Multiplicity distributions in $e^+e^-$ annihilation into hadrons and the extended modified negative binomial

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Abstract

It is shown that simple extension of the modified negative binomial distribution describes negatively charged particle multiplicity distributions in $e^+e^-$ annihilation, measured in the whole phase space, as well as the modified negative binomial.

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1This work was supported in part by INTAS, contract INTAS-93-3602
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It has been shown recently that negatively charged particle multiplicity distributions in $e^+e^-$ annihilation into hadrons [1-5] and in the lepton-nucleon scattering [6] are well described by the modified negative binomial distribution (MNBD). It has been shown also [7] that the MNBD and its simple extension EMNBD quite well describe charged particle multiplicity distributions in restricted (pseudo)rapidity intervals in $e^+e^-$ annihilation into hadrons and in $e^+p$ collisions at HERA energies. The aim of this paper is to show that the EMNBD describes negatively charged particle multiplicity distributions in $e^+e^-$ annihilation, measured in the whole phase space, as well as the MNBD.

Let us remind that the MNBD can be defined by the probability generating function

$$M(x) = \sum_n P_n x^n = \left( \frac{1 + \Delta (1 - \varphi(x))}{1 + r (1 - \varphi(x))} \right)^k,$$

where $P_n$ is the probability to produce $n$ particles,

$$\varphi(x) = 1 - p (1 - x)$$

and $k$, $\Delta$, $p$ and $r$ are parameters connected to the mean multiplicity $<n>$ by the relation

$$<n> = k \ p \ (r - \Delta).$$

It has been assumed in the toy model proposed in [1,3,4] that the parameter $k$ is the number of sources of particle production at some initial stage of the interaction; these sources develop independently of each other according to some branching process (characterized by the parameters $r$ and $\Delta$) and during the branching process intermediate neutral clusters are produced. The parameter $p$ is equal to the cluster decay probability into a charged hadron pair and $(1 - p)$ is the probability of cluster decay into pair of neutral hadrons. The energy independence of the parameter $\Delta$ (or the product $\Delta p$), observed in the papers [1,3,4,5] can indicate that the branching process is the pure birth branching process and $\Delta$ can be fixed at $-1$. The probability generating function for the EMNBD has also the form (1) with $\varphi(x)$ replaced by

$$\varphi_2(x) = 1 - \varepsilon_1 (1 - x) - \varepsilon_2 (1 - x^2),$$

where the parameters $\varepsilon_1$ and $\varepsilon_2$ can be considered as the cluster decay probabilities into one and two pairs of charged hadrons respectively. The EMNBD
transforms into the MNBD when $\varepsilon_2 = 0$, in this case $\varepsilon_1 = p$. The probabilities $P_n$ for the EMNBD can be calculated using the formulae given in [7].

The results of the EMNBD fits to the negatively charged particle multiplicity distributions in $e^+e^-$ annihilation into hadrons [8-19] are given in the table 1. The parameter $\Delta$ was fixed at the value $\Delta = -1$ and the parameter $r$ was calculated from the mean charged multiplicity $<n>$ using the relation

$$r = \Delta + \frac{<n>}{k(\varepsilon_1 + 2\varepsilon_2)}.$$  \hspace{1cm} (5)

The integer parameter $k$ has been tested in the interval from 1 to 9, and the parameters $\varepsilon_1$ and $\varepsilon_2$ have been assumed to be nonnegative. The errors for the parameter $k$ were calculated using the quadratic interpolation for the $\chi^2$ dependence on $k$ on both sides from the $\chi^2_{\text{min}}$. One can see from the table 1 that the quality of the EMNBD fits is good. The $\chi^2$ for the EMNBD fits are in general smaller than the $\chi^2$ for the MNBD fits [4,5], shown in the last column of the table 1. It is necessary to note that the $\chi^2/NDF$ values for the fits should be considered just indicative, since the full covariance matrix is not given in the experimental publications and therefore the proper treatment of the correlations between measurements of the neighbour multiplicities is not possible.

The energy dependence of the parameters $\varepsilon_1$ and $\varepsilon_2$ is presented in the fig. 1. These parameters appear to be energy independent, if one excludes first and last energy points with $\sqrt{s}$ equal 3 and $\simeq 133$ GeV. The big values $\varepsilon_2$ at these energies can be explained by statistical fluctuations. One should note also that for these energies practically the same $\chi^2/NDF$ values are obtained for the EMNBD fits with the parameters $\varepsilon_1$ and $\varepsilon_2$ fixed at the average values $\simeq 0.65$ and $\simeq 0.16$ respectively (not shown).

The energy dependence of the parameter $k$ for the EMNBD fits is compared in the fig. 2 with the energy dependence of the parameter $k$ for the MNBD fits. The $k$ for both parametrizations rise almost linearly with $\log(\sqrt{s})$ at energies below $\simeq 30$ GeV and seem to approach some asymptotic value $\sim 7$ or 6 at higher energies.

At the present level of the experimental precisions in the multiplicity measurements both the MNBD and EMNBD parametrizations look more or less the same, this is explained by the smallness of the $\varepsilon_2$ with respect to the $\varepsilon_1$. The good quality of fits is expected also for the next iteration when one adds the term $\varepsilon_3 (1 - x^3)$, responsible for the cluster decay into
three charged hadron pairs, to the function $\varphi(x)$; indeed the probability $\varepsilon_3$ is expected to be smaller than the $\varepsilon_2$. These iterations remind the Padé approximants (ratios of the polynomials) of the increasing order, used in the calculational mathematics for the function approximation. The better precision of the measurements is needed in order to clarify whether the MNBD and EMNBD parametrizations are simply the successive approximations to the genuine multiplicity distribution, given by Nature or no additional iterations is needed.

In conclusion, it is shown that the EMNBD describes negatively charged particle multiplicity distributions in $e^+e^-$ annihilation into hadrons as well as the MNBD. The energy dependence of the parameter $k$, assumed to be the number of particle production sources, is similar for both parametrizations. The energy independence of the EMNBD parameters $\varepsilon_1$ and $\varepsilon_2$ supports the toy model proposed in [1,3,4]. Better precision in multiplicity measurements is needed to discriminate between the MNBD and the EMNBD.

**Acknowledgements**

I am indebted to O. L. Kodolova for reading the manuscript and critical comments.
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Figure Captions

Fig. 1 The energy dependence of the parameters $\varepsilon_1$ and $\varepsilon_2$ obtained from the EMNBD fits to the negative charged particle multiplicity distributions in the $e^+e^-$ annihilation.

Fig. 2 The energy dependence of the parameter $k$ for the EMNBD fits compared to the similar dependence for the MNBD fits.
Table 1: Results of the EMNBD fits to the negatively charged particle multiplicity distributions. The last column gives the $\chi^2$ values for the MNBD fits taken from [4,5].

| Experiment | $\sqrt{s}$ (GeV) | $k$ | $\varepsilon_1$ | $\varepsilon_2$ | $\chi^2$/NDF | $\chi^2$(MNBD) |
|------------|------------------|-----|----------------|----------------|----------------|----------------|
| MARKII[8]  | 3.0   | 1±0.67 | 0.462±0.049 | 0.510±0.031 | 5.6/2 | 3.3 |
|            | 4.0   | 2±0.47 | 0.612±0.029 | 0.211±0.029 | 3.1/3 | 7.6 |
|            | 4.8   | 2±0.42 | 0.622±0.019 | 0.281±0.018 | 8.2/3 | 14  |
|            | 7.4   | 3±3.8 | 0.544±0.074 | 0.159±0.069 | 5.1/4 | 5.2 |
| ARGUS[9]   | 9.36  | 3±3.8 | 0.559±0.036 | 0.339±0.033 | 2.6/7 | 5.1 |
|            | 10.58 | 4±2.76 | 0.596±0.119 | 0.308±0.102 | 1.3/7 | 1.6 |
| TASSO[10]  | 14.0  | 4±0.512 | 0.664±0.024 | 0.165±0.021 | 5.8/11 | 12  |
|            | 22.0  | 5±0.58 | 0.655±0.029 | 0.141±0.026 | 5.2/12 | 8.2 |
|            | 34.8  | 6±0.34 | 0.653±0.018 | 0.111±0.018 | 7.8/16 | 16  |
|            | 43.6  | 8±2.13 | 0.567±0.030 | 0.086±0.034 | 12.7/17 | 13 |
| HRS[11]    | 29    | 6±0.98 | 0.643±0.025 | 0.141±0.23 | 6.5/12 | 7.8 |
| AMY[12]    | 50    | 6±1.21 | 0.665±0.079 | 0.184±0.077 | 2.1/17 | 2.3 |
|            | 52    | 7±0.99 | 0.724±0.143 | 0.041±0.181 | 6.1/17 | 6.1 |
|            | 55    | 6±1.10 | 0.680±0.079 | 0.170±0.081 | 3.8/17 | 4.6 |
|            | 56    | 7±1.52 | 0.647±0.061 | 0.136±0.063 | 11.4/17 | 12 |
|            | 57    | 6±1.10 | 0.670±0.077 | 0.181±0.079 | 6.6/17 | 7.5 |
|            | 60    | 7±1.84 | 0.653±0.092 | 0.120±0.101 | 6.1/18 | 6.4 |
|            | 60.8  | 7±0.87 | 0.648±0.053 | 0.167±0.053 | 15.8/18 | 16 |
|            | 61.4  | 6±0.98 | 0.691±0.073 | 0.171±0.073 | 9.1/18 | 10  |
| ALEPH[16]  | 91.2  | 8±1.16 | 0.657±0.159 | 0.039±0.225 | 10.9/21 | 11 |
| ALEPH[17]  | 91.2  | 7±1.20 | 0.736±0.356 | 0.035±0.444 | 3.8/24 | 3.8 |
| DELPHI[13] | 91.2  | 7±0.55 | 0.754±0.203 | 0.334±0.252 | 18.6/19 | 30 |
| DELPHI[14] | 91.2  | 6±0.23 | 0.639±0.022 | 0.228±0.080 | 94.2/23 | 14 |
| L3[18]     | 91.2  | 7±0.75 | 0.696±0.107 | 0.074±0.119 | 14.0/21 | 14 |
| OPAL[15]   | 91.2  | 7±0.58 | 0.678±0.065 | 0.101±0.066 | 4.2/23 | 5  |
| OPAL[19]   | 133   | 5±1.75 | 0.433±0.150 | 0.493±0.113 | 3.7/21 | 5.3 |
