Determining $\gamma$ using $B \to D^{**}K$

Nita Sinha

The Institute of Mathematical Sciences,
C.I.T Campus, Taramani, Chennai 600113, India.

(Dated: March 26, 2022)

Abstract

We propose the use of $B^\pm \to \bar{D}^{\pm*0}K^\pm$ decay modes for a theoretically clean determination of the weak phase $\gamma$. The self tagging decays of the neutral $D^{**}$ mesons, makes a measurement of the $b \to u\bar{c}s$ amplitude feasible. This overcomes the problem with the Gronau-London-Wyler proposal. Even an upper limit on the $B(B^- \to \bar{D}^{**0}K^-)$ will place an assumption free, lower bound on $\gamma$.  

*Electronic address: nita@imsc.res.in
It is hoped that theoretically clean, precise measurements of all the angles of the unitarity triangle will provide us with a testing ground for the Standard Model (SM) parameterization of $CP$ violation \cite{1}. A time dependent $CP$ asymmetry in the golden mode $B^0 \rightarrow J/\psi K_s$ has been successfully used to measure $\sin(2\beta)$ \cite{2}. Clean methods to determine all the $CP$ violating phases from a variety of final states are crucial in a search for physics beyond the SM. A clean extraction of the weak phase $\gamma$ has been an experimental challenge. While first estimates of the angle $\gamma$ are provided by methods based on approximations like SU(3) \cite{3}, precise measurements have to be free from such assumptions. A promising theoretically clean method was proposed by Gronau, London and Wyler (GLW) \cite{4}. The method relies on the interference of the $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ tree amplitudes. The former appears in the decay of $B^- \rightarrow D^0 K^-$, while the latter in $B^- \rightarrow \bar{D}^0 K^-$. If one observes the decay $B^- \rightarrow D_{CP} K^-$, where $D_{CP}$ is a $CP$ eigenstate into which the neutral $D$ decays, then the interference achieved allows determination of $\gamma$. The technique requires a measurement of the branching ratios: $\mathcal{B}(B^- \rightarrow D^0 K^-)$, $\mathcal{B}(B^- \rightarrow \bar{D}^0 K^-)$, $\mathcal{B}(B^- \rightarrow D_{CP} K^-)$ and $\mathcal{B}(B^+ \rightarrow D_{CP} K^+)$. However, the measurement of $\mathcal{B}(B^- \rightarrow \bar{D}^0 K^-)$ poses an experimental problem \cite{5}. The branching ratio $\mathcal{B}(B^- \rightarrow \bar{D}^0 K^-)$ measured by reconstructing the $\bar{D}^0$ in a hadronic mode is contaminated by $\mathcal{B}(B^- \rightarrow D^0 K^-)$, where the $D^0$ decays via a doubly-Cabibbo suppressed decay mode. The $\bar{D}^0$ cannot be tagged through a semileptonic mode either, due to the background from direct decays of the $B^-$. A resolution to this problem was provided by Atwood, Dunietz and Soni \cite{5}, who considered decays into two final states of the neutral $D$, with at least one of the final states being not a $CP$ eigenstate. The $\mathcal{B}(B^- \rightarrow D^0 K^-)$ is treated as a parameter that can be solved. The method has the advantage that the $CP$ asymmetry for the non-$CP$ eigenstate is larger, since the final state is chosen to be a doubly Cabibbo suppressed mode of $D^0$. However, a precise measurement of $\gamma$, from this method requires at least one precise determination of a doubly Cabibbo suppressed branching ratio of $D^0$. Another method using the vector-vector $D^* K^*$ final states was proposed \cite{6}. An angular analysis results in a large number of observables that allow determination of $\gamma$ as well as other parameters, including the doubly Cabibbo suppressed branching ratio. Several other variations of the $DK$ method have also been presented \cite{7}. Recently, alternate methods using either multibody final states or multiple final states of the $D^0$ have been investigated \cite{8}. While the sensitivity of most $DK$ methods is expected to be similar, larger number of methods are useful for increasing
statistics and providing consistency checks \cite{9}.

In this brief report, we propose the use of $B^\pm \to \bar{D}^{\ast\pm0}K^\pm$ decays, as these will allow an implementation of the GLW method. Since the excited $D$-meson states, $D^{\ast0}$ ($\bar{D}^{\ast0}$), decay into a charged $D^+\pi^-/D^+\pi^-$ ($D^-\pi^+/D^-\pi^+$), the branching ratios for both $B^- \to D^{\ast0}K^\pm$ as well as $B^- \to \bar{D}^{\ast0}K^\pm$ can be measured. Hence, the difficulty of the GLW method is overcome. Note that the vector $D^{*0}$ meson cannot decay into charged $D\pi$ modes and hence, using $D^*K^\pm$ final states does not resolve the problem.

Various $D^{\ast0}$ mesons have been observed by many collaborations \cite{10}. The Belle collaboration has recently measured the product of branching fractions: $\mathcal{B}(B^- \to D^{\ast0}\pi^-) \times \mathcal{B}(D^{\ast0} \to (D/D^*)^+\pi^-) \approx O(10^{-4})$, for four of the D-meson excited states \cite{11}. With larger number of $B-\bar{B}$ pairs expected, it should soon be possible to measure the branching ratios for the $D^{\ast0}K^-$ final state as well. The high luminosity at the super-B factories and other planned B-physics experiments, should enable a measurement of the branching fractions for $\mathcal{B}(B^- \to \bar{D}^{\ast0}K^-) \times \mathcal{B}(\bar{D}^{\ast0} \to (D/D^*)^-\pi^+)$, allowing $\gamma$ to be cleanly extracted. Even if only an upper limit on $\mathcal{B}(B^- \to \bar{D}^{\ast0}K^-)$ is available, it will still allow us to obtain a bound on $|\sin \gamma|$, free of any theoretical assumptions.

The decay amplitudes for $B^-$ may be defined as:

\begin{align}
\mathcal{A}(B^- \to D^{\ast0}K^- \to D^+\pi^-K^-) & \equiv a_c e^{i\delta_c} \\
\mathcal{A}(B^- \to D^{\ast0}K^- \to D^-\pi^+K^-) & \equiv a_u e^{i\delta_u} e^{-i\gamma},
\end{align}

(1)

where, $a_c, a_u$ are the decay amplitudes involving the $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions and $\delta_c, \delta_u$ are the corresponding strong phases. In the Wolfenstein parameterization \cite{12}, while the amplitude for $B^- \to D^{\ast0}K^-$ has no weak phase, that for $B^- \to \bar{D}^{\ast0}K^-$ has the weak phase $\gamma$. Interference of these amplitudes is achieved by looking at the decays into $CP$ eigenstates of the neutral $D$, $B^- \to D^{\pm}_{CP}\pi^0K^-$,

\begin{align}
\mathcal{A}(B^- \to [D^{\ast0}K^- \pm \bar{D}^{\ast0}K^-] \to D^{\pm}_{CP}\pi^0K^-) = \frac{r}{\sqrt{2}}[a_c e^{i\delta_c} \pm a_u e^{i\delta_u} e^{-i\gamma}],
\end{align}

(2)

where, $r$ is the ratio of the amplitudes of $D^0$ to a $CP$ eigenstate to that of the $D^+$ to a Cabibbo-allowed mode (or any mode in which it is reconstructed) and the $CP$ even (odd) eigenstates of $D^0$ are defined as: $D^{\pm}_{CP} = \frac{1}{\sqrt{2}}[D^0 \pm D^0]$. Using these and the amplitudes for the $CP$ conjugate modes, we have:

\begin{align}
\mathcal{B}_{sum} \equiv \mathcal{B}(B^- \to [D^{\pm}_{CP}\pi^0]_{D^{*0}}K^-) + \mathcal{B}(B^+ \to [D^{\pm}_{CP}\pi^0]_{D^{*0}}K^+) 
\end{align}
\[ r^2 (a_c^2 + a_u^2 + 2a_c a_u \cos \delta \cos \gamma) \] (3)

and the \( CP \) asymmetry,

\[
A_{CP} \equiv \frac{\mathcal{B}(B^- \to [D^{\pm}_{CP}\pi^0]_{D^*}K^-) - \mathcal{B}(B^+ \to [D^{\pm}_{CP}\pi^0]_{D^*}K^+)}{\mathcal{B}(B^- \to [D^{\pm}_{CP}\pi^0]_{D^*}K^-) + \mathcal{B}(B^+ \to [D^{\pm}_{CP}\pi^0]_{D^*}K^+)} \]

\[ = \mp 2a_u a_c \sin \delta \sin \gamma \]

\[ = \frac{a_c^2 + a_u^2 \pm 2a_c a_u \cos \delta \cos \gamma}{a_c^2 + a_u^2} \] (4)

where, \( \delta = \delta_c - \delta_u \).

The measured values for \( \mathcal{B}(B^- \to D^{*0}K^-) \) and \( \mathcal{B}(B^- \to \bar{D}^{*0}K^-) \) determine \( a_c \) and \( a_u \) respectively. While the corresponding amplitude \( a_c \) could be determined in the original GLW method, a measurement to determine their corresponding \( a_u \), was not feasible. Our choice of final states with excited neutral \( D \) mesons that decay into self tagging charged \( D\pi \) modes, has made the determination of \( a_u \) possible. Knowing \( a_c \) and \( a_u \) and the two observables in Eqs.(3 and 4), the phases, \( \delta \) and \( \gamma \) can be determined. \( |\sin \gamma| \) is determined up to a two-fold ambiguity from the relation:

\[
\sin^2 \gamma = \frac{4r^4a_u^2a_c^2 + A_{CP}^2 \mathcal{B}_{sum}^2 - X^2 \mp \sqrt{(4r^4a_u^2a_c^2 + A_{CP}^2 \mathcal{B}_{sum}^2 - X^2)^2 - 16r^4a_u^2a_c^2A_{CP}^2 \mathcal{B}_{sum}^2}}{8r^4a_u^2a_c^2} \]

(5)

where, \( X \equiv \mathcal{B}_{sum} - r^2a_c^2 - r^2a_u^2 \).

Note that the technique presented is also applicable to the decay of the neutral \( B \) mesons, \( \bar{B}^0 \to D^{*0}\bar{K}^*(0)(B^0 \to D^{*0}K^{*0}) \). The decay of \( K^* \) into self tagging modes render the neutral \( B \) decay modes to be treated exactly like the charged \( B \) decays. While the branching ratios in case of the neutral \( B \) decays are expected to be smaller, the \( CP \) asymmetry will be larger as both the \( b \to c \) as well as \( b \to u \) contributions will be colour suppressed and therefore comparable. Only inclusive data in the \( \bar{B}^0 \to D^{*0}\bar{K}^*(0)(B^0 \to D^{*0}K^{*0}) \) modes will be required; an angular analysis to obtain individual partial wave amplitudes need not be performed. In the decays of neutral \( B \) mesons, the usefulness of \( D^{*0} \) states, had been pointed out in Ref. [13], where a method to extract \( (2\beta + \gamma) \), using a time dependent study was presented.

Several of the \( D^{*0} \) modes have been observed and the product of branching ratios for production of \( D^{*0}\pi \) and the decay of \( D^{*0} \to D\pi(D^*\pi) \) have been measured. In principle, any of the \( D^{*0} \) modes could be used. Since the \( CP \) asymmetry will be larger for the mode with larger strong phase difference \( \delta \), this variety of \( D^{*0} \) states will allow one to choose
the state with the largest $CP$ asymmetry, large branching ratio and high reconstruction efficiency. For our numerical estimations below, as an example, we choose the $D_2^0$ resonance, since this has a narrow width and decays to a $D^+\pi^-$. The axial vector mesons, $D_1^0, D_{1}^{*0}$ are allowed to decay only into $D^{*+}\pi^-$. Hence, the reconstruction efficiency for the axial vector mesons might be expected to be lower than that for the tensor or the scalar. Moreover there is also the problem of mixing in case of the axial vector mesons. The scalar $D_0^{*0}$ is rather broad. A possible complication is that both the $D_0^{*0}$ and the $D_{2}^{*0}$ have overlapping Breit Wigner shapes. However, due to the narrow width of $D_2^{*0}$, it may be possible to select the region of interference and extract the region corresponding entirely to $D_0^{*0}$ \[14\].

In the following, we show, that even if the $B(B^- \to \bar{D}^{*0}K^-)$ is not exactly measured but only an upper limit is available, one can already start putting bounds on $|\sin \gamma|$. For this numerical analysis we use the following \[15\]:

$$a_c^2 = \lambda^2 B(B^- \to D_2^{*0}\pi^-) B(D_2^{*0} \to D^+\pi^-) B(D^+ \to K^-\pi^+\pi^+) = 2.69 \times 10^{-6}$$

$$r^2 = \frac{B(D^0 \to K_S\pi^0)}{B(D^+ \to K^-\pi^+\pi^+)} = 0.125$$

$$B_{sum} = 3.45 \times 10^{-7}. \quad (6)$$

Naively, we expect, $\frac{a_u^2}{a_c^2} = \left[\frac{|V_{ub}V_{cs}|}{|V_{cb}V_{us}|}\right]^2 a_2^2 a_1^{-2} = 0.030$, where, to obtain the colour suppression factor, we use $(\frac{a_2}{a_1})^2 = 2 \frac{B^0 \to D_0^{*0}}{B^0 \to D^-\pi^+} = 0.194$.

Using the numerical values of the parameters given by Eq. \[14\], an extremization of the expression for $\sin^2 \gamma$ given in Eq. \[15\] results in bounds on $|\sin \gamma|$ for limiting values of $a_u^2$. We find that while the first solution (with the -ve sign for the discriminant) gives a lower bound on $|\sin \gamma|$ for $a_u^2 \lesssim 5.4 \times 10^{-6}$, the second solution (with the +ve sign for the discriminant) provides an upper bound on $|\sin \gamma|$ for $a_u^2 \lesssim 7.0 \times 10^{-8}$. The first solution vanishes for $A_{CP} = 0$, on the other hand, the second is non-vanishing for zero $CP$ asymmetry. The $|\sin \gamma|$ contours corresponding to the first solution are shown in Fig.\[14\]. From the figure, lower bounds on $|\sin \gamma|$, for a measured upper limit on $a_u^2$ and non-vanishing $CP$ asymmetry can be easily estimated. For example, if we assume that the measured $a_u^2$ is bounded by $B(B^- \to \bar{D}^{*0}K^- \to D^-\pi^+K^-) < 5.0 \times 10^{-8}$, then for a measured $CP$ asymmetry $|A_{CP}| = 0.20$, we have the limit $|\sin \gamma| > 0.75$. For larger $CP$ asymmetries, the bound becomes tighter. The second solution results only in larger values of $|\sin \gamma|$, consistent with the bound obtained from the first solution. Once, the branching ratio, $B(B^- \to \bar{D}^{*0}K^- \to D^-\pi^+K^-)$ and the
FIG. 1: $|\sin \gamma|$ contours corresponding to various values of the $CP$ asymmetry, $A_{CP}$ and the branching ratio, $B(B^- \to D^{*0}K^- \to D^-\pi^+K^-)$.

direct $CP$ asymmetry are both measured, the value of $|\sin \gamma|$ corresponding to each of the solutions can be extracted.

In the above estimates, we used the branching ratio for the decay of $D^0$ to the particular $CP$ eigenstate $K_S\pi^0$. To improve statistics we could add either all $CP$ even or all $CP$ odd states. In fact, since the solution for $|\sin \gamma|$, involves only squares of terms that switch sign from $CP$ even to $CP$ odd, it is possible to achieve even higher statistics (increase $r^2$), by combining all possible $CP$ eigenstate modes. Also, $D^+ (D^-)$ could be reconstructed using additional final states, which could increase $a_u^2 (a_u^2)$.

To conclude, we have suggested the use of $B \to D^{*0}K$ modes for determination of $\gamma$. The $D^{*0} (D^{*0})$ decay to flavour specific modes $D^+/D^{*-}\pi^- (D^-/D^{*-}\pi^+)$, allowing measurement of the $b \to u$ amplitude. This overcomes the problem of the Gronau-London-Wyler proposal and a clean extraction of $\gamma$ is possible. The Belle collaboration has already measured the product of branching fractions: $B(B^- \to D^{*0}\pi^-) \times B(D^{*0} \to (D/D^*)^+\pi^-) \approx O(10^{-4})$, for four of the D-meson excited states. A measurement of the branching ratios for the $D^{*0}K^-$...
final state should be feasible very soon. The large number of $B - \bar{B}$ pairs expected at the high luminosity super-B factories and other planned B-physics experiments, should enable a measurement of the branching fractions for $\mathcal{B}(B^{-} \to \bar{D}^{*0}K^{-}) \times \mathcal{B}(\bar{D}^{*0} \to (D/D^{*})^{-}\pi^{+})$, allowing $\gamma$ to be cleanly extracted. Even an upper limit on the $\mathcal{B}(B^{-} \to \bar{D}^{*0}K^{-})$ will place an assumption free, lower bound on $\gamma$.

Acknowledgements: The author thanks G. Rajasekaran, A. Soffer, O. Long and D. London for their comments. This work was supported by a project under the Department of Science and Technology, India.

[1] For a review, see for example, P. F. Harrison and H. R. Quinn (Eds.), The BaBar Physics Book, SLAC Report 504, October 1998.

[2] T. E. Browder, to appear in the proceedings of 21st International Symposium on Lepton and Photon Interactions at High Energies (LP 03), Batavia, Illinois, August 11-16, 2003 [http://conferences.fnal.gov/lp2003/program/papers/browder.pdf]. arXiv:hep-ex/0312024

[3] M. Gronau and J. L. Rosner, arXiv:hep-ph/0311280

[4] M. Gronau and D. London., Phys. Lett. B 253, 483 (1991); M. Gronau and D. Wyler, Phys. Lett. B 265 (1991) 172.

[5] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3257 (1997) [arXiv:hep-ph/9612433];

[6] N. Sinha and R. Sinha, Phys. Rev. Lett. 80, 3706 (1998) [arXiv:hep-ph/9712502].

[7] I. Dunietz, Phys. Lett. B 270, 75 (1991); D. Atwood, G. Eilam, M. Gronau and A. Soni, Phys. Lett. B 341, 372 (1995); J. H. Jang and P. Ko, Phys. Rev. D 58, 111302 (1998); M. Gronau and J. L. Rosner, Phys. Lett. B 439, 171 (1998); M. Gronau, Phys. Rev. D 58, 037301 (1998) arXiv:hep-ph/9802315; D. Atwood and A. Soni, arXiv:hep-ph/0312100.

[8] Y. Grossman, Z. Ligeti and A. Soffer, Phys. Rev. D 67, 071301 (2003) [arXiv:hep-ph/0210433]; R. Aleksan, T. C. Petersen and A. Soffer, Phys. Rev. D 67, 096002 (2003) [arXiv:hep-ph/0209194]; A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D 68, 054018 (2003) [arXiv:hep-ph/0303187].

[9] Talk given by A. Soffer at Super B Factory Workshop in Hawaii, Honolulu, January 19-22, 2004 [http://www.phys.hawaii.edu/~superb04/talks/Soffer.ppt].

[10] Ref. 11 and references therein.
[11] K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0307021.

[12] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).

[13] B. Kayser and D. London, Phys. Rev. D 61, 116013 (2000) [arXiv:hep-ph/9909561].

[14] A. Soffer, private communication.

[15] The product $\mathcal{B}(B^- \to D_s^{*0}\pi^-)\mathcal{B}(D_s^{*0} \to D^+\pi^-)$ has been measured by the Belle Collaboration and is given in [11]. For all other branching ratios, we use the values listed in [16].

[16] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66, 010001 (2002).