Representation, representational transformation and spatial reasoning hierarchical in spatial thinking

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Abstract: The aim of this study is to find out the spatial thinking process of students in transforming 2-dimensional (2D) object image to 3-dimensional (3D) object image representation. Spatial thinking is needed to build how to transforming 2- dimensional (2D) object image to 3- dimensional (3D) object image. There is application of spatial thinking in our daily activities for examples in using maps, planning routes, designing floor plans, and creating art. The students of 5\textsuperscript{th} grade have concrete operation level’s, it is critical period of concrete operation toward early abstract period. In preliminary study researcher found that they have problem to drawing 2-dimensional image object to 3-dimentional image object. They had said that side of cube is not square after they had drawn it but it was parallelogram. It is a problem and important thing, the researcher needs to explore how student using their spatial thinking. It is included how to student thinking process specially in case transforming 3-dimensional (3D) object image to 2- dimensional (2D) object image in critical period. Spatial thinking in this study is identified through geometrical problems of transforming a 2-dimensional object image into a 3-dimensional object image. Subject in this study are a boy and a girl who have visual learning style in middle level of mathematical ability. This is explorative descriptive study through qualitative approach. The result are: 1) there are different internal representations and spatial reasoning of spatial thinking between a boy and a girl, 2) there are different external representations and spatial reasoning of spatial thinking between a boy and a girl, 3) there are different representational transformation spatial objects between a boy and a girl.

1. Introduction
Spatial thinking is a fundamental of cognitive skill. Space has always been a key organizing concept for our discipline. Spatial thinking has supported through representation, representational transformation and spatial reasoning. These elements have related each other. Finding a location, reading the maps are example of spatial thinking in our daily activities. When we are finding a location, there are included representation, representational transformation and spatial reasoning. There are any differences of thinking processes among individuals to build their information for finding a location, reading maps, etc. The spatial thinking is important across disciplines, but in this paper explores the current position of spatial thinking in education especially in geometry. It is one of tasks of spatial thinking to provide a process that may be made toward answering other important questions: how is spatial thinking organized become a hierarchical through representation, representational transformation and spatial reasoning in individual. Although the geometrical object is
really close to human activity, the authors found two cases from the pilot study from 3 district elementary schools Kediri area and researchers take This research is follow-up study from the previous year. The findings of the previous research obtained two cases and by developing indicators in previous studies, we conducted a preliminary study which found the problem finding that students had difficulty in transforming two dimensional (2D) images into three-dimensional (3D) images. There were as many as six students from ten students in three different school of Kediri city still have difficulty in classifying four sides object such as square, rectangular, parallelogram and rhombus. They have difficulty to point differences between square, parallelogram and rhombus. It is mean that students have difficulty in 2D geometry objects especially square, parallelogram and rhombus in four sides object topic. The authors also found that as many as eight from ten students in three different school of Kediri city still have difficulty to make model of cube nets. There found different characteristics between boys and girls to answers the questions. Four fifth grade of elementary students were asked that no other model of cube nets can be formed. The students did not attempt to explore further by manipulating the cube model that has been given. Hence, the exploration of students’ spatial thinking is needed to be done. It is not only to enhance students’ knowledge in mathematics but spatial thinking is needed to development of science and technology.

2. Theoretical Frame Work

2.1. Spatial thinking

Understanding what is spatial thinking in the terms of human development and learning is important to discuss. Spatial thinking can be defined a constructive combination of cognitive skills comprised of knowing concepts of space, using tools of representation and applying processes of reasoning [1]. People have been using spatial thinking without aware of it. Space is part of spatial, and spatial thinking is required to solve problem related to space. People have used space to models of the world, find answers of problem solving, structure of answer of question, and communicate solutions. Spatial thinking is a fundamental of cognitive skill. Space has always been a key organizing concept for our discipline. Spatial thinking is used in any disciplines and important across discipline. This paper explores the current position of spatial thinking in education especially in geometry. One of the most important debates informing spatial thinking, especially in education concerns the extent to which spatial thinking is, in some sense and personal innate. Three components of spatial thinking are representation, representational transformation and spatial reasoning. The components are used to describe how is spatial thinking organized become a hierarchical in individual.

2.1.1. Representation

Representation in most general sense seems to mean “something” that substitutes something else. The representation is not always used to refer to ‘something” but also to some activities or operations, and the representation may be either the act of representation or the product of representation. This distinction is a unity that cannot be separated from each others, product of representation and process of representation. Process of representation is mental process, that happened only in human mind and we called internal representation. Human describes their ideas become words, images or kinesthetic, there are products of representing and we called external representation. This paper is using two kinds of representation, that are internal representation and external representation. There are two kinds of representation i.e. internal representation and external representation [2]. Internal representations are defined as individual cognitive configuration inferred from human behavior describing some aspect of the process of mathematical thinking and problem solving and external representations can be described as structured physical situations that can be seen as embodying mathematic ideas [3]. Internal representation also called mental representation [4-6]. There are abstraction and distortion process in internal representation. An abstraction is remove the real object form, there is omitting the conditions because an abstraction involves the omission of a truth [7]. In the abstraction process, we do not describe the system in it is entirety, so there is not telling the whole truth. An abstraction is include “freedom” a distorted, irrelevant information is not to use. Omission necessarily results in
distortion [8]. Thus, an abstraction can result the distortion, an abstraction and a distortion are very different in each case. The result of abstraction is described in verbal or visual namely external representation. This paper is limited to abstractions and distortions that they are a part of internal representations that can be observed and only refers to information that can be explored through the interview guidelines and the task of solving spatial problems.

2.1.2. Transformation of Spatial Object Representation

Transformation and representation is a part of spatial thinking that is not be separated each other. Transformation is a tool for describing representation, especially in spatial objects. The transformation process of representation of spatial objects occurs in the mind [9]. Transformation of spatial object representation are divided become two types, they are object-based spatial representations and egocentric perspective transformation. Object-based transformations are imagined rotation or translations of objects relative to reference frame of environment. Egocentric perspective transformations are imagined rotations and transformation of one’s point-of-view relative to that reference frame. Both of them are imagined rotations or translation of mental process with different aspect. Transformation of object representation, encoding operations establish mental representation of the spatial of the spatial word by mentally rotating object, mentally extrapolating, mentally extending [2]. Mental representation is the internalization of physical change, that is, there is an internal process that is done when we interact with the world or physical objects. The interaction performed by each individual is different, including to solve the same problem. Transformation of spatial object representation in this study is referred to an operation used to change perspective (frame of reference), change orientation (rotation), change shape, move thoroughly, and reconfigure parts of an object in mind.

2.1.3. Spatial Reasoning Process

Reasoning means changing the information obtained so that generate become conclusions. Make a decision refers to estimating and choosing between several alternative. Spatial reasoning, on the other hand, consists of the set of cognitive processes by which mental representations for spatial object, relationships, and transformations are constructed and manipulated [10]. Spatial reasoning process is a process when an information of the object of space and information associated with various ways such as measurement, observation, and conclusion. The conclusion is used up to a valid conclusion about an object, relationship, and how to determine the completion of a particular task. The process of spatial reasoning is used in the inference of all possible spatial relationships between one group of objects using a particular subgroup. The distortion of spatial thinking and abstract spatial thinking are two things that play a role in the process of spatial reasoning.

3. Methods

Exploratif descriptive through qualitative approach was used in this study.

3.1 Subject

The researchers given learning styles test to the 5th grade student and classified to visual, auditory and kinesthetic. Visual students did gender test and the researches classified to girls type A, girls type B, boys type A and boy type B. Girls and boys type A who gets strongest score are subjects of this research. Girls and boy type A have strong characteristic of gender. The subjects of this study were the 5th grade of visual learning style of both boy and girl students. The 5th-grade students were chosen because they have learned space matter in school and still in concrete operation phase’s, also it is important to study this subject in the earliest of cognitive development. By choosing a visual subject, students are expected to optimize spatial thinking that is closely related to visualization. The study result shows that the gender influence students’ performance and concept in mathematics and that boy’s do better in space [11-13] than the girls. Thus, besides exploring the spatial thinking of students, the author also distinguishes between boys’ and girls’ work to provide comprehensive information.
3.2 Technique
Subjects did geometry test for three days with one problem in each day and they did interview test for three days, we called data I. Totally six days to get the data I. The data I needs to check through triangulation of time and method to get the valid data. Totally twelve days to get data valid from data I into data II. The subjects’ work in solving geometry test were explored and analyzed through an unstructured interview where the interviewer follows the interviewees’ narration and generates questions spontaneously based on his or her reflections on that narration. A deep interview conducted to the subject regarding what they thought, done, written and spoken while doing the test. The audio and video format were used to record subjects along the research process from solving geometry test until interview section.

3.3 Instrument
Two types of instruments were used in this study: main instrument and tracer instrument. The main instrument is researchers and the tracer instruments are interview and geometry problem solving test. The tracer instruments have validity 0.7 and reliability 0.8. The tracer instruments are learning style, gender test, interview and geometry problem solving test. The tracer instrument contained statements about the spatial concepts with which the students could explore the cube nets from the given cube object. The learning style was used to classify student become visual, auditory and kinesthetic learning style. The geometry problem solving test was used to explore students’ spatial concepts to transforming 2D images to 3D images. The interview instrument was used to explore students’ statements about spatial concepts to transforming 2D images to 3D images. The results will be check using validity test. Four validity test data were used in this study: 1) credibility test (internal validity), 2) test of dependency (reliability), 3) confirmation test (objectivity) and 4) transferability test.

4. Result and Discussion
4.1 Internal Representation
Internal activity begins when the senses of the subject capture cube net image that is in front of him, the subject was looking at cube net image in front of him. The subject encoded the information obtained when viewing the cube net image into his mind, the subject attributing information possessed to the current information that is the subject of reconstructing the cube net image into model cubes image in his mind. Representation internal shown when the subjects using abstraction and distortion that he imagined folding the cube nets image become a model cube in his mind. He imagined like a transformer robot changes into a car, it has shown he has using the same characteristic on a cube net and a transformer robot’s, they have the same object to fold. He has used the same characteristic of two objects. It has shown abstraction in his internal representation. He omitted the truth characteristics of two objects. It has shown distortion in his internal representation. The subjects both boy and girl imagine to fold a cube net’s but they have different manner to sketch a cube net’s become sketch a cube model. The following text represented the explanation of students related to how they imagine the cube net image to the cube image.

Question : Can you tell me, how did you draw a sketch of cube nets become a sketch of cube model?
Boy : Firstly, I sketched the cube nets, then I imagined folding the cube nets image become a model cube in my mind, like a transformer robot changes into a car, then I make the net’s sketch...
Girl : ... I try to start drawing a sketch of cube nets, and gave a mark in every pieces of square, then I imagined folding them base on the position of the mark...one mark pieces of a square was in front side and six mark pieces of a square was in back side, three mark pieces of square was in right side and four mark pieces of square was in left side, two mark pieces of a square was in top side and five mark pieces of a square in under side....
4.2 External Representation
Both the female and male subjects are able to represent what they have understood by using words, using a new term for ease of completion, mention the steps of drawing a sketch of cube net’s to a sketch of cube model’s. When the subject was asked whether there is another way to find out the cube nets, the subject can easily explain. The subject mentions another way of determining the sketch of the cube net’s to a sketch of cube model.

Boy : The cube net will become a cube model, then I drew the sketch. But I prefer to imagine it because the cube net is only one, meaning it can only be used to make a cube model only. I will draw it base in my imagination, like a transformer robot changes into a car. It is easy to do when you are familiar with a transformer robot toys.

Girl : you just marked the cube nets and it will form the cube models, follow the mark to make a sketch cube model.

This shows the verbal external representations that have been performed by the subject. The subjects state what they thought in a sketch of cube nets based on the cube nets comes-up in their mind. This indicated the subject expressing what is thought in an external representation of images. They sketched the cube object as a reference to determine the dots on each square in the cube net. They marked dots on every square on cube nets corresponding to many points on each side of the cube object. This shows the visual external representation performed by the subject. Subjects have illustrated their thinking like figure 1 above.

![Figure 1: Boy’s [left] and girl’s [right] cube net drawing](image)

4.3 Transformation of Spatial Object Representation
In transformation representation of this spatial object happened dimension transformation that is a change of way of view/orientation of the subject in transforming 2-dimensional object image to 3-dimensional object image.

Boy : With a bit of imagining the changing of the shape of the cube object into something like I’ve ever seen, I split the cube object in my mind until it finally transforms into a cube model ...

The process of changing the shadow cubes shows the occurrence of the cube shadow webs indicating a change of frame of reference from viewing 2-dimensional abstract objects to 3-dimensional abstract objects. Then, the subject changes the shadow cubes with a particular orientation /viewpoint. Changes in the way of the view of 2-dimensional objects are shown by using the benchmark and the opponent as a reference transforming the dimension into a shadow cube net, from the shadow cube net to the shadow cube webs and then declaring it into a sketch of the image of the cube models.

Girl : In order not to think anymore, I make the cube nets based on the first image I just shift the front side with 5 points to the right of a square so that it is under a square with a point 4, in shear mean that one square is moved to the left or right most Many 3 squares because at most 4 rail shear

Question : What is ‘rail shear’?

Girl : It’s a term that makes it easy for me, the sliding rails I mean are four rectangles that still can not be shifted, if it shifted and folded then it is not cube, which may shift it...
The subject shifts the square on the cube nets to obtain a model of different cube nets than before. The subject changed the shape of the old cube webs into new cube webs of how to "shift" a square on the cube nets. Subjects imagine "shifting" square on the cube nets to acquire the shape of new cube webs. This shows the subject of transforming the representation of spatial objects.

4.4 Spatial Reasoning Process
Manipulation processes that occur in the subject both boy and girl included several things: 1) trying to make a cube nets by imagining familiar objects and can be made into cube models. 2) plotting location side, 3) placing the dots on the two-dimensional object image to the corresponding side of the cube model image. This manipulation is done by observing the cube forms elements and imagining familiar objects to be made into cube models. After manipulation of the 2-dimensional object into a 3-dimensional object, the subjects started to determine the side pair i.e. determining which side is in front, back, left or right side.

Boy: I mark all the squares on the cube nets as the same number as the cube object. First I mark an arbitrary square on the cube nets. I made it as a standard, then marked its opponent. The process was going on until all the squares on the cube nets all marked. I set the first benchmark 1 dot on the bottom side, then 6 dots on the top side, 5 dots on the front side, 2 dots on the back side, 3 dots on the right side and 4 nodes on the left side.

Girl: ... then I defined the pair of sides which match with the cube net I made. Then, I marked it with a dot in the left-right side, up-down side, and front-back side by looking at the cube object. The pair sides I marked on the square must be a pair of opposing side if the net is converted into a cube.

This process requires consideration and leads to the decision that is obtained by the side pair. This indicates the subject is doing the reasoning process. When marking the cube nets corresponding to the number of nodes on the side of the cube whose position corresponds to each square in the cube net. There is a different way between boys and girls about how they placed the dots to the cube net. In this activity, the subject uses a scanning technique to check the square in the cube nets whose position corresponds to the sides of the dotted cube. The process of sketching the cube nets requires manipulation of matching such that each square in the cube net is marked with dots as many as dots on the cube object. This process requires consideration of some information i.e. using side pairs and placing dots. This shows that there is a reasoning process in this activity. When they asked to make different cub net, subject solved a similar problem by splitting the cube, predicting the direction of movement, cutting the cube and arranging parts. This creates a consistent attitude in using the sketching steps of the cube webs.

The subject explained the reason for choosing the method they used. The process of choosing involves grouping things according to the criteria, taking into consideration some of the ways it knows, comparing, interpreting and deciding which way to choose and deciding in a conclusion. This indicates that the subjects have made the process of reasoning while expressed the reasons why choose it. Drawing different net is a manifestation of reasoning process because there is a processing information, comparing manipulations, considering making decisions before draw cube net until understanding the way. Revealing a different way to obtain different cube net is one of the reasoning processes. This indicates the subject has done the reasoning process. The subjects are able to make a conclusion of what they have done. The subjects revealed what need to be done in solving the problem and in making a conclusion. This shows that the subjects have done the reasoning process.

5. Conclusion
In this research we have obtained research results that reveal the spatial thinking profile of elementary students in solving geometric problems based on gender: 1) male students generally use mental activities such as imagining and using subjects experience to solve problems. The imagining process
2D to 3D representation is similar to the process of transforming robot transformers into cars. This shows the subject using an analogy in solving geometrical problems. 2) Female students in general use visual activities such as drawing sketch and creating schemes in solving geometric problems. 3) Male students tend to use mental activity by imagining and imagining spatial objects. 4) Female Students find a fast way to complete the marking cubes using pair formulation points 2-5, 3-4 and 1-6. 5) Female students use more stages of dimensional transformation. The subjects have used steps of transformation that every single component of spatial thinking has used reasoning process. The components of spatial thinking become a hierarchical that it is not separated each others. Firstly, internal representation become based external representation and in the process of them included transformation representation. On they other way, all the process connected by reasoning. Finally, we do hope that this study can contributing the theories about spatial thinking profiles of elementary students in solving geometric problems as a foundation for the development of learning models, particularly in related to the improvement of spatial thinking ability in geometric problem solving. Also, as a reference for teachers to identify the spatial thinking profile of elementary students in geometry-based problem solving.

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