$e^+e^- \rightarrow t\bar{t}H$ including decays: 
on the size of background contributions

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Abstract

We present results for the lowest order cross sections, calculated with the complete set of the standard model Feynman diagrams, of all possible detection channels of the associated production of the top quark pair and the light Higgs boson, which may be used for determination of the top–Higgs Yukawa coupling at the future $e^+e^-$ linear collider. We show that, for typical particle identification cuts, the background contributions are large. In particular, the QCD background contributions are much bigger than could be expected when taking into account a possibly low virtuality of exchanged gluons. Moreover, we include the initial state radiation effects and discuss the dependence of the cross sections on the Higgs boson and top quark masses.

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1 Introduction

A reaction of associated production of a top quark pair and a Higgs boson at the future $e^+e^-$ linear collider

$$e^+e^- \rightarrow t\bar{t}H$$  (1)

can be used to determine the top–Higgs Yukawa coupling [1]. In the Standard Model (SM), after taking into account decays of the top quark, $t \rightarrow bW^+$, of the antitop quark, $\bar{t} \rightarrow \bar{b}W^-$, together with the subsequent decays of the $W$ bosons, and of the Higgs boson that, if its mass is less than about 140 GeV, decays preferably into a $b\bar{b}$-quark pair, reaction (1) takes the following form

$$e^+e^- \rightarrow b\bar{b}f_1f_2^f_1^f_2.$$  (2)

If we neglect the Cabibbo–Kobayashi–Maskawa (CKM) mixing in (2) then $f_1, f_2 = \nu_e, \nu_{\mu}, \nu_{\tau}, u, c$ and $f_1^f_2 = e^-, \mu^-, \tau^-, d, s$, respectively. For the heavier Higgs boson, with mass $m_H > 140$ GeV, the decay into electroweak boson pair will dominate and the detection channels of (1), will have 10 fermions in the final state. This option is much more complicated and it will not be addressed in the present work.

Reactions (2) receive contributions from tens thousands of Feynman diagrams already in the lowest order of SM. For example, reactions

$$e^+e^- \rightarrow b\bar{b}\nu_e e^+\mu^-\bar{\nu}_\mu,$$  (3)

$$e^+e^- \rightarrow b\bar{b}b\mu^-\bar{\nu}_\mu,$$  (4)

$$e^+e^- \rightarrow b\bar{b}b\bar{c}\bar{s}s\bar{c},$$  (5)

which correspond to the leptonic, semileptonic and hadronic decays of the $W$ bosons, receive contributions, respectively, from 37 868, 26 816 and 240 966 Feynman diagrams in the unitary gauge, with the neglect of the Yukawa couplings of the fermions lighter than the $c$ quark. An overwhelming majority of the diagrams comprise background to the resonant production and decay of the top quark pair and Higgs boson that is mediated by 20 signal Feynman diagrams which contain the propagators of the top, antitop and Higgs at a time. The representative signal Feynman diagrams are shown in Fig. 1.

![Feynman diagrams](image)

Figure 1: Representative signal Feynman diagrams of reaction (2) in the unitary gauge. The remaining diagrams are obtained by 4 permutations of the two $b$ and two $\bar{b}$ lines. The Higgs boson coupling to electrons has been neglected.
To which extent the background contributions may affect the associated production of the top quark pair and Higgs boson has been discussed in \[2\] for a few selected reactions (2). A similar issue was also discussed in \[3\], where processes of the form 
\[
e^+e^- \rightarrow b\bar{b}bW^+W^- \rightarrow b\bar{b}b\bar{b}\pm\nu qq'
\]
accounting for the signal of associated Higgs boson and top quark pair production, as well as several irreducible background reactions, were studied and in \[4\], where pure electroweak (EW) contributions to the leptonic and semileptonic reactions (2) were computed. The off resonance background contributions in 
\[
e^+e^- \rightarrow Hbbd\mu^-\bar{\nu}_\mu
\]
have been calculated in \[5\].

In the present work, we will present results for the lowest order cross sections of all reactions (2) possible in the SM calculated with the complete set of the Feynman diagrams with cuts on angles and energies of the final state particles that should allow for identification of the corresponding jets and/or separate charged leptons \[6\]. Such results for reaction (4) were already shown in our former work \[2\]. The size of the background contributions can be then inferred from comparison of these cross sections with the signal cross sections of the associated production of the top quark pair and Higgs boson. To illustrate the size of the QCD background we present also the cross sections calculated with the neglect of the gluon exchange contributions. The calculation has been performed with **carlomat**, a recently released program for automatic computation of lowest order cross sections \[7\]. For testing purposes, the lowest order cross sections calculated with the complete set of the Feynman diagrams have been also computed with another multipurpose Monte Carlo (MC) generator **WHIZARD/OMEGA** \[8\]. Moreover, we will include the initial state radiation (ISR) effects and discuss the dependence of the cross sections on the Higgs boson and top quark masses.

2 Calculational details

The results presented in Section 3 have been mostly obtained with **carlomat** \[7\], a new multipurpose program for automatic computation of the lowest order cross sections whose main goal is to provide the reliable description of multiparticle reactions. Both **carlomat** and the routines for the MC computation of cross sections it generates are written in Fortran 90/95.

The routine for matrix element calculation utilizes the helicity amplitude method. Poles in the propagators of unstable particles are regularized with constant particle widths \(\Gamma_a\) which are introduced through the complex mass parameters \(M_a^2\) by making the substitution
\[
m_a^2 \rightarrow M_a^2 = m_a^2 - im_a\Gamma_a, \quad a = Z, W, H, t.
\]  
(6)

Substitution (6) is made both in the \(s\)- and \(t\)-channel propagators. The calculation is performed in the complex mass scheme (CMS) \[9\], where the electroweak mixing parameter \(\sin^2\theta_W\) is defined as a complex quantity
\[
\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2},
\]  
(7)

with \(M_W^2\) and \(M_Z^2\) given by (6). The widths \(\Gamma_a\) of (6), except for \(\Gamma_Z\), whose actual value is rather irrelevant in the context of associated top quark pair and Higgs boson production and
decay, are calculated in the lowest order of SM. This, combined with the use of CMS, which preserves the Ward or Slavnov–Taylor identities and hence the gauge invariance, should minimize the unitarity violation effects. The latter are related to the fact that substitution (6) introduces spurious terms of order $O(\Gamma_a/m_a) = O(\alpha)$ relative to the lowest order in places other than the resonant propagators. These higher order unitarity violating terms cannot be enhanced due to the exact preservation of the Ward or Slavnov–Taylor identities, however, the use of other widths in (6) than the lowest order ones would have increased them, as we do not include any higher order corrections to the unstable particle decays.

As in the next to leading order the number of the Feynman diagrams increases dramatically, calculation of the full $O(\alpha)$ radiative corrections to any of reactions (2) is not feasible either at the moment or even in the foreseeable future. However, precision of theoretical predictions for such reactions can be improved by the inclusion of the ISR effects in the leading logarithmic (LL) approximation in the structure function approach. The most recent version of carlomat allows to compute the ISR corrected cross section in LL approximation $d\sigma^{LL}(p_1, p_2)$ according to the following formula

$$d\sigma_{\text{Born+ISR}}(p_1, p_2) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}^{LL}(x_1, Q^2) \Gamma_{ee}^{LL}(x_2, Q^2) d\sigma_{\text{Born}}(x_1 p_1, x_2 p_2),$$

where $p_1$ ($p_2$) is the four momentum of a positron (electron), $x_1$ ($x_2$) is the fraction of the initial momentum of the positron (electron) that remains after emission of a collinear photon and $d\sigma_{\text{Born}}(x_1 p_1, x_2 p_2)$ is the lowest order cross section calculated at the reduced four momenta of the positron and electron. The structure function $\Gamma_{ee}^{LL}(x, Q^2)$ is given by Eq. (67) of [10], with ‘BETA’ choice for non-leading terms. The splitting scale $Q^2$, which is not fixed in the LL approximation is selected to be equal $s = (p_1 + p_2)^2$.

carlomat generates also dedicated phase space parametrizations which take into account mappings of peaks in the matrix element caused by propagators of massive unstable particles, of a photon, or a gluon in each Feynman diagram. This means that a number of different phase space parametrizations generated is equal to a number of the Feynman diagrams. The phase space parametrizations are automatically implemented into a multichannel MC integration routine that performs integration over a 20-dimensional phase space of reactions (2). The integration is performed in several iterations, with the weights of different kinematical channels determined anew after each iteration. In this way, the kinematical channels with small weights are effectively not used in the subsequent iterations.

### 3 Results

We use the following set of initial physical parameters: the Fermi coupling, fine structure constant in the Thomson limit and strong coupling

$$G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha_0 = 1/137.0359991, \quad \alpha_s(m_Z) = 0.1176,$$

the $W$- and $Z$-boson masses

$$m_W = 80.419 \text{ GeV}, \quad m_Z = 91.1882 \text{ GeV},$$

\[9\]
the top quark mass and the heavy external fermion masses

\[ m_t = 174.3 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad m_c = 1.3 \text{ GeV}, \quad m_\tau = 1.77699 \text{ GeV}. \quad (11) \]

The same masses are used both in the matrix elements and phase space parametrizations. We would like to stress that, in the CMS, the complex top quark mass calculated according to Eq. (6) enters both the top quark propagator, including the numerator, as well as the top–Higgs Yukawa coupling. In order to speed up the computation fermion masses smaller than those of the c quark and τ lepton have been neglected.

The value of the Higgs boson mass is assumed at \( m_H = 130 \text{ GeV}. \) To avoid unitarity violation the widths of \( t \)-quark, \( W^- \) and Higgs bosons are calculated to the lowest order of SM resulting in the following values:

\[ \Gamma_t = 1.53088 \text{ GeV}, \quad \Gamma_W = 2.04764 \text{ GeV}, \quad \Gamma_H = 8.0540 \text{ MeV}. \quad (12) \]

The \( Z \) boson width, whose actual value is not relevant in the calculation, is fixed at its experimental value \( \Gamma_Z = 2.4952 \text{ GeV}. \)

We identify jets with their original partons and define the following basic cuts which should allow to detect events with separate jets and/or isolated charged leptons:

- cuts on an angle between a quark and a beam, an angle between two quarks and on a quark energy:

  \[ 5^\circ < \theta(q, \text{beam}) < 175^\circ, \quad \theta(q, q') > 10^\circ, \quad E_q > 15 \text{ GeV}, \quad (13) \]

- cuts on angles between a charged lepton and a beam, a charged lepton and a quark and on energy of the charged lepton, \( l = \mu, \tau, \) in the semileptonic detection channels of (2)

  \[ 5^\circ < \theta(l, \text{beam}) < 175^\circ, \quad \theta(l, q) > 10^\circ, \quad E_l > 15 \text{ GeV}, \quad (14) \]

- a cut on an angle between the two charged leptons in the leptonic detection channels of (2)

  \[ \theta(l, l') > 10^\circ, \quad (15) \]

- a cut on the missing transverse energy in the hadronic and semileptonic detection channels of (2)

  \[ E_T > 15 \text{ GeV}. \quad (16) \]

The results for the lowest order cross sections of the different channels of (2), which correspond to different decay modes of the \( W^+ \) and \( W^- \) bosons resulting from \( t \) and \( \bar{t} \) decays, are shown in Tables 1–3 for \( \sqrt{s} = 500 \text{ GeV}, 800 \text{ GeV}, 1 \text{ TeV} \) and \( 2 \text{ TeV}, \) the first two centre of mass energies being characteristic for the International Linear Collider (ILC) project \cite{11} and the second two for the Compact Linear Collider (CLIC) design \cite{12}. We present cross sections of specific reactions, calculated with the complete set of the lowest order Feynmann diagrams: \( \sigma_{\text{all}}^\text{Whiz}, \) computed with WHIZARD/OMEGA and \( \sigma_{\text{all}}, \) computed with carlomat. Both
\[ \sigma_{\text{Whiz}} \] and \[ \sigma_{\text{all}} \] have been computed with the same cuts and initial input parameters. The comparison shows agreement within 1–3 standard deviations of the MC integration that is indicated in parentheses. In order to illustrate the size of the QCD background for each reaction we present also the cross sections \[ \sigma_{\text{no QCD}} \] with the neglect of the gluon exchange contributions, computed with carlomat. Comparison of \[ \sigma_{\text{all}} \] and \[ \sigma_{\text{no QCD}} \] shows that the QCD contributions are sizeable, amazingly enough also for the leptonic channels of Table 1. They are particularly large at \( \sqrt{s} = 500 \text{ GeV} \), where the signal is reduced because of the limited phase space volume available for reaction (1). However, the QCD background is sizeable also for higher centre of mass energies. This is somewhat surprising if one takes into account a possibly low virtuality of the exchanged gluons. Let us note that, despite quite different numbers of the Feynman diagrams contributing, the cross sections in Tables 1–3 are practically flavor independent within each of the detection channels. This means that, due to the cuts enforced, there is practically no dependence on \( m_b, m_c \) and \( m_\tau \) in the gluon and/or photon exchange background contributions.

The full size of the background contributions can be inferred from comparison of the cross sections \[ \sigma_{\text{all}} \] of Tables 1–3 with the corresponding signal cross sections presented in Table 4. The background contributions are large, but they can be efficiently reduced by imposing cuts on invariant masses allowing for reconstruction of the top, antitop and Higgs boson. In particular, a narrow cut, of the order of 1 GeV, on the invariant mass of the two \( b \)-jets that reconstruct the Higgs boson mass reduces the background very efficiently. This issue has been already extensively discussed in [2] so we will not address it here. The signal cross sections are also the same for all the channels within a given type: leptonic, semileptonic and hadronic one, as we have neglected the light fermion masses. However, even if the light fermion masses had been kept non zero, the flavor dependence of the cross sections would have been negligibly small, as amplitudes of the signal diagrams of the associated production of the top quark pair and Higgs boson are almost completely independent of the external fermion masses which always couple to the propagators of the heavy particles.

Let us note that, in spite of somewhat different cuts imposed, the cross sections of the semileptonic channels in Tables 1–3 are roughly a factor 3 bigger than those of the leptonic channels. A similar relation holds between the cross sections of the hadronic and semileptonic channels.

It is interesting to look at the dependence of the cross sections on the Higgs boson and top quark masses. In order to illustrate this we show in Table 5 the cross sections \[ \sigma_{\text{all}}, \sigma_{\text{no QCD}} \] and \[ \sigma_{\text{signal}} \] of reaction (4) with cuts given by (13), (14) and (16) that have been calculated with 2 different hypothetical values of the Higgs boson mass: \( m_H = 115 \text{ GeV} \) and \( m_H = 130 \text{ GeV} \), and 3 values of the top quark mass: \( m_t = 171 \text{ GeV} \), \( m_t = 174.3 \text{ GeV} \) and \( m_t = 176 \text{ GeV} \). The top quark mass has been chosen within roughly 2\( \sigma \) range of the combined Tevatron result \( m_t = 173.1 \pm 1.3 \text{ GeV} \) [13]. The change of particle masses affects the corresponding lowest order SM widths of (12) which become

\[
\Gamma_t(171 \text{ GeV}) = 1.43152 \text{ GeV}, \quad \Gamma_t(176 \text{ GeV}) = 1.58351 \text{ GeV},
\Gamma_H(115 \text{ GeV}) = 6.0223 \text{ MeV}.
\] (17)

The cross sections for \( m_H = 115 \text{ GeV} \) are bigger than those for \( m_H = 130 \text{ GeV} \). The relative difference is largest at \( \sqrt{s} = 500 \text{ GeV} \), where it amounts to a factor 3–4 for the signal cross
Table 1: Cross sections in ab of leptonic detection channels of (2) calculated with WHIZARD/OMEGA (first column) and carlomat (second column) with cuts given by (13), (14) and (15). The numbers in parenthesis show the MC uncertainty of the last decimal.

| √s [GeV] | \(\sigma_{\text{all}}^{\text{Whiz.}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) | \(\sigma_{\text{all}}^{\text{Whiz.}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 500      | 8.71(15)        | 8.26(6)         | 2.889(7)        | 8.47(5)         | 8.28(6)         | 2.882(7)        |
| 800      | 36.2(1)         | 35.6(2)         | 24.41(4)        | 36.1(1)         | 36.0(1)         | 24.42(4)        |
| 1000     | 34.1(1)         | 34.3(2)         | 22.50(4)        | 34.2(1)         | 34.3(1)         | 22.57(5)        |
| 2000     | 18.6(2)         | 18.4(2)         | 11.15(7)        | 18.2(2)         | 18.3(1)         | 11.06(7)        |

| √s [GeV] | \(\sigma_{\text{all}}^{\text{Whiz.}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) | \(\sigma_{\text{all}}^{\text{Whiz.}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 500      | 8.43(4)         | 8.43(6)         | 2.895(8)        | 8.43(1)         | 8.49(7)         | 2.890(8)        |
| 800      | 36.1(1)         | 36.1(1)         | 24.57(4)        | 35.8(1)         | 35.9(1)         | 24.48(4)        |
| 1000     | 34.2(1)         | 34.3(1)         | 22.55(4)        | 34.3(2)         | 34.0(1)         | 22.54(4)        |
| 2000     | 19.3(2)         | 20.0(2)         | 12.28(14)       | 17.6(1)         | 17.7(1)         | 10.52(3)        |

| √s [GeV] | \(\sigma_{\text{all}}^{\text{Whiz.}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) | \(\sigma_{\text{all}}^{\text{Whiz.}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 500      | 8.44(1)         | 8.43(5)         | 2.907(6)        | 8.40(3)         | 8.46(3)         | 2.885(6)        |
| 800      | 35.8(1)         | 35.8(1)         | 24.49(4)        | 36.2(2)         | 35.9(1)         | 24.49(4)        |
| 1000     | 34.0(2)         | 34.1(1)         | 22.48(4)        | 33.8(1)         | 34.2(1)         | 22.42(4)        |
| 2000     | 17.5(1)         | 17.7(1)         | 10.50(3)        | 17.3(1)         | 17.7(1)         | 10.56(4)        |

section, a factor 2 for pure EW contributions and a factor 1.3–1.5 for the full cross section. The change of a top quark mass in Table 5 is smaller than that of the Higgs mass, however, it has proportionally the same effect on the cross section at \(\sqrt{s} = 500\) GeV. This can be easily traced back to the phase space volume limitations that are much stronger for the
| \( \sqrt{s} \) [GeV] | \( \sigma^{\text{Whiz.}}_{\text{all}} \) | \( \sigma_{\text{all}} \) | \( \sigma_{\text{no QCD}} \) | \( \sigma^{\text{Whiz.}}_{\text{all}} \) | \( \sigma_{\text{all}} \) | \( \sigma_{\text{no QCD}} \) |
|-----------------|---------------|-----------|---------------|---------------|-----------|-----------|
| 500             | 26.8(1)       | 26.9(1)   | 7.91(2)       | 26.8(1)       | 26.9(1)   | 7.93(2)   |
| 800             | 99.9(3)       | 100.6(3)  | 67.3(1)       | 99.7(5)       | 100.5(3)  | 67.1(1)   |
| 1000            | 94.0(3)       | 95.0(3)   | 61.9(1)       | 92.4(4)       | 94.2(3)   | 61.9(1)   |
| 2000            | 49.6(7)       | 49.1(3)   | 29.4(1)       | 47.8(4)       | 48.2(3)   | 29.8(2)   |

Table 2: Cross sections in ab of semileptonic detection channels of (2) calculated with WHIZARD/OMEGA (first column) and carlomat (second column) with cuts given by (13), (14) and (16). The numbers in parenthesis show the MC uncertainty of the last decimal.

heavier Higgs boson and top quark. At higher energies, where the relative differences in cross sections are smaller, the mass dependence becomes more involved as the phase space effect interferes with the shift of the maximum of the cross section dependence on \( \sqrt{s} \) that is caused by changes in \( m_H \) and/or \( m_t \). For example, the signal cross section at \( \sqrt{s} = 800 \text{ GeV} \)
\[ e^+ e^- \rightarrow b\bar{b}budd\bar{u} \quad e^+ e^- \rightarrow b\bar{b}c\bar{s}s\bar{c} \]

| \( \sqrt{s} [\text{GeV}] \) | \( \sigma_{\text{all}}^{\text{Whiz.}} \) | \( \sigma_{\text{all}} \) | \( \sigma_{\text{no QCD}} \) | \( \sigma_{\text{all}}^{\text{Whiz.}} \) | \( \sigma_{\text{all}} \) | \( \sigma_{\text{no QCD}} \) |
|---|---|---|---|---|---|---|
| 500 | 92.8(4) | 94.1(3) | 24.10(7) | 93.5(6) | 93.1(2) | 24.16(5) |
| 800 | 318(2) | 314(1) | 205.7(3) | 308(2) | 314(1) | 205.7(3) |
| 1000 | 284(2) | 291(1) | 187.4(3) | 286(2) | 291(1) | 187.6(3) |
| 2000 | 137.1(5) | 139.2(7) | 83.1(3) | 137(1) | 139.2(5) | 83.0(3) |

Table 3: Cross sections in ab of hadronic detection channels of \( (2) \) calculated with \textsc{Whizard}/\textsc{Omega} (first column) and \textsc{carlomat} (second column) with cuts given by \( (13) \) and \( (16) \). The numbers in parenthesis show the MC uncertainty of the last decimal.

| \( \sqrt{s} [\text{GeV}] \) | \( \sigma_{\text{leptonic}} \) | \( \sigma_{\text{semileptonic}} \) | \( \sigma_{\text{hadronic}} \) |
|---|---|---|---|
| 500 | 1.12(1) | 3.095(3) | 9.40(1) |
| 800 | 16.86(4) | 46.27(2) | 141.4(1) |
| 1000 | 14.67(4) | 40.18(2) | 122.5(1) |
| 2000 | 5.67(7) | 15.14(3) | 44.58(3) |

Table 4: Signal cross sections in ab of different detection channels of \( (2) \). The cuts are given by \( (13) \), \( (14) \) and \( (15) \) for the leptonic, by \( (13) \), \( (14) \) and \( (16) \) for the semileptonic and by \( (13) \) and \( (16) \) for the hadronic channels. The numbers in parenthesis show the MC uncertainty of the last decimal.

for \( m_H = 115 \text{ GeV} \) is about 50\% bigger than that for \( m_H = 130 \text{ GeV} \), while the cross section for \( m_t = 171 \text{ GeV} \) is smaller smaller than that for \( m_t = 176 \text{ GeV} \).
Table 5: Cross sections in ab of (4) calculated with carlomat with different values of the top quark mass \(m_t\) and the Higgs boson mass \(m_H\) with cuts given by (13), (14) and (16). The numbers in parenthesis show the MC uncertainty of the last decimal.

| \(\sqrt{s} [\text{GeV}]\) | \(m_t [\text{GeV}]\) | \(m_H = 115 \text{ GeV}\) | \(m_H = 130 \text{ GeV}\) |
|-----------------|-----------------|-----------------|-----------------|
|                 | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) | \(\sigma_{\text{signal}}\) | \(\sigma_{\text{all}}\) | \(\sigma_{\text{no QCD}}\) | \(\sigma_{\text{signal}}\) |
| 500             | 39.8(1)         | 20.21(5)        | 14.61(1)        | 29.9(1)         | 10.29(3)        | 4.670(4)        |
| 171             | 35.3(1)         | 16.41(4)        | 11.69(1)        | 26.9(1)         | 7.91(2)         | 3.095(3)        |
| 176             | 33.3(1)         | 14.52(4)        | 10.19(1)        | 25.4(1)         | 6.75(2)         | 2.365(2)        |
| 800             | 118.2(2)        | 84.8(1)         | 63.17(3)        | 99.2(2)         | 66.0(1)         | 44.62(2)        |
| 171             | 119.9(3)        | 86.8(1)         | 65.61(3)        | 100.6(2)        | 67.0(1)         | 46.27(2)        |
| 174.3           | 120.3(3)        | 88.0(1)         | 66.83(3)        | 100.4(2)        | 67.8(1)         | 47.08(2)        |
| 176             | 105.8(3)        | 73.8(1)         | 51.73(2)        | 92.1(2)         | 59.8(1)         | 38.34(2)        |
| 1000            | 108.6(3)        | 75.7(1)         | 54.22(3)        | 94.4(3)         | 61.5(1)         | 40.18(2)        |
| 174.3           | 110.2(3)        | 77.7(1)         | 55.51(3)        | 94.8(2)         | 62.3(1)         | 41.07(2)        |
| 176             | 49.9(2)         | 31.37(9)        | 18.17(5)        | 46.2(2)         | 27.31(7)        | 14.34(4)        |
| 2000            | 51.5(2)         | 32.64(9)        | 19.20(5)        | 47.6(2)         | 28.35(8)        | 15.14(3)        |
| 174.3           | 51.8(2)         | 33.21(9)        | 19.78(5)        | 47.5(2)         | 28.75(9)        | 15.60(3)        |
| 176             |                 |                 |                 |                 |                 |                 |

In order to illustrate the potential effect that the unitarity violating terms discussed in Section 2 may have on the presented results we show in Table 6 the lowest order cross sections of (4) calculated with carlomat for two different values of the Higgs boson mass \(m_H\) and \(m_t = 174.3 \text{ GeV}\) in the CMS and fixed width scheme (FWS). The latter differs from the CMS in the definition of the EW mixing parameter that instead of by Eq. (7) is defined by

\[
\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2},
\]

with \(m_W\) and \(m_Z\) being physical masses of the EW bosons. As the cross sections agree within one standard deviation of the MC integration, we see that the unitarity violation terms are not enhanced in the FWS at the presented values of \(\sqrt{s}\), even though the Ward or Slavnov–Taylor identities are not satisfied in this scheme.

The impact of the ISR is illustrated in Table 7 where we show the cross sections \(\sigma_{\text{all+ISR}}, \sigma_{\text{no QCD+ISR}}\) and \(\sigma_{\text{signal+ISR}}\) for reaction (4) including ISR according to Eq. (8) with the splitting scale \(Q^2 = s\). They have been calculated with the Higgs boson masses \(m_H = 115 \text{ GeV}\) and \(m_H = 130 \text{ GeV}\), and the top quark mass \(m_t = 174.3 \text{ GeV}\). These cross sections should be compared with the Born cross sections at the corresponding centre of mass energy and \(m_t = 174.3 \text{ GeV}\) in Table 5. We see that, as could be expected from Eq. (8), ISR decreases substantially the signal and pure EW cross sections at \(\sqrt{s} = 500 \text{ GeV}\), as due to the photon emission the actual centre of mass energy of \(e^+e^-\) scattering may fall down below the \(t\bar{t}H\) threshold, where the signal cross section \(\sigma_{\text{signal}}\) becomes very small. Note that the
Table 6: Cross sections in ab of (4) in the CMS and FWS calculated with carlomat for two different values of the Higgs boson mass $m_H$ and $m_t = 174.3$ GeV. Cuts are given by (13), (14) and (16). The numbers in parenthesis show the MC uncertainty of the last decimal.

Table 7: Cross sections in ab of (4) including ISR in the LL approximation calculated with carlomat for two different values of the Higgs boson mass $m_H$ and $m_t = 174.3$ GeV. Cuts are given by (13), (14) and (16). The numbers in parenthesis show the MC uncertainty of the last decimal.

4 Summary

We have presented results for the lowest order cross sections, calculated with the complete set of the standard model Feynman diagrams, of all reactions (2) relevant for the associated production of the top quark pair and Higgs boson that can be used for determination of the top–Higgs Yukawa coupling at the $e^+e^-$ linear collider. A comparison with the corresponding
signal cross sections of (1) has shown that the background contributions are large for typical particle identification cuts. In particular, the QCD background contributions are much bigger than it could have been expected taking into account a possibly low virtuality of exchanged gluons. As we have shown elsewhere [2] for a few representative channels of (2) the background can be efficiently reduced by imposing invariant mass cuts allowing for the top and antitop quark, and Higgs boson identification. We have shown that the unitarity violating terms discussed in Section 2 have practically no effect on the lowest order cross sections of (4) at the considered range of $\sqrt{s}$ by comparing the cross sections calculated in the CMS and FWS, where the latter violates gauge invariance.

Moreover, we have included the ISR and illustrated its effects concentrating on a semileptonic reaction (4) for which we have have also discussed the dependence of the cross sections on the Higgs boson and top quark masses. Taking into account cross sections of the different $t\bar{t}H$ detection channels presented in Tables 1–7 the best place to measure the top–Higgs Yukawa coupling seem to be a linear collider operating at the centre of mass energy of about 800 GeV.

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References

[1] A. Djouadi, J. Kalinowski, P.M. Zerwas, Mod. Phys. Lett. A7 (1992) 1765; A. Djouadi, J. Kalinowski, P.M. Zerwas, Z. Phys C54 (1992) 255.

[2] K. Kołodziej, S. Szczypiński, Nucl. Phys. B801 (2008) 153.

[3] S. Moretti, Phys. Lett. B452 (1999) 338.

[4] C. Schwinn, arXiv:hep-ph/0412028.

[5] K. Kołodziej, S. Szczypiński, Acta Phys. Pol. B38 (2007) 3609.

[6] H. Baer, S. Dawson, L. Reina, Phys Rev. D61 (2000) 013002; A. Juste, G. Merino, arXiv:hep-ph/9910301; A. Juste, ECONF C0508141:ALCPG0426, 2005, arXiv:hep-ph/0512246; A. Gay, arXiv:hep-ph/0604034.

[7] K. Kołodziej, Comput. Phys. Commun. 180 (2009), 1671; arXiv:0903.3334. The program can be downloaded from http://kk.us.edu.pl/.

[8] M. Moretti, T. Ohl, J. Reuter, IKDA 2001/06-rev, LC-TOOL-2001-040-rev, hep-ph/0102195-rev; W. Kilian, T. Ohl, J. Reuter, arXiv:0708.4233.
[9] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Nucl. Phys. B560 (1999) 33 and Comput. Phys. Commun. 153 (2003) 462.

[10] W. Beenakker et al., in G. Altarelli, T. Sjöstrand and F. Zwirner (eds.), Physics at LEP2 (Report CERN 96-01, Geneva, 1996), Vol. 1, p. 79, [hep-ph/9602351](https://arxiv.org/abs/hep-ph/9602351).

[11] James Brau, Yasuhiro Okada, Nicholas Walker, et al. [ILC Reference Design Report Volume 1 - Executive Summary], [arXiv:0712.1950](https://arxiv.org/abs/0712.1950).

J.A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group Collaboration], [arXiv:hep-ph/0106315](https://arxiv.org/abs/hep-ph/0106315);

T. Abe et al., [American Linear Collider Working Group Collaboration], [arXiv:hep-ex/0106056](https://arxiv.org/abs/hep-ex/0106056);

K. Abe et al., [ACFA Linear Collider Working Group Collaboration], [arXiv:hep-ph/0109166](https://arxiv.org/abs/hep-ph/0109166).

[12] R.W. Assmann et al. [CLIC Study Team], CERN 2000–008;

H. Braun it et. al. [CLIC Study Group], CERN-OPEN-2008-021, CLIC-Note-764.

[13] Tevatron Electroweak Working Group and CDF Collaboration and D0 Collaboration, [arXiv:0903.2503](https://arxiv.org/abs/0903.2503).