It is argued that distribution over the number of charged and neutral soft chiral pions are very broad if they are emitted coherently.

1 Introduction

Charge distribution of pions in multiple production processes drew much attention recently. The growth of interest in this subject is due to expectations to detect the disoriented chiral condensate (DCC) formation in high energy collisions. The simplest picture of the process is given by "Baked Alaska scenario", where coherent pulses of semiclassical pion field are emitted leading to anomalously large fluctuations in the ratio of neutral to charge pions produced. In particular, the probability to produce $n_0$ neutral pions (for large total number of pions $n$) is given by inverse square root formula,

$$w(n_0) \sim 1/\sqrt{n_0n},$$

being very flat and so quite different from usual binomial-like distributions. This mechanism may be relevant for description of "Centauro" (and "Anti-Centauro") type events found in cosmic ray experiments, see and references therein, in which the number of charged particles drastically exceeds the number of neutral ones (or vice versa).

2 Coherent pion production with isospin conservation

Now the problem arises – to what extent the behaviour in Eq. (1) can be considered as a signature of DCC formation. Let us remind in this connection that the distribution of the form of Eq. (1) was found long ago in a model of independent pion production when isotopic spin conservation was taken into account. The role of isotopic spin effects was analyzed within the framework of the model for production of the coherent state of pions. If the different kinds of pions were not coupled to each other, then we would have the following expression for the final state:

$$|f> = e^{-c/2} \exp[\int d^3k \sum_{i=1}^{3} f_i(k) a_i^\dagger(k)] |0>,$$

$$c = \sum_i c_i = \sum_i \int d^3k |f_i(k)|^2$$

(2)
where $a_i^\dagger$ are the creation operators for $i$-type pions, $|0>\text{ is the pion vacuum and}\ |f(k)|^2\text{ determines the momentum distribution of pions }|f_i(k)|\text{ is the effective density of the pion source}\text{. The probability for production of }\pi^\pm\text{ and }\pi^0\text{ mesons in this case would have the same form as in the model of uncorrelated particle production and a Poisson distribution with respect to the number of pions of each type would exist.}\text{ Let us now take into account that }f_i\text{ and }a_i\text{ are components of vectors in the isotopic space}\text{. The state }|f>\text{ in Eq. 2 does not have a specific isospin }I\text{ (or electric charge), but contains states with large values of }I\text{ which increase with increasing number of particles}\text{. This is not consistent with the conservation of total isospin in the collision process}\text{. For example, the isospin of a pion system cannot be higher than 2 in the collision of two nucleons with production of pions}\text{. The state with isospin }I = 0\text{ appears as a result of averaging }|f>\text{ over directions of the }f\text{ vector in the isospace}\text{,}

$$|f; I = 0> = e^{-c/2} \int d\Omega \exp\left(\int d^3k f(k) a_i^\dagger(k)\right)|0>$$

(3)

This state may be considered as a result of generating of isoscalar pairs of pions:

$$|f; I = 0> = e^{-c/2} \frac{1}{\sqrt{4\pi}} \sum_{m=0}^{\infty} \frac{1}{(2m + 1)!} \left(\sum_{j=1}^{3} \int d^3k f(k) a_j^\dagger(k)\right)^m|0>$$

(4)

It is the proper state for annihilation operator of isoscalar pair and it may be considered as a coherent state of isoscalar pairs.

The important point is that the distributions with respect to the number of neutral pions $w_0(n_0)$ and charged pions $w_{ch}(n_{ch})$ for $I = 0$ are much broader than those obtained by ignoring the constraints imposed on the isospin of the pion system\text{, if the total average number of pions is large, }<n>\gg 1\text{. In the main region of variation of }n_0\text{ we obtain}

$$w_0(n_0, I = 0) \approx \frac{1}{\sqrt{2c}} \frac{\Gamma(n_0/2)}{\Gamma(n_0/2 + 1)}$$

(5)

i.e., for example, the probability for occurrence of events without $\pi^0$ is high enough, amounting here to more than 10 per cent for $<n>\approx c = 100$. The probability $w_0(n_0, I = 0)$ decreases slowly, in accordance with Eq. 5 up to $n_0 \sim c$ with increasing $n_0$\text{, where }w_0(c, I = 0) = 1/2c\text{ and then decreases to zero in the region }|n_0 - c| \sim \sqrt{c}\text{. Such behaviour of }w_0\text{ can explain an essential fraction of Centauro-type events in which the }\pi^0\text{ mesons are missing at }<n>\gg 1\text{. The probability for the occurrence of such events would be}
\[ e^{-c/3} \sim e^{-33} < 10^{-14} \] for the original Poisson distribution, i.e., they would not be observed.

Analogously we can see that for \( c - n_{ch} \gg \sqrt{c} \) the distribution of charged pions is given by

\[ w_{ch}(n_{ch}, I = 0) \approx 1/\sqrt{c(n_{ch})}, \quad <n_{ch}> \approx 2c/3, \quad (6) \]

we again obtain a slowly varying function of \( n_{ch} \). The probability \( w_{ch}(n_{ch}, I = 0) \) increases slowly with increasing \( n_{ch} \), reaching a maximum at \( n_{ch} = c \)

\[ w_{ch}(c, I = 1) = \frac{\Gamma(1/4)}{\sqrt{2\pi(2c)^{3/4}}} \quad (7) \]

and then decreases fastly in the interval \(|n_{ch} - c| \sim \sqrt{c}\). The events without charged pions also comprise a sizable fraction, \( w_{ch}(0, I = 0) \approx 1/c = 0.01 \).

The same qualitative results were obtained when the states with \( I = 1 \) were selected. More recently similar results were obtained for squeezed states.

### 3 Soft chiral pion bremsstrahlung

Below we consider soft chiral pion bremsstrahlung accompanying some basic high-energy process and estimate charge distribution of the pions. The quantum charge states of chiral pions emitted from simple vertices will be explicitly calculated. It will be found that neutral pion number distribution again has the form of Eq. (1). That is, such flat charge distributions are typical for soft chiral pions and do not indicate directly on DCC formation.

The soft chiral pion bremsstrahlung was studied many years ago. Similar to photons, soft pions are emitted from external lines of diagrams representing the basic process (to be definite, we shall take external particles to be spin 1/2 fermions (nucleons)). The complications arising due to non-commutative pion-nucleon vertices and nonlinear pion-nucleon coupling were shown to be mutually cancelled. Nonlinear pion-pion coupling can be taken into account but its effect vanishes in the limit of small pion momenta. Soft virtual pion exchange changes normalization and does not influence the pion number distributions. The net result for soft pion emission is given by substitution:

\[ \psi_j \rightarrow \exp(-i\gamma_5\tau_i\phi_i/2)\psi_j, \quad (8) \]

where \( \psi_j \) is the fermion field for every incoming or outgoing nucleon in the skeleton diagram of a basic process, \( \phi_i = \pi_i/f_\pi, \pi_i \) being the pion field, \( f_\pi = 93 \) MeV is the pion decay constant.
As the simplest example consider the scalar vertex $\bar{\psi}\psi$ ($\Gamma$ is the identity matrix). Its chiral-invariant extension has the form

$$V_s = \bar{\psi}\exp(-i\gamma_5\tau_i\phi_i)\psi(x).$$

(9)

It coincides formally with the modified nucleon mass term in the chiral lagrangian. We neglect pion momenta and for the fields $\phi_i$ use the decomposition

$$\phi_i \rightarrow \phi_i(0) = \phi_i^+ + \phi_i^- = \int d^3k f(k) [a_i^+(k) + a_i^-(k)],$$

(10)

where creation and annihilation operators $a_i^+(k), a_i^-(k)$ obey canonical commutation relations. In the free field approximation

$$f(k) = (2\pi)^{-3/2}(2k_0)^{-1/2}f_\pi^{-1}, \quad k_0 \leq k_m$$

(11)

where $k_m$ is an upper limit of pion softness. Then we must calculate the matrix elements of $\pi^0, \pi^+, \pi^-$ production

$$M_s = \langle n^+, n^-, n_0 | \exp(-i\gamma_5\tau_i\phi_i(0)) | 0 \rangle$$

(12)

The total isotopic spin of the pions produced can be zero or one. Consider the first case. If the average number of pions is large (it is the most interesting case) then

$$\langle n \rangle \approx c, \quad c = \int d^3k |f(k)|^2 \gg 1.$$ 

(13)

To estimate it take the free field approximation (11); then

$$\langle n \rangle = \frac{1}{f^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k_0} \approx \frac{k_m^2}{8\pi^2f^2}$$

(14)

A prominent feature of the model is the distribution over number of charged and neutral pions produced. It can be obtained from matrix elements (12) and has the multiplicative form with respect to the total number of pions $n = n_0 + n_{ch}$ and the number of neutral pions $n_0$,

$$w(n, n_0) = w(n)w_n(n_0)$$

(15)

where

$$w_n(n_0) = \frac{1}{n + 1} \frac{(n/2)! \cdot \Gamma(n_0 + 1/2)}{\Gamma(n + 1)! (n_0/2)!} \approx \frac{1}{\sqrt{n_0}}$$

(16)
is the probability to produce \( n_0 \) neutral pions for the given total number of pions (\( \Gamma(n) \) is the Euler \( \Gamma \)-function, \( n_0 \) and \( n \) are even, \( n_0 \leq n \)), and

\[
w(n) = \frac{(n - c + 1)^2}{N_s} \frac{e^{-c c_n}}{(n + 1)!}
\]

(17)

is the distribution over the total number of pions produced.

The distribution \( w_2(n) \) over the number of neutral pions \( n_0 \) and corresponding distribution over the number of charged pions are very broad. These distributions appear to be very similar to those given by Eqs. (14). The distribution \( w_2(n) \) over the total number of pions is Poisson-like (though with an additional central dip at \( n \sim \langle n \rangle \)) and it is much more narrow than (16).

As a case of immediate physical interest consider now the soft pion emission for electromagnetic scattering of strongly interaction fermions. The skeleton vertex has now the form

\[
V_0 = e \bar{\psi} \gamma_\mu Q \psi = e \bar{\psi} \gamma_\mu \frac{\tau_3 + N_B}{2} \psi
\]

(18)

where the baryon number \( N_B = 1 \) for nucleons and \( Q \) is electric charge in units of \( e \). Chiral extension of the vertex is taken as

\[
V = e \bar{\psi} \exp(-i \gamma_5 \tau_k \phi_k/2) \gamma_\mu \frac{\tau_3 + N_B}{2} \exp(-i \gamma_5 \tau_k \phi_k/2) \psi
\]

(19)

We consider diagonal transitions with high average multiplicity, \( c \gg 1 \). Then the average number of pions is

\[
\langle n \rangle = \frac{1}{3} c, \quad c \gg 1
\]

(20)

and the distribution over the number of neutral and charged pions is given by

\[
w_2(n, n_0) \equiv \frac{3(c - n_0)^2}{2N_c e^2 \sqrt{2c}} \frac{\Gamma(n_0 + 1)}{n_0!} w_2(n) \]

(21)

\[
w_2(n) \equiv \frac{2}{3e \sqrt{2\pi c}} (n - c)^2 \exp \left( -\frac{(n - c)^2}{2c} \right)
\]

(22)

where \( w_2(n) \) is the probability to find \( n = n_0 + n_c \) pions, \( n \) and \( n_0 \leq n \) are even.

The distribution over the number of neutral (or charged) pions in Eq. (21) is again very broad ensuring a sizable number of events, in which almost all pions are neutral (or charged). The distribution \( w_2(n) \) over the total number
of pions is again narrow and in fact coincides with Eq. (17) for \( c \gg 1, n \gg 1 \)
up to a normalization factor. The total probability of high multiplicity events
for electromagnetic vertex is

\[
\sum_{n=2k} w_2(n) = \frac{1}{3}
\]

just corresponding to factor 1/3 in Eq. (20).

4 Discussion

Examples of coherent soft pion emission considered above show very broad
distributions over the number of neutral and charged pions in high multiplicity
events. The neutral pion distributions in all cases are given essentially by the
function of Eq. \( \ref{eq:5} \) leading to inverse square root behaviour of Eq. \( \ref{eq:6} \) characteristic
for DCC formation. So we conclude that one has to look for more delicate
criteria of DCC formation. Here the momentum distribution of pions produced
may appear helpful.

The bremsstrahlung spectrum of pseudoscalar pions has the form \( dn \sim kdk \) (contrary to photon spectrum \( dk/k \)) and so very small momenta \( k \) are
inefficient for this mechanism. It was necessary (as in Bloch-Nordsiek model)
to introduce an upper limit of pion softness, \( k < k_m \) and the total number
of pions produced by this mechanism is proportional to \( k_m^2 \). The value of \( k_m \)
is not quite definite (the most severe possible estimation is around rho-meson
mass) but it does not exceed the momentum transfer \( \Delta p \) in the baryonic vertex \( \Gamma \). Anyhow it is clear that the presence of large baryonic momentum transfer
\( \Delta p \) (and so the presence of high \( p_T \) baryons) is highly favourable for copious
production of pions by the bremsstrahlung mechanism. At the same time the
soft pions are expected to be present in lower \( p_T \) region. In the last region the
pion spectrum of the form \( kdk \) by itself can be used for identification of the
process of pion bremsstrahlung. It can be seen in future experiments when it
will be possible to look at narrow windows of \( p_T \).

The conditions for such broad distributions to appear in high multiplicity
events are small isotopic spins of the pion system and many particle matrix
elements symmetric with respect to pion momenta, thus ensuring a constructive
interference. This can be seen already from an early paper by A. Pais \( \ref{ref:14} \) and
was explicitly demonstrated more recently in paper \( \ref{ref:15} \). Both of these conditions
are fulfilled in our model examples.

In conclusion, it thus appears that inverse square root distributions over
number of neutral and charged pions are of very general nature being character-
istic for coherent soft pion radiation.
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