An Integer Programming Model For Solving Heterogeneous Vehicle Routing Problem With Hard Time Window considering Service Choice

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Abstract. Generally a Vehicle Routing Problem with time windows (VRPTW) can be defined as a problem to determine the optimal set of routes used by a fleet of vehicles to serve a given set of customers with service time restrictions; the objective is to minimize the total travel cost (related to the travel times or distances) and operational cost (related to the number of vehicles used). In this paper we address a variant of the VRPTW in which the fleet of vehicle is heterogenic due to the different size of demand from customers. The problem, called Heterogeneous VRP (HVRP) also include service levels. We use integer programming model to describe the problem. A feasible neighbourhood approach is proposed to solve the model.

Keywords: Vehicle routing problem, Heterogenic fleet, time restricted, integer programming, feasible neighbourhood search

1. Introduction
Vehicle Routing Problem (VRP) is a key element of many distribution systems which involve routing and scheduling of vehicles through a set of nodes. This is a well known combinatorial optimization problem which requires the determination of an optimal set of routes used by a fleet of vehicles to serve a set of customers, taking into account various operational constraints. VRP was first introduced by Dantzig and Ramser (1959). Since then many researchers have been working in this area to discover new methodologies in selecting the best routes in order to find the better solutions. Studies on this NP-hard problem have resulted in several exact and heuristic techniques of general applicability (Cordeau et al., 2007; El-Sherbeny, 2010; Yeun et al., 2008; Cordeau et al., 2001). Drexl (2011) presents a compact review of vehicle routing literature.

The VRPTW is a generalisation of VRP. It can be reviewed as a combined vehicle routing and scheduling problem which often arises in many real-world applications. It is to optimise the use of a fleet of vehicles that must make a number of stops to serve a set of customers, and to specify which customers should be served by each vehicle and in what order to minimise the cost, subject to vehicle capacity and service time restrictions (Ellabib et al. 2002). The problem involves assignment of vehicles to trips such that the assignment cost and the corresponding routing cost are minimal.

In terms of graph the VRPTW can be defined as follows: Let $G = (V, E)$ be a connected digraph consisting of a set of $n + 1$ nodes, each of which can be reached only within a specified time interval or time window, and a set $E$ of arcs with non-negative weights representing travel distances and
associated travel times. Let one of the nodes be designated as the depot. Each node $i$, except the depot, requests a service of size $q_i$.

The VRPTW has been the subject of intensive research efforts for both heuristic and exact optimisation approaches. Early surveys of solution techniques for the VRPTW can be found in Golden and Assad (1986), Desrochers et al. (1990). The main focus in Desrosiers et al. (1995) and Cordeau et al. (2001) were exact solution techniques. Further details on these exact methods can be found in Larsen (1999) and Cook and Rich (1999). Because of the high complexity level of the VRPTW and its wide applicability to real-life situations, solution techniques capable of producing high quality solutions in limited time, i.e., heuristics, are the most popular. Shi et al. (2012) propose a modified artificial bee colony algorithm to solve VRPTW. A hybrid metaheuristic based on Large Neighbourhood Search approach and Variable Neighbourhood Search technique was proposed by Rincon-Garcia et al. (2017) to tackle the VRPTW.

The company focused in this paper has various type of vehicle in its operation to deliver goods for customers. Each type of vehicle has different capacities. The variant of VRP which considers mixed fleet of vehicles is called Heterogeneous VRP (HVRP), introduced firstly by Golden et al. (1984). This generalization is important in practical terms, for most of customers demand are served by several type of vehicles (Hoff et al., 2010; Koc, 2016). The objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

As this is a combinatorial problem it is not surprising that the most approach addressed for solving the HVRP is heuristics. Liu and Shen (1999) were the first to tackle the HVRPTW and developed a number of parallel insertions heuristics based on the insertion scheme of (Solomon, 1987) and embedding in the calculations of the relevant criteria the acquisition costs of (Golden et al. 1984). Brasy et al. (2008) presented a deterministic annealing meta heuristic for the HVRPTW, outperforming the results of (Liu and Shen 1999), and then Brasy et al. (2009) developed a linearly scalable hybrid threshold-accepting and guided local search meta heuristic for solving large scale HVRPTW instances. Coelho et al. (2016) designed a trajectory search heuristic to solve a large-scale HVRPTW considering multi-trips. Repousis and Tarantillas (2010) presented an Adaptive Memory Programming solution approach for the VRPTW that provides very good results in the majority of the benchmark instances examined. Subramaniam et al. (2012) proposed a hybrid algorithm for the problem. Their algorithm is composed by an Iterrated Local Search (ILS) based heuristic and a Set Partitioning (SP) formulation. Yousefikhoshbakht et al. (2016) present a modified column generation to solve HVRPTW.

This paper concerns with HVRP with hard time windows and service choice. The basic framework of the vehicle routing part can be viewed as a Vehicle Routing Problem with Time Windows (VRPTW) in which there are a limited number of vehicles, characterized by different capacities are available and the customers have a specified time windows for services. We address a large-scale mixed integer programming formulation to model the problem. A feasible neighbourhood heuristic search is proposed to get the sub-optimal integer feasible solution.

2. Problem Description of The Hvrtw-SC

This paper considers a problem faced by a service company located in Medan city, North Sumatra Province, Indonesia. The basic framework of the VRPTW can be defined graphically as follows. Let $G = (V, A)$ be a complete directed graph, where $V = \{0, 1, \ldots, n\}$ is the vertex set and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of route. For each route $(i, j) \in A$ a distance (or travel) cost $c_{ij}$ is defined. Vertex 0 ($i = 0$) is the depot vertex, center of service, where the vehicle fleet is located. Define $V_c$ is the set of customers’ vertex. Each vertex $i \in V_c$ has a known fixed daily demand $w_i \geq 0$, a service time $s_i \geq 0$, and a service time windows $[a_i,b_i]$. In particular, at depot the demand $w = 0$ and service time $t = 0$.

As mentioned before this is a heterogeneous problem, therefore the fleet of K vehicles is composed by m different type of vehicles, each with capacity $Q_m$. The number of vehicles available to be used for vehicle type $m$ is $n_m$. Let $K_m$ is the set of vehicle type $m$ then each customer is served
accordingly by exactly one vehicle. At the depot \((i = 0)\), define a time window for vehicles to leave and to return to depot with \([a_0, b_0]\). We then can say that the arrival time of a vehicle at customer \(i\) is \(a_i\) and its departure time is \(b_i\). Each type of vehicle is imposed with a fixed cost, \(f_m\). Further more, a fixed acquisition cost \(f_k\) is incurred for each of vehicle \(k\) in the routes. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer \(i\) before \(a_i\) and after \(b_i\) however, the vehicle can arrive before \(a_i\) and wait for service.

Each customer node \(i \in V_c\) has a known daily demand, \(W_i\), service frequency, \(\sigma_i\) and a minimum service frequency, \(F_i\), measured in days, \(t\), per period. The demand accumulated between visits, \(w_i\), is a function of the daily demand of the node. The stopping cost at a node \(i\), \(\tau_i\), is a function of the frequency of the schedule since more items accumulate with less frequent service and, therefore, require more time to load/unload. Associated with each arc \((i, j) \in A\) is a known travel cost, \(c_{ij}\). There is a set \(K\) of vehicles, each with capacity \(C\), and \(T\) is the set of workdays in the planning horizon.

To associate with service choice, it is reasonable to define a service gain (in terms finance), \(\delta_i\), as an incentive to offer more frequent service to customers. Therefore we can say that service gain is a function demand’s rate \(w_i\).

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- \(x_{0j}^k = \begin{cases} 1 & \text{if vehicle type } k \in K \text{ to deliver from depot to customer } j \in V_c; \\ 0 & \text{otherwise;} \end{cases}\)
- \(x_{ij}^m = \begin{cases} 1 & \text{if vehicle type } m \in K_m \text{ to deliver for } (i, j) \in V_c, i \neq j; \\ 0 & \text{otherwise;} \end{cases}\)
- \(z_0^m = \begin{cases} 1 & \text{if vehicle type } m \in K \text{ is available and active at depot;} \\ 0 & \text{otherwise;} \end{cases}\)
- \(l_i^m\) Arrival time for vehicle type \(m \in K_m\) at customer \(i \in V_c\) (non-negative continuous variable)
- \(u_i^m\) Duration of service of vehicle type \(m \in K_m\) at customer \(i \in V_c\) (non-negative continuous variable)

### 3. The mathematical model

First we formulate the objective function. The decision to be made is to choose the route of vehicles to serve customers demand, such that to minimize cost. Expression (1) is the objective function which describes the minimum of the travel costs.

In this basic framework the manager of the catering company wants to use the available vehicle for each type efficiently, such that the total cost is minimized. The total cost consists of traveling cost of all vehicle used and the cost for the availability of vehicle in the planning horizon time of a day.

\[
\text{Minimize} \quad \sum_{j \in V_c} c_{oj} \sum_{k \in K} x_{0j}^k + \sum_{(i, j) \in V_c} \sum_{m \in K_m} \sum_{t \in T} (r_{ij} \sigma_{ij} - w_{ij} \delta_{ij}) x_{ijm} + \sum_{m \in K_m} f_m z_0^m
\]  

(1)
Subject to

\[ \sum_{k \in K} x_{0j}^k = 1, \quad \forall j \in V_c \quad (2) \]

\[ \sum_{k \in K} \sum_{j \in V} x_{kj}^k = 1, \quad \forall i \in V_c \quad (3) \]

\[ \sum_{i \in V} x_{ij}^k - \sum_{i \in V} x_{ji}^k = 1; \quad \forall j \in V_c, \forall k \in K \quad (4) \]

\[ x_{ij}^m \leq z_0^m, \quad (i, j) \in V_c, \forall m \in K_m \quad (5) \]

\[ \sum_{j \in V_c} x_{ij}^k \leq 1; \quad \forall k \in K \quad (6) \]

\[ \sum_{i \in V_c, j \in V} x_{ij}^k \leq 1; \quad \forall k \in K \quad (7) \]

\[ \sum_{i \in V_c} d_i \sum_{j \in V_c} x_{ij}^m \leq Q_m; \quad \forall m \in K_m \quad (8) \]

\[ x_{ij}^m (l_i^m + u_i^m + s_i + t_{ij} - l_j^m) = 0; \quad \forall m \in K_m, (i, j) \in A \quad (9) \]

\[ l_i^m \leq a_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c \quad (10) \]

\[ a_i \sum_{j \in V_c} x_{ij}^m \leq l_i^m + u_i^m \leq b_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c \quad (11) \]

\[ \sum_{j \in V_c} w_{ij} x_{ij}^m \leq n_m^k; \quad \forall m \in K_m \quad (12) \]

\[ x_{0j}^k, x_{ij}^m, z_0^m \in \{0,1\}; \quad \forall i \in V, \forall j \in V_c, \forall k \in K, \forall m \in K_m \quad (13) \]

\[ l_i^m, u_i^m \geq 0; \quad \forall i \in V_c, \forall m \in K_m \quad (14) \]

Constraints (2) and (3) are to guarantee that only one vehicle regardless their type are allowed to enter and depart from every customer node and comes back to the depot. Constraint (4) describes a flow conservation equation to maintain the continuity of each vehicle route on each period of time. Constraint (5) states that each customer is served only by the available and active vehicle of the corresponding type. Constraints (6) and (7) are to check the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively. Constraints (8) ensure that each delivery does not exceed the capacity of each type of vehicle. Constraints (9) establishes the equilibrium among the arrival time, duration of service, service time and travel time at customers in the routes assigned. Constraints (10) and (11) present strict time window for each customer. Constraint (12) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot. Constraint (13) to state the binary variables, and Constraint (14) to define the continuous variables.
4. The Method Proposed

Stage 1.
Step 1. Solve the relaxed problem. If the result is a feasible integer solution. Stop.
    The original problem has been solved.
    Otherwise go to step 2.
Step 2. Get row $i^*$ such that
    \[ \delta_i = \min\{f_i, 1 - f_i\} \]
Step 3. Perform a pricing operation
    \[ v^*_i = c^*_iB^{-1} \]
Step 4. Calculate the maximum movement of nonbasic $j$, \( \alpha_j = v^*_i \alpha_j \)
    With $j$ corresponds to
    \[ \min_j \begin{bmatrix} d_j & \alpha_j \end{bmatrix} \]
    Eventually the column $j^*$ is to be increased from LB or decreased from UB. If none go to next $i^*$.
Step 5. Solve \( B\alpha_i = \alpha_j \) for \( \alpha_j \)
Step 6. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic $j^*$
    from its bounds.
Step 7. Exchange basis
Step 8. If row $i^* = \emptyset$ go to Stage 2, otherwise Repeat from step 2.

Stage 2.
Step 1 adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.
Step 2 adjust integer feasible superbasics. The objective of this phase is to conduct a neighbourhood search to verify local optimality.

5. Conclusion

This paper was intended to present a solution for one of the most important problems in Supply Chain Management and Distribution problems. The aim of this paper was to develop a model of heterogeneous vehicle routing problem (HVRP) with time windows considering service-choice. This problem has imposed a benefit that could be gained as incentive for offering serviced to customers. The proposed algorithm employs nearest neighbor heuristic algorithm for solving the model.

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