Neutrino CP phases and lepton electric dipole moments in supersymmetric theories

G.C. Branco\textsuperscript{a1}, D. Delépine\textsuperscript{a2} and S. Khalil \textsuperscript{b,c 3}

\textsuperscript{a} Centro de Física das Interacções Fundamentais, Departamento de Física, Instituto Superior Técnico, P-1049-001 Lisboa, Portugal
\textsuperscript{b} IPPP, University of Durham, South Rd., Durham DH1 3LE, U.K.
\textsuperscript{c} Ain Shams University, Faculty of Science, Cairo, 11566, Egypt.

Abstract

We analyse the dependence of the electron electric dipole moment (EDM) on neutrino CP violating phases in the context of supersymmetric models. We start by studying the supersymmetric contributions to the lepton EDM and lepton flavour violation processes $\tau \to \mu\gamma$ and $\mu \to e\gamma$, in the framework of the mass insertion approximation, showing that, due to the large neutrino mixing, $\mu \to e\gamma$ leads to severe constraints on the relevant mass insertions. We derive model independent bounds on these mass insertions and show that once these bounds are satisfied, the present experimental limits on electron EDM do not constraint the neutrino phases.
1 Introduction

The Standard Model (SM) of electroweak and strong interactions has had an impressive success when confronted with experiment. So far, we have only two pieces of evidence favouring the presence of physics beyond the SM, namely, the experimental evidence for neutrino oscillations [1–5] pointing towards non-vanishing neutrino masses and mixings and the observed size of the baryon asymmetry of the Universe (BAU). It has been established that the strength of the CP violation in the Standard Model is not sufficient to generate the cosmological baryon asymmetry of our universe, thus requiring the presence of new sources of CP violation [6]. The most attractive mechanisms to generate the observed BAU are leptogenesis and electroweak baryogenesis in supersymmetric extensions of the SM. It was shown that the supersymmetric extensions of the SM have all the necessary requirements to generate enough BAU. In particular, the SUSY models have new sources for CP violation and with a light stop, the phase transition becomes much stronger [7]. However, the bound on the neutron EDM imposes severe constraints on the flavour diagonal phases and they could rule out the electroweak baryogenesis scenarios [8]. A possible way to overcome this problem and to generate enough BAU, without overproducing the EDMs, is to assume that SUSY CP violation has a flavour character as in the SM [9]. In the leptogenesis scenario, baryon asymmetry is generated by a lepton asymmetry arising from the out of equilibrium decay of heavy Majorana neutrinos [10], which is then converted into a baryon asymmetry through sphaleron interactions. The leptogenesis mechanism is specially attractive due to its simplicity and to the recent experimental results confirming neutrino oscillations and hence non-vanishing neutrino masses. The relation between leptogenesis and CP violation observable at low energy (in lepton EDMs or in neutrino oscillation asymmetries) remains an open and interesting question [11].

Until now, no CP violation effects have been observed in the leptonic sector. However, a new method for measuring lepton EDMs [12] and the prospects of ν-factories provide the hope of having a drastic improvement in our knowledge of CP violation in the leptonic sector, within a few years.

In this paper, we study the impact of CP violating phases in the neutrino sector, on CP violating low energy observables, in general SUSY models. In particular, we focus on their effect on the EDM of electron and muon. In order to perform a model independent analysis, we use the mass insertion approximation method which allows to parametrize the main source of the CP and flavour violation in SUSY model. In this framework, the neutralino and chargino exchanges give the dominant contributions to the lepton flavour violation (LFV) processes and to the EDMs. We derive model independent bounds on these mass insertions and we discuss possible constraints on the neutrino CP phases from these bounds, in the case of non-universal SUSY soft-breaking terms. The bounds on the leptonic mass insertions provide useful tests on SUSY models and are complementary to those obtained in the quark sector [13, 14]. To illustrate these constraints, we shall focus our interest on models where all the CP violation is induced by the CP violating phases of the leptonic mixing matrix ($U_{MNS}$), appearing in $W$-mediated charged current interactions.
In Ref.[15], the lepton EDMs have been studied in a minimal supersymmetric seesaw model with universality of soft SUSY breaking terms. It was shown that, through the renormalization group equations (RGE), \( CP \) violating phases are induced in the off-diagonal elements of slepton mass matrix \( m_L^2 \) and the trilinear coupling \( A_e \). Some of these \( CP \) phases are related to the \( CP \) phases of the lepton mixing matrix through the RGE. Also, it was emphasized that in the case of non-degenerate heavy Majorana neutrinos the EDM of the muon and the electron could be enhanced.

The paper is organized as follows. In section II, we shall introduce our notation and convention. The dependence of the soft SUSY breaking terms in the \( CP \) violating phases of the leptonic mixing matrix is extensively discussed. In section III, the analytical expressions for the lepton EDMs and for lepton flavour violating processes are given in terms of the mass insertions approach. In section IV, the bounds on the mass insertions coming from \( Br(\mu \rightarrow e\gamma) \) and \( d_e \) are given and their dependence on SUSY parameters is discussed. The recent determination of the elements of the lepton mixing matrix is also used to get a strong limit on the chargino contribution to lepton flavour violating processes such as \( Br(\tau \rightarrow \mu, e\gamma) \). In the section V, we shall discuss the dependence of the lepton EDM on the \( CP \) violating phases appearing in the leptonic mixing matrix. Particular emphasis is given to the Majorana phases dependence. We show that if there is no SUSY \( CP \) violating phases, whatever is the structure of the soft SUSY breaking terms, the lepton EDMs only depend on the Dirac \( CP \) violating phase of \( U_{MNS} \). As an illustrative example, we studied the case of Hermitian Yukawa couplings. In section VI, we summarize our main results and present our conclusions.

2 Supersymmetric model with right–handed neutrinos

The seesaw mechanism[16] provides a natural explanation for the smallness of neutrino masses which are of order \( v^2/M_R \) where \( v \) stands for the scale of electroweak symmetry breaking and \( M_R \) denotes the right-handed neutrino mass. Since the right-handed Majorana neutrino mass term is \( SU(2)_L \otimes U(1)_Y \otimes SU(3)_{QCD} \) invariant, \( M_R \) can have a value much larger than \( v \). Supersymmetry can play an important rôle in ensuring the stability of the hierarchy between the weak scale and right–handed neutrino scale. We consider the supersymmetric standard model with right–handed neutrinos, which is described by the superpotential

\[
W = -\mu H_1 H_2 + Y_{eij} E^c_i L_j H_1 + Y_{\nu ij} N^c_i L_j H_2 + \frac{1}{2} Y_{\nu rij} N^c_i N^c_j R 
\]

where \( i,j = 1 \ldots 3 \) are generation indices and the superfields \( E^c, L = (N, E), N^c \) contain the leptons \( e^c_R, (\nu_L, e_L), \nu^c_R \), respectively. The expectation values of the Higgs multiplets \( H_1 \) and \( H_2 \) generate ordinary Dirac mass terms for quarks and leptons, and the expectation value of the singlet Higgs field \( R \) yields the Majorana mass matrix of the right-handed neutrinos, \( M_{Rij} = Y_{\nu rij}\langle R \rangle \). In general, the Majorana neutrino mass matrix is a complex
symmetric matrix. At low energy and after the decoupling of the heavy neutrinos, the effective superpotential is given by

$$W_{\text{eff}} = (Y_{\nu})_{\text{eff}} L_i L_j H_2^2 + Y_{eij} E^c_i L_j H_1,$$

(2)

where \((Y_{\nu})_{\text{eff}} = -Y^{\dagger}_{\nu} M^{-1}_{\nu} Y_{\nu}\) and the light neutrino masses are given by \(M_\nu = (Y_{\nu})_{\text{eff}} \langle H_0 \rangle^2\). Since \(M_\nu\) is a symmetric matrix, it can be diagonalized by a unitary transformation. In addition, the relevant soft SUSY breaking terms are in general given by

$$L_{\text{soft}} = -\tilde{m}_{ij} L_i L_j - \tilde{m}_{eij} E^c_i E_j + Y^{A}_{eij} E^c_i E^c_j H_1 + Y^{A}_{\nu ij} N^c_i N^c_j H_2 + \text{c.c.} + \ldots,$$

(3)

where \(L = (N_L, E_L)\) and \(E^c \equiv E^*_R\) refer to the scalar partners of \((\nu_L, e_L)\) and \(e^*_R\) respectively and \(Y^{A}_{e,\nu ij} \equiv A_{e,\nu ij} Y_{e,\nu ij}\). Using the seesaw mechanism to explain the smallness of neutrino masses, we assume that the right-handed neutrino masses \(M_R\) are much larger than the Fermi scale \(v\). One then easily verifies that all mixing effects on light scalar masses caused by the right-handed neutrinos and their scalar partners are suppressed by \(O(v/M_R)\), and therefore negligible.

Now we present the general expressions for the slepton mass matrices in the super-MNS basis which, in analogue to the super–CKM basis in the quark sector, is defined as follows. Given the Yukawa matrices, we perform unitary transformations of the lepton superfields \(N_L\) and \(E_{L,R}\) such that the lepton mass matrices take diagonal forms:

$$N_L \to V^\dagger_{\nu L} N_L,$$

$$E_{L,R} \to V^\dagger_{e L,R} E_{L,R},$$

(4)

with \(Y^{\nu}_{\text{eff}} \to (V^\dagger_{\nu L})^T Y^{\nu}_{\text{eff}} V^\dagger_{\nu L} = \text{diag}(h_e, h_\mu, h_\tau)\) and \(Y^e \to (V^e_R)^\dagger Y^e V^e_R = \text{diag}(h_e, h_\mu, h_\tau)\). In this basis, the lepton charged current interactions is given by

$$-\frac{g}{\sqrt{2}} \left( L_{\mu} \gamma^\mu (V^\dagger_{\nu L})_{ij} \nu_j W_\mu + \text{h.c.} \right)$$

(5)

with \(g\), the weak \(SU(2)_L\) gauge coupling. The lepton flavour mixing matrix is then given by

$$U_{\text{MNS}} = V^\dagger_{\nu L} V^\nu_{\nu L}.$$  

(6)

Both \(V^\nu_{\nu L}\) and \(V^\dagger_{\nu L}\) are unitary matrices which can be parametrised as

$$V^\nu_{\nu L} \equiv P_{e,\nu} V^{e,\nu}_\delta Q_{e,\nu}$$

(7)

where \(P_{e,\nu} \equiv \text{diag}(e^{i\alpha_{e,\nu}}, e^{i\alpha_{e,\nu}}, e^{i\alpha_{e,\nu}})\), \(Q_{e,\nu} \equiv \text{diag}(1, e^{i\beta_{e,\nu}}, e^{i\beta_{e,\nu}})\) and \(V^{e,\nu}_\delta\) are unitary matrices which only contain one \(CP\) violating phase. So, \(U_{\text{MNS}}\) can be rewritten as

$$U_{\text{MNS}} = Q^\dagger_{e} P^\dagger_{\nu} V^{e,\nu}_{\delta} Q_{\nu}$$

(8)

$$= Q^\dagger_{e} (P^\dagger U_{\delta} Q) Q_{\nu}$$

(9)

$$\equiv P^\dagger_{L} U_{\delta} P_M$$

(10)
where \( U_\delta \) is a unitary transformation parametrised by three angles and one \( CP \) violating phase, similar to \( V_{CKM} \) quark mixing matrix. \( P \) and \( Q \) are diagonal phase matrices similar to \( P_{e,\nu} \) and \( Q_{e,\nu} \), respectively. As one can see from eq.(10), in general \( U_{\text{MNS}} \) contains six \( CP \) violating phases but as it is the case in the SM, it is always possible to redefine the \( E_L \) superfields by a diagonal unitary transformation such that three phases contained in \( Q^\dagger P \equiv P_L^\dagger \) are removed and an equivalent transformation on \( E_R \) superfields in order to keep the charged lepton masses real. So, only three phases of \( U_{\text{MNS}} \) are physical: one coming from the unitary matrix \( U_\delta \) which contains one \( CP \) violating phase (this phase is usually called the Dirac phase), and a diagonal unitary matrix \( P_M \equiv (1, e^{i\alpha}, e^{i\beta}) \equiv QQ_\nu \) which contains the two Majorana phases. In this basis, \( U_{\text{MNS}} \) is given by

\[
U_{\text{MNS}} = U_\delta P_M
\]

with \( P_M \equiv \text{diag}(e^{i\phi_j^M})_{j=1,3} \) with \( \phi_1^M = 0 \) and \( \phi_{2,3}^M \) are the usual Majorana phases.

In this super–MNS basis, the low-energy sneutrino and charged slepton mass matrices are given by

\[
M_\tilde{\nu}^2 = \begin{pmatrix}
(M_\tilde{\nu}^2)_{LL} & (M_\tilde{\nu}^2)_{LR} \\
(M_\tilde{\nu}^2)_{RL} & (M_\tilde{\nu}^2)_{RR}
\end{pmatrix},
\]

where

\[
(M_\tilde{\nu}^2)_{LL} = P_L V_L^\dagger \hat{m}_l^2 V_L^\dagger + m_l^2 + \frac{m_Z^2}{2}(1 - 2 \sin^2 \theta_W) \cos 2\beta,
\]

\[
(M_\tilde{\nu}^2)_{RR} = P_L V_R^\dagger \hat{m}_l^2 V_R^\dagger + m_l^2 - \frac{m_Z^2}{\sin^2 \theta_W} \cos 2\beta,
\]

\[
(M_\tilde{\nu}^2)_{RL} = (M_\tilde{\nu}^2)_{LR} = -\mu m_l \tan \beta + v \cos \beta P_L (V_L^\dagger Y_\nu^e V_L^\dagger P_L^\dagger),
\]

and

\[
(M_\tilde{\nu}^2)_{LL} = V_L^\dagger \hat{m}_l^2 V_L^\dagger + \frac{m_Z^2}{2} \cos 2\beta + v \sin \beta V_L^\dagger (Y_{\nu}^{\nu*} Y_{\nu}^\nu) V_L^\dagger,
\]

where \( m_l \) is the diagonal charged lepton mass matrices, \( m_Z \) is the mass of the \( Z \) gauge boson, \( \theta_W \) is the usual Weinberg angle of the weak interactions and \( \tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle \).

In the last equation, we have kept the contribution to \( M_{\tilde{\nu}}^2 \) proportional to the Dirac mass of the neutrinos because in general the unitary transformation which diagonalised \( Y_{\nu}^{\nu*} Y_{\nu}^\nu \) doesn’t necessarily diagonalise \( Y_{\nu}^{\nu*} Y_{\nu}^\nu \). Recall that, due to very heavy Majorana masses, the \( RR \) and \( LR \) contributions to the sneutrino mass matrix are suppressed. As it can be seen from eqs.(12-13), for non-universal soft breaking terms, the dependence on the \( CP \) violating phases coming from \( Y_{\nu}^{\nu*} \) and \( Y_e^\nu \) is very involved and in general, their contributions can not be distinguished from the \( CP \) violating phases arising from the soft breaking terms. Of course, we could have started to work from the beginning in the mass eigenstates basis for the leptons, where the \( U_{\text{MNS}} \) matrix contains only three phases. But usually, in models with non-universal SUSY soft breaking terms and flavour symmetry, the textures of the soft-breaking terms (\( \hat{m}_l^2, \hat{m}_e^2 \) and \( Y_e^A \)) are given in a weak basis where in general \( Y_e^\nu \) and \( Y_{\nu}^{\nu*} \) are not diagonal (see for instance, refs.[17]).

In this paper, we shall be interested in studying the effects of \( CP \)-violating phases arising from the lepton mass matrices (\( \approx Y_e^\nu \) and \( Y_{\nu}^{\nu*} \)) on charged leptons EDM’s taking into account the bounds on flavour changing lepton decays.
Since charged leptons are in and out states in $l_i^- \rightarrow l_j^+ \gamma$ or in lepton EDM’s, the computation of the contribution to these processes involving the supermultiplets $E_{L,R}$ has to be performed in the charged lepton mass eigenstates basis. But for supermultiplets $N_L$, as they only contribute to charged lepton decays and EDM’s through loop contribution, the computation of their contribution to these processes can be done in any weak basis for the supermultiplets $N_L$. As a corollary, it means that these processes are independent of $V_L^{\nu}$. Thus, we can directly conclude that if the textures for $Y^e$ and $Y^\nu_{eff}$ are such that $U_{MNS}$ is dominated by $V_L^{\nu}$, there is no effect of the neutrino $CP$ phases on these different processes for any kind of textures for the SUSY soft breaking terms. But the situation is completely different if the textures for the Yukawa couplings give $V_L^{\nu} \simeq V_L^\nu$ or if $V_L^{\nu} \simeq 1$. Indeed, in such a case, as one can see from eqs.(12-13), the neutrino $CP$ phases dependence of the SUSY soft breaking terms can be explicitly studied. In the following part of the paper, we shall assume that $V_L^{\nu} \simeq 1$. This means that we shall assume that the origin of the large lepton mixing angles comes from the charged lepton Yukawa couplings.

In the “super-MNS” basis where all lepton masses are real and $U_{MNS}$ only contains three phases, one has for $V_L^{\nu} \simeq 1$,

$$U_{MNS} \simeq P_L V_L^{\nu^\dagger}$$

(14)

with $P_L \simeq Q_e$.

Moreover in the “super-MNS” basis, the couplings of lepton and slepton states to the neutralinos are flavour diagonal and all the source of flavour mixing are inside the off-diagonal terms of the slepton mass matrix. These terms are denoted by $(\Delta^I_{AB})^{ij}$, where $A,B = (L,R)$ for $l \equiv e$ and $A,B = L$ for $l \equiv \nu$ and $i,j = 1,2,3$ denote the flavour indices. The slepton propagator is expanded as a series of the dimensionless quantity $(\delta^I_{AB})^{ij} = (\Delta^I_{AB})^{ij}/\bar{m}^2$, where $\bar{m}^2$ is an average slepton mass.

$$\langle \tilde{\nu}_{A,B} \tilde{\nu} \rangle = i \left(k^2 \mathbf{1} - \bar{m}^2 \mathbf{1} - \Delta^I_{AB} \right)^{-1}_{ij} \simeq \frac{i \delta^{ij}}{k^2 - \bar{m}^2} + \frac{i (\Delta^I_{AB})^{ij}}{(k^2 - \bar{m}^2)^2} + O(\Delta^2),$$

(15)

where $l = \nu,e$ denote the neutrino and charged lepton sectors respectively. As mentioned, $A,B$ stand for $L$ with neutrino sector and $L,R$ for the charged sector, $i,j = 1,2,3$ are the flavour indices, $\mathbf{1}$ is the unit matrix, and $\bar{m}$ is the average slepton mass. This method, known as mass insertion approximation, allows to parametrize, in a model independent way, the flavour violation in supersymmetric theories. It is also worth mentioning that since $\bar{m}^2$ is Hermitian, the mass matrices $(M^2_{\tilde{\nu},\tilde{\nu}})_{LL}$ and $(M^2_{\tilde{\nu},\tilde{\nu}})_{RR}$ in eqs.(12, 13) are also Hermitian in the “super-MNS” basis. Therefore, the LL mass insertions $(\delta^I_{LL}) = 1/\bar{m}^2(M^2_{\tilde{\nu},\tilde{\nu}})_{LL}$ and $(\delta^I_{RR}) = 1/\bar{m}^2(M^2_{\tilde{\nu},\tilde{\nu}})_{RR}$ are also Hermitian.

In the “super-MNS” basis and assuming the decoupling of the right-handed neutrino scalars, the Lagrangian describing the interaction between the charginos and the leptons and their partners needed to compute the chargino contributions to the lepton EDM and LFV is given by

$$\mathcal{L}_{e\tilde{\nu}\chi^\pm} = \sum_k \sum_{a,b} \left( - g V_{k1}(U_{MNS})_{ab} \tilde{e}^a_L (\chi^+_k)^* \tilde{\nu}^b_L + U_{k2} [Y^\nu_{\text{diag}} U_{MNS}]_{ab} \tilde{e}^a_R (\chi^+_k)^* \tilde{\nu}^b_L \right),$$

(16)
where the indices \(a, b\) and \(k\) label flavour and chargino mass eigenstates respectively and \(V, U\) are the chargino mixing matrices defined by

\[
U^* M_{\chi^+} V^{-1} = \text{diag}(m_{\chi^+_1}, m_{\chi^+_2}),
\]

and

\[
M_{\chi^+} = \begin{pmatrix}
  M_2 & \sqrt{2} M_W \sin \beta \\
  \sqrt{2} M_W \cos \beta & -\mu
\end{pmatrix}.
\]

The relevant Lagrangian for the neutralino contributions is given by

\[
L_{\bar{e}\tilde{e}\chi^0} = \sum_k \sum_a \left( g \frac{(N_{k2} + \tan \theta_W N_{k1})}{\sqrt{2}} \bar{e}_L^{a} (\tilde{\chi}^0_k)^* \tilde{e}_R^{a} - N_{k3} (Y^\text{diag}_e)_{aa} \bar{e}_L^{a} (\tilde{\chi}^0_k)^* \tilde{e}_L^{a}
\right.
\]

\[
- g \sqrt{2} \tan \theta_W N_{k1} \bar{e}_L^{a} (\tilde{\chi}^0_k)^* \tilde{e}_R^{a} - N_{k3}^* (Y^\text{diag}_e)_{aa} \bar{e}_R^{a} (\tilde{\chi}^0_k)^* \tilde{e}_L^{a} \right).
\]

where the matrix \(N\) is defined as the 4 × 4 rotation matrix which diagonalized the neutralino mass matrix \(M_N\),

\[
N^* M_N N^{-1} = \text{diag}(m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}).
\]

\(M_N\) is given by

\[
M_N = \begin{pmatrix}
  M_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\
  0 & M_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\
  -m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & \mu \\
  m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & \mu & 0
\end{pmatrix},
\]

with \(M_{1,2}\) are respectively the \(U(1)_Y\) and \(SU(2)_L\) gaugino soft masses.

### 3 Supersymmetric contributions to EDM and LFV

#### 3.1 Electric dipole moment of charged leptons

The effective Hamiltonian for the EDM of the charged leptons \(l\) can be written as

\[
H_{\text{eff}}^{\text{EDM}} = C_1 \mathcal{O}_1 + \text{h.c.},
\]

where \(C_1\) and \(\mathcal{O}_1\) are the Wilson coefficient and the electric dipole moment operator respectively. The operator \(\mathcal{O}_1\) is given by

\[
\mathcal{O}_1 = -\frac{i}{2} \bar{l} \sigma_{\mu\nu} \gamma_5 l F^\mu\nu.
\]

The supersymmetric contributions to the Wilson coefficient of the charged lepton result from the one loop penguin diagrams with neutralino and chargino exchange (figs.(1.a),(1.b)).
Figure 1: The neutralino (fig.1.a) and chargino contributions (fig.1.b) to the charged lepton EDM in “Super-MNS” basis. For neutralino (chargino) diagram, the photon line has respectively to be attached to the scalar (fermion) line of the loop. The cross represents the mass insertions.

The lepton EDM is given by
\[ d_l/e = \text{Im} \left[ C_{\chi^0_l} + C_{\tilde{\chi}^0_l} \right], \quad (24) \]
where \( e \) is the electron electric charge. In the framework of mass insertion approximation, we find that the neutralino contribution to the above Wilson coefficient is given by
\[
C_{\chi^0_l} = \frac{\alpha_W}{4\pi} \sum_{a=1}^{4} \frac{1}{m_{\tilde{\chi}_a^0}} f_1(x_a) \left[ \tan \theta_W N^*_{a1} (N^*_{a2} + \tan \theta_W N^*_{a1}) \left( \delta^I_{RL} \right)_{ii} 
- N_{a3}^* N_{a1}^* \tan \theta_W \frac{m_{\tilde{\chi}_a^0}}{m_W \cos \beta} \left( \delta^I_{RR} \right)_{ii} 
- N_{a3}^* (N_{a2}^* + \tan \theta_W N_{a1}^*) \frac{m_{\tilde{\chi}_a^0}}{2m_W \cos \beta} \left( \delta^I_{ll} \right)_{ii} \right], \quad (25)\]
where \( x_a = \tilde{m}^2 / m_{\tilde{\chi}_a^0} \) and the function \( f_1(x) \) is given by
\[
f_1(x) = \frac{x (5 - 4x - x^2 + 2(1 + 2x) \log x)}{2(1 - x)^4}. \quad (26)\]

It is worth mentioning that in minimal supergravity model, the lightest neutralino leads to the dominant contribution to \( C_{\chi^0_l} \). However, in general supersymmetric models other neutralino exchanges could give also significant contributions to \( C_{\chi^0_l} \).

Calculating the chargino contribution to the Wilson coefficient of the charged lepton \( l_i \), in the mass insertion approximation, one obtains the following expression for the Wilson coefficient \( C_{\tilde{\chi}^0_l} \):
\[
C_{\tilde{\chi}^0_l} = \frac{\alpha_W}{4\pi} \frac{m_{\tilde{\chi}_a^0}}{\sqrt{2} m_W \cos \beta} \sum_{j,k=1}^{3} \left( (U_{MNS})_{ij} (\delta^\nu_{LL})_{jk} (U^\dagger_{MNS})_{kl} \right) \times \sum_{a=1}^{2} \frac{1}{m_{\tilde{\chi}_a^+}} U_{a2}^* V_{a1}^* f_2(x_a) \quad (27)\]
where \( m_{\tilde{\chi}_a} \) is the chargino mass and \( x_a = \hat{m}^2/m_{\tilde{\chi}_a}^2 \). The loop function \( f_2(x) \) is given by

\[
f_2(x) = \frac{x}{2(1-x)^4}(5x^2 - 4x - 1 - 2x(2 + x) \ln x)
\]

(28)

### 3.2 Lepton flavour violation, \( l_i \to l_j \gamma \)

The experimental bounds on the lepton flavour violating decays of charged leptons, in particular the \( \mu \to e\gamma \), impose strong constraints on the absolute values of the relevant mass insertions. As we will show in the next section, these constraints have important consequences on the prediction of the lepton EDM results. Therefore in our analysis we have to take the effect of these decays into account. These processes receive contributions from chargino and neutralino exchanges. Assuming \( m_{\mu} \gg m_{l_j} \), the amplitude to \( l_i \to l_j \gamma \) can be written as

\[
M_{l_i \to l_j \gamma} = e_e e_\alpha(q) \left( \overline{u}_l i \sigma^{\alpha\beta} q_R u_l \right) (A_{Rji}^C + A_{Rji}^N) + (L \leftrightarrow R)
\]

(29)

where \( A_{C,N}^{C,N} \) denote the chargino and neutralino contributions, respectively. The neutralino contributions are given by

\[
A_{N Rji}^N = \frac{\alpha_W}{4\pi} m_{l_i} \sum_{a=1}^4 \frac{1}{m_{\tilde{\chi}_a}^2} \left[ \left( \frac{m_{\tilde{\chi}_a}^2}{2m_W \cos \beta} \right) N_{a3}(N_{a2} + \tan \theta_W N_{a1}) f_1(x_a) + \frac{(N_{a2} + \tan \theta_W N_{a1})^2}{2} f_3(x_a) \right] \left( \delta_{LL}^{Rji} \right)_{ji} + \frac{m_{\tilde{\chi}_a}^2}{m_{l_i}} \tan \theta_W N_{a1}(N_{a2} + \tan \theta_W N_{a1}) f_1(x_a) \left( \delta_{LR}^{Rji} \right)_{ji}
\]

(30)

\[
A_{N Lji}^N = \frac{\alpha_W}{4\pi} m_{l_i} \sum_{a=1}^4 \frac{1}{m_{\tilde{\chi}_a}^2} \left[ \left( \frac{m_{\tilde{\chi}_a}^2}{m_W \cos \beta} \right) \tan \theta_W N_{a1}^* N_{a2} f_1(x_a) + 2 \tan^2 \theta_W |N_{a1}|^2 f_3(x_a) \right] \left( \delta_{RR}^{Lji} \right)_{ji} + \frac{m_{\tilde{\chi}_a}^2}{m_{l_i}} \tan \theta_W N_{a1}^* (N_{a2}^* + \tan \theta_W N_{a1}^*) f_1(x_a) \left( \delta_{RL}^{Lji} \right)_{ji}
\]

(31)

with \( f_3(x) \) given by

\[
f_3(x) = \frac{x(-17 + 9x + 9x^2 - x^3 - 6(1 + 3x) \ln x)}{12(x - 1)^5}
\]

(32)

For the chargino contribution one obtains:

\[
A_{C Rji}^C = \frac{\alpha_W}{4\pi} m_{l_i} (U_{MNS})_{j h} \left( \delta_{LL}^{Rji} \right)_{hk} (U_{MNS}^\dagger)_{ki} \times
\]

\[
\sum_{a=1}^2 \frac{1}{m_{\tilde{\chi}_a}^2} \left( \frac{m_{\tilde{\chi}_a}^2}{\sqrt{2} m_W \cos \beta} \right) U_{a2} V_{a1} f_4(x_a) - |V_{a1}|^2 f_5(x_a)
\]

(33)
\[ A_{Lji}^C = O(m_{l_j}) \] (34)

The one-loop functions are given as

\[ f_4(x) = \frac{x}{2(1-x)^4}(5x^2 - 4x - 1 - 2x(2 + x) \ln x) \] (35)

\[ f_5(x) = \frac{x}{6(1-x)^5}(x^3 + 9x^2 - 9x - 1 - 6x(1 + x) \ln x). \] (36)

The branching ratio of \( \mu \rightarrow e\gamma \) can be expressed as

\[ Br(\mu \rightarrow e\gamma) = 384 \pi^3 \alpha \frac{v^4}{m_\mu^2} \left( |A_{R12}^C + A_{R12}^N|^2 + (R \leftrightarrow L) \right) \] (37)

with \( v = (8G_F^2)^{-1/4} \simeq 174 \text{ GeV} \) and \( \alpha = e^2/4\pi \). Based on these results on \( \mu \rightarrow e\gamma \), one can immediately write down the rate for the process \( \tau \rightarrow \mu, e\gamma \). Using \( \Gamma_\tau \simeq 5(m_\tau/m_\mu)^5 \Gamma_\mu \), one obtains for the branching ratios,

\[ Br(\tau \rightarrow \mu\gamma) = \frac{384}{5} \pi^3 \alpha \frac{v^4}{m_\tau^2} \left( |A_{R23}^C + A_{R23}^N|^2 + (R \leftrightarrow L) \right) \] (38)

\[ Br(\tau \rightarrow e\gamma) = \frac{384}{5} \pi^3 \alpha \frac{v^4}{m_\tau^2} \left( |A_{R13}^C + A_{R13}^N|^2 + (R \leftrightarrow L) \right) \] (39)

4 Constraints from \( BR(\mu \rightarrow e\gamma) \) and electron EDM

In this section we present our results for the bounds on \( (\delta_{AB})_{ij} \) and \( (\tilde{\delta}_{LL})_{ij} \) which come respectively from the neutralino and chargino contributions to \( BR(\mu \rightarrow e\gamma) \) and electron EDM. As it is well known, until now, no lepton flavour violating processes or electric dipole moments of lepton have been experimentally observed. So we have only bounds on these different processes. As can be seen from table I, where we summarize the present experimental status, the strongest bounds are related to the electron EDM and the \( BR(\mu \rightarrow e\gamma) \).

| Process          | Bound |
|------------------|-------|
| \( d_e \)       | \(< 4.3 \times 10^{-27} e \text{ cm}^{[19]} \) |
| \( d_\mu \)     | \((3.7 \pm 3.4) \times 10^{-19} e \text{ cm}^{[20]} \) |
| \( d_\tau \)    | \(< 3.1 \times 10^{-15} e \text{ cm}^{[21]} \) |
| \( Br(\mu \rightarrow e\gamma) \) | \(< 1.2 \times 10^{-11}^{[22]} \) |
| \( Br(\tau \rightarrow \mu\gamma) \) | \(< 1.1 \times 10^{-6}^{[23]} \) |
| \( Br(\tau \rightarrow e\gamma) \) | \(< 2.7 \times 10^{-6}^{[24]} \) |

Table 1: The current experimental bounds on the LVF processes and lepton EDMs.

An experiment aimed to reach the sensitivity for \( Br(\mu \rightarrow e\gamma) \) of \( 10^{-14} \) has been proposed at PSI\[^{[25]}\], and the stopped-muon experiment that could take place at neutrino factories could reach \( Br(\mu \rightarrow e\gamma) \sim 10^{-15}^{[26]} \). The B-factories as Belle and LHC should be able to improve \( Br(\tau \rightarrow \mu\gamma) \) by typically one order of magnitude. But the most
significant experimental improvements could be expected from the electric dipole moment for the electron and the muon. Indeed, recently, it has been proposed that one could improve by six orders of magnitude the measurement of $d_e$ using a new technical method [12]. At BNL, a new experiment has been proposed with the objective to reach a sensitivity of $10^{-24} \text{cm}$ for $d_\mu$ [27]. Neutrino factories or PRISM should be able to reach a sensitivity of $10^{-26} \text{cm}$ [26].

The lepton mixing matrix $U_{MNS}$ is also constrained by solar neutrino and atmospheric neutrino data. Indeed, in case of LMA solution to the solar neutrino problem, the mixing angles as defined in the standard parametrisation for $U_\delta$ [24] are typically in the ranges

\begin{align*}
0.24 \lesssim & \tan^2 \theta_{12} \lesssim 0.89 \quad [3], \quad 0.40 \lesssim \tan^2 \theta_{23} \lesssim 3.0 \quad [1], \quad \text{and} \quad |\sin \theta_{13}| \lesssim 0.2 \quad [2].
\end{align*}

Here we use the following values for the lepton mixing angles, \( \theta_{12} = 0.59, \theta_{23} = 0.78, \) and \( \theta_{13} = 0.2 \)

### 4.1 Constraints from neutralino contributions

First we consider the upper bounds on the relevant mass insertions in the charged lepton sector, mediated by neutralino exchange. In Ref.[13] bounds on these mass insertions have been presented but only in a very special case, where the lightest neutralino is assumed to be photino like and of course, in that case, it also gives the dominant contribution to $C_{l_i}^N$ and $A_{R,Lij}^N$. With these assumptions, the bounds on the mass insertions depend only on the ratio $x = m_\gamma^2/m_1^2$, where $m_\gamma$ is the photino mass. However, in a general SUSY model the bounds depend on the gaugino masses, \( \mu \)-term and $\tan \beta$. As we will show the bounds of some mass insertions are sensitive to some of these parameters, in particular $\tan \beta$.

In table 2, we present the upper bounds on the absolute values of the mass insertions \( (\delta_{AB}^l)_{12} \) (with $A, B = (L, R)$) from neutralino contributions to the \( BR(\mu \rightarrow e\gamma) \). We consider some representative value of the ratio $x_{12} = M_1/M_2$ and fixed values of $\mu = \tilde{m} = 200 \text{ GeV}, M_2 = 100 \text{ GeV}$ and $\tan \beta = 5$.

| $x_{12}$ | $(|\delta_{LL}^l|)_{12}$ | $(|\delta_{LR}^l|)_{12}$ | $(|\delta_{RR}^l|)_{12}$ |
|---------|-----------------|-----------------|-----------------|
| 0.25    | $8.4 \times 10^{-4}$ | $2.7 \times 10^{-6}$ | $4.2 \times 10^{-3}$ |
| 0.5     | $1 \times 10^{-3}$ | $1.8 \times 10^{-6}$ | $1.7 \times 10^{-3}$ |
| 1       | $1.3 \times 10^{-3}$ | $1.5 \times 10^{-6}$ | $1.2 \times 10^{-3}$ |
| 2       | $1.4 \times 10^{-3}$ | $1.8 \times 10^{-6}$ | $1.8 \times 10^{-3}$ |

Table 2: Upper Bounds on $(|\delta_{LL}^l|)_{12}$ from $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ for $\mu = \tilde{m} = 200 \text{ GeV}, M_2 = 100 \text{ GeV}$, and $\tan \beta = 5$.

From the result in table 2, it is remarkable that the strong bounds on $(|\delta_{LR}^l|)_{12}$ are the same for $x_{12}$ and $1/x_{12}$, i.e., they are insensitive to the nature of the lightest neutralino, whether it is bino–like or wino–like. The dependence of the absolute values of the mass insertions $(\delta_{AB}^l)_{12}$ on $\tan \beta$ is given in table 3 for $M_1 = M_2 = 100 \text{ GeV}$ and $\mu = \tilde{m} = 200 \text{ GeV}$.
**Table 3:** Upper Bounds on $|\langle \delta_{LL} \rangle_{12}|$ from $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ for $\mu = \tilde{m} = 200$ GeV and $M_1 = M_2 = 100$ GeV.

As can be seen from this table the bounds on the LR mass insertion are essentially independent on the values of $\tan \beta$. We have also found that the results are independent on the values $\mu$. Next, we consider the bounds on the imaginary parts of the relevant $LR$ mass insertions from the experimental limit of the electron EDM. As mentioned above, the $LL$ and $RR$ mass insertions are Hermitian, so that $\text{Im}(\delta_{LL,RR})_{ij} = 0$ and only LR transitions contribute to the EDM. In table 4, we present the upper bounds on $\text{Im}(\delta_{LR})_{11}$ as function of the bino-wino ratio $x_{12}$ and $\mu$, from the experimental bound on electron EDM, $d_e < 4.3 \times 10^{-27}$ e cm.

**Table 4:** Upper Bounds on $\text{Im}(\delta_{LR})_{11}$ from electron EDM, $d_e < 4.3 \times 10^{-27}$ e cm for $\tan \beta = 5$, $\tilde{m} = 200$ GeV and $M_2 = 100$ GeV.

### 4.2 Constraints from chargino contributions

Now we turn to the constraints on the LL mass insertion in the sneutrino sector due to the chargino contributions to the LFV process $\mu \rightarrow e\gamma$ and the electron EDM. From eq.(33) and using the experimental bound given above for $BR(\mu \rightarrow e\gamma)$ together with the fact that $f_4(x) \ll f_5(x)$ for $x \simeq 1$, one can easily find a limit on $\delta_{LL}^{\nu}$. Indeed, one gets

$$
\sum_{i,j} \left( \frac{100 \text{ GeV}}{m_a \cos \beta} \right) U_{a2}^* V_{a1}^*(U_{MNS})_{e_j} (\delta_{LL}^{\nu})_{ji} (U_{MNS})_{\mu_i} = 6.4 \times 10^{-4} \sqrt{\frac{BR(\mu \rightarrow e\gamma)}{1.2 \times 10^{-11}}} \lesssim 6.4 \times 10^{-4}
$$

As usually done in the mass insertion method, we assume that there is no cancellation between contributions involving different $\delta_{LL}^{\nu}$ elements. So the bound given in eqs.(40) has to be applied to each contribution $(\delta_{LL}^{\nu})_{ji}$. We can proceed in the same way for $d_e$.
and using the experimental limit on $d_e$, one gets

$$\text{Im} \left( \sum_{i,j} \left( \frac{100 \text{ GeV}}{m_a \cos \beta} \right) U^*_{a_2} V^*_{a_1} (U_{MNS})_{ej} (\delta^\nu_{LL})_{ji} (U^\dagger_{MNS})_{ie} \right)$$

$$= \sum_{i,j} \left( \frac{100 \text{ GeV}}{m_a \cos \beta} \right) \text{Im} (U^*_{a_2} V^*_{a_1}) \text{ Re} \left( (U_{MNS})_{ej} (\delta^\nu_{LL})_{ji} (U^\dagger_{MNS})_{ie} \right)$$

$$= 2 \times 10^{-2} \frac{d_e}{4.3 \times 10^{-27} e \text{ cm}}$$

\[ \lesssim 2 \times 10^{-2} \] (42)

where to get eqs(41), we use the hermiticity of $\delta^\nu_{LL}$. From eq.(42), it is clear that the chargino contribution to the electron EDM do not lead to any significant constraint on the LL mass insertions and as mentioned above the source of CP violation in this case is the SUSY phase $\phi_\mu$. The most significant constraints on the LL mass insertions come as usually from the $\mu \rightarrow e\gamma$ experimental bound. In order to illustrate the dependence of the bounds given in eq. (40) on SUSY parameters, we present in tables 5 and 6 the upper bounds on the magnitude of the relevant LL mass insertions obtained from the experimental limits of LFV process $\mu \rightarrow e\gamma$. To get these bounds, we used the values given in the beginning of this section for the elements of $U_{MNS}$ lepton mixing matrix.

| $m$ | $(|\delta^\nu_{LL}|)_{11}$ | $(|\delta^\nu_{LL}|)_{12}$ | $(|\delta^\nu_{LL}|)_{13}$ | $(|\delta^\nu_{LL}|)_{22}$ | $(|\delta^\nu_{LL}|)_{23}$ | $(|\delta^\nu_{LL}|)_{33}$ |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| 100 | $6.3 \times 10^{-4}$ | $6.5 \times 10^{-4}$ | $4.8 \times 10^{-4}$ | $9.7 \times 10^{-4}$ | $7.3 \times 10^{-3}$ | $1.9 \times 10^{-5}$ |
| 200 | $6 \times 10^{-4}$ | $6.2 \times 10^{-4}$ | $4.6 \times 10^{-4}$ | $8.1 \times 10^{-4}$ | $6.8 \times 10^{-3}$ | $1.8 \times 10^{-3}$ |
| 300 | $8 \times 10^{-4}$ | $8.1 \times 10^{-4}$ | $6.1 \times 10^{-4}$ | $1.2 \times 10^{-3}$ | $9 \times 10^{-4}$ | $2.3 \times 10^{-3}$ |
| 400 | $1.1 \times 10^{-3}$ | $1.2 \times 10^{-3}$ | $8.6 \times 10^{-4}$ | $1.4 \times 10^{-3}$ | $1.3 \times 10^{-4}$ | $3.4 \times 10^{-3}$ |
| 500 | $1.7 \times 10^{-3}$ | $1.7 \times 10^{-3}$ | $1.3 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $1.9 \times 10^{-4}$ | $5 \times 10^{-3}$ |

Table 5: Upper Bounds on $(|\delta^\nu_{LL}|)_{ij}$ from $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ for $M_2 = \mu = 200$ GeV and $\tan \beta = 5$.

| $\tan \beta$ | $(|\delta^\nu_{LL}|)_{11}$ | $(|\delta^\nu_{LL}|)_{12}$ | $(|\delta^\nu_{LL}|)_{13}$ | $(|\delta^\nu_{LL}|)_{22}$ | $(|\delta^\nu_{LL}|)_{23}$ | $(|\delta^\nu_{LL}|)_{33}$ |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 5           | $6.3 \times 10^{-4}$ | $6.7 \times 10^{-4}$ | $5 \times 10^{-4}$ | $9.9 \times 10^{-3}$ | $7.3 \times 10^{-3}$ | $1.7 \times 10^{-5}$ |
| 15          | $2.5 \times 10^{-4}$ | $2.7 \times 10^{-4}$ | $2 \times 10^{-4}$ | $3.9 \times 10^{-3}$ | $3 \times 10^{-4}$ | $1.7 \times 10^{-3}$ |
| 25          | $1.6 \times 10^{-4}$ | $1.7 \times 10^{-4}$ | $1.2 \times 10^{-4}$ | $2.5 \times 10^{-3}$ | $1.8 \times 10^{-4}$ | $4.4 \times 10^{-4}$ |
| 35          | $1.1 \times 10^{-4}$ | $1.2 \times 10^{-4}$ | $9 \times 10^{-5}$ | $1.8 \times 10^{-3}$ | $1.3 \times 10^{-4}$ | $3 \times 10^{-4}$ |
| 45          | $9 \times 10^{-5}$ | $1.4 \times 10^{-4}$ | $7 \times 10^{-5}$ | $9.9 \times 10^{-3}$ | $1 \times 10^{-4}$ | $2.5 \times 10^{-4}$ |

Table 6: Upper Bounds on $(|\delta^\nu_{LL}|)_{ij}$ from $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ for $M_2 = \mu = 200$ GeV and $m = 100$.

Now, let us illustrate the relation between $Br(\mu \rightarrow e\gamma)$ and $d_e$ chargino contribution and what our experimental knowledge on neutrino mixing matrix $U_{MNS}$ can tell us on
relation between EDM’s and flavour changing processes. For that, one can define (without summation on repeated indices)

$$(U_{MNS})_{kj} (\delta_{LL})_{ji} (U_{MNS}^\dagger)_{il} \equiv \rho_{kl} e^{i\phi_{kl}}$$

(43)

In particular, one has

$$|\rho_{e\mu}| = |\rho_{e\tau}| \left| \frac{(U_{MNS})_{i\mu}}{(U_{MNS})_{i\tau}} \right|$$

(44)

But as we can see from eqs(40), $|\rho_{e\mu}|$'s are strongly constrained by $Br(\mu \rightarrow e\gamma)$. Using the experimental knowledge on $U_{MNS}$, one gets an upper limit on chargino contribution to $d_e$,

$$\frac{d_e}{e} \lesssim 1.37 \times 10^{-28} \sin \phi_\mu \sqrt{\frac{Br(\mu \rightarrow e\gamma)}{1.2 \times 10^{-11}}} \text{ cm}$$

(45)

$$\frac{d_\mu}{e} \lesssim 2.82 \times 10^{-26} \sin \phi_\mu \sqrt{\frac{Br(\mu \rightarrow e\gamma)}{1.2 \times 10^{-11}}} \text{ cm}$$

(46)

Proceeding in the same way for $Br(\tau \rightarrow \mu\gamma)$, one has

$$|\rho^{ij}_{\mu\tau}| = |\rho^{ij}_{e\mu}| \left| \frac{(U_{MNS})_{i\tau}}{(U_{MNS})_{e\tau}} \right| \left| \frac{(U_{MNS})_{ij}}{(U_{MNS})_{je}} \right|$$

(47)

Due to atmospheric neutrino data, one has that

$$|\rho^{ij}_{\mu e}| = |\rho^{ij}_{e\mu}| \left| \frac{(U_{MNS})_{ij}}{(U_{MNS})_{je}} \right|$$

(48)

Thus the chargino contribution to $Br(\tau \rightarrow \mu\gamma)$ is given by

$$Br(\tau \rightarrow \mu\gamma) \simeq \frac{1}{5} \frac{m_\mu^2}{m_\tau} Br(\mu \rightarrow e\gamma)$$

(49)

$$\lesssim 7.8 \times 10^{-15}$$

The same discussion can be done for $Br(\tau \rightarrow e\gamma)$ and one gets similar results.

5 Neutrino $CP$ phases and EDM.

In this section, we shall discuss the dependence of lepton EDM’s on neutrino $CP$ phases appearing in $U_{MNS}$. It is clear that the running of the soft breaking terms from the GUT to the weak scale induce a dependence on $U_{MNS}$ (and particularly on its phases). However, their effects on the soft breaking terms are usually very small$^4$. So, in the next discussion, we shall neglect this dependence and we shall study two extreme cases.

$^4$The effect of the running of the SUSY soft breaking terms from GUT to weak scale in the universal scenario for SUSY soft breaking terms has been recently studied in ref.[15].
First, we shall assume that all the SUSY soft-breaking terms are real and that the only source of CP violation arises from the charged lepton Yukawa couplings. The second case corresponds to assuming that $\phi_\mu$, the phase of the $\mu$-term, is different from zero and then checking what kind of textures for the SUSY soft-breaking terms may depend on the neutrino Majorana phases.

Let us consider the case where there is no CP violation coming from the diagonalisation of the charginos and neutralinos mass matrices. In this case, the lepton EDM has a very simple form,

$$d_l/e = \frac{\alpha_W}{4\pi} \sum_{a=1}^{4} \frac{1}{m_{\chi_a^0}} f_1(x_a) \tan\theta_W N^*_a(N^*_a + \tan\theta_W N^*_a) \Im(\delta^l_{RL})_{ii}$$

(50)

At first sight, it seems to be independent of the low-energy neutrino CP phases. But when $V^\nu_L \simeq 1$, the effects of low energy neutrino CP phases appear through the definition of $\delta^l_{RL}$. To illustrate this point, let us recall that for $V^\nu_L \simeq 1$, $U_{MNS} \simeq P_L V^\nu_L$. In this case, the definition of $\delta^l_{RL}$ can be written as,

$$\delta^l_{RL} \equiv \frac{1}{m^2} v \cos\beta P_L (V^\nu_R)^\dagger Y^e V^e_L P_L$$

(51)

where to simplify the notation, we defined $U^e_R \equiv P_L V^\nu_L$. It is clear that the low energy neutrino CP phases can strongly affect the EDM’s through the definition of $Y^e_A$. Indeed, one has

$$(Y^e_A)_{ij} \equiv (A)_{ij} (Y^e)_{ij}$$

$$\equiv (A)_{ij} (V^e_R \text{ diag}(h_e, h_\mu, h_\tau)V^e_L)^\dagger_{ij}$$

$$= (A)_{ij} (U^e_R \text{ diag}(h_e, h_\mu, h_\tau)U_{MNS})_{ij}$$

(53)

(54)

To get the last line, we used the fact that $P_L$ is a diagonal unitary matrix and commutes with $\text{ diag}(h_e, h_\mu, h_\tau)$. This result depends on the texture for the trilinear terms $(A)_{ij}$, but it is possible to extract some general properties. Indeed, any $A$ matrix can be written as follows,

$$A \equiv a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \sum_{p,q} c^{pq} b_{pq}$$

(55)

where $b_{pq}$ is define as a matrix with all entries equal to zero except for the element $(p, q)$ which is equal to 1 and $c^{pq}$ are numerical coefficients. The first term corresponds to the usual universal trilinear terms where $Y^e_A = a Y^e$. Im$(\delta^l_{RL})_{11}$ can now be rewritten as

$$\text{Im}(\delta^l_{RL})_{11} = \sum_{p,q,j=1}^{3} \frac{1}{m^2} v \cos\beta \text{ Im} \left( c^{pq} (U^e_R)_{1p} (U^e_R)_{jq} h_{jj} (U_{MNS})_{jq} (U^*_{MNS})_{1q} \right)$$

$$= \sum_{p,q,j=1}^{3} \frac{\sqrt{2m_j}}{m^2} \text{ Im} \left( c^{pq} (U^e_R)_{1p} (U^e_R)_{jq} (U_{MNS})_{jq} (U^*_{MNS})_{1q} \right)$$

(56)

(57)
with $m_j$, the charged lepton masses ($m_{1,2,3} = m_{e,\mu,\tau}$). We can directly see from eqs (57) that with universal trilinear couplings, the $\text{Im}(\delta_{RL}^{l})_{11}$ is independent of the Majorana or Dirac neutrino phases. But for a texture for the $A$ matrix different from the universal case, $\text{Im}(\delta_{RL}^{l})_{11}$ depends on the neutrino Dirac phase.

An interesting limit is to consider the case of Hermitian Yukawa coupling for the charged leptons ($U_{MNS} = U_{R}^{e}$). In that case, eq.(57) reads as

$$\text{Im}(\delta_{RL}^{l})_{11} = \sum_{p,q,j=1}^{3} \frac{\sqrt{2}m_j}{m^2} \text{Im} (c^{pq} (U_{MNS}^{1p})(U_{MNS}^{*})_{jp}(U_{MNS})_{jq}(U_{MNS}^{*})_{1q})$$  \hspace{2cm} (58)

It is important to notice that, in the Hermitian case, there is no contribution to $\text{Im}(\delta_{RL})$ coming from the Dirac CP violating phase of the mixing matrix $U_{MNS}$ for $p = q$, and that the $A$ matrix has to be not Hermitian. Otherwise, $\text{Im}(\delta_{RL}^{l})_{11} = 0$.

In case of no $CP$ violation coming from the $A$ matrix (all $c^{pq}$ are real $^5$), one has

$$\text{Im}(\delta_{RL}^{l})_{11} = \sum_{p,q,j=1}^{3} \frac{\sqrt{2}m_j}{m^2} \text{Im} ((U_{MNS})_{1p}(U_{MNS}^{*})_{jp}(U_{MNS})_{jq}(U_{MNS}^{*})_{1q})$$  \hspace{2cm} (59)$$

$$\simeq \sum_{p,q=1}^{3} \frac{\sqrt{2}m_\tau}{m^2} c^{pq} \text{Im} ((U_{MNS})_{1p}(U_{MNS}^{*})_{3p}(U_{MNS})_{3q}(U_{MNS}^{*})_{1q})$$  \hspace{2cm} (60)

where the imaginary part which appears in eq.(59) is the usual rephasing invariant measure of Dirac $CP$ violation$^6$. Indeed, in neutrino oscillations, the $CP$ asymmetry defined as the difference of the $CP$ conjugated neutrino oscillation probabilities $P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ is proportional to the imaginary part of an invariant quartet $J_{CP}$ defined as

$$J_{CP} \equiv \text{Im} ((U_{MNS})_{11}(U_{MNS}^{*})_{21}(U_{MNS})_{22}(U_{MNS}^{*})_{12})$$  \hspace{2cm} (61)

As all the rephasing invariant quartets are equal up to their sign, $\text{Im}(\delta_{RL}^{l})_{11}$ can be written as

$$|\text{Im}(\delta_{RL}^{l})_{11}| \simeq |J_{CP}| \sum_{p,q=1; p \neq q}^{3} \frac{\sqrt{2}m_\tau}{m^2} c^{pq}$$  \hspace{2cm} (62)$$

$$\lesssim 10^{-6}$$  \hspace{2cm} (63)

where the last inequality is obtained using table 4. This means that if the lepton large mixings and $CP$ violation has its origin in the charged lepton Yukawa coupling, in case of Hermitian Yukawa coupling and for a given texture of the trilinear $A$ terms, one has a simple correlation between the measure of electron EDM and $CP$ asymmetries in neutrino

$^5$Note that even if the trilinear couplings are real at GUT scale, they receive small complex contributions from the complex Yukawa through the running to the electroweak scale. Here we neglect this effect.

$^6$By Dirac $CP$ violation, we mean $CP$ violation arising from the Dirac phase of the $U_{MNS}$ lepton mixing matrix.
oscillation. For instance, assuming that $c^{pq} \sim \tilde{m} \sim v$, using eqs.(62-63), one gets a limit on $|J_{CP}|$,

$$|J_{CP}| \lesssim 7 \times 10^{-5}$$

(64)

It is amazing to note that this value is very close to the experimental measure of the rephasing invariant quartet of the quark sector, $|J^q_{CP}|$. Indeed, one has[24],

$$|J^q_{CP}| = (3.0 \pm 0.3) \times 10^{-5}$$

(65)

Before concluding, let us discuss the case where the $CP$ violation arise from both the Yukawa couplings and the SUSY parameters, in particular, if the diagonalisation of the chargino or the neutralino mass matrices. As an illustrative example, we shall discuss the case of the chargino contribution. The discussion for the neutralino contribution can be extended in a straighforward way. In this case, the chargino contribution to electron EDM is given by

$$d_e \quad \frac{4.3 \times 10^{-27} e \text{ cm}}{4.3 \times 10^{-27} e \text{ cm}} = 50 \times \sum_{i,j} \left( \frac{100 \text{ GeV}}{m_a \cos \beta} \right) \Im (U^a_2 V^*_a) \Re (U^*_{MNS})_{ej} (\delta^\nu_{LL}^j)_{ji} (U^\dagger_{MNS})_{ie}$$

(66)

The neutrino $CP$ violating phases dependence can appear directly through $U_{MNS}$ but also through the definition of $\delta_{LL}^\nu$ as given by eqs.(13). In case of $V'_L \simeq 1$, one has $(\delta_{LL}^\nu)_{ji} \simeq (\tilde{m}_i^2)_{ji}/\tilde{m}_i^2$. By neglecting the Dirac CP-violating phase of $U_{MNS}$ and using eqs(11), one gets a simple expression for the EDM in terms of the Majorana CP-violating phases,

$$d_e \quad \frac{4.3 \times 10^{-27} e \text{ cm}}{4.3 \times 10^{-27} e \text{ cm}} = 50 \times \sum_{i,j} \left( \frac{100 \text{ GeV}}{m_a \cos \beta} \right) \Im (U^a_2 V^*_a) \times \cos (\phi^M_j - \phi^M_i) \Re (U^\dagger_{6})_{ej} (\delta^\nu_{LL}^j)_{ji} (U^\dagger_{6})_{ie}$$

(67)

It is clear from eq.(67) that, in the case of a texture for $(\delta^\nu_{LL}^j)_{ji}$ different from the universal case, the electron EDM is a function of the cosinus of the Majorana phases and could be used as a way to probe the Majorana phases for a given texture for the SUSY soft-breaking terms.

6 Conclusion

In this paper, we have analyzed the constraints obtained from the chargino and neutralino contributions to the lepton EDM and LFV. We have adopted the mass insertion method which implies a model independent parametrization. We have provided analytical results for these contributions as functions of the leptonic mass insertions. We also derived model independent upper bounds on the relevant mass insertions by requiring that the pure chargino or neutralino contribution do not exceed the experimental limit of the lepton EDM and LFV (in particular the electron EDM and the branching ratio of $\mu \rightarrow e\gamma$, which give the most strainget bounds).
It was emphasized that once the bounds from $\mu \rightarrow e\gamma$ are imposed, the current experimental limits on the EDMs can be satisfied for any value of neutrino phases, whatever is the structure of the soft SUSY breaking terms.

Acknowledgements

This work was partially supported by the Nato Collaborative Linkage Grants. The work of D.D. was supported by Fundação para a Ciência e a Tecnologia (FCT) through the project POCTI/36288/FIS/2000. The work of S.K. was supported by PPARC.

References

[1] S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 86 (2001) 5656 [arXiv:hep-ex/0103033]. S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 86 (2001) 5651 [arXiv:hep-ex/0103032]. S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Lett. B 539 (2002) 179 [arXiv:hep-ex/0205075].

[2] M. Apollonio et al. [CHOOZ Collaboration], Phys. Lett. B 466 (1999) 415 [arXiv:hep-ex/9907037].

[3] Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 87 (2001) 071301 [arXiv:nucl-ex/0106015]. Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 89 (2002) 011301 [arXiv:nucl-ex/0204008].

[4] M. Ambrosio et al. [MACRO Collaboration], Phys. Lett. B 517 (2001) 59; G. Giacomelli and M. Giorgini [MACRO Collaboration], arXiv:hep-ex/0110021.

[5] K. Eguchi et al. [KamLAND Collaboration], arXiv:hep-ex/0212021.

[6] G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. Lett. 70, 2833 (1993) [Erratum, ibid. 71, 210 (1993)]; M. B. Gavela, P. Hernandez, J. Orloff, O. Pene and C. Quimbay, Nucl. Phys. B 430, 382 (1994); M. B. Gavela, M. Lozano, J. Orloff and O. Pene, Nucl. Phys. B 430, 345 (1994); P. Huet and E. Sather, Phys. Rev. D 51, 379 (1995).

[7] D. Delepine, J. M. Gerard, R. Gonzalez Felipe and J. Weyers, Phys. Lett. B 386 (1996) 183 [arXiv:hep-ph/9604440]; D. Delepine, arXiv:hep-ph/9609346; M. Carena, M. Quiros and C. E. Wagner, Phys. Lett. B 380, 81 (1996); M. Carena, M. Quiros and C. E. Wagner, Nucl. Phys. B 524, 3 (1998); J. R. Espinosa, Nucl. Phys. B 475, 273 (1996); J. M. Cline and K. Kainulainen, Nucl. Phys. B 482, 73 (1996); J. M. Cline, M. Joyce and K. Kainulainen, Phys. Lett. B 417, 79 (1998) [Erratum, ibid. B 448, 321 (1999)].

[8] S. Pokorski, J. Rosiek and C. A. Savoy, Nucl. Phys. B 570, 81 (2000); S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B 606, 151 (2001) and references therein;
D. Chang, W. Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999) [Erratum, ibid. 83, 3972 (1999)]; D. Chang, W. F. Chang and W. Y. Keung, Phys. Lett. B 478, 239 (2000); D. Chang, W. F. Chang and W. Y. Keung, hep-ph/0205084; A. Pilaftsis, hep-ph/0207277.

[9] D. Delepine, R. Gonzalez Felipe, S. Khalil and A. M. Teixeira, Phys. Rev. D 66 (2002) 115011 [arXiv:hep-ph/0208236].

[10] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45; L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996) 169 [arXiv:hep-ph/9605319]; M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B 345 (1995) 248 [Erratum-ibid. B 382 (1996) 447] [arXiv:hep-ph/9411366]; W. Buchmuller and M. Plumacher, Phys. Lett. B 389 (1996) 73 [arXiv:hep-ph/9608308].

[11] G. C. Branco, L. Lavoura and M. N. Rebelo, Phys. Lett. B 180 (1986) 264; A. Pilaftsis, Phys. Rev. D 56 (1997) 5431 [arXiv:hep-ph/9707235]; G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B 617 (2001) 475 [arXiv:hep-ph/0107164]; W. Rodejohann and K. R. Balaji, Phys. Rev. D 65 (2002) 093009 [arXiv:hep-ph/0201052]; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B 640 (2002) 202 [arXiv:hep-ph/0202030]; J. R. Ellis and M. Raidal, Nucl. Phys. B 643 (2002) 229 [arXiv:hep-ph/0206174]; S. Davidson and A. Ibarra, arXiv:hep-ph/0206304; P. H. Frampton, S. L. Glashow and T. Yanagida, Phys. Lett. B 548 (2002) 119 [arXiv:hep-ph/0208157]; T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi and M. Tanimoto, Phys. Rev. Lett. 89 (2002) 231601 [arXiv:hep-ph/0209020]; G. C. Branco et al., arXiv:hep-ph/0211001;

[12] S. K. Lamoreaux, arXiv:nucl-ex/0109014.

[13] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996) [arXiv:hep-ph/9604387].

[14] S. Khalil, T. Kobayashi and A. Masiero, Phys. Rev. D 60, 075003 (1999); S. Khalil and O. Lebedev, Phys. Lett. B 515, 387 (2001); E. Gabrielli and S. Khalil, Phys. Rev. D 67, 015008 (2003).

[15] J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B 528 (2002) 86 [arXiv:hep-ph/0111324].

[16] M. Gell-Mann, P. Ramond and R. Slansky, in Supersymmetry, eds. D. Freedman and P. van Nieuwenhuizen North Holland, Amsterdam, p.315 (1979); T. Yanagida, in Proceedings of the workshop on unified theory and baryon number in the universe, eds. O. Sawada and A. Sugamoto, KEK, Tsukuba, Japan (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912; R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 23 (1981) 165.

[17] see e.g., P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. B 477 (1996) 353 [arXiv:hep-ph/9601243]; M. E. Gomez, G. K. Leontaris, S. Lola and J. D. Vergados,
Phys. Rev. D 59 (1999) 116009 [arXiv:hep-ph/9810291]; S. Lola and G. G. Ross, Nucl. Phys. B 553 (1999) 81 [arXiv:hep-ph/9902283]; W. Buchmuller, D. Delepine and F. Vissani, Phys. Lett. B 459 (1999) 171 [arXiv:hep-ph/9904219]; W. Buchmuller, D. Delepine and L. T. Handoko, Nucl. Phys. B 576 (2000) 445 [arXiv:hep-ph/9912317].

[18] J. Hisano and D. Nomura, Phys. Rev. D 59 (1999) 116005 [arXiv:hep-ph/9810479].

[19] E. D. Commins, S. B. Ross, D. DeMille and B. C. Regan, Phys. Rev. A 50 (1994) 2960.

[20] H. N. Brown et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 86 (2001) 2227 [arXiv:hep-ex/0102017]. G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 89 (2002) 101804 [Erratum-ibid. 89 (2002) 129903] [arXiv:hep-ex/0208001].

[21] M. Acciarri et al. [L3 Collaboration], Phys. Lett. B 434 (1998) 169.

[22] M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83 (1999) 1521 [arXiv:hep-ex/9905013].

[23] S. Ahmed et al. [CLEO Collaboration], Phys. Rev. D 61 (2000) 071101 [arXiv:hep-ex/9910060].

[24] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66 (2002) 010001. Rev. D 66 (2002) 010001.

[25] L.M. Barkov et al., Research Proposal for experiment at PSI, 1999.

[26] J. Aysto et al., arXiv:hep-ph/0109217.

[27] Y. K. Semertzidis et al., arXiv:hep-ph/0012087.

[28] I. Masina, C.A. Savoy, hep-ph/0211283