THE INTERNAL GEOMETRY OF AN EVAPORATING
BLACK HOLE

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Abstract

We present a semi-classical model for the formation and evaporation of a
four dimensional black hole. We solve the equations numerically and obtain
solutions describing the entire the space-time geometry from the collapse to
the end of the evaporation. The solutions satisfy the evaporation law: \( \dot{M} \propto -M^{-2} \) which confirms dynamically that black holes do evaporate thermally.

We find that the evaporation process is in fact the shrinking of a throat
that connects a macroscopic interior “universe” to the asymptotically flat
exterior. It ends either by pinching off the throat leaving a closed universe and
a Minkowskian exterior or by freezing up when the throat’s radius approaches
a Planck size. In either case the macroscopic inner universe is the region where
the information lost during the evaporation process is hidden.

Introduction

Hawking’s discovery [1] of the thermal emission provoked by a gravitational collapse has
provided quantum mechanical grounds [2] for the Bekenstein entropy [3]. Alas, it has also
lead to an apparent breakdown of predictability [4] precisely because of the thermal character
of the emitted flux. In addition, if the black hole completely evaporates, this breakdown of
predictability results in a loss of information since one is left with uncorrelated quanta [5].

Confronted with this potential violation of the unitary evolution of Quantum Mechanics,
three scenarii have been proposed [6]: (i) The loss of information, derived from the original
semi-classical treatment of Hawking is real [1,7]; (ii) The evaporation process stops, leaving
a Planckian remnant \[8\] which detains the lost information. (iii) A fully quantum description will reveal that the information is properly recovered within the detailed structure of the emitted quanta \[9\]. In order to address these issues in a simpler context, a two dimensional (2D) dilaton gravity model \[10\] was introduced and analyzed intensively. This model has indeed sharpened the problem \[11–13\] but has not settled the question yet.

The resolution of this problem clearly requires a full dynamical description of the evaporating black hole space-time. At present a quantum mechanical description is beyond our reach. We present here a four dimensional (4D) semi-classical model which describes the formation and evaporation of a spherically symmetric black hole. We solve the dynamical equations numerically and obtain the space-time geometry till the Plancking regime where the approximation breaks down and the final product depends on the regularization scheme.

The solution confirms the generally accepted features of black hole evaporation, which were anticipated assuming the quasi stationarity character of the external geometry \[14\]. But it also affords an explicit description of the interior geometry and the emergence of the fluxes. In term of the external retarded time, the interior region is static because the outgoing flux rises in the exterior part only. Hence, at the end of the evaporation one is left with a macroscopic interior region describing a closed universe wherein the curvature is weak. Within the semiclassical treatment, this closed region detains all the correlations with the asymptotic Hawking quanta as well as the quantum specification of the infalling matter.

**The Model**

We use the spherical symmetric metric:

\[
ds^2 = e^{2f(U,V)}dUdV - r^2(U,V)d\Omega^2.
\]

(1)

$U$ and $V$ are outgoing and ingoing null coordinates. They are specified on the boundaries $U = U_\text{\(\)}$ for $V > 0$ and $V = 0$ for $U > U_\text{\(\)}$ by the standard coordinate choice, $r = (V - U)/2$:

\[
r(U,V = 0) = -U/2; \quad r(U_\text{\(\)}, V) = (V - U_\text{\(\)})/2.
\]

(2)

$U_\text{\(\)}$ is taken large and negative in order to approximate null past infinity ($\mathcal{I}^-$).
We introduce two matter sources: a classical flux and a quantum field. The first one is a radially infalling null dust \( T^{Cl}_{\mu \nu} = \delta^V_{\mu} \delta^V_{\nu} F(V)/4\pi r^2 \) carrying a mass \( M_0 \). We choose \( F(V) \) as a Gaussian centered around \( V_i \) well inside our \( V \)-domain. Without a quantum backreaction, it would create an eternal black hole of radius \( 2M_0 \) if the width, \( \sigma \), is small enough (i.e. \( \sigma < M_0 \)) that the reflection at \( r = 0 \) can be neglected. The second source is the expectation value of the energy momentum tensor \( T^{Q}_{\mu \nu} \). In the absence of quantum on the boundaries it is entirely determined by the geometry. This tensor engenders, in turn, the Hawking flux and the black hole evaporation. As a model, we take the expectation value of the tensor \[15\] which arises from the quantization of a 2D massless field in the 2D curved background described by \( ds^2 = e^{2f(U,V)}dUdV \) (see Eq. \[1\]) divided by \( 4\pi r^2 \) to mimic the four dimensional radial dependence. It reads simply:

\[
T^{Q}_{UU} = -\alpha f.uv \frac{4\pi r^2}{4\pi r^2} ,
\]

\[
T^{Q}_{UU} = \frac{\alpha[f.uv - (f.u)^2]}{4\pi r^2} ; \quad T^{Q}_{VV} = \frac{\alpha[f.vv - (f.v)^2]}{4\pi r^2} .
\]

The normalization constant \( \alpha \) depends linearly on the number of massless fields and will control the rate of evaporation (see Eq. \[10\] below). In 2D, this expectation value (times \( 4\pi r^2 \)) gives an asymptotic flux which coincides with the flux obtained from the Bogoliubov transformation \[16\] relating the scattered modes to the asymptotic ones. In 4D, it offers a good approximation since most of the energy is carried away by s-waves quanta \[17\].

In the metric Eq. \[1\], the Einstein equations split into two dynamical equations and two constraints. The classical source \( T^{Cl}_{\mu \nu} \) appears in one constraint only while \( T^{Q}_{UV} \) modifies the dynamical Einstein equations. On the left of the infalling matter, for \( V \ll V_i \), one has Minkowski space-time. Indeed, on \( V = 0 \), the first constraint gives \( \partial f(U,0)/\partial U = 0 \). Then by choosing \( f(U,0) = 0 \), one equates \( (U + V)/2 \) to the proper time at rest with respect to the spherical infalling shell. On \( U = U_- \), the second constraint gives

\[
f.u(U_-, V) = 8\pi r(T^{Cl}_{VV}) .
\]

3
A simple integration of Eq. 5 gives the Cauchy date for the dynamical integration, \( f(U, V) \).

The modified dynamical equations read:

\[
f_{,UV}(1 - \alpha e^{2d}) = d_{,U}d_{,V} + \frac{e^{2(d+f)}}{4} \tag{6}
\]

\[
d_{,UV}(1 - e^{2d}) = d_{,U}d_{,V}(2 - \alpha e^{2d}) + \frac{e^{2(d+f)}}{4} \tag{7}
\]

where we define \( d \equiv -\ln(r) \). The inclusion of the quantum matter term (Eq. 3) leads, as in dilatonic gravity \[10\], to singular equations at the critical radius \( r_\alpha = \sqrt{1/\alpha} \). In 4D this singularity is an artifact of our model which assumes the validity of expectation values (Eq. 3) in the Plankian domain. Still this method offers \[18\] a reliable approximation during most of the evaporation process when the prefactor \( P = 1 - \alpha e^{2d} \) is still close to 1. If one wishes to carry out the integration into the Planckian domain one can eliminate this singularity by regularizing the prefactor \( P \) at small radii in the following way:

\[
P_n = 1 - \frac{\alpha e^{2d}}{1 + (\alpha e^{2d})^n}. \tag{8}
\]

The final stage of the evaporation, where the mass approaches the Plank mass, will depend then on \( n \).

**Geometrodynamics**

We integrate numerically the dynamical equations using the Cauchy data and obtain the geometry of the collapse and the subsequent evaporation. The evaporation stage is contained in a tiny \( U \) lapse (Recall that the \((U, V)\) coordinates constitute the inertial frame of an observer at rest at the origin). We make, therefore, a coordinate transformation to \((u, v)\), the inertial asymptotic system at rest with respect to the black hole. \((u, v)\) are defined by \( \frac{dr}{du} \big|_v \to -\frac{1}{2}, \frac{dr}{dv} \big|_u \to \frac{1}{2} \) for \( r \to \infty \). With this transformation the tiny \( U \) lapse is spread out since any inertial collapse results in a Doppler shift given by:

\[
\frac{dU}{du} = e^{-u/4M_0} \tag{9}
\]

wherein the imaginary frequency determines the initial Hawking temperature \( 1/8\pi M_0 \).
Fig. 1 depicts the geometry. On the left one finds the inner boundary of the trapped region (defined by $\partial r/\partial v = 0$). It is space-like because of the positivity of the classical infalling energy [13]. This boundary starts at $r = 0$ (outside of the figure) and ends when the negative energy flux, that causes the evaporation, equals the tail of the classical flux (at $u = 208$). The outer part of the trapped region is the apparent horizon, $r_{ah}$. Since the infalling quantum flux is negative, $r_{ah}(v)$ is time-like [14]. It ends upon reaching (at $v = 345$) the critical radius $r_{\alpha}$ (not drawn) at which our dynamical equations break down.

We note that the apparent horizon is almost a “static” line: $v = u + \text{const}$. This proves that the inwards flux ($T_{vv}$) on this horizon is equal and opposite to the outgoing flux ($T_{uu}$) at infinity. The mass loss can be calculated from either flux. In practice we measure it from $dr_{ah}/dv$ using $r_{ah} = 2M_{ah}$. When the mass is still large (compared to the critical mass $M_{\alpha} = 2r_{\alpha}$) our dynamical model yields:

$$\frac{dM_{ah}}{dv} = -\alpha \frac{1}{32} \frac{1}{M_{ah}^2(v)}.$$  \hspace{1cm} (10)

This demonstrates that “black holes do evaporate thermally” in perfect agreement with Bardeen [14] whose original derivation assumes a quasi stationary character of the external geometry. Naturally, when the mass approaches $M_{\alpha}$ the mass loss depends on the regularization scheme (see Eq. 8 and Fig. 2). For $n = 1$ we find that $\dot{M}$ increases sharply while for $n = 2$, a rapid drop in $\dot{M}$ and in the temperature appears upon approaching the critical mass.

In order to illustrate the internal geometry we have depicted, in Fig. 3, the radii of the spheres encountered by successive outgoing light rays (i.e. along $u = \text{const}$) in the Eddington-Finkelstein coordinates $(r, v)$. When the matter is far away from its Schwarzschild radius, the evolution begins with $r_a(v) = (v - u)/2$. In the absence of backreaction it would have ended with the asymptotic line which starts at $r = 0$ and approaches asymptotically $r = 2M_0$, forming the event horizon. When back reaction is included the null rays still spread out from $r = 0$ to $r = 2M_0$ but then contract within the trapped region reaching a minimal radius at $r_{ah}$ before spreading out again. As the black hole losses it mass $r_{ah}$
diminishes accordingly. This geometry describes a shrinking “throat” that separates the inner “universe” around the infalling mass from the exterior space-time. Since the inner universe is static (see Fig 1b) during the whole evaporation, it remains macroscopic. Thus, what seems to be, for an external observer, the evaporation of the black hole, is in fact the shrinking of the throat that connects the internal region to the rest of the world \[20\].

**Implications for the Loss of Information**

Before analyzing what implications to the loss of information paradox this geometry might give, we recall that the geometry was obtained using a semi-classical approximation. Thus, the forthcoming analysis is meaningful only if the mean geometry is correctly approximated by the semi-classical method (beware that, for instance, the “trans-Planckian” fluctuations \[18\] might completely ruin it). Then, in order to address the problem of information loss, which deals with quantum correlations, one has to analyze the quantum fluctuations on the resulting background.

Before the pinching off the Schrödingerian evolution of the quantum matter state on all our slices \[21\] can be used to evaluate the correlations between the field configurations inside and outside the throat. As the evaporation approaches the Planckian regime, a region of high curvature appears *only* at the throat, separating “practically” the left unaffected interior region from the exterior space-time. Hence whether or not the evaporation stops, does not change significantly the situation. If the black hole completely disappears, one finds two disconnected macroscopic regions: a quite big “baby universe” \[20\] and a Minkowskian exterior. On the other hand, if a remnant characterized by a Planckian size throat is left, there is still a tiny connection with the macroscopic internal region. In either case, the quantum matter field configurations in the internal region are still correlated to external configurations as they were just before the pinching off. The loss of information in the exterior part of the space-time is analogous \[7\] to the loss of the quantum correlations which occurs in any subsystem upon tracing over the quantum states belonging to its complementary subsystem. The “new” behavior that appears to an outside observer occurs because The dynamics of the evaporation force quantum mechanics to operate in the realm of space-time with a varying
topology.

This proposal requires, nevertheless, a drastic modification of the interpretation of the Bekenstein entropy. The area of the black hole does not count the (log of the) number a microscopic states contained in the internal region but only the number of them which are still coupled to the external world. (For instance, this entropy reflects the potential increase in the external entropy due to a complete evaporation of the black hole and has to be used upon enclosing the black hole in a cavity and searching for thermodynamical equilibrium since only the states coupled to the external world states could participate to the thermalisation.) The universality of this area-entropy comes now from the universality of the characteristics of the throat (the throat has no hair) that connects the interior region to the rest of the world. Within the interior region the number of available micro-states depends explicitly on the history of the collapse since the characteristic size of this region has nothing to do with the actual throat radius.

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Fig. 1a: Contour lines of $r = \text{const}$ from $r = 3$ (on the upper left corner) to $r = 80$ (in the lower right corner). The heavy line depicts the apparent horizon $r_{ah}$. The infalling matter is centered initially at $v_i = 14$ and it has a width of $\sigma = 4.5$. For $v \ll v_i$ the spacetime is Minkowski. For $v \gg v_i$ it has a Schwarzschild geometry characterized by the time dependent residual black hole mass.

Fig 1b: Contour lines of $f = \text{const}$ of the same evaporating geometry. The heavy dashed line designates the locus where $T_{uu}^Q$ reaches half of its asymptotic value. It coincides with the places where $f_{VU}$ is maximum and it is always outside the trapped region. The fact that $f_{VU}$ vanishes inside the trapped region indicates that $T_{vu}^Q$ is constant and $T_{uu}^Q$ is zero. This implies that the infalling matter keeps collapsing under its own gravitational field ignoring completely the Hawking flux which develops outside its past light-cone.

Fig. 2. The logarithm of the rate of evaporation vs. the logarithm of the residual mass for different regularization schemes (see Eq. 8): $n = 1$ (solid line) with an exploding solution and $n = 2$ (dashed line) with a decreasing evaporation rate. The non regularized solution ($n = 0$) follows the $n = 2$ curve and stops at the critical mass around the intersection of the curves. Note that the slope, being -2 gives Eq. 10.

Fig. 3: The radii of spheres $(r_u(v))$ on successive $u = \text{const}$ slices in advanced Eddington-Finkelstein coordinates $(r, v)$. The bold line denotes the center of the infalling matter $v = v_i$. 
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-th/9405007v2