Optimization of Full Trajectory Pitch Tracking Control for Low Altitude Penetration of UAV

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Abstract. The low altitude penetration stage of UAV is easily affected by parameter perturbation, which leads to poor anti-damage in the process of penetration. In order to improve the stability of low altitude penetration of UAV, a full trajectory pitching control algorithm based on adaptive dynamic feedback tracking and parameter fusion is proposed. The control objective function of low altitude penetration of UAV is constructed at the constraint cost of minimum damage probability, and the parameters such as pitch angle, steering angle and acceleration are used as the control constraint parameters, and the adaptive learning training for tracking control of the whole trajectory is carried out. The dynamic inverse feedback parameter adjustment model of low altitude penetration is established, the whole trajectory control variable is qualitatively treated, the equivalent mathematical model of anti-interception of UAV is constructed, and the yaw angle is corrected by adaptive dynamic feedback tracking method. The whole trajectory steady-state tracking control of UAV with low altitude penetration is realized. The simulation results show that the algorithm has good steady-state property and strong yaw correction ability in the whole trajectory pitch tracking control of UAV at low altitude penetration stage, which improves the ability of anti-damage in low-altitude penetration stage, it has good control stability and robustness.

1. Introduction
Unmanned aerial vehicles (UAVs) are use remote control and program instructions to control and navigate flight. UAVs play an important role in replacing manned aircraft in carrying out special missions. The military UAV can realize the functions of intelligence reconnaissance, battlefield detection and missile attack. The future UAV system has the ability to perceive the battlefield situation [1]. The UAV can carry the autonomous detection system and missile attack system. With more comprehensive environmental awareness, UAVs can attack the changes in the environment and achieve autonomous control. In the process of low altitude penetration, military UAV is vulnerable to the attack of local air interception system, which leads to the decline of survivability. In order to improve the low altitude penetration capability of UAV and improve the low altitude anti-damage capability of UAV, military UAV is vulnerable to attack by local air interception system. In order to improve the survivability and stability control ability of UAV, it is important to study the low altitude penetration control law of UAV [2].

UAVs are vulnerable to ground and control interference and interception in low-altitude penetration, especially in low-altitude navigation, they are vulnerable to electronic radar detection and electromagnetic interference, flight parameters are prone to perturbation, resulting in poor flight stability. In traditional methods, low-altitude penetration control algorithms for UAVs are mainly...
adaptive inversion control [3]. Fuzzy control, integral control and PID control are used to design the robust control constrained parameter model of low altitude penetration of UAV. Combining fuzzy control method, the flight trajectory optimization of UAV is realized, the anti-missile capability and anti-damage capability of UAV are improved. The relevant literatures have studied the stabilization control of low altitude penetration of UAV and obtained certain results. Among them, a longitudinal robust dynamic inversion control model of thrust vector aircraft is proposed in reference [4]. The longitudinal model of UAV is obtained by coordinate transformation of reasoning vector. The nonlinear dynamic inversion control of low altitude flight of UAV is carried out in linear uncertain system to improve the flight control robustness of UAV. It is sensitive to the perturbation of UAV’s flight parameters, which will lead to flight instability under small disturbances. In reference [5], an anti-jamming control algorithm for UAV based on attitude fusion filter is proposed, and an extended Kalman filter is designed to filter the disturbance and fuse the uncertain flight delay and unknown disturbance error. Filtering and adaptive correction can improve the Penetration Stability of UAV at low altitude, but the control time delay of this method is large, the real-time performance is not good, and the anti-damage performance is not strong [6].

To solve the above problems, this paper presents a full trajectory pitching tracking control algorithm based on adaptive dynamic feedback tracking and parameter fusion for low altitude penetration of UAV. Firstly, the control objective function of low altitude penetration of UAV is constructed, and the parameters such as pitch angle, steering angle and acceleration are taken as the control constraint parameters, and then the adaptive learning training for tracking control of the whole trajectory is carried out. The dynamic inverse feedback parameter adjustment model of low altitude penetration is established, the whole trajectory control variable is qualitatively treated, the equivalent mathematical model of anti-interception of UAV is constructed, and the yaw angle is corrected by adaptive dynamic feedback tracking method. The whole trajectory steady-state tracking control of unmanned aerial vehicle (UAV) with low altitude penetration is realized. Finally, the simulation results show that the proposed method can improve the low altitude penetration stability of UAV.

2. Control object and flight kinematics model

2.1. Control object description

In order to control the whole ballistic pitch tracking stability of UAV in low altitude penetration, it is necessary to construct the flight control object model and the control constraint parameter model:

\[
\dot{x} = V \cos \theta \cos \psi, \\
\dot{y} = V \sin \theta, \\
\dot{z} = -V \cos \theta \sin \psi, \\
\dot{\psi} = \omega_\psi \sin \gamma + \omega_\gamma \cos \gamma, \\
\dot{\gamma} = (\omega_\psi \cos \gamma - \omega_\gamma \sin \gamma) / \cos \theta, \\
\dot{\theta} = -\tan \theta (\omega_\psi \cos \gamma - \omega_\gamma \sin \gamma) \\
\]

The vertical plane \(x_1Oy_1\), and space coordinate system \(Ox_2y_2z_2\), ground coordinate system of ballistic coordinate system \(Ax_3y_3z_3\) are constructed, the relation of low altitude penetration trajectory UAV is described as follows:
\[
\sin \beta = \cos \theta \left( \cos \gamma \sin (\psi - \psi_r) + \sin \vartheta \sin \gamma \cos (\psi - \psi_r) \right) - \sin \theta \cos \vartheta \sin \gamma \tag{7}
\]

\[
\sin \alpha = \frac{\left( \cos \theta \left( \cos \gamma \sin (\psi - \psi_r) - \sin \vartheta \sin \gamma \sin (\psi - \psi_r) \right) - \sin \theta \cos \vartheta \sin \gamma \right)}{\cos \beta} \tag{8}
\]

\[
\sin \gamma_r = \frac{\left( \cos \alpha \sin \beta \sin \vartheta - \sin \alpha \sin \beta \cos \gamma \cos \vartheta \sin \gamma \right)}{\cos \theta} \tag{9}
\]

Where, \(x\), \(y\), \(z\) are the center of mass of UAV, \(m\) is the mass of UAV fuselage. \(\theta\) is the inclination angle between the longitudinal axis (X axis) and horizontal plane (Y plane) of UAV, which is expressed as the angle between velocity vector (OX axis) and horizontal plane, \(OY\) is the steady flight angle of low altitude penetration section, and DS is the UAV relative body coordinate system \(\psi_r\). If \(\alpha\) angle is very small, adjusting the weight moment to make the center of gravity move down to improve the steady-state of low altitude penetration of UAV, \(\gamma\) is differential yaw angle [7].

In order to obtain the desired steady state of flight, the Inertia reference angle is chosen as follows:

\[
\vartheta_r = -\hat{\vartheta}(\alpha V^2 + mg(\sin \vartheta + V \omega_z) + m(\cos \vartheta + V \omega_x) + c_1 e_1 + \lambda e_1 - c_1 \zeta_1 - \ddot{\gamma}) \tag{10}
\]

While \(\vartheta = \pm 90^\circ\), \(m(\cos \vartheta + V \omega_z) = 0\). Combined with adaptive law, the control objective function of UAV low altitude penetration is constructed at the constraint cost of minimum damage probability of whole route. The stability function is obtained as follows:

\[
\omega_{\vartheta} = -c_1 e_1 + \dot{\vartheta} - \lambda_1 \zeta_1 \tag{11}
\]

The \(c_1, \lambda_1\) are larger than zero. According to the analysis of the control object above, the pitch tracking control of the low altitude penetration of UAV is carried out.

### 2.2. Construction of Control objective function and Adaptive learning training

Based on the above control object model of UAV’s low altitude penetration, the control objective function of UAV’s low altitude penetration is constructed at the constraint cost of minimum damage probability of the whole route, and the pitch angle and steering angle are used to construct the control objective function of UAV’s low altitude penetration [8-10]. Acceleration and other parameters are control constraint parameters. The characteristic value of steady-state control \(s \geq 0\), If the static parameters of the low altitude penetration section of \(\frac{1}{q} = \frac{d}{r} = \frac{s}{2}\), UAV are asymptotically stable in the \(H_\infty\) - order, the margin function of full trajectory pitching tracking is constructed at the stable critical point (-1 0):

\[
\|u\|_u = \|u\|_{\mathcal{U}_s} + \|\nabla^5 u\|_{\mathcal{L}_5} + \|\nabla^1 u\|_{\mathcal{L}_1} \tag{12}
\]

The adaptive learning training of the tracking control of the whole trajectory is carried out by using the weighted algorithm for the adaptive adjustment coefficient \(a, b\), satisfying the \(\Phi(B) \subset B\). The dynamic inverse feedback parameter regulation model of the low altitude penetration is established, and the small disturbance coupling component is described as:
The initial conditions are satisfied:

$$X^{(G)} = [-1, 2], [-1, 2]$$

(14)

In order to reduce the initial center shift in the low altitude penetration process, an adaptive learning function is introduced:

$$J = \sum_{i=1}^{N_i}[z^i(k)z(k) - \gamma^i w^i(k)]$$

(15)

The pitching stability conditions of low altitude penetration of UAV are obtained as follows:

$$U = \left\{ \begin{array}{ll}
v \geq \frac{\rho_2 - \rho_1 A_e}{\delta} \\
v < \frac{\rho_1 - \rho_2 + \rho_1 A_e - \rho_1 A_e}{1 - \delta} \end{array} \right.$$  

(16)

The stability discriminant characteristic of incremental dynamic inverse is described as follows:

$$P(Y) = \frac{\exp\left\{-\beta \sum_{c \in C} V_c(Y)\right\}}{\sum_{r} \exp\left\{-\beta \sum_{c \in C} V_c(Y)\right\}}$$  

(17)

Taking the above statistical characteristic as the control objective function and combining the principle of damage minimization, the optimal solution of flight steady-state control is obtained [11].

3. Control algorithm optimization

3.1. Dynamic inverse feedback parameter regulation model and parameter processing

Based on the analysis of the control objective function and the control constraints parameters of the UAV low-altitude penetration at the constraint cost of minimizing the damage probability of the whole flight path, the optimal design of the control algorithm is proposed [12]. A Full-ballistic pitch tracking control algorithm for UAV low-altitude penetration based on adaptive dynamic feedback tracking and parameter fusion is proposed. The adaptive learning function of UAV in low altitude penetration section is:

$$P(x_n | x_n, \theta, \beta) = P(x_n | x_n, \theta) p(x_n | \beta)$$  

(18)

The dynamic inverse feedback control model of UAV under interdiction is described as follows:
Dynamic inverse control allocation method is used to adjust steady-state, and the unstable feedback variable \( x_1, x_2, \ldots, x_n \), is obtained to construct one-dimensional p-Laplace equation for qualitative treatment of steady-state parameters\([13]\). The output is obtained as follows:

\[
\dot{x}(t) = Ax(t) + Bx(t - d_1(t) - d_2(t)) \\
x(t) = \phi(t) \quad t \in [-\tau, 0]
\]  \hspace{1cm} (20)

Where, \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \) is a one-dimensional p-Laplace learning function, from which the dynamic inverse feedback parameter adjustment model of low altitude penetration is established, and the whole ballistic stabilization of flight control variables is processed to improve the yaw correction ability of flight control \([14]\).

### 3.2. Full trajectory pitching tracking control

In the low altitude penetration stage of UAV, the full missile pitching tracking control is carried out in combination with finite time stability, and the control differential equation is introduced as follows:

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]  \hspace{1cm} (21)

The state feedback adjustments for flight trajectory are:

\[
u_k(t) = Kx_k(t)
\]  \hspace{1cm} (22)

The gradient differential term of unmanned aerial vehicle (UAV) at low altitude penetration path is obtained by using continuous time-delay compensation method for closed-loop adjustment:

\[
\dot{x}(t) = Ax(t) + BKx(t - d_1(t) - d_2(t))
\]  \hspace{1cm} (23)

The Lyapunov functional is constructed to identify the flight state \([15]\), and the parameter fusion model is obtained as follows:

\[
\begin{bmatrix}
\dot{m}(t) = -Am(t) + Wg(p(t - \sigma)) + u \\
\dot{p}(t) = -Cp(t) + Dm(t - \tau)
\end{bmatrix}
\]  \hspace{1cm} (24)

Where

\[
m(t) = [m_1(t), m_2(t), \ldots, m_n(t)]^T
\]  \hspace{1cm} (25)

\[
p(t) = [p_1(t), p_2(t), \ldots, p_n(t)]^T
\]  \hspace{1cm} (26)

\[
A = \text{diag}\{a_1, a_2, \ldots, a_n\}
\]  \hspace{1cm} (27)

\[
C = \text{diag}\{c_1, c_2, \ldots, c_n\}
\]  \hspace{1cm} (28)

\[
D = \text{diag}\{d_1, d_2, \ldots, d_n\}
\]  \hspace{1cm} (29)
### 4. Analysis of simulation experiment

In order to verify the application performance of this method in the steady-state pitch tracking control of unmanned aerial vehicle (UAV) with low altitude penetration, the simulation experiments are carried out, and the control model simulation design is carried out in Matlab Simulink and Vega Prime simulation models, and the control model is designed in VC++. In the environment of 6.0, the Vega application program is written to collect the inertial parameters and process the information of UAV. The altitude of low altitude penetration of UAV is 200m, the longitudinal velocity of flight is 120m/s, and the yaw angle of direct section is $\psi = 12.5^\circ$, adaptive adjusting parameter $\varepsilon_1 = 0.25$. The sampling length of flight attitude parameter is 1024, the acquisition time is 25 s, the interval time between two interceptions is 2.5 s, and the distributed azimuth angles of low altitude penetration interception for UAV are 10° and 30°, respectively. According to the above the environment and parameters, simulating the whole trajectory pitching control of UAV's low altitude penetration and flying speed are calculated. The output results of acceleration, yaw and steering angle are shown in Figure 1.

**Figure 1.** Output results of flight attitude parameters and control parameters

Figure 1 shows that the attitude parameter steady-state adjustment ability of the whole trajectory control of UAV at low altitude penetration using this method is good. The yaw amplitude response of UAV low altitude penetration is measured and the result is shown in Figure 2.

Figure 2 shows that the proposed method has good convergence and strong output steady-state tracking control ability for UAV yaw correction. The performance of UAV low altitude penetration missile control is tested by different methods. The comparison results are shown in figure 3. The analysis shows that the precision of this method for UAV low altitude penetration control is higher and the pitching angle tracking is wrong. Small difference, superior performance.
5. Conclusions
In order to improve the low altitude penetration ability of UAV and improve its ability of resisting damage at low altitude, the stability control of low altitude penetration is needed. In this paper, a UAV based on adaptive dynamic feedback tracking and parameter fusion is proposed. A full trajectory pitching tracking control algorithm for low altitude penetration. The control objective function of unmanned aerial vehicle (UAV) low altitude penetration is constructed at the constraint cost of the minimum damage probability of the whole route, and the pitch angle and steering angle are used to construct the control objective function of the UAV's low altitude penetration. Acceleration and other parameters are control constraint parameters. The adaptive learning training of tracking control of whole trajectory is carried out, the dynamic inverse feedback parameter adjustment model of low altitude penetration is established, and the whole trajectory control variable of flight control is treated qualitatively. The equivalent mathematical model of UAV anti-interception is constructed, and the adaptive dynamic feedback tracking method is used to correct the yaw angle to realize the full trajectory steady-state tracking control of UAV at low altitude penetration. The research results show that the proposed method has good steady-state stability, strong yaw correction ability and good control robustness for full trajectory pitch tracking control of UAV at low altitude penetration.

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Figure 2. Yaw amplitude response of low altitude penetration of UAV

Figure 3. Control performance comparison
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