Sommerfeld corrections to
type-II and III leptogenesis

Alessandro Strumia

Dipartimento di Fisica dell’Università di Pisa and INFN, Italia

Abstract

We study thermal leptogenesis from decays of the electroweak triplets that mediate neutrino masses in type-II and type-III seesaw. We find that Sommerfeld corrections reduce the baryon asymmetry by $\sim 30\%$, and that successful leptogenesis needs triplets heavier than 1.6 TeV, beyond the discovery reach of LHC.

1 Introduction

Leptogenesis might be produced by decays of scalar (type-II seesaw) or fermion (type-III seesaw [1]) SU(2)$_L$ triplets with mass $M$. The Boltzmann equation for their abundance contains their annihilation rate $\gamma_A$ (number of annihilations per spacetime volume at temperature $T$): the final baryon asymmetry depends on the value of $\gamma_A$ in the non-relativistic limit, at $T \sim M/\ln(M_{Pl}/M)$. Non-relativistic scatterings among gauge-charged particles are affected by non-perturbative corrections: the states involved in the annihilation no longer are plane waves when the kinetic energy is comparable to the electroweak potential energy [2]. In this paper we study how such Sommerfeld corrections affect the final baryon asymmetry.

In section 2 we use group theory to derive a simple formula for non-abelian Sommerfeld corrections. In section 3 we apply it to type-III see-saw and in section 4 to type-II see-saw.
In both cases we also extend computations of thermal leptogenesis down to values of $M \sim \text{TeV}$ accessible to the LHC collider, motivated by recent papers that studied how type-II or III see-saw can be searched for at LHC. However, we find a model-independent lower bound on $M$ that limits this possibility. In section 5 we discuss our results and conclude.

2 Sommerfeld corrections

We denote as $T$ the scalar or fermion triplet that decays producing the lepton asymmetry. Following [3, 4] the final baryon asymmetry is parameterized as $n_B/n_\gamma = -0.029\eta\varepsilon$, where the efficiency $\eta$ encodes the dynamics and $\varepsilon$ the amount of CP violation. The Boltzmann equation that dictates the evolution of the triplet abundance contains the $T$ decay rate $\gamma_D$ and the $TT^*$ annihilation rate $\gamma_A$:

$$sHz \frac{dY}{dz} = -\left( \frac{Y}{Y_{eq}} - 1 \right) \gamma_D - 2 \left( \frac{Y^2}{Y_{2eq}} - 1 \right) \gamma_A. \quad (1)$$

where $z \equiv M/T$, $H$ is the Hubble rate at temperature $T$; $Y \equiv n_T/s$ for the real fermion triplet and $Y \equiv (n_T + n_{T^*})/s$ for the complex scalar triplet; $Y_{eq}$ is the value that $Y$ would have in thermal equilibrium; $n$ is the number density; $s = 2\pi^2 g_{ss} T^3/45$ with $g_{ss} = 106.75$ is the entropy density of SM particles; $\gamma_A$ is the thermal average of the annihilation cross section summed over all initial- and final-state Lorentz and gauge indices:

$$\gamma_A = \frac{T}{64\pi^4} \int_{4M^2}^{\infty} ds \frac{s^{1/2}}{s} K_1 \left( \frac{\sqrt{s}}{T} \right) \hat{\sigma}_A(s) \quad \text{where} \quad \hat{\sigma}_A \equiv \int dt \sum_{\text{all}} \frac{|A|^2}{8\pi s}, \quad (2)$$

and $K_1$ is a Bessel function;

Explicit expressions for $\hat{\sigma}_A$ are given in appendix A of [5] for a generic SU(2)$_L \otimes$U(1)$_Y$ multiplet. Similarly to the case of Cold Dark Matter freeze-out, the value of $\gamma_A$ is relevant in the non relativistic limit, $T/M \approx 1/\ln M_{Pl}/M < 1$, so that $\hat{\sigma}_A$ can be approximated as the sum of $s$-wave, $p$-wave, etc contributions:

$$\hat{\sigma}_A = c_s \beta + c_p \beta^3 + \mathcal{O}(\beta^5) \quad (3)$$

where $\beta = \sqrt{1 - 4M^2/s}$ is the triplet velocity in the $TT^*$ center of mass frame. The corresponding annihilation rate is

$$\gamma_A = \frac{MT^3 e^{-2M/T}}{32\pi^3} \left[ c_s + \frac{3T}{2M} (c_p + \frac{c_s}{2}) + \mathcal{O}(\frac{T}{M})^2 \right]. \quad (4)$$

For the fermion triplet one has

$$c_s = \frac{111g_f^2}{8\pi}, \quad c_p = \frac{51g_f^2}{8\pi}. \quad (5)$$
For the scalar triplet \( \gamma_A \) is well approximate by just its s-wave coefficient

\[
c_s = \frac{9g_4^2 + 12g_2^2 Y^2 + 3g_Y^4}{2\pi} \tag{6}
\]
(hypercharge is normalized such that \( Y = 1/2 \) over lepton doublets).

We now compute the non-perturbative electroweak Sommerfeld corrections to \( c_s \). Scatterings among charged particles are distorted by the Coulomb force, when their kinetic energy is low enough that the electrostatic potential energy is relevant. This leads e.g. to significant enhancements of the \( \mu^- \mu^+ \) annihilation cross section (attractive force) or to significant suppressions of various nuclear processes (repulsive force). The Sommerfeld correction to s-wave point-like annihilations (e.g. \( \mu^- \mu^+ \to \gamma\gamma \)) can be computed as [2]

\[
S = |\psi(\infty)/\psi(0)|^2, \quad \psi(r) \text{ is the (reduced) s-wave-function for the two-body state with kinetic energy } K = M^2, \text{ that in the non-relativistic limit satisfies the Schrödinger equation}
\[
-\frac{1}{M} \frac{d^2\psi}{dr^2} + V \cdot \psi = K\psi \tag{7}
\]
with outgoing boundary condition \( \psi'(\infty)/\psi(\infty) \simeq iM\beta \). For a single abelian massless vector with potential \( V = \alpha/r \) the result is \( \sigma = S\sigma_{\text{perturbative}} \) where the Sommerfeld correction is [2]

\[
S(x) = \frac{-\pi x}{1 - e^{-\pi x}}, \quad x = \frac{\alpha}{\beta}. \tag{8}
\]
Here \( \alpha < 0 \) describes an attractive potential that leads to an enhancement \( S > 1 \), and \( \alpha > 0 \) describes a repulsive potential that leads to \( S < 1 \). We will need the thermally averaged value of the Sommerfeld correction:

\[
S_T(y) = \frac{\int_0 \beta^2 e^{-\beta^2 y} S(\alpha/\beta) d\beta}{\int_0 \beta^2 e^{-\beta^2 y} d\beta}, \quad y \equiv \frac{\alpha}{\beta_T}
\tag{9}
\]
where \( \beta_T = \sqrt{T/M} \) is the characteristic velocity in the thermal bath and the upper integration limit on \( \beta \) is any value much larger that \( \beta_T \). The function \( S_T \) is computed numerically, with the qualitative result \( S_T(x) \approx S(x) \).

The generalization to non-abelian massive vectors was discussed in [6, 5] and needs a long list of potential and annihilation matrices, such that the matrix Schrödinger equation (7) can be solved only numerically. We here show how in the SU(2)\(_L\)-invariant limit \( M \gg M_{W,Z} \) the Sommerfeld correction can be analytically computed as a sum of abelian-like cases. For any simple gauge group with coupling \( \alpha = g^2/4\pi \), the potential between two particles in representations \( R \) and \( R' \) at distance \( r \) is

\[
V = \frac{\alpha}{r} \sum_a T_R^a \otimes T_{R'}^a \tag{10}
\]
where $a$ runs over the adjoint, $T_{aR}^R$ are the generators in the representation $R$, and the tensor product $\otimes$ indicates that $V$ has 4 gauge indices. Unlike in [5] we do not include (anti)symmetrizations and their corresponding normalization of the 2-body states: their only effect is enforcing $V = 0$ for the 2-fermion states forbidden by the Pauli exclusion principle, which can be less formally imposed by hand. Group theory allows to decompose the tensor product of the representations $R$ and $R'$ into a sum of irreducible representations $Q$:

$$ R \otimes R' = \bigoplus_Q Q, \quad T_{aR}^R \otimes 1_{R'} + 1_R \otimes T_{aR'}^R = \sum_Q T_Q^a $$

so that, recalling the definition $T_{aR}^R \cdot T_{bR}^R \equiv C_{R1}$ of the quadratic Casimir $C_{R1}$, gives:

$$ V = \frac{\alpha}{2r} \sum_A (T_{aR\otimes R'}^a \cdot T_{bR\otimes R'}^b - T_{aR}^a \cdot 1_{R'} - 1_R \otimes T_{aR'}^A \cdot T_{bR'}^A) = \frac{\alpha}{2r} (\sum_Q C_{Q1} - C_{R1} - C_{R'1}) $$

where $1_Q$ is the projector along the subspace $Q$, so that the matrix $V$ is diagonal along each irreducible representation $Q$. The Casimir of the SU(2) irreducible representations with dimension $n$ is $C_n = (n^2 - 1)/4$, so that a two-body state $n \otimes \bar{n}$ with total isospin $N \leq 2n - 1$ has potential $V = (N^2 + 1 - 2n^2)\alpha_2 / 8r$. The two-body state of the (scalar or fermion) triplets involved in leptogenesis decompose as $3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S$, and the potentials are $V = -2\alpha_2 / r$ for the singlet state, $V = -\alpha_2 / r$ for the triplet state, and $V = \alpha_2 / r$ for the quintuplet state.

Finally, one might worry that the Sommerfeld correction, computed for scattering in vacuum, does not apply for annihilations in the cosmological plasma. This is not the case because $\gamma_A$ is only needed when the $TT^*$ annihilation rate becomes smaller than the expansion rate $H \sim T^2 / M_{Pl}$: view of the Planck suppression $H$ and thereby $\gamma_A$ is so small that annihilations are rare enough that the vacuum approximation holds. Furthermore, the $T$ life-time $\tau \sim 1/ (\lambda^2 M)$ is longer enough than the Coulomb time-scale $\sim 1/(M\beta^2)$ provided that $\lambda \ll \beta$, where $\lambda$ denotes the small coupling(s) present in type-III or type-II see-saw. Finally, thermal effects generate a Debye mass for the vectors ($m^2 = 11g_2^2 T^2 / 6$ for the SU(2)$_L$ vectors), screening the Coulomb potential into a Yukawa potential, $e^{-mr}/r$: the results of [5] show that the Debye mass $m$ is small enough that we can neglect it, approximating vectors as massless.

### 3 Type-III see-saw: leptogenesis from a fermion triplet

The two body $TT$ states can be classified according to their quantum numbers $(I, S, L)$, where $I = \{1, 3, 5\}$ is the total isospin, $S = \{0, 1\}$ is the total spin, $L$ is total orbital angular momentum. Restricting to the dominant $s$-wave annihilations, $L = 0$, the states allowed by quantum statistics are $(I, S) = (1, 0), (3, 1) \text{ and } (5, 0)$. Their annihilations
Figure 1: Annihilation rate $\gamma_A$ for the fermion and scalar triplet for $M = 10^5$ GeV. The ‘tree’ line shows the tree-level result, the dashed line shows the s-wave approximation, and the ‘Sommerfeld’ line includes Sommerfeld corrections. Leptogenesis is dominantly produced at $z \equiv M/T \sim 20$.

into couples of SM vectors ($W^a$ and $Y$ of SU(2)$_L$ and hypercharge), of SM fermions $\Psi$ and of SM Higgs doublets $H$ are restricted by their $(I,S,L)$ quantum numbers. Taking into account the potentials computed in section 2, the total Sommerfeld-corrected s-wave annihilation rate is:

$$c_s = \frac{2g_4^2}{\pi} S_T(-2\alpha_2 \sqrt{z}) + \frac{5g_4^2}{2\pi} S_T(\alpha_2 \sqrt{z}) + (1 + \frac{1}{24}) \frac{9g_4^2}{\pi} S_T(-\alpha_2 \sqrt{z})$$

(13)

\[^1\text{We compare with previous results. Neglecting Sommerfeld corrections, } S_T \rightarrow 1: \text{ i) eq. (13) agrees with eq. (29c) of [3] (where a factor 2 was however missed in the Boltzmann equation in front of } \gamma_A; \text{ the factor 1/24 coming from annihilations into } HH^* \text{ was neglected; the } g_4^2 \text{ factor was unproperly typed as } g_8^2 \text{ in the numerical code); ii) eq. (13) differs by order one factors from eq. (4.9) of [7]; iii) eq. (13) agrees with analogous computations of the relic abundance of wino-like dark matter. Some dark matter studies also included Sommerfeld corrections [6, 5], but a direct comparison is not immediate (dark matter studies included SU(2)$_L$-breaking effects and thereby used electric charge instead of isospin to classify states, obtaining complex expressions); the numerical results for } \gamma_A \text{ can be compared in the limit } M \gg M_Z/\alpha_2 \text{ and agree. Some recent experimental results suggested the possibility of Sommerfeld enhancements from new gauge interactions in the Dark Matter sector [8]: our result in eq. (11) holds for any gauge group and allows to study such effects.} \]
where the text under each contribution to the rate indicates the corresponding annihilation processes. Eq. (13) differs from the perturbative result of eq. (5) by an order one factor at $T \lesssim M g_2^2$.

Fig. 1a shows our result for the adimensional combination $\frac{\gamma_A e^{2z^3} z^3}{M^4}$, that becomes constant in the non-relativistic limit $z \equiv M/T \gg 1$, where $s$-wave annihilations dominate. The final baryon asymmetry roughly depends on $\gamma_A$ at $z \sim 20$: the Sommerfeld enhancement to the $s$-wave contribution is more important than including $p$-wave and all other $L \neq 0$ annihilations.

Fig. 2a shows the contour plot of the efficiency $\eta$ as function of $M$ and of $\tilde{m}_1$, which, as usual, is the lightest triplet contribution to neutrino masses. Considering e.g. the point $M = 10^5$ GeV and $\tilde{m}_1 = 10^{-5}$ eV, Sommerfeld corrections reduce the efficiency from $2.6 \times 10^{-8}$ to $2.0 \times 10^{-8}$.

For large enough $\tilde{m}_1$ the decay rate $\gamma_D$ is larger than the annihilation rate $\gamma_A$ in the relevant temperature range $T \sim M/20$, so that the efficiency $\eta$ no longer depends on $\gamma_A$, and gets the value typical of type-I see-saw, largely independent on $M$. In this region Sommerfeld corrections, that enhance $\gamma_A$, do not affect the final asymmetry.
Figure 3: Values of the CP asymmetry $|\varepsilon_1| \leq 1$ needed to get the observed baryon asymmetry from decays of a fermion triplet (left) and a scalar triplet (right), as function of its mass $M$ for the values of $\tilde{m}_1$ superimposed to the curves. We see that successful leptogenesis needs $M > 1.6$ TeV.

As in [3] we shaded in green the region where thermal leptogenesis can produce the observed baryon asymmetry, assuming that the CP asymmetry is generated by two other triplets so heavy that their effects are fully encoded in the dimension-5 operator $(LH)^2$, constrained by neutrino masses. We see that the resulting bound on the CP asymmetry [3] implies the model-dependent lower bound $M > 3 \times 10^{10}$ GeV.

Various recent papers considered type-III and type-II see-saw with $M$ so small that triplets can be probed by colliders: LHC experiments are going to probe the region $100$ GeV $\lesssim M \lesssim 1$ TeV. We here find that this range of $M$ is not compatible with thermal leptogenesis, that needs $M > 1.6$ TeV because at lower $M$ the efficiency becomes so small that the baryon asymmetry $n_B/n_\gamma = -0.029\eta\varepsilon_1$ [3] remains smaller than the observed value, $n_B/n_\gamma \approx 6.15 \times 10^{-10}$ [9], even if the CP asymmetry is maximal, $|\varepsilon_1| = 1$ (resonant leptogenesis [10] allows to realize an order one CP-asymmetry, although unity is not reached). This issue is better illustrated by fig. 3a, that also allows to see how the lower bound on $M$ depends on $\tilde{m}_1$.

Let us now discuss why the efficiency strongly decreases when $M$ is below 3 TeV. The Higgs starts to acquire its vev via a first order phase transition at the critical temperature $T_{cr} \approx 1.2m_h$. Sommerfeld corrections have to be computed as in [6, 5], but this is a minor detail. The key issue is that SU(2)$_L$-breaking suppresses the sphaleron rate $\gamma_{\text{sphaleron}}$ [11], so that the lepton asymmetry ceases to be converted into the baryon asymmetry below the temperature $T_{\text{dec}} \approx 80$ GeV $+ 0.45m_h$ [11]. The approximations for $T_{cr}$ and $T_{\text{dec}}$ hold for the light Higgs mass suggested by present data, $115$ GeV $\lesssim m_h \lesssim 200$ GeV [12]; we here
assume $m_h = 120$ GeV. In order to quantitatively include the sphaleron decoupling effect it is convenient to write the Boltzmann equations for the $B - L$ and the $B$ asymmetries:

\[
\begin{align*}
    sHz \frac{dY_{B-L}}{dz} &= -\gamma_D \varepsilon_1 \left( \frac{Y}{Y_{eq}} - 1 \right) - \frac{Y_{B-L}}{Y_{eq}} \frac{\gamma_D}{2} \tag{14a} \\
    sHz \frac{dY_B}{dz} &= -\gamma_{\text{sphaleron}} (Y_B c - (1 + c) Y_{B-L}) \quad c \approx 0.52 \tag{14b}
\end{align*}
\]

The first equation is the standard one [3], as sphalerons conserve $B - L$. The second equation tells the final baryon asymmetry, and is approximatively solved by

\[
Y_B(T < T_{\text{dec}}) \approx (1 + 1/c) Y_{B-L}(T_{\text{dec}}). \tag{15}
\]

The evolution of the various asymmetries is illustrated in fig. 4a, where we see that the observed baryon asymmetry is reproduced with a maximal CP asymmetry $|\varepsilon_1| = 1$ for $M = 1.6$ TeV and $\tilde{m}_1 = 0.06$ eV, and it is dominantly produced at low temperature $T \sim M/10$, where neglected higher order processes are expected correct it by $\lesssim 10\%$.

Fig. 3a shows how the CP asymmetry $\varepsilon_1$ needed to get the observed baryon asymmetry depends on $M$ for several values of $\tilde{m}_1$. Since the lepton asymmetry is dominantly generated by triplet decays at $T \sim M/20$, the sphaleron decoupling effect becomes significant at $M \gtrsim 20T_{\text{dec}} \sim \text{few TeV}$, preventing successful baryogenesis if $M > 1.6$ TeV.

### 4 Type-II see-saw: leptogenesis from a scalar triplet

The two body $T T^*$ states have total isospin $I = \{1, 3, 5\}$, total spin $S = 0$; and we focus on $s$-wave annihilations so that $L = 0$: at tree level such states can only annihilate into two SM vectors, that can have isospin $I = \{1, 3, 5\}$.

\[
c_s = \left( \begin{array}{c}
\frac{2g_2^4 + 3g_Y^4}{2\pi} S_T(-2\alpha_2 + \alpha_Y \sqrt{z}) + \frac{5g_2^4}{2\pi} S_T((\alpha_2 - \alpha_Y) \sqrt{z}) \\
\frac{6g_2^2g_Y^2}{\pi} S_T(-\alpha_2 + \alpha_Y \sqrt{z})
\end{array} \right)
\]

In the limit $S_T \rightarrow 1$ this expression for $\gamma_A$ reduces to the one in [4].

Our results for the type-II see-saw are presented in fig. 1b, 2b, 3b, that closely resemble the corresponding figures 1a, 2a, 3a already presented in the type-III section.

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2 Including flavor details the first equation gets replaced by three equations for the partial $B/3 - L_{e,\mu,\tau}$ asymmetries [13]. If fermion triplet(s) will be discovered at LHC, it should be possible to reconstruct the flavor structure of their couplings [14] and a more precise analysis will become worthwhile.
Fig. 4: Evolution of $Y_L$, $Y_B$, $Y - Y_{\text{eq}}$ for $M = 1.6$ TeV, $\tilde{m}_1 = m_{\text{atm}} = 0.06$ eV, $|\varepsilon_1| = 1$.
5 Conclusions

Type-II and type-III see-saw generate neutrino masses from particles with electroweak gauge interactions: scalar or fermion SU(2)$_L$ triplets. Previous works showed that these triplets can lead to successful thermal leptogenesis [3, 4, 7] as $n_B/n_\gamma = -0.029\varepsilon \eta = 6 \cdot 10^{-10}$. The triplet annihilation rates (that keep triplet abundances close to thermal equilibrium apparently violating the out-of-equilibrium Sakharov condition for baryogenesis) can be small enough to allow a large enough efficiency $\eta$.

In the present work we showed that Sommerfeld corrections, that account for long-range non-abelian electrostatic interactions, enhance the annihilation rate, decreasing the efficiency by $\sim 30\%$. Fig. 2a and b show the efficiency as function of the lightest triplet mass $M$ and of the lightest triplet contribution $\tilde{m}_1$ to neutrino masses.

The efficiency steeply decreases at $M \lesssim 3$ TeV because, when at $T \sim M/20$ triplet decays produce the lepton asymmetry, sphalerons no longer convert it into the baryon asymmetry. Indeed at $T \lesssim m_h$ the electroweak symmetry starts to be broken suppressing the sphaleron interaction rate. We find that baryogenesis via type-II or III thermal leptogenesis is only possible at $M \gtrsim 1.6$ TeV. This bound on $M$ becomes stronger if $\tilde{m}_1$ is smaller than 1 eV, or if the Higgs is heavier than its minimal value, $m_h = 115$ GeV.

Previous studies [15, 14] found that see-saw triplets lighter than about 1 TeV give detectable effects at the LHC collider. It is therefore interesting to study if/how the absolute leptogenesis lower bound on $M$ can be evaded. One possibility is a new source of baryogenesis at the weak scale, for example new physics that makes the electroweak phase transition of second order and provides a new source of CP violation [16], a scenario which is already strongly constrained but not yet excluded. The alternative possibility is another heavier baryogenesis mechanism: here the problem is that the baryon asymmetry gets washed out by triplets unless their $L$-violating interactions are slower than the expansion rate. In the fermion triplet case, this happens if $\tilde{m}_1 \ll 10^{-2}$ eV (which implies significantly displaced decay vertices at LHC [14]) or if the triplet is negligibly coupled to some lepton flavor. However, neutrino oscillations need neutrino masses heavier than $10^{-2}$ eV and with large mixings among flavors. Scalar triplets offer one more possibility [4]: $L$-violating rates are suppressed if triplet decay rates into $LL$ or $HH$ are much different, but this suppression would hold in cosmology as well as at LHC.

We obtained the bound $M > 1.6$ TeV in type-II and type-III see-saw, and we expect a similar lower bound on $M$ in any model where the particle responsible for leptogenesis has interactions large enough to be detectably produced at LHC. In particular, in type-I see-saw models where baryogenesis is produced by decays of a right handed neutrino charged under new massive vectors, one can maybe find a situation where the new vectors are light enough to lead to detectable signals at LHC, and heavy enough to suppress their contribution to $\gamma_A$ [17].
We conclude discussing the bound on $M$ from the string-anthropic-multiverse point of view that recently received significant attention [18]. Assuming that baryogenesis is impossible in the vast majority of string models because they predict $M \ll \langle h \rangle$, one could argue that it is more likely to find a $M$ close to the anthropic bound that makes baryogenesis possible. If this possibility will be confirmed by future data, one might even argue that the weak scale is much below the Planck scale because it cannot be larger than $M$, which is generated exponentially below the Planck scale by string instanton effects [19].

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