Colloidal Dynamics on Disordered Substrates

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Using Langevin simulations we examine driven colloids interacting with quenched disorder. For weak substrates the colloids form an ordered state and depin elastically. For increasing substrate strength we find a sharp crossover to inhomogeneous depinning and a substantial increase in the depinning force, analogous to the peak effect in superconductors. The velocity versus driving force curve shows criticality at depinning, with a change in scaling exponent occurring at the order to disorder crossover. Upon application of a sudden pulse of driving force, pronounced transients appear in the disordered regime which are due to the formation of long-lived colloidal flow channels.

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Colloidal crystals are an ideal system in which to study the general problem of ordering and dynamics in 2D, since the particle size permits direct imaging of the particle locations and motion. A considerable amount of work has been conducted on the melting of 2D colloidal crystals in the absence of a substrate [1–3]. In addition, a number of experimental and theoretical studies have considered colloidal crystallization and melting in 2D systems with periodic 1D and 2D substrates [4,5], where a rich variety of crystalline states can be stabilized.

Colloidal crystals are also ideal for studying the ordering and dynamics of an elastic media interacting with random substrates, a problem that is relevant to a wide variety of systems such as superconducting vortices, Wigner crystals, and charge-density waves (CDWs). Open issues include the nature of the dynamical response to applied forces, as well as whether an order to disorder transition occurs as the strength of the random substrate increases. Recently, LeDoussal and Carpentier have theoretically investigated the effects of quenched disorder on the order and melting of 2D lattices and find a sharp crossover from the ordered Bragg glass (where defects are absent) to a disordered or molten state [6–8]. They predict that the depinning threshold increases at this crossover due to the softening of the lattice, which allows the particles to better adjust to the substrate. A similar mechanism could account for the peak effect observed in vortex matter in superconductors [9–14], in which the depinning threshold rises dramatically when the applied magnetic field is increased. In low temperature superconductors, where the vortices are fairly stiff so that their behavior can be considered as effectively 2D, recent small angle neutron scattering experiments have shown that the peak effect is associated with a sharp disordering or melting transition [15].

In addition to static properties, the dynamics of elastic media interacting with quenched disorder in 2D is a topic of intense study. In the disordered region the driven system may break up into pinned and flowing regions, as observed in experiments [16] and simulations [17–19] of vortices in superconductors. Conversely, for weak substrate disorder the elastic media is defect free and undergoes elastic depinning, in which the particles keep the same neighbors as they move. Fisher predicted that elastic depinning would show criticality [20], and that the velocity vs. force curves would scale as \( \nu = (f - f_c)^{\beta} \), where \( f_c \) is the depinning threshold force. This scaling has been studied extensively in 2D CDW systems where \( \beta = 2/3 \) [21–23]. It is, however, not known whether this exponent occurs in other systems undergoing elastic flow. Another intriguing dynamical phenomena is the pronounced transient behavior exhibited by vortices under a sudden applied current pulse at magnetic fields near the peak effect regime [11,12]. Due to surface barrier effects in the vortex sample, it is not clear whether these transient effects relate to the plasticity of the vortex dynamics or to contamination of the vortex lattice by disorder from the sample edges [14]. Recently, Pertsinidis and Ling [24] have studied colloids in 2D driven by an electric field and interacting with a disordered substrate. They observe plastic depinning with filamentary or river-like flow of colloids and a velocity-force curve scaling with \( \beta = 2.2 \). Under a pulsed drive the system shows very long time transients that fit to a stretched exponential.

Motivated by the recent colloidal experiments as well as the pulse drive experiments in vortex matter, we have conducted Langevin simulations of colloidal particles interacting via a Yukawa potential in 2D systems with random disorder. In simulation, the strength of the disorder can be carefully tuned, which is difficult to achieve in experiments. In addition, the initial conditions of the colloidal arrangements are easily controlled, whereas in experiments, defects generated in the colloidal lattice during preparation may become frozen in by the disorder.

Our simulations show that for weak substrates the colloids form an ordered triangular array which depins elastically without the generation of defects. For increased substrate strength, there is a sharp crossover to a disordered phase where the colloids depin plastically into riverlike structures. This crossover is accompanied by a sharp increase in the depinning threshold, analogous to the peak effect phenomena in superconductors. We
find scaling of the velocity vs. applied drive with an exponent of $\beta = 0.67$ in the elastic regime, in agreement with studies in 2D CDW’s. In the plastic regime we find $\beta = 1.97$, close to the value observed in experiments [3]. In the disordered region, long time transients that fit to a stretched exponential occur in response to a sudden applied drive pulse, as also observed in the colloidal and vortex experiments.

The colloids are simulated using Langevin dynamics in 2D [3]. The colloids interact via a Yukawa or screened Coulomb interaction potential $V(r_{ij}) = (Q^2/|r_i - r_j|) \exp(-\kappa|r_i - r_j|)$. Here $Q$ is the charge of the particles, $1/\kappa$ is the screening length, and $r_{i(j)}$ is the position of particle $i(j)$. The length of the system is measured in units of the lattice constant $a_0$ and a screening length of $\kappa = 2/a_0$ is used. The quenched disorder on the substrate is modeled as randomly placed parabolic traps with radius $r_p < a_0$ and a maximum force $f_p$. This same type of model for pinning has been used previously to model quenched disorder in superconducting vortex systems [14]. Other types of pinning potentials used in vortex simulations produce results similar to the parabolic pinning [16-18]. The equation of motion for colloid $i$ is

$$\frac{dr_i}{dt} = f_{ij} + f_p + f_T + f_d$$

(1)

Here $f_{ij} = \sum_{j=1}^{N_c} \nabla_i V(r_{ij})$ is the interaction force from the other colloids, $f_p$ is the pinning force, $f_T$ is a randomly fluctuating force due to thermal kicks, and $f_d$ is the force due to an applied driving field. We start the system at a temperature above melting as determined from the diffusion, and gradually cool below $T_m$ to $T/T_m = 0.4$ for most of the data presented here. To measure velocity $v$ vs force curves, care must taken to average over substantial amounts of time to avoid transient effects. The driving force is increased from zero by small increments and the velocity is averaged for $5 \times 10^4$ time steps at each increment, with typical simulations running for $10^7$ time steps. In this model we do not take into account hydrodynamic effects or long-range attractions between colloids.

In Fig. 1(a) we show the depinning force $f_c$ vs. substrate strength $f_p$ from a series of simulations. For $f_p < 0.18$ the depinning force increases as a power law, $f_c \propto f_p^{-1.9 \pm 0.1}$. To compare the depinning force to the order in the system, in Fig. 1(b) we show the the percentage of defects or non-six fold coordinated particles $P_d$ as calculated from a Delaunay triangulation. This measure indicates that the colloidal crystal is in an ordered state ($P_d = 0$) for $f_p < 0.18$, and that there is a crossover to a disordered state ($P_d \neq 0$) at $f_p = 0.18$. In Fig. 1(c) we show a representative Delaunay triangulation for the ordered state where there are no defects but small distortions in the particle positions can be seen, and in Fig 1(d) we show the disordered state where defects are present. The crossover to the disordered state coincides with a rapid increase in the depinning force as seen in Fig. 1(a) and in the inset of Fig. 1(a) which shows a peak in $df_c/df_p$ at the crossover. This behavior is consistent with the recent experiments in superconductors which find that at the peak effect there is an increase in the pinning with a simultaneous disordering of the lattice [13]. For $f_p > 0.2$, the depinning scales as $f_c \propto f_p$, as expected for the single particle pinning regime. The sudden increase in the depinning force results from the fact that the defected colloid lattice is much softer than the ordered lattice, allowing the colloids to adjust their positions to accommodate to the optimal pinning sites. We have also investigated this transition for different colloidal densities and disorder strengths. For increasing $T$ the order to disorder transition is shifted to lower values of $f_p$. We have also investigated finite size effects for increasing system sizes, and find that the order-disorder crossover shifts only a small amount before saturating, while the sharpness of the transition persists with increased system size.
It is beyond the scope of this paper to determine whether the order to disorder crossover is a first order transition. Although the sharpness suggests a possible first order transition, Carpentier and LeDoussal show that for 2D systems with quenched disorder, a sharp disordering crossover, rather than transition, occurs [8]. In addition, a first order transition is not expected since the Bragg glass in 2D has been shown to have dislocations on large scales at all temperatures. The distance between these dislocations can be arbitrarily large [28].

In Fig. 2 we show that the order-disorder crossover coincides with the onset of plastic flow above depinning. In Fig. 2(b) the elastic colloid flow is shown for $f_p = 0.12$ above depinning ($f_d/f_c = 1.1$). Here each colloid keeps the same neighbors as it moves. In Fig. 2(a) the inhomogeneous or plastic colloidal flow is shown for $f_d/f_c = 1.1$ for $f_p = 0.25$. Here, only a portion of the colloids are moving at any one time, and the motion occurs in channels or rivers between pinned regions. The colloid velocities show a bimodal distribution in this regime, split between the stationary and moving colloids. In addition, the channels seen in Fig. 2(a) are not static but change over time, so that any one colloid is only temporarily trapped in a pinning site. These features of the plastic flow are in agreement with observations in colloidal experiments [7] and in vortex simulations of the strongly pinned regime [6, 13].

In order to correlate the different types of flow observed in Fig. 2 with properties of bulk measurements, we show in Fig. 3 the scaling of the velocity vs driving force. For elastic depinning in the ordered regime [Fig. 3(a)], the velocity vs force curves fit well to $v = (f_d - f_c)\beta$ with $\beta = 0.66 \pm 0.02$, as illustrated in Fig. 3(b). These results are in good agreement with theoretical predictions [2] and simulation results [25, 28] for CDW’s in 2D where the depinning is elastic. In contrast, in driven 2D vortex matter, Higgins and Bhattacharaya [10] found an exponent of $\beta = 1.2$ below the peak effect where elastic flow is expected to occur. This may be due to the effects of surface barriers disordering the lattice. In Fig. 3(c,d) the $v - f_d$ scaling for the plastic regime is presented, with $\beta = 1.94 \pm 0.03$, close to the value of 2.2 found in the colloid experiments [7]. For larger system sizes, we find that the scaling region is expanded but the exponent is unchanged. Other studies in the plastic flow regime found $\beta = 5/3$ for electron flow in metallic dots [22] and $\beta = 2.22$ for vortex flow in Josephson-junction arrays [21].

In Fig. 4 we show the response of colloids prepared in an ordered state to the application of a sudden pulse of driving force of different strengths in the plastic flow regime. Since the pulse strength is chosen to be below the depinning threshold value $f_c$, the initial colloid velocity is high, and then gradually decreases. We find that a simple functional form cannot be fit to the curves. Instead, we use a stretched exponential fit as performed in experiments [3]: $v(t) = v_0 \exp\left(-\frac{t}{t_0}\right)^\alpha + v_1$. The values of $t_0$ and $\alpha$ depend on the magnitude of the drive. For the parameters investigated here, $\alpha$ falls between 0.08 and 0.4, in agreement with the values found in experiment. A similar stretched exponential decay was also found in vortex matter for the transient response to pulses [12]. We find that in the long time limit, the colloid flow occurs only through a few long-lived channels. In the elastic regime, the decay of $v$ is much faster and fits to an initial pure exponential with the velocities going to zero. We note that in the elastic regime the colloids move less than a lattice constant after a pulse is applied, whereas in the plastic regime, colloids in the moving channels can move the entire length of the system. For increased system sizes, the transient times are enhanced in the plastic flow regime but are unchanged in the elastic regime.
lived transients in the plastic regime are responsible for the very slow velocity-force sweep necessary to measure an accurate depinning threshold. This sweep-rate dependence is also consistent with the experimentally observed sweep rate dependent critical currents in the peak regime, where slow rates produce larger measured critical currents.

To summarize, we investigated the behavior of 2D colloids interacting with random disorder using Langevin simulations. For weak disorder the colloids form an ordered lattice which depins elastically and shows critical scaling in the velocity vs force curves, with $\beta = 0.67$, in agreement with studies of 2D CDW's. For increasing disorder strength, we find a sharp crossover to a disordered state, accompanied by a sharp increase in the depinning force, analogous to the peak effect observed for vortex matter in superconductors. In the disordered region, the colloids depin inhomogeneously into fluctuating channels and the $v-f$ curve scaling gives $\beta = 1.97$, in agreement with experiments. In the disordered flow regime, pronounced transients occur in response to a sudden pulse, with the late time dynamics determined by a few long lived channels.

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