The Meta Distribution of the SIR in Linear Motorway VANETs

Konstantinos Koufos and Carl P. Dettmann

Abstract—The meta distribution of the signal-to-interference-ratio (SIR) is an important performance indicator for wireless networks because, for ergodic point processes, it describes the fraction of scheduled links that achieve certain reliability, conditionally on the point process. The calculation of the moments of the meta distribution requires the probability generating functional (PGFL) of the point process. In vehicular ad hoc networks (VANETs) along high-speed motorways, the Poisson point process (PPP) is a poor deployment model, because the drivers, due to the high speeds, maintain large safety distances. In this paper, we model the distribution of inter-vehicle distance equal to the sum of a constant hardcore distance and an exponentially distributed random variable. We design a novel discretization model for the locations of vehicles which can be used to approximate well the PGFL due to the hardcore point process and the meta distribution of the SIR generated from synthetic motorway traces. On the other hand, the PPP overestimates significantly the coefficient-of-variation of the meta distribution due to the hardcore process, and its predictions fail. In addition, we show that the calculation of the meta distribution becomes especially meaningful in the upper tail of the SIR distribution.

Index Terms—Headway distance models, probability generating functional, reduced Palm measure, synthetic mobility traces.

I. INTRODUCTION

The long-term vision of having vehicles communicating with each other for improving traffic flow, enabling automated driving, etc. is not far from reality [1]. The first standardization actions started in 1999, once the Federal Communication Commission in U.S. allocated 75 MHz of spectrum in the 5.9 GHz band for dedicated short-range communication [2]. In 2008, the European Commission set aside 30 MHz for cooperative intelligent transport systems. The technology amendment IEEE 802.11p will be the basis for world-wide standards supporting Vehicle-to-Vehicle (V2V) communication in the 5.9 GHz band. In addition, the V2V communication will be secured under the umbrella of cellular networks [3].

The performance of Vehicular ad hoc networks (VANETs) has been extensively studied during the past three decades using measurements and advanced simulators, see for instance [4], [5] and the references therein. Recently, analytical tools, like spatial point processes [6], have been employed to reduce the simulation load and gain quick insights into the system performance [7]. The classical analysis of wireless networks using stochastic geometry assumes a spatial model for the network elements, and it averages the performance indicator (mostly outage probability) over all network states [8]. Unfortunately, the average does not represent well the reliability of each individual link, when the standard deviation (of the indicator) is comparable to the mean. Because of that, the meta distribution of the Signal-to-Interference Ratio (SIR) has been recently proposed to assess the distribution of the outage probability across the network, conditioned on the realization of the point process [9]. Thus far, the meta distribution of bipolar, cellular and heterogeneous wireless networks has been investigated [10], [11].

The spatial distribution of vehicles requires a combination of two models: one for the road infrastructure and another for the locations of vehicles conditionally on the roads. The Manhattan Poisson line process (with horizontal and vertical layout of streets) and the Poisson line process (for streets with random orientation) have so far been used in urban vehicular communication studies. For analytical tractability, they have been coupled with one-dimensional (1D) Poisson Point Processes (PPPs) for the locations of vehicles along the streets. The resulting point process for the vehicles is commonly referred to as a Cox process in the plane. The study in [12] shows that the distribution of interference level is discontinuous at the intersections, the study in [13] brings up the trade-off between the intensities of streets and vehicles in the coverage probability of the typical receiver, and the study in [14] enhances the model of [13] assuming not only vehicular but also macro-base stations. Simpler models for the urban road network, e.g., two orthogonal streets in [15] and a grid of roads in [16] coupled with 1D PPPs, have highlighted the fact that the coverage probability becomes lower near intersections.

The roadway infrastructure becomes (at least) less relevant for vehicular communication along a motorway. A simple 1D setup with single or multiple lanes should suffice. The 1D simplification allows incorporating very realistic deployment models for the vehicles, e.g., flow of platoons, into connectivity analysis without interference [17]. If the fading is also neglected, more network properties can be analytically evaluated, e.g., expected number of connected clusters in [18]. In this paper, however, we are interested in V2V communication under interference and fading channels. In [19], a modified Matérn hardcore type-II process is considered for the intensity of concurrent interferers per lane, and the average multihop packet transmission time is calculated. In [20], the 1D Matérn type-II is enhanced with discrete marks modeling non-saturated data traffic, and the transmission success probability is evaluated. In [21], it is shown that with low transmission probability, the outage due to 1D Bernoulli lattice converges to that due to a PPP of equal intensity.

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While the studies [19]–[21] are pertinent to motorways, and they also incorporate higher layer features (multi-hop transmission, queueing and application to automotive radars respectively), they calculate the average of the performance indicator. To the best of our knowledge, the meta distribution of the SIR in VANETs has so far been studied only in [22]. Assuming 1D PPPs for the vehicles located over a regular grid, this (simulation-based) study indicates that the meta distribution is bimodal. Intuitively, the V2V communication in line-of-sight experiences much higher reliability than that over an intersection, making the performance of a randomly selected link either extremely reliable or totally unreliable.

Unlike the urban scenario in [22], we would like to shed some light on the meta distribution of the SIR along motorways. Naturally, the drivers maintain large safety distances in motorways, and the PPP may not model accurately the locations of vehicles [23]. In order to maintain some degree of analytical tractability, while introducing more realistic deployment, we have adopted the shifted-exponential distribution for the inter-vehicle distance [23]–[25]. The Probability Generating Functional (PGFL) of shifted-exponential (or hardcore) point process is unknown. In order to approximate the outage probability, we calculated the moments of interference under Palm (and reduced Palm) measure with respect to (w.r.t.) the shift (or hardcore distance). Then, we selected suitable Probability Distribution Function (PDF) modeling the inter-vehicle distance. While this approach gave good predictions for the outage probability due to the hardcore process, it is not straightforward to extend it to calculate meta distributions.

Instead of pursuing interference modeling, we will deal directly with the PGFL of the hardcore point process. Unfortunately, the bounds using first-order factorial moment expansion and conditional Papangelou intensity for Gibbs processes, see [25, Thereom 1], are not tight in the upper tail of the SIR Cumulative Distribution Function (CDF). In order to approximate the PGFL, we will split the contributions into near- and far-field. For the far-field, we model the interferers with a PPP. For the near-field, we discretize the lane into intervals equal to the hardcore distance, and we allow at most one vehicle per interval. Let us call this model, the discretization model. The main contributions of this paper are:

- Using the discretization model, we devise accurate approximations for the conditional PGFL and the meta distribution of the SIR due to the hardcore point process.
- Furthermore, we illustrate that the hardcore process (and subsequently the discretization model) approximate well the meta distribution generated from synthetic motorway traces [11, 13], while the conventional PPP fails.
- We show that introducing hardcore distance, while keeping the intensity of vehicles fixed, reduces the Coefficient-of-Variation (CoV) of the meta distribution. As a result, the conventional PPP predicts larger disparity in the performance of different links along the motorway, and it incurs large errors in the estimation of the meta distribution generated from the traces.
- We show that the CoV of the meta distribution increases for increasing SIR threshold, while all other parameters remain fixed. As a result, the calculation of the meta distribution becomes particularly meaningful in the upper tail of the SIR CDF.

The rest of this paper is organized as follows: In Section [II] we present the system model and the discretized approximation to the hardcore process. In Section [III] we calculate the PGFL for the discretization model and in Section [IV] its meta distribution. In Section [V] we devise simple approximations for the first two moments of the meta distribution. In Section [VI] we validate the models against real traces. Finally, in section [VII] we summarize the main findings and outline relevant topics for future work.

II. SYSTEM MODEL

We consider 1D point process of vehicles $\Phi_c$, where the inter-vehicle distance follows the shifted-exponential PDF. The shift is denoted by $\mu > 0$, and the rate of the exponential by $\lambda > 0$. The intensity of vehicles is $\lambda^{-1} = \frac{\mu}{\lambda + \mu}$, or $\lambda = \frac{\mu}{\mu + \lambda}$. We condition on the location of a transmitter at the origin. The receiver associated to it is the nearest vehicle ahead, at distance $d$, see Fig. [1]. We assume that only the vehicles behind the transmitter generate interference. Other vehicles may also interfere due to antenna backlobes radiation, but this would not dominate the interference level, and it is currently neglected. Hereafter, the process $\Phi_c$ denotes the points with non-negative coordinate, see Fig. [1].

Since the transmitter is placed at the origin $o$, the probability to find a vehicle at $x = r > 0$ follows from the Pair Correlation Function (PCF) of $\Phi_c$, $\rho^{(2)}(r) = \sum_{k=1}^{\infty} \rho^{(2)}_k(r)$, where

$$\rho^{(2)}_k(r) = \begin{cases} \frac{k!}{(j-1)!} \Gamma(j) e^{-\gamma_j(r)}, & r \in (kc, (k+1)c) \\ 0, & \text{otherwise}, \end{cases}$$

$k \geq 1$ and $\Gamma(j) = (j-1)!$ [27, equation (32)].

The transmit power level is normalized to unity. The distance-based pathloss follows power-law, $r^{-\eta}$, with exponent $\eta > 2$. The fading power level is independent and identically distributed (i.i.d.) over different links, following the exponential PDF with mean unity. Each interferer is active with probability $\xi$, independently of the activity of others.

We will now describe our novel discretization model which will be used to approximate the CDF and the meta distribution of the SIR due to the hardcore process. The model splits the interferers into near- and far-field depending on their locations, see Fig. [1B]. The separation threshold is denoted by $R$; the details on how to select $R$ are presented in the next section. The locations of vehicles for $x > R$ is approximated by a PPP $\Phi_p$ of intensity $\lambda$ without introducing much error, because these vehicles do not dominate the interference statistics. On the other hand, the approximating distribution for the near-field interferers considers some of the deployment constraints introduced by $\Phi_c$: Firstly, we discretize the line segment $x \in [c, R]$ into intervals of length $c$, where $R = Kc$, $K \in \mathbb{N}_+$. Secondly, taking into account that the minimum distance separation between successive vehicles is $c$, we allow at most one vehicle inside each interval. We assume that whether an interval contains a vehicle or not is independent of other intervals. Even though this approximation
may not satisfy the hardcore constraint for all vehicles, it will suffice to approximate well the PGFL of the point process \( \Phi_c \) for realistic parameter settings. Furthermore, while the PDF of the location of a vehicle inside the \( k \)-th interval, \((kc, (k+1)c)\), is available from the PCF, we approximate it by the uniform distribution \( U_k \) \( k \in \{1, 2, \ldots, K-1\} \), to reduce the computational complexity at the cost of small accuracy loss.

Let us denote by \( P_k \) the Bernoulli-distributed Random Variable (RV) with parameter \( p_k \), equal to the probability that the \( k \)-th interval contains a vehicle. The parameter \( p_k \) can be calculated as the integral of the PCF within \([kc, (k+1)c]\). For instance, for \( k=1 \), we have \( p_1 = \int_{c}^{c(1-c)} \mu e^{-\mu(r-c)}d \tau = 1-e^{-\mu c} \).

For large \( k \), the following simplification might be of use

\[
p_k = \int_{kc}^{(k+1)c} \frac{p_k^2}{2\pi c} \frac{r}{\tau^2} dr,
\]

where \( \Gamma(x, a) = \int_{x}^{\infty} e^{-a s} s^{a-1} ds \) is the incomplete Gamma function. 
(a) follows after substituting equation (1) and-interchanging the order of integration and summation, and (b) uses that for large \( k \) the function \( t(x) = \frac{\int_{x}^{\infty} e^{-\mu(r-c)}d \tau}{\int_{1-c}^{\infty} e^{-\mu r} d \tau} \) is negligible for small \( x \) and ramps up to unity after some point \( x_0 \). Therefore we may approximate the sum \( \sum_{j=1}^{k} t(j) \) by the integral of a unit pulse between \( x_0 \) and \( k \). Without introducing much error in the calculation of the integral, we define \( x_0 \) to be the point where \( t(x_0) = \frac{1}{2} \). In order to approximate \( x_0 \), we know that \( \lim_{x \to \infty} \frac{\Gamma(x, x)}{\Gamma(x)} = \frac{1}{2} \). We use this property and solve for \( x_0 = c \mu (k-x_0) \) or \( x_0 = \frac{c \mu (k+1)}{1+c \mu} \) in the first term and \( x_0 = \frac{c \mu (k+1)}{1+c \mu} \) in the second, and the result follows.

III. APPROXIMATING THE CONDITIONAL PGFL OF \( \Phi_c \)

Under i.i.d. Rayleigh fading, the outage probability, \( P_{out}(\theta) = \mathbb{P}(\text{SIR} \leq \theta) \), due to the point process \( \Phi_c \) and conditioned on the link distance \( d \) is, see also [8, Theorem 1],

\[
P_{out}(\theta) = 1 - \mathbb{E}^{o\theta} \left\{ \prod_{x_k \in \Phi \setminus \{o\}} \frac{1}{1+\xi_k s(\theta) (x_k + d)^{-\eta}} \right\},
\]

where \( s \equiv s(\theta) = \theta d^\eta \), the RVs \( \xi_k \) describe the activity of the \( k \)-th vehicle and the contribution to the interference due to the transmitter at \( o \) must be excluded from the PGFL.

The RVs \( \xi_k \) are i.i.d. following the Bernoulli PDF with parameter \( \xi \), and thus

\[
P_{out}(\theta) = 1 - \mathbb{E}^{o\theta} \left\{ \prod_{x_k \in \Phi \setminus \{o\}} \left( 1 - \xi (1 + s(x_k + d)^{-\eta})^{-1} \right) \right\}, \tag{4}
\]

After splitting the contributions to the PGFL into near- and far-field terms, and using the approximation for the point process \( \Phi_c \) discussed in Section III we have

\[
P_{out}(\theta) \approx 1 - \mathbb{E}^{o\theta} \left\{ \prod_{x_k \in U_k} \left( 1 - \xi (1 + s(x_k + d)^{-\eta})^{-1} \right) \right\} \mathbb{E}^{o\theta} \left\{ \prod_{x_k \in \Phi \setminus \{o\}} G_f(x_k) \right\}, \tag{5}
\]

where \( G_f(x_k) = 1 - \xi p_k + \xi p_k (1 + s(x_k + d)^{-\eta})^{-1} \) for the near-field and \( G_f(x_k) = 1 - \xi + \xi (1 + s(x_k + d)^{-\eta})^{-1} \) for the far-field. For the near-field we have scaled the activity \( \xi \) with the probability the \( k \)-th interval contains a vehicle. This is valid because the RVs \( P_k \) are independent of each other and of \( \xi_k \).

The expectation over the far-field is straightforward to compute from the PGFL of \( \Phi_c \) within \( (R, \infty) \).

\[
\mathbb{E} \left\{ \prod_{x_k \in \Phi \setminus \{o\}} G_f(x_k) \right\} = \exp \left\{ -\lambda_\xi \int_{R+d}^{\infty} \frac{1}{s^{\eta+1}} ds \right\}, \tag{6}
\]

where the integral can be expressed in terms of the hypergeometric \( F_1 \) function [29, p. 556].

The expectation for the near-field, \( J_n = \mathbb{E} \left\{ \prod_{x_k \in U_k} G_n(x_k) \right\} \) requires to average over a uniform distribution for the location of a vehicle inside each interval. After bringing the expectation operator inside the product we have

\[
J_n = \frac{1}{c} \sum_{k=1}^{K-1} \left( 1 - \xi p_k + \frac{\xi p_k}{1 + s(x_k + a_k)^{-\eta}} \right) dx, \tag{7}
\]

\[
= \frac{1}{c} \left( 1 - \xi p_k + \frac{\xi p_k}{cs(\eta+1)} (f(a_{k+1}) - f(a_k)) \right), \tag{8}
\]

where \( a_k = d + ck \), \( f(a_k) = \frac{\eta+1}{\eta} \), \( a_k^2 = \frac{\eta}{\eta-2} \), and \( p_k \) is given in (2). After substituting (5) and (3) into (4), and average over the link distance \( d \), we get an approximation for the outage probability due to the hardcore process \( \Phi_c \). We will compare the discretization model with a few other models. The first one uses a PPP with intensity \( \lambda \) in \((c, \infty)\) for the interferers and
The calculation of $E^{\theta}(I)$ involves the PCF in (1) which has a complicated form. Due to the fact that $\rho^{(2)}(x) \geq \mu e^{-\mu c}$ for $x \geq c$ the mean interference can be bounded as

$$E^{\theta}(I) = \xi \int_{c}^{\infty} \int_{c}^{\infty} (r + x)^{-\eta} \rho^{(2)}(x) \mu e^{-\mu(r-c)} dx dr$$

$$\geq \xi \int_{c}^{\infty} \int_{c}^{\infty} \mu e^{-\mu(r-c)} dx dr$$

$$= \frac{\xi \mu^\eta}{\eta - 1} e^{\mu} \Gamma(2-\eta, 2\mu c).$$

The above inequality cannot be solved in closed-form w.r.t. $R$. Due to the fact that $R$ is expected to be much larger than $\mu^{-1}$, we will expand the incomplete Gamma function of the left-hand side for a large argument $\mu R \gg 1$ obtaining:

$$e^{\mu(c+R)} \Gamma(2-\eta, \mu (c+R)) \leq (\mu R)^{1-\eta}.\] After substituting this in the left-hand side of (14), we get a bound on the threshold

$$R \geq R_{\text{min}} \geq R^*$$

satisfying the constraint $E^{\theta}(I)_{x>R} \leq qE^{\theta}(I)$. After cancelling out common terms, we end up with

$$\min_{R \geq R^*} \left\{ \mu e^{\mu(c+R)} \Gamma(2-\eta, \mu (c+R)) \leq \frac{q}{1-\mu c} \Gamma(2-\eta, 2\mu c) \right\}.$$

Let us now assume that we increase the intensity of the hardcore process and at the same time we impose stronger thinning, keeping constant the intensity of retained transmitters $\lambda \xi$. Under this transformation, the hardcore process converges in distribution to a PPP. Simply thinning the process maintains some degree of correlation in the locations of

The discretization model is a very tight approximation to the simulated outage probability due to the hardcore process $\Phi_c$ with $\lambda = 0.025 m^{-1}$, $c = 16 m$ and approximations. Pathloss exponent $\eta = 3$, $10^6$ simulation runs. Independent spatial, fading and activity realization in each run. In the discretization model, $R_{\text{min}} \approx 500 m$ for $q = 2\%$, see (15).

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vehicles. Nevertheless, it is natural to assume that the strongly thinned hard-core process will generate interference statistics similar to those of the thinned PPP. In other words, a PPP with parent intensity \( \lambda \) in \((c, \infty)\) will predict more accurately the interference field due to the point process \( \Phi_c \) of equal intensity for smaller \( \xi \), see Fig. 2. Next, we show that the discretization model is consistent with this behaviour; for low activity, its PGFL converges to the PGFL due to a PPP.

**Lemma 1.** For low activity \( \xi \) such that \( \lambda \xi (R-c) \ll 1 \) and \( \lambda c \ll 1 \), the outage probability due to the discretization model converges to that of a PPP given in \((9)\) for all thresholds \( \theta \).

**Proof.** For small \( \xi \), the near-field term in \((7)\) can be approximated as

\[
J_n = \prod_{k=1}^{K-1} \left( 1 - \frac{p_k}{e} \int_0^c \frac{\lambda \xi s \, ds}{s + (x + a_k)} \right) \approx \prod_{k=1}^{K-1} \left( 1 - \frac{\lambda \xi s \, ds}{s + (x + a_k)} \right).
\]

The approximation in \((a)\) follows from \( p_k \approx \lambda c \forall k \). This is valid for large \( K \), see \((3)\), and it can also be valid for small \( K \) under the condition \( \lambda c \ll 1 \). For instance, \( p_1 = 1 - e^{-c \lambda (\frac{\lambda c^2}{2})} = \lambda c + o(\lambda^2 c^2) \). The inequality in \((b)\) is the Weierstrass product inequality \( \prod_{k=1}^{K-1} (1 - y_i) \geq 1 - \sum_{k=1}^{K-1} y_i \), \( y_i \in [0,1] \), and \((c)\) follows from the expansion \( e^{-s} \approx 1 - s, x \to 0 \). These approximations hold for \( \lambda \xi \ll 1 \). We get \((d)\), which is the expression for the PGFL of a PPP in \((c, R)\), after adding up the \((K-1)\) integral terms. After multiplying the above simplification for \( J_n \) with the far-field term in \((6)\), we end up with the desired result. \( \square \)

**IV. THE META DISTRIBUTION OF THE SINR FOR \( \Phi_c \)**

The meta distribution of the SINR is the PDF of the probability of successful reception, \( P_s(\theta) \), conditioned on the spatial realization, i.e., fixed but unknown locations for the transmitter-receiver link and the interferers. \( P_s(\theta) = \mathbb{P}(\text{SINR} > \theta | \Phi_c) \). The conditional probability is computed, in our case, over fast fading and ALOHA, see \((2)\).

\[
P_s(\theta) = \prod_{x_k \in \Phi_c} \left( 1 - \xi + \xi \left( 1 + s(x_k + d)^{-\eta} \right)^{-1} \right).
\]

The PDF of the RV \( P_s(\theta) \) is an important indicator of the network performance, because due to the ergodicity of the point process \( \Phi_c \), it is equivalent to the spatial distribution of the probability of successful reception (or outage) given a realization of the point process. In particular, the complementary CDF \( \mathbb{P}(P_s(\theta) > u), u \in [0,1] \), indicates in each realization of \( \Phi_c \), the fraction of scheduled links that experience a SINR higher than \( \theta \) with probability at least \( u \). The calculation of the PDF of \( P_s(\theta) \) is not easy, but it can be calculated using \((10)\) Appendix. The \( b \)-th moment, \( M_b(\theta) = M_b(\Phi_c) \), of the meta distribution is computed by raising \((16)\) to the \( b \)-th power and taking its expectation over \( x_k \)

\[
M_b = \mathbb{E} \left\{ \prod_{x_k \in \Phi_c} \left( 1 - \xi + \xi \left( 1 + s(x_k + d)^{-\eta} \right)^{-1} \right)^b \right\}.
\]

Using the discretization model, the contribution to the moment \( M_b \) from the near-field can be written as

\[
M_{b,n} = \mathbb{E} \left\{ \prod_{x_k \in U_k} \left( \frac{1}{1 + s \xi_k P_k(x_k + d)^{-\eta}} \right)^b \right\}.
\]

Taking the average w.r.t. the Bernoulli RVs \( P_k \) yields

\[
M_{b,n} = \mathbb{E} \left\{ \prod_{x_k \in \Phi} \left( 1 - \xi + \xi \left( 1 + s(x_k + d)^{-\eta} \right)^b \right) \right\}.
\]

Note that \( M_{1,n} = J_n \), see \((7)\), as it should. Similar to Lemma \( I \) it is possible to show that for \( b > 1 \) the term \( M_{b,n} \), under certain conditions, gives approximately equal contribution to that due to a PPP in \((c, R)\).

**Lemma 2.** For low activity \( \xi \) such that \( \lambda \xi (R-c) \ll 1 \) and \( \lambda c \ll 1 \), \( M_{b,n} \approx \exp(\lambda c (1 - (1 - (\frac{\lambda c}{\lambda c}))) \, dx) \).

**Proof.** Firstly, we take the expectation over \( x_k \) for each term of the product in \((17)\) obtaining

\[
M_{b,n} = \prod_{k=1}^{K-1} \left( 1 - \frac{1}{c_j^b} \left( 1 - \frac{\xi s}{s + (x + a_k)} \right)^b \right) \, dx.
\]

Secondly, we follow exactly the same steps as in the proof of Lemma \( I \). After substituting \( p_k \approx \lambda c \forall k \) in \( M_{b,n} \), the sufficient condition for the lemma to hold is

\[
\lambda c \sum_{k=1}^{K-1} \left( 1 - \frac{1}{c_j^b} \left( 1 - \frac{\xi s}{s + (x + a_k)} \right)^b \right) \, dx \leq \lambda c \sum_{k=1}^{K-1} \left( 1 - (1 - \xi) c_j^b \right) \approx \lambda c (K-1) \xi b = \lambda \xi b (R-c) \ll 1,
\]

where \((a)\) holds \( \forall s \), and \((b)\) is true for \( \xi b \ll 1 \) which is met under the condition \( \lambda \xi (R-c) \ll 1 \) for realistic values of \( \lambda, R, c \). By realistic values we mean \( \lambda (R-c) \gg 1 \), or more than a single vehicle (on average) in the near-field.

The numerical calculation of \((13)\) can be simplified using binomial expansion in the integrand yielding

\[
M_{b,n} = \prod_{k=1}^{K-1} \left( 1 - \frac{1}{c_j^b} \left( 1 - \frac{\xi s}{s + (x + a_k)} \right)^b \right) \, dx = \prod_{j=1}^{b} F_j \left( j - \frac{1}{\eta}, j - \frac{1}{\eta} + 1; -s (R+d)^{-\eta} \right).
\]

The moment \( M_b \) for the discretization model is finally calculated by multiplying \((19)\) with \((20)\) before averaging.
over the shifted-exponential distribution for the link distance $d$. Due to the fact that the integration becomes computationally demanding for high $b$, we will calculate only the first two moments, and we will match them to a Beta PDF $g(z) = \frac{1}{\Gamma(\alpha, \beta)} z^{\alpha-1} (1-z)^{\beta-1}$, $\alpha, \beta > 0$, where $\Gamma(\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)}$. The Beta approximation has been widely adopted for meta distribution modeling [10, 11]. Its parameters are $\alpha = \frac{c_1 (1-c_1) - c_2}{c_1 (1-c_1) c_2}$ and $\beta = c_1 - 1 + \frac{c_1 (1-c_1) c_2}{c_1 (1-c_1) c_2}$, where $c_1 = M_{1,1_1} M_{1_2}$ and $c_2 = M_{2,1} M_{2_2}$. [10, Sec. II-F].

We will also approximate the meta distribution due to the models M1 and M2. In Fig. 2, we have seen that the model M3 involves much more complicated numerical integration than M1 without obvious approximation benefits. The model M4 improves the prediction as compared to the conventional model M2 but not as much as the model M1. For the model M1 the moments $M_b$ are calculated after substituting $R = c$ in (20) and averaging over $\mu e^{-\mu (c-r)}$, $r > c$. For the model M2 the moments are computed after substituting $R = 0$ in (20) and averaging over $\lambda e^{-\lambda r}$, $r > c$. For all models we will apply the method of moments with Beta approximation.

The simulated meta distribution due to the hard-core process and the various approximations are depicted in Fig. 3, for few thresholds $\theta$. The first point to remark is that the model M2 is very unreliable. Its approximation error can be very large especially for high thresholds, because it significantly overestimates both moments, see also Fig. 2. Correcting M2 using model M1 already brings a major improvement, which can be enhanced further using the discretization model. Reading from the figure, the model M2 predicts that 70% of scheduled links achieve a SIR of 0 dB with probability at least 0.8 while the two other models predict very accurately the correct fraction 83% of links. Another remark from Fig. 3 is that for low thresholds $\theta$, the CoV of the meta distribution decreases, which means that most of the scheduled links will experience about the same reliability. The meta distribution for lower activity $\xi = 0.1$ is depicted in Fig. 4.

V. PROPERTIES OF THE META DISTRIBUTION

In this section we study the behaviour of the moments $M_1, M_2$ and the CoV of the meta distribution w.r.t. the activity and the SIR threshold. In addition, we devise low-complexity approximations for these terms, which turn out to be much more accurate than the predictions using the model M2.

Lemma 3. For $\lambda c \ll 1$, the CoV of the meta distribution for the discretization model as $\xi \to 0$, $\theta \to \infty$ such that $\xi \theta^\eta = T$, where $T > 0$ is constant, converges to $\lambda^2 \theta^\eta \sqrt{\nu} = \nu = \frac{\lambda T}{\nu^\eta}$.

Proof. For $\xi \to 0$, the condition $\lambda c b (R-c) \ll 1$, $b \in \{1, 2\}$ is satisfied for realistic $\lambda, c, R$. In addition $\lambda c \ll 1$ and thus, the assumptions in Lemma 1 and Lemma 2 hold. Therefore the first two moments of the meta distribution due to the discretization model can be well-approximated using the model M1. The first moment due to the model M1, see (9), is

$$M_1 = \int_{c}^{\infty} e^{-\lambda x} \rho \frac{\eta^\eta e^{-\eta x}}{\eta^\eta} \mu e^{-\mu (r-c)} dr.$$ (21)

For large $\theta$, the integral w.r.t. $x$ in (21) accepts the following approximation.

$$\int_{c+\theta}^{\infty} \frac{e^{-\eta x}}{\eta^\eta} \frac{\eta^\eta e^{-\eta x}}{\eta^\eta} \mu e^{-\mu (r-c)} dr \approx \int_{c+\theta}^{\infty} \left(1 - \frac{x^\eta}{\eta^\eta} \right) dx + \int_{c+\theta}^{\infty} \theta r^\eta x^{-\eta} dr = \frac{\eta^\eta}{\eta^\eta} \theta^\eta r^\eta - (c+r) + \frac{\eta^\eta}{\eta^\eta} \frac{\eta^\eta}{\eta^\eta} \theta^\eta r^\eta - (c+r),$$

where $(a)$ follows from expanding the fraction (up to first-order) for small $x < \eta^\eta/\eta^\eta$ in the first term, and large $x > \theta^\eta/\eta^\eta$ in the second term.

After substituting the above approximation in (21), and carrying out the integration w.r.t. $r$, we end up with

$$M_1 \approx e^{\lambda c} \frac{\mu e^{-\nu_1}}{\mu + \nu_1}, \quad \nu_1 = \lambda \frac{(\eta^\eta + 1)}{\eta^\eta - 1} \theta^\eta - 1.$$ (22)
In a similar fashion, we can obtain for the second moment the following approximation

\[
M_2 \approx \exp^{2\lambda \xi-\lambda \xi e^{-\mu e^{-\nu c}}} \cdot \frac{\nu c}{\mu+\nu c}.
\]

After substituting \(\tilde{\xi} = T \xi^{-1}\) in \(\nu_1, \nu_2\) and taking the limit of \(M_1, M_2\) for \(\xi \to 0\), we get

\[
\lim_{\xi \to 0} M_1 = \frac{\mu e^{-\nu c}}{\mu + \nu}, \quad \lim_{\xi \to 0} M_2 = \frac{\mu e^{-2\nu c}}{\mu + 2\nu}.
\]  \(\text{(24)}\)

Finally, writing the CoV as \(\sqrt{M_2 / M_1} - 1\), and substituting the limits from \(\text{(24)}\) yields the desired result. The limits depend on \(\xi, \theta\) through the product \(\xi \theta^{1/5}\), while the approximations \(\text{(22)}\) and \(\text{(23)}\) depend also individually on \(\xi\).

In order to illustrate the usefulness of Lemma 3 let us assume early penetration of VANETs, with activity \(\xi = 10^{-2}\). Let us also consider a high-rate data communication at \(\theta = 30\) dB, yielding \(T = 0.1\) for \(\eta = 3\). The numerical calculation of the moments \(M_1, M_2\) and the CoV using various models are presented in Table I. The model M1 estimates accurately the moments due to the discretization model because the activity is low. The approximations \(\text{(22)}\) and \(\text{(23)}\) follow closely the results due to model M1, because the threshold \(\theta\) is large. In addition, the limit \((\xi \to 0)\) for the CoV in Lemma 3 works well yielding, \(\sqrt{\mu/\nu} = 0.070\). Neglecting the hard core distance, i.e., \(\mu = \lambda\) and \(\sqrt{\lambda/\nu} = 0.112\) incurs large overestimation error. This is because the link gain experiences much higher variability for \(c = 0\) than for \(c = 16\), while \(\mu\) is fixed. In Fig. 3 it is illustrated that the approximations in \(\text{(22)}\) and \(\text{(24)}\) are reliable also for realistic activity values, e.g., up to \(\xi = 0.2\). For \(\xi = 0.5\) we obtain \(\theta = 1\) for \(T = 0.5\), and the approximations in \(\text{(22)}\) and \(\text{(23)}\), as expected, break down.

The limit of the CoV in Lemma 3 increases in \(\nu\) and thus in \(\theta\), while \(\xi\) is kept constant, see Fig. 4 for an illustration. This is also evident by visual inspection from Fig. 3 where the variance of the meta distribution is much less for \(\theta = 1\) than for \(\theta = 100\). The large CoVs (for increasing thresholds \(\theta\)) mean that the average success probability \(M_1\) does not represent well the performance of different links across a snapshot of the network. Furthermore, Lemma 3 indicates that for low and decreasing activity \(\xi \to 0\), while keeping constant \(M_1\) by increasing \(\theta\), the variance of the conditional success probability across the network does not become zero. This is in accordance with 11 Corollary 7 for cellular networks with random activity. In Fig. 3 for \(M_1 \approx 0.506\) and activity \(\xi \to 0\), the standard deviation of the conditional success probability converges to 0.16. Finally, according to \(\text{(24)}\), for \(\eta \geq 2\) and \(\theta \geq 1\), \(M_1, M_2\) increase for increasing \(\eta\) but the CoV decreases. Intuitively, higher pathloss means that links scheduled at the same time are better isolated from each other, and the fraction of links achieving certain reliability should increase. We will illustrate this with real traces in Fig. 8 and Fig. 9.

Next, we present approximations for \(M_1, M_2\) and CoV for \(\theta = 0\). For small \(\theta\), the model M1 approximates well the discretization model even for large activity, see Fig. 2 and

| Table I | Discretization model | Model M1 | Model M2 | CoV |
|--------|----------------------|----------|----------|-----|
|        | 0.89968              | 0.81293  | 0.6680   |     |
| Model M1 | 0.90050              | 0.81456  | 0.6729   |     |
| Lemma 3 | 0.80988              | 0.80846  | 0.66546  |     |
| Model M2 | 0.90015              | 0.81894  | 0.66728  |     |
| Simulations | 0.89811              | 0.81010  | 0.66578  |     |

Fig. 5. The moments \(M_1, M_2\) of the meta distribution for pairs \((\xi, \theta)\) satisfying \(T = \xi^{1/5} = 0.5\). The discretization model, the model M1 and the approximations \(\text{(22)}\) and \(\text{(23)}\) in Lemma 3 estimate accurately the moments due to the simulated hardcore process. The model M2 fails in the estimation of both moments. See the caption of Fig. 3 for other parameter settings.

Fig. 6. The first two moments of the meta distribution and the CoV for fixed activity \(\xi = 0.05\) and large thresholds \(\theta\). The discretization model, the model M1 and the approximations \(\text{(22)}\) and \(\text{(23)}\) in Lemma 3 estimate very good predictions to the simulated hardcore process, while the model M2 fails. The limit in Lemma 3 for the CoV increases approximately by \(\theta^{1/2n}\) for large \(\theta\). See the caption of Fig. 3 for other parameter settings.

Fig. 3 for example illustrations. Because of that, we can directly work with the model M1 without introducing any constraints on the activity as we did in Lemma 3.

**Lemma 4.** For \(\theta \ll 1\), the CoV of the meta distribution for
the discretization model is approximated by \( \nu^*/(\mu^2 + 2\nu^*) \) where \( \nu^* = (\eta^+1)^\lambda^c/n^+ 2 \eta^+ \lambda^c/n^+ - 1 \) and \( t = \xi \theta \ll 1 \) is a positive constant.

Proof. For small \( \theta \), the first moment of the meta distribution in (21) can be approximated as

\[
M_1 \approx f_c e^{-\lambda^c \int_{-c}^{c} x^\eta x^{-\eta} \mu e^{-\mu(x-c)} dx} = \int_c e^{-\lambda^c \int_{0}^{\infty} r^c(r+c)^{1-\eta} \mu e^{-\mu(r-c)} dr}.
\]

For small \( \theta \), we get \( \mu > \lambda^c / \eta^+ \), which means that the term \( e^{-\mu r^c} \) will dominate the integrand in the above expression. In addition, the main contribution to the integral is given by the vicinity of \( c \). Because of these reasons, we can expand the term \( r^c(r+c)^{1-\eta} \) in the exponent around \( c \), without introducing much error. After expanding up to the first order and carrying out the integration, we end up with

\[
M_1 \approx e^{\lambda^c c \mu} \frac{\mu e^{-\mu c}}{\mu + \nu^*}.
\]

Similarly, the approximation of the second moment for \( \theta \ll 1 \), keeping only the leading-order term w.r.t. \( t \), is

\[
M_2 \approx e^{\lambda^c c \mu} \frac{\mu e^{-2\mu c}}{\mu + 2\nu^*}.
\]

The approximations (25) and (26) depend on \( \xi \) and \( \theta \) only through the product \( \xi \theta \). After substituting \( \xi \theta \approx \theta \) and (25) into \( \sqrt{M_2} - 1 \) for the CoV, the result of the lemma follows.

The approximations (25) and (26) are validated in Fig. 7. Summing up, according to Lemma 4 if we reduce the activity by 10 dB, we can increase the threshold \( \xi \) by 10 \( \eta^+ \) dB to maintain the same probability of success \( M_1 \) for large \( \theta \) and \( \xi \rightarrow 0 \). According to Lemma 3 in the low \( \theta \) regime, we can increase the threshold only by 10 dB. For instance, in Fig. 2 the probability of outage at \(-10 \) dB is 0.014 with activity \( \xi = 0.5 \). The same outage with activity \( \xi = 0.1 \) occurs around \(-3 \) dB, confirming the approximation (25) in Lemma 4. Note also that according to (26), the two pairs \((\xi, \theta) \in \{(0.5, -10 \text{dB}), (0.1, -3 \text{dB})\}\) result in the same second moment \( M_2 \) too. On the other hand, the probability of outage at 10 dB is 0.45 with activity \( \xi = 0.5 \) in Fig. 2, while with activity \( \xi = 0.1 \) the same outage occurs at 24.5 dB, instead of 31 dB predicted by (24) in Lemma 3. This is because (24) is valid for \( \xi \rightarrow 0 \), and (22) should be used instead. Finally, the CoVs calculated in Lemma 3 (and Lemma 4) decrease in \( \theta \), while the intensity \( \lambda \) is kept fixed. This can be seen after writing the CoV as \( \nu(\mu^2 + 2\nu^*)^{-1} \), where \( \nu \mu^{-1} \ll (1-\lambda^c) \), see Fig. 9 and the inset in Fig. 7 for related illustrations.

VI. VALIDATION WITH REAL TRACES

In (23) we have analyzed synthetic traces for three-lane one-directional motorway traffic (4), using summary statistics, i.e., the J-function and the Ripley’s K-function (31, Ch. 2.8). We have illustrated that the PPP cannot capture well the distribution of inter-vehicle distances, because it permits unrealistically small distances with high probability. The PPP becomes more problematic for the left lane of the motorway because due to the high speeds over there, the drivers maintain large safety distances. We have illustrated in (23) that the envelope of the J-function for the fitted hardcore process \( \Phi_c \) can capture the J-function of the real snapshot, see (23, Sec. IV) for a detailed description of the fitting method. In the current paper, we use the fitted parameters \( \lambda, c \) to assess which model (discretization, M1 and M2) can predict well the simulated outage probability and the meta distribution of snapshots for the left lane. We see in Fig. 8 that for the selected snapshot, \( \lambda_c \approx 0.3 \), the discretization model outperforms the model M1, especially for large thresholds, while the model M2 incurs very large errors. Higher spatial regularity, \( \lambda_c \approx 0.4 \) in Fig. 9 makes clear the benefit of using discretization. Summing up, Fig. 8 and Fig. 9 highlight the hardcore process (and its discretized approximation) as reasonable choices for modeling the statistics of outage probability along motorway VANETs. The extension to multiple lanes should be straightforward by discretizing with lane-specific parameter \( c \). It is omitted due to lack of space. The reader may refer to (23 Section VII) for the spatial modeling of interferers located at a different lane from the link under consideration.

VII. CONCLUSIONS

In practice vehicles have non zero size, and in high-speed motorways they maintain large safety distances from the vehicle ahead. Because of that, a shifted-exponential PDF captures the distribution of headway distance along a lane much better than the PPP. In order to approximate the PGFL of the shifted-exponential (or hardcore) process, we used a shifted-exponential PDF for the transmitter-receiver link coupled with a guard zone behind the transmitter. This model predicts the moments of the SIR much better than the PPP. Nevertheless, for very regular deployments, and/or large SIR threshold and transmission probability, it starts to lose some of its power. Because of that, we have also devised a discretized deployment for the near-field. This is more tailored to the hardcore constraints, hence it approximates better the PGFL of the hardcore process and its meta distribution. The
The simulated probability of outage using real trace and the approximations using the discretization model, and the models M1 and M2. For the discretization model and M1, the (fitted) shifted-exponential PDF has $\lambda \approx 0.0205 \text{ m}^{-1}$ and $c \approx 14.82 \text{ m}$, yielding $\lambda c \approx 0.3037$. For M2, the (fitted) exponential PDF has $\lambda \approx 0.0195 \text{ m}^{-1}$. For $\eta = 2\%$, we get $R_{\text{min}} \approx 442 \text{ m}$ from (15), and we calculate $K = 2000$ and $R = K c \approx 445 \text{ m}$. $10^5$ simulations. The approximation (22) in Lemma 3 for large thresholds is accurate for $\theta > 10$, and the approximation (25) in Lemma 4 for small thresholds is accurate for $\theta < 0.5$. (b)-(c) The simulated meta distribution and the approximations using the same models. $10^4$ spatial configurations and $10^4$ realizations of fading and activity per configuration. The approximations (22) and (25) coupled with the Beta approximation are depicted for large thresholds $\theta = 10$ and $\theta = 100$. For $\theta = 1$, we used the approximations (25) and (26), which are not accurate because Lemma 4 assumes small $\theta \ll 1$. Activity $\xi = 0.5$. We have selected the 1000-th snapshot from motorway M40, May 7 2010, busy hour [4], [5]. The empirical CDF of inter-vehicle distances, obtained from the trace, is used to generate the locations of interferers and the useful link distance.

Fig. 9. Fitting the models to the simulations using the 1200-th snapshot from motorway M40, May 7 2010, busy hour [4], [5]. For the discretization model and the model M1 we use the estimates $\lambda \approx 0.0203 \text{ m}^{-1}$, $c \approx 19.76 \text{ m}$, resulting to $\lambda c \approx 0.4017$. For the model M2 we estimate $\lambda \approx 0.0191 \text{ m}^{-1}$. See the caption of Fig. 8 for explanation of the legend and other parameter settings.

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