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Impact of temperature dependent viscosity and thermal conductivity on MHD blood flow through a stretching surface with ohmic effect and chemical reaction

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Abstract: A study has been carried for a viscous, incompressible electrically conducting MHD blood flow with temperature-dependent thermal conductivity and viscosity through a stretching surface in the presence of thermal radiation, viscous dissipation, and chemical reaction. The flow is subjected to a uniform transverse magnetic field normal to the flow. The governing coupled partial differential equations are converted into a set of non-linear ordinary differential equations (ODE) using similarity analysis. The resultant non-linear coupled ordinary differential equations are solved numerically using the boundary value problem solver (bvp4c) in MATLAB with a convincible accuracy. The effects of the physical parameters such as viscosity parameter ($\mu(\tilde{T}_b)$), permeability parameter ($\beta$), magnetic field parameter ($M$), Local Grashof number ($Gr$) for thermal diffusion, Local modified Grashof number for mass diffusion ($Gm$), the Eckert number ($Ec$), the thermal conductivity parameter ($K(\tilde{T}_b)$) on the velocity, temperature, concentration profiles, skin-friction coefficient, Nusselt number, and Sherwood number are presented graphically. The physical visualization of flow parameters that appeared in the problem is discussed with the help of various graphs to convey the real life application in industrial and engineering processes. A comparison has been made with previously published work and present study reveals the good agreement with the published work. This study will be helpful in the clinical healing of pathological situations accompanied by accelerated circulation.

Keywords: heat transfer, mass transfer, MHD, variable viscosity, ohmic effect, chemical reaction

1 Introduction

The analysis of boundary layer flow over a stretching surface has received so much attention from researchers and scientists due to its wide usage in the chemical, food, petroleum industry etc. It also plays a crucial role in the bio-medical field because there is a significant variation in human anatomy, physiology and stenosis. Therefore, predicting accurate blood velocity, shear stress, etc., is more important for clinical healing. Misra et al.[1] introduce the theory of the formation of boundary layer at the entry section of the vessel wall. They also described that this is happening due to the presence of a plasma layer in the vessel wall. A theoretical analysis of wall shear stress (WSS) in the stenosed coronary artery with the help of laminar boundary layer theory has been proposed by Back et al. [2]. This study reveals that results for WSS obtained through boundary layer theory are in good agreement with the results obtained from the Navier stokes equation.

The fundamental study of the magnetohydrodynamic flow is to understand the dynamical aspects of the magnetic field in the circulation of the conductive fluids owing to its induced current. The presence of plasma in the blood instigates the magnetofluid properties and the movement of the plasma particles affects the flow field in the circulatory system. The utilization of MHD in the diverse areas is to repair the nervous tissues or regeneration of the cells product [3], bone graft [4], and fracture healing [5]. The mixed convective heat transfer through the porous medium was studied by Makinde et al. [6] and transporting the mass in the presence of a magnetic field with constant wall suction and heat absorption through fluid past a vertical porous plate. Hamad et al. [7] pioneered the study investigating the free convection of nanofluid through a semi-infinite vertical plate. Bhattacharyya and

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Pop [8] employed the shooting method to solve the equations governing the boundary layer flow deals with the MHD due to an exponentially shrinking sheet. To stimulate the future experimental work, the theoretical investigations were undertaken by Sharma et al. [9, 10], which examined the transfer of heat and mass in magnetofluid flow of blood under the influence of first-order chemical reaction, magnetic field through tapered/non-tapered blood vessels with constriction. Adopting the strategy of boundary layer approximation and similarity transformation, Misra and Shit [11] formulated the equations governing the unsteadiness of MHD flow and heat transfer through a capillary with permeable walls and perceived that the introduction of the external magnetic field controls the flow velocity, which may be useful in the treatment of the vascular diseases. The transportation of heat and mass on the MHD blood flow through the permeable stretching surface was discussed by Reddy et al. [12] due to the presence of a magnetic field in the transverse direction of the flow pattern of the fluid. They reported that the increment in the value of Hartmann number leads to growth in Lorentz force and hence results decay in the flow velocity. A Taylor-series based method (differential transformation technique) was introduced by Kumar [13] to simplify the non-linear governing equation of MHD Jeffery fluid, passes through the vertical stretching surface.

In the current scenario, due to the growth of pollution and harmful materials in our environment, humans are harshly infected from cancer and other respiratory diseases. The heat transfer aspect is used to produce radiation at the forefront of the infected area. Heat transfer plays a vital role in many industrial and chemical processes such as concrete heating, die temperature control, steam generators, batch reactors etc. Apart from these applications, transfer of heat also useful in treatment techniques for diseases such as hyperthermia, laser therapy, physiotherapy, and cryopreservation. Barozzi and Dumas [14] employed the numerical approach to compare convective heat transfer with experimental data in the blood vessel of the circulatory system and reported that the heat transfer rate is high for small blood vessels. They also observed that the pulsatile nature of flow increases the heat transfer rate compared with a steady flow. A theoretical observation of heat transfer for unsteady boundary layer flow with stretching permeable surface and the time-dependent surface temperature has been completed by Ishak et al. [15] and reported that the heat transfer rate enhances with rising in unsteadiness parameter. Tsai et al. [16] examined the influence of space-dependent heat source for a quiescent fluid medium passes through an unsteady stretching sheet. They found that non-uniform heat source plays a significant role for heat transfer rate. An analytic study of thermal radiation over a stretching surface has been proposed by Ali et al. [17]. In this study, the behaviour of unsteady third-grade fluid observed, and state that the oscillatory frequency ratio of the sheet is responsible for diminishes in the amplitude of fluid velocity. Many Researcher [18–27] described the heat transfer along various type of flow problems with stretching surface.

In the lubricating fluid system, viscosity of the fluid is affected by heat generated due to internal friction of fluid particles. Therefore, to predict the precise flow behavior, variation in viscosity is allowable. Initially, Rand et al. [28] experimental examined the effect of temperature on the human blood viscosity with normothermic and hypothermic conditions, and reported that enhancement in temperature leads decaying in viscosity and hence blood velocity. Further, this study was carried forward by Snyder [29] to analyzed the combined effect of temperature and hematocrit on the blood viscosity. In this study, they select lizards due to having a natural ability to control fluctuation in body temperature. They reported that with increment in hematocrit, both oxygen capacity and viscosity of blood rises also find that reduction in body temperature leads to the decaying erythrocytes in the blood. Makinde et al. [30] proposed a mathematical model for steady, fully developed Couette flow with temperature-dependent viscosity and state that after enhancement in viscosity exponent, the motion of fluid getting slow. Impact of varying thermal conductivity over unsteady boundary layer flow on vertical surface was examined by Vajravelu et al. [31]. In the absence of magnetic field and ohmic heating, a comparative study to identify the combined effect of viscosity and thermal conductivity in a variable form on the fluid flows through a vertical stretch surface is proposed by Cai wenli et al. [32]. Further, this study is carried forward by Mekheimer et al. [33] for the peristaltic flow, flows in the vertical asymmetric channel. In this study, they adopted the perturbation technique to find the solution of governing equation by considering both viscosity and thermal conductivity parameters as a perturbation parameter. This study also reveals that the variation in the velocity profile becomes high for non-zero positive values of the viscosity variation parameter.

Ohmic heating is a process of conversion of an electric current into thermal energy, which arise due to friction of fluid particles. There are so many industrial and medical applications of ohmic heating such as portable fan heater, incandescent bulb, PCR reactors and electroporation (electropermeabilization). Pliquett [34] has done the experimental study to analyze the impact of ohmic heating on solid tissue during eletroporation therapy and
found that, ohmic heating elevates the temperature that plays a significant role to develop stable permeable structure. El-Amin [35] developed the mathematical model for the flow through a non-isothermal horizontal cylinder to examined the combined effect of joule heating and viscous dissipation. This work considered that wall temperature is not constant, and it varies with the non-isothermal exponent parameter. This study is extended by the Aydin and kaya [36] for fluid flows through the permeable vertical flat plate. The Keller box method is used to solve boundary layer equations. This work gives the Richardson number concept and states that both flow velocity and temperature show the reverse effect with it. Further, Pal and Talukdar [37], extended the study of [36], and analyzed the impact of chemical reaction on mass transport with thermal radiation. They assumed that free stream velocity rises exponentially. A numerical study to observe the physical importance of ohmic heating on the MHD nanofluid which flows through a horizontal stretching surface was discussed by Elazem [38]. In this study Chebyshev pseudospectral technique was used to find non-dimensional governing equations solution and states that this technique is more accurate than the finite difference method. Sharma et al. [39, 40] discussed MHD blood flow considering joule effect and porous medium.

The occurrence of mass transport is recognizable in daily life and also have so many industrial applications such as a drop of a dye in water, distillation, liquid extracting, and drying. Bio-engineering design such as blood oxygenators, respirators, and artificial kidneys involves mass transport. Abel et al. [41] observed the Non-Newtonian flow along the stretching surface with mass transfer, also they analyzed the heat transfer for two different models. Rashidi et al. [42] observed the mass transfer through the vertical sheet in presence of porous medium along with buoyancy effect. A chemical reaction is generally considered as either homogenous or heterogenous on whether the reaction appears as single-phase reaction or at surface where a reaction occurs. The modified homogeneous-heterogeneous reactions were reported by Khan et al. [43] for MHD stagnation flow. Tripathi and Sharma [44–46] analyzed the effects of first-order chemical reaction with concentration profile over two-phase model of blood flow.

All the above-published work neglected the combined effect of ohmic heating, temperature-dependent viscosity and thermal conductivity on the blood flow passes through the stretching surface. Therefore, to fulfill the gap in existing research, an effort has been made to analyze the physical significance of variable thermal conductivity, viscosity, and ohmic heating on the stretching surface in the present work. To demonstrate the phenomena of mass transport, effect of the chemical reaction has been considered. The governing equations are transformed into system of ODE’s by the similarity transformation, and solved numerically using "MATLAB BVP4C Solver". Influences of physical parameters (for e.g. magnetic field, permeability, viscosity, chemical reaction, thermal conductivity etc) on velocity, temperature and concentration profile are analyzed graphically. A comparison with the previously published work has been demonstrated graphically and gets the acceptable agreement with published work. Present study has many practical application in the industrial and medical field.

2 Mathematical formulation

Considered heat and mass transfer convective unsteady, incompressible two-dimensional blood flow over permeable vertical stretching artery in the presence of porous medium. Generally, blood exhibits both type of Newtonian and non-Newtonian nature which depends upon the length of artery and shear stress. In the present work, length of the artery is considered sufficient large, so that blood is considered as an incompressible Newtonian fluid. The Cartesian coordinate system has been consider as shown in figure 1. The x-axis is taken along the flow direction and y-direction is normal to the flow. Let us consider a region of flow which barred for y > 0. The magnetic Reynolds number is considered very small in comparison with acting magnetic field, so that the induced magnetic field can be neglected. Magnetic field applied in the positive y-direction is taken in the form of $B_y^* = B_0^* \sqrt{(1-\gamma t)}$, where $B_0^*$ is constant.

The ohmic heating and magnetic field along with chemical reaction also studied in fluid flow. All the variables are independent of the y-direction. Stretched artery with the velocity $\tilde{U}_w^*$ along the x-axis is

![Figure 1: A schematic representation of mathematical model](image-url)
\[ \dot{U}_w = dx(1 - \gamma t)^{-1}, \]

where \( d \) and \( \gamma \) are constants and dimensions of these constants are \((sec)^{-1}\) with the condition \( d > 0, \gamma \geq 0 \) and \( \gamma t < 1 \).

Temperature and Concentration of the arterial wall are

\[ T_w^* = T_w^0 + T_0^0 \left( \frac{dx}{dV} \right) (1 - \gamma t)^{-2}, \]

\[ \tilde{C}_w^* = C_w^0 + C_0^0 \left( \frac{dx}{dV} \right) (1 - \gamma t)^{-2}, \]

Where \( T_w^0 \) and \( C_w^0 \) represents temperature and concentration of the outside the boundary layer, respectively and \( T_0^0 \) and \( C_0^0 \) are constants.

\( \nu = \frac{\mu}{\rho} \) and \( \mu^\infty \) are the kinematic and dynamic viscosity of ambient fluid respectively. \( \tilde{V}_w^* \) indicates velocity of mass transfer of arterial wall at surface and represented as

\[ \tilde{V}_w^* = v_0(1 - \gamma t)^{-\frac{1}{2}}, \]

for \( v_0 < 0, \tilde{V}_w^* \) is suction velocity and for \( v_0 > 0, \tilde{V}_w^* \) is the injection velocity.

It is assume that the temperature-dependent viscosity \( \mu(T_b) \), changes linearly with temperature and is written as

\[ \mu(T_b) = \mu^\infty \left[ 1 + \frac{E_1}{AT^*}(T_w^* - T_b) \right], \]

\( \Delta T^* = T_w^* - T^\infty \) and \( T_b \) represents blood temperature, \( \epsilon_1 \) is small viscosity variation parameter.

The thermal conductivity of the fluid is assumed as temperature dependent and, vary linearly with temperature as

\[ K(T_b) = K^\infty \left[ 1 + \frac{E_2}{AT^*}(T_b - T^\infty) \right], \]

where \( \epsilon_2 \) is small thermal conductivity variation parameter, \( K^\infty \) is thermal conductivity of ambient fluid.

Under Boussinesq’s approximation and using above assumptions the governing equations for blood flow are .

\[ \frac{\partial \tilde{u}_b}{\partial x} + \frac{\partial \tilde{v}_b}{\partial y} = 0, \]

\[ \frac{\partial \tilde{u}_b}{\partial t} + \tilde{u}_b \frac{\partial \tilde{u}_b}{\partial x} + \tilde{v}_b \frac{\partial \tilde{u}_b}{\partial y} - \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu(T_b) \frac{\partial \tilde{u}_b}{\partial y} \right) + g \beta_T^* (\tilde{T}_b - T^\infty) - \frac{\mu(T_b)}{\rho k^*} \tilde{u}_b + g \beta_C^* (\tilde{C}_b - C^\infty) - \frac{(B')^2 \gamma}{\rho} \tilde{u}_b, \]

\[ \rho C_p \left( \frac{\partial \tilde{T}_b}{\partial t} + \tilde{u}_b \frac{\partial \tilde{T}_b}{\partial x} + \tilde{v}_b \frac{\partial \tilde{T}_b}{\partial y} \right) = \frac{\partial}{\partial y} \left( K(T_b) \frac{\partial \tilde{T}_b}{\partial y} \right) - \frac{\partial \tilde{q}_r^*}{\partial y} + (B')^2 \sigma \tilde{u}_b^2 + \mu(T_b) \left( \frac{\partial \tilde{u}_b}{\partial y} \right)^2, \]

\[ \frac{\partial \tilde{C}_b}{\partial t} + \tilde{u}_b \frac{\partial \tilde{C}_b}{\partial x} + \tilde{v}_b \frac{\partial \tilde{C}_b}{\partial y} = D_m^* \frac{\partial^2 \tilde{C}_b}{\partial y^2} - R^* (\tilde{C}_b - C^\infty), \]

and Boundary conditions for the flow are

\[ \tilde{u}_b = \tilde{U}_w^*(x, t), \quad \tilde{v}_b = \tilde{V}_w^*(t), \]

\[ \tilde{T}_b = \tilde{T}_w^*(x, t), \quad \tilde{C}_b = \tilde{C}_w^*(x, t), \quad at \ y = 0, \]

and

\[ \tilde{u}_b = 0, \quad \tilde{T}_b = T^\infty, \quad \tilde{C}_b = C^\infty, \quad at \ y = \infty, \]

Where \( \tilde{u}_b \) & \( \tilde{v}_b \) represent velocity components in x and y direction, respectively. \( \beta_T^* \) and \( \beta_C^* \) represent coefficient of thermal and concentration expansion. \( k \) is porous medium permeability and represent as \( k = k_1(1 - \gamma t) \). \( k_1 \) is initial permeability, \( g \) is the gravitational acceleration, \( C_p \) is specific heat at constant pressure, \( D_m^* \) is molecular diffusivity and
\( R^* \) represents parameter of chemical reaction.

\( q_r^* \) expresses radiative heat flux which characterized by Rosseland approximation \([32]\):

\[
q_r^* = -\frac{4\alpha^*}{3k^*} \frac{\partial T_b^*}{\partial y},
\]

where \( \alpha^* \) represents Stefan-Boltzman constant and \( k^* \) indicates Rosseland mean absorption coefficient. Temperature difference inside flow is assumed adequately meager, so by Taylor series expansion of \( T_b^* \) about \( T_\infty^* \) and avoid higher order terms, then we get

\[
T_b^* = 4(T_\infty^*)^3 T_b^* - 3(T_\infty^*)^4,
\]

Now introducing similarity transformations are -

\[
\begin{align*}
\psi^* &= (d(1-\gamma)\frac{1}{2}(1-\gamma t)^{-1/2})y, \\
\theta(\eta) &= \left( \frac{\tilde{y} - T_\infty^*}{T_b^* - T_\infty^*} \right), \\
\phi(\eta) &= \left( \frac{\tilde{C}_w - C_w}{C_w - C_g} \right),
\end{align*}
\]

Here \( \psi^*(x, y, t) \) represents stream function and expressed in form of \( \tilde{u}_b^* = \frac{\partial \psi^*}{\partial y} \) and \( \tilde{v}_b^* = -\frac{\partial \psi^*}{\partial x} \).

\( \tilde{u}_b^* = \tilde{U}_w^*(\eta) \) where prime indicates the differentiation with respect to \( \eta \).

Here \( f^* = \tilde{u}_b^*/\tilde{U}_w^* \), \( \theta^* & \phi \) are the dimensionless velocity, temperature and concentration, respectively. After substituting the similarity transformation into equations (4)-(6), we get

\[
\begin{align*}
f''' &= \frac{1}{(1+\epsilon_1(1-\Theta))} \left[ A f'' + \frac{\eta}{2} A f'' + f' - f'' - \epsilon_1 f'' f' - \lambda \theta - \delta \phi + M f' - (-1 + e_1 \theta - e_1) \beta f' \right], \\
\theta'' &= \frac{1}{(1+\eta(1-\Theta))} \left[ \frac{1}{2} A \theta' + \frac{\eta}{2} A \theta' + f' \theta' - M Ec f'' - (1 - e_1 \theta + e_1) Ec f'' - f \theta' - \epsilon_\beta \theta' \right], \\
\phi'' &= Sc \left[ 2A \phi + \frac{\eta}{2} A \phi + f' \phi - f \phi' + Re_x \xi \phi \right],
\end{align*}
\]

The boundary conditions corresponding equations (12)-(14) are

\[
\begin{align*}
f(0) &= \frac{v_0}{(\nu(1-\Theta))} = f_w, & f'(0) &= 1, & f'(\infty) &= 0, \\
\theta(0) &= 1, & \theta(\infty) &= 0, \\
\phi(0) &= 1, & \phi(\infty) &= 0,
\end{align*}
\]

In above equation if \( f_w < 0, f_w > 0 \) and \( f_w = 0 \) symbolize the injection, suction and impermeable nature of sheet, respectively. Introduce following non-dimensional parameters-

\( A = \frac{\lambda}{\alpha} \) (Unsteady Parameter), \( Gr = \frac{\tilde{U}_w^*(T_\infty^* - T_b^*)}{\nu^3} \) (Local Grashof number),

\( Gm = \frac{\tilde{U}_w^*(C_w - C_g)}{\nu^3} \) (local Grashof number for mass diffusion),

\( M = \frac{\sigma(K^*)^2}{\nu} \) (Magnetic number), \( \beta = \frac{k^*}{\nu} \) (Permeability parameter),

\( Pr = \frac{\nu C_w}{K_w} \) (Prandtl number), \( Nr = \frac{16(T_\infty^*)^3 \nu}{\nu^2 K_w} \) (Thermal Radiation parameter)

\( Ec = \frac{(C_w')}{C_w(T_\infty^* - T_b^*)} = \frac{2x dv}{cp_0 \nu} \) (Eckert number), \( Sc = \frac{\nu}{K_w} \) (Schmidt number),

\( Re_x = \frac{U_w^* x}{\nu} \) (Local Reynolds number), \( \xi = \frac{\nu^2}{(U_w^*)^3} \) (Scaled chemical reaction parameter),

\( \lambda = \frac{Gr}{Re_x}, \delta = \frac{Gm}{Re_x}, \delta = \frac{Gm}{Re_x} \).

There are three type of physical quantities of our interest Skin-friction coefficient \( C_f \), rate of heat transfer and mass transfer which are given by

\[
\begin{align*}
C_f &= \frac{\nu v_0}{(\rho(\nu^2))^{1/2}}, \\
Nu_x &= \frac{q_w}{\nu(\tilde{T}_b^*)^3(T_w^* - T_\infty^*)}, \\
Sh_x &= \frac{m_{nx}}{\rho \nu^2 (C_w - C_g)}.
\end{align*}
\]
where \( \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \), \( \tilde{q}_w = -K(T_b) \left( \frac{\partial T}{\partial y} \right)_{y=0} \) and \( m_w = -\rho D_m^* \left( \frac{\partial \tilde{c}}{\partial y} \right)_{y=0} \) are the skin friction, heat flux and mass flux. After substituting these coefficient, then equation (16) can be written as,

\[
\begin{align*}
C_f Re_x^{1/2} & = -2f''(0), \\
Nu_x Re_x^{-1/2} & = -\theta'(0), \\
Sh_x Re_x^{-1/2} & = -\phi'(0),
\end{align*}
\]

(17)

3 Numerical solution

In order to analyze the influence of variable viscosity and thermal conductivity on blood flow through the permeable vertical stretching surface, the non-dimensionalize boundary layer equations (12)-(14) subjected to the boundary conditions (15) are solved numerically by using “BVP4C Solver” under Matlab. Equations (12)-(14) are non-linear coupled ODE’s, so to apply “BVP4C solver”, first these will be converted into the system of initial value problem (IVP). Let us assume that-

\[
\begin{align*}
h_1 &= f, & h_2 &= f', & h_3 &= f'' \\
h_4 &= \theta, & h_5 &= \theta' \\
h_6 &= \phi, & h_7 &= \phi'
\end{align*}
\]

(18)

By introducing new variables the equations (12)-(14) will be transformed into IVP as

\[
\begin{align*}
h_1' &= h_2, \\
h_2' &= h_3, \\
h_3' &= \frac{1}{(1+\epsilon_1(1-h_3))} \left[ Ah_2 + \frac{1}{2}Ah_3 + h_2^2 - h_1h_3 + \epsilon_1 h_3 h_5 - \lambda h_6 - \delta h_6 + Mh_2 - (-1 + \epsilon_1 h_4 - \epsilon_1\beta)h_2 \right], \\
h_4' &= h_5, \\
h_5' &= \frac{1}{(1+\epsilon_2 h_4+M\eta)} \left[ Pr \left( 2Ah_4 + \frac{1}{2}Ah_5 + h_2h_4 - h_1h_5 - MEch_2^2 - (1 - \epsilon_1 h_4 + \epsilon_1\epsilon)Ech_3^2 \right) - \epsilon_2 h_5^2 \right], \\
h_6' &= h_7, \\
h_7' &= Sc \left[ 2Ah_6 + \frac{1}{2}Ah_7 + h_2h_6 - h_1h_7 + Re_x \xi h_6 \right],
\end{align*}
\]

(19)

and the boundary conditions (15) for the above IVP are converted in the following form:

\[
\begin{align*}
h_1(0) &= f_w, & h_2(0) &= 1, & h_2(\infty) &= 0, \\
h_4(0) &= 1, & h_4(\infty) &= 0, \\
h_6(0) &= 1, & h_6(\infty) &= 0,
\end{align*}
\]

(20)
4 Results and discussion

In the present study, the effects of temperature-dependent viscosity, thermal conductivity and ohmic heating for blood flow through the vertical stretching surface have been discussed. Effects of the physical parameters \( (M, \delta, Ec, \xi \& Sc) \) have been neglected for validating the current study with Wenli et al. [32]. Wenli et al. [32] used the shooting method to simplify the dimensionless governing equation. Therefore in this comparison, the shooting method has used for [32] work and Matlab BVP4C solver for the present study. This BVP4C solver is based on the Runge-Kutta method of order 4. The iterative process will be terminated when the error involved is \( <10^{-6} \). Figures 2-3 have been drawn for validating the velocity and temperature profile of present work with the previous study [32], and these figures replicates the good agreement between present work and previous published work [32] for the fluid velocity and temperature. The numerical results are getting by solving equations (12)-(20) by using BVP4C solver in Matlab. The default values for non-dimensional parameters are considered as \( M = 1.5, Gr = 1, Gm = 1, \beta = .5, Nr = 1, e_1 = .1, e_2 = .1, A = 0, Pr = 25, Ec = .1, Sc = 1, Re = 1, \xi = 1, f_w = 0 \).

Figures 4-5 replicate the variation of both velocity and temperature with parameter \( \eta \) for the different values of the magnetic number. Figure 4 perceived that a non-linear decay in velocity with the parameter \( \eta \) for all the values of \( M \). A rising in magnetic field leads to decay in velocity profile owing to its resistivity occurs due to Lorentz force on the flow of fluid. Figure 5 depicts the impact of magnetic field on the temperature profile, due to applying ohmic heating in the energy equation. Physically, with the applying ohmic heating in the blood flow, an electromagnetic force appears in the flow, which produces a rise in the blood temperature. Therefore a decay in the surface temperature is observed. Further, it is also concluded that the presence of a magnetic field generates viscous heating in the flow, which leads to an increase in the flow temperature, which is in good agreement with the published work [38].

Figures 6-7 replicate the variation of velocity with \( \eta \) for the different values of local Grashof number \( (Gr) \) and local Grashoff number for mass transfer \( (Gm) \), respectively. From the both figures it can be analyzed that velocity profile approaches gradually to zero in non-linear way for all values of \( Gr \) and \( Gm \). Also, it can be observed that the velocity profile enhances due increment in both \( Gr \) and \( Gm \). As Grashoof number stated the relation between buoyancy force and viscous force, so it is concluded that
Figure 4: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $M$.

Figure 5: Deviation of temperature profile $\theta(\eta)$ with $\eta$ for different values of $M$.

Figure 6: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $Gr$.

Figure 7: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $Gm$. 
Figure 8: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $\epsilon_1$.

Figure 9: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $A$.

Figure 10: Deviation of temperature profile $\theta(\eta)$ with $\eta$ for different values of $A$.

Figure 11: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $\beta$.

Figure 12: Deviation of velocity profile $f'(\eta)$ with $\eta$ for different values of $f_w$. 
Figure 13: Deviation of temperature profile $\theta(\eta)$ with $\eta$ for different values of $Pr$.

Figure 14: Deviation of temperature profile $\theta(\eta)$ with $\eta$ for different values of $\varepsilon_2$.

Figure 15: Deviation of temperature profile $\theta(\eta)$ with $\eta$ for different values of $Ec$.

Figure 16: Deviation of temperature profile $\theta(\eta)$ with $\eta$ for different values of $Nr$. 
Figure 17: Deviation of concentration profile $\phi(\eta)$ with $\eta$ for different values of $Sc$.

Figure 18: Deviation of concentration profile $\phi(\eta)$ with $\eta$ for different values of $\zeta$.

Figure 19: Deviation of skin-friction coefficient with $\epsilon_1$ for different values of $M$.

Figure 20: Deviation of skin-friction coefficient with $\epsilon_1$ for different values of $\beta$. 
Figure 21: Deviation of skin-friction coefficient with $c_1$ for different values of $Gr$.

Figure 23: Deviation of Nusselt number with $c_2$ for different values of $Ec$.

Figure 22: Deviation of skin-friction coefficient with $c_1$ for different values of $Gm$.

Figure 24: Deviation of Nusselt number with $c_2$ for different values of $Pr$. 
Figure 25: Deviation of Nusselt number with $\varepsilon$ for different values of $M$.

Figure 26: Deviation of Nusselt number with $\varepsilon$ for different values of $Nr$.

Figure 27: Deviation of Sherwood number with unsteady parameter $A$ for different values of $Sc$.

Figure 28: Deviation of Sherwood number with unsteady parameter $A$ for different values of $\xi$. 
rising in Grashof number leads to less dominance of viscous force, i.e., the resistivity of the flow decreases, and the velocity of fluid flow increases.

Figure 8 demonstrates the impact of the variation of temperature-dependent viscosity on the velocity profile for the different values of ($\varepsilon_1$). An increment in viscosity parameter ($\varepsilon_1$) leads to enhancement in the velocity of fluid due to increment in thickness of boundary layer. From the Figure 8, it can be seen that as the parameter $\eta$ increases, the velocity decaying monotonically and approaches to 0. As increasing $\varepsilon_1$, initially near the surface of wall velocity decreases, then after velocity increases away from the wall surface due to increment in the thickness of the boundary layer, which is quite similar with the previous study [32].

Figures 9-10 exhibit the effect of an unsteady parameter (A) with $\eta$ on the fluid velocity, and temperature. Variation of fluid velocity for different values of A can be analyzed from the figure 9 and noticed that thickness of the boundary layer decreases with an increment in unsteady parameter. As can be seen from this figure, the fluid velocity is decaying after enhancing the value of A. Figure 10 describes the variation of temperature profile for different values of A and it can be noted as ($\eta$) increases from 0, a non-linear decay occurs in fluid temperature, and it approaches to 0. A rise in unsteady parameter leads to diminishes in the thickness of the thermal boundary layer, resultant a downfall in the the temperature profile can be observed.

Figure 11 expresses the variation in flow velocity with parameter $\eta$ for the different values of permeability parameter ($\beta$) and it is noted that for all values of ($\beta$) velocity decreasing in monotonic way and approaches to 0. It can be perceived that as the permeability parameter enhances, fluid velocity decreases, which is relatively similar to the earlier study [47]. Figure 12 described variation in velocity for different values $f_w$. In this figure, the velocity profile is displayed for injection, impermeable, and suction sheet, respectively, which depends on the nature of $f_w$. It is observed that thickness of the boundary layer is high for the injection in comparison with suction and hence, the velocity profile decreases with enhancing $f_w$. It is also noted that for injection, fluid velocity is high in comparison with suction.

Figures 13-14 explained the variation in fluid temperature with parameter ($\eta$) under the different values of Prandtl number ($Pr$) and thermal Conductivity variation parameter ($\varepsilon_2$), and these figures depict that for $Pr$ and $\varepsilon_2$ temperature shows a non-linear decaying profile and approaches to 0 with increasing in $\eta$. Prandtl number illustrates the relation between momentum diffusivity and thermal diffusivity. High Prandtl number replicates the dominance of momentum diffusivity over the thermal diffusivity, and hence thickness of the thermal boundary layer reduced. The graphical analysis through the Figure 13 reveals that fluid temperature decay slightly with enhancement in $Pr$. Figure 14 replicates the variation in temperature profile for different values of thermal conductivity variation parameter $\varepsilon_2$. Rise in $\varepsilon_2$ leads to slightly growth in fluid temperature as depicted in Figure 14, owing to enhancement in the thickness of thermal boundary layer.

Figure 15-16 depict the effects of Eckert number and radiation parameter on the fluid temperature with the parameter $\eta$, respectively. A non-linear decreasing profile in fluid temperature for all values of $Ec$ and $Nr$ observed. Form Figure 15 it is observed that for high values of $Ec$, initially temperature increases and after that temperature profile decreases and tends to 0. A rise in $Ec$ leads to enhancement in fluid temperature owing to heat energy is stored in the fluid due to friction heating. Figure 16 reveals the effect of radiation on the fluid temperature and noted that with rise in radiation parameter, fluid temperature enhances. This is happening due to the release of heat energy in the radiation cycle process, which absorbed by the fluid. Therefore, a rise in the fluid temperature and thermal boundary layer thickness occurs.

Figures 17-18 depict the influences on concentration profile for different values of Schmidt number and chemical reaction parameter, respectively. It is observed that a decay in concentration profile with parameter $\eta$ for all values numeric of Sc and $\xi$. Schmidt number gives the relationship between the viscous diffusion rate and molecular diffusion rate. A rising in Schmidt number leads to decline in the thickness of mass-transfer boundary layer, resultant the concentration profile diminishes. The effect of chemical reaction on the concentration profile can be observed through Figure 18. It interpreted that in the presence of chemical reaction ($\xi \neq 0$), concentration profile decreases rapidly in comparison with the absence of chemical reaction ($\xi = 0$). It is also analyzed that an enhancement in the chemical reaction parameter diminishes the concentration of species in the boundary layer, that why the thickness of the mass transfer boundary layer decreases. This is happening because flow, consumes the chemical in the chemical reaction process, resulting a decay in the concentration profile occurs.

Figures 19-22 delineate the deviation of skin-friction coefficient with viscosity variation parameter ($\varepsilon_1$) for the different values of parameters $M$, $\beta$, $Gr$ and $Gm$, respectively. An enhancement in the values of viscosity variation parameter $\varepsilon_1$ exhibits the rising in fluid viscosity.
Therefore, the skin-friction coefficient of fluid enhances linearly near to the wall. A rise in $M$ and $\beta$, reveal the enhancement in $C_f$ and reverse effect is observed with increment in both $Gr$ and $Gm$, respectively. From Figure 19 it can be also analyzed that growth rate of $C_f$ becomes high when $M$ enhances from 0 to 1, in comparison with 1 to 2.

Figures 23-26 delineate the variation in heat transfer coefficient (nusselt number) with thermal conductivity variation parameter $\epsilon_2$ for different values of Eckert number, Prandtl number, magnetic number, and radiation parameter, respectively. An enhancement in $\epsilon_2$, contributes to rising in the thermal conductivity of the fluid, resulting nusselt number decreases monotonically. From these figures, it is observed that the rate of heat transfer approaches to 0 for a considerable value of $\epsilon_2$. A rising in $Pr$ demonstrate the enhancement in heat transfer rate and reverse effect is noted with rising in $Ec$, $M$ and $Nr$, respectively.

Deviation in rate of mass transfer coefficient (Sherwood number) with unsteady parameter $(A)$ for different values of $Sc$ and $\xi$ are exhibited in Figures 27-28, respectively. A rising in unsteady parameter $(A)$, contributes towards non-linear enhancement in Sherwood number near the wall. A rise in $Sc$ and $\xi$, reveals the improvement in $Sh_x$ and also observed from Figure 27 that growth rate of $Sh_x$ becomes higher when $Sc$ enhances form 0.5 to 1, in comparison with 1 to 1.5.

- The thickness of boundary layer enhances by increasing in value of $\epsilon_1$ while viscosity of blood shows reverse effect with it.
- A rise in thermal conductivity variation parameter $(\epsilon_2)$ leads to slightly growth in blood temperature and thickness of thermal boundary layer.
- By increasing unsteady parameter $A$, both the velocity and temperature of flow diminish.
- The numerical value of heat transfer coefficient (Nusselt number) diminishes with increment in thermal conductivity variation parameter $\epsilon_2$ and magnetic field $(M)$ while the reverse effect is observed when $Ec$ and $Nr$ enhances.
- The numerical value of coefficient of skin-friction increases with increment in both $\epsilon_1$ and magnetic field $M$.
- A rise in both unsteady parameter $(A)$ and chemical reaction parameter $(\xi)$, symbolizes the enhances the rate of mass transfer coefficient.

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**5 Conclusion**

Two-dimensional unsteady MHD convection blood flow with temperature-dependent thermal conductivity and viscosity through a vertical stretching surface is numerically investigated in this paper. The governing equations are reduced in the system of ODE’s, and then analyzed numerically with the help of ‘MATLAB BVP4C solver’. The physical insight analysis for flow parameters is graphically performed. The present mathematical study of boundary layer flow for blood will be helpful for researchers and clinical engineers to estimate the realistic behaviour of blood flow velocity, temperature etc., in the process of clinical healing. The main results of this study are as follows:

- Ohmic heating in the flow, enhance the temperature and thickness of thermal boundary layer.

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