Accretion onto Stars with Octupole Magnetic Fields: Matter Flow, Hot Spots and Phase Shifts

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ABSTRACT

Recent measurements of the surface magnetic fields of classical T Tauri stars (CTTSs) and magnetic cataclysmic variables show that their magnetic fields have a complex structure. The magnetic field associated with the octupole moment may dominate the magnetic field associated with other moments in some stars, such as the CTTS V2129 Oph. Previously, we studied disc accretion onto stars with magnetic fields described by a superposition of aligned or misaligned dipole and quadrupole moments. In this paper, we present results of the first simulations of disc accretion onto stars with an octupole field. As examples, we consider stars with a superposition of octupole and dipole fields of different strengths and investigate matter flow around them, the shapes of hot spots on their surfaces, and the light curves produced by their rotation. We investigate two possible mechanisms for producing phase shifts in the light curves of stars with complex fields: (1) change of the star's intrinsic magnetic field and (2) variation of the accretion rate, causing the disc to interact with the magnetic fields associated with different stellar magnetic moments. We also discuss the applicability of the potential approximation for extrapolation of the surface magnetic field to larger distances.

Key words: accretion, accretion discs - magnetic fields - MHD - stars: magnetic fields.

1 INTRODUCTION

Strong magnetic fields have been observed in a number of stars, including classical T Tauri stars (CTTSs) (Basri, Marcy & Valenti 1992; Johns-Krull 2007), magnetic white dwarfs (e.g., Warner 1995; Euchner et al. 2002), and various types of neutron stars (e.g., Ghosh & Lamb 1978; Ghosh 2007). In most stars the structure of the field is unknown.

It is usually suggested that the magnetic fields of gaseous stars are generated and supported by some type of the dynamo mechanism that operates in the stellar interior or in the surface layers of the star. Numerical simulations of uniformly rotating, fully convective stars show the formation of complex, non-axisymmetric fields that can be represented only by a superposition of different multipoles (e.g., Chabrier & Küker 2006). However, in simulations of differentially rotating stars, an additional, ordered component appears (e.g., Dobler, Stix & Brandenburg 2006).

Measurements of the surface magnetic fields of CTTSs using different techniques indicate that they have a complex structure (Johns-Krull, Valenti & Koresko 1999; Johns-Krull 2007). Measurements of the magnetic fields of nearby low-mass stars using the Zeeman-Doppler technique show that their fields are often complex (Donati & Cameron 1997; Donati et al. 1999; Jardine et al. 2002). Recent observations of two CTTSs have shown that in one star (V2129 Oph) the surface magnetic field associated with the octupole moment dominates the fields associated with other moments (Donati et al. 2007), whereas in the other star (BP Tau) the fields associated with the dipole and octupole moments are both significant and dominate the fields associated with other multipoles (Donati et al. 2008).

Recently, the magnetic field structure of gaseous stars with complex fields has been studied analytically. It was found that the fraction of the flux through the stellar surface in open field lines is smaller in stars with complex fields than in those with purely dipolar fields (Gregory et al. 2008; Mohanty & Shu 2008). The potential approximation is usually used to extrapolate the surface magnetic field to larger distances. Gregory et al. (2006) calculated possible gas flows around stars...
with magnetic fields constructed from measurements using the potential approximation. Recently, we were able to compute the flow of gas around V2129 Oph and BP Tau in global 3D MHD simulations in which the magnetic field is not assumed to be given by a potential but is instead calculated as part of the simulation (Long et al. 2009).

Zeeman tomography of magnetic white dwarfs has shown that they also have complex magnetic fields (Euchner et al. 2002). In some, such as HE 1045−0908, the magnetic field associated with the star's quadrupole moment dominates whereas the fields associated with its dipole and octupole moments are much weaker (Euchner et al. 2005). In others, representation of the field requires inclusion of misaligned dipole, quadrupole, octupole, and other multipoles up to $n = 4$ or $n = 5$ (Euchner et al. 2006; Beuermann et al. 2007). These results are in accord with earlier indications of field complexity, such as asymmetric distribution of spots on the stellar surface (Meggitt & Wickramasinghe 1989; Pirola et al. 1987; see Wickramasinghe & Ferrario 2000 for a review).

Neutron stars also have dynamically important magnetic fields. Soon after they have formed, the magnetic fields of some may be enhanced by a dynamo mechanism (e.g., Thompson & Duncan 1993) and may have a complex structure. There is evidence that at least some accretion-powered X-ray pulsars have complex magnetic field structures (Elslner & Lamb 1976; Gil et al. 2002; Nishimura 2005) or that the dipole moment is off-center (e.g., Coburn 2001; see also Ruderman 1991; Chen & Ruderman 1993). Accreting millisecond X-ray pulsars have relatively weak dipole magnetic fields (e.g., Psaltis & Chakrabarty 1999), but the details of their fields are unknown. They have almost sinusoidal light curves that are consistent with a dipole magnetic field with a small inclination to the spin axis (e.g., Gierliński & Poutanen 2005; Lamb et al. 2008a,b). However, this does not exclude the presence of higher-order multipole fields near the star. An unusual phenomenon — rapid, relatively large shifts in the phase of the light curve — has been observed in many millisecond pulsars (e.g., Morgan et al. 2003; Markwardt 2004; Burderi et al. 2006; Hartman et al. 2009; Patruno et al. 2009). Several mechanisms have been proposed to explain this phenomenon.

Lamb et al. (2008a,b) have explained several different properties of accreting millisecond pulsars, including the phase shifts of their light curves, with the “nearly aligned moving spot model”. Motion of emitting areas located close to the spin axis alters both the shape and the arrival time of the pulse in a complex way. These movements could be caused by changes in the accretion rate. Earlier 3D MHD simulations of accretion onto stars with misaligned dipole magnetic fields (Romanova et al. 2003, 2004) have shown that if the dipole is nearly aligned with the spin axis, the place where the accreting matter impacts the stellar surface can move prograde or retrograde relative to the surface, which favors the moving spot idea. In another model, the magnetic field of the neutron star is assumed to change in time (Burderi et al. 2006). Here we consider stars with complex magnetic fields and investigate both of these mechanisms for producing phase shifts.

In our earlier studies, we were able to perform global 3D MHD simulations of accretion onto stars with a dipole magnetic field (Koldoba et al. 2002; Romanova et al. 2003, 2004; Kulkarni & Romanova 2005) and onto stars with a combination of dipole and quadrupole fields (Long, Romanova & Lovelace 2007, 2008). 3D simulations of accretion onto stars with multipolar fields are very challenging due to the very steep gradients of the magnetic field. In this paper, we report on the first simulations of accretion onto stars with an octupole field component. Our methods allow us to investigate the general case of a magnetic field produced by superposition of dipole, quadrupole, and octupole fields oriented in different directions. However, even the simpler case of a magnetic field produced by superposition of misaligned dipole and quadrupole fields creates very complex field structures (e.g., Long et al. 2008). Consequently, for clarity we omit quadrupole fields in the present work, restricting consideration to the simpler case of a superposition of octupole and dipole fields. We investigate matter flows onto stars with such fields, the shapes of the resulting hot spots, and the light curves produced by these hot spots. We also investigate mechanisms for producing phase shifts in the light curves of stars with complex magnetic fields. Finally, we discuss the validity of the potential approximation for computing the magnetic field outside such stars.

Section 2 describes the simulation method and the magnetic field configurations. Section 3 presents our results for different combinations of dipole and octupole fields. Section 4 shows examples of the phase shifts that are possible in the light curves of stars with complex fields. Section 5 summarizes and discusses our most important results.

2 NUMERICAL MODEL AND MAGNETIC FIELD CONFIGURATIONS

2.1 Model

Our 3D MHD model has been described in a series of papers (Koldoba et al. 2002; Romanova et al. 2003, 2004; Kulkarni & Romanova 2005; Long et al. 2007, 2008), where disc accretion onto stars with dipole and more complex magnetic fields has been investigated. Here, we briefly summarize different aspects of the model.

2.1.1 Initial Conditions

We consider a rotating magnetic star surrounded by an accretion disc and a corona. The disc is cold and dense, while corona is hot and rarefied, and at the reference point (the
inner edge of the disc in the disc plane \( \ell_c = 100T_a \), \( \rho_c = 0.01\rho_d \), where the subscripts \( d \) and \( c \) denote the disc and the corona. The disc and corona are initially in rotational hydrodynamic equilibrium, where the sum of the gravitational, centrifugal, and pressure gradient forces is zero at each point in the simulation region. To avoid an initial magnetic field discontinuity at the disc-corona boundary, the corona is set to rotate with the Keplerian angular velocity at each point in the simulation region. To avoid an initial magnetic and the corona. The disc and corona are initially in rotational

\[ T^\rho_{\ell_c} \]

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At the inner boundary (the surface of the star), most of variables are set to have free boundary conditions, \( \partial A/\partial r = 0 \). The initial magnetic field on the surface of the star is taken to be a superposition of misaligned dipole and octupole fields (see \( B_1 \) and \( B_3 \) in Eqn. 2). As the simulation proceeds, we assume that the normal component of the fields remain unchanged, i.e., the magnetic field is frozen in the surface of the star. We neglect possible changes of the magnetic field structure inside the star due to, e.g., dynamo processes, because in the most cases the time scale of such variation is much longer than the length of the simulation time.

At the outer boundary, free conditions are taken for all variables. In addition, matter is not permitted to flow back from the corona into the region. The simulation region is large enough (\( r_{\text{max}} \approx 36R_\star \)), and the disc is massive enough, to supply matter for the entire duration of the simulations.

2.1.2 Boundary Conditions

At the inner boundary (the surface of the star), most of variables \( A \) are set to have free boundary conditions, \( \partial A/\partial r = 0 \). The initial magnetic field on the surface of the star is taken to be a superposition of misaligned dipole and octupole fields (see \( B_1 \) and \( B_3 \) in Eqn. 2). As the simulation proceeds, we assume that the normal component of the fields remain unchanged, i.e., the magnetic field is frozen in the surface of the star. We neglect possible changes of the magnetic field structure inside the star due to, e.g., dynamo processes, because in the most cases the time scale of such variation is much longer than the length of the simulation time.

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2.1.3 The “Cubed sphere” Grid

The 3D MHD equations are solved with a Godunov-type code on the “cubed sphere” grid (Koldoba et al. 2002; see also Putman & Lin 2007). The grid consists of \( N_r \) concentric spheres, where each sphere represents an inflated cube. Fig. [1] shows that the grid consists of six sectors corresponding to six sides of the cube with \( N \times N \) curvilinear Cartesian grids on each side. The whole grid consists of \( 6 \times N_r \times N^2 \) cells. For modeling of octupole fields, a higher radial grid resolution is needed near the star compared with the pure dipole cases. We do this by choosing the radial size of the grid cells to be 2.5 times smaller than the angular size in the region \( r < (5 - 7)R_\star \) (see (2.1.4)), while it is equal to the angular size in the outer region as in all our previous work. The typical grid used in simulations has \( 6 \times N_r \times N^2 = 6 \times 200 \times 51^2 \) grid cells. Simulations with higher/lower grid resolutions were performed for comparisons.

2.1.4 Reference Units

The MHD equations are solved using dimensionless variables \( \tilde{A} \). To obtain the physical dimensional values \( A \), the dimensionless values \( \tilde{A} \) should be multiplied by the corresponding reference units \( A_0 \); \( A = \tilde{A}A_0 \). To choose the reference units, we first choose the stellar mass \( M_\star \) and radius \( R_\star \). The reference units are then chosen as follows: mass \( M_0 = M_\star \), distance \( R_0 = R_\star/0.35 \), velocity \( v_0 = (GM_\star/R_0)^{1/2} \), time scale \( \tau_0 = 2\pi R_0/v_0 \), angular velocity \( \Omega_0 = v_0/R_0 \). The reference magnetic field \( B_0 \) can be obtained by choosing a reasonable fiducial value for the surface dipole field strength \( B_{1\ell} \). Then \( B_0 \) is the fiducial dipole field strength at \( R_0 \): \( B_0 = B_{1\ell}(R_\star/R_0)^3 \). We then define the reference dipole moment \( \mu_{1,0} = B_0R_0^2 \), quadrupole moment \( \mu_{2,0} = B_0R_0^4 \), octupole moment \( \mu_{3,0} = B_0R_0^6 \), density \( \rho_0 = B_0^2/v_0^2 \), pressure \( p_0 = \rho_0v_0^2 \), mass accretion rate \( \dot{M}_0 = \rho_0v_0R_0^2 \), angular momentum flux \( L_0 = \rho_0v_0^2R_0^2 \) (the radiation flux \( J \) is also in units of \( E_0 \)), temperature \( \theta_0 = \tau_0p_0/\rho_0 \), where \( \tau \) is the gas constant, and the effective blackbody temperature \( T_{\text{eff},0} = (\rho_0v_0^2/\sigma)^{1/4} \), where \( \sigma \) is the Stefan-Boltzmann constant.

Therefore, the dimensionless variables are \( \tilde{r} = r/R_0 \), \( \tilde{v} = v/v_0 \), \( \tilde{t} = t/\tau_0 \), \( \tilde{B}_{1\ell} = B_{1\ell}/B_0 \), \( \tilde{\mu}_n = \mu_n/\mu_{0,n} \) (\( n = 1, 2, 3 \) for dipole, quadrupole and octupole components) and so on. In the subsequent sections, we show dimensionless values for all quantities and drop the tildes (~). Our dimensionless simulations are applicable to different astrophysical objects with different scales. We list the reference values for typical CTTs, cataclysmic variables, and millisecond pulsars in Tab. [1].

2.2 Magnetic Field Configurations

When electrical currents outside the star can be neglected, the magnetic field there can be described by a magnetic scalar potential \( \varphi(h) \) and can be represented as a sum of multipoles. In this work we consider intrinsic stellar magnetic fields that can be described outside the star by a superposition of symmetric dipole, quadrupole, and octupole fields. The total magnetic field can then be written

\[ \mathbf{B}(r) = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 \, . \]
of \( \Theta \) Sketch of the directions of the multipole magnetic moments. The rotation axis \( \Omega \) is in the \( z \) direction. The magnetic axes of the dipole \( \mu_1 \), quadrupole \( \mu_2 \) and octupole \( \mu_3 \) are inclined at angles of \( \Theta_1 \), \( \Theta_2 \) and \( \Theta_3 \) from the \( z \)-axis. The dipole moment \( \mu_1 \) is placed in the \( xz \) plane. The angles between \( \Omega - \mu_2 \), \( \Omega - \mu_3 \) and \( xz \) planes are \( \phi_2 \) and \( \phi_3 \) respectively.

\[
\begin{align*}
B_1 & = \frac{3\mu_1(\hat{\mu}_1 \cdot \hat{r})}{r^3} \hat{r} - \frac{\mu_1}{r^3} \hat{\mu}_1 \\
B_2 & = \frac{3\mu_2}{4r^4} \left( 5(\hat{\mu}_2 \cdot \hat{r})^2 - 1 \right) \hat{r} - \frac{3\mu_2}{2r^2} (\hat{\mu}_2 \cdot \hat{r}) \hat{\mu}_2 \\
B_3 & = 5\left( \frac{\mu_3}{2r^7} \right) (\hat{\mu}_3 \cdot \hat{r}) \left[ 7(\hat{\mu}_3 \cdot \hat{r})^2 - 3 \right] \hat{r} \\
& - 3\left( \frac{\mu_3}{2r^7} \right) \left[ 5(\hat{\mu}_3 \cdot \hat{r})^2 - 1 \right] \hat{\mu}_3.
\end{align*}
\]

Here \( B_1 \), \( B_2 \), and \( B_3 \) are the magnetic fields produced by the symmetric dipole, quadrupole and octupole moments and \( \hat{r} \), \( \hat{\mu}_1 \), \( \hat{\mu}_2 \), and \( \hat{\mu}_3 \) are the unit vectors describing the position and the symmetric dipole, quadrupole, and octupole moments, respectively. The magnetic moments may be inclined at angles \( \Theta_1 \), \( \Theta_2 \), and \( \Theta_3 \) relative to the rotation axis \( \Omega \). They may also be in different meridional planes \( \Omega - \mu_2 \) and \( \Omega - \mu_3 \), with at azimuthal angles \( \phi_2 \) and \( \phi_3 \) relative to the \( \Omega - \mu_1 \) plane defined by the dipole moment and the rotation axis.

Fig. 2 illustrates the geometry by an example in which all three magnetic moments have different orientations.

### 2.3 The Alfvén Surface and Magnetospheric Radius

The magnetospheric radius, \( r_m \), is a characteristic radius where the inflowing matter is stopped by the magnetosphere. Several expressions for this radius have been derived since the 1970s for stars with dipole fields. The first estimations were done for spherically-accreting matter. It was suggested that the radially-falling matter is stopped by the magnetic pressure of the magnetosphere (e.g., Lamb et al. 1973):

\[
B(r_A)^2/8\pi = \rho(r_A)v(r_A)^2,
\]

where \( r_A \) is the Alfvén radius and \( k \) is a dimensionless coefficient of the order of unity (Elsner & Lamb 1977; Ghosh et al. 1977). For disc accretion, a similar criterion can be used but for stresses. The disk rotates in the azimuthal direction and hence main stresses are connected with the azimuthal components of the stress tensor: \( T_{\phi\phi} = [p + \rho v_\phi^2] + [B^2/8\pi - B_\phi^2/4\pi] \) (here we neglected the viscous stress which is much smaller than the matter stress). The motion of the disk is disturbed by the rotating magnetosphere, when the matter stresses become comparable with the magnetic stresses, or \( p + \rho v_\phi^2 = B^2/8\pi - B_\phi^2/4\pi \). The dominant component of the dipole magnetic field is the poloidal component, and hence \( B_\phi \ll B \) and we obtain the condition for stresses as \( p + \rho v_\phi^2 = B^2/8\pi \).

Different criteria were proposed by other authors (Cameron & Campbell 1993; Armitage & Clarke 1996; Matt & Pudritz 2005). Analysis of these criteria shows that most of them give a radius similar to that given by Eqs. 3 but with different coefficients \( k \) (Bessolaz et al. 2008). A comparison of that formula with simulations gives \( k \approx 0.5 \) (Long, Romanova & Lovelace 2005), which is very close to the coefficient estimated by Ghosh & Lamb (1979).

For multipolars, we use the generalized formula, such that the \( n \)-th component of the field is \( B_n \sim \mu_n/r_n^{n+2} \), (see details in Appendix A2), and the magnetospheric radius

\[
r_m = k_n \mu_n/r_n, \quad r_n = \mu_n^{4/7} M^{-2/7} (GM)^{-1/7}.
\]

### 3 ACCRETION ONTO STARS WITH AN OCTUPOLE AND DIPOLE FIELD

#### 3.1 Pure octupole

First, we consider accretion onto a star with an octupole magnetic field with \( \mu_3 = 0.5 \). The magnetic axis of the octupole field is inclined (misaligned) from the stellar spin axis at a small angle, \( \Theta_3 = 10^\circ \).

Fig. 3 shows the magnetic field distribution on the stellar surface. There are two polar regions with strong positive (red) and negative (blue) polarities, as seen in the middle and right panels. There are also two octupolar belts with opposite polarities as seen in the left panel. A meridian line which goes from the north to the south magnetic poles will pass through the positive northern pole, negative northern belt, positive southern belt and negative southern pole with zero magnetic field at the magnetic equator. This distribution differs from the dipole field which has only positive and negative poles.

Fig. 4 (top panel) shows 3D views of matter flow to the...
star at time $t = 3.5$. The magnetic field near the star has an octupolar shape with three sets of closed loops of field lines connecting regions of different polarities on the star, which we call northern, southern and equatorial sets of loops. Some field lines are closed, while others are dragged by the disc, wrapped around the rotation axis and inflate into the corona. Matter of the disc is lifted above the equatorial set of loops and forms a low-density gap around the star.

Fig. 6 shows the density distribution on the star with an octupole magnetic field, $\mu_3 = 0.5, \Theta_3 = 10^\circ$, as seen from the equatorial plane (left panel), the north pole (middle panel), and the south pole (right panel).

The bold red line represents the magnetospheric surface, where the matter stresses equal the magnetic stresses,

$$\beta = \frac{p + \rho c^2}{B^2/8\pi} = 1.$$  \hspace{1cm} (6)

The magnetic field lines stay closed inside the magnetospheric surface, where the magnetic stresses dominate.

The initial octupolar field shown in the panel (a) is a potential field, that is, a field that has not been disturbed by the surrounding plasma. The potential approximation is often used in extrapolation of the observed surface fields to larger distances (e.g., Jardine et al. 2002; Gregory et al. 2006; Donati et al. 2007). Panels (b)-(d) show that the potential approximation is sufficiently good inside the Alfvén surface, where $\beta < 1$. However, the field strongly departs from the potential one at larger distances.

Fig. 7 shows that the hot spots on the stellar surface represent two rings which are located in the planes parallel to the magnetic equator. Each ring has a density enhancement on one side, where disc accretion is higher due to the inclination of the magnetic axis. The middle and right panels show that the density enhancements in the northern and southern rings are antisymmetric relative to the magnetic equatorial plane. That is why the left panel shows only a part of the southern ring.

For the calculation of the light curves, we use the same approach as Romanova et al. (2004). The total energy of the accreting matter is assumed to be converted into isotropic blackbody radiation on the surface of the star. The specific intensity of radiation from a position $\mathbf{R}$ on the stellar surface into a solid angle $d\Omega$ in the direction $\hat{k}$, is $I(\mathbf{R}, \hat{k}) = (1/\pi)F_c(\mathbf{R})$, where $F_c(\mathbf{R})$ is the total energy flux of the inflowing matter, $\theta = \arccos (\hat{\mathbf{R}} \cdot \hat{k})$. Therefore we obtain the radiation energy received per unit time in the direction $\hat{k}$,
\( J = r^2 F_{obs} = \int I(R, \hat{k}) \cos \theta dS \), where \( r \) is the distance between the star and the observer, \( F_{obs} \) is the observed flux, and \( dS \) is the surface area element. Simulations show that the spots usually “choose” their favorite position on the surface of the star, which does not vary much with time. That is why we fix the spots at some moment of time and rotate the face of the star, which does not vary much with time. That is why in Fig. 8, the magnetic field distribution, shows similar features to that in pure octupole case (Fig. 3).

Fig. 8 shows 3D views of accretion flow onto the star at time \( t = 5 \). The disc matter comes close to the star and mainly interacts with the octupolar component and accretes onto the octupolar belts on the stellar surface. External field lines are dragged by the disc and inflate. This case shows similar features to that of the pure octupole, because in both cases the octupole determines matter flow.

Fig. 10(a) shows that initially, at \( t = 0 \), the octupole and dipole components are equal in the equatorial and polar directions:

\[
\begin{align*}
\mu_{eq} &= \left( \frac{3}{2} \frac{\mu_3}{\mu_1} \right)^{\frac{1}{2}}, \\
\mu_{pole} &= \left( \frac{2 \mu_3}{\mu_1} \right)^{\frac{1}{2}}.
\end{align*}
\]

Substituting for \( \mu_1 \) and \( \mu_3 \), we obtain \( \mu_{eq} = 1.5 \) and \( \mu_{pole} = 1.4 \). At smaller/larger distances the octupole/dipole field dominates. It is evident that near the star, \( r \sim R_* = 0.35 \), the octupolar component is much stronger than the dipole component. That is why in Fig. 8, the magnetic field distribution, shows similar features to that in pure octupole case (Fig. 3).

3.2 Strong octupole and weak dipole

Next, we consider accretion onto a star with a superposition of a relatively strong octupole and a much weaker dipole component on the stellar surface: \( \mu_1 = 0.2, \mu_3 = 0.3, \Theta_1 = 30^\circ, \Theta_3 = 20^\circ, \phi_3 = 70^\circ \). Here, the rotation axis, dipole and octupole moments are all misaligned.

First, we estimate the radius at which the magnetic fields of the dipole and octupole components are equal. Using Eqn. 2 and suggesting for simplicity that the dipole and octupole moments are aligned, we obtain approximate formulae for the equatorial and polar magnetic fields: (1) Equatorial field: \( B_1 = \mu_1/r^3, B_3 = (3/2)\mu_3/r^3 \), and (2) Polar field: \( B_1 = 2\mu_1/r^3, B_3 = 4\mu_3/r^3 \). By equating \( B_1 \) and \( B_3 \) we obtain the distances where the dipole and octupole components are equal in the equatorial and polar directions:

\[
\mu_{eq} = \left( \frac{3}{2} \frac{\mu_3}{\mu_1} \right)^{\frac{1}{2}}, \mu_{pole} = \left( \frac{2 \mu_3}{\mu_1} \right)^{\frac{1}{2}}.
\]
component dominates near the star, while the dipole dominates at larger distances, such as the inner edge of the disc, which is different from the pure octupole case. Panels (b)- (d) show that the disc is stopped by the magnetosphere at $r_m \approx 0.9 \approx 2.6 R_\star$. This radius is smaller than $r_{eq} \approx 1.5$, and therefore the octupolar field dominates at the disc-magnetosphere boundary. The accretion flow is similar to that in the pure octupole case (see Fig. 5), though here the misalignment angle $\Theta = 20^\circ$ is larger and matter flows more easily above the equatorial set of loops. On the other hand, the octupole is inclined more strongly (than in the pure octupole case), and part of the funnel stream is located in the equatorial plane, due to which the magnetospheric gap is not empty (panel d).

Comparison of panel (a) with the other panels shows the difference between the initial potential field (panel a) and the non-potential fields obtained in the simulations. The magnetic field lines threading the disc and corona inflate, are wrapped around the rotation axis and form some kind of a magnetic tower (Romanova et al. 2009). The field strongly differs from the potential one at $r > r_m$, where the disc and corona matter strongly disturb the magnetosphere. The potential approximation, however, is reasonably good inside the magnetospheric radius, $r < r_m$, where $\beta < 1$ (inside the red line in the figure).

Fig. 11 shows the hot spots on the surface of the star. The spots are similar to those in the pure octupole case, but the rings have a higher inclination relative to the equatorial plane due to the larger misalignment angle.

Fig. 12 shows the light curves associated with rotation of the star. The shapes of the light curves strongly depart from sinusoids, in contrast with the pure octupolar case. However, the reason is not the small dipole component, but instead the fact that the misalignment angle ($\Theta_3 = 20^\circ$) is higher than that in pure octupolar case ($\Theta_3 = 10^\circ$), and the southern ring is seen by the observer even at small inclination angles $i$. At very high $i$, the observer sees both rings in similar proportions, and the light curve becomes sinusoidal with two peaks per period.

### 3.3 Strong dipole and weak octupole

We now consider the case where the dipole component ($\mu_1 = 2.0$) is much stronger than the octupole component ($\mu_3 = 0.2$). The misalignment angles are $\Theta_1 = 20^\circ$, $\Theta_3 = 10^\circ$, the phase angle is $\phi_3 = 180^\circ$.

From Eqn. 7 we find the critical radii where the dipole and octupole components are equal: $r_{eq} = 0.39$ and $r_{pole} = 0.45$. These radii are only slightly larger than the radius of the star, $R_\star = 0.35$, and hence the dipole determines the dynamics of the flow in the entire vicinity of the star.

Fig. 13 shows the magnetic field on the surface of the star. The field distribution is complex and different from a dipole or octupole field. There are two magnetic poles with different polarities near the octupole axis, and two belts of different polarities caused by the octupole component. The field in the belts, however, is strongly distorted by the dipole component.

Fig. 14 shows the 3D views of the accretion flow to the star at $t = 9$. The magnetic field looks like a dipole at all distances. The matter is stopped by the dipole component of the magnetosphere and is channeled to the polar regions in two funnel streams which is typical for accretion onto a star with a dipole field (Romanova et al. 2003). In the disc plane (bottom panel), the matter is stopped by the dipole-like field, and no equatorial accretion is observed.

Fig. 15 (panel a) shows that initially, at $t = 0$, the mag-
Figure 15. Density distribution in different slices (color background) and 3D magnetic field lines (yellow lines) for the case of a strong dipole and a very weak octupole field with parameters $\mu_1 = 2.0$, $\mu_3 = 0.2$, $\Theta_1 = 20^\circ$, $\Theta_3 = 10^\circ$, $\phi_3 = 180^\circ$. Panel (a) shows an $xz$ slices at $t = 0$; panels (b), (c), (d) show $xz$, $yz$ and $xy$ slices at $t = 9$. The red lines show the magnetospheric surface, where $\beta = 1$. The thick cyan, white and orange lines represent the rotation, dipole and octupole axes respectively.

The magnetic field has a dipole shape in the whole simulation region excluding the close vicinity of the star. Panels (b) and (c) show that the disc matter is stopped by the dipole-like magnetosphere and accretes towards the poles in two funnel streams. Panel (d) shows that a low-density magnetospheric gap forms around the star in the disc plane. Comparison of panel (a) with panels (b)-(d) shows that the potential approximation of the initially dipole magnetic field shown in panel (a) stays valid at later times only inside the magnetospheric surface, $r < r_{eq}$, where $\beta < 1$. At larger distances, the field lines are dragged by the disc and corona, and the field strongly departs from the initially potential dipole field.

The hot spots are shown in Fig. 16. As we expect, there are only two polar spots, which is typical for accretion onto a star with a dipole field. No ring-like spots are observed because the dipole component dominates almost everywhere and matter is not channeled by the octupole-like field. The hot spots are located near the northern and southern magnetic poles.

Fig. 17 shows the light curves associated with the rotation of the star at $t = 9$. The shapes of the light curves are quite sinusoidal for almost all inclination angles, which is also typical for stars with slightly misaligned dipole fields (Romanova et al. 2004). At large $i$, two peaks per period are observed, because the spot near the southern magnetic pole becomes visible and contributes to the light curves.

### 3.4 Dipole and octupole of comparable strength

We now consider an interesting case where both dipole and octupole components disrupt the disc and channel matter. The parameters are: $\mu_1 = 1$, $\mu_3 = 0.3$, $\Theta_1 = 30^\circ$, $\Theta_3 = 20^\circ$, $\phi_3 = 70^\circ$. For these parameters we find from Eqs. $r_{eq} = 0.67$, $r_{pole} = 0.77$. However, the inclination of the dipole and octupole axes could also influence the result. Fig. 18 shows a close view of the matter flow. In the $xz$ plane, the funnel streams are first channeled by the dipole-like field. When the matter flows close to the star, at about $2R_\star$, the octupole determines the flow and converts each funnel stream into three small accretion streams between the loops of field lines. In the $yz$ plane, the disc is stopped by the dipole component, and only a small amount of matter flows along the dipole-like field lines and is then governed by the octupole. The $xy$ plane shows that the matter flow is strongly non-axisymmetric: in some directions the disc matter is stopped by the dipole, and in other directions matter flows much closer to the star and is stopped by the octupolar component.

Figure 19 shows the hot spots in this case. Most of the matter is governed by the octupolar field and forms ring-like spots. Some matter flows into the polar regions located near the dipole magnetic poles.

### 4 PHASE SHIFTS IN LIGHT CURVES OF STARS WITH COMPLEX FIELDS

In stars with fixed complex magnetic fields, and with a constant accretion rate, the emitting region may have complex shapes, but the location of these hot spots is approximately fixed on the stellar surfaces. Rotation of these stars leads to a light curve with a pattern that repeats every rotation, and no variations like phase shift of the light curves are expected. That is what we see in the light curves shown in Fig. 7, 12 and 17 where oscillations occur with the fixed, initial phase. However, the pulse from the emitting region may change in a complex way and produce the phase shift, if (1) the magnetic
field of the star reconstructs, or (2) accretion rate varies with time. Below we discuss these two possibilities.

4.1 Phase shift due to field reconstruction

The magnetic field of a star may vary with time due to internal dynamo processes. Such variations may lead to reconstruction of the field, and different multipoles may dominate after the

Figure 11. The surface density distribution on the star with a strong octupole and weak dipole magnetic field, $\mu_1 = 0.2, \mu_3 = 0.3, \Theta_1 = 30^\circ, \Theta_3 = 20^\circ, \phi_3 = 70^\circ$, at $t = 5$, as seen from the equatorial plane (left panel), the north pole (middle panel), and the south pole (right panel).

Figure 12. The light curves in the case of a strong octupole and a weak dipole, $\mu_1 = 0.2, \mu_3 = 0.3$ and $\Theta_1 = 30^\circ, \Theta_3 = 20^\circ, \phi_3 = 70^\circ$, for different inclination angles $i$.

Figure 13. The surface magnetic field strength of the star with a strong dipole and weak octupole magnetic field, $\mu_1 = 2.0, \mu_3 = 0.2, \Theta_1 = 20^\circ, \Theta_3 = 10^\circ, \phi_3 = 180^\circ$, as seen from the equatorial plane (left panel), the north pole (middle panel) and the south pole (right panel).
Figure 14. 3D views of matter flow to a star with a strong dipole and weak octupole magnetic field, $\mu_1 = 2.0, \mu_3 = 0.2, \Theta_1 = 20^\circ, \Theta_3 = 10^\circ, \phi_3 = 180^\circ$, at $t = 9$. The disc is shown by a constant density surface in green with $\rho = 0.25$ in the top panel; different density levels in the disc plane are shown in the bottom panel. The colors along the field lines represent different polarities and strengths of the field. The thick cyan, white and orange lines represent the rotation, dipole and octupole axes respectively.

Figure 16. The hot spots viewed from different angles for the case of $\mu_1 = 2.0, \mu_3 = 0.2, \Theta_1 = 20^\circ, \Theta_3 = 10^\circ$, $\phi_3 = 180^\circ$: from the equatorial plane (left panel); from the north pole (middle panel); and from the south pole (right panel). The color represents the density.

Figure 18. The octupole-like field close to the star (like in §3.1) and the hot spots represent two octupolar rings. However, the orientation of the rings and their brightness distribution are different due to the different axis directions. The most important is that the octupole axes have different directions relative to the meridional plane, which leads to changes in hot spots and that determines the phase shift. Fig. 20 shows that the light curves in states (a) and (b) have different phases with a phase shift of $\Delta \Phi = \Phi_b - \Phi_a = 170^\circ$, where the amplitudes are normalized to the same values. The shapes of the light curves are different because the angle $\Theta_3$ is different in these states, although this difference may not always lead to changes in the pulse shapes.

In most cases it is not well known how fast the dynamo-driven magnetic field reconstruction is. Observations of the CTTSs type young stars show that the field may possibly vary on the scales of months or even weeks (Smirnov et al. 2004; Donati et al. 2007, 2008). It is less clear what the time-scale of magnetic field variation in neutron stars and white dwarfs is.

We should mention that such phase shifts could occur during reconstruction of a field with any combination of multipoles, including a purely dipole configuration.

4.2 Phase shift due to accretion rate variation

In another example we consider a star with a fixed complex field where the different multipoles are oriented at different meridional angles, $\phi_n$. Then, at high accretion rates, the disc comes closer to the star and interacts with the higher order multipoles, while at lower accretion rates, the truncation dis-
dimensional units into a different form. In order to do that, we note that the dimensionless parameters $\mu_1$ and $\mu_3$ control the magnetic moments of the star. However, they can also be used to represent the lower order multipoles. Now, however, we keep $\mu_1$ fixed. This brings us the convenience of using $\mu_1$ to represent the accretion rate while, which is why varying $\mu_1$ corresponds to varying the dipole moment. Now, however, we keep $\mu_1$ fixed. It is then convenient to express $\mu_{1,0}$ in terms of $\mu_{1,0}$ as $\mu_{1,0} = \mu_{1,0}/\mu_1$. Then, $\mu_{1,0} = B_0 R_0^2$ gives $B_0 = \mu_{1,0}/R_0^2 = \mu_{1,1}/\mu_1 R_0^2$, and $\mu_{3,0} = B_0 R_0^3$ becomes $\mu_{3,0} = \mu_{1,1} R_0^2/\mu_1$. The dimensional dipole moment then is $\mu_{1,1} = \mu_{3,0}/\mu_1 = (\mu_3/\mu_1) \mu_{1,1} R_0^3$. The reference accretion rate is $\dot{M}_0 = \rho_0 v_{\theta} R_0^2 = B_0^2 R_0^5/\sqrt{GM} = \mu_{1,1}^2/(\mu_3^2 R_0^{7/2} \sqrt{GM})$. The dimensionless accretion rate $\dot{M}_{dim}$ is then given by

$$\dot{M}_{dim} \approx \frac{\dot{M}}{\mu_1^2} \frac{\mu_3^2}{(GM_c)^{1/2} R_0^{7/2}}. \quad (8)$$

It is now clear that changing the dimensionless parameter $\mu_1$ has the effect of changing the accretion rate $\dot{M}_{dim}$, while keeping $\mu_3$ fixed as mentioned above. To keep $\mu_3$, fixed as well, we simply have to change $\mu_1$ such that the ratio $\mu_{1,1}/\mu_{1,0}$ is fixed. This allows us to change the accretion rate while keeping the stellar magnetic field fixed.

The physical meaning of this recasting is the following. The most natural interpretation of changing $\mu_1$ and $\mu_3$ is changing the stellar magnetic field, the result of which is a decrease in the magnetospheric radius. However, we can also decrease the magnetospheric radius by fixing the stellar magnetic field and increasing the accretion rate instead, which is exactly what Eqn. (8) shows. The dimensionless parameters $\mu_1$ and $\mu_3$ are therefore best thought of as controlling the magnetospheric size.

This brings us the convenience of using $\mu_1$ to represent two states of a star with different accretion rates to investigate the phase shifts between them. As an example, we take a star with a superposition of dipole and octupole fields, where they have parameters, $\Theta_1 = 30^\circ$, $\Theta_3 = 20^\circ$, $\phi_3 = 70^\circ$ and fixed ratio of $\mu_3/\mu_1$. It is important that the dipole and octupole have different phase angles $\phi_n$.

We choose two states with the above parameters for the dipole and octupole but different $\mu_1$; state (a) with $\mu_1 = 2$; and state (b) with $\mu_1 = 0.2$. We observe from simulations that in state (a) the disc mainly interacts with the dipole component, while in state (b) it mainly interacts with the octupole component. Fig. 21 shows a phase shift of peaks $\Delta \Phi = \Phi_b - \Phi_a = 120^\circ$ between the states (a) and (b), where the amplitudes are normalized to the same values. The phase shift occurs because the octupole and dipole axes are located in different meridional planes and have different angles $\phi$, due to which the matter flows to different places of the star at different accretion rates.

This type of phase shift is expected during periods of accretion rate variation. What is the accretion rate "jump" between states (a) and (b)? We can give a simple estimate, $(\dot{M})_b/(\dot{M})_a = ((\mu_1)_a/((\mu_1)_b)^2 = (2/0.2)^2 = 100$. However, we should notice that in state (b) octupole strongly dominates and Eqn. (8) may not be used directly. In addition, the transition may occur at smaller values of $\mu_1$ in state (a). So, it is clear that this type of phase shift may be produced by accretion rate (luminosity) variation.

Another thing that should be mentioned is that the phase shifts would be expected to be accompanied by changes in the light curve pulse profile, because the shape and position of the hot spots is very different at different field configurations.
5 CONCLUSIONS AND DISCUSSIONS

We performed, for the first time, global 3D MHD simulations of accretion onto stars with predominantly octupolar magnetic fields, and for a combination of dipole and octupole fields. The calculation becomes possible due to enhancement of the grid resolution near the star. Simulations have shown that:

(i) If the disc interacts predominantly with the octupolar field, then matter flows into two octupolar rings on the surface of the star. In the case of an inclined octupole field, the hot spots are inclined and have a brightness amplification on one side of the ring.

(ii) At small misalignment angles of the octupole, $\Theta = 10^\circ$, the light curves are quite sinusoidal and “mimic” the shape of the light curves of the pure dipole field, in particular at small inclination angles $i$. However, at higher $\Theta$ (e.g., $\Theta = 20^\circ$), the light curves strongly depart from sinusoidal even for small $i$.

(iii) If the dipole component strongly dominates, then matter flows in two funnel streams governed by the dipole field, and two round spots form in the vicinity of the dipole magnetic poles. If the dipole and octupole have comparable strength at the truncation radius, then both components channel matter to the star. Usually both octupolar rings and polar spots form on the surface of the star.

(iv) The potential field approximation is valid only in the region around the star where the magnetic stresses are higher than the matter stresses. At larger distances, the field is dragged by the disc and corona and strongly departs from being potential. The external magnetic field usually acquires a significant azimuthal component and inflates. A number of field lines wrap around the rotation axis, forming a magnetic tower which may propagate to larger distances due to magnetic force (e.g., Romanova et al. 2009).

(v) Accretion onto stars with multipolar fields may lead to phase shifts in the light curves. This may occur either (a) during dynamo-generated internal field reconstruction, or (b) during variation of the accretion rate when the complex magnetic field is fixed and has different phase angles $\phi_n$ for each multipole component, but the disc interacts with multipoles of different orders at different accretion rates. The phase shifts would be expected to be accompanied by changes in the light curve profile, because the shape and position of the hot spots is very different at different field configurations.

These new challenging 3D simulations helped us understand accretion onto stars with octupolar fields. Recent measurements by Donati et al. (2007, 2008) of the two CTTSs V2129 Oph and BP Tau have shown that their fields have a significant octupolar component. In our next paper (Long et al. 2009) we compare our numerical model with observations of these stars.

Magnetic fields in young stars of the CTTS type may have a complex structure and may vary with time due to internal dynamo processes. The recently measured magnetic fields of the accreting T Tauri stars CV Cha and CR Cha (Hussain et al. 2009) show a complex structure consisting of a number of multipoles. In another CTTS, V2247 Oph, the magnetic field is complex and varies on the very short time-scale of a week (Donati et al. 2009). Frequent phase shifts are expected in this star due to internal field reconstruction, as discussed in this paper. In all these stars the phase shifts may also occur due to accretion rate variation.

A phase shift of 0.2 was recently observed in the millisecond pulsar SAX J1808.4-3658 on the 14th day of its outburst (Burderi et al. 2006). This phase jump was observed only in the fundamental component of the almost sinusoidal light curve, and much less so in the 1st harmonic. Burderi et al. (2006) suggested that this phenomenon may be connected with fast spin-down of the millisecond pulsar or with some magnetic field reconstruction during a stage of enhanced accretion. We suggest that it raises the possibility that the magnetic field of the millisecond pulsar may be more complex than a dipole, and therefore during enhanced accretion the disc may interact with deeper layers of the magnetosphere where quadrupolar or other components might possibly influence the flow. We also suggest that the dipole may be slightly off-center (Ruderman 1991; Long et al. 2008), and hence the disc matter accretes to both magnetic poles when the accretion rate is high, or to only one pole if it is low. Superposition of dipoles may also be a possible reason.

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APPENDIX A: THE MAGNETOSPHERIC RADIUS

The magnetospheric radius $r_m$ is the radius where the disc is truncated by the magnetosphere. It is also used to depict the characteristic size of the magnetosphere. Here we estimate the magnetospheric radii for various multipoles. For simplicity we assume that they are aligned with the rotation axis.

A1 The Magnetospheric Radius for A Dipole Field

Here we derive the general formula for the truncation (magnetospheric) radius $r_m$ in the cases of the dipole fields. The disc is truncated where the magnetic stresses equal the matter stresses, $B^2 / 8\pi = \rho + \rho v^2$. We assume that the disc is cold, $\rho << \rho v^2$, hence $B^2 / 8\pi = \rho v^2$.

We also assume that the system is in a stationary state and matter is supplied from the viscous disc with a constant accretion rate $\dot{M} = 4\pi a h v_r \rho$, where $r$ is the distance to the star, $h = (c_s / v_K) r$ is the disc scale height, $v_r = \alpha c_s / v_K$ is the radial flow velocity, $c_s$ is the sound speed, $v_K = (GM / r)^{1/2}$ is the Keplerian velocity, $\alpha$ is the Shakura-Sunyaev viscosity parameter. We obtain,

$$\rho v^2 \approx \rho v_K^2 = (4\pi / 3) (v_K/c_s)^3 \dot{M} (GM)^{1/2} r^{-5/2}. \quad (A1)$$

For the equatorial component of the dipole field the magnetic field is $B = \mu_1 / r^3$. Substituting into the equation...
$B^2/8\pi = \rho v^2_0$ and using Eqn. A1 we obtain,

$$r_m = k_1 r_m^{(0)}, \quad k_1 = (\alpha^2/2)^{1/7} \left( \epsilon_s/v_K \right)^{6/7},$$

$$r_m^{(0)} = \mu_1^{4/7} \dot{M}^{-2/7} (GM)^{-1/7}. \quad (A2)$$

The main term, $r_m^{(0)} = \mu_1^{4/7} \dot{M}^{-2/7} (GM)^{-1/7}$, exactly coincides with the main term for spherical accretion (see also Bessolaz et al. 2008). However, here the coefficient $k_1$ depends on $\alpha$ and the ratio $\epsilon_s/v_K$. We should note that the formula for $\dot{M}$ used in this derivation is applicable only for thin discs, and is only approximately valid near the magnetosphere, where the disc matter is accumulated, the disc becomes thicker and has an excess of matter pressure. That is why the coefficient $k_1$ can be considered to be approximate and can be substituted with some number. Comparison of numerical results for $r_m$ with Eqn. A2 made in Long et al. (2005) shows that for their set of simulations, $k_1 \approx 0.5$ (see more comparisons in Bessolaz et al. 2008).

### A2 The Magnetospheric Radius for A Multipolar Field

Next, we derive the magnetospheric radius for multipolar field. For simplicity, we assume that the main axis of the multipolar field is aligned with the rotation axis, and therefore the $n-$th multipolar component of the magnetic field can be presented in the form $B_n \sim \mu_n/r_n^{n+2}$. Similarly, we obtain,

$$r_{m,n} = k_n r_{m,n}^{(0)}, \quad k_n = (\alpha^2/2)^{1/7} \left( \epsilon_s/v_K \right)^{6/7},$$

$$r_{m,n}^{(0)} = \mu_n^{4/7} M^{-2/7} (GM)^{-1/7}. \quad (A3)$$

This formula can be applied in the cases of aligned multipoles, or can be used for estimates of the magnetospheric radius in general, misaligned cases.

We should note that the $B_z$ equals 0 in the disc plane for aligned fields of $2n$-th order multipoles, such as quadrupole (ref. Long et al. 2007), and matter could flow directly to the star in the disc plane. So $r_{m,n}$ does not reflect where the inflowing matter stops, but only the size of magnetosphere.

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