Gaugino-pair production in polarized and unpolarized hadron collisions

Jonathan Debove\textsuperscript{a}, Benjamin Fuks\textsuperscript{b}, and Michael Klasen\textsuperscript{c}\textsuperscript{∗}

\textsuperscript{a} Laboratoire de Physique Subatomique et de Cosmologie, Universit\textsuperscript{e} Joseph Fourier/CNRS-IN2P3/INPG, 53 Avenue des Martyrs, F-38026 Grenoble, France
\textsuperscript{b} Physikalisches Institut, Albert-Ludwigs-Universit\textsuperscript{a} Freiburg, Hermann-Herder-Straße 3, D-79106 Freiburg i.Br., Germany

(Dated: April 14, 2021)

We present an exploratory study of gaugino-pair production in polarized and unpolarized hadron collisions, focusing on the correlation of beam polarization and gaugino/Higgsino mixing in the general Minimal Supersymmetric Standard Model. Helicity-dependent cross sections induced by neutral and charged electroweak currents and squark exchanges are computed analytically in terms of generalized charges, defined similarly for chargino-pair, neutralino-chargino associated, and neutralino-pair production. Our results confirm and extend those obtained previously for negligible Yukawa couplings and nonmixing squarks. Assuming that the lightest chargino mass is known, we show numerically that measurements of the longitudinal single-spin asymmetry at the existing polarized \textit{pp} collider RHIC and at possible polarization upgrades of the Tevatron or the LHC would allow for a determination of the gaugino/Higgsino fractions of charginos and neutralinos. The theoretical uncertainty coming from factorization scale and squark mass variations and the expected experimental error on the lightest chargino mass is generally smaller than the one induced by the polarized parton densities, so that more information on the latter would considerably improve on the analysis.

I. INTRODUCTION

Weak-scale Supersymmetry (SUSY) continues to be a theoretically attractive extension of the Standard Model (SM) of particle physics. If \textit{R}-parity is conserved, it provides in particular a convincing candidate for the large amount of cold dark matter observed in the Universe. In the Minimal Supersymmetric SM (MSSM) \textsuperscript{1}, this is generally the lightest neutralino, one of the spin-1/2 SUSY partners of the electroweak gauge bosons (gauginos) and of the Higgs bosons (Higgsinos), which mix to form four neutral (neutralino) and two charged (chargino) mass eigenstates. The gaugino/Higgsino decomposition of the neutralinos/charginos contains important information about the SUSY-breaking mechanism and plays a crucial role in the determination of the dark matter relic density \textsuperscript{2, 3}.

Unfortunately, SUSY particles have yet to be found at high-energy accelerators. The LEP and Tevatron colliders have constrained the gauginos and scalar partners of the fermions (squarks/sleptons) to be heavier than a few tens and hundreds of GeV, respectively, and the search for SUSY particles has thus become one of the defining tasks of the LHC. At the same time of the LHC start-up, RHIC is scheduled to operate in the years 2009 through 2012 in its polarized \textit{pp} mode at an increased center-of-mass energy of $\sqrt{s} = 500$ GeV and with a large integrated luminosity of \textit{266 pb$^{-1}$} during each of the ten-week physics runs. It is therefore interesting to investigate the influence of proton beam polarization on production cross sections and longitudinal spin asymmetries for SUSY particle production at the existing polarized \textit{pp} collider RHIC \textsuperscript{4} and at possible polarization upgrades of the Tevatron \textsuperscript{5} or the LHC \textsuperscript{6}.

While electron beams in high-energy accelerators can polarize automatically due to the emission of spin-flip synchrotron radiation via the Sokolov-Ternov effect, electrons of lower energy and protons have to be polarized in a source. These beams then have to be accelerated and stored with little loss of polarization. Today polarized proton beams can be produced either by a polarized atomic beam source or in an optically pumped polarized ion source. Pulsed beams with polarization of up to 87\% for a 1 mA $H^-$ beam current and up to 60\% for 5 mA, respectively, have been achieved with these sources \textsuperscript{7}. Crossing resonances while accelerating the beam can, however, in principle lead to a reduction of the polarization, but this can be avoided with a spin rotation using Siberian snakes.

In a previous paper, we studied the correlation of beam polarization and the mixing of the scalar partners of left- and right-handed leptons produced in polarized hadron collisions \textsuperscript{8}. Unpolarized production cross sections for sleptons are known at next-to-leading order (NLO) of perturbation theory \textsuperscript{9} and have recently been resummed at next-to-leading logarithmic (NLL) accuracy \textsuperscript{10, 11, 12}. Here, we present an exploratory study of gaugino-pair production in polarized hadron collisions, focusing on the correlation of beam polarization and gaugino/Higgsino mixing in the general MSSM. Unpolarized cross sections for gaugino pairs are also known at NLO \textsuperscript{9} and have been re-examined

\textsuperscript{∗}klasen@lpsc.in2p3.fr
recently for possible signals of non-minimal flavor violation [13]. The impact of gaugino/Higgsino mixing on beam polarization asymmetries in chargino and neutralino production has been considered previously, albeit only recently for possible signals of non-minimal flavor violation [13]. The impact of gaugino/Higgsino mixing on beam polarization asymmetries in chargino and neutralino production has been considered previously, albeit only relatively recently for possible signals of non-minimal flavor violation [13]. The impact of gaugino/Higgsino mixing on beam polarization asymmetries in chargino and neutralino production has been considered previously, albeit only recently for possible signals of non-minimal flavor violation [13].

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In Sec. III, we show numerically that measurements of the longitudinal single-spin asymmetry would allow for a determination of the gaugino/Higgsino fractions of charginos and neutralinos, assuming that the lightest chargino mass is known. We also estimate the theoretical uncertainties coming from variations of the unphysical factorization scale, the yet unknown squark masses, the expected experimental error on the lightest chargino mass, and our limited knowledge of the polarized parton densities. We summarize our results in Sec. IV.

II. ANALYTICAL RESULTS

We start by computing analytically the partonic cross sections for the pair production of gauginos and Higgsinos, whose mixing to neutralinos and charginos $\chi_i^{0,\pm}$ is described in App. A in terms of generalized electroweak couplings, which are defined in App. B. The process

$$q(h_a, p_a) \bar{q}'(h_b, p_b) \to \chi_i^{0,\pm}(p_1) \bar{\chi}_j^{0,\mp}(p_2)$$

is induced by initial quarks $q$ and antiquarks $\bar{q}'$ with definite helicities $h_a, h_b$ and momenta $p_a, p_b$, and is mediated by $s$-channel electroweak gauge-boson and $t$- and $u$-channel squark exchanges (see Fig. 1). Its cross section can be expressed generically as

$$\frac{d\sigma_i^{q\bar{q}'}_{h_a h_b}}{dt} = \frac{\alpha^2}{3s^2} \left[ (1 - h_a)(1 + h_b) \left[ |Q_{LL}^u|^2 u_{\tilde{\chi}_i} u_{\bar{\chi}_j} + |Q_{LL}^t|^2 t_{\tilde{\chi}_i} t_{\bar{\chi}_j} + 2 \text{Re}[Q_{LL}^u Q_{LL}^t] |m_{\tilde{\chi}_i} m_{\bar{\chi}_j}| \right] 
+ \frac{\alpha^2}{3s^2} (1 + h_a)(1 - h_b) \left[ |Q_{RL}^u|^2 u_{\tilde{\chi}_i} u_{\bar{\chi}_j} + |Q_{RL}^t|^2 t_{\tilde{\chi}_i} t_{\bar{\chi}_j} + 2 \text{Re}[Q_{RL}^u Q_{RL}^t] |m_{\tilde{\chi}_i} m_{\bar{\chi}_j}| \right] 
+ \frac{\alpha^2}{3s^2} (1 - h_a)(1 - h_b) \left[ |Q_{LR}^u|^2 u_{\tilde{\chi}_i} u_{\bar{\chi}_j} + |Q_{LR}^t|^2 t_{\tilde{\chi}_i} t_{\bar{\chi}_j} + 2 \text{Re}[Q_{LR}^u Q_{LR}^t] |(ut - m_{\tilde{\chi}_i}^2, m_{\bar{\chi}_j}^2)| \right] 
+ \frac{\alpha^2}{3s^2} (1 + h_a)(1 + h_b) \left[ |Q_{RR}^u|^2 u_{\tilde{\chi}_i} u_{\bar{\chi}_j} + |Q_{RR}^t|^2 t_{\tilde{\chi}_i} t_{\bar{\chi}_j} + 2 \text{Re}[Q_{RR}^u Q_{RR}^t] |(ut - m_{\tilde{\chi}_i}^2, m_{\bar{\chi}_j}^2)| \right], \right]$$

i.e. in terms of generalized charges $Q_{LL}^{u,t}$, the conventional Mandelstam variables

$$s = (p_a + p_b)^2, \quad t = (p_a - p_1)^2, \quad u = (p_a - p_2)^2,$$

the gaugino and squark masses $m_{\tilde{\chi}_i, \tilde{\chi}_j}$ and $m_{\tilde{q}_a, \tilde{q}'_b}$, and the masses of the neutral and charged electroweak gauge bosons $m_Z$ and $m_W$. Propagators appear as mass-subtracted Mandelstam variables,

$$s_z = s - m_Z^2, \quad s_w = s - m_W^2,$$
$$t_{\tilde{q}} = t - m_{\tilde{q}}^2, \quad u_{\tilde{q}} = u - m_{\tilde{q}}^2,$$
$$t_{\tilde{\chi}} = t - m_{\tilde{\chi}}^2, \quad u_{\tilde{\chi}} = u - m_{\tilde{\chi}}^2,$$

FIG. 1: Tree-level Feynman diagrams for the production of gaugino pairs.
and the weak interaction is defined through the square of its coupling constant $g^2 = e^2 / \sin^2 \theta_W$ in terms of the electromagnetic fine structure constant $\alpha = e^2 / (4\pi)$ and the squared sine of the electroweak mixing angle $x_W = \sin^2 \theta_W = 1 - \cos^2 \theta_W = 1 - c_W^2$. Unpolarized cross sections, averaged over initial spins, can easily be derived from the expression

$$d\tilde{\sigma} = \frac{d\tilde{\sigma}_{1,1} + d\tilde{\sigma}_{1,-1} + d\tilde{\sigma}_{-1,1} + d\tilde{\sigma}_{-1,-1}}{4},$$

while single- and double-polarized cross sections, including the same average factor for initial spins, are given by

$$d\Delta\tilde{\sigma}_L = \frac{d\tilde{\sigma}_{1,1} \pm d\tilde{\sigma}_{1,-1} \pm d\tilde{\sigma}_{-1,1} \mp d\tilde{\sigma}_{-1,-1}}{4} \quad \text{and} \quad d\Delta\tilde{\sigma}_{LL} = \frac{d\tilde{\sigma}_{1,1} - d\tilde{\sigma}_{1,-1} - d\tilde{\sigma}_{-1,1} + d\tilde{\sigma}_{-1,-1}}{4},$$

where the upper (lower) signs refer to polarized (anti-)quarks. The partonic single- and double-spin asymmetries then become

$$\hat{A}_L = \frac{d\Delta\tilde{\sigma}_L}{d\tilde{\sigma}} \quad \text{and} \quad \hat{A}_{LL} = \frac{d\Delta\tilde{\sigma}_{LL}}{d\tilde{\sigma}}.$$

For $\tilde{\chi}_i^{+}\tilde{\chi}_j^{-}$-production, the general charges are given by

$$Q_{LL}^{u+} = \frac{1}{x_W (1 - x_W)} \left[ \frac{L_{Zqq}^0 O_{ij}^{LL}}{s_z} + \sum_{k=1}^{6} \frac{L_{\tilde{\chi}_i^0}^* \tilde{u}_{k} q \tilde{L}_{\tilde{\chi}_j^0}^* \tilde{u}_{k}}{u_{\tilde{u}_k}} \right] - \frac{e_q \delta_{ij} \delta_{qq'}}{s},$$

$$Q_{LL}^{l+} = \frac{1}{x_W (1 - x_W)} \left[ \frac{L_{Zqq}^0 O_{ij}^{LR}}{s_z} - \sum_{k=1}^{6} \frac{L_{\tilde{\chi}_i^0}^* \tilde{d}_{k} q \tilde{L}_{\tilde{\chi}_j^0}^* \tilde{d}_{k}}{t_{\tilde{d}_k}} \right] - \frac{e_q \delta_{ij} \delta_{qq'}}{s},$$

$$Q_{RR}^{u+} = \frac{1}{x_W (1 - x_W)} \left[ \frac{R_{Zqq}^0 O_{ij}^{LR}}{s_z} - \sum_{k=1}^{6} \frac{R_{\tilde{\chi}_i^0}^* \tilde{u}_{k} q R_{\tilde{\chi}_j^0}^* \tilde{u}_{k}}{u_{\tilde{u}_k}} \right] - \frac{e_q \delta_{ij} \delta_{qq'}}{s},$$

$$Q_{RR}^{l+} = \frac{1}{x_W (1 - x_W)} \left[ \frac{R_{Zqq}^0 O_{ij}^{RL}}{s_z} + \sum_{k=1}^{6} \frac{R_{\tilde{\chi}_i^0}^* \tilde{d}_{k} q R_{\tilde{\chi}_j^0}^* \tilde{d}_{k}}{t_{\tilde{d}_k}} \right] - \frac{e_q \delta_{ij} \delta_{qq'}}{s},$$

$$Q_{LL}^{u-} = \frac{1}{x_W (1 - x_W)} \left[ \frac{L_{Zqq}^0 O_{ij}^{LL}}{s_z} + \sum_{k=1}^{6} \frac{L_{\tilde{\chi}_i^0}^* \tilde{u}_{k} q \tilde{L}_{\tilde{\chi}_j^0}^* \tilde{u}_{k}}{u_{\tilde{u}_k}} \right],$$

$$Q_{LL}^{l-} = \frac{1}{x_W (1 - x_W)} \left[ \frac{L_{Zqq}^0 O_{ij}^{LR}}{s_z} - \sum_{k=1}^{6} \frac{L_{\tilde{\chi}_i^0}^* \tilde{d}_{k} q \tilde{L}_{\tilde{\chi}_j^0}^* \tilde{d}_{k}}{t_{\tilde{d}_k}} \right],$$

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$$Q_{RR}^{l-} = \frac{1}{x_W (1 - x_W)} \left[ \frac{R_{Zqq}^0 O_{ij}^{RL}}{s_z} + \sum_{k=1}^{6} \frac{R_{\tilde{\chi}_i^0}^* \tilde{d}_{k} q R_{\tilde{\chi}_j^0}^* \tilde{d}_{k}}{t_{\tilde{d}_k}} \right].$$

Note that there is no interference between $t$- and $u$-channel diagrams due to (electromagnetic) charge conservation. After accounting for our harmonization of generalized charge definitions, which are now similar for all gaugino channels, our results agree with those published in Ref. [13] for $\tilde{\chi}_i^{+}\tilde{\chi}_j^{-}$-production. The cross section for chargino-pair production in $e^+e^-$-collisions can be deduced by setting $e_q \rightarrow e_l = -1$, $L_{Zqq} \rightarrow L_{Zee} = T_{ij}^3 - e_l x_W$ and $R_{Zqq} \rightarrow R_{Zee} = -e_l x_W$. Neglecting all Yukawa couplings, we can then reproduce the calculations of Ref. [13].

The changes of the neutralino-chargino associated production are given by

$$Q_{LL}^{o-} = \frac{1}{x_W (1 - x_W)} \left[ \frac{L_{WW}^O O_{ij}^{LL}}{s_w} + \sum_{k=1}^{6} \frac{L_{\tilde{\chi}_i^0}^* \tilde{u}_{k} q \tilde{L}_{\tilde{\chi}_j^0}^* \tilde{u}_{k}}{u_{\tilde{u}_k}} \right],$$

$$Q_{LL}^{o-} = \frac{1}{x_W (1 - x_W)} \left[ \frac{L_{WW}^O O_{ij}^{LR}}{s_w} - \sum_{k=1}^{6} \frac{L_{\tilde{\chi}_i^0}^* \tilde{d}_{k} q \tilde{L}_{\tilde{\chi}_j^0}^* \tilde{d}_{k}}{t_{\tilde{d}_k}} \right],$$

$$Q_{RR}^{o-} = \frac{1}{x_W (1 - x_W)} \left[ \frac{R_{WW}^O O_{ij}^{LR}}{s_w} + \sum_{k=1}^{6} \frac{R_{\tilde{\chi}_i^0}^* \tilde{u}_{k} q R_{\tilde{\chi}_j^0}^* \tilde{u}_{k}}{u_{\tilde{u}_k}} \right].$$
The related charge-conjugate process $qq' \rightarrow \tilde{\chi}_{i}^{+} \tilde{\chi}_{j}^{0}$ is obtained by complex conjugation and making the replacements $i \leftrightarrow j$, $q \leftrightarrow q'$, and $LR \leftrightarrow RL$ in these charges. In the case of nonmixing squarks with neglected Yukawa couplings, we agree with the results of Ref. [13] provided we correct a sign in their Eq. (2) as described in the Erratum.

Finally, the charges for neutralino-pair production are given by

$$Q_{L0}^{00} = \frac{1}{x_{W}(1 - x_{W})} \sqrt{1 + \delta_{ij}} \left[ \frac{L_{q q}^{o} O_{q q}^{o L}}{s_{W}} + \sum_{k=1}^{6} \frac{L_{\chi_{q}^{0} q_{k} q}^{o R} L_{\chi_{j}^{0} q_{k} q}^{o L}}{u_{q_{k}}} \right],$$

$$Q_{L0}^{00} = \frac{1}{x_{W}(1 - x_{W})} \sqrt{1 + \delta_{ij}} \left[ \frac{L_{q q}^{o} O_{q q}^{o L}}{s_{W}} - \sum_{k=1}^{6} \frac{L_{\chi_{q}^{0} q_{k} q}^{o R} L_{\chi_{j}^{0} q_{k} q}^{o L}}{u_{q_{k}}} \right],$$

$$Q_{R0}^{00} = \frac{1}{x_{W}(1 - x_{W})} \sqrt{1 + \delta_{ij}} \left[ \frac{R_{q q}^{o} O_{q q}^{o L}}{s_{W}} + \sum_{k=1}^{6} \frac{R_{\chi_{q}^{0} q_{k} q}^{o R} R_{\chi_{j}^{0} q_{k} q}^{o L}}{t_{q_{k}}} \right],$$

$$Q_{R0}^{00} = \frac{1}{x_{W}(1 - x_{W})} \sqrt{1 + \delta_{ij}} \left[ \frac{R_{q q}^{o} O_{q q}^{o L}}{s_{W}} - \sum_{k=1}^{6} \frac{R_{\chi_{q}^{0} q_{k} q}^{o R} R_{\chi_{j}^{0} q_{k} q}^{o L}}{t_{q_{k}}} \right],$$

which agrees with the results of Ref. [13] and also with those of Ref. [16] in the case of nonmixing squarks.

### III. NUMERICAL RESULTS

We now present numerical predictions for the cross sections and single- and double-spin asymmetries of gaugino-pair production at the polarized $pp$ collider RHIC [4] and possible polarization upgrades of the $p\bar{p}$ and $pp$ colliders Tevatron [3] and LHC [6]. Thanks to the QCD factorization theorem, total unpolarized hadronic cross sections

$$\sigma = \int_{4m^{2}/S}^{1} dx a_{A}(x_{a}, \mu_{F}) b_{B}(x_{b}, \mu_{F}) \frac{dt}{dt} \int_{t_{\min}}^{t_{\max}} f_{\alpha/\beta}(x_{a}, \mu_{F}) f_{\beta/\alpha}(x_{b}, \mu_{F}) \frac{d\phi}{d\phi}$$

(11)
can be calculated by convolving the relevant partonic cross sections \(d\sigma/dt\), computed in Sec. III with universal parton densities \(f_{a/A}\) and \(f_{b/B}\) of partons \(a, b\) in the hadrons \(A, B\), which depend on the longitudinal momentum fractions of the two partons \(x_{a,b} = \sqrt{t}e^{\mp y}\) and on the unphysical factorization scale \(\mu_F\). Polarized cross sections are computed similarly by replacing one or both of the unpolarized parton densities \(f_{a,b}(x_{a,b}, \mu_F)\) with their polarized equivalents \(\Delta f_{a,b}(x_{a,b}, \mu_F)\) and the unpolarized partonic cross section \(d\sigma\), given in Eq. (5), with its single- or double-polarized equivalent given in Eq. (4).

Efforts over the past three decades have produced extensive data sets for polarized deep-inelastic scattering (DIS), resulting in a good knowledge in particular of the polarized valence-quark (non-singlet) distributions. For consistency with our leading order (LO) QCD calculation in the collinear approximation, where all squared quark masses (except for the top-quark mass) \(m_q^2 \ll s\), we employ related sets of unpolarized [17] and polarized [18] LO parton densities. We estimate the theoretical uncertainty due to the less well known polarized parton densities by showing our numerical predictions for both the GRV2000 LO standard (STD) and valence (VAL) parameterizations, which treat the polarized sea-quarks in a flavor-symmetric or flavor-asymmetric way. The polarized gluon density could not be constrained very well in the fits to the DIS data, but it fortunately does not enter directly in our analysis.

Results from semi-inclusive DIS with an identified hadron in the final state have the promise to put individual constraints on the various quark flavor distributions in the nucleon. In addition, precise asymmetry measurements from RHIC are expected to put significant constraints on the polarized gluon distribution. A first step in this direction has been undertaken very recently by including semi-inclusive DIS data from the SMC, HERMES and COMPASS experiments and \(\pi^0\) and jet production data from the PHENIX and STAR collaborations in a global analysis [19].

If not stated otherwise, we set the factorization scale \(\mu_F\) to the average mass of the final state SUSY particles. The bottom- and top-quark densities in the proton are small and absent in the GRV and GRSV parameterizations, as is the charm-quark density. We therefore consider for squark exchanges only the SUSY-partners of the light quark flavors without mixing and all degenerate in mass. The corresponding uncertainty is estimated by giving predictions for two different squark masses, one at the mass limit set by the D0 collaboration at 325 GeV [20] and one for a typical SUSY-breaking scale of 1 TeV.

A. Gaugino masses and mixings

We wish to study the correlations of beam polarizations and the gaugino/Higgsino fractions of charginos and neutralinos without referring to a particular SUSY-breaking model. Furthermore, we wish to keep the physical gaugino masses as constant as possible, since the absolute cross sections depend strongly on them through trivial phase space effects. We start therefore by fixing the lightest chargino mass \(m_{\tilde{\chi}^+_1}\) to either 80 GeV (for our RHIC predictions) or 151 GeV (for our Tevatron and LHC predictions). The relatively strong limit of 151 GeV has recently been obtained by the CDF collaboration at Run II of the Tevatron and holds for a constrained MSSM with light nonmixing sleptons [21]. On the other hand, charginos with a mass as low as 80 GeV may still be allowed, if they are gaugino-like, their mass difference with the lightest neutralino is very small, and if the sneutrinos are light [22, 23].

The second-lightest neutralino usually stays close in mass to the lightest chargino (see below) and must be heavier than 62.4 GeV, while the lightest neutralino can be half as heavy and is constrained to masses above 32.5-46 GeV, depending again on the sfermion masses [24]. The associated production of the second-lightest neutralino with the lightest chargino is usually experimentally easily identifiable through the gold-plated tri-lepton decay. It has been pointed out that the electroweak precision fits improve when including heavy sfermions as proposed by split-SUSY scenarios, but light gauginos or Higgsinos with masses close to the current exclusion limits [25].

In the MSSM, the gaugino masses and mixings depend on the \textit{a priori} unknown SUSY-breaking parameters \(M_1, M_2, \mu,\) and on \(\tan \beta\) (see App. A). Taking \(\tan \beta = 10\) and assuming \(B\)-ino and \(W\)-ino mass unification at the GUT scale, so that \(M_1 = \frac{3}{2} \tan^2 \beta W M_2 \simeq 0.5 M_2\) at the electroweak scale, we can compute the Higgsino mass parameter \(\mu\) from Eq. (11),

\[
\mu = \frac{m_{\tilde{\chi}^+_1}^2 M_2 s_\beta \pm m_{\tilde{\chi}^+_1} \sqrt{\left(m_{\tilde{\chi}^+_1}^2 - M_2^2 - m_{\tilde{\chi}^+_1}^2 - m_{\tilde{\chi}^-_1}^2\right)^2 - m_{\tilde{\chi}^+_1}^2 c_\beta^2}}{M_2^2 - m_{\tilde{\chi}^+_1}^2},
\]  

(12)

as a function of the only remaining parameter \(M_2\), once the lightest chargino mass \(m_{\tilde{\chi}^+_1}\) is fixed. Since the one-loop contribution to the anomalous magnetic moment \(a_\mu = (g\mu - 2)/2\) of the muon induced by gauginos and sleptons of common mass \(M_{\text{SUSY}}\) is approximately given by [20]

\[
a_{\mu}^{\text{SUSY, 1-loop}} = 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}}\right)^2 \tan \beta \text{sgn}(\mu),
\]  

(13)
negative values of $\mu$ would increase, not decrease, the disagreement between the recent BNL measurement and the theoretical SM value of $a_\mu$ [24]. The region $\mu < 0$ is therefore disfavored, and we take $\mu > 0$ unless noted otherwise. As the off-diagonal matrix elements of the gaugino mass matrices depend on $\sin\beta$ and $\cos\beta$ (see App. A), one might be tempted to fix $M_2$, e.g. to $2m_{\tilde{\chi}_1^\pm}$, and study rather the variation of the chargino/neutralino masses and gaugino/Higgsino fractions with $\tan\beta$. However, this parameter can often be constrained from the Higgs sector alone [27], at least if it is large [28]; otherwise measurements from the sfermion or neutralino sector may still be necessary [29]. Furthermore, $\sin\beta$ and $\cos\beta$ vary significantly only for low $\tan\beta = 2\ldots 10$. In this range, the gaugino fraction of the lightest negative chargino decreases, e.g., from 40% to 20% in the optimal case of $M_2 = 2m_{\tilde{\chi}_1^\pm} = 160$ GeV.

In Fig. 2 we show the physical masses of the two charginos and the four neutralinos as a function of $M_2$ for $m_{\tilde{\chi}_1^\pm} = 80$ GeV (left) and 151 GeV (right). The lightest chargino mass (short-dashed line) is, of course, constant in both cases. As mentioned above, the mass of the second-lightest neutralino stays close to it, except around $M_2 = 190$ GeV (320 GeV), where an avoided crossing with $m_{\tilde{\chi}_1^\pm}$ occurs, which is typical of Hermitian matrices depending continuously on a single parameter. At this point, these two neutralino eigenstates change character, as can clearly be seen from the gaugino fractions plotted in Fig. 3. While for small values of $M_2 \ll |\mu|$ the lighter neutralinos, diagonalized by the matrix $N$, are gaugino-like, they become Higgsino-like for large values of $M_2 \gg |\mu|$. Furthermore, in this region the mass difference between the lightest neutralino and chargino becomes small (see Fig. 2). It can also be seen from this figure that the heavier chargino and the heaviest neutralino are mass-degenerate for all values of $M_2$ and that their mass grows linearly with $M_2$, when $M_2 \gg |\mu|$. The gaugino fractions of the negative and positive charginos, diagonalized by the matrices $U$ and $V$, are shown in Fig. 4. They behave similarly to those of the lightest and heaviest neutralinos. We will frequently refer to these well-known variations of the neutralino/chargino masses and gaugino/Higgsino fractions in the subsequent sections when discussing the behavior of cross sections and asymmetries.

### B. RHIC cross sections and asymmetries

RHIC is scheduled to operate in the years 2009 through 2012 in its polarized $pp$ mode at an increased center-of-mass energy of $\sqrt{s} = 500$ GeV and with a large integrated luminosity of $266$ pb$^{-1}$ during each of the ten-week physics runs [4]. It has been demonstrated that polarization loss during RHIC beam acceleration and storage can be kept small, so that a polarization degree of about 45% has already been and 65%-70% may ultimately be reached [30]. It is therefore interesting to investigate the influence of proton beam polarization on production cross sections and longitudinal spin asymmetries for SUSY particle production at the existing polarized $pp$ collider RHIC.

In the left part of Fig. 4 we show the total unpolarized cross section for the pair production of the lightest chargino of mass 80 GeV (short-dashed line) and the one for its associated production with the second-lightest neutralino (dotted-dashed line) at the $pp$ collider RHIC, expected to produce a total integrated luminosity of about $1$ fb$^{-1}$ during the next...
FIG. 3: Gaugino and Higgsino fractions of the four neutralinos as a function of the SUSY-breaking parameter $M_2$ for a fixed lightest chargino mass of $m_{\tilde{\chi}_1^\pm} = 80$ GeV (left) and 151 GeV (right). We choose $\tan \beta = 10$, $\mu > 0$ using Eq. (12), and fix $M_1 = \frac{3}{4} \tan^2 \theta_W M_2$.

FIG. 4: Gaugino and Higgsino fractions for charginos as a function of the SUSY-breaking parameter $M_2$ for a fixed lightest chargino mass of $m_{\tilde{\chi}_1^\pm} = 80$ GeV (left) and 151 GeV (right). We choose $\tan \beta = 10$, $\mu > 0$ using Eq. (12), and fix $M_1 = \frac{3}{4} \tan^2 \theta_W M_2$.

four years [4]. Both cross sections exceed 1 fb (short-dashed horizontal line, corresponding to one produced event) in most of the $M_2$ range shown and depend little on the squark mass, indicating that $s$-channel gauge-boson exchanges dominate. From Eqs. (8) and (9) and App. B we learn indeed that, in the absence of heavy bottom- and top-quarks, squark exchanges contribute only to $Q_{t,u}^{LL}$ for chargino pairs and in addition to $Q_{u}^{LR}$ and $Q_{t}^{RL}$ for the associated channel. For the latter, we sum both charge conjugate processes, even though it might be interesting to identify the chargino charge, given that the dependence of its gaugino fraction on $M_2$ is slightly different for the two charges (see Fig. 4). The pair production of the second-lightest neutralino (not shown) does receive squark contributions from all generalized charges, but the corresponding cross section lies below $10^{-2}$ fb and is therefore invisible at RHIC. As our cross sections are computed at LO, they depend to some extent on the factorization scale $\mu_F$. Since this scale is unphysical and unknown, we vary it in the traditional way by a factor of two around the average final state mass,
representing the large perturbative scale in the partonic cross section (shaded bands).

Among the bosons exchanged in the s-channel, the W-boson is most sensitive to the polarization of the initial quarks and antiquarks, and consequently the single-spin asymmetry for the associated channel, shown in the right part of Fig. 5, reaches large values of around -20%. Note that polarization of the proton beam(s) will not be perfect, so that all calculated single-spin (double-spin) asymmetries should be multiplied by the degree of beam polarization. The single-spin asymmetry for the associated channel, shown in the right part of Fig. 5, reaches large values of around -20%. Note that polarization of the proton beam(s) will not be perfect.

The same applies to the single-spin asymmetry for realistic polarizations are in principle rather complex, e.g.

\[ N_{1,1} = \frac{1}{2}(1 + P_A)\frac{1}{2}(1 + P_B)\sigma_{1,1} + \frac{1}{2}(1 + P_A)\frac{1}{2}(1 - P_B)\sigma_{1,1} \]
\[ + \frac{1}{2}(1 - P_A)\frac{1}{2}(1 + P_B)\sigma_{-1,1} + \frac{1}{2}(1 - P_A)\frac{1}{2}(1 - P_B)\sigma_{-1,-1}, \]

where \(\sigma_{i,j}\) mean the ideal cross sections and \(P_A\) and \(P_B\) the beam polarizations, the double-spin asymmetry

\[ A_{LL} = \frac{N_{1,1} + N_{-1,-1} - N_{1,-1} - N_{-1,1}}{N_{1,1} + N_{-1,-1} + N_{1,-1} + N_{-1,1}} \]

simplifies to give \(A_{LL} = P_A P_B A'_{LL}\) with \(A'_{LL}\) composed of \(\sigma_{i,j}\). The same applies to the single-spin asymmetry \(A_L\) with just multiplication by \(P_A\) (or \(P_B\)).

As the mass of the neutralino increases and the gaugino fractions of the chargino and neutralino fall up to \(M_2 \leq 200\) GeV, the cross section and the absolute value of the asymmetry decrease, too. For these values of \(M_2\), the conditions of the LEP chargino mass limit still apply. The uncertainty in the scale variation is with 0.5% considerably smaller than the variation in the asymmetry of 2%, while the uncertainty coming from the polarized parton densities is with 1.5% of almost comparable size. Single-spin asymmetry measurements for associated chargino-neutralino production at the only existing polarized hadron collider RHIC could therefore be used to determine the gaugino and Higgsino components of charginos and neutralinos, provided the polarized quark and antiquark densities at momentum fractions of \(x_{a,b} \approx 2 \times 80\) GeV / 500 GeV = 0.32 are slightly better constrained. For the double-spin asymmetry (not shown), the parton density uncertainty exceeds the variation and leads to a sign change of the relatively small asymmetry (+6%/-3%), so that in this case no useful information on the gaugino/Higgsino mixing can be extracted.

The single- and double-spin asymmetries for neutralino pairs reach similar sizes as those for the associated channel, since the left- and right-handed couplings of the Z-boson exchanged in the s-channel are also different. However, we do not show them here, since the corresponding cross section is unfortunately too small at RHIC, as mentioned above. The variation of the asymmetries would, indeed, be quite dramatic: \(A_L\) changes its sign from -20% to +20% for \(M_2 \leq 200\) GeV, and \(A_{LL}\) falls from -5% to -20%.

FIG. 5: Unpolarized gaugino-pair production cross sections (left) and single-spin asymmetries for chargino-neutralino associated production (right) with \(m_{\tilde{g}} \approx m_{\tilde{g}}^\pm \approx 80\) GeV in \(pp\) collisions at RHIC and \(\sqrt{s} = 500\) GeV using LO GRV [17] and GRSV [18] parton densities. We choose \(\tan\beta = 10, \mu > 0\) using Eq. (12), and fix \(M_1 = \frac{\mu}{\tan\beta} M_2\).
For chargino pairs, massless photons can be exchanged in the s-channel, see Eq. (8), which leads to single- and double-spin asymmetries (not shown) that vary very little with \( M_2 \) and that can therefore not be used to extract information on gaugino/Higgsino mixing. In addition, these asymmetries depend strongly on the polarized parton densities.

C. Tevatron cross sections and asymmetries

The \( pp \) collider Tevatron will continue running in 2009 and possibly in 2010, and the future accelerator program at Fermilab is currently less clear than ever. The feasibility of polarizing the proton beam has been demonstrated many years ago [5]. It would require replacing some of the dipoles with higher-field magnets to gain space to install the six required Siberian snakes at a very moderate cost [31] and would represent an interesting possibility for QCD studies as well as new physics searches. Given the recent impressive achievements at RHIC, the degree of polarization should be comparable, i.e. about 65%-70%. Polarization of the antiproton beam is, however, much more challenging.

In the upper left part of Fig. 6 we show the total unpolarized cross sections for gaugino production with \( m_{\tilde{\chi}_1^+} = 151 \) GeV at the Tevatron, which is currently running at \( \sqrt{s} = 1.96 \) TeV and expected to produce a total integrated luminosity of 4–8 fb\(^{-1}\) up to 2009. Therefore, besides the pair production of the lightest chargino (short-dashed line) and its associated production with the second-lightest neutralino (dot-dashed line), also pair production of the latter might be visible (long-dashed line), at least for low values of \( M_2 \leq 300 \) GeV, where the gaugino component is still large (see Figs. 3 and 4 and the cross section exceeds 1 fb (short-dashed horizontal line). The influence of squark exchanges and the dependence on the squark mass are clearly visible in this channel, whereas they are again much smaller (but slightly larger than at RHIC) for the other two channels. The factorization scale dependence (shaded bands) remains modest (10%-13%) at the Tevatron.

The single-spin asymmetry for chargino-pair production (upper right part of Fig. 6) at a possible proton beam polarization upgrade of the Tevatron could be very large and reach -40%. Since the physical mass has been fixed at 151 GeV and the unpolarized cross section stays almost constant, the reduction in absolute value by about 6% for any given curve is directly related to the reduction of the gaugino fraction, as \( M_2 \) increases. The parton density (and factorization scale) uncertainties are (much) smaller than this variation, i.e. 2% (or 1%), so that significant information could be extracted from this asymmetry. On the other hand, the double-spin asymmetry (not shown), although large with about -20%, is almost insensitive to the gaugino fraction and would furthermore require polarization of the antiproton beam, which is a technical challenge.

In contrast to our results for RHIC, the associated channel (not shown) is not very interesting at the Tevatron. While the single- and double-spin asymmetries may be large (about -10% and +15%, respectively), they are almost constant and would not yield new information on the gaugino fractions.

The single- (lower left part of Fig. 6) and double-spin asymmetries (lower right part of Fig. 6) for the pair production of the second lightest neutralino are most sensitive to its gaugino component and (relatively modest) mass variation, in particular for the low values of \( M_2 \leq 300 \) GeV, where the cross section should be visible. Here, \( A_L \) changes sign from -50% to almost +30% and the theoretical uncertainties are extremely small. In the same region, the absolute value of \( A_{LL} \) increases by about 5% and can almost reach -35% for the standard GRSV parameterization of the polarized parton densities. The parton density uncertainty remains modest with about 3%. For large \( M_2 \geq 300 \) GeV, both asymmetries are constant in this channel.

D. LHC cross sections and asymmetries

As the LHC is nearing completion, different upgrade scenarios are emerging, concerning foremost higher luminosity and beam energy [32], but also beam polarization [6]. It is interesting to remember that a detailed study has been performed some time ago for the SSC, resulting in a design that had reserved 52 lattice locations for the future installation of Siberian snakes [31]. Since this is currently not the case at the LHC, its polarization upgrade would require replacing some of the dipoles with higher-field magnets to create these locations, just as in the case of the Tevatron. The number of resonances to be crossed during acceleration would be considerably larger due to the higher energy of the LHC, requiring longer tuning before ultimately reaching polarizations of up to 65%-70%.

For \( pp \) collisions of 14 TeV center-of-mass energy at the LHC, we show the unpolarized total cross sections for a chargino of mass 151 GeV in the upper left part of Fig. 7. Low-luminosity LHC runs of 10 fb\(^{-1}\) per year are currently scheduled for the years 2009 to 2011, and the high-luminosity phase of 100 fb\(^{-1}\) per year should start in 2011. With these luminosities, pair production of the lightest chargino (short-dashed line), its associated production with the second-lightest neutralino (dot-dashed line), and pair production of the latter (long-dashed line) should all be well visible. Whereas the cross sections for the first two channels are again almost constant and fairly independent of the
FIG. 6: Unpolarized gaugino-pair production cross sections (top left), single-spin asymmetries for chargino- (top right) and neutralino-pair production (bottom left), and double-spin asymmetries for neutralino-pair production (bottom right) with $m_{\tilde{\chi}_0^2} \approx m_{\tilde{\chi}_1^\pm} = 151$ GeV in $p\bar{p}$ collisions at the Tevatron and $\sqrt{S} = 1.96$ TeV using LO GRV [17] and GRSV [18] parton densities. We choose $\tan \beta = 10$, $\mu > 0$ using Eq. (12), and fix $M_1 = \frac{3}{4} \tan^2 \theta_W M_2$.

squark mass, at least for the Higgsino-like region of $M_2 \geq 300$ GeV, the neutralino-pair production cross section is again quite sensitive to squark exchanges in the gaugino-like region below that value and stays almost constant above. The factorization scale dependence is very small at the LHC and included in the line width of the upper left part of Fig. 7. Note, however, that with 100 fb$^{-1}$ of data, the mass of the lightest chargino will only be measured with an uncertainty of $\pm 11\%$ [33]. This induces a very visible uncertainty (shaded bands) in the total cross sections (lower left part of Fig. 7).

For a possible polarization upgrade of the LHC [6], we show the single-spin asymmetry for neutralino-pair production in the right parts of Fig. 7 again with the scale (line width, top) and chargino mass (shaded bands, bottom) uncertainty. At $M_2 \geq 300$ GeV, where the gaugino fraction is small, the asymmetry is not very interesting, as it is almost constant and smaller than 5%. In the gaugino-like region at $M_2 \leq 300$ GeV, it changes sign from -20% to almost +10%, a variation, that is considerably larger than the parton density uncertainty of at most 7%, the squark mass dependence of at most 2%, the almost invisible scale dependence, and also the chargino mass uncertainty of 3% to 10%. At a polarized LHC, a measurement of the single-spin asymmetry for neutralino-pair production would therefore yield interesting information about its gaugino fraction.
While the cross sections vary very little when changing the sign of $\mu$ or varying $\tan \beta$, it is interesting to study further the single-spin asymmetries for neutralino pairs in these alternative scenarios. When comparing the asymmetry for $\mu < 0$, shown in the upper left part of Fig. 8, to the one for $\mu > 0$, shown in the upper right part of Fig. 7, one notices an even steeper rise in the former to more than +15%, as $M_2$ approaches the critical value of $2m_{\tilde{\chi}_1^\pm}$, the neutralino changes its character from gaugino to Higgsino, and the sign and (smaller) absolute value of the Higgsino mass parameter $\mu$ become of particular importance.

A similar effect is observed when comparing for $\mu > 0$ the asymmetry with a lower value of $\tan \beta = 2$ in the upper right part of Fig. 8 to the one for the standard value of $\tan \beta = 10$ in the upper right part of Fig. 7. In this case, the asymmetry rises less (barely above zero) towards $M_2 = 2m_{\tilde{\chi}_1^\pm}$, where the gaugino/Higgsino decomposition is flipped, the ratio of the two Higgs vacuum expectation values $\tan \beta$ is particularly important, and the absolute value of the Higgsino mass parameter $\mu$ is effectively larger than in the standard scenario.

The dependence on $\tan \beta$ at the critical point $M_2 = 2m_{\tilde{\chi}_1^\pm}$ can be seen more clearly in the lower left part of Fig. 8 and indeed the asymmetry decreases from large to small $\tan \beta$ from distinctively positive values to values at or
below zero for all choices of squark masses (1 TeV or 325 GeV) and parton density functions (standard or valence GRSV parameterizations). This decrease is correlated with a similar decrease in the $B$-ino fraction $|N_{21}|^2$ and with an increase in the $W$-ino fraction $|N_{22}|^2$ of the second-lightest neutralino, while the Higgsino fractions $|N_{23}|^2$ and $|N_{24}|^2$ stay almost constant, as can be seen in the lower right part of Fig. 8.

The double-spin asymmetry for neutralino pairs, as well as the one for chargino pairs and the associated channel, are always smaller than 4% and 2%, respectively. Furthermore, they vary by less than 2% and are therefore not shown here. The single-spin asymmetry for chargino pairs (not shown) can reach a slightly larger value of -12%, but again it varies by less than 3% as a function of $M_2$, which is almost of the same size as the parton density uncertainty (2%). The situation for the single-spin asymmetry of the associated channel (not shown, either) is similar with a maximum of -10%, a variation with $M_2$ of about 1% and a parton density uncertainty of less than 1%.

**FIG. 8:** Single-spin asymmetries for neutralino-pair production with $\tan \beta = 10$ and $\mu < 0$ (top left), $\tan \beta = 2$ and $\mu > 0$ (top right), and $M_2 = 2m_{\tilde{\chi}^1_1}$ with $\mu > 0$ as a function of $\tan \beta$ (bottom left) in $pp$ collisions at the LHC and $\sqrt{s} = 14$ TeV. For the third scenario, we show also the gaugino and Higgsino fractions of the second-lightest neutralino (bottom right). We fix $\mu$ using Eq. (12) and $M_1 = \frac{1}{2} m_{\tilde{q}} \tan^2 \theta_W M_2$. 

$p p \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_2$ at LHC, $\sqrt{s} = 14$ TeV

$p p \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_2$ at LHC, $\sqrt{s} = 14$ TeV

$p p \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_2$ at LHC, $\sqrt{s} = 14$ TeV

$p p \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_2$ at LHC, $\sqrt{s} = 14$ TeV
IV. CONCLUSION

In summary, we have presented an exploratory study of gaugino-pair production in polarized hadron collisions at RHIC and possible upgrades of the Tevatron and the LHC, focusing on the correlation of beam polarization and gaugino/Higgsino mixing in the MSSM. Assuming gaugino mass unification at the GUT scale, a typical value of \( \tan \beta = 10 \) (or 2), a known lightest chargino mass and restricting ourselves (mostly) to positive values of \( \mu \), favored by the anomalous magnetic moment of the muon, we computed gaugino cross sections and beam polarization asymmetries as a function of the SUSY-breaking mass parameter \( M_2 \) without imposing a particular SUSY-breaking model.

While the unpolarized cross sections were held almost constant by imposing a fixed chargino mass, the single-spin asymmetries were found to be strongly correlated with \( M_2 \) and therefore the gaugino fractions of the lightest chargino and second-lightest neutralino in their associated production at RHIC, where the asymmetries could reach -20%. Even larger asymmetries of up to -40% would be obtained at a polarization upgrade of at least the proton beam of the Tevatron, where the single-spin asymmetry of chargino-pair production and the single- and double-spin asymmetries of neutralino-pair production would be the most promising observables. At the LHC, proton beam polarization would make it possible to measure in particular the single-spin asymmetry of neutralino pairs, which changes sign as \( M_2 \) grows and the gaugino fraction of the second-lightest neutralino falls.

For the channels mentioned above, the theoretical uncertainties coming from parton density, factorization scale, and squark mass variations and a realistic experimental uncertainty on the lightest chargino mass were found to be smaller than the differences induced in the asymmetries by variations of the gaugino fractions. However, more information on the so far poorly constrained polarized parton densities would clearly increase the impact of an analysis of gaugino-pair production in polarized hadron collisions.

Acknowledgments

We thank A. de Roeck, M. Drees, S. Kraml, P. Newman, T. Roser, I. Schienbein and W. Vogelsang for useful discussions. This work has been supported by a Ph.D. fellowship of the French ministry for education and research and by the Theory-LHC-France initiative of the CNRS/IN2P3.

APPENDIX A: GAUGINO AND HIGGSINO MIXING

The soft SUSY-breaking terms in the minimally supersymmetric Lagrangian include a term [1]

\[ \mathcal{L} \supset - \frac{1}{2} (\psi^0)^T Y \psi^0 + \text{h.c.}, \]  

(A1)

which is bilinear in the (two-component) fermionic partners

\[ \psi^0_j = (-i \tilde{B}, -i \tilde{W}^3, \tilde{H}^0_1, \tilde{H}^0_2)^T \quad \text{with} \quad j = 1, \ldots, 4 \]  

(A2)

of the neutral electroweak gauge and Higgs bosons and proportional to the, generally complex and necessarily symmetric, neutralino mass matrix

\[ Y = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}. \]  

(A3)

Here, \( M_1, M_2, \) and \( \mu \) are the SUSY-breaking \( B \)-ino, \( W \)-ino, and off-diagonal Higgsino mass parameters with \( \tan \beta = s_\beta/c_\beta = v_u/v_d \) being the ratio of the vacuum expectation values \( v_u, v_d \) of the two Higgs doublets, while \( m_Z \) is the SM \( Z \)-boson mass and \( s_W, c_W \) the sine (cosine) of the electroweak mixing angle \( \theta_W \). After electroweak gauge-symmetry breaking and diagonalization of the mass matrix \( Y \), one obtains the neutralino mass eigenstates

\[ \chi^0_i = N_{ij} \psi^0_j, \quad i = 1, \ldots, 4, \]  

(A4)

where \( N \) is a unitary matrix satisfying the relation

\[ N^* Y N^{-1} = \text{diag} (m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}). \]  

(A5)
In four-component notation, the Majorana-fermionic neutralino mass eigenstates can be written as
\[
\tilde{\chi}_i^0 = \left( \begin{array}{c} \tilde{\psi}_i^0 \\ \tilde{\chi}_i \end{array} \right).
\] (A6)

The application of projection operators leads to relatively compact analytic expressions for the mass eigenvalues
\[ m_{\tilde{\chi}_i^0} < m_{\tilde{\psi}_i^0} < m_{\tilde{\chi}_2^0} \] [34]. As we choose them to be real and non-negative, our unitary matrix \( N \) is generally complex [35]. For the MSSM with additional \( CP \)-violating phases see Ref. [36] and the references therein.

The chargino mass term in the SUSY Lagrangian [1]
\[
\mathcal{L}^c \supset - \frac{1}{2} (\psi^+ \psi^-) \left( \begin{array}{cc} 0 & X^T \\ X & 0 \end{array} \right) \left( \begin{array}{c} \psi^+ \\ \psi^- \end{array} \right) + \text{h.c.}
\] (A7)
is bilinear in the (two-component) fermionic partners
\[
\psi_j^\pm = (-i \tilde{W}^\pm, \tilde{H}_2^\pm)^T \quad \text{with} \quad j = 1,\ldots, 2
\] (A8)
of the charged electroweak gauge and Higgs bosons and proportional to the, generally complex, chargino mass matrix
\[
X = \left( \begin{array}{cc} M_2 & m_W \sqrt{2} s_\beta \\ m_W \sqrt{2} c_\beta & \mu \end{array} \right),
\] (A9)
where \( m_W \) is the mass of the SM \( W \)-boson. Since \( X \) is not symmetric, it must be diagonalized by two unitary matrices \( U \) and \( V \), which satisfy the relation
\[
U^* X V^{-1} = \text{diag} \left( m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm} \right)
\] (A10)
and define the chargino mass eigenstates
\[
\chi_i^\pm = V_{ij} \psi_j^+, \quad \chi_j^- = U_{ij} \psi_j^-, \quad i, j = 1, 2.
\] (A11)

In four-component notation, the Dirac-fermionic chargino mass eigenstates can be written as
\[
\tilde{\chi}_i^\pm = \left( \begin{array}{c} \tilde{\psi}_i^\pm \\ \tilde{\chi}_i \end{array} \right).
\] (A12)

As Eq. (A10) implies
\[
V X^T V^{-1} = \text{diag} \left( m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm} \right),
\] (A13)
the hermitian matrix \( X^T X \) can be diagonalized using only \( V \), and its eigenvalues
\[
m_{\tilde{\chi}_1^\pm} = \frac{1}{2} \left\{ |M_2|^2 + |\mu|^2 + 2 m_W^2 \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2 m_W^2)^2 - 4 |\mu M_2 - m_W s_\beta|^2} \right\}
\] (A14)
are always real. If we take also the mass eigenvalues \( m_{\tilde{\chi}_1^\pm} \leq m_{\tilde{\chi}_2^\pm} \) to be real and non-negative, the rotation matrix
\[
V = \left( \begin{array}{cc} \cos \theta_+ & \sin \theta_+ e^{-i \phi^+} \\ -\sin \theta_+ e^{i \phi^+} & \cos \theta_+ \end{array} \right)
\] (A15)
can still be chosen to have real diagonal elements, but the off-diagonal phase \( e^{i \phi^+} \) is needed to rotate away the imaginary part of the off-diagonal matrix element in \( X^T X \),
\[
\Im \left[ (M_2^2 s_\beta + \mu c_\beta) e^{i \phi^+} \right] = 0.
\] (A16)
The rotation angle \( \theta_+ \in [0; \pi] \) is uniquely fixed by the two conditions
\[
\tan 2 \theta_+ = \frac{2 \sqrt{2} m_W (M_2^2 s_\beta + \mu c_\beta) e^{i \phi^+}}{|M_2|^2 - |\mu|^2 + 2 m_W^2 c_\beta}
\] (A17)
\[
\sin 2 \theta_+ = \frac{-2 \sqrt{2} m_W (M_2^2 s_\beta + \mu c_\beta) e^{i \phi^+}}{\sqrt{(|M_2|^2 - |\mu|^2 + 2 m_W^2 c_\beta)^2 + 8 m_W^2 [(M_2^2 s_\beta + \mu c_\beta) e^{i \phi^+}]^2}}.
\] (A18)

Once \( V \) is known, the unitary matrix \( U \) can be obtained from
\[
U = \text{diag} \left( m_{\tilde{\chi}_1^\pm}^{-1}, m_{\tilde{\chi}_2^\pm}^{-1} \right) V^* X^T.
\] (A19)

For the MSSM with additional \( CP \)-violating phases see again Ref. [36] and the references therein.
APPENDIX B: FEYNMAN RULES

For the electroweak interaction, we define the square of the weak coupling constant 
\[ g^2 = e^2 / \sin^2 \theta_W \]
in terms of the electromagnetic fine structure constant \( \alpha = e^2 / (4\pi) \) and the squared sine of the electroweak mixing angle \( x_W = \sin^2 \theta_W = s_W^2 = 1 - \cos^2 \theta_W = 1 - c_W^2 \). Following the standard notation, the \( \gamma \) and \( \chi^\pm \) interaction vertices shown in Fig. 9 are proportional to \( \delta_{ij} \) and \( [1] \):

\[
O_{ij}^L = -\frac{c_W}{\sqrt{2}} N_{i4} V_{j2}^* + c_W N_{i2} V_{j1}^* \quad \text{and} \quad O_{ij}^R = \frac{c_W}{\sqrt{2}} N_{i3} U_{j2} + c_W N_{i2} U_{j1},
\]

\[
O_{ij}^{L'} = -V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^* + \delta_{ij} x_W \quad \text{and} \quad O_{ij}^{R'} = -U_{i1} U_{j1} - \frac{1}{2} U_{i2} U_{j2} + \delta_{ij} x_W,
\]

\[
O_{ij}^{L''} = -\frac{1}{2} N_{i3} N_{j3}^* + \frac{1}{2} N_{i4} N_{j4}^* \quad \text{and} \quad O_{ij}^{R''} = \frac{1}{2} N_{i3}^* N_{j3} - \frac{1}{2} N_{i4}^* N_{j4}.
\]

The interaction vertices of left- and right-handed quarks with electroweak gauge bosons shown in Fig. 10 are
violating interactions are treated using the rules in Ref. [39].

where the matrices

\[ L_{\bar{\chi}^0 L} \] and \( R_{\bar{\chi}^0 L} \) relate to the gaugino/Higgsino mixing (see App. A). All other couplings vanish due to (electromagnetic) charge conservation (e.g. \( L_{\bar{\chi}^0 L} \)). These general expressions can be simplified by neglecting the Yukawa couplings except for the one of the top quark, whose mass is not small compared to \( m_W \). Fermion number violating interactions are treated using the rules in Ref. [39].
In general SUSY models with non-minimal flavor violation \cite{40}, the diagonalization of the mass squark matrices $M_u^2$ and $M_d^2$ requires the introduction of two additional $6 \times 6$ matrices $R^u$ and $R^d$ with

$$\text{diag} (m_{\tilde{u}_1}^2, \ldots, m_{\tilde{u}_6}^2) = R^u M_u^2 R^{u\dagger} \quad \text{and} \quad \text{diag} (m_{\tilde{d}_1}^2, \ldots, m_{\tilde{d}_6}^2) = R^d M_d^2 R^{d\dagger}. \quad (B4)$$

By convention, the masses are ordered according to $m_{\tilde{u}_1} < \cdots < m_{\tilde{u}_6}$. The physical mass eigenstates are given by

$$\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \\ \tilde{u}_4 \\ \tilde{u}_5 \\ \tilde{u}_6 \end{pmatrix} = R^u \begin{pmatrix} \tilde{u}_L \\ \tilde{c}_L \\ \tilde{t}_L \\ \tilde{u}_R \\ \tilde{c}_R \\ \tilde{t}_R \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{d}_3 \\ \tilde{d}_4 \\ \tilde{d}_5 \\ \tilde{d}_6 \end{pmatrix} = R^d \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_L \\ \tilde{d}_R \\ \tilde{s}_R \\ \tilde{b}_R \end{pmatrix}. \quad (B5)$$

In the limit of vanishing off-diagonal parameters in the mass matrices, the matrices $R^q$ become flavor-diagonal, leaving only the well-known helicity mixing already present in constrained minimal flavor violation. Since we consider numerically only the exchange of nonmixing squarks, we take there $R^d = 1$.

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