Integrable Impurity Model with Spin and Flavour: Model Inspired by Resonant Tunneling in Quantum Dot

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Abstract

We introduce an integrable impurity model in which both electrons and impurity have spin and flavour degrees of freedom. This model is a generalization of the multi-channel Kondo model and closely related with resonant tunneling through quantum dot. The Hamiltonian is exactly diagonalized by means of the Bethe ansatz.

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1 Introduction

Kondo type integrable impurity models have been studied extensively since the discovery of the exact solution to the single-channel Kondo problem by Andrei and Wiegmann[1][2]. There are many generalizations of the original Kondo model. Among them one of the most important generalization is the multi-channel Kondo model[3][4][5][6]. In this model electrons have flavour (channel) degrees of freedom in addition to the spin (color) degrees of freedom whereas impurity has only spin degrees of freedom.

On the other hand, in the last few years, transport properties of interacting electrons in one-dimensional (1D) wire, which are also interacting with a single or double barrier, are studied by various methods[7][8]. These electrons are believed to be described by a Luttinger liquid, and resemblance between 1D electron systems with double barrier, i.e., quantum dot, and the Kondo model was recognized in various aspects. For example, the number of electrons in the dot is severely restricted because of small electric capacitance of the dot and only their spin degrees of freedom is a dynamical variable. Resonant tunneling of electrons through dot as a result of the Kondo type interaction was predicted theoretically[9] and it was observed quite recently by experiments[10].

There is another interesting electron system which is described by 1D Luttinger liquid interacting with quantum barrier, i.e., quantum Hall states connected by point contacts. In most of 1D electron systems, localization effect is very strong and backward scattering by impurities makes low-energy excitations are gapfull. In this sense chiral Luttinger liquid[11] is more stable than the right-left symmetric one and experimentally easy to be observed. Actually two quantum Hall edge states connected by a point contact were investigated intensively, and theoretical studies in terms of chiral Luttinger liquid and experiments are in good agreement. In this study exactly integrable model and its solutions play a very important role[8][12].

In this paper, inspired by the above mentioned resonant tunneling through quantum dot in the (chiral) Luttinger liquid, we shall introduce an impurity model which is a kind of generalization of the multi-channel Kondo model. In our model both electrons and impurity have $f$ species of flavour degrees of freedom in addition to the $N$ species of spin degrees of freedom. Related model has been already introduced for the other impurity problems and studied using renormalization group and conformal field theory[13]. In this paper integrability of the model (with more general interactions) is proved and we show that the model can be analysed using the Bethe ansatz solutions.

In the Kondo type models, we are mostly interested in low-energy phenomena
and excitations near the Fermi surface. In most of studies, the linearized dispersion relation is employed for electrons. But in certain cases, to study the physical properties of the model correctly, we must introduce second-derivative terms with momentum cutoff which is sent to infinity after the calculation and additional electron-electron interaction terms in the Hamiltonian. Especially in the multi-channel Kondo model, by introducing second-derivative terms and electron-electron interaction terms we get bound states of electrons which are flavour singlet. In this model flavour degrees of freedom of electrons influence the physical properties of impurity via these bound states.

Let us briefly review the integrability of the Kondo model with ultraviolet cutoff following the studies in Refs.\[3\][14]. In the (SU($N$)) Kondo model, integrability is guaranteed by the fact that $S$ matrices are rational solutions to the Yang-Baxter equations. Electron-electron $S$ matrices of the model with cutoff $\Lambda$ are given by

$$S_{ij} = \frac{\lambda_i - \lambda_j - icP_{ij}}{\lambda_i - \lambda_j - ic},$$

(1)

and electron-impurity $S$ matrices are given as

$$S_{j0} = \frac{\lambda_j - \alpha - icP_{0j}}{\lambda_j - \alpha - ic},$$

(2)

where $i,j = 1, \cdots, N_e$, $S_{ij}$ ($S_{j0}$) acts to the tensor product of the spin spaces of the $N_e + 1$ electrons and operates nontrivially only on $p$ spins of $i$-th and $j$-th electrons $C^N \otimes C^N$ (resp. of electron $j$ and impurity), i.e., $P_{ij}$ exchange spin coordinates of $i$-th and $j$-th electrons. Parameter $\lambda_j$ is given by $\lambda_j = k_j/\Lambda$, where $k_j$ is the rapidities of electron $j$. The impurity is denoted by the subscript 0 and from (2) $\alpha$ can be interpreted as the rapidity of the impurity. Hereafter we shall call the parameter $c$, in the above form of $S$ matrices, coupling constant of $S$ matrices. Please notice that as the cutoff is sent to infinity the above $S$ matrices become ordinary $S$ matrices of the Kondo model\[1\][2]. The above $S$ matrices (1) and (2) satisfy the Yang-Baxter equations

$$S_{ij}S_{i0}S_{j0} = S_{j0}S_{i0}S_{ij},$$

$$S_{ij}S_{ik}S_{jk} = S_{jk}S_{ik}S_{ij}.$$  

(3)

For the multi-channel Kondo model electron-electron $S$ matrices are given by

$$S_{ij} = \frac{\lambda_i - \lambda_j - icP_{ij} \lambda_i - \lambda_j + icP_{ij}}{\lambda_i - \lambda_j - ic}.$$  

(4)

\footnote{For the model with finite cutoff, the model is integrable only when the electron-electron coupling is some given value determined by electron-impurity coupling. After removing the cutoff the model is integrable for arbitrary strength of the electron-electron coupling.}
where these $S$ matrices act to the tensor product of the spin and flavour spaces of the $N_e + 1$ electrons, $(C^N \otimes C^I) \otimes (C^N \otimes C^I)$, and $S_{ij}$ operates on the $i$-th and $j$-th electrons, i.e., $P_{ij}$ exchanges the flavour coordinate of the electrons $i$ and $j$. Electron-impurity $S$ matrices are the same with (2), i.e., flavour parts of electron-impurity $S$ matrices are trivial.

This paper is organized as follows. In Sect.2, we shall introduce a 1D electron-impurity model which is a kind of generalization of the multi-channel Kondo model. There are various parameters in the model. It is shown that if these parameters satisfy certain relations, the model is integrable and electron-electron and electron-impurity $S$ matrices have factorized forms similar to (4). In Sect.3, Bethe ansatz equations are given. In Sect.4, its relationship to quantum dot is explained, and section 5 is devoted for conclusion.

Thermodynamics of this model and detailed studies of Bethe ansatz equation for arbitrary $f$ and $N$ will appear elsewhere[15]. Actually, one can consider new "category" of integrable models whose simplest example is the model given in this paper. Some of generalization of the present model will be suggested in later discussions. We shall report studies of them in future publications. It is also interesting to study (solvable) field theory models which correspond to the present model of the first quantization. They are useful and can predict physical properties of resonant tunneling through dot as varying gate voltage, etc.

2 The integrability of the model

2.1 The model with finite cutoff

In our model the electrons and impurity are in the fundamental representation of spin $SU(N)$ and flavour $SU(f)$. Impurity is located at the origin. Wave functions of the model $f_{\{\lambda_N\}_{l=1}^{N_e}}(\{x_i\})$ are functions of the coordinates of the conduction electrons $x_1, \cdots, x_{N_e}$, spin coordinates of the electrons (and impurity) $\lambda_1, \cdots, \lambda_{N_e}$ (resp. $l$) and flavour coordinates of the electrons (and impurity) $\alpha_1, \cdots, \alpha_{N_e}$ (resp. $a$) with $\{\lambda_1, \cdots, \lambda_{N_e}, l\} \in \{1, \cdots, N\}$ and $\{\alpha_1, \cdots, \alpha_{N_e}, a\} \in \{1, \cdots, f\}$.

Model is defined by the following first quantized Hamiltonian with finite cutoff (hereafter we call this model A)

$$H = H_{ee} + H_{ei} + \tilde{H}_{ee} + \tilde{H}_{ei},$$  \hspace{1cm} (5)

where

$$H_{ee} = -i \sum_{j=1}^{N_e} \partial_j + \sum_{l<j} \delta(x_l - x_j)(U_c P_{jl} - U_l P_{jl})$$
\[ H_{ee} = \sum_{j=1}^{N_e} \delta(x_j)(J_c P_{0j} + J_f P_{0j}) \]

\[ \tilde{H}_{ee} = -\frac{1}{2\Lambda} \sum_{j=1}^{N_e} \partial_j^2 \]

(6)

\[ H_{ei} \]

\[ H_{ee} \]

\[ \tilde{H}_{ee} \]

\[ \Lambda \]

\[ U_c, U_f, J_c, J_f \]

\[ \xi \{ \alpha_k \}_{l}^{a} \{ \lambda_j \}_{l}^{a} (Q) \]

\[ \theta(x_Q) \]

\[ \xi_{(i,j)Q} = S_{ij} \xi(Q), \]

(8)

\[ S_{ij} = S_{ij}^c S_{ij}^f, \]

(9)

\[ S_{ij}^c = \frac{\lambda_i - \lambda_j - i(U_c + U_f)P_{ij}}{\lambda_i - \lambda_j - i(U_c + U_f)} \]

\[ S_{ij}^f = \frac{\lambda_i - \lambda_j + i(U_c + U_f)P_{ij}}{\lambda_i - \lambda_j + i(U_c + U_f)}, \]

(10)

\( H_{ee} \) consists of the kinetic term of the electrons with the linearized dispersion relation and electron-electron interactions. \( H_{ei} \) is the electron impurity interaction term. \( \tilde{H}_{ee} \) is the second-derivative term which introduces curvature in the dispersion relation and \( \Lambda \) is the cutoff which will be sent to infinity at the final stage of the calculation.

\( U_c, U_f, J_c, J_f \) are all parameters of the present model, and their physical meaning is explained in Sect.4 (see also Fig.2). Note that for the infinite cutoff limit the model is integrable for more general interactions (see Sect.2.2).

We shall diagonalize the Hamiltonian by the Bethe ansatz. The Bethe ansatz wave functions for the model with \( N_e \) conduction electrons are given by

\[ F_{\{ \lambda_j \}_{l}^{1}}^{\{ \alpha_k \}_{l}^{a}} (\{ x_i \}) = A \sum_{Q \in S_{N_e+1}} \sum_{j=1}^{N_e} e^{ik_j x_j} \xi_{\{ \alpha_k \}_{l}^{a}} (Q) \theta(x_Q), \]

(7)

where \( A \) is the antisymmetrizer, \( Q \) runs over all the permutations of electrons and impurity and \( \theta(x_Q) \) is unity if \( x_{Q(0)} < x_{Q(1)} < \cdots < x_{Q(N_e)} \) and is vanishing otherwise.

\( \xi_{\{ \alpha_k \}_{l}^{a}} (Q) \) are the spin and flavour dependent amplitudes in the region \( Q \). \( \xi_{\{ \alpha_k \}_{l}^{a}} (Q) \) can be interpreted as the components of the vectors \( \xi(Q) \) in the spin and flavour space \( \otimes_{j=1}^{N_e+1} (C^N \otimes C^f) \). \( \xi(Q) \)'s in different regions are related by the \( S \) matrices as

\[ \xi_{(i,j)Q} = S_{ij} \xi(Q), \]

where \( Q = (Q(0), \cdots, Q(l) = i, Q(l+1) = j, \cdots, Q(N_e)) \) and \( (i,j)Q = (Q(0), \cdots, Q(l+1), Q(l), \cdots, Q(N_e)) \). Consistency of the Bethe ansatz wave functions is guaranteed if the \( S \) matrices satisfy the Yang-Baxter equations.

The electron-electron \( S \) matrices are factorized to the spin part and flavour part,
and $\lambda_j = k_j/\Lambda$. Note that for the ordinary Kondo Hamiltonian, where only first-derivative terms in $H_{ee}$ exist without the second-derivative terms in $\tilde{H}_{ee}$ and the electron-electron interaction, electron-electron $S$ matrices cannot be determined by the Hamiltonian itself. $S$ matrices are determined by demanding that $S$ matrices satisfy the Yang-Baxter equations. But in the present case, $S$ matrices are determined by the Hamiltonian uniquely, and by removing the cutoff we reconstruct the well known electron-electron $S$ matrices of the Kondo model\cite{3}\cite{14}.

Note also that with any reasonable choice of interactions the spin part and flavour part of the $S$ matrices have the same form up to the sign of the coupling constant term $(U_c + U_f)$ in (10).

Let us turn to the electron-impurity $S$ matrices. For the electron-impurity interaction of the form given by Hamiltonian (6) electron-impurity $S$ matrices are factorized. Furthermore, if the coupling constants satisfy the equation

$$J_f = -J_c,$$

then the coupling constants of the $S$ matrices in the spin part and in the flavour part are same (this condition is necessary for the $S$ matrices to satisfy the Yang-Baxter equation because the coupling constants of the $S$ matrices in the spin part and in the flavour part of the electron-electron $S$ matrices are the same).

When the coupling constants satisfy Eq.(11), electron-impurity $S$ matrices are given by

$$S_{j0} = S_{j0}^c S_{j0}^f,$$

$$S_{j0}^c = \frac{\lambda_j - \alpha - icP_{0j}}{\lambda_j - \alpha + ic},$$

$$S_{j0}^f = \frac{\lambda_j - \alpha + icP_{0j}}{\lambda_j - \alpha - ic}.$$  

Here $\alpha$ and $c$ are functions of coupling constants ($J_c, J_f$), and for the model to be integrable electron-electron coupling constants $(U_c, U_f)$ must satisfy the condition

$$U_c + U_f = c.$$  

Notice that there is another case in which the $S$ matrices of the model is the solution to the Yang-Baxter equations, that is, the coupling constants satisfy the following conditions

$$J_f = 0 \quad \text{and} \quad J_c \text{ is finite},$$

(or the case with the spin and flavour exchanged in the above). In the above case, the model is nothing but the multi-channel Kondo model.
Integrable region of the model in the space of electron-impurity coupling constants is given by two regions (crossing at the trivial point $J_c = J_f = 0$) characterized by Eqs.(11) and (15).

2.2 The model with infinite cutoff

We shall study another model (which we call model B) whose Hamiltonian is given by

$$H = -i \sum_{j=1}^{N_e} \partial_j + \sum_{l<j} \delta(x_l - x_j)(U_c P_{jl} - U_f P_{jl}) + \sum_{j=1}^{N_e} \delta(x_j)(J_{cf} P_{0j} P_{0j} + J_c P_{0j} + J_f P_{0j} + J)$$

(16)

This model is obtained by taking the cutoff to infinity in the model A and also the electron-impurity interactions of the model B are more general.

To obtain the correct results, we must first study the model A and classify the complex solutions (string) of the Bethe ansatz equations, and then study the model B for more general cases.

In this model electron-electron $S$ matrices are given by

$$S_{ij} = P_{ij} P_{ij},$$

(17)

and electron-impurity $S$ matrices have the following form

$$S_{j0} = f P_{j0} P_{j0} + g P_{j0} + h P_{j0} + l,$$

(18)

where $f, g, h$ and $l$ are complicated functions of $J_{cf}, J_c, J_f, J$, and for arbitrary coupling constants the model is integrable (at least) formally (i.e., $S$ matrices satisfy the Yang-Baxter equations). But in practice only when the electron-impurity $S$ matrices are factorized to the spin part and flavour part, we can diagonalize the eigenvalue problem which appears by imposing the periodic boundary condition on the wave function. The condition that $S$ matrices are factorized is given by

$$J_{cf}^3 - J_{cf}(J_c^2 + J_f^2) + J_{cf}(-4 - J^2) + 2 JJ_c J_f = 0.$$  

(19)

When the coupling constants satisfy Eq.(19) electron-impurity $S$ matrices are given by

$$S_{j0} = S_{j0}^c S_{j0}^f,$$

(20)

---

2 In this paper factorization means that $S$ matrices are factorized to spin part and flavour part.
\[
\begin{align*}
S_{j0}^c &= -\frac{\alpha_c - ic_c P_{0j}}{\alpha_c - ic_c} \\
S_{j0}^f &= -\frac{\alpha_f + ic_f P_{0j}}{-\alpha_f + ic_f}.
\end{align*}
\] (21)

In the model B even if \(c_c \neq c_f\), the Hamiltonian can be diagonalized using Bethe ansatz as seen from the form of the electron-electron \(S\) matrices [17]. This broadness of the interaction is important when the model is applied to experimentally accessible physical systems like the resonant tunneling in the quantum dot (see discussion in Sect. 4).

There is a simple nontrivial solution to the factorizability condition of the electron-impurity \(S\) matrices, i.e., Eq. (19). That is

\[J_{cf} = J = 0.\] (22)

The electron-impurity interactions of the model A have the form of this solution except that for the model A there exist additional condition for the model to be integrable that is \(J_c = -J_f\). Parameter region in which the model A is integrable and the special solution for the model B is given in Fig. 1.

For the solution (22) \(S\) matrices are expressed explicitly as

\[
\begin{align*}
S_{j0}^c &= 2 + i \frac{8J_c}{J_c^2 - J_f^2 - 4} P_{0j} \\
S_{j0}^f &= 2 - i \frac{8J_f}{J_c^2 - J_f^2 + 4} P_{0j} \\
S_{j0}^f &= 2 + i \frac{8J_f}{J_c^2 - J_f^2 + 4} P_{0j}.
\end{align*}
\] (23)

3 Bethe ansatz equations of the models

In this section we shall diagonalize the Hamiltonian of the model for the special case \(N = f = 2\) by the Bethe ansatz.

Imposing the periodic boundary conditions for the wave function (2), we obtain eigenvalue problems

\[e^{ikjL} \xi(Q) = Z_j \xi(Q),\] (24)

where

\[Z_j = S_{jj-1} \cdots S_{ji} S_{j0} S_{jN} \cdots S_{jj+1}\] (25)

are matrices which operate to vectors in \(\otimes_{j=1}^{N+1} (C^N \otimes C^f)\) and each factor comes from the scattering of \(j\)-th electron by another electron and the impurity.
Since $p_{ij}$ and $\mathcal{P}_{ij}$ commute with each other, $Z_j$ can be factorized to spin part and flavour part as $Z_j = Z_j^f Z_j^c$,

$$
Z_j^f = S_{jj-1}^f \cdots S_{j1}^f S_{j0}^f S_{jN}^f \cdots S_{jj+1}^f
$$

$$
Z_j^c = S_{jj-1}^c \cdots S_{j1}^c S_{j0}^c S_{jN}^c \cdots S_{jj+1}^c.
$$

(Eq. 26)

Eigenvectors of $Z_j$ are simultaneous eigenvectors of $Z_j^c$ and $Z_j^f$, and then we can diagonalize the spin and flavour degrees of freedom independently\[2\][16]. Operators $Z_j^c$ and $Z_j^f$ are transfer matrices of the inhomogeneous XXX model at the special values of the rapidities and they can be diagonalized by the algebraic Bethe ansatz\[2\][16].

Resultant Bethe ansatz equations in the model A are given by

$$
e^{ik_j L} = \prod_{\gamma=1}^M \frac{\omega_\gamma^c - \lambda_j - \frac{ic}{2}}{\omega_\gamma^c - \lambda_j + \frac{ic}{2}} \prod_{\gamma=1}^M \frac{\omega_\gamma^f - \lambda_j + \frac{ic}{2}}{\omega_\gamma^f - \lambda_j - \frac{ic}{2}},
$$

$$
\prod_{\delta(\neq \gamma)} \frac{\omega_\delta^c - \omega_\gamma^c - ic}{\omega_\delta^c - \omega_\gamma^c + ic} = \prod_{i=1}^N \frac{\omega_i^c - \lambda_i + \frac{ic}{2}}{\omega_i^c - \lambda_i - \frac{ic}{2}} \omega_\gamma^c - \alpha - \frac{ic}{2},
$$

$$
\prod_{\delta(\neq \gamma)} \frac{\omega_i^f - \omega_i^f - ic}{\omega_i^f - \omega_i^f + ic} = \prod_{i=1}^N \frac{\omega_i^f - \lambda_i + \frac{ic}{2}}{\omega_i^f - \lambda_i - \frac{ic}{2}} \omega_\gamma^f - \alpha - \frac{ic}{2}.
$$

(Eq. 27)

where $\omega_\gamma^c$'s are spin rapidities, $\omega_i^f$'s are flavour rapidities and $M(\bar{M})$ is the number of down(up) spins of the flavour $SU(2)$. Energy of the state is give by

$$
E = \sum_{j=1}^{N_c} \left( k_j + \frac{1}{2\Lambda} k_j^2 \right).
$$

(Eq. 28)

The Bethe ansatz equations (Eq. 27) are related to those for the ordinary Kondo model. The spin part and flavour part of the equations (Eq. 27) have the same form with the spin part of the ordinary Kondo model (with the cutoff). Then the classification of the string solution for the Bethe ansatz equations is very similar to that of the Kondo model. Classification of string solutions and study of the finite temperature thermodynamics will appear elsewhere\[15\].

4 Quantum dot

In this section, we explain the relationship between the present model (with $N = f = 2$) and electron transport phenomena through quantum dot. There are two cases to which our model is relevant, one is the system of quantum dot in 1D interacting electrons (Luttinger liquid) and the other is the double-layer quantum Hall edges which are contacted by a quantum dot.
4.1 Electrons and dot in one-dimensional wire

For the system of quantum dot in the Luttinger liquid, we shall interpret the flavour degrees of freedom in the present model as chirality of electrons. Strictly speaking, signature of the linear momentum term in the electron Hamiltonian depends on the chirality of the electron, i.e.,

$$-i \sum_{j=1}^{N_e} \alpha_j \partial_j,$$

(29)

where $\alpha_j = +1(-1)$ if $j$-th electron is rightmover(leftmover). However we think that chirality does not give substantial influences in the present case because interaction between electrons and the dot occurs only at the point $x = 0$ (see similar discussion in Refs. [8][12]). Explicitly in quantum field language left and right-moving fermions, $\psi_L(x)$ and $\psi_R(x)$, can be regarded as flavour-two-component left mover by $\psi_1(x) = \psi_L(x)$, $\psi_2(x) = \psi_R(-x)$. Flavour change at $x = 0$ is therefore nothing but reflection or backscattering at $x = 0$ (see Fig.2).

As we explained in the introduction, charge of quantum dot is strictly restricted by small electric capacitance of the dot. Therefore if electron number in the dot is restricted to be odd, only spin degrees of freedom of the dot is dynamical and the dot can be regarded a spin-$\frac{1}{2}$ impurity as in the Kondo model.

Let us study the above system in more detail and “derive” the Hamiltonian in this paper from microscopic point of view. Coulomb interaction in the dot is described by the term like

$$H_C = U_{\rho} (n_{\rho} - n_{0\rho})^2,$$

(30)

where $U_{\rho} \sim e^2/l_d$ with linear magnitude of the dot $l_d$, $n_{\rho}$ is the electron number in the dot and $n_{0\rho}$ is its mean value controlled by gate voltage. Let us consider the case in which $n_{0\rho}$ is an odd integer and lower energy levels are filled with spin up and down electrons, and therefore we focus on the the lowest unfilled energy level (LUEL) inside the dot.

It is helpful to consider discrete version of the above system and let us denote electron operator in the LUEL in the dot by $c_{0\sigma}$ where $\sigma = \pm 1$ is the spin index. Then the Hamiltonian is given as

$$H_{DV} = H_0 + H_V + H_C,$$

(31)

\footnote{Chirality, of curse, plays an important role in the localization phenomenon by random impurities. Because of this, chiral Luttinger liquid can be realized in real materials and experiments more easily.}
\[ H_0 = -t \sum_{\sigma} \left[ \sum_{i \geq 1} (c_{i\sigma}^\dagger c_{i+1\sigma} + \text{H.c.}) + \sum_{i \leq -2} (c_{0\sigma} c_{i\sigma} + 1 + \sigma + \text{H.c.}) \right], \]

\[ H_{t'} = -t' \sum_{\sigma} (c_{0\sigma} c_{1\sigma} + c_{-1\sigma}^\dagger c_{0\sigma} + \text{H.c.}), \]

where \( i \) is the site index and \( H_C \) is given by (30) with \( n_\rho = \sum_{\sigma} c_{0\sigma}^\dagger c_{0\sigma} \) and \( n_{0\rho} = 1 \).

In the present case \( U_\rho \gg t' \) and the electron transmission between the dot and the leads is treated by perturbation. In the dot, the double-occupied state has large energy \( U_\rho \) compared to the single-occupied states, and therefore we shall focus on the subspace of single occupancy in the LUEL. Then it is rather straightforward to derive an effective Hamiltonian for the subspace by mean of the perturbative calculation for \( H_{t'} \), e.g.,

\[ R \langle \downarrow | 0 \langle \uparrow | H_{t'} (E_0 - H_0 - H_C)^{-1} H_{t'} | \uparrow \rangle_0 | \downarrow \rangle_R = -2t'^2 / U_\rho, \]

\[ R \langle \downarrow | 0 \langle \uparrow | H_{t'} (E_0 - H_0 - H_C)^{-1} H_{t'} | \downarrow \rangle_0 | \uparrow \rangle_R = 2t'^2 / U_\rho, \]

etc., where intermediate state of the dot is the double-occupied state \( c_{0\uparrow}^\dagger c_{0\downarrow} |0\rangle_0 \) and \( | \cdot \rangle_R \) is a state of free electron in the right side of the dot. Similarly conduction electron can be transferred across the dot with or without flipping its own spin and that of the dot. Therefore there appear Kondo-type interactions between spin of the dot and that of conduction electrons. In the first quantization, these interactions between spin degrees of freedom of the dot and conduction electrons are described by the terms proportional to the spin and flavour exchange operators \( P_{0j} \), \( P_{0j} \) etc.

The various parameters in the Hamiltonian of the present model (16) have the following physical meanings in the electron transport properties through quantum dot (see Fig.2). \( U_c \) is the spin flip interaction between electrons, whereas \( U_f \) is the strength of backscattering of electrons. For clean samples, these parameters become small. Similarly \( J_c \) represents the electron transmission through the dot with spin flip. Flavour exchange operator \( P_{0j} \) is related to flavour angular momentum operators of the dot \( \vec{S}_f \) and electron flavour current \( \psi^\dagger \vec{\tau} \psi(0) \) as

\[ P_{0j} \propto \vec{S}_f \cdot \psi^\dagger \vec{\tau} \psi(0) + C, \]

where \( C \) is some constant. As explained above, flavour change at \( x = 0 \) \( \psi_1 \leftrightarrow \psi_2 \) is \( \psi_L(x) \leftrightarrow \psi_R(-x) \). Therefore \( J_f \) and \( J_{cf} \) correspond to electron “backscattering” by the dot with and without spin flip, respectively. In experiments these backscatterings by dot are controlled by varying, e.g., gate voltages, in order to observe resonant tunnelling. Actually condition of resonant tunneling is \( J_f = 0 \) [17], i.e., vanishing of electron backscattering by potential barrier by the dot.

We should also remark here that in order to describe 1D electron system with quantum dot faithfully flavour of the dot must be sufficiently large. Otherwise
backscatterings of electrons by the dot alternate in direction. We can employ cyclic representation of quantum group $SU(2)_q$ for flavour degrees of freedom of impurity for the above purpose. More systematic studies are possible by using (solvable) field theory model corresponding to the present model in the first quantization. This problem is under study and results will be reported in future publication.[15]

4.2 Quantum Hall edges in double-layer system

Another related system to application of the present model is quantum Hall state(QHS) in a double layer electron system and spin-singlet QHS. In these cases, low-energy excitations are chiral edge states with (pseudo-)spin degrees of freedom. If two edges states in double-layer QHS are connected by dot, which is also double-layer system, two edge states are labeled by flavour indices and layers are labeled by pseudo-spin indices (we assume here that real spins of electrons are polarized by an external magnetic field). It is well known that the pseudo-spin description is useful for study of low-energy excitations in the double-layer QHS[19]. In double-layer dot, state with a “movable” electron in the upper(lower) layer has pseudo-spin $+\frac{1}{2}(-\frac{1}{2})$ (see Fig.3). As in the previous case, Coulomb interaction energetically prefers the single-occupied states in the quantum dot, and because of that Kondo-type interactions effectively appear in the low-energy subspace. All relevent interactions between fermions and dot in the above system are obviously included in the Hamiltonian. Here again field theory model is possible to be constructed for studying this system and especially its bosonized form is quite useful for the calculation of conductivity, etc., as in the $\nu = \frac{1}{3}$ single layer case[12]. There are more parameters in the double layer case compared with the single-layer case and therefore we can expect interesting phenomena. This problem is under study and we shall report results in future publication.

5 Conclusion

In this paper we have introduced the Kondo type integrable impurity model with spin and flavour degrees of freedom in both electrons and impurity, and we have diagonalized the Hamiltonian by the Bethe ansatz for the case of $N = f = 2$. Among the low-energy excitations, there exists what we call flavouron in addition to the holon and spinon excitations. Detailed study of the model for arbitrary $N$ and $f$ and finite-temperature thermodynamics of the model will appear elsewhere[15].

If we interpret impurity as quantum dot, flavour degrees of freedom in impurity should not be the spin 1/2 representation of $SU(2)$, as we explained in Sect.4. In
a faithful model which describes quantum dot, flavour degrees of freedom of the impurity is given by a cyclic representation of $SU(2)_q$ or spin $j/2$ representation of $SU(2)$ with $j \to \infty$. Then it is very interesting to study the model in which spin degrees of freedom of both electrons and the impurity belong to the fundamental representation of $SU(N)$ whereas flavour degrees of freedom of the impurity belong to the spin $j/2$ representations of $SU(2)$.

The model in which flavour degrees of freedom is in the cyclic representation of $SU_q(2)$ for the impurity can be more easily studied by bosonizing the model and map the problem to the integrable field theory with boundary. To study the transport properties of the model, the boson model is more powerful than the original model.

There are some generalizations of the model in this paper. In this paper only isotropic case (corresponding to the rational solution of Yang-Baxter equations) was studied. For the application to the resonant tunneling in quantum dot, study of anisotropic case is interesting, and study of the model in terms of the boundary conformal field theory is also important. As we stated in Sect.4, in order to interpret flavour in our model as chirality the kinetic term which is linear in momentum should be modified. This problem is under study.

The model is related to the resonant tunneling through quantum dot in (chiral) Luttinger liquid. There is another interesting system of quantum dot which is described by an integrable impurity model with flavour degrees of freedom, that is, multi-channel quantum dot. We found that the following Hamiltonian which represents this phenomenon is integrable,

$$H = -\sum_{j=1}^{N_c} \partial_j^2 + \sum_{l<j} \delta(x_l - x_j)(U_c P_{jl} - U_f P_{jl})$$
$$+ \sum_{j=1}^{N_c} \delta(x_j) (J_c P_{0j} + J_f P_{0j}),$$

where there are some constraints between the coupling constants (there is relation between electron-electron coupling constant and electron-impurity coupling constant). Detailed will be reported in near future.

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Figure 1: The region of the model to be integrable (if $J_{cf} = J = 0$). For the model A only on the solid lines $J_c = -J_f$ and $J_f = 0$ (multi-channel Kondo model) the model is integrable. For the model B on the entire space the model is integrable.

Figure 2: The relation between Hamiltonian (16) for $f = 2$ and the resonant tunneling of Luttinger liquid in quantum dot. The flavour degrees of freedoms are interpreted as the right or left movers.

Figure 3: Two layer quantum Hall states are connected by the quantum dot which has also two layers.
