A non-Hermitian analysis of strongly correlated quantum systems

Yuichi Nakamura a,∗, Naomichi Hatano b

a Department of Physics, University of Tokyo, Komaba, Meguro, Tokyo, 153-8505, Japan
b Institute of Industrial Science, University of Tokyo, Komaba, Meguro, Tokyo, 153-8505, Japan

Abstract

We study a non-Hermitian generalization of strongly correlated quantum systems in which the transfer energy of electrons is asymmetric. It is known that a non-Hermitian critical point is equal to the inverse localization length of a Hermitian non-interacting random electron system. We here conjecture that we can obtain in the same way the correlation length of a Hermitian interacting non-random system. We confirm the conjecture using exact solutions and numerical finite-size data of the Hubbard model and the antiferromagnetic XXZ model in one dimension.

Key words: non-Hermitian quantum mechanics, correlation length, Hubbard model, antiferromagnetic XXZ model

We study a non-Hermitian generalization of strongly correlated quantum systems by adding an imaginary vector potential ig (where g is a real constant) to the momentum operator. Hatano and Nelson [1] analyzed this kind of non-Hermitian generalization of the one-electron Anderson model. Their purpose of the non-Hermitian generalization was to obtain a length scale inherent in the wave function of the Hermitian model, namely the localization length, only from the non-Hermitian energy spectrum. We here conjecture that we can obtain in the same way the correlation length of a Hermitian interacting non-random model.

The non-Hermitian one-electron Anderson model is given by [1]

\[ \mathcal{H} = -t \sum_{x=1}^{L} (e^{g}|x+1\rangle \langle x| + e^{-g}|x\rangle \langle x+1|) + \sum_{x=1}^{L} V_{x}|x\rangle \langle x| \]  (1)

in one dimension, where g is the parameter that generates the non-Hermiticity, corresponding to the imaginary vector potential, and \( V_{x} \) is a random potential at site \( x \). At the Hermitian point \( g = 0 \), all eigenvalues of the Hamiltonian (1) are real. As we increase the non-Hermiticity \( g \), a pair of eigenvalues collide at a point \( g = g_{c} \) and become complex. It was revealed [1] that the non-Hermitian critical point \( g_{c} \) is equal to the inverse localization length of the eigenfunction of the original Hermitian Hamiltonian.

The above study was for a non-interacting random electron system. What length scale emerges in the energy spectrum if we consider the non-Hermitian generalization of an interacting non-random system? We conjecture here that the answer is the correlation length. We show for the Hubbard model and the antiferromagnetic XXZ model in one dimension that the non-Hermitian critical point of the ground state where the energy gap vanishes is equal to the inverse correlation length of the Hermitian point. We also analyze numerical data of non-Hermitian quantum systems of finite size, confirming the conjecture.

We first consider a one-dimensional non-Hermitian Hubbard model in the form

\[ \mathcal{H} = -t \sum_{l, \sigma = \uparrow, \downarrow} (e^{g} c_{l+1, \sigma}^\dagger c_{l, \sigma} + e^{-g} c_{l, \sigma}^\dagger c_{l+1, \sigma}) + U \sum_{l} c_{l, \uparrow}^\dagger c_{l, \uparrow} c_{l, \downarrow}^\dagger c_{l, \downarrow}. \]  (2)
First, the inverse correlation length \(1/\xi\) of the charge excitation at the Hermitian point \(g = 0\) was obtained by Stafford and Millis [3] in the form

\[
\frac{1}{\xi} = \text{arcsinh}(U/4t) - 2 \int_0^\infty \frac{J_0(\omega) \sinh(U/4t)\omega}{\omega(1 + e^{2(U/4t)\omega})} d\omega, \tag{3}
\]

where \(J_0(\omega)\) is the Bessel function of the first kind. Next, Fukui and Kawakami [2] solved the non-Hermitian Hubbard model (2) exactly. They showed that, as we increase the non-Hermiticity \(g\), the Hubbard gap due to the charge excitation vanishes at a point \(g_c\) given in the form

\[
g_c = \text{arcsinh}(U/4t) + 2i \int_{-\infty}^\infty \text{arctan} \left( \frac{\lambda + iU/4t}{U/4t} \right) \sigma(\lambda) d\lambda, \tag{4}
\]

where \(\sigma(\lambda)\) is given by

\[
\sigma(\lambda) = \frac{1}{2\pi} \int_0^\infty \text{sech} \left( \frac{U}{4t} \right) \cos(\lambda \omega) J_0(\omega) d\omega. \tag{5}
\]

We can show after some algebra that the expressions of the inverse correlation length \(1/\xi\) in Eq. (3) and the non-Hermitian critical point \(g_c\) in Eq. (4) are equal.

We next consider the one-dimensional \(S = 1/2\) antiferromagnetic non-Hermitian XXZ model in the form

\[
\mathcal{H} = J \sum_i \left[ \frac{1}{2} (e^{g_S} S_i^+ S_{i+1}^- + e^{-g_S} S_i^- S_{i+1}^+) + \Delta S_i^z S_{i+1}^z \right]. \tag{6}
\]

The non-Hermitian Hamiltonian (6) for \(\Delta = 1\) is derived by considering the second-order perturbation of the non-Hermitian Hubbard Hamiltonian

\[
\mathcal{H} = -t \sum_i (e^{g_{\uparrow\uparrow}} c_{i+1,\uparrow} c_{i,\uparrow} + e^{-g_{\uparrow\uparrow}} c_{i,\uparrow}^\dagger c_{i+1,\uparrow} + e^{-g_{\downarrow\downarrow}} c_{i+1,\downarrow}^\dagger c_{i,\downarrow} + e^{g_{\downarrow\downarrow}} c_{i,\downarrow} c_{i+1,\downarrow}) + U \sum_i c_{i,\uparrow}^\dagger c_{i,\uparrow} c_{i,\downarrow}^\dagger c_{i,\downarrow} \tag{7}
\]

with respect to \(t\). As we increase the non-Hermiticity \(g\) in Eq. (7), we can eliminate the spin gap rather than the charge gap; see the difference between Eqs. (2) and (7). We then generalize the model to an arbitrary \(\Delta\). Albertini et al [4] calculated for \(\Delta > 1\) the non-Hermitian critical point \(g_c\) at which the spin gap vanishes:

\[
g_c = \frac{\gamma}{2} + \sum_{n=1}^\infty (-1)^n \tanh(n\gamma), \tag{8}
\]

where \(\gamma = \text{arccosh}\Delta\). The expression (8) is actually well known [5] as the inverse correlation length at the Hermitian point \(g = 0\). We have thus confirmed our conjecture for two solvable models.

Unfortunately, it is difficult to know the ground-state properties of the non-Hermitian models (2) and (6) for \(g > g_c\). However, we expect that the ground-state energy becomes complex in the region \(g > g_c\) on the basis of finite-size data, which we discuss below.

References

[1] N. Hatano and D. R. Nelson, Phys. Rev. Lett. 77 (1996) 570; Phys. Rev. B 56 (1997) 8651.
[2] T. Fukui and N. Kawakami, Phys. Rev. B 58 (1998) 16051.
[3] C. A. Stafford and A. J. Millis, Phys. Rev. B 48 (1993) 1409.
[4] G. Albertini, S. R. Dahmen and B. Wehefritz, Nucl. Phys. B 493 (1997) 541.
[5] R. J. Baxter, Exactly Solved Models in Statistical Mechanics (Academic Press, New York, 1982) p.155.
[6] C. K. Majumdar and D. P. Ghosh, J. Math. Phys. **10** (1969) 1388.