotropy

The same fluctuations that are responsible for biases in the catalogue selection and source counts (Eddington 1913) will bias the source profiles. For the WMAP catalogues, the sources are mostly selected at lower frequency, where they tend to be brightest, and there is a significant mismatch between the beam scales at the K band (49 arcmin FWHM) and the Q, V and W bands (29, 20 and 13 arcmin FWHM, respectively; Page et al. 2003). Although the noise is distinct, the same CMB is seen by each of the DAs (up to beam smoothing). The profiles in the higher frequency DAs can therefore be broadened by CMB fluctuations that cannot be distinguished from source flux in the lower frequency maps.

To roughly quantify the bias as a function of source flux density, we performed Monte Carlo simulations, injecting sources of known flux into synthetic realizations of the CMB and detector noise, and then finding these sources.

Following the WMAP source detection procedure (Wright et al. 2009), the maps are weighted by \( (N_{\text{obs}})^{1/3} \), and then filtered in harmonic space by \( b_i/\sqrt{2} N^2 \sum \bar{C}_{\text{cmb}} + N_i \) to maximize the signal-to-noise ratio of the recovered sources. The noise spectrum is computed from the pixel noise, pixel area and the mean over the sky of the inverse number of observations: \( N_i = \sigma_i^2 \Omega_{\text{pix}} N_{\text{obs}} \). The pixel noise factors \( \sigma_i \) are given by the WMAP data release for each DA.

The synthetic sources that exceed \( 5\sigma \) in the filtered map are taken to exceed the catalogue threshold. Our synthetic source selection is slightly simpler than the WMAP selection. First, we select only in \( k \) (where most sources are brightest), instead of the five-band selection used by WMAP. Secondly, we use the centre of the local maximum pixel as the source position, rather than fitting for the position of each source candidate. We stack the unfilted maps around the recovered sources and examine the source profiles. We show the bias for several input source flux densities in Fig. 7. For a 1.0 Jy source the bias is a little less than 20 \( \mu \text{K} \) at the source position and falls to half-maximum at about 0.5. For faint sources,

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3 http://cosmologist.info/cosmomc/
only those that fall on background peaks are recovered, so the bias is large. Almost all bright sources are found, but slight biases remain because of errors in the source positions, which are shifted to favor background peaks.

After evaluation at several flux densities, we can interpolate to approximate $P_{\text{bias}}(\theta, S)$, the bias in the profile as a function of flux density. Then the expected bias in a catalogue can be estimated with

$$P_{\text{bias}}(\theta) = \frac{1}{N_{\text{cat}}} \int dS \frac{dN_{\text{cat}}}{dS} P_{\text{bias}}(\theta, S),$$

where $dN_{\text{cat}}/dS$ gives the distribution of fluxes in the catalogue, accounting both for the intrinsic source counts and the selection function, and $N_{\text{cat}}$ is the total number of sources. We estimate this integral by evaluating a sum over sources in the WMAP CMB-free catalogue, which is free from CMB contamination and lists noise-bias corrected source fluxes:

$$P_{\text{bias}}(\theta) \approx \frac{1}{N_{\text{cat}}} \sum_i P_{\text{bias}}(\theta, S_i).$$

The median $K$-band flux in this catalogue is 0.9 Jy and the minimum de-boosted flux is 0.1 Jy. The resulting expected bias is about 25 μK at the peak and quite broad. Despite the crudeness of our estimate, which is slightly larger than the residual for the WMAP standard catalogue profile in Fig. 5, where the fit takes up some of the residual, the bias due to source selection appears to be the probable explanation for the broad profiles found here and in Sawangwit & Shanks (2010a) for the WMAP standard catalogue.

4 CONCLUSIONS

We stacked point sources from three different catalogues on the maps from eight of WMAP’s $K$ band. For the WMAP standard catalogue, we see evidence for residual CMB fluctuations that bias the profiles. Some complications remain in the profiles for CMB-free catalogue sources, but for the $V$ and $W$ bands the fits are reasonable. For NVSS sources, the fits are reasonable for all DAs. Therefore, when sources are selected from data that contain no significant CMB contribution, we find no compelling evidence that the beams for WMAP differ substantially from the Jupiter-based models. These conclusions directly contradict those of Sawangwit & Shanks (2010a) and Whitbourn et al. (2011), although they do not report the statistical significance of their result. The reason for the discrepancy, especially for the CMB-free and NVSS catalogues, is not clear. One possibility relates to the culling of close pairs, where we cut at $2^\circ$, while Whitbourn et al. (2011) cuts NVSS pairs at $1^\circ$ and WMAP CMB-free pairs not at all. At the same time, they follow the profile out further than we do, beyond $2^\circ$. Their method for background subtraction also differs from our offset fit.

The biases in the profiles erroneously boost the inferred source flux, especially for the smaller beams, and this affects the source spectral index. Using fluxes from the CMB-free catalogue, the spectral indices are significantly steeper and show spectral steepening at high frequency. This in turn can affect the point source subtraction for the power spectrum.

The issues we confronted in this work (pixel and binning effects, positional uncertainty, foreground subtraction, selection bias) are common to all microwave experiments that construct point source catalogues. Recent catalogues from ACT (Marriage et al. 2011) and SPT (Vieira et al. 2010) are much less susceptible to the CMB fluctuations because the power in the CMB falls so rapidly on arcminute angular scales. These catalogues take care to de-boost their measured flux densities for the biases due to noise and other sources. However, at these frequencies and angular scales, the background of dusty galaxies can play a role similar to that the CMB plays here. Subsequent analysis of these catalogues could run into similar biases, for example, when stacking 220 GHz maps on 150 GHz source positions or vice versa, or stacking ACT 220 GHz sources in the SPT data where the beam is smaller.

Planck data (Planck Collaboration et al. 2011a), with larger beams (5–30′′ or arcmins), is more susceptible to CMB fluctuations, so the same cautions apply as for WMAP, both for the recent Early Source Catalogue (Planck Collaboration et al. 2011b) and for the final band-merged catalogue.

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APPENDIX A: COVARIANCE FOR STACKED PROFILES

The stacking procedure is equivalent to the linear operation

\[ P_i = \sum_q S_{iq} M_q. \tag{A1} \]

where \( P_i \) is the stacked profile in angular bin \( i \), \( M_q \) is the \( q \)th pixel’s value in the map, and a stacking operator \( S_{iq} \) is

\[ S_{iq} = \begin{cases} 1 & \text{if pixel } q \text{ is in annulus } i \text{ about a source,} \\ 0 & \text{otherwise.} \end{cases} \tag{A2} \]

The catalogue point source positions are implicit in the \( S_{iq} \) matrix; thus, the contraction of the map with the stacking operator separately accumulates the contribution for each profile bin.

If the map \( M \) is zero mean, as for CMB and detector noise, then the covariance of profile bins is a simple function of the covariance of maps,

\[ \text{Cov}(P_i, P_j) = \frac{\sum_{qq'} S_{iq} S_{jq'} \langle M_q M_{q'} \rangle}{\left(\sum_q S_{iq}\right) \left(\sum_q S_{jq}\right)}. \tag{A3} \]

For a map with only white detector noise, \( n_q \), we have \( \langle n_q n_{q'} \rangle = \delta_{qq'} \sigma_n^2 / N_{\text{obs},q} \). Therefore,

\[ \text{Cov}(P_i, P_j) = \sigma_n^2 \frac{\sum_q S_{iq} S_{jq'} \delta_{qq'} N_{\text{obs},q}^{-1}}{\left(\sum_q S_{iq}\right) \left(\sum_q S_{jq}\right)} \]

\[ = \sigma_n^2 \frac{\sum_q S_{iq} S_{jq'} N_{\text{obs},q}^{-1}}{\left(\sum_q S_{iq}\right) \left(\sum_q S_{jq}\right)}. \tag{A4} \]

If additionally we require that the sources are well separated, that is, that the annuli of different sources never overlap (and the annuli around single sources naturally never overlap), then the bins are uncorrelated and the covariance simplifies:

\[ \text{Cov}(P_i, P_j) = \delta_{ij} \sigma_n^2 \frac{\sum_q S_{iq} S_{jq'} N_{\text{obs},q}^{-1}}{\left(\sum_q S_{iq}\right) \left(\sum_q S_{jq}\right)}. \tag{A5} \]

Since \( S_{iq} \) has value 0 or 1, then \( S_{iq} S_{jq'} = S_{iq} \), so

\[ C_{ij} \equiv \text{Cov}(P_i, P_j) = \delta_{ij} \sigma_n^2 \frac{\sum_q S_{iq} N_{\text{obs},q}^{-1}}{\left(\sum_q S_{iq}\right)^2}, \tag{A6} \]

where \( \sigma_n^2 N_{\text{obs},q}^{-1} \) is the noise variance in pixel \( q \), provided by the WMAP team for each DA’s map. Noting the similarity to the stacking operation (equation A1), we compute this quantity using a slight modification to the stacking code, but applied to the \( N_{\text{obs},q}^{-1} \) map with a different normalization, and verify with Monte Carlo noise simulations.

For the CMB, the term in angle braces in (A3) is simply the angular correlation function, obtained from the theory angular power spectrum \( C_l \) by a sum over Legendre polynomials:

\[ \langle M_q M_{q'} \rangle = \omega(\theta_{qq'}) = \frac{1}{4\pi} \sum_l (2l + 1) b_l^2 u_l^2 C_l(\cos \theta_{qq'}). \tag{A7} \]

The beam window function for the map is represented by \( b_l^2 \) and the pixel window function by \( u_l^2 \). In practice, however, it is computationally inconvenient to keep track of all possible pixel separations \( \theta_{qq'} \), and more convenient to compute this portion of the covariance using a Monte Carlo of CMB realizations.