Multiparticle entanglement for entanglement teleportation

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The scheme for entanglement teleportation is proposed to incorporate multipartite entanglement of four qubits as a quantum channel. Based on the invariance of entanglement teleportation under arbitrary two-qubit unitary transformation, we derive relations of separabilities for joint measurements at a sending station and for unitary operations at a receiving station. From the relations of separabilities it is found that an inseparable quantum channel always leads to a total teleportation of entanglement with an inseparable joint measurement and/or a nonlocal unitary operation.

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I. INTRODUCTION

Quantum teleportation is one of the most striking features emerging from quantum entanglement which is inherent in quantum mechanics [1]. Entangled systems divided into two parts enable to transfer the quantum information of an unknown quantum state to a remote place while the original state is destroyed. No information of the unknown state is ever revealed during teleportation process. Quantum teleportation has been especially interested in single body systems of two-level, N-dimensional, and continuous variable states [2, 3, 4, 5].

Entanglement teleportation is to transfer the entanglement initially imposed on an unknown multipartite state to a multipartite state at a remote place [6]. The entanglement is transferred onto the composite system of subsystems which have never directly interacted. In this sense, entanglement teleportation is similar to entanglement swapping [7]. However, entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure (entangled state itself). Entanglement teleportation of two qubits has recently been studied for pure and noisy quantum channels [8, 9, 10]. It is closely related to quantum computation as the two-qubit teleportation together with one-qubit unitary operations is sufficient to implement universal gates required for quantum computation [11].

In the earlier protocols for two-qubit teleportation, separate Einstein-Podolsky-Rosen (EPR) pairs are utilized for the quantum channel so that the joint measurement is decomposable into two independent Bell-state measurements and the unitary operation into two local one-qubit operations. This implies that entanglement teleportation can be implemented by a series of single-qubit teleportations [12, 13] which we call “a series teleportation of entanglement”. It is desirable to ask the following questions: whether a quantum channel is restricted only to EPR entanglement, if not, what other types of entanglement are possible, and how they play a role in entanglement teleportation. These questions have been addressed in part by employing Greenberger-Horne-Zeilinger (GHZ) entanglement of three and four qubits as a quantum channel [14, 15]. However, the investigations have been restricted thus far to partially unknown entangled states such as $a|01⟩ + b|10⟩$ and do not cover all possible states of a two-qubit system.

In this paper we consider entanglement teleportation of completely unknown entangled states such as

$$a|00⟩ + b|01⟩ + c|10⟩ + d|11⟩$$

where $a$, $b$, $c$, and $d$ are complex numbers and $\{|ij⟩\}$ is an orthonormal basis set. The present scheme is formulated so as to employ multipartite entanglement of four qubits as a quantum channel; the composite system of four qubits may have various types of entanglement, for example, two EPR pairs, four GHZ triads, etc. We show that entanglement teleportation has an invariance under arbitrary two-qubit unitary transformation and variant protocols are available. By the invariance of entanglement teleportation, we derive relations of separabilities for joint measurements at a sending station and for unitary operations at a receiving station. Due to the relations of separabilities, we show that an inseparable quantum channel always leads to a “total teleportation of entanglement”, which employs an inseparable joint measurement and/or a nonlocal unitary operation, as opposed to a series teleportation of entanglement.

II. TWO-QUBIT TELEPORTATION

In the original proposal [1], quantum teleportation utilizes an EPR pair as a quantum channel which is shared by a sender, Alice, and a receiver, Bob. After she receives
a particle in an unknown state and one of the entangled pair, Alice performs a joint measurement on their composite state. She transmits the outcome to Bob through a classical channel. Bob applies a suitable unitary operation on his particle of the entangled pair, which is chosen in accordance with the outcome of the joint measurement. The final state of Bob’s particle is completely equivalent to the original unknown state if the quantum channel is maximally entangled.

A completely unknown state of two qubits $U_1$ and $U_2$ to teleport can be represented by

$$|\phi_U\rangle_U = \sum_{i,j=0}^1 c_{ij}|i,j\rangle_U$$

where the subscript $U$ denotes the composite system of two qubits $U_1$ and $U_2$, $c_{ij}$ is a complex number and $\{|i,j\rangle_U\}$ is an orthonormal basis set; $|i,j\rangle_U = |i\rangle_{U_1} \otimes |j\rangle_{U_2}$ with the basis set $\{\{0\rangle_q,\{1\rangle_q\}$ of qubit $q$. Note that the unknown state in Eq. (3) is entangled unless the coefficient matrix $c_{ij}$ is decomposable such that $c_{ij} = d_i e_j$ for some complex vectors $d_i$ and $e_j$.

We consider a quantum channel of four qubits which are divided into two parts, i.e., two qubits are sent to Alice and the others to Bob as shown in Fig. 1. Alice’s two qubits $A_1$ and $A_2$ are denoted by $A$ and Bob’s two qubits $B_1$ and $B_2$ by $B$. A perfect teleportation requires that two parts of $A$ and $B$ be in a maximally entangled pure state, that is, the state $|\phi_c\rangle_{AB}$ of the quantum channel satisfies the relation,

$$\text{Tr}_B(A)\langle |\phi_c\rangle_{AB}|\phi_c\rangle = \frac{1}{4} |\mathbb{I}_{A(B)}\rangle,$$

where $\text{Tr}_i$ is a partial trace over subsystem $i$ and $\mathbb{I}_i$ an identity operator of part $i$. The channel state $|\phi_c\rangle_{AB}$ can be written by Schmidt decomposition as

$$|\phi_c\rangle_{AB} = \frac{1}{2} \sum_{i,j=0}^1 |\psi_{ij}\rangle_A \otimes |\varphi_{ij}\rangle_B,$$

where $|\psi_{ij}\rangle_A = \hat{U}|i,j\rangle_A$, $|\varphi_{ij}\rangle_B = \hat{V}|i,j\rangle_B$, and $\hat{U}$ and $\hat{V}$ are two-qubit unitary operators. Note that $|\psi_{ij}\rangle_A$ is an entangled state of Alice’s two qubits if the unitary operator $\hat{U}$ is nonlocal. Similarly, $|\varphi_{ij}\rangle_B$ is an entangled state of Bob’s two qubits if $\hat{V}$ is nonlocal.

The channel state $|\phi_c\rangle_{AB}$ in Eq. (4) can be represented in the more convenient form of

$$|\phi_c\rangle_{AB} = (\mathbb{I}_A \otimes \hat{V}\hat{U}^T)|\phi_c\rangle_{AB}$$

where

$$|\phi_c\rangle_{AB} = \frac{1}{2} \sum_{i,j=0}^1 |i,j\rangle_A \otimes |i,j\rangle_B.$$

The state $|\phi_c\rangle$ is also a maximally entangled state of the two parts $A$ and $B$. On the other hand, it is separable in $(A_1, B_1)$ and $(A_2, B_2)$ such that $|\phi_c\rangle_{AB} = |\text{EPR}\rangle_{A_1B_1} \otimes |\text{EPR}\rangle_{A_2B_2}$ where $|\text{EPR}\rangle = \sum_i |i,i\rangle / \sqrt{2}$. The state $|\phi_c\rangle$ has been used as a quantum channel for entanglement teleportation $[11, 13]$.

A pure generalized GHZ state of four qubits is defined by using generalized Schmidt decomposition as $[11]$

$$|\phi_4\rangle_{AB} = \sum_{i=0}^3 \lambda_i |\alpha_i\rangle_{A_1} \otimes |\beta_i\rangle_{A_2} \otimes |\gamma_i\rangle_{B_1} \otimes |\delta_i\rangle_{B_2}$$

where $\{\{\alpha_i\}, \{\beta_i\}, \{\gamma_i\}, \{\delta_i\}\}$ are orthonormal vector sets and $\lambda_i$’s are positive. A pure generalized GHZ state is not a good candidate for a quantum channel because it does not fulfill the requirement $[10]$ of maximal entanglement of two parts $A$ and $B$. More explicitly,

$$\text{Tr}_B(|\phi_4\rangle_{AB}\langle\phi_4|) = \sum_{i=0}^3 |\lambda_i|^2 |\alpha_i,\beta_i\rangle_A \langle\alpha_i,\beta_i|,$$

which is not proportional to $\mathbb{I}_A$. In fact it is proportional to a projector which projects a state into a subspace spanned by $\{\{\alpha_i\},\beta_i\}_A$. We note however that a single-qubit teleportation can be performed via a quantum channel of three qubits which is in a maximal GHZ state $[12]$ and thus there could be a possibility that a GHZ state of more than two times of the teleporting qubits may lead to a perfect teleportation.

Alice performs a joint measurement on the four qubits, $A_1$, $A_2$, $U_1$, and $U_2$. The joint measurement is constructed using a set of sixteen projectors $\{\hat{M}_{\alpha\beta} = |\mu_{\alpha\beta}\rangle\langle\mu_{\alpha\beta}|\}$ where

$$|\mu_{\alpha\beta}\rangle\langle\mu_{\alpha\beta}| = (\mathbb{I}_A \otimes \hat{U}_{\alpha\beta}) |\phi_c\rangle_{AU}.$$

Here $|\phi_c\rangle_{AU}$ is the same as the state given in Eq. (3) and $\hat{U}_{\alpha\beta} = \hat{\sigma}_\alpha \otimes \hat{\sigma}_\beta$ is a local unitary operator with Pauli spin operators $\hat{\sigma}_\alpha = \mathbb{I}_z, \hat{\sigma}_x, \hat{\sigma}_y$, and $\hat{\sigma}_z$. The set $\{\hat{M}_{\alpha\beta}\}$ satisfies a completeness relation as

$$\sum_{\alpha,\beta=1}^4 \hat{M}_{\alpha\beta} = \mathbb{I}_A \otimes \mathbb{I}_U.$$

Further the sixteen projectors are orthogonal such that

$$\hat{M}_{\alpha\beta}\hat{M}_{\alpha'\beta'} = \text{Tr} (\hat{U}_{\alpha\beta}^\dagger \hat{U}_{\alpha'\beta'}^\dagger \mathbb{I}_\gamma) |\mu_{\gamma\delta}\rangle\langle\mu_{\gamma\delta}| = \delta_{\alpha\alpha'}\delta_{\beta\beta'}\hat{M}_{\alpha\beta}\mathbb{I}_{\gamma\delta} \hat{M}_{\alpha\beta} \mathbb{I}_{\gamma\delta} \hat{M}_{\alpha\beta}.\] (11)
This implies that the joint measurement represented by 
\{M_{\alpha\beta}\} is an orthogonal measurement on the composite 
system of A and U.

A key step is to evaluate a partial inner product 
\[ AU|\alpha\beta\rangle |\phi_c\rangle AB = \sum_{i,j} |i,j\rangle U(i,j) \] on the right side:
\[
AU|\alpha\beta\rangle |\phi_c\rangle AB = \frac{1}{4} \hat{T}_{BU} \hat{U}_{\alpha\beta} \hat{T}_{BU}, \tag{12}
\]
where \( \hat{T}_{BU} = \sum_{i,j} |i,j\rangle \langle i,j|_{BU} \) is a transfer operator 
from a state of U to that of B such that \( \hat{T}_{BU} |\phi\rangle U = |\phi\rangle B \).
The form of \( \hat{T}_{BU} \) plays a crucial role in revealing an 
invariance of entanglement teleportation which will be discussed 
in the next section.

The state \(|\Psi\rangle_{UAB}\) of the whole composite system of U, 
A, and B can be represented with respect to the basis set 
\{\{M_{\alpha\beta}\}_{AU}\} of the joint measurement as follows
\[
|\Psi\rangle_{UAB} = |\phi_u\rangle U \otimes |\phi_c\rangle AB
= \left( \sum_{\alpha,\beta=1}^4 M_{\alpha\beta} \right) |\phi_c\rangle AB \otimes |\phi_u\rangle U
= \frac{1}{4} \sum_{\alpha,\beta=1}^4 |\mu_{\alpha\beta}\rangle_{AU} \otimes U_{\alpha\beta} \hat{T}_{BU} |\phi_u\rangle U. \tag{13}
\]
Suppose Alice obtains an outcome \((\alpha, \beta)\) when she per-
forms the joint measurement on the composite system 
of A and U. Bob’s two qubits come to be in the state 
of \( |\phi_{UAB}\rangle = U_{\alpha\beta} |\phi_c\rangle AB \). When he receives through a classical 
communication the four-bit message concerning the outcome 
\((\alpha, \beta)\), Bob applies the corresponding unitary operation 
\( \hat{U}_{\alpha\beta} \) on his qubits, which completes the two-qubit 
teleportation process.

### III. RELATIONS OF SEPARABILITIES FOR JOINT MEASUREMENTS AND FOR UNITARY OPERATIONS

In the proposed protocol of two-qubit teleportation, we 
employed an orthogonal measurement for the joint mea-
surement. We may consider a positive operator valued 
measurement for a joint measurement, such that for a set 
of unitary operators \( \{\hat{U}_g\} \) with the order 
\[
\frac{1}{G} \sum_{g=1}^G \hat{U}_g |\phi\rangle AU \langle \phi| \hat{U}_g^\dagger = \frac{1}{4^2} \hat{U}_{A} \otimes \hat{U}_{U} \tag{14}
\]
where \(|\phi\rangle_{AU}\) is a maximally entangled state of A and U.
This type of a positive operator valued measurement was 
studied for universal teleportation \[13\]. If \( \hat{U}_g = \hat{\sigma}_k \otimes \hat{\sigma}_\beta \), 
this measurement is simply equal to the orthogonal joint 
measurement represented by the bases in Eq. \[14\].

We shall show an invariance of entanglement telepor-
tation under arbitrary two-qubit unitary transformation. 
For a maximally entangled state \(|\phi\rangle\) of two parts, let 
\(|\mu_{\alpha\beta}\rangle_{AU} = \hat{A}_{\alpha} \otimes \hat{U}_g |\phi\rangle_{AU} \} be a set of joint measurement 
bases and \(|\phi_{\alpha\beta}\rangle_{AB} = \hat{A}_{\alpha} \otimes \hat{U}_g |\phi\rangle_{AB} \} be a set of unitarily 
transformed channel states. The partial inner product of 
\(|\mu_{\alpha\beta}\rangle_{AU} and \{|\phi_{\alpha\beta}\rangle_{AB}\} is obtained as 
\[
AU|\alpha\beta\rangle |\phi_{\alpha\beta}\rangle AB = \frac{1}{4} \hat{U}_g \hat{U}_{\alpha\beta} \hat{T}_{BU} \tag{15}
\]
when \( g = g' \), this is just a transfer operator. The tele-
portation is completely specified by \( G \) pairs of joint mea-
surement bases and their corresponding channel states, 
\( \{|\mu_{\alpha\beta}\rangle, |\phi_{\alpha\beta}\rangle\} \). The partial inner product in Eq. \[15\] is 
invariant under the transformation of
\[
|\mu_{\alpha\beta}\rangle_{AU} \rightarrow \hat{W}_r^T \otimes \hat{W}_l |\mu_{\alpha\beta}\rangle_{AU} \tag{16}
\]
and
\[
|\phi_{\alpha\beta}\rangle_{AB} \rightarrow \hat{W}_r^T \otimes \hat{W}_l |\phi_{\alpha\beta}\rangle_{AB}. \tag{17}
\]
for each \( g \) with some two-qubit unitary operators \( \hat{W}_l \) and 
\( \hat{W}_r \). Thus one may have variant protocols of entan-
glement teleportation under the transformation in Eqs. \[16\] 
and \[17\], due to the arbitrariness of \( \hat{W}_l \) and \( \hat{W}_r \). We 
ote that the invariance of entanglement teleportation 
may be extended further with respect to a rather 
general completely positive operation \[14\].

The invariance of entanglement teleportation raises re-
lations of separabilities for joint measurements and for 
unitary operations. In particular, an inseparable joint 
measurement may be transformed into two independent 
Bell-state measurements and/or a nonlocal unitary op-
eration into a local operation. A joint measurement is 
said to be separable when each measurement basis can 
be decomposed into a product state of either \((A_1, U_1)\) and 
\((A_2, U_2)\) or \((A_1, U_2)\) and \((A_2, U_1)\). Further a protocol of 
entanglement teleportation is called a series teleportation 
of entanglement when its joint measurement is separable 
and the corresponding unitary operation is local. The 
series teleportation of entanglement consists of indepen-
dent Bell-state measurements and local unitary operations \[13\]. 
Otherwise, it is called a total teleportation of 
entanglement in the sense that it is not decomposable 
into a series of single-qubit teleportation \[13\].

In Sec. \[14\] we presented a protocol of total teleportation 
of entanglement with an inseparable joint measure-
ment and a local unitary operation when the quantum 
channel state in Eq. \[14\] is inseparable. One may try to 
construct a series teleportation of entanglement by us-
ing the invariance of entanglement teleportation under the 
transformation of Eqs. \[16\] and \[17\]. Suppose that a 
joint measurement becomes separable in \((A_1, U_1)\) and 
\((A_2, U_2)\) for some \( \hat{W}_l \) and \( \hat{W}_r \) such that
\[
|\mu_{\alpha\beta}\rangle_{AU} \rightarrow \hat{U}_{\alpha\beta} |\phi_{\alpha\beta}\rangle_{AU} = \hat{A}_{\alpha} \otimes \hat{U}_g |\phi_{\alpha\beta}\rangle_{AU} \tag{18}
\]
where \( \hat{U}_{\alpha\beta} = \hat{W}_l \hat{U}_{\alpha\beta} \hat{V}^T \hat{W}_r = \hat{\sigma}_\alpha \otimes \hat{\sigma}_\beta \). Then, the 
corresponding unitary operators are transformed as
\[
\hat{U}_{\alpha\beta} \rightarrow \hat{U}_{\alpha\beta} (\hat{V}^T)^\dagger. \tag{19}
\]
The transformed unitary operators are clearly nonlocal since $\hat{V}\hat{U}^T$ is nonlocal due to the inseparability of $|\phi_c\rangle$. The new protocol consists of the separable joint measurement and the nonlocal unitary operation, which is the opposite case to the untransformed protocol of the inseparable joint measurement and local unitary operation. However, the altered protocol is a total teleportation as well. An inseparable quantum channel always leads to a total teleportation of entanglement.

It is possible to obtain two EPR pairs by applying some two-qubit unitary operation to an inseparable quantum channel, which enable a series teleportation of entanglement with rather simple Bell-state measurement and local unitary operation. Unless a quantum channel is likely to suffer from a reservoir, it may be the simplest protocol that employs EPR pairs as a quantum channel. However, when a reservoir is present, it is important to study inseparable quantum channels because some inseparable channel can be robust against the decoherence than EPR pairs. It is known that some particular state is robust against the decoherence once the interaction with a reservoir is known. For example, the decoherence-free state, an eigenstate with zero eigenvalue of the interaction Hamiltonian, is never decohered in the given reservoir. We will not further discuss the effects of the decoherence, which is beyond the scope of this paper.

### IV. MANY-QUBIT ENTANGLEMENT OF INSEPARABLE QUANTUM CHANNEL

Any quantum channel in a maximally entangled state of two parts $A$ and $B$ can be employed for a perfect teleportation of entanglement. Entanglement of four qubits may be classified into two-qubit entanglement, tree-qubit entanglement, and four-qubit entanglement. A state of four qubits is said to have two-qubit entanglement when some two qubits among the four qubits are in an entangled state, three-qubit entanglement when some three qubits are in a three-qubit GHZ state, and four-qubit entanglement when the four qubits are in a four-qubit GHZ state. Note that W-class states and biseparable states belong to two-qubit entanglement by our definition. As shown in Sec. II a four-qubit GHZ state is not a good candidate for a perfect teleportation of entanglement.

The entanglement structure of a possible quantum channel state depends on two-qubit unitary operator $\hat{V}\hat{U}^T$. A quantum channel of two EPR pairs is in the state $|\phi_c\rangle_{AB}$, which is separable in $(A_1, B_1)$ and $(A_2, B_2)$, and it has only two-qubit entanglement. We shall present an example of an inseparable quantum channel which has many-qubit entanglement; the channel state is written as

$$|\phi_c\rangle_{AB} = \frac{1}{\sqrt{2}} \left( |0000\rangle - |0111\rangle + |0101\rangle - |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle \right)_{A_1A_2B_1B_2}$$

This state is obtained from Eq. (1) with the two-qubit unitary operator,

$$\hat{V}\hat{U}^T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix},$$

which is represented in a product basis set of $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. The operator $\hat{V}\hat{U}^T$ transforms the product bases to Bell bases.

The reduced density operator of each qubit $i$ is proportional to an identity operator $\rho_i = \mathbb{1}/2$. Noting $|\phi_c\rangle$ is pure, this implies that $|\phi_c\rangle$ has no individual information but it contains entanglement of a given qubit and the rest.

To investigate two-qubit entanglement, we employ the Peres-Horodecki criterion for two qubits that their density operator $\rho$ is entangled if and only if its partial transposition has any negative eigenvalue. The partial transposition of $\rho$ is defined as

$$\rho^T_{ij} = \sum_{ijkl} \rho_{ijkl} |i\rangle \langle j| \otimes |k\rangle \langle l|$$

when $\rho = \sum_{ijkl} \rho_{ijkl} |i\rangle \langle j| \otimes |k\rangle \langle l|$. As an example, consider a reduced density operator of a pair among four qubits which is in a symmetric W-state,

$$|W\rangle = \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle).$$

The partial transposition of the reduced density operator has a negative eigenvalue $(1 - \sqrt{2})/4$ for all pairs.

We shall show below that all pairs, which can be selected out of four qubits in the state $|\phi_c\rangle$, are in separable states. Every pair $(i, j)$ except $(A_1, B_1)$ and $(A_2, B_2)$ has the reduced density operator $\rho_{ij} = \mathbb{1}/2$ and it is disentangled. In addition, the reduced density operator of $(A_1, B_1)$ or $(A_2, B_2)$ is given as

$$\rho_{A_1B_1}(A_2B_2) = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$ 

The partial transposition of $\rho_{A_1B_1}(A_2B_2)$ has only positive eigenvalues of $(0, 0, 1/2, 1/2)$. These results imply that the state $|\phi_c\rangle$ in Eq. (23) has no two-qubit entanglement. However, the state $|\phi_c\rangle$ is entangled as shown in the consideration of the reduced density operators for single qubits and it has three-qubit entanglement.

A reduced density operator $\rho$ of each triad is obtained by tracing over the other qubit and it is in the form of

$$\rho = \frac{1}{2} |\phi_0\rangle \langle \phi_0| + \frac{1}{2} |\phi_1\rangle \langle \phi_1|$$

where $|\phi_0\rangle = |000\rangle + \lambda_1 |011\rangle + |101\rangle + |110\rangle$ and $|\phi_1\rangle = \lambda_3 |001\rangle + \lambda_4 |010\rangle + |100\rangle + |111\rangle$ with $\lambda_i$ given in Tab. I. By generalized Schmidt decomposition it is found

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1.$$
TABLE I: Amplitudes $\lambda_i$ of two orthogonal GHZ states $|\phi_0\rangle$ and $|\phi_1\rangle$ in Eq. (25).

| triad    | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|----------|--------------|--------------|--------------|--------------|
| $(A_1, A_2, B_1)$ | $-1$         | $1$          | $1$          | $1$          |
| $(A_1, A_2, B_2)$ | $1$          | $1$          | $-1$         | $-1$         |
| $(A_1, B_1, B_2)$ | $-1$         | $1$          | $1$          | $-1$         |
| $(A_2, B_1, B_2)$ | $-1$         | $-1$         | $1$          | $1$          |

that both $|\phi_0\rangle$ and $|\phi_1\rangle$ are maximal three-qubit GHZ states.

To investigate three-qubit entanglement explicitly, one may employ an entanglement witness scheme that a density operator of three qubits $\rho$ has three-qubit entanglement if $\text{Tr}(W\rho) < 0$ for some three-qubit GHZ entanglement witness $W$. However, it is nontrivial to find such an entanglement witness for a given density operator while a typical entanglement witness is known as

$$W = \frac{3}{4} I - |\phi\rangle \langle \phi|$$  \hspace{1cm} (26)

where $|\phi\rangle = \frac{1}{\sqrt{3}} \sum_{i=0}^{1} |\alpha_i, \beta_i, \gamma_i\rangle$ is a maximal three-qubit GHZ state. We perform numerical calculations with steepest decent method to search some local trilateral rotation for the typical witness (26) to minimize $\text{Tr}(W\rho)$ and we find $\text{Tr}(W\rho) \geq 1/4$. It implies that the typical entanglement witness can not detect three-qubit entanglement of triads.

V. REMARKS

We proposed the scheme for entanglement teleportation of a completely unknown state so as to incorporate a multipartite entangled state as the quantum channel. Deriving the relations of separabilities for joint measurements and for corresponding unitary operations, it was found that an inseparable quantum channel always leads to a total teleportation of entanglement. We gave the example of inseparable quantum channel with each triad in three-qubit entanglement.

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