Hybrid Natural Low Scale Inflation

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Abstract

We discuss the phenomenological implications of hybrid natural inflation models in which
the inflaton is a pseudo-Goldstone boson but inflation is terminated by a second scalar field.
A feature of the scheme is that the scale of breaking of the Goldstone symmetry can be lower
than the Planck scale and so gravitational corrections are under control. We show that, for
supersymmetric models, the scale of inflation can be chosen anywhere between the Lyth upper
bound and a value close to the electroweak breaking scale. Unlike previous models of low scale
inflation the observed density perturbations and spectral index are readily obtained by the choice
of the free parameters.

1 Introduction

Even though the concept of inflation was proposed almost thirty years ago \cite{1, 2, 3} only a handful
of models implementing it can be considered to have a compelling physical basis. Essentially the
problem arises due to the fact that the inflaton has to be a very light field with mass less than
the Hubble expansion parameter during inflation. Quantum or gravitational contributions to the
inflaton mass are typically too large unless there is an underlying symmetry protecting the inflaton
from such corrections. Although supergravity can protect against large radiative contributions the
gravitational corrections are typically of order the Hubble scale and too large (the “eta” problem);
only a Goldstone (shift) symmetry can avoid such contributions. In this case the inflaton is a
pseudo Goldstone boson, its small mass being due to the breaking of the continuous Goldstone
symmetry by an anomaly at the quantum level or by explicit breaking.

The original inflationary model based on a pseudo Goldstone inflaton was the Natural Inflation
model of Freese \textit{et al.} \cite{4, 5, 6}. It was based on an anomalous Abelian symmetry such that the
quantum anomaly generated mass for the would-be Goldstone mode. A potential problem for
the model is due to the fact that, in order to generate sufficient inflation, the scale of symmetry
breaking must be bigger than the Planck scale and in this case there may be large Quantum Gravity corrections of \( O(f/M_{\text{Planck}}) \). \(^1\) Models of Natural Inflation with explicit breaking of the Goldstone symmetry have received relatively little attention. Small terms explicitly breaking the continuous symmetry arise if the underlying symmetry is a discrete symmetry that gets promoted to a continuous one if only the low dimension (renormalisable) terms are kept in the Lagrangian. In this case the higher dimension terms explicitly breaking the continuous symmetry are suppressed by a large inverse mass scale and are naturally small. In these models to avoid the need for a super-Plankian scale, \( f \), the end of inflation must be triggered by a second scalar field \(^9\). In this case the number of e-folds of inflation depends on an additional parameter allowing for viable models with a sub-Plankian value for \( f \) \(^10\). It proves to be impossible to arrange for such hybrid natural inflation based on an Abelian global symmetry but models of hybrid natural inflation have been developed with an underlying non-Abelian discrete symmetry \(^11\) \(^12\). In this letter we explore the phenomenology of these models in detail, concentrating on the possibility that the scale of inflation may be very low. To achieve low scale inflation the hybrid field (which is not a pseudo Goldstone boson) must be light and this in turn requires that the underlying theory should be supersymmetric to protect the mass of the hybrid field from large radiative corrections.

We start with a model independent discussion of the structure and phenomenological implications of hybrid natural inflation before turning to a more detailed discussion of a particular supersymmetric model.

\section{Slow-roll parameters and observables}

As discussed in \(^12\), the terms of the (hybrid) inflaton potential relevant when density perturbations are being produced have a simple universal form corresponding to the slow roll of a single inflaton field \( \phi \):

\[ V \simeq V_0 \left( 1 + a \cos \left( \frac{\phi}{f} \right) \right), \]

where \( a \) is a constant. Natural Inflation corresponds to the case \( a = 1 \) while Hybrid Natural Inflation has \( a < 1 \). We first list the detailed expressions for the observables, the spectral index \( n_s \), the density perturbation at wave number \( k \), the tensor to scalar perturbations ratio, \( r \), and the “running” index \( n_r \). These are described in terms of the scale \( \phi_H \) at which the perturbations are produced, some 40–60 e-folds before the end of inflation, together with the scale of inflation given in terms of \( V_0 \) and the parameters \( a \) and \( f \). We will discuss later the model constraints on these parameters. Hybrid Natural Inflation is consistent with recent observational bounds coming from the five years data of the WMAP team \(^13\), \(^14\) and those from BAO \(^15\) and SN surveys \(^16\), \(^17\), \(^18\). A Table displaying these results can be found in \(^12\).

The observables are given in terms of the usual slow-roll parameters \(^19\).

\(^1\)In models with additional scalar fields it is possible to avoid such super Plankian scales\(^7\) \(^8\) while maintaining the Natural Inflation form of the inflaton potential with an effective super Plankian scale \( f \).
\[\epsilon \equiv \frac{1}{2} \left( \frac{V'}{V} \right)^2 \simeq \frac{1}{2} \left( \frac{M^2 \alpha^2 \sin^2 \left( \frac{\phi}{f} \right)}{f^2} \right),\]  
\[\eta \equiv -2M^2 \frac{V''}{V} \simeq -M^2 \frac{\alpha^2}{f^2} \cos \left( \frac{\phi}{f} \right),\]  
\[\xi \equiv -2M^4 \frac{V'''}{V} \simeq -M^2 \frac{\alpha^2}{f^2} \sin^2 \left( \frac{\phi}{f} \right) = -2 \left( \frac{M^2}{f} \right)^2 \epsilon,\]

where \(M\) is the reduced Planck scale, \(M = 2.44 \times 10^{18}\) GeV.

In terms of these the observables are given by

\[r = 16\epsilon,\]  
\[n_s = 1 + 2\eta - 6\epsilon,\]  
\[n_r = 16\epsilon \eta - 24\epsilon^2 - 2\xi,\]  
\[\delta^2_H(k) = \frac{1}{150\pi^2 \epsilon_H M^4}.\]

2.1 Natural Inflation.

For the case of Natural Inflation \(\phi\) is the only scalar field and the end of inflation is determined by the potential of Eq.(1) with \(a = 1\). Thus the inflaton value, \(\phi_e\), at the end of inflation is determined and, up to an uncertainty about the number of intermediate e-folds of inflation, hence so too is the inflaton value, \(\phi_H\), at the time the density perturbations relevant today leave the horizon. As a result Natural Inflation has only two free parameters, the scale of inflation \(\Delta\) and \(f\). The spectral index is always less than one and using the WMAP 3-year data gives the bound \(f > 3.5M\). For \(f\) close to this bound the density fluctuations constrain the scale of inflation to lie in the range \(\Delta = 10^{15}\text{GeV} - 10^{16}\text{GeV}\). With this \(r\) and \(n_r\) are determined

\[r = 4 \left( \delta_{ns} - \frac{M^2}{f^2} \right),\]  
\[n_r = -\frac{1}{2} \left( \frac{\delta_{ns} - M^4}{f^4} \right),\]

where \(\delta_{ns} \equiv 1 - n_s\). This implies \(r \leq 0.2\) and \(|n_r| < 10^{-3}\), where we have used the value \(n_s \geq 0.947\) \([13], [14]\).

2.2 Hybrid Natural Inflation.

For the case of Hybrid Natural Inflation \(\phi_e\) is determined by the hybrid sector of the theory so \(\phi_e\), or more conveniently \(\phi_H\), is essentially a free parameter. Thus Hybrid Natural Inflation has four free parameters namely \(\Delta\), \(f\), \(a\) and \(\phi_H\). Using the equation for the spectral index Eq.(6) at \(\phi_H\) we find

\[\delta_{ns} = \frac{2a}{f^2 (1 + a c_H)} + \frac{3a^2}{f^2 (1 + a c_H)^2},\]  
\[f^2 \delta_{ns} \equiv z = \frac{a(2c_H + a(3 - c_H)^2)}{(1 + a c_H)^2},\]

where \(c_H \equiv \cos \left( \frac{\phi_H}{f} \right)\). Then

\[\delta^2_H(k) = \frac{1}{150\pi^2 \epsilon_H M^4}.\]
showing that $f \propto \sqrt{a}$ for small $a$.

At $\phi_H$ the solutions to Eq. (12) are given by

$$c_{1H} = \frac{1 - z + \sqrt{1 + 3a^2 - 3(1 - a^2)z}}{a(1 + z)}, \quad a \geq \frac{1}{3}$$

and

$$c_{2H} = \frac{1 - z - \sqrt{1 + 3a^2 - 3(1 - a^2)z}}{a(1 + z)}, \quad a < 1.$$  

The first solution in the limit $a = 1$ corresponds to Natural Inflation. However to avoid the possibility of large gravitational corrections to the potential we will concentrate on the case $f < M$. Then it follows from Eq. (11) that, in order to obtain a small $\delta_{\text{ns}}$, the parameter $a$ must be small ($a < 0.026$ for $n_s \geq 0.947$) and so the relevant solution is the second one.

We are now able to discuss the phenomenological implications of Hybrid Natural Inflation. Eq. (12) implies

$$\delta_{\text{ns}} \simeq 2a \left(\frac{M}{f}\right)^2 c_H,$$  

where now and in what follows $c_H$ corresponds to $c_{2H}$ of Eq. (14). The hybrid sector triggers the end of inflation through the growth of a term proportional to $\sin (\phi/f)$ driving the mass squared of the hybrid field to be negative. To avoid introducing a fine tuning between terms in this sector it is necessary that $\sin (\phi/f)$ should be varying rapidly for $\phi \approx \phi_e$ corresponding to $\phi/f \ll \pi/2$, $c_H \simeq 1$. Thus, $c.f.$ Eq. (15), fitting the spectral index essentially fixes the ratio $a/f^2$. The remaining observables are given by

$$r = \frac{8a^2(1 - c_H^2)}{f^2 + (ac_H)^2} \simeq 4a \delta_{\text{ns}} (1 - c_H^2) \simeq 4a \delta_{\text{ns}} \left(\frac{\phi_H}{f}\right)^2 < 2\delta_{\text{ns}}^2 \left(\frac{M}{f}\right)^2,$$

$$n_r = \frac{2a^2(1 - 3a^2 - 2ac_H)(1 - c_H^2)}{f^4(1 + ac_H)^4} \simeq \frac{1}{2} \delta_{\text{ns}}^2 (1 - c_H^2) \simeq \frac{1}{2} \delta_{\text{ns}}^2 \left(\frac{\phi_H}{f}\right)^2 < \frac{1}{2} \delta_{\text{ns}}^2,$$

$$A_H^2 = 75\pi^2 \delta_{\text{H}}^2 = 8V H \frac{1}{M^4} \equiv \left(\frac{\Delta}{M}\right)^4 \frac{1}{r},$$  

where the observed magnitude of the density perturbations corresponds to $A_H \equiv \sqrt{75} \pi \delta_H$ and $\delta_H \simeq (1.91 \times 10^{-5})$. From this we see that the tensor to scalar ratio is small bounded by $r < 5.6 \times 10^{-3}$ but is typically much smaller since $c_H \simeq 1$ and $a$ may be much smaller than 0.026. The running index is bounded as $n_r < 1.4 \times 10^{-3}$ and the scale of inflation is also bounded, $\Delta^4 = \frac{1}{8} M^4 A_H^2 r$ so $\Delta < 9 \times 10^{15} \left(\frac{f}{M}\right)^{1/2}$ GeV. However much lower scales of inflation through the choice of $a (1 - c_H^2)$ can be obtained. For small $\phi_H/f$ we have

$$\Delta = M \sqrt{\frac{A_H \delta_{\text{ns}}}{2}} \left(\frac{\phi_H}{M}\right) \simeq 9 \times 10^{15} \sqrt{\frac{\phi_H}{M}} \text{GeV}. $$

From this it is clear that to achieve low scale inflation it is necessary that $\phi_H$ should be small. As mentioned above $\phi_H$ is determined by the field value $\phi_e$ at the end of inflation and this is in turn determined by the hybrid sector of the theory. In non-supersymmetric hybrid models $\Delta \geq m_\chi$ in order to have zero cosmological constant after inflation and so low scale inflation requires low hybrid mass ($m_\chi$ is the mass of the hybrid field $\chi$ defined below). This in turn points to a supersymmetric model of inflation since the hybrid field, which is not a pseudo-Goldstone boson,
needs supersymmetric protection against large radiative corrections to its mass (the hierarchy problem again). Thus we turn to a specific supersymmetric example of a hybrid sector in order to illustrate the expectation for the range of $\phi_e$ and hence of the scale of inflation.

### 3 A supersymmetric example

The field content of the supersymmetric model \cite{12} consists of chiral supermultiplets of the $N = 1$ supersymmetry which transform non-trivially under a $D_4$ non-Abelian discrete symmetry. The supermultiplets consist of a $D_4$ doublet $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ and three singlet representations $\chi_{1,2}$ and $A$ transforming as $1^+, 1^-$ and $1^{++}$ respectively where the superscripts refer to the transformation properties under the $Z_2$ semidirect product factors of $D_4$ ($D_4 = Z_2 \ltimes Z'_2$). The interactions are determined by the superpotential given by

$$P = A \left( \Delta^2 - \lambda_3 \frac{1}{M^2} \varphi_1 \varphi_2 \chi_1 \chi_2 \right),$$

(20)

where $\Delta$ is a constant with units of mass. These are the leading order terms consistent with an additional $R-$symmetry under which the fields $A, \varphi, \chi$ have charges $2, -2$ and $2$ respectively.

The field $\varphi$ has the form

$$\varphi = e^{i\phi \sigma} \begin{pmatrix} 0 \\ \rho + v \end{pmatrix} = \frac{\rho + v}{\phi} \begin{pmatrix} (\phi_2 + i\phi_1) \sin \left( \frac{\phi}{\tau} \right) \\ \phi \cos \left( \frac{\phi}{\tau} \right) - i\phi_3 \sin \left( \frac{\phi}{\tau} \right) \end{pmatrix},$$

(21)

where $\sigma_i$ are the Pauli spin matrices, $\phi_i$ are the pseudo Goldstone fields. The field $\rho$ acquires a mass of $O(m_\varphi)$ and plays no role in the inflationary era.

Inflation occurs for small $\frac{\phi_1}{v}$. In this region, for positive $\lambda_2$, the field $\phi_3$ has a positive mass squared while $\phi_{1,2}$ have negative mass squared. Thus $\phi_3$ does not develop a vev. The full potential for the fields acquiring vevs then has the form

$$V(\phi, \chi) \approx \Delta'^4 + 64\lambda_2 \frac{m^2_{\varphi} f^4}{M^2} \cos \left( \frac{\phi}{f} \right) + \sum_{i=1}^2 m^2_{\chi_i} |\chi_i|^2 - 8\lambda_3 e^{i\alpha} \Delta^2 \frac{f^2}{M^2} \sin \left( \frac{\phi}{2f} \right) \chi_1 \chi_2 + h.c. + O \left( \frac{m^2_{\chi_i}}{M^2} \chi_i^4 \right),$$

(22)

where $\phi^2 = \phi_1^2 + \phi_2^2$, $\Delta' = \Delta + 16m^2_{\varphi} f^2 + 192\lambda_2 \frac{m^2_{\varphi} f^4}{M^2}$ and $\alpha = \tan^{-1} \left( \frac{\phi_1}{\phi_2} \right)$. The second term is a $D-$term which only arise when supersymmetry is broken and hence is proportional to the SUSY breaking scale, $m^2$. It can come from radiative corrections with a messenger field, $M$, of mass $M_M$ in the loop.

A particularly simple case to analyse and one that allows for the lowest scale of inflation is the case that the hybrid fields have only soft supersymmetry breaking mass, $m_{\chi_i} = O(\Delta^2/M)$. The condition for the end of inflation is

$$8\lambda_3 \Delta^2 \frac{f^2}{M^2} \sin \left( \frac{\phi_e}{2f} \right) \approx m^2_{\chi_i},$$

(23)

where $\lambda_3$ is a coefficient expected to be of $O(1)$ and $m_{\chi_i}$ is a soft supersymmetry breaking mass of $O(\Delta^2/M)$. Thus in this case, taking $\lambda_3 = O(1)$, $\phi_e$ is determined by two of the remaining inflationary
parameters
\[ \phi_e = O \left( \frac{\Delta^2}{4f} \right). \] (24)

Since \( \phi_H \approx \phi_e \) one sees that the dependence on \( \Delta \) cancels in Eq. (19) and the density fluctuations are determined by \( f \). To get the observed density fluctuations
\[ 4 \frac{f}{M} \approx 10^{-5}. \] (25)

Using this Eq. (15) gives
\[ a \approx \frac{1}{2} \delta_{ns} \left( \frac{f}{M} \right)^2 \approx 10^{-13}. \] (26)

For the supersymmetric model we have from Eq. (22)
\[ a = 64 \lambda_2 \frac{m^2}{M_f} \frac{f^4}{\Delta^4} = 64 \lambda_2 \left( \frac{f}{M} \right)^2 \left( \frac{f}{M_M} \right)^2, \] (27)

so Eq. (26) can be satisfied by the choice of the messenger mass scale, \( f/M_M \approx 10^{-2}/\sqrt{\lambda_2} \).

From Eqs. (16), (17) and Eq. (25) we get
\[ r < 2 \delta_{ns}^2 \left( \frac{f}{M} \right)^2 \approx 6 \times 10^{-14}, \] (28)
\[ n_r < \frac{1}{2} \delta_{ns}^2 \approx 1.4 \times 10^{-3}. \] (29)

Since \( \Delta^4 = \frac{1}{8} M^4 A_H^2 \tau \) we obtain the bound for \( \Delta < 1.6 \times 10^{13} \text{GeV} \).

To summarise, for the case the hybrid fields have only soft supersymmetry breaking masses, fitting the observed values of the spectral index and magnitude of the density perturbations give two constraints on the three hybrid inflation parameters. The remaining parameter, that can be taken to be the scale of inflation, \( \Delta \), is undetermined. Its upper bound is \( 10^{13} \text{GeV} \). It has a lower bound because gravitational corrections give an irreducible mass of \( O(\Delta^2/M) \) to the \( \chi \) field where \( \Delta \) is the zero temperature supersymmetry breaking scale after inflation has ended. Keeping only this contribution to \( m^2_{\chi i} \) and solving Eq. (23) gives \( \phi_e = O \left( \Delta^4/(4f \Delta^2) \right) \) and using this in Eq. (19) one finds \( \Delta^4 \approx 10^{-5} \left( \frac{M}{M_f} \right) \Delta^4 \). To avoid large gravitational corrections we require \( f/M < 1 \) which implies \( \Delta > 4 \times 10^{-2} \Delta \). In gravity mediated schemes the need to split the superpartners from the Standard Model states requires \( \Delta = O(10^3 \text{GeV}) \) but in gauge mediated schemes \( \Delta \) can be as low as \( 10^4 \text{GeV} \) implying \( \Delta > 400 \text{GeV} \), close to the electroweak breaking scale.

4 Initial conditions for inflation

We have argued that hybrid natural inflation with no fine tuning of parameters requires a small value for \( \frac{\phi_H}{f} \). Moreover one may see from Eq. (21) that this ratio becomes very small indeed for the scale of inflation near the lower bound just discussed. This immediately raises the question of initial conditions and how a very small initial value for \( \phi \) can be achieved. To answer this question consider the thermal effects present before inflation. In general \( \phi \) will have coupling to other fields with a coupling that is \( D_4 \) symmetric but not \( SU(2) \) symmetric. For example if there is a second doublet field \( \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \) there is a \( D_4 \) invariant superpotential coupling \( (\phi_1 \psi_1 - \phi_2 \psi_2) \chi^+_i \chi^-_i \).
that is not $SU(2)$ invariant. From Eq. (21), one may see that this includes a term of the form $\psi_1 x_1^+ \sin(\phi/f)$. Such terms can maintain the fields in thermal equilibrium and generate a term of the form $\sin(\phi/f)^2 T^2$ in the effective scalar potential at temperature $T$. This in turn drives the initial value of $\phi$ to be small. To quantify this we suppose that at temperature $T_1$ the phase transition associated with the field $\phi$ occurs. As this occurs the vev of the angular variable $\phi_{T_1}/f$ is undetermined so we expect $\phi_{T_1}/f = O(1)$. The start of inflation occurs at a temperature $T_2$ of $O(\Delta)$ and subsequently the fields fall out of thermal equilibrium. In the intervening period the vev of the field is reduced $\phi_{T_2} = \phi_{T_1} e^{-m_{\phi}/H} \approx \phi_{T_1} e^{-M/\Delta}$ where the Hubble parameter $H = T^2/M$ and the final estimate follows using the thermal mass. One sees that such thermal effects readily set the necessary initial conditions for low scale natural hybrid inflation. One may worry that thermal fluctuations $\delta\phi = O(T)$ negates this conclusion, destabilising the hybrid field through the fourth term on the right hand side of Eq. (22). However this is not the case as it is the mean field values that are relevant to the phase transitions and the thermal fluctuations do not contribute to the mean value of the $\sin(\phi/f)$ term, being odd in $\phi$.

5 Summary and conclusions

In summary, we have explored the phenomenological aspects of Hybrid Natural Inflation in which the inflaton is a pseudo Goldstone boson and hence does not suffer from the “eta” problem. The end of inflation is driven by a hybrid sector and as a result the scale of symmetry breaking associated with the pseudo Goldstone boson inflaton can be smaller than the Planck scale. In contrast with Natural Inflation there is no conflict of such a sub-Plankian value with the requirement of generating enough inflation or with bounds imposed by the spectral index. For the supersymmetric model discussed here the inflaton potential relevant at the time density perturbations are produced is governed by three parameters. Two of the parameters are determined by fitting the observed spectral index and the magnitude of density perturbations. The scale of inflation is set by the remaining parameter and can be anywhere in the range $400\text{GeV} < \Delta < 10^{16}\text{GeV}$ and thermal effects can set the initial conditions necessary for low scale inflation provided there is a period of thermal equilibrium after the initial phase transition associated with the Goldstone boson. Thus Hybrid Natural Inflation provides another example of inflationary models capable of generating very low scales of inflation. However in contrast with previous examples, there is no difficulty in fitting the observed spectral index for even the lowest scale of inflation.

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