Localized Gravity in String Theory
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We propose a string realization of the AdS$_4$ brane in AdS$_5$ that is known to localize gravity. Our theory is $M$ D5 branes in the near horizon geometry of $N$ D3 branes, where $M$ and $N$ are appropriately tuned.

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String theories are only consistent with many additional dimensions. Ever since the notion of extra dimensions was introduced, it was believed that consistency with observed four dimensional gravity requires these dimensions are of finite extent, so it has been held that a viable string theory background requires the compactification of the additional dimensions. Gravity localization provides an alternative to this viewpoint; it is only necessary that a bound state graviton mode dominates over the Kaluza-Klein modes associated with the additional dimensions over the length scales for which we have observed four-dimensional gravity. In an effective five-dimensional Lagrangian was given which exhibits this phenomenon. However, it seemed difficult to realize this possibility in string theory or even a supersymmetric theory. One of the major difficulties in the attempted constructions was the attempt to eliminate the infinite volume region of space; that is geometry had to satisfy a finite “volume” condition; alternatively, there were stringent conditions on the regions of space far from the localizing brane, including the boundary. Attempts to realize the warped geometry in string theory dealt with this through explicit compactification so the is manifestly four-dimensional at long distances. However, it was recently shown that the localization idea is far more general, and does not require anything about the regions of space far from the brane. One important point is that gravity needs only be localized in a region of finite extent; the full space might truly reflect the higher dimensional geometry. This is a dramatically different picture from compactification, which requires global properties of the space-time. It is also a much weaker requirement on the geometry than the original model seemed to indicate. Furthermore, the supergravity solution is only known explicitly for a very limited class of objects within string theory; it seems quite credible that the required geometric features could be realized in a string background. We initiate an exploration of this possibility in this paper.

Our proposal is that a set-up with $N$ D3 branes (say along 012345) intersecting $M$ D5 branes (say along 012345) over a common 3-dimensional worldvolume can lead to localized gravity in a 4-dimensional AdS space. The fact that this set-up preserves supersymmetry (in fact 16 supersymmetries) guarantees stability.

A special property of the AdS brane is that it lends itself to a very natural holographic interpretation. As shown by Cardy, SO(3, 2) is the subgroup of the 4d conformal group SO(4, 2) that preserves a given boundary. This means the symmetry of AdS$_4$ is preserved.

As we showed in another paper (see also [1]) the AdS$_5 \times S^5$ near horizon geometry set-up by the N D3 branes, the 5-branes span an AdS$_4 \times S^2$ worldvolume. This is very easy to see already from the way AdS$_5 \times S^5$ arises as the D3 near horizon geometry: the dictionary of AdS/CFT comes with an identification of flat space embedding coordinates of the D3 brane and the near horizon geometry. When AdS$_5$ is written in Poincare patch coordinates, the 4 Minkowski directions are the 4 worldvolume directions of the D3 (0126). The transverse directions are written in spherical coordinates. The radial direction of this flat transverse space becomes the radial direction in AdS$_5$ and the sphere surrounding the D3 brane is the $S^5$ of the near horizon geometry. The D5 brane, whose defining equation in the flat embedding space coordinates is $x_6 = x_7 = x_8 = x_9 = 0$ hence has an AdS$_4 \times S^2$ worldvolume, both of curvature radius $L$. The latter is given by $x_7 = x_8 = x_9 = 0$ and the former by $x_6 = 0$.

In this paper, we show that we expect such a brane to be a viable candidate for localized gravity. One can already argue from the gravity side why such a wrapped 5-brane in the near horizon geometry of the D3 branes should describe localized gravity. Forgetting for the moment the issue of stability, consider a probe brane inside the near horizon geometry of a stack of $N$ D3-branes. The near horizon geometry is AdS$_5 \times S^5$. A 5-brane...
wrapped over an $S^2$ inside the $S^5$ effectively becomes a 3-brane in AdS$_5$. A 3-brane living in the AdS$_5$ creates a warped geometry of the sort considered in [11,12], that is an AdS$_4$ brane. This is possible because five of the dimensions in the near horizon region have already been compactified, so a 3-brane is adequate to provide a warped AdS geometry.

We now argue that the corresponding near horizon solution of the intersecting D3-D5 configuration should localize gravity along the lines of [1,11,13,14]. The reasons for this are:

- **The conformal symmetry:** As shown in [1], the geometry set-up by the intersecting D3-D5 configuration has a dual CFT description in terms of a CFT with boundary. This dual makes it clear, that $SO(3,2)$ symmetry is preserved, so our space time has an AdS$_4$ slicing.

- **Asymptotics:** Far away along the 6 direction the geometry is still that of the D3 branes alone. So the geometry asymptotes to AdS$_5$ × $S^5$ for $r$ going to ±∞, where $r$ is the transverse coordinate appearing in the warp factor.

- **Discrete symmetry:** The set-up with the intersecting branes has a discrete $Z_2$ symmetry, $x_6 \rightarrow -x_6$. Hence the warp factor will have to have the same symmetry, $r \rightarrow -r$.

- **Positive tension:** Since the energy density on the 5brane is positive, the jump in extrinsic curvature has to be reflected by a jump in the warp factor that has the standard “up-down” shape.

At the moment the full localized supergravity solution is not known even though the results from [11] should help to construct one. Instead, we consider the back-reaction from a five-dimensional effective theory point of view. In the 5d string frame the Einstein-Hilbert term is given by

$$\frac{L^5}{g_s^2}R$$

while the tension of $M$ D5 branes wrapping the $S^2$ of radius $L^2$ is given by

$$4\pi T_{D5}ML^2$$

where we need to recall that $T_{D5} \sim \frac{1}{g_s^2}$. So the jump in extrinsic curvature is proportional to $Mg_s$.

First note that for $g_s \rightarrow 0, g_sN$ fixed, but $g_sM \rightarrow 0$, the backreaction can be neglected. It was in this limit that the probe calculation of [11] established that the D5 brane worldvolume is AdS$_4$ × $S^2$. Clearly we need to get away from this limit to get a significant backreaction and a localized graviton.

![FIG. 1. Turning on $g_sM$ at $q = 0$.](image-url)
is a necessary (but not sufficient) condition for gravity localization. This was not satisfied, for example, in a theory where \( q \) D3 branes end on a single D5 brane.

In the language of the boundary CFT, for \( M \geq 2 \) there exists an interacting 3d CFT on the boundary whose degrees of freedom grow like \( MN^2 \). To see this, replace for a moment the D5 branes with NS5 branes and keep them at finite separation. The gauge theory realized by this brane configuration is 4d SU(N) with a “fat” defect which has a 3d \( U(N)^{M-1} \) gauge theory with bifundamentals living on it, the A-type quiver, as one can determine by the usual rules [13], see Fig. 2. This theory flows to an interacting CFT at the origin of its moduli space, when we take the couplings to infinity, that is take the NS5 branes to coincide. D5 branes give a mirror realization of the same CFT. The stress energy tensor of this 3d CFT should couple to a mode of the 5d graviton localized on the brane. This serves as a further check that this theory is a reasonable candidate for gravity localization.

Now we can ask about going beyond the near horizon region. It is important that localization is a local phenomenon; even if asymptotically the three branes give flat space, one doesn’t lose the graviton and its associated four-dimensional gravitational effects. All that happens is that in the regions where we don’t expect four-dimensional gravity, the KK contribution is suppressed relative to the graviton contribution. This is similar to what happens for a positive tension brane in flat space; there still is a graviton, but the KK modes are not suppressed since there is no barrier in the volcano potential. What is important is that there is some region with a barrier; the probe calculation argues this is the case. This suffices for gravity for length scales that are not too large.

The localized gravity set-up is still relatively young and we have yet to realize the full range of possibilities. What is clear is that one can localize gravity in such a way that only local regions see the localized graviton as dominating the gravitational force. This means one can have a set-up where four-dimensional gravity only applies in specific regions of space. In some ways, this is a rather compelling picture in that one doesn’t need to
assume radical global properties of the full higher dimensional space. What we have demonstrated in this paper is that intersecting branes are sufficient to realize four-dimensional gravity.

It is nice that this construction involves multiple branes, so that there is a realistic possibility that one can realize gauge configurations, and in particular, the standard model, in such a set-up.

Of course, the set-up as it now stands seems to rely on the fact that we have AdS space, where current evidence is that we live in dS space with small cosmological constant. However, it should be borne in mind that the set-up as described is completely supersymmetric. It is not clear how the geometry will be modified with new sources of energy on the brane.

We have argued from an effective theory perspective that we expect a range of $M$ where gravity localization is valid. What is really required to verify this is the full supergravity solution. The fact that this is difficult should not be seen as an argument against localization. After all, we have rigorously demonstrated the AdS nature of the brane, so there must exist some supergravity solution. It is an important and challenging problem to find it.

We conclude that localized gravity is most likely a viable alternative to compactification. Because all curvature scales are set by parameters of the theory (gauge charge and tension), many of the difficult moduli problems should not be present. Of course, there is a multiplicity of possible vacua corresponding to the many possible brane set-ups. Perhaps it is hopeful that a four-dimensional world can exist inside many possible brane structures, since we are only sensitive to the local geometry.

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