Pion and kaon parton distribution functions from basis light front quantization and QCD evolution

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We investigate the parton distribution functions (PDFs) of the pion and the kaon by combining QCD evolution with the basis light front quantization (BLFQ). The initial PDFs result from the light front wave functions obtained by diagonalizing the effective Hamiltonian consisting of the holographic quantum chromodynamics (QCD) confinement potential, a complementary longitudinal confinement potential, and the color-singlet Nambu–Jona-Lasinio (NJL) interactions. The valence-quark PDF of the pion, after QCD evolution, is consistent with the experimental data from the E-0615 experiment at Fermilab. Meanwhile, the calculated pion structure function agrees with the ZEUS and the H1 experiments at DESY-HERA for large x. Additionally, the ratio of the up quark PDF of the kaon to that of the pion is in agreement with the NA-003 experiment at CERN. We also present the cross section for the pion-nucleus induced Drell-Yan process with the obtained pion PDFs supplemented by the PDFs of the target nuclei.

I. INTRODUCTION

Parton distribution functions (PDFs) encode the non-perturbative structure of a hadron by specifying the distribution of longitudinal momentum by quarks and gluons as its constituents. The determination of PDFs from the analysis of hard scattering processes is one of the main topics of hadron physics [1–19]. The structure of hadrons including their PDFs is expected to be described by quantum chromodynamics (QCD) in the low energy region where quarks are confined. In addition to color confinement, the explicit and the dynamical breaking of the QCD evolution with the basis light front quantization (BLFQ). The initial PDFs result from the light front wave functions obtained by diagonalizing the effective Hamiltonian consisting of the holographic quantum chromodynamics (QCD) confinement potential, a complementary longitudinal confinement potential, and the color-singlet Nambu–Jona-Lasinio (NJL) interactions. The valence-quark PDF of the pion, after QCD evolution, is consistent with the experimental data from the E-0615 experiment at Fermilab. Meanwhile, the calculated pion structure function agrees with the ZEUS and the H1 experiments at DESY-HERA for large x. Additionally, the ratio of the up quark PDF of the kaon to that of the pion is in agreement with the NA-003 experiment at CERN. We also present the cross section for the pion-nucleus induced Drell-Yan process with the obtained pion PDFs supplemented by the PDFs of the target nuclei.

One of the available experiments with access to the pion PDFs is the Drell-Yan dilepton production in π− tungsten reactions [1,3]. Several next-to-leading order (NLO) analyses of this Drell-Yan process have been performed by Refs. [3,5]. The subsequent determination of the nucleon and the light meson PDFs with associated uncertainties from the experiment is available in Refs. [8,18,26]. The pion PDF has also been the subject of detailed analyses in the phenomenological models in Refs. [27,28], also including the chiral quark model [29] and anti-de Sitter (AdS)/QCD models [30,31]. The pion PDFs have also been investigated within lattice QCD [12,34–37]. See Ref. [38] for the corresponding review of lattice QCD results. Additionally, the first global fit analysis of PDFs in the pion has been performed in Ref. [26].

Although meson PDFs are expected to be universal, tension exists regarding the behavior of the pion valence PDF. On the one hand, from the analyses of the Drell-Yan data [1,3], the large-x behavior of the pion valence PDF is expected to fall off linearly or slightly faster, which is supported by the constituent quark models [27,28], the Nambu–Jona-Lasinio (NJL) model [29], and duality arguments [39]. However, the reanalysis of the data for the Drell-Yan process [8] including the next-to-leading logarithmic threshold resummation effects shows a considerably softer valence PDF at high x when compared to the NLO analysis [3,5]. Both observations disagree with perturbative QCD where the behavior of the same function has been predicted to be $(1-x)^2$ [40,43], which is further supported by the Bethe-Salpeter equation (BSE) approach [10].

Information from experiments on the light-quark PDF of the kaons exists in the form of the ratio of the up (u) quark PDF in the kaon to that in the pion [6,7]. Theoretically, the kaon’s PDF from the BSE approach has been investigated in Ref. [44]. More recent study of the pion and kaon valence PDFs from the BSE with a beyond-rainbow-ladder truncation of QCD shows a good agreement with the experimental valence PDF of the pion [45]. The kaon’s PDF has also been studied in several quark models such as the gauge-invariant nonlocal chiral-quark model [11], the dressed quark model [46], and the NJL...
Our theoretical calculation of meson structures is based on the basis light front quantization (BLFQ) approach, which has been developed for solving many-body bound state problems in quantum field theories \[52--54\]. BLFQ is a Hamiltonian-based formalism incorporating the light front dynamics \[55\]. This formalism has been successfully applied to the quantum electrodynamics (QED) systems including the electron self-energy \[56\] and positronium with strong coupling \[53\]. It has also been applied to heavy quarkonia \[57] and \(B_c\) mesons \[58\] both as QCD bound states. Recently, the BLFQ approach using a Hamiltonian that includes the color-singlet NJL interaction to account for the chiral dynamics has been applied to the light mesons \[59\]. Furthermore, the BLFQ formalism has been extended to time-dependent strong external field problems such as those in non-linear Compton scattering \[60\]. (For the reviews of BLFQ and its application, see Refs. \[52\]--\[57\], \[60\]--\[65\].) With light front kinematics, the PDFs can also be calculated using the microcanonical ensemble \[69\].

In this work, we elaborate on Ref. \[70\] in the determination of the valence quark PDFs of the pion and the kaon at independent initial scales using the light front wave functions (LFWFs). These wave functions were obtained within the framework of BLFQ by diagonalizing the effective light front Hamiltonian whose interactions include the light front holographic QCD (LFHQCD) confinement potential in the transverse direction \[71\], a longitudinal confinement potential \[54\], and the NJL interactions \[72\]. These LFWFs have been successfully applied to compute the parton distribution amplitudes and the electromagnetic form factors for the pion and the kaon \[59\]. We then evolve our initial valence quark PDFs of the pion and the kaon utilizing the next-to-next-to-leading order (NNLO) Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations \[73--75\] to the relevant scales in order to compare with the data of PDFs from the E615 experiment at Fermilab, with the pion structure function from the ZEUS and the H1 experiments at DESY-HERA, and with the ratio \(u_{\pi}^F(x, \mu^2)/d_{\pi}^F(x, \mu^2)\) from the NA-003 experiment at CERN. Using our pion PDFs in conjunction with the nuclear PDFs from the nuclear Coordinated Theoretical-Experimental Project on QCD (nCTEQ) 2015 global fit \[76\], instead of the collection of free nucleon PDFs used in Ref. \[70\], we further calculate the cross section for the pion-nucleus induced Drell-Yan process, to show that our PDFs consistently describe the measured data from a variety of experiments.

The paper is organized as follows. Section II is the introduction. The valence PDFs for the pion and the kaon from the BLFQ-NJL model are given in Section III. Section IIIA discusses results of these PDFs following DGLAP evolution. Specifically, in Subsection IIIA we present the pion and the kaon PDFs at various scales as well as the implied structure function for the pion. Based on these pion PDFs, the cross section for the unpolarized Drell-Yan process is calculated in Subsection III B. Section IV is the summary.

II. BLFQ-NJL Model for the Light Mesons

A. The light front confinement and NJL interactions for the light mesons

Let us start with an overview of the BLFQ-NJL model for the light mesons following Ref. \[59\]. In the approach of BLFQ, the structures of the bound states are embedded in the LFWFs obtained as the solutions of the time-independent light front Schrödinger equation

\[
H_{\text{eff}} | \Psi \rangle = M^2 | \Psi \rangle,
\]

(1)

where \(H_{\text{eff}}\) is the effective Hamiltonian of the system with the mass squared \(M^2\) being the eigenvalue of \(| \Psi \rangle\). In general, \(| \Psi \rangle\) is the vector in the Hilbert space spanning into all Fock sectors. In the valence Fock sector, the effective Hamiltonian for the light mesons with non-singlet flavor wave functions is given by \[59\]

\[
H_{\text{eff}} = \frac{k_{\perp}^2 + m_q^2}{x} + \frac{k_{\parallel}^2 + m_q^2}{1-x} + \kappa^2 \zeta_{\perp}^2 - \frac{\kappa^4 (m_q + m_{\bar{q}})^2}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x) \partial_x) + H_{\text{NJL}}^\text{eff},
\]

(2)

where \(m_q (m_{\bar{q}})\) is the mass of the quark (anti-quark), and \(\kappa\) is the strength of the confinement. \(\zeta_{\perp}\) is the holographic variable \[71\], with \(k_{\perp}\) being the conjugate variable of \(\zeta_{\perp}\). The first two terms in Eq. (2) are the light front kinetic energy for the quark and the antiquark. The third and the fourth terms are the confining potential in the transverse direction based on the LFHQCD \[71\] and a longitudinal confining potential \[54\] that reproduces 3D confinement in the nonrelativistic limit. Additionally, the \(H_{\text{NJL}}^\text{eff}\) is the color-singlet NJL interaction to account for the chiral dynamics \[72\].

The NJL interaction for the positively-charged pion is

\[
H_{\text{NJL}}^\pi = G_\pi \left\{ \bar{u}_{u_{\pi}^u} (p_{1u}^u) u_{u_{\pi}^u} (p_{1u}^u) \right\} \epsilon_{2u} \epsilon_{2u} (p_{2u}^u) v_{d_{\pi}^d} (p_{2d}^d) v_{d_{\pi}^d} (p_{2d}^d) + 2 \bar{u}_{u_{\pi}^u} (p_{1u}^u) \gamma_5 u_{u_{\pi}^u} (p_{1u}^u) \epsilon_{2u} \epsilon_{2u} (p_{2u}^u) \epsilon_{2u} \epsilon_{2u} (p_{2u}^u)
\]

(3)

which can be derived from the NJL Lagrangian after the Legendre transform in the two-flavor NJL model \[72\]. Here, only the combinations of Dirac bilinears relevant to the valence Fock sector LFWFs of the \(\pi^+\) are included. For the positively charged kaon, the interaction...
is given by
\[
H_{\text{NJL},K}^{\text{eff}} = G_K \left\{ -2 \pi u_{a1}(p'_1)\psi_{s2}(p_2)\tau_{a2}(p_2)u_{a1}(p_1) + 2 \pi u_{a1}(p'_1)\gamma_5\psi_{s2}(p_2)\tau_{a2}(p_2)\gamma_5 u_{a1}(p_1) \right\},
\]
(4)

obtained similarly from the Lagrangian of the three-flavor NJL model. Here \( u_a(p) \) and \( \psi_s(p) \) are solutions of the free Dirac equation, with the nonitalic subscripts representing the flavors while the italic subscripts designate the spins. Meanwhile, \( p_1 \) and \( p_2 \) are the momenta of the valence quark and the valence antiquark, respectively. \( G_n \) and \( G_K \) are independent coupling constants of the theory. We have ignored the instantaneous terms due to the NJL interactions in deriving Eqs. (3) and (4). Explicit expressions and the detailed calculations of the matrix elements of the NJL interactions in the basis function representation we adopt can be found in Ref. [59].

Parameters in the BLFQ-NJL model are adjusted to reproduce the ground state masses of the pseudoscalar and vector mesons with light-light and light-strange non-singlet flavor components. Meanwhile, the confining strengths are determined by the experimental charge radii of the \( \pi^+ \) and the \( K^+ \) [59].

**B. Valence quark PDFs in the pion and the kaon from BLFQ**

The LFWFs of the valence quarks in the \( \pi^+ \) meson and the \( K^+ \) meson have been solved in the BLFQ framework using the NJL interactions discussed in the previous subsection [59]. In the leading Fock sector, the LFWF for the mesons is written as

\[
\begin{aligned}
|\Psi(p^+, \vec{b}^\perp)\rangle & = \sum_{r,s} \int_0^1 d\kappa \int 4\pi x(1-x) \int \frac{d\kappa^\perp}{(2\pi)^2} \\
& \times \psi_{rs}(x, \kappa^\perp) b_r^\perp(xP^+, \kappa^\perp + x\vec{b}^\perp) \\
& \times d_{\perp}^l((1-x)P^+, -\kappa^\perp + (1-x)\vec{P}^\perp)|0\rangle,
\end{aligned}
\]
(5)

where \( P = k + p \) is the light front 3-momentum of the meson, \( x = k^+/P^+ \) is the longitudinal momentum fraction carried by the valence quark, and \( \kappa^\perp = \vec{k}^\perp - x\vec{b}^\perp \) is the relative transverse momentum. The valence wave function is then expanded in the following orthonormal basis:

\[
\psi_{rs}(x, \kappa^\perp) = \sum_{nml} \psi(n, m, l, r, s) \phi_{nm}
\]
\[
\left( \frac{\kappa^\perp}{\sqrt{x(1-x)}} \right) \chi_l(x),
\]
(6)

where \( \phi_{nm} \) is the two-dimensional (2D) harmonic oscillator (HO) function, and \( \chi_l \) is the longitudinal basis function. Here \( n, m, \) and \( l \) are basis quantum numbers corresponding to the radial excitation, the orbital angular momentum projection, and the longitudinal excitation, respectively. Explicitly, \( \phi_{nm} \) is given by

\[
\phi_{nm}(q^\perp; b) = \frac{1}{b} \sqrt{\frac{4\pi n!}{(n + |m|)!}} \left( \frac{q^\perp}{b} \right)^{|m|} \exp\left( -\frac{q^\perp^2}{2b^2} \right) \times L_{n}^{|m|}\left( \frac{q^\perp^2}{b^2} \right) e^{im\phi},
\]
(7)

with \( \tan(\phi) = q^2/b^2 \) and \( L_n^{|m|} \) being the associated Laguerre function. Meanwhile, the longitudinal basis \( \chi_l(x) \) is defined as

\[
\chi_l(x; \alpha, \beta) = \sqrt{\frac{4\pi(2l + \alpha + \beta + 1)}{\Gamma(l + 1)\Gamma(l + \alpha + 1)\Gamma(l + \beta + 1)}} \times x^{\beta/2}(1-x)^{\alpha/2} \frac{1}{P(\alpha, \beta)}(2x - 1),
\]
(8)

with \( P(\alpha, \beta) \) being the Jacobi polynomial and \( \alpha = 2m_q(m_q + m_{\pi})/\kappa^2, \beta = 2m_q(m_q + m_{\pi})/\kappa^2. \) Here \( m_q \) and \( m_{\pi} \) are the masses of the valence quark and the valence antiquark, respectively.

In order to numerically diagonalize \( H_{\text{eff}} \), the infinite dimensional basis must be truncated. Because the NJL interactions do not couple to \( |m| \geq 3 \) basis states, we have a natural truncation for \( m \geq 59. \) Specifically, we apply the following truncation to restrict the quantum numbers [54, 57, 59]:

\[
0 \leq n \leq N_{\text{max}}, \quad -2 \leq m \leq 2, \quad 0 \leq l \leq L_{\text{max}},
\]
(9)

where \( L_{\text{max}} \) is the basis resolution in the longitudinal direction whereas \( N_{\text{max}} \) controls the transverse momentum covered by 2D HO functions. Notice that our definition of \( N_{\text{max}} \) in Eq. (9) is different from that in Refs. [57, 58].

The probability of finding a quark inside the meson carrying the momentum fraction \( x \) is then given by [57]

\[
f(x) = \frac{1}{4\pi x(1-x)} \int \frac{d\kappa^\perp}{(2\pi)^2} \psi_{rs}(x, \kappa^\perp) \psi_{rs}(x, \kappa^\perp)
\]
(10a)

\[
= \frac{1}{4\pi} \sum_{n,m,l,r,s} \psi^*(n, m, l', r, s) \psi(n, m, l, r, s) \chi_{l'}(x) \chi_{l}(x),
\]
(10b)

which is interpreted as the PDF for the valence quark. Correspondingly, the PDF for the valence antiquark is given by \( f(1-x) \). In obtaining Eq. (10b) from Eq. (10a), the transverse integrals are evaluated exactly using the orthonormal property of the 2D HO functions. Equation (10) implies the following momentum sum rule:

\[
\int_0^1 x f(x) dx + \int_0^1 x f(1-x) dx = 1,
\]
(11)
FIG. 1. The PDFs for the valence quarks of the $\pi^+$ and $K^+$ mesons. The top-left panel shows the $\pi^+$ valence PDFs calculated from the LFWFs in the BLFQ-NJL model with different $L_{\text{max}}$, together with the extrapolation to $L_{\text{max}} \to +\infty$. The blue, red, and orange dashed lines correspond to the PDFs obtained from $N_{\text{max}} = 8$ and $L_{\text{max}} = 8, 16, 32$, respectively. The purple solid line represents Eq. (12) using the extrapolated parameters in Table I. The top-right panel presents the corresponding results for the $K^+$. The bottom-left panel shows the extrapolation of the fitting parameters in Eq. (12) for the $\pi^+$ valence PDF. Because the $u$ and the $d$ quarks have the same mass in Ref. [59], the parameter $a$ is always identical to $b$ for a fixed $L_{\text{max}}$ for the pion. The bottom-right panel displays the extrapolations of the fitting parameters for the $K^+$ valence PDF. The yellow stars and the purple pluses are the fitting parameters $a$ and $b$ respectively for different $L_{\text{max}}$. The blue solid line and the red dashed line are quadratic functions of $L_{\text{max}}^{-1}$ as the best fits to the data points.

which indicates that, at our model scale, the valence quarks carry the entire momentum of the meson. Our normalization of the LFWF ensures that the normalizations of the PDFs for both valence quarks are 1.

We then substitute the valence wave functions given by Eq. (6) obtained from Ref. [59] into Eq. (10b) to calculate the valence PDFs for the $\pi^+$ and the $K^+$. We show in the upper panels of Fig. 1 that with a fixed $L_{\text{max}}$ the numerical PDFs oscillate about a single-peaked function, with the amplitude of the oscillation decreasing with increas-
functions on the normalization of Eq. (12). Subsequently, we fit the initial input using Eq. (12) with parameters extrapolated for each $L_{\text{max}}$. The solid black, brown dot-dashed, and pink long-dashed lines are the accompanying valence quark, the sea quark, and the gluon distributions respectively all at $\mu^2 = 16 \text{ GeV}^2$. Our valence PDF is compared with the original analysis of the FNAL-E-0615 experiment data 33 as well as with the reanalysis of the FNAL-E-0615 experiment data 33. The red band corresponds to the LFHQCD prediction 33.

TABLE I. Dependence of the PDF fitting parameters on the longitudinal basis cutoff $L_{\text{max}}$. With $N_{\text{max}} = 8$, the extrapolations are carried out by fitting to quadratic functions of $L_{\text{max}}^{-1}$.

| $L_{\text{max}}$ | 8 | 12 | 16 | 20 | 24 | 28 | 32 | Extrapolated to $+\infty$ |
|-----------------|---|----|----|----|----|----|----|--------------------------|
| $\pi^+$         | $a = b = 0.8045$ | $0.6978$ | $0.6549$ | $0.6351$ | $0.6249$ | $0.6195$ | $0.6163$ | $0.5961$ |
| $K^+$           | $a = 0.7415$ | $0.6823$ | $0.6611$ | $0.6500$ | $0.6500$ | $0.6403$ | $0.6414$ | $0.6337$ |
| $K^+$           | $b = 1.0002$ | $0.9193$ | $0.8907$ | $0.8757$ | $0.8761$ | $0.8625$ | $0.8643$ | $0.8546$ |

III. PDFS, STRUCTURE FUNCTION, AND CROSS SECTIONS

A. PDFs and structure function

By performing the QCD evolution, the valence quark distributions at high $\mu^2$ scale can be determined with the initial input using Eq. (12) with parameters extrapolated to the infinite longitudinal basis cutoff as given in the last column of Table I. Specifically, we evolve our input PDFs to the relevant experimental scales $\mu^2 = 16 \text{ GeV}^2$ and $\mu^2 = 20 \text{ GeV}^2$ with independently adjustable initial scales of the pion and the kaon PDFs using the DGLAP equations 73–75. Here, we use the higher order perturbative parton evolution toolkit (HOPPET) to numerically solve the DGLAP equations 71. We find that the initial scales increase when we progress from the leading order (LO) to NNLO. Meanwhile the evolved PDFs fit better to the experimental data demonstrated by smaller values of $\chi^2$ per degree of freedom (d.o.f.) at higher orders, as shown in Table II. Since the results from the higher order DGLAP equation appears more reliable due to higher initial scales, only the results for the PDFs at NNLO are presented in this paper.

We adopt $\mu^2_{\text{fit}} = 0.240 \pm 0.024 \text{ GeV}^2$ for the initial scale of the pion PDF and $\mu^2_{\text{fit}} = 0.246 \pm 0.024 \text{ GeV}^2$ for the initial scale of the kaon PDF which we determine by requiring the results after NNLO DGLAP evolution to fit both the pion PDF data from the FNAL-E-0615 experiment 33 and the ratio $u_\pi / u_\pi^c$ data from the CERN-NA-003 experiment 71. At our central value of the initial scales, the $\chi^2$ per d.o.f. for the fit of the pion PDF is 3.64, whereas for the ratio $u_\pi / u_\pi^c$, the corresponding
s quark carries larger momentum than the s quark distribution in the kaon. The data in (a) are taken from the CERN-NA-003 Drell-Yan experiment [7]. The blue dashed and magenta dashed dotted lines in (a) correspond to the results obtained in the NLO Glück-Reya-Stratmann (GRS) model [80] and the prediction from the BSE [44], respectively.

In Fig. 2, we show our result for the valence quark distribution of the pion. We compare the valence quark distribution after QCD evolution with the data from the FNAL-E-0615 experiment [6] and with the reanalysis of the same data including soft gluon resummation [8]. The error band in the valence quark distributions is due to the spread in the initial scale of Ref. [33] at \( \mu_0^2 = (1.12 \pm 0.32) \text{ GeV}^2 \). We find that at this scale the valence quarks carry 57% of the pion’s momentum from our model, close to the 54% given by Ref. [33]. At the same scale, in contrast to the absence of gluon contributions in Ref. [33], our model allocates 35% of the pion’s momentum to the gluons and 8% to the sea quarks.

Another comparison can be made for the pion PDFs at the initial scale of Ref. [33] at \( \mu_0^2 = (1.12 \pm 0.32) \text{ GeV}^2 \). We find that at this scale the valence quarks carry 57% of the pion’s momentum from our model, close to the 54% given by Ref. [33]. At the same scale, in contrast to the absence of gluon contributions in Ref. [33], our model allocates 35% of the pion’s momentum to the gluons and 8% to the sea quarks.

In Fig. 2 (a), we present the ratio of the u quark distributions in the kaon to that in the pion, \( u^+_K/u^+_\pi \) as a function of x and; (b) comparison of the valence quark distributions in the kaon and the pion. In (b) the blue dashed and red long-dashed lines correspond to the u and the \( \bar{s} \) quark distributions in the kaon, respectively. The black solid line represents the valence quark distribution in the pion. The grey band in (a) corresponds to the QCD evolution from the initial scales \( \mu_0^2 = 0.240 \pm 0.024 \text{ GeV}^2 \) for the pion and \( \mu_0^K = 0.246 \pm 0.024 \text{ GeV}^2 \) for the kaon. The data in (a) are taken from the CERN-NA-003 Drell-Yan experiment [7]. The blue dashed and magenta dashed dotted lines in (a) correspond to the results obtained in the NLO Glück-Reya-Stratmann (GRS) model [80] and the prediction from the BSE [44], respectively.

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In Fig. 3 (a), we present the ratio of the u quark distributions in the kaon to that in the pion, with the valence quark PDFs in the kaon shown in Fig. 3 (b). We observe that at \( \mu^2 = 20 \text{ GeV}^2 \), our center value of \( u^+_K/u^+_\pi \) is in good agreement with the data from CERN-NA-003 experiment [7] as well as with a phenomenological quark model (GRS, NLO) [80] and the BSE approach [44]. One notices that the ratio decreases as x increases. This phenomena is rooted in the results shown in Fig. 3 (b) where we compare the valence quark distributions of the kaon and the pion. We find that at the scale of \( \mu^2 = 20 \text{ GeV}^2 \) the distribution of the u quark PDF at high x in the pion is above that in the kaon. This can be understood since \( m_u > m_d \) the peak of the \( \bar{s} \) quark distribution in the kaon appears at higher x compared to the u quark distribution. Therefore the \( \bar{s} \) quark carries larger momentum.

Another comparison can be made for the pion PDFs at

| Order | Initial scale of pion \( \mu_0^2 \) | Initial scale of kaon \( \mu_0^K \) | E-0615 \( \chi^2/(d.o.f.) \) | NA-003 \( \chi^2/(d.o.f.) \) |
|-------|-------------------------------|-------------------------------|--------------------|--------------------|
| LO    | 0.120 ± 0.012 GeV\(^2\)        | 0.133 ± 0.013 GeV\(^2\)        | 6.71               | 0.88               |
| NLO   | 0.205 ± 0.020 GeV\(^2\)        | 0.210 ± 0.021 GeV\(^2\)        | 4.67               | 0.56               |
| NNLO  | 0.240 ± 0.024 GeV\(^2\)        | 0.246 ± 0.024 GeV\(^2\)        | 3.64               | 0.50               |
Comparisons are made with results from Refs. [46, 82].

| Flavor | $\mu^2$ GeV$^2$ | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|--------|----------------|-----------------|-----------------|-----------------|-----------------|
| $s^K$  | BLFQ-NJL       | 1.69            | 0.268           | 0.127           | 0.074           | 0.048           |
| $u^K$  | BLFQ-NJL       | 0.268           | 0.125           | 0.076           | 0.054           |
| $s^K$  | BLFQ-NJL       | 0.268           | 0.114           | 0.059           | 0.037           |
| $u^K$  | BLFQ-NJL       | 0.271           | 0.124           | 0.069           | 0.044           |

Hecht (2001) [10]

| Flavor | $\mu^2$ GeV$^2$ | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|--------|----------------|-----------------|-----------------|-----------------|-----------------|
| $s^K$  | BLFQ-NJL       | 4               | 0.24            | 0.098           | 0.049           |
| $u^K$  | BLFQ-NJL       | 0.26            | 0.11            | 0.052           |
| $s^K$  | BSE (2018)     | 0.24            |
| $u^K$  | JAM global fit | 0.28            | 0.11            | 0.048           |

TABLE IV. Lowest four moments of valence quark distributions in the kaon based on the initial PDF from BLFQ-NJL model with the results from the global fit, lattice QCD, and phenomenological models at various scales. Results tabulated here at $\mu^2 \geq 4$ GeV$^2$ are also presented in Fig. 1(a).

| Flavor | $\mu^2$ GeV$^2$ | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|--------|----------------|-----------------|-----------------|-----------------|-----------------|
| $s^K$  | BLFQ-NJL       | 27              | 0.23            | 0.094           | 0.048           |
| $u^K$  | BLFQ-NJL       | 0.21            | 0.056           |

Sutton (1992) [9]

| Flavor | $\mu^2$ GeV$^2$ | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|--------|----------------|-----------------|-----------------|-----------------|-----------------|
| $s^K$  | BLFQ-NJL       | 49              | 0.200           | 0.080           | 0.007           |
| $u^K$  | BLFQ-NJL       | 0.23            | 0.090           |

Martinell (1988) [35]

| Flavor | $\mu^2$ GeV$^2$ | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|--------|----------------|-----------------|-----------------|-----------------|-----------------|
| $s^K$  | BLFQ-NJL       | 1.023           | 0.015           | 0.079           | 0.009           | 0.040           | 0.005           | 0.023           | 0.003           |
than the u quark does, reducing the probability of finding a u quark with high x in the kaon. Specifically, the u quark PDF in the kaon falls off at large x as \((1 - x)^{0.60}\) whereas the same behavior in the pion is \((1 - x)^{1.49}\). We also observe that in our model, the \(\overline{s}\) quark PDF in the kaon falls off as \((1 - x)^{1.32}\).

We further evaluate the lowest four nontrivial moments of the valence quark PDF defined as

\[
\langle x^n \rangle = \int_0^1 dx \ x^n f_{v/K}^2(x, \mu^2), \quad n = 1, 2, 3, 4. \tag{13}
\]

The corresponding moments of the pion PDF at different scales are shown in Fig. 4 (a), together with the results from the global fit [26], lattice QCD [12, 34, 37], and several phenomenological models [3, 5, 11]. Our predictions are in good agreement with Refs. [3, 5, 11, 12, 26, 35]. The numerical values of the lowest four moments of the pion PDF at various scales are presented in Table III.

The scale dependence of the lowest four moments of the pion valence quark PDF is presented in Fig. 4 (b). These moments decrease uniformly as the scale \(\mu^2\) increases, compensated by the increase in contributions from sea quarks and gluons. We find good agreement of our calculated \(\langle x \rangle\) with the Jefferson Lab Angular Momentum Collaboration (JAM) global fit [26] over nearly 3 decades of the \(\mu^2\) scale within our uncertainty and close to the central value. Additionally, the numerical values of the lowest four moments of the valence quark PDFs in the kaon at various scales are presented in Table IV.

With the pion PDFs known over a wide range of scales, we can calculate the pion structure function \(F_2^\pi(x, \mu^2)\). According to the parton model, the pion structure function is related to its PDFs by

\[
F_2^\pi(x, \mu^2) = \sum_a e_a^2 \ x f_a^\pi(x, \mu^2), \tag{14}
\]

where \(a\) is the flavor index and \(e_a\) is the electric charge of the quark flavor \(a\) in the units of the elementary charge. Here we have included heavy flavor contributions relevant to the scale of the structure functions. Our results for the structure function \(F_2^\pi(x, \mu^2)\) are shown in Fig. 5 and Fig. 6 in comparison with DESY-HERA-ZEUS [83] and DESY-HERA-H1 data [84], respectively, at the experimental scales. Both the ZEUS and the H1 data were determined from the neutrons production in \(ep\) collisions, \(ep \rightarrow e'Xn\) process. The two different sets of ZEUS data in Fig. 5 correspond to different pion fluxes used to determine \(F_2^\pi\). One of them was obtained using the additive quark model (AQM) whereas the other is obtained using the effective one-pion-exchange flux (EF) in hadron-hadron charge-exchange reactions. The difference between these two results are attributed to the model dependence of the experimental analysis. Our result appears to favor the AQM for \(\mu^2 \leq 240\) GeV$^2$.

We notice from Fig. 5 and Fig. 6 that our results deviate from the data at very low \(x\). We expect that at low initial scale the DGLAP evolution with leading twist is not sufficient at low \(x\) [87, 88] and one needs to take into account of the higher twist corrections [89–95]. On the other hand, our \(F_2^\pi(x, \mu^2)\) shows better agreement with data as \(x\) increases. The various valence, sea, and gluon contributions to the total structure function \(F_2^\pi\) of the...
pion at 55 GeV$^2$ are shown in Fig. 7. We observe that at low $x$ the sea quark contribution dominates. However, at large $x$ the distribution is mostly accounted for by the valence quarks. Since gluons carry no electromagnetic charge, their contribution to the structure function vanishes.

### B. Cross section of the unpolarized Drell-Yan process

In this section we present the cross section of the Drell-Yan process using our BLFQ-NJL model for the pion PDF. The momenta of the incoming hadrons are denoted by $p_{1,2}$. We define $l, l'$ as the momenta of the two outgoing leptons. The kinematics of the process are described by the invariant mass of the lepton pair $m$, center of mass energy square $s$, rapidity $y$ or the Feynman variable $x_F$, and the variable $\tau$. These variables are defined and related to each other by

\begin{align}
  s &= (p_1 + p_2)^2, \\
  q &= l + l', \\
  m^2 &= q^2, \\
  y &= \frac{1}{2} \ln \frac{p_2 \cdot q}{p_1 \cdot q} = \frac{1}{2} \ln \frac{x_1}{x_2}, \\
  x_F &= x_1 - x_2, \\
  \tau &\equiv \frac{m^2}{s} = x_1 x_2. 
\end{align}

In the parton model, the $x_i$ denotes the fraction of the hadron momentum $p_i$ carried by the annihilating parton (or anti-parton) and is given by

\begin{align}
  x_{1,2} = \pm \frac{x_F}{2} + \sqrt{\frac{x_F^2}{4} + \tau} = \sqrt{\tau} e^{\pm y},
\end{align}

where the + signs refer to $x_1$, while the − signs refer to $x_2$. The produced lepton pair will in general have a
non-vanishing three-momentum $\vec{q} = \vec{l} + \vec{l}'$. In the leading order in QCD, the cross section for the Drell-Yan process $\pi^- \text{Nucleus} \rightarrow \mu^+ \mu^- X$ is related to the pion and the nuclear PDFs by \cite{97, 98}

$$
\frac{m^3 d^2 \sigma}{dm dx_F} = \frac{8 \pi \alpha^2}{9} \frac{x_1 x_2}{x_1 + x_2} \sum_a e_a^2 f_{a}^{\pi^\pm}(x_1) f_{a}^{N}(x_2),
$$

(17)

where the summation goes over $a = u, \bar{u}, d, \bar{d}, s, \bar{s}, c,$ and $\bar{c}$. Here $\alpha$ is the coupling constant of QED, leaving $e_a$ in the units of the elementary charge. In order to evaluate Eq. (17), we adopt the nuclear PDFs from the nCTEQ 2015 \cite{76} at the experimental scale $\mu^2 = 16$ GeV$^2$ in conjunction with our pion PDFs at the same scale. While the PDFs for the tungsten and the beryllium nuclei are readily available in Ref. \cite{76}, we approximate the bound nucleon PDFs in the platinum nucleus by the corresponding bound nucleon PDFs in the gold nucleus.

After integrating out the $x_F$ dependence of the differential cross section $m^3 d^2 \sigma/dm dx_F$, we obtain our results plotted as a function of either $\sqrt{\tau}$ in Fig. 8 or $\tau$ in Fig. 9 to compare with the experimental data. The FNAL-E-0615 and the CERN-NA-003 data in Fig. 8(a) correspond to a tungsten and a platinum targets, respectively. Whereas the data in Fig. 8(b) correspond to a tungsten target. In Fig. 8(a), we show the results evaluated using the tungsten and the platinum nuclear PDFs. We employ the tungsten nuclear PDF to compute the cross section in Fig. 8(b). We find that the cross sections per nucleon obtained by considering the tungsten and the platinum nuclear PDFs are very close. In Fig. 9, the CERN-WA-011 data represent a beryllium target and the same target nuclear PDF has been used by our approach to evaluate the cross section. In Fig. 10, we show

FIG. 6. (Color online) Structure function $F_2^\pi(x, \mu^2)$ for the pion as a function of $x$ at fixed experimental values of $\mu^2$. The data are taken from Ref. \cite{84} by the H1 Collaboration in DESY-HERA. The error bands are results of the present work taking the uncertainty in the initial scale $\mu_0^2 = 0.240 \pm 0.024$ GeV$^2$ into account.
FIG. 7. (Color online) Contributions to the $F_2^\pi$ from the valence quark and sea quarks in different flavors. The data are taken from Ref. [84] by the H1 Collaboration in DESY-HERA. The error bands are results of the present work taking the uncertainty in the initial scale $\mu_0^2 = 0.240 \pm 0.024$ GeV$^2$ into account.

FIG. 8. (Color online) The cross section $m^3 \, d\sigma/dm$ for the $\pi^-$-nucleus Drell-Yan process as a function of $\sqrt{\tau}$ in the regions (a) $0 < x_F < 1$ and (b) $0 < x_F < 0.5$. The data of FNAL-E-0615 experiment with 252 GeV pions and CERN-NA003 with 200 GeV pions as well as CERN-NA-010 with 194 GeV pions are taken from Ref. [6] and Refs. [7, 85], respectively. The error-bands are the cross section calculated from the BLFQ-NJL model taking the uncertainty in the initial scale $\mu_0^2 = 0.240 \pm 0.024$ GeV$^2$ into account. The FNAL-E-0615 and the CERN-NA-010 data both correspond to a tungsten target while the CERN-NA-003 data correspond to a platinum target. The black solid and the red dashed lines in (a) represent the cross sections evaluated using the tungsten and the platinum nuclear PDFs, respectively.

the cross section $d\sigma/dm$ as a function of $m$ and compare with the data of the FNAL-E-0326 experiment [99] and the FNAL-E-0444 experiment [100] with 225 GeV pions, as well as with the data of CERN-WA-039 experiment with 39.5 GeV pions [101]. Notice the FNAL-E-0444 data correspond to a carbon target whereas the FNAL-E-0326, and the CERN-WA-039 data represent a tungsten target. We use the corresponding target nuclear PDFs to calculate the cross section displayed in Fig. 10. Based on Figs. 8-10 we find that our results are in acceptable agreement with data from widely different experimental conditions [6, 7, 85, 86, 99, 102].

IV. SUMMARY AND CONCLUSION

We calculated the valence quark PDFs of the pion and the kaon in the framework of the basis light front quantization from their light front wave functions. These wave functions were obtained as the eigenfunctions of the effective Hamiltonian, consisting of confinement potentials and the color-singlet Nambu–Jona-Lasinio interactions. The parameters in the BLFQ-NJL model were adjusted to reproduce the experimental mass spectrum and the charge radii of the light mesons [59]. The initial scales of our PDFs, the only adjustable parameters in this
work, have been obtained by consistently fitting both the evolved valence pion PDFs to the FNAL-E-0615 data \cite{6} and the evolved ratio of the up quark PDFs in the kaon to that in the pion to the CERN-NA-003 experimental data \cite{7}. The moments of the pion PDF have been found in agreement with the JAM global fit \cite{26}, with lattice QCD \cite{12,35}, as well as with phenomenological quark models \cite{3,5,11} across various scales.

We have subsequently calculated the structure function $F_2^\pi(x,Q^2)$ for the pion, the large $x$ behavior of which is consistent with the DESY-HERA experiment. However, the discrepancies at small $x$ for the structure function suggest the need to include the higher-twist corrections and a non-vanishing initial gluon distribution to the DGLAP evolution. We have also studied the cross sections of the pion-nucleus induced Drell-Yan process in comparison with Refs. \cite{6,7,86,99,100,102}, finding reasonable agreement with these various experimental data. These comparisons affirm the robustness of the BLFQ-NJL model with QCD evolution as a theoretical method to describe the structures of the pion and the kaon in the language of parton distribution functions.
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