$B$ decays into a scalar/tensor meson in pursuit of determining the CKM angle $\gamma$

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In this work, I suggest a new way for determining the CKM angle $\gamma$ via $B$ decays into a scalar/tensor meson without any hadronic uncertainty. The proposed idea makes profits of the two triangles formed by the $B^\pm \rightarrow (D^0, \bar{D}^0, D_{CP}^0) K^{\pm}_{0(2)}(1430)$ decay amplitudes. The advantages in it are large CP asymmetries and the avoidance of the use of doubly Cabibbo-suppressed $D$ decays. Branching ratios of $B^\pm \rightarrow (D^0, \bar{D}^0, D_{CP}^0) K^{\pm}_{0(2)}(1430)$ are estimated to have the order $10^{-6} - 10^{-5}$ and therefore measurable by the ongoing LHCb experiment and future experimental facilities. The usefulness of other related channels, for instance the neutral $B_d$ decays into $DK_{0(2)}^*$ (1430) and $B_s \rightarrow (D^0, \bar{D}^0) M$ ($M = f_0(980), f_0(1370), f_2^0(1525), f_1(1285), f_1(1420), h_1(1180)$), the $B \rightarrow \bar{D}^0 a_{0,2}^+$ for the extraction of $\gamma + 2\beta$ and the $B_s \rightarrow D^+_s K_{0,2}^{*+}$ to access $\gamma + 2\beta_s$, is also discussed in brevity.

I. INTRODUCTION

The standard model description of CP violation is encoded in a single, irreducible phase in the $3 \times 3$ quark mixing matrix called the Cabibbo-Kobayashi-Maskawa (CKM) matrix. One of the foremost tasks in the past decades has been to study the $(bd)$ unitarity triangle, the graphical representation of the condition stemming from the unitarity of the CKM matrix: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, whose angles ($\alpha, \beta, \gamma$) satisfy $\alpha + \beta + \gamma = 180^\circ$. In contrast with the precise results on $\alpha$ and $\beta$ [1, 2], our knowledge of the angle $\gamma$ is rather uncertain, with a precision of roughly $10^\circ$. This is one of the dominant sources of the current uncertainties on the apex of the $(bd)$ unitary triangle. One of the major efforts in flavor physics by LHCb and the future SuperB factory experiments will be devoted to reduce the errors in $\gamma$.

In this work [3], we propose a new method based on $B \rightarrow DM$ decays with $M$ being a light scalar/tensor meson. Among the various modes to be discussed, of special interest are the $B^\pm \rightarrow (D^0, \bar{D}^0, D_{CP}^0) K^{\pm}_{0(2)}(1430)$ modes, where $K^{\pm}_{0(2)}(1430)$ is a $J^P = 0^+(2^+)$ scalar (tensor) resonance. The small (zero) decay constant of $K^0_3(1430)/(K^0_2(1430))$ compensates the large Wilson coefficient in the color-allowed amplitude, leading to comparable decay amplitudes of $B^\pm \rightarrow D^0 K^\pm_{0(2)}(1430)$ and $B^\pm \rightarrow \bar{D}^0 K^{*\pm}_{0(2)}(1430)$. As a consequence, large CP asymmetries are predicted. Measurements of branching ratios (BRs) of the neutral $B_d$ decays into $DK_{0(2)}^*$ (1430) and time-dependent CP asymmetries in $B_s \rightarrow DM$ ($M = f_0(980), f_0(1370), f_2^0(1525), f_1(1285), f_1(1420), h_1(1180)$) are also useful to extract the angle $\gamma$. In addition, the combination $\gamma + 2\beta$ and $\gamma + 2\beta_s$ could be extracted via time-dependent measurements of the $B \rightarrow \bar{D}^0 a_{0,2}^+$ and the $B_s \rightarrow D^+_s K_{0,2}^{*+}$ which may have large CP asymmetries.

II. $\gamma$ FROM $B \rightarrow D^0(\bar{D}^0) K^*_{0,2}(1430)$

The angle $\gamma \equiv \arg(-V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*))$ is the relative weak phase of the decays induced by the $b \rightarrow c\bar{s}s$ and $b \rightarrow u\bar{c}c$ transition. The proposed method in this work uses the information that the six decay amplitudes of $B^\pm \rightarrow (D^0, \bar{D}^0, D_{CP}^0) K^{\pm}_{0,2(2)}$ form two triangles in the complex plane, graphically manifesting the identities

$$\sqrt{2}A(B^+ \rightarrow D^0_{\pm} K^+_{0,2}) = A(B^+ \rightarrow D^0 K^+_{0,2}) \pm A(B^+ \rightarrow \bar{D}^0 K^+_{0,2}),$$

$$\sqrt{2}A(B^- \rightarrow D^0_{\pm} K^-_{0,2}) = A(B^- \rightarrow D^0 K^-_{0,2}) \pm A(B^- \rightarrow \bar{D}^0 K^-_{0,2}),$$

in which we have adopted the convention $CP|D^0_\pm = |\bar{D}^0_\pm|$ and $D^0_\pm = (|D^0_\pm|\pm |\bar{D}^0_\pm|)/\sqrt{2}$ are the CP even and odd eigenstates. Measurements of the decay rates of the six processes have the potential to completely determine the sides and apexes of the two triangles, more particularly the relative phase between $A(B^- \rightarrow D^0 K^+_{0,2})$ and its CP conjugate $A(B^+ \rightarrow D^0 K^+_{0,2})$ is $2\gamma$.

The ratio of the sides and the relative strong phase difference govern the shape of the two triangles

$$r^{K_j}_{\alpha} \equiv |A(B^- \rightarrow \bar{D}^0 K^+_j)/A(B^- \rightarrow D^0 K^-_j)|, \delta^{K_j}_{\alpha} \equiv \arg(e^{i\gamma}A(B^- \rightarrow \bar{D}^0 K^+_j)/A(B^- \rightarrow D^0 K^-_j)),$$

with $K_j = K^*_{0,2}$. In fact physical observables to be experimentally measured are defined in terms of these two

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where the color-allowed amplitude is comparable to the color-suppressed one giving $K^*_0$. The ratio of decay amplitudes is small and thereby the two triangles formed by decay amplitudes are very squashed. In this figure, the solid (green), dashed (black), dotted (blue) and dot-dashed (orange) lines in diagrams (a,c,e) are obtained according to four choices $\delta^K_B = (30, 60, 120, 150)\circ$ respectively, while the corresponding curves in panels (b,d,f) correspond to $\delta^K_B = (30, 60, -30, -60)\circ$. The shadowed (light-green) region denotes the current bounds on $\gamma = (68^{+10}_{-11})\circ$ from a combined analysis of $B^\pm \to DK^\pm$ [2], in which the vertical (red) line reflects the central value.

Up to now, experimental results on $\gamma$ mostly come from the $B^\pm \to DK^\pm$ [2, 4], in which the $B^- \to D^0 K^-$ is suppressed by both CKM factor and color factor. The ratio of decay amplitudes $r_B^2 \sim |V_{ub}V_{cs}^\ast|/|V_{cb}V_{us}^\ast|a_2/a_1| \sim 0.1$ is small and thereby the two triangles formed by decay amplitudes are very squashed. $R_{CP\pm}^K$ and $A_{CP\pm}^K$ have a mild sensitivity to the angle $\gamma$, inducing large experimental uncertainties in $\gamma$ [2, 4].

Here we wish to stress that the low sensitivity problem is highly improved in $B^\pm \to DK^\pm$ due to $r_{K^*_0,2} \sim 1$ and in particular large CP asymmetries are expected. Although the color-allowed diagram has a large Wilson coefficient $a_1 \sim 1$, the emitted $K^*_0,2$ meson is produced from a local vector or axial-vector current (at the lowest order in $\alpha_s$), whose matrix element between the QCD vacuum and the $K^*_0(K^*_2)$ state is small (identically zero). Due to the suppression from decay constant, the color-allowed amplitude is comparable to the color-suppressed one giving $r_{K^*_0,2} \sim 1$.

An estimate of decay amplitudes can be made under the factorization assumption

$$A(B^- \to D^0 K_0^{* -}) = -V_{ub}V_{cs}^\ast C, \quad A(B^- \to D^0 K_2^{* -}) = -V_{cb}V_{us}^\ast (C - T), \quad (2)$$

where

$$C = G_F f_{D^0} a_2 (m_{B^0}^2 - m_{K_0^*}^2) f_{K_0^*}^{BD}(m_{K_0^*}^2)/\sqrt{2}, \quad T = G_F f_{K_0^*} a_1 (m_{B^0}^2 - m_{D^0}^2) f_{D^0}^{BD}(m_{K_0^*}^2)/\sqrt{2},$$

and $G_F$ is the Fermi constant. The $K_0^*$ decay constant vanishes in the flavor SU(3) symmetry limit and may acquire a small value arising from the symmetry breaking effects. Adopting the results in QCD sum rules [3]

$$f_{K_0^*} = -24\text{MeV}, \quad \text{or} \quad f_{K_0^*} = 36\text{MeV},$$

FIG. 1: The sensitivity of $R_{CP\pm}^{K^*_0}$ and $A_{CP\pm}^{K^*_0}$ to $\gamma$. In panels (a,b), $r_B^{K^*_0} = 2$ is employed, while in panels (c,d) $r_B^{K^*_0} = 1$. In the last two panels (e,f), we consider the case in which the ratio is not enhanced sizably $r_B^{K^*_0} = 0.3$. Such choice corresponds to the sign of the Wilson coefficient $a_2$ reversed namely $a_2 = -0.2$, resulting in a large branching ratio: $B(B^- \to D^0 K_0^{* -}) \sim 10^{-5}$. The analysis of $\gamma$ [2, 4] results in $\Delta = 90^{+30}_{-15}\circ$ with a central value $\gamma = (68^{+10}_{-11})\circ$. In the last two panels (e,f), we consider the case in which the ratio is not enhanced sizably $r_B^{K^*_0} = 0.3$. Such choice corresponds to the sign of the Wilson coefficient $a_2$ reversed namely $a_2 = -0.2$, resulting in a large branching ratio: $B(B^- \to D^0 K_0^{* -}) \sim 10^{-5}$.
we obtain the relation $2a_1|f_{K^*_1}| \sim a_2 f_D$. Using the $B \rightarrow K^*_0$ form factors computed in the perturbative QCD approach [6], the $B \rightarrow D$ form factors from Ref. [7] and $a_2 = 0.2, a_1 = 1$ we get $C/T \sim 1.2$ and

$$r_B^{K^*_0} = |CV_{ub}V_{cs}^*|(V_{cb}V_{us}^*(C-T))| \sim 2, \quad B(B^+ \rightarrow \bar{D}^0 K^{*+}_0) \sim 4 \times 10^{-6}. \quad (3)$$

Since the strong phase cannot be computed by perturbation theory, we shall take several benchmark values to illustrate the dependence of $R_{CP+}$ and $A_{CP+}$ in Fig. 1. In panels (a,b), $r_B^{K^*_0} = 2$ is employed, while in panels (c,d) $r_B^{K^*_0} = 1$. In the last two panels (e,f), we consider the case in which the ratio is not enhanced sizably $r_B^{K^*_0} \approx 0.3$. Such choice corresponds to the sign of the Wilson coefficient $a_2$ reversed namely $a_2 = -0.2$, resulting in a large branching ratio: $B(B^+ \rightarrow D^0 K^{*+}_0) \sim 10^{-9}$. In this figure, the solid (green), dashed (black), dotted (blue) and dot-dashed (orange) lines in diagrams (a,c,e) are obtained according to four choices $\delta_B^{K^*_0} = (30, 60, 120, 150)^\circ$ respectively, while the corresponding curves in panels (b,d,f) correspond to $\delta_B^{K^*_0} = (30, 60, -30, -60)^\circ$. The shadowed (light-green) region denotes the central bounds on $\gamma = (68^{+10}_{-11})^\circ$ from a combined analysis of $B^+ \rightarrow DK^{*+}$ [2], in which the vertical (red) line reflects the central value. CP odd quantities can be obtained via the relation in Eq. (2), for instance $P_{CP-} = (P_{CP+})_{\delta_B^{K^*_0}} \rightarrow 180^\circ - \delta_B^{K^*_0}$.

III. $\gamma$ FROM OTHER USEFUL CHANNELS

The above method to use the two triangles formed by the six decay amplitudes for determining $\gamma$ is also valid in $B_s \rightarrow D K^{*0}_{0,2}$, in which the tree amplitude $T$ vanishes. No time-dependent measurement is required since $K^{*0}_{0,2}$ is self-tagging. The color-suppressed decay amplitudes involving $D^0$ and $D^{*0}$ arise from the same type of Feynman diagram, thereby one may expect that $\delta_B^{K^*_0} \sim 0$. If true, the CP asymmetries $A_{CP\pm}^{K^*}$ would be roughly 0 but $P_{CP\pm}$ can sizably deviate from 1.

Following the similar strategy, we collect in Tab. III other $B$ decay channels into a scalar/tensor meson useful for the extraction of CKM angle $\gamma$ or its combination. All these decay modes are expected to have larger $r_f$, ratios of decay amplitudes, compared to the corresponding channels in which the scalar/tensor meson is replaced by a pseudoscalar pion/kaon. For earlier discussions on $B$ decays into a scalar/tensor meson and the role in extracting $\gamma + 2\beta$, please see Ref. [8]. The branching fractions and ratios of decay amplitudes of the $B \rightarrow DT$ modes are taken from the recent calculation in the perturbative QCD approach [9] while the rest entries when available are obtained in the factorization approximation [3] with inputs from Refs. [3,10,11]. Since the branching ratios obtained in naive factorization method are very small and usually suffer large theoretical uncertainties, an analysis in which the QCD/power corrections are taken into account is called for.

To illustrate how to extract the CKM angle, we consider the example of $B_s \rightarrow D f_0$ with the amplitudes

$$A(\bar{B}_s \rightarrow \bar{D}^0 f_0) = V_{ub}V_{cs}^*A_1, \quad A(B_s \rightarrow D^0 f_0) = V_{ub}V_{cs}^*A_1,$$

$$A(\bar{B}_s \rightarrow D^0 f_0) = V_{ub}V_{us}^*A_2, \quad A(B_s \rightarrow \bar{D}^0 f_0) = V_{ub}V_{us}^*A_2. \quad (4)$$

For each amplitude, there is only one weak phase, and therefore no direct CP asymmetry is predicted. For simplicity we use the notation for the relative size and strong phase of the two amplitudes

$$r_{B_s}^f = |V_{ub}V_{cs}^*A_1/(V_{ub}V_{us}^*A_2)|, \quad \delta_{B_s}^f = arg(A_1/A_2). \quad (5)$$

The normalized time-dependent decay widths of $B_s \rightarrow D f_0$ are given by [12]

$$\Gamma(B_s^0(t) \rightarrow D^0(\bar{D}^0)f_0) = e^{-t/\tau_{B_s}}\tilde{\Gamma}\left[1 + \cos(\Delta m t)C_{D^0(\bar{D}^0)f_0} + \sin(\Delta m t)S_{D^0(\bar{D}^0)f_0}\right], \quad (6)$$

where $\tilde{\Gamma}$ is the averaged decay width and we have neglected the width difference in the evolution. For the corresponding $\bar{B}_s^0$ decays, the signs in front of cosine and sine terms should be reversed. Substituting the amplitudes defined in Eq. (4), we have

$$C_{D^0 f_0} = -C_{\bar{D}^0 f_0} = [1 - (r_{B_s}^f)^2]/[1 + (r_{B_s}^f)^2],$$

$$S_{D^0 f_0} = 2r_{B_s}^f \sin(\gamma + \delta_{B_s}^f + 2\beta_s)/[1 + (r_{B_s}^f)^2],$$

$$S_{\bar{D}^0 f_0} = 2r_{B_s}^f \sin(\gamma - \delta_{B_s}^f + 2\beta_s)/[1 + (r_{B_s}^f)^2]. \quad (7)$$

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with the phase $\beta_s$ being the $B_s - \bar{B}_s$ mixing phase $q/p = V_{tb} V_{ts}^*/(V_{tb}^* V_{ts}) = e^{-2i\beta_s}$. In the SM, $\beta_s \simeq -0.019$ rad. The equality $C_{D^0 f_0} = -C_{D^0 f_0}$ is due to the fact that there is one weak phase in decay amplitudes. Measuring the time-dependent decay widths, we will be able to determine the coefficients $C_{D^0(D^0) f_0}$ and $S_{D^0(D^0) f_0}$, and thus the three quantities, $r_{f_0}^2$, $\delta_{f_0}$, and $\gamma + 2\beta_s$, can be extracted cleanly modulo a discrete ambiguity.

### IV. CONCLUSIONS

Much progress has been made in recent years in testing the CKM description of the quark mixing and the CP violation. In this work we have explored the possibility to extract the CP violation angle $\gamma$ or the combination $\gamma + 2\beta_s$ with $B$ decays into a scalar/tensor meson. A clean method is to use the two triangles formed by the decay amplitudes of $B^\pm \to (D^0, D^0)^* f_0^\pm K_{0(2)}^*$ (1430). We expect that $B^\pm \to D^0 K_{0(2)}^*$ (1430) and $B^\pm \to D^0 K_{0(2)}^*$ (1430) have similar decay rates and the CP asymmetries have a strong correlation with $\gamma$. The analysis is also supported by the recent study of $B\to DT$ in perturbative QCD approach. Our method does not require the reconstruction of the $D$ meson via its doubly Cabibbo-suppressed decays, which are usually buried under the combinatorial background. Using the factorization assumption and the relevant experimental data we have estimated he branching ratios of these modes and find them to be of order $10^{-5} - 10^{-6}$. The usefulness of other related channels, for instance the neutral $B_d$ decays into $D K_{0(2)}^*$ (1430) and the time-dependent CP asymmetries in $B_s \to (D^0, D^0)^* M (M = f_0(980), f_0(1370), f_2^*(1525), f_1(1285), f_1(1420), h_1(1180))$, the $B \to D^\mp a_{0,2}^\mp$ for the extraction of $\gamma + 2\beta$ and $B_s \to D^\mp K_{0,2}^*$ to access $\gamma + 2\beta_s$, is also discussed in brevity.

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