Family symmetries

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Abstract. The hierarchical structure of quark and charged lepton masses and the small mixing angles in the quark sector is in stark contrast with the structure of neutrino masses and mixing angles. I discuss how these apparently disparate structure can be elegantly explained through an underlying discrete non-Abelian family symmetry.

1. Introduction
The pattern of quark and lepton masses and mixing angles is shown in figure 1. The structure shown assumes that, like the quarks and charged leptons, there are just three neutrino species and an hierarchical mass structure, although the data is consistent too with a near degenerate structure with mean mass slightly less than 1 eV. Clearly the lepton structure is quite different from the quarks. Apart from the fact the neutrinos are extremely light the mixing angles in the lepton sector are large [1], in contrast to the small mixing angles observed in the quark sector.

Figure 1. Quark and lepton masses and mixing angles.

This is illustrated more clearly in figure 2 where one may see that for the normal mass hierarchy the heaviest state, the atmospheric neutrino, is approximately an equal admixture of
The composition of the neutrino mass eigenstates for the case of a normal and an inverted hierarchy showing near tri-bi-maximal mixing and the form of the mixing matrix for this case. The shading indicates the neutrino flavour with $\nu_\mu$, $\nu_e$, and $\nu_\tau$ being represented by progressively darker shades and ordered left to right.

$\nu_\mu$ and $\nu_\tau$ while the solar neutrino is a nearly equal mixture of $\nu_e$, $\nu_\mu$ and $\nu_\tau$. The form of the MNS mixing matrix shown for this precise pattern is also shown and corresponds to what is known as tri-bi-maximal mixing [2].

Given these apparent differences, at first sight it seems unlikely that there is an underlying symmetry relating quarks and neutrinos. However if neutrino masses are due to the see-saw mechanism a comparison between the sectors is not straightforward. As I will discuss in this case the neutrino Dirac mass matrix which couples the left handed-neutrino states to the right-handed neutrino states has a similar form to the quark and charged lepton (Dirac) mass matrices. This opens the possibility that the phenomenologically successful Grand Unified (GUT) symmetry relations between down quarks and leptons may be extended to the up quarks and neutrinos. It also suggests that there may be an underlying spontaneously broken family symmetry in which the hierarchical structure of masses and mixings is related to an expansion in the family symmetry breaking scale to the mediator mass scale. The near tri-bi-maximal mixing strongly suggests that the family symmetry should be non-Abelian so that the Yukawa couplings involving different families may be related. Moreover I will argue that the apparent simple quantisation of the neutrino mixing angles suggests that the non-Abelian symmetry should be discrete rather than continuous.

2. Quark and charged lepton Dirac mass matrices

The structure of the quark and charged lepton masses is generated in the Standard Model after spontaneous symmetry breakdown by the Yukawa couplings:

$$L_{Yukawa} = Y_{ij}^u Q^i u^{c-j} H + Y_{ij}^d Q^i d^{c-j} \overline{H}$$

$$M_{ij}^u = Y_{ij}^u < H^0 >, \ M_{ij}^d = Y_{ij}^d < \overline{H}^0 >$$

Theoretical ideas for determining fermion masses relate to the underlying Yukawa Lagrangian. Unfortunately we are not able unambiguously to determine it because all we can measure are the CKM mixing matrix and the eigenvalues of the mass matrix - there is no information at all
about the unitary matrices acting on the right handed fields needed to diagonalise the fermion mass matrix and only incomplete information about the left-handed matrices. However it is interesting to note that the data for quarks and charged leptons is consistent with a very left-right symmetric structure of the form

$$\frac{M^{d,l,u}}{m_{b,e,d}} \simeq \begin{pmatrix} e^d & e^3 & -e^3 \\ e^3 & ae^2 & -ae^2 \\ -e^3 & -ae^2 & 1 \end{pmatrix}, \quad e^d = 0.15, \quad e^l = 0.15, \quad e^u = 0.05$$

(2)

where the constants $a^{u,d} = 1$ and $a^l = -3$. The hierarchical structure of the mass matrix is determined in terms of powers of the small parameters $e^{u,d,l}$. The “texture zero” in the $(1,1)$ element that may result from an underlying family symmetry leads to a successful prediction for the Cabibbo angle and a good relation between the down quark and charged lepton masses (see eq. (9) below).

2.1. Grand Unified symmetries

The structure of the down quark and charged leptons in eq. (2) is consistent with an underlying GUT symmetry such as $SU(5)$ or $SO(10)$ relating quarks and leptons. In what follows I shall assume that the theory is supersymmetric up to soft supersymmetry breaking terms of electroweak breaking scale. This is needed in a GUT to stabilise the mass hierarchy and leads to the remarkably precise prediction of gauge coupling unification. There are several good mass relations that may be derived from the underlying GUT. These are illustrated in figure 3 for the case the down quark and charged lepton (Dirac) mass matrices are symmetric. The natural expectation is that the mass matrix elements of the down quark and charged lepton mass matrices should be equal and this works well for the elements in the $(3,3)$, $(1,2)$ and $(2,1)$ positions leading to the approximate relations $m_b(M_X) = m_e(M_X)$ and $Det(M^l) \big|_{M_X} = Det(M^d) \big|_{M_X}$ where $M_X$ is the GUT scale. After including radiative corrections these relations can be in good agreement with the measured masses [7]. In the supersymmetric case the corrections are sensitive to the SUSY spectrum and $\tan \beta$. Continuing the measured masses up to the GUT scale a recent analysis [7] gives $m_b(M_X)/m_e(M_X) = \{1.0, 0.6, 1.0\}$, $Det(M^l) \big|_{M_X}/Det(M^d) \big|_{M_X} = \{0.6, 0.3, 1.04\}$ for a SUGRA spectrum with $\tan \beta = 2$, $\tan \beta = 30$ and for an anomaly mediated spectrum respectively. However equality for the down quark and charged lepton $(2,2)$ matrix elements is not consistent with the measured masses. What is required is for the lepton element to be a factor of 3 greater than the down quark element. As first shown by Georgi and Jarlskog [8] this is possible in specific GUT theories if a different group invariant dominates the $(2,2)$ element with the factor of 3 being just the number of colours. It leads to the relation $3m_s/m_\mu(M_X) = 1$ to be compared to the measured result $3m_s/m_\mu(M_X) = \{0.9, 0.82, 1.05\}$.

3. The see-saw mechanism and Neutrino mass matrices

The effective Lagrangian structure of the light neutrino masses describing near tri-bi-maximal mixing is given by

$$L'_{Majorana, eff} = \beta \nu_i \theta_{23}^j \nu_j \theta_{23}^i + \gamma \nu_i \theta_{123}^j \nu_j \theta_{123}^i$$

(3)

where $\theta_{23} = (0, 1, -1)$ and $\theta_{123} = (1, 1, 1)$ and $\beta$ and $\gamma$ are constants. This is to be compared to the leading term for quarks and charged leptons responsible for generating the heavy third generation masses

$$L'^{l}_{Dirac} = \alpha \nu_i \theta_3^j \nu_j \theta_3^i + \ldots$$

(4)

where $\theta_3 = (0, 0, 1)$. One sees that if there is a simple relation between the masses of quarks and leptons one must explain why the dominant breaking in the third direction does not apply to the
neutrinos and explain what forces the elements of the \( \theta_s \) to have their special form. Surprisingly the first problem has an easy answer if the see-saw mechanism is responsible for the neutrino effective Lagrangian. To see how this works consider the following Lagrangian which generates Dirac neutrino masses

\[
L_{\nu_{\text{Dirac}}} = \alpha \nu_i \theta_i^3 \nu_j \theta_j^3 + \beta \nu_i \theta_i^{23} \nu_j \theta_j^{23} + \gamma \nu_i \theta_i^{123} \nu_j \theta_j^{123}
\]  

(5)

where, with \( \alpha > \beta > \gamma \), the largest coupling is to \( \theta_3 \) as is the case for the quarks and charged leptons. This is of the form shown in eq. (2) for the quarks and charged leptons with \( \epsilon^\nu = 0.05 \) and \( a^\nu = 0 \). We also allow the Majorana mass matrix, \( M_M \), for the right handed neutrinos to have an hierarchical form. To illustrate the point we take \( M_M \) diagonal, \( M_M = \text{Diag}(M_1, M_2, M_3) \) with \( M_3 \gg M_2, M_1 \). With this the effective Lagrangian has the form

\[
L_{\nu_{\text{Majorana,eff}}}^\nu \simeq \frac{\alpha^2}{M_3} \nu_i \theta_i^3 \nu_j \theta_j^3 + \frac{\beta^2}{M_2} \nu_i \theta_i^{23} \nu_j \theta_j^{23} + \gamma^2 \left( \frac{1}{M_1} + \frac{1}{M_2} \right) \nu_i \nu_j \left( \theta_i^{123} \theta_j^{23} + \theta_i^{123} \theta_j^{23} \right)
\]  

(6)

The important point is that the see-saw mechanism has the Majorana mass in the denominator so that if the third generation dominance should be much greater for the Majorana masses than for the Dirac masses than the third term in eq. (6) will be small [5]. In this case the first two terms dominate and one gets the desired mixing pattern of eq. (3).

This demonstrates that there may be an underlying connection between quarks, charged leptons and neutrinos that is only revealed once the see-saw is unravelled. Given the similarity of the Dirac mass matrices it is straightforward to extend the GUT relations to the up quarks with the Georgi Jarlskog mechanism leads to the prediction \( a^\nu = 0 \) as required by eq. (5) [9].

3.1. Family symmetries

In addition to an underlying GUT the structure of the mass matrices also suggests the existence of an underlying spontaneously broken family symmetry. For example consider the case that the family symmetry forces the \( (1,1) \) matrix element to be very small as in eq. (2). In the down quark sector this, with the symmetric structure assumed here, leads to a precise prediction [10] for the Cabibbo angle in terms of the first two generation down quark masses that is in remarkable agreement with experiment. As we shall discuss such a texture zero in the neutrino mass matrix also plays an important role.
Can family symmetries explain the difference in mixing angles in the quark and lepton sectors? As emphasized in figure 2 the most significant feature of the neutrino mixing is the near bi-tri-maximal mixing. This strongly suggests that a family symmetry is non-Abelian in order to relate the proportion of different family neutrino components in a mass eigenstate. Moreover the quantization of the mixing angles suggests that the non-Abelian symmetry be discrete!

3.1.1. Non-Abelian discrete symmetries

To illustrate why discrete non-Abelian symmetries may be relevant to generating neutrino mass structure consider the group generated by the semi-direct product $Z_3 \ltimes Z_n$. Acting on a triplet of fields $\phi$, the generators have the action shown in Table 1 where $\alpha^n = 1$. For the case $n = 2$ the discrete group is $\Delta(12) \equiv A_4$, and for $n = 3$ the discrete group is $\Delta(27)$. Note that one cannot simultaneously diagonalise the $Z_3$ and $Z_n$ groups, showing that the group is non-Abelian.

Many discrete groups have been analysed [11] so it is obviously important to consider what is the best choice. There are various criteria:

**Vacuum structure**

Spontaneous breaking of the discrete symmetry readily leads to an alignment of the vacuum structure consistent with the requirements of tri-bi-maximal mixing. As is the case for continuous symmetries symmetry breaking often leaves an unbroken subgroup. For the $Z_3 \ltimes Z_n$ symmetry the potential consistent with the discrete symmetry has the form

$$V(\phi) = -m^2 \sum_i \phi_i^\dagger \phi_i + \lambda \left( \sum_i \phi_i^\dagger \phi_i \right)^2$$  \hspace{1cm} (7)

$$+ \alpha \frac{m^2}{M^2} \sum_i \phi_i^\dagger \phi_i \phi_i^\dagger \phi_i + ...$$  \hspace{1cm} (8)

where $m$ is a supersymmetry breaking mass and $M$ is the mediator mass. The first two terms are invariant under the larger $SU(3)$ continuous symmetry. The last term breaks the continuous symmetry to the discrete subgroup. Due to it the possible vacuum structure is quantised and depends on the sign of the coefficient $\alpha$. For $\alpha > 0$ we have $\langle \phi \rangle = v(1,1,1)/\sqrt{3}$ and the discrete group is broken to $Z_3$ while for $\alpha < 0$ we have $\langle \phi \rangle = v(0,0,1)$ and the discrete group is broken to $Z_n$. The structure of these vevs is what is required for tri-bi-maximal mixing. It is relatively easy to arrange for a further stage of breaking to give $\langle \phi_{23} \rangle \propto (0,1,-1)/\sqrt{3}$.

**Quasi degenerate neutrinos**

The choice of the discrete family group is restricted to be a subgroup of $SO(3)$ [12] if one wants to implement it for the case of quasi degenerate neutrinos for it allows the degenerate mass term $m_{\nu}^i \nu^i$. An example based on $A_4$ was presented in [13].

**Quark lepton symmetry**

As we have discussed there are several phenomenologically successful relations between quark and lepton masses that follow from an underlying GUT. The requirement that such structure is compatible with the family symmetry severely restricts the possible family symmetry. To date the only family symmetry explored that can achieve this is based on an underlying $SO(10) \times \Delta(27)$ symmetry [14].

3.2. A complete model.

As we have just discussed a combination of a sequential see-saw origin for neutrino masses and a discrete non-Abelian symmetry can give rise to near tri-bi-maximal mixing. However it is necessary to check that it is indeed possible in a complete model to achieve the necessary
hierarchical structure for the Majorana masses in a manner consistent with the discrete non-Abelian symmetry. An explicit example has been constructed [14] consistent with an underlying symmetry group \( \Delta(27) \times SO(10) \times G \) where \( G \) are additional symmetries used to limit the allowed terms in the Lagrangian to those needed to implement the mechanism discussed above. Various possible choices for \( G \) were identified, for example \( G = U(1)_R \times U(1) \) where the first \( U(1) \) is an \( R \)–symmetry. In these models \( G \) restricts the couplings in the Dirac matrices to be just those of eq. (5) and also restricts the form of the Majorana mass matrix to have a sufficient hierarchy between \( M_1 \) and \( M_2 \) and \( M_3 \) so that the last term of eq. (6) is negligible while allowing \( M_1 \) and \( M_2 \) to be nearly equal so that the atmospheric and solar masses are correctly reproduced. Given space limitations I will not give further details of the complete model but turn to the interesting predictions of the scheme.

4. Experimental implications

4.1. Neutrino hierarchy

The model just discussed generated tri-bi-maximal mixing for the normal hierarchy of neutrino masses as shown in figure 2. Recently, using the family group \( A_4 \), a limited version describing charged lepton and neutrino masses only has been constructed for the case of quasi degenerate neutrinos [13]. In this case, too, the ordering of the states is as shown for the normal hierarchy case but the mean mass of the states is much larger than the splitting between the states.

What about the remaining possibility, the inverted hierarchy of figure 2? When one determines the light neutrino eigenstates following from the see-saw mechanism, c.f. eq. (6), the heaviest two states determine the mixing and the lightest state is then the one orthogonal to the heavier states. However for the inverted hierarchy the bi-maximal mixed state is the lightest while the intermediate state has no particular relation between the mixing angles. For this reason the inverted hierarchy is disfavoured in the discrete non-Abelian family symmetry models.

4.2. Mixing angles

By design, in leading order, diagonalisation of the neutrino mass matrix gives tri-bi-maximal mixing, \( \theta_{12} = 35.26^0, \theta_{23} = 45^0 \) and \( \theta_{13} = 0 \). However there are significant corrections to this coming from the charged lepton sector and the form of these is largely determined by the texture zero structure which follows from the underlying family symmetry.

4.2.1. The (1,1) texture zero. A very important aspect of the family symmetry constraint is the prediction of a (1, 1) texture zero in the Dirac mass matrices, c.f. eq. (2), which applies to the up and down quarks and the charged leptons and neutrinos. For the quarks the (1, 1) texture zero, plus the symmetric form of the mass matrices, gives a prediction[10] for the Cabibbo angle

\[
|V_{us}| = |\sqrt{\frac{m_d}{m_s}} e^{i\delta} \sqrt{\frac{m_u}{m_c}}| (9)
\]

which continues to be in excellent agreement with experiment for a choice of the relative phase near \( 90^0 \).

In the neutrino sector one may see from eq. (6) that the (1, 1) texture zero leads to bi-maximal mixing for the atmospheric neutrino sector. There is a correction to pure bi-maximal mixing coming from the charged lepton mixing and the (1, 1) in the lepton sector leads to the analagous prediction to that of eq. (9), giving

\[
\theta_{12} = 35.26^0 + \sqrt{\frac{m_e}{m_\mu}} \pm 2^0 (10)
\]
The ±20 error is included to take account of the next to leading term in the expansion in terms of $\epsilon^\nu$, which we have taken to be 0.05, the same as $\epsilon^u$.

For the case of $\theta_{13}$ the neutrino contribution to $\theta_{13}$ vanishes. However there is a contribution coming the diagonalization of the charged lepton mass matrix. For it the $(1,1)$ texture zero plus the symmetric form leads to the prediction

$$\theta_{13} \simeq \frac{\theta_{12}}{\sqrt{2}} \simeq \sqrt{\frac{m_e}{2m_\mu}} \simeq 3^0. \quad (11)$$

Again there may be corrections to $\theta_{13}$ of order $20^0$ coming from subleading terms in the $\epsilon^\nu$ expansion.

4.3. SUSY spectrum and FCNC

In supersymmetric models the SUSY spectrum will be constrained by the family symmetries too. Measurement of this spectrum will give important tests of the existence of such a family symmetry. For example for the symmetries discussed above the leading soft mass term is constrained to have the family independent form $m_\phi^2 \sum_{i=1,2,3} (\phi_i)\dagger \phi_i$ where $\phi_i$ is a scalar partner of a quark or a lepton and $m_\phi^2$ is a SUSY breaking mass. This is the structure needed to suppress the new SUSY contributions to flavour changing neutral currents (FCNC) and provides an elegant origin for the degeneracy. We know the family symmetry must be strongly split to get the heavy third family. The leading effect of such breaking is suppressed relative to the leading term by powers of the familon field vevs, $m_\phi^2 (\phi_3 \theta_3 \dagger \phi_3 \theta_3)$ [16]. If the family symmetry is continuous there will also be $D-$term contributions to the soft masses. These have the potential to generate unacceptably large FCNC but are under control for a subset of SUGRA and gauge mediated models [17]. In the case of a discrete family symmetry these terms are absent.

In addition to soft masses there are other soft terms, in particular the trilinear $A$-terms. In a theory with family symmetries these are also potentially dangerous as in general they are not diagonalized by the same rotations that diagonalize the fermion masses and so can lead to FCNC processes. Detailed studies [20] show that the constraints on FCNC strongly limit the (poorly constrained) quark and lepton mass matrix elements giving rise to mixing in the right handed quark sector (such mixing is unobservable in non-SUSY models but generates observable effects in the squark and slepton sector). In the models discussed above the symmetry ensures such FCNC terms are consistent with present bounds, although in the most sensitive case, $\mu \rightarrow e\gamma$, the predicted rate is close the present bounds. It is convenient to quote the results in the mass insertion notation [21] giving for the model discussed above [22]

$$|\langle \delta^e_{LR} \rangle_{12}| \lesssim |\langle \delta^\ell_{LR} \rangle_{12}| \approx 3 \times 10^{-5} \frac{A_0}{100 \text{ GeV}} \times \frac{(200 \text{ GeV})^2}{\langle m_\ell \rangle_{LR}^2} \frac{10}{\tan \beta} \left(\frac{\sigma}{0.15}\right)^4 \quad (12) \quad (13)$$

This is to be compared to the current experimental limit $|\langle \delta^e_{LR} \rangle_{12}| \lesssim |\langle \delta^\ell_{LR} \rangle_{12}| < 10^{-5}$ showing that for a slepton mass of $O(200 \text{ GeV})$ $\mu \rightarrow e\gamma$ should be at the present bound. Given that the little hierarchy problem suggests that sleptons should not be much heavier than this one sees that $\mu \rightarrow e\gamma$ provides a very sensitive test of the model and that even a small improvement in the experimental sensitivity will be very significant.

4.4. CP violation

In SUSY models there is a potential SUSY CP problem because new CP violating phases associated with the soft SUSY breaking terms can be present and must be very small ($< O(10^{-2})$)
if they are to be consistent with the bounds on dipole electric moments [19]. The family symmetry models offer an attractive way to solve this problem because in them CP violation may arise only through spontaneous breaking once the flavon fields acquire their vevs [20]. In this case CP violation occurs in the flavour changing sector where it is observed. CP violating effects in the flavour conserving sector arise only via two flavour changing vertices suppressed by the square of a small mixing angle. A recent study has shown that, for a reasonable SUSY spectrum, these CP violating effects are just consistent with present bounds [22]:

\[
|\text{Im}(\delta_{LR}^{\ell})_{11}| \lesssim 2 \times 10^{-8} \frac{A_0}{100 \text{ GeV}} \left(\frac{500 \text{ GeV}}{\langle \tilde{m}_u \rangle_{LR}}\right)^2 \times \\
\left(\frac{\varepsilon}{0.15}\right)^6 \left(\frac{\varepsilon_{0.05}}{\kappa}\right)^3 \sin \phi_1
\]  \tag{14}

\[
|\text{Im}(\delta_{LR}^{d})_{11}| \approx 2 \times 10^{-7} \frac{A_0}{100 \text{ GeV}} \left(\frac{500 \text{ GeV}}{\langle \tilde{m}_d \rangle_{LR}}\right)^2 \times \\
\left(\frac{\varepsilon}{0.15}\right)^6 \frac{10}{\tan \beta} \sin \phi_1
\]  \tag{15}

\[
|\text{Im}(\delta_{LR}^{\nu})_{11}| \approx 6 \times 10^{-8} \frac{A_0}{100 \text{ GeV}} \left(\frac{200 \text{ GeV}}{\langle \tilde{m}_e \rangle_{LR}}\right)^2 \times \\
\left(\frac{\varepsilon}{0.15}\right)^6 \frac{10}{\tan \beta} \sin \phi_1
\]  \tag{16}

\[
|\text{Im}(\delta_{LR}^{\ell})_{11}| \approx 6 \times 10^{-8} \frac{A_0}{100 \text{ GeV}} \left(\frac{200 \text{ GeV}}{\langle \tilde{m}_e \rangle_{LR}}\right)^2 \times \\
\left(\frac{\varepsilon}{0.15}\right)^6 \frac{10}{\tan \beta} \sin \phi_1
\]  \tag{17}

to be compared with the experimental bounds of $10^{-6}$, $10^{-6}$ and $10^{-7}$ respectively. As for the process $\mu \rightarrow e\gamma$ these rates are not far from the experimental limits for the expected masses of supersymmetric particles and a sizeable CP violating phase $\phi_1$.

5. Summary and Conclusions
It is remarkable that the see-saw mechanism together with a strongly hierarchical Majorana mass spectrum explains the significant differences observed between the quark and lepton mixing angles. In this case the Dirac mass matrix structure of the quarks, charged leptons and neutrinos are consistent with an underlying quark-lepton symmetry relating the mass matrices as follows from an underlying Grand Unified structure.

If there is also an underlying discrete non-Abelian family symmetry the near tri-bi-maximal mixing observed in neutrino oscillations is readily explained giving rise to a unified description of all the quark and lepton masses and mixing. In an explicit model implementing such a symmetry the mixing angles are predicted to be near tri-bi-maximal with and accuracy of $O(2^0)$. The squarks and sleptons have a near degenerate mass spectrum split by small fermion mass related terms. As a result the family symmetry solves the SUSY flavour changing problem and, if CP is spontaneously broken in the familon sector, the family symmetry also solves the SUSY CP problem. If such a scenario is correct there will be many associated measurable signals in flavour changing and CP violating processes and in the SUSY scalar spectrum. Indeed $\mu \rightarrow e\gamma$ and the mercury electric dipole moment are within a factor of 10 of the present bounds.

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