Low-energy features of SU(2) Yang-Mills theory with light gluinos

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We report on the latest results of the low-lying spectrum of bound states in SU(2) Yang-Mills theory with light gluinos. The behavior of the disconnected contributions in the critical region is analyzed. A first investigation of a three-gluino state is also discussed.

1. Introduction

The numerical simulation we report on aims at a better understanding of the non-perturbative low-energy features of supersymmetric gauge theories. We concentrate on the simplest supersymmetric gauge theory, namely SU(2),\textit{N} = 1 super-Yang-Mills. This model contains, in addition to the gauge field a massless Majorana fermion in the adjoint representation (called gluino). For the theoretical motivation of this investigation see \cite{1}–\cite{3} and references therein.

2. Lattice formulation

We regularize the theory by the Wilson action as proposed in \cite{3}. Supersymmetry is broken, both by the lattice regularization and the introduction of a mass term for the gluino. The action contains two bare parameters: the gauge coupling \( \beta \) and the hopping parameter \( K \) (bare gluino mass). Supersymmetry is expected to be restored by tuning the bare parameters to their critical values \cite{3}. The path-integral for Majorana fermions is a Pfaffian

\[
\int [d\psi] e^{-\frac{1}{2} \psi_a (CQ)_{ab} \psi_b} = \text{Pf}(CQ),
\]

where \( Q \) is the Wilson fermion matrix in the adjoint representation (see for example \cite{3}), and \( C \) the charge conjugation matrix. The Pfaffian satisfies

\[
\text{Pf}(CQ)^2 = \det(CQ) = \det Q = \det(\tilde{Q}).
\]  

\( \tilde{Q} \) is the hermitean fermion matrix \( \tilde{Q} = \gamma_5 Q \) with doubly degenerate real eigenvalues, \( \det(Q) \geq 0 \). In practice we have simulated with weight \( \det(Q)^{\frac{1}{2}} \). This may lead to a sign problem. However, in \cite{3} it is found that sign flips are rare.

3. The low-lying spectrum

A basic assumption about the low-energy dynamics of super-Yang-Mills theory is confinement, as supported by the non-vanishing string tension \cite{3}. Therefore the low-lying spectrum consists of color singlets as in QCD. In the SUSY-limit of zero gluino mass the states should be organized in degenerate multiplets. In analogy to QCD we consider scalar and pseudoscalar mesons and glueballs. To complete the supermultiplet a spin \( \frac{1}{2} \) gluino-glue particle is also considered. In detail these particles and some of the corresponding operators are:

- Scalar meson (\( a\text{-f0} \)): \( \phi_s = \bar{\psi}\psi \),
- Pseudoscalar meson (\( a\text{-f'} \)): \( \phi_p = \bar{\psi}\gamma_5\psi \),

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Figure 1. The lightest bound state masses in lattice units as function of the bare gluino mass parameter $1/K$. The shaded area at $K = 0.1955(5)$ is where zero gluino mass and supersymmetry are expected.

- Gluino-glue state: $\chi_\alpha = \sum_{kl} Tr(\tau_i U_{kl}) \psi^r_\alpha$,
- $0^+$ glueball,
- $0^-$ glueball.

For the gluino-glue state and the glueball masses blocking and smearing was used. The results are displayed in fig.1. The presumable existence of a second multiplet requires yet another spin $\frac{1}{2}$ particle. The search for this state is an open issue.

4. A look at the $a-\eta'$ in the critical region

The correlator of the $a-\eta'$ consists of a disconnected and a connected part,

$$C(t) = -2C(t)_{\text{conn}} + C(t)_{\text{disconn}}.$$  

In QCD, $C(t)_{\text{conn}}$ gives rise to the $\pi$-mass and $C(t)$ to the $\eta'$-mass, so that

$$R(t) = C(t)/C(t)_{\text{disconn}}$$

is expected to decrease as we approach the chiral limit. In order to investigate whether this is also true in our case, we plot $R(t)$ in fig.2. For $K = 0.1925$ and $K = 0.196$ we observe that indeed $R(t)$ demonstrates a QCD-like behavior.

5. Investigation of a three-gluino state

Three-gluino states can also be constructed in analogy to QCD baryons. This holds also for SU(2) since the fermions are in the adjoint representation. In this case a possible choice for the wave function is

$$\phi^a(x) = \epsilon_{abc}(C\gamma_4)_{\beta\gamma} \psi(x)^a_\beta \psi(x)^c_\gamma.$$  

(3)

This wave function which is antisymmetric in color and symmetric in spin, carries spin $\frac{3}{2}$.

For SU(3) color additional possibilities are obtained by a symmetric color coupling

$$\phi'^a(x) = d_{abc}(C\gamma_5)_{\beta\gamma} \psi(x)^a_\beta \psi(x)^c_\gamma,$$

$$\phi'^{\text{tras}}(x) = d_{abc}(C)_{\beta\gamma} \psi(x)^a_\beta \psi(x)^c_\gamma.$$  

The propagator of such a state has basically two contributions displayed in fig.3. The correlation function $\langle \phi'^a \phi'^a \rangle$ for the wave function eq.3 has the following form:

$$C(x, y) = -\epsilon_{\alpha'\beta'\gamma'} \epsilon_{abc}(C\gamma_4)_{\beta'\gamma'}(C\gamma_4)_{\beta\gamma} *$$

$$\{ 2 \Delta x_{\alpha a} \Delta y_{\beta b} \Delta y_{\gamma c} + 4 \Delta x_{\alpha a} \Delta y_{\beta b} \Delta y_{\gamma c} \}$$

\footnote{We would like to thank A. González-Arroyo for a clarifying discussion on the spin content of these particles.}
three-gluino propagator

Figure 3. Contributions to the propagator of a three-gluino state. The second contribution arises, since contractions of the form $\psi(x)\psi(x)$ are allowed for Majorana fermions.

\begin{align*}
+2\Delta_{xab}^{xb} & \Delta_{xc}^{ya\prime\alpha} \Delta_{yc}^{yb\prime\beta} C_{\gamma\delta} C_{\delta\gamma} + 4\Delta_{x\alpha\gamma}^{y\gamma} \Delta_{y\beta\gamma}^{x\gamma} \\
+\Delta_{x\alpha\gamma}^{y\gamma} & \Delta_{x\beta\gamma}^{y\gamma} C_{\gamma\delta} C_{\delta\gamma} \\
+2\Delta_{x\alpha\gamma}^{y\gamma} & \Delta_{x\beta\gamma}^{y\gamma} C_{\gamma\delta} C_{\delta\gamma} 
\end{align*}

where $\Delta = Q^{-1}$ is the gluino propagator. The last four terms pertaining to the second “spectacles” graph can be evaluated by “gauge averaging” in analogy to the volume source method.

5.1. Evaluation of the spectacles graph

We now show how to evaluate the second graph of fig. 3. With $\Omega_x$ the gauge transformation in the fundamental representation, we see that the gauge transformation in the adjoint, defined as $G_{x,ab}(\Omega) = [G_{x,ab}^{-1}]^T = 2Tr[\tau_a\Omega^{-1}(x)\tau_b\Omega(x)]$, obeys

\begin{align*}
\int d\Omega G_{a_1b_1} & = 0, \\
\int d\Omega G_{a_1b_1} G_{a_2b_2} G_{a_3b_3} & = \frac{1}{6} \epsilon_{a_1a_2a_3} \epsilon_{b_1b_2b_3}. \tag{4}
\end{align*}

The propagator $\Delta$ transforms under a gauge transformation as

$\Delta_{x\alpha}^{y\gamma} \rightarrow G_{x,\alpha\gamma}^{-1} \Delta_{x\alpha}^{y\gamma} C_{\gamma\delta} C_{\delta\gamma}$. \tag{5}$

These are the necessary ingredients for an evaluation of the second graph. We have to calculate for example (spinor indices are left out for simplicity)

$\tilde{C}(x, y) \equiv \Delta_{yc}^{y\gamma} \Delta_{x\beta}^{xb} C_{\gamma\delta} C_{\delta\gamma} \epsilon_{a'b'c'}$.

First we compute the vector

$W_{zb',x} = \Delta_{zb'}^{x\gamma} \Delta_{za}^{xb} C_{\gamma\delta} C_{\delta\gamma}$,

for a fixed site $x$ and all sites $z$. Next we observe that, with the help of eqs. (4) and (5), we find the identity

$\langle \Delta_{yc}^{y\gamma} W_{zb',x} \rangle = \frac{1}{6} \delta_{zy} \epsilon_{a'b'c'} \epsilon_{abc} \langle \Delta_{yc}^{y\gamma} W_{yb,x} \rangle$.

Composing the “shifted” vector $W_{xb'}^{\text{shifted}}$, $W_{xb'}^{\text{shifted}} = W_{zb'-1,x} - W_{zb'+1,x}$

(with $W_{x4,y} = W_{x1,y}$, $W_{x0,y} = W_{x3,y}$) it can be shown that

$\sum_{y',c'} < \Delta_{yc}^{y\gamma} W_{zb',x}^{\text{shifted}} >= < \tilde{C}(x, y)$.

To evaluate the l.h.s. of this relation numerically only one additional inversion is needed with $W_{xb'}^{\text{shifted}}$ as the source. In this way $\langle \tilde{C}(x, y) \rangle$ is obtained from a given $x$ to all $y$ by two inversions of the fermion matrix $Q$. An analysis of the mass of the particle characterized by eq. (8) is currently under way.

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