Towards a Music Algebra: Fundamental Harmonic Substitutions in Jazz
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Abstract—In this paper the most common harmonic substitutions, at least as far as jazz music is concerned, are unconventionally addressed. The novelty consists in introducing a new method finalized to formally defining and logically applying all the fundamental harmonic substitutions, by exploiting an unusually rigorous notation. After defining the substitutions and discussing their applicability, we resort to them in order to modify some simple harmonic progressions substantially based upon a banal major turnaround. As explicitly suggested by the title, the modifications are carried out by following an extremely formal line of reasoning: all the logic passages are accurately described by resorting to a notation so similar to the one commonly employed in mathematics and physics, that the harmonic analysis of a song turns out to be de facto comparable to the demonstration of a theorem.

Keywords—Music Algebra, Unconventional Notation, Harmonic Substitutions, Ionian Scale, Turnaround.

I. INTRODUCTION
In this paper, all the fundamental harmonic substitutions [1] [2] are discussed by resorting to a notation, generally employed in fields such as mathematics and physics, extremely formal and rigorous. We herein exclusively refer to a single harmonization, the Ionian one: consequently, for the sake of clarity, we reveal in advance that the so-called "Modal Interchange" is not addressed in this article. Obviously, the line of reasoning we exploit can be equally followed starting from other scales, such as the Natural Minor Scale (that can be obtained from the Ionian Scale by means of a banal translation), the Ipoionian Scale (Bach Minor Scale), the Harmonic Scales (Major and Minor).

II. DIATONIC SUBSTITUTIONS
Two chords that arise from the harmonization of the same scale are interchangeable if the distance between them (between the roots) is equal to a diatonic third (both ascending and descending). [1] [2] Exclusively referring to the Ionian Harmonization, the Diatonic Substitutions in their entirety can be effectively summarized by means of the following relations:

\[ VI_{7}\leftrightarrow I_{7}\leftrightarrow II_{7}\leftrightarrow I_{7} \] (1)
\[ VII_{7}\leftrightarrow I_{7}\leftrightarrow IV_{7}\leftrightarrow I_{7} \] (2)
\[ I_{7}\leftrightarrow IV_{7}\leftrightarrow V_{7}\leftrightarrow V_{7} \] (3)
\[ II_{7}\leftrightarrow V_{7}\leftrightarrow IV_{7}\leftrightarrow V_{7} \] (4)
\[ IV_{7}\leftrightarrow V_{7}\leftrightarrow I_{7}\leftrightarrow I_{7} \] (5)
\[ V_{7}\leftrightarrow VII_{7}\leftrightarrow I_{7}\leftrightarrow I_{7} \] (6)

If we set, for example, \( I = C \), from the foregoing relations we immediately obtain the following:

\[ Am_{7}\leftrightarrow C_{7}\leftrightarrow Em_{7} \] (8)
\[ B_{7}\leftrightarrow D_{7}\leftrightarrow F_{7}\leftrightarrow I_{7} \] (9)
\[ C_{7}\leftrightarrow Em_{7}\leftrightarrow G_{7} \] (10)
\[ D_{7}\leftrightarrow F_{7}\leftrightarrow Am_{7} \] (11)
\[ Em_{7}\leftrightarrow G_{7}\leftrightarrow B_{7} \] (12)
\[ F_{7}\leftrightarrow Am_{7}\leftrightarrow C_{7} \] (13)
\[ G_{7}\leftrightarrow B_{7}\leftrightarrow D_{7} \] (14)

The Diatonic Substitutions can be described, with obvious meaning of the notation, by resorting to a matrix of chords (that represents the Diatonic Tensor):

\[
\text{Diat}_{\text{Ion}}(I) = \begin{bmatrix}
I^\Delta & 0 & II_{7} & 0 & 0 & VI_{7} & 0 \\
0 & II_{7} & 0 & IV^\Delta & 0 & 0 & VII_{7} \\
I^\Delta & 0 & II_{7} & 0 & 0 & VI_{7} & 0 \\
0 & II_{7} & 0 & IV^\Delta & 0 & 0 & VII_{7} \\
0 & 0 & II_{7} & 0 & V_{7} & 0 & VII_{7} \\
I^\Delta & 0 & 0 & IV^\Delta & 0 & 0 & VII_{7} \\
0 & II_{7} & 0 & 0 & V_{7} & 0 & VII_{7}
\end{bmatrix}
\] (15)
Evidently, the Diatonic Substitutions can be summarized exploiting the following simple relation:

\[ \text{Diaton} \leftrightarrow \text{Diaton}^\Delta \]  \hspace{1cm} (16)

From (15), by setting, for example, \( I = C \), we obtain:

\[
\text{Diaton}^\Delta(C) = \begin{bmatrix}
C^\Delta & 0 & Em7 & 0 & 0 & Am7 & 0 \\
0 & Dm7 & 0 & F^\Delta & 0 & 0 & B\emptyset \\
C^\Delta & 0 & Em7 & 0 & G^7 & 0 & 0 \\
0 & Dm7 & 0 & F^\Delta & 0 & Am7 & 0 \\
0 & 0 & Em7 & 0 & G^7 & 0 & B\emptyset \\
C^\Delta & 0 & 0 & F^\Delta & 0 & Am7 & 0 \\
0 & Dm7 & 0 & 0 & G^7 & 0 & B\emptyset 
\end{bmatrix}
\]  \hspace{1cm} (17)

In Figure 1 a useful graphic representation of the Diatonic Substitutions is provided.

![Figure 1. Diatonic Substitutions](image)

### III. SECONDARY DOMINANTS

Any chord, even if it arises from a previous harmonic substitution, can be converted into a Dominant Seventh Chord. [1] [2] [3] [4] Referring, once again, to the chords obtained from the Ionian Harmonization, we have:

\[ I\text{maj7} \rightarrow I^7 = V(IV) \]  \hspace{1cm} (18)

\[ I\text{m7} \rightarrow I^7 = V(V) \]  \hspace{1cm} (19)

\[ III\text{m7} \rightarrow III^7 = V(VI) \]  \hspace{1cm} (20)

\[ IV\text{maj7} \rightarrow IV^7 = V(bVII) \]  \hspace{1cm} (21)

\[ V^7 = V(I) = \text{Primary Dominant}(I) \]  \hspace{1cm} (22)

\[ VI\text{m7} \rightarrow VI^7 = V(II) \]  \hspace{1cm} (23)

\[ VII\phi \rightarrow VII^7 = V(III) \]  \hspace{1cm} (24)

If we set, for example, \( I = C \), from the foregoing relations we immediately obtain the following:

\[ C\text{maj7} \rightarrow C^7 = V(F) \]  \hspace{1cm} (25)

\[ D\text{m7} \rightarrow D^7 = V(G) \]  \hspace{1cm} (26)

\[ E\text{m7} \rightarrow E^7 = V(A) \]  \hspace{1cm} (27)

\[ F\text{maj7} \rightarrow F^7 = V(B^b) \]  \hspace{1cm} (28)

\[ G^7 = V(C) = \text{Primary Dominant}(I) \]  \hspace{1cm} (29)

\[ A\text{m7} \rightarrow A^7 = V(D) \]  \hspace{1cm} (30)

\[ B\emptyset \rightarrow B^7 = V(E) \]  \hspace{1cm} (31)

From (28) we immediately deduce that the dominant seventh chord obtained from the fourth degree (\( F^7 \), in the specific case) leads towards a note, to extremely simplify, that does not belong to the \( C \) Ionian Scale (\( B\emptyset \), in the specific case). Consequently, the Secondary Dominant that arises from the forth degree is to be regarded as non-functioning (or non-functional).

### IV. TRITONE SUBSTITUTION

Any Dominant Seventh Chord, especially in case it is altered and even if it arises from a previous harmonic substitution, can be replaced by a chord of the same kind (a dominant seventh chord) distant three whole tones from the initial chord. [1] [2] [3] [4] [5] Generally, if we denote with \( X \) a generic note belonging to the Chromatic Scale, and with \( t \) a Whole Tone Interval, we can write:

\[ X^7 \rightarrow Y^7 \]  \hspace{1cm} (32)

\[ Y = X + 3t \]  \hspace{1cm} (33)

If we set, for example, \( X = C \), from the foregoing relations we immediately obtain the following:

\[ C^7 \rightarrow G^b^7 \equiv F^\#^7 \]  \hspace{1cm} (34)

### V. DIMINISHED SUBSTITUTION

Any Dominant Seventh Chord, especially if it is provided with the flat ninth and even if it arises from a previous harmonic substitution, can be replaced by a Diminished Chord distant a major third, a perfect fifth, a minor seventh or a flat ninth from the initial chord. [2] [3] [4] [5] [6] [7] All the above-mentioned intervals are implicitly regarded as ascending. In other terms, we can concisely write, with obvious meaning of the notation, as follows:

\[ X^{7(b)} \rightarrow Y^\text{dim7} \]  \hspace{1cm} (35)

\[ Y = X + 2t + \frac{3n}{2} t \quad n = 0,1,2,3 \]  \hspace{1cm} (36)

More explicitly, we have:
Let’s consider the following simple harmonic progression:

\[
X'(b) \rightarrow \begin{cases} 
(X + 2t)dim7 & n = 0, \text{major third} \\
(X + 7t)dim7 & n = 1, \text{perfect fifth} \\
(X + 5t)dim7 & n = 2, \text{minor seventh} \\
(X + 13t)dim7 & n = 3, \text{flat ninth}
\end{cases}
\]  

(37)

By setting, for example, \(X = C\), from (37) we obtain:

\[
C'(b) \rightarrow \begin{cases} 
Edim7 & n = 0, \text{major third} \\
Gdim7 & n = 1, \text{perfect fifth} \\
B ^b dim7 & n = 2, \text{minor seventh} \\
D ^b dim7 & n = 3, \text{flat ninth}
\end{cases}
\]  

(38)

VI. EXPANSION

The Dominant 9sus4 and b9sus4 Chords can be expressed by resorting to the so-called Slash Chords. In very formal terms, with obvious meaning of the notation, we can write:

\[
V7^{\text{9sus4}}(X) = \frac{11 - 7(X)}{(X + 2t)} 
\]  

(39)

\[
V7^{\text{b9sus4}}(X) = \frac{11 - 7b5(X)}{(X + 7t)} 
\]  

(40)

By virtue of (39) and (40), we can state that, in a certain measure, any Dominant Seventh Chord can be imagined as being preceded by a Minor Seventh Chord or a Half-Diminished Chord distant a descending perfect fourth. [3]

Consequently, employing a vertical line to separate two consecutive beats or bars, we have:

\[
V7 \rightarrow \{11 - 7\} \bigg| V7
\]  

(41)

If we set, for example, \(X = C\), from (39) and (40) we obtain:

\[
G7^{\text{9sus4}} = \frac{Dm7}{G} 
\]  

(42)

\[
G7^{\text{b9sus4}} = \frac{D^b}{G} 
\]  

(43)

Coherently with the setting \((X = C)\), from (41) we have:

\[
G7 \rightarrow \{Dm7 \bigg| G7, D^b \bigg| G7
\]  

(44)

VII. SOME NOTEWORTHY CASES

Case 1: Rhythm Changes (Bridge)

Let’s consider the following simple harmonic progression:

\[
\begin{array}{cccc}
\text{Imaj7} & \text{Imaj7} & \text{VIm7} & \text{V7} \\
\text{Im7} & \text{Im7} & \text{V7} & \text{V7}
\end{array}
\]

Harmonic Progression 1.1

If we set \(I = B^b\), we immediately obtain:

\[
\begin{array}{cccc}
\text{B^b maj7} & \text{B^b maj7} & \text{Gm7} & \text{Gm7} \\
\text{Cm7} & \text{Cm7} & \text{F7} & \text{F7}
\end{array}
\]

Harmonic Progression 1.2

Taking into account the previous progression, let’s carry out the following substitutions:

\[
\begin{array}{c}
\text{bars 1,2: } B^b \text{ maj7 } \rightarrow \text{ Dm7 } \rightarrow \text{ Gm7 } \rightarrow \text{ Cm7 } \rightarrow \text{ F7 } \\
\text{bars 3,4: } \text{ Gm7 } \rightarrow \text{ G7 } \\
\text{bars 5,6: } \text{ Cm7 } \rightarrow \text{ C7 }
\end{array}
\]  

(45)

(46)

(47)

From the Harmonic Progression 1.2, by virtue of (45), (46) and (47), we finally obtain:

\[
\begin{array}{cccc}
\text{D7} & \text{D7} & \text{G7} & \text{G7} \\
\text{C7} & \text{C7} & \text{F7} & \text{F7}
\end{array}
\]

Harmonic Progression 1.3

Case 2: I’m Getting Sentimental Over You (first 8 bars)

Let’s now consider the following harmonic progression:

\[
\begin{array}{cccc}
\text{Imaj7} & \text{V7} & \text{Imaj7} & \text{IVm7} \\
\text{Im7} & \text{V7} & \text{Imaj7} & \text{IVm7}
\end{array}
\]

Harmonic Progression 2.1

By setting \(I = F\), we immediately obtain:

\[
\begin{array}{cccc}
\text{Fmaj7} & \text{C7} & \text{Fmaj7} & \text{Dm7} \\
\text{Gm7} & \text{C7} & \text{Fmaj7} & \text{Dm7} & \text{Gm7} & \text{C7}
\end{array}
\]

Harmonic Progression 2.2

Taking into account the foregoing progression, let’s carry out the following substitutions:

\[
\begin{array}{c}
\text{bar 2: } C7 \rightarrow \text{ E0 } \rightarrow \text{ E7 } \rightarrow \text{ Bm7 } \rightarrow \text{ E7 } \\
\text{bar 3: } \text{ Fmaj7 } \rightarrow \text{ F7 } = V(B^b) \rightarrow \text{ A0 } \\
\text{bars 4,6: } \text{ Dm7 } \rightarrow \text{ D7 } \\
\text{bars 5,8: } \text{ Gm7 } \rightarrow \text{ G7 }
\end{array}
\]  

(48)

(49)

(50)

(51)
From the Harmonic Progression 2.2, by virtue of (48), (49), (50) and (51), we immediately obtain:

\[
\begin{array}{cccc}
\text{Fmaj7} & \text{Bm7} & \text{E7} & \text{Am7} \\
\text{G7} & \text{C7} & \text{Fmaj7} & \text{D7} \\
\end{array}
\]

Harmonic Progression 2.3

It is worth underlining that, in some cases, a desired chord (or a harmonic progression in its entirety) can be obtained by following various lines of reasoning. Just to provide an example, it is easy to verify how the third bar of the Harmonic Progression 2.3 could have been alternatively deduced by resorting to the following simple substitutions:

\[
\begin{align*}
\text{bar 3: } & \text{Fmaj7} \xrightarrow{\text{dial.}} \text{Dm7} \xrightarrow{\text{sec. dom.}} \text{D7} \quad (52) \\
\text{bars 4: } & \text{Dm7} \xrightarrow{\text{sec. dom.}} \text{D7} \quad (53) \\
\text{bars 3,4: } & \text{D7} \xrightarrow{\text{exp}} \text{A0} \mid \text{D7} \quad (54)
\end{align*}
\]

Case 3: Stella by Starlight (first 8 bars)

Let’s consider the following harmonic progression:

\[
\begin{array}{cccc}
\text{Hm7} & \text{Hm7} & \text{V7} & \text{V7} \\
\text{Imaj7} & \text{Imaj7} & \text{Hm7} & \text{V7} \\
\end{array}
\]

Harmonic Progression 3.1

If we set \( I = B^b \), we immediately obtain:

\[
\begin{array}{cccc}
\text{Cm7} & \text{Cm7} & \text{F7} & \text{F7} \\
\text{B^b maj7} & \text{B^b maj7} & \text{Cm7} & \text{F7} \\
\end{array}
\]

Harmonic Progression 3.2

Taking into account the progression we have just obtained, let’s carry out the following substitutions:

\[
\begin{align*}
\text{bars 1,2: } & \text{Cm7} \xrightarrow{\text{dial.}} \text{A0} \xrightarrow{\text{sec. dom.}} \text{A7} \xrightarrow{\text{exp.}} \text{E5} \mid \text{A7} \quad (55) \\
\text{bars 3,4: } & \text{F7} \xrightarrow{\text{exp}} \text{Cm7} \mid \text{F7} \quad (56) \\
\text{bars 5,6: } & \text{B^b maj7} \xrightarrow{\text{sec. dom.}} \text{B^b7} \xrightarrow{\text{exp}} \text{Fm7} \mid \text{B^b7} \quad (57) \\
\text{bar 7: } & \text{Cm7} \xrightarrow{\text{dial.}} \text{E^b maj7} \quad (58) \\
\text{bar 8: } & \text{F7} \xrightarrow{\text{dial.}} \text{Dm7} \xrightarrow{\text{sec. dom.}} \text{D7} \xrightarrow{\text{trit.}} \text{A^b7} \quad (59)
\end{align*}
\]

From the Harmonic Progression 3.2, exploiting (55), (56), (57), (58) and (59), we finally obtain:

\[
\begin{array}{cccc}
\text{Em7} & \text{F7} & \text{B^b7} & \text{E^b maj7} \\
\text{Fm7} & \text{B^b maj7} & \text{G^7} \\
\end{array}
\]

Harmonic Progression 3.3

The chord obtained in the eight bar (A\(^b\)7) leads towards B\(^b\)maj7 (ninth bar, not displayed in the Harmonic Progression 3.3). The progression B\(^b\)VII7 | Imaj7 is commonly named “Back-Door Solution”. [5]

Case 4: Easy Living (first 8 bars)

Let’s consider the following banal harmonic progression:

\[
\begin{array}{cccc}
\text{Imaj7} & \text{VImaj7} & \text{V7} & \text{V7} \\
\text{Imaj7} & \text{VImaj7} & \text{VImaj7} & \text{VImaj7} \\
\text{Imaj7} & \text{VImaj7} & \text{Hm7} & \text{V7} \\
\end{array}
\]

Harmonic Progression 4.1

If we set \( I = A^b \), from the foregoing progression we obtain:

\[
\begin{array}{cccc}
\text{A^b maj7} & \text{Fm7} & \text{B^m7} & \text{E7} \\
\text{A^b maj7} & \text{Fm7} & \text{B^m7} & \text{E7} \\
\text{A^b maj7} & \text{Fm7} & \text{B^m7} & \text{E7} \\
\end{array}
\]

Harmonic Progression 4.2

Taking into account the previous harmonic progression, let’s carry out the following substitutions:

\[
\begin{align*}
\text{bar 1: } & \text{Fm7} \xrightarrow{\text{sec. dom.}} \text{F7} \xrightarrow{\text{dim.}} \text{Adim7} \quad (60) \\
\text{bar 2: } & \text{E^b7} \xrightarrow{\text{dial.}} \text{G0} \xrightarrow{\text{sec. dom.}} \text{G7} \xrightarrow{\text{dim.}} \text{Bdim7} \quad (61) \\
\text{bars 3,6: } & \text{A^b maj7} \xrightarrow{\text{dial.}} \text{Cm7} \quad (62) \\
\text{bar 3: } & \text{Fm7} \xrightarrow{\text{dial.}} \text{A^b maj7} \xrightarrow{\text{sec. dom.}} \text{A^b7} \quad (63) \\
\text{bar 4: } & \text{B^m7} \xrightarrow{\text{dial.}} \text{D^b maj7} \quad (64) \\
\text{bar 4: } & \text{E^b7} \xrightarrow{\text{dial.}} \text{Cm7} \xrightarrow{\text{sec. dom.}} \text{C7} \xrightarrow{\text{trit.}} \text{G^b7} \quad (65) \\
\text{bar 7: } & \text{Fm7} \xrightarrow{\text{sec. dom.}} \text{F7} \quad (66)
\end{align*}
\]

From the Harmonic Progression 4.2, taking into account (60), (61), (62), (63), (64), (65) and (66), we finally obtain:

\[
\begin{array}{cccc}
\text{A^b maj7} & \text{Ao7} & \text{B^m7} & \text{Bo7} \\
\text{A^b maj7} & \text{Fm7} & \text{B^m7} & \text{E7} \\
\text{A^b maj7} & \text{Fm7} & \text{B^m7} & \text{F7} \\
\end{array}
\]

Harmonic Progression 4.3
The second chord in the fourth bar (G7) leads towards A’maj7; once again, a typical “Back-Door Solution”.

**Case 5: Giant Steps**

Let’s now consider the following harmonic progression:

| Gmaj7 | VIm7 | Imaj7 | VIm7 |
|-------|------|-------|------|
| Imaj7 | V7   | Vlma7 | V7   |

**Harmonic Progression 5.1**

By setting I = B, we immediately obtain:

| Bmaj7 | Gm7 | Cm7 | F7 | Bmaj7 | Gm7 |
|-------|-----|-----|----|-------|-----|
| Cm7   | F7  | Bmaj7 | F7 | Bmaj7 | F7 |
| Bmaj7 | Gm7 | Cm7 | F7 | Bmaj7 | F7 |

**Harmonic Progression 5.2**

Taking into account the previous progression, let’s carry out the following substitutions:

- **bar 1:** Gm7 \(\text{sec. dom.} \rightarrow \text{G}^\text{m7} \equiv A^\text{b7} \rightarrow \text{D}^\text{7} \) (67)
- **bars 2,5,11:** Cm7 \(\text{sec. dom.} \rightarrow \text{C}^\text{m7} \equiv D^\text{b7} \rightarrow \text{G}^\text{7} \) (68)
- **bars 2,5:** F7 \(\text{diat.} \rightarrow \text{A}^\text{b7} \equiv \text{B}^\text{b7} \) (69)
- **bars 3,6,9,15:** Bmaj7 \(\text{sec. dom.} \rightarrow \text{D}^\text{m7} \equiv \text{E}^\text{b7} \) (70)
- **bars 4,10:** Gm7 \(\text{sec. dom.} \rightarrow \text{A}^\text{b7} \rightarrow \text{D}^\text{7} \rightarrow \text{Am}^\text{7} \rightarrow \text{D}^\text{7} \) (71)
- **bars 8,14:** F7 \(\text{diat.} \rightarrow \text{B}^\text{b7} \rightarrow \text{Fm}^\text{7} \rightarrow \text{B}^\text{b7} \) (72)
- **bars 12,16:** F7 \(\text{exp.} \rightarrow \text{C}^\text{m7} \rightarrow \text{F}^\text{7} \) (73)

From the foregoing harmonic progression, by virtue of (67), (68), (69), (70), (71), (72) and (73), we obtain:

| Bmaj7 | D7   | G7   | B7   | E7   | Am7 | D7 |
|-------|------|------|------|------|-----|-----|
| G7    | B7   | E7   | F7   | Bmaj7 | Fm7 | B7 |
| E7    | Am7  | D7   | G7   | Cm7  | F7  |
| Bmaj7 | Fm7  | B7   | E7   | Cm7  | F7  |

**Harmonic Progression 5.3**

The Harmonic Progression 5.3, clearly, does not represent the desired result yet. In order to obtain the original Coltrane progression, we have to further modify all the chords written in red. On this purpose, we introduce a banal harmonic substitution (often simply named “Quality Substitution”) that allows to instantly transform a Dominant Seventh Chord into a Major Seventh, without resorting to the so-called “Modal Interchange”. In this regard, suffice it to bear in mind that the two above-mentioned chords are characterized by the same triad (the same chordal notes); in other terms, they only differ in the seventh (that can be considered, at least in a certain sense, as being a fundamental tension). It is worth highlighting how, by exploiting this very method, the well-known “Tadd Dameron (or Coltrane) Turnaround” [4] [8] can be obtained. Ultimately, we can write:

- **bars 2,5,11:** G7 \(\text{dom. to maj.} \rightarrow \text{Gmaj7} \) (74)
- **bar 3,6,9,15:** E7 \(\text{dom. to maj.} \rightarrow \text{Ebmaj7} \) (75)

Taking into account (74) and (75) we finally obtain:

| Bmaj7 | D7   | Gmaj7 | B7   | E'maj7 | Am7   | D7 |
|-------|------|-------|------|--------|-------|-----|
| Gmaj7 | B7   | E'maj7 | F7   | Bmaj7  | Fm7  | B7 |
| E'maj7 | Am7   | D7   | Gmaj7 | Cm7  | F7 |
| Bmaj7 | Fm7  | B7   | E'maj7 | Cm7  | F7 |

**Harmonic Progression 5.4**

### VIII. FINAL REMARKS

Let’s start from a banal sequence of four bars (or beats, or sets composed of an equal number of beats) characterized by the same chord (in our case, a Major Seventh Chord):

| Imaj7 | Imaj7 | Imaj7 | Imaj7 |

**Harmonic Progression 6.1**

Although it cannot be formally regarded as a harmonic substitution, we are clearly allowed to resort to the so-called “Tonicization” [9] [10] [11] [12] [13] in order to create a first, simple harmonic motion (nothing but an Authentic Cadence):

| Imaj7 | Imaj7 | V7 | V7 |

**Harmonic Progression 6.2**

Let’s now carry out the following substitutions:
bars (beats) 3: A: V7 [Exp] | IIm7 | V7  (76)

bar (beat) 2: IImaj7 [diat] | VI7m7  (77)

From the Harmonic Progression 6.2, by virtue of (76) and (77), we immediately obtain:

| IImaj7 | VI7m7 | IIm7 | V7 |

Harmonic Progression 6.3

It is evident that, starting from a single Major Seventh Chord, we can easily obtain a Major Turnaround. In other terms, we are implicitly stating that all the harmonic progressions we have obtained in the previous section may be imagined as arising from a single chord. In a certain sense, we could even state that, net of some progressions explicitly based upon the so-called “Pure Plagal Cadence” (IVmaj7 | IImaj7), all the Popular Jazz Songs, as far as the harmonic structure is concerned, may be considered as originating from a single chord (a Major Seventh Chord or, exploiting the harmonization of whatever minor scale, a Minor Seventh). It is worth underlining that we have explicitly referred to the “Pure Plagal Cadence”, since the “Authentic” or “Extended” one (IVmaj7 | V7 | IImaj7) can be immediately deduced, by simply resorting to a banal Diatonic Substitution, from a IIIm7 | V7 | IImaj7 harmonic progression which, in turn, can be obtained from a single Major Seventh Chord by exploiting a “Tonicization” and, taking into account (76), an Expansion.

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