Profiles’ classifier of hot-rolled rolling

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Abstract. The geometric parameters describing the features of the cross-sectional profile of the hot-rolled strips do not give a complete picture of the flatness acquired by the cold-rolled strips rolled from these strips. An additional analysis showed that there are four characteristic classes of cross-sectional profiles of hot rolled strips that have a significant effect on the shape of the strips during cold rolling, three of which negatively affect the flatness of the cold rolled strips. The cross-sectional profiles of hot-rolled strips with a concave middle part and (or) marginal thickenings lead to the appearance of edge waviness, peak-like cross-sectional profiles cause central warping. Therefore, the actual task is to determine the factual shape of cross-sectional profile. Sixth order polynomials were used to digitalize and parameterize hot-rolled profile. As a result, we developed analytic function of the transverse profile, which keeps important information about its near-edge areas and features in the middle part. To assign a specific cross-sectional profile of a hot-rolled strip to one of four characteristic classes of cross-sections, mathematical software was developed, called a classifier, and implemented with the programming environment R. To classify the profiles of the hot-rolled cross-section according to characteristic classes, a linear discriminant method was used as a machine learning method analysis. The result is an adequate mathematical model for recognizing the shape of the cross-sectional profile.

1. Introduction
The peculiarities of the cross-section profile of a hot-rolled rolling significantly influence on the flatness of the cold-rolled strips, rolled from this rolling [1–35]. Taking into account only geometric parameters does not give a complete picture of flatness which the cold-rolled strips acquire [5, 9, 14, 17, 21, 26]. For this purpose, it is necessary to take into account not only the geometric parameters, describing the peculiarities of the cross-section profile of the hot-rolled rolling, but also the specific shape of the contour of the cross-section profile [6, 10, 13, 18, 22, 27].

The shape of the cross-section profile can be uniquely described using a finite vector $V$, whose components are the classification features. Four classes of profiles we define as a set $\Omega = (\Omega_r)$, where $r = 1, \ldots, N$ (in our case, $N = 4$); $\Omega_r$ is the number of $r$-th class; $N$ is the number of classes. To assign a real cross-section profile to one of four characteristical classes, you need to construct a function $f: V \rightarrow \Omega$, called a classifier, which assigns a set of vectors of the classificational features $V$ corresponding to the real cross-section profiles of strips, to the selected classes $\Omega$.

To parameterize the contour of the cross-section profile and to construct a vector of the classificational features $V$, we apply a polynomial regression. To exclude the influence of the thickness of the hot-rolled strip on the polynomial’s coefficients, we normalize the measurements of the cross-
section profiles so that the maximum relative height of the section is equal to one:

\[ h(x) = \frac{h[x_i] - h_{\text{min}}}{h_{\text{max}} - h_{\text{min}}}, \]

where \( h_{\text{min}} \) and \( h_{\text{max}} \) are respectively the minimum and maximum strip’s thickness.

Let’s write the approximating polynomial in the generalized form:

\[ h(x) = a_0g_0(x) + a_1g_1(x) + \ldots + a_ng_n(x), \tag{1} \]

where \( n \) is the order of the polynomial; \( a_0, \ldots, a_n \) are the parameters of the generalized polynomial.

If \( g_n(x) = x^n \), then the approximating function (1) has the form of a multidimensional regression model, whose coefficients are calculated by means of the least squares method. Increasing the order of the polynomial does not lead to a significant increasing in accuracy, and at \( n > 5 \) there are oscillations in the edge zones. Therefore, in order to avoid these disadvantages, we used the Chebyshev’s polynomials for an approximation.

The Chebyshev’s polynomials are the orthogonal algebraic polynomials defined by the recurring relations, where each subsequent \( k \)-th coefficient is calculated by \((k-1)\) previous coefficients:

\[
\begin{align*}
\lambda_1 &= \frac{1}{m} \sum_{i=1}^{m} x_i, \\
\lambda_k &= \frac{\sum_{i=1}^{m} x_i g_{k-1}(x_i)}{\sum_{i=1}^{m} g_{k-1}^2(x_i)}, \\
\beta_k &= \frac{\sum_{i=1}^{m} x_i g_{k-2}(x_i) g_{k-1}(x_i)}{\sum_{i=1}^{m} g_{k-2}^2(x_i)}.
\end{align*}
\]

**2. Checking adequacy of mathematical model**

To check the adequacy of the model (1), we calculate the value of the average approximation’s error \( \langle A \rangle \). According to [1, 5, 6] the model can be considered adequate at \( \langle A \rangle < 15\% \):

\[ \langle A \rangle = \frac{1}{m} \sum_{i=1}^{m} \frac{|H[x_i] - h[x_i]|}{H[x_i]} \cdot 100\%. \]

As a recognition method of images, that allows you obviously to set the function \( f \), we use a linear discriminant analysis [2, 13, 14]. The essence of the method is to find such the linear combinations of parameters of the classification feature vectors, called the canonical discriminant functions, whose values differ as much as possible from each other for the objects of the different classes:

\[ f_u = w_0^u a_0 + w_1^u a_1 + \ldots + w_n^u a_n, \]

where \( u = 1, \ldots, (N - 1) \) is the number of discriminant functions; \( w_0^u, \ldots, w_n^u \) are the coefficients of \( u \)-th discriminant function; \( a_0, \ldots, a_n \) are the parameters of Chebyshev’s polynomial (1).

Let’s write a system of discriminant functions in a matrix form:

\[ F = W^T V, \tag{2} \]

\[ F_{(N-1)\times 1} = [f_1 \ldots f_{N-1}]^T, \quad V_{(n+1)\times 1} = [a_0, \ldots, a_n]^T, \quad W_{(n+1)\times (N-1)} = \begin{bmatrix} w_0^1 & w_0^2 & \ldots & w_0^{N-1} \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
w_n^1 & w_n^2 & \ldots & w_n^{N-1} \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & \ldots & w_{N-1} \end{bmatrix}. \]

The matrix \( W \) in the model (2), called the linear Fischer’s discriminant [3, 21, 22], is chosen so that the linear functional \( J(W) \) reaches a maximum:
\[
J(W) = \frac{W^T S_B W}{W^T S_B W} \rightarrow \max,
\]
where \( S_B \) is the inter-class scattering matrix; \( S_W \) is the inside-class scattering matrix.

The scattering matrices \( S_B \) and \( S_W \) are calculated as follows:

\[
S_B = \sum_{r=1}^{N} N_r (\mu_r - \mu)(\mu_r - \mu)^T, \quad S_W = \sum_{r \in \Omega} S_r, \quad S_r = \sum_{v \in \Omega_r} (V - \mu)(V - \mu)^T,
\]

\[
\mu_r = \frac{1}{M_r} \sum_{v \in \Omega_r} V, \quad \mu = \frac{1}{M} \sum_{v \in \Omega} V = \frac{1}{M} \sum_{v \in \Omega} M \mu_v,
\]

where \( M \) is the analyzed number of the hot-rolled profiles; \( M_r \) is the number of the cross-section’s profiles belonging to the class \( \Omega_r \).

To find the maximum of the functional \( J(W) \), it is necessary to differentiate it by the variable \( W \) and equate the result to zero. Whence we get the system of equations:

\[
S_W^{-1} S_B w = k_s w_s,
\]

where \( k_s \) and \( w_s \) are the eigenvalues and eigenvector of the matrix \( S_W^{-1} S_B \).

To check the adequacy of the model (2) and visually the representation of an application of a discriminant analysis, we construct a classification matrix – the square matrix of an order \( N \), whose columns represent the predicted solutions, and whose rows are a known class of the object. On the diagonal of the matrix is placed the number of correctly recognized the cross-section profiles, and outside the diagonal is placed the number of the errors of the first and second kind:

\[
CostM = \begin{bmatrix}
\text{Известные Predsказанные} & \Omega_1 & \Omega_2 & \cdots & \Omega_N \\
\Omega_1 & t_{11} & e_{12} & \cdots & e_{1N} \\
\Omega_2 & e_{21} & t_{22} & \cdots & e_{2N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\Omega_N & e_{N1} & e_{N2} & \cdots & t_{NN}
\end{bmatrix},
\]

where \( t_{rs} \) is the number of correctly classified the objects of class \( r \); \( e_{rs} \) is the number of the objects of class \( r \) assigned to class \( s \).

3. Numerical evaluation of classifier’s quality

To numerically evaluate the quality of the classifier, we calculate the accuracy \( Acc \), which shows the proportion of the hot-rolled profiles for which the classifier made the right decision:

\[
Acc = \frac{1}{M} \sum_{r=1}^{N} t_{rr}.
\]

As a criterion for how well the model (2) is able to work with different data, we use the cross-verification method (the cross-validation method). The essence of the method is to divide the initial selection of the classified objects into training (most of the data) and test (a smaller part of the data) samples. The training sample is used for parameterizing the classifier, and the test sample is used for checking the resulting model. If the accuracy of the classification \( Acc \) for the test sample \( Acc_{\text{test}} \) and the training sample \( Acc_{\text{train}} \) is approximately equal (\( Acc_{\text{test}} \approx Acc_{\text{train}} \)), then the worked out model is considered to have passed cross-checking.

For the model (1), the approximations of the cross-section profiles were performed by means of the Chebyshev’s polynomials of the sixth degree, which preserve significant information as about the middle part of the profile and fairly accurately model the edge thickening (Figure 1).
Figure 1. Examples of approximation of real cross-section’s profiles by Chebyshev’s polynomials of the sixth order:

- **a** – without edge thickenings (class 1);
- **b** – with a peak-shaped convex of the middle (class 2);
- **c** – with edge thickenings (class 3);
- **d** – concave profile with edge thickenings (class 4).

To quantify the adequacy of the model (1), we will find the average approximation error \( \langle A \rangle \) for each of the 520 analyzed hot-rolled rolling profiles and present the obtained results as a diagram in Figure 2. We note that \( \langle A \rangle \) does not exceed 15 %, which means that model (1) is adequate.

Figure 2. Histogram of values \( \langle A \rangle \).

We performed the parametrical identification of the model (2) in the programming environment R [4, 6, 14, 22]. The number of the canonical discriminant functions is one less than the number of classes, so there are four characteristic profile classes for which the functions were calculated:

- \( f_1 = -5,9a_0 + 0,2a_1 + 5,5a_2 - 0,1a_3 - 3,5a_4 - 0,1a_5 + 2,1a_6, \)
- \( f_2 = 0,9a_0 - 1,5a_1 + 4,1a_2 + 0,1a_3 - 3,8a_4 + 0,5a_5 + 0,2a_6, \)
- \( f_3 = 4,3a_0 - 9,2a_1 - 10,8a_2 - 0,01a_3 - 2,2a_4 - 0,2a_5 - 0,1a_6, \)

where \( f_1, f_2, f_3 \) are the discriminant functions.

To determine the shape of the specific cross-section profile, that has a certain vector of the classification features \( \hat{V} = [\hat{a}_0, ..., \hat{a}_n] \), it is necessary to find the projection of this vector on the first
discriminant axis $\hat{f}_1 = w_i \hat{V}$. The resulting value $\hat{f}_1$ is compared with the values of $f_1(\mu)$. The cross-section profile of a hot-rolled rolling belongs to the class where the calculation result $|f_1(\mu, \Omega) - \hat{f}_1|$ is minimal. If the found minimum values are equal, then we construct a projection on the next discriminant axis. The classification accuracy reaches about 90%, so this classifier is adequate.

4. Conclusions
The analysis of the cross-section profile of hot-rolled strips and the flatness of the cold-rolled strips has shown that there are four main forms of the cross-section profile of hot-rolled rolling, which have a significant impact on the formation of the cold-rolled strips. The mathematical models have been developed: 1) the description of the profile shape, including the edge thickening and peculiarities of the middle part, 2) the recognition and classification of the profile shape. Taking into account the class of the profile contour allows you to adjust the cold rolling technology at the stage of forming the assembly party.

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