Electromagnetic Masses of the Massive Yang-Mills Particles $K^*(892)$

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Abstract

Electromagnetic mass difference between neutral $K^*$ and charged $K^*$ has been calculated in the $U(3)_L \times U(3)_R$ chiral fields theory of mesons. It has been revealed that the non-abelian gauge structure of the massive Yang-Mills lagrangian obeyed by $K^*$ plus VMD (vector meson dominance) causes the EM-mass of neutral one larger than charged one. Experiment supports this effect.

Since the Yang-Mills (YM) fields theory was discovered\cite{1}, several important concepts and pictures based on the non-abelian gauge symmetry structure of YM theory have been deeply rooted in the particle physics. In this present letter we try to illustrate another unusual effect of the massive Yang-Mills (MYM) theory obeyed by the mesons with spin one\cite{2}. Specifically, it will be revealed below that the non-abelian gauge structure of MYM with gauging Wess-Zumino-Witten anomaly (GWZW)\cite{3} (or Bardeen anomaly \cite{4}) and VMD (vector meson dominance)\cite{5} makes the electromagnetic mass (i.e., electromagnetic self-energy) of neutral $K^*(892)$ (i.e., $K^{*0}$ or $\bar{K}^{*0}$) larger than one of charged $K^*$ (i.e., $K^{*\pm}$).

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namely
\[ m(K^{*0})_{EM} > m(K^{*+})_{EM} \]  
(1)

where the subscript \( EM \) denotes electromagnetic mass. This is really unusual because it is contrary to the common knowledges of the hadron’s EM-masses, such as \( m(\text{neutron})_{EM} < m(\text{proton})_{EM} \)\(^6\), \( m(\pi^0)_{EM} < m(\pi^+)_{EM} \)\(^7\), and \( m(K^0)_{EM} < m(K^+)_{EM} \)\(^8\). The practical calculations (see below) will indicate that the contributions coming from MYM lagrangian of \( K^* \) plus VMD are significantly larger than one from GWZW plus VMD. So the claim (1) is due to the non-abelian gauge structure of YM theory basically. Using the result of \( m(K^{*0})_{EM} - m(K^{*+})_{EM} \) calculated in this letter and a known estimation of \( (m(K^{*0}) - m(K^{*+}))_{\text{non-EM}} \)\(^9\), the total mass difference between \( K^{*0} \) and \( K^{*+} \) is predicted. The result is in good agreement with the data, so eq.(1) has been supported experimentally.

The physics for low-lying mesons (\( \pi, K, \eta, \eta', \rho, \omega, a_1, f, K^*, \phi, K_1, f_s \)) is very rich. The dynamics guiding them can be constructed in terms of the principles of non-perturbative QCD theory. A reliable and satisfactory theory for these mesons is required to meet all constraints coming from both the experimental data and the theoretical considerations, and to pay the parameters as few as possible. Recently such a theory called \( U(3)_L \times U(3)_R \) chiral fields theory of pseudoscalar, vector, and axial-vector mesons\(^{10}\) has been proposed and studied for various mesonic processes. This theory provides an unified description of meson physics at low energies. The basic inputs for it are the cutoff \( \Lambda \) (or \( g \) in [10]) and \( m \) (related to quark condensate). The phenomenologies of the theory are quite encouraging. The error bars for most of the processes are less than 20 percent, which are consistent with the approximation of large \( N_c \) expansion. Theoretically, this theory has many attractive features. We would like to emphasize some of them here as follows:

1. The VMD is a natural consequence of this theory instead of an input. According
to ref.[10], VMD reads

\[ \mathcal{L}_{\rho \gamma} = -\frac{e}{f_\rho} \partial_\mu \rho_\nu (\partial^\mu A^\nu - \partial^\nu A^\mu) \]  
\[ \mathcal{L}_{\omega \gamma} = -\frac{e}{f_\omega} \partial_\mu \omega_\nu (\partial^\mu A^\nu - \partial^\nu A^\mu) \]  
\[ \mathcal{L}_{\phi \gamma} = -\frac{e}{f_\phi} \partial_\mu \phi_\nu (\partial^\mu A^\nu - \partial^\nu A^\mu) \]

The direct couplings of photon to other mesons are obtained through the substitutions

\[ \rho_\mu^0 \rightarrow \frac{e}{f_\rho} A_\mu, \quad \omega_\mu \rightarrow \frac{e}{f_\omega} A_\mu, \quad \phi_\mu \rightarrow \frac{e}{f_\phi} A_\mu \]  

where

\[ \frac{1}{f_\rho} = \frac{1}{2} g, \quad \frac{1}{f_\omega} = \frac{1}{6} g, \quad \frac{1}{f_\phi} = -\frac{1}{3\sqrt{2}} g. \]

We choose the parameter \( g = 0.38 \pm 0.01 \) in this letter, which corresponds to taking the experimental value \( m_a = 1.23 \pm 0.04 GeV \) as an input [11].

2. This theory starts with a chiral lagrangian of quantum quark fields within mesonic background fields, and the chiral dynamics of the mesons comes from the path integration over quark fields. Thus a cutoff \( \Lambda \) due to quark loop calculations has to be introduced into this truncated effective fields theory as follows,

\[ g^2 = \frac{8}{3} \frac{N_c}{(4\pi)^{D/2}} \frac{D}{4} \left( \frac{\Lambda^2}{m^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \]

\( (D = 4 - \epsilon) \) or in terms of cut-off \( \Lambda \)

\[ g^2 = \frac{8}{3} \frac{N_c}{(4\pi)^2} \left\{ \log(1 + \frac{\Lambda^2}{m^2}) + \frac{1}{1 + \frac{\Lambda^2}{m^2}} - 1 \right\}. \]

Here the \( g \) (or \( \Lambda \)) emerges as an intrinsic parameter and serves to describe the ultraviolet logarithm divergence in the theory. Therefore, it is legitimate to use the \( g \) (or \( \Lambda \)) to factorize the logarithm divergence in the loop calculations of this truncated fields theory.
3. Due to the universality of couplings in this theory, all the couplings in the lagrangian
are fixed by $g$ and $m$. Therefore, no new parameter needs to be introduced into the GWZw
anomaly part of this theory.

Thus, the electromagnetic interactions of mesons have been constructed by VMD
naturally and the divergences of photon-meson loops can be factorized consistently, then
the calculations of meson’s EM-masses become practicable and reliable. We will present
systematical studies to all low-lying meson’s EM-masses in detail elsewhere. In this letter
we focus on the EM-masses of $K^*$. 

Generally, in order to get the virtual photon contributions to the masses of mesons,
we use $\mathcal{L}_i(\Phi, \gamma, ...)|_{\Phi=\pi, K, K^*,...}$ to calculate the following S-matrix

$$S_\Phi = \langle \Phi | T \exp \left[ i \int dx^4 \mathcal{L}_i(\Phi, \gamma, ...) \right] - 1 | \Phi \rangle |_{\Phi=\pi, K, K^*,...}.$$  \hspace{1cm} (9)

On the other hand $S_\Phi$ can also be expressed in terms of the effective lagrangian of $\Phi$ as

$$S_\Phi = \langle \Phi | i \int d^4x \mathcal{L}_{\text{eff}}(\Phi) | \Phi \rangle.$$ 

Noting $\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} m_\Phi^2 \Phi^2$, then the electromagnetic interaction correction to the
mass of $\Phi$ reads

$$\delta m_\Phi^2 = \frac{2i S_\Phi}{\langle \Phi | \Phi^2 | \Phi \rangle},$$  \hspace{1cm} (10)

where $\langle \Phi | \Phi^2 | \Phi \rangle = \langle \Phi | \int d^4x \Phi^2(x) | \Phi \rangle$.

We adopt dimensional regularization to do loop-calculations, and take the most generic
linear gauge condition for electromagnetic fields to all diagram calculations in this letter.
Namely, the $A_\mu$-propagator with an arbitrary gauge constant $a$ is taken to be

$$\Delta_F^{(\gamma)}_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \Delta_F^{(\gamma)}(k)e^{-ik(x-y)},$$  

$$\Delta_F^{(\gamma)}_{\mu\nu}(k) = \frac{-i}{k^2} [g_{\mu\nu} - (1 - a) \frac{k_\mu k_\nu}{k^2}].$$  \hspace{1cm} (11)
The calculations in this letter are of $O(\alpha_{em})$ and one-loop. The interactions between $K^*$ and other mesons consist three parts: gauging non-linear $\sigma$ model, MYM lagrangian and GWZW lagrangian [10]. There are two sorts of vertices which contribute to $m^2(K^{*0})_{EM} - m^2(K^{*+})_{EM}$: three points vertices and four points vertices. Due to VMD, the former has to be the coupling of $K^*$ to a neutral vector meson plus other field, and the latter is the interaction of two $K^*$ with two neutral vector-mesons (Here, the neutral vector mesons are $\rho^0$, $\omega$ and $\phi$). It is easy to be sure that all contributions to $m^2(K^{*0})_{EM} - m^2(K^{*+})_{EM}$ come from MYM- and GWZW- parts of $K^*$-lagrangian, and no contributions are from gauging $\sigma$-model. In the following, we calculate them separately.

From[10], MYM lagrangian related to $K^*$ reads

$$\mathcal{L}_{MYM} = -\frac{g^2}{8} \text{Tr}(\partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu])^2 + \text{mass terms of } v, \quad (12)$$

where $v_\mu = \tau_i \rho_\mu^i + \lambda_a K_\mu^a + \left(\frac{2}{3} + \frac{1}{\sqrt{3}} \lambda_8\right)\omega_\mu + \left(\frac{1}{3} - \frac{1}{\sqrt{3}} \lambda_8\right)\phi_\mu$, and $a_\mu$ is axial vector field (notations of [10] are used). The three-point MYM vertex contributing to $K^*$’s EM masses and four-point one read from eq.(12)

$$\mathcal{L}^{(3)}_{MYM}(K^{*\pm}) = -\frac{i}{g}(\rho_\mu^0 + \omega_\mu - \sqrt{2}\phi_\mu)(K^{*+\mu} K^{*-\nu} - K^{*-\mu} K^{*+\nu}), \quad (13)$$

$$\mathcal{L}^{(3)}_{MYM}(K^{*0}) = -\frac{i}{g}(-\rho_\mu^0 + \omega_\mu - \sqrt{2}\phi_\mu)(K^{*0\mu} K^{*0\nu} - K^{*0\nu} K^{*0\mu}), \quad (14)$$

$$\mathcal{L}^{(4)}_{MYM}(K^{*\pm}) = -\frac{1}{g^2}(\omega^\mu - \sqrt{2}\phi^\mu)\{2\rho_\mu^0 K^{*+\nu} K^{*-\nu} - \rho_\mu^0 (K^{*+\nu} K^{*-\nu} + K^{*-\nu} K^{*+\nu})\}, \quad (15)$$

$$\mathcal{L}^{(4)}_{MYM}(K^{*0}) = \frac{1}{g^2}(\omega^\mu - \sqrt{2}\phi^\mu)\{2\rho_\mu^0 K^{*0\nu} K^{*0\nu} - \rho_\mu^0 (K^{*0\nu} K^{*0\nu} + K^{*0\nu} K^{*0\nu})\}, \quad (16)$$
where \( v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu \) (\( v = \rho, \omega, \phi, K^* \)). The corresponding direct couplings of \( K^* \) to photon, \( \mathcal{L}^{(3)}_{\text{MYM}}(K^{*\pm}, \gamma), \mathcal{L}^{(4)}_{\text{MYM}}(K^{*\pm}, \gamma), \mathcal{L}^{(4)}_{\text{MYM}}(K^*v\gamma) \) |\( v = \rho, \omega, \phi \) and \( \mathcal{L}^{(3)}_{\text{MYM}}(K^{*0}, \gamma) = \mathcal{L}^{(4)}_{\text{MYM}}(K^{*0}, \gamma) = 0 \), can be obtained through the VMD substitutions eq.(5) respectively. Thus the S-matrix is

\[
S = \langle K^* | T \{ \exp i \int d^4x [\mathcal{L}_{\text{MYM}}(K^*) + \mathcal{L}_{\text{MYM}}(K^*\gamma) + \mathcal{L}_{\rho\gamma} + \mathcal{L}_{\omega\gamma} + \mathcal{L}_{\phi\gamma} - 1] \} | K^* \rangle \tag{17}
\]

which can be calculated in the standard way. Then the EM-mass is given by formula of (10).

The Feynman diagrams corresponding to \( \mathcal{L}^{(3)}_{\text{MYM}} \) and \( \mathcal{L}^{(4)}_{\text{MYM}} \) are shown in Fig.(1) and Fig.(2) respectively.

Firstly, let’s check the gauge-independence of the S-matrix. The gauge-dependent part of the S-matrix of Fig.(1) has been derived from eqs.(11), (17) and \( \mathcal{L}^{(3)}_{\text{MYM}}(K^{*\pm}\gamma) \), which is

\[
S^{(3)G}_{\text{MYM}} = \frac{3a_4}{4} e^2 \langle K^{*+} K^{*-} \rangle \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}, \tag{18}
\]

where \( \langle K^{*+} K^{*-} \rangle = \int d^4x (K^{*+} |\frac{1}{2} (K_1^2 + K_2^2)| K^{*-}) \). Similarly, the gauge-dependent part of Fig.(2) (corresponding to \( \mathcal{L}^{(4)}_{\text{MYM}}(K^{*\pm}\gamma) \)) is

\[
S^{(4)G}_{\text{MYM}} = -\frac{3a_4}{4} e^2 \langle K^{*+} K^{*-} \rangle \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}, \tag{19}
\]

From eqs.(18),(19), we have \( S^{(3)G}_{\text{MYM}} + S^{(4)G}_{\text{MYM}} = 0 \). Therefore the EM-mass calculations of MYM part is gauge-independent. Furthermore, when one takes the dimensional regularization, \( S^{(3)G}_{\text{MYM}} = S^{(4)G}_{\text{MYM}} = 0 \), then \( S^{(3)}_{\text{MYM}} \) and \( S^{(4)}_{\text{MYM}} \) become gauge-independent individually. To the GWZW anomaly part, since there is an antisymmetric tensor \( \epsilon^{\mu\nu\lambda\rho} \) in the lagrangian (see below), the gauge-independency is obvious. So we conclude that our EM-mass calculations in this letter is gauge-independent.

From Fig.(1), the EM-mass square difference corresponding to the contributions of
three-point MYM vertices theory can be calculated directly. The result is

\[
(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))^{(3)}_{MYM} = \frac{-ie^2}{3} \int \frac{d^4k}{(2\pi)^4} m^2_\rho \times \frac{1}{-k^2(-k^2 + 2p \cdot k)(-k^2 + m^2_\rho)}
\]

\[
\times \left( m^2_\omega - \frac{2m^2_\phi}{-k^2 + m^2_\phi} \right)
\times \{ 3k^2 + 4m^2_{K^*} - \frac{k^4}{m^2_{K^*}} - 4 \left( \frac{k \cdot p}{k^2} \right)^2 + 2k \cdot p 
\quad + \frac{\langle (k \cdot K^{*-}) \{k \cdot K^{*-} \rangle}{9 + \frac{k^2}{m^2_{K^*}} - \frac{2k \cdot p}{k^2}} \}}{(9 + \frac{k^2}{m^2_{K^*}}) - \frac{2k \cdot p}{k^2}} \},
\]

where \( p \) is 4-momentum of \( K^* \), i.e., \( p^2 = m^2_{K^*} \). It is standard to carry the Feynman integration in eq.(20) through to the end. The logarithmic divergences in it are factorized by eq.(7). Then we have

\[
(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))^{(3)}_{MYM} = \Delta_\omega + \Delta_\phi
\]

where

\[
\Delta_\omega = \frac{e^2 m^2_\rho m^2_\omega}{12\pi^2} \left\{ \frac{45m^2_{K^*}}{16m^2_\rho m^2_\omega} - \frac{49m^4_{K^*}}{180m^4_\rho m^4_\omega} \right\} + \int_0^1 dx \frac{1}{m^2_\omega - m^2_\rho} \left\{ \frac{m^2_{K^*}}{m^2_\omega}(Y_\omega(\frac{5}{2}x - \frac{7}{2}) - 1 - 2x^2 + x^3) \log Y_\omega 
\quad - \frac{m^2_{K^*}}{m^2_\rho}(Y_\rho(\frac{5}{2}x - \frac{7}{2}) - 1 - 2x^2 + x^3) \log Y_\rho 
\quad + \frac{m^4_{K^*}}{m^4_\omega} \left( \frac{1}{8}Y^2_\omega + \frac{1 + x^2}{2} Y_\omega + x^2 \right) \log Y_\omega 
\quad - \frac{m^4_{K^*}}{m^4_\rho} \left( \frac{1}{8}Y^2_\rho + \frac{1 + x^2}{2} Y_\rho + x^2 \right) \log Y_\rho \right\} 
\quad - \frac{e^2 m^2_\rho m^2_\omega}{m^2_{K^*}} \left( \frac{1}{32}g^2 + \frac{1}{64\pi^2} \log \frac{f_\pi^2}{6(g^2 m^2_\rho - f_\pi^2)} \right)
\]
\[
+ \frac{1}{48\pi^2} - \frac{1}{64\pi^2} \frac{m_\omega^2}{m_\rho^2 - m_\rho^2 \log \frac{m_\rho^2}{m_\omega^2}} \\
+ \frac{e^2}{64\pi^2} \frac{m_\rho^2 m_\omega^2}{m_\omega^2 - m_\rho^2} \log \frac{m_\rho^2}{m_\omega^2},
\]
\[
\Delta \phi = 2 \Delta \omega (m_\omega^2 \to m_\phi^2),
\]

where

\[
Y_i = x^2 + \frac{m_i^2}{m_{K^*}^2} (1 - x), \quad i = \rho, \omega, \phi.
\]

Substituting the experimental values of \( f_\pi \) (= 0.186 GeV), \( m_\rho \), \( m_\omega \), \( m_\phi \) and \( m_{K^*} \) into eqs.(21) (22) and (23), we get the numerical result

\[
(m_{EM}^{2}(K^{*0}) - m_{EM}^{2}(K^{*\pm}))_{MYM}^{(3)} = 1.57 \pm 0.02 \times 10^{-3} GeV^2.
\]

Similarly, the four-point MYM-vertex contributions to EM-mass difference of \( K^* \) (Fig.(2)) can be calculated by using \( \mathcal{L}_{MYM}^{(4)}(K^*\gamma), \mathcal{L}_{MYM}^{(4)}(K^{*+}\gamma), \mathcal{L}_{MYM}^{(4)}(K^{*0}\gamma) \) and \( \mathcal{L}_{\rho\gamma}, \mathcal{L}_{\omega\gamma}, \mathcal{L}_{\phi\gamma} \). It is straightforward to get the result as follows

\[
(m_{EM}^{2}(K^{*0}) - m_{EM}^{2}(K^{*\pm}))_{MYM}^{(4)} = \frac{3e^2}{64\pi^2} \left\{ \frac{m_\rho^2 m_\omega^2}{m_\omega^2 - m_\rho^2} \log \frac{m_\rho^2}{m_\omega^2} + 2 \frac{m_\rho^2 m_\phi^2}{m_\phi^2 - m_\rho^2} \log \frac{m_\phi^2}{m_\rho^2} \right\}.
\]

Numerically, we have

\[
(m_{EM}^{2}(K^{*0}) - m_{EM}^{2}(K^{*\pm}))_{MYM}^{(4)} = 9.38 \times 10^{-4} GeV^2.
\]

In the GWZW anomaly part of the \( U(3)_L \times U(3)_R \) chiral theory of mesons there are vertices of \( K^* - (\rho, \omega, \phi) - K \). So this part makes contributions to the EM-masses of \( K^* \), which should be taken into account. The anomaly lagrangians for \( K^{*\pm} \) and \( K^{*0} \) read respectively

\[
\mathcal{L}_{GWZW}(K^{*\pm}) = -\frac{N_c}{2\pi^2 g^2 f_K} \epsilon^{\mu\nu\omega\beta} (K_{\mu}^{*\pm} \partial_{\nu} K^\pm + K_{\nu}^{*\pm} \partial_{\mu} K^\pm).
\]
\[ L_{GWZW}(K^{*0}) = - \frac{N_c}{2\pi^2 g^2 f_K} \epsilon^{\mu\nu\alpha\beta}(K^* \partial_{\mu} K^0 + K^* \partial_{\nu} K^0) \times (-\frac{1}{2} \partial_{\nu} \rho_{\alpha} + \frac{1}{2} \partial_{\nu} \omega_{\alpha} + \frac{\sqrt{2}}{2} \partial_{\nu} \phi_{\alpha}), \tag{27} \]

where \( f_K \) is determined by the following equation,

\[ f^2_K \frac{1}{1 - \frac{f^2_{K^*}}{g^2 m_{K^*}^2}} = f^2_{\pi} \frac{1}{1 - \frac{f^2_{a^*}}{g^2 m_{a^*}^2}}. \]

The corresponding direct couplings of \( K^* \) to photon, \( L_{GWZW}(K^{*\pm}, \gamma) \) and \( L_{GWZW}(K^{*0}, \gamma) \), can be obtained through the VMD substitutions of eq.(5). Then, the GWZW’s contribution to the EM-mass difference of \( K^* \) can be computed directly. The result is

\[ (m_{EM}^2(K^{*0}) - m_{EM}^2(K^{*\pm}))_{GWZW} = \frac{-9e^2}{2\pi^4 g^2 f_K^2} \int_0^1 dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \times m_{K^*}^2 \frac{m_{a^*}^2}{48\pi^2} (\frac{m_{K^*}^2}{M_a^2} - \frac{2m_{a^*}^2}{M_b^2}), \tag{29} \]

where

\[ M_a^2 = m_{a^*}^2 x_3 + m_{a}^2 (x_2 - x_3) + m_{K^*}^2 (1 - x_1) - m_{K^*}^2 x_1 (1 - x_1), \]

\[ M_b^2 = m_{b^*}^2 x_3 + m_{b^*}^2 (x_2 - x_3) + m_{K^*}^2 (1 - x_1) - m_{K^*}^2 x_1 (1 - x_1). \]

Numerically, we have

\[ (m_{EM}^2(K^{*0}) - m_{EM}^2(K^{*\pm}))_{GWZW} = 6.95 \pm 0.33 \times 10^{-4} GeV^2. \tag{30} \]

The full EM-mass difference between \( K^{*0} \) and \( K^{*\pm} \) is the sum of eqs.(24),(25) and (30), which reads

\[ (m(K^{*0}) - m(K^{*\pm}))_{EM} = 1.79 \pm 0.03 MeV. \tag{31} \]

In above computations, there are no any adjustable parameters. In other words, when one takes the experimental value of \( m_\rho, m_a, m_{K^*}, m_K, m_\omega, m_\phi \) and \( f_\pi \) as inputs, then
\((m(K^{*0}) - m(K^{*\pm}))_{EM}\) is fixed. The fact that this value is positive is just the claim of eq.(1). We can see also that the contribution of MYM part is significantly larger than one of GWZW part,

\[ (m(K^{*0}) - m(K^{*\pm}))_{EM}^{MYM} : (m(K^{*0}) - m(K^{*\pm}))_{EM}^{GWZW} = 3.75 : 1. \] (32)

Therefore, under VMD, this EM-mass difference is dominated by the MYM lagrangian of \(K^*\).

The total mass difference between \(K^{*0}\) and \(K^{*\pm}\) should be expressed as

\[
(m(K^{*0}) - m(K^{*\pm}))_{total} = (m(K^{*0}) - m(K^{*\pm}))_{non-EM} + (m(K^{*0}) - m(K^{*\pm}))_{EM},
\] (33)

where \((m(K^{*0}) - m(K^{*\pm}))_{non-EM}\) is non-electromagnetic contribution to the mass difference. As the quark masses enter the effective lagrangian, this non-electromagnetic contribution can be estimated. In ref.[9], such an estimation has been done and the authors have made the best recommendation to the value of this quantity as follows

\[
(m(K^{*0}) - m(K^{*\pm}))_{non-EM} = 4.47 MeV.
\] (34)

Then the prediction is

\[
(m(K^{*0}) - m(K^{*\pm}))_{total} = 6.26 \pm 0.03 MeV.
\] (35)

which is in good agreement with the experimental data \(6.7 \pm 1.2 MeV\) [11]. This fact means that the experiment favors to support the theoretical prediction of eq.(31) even though the experimental error is still large so far.

To conclude: In terms of the \(U(3)_L \times U(3)_R\) chiral fields theory of mesons, all one-loop diagrams contributing to the electromagnetic mass difference of \(K^*\) have been calculated. The calculations are gauge independent. The gauging non-\(\sigma\) model part of the theory has
no contribution to EM-masses of $K^*$ while MYM- and GWZW- parts make contributions. MYM- and GWZW-parts make the EM-mass of $K^{*0}$ larger than one of $K^{*\pm}$ respectively, but MYM-part is dominant. Then we conclude that in this reliable theory of mesons the non-abelian gauge fields structure of Yang-Mills theory causes the EM-mass of neutral $K^*$ larger than one of charged $K^*$. This effect of YM theory is remarkable. It is interesting that the experiment supports it.

Similar studies can be done to other vector mesons and such an effect will also emerge. However, the things will not be so clear as in the $K^*$ case because the non-linear $\sigma$ model part of the meson’s dynamics will be involved in the evaluations. We will provide such an investigation elsewhere.

ACKNOWLEDGMENTS

We would like to thank Bing An Li for helpful discussions. This work was supported in part by the National Science Funds of China through Chen Ning Yang.

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Caption

Fig. 1 $S^{(3)}_{MYM}$-Feynman diagrams corresponding to $L^{(3)}_{MYM}(K^\ast)$ (eqs. (13)-(14)), $L^{(3)}_{MYM}(K^\ast \gamma)$, and $L_{\gamma\rho}, L_{\gamma\omega}, L_{\gamma\phi}$ (eqs. (2)-(4)), the curly line is photon-line, $v$ denotes neutral vector mesons $\rho^0, \omega$ and $\phi$.

Fig. 2 $S^{(4)}_{MYM}$-Feynman diagrams corresponding to $L^{(4)}_{MYM}(K^\ast)$ (eqs. (15)-(16)) $L^{(4)}_{MYM}(K^\ast v\gamma)|_{v=\rho,\omega,\phi}$, $L^{(4)}_{MYM}(K^\ast \gamma)$, and $L_{\gamma\rho}, L_{\gamma\omega}, L_{\gamma\phi}$ (eqs. (2)-(4)), the curly line is photon-line, $v$ denotes neutral vector mesons $\rho^0, \omega$ and $\phi$.

Fig. 3 $S_{GWZW}$-Feynman diagrams corresponding to $L_{GWZW}(K^\ast)$ (eqs. (27)-(28)), $L_{GWZW}(K^\ast \gamma)$, and $L_{\gamma\rho}, L_{\gamma\omega}, L_{\gamma\phi}$ (eqs. (2)-(4)), the curly line is photon-line, $v$ denotes neutral vector mesons $\rho^0, \omega$ and $\phi$. 
Fig. 1

Fig. 2

Fig. 3