Starspot variability as an X-ray radiation proxy

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ABSTRACT
Stellar X-ray emission plays an important role in the study of exoplanets as a proxy for stellar winds and as a basis for the prediction of extreme ultraviolet (EUV) flux, unavailable for direct measurements, which in turn are important factors for the mass-loss of planetary atmospheres. Unfortunately, the detection thresholds limit the number of stars with the directly measured X-ray fluxes. At the same time, the known connection between the sunspots and X-ray sources allows using of the starspot variability as an accessible proxy for the stellar X-ray emission. To realize this approach, we analysed the light curves of 1729 main-sequence stars with rotation periods 0.5 < P < 30 d and effective temperatures 3236 < T_{\text{eff}} < 7166 K observed by the Kepler mission. It was found that the squared amplitude of the first rotational harmonic of a stellar light curve may be used as a kind of activity index. This averaged index revealed practically the same relation with the Rossby number as that in the case of the X-ray to bolometric luminosity ratio R_x. As a result, the regressions for stellar X-ray luminosity \( L_x(P, T_{\text{eff}}) \) and its related EUV analogue \( L_{\text{EUV}} \) were obtained for the main-sequence stars. It was shown that these regressions allow prediction of average (over the considered stars) values of \( \log(L_x) \) and \( \log(L_{\text{EUV}}) \) with typical errors of 0.26 and 0.22 dex, respectively. This, however, does not include the activity variations in particular stars related to their individual magnetic activity cycles.

Key words: stars: activity – starspots – ultraviolet: stars – X-rays: stars.

1 INTRODUCTION
The stellar X-ray luminosity is widely used as an index of stellar magnetic activity (Wright et al. 2011). Moreover, the X-ray surface flux is considered as a proxy for mass-loss rate (i.e. stellar wind) for main-sequence stars (Wood et al. 2014). In addition, the stellar X-ray emission plays an important role in exoplanetary science as a basis for prediction of extreme UV flux, which is a crucial impacting factor for the planetary ionospheres and upper atmosphere mass-loss (Shaikhislamov et al. 2014, 2016; Khodachenko et al. 2015, 2017).

Unfortunately, only 32.9 per cent of known stars at distances up to ∼25 pc from the Sun were detected as X-ray sources (Hünsch et al. 1999). This problem is especially obvious for more distant stars, where the Kepler mission found most exoplanets. Such stars with undetected X-ray fluxes need a kind of accessible X-ray proxy.

Apparently, the stellar starspot variability at optical wavelengths could give such a proxy.

For example, solar X-ray emission is associated with active regions (Wagner 1988), hence, with sunspots. Correspondingly, there are high (>0.95) linear correlations between the sunspot number or the total spot area and the monthly averaged solar X-ray background flux (Ramesh & Rohini 2008). An analogous relation was found in other main-sequence stars using spot-induced brightness variations (Messina et al. 2003). To show this, Messina et al. (2003) used the maximum amplitude \( (A_{\text{max}}) \) of rotational variations of the stellar light curve. It has been shown that this simple activity index is non-linearly related with X-ray luminosity \( L_x/L \propto A_{\text{max}}^b \), where \( b \approx 2 \), and \( L_x \) is the stellar bolometric luminosity.

In our previous studies, we proposed and advocated another activity index – the squared amplitude \( A_1^2 \) of the first (fundamental) rotational harmonics of the stellar light curve (Arkhypov et al. 2015a,b, 2016, 2018). Its major advantage consists in the statistical proportionality to the starspot number \( N_s \) (see arguments in Arkhypov et al. 2016), and hence, the presumable proportionality to the X-ray emission: \( L_x/L \propto A_1^2 \). In fact, the analogous proportionality...
extend the previously analysed data set (Arkhypov et al. 2016), tested below for the main-sequence stars. For this purpose, we used the empirical relation between \( L_x / L \) and \( A_1^2 \) to predict the stellar X-ray luminosity \( L_x(P, T_{\text{eff}}) \) as a function of stellar rotation period \( P \) and effective temperature \( T_{\text{eff}} \). This could be useful for the modelling of exoplanetary environments’ evolution including the systems of non-solar-like stars. Hitherto the atmosphere-magnetosphere modelling is mainly based on the assumption of a solar-like X-ray emission (e.g. Trammell, Arras & Li 2011; Koskinen et al. 2013, Shaikhislamov et al. 2014, 2016; Khodachenko et al. 2015, 2017). However, many of the Kepler stars are distant objects of non-solar type with an enhanced luminosity, whereas the majority of the stellar population is composed of the faint red dwarfs. This makes the importance of the characterization of X-ray luminosity for a broader, than just solar type, class of stars.

In Section 2, we outline our approach using an extended stellar sample and our time-tested processing method (Arkhypov et al. 2015a,b, 2016, 2018). In Section 3, we justify that the used activity index can play a role of X-ray proxy. As a result, we obtain in Section 4 an approximating regression for \( L_x(P, T_{\text{eff}}) \), which is tested in comparison to observations in Sections 5. The related extreme ultraviolet (EUV) luminosity of stars is considered in Section 6. Section 7 summarizes the obtained results and their applicability.

2 STELLAR SET AND LIGHT-CURVE PROCESSING

As in our previous studies (Arkhypov et al. 2015a,b, 2016, 2018), we consider here the squared amplitude \( A_1^2 \) of the light curve’s first (fundamental) harmonic with period \( P \) of stellar rotation. This squared amplitude is taken because of its suspected and confirmed statistical proportionality to the number of starspots, at least for the solar-like stars (Arkhypov et al. 2016). Moreover, the amplitude of the first harmonic \( A_1 \), in differ to the light-curve variance, is practically insensitive to the photon noise. In contrast with other rotational harmonics, it depends minimally on flares and short-period pulsations. Note that since the light integration over the stellar disc reduces the amplitude of harmonics progressively with the increasing harmonic number, the value of \( A_1 \) can be measured with the maximal accuracy.

To calculate the activity index \( A_1^2 \) with a maximal time resolution, while focusing on the rotational variability of a star, we divided the stellar light curve on to consecutive fragments that have durations of one stellar rotation period \( P \) each and removed contaminating signals/features such as sporadic flares and linear trend, as well as performed interpolation of the light curve in short (<0.2 P) gaps (see details in Arkhypov et al. 2015a,b, 2016, 2018). The standard Fourier analysis applied is prepared in such a way that one-period fragments of the light curve give the varying (from one fragment to another) index \( A_1^2 \).

The aforementioned expectations regarding the proportionality between the X-ray emission and the measured parameter \( A_1^2 \) are tested below for the main-sequence stars. For this purpose, we extend the previously analysed data set (Arkhypov et al. 2016), which contains the light curves of the main-sequence stars observed by the Kepler space observatory, to include the light curves of an additional set of 637 slow rotators (Arkhypov et al. 2018). In Fig. 1(a), the analysed extended sample (see the electronic version of Table 1) that includes 1729 stars with \( 0.5 < P < 30 \) d (according to measurements in Nielsen et al. 2013; McQuillan et al. 2014) and the HR-diagram with respect to surface gravity log(\( g \)) versus \( T_{\text{eff}} \) analogous to Mathur et al. (2017).

![Figure 1](https://academic.oup.com/mnras/article-abstract/476/1/1224/4839014)

Figure 1. Stellar set properties: (a) the distribution of 1729 selected stars over the effective temperature \( T_{\text{eff}} \) (Mathur et al. 2017) and the rotation periods \( P \) (Nielsen et al. 2013; McQuillan et al. 2014) and (b) the HR-diagram with respect to surface gravity log(\( g \)) versus \( T_{\text{eff}} \).
A 476, R. Arkhypov et al. (2016) that the squared amplitude reflects the longitudinal distribution of spots. It has been shown in Rx of X-ray ratio sample and in the most complete nowadays catalogue of estimates because of the absence of common objects in the analysed stellar duration for every star in our set.

3 JUSTIFICATION OF THE OPTICAL PROXY FOR X-RAY EMISSION

We analyse the rotational modulation of the stellar radiation flux F (PDCSAP_Flux from the Kepler mission archive), which reflects the longitudinal distribution of spots. It has been shown in Arkhypov et al. (2016) that the squared amplitude $A_2^2$ of the first (or fundamental) Fourier harmonic with period $P$ of the stellar one-period light curve is proportional to a spot number. Following this, we used the value $A_2^2$ as an activity index. To exclude the temporal oscillations of stellar activity due to magnetic cycles, we measured a value for $\langle A_2^2 \rangle$ that was then averaged over the whole light-curve duration for every star in our set.

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Because of the absence of common objects in the analysed stellar sample and in the most complete nowadays catalogue of estimates of X-ray ratio $R_x \equiv L_x / L$ by Wright et al. (2011), the direct comparison between $R_x$ and the value $\langle A_2^2 \rangle$ is impossible. Since the ratio $R_x$ is commonly considered as a function of the Rossby number $Ro \equiv P / T_{MLT}$, where $T_{MLT}$ is the turnover time in the mixing length theory (Wright et al. 2011 and therein), instead of direct comparison of the activity indexes $R_x$ and $\langle A_2^2 \rangle$ themselves, we compare their dependencies on $Ro$. Following the arguments by Mamajek & Hillenbrand (2008), we use in this study the classical version of $T_{MLT}$ from Noyes et al. (1984). This approach is valid for $T_{eff} \gtrsim 4000$ K corresponding to the colour index $(B - V)_0 < 1.4$ of stars used in Noyes et al. (1984). An alternative turnover time approximation by Wright et al. (2011), assuming the linear relation between log $(T_{MLT})$ and the colour index $V - K_s$, apparently overestimates $T_{MLT}$ for the stars hotter than the Sun, because theoretically $\log (T_{MLT}) \rightarrow -\infty$ in vanishing convection zones for $T_{eff} \approx 8200$ K (Simon et al. 2002), i.e. for $V - K_s \sim 0.5$. Equation (11) in Wright et al. (2011), which describes the relation between log $(T_{MLT})$ and stellar mass, gives an unrealistic independence of the stellar activity on $Ro > 1$, when the mass estimates for the Kepler stars (Mathur et al. 2017) are used. Another argument for using the mixing time $T_{MLT}$ from Noyes et al. (1984) is provided in Section 5.

Since $T_{MLT}$ in Noyes et al. (1984) is defined via the colour index $(B - V)_0$, we use equation (26) from Arkhypov et al. (2016) for the transformation $T_{eff} \rightarrow (B - V)_0$ to make use of the data from the X-ray catalogue by Wright et al. (2011). Analogously, for the analysed set of Kepler Input Catalog (KIC) stars with a slightly different temperature scale, we found the following regression using the bright stars from SIMBAD data base considered in our previous study (Arkhypov et al. 2016):

$$ (B - V)_0 = 0.6369Y^3 - 2.262Y^2 - 14.38Y + 52.84, \quad (1) $$

were $Y = \log (T_{eff}/1K)$.

Table 1. The analysed stellar set, applied parameters, and predictions (sampled from the whole electronic version).

| KIC number | $T_{eff}$ | $p$ | $\log(A_2^2)$ | $\sigma$ | $(B - V)_0$ | $\tau_{MLT}$ | $\log(R_x)$ | $\log(L_x)$ |
|------------|----------|----|---------------|-------|-------------|-------------|------------|------------|
| 892834     | 4940     | 13.765 | -4.94         | 0.04  | 0.96        | 22.60       | -4.95      | 28.29      |
| 1026146    | 4419     | 14.891 | -5.24         | 0.04  | 1.21        | 24.62       | -4.94      | 27.82      |
| 1160947    | 4583     | 25.155 | -5.51         | 0.06  | 1.12        | 23.95       | -5.59      | 27.60      |
| 1161620    | 5977     | 6.636  | -5.16         | 0.03  | 0.57        | 7.60        | -5.37      | 28.62      |
| 1162635    | 3758     | 15.678 | -5.57         | 0.04  | 1.62        | 28.09       | -4.85      | 27.41      |
| 1163579    | 5721     | 5.429  | -4.98         | 0.03  | 0.65        | 12.27       | -4.57      | 29.40      |
| 1239993    | 4996     | 21.216 | -6.00         | 0.05  | 0.93        | 22.33       | -5.47      | 27.87      |
| 1295069    | 5418     | 17.187 | -5.64         | 0.05  | 0.76        | 17.85       | -5.49      | 28.37      |

Notes. a Stellar effective temperature from Mathur et al. (2017).

b Rotation period from McQuillan et al. (2014) or Nielsen et al. (2013) if absent in the first source.

Our active index, averaged over a whole light curve.

Estimated error of $\log(A_2^2)$.

Colour index according to the transformation $T_{eff} \rightarrow (B - V)_0$ in equation (1).

Turnover time according to equation (4) in Noyes et al. (1984).

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were $Y = \log (T_{eff}/1K)$.

Figure 2. Typical light curve (fragment) with a starspot variability from the Kepler archive (KIC 2283703). The time BJD here is the differential barycentric Julian days, counted from the mission starting time. The plot was prepared using NASA Exoplanet Archives service (http://exoplanetarchive.ipac.caltech.edu/).
The estimates of log(⟨A₂⟩) for individual stars were used to calculate the converging regression lines for both saturated and non-saturated activity regions with respect to Ro

\[
\log(⟨A₁²⟩) \approx y_{1,2} \log \left( \frac{P}{\tau_{MLT}} \right) + \delta_{1,2} \equiv \log((A₁²)_{\text{reg}}),
\]

where \( y_1 = -0.00 \pm 0.18 \) and \( \delta_1 = -3.99 \pm 0.23 \) are the fitted coefficients for \( \log(P/\tau_{MLT}) < \log(Ro_{\text{sat}}) = -0.89 \), whereas \( y_2 = -1.86 \pm 0.04 \) and \( \delta_2 = -5.63 \pm 0.02 \) correspond to the case of \( \log(P/\tau_{MLT}) > \log(Ro_{\text{sat}}) = -0.89 \). The saturation Rossby number \( Ro_{\text{sat}} = 10^{-0.89} = 0.13 \), at which both regressions converge, is the same as one found in Wright et al. (2011).

Using \( R_s \equiv L_s/L \) as the activity index from the stellar catalogue by Wright et al. (2011), we found the similar pattern in Fig. 3(b) with the following regression:

\[
\log(R_s) \approx y'_{1,2} \log \left( \frac{P}{\tau_{MLT}} \right) + \delta'_{1,2} \equiv \log((R_s)_{\text{reg}}).
\]

with close to equation (8) parameters: \( y'_1 = -0.13 \pm 0.04 \) and \( \delta'_1 = -3.33 \pm 0.06 \) in the saturation regime and \( y'_2 = -2.04 \pm 0.08 \) and \( \delta'_2 = -4.85 \pm 0.04 \) in the non-saturated case. The similarities \( y_1 \approx y'_1 \), \( y_2 \approx y'_2 \), \( \delta_1 \approx \delta'_1 \), and \( \delta_2 \approx \delta'_2 \) are understandable, because the X-ray emission is associated with the active regions, i.e. with the starspots. In particular, the solar X-ray flux correlates with both sunspot number and their total area (Ramesh & Rohini 2008), which control our index \( A_2 \).

Fig. 3(c) shows the comparison between the regressions’ prediction for both activity indexes in the region of measured log(\( R_s \)). After the correction \( \log((A_1²)_{\text{reg}} - \Delta \) with an average difference \( \Delta = -0.82 \) from log(\( R_s \)reg), we arrive at the approximation

\[
\log(R_s)_{\text{reg}} \approx \log((A_1²)_{\text{reg}}) + 0.82.
\]

The slight deviations (\( \lesssim 0.15 \) dex) from the equality \( \log(R_s)_{\text{reg}} = \log((A_1²)_{\text{reg}}) + 0.82 \) are consistent with the regression standard errors which are

\[
\sigma_{\text{reg}} = \sqrt{\sigma_2^2[\log(Ro)]^2 + \sigma_1^2},
\]

where \( Ro = P/\tau_{MLT} \) is the Rossby number, and the standard errors of the regression coefficients are as follows: \( \sigma_{y_1} = 0.18 \) and \( \sigma_{\delta_1} = 0.23 \) for saturated \( (A_1²) \); \( \sigma_{y_2} = 0.04 \) and \( \sigma_{\delta_2} = 0.02 \) for unsaturated \( (A_1²) \); \( \sigma_{y_3} = 0.04 \) and \( \sigma_{\delta_3} = 0.06 \) for saturated \( R_s \); and \( \sigma_{y_4} = 0.08 \) and \( \sigma_{\delta_4} = 0.04 \) for unsaturated \( R_s \). The corresponding error bars are shown in Fig. 2(c) for the equidistantly selected values of log(\( Ro \)).

The approximate relation \( \log(R_s)_{\text{reg}} \approx \log((A_1²)_{\text{reg}}) + 0.82 \) means that the regression \( \log((A_1²)_{\text{reg}} \) and the related activity index \( \log((A_1²) \) can be used as a quasi-proportional proxy for \( \log(R_s) \).

4 FROM THE OPTICAL PROXY TO THE PREDICTION OF STELLAR X-RAY LUMINOSITY

Here, we apply the optical proxy approach to predict stellar average X-ray luminosity, i.e. obtain the regression \( L_s(P, T_{\text{eff}}) \). Theoretically, using equation (4) and the approximation \( \log((A_1²) \approx \log((A_1²)_{\text{reg}} \), following equation (2), the proxy \( \log((A_1²)_{\text{reg}} \) can be transformed to \( R_s \). However, in practice, an empirical coefficient \( K \) should be included in this transformation to take into account a selection effect, i.e. the ignoring of stars with undetected X-ray radiation or low-amplitude light curves, so that

\[
\log(R_s)_{\text{reg}} \approx \log((A_1²) + K).
\]
where
\[ \log(K) = \log\left(R^\text{unb}_x\right) - \log\left(\langle A^2_1 \rangle\right)_{\text{reg}}. \] (7)

Here, \(R^\text{unb}_x\) is the regression for \(R_x\), which was obtained in Wright et al. (2011) using the ‘unbiased sample’ of stars
\[ \log(R^\text{unb}_x) \approx \gamma^\text{unb}_{1,2} \log\left(\frac{P}{T^\text{eff}_M L}\right) + \delta^\text{unb}_{1,2}, \] (8)
where \(\gamma^\text{unb}_{1,2} = 0\) (assumed) and \(\delta^\text{unb}_{1,2} = -3.13 \pm 0.08\) are the fitting coefficients for \(\log(R_0) < \log(R_{\text{unb}}) = -0.89\), whereas \(\gamma^\text{unb}_{1,2} = -2.70 \pm 0.13\) and \(\delta^\text{unb}_{1,2} = \delta^\text{unb}_{1,2} - \gamma^\text{unb}_{1,2} \log(R_{\text{unb}}) = -5.53 \pm 0.14\) correspond to the case of \(\log(R_0) > \log(R_{\text{unb}}) = -0.89\).

It follows from the definition \(R_x = L/L\) that in order to estimate \(L_x\), one needs a regression for the bolometric luminosity \(L(T^\text{eff})\). While the majority of astrophysical studies were focused on the universal mass-luminosity relation, we base our derivation of the needed relation on the apparently most complete compilation of the published values for \(L\) and \(T^\text{eff}\) for main-sequence stars (Eker et al. 2015):
\[ \log(L) = a_L Y^3 + b_L Y^2 + c_L Y + d_L, \] (9)
where \(L\) is the luminosity in solar units, \(Y = \log(T^\text{eff}/1K)\), and the fitting coefficients are \(a_L = 3.801, b_L = -47.396, c_L = 202.329,\) and \(d_L = -292.539\). The standard deviation is \(s_L = 0.20\) dex for all used stars (265 objects excluding 31 stars with problematic main-sequence status and \(>3\sigma\) outliers). Higher polynomial power has practically no decreasing effect on \(s_L\).

Using equations (6)–(9), we calculate \(L_x = R_x L\) for every star from our set. In fact, we replace in equation (6) the averaged \(\langle A^2_1 \rangle\) with the values \(\log(R^\text{unb}_x)\) for individual stars. This substitution is justified, since the further calculation of the regression \(L_x(P, T^\text{eff})_{\text{reg}}\) is equivalent to an averaging over stars. This regression has the following form:
\[ \log(L_x) \approx a_{1,0} X^3 + b_{1,0} X^2 + c_{1,0} X + d_{1,0} = \log(L_x)_{\text{reg}}, \] (10)
\[ a_{1,0} = a_L Y^3 + b_L Y^2 + c_L Y + d_L, \] (11)
\[ b_{1,0} = a_L Y^3 + b_L Y^2 + c_L Y + d_L, \] (12)
\[ c_{1,0} = a_L Y^3 + b_L Y^2 + c_L Y + d_L, \] (13)
\[ d_{1,0} = a_L Y^3 + b_L Y^2 + c_L Y + d_L, \] (14)
where \(X = \log(P/1\text{d}), Y = \log(T^\text{eff}/1K)\), and the fitting coefficients \(a_{1,0}, b_{1,0}, c_{1,0}, d_{1,0}\), found with the least square method, are listed in Table 2.

Fig. 4 shows the deviation \(\varepsilon_{\text{reg}} = \log(L_x) - \log(L_x)_{\text{reg}}\) between the regression and the \(\log(L_x)\), calculated for individual stars (equations 6–9) and averaged over the considered sample in a sliding window \(\log(P_x) = 0.2 < \log(P) < \log(P_x) + 0.2\) and \(\log(T^\text{eff}_x) = 0.05 < \log(T^\text{eff}) < \log(T_x) + 0.05\) with the central values of stellar rotation period \(P\), and effective temperature \(T^\text{eff}\). On average, 104 (up to 403) individual estimates of \(L_x\) appeared within this sliding window. Fig. 4(b) demonstrates a histogram of \(\varepsilon_{\text{reg}}\) with the standard deviation \(s_{\text{reg}} = 0.14\). Hence, the total standard error of the regression \(\log(L_x)_{\text{reg}}\) can be estimated as a combination of the standard errors of the involved regressions, i.e. \(s_{\text{reg}} = (s^2_{\text{reg}} + s^2_{\text{reg}} + s^2_{\text{reg}})/2 \approx 0.26\), where \(s_{\text{reg}} \approx 0.1\) dex for \(\log(R_x)_{\text{reg}}\) (see equation 5 and Fig. 4c for \(R_x\) errors), \(s_{\text{reg}} = 0.14\) dex for \(\log(L_x)_{\text{reg}}\), and \(s_L = 0.20\) dex for \(\log(L)\). Therefore, the derived regression (equations 10–14) predicts the average logarithm of stellar X-ray luminosity with typical error 0.26 dex.

### 5 COMPARISON OF THE PREDICTIONS VERSUS OBSERVATIONS

For the verification of our predictions, for all 824 objects from the X-ray catalogue by Wright et al. (2011), considered in this paper, we provide in Table 3 the values of \(\log(L_{\text{obs}})\), calculated using equations (10)–(14), as well as the observed values of \(\log(L_{\text{obs}})\). One can compare these values for different stellar clusters using the associated electronic version of Table 3. However, we would like...
to note that the regressions for luminosity \( \log(L) \) and activity index \( \log(A_1) \) (equations 9 and 2, respectively) were obtained for the main-sequence stars only. Hence, the pre-main-sequence objects in young clusters might appear a wrong example for controlling our estimates. At the same time, the mainly old field stars, which are more numerous than members of any cluster in the used catalogue, are most suitable for the verification of the obtained regressions.

Fig. 5 shows the comparison of the predicted X-ray luminosity \( \log(L_{\alpha}) = \log(L_{\alpha})_{\text{log}} \) (see equation 10) and the observed luminosity \( \log(L_{\alpha})_{\text{obs}} \) for 443 field stars from the catalogue by Wright et al. (2011). We are focused on the field stars because of the limited and distorted X-ray statistics in more distant stellar clusters, where many of the detections are just above the detection threshold. In Fig. 5(a), one can see the general agreement between the predicted and observed stellar distributions in relation to \( T_{\text{eff}} \). However, Fig. 5(b) reveals that some stars show \( \log(L_{\alpha})_{\text{obs}} \) \( \gg \) \( \log(L_{\alpha})_{\text{log}} \) at \( \log(L_{\alpha})_{\text{obs}} \) \( \lesssim 27 \) erg s\(^{-1}\). This effect disappears when the faint stars with \( T_{\text{eff}} \lesssim 4000 \) K are omitted in Fig. 5(c). Fig. 5(d) demonstrates that the stars with \( T_{\text{eff}} \lesssim 4000 \) K show a clear cut-off of the observed X-ray flux \( F_{\text{w}} \) at the detection threshold of \( \sim 10^{-13} \) ergs s\(^{-1}\) cm\(^{-2}\). The stars with \( F_{\text{w}} \) above this threshold are seen in Fig. 5(b) as a specific population of objects with \( \log(L_{\alpha})_{\text{obs}} \) \( \gg \) \( \log(L_{\alpha})_{\text{log}} \). However, at \( T_{\text{eff}} > 4000 \) K (Fig. 5(c)), the X-ray luminosity values for the considered stars are clustered along the equality line \( \log(L_{\alpha})_{\text{log}} = \log(L_{\alpha})_{\text{obs}} \) with a negligible average difference \( \log(L_{\alpha})_{\text{obs}} - \log(L_{\alpha})_{\text{log}} = 0.04 \pm 0.04 \), and the standard deviation of \( \log(L_{\alpha})_{\text{obs}} - \log(L_{\alpha})_{\text{log}} \) for an individual star is \( \sigma_{\text{w}} = 0.60 \) dex. Apparently, the \( L_{\alpha} \) variability in time gives the main contribution to \( \sigma_{\text{w}} \).

In Fig. 6, we test an alternative possible explanation of the aforementioned deviations at \( \log(L_{\alpha})_{\text{obs}} \gg \log(L_{\alpha})_{\text{log}} \) in Fig. 5(b) as a result of underestimated \( \tau_{\text{MLT}} \) for the red dwarfs outside of the temperature region, for which the used approximation of \( \tau_{\text{MLT}} \) was found (Noyes et al. 1984). To do that, we calculated the average values \( \log(A_1) \) for different Rossby numbers \( \text{Ro} = P/\tau_{\text{MLT}} \) in two temperature domains \( T_{\text{eff}} > 4000 \) K and \( T_{\text{eff}} < 4000 \) K using the different definition versions of \( \tau_{\text{MLT}} \) according to Noyes et al. (1984) and equation (10) in Wright et al. (2011), respectively. One can see in Fig. 6(a) that \( \tau_{\text{MLT}} \) from Noyes et al. (1984) gives the unified sequence of estimates that are independent on the temperature domain. However, the increased \( \tau_{\text{MLT}} \) from Wright et al. (2011) shifts the low-temperature points towards the lower \( \text{Ro} \), destroying the similarity between the estimates in Fig. 6(b).


table 3

| RA 2000\(^a\) (deg.) | DE 2000\(^b\) (deg.) | Affiliation | \( T_{\text{eff}} \) (K) | \( P \) (days) | \( \tau_{\text{MLT}} \) (days) | \( \log(R_{\text{b}}) \)\(^f\) \( \log((L_{\alpha})_{\text{obs}}) \)\(^g\) | \( \log((L_{\alpha})_{\text{log}}) \)\(^h\) |
|-----------------|-----------------|------------|----------------|----------------|----------------|----------------|----------------|
| –               | –               | Sun        | 5700           | 26.09          | 9.97           | –6.66          | 27.31           | 27.35          |
| 2.02679         | 47.9507         | Field      | 3201           | 4.38           | 32.85          | –3.17          | 28.84           | 28.21          |
| 2.84350         | 30.4096         | Field      | 5460           | 6.05           | 15.21          | –4.45          | 29.96           | 29.14          |
| 5.49087         | 49.2106         | Field      | 3405           | 6.17           | 30.69          | –3.65          | 28.58           | 28.25          |
| 5.71579         | –12.2094        | Field      | 5687           | 7.78           | 11.43          | –5.08          | 29.00           | 29.15          |
| 7.22467         | 50.3758         | Field      | 3163           | 1.09           | 33.30          | –3.13          | 27.25           | 28.29          |
| 9.14317         | 55.6267         | Field      | 3654           | 8.35           | 28.54          | –4.09          | 28.17           | 28.61          |
| 9.50008         | 43.8959         | Field      | 3619           | 0.55           | 28.82          | –3.13          | 28.23           | 29.02          |

Notes: \(^a\) Right ascension for epoch 2000.0 from Wright et al. (2011). \(^b\) Declination for epoch 2000.0 from Wright et al. (2011). \(^c\) Effective temperature from Wright et al. (2011). \(^d\) Turnover time according to equation (4) in Noyes et al. (1984) and transform \( T_{\text{eff}} \rightarrow (B - V)_0 \) (equation 26 in Arkhypov et al. 2016). \(^e\) From equations (6)–(8). \(^f\) Logarithm of the predicted X-ray luminosity using equations (10)–(14). \(^g\) Logarithm of the measured X-ray luminosity from Wright et al. (2011).

6 EUV APPLICATION

We demonstrate below the application potential of the proposed X-ray proxy for the prediction of related EUV radiation. The EUV radiation at wavelengths from 124 to 912 Å plays an important role in planetary science as a crucial impacting/heating factor for the upper atmospheres. However, in the case of exoplanets it is...
unobservable because of significant interstellar extinction. That is why there is a common practice to use the observable X-ray radiation as a proxy for the EUV flux. Apparently, the best results were obtained by Chadney et al. (2015) using the empirical relation

\[
\log(F_{\text{EUV}}) = 2.63 + 0.58 \log(F_x), \tag{15}
\]

where \( F_x = L_x/(4\pi R^2) \) (mW m\(^{-2}\)) is the stellar surface flux in the ROSAT band 6–124 Å, and \( F_{\text{EUV}} \) is the EUV surface flux at 124–912 Å. Correspondingly, the stellar EUV luminosity \( L_{\text{EUV}} \) can be found using the stellar radius \( R \stellar \) from the reference catalogue of KIC stellar data by Mathur et al. (2017)

\[
\log(L_{\text{EUV}}) = \log(F_{\text{EUV}}) + \log(4\pi R^2) + 4, \tag{16}
\]

where term 4 is added for the unit transformation (mW) \( \rightarrow \) (erg s\(^{-1}\)).

Using equations (15) and (16) in combination with equations (6)–(9), one can calculate \( L_{\text{EUV}} \) for every KIC star in our data set. These estimates were used to obtain the regression \( L_{\text{EUV}}(P, T_{\text{eff}}) \) as follows:

\[
\log(L_{\text{EUV}}) = a_{0} X^3 + b_{0} X^2 + c_{0} X + d_{0} \equiv \log(L_{\text{EUV}})_{\text{reg}}, \tag{17}
\]

\[
a_{0} = a_{1} Y^3 + b_{1} Y^2 + c_{1} Y + d_{1}, \tag{18}
\]

\[
b_{0} = a_{2} Y^3 + b_{2} Y^2 + c_{2} Y + d_{2}, \tag{19}
\]

\[
c_{0} = a_{3} Y^3 + b_{3} Y^2 + c_{3} Y + d_{3}, \tag{20}
\]

\[
d_{0} = a_{4} Y^3 + b_{4} Y^2 + c_{4} Y + d_{4}. \tag{21}
\]

where the fitting coefficients \( a_{0i}, b_{0i}, c_{0i}, d_{0i}, \) obtained with the least square method, are listed in Table 4.

Fig. 8 shows the deviation \( \varepsilon_{\text{EUV}} = \log(L_{\text{EUV}}) - \log(L_{\text{EUV}})_{\text{reg}} \) between the regression and \( \log(L_{\text{EUV}}) \), calculated for individual stars (equations 15–16) and averaged over the considered sample in a sliding window \( \log(P_{\text{c}}) - 0.2 < \log(P) < \log(P_{\text{c}}) + 0.2 \) and \( \log(T_{\text{c}}) - 0.05 < \log(T_{\text{eff}}) < \log(T_{\text{c}}) + 0.05 \) with the central
values of stellar rotation period $P$, and effective temperature $T$. On average, 104 (up to 403) individual estimates of $L_{\text{EUV}}$ appeared within this sliding window. Fig. 8(b) shows a histogram of $\delta_{\text{reg}}$ with the standard deviation $\sigma_{\text{reg}} = 0.08$. The deviation of the regression (17) from the true value $\log (L_{\text{EUV}})_0$ is 

$$d_{\text{tot}} = \log (L_{\text{EUV}})_0 - \log (L_{\text{EUV}})_{\text{reg}} = \delta_{\text{reg}} + \Delta,$$

where $\delta_{\text{reg}} = \log (L_{\text{EUV}}) - \log (L_{\text{EUV}})_{\text{reg}}$ and $\Delta = \log (L_{\text{EUV}}) - \log (L_{\text{EUV}})_0$. Here, $\log (L_{\text{EUV}})$ is an estimate obtained using equations (15) and (16). Correspondingly, the standard error of the regression (17) is 

$$\sigma_{\text{tot}} = \sqrt{\langle \delta_{\text{reg}}^2 \rangle + \langle \Delta^2 \rangle},$$

where $\langle \delta_{\text{reg}}^2 \rangle \approx \sigma_{\text{reg}}^2 = 0.08$, and $\langle \Delta^2 \rangle$ can be obtained from equations (15) and (16) by substitution of variables in the form of a sum of average values with index $o$ plus fluctuation marked with $\Delta$, i.e. $\log (L_o) = \log (L_0) + \Delta \log (L_o)$ and $\log (R_o) = \log (R_o) + \Delta \log (R_o)$. Then the following expression can be obtained:

$$\langle \Delta^2 \rangle = 0.34\langle \Delta \log (L_o) \rangle^2 + 0.71\langle \Delta \log (R_o) \rangle^2 + \rho^2.$$

Figure 7. Predicted X-ray luminosity versus $P$ is shown as solid curve according to equations (10–14) for the solar effective temperature ($T_{\text{eff}} = 5770$ K) in comparison with averaged predictions for individual stars (equations 6–9) with 5500 < $T_{\text{eff}}$ < 6000 K (solid squares with error bars). Here, the opened square is the average solar X-ray luminosity from the catalogue by Wright et al. (2011). The dependence by Ribas et al. (2005) is transformed in the $P$-scale using the gyrochronological relation from Garcia et al. (2014) (dashed line) as well as in Mamajek & Hillenbrand (2008) (dotted line). The long-dashed line corresponds to the regressions in Mamajek & Hillenbrand (2006). The standard error $\sigma_{\text{tot}}$ of our prediction is depicted as a left-bottom bar.

Table 4. Fitting coefficients for equations (17)–(21).

| $T_{\text{eff}}$ | $i$ | $a_i$ | $b_i$ | $c_i$ | $d_i$ |
|------------------|-----|-------|-------|-------|-------|
| <5500 K          | 1   | -9.606| 42.81 | 72.39 | -366.5|
|                  | 2   | 8.348 | -100.3| 394.1 | -510.2|
|                  | 3   | 8.862 | 64.78 | -822.6| 1708  |
|                  | 4   | -1.152| -55.47| 455.7 | -838.7|
| >5500 K          | 1   | -0.756| 19.33 | -108.2| 174.4 |
|                  | 2   | 10.97 | -65.78| 36.82 | 206.3 |
|                  | 3   | -2.667| -71.48| 629.6 | -1215|
|                  | 4   | -21.71| 75.70 | 349.5 | -1201|

where $\langle \Delta \log (L_o) \rangle^2 = \sigma_{\text{tot}}$, $\langle \Delta \log (R_o) \rangle^2 \sim 0.1$, which corresponds to the typical error $\sim 27$ percent in Mathur et al. (2017), and $\rho \sim 0.1$ is the typical uncertainty of prediction with equation (15) caused by the coefficient errors, which was estimated as a typical deviation of stellar estimates from the regression in Fig. 2 in Chadney et al. (2015). In summary, the total standard error of the regression $\log (L_{\text{EUV}})_{\text{reg}}$, i.e. equation (17), is

$$\sigma_{\text{EUV}} = \sqrt{\langle \delta_{\text{reg}}^2 \rangle + 0.34\sigma_{\text{reg}}^2 + 0.71\langle \Delta \log (R_o) \rangle^2 + \rho^2}.$$

It follows from this equation that the obtained regression (equations 17–21) predicts the average logarithm of stellar EUV luminosity with a typical error $\sigma_{\text{EUV}} \approx 0.22$ dex.

For testing of this prediction we use the best calibrated and studied case of the solar-type stars. Fig. 9 shows our $L_{\text{EUV}}(P)$ prediction for the stars with solar $T_{\text{eff}} = 5770$ K (Allen 1973) in comparison with the relations modelled by Ribas et al. (2005) for the solar analogues with estimated ages. Similarly to Section 5, we...
transformed the stellar ages to the $P$-scale using two versions of gyrochronological relation: (a) equations (12)–(14) and Table 10 in Mamajek & Hillenbrand (2008) (dot line) and (b) equation (1) in García et al. (2014) (dashed line). Both curves are mainly inside $\pm \sigma_{EUV}$ confidence interval of the prediction. Finally, the average EUV-flux of the Sun (Ribas et al. 2005), indicated in Fig. 9 with an opened square, sufficiently well corresponds to our solid curve.

Figure 8. Distributions of the deviation $\epsilon_{EUV \text{ reg}}$: (a) in relation to stellar parameters $P$ and $T_{\text{eff}}$ and (b) in form of a histogram, where the value $n$ is the number of $\epsilon_{\text{reg}}$ estimates in one bin of the histogram, and $N$ is the total number of such estimates in a sliding window.

7 CONCLUSIONS

Since our activity index $\langle A^2_1 \rangle$ gives a realistic prediction for $L_x$ and related $L_{\text{EUV}}$, it may be considered as a practical proxy for the stellar X-ray emission. In contrast with the spectral line indexes (e.g. $S_{\text{HK}}$, $R_{\text{HK}}$, and $R'_{\text{HK}}$ in Noyes et al. 1984; Mamajek & Hillenbrand 2008), our approach is based on the optical broad-band photometry. Hence, the index $\langle A^2_1 \rangle$ is applicable for more faint and numerous stars.

Fig. 10 shows $T_{\text{eff}}$, $P$-patterns of individual predictions for $\log (L_x)$ and $\log (L_{\text{EUV}})$, averaged in the same sliding window as in Figs 4 and 8 with the dimensions $\log (T_{\text{eff}}) \pm 0.05$ and $\log (P) \pm 0.2$. One can see the similar bright areas in the both plots at $3.6 \leq \log (T_{\text{eff}}) \leq 3.76$ and $0 \leq \log (P) \leq 0.7$. The stars in the corresponding intervals $4000 \leq T_{\text{eff}} \leq 5800$ K and $1 \leq P \leq 5$ d have the enhanced X-ray and EUV luminosities. Therefore, the exoplanets orbiting such stars should experience higher radiative impact that makes of crucial importance the study and an appropriate account of the processes of erosion of upper atmospheres as well as their related

Figure 10. Average predictions for (a) $\log (L_x)$ using equations (6)–(9) and (b) $\log (L_{\text{EUV}})$ using equations (15) and (16). Both plots depict the estimates for individual stars, averaged in the sliding window.
magnetospheric features (Khodachenko et al. 2012, 2015; Shaikhislamov et al. 2016).

The obtained regressions (equations 10 and 17) allow characterizing of X/EUV radiation at the distant objects below the sensitivity thresholds of X-ray detectors. This opens the way for statistical studies of exoplanetary environments as well as for exobiological applications. For example, the X/EUV radiation is a crucial factor for (pre)biological evolution and interplanetary panspermia.

In summary, the approach we have developed using starspot variability seems to be a useful tool for a broad range of astrophysical studies.

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SUPPLEMENTARY INFORMATION

Supplementary data are available at MNRAS online.

Table 1. The analysed stellar set, applied parameters, and predictions.

Table 3. Parameters and predictions for stars in the catalogue by Wright et al. (2011).

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