Application of two-dimensional mathematical model in backwater calculation for flood control evaluation of mountainous rivers

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Abstract: Flood control impact assessment of construction projects within the scope of river course management shall be conducted in accordance with relevant laws and regulations. The calculation of backwater is an important content of flood control evaluation of Bridges across rivers. Two-dimensional mathematical model is a main method for flood control evaluation of backwater calculation. This paper introduces the basic principle of the two-dimensional mathematical model and applies the two-dimensional mathematical model in the actual application of flood control evaluation of the great Darui Railway Bridge of Ruili River, analyzes the changes of water level, flow rate and flow direction before and after the construction of the bridge, and uses the standard formula method to check. The results show that the two-dimension mathematical model has a high accuracy in the calculation of backwater of flood control evaluation of Bridges over mountainous rivers, which can provide scientific basis for flood control evaluation of projects.

1.Introduction
With the rapid development of China's economy and society, especially since the beginning of the 21st century, the country has been increasing the investment in infrastructure construction, resulting in an increasing number of buildings across, through or blocking the rivers. These buildings play their own role but also have a negative impact on flood control of the river. If the design or construction of these buildings does not meet the flood control requirements, it will increase the water resistance area of the river, occupy the flood discharge section of the river, increase the water level of the river, change the local flow pattern of the water flow, and affect the flood control safety of the river [1]. Especially in mountainous rivers in Yunnan province, due to frequent local rainstorms in mountainous areas, the construction of buildings across, through or blocking the rivers has more and more prominent impact on safe flood movement in mountainous areas.

In order to strengthen the management of the new buildings in the channel, ensure the river flood control safety, protect of the local national economy development and people's life and property security, in accordance with the relevant laws and regulations, flood control impact assessment of construction projects within the scope of river channel management shall be carried out, and the backwater calculation is an important part of flood control evaluation of bridges across rivers.
According to the investigation, mathematical model and empirical formula are often used to calculate the backwater of bridge. Among them, the empirical formula can only be reflected in the change of section flow rate, water level and other factors. However, with the development of computing technology, numerical model has been widely used in the research, planning and design of river engineering, and has become an effective means of engineering optimization design, which has been recognized by the engineering community. Generally, the numerical calculation model is two-dimensional plane model[2].

2. Introduction to two-dimensional mathematical model

2.1 Basic equations and calculation methods of the model

(1) Basic equation

Flow continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial h\tilde{u}}{\partial x} + \frac{\partial h\tilde{v}}{\partial y} = hs$$

Momentum equation of water flow:

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial \tilde{u}^2}{\partial x} + \frac{\partial \tilde{u}\tilde{v}}{\partial y} = f\tilde{v}h - gh\frac{\partial \eta}{\partial x} - i\tilde{u}\frac{\partial \rho_a}{\partial x} - \frac{gh\tilde{v}}{\rho_0} - \frac{\tau_{ux}}{\rho_0} - \frac{\tau_{uy}}{\rho_0} -$$

$$\frac{1}{\rho_0}\left(\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{yx}}{\partial y}\right) + \frac{\partial}{\partial x}\left(hT_{xx}\right) + \frac{\partial}{\partial y}\left(hT_{xy}\right) + \frac{\partial}{\partial x}\left(h\rho_a\right) + \frac{\partial}{\partial y}\left(h\rho_a\right) -$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial \tilde{u}\tilde{v}}{\partial x} + \frac{\partial \tilde{v}^2}{\partial y} = f\tilde{u}h - gh\frac{\partial \eta}{\partial y} - i\tilde{v}\frac{\partial \rho_a}{\partial y} - \frac{gh\tilde{u}}{\rho_0} - \frac{\tau_{vx}}{\rho_0} - \frac{\tau_{vy}}{\rho_0} -$$

$$\frac{1}{\rho_0}\left(\frac{\partial s_{yx}}{\partial x} + \frac{\partial s_{yy}}{\partial y}\right) + \frac{\partial}{\partial x}\left(hT_{yx}\right) + \frac{\partial}{\partial y}\left(hT_{yy}\right) + \frac{\partial}{\partial x}\left(h\rho_a\right) + \frac{\partial}{\partial y}\left(h\rho_a\right) -$$

$$\tilde{u} = \int_d \eta dz \quad \tilde{v} = \int_d \eta dz$$

Where, $\tilde{u}, \tilde{v}$ is the velocity based on the average depth of water; $\eta$ is the river bottom elevation; $d$ is the static depth; $h=\eta+d$ is the total head; $t$ is time; $x, y$ and $z$ are Cartesian coordinates; $u$ and $v$ are the velocity component of the direction $x$ and $y$; $g$ is gravity acceleration; $\rho$ is the density of water; $s_{xx}, s_{xy}, s_{yx}$ and $s_{yy}$ are the component of radiation stress; $p_a$ is atmospheric pressure; $\rho_0$ is the relative density of water; $S$ is the flow rate of point source; $u$ and $v$ are the flow rate of source and sink item; $T_{xx}, T_{xy}$ and $T_{yy}$ are lateral stress terms including viscous friction, turbulent friction and differential advection.

(2) Calculation method

① Staggered grid and time integration method

Finite Volume Method is used to calculate the spatial dispersion of the region [3-4], which subdivides the continuum into non-overlapping units. When the governing equations are discrete, the variables are interlaced on the grid, the water level is defined on the grid nodes, and the single width flow is defined in the middle of the adjacent grid in their respective directions. ADI was used to solve the equation, and Double Sweep algorithm was used to solve the equation matrix, which has second order accuracy.

② Definite solution conditions of the model
The initial conditions:
\[ \zeta(\xi, \eta, t|_{t=0}) = \text{const} \]
\[ u(\xi, \eta, t|_{t=0}) = v(\xi, \eta, t|_{t=0}) = 0 \]

It is generally assumed that the model is initially static ("cold start"), that is, when \( t=0 \), the flow field is static. The initial water level is a certain value.

3. The boundary conditions

The open boundary, flow boundary condition and water level boundary condition are used for calculation.

Water level: \( \zeta = f_\zeta(t) \)

Traffic (total or per unit): \( Q = f_Q(t) \)

3. Application examples of the model in engineering practice

3.1 Project overview

The section of the great Ruili River Bridge crossing the downstream of the river is planned to build, located in nearby 97°58' E, 24°4' N. About 900m upstream of the bridge is Wanrui Highway Bridge, about 700m upstream is G56 Hangrui Highway Bridge, the left bank is the Ruili River Resort, and the right bank is farmland (see figure 1). The average bridge across the river section is about 10m high with a total length of 1581.41m. Ordinary simply supported beams are used to cross the river with a span of 48x32. There are 49 piers in total. Among them, No. 26 ~ 34 bridge piers are located in the main channel of Ruili River, and the rest are located on both sides of the channel without encroaching on the channel. Among them, No. 26 ~ 36 piers are solid piers with double columns and round ends. Each column is 0.8m wide, 1.5-2.97m apart and 5.6m long. Other piers are elliptic solid pier with a width of 2.42 ~ 2.92m and a length of 4.13 ~ 5.03m. The bridge line is basically perpendicular to the flow direction. The bored piles are used for the foundation. The left and right bank of the bridge section has been embankment, and the bridge is about 340m wide across the river. The flood control standard of the bridge is 100 years for design, 300 years for check and 50 years for construction. Based on the above method, the backwater of Ruili River Bridge is analyzed and calculated.

3.2 Research scope and grid division

The two-dimensional model is calculated for the local section of the Ruili river trunk stream across the Darui Railway Bridge. The upstream boundary is 970m at the upstream of Wanrui Highway Bridge, where the highest flood peak flow in 20 years is 2460m3/s. The lower boundary is 2348m at the downstream of Ruili River Railway Bridge, where the highest flood level in 20 years is 769.82m. The total length of the river section for this two-dimensional calculation is 4379m.

The calculated region is divided by triangles, and the shoreline and local river channel abrupt changes are well fitted. The grid resolution gradually increases from the boundary to the bridge site, with the lowest resolution of 10m and the highest resolution of 2m. Before the construction of the bridge, the whole computing area has a total of 3,907 nodes and 11,398 triangular grids. After the bridge was built, there were 6392 nodes and 11,597 triangular grids. See fig.2 for grid section near the river section and pier where the bridge is located.

Topography generalization: the underwater topography data of Ruili River section of Darui Railway adopts the measured riverbed elevation data in February 2017, and is inserted into the grid nodes by linear interpolation, and the vicinity of the bridge pier is encrypted.
3.3 Model validation

The verification of the model adopts the data of velocity distribution of 3 sections of Wanrui road bridge, the great Ruili River Bridge, and middle section of engineering area measured in December, 2016, and simultaneous water level observation of 5 temporary water meters. The verification results of water level are shown in FIG. 3. It can be seen from the figure that the calculated value is in good agreement with the measured value. According to statistics, the deviation of water level calculation is generally within 0.05m. FIG. 4-6 shows the comparison between the calculated velocity distribution and the measured velocity distribution of the three flow measurement sections in the calculated area. As can be seen from the figures, the calculated section velocity distribution is in good agreement with the measured data. Except for some individual points, the calculated velocity distribution trend along the section is basically consistent with the measured data, the calculated velocity result is not far from the measured value, and the deviation between the calculated velocity and the measured value is generally within 0.15m/s.

It can be seen from the verification results of cross-section velocity distribution and synchronous water surface line that the mathematical model adopted this time basically reflects the water flow movement rule of the engineering river section. The accuracy of model validation meets the requirements of the Technical Specification for Flow and Sediment Simulation of Inland Waterway and Ports (JTJ/ T232-98), which can be used to analyze the impact of the proposed project on the river level and flow field.
3.4 Analysis of calculation results

3.4.1 Height analysis of backwater

As the newly built Ruili river bridge has several piers in the Ruili River channel, it occupies the area of the river channel, which affects the water level of the river where the bridge is located, and may change the top elevation of the planned embankment on both sides. Therefore, under the highest flood level in 20 years, the changes of river water level before and after the construction of the bridge are analyzed respectively. The calculation scheme is as follows:

Plan 1: Under the current working condition, there is no bridge in the calculation area.

Plan 2: Under the planned working condition (the highest flood level in 20 years), a new great bridge over Ruili River is built.

In order to analyze the variation of backwater and velocity before and after the bridge construction, the comparison is made at the selected feature points of Darui Railway Bridge, such as upstream Wanrui Road, upstream Hangrui Road, upstream 400m, upstream 200m, bridge position, downstream 200m and downstream 400m. Before and after the construction of the bridge, the height changes of the water level and its influence range are shown in Table 1.

Table 1  Achievements of Backwater Level before and after Bridge Construction in 20 Years

| Setting       | Before bridge construction | After bridge construction | D-value |
|---------------|---------------------------|---------------------------|---------|
| Upstream 900m (Wanrui Road) | 774.6                     | 774.6                     | 0       |
| Upstream 700m (Hangrui Road) | 774.24                    | 774.24                    | 0       |
| Upstream 400m | 773.87                    | 773.88                    | 0.01    |
| Upstream 200m | 773.58                    | 773.6                     | 0.02    |
The dyke standard of the Great Ruili River Bridge section of Ruili River is IV standard, and the corresponding height of the wave wall is 0.6m. According to the technical regulations on river-related Bridges in China, for the river (sea) embankments that are not allowed to cross waves, the maximum height of backwater caused by blocked water on the bridge piers should be controlled within 10% of the safety ultra high value on the top of the embankment, and the maximum height of backwater caused by the biggest flood in 20 years should be controlled within 0.06m. As can be seen from table 1, after the completion of the Ruili River Bridge, the maximum accumulation of flood occurred once every 20 years is 0.05m, meeting the requirement. As for Wanrui Road Bridge and Hangrui Highway Bridge, after the Ruili River Bridge being built, the height of back water is invariable, and meets the requirement.

As can be seen from the contour map of water level choking up high, the part of water level choking up high in the river area is mainly concentrated near the bridge piers, with little influence in the center and on both sides of the river. This is because the flood formed resistance when the bridge piers, and more water flow through the main channel, but at the same time, because of the compression velocity of the bridge piers, the main channel is also relatively increased, the water level rises mainly in the vicinity of the bridge position, and has little impact on the water level of the main channel and the river bank.

### 3.4.2 Analysis of flood flow field of the biggest flood in 20 years

(1) Flow rate analysis

FIG. 7 and FIG. 8 are the flow charts of the water near the bridge position and piers when the bridge is subject to the biggest flood in 20 years before and after the construction of the bridge. Before the bridge is built in the project area, the flow direction is from north to south, and the flow velocity on the left and right sides of the river is between 0.4 ~ 3.4 m/s, the flow velocity on the left bank is 1.0 ~ 2.85m/s, and the flow velocity on the right bank is 0.4 ~ 2.75m/s.

After the construction of the bridge, the flow direction is less affected by the overall action of the bridge pier, except that the local water area is affected by the "overhanging flow" of the bridge pier. However, the flow velocity of the bridge pier and the surrounding area has a certain change. The "backwater" of the flood on the upstream side of the bridge pier leads to a decrease in the flow velocity of 0.15m/s ~2.05m/s, and the "blockage" of the bridge pier on the backflow side leads to a decrease in the flow velocity of 0.25m/s~2.45m/s. In addition, under the same flow condition, the decrease of the water area occupied by the bridge piers results in the increase of the flow between the piers and the increase of the flow rate at the hole from 0 m/s to 0.76m/s.

![Fig 7 Velocity contour map near bridge location after bridge construction](image)

![Fig 8 Velocity contour map near bridge location before bridge construction](image)

(2) Flow analysis

FIG. 9 and FIG. 10 are the flow charts of the water near the bridge position and pier when the bridge is subject to the flood once every 20 years before and after the construction of the bridge. The current flow direction of the Darui Railway Bridge is from north to south. After the Darui Railway
Bridge was built, due to the backwater effect of the piers, the flow direction near the piers was deflecting, the flow direction of the bridge bore hardly changed, and the flow direction of the upstream and downstream didn't change either.

(3) Analysis of flow field changes around the beach before and after the construction of the bridge

According to the technical regulations on river-related bridge in China, the scour of embankment foot (general scour and local scour of bridge pier) caused by bridge construction under design flood conditions should be controlled within 0.5m. When the flow rate changes within 5%, the moving bed experiment and empirical formula calculation show that, in most cases, the beach erosion is within 0.3m, generally not more than 0.5m. After the completion of the bridge, the changes in the flow velocity and direction of both sides of the river are as follows:

① Flow rate change

After the completion of the Ruili River Bridge, when the biggest flood in 20 years hits, the velocity of the right bank beach increases within the range of 0~0.15m/s, while the current velocity of the right bank is 1.15m/s, with a variation range of more than 5%. The increase range of the left bank's flow velocity is 0~0.24m/s, while the current flow velocity is 1.65m/s, with a change range of more than 5%. Therefore, the changes of the left and right bank's flow velocity do not meet the requirements. It is necessary to reinforce the embankment foot with anti-scour sheet pile on the water-facing slope of the embankment. The reinforcement scope is 100m in the upstream and downstream of the axle axis.

② Flow direction change

After the bridge construction, when the biggest flood in 20 years hit, the water flow around the bridge pier was deflected, but the left and right bank flow direction remained unchanged.

### 3.4.3 Calculation results recheck

The calculation of the depth rise adopts the formula (3.5.1) in the Code for Hydrological Survey and Design of Railway Engineering (TB1017—99)[5]. Recheck the backwater height before and after the bridge construction, and the calculation results are shown in table 2.

Table 2 Specification formula for calculating backwater height and length before and after bridge construction

| Frequency | flow (m³/s) | Water blocking rate (%) | Mean velocity (m/s) | Water surface gradient (%) | Backwater height (m) | Backwater length (m) |
|-----------|------------|------------------------|---------------------|----------------------------|---------------------|---------------------|
|           | Before     | After                  | Before              | After                      |                     |                     |
| 5%        | 2460       | 4.94                   | 1.44                | 1.55                       | 0.6                 | 0.05                | 166.7               |

It can be seen from the comparison between table 1 and table 2 that the calculation results of the two methods are relatively close, but the calculation results of the standard formula are smaller than that of the two-dimensional model. As the standard formula is a simplified calculation formula for complex engineering practice, the accuracy is not high, and it is not as close to the actual working condition as the two-dimensional model, so the results of backwater analysis in this evaluation calculated by the two-dimensional model are adopted.
4. Conclusion and suggestions

The standard formula method requires fewer known parameters, and the calculation process is simple, but the calculation accuracy is not high, which cannot better reflect the actual working condition. On the other hand, the 2-d mathematical model requires more detailed hydrological data in parameter calibration and model validation, and the calculation results are also affected by engineering generalization, grid division and other factors. But it became the main method in the flood control evaluation analysis because it has been relatively mature, easy to operate, and can simulate the movement process of water flow simulation engineering area, which will give us a better understanding of the water level, flow velocity and pattern of flow condition changes before and after the project.

In practical application, the model should be constantly improved and optimized to make the calculation results of the model more reasonable and reliable, so that the two-dimensional mathematical model can play a better role in flood control evaluation. For engineering projects with complex topography, numerical simulation is suggested when the backwater analysis calculation is carried out, and the calculation results should be compared with the standard and empirical formulas, so as to get a more practical conclusion.

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