Multifractal Behavior of the Korean Stock-market Index KOSPI

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Abstract

We investigate multifractality in the Korean stock-market index KOSPI. The generalized \(q\)th order height-height correlation function shows multiscaling properties. There are two scaling regimes with a crossover time around \(t_c = 40\) min. We consider the original data sets and the modified data sets obtained by removing the daily jumps, which occur due to the difference between the closing index and the opening index. To clarify the origin of the multifractality, we also smooth the data through convolution with a Gaussian function. After convolution we observe that the multifractality disappears in the short-time scaling regime \(t < t_c\), but remains in the long-time scaling regime \(t > t_c\), regardless of whether or not the daily jumps are removed. We suggest that multifractality in the short-time scaling regime is caused by the local fluctuations of the stock index. But the multifractality in the long-time scaling regime appears to be due to the intrinsic trading properties, such as herding behavior, information outside the market, the long memory of the volatility, and the nonlinear dynamics of the stock market.

Key words: Econophysics, multi-scaling, multifractal, stock market

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1 Introduction

In recent years, concepts and techniques from statistical physics have been widely applied to economics [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19], and the complex behaviors of economic systems have been found to be very similar to those of complex systems customarily studied in statistical physics. Stock-market indexes around the world have been accurately recorded for many years and therefore represent a rich source of data for quantitative analysis, and the statistical behaviors of stock markets have been studied by various methods, such as distribution functions [10,11,12,19], correlation functions [12,13,14], multifractal analysis [19,20,21], and network analysis of the market structure [15,16].

Multiscaling properties have been reported for many economic time series [22,23,24,25,26,27,28,29,30,31]. Multifractality has been observed in stock markets [20,29,32,33,34], the price of crude oil [25], the price of commodities [34], and foreign exchange rates [24,35]. In daily stock indexes and foreign exchange rates, the generalized Hurst exponent $H_q$ decreases monotonically with $q$ [24,29,32,33,34,35] (see formal definitions of $H_q$ and $q$ in Eqs. (1) and (2) below). In the U.S. NASDAQ index, two scaling regimes have been reported: one quasi-Brownian and the other multifractal [33]. Two scaling domains have also been reported in the fluctuations of the price of crude oil: $H_q$ increases with $q$ in the short-time domain, but decreases with $q$ in the long-time domain [25]. The existence of multiple scaling regimes seems to depend on the resolution of the data sets in the economic time series.

Although many economic time series display multifractality (in the theory of surface scaling analysis referred to as multiaffinity), the origins of multifractality in the stock market are not well understood. It has been suggested that herding behavior and nonlinear complex dynamics of the stock market induce multiscaling [10]. However, it is very difficult to quantify herding behavior and complex dynamics in the stock market. Buendía et al. observed multiaffinity in a frustrated spring-network model simulating the surface structure of cross-linked polymer gels [36]. Removing vertical discontinuities from the rough surface by convolution with a Gaussian, they observed that the multiaffine surface changed into a self-affine one [37]. They concluded that vertical discontinuities can be one cause of multiaffinity. Mitchell conversely introduced artificial vertical discontinuities into a self-affine surface and observed that the surface became multiaffine [38].

In the present paper we investigate multifractality in the Korean stock-market index KOSPI (Korean Composite Stock Price Index). We observe that local fluctuations, including jumps, are responsible for multifractality in the short-time scaling regime. However, multifractality in the long-time scaling regime
is not removed by smoothing of the time series.

2 Method and Results

We consider a set of data recorded every minute of trading from March 30, 1992, through November 30, 1999. We count the time during trading hours and remove closing hours, weekends, and holidays from the data. Denoting the stock-market index as $x(t)$, the generalized $q$th order height-height correlation function (GHCF) $F_q(t)$ is defined by

$$F_q(t) = \langle |x(t' + t) - x(t')|^q \rangle^{1/q},$$  \hspace{1cm} (1)

where the angular brackets denote a time average over the time series. The GHCF $F_q(t)$ characterizes the correlation properties of the time series $x(t)$, and for a multiaffine series a power-law behavior like

$$F_q(t) \sim t^{H_q}$$  \hspace{1cm} (2)

is expected, where $H_q$ is the generalized $q$th order Hurst exponent [39]. If $H_q$ is independent of $q$, the time series is monofractal. If $H_q$ depends on $q$, the time series is multifractal. Multifractality is a distinctive property of the stock market index observed.

In Fig. 1(a) we present the GHCF as a function of the time interval $t$. We observe clear multifractal behavior in the time series of the stock index. There are two different scaling regimes separated by a crossover time of about $t_c = 40$ min. The slopes of the log-log plot depend on $q$ in each scaling regime. As shown in Fig. 1(b), $H_q \sim 1/q$ for large $q$ in both scaling regimes, while it saturates for small $q$. The $1/q$ behavior is consistent with numerical observations in Refs. [37,38] and analytical results for functions that include discontinuities in Ref. [38]. These behaviors are different from the multifractality in the price of crude oil [25]. Alvarez-Ramirez et al. reported the existence of two scaling regimes for crude-oil prices [25], but $H_q$ increased with $q$ in the long-time scaling regime. Kim et al. did not observe a short-time scaling regime for the daily Yen-Dollar exchange rate [22].

2.1 Removal of daily jumps

We analyzed the KOSPI time series to find the origin of multifractality in the Korean stock market. Inspired by Refs. [36,37,38], we removed the daily jumps
Fig. 1. (Color online.) (a) Log-log plot of the generalized height-height correlation function $F_q(t)$ versus time $t$ for the prices of the Korean stock-market index, KOSPI with $q = 0.5, 1, 2, \cdots, 10$ from bottom to top. (b) The generalized Hurst exponents $H_q$ versus $1/q$ for the price of KOSPI in the scaling regimes $t < t_c$ (○) and $t > t_c$ (□) where $t_c \simeq 40$ min. The error bars here and in Figs. 3(b) and 4(c) are standard deviations in the slopes of least-square fits to log-log plots of $F_q(t)$.

of the stock index due to the difference between the closing and opening index. In KOSPI, there was no trading after closing and before opening the market until 1995. So, the changes of the index associated with the daily jumps are small during those years. Since 1995, after-market trading and before-market
Trading have been allowed for one hour each, leading to substantial overnight jumps. There are additional big daily changes during the period of the Asian financial crisis around November, 1997. In Fig. 2, we present three time series: the original time series, the time series with the daily jumps removed, and the time series of the daily jumps only. In the period between 1992 and 1999, there were bubbles in the Korean stock market around 1995, and there were big crashes around 1997 after the Asian financial crisis. Larger daily changes can be observed around the period of the Asian financial crisis.

We show the GHCF for the original KOSPI, the stock index with the daily jumps removed, and the time series of the daily jumps only in Fig. 3(a). In the short-time scaling regime \( t < t_c \), the deviations between the original and the modified data sets increase with increasing \( q \). But in the long-time scaling regime \( t > t_c \), the deviations are larger for small \( q \). For large \( q \), the GHCF have large fluctuations, and there may be additional scaling regimes for very long times. However, practically no change is observed in \( H_q \), as shown in Fig. 3(b). Removing the daily jumps thus does not delete the multifractal properties of the stock index and only slightly changes the generalized Hurst
Fig. 3. (Color online.) (a) Log-log plots of the generalized height-height correlation function $F_q(t)$ versus time in the time series of the original index (solid lines) and the index with the daily jumps removed (dotted lines) for $q = 0.5, 1, 2, \cdots, 10$ from bottom to top. Inset: the generalized height-height correlation function versus time for the daily jumps. (b) The generalized Hurst exponents $H_q$ versus $1/q$ for the index with the daily jumps removed in the scaling regimes $t < t_c (\times)$ and $t > t_c (*)$ where $t_c \simeq 40$ min. For easy comparison we also include the original index in the scaling regimes $t < t_c (\circ)$ and $t > t_c (\Box)$ (identical to Fig. 1(b)).
exponents. In the inset in Fig. 3(a) we present the GHCF of the time series of the daily jumps. We observe obvious multifractality, and there are several scaling regimes.

2.2 Gaussian smoothing

Next, as simply removing the discontinuities has little effect, we consider the effects of smoothing the stock-index time series by convolution with a Gaussian of standard deviation 5 minutes. The convolution of two functions $f(t)$ and $g(t)$ is defined by

$$
\tilde{f} = \int_{-\infty}^{\infty} f(t)g(t-u)du,
$$

(3)

where $g(t)$ is a Gaussian convolution kernel [40]. In discrete time series the convolution is a sum instead of an integral

$$
\tilde{f}_i = \sum_{j=1}^{m} f_j g_{i-j}.
$$

(4)

In Fig. 4(a) we show a short segment of the original time series and the time series smoothed by convolution with a Gaussian. Although this procedure removes short-time fluctuations of the stock index, global trends remain. In Fig. 4(b) we show the GHCF before and after the convolution. Before the convolution, the GHCF shows multifractality. After the convolution we observe monofractal behavior in the short-time scaling regime, $t < t_c$. In this scaling regime, the lines of the log-log plots are all parallel, with the same slope, $H_q(t < t_c) \approx 0.986(4) \simeq 1$, independent of $q$, as shown in table 1. $H_q = 1$ means that the convoluted time series is a smooth curve. The result of $H_q \approx 1$ for $t < t_c$ and unchanged $H_q$ for $t > t_c$ agrees with those obtained from Gaussian smoothing of simulated hydrogel surfaces in Ref. [37]. For $t > t_c$, the convoluted time series yield the same slopes as the original time series. The corresponding values of $H_q$ are shown vs $1/q$ in Fig. 4(c).

We also checked the effects of varying the standard deviation in the Gaussian function between 1 min and 100 min. When the standard deviation is close to the time interval of the stock-index time series, we found that the multifractal behaviors are the same as those of the original time series. When the standard deviation is greater than 4 minutes and less than the crossover time $t_c = 40$ minutes, the multifractal behavior is the same as the results obtained with a standard deviation of 5 minutes in Fig. 4: $H_q$ is close to one for $t < t_c$, and it depends on $q$ for $t > t_c$. When the standard deviation is greater than 40
Fig. 4. (Color online.) (a) A portion of the original stock index (black) and the time series convoluted with a Gaussian (gray, green online). (b) Log-log plots of the generalized height-height correlation function $F_q(t)$ versus time for the original time series (solid curves) and the time series smoothed by Gaussian convolution (dotted curves) for $q = 0.5, 1, 2, \ldots, 10$ from bottom to top. The same gray scales (colors online) correspond to the same values of $q$. (c) The generalized Hurst exponents $H_q$ versus $1/q$ for the time series smoothed by Gaussian convolution in the scaling regimes $t < t_c$ ($\times$) and $t > t_c$ ($\ast$), where $t_c \approx 40$ min. For easy comparison we also include the original index in the scaling regimes $t < t_c$ ($\circ$) and $t > t_c$ ($\Box$) (identical to Fig. 1(b)).
In summary, we have studied multifractality of the Korean KOSPI stock index. Multifractality is observed in two scaling regimes in the original time series of the stock index. However, multifractality in the short-time regime can be removed by convoluting the time series by a Gaussian. In the long-time regime the multifractality is unchanged by smoothing the time series. We propose that
multifractality in the short-time regime is caused by the local fluctuations in
the stock index. However, multifractality in the long-time regime appears to
be a result of the complex behaviors in the stock market, such as herding
behavior, information outside the market, the long memory of the volatility,
and the intrinsic nonlinear dynamics of the market. Understanding the origins
of multifractality in the long-time scaling regime remains an active research
topic.

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References

[1] R. N. Mantegna and H. E. Stanley, An Introduction to Econophysics:
Correlations and Complexity in Finance, Cambridge University Press, Cambridge,
1999.
[2] B. Mandelbrot, Fractals and Scaling in Finance, Springer, New York, 1997.
[3] J. P. Bouchaud and M. Potters, Theory of Financial Risk, Cambridge University
Press, New York, 2000.
[4] D. Sornette, Phys. Rep. 378 (2003) 1.
[5] R. N. Mantegna and H. E. Stanley, Nature 376 (1995) 46.
[6] J.-P. Bouchaud and D. Sornette, J. Phys. I France 4 (1994) 863.
[7] L. Bachelier, Ann. Sci. École Norm. Sup. 3 (1900) 21.
[8] B. Mandelbrot, J. Business 36 (1963) 294.
[9] E. F. Farma, J. Business 36 (1963) 420.
[10] P. Gopikrishnan, V. Plerou, L. A. N. Amaral, M. Meyer, and H. E. Stanley,
Phys. Rev. E 60 (1999) 5305.
[11] P. Gopikrishnan, M. Meyer, L. A. N. Amaral, and H. E. Stanley, Eur. Phys. J.
B. 3 (1999) 139.
[12] Y. Liu, P. Gopikrishnan, P. Cizeau, M. Meyer, C.-K. Peng, and H. E. Stanley,
Phys. Rev. E 60 (1999) 1390.
[13] P. Gopikrishnan, V. Plerou, Y. Liu, L. A. N. Amaral, X. Gabaix, and H. E. Stanley, Physica A 287 (2000) 362.

[14] H. E. Stanley, L. A. N. Amaral, P. Gopikrishnan, and V. Plerou, Physica A 283 (2000) 31.

[15] H.-J. Kim, I.-M. Kim, and B. Khang, J. Kor. Phys. Soc. 40 (2002) 1105.

[16] G. Bonanno, G. Caldarelli, F. Lillo, and R. N. Mantegna, Phys. Rev. E 68 (2003) 046130.

[17] Y. Nakajima, J. Kor. Phys. Soc. 40 (2002) 1096.

[18] S. Y. Park, S. J. Koo, K. E. Lee, J. W. Lee, and B. H. Hong, New Physics 44 (2002) 293.

[19] K. E. Lee and J. W. Lee, J. Kor. Phys. Soc. 44 (2004) 668.

[20] A. Z. Górski, S. Drodz, and J. Septh, Physica A 316 (2002) 496.

[21] B. H. Wang and P. M. Hui, Eur. Phys. J. B. 20 (2001) 573.

[22] K. Kim, J.-S. Choi, and S.-M. Yoon, J. Kor. Phys. Soc. 44 (2004) 643.

[23] J. A. Skjeltorp, Physica A 283 (2000) 486.

[24] K. Kim and S. M. Yoon, Fractals 11 (2003) 131.

[25] J. Alvarez-Ramirez, M. Cisneros, C. Ibarra-Valdez, A. Soriano, and E. Scalas, Physica A 313 (2002) 651.

[26] Z. Xin-Tian, H. Xiao-Yuan, and S. Yan-Li, Physica A 333 (2003) 293.

[27] Z. Eisler and J. Kertesz, Physica A 343 (2004) 603.

[28] A. Bershadskii, Physica A 317 (2003) 591.

[29] M. Ausloos and K. Ivanova, Comp. Phys. Comm. 147 (2002) 582.

[30] K. Ivanova and M. Ausloos, Eur. Phys. J. B 8 (1999) 665.

[31] A. Turiel and C. J. Perez-Vicente, Physica A 322 (2003) 629.

[32] H. Katuragi, Physica A 278 (2000) 275.

[33] A. Bershadskii, J. Phys. A 34 (2001) L127.

[34] K. Matia, Y. Ashkenazy, and H. E. Stanley, Europhys. Lett. 61 (2003) 422.

[35] Z. Xu and R. Gencay, Physica A 323 (2003) 578.

[36] G. M. Buendía, S. J. Mitchell, and P. A. Rikvold, Phys. Rev. E 66 (2002) 046119.

[37] G. M. Buendía, S. J. Mitchell, and P. A. Rikvold, Microelectronics J. in press.

[38] S. J. Mitchell, cond-mat/0210239
[39] P. Meakin, Fractals, scaling and growth far from equilibrium, Cambridge University Press, Cambridge, 1998.

[40] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, Numerical Recipes in Fortran: The art of scientific computing, Cambridge University Press, Cambridge, 1986, pp 492.

[41] A. Krawiecki, J. A. Holyst, and D. Helbing, Phys. Rev. Lett. 89 (2002) 158701.

[42] M. Barotolozzi and A. W. Thomas, Phys. Rev. E 69 (2004) 046112.

[43] The performance of our analysis code was checked with a synthetic, monofractal time series, for which it showed no indication of multifractality.