Real-time crankshaft angular speed tracking and indicated torque estimation via optimized Luenberger sliding mode observer

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Abstract. The interest in engine indicated torque estimation plays an important role in the automotive industry. In this study, an optimized Luenberger sliding mode observer is proposed based on easily available crankshaft angular speed of a four-cylinder spark ignition (SI) engine. Especially, the new observer is applied to track crankshaft angular speed and estimate engine indicated torque in New European Drive Cycle (NEDC). Convergence is proven through Lyapunov stability theory. The experimental results show that the proposed estimated technique can effectively track speed and has a higher accuracy in steady state.

1. Introduction
The engine indicated torque is one of the most significant performance parameters of internal combustion engines (ICEs). The real-time indicated torque monitoring can grasp the running condition of the vehicle and improve the working performance of engine [1]. The engine indicated torque has been widely applied in diagnostic field including misfire detection, combustion analysis and torque balancing [2-3], etc. Moreover, the importance of engine indicated torque can be discovered by the projected increase in vehicle related emissions worldwide [4]. Actual engine torque is usually hard to be measured due to the deficiencies of high cost and inconvenient of torque sensors. Hence, in order to reconstruct indicated torque based on crankshaft angular speed which is easier and cheaper to measure, an observer can be used instead of expensive sensors [5].
Numerous torque estimation techniques have been addressed previously in the literature [3, 6-7]. Most of the proposed approaches have their foundations on observer design including discrete non-stationary linear approximation observer [3], discrete angular domain observer [6], nonlinear torque observer [7], etc. Further, sliding mode observer has a good robustness and accuracy in real-time torque estimation [1]. However, the problem in dealing with dynamic torque estimation is needed to solve. Other focuses on filtering algorithm by using crankshaft speed [5], such as Kalman filtering [2]. For the design of an observer of engine indicated torque, a stiff crankshaft model is used based on tracking error of crankshaft angular speed. An optimized Luenberger sliding mode observer (OLSMO) is proposed and validated both theoretically (the convergence of observer) and experimentally (on the four-cylinder engine). Compared with sliding mode observer (SMO) and Luenberger sliding mode observer (LSMO), OLSMO has a higher accuracy in tracking speed and it can also effectively improve
the dynamic performance of estimation of torque in transient condition. Additionally, the designed OLSMO greatly weakens system chattering.

2. Stiff crankshaft model
In this part, the dynamic system of engine stiff crankshaft model is briefly described. According to the experience equation of SI engine proposed by Crossley and Cook [8], the engine torque, \( T_{\text{eng}} \), produced by the engine cylinders can be expressed as

\[
T_{\text{eng}} = -181.3 + 379.36m + 21.91(A/F) - 0.85(A/F)^2 + 0.26A_f + 0.0028A_f^2 + 0.027\dot{\theta} - 0.000107\dot{\theta}^2 + 0.00048\dot{A}_f + 2.55A_m - 0.05A_f^2 m_u
\]  

(1)

where \( A/F \) is air to fuel ratio, \( A_f \) is spark advance (degrees before top-dead-center), \( \theta \) is the crank angle and \( m_u \) is the mass of air entering into intake manifold for combustion in cylinder. The engine torque less the net load torque results in acceleration.

\[
J_e\ddot{\theta} = T_{\text{eng}} - T_{\text{load}}
\]  

(2)

where \( J_e \) is engine crankshaft effective rotational moment of inertia. In this study, \( T_{\text{load}} \) is engine load torque, which can be approximately described as three main factors including aerodynamic resistance torque \( T_{\text{aero}} \), road grade and rolling resistance torque \( T_{\beta} \), and friction brake torque \( T_{f\beta} \) [9]. Further, the rotation dynamic equation of the crankshaft can also be described as

\[
J_e\ddot{\theta} = T_{\text{ind}} - T_i - T_{\text{pre}} - T_{\text{load}}
\]  

(3)

where \( T_{\text{ind}} \) is engine indicated torque, \( T_i \) is reciprocating inertia torque of piston and part of the connecting rod, \( T_{\text{pre}} \) is the average friction torque created by sliding friction of piston and ring and the pumping action of the engine. If we define \( x_1 = \theta \), \( x_2 = \dot{\theta} \), then state space can be obtained

\[
\dot{x}_1 = x_2
\]  

(4)

\[
J_e\dot{x}_2 = T_{\text{eng}} - T_{\text{load}}
\]  

(5)

Substitute equation (1) into the state space, and the formula (5) can be represented as

\[
J_e\dot{x}_2 = -181.3 + 379.36m_u + 21.91(A/F) - 0.85(A/F)^2 + 0.26A_f + 0.0028A_f^2 + 0.027x_2 - 0.000107x_2^2 + 0.00048x_1A_f + 2.55A_m - 0.05A_f^2 m_u - T_{\text{load}}
\]  

(6)

3. Engine indicated torque estimation
In this study, the sliding surface is chosen as \( s = x_2 - \dot{x}_2 \), where \( \dot{x}_2 \) is the estimation of the measured speed \( x_2 \). Based on the principle of sliding mode variable structure, sliding surface should meet the accessibility when \( ss < 0 \). In order to satisfy the sliding mode condition, the effective injection signal is necessarily chosen. During the sliding mode motion, discontinuous injection signals can make the well-chosen switching function equal to zero. Therefore, the system yields excellent robustness to disturbance signals and uncertainty. Additionally, the better sharp saturation function is applied to weaken chattering of switching function in tracking crankshaft angular speed. Meanwhile, in order to ensure that the switching function has a good performance of fast convergence, sliding mode gain should be as big as possible. However, overlarge value of sliding mode gain will intensify chattering of switching function and decrease accuracy in tracking speed. Hence, a Luenberger observer is further introduced, which can accelerate estimating speed of estimated states. Then, we can improved injection signal from the derivative of switching function

\[
J_e\dot{s} = \dot{x}_2 - \dot{x}_2 = E(s) - [Ls + Ks\text{sat}(s/\varepsilon)]
\]  

(7)

where \( E(s) \) represents error function between designed observer and stiff crankshaft model. \( K \) and \( L \) are sliding mode gain and Luenberger gain respectively. The second item \( Ls + Ks\text{sat}(s/\varepsilon) \) is a designed injection signal. \( \text{sat}(\bullet) \) is a saturation function. And \( \varepsilon \) is a positive constant, the slope of the linear part
in \( sat(s/e) \) is \( 1/e \) that should be selected large enough to approach to sign function. According to the estimated speed \( \hat{x}_2 \), the crankshaft dynamics in equation (6) can be rewritten as

\[
J_2 \ddot{x}_2 = -181.3 + 379.36 \dot{x}_2 + 21.91(A/F) - 0.85(A/F)^2 + 0.26A_f + 0.0028A_f^2 + 0.027\dot{x}_2
- 0.000107\dot{x}_2^2 + 0.00048\dot{x}_2A_f + 2.55A_f \dot{\dot{x}}_2 - 0.05A_f \dot{\dot{x}}_2 - T_{load} + L_s + K_{sat}(s/e)
\]  

(8)

Substitute equation (8) into the equation (3), estimated indicated torque can be obtained

\[
\hat{T}_{ind} = -181.3 + 379.36 \dot{x}_2 + 21.91(A/F) - 0.85(A/F)^2 + 0.26A_f + 0.0028A_f^2 + 0.027\dot{x}_2
- 0.000107\dot{x}_2^2 + 0.00048\dot{x}_2A_f + 2.55A_f \dot{\dot{x}}_2 - 0.05A_f \dot{\dot{x}}_2 + \hat{T}_r + \hat{T}_{fric} + L_s + K_{sat}(s/e)
\]

(9)

Based on Lyapunov stability theory, if characteristic matrix of Luenberger observer satisfies Hurwize condition, the gradual stability is assured. Additionally, a Lyapunov function exists when error dynamic system of sliding mode observer meets \( \dot{x} < 0 \), it is concluded that the error dynamics of designed observer is stable and model states can ultimately converge to the actual system states.

4. Experimental results and discussion

In the experiment, a four-cylinder SI engine (details listed in Table 1) is applied in real-time torque estimation study. In order to verify the proposed observer, a dynamic engine platform is conducted in an unsteady condition and it can be used to measure crankshaft angular speed. The full running condition of vehicle is set as NEDC, which has been widely applied to exam transient emission. Additionally, the crankshaft speed signal is extracted by crankshaft position sensor, and sampling frequency of signal is chosen to 20 kHz. The values of observer gain are chosen as \( K = 60, \ L = 120 \).

**Table 1.** Features and specifications of turbocharged SI engine

| Parameter             | Value and Unit | Parameter             | Value and Unit |
|-----------------------|----------------|-----------------------|----------------|
| Engine Type           | 4-stroke in-line| Engine Capacity       | 2.0 L          |
| Compression Ratio     | 10.5:1         | Bore                  | 83 mm          |
| Crank Radius          | 93/2 mm        | Connecting Rod Length | 145 mm         |

According to previous study, the comparison of performance of the designed observer (OLSMO, LSMO, SMO) is presented in next few figures. The tracking results of crankshaft angular speed are given in Figure 1, which displays a good overall performance of OLSMO in speed tracking during 25 seconds under NEDC condition and estimated speed can follow the actual speed signal well without strong chattering. However, dynamic speed tracking result of SMO is bad. Additionally, the speed tracking error of OLSMO is given in Figure 2. The steady state error (during 7 to 10 seconds) is closed to zero, and the speed tracking error is only 3% in transient state (such as 11 to 15 seconds). Therefore, Figure 2 indicates that the proposed observer possesses good stability.

**Figure 1.** Speed tracking results

**Figure 2.** Speed tracking error of OLSMO

It is observed that the SMO cannot reconstruct actual engine indicated torque in transient working condition in Figure 3, but the estimated indicated torque of the SI engine has a better effect using OLSMO than SMO. Compared with LSMO and SMO, the designed OLSMO in Figure 4 can effectively estimate the indicated torque in steady condition. Further, OLSMO greatly weakens the
severe chattering existed in the LSMO and SMO. So it is concluded that the proposed OLSMO has better dynamic performance in estimating indicated torque of engine than LSMO and SMO.

![Figure 3. Indicated torque estimation results](image1)

![Figure 4. Detail in torque estimation](image2)

5. Conclusion
This study presents an optimized Luenberger sliding mode observer that can be applied to estimate indicated torque of SI engine. Compared with LSMO and SMO, OLSMO achieves a higher accuracy in tracking crankshaft angular speed and it can also effectively reconstruct engine indicated torque in non-stationary working condition. Moreover, the designed OLSMO greatly weakens system chattering. Furthermore, we plan to develop the estimated torque which can be used to misfire detection, torque balancing and combustion analysis. Additionally, the estimation accuracy of engine indicated torque is also needed to be considered.

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