Deeply Inelastic Hadronic Final States: QCD Corrections

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ABSTRACT

The status of the calculation of next-to-leading-order QCD corrections to hadronic final states in deeply inelastic scattering is reviewed, and an overview of the phenomenology (including the measurement of the strong coupling constant and the gluon density via jet rates) is given. We also describe a new universal program (DISASTER++) for the calculation of (2+1)-jet observables.

1. Introduction

The electron–proton collider HERA has started to accumulate large luminosity. This opens up the possibility to study the hadronic final state in deeply inelastic scattering to high precision. The motive for this is twofold: QCD can be tested at large scales in the spacelike regime, and an extraction of physical quantities, in particular the running strong coupling constant $\alpha_s(Q^2)$ and the gluon density $f_g(\xi, Q^2)$, is possible. The aim of this paper is to give an overview of the status of next-to-leading-order calculations both for jet-like quantities and for one-particle-inclusive processes. We will also discuss the phenomenology of processes with (2+1) jets\(^a\) in the final state, and of inclusive particle spectra.

In the last few years, the technology for the calculation of QCD corrections in next-to-leading order of perturbation theory has developed considerably. There are explicit algorithms available which permit the calculation to be done in a “universal” way: the infrared singularities are subtracted in such a way that arbitrary infrared-safe observables can be calculated numerically. In principle, all existing algorithms are variations on a common theme, namely the interplay of the factorization theorems of perturbative QCD and the infrared-safety of the observables under consideration. This will be reviewed in Section 2.

The practical implementations of this principle can be quite different. In Section 3, we will discuss the two possible ways to do this: the phase-space-slicing method and the subtraction method. The extraction of the singular terms can be done in different

\(^a\) WWW URL: [http://www.hep.psi.ch/graudenz/index.html](http://www.hep.psi.ch/graudenz/index.html)

\(^a\) Here \((n + 1)\) stands for \(n\) hard jets and the proton remnant jet.
ways, and we will describe in some detail a new calculation based on the subtraction formalism and on a general partial fractions formula.

An overview of applications for (2+1)-jet-like observables will be given in Section 4, where we state some results for the cut dependence of jet cross sections and for transverse momentum spectra. One-particle-inclusive processes will be discussed in Section 5. The theoretical predictions in the framework of the fragmentation function picture fail to describe the experimental data for small values of the photon virtuality $Q^2$ and for small values of the energy $E_h$ of the observed hadron, although there is excellent agreement at large $Q^2$ and large $E_h$. We will give a semi-quantitative estimate of the region where the fragmentation function picture should be applicable. The paper closes with a summary and an outlook.

2. Infrared-safe Observables and Factorization in QCD

Perturbative QCD permits the calculation of processes with partons in the final state. The gap to experimental data with final-state hadrons is bridged by means of infrared-safe observables $O$. Examples are jet cross sections for various jet definitions, and event shape variables such as Thrust. The expectation value of an observable $O$ for experimental data is given by

$$\langle O \rangle_{\text{exp}} = \frac{1}{N} \sum_{I=1}^{N} O^{(n_I)} \left(h_1^{(I)}, \ldots, h_{n_I}^{(I)}\right),$$

where $N$ is the number of events, $n_I$ is the number of final-state hadrons of event $I$, and the $h_i^{(I)}$ are the momentum vectors of the hadrons. This is to be compared with the expectation value for parton final states:

$$\langle O \rangle_{\text{th}} = \sum_{n} \int d\text{PS}^{(n)} \sigma^{(n)} (p_1, \ldots, p_n) \ O^{(n)} (p_1, \ldots, p_n).$$

Here $n$ is the number of final-state partons with momenta $p_1, \ldots, p_n$, $d\text{PS}^{(n)}$ is the phase space measure, and $\sigma^{(n)}$ is the hard scattering cross section for partons in the final state, calculated in perturbation theory. For the next-to-leading-order calculation of the (2+1)-jet-production cross section to be described later on, the relevant values are $n = 2$ for the Born term and the virtual corrections, and $n = 3$ for the real corrections. Both virtual and real corrections exhibit infrared singularities. The soft singularities and the collinear singularities in the final state cancel, owing to the Kinoshita–Lee–Nauenberg mechanism. The collinear singularities from the initial state can be consistently absorbed into redefined parton densities. The resulting hard scattering cross section is finite.

We have not indicated technical complications such as different types of final-state partons (quarks, gluons) and the convolution with parton densities if there are hadrons in the initial state.
The factorization theorems of perturbative QCD (see Ref. [1] and references therein) show that parton cross sections $\sigma$ have a very simple behaviour in collinear and soft phase space regions:

- In the collinear limit, for the partons with labels $j$ and $k$ being collinear, the limit for $\sigma$ is
  \[ \sigma \sim \frac{\alpha_s}{2\pi} \frac{1}{s_{jk}} \hat{P}_{kj\rightarrow i}(u) \sigma_{\text{Born}}, \]  
  where $u$ is the momentum fraction of the parent parton $i$ carried by parton $j$, $s_{jk} = 2p_jp_k$ and $\sigma_{\text{Born}}$ is the Born cross section corresponding to the real correction $\sigma$ in the limit under consideration. The $\hat{P}_{kj\rightarrow i}(u)$ are the unsubtracted Altarelli–Parisi splitting functions (see, for example, Ref. [2]). There is a slight technical complication: factorization in this form holds only for polarized cross sections. The sum over the parton polarizations introduces residual azimuthal correlations, even in the collinear limit. These have to be taken into account in the construction of the subtraction terms, see below. After integration over the azimuthal angle, factorization of the form of Eq. (3) holds also in the unpolarized case. The splitting functions $\hat{P}_{kj\rightarrow i}(u)$ are universal, process-independent functions. This has the important consequence that physical parton densities and fragmentation functions can be defined in a process-independent way.

- In the soft limit, the situation is slightly more complicated. For a gluon\(^c\) with label $k$ becoming soft, the factorization formula now reads
  \[ \sigma \sim \frac{\alpha_s}{2\pi} \sum_{i,j \neq k} C_{ijk} \frac{p_ip_j}{(p_ip_k)(p_jp_k)} \sigma_{\text{Born}}, \]  
  The sum runs over all pairs of partons \{ $i$, $j$ \}, and the $C_{ijk}$ are constant coefficients. The structure of this formula can be easily understood in terms of an eikonal approximation, where the matrix element $\mathcal{M}$ factorizes as
  \[ \mathcal{M} \sim \frac{p_i\epsilon(\lambda)}{p_ip_k} \mathcal{M}_{\text{Born}}. \]
  The sum over the gluon polarization $\lambda$, $\epsilon(\lambda)$ being the gluon polarization vector, leads to the form given in Eq. (4).

In order not to spoil the cancellation of soft and collinear singularities from the real corrections against those of the virtual corrections, the observables $\mathcal{O}$ have to fulfill certain conditions. This is the topic we now turn to.

\(^c\)Soft quarks do not lead to a soft singularity. The matrix element is singular as $1/E$, but this singularity is compensated by a factor $E$ in the phase space volume.
An observable $\mathcal{O}$ is called *infrared-safe*, if the functions $\mathcal{O}^{(n)} (p_1, \ldots, p_n)$ of the parton momenta respect soft and collinear limits:

$$
\mathcal{O}^{(n)} (p_1, \ldots, p_i, \ldots, p_n) \xrightarrow{p_i \to 0} \mathcal{O}^{(n-1)} (p_1, \ldots, \hat{p}_i, \ldots, p_n),
$$

$$
\mathcal{O}^{(n)} (p_1, \ldots, p_i, \ldots, p_j, \ldots, p_n) \xrightarrow{p_i \parallel p_j} \mathcal{O}^{(n-1)} (p_1, \ldots, \hat{p}_i, \ldots, \hat{p}_j, \ldots, p_n, p_i + p_j).
$$

Momenta denoted by $\hat{p}$ are to be omitted. The property of infrared safety for observables has the consequence that the factorization from Eqs. (3) and (4) also works if $\sigma$ is replaced by $\sigma \mathcal{O}$. This ensures the cancellation of the infrared singularities for the case of a convolution of the parton cross section with an observable, as in Eq. (2).

3. Universal Calculations in Next-to-Leading Order for Jet Quantities

The factorization theorems of perturbative QCD make it possible to perform calculations for a particular process in such a way that arbitrary infrared-safe observables can be evaluated numerically. For this it is necessary to extract and cancel the infrared singularities in an observable-independent way. Technically, there are two different procedures to achieve this: the phase-space-slicing method \[3\] and the subtraction method \[4\]. These techniques can be illustrated by means of a simple example \[5\].

Assume that the integral

$$
I = \int_0^A \mathrm{d}x \, x^{-\epsilon} \frac{1}{x} \, f(x)
$$

is to be evaluated. The $x$-integration stands for the phase-space integral, the factor $x^{-\epsilon}$ is the regulator$^e$, the term $1/x$ is the infrared singularity (a “propagator term”), and the integrable function $f(x)$ contains regular parts of the cross section and of the observable.

- In the case of the *phase-space-slicing method*, the range of integration is split into two parts $[0, a]$ and $[a, A]$ by means of an arbitrary small technical cut-off parameter $a$. For the lower interval, the function $f(x)$ can be approximated by $f(0)$ up to terms $\mathcal{O}(x)$, which regulate the $1/x$ singularity, and, after integration, give rise to terms $\mathcal{O}(a)$. For the upper interval, the regulator $\epsilon$ can be set to zero. The final result for the integral is thus

$$
I = -\frac{1}{\epsilon} f(0) + f(0) \ln a + \int_0^A \mathrm{d}x \, \frac{1}{x} f(x) + \mathcal{O}(a) + \mathcal{O}(\epsilon).
$$

The first term is the singular part, which would cancel against a similar singular part of opposite sign from the virtual corrections. The remaining terms

$^d$ Reference \[5\] actually contains the first fully general algorithm to calculate subtraction terms.

$^e$ The calculations are performed in $d = 4 - 2\epsilon$ space-time dimensions, so that UV and IR singularities are regulated and made explicit in the form of poles in $\epsilon$. Infrared singularities are regulated for $\epsilon < 0$. 
are finite. The integral in the third term can be evaluated numerically. The method is exact only in the limit \( a \to 0 \) (in practice, very small values for \( a \) are sufficient). The large logarithm in the second term is compensated by a similar logarithm from the numerical integration in the third term.

- The subtraction method requires the definition of a subtraction term, such that a splitting of the integral into a finite and a singular part is possible. As an example we give a specific way to define the subtraction term:

\[
I = \int_0^A dx \ x^{-\epsilon} \left( f(x) - f(0) \right) + \int_0^A dx \ x^{-\epsilon} \frac{1}{x} f(0).
\]  

(9)

The subtraction ("local in phase space") in the first integral renders the integrand integrable for \( \epsilon \to 0 \), and the final result is thus

\[
I = -\frac{1}{\epsilon} f(0) + f(0) \ln A + \int_0^A dx \ x^{-\epsilon} \frac{1}{x} \left( f(x) - f(0) \right) + O(\epsilon).
\]

(10)

The integral in the third term can again be performed numerically. Because \( f(0) \), the subtracted term in the integrand, is the residue of \( f(x)/x \) at \( x = 0 \), this method is also called the residue method.

The advantage of the phase-space-slicing method is that it can be implemented in a straightforward way: all that is required is the integral of the real corrections over small cones in phase space (corresponding to the approximation \( x \approx 0 \)) and a numerical implementation in the form of a Monte-Carlo program of the cross section itself. The disadvantage is that the cancellation of the term proportional to \( \ln a \) is delicate; the statistical fluctuations require a large number of Monte-Carlo events. Moreover, the convergence of the final result in the limit \( a \to 0 \) has to be checked, in principle, for every calculated expectation value. These technical problems are not present in the case of the subtraction method, because no small cut-off has to be used\(^f\). However, the implementation of the method requires the construction of the subtraction term, which is a non-trivial task. If this can be afforded, the subtraction method is the method of choice.

For the particular case of \((1+1)\)- and \((2+1)\)-jet production in deeply inelastic scattering, there are by now several calculations in the form of weighted Monte-Carlo programs available:

- **PROJET** \([6]\): The jet definition is restricted to the modified JADE scheme; the program is based on the calculation in Refs. \([7\text{-}9]\).

- **DISJET** \([10]\): Again the jet definition is restricted to the modified JADE scheme; the program is based on the calculation in Refs. \([11\text{-}12]\).

\(^f\)Strictly speaking, a tiny cut-off of the order of \( 10^{-8} \) to \( 10^{-10} \) has to be introduced in order to make the cancellation \((f(x) - f(0))/x \) work, because there is only a finite number of significant digits available.
• **MEPJET** [13]: This is a program for the calculation of arbitrary observables which uses the phase-space-slicing method. The calculation [14] uses the Giele–Glover formalism [15] for the analytical calculation of the IR-singular integrals of the real corrections, and the crossing-function technique [16] to handle initial-state singularities. The latter requires the calculation of “crossing functions” for each set of parton densities.

• **DISENT** [17]: This program is based on the subtraction method. The subtraction term is defined by means of the dipole formalism\(^g\) [18, 19].

• **DISASTER++** [20]: This is a C++ class library\(^h\). The subtraction method is employed, and the construction of the subtraction term resembles the method of Ref. [3], i.e. it is obtained by the evaluation of the residues of the cross section in the soft and collinear limits. Double counting of soft and collinear singularities is avoided by means of a fully general partial fractions method.

Why a new calculation? There are two reasons: (a) The existing programs have the restriction that the number of flavours is fixed \((N_f = 5\) in the case of MEPJET and \(N_f\) fixed, but arbitrary for DISENT). For studies of the scale-dependence it is necessary to have a variable number of flavours, in order to be consistent with the scale evolution of the strong coupling constant and the parton densities. DISASTER++ makes the \(N_f\) dependence explicit in the “user routine” on an event-by-event basis, and thus results for arbitrary renormalization and factorization scales can be binned simultaneously.

(b) DISASTER++ is already set up such that the extension to one-particle-inclusive processes will be possible without the necessity of re-coding the contributions which are already present for the jet-type observables.

In the following we will illustrate how the subtraction term for the real corrections is constructed. We will be concerned with infrared-safe observables that are zero if more than one parton is soft, or if more than two partons are collinear, or combinations thereof. These are the most relevant ones in practical applications. In principle, the procedure is simple. In energy and angle variables, soft singularities are of the form \(1/E_i^2\), for parton \(i\) being soft, and collinear singularities are given by terms \(1/v_{ij}\), for partons \(i, j\) being collinear, where \(v_{ij} = (1 - \cos \vartheta_{ij})/2\), and \(\vartheta_{ij}\) is the angle between the two partons. The singularities can be extracted by performing the limits \(E_i \to 0\) and \(v_{ij} \to 0\) of the terms \(E_i^2 \sigma, v_{ij} \sigma\) and \(E_i^2 v_{ij} \sigma\), and the subtraction terms

\(^g\) The subtraction term is written as a sum over dipoles (an “emitter” formed from two of the original partons and a “spectator” parton). Besides the factorization theorems of perturbative QCD, the main ingredient is an exact factorization formula for the three-particle phase space, which allows for a smooth mapping of an arbitrary 3-parton configuration onto the various singular contributions.

\(^h\) The acronym stands for “Deeply Inelastic Scattering: All Subtractions Through Evaluated Residues”. Most of the program is written in C++. A FORTRAN interface is in preparation; thus there will not be any problem to interface the class library to existing FORTRAN code.
will consequently be given by
\[
\frac{1}{E_i^2} \left[ E_i^2 \sigma \right]_{E_i=0}, \quad \frac{1}{v_{ij}} \left[ v_{ij} \sigma \right]_{v_{ij}=0}, \quad \frac{1}{E_i^2 v_{ij}} \left[ E_i^2 v_{ij} \sigma \right]_{E_i=0, v_{ij}=0}. \tag{11}
\]

There is, however, the problem of overlapping singularities, or double counting. Consider a term of the form \(1/(E_1^2 v_{12} v_{13})\). Here the problem is that if parton 1 is soft, then there are two regions which give rise to an additional collinear singularity: \(v_{12} = 0\) and \(v_{13} = 0\). In order to include every possible singular configuration only once in the subtraction term, it is convenient to separate the singular terms. This can be done in two ways: (\(\alpha\)) division of the phase space, or (\(\beta\)) separation of the singularities in the matrix element. In Ref. \[5\] a combination is used: \(1/(v_{12} v_{13})\) is written in terms of partial fractions as \(1/(v_{12} + v_{13}) + 1/(v_{12} v_{13})\), and the energy of the \(E_1\)-integration is restricted to be smaller than \(E_2/2\). In principle, phase-space cuts of this kind do not pose a problem in Monte-Carlo programs. However, it is preferable that the integrand be a smooth function. We achieve this by using the following general formula for partial fractions:
\[
\frac{1}{x_1 x_2 \cdots x_n} = \sum_{\sigma \in S_n} \frac{1}{x_{\sigma_1} (x_{\sigma_1} + x_{\sigma_2}) \cdots (x_{\sigma_1} + \ldots + x_{\sigma_n})}. \tag{12}
\]

The sum runs over all \(n!\) permutations of \(n\) objects\(^i\). The most straightforward way to apply this formula to the case at hand \([(2+1)\)-jet production\)] would be to set the set of variables \(\{x_1, \ldots, x_9\}\) to \(\{E_1^2, E_2^2, E_3^2, v_{01}, v_{02}, v_{03}, v_{12}, v_{13}, v_{23}\}\) (\(p_0\) is the momentum of the incident parton), where the energies are rescaled such that they are dimensionless. The product of the cross section and the observable \(\sigma O\) is then rewritten as \([x_1 \cdots x_n \sigma O]/(x_1 \cdots x_n)\), and the partial fractions identity (12) is applied to the denominator. The numerator is regular: the two-parton singularities are regulated by the \(x_i\), and the remaining singularities are regulated by the observable (recall that we restrict the discussion to this kind of observables). The general partial fractions formula now gives us a “hierarchy” of singularities: the “leading” one \(1/x_{\sigma_1}\), the “subleading” one \(1/(x_{\sigma_1} + x_{\sigma_2})\), and so forth. It turns out that for the observables under consideration, we have to consider only the leading and subleading singularities. Let us write this singular part, for a specific term in the sum, as \(k_A\), and the remaining terms from the product \(\sigma O\) as \(\tau_A\); the index \(A\) parametrizes the set of permutations. We then have \(\sigma O = \sum_A k_A \tau_A\). Each of the terms is singular only if the “leading variable” is zero. The subtraction term can therefore be constructed with respect to the leading and subleading variable in \(k_A\). More precisely this is done in the following way. For each index \(A\) there is a particular parton label \(i_A\) related

\(^i\) This decomposition has a nice technical property. Holding \(\sigma_1\) fixed, excluding \(1/x_{\sigma_1}\) from the sum, and setting \(x_{\sigma_1}\) to zero in the remainder, the sum over the restricted set of permutations yields \(1/(x_1 \cdots \hat{x}_{\sigma_1} \cdots x_n)\). This property is useful to recombine terms after the soft and collinear limits have been performed.
to it (for example, for $1/[E_1^2 (E_1^2 + v_{12})]$ or $1/[v_{12} (E_1^2 + v_{12})]$ this would be $i_A = 1$). The phase space $\text{dPS}^{(n)}$ is factorized according to $\text{dPS}_{i_A} \text{dPS}^{(n-1)}$, i.e. the one-parton phase space corresponding to parton $i_A$ is pulled out of the full phase space. Finally the phase space integration from Eqn. (2) is rewritten as

$$\int \text{dPS}^{(n)} \sigma O = \sum_A \int \text{dPS}_{i_A} k_A \left( \int \text{dPS}^{(n-1)} \tau_A - \left[ \int \text{dPS}^{(n-1)} \tau_A \right]_{\text{soft/coll. limit}} \right)$$

$$+ \sum_A \int \text{dPS}_{i_A} k_A \left[ \int \text{dPS}^{(n-1)} \tau_A \right]_{\text{soft/coll. limit}}. \quad (13)$$

The first integral is finite and can be calculated numerically. The second integral contains all infrared singularities. The term in the square bracket is simple because of the factorization theorems of QCD, and the one-particle integral over the kernel $k_A$ and the factorization contribution from the term in the square brackets can be performed easily$^j$.

4. Numerical Results and Applications: $\alpha_s$ and the Gluon Density

We now turn to some applications of $(2+1)$-jet observables. Loosely speaking, observed jets of hadrons can be identified with partons at hard scales. The subsequent fragmentation process is assumed to be sufficiently soft in order to keep non-perturbative effects small. Jets cannot be defined in a canonical way, rather they are objects by definition. It is convenient to define jets in terms of iterative cluster algorithms$^k$. There are three ingredients in the definition of a cluster algorithm: (i) a distant measure which determines the relative distance of two clusters in momentum space (for example their invariant mass $s_{ij} = (p_i + p_j)^2$, (ii) a mass scale $M^2$ that determines whether two clusters are to be combined into a single one (for example if $s_{ij} < M^2$), and (iii) a recombination procedure that prescribes how two clusters $p_i$ and $p_j$ are to be merged into a single cluster $p_*$ (for instance $p_* = p_i + p_j$). There are various choices for (i), (ii) and (iii). The example given is the JADE algorithm$^j$ in the so-called E-scheme. This particular algorithm is known for large hadronization corrections. This problem can be reduced by explicitly forcing clusters to behave as if they were massless (JADE and P-schemes). The large corrections partly come from the fact that for JADE-type algorithms, soft partons are combined first, even if they

$^j$There are a few complications, though. The first one is that the required sum of $9!$ terms, which has to be done numerically, contains too many terms to be efficient (or even possible). The solution is to perform the partial fractions decomposition separately for the energy and angle terms. The second complication is the presence of correlations in azimuthal angle for collinear singularities, which have been mentioned above. They can be dealt with by introducing a fictitious azimuthal variable for the collinear configurations.

$^k$“Non-cluster” algorithms of the cone-type suffer from the problem that the outcome may depend on the choice of a seed in the jet-finding process. This effect is particularly large at small transverse momentum. Cluster algorithms are well-defined without any ambiguity in the procedure.
differ considerably in their direction. Intuitively, such partons should possibly be combined with other partons nearby, even if they are hard. This can be achieved by means of a distance measure based on relative transverse momentum, leading to the $k_T$ algorithm, defined in the Breit frame of reference. For deeply inelastic scattering, it has been introduced in Ref. [22].

In the case of hadrons in the initial state, the corresponding remnants need a special treatment, because partons in the very forward direction lead to initial-state singularities. This problem is taken care of by either including the remnant jet in the clustering procedure (as in the case of the modified JADE algorithm) or by discarding partons which are too close to the remnant (as for the $k_T$ algorithm). In any case a well-defined prediction for the (2+1)-jet cross section is obtained.

![Cut-dependence of $R_{(2+1)}$ for the modified JADE cluster algorithm (a) and for the $k_T$ algorithm (b). The jet resolution mass is $M^2 = cW^2$ for the JADE algorithm and $M^2 = cQ^2$ for the $k_T$ algorithm. Leading order [ ], next-to-leading order: $E$-scheme [ ], $P$-scheme [ ], JADE-scheme [ ]. The recombination scheme dependence of the $k_T$ algorithm is small, therefore only the $E$-scheme is shown in this case.](image)

Figure 1: Cut-dependence of $R_{(2+1)}$ for the modified JADE cluster algorithm (a) and for the $k_T$ algorithm (b). The jet resolution mass is $M^2 = cW^2$ for the JADE algorithm and $M^2 = cQ^2$ for the $k_T$ algorithm. Leading order [ ], next-to-leading order: $E$-scheme [ ], $P$-scheme [ ], JADE-scheme [ ]. The recombination scheme dependence of the $k_T$ algorithm is small, therefore only the $E$-scheme is shown in this case.

In order to reduce systematic errors, it is convenient to consider the (2+1)-jet rate $R_{(2+1)} = \sigma_{2+1}/\sigma_{\text{tot}}$. Figure 1 shows the cut-dependence of $R_{(2+1)}$ for two different jet

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1 These algorithms have the property of factorization, which means that the $(n+1)$-jet cross section factorizes in exactly the same way into a product of a coefficient function and a parton density as a structure function does, without any residual explicit $x_B$-dependence. This kind of factorization is technically convenient for the calculation of resummed cross sections. We wish to stress that any acceptance cut on the final-state jets destroys this factorization property. What is important to have in a well-defined perturbative prediction is that the singular parts factorize in a universal way, which permits the definition of process-independent parton densities. This is, of course, the case for all infrared-safe observables, including the non-factorizing kind of jet algorithms.
The recombination scheme dependence of the modified JADE algorithm is large, as shown in Fig. 1a. For the E-scheme the QCD corrections may be as large as 100%; they are considerably smaller for the “massless” recombination schemes. The perturbative stability of the $k_T$ algorithm is better, here the QCD corrections are of the order of a few per cent (Fig. 1b).

Deeply inelastic scattering is a multi-scale problem. Besides the photon virtuality $Q^2$ there are other hard scales related to the scattering process, for example the transverse momentum $p_T$ of partons or jets. These hard scales can be very different from each other. The transverse momentum distribution of the jets for fixed lepton variables of $x_B = 0.01$ and $Q^2 = (20 \text{ GeV})^2$ is shown in Fig. 2a for the JADE PSscheme with a recombination scale of 0.02 $W^2$ in the hadronic CMS (a) and for the $k_T$ algorithm with a recombination scale of 0.5 $Q^2$ in the Breit frame (b). The transverse momentum distribution is quite broad, and therefore $p_T^2$ is not necessarily of the order

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$n$ The centre-of-mass energy is 300 GeV, the photon virtuality is restricted by $(10 \text{ GeV})^2 < Q^2 < (20 \text{ GeV})^2$, the total hadronic energy $W$ is assumed to be larger than 70 GeV and the lepton variable $y$ is restricted to values smaller than 0.7. The parton densities are MRS A' [23], and the renormalization and factorization scales are set to $Q^2$.

$n$ The large recombination scheme dependence of the JADE algorithm sometimes leads to the conclusion that the algorithm is ill-defined. This is a misconception. Different recombination schemes define different observables, and there is no reason why different observables should not lead to different predictions. However, the strong dependence of the jet rate on the recombination scheme points to the fact that jet masses are actually important.
of $Q^2$. This introduces a theoretical uncertainty, because it is not clear whether $Q^2$ or $p_T^2$ is the right scale to be used in the renormalized coupling $\alpha_s(\mu_r^2)$ and in the parton densities $f_i(\xi, \mu_f^2)$. As long as no resummed calculation is available, this uncertainty has to be considered as a contribution to the systematic error in case that physical quantities are extracted by using fixed-order next-to-leading-order calculations. 

As an example for an event shape variable we show a Thrust distribution in Fig. 2c. The phase space region is the same as the one of the jet rates shown before. The Current Thrust variable is defined by $T_z = 2 \sum_i p_{i\parallel}/Q [24]$, where the sum is over all partons in the current hemisphere (the hemisphere containing the virtual photon). The quantity $p_{i\parallel}$ is the longitudinal component of the parton momentum $p_i$ along the virtual photon direction in the Breit frame of reference. The Thrust variable goes to one for a (1+1)-jet-like configuration. Values of $T_z < 1$ are therefore obtained from processes of $O(\alpha_s)$. The QCD corrections are moderate. It turns out that the NLO calculations do not describe the experimental data well (see, for example, Ref. [25]). The inclusion of a power suppressed term of the form $A/Q$ with a large coefficient $A$ brings data and the theoretical prediction in good agreement. A quantitative derivation for the coefficient $A$ in terms of an effective coupling constant at low scales is given in Ref. [26].

Applications of NLO calculations in deeply inelastic scattering include the measurement of the strong coupling constant [27, 28] and a direct fit of the gluon density [29]. The $\alpha_s$ measurements show a clear evidence for the running of the coupling constant as a function of the scale $Q^2$. To complement the measurement via jet rates, measurements of $\alpha_s$ based on event shape variables are currently under way [25]. The gluon density has been determined via jet rates by means of a Mellin transform method [30, 31] which makes the repeated evaluation of the NLO cross section for the purpose of the fitting procedure feasible. The quark distributions are well known, and therefore the quark-initiated contribution can be subtracted. The obtained direct

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\[ q \]

Not shown here because of lack of space is a comparison of the scale dependence in leading and next-to-leading order. For the jet cross sections shown in Fig. 1 the scale dependence is reduced by about a factor of 2, if the renormalization and factorization scales are varied between 0.5 $Q$ and 2 $Q$. 

\[ p \]

For the Thrust variable, the requirement of a power-suppressed term $A/Q$ makes an $\alpha_s$ measurement conceptually difficult, because the coefficient $A$ is parametrized by a universal effective coupling constant $\alpha_{s,\text{eff}}$, whereas the perturbative contribution has the renormalized coupling constant as a coefficient. There are matching procedures to disentangle the two terms, but the size (about the same order of magnitude as the perturbative contribution) of the non-perturbative contribution is disturbing.

\[ q \]

This method is actually quite general and permits the calculation of arbitrary observables, the only limitation being that the factorization scale has to be fixed.
fit is in good agreement with gluon density parametrizations from global fits\(^r\).

5. One-Particle-Inclusive Processes

One-particle-inclusive processes are another very promising field. The corresponding cross sections can be considered to be related to a special type of observable \(O\), the fragmentation functions \(D_i(\xi, \mu_F^2)\). In the terminology used above, they are not infrared-safe, so that collinear singularities remain, which do not cancel. However, the singularities are of a universal form, and can be absorbed into the fragmentation functions. The redefined fragmentation functions are finite and universal, i.e. process-independent.

![Preliminary ZEUS 1994](image)

Figure 3: **Theoretical prediction vs. experimental data for the** \(Q^2\)-**dependence of** \(\sigma^h/\sigma_{tot}\) **for various bins of** \(x_p\). The kinks in the theoretical curves come from the overlay of several bins in \(x_B\) for the same bin in \(Q^2\). The region within the triangle cannot be described by means of the fragmentation function picture; see text.

Figure 3 shows a comparison of recent preliminary experimental data for charged hadron production from the ZEUS collaboration with a next-to-leading order calculation. The employed variable is \(x_p = 2P_h/Q\), where \(P_h\) is the momentum of the

\(^r\) For a specific value of the momentum fraction, the error bands show a cross-over. This phenomenon is well known from global fits \(^{32}\) and is a consequence of the limited number of parameters in the ansatz for the gluon density.
observed charged hadron $h$ in the current hemisphere of the Breit frame. The cross section $\sigma^h$ is a convolution of parton densities, fragmentation functions and perturbatively calculated coefficient functions. The parton densities are from the MRS A' set and the fragmentation functions are from Refs. [33, 34]. The coefficient functions have been calculated in Ref. [35]. The numerical results have been obtained by using the implementation in the program CYCLOPS of a recent recalculation [36]. The comparison of the theoretical prediction with experimental data shows a very good agreement, except for small values of $Q^2$ and small values of $x_p$.

What is the reason for this discrepancy? It is instructive to consider the situation in terms of the rapidity variable $y_h$ of the observed hadron. We consider the leading-order process, with a single parton fragmenting into the hadronic final state. Under the assumption that the hadron has a typical transverse mass $m_T$ of $\mathcal{O}(500\text{ MeV})$, the relation between $x_p$ and $y_h$ can be calculated. In the Breit frame, a positive value of $y_h$ stands for production in the current direction, and a negative value for production in the target direction. In order to make the fragmentation function picture a valid description, the hadron should be produced closely in rapidity to the parent parton, which means that $y_h$ should be larger than about $y_{\text{min}} = 1$ units of rapidity. This can be translated into a lower bound on $x_p$:

$$x_{p,\text{min}} = \frac{2m_T}{Q} \frac{1}{\sqrt{1 - \left(\tanh y_{\text{min}}\right)^2}}$$

(14)

This excludes the region roughly indicated by the triangle in the figure. The theoretical prediction thus fails in this region because the fragmentation function picture is not applicable: the hadron is not produced sufficiently close to the parent parton. Another reason is that mass effects (which are not included in the fragmentation function formalism) become important at small $Q^2$ and small $x_p$ for $x_p \approx 2m_\pi/Q$. This excludes a similar region in the plot. Outside of this critical region, in particular at large values of $Q^2$, the good agreement shows the universality of fragmentation functions.

A future application might be the determination of $\alpha_s$ via scaling violations of the fragmentation functions. This has been studied in some detail in Ref. [37]. It turns out that the main uncertainty is the dependence on a parametrization of the parton densities. This dependence is reduced at large values of $Q^2$, but then the available luminosity is the limiting factor. Another possibility might be the measurement of $\alpha_s$ from the hard scattering matrix element by means of $p_T$ spectra of charged particles. For this the NLO calculation is not yet available; however, as indicated above, the implementation of the matrix elements in DISASTER++ is already such that it may become available in the near future. The advantage of such a measurement would be that the fragmentation process is no longer modelled by means of event generators as in the case of jet cross sections or event shape variables, but described by a universal
parametrization of fragmentation functions with a QCD-predicted scale evolution.

Summary and Outlook

We have reviewed the status and the applications of next-to-leading-order calculations in deeply inelastic lepton–nucleon scattering. In the last few years, programs have become available that permit the calculation of arbitrary infrared-safe observables of the (2+1)-jet type. In next-to-leading order, these programs are still restricted to the case of photon exchange. A natural extension would be the case of $Z$ exchange, and the inclusion of charged-current processes. Moreover, the next-to-leading order is not yet available for polarized initial states, and for the $p_T$ spectra of identified hadrons.

Concerning the phenomenology of hadronic final states, the main problem is to understand the phase space region of small $Q^2$ (where, for example, the Thrust distribution is not well described in next-to-leading order), and the region of forward jets. With high-statistics data, the extraction of physical quantities can be improved by hard cuts to remove these dangerous regions, but a better theoretical understanding is certainly desirable. The physics of deeply inelastic hadronic final states will thus continue to be a very interesting topic in the future.

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