Hund nodal line semimetals: The case of twisted magnetic phase in the double-exchange model

R. Matthias Geilhufe\textsuperscript{1}, Francisco Guinea\textsuperscript{2,3}, and Vladimir Juričić\textsuperscript{1}
\textsuperscript{1}Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 10691 Stockholm, Sweden
\textsuperscript{2}IMDEA Nanoscience, Faraday 9, 28015 Madrid, Spain
\textsuperscript{3}School of Physics and Astronomy. University of Manchester, Manchester M13 9PY
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We propose a class of topological metals, which we dub Hund nodal line semimetals, arising from the strong Coulomb interaction encoded in the Hund’s coupling between itinerant electrons and localized spins. We here consider a particular twisted spin configuration, which is realized in the double exchange model which describes the manganite oxides. The resulting effective tetragonal lattice of electrons with hoppings tied to the local spin features an antiunitary non-symmorphic symmetry that in turn, together with another non-symmorphic but unitary, glide mirror symmetry, protects crossings of a double pair of bands along a high-symmetry line on the Brillouin zone boundary. We also discuss symmetry breaking arising from various perturbations of the twisted phase. Our results may motivate further studies of other realizations of this state of matter, for instance in different spin backgrounds, properties of its drumhead surface states, as well as its stability to disorder and interactions among the itinerant electrons.

\textit{Introduction.} Weyl semimetals (WSMs), paradigmatic representatives of topological metals, have recently attracted considerable attention in both theoretical and experimental condensed matter communities due to their intriguing properties, such as unusual Fermi arc surface states and chiral anomaly [1, 2]. Due to a broken either time-reversal or inversion symmetry, they host chiral Weyl fermions at crossings of valence and conduction bands that occur at an even number of isolated points in the Brillouin zone (BZ). These crossings are (anti-)monopole defects of the Abelian Berry curvature, with the corresponding topological charge defining the topological invariant of a WSM. In contrast to WSM, Dirac semimetals host four-fold degenerate Dirac points, at which Kramers doublets of bands cross. Such a band crossing requires both, time-reversal and inversion symmetry [3, 4].

Band crossings can also occur on a manifold of higher dimensionality in the BZ, such as a line, giving rise to nodal line semimetals (NLSMs) [5]. NLSMs feature localized drumhead surface states forming flat electronic bands thereby providing a platform for a possible realization of interaction driven states, such as superconductors, magnetic phases, charge density waves, and others [6–8]. As their point-like analogs, NLSMs can be of a Dirac or a Weyl type, depending on the degeneracy of the bands crossing at the nodal line. These states are protected by the topology and various symmetries including a combination of inversion and time-reversal [9] and non-symmorphic lattice symmetries [10, 11], both with and without spin-orbit coupling; for a recent review, see Ref. [12]. Recently, NLSMs have been theoretically proposed and experimentally realized in, for instance, ZrSiS [13, 14], HfSiS [15, 16], and TTbSe\textsubscript{2} [17]; for a recent review on the materials aspects of NLSMs, see Ref. [18].

The emergence of topologically nontrivial states in gapless electronic states in conjunction with the space-group non-magnetic lattice and time-reversal symmetries is by now rather well understood [19–22]. In contrast, topological metallic states in magnetic lattices have just begun to be explored [23–27]. In that respect, the questions regarding possible topological phases emerging out of magnetic lattices with non-collinear spins and the specific mechanisms are still pertinent, and we address these in the present paper.

In the following, we will study magnetic phases in a broad class of materials, manganite oxides or Colossal Magneto Resistance systems (CMR’s) [28]. These materials exhibit a large number of magnetic phases[29], which can be tuned by the magnetic field. Most of them are metals, and are described by the so called double exchange model [30–32]. A typical composition is La\textsubscript{1–x}Ca\textsubscript{x}MnO\textsubscript{3}, although the Ca ions can be replaced by other ions with the same valence, such as Sr or Ba. As function of the doping, \textit{x}, the charge state of the Mn ion fluctuates between Mn\textsuperscript{3+} and Mn\textsuperscript{4+}. These atoms contain localized spins, which arise from core orbitals, and delocalized carriers, whose bands are also derived from Mn orbitals. The localized spins are due to the spin alignment of the three \textit{t}\textsubscript{2g} orbitals in the \textit{d} band of Mn (in a cubic lattice), and they give rise to a spin \( S = 3/2 \). Due to a strong intra-atomic Hund’s coupling, the carriers’ spins are aligned parallel to the core spin, so that the magnetic phase influences the hopping of the delocalized carriers between neighboring Mn ions. The rich variety of magnetic phases that these compounds show as function of doping of the conduction band and magnetic field was initially assigned to canting of the core spins. It was shown later that electronic phase separation is also likely [33, 34], which explains the observed hysteretic behavior of many CMR systems.

In this paper, we show that the properties of double exchange materials allow us to define a class of NLSMs, which we dub \textit{Hund nodal line semimetals}, which arise from a strong Hund’s coupling between itinerant and localized electrons that form a particular non-collinear...
spin configuration [41]. We here consider the so-called twisted magnetic phase [35], which is realized in the double exchange model as discussed in the context of manganites [36, 37]. Such a strong coupling leads to the emergence of an effective spin lattice, which, as we show, features a particular antiunitary non-symorphic symmetry, that, together with the unitary glide mirror symmetry, protects the Hund nodal line semimetallic state. To corroborate this, we consider various perturbations of the twisted phase, and explicitly show that the nodal line is stable as long as the resulting Hamiltonian on the BZ boundary respects the protecting symmetries. Finally, our results open up the path to study realizations of Hund nodal semimetals when itinerant electrons are strongly coupled to the spins forming other configurations, as well as the stability of this phase to impurities and interactions among the itinerant electrons.

The Model. As mentioned in the introduction, the double exchange model is characterized by a strong Hund’s coupling between the localized spin \( S_i \) at a site \( i \) and the itinerant electron spin at the same site \( s_i = \sum_{\alpha,\beta \neq e} c_{i,\alpha}^\dagger \epsilon_{\alpha\beta} G_{i,\alpha} \beta \), with \( c_{i,\alpha} \) as the annihilation operator for the electron with the spin projection \( \alpha = \uparrow, \downarrow \), and \( \epsilon \) are Pauli matrices, forces the electron to align with the localized spin. After projecting out the electrons’ spin component antiparallel to the localized spins, an effective tight-binding model is obtained for itinerant electrons but now with the spin parallel to \( S_i \).

\[
H = t \sum_{\langle ij \rangle} \left[ (\theta_i \phi_i) (\theta_j \phi_j) c_i^\dagger c_j + h.c. \right].
\]  

The hopping energies are given in terms of the overlap between the localized spins

\[
\langle \theta_i \phi_i \mid \theta_j \phi_j \rangle = \cos \frac{\theta_i}{2} \cos \frac{\theta_j}{2} + \sin \frac{\theta_i}{2} \sin \frac{\theta_j}{2} e^{-i(\phi_i - \phi_j)},
\]

and \( c_i \) is the annihilation operator for an electron (after the projection) at site \( i \). Here, the localized spin \( S \) is described as a classical three-dimensional unit vector, \( |S| = 1 \), with spherical angles \( (\theta, \phi) \). In addition, the localized spins are coupled through the superexchange interaction, which together with the effective hopping Hamiltonian in Eq. (1) defines the double exchange model. A competition between the kinetic energy of itinerant electrons and the super-exchange energy of the localized spins yields a rich phase diagram of this model at a finite temperature and chemical potential [35].

Twisted phase. We consider the itinerant electrons hopping in the background of the localized spins in the twisted phase given by [35]

\[
S_i = \cos \phi (-1)^{x+y} e_x + \sin \phi (-1)^{x} e_y,
\]

where \((x, y, z)\) are the Cartesian coordinates of the site \( i \) of a cubic lattice in units of the lattice spacing \( a \) and \( e_j \), \( j = x, y, z \), is the unit vector in the lattice direction \( j \). Since the spin is confined to the \( x-y \) plane, it can be parametrized by a polar angle \( \phi \). This spin configuration spans two sublattices in the \( x-y \) plane and also the two adjacent planes feature inequivalent spin configurations, as shown in Fig. 1. We therefore take the unit cell with four sites labeled as \( \alpha_k \), with \( \alpha = a, b \) as the sublattice index in the \( x-y \) plane and \( k = 1, 2 \) denotes the two adjacent inequivalent \( x-y \) planes. The lattice translations are generated by \( x_i = a \sqrt{2}(e_x \pm e_y) = \tilde{a}(e_x \pm e_y) \) and \( x = 2ae_z \). The Brillouin zone is now diamond shaped in the \( x-y \) plane: \(-\frac{\pi}{2} \leq k_x \leq \frac{\pi}{2}, -\frac{\pi}{2} \leq k_y \leq \frac{\pi}{2}, \) with \( k_{x \pm} = k_x \pm k_y \). The four distinct sites within the unit cell exhibit the four spin states \( | \pm \phi \rangle \) and \( | \pm \phi + \pi \rangle \), given by

\[
| \pm \phi \rangle = \frac{1}{\sqrt{2}} \left( e^{\pm i\phi} \right),
\]

\[
| \pm \phi + \pi \rangle = \frac{i}{\sqrt{2}} \left( e^{\pm i\phi} \right).
\]

In this basis the nearest-neighbor hoppings according to Eq. (2) read

\[
t_{a_{1b1}} = \langle \phi | \pi - \phi \rangle = \frac{i}{2} (1 - e^{-2i\phi}),
\]

\[
t_{a_{2b2}} = \langle -\phi | \pi + \phi \rangle = \frac{i}{2} (1 - e^{-2i\phi}),
\]

\[
t_{a_{1a2}} = \langle \phi | -\phi \rangle = \frac{1}{2} (1 + e^{-2i\phi}),
\]

\[
t_{b_{1b2}} = \langle \pi | \phi + \phi \rangle = \frac{1}{2} (1 + e^{2i\phi}),
\]

and the overall energy scale \( t \) is set to unity. Explicitly the form of the tight-binding model is

\[
H_{\text{tw}} = \sum_{\langle \langle \rangle \rangle} \left( t_{a_{1b1}} a_{1\downarrow}^\dagger b_{1\uparrow} + t_{a_{2b2}} a_{2\uparrow}^\dagger b_{2\downarrow} + h.c. \right)
\]  

\[
+ \sum_{\langle \langle \rangle \rangle} \left( t_{a_{1a2}} a_{1\uparrow}^\dagger a_{2\downarrow} + t_{b_{1b2}} a_{2\uparrow}^\dagger b_{1\downarrow} + h.c. \right),
\]

with \( \langle \langle \rangle \rangle \) and \( \langle \rangle \) denoting the sum over nearest-neighboring sites in the \( x-y \) plane and perpendicular to it, and \( \alpha_{\pm} \) is the annihilation operator for an electron at a site \( i \) belonging to a sublattice \( \alpha = a, b \) in the one of the two inequivalent \( x-y \) planes \( s = 1, 2 \). Then this Hamiltonian takes the following form in the momentum space

\[
H_{\text{tw}} = \sum_k \left\{ (\cos k_x + \cos k_y) \left[ i (1 - e^{-2i\phi}) a_{1\uparrow,k}^\dagger b_{1\downarrow,k}^\dagger + i (1 - e^{-2i\phi}) a_{2\downarrow,k}^\dagger b_{2\uparrow,k} + \cos k_x \times \right] \left[ (1 + e^{-2i\phi}) a_{1\uparrow,k}^\dagger a_{2\downarrow,k} + (1 + e^{2i\phi}) b_{1\downarrow,k}^\dagger b_{2\uparrow,k}^\dagger + h.c. \right] \right\},
\]  

with the lattice spacing of the original cubic lattice set to unity, \( a = 1 \). In the spinor basis defined as \( \psi_k = (a_{1\uparrow,k}, b_{1\downarrow,k}, a_{2\downarrow,k}, b_{2\uparrow,k})^\dagger \), the above Hamiltonian can
be conveniently rewritten as $H_{NN}^{\text{tw}} = \sum_k \psi_k^\dagger H(k) \psi_k$ with

$$H(k) = (\cos k_x + \cos k_y)(1 - \cos 2\phi)\sigma_0 \otimes \tau_2$$

$$- \sin 2\phi \sigma_3 \otimes \tau_1 + \cos k_z \left[ (1 + \cos 2\phi)\sigma_1 \otimes \tau_0 + \sin 2\phi \sigma_2 \otimes \tau_3 \right].$$

(12)

Here, the Pauli matrices $\sigma_i$ and $\tau_i$ act in the (1, 2) and $(a, b)$ spaces, while $\sigma_0$ and $\tau_0$ are the corresponding $2 \times 2$ unity matrices. This Hamiltonian yields twofold degenerate valence and conduction bands

$$E_{\pm,k} = \pm 2 \sqrt{(\cos k_x + \cos k_y)^2 \sin^2 \phi + \cos^2 k_z \cos^2 \phi}.$$  

(13)

The obtained band structure features diamond shaped nodal line given by $k_x \pm k_y = \pm\pi$ in each of the Brillouin zone boundary planes $k_z = \pm\pi/2$. We will next show that this nodal line is protected by a combination of an anti-unitary and a unitary lattice symmetry.

**Symmetry group of the system and protection of the line nodes.** The four sites in the unit cell are associated to the spin states $|\phi\rangle$, $|-\phi\rangle$, $|\phi + \pi\rangle$, and $|\pi - \phi\rangle$. Hence, starting for example from $|\phi\rangle$, we construct three symmetry elements that realize the mapping to the the remaining three sites (see Table I). First, consider the complex conjugation $K$ which acts as $\phi \rightarrow -\phi$. In the basis $\psi_k$ this operation corresponds to the permutations $a_1 \rightarrow a_2$ and $b_1 \rightarrow b_2$, which can be expressed as $(\sigma_2 \otimes \tau_0)K$. We therefore conclude that the combination of complex conjugation accompanied by a fractional shift in the unit cell along the $z$-axis is a symmetry of the Hamiltonian,

$$\{K|0,0,1/2\} : (x_+, x_-, z, \phi) \rightarrow (x_+, x_-, z+a, -\phi).$$

(14)

Second, consider the twofold rotation about the $z$-axis, $C_{2z} : (x_+, x_-, z) \rightarrow (-x_+, -x_-, z)$, which at the same time operates in spin space as $\exp(i\xi_3 \pi/2) = i\xi_3$. Therefore, under $C_{2z}$, the spin state transforms as $|\pm\phi\rangle \rightarrow |\pi \pm \phi\rangle$, which corresponds to the permutation $a1 \leftrightarrow b2, a2 \leftrightarrow b1$. According to Fig. 1, this implies that the combined operation of $C_{2z}$, inversion $I$ (swapping the planes $1 \leftrightarrow 2$) and an improper translation by half a lattice constant in $x_+$ and $x_-$ direction is a symmetry of the Hamiltonian,

$$\{IC_{2z}|1/2, -1/2, 0\} :$$

$$(x_+, x_-, z, \phi) \rightarrow (x_+ + \frac{a}{2}, x_- - \frac{a}{2}, -z, \phi).$$

(15)

This operation corresponds to a glide-mirror symmetry and can be expressed as $i\sigma_1 \otimes \tau_2$ ($i$ is included to satisfy double group algebra, i.e. $IC_{2z}^2 = E$). Note, that spin is a pseudovector and therefore not affected by an inversion. As the system is lattice periodic, the operations represent the generators of a factor group $\mathcal{G}/\mathcal{T}$, where $\mathcal{G}$ denotes the full space group and $\mathcal{T}$ the group of pure lattice translations which is an Abelian normal subgroup of $\mathcal{G}$. It follows that there has to be a third nontrivial symmetry element given by the composition of $\{K|0,0,1/2\}$ and $\{IC_{2z}|1/2, -1/2, 0\}$,

$$\{KIC_{2z}|1/2, -1/2, 1/2\} :$$

$$(x_+, x_-, z, \phi) \rightarrow (x_+ + \frac{\tilde{a}}{2}, x_- - \frac{\tilde{a}}{2}, -z + a, -\phi).$$

(16)

This operation corresponds to the map $|\phi\rangle \rightarrow |\pi - \phi\rangle$, re-
alized by the permutations $a_1 \leftrightarrow b_1$ and $a_2 \leftrightarrow b_2$, which is represented as $-i\sigma_3 \otimes \tau_2$ in the basis defined by $\psi_k$. Furthermore, since spin is involved, each of the elements comes with a respective double group partner, with a corresponding sign change. The factor group itself is isomorphic to the magnetic double group

$$G/T \simeq \mathcal{C}_S \oplus \{K|0,0,1/2\} \circ \mathcal{C}_S,$$

where $\oplus$ denotes the set sum and $\mathcal{C}_S = \{E, T, IC_{2z}, IC_{2z}^\dagger\}$. In the double exchange model for the twisted phase, time-reversal symmetry is broken. Instead the antiumilary symmetries $\{K|0,0,1/2\}$ and $\{IC_{2z}|\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\}$ are present. Furthermore, $\{K|0,0,\frac{1}{2}\}^2 = 1$ and $\{IC_{2z}|\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\}^2 = -1$, which indicates that $\{IC_{2z}|\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\}$ can be regarded as an effective time-reversal operation. However, as the spin degree of freedom is frozen into the lattice, this effective time-reversal operation acts in a pseudo-spin space spanned by the pseudo-spinor $\psi_k$. Additionally, $\{IC_{2z}|\frac{1}{2}, -\frac{1}{2}, 0\}$ is a unitary glide-mirror. A combination of effective time-reversal and the unitary glide-mirror protects the line nodes on the BZ boundary as follows.

We first notice that the product operation acts on the spatial coordinates $(x_+, x_-, z)$ as

$$\begin{align*}
\{KIC_{2z}|\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\} \ast \{IC_{2z}|\frac{1}{2}, -\frac{1}{2}, 0\} : \\
(x_+, x_-, z) \rightarrow (x_+ + \bar{a}, x_- - \bar{a}, z + a),
\end{align*}$$

while

$$\begin{align*}
\{IC_{2z}|\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\} \ast \{KIC_{2z}|\frac{1}{2}, -\frac{1}{2}, 0\} : \\
(x_+, x_-, z) \rightarrow (x_+ + \bar{a}, x_- - \bar{a}, z - a).
\end{align*}$$

Therefore,

$$KIC_{2z} \ast IC_{2z} = e^{2i\pi x_{\alpha}} KIC_{2z} \ast KIC_{2z},$$

implying that these two operators commute and anticommute, respectively at $k_z = 0$ and at the BZ boundary plane, located at $k_z = \pm \pi/2a$ plane. We use here short notation where we have removed the explicit reference to partial translations. Taking into account that the unitary operator IC $IC_{2z}^2 = -1$, its eigenvalues are $g_{\pm} = \pm i$. Antiumilary operator $KIC_{2z}^2 = -1$ and transforms the momentum as $(k_+, k_-, k_z) \rightarrow (-k_+, -k_-, k_z)$ and therefore each band is Kramers degenerate at $k_z = (k^0_+, k^0_-, k^0_z)$, with $k^0_z = \pm \pi/\bar{a}$ and $k^0_\perp = \pm \pi/\bar{a}$, which forms a surface in the BZ. In addition, on this surface we now take a line defined by $k_z = k^0_z$, with $k^0_z = 0$ ($k^0_z = \pm \pi/2a$) since the two operations (anti)commute at $k^0_z$, and denote the line by $\ell$, as shown in Fig. 2c. Consider a Bloch state on $\ell$, $|\Psi_k\rangle$, such that $IC_{2z}|\Psi_k\rangle = g_+|\Psi_k\rangle$. Then,

$$IC_{2z}|\Psi_k\rangle = e^{2i\ell_\alpha} KIC_{2z}|\Psi_k\rangle = e^{2i\ell_\alpha} g_-|KIC_{2z}|\Psi_k\rangle.$$  

Therefore, for the line $\ell$ in the middle of the BZ for which $k_{\ell}^0 = 0$, the Kramers partners of bands at the same momentum have opposite eigenvalues of the unitary operator and thus anticross. On the other hand, for the line $\ell$ that lies on the BZ boundary, and defines the edge of the surface BZ, the eigenvalues of the Kramers partners are the same and the bands can cross along this line. Most importantly, as we just showed, the crossing is protected by the combination of these two symmetries, i.e. it cannot be removed as long as these symmetries are operative.

Having established the underlying symmetries for the twisted phase, it is possible to construct a general lattice tight binding Hamiltonian. Such a general Hamiltonian can be written as follows,

$$H(k, \Phi) = \sum_{j=1}^{4} H^{(j)}(k, \Phi),$$

with

$$H^{(j)}(k, \Phi) = \sum_{\alpha = e, o} \sum_{\mu \nu} f_{\mu \nu}^{(j)}(k) F_{\mu \nu}^{\alpha, \alpha}(\Phi) \mathcal{S}_{\mu \nu}^{\alpha, \alpha}.$$  

Here, $F_{\mu \nu}^{\alpha, \alpha}(\Phi)$ are respectively even and odd functions of $\Phi$, $F_{\mu \nu}^{\alpha, \alpha}(\Phi) = \pm F_{\mu \nu}^{\alpha, \alpha}(-\Phi)$, and index $j$ labels the four combinations of the parities of the function $f^{(j)}$ under the change of sign of the $x - y$ (in-plane) and the $z$ components of the momentum, which appear due to the form of the symmetry transformations given by Eqs. (32) and (33) in Appendix. The form of the matrices appearing in the Hamiltonian is given in the Table III in Appendix.

**Symmetry breaking perturbations.** In the twisted phase the localized spins are aligned in plane as described by Eq. (3) and shown in Fig. 1. The fourfold degenerate line node at the edges of the $k_z = \pm \pi/2$ planes are protected by a combination of $IC_{2z}$ and $KIC_{2z}$. In the following, we consider alternations in the spin structure of the material, which can break these symmetries.
we introduce a slight tilt of the magnetic moments by an angle $\theta$ in $z$-direction with a staggering between the two layers in the unit cell. The resulting localized spins read

$$S_i = \cos \phi \sin \theta (-1)^{x+y} e_x + \sin \phi \sin \theta (-1)^x e_y + \cos \theta (-1)^y e_z. \quad (24)$$

Consequently, IC$_{2z}$ and K symmetries of the system are broken, while the KIC$_{2z}$ symmetry is kept intact. This spin configuration, using Eq. (1), yields a perturbation of the form,

$$V_{KIC2} = -(\cos k_x + \cos k_y)$$

$$\times \left[ \frac{\cot \theta}{\sin \theta} \sigma_3 \otimes \tau_2 + \cot^2 \theta \sigma_0 \otimes \tau_2 \right]. \quad (25)$$

Notice that $V_{KIC2}$ vanishes for $\theta = \pi/2$, which represents the twisted phase. For small derivations $\theta = \pi/2 + \delta$,

$$V_{KIC2} \approx -\delta (\cos k_x + \cos k_y) \sigma_3 \otimes \tau_2. \quad (26)$$

$V_{KIC2}$ vanishes on the BZ boundary plane $k_x \pm k_y = \pm \pi$, and consequently preserves the degeneracy of the line node, as shown in Fig. 3a. Otherwise, it splits the twofold degeneracy of the bands in the $k_x = 0$ and $k_z = \pi/2$ planes, since it breaks IC$_{2z}$ symmetry. Furthermore, the symmetry KIC$_{2z}$ allows for a perturbation of the form

$$V_{KIC2} \approx -\delta \left( \cos \frac{k_x}{2} + \cos \frac{k_y}{2} \right) \sigma_3 \otimes \tau_2. \quad (27)$$

which also breaks both IC$_{2z}$ and K symmetries but in contrast gives rise to a mass term which is non-vanishing along the edge of the BZ boundary at $k_z = \pi/2$. The nodal line is then gapped, but the Kramers degeneracy is preserved, leading to two pairs of nodal lines symmetrically split about the zero energy, as shown in Fig. 3b.

To further illustrate the symmetry protection of the line node, we consider slightly twisted spins in $z$-direction that stagger between $a$- and $b$-sites, with the form

$$S_i = \cos \phi \sin \theta (-1)^{x+y} e_x + \sin \phi \sin \theta (-1)^x e_y + \cos \theta (-1)^y e_z. \quad (28)$$

For this perturbation, the IC$_{2z}$ and KIC$_{2z}$ symmetries are broken and only the K symmetry is kept. The resulting perturbation reads

$$V_K = \cos \frac{k_z}{2} \left[ \frac{\cot \theta}{\sin \theta} \sigma_1 \otimes \tau_3 + \cot^2 \theta \sigma_1 \otimes \tau_0 \right]. \quad (29)$$

For very small deviations from the twisted phase we obtain

$$V_K \approx \delta \cos \frac{k_z}{2} \sigma_1 \otimes \tau_3. \quad (30)$$

Similarly to $V_{KIC2}$, $V_K$ leads to a fully gapped band structure (Fig. 3b).

**FIG. 3**: Lifting the four-fold degenerate line node via symmetry breaking perturbations. (a) A perturbation introduced by slightly tilting the spins in the $z$-direction, as given by Eq. (24), preserves Hund NLSM state. (b) The mass term respecting only KIC$_{2z}$ symmetry splits the four-fold line node, yielding two pairs of line nodes symmetrically split about $E = 0$. A similar effect is obtained when the spins are slightly tilted in the $z$ direction but staggered in the $x - y$ plane, as in Eq. (28). In this case, Kramers degeneracy is ensured by the K symmetry, see also Table I.

**Discussion and Conclusions.** We show that nodal line semimetals protected by non-symmorphic unitary and anti-unitary symmetries can emerge out of a strong Hund’s coupling between the localized spins forming a noncollinear magnetic phase and the itinerant electrons. This situation is likely to exist in systems described by the so called double exchange model, like the Colossal Magneto Resistance manganite oxides [28, 31]. Furthermore, breaking of these symmetries through, for instance, perturbing the background spin configuration, may give rise to yet other topological metals. In addition, the rich phase diagram of the double exchange materials allow for electronic phase separation [33, 34], and the coexistence of phases with different topologies. Non trivial interface states are expected to emerge. Our findings therefore open up a route for studying the emergence of exotic
states of matter in materials where localized spins and itinerant electrons are strongly coupled.

This should motivate further studies of the experimental imprints of this class of topological metals, through for instance, the drumhead surface states, in the tunneling experiments and transport. Finally, the role of interactions [38] and different types of disorder [39] in this context are still open problems.

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Appendix: Tight binding Hamiltonian constrained by the lattice symmetries. We now construct a general lattice tight binding Hamiltonian in the twisted phase (1) consistent with the symmetries discussed above. The form of the Hamiltonian in the momentum space is

$$H(k, \Phi) = \sum_{\mu\nu} f_{\mu\nu}(k) F_{\mu\nu}(\Phi) \sigma_\mu \otimes \tau_\nu,$$

with $\mu, \nu = 0, 1, 2, 3$.

First, consider non-symmorphic unitary operation $IC_{2z}$, which constraints the above Hamiltonian as

$$H(k_\perp, k_z, \Phi) = (i\sigma_1 \otimes \tau_2) H(k_\perp, -k_z, \Phi) i\sigma_1 \otimes \tau_2,$$

since under this transformation $(k_\perp, k_z) \rightarrow (k_\perp, -k_z)$, and $k_\perp \equiv (k_+, k_-)$. Second, the invariance of the Hamiltonian under non-symmorphic antiunitary operation $KIC_{2z}$ yields the following condition

$$H(k_\perp, k_z, \Phi) = -(-i\sigma_3 \otimes \tau_2) H(-k_\perp, k_z, \Phi)^\ast (-i\sigma_3 \otimes \tau_2),$$

since under this operation momentum transforms as $(k_\perp, k_z) \rightarrow (-k_\perp, k_z)$, and the additional minus sign arises from the fact that this operation involves an odd number of partial translations, as opposed to $IC_{2z}$ with two half-translations.

Moreover, Hamiltonian has to be invariant under the mirror symmetry through the $z = 0$ plane accompanied by $\Phi \rightarrow -\Phi$, which is represented by the matrix $\sigma_1 \otimes \tau_0$,

$$H(k_\perp, k_z, \Phi) = (\sigma_1 \otimes \tau_0) H(k_\perp, -k_z, -\Phi) \sigma_1 \otimes \tau_0,$$

This operation lifts the degeneracy between even and odd functions $F_{\mu\nu}(\Phi)$ of the angle $\Phi$ appearing in the general Hamiltonian given by Eq. (31).

These three symmetry operations combined, using transformation properties of the Pauli matrices in Table II, yield the following form of the general Hamiltonian

$$H(k, \Phi) = \sum_{j=1}^{4} H^{(j)}(k, \Phi),$$

with

$$H^{(j)}(k, \Phi) = \sum_{\alpha=e,o} \sum_{\mu\nu} f^{(j)}_{\mu\nu}(k) F_{\mu\nu}^{j,\alpha}(\Phi) \Sigma_{\mu\nu}^{j,\alpha}.$$

Here, $F_{\mu\nu}^{j,\alpha}(\Phi)$ are respectively even and odd functions of $\Phi$, $F_{\mu\nu}^{j,\alpha}(\Phi) = \pm F_{\mu\nu}^{j,\alpha}(-\Phi)$, and index $j$ labels the four combinations of the parities of the function $f^{(j)}$ under the change of sign of the $x-y$ (in-plane) and the $z$ components of the momentum, which appear due to the form of the symmetry transformations given by Eqs. (32) and (33). The form of the matrices appearing in the Hamiltonian is given in the Table III.

It is worthwhile noticing that the nearest-neighbor Hamiltonian in Eq. (12) realizes the case with $j = 1$, see also Table III. Namely, the corresponding functions of the momentum are even under both momentum exchange operations, and symmetry allowed matrices off-diagonal in only one of the subspaces $(1, 2$ or $a, b$) appear in the Hamiltonian in Eq. (12) since the lattice Hamiltonian involves only nearest-neighbor hopping.
TABLE II: Commutation (+) and anticommutation (-) of $\sigma_\mu \otimes \tau_\nu$ with respect to the antiunitary operation $\{ \text{KIC}_2 | \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \} = -i\sigma_3 \otimes \tau_2 \hat{K}$ (black) and the unitary operation $\{ \text{IC}_2 | \frac{1}{2}, -\frac{1}{2}, 0 \} = i\sigma_1 \otimes \tau_2$ (red).

| $\sigma_\mu$ | $\tau_\nu$ | 0 | 1 | 2 | 3 |
|------------|------------|---|---|---|---|
| 0          | +          | + | - | - | + |
| 1          | +          | + | + | + | - |
| 2          | +          | - | + | - | - |
| 3          | -          | + | - | - | + |

TABLE III: Elements of the symmetry constrained Hamiltonian in Eq. (22). First column: Parity of the function $f^{(\mu \nu)}(k_\perp, k_z)$ under $k_\perp \rightarrow -k_\perp$ (black) and $k_z \rightarrow -k_z$ (red), with $k_\perp \equiv (k_+, k_-)$. Second and third columns: Symmetry allowed matrices $\Sigma^{\mu \nu}$, with $\alpha = e, o$ multiplying the corresponding functions $f^{(\mu \nu)}(k)$ in the Hamiltonian in Eq. (22).

| $f^{(\mu \nu)}(k_\perp, k_z)$ | $\Sigma^{\mu \nu}$ | $\Sigma^{\mu \nu}$ |
|-----------------------------|-------------------|-------------------|
| 1 +                         | $\sigma_0 \otimes \tau_2, \sigma_1 \otimes \tau_0$ | $\sigma_2 \otimes \tau_1, \sigma_2 \otimes \tau_3, \sigma_3 \otimes \tau_3$ |
| 2 +                         | $\sigma_2 \otimes \tau_2, \sigma_3 \otimes \tau_2$ | $\sigma_0 \otimes \tau_1, \sigma_0 \otimes \tau_3$ |
| 3 -                         | $\sigma_0 \otimes \tau_0, \sigma_1 \otimes \tau_2$ | |
| 4 -                         | $\sigma_2 \otimes \tau_0, \sigma_3 \otimes \tau_0$ | $\sigma_1 \otimes \tau_1, \sigma_1 \otimes \tau_3$ |