A Jet Source of Event Horizon Telescope Correlated Flux in M87

Brian Punsly

1415 Granvia Altamira, Palos Verdes Estates, CA 90274, USA
ICRANet, Piazza della Repubblica 10 Pescara I-65100, Italy; brian.punsly1@cox.net

Received 2017 August 3; revised 2017 October 21; accepted 2017 October 23; published 2017 December 1

Abstract

Event Horizon Telescope (EHT) observations at 230 GHz are combined with Very Long Baseline Interferometry (VLBI) observations at 86 GHz and high-resolution Hubble Space Telescope optical observations in order to constrain the broadband spectrum of the emission from the base of the jet in M87. The recent VLBI observations of Hada et al. provide much stricter limits on the 86 GHz luminosity and component acceleration in the jet base than were available to previous modelers. They reveal an almost hollow jet on sub-mas scales. Thus, tubular models of the jet base emanating from the innermost accretion disk are considered within the region responsible for the EHT correlated flux. There is substantial synchrotron self-absorbed opacity at 86 GHz. A parametric analysis indicates that the jet dimensions and power depend strongly on the 86 GHz flux density and the black hole spin, but depend weakly on other parameters, such as jet speed, 230 GHz flux density, and optical flux. The entire power budget of the M87 jet, \(<10^{44} \text{ erg s}^{-1}\), can be accommodated by the tubular jet. No invisible, powerful spine is required. Even though this analysis never employs the resolution of the EHT, the spectral shape implies a dimension transverse to the jet direction of 12–21 \(\mu\text{as} (\sim 24–27 \mu\text{as})\) for 0.99 > \(a/M\) > 0.95 (\(a/M \sim 0.7\)), where \(M\) is the mass and \(a\) is the angular momentum per unit mass of the central black hole.

Key words: accretion, accretion disks – black hole physics – galaxies: active – galaxies: jets – quasars: general

1. Introduction

The Event Horizon Telescope (EHT) is a global Very Long Baseline Interferometer (VLBI) that can achieve \(\sim 25 \mu\text{as}\) resolution at 230 GHz (Krichbaum et al. 2015). The 86 GHz VLBI has far superior imaging capabilities at the expense of lower resolution, \(\sim 60 \mu\text{as}\) (Kim et al. 2016; Akiyama et al. 2017). In this analysis, the capabilities of EHT to describe the base of the jet in M87 is enhanced by combining these observations with 86 GHz VLBI and high-resolution optical observations with the Hubble Space Telescope (HST). 86 GHz VLBI observations reveal a hollow jet on sub-mas scales (Hada et al. 2016). Thus, squat tubular models of the jet base within the compact region producing the EHT correlated flux (referred to as the EHT core, or the EHTC) are studied in this article. By constraining the broadband spectrum with 86 GHz VLBI and optical images of the M87 jet, \(\sim 10^{44} \text{ erg s}^{-1}\) is the mass and \(\mu\) is the angular momentum per unit mass of the central black hole.

2. Tubular Jet Models

This section describes physics relevant to the tubular jet model. The basic model is a tubular geometry with an inner radius at the innermost stable orbit (ISCO) and an outer radius, \(R\), with a fiducial height \(H = 2R\) that is allowed to vary in some of the models (see Figure 1). The rest-frame evaluated number density and the vertical poloidal magnetic field that is anchored in the equatorial plane (\(N\) and \(B^\phi\), respectively) are both constant throughout the volume.

2.1. Synchrotron Emission and Absorption

The underlying power law for the flux density is defined by

\[
F_\nu (\nu = \nu_0) = F_\nu^0 = \frac{3e}{2\pi m_e c^2} \alpha \left(\frac{\nu}{\nu_0}\right)^{1+\alpha},
\]

\[
g(n) = \sqrt{3\pi} \frac{\Gamma[(3n+2)/12] \Gamma[(3n+2)/12] \Gamma[(n+6)/4]}{\Gamma[(n+8)/4]},
\]

\[
N = \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} N_f \Gamma^{-n} d\nu, \quad n = 2\alpha + 1,
\]

where \(\Gamma\) is the gamma function. \(B\) is the magnitude of the total magnetic field. The
power law spectral index for the flux density is \( \alpha = (n - 1)/2 \). The low-frequency VLBI data do not strongly constrain the low-energy cutoff, \( E_{\min} = \Gamma_{\text{min}} m_e c^2 \), due to insufficient spatial resolution. Thus, no results that depend on \( \Gamma_{\text{min}} \) are discussed in this paper. There are still many interesting conclusions that can be drawn from the models. The high-energy cutoff, \( E_{\max} = \Gamma_{\text{max}} m_e c^2 \), might be revealed by the ultraviolet (UV) HST observations (see Section 3.1), but this information is never needed in the analysis. The conversion to the observer’s frequency, \( n_o \), is given by 
\[
    n_o = n_{nd} \delta,
\]
where \( \delta \) is the total Doppler factor that includes gravitational redshift and relative motion. The SSA opacity in the observer’s frame, \( \mu(n_o) \), is obtained with the direct substitution of \( n_{nd} = n_o \) into Equation (1). The homogeneous approximation yields a simplified solution to the radiative transfer equation for the intensity, \( I_n \), from the SSA source (Ginzburg & Syrovatskii 1965),
\[
    I_n(v) = J_n(v) \frac{\mu(v)}{1 - e^{-\mu(v)L}},
\]
where \( L \) is the distance traversed by the radiation through the plasmoid and the synchrotron emissivity is given in Tucker (1975) as
\[
    J_n = 1.7 \times 10^{-21}[4\pi N_f] a(n) B^{1+\alpha} \left( \frac{4 \times 10^6}{\nu} \right)^\alpha,
\]
where \( n_o = \frac{2^{n+1}/\sqrt{3} \Gamma(3n-1)/12 \Gamma(3n+19)/12 \Gamma(n+5/4)}{8\sqrt{\pi} (n+1) \Gamma(n+7/4)} \).

Figure 1. Details of the uniform tubular plasmoid model. The example chosen here is the fiducial model with \( H/R = 2 \) and \( a/M = 0.95 \).

One can transform this to the observed flux density, \( S(\nu_o) \), in the optically thin region of the spectrum (for M87 in the infrared (IR) and optical), using the relativistic transformation relations from Lind & Blandford (1985),
\[
    S(\nu_o) = \frac{\delta^{(3+\alpha)}}{4\pi D_L^2} \int \nu'_i dV',
\]
where \( D_L \) is the luminosity distance, and in this expression, the primed frame is the rest frame of the plasma.

### 2.2. Relativistic Considerations

Calculations are computed on the background of the Kerr metric (that of a rotating uncharged BH), with mass, \( M \), and angular momentum per unit mass, \( a \). In Boyer–Lindquist coordinates, \( g_{\mu\nu} \) is given by the line element
\[
    ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 + \rho^2 d\theta^2 \
    + \left(\frac{\rho^2}{\Delta}\right)dr^2 + \frac{4Mra}{\rho^2} \sin^2 \theta d\phi dt + \left[(r^2 + a^2) + \frac{2Mra}{\rho^2} \sin^2 \theta\right] \sin^2 \theta d\phi^2,
\]
where \( \rho^2 = r^2 + a^2 \cos^2 \theta \) and \( \Delta = r^2 - 2Mr + a^2 \). The event horizon is defined by \( r = M + \sqrt{M^2 - a^2} \). The magnetic field, \( B^\phi \), is vertical and anchored in the equatorial plane. It is assumed to rotate with an angular velocity as viewed from asymptotic infinity, \( \Omega_e \approx \Omega_{\text{kep}} \), the Keplerian angular
velocity at the foot point (at \( r = r_0 \)) of the field lines in the equatorial plane,
\[
\Omega_{\text{kep}}(r_0) = \frac{M^{0.5}}{r_0^{1.5} + aM^{0.5}}.
\]  
(9)

It should be noted in the following that the system might rotate slightly slower due to magnetic torques. Each field line is defined as vertical in terms of the Boyer–Lindquist coordinate system. There is a constant coordinate displacement from the vertical axis that is expressed by \( r \sin \theta = r_0 \).

It is useful to define an orthonormal “corotating” frame (corotates with the foot point, which is designated with a prime) for ease of calculation with a 4-velocity
\[
e_0' = \alpha_{\text{kep}}^{-1} \left( \frac{\partial}{\partial t} + \Omega_{\text{kep}}(r_0) \frac{\partial}{\partial \phi} \right)
\]
\[
\alpha_{\text{kep}} = \sqrt{-g_{tt} - 2 \Omega_{\text{kep}}(r_0) g_{t\phi} - \Omega_{\text{kep}}(r_0)^2 g_{\phi\phi}},
\]  
(10)
where \( \alpha_{\text{kep}} \) is the gravitational redshift of the corotating frame, with respect to the stationary frames at asymptotic infinity. For global calculations, we use the hypersurface orthogonal, orthonormal Zero Angular Momentum Observer (ZAMO) frames
\[
\hat{e}_r = a Z \left( \frac{\partial}{\partial t} + \Omega_Z \frac{\partial}{\partial \phi} \right), \quad \hat{e}_\theta = \frac{\Delta \sin \theta}{\sqrt{g_{\theta\theta}}},
\]
\[
\hat{e}_\phi = \frac{1}{\sqrt{g_{\phi\phi}}} \frac{\partial}{\partial \phi}, \quad \hat{e}_r = \left( \frac{1}{\rho} \right) \frac{\partial}{\partial r}, \quad \hat{e}_\theta = \left( \frac{1}{\rho} \right) \frac{\partial}{\partial \theta},
\]  
(11)

The boost to the orthonormal corotating frame is
\[
\gamma_{\text{Kep}}^\phi = \left[ \Omega_{\text{kep}}(r_0) - \Omega_Z \right] \sqrt{g_{\theta\theta}} / \alpha_Z,
\]
\[
\gamma_{\text{Kep}} = \alpha_Z / \alpha_{\text{kep}} = \left[ 1 - \left( \nu_{\text{Kep}}^\phi / c \right)^2 \right]^{-0.5},
\]  
(12)
i.e., \( \gamma_{\text{Kep}}B_{\text{ZAMO}}^\phi = B_{\text{ZAMO}}^\phi \) and in Equation (7), \( dV' \approx \gamma_{\text{Kep}} dV_{\text{ZAMO}} \).

Consider the frozen-in condition applied to the toroidal magnetic field in the ZAMO frame,
\[
B_{\text{ZAMO}}^\phi / B_{\text{ZAMO}}^p = (\nu_{\text{ZAMO}}^\phi - \nu_F^\phi) / \nu_{\text{ZAMO}}^\phi.
\]  
(13)
where \( \nu_F^\phi \) is the azimuthal velocity of the field, and \( \nu_{\text{ZAMO}}^\phi \) and \( \nu_{\text{ZAMO}}^\phi \) are the azimuthal velocity and vertical velocity of the bulk flow of plasma (Punsly 2008). In the region of jet initiation, \( \nu_{\text{ZAMO}}^\phi \) is considered to be nonrelativistic. This is motivated theoretically as a boundary condition in the equatorial plane and in VLBI observations that indicate apparent velocities, \( \nu_{\text{app}} = 0.1-0.4c \), which are \( < 2 \) mas from the core. Furthermore, \( \nu_{\text{app}} \) increases from \( \sim 0.15c \) to \( 1.5-2.0c \) in the first 10 mas, indicating strong magnetic forces and requiring the jet base to be magnetically dominated (Hada et al. 2016, 2017; Mertens et al. 2016). In this Poynting flux dominated regime, the conservation of angular momentum condition provides a constraint on the azimuthal magnetic field,
\[
\alpha_Z \sqrt{g_{\theta\theta}} B_{\text{ZAMO}}^\phi \approx - \frac{\Omega_{\text{kep}} \Phi}{k_F c} = \text{constant},
\]  
(14)
where \( \Phi \) is the total poloidal flux contained within the cylindrical radius, and \( k_F \) is a geometrical factor that equals \( \pi \) for the assumed uniform cylindrical asymptotic jet (Punsly 2008). From Equations (8), (11), and (14),
\[
B_{\text{ZAMO}}^\phi \approx - \frac{\Omega_{\text{kep}}(r_0)\Phi}{\Delta / \sin \theta \pi c}.
\]  
(15)

From Equations (13)–(15) and assuming that \( \nu_F \sim 0.1c \), the angular velocity of the plasma, as viewed from asymptotic infinity, \( \Omega_p = \phi / dt \), in the tubular plasmoid is in approximation corotation with the field lines,
\[
\left| \frac{\Omega_p - \Omega_{\text{kep}}(r_0)}{\Omega_{\text{kep}}(r_0)} \right| \approx \left| - \frac{v_{\text{ZAMO}}^\phi / c}{\alpha_Z} \right| \sim \left| -0.1 \alpha_Z \right| \ll 1,
\]  
(16)
where \( \alpha_Z \) is the cross-sectional area of the plasmoid.

The Doppler factor is computed in a two-step process (Lightman et al. 1975). Consider a plasmoid that is moving along the vertical axis (perpendicular to the plane of rotation), with a velocity \( \nu_F^\phi \), as measured in the corotating frame and a bulk Lorentz factor, \( \gamma \). First, compute the Doppler shift in the corotating frame due to relative motion if a photon is emitted from the plasmoid at an angle, \( \psi \), along the line of sight (LOS), where \( \psi \) is measured relative to the direction of bulk motion in the corotating frame. Second, consider the gravitational redshift of corotating frame in Equation (10) to find
\[
\delta = \alpha_{\text{kep}} / \gamma (1 - (\nu_F^\phi / c) \cos \psi).
\]  
(17)
In the following, we will assume a value of \( \psi = 15^\circ \), which is consistent with most of the observed jet motion (Stawarz et al. 2006). This exact value does not affect the results significantly because the putative plasma flow is subrelativistic.

The MHD Poynting flux in the magnetically dominated limit is
\[
\int S^\phi dA_{\perp} = \frac{k_T^2 \Phi^2}{2 \pi^2 c}.
\]  
(18)
where \( k_T \) is a geometrical factor that equals 1 for a uniform, highly collimated jet (Punsly 2008).

The simple parametric form of the spectrum in Equation (4) cannot be produced by a homogeneous distribution of plasma due to the spatial gradients in the gravitational redshift in the expression for \( \delta \) in Equation (17). The value of \( \alpha_{\text{kep}} \) varies throughout the volume. Homogeneity is regained by implementing a single value of \( \alpha_{\text{kep}}^{(2.5+\alpha)} \) that occurs in Equation (1) when it is written in terms of \( \nu_0 \) and a single value of \( \gamma_{\text{Kep}} P_{\text{Kep}}^{(3+\alpha)} \) in Equation (7) throughout the compact calculation volume. Employing the volumetric average of these values instead of the exact coordinate dependent values is a major simplifying approximation used in the calculation. The volumetric average of \( B^\phi \) is also implemented in the computation of \( \mu \) and \( J_z \). These averages and constant plasma parameters result in simple radiative transfer solutions as in Equation (4).

### 3. Constructing Models of the Jet Base

In this section, the formalism described in the last section is used to construct tubular models of the jet base. The first subsection describes the observational data that is used to constrain the models. The second subsection describes the
models in detail and the resultant plasma state of the jet base. Table 1 describes the parametric analysis of various fits to the data and BH states.

### 3.1. Constraining the Broadband Spectrum

In this section, the broadband spectrum of the jet base is constrained. There are three portions of the spectrum to consider: the EHT data that is located near the peak of the spectrum, the 86 GHz VLBI data that constrains the SSA opacity, and the high-frequency synchrotron tail is constrained by $HST$ high-resolution optical/IR photometry. The 86 GHz and optical data are of much lower resolution and are considered as upper limits on the flux from the jet base.

There have been two published EHT detections of correlated flux of M87, which are shown in Figure 2. The observations in 2009 and 2012 were fit with $980 \pm 40$ mJy and $980 \pm 50$ mJy of correlated flux within a circular Gaussian component of FWHM of 40 and 330 $\mu$as, respectively (Doeleman et al. 2012; Akiyama et al. 2015). An exact value of correlated flux is not utilized in the following text (see Section 3.2).

The only published fit to the core with the 86 GHz VLBI is 669 mJy in an elliptical Gaussian fit of 0.081 mas $\times$ 0.062 mas from the 2014 observations (Hada et al. 2016). These dimensions are larger than the EHT fit, and the observation is not contemporaneous. Furthermore, it is not clear if it arises from the same region as the EHTC due to SSA opacity. There is likely a significant fraction of $F_c (\nu_0 = 86$ GHz) located within the EHTC, since an extrapolation of the SSA core shift analysis of Hada et al. (2011) indicates that the EHTC is only $\sim 10\mu$as from the 86 GHz core. The precise fraction of $F_c (\nu_0 = 86$ GHz) within the EHTC cannot be determined, as long as the 86 GHz VLBI baselines are restricted to Earth. Thus, various values are chosen in the nine fits in Figure 2 and Table 1 in order to explore the dependence on the plasma composition at the base of the jet on $F_c (\nu_0 = 86$ GHz). Due to the uncertainty, 669 mJy is considered only as a crude upper limit from which to start the parametric variation of $F_c (\nu_0 = 86$ GHz) from the EHTC.

The highest-resolution $HST$ optical/UV imaging achieves a resolution of $\sim 100$–130 mas (Chiaberge et al. 1999). The published results, corrected for Galactic extinction, are plotted in Figure 2 (Chiaberge et al. 1999, 2002; Prieto et al. 2016).

The data is nonsimultaneous and is distributed from 1995 to 2003. The variability implied by the scatter indicates a factor of $<2$ variability over time. Compared to the 0.040 mas EHTC, 130 mas is large. However, inspections of 5 GHz (Hada et al. 2014), 15 GHz (Lister et al. 2013), 43 GHz (Hada et al. 2014; Mertens et al. 2016), and 86 GHz (Kim et al. 2016; Hada et al. 2016) VLBI images indicate that there is no strong optically thick component between 0.06 mas and 100 mas that could produce a significant contribution to the optical/UV flux density: the unresolved 86 GHz core is the only possible source of the $HST$ detected flux. We can get a tight bound on the high-frequency synchrotron tail by assuming that most of this emission is associated with the smaller EHTC. This assumption has been made previously (Dexter et al. 2012). These data are plotted in Figure 2, as well as the 11.7$\mu$ Mid-IR flux density, with 400 mas resolution that provides a loose upper bound on the synchrotron tail in the gap in spectral coverage (Whysong & Antonucci 2004).

### 3.2. Explicit Models for the Spectral Fits to the EHTC

The homogeneity produced by the volumetric averages described at the end of Section 2 simplifies the radiative transfer equation, allowing for a solution of the form of Equation (4) for every path through the plasmoid with the same $\mu (\nu)$, $I_\nu$, and $\delta$. The various models are described in Table 1. Each “model” has a preassigned $\alpha$, $a$, $v^2$, and $H/R$ (the last four columns in Table 1), plus the LOS, $\psi = 15^\circ$. Each model has a corresponding fit to the data in Figure 2, indicated in columns 2–4 in Table 1 by the three flux densities. The term model represents an infinite number of degenerate solutions as indicated by the curves in Figures 3 and 4. Due to the homogeneous approximation and the volumetric averages described at the end of Section 2, the spectrum will depend on the uniform values of $\mu (\nu)$, $I_\nu (\nu)$, $\delta$, $\psi$, $\alpha$, $H$, and $R$; there are seven parameters. The models have the five preassigned values and three free variables $N$, $B^p$, and $R$. Thus, eight model values are used to solve for the seven parameters that

| Model | 230 GHz $E_c$(mJy) | 86 GHz $E_c$(mJy) | $3.72 \times 10^{14}$ Hz $F_c$(mJy) | $\alpha$ | $a/M$ | $v^2/c$ | Jet Length ($H$) |
|-------|-------------------|------------------|--------------------------------|--------|-------|---------|-----------------|
| A     | 830               | 450              | 0.5                            | 1.01   | 0.99  | 0.1     | 2R              |
| B     | 830               | 450              | 0.5                            | 1.01   | 0.95  | 0.1     | 2R              |
| C     | 830               | 450              | 0.5                            | 1.01   | 0.95  | 0.1     | 2R              |
| D     | 830               | 450              | 0.5                            | 1.01   | 0.95  | 0.1     | 2R              |
| E     | 830               | 450              | 0.5                            | 1.01   | 0.95  | 0.1     | 2R              |
| F     | 830               | 450              | 0.75                           | 0.96   | 0.95  | 0.1     | 2R              |
| G     | 830               | 450              | 0.37                           | 1.06   | 0.95  | 0.1     | 2R              |
| H     | 830               | 550              | 0.5                            | 1.01   | 0.95  | 0.1     | 2R              |
| I     | 830               | 350              | 0.5                            | 1.02   | 0.95  | 0.1     | 2R              |
| J     | 980               | 450              | 0.5                            | 1.04   | 0.95  | 0.1     | 2R              |
| K     | 630               | 450              | 0.5                            | 0.98   | 0.95  | 0.1     | 2R              |
| L     | 500               | 350              | 0.5                            | 0.94   | 0.95  | 0.1     | 2R              |
| M     | 330               | 350              | 0.5                            | 0.88   | 0.95  | 0.1     | 2R              |
| N     | 820               | 540              | 0.5                            | 1.01   | 0.95  | 0.1     | 4R              |
| O     | 820               | 560              | 0.5                            | 1.01   | 0.95  | 0.1     | 8R              |

1 Formally, the variable $N$ that is defined in Equation (3) is a surrogate for the variable $N$, since Equations (1) and (5) only depend on $N$. As discussed in reference to Equation (3), since the low-energy cutoff cannot be determined, there is a significant uncertainty in $N$ and results depending on $N$ are not well constrained and therefore are not considered in this study.
Figure 2. Thirteen distinct family of models in Table 1 (Models A–M) produce 9 distinct spectral fits to the data. The fits are displayed in four frames for clarity. Frame (a) (top left) compares different choices for $E_{\nu_0} (\nu_0 = 86 \text{ GHz})$ with $E_{\nu_0} (\nu_0 = 230 \text{ GHz})$ held constant. Frames (b) and (c) compare different choices for $E_{\nu_0} (\nu_0 = 230 \text{ GHz})$, with $E_{\nu_0} (\nu_0 = 86 \text{ GHz})$ held constant. Frame (d) explores variations in the strength of the synchrotron tail (note the spectral break in the UV).

determine the tubular jet spectrum. Eliminate the preassigned values of the models that are common for the description of the jet spectrum ($\psi$, $\alpha$, and $H/R$). The problem reduces to five model values ($N$, $B^*$, $R$ are free to vary, and $\alpha$ and $v^*$ are fixed) that determine the four parameters required to generate the spectrum ($\mu(\nu)$, $j_0(\nu)$, $\delta$, and $R$). Thus, in each model class, there are actually an infinite number of physical solutions for the same fit, as shown by the 1D curves in Figures 3 and 4. There are 15 models and 9 different fits. As a consequence of the single values of $\mu(\nu)$, $j_0(\nu)$, and $\delta$ throughout the calculational volume, the models A–E have been chosen to have exactly the same fit to the data. This fit is a control variable in the numerical experiments to follow. The data are upper limits. Thus, the fits in Figure 2 explore a wide range of the excess of the data relative to the actual flux produced by the forward jet base located inside the EHTC.

In order to interpret $F_{\nu_0} (\nu_0 = 230 \text{ GHz})$, note that simulated models indicate that a gravitationally lensed counter jet and/or the accretion disk itself can produce the observed $F_{\nu_0} (\nu_0 = 230 \text{ GHz})$ (Dexter et al. 2012; Moscibrodzka et al. 2016). In the models, a luminous disk will have an opacity that is sufficient to absorb the lensed counter-jet emission (Dexter et al. 2012). In these models, the pressure-driven “funnel wall jet” initiates in a distributed region suspended along the interface between the accretion vortex and the disk, i.e., at larger cylindrical radii than the ISCO and above the equatorial plane (Hawley & Krolik 2006; Moscibrodzka et al. 2016). By contrast, in the present model, the jet initiates just outside the ISCO in the equatorial plane, assuming the role of the luminous inner disk, thereby plausibly absorbing the lensed counter-jet emission. Thus, the forward tubular jet base can be the predominant source of $F_{\nu_0} (\nu_0 = 230 \text{ GHz})$. However, because of the uncertainty in the source of EHT correlated flux, the parametric study presented here allows for the disk and counter jet to produce a wide range of $F_{\nu_0} (\nu_0 = 230 \text{ GHz})$, from 0 to 2/3 of the total.

Table 1 lists the 15 models that are used to analyze the tubular jet model in this study. There are four separate issues that are being investigated by the parametric study:

1. the BH spin parameter, $a/M$;
2. the axial velocity of the jet, $v^*$;
3. the spectral fit to the uncertain broadband spectrum of the jet base discussed in the last subsection;
4. and the length, $H$, of the jet base responsible for the broadband spectrum given by the aspect ratio $H/R$.

In order to explore each of these items, three items should be held fixed, with the fourth allowed to vary. Most of the models have $H/R = 2$. A larger $H/R$ might be more physically reasonable for a jet base, but a simple uniform right circular tube plasmoid model is less justified. In the models, the tubular plasmoids have a length of $H \approx 5M \approx 4.2 \times 10^{15} \text{ cm}$. 
For an average jet propagation speed of 0.1c, this corresponds to a propagation time for an element of plasma to traverse a jet base length, \(H\), of \(t_{\text{jet}} = 1.4 \times 10^6\) s. Based on the gravitational redshift, \(\nu_0 = 230\) GHz corresponds to \(\nu \approx 300\) GHz. The synchrotron lifetime at the peak emission frequency, \(\nu_m\), in the plasma rest frame is (Tucker 1975)

\[
t_{\text{sy}} \approx \frac{5 \times 10^{11}}{(B^3 \nu_m)^{1/2}} \text{ s.} \quad (19)
\]

Using \(B = 25\) G, \(t_{\text{jet}}/t_{\text{sy}} \approx 190\). Thus, \(H/R = 2\) seems short, but dynamically, it is a long time. The basic premise of this analysis is that when the jet is ejected from the hot denser accretion flow, it is highly luminous, and it is this jet base that is being modeled. The parametric analysis below will consider the possibility that much of the 86 and 230 GHz emission might arise farther out in the jet. Addressing the uncertainty in the flux density of the jet base is the basic principle of this parametric study. It does not make sense to model a large region of the jet by a single-zone model and that is not the intent here.

### 3.2.1. Exploring Spin Variation, Fiducial Fit, and Fiducial Model

Models A–C explore changes in the jet as the spin is varied. Per point 2 above, the axial velocity is fixed at \(v^2 = 0.1c\). The nonrelativistic value is motivated by high-resolution VLBI component motion discussed in the last section (Mertens et al. 2016; Hada et al. 2017). Per point 3 above, a fiducial fit is chosen for comparison purposes. It is arbitrary, since the data do not constrain the choice that strongly. The fit assumes that most of the EHTC \(F_2(\nu_0 = 230\) GHz) is attributed to the base of the forward jet, \(F_1(\nu_0 = 230\) GHz) = 830 mJy. Second, due to the core shift analysis discussed in the previous subsection, the 230 GHz core is only \(\sim 10\) \(\mu\)as from the center of the Gaussian fit to the 86 GHz core. Combining this with \(F_2(\nu_0 = 230\) GHz) = 830 mJy, and assuming that the spectrum has only weak SSA absorption at 230 GHz, a substantial flux density at 86 GHz is expected. It must be less than the total flux fit in Hada et al. (2016), thus a value of \(F_2(\nu_0 = 86\) GHz) = 450 mJy is chosen. The other constraint on the fit comes from the \(HST\) observations, where we used VLBI observations to argue that it is a tight upper limit to the EHTC flux: \(F_2(\nu_0 = 3.72 \times 10^{14}\) Hz) = 0.5 mJy.

Per point 4 above, a fiducial value of \(H/R = 2\) is chosen based on the simplifying assumption of a uniform small region, yet it still provides an elongated aspect of a jet. With Equation (19), this a dynamically significant length of jet to consider.

Note that by their construction, Models A–C have the same spectrum (see Figures 2(a) and 3(a)). For most of the comparative analysis to follow, Model B is utilized as a fiducial model of the physical state of the system, \(v^2 = 0.1c\), \(a/M = 0.95\), and \(H/R = 2\). Figure 3(a) shows a larger radius for a lower spin. This is expected since the ISCO is farther out. The Poynting flux is larger at a fixed \(B^P\) for higher spin. This is primarily because Equation (18) indicates a quadratic dependence of the Poynting flux on \(\Omega_F \approx \Omega_{\text{kep}}(\nu_0)\), and with Equation (9), the smaller ISCO for the higher spin indicates a much larger \(\Omega_{\text{kep}}(\nu_0)\) throughout the base of the tubular jet.

![Figure 3. Exploration of the different physical parameters that produce the fiducial fit in Table 1 and Figure 2, as a consequence of varying the assumptions on \(a\) and \(v^2\). The top frame shows the dependence on spin. The bottom frame shows the minor effect of varying the velocity, \(v^2\).](image)

### 3.2.2. Exploring Axial Velocity Variation

The next set of models, D and E, explore the effects of varying the axial velocity, \(v^2\). In this case, not only must the fit and \(H/R\) be fixed, but also the spin. In this regard, \(a/M = 0.95\) is chosen as the fiducial physical state of the BH, and Model B is the fiducial model for comparison. Note that Models A–E will have the same spectrum by construction since this is a control variable in the numerical experiment (see Figures 2(a) and 3(b)).

The axial velocity is necessarily nonrelativistic if the outflow has bilateral symmetry; this implies \(v^2 = 0\) at the equator. This is consistent with the VLBI observations of component motion within 2 mas of the BH that indicates speeds on the order of \(0.1c–0.4c\) (Hada et al. 2016, 2017; Mertens et al. 2016). The slowest jet has the smallest radius, yet the overall variation between \(v^2 = 0.05\) and \(v^2 = 0.2c\) is only 2%–3%. The Poynting flux is smaller for \(v^2 = 0.05\) at a fixed \(B\) and \(a\) due to the smaller radius, and therefore there is less cross-sectional surface area in the integral of Equation (18). The variation between \(v^2 = 0.05\) and \(v^2 = 0.2c\) is minimal.

### 3.2.3. Exploring Variation in the High-frequency Synchrotron Tail

Models F and G hold properties 1, 2, and 4 above constant. The experiment considers variations in the fit if
This is essentially an exploration of the dependence of the physical parameters of the tubular jet model solely based on the variation in the synchrotron tail (see Figures 2(d) and 4(d)). There is very little change in $R$ and the Poynting flux, even with a factor of 2 change in the optical flux. Thus, exact knowledge of the optical flux from the EHTC is not necessary for an accurate estimate of the size and jet power in the tubular jet model.

3.2.4. Exploring the Uncertainty in the 86 GHz Flux Density

Models H and I hold properties 1, 2, and 4 above constant. This numerical experiment considers variations in the fit to the data if $F_\nu(\nu_0 = 86 \text{ GHz})$ is allowed to vary, with $F_\nu(\nu_0 = 86 \text{ GHz}) = 450 \text{ mJy}$ and $F_\nu(\nu_0 = 230 \text{ GHz}) = 830 \text{ mJy}$ held constant. When combined with the fiducial Model B, this is essentially an exploration of SSA opacity variation when combined with the fiducial Model B. It is motivated by the fact that the amount of $F_\nu(\nu_0 = 86 \text{ GHz})$ that is attributable to the jet base is uncertain as discussed in Section 3.1 (see Figures 2(a) and 4(a)).

A large variation in the tubular jet model is seen based on the assumed value of $F_\nu(\nu_0 = 86 \text{ GHz})$ from the jet base. Lower $F_\nu(\nu_0 = 86 \text{ GHz})$ with $F_\nu(\nu_0 = 230 \text{ GHz}) = 830 \text{ mJy}$ held fixed means a higher opacity. At a fixed $B^p$, by Equation (1), this can be achieved with a higher $N$ or $R$: $\mu(\nu) \sim NHR \sim NR$. However, the luminosity of the synchrotron tail $F_\nu(\nu_0 = 3.72 \times 10^{14} \text{ Hz}) \sim NHR^2 \sim NR^3$. This is held fixed, so the only solution is an increase in $N$ and a decrease in $R$. The higher opacity solutions have larger $N$ and smaller $R$. For a fixed $B^p$, $v^2 c^2$ and $a$, Equation (18) indicates a smaller Poynting flux as well, due to the smaller cross-sectional area of the jet associated with the smaller $R$ values. The conclusion is that the uncertainty in $F_\nu(\nu_0 = 86 \text{ GHz})$ presents significant uncertainty in the tubular jet models.

3.2.5. Exploring the Uncertainty in the 230 GHz Flux Density

Models J and K hold properties 1, 2, and 4 above constant. This numerical experiment considers variations in the spectral fit if $F_\nu(\nu_0 = 230 \text{ GHz})$ is allowed to vary, with $F_\nu(\nu_0 = 3.72 \times 10^{14} \text{ Hz}) = 0.5 \text{ mJy}$ and $F_\nu(\nu_0 = 86 \text{ GHz}) = 450 \text{ mJy}$ held fixed. This is essentially an exploration of peak spectral flux density variation when combined with Model B. It is motivated by the fact that $F_\nu(\nu_0 = 230 \text{ GHz})$ from the jet base...
is uncertain based on the discussion at the beginning of Section 3.2 (see Figures 2(b) and 4(b)).

Surprisingly, changing $F_\nu$($\nu_0 = 230$ GHz) has little effect on the tubular jet models. Ostensibly, this appears to be another opacity study, but with different results than Section 3.2.4. There is a significant difference from Section 3.2.4. The synchrotron luminosity near the peak and throughout the sub mm is not held fixed. Every increase in opacity corresponds to an increase in the synchrotron luminosity. The opacity and total luminosity affect $R$ in an opposite sense. The effects on the tubular jet model are opposite and tend to cancel out.

3.2.6. Exploring a Small Jet Contribution to the 230 GHz Flux Density

Models L and M hold properties 1, 2, and 4 above constant. This study considers the possibility that the forward jet base flux density is a small fraction of the total observed EHTC flux density. For example, the majority of the flux density might be from the jet much farther out than the model of the jet base, the disk, or the counter jet. Combining these models in combination with Model I gives a second parametric study of the variation with Model I gives a second parametric study of the disk, or the counter jet. Combining these models in combination with Model I gives a second parametric study of the disk, or the counter jet.

Extrapolating a single-zone model that is designed to explore basic parameter changes. For example, the majority of the forward jet base. The surprising result of these two experiments indicates that exact knowledge of the 230 GHz flux density is not attributable to the EHTC model as the 230 GHz flux density is varied. It shows that the tubular jet model is not strongly perturbed even if the majority of the EHTC model are opposite and tend to cancel out.

3.2.7. Exploring Variations in the Length of the Jet Base

The final parametric study holds properties 1–3 above fixed, but vary the aspect ratio of the jet, $H/R$. Models N and O are identical from module to module.

The fiducial jet length of $2R$ is motivated by considering the plausibility of a uniform tubular geometry. As the tube gets longer, the right cylindrical shape and uniformity become less accurate descriptions and the model becomes more complicated. Besides the shape and uniformity, major concerns are posed by the effects caused by the change in the redshift of Equation (17), as the jet gets farther from the BH. The short $H/R = 2$ tube is a crude one-zone model used as an approximation in order to explore basic parameter changes. Extrapolating a single-zone model that is designed to explore the base of the jet to a many-fold longer jet length is not justified. Thus, a longer jet is built up by connecting shorter $H/R = 2$ length modules end to end. Each module has its own volumetric averages of $\delta^{2.5+\alpha}$ and $\gamma_{\nu B0} (3^{3+\alpha})$ per the strategy described at the end of Section 2. The plasma properties, $N$, $B^p$, $\alpha$, and $R$ are identical from module to module.

The top frame of Figure 5 shows the dependence of $R$ and Poynting flux on the aspect ratio $H/R$. All models have $a/M = 0.95$, $v^2 = 0.1$, $\alpha = 1.01$, and the same optical flux. The larger the aspect ratio, the narrower the tubular jet. $R$ must decrease in order to maintain a similar volume, otherwise the jet will over-produce optical emission relative to the fiducial Model B (bottom frame). The smaller $R$ also reduces the cross-sectional area of the jet, and therefore the Poynting flux at fixed $B^p$.
to maintain a similar volume. The long, thin-walled tube when $H/R = 8$ seems unrealistic. One is probably not exploring physical changes in the long tubes, but seeing a break down of the assumptions of the model.

The bottom frame of Figure 5 shows the resultant spectra from the segmented models. Since the opacity is different in every segment, the fiducial fit of Model B cannot be attained. The models were an attempt to get close to the same spectrum as Model B, so that a comparison can be made. The lower opacity of the base results in an excess of flux density at 86 GHz for Models N and O relative to Model B if the optical flux and the 230 GHz flux density are held approximately equal to that of Model B. Note that an exact value of $B^0$ is chosen for the spectra of Models N and O. Each value of $B^0$ produces a slightly different spectra, so a particular representative value was plotted.

In spite of not being able to produce identical spectral fits, the models demonstrate one basic conclusion: the largest angular size transverse to the jet direction occurs for the models demonstrated one basic conclusion: the largest angular size transverse to the jet direction occurs for the models. Since the opacity is different in every segment, the fiducial fit of Model B cannot be attained. The models were an attempt to get close to the same spectrum as Model B, so that a comparison can be made. The lower opacity of the base results in an excess of flux density at 86 GHz for Models N and O relative to Model B if the optical flux and the 230 GHz flux density are held approximately equal to that of Model B. Note that an exact value of $B^0$ is chosen for the spectra of Models N and O. Each value of $B^0$ produces a slightly different spectra, so a particular representative value was plotted.

In spite of not being able to produce identical spectral fits, the models demonstrate one basic conclusion: the largest angular size transverse to the jet direction occurs for the models. Since the opacity is different in every segment, the fiducial fit of Model B cannot be attained. The models were an attempt to get close to the same spectrum as Model B, so that a comparison can be made. The lower opacity of the base results in an excess of flux density at 86 GHz for Models N and O relative to Model B if the optical flux and the 230 GHz flux density are held approximately equal to that of Model B. Note that an exact value of $B^0$ is chosen for the spectra of Models N and O. Each value of $B^0$ produces a slightly different spectra, so a particular representative value was plotted.

In spite of not being able to produce identical spectral fits, the models demonstrate one basic conclusion: the largest angular size transverse to the jet direction occurs for the models. Since the opacity is different in every segment, the fiducial fit of Model B cannot be attained. The models were an attempt to get close to the same spectrum as Model B, so that a comparison can be made. The lower opacity of the base results in an excess of flux density at 86 GHz for Models N and O relative to Model B if the optical flux and the 230 GHz flux density are held approximately equal to that of Model B. Note that an exact value of $B^0$ is chosen for the spectra of Models N and O. Each value of $B^0$ produces a slightly different spectra, so a particular representative value was plotted.

3.2.8. Exploring Variations in the Elevation of the Jet Initiation Point

Another variable in the basic configuration is the elevation of the launch point above the equatorial plane. The outgoing jet might be lower density material that initiates at the top boundary of a denser accretion disk, as depicted in Figure 6. The $H/R = 2$ jet that initiates $2R$ above the equator has the same proper motions 1–4 as Model B. The lone difference is the elevation of the initiation point. This results in a less than 1% decrease in $R$ and the Poynting flux. This negligible change arises from a near cancelation of effects. Note that by Equation (1), $\mu(\nu_o) \sim N H^{2.5+\alpha}$, and from Equations (5), (7), and (12), the synchrotron luminosity is $L(\nu_o) \sim N H^{2.5+\alpha}$. The change in elevation results in a change to the gravitational redshift contribution to $\delta$ in Equation (17), since the volume is farther from the BH with less redshift. This effect is surprisingly small since $\gamma_{Kepz}(\delta^{3+\alpha})/\gamma_{Kepz}(\delta^{5.5+\alpha}) \approx \gamma_{Kepz}(\delta^{0.5}) \approx 1$ throughout the volume of both the elevated jet and the equatorial plane launched jet, and likewise so are the volumetric averages that are used.

Properties related to the number density are not plotted because there is a huge uncertainty since the low-energy cutoff in Equation (3) is unconstrained by the insufficient resolution of VLBI at cm wavelengths. However, the change in elevation of the initiation point and the resultant smaller gravitational redshift actually decreases the proper number density, $N$, by a factor of $\sim 4$. Elevated jet initiation might be physically more reasonable and more conducive to establishing jet solutions that are magnetically dominated. Recall the discussion that motivates Equation (14), indicating that the jet begins highly magnetically dominated in order to explain the observed acceleration on sub-mas scales.

4. Conclusion

EHT observations at 230 GHz were combined with 86 GHz VLBI observations in order to constrain the SSA opacity. Considering 0′′1 resolution HST optical photometry in the context of VLBI images of the jet on scales $\lesssim 100$ mas indicates that the EHTC is the most plausible source of the HST flux. These data indicate a large SSA opacity at $\sim 100$ GHz and a modest IR/optical synchrotron (HST) luminosity. These constraints are applied to the tubular jet base models illustrated in Figure 1.

Section 3 is a parametric analysis of possible tubular jet models that are consistent with the data. Fifteen models are considered corresponding to nine different fits to the data (see Table 1 and Figure 2). Due to insufficient resolution and/or lack of imaging, the observations do not tightly constrain the the flux density of the EHTC. To compensate for the uncertainty, many fits to the data and models were explored. For each model, $R$ and the Poynting flux are plotted as a function of the proper poloidal magnetic field, $B^0$, in Figures 3 and 4. In order to interpret the results displayed in Figures 3–4, it is useful to have an expectation on the Poynting flux. Isotropic estimates of jet power yield $Q = 0.75 – 6 \times 10^{43}$ erg s$^{-1}$ (Willett et al. 1999; Punsly 2005; McNamara et al. 2011). Estimates based on the brightest features in the interior jet are biased toward the more energetic episodes in the jet history and find $Q \gtrsim 10^{44}$ erg s$^{-1}$ (Stawarz et al. 2006; Owen et al. 2000). A reasonable range is $Q \sim 10^{43} – 10^{44}$ erg s$^{-1}$. The parametric analysis of the jet base indicates the following.

1. The entire power budget of the M87 jet can be accommodated for $0.99 > a/M > 0.95$ (a/M $\sim 0.7$) if the outer radius of the emitting region is $\sim 1.8–3.0 M$ ($\sim 3.5–3.9 M$) and the vertical magnetic field is 8–40 G (20–50 G).

2. The dimension of the jet transverse to the direction is $12–21$ $\mu$as ($\sim 24–27$ $\mu$as) for $0.99 > a/M > 0.95$ (a/M $\sim 0.7$), or $\sim 1/2$ the EHTC size in the numerical models of Dexter et al. (2012), Moscibrodzka et al. (2016). If the notion of a compact luminous jet base near the BH is abandoned and aspect ratios, $H/R > 2$, are considered, then the minimum transverse sizes are decreased. EHT imaging might be able to discriminate between the models.

3. The jet base dimensions and power depend strongly on $F_o(\nu_o = 86$ GHz) and weakly on other parameters, such as jet speed, $F_o(\nu_o = 230$ GHz), and optical flux.
This analysis is not a full radiative transfer calculation that captures effects, such as gravitational lensing (Dexter et al. 2012; Moscibrodzka et al. 2016). It relies on the simplified solution of Equation (4). The analysis does incorporate gravitational redshift and transverse Doppler shift. The preferred configuration was argued in Section 3.2.8 to be a jet that initiates at the top of the disk as illustrated in Figure 6. It would be interesting to see if there are some significant changes to the apparent transverse size in this preferred configuration if the potential effects of gravitational lensing are included.

The leptons responsible for the EHTC emission will synchrotron and possibly self-Compton cool (see the discussion of Equation (19)) near the BH and must be reheated in order to explain the almost hollow jet detected in Hada et al. (2016) on 0.1–0.5 mas scales at 86 GHz. The tubular jet model provides a natural explanation of this emission at a de-projected distance of ~120M–600 M from the central BH. The jet is Poynting flux dominated. It carries a large, ~10^{44} erg s^{-1}, energy flux outward in a tubular conduit (i.e., the tubular jet). It only takes a negligible fraction of the Poynting flux density at 230 GHz in current models. These jets can be stronger sources of Poynting flux than jets that form outside of the ISCO.

This study shows that a tubular jet from the inner accretion flow can be the source of energy that powers both the EHT core and the large scale jet, including the energy flux needed to power extreme dissipation sites, such as HST-1 (Stawarz et al. 2006). As such, there is no energetic requirement for a powerful spine, however it does not disprove the possibility of a powerful spine. The observational advantage over considering a powerful spine is that models that drive the spine from the event horizon produce an invisible forward jet due to the lack of energetic plasma at the jet base. The strong jet spine models initiate within the accretion vortex that is almost devoid of plasma. Thus, the claim that there is insufficient energy contained within the particles in the jet base that can be released as radiation (Moscibrodzka et al. 2016). Thus, the powerful jet spine is never a significant contributor to the correlated EHTC flux density at 230 GHz in current models. This is in contradistinction to the tubular jet models that emanates from the inner disk, which is rich in hot plasma.

It is also noted that the spine might be relatively weak, as in the the older spine-sheath type models Punsly (1996, 2008). There is currently no direct observational evidence indicating that most of the jet energy is in the spine. Observational evidence of a spine has been seen with VLBI at 5–15 GHz (Asada et al. 2016; Hada 2017). A clear central ridge was resolved with 15 GHz VLBI at a distance of ~13.5–30 mas, a de-projected distance of >17,000 M from the BH (Hada 2017). The central ridge is less luminous than the outer sheath, especially the southern ridge. It cannot be traced back to the source and seems to merge with the southern ridge ~13.5 mas from the BH. Furthermore, the central ridge does not seem to “light up” until right after (based on distance from the BH) the outer ridges brighten, suggesting that the feature might be generated by the outer ridges themselves. The total fraction of the 15 GHz luminosity of the jet within 30 mas that is produce by the central ridge is negligibly small. In summary, there is no observational evidence indicating a powerful spine of plasma being emitted from the BH. There is an explanation of the faint ridge arising from the tubular jet. The central axis is a natural place for shocks from the outer boundary to coalesce creating an axial region of enhanced dissipation (Sanders 1983).

The tubular jet model offers observers a more tangible set of predictions than an invisible powerful spine jet. This is a strong motivation for pursuing models in which the outer sheath is the major source of energy flux from the BH accretion system. The flux detected by observers can then be used to directly constrain the energetics and microphysics of the jet form the EHTC to large distances.

I would like to thank the anonymous referee for many valuable and insightful comments.

**ORCID iDs**

Brian Punsly © https://orcid.org/0000-0002-9448-2527
References

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009, ApJ, 707, 55
Akiyama, K., Kuramochi, K., Ikeda, S., et al. 2017, ApJ, 838, 1
Akiyama, K., Lu, R., Fish, V., et al. 2015, ApJ, 807, 150
Asada, K., Nakamura, M., & Pu, H.-Y. 2016, ApJ, 833, 56
Blandford, R., East, W., Nalewajko, K., Yuan, Y., & Zrake, J. 2015, arXiv:1511.07515
Broderick, A. E., & Loeb, A. 2009, ApJ, 697, 1164
Chiaberge, M., Capetti, A., & Celotti, A. 1999, A&A, 349, 77
Chiaberge, M., Macchetto, F. D., Sparks, W. B., et al. 2002, ApJ, 571, 247
Dexter, J., McKinney, J. C., & Agol, E. 2012, MNRAS, 421, 151
Doeleman, S., Fish, V., Schenck, D., et al. 2012, Sci, 338, 355
Gebhardt, K., Adams, J., Richstone, D., Lauer, T. R., & Faber, S. M. 2011, ApJ, 729, 119
Ghisellini, G., Tavecchio, F., & Chiaberge, M. 2005, A&A, 432, 401
Ginzburg, V., & Syrovatskii, S. 1965, ARA&A, 3, 297
Ginzburg, V., & Syrovatskii, S. 1969, ARA&A, 7, 375
Hada, K. 2017, Galax, 5, 2
Hada, K., Doi, A., Kino, M., et al. 2011, Natur, 477, 185
Hada, K., Giroletti, M., Kino, M., et al. 2014, ApJ, 788, 165
Hada, K., Kino, M., Doi, A., et al. 2016, ApJ, 817, 131
Hada, K., Park, J., Kino, M., et al. 2017, PASJ, 69, 71
Hawley, J., & Krolik, J. 2006, ApJ, 641, 103
Kennel, C., & Coroniti, F. 1984, ApJ, 283, 694
Kim, J.-Y., Lu, R.-S., Krichbaum, T., et al. 2016, Blazars through Sharp Multi-Wavelength Eyes, ed. J. L. Geomez, A. P. Marscher, & S. G. Jorstad, arXiv:1609.07896
Kino, M., Takahara, F., Hada, K., et al. 2015, ApJ, 803, 30
Krichbaum, T. P., Roy, A., Lu, R.-S., et al. 2015, in Proc. 12th European VLBI Network Symposium and Users Meeting, ed. A. Tarchi, M. Giroletti, & L. Feretti (Trieste: POS), 13, http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=230
Lazarian, A., Eivink, G., Visnia, E., & Kowal, G. 2015, RSPTA, 373, 20140144
Lightman, A., Press, W., Price, R., & Teukolsky, S. 1975, Problem Book in Relativity and Gravitation (Princeton, NJ: Princeton Univ. Press)
Lind, K., & Blandford, R. 1985, ApJ, 295, 358
Lister, M. L., Aller, M. F., Aller, H. D., et al. 2013, AJ, 146, 120
Matsumoto, Y., Amano, T., Kato, T., & Hoshino, M. 2017, PhIoL, 119, 105101
McNamara, B., Rohanizadegan, M., & Nulsen, P. 2011, ApJ, 727, 39
Mertens, F., Lobanov, A., Walker, R., & Hardee, P. 2016, A&A, 595, 54
Moscbrodzka, M., Falcke, H., & Shiokawa, H. 2016, A&A, 586, 38
Owen, F. N., Eilek, J. A., & Kassim, N. E. 2000, ApJ, 543, 611
Prieto, M. A., Fernandez-Ontiveros, J. A., Markoff, S., Espada, D., & Gonzalez-Martin, O. 2016, MNRAS, 457, 3801
Punsly, B. 1996, ApJ, 473, 178
Punsly, B. 2005, ApJL, 623, 9
Punsly, B. 2008, Black Hole Gravitohydromagnetics (2nd ed.; New York: Springer)
Punsly, B., Igumenshchev, I. V., & Hirose, S. 2009, ApJ, 704, 1065
Sanders, R. 1983, ApJ, 266, 73
Stawarz, L., Aharonian, J., Kataoka, J., et al. 2006, MNRAS, 370, 981
Tavecchio, F., & Ghisellini, G. 2008, MNRAS, 385, 98
Tucker, W. 1975, Radiation Processes in Astrophysics (Cambridge, MA: MIT Press)
Whysong, D., & Antonucci, R. 2004, ApJ, 602, 116
Willott, C., Rawlings, S., Blandell, K., & Lacy, M. 1999, MNRAS, 309, 1017
Wilmot-Smith, A. L., Pontin, D. I., & Hornig, G. 2010, A&A, 516, 5