Vacua by Derivative Corrections in $\mathcal{N} = 1$ Supergravity with Matter Multiplets

Atsuki Inoue$^\dagger$ and Shin Sasaki$^\ddagger$

Department of Physics, Kitasato University, Sagamihara 228-8555, Japan

We study the vacuum structures of four-dimensional $\mathcal{N} = 1$ old minimal supergravity with higher derivative corrections. We find that $\mathcal{N} = 1$ supergravity with Riemann curvature square corrections and higher derivative matter chiral multiplets induces a non-trivial de Sitter vacuum, even in the absence of superpotentials. This vacuum generically breaks supersymmetry. We show that the auxiliary fields in the gravity and the chiral multiplets play important roles to generate a potential in supersymmetric higher derivative theories.

I. INTRODUCTION

Vacuum structures in supergravity theories have been intensively studied due to their phenomenological importance. They are highly constrained because of the restricted scalar potential properties. For example, four-dimensional $\mathcal{N} = 1$ supergravities exhibit scalar potentials of the generic form $V = e^K (g^{ij} D_i W D_j W - 3 |W|^2)$ possibly with D-term contributions. Here $K, g_{ij}$ are the Kähler potential and the Kähler metric, $D_i$ is the covariant derivative in the Kähler geometry and $W$ is a superpotential. A widely known fact that any de Sitter vacuum in $\mathcal{N} = 1$ supergravity break supersymmetry is based on this potential structure. In order to admit (meta)stable de Sitter vacua, which are supposed to describe our universe, appropriate Kähler and superpotentials have to be prepared. However, this is not always possible when supergravities are viewed as low-energy effective theories of UV theories, like string theories [1, 2].

On the other hand, low-energy effective theories generally receive derivative corrections. For example, the fourth order derivative corrections to the gravity sector are given by the curvature square terms. It is known that the bosonic $\mathcal{R} + \mathcal{R}^2$ gravity contains a real propagating degree of freedom known as the scalaron [3]. Here $\mathcal{R}$ is the Ricci scalar. Due to the new scalar degree of freedom appearing in the derivative corrections, there are room for the modifications of the scalar potentials. Indeed, the bosonic $\mathcal{R} + \mathcal{R}^2$ gravity is equivalently dual to the Einstein gravity with a real scalar field accompanied by a non-trivial potential. This was used to realize an inflation model [7]. In the supersymmetric counterpart, this was first discussed in [8] and supersymmetric completions of the $\mathcal{R} + \mathcal{R}^2$ gravity in the old minimal [9] and the new minimal frameworks [10] have been studied. For the former, the scalar and the vector auxiliary fields $M, \nabla^n b_m$ become propagating. In the dual picture they together with the scalaron form two chiral multiplets $S$ and $T$ while the dual for the latter contains a chiral multiplet. The difference of the auxiliary fields in $\mathcal{N} = 1$ supergravity traces back to the gauge fixing in the superconformal tensor calculus [11]. They are a class of $f(\mathcal{R}, \nabla \mathcal{R})$ supergravities, where $f$ is an arbitrary function of the curvature superfields $\mathcal{R}, \nabla \mathcal{R}$. Vacuum structures of $f(\mathcal{R}, \nabla \mathcal{R})$ pure supergravity [12] and with the chiral matter multiplets [13] have been studied.

Things get more involved when higher derivative corrections in chiral matter sectors are introduced. It is known that supersymmetric completions of higher derivative matter terms, even in the absence of the gravity sector, are generically cumbersome issue. This is mainly due to the auxiliary field problem [14, 15] and the presence of the Ostrogradski’s ghosts [16]. The former stems from the fact that the equation of motion for the auxiliary field $F$ in the chiral multiplet ceases to be algebraic. The latter is a fate of higher spacetime derivatives of fields of the form $\partial^n \varphi$ ($n \geq 2$). The higher derivative chiral model [17, 18] is a supersymmetric completion of higher derivative scalar models that does not suffer from these problems. This is given by the form

$$U(\Phi, \bar{\Phi}) D_\alpha \Phi D^{\alpha} \bar{\Phi} D_\beta \Phi D^{\beta} \bar{\Phi}$$

in the D-term. Here $D, \bar{D}$ are the $\mathcal{N} = 1$ supercovariant derivatives and $\Phi$ is a chiral superfield, $U(\Phi, \bar{\Phi})$ is an arbitrary function of $\Phi, \bar{\Phi}$ and the spacetime derivatives of them. This has been utilized to write down a supersymmetric Dirac-Born-Infeld model [20, 21], Skyrme models [22, 25] and to study nonlinear realizations [26], BPS states [27, 29] and so on.

It is remarkable that the higher derivative chiral model admits non-trivial scalar potentials even in the absence of superpotentials. A rich structure of scalar potentials arises from the non-trivial solutions to the equation of motion for the auxiliary field. As we will show in below, this mechanism is in contrast to the situation in the gravity sector in which explicit scalar potentials appear in the dual frame. The higher derivative chiral model coupled with the Einstein supergravity has been studied [30, 31]. It is shown that the scalar potential induced by the higher derivative chiral model is negative semi-definite which results in anti-de Sitter or Minkowski vacua. They are uplifted by the D-term contributions from the gauge sector.

In this letter, we study vacuum structures of a four-dimensional $\mathcal{N} = 1$ model including the fourth order derivative corrections both in the gravity and the chiral matter sectors. We show that supergravity with curvature square terms coupled with the higher derivative chiral model induces non-trivial scalar potential and exhibits a de Sitter vacuum even in the absence of super-
potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.

\[ \mathcal{L} = \frac{3}{8} \int d^3 \Theta 2 \mathcal{E} (D^2 - 8R) \left[ e^{-\frac{\kappa}{2}} + \alpha \left( R\tilde{R} - \frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \right) \right] \\
- \frac{1}{3} U(\Phi, \bar{\Phi}) \mathcal{D}^\alpha \Phi \mathcal{D}^\beta \bar{\Phi} \mathcal{D}^\alpha \bar{\Phi} + h.c., \]  

(2)

where the curvature superfields \( G_{\alpha\beta} G^{\alpha\beta} \) and \( W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \) contain the Ricci and the Weyl tensor squares, respectively. For simplicity, we focus on a model where \( \mathcal{L} = 1 \) in the following. As is well-known, the potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.

\[ \mathcal{L} = \frac{3}{8} \int d^3 \Theta 2 \mathcal{E} (D^2 - 8R) \left[ e^{-\frac{\kappa}{2}} + \alpha \left( R\tilde{R} - \frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \right) \right] \\
- \frac{1}{3} U(\Phi, \bar{\Phi}) \mathcal{D}^\alpha \Phi \mathcal{D}^\beta \bar{\Phi} \mathcal{D}^\alpha \bar{\Phi} + h.c., \]  

(2)

where the curvature superfields \( G_{\alpha\beta} G^{\alpha\beta} \) and \( W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \) contain the Ricci and the Weyl tensor squares, respectively [33]. For simplicity, we focus on a model where \( \mathcal{L} = 1 \) in the following. As is well-known, the potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.

\[ \mathcal{L} = \frac{3}{8} \int d^3 \Theta 2 \mathcal{E} (D^2 - 8R) \left[ e^{-\frac{\kappa}{2}} + \alpha \left( R\tilde{R} - \frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \right) \right] \\
- \frac{1}{3} U(\Phi, \bar{\Phi}) \mathcal{D}^\alpha \Phi \mathcal{D}^\beta \bar{\Phi} \mathcal{D}^\alpha \bar{\Phi} + h.c., \]  

(2)

where the curvature superfields \( G_{\alpha\beta} G^{\alpha\beta} \) and \( W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \) contain the Ricci and the Weyl tensor squares, respectively [33]. For simplicity, we focus on a model where \( \mathcal{L} = 1 \) in the following. As is well-known, the potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.

\[ \mathcal{L} = \frac{3}{8} \int d^3 \Theta 2 \mathcal{E} (D^2 - 8R) \left[ e^{-\frac{\kappa}{2}} + \alpha \left( R\tilde{R} - \frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \right) \right] \\
- \frac{1}{3} U(\Phi, \bar{\Phi}) \mathcal{D}^\alpha \Phi \mathcal{D}^\beta \bar{\Phi} \mathcal{D}^\alpha \bar{\Phi} + h.c., \]  

(2)

where the curvature superfields \( G_{\alpha\beta} G^{\alpha\beta} \) and \( W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \) contain the Ricci and the Weyl tensor squares, respectively [33]. For simplicity, we focus on a model where \( \mathcal{L} = 1 \) in the following. As is well-known, the potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.

\[ \mathcal{L} = \frac{3}{8} \int d^3 \Theta 2 \mathcal{E} (D^2 - 8R) \left[ e^{-\frac{\kappa}{2}} + \alpha \left( R\tilde{R} - \frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \right) \right] \\
- \frac{1}{3} U(\Phi, \bar{\Phi}) \mathcal{D}^\alpha \Phi \mathcal{D}^\beta \bar{\Phi} \mathcal{D}^\alpha \bar{\Phi} + h.c., \]  

(2)

where the curvature superfields \( G_{\alpha\beta} G^{\alpha\beta} \) and \( W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \) contain the Ricci and the Weyl tensor squares, respectively [33]. For simplicity, we focus on a model where \( \mathcal{L} = 1 \) in the following. As is well-known, the potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.

\[ \mathcal{L} = \frac{3}{8} \int d^3 \Theta 2 \mathcal{E} (D^2 - 8R) \left[ e^{-\frac{\kappa}{2}} + \alpha \left( R\tilde{R} - \frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} \right) \right] \\
- \frac{1}{3} U(\Phi, \bar{\Phi}) \mathcal{D}^\alpha \Phi \mathcal{D}^\beta \bar{\Phi} \mathcal{D}^\alpha \bar{\Phi} + h.c., \]  

(2)

where the curvature superfields \( G_{\alpha\beta} G^{\alpha\beta} \) and \( W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \) contain the Ricci and the Weyl tensor squares, respectively [33]. For simplicity, we focus on a model where \( \mathcal{L} = 1 \) in the following. As is well-known, the potentials and gauge sectors. This is in contrast to the models in Refs. [30, 31] and shows that derivative corrections to the gravity sector can uplift the anti-de Sitter vacua in a natural way.
0 and the vanishing fermions in the following. Then the solutions to the equation \[ F \ddot{F} + \frac{d}{dx} \left( \frac{d}{dx} \right) \left( \frac{2}{3} \right) G_{A \bar{A}} \] are given by
\[
F \ddot{F} = \omega^3 \left( -\frac{q}{2} + \sqrt{\left( \frac{q}{2} \right)^2 + \left( \frac{p}{3} \right)^3} \right) \\
+ \omega^3 \left( -\frac{q}{2} - \sqrt{\left( \frac{q}{2} \right)^2 + \left( \frac{p}{3} \right)^3} \right) + 2 \frac{e^{-\frac{\kappa}{3}}}{32U} G_{A \bar{A}},
\]
where \( \omega^3 = 1, a = 0, 1, 2 \) and we have defined
\[
G_{A \bar{A}} = g_{A \bar{A}} - \frac{1}{3} K_A K_{\bar{A}}, \\
p = -\frac{1}{3} \left( \frac{e^{-\frac{\kappa}{3}}}{32U} \right)^2 G_{A \bar{A}}^2, \\
q = -\frac{1}{9} \left( \frac{e^{-\frac{\kappa}{3}}}{32U} \right)^2 K_A K_{\bar{A}} M \bar{M} - \frac{2}{27} \left( \frac{e^{-\frac{\kappa}{3}}}{32U} \right)^3 G_{A \bar{A}}^3.
\]
We note that the formal expression \[ (5) \] is justified only when \( F \ddot{F} \geq 0 \), otherwise there are no solutions. The fact that there are generically three independent solutions \[ (5) \] for \( a = 0, 1, 2 \) causes several consequences in the on-shell Lagrangian. First, there are three distinct on-shell branches associated with the three solutions \( a = 0, 1, 2 \) [27,29]. Second, the fourth order term of \( F \) in the Lagrangian induces non-trivial scalar potential even in the absence of superpotentials \( W \). Indeed, we find that the scalar potential is given by
\[
V(x, y) = \frac{1}{3} \pi F(x, y) + \frac{1}{3} K_A K_{\bar{A}} F(x, y) + 48U e^{\frac{\kappa}{3}} (F \ddot{F}(x, y))^2.
\]
Here \( x = M \bar{M}, y = A \bar{A} \) and \( F \ddot{F}(x, y) \) is a solution in \[ (5) \]. Although the scalar potential \[ (7) \] is independent of \( \alpha \) and \( \gamma \), it vanishes when the fourth derivative interactions are absent. Namely, when \( \alpha = \gamma = U = 0 \), we have \( F = M = 0 \) and \( V \) becomes trivial. For \( \alpha = \gamma = 0 \) but \( U \neq 0 \) case, we have a non-trivial \( F \) but the scalar field \( M \) remains to be auxiliary and it is integrated out. The resulting scalar potential is a negative semi-definite and allows only for anti-de Sitter (or Minkowski) vacua [31].

### III. VACUUM STRUCTURES

#### A. Potential minima

We now examine a minimum of the scalar potential \[ (7) \]. In the following, we consider the canonical Kähler potential of the simplest form \( K = A \bar{A} \) and \( U = \text{const.} = \beta \). Even in such a case, the scalar potential still possesses non-trivial structure. Since \( M \) is not an auxiliary field anymore, vacua are defined by minima both in the \( x = \)

![FIG. 1: The scalar potential in the \( a = 0 \) branch. \( \beta = 1 \). The red point indicates a minimum.](image-url)

\( M \bar{M} \) and \( y = A \bar{A} \) directions. We also note that the solutions \[ (5) \] are allowed when \( F \ddot{F} \) is positive semi-definite. This means that the range of VEVs of the scalar fields \( (x, y) \) is restricted. Since the functional form \( F \ddot{F}(x, y) \) is different for \( a = 0, 1, 2 \) branches, we study each branch separately.

1. \( a = 0 \) branch

First we focus on the \( a = 0 \) branch. This is always a real solution. For a fixed \( x \) and large values of \( y \), the dominant contribution comes from the first term in \[ (7) \] and the potential grows to infinity:
\[
V(x, y) \sim \left( \frac{1}{\beta^2} \right)^{\frac{1}{4}} e^{\frac{\kappa}{4}} (yx)^{\frac{1}{4}} \to \infty, \quad (x \neq 0, y \to \infty).
\]

Thus for \( \beta > 0, 5 - 40x < 0 \) or for \( \beta < 0, 5 - 40x > 0 \), the potential decreases along the \( y \)-direction and then it increases again as we go to \( y \to \infty \). Then we expect that there are minima in the \( a = 0 \) branch. A numerical analysis helps us to find a minimum of the potential.

For \( \beta > 0 \) case, the result is given in Fig 1. We find a minimum where the vacuum energy is negative, \( \Lambda = -1.1153 \times 10^{-4} \). The eigenvalues of the mass matrix
\[
M^2 = \frac{1}{2} \begin{pmatrix} \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j} & \frac{\partial^2 V}{\partial \varphi_i \partial \bar{\varphi}_j} \\ \frac{\partial^2 V}{\partial \bar{\varphi}_i \partial \varphi_j} & \frac{\partial^2 V}{\partial \bar{\varphi}_i \partial \bar{\varphi}_j} \end{pmatrix}, \quad \varphi_i = (A, M),
\]

where \( \varphi_i = (A, M) \) is an auxiliary field.
at the vacuum are given by \( m^2 = (5.2979, 4.3826 \times 10^{-3}, 0, 0) \) which guarantees that there are no tachyonic modes. The two zero-eigenvalues correspond to the Nambu-Goldstone modes associated with the phase directions of \( M \) and \( A \). Since the supersymmetry transformation of the chiral fermion is proportional to the auxiliary field \( F \) and \( |F| = 4.8905 \times 10^{-4} \neq 0 \), we find that the vacuum breaks supersymmetry. We also note that the minimum is in the region where the solution \( F \bar{F} \geq 0 \) is justified. See FIG. 2 for the allowed region of VEVs \((x, y)\).

![FIG. 2: The allowed region of the VEV \((x, y)\) for \( F \bar{F}(x, y) \geq 0 \). The red point represents the minimum of the potential (see Fig. 1).](image)

For \( \beta < 0 \), however, we find that the scalar potential is unbounded from below. It becomes infinitely small along the \( x \)-axis (FIG 3). Therefore there are no vacua in the \( \beta < 0 \) case.

2. \( a = 1, 2 \) branches

For the \( a = 1, 2 \) branches, the condition \( D = \left( \frac{q}{2} \right)^2 + \left( \frac{q}{2} \right)^3 < 0 \) is necessary for \( F \bar{F} \) to be real numbers. We find that for \( \beta > 0 \), this is not the case. In the case of \( \beta < 0 \), we find \( D < 0 \) and \( F \bar{F} \geq 0 \) provided \( y \geq 3 \). When this condition is satisfied, the solutions are given by

\[
F \bar{F} = 2 \sqrt{-\frac{p}{3}} \cos \left( \frac{2 \alpha \pi}{3} \right) + \frac{2 e^{-\frac{q}{3}}}{32U}, \quad (a = 1, 2)
\]

where

\[
\tan \theta = -\frac{2\sqrt{-D}}{q}.
\]

The plots of the scalar potentials are found in (FIG. 4). We find that there are infinitely negative directions along \( x = 0 \) both in the \( a = 1, 2 \) branches. Therefore they are unbounded from below and there are no minima in these branches.

![FIG. 3: The scalar potential in the allowed region of \( F \bar{F} \geq 0 \). The \( a = 0 \) branch for \( \beta = -1 \).](image)

![FIG. 4: The scalar potentials in the \( a = 1 \) (left) and the \( a = 2 \) (right) branches. The parameter is fixed to \( \beta = -1 \).](image)

**B. Einstein equation**

Given the vacuum energy, we solve the Einstein equation and determine the spacetime structures. The equation of motion for the metric is found to be

\[
\begin{align*}
R_{mn} - \frac{1}{2} g_{mn} \Lambda &= 0, \\
+ \frac{3\alpha + 2\gamma}{24} \left( 2R R_{mn} - \frac{1}{2} g_{mn} R^2 - 2 (\nabla_m \nabla_n - g_{mn} \nabla^2) R \right) \\
+ \frac{3\alpha + 4\gamma}{8} \left( \frac{1}{2} g_{mn} R_{pq} R^{pq} - 2 g_{pq} R_{mp} R_{nq} + \nabla_n \nabla_p R_{mq} g^{kp} \\
- \nabla_m \nabla_p R_{kn} g^{kp} + \nabla_k \nabla_l R_{mn} g^{kl} + \nabla_q \nabla_p R_{kl} g^{kp} g_{mn} \right) \\
+ \frac{\gamma}{4} \left( -\frac{1}{2} g_{mn} R_{klpq} R^{klpq} + 2 R_{mk} R_{n klp} + 2 R_{klnp} R^{kl}_{n p} \\
+ 4 \nabla_k \nabla_l R_{mlnk} \right) - \Lambda g_{mn} = 0,
\end{align*}
\]

where \( \Lambda \) is the vacuum energy. Assuming the de Sitter space ansatz;

\[
R_{mn} g_{pq} - g_{mp} g_{nq} = \frac{1}{\Lambda^2} (g_{mp} g_{nq} - g_{mq} g_{np}),
\]
where $\lambda$ is the de Sitter radius, we have the equation $\Lambda^4 - 3\lambda^2 + 3\gamma = 0$ from (13). The solutions are found to be
\[
\lambda^2 = \frac{3 \pm \sqrt{9 - 12\Lambda^2\gamma}}{2\Lambda}.
\] (15)

From this expression, we find that there is a solution $\lambda^2 > 0$ when $\gamma > 0$ and $\Lambda < 0$, namely, a de Sitter space is allowed even for the negative vacuum energy $\Lambda < 0$ (the minus sign in (15)). Indeed, for $\gamma > 0$ (cf. Eq. (3)), a de Sitter space $\lambda^2 > 0$ is allowed in the $\beta > 0$, $\alpha = 0$ branch with the vacuum energy $\Lambda = -1.1533 \times 10^{-4}$. Note that the solution (15) is independent of the coefficient $\alpha$.

IV. CONCLUSION

In this letter, we studied the four-dimensional $\mathcal{N} = 1$ old minimal supergravity coupled with the curvature square terms and the higher derivative chiral model. This is a natural model that contains fourth order spacetime derivatives both in the gravity and the chiral multiplets.

The auxiliary fields play an important role both in the gravity and the chiral multiplets. A well-known fact that the auxiliary field $M$ in the gravity multiplet becomes propagating in the presence of the curvature square terms, and the non-trivial solutions for $F$ in the higher derivative chiral matter sector, allows us to generate a non-vanishing scalar potential. We explicitly showed that the extra scalar field $\bar{F}$ together with the higher derivative chiral model generates a scalar potential even in the absence of superpotentials. The non-trivial scalar potentials are generated through each solution to the equation of motion of $F$. Although they are complicated, the explicit forms of the potentials enable us to find a minimum and the vacuum energy in the $\alpha = 0$ branch. We found that the vacuum generically breaks supersymmetry since the auxiliary field $\bar{F}$ at the vacuum is non-zero in general. Given the vacuum energy, we solved the Einstein equation including the curvature squares and found that a (meta) stable de Sitter space is allowed even for the negative vacuum energy. This is in contrast to the previous works [30, 31] where no curvature square terms are present. Our analysis showed that the de Sitter radius depends on the coefficient $\gamma$ of the Riemann curvature squares. This would be relevant in the heterotic supergravities [35, 36] where the $R_{mnkl}R^{mnkl}$ term plays an important role to cancel the anomaly.

In this letter, we focused on the simplest case, namely, the flat Kähler potential $K = AA$ and the constant $U$. Even such a case, we found rich structures of scalar potentials. This indicates that our findings will be useful for model building without superpotentials. Since low-energy effective theories generically have non-trivial Kähler potentials and the function $U$, it would be interesting to study vacuum structures in specific setups like the Skyrme matters [22, 25], the D-brane worldvolume theory [20, 21], string compactifications [2] and so on. In [31], anti-de Sitter vacua are uplifted to de Sitter ones by the D-term contributions of gauge sectors. It is natural to incorporate the derivative corrections to gauge sectors and study their roles in vacuum structures. It has been shown that the higher derivative chiral model admits non-standard supersymmetry breaking vacua such as modulated ground states [37–40]. It would also be interesting to study the corresponding spacetime structures in these vacua.

An inflation model based on the higher derivative chiral model has been studied in [41]. Their analysis shows that there is an intrinsic singularity of the speed of sound in the $\alpha = 2$ on-shell branch in the Einstein supergravity. It would be interesting to study the inflationary dynamics including the curvature squared corrections. We also expect that our analysis may shed light on the complete understanding of the string landscape and the swampland program [32, 49]. We will come back to these issues in future works.

Acknowledgments

The work of S.S. is supported in part by Grant-in-Aid for Scientific Research (C), JSPS KAKENHI Grant Number JP20K03952.

[1] S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B 584 (2000), 69-108 [arXiv:hep-th/9906070 [hep-th]].
[2] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D 68 (2003), 046005 [arXiv:hep-th/0301240 [hep-th]].
[3] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, JCAP 10 (2003), 013 [arXiv:hep-th/0308055 [hep-th]].
[4] P. Denef and M. R. Douglas, JHEP 05 (2004), 072 [arXiv:hep-th/0404116 [hep-th]].
[5] P. Denef and M. R. Douglas, JHEP 03 (2005), 061 [arXiv:hep-th/0411183 [hep-th]].
[6] B. Whitt, Phys. Lett. B 145 (1984), 176-178.
[7] A. A. Starobinsky, Phys. Lett. B 91 (1980), 99-102.
[8] S. Ferrara, M. T. Grisaru and P. van Nieuwenhuizen, Nucl. Phys. B 138 (1978), 430-444.
[9] S. Cecotti, Phys. Lett. B 190 (1987), 86-92.
[10] S. Cecotti, S. Ferrara, M. Porrati and S. Sabharwal, Nucl. Phys. B 306 (1988), 160-180.
[11] T. Kugo and S. Uehara, Nucl. Phys. B 226 (1983), 49-92.
[12] A. Hindawi, B. A. Ovrut and D. Waldram, Nucl. Phys. B 476 (1996), 175-199 [arXiv:hep-th/9511223 [hep-th]].
[13] A. Hindawi, B. A. Ovrut and D. Waldram, Phys. Lett. B 381 (1996), 154-162 [arXiv:hep-th/9602075 [hep-th]].
[14] S. J. Gates, Jr., Phys. Lett. B 365 (1996) 132 [hep-
\[\text{[15] S. J. Gates, Jr., M. T. Grisaru, M. E. Knutt and S. Pecati, Phys. Lett. B 503 (2001) 349 [hep-ph/0012301]; S. J. Gates, Jr., M. T. Grisaru, M. E. Knutt, S. Pecati and H. Suzuki, Nucl. Phys. B 596 (2001) 315 [hep-th/0009192], S. J. Gates, Jr., M. T. Grisaru and S. Pecati, Phys. Lett. B 481 (2000) 397 [hep-th/0002045].}\]

\[\text{[16] M. Ostrogradski, Mem. Ac. St. Petersbourg VI (1850) 385.}\]

\[\text{[17] J. Khoury, J. L. Lehners and B. Ovrut, Phys. Rev. D 83 (2011), 125031 [arXiv:1012.3745 [hep-th]].}\]

\[\text{[18] J. Khoury, J. L. Lehners and B. Ovrut, Phys. Rev. D 84 (2011), 043521 [arXiv:1103.0003 [hep-th]].}\]

\[\text{[19] M. Koehn, J. L. Lehners and B. Ovrut, Phys. Rev. D 87 (2013) no.6, 065022 [arXiv:1212.2185 [hep-th]].}\]

\[\text{[20] M. Rocek and A. A. Tseytlin, Phys. Rev. D 59 (1999), 106001 [arXiv:hep-th/9811232 [hep-th]].}\]

\[\text{[21] S. Sasaki, M. Yamaguchi and D. Yokoyama, Phys. Lett. B 718 (2012), 1-4 [arXiv:1205.1353 [hep-th]].}\]

\[\text{[22] C. Adam, J. M. Queiruga, J. Sanchez-Guillen and A. Wereszczynski, Phys. Rev. D 84 (2011), 025008 [arXiv:1105.1168 [hep-th]].}\]

\[\text{[23] C. Adam, J. M. Queiruga, J. Sanchez-Guillen and A. Wereszczynski, JHEP 05 (2013), 108 [arXiv:1304.0774 [hep-th]].}\]

\[\text{[24] S. B. Gudnason, M. Nitta and S. Sasaki, JHEP 02 (2016), 074 [arXiv:1512.07557 [hep-th]].}\]

\[\text{[25] S. B. Gudnason, M. Nitta and S. Sasaki, JHEP 01 (2017), 014 [arXiv:1608.03526 [hep-th]].}\]

\[\text{[26] M. Nitta and S. Sasaki, Phys. Rev. D 90 (2014) no.10, 105002 [arXiv:1408.4210 [hep-th]].}\]

\[\text{[27] M. Nitta and S. Sasaki, Phys. Rev. D 90 (2014) no.10, 105001 [arXiv:1406.7647 [hep-th]].}\]

\[\text{[28] M. Nitta and S. Sasaki, Phys. Rev. D 91 (2015), 125025 [arXiv:1504.08123 [hep-th]].}\]

\[\text{[29] M. Nitta and S. Sasaki, Phys. Rev. D 103 (2021) no.2, 025001 [arXiv:2110.07973 [hep-th]].}\]

\[\text{[30] M. Koehn, J. L. Lehners and B. A. Ovrut, Phys. Rev. D 86 (2012), 085019 [arXiv:1207.3798 [hep-th]].}\]

\[\text{[31] F. Farakos and A. Kehagias, JHEP 11 (2012), 077 [arXiv:1207.4767 [hep-th]].}\]

\[\text{[32] J. Wess and J. Bagger, Princeton University Press, 1992.}\]

\[\text{[33] S. Theisen, Nucl. Phys. B 263 (1986), 687.}\]

\[\text{[34] K. S. Stelle, Gen. Rel. Grav. 9 (1978), 353-371.}\]

\[\text{[35] E. A. Bergshoeff and M. de Roo, Nucl. Phys. B 328 (1989), 439-468.}\]

\[\text{[36] B. Zwiebach, Phys. Lett. B 156 (1985), 315-317.}\]

\[\text{[37] M. Nitta, S. Sasaki and R. Yokokura, Eur. Phys. J. C 78 (2018) no.9, 754 [arXiv:1706.02938 [hep-th]].}\]

\[\text{[38] M. Nitta, S. Sasaki and R. Yokokura, Phys. Rev. D 96 (2017) no.10, 105022 [arXiv:1706.05232 [hep-th]].}\]

\[\text{[39] S. B. Gudnason, M. Nitta, S. Sasaki and R. Yokokura, Phys. Rev. D 99 (2019) no.4, 045011 [arXiv:1810.11361 [hep-th]].}\]

\[\text{[40] S. Bjarke Gudnason, M. Nitta, S. Sasaki and R. Yokokura, Phys. Rev. D 99 (2019) no.4, 045012 [arXiv:1812.00978 [hep-th]].}\]

\[\text{[41] R. Gwyn and J. L. Lehners, JHEP 05 (2014), 050 [arXiv:1402.5120 [hep-th]].}\]

\[\text{[42] L. Susskind, [arXiv:hep-th/0302219 [hep-th]].}\]

\[\text{[43] C. Vafa, [arXiv:hep-th/0509212 [hep-th]].}\]