Influence of Heat and Mass Transfer on Peristaltic Transport of Viscoplastic Fluid in Presence of Magnetic Field through Symmetric Channel with Porous Medium

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Abstract. In the present paper, we discussed the influence of heat and mass transfer on peristaltic transport of viscoplastic fluid in presence of magnetic field through symmetric channel with porous medium. The constitutive equation of Bingham plastic model is chosen to describe viscoplastic material. The nonlinear partial differential equations that described the motion of flow are simplified under assumptions of low Reynolds number and long wavelength. These equations are solved by mean of the regular perturbation method which is restricted to the smaller values of Bingham and Grashof numbers. Series solution for the axial velocity, temperature and concentration distribution have been computed. The flow quantities have been illustrated graphically for different interesting parameters. The pressure rise and trapping phenomena are also examined graphically. MATHEMATICA software is used to plot all figures.

Keywords: Magnetic field, Viscoplastic fluid, Bingham plastic model, Peristaltic transport.

1. Introduction

Peristalsis is a well-known process in which a progressive wave of contraction or expansion moves along the walls of channel causing the movement of contents of channel. This phenomena widely accrue in several applications to biological, medical and engineering; like urine motion from kidney to bladder through ureter, transportation of lymph from lymphatic vessels, heart-lung machine and many others. Many works on peristaltic flow in various geometries shown that the non-Newtonian behavior and the non-Newtonian fluid flows have many applications in engineering and medicine. So Bingham fluid is one of the non-Newtonian fluid models which is chosen for our study. Heat transfer is an important principle in biological system. There are three mechanisms of heat transfer but the convection is the most applicable heat transfer modality within the circulation of fluid in human body [1-3]. There are many application of MHD flows in the biomedical sciences such as cancer tumor treatment, bleeding reduction during surgeries, power generation development of magnetic devices and many others. Therefore many researcher discussed the peristaltic transport with magnetic field effects [4-6]. Currently, the combined effects of heat and mass transfer receive considerable attentions due to its application in the salty springs in the sea, in reservoir engineering in connection with thermal recovery, in the chemical industry etc. [7]. Srinivas and Kothandapani [8] investigated the influence of heat and
mass transfer on MHD peristaltic flow through porous space with compliant walls. Effect of heat and mass transfer on peristaltic flow of a Bingham fluid in the presence of inclined magnetic field and channel with different wave forms is studied by Akram et al. [9]. Ramesh [10] discussed the influence of heat and mass transfer on peristaltic flow of a couple stress fluid through porous medium in the presence of inclined magnetic field in an inclined asymmetric channel.

In this paper, the influence of heat and mass transfer on peristaltic transport of viscoplastic fluid in presence of magnetic field through symmetric channel with porous medium has been investigated. By using the perturbation method the non-linear governing equations of flow are solved analytically. The effect of all parameters on the flow regime are explained graphically.

2. Mathematical Formulation
Consider a peristaltic transport of an incompressible magnetohydrodynamic (MHD) viscoplastic fluid in a two dimensional symmetric channel of width \(2a\) with porous medium as shown in figure 1. The flow is generated by propagation of wave on the channel walls train moving ahead with constant speed \(c\). The uniform magnetic field is applied in \(Y\)-direction to study the effect of it on the fluid flow. Electric field is absent. Heat and mass transfer studied through convective condition. The geometries of the channel wall is given by [1,11]

\[
Y = H(\tilde{X}, \tilde{t}) = a + \phi \cos \left(\frac{2\pi}{\lambda} (\tilde{X} - \tilde{c}\tilde{t})\right)
\]

where \(\phi\) is the wave amplitudes, \(\lambda\) is the wavelength, \(\tilde{t}\) is the time and \((\tilde{X}, \tilde{Y})\) is the rectangular coordinates in the fixed frame of reference.

3. Basic and Constitutive Equations
Based on the above consideration, the basic governing equations that describe the flow in the present problem are given by [5,12]

- equation of mass conservation
  \[
  \nabla \cdot \mathbf{V} = 0,
  \]
  \[\text{(2)}\]

- motion equations (Navier-Stokes equations)
  \[
  \rho \frac{d\mathbf{V}}{dt} = \nabla \cdot \mathbf{\sigma} + \rho \mathbf{g} a(T - T_0) + J \times \mathbf{B} - \frac{\eta}{k_0} \mathbf{V},
  \]
  \[\text{(3)}\]

- energy equation
  \[
  \rho c_p \frac{dT}{dt} = \kappa \nabla^2 T + \mathbf{\sigma} \cdot (\nabla \mathbf{V}),
  \]
  \[\text{(4)}\]

- concentration equation
  \[
  \frac{dc}{dt} = D_m \nabla^2 \tilde{C} + \frac{D_m K_T}{\theta_m} \nabla^2 \tilde{T},
  \]
  \[\text{(5)}\]
in which $\mathbf{V}$ is the velocity, $\rho$ is the density, $\frac{d}{dt}$ is the material time derivative, $\alpha$ is the coefficient of thermal expansion, $\mathbf{\sigma}$ is the Cauchy stress tensor, $g$ is the acceleration due to gravity, $\mathbf{j} = \sigma' (\mathbf{V} \times \mathbf{B})$ is the current density, $\mathbf{B} = (0, B_0, 0)$ is the magnetic field, $\sigma'$ is the electrical conductivity, $\eta$ is the viscosity, $k_0$ is the permeability parameter of porous medium, $\nabla^2$ is the Laplace operator, $T$ is the temperature, $\kappa$ is the thermal conductivity, $C_p$ is the specific heat, $\bar{C}$ is the mass concentration, $D_m$ is the coefficient of mass diffusion, $K_T$ is the thermal diffusion ratio and $T_m$ is the mean temperature.

The term $(\mathbf{\sigma} \cdot (\nabla \mathbf{V}))$ in equation (4) can be compute from the definition of dot product of two tensor (if $\mathbf{B}$ and $\mathbf{B}$ are any two tensor then $\mathbf{B} \cdot \mathbf{B} = tr(\mathbf{B} \mathbf{B})$). Let $\mathbf{U}$ and $\mathbf{V}$ be the velocity components along the $\mathbf{X}$ and $\mathbf{Y}$-directions respectively in the fixed frame, the velocity vector $\mathbf{V}$ can be written as

$$\mathbf{V} = (\mathbf{U}(\mathbf{X}, \mathbf{Y}, t), \mathbf{V}(\mathbf{X}, \mathbf{Y}, t), 0).$$ (6)

The Bingham plastic fluid is considered and the constitutive equations can be defined as [1,5]

$$\mathbf{\sigma} = -\mathbf{P} I + \mathbf{\tau},$$ (7)

$$\mathbf{\tau} = 2\eta \mathbf{D} + 2\tau_0 \mathbf{\bar{D}},$$ (8)

in above equations, $\mathbf{\tau}$ is the extra tensor, $I$ is the identity tensor, $\mathbf{P}$ is the pressure while the rate of deformation tensor $D$ and the tensor $\mathbf{\bar{D}}$ are defined by

$$D = \frac{1}{2} \{ (\nabla \mathbf{V}) + (\nabla \mathbf{V})^T \},$$

$$\mathbf{\bar{D}} = \frac{\mathbf{P}}{\sqrt{2\tau_0}}.$$ (9)

From equations (2)-(7), the governing equation in the fixed frame are given by

$$\rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \frac{\partial \mathbf{U}}{\partial \mathbf{X}} + \mathbf{V} \frac{\partial \mathbf{U}}{\partial \mathbf{Y}} \right) = -\frac{\partial \mathbf{P}}{\partial \mathbf{X}} + \frac{\partial \mathbf{\tau}_{XX}}{\partial \mathbf{Y}} + \frac{\partial \mathbf{\tau}_{XY}}{\partial \mathbf{Y}} + \rho g (T - T_0) - \sigma' \mathbf{B}_0^2 \mathbf{U} - \frac{\eta}{k_0} \mathbf{\bar{U}},$$ (11)

$$\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{U} \frac{\partial \mathbf{V}}{\partial \mathbf{X}} + \mathbf{V} \frac{\partial \mathbf{V}}{\partial \mathbf{Y}} \right) = -\frac{\partial \mathbf{P}}{\partial \mathbf{Y}} + \frac{\partial \mathbf{\tau}_{YY}}{\partial \mathbf{X}} + \frac{\partial \mathbf{\tau}_{XY}}{\partial \mathbf{X}} - \frac{\eta}{k_0} \mathbf{\bar{V}},$$ (12)

$$\left( \frac{\partial \mathbf{F}}{\partial t} + \mathbf{U} \frac{\partial \mathbf{F}}{\partial \mathbf{X}} + \mathbf{V} \frac{\partial \mathbf{F}}{\partial \mathbf{Y}} \right) = \kappa \left( \frac{\partial^2 \mathbf{F}}{\partial \mathbf{X}^2} + \frac{\partial^2 \mathbf{F}}{\partial \mathbf{Y}^2} \right),$$ (13)

$$\left( \frac{\partial \mathbf{F}}{\partial t} + \mathbf{U} \frac{\partial \mathbf{F}}{\partial \mathbf{X}} + \mathbf{V} \frac{\partial \mathbf{F}}{\partial \mathbf{Y}} \right) = \mathbf{D}_m \left( \frac{\partial^2 \mathbf{F}}{\partial \mathbf{X}^2} + \frac{\partial^2 \mathbf{F}}{\partial \mathbf{Y}^2} \right) + \mathbf{D}_m \frac{\partial \mathbf{K}_T}{\partial \mathbf{m}} \left( \frac{\partial^2 \mathbf{F}}{\partial \mathbf{X}^2} + \frac{\partial^2 \mathbf{F}}{\partial \mathbf{Y}^2} \right).$$ (14)

The corresponding boundary conditions are

$$\mathbf{U} = 0 \quad \mathbf{F} = \mathbf{F}_1, \quad \mathbf{C} = \mathbf{C}_1 \quad \text{at} \quad \mathbf{Y} = \mathbf{H} \),$$

$$\mathbf{U} = 0 \quad \mathbf{F} = \mathbf{F}_0 \quad \mathbf{C} = \mathbf{C}_0 \quad \text{at} \quad \mathbf{Y} = -\mathbf{H} \).$$ (15)

In view of equations (8) and (9), the components of extra stress tensor in the fixed frame becomes

$$\tau_{XX} = 2\eta \frac{\partial \mathbf{U}}{\partial \mathbf{X}} + \frac{2\tau_0 \frac{\partial \mathbf{U}}{\partial \mathbf{X}}}{(2\frac{\partial^2 \mathbf{U}}{\partial \mathbf{X}^2} + (\frac{\partial \mathbf{U}}{\partial \mathbf{X}})^2 + (\frac{\partial \mathbf{V}}{\partial \mathbf{Y}})^2)^2},$$

$$\tau_{YY} = \tau_{XX} \eta \frac{\partial \mathbf{U}}{\partial \mathbf{Y}} + \frac{2\tau_0 \frac{\partial \mathbf{U}}{\partial \mathbf{Y}}}{(2\frac{\partial^2 \mathbf{U}}{\partial \mathbf{X}^2} + (\frac{\partial \mathbf{U}}{\partial \mathbf{X}})^2 + (\frac{\partial \mathbf{V}}{\partial \mathbf{Y}})^2)^2},$$

$$\tau_{XY} = \tau_{XX} \eta \frac{\partial \mathbf{V}}{\partial \mathbf{Y}} + \frac{2\tau_0 \frac{\partial \mathbf{V}}{\partial \mathbf{Y}}}{(2\frac{\partial^2 \mathbf{U}}{\partial \mathbf{X}^2} + (\frac{\partial \mathbf{U}}{\partial \mathbf{X}})^2 + (\frac{\partial \mathbf{V}}{\partial \mathbf{Y}})^2)^2}.$$ (16)

Peristaltic motion is unsteady phenomenon in nature but it can be assumed steady by using the transformation from the laboratory frame (fixed frame) $(\mathbf{X}, \mathbf{Y})$ to the wave frame (move frame) $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ which defined as [1,11]

$$\bar{\mathbf{x}} = \mathbf{x} - ct, \quad \bar{\mathbf{y}} = \mathbf{y}, \quad \bar{\mathbf{u}}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \bar{\mathbf{U}}(\mathbf{X}, \mathbf{Y}, t) - \mathbf{c}, \quad \bar{\mathbf{v}}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \bar{\mathbf{V}}(\mathbf{X}, \mathbf{Y}, t),$$

$$\bar{\mathbf{p}}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \bar{\mathbf{P}}(\mathbf{X}, \mathbf{Y}, t), \quad \bar{\mathbf{T}}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \bar{\mathbf{T}}(\mathbf{X}, \mathbf{Y}, t), \quad \bar{\mathbf{C}}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \bar{\mathbf{C}}(\mathbf{X}, \mathbf{Y}, t),$$ (17)

where $\bar{\mathbf{u}}, \bar{\mathbf{v}}$ and $\bar{\mathbf{p}}$ are the velocity components and the pressure in the wave frame, respectively.

Now, we transform equations (1) and (10)-(16) in wave frame with the help of equation (17) and normalize the resulting non-dimensional quantities [1,11]
\( \ddot{x} = \lambda x \), \( \ddot{y} = a_1 y \), \( \ddot{u} = cu \), \( \ddot{v} = cv \), \( \ddot{\psi} = \frac{\dot{\lambda}}{c} t \), \( \ddot{p} = \frac{cn^2}{a_1^2} p \),

\[
\begin{align*}
Re &= \frac{a_1c}{\eta}, \\
\ddot{q} &= a_1cF, \\
\ddot{k}_0 &= \frac{a_1^2 k_0}{c_1 - c_0}, \\
\ddot{\tau}_{ij} &= \frac{c}{a_1} \ddot{\tau}_{ij}, \\
\theta &= \frac{T_1 - T_0}{T_{11} - T_0}, \\
Pr &= \frac{c}{c_0}, \\
Sc &= \frac{\eta}{\rho d_m}, \\
Sr &= \frac{c_0 (T_1 - T_0) k_T}{\eta T_m (c_1 - c_0)}, \\
Bn &= \frac{a_1 T_0}{c \eta}, \\
M^2 &= \frac{\sigma B^2 a_1^2}{c \eta}, \\
Gr &= \frac{\rho g \alpha a_1^2 (T_1 - T_0)}{c \eta},
\end{align*}
\]


\]

which gives

\[
\ddot{y} = H(x) = 1 + \phi \cos 2\pi x, \\
\ddot{\psi} = \frac{\ddot{\psi}}{\ddot{\psi}} + \frac{\ddot{\psi}}{\ddot{\psi}} = 0,
\]

\[
\begin{align*}
Re \left( (u + 1) \delta \frac{\ddot{u}}{\ddot{x}} + v \frac{\ddot{u}}{\ddot{y}} \right) &= -\frac{\partial p}{\partial x} + \delta \frac{\ddot{x}_y}{\ddot{x}} + \delta \frac{\ddot{x}_y}{\ddot{y}} + \frac{Gr \ddot{\theta} - A^2 (u + 1)}{2}, \\
Re \left( (u + 1) \delta \frac{\ddot{v}}{\ddot{x}} + v \frac{\ddot{v}}{\ddot{y}} \right) &= -\frac{\partial p}{\partial y} + \delta \frac{\ddot{x}_y}{\ddot{x}} + \delta \frac{\ddot{x}_y}{\ddot{y}} - \frac{1}{k_0} v, \\
Re Pr \left( (u + 1) \delta \frac{\ddot{u}}{\ddot{x}} + v \frac{\ddot{u}}{\ddot{y}} \right) &= \left[ \delta^2 \frac{\ddot{u}_x}{\ddot{x}} + \frac{\ddot{u}_y}{\ddot{y}} \right]^2 + Ec Pr \left[ \delta \frac{\ddot{u}_x}{\ddot{x}} + \frac{\ddot{u}_y}{\ddot{y}} \right], \\
Re \left( (u + 1) \frac{\ddot{u}}{\ddot{x}} + v \frac{\ddot{u}}{\ddot{y}} \right) &= \frac{1}{Sc} \left[ \delta^2 \frac{\ddot{u}_x}{\ddot{x}} + \frac{\ddot{u}_y}{\ddot{y}} \right]^2 + Sr \left[ \delta^2 \frac{\ddot{u}_x}{\ddot{x}} + \frac{\ddot{u}_y}{\ddot{y}} \right],
\end{align*}
\]

\[
\begin{align*}
\ddot{u} &= -1, \quad \ddot{\theta} = 1, \quad \ddot{\Omega} = 1 \text{ at } y = H, \\
\ddot{u} &= -1, \quad \ddot{\theta} = 0, \quad \ddot{\Omega} = 0 \text{ at } y = -H,
\end{align*}
\]

\[
\ddot{\tau}_{xx} = 2 \delta \ddot{\tau}_{xx} + \frac{2 Bn \delta^2 \ddot{u}}{(2 \delta^2 \ddot{u}_x^2 + (\ddot{u}_x + \delta \ddot{u}_x)^2 + (\ddot{u}_y + \delta \ddot{u}_y)^2)^2},
\]

\[
\ddot{\tau}_{xy} = \ddot{\tau}_{yx} = \frac{\ddot{\tau}_{xy}}{\ddot{\tau}_{yx}} = \frac{\ddot{u}_x + \delta \ddot{u}_x}{(2 \delta^2 \ddot{u}_x^2 + (\ddot{u}_x + \delta \ddot{u}_x)^2 + (\ddot{u}_y + \delta \ddot{u}_y)^2)^2},
\]

\[
\ddot{\tau}_{yy} = 2 \delta \ddot{\tau}_{yy} + \frac{2 Bn \delta^2 \ddot{u}}{(2 \delta^2 \ddot{u}_x^2 + (\ddot{u}_x + \delta \ddot{u}_x)^2 + (\ddot{u}_y + \delta \ddot{u}_y)^2)^2},
\]

in the above expressions, \( T_1 \) and \( T_0 \) are the temperature at the right side wall and the left side wall respectively whereas \( C_1 \) and \( C_0 \) denoted the concentration at the right side wall and the left side wall of the channel respectively. \( \ddot{\psi} \) is the stream function, \( \ddot{\delta} \) is the dimensionless wave number, \( Re \) is the Reynolds number, \( Ec \) is the Eckert number, \( Pr \) is the Prandtl number, \( Sr \) is the Soret number, \( Sc \) is the Schmidt number, \( M \) is the Hartman number, \( Bn \) is the Bingham number, \( Gr \) is the Grashof number, \( \ddot{\Omega} \) is the non-dimensional the concentration and \( \ddot{\theta} \) is the temperature in the non-dimensional form.

Introduction of dimensionless stream function (\( \ddot{\psi} \)) by the relations \( u = \ddot{\psi}_y \) and \( v = -\delta \ddot{\psi}_x \) in equations (20)-(26) shows that the continuity equation satisfies identically while other equations subjected to low Reynolds number and long wave length approximation (i.e. neglecting wave number (\( \ddot{\delta} \ll 1 \)) and Reynolds number (\( Re \ll 1 \)), yields

\[
\begin{align*}
\ddot{\psi} &= \frac{\ddot{x}_y}{\ddot{\tau}_{xy}} + Gr \ddot{\theta} - A^2 (\ddot{\psi}_y + 1), \\
\ddot{\psi} &= 0,
\end{align*}
\]
\[
\frac{\partial^2 \theta}{\partial y^2} = Br \tau_{xy} \psi_{yy}, \tag{29}
\]
\[
\frac{\partial \Omega}{\partial y^2} = -Sc \tau_{\Omega y} \frac{\partial^2 \theta}{\partial y^2}, \tag{30}
\]
\[
\begin{align*}
\psi_y &= -1, \quad \theta = 1, \quad \Omega = 1 & \text{at } y = H \\
\psi_y &= -1, \quad \theta = 0, \quad \Omega = 0 & \text{at } y = -H
\end{align*}
\tag{31}
\]
and
\[
\tau_{xx} = \tau_{yy} = 0, \quad \tau_{xy} = \psi_{yy} + B_n, \tag{32}
\]
where, \(Br = EcPr\) is the Brinkman number.

By using equation (32) into equation (27) and deriving the result with respect to \(y\), in view of equation (28), yields
\[
\psi_{yy} + Gr \theta_y - A^2 \psi_{yy} = 0. \tag{33}
\]
From equation (29) and (32) we have
\[
\theta_{yy} = Br(\psi_{yy}^2 + Bn \psi_{yy}). \tag{34}
\]

4. Rate of Volume Flow

At any instant the volume flow rate in the fixed frame reference \((\bar{X}, \bar{Y})\) is given by
\[
Q(\bar{X}, \bar{Y}) = \int_{-H(X, \bar{Y})}^{H(X, \bar{Y})} \bar{U}(\bar{X}, \bar{Y}, \bar{t}) d\bar{Y}, \tag{35}
\]
while the expression for the volumetric flow rate in the wave frame of reference \((\bar{x}, \bar{y})\) is defined as
\[
\bar{q}(\bar{x}) = \int_{-H(\bar{x})}^{H(\bar{x})} \bar{U}(\bar{x}, \bar{y}, \bar{t}) d\bar{y}. \tag{36}
\]

Using equation (17) into equation (35) and making use of equation (36) we obtain the relation of the two fluxes as follows
\[
Q = \bar{q} + 2cH. \tag{37}
\]
The average volume flow rate over the period time \(T = \left(\frac{\lambda}{c}\right)\) of the peristaltic wave at a fixed position \(\bar{x}\) is defined as
\[
Q^* = \frac{1}{T} \int_0^T Q d\bar{t}. \tag{38}
\]
Substituting equation (37) into equation (38), we have
\[
Q^* = \bar{q} + 2ca. \tag{39}
\]

By using equation (18), \(\Theta = \frac{Q}{ca_1}\) and \(F = \frac{q}{ca_1}\), into equation (39) we have
\[
\Theta = F + 2, \tag{40}
\]
where \(F\) is the dimensionless volume flow rate in the wave frame defined by
\[
F = \int_{-H(x)}^{H(x)} \frac{\partial \psi}{\partial y} dy = \psi(H(x)) - \psi(-H(x)), \tag{41}
\]
then \(\psi = \frac{F}{2}\) at the right wall and \(\psi = -\frac{F}{2}\) at the left wall of the channel.

The non-dimensional expression for pressure rise over one cycle of the wave is given
\[
\Delta P = \int_0^1 \frac{\partial \psi}{\partial x} dx, \tag{42}
\]
which is difficult to evaluate directly, so we used MATHEMATICA software to compute it numerically.

5. Solution of the Problem

In above equations, we have a system of non-linear partial differential equations which is difficult to solve it exactly. So had to resort to the application of an approximation method, via the perturbation method to solve it. Let us expand stream function and the concentration for small values of Bingham number \((Bn \ll 1)\) and the temperature for small values of Grashof number \((Gr \ll 1)\) as follows
\[
\psi = \sum_{i=0}^\infty (Bn)^i \psi_i + O(Bn^2), \tag{43}
\]
\[
\theta = \sum_{i=0}^\infty (Gr)^i \theta_i + O(Gr^2), \tag{43}
\]
\[
\Omega = \sum_{i=0}^\infty (Bn)^i \Omega_i + O(Bn^2). \tag{43}
\]
Inserting equation (43) into equations (30), (33) and (34) with the corresponding boundary conditions (equation (31)) and then collecting the coefficients of like power of $Bn$ and $Gr$ yields the zeroth and the first order systems.

5.1. Zeroth Order System

\[
\begin{align*}
\psi_{0y} - A^2 \psi_{0y} &= 0, \\
\theta_{0y} + Br \psi_{0y} &= 0, \\
\Omega_{0y} + ScSr \theta_{0y} &= 0,
\end{align*}
\]

with the corresponding boundary conditions

\[
\begin{align*}
\psi_0 &= \frac{\epsilon}{2}, \quad \psi_0 = -\frac{\epsilon}{2}, \quad \theta_0 = 1, \quad \Omega_0 = 1 \quad \text{at} \quad y = H, \\
\psi_0 &= \frac{\epsilon}{2}, \quad \psi_0 = -\frac{\epsilon}{2}, \quad \theta_0 = 0, \quad \Omega_0 = 0 \quad \text{at} \quad y = -H.
\end{align*}
\]

5.2. First Order System

\[
\begin{align*}
\psi_{1y} + Br \psi_{1y} - A^2 \psi_{1y} &= 0, \\
\theta_{1y} + 2Br \psi_{0y} \psi_{1y} + \psi_{1y} &= 0, \\
\Omega_{1y} + ScSr \theta_{1y} &= 0,
\end{align*}
\]

with the corresponding boundary conditions

\[
\begin{align*}
\psi_1 &= 0, \quad \psi_1 = 0, \quad \theta_1 = 0, \quad \Omega_1 = 0 \quad \text{at} \quad y = H, \\
\psi_1 &= 0, \quad \psi_1 = 0, \quad \theta_1 = 0, \quad \Omega_1 = 0 \quad \text{at} \quad y = -H.
\end{align*}
\]

MATHEMATICA software is used for all calculation and the solution of zeroth system with the corresponding boundary conditions is

\[
\begin{align*}
\psi_0 &= \frac{e^{Ay}c_1 + e^{-Ay}c_2}{A} + c3 + yc4, \\
\theta_0 &= \frac{Br(-c2e^{-2Ay} - c1e^{2Ay} - 2Ac1c2y^2)}{2A} + c5 + yc6, \\
\Omega_0 &= \frac{1}{2}BrScSr\left(\frac{c2e^{-2Ay}}{2A^2} + \frac{c1e^{2Ay}}{2A^2} + 2c1c2y^2\right) + c7 + yc8,
\end{align*}
\]

where $c_i, i = 1, 2, \ldots, 8$ are constants which are found by using the boundary conditions.

By using the solution of zeroth order, the solution of first system with the corresponding boundary conditions is

\[
\begin{align*}
\psi_1 &= -\frac{1}{24A^5}(Br(c2e^{-2Ay} - c1e^{2Ay} + 8A^3c1c2y^3) - 12A^3e^{-Ay}(c6e^{Ay}c^2 + 2(e^{2Ay}c9 + c10)) + c11 + yc12, \\
\theta_1 &= -\frac{1}{216A^5}e^{-3Ay}(108A^3e^{2Ay}(2c10 + 2c9e^{2Ay} + c6c^2e^{2Ay} - 8Br^2(c2 - c1c^2e^{2Ay} + 9c1c2e^{4Ay}(-25 + 12Ay) + 9c1c2e^{2Ay} (25 + 12Ay)) + 9Br^2(-c2 + c2e^{2Ay} + 48Ac6e^{2Ay} (c2 + c1e^{2Ay}) + 24A^5(c1c10 + c2c9)e^{2Ay}y^2 + 4A^3(3c10c2 + 3c1c9e^{2Ay} - 2c1c2e^{2Ay}y^3)) + c13 + yc14, \\
\Omega_1 &= \frac{1}{216A^5}e^{-3Ay}ScSr(108A^3e^{2Ay}(2c10 + 2c9e^{2Ay} + c6c^2e^{2Ay}y^2) - 8Br^2(c2 - c1c^2e^{2Ay} + 9c1c2e^{4Ay}(-25 + 12Ay) + 9c1c2e^{2Ay} (25 + 12Ay)) + 9Br^2(-c2 + c2e^{2Ay} + 48Ac6e^{2Ay} (c2 + c1e^{2Ay}) + 24A^5(c1c10 + c2c9)e^{2Ay}y^2 + 4A^3(3c10c2 + 3c1c9e^{2Ay} - 2c1c2e^{2Ay}y^3)) + c15 + yc16,
\end{align*}
\]
where \( c_i, i = 8, 9, \ldots, 16 \) are constants which are found by using the boundary conditions.

6. Results and Discussion

To study the effect of physical parameters such as Hartman number (magnetic parameter) \( M \), permeability parameter \( k_0 \), amplitude wave \( \phi \), Bingham number \( Bn \), Brinkman number \( Br \), Grashof number \( Gr \), flow rate \( F \), Schmidt number \( Sc \), and Soret number \( Sr \), we have plotted the axial velocity \( u \), temperature \( \theta \), concentration \( \Omega \), pressure rise \( \Delta P_a \) and trapping phenomenon in figures 2-33. MATHEMATICA software is used to plot all figures.

6.1. Velocity Distribution \( u \)

Graphical results are displayed in order to see the behavior of parameters involved in the axial velocity \( u \). The effect of different values of \( M, k_0, Bn, Br, \phi \) and \( F \) on the axial velocity \( u \) are explained in figures 2-7. The behavior of velocity distribution is parabolic as seen in figures. Figure 2 explained the effect of \( M \) on the axial velocity \( u \). This result agree with the result of Adnan and Abdulhadi [5]. It is noticed that with an increase of \( M \), the axial velocity increases at the walls of the channel, however, it decreases at the central part of the channel. Figure 3 displayed the influence of \( k_0 \) on the axial velocity. It is noticed that at the walls of the channel the axial velocity decreases slowly with an increase of \( k_0 \), however it increases at the center of the channel. Figure 4 shown the effect of \( Bn \) on the axial velocity \( u \). It is observed that the increase in \( Bn \) lead to \( u \) decreases at the left wall of the channel, while \( u \) increasing at the middle portion and then gradually disappear as there is no effect on axial velocity near the right wall of the channel. From figure 5 noted that the axial velocity do not change at increasing in \( Br \). Figures 6 and 7 displayed that the axial velocity increases with an increase in \( \phi \) and \( F \).

![Figure 2. Effect of \( M \) on the axial velocity \( u \) at \( k_0 = 2, Bn = 0.001, Br = 6, F = 1.4, \phi = 3, x = 0.1 \).](image1)

![Figure 3. Effect of \( k_0 \) on the axial velocity \( u \) at \( M = 0.5, Bn = 0.001, Br = 6, F = 1.4, \phi = 3, x = 0.1 \).](image2)

![Figure 4. Effect of \( Bn \) on the axial velocity \( u \) at \( M = 0.5, k_0 = 2, Br = 6, F = 1.4, \phi = 3, x = 0.1 \).](image3)

![Figure 5. Effect of \( Br \) on the axial velocity \( u \) at \( M = 0.5, k_0 = 2, Bn = 0.001, F = 1.4, \phi = 3, x = 0.1 \).](image4)
6.2. Temperature Distribution $\theta$

The variation in temperature profile for different values of involved parameters are displayed in figures 8-13. Figures 8 and 9 are shown that the impact of $M$ and $\phi$ on the temperature profile $\theta$. It is noticed that the temperature distribution decreases in the central region and increases near the channel walls with increasing in $M$ and $\phi$. Figures 10-13 are explained that the temperature increases by increasing in $k_0$, $Br$, $Gr$ and $F$. The effects of $Br$, $Gr$ and $F$ are consistent with the results analyzed in previous studies (Adnan and Abdulkhadi [5] and Ali and Asghar [1]).
6.3. Concentration Distribution $\Omega$

The graphical results for concentration profile are illustrated in figures 14-21. Opposite behaviour for concentration distribution is noticed compare with the temperature distribution. Figures 14 and 15 are illustrated that the effect of $M$ and $\phi$ on the concentration profile $\Omega$. It is noticed that the concentration distribution increases in the central region and decreases near the channel walls with increasing in $M$ and $\phi$, but opposite behaviour is appearing with the increase in $Sr$ and $Sc$ as displayed in figures 16 and 17. Figures 18-21 are shown that the concentration decreases by increasing in $k_0, Bn, Br$ and $F$. Ali and Asghar [1] is also of the same opinion for the results of Brinkman number $Br$.

Figure 12. Effect of $Gr$ on the temperature profile $\theta$
at $M = 0.5, k_0 = 2, Br = 6, F = 1.4, \phi = 3, x = 0.1$.

Figure 13. Effect of $F$ on the temperature profile $\theta$
at $M = 0.5, k_0 = 2, Gr = 0.001, Br = 6, \phi = 3, x = 0.1$.

Figure 14. Effect of $M$ on the concentration profile $\Omega$
at $k_0 = 2, Bn = 0.001, Br = 6, F = 1.4, \phi = 3, Sc = 0.4, Sr = 0.8, x = 0.1$.

Figure 15. Effect of $\phi$ on the concentration profile $\Omega$
at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 6, F = 1.4, Sc = 0.4, Sr = 0.8, x = 0.1$. 
Figure 16. Effect of $Sr$ on the concentration profile $\Omega$ at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 6, F = 1.4, \phi = 3, Sc = 0.4, x = 0.1$. 

Figure 17. Effect of $Sc$ on the concentration profile $\Omega$ at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 6, F = 1.4, \phi = 3, Sr = 0.8, x = 0.1$. 

Figure 18. Effect of $k_0$ on the concentration profile $\Omega$ at $M = 0.5, Bn = 0.001, Br = 6, F = 1.4, \phi = 3, Sc = 0.4, Sr = 0.8, x = 0.1$. 

Figure 19. Effect of $Bn$ on the concentration profile $\Omega$ at $M = 0.5, k_0 = 2, Br = 6, F = 1.4, \phi = 3, Sc = 0.4, Sr = 0.8, x = 0.1$. 

Figure 20. Effect of $Br$ on the concentration profile $\Omega$ at $M = 0.5, k_0 = 2, Bn = 0.001, F = 1.4, \phi = 3, Sc = 0.4, Sr = 0.8, x = 0.1$. 

Figure 21. Effect of $F$ on the concentration profile $\Omega$ at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 6, \phi = 3, Sc = 0.4, Sr = 0.8, x = 0.1$. 
6.4. Pressure Rise $\Delta P_\lambda$

The variation of pressure rise $\Delta P_\lambda$ per wave length against the mean flow rate $\Theta$ of a symmetric channel are explained in figures 22-27 and the influence of pertinent parameters on the pressure rise $\Delta P_\lambda$ are illustrated. The entire pumping region consist of three zones, which are retrograde pumping ($\Theta < 0, \Delta P_\lambda > 0$), co-pumping ($\Theta < 0, \Delta P_\lambda < 0$) and augmented pumping ($\Theta > 0, \Delta P_\lambda < 0$) (see Adnan and Abdulhadi [5] and Misra, et al. [11]). Figure 22 highlights the variation of $\Delta P_\lambda$ for different values of Hartman number $M$, from this figure we observed that $\Delta P_\lambda$ increases by increasing $M$ in retrograde pumping while revers tend is noticed in the co-pumping and augmented regions. The opposite results are revealed for rescinding values of permeability parameter $k_0$ as displayed through figure 23. From figure 24 we noted that $\Delta P_\lambda$ decreases with increase in Bingham number $Bn$ in all three region (retrograde pumping, co-pumping and augmented pumping regions). Figure 25 explained that $\Delta P_\lambda$ not change by increasing of $Br$. It is visualized from figure 26 that an increase in $Gr$ causes increases $\Delta P_\lambda$ only in augmented pumping region and the others stay in rest. Figure 27 is plotted to see the effect of $\phi$ on the pressure rise $\Delta P_\lambda$. It is noted that $\Delta P_\lambda$ decreases in retrograde pumping and co-pumping regions and it increases in the augmented pumping region.

![Figure 22](image1.png)  
Figure 22. Effect of $M$ on the pressure rise $\Delta P_\lambda$ at $k_0 = 2, Bn = 0.001, Br = 6, Gr = .01, \phi = 3, y = 0.1$.  

![Figure 23](image2.png)  
Figure 23. Effect of $k_0$ on the pressure rise $\Delta P_\lambda$ at $M = 0.5, Bn = 0.001, Br = 6, Gr = .01, \phi = 3, y = 0.1$.  

![Figure 24](image3.png)  
Figure 24. Effect of $Bn$ on the pressure rise $\Delta P_\lambda$ at $M = 0.5, k_0 = 2, Br = 6, Gr = .01, \phi = 3, y = 0.1$.  

![Figure 25](image4.png)  
Figure 25. Effect of $Br$ on the pressure rise $\Delta P_\lambda$ at $M = 0.5, k_0 = 2, Bn = 0.001, Gr = .01, \phi = 3, y = 0.1$.  

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6.5. Trapping Phenomenon

Trapping is one of the interesting phenomena of peristaltic flow in which an internally circulation bolus of fluid is made up by closed stream lines. The behavior of stream function is illustrated in figures 28-33 and the stream lines near the channel walls do nearly strictly follow the wall waves, which are mainly developed by the relative movement of the walls. We observed that the size of trapped bolus reduces with increasing $M$ whereas it enhances with increasing $k_0$, $\phi$ and $F$ as displayed through figures 28-31 respectively. By increasing $Bn$ the trapped bolus increases in size at the left wall of the channel but opposite behavior occur at the right wall as shown in figure 32. While figure 33 explained that the increasing values of $Br$, it has the slight effect always negligible at the size of the trapped bolus one can see it clear of a diagram streamline. The result of $M$ is similar to this given by Satyanarayana et al. [3].

![Figure 26. Effect of $Gr$ on the pressure rise $\Delta P_a$ at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 6, \phi = 3, y = 0.1.$](image1)

![Figure 27. Effect of $\phi$ on the pressure rise $\Delta P_a$ at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 6, Gr = .01, y = 0.1.$](image2)

![Figure 28. Effect of $(a) M = 0.5, (b)M = 1.5, (c)M = 2$ on the streamline at $k_0 = 2, Bn = 0.001, Br = 0.6, F = 1.8, \phi = 0.3.$](image3)
Figure 29. Effect of $(a) k_0 = 0.2$, $(b) k_0 = 0.5$, $(c) k_0 = 2$ on the stream line at $M = 0.5, Bn = 0.001, Br = 0.6, F = 1.8, \phi = 0.3$.

Figure 30. Effect of $(a) \phi = 0.2$, $(b) \phi = 0.3$, $(c) \phi = 0.4$ on the stream line at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 0.6, F = 1.8$.

Figure 31. Effect of $(a) F = 1.5$, $(b) F = 1.6$, $(c) F = 1.8$ on the stream line at $M = 0.5, k_0 = 2, Bn = 0.001, Br = 0.6, \phi = 0.3$.

Figure 32. Effect of $(a) Bn = 0.2$, $(b) Bn = 0.5$, $(c) Bn = 0.65$ on the stream line at $M = 0.5, k_0 = 2, Br = 0.6, F = 1.8, \phi = 0.3$. 
7. Conclusions

In the present paper, the influence of heat and mass transfer on peristaltic transport of viscoplastic fluid in presence of magnetic field through symmetric channel with porous medium has been investigated. The flow problem is transformed from laboratory frame to move frame by using appropriate transformation. Low Reynolds number and long wavelength are used to simplify the problem. The method of perturbation is employed to solve the governing equations of flow. In view of this study, some of the interesting outcomes are summarized as follows:

- The axial velocity decreases in the central region and it increases near the boundaries of the channel with increasing $M$ but the opposite occur for increasing $k_0$. Moreover, it increases over the whole cross-section with increasing $\phi$ and $F$.
- By increasing $Bn$, the axial velocity decreases near the left wall while it increases at the center of the channel. Further $Br$ has not effected on the axial velocity.
- It is noted that, the temperature profile decreases over the whole cross-section of the channel while it increases at the boundaries with increasing $M$ and $\phi$. Further the temperature enhances with increasing $k_0$, $Gr$, $Br$ and $F$.
- Opposite behavior for concentration distribution is noted compared to temperature profile. Furthermore, the concentration decreases over the whole cross-section except near the boundaries of the channel it is increases with increasing $Sr$ and $Sc$.
- The impacts of pertinent parameters on the pumping rate are different for different pumping region.
- The volume and number of the trapped bolus decreases by increasing $M$ but it increases for increasing $k_0$, $F$ and $\phi$. Further by increasing $Bn$ the trapped bolus increases in number and size at the left wall of the channel but opposite behavior occur at the right wall.

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