Three-loop corrections to the quark and gluon decomposition of the QCD trace anomaly and their applications

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In the QCD energy-momentum tensor $T_{\mu\nu}$, the terms that contribute to physical matrix elements are expressed as the sum of the gauge-invariant quark part and gluon part. Each part undergoes the renormalization due to the interactions among quarks and gluons, although the total tensor $T_{\mu\nu}$ is not renormalized thanks to conservation of energy and momentum. We show that, through the renormalization, each of the quark and gluon parts of $T_{\mu\nu}$ receives a definite amount of anomalous trace contribution, such that their sum reproduces the well-known QCD trace anomaly. We provide a procedure to derive such anomalous trace contribution for each quark/gluon part to all orders in perturbation theory, and obtain the corresponding explicit formulas up to three-loop order in the MS scheme in the dimensional regularization. We apply our three-loop formulas of the quark/gluon decomposition of the trace anomaly to calculate the anomaly-induced mass structure of nucleons as well as pions. Another application of our three-loop formulas is a quantitative analysis for the constraints on the twist-four gravitational form factors of the nucleon, $\bar{C}_{q,g}$.

KEYWORDS: trace anomaly, energy-momentum tensor, QCD, gravitational form factors

1. Introduction

The QCD energy-momentum tensor $T_{\mu\nu}$ is known to receive the trace anomaly [1] as

$$T_{\mu} = \eta_{\mu\nu} T^{\mu\nu} = \frac{\beta(g)}{2g} F^2 + (1 + \gamma_m(g)) m \bar{\psi} \gamma_{\mu} \gamma_5 \psi,$$  \hspace{1cm} (1)

representing the broken scale invariance due to the quantum loop effects, with the beta-function $\beta$ for the QCD coupling constant $g$ and the anomalous dimension $\gamma_m$ for the quark mass $m$. Here, $\eta_{\mu\nu}$ is the metric tensor, and $F^2$ (= $F_{\mu\nu}^a F_{a\mu\nu}$) and $\bar{\psi} \gamma_{\mu} \gamma_5 \psi$ denote the renormalized composite operators dependent on a renormalization scale. The symmetric energy-momentum tensor $T^{\mu\nu}$ is expressed as the sum of the gauge-invariant quark part $T_q^{\mu\nu}$ and gluon part $T_g^{\mu\nu}$, as $(D^\mu = \partial^\mu + i g A^\mu, R^{\mu\nu} = (R^{\rho\nu} S^\rho + R^{\rho\rho} S^\nu) / 2)$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma_{\mu\nu} \gamma_5 \psi + \left( F_{\mu\nu}^{\alpha\beta} F_{\alpha\beta} + \frac{g}{4} F_{\mu
u}^{\alpha\beta} F_{\alpha\beta} \right) \equiv T_q^{\mu\nu} + T_g^{\mu\nu},$$  \hspace{1cm} (2)

using the QCD equations of motion (EOM), up to the ghost and gauge-fixing terms that are not relevant for the following discussion. Classically, we have, $\eta_{\mu\nu} T_q^{\mu\nu} = m \bar{\psi} \gamma_{\mu} \gamma_5 \psi$ and $\eta_{\mu\nu} T_g^{\mu\nu} = 0$, up to the terms that vanish by the EOM, but (1) does not coincide with the quantum corrections to the $m \bar{\psi} \gamma_{\mu} \gamma_5 \psi$ operator, reflecting that renormalizing the quantum loops and taking the trace do not commute. We note that the total tensor $T^{\mu\nu}$ of (2) is not renormalized; it is a finite, scale-independent operator, because of the energy-momentum conservation, $\partial_\nu T^\mu_\nu = 0$, while $T_q^{\mu\nu}$ and $T_g^{\mu\nu}$ are not conserved.
separately and \( T^{\mu \nu}_q \) as well as \( T^{\mu \nu}_g \) is subject to regularization and renormalization. This fact suggests that each of \( T^{\mu \nu}_q \) and \( T^{\mu \nu}_g \) should receive a definite amount of anomalous trace contribution, such that their sum reproduces (1). The corresponding trace anomaly for each quark/gluon part is derived up to two-loop order in [2]. The extension to the three-loop order is worked out in [3], demonstrating in the MS-like schemes in the dimensional regularization, we obtain

\[
\eta_{\mu \nu} T^{\mu \nu}_q = m \bar{\psi} \psi + \frac{\alpha_s}{4\pi} \left( \frac{n_f}{3} F^2 + \frac{4 C_F}{3} m \bar{\psi} \psi \right) + \cdots, \quad \eta_{\mu \nu} T^{\mu \nu}_g = \frac{\alpha_s}{4\pi} \left( -\frac{11 C_A}{6} F^2 + \frac{14 C_F}{3} m \bar{\psi} \psi \right) + \cdots, \tag{3}
\]

for \( n_f \) flavor and \( N_c \) color with \( C_F = (N_c^2 - 1)/(2N_c) \) and \( C_A = N_c \), where the ellipses stand for the two-loop (\( O(\alpha_s^2) \)) as well as three-loop (\( O(\alpha_s^3) \)) corrections, whose explicit formulas are presented in [2, 3]. The sum of the two formulas of (3) coincides with (1) at every order in \( \alpha_s (\approx g^2/(4\pi)) \).

2. Renormalization mixing at three loop

We sketch how the formulas (3) are obtained. First of all, the renormalization of \( T^{\mu \nu}_q, T^{\mu \nu}_g \) of (2) is not straightforward. Indeed, \( T^{\mu \nu}_q, T^{\mu \nu}_g \) are composed of the twist-two (traceless part) and twist-four (trace part) operators and the renormalization mixing between the quark part and gluon part also arises. To treat them, we define a basis of independent gauge-invariant operators up to twist four,

\[
O_q = i \bar{\psi} \gamma^\mu D^\nu \psi, \quad O_{q(4)} = \eta^{\mu \nu} m \bar{\psi} \psi, \quad O_g = - F^{\mu \lambda} F_{\nu \lambda}, \quad O_{g(4)} = \eta^{\mu \nu} F^2, \tag{4}
\]

and the corresponding bare operators, \( O_k^B \). The renormalization constants are introduced as

\[
\begin{align*}
O_g &= Z_T O_g^B + Z_M O_{g(4)}^B + Z_L O_q^B + Z_S O_{q(4)}^B, \quad O_{g(4)} = \frac{Z_F O_{g(4)}^B + Z_C O_{q(4)}^B}{Z_Q O_q^B + Z_B O_{g(4)}^B}, \tag{5} \\
O_{q(4)} &= \frac{Z_F O_{q(4)}^B + Z_C O_{q(4)}^B}{Z_Q O_q^B + Z_B O_{q(4)}^B}, \quad O_q = Z_Q O_q^B + Z_K O_{q(4)}^B + Z_Q O_{g(4)}^B + Z_B O_{g(4)}^B, \tag{6}
\end{align*}
\]

where, for simplicity, the mixing with the EOM operators as well as the BRST-exact operators is not shown, as their physical matrix elements vanish and they do not affect our final result [4]. Here, \( O_g \), as well as \( O_q \), is a mixture of the twist-two and -four operators, and the corresponding twist-four components receive the contributions of the twist-four operators \( O_{g(4)} \) and \( O_{q(4)} \). The two formulas of (6) reflect, respectively, that the twist-four operator \( O_{g(4)} \) mixes with itself and another twist-four operator \( O_{q(4)} \), and that \( O_q \) is renormalization group (RG)-invariant (see [2, 3, 5]).

Subtracting the traces from both sides of the equations (5), \( O_k \) and \( O_{k(2)}^B \) are, respectively, replaced by the corresponding twist-two parts, \( O_{k(2)} \) and \( O_{k(2)}^B \), such that the twist-four contributions drop out. The renormalization constants \( Z_T, Z_L, Z_Q \) and \( Z_O \) remain in the resulting equations that represent the flavor-singlet mixing of the twist-two spin-2 operators, and thus can be determined by the second moments of the DGLAP splitting functions which are known up to the three-loop accuracy [6].

For the renormalization mixing (6) at twist four, the Feynman diagram calculation of \( Z_T \) and \( Z_C \) is available to the two-loop order [5]. Moreover, it is shown [3] that the constraints imposed by the RG invariance of (1) allow to determine the form of \( Z_T \) as well as \( Z_C \) in the MS-like schemes, completely from \( \beta(g) \) and \( \gamma_m(g) \), which are known to five- and four-loop order in the literature, respectively.

Therefore, six renormalization constants \( Z_T, Z_L, Z_Q, Z_O, Z_F \) and \( Z_C \) among ten constants arising in (5), (6) are available to a certain accuracy in the MS-like schemes, and they take the form,

\[
Z_X = \left( \delta_X T + \delta_X g + \delta_X F \right) + \frac{ax}{e} + \frac{bx}{e^2} + \frac{cx}{e^3} + \cdots, \tag{7}
\]

in the \( d = 4 - 2\epsilon \) spacetime dimensions with \( X = T, L, \psi, Q, F, \) and \( C \); here, \( a_X, b_X, c_X, \ldots \), are the constants given as power series in \( \alpha_s \), and \( \delta_X X' \) denotes the Kronecker symbol. However, \( Z_M, Z_S, \)
ZK and ZB still remain unknown. It is shown [3] that these four renormalization constants can be determined to the accuracy same as the renormalization constants (7), by invoking that they should also obey (7) with X = M, S, K, B, and that the RHS of the formulas of (5) are, in total, UV-finite. Thus, all the renormalization constants in (5), (6) are determined up to the three-loop accuracy, and this result allows us to derive the three-loop formulas [3] for (3), by calculating the trace part of (5).

3. Anomaly-induced mass structure of hadrons

The QCD trace anomaly (1) signals the generation of a nonperturbative mass scale, say, the nucleon mass mN: Taking the matrix element of (1) in terms of a hadron state |h(p)⟩ with the 4-momentum pμ as p2 = m2, and using the fact that ⟨h(p)|Tμν|h(p)⟩ = 2pμpν, we obtain

\[ 2m_\pi^2 = \langle h(p)|T_\mu^\mu|h(p)\rangle = \langle h(p)\left(\frac{\beta(g)}{2g}F^2 + (1 + \gamma_m(g))m\bar{\psi}\psi\right)|h(p)\rangle, \]

so that almost all of the hadron mass mH could be attributed to the quantum loop effects in QCD responsible for the trace anomaly. Based on (8) for the nucleon (h = N), it is frequently argued that the entire mass mN comes from gluons in the chiral limit. However, the partition of QCD loop effects as (3) shows that the latter statement would not be suitable: Indeed, (3) allows us to separate (8) as

\[ 2m_\pi^2 = \langle h(p)|\eta_\nu T_\gamma^\nu(\mu)|h(p)\rangle + \langle h(p)|\eta_\nu T_\gamma^\nu(\mu)|h(p)\rangle, \]

and evaluating (3) with Nc = 3, nf = 3 at the renormalization scale µ, one finds [3]

\[ \eta_\nu T_\gamma^\nu(\mu) = \left(-0.437676\alpha_s(\mu) - 0.261512\alpha_s^2(\mu) - 0.183827\alpha_s^3(\mu)\right)F^2(\mu) + \cdots, \]

\[ \eta_\nu T_\gamma^\nu(\mu) = \left(0.0795775\alpha_s(\mu) + 0.0588695\alpha_s^2(\mu) + 0.0216037\alpha_s^3(\mu)\right)F^2(\mu) + \cdots, \]

with the ellipses associated with the operator m\bar{\psi}\psi. The nucleon (h = N) in the chiral limit gives

\[ \frac{\langle N(p)|\eta_\nu T_\gamma^\nu(\mu)|N(p)\rangle}{\langle N(p)|\eta_\nu T_\gamma^\nu(\mu)|N(p)\rangle} = -0.181818 - 0.0258682\alpha_s(\mu) + 0.0424613\alpha_s^2(\mu), \]

where \(\frac{\alpha_s}{\pi}/(-\frac{11}{6}) \equiv -0.181818\). Eqs. (9)-(12), combined with \(\langle N(p)|F^2|N(p)\rangle < 0\), show that the gluon- and quark-loop effects make the nucleon mass heavy and light, respectively, with the magnitude of the former being five times larger than that of the latter. From (12), the \(\mu\)-dependence of this result for the relative size of the gluon/quark loop effects in the chiral limit is rather weak. It is also worth noting that the total sum (9) of (10) and (11) allows us to constrain the matrix element of \(F^2\) as \(\langle N(p)|F^2(\mu = 1\ GeV)|N(p)\rangle \approx -8.61m_\pi^2\), using \(\alpha_s(1\ GeV) = 0.47358\ldots\), as the three-loop running coupling constant in the \(\overline{\text{MS}}\) scheme with \(\alpha_s(M_Z) = 0.1181\). We note that the neglected four-loop contributions are expected to produce corrections less than ten percent because \(\alpha_s^2(1\ GeV) \approx 0.1\).

Next, we consider the pion case, for which the PCAC relation, \(-\langle m_u + m_d\rangle|\langle \bar{u}u + \bar{d}d\rangle|0\rangle = 2f_\pi^2m_\pi^2\), with \(f_\pi\) the pion decay constant, indicates \(m_\pi^2 \approx m\) as \(m \to 0\). Eq. (8) for the pion (h = \(\pi\)) implies, \(\langle \pi(p)|F^2|\pi(p)\rangle|_{m\to0} = 0\), in the chiral limit \(m \to 0\). Eq. (8) also gives the relation among the \(O(m)\) terms: When the substitution, \(\pi(p) \to \pi(p)_0 + \pi(p)_1 + \ldots\), is made, where \(\pi(p)_0 \equiv |\pi(p)|,|\pi(p)|\) is the \(O(1/m)\)-term, we have, \(\langle \pi(p)|m\overline{\psi}\psi|\pi(p)\rangle \to 0\langle \pi(p)|m\overline{\psi}\psi|\pi(p)\rangle_0\) and \(\langle \pi(p)|F^2|\pi(p)\rangle \to 0\langle \pi(p)|F^2|\pi(p)\rangle_0 + 1\langle \pi(p)|F^2|\pi(p)\rangle_1\), up to the corrections of \(O(m^2)\). The pion mass can also be calculated as the mass shift due to the ordinary first-order perturbation theory in the quark mass term in the QCD Hamiltonian, as [7]

\[ m_\pi^2 = 0\langle \pi(p)|m\overline{\psi}\psi|\pi(p)\rangle_0, \]
and, combining this with the above results for the $O(m)$ terms in (8), we obtain

\[
(1 - \gamma_m(g))m_n^2 = \langle \pi(p) | \frac{B(g)}{2g} F^2 | \pi(p) \rangle ,
\]

to the $O(m)$ accuracy. Therefore, up to the corrections of $O(m^2)$, the terms associated with the $F^2$ operator and the $m_\pi Q$ operator in the RHS of (8) contribute to $m_n^2$ according to the relative weights, 

\[
(1 - \gamma_m(g)) \text{ and } (1 + \gamma_m(g)), \text{ respectively; here, } \gamma_m(g) = 0.63662 \alpha_s + 0.768352 \alpha_s^3 + 0.801141 \alpha_s^3 \approx 0.559, \text{ at the three-loop accuracy. Substituting (13) and (14) into (9) with (3), we find}
\]

\[
\frac{1}{2m_\pi} \langle \pi(p) | \eta_{\nu T_\pi}^{\nu \lambda}(\mu) | \pi(p) \rangle = 0.611111 - 0.12215 \alpha_s(\mu) - 0.124659 \alpha_s^2(\mu) - 0.0430357 \alpha_s^3(\mu) ,
\]

\[
\frac{1}{2m_\pi} \langle \pi(p) | \eta_{\nu T_\pi}^{\nu \lambda}(\mu) | \pi(p) \rangle = 0.388889 + 0.12215 \alpha_s(\mu) + 0.124659 \alpha_s^2(\mu) + 0.0430357 \alpha_s^3(\mu) ,
\]

and, using $\alpha_s(1 \text{ GeV}) = 0.47358 \ldots$, we obtain

\[
\frac{1}{2m_\pi} \langle \pi(p) | \eta_{\nu T_\pi}^{\nu \lambda}(\mu = 1 \text{ GeV}) | \pi(p) \rangle = 0.521 ,
\]

\[
\frac{1}{2m_\pi} \langle \pi(p) | \eta_{\nu T_\pi}^{\nu \lambda}(\mu = 1 \text{ GeV}) | \pi(p) \rangle = 0.479 ,
\]

which hold to the $O(m)$ accuracy. Again, the $\mu$-dependence of the result is rather weak, but (17) shows the structure different from the nucleon case: Both the gluon- and quark-loop effects produce roughly half of the pion mass. This may be a particular nature as a Nambu-Goldstone boson.

4. **Anomaly constraints on the nucleon’s twist-four gravitational form factor**

The nucleon matrix element of each term in (2) is parameterized as $(\langle p | \equiv | N(p) \rangle)$

\[
\langle p'| T_{q,q}^{\nu \lambda} | p \rangle = \bar{u}(p') \left[ A_{q,q}(t) \gamma^\mu \bar{P}^\nu + B_{q,q}(t) T_{q,q}^{\nu \lambda} \right] \frac{\bar{P}^{\mu} i \sigma^{\nu \lambda} \Delta t}{2m_N} + D_{q,q}(t) \frac{\Delta^\nu \eta^{\mu \nu}}{4m_N} + \bar{C}_{q,q}(t)m_N \eta^{\mu \nu} | u(p) \rangle ,
\]

in terms of the gravitational form factors $A_{q,q}(t), B_{q,q}(t), D_{q,q}(t),$ and $\bar{C}_{q,q}(t) [9, 10]$, where $\Delta = t^2 - p^2$, $\bar{P} = (p + p')/2$, $t = \Delta^2$, $B^2 = m_N^2 - t/4$, and $u(p)$ is the nucleon spinor. $A_{q,q}(t)$ and $B_{q,q}(t)$ are familiar twist-two form factor, obeying the forward ($t \rightarrow 0$) sum rules, $A_{q,q}(0) + B_{q,q}(0) = 1$, $[A_{q,q}(0) + B_{q,q}(0) + A_q(0) + B_q(0)]/2 = 1/2$, representing a sharing of the total momentum and total angular momentum by the quarks/gluons. $D_{q,q}(t), \bar{C}_{q,q}(t)$ have also received considerable attention recently [8-11]; $D_{q,q}(t)$ are related to the so-called D term, $D \equiv D_q(0) + D_{q}(0)$ [9]. For $\bar{C}_{q,q}(t)$, exact manipulations for the divergence of (2) yield the operator identities, $\partial_i T_{q,q}^{\mu \nu} = \bar{q} g F^{\mu \nu} \gamma_i \psi$, $\partial_i T_{q,q}^{\mu \nu} = -F_{ab}^{\mu \nu} \partial_i F_{ab}^{\rho \nu}$ up to the terms that vanish by the EOM, so that $\bar{C}_{q,q}(t) = \bar{D}_{q,q}(t), \bar{C}_{q,q}(t) \propto \langle \rho | \bar{q} i g F_{\mu \nu} \gamma_i \psi | p \rangle$, and $\bar{C}_{q,q}(t) \propto \langle \rho | F_{\mu \nu} i D_{ab}^{\rho \nu} F_{ab}^{\rho \nu} | p \rangle$ hold [10], showing that $\bar{C}_{q,q}(t)$ are of twist four. We note that $\bar{C}_{q,q}(t)$ are relevant to the force distribution inside the nucleon [9, 12] and the nucleon’s transverse spin sum rule [13].

To see the consequence of (3) on $\bar{C}_{q,q}(t)$, we take the trace of the forward limit, $\Delta^\mu \rightarrow 0$, of (18):

\[
\bar{C}_{q}(t = 0) = -\tilde{C}_{q}(t = 0) = -\frac{1}{4} A_q(t = 0) + \frac{1}{8} \frac{\langle p | \eta_{\nu T_\pi}^{\nu \lambda} | p \rangle}{m_N} ,
\]

and substitute (3) into the second term, and the three-loop DGLAP evolution into $A_q(0) \equiv A_q(t = 0, \mu)$ as a flavor-singlet spin-2 operator renormalized at the scale $\mu$. $\bar{C}_{q}(t = 0, \mu)$ of (19) reads [2]

\[
\bar{C}_{q}(\mu) = -\frac{1}{4} \left( \frac{n_f}{4 C_F + n_f} + \frac{2n_f}{3 \beta_0} \right) + \left( \frac{2n_f}{3 \beta_0} + 1 \right) \frac{\langle p | \bar{q} \gamma_5 \bar{\psi} \gamma_i | p \rangle}{8m_N^2} - \frac{4 C_F A_q(0) + n_f (A_q(0) - 1)}{4 (4 C_F + n_f)} \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{4 C_F + 2n_f} + \cdots ,
\]
with $\beta_0 \equiv (11C_A - 2n_f)/3$, where $\mu_0$ is a certain starting scale, $A_{q0} \equiv A_q(\mu_0)$, and $\langle p| F^2 |p \rangle$ has been eliminated in favor of the nucleon mass $m_N$ using (8). Here, explicitly shown are the leading order (LO) terms that are derived from the contributions at the one-loop accuracy in (19); the ellipses denote the NLO and NNLO terms derived from the two- and three-loop contributions in (19), respectively. The first few terms independent of $\mu$ in (20) represents the asymptotic value which is completely determined by the values of $N_c$ and $n_f$ in the chiral limit. Substituting $N_c = 3$ and $n_f = 3$, we obtain
\[
\begin{multline}
\hat C_q(\mu)|_{n_f=3} = -0.146 + 0.306 \frac{\langle p|m\bar q q|p \rangle}{2m_N^2} + \left(0.09 - 0.25A_{q0}\right) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{31}} + \alpha_s(\mu) \left\{0.006 + 0.08 \frac{\langle p|m\bar q q|p \rangle}{2m_N^2} + \left(0.013 - 0.035A_{q0}\right) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{31}} + \frac{0.035A_{q0} - 0.028}{\alpha_s(\mu_0)} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{50}{31}}\right\} + \cdots, \quad (21)
\end{multline}
\] where the NLO as well as LO terms are explicitly shown, and the ellipses stand for the NNLO terms. For illustration, we plot (21) as a function of $\mu$ in the chiral limit: Fig. 1(a) shows the results up to the LO, NLO, and NNLO accuracy; the NLO as well as NNLO corrections give a few percent level effects, reflecting the small numerical coefficients arising in (21), and the NLO and NNLO corrections tend to cancel; the approach to the asymptotic value, $-0.146$, is quite slow. The NNLO result is separated, in Fig. 1(b), into the individual contributions of each term in (19), the first (twist-2) term and the second (anomaly) term; both twist-2 and anomaly terms produce the important effects.

To summarize, the quark/gluon decomposition of the QCD trace anomaly allows us to study the hadrons from new aspects, revealing e.g., quite different pattern between the nucleon and the pion.

Fig. 1. The nucleon’s gravitational form factor $\hat C_q(t = 0, \mu)$ at the NNLO (3-loop) accuracy: (a) the LO, NLO, and NNLO calculations; (b) the contributions of the first (twist-2) and second (anomaly) terms of (19).

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