Time-varying formation control for unmanned intelligent vehicle (UIV) with mobile reference center

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Abstract. This paper studies a time-varying formation control algorithm based on the unmanned intelligent vehicle (UIV) system. First, modeling the multi-agent system. Reference [4] gives the time-varying formation control algorithm of multi-agent under changing communication topology and necessary and sufficient conditions for the establishment of the algorithm. The simulation verifies the effectiveness of the algorithm. On this foundation, based on the consensus theory, this paper proposes a formation algorithm in the case of a formation reference center movement and realizes multiple time-varying formations of a multi-agent system through simulation. The simulation results show that the algorithm proposed in this paper can quickly realize formation under different formation requirements and the algorithm has better control effect and robustness.

Keywords: Time-varying formation, Consensus theory, Multi-agent system.

1. Introduction
Currently, robotics technology has been widely popularized, which application fields include scientific research, industry and even human life. However, a single robot has not achieved the desired effect in the face of heavy work tasks. Therefore, the multi-agent system came into being. Compared with one agent, a multi-agent system which has stronger practicability can not only complete the tasks of a single agent in most cases, but also deal with high-complexity tasks that a single robot cannot complete. Moreover, the multi-intelligence system also has higher work efficiency. The workload of each individual in the system is relatively reduced. At the same time, the increase in number makes the entire system more comprehensive in obtaining information, which is convenient for making more efficient allocation plans. The above advantages make the multi-intelligence system widely used in many fields, especially for exploration, surveillance, search and rescue.[1]. This paper considers the problem of formation control using such a system of cooperative agents.

Formation control is one of the most active research topics in the realm of multi-agent systems and its general goal is to drive multiple agents to achieve prescribed constraints on their states.[1] There are three classic methods to solve the problem of multi-agent formation control. The most commonly used method is leader-follower approaches. Choose one agent as the leader, and the other agents maintain a fixed distance and angle movement from the leader to achieve the overall formation. Another method is virtual structure which is similar to the leader–follower structure. However, the leader is virtual. The leader never fails and the stability of the whole system is not depending on the leader. The third method
is behavior-based method, prescribing several desired behaviors for each agent and to make the control action of each agent a weighted average of the control for each behavior. Possible behaviors include collision avoidance, goal seeking and formation keeping.[3] The above three methods also have their shortcomings. For instance, in the leader-follower, problems such as leader fault and communication interruption will cause errors in the formation of the whole system. As the number of agents increases, virtual structure and behavior-based method often require more powerful computing power as support.

The development of consensus theory has made it widely used in formation problems. In a multi-agent system, consensus refers to the system reach an agreement regarding a certain quantity of interest that depends on the state of all agents.[6] In [7], the problem of quadrotor indoor formation was studied. [8] discussed a leader follower consensus-based formation method in the case of changing the topology of the system and non-periodic sampling data. But the formation of the two does not change over time. In [5], paper studies the time-varying formation problem, but the topological structure between agents is fixed. In order to enable the formation of multi-agents to have a time-varying formation and at the same time to have a transformable topological structure. [4] proposed the corresponding control algorithm for the UAVs system and obtained its sufficient and necessary conditions, but it did not give an explanation of the movement of the reference center under the overall movement of the formation. This paper mainly studies the formation control problem of the unmanned intelligent vehicle (UIV) system, establishes the corresponding model and control algorithm, then simulates its algorithm on the basis of [4]. Moreover, the system movement condition under the condition of the reference center movement is simulated. The paper is arranged as follows. The second part analyzes and models the formation problem and gives the corresponding control algorithm. The third part provides simulation results for the control algorithm of the second part. The fourth part optimizes the control algorithm based on the second part and provides simulation results. The final part summarizes the whole work.

2. Modeling and control algorithm

With the development of computer science and wireless communication technology, it has become possible for vehicles to coordinate and cooperation among vehicles. In recent years, multiple vehicles formation becomes a focal problem in academia.[9] This paper will study the formation problem through the UIV system. Assuming that the number of individuals in UIV system is $N$, for each individual, the task can be decomposed into individual attitude control and the formation control. To facilitate the analysis and solution of the problem, the control of a single smart car can be divided into two loops: outer-loop and inner-loop.[4][1] As exhibited in Fig. 1, where $r(t)$, $\psi_e(t)$, $\psi(t)$, $\rho(t)$ and $c(t)$ represent the expected formation, expected attitude, attitude, control torque and actual state vectors, respectively. This paper mainly focuses on the study of the control problem of the outer loop, that is, the formation control. For the attitude control of the inner loop, please refer to the use of PID controller in [10][11]. For a single intelligent vehicle $i$, $i \in \{1,2,\cdots,N\}$, the outer loop control can be expressed as:

$$\dot{x_i}(t) = v_i(t),$$

$$\dot{v_i}(t) = u_i(t),$$

where $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^n$ denote the position, velocity and control input vectors of UIV $i$ respectively. [4]

Fig. 1 Structure diagram of double-loop system.
The entire interactive system of the unmanned intelligent vehicle can be expressed as an undirected graph \( G = (V, E) \), \( V \) represents a collection of \( n \) nodes and each node corresponds to an individual intelligent vehicle. \( E \) represents the set of connections between two nodes, that is, the interactive relationship between two intelligent vehicles. For \( \forall i, j \in \{1,2,\cdots,N\} \), use \( w_{ij} \) to represent the communication and connection between the UIV \( i \) and the UIV \( j \), thus forming a symmetric matrix \( W \). Define \( L = D - W \), where \( D = \text{diag}\left\{ \sum_{j=1}^{N} w_{ij}, i = 1,2,\cdots,N \right\} \) as the in-degree matrix. Let \( \theta(t) \) represents the switching signal the value of which is the index of the topology at \( t \) correspondingly, \( \Gamma_{\theta(t)} \), \( L_{\theta(t)} \) are the topological structure graph and Laplacian matrix at time \( t \). Use \( \lambda_{\theta(t)}^{\pm} \) represents the eigenvalues of the Laplacian matrix, assuming \( \lambda_{\theta(t)}^{1} \leq \lambda_{\theta(t)}^{2} \leq \cdots \leq \lambda_{\theta(t)}^{N} \). Let \( \Delta_{\theta(t)} = \text{diag}\{ \lambda_{\theta(t)}^{1} \lambda_{\theta(t)}^{2} \cdots \lambda_{\theta(t)}^{N} \} \).

Define \( c(t) = [c_{t1}^{T}, c_{t2}^{T}, \cdots, c_{tN}^{T}]^{T} \), where \( c_{ti}(t) = [x_{ti}(t), v_{ti}(t)]^{T}, r(t) = [r_{t1}^{T}, r_{t2}^{T}, \cdots, r_{tN}^{T}]^{T} \), where \( r_{ti}(t) = [r_{ix}(t), r_{iy}(t)]^{T} \) is a continuous differentiable vector. The paper [4] puts forward the control objectives of the system formation as:

\[
\lim_{t \to \infty} (c_{t}(t) - r_{t}(t)) = 0 \quad (i = 1,2,\cdots,N) \quad (2)\text{in formula (2)}
\]

\[
\lim_{t \to \infty} (c_{t}(t) - r_{t}(t)) = 0 \quad (i = 1,2,\cdots,N)
\]

For the UIV \( i \), the corresponding control output is proposed in [4] as follows:

\[
\text{uit} = \dot{G}_{1}\text{cit} - \dot{r}_{it} - \dot{R}(t) = G_{1}(c_{t}(t) - r_{t}(t)) + G_{2} \sum_{j\in N_{i}(t)} w_{ij} \left( (c_{j}(t) - r_{j}(t)) - (c_{i}(t) - r_{i}(t)) \right) + \dot{r}_{iv}(t) \quad (3)
\]

In formula (3), \( G_{1} \in \mathbb{R}^{1 \times 2} \) and \( G_{2} \in \mathbb{R}^{2 \times 2} \) are the gain matrices. The specific formula derivation can refer to [4].

Combining formulas (1)(3) and using Kronecker product can gain the control algorithm for the entire UIV system:

\[
\dot{c}(t) = \left( I_{N} \otimes (K_{1}\dot{G}_{1} + K_{2}K_{2}^{T} - L_{\theta(t)} \otimes (K_{2}\dot{G}_{2} + I)) \right) C(t) - I_{N} \otimes K_{2}G_{1} - L_{\theta(t)} \otimes K_{2}G_{2} \lim_{t \to \infty} (c_{t}(t) - r_{t}(t) - R(t)) = 0 \quad (i = 1,2,\cdots,N) \quad (4)
\]

Among them, \( I_{N} \) is the \( N \) row unit column vector, \( \otimes \) is the Kronecker product, \( K_{1} = [1,0]^{T}, K_{2} = [0,1]^{T} \).

The time-varying expected formation form \( r_{i}(t) \) needs to meet the sufficient and necessary conditions proposed in [4] as follows:

1. For the agent \( i \) in the system satisfies:

\[
\lim_{t \to \infty} (r_{ix}(t) - r_{iv}(t)) = 0
\]

Where \( i \in \{1,2,\cdots,N\}, j \in N_{i}(t) \).

2. Linear systems need to have asymptotic stability,

\[
\dot{\sigma}(t) = (I_{N} - I) \otimes (K_{2}\dot{G}_{2} + K_{1}K_{2}^{T} - L_{\theta(t)} \otimes K_{2}G_{2})\sigma(t)\quad (6)
\]

Where \( \sigma(t) \) indicates the system status. In order to determine the gain matrix \( G_{1}, G_{2} \), [4] proposed three steps as follows:

1: For a particular formation, the target formation form needs to meet the necessary and sufficient conditions (5), (6), otherwise, the target formation form needs to be re-selected.

2: Select \( G_{1} \) and the eigenvalues of \( K_{2}G_{1} + K_{1}K_{2}^{T} \) are used to determine the formation reference function.

3: Solve the algebraic Riccati equation in [4] to obtain \( P \), use the formula \( G_{2} = (2\lambda_{\min})^{-1}K_{2}^{T}P \) to solve the gain matrix \( G_{2} \).
3. Formation simulation
The motion plane of the UIV system is a two dimensional plane, that is, \( n = 2 \). Four intelligent vehicles are selected as the research objects and the desired formation form is selected which is decomposed to the X and Y axes as shown in (7):

\[
\mathbf{r}_i(t) = \begin{bmatrix}
  r \cos \left( \omega t + \frac{(i-1)\pi}{2} \right) \\
  -\omega r \sin \left( \omega t + \frac{(i-1)\pi}{2} \right) \\
  r \sin \left( \omega t + \frac{(i-1)\pi}{2} \right) \\
  \omega r \cos \left( \omega t + \frac{(i-1)\pi}{2} \right)
\end{bmatrix} (i = 1,2,3,4),
\]

Where \( w = 10 \text{ m}, \omega = 0.1 \text{ rad/s} \). Choose \( K_1 = I_2 \otimes [-1, -0.8] \), According to the steps, the corresponding \( K_2 = I_2 \otimes [0.3535, 0.6582] \) can be obtained. The set of topology graphs between intelligent vehicle is shown in Fig. 2.

Choose the initial states of four intelligent vehicles as \( c_1(0) = [9.84, -0.11, 0.19, 0.07]^T \), \( c_2(0) = [-0.41, 0.04, 10.51, 0.22]^T \), \( c_3(0) = [-10.47, 0.08, 0.48, 0.02]^T \), \( c_4(0) = [-0.93, -0.08, -9.11, -0.25]^T \). Fig. 3 displays the state trajectories of the four UIVs in the simulation, where the initial states of the four UIVs are marked with pentagonal, square, circle, hexagon respectively. It can be seen that the position trajectory and speed trajectory of the four intelligent vehicles are close to the theoretical value after time \( t \) is a certain value, which indicates the UIV system can realize formation. Define the error function \( \delta_i(t) = c_i(t) - r_i(t) - R(t) \), Fig. 4 exhibits the error curve of each UIV including the error of position as well as velocity. (a), (b), (c), (d) represent the error curves of four intelligent vehicles. It is easy to see that the system’s speed as well as position errors are close to zero in a short time. The algorithm is effective.
Fig. 4 Error curve $\delta (t)$. (a) UIV1, (b) UIV2, (c) UIV3 and (d) UIV4.

4. Algorithm improvement and simulation

4.1. Algorithm

In [4], the corresponding formation control algorithm was proposed for the time-varying system as shown in (3), (2) proposes the control purpose of the control algorithm. However, no reference function form is given in the case of the overall movement of the formation form. On this basis, make the following amendments to the control purpose:

$$\lim_{t \to \infty} (c_i(t) - h_i(t) - R) = 0 \quad (i = 1, 2, \cdots, N).$$

(8)

In the formula, $R$ replaces $R(t)$ to become a fixed reference center, that is, determine the reference center to be a fixed point. And then consider the movement of the entire formation on the basis of $r_i(t)$.

Moreover, change the expected formation form function to $h_i(t), h_i(t) = r_i(t) + f(t)$, where $f(t)$ represents the motion function of the entire formation.

From (3), the control algorithm (3), (4) can be changed to:

$$u_i(t) = G_1(c_i(t) - h_i(t)) + G_2 \sum_{j \in N_{h_i(t)}} w_{ij} \left( (c_j(t) - h_j(t)) - (c_i(t) - h_i(t)) \right) + \dot{h}_i(t).$$

(9)

$$\dot{c}(t) = (I_N \otimes (K_2G_1 + K_1K_2^2) - L_{B(t)} \otimes (K_2G_2)) \dot{c}(t) - (I_N \otimes (K_2G_1) - L_{B(t)} \otimes (K_2G_2)) h(t) + (I_N \otimes K_2) \dot{h}_i(t).$$

(10)
4.2. Simulation

On the foundation of the third section, we fix the coordinates of the control center $R$ to $(0, 0)$, and make the movement form of the formation on the X and Y axes as shown in (11):

$$r_i(t) = \begin{bmatrix}
    r \cos\left(\omega t + \frac{(i-1)\pi}{2}\right) + Vt \\
    -\omega r \sin\left(\omega t + \frac{(i-1)\pi}{2}\right) + V \\
    r \sin\left(\omega t + \frac{(i-1)\pi}{2}\right) + Vt \\
    \omega r \cos\left(\omega t + \frac{(i-1)\pi}{2}\right) + V
\end{bmatrix} \quad (i = 1, 2, 3, 4),$$

In the formula, $V = 1 \text{m/s}$, which is equivalent to the uniform movement of the entire formation to the upper right corner at the speed of $V$. Fig. 5 show the state trajectories of the four UIVs with mobile reference center, where the initial velocity of the four UIVs are marked with pentagonal, square, circle, hexagon. It is noticeable that the four UIVs move straight to the upper right corner at a constant speed. With realizing the basic circular formation, the velocity trajectory also approaches a circle after a certain time $t$, which meets the formation requirements. Fig. 6 displays the movement of the formation system in different time periods, and the circle presented in the form of a dotted line is used to detect whether the system at time $t$ satisfies the circular formation form. The selected time nodes are: $t_1 = 20s, t_2 = 40s, t_3 = 60s, t_4 = 80s$ respectively corresponding to the four graphs (a), (b), (c), (d). The formation form at a particular moment still satisfies the circular shape and satisfies the formation requirements. Define the error function $\delta_{ix}(t) = c_{ix}(t) - h_{ix}(t), \delta_{iv}(t) = c_{iv}(t) - h_{iv}(t)$ Fig. 7 exhibits the error curve of each UIV including the error of position as well as velocity. The figure (a) corresponds to the position error curve of four intelligent vehicles, and the figure (b) corresponds to the velocity error. It is easy to see that the speed as well as position errors of the system approach zero in a short time, and the control algorithm is effective for the formation system.

Fig. 5 State trajectories of four UIVs with mobile reference center. (a) Positions and (b) velocities.
5. Conclusion
This paper discusses the formation control of unmanned intelligent vehicles. The formation model and the formation goal proposed in [4] are given. The corresponding control algorithm and the necessary and sufficient conditions for formation are presented. And simulation are given to prove the effectiveness of the algorithm. On this basis, the control algorithm proposed in [4] is modified to make it able to deal with the entire formation movement. The formation control algorithm when the reference center is the point of movement is proposed and simulation is given. Under different target formations, intelligent vehicles can quickly achieve the target formation, with faster convergence speed, good control effect, and strong algorithm robustness.

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