Teaching “surface area of a sphere and volume of a ball” using an inquiry approach

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Abstract. This study aimed to investigate how to teach the surface area of a sphere and volume of a ball to students of elementary school teacher education using the inquiry approach. The study used a single-subject research approach for students of elementary school teacher education, Universitas Kanjuruhan Malang, Indonesia, who have been permanent teachers in elementary schools. They were to teach the surface area of the volume using experimental demonstration method. The data analysis used in this study was a combination of quantitative and qualitative description. The results of the study showed that students found the surface area of a sphere was $4\pi r^2$ using the approach of circle area and the volume of a ball was $\frac{4}{3} \pi r^3$ using the approach of a cylinder volume. It had been proven that the approach was effective to improve students’ understanding, particularly on the surface area and the volume. In addition, they were also able to implement it to their students at schools.

1. Introduction
Learning Geometry consists of material two-dimensional shape and three-dimensional shape. Balls are a geometry which are frequently used in games and sports. They are a three-dimensional shape of unlimited circles with the same length of the radius, which is centered in a certain spot [1] with one side. In Indonesia, geometry is taught since elementary schools and is often considered interesting and fun [2]. It is commonly known that mathematics is an abstract concept [3] which does not involve a lot of memorizing [4]. It is a subject promoting logical, analytical, systematic, critical, creative thinking [5], and reasoning ability [6] which are some of the most important aspects of students’ success [5]. This fact demands teachers to be able to teach the concepts of mathematics to their students well.

It has been shown that most elementary school teachers use procedural method when solving a problem in mathematics without transferring its conceptual understanding [6]. The surface area of a sphere and the volume of a ball, for instance, are limitedly taught at the level of memorizing the formula [3,7]. This condition makes the students solely memorize the formula through communication [8]. For students of elementary school teacher education, an inquiry approach – based learning is strongly recommended since it is considered efficient to enhance curiosity and motivation and is able to develop students’ critical thinking [9,10].

The inquiry approach is designed to give students direct experiences of scientific process in stimulating critical thinking, analyzing and solving a particular problem [11]. This approach is widely used and getting more well-known particularly in science curriculum, research, and teaching [12]. One
of its underlining reasons is that the earning success is improved significantly due to the development which is supported by the surroundings.

It is expected that students require a set of knowledge and skills in the form of understanding from their own discoveries instead of memorizing a series of facts [13]. Knowledge acquired from one’s discoveries are proven to last longer in comparison with that obtained with other ways [14]. In general, learning through discoveries is able to improve reasoning ability and free thinking [12]. Previous inquiry learning has been applied to physics learning showing significant results in conceptual understanding [15] and on computer-based learning shows a very structured step-by-step [16]. In this study, students are expected to understand the concepts of the surface area of a sphere and the volume of a ball through their own discoveries.

2. Method
This research is experimental research, which focuses on the process of proving surface area of a sphere and volume of a ball. Participants of the study were eight students of elementary school teacher education at Universitas Kanjuruhan Malang, Indonesia, who are permanent teachers in elementary schools. Participants are chosen because they already know the material and how students are in learning. In the teaching and learning process, those students teach using demonstrative and experimental methods to understand the concepts of the surface area of a sphere and the volume of a ball utilizing concrete media. This study used several instruments comprising a. A pre-test to find out students’ initial skills; b. A post-test to know students’ final skills; and c. An in-depth interview to dig out more on the subject being studied. Data analysis used qualitative data about the learning process surface of a sphere and volume of a ball and quantitative data on the results of student pretest and posttest.

The teaching and learning inquiry processes had been carried out for three sessions which lasted 100 minutes each. The concrete media used were an orange which resembled a ball. First of all, we peeled them to teach them the surface area using the approach of the surface area of a circle. In the next meeting, we used a cylinder to teach the students the concept of volume of a ball.

3. Results and discussion
The results of inquiry-based learning research in this study consists of five phases including orientation, conceptualization, investigation, conclusion, and discussion [12]. In the orientation phase, we tried to stimulate students’ period understanding on a ball by identifying their nature, volume, and surface area. It was found that only five out of eight students who had solid understanding and were able to identify the aforementioned nature of a ball. They mostly knew the formula of both the surface area and the volume without knowing how to discover it. This leads to the fact that most elementary school teachers teach using a procedural method and neglect the conceptual understanding [17].

In the conceptualization and investigation phases, we directed the students to understand how to discover the formula of both the surface area and the volume. Students were asked to make groups of four and discuss the conceptual understanding of both the formulas. Following are the results of the discussion.

3.1. The surface area of a sphere
To find out the surface area of a sphere, we used an orange which would also be related to the area of a circle. First, the orange was cut into two equal sizes whose diameters were measured. Students were then asked to make more than one circle with the same size of diameter as that of the orange. The orange was peeled and the peel was put into the circles made. All the circles had to be covered by all the orange peels. The results of this investigation are shown in Figure 1.
3.2. The volume of a ball

To find out the volume of a ball, we used an interactive video using a cylinder to illustrate the volume of a ball. The results of the experiment are illustrated in Figure 2.

Figure 2 shows that the balls in the cylinder have the same diameter size as the cylinder. Meanwhile, the height of the cylinder is twice as high as the radius of the ball. It can be concluded that:

\[ V_{cylinder} = \pi r^2 t = \pi r^2 \times 2r = 2\pi r^3 \quad (1) \]

To discover the volume of a ball, the ball is cut into two same sizes. The cylinder was then filled with water poured into the half-cut ball. The cylinder was fully filled with water when there were three half-cut balls inserted. Thus, it can be concluded that:

\[ V_{cylinder} = 3 \times V \frac{1}{2} ball \quad (1) \]

\[ V \frac{1}{2} ball = \frac{1}{3} V_{cylinder} = \frac{1}{3} \times 2\pi r^3 = \frac{2}{3} \pi r^3 \quad (2) \]

\[ V \text{ ball} = 2 \times V \frac{1}{2} \text{ ball} = 2 \times \frac{2}{3} \pi r^3 = \frac{4}{3} \pi r^3 \quad (3) \]

Based on the investigation, it can be inferred that the surface area of a sphere is \(4\pi r^2\) and the volume of a ball is \(\frac{4}{3}\pi r^3\).

The activity proves that the discovery of the formulas of both the surface area and the volume using an orange is effective to maximize individual potentials which is not only able to find problems, but also to solve them effectively [18]. Additionally, the approach is also able to motivate, train, and construct scientific knowledge [19]. This experimentation also enables the students to implement it to their
students in elementary schools since it can enhance their students’ understanding as well [20], Wartono and student learning outcomes [21]. Statistically, the inquiry-based learning of the surface area of a sphere and the volume of a ball is effective to enhance students’ understanding on how to discover both of the formulas. It has been shown by the means of the pre-test and post-test which are 64 and 89 in order. This indicates that the inquiry-based learning enhances students’ understanding by 40%. This is in line with inquiry learning in physics [15] and computer learning [16] that can describe step by step and can improve student understanding.

4. Conclusion
Teaching mathematics, particularly teaching the surface area of a sphere and the volume of a ball based on an inquiry approach can be conducted by utilizing either concrete or electronic media. The results of the inquiry-based learning using an orange as concrete media proved that the surface area of ball is equal to four times of circle surface area $4\pi r^2$ while half of the volume of a ball is equal to two-third of a cylinder volume, which means $\frac{4}{3}\pi r^3$. The experimentation also emphasized that inquiry-based learning is able to improve students’ understanding and is able to be implemented in teaching elementary school students.

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