Realistic error estimates on kinematic parameters

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ABSTRACT

Current error estimates on kinematic parameters are based on the assumption that the data points in the spectra follow a Poisson distribution. For realistic data that have undergone several steps in a reduction process, this is generally not the case. Neither is the noise distribution independent in adjacent pixels. Hence, the error estimates on the derived kinematic parameters will (in most cases) be smaller than the real errors. In this paper, we propose a method that makes a diagnosis of the characteristics of the observed noise. The method also offers the possibility to calculate more realistic error estimates on kinematic parameters. The method was tested on spectroscopic observations of NGC 3258. In this particular case, the realistic errors are almost a factor of 2 larger than the errors based on least squares statistics.

Key words: methods: data analysis – methods: numerical – methods: statistical – galaxies: kinematics and dynamics

1 INTRODUCTION

The bulk of kinematic information available on elliptical galaxies is nowadays retrieved through observation and analysis of the line-of-sight velocity distributions (LO SVD’s) of the stars in these galaxies.

The basic idea behind the study of LO SVD’s is that a galaxy spectrum can be generated by the spectrum of a star of similar type that has been redshifted by the rotation of the galaxy and smeared out by the velocity dispersion of the stars [Minkowski 1952].

The study of LO SVD’s as we know it now started about 30 years ago, when near the end of the seventies, Sar gent et al. [1977] and Tonry & Davis [1973] published surveys of galaxy redshifts and velocity distributions. The methods they presented in their papers were widely used afterward:

Fourier quotient method [Sargent et al. 1977]: The LO SVD is obtained by dividing the Fourier transform of the galaxy spectrum by the Fourier transform of the template spectrum, and transform the result back into pixel space. Although it seems to be natural to use Fourier space, this has also some disadvantages. The error calculation becomes complicated due to the correlations in the quotient and the method is very sensitive to template mismatch.

Cross-correlation method [Tonry & Davis 1973]: The peak in the galaxy-template correlation function is fitted by a Gaussian function. The method also provides a way to obtain error estimates on the kinematic parameters.

These techniques assume a Gaussian form for the broadening functions for mainly practical reasons: it provides a way of fitting the noise in the spectra and it allows to calculate convolutions analytically. The mean and variance estimation of the Gaussian function were considered as parameters describing the mean streaming and stellar velocity dispersion.

About ten to fifteen years later, the quality of the data had improved a lot and new techniques were developed trying to get more information out of the observed spectra than just two parameters. The main problem was to avoid contamination of the spectrum of a Gaussian form for the LO SVD. There are mainly two criteria that can be used to classify these techniques: (a) whether a non-parametric LO SVD or a parametricized LO SVD is derived. In the case of high S/N data, a non-parametric LO SVD yields all information you can get out of a spectrum. But because of the presence of noise there is always a need for some sort of smoothing. There are several ways to do this, either by applying some filter or by using a smoothing parametric error. For these non-parametric fitting methods, one or both criteria can be used to estimate an error estimate. (b) whether the fitting method is performed in Fourier space or in pixel space. A treatment of the problem in pixel space has the advantage that parts of the spectrum can be easily eliminated and allows an easier error estimate.

A lot of methods have been proposed:

Fourier fitting method [ Franx et al. 1983]: (parametric in Fourier space) A convolution of the LO SVD with the template is fitted to the galaxy spectrum in Fourier space. Error estimates are derived from a least squares fit.

Fourier correlation quotient method [Bender 1990]:
It is generally believed that, if the data are good enough to apply non-parametric tting, such an approach implies less bias than a parametric t. However, having a non-parametric LOSVD, it is still very useful to describe the obtained pro le by just a few characteristic numbers. Higher order information can be obtained either by calculating the m om ents of the velocity distribution or by tting a param et erized function to x. Since there is no danger of confusion when dealing with the m om ents of the LOSVD, we denote them here explicitly:

\[
Z = \left( \begin{array}{c} h_{1} \\ \vdots \\ h_{m} \end{array} \right) = \left( \begin{array}{c} \sigma_{1} \\ \vdots \\ \sigma_{m} \end{array} \right),
\]

and

\[
Z = \left( \begin{array}{c} h_{1} \\ \vdots \\ h_{m} \end{array} \right) = \left( \begin{array}{c} \sigma_{1} \\ \vdots \\ \sigma_{m} \end{array} \right),
\]

with L(x) the LOSVD. The kinematic m om ents are the mean projected rotation \( h_{1} \), the projected velocity dispersion \( \sigma_{1} \), the skewness \( \beta = \frac{1}{\sigma_{1}^{2}} \) and the kurtosis

\[
\gamma_{4} = \frac{\sigma_{4}}{\sigma_{1}^{4}}.
\]

The calculation of higher order m om ents out of the LOSVD model may be troublesome because of the strong dependence of the m om ents on the wings of the pro le. These wings unfortunately are the hardest to retrieve from observations. Therefore, in most practical cases, a param et erized function is tted to the LOSVD. A truncated Gaussian form is often used (Gerhard 1993; van der Marel & Franx 1993):

\[
L_{\theta}(v) = \exp \left( -\frac{(v-v_{0})^{2}}{2\sigma^{2}} \right) I_{\theta}(v),
\]

where \( I_{\theta}(v) \) is the LOSVD in the form of a Gaussian function.

In this expression, \( v_{0} \) and \( \sigma \) are free param eters, they are also called 'mean velocity' and 'velocity dispersion'. The functions \( I_{\theta}(x) \) are the \( J \) th order Gaussian form in the form of a Gaussian function.

In some cases it can be noticed that the error estim ates on the param eters are smaller than the scatter among neighbouring data points in the kinematic pro le. This is especially striking for the so-called higher order m om ents (third and fourth order m om ents). Examples of this behaviour can be seen in some of the data presented by Bender et al. (1994), Kormendy et al. (2000), Halkola et al. (2001). A possible interpretation of this fact is that the error estim ates given by the param et erization method are larger than the error estim ates obtained by the param erization method.
The derivation of these error estimates relies on assumptions that are generally not completely met. Though people are aware of this, it is generally unclear how large the discrepancy between reality and assumptions is.

One can come up with at least two good reasons why it is important to have realistic error estimates for dynamical modeling. First, for the construction of dynamical models, it is convenient to work with a goodness of fit indicator, which is meant to be an objective indicator of how good the model fits the data. Such goodness of fit indicators need error estimates on the data. However, if these are not realistic errors, it is not straightforward to interpret the values of this goodness of fit indicator, let alone derive meaningful confidence intervals for the models. In some cases, awareness of this problem may prompt one to adopt model error estimates to use in the goodness of fit indicator (e.g., Kochanek 1994, Cretton et al. 2000, Gerhard et al. 1995, Saglia et al. 2000, Kronawitter et al. 2000). However, playing with the error estimates on individual data points is equivalent to using different relative weights for the data points, and this is likely to complicate the interpretation of a goodness of fit indicator rather than simplify it.

A second issue of concern is that these parameters are often used to determine realistic dynamical models and to constrain the potential of a galaxy at the same time. In particular the velocity dispersion and the fourth order parameter play an important role in inferring the existence of central black holes or dark matter haloes around elliptical galaxies. In practice this is done by assuming a potential for the system and trying to fit a dynamical model based on the selected potential to simultaneously fit the velocity dispersion and fourth order parameter within the given error estimates. If no such model can be found, it is concluded that the assumed potential is not the right one, and the amount of dark matter is changed. If the error estimates are smaller than the real errors and the conclusions on the potential critically depend on them (e.g., Rix et al. 1991, Kronawitter et al. 2000, Cretton & van den Bosch 1999), some potential fits on the data cannot be rejected.

An inevitable consequence is that it is possible in some cases the use of realistic error estimates in dynamical modeling will weaken the dynamical evidence for dark matter in elliptical galaxies.

The method to derive realistic error estimates on kinematics parameters that is presented in this paper reaches out towards a need on the observational side and the modeling side. The method makes an implicit diagnosis of the noise distribution and uses the characteristics of this distribution as the basis of a new error estimate.

In section 2 a diagnosis of the problem with error estimates is given. An outline of the new method is presented in section 3, this is elaborated and illustrated in section 4. A discussion and conclusions can be found in sections 5 and 6 respectively.

2 DIAGNOSIS OF THE PROBLEM

In the absence of external sources of errors, a raw spectral image contains data points that follow a Poisson distribution, the variance of which is denoted by "Poisson noise". Another characteristic of a raw spectral image is that the data points in the image are independent. Hence, the power spectrum of the raw signal would match the definition of white noise: it has constant power in the frequency domain.

Before kinematic parameter errors can be calculated, the spectra have to be brought in a state that is referred to as "cleaned" and "calibrated". Fitting the spectra in such a condition requires a number of image processing steps, the so-called "standard reduction techniques", for which adequate tools are available in image analysis packages such as MIDAS or IRAF. But besides bringing the observed spectra in good condition, these reduction steps also have a non-negligible influence on the noise distribution.

For example, the removal of cosmic rays or pixels in ray events, where the values of two or more adjacent pixels are replaced by an average over a surrounding region, clearly introduce dependencies in the data. Likewise, dependencies are introduced by any operation where some kind of interpolation is involved, like e.g. calibration. Moreover, a sky subtraction does not guarantee a perfect elimination of the sky in the data, regardless of the level of sophistication that is used to perform the operation. The residual of this correction also adds up to the noise distribution. In some cases the above effects can be the source of considerable uncertainties on the derived kinematic parameters, while this is not expected by error estimates based on Poisson noise.

In some cases, a least squares technique is used to derive the kinematic parameters and error estimates from the data. The statistical interpretation of this method relies on the assumption that (1) the noise is independent and (2) Gaussian distributed on the input data. For large numbers, a Gaussian distribution is a fairly good approximation for a Poisson distribution and therefore the second assumption is correct. It is clear from the above arguments that the first condition generally is not met after data reduction. As a consequence, the errors derived from standard statistics will in many cases differ from the real errors on the kinematic parameters.

Sometimes, Monte Carlo simulations are used (e.g., Bender et al. 1994, Stark et al. 1999) to estimate the uncertainties on the derived parameters. For the realization of synthetic galaxy spectra, a Gaussian noise distribution with given S/N is used. So also this method may show a smearing tendency to underestimate the errors.

A coping with the fact that the cleaned and calibrated spectra in ages come as they are, it is possible to obtain error estimates that are more realistic estimates than those currently found in literature. This is the purpose of the present paper.

3 OUTLINE OF THE METHOD

To get an idea of more realistic errors on the parameters and to overcome the above problems we propose the following scheme. The steps are described in more detail in the following sections.

Perform a test of a model galaxy spectrum to the observed galaxy spectrum and determine the residual of this test. The model galaxy spectrum is a convolution of the selected template spectrum with a LOSVD. For this step it is important to use an expression for the LOSVD that offers enough freedom. The purpose is to test every part of the...
galaxy spectrum that can reasonably be described by a convolution of the template spectrum with a general and smooth LO SVD.

The residual of this template contains (1) the Gaussian noise on the raw spectrum, (2) the features left or introduced by several reduction steps. The errors estimated with the proposed method take into account artifacts from, at editing, cosmic rays, natal, effects of rebinning and remnants of sky removal, for as far as the latter has been removed reasonably well. On the other hand, the method does not account for errors in continuum subtraction nor template mismatch. Hence, the residual of the template can be considered as a realization of the observed noise involved in the problem: the term "observed noise" applies to "cleaned and calibrated" spectra and is used to make a distinction with Poisson noise. Where pure white noise is featureless and transforms into a power spectrum, the power spectrum of the residual will be different. A smooth representation of the power spectrum of the residual will be taken as significant information on the main characteristics of the observed noise.

The next step is to generate a number of synthetic galaxy spectra, that have a noise distribution with the same power spectrum as the observed noise, and to use these synthetic spectra to determine the spread on the kinematic parameters. These equivalent noise distributions that carry the characteristics of the original observed noise can be obtained by multiplying the Fourier transform of a white noise profile with a smooth representation of the power spectrum of the residual, and transforming this back.

The idea of using Monte Carlo simulations to determine the error estimates of course is not new, but the main point is that in the present method one tries to model the real characteristics of the noise.

A demonstration of the method on realistic galaxy spectra is given in section 4.1.

4 NGC 3258

4.1 Observations

NGC 3258 is an E1 galaxy and was observed with the ESO-NTT telescope in the nights of 27-28/2/2001 (64N-192). Spectra of the major axis were taken using the red arm of EM MI, covering the Ca II triplet. Grating # 7 was used, having a dispersion of 0.66 A/pk. The detector was a Tektronix CCD with 2048 x 2047 pixels, 24 m x 24 m in size and with a pixel scale of 0.27 A/pixel. A slit width of 15 A thus yields a spectral resolution of 3.67 A FWHM, resulting in an instrumental dispersion of about 54 km/s in the region of the Ca II triplet. For NGC 3258, several exposures of 3600 sec were taken (in total 11 hours). A number of standard stars (G dwarfs and K and M giants) were also observed.

4.1.1 Standard data reduction steps

Standard reduction steps were applied to these spectra. Out of the bias and dark exposures a correction term was derived. Several dome flats at R ages were used to compute an appropriate 

A After rebinning, the im ages were inspected carefully and remnants of cosmic rays were also removed by hand.

For the wavelength calibration, lamp spectra were taken just before or after each of the spectroscopic observations of the galaxy or the stellar template stars. For each row in the lamp spectra ages, a polynomial was constructed to transform the pixel scale into wavelength scale. The same coordinate transformation was then applied to the galaxy or star spectra. The calibrated images have a step of 0.3 A .

For the aim axis correction, the mean value of the aim mass at the beginning and end of the exposure was used. The contribution of the sky to the spectra was estimated from an upper and lower region of the image, where there was no contribution of the light of the galaxy or template star. Several galaxy spectra were reduced separately, aligned and combined into one galaxy spectra image.

Spatial rebinning of the galaxy spectra resulted in data with a S/N 50/bin (about 70 in the center). Only a small part of the spectrum, from 8500 A to 8750 A was used to determine the kinematic parameters.

4.2 The observed noise on the data

The idea is that the residual of the observed galaxy spectrum with a sufficiently excitable model spectrum can be attributed to some sort of noise involved in the problem. For a given stellar template spectrum it is important to consider a representation for the LO SVD that contains enough degrees of freedom to allow for the best fitting model spectrum. For this reason, nonparametric LO SVD's in combination with a flexible method to generate them are chosen to perform this. We have already chosen not to restrict ourselves to positive LO SVD's, because this limits the freedom of the method and at this point there are no physical restrictions on the LO SVD's. The method adopted here performs a least squares fit using a set of cubic spline basis functions to represent the LO SVD. An outline of the method can be found in Appendix A. However, for this step, any non-parametric fitting method with enough degrees of freedom will do. The model spectrum was a linear combination of convolutions with a number of observed template stars in which the coefficients were chosen on the basis of a goodness of fit estimator.

The non-parametric fitting is only used to get an idea of the fraction of the signal in the galaxy spectrum that cannot be accounted for by a model spectrum with the chosen template, and hence that will be part of the residual. Furthermore, also traces of sky lines, or incompletely removed cosmic rays will contribute to the residual of this fitting.

As an illustration, the results of the method for two different positions along the major axis of NGC 3258 are shown in figure 4.1. For r = 0.65 A and r = 11.0 A, the upper panel displays the LO SVD obtained from a non-parametric fitting with 10 degrees of freedom. The middle panel shows the observed galaxy spectrum and the residual. The lower panel shows the residual of the LO SVD. From these lower panels it is clear that the residual at 11.0 A (values mostlly between -100 and 100) is much larger than the residual at 0.65 A (values mostly between -60 and 60). This is misleading, since it shows that the Poisson noise is not the only source of errors. The galaxy spectrum at 0.65 A has much more counts in absolute value than the galaxy spectrum at 11.0 A, hence pure Poisson noise is higher at 0.65 A than at 11.0 A. The non-Poisson contribution to the residual at 11.0 A is likely to be due mostly from an imperfect sky subtraction.
4.3 The power spectrum of the residual

To see what the magnitudes of the various frequency components in these residuals are, their power spectra have been calculated. They are presented in Figure 3, the residual at 0\(\text{arcsec}\) in the upper panel, the residual at 11\(\text{arcsec}\) in the lower panel. The scale on the horizontal axis is the period of the power spectrum, instead of the more widely used frequency. This period (labeled \(\lambda\)) is directly related to the wavelength scales of the features in the original spectrum. Because of the wide range in values for \(\lambda\), a logarithmic axis is plotted. On top of the residual power spectra in Figure 3, a sample power spectrum for the expected white noise at these radii is presented. There is a clear difference between both profiles.

Having these power spectra representations, one would like to know which part is playing an important role for the determination of real errors on the kinematic parameters. In other words, which frequencies have an important impact on the derived kinematic parameters?

This can be investigated by means of some simulations. Galaxy spectra with various noise characteristics can be created starting from a galaxy spectrum obtained after convolution of a template spectrum with a known LOSVD and adding wave functions (sine and cosine functions) with different frequencies. For these artificial spectra, a best fitting model spectrum was determined using a least-squares minimization. The model spectrum was compared with the template spectrum and a LOSVD expressed as a truncated Gaussian-Hermite series. This analysis of the artificial spectra gives new values for the kinematic parameters, that will differ from the original kinematic parameters. The results of such simulations as a function of the scale size of the waves can be seen in Figure 4 (upper panels for d\(_{\text{HI}}/\text{v}\) and d\(_{\text{HI}}/\lambda\), lower panels for d\(_{\text{Hg}}/\text{v}\) and d\(_{\text{Hg}}/\lambda\)).

It is clear that the measurement of these kinematic parameters is sensitive to superpositions of wave functions with wavelengths (\(\lambda\)) between 10 A and 400 A (shaded regions in Figure 4). This range is independent of the wavelength...
range used in the analysis, but scales with \( \lambda = 325 \) km/s in this case.

This explains that the region of interest in the power spectrum of the residuals lies between \( \lambda = 10 \) A and \( \lambda = 400 \) A. It is also true (see figure 3) that the residual power spectra overshoot the white noise.

For short wavelengths, the residual power spectrum seems to have lower values than the Poisson noise power spectrum. At first sight this seems to be puzzling. However, this is a result of the interpolations that took place in the course of the data reduction process. Interpolation always implies some sort of smoothing, which means that features with a small wavelength scale disappear from the image.

Moreover, oversampling lies at the origin of the transfer of an amount of noise coming from small wavelength scale to larger wavelength scale. The stronger the oversampling, which has the artificially enlarging of feature wavelength scales as a result, the stronger this smearing effect. In the case of NGC 3258, the original spectrum had a step of 0.66 A/pixel, while the spectra after calibration have a step of 0.3 A/pixel.

4.4 Realistic error estimates

The power spectra of the residuals clearly show that the assumption of independent (white) noise is not a valid one. This implies that the classical statistical tools for calculating error estimates out of least squares fitting techniques cannot be used.

The only solution that is left, is to determine the uncertainties on the parameters through Monte Carlo simulations. In this case it is important to work with simulated galaxy spectra that have the same noise characteristics as the original galaxy spectrum. Our method proposes achieving this by using simulated galaxy spectra that give residuals that follow the same power spectrum as the initially determined residual.

To realize this, a smooth representation of the residual power spectrum is created. This is illustrated in figure 4.
The best setting values for decomposition were used, following van der Marel & Franx templates spectrum and a parameterized expression for the Carlos simulation are constructed.

In this way, the simulated galaxy spectra for the Monte Carlo simulation are constructed.

To these spectra, a model spectrum composed with the template spectrum and a parameterized expression for the LOSVD is fitted. For the paramaterization, a Gaussian template decom position was used, following van der Marel & Franx (1993). The best-fitting values for $h_3$, $h_4$, and $h_5$ were obtained using the Levenberg-Marquardt method as described in Press et al. (1989).

The error estimates are the root mean squares of the kinematic parameters that were derived from 100 realizations of the galaxy spectra. The results are shown in Figure 5 from upper to lower panel: the mean rotation velocity, the velocity dispersion, $h_5$ and $h_6$. The vertical error estimates present realistic errors. The horizontal lines correspond to error estimates based on least squares statistics.

The mean velocity reaches a maximum value of about 56 km/s and the realistic error estimates lie between 5.7 km/s and 11.3 km/s (with a mean error of 7.5 km/s and a mean error from least squares statistics of 4.9 km/s). The central velocity dispersion is 344 km/s, with an error of 8 km/s. The mean error on $h_3$ is 10.4 km/s, the mean error on $h_5$ based on least squares statistics is 3.8 km/s. For large radii, the profile is antisymmetric.

The profile for $h_3$ goes through the origin, apparently there is no serious problem with template mismatch. The mean error on $h_3$ is 0.02, the lowest value is about 0.013, the highest value is 0.039 (measured in an outer part). From least squares statistics, the mean error on $h_3$ is 0.01.

The $h_4$ profile is approximately symmetric, with lower values in the centre and higher values for the outer data points. The central data points show some scatter, but this is not inconsistent with the errors (at least 0.014) on the data in that region. The errors increase outward, the outer data points have an error of 0.049 (positive side) and 0.03 (negative side). The mean error on $h_4$ is 0.023, the mean error on $h_5$ following least squares statistics is 0.01.

For the $h_3$ and $h_4$ profiles, we compared the data points at either side of the centre. The amount of scatter in the profiles can be expressed as the mean distance between corresponding data points at positive and negative radius,

$\langle \sum_{i=1}^{n} (\sqrt{y_i^2 + y_i^2})^2 \rangle$,

with $n$ the number of data points at positive or negative radius, and $y_i$ and $y_i'$ the values for the parameters in corresponding points at positive and negative side of the centre. From $h_3$, the value of this scatter indicator is 0.019 and for $h_4$, this is 0.031. This means that for both parameter sets, the mean errors derived with the proposed method are close to this scatter indicator, while the mean errors following from least squares statistics are clearly smaller.

5 Discussion

To verify and validate the algorithm and its implementation, we now explore a few situations in which (following the discussion in Section 4) conventional and more realistic error bars based on the method described in this paper are expected to give different results by means of simulations. Using simulations has the advantage that also the true errors can be calculated and hence the error estimates can be compared.

5.1 Impulse noise

A fraction of the noise with non-Poissonian character is mentioned as spikes that overshoot the Poisson noise and that are relatively small on wavelength scale. This situation can
be mimicked in a simulation by adding pulse noise to a spectrum with otherwise pure Poisson noise.

Synthetic model spectra were obtained by convolving a LO SVD characterized by $h_1 = h_3 = 0$ and $= 250 \text{ km/s}$ with a K3 giant and adding Poisson noise. Different types of model spectra were then created by convoluting $10\%$ of the pixels in the spectrum with pulse noise with an amplitude of $3, 6, 9, 12$ or $15$ times the amplitude of the Poisson noise.

The RMS deviation on the kinematic parameters calculated out of $100$ equivalent realizations of one type of synthetic spectrum were considered to be the true errors on the kinematic parameters in that specific situation. For each type of spectrum, the analyses and models outlined in section 5.2 were applied (these results will be referred to as realistic errors), the realistic errors were calculated with a standard least squares technique. The results are presented in figure 6. The true errors are indicated by asterisks, the realistic errors in squares and the conventional errors in diamonds.

It is clear that the conventional errors returned by a standard technique are independent of the degree of contamination and can underestimate the true errors considerably. For the pulse noise with an amplitude of $3$ times that of the Poisson noise the conventional error underestimates the true error by $27\%$ for $h_1$, $37\%$ for $h_2$ and $31\%$ for $h_3$. The realistic errors on the other hand follow the true errors much better. For the same situation the difference between realistic errors and true errors is $10\%$ for $h_1$, $5\%$ for $h_2$, $0\%$ for $h_3$ and $14\%$ for $h_4$. For serious contamination, with an impulse noise with an amplitude of $15$ times that of the Poisson noise, the realistic error bars are slightly overestimated, but this is hopefully no longer a realistic situation.

5.2 Sampling

Changing the original sampling step of a spectrum has an impact on the error propagation that is completely neglected when Poisson noise is used as basic assumption for the error calculation.

For the simulations, a number of synthetic spectra were obtained with the same LO SVD and template stars as above, but the number of pixels was increased by $n$. The synthetic spectra were resampled in order to end up with spectra with $N$ pixels. This means that for the initial spectra, the wavelength scale was $n$ times the original wavelength scale, with $n$ equal to $1.5$, $2$, $3$, or $4$, in order to mimic oversampling and also $n$ equal to $0.5$ and $0.25$, thus mimicking undersampling.

These sampling factors are chosen only for the sake of illustrating the method. These resampled spectra were analyzed using the technique described in section 5.2, yielding realistic errors, and a standard technique for the conventional errors. Again, the realistic errors were calculated out of Monte Carlo simulations with $100$ spectra. The resulting error estimates are presented in squares for the realistic errors and diamonds for the conventional errors. The true errors are indicated by asterisks. Again, the conventional errors are independent of any change in sampling, while the realistic errors mainly follow the true errors. The change in errors scales roughly with $n$, with $n$ the sampling factor.

This result shows that care should be taken if kinematic parameters and conventional errors are estimated from spectra that are logarithmically rebinned. In that case the wavelength scale is clearly changed, but in such a way that some regions are more compressed and other regions are stretched.

5.3 Template mismatch

We do not claim that the proposed method is able to cope with the errors coming from template mismatch. But also in this case, the errors estimated with the method presented in this paper are closer to the true errors than the conventional errors obtained by standard techniques. The synthetic spectrum, created with a K3 giant, was also analyzed with the following templates: G2 dwarf, G5 dwarf, K1 giant, K4 giant.
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Figure 7. The results of simulations with rebinned spectra, where the wavelength scale after rebinning is n times the wavelength scale of the original spectrum. The sampling factor n is on the horizontal axis, the vertical axes represent the errors. The true errors are indicated in asterisks, the realistic errors in squares and the conventional error in diamonds.

Figure 8. The results of simulations with other template stars. The horizontal axis shows the sequence of template stars, the bars indicate the errors: black for the true error, dark gray for the conventional error and light gray for the "realistic" error (indicating that the error is obtained using the technique presented in this paper). The results for are in the upper panel, the results for h3 in the middle panel and the results for h4 in the lower panel. The results for h1 are not shown because the correction for the radial velocity of the template star would introduce an additional uncertainty.

It is clear that the true errors (black bars) are larger than the conventional errors and the realistic errors. For all three kinematic parameters, it is remarkable that the conventional errors show little variation if different stellar templates are used, whereas the differences in values for the "realistic" errors are larger. Moreover, in cases with large true errors, the "realistic" errors are closer to these values than the conventional errors. It seems that the errors obtained with this method can be used as indicators for template mismatch in the sense that larger errors indicate a poorer matching template, whereas the conventional errors clearly cannot be used as such.

6 CONCLUSIONS

As the technical limitations on the quality of observed data are diminishing, the numerical signal processing starts putting limitations on the information content of the observations. Therefore, it is important to have realistic estimates of the errors on the data.

In this paper, we propose a new method to calculate error estimates on kinematic parameters derived from spectroscopic observations. The starting point is the realization that the data points in the "reduced and calibrated" spectra in ages do not follow a Poisson distribution and are not
The proposed method first determines the characteristics of the real noise on the data. For this, a suitable fitting method requiring enough degrees of freedom is used. The residual of the fit is considered as observed noise. It is shown that the power spectrum of this observed noise can be very different from white noise. Realistic error estimates are obtained through Monte Carlo simulations with synthetic galaxy spectra, where the noise distribution follows the same power spectrum as the observed noise. In this way, errors are estimated taking the real characteristics of the noise into account, instead of relying on a Gaussian (or Poisson) noise distribution. Moreover, the proposed scheme can be applied in combination with several methods that are currently used to derive kinematic parameters from spectroscopic data.

The method was tested on spectroscopic observations of NGC 3258. In this particular case, the realistic errors are almost twice as large as the errors based on least squares statistics. What is most important, is that the realistic error estimations are more consistent with the scatter among neighbouring data points in the kinematic profile.

Simulations with spectra containing Poisson noise and impulse noise confirm that the proposed method estimates error variances that are closer to the true errors than conventional error estimations. Moreover, from simulations with synthetic spectra, it becomes clear that an oversampling of the spectra results in an underestimate of the errors on kinematic parameters when simple least squares statistics are used. This may be an important consideration when kinematic parameters are calculated from logarithmically rebinned spectra. Although this method is not able to calculate realistic errors on kinematic parameters obtained with an ill-fitting template star, the real errors do trace the trajectory of the template in a smooth way, i.e., in the sense that they become larger with increasing template length, unlike the usual error estimates.

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A P P E N D I X A : C U B I C S P LI N E F I T

In this paper, a new method is used to perform a non-parametric fit of a model spectrum to an observed galaxy spectrum. The method performs a least squares fit, using a set of cubic spline basis functions to represent the LO SVD. Each basis function is a third-degree cubic spline. There are two types of cubic spline basis functions, each a composition of two polynomial basis functions, as illustrated in Figure A1.

In this case, the cubic splines are defined in the interval [1,1]. The first third-degree basis function (left in Figure A1) is a composition of polynomial basis I and II. The second basis function (right in Figure A1) is a composition of polynomial basis II.
III and IV. These polynomials are:

I = 1 + 2x + 3x^2 (A1)
II = 1 + 2x + 3x^2 (A2)
III = x + 2x^2 + x^3 (A3)
IV = x + 2x^2 + x^3 (A4)

Function I is defined in the interval [0,1] and meets the requirements that \( y(1) = 0 = y'(1), y(0) = 1, y'(0) = 0 \) (prime denoting first order derivative). This behaviour is mirrored with respect to the vertical axis into the region \([-1,0]\), resulting in function II. The composition of I and II gives a symmetric cubic spline basis function. The first two conditions assure that the function declines smoothly to 0 at the edges. The other two conditions give the profile at peak on the y-axis.

Function IV is defined in the interval [0,1] and meets the requirements that \( y(1) = 0 = y'(1), y(0) = 0, y'(0) = 1 \). This behaviour is mirrored with respect to the centre into the region \([-1,0]\), resulting in function III. The composition of III and IV is an antisymmetric cubic spline basis function. Again, the first two conditions assure that the function declines smoothly to 0 at the edges. The other conditions make sure the function goes through the centre and gives a steeply declining or inclining profile near the centre.

For the practical implementation, the interval where the fitting is performed is divided in \( n \) sub-intervals, using \( n+1 \) points. This is illustrated in Figure A2. Each triple of adjacent points \( p_i, p_{i+1}, p_{i+2} \) is used to define a set of cubic splines that is included in the fitting.

The choice of the number of sub-intervals or basis functions is free. The less sub-intervals are considered, the more the line profile is smoothed. There is no a priori criterion that can be used to decide how many of these sub-intervals have to be used to obtain a good fit. Instead, a number of fits with an increasing number of sub-intervals (hence basis functions) is performed and the values for \( \tau \) indicate when the number of cubic spline pairs gives enough degrees of freedom to come to a good fit. This is illustrated in Figure A3 once a sufficient degree of freedom is reached, the values for \( \tau \) start decreasing only very slowly when the number of nodes is increased. If \( n \) points are used, \( n+1 \) sub-intervals are used for the fitting, yielding 2n degrees of freedom.

Kuijken & Merrild (1993) came up with a similar idea, but they used Gaussian functions for the decomposition of the LO SVD. The cubic splines used here are for the same degrees of freedom slightly more flexible in fitting. The fit is not restricted to strictly positive results. If one wants to obtain physical solutions with this method, linear constraints can be added to the fitting.

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Figure A3. The $\chi^2$ values of a number of $\omega$s with an increasing number of sub-intervals, for the spectra at $r = 0.8^{\circ}$ (filled squares) and $r = 11^{\circ}$ (filled triangles). For both spectra, using four nodes or more gives about the same values for $\chi^2$. 