INS/CNS integrated filtering technique based on celestial calibration

Fei Yan¹, Wangyang Zhao²*, Hao Hong², Fenghao Zhao², Juzhong Liu²
¹The Navy Force Representative Office in Tianjin Area, Tianjin 300131, China
²Tianjin Navigation Instrument Research Institute, Tianjin 300131, China

*Corresponding author e-mail: zhaowangyang@163.com

Abstract. This paper analyzes the filtering mode and working characteristics of INS/CNS system according to application features of celestial equipment. In view of combination mode of gyro drift calibrated by CNS, this paper establishes corresponding filtering model according to different measurement information selected by INS/CNS system. The error of inertia component is compensated by virtue of star vector estimate value so as to guarantee the long-term precision of inertial navigation. Simulation analysis verifies that this method may improve adaptability of INS/CNS system to environment, and effectively guarantee the navigation precision in long-time operation of the system.

1. Introduction

In INS/CNS integrated system (hereinafter referred to as INS/CNS system), different equipment forms of star light navigation system decide different operating modes, and the difference of application environment and fields. Therefore, there are several classification methods for celestial navigation equipment. They could be divided to celestial navigation and radio celestial navigation according to bands of sensitive celestial, and in which the former may contain visible celestial navigation, infrared celestial navigation and ultraviolet CNS; divided to tiny field, wide field and radio CNS according to star measurement equipment [1]; divided to space and near-surface CNS according to different space of application [2]. Regardless of different equipment types and application odes of CNS, the essence in combination mode is to integrate the output parameters of the two sets of equipment, apply Kalman Filter, use the error equation of INS as the status equation of the integrated system, and use the navigation parameter error of INS and CNS as system measurement according to the carrier navigation information acquired by CNS, and then estimate and compensate INS error through filtering so as to improve adaptability and navigation precision of INS/CNS of long endurance.

2. Operating principle of gyro drift calibrated by CNS

CNS equipment is a highly precise attitude measuring device. It could be used to calibrate gyro drift of INS measurement element with its high-precision attitude information $C_i^s$. The basic principle is to establish a Kalman Filter model with inertia coordinate system as reference coordinate system, rotation quaternion of CNS carrier toward INS as the true value of observed quantity, and rotation quaternion of INS as actual observed quantity, and estimate gyro drift and attitude error of INS so as to realize calibration of gyro drift and correct attitude.
quaternion error.

The INS/CNS integrated mode based on gyro draft calibrated by CNS can be seen in Fig.1.

![INS/CNS integrated mode based on gyro draft calibrated by CNS](image)

**Fig 4.1** INS/CNS integrated mode based on gyro draft calibrated by CNS

3. Filtering model of INS/CNS integrated system

3.1 Calculation of additive quaternion error

Suppose the installation error has been compensated before the use of CNS, the device may directly output the matrix $C^i_b$ of carrier relative to INS after star atlas shooting, mass point attraction, atlas matching and attitude computation.

$$C^i_b = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Suppose $q_{i/b} = [q_0 \ q_1 \ q_2 \ q_3]$ and attitude matrix $C^i_b$ shall be:

$$\begin{align} q_0 &= \sqrt{1 + C_{11} + C_{22} + C_{33}} \\
q_1 &= \frac{1}{2} \text{sign}(C_{32} - C_{23}) \sqrt{1 + C_{11} - C_{22} - C_{33}} \\
q_2 &= \frac{1}{2} \text{sign}(C_{13} - C_{31}) \sqrt{1 - C_{11} + C_{22} - C_{33}} \\
q_3 &= \frac{1}{2} \text{sign}(C_{21} - C_{12}) \sqrt{1 - C_{11} - C_{22} + C_{33}} \end{align} \quad (1)$$

In the equation, sign is sign function, and calculated as follows:

$$\text{sign}(x) = \begin{cases} 1 & (x > 0) \\
0 & (x = 0) \\
-1 & (x < 0) \end{cases} \quad (2)$$

According to equations (1) and (2), the rotation quaternion $\tilde{q}_{i/b}$ of CNS corresponding to INS can be acquired. The rotation quaternion $\tilde{q}_{i/b}$ corresponding to INS can be acquired directly based on the angular velocity outputted by gyro. INS rotation quaternion $\tilde{q}_{i/b}$ will be used as the actual observed value, while CNS rotation quaternion $\tilde{q}_{i/b}$ could be used as true value due to its relatively high precision. Their difference will be the error of INS additive quaternion \[^3\]. Kalman filter algorithm is applied to INS/CNS data processing, i.e., with INS quaternion error equation as the state equation and INS quaternion error as the quantity, the
measurement equation will be established.

3.2 Establish state and measurement equation

The error of INS quaternion shall be:

$$\delta \hat{q}_{i/b} = \hat{q}_{i/b} - \hat{q}_{i/b}$$  \hspace{1cm} (3)

The quaternion error and gyro drift are used as state variables, i.e., the quantity of state shall be:

$$X = [\delta \hat{q}_0, \delta \hat{q}_1, \delta \hat{q}_2, \delta \hat{q}_3, e_{ibx}, e_{iby}, e_{ibz}, e_{rx}, e_{ry}, e_{rz}]$$

In INS, the gyro is fixed to a carrier, and senses the projection $\omega^{b}_{i/b}$ of moving angular velocity vector of carrier system relative to INS to the carrier system. The quaternion can be represented by $\omega^{b}_{i/b} = \begin{bmatrix} 0 & \omega^{b}_{ibx} & \omega^{b}_{iby} & \omega^{b}_{ibz} \end{bmatrix}$, and the rotation quaternion $\hat{q}_{i/b}$ of carrier system corresponding to INS can be acquired[4]:

$$\hat{q}_{i/b} = \frac{1}{2} \omega^{b}_{i/b} \otimes \hat{q}_{i/b}$$  \hspace{1cm} (4)

While,

$$\omega^{b}_{i/b} = \hat{q}_{i/b} \otimes \omega^{b}_{i/b} \otimes \hat{q}_{i/b}$$  \hspace{1cm} (5)

Put equation (5) into (4):

$$\hat{q}_{i/b} = \frac{1}{2} \hat{q}_{i/b} \otimes \omega^{b}_{i/b} \otimes \hat{q}_{i/b}$$  \hspace{1cm} (6)

Write equation (6) to matrix vector form:

$$Q(\hat{q}_{i/b}) = \frac{1}{2} M(\hat{q}_{i/b}) \otimes Q(\omega^{b}_{i/b}) = \frac{1}{2} M^{*}(\omega^{b}_{i/b}) Q(\hat{q}_{i/b})$$  \hspace{1cm} (7)

In the equation:

$$M^{*}(\omega^{b}_{i/b}) = \begin{bmatrix} 0 & -\omega_{ibx} & -\omega_{iby} & -\omega_{ibz} \\ \omega_{ibx} & 0 & -\omega_{iby} & -\omega_{ibz} \\ -\omega_{iby} & \omega_{ibx} & 0 & -\omega_{ibz} \\ -\omega_{ibz} & -\omega_{iby} & \omega_{ibx} & 0 \end{bmatrix}, \quad Q(\hat{q}_{i/b}) = \begin{bmatrix} \hat{q}_0 & -\hat{q}_1 & -\hat{q}_2 & -\hat{q}_3 \\ \hat{q}_1 & \hat{q}_0 & -\hat{q}_3 & \hat{q}_2 \\ \hat{q}_2 & -\hat{q}_3 & \hat{q}_0 & -\hat{q}_1 \\ -\hat{q}_1 & \hat{q}_2 & -\hat{q}_0 & \hat{q}_3 \end{bmatrix}$$

Differentiate both sides of equation (7) simultaneously, and the error equation with quaternion as state variable shall be:

$$Q(\delta \hat{q}_{i/b}) = \frac{1}{2} M^{*}(\omega^{b}_{i/b}) Q(\delta \hat{q}_{i/b}) + \frac{1}{2} M^{*}(\delta \hat{q}_{i/b}) Q(\delta \omega^{b}_{i/b})$$  \hspace{1cm} (8)

In the equation, $\delta \hat{q}_{i/b}$ is quaternion error; $\delta \omega^{b}_{i/b}$ is gyro angular velocity error.

The calculation equation of gyro angular velocity error $\delta \omega^{b}_{i/b}$ shall be:
\[
\begin{align*}
\dot{\varepsilon}_b &= 0 \\
\dot{\varepsilon}_r &= -\frac{1}{T_g} \varepsilon_r + w_r \\
\delta \omega_{th}^b &= \varepsilon_b + \varepsilon_r + \varepsilon_g
\end{align*}
\] (9)

In the equation, \( \varepsilon_b \) is gyro constant drift, \( \varepsilon_r \) is gyro random drift, \( \alpha_w \) is white noise, and \( T_g \) is relative time of first-order Markov process for gyro drift.

State equation with gyro drift as state variable shall be:
\[
\dot{X}(t) = F(t)X(t) + G(t)W(t)
\] (10)

In the formula, \( F(t) \) is system state transform matrix, and \( G(t) \) is system noise transform matrix, which are defined respectively as follows:
\[
F(t) = \begin{bmatrix}
0.5M'(\alpha_w^b)_{4 \times 4} & 0.5M'(\vec{\omega}_{l/b})_{4 \times 3} & 0.5M'(\vec{\omega}_{l/b})_{4 \times 3}
\end{bmatrix}_{10 \times 10}
\]
\[
G(t) = \begin{bmatrix}
0.5M'(\vec{\omega}_{l/b})_{4 \times 4} & 0_{4 \times 3} & 0_{4 \times 3} \\
0_{3 \times 4} & 0_{3 \times 3} & I_{3 \times 3} \\
0_{3 \times 4} & I_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix}_{10 \times 10}
\]

\[
M'(\vec{q}_{l/b})_{4 \times 3} = \begin{bmatrix}
-q_{l/b1} & q_{l/b2} & -q_{l/b3} \\
q_{l/b0} & -q_{l/b3} & q_{l/b2} \\
q_{l/b3} & q_{l/b0} & -q_{l/b1} \\
-q_{l/b2} & q_{l/b1} & q_{l/b0}
\end{bmatrix}
\]
\[
F_M = \text{diag} \left[ \frac{1}{T_{gx}}, \frac{1}{T_{gy}}, \frac{1}{T_{gz}} \right]
\]

In the equation, \( \dot{X}(t) = F(t)X(t) + G(t)W(t) \)

Establish measurement equation

\[
Z(t) = H(t)X(t) + V(t)
\] (11)

In the equation, \( H = [I_{4 \times 4} \ 0_{4 \times 6}] \) is measurement transform matrix, and \( V \) is system measurement noise.

3.3 Simulation analysis

In the mode of gyro draft calibrated by CNS, INS device adopts single-axial rotating strapdown INS. The rotation modulation mode is “forward/backward rotation and stop”. Under static conditions, the celestial sensor output frequency shall be 1Hz, and attitude calculation method shall be QUEST algorithm.
### Table 1 Simulation conditions of gyro draft calibrated by CNS mode

| Simulation parameters               | Order of magnitude | Remarks                                    |
|------------------------------------|--------------------|--------------------------------------------|
| X, Y, Z Gyro drift stability       | 0.008°/h           | Description of first-order Markov process  |
| X, Y, Z Gyro drift constant error  | 0.005°/h           | -                                          |
| X, Y, Z Gyro installation error    | (-23", 17"); (19", -17"); (24", 11") | -                                          |
| X, Y, Z Accelerometer zero offset error | 8×10⁻⁵g         | -                                          |
| X, Y, Z Accelerometer installation error | (-21", -17"); (25", -14"); (13", 19") | -                                          |
| Forward/backward rotation and stop method | Speed: 9°/s; stop time: 5s | -                                          |
| Celestial sensor precision         | Optic axis direction precision: 5", lateral axis direction precision: 7" | -                                          |
| Co-base calibration residue        | (-6", -7"); (10", -11"); (9", 8") | -                                          |
| Celestial sensor output frequency  | 1Hz                | -                                          |

#### Fig 2 Gyros and platform rotation modulation period

#### Fig 3. Vertical gyro drafts estimation

### 4. Conclusion

This paper elaborates the basic operating principle of INS/CNS integrated system based on gyro draft calibrated by CNS, and analyzes the attitude determination and positioning precision of CNS. It establishes an INS/CNS filtering mode according to celestial vector characteristics, studies celestial vector-based rapid inertial element error estimation method, and guarantees long-term precision of CNS.
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