Visual study of variation of flow patterns around a disk in a casing: effects of edge of the disk

H Furukawa¹, H Tsutsui¹, K Aoi², T Watanabe³ and I Nakamura¹

¹ Department of Mechanical Engineering, Meijo University, Nagoya 468-8502, Japan
² Noritake Engineering Co., Ltd., Nagoya 451-8501, Japan
³ EccoTopia Science Institute, Nagoya University, Nagoya, 464-8603, Japan

E-mail: furukawa@ccmfs.meijo-u.ac.jp

Abstract. This report presents the result of visualization of versatile patterns of flow around a rotating disk in a casing. Investigation of this type of flow has rather long history and many works have been performed from experimental, theoretical and numerical points of views. In this study, we concern the effect of the clearance between a rim wall of a rotating disk and the shroud of the outer casing on the flow structure. The radii of the disks were \( r_d = 141.8\, \text{mm}, 135.5\, \text{mm}, 127.0\, \text{mm} \) and \( 112.0\, \text{mm} \), and the inner radius of a fixed casing is \( r_e = 142\, \text{mm} \). Flow pattern was visualized using aluminium powder. The Reynolds number based disk radius was varied and its critical value was determined at various bifurcation points. Qualitative structures were determined from time variations of the brightness at fixed pixels of CCD camera used to capture the spiral rolls of flow patterns. The sectional views of clearance space of the disk and the casing unveiled oscillating end flow pattern according to the spiral-roll of plan view.

1. Introduction
In 1905, which is exactly one hundred years ago, a pioneering work of Ekman[1] appeared which unveiled the spiral velocity field induced by wind over the ocean and Coriolis force produced by earth’s rotation. And since von Karman's finding of the similar solution of swirling flow on an infinite rotating plane[2], and Bödewadt’s work on the flow on the flow over a stationary infinite plane produced by the solid body rotating flow at infinity[3], the rotating flow induced by a spinning disk has attracted many researchers and number of papers appeared. From an engineering point of view, flows around a rotating disk relates to many types of fluid machinery such as pumps, compressors, water turbines, gas turbines, thrust bearings. Therefore, following the works by Schultz-Grunow[4], Zumbush et al[5], Panetell[6], Stepanoff[7], Dailey and Nice[8] and Itoh[9, 10], many practical studies on a variety of situations were performed on the rotating disk flows (see reference [11]). Another side of this problem is purely fluid mechanical, and Bachelor’s seminar work[12] of the similar solution of the flow between two parallel rotating infinite planes (or disks) sparked the interest of many fluid dynamics researchers for this type of variety of rotating flow. Detailed review about this problem is given by Zandbergen and Dijkstra[13].

Classification of this type of flow is not as easy as at first sight one may considers. Here we pay attention to four types of topological configurations restricted by one wall or more than one walls.
The first is Kármán-Bödewadt type, and the fluid flow occurs upper half space of an infinite plane or disk, and the original Ekman type flow may be also included. The second is Schultz-Grunow – Batchelor type in which the flow is limited by two parallel infinite disks which can rotate at each own angular velocity. It is noticeable that already Schultz-Grunow reported from their experimental results the three layer structure in the two disk type flow that consists from the rotating disk layer, the stationary disk layer and the core flow region between them. The third is closed simply connected region type, that is the fluid is swirling in a cylindrical container, and the forth final type is closed but doubly connected region type whose typical flow is Taylor-Couette vortex occurring in a toroidal space in topological sense. Our problem is the last type but rotating body is not a simple cylinder but it is a thin axle having a circular disk, (experimental facility of Zumbusch et al. was this type). The related type is the multiple disk type [14, 15]. Serre et al [16] called the third type as cylindrical cavity and the fourth type without a disk as annular cavity respectively. Historical development of the study of rotating flow related to the present research is well described by them. Also Schouveiler L, Le Gal P and Chauve M [20] obtained many interesting points of the disk-casing flow structure. They examined the effect of boundary conditions at disk’s periphery which is related to the present experiment. They noticed the large effect of this condition on the flow field, but their “radial gap type case” is limited only one case, and we consider that more experiments are needed to clarify this effect. Main result of present study is the instability of shear flow in the edge clearance and its effects to the Bödewadt layer over the stationary casing disk. References [22,23] are important recent works which study the instability of rotating shear disk type layer appearing in a cylinder having rotating top and bottom endwalls. Excellent experimental velocity distributions are given in [24].

2. Experimental facility and method

Sectional view of present experimental apparatus is shown in figure 1, which is constructed by a short cylindrical casing enclosing revolving disk fixed to a thin shaft. Four types of disks were used and their radii are \( r_d = 141.8 \text{ mm}, 135.5 \text{ mm}, 127.0 \text{ mm} \) and 112.0 mm. The inner radius of the casing is \( r_e = 142 \text{ mm} \) and the minimum edge clearance between the disk and the casing is \( r_g = 0.2 \text{ mm} \) and the maximal clearance is 30mm. The disk is centered in the casing and its thickness is \( h_d = 10.0 \text{ mm} \), then the distance from the disk surface to the casing surface is constant value of \( h = 10.0 \text{ mm} \). Disks are made of duraluminum and coated with black lacquer and the casing is made of acrylic resin. Mixture of distilled water and glycerol is working fluid whose viscosity was measured by a cone-plate type viscometer. Aluminum powder sieved under 20 μm mesh was used for visualization. A CCD camera having 3.2 million pixels and suitable photo editor was used. Light source used was xenon slit light whose width is 2mm. Reynolds number is defined as: \( R_e = \frac{\omega r_d^2}{\nu} \), where \( \nu \) is kinematic viscosity of working fluid and \( \omega \) is angular velocity of the disk. The number of geometrical parameters of this flow field is rather many but in this experiment, effect of the edge clearance \( r_g \) is examined while keeping other dimensions as constant. Therefore the value of \( r / h \) is a candidate of geometrical parameter and it was varied from 0.02 to 3. Also \( (\omega r_d) r / r_g = Re(r / r_g) \) may be used. Two configurations of light slit plane were used for the visualization. The one was perpendicular to the rotating axis and the plane is parallel to the disk plane. The other contained the rotating axis, which is a coordinate plane of the \( r-z \) of the cylindrical coordinate used here. The rotating speed of the disk was controlled by an inverter and monitored by a digital tachometer. Mixture ratio of aluminum powder to the working fluid was 110mg per 4l. A number of trials confirmed that this mixture is the best for the visualization. As a surfactant to avoid
clustering of aluminum powder, a home-use detergent was found a useful matter then amount of 1.5 ml of it was added to the working fluid.

3. Experimental results

3.1. Fundamental flow pattern
In this experiment we concern mainly flow field over the stationary disk that is the bottom surface of the casing and that the flow field neighbor on the rotating disk edge region, since the Ekman layer or Bödewadt layer is more unstable than the Kármán layer over the rotating disk. Then it shows rich flow pattern within the present experimental condition. The structure of the Kármán layer is a future problem.

Figure 2 presents the case of \( r_s = 127.0 \text{ mm} \) (\( r_s/h = 0.5 \)), where the photos in the left column show the plane views of the flow structure near the bottom stationary casing surface. The bottom of the light slit locates at \( z=2 \text{ mm} \), and the slit width is 2mm. Radial lines and circles are drawn on a OHP (Over Head Projector) sheet for fixing the polar coordinate system of which radii are \( r = 75 \text{ mm}, 100 \text{ mm} \) and \( 125 \text{ mm} \), and the angle between radial lines is \( \pi/8 \) radian. Two inclined shadows appearing plan views are due to refraction of the light and the central black bar is the shadow of the rotating shaft. Direction of rotation of the disk is anticlockwise and also spiral-rolls are rotating in the same direction. The right hand side column shows sectional views including rotating axis. Reynolds number increases from upper figure to lower figure.

Figure 2. Visualization of transition patterns in the stationary layer and in the radius direction gap. (a) \( Re = 4.4 \times 10^3 \): Circular-roll
(b) \( Re = 5.9 \times 10^3 \): Mixed state with Circular-roll and Spiral-roll
(c) \( Re = 11.0 \times 10^3 \): Spiral-roll
(d) \( Re = 14.0 \times 10^3 \): Turbulent-spiral-roll
(e) \( Re = 15.3 \times 10^3 \): Turbulent-roll
The flow near the stationary casing is resembles to Bödewadt layer. We can see that circular roll waves emerged near the cylindrical side wall of the casing propagate inward, while they fade out with the region about $r = 100$ mm from the axis (figure 2(a)). And in this Reynolds number case, right hand sectional view shows that the flow structure is symmetric with respect to the mid circular plane of the disk. Notice the black vague band emerging from the rotor edge side to the casing side wall, which indicates the border of the streams starting from upside and downside of the disk. The fluid near the disk is accelerated to the peripheral direction and on the other hand the fluid near the stationary disk that is the casing bottom wall is flowing to inward and decelerated. In the region $z = 8$ to $10$ mm, the velocity field has no radial component so it is the core region flow having only azimuthal velocity component. In figure 2(b) typical well known spiral pattern of this flow appears, but its strength is weak and within the region of $r = 75$ mm, the flow has weak ring-roll. The flow in the sectional plane shown in the right column has characteristics of unsteady nature. The stationary black band in right hand picture of figure 2(b) oscillates around a fixed point on the edge of the disk, so the up and down fluctuation of the band on the casing side wall appears. In our case, it is possible that first instability occurs from the shear flow existing in the clearance and it induces spiral-rolls in the Bödewadt layer on the casing surface.

When Reynolds number increases, the arm of the spiral invades in the central region and number of the spiral-roll increases as shown in figure 2(c). In corresponding $r$-$z$ sectional view, we see the number of stripes also increases. In figure 2(d) at $Re=14.0 \times 10^3$, we see the turbulent spiral roll and in $r$-$z$ sectional view, pattern becomes multi-layer structure. In figure 2(e) the flow is turbulent state but some structure is still discernible, and rough spiral, spiral-dislocation, spiral-roll merging, and inner vague rings are clearly seen.

3.2. The effect of radial edge clearance

In order to clarify the effect of disk radius or edge clearance between disk side and casing cylindrical side wall, representative figure are shown in figures 3(a) to 3(d).
Sectional view of smallest clearance case is shown in figure 4 where we can not see oscillating flow structure even spiral-roll appears in the plan view as seen in figure 3(a).

The radial clearance value \( r_g \) (see figure 1) increases from figures 3(a) to (d) as shown in the capture. Transition Reynolds numbers at which the flow became spiral-roll were about \( 16.0 \times 10^3 \), \( 7.2 \times 10^3 \), \( 6.7 \times 10^3 \), \( 2.3 \times 10^3 \), respectively. In every flow, the spiral arm emerges at first near the side wall of the cylindrical casing, and then it grows inwards with increasing Reynolds number as shown in figures 3. Also the clearance increase causes the widening of the spacing of neighbour spiral arm. In the case of figure 3(d) spiral-rolls do not contact with the casing. The rotational direction of the disk is anticlockwise in these photos, and curves of the spiral arms of the cases in figures 3(b),(c),(d) are same nature but that of figure 3(a) case shows the direction opposite to others' direction. According to Faller who studies rotating disk flow for more than forty years, traditionally classified as type 2 spiral vortices in Ekman layer has properties that the first 1): they has opposite angle relative to circles on the disk with respect to type 1 spiral vortices and the second 2): the vortices move rapidly outward amplifying as they progress. With this respect the difference between figure 3(a) and others seems the result of above property 1), but spiral-rolls appeared in figures 3(b, c, d) do not show the property 2). It must be pointed out that in our experiments the edge clearance has large effect to transition to the rotating flow of a rotor and enclosing casing system. The fluid flow similarity requires geometrical similarity, it is a most important point, and then requires Reynolds number similarity. Our disk-casing geometry has rather large radial gap between the rotating disk edge and stationary casing and there is a rotating shaft as shown in the figure 1. So the complete geometrical similarity is lost between present facility and the one used in the reference [20], but some point is resembles with two experiments, as one can recognize it by comparing the photographs in the [20] and present figures. Specifically the change of spiral roll direction induced by radial gap was also found in [20]. With this respect one may say that the various rotating disk flow has global similarity. We consider that the most important effect of large radial gap is the shear flow instability emerged from the edge into the clearance space. Such a rotating radial jet is treated earlier in [21],(cf. page 218 of the book), but instability is not discussed in it. Related flow appears at the cell boundary of Taylor-Couette vortex and the instability of such a flow is examined recent papers, [22],[23] both studies treat the flow in a cylinder having rotating end disks.

3.3. Digital pattern analysis of pictures.

Digital video camera data have information from which we can obtain quantitative structure of spiral-rolls. Every pixel of an image has color cord of R(red), G(green) and B(blue) respectively from 0 to 255, and brightness is defined as the mean value of color cord of respective R,G and B. The brightness of a small region fixed relative to the stationary disk has been analyzed using Fourier transform with respect to time. Figures 5(a), (b) show an example of measuring region and corresponding power spectrum in which we can see clear peak.
Figure 5. Measuring region closed by a red rectangle and power spectrum of brightness.

In (b) the ordinate has an arbitrary linear scale. $\text{Re} = 1.1 \times 10^3$

A relationship of Reynolds number and peak frequency of brightness time series is shown in figure 6. The data were sampled at $r = 110\text{mm}$. The disk radius was $r_d = 127.0 \text{mm}$ and sampling spot was $5 \times 5 \text{pixels}$, and brightness was calculated as the mean value of $25 \text{pixels}$. Measuring time was $8.5 \text{sec}$, and the number of frame was 256. In this case spiral-roll appeared evidently. At the Reynolds number above or below the range shown in figure 6, it was difficult whether the peak is clearly existing or not. At lower Reynolds number, ring rolls appeared, while two spectrum peaks appeared and the flow became turbulent spiral roll at higher Reynolds number.

The figure means that the rotating speed of spiral-roll increases with increasing Reynolds number which is in this case in proportion to rotating speed. It means that wave length is constant. The standard deviation of brightness obtained on the same radius versus measuring spot position is an indicator of the spiral structure. The figure 7 presents a sample figure of this distribution for the case of as the same disk as in the case of figure 6. From the figure we can see that standard deviation is large in the range of $r = 100$ to $125 \text{mm}$ and it correspond to the emerging region of spiral-roll. So we consider that the spiral-roll in this case appears at the edge of the disk. The Reynolds number range at where spiral exists is an important factor to elucidate the effect of the edge clearance on the flow structure.

Figure 8 shows the average radial range of three cases indicated in the figure.

Figure 6. Peak frequency

Figure 7. Standard deviation distribution
Big arrows correspond to the average value and black bar shows individual experiment. From the figure we can see that the smaller the disk diameter at the smaller $Re$ the spiral-roll emerges. It means that the shear layer stability in the clearance region has large influence of the spiral-roll formation.

4. Concluding remarks

In many swirling flow problems, rotor-casing system is important not only in an engineering point of view but also academic interest. In this report, we examined a flow filed produced by a disk having an axle through it and is rotating in the mid position of a stationary cylindrical casing. Some discussion is given about topological nature of the flow field which have been studied by many researchers. Simple visualization technique by use of aluminium powder was used. Disk diameter was varied and clearance between disk and casing cylindrical side wall was changed and its effect, as well as the effect of the Reynolds number, on the flow structure was investigated.

Mainly visualized was Bödewadt layer stability and sectional view of flow field in the clearance region which consists of shear flow spring out from disk. Large effect of edge clearance was found and in our experimental situation it is this shear flow instability that induces the spiral-roll structure in the Bödewadt layer. Spiral-roll direction showed the change of arm direction in the case of very small clearance and large clearance case. The time series of brightness obtained from CCD camera image was analyzed to obtain quantitative value of the structure of spiral-roll. Kármán layer over the surface of rotating disk was found to be very stable as usual. It needs more detailed study to clarify the three dimensional and time varying and complete turbulence structure of this flow.

Acknowledgement

Authors gratefully acknowledge reviewers for their critical comments and suggestions which made the paper improved appreciably.

References

[1] Ekman V W 1905 *Arkiv För Matematic,Astronomi och Fisik* 2-11 1
[2] Von Kármán 1921 *Z. Angew. Math. Mech* 1 233
[3] Bödewadt U T 1940 *Z. Angew. Math. Mech*, 20 241
[4] Von Schultz-Grunow F 1935 *Z. Angew. Math. Mech*, 15 191
[5] Zumbush O, Behrens H and Beer H 1935 *Z. Angew. Math. Mech*, 15 356
[6] Pantel K 1949 *Forshung auf den Gebiete des Ingenieurwesens* 16-4 97
[7] Stepanoff A J 1940 *Centrifugal and axial flow pumps* Wiley and Sons
[8] Daily J W and Nese R E 1960 *Trans ASME J. Basic Eng*. 82 217
[9] Itoh M and Yammada Y and Nishioka K 1985 *Trans.JSME* 52 452 (in Japanese)
[10] Itoh M 1991 *ASME Boundary Layer Stability and Transition to Turbulence* 114 83
[11] Owen J M and Rogers R H 1989 *Flow and heat transfer in rotating disc systems, vol. 1: Rotar-Stator systems* Somerest, England; Res. Studies Press
[12] Batchelor G K 1951 *Quart.J.Mech.and Appl.Math.* IV-1 29
[13] Zandbergen P J and Dijkstra D 1987 *Ann. Rev. fluid Mech.* **19** 465
[14] Schuler C A, Usry W, Weber B, Humphrey J A C and Greif R 1990 *Physics of Fluids* **A2-10** 1760
[15] Herrero J, Giralt F and Humphrey J A 1999 *Physics of Fluids* **11-1** 88
[16] Serre E, Del Alco E C and Bontoux P 2001 *J. Fluid Mech.* **434** 65
[17] Faller A J 1991 *J. Fluid Mech.* **230** 245
[18] Shouveiler L, Le Gal P, Cauve M P and Takeda Y 1999 *Exp. Fluids* **26** 179
[19] Gauthier G, Gondret P, Moisy F and Babaud M 2002 *J. Fluid Mech.* **473** 1
[20] Shouveiler L, Le Gal P, Cauve M P 2001 *J. Fluid Mech.* **443** 329
[21] Loitsianski L G, 1967 *Laminare Grenzschichten* tr Szablewski W Akademie-Verlag Berlin
[22] Nore C, Tuckerman L S, Daube O and Xin S 2003 *J. Fluid Mech.* **477** 51
[23] Lopez J M and Marques F 2004 *J. Fluid Mech.* **507** 265
[24] di Leonardo R, Ianni F and Ruocco G 2005 *J. Fluid Mech.* **525** 27