The influence of the static-pre-stress and mechanical damage variable in the thermal quality factor of two-temperature viscothermoelastic resonators

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Abstract
The mechanical damage variable, as well as the thermal and mechanical relaxation times, plays essential roles in the thermal quality factor of the resonators, where controls energy damping through the coupling of mechanical and thermal behavior. In this article, we developed a mathematical model in which a static-pre-stress and mechanical damage variable in the context of a two-temperature viscothermoelasticity of silicon resonator has been considered. The effects of static-pre-stress, thermal relaxation time, mechanical relaxation time, mechanical damage variable, isothermal frequency, and length-scale on the quality factor have been discussed in the context of a one-temperature and two-temperature models. The model predicts that significant improvement in terms of quality factors is possible by tuning the static-pre-stress, isothermal frequency, and length-scale of the resonator. Moreover, the thermal and mechanical relaxation times and the mechanical damage variable have impacts on the thermal quality factor.

Keywords
Resonator, thermal quality factor, viscothermoelasticity, two-temperature, mechanical damage variable

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Introduction
The most critical parameter of viscothermoelastic resonators is its thermal quality factor $Q$. Higher $Q$-factor indicates the less energy is dissipated during vibrations and low damping. The study of the energy dissipation mechanism is significant for the improvement of the design of micro-nano electromechanical resonators.¹⁻⁴

The first who introduced the $Q$-factor in thermoelastic dissipation is Zener,⁵⁻⁷ where he introduced an approximate analytical form of it and studied the thermoelastic damping in beams by treating the viscoelastic material. Many authors calculated the thermoelastic damping using the classical theory of thermoelasticity based on the Fourier heat law of heat conduction. Lifshitz and Roukes⁸ developed an exact expression for thermoelastic damping. Sun et al.⁹ discussed thermoelastic damping of a beam resonator based on a non-Fourier heat equation. Sharma and Sharma¹⁰ studied

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the energy dissipation in scale circular plate resonators using the Lord-Shulman theory of generalized thermoelasticity (L-S).

Tzou\textsuperscript{11,12} proposed a mathematical model to study heat conduction known as dual-phase lag (DPL). In this model, he established the temperature gradient and heat flux. Many authors used his model in heat transfer applications and physical systems,\textsuperscript{13–18} and thermoelastic damping vibration.\textsuperscript{19,20} Guo et al.\textsuperscript{21,22} introduced the thermoelastic damping theory of micro- and nanomechanical resonators using the DPL model.

The study of viscothermoelastic materials has become essential in mechanics. Biot\textsuperscript{23,24} discussed the theory of viscothermoelasticity in thermodynamics. A thermoviscoelasticity model of polymers at finite strains was derived by Drozdov.\textsuperscript{25} A new model of thermoviscoelasticity for isotropic media was established by Ezzat and El-Karamany.\textsuperscript{26}

As the size of a flexural resonator is reduced, its natural frequency increases, and thermoelastic damping also increases in the process. The natural frequency of beams can also be changed by the application of an axial force.\textsuperscript{3,27} A compressive force decreases in the natural frequency, whereas a tensile force increases in the natural frequency. Experimental results have been utilizing the frequency change to tune resonators. Furthermore, these experiments also suggest an increase in the $Q$-factor with the application of tensile stress.\textsuperscript{3,28} Youssef constructed a model of thermoelasticity with two-temperature heat conduction equations. This model is based on the conductive temperature and the dynamical temperature. The difference between the values of the two types of temperatures is proportional to the heat supply.\textsuperscript{29} Youssef and many other authors solved applications in the context of the two-temperature model of thermoelasticity.\textsuperscript{30–33}

Damage variables can be viewed in different ways. In a cross section of the damaged body, we thus consider an area element $dA$ with unit normal vector $n$. The area of the defects in this element is denoted by $dA_D$, and the amount of damage can be represented by the area fraction \textsuperscript{34}

$$ D_n = \frac{dA_D}{dA}, \quad 0 \leq D_n \leq 1 \quad (1) $$

where $D_n = 0$ corresponds to the undamaged material and $D_n = 1$ formally describes the totally damaged material with a complete loss of stress carrying capacity. In the real materials, at values of $D_n \approx 0.2 \ldots 0.5$ processes take place, which leads to total failure. If the damage is constant across a finite area, for instance, under uniaxial tension, the relation (1) reduces to

$$ D_n = A_D / A \quad (2) $$

The influence of microcracks that are inclined to the cross section cannot be described correctly in the same way. Correspondingly, in case of isotropic damage $D$ independent of $n$, hence, the effective stresses are given by \textsuperscript{34}

$$ \sigma_{ij} = (1 - D)\bar{\sigma}_{ij} \quad (3) $$

where $\bar{\sigma}_{ij}$ are the average stresses in the undamaged material.

Many applications and problems have been published under this definition of damage mechanics.\textsuperscript{35–39} Therefore, the present work is dealing with the mechanical damage variable, as well as the thermal and mechanical relaxation times, which plays essential roles in the thermal quality factor of the resonators. Hence, we will develop a mathematical model in which a static-pre-stress and mechanical damage variable will be applied in the context of a two-temperature viscothermoelastic model and resonator of silicon will be considered. The effects of the static-pre-stress, thermal relaxation time, mechanical relaxation time, mechanical damage variable, isothermal frequency, and length-scale on the quality factor will be discussed based on one-temperature and two-temperature models.

**Basic equations and model formulation**

The constitutive relationship for an Euler–Bernoulli’s beam with dimensions length $L$, width $b$, and thickness $h$ along the $x$, $y$, and $z$-axes, respectively, as in Figure 1. A moment of inertia $I$, and applied axial force $F$ and flexural displacement $w(x,y,z,t)$ can be written as\textsuperscript{9–12}

$$ (1 - D)EI \frac{d^4 w}{dx^4} - (1 - D)F \frac{d^2 w}{dx^2} + (1 - D)E\alpha_T \frac{\partial^2 T}{\partial x^2} + \rho I \frac{d^2 w}{dt^2} = 0. \quad (4) $$

where $E$ is the Young’s Modulus, and $\alpha_T$ is the coefficient of thermal expansion.

By deriving the equation of motion for lateral deflection, we obtain

![Figure 1. The beam resonator.](image-url)
\[ I = \int_{-b/2}^{b/2} \int_{-h/2}^{h/2} y^2 dy dz = \frac{bh^3}{12} \]  

(5)

and

\[ I_T = \int_{-b/2}^{b/2} \int_{-h/2}^{h/2} y \theta dy dz \]  

(6)

where \( I_T \) is the thermal moment about the x-axis.

Equation (4) takes the form

\[
(1 - D)E \frac{\partial^4 w}{\partial x^4} - \frac{12}{bh^3} (1 - D) F \frac{\partial^2 w}{\partial x^2} + (1 - D)E \alpha_T \frac{12 \frac{\partial^2 w}{\partial x^2}}{h^3} 
\]

\[
+ \int_{-h/2}^{h/2} y \theta dy + \frac{12 \rho \frac{\partial^2 w}{\partial t^2}}{h^2} = 0
\]

(7)

The two-temperature heat equations with one-relaxation time take the forms\(^{19,20,29}\)

\[
K \nabla^2 \phi = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \rho C_v \theta + \frac{(1 - D)\alpha_T T_0}{(1 - 2\nu)} E \theta \right)
\]  

(8)

and

\[
\theta = \phi - \alpha \nabla^2 \phi
\]  

(9)

where \( T_0 \) is the equilibrium temperature of the beam, \( \phi \) is the conductive temperature increment, \( \theta \) is the dynamical temperature increment, \( \alpha \) is non-negative constant and is called the two-temperature parameter, \( K \) is the thermal conductivity, \( \nu \) is the Poisson’s ratio, \( C_v \) is the specific heat at constant strain, and \( \tau_0 \) is the thermal relaxation time.

The volumetric deformations as follows

\[ e = e_{xx} + e_{yy} + e_{zz} \]  

(10)

and

\[
E e_{xx} = \sigma_0 - y E \frac{\partial^2 w}{\partial x^2}, \quad E e_{yy} = E e_{zz} = \]  

(11)

\[ = - \nu \sigma_0 + y y E \frac{\partial^2 w}{\partial x^2} + (1 + \nu) \alpha_T E \theta \]

where \( \sigma_0 = (F/bh) \) is the stress due to the applied axial force.

Then, we have

\[ E e = (1 - 2\nu)\sigma_0 - (1 - 2\nu)y E \frac{\partial^2 w}{\partial x^2} + 2(1 + \nu)\alpha_T E \theta \]  

(12)

The stress component in x-axis takes the form

\[
\sigma_{xx} = (1 - D) \left( \sigma_0 - E y \frac{\partial^2 w}{\partial x^2} - E \alpha_T \theta \right)
\]  

(13)

Substituting from equation (11) into equation (8), we obtain

\[
\nabla^2 \phi = \eta \left( \frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2} \right) \left( \frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2 w}{\partial x^2}
\]

\[
+ \alpha_T T_0 (1 - D) E \left( \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2 w}{\partial x^2}
\]

(14)

which gives

\[
\nabla^2 \phi + \frac{\alpha_T T_0 (1 - D) E}{K} \eta \left( \frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left( \frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2 w}{\partial x^2}
\]

\[
= \left( \frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left( \eta + 2 \frac{(1 - D)E \alpha_T (1 + \nu)}{K(1 - 2\nu)} \right) \theta
\]

(15)

where \( \eta = (\rho C_v / K) \).

For viscothermoelastic materials, we consider Young’s modulus in form\(^{19,23-25}\)

\[ E = E_0 \left( 1 + E_1 \frac{\partial}{\partial t} \right) \]  

(16)

where \( E_0 \) is Young’s modulus for the usual case, while \( E_1 \) is the mechanical relaxation time. Hence, equation (15) takes the form

\[
\nabla^2 \phi + \frac{\Delta E (1 - D)}{\alpha_T} \left( 1 + E_1 \frac{\partial}{\partial t} \right) \left( \frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) y \frac{\partial^2 w}{\partial x^2}
\]

\[
= \left( \frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left( \eta + 2 \Delta E \left( 1 - D \right) \left( 1 + \nu \right) \left( 1 + E_1 \frac{\partial}{\partial t} \right) \right) \theta
\]

(17)

where \( \Delta E = \left( T_0 \alpha_T^2 E_0 / K \right) \).

Because of no gradient exists in the z-direction, then, \( \nabla^2 \phi \approx (\partial^2 \phi / \partial x^2) \), hence, we have

\[
(1 - D)E_0 \left( 1 + E_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 w}{\partial x^2} - \frac{12}{bh^3} (1 - D) F \frac{\partial^2 w}{\partial x^2} + \]

\[
\frac{12 \alpha_T}{h^2} (1 - D)E_0 \left( 1 + E_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 w}{\partial x^2} \int_{-h/2}^{h/2} y \theta dy + \frac{12 \rho \frac{\partial^2 w}{\partial t^2}}{h^2} \frac{\partial}{\partial t^2} = 0
\]  

(18)
\[
\begin{align*}
\frac{\dd^2 \phi}{\dd t^2} + \frac{\Delta E}{\alpha T} (1 - D) y \left( 1 + E_1 \frac{\dd}{\dd t} \left( \frac{\dd}{\dd t} + \tau_0 \frac{\dd^2}{\dd x^2} \right) \frac{\dd^2 w}{\dd x^2} \right) \\
= \left( \frac{\dd}{\dd t} + \tau_0 \frac{\dd^2}{\dd t^2} \right) \left( \eta + 2 \Delta E (1 - D)(1 + v)/(1 - 2v) \right) \left( 1 + E_1 \frac{\dd}{\dd t} \right) \theta \\
\end{align*}
\]

and

\[
\theta = \phi - A \frac{\dd^2 \phi}{\dd y^2} 
\]

We consider the following functions

\[
w(x, t) = W(x)e^{i\omega t}, \quad \theta(x, y, t) = \theta(x, y)e^{i\omega t},
\]

(21)

Hence, equations (18)–(20) will be in the forms

\[
(1 - D)(1 + i E_1 \omega) \frac{\dd^2 W}{\dd x^2} - \frac{12}{h^4} E_0 (1 - D) F \frac{\dd^2 W}{\dd x^2} \\
+ \frac{12 \alpha T}{h^2} (1 - D)(1 + i E_1 \omega) \frac{\dd^2}{\dd x^2} \int_{-h/2}^{h/2} \psi \dd y - \frac{12 \rho \omega^2}{E_0 h^2} W = 0
\]

(22)

\[
\frac{\dd^2 \psi}{\dd y^2} + \frac{\Delta E}{\alpha T} (1 - D)(1 + i E_1 \omega)(i \omega - \tau_0 \omega^2) y \frac{\dd^2 W}{\dd x^2} \\
= (i \omega - \tau_0 \omega^2) \left( \eta + 2 \Delta E \frac{1 - D}{1 - 2v}(1 + i E_1 \omega) \right) \theta
\]

(23)

and

\[
\theta = \phi - A \frac{\dd^2 \phi}{\dd y^2}
\]

(24)

Eliminating \( \psi \) between equations (23) and (24), we get

\[
\frac{\dd^2 \phi}{\dd y^2} - \lambda^2 \phi = -A \frac{\dd^2 W}{\dd x^2}
\]

(25)

where

\[
\lambda^2 = \left( i \omega - \tau_0 \omega^2 \right) \left( \eta + 2 \Delta E \frac{1 - D}{1 - 2v}(1 + i E_1 \omega) \right) \left[ 1 - a(i \omega - \tau_0 \omega^2) \right] \left( \eta + 2 \Delta E \frac{1 - D}{1 - 2v}(1 + i E_1 \omega) \right)
\]

and

\[
\alpha = \frac{\Delta E}{\alpha T} (1 - D)(1 + i E_1 \omega)(i \omega - \tau_0 \omega^2) \left[ 1 - a(i \omega - \tau_0 \omega^2) \right] \left( \eta + 2 \Delta E \frac{1 - D}{1 - 2v}(1 + i E_1 \omega) \right)
\]

The general solution of the differential equation (25) takes the form

\[
\phi(x, y) = A \cos(\lambda y) + B \sin(\lambda y) + \frac{\alpha}{\lambda^2} y \frac{\dd^2 W(x)}{\dd x^2}
\]

(26)

The boundary conditions are as follows

\[
\left. \frac{\dd \phi(x, y)}{\dd y} \right|_{y = \pm h/2} = 0
\]

(27)

Hence, we obtain

\[
\phi(x, y) = \left[ y - \frac{\sin(\lambda y)}{\lambda \cos(\lambda h/2)} \right] \frac{\alpha}{\lambda^2} \frac{\dd^2 W(x)}{\dd x^2}
\]

(28)

From equations (4), (6), and (21), we obtain

\[
\frac{I}{\rho A} E_0 (1 - D)(1 + i E_1 \omega)f(\omega) \frac{\dd^2 W(x)}{\dd x^2} \\
- \frac{E_0}{\rho A} (1 - D) F \frac{\dd^2 W(x)}{\dd x^2} = \omega^2 W(x)
\]

(29)

where

\[
f(\omega) = 1 + \frac{\alpha T \beta}{\lambda^2} \left( \frac{h^3}{12} + \frac{h}{\lambda^2} - \frac{2 h}{\lambda^2} \tan(\lambda h/2) \right)
\]

Hence, we have

\[
\frac{I E_0}{\rho A} \left( \frac{\dd^2 W(x)}{\dd x^4} - \frac{F}{I E_0} \frac{\dd^2 W(x)}{\dd x^2} \right) = \omega^2 W(x)
\]

(30)

where \( E_0 = E_0(1 - D)(1 + i \omega_0 E_1)f(\omega_0) \).

For a simply supported beam, the exact analytical solution for the natural frequency is given as \(^9,10,27\)

\[
\omega = \frac{\pi}{L \sqrt{\rho A}} \sqrt{\frac{\pi^2 I E_0}{L^2} (1 - D)(1 + i \omega_0 E_1)f(\omega_0) + F}
\]

(31)

where \( \omega_0 \) is the isothermal value of frequency given by \(^27\)

\[
\omega_0 = \frac{q_n h}{12 \rho} \sqrt{\frac{E_0}{L}} \quad q_n \approx \frac{1}{L} (4.73, 7.853, 10.996, \ldots),
\]

(32)

In general, the frequencies are complex, the real part \(|\text{Re}(\omega)|\) giving the new eigenfrequencies of the beam in the presence of thermoelastic coupling, and the imaginary part \(|\text{Im}(\omega)|\) giving the attenuation of the vibration. The amount of thermoelastic damping, expressed in terms of the inverse of the quality factor, will then be given by \(^4-7,40\)
**Numerical results and discussions**

Now, the relationships between the variation of the thermoelastic damping with the beam height \( h \), and the beam length in different values of the applied axial force \( F \) on the microbeam resonator, which is made of silicon and clamped at two ends, will be explored. Material properties of silicon (Si) have been taken as follows:

\[
T_0 = 300(K), \rho = 2330(\text{kg m}^{-3}), \\
K = 141.04(\text{Wm}^{-1} \text{k}^{-1}), C_v = 1.64 \times 10^6(\text{Jm}^{-3} \text{K}^{-1}), \\
E = 165(\text{GPa}) \\
\alpha = 2.6 \times 10^{-6}(\text{K}^{-1}), a = 1.0 \times 10^{-10}(\text{m}^2), \\
E_1 = 1.0 \times 10^{-10}(\text{s}), F = 2.0(\text{GPa m}^2), \tau_0 = 0.001(\text{s})
\]

The aspect ratios of the beam are fixed \( \ell/h = 20, b = 2h \). For the scale of a microbeam, we will take the beam’s thickness \( h = 0.0002 \text{m} \).

Figure 2 represents the Q-factor per unit area \((Q^{-1}/A)\) with various values of the two-temperature parameter \( a = (0.0, \text{2.0}) \times 10^{-10} \text{m}^2 \) and force \( F = (0.0, 2.0) \) due to static-pre-stress when \( \omega_0 = 10.0, D = 0.2, L/h = 20.0, E_1 = 1.0 \times 10^{-10}, \tau_0 = 0.001 \). It is noted that the two-temperature parameter has a significant effect on the Q-factor for the different values of the force due to the static-pre-stress. The values of the Q-factor decrease when the value of the two-temperature parameter increases. The static-pre-stress has a significant effect on the thermal quality factor in the context of the one-temperature and two-temperature models, and increase in the value of the static-pre-stress leads to decrease in the thermal quality of the resonator. The maximum values of the thermal quality satisfy the following order

\[
\text{Max}Q^{-1}(F = 0.0, \text{one-tempe}) > \text{Max}Q^{-1}(F = 0.0, \text{two-tempe}) \]
\[
> \text{Max}Q^{-1}(F = 0.2, \text{one-tempe}) > \text{Max}Q^{-1}(F = 0.2, \text{two-tempe})
\]

(34)

Figure 3 represents the Q-factor per unit area \((Q^{-1}/A)\) with various values of the two-temperature parameter \( a = (0.0, 2.0) \times 10^{-10} \text{m}^2 \) and thermal relaxation time \( \tau_0 = (0.0, 0.001) \) when \( \omega_0 = 10.0, D = 0.2, L/h = 20.0, E_1 = 1.0 \times 10^{-10}, F = 2.0 \). It is noted that the two-temperature parameter has a significant effect on the Q-factor for the different values of the thermal relaxation time. The values of the Q-factor decrease when the value of the two-temperature parameter increases. The thermal relaxation time has a significant effect on the thermal quality factor in the context of the one-temperature and two-temperature models. The increase in the value of the thermal relaxation time leads to a decrease in the thermal quality of the resonator. The maximum values of the thermal quality satisfy the following order

\[
\text{Max}Q^{-1}(\tau = 0.0, \text{one-tempe}) > \text{Max}Q^{-1}(\tau = 0.0, \text{two-tempe}) \]
\[
> \text{Max}Q^{-1}(\tau = 0.2, \text{one-tempe}) > \text{Max}Q^{-1}(\tau = 0.2, \text{two-tempe})
\]
$$\max Q^{-1}(\tau_0 = 0, \text{one-temp.}) > \max Q^{-1}(\tau_0 \neq 0, \text{two-temp.})$$

$$(35)$$

Figure 4 represents the Q-factor per unit area ($Q^{-1}/A$) with various values of the two-temperature parameter $a = (0.0, 2.0) \times 10^{-10} \text{ m}^2$ and mechanical relaxation time $E_1 = (0.0, 1.0) \times 10^{-10}$ when $\omega_0 = 10.0$, $D = 0.2$, $L/h = 20.0$, $\tau_0 = 0.001$, $F = 2.0$. It is noted that the two-temperature parameter has a significant effect on the Q-factor for the different values of the mechanical relaxation time. The values of the Q-factor decrease when the value of the two-temperature parameter increases. The mechanical relaxation time has a significant effect on the thermal quality factor in the context of the one-temperature and two-temperature models. The values of the thermal quality factor decrease when the mechanical damage variable increases. The maximum values of the thermal quality satisfy the following order

$$\max Q^{-1}(D = 0.0, \text{one-temp.}) > \max Q^{-1}(D = 0.0, \text{two-temp.})$$

$$\max Q^{-1}(D = 0.2, \text{one-temp.}) > \max Q^{-1}(D = 0.2, \text{two-temp.})$$

$$(37)$$

Figure 5 represents the Q-factor per unit area ($Q^{-1}/A$) with various values of the two-temperature parameter $a = (0.0, 2.0) \times 10^{-10} \text{ m}^2$ and isothermal frequency $\omega_0 = (10.0, 15)$ when $D = 0.2$, $L/h = 20.0$, $\tau_0 = 0.001$, $E_1 = 1.0 \times 10^{-10}$, $F = 2.0$. It is observed that the two-temperature parameter has a significant effect on the Q-factor for the different values of the isothermal frequency. The values of the Q-factor decrease when the value of the two-temperature parameter increases. The value of isothermal frequency has a significant effect on the thermal quality factor in the context of the one-temperature and two-temperature models. The values of the thermal quality factor increase when the value of isothermal frequency increases. The maximum values of the thermal quality satisfy the following order

$$\max Q^{-1}(\omega_0 = 15, \text{one-temp.}) > \max Q^{-1}(\omega_0 = 15, \text{two-temp.})$$

$$\max Q^{-1}(\omega_0 = 10, \text{one-temp.}) > \max Q^{-1}(\omega_0 = 10, \text{two-temp.})$$

$$(38)$$

Figure 6 represents the Q-factor per unit area ($Q^{-1}/A$) with various values of the two-temperature parameter $a = (0.0, 2.0) \times 10^{-10} \text{ m}^2$ and mechanical damage variable $D = (0.0, 0.2)$ when $\omega_0 = 10.0$, $L/h = 20.0$, $\tau_0 = 0.001$, $E_1 = 1.0 \times 10^{-10}$, $F = 2.0$. It is noted that the two-temperature parameter has a significant effect on the Q-factor for the different values of the mechanical damage variable. The values of the Q-factor decrease when the value of the two-temperature parameter increases.
Figure 7 represents the Q-factor per unit area with various values of the two-temperature parameter. The values of the Q-factor decrease when the value of the two-temperature parameter increases. The value of the length-scale ratio of the resonator has a significant effect on the thermal quality factor in the context of the one-temperature and two-temperature models. The values of the thermal quality factor decrease when the value of the length-scale ratio of the resonator $L/h$ increases. The maximum values of the thermal quality factor satisfy the following order

$$\text{Max}Q^{-1}|_{L/h = 10, \text{one-\text{temp.}}} > \text{Max}Q^{-1}|_{L/h = 10, \text{two-\text{temp.}}}$$
$$> \text{Max}Q^{-1}|_{L/h = 20, \text{one-\text{temp.}}} > \text{Max}Q^{-1}|_{L/h = 20, \text{two-\text{temp.}}}
$$

(39)

Conclusion

A mathematical model of a static-pre-stress and mechanical damage variable in the context of a two-temperature viscothermoelastic silicon resonator has been developed. The effects of static-pre-stress, thermal relaxation time, mechanical relaxation time, mechanical damage variable, isothermal frequency, and length-scale on the thermal quality factor have been discussed in the context of the one-temperature and two-temperature models.

The results conclude the following:

- The thermal and mechanical relaxation times increase the thermal quality factor.
- The model predicts that significant improvement in terms of quality factors is possible by tuning the static-pre-stress, isothermal frequency, and length-scale of the resonator.
- The mechanical damage variable and two-temperature parameter have impacts on the thermal quality factor.
- Considering two-temperature model decreasing the value of the thermal quality factor.

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