On Heat Transfer of a Convective Radiating Cylinder

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Abstract. The study of heat transfer in a stockpile of reactive materials, modeled in a cylindrical domain, is carried out in this article. The reactive cylinder is assumed to lose heat to the surrounding environment by convection and radiation. The complicated combustion process results with nonlinear interactions, and therefore, the ordinary differential equation (ODE) governing the problem cannot be solved exactly, but numerically. The Runge-Kutta-Fehlberg (RKF45) numerical method is applied to solve the ODE. The solutions are expressed graphically and discussed accordingly. The results generally show that the reaction rate parameter increases the temperature of the system, whereas the radiation parameter reduces the temperature profiles thereof.

1. Introduction

The transfer of heat in a stockpile of reactive materials has sparked a significant interest to many researchers. For example, the spontaneous combustion of a stockpile of coal in the mining industries, encouraged many scientists to conduct experiments to investigate the causes of this phenomenon. Researchers like Hensel et al. [1], Lohrer et al. [2], and Schmidt et al. [3], carried out experiments to determine the effects of physical parameters on the spontaneous combustion of coal stockpiles. The physical parameters studied include material humidity, coarseness of the material, temperature of the environment, and the stockpile oxygen concentration, just to mention a few. The main cause of spontaneous combustion in a stockpile of reactive materials, is low-temperature oxidation reaction. This process takes place when the carbon component of the material in a stockpile reacts automatically with the oxygen trapped within the stockpile. The spontaneous combustion process, due to the low-temperature oxidation process, also called the exothermic chemical reaction, results in heat and greenhouse gases as products [4,5]. As the combustion takes place, the release of heat increases with time. If the heat keeps on increasing within the stockpile, it may raise the temperature to a level where self-ignition takes place, unless there is sufficient release of heat to the surrounding environment [6,7].

The investigation of heat transfer in this article is executed theoretically, using a mathematical approach. This approach considers thermo-physical parameters embedded in the differential equation governing the problem. The parameters are varied to study their effects on the temperature of the combusting stockpile. The parameters which enhance the low-temperature oxidation process, will facilitate the release of more heat as a product, and consequently increase in the temperature of the stockpile is experienced.

This type of theoretical investigation of heat transfer in combustible materials was carried out by several researchers [8,9,10,11]. The previous studies mentioned above, did not concentrate mainly on the heat loss to the environment by both convection and radiation, which is investigated in this article. The mathematical model and
the numerical approach are discussed in section 2 and 3 respectively, and the results are shown in section 4 and discussed accordingly.

2. Mathematical formulation

A complete irreversible one-dimensional combustion process is described by Equation (1) below.

\[ C_5H_j + \left( l + \frac{j}{4} \right) O_2 \rightarrow iCO_2 + \left( \frac{j}{2} \right) H_2O + \text{heat}. \]  

The cylinder is assumed to undergo an \( n \)-dimensional exothermic chemical reaction with a constant thermal conductivity. Heat loss by convection follows Newton’s law of cooling, which is described by

\[ - \frac{dh}{dr} \theta = h \left( T - T_\infty \right), \]

where \( h \) is the heat transfer coefficient, \( k \) is the thermal conductivity of the material, \( T \) is the absolute temperature of the cylinder, and \( T_\infty \) is the ambient temperature. Radiative heat loss to the environment is described by Boltzmann’s law and it is expressed as

\[ q = \mu \sigma (T^4 - T_\infty^4), \]

where \( \mu \) is the object’s emissivity such that \( 0 < \mu < 1 \), and, \( \sigma \) is the Stefan-Boltzmann constant, approximated as \( 5.6703 \times 10^{-8} \text{W/m}^2\text{K}^4 \). The geometry of the problem is illustrated in Figure 1 below.

![Figure 1: Geometry of the problem](image)

To simplify the problem, a one-dimensional ODE is applied, and it is expressed as follows [8,910]:

\[ \frac{k}{r} \frac{d}{dr} \left( r \frac{d\theta}{dr} \right) + QA \left( \frac{RT}{vI} \right)^m \exp \left( - \frac{E}{RT} \right) - \mu \sigma \left( T^4 - T_\infty^4 \right) = 0. \]  

The corresponding boundary conditions are:

\[ r = 0; \quad \frac{dr}{dr} = 0 \]  

\[ r = a; \quad \frac{dr}{dr} = - \frac{h}{k} \left[ T - T_\infty \right]. \]  

Here, \( Q \) is the heat of reaction, and, \( A \) is the rate constant. \( K \) is the Boltzmann’s constant, \( v \) is the vibration frequency, \( l \) is the Planck’s number, \( E \) is the activation energy, and, \( R \) is the universal gas constant. \( m \) represents the kinetics type, which is assigned the following values, -2 for sensitized kinetics, 0 for Arrhenius kinetics, and 0.5 for bimolecular kinetics. Lastly, \( \tau \) is the radius of the cylinder.

The governing equation is transformed into a non-dimensional form to allow application of the RKF45 method to get the solutions. The process is done as follows:

\[ \theta = \frac{E}{RT_\infty} \left( T - T_\infty \right), \quad r = \frac{\tau}{a}, \quad \varepsilon = \frac{RT_\infty}{E}, \quad Ra = \frac{\mu \sigma a^2 T_\infty^2}{k R}, \]

\[ \lambda = \left( \frac{kT_\infty}{vI} \right) \left( \frac{QAEa^2}{kR} \right) \exp \left( - \frac{E}{RT_\infty} \right). \]  

Equations (2) to (4) now take the forms

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d\theta}{dr} \right) + \lambda (1 + \varepsilon \theta)^m e^{\theta/(1+\varepsilon \theta)} - Ra((\varepsilon \theta + 1)^4 - 1) = 0, \]
with the boundary conditions

\[ r = 0; \quad \frac{d\theta}{dr} = 0 \]  \hspace{1cm} (7)

\[ r = 1; \quad \frac{d\theta}{dr} = -Bi\theta. \]  \hspace{1cm} (8)

Here, \( \theta \) is the dimensionless temperature, \( \lambda \) is the Frank-Kamenetskii (reaction rate) parameter, \( Bi \) is the thermal Biot number, \( \varepsilon \) is the activation energy parameter, \( r \) is the cylinder’s radius, and \( Ra \) is the radiation parameter.

3. Numerical Approach

The governing equation was solved using the RKF45 method coupled with the Shooting technique, which are embedded in Maple software that was used in this case, to solve the dimensionless Equations (6) – (8). The algorithm in this numerical procedure reduces the higher orders of the ODE to the first order form. Then, it follows that, \( \dot{\theta} = p_1, \quad \dot{p}_1 = p_2 \). Therefore, Equations (6) – (8) are then modified to

\[ p_1' = p_2 \]

\[ p_2' = -p_2 - \lambda(1 + \varepsilon p_1)^m e^{p_1/(1+\varepsilon p_1)} + Ra((\varepsilon p_1 + 1)^4 - 1), \]  \hspace{1cm} (9)

with the boundary conditions

\[ p_2(0) = 0, \quad p_2(1) = -Bi\theta. \]  \hspace{1cm} (10)

4. Result and discussion

The numerical solutions to Equations (6) – (8) are given graphically in this section. The solutions illustrate the effects of the kinetic parameters on the temperature during the combustion process. The following parameters values were used:

\[ \lambda = 0.1, m = 0.5, Bi = 1, \varepsilon = 0.1, \text{ and } Ra = 1. \]

The effects of the kinetic parameters on the temperature are illustrated in Figures 2 – 6. Figures 2 – 3, show that an increase in the reaction rate parameter (\( \lambda \)) and the kinetics type parameter (\( m \)), gives a corresponding increase in the temperature profiles. The increase in \( \lambda \) indicates that the low-temperature oxidation process is accelerated, giving more heat as the product, hence the increase in the temperature. The increase in \( m \) means that the low-temperature oxidation process is quickest in biomolecular kinetics (\( m = 0.5 \)), compared to the Arrhenius type (\( m = 0 \)), and that is slow in the sensitized (light induced) kinetics (\( m = -2 \)), which is the lowest profile in Figure 3. A different scenario is illustrated in Figures 4 – 6, where an increase in the parameters \( \varepsilon \) (the activation energy), \( Ra \) (radiation), and \( Bi \) (thermal Biot number), show a decrease in the profiles of temperature. These parameters tend to slow down the rate of the low-temperature reaction, which means that the release of heat in the system is minimized, hence the decrease in the temperature. The effect of \( \varepsilon \) means that the activation energy of the materials in a cylindrical domain is not so higher to stimulate the combustion process. The decrease in the temperature due to \( Ra \), means that more heat can escape the cylinder due to radiation, and lastly, the effect of \( Bi \) on the temperature indicates that more heat is also allowed to escape the system by convection. The last three parameters are helpful to reduce the rate of the low-temperature oxidation process which when accelerated, produces more heat and greenhouse gases, which are detrimental to the environment.
Figure 2: $\lambda$ effect on temperature

Figure 3: $m$ effect on temperature

Figure 4: $\epsilon$ effect on temperature

Figure 5: $Ra$ effect on temperature

Figure 6: $Bi$ effect on temperature
5. Conclusion
In this article, the transfer of heat in a stockpile of combustible materials modeled in a cylindrical domain was investigated. The heat production in reactive materials is due to the low-temperature oxidation process, and it was demonstrated by the graphical solutions that the kinetic parameters such as the reaction rate and the kinetics type, accelerate the production of heat, which contributes extensively to global warming. On the other hand, it was discovered that the following parameters, activation energy, radiation and thermal Biot number, decelerate the low-temperature oxidation process, and consequently less heat is released. These parameters are important to keep the environment safe, and appropriate control thereof can help to reduce veld fires that are caused sometimes by self-ignition process in stockpiles of combustible material left out in the open space, like the dumped rubbish of used plastics, papers and clothes. This study can be extended to multi-step combustion processes, such as the combustion process of fuel in an automobile engine.

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