EVOLUTION OF GALAXY CLUSTERS IN Λ MDM COSMOLOGIES

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The time evolution of galaxy cluster abundance is used to constrain cosmological parameters in dark matter models containing a fraction of hot particles (massive neutrino). We test the modified MDM models with cosmic gravitational waves which are in agreement with observational data at $z = 0$, and show that they do not pass the cluster evolution test and therefore should be ruled out. The models with a non-zero cosmological constant are in better agreement with the evolution test. We estimate $\Omega_\Lambda$ and find that it is strongly affected by a small fraction of hot dark matter: $0.4 < \Omega_\Lambda < 0.8$ for $\Omega_H/\Omega_M < 0.2$.

1 Introduction

Any realistic cosmological model should be consistent with observation data on large scale structure (LSS) in the Universe in the range of scales from galaxy ($\sim 1 h^{-1} \text{Mpc}$) up to large scale CMB anisotropy (CMBA) ($\sim 1000 h^{-1} \text{Mpc}$).

Nowadays, the most popular cosmological models are cold dark matter with non-zero cosmological constant ($\Lambda$CDM) and mixed dark matter without and with cosmological constant, where MDM is in the form of non-baryonic hot and cold collisionless particles.

All cosmological models were re-addressed after CMBA experimental detection to reveal their positive and negative features. Both $\Lambda$CDM and MDM models have met several difficulties.

As far as $\Lambda$CDM models are concerned, they demand a high value of the cosmological constant ($\Omega_\Lambda \geq 0.7$). In this case $\Lambda$CDM is able to fit a set of LSS observational constraints whereas at small scales it overproduces the number of collapsed objects by a factor 2 in comparison with the corresponding number of gravitationally bounded objects in galaxies catalogues.

Regarding standard MDM models the difficulties are related with late galaxy (quasar) formation and too high number density of galaxy clusters at $z = 0$: standard MDM is ruled out at $2\sigma$ CL.

One of the possibilities to overcome the sMDM difficulties is a consideration of MDM with some amount of cosmic gravitational waves (CGW). The importance of fundamental CGW has been discussed earlier in papers. Although $\Lambda$CDM models with $\Omega_M \leq 0.3$ normalized by COBE 4-year data are consistent with the cluster number density test, in order to archive an agreement with CMBA and cluster abundance data in MDM models (for both blue $(n > 1)$ and red $(n < 1)$ scalar perturbation spectra) it is necessary to take into account the CGW contribution in the derived value of $\Delta T/T$ at $10^6$ angular scale. The latter is estimated by parameter $T/S$, the ratio of the tensor to the scalar mode contributions.

\[n\] is the slope-index of post-inflationary density perturbation power spectrum.
As an alternative to ΛCDM and MDM, the MDM models with a non-zero cosmological constant (ΛMDM) have been proposed in 19, 20, 21. The advantage of these models is in retaining the inflationary paradigm and flat Harrison-Zel’dovich spectrum \((n = 1)\) with a smaller value (comparing with ΛCDM) of cosmological Λ-term. Other notable features of ΛMDM are the possibility of negligible CGW contribution \((T/S = 0)\) and a small fraction of hot particles: even 10% of massive neutrinos \((\Omega_H/\Omega_M \sim 0.1)\) could change the value of cosmological constant, being however in good agreement with other independent tests (CMB data 22, QSO lensing 23, Hipparcos data 24, SNIa 25).

In this paper we consider MDM (with CGW) and ΛMDM (with \(T/S=0\)), applying to these models the cluster evolution test. The latter test constraints effectively parameter \(\Omega_M\) (and hence \(\Omega_\Lambda\) in flat models)\(^7\),\(^26\),\(^27\). Our aim is to demonstrate how the presence of hot component could influence the \(\Omega_\Lambda\) limits.

We describe our models in Section 2 and the evolution test in Section 3, the results are summarized in Section 4.

### 2 Cosmological Models

We assume that DM is given by mixture of CDM and HDM components in the flat background space. The free model parameters are:

- \(\Omega_M\), the total matter density in the Universe \((\Omega_M = 1 - \Omega_\Lambda = \Omega_b + \Omega_H + \Omega_C)\), the latter are density parameters of baryons, hot, and cold particles, respectively,
- \(\Omega_H/\Omega_M\), the fraction of hot DM,
- \(N_H\), the number of massive neutrino species,
- \(h\), the Hubble constant in units \(100 \text{ km s}^{-1}\text{Mpc}^{-1}\).

The massive and massless neutrinos are described by the corresponding distribution functions which are evaluated from the Boltzmann-Vlasov collisionless equations. Cold particles (neutralino or hypothetical axions) are described as a pressureless fluid \((p_C = 0)\). Baryons and photons are treated as an ideal hydrodynamic fluid satisfying the Euler equations of motion (we choose the fixed value for baryon density parameter, \(\Omega_b = 0.015/h^2\)). All components interact with each other only gravitationally.

Assuming the power low post-inflationary density perturbations spectrum \((P_0(k) \propto k^n)\), the final total density power spectrum can be expressed as:

\[
P(k, z) = A k^n T^2(k, z) \left( \frac{g(\Omega_M(z))}{(1+z)g(\Omega_M)} \right)^2
\]

where \(A\) is the normalization constant, \(T(k, z)\) is the total transfer function, and \(g(\Omega_M(z))\) is the suppression coefficient. According to 28, \(g(\Omega_M(z))\) can be approximated as

\[
g(\Omega_M(z)) = 2.5 \Omega_M(z) \left( \frac{1}{70} + \frac{209 \Omega_M(z)}{140} - \frac{\Omega_M^2(z)}{140} + \Omega_M^4(z) \right)^{-1}
\]

where the current matter abundance \(\Omega_M(z)\) can be written as follows:

\[
\Omega_M(z) = \Omega_M \frac{(1+z)^3}{1 - \Omega_M + (1+z)^3\Omega_M}, \quad \Omega_M \equiv \Omega_M(0).
\]

We use the transfer function approximations for sCDM 30 and ΛMDM 31 models.
3 Cluster Evolution Test

The mass function for the gravitationally bounded halos of mass greater than \( M \) formed in the flat Universe by redshift \( z \) is given by

\[
N(> M, z) = \int_{M}^{\infty} n(M', z) dM',
\]

where \( n(M, z) dM \) is the comoving number density of collapsed objects with masses lying in the interval \((M, M + dM)\):

\[
n(M, z) = \sqrt{\frac{2}{\pi M}} \frac{\rho_c}{\sigma^2(R, z)} \left| \frac{d\sigma(R, z)}{dM} \right| e^{-\frac{\delta_c^2}{2\sigma^2(R, z)}}.
\]

\( M = \frac{4}{3} \rho R^3 \), \( \rho \) is the mean matter density, and \( \delta_c \) is the critical density contrast for a linear overdensity able to collapse. The rms amplitude of density fluctuation in the spheres of radius \( R \) at redshift \( z \) is related to the power spectrum as

\[
\sigma^2(R, z) = \frac{1}{2\pi^2} \int_{0}^{\infty} P(k, z) |W(k, R)|^2 k^2 dk,
\]

where \( W(kR) \) is a Fourier component of the top-hat window function: \( W(x) = \frac{2}{\pi} (\sin x - x \cos x) \).

For the matter dominated Universe (\( \Omega_M = 1 \)) \( \delta_c = 1.686 \) (e.g. [28]). For the flat models with \( \Lambda \)-term \( \delta_c \) depends weakly on the current matter abundance. The theoretical mass functions obtained with the help of the Press-Schechter formalism (eqs.5,6) are in a good agreement with other methods including numerical simulations [32,34]. Due to the exponential dependence (see eq.5) the cluster number density is very sensitive to the value of \( \sigma_8 \).

Rich galaxy clusters are strong X-rays sources characterised by the gas temperature (see the review papers at this Conference). With help of numerical simulation it was confirmed that \( T_g \propto M^{2/3} \), the proportionality coefficient depends slightly on cosmological models. Here, we make use the \( T - M \) relation for isothermal gas given in [28]:

\[
T_g = \frac{7.75}{\beta} \left( \frac{6.8}{5X + 3} \right) \left( \frac{M}{10^{15} h^{-1} M_{\odot}} \right)^{\frac{4}{3}} \left( \frac{\Omega_M}{\Omega_M(z)} \right)^{\frac{1}{3}} \left( \frac{\Delta_{cr}}{178} \right)^{\frac{2}{3}} (1 + z) \text{ Kev}
\]

(7)

where \( \beta (\approx 1) \) is the ratio of the galaxy kinetic energy to the gas thermal energy, \( X (\approx 0.76) \) is the hydrogen mass fraction. The value \( \Delta_{cr} \) is the ratio of a mean halo density (within the virial radius of collapsed object) to the critical density of the Universe at the corresponding redshift. For \( \Omega_M \leq 1 \), \( \Delta_{cr} \) can be derived analytically and approximated as \( \Delta_{cr} = 178 \Omega_M^{0.45} \).

As well as the cluster mass function \( N(> M) \equiv N(> M, 0) \), the X-cluster temperature function \( N(> T) \equiv N(> T, 0) \) is also sensitive to \( \sigma_R \equiv \sigma(R, 0) \). The comparison of the observed temperature function [35] with cluster mass function [34] shows an agreement between two approaches. Confronting with observational data both tests provide a powerful constraint on \( \sigma \)-value in different DM models [13,16].

The cluster mass(temperature) functions and evolution tests in \( \Lambda CDM \) has been discussed in details (e.g. [13,40,41]). It has been shown that flat cosmological models are preferable in comparison with open ones. To archive a better consistency with cluster evolution observations the value of \( \Omega_M \) should be lower than that obtained from the cluster number density test at \( z = 0 \).

The dependence of the cluster mass(temperature) function on \( z \) is model dependent due to the differences in growth rate of density perturbations. This is reflected in \( \sigma \) time evolution related to the suppression coefficient \( g(\Omega_M(z)) \):

\[
\sigma(R, z) = \sigma_R \frac{g(\Omega_M(z))}{g(\Omega_M)} \frac{1}{1 + z},
\]

(8)
where \( g(\Omega(z)) \) is given by eq. (3).

We consider below the cosmological models containing massive neutrino. In this case \( z \)-dependence also appears in the transfer function.

## 4 Results and Discussion

### 4.1 MDM Models with Non-Zero Tensor Mode

To test MDM models with zero cosmological constant we follow the normalization procedure given in [2]. All models are normalized by \( \sigma_8 \) by the best fit of observational present day cluster mass \( M_h \) and temperature \( T/S \) functions. The agreement with \( \Delta T/T \) data leads to a non-zero derived parameter \( T/S \). The required value of \( T/S \) is mainly determined by spectral index \( n \), abundance of hot matter component \( \Omega_H \), and Hubble constant \( h \).

Fig. 1a presents the present day cluster temperature functions \( N(> T) \) for different values of \( \Omega_H \) in MDM models with CGW normalized by \( \sigma_8 \simeq 0.52 \). The normalization does not depend on \( \Omega_H \), therefore all curves cross each other at some fixed point (corresponding to the mass \( M \) in the sphere of radius \( 8h^{-1}\text{Mpc} \)).

A significant contribution of both CGW and massive neutrino is needed to fit the CMBA data. For \( h = 0.6 \) and \( \Omega_H = 0.2 \) models with flat spectra \( n = 1.1 \), \( T/S=3.6 \); for red spectra \( n = 0.9 \), \( T/S=0.6 \). An increase of \( \Omega_H \) decreases the parameter \( T/S \), however the problem arises with small-scale clustering and \( L_y \) cloud formation test [1]. At low \( \Omega_H < 0.2 \), neither possible changes of \( h \) nor models with three species of massive neutrino can suppress high contribution of CGW to CMBA. Even neglecting the observational problem at high \( \Omega_H \geq 0.2 \) cannot help to achieve an agreement with cluster evolution test in MDM matter dominated models.

Figs. 1b, 2a, 2b present the cluster evolution in MDM models. None of the considered models can fit the data at high \( z > 0.3 \). We varied \( \Omega_H \) (Fig. 1b.), \( h \) (Fig. 2a.), and post-inflationary spectral index \( n \) (Fig. 2b). All the models predict the number of galaxy clusters at high redshifts at least two orders of magnitude smaller than the observed one. This fact indicates that the evolution should be slower than that found in MDM models without a cosmological constant.

### 4.2 \( \Lambda \)MDM Models without CGW

Let us consider flat \( \Lambda \)MDM models without CGW normalized by the COBE 4-year data.

The best fit \( \sigma_8 \) obtained from nearby cluster observational abundance depends on \( \Omega_M \) as:

\[
\sigma_8(\text{cl}) = 0.52 \Omega_M^{0.52+0.13\Omega_M}.
\]

The coincidence of \( \sigma_8(\text{cl}) \) with \( \sigma_8(\text{cmb}) \) derived in COBE normalized \( \Lambda \)CDM model would limit the parameters \( \Omega_M \) (\( \Omega_\Lambda \)) by the corresponding values \( \sim 0.3(0.7) \) (e.g. [2]). Cluster evolution test bounds these parameters even stronger [3], \( \sim 0.2(0.8) \).

As an example, we show the cluster evolution functions in \( \Lambda \)CDM model with \( \Omega_\Lambda = 0.7 \) for the cluster masses \( M \geq 8 \cdot 10^{14}M_\odot \) in the comoving radius \( 1.5h^{-1}\text{Mpc} \), as functions of \( h \) (Fig. 3a) and \( n \) (Fig. 3b). The cluster evolution is not practically affected by changes of the spectral index. Regarding parameter \( h \), the models with \( h \geq 0.65 \) are preferable.

While the requested value of cosmological constant in \( \Lambda \)CDM models is quite high (\( \Omega_\Lambda \geq 0.7 \)), a small amount of hot particles (\( \Omega_H/\Omega_M < 0.2 \)) in \( \Lambda \)MDM models allows a consistency with observations for smaller \( \Omega_\Lambda \).

We performed the calculations of function \( N(> T) \) for different parameters \( \Omega_\Lambda \) and \( \Omega_H/\Omega_M \) (Figs. 4a,b). Increasing \( \Omega_H/\Omega_M \) reduces the needed value of cosmological constant. Taking into account the \( L_y \) forest test, models with \( \Omega_H/\Omega_M \simeq 0.1 \) are preferable. Parameter \( \Omega_M \) remains in the range \( 0.4 \leq \Omega_M \leq 0.6 \) in agreement with other results (cf. [4]).

Figs. 5-6 present the cluster evolution in \( \Lambda \)MDM models as functions of the parameters \( \Omega_\Lambda \), \( \Omega_H/\Omega_M \), and \( h \). Increasing \( \Omega_H/\Omega_M \) raises the needed value of \( \Omega_\Lambda \) (in agreement with the cluster
number density test). Changing $h$ does not significantly influence the $\Omega_\Lambda$ bounds.

Our investigation shows that cluster evolution test clearly indicate the existence of non-zero cosmological constant for a set of spatially flat AMDM models with negligible abundance of CGW. The presence of a small amount of hot particles like massive neutrino reduces the required value of the cosmological constant. For example, if massive neutrinos constitute only $\sim 10\%$ of the total DM density, the models with $\Omega_\Lambda \simeq 0.5 \pm 0.1$ and $h \simeq 0.65 \pm 0.05$ satisfy the observational data best.

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Figure 1: (a) The present day cluster abundance \(N(> T)\) (data from Henry and Arnaud 1991), and (b) the cluster evolution \(N(> M = 8 \cdot 10^{14} M_\odot, z)\) (data from Bahcall and Fan 1998), in MDM models with CGW, normalized as \(\sigma_8 = 0.52\), with \(n = 1.0\), \(h = 0.6\), \(\Omega_b = 0.015/h^2\), for \(\Omega_H = 0, 0.1, 0.2, 0.3\) (solid, dot, short-dash, long dash lines, resp.). The needed T/S is shown.

Figure 2: The cluster evolution \(N(> M = 8 \cdot 10^{14} M_\odot, z)\) in MDM models normalized as \(\sigma_8 = 0.52\) with \(\Omega_H = 0.2\), \(\Omega_b = 0.015/h^2\), for (a) \(n = 1\), \(h = 0.5, 0.6, 0.7\) (solid, dot, short-dash lines, resp.), and (b) \(h = 0.6\), \(n = 0.9, 1.0, 1.1\) (solid, dot, short-dash lines, resp.). The needed T/S is shown. The points correspond to Bahcall and Fan 1998.
Figure 3: The cluster evolution $N(> M = 8 \cdot 10^{14} M_\odot, z)$ in ΛCDM models normalized as $\sigma_8 = 0.52 \Omega_0^{0.52} \Omega_{M}^{0.13}$ with $\Omega_b = 0.015/h^2$, $\Omega_{\Lambda} = 0.7$, for (a) $n = 1$ and $h = 0.65, 0.70, 0.75$ (solid, dot, short-dash, long-dash lines, resp.), and (b) $h = 0.7$, and $n = 0.8, 0.9, 1.0, 1.1$ (solid, dot, short-dash, long-dash lines, resp.). The data points correspond to Bahcall and Fan 1998.

Figure 4: The present day cluster abundance $N(> T)$ in ΛMDM models normalized by COBE 4-year data with $h = 0.6$, $\Omega_M = 0.015/h^2$, $n = 1$, for $\Omega_{\Lambda} = 0.31, 0.45, 0.65, 0.74$ (solid, dot, short-dash, long-dash, dot-dash lines, resp.), and (a) $\Omega_H/\Omega_M = 0.1$, (b) $\Omega_H/\Omega_M = 0.2$. The points correspond to Henry and Arnaud 1991.
Figure 5: The cluster evolution \( N(> M = 8 \cdot 10^{14} M_\odot, z) \) in ΛMDM models normalized as \( \sigma_8 = 0.52 \Omega_M^{(-0.52+0.13\Omega)} \) with \( \Omega_b = 0.015/h^2 \), \( h = 0.6 \), \( n = 1 \), for \( \Omega_\Lambda = 0, 0.31, 0.45, 0.55, 0.65 \) (solid, dot, short-dash, dot-dash lines, long-dash, resp.), and (a) \( \Omega_H/\Omega_M = 0.1 \), (b) \( \Omega_H/\Omega_M = 0.2 \). The data points correspond to Bahcall and Fan 1998.

Figure 6: The cluster evolution \( N(> M = 8 \cdot 10^{14} M_\odot, z) \) in ΛMDM models normalized as \( \sigma_8 = 0.52 \Omega_M^{(-0.52+0.13\Omega)} \) with \( \Omega_b = 0.015/h^2 \), \( h = 0.6 \), \( n = 1 \), for (a) \( h = 0.6 \), \( \Omega_H/\Omega_M = 0.1 \), (b) \( h = 0.65 \), \( \Omega_H/\Omega_M = 0.1 \) (dot, short dash and long dash lines, resp.). The data points correspond to Bahcall and Fan 1998.
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\( \sigma_8 \) normalization: \( (\sigma_8)_{z=0} = 0.52 \ \Omega_M^{(-0.52 + 0.13 \Omega_w)} \)