Gap Symmetry of Superconductivity in UPd$_2$Al$_3$

H. Won,$^1$ D. Parker,$^2$ K. Maki,$^2$ T. Watanabe,$^3$ K. Izawa,$^3$ and Y. Matsuda$^3$

$^1$Department of Physics, Hallym University, Chuncheon 200-700, South Korea
$^2$Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484 USA
$^3$Institute for Solid State Physics, University of Tokyo, Kashiwanoha 5-1-5, Kashiwa, Chiba 277-8581, Japan

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Abstract

The angle dependent thermal conductivity of the heavy-fermion superconductor UPd$_2$Al$_3$ in the vortex state was recently measured by Watanabe et al. Here we analyze this data from two perspectives: universal heat conduction and the angle-dependence. We conclude that the superconducting gap function $\Delta(k)$ in UPd$_2$Al$_3$ has horizontal nodes and is given by $\Delta(k) = \Delta \cos(2\chi)$, with $\chi = ck_z$.

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1. Introduction

Since the discovery of the heavy-fermion superconductor CeCu$_2$Si$_2$ in 1979[1] the gap symmetries of unconventional superconductors have become a central issue in condensed-matter physics[2]. In the last few years, the angle-dependent magnetothermal conductivity in the vortex state of nodal superconductors has been established as a powerful technique to address the gap symmetry. This is in part due to the theoretical understanding of the quasiparticle spectrum in the vortex state of nodal superconductors, following the path-breaking work by Volovik[3, 4, 5]. Using this approach, Izawa et al have succeeded in identifying the gap symmetries of superconductivity in Sr$_2$RuO$_4$, CeCoIn$_5$, κ-(ET)$_2$Cu(NCS)$_2$, YNi$_2$B$_2$C, and PrOs$_4$Sb$_{12}$[6, 7, 8, 9, 10].

Superconductivity in UPd$_2$Al$_3$ was discovered by Geibel et al[11] in 1991. The reduction of the Knight shift in NMR[12] and the Pauli limiting of $H_{c2}$[13] indicate spin singlet pairing in this compound. Nodal superconductivity with horizontal nodes has been suggested from the thermal conductivity data[14] and from the c-axis tunneling data of thin film UPd$_2$Al$_3$ samples[15]. Very recently, McHale et al[16] have proposed $\Delta(k) = \Delta \cos(\chi)$ (with $\chi = ck_z$) based on a model where the pairing interaction arises from antiparamagnon exchange with $Q = (0, 0, \pi_c)$. Furthermore, the thermal conductivity data of UPd$_2$Al$_3$ for a variety of magnetic field orientations have been reported[18]. At first glimpse the experimental data appeared to support the model proposed by McHale et al.

The object of the present paper is to show that an alternative model, i.e. $\Delta(k) = \Delta \cos(2\chi)$, describes the thermal conductivity data more consistently. For this purpose we first generalize the universal heat conduction initially proposed in the context of d-wave superconductivity[19, 20] to a variety of nodal superconductors. We limit ourselves to quasi-2D systems with $\Delta(k) = \Delta f$ and $f = \cos(2\phi), \sin(2\phi), \cos \chi, e^{i\phi} \cos \chi, \cos(2\chi) \sin \chi$, and $e^{i\phi} \sin \chi$. It is found that the in-plane thermal conductivity $\kappa_{xx}$ is independent of $f$. On the other hand, the out-of-plane thermal conductivity $\kappa_{zz}$ can discriminate different $f$’s. Second, we extend an early study of the angle-dependent thermal conductivity[21] for $\kappa_{yy}$ in a magnetic field rotated in the z-x plane. The comparison of these results with experimental data indicates $\Delta(k) = \Delta \cos(2\chi)$.

2. Universal Heat Conduction

Here we consider the thermal conductivity $\kappa$ in the limit $T \to 0K$ in the presence of disorder. It is assumed that the impurities are in the unitary scattering limit[20]. We consider
the quasi-2D gap functions $\Delta(k) = \Delta f$ with $f = \cos(2\phi), \sin(2\phi)$ [d-wave superconductor as in the high-$T_c$ cuprates], $\cos \varphi, e^{i\phi} \cos \varphi$ (f-wave superconductor as proposed for Sr$_2$RuO$_4$), $\cos(2\chi), \sin \chi$, and $e^{i\phi} \sin \chi$. Following Ref. [20], the thermal conductivity within the conducting plane is given by

$$\kappa_{xx}/\kappa_n = \kappa_{yy}/\kappa_n = \frac{\Gamma_0}{\Delta} \left( (1 + \cos(2\phi)) \frac{C_0^2}{(C_0^2 + |f|^2)^{3/2}} \right)$$

$$= \frac{2\Gamma_0}{\pi \Delta \sqrt{1 + C_0^2}} E\left( \frac{1}{\sqrt{1 + C_0^2}} \right) = I_1(\Gamma/\Gamma_0)$$

where $\kappa_n$ is the thermal conductivity in the normal state when $\Gamma = \Gamma_0$, and $\Gamma$ is the quasi-particle scattering rate in the normal state. Here $\langle \ldots \rangle$ denotes the average over $\phi$ and $\chi$, and Eq.(1) tells us that the planar thermal conductivity is independent of the gap functions given above. Also $\Gamma_0 = \frac{\pi}{27} T_c = 0.866 T_c$ and $T_c$ is the superconducting transition temperature of the pure system. However, the quasi-particle scattering rate at $E = 0$ is given by $\Delta C_0$, and $C_0$ is determined by [20]

$$\frac{C_0^2}{\sqrt{1 + C_0^2}} K\left( \frac{1}{\sqrt{1 + C_0^2}} \right) = \frac{\pi \Gamma}{2\Delta}$$

and $\Delta = \Delta(0, \Gamma)$ has to be determined self-consistently as in [20]. Here $K(k)$ and $E(k)$ are the complete elliptic integrals. We show $I_1(\Gamma/\Gamma_0)$ in Fig.1. Now let us look at the out-of-plane thermal conductivity $\kappa_{zz}$. This is given by

$$\kappa_{zz}/\kappa_n = \frac{\Gamma_0}{\Delta} \left( (1 - \cos(2\chi)) \frac{C_0^2}{(C_0^2 + |f|^2)^{3/2}} \right)$$

$$= I_1(\Gamma/\Gamma_0)$$

for $f = \cos(2\phi), \sin(2\phi)$ and $\cos(2\chi)$, but

$$\frac{\kappa_{zz}}{\kappa_n} = \frac{4\Gamma_0}{\pi \Delta \sqrt{1 + C_0^2}} \left( E\left( \frac{1}{\sqrt{1 + C_0^2}} \right) - C_0^2 (K\left( \frac{1}{\sqrt{1 + C_0^2}} \right) - E\left( \frac{1}{\sqrt{1 + C_0^2}} \right)) \right)$$

$$= I_2(\Gamma/\Gamma_0)$$

for $f = \cos \chi, e^{\pm i\phi} \cos \chi$, and

$$\frac{\kappa_{zz}}{\kappa_n} = \frac{2\Gamma_0 \Gamma}{\Delta} \left( 1 - \frac{E\left( \frac{1}{\sqrt{1 + C_0^2}} \right)}{K\left( \frac{1}{\sqrt{1 + C_0^2}} \right)} \right) \equiv I_3(\Gamma/\Gamma_0)$$

for $f = \sin \chi, e^{i\phi} \sin \chi$. These functions are shown in Fig. 1.
In Fig. 2 we show $\kappa_{yy}(H)$ and $\kappa_{zz}(H)$ for $\mathbf{H} \parallel \hat{z}$ taken for UPd$_2$Al$_3$ [22]. In particular $(\kappa_{00})_{yy} = (\kappa_{00})_{zz}$ indicates $\Delta(k) \sim \cos(2\chi)$. Of course the effect of the magnetic field is not equivalent to the effect of impurities. But this comparison points to $\Delta(k) \sim \cos(2\chi)$ for UPd$_2$Al$_3$. We note also that for $f = \sin \chi$ and $e^{i\phi} \sin \chi$, there will be no universal heat
3. Angle-dependent magnetothermal conductivity

First let us recapture the quasiparticle density of states in the vortex state of nodal superconductors. \[ f = e^{i\phi}\cos \chi, \cos \chi, \cos 2\chi, \sin \chi \text{ and } e^{i\phi}\sin \chi \] Then the first two f’s have nodes at $\chi_0 = \pm \frac{\pi}{2}$, whereas $f = \cos 2\chi$ at $\chi_0 = \pm \frac{\pi}{4}$ and $f = \sin \chi$ and $e^{i\phi}\sin \chi$ at $\chi_0 = 0$.

In an arbitrary field orientation we obtain the quasiparticle density of states \[ G(H) \equiv \frac{N(0,H)}{N_0} = \frac{2v_a\sqrt{eH}}{\Delta}I_1(\theta) \] for the superclean limit and \[ G(H) \approx \left(\frac{2\Gamma}{\pi\Delta}\right)^{1/2}\log\left(\frac{2\Delta}{\pi\Gamma}\right)^{1/2}(1 + \frac{v_a^2eH}{8\pi^2\Delta\Gamma}\log(\frac{\Delta}{v_a\sqrt{eH}})I_2(\theta)) \] for the clean limit, where

\[ I_1(\theta) = (\cos^2 \theta + \alpha \sin^2 \theta)^{1/4}\frac{1}{\pi} \int_0^{\pi} d\phi \left( \cos^2 \theta + \sin^2 \phi (\sin^2 \phi + \alpha \sin^2 \chi_0) + \sqrt{\alpha \sin(\chi_0) \cos \phi \sin(2\theta)} \right)^{1/2} \]

\[ \simeq (\cos^2 \theta + \alpha \sin^2 \theta)^{1/4}(1 + \sin^2 \theta(\frac{1}{2} + \alpha \sin^2 \chi_0))^{1/2}\left(1 - \frac{1}{64}\frac{\sin^2 \theta(\sin^2 \theta + 16\alpha \sin^2 \chi_0 \cos^2 \theta)}{(1 + \sin^2 \theta(\frac{1}{2} + \alpha \sin^2 \chi_0))^2} \right) \]

and

\[ I_2(\theta) = (\cos^2 \theta + \alpha \sin^2 \theta)^{1/2}(1 + \sin^2 \theta(\frac{1}{2} + \alpha \sin^2 \chi_0)) \].

Here $\alpha = (v_c/v_a)^2$ and $\theta$ is the angle $H$ makes from the z-axis. Then the specific heat, the spin susceptibility and the planar superfluid density in the vortex state in the limit $T \to 0K$ are given by

\[
C_s/\gamma_NT = G(H), \frac{\chi_S}{\chi_N} = G(H),
\]

\[
\frac{\rho_{S\parallel}(H)}{\rho_{S\parallel}(0)} = 1 - G(H)
\]

Similarly the thermal conductivity $\kappa_{yy}$ when the magnetic field is rotated in the z-x plane is given by

\[
\kappa_{yy}/\kappa_n = \frac{2v_a^2eH}{\Delta^2}F_1(\theta)
\]

in the superclean limit and

\[
\kappa_{yy}/\kappa_{00} = 1 + \frac{v_a^2(eH)}{6\pi^2\Gamma\Delta}F_2(\theta)\log(2\sqrt{\frac{2\Delta}{\pi\Gamma}})\log(\frac{2\Delta}{v_a\sqrt{eH}})
\]
in the clean limit where

\[
F_1(\theta) = \sqrt{\cos^2 \theta + \alpha \sin^2 \theta (1 + \sin^2 \theta (\frac{3}{8} + \alpha \sin^2 \chi_0))}
\]

(16)

\[
F_2(\theta) = \sqrt{\cos^2 \theta + \alpha \sin^2 \theta (1 + \sin^2 \theta (\frac{1}{4} + \alpha \sin^2 \chi_0))}
\]

(17)

We show in Fig. 3 \(F_1(\theta)\) and \(F_2(\theta)\) for \(\alpha = 0.69\) (the value appropriate for UPd\(_2\)Al\(_3\)) and

\[\chi_0 = 0, \frac{\pi}{4}, \text{ and } \frac{\pi}{2},\]

which is compared with the experimental data \[18\] taken at \(T = 0.4K\) shown in Fig. 4. Except for the data taken for \(H = 2.5T\), the data for \(H = 0.5T, 1T\) and \(2T\) are consistent with \(\chi_0 = \frac{\pi}{4}\), indicating again \(f = \cos 2\chi\). We note also the sign of the twofold term in \(\kappa_{yy}\) at \(T = 0.4K\) changes sign at \(H = 0.36T\). This is consistent with the fact that for \(T < v\sqrt{eH}\) the nodal excitations are mostly due to the Doppler shift while for \(T > v\sqrt{eH}\) the thermal excitations dominate \[24\].

4. Concluding Remarks

We have analyzed recent thermal conductivity data \[18\] of UPd\(_2\)Al\(_3\) from 2 perspectives: universal heat conduction and the angle-dependence. The present study indicates \(\Delta(k) = \Delta \cos(2\chi)\). This is different from the conclusion reached in Ref. \[18\]. Also we have extended the universal heat conduction for a class of superconducting order parameters \(\Delta(k)\), which will be useful for identifying the gap symmetry of new superconductors such as URu\(_2\)Si\(_2\) and UNi\(_2\)Al\(_3\).

Furthermore, we have worked out the expressions for \(\kappa_{yy}\) when the magnetic field is rotated within the z-x plane. The angle dependence of \(\kappa_{yy}\) is extremely useful to locate the nodal lines when all nodal lines are horizontal. Perhaps \(\kappa_{yy}\) in Sr\(_2\)RuO\(_4\) will help to identify the precise position of the horizontal nodal lines in \(\Delta(k)\), if a further study of
nodal lines is necessary. Also after UPt$_3$ and UPd$_2$Al$_3$ we expect many of the U-compound superconducting energy gaps have horizontal lines.

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[1] F. Steglich, J. Aarts, C.D. Bredl, W. Lieke, D. Meschede, W. Franz and H. Schäfer, Phys. Rev. Lett. 43, 1892 (1979).
[2] M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
[3] G.E. Volovik, JETP Lett. 58, 469 (1993).
[4] H. Won and K. Maki, cond-mat/0004105.
[5] T. Dahm, H. Won and K. Maki, cond-mat/0006301.
[6] K. Izawa et al, Phys. Rev. Lett. 86, 2653 (2001).
[7] K. Izawa et al, Phys. Rev. Lett. 87, 57002 (2001).
[8] K. Izawa et al, Phys. Rev. Lett. 88, 27002 (2002).
[9] K. Izawa et al, Phys. Rev. Lett. 89, 137006 (2002).
[10] K. Izawa et al, Phys. Rev. Lett. 90, 11701 (2003).
[11] C. Geibel et al, Z. Phys. B 84, 1 (1991).
[12] H. Tou et al, J. Phys. Soc. Jpn. 64,725 (1995).
[13] J. Hessert et al, Physica B 230-232, 373 (1997).
[14] May Chiao, B. Lussier, E. Elleman and L. Taillefer, Physica B 230, 370 (1997).
[15] M. Jourdan, M. Huth and H. Adrian, Nature 398, 47 (1999).
[16] P. MacHale, P. Thalmeier and P. Fulde, cond-mat/0401520.
[17] N. Bernhoeft, Eur. Phys. J. B, 685 (2000).
[18] T. Watanabe et al, cond-mat/0405211.
[19] P. A. Lee, Phys. Rev. Lett. 71, 1887 (1993).
[20] Y. Sun and K. Maki, Europhys. Lett. 32, 335 (1995).
[21] P. Thalmeier and K. Maki, Europhys. Lett. 58, 119 (2002).
[22] T. Watanabe et al (unpublished).
[23] H. Won and K. Maki, Europhys. Lett. 56, 729 (2001).
[24] H. Won and K. Maki, Current Appl. Phys. 1, 291 (2001); also in “Vortices in Unconventional Superconductors and Superfluids”, edited by G. E. Volovik, N. Schopohl and R. P. Huebener (Springer, Berlin 2002).