Quantitative complementarity in two–path interferometry

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Abstract

The quantitative formulation of Bohr’s complementarity proposed by Greenberger and Yasin is applied to some physical situations for which analytical expressions are available. This includes a variety of conventional double–slit experiments, but also particle oscillations, as in the case of the neutral–kaon system, and Mott scattering of identical nuclei. For all these cases, a unified description can be achieved including a new parameter, $\nu$, which quantifies the effective number of fringes one can observe in each specific interferometric set–up.

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I. INTRODUCTION

Bohr’s complementarity principle and the closely related concept of duality in interferometric devices are fundamental aspects of quantum mechanics. The well known statement that “the observation of an interference pattern and the acquisition of which-way information are mutually exclusive”, as recently rephrased by Englert [1], has been discussed for many years at a qualitative level. Only recently, quantitative statements for this long known “interferometric duality” [1] have become available. The first successful step in this direction, which is the one to be considered in the present paper, is remarkably simple and due to Greenberger and Yasin [2]. Further extensions and refinements, such as those in the previously mentioned Ref. [1], together with experimental analyses, have appeared more recently and reviewed, for instance, in Refs. [3–5].

The quantitative expression for “interferometric duality” proposed by Greenberger and Yasin [2] reads:

$$P^2 + V_0^2 \leq 1,$$

where the equal sign is valid for pure quantum mechanical states. In the previous expression, $V_0$ is the fringe visibility which quantifies the sharpness or contrast of the interference pattern (a wave–like property), whereas $P$ is the path “predictability” quantifying the a priori knowledge one can have on the path taken by the interfering system (its complementary particle–like property). Since we restrict our analysis to two–path interferometers, $P$ is defined by [2]:

$$P = |w_I - w_{II}|,$$

where $w_I$ and $w_{II}$ are the probabilities for taking each path, $w_I + w_{II} = 1$. As already stated, these are a priori probabilities which depend exclusively on the state of the interfering system and the specific parameters of the experimental set–up; in other words, we discard information improving measurements as contemplated in Ref. [1]. The fringe visibility in Eq. (1.1) is defined in the standard way. It appears in the oscillatory factor of the intensity, $I(y)$, of a generic interference pattern:

$$I(y) = F(y) \{1 + V_0(y) \cos[\phi(y)]\},$$

where $\phi(y)$ is the phase–difference between the two paths, $y$ characterizes the detector position in the interferometric set–up and $F(y)$ is specific for each set–up.

In the most simple analyses, $V_0$ is taken as $y$–independent. But this is often a too idealized assumption because the probabilities $w_I$ and $w_{II}$ for each path generally depend on the detector position. The path predictability is then $y$–dependent, $P(y)$, and so is the fringe visibility, $V_0(y)$, via the generalized version of the complementarity relation (1.1):

$$P^2(y) + V_0^2(y) \leq 1.$$
The purpose of the present paper is to investigate the physical situations for which the expressions for $P(y), V_0(y)$ and $\phi(y)$ can be analytically computed. As far as we know, this includes interference patterns of various types of double–slit experiments, the oscillations due to particle mixing shown, among others, by the neutral–kaon system and the differential cross section for Mott scattering of identical particles or nuclei.

These three kinds of phenomena, pertaining to distinct branches of physics, and the meaning of the variable $y$, linked respectively to a linear position, a time variable or a scattering angle, are remarkably different from each other. In spite of this, we can always obtain a unified description in terms of the same $y$–dependent expressions:

$$V_0(y) = \frac{1}{\cosh(Ay)}, \quad P(y) = |\tanh(Ay)|, \quad \phi(y) = By,$$

where $A$ and $B$ are constants. In deriving Eqs. (1.5) we have assumed that the state entering the interferometer is a pure state. As well known, expression (1.4) is then fulfilled with the equal sign, $\cosh^{-2}(Ay) + \tanh^2(Ay) = 1$, for all $y$. Mott scattering experiments of nuclei or particles with spin $S \neq 0$ are usually performed with unpolarized beams described by a density operator proportional to the identity matrix. In this case, the previous results are modified to:

$$V_0(y) = K \frac{1}{\cosh(Ay)}, \quad P(y) = 1 - K + K |\tanh(Ay)|, \quad \phi(y) = By,$$

with the new constant $K$ ($0 < K < 1$) depending on the mixed state. The expression (1.4) is now satisfied as an inequality, $1 + 2K(1 - K)[|\tanh(Ay)| - 1] < 1$, for all $y$ values.

An interesting consequence of the linear dependence on $y$ of the two arguments $Ay$ [in $V_0(y)$] and $By$ [in the phase $\phi(y)$] is that the oscillatory factor in Eq. (1.3), $I(y)/F(y) = 1 + V_0(y) \cos(By)$, allows for a full characterization of each one of the cases we consider in terms of a single ratio $R \equiv |A/B|$. This permits an easy comparison of quite distinct interferometric experiments (see below) and the definition of a new index:

$$\nu \equiv 0.264 \frac{|B|}{A} = 0.264 \frac{R}{R},$$

specific for each set–up. This index has been defined to estimate the effective number of fringes, i.e., how many of them appear in a given set–up before the visibility decreases by the usual factor of $e$.

II. DOUBLE–SLIT EXPERIMENTS

As this is the best known case, a brief analysis should be sufficient for our present purposes. We consider the set–up of Fig. 1, where a monochromatic plane–wave with wavelength $\lambda = 2\pi/k$ is perpendicularly directed towards a double–slit in close contact with a convergent lens of focal length $f$. The light intensity, $I(y)$, is then detected along the $y$–axis of
a screen perpendicular to the optical axis of the lens and placed at a distance $l$ from the slits–plus–lens assembly. Assuming a perfect transparency through the two identical slits (step–function transmission), the analytic expression for $I(y)$ is well known but it exists only in the Fraunhofer limit, $l = f$, where the beams diffracted from each slit overlap and $V_0 = 1$. Since we are interested in the analytic $y$–dependence of $V_0(y)$, we have to discard this simplest case and follow Bartell’s analysis [6]. This requires to consider identical slits incorporating Gaussian transmission filters, $T(x) = \exp(-x^2/2x_0^2)$, where $x_0$ is the effective width of each slit. If their centers are separated by a distance $d$, the intensity $I(y)$ along the screen coordinate $y$ is given by [6]:

$$I(y) = N e^{-y^2/\sigma^2} \cosh(Ay) \left[ 1 + \frac{1}{\cosh(Ay)} \cos(By) \right], \quad (2.1)$$

where $\sigma^2 \equiv x_0^2(1 - l/f)^2 + l^2/(k^2x_0^2)$ (the second term here accounts for the spreading of the beam) and

$$A = \frac{d}{\sigma^2} \left( 1 - \frac{l}{f} \right), \quad B = \frac{d}{\sigma^2} \frac{l}{kx_0^2}, \quad R = \frac{kx_0^2}{l} \left( 1 - \frac{l}{f} \right). \quad (2.2)$$

As anticipated, $A$ and $B$ are constants and (1.4) is fulfilled with the equal sign since we are dealing with pure states. For future reference and illustrative purposes, we have plotted the oscillatory factor (inside square brackets) of Eq. (2.1) in Fig. 2 for the following set of parameters: $k = 10^7 m^{-1}$, $x_0 = 10^{-4}$ m, $d = 3 \cdot 10^{-3}$ m, $l = 0.1$ m and $f = 0.11$ m. These values imply $R \approx 0.10$ and an effective number of fringes $\nu \approx 2.6$, as shown in Fig. 2.

![Fig. 1. Schematic double–slit set–up considered by Bartell in Ref. [6] producing the interference pattern of Eq. (2.1). For details, see the main text.](image)

The need to consider a Gaussian profile for the beams suggests the use of laser light. In Fig. 3 we have sketched a possible experimental set–up with a symmetrical beam–splitter and two mirrors recombining the beams on a screen. When the latter is at $L = 0$, the two beams completely overlap and produce interference fringes centered at $y = 0$ of maximal visibility $V_0(y) = 1$. When the screen is displaced ($L \neq 0$) the centers of the two light spots separate symmetrically along the $y$–axis and the intensity $I(y)$ is given by Eq. (2.1) with $\sigma^2 = x_0^2$, $A = 2L \sin \theta/x_0^2$, $B = 2k \sin \theta$ and $R = L/(kx_0^2)$. With $k = 10^7 m^{-1}$, $x_0 = 10^{-4}$ m and $L = 0.01$ m one obtains again the same $R \approx 0.10$ and $I(y)$ as before.
FIG. 2. The function $I(x)/F(x) = 1 + \cos x / \cosh(Rx)$ is plotted for different values of $R = |A/B|$. The value $R = 1$ illustrates the neutral–kaon case of Section III. The value $R = 0.3$ refers to Mott scattering of $\alpha$ particles of Section IV. $R = 0.1$ corresponds to the double–slit examples of Section II, as well as to Mott scattering for the nuclei considered in Section IV. Note that any value of $R$ can be obtained with the double–slit set–ups by appropriate choices of the experimental parameters. Values of $I(x)/F(x)$ between the horizontal lines at $1 - 1/e$ and $1 + 1/e$ correspond to $|\cos x| / \cosh(Rx) \leq 1/e$. The vertical arrows correspond to the values of $\nu = 0.264/R$ estimating the number of observable oscillations and $x$ is in units of $2\pi$.

In principle, neutron interferometric experiments with a double–slit, such as those described in Ref. [7], could also be considered except for the fact that the beam profile is not necessarily Gaussian and so no analytic expression for $I(y)$ can be derived away from the Fraunhofer limit. In practice, there is also the problem of correcting our idealized predictions by the various effects present in the real experiment. As a first attempt, we have considered that all these corrections can be simulated by a convolution of our predictions with an additional Gaussian distribution. Once the single–slit data of Ref. [7] are adjusted in this way, the same convolution is seen to correct our ideal prediction for the double–slit and a semiquantitative agreement with the data of Ref. [7] is obtained. Quantitative agreement can also be achieved as recently shown in Ref. [8], but such an analysis goes beyond the scope of the present paper. Other neutron interferometric experiments, such as those recently reviewed by Rauch and Werner [9], could be treated along the same lines once the corresponding analytical expressions become available.
III. PARTICLE OSCILLATIONS

Particle–antiparticle oscillations are known to take place, among others, in the $K^0$–$\bar{K}^0$ system. We restrict our discussion to this case because it has been considered as the archetypal example [10] and studied in detail (for a classical review, see Kabir’s book [11]; for recent quantum mechanical results, see Ref. [12]). However, our findings can be immediately extrapolated to the $B^0$–$\bar{B}^0$, $D^0$–$\bar{D}^0$, . . . systems. Neutral kaons are copiously produced by strangeness conserving strong interactions and so initially appear either as $K^0$’s (strangeness +1) or $\bar{K}^0$’s (strangeness −1). But time–evolution in free space is governed by the $\{K_S, K_L\}$ basis states diagonalizing the weak Hamiltonian. For these short– and long–lived states one has:

\begin{equation}
|K_S\rangle = e^{-i\lambda_S t}|K_S\rangle, \quad |K_L\rangle = e^{-i\lambda_L t}|K_L\rangle,
\end{equation}

where $\lambda_{S,L} \equiv m_{S,L} - (i/2)\Gamma_{S,L}$ and $m_{S,L}$ and $\Gamma_{S,L}$ are the kaon masses and decay widths.

The time–evolution of initial $K^0$ or $\bar{K}^0$ states is then given by:

\begin{align}
|K^0\rangle &\rightarrow |K^0(t)\rangle = \frac{1}{\sqrt{2(1 + \epsilon)}} \left[ e^{-i\lambda_S t}|K_S\rangle + e^{-i\lambda_L t}|K_L\rangle \right], \\
|\bar{K}^0\rangle &\rightarrow |\bar{K}^0(t)\rangle = \frac{1}{\sqrt{2(1 - \epsilon)}} \left[ e^{-i\lambda_S t}|K_S\rangle - e^{-i\lambda_L t}|K_L\rangle \right].
\end{align}

The CP–violation effects can safely be neglected ($\epsilon = 0$) for our purposes thus obtaining $\langle K_S|K_L\rangle = 0$ and

\begin{align}
|K^0(t)\rangle &= \frac{1}{\sqrt{2}} \left[ \frac{1 + e^{-\frac{1}{2}\Delta\Gamma t}e^{-i\Delta m t}}{\sqrt{1 + e^{-\Delta\Gamma t}}} |K^0\rangle + \frac{1 - e^{-\frac{1}{2}\Delta\Gamma t}e^{-i\Delta m t}}{\sqrt{1 + e^{-\Delta\Gamma t}}} |\bar{K}^0\rangle \right], \\
|\bar{K}^0(t)\rangle &= \frac{1}{\sqrt{2}} \left[ \frac{1 - e^{-\frac{1}{2}\Delta\Gamma t}e^{-i\Delta m t}}{\sqrt{1 + e^{-\Delta\Gamma t}}} |K^0\rangle + \frac{1 + e^{-\frac{1}{2}\Delta\Gamma t}e^{-i\Delta m t}}{\sqrt{1 + e^{-\Delta\Gamma t}}} |\bar{K}^0\rangle \right],
\end{align}

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when reverting to the \{K^0, \bar{K}^0\} basis. This is necessary to discuss strangeness measurements projecting into one of these two basis states. In Eqs. (3.3) we have defined \( \Delta m = m_L - m_S \), \( \Delta \Gamma = \Gamma_L - \Gamma_S \) and normalized to undecayed states at time \( t \).

Strangeness oscillations in \( t \) of the \(|K^0(t)\rangle\) and \(|\bar{K}^0(t)\rangle\) states are easily deducible from Eqs. (3.3). The oscillation phase is given by \( \phi(t) = \Delta m t \) and the time dependent visibility by:

\[
V_0(t) = \frac{1}{\cosh \left( \frac{1}{2} \Delta \Gamma t \right)}. \tag{3.4}
\]

We then recover the structure of Eqs. (1.3) and (1.5) with:

\[
A = \frac{1}{2} \Delta \Gamma, \quad B = \Delta m, \quad R = \frac{|\Delta \Gamma|^2}{2 \Delta m}, \tag{3.5}
\]

and from the experiment one has \( |\Delta \Gamma|/(2 \Delta m) \simeq 1.05 \). These results and the discussion in the previous paragraph clearly show the interferometric characteristics of neutral kaon propagation in free space, where the \( K_S \) and \( K_L \) propagating components play the role of the two interferometric paths \([13,14]\). The path–predictability \( \mathcal{P}(t) \) can be computed once one knows that the state has survived up to time \( t \). The larger is \( t \), the more probable is the propagation of the \( K_L \) component (\( \Gamma_S \simeq 579 \Gamma_L \)). One thus gets:

\[
\mathcal{P}(t) = \left| \frac{1}{1 + e^{-\Delta \Gamma t}} - \frac{1}{1 + e^{\Delta \Gamma t}} \right| = \left| \tanh \left( \frac{1}{2} \Delta \Gamma t \right) \right|. \tag{3.6}
\]

Again, one satisfies Eq. (1.4), \( \mathcal{P}^2(t) + V_0^2(t) = 1 \), as expected for pure states like \(|K^0(t)\rangle\) and \(|\bar{K}^0(t)\rangle\).

\( B^0 - \bar{B}^0 \) oscillations have also been observed in recent experiments leading to compatible values for \( \Delta m_B \) (see, for instance, Ref. [15]). The available data are consistent with \( \Delta m_B >> \Delta \Gamma_B \), so that \( R << 1 \) and \( \nu >> 1 \).

**IV. MOTT SCATTERING**

For energies below the Coulomb barrier, the scattering of two identical nuclei is elastic and exclusively due to electrostatic interactions. The differential cross section can be analytically computed by suitably modifying Rutherford’s formula. This leads to the well known Mott’s differential cross section:

\[
d\sigma/d\Omega = \left( \frac{Z^2 e^2}{4E} \right)^2 \left\{ \frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} + C_S \frac{2}{\sin^2(\theta/2) \cos^2(\theta/2)} \cos[\eta \ln \tan^2(\theta/2)] \right\}, \tag{4.1}
\]

where \( Ze \) and \( S \) are the nuclear charge and spin, whereas \( E \) and \( \theta \) are the center–of–mass energy and scattering angle. The first two terms inside brackets correspond to the squared moduli of the Rutherford scattering amplitudes at angles \( \theta \) and \( \pi - \theta \), respectively. The
third term comes from the interference of these two amplitudes and contains the factor $C_S$ accounting for spin effects and the opposite sign from boson and fermion statistics. The two amplitudes can be associated with the direct and crossed Feynman diagrams, which play the same role as the two paths in conventional double-slit interferometry. The phase difference appears in the interfering term and depends on the Sommerfeld parameter:

$$\eta = Z^2 \alpha \sqrt{\frac{Mc^2}{2E}},$$  

with $M$ being the mass of the nucleus and $\alpha$ the fine structure constant.

In spite of the obvious differences between this situation and those previously discussed, it is quite easy to express the $\theta$–dependent differential cross section $d\sigma/d\Omega$ in the form of Eq. (1.3). One needs the change of variable:

$$e^x \equiv \tan^2(\theta/2) = \frac{1 - \cos \theta}{1 + \cos \theta},$$  

with $x$ ranging from 0 to $\infty$, as $\theta$ runs from $\theta = \pi/2$ to $\theta = \pi$. Because of the obvious symmetry of $d\sigma/d\Omega$, negative values of $x$ cover the range $0 \leq \theta < \pi/2$. The new variable $x$ allows one to rewrite the relevant part of Eq. (4.1):

$$\frac{d\sigma}{d\Omega} \propto 1 + \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} C_S \cos[\eta \ln \tan^2(\theta/2)],$$  

as:

$$I(x) \propto 1 + \frac{C_S}{\cosh x} \cos(\eta x),$$  

which has the same structure as Eqs. (1.3) and (1.5) with:

$$A = 1, \quad B = \eta, \quad R = 1/\eta.$$  

The remaining parameter $C_S$ depends on the nuclear spin $S$ and is discussed in the following two paragraphs.

The simplest case to analyze corresponds to the elastic scattering of spin–zero nuclei, for which one obtains $C_0 = 1$ in Eq. (4.1). The same result holds for beams of spin–$S$ nuclei polarized in the same direction. The equal spin components of the two colliding nuclei do not alter their indistinguishability. Thus for $\theta = \pi/2$ ($x = 0$) the two amplitudes contribute with the same probability $w_I = w_{II} = w(\pi/2) = 1/2$, $\mathcal{P}(x = 0) = 0$ and one has maximal visibility, $\mathcal{V}_0(x = 0) = 1$. For $\theta > \pi/2$ ($x > 0$), $w_I = w(\theta)$ and $w_{II} = w(\pi - \theta)$ are given by the Rutherford formula and imply $\mathcal{P}(\theta) = 2|\cos \theta|/(1 + \cos^2 \theta)$ or $\mathcal{P}(x) = \tanh x$. In Fig. 4 we show the dependence of $\mathcal{P}^2(\theta)$ and $\mathcal{V}^2_0(\theta) = (1 - \cos^2 \theta)^2/(1 + \cos^2 \theta)^2$ on $\cos \theta$. These cases admit thus a complete description in terms of pure states [see Eq. (1.5)].

This is not the case for unpolarized beams of spin–$S$ nuclei. The $C_S$ factor appearing in the Mott cross section (4.1) is known to be given by $C_S = (-)^{2S}/(2S + 1)$ and the
path predictability can be easily computed. Indeed, among the $(2S + 1)^2$ equally probable spin combinations one can have, $(2S + 1)$ of them correspond to cases where both nuclei have identical spin components; this implies no additional which–path information and the predictability is the same as in the spinless case. For the remaining $2S(2S + 1)$ cases, the paths of either nuclei are “marked” by their distinct spin components and the predictability is one. One thus obtains $P(x) = (2S + |\tanh x|)/(2S + 1)$ and $K = |C_S| = 1/(2S + 1)$ as required [see Eq. (1.6)].

Accurate data for Mott scattering below the Coulomb barrier have been obtained in several experiments long time ago. In Ref. [16], $\alpha-\alpha$ scattering was measured at center–of–mass energies $E = 75, 150$ and 200 keV. Those by Bromley et al. [17] refer to $^{12}C + ^{12}C$ at $E = 3$ and 5 MeV and to $^{16}O + ^{16}O$ at $E = 7, 8.8$ and 10 MeV. In both cases one has $S = 0$ and the data are perfectly fitted by the theoretical curves. The oscillatory factors for some of these curves are plotted in Fig. 5. For $S = 1/2$ nuclei we have recent data on $^{13}C + ^{13}C$ scattering at $E = 75$ keV [18]. These data agree with $V_0(x) = 1/(2\cosh x)$ as required by Eq. (1.6) for $K = 1/2$.

V. CONCLUSIONS

We have explored the possibility of achieving a unified description for a series of phenomena belonging to distinct fields of physics but admitting an analogous treatment in terms of basic concepts of two–path interferometry. This includes the quantitative expression for the interferometric duality (1.1), originally proposed by Greenberger and Yasin, which has been generalized here to account for the dependence of the fringe visibility $V_0(y)$ and the path
FIG. 5. Plot of the function $I(x)/F(x) = 1 + \cos x / \cosh(Rx)$ for values of $R = 1/\eta$ corresponding to some of the experiments reported in Refs. [16,17], where Mott formula was successfully tested for $\text{He}^4$, $\text{C}^{12}$ and $\text{O}^{16}$ spin–zero nuclei. Values of $I(x)/F(x)$ between the horizontal lines at $1 - 1/e$ and $1 + 1/e$ correspond to $|\cos x| / \cosh(Rx) \leq 1/e$. The vertical arrows correspond to the quantity $\nu = 0.264/R$ giving the number of observable oscillations.

predictability $\mathcal{P}(y)$ on the detector position $y$. We have achieved this unified description for the cases where the $y$–dependence in $V_0(y)$ and $\mathcal{P}(y)$, as well as the one in the relative phase $\phi(y)$, can be analytically computed. Besides conventional double–slit phenomena, this includes particle oscillations —as seen, among others, in the neutral kaon system— and the interference effects in the differential cross section of Mott scattering for identical nuclei. The relevant aspects of the interferometric behaviour of these various types of phenomena can then be described within a global framework and in terms of a unified expression containing a new parameter, $\nu$. This parameter characterizes every specific experimental set–up and estimates the effective number of visible fringes.

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