A project optimization for small watercourses restoration in the northern part of the Volga-Akhtuba floodplain by the geoinformation and hydrodynamic modeling

Alexander Voronin, Ann Vasilchenko, Alexander Khoperskov
Mathematics and Information Technology Institute, Volgograd State University, Volgograd, 400062, Russia
E-mail: *khoperskov@volsu.ru

Abstract. The project of small watercourses restoration in the northern part of the Volga-Akhtuba floodplain is considered together with the aim of increasing the watering of the territory during small and medium floods. The topography irregularity, the complex structure of the floodplain valley consisting of large number of small watercourses, the presence of urbanized and agricultural areas require careful preliminary analysis of the hydrological safety and efficiency of geographically distributed project activities. Using the digital terrain and watercourses structure models of the floodplain, the hydrodynamic flood model, the analysis of the hydrological safety and efficiency of several project implementation strategies has been conducted. The objective function values have been obtained from the hydrodynamic calculations of the floodplain territory flooding for virtual digital terrain models simulating alternatives for the geographically distributed project activities. The comparative efficiency of several empirical strategies for the geographically distributed project activities, as well as a two-stage exact solution method for the optimization problem has been studied.

1. Introduction
The hydrographic network of the Volga-Akhtuba floodplain (VAF) is an extremely complex system, and its properties are largely determined by seasonal flooding of the area [1, 2]. The aim of our study is the research of northern part of the Volga-Akhtuba floodplain with an area of approximately 900 km². Volzhskaya HPP fully controls the hydrological regime of the Volga River following the primary hydropower interests [3]. The latter leads to significant decrease in the spring floods volume in comparison with the natural volume in absence of the large dams on the Volga River.

The limitations of flood peaks due to the hydrological safety requirements for expanding agricultural and urbanized areas, the anthropogenic degradation of riverbeds of the Volga and Akhtuba Rivers, as well as numerous small channels, are the reasons of the aridization of the interfluve territory [3]. The interfluve flooding does not occur in the case of low spring water on the Volga River, that leads to an ecological catastrophe [4]. The most recent examples of the such years are 2006 and 2015 when in the absence of spring flooding the wetland landscapes transformed into the arid areas without vegetation and reproduction of fish stocks.
Figure 1. The examples of the Volga River HPP dam hydrographs for low (1976, 1996, 2006, 2015) and medium (1981, 1990, 1995, 2013) peaks of water discharge [2, 5] (Figure 1).

One of the most important factors is the natural clean up of the interfluve territory due to very high water levels (with the maximum peak of water discharge $Q_{\text{max}} > 32000 \text{m}^3/\text{sec}$) and long-lasting (more 40 days) flood peak which occurs every few years. Such spring super floods have not been observed during the last 25 years. The implementation of hydrotechnical projects increasing the territory watering level in the case of small and medium floods is perhaps the only way to preserve the ecosystem of the VAF. One of the such projects is the restoration of small watercourses, about 884 km long, in the northern part of the VAF. The branching of the hydrographic network together with the irregularity of the terrain leads to a multidirectional dependence of the flooded area on local changes in the watercourses depths [6]. Therefore, an urgent task aimed at improving the hydrological state and the ecological situation as a whole is the search for the optimal geographically distributed types of activities for the restoration (deepening and cleaning) of small watercourses in the interfluve.

2. Geoinformation and hydrodynamic modeling of floods

The topography irregularity, the complex structure of the floodplain valley consisting of a large number of small watercourses, the presence of urbanized and agricultural areas require a careful preliminary analysis of the hydrological safety and efficiency of geographically distributed project activities. On the basis of digital terrain models and watercourses structure, a discretization of the projects optimization task in the geographically distributed project activities conditions has been carried out. Using the digital terrain and watercourses structure models of the floodplain, and the hydrodynamic flood model, the analysis of the hydrological safety and efficiency of the several project implementation strategies has been conducted [7].

Our objective function has shown the improvement of the interfluve hydrological regime due to clearing of small natural watercourses in this territory. The objective function values have been obtained from the hydrodynamic calculations of the floodplain territory flooding for virtual digital terrain models for simulation of the alternatives for the geographically distributed project
The comparative efficiency of the several empirical strategies for the geographically distributed project activities, as well as a two-stage exact solution method for the optimization problem, has been explored.

The satellite data from ASTER GDEM [8], SRTM [9], Landsat and Sentinel are utilized in our Digital Elevation Model for the northern part of the VAF $b(x_i, y_j) \ (i = 1, ..., N_x; \ j = 1, ..., N_y)$. Using approximation and excluding artifacts in the original data, in the geoinformation system Professional GIS “Panorama” we construct basic grids with a cell size varying from 25 to 10 m in the SK-42 reference system (Krasovsky ellipsoid). The digital map of the riverbeds and river bottoms of the Volga, Akhtuba Rivers and small watercourses in the floodplain valley has been built combined with vectorization of space images, topographic maps, and pilot charts coupled with the results obtained from our field measurements [7, 10].

An important feature of the VAF hydrological regime is the submergence of about $25 - 40\%$ of the interfluve area in spring and drying up of reservoirs in summer. Monitoring the changes in the coastlines of such seasonal reservoirs allows us building up a system of topographical isolines by the actual digital terrain model for the most key-zones of floodplain [10, 11]. Current VAF vector map includes the layer of hydraulic system consisting of 1542 small riverbed objects of 1–3 m depth, the infrastructure layer including 118 settlements and the relief layer comprising more than 15000 relief objects.

To solve the main aim we conducted a hydrological structuring of the territory (Figure 2). The boundaries of 58 zones formed by the local watercourses systems are shown in Figure 3. For example the watercourses of zone 37 are also depicted there. We identified 58 of such zones ($i = 1, 2, ..., 58$) using geoinformation analysis tools for our topography.

The edges of a hierarchical graph correspond to the nodes of the digital terrain model grid denoting the watercourses. Three main watercourses (Volga River, Akhtuba River and Erik Gniloy) form the first (highest) level of the hierarchical graph (See Figure 2). The second level of the graph contains 58 main watercourses about 2–3 m deep for each $i$-th zone. The third level is formed by watercourses of 1 to 2 m deep connected to the riverbeds of the second-level watercourses.

![Figure 2](image_url)
We assume a numerical 2D shallow water dynamics model taking into the account all the principal aspects of the flooding: the inhomogeneous relief of the terrain $b(x, y)$, the bottom friction force $f_{fric}$ for the inhomogeneous Manning coefficient $n_M(x, y)$, the evaporation $\sigma_e$, the infiltration $\sigma_{in}$, the Coriolis force $f_{Cor}$, and the viscous force $f_{visc}$ (this is described in detail in Refs. [12, 13, 11] for Combined Smoothed Particle Hydrodynamics — Total Variation Diminishing method). Our parallel OpenMP-CUDA software has been described in the work [14].

Calculation error estimates of the VAF flooded zones’ areas appearing due to the inaccuracy of the digital terrain model have been evaluated by comparison of calculation results with GPS measurement data with special imitation hydrodynamic flood simulations accounting for a random height change at the grid nodes. The safety control has been carried out by the union, while the calculation of the objective function (the area of flooded zones) has been fulfilled by the intersection of the digital flood maps obtained from hydrodynamic simulations. The calculation of the objective function determined by the increment of the area of the flooded territory, has been conducted matching the intersection of these maps. The largest relative error in the VAF zones objective function calculation, $\epsilon$, defined as the area difference in the union and intersection of the digital flood maps, amounts 5%. The total area of zones 19, 25, 31, and 38 excluded from the general analysis due to the excess of the $\epsilon$ value is 26 square kilometers. These territories are highlighted by a dark tint in Figure 3.

The analysis of space images and results of hydrodynamic simulations shows that the area of flooding territory $S_F$ is determined by the flow rate constant $Q_{max}$ of the VHPP hydrograph forming the first phase of the flood and flood duration $t$. The distributions of the parameters $Q_{max}$ and $t$ since 1990 and the approximate boundaries of small (A, B), mean (C, D) and large (E) floods are represented in Figure 4.
Figure 4. The peak values of water discharge $Q_{\text{max}}$ versus the duration of maximum discharge ($t$) for 1995–2016 are represented by points. The boundaries of small ($A$, $B$), mean ($C$, $D$) and large ($E$) flood areas are shown by lines.

3. The optimization problem
Searching for a set of sections of each of the 1542 watercourses maximizing the safe flooding area, $\Delta S(L, Q, t)$ provides a parametric optimization of the project ($L \leq L_{\text{max}}$, where $L$ and $L_{\text{max}}$ are the actual and limiting total lengths of the restored watercourses, respectively). If the solutions depend on these parameters significantly, the choice of the restored watercourses final sections should be approved by the decision-making experts [15, 16]. The latter requires attraction of an additional information.

The two-stage method for solving a discrete version of the optimization problem is described below. Its accuracy for the territories with an insignificant inter-zone water flows effect is limited by the accuracy of the geoinformation and hydrodynamic models. Let us consider a two-stage algorithm for solving the discrete parametric project optimization problem in the territory with a small inter-zone water flows effect. The project optimization for each zone is the first stage. Alternatives to the first stage of optimization in each of the zones are continuous fragments of the local watercourse systems connected to the main watercourses. The edges of the watercourses graph of the $i$-th zone ($i = 1, \ldots, N$, where $N = 42$ for the domain $B$, and $N = 28$ for the domain $C$) are numbered sequentially in the direction of increasing of the hierarchical level from 1 to $k_i$.

The discrete alternatives are continuous sequences of edges of the watercourses graph with monotonically increasing numbers. Thus, each alternative $a_{ij}$ ($j = 1, \ldots, n_i$) of the $i$-th zone contains from 1 to $k_i$ edges for which the sum of the lengths is $L_{ij}$. To reduce the discretization error, the long edges of the graphs can be replaced by a sequence of edges which length does not exceed a given value $\ell$. This parameter determining the discretization error and simultaneously the number of necessary hydrodynamic simulations is limited by the available computational resource. Thus, formed alternatives constitute the vertices of the hierarchical graphs the edges
Figure 5. The alternatives graph $a_{37,j} (j=1,...,20)$ for the watercourses graph of 37-th zone for $\ell = 1$ km.

of which are directed from each alternative vertex (excluding the last) to the other vertex-alternative obtained from it by adding an edge. For example, Figure 5 shows the 9-level graph of hierarchical alternatives of the watercourses graph for the 37th zone with $\ell = 1$ km, including 20 vertices. The numbers of edges are indicated at Figure 2.

In the case of $\ell = 0.5$ km or less, the graph becomes rather complicated and inconvenient for visual analysis. The lengths of alternatives $L_{37,j}$ (km) are equal to the number of the corresponding hierarchical level of the graph $G_{37}$.

Thus, the first stage task is a conditional optimization of the project in each of the zones where the common lengths of the reconstructed watercourses in the zones ($L_i$) are used as the parameters. It can be written as follows:

$$\Delta S_i(a_{ij}) \rightarrow \max_{a_{ij}}, \quad L_{ij} = L_i, \quad 0 \leq L_i \leq L_i^{\max},$$

$$(i = 1,...,N; \quad j = 1,...,n_i), \quad (1)$$

where $L_i^{\max}$ is the total length of the watercourses in the zone. To solve the problem in the $i$th zone of the parameter plane ($\Delta S_{ij}, L_{ij}$), one can construct a metrized graph $G_i$ which vertices ($a_{ij}$) are located at points ($\Delta S_{ij}, L_{ij}$), while the edges are linear interpolations of the grid functions $\Delta S_{ij}(a_{ij}) (j = 1,...,n_i)$. Increasing non-negative part of the upper boundary of the graph edges corresponds to the parametric dependence $\Delta S_i^*(L_i)$ for the solution of problem (1) in the $i$-th zone.

The result of problem solution (1) is the numbers of the optimal alternatives $L_i$ for the zones $j_i^*(L_i)$, as well as the values of the grid function:

$$\Delta S_i^*(L_i) \left( L_i = k_i \ell; \quad k_i = 1,..., \frac{L_i^{\max}}{\ell}; \quad i = 1,...,N \right). \quad (2)$$

The values of the function between the grid nodes can be evaluated by the linear or cubic spline-interpolation.

At the second stage, the problem of conditional optimization of the flooding area total increment with the total length of the restored watercourses as a parameter is solved using the dependencies obtained at previous stage:

$$\Delta S = \sum_{j=1}^{n_i} \Delta S(L_i) \rightarrow \max_{L_i},$$

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Figure 6. The metrized graph. The thick red line shows the solution of the problem (1) for the considered zone.

\[ \sum_{i=1}^{N} L_i = L \quad (i = 1, ..., N), \quad L \leq L_{\text{max}}, \tag{3} \]

where \( L \) is total length of restored watercourses, \( L_{\text{max}} \) is the limiting value of the total length \( L \). A multi-step dynamic programming method is used for the problem (3) solution. The corresponding Bellman equation can be represented in the following form:

\[
\int_{-\infty}^{\infty} W_k = \max_{0 \leq L_k \leq \min(L_{\text{max}}^{\text{max}}, C_k)} \left( \Delta S_k(L_k) + W_{k+1}(C_k - L_k) \right)
\]

\[ (k = N - 1, ..., 0), \quad 0 \leq C_k \leq L, \]

\[ 0 \leq L \leq L_{\text{max}}, \quad W_N = \Delta S_N, \quad W_0 = \Delta S. \tag{4} \]

The numerical solution has been obtained with steps \( \Delta L_k = 0.1 \ell, \Delta C_k = \ell \).

The results of numerical solution of the problem (1)—(3) has been obtained in the territory with a negligible effect of inter-zonal water flows for the parameters \( Q = 22000 \text{ m}^3/\text{sec}, \ t = 5 \text{ days} \) (See domain \( B \) in Figure 4), \( \ell = 1 \text{ km} \). The values of \( \Delta S_i(a_{ij}) \) (\( i = 1, ..., N; \ j = 1, ..., n_i \)) for all the zones has been calculated simultaneously by the numerical hydrodynamic simulations. Hence, the total number of calculations has not exceeded the value of \( \max \{ n_i \} \) (\( i = 1, ..., N \)).

The actual number of calculations has been even less than this value, since the alternatives corresponding to the negative effect of the deepening of the first sections of the long watercourses have been excluded from consideration. Figure 6 shows a metrized graph (See Figure 5 for comparison), where the solution of problem (1) in this zone coincides with selected part of the graph. Our problem is more complex than simple distributed hydrological systems studied in the works \([17, 18, 19, 20]\).

The equation (4) has been solved numerically with steps \( \Delta L_k = 0.1 \ell, \Delta C_k = \ell \). The functions \( \Delta S^*(L) \) corresponding to the solution of problem (1), (3) are shown in Figure 7.

The efficiency of the specified algorithm (strategy I) is essentially limited by the condition of smallness of the interband flooding effect. In addition a study of the project limited optimization
algorithm with several empirical strategies of project activities in the VAF zones has been carried out: the restoration of only the second level watercourses (strategy II); the restoration of only sections of watercourses of the second level with a length of 1 km (strategy III); the restoration of all watercourses of each zone at a distance of 1 km from the beginning of the second-level watercourses (strategy IV).

The first stage for each strategy has been implemented on the whole VAF territory by parallel calculation of the project effect values in each zone. At the second stage, the zones have been ordered by the effect magnitude, and the integral effect $\Delta S(L)$ parametric curve has been built with the cumulative total number. The verification of the results by direct hydrodynamic calculations has showed that the error of the parallel calculation of the zonal effects does not exceed the simulation error.

Utilizing the obtained data, two composite strategies for two-stage optimization have been proposed combining, at the first stage, the results of the problem (1) solution in the territories with insignificant effect of interband flooding and strategies II and III in the territories with significant effect of interband flooding. Using obtained data two composite strategies for two-stage optimization combining the results of problem (1) solution at the first stage for the territories with insignificant effect inter-zonal flooding with strategies II and III for the territories with significant effect inter-zonal flooding have been suggested. At the second stage the problem (3) has been solved for the whole floodplain territory. The dependencies $\Delta S^*(L)$ for the solutions of these problems (curves V and VI, respectively) are constructed.

In general, as calculations have shown, the largest effect of the project is achieved in the area of small floods ($B$ and $C$ in the Figure 4). Its efficiency decreases sharply in zones $A$ and $D$. The latter can be explained by the fact that during small floods the localization of flood waters in the deepened watercourses forms a significant fraction in their total volume, while during large floods small watercourses do not significantly affect the size of the flooded area.
4. Conclusion
We have considered a new approach to the hydrotechnical project optimization for small watercourses deepening in the interfluve to improve the efficiency of flood water use in the flooded area. A distinctive feature of current project is the project parameters spatial distribution and its optimal implementation at the inhomogeneous territory. The presence of global external parameters such as peak and duration of floods, the amount of project financing and etc. make our problem complicated and we somehow should overcome this uncertainty. Such projects may be implemented in valleys of large rivers to improve the conditions of agriculture, and mitigate the negative environmental consequences of the hydroelectric power stations construction. A discrete optimization combining precise and heuristic methods for the territory where the preliminary zoning is carried out underlies at the heart of our approach.

The possibility of application of exact two-stage optimization method is caused by smallness of the effect of the inter-zone (transboundary) flooding of the territory. The discretization error for small watercourses restricts this method due to the limitations of the computational complexity of the problem, as well as the digital elevation model (DEM) error. We include these errors in the algorithm of problem solution. The zoning procedure allows selecting areas for each of which an independent mini-project is considered. It enables to calculate their effectiveness simultaneously within the framework of one hydrodynamic calculation. At the second stage, the overall optimization of the whole project has been carried out. In addition to the exact method, we have proposed several empirical and composite optimization strategies useing the exact method for zones set with small transboundary flooding and the empirical strategies for rest of the territory.

The efficiency and specific features of the approach application have studied using the VAF Northern part of about 900 sq km area as an example. Our analysis for VAF has revealed that the efficiency is the best for the mean values of the flood peaks. In the case of small and large flood peaks the efficiency of the entire project has dramatically decreased. However, the project efficiency for individual zones can be very high, achieving 100 %. In the current paper we have limited the objective function by the flooding area. For more accurate estimates of the objective function it is necessary to take into account the data on the ecological and economic significance of different zones, as well as the control mechanisms based on regional co-financing [21, 22].

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References
[1] Gorski K, Bosch L, Wolfsaar K, Middelkoop H, Nagelkerke L, Filippov O, Zolotarev D, Yakovlev S, Minin A, Winter H et al. 2012 River Research and Applications 28 1121–1134
[2] Gorski K, Buijse A, Winter H, De Leeuw J, Compton T, Vekhov D, Zolotarev D, Verreth J and Nagelkerke L 2013 River Research and Applications 29 1226–1236
[3] Kozlov A, Kozlova M and Skorik N 2016 Remote Sensing 8 762
[4] Kuzmina Z V, Treshkin S and Karimova T Y 2015 Arid Ecosystems 5 230–242
[5] Gorski K, De Leeuw J, Winter H, Khoruzhaya V, Boldyrev V, Vekhov D and Nagelkerke L 2016 Inland Waters 6 105–110
[6] Vasilchenko A, Voronin A, Svetlov A and Antonyan N 2016 International Journal of Pure and Applied Mathematics 110 183–192

[7] Voronin A A, Vasilchenko A, Pisarev A V, Khrapov S S and Radchenko Y E 2016 Science Journal of Volgograd State University. Mathematics. Physics 1 (32) 24–37

[8] Suwandana E, Kawamura K, Sakuno Y and Kustiyanto E 2012 Remote Sensing Letters 3 423–432

[9] Rabus B, Eineder M, Roth A and Bamler R 2003 ISPRS Journal of Photogrammetry and Remote Sensing 57 241–262

[10] Presnyakova A N, Pisarev A V and Khrapov S S 2017 Science Journal of Volgograd State University. Mathematics. Physics 38 66–74

[11] Khrapov S S, Pisarev A V, Kobelev I A, Zhumaliev A G, Agafonnikova E O, Losev A G and Khoperskov A V 2013 Advances in Mechanical Engineering 5 787016

[12] Khoperskov A and Khrapov S 2017 Numerical Simulations, InTech 20

[13] Khrapov S S, Kuz’min N M and Butenko M A 2016 Science Journal of Volgograd State University. Mathematics. Physics 6 (37) 166–173

[14] Dyakonova T, Khoperskov A and Khrapov S 2016 Communications in Computer and Information Science 687 132–145

[15] Voronin A, Isaeva I, Khoperskov A and Grebenjuk S 2017 Communications in Computer and Information Science 754 419–429

[16] Ahmad S and Simonovic S P 2006 Water Resources Management 20 391–410

[17] Gupta V, Puig V and Blesa J 2017 A methodology for distributed fault diagnosis Journal of Physics: Conference Series vol 783 (IOP Publishing) p 012005 URL http://stacks.iop.org/1742-6596/783/i=1/a=012005

[18] Hartanto I M, van der Kwast J, Alexandridis T K, Almeida W, Song Y, van Andel S and Solomatine D 2017 International Journal of Applied Earth Observation and Geoinformation 57 123–135

[19] Hahti K, Warsta L, Kokkonen T, Younis B A and Koivusalo H 2016 Water Resources Research 52 246–263

[20] Khalid K, Ali M, Rahman N A, Mispán M, Rasid M, Haron S and Mohd M 2015 ARPN Journal of Engineering and Applied Sciences 10 6628–6633

[21] Burkov V N, Goubko M, Korgín N and Novikov D 2017 Introduction to theory of control in organizations (CRC Press)

[22] Ougolnitsky G A, Rokhlin D B and Usov A B 2017 Far East Journal of Mathematical Sciences 102 1319–1570