The spin of accreting stars: dependence on magnetic coupling to the disc

Sean Matt*† and Ralph E. Pudritz†

Physics & Astronomy Department, McMaster University, Hamilton ON, Canada L8S 4M1

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ABSTRACT

We formulate a general, steady-state model for the torque on a magnetized star from a surrounding accretion disc. For the first time, we include the opening of dipolar magnetic-field lines due to the differential rotation between the star and disc, so the magnetic topology then depends on the strength of the magnetic coupling to the disc. This coupling is determined by the effective slip rate of magnetic-field lines that penetrate the diffusive disc. Stronger coupling (i.e. lower slip rate) leads to a more open topology and thus to a weaker magnetic torque on the star from the disc. In the expected strong coupling regime, we find that the spin-down torque on the star is more than an order of magnitude smaller than calculated by previous models. We also use our general approach to examine the equilibrium (‘disc-locked’) state, in which the net torque on the star is zero. In this state, we show that the stellar spin rate is roughly an order of magnitude faster than predicted by previous models. This challenges the idea that slowly-rotating, accreting protostars are disc locked. Furthermore, when the field is sufficiently open (e.g. for mass accretion rates \( \geq 5 \times 10^{-9} \) \( \text{M}_\odot \text{yr}^{-1} \), for typical accreting protostars), the star will receive no magnetic spin-down torque from the disc at all. We therefore conclude that protostars must experience a spin-down torque from a source that has not yet been considered in the star–disc torque models – possibly from a stellar wind along the open field lines.

Key words: accretion, accretion discs – MHD – stars: formation – stars: magnetic fields – stars: pre-main-sequence – stars: rotation.

1 INTRODUCTION

Accretion discs are responsible for some of the most energetic and spectacular phenomena in many classes of astrophysical objects, including protostars, white dwarfs (cataclysmic variables and intermediate polars), neutron stars (binary X-ray pulsars), and black holes (both stellar mass X-ray transients and supermassive active galactic nuclei). Gravitational potential energy liberated by the accretion process gives rise to exceptional luminosity excesses and can drive powerful jets and outflows. Accretion on to the central object can occur only as quickly as angular momentum can be transported away from the system. Furthermore, the accretion of disc material, which has high specific angular momentum, spins up the central object, if the object rotates at less than the break-up rate. It is therefore surprising that the central objects (hereinafter ‘stars’) are often observed to spin far below their break-up rates, in spite of long-lived accretion. Why does this happen?

There is good evidence that accretion on to magnetized stars occurs along closed magnetospheric field lines that connect the star to the inner edge of the disc. Theoretical models of this sort have been successful in explaining numerous observed features in accreting protostars (e.g. Hayashi, Shibata & Matsumoto 1996; Goodson, Böhm & Winglee 1999; Muzerolle, Calvet & Hartmann 2001), intermediate polars (e.g. Patterson 1994), and X-ray pulsars (e.g. Joss & Rappaport 1984; Aly & Kuijpers 1990; Kato et al. 2001; Kato, Hayashi & Matsumoto 2004). In some cases, there is even direct evidence that the stars are magnetized, namely for accreting protostars (Johns-Krull et al. 1999), intermediate polars (Piirona, Hakala & Coyne 1993), and X-ray pulsars (Makishima et al. 1999).

Magnetic fields can also be effective at transferring angular momentum away from the star, possibly explaining the observed rotation rates. Torques on the star that are exerted by magnetic-field lines anchored to the star and that are also connected to the disc have been calculated by several authors (e.g. Ghosh & Lamb 1979; Cameron & Campbell 1993; Lovelace, Romanova & Bisnovatyi-Kogan 1995; Wang 1995; Yi 1995; Armitage & Clarke 1996, hereinafter AC96; Rappaport, Fregeau & Spruit 2004). Under certain circumstances, this torque can counteract the angular momentum deposited by accretion, leading to a net spin-down of the star (possibly explaining spin-down episodes observed in X-ray pulsars; Ghosh & Lamb 1978; Lovelace et al. 1995) or giving rise to an equilibrium state, in which the net torque on the star is zero, possibly explaining the slow spin of some accreting protostars (Königl 1991, hereinafter K91).
In this equilibrium state, the spin rate of the central object depends on the accretion rate in the disc, and so a system is then considered to be 'disc locked'. Because these models for the magnetic star–disc interaction show that accreting stars can spin more slowly than the break-up rate, there is a general perception that the presence of an accretion disc in any system leads to slow rotation rates. This idea of disc locking has been applied to a variety of problems. As an example, in systems where the moment of inertia of the star is changing (e.g. during contraction), some authors have assumed that disc-locking keeps the star at a constant spin rate (e.g. as applied to protostars by Bouvier, Forestini & Allain 1997; Sills, Pinsonneault & Terndrup 2000; Barnes, Sofia & Pinsonneault 2001; Tinker, Pinsonneault & Terndrup 2002; Rebull, Wolff & Strom 2004; and suggested for stellar collision products by Leonard & Livio 1995; Sills et al. 2001; De et al. 2004).

There is a nagging problem with this physical picture, however, because the magnetic torque calculations discussed above (with the exception of Lovelace et al. 1995) assume that the stellar magnetic field remains largely closed and that field lines connect to a large portion of the disc. This assumption is questionable because closed magnetic structures tend to open when enough energy is added to them, and these systems possess a natural source of energy in the form of gravitational potential energy that is released during disc accretion. This energy release can drive outflows and twist field lines, thereby adding energy to the magnetic field. Thus, the general surplus of energy in accreting systems suggests that associated magnetic fields should be dominated by open, rather than closed, topologies. How are low spin rates achieved in this case?

In this paper, we generalize the star–disc interaction model to include the effect of varying field topology (i.e. connectedness). We consider the mechanical energy that is added to the field via differential rotation between the star and disc as the only mechanism responsible for opening the field (though our formulation is easily adaptable for other mechanisms). The time-dependent behaviour of a dipole stellar field attached to a rotating, conducting disc has been studied, using an analytic approach, by several authors (e.g. Lynden-Bell & Boily 1994; Agapitou & Papaloizou 2000; Uzdensky, Königl & Litwin 2002a, hereinafter UKL). They have shown that, as the differential twist angle between the star and disc (\(\Delta \Phi\)) monotonically increases, the torque exerted by field lines first reaches a maximum value, then decreases. This occurs because azimuthal twisting of the dipole field lines generates an azimuthal component to the field, and the magnetic pressure associated with this component acts to inflate the field, which then balloons outward at an angle of \(\sim 60^\circ\) from the rotation axis, causally disconnecting the star and disc (see also Aly 1985; Aly & Kuijpers 1990; Newman, Newman & Lovelace 1992; Lovelace et al. 1995; Bardou & Heyvaerts 1996). Typically, this inflation or opening of the field occurs when a critical differential rotation angle of \(\Delta \Phi \approx \pi\) has been reached, and the amount of flux that opens depends on the strength of the magnetic coupling of the field to the disc (UKL). This analytic work on the field opening has been corroborated by time-dependent, numerical magnetohydrodynamic simulations of the stellar dipole–disc interaction (Hayashi et al. 1996; Goodson, Winglee & Böhm 1997; Miller & Stone 1997; Goodson et al. 1999; Kato et al. 2001; Matt et al. 2002; Romanova et al. 2002; Küker, Henning & Rüdiger 2003; Kato et al. 2004).

Our primary goal in this paper is to determine the effect of the topology of the magnetic field on the torques in the steady-state, star–disc interaction model. In a previous paper (Matt & Pudritz 2004), we gave a brief outline of this theory and showed that a more open (i.e. less connected) field topology results in a spin-down torque on the star that is less than for the closed-field assumption. Consequently, the equilibrium state (with a net zero torque) features a faster spin than predicted by previous models, which calls into question the general belief that accretion discs necessarily lead to slow rotation. The present paper contains a more detailed presentation of the theory and our assumptions, and we consider all possible spin states of the system (not just the equilibrium state). We also extend our analysis to show that there are at least three different modes in which a magnetic star–disc system can operate. Our analysis is applicable to all classes of magnetized objects that accrete from Keplerian discs. However, because an abundance of observational data exists for accreting protostars, in particular for classical T Tauri stars (CTTSs), we adopt a set of fiducial parameters that are appropriate for these systems and discuss various aspects of the model in this context.

Section 2 contains a formulation of the general model. The special case of a disc-locked system is the topic of Section 3. The final Section (Section 4) contains a summary of our results and includes a list of problems with using the disc-locking scenario to explain CTTS spins, plus a discussion of three possible configurations of the general system.

2 STAR–DISC INTERACTION MODEL

Magnetic, star–disc interaction models in the literature differ in their various assumptions, adopted parameter values, and in the introduction of ‘fudge factors,’ but they are quite similar on the whole (for a review, see Uzdensky 2004). We formulate a general model that builds upon this previous work (mostly following AC96), by including the effect of varying magnetic-field topology, via the introduction of the physical parameters \(\beta\) and \(\gamma\) (defined below).

According to the usual model assumptions, a rotating star is surrounded by a thin, Keplerian accretion disc. The angular momentum vector of the disc is aligned with that of the star, which rotates as a solid body and at a rate that is some fraction of break-up speed, defined by

\[
f \equiv \frac{\omega_s}{\Omega_s} \sqrt{\frac{R_*^3}{G M_*}} \tag{1}
\]

where \(\omega_s\), \(R_*\), and \(M_*\) are the angular rotation rate, radius and mass of the star, respectively. Note that \(f\) is always within the range zero to one.\(^2\) The disc rotates at an angular rate different from that of the star at all radii, except at the singular corotation radius given by

\[
R_{\text{co}} = f^{-2/3} R_* \tag{2}
\]

For \(r < R_{\text{co}}\), the disc rotates faster than the star, while for \(r > R_{\text{co}}\), the angular rotation rate of the star is greater than that of the disc.

\(^1\) Note that the X-wind model of Shu et al. (1994, and subsequent works) is unique in star-disc interaction theory in the literature, as it assumes that a system will always accrete very near its disc-locked state. The magnetic-field geometry employed by the X-wind model is also unique and was designed, in part, to avoid the problem of field opening due to differential twisting, as considered in this paper. Therefore, much of our discussion does not apply to the X-wind.

\(^2\) Disc accretion solutions do exist for \(f > 1\) (Paczynski 1991; Popham & Narayan 1991), in which the star is actually spun down by accretion toward \(f = 1\), even without any magnetic torques. However, we only consider cases with \(f \lesssim 1\) in this paper, because this characterizes the spin of observed protostellar systems.
The disc is assumed to be in a steady-state wherein the mass accretion rate $\dot{M}_d$ is constant in time and at all radii. In a real disc from which winds are launched, $\dot{M}_d$ may have a weak radial dependence, but we assume this has a negligible effect on the model. A rotation-axis-aligned dipole magnetic field, anchored into the stellar surface, also connects to the disc. The field is strong enough to truncate the disc at some inner location $R_t$, from where disc material is subsequently channelled along magnetic-field lines as it accretes onto the star. In general, the disc may have its own magnetic field (either generated in a disc dynamo or carried in by the disc from larger scales). We do not specifically include this field in the model, though it may be responsible for angular momentum transport in the disc (providing $\dot{M}_d$) and may also aid in the connection of the stellar field to the disc. Within the disc, the kinetic energy of the gas is much greater than the magnetic energy of the stellar field, but the region above the star and disc (the corona) is filled with low-density material, and so the corona is magnetically dominated.

In this configuration, the magnetic field connects the star and disc by conveying torques between the two. Torques are conveyed on an Alfvén wave crossing time, which is much shorter than the Keplerian orbital time. Everywhere that the magnetic field connects the star to the disc, except at $R_{co}$, the magnetic field is twisted azimuthally by differential rotation between the two. Inside $R_{co}$ the field is twisted such that field lines ‘lead’ the stellar rotation, so torques from field lines threading the region $r < R_{co}$ act to spin up the star (and spin down the disc). Conversely, torques from field lines threading $r > R_{co}$ act to spin the star down (and spin the disc up). The accretion of disc matter on to the star also deposits angular momentum on to the star.

In order for disc material to accrete, $R_t$ must be less than $R_{co}$ so that accreting material loses angular momentum to the star as it falls inward. In order for the star with $f \leq 1$ to feel any spin-down torques from the disc, the stellar field must connect to the disc beyond $R_{co}$. Under this condition, the field that connects outside $R_{co}$ transfers angular momentum from the star to the disc. To maintain a steady accretion rate, the disc must then transport this excess angular momentum outward, resulting in an altered disc structure (Sunyaev & Shakura 1977a,b; Spruit & Taam 1993; Rappaport et al. 2004).

We will define $R_{out}$ as the outermost radial extent of the magnetic connection, and the usual assumption is that $R_{out} \gg R_{co}$. Fig. 1 illustrates the basic picture, and shows the locations of $R_t$, $R_{co}$ and $R_{out}$ for a possible configuration of the star–disc interaction model.

The assumption of a dipole field refers to the poloidal component of the field, $B_p = (B_\phi^2 + B_z^2)^{1/2}$, where $B_\phi$ and $B_z$ are the cylindrical $r$- and $z$-components of the magnetic field, and the closed field only exists in the region interior to $R_{out}$. The dipole field is used for simplicity and because it has the weakest radial dependence ($B_r \propto r^{-3}$ along the equator) of any natural magnetic multipole. In reality, the twisting of dipole field lines alters the poloidal field, but this perturbation should be slight in the region where the magnetic field remains closed (as justified by the work cited in Section 1, e.g. UKL, and see also Livio & Pringle 1992).

For discussion throughout this paper, it is often instructive to use physical units, especially for comparison with observations. For this purpose, we adopt a set of observationally determined ‘fiducial parameters’ that are appropriate for CTTSs (e.g. see Johns-Krull & Gafford 2002):

$$\dot{M}_d = 5 \times 10^{-8} M_\odot \text{yr}^{-1},$$

$$M_* = 0.5 M_\odot,$$

$$R_* = 2 R_\odot,$$

$$B_\phi = 2 \times 10^8 \text{G},$$

where $B_\phi$ is the stellar magnetic-field strength at the equator. However, our formulation of the problem is applicable to any magnetic star–disc system.

2.1 Twisting and slipping of magnetic-field lines

The torque exerted by magnetic-field lines threading an annulus of a disc of radial width $dr$ is given by (e.g. AC96)

$$d\tau = -\gamma \frac{\mu^2}{r^4} dr,$$

where

$$\gamma = B_\phi/B_z.$$

Here, $\mu$ is the dipole moment and $B_\phi$ is the azimuthal component of the magnetic field. The radial component, $B_z$, is assumed to be negligible within the disc, the torque has been vertically integrated through the disc, and $B_\phi$ refers to the value at the disc surface. Here, and throughout this paper, we choose the sign of the torque to be relative to the star such that a positive torque spins the star up, and consequently spins the disc down (and vice versa for a negative torque).

The differential magnetic torque of equation (3) depends not only on $\mu$, but also $\gamma$, which is the ‘twist’, or pitch angle, of the field. The total (integrated) magnetic torque will also depend on the size and radial location of the magnetically connected region in the disc. While $\mu$ is a constant parameter of the system, the radial dependence of $\gamma$ depends on the physical coupling of the magnetic field to the disc.

In general, the coupling is not perfect. Magnetic forces act to resist the twisting of the field, and so the field will ‘slip’ backward through the disc at some rate $v_d$ proportional to $\gamma$. The exact slipping rate depends upon which physical mechanism is at work. In the literature, there are generally three mechanisms discussed (e.g. see Wang 1995): (1) magnetic reconnection in the disc, (2) reconnection outside the disc, and (3) turbulent diffusion of the magnetic field through the disc. We adopt the latter mechanism, but we further discuss the other two, below.

The magnetic field slips azimuthally at a speed (e.g. Lovelace et al. 1995; UKL)

$$v_d = \frac{n_i}{\gamma} = \beta v_b \gamma,$$

3 These fiducial parameters are slightly different from those used for figs 2 and 3 of Matt & Pudritz (2004), who considered the specific case of the CTTS BP Tau.

4 This should not be confused with the ‘twist angle’ of the footpoints of the field, $\Delta \Phi$, as discussed by UKL, though $\Delta \Phi$ and $\gamma$ are intimately related.

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Figure 1. Magnetic star–disc interaction. The stellar field connects to a region of the disc, from $R_t$ to $R_{out}$, reaching beyond $R_{co}$ in this case. The stellar field dominates the accretion flow on to the star (arrow).
where \( \eta \) is the turbulent magnetic diffusivity, \( h \) is the local disc scale height, \( v_d \) is the Keplerian orbital speed, and we have introduced the dimensionless ‘diffusion parameter’ \( \beta \equiv \eta h (h v_d)^{-1} \). The variable \( \beta \) simply parameterizes the coupling of the stellar magnetic field to the disc such that \( \beta \gg 1 \) corresponds to weak coupling, and \( \beta \ll 1 \) to strong coupling.

Generally, \( \beta \) is a scale factor that compares \( v_d \) with \( v_c \), and we have chosen this generic formulation so that the system behaviour is largely independent of any particular disc model – as long as the disc obeys Keplerian rotation and provides a steady accretion rate. However, if we temporarily consider a standard \( \alpha \)-disc (Shakura & Sunyaev 1973), we may rewrite our diffusivity parameter \( \beta \) in a more physically revealing way:

\[
\beta = \frac{\alpha}{r},
\]

where \( \alpha \) has its usual meaning and \( P_i \) is the turbulent Prandtl number, equal to the turbulent viscosity divided by \( \eta \). The disc turbulence is likely to be driven by the magneto-rotational instability (MRI; Balbus & Hawley 1991), which follows the general behaviour of an \( \alpha \)-disc. Because both \( \alpha \) and \( r / h \) typically have weak radial dependences, and the value of \( P_i \) is unknown, we assume that \( \beta \) is constant in the region of the disc connected to the stellar field.

The value of \( \beta \) is not well constrained (AC96 used \( \beta = 1 \)), but extreme \( \alpha \)-disc parameters give an upper limit of \( \beta \lesssim 1 \). For a more reasonable estimate, note that a thin disc usually means \( h / r \gtrsim 0.1 \), and \( \alpha \) is typically in the range 0.001 to 0.1 (Sano et al. 2004). So, assuming \( P_i \) is of order unity, \( \beta \lesssim 10^{-2} \). We get a similar estimate using equation (5) and reasonable guesses for CTTS disc parameters:

\[
\beta \approx 10^{-2} \left( \frac{\eta}{10^{20} \text{ cm}^2 \text{ s}^{-1}} \right)^{-1} \left( \frac{h}{R_\odot} \right)^{-1} \left( \frac{v_c}{100 \text{ km s}^{-1}} \right)^{-1}.
\]

However, given the uncertainties and possible variation among different systems, and to assess the effect of the coupling of the field to the disc, we retain \( \beta \) as a free parameter.

In the disc-connected region, if \( v_d \) anywhere less (greater) than the local differential rotation speed between the star and disc, \( \gamma \) will increase (decrease) on an orbital time-scale. Thus the magnetic field will quickly achieve a steady-state configuration in which \( v_d \) equals the local differential rotation rate (e.g. UKL), which gives

\[
\gamma = \beta^{-1} \left[ \left( \frac{r}{R_\odot} \right)^2 - 1 \right].
\]

The solid line in Fig. 2 shows the quantity \( \beta \gamma \) as a function of radius (normalized to \( R_\odot \)) along the surface of the disc. The magnetic twist is zero at \( R_\odot \), and is oppositely directed on either side of \( R_\odot \). Also, at a given radius, the twist will be larger for smaller values of \( \beta \) (and vice versa).

We have assumed that the field coupling is determined by turbulent diffusion and, when \( \beta \) is constant, we find that \( \gamma \propto r^{1.5} \) (equation 8, for \( r \gg R_\odot \)). Other coupling mechanisms (as discussed above) result in a different radial dependence of \( \gamma \). For example, Wang (1995) showed that if the twist is limited by reconnection in the disc, \( \gamma \propto r^{1.6} \) (for \( r \gg R_\odot \)), while for reconnection outside the disc, \( \gamma \) approaches a constant value (for \( r \gg R_\odot \)). On the other hand, Livio & Pringle (1992) and AC96, assumed \( \gamma \) was limited by reconnection in the stellar corona, and they used the same formulation as equation (8) (with \( \beta = 1 \)). In any case, note that the radial dependence of the differential magnetic torque is dominated by the fall-off of the dipole magnetic field (which results in the \( r^{-1} \) dependence of equation 3). Therefore, the choice of magnetic coupling mechanism will not much influence our results (AC96). Similarly, a small radial dependence of \( \beta \) (which we take as constant) will not introduce a large error.

### 2.1.1 Maximum twist for dipole field

As discussed in Section 1, several authors have shown that dipole field lines will transition from a closed to an open topology when a critical differential rotation angle of \( \Delta \Phi = \pi \) has been reached. This corresponds to a critical field twist of \( \gamma_c \approx 1 \). Because the twisting of field lines does not significantly alter the poloidal configuration of the field lines that remain closed, and as an approximation we will assume that the opening of field lines is only important for the determination of the size of the connected region (i.e. to determine \( R_{\odot} \)), we include the effect of field line opening in the steady-state torque theory in the following manner: we will use equation (8) only where \( \gamma < \gamma_c \) and assume that the field will be open everywhere else (a similar approach was used by Lovelace et al. 1995 and justified by the work of UKL). In other words, wherever equation (8) predicts \( \gamma \geq \gamma_c \), the magnetic connection is assumed to be severed, so the star and disc are causally disconnected, such that no torques can be conveyed between the two. The size of the connected region in a Keplerian disc is then limited to a finite radial extent near \( R_{\odot} \), where the differential rotation between the star and disc is the smallest.

Will field lines, once opened by differential rotation, remain open? It has been suggested that such field lines could reconnect in the current sheet formed during the opening (Aly & Kuiper 1990; Uzdensky, Königl & Litwin 2002b). In order for this to be important, the time-scale for reconnection should be comparable with or shorter than that for field line opening. It is not clear whether this is the case in these systems (Matt et al. 2002; Uzdensky et al. 2002b), but even if it is, there are other considerations. First, due to the topology of the field, reconnection must initially occur between open field lines at the smallest radii (connecting to the lowest latitude on the star). It is possible that, if reconnection does occur, only a small amount of flux will be able to reconnect before this newly connected field again begins to open (as in the simulations of Goodson et al. 1999, and see Uzdensky et al. 2002b). In this case, the size of the connected region (in a time-averaged sense) will be only slightly larger than if the reconnection were never to occur. Secondly, the configuration of the opened field is favourable to launch magnetocentrifugal flows from the disc (Blandford & Payne 1982). We ignore such outflows.
in our model, but in a real system, they could help to maintain an open magnetic-field configuration. Thus we conclude that, once the field has opened, reconnection along the current sheet is unlikely to affect the size of the connected region significantly.

2.1.2 Maximal spin-down torque for maximal twist

It is instructive to look at the maximum possible spin-down torque in this system. Regardless of any disc model or any magnetic coupling physics, the largest possible magnetic torque on a star that connects to a disc via a dipole magnetic field occurs when the field is maximally twisted ($\gamma = \gamma_c$) at all radii along the surface of the disc. Spin-down torques on the star only occur along field lines threading the disc outside $R_{co}$. Also, the accretion of mass from a Keplerian disc always adds angular momentum to the star. Therefore, the largest net spin-down torque on the star occurs when the disc is truncated exactly at the corotation radius ($R_c = R_{co}$), the field threads the disc to $R_{out} \rightarrow \infty$, and the disc does not accrete ($M_a = 0$). One then integrates equation (3) from $R_{co}$ to $\infty$ to get

$$\tau_{\text{max}} = -\frac{\gamma_c}{3} \frac{\mu^2}{R_{co}^3}. \tag{9}$$

This is the absolute maximum spin-down torque that a star can undergo from a disc that exists in the equatorial plane and to which the star is connected via a dipole magnetic field. It is even independent of the rotation profile of the disc, except that the angular rotation rate of the disc is slower than the star outside some radius $R_{co}$. It is also independent of the angular momentum transport mechanism within the disc.

To achieve this maximal torque requires: (a) the field twist has no radial dependence; and (b) the twist is very nearly equal to the maximum allowed value of $\gamma_c$. If the coupling of the field to a Keplerian disc is determined by turbulent diffusion, a constant $\gamma$ can only be achieved in the unlikely event that $\eta_l$ decreases with radius to exactly counteract the increase in differential rotation rate. Alternatively, reconnection in the stellar or disc corona may also lead to a constant $\gamma$ (Aly & Kuiper 1990; Wang 1995, but see the discussion in Section 2.1.1). However, in either case, it is not clear why the value of the constant twist would necessarily be near the maximal value $\gamma_c$ (instead of, for example, 0.1 $\gamma_c$). Though this torque may not be very realistic, it is similar in strength to the spin-down torque used in previous models (e.g. it is the equivalent to the solution of AC96 for $\gamma_c = 1$).

2.1.3 Determination of $R_{in}$ and $R_{out}$

To derive a more realistic magnetic torque, we adopt equation (8) for the radial dependence of $\gamma$. Following Lovelace et al. (1995), we assume that this equation is only valid where $|\gamma| \leq \gamma_c$, and that the field is open everywhere else. Thus, equation (8) predicts that the outer radius of the magnetically connected region in the disc is

$$R_{\text{out}} = (1 + \beta \gamma_c)^{3/2} R_{co}. \tag{10}$$

There is a corresponding location inside $R_{co}$ at which the twist formally exceeds the critical value, given by

$$R_{in} = (1 - \beta \gamma_c)^{3/2} R_{co}. \tag{11}$$

The dotted lines in Fig. 2 indicate these radii for $\beta \gamma_c = 0.5$, in which case $R_{in} \approx 0.63 R_{co}$ and $R_{out} \approx 1.31 R_{co}$. It is evident that the field topology is a function of $\beta \gamma_c$ such that more diffusion in the disc allows for a larger connected region. Note that, if $\beta \gamma_c > 1$, the field can remain connected to the disc at any $r < R_{co}$ (as $R_{in}$ is then not defined).

The typical assumption of a closed magnetic topology, corresponding to $R_{out} \rightarrow \infty$ (e.g. Yi 1995; AC96), is equivalent to $\gamma_c \rightarrow \infty$ – the field is allowed to twist to arbitrarily large values without opening. In order to consider the effect of varying topology (i.e. where the field is open beyond some finite $R_{out}$), we adopt a value of $\gamma_c = 1$ (as justified by e.g. UKL). However, we will retain $\gamma_c$ as a parameter in all of our formulae for a comparison between the two cases ($\gamma_c \rightarrow \infty$ and $\gamma_c = 1$) and so that different values of $\gamma_c$ may be considered by the reader. The combined parameter $\beta \gamma_c$ appears throughout our formulation. We generally think of this parameter in two ways. First, when $\gamma_c = \infty$, the stellar field is closed and connects to the entire disc, and this represents the ‘standard’ star–disc interaction model. Secondly, for the more realistic case of $\gamma_c = 1$, the field topology is partially open, and $R_{out}$ then depends on $\beta$.

2.2 Three possible states of the system

The inner edge of the disc is delimited by $R_c$ (discussed in Section 2.1.1), and there are two possible magnetic configurations for an accreting system, depending on the location of $R_c$, relative to $R_{in}$. First, if $R_c < R_{in}$, the stellar field will be largely open, and equation (8) is not valid anywhere. The outer radius of the magnetically connected region, $R_{out}$ (which is not then determined by equation 10) will be near the inner edge of the disc ($R_{out} \sim R_c$). We will refer to this situation as ‘state 1’ of the system. In state 1, the star receives no spin-down torques from the disc.

In ‘state 2’, $R_{in} < R_c < R_{co}$ and the star is magnetically connected to the disc from $R_c$ to $R_{out}$. State 2 represents the typical configuration considered in many models and was discussed at the beginning of this section. Also, systems near their disc-locked state (Section 3) are always in state 2.

Finally, there exists a third possible, non-accreting state, ‘state 3’, that occurs when the disc is powered over by the magnetic field (e.g. for fast rotation, large $\mu$ or small $M_*$ and the disc becomes truncated outside $R_{co}$ (e.g. Illarionov & Sunyaev 1975). In state 3, there are no positive (spin-up) torques on the star, so it can never be in spin equilibrium (Sunyaev & Shakura 1977b).

Fig. 3 illustrates the basic magnetic configuration of each state. One can think of this figure, for example, as a sequence (from top to bottom) of decreasing $M_*$ (or increasing $\mu$). A steadily decreasing $M_*$ may represent an evolutionary sequence (e.g.) for protostars as one goes from class 0 sources to weak-lined T Tauri stars (class 3; as shown in Reid, Pudritz & Wadsley 2002). As $M_*$ decreases from a state in 1, the disc truncation radius (which is at the inner edge of the disc in the figure.) moves outward and eventually crosses the location of $R_{in}$ (entering state 2) and then $R_{co}$ (state 3). The conditions for which a system transitions from an accreting state to state 3 is unknown (see Rappaport et al. 2004 and Section 4.2.3). In the current work, we do not consider state 3, other than to note that it occurs somewhere below the lower $M_*$ limit of accreting systems. Instead, we focus most of our attention on state 2 and discuss state 1, where appropriate.

2.3 Torques between the star and disc

A combination of equations (3) and (8) gives the full radial dependence of the differential magnetic torque in the system.

$$\frac{d\tau}{dr} = \frac{\beta^2 \mu^2}{R_{co}^2} \left( \frac{r}{R_{co}} \right)^{-4} \left[ 1 - \left( \frac{r}{R_{co}} \right)^{3/2} \right], \tag{12}$$
where, for convenience, we have used equation (2) to express the radius in units of $R_{co}$. Furthermore, the angular momentum carried by accreting material through each annulus of a Keplerian disc (of width $dr$ and vertically integrated) equals $\frac{d\tau_i}{dr} = 0.5\dot{M}_i (GM_*/r)^{-1/2} dr$ (e.g. Clarke et al. 1995). This can be combined with equation (2) to give the differential accretion torque, as a function of $r/R_{co}$.

$$\frac{d\tau_i}{dr} = \frac{1}{2} \dot{M}_i r^{1/3} \left( \frac{GM_*/R_{co}}{r} \right)^{1/2} \left( \frac{r}{R_{co}} \right)^{-1/2}. \quad (13)$$

The assumption that $\dot{M}_i$ is constant at all radii in the disc requires that the net angular momentum transported away from each annulus in the disc equals $d\tau_i$. The disc must therefore be structured in such a way that the differential torques internal to the disc, $d\tau_i$, satisfy

$$d\tau_i = d\tau_0 + d\tau_m. \quad (14)$$

These internal torques could result from angular momentum transport via (e.g.) turbulent viscosity (Shakura & Sunyaev 1973), MRI (Balbus & Hawley 1991) or disc winds (see review by Königl & Rappaport et al. 2004). If one assumes a particular angular momentum transport mechanism in the disc, the solution to equation (14) determines the structure of the disc. As an example, for the case of $\alpha$ viscosity, the disc responds to external magnetic torques by increasing its surface density in order to transport the additional angular momentum outward. A detailed treatment of the disc adjustment is not necessary here, as we are presently concerned with torques on the star, and we simply assume that the disc structures itself such that equation (14) is valid.

Fig. 4 shows the differential torques ($d\tau/dr$) as a function of $r/R_{co}$ for the adopted fiducial parameters. The solid, dash–dotted, and dashed lines in Fig. 4 represent the differential torques from equations 12, 13 and 14, respectively. The system shown has $\beta = 1$ and $\gamma \rightarrow \infty$, so that the field is connected to the entire surface of the disc, and so that the figure represents the closed topology of several models in the literature (e.g. AC96).

The differential magnetic torque ($d\tau_m$; solid line in Fig. 4) is strongest near the star, where the dipole field is strongest, and it acts to spin up the star (and thus spins down the disc) for $r < R_{co}$. At $R_{co}$, $d\tau_m$ goes to zero, as the field is not twisted ($\gamma = 0$) there. Outside $R_{co}$, the magnetic torque becomes stronger again, as the twist increases, though now acting to spin the star down (and the disc up). The dipole field strength falls off faster with distance ($B_i \propto r^{-3}$) than the magnetic twist increases ($\gamma \propto r^{-3}$), so $d\tau_m$ has a minimum value at $r/R_{co} \approx 1.37$ and then goes to zero as $r \rightarrow \infty$. The dashed line in the figure ($d\tau_i$) gives us some information about the disc structure. In this case, the structure will be significantly different from a case with $d\tau_m = 0$. For a very large $d\tau_m$ outside $R_{co}$, the assumption that the disc can counteract it (via an increase in $d\tau_i$) must eventually break down, and the system would then be in state 3 (Section 2.2).

2.3.1 Truncation radius

Inside the corotation radius, the stellar magnetic torque acts to extract angular momentum from the disc, further enabling accretion (not hindering accretion, as for $r > R_{co}$). The differential magnetic torque increases rapidly as one moves toward the star, and at some point, $d\tau_m = d\tau_i$, the external magnetic torque alone is capable of maintaining $M_i$. Consequently, $d\tau_i$ goes to zero at the same radius and formally becomes negative for smaller $r$ (see Fig. 4). Negative $d\tau_i$ is unrealistic, however, because it would require angular momentum transfer inward through the disc, from slower spinning material to faster spinning material, so the Keplerian disc does not

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**Figure 3.** Three possible configurations of the magnetic star–disc interaction. There are two possible accreting states: either the stellar field connects only to the inner edge of the disc (top panel) or it connects to an extend region, reaching beyond $R_{co}$ (middle panel). In either case, the stellar field dominates the accretion flow (arrows) onto the star. Under certain conditions (e.g. low $M_*$), accretion on to the central star will cease, defining the third, non-accreting state (bottom panel).

**Figure 4.** Differential torques in the fiducial CTTS system (see discussion above Section 2.1) for $\beta = 1$ and $\gamma \rightarrow \infty$, where the field is assumed to remain connected over the entire disc. The truncation radius, $R_{co}$, is where $d\tau_m = d\tau_i$ and $d\tau_i = 0$, indicated by the vertical dotted line ($R_i \approx 0.91 R_{co}$). The system is shown in its equilibrium state, where the net torque on the star is zero, requiring a stellar spin period of 6.0 days ($R_{co} \approx 5.5 R_i$).
exist where where \( dr_t \leq 0 \). Thus, the disc truncation radius, \( R_t \), is where \( dr_t = 0 \).

At \( R_t \), the stellar magnetic field will quickly spin down the disc material, forcing it into corotation with the star. Sub-Keplerian rotation leads to a free-fall of disc material on to the surface of the star in a ‘funnel flow’ along magnetic-field lines (e.g. K91; Romanova et al. 2002). Whether or not the funnel flow originates exactly from \( R_t \) or from inside that radius is subject to debate (e.g. Aly & Kuijpers 1990). However, for the present discussion of angular momentum transport, the most important thing is that \( R_t \) defines the location where the stellar magnetic field completely dominates over the disc internal stresses, and so all the angular momentum of disc material at \( R_t \) will end up on the star.

By setting equations (12) equal to (13), we derive a relationship defining \( R_t \),

\[
\left( \frac{R_t}{R_{\infty}} \right)^{-7/2} \left[ 1 - \left( \frac{R_t}{R_{\infty}} \right)^{3/2} \right] = \frac{\beta}{\psi} f^{-7/3},
\]

where

\[
\psi = 2\mu^2 M_{\infty}^{-1} (GM_*)^{-1/2} R_{\infty}^{-7/2}
\]

is a dimensionless parameter relating the strength of the magnetic field to the strength of accretion. This formula was also derived by Yi (1995), but with different disc parameters in place of our \( \beta \). For any given \( \beta \), \( f \) and \( \psi \), there is only one solution to equation (15) such that \( R_t < R_{\infty} \). A real system may deviate slightly from our simple picture (e.g. of an unperturbed dipole field), leading to an uncertainty in the exact value of \( R_t \). However, due to the steep radial dependence of \( dr_t \) relative to \( dR_t \), the location of \( R_t \) should not be significantly affected. For the system plotted in Fig. 4, the solution to equation (15) is \( R_t/R_{\infty} \approx 0.915 \), represented by the vertical dotted line in the figure.

For a system in state 1 (with \( R_t < R_{\infty} \)), equation (15) is not valid because the field will open (see Section 2.1.3). A substitution of \( R_t < R_{\infty} \) in equation (15) indicates that the system will be in state 1 if

\[
f < (1 - \beta \gamma) \gamma \psi^{-3/7},
\]

and it will be in state 2 for any larger \( f \). Note that condition (17) can never be satisfied if \( \beta \gamma \gg 1 \) (as \( R_{\infty} \) is then undefined), and so the system would then always exist in state 2. For the more probable case that \( \beta \gamma \ll 1 \), state 1 is a possible configuration of any system.

To determine \( R_t \) in state 1, instead of using equation (12) for the differential magnetic torque one must consider the maximum possible \( dr_t \) in order that the field remains closed. This is determined by using \( \gamma = -\gamma_c \) in equation (3). By setting this equal to equation (13), one finds

\[
R_t = (\gamma_c \psi^{2/7} R_{\infty} = (2\gamma_c \gamma^{-2/7} (GM_*)^{-1/2} (M_{\infty}^{2/7} \mu)^{4/7}.
\]

Note that this equation does not depend on the rotation rate or radius of the star and has the same dependences on other system parameters as in many previous theoretical works (e.g. Davidson & Ostriker 1973; Ghosh & Lamb 1979; Shu et al. 1994). Because \( R_t < R_{\infty} \) in state 1, the field lines will be open inside \( R_{\infty} \). Thus, in state 1, the stellar field connects only to a small portion of the disc near \( R_t \), from where a funnel flow originates, and all exterior field lines are open, as shown in the top panel of Fig. 3. State 1 is discussed further in Section 4.2.

Fig. 5 shows the predicted location of \( R_t/R_{\infty} \) as a function of \( \log (f) \), for the fiducial CTTS system. The dashed lines represent the prediction from equation (15) for \( \beta = 0.1, 0.5, 1.0 \) and 2.0, as indicated, and all have \( \gamma_c \to \infty \) so that the magnetic field is assumed to remain closed. The solid line is for \( \beta = 0.1 \), but with \( \gamma_c = 1 \), so that the field is partially open. The system with \( \gamma_c = 1 \) switches to state 1, for \( \log (f) \lesssim -1.5 \), and \( R_t \) is then predicted by equation (18). The dotted line shows the location of \( R_{\infty} \) (equation 2).

2.3.2 Accretion torque

We assume that accreted disc material is quickly integrated into the structure of the star and the accreted angular momentum is redistributed into the stellar rotation profile. So there is a torque on the steadily accreting star that is given by \( \tau_a = M_t (l_t(R_*) - l_*) \), where \( l_t(R_*) \) is the specific angular momentum of the disc material at \( R_t \) and \( l_* \) is that of the star. Combined with equation (1), and assuming solid body rotation of the star, this becomes

\[
\tau_a = M_t (G M_* R_t)^{1/2} \left[ (R_t/R_*)^{1/2} - k^2 \right],
\]

where \( k \) is the normalized radius of gyration (\( k^2 \approx 0.2 \) for a fully convective star; AC96). This formula is valid for a system in either state 1 or state 2.

The term in square brackets in equation (19) is dimensionless and compares the accreting angular momentum (first term) with how much the star already has (second term). Note that the first

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term will always be greater than or equal to one, while the second term has a maximum value of $k^2$ (when $f = 1$). Thus, the second term is usually negligible (particularly when $f \ll 1$).

### 2.3.3 Magnetic torque

When in state 2 (i.e. when $R_t > R_m$), the stellar field connects to a significant portion of the disc, and one can integrate equation (12) over the connected region, from $R_t$ to $R_{out}$, to obtain the total magnetic torque on the star,

$$
\tau_m = \frac{1}{3} \frac{\mu^2}{R_{co}^3} \left[ 2\left(\frac{R_{co}}{R_{out}}\right)^{3/2} - \left(\frac{R_{co}}{R_{out}}\right)^3 \right] - 2\left(\frac{R_{co}}{R_{t}}\right)^{3/2} + \left(\frac{R_{co}}{R_{t}}\right)^3 \right].
$$

This torque is independent of the detailed structure of the Keplerian disc. Also, equation (20) includes the dependence of the magnetic torque on the field topology via the variable $R_{out}$. For example, the spin-down torque (found by setting $R_t = R_{co}$) exerted by field lines connected out to $R_{out}$ is one half of the spin-down torque for $R_{out} \to \infty$. Similarly, for $R_{out} \approx 7.2 R_{co}$ or $R_{out} \approx 34 R_{co}$, the spin-down torque is 90% or 99% per cent (respectively) of the spin-down torque for $R_{out} \to \infty$. It is evident that, even when $R_{out}$ is large, most of the spin-down torque comes from field lines connected not too far from $R_{co}$. This is simply because the differential magnetic torque (equation 12) becomes very weak far from the star. Thus the typical assumption of $R_{out} \to \infty$ is not significantly affected by the fact that real discs have finite radial extents, so long as they reach to several times $R_{co}$.

Above, we have taken $R_{out}$ as arbitrary, but our goal in this paper is to consider the opening of the field from differential rotation, so $R_{out}$ is then given by equation (10), and the preferred formulation of the magnetic torque becomes

$$
\tau_m = \frac{1}{3} \frac{\mu^2}{R_{co}^3} \left[ 2\left(1 + \beta \gamma \right)^{-1} - \left(1 + \beta \gamma \right)^2 \right] - 2\left(\frac{R_{co}}{R_{t}}\right)^{3/2} + \left(\frac{R_{co}}{R_{t}}\right)^3 \right].
$$

This is exactly the solution found by AC96 for the special case of $\beta = 1$ and $\gamma \to \infty$ (so that $R_{out} \to \infty$), but our formulation includes the effect of field opening via a differential rotation, in which case the field topology depends on the diffusion parameter $\beta$. The total magnetic torque on the star can be either positive or negative, depending on the size of the connected region inside $R_{co}$, compared with the connected region outside $R_{co}$. In the next section (Section 2.4), we shall show that, for reasonable values of $\beta$ and $\gamma$, $R_{out}$ is very close to $R_{co}$, and the spin-down torque is significantly affected.

### 2.4 Effect of opened field

Fig. 6 shows the differential torques in the fiducial CTTS system with $\beta = 1$, as in Fig. 4. However, unlike Fig. 4, here we show the system for $\gamma \to 1$, so that the field is open beyond $R_{out} \approx 1.59 R_{co}$. The star now rotates significantly faster, with a period of 3.3 d ($f \approx 0.14$) and $R_t \approx 0.974 R_{co}$. A comparison between Figs 4 and 6 illustrates the effects of varying field topology on the differential torques in the star–disc system.

There are some interesting differences between Figs 4 and 6. Most notably, the differential magnetic torque in Fig. 6 abruptly goes to zero at the location of $R_{out}$, due to the assumption that there is no torque on the star from the disc where the field lines are open. It is evident that a more open topology results in a smaller connected region, which leads to a net (integrated) spin-down torque that is smaller than for the completely closed topology. Thus, a more open topology results in a faster equilibrium spin rate, as can be seen by comparing the stellar spin rate of 6.0 d for Fig. 4 with 3.3 d for Fig. 6.

To quantify the effect of a more open topology on the magnetic torque and to determine the dependence on $\beta$, we first consider only the portion of the magnetic torque that acts to spin down the star by setting $R_t = R_{co}$ in equation (21). For normalization, we use the maximum spin-down torque, $\tau_{max}$ (equation 9). We define the ratio of the true spin-down torque to this maximum torque as $\chi \equiv \tau_m(R_t = R_{co})/\tau_{max}$, which is given by

$$
\chi = (\beta \gamma)^{-1} \left[ 1 + \left(1 + \beta \gamma \right)^{-2} - 2\left(1 + \beta \gamma \right)^{-1} \right]
$$

and only depends on the parameter $\beta \gamma$. It is immediately evident that for $\beta \gamma = 1, \chi = 1/4$, so the spin-down torque is four times less than that used by AC96, when one considers a more realistic magnetic topology.

Fig. 7 illustrates the dependence of the torque ratio $\chi$ on $\beta \gamma$. It decreases as $(\beta \gamma)^{-1}$ for $\beta \gg 1$ and increases as $\beta \gamma$, for $\beta \ll 1$ (as revealed by Taylor expansion of equation 22). The limiting behaviour of $\chi$ is indicated by the two dotted lines in the figure. This behaviour can be understood as a competition between two effects: in the strong magnetic coupling limit ($\beta \ll 1$), the...
field topology becomes more open for smaller $\beta \gamma_c$, reducing the spin-down torque. In the weak coupling limit ($\beta > 1$), the topology is largely closed, but the twisting of the field lines is smaller for larger $\beta$, which reduces the differential magnetic torque at all radii ($d\tau_m \propto \beta^{-1}$). These two effects conspire to give a maximal value of $\chi$ for the special case of $\beta \gamma_c = 1$. For the more probable value of $\beta = 10^{-2}$, the spin-down torque is two orders of magnitude lower than that used by AC96. While it may at first seem surprising that strong magnetic coupling leads to weaker spin-down torques on the star, further reflection reveals that this is necessarily true, as stronger coupling leads to stronger twisting, which further disconnects the star from the disc.

Finally, equations (19) and (21) can be used to calculate the net torque on the star. Fig. 8 shows this net torque as a function of $\log(f)$ for the fiducial CTTS parameters and for different values of $\beta$ and $\gamma_c$. For each case, we calculate the net torque as follows: first, for a given value of $\beta$ and $\gamma_c$, and for each value of $f$, we use equation (17) to determine whether the system is in state 1 or 2. We then find the location of the truncation radius, using equation (15) if the system is in state 2, or equation (18) if in state 1. Finally, we calculate the integrated torques $\tau_s$ (equation 19) and $\tau_m$ (equation 21, if in state 2; $\tau_m = 0$, if in state 1). As discussed in Section 2.3.1, only cases with $\beta \gamma_c < 1$ can be in state 1, so only those cases show a transition at $\log f \approx -1.45$ in Fig. 8. The figure also shows the effect of the field topology on the net torque on the star from the disc. It is evident that, when the magnetic field is partially open ($\gamma_c = 1$), the net torque is larger than for the case where the field is everywhere closed ($\gamma_c = \infty$). The spin rate at which the net torque on the star is zero indicates the equilibrium spin state, which is the topic of the next section.

3 THE DISC-LOCKED STATE

The general theory presented in Section 2 enables one to calculate the net torque ($\tau_s + \tau_m$) on the star for any accreting system with known $M_\star, R_\star, \mu, M_i$ and $\Omega_i$, (one must also adopt values for $\gamma_c$ and $\beta$). The system is stable, in that a positive torque spins the star up, and a faster spin reduces the total torque. Conversely, a negative torque spins the star down, and the torque increases (becoming less negative) for slower spin. Therefore, in a system where the other parameters are relatively constant, the spin rate of the star naturally adjusts to an equilibrium state in which $\tau_s + \tau_m = 0$, known as the ‘disc-locked’ state (e.g. K91; Cameron & Campbell 1993; Shu et al. 1994; AC96). Because the only torques that spin the star down originate along field lines that connect to the disc outside $R_{co}$, systems in their equilibrium state must be in state 2 ($R_i > R_{co}$, see Fig. 3). In this section, we show that both $R_i$ and $\Omega_i$ in the disc-locked state are significantly affected by the field topology and thus have a strong dependence on the magnetic diffusion parameter $\beta$.

3.1 Truncation radius in the disc-locked state

The disc-locked state is defined by the condition $\tau_s = -\tau_m$. Thus, by combining equations (19) and (21), and using equations (2) and (15) to eliminate $f$, this condition can be rearranged to be

$$K(\beta \gamma_c) - \frac{\langle R_{co}/R_i \rangle_{eq}^{1/2}}{(\langle R_{co}/R_i \rangle_{eq}^{1/2} - 1) - \langle R_{co}/R_i \rangle_{eq}^{1/2}} = 7, \quad (23)$$

where

$$K(\beta \gamma_c) = 2(1 + \beta \gamma_c)^{-1} - (1 + \beta \gamma_c)^{-2}, \quad (24)$$

and the subscript ‘eq’ refers to the value in the disc-locked state. In deriving equation (23), we have ignored the term proportional to $k^2 f$ in equation (19), as justified in the discussion following that equation. The function $K(\beta \gamma_c)$ characterizes the topology of the field in the sense that, when $\beta \gamma_c$ varies between 0 and $\infty$, $K$ varies between 1 (completely open field) and 0 (completely closed). Equation (23) has exactly one solution such that $(R_i/R_{co})_{eq} < 1$ (which is the only physical solution) for any given $\beta \gamma_c > 0$.

The location of $R_i$ for accreting systems is, in principle, an observable parameter. For example, Kenyon, Yi & Hartmann (1996) used a magnetic accretion model to predict infrared excesses in CTTSs and then to determine the value of $R_i/R_{co}$ for a sample of stars in the Taurus-Auriga molecular cloud. The value of $(R_i/R_{co})_{eq}$ predicted by equation (23) represents the value for a system that is disc-locked.

The solid line in Fig. 9 shows $(R_i/R_{co})_{eq}$ as a function of $\log(\beta \gamma_c)$. When a closed-field topology is assumed ($\gamma_c \to \infty$), $(R_i/R_{co})_{eq} \approx 0.915$. However, for the more reasonable value of $\gamma_c = 1$, $(R_i/R_{co})_{eq}$ increases (approaching unity) as the magnetic...
coupling becomes stronger (smaller $\beta$). This confirms the conclusion of \citet{Wang1995} and see \citet{Cameron1993, Yi1994, Yi1995} that any disc-locked system will have $(R_c/R_{co})_{eq} \gtrsim 0.9$, and we find that the effect of a more open field topology is to increase this value significantly.

The figure also indicates the three possible states of the system (discussed further in Section 4.2), determined by the location of $R_1$, relative to $R_{co}$ and $R_{eq}$. A system that is disc-locked is always in state 2. If a system is observed with $R_c/R_{co}$ larger than the solid line in the figure, the star should be spinning down. Conversely, if $R_c/R_{co}$ is smaller than the solid line in the figure, the net torque from accretion and from field lines connecting the star and disc will act to spin the star up. It is interesting that Kenyon et al. (1996) found typical values of $R_c/R_{co}$ in the range 0.6 to 0.8 for the stars in their sample. If true, these stars cannot be in spin equilibrium, unless they feel significant spin-down torques other than those from field lines connecting them to their discs. Furthermore, if $\beta \gamma < 0.3$ is appropriate, the stars in their sample should exist in state 1.

### 3.2 Stellar spin rate in equilibrium

Now that we can calculate $(R_c/R_{co})_{eq}$ via equation (23), we rewrite equation (15) to find the fractional spin rate of the star in equilibrium:

$$ f_{eq} = C(\beta, \gamma_c)(2/\psi)^{3/7}, $$(25)

where

$$ C(\beta, \gamma_c) = \frac{2}{\beta} \left( \frac{R_{co}}{R_1} \right)_{eq}^{2} \left[ \left( \frac{R_{co}}{R_1} \right)_{eq}^{3/2} - 1 \right]^{-3/7}. $$

(26)

Because $(R_{co}/R_1)_{eq}$ depends only on $\beta \gamma_c$ (via equations 23 and 24), the dimensionless function $C(\beta, \gamma_c)$ depends only on $\beta$ and $\gamma_c$. We can also combine equations (1), (16) and (25) to find the equilibrium angular spin rate of the star:

$$ \Omega_{eq} = C(\beta, \gamma_c) M_*/G M_\odot^{3/7} \mu^{-6/7}. $$

(27)

This equation has the same dependence of $\Omega_{eq}$ on $M_*$, $M_\odot$, and $\mu$ as equation (3) of K91, and as in the theory of Shu et al. (1994) and Ostriker & Shu (1995). The only difference is the factor $C$ used in the various theories. K91 used $C \approx 1.15$, and Ostriker & Shu (1995) found $C \approx 1.13$. However, our formulation of the problem allows us to determine the effect of the field topology on the equilibrium spin rate, via the function $C(\beta, \gamma_c)$, for arbitrary values of the diffusion parameter $\beta$.

Fig. 10 reveals the dependence of $C(\beta, \gamma_c)$ on $\beta$ for the two values of $\gamma_c$ we have considered throughout. For $\gamma_c \to \infty$, $C(\beta) \approx 1.59 \beta^{1/7}$, which is represented by the dotted line in the figure. The solid line shows $C(\beta, \gamma_c)$ for the more realistic value of $\gamma_c = 1$ and illustrates the effect of a reduced magnetic connection to the disc. For comparison, the dashed line shows the spin rate factor used by K91, which also roughly represents the typical factors of order unity used in most star-disc interaction models.

The dotted line in Fig. 10 represents the assumption that the magnetic field everywhere connects to the disc, regardless of the field twist. In that case, the magnetic torque increases with with decreasing $\beta$, as the field then becomes highly twisted, and so the prediction is that $\Omega_{eq} \propto \beta^{-3/7}$. However, when one considers that dipole field lines will become open when largely twisted, the torque has a maximum value for $\beta = 1$ and decreases for any other $\beta$ (see Section 2.4). Correspondingly, the solid line in Fig. 10 has a minimum value for $\beta = 1$. This minimum value represents the ‘best case’ for disc locking, and even at this location $C$ is a factor of 1.8 larger than the value for $\gamma_c \to \infty$ and 2.5 times larger than used by K91. For the more probable value of $\beta = 10^{-2}$, the predicted equilibrium spin rate of the star is more than an order of magnitude faster than predicted by any other model. Note that the spin rates of the system plotted in Figs 4 and 6 were chosen to be in equilibrium, and a comparison between the two figures shows the effect of field topology on the $\beta = 1$ ‘best case’.

Matt & Pudritz (2004) applied the analysis presented in this section to the CTTS BP Tau, which is one of the few stars for which all of the relevant system parameters are known (or well-constrained). They argued that the existence of slowly rotating, accreting stars, such as BP Tau, cannot be explained by a disc-locking scenario. To further illustrate the effect of field topology on the predictions of disc-locking, and to apply the analysis to all CTTSs, we have plotted Fig. 11. The figure shows the predicted spin period for a wide range of observable parameters. The spin period is given by $2 \pi/\Omega_{eq}$, and we have used the relationship $\mu = B_1 R_1^3$.

The solid lines are for $\gamma_c \to \infty$ and $\beta = 1$, and so they represent the ‘standard prediction’ by previous models. The broken lines take into account that some of the field should be open ($\gamma_c = 1$) for the three different values of $\beta = 1, 0.1$ and 0.01. Note that $\beta = 0.1$ predicts the same period as for $\beta = 10$ (owing to the approximate symmetry of the solid line in Fig. 10), and $\beta = 0.01$ corresponds to $\beta = 100$. It is evident that, even in the ‘best case’ ($\beta = 1$) for the disc-locking scenario, the effect of a more open topology is to reduce the equilibrium spin period by a factor of two, compared with the closed-field assumption. Given the uncertainties in some of the observed parameters, it may not yet be possible to constrain the predicted period to within a factor of two. In particular, a difference of a factor of two in the predicted spin period could result from an error of a factor of 5.0, 2.2, 2.6 or 1.3 in the observed parameters $M_*, B_1$, or $R_*$, respectively. However, for the more probable value of $\beta = 10^{-2}$ (dotted lines in Fig. 11), the predicted spin period is an order of magnitude lower than the ‘standard prediction’, which cannot be reconciled by observational errors.
The spin of accreting stars

3.3 Time to reach equilibrium

It is important to determine how quickly the star will spin up or down to reach the equilibrium state. Rather than fully solving the time-dependent problem, which should also include the spin-up due to the contraction of the protostar (e.g. Yi 1994), one typically estimates a characteristic spin-down time using the angular momentum of the star, $L^\ast$, divided by the net torque on the star. To be more precise, and for arbitrary spin rates, one should replace $L^\ast$ with the difference between the current $L^\ast$ and the value of $L^\ast$ for the equilibrium spin rate. Assuming solid body rotation of the star, the characteristic time to reach spin equilibrium is then

$$t_{\text{spin}} = M^\ast k^2 R^2_{\ast} \left( \frac{\Omega^\ast_{\text{eq}} - \Omega_{\ast}}{\tau_a + \tau_m} \right),$$

which corresponds to a spin-up (down) time for a star currently spinning slower (faster) than $\Omega^\ast_{\text{eq}}$. If $t_{\text{spin}}$ is long compared with the time-scale for other system parameters to change (e.g. compared with the lifetime of the disc), the star is unlikely ever to be in a spin equilibrium state.

Fig. 12 shows $t_{\text{spin}}$ for the adopted fiducial parameters, as a function of the spin rate fraction $f$ and for different values of $\beta$ and $\gamma_c$. It is evident from the figure that $t_{\text{spin}}$ generally decreases for increasing $f$, because faster spin means $R_{co}$ is closer to the star so the magnetic torque will be stronger (whether it spins the star up or down). There is an exception to this when the star spins much slower than $f_{eq}$. For example, for the case with $(\beta, \gamma_c) = (0.01, 1)$, when $f \lesssim 0.2$, $t_{\text{spin}}$ decreases with decreasing $f$. This can be understood, as then

$$R_{co}/R_\ast$$

decreases rapidly with decreasing $f$ (see Fig. 5), leading to much stronger magnetic spin-up torques. Note also that the cases with $(\beta, \gamma_c) = (0.01, 1)$ and $(0.1, 1)$ are in state 1 for $f \lesssim 0.036$ and 0.033, respectively, in which $\tau_m = 0$.

For the ‘standard’ prediction with $(\beta, \gamma_c) = (1, \infty)$, $t_{\text{spin}}$ is always less than $5 \times 10^4$ yr, which is much shorter than the expected disc lifetime of more than $10^6$ yr (Muzerolle et al. 2000). However, when
one considers the effect of a more open field topology ($\gamma_c = 1$), the magnetic torque is reduced, and so $t_{\text{spin}}$ is longer and increases when $\beta$ decreases. For the probable case with $(\beta, \gamma_c) = (0.01, 1)$, $t_{\text{spin}} \sim 4 \times 10^3$ yr. This is still relatively short compared with the expected disc lifetime. Therefore, we agree with previous authors (e.g. K91; AC96) that systems such as those considered above should exist near their equilibrium spin states throughout most of their accretion lifetimes, but only if $\beta \gtrsim 0.01$. However, the effect of a more open field topology is that the equilibrium spin rate is much faster than previously predicted.

We note in passing that the characteristic time $t_{\text{spin}}$ we have calculated here is significantly shorter than recently calculated by Hartmann (2002). Hartmann’s estimate assumes that the upper limit over the region from $\beta \gtrsim 0.01$ to $\beta \lesssim 0.01$. However, as discussed in Section 2.3, this is the torque necessary to provide a steady accretion rate of $M_\ast$ through the radius $R_{\text{out}}$. In order for the disc to exert a spin-down torque on the star (via the magnetic connection), it must provide a torque in addition to $M_\ast (GM_\ast R_{\text{out}})^{1/2}$, requiring the disc to have a different structure from that in the absence of a stellar field (see Rappaport et al. 2004). It is not yet clear to what extent the disc can be restructured (before accretion will cease), and so Hartmann’s estimate does not represent a true limit.

4 DISCUSSION AND CONCLUSIONS

We extended the standard picture of the interaction of a magnetized star with a steady-state accretion disc. Our more comprehensive formulation of this problem allows us to determine the location of the disc truncation radius $R_\ast$ and calculate the torque on the star for a system with arbitrary values of $M_\ast, \Omega_*, M_\ast, R_\ast, B$, and $\gamma_c$, which parametrizes the coupling of the magnetic field to the disc. We consider only two sources for the torques: (a) torque from the angular momentum deposited by accretion of disc material from $R_\ast$; and (b) torques exerted by field lines connecting the star to the disc over the region from $R_\ast$ to $R_{\text{out}}$.

Our model resembles several previous studies (e.g. AC96), except that we have now determined the dependence of the torques on the magnetic coupling to the disc. Specifically, the differential rotation between the star and disc results in a largely open topology (e.g. UKI), so the size of the region of the disc that is magnetically connected to the star is smaller (i.e. $R_{\text{out}}$ is smaller). Thus, when one considers this effect, the magnetic spin-down torque on the star is less than if one assumes the field remains everywhere closed. The strongest spin-down torque occurs for intermediate magnetic-field coupling ($\beta = 1$), in which case the spin-down torque is a factor of four less than for the closed-field assumption. For strong magnetic coupling ($\beta \ll 1$), as expected near the inner edge of an accretion disc, $R_{\text{out}}$ is very close to $R_{\text{co}}$, and the spin-down torque then is proportional to $\beta$. For the probable value of $\beta = 0.01$, the spin-down torque is 100 times less than for the closed-field assumption! Furthermore, the possibility that field lines may open inside the region of the disc that is magnetically connected to the star has been carried out for three CTTSs: BP Tau (Johns-Krull et al. 1999), TW Hya (Johns-Krull & Valenti 2001), and T Tau (Smirnov et al. 2001, 2004), and all measurements give an upper limit of roughly 200 G for the strength of the dipole component of the stellar magnetic field.

The measured mean fields of 2 kG thus represent a field that is disordered or characterized by multipoles of higher order than a dipole (Johns-Krull et al. 1999). Such high-order fields, even if very strong on the stellar surface, decrease in strength too quickly with unknown system parameter(s) to satisfy equation (27). The measured mean fields of 2 kG thus represent a field that is disordered or characterized by multipoles of higher order than a dipole (Johns-Krull et al. 1999). Such high-order fields, even if very strong on the stellar surface, decrease in strength too quickly with unknown system parameter(s) to satisfy equation (27). The measured mean fields of 2 kG thus represent a field that is disordered or characterized by multipoles of higher order than a dipole (Johns-Krull et al. 1999). Such high-order fields, even if very strong on the stellar surface, decrease in strength too quickly with unknown system parameter(s) to satisfy equation (27).
increasing radius to exert significant spin-down torques, especially for slow rotators in which $R_{\infty}$ is at several stellar radii. Furthermore, a 200-G dipole field cannot exert a significant spin-down torque on a CTTS, even if the field connects to the disc everywhere outside $R_{\infty}$. This is evident, for example, in the upper right panel of Fig. 11, which indicates that the equilibrium spin period for a star with such a field is less than one day. For the cases with $\gamma_c = 1$ and $\beta \lesssim 0.1$, there is no equilibrium possible, as the magnetic spin-down torques are not strong enough to counteract the angular momentum added by accretion, even for maximal stellar spin ($f = 1$). Also, the time to reach equilibrium (for cases in which equilibrium is possible, as discussed in Section 3.3) increases by an order of magnitude when $B_\star = 200$ G, compared with 2 kG.

Second is the issue pointed out by Wang (1995) and discussed in Section 3.1 that stars in their disc-locked state must have $R_1/R_{\infty} \gtrsim 0.9$, for a wide range of possible assumptions in the model. Interestingly, Kenyon et al. (1996) concluded that, for their CTTS sample, the typical value of $R_1/R_{\infty}$ was in the range 0.6 to 0.8, well below the disc-locked value. If true, these stars cannot be disc-locked, because the sum of the accretion torque and the torque carried by field lines connected to the disc will be positive (spinning the stars up; see Sections 2.3.3 and 3.1). Thus, these stars can only be in spin equilibrium if they feel significant spin-down torques other than those from field lines connecting them to their discs. This conclusion does not depend on whether or not the stellar field can open, as the calculation of $R_1$ in equilibrium also does not.

Finally, CTTSs may drive stellar winds (Safier 1998), and outflows from the disc are known to explain several aspects of observed protostellar outflows (e.g. Königl & Pudritz 2000). Winds escape from regions with open magnetic-field lines, or they can themselves open the field (e.g. as in the solar wind; Parker 1958), disconnecting the star and disc. Safier (1998) concluded that CTTSs’ winds should open all stellar field lines beyond roughly 3$R_\star$. Furthermore, a recent measurement of rotation in the jet from the CTTS DG Tau (Bacciotti et al. 2002, and see Testi et al. 2002) suggests that the low-velocity component ($\sim 70$ km s$^{-1}$) originates in the disc from as close as 0.3 au from the star (Anderson et al. 2003). This is probably an upper limit (Pesenti et al. 2004), and the more tightly collimated, high-velocity component ($\sim 220$ km s$^{-1}$; Pyo et al. 2003) must then originate from well within this radius in the disc (Anderson et al. 2003). Theoretical disc-wind models (e.g. Königl & Pudritz 2000) predict jet speeds of the order of the Keplerian velocity from where the wind is launched, and observed protostellar jets typically travel with speeds of a few hundred km s$^{-1}$ (e.g. Reipurth & Bally 2001). Considering a star with $M_\star = 0.5 M_\odot$ and $R_\star = 2 R_\odot$, and assuming $v_\perp = 100$ km s$^{-1}$ at the launch point, this requires the disc winds to originate from near $GM_\star/v_\perp^2 \approx 4.8 R_\star$. These ionized disc winds prevent the star from being magnetically connected to the disc beyond the innermost location of the wind launching point (effectively giving an upper limit to $R_{\text{out}}$), and these winds carry angular momentum from the disc, not from the star. As calculated in this paper, a reduced size of the connected region leads to a reduced spin-down torque on the star, regardless of the cause of the field opening. Protostellar outflows thus provide an independent and stringent constraint on disc-locking models.

We conclude that spin-down torques exerted by field lines connecting the star to the disc outside $R_{\infty}$ are likely to be much weaker than usually assumed. Therefore, the existence of slowly rotating ($f \lesssim 0.1$, or perhaps higher) CTTSs probably cannot be explained by a disc-locking scenario. Either these stars are all in the process of spinning up, or the stars feel torques other than those related to a magnetic connection to the accretion disc. Given that the typical spin-up times for these systems are short (see Section 3.3), the latter possibility appears the most likely.

### 4.2 Three states of the system

We identified three possible configurations of the system, determined by the location of $R_1$ relative to the two key radii $R_m$ (equation (11)) and $R_{\infty}$. There are two accreting configurations, which we call states 1 and 2, and one non-accreting configuration, state 3. Here, we summarize the conditions that determine and characterize each state.

Fig. 3 illustrates the basic magnetic configuration of the three states, which may even represent an evolutionary sequence for a system with (e.g.) an evolving $M_\star$ (see Section 2.2). Fig. 9 shows the location of $R_{\text{in}}/R_{\infty}$ (curved, dashed line) for various values of $\beta\gamma_c$, and the horizontal dashed line represents the location of $R_{\infty}$. For a system with a given value of $\beta\gamma_c$ and for which $R_1$ is determined, the figure indicates which state the system will be in and whether the star should be spinning up or down.

#### 4.2.1 State 1: $R_1 < R_{\infty}$ (or $R_{\text{out}} < R_{\infty}$)

This state is a direct consequence of the opening of field lines via the differential rotation between the star and disc. When $R_1$ is sufficiently less than $R_{\infty}$ (i.e. when $R_1 < R_{\infty}$), the magnetic field becomes highly twisted there, opening the field inside $R_{\infty}$ and resulting in a highly open field topology. Note that state 1, in this context, is only possible for $\beta\gamma_c < 1$, because otherwise $R_{\text{in}}$ is undefined (see Fig. 9 and discussion in Section 2.1.3).

As illustrated in the top panel of Fig. 3 and discussed in Section 2.3.1, the stellar field in state 1 connects only to a very small, innermost region of the disc near $R_\star$, and all exterior field lines are open. The size of the small connected region is likely to be determined by dissipative processes within the disc, but we do not attempt to calculate this here. The location of $R_1$ is determined by equation (18) (and see Fig. 5), and accretion on to the star occurs from there.

A star in this state will always feel a net positive torque from the disc, because no field connects outside $R_{\infty}$. Specifically, the star is spun up by the accretion torque ($\tau_a$; equation 19), and equation (21) for the magnetic torque is not applicable. In fact, because $R_1$ increases with stellar field strength, and as $\tau_a \propto R_1^{1/2}$, the presence of a stellar field actually increases the spin-up torque on the star, relative to non-magnetic accretion. In any case, a system in state 1 cannot be in spin equilibrium, unless it receives torques other than those considered in this paper.

Is state 1 a probable, or even common, configuration for accreting systems? Fig. 9 indicates that, when the magnetic coupling to the disc is strong, the range of possible values of $R_1$ for which a system can be in state 2 is significantly reduced. For example, if $\beta\gamma_c = 0.01$, any system with $R_1 < 0.993R_{\infty}$ should be in state 1. The specific conditions under which any given system will be in state 1 is given by equation (17). For illustrative purposes, we can solve this equation (and use equation 16) for the mass accretion rate. Assuming $\gamma_c = 1$, $\beta = 0.01$ and $f = 0.1$, and using the fiducial CTTS values (see discussion above Section 2.1), one finds that a system should be in state 1 if

$$M_\star > 5.4 \times 10^{-7} \left( \frac{\gamma_c}{1} \right) \left( \frac{1 - \beta\gamma_c}{0.99} \right)^{-7/3} \left( \frac{M_\star}{0.5 M_\odot} \right)^{-1/2} \times \left( \frac{B_\star}{2 \text{ kG}} \right)^2 \left( \frac{R_\star}{2 R_\odot} \right)^{3/2} \left( \frac{R_{\text{out}}}{0.1} \right)^{7/3} M_\odot \text{ yr}^{-1}. \quad (29)$$

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This threshold value of \( M_a \) is an order of magnitude larger than the fiducial value, suggesting that CTTS slow rotators will most commonly exist in state 2.

However, equation (29) assumes a magnetic-field strength of 2 kG, and, as discussed in Section 4.1, the stars probably have surface dipole field strengths of less than 200 G. This consideration decreases the threshold value of \( M_a \) by at least two orders of magnitude, suggesting that typical CTTS systems may exist in state 1. We can also look at this from the standpoint of stellar spin, using equation (17), which indicates that a system with the adopted fiducial parameters (but with \( B_s = 200 \) G) will be in state 1 if it spins more slowly than 26 per cent of break-up speed. (As discussed in Section 2.3.1, this also corresponds to an upper limit of \( R_c \lesssim 2.4 R_a \).) Thus, if the dipole fields are indeed weak, it is more probable that slow rotators, and even some fast rotators, will be in state 1. Furthermore, the conclusion of Kenyon et al. (1996), that \( R_t/R_{co} \) typically ranges from 0.6 to 0.8, suggests that CTTSs will be in state 1, as long as \( \beta \gamma_c < 0.3 \) (see Section 3.1).

We have thus far considered the opening of field lines via the differential rotation, so the existence of state 1 requires that \( \beta \gamma_c \) is significantly less than unity. Given the significant uncertainty in the value of \( \beta \) in real systems, it is still not clear whether or not state 1 should be common. From the standpoint of torques on the star, the most important feature of state 1 is that the stellar field never connects to the disc outside \( R_{co} \). Thus, for the following discussion, we will generalize the definition of state 1 to include any magnetic configuration in which the star does not connect outside \( R_{co} \).

State 1 is characterized by a large amount of open stellar field, so it is natural to consider the effects of a stellar wind in the magnetically open region. As discussed in Section 4.1, a wind can even be responsible for opening the field (which does not depend on our parameters \( \beta \) and \( \gamma_c \)). Thus, if a wind (or any other process) keeps the stellar field open beyond some radius \( R'_{out} \), and if \( R'_{out} < R_{co} \), the system will be in state 1. For example, Safier (1998) concluded that stellar winds from CTTSs could result in \( R'_{out} \lesssim 3 R_a \). If true, this means that any system rotating more slowly than 19 per cent of break-up speed will have \( R'_{out} < R_{co} \) (equation 1) and be in state 1.

There is empirical evidence for systems in state 1 from some numerical simulations of the star-disc interaction, which usually represent systems with \( \beta \ll 1 \). In the simulations of Goodson & Winglee (1999) and von Rekowski & Brandenburg (2004), as an example, after the initial state, the stellar field never connects to the disc outside \( R_{co} \), even immediately following reconnection events. These authors report that the only significant spin-down torques on the star come from the open field regions (though a stellar wind was not properly included), rather than along field lines connecting the stars to their discs (but also see Romanova et al. 2002). It seems that state 1 is a probable configuration for accreting stars, particularly among slow rotators. Because, in this state, the net torque from the interaction with the accretion disc only acts to spin up the star, stars with long-lived accretion phases must somehow rid themselves of this excess angular momentum. Stellar winds can exert spin-down torques on the star, and if these torques are significant (e.g. Tout & Pringle 1992), the equilibrium spin rate may be simply determined by a balance between this torque and \( \tau_a \). In this situation, state 1 could actually represent the expected configuration for accreting systems in spin equilibrium.

4.2.2 State 2: \( R_{in} < R_t < R_{co} \)

In this state, the stellar field connects to a finite region of the disc between \( R_t \) and \( R_{out} \), as illustrated in the middle panel of Fig. 3. This represents the typical configuration in many star-disc interaction models, except that the determination of \( R_{out} \) varies between models. The location of \( R_t \) is determined by equation (15) (and see Fig. 5), and accretion on to the star occurs from there.

The star is spun up by the accretion torque (equation 19) and magnetic torques (equation 21) from field lines connected to the region of the disc between \( R_t \) and \( R_{co} \) and spun down by magnetic torques from field lines connected between \( R_{in} \) and \( R_{out} \). Therefore, a system can exist in an equilibrium, disc-locked state, in state 2, in which the net torque on the star is zero, and the spin rate of the star then correlates with accretion parameters (see Section 3).

When one considers that the differential rotation determines \( R_{out} \) (via equation 10), both \( \Omega_{1}^{\ast} \) and \( (R_t/R_{co})_{eq} \) are larger for smaller \( \beta \). Also, as shown in Fig. 9, the range of (non-equilibrium) values of \( R_t \) that exist in state 2 becomes narrower as \( \beta \) decreases. For the strong coupling case of \( \beta \gamma_c = 0.01 \), a system can only be in state 2 if \( 0.993 < R_t/R_{co} < 1.0 \). So for strong coupling, it is unlikely that a given system will exist in state 2 unless it is very near its disc-locked state, which then requires a fast stellar spin.

Another intriguing effect of a largely open field topology is that, in order for a system to be disc-locked, the differential magnetic torque in the disc \( \Delta \tau_{in} \) must be stronger (compared with the completely closed assumption) in order to make up for the decreased size of the connected region (compare Figs 4 and 6). When the connected region is very small (i.e. for small \( \beta \)), \( \Delta \tau_{in} \) is very large, which ought to have a significant effect on the disc structure there. There may be a physical limit beyond which the disc cannot respond and accretion will cease (see discussion of state 3, below). This could possibly lead to a time-dependent process (e.g. Spruit & Taam 1993), perhaps analogous to the simulations of (e.g.) Goodson et al. (1999), though their stellar field does not connect outside \( R_{co} \), and in which there may still be a time-averaged net torque of zero.

4.2.3 State 3: \( R_t > R_{co} \)

Because no accretion on to the star occurs, state 3 may characterize the non-accreting, weak-line T Tauri phase of pre-main-sequence stellar evolution. Also, as the disc does not extend inside \( R_{co} \), the star feels no spin-up torques, only spin-down torques, and so it cannot exist in spin equilibrium (Sunyaev & Shakura 1977b). A possible magnetic-field configuration of this state is illustrated in the bottom panel of Fig. 3.

It is not yet clear under which conditions a system will be in this state, though the relative values of differential torques (or stresses) in the disc are certainly important. Some authors (e.g. Wang 1995; Clarke et al. 1995) have speculated that state 3 occurs when \( | \Delta \tau_{in}/\Delta \tau_{out} | \) becomes greater than one (Cameron & Campbell 1993, suggested a value of 2) anywhere outside \( R_{co} \). In this work, we have assumed that the disc will be structured such that equation (14) is satisfied (see Section 2.3 and Rappaport et al. 2004). However, this assumption must eventually break down for large enough \( f \), large enough \( \mu \), or small enough \( M_a \).

The general question of what conditions govern a system in state 3, to our knowledge, remains an unanswered astrophysical problem. It is not clear what will determine the location of \( R_t \) in state 3 (as neither of equations 15 or 18 is then valid). In addition to magnetic torques, outflows and/or radiation from the star may be important (Johnstone 1995). Understanding this state is probably relevant to understanding the transition from classical to weak-line T Tauri phases, and it may even have further implications for gas giant planet formation/migration (Lin, Bodenheimer & Richardson...
4.3 Conclusions

We have considered that the opening of magnetic-field lines expected from differential rotation in the star–disc interaction results in a largely open field topology. This significantly alters the torque that a star receives from its accretion disc, compared with previous models that assume a closed field. Our main conclusions from this work are the following.

(1) This more open field topology results in a weaker spin-down torque felt by the star from the disc (Section 2.4). The strongest possible torque occurs for intermediate magnetic coupling to the disc. Stronger coupling, as expected near the inner edge of the disc, results in a spin-down torque that is more than an order of magnitude below the torque found for the closed-field assumption.

(2) In the disc-locked, spin equilibrium state, this results in a stellar spin rate that is much faster than predicted by previous models (Section 3.2).

(3) We have identified and discussed three possible magnetic-field configurations in magnetic star–disc systems (Section 4.2). The three configurations could represent, for example, an evolutionary sequence for a system with a gradually decreasing mass accretion rate (or e.g. a gradually increasing stellar spin rate). Our conclusions about each state, in the context of T Tauri stars, are the following.

(i) Assuming strong magnetic coupling to the disc, slowly rotating CTTSs should be in state 1 if $\dot{M}_i \gtrsim 5 \times 10^{-9} M_\odot \text{yr}^{-1}$ (equation 29 for $B_* = 200$ G). Because typical accretion rates are higher than this, state 1 may represent a common configuration in these systems. In this state, the star feels no spin-down torques from the disc. However, if spin-down torques from (e.g.) a stellar wind are significant, stars may be in spin equilibrium in state 1, though they should not then be considered ‘disc locked’.

(ii) State 2 is the typical configuration assumed in star–disc interaction models. For strong magnetic coupling in the disc, or if stellar or disc winds are important, we find that a system can only be in state 2 under special circumstances. In particular, the accretion rate must be lower than for state 1.

(iii) State 3 probably represents the non-accreting, weak-line T Tauri phase. Given that the accretion disc can restructure itself in response to (e.g.) external magnetic torques, it is not yet clear when a system will transition into this state. This important evolutionary phase requires more theoretical study.

(4) These considerations, and additional issues from the literature, suggest that slowly rotating CTTSs probably cannot be explained by a disc-locking scenario (Section 4.1).

(5) If slowly rotating CTTSs are in spin equilibrium, then another spin-down torque must be active in the system. We suggest that this might arise from magnetized stellar winds.

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