Testing $\nu$MSM with indirect searches

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Abstract

We consider neutrino Minimal extension of the Standard Model ($\nu$MSM), which by introducing only three sterile neutrinos in sub electroweak region can explain active neutrino oscillations (via seesaw type I mechanism), baryon asymmetry of the Universe (leptogenesis via oscillations) and dark matter phenomena (with keV-scale sterile neutrino forming dark matter). We estimate sterile neutrino virtual contributions to various lepton flavor and lepton number violating processes. The contributions are too small, giving no chance for indirect searches to compete with direct measurements in exploring $\nu$MSM.

Introduction. The Standard Model of elementary particle physics (SM) is an extremely successful model, which has passed countless precision tests, and whose main predictions have been confirmed. Its last but crucial missing ingredient, required for the theory to be consistent — the Higgs boson — was discovered at the LHC in 2012 [1, 2].

The neutrino flavor oscillations — transitions between neutrinos of different flavors (see e.g. [3] review) — are among the few firmly established phenomena beyond the Standard Model. The direct coupling of the neutrino species (in the form $\bar{\nu}_\alpha \nu_\beta$) is prohibited by the SU(2) gauge symmetry. The oscillation phenomena can be described by a non-renormalizable operator of “dimension 5”:

$$\mathcal{L}_{\text{osc}} = \mathcal{L}_{\text{SM}} + c_{\alpha\beta} \frac{(\bar{L}_\alpha \cdot \tilde{\Phi})(L^C_\beta \cdot \tilde{\Phi}^*)}{\Lambda},$$

where $L_\alpha$ are left SU(2) doublets of leptons of different flavors, $\alpha = e, \mu, \tau$, subscript $C$ refers to the charge conjugation, $L^C_\alpha = i\sigma^2L^*_\alpha$ and $\Phi$ is the Higgs SU(2) doublet, $\Phi_i = \epsilon_{ij}\Phi^*_j$; $c_{\alpha\beta}$ is a dimensionless $3 \times 3$ matrix. If some of its elements are $\mathcal{O}(1)$, the scale of new physics where the interaction [1] must

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We use notations where $\nu_\alpha = \{\nu_e, \nu_\mu, \nu_\tau\}$ are neutrinos interacting with $W$-boson, i.e. weak charge eigenstates.
be replaced with a renormalizable model is $\Lambda \sim v^2/m_{\text{atm}} \sim 10^{15}$ GeV, where $v = \sqrt{2\langle \Phi \rangle} = 246$ GeV.

The neutrino mixing term (1) can be mediated via exchange of some new particles (for example: gauge singlet fermions; a scalar in an adjoint representation of the SU(2) gauge group; a fermion in the adjoint representation of the SU(2); etc., see [3] for a review) interacting with left lepton doublet $L_\alpha$ and with the Higgs doublet $\Phi$. A traditional explanation of the smallness of neutrino masses ($m_{\text{atm}} \ll m_{\text{electron}}$) is provided by the see-saw mechanism. The small number is the ratio of the Dirac mass of the neutrino to the mass of the new particle. The see-saw mechanism does not predict, however, the mass of the new particles that can have any value.

If the new particles carry quantum number of the SU(2) gauge group, their non-detection means that they should be heavier than the reach of modern accelerators. The situation is different, however, if the term (1) is mediated via exchange of the gauge-singlet fermion (the so-called Type-I see-saw mechanism [4–7], see [8] and refs. therein). In this case the SM Lagrangian $L_{\text{SM}}$ is extended by introducing $N$ right-handed fermions $N_I, I = 1, \ldots, N$:

$$L = L_{\text{SM}} + i N_I \gamma^\mu \partial_\mu N_I - \left( F_{\alpha I} \bar{L}_\alpha \Psi N_I - \frac{M_I}{2} \bar{N}_I^C N_I + \text{h.c.} \right),$$

(2)

where $F_{\alpha I}$ are new Yukawa couplings. The Yukawa interaction terms dictate the SM charges of the right-handed particles: they turn out to carry no electric, weak and strong charges; therefore they are often termed “singlet fermions” or “sterile neutrinos”. Sterile neutrinos can thus have Majorana masses, $M_I$, consistent with the gauge symmetries of the SM. The number of these singlet fermions must be $N \geq 2$ to explain the data on “active” neutrino oscillations. In the case of $N = 2$ there are 11 new parameters in the Lagrangian (2), while the neutrino mass/mixing matrix $M_\nu$ has 7 parameters in this case (including two active neutrino masses, one of the three active neutrinos remains massless). The situation is even more relaxed for $N > 2$.

Generically the remaining free parameters $F_{\alpha I}$ may take any values (consistent with perturbativity), including those which make sterile neutrinos even more valuable in particle physics. Indeed, the lightest sterile neutrino can serve as a dark matter candidate, while violating lepton number Majorana mass term and CP-violating complex phases of Yukawa matrix in (2) provide necessary conditions for leptogenesis via active-sterile neutrino oscillations in the early Universe. If happened before the electroweak phase transition, it can yield the baryon asymmetry of the plasma thus explaining the matter-antimatter asymmetry in the later Universe. Successful solution of both problems (dark matter and baryon asymmetry of the Universe) requires $N \geq 3$ sterile neutrinos.

$\nu$MSM: description and phenomenology. The most economic model of this type, which recruits $N = 3$ sterile neutrinos, is known as $\nu$MSM [9,10] that is \textit{neutrino Minimal extension of the Standard Model}. It turns out that sterile neutrino dark matter particle mass is confined in $M_1 \sim 1 \text{ keV} - 50 \text{ keV}$ region, see [11] for review. The requirement of dark matter stability on cosmological timescales
makes its coupling with the SM species so feeble, that it does not contribute significantly to the neutrino oscillation pattern \[12\]. In this sense the \(\nu\)MSM setup is close to that of \(N = 2\) seesaw scheme: one of the active neutrino is (almost) massless. The two heavier sterile neutrinos are responsible for both active neutrino masses and baryon asymmetry of the Universe (BAU). Actually, to produce enough lepton asymmetry before the electroweak phase transition and yet not equilibrate sterile neutrino in the primordial plasma one must resort to a resonance enhancement of active-sterile oscillations in the early Universe \[10, 13\], achieved with almost degenerate neutrinos,

\[
|M_3 - M_2| \equiv \Delta M \ll M_{2,3}.
\]  

Moreover, the particular mechanism of the leptogenesis works for sterile neutrinos below the electroweak scale, though an exact upper limit on the sterile neutrino mass has not been settled yet.

In all the relevant dynamics the active-sterile mixing rising from Yukawa terms in (2) plays the major role. Fermions \(N_I\) in (2) are charge eigenstates of weak interactions and they are truly neutral. However, due to the Yukawa mixing the mass eigenstates have small admixture of \(\nu_\alpha\) characterized by small active-sterile mixing angle

\[
U_{\alpha I} \equiv \frac{v F_{\alpha I}}{\sqrt{2} M_I},
\]

\(|U_{\alpha I}| \ll 1\), and as a result carry small weak charge. For a given sterile neutrino its smallness is characterized by the following dimensionless number:

\[
U_I^2 \equiv \sum_\alpha \frac{v^2 |F_{\alpha I}|^2}{2 M_I^2}.
\]

This means that from a phenomenological point of view particles \(N_I\) behave like heavy neutral leptons. In what follows we denote the heavy mass eigenstates in the same way as the charge eigenstates, \(N_I\), because in \(\nu\)MSM the admixture of \(\nu_\alpha\) is very small indeed. These particles can be created in decays of other particles instead of the usual neutrino \(\nu_\alpha\) (if kinematics allows it), and in turn decay into the SM particles, see e.g. Fig. [1]. Both production and decay rates are proportional to the squared mixing angles \([4]\): the smaller the mixing the lower the rates. Based on this phenomenology, a number of searches of heavy neutral leptons have been performed in the past, yielding exclusion regions in the \((U_{\alpha I}, M_I)\) parameter space, for review see e.g. \([14]\).

In \(\nu\)MSM with almost decoupled dark matter neutrino and degenerate two heavy sterile neutrinos the direct searches place limits on the flavor mixing angles

\[
U_\alpha^2 \equiv \sum_{I=2,3} \frac{v^2 |F_{\alpha I}|^2}{M_I^2},
\]

while cosmology constrains mostly the sum of the angles \([6]\) over all relevant flavors:

\[
U^2 = \sum_{I=2,3} U_I^2 = \sum_{\alpha=e,\mu,\tau} U_\alpha^2,
\]
that is the overall mixing strength. Sterile neutrinos produced in the early Universe can decay and destroy the primordial chemical elements. For successful Big Bang Nucleosynthesis (BBN) the lifetime $\tau_N$ of heavy sterile neutrinos $N_{2,3}$ is restricted to be shorter than $0.1 \text{s}$ \cite{15}. This restriction yields a lower bound on overall mixing $U^2$ \cite{16}. Then, successful generation of the lepton asymmetry asks for the sterile neutrinos to be out-of-equilibrium in the plasma, which places an upper limit on the mixing \cite{17}

$$U^2 < \kappa \times 10^{-6} \left( \frac{\text{GeV}}{M_N} \right)^2,$$

where $\kappa = 1(2)$ refers to the normal(inverted) hierarchy in active neutrino sector. Leptogenesis gives also lower limits on mixing referring to the minimal connection between active and sterile neutrino sectors which still provides enough baryon asymmetry in the end. Finally, a lower bound on mixing \cite{16}

$$U^2 > 5 \kappa \times 10^{-11} \frac{\text{GeV}}{M_N}$$

is inherited in the seesaw mechanism: at a given sterile neutrino masses the mixing can not be arbitrary small, as it determines the active neutrino masses, and two of them must exceed 0.05 eV and 0.008 eV to be consistent with atmospheric and solar neutrino oscillation data \cite{18}, respectively. The aforementioned constraints are presented in Fig.2 There are also limits on the model parameter space associated with dark matter, however they are largely depend on the mechanism of dark matter production in the early Universe (for working examples and discussion see e.g. \cite{17, 21, 24}), hence we ignore them for the sake of generality.

The best way to explore the viable region of model parameter space outlined in Fig.2 is direct searches for sterile neutrino production and decays, see
Figure 2: Viable region of \( \nu \)MSM parameter space for the cases of normal (left panel) and inverted (right panel) hierarchies in active neutrino sector, adopted from [19]. Upper limits are from successful baryogenesis (BAU) and from direct searches (PS191, CHARM, NuTeV, and very recent E949 [20]), lower limits are from seesaw mechanism (seesaw), from Big Bang Nucleosynthesis (BBN) and from successful baryogenesis (BAU). There is also an unnamed line indicating a possible sensitivity of a dedicated fixed target experiment proposed in [19].

[16] for details. For light neutrino masses, \( M < 5 \) GeV, these searches can be performed with a fixed target experiment operating on high energy and high intensity proton beam, which produces heavy hadrons on target and the latter decay into sterile neutrinos as shown in Fig. 1. The suitable experimental setup was sketched in Ref. [19] and evolved to the proposal of a new beam dump experiment at CERN [25], later called SHiP. For heavier mass the only option is high-energy colliders, and \( e^+e^- \) machines producing many billions of \( Z \)-bosons (which immediately decaying into sterile neutrinos) are advocated [26] to be very sensitive to \( \nu \)MSM.

At the same time, \( \nu \)MSM can be tested indirectly due to virtual sterile neutrino contribution to particular processes with lepton number or lepton flavor violation. In this Letter we obtain the \( \nu \)MSM predictions for the bunch of relevant processes. As we found, the expected rates, except neutrinoless double \( \beta \)-decay, are so tiny, that they leave no chance at all to find any hint of \( \nu \)MSM physics indirectly. These findings confirm that the direct searches are not only superior but the only realistic way to explore the model.

Even with the seesaw, BBN and BAU constraints satisfied, there is a lot of freedom in relations between different Yukawa couplings \( F_{\alpha I} \). To present quantitative predictions three different sets of couplings were considered in [16]. These “extreme models” are formulated in such a way that coupling to a single active neutrino flavor dominates:

- model I \( : |U_{eI}|^2 : |U_{\mu I}|^2 : |U_{\tau I}|^2 \approx 52 : 1 : 1 \), \( \kappa = 2 \),
- model II \( : |U_{eI}|^2 : |U_{\mu I}|^2 : |U_{\tau I}|^2 \approx 1 : 16 : 3.8 \), \( \kappa = 1 \),
- model III \( : |U_{eI}|^2 : |U_{\mu I}|^2 : |U_{\tau I}|^2 \approx 0.061 : 1 : 4.3 \), \( \kappa = 1 \).

\(^2\)Which is almost always (except narrow mass interval \( M_N \approx 150 \) MeV \(-250 \) MeV) saturated by two active massive neutrinos, and hence insensitive to the model.
here and below dark matter \( N_1 \) is ignored and index \( I \) runs through 2 and 3 only.

Recall that with effective framework of 2 sterile neutrinos relevant for \( \nu MS\)M the type I seesaw model brings 11 extra parameters while neutrino oscillation data can fix at most 7 (one eigenstate is massless then). As a result, 4 parameters remain free, including one CP-violating phase in the active-sterile Yukawa matrix. For some special values of this phase accidental cancellations of sterile neutrino contribution in certain processes are possible, see examples in [27–29]. We did not consider this effect hereafter, as it can only make the effects of sterile neutrinos weaker.

We start with lepton flavor violating processes.

\[ l \rightarrow l' \gamma. \] Decay rate of \( l \rightarrow l' \gamma \) is given by (c.f. [14]):

\[
\Gamma(l \rightarrow l' \gamma) = \frac{3 \alpha G_F^2 m_l^3}{8 \pi 192 \pi^3} \left| \sum_I U_{\alpha I} U_{\mu I} g \left( \frac{M_{N_I}^2}{m_W} \right) \right|^2,
\]

where \( G_F \) is the Fermi constant, \( \alpha \) is the fine structure constant, \( m_W \) is the mass of \( W \)-boson and

\[
g(x) = \frac{x(1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x)}{2(1 - x)^4}.
\]

Inserting the limits on \( U_{\alpha f}(M_N) \) discussed above [8], [9] into eq. (10) one can compare the expected in \( \nu MS\)M branching ratios of \( l \rightarrow l' \gamma \) decays with existing experimental limits. We present the result for \( \mu \rightarrow e \gamma \) on the left top panel in Fig. 3. Solid line shows the experimental limit [18] \( \text{Br}(\mu \rightarrow e \gamma)^{\exp} < 5.7 \times 10^{-13} \). One can see that it is much less stringent than cosmological and seesaw constraints. Experimental limits for two other relevant processes are even weaker [18], \( \text{Br}(\tau \rightarrow e \gamma)^{\exp} < 3.3 \times 10^{-8} \) and \( \text{Br}(\tau \rightarrow \mu \gamma)^{\exp} < 4.4 \times 10^{-8} \), while the \( \nu MS\)M predictions are lower.

\( \mu - e \) conversion. Another potentially interesting process is \( \mu - e \) conversion in nuclei. As was shown in [30] for the quasi-degenerate spectrum of sterile neutrinos, which is the case for \( \nu MS\)M, the ratio

\[
R_{\mu e}^{\mu - e \gamma} = \frac{R(\mu Z - e Z)}{\text{Br}(\mu \rightarrow e \gamma)},
\]

where \( R(\mu Z - e Z) \) is the conversion rate in the nuclei of electric charge \( Z \), could be sufficiently large, \( \sim 10^5 \), for light, \( M_N < 10 \text{ GeV} \), sterile neutrinos. This fact and expecting experimental progress [31] make \( \mu - e \) conversion much more promising in testing \( \nu MS\)M as compared to other lepton flavor violating processes. However, the \( \nu MS\)M predictions are still some 5 orders of magnitude below the sensitivity of future experiments, that is in agreement with Ref. [32]. The results based on calculations performed in Ref. [30] are presented on left bottom panel in Fig. 3.
Figure 3: Decay branching ratios $\text{Br}(\mu \rightarrow e\gamma)$ (top left panel), $\text{Br}(\mu \rightarrow eee)$ (top right panel), conversion rate $R(\mu Ti \rightarrow eTi)$ (bottom left panel) and neutrino transition dipole moment $\mu_{tr}$ (bottom right panel) as functions of the sterile neutrino mass $M_N$ in the model I (which exhibits the highest rate): white region is allowed. The region above solid line is excluded by experimental searches.

$l \rightarrow l' l'' l'''$. Branching ratio of the process $l \rightarrow l' l'' l'''$ was calculated in \[33\]. Present experimental limit on the branching ratio of $\mu^- \rightarrow e^- e^+ e^-$ is \[18\] $\text{Br}(\mu^- \rightarrow e^- e^+ e^-)_{\text{exp}} < 1.0 \times 10^{-12}$. Predictions of $\nu$MSM for the branching ratio of this process are shown on right top panel in Fig.3. Present experimental limits on similar $\tau$-lepton decays are typically weaker by four orders of magnitude (i.e. $\text{Br}(\tau^- \rightarrow e^- \mu^+ e^-)_{\text{exp}} < 1.5 \times 10^{-8}$), while $\nu$MSM predictions are lower, than for the muon decay.

We proceed with lepton number violating processes relevant for probing $\nu$MSM. There are three related types of these processes: neutrinoless double beta decay, decays of $\tau$-lepton and meson decays into the same-sign leptons.
Neutrinoless double beta decay. The process of neutrinoless double beta decay is on the keen interest both from the experimental and theoretical points of view since it is naturally dominated by active neutrinos. The effective mass $m_{\beta\beta}$ standing in front of the decay amplitude depends on 7 out of the 9 parameters of active neutrino sector:

$$m_{\beta\beta}^\nu = \left| \sum_i m_i U_{ei}^2 \right|,$$

where $m_i$ are active neutrino masses and $U_{ei}$ are elements of the Pontecorvo–Maki–Nakagawa–Sakata matrix describing active neutrino mixing. As we have already explained, in the $\nu$MSM model the lightest active neutrino is nearly massless: $m_1 < 10^{-5}$ eV [9]. This fact leads to specific bounds on the effective mass since existing limits depend on the mass of the lightest neutrino [34].

Sterile neutrinos themselves also contribute to (12). As shown in [28] their contribution to the effective neutrino mass is destructive:

$$m_{\beta\beta} = [1 - f(M_N)] m_{\beta\beta}^\nu,$$

where function $f(M_N)$ describes the resulting suppression of the decay amplitude, which depends on the sterile neutrino mass. Following analysis in [28] we assume that $f(M_N) = 1$ for $M_N < \Lambda_\beta$ and for heavier sterile neutrinos it decreases as $f(M_N) = (\Lambda_\beta/M_N)^2$, where the typical energy scale of the process is $\Lambda_\beta = 100$ MeV. Constraints on the effective mass $m_{\beta\beta}$ as function of the mass of heavier sterile neutrinos are shown in Fig. 4. At large sterile neutrino masses and for the present central values of the known active neutrino sector parameters the allowed intervals in case of normal (NH) and inverted hierarchies (IH) are

$$1.5 \text{ meV} < m_{\beta\beta}^{\text{NH}} < 3.9 \text{ meV,} \quad 17 \text{ meV} < m_{\beta\beta}^{\text{IH}} < 49 \text{ meV}.$$  

If neutrinoless double beta decay will be found with the rate corresponding to the effective mass above the upper bound in (14) it would imply additional (to $\nu$MSM) new physics in the neutrino sector.
Decays into the same sign pairs. Branching ratios for lepton number violating $\tau$-lepton decays (e.g. $\text{Br}(\tau^- \to e^+\pi^\mp\pi^-)$) and for meson decays into the same sign charged leptons (e.g. $K^- \to e^+e^+\pi^-\pi^-$) can be obtained by making use of formulas in Appendix B of Ref. [14] without adopting the narrow width approximation for integration over the phase space$^3$ for sterile neutrino heavier than decaying particles the approximation is not applicable, and for lighter sterile neutrino it is produced in the decay and can subsequently decay within a detector, for $\nu$MSM both processes are described in [16] and are naturally associated with direct searches. Decay widths for these processes with heavy neutrino are proportional again to the $U^4$ and hence resulting bounds on branching ratios are far beyond the grasp of future experiments. To illustrate this statements we have calculated the decay rate of 3-body $\tau$-lepton decay and found numerically for the branching ratio $\text{Br}(\tau^- \to e^+\pi^\mp\pi^-) \simeq 6 \times 10^{-31} \times \left\{ \frac{U^2}{10^{-8}} \right\}^2 \left( \frac{5 \text{ GeV}}{M_I} \right)^2$. 

Quite the same situation is realized for other similar decay modes and decaying mesons. One concludes that the number of weakly decaying heavy particles (leptons and mesons) collected to check $\nu$MSM predictions must be unrealistically big.

Neutrino transition dipole moments. Heavy sterile neutrino can radiatively decay into active neutrino and photon (one-loop level process). Radiative decay width reads [36]:

$$\Gamma(N_I \to \gamma\nu_\alpha) = \frac{9\alpha G_F^2}{1024\pi^4} \sin^2(2U_{\alpha I})M_I^5 \approx 2.6 \times 10^{-16} \times U_{\alpha I}^2 \left( \frac{M_I}{\text{GeV}} \right)^5 \text{ GeV},$$

(15)

Since both neutrinos are electrically neutral, this process can be effectively described by the transition dipole moments $\mu_{\text{tr} \alpha I}$, see [37]. From (15) we obtain the following estimate

$$\mu_{\text{tr} \alpha I} \approx 2.7 \times 10^{-10} \times \mu_B \times \frac{M_I}{\text{GeV}} \times U_{\alpha I}$$

where $\mu_B$ is the Bohr magneton. Transition dipole moments contribute to neutrino scattering off nuclei and can be probed with large neutrino flux. Since the sterile neutrino is not observed in this type of searches, we consider it among indirect searches for sterile neutrinos. Given the smallness of mixing predicted in $\nu$MSM, see eq. (9), the transition dipole moments are too small to be tested in future experiments. The corresponding results are presented in Fig. [3].

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$^3$Results obtained within that approximation, including many presented in Ref. [14], are often very confusing, as for a really narrow resonance (hence long lived) they are applicable only for describing of signal events in a very large detector, which size exceeds the resonance decay length.
Conclusions. To summarize, we have obtained $\nu$MSM predictions for lepton flavor and lepton number violating processes and observed that they are far below the expected sensitivity of present and foreseeable future experiments. Thus to probe $\nu$MSM one must resort to direct searches only, see [16, 26] for examples.

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