Relic gravitational waves in the frame of slow-roll inflation with a power-law potential, and their detection

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Abstract

We obtained the analytic solutions of relic gravitational waves (RGWs) for the slow-roll inflation with a power-law form potential of the scalar field, \( V = \lambda \phi^n \). Based on a reasonable range of \( n \) constrained by cosmic microwave background (CMB) observations, we give tight constraints of the tensor-to-scalar ratio \( r \) and the inflation expansion index \( \beta \) for the fixed scalar spectral index \( n_s \). Even though, the spectrum of RGWs in low frequencies hardly depends on any parameters, the high frequency parts will be affected by several parameters, such as \( n_s \), the reheating temperature \( T_{RH} \) and the index \( \beta_s \) describing the expansion from the end of inflation to the reheating process. We analyzed in detail all the factors which would affect the spectrum of RGWs in high frequencies including the quantum normalization. We found that the future GW detectors SKA, eLISA, BBO and DECIGO are promising to catch the signals of RGWs. Furthermore, BBO and DECIGO have the potential not only to distinguish the spectra with different parameters but also to examine the validity of the quantum normalization.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The validity of general relativity and quantum mechanics makes sure the generation of a stochastic background of relic gravitational waves (RGWs) [1–6] during the early inflationary stage, whose primordial amplitude could be determined by the quantum normalization at the time of the wave modes crossing the horizon during the inflation. Since the interaction of RGWs with other cosmic components is very weak, the evolution of RGWs are mainly determined by
the behavior of cosmic expansion including the current acceleration [7, 8]. Therefore, RGWs could serve as a unique tool to study the very early Universe earlier than the recombination stage when the cosmic microwave background (CMB) radiation was generated. As an interesting source for gravitational wave (GW) detectors, RGWs exist everywhere and anytime unlike GWs radiated by the usual astrophysical process. Moreover, RGWs spread a very broad range of frequency, $10^{-18}$–$10^{10}$ Hz, making themselves become one of the major scientific goals of various GW detectors with different response frequency bands. The current and planed GW detectors contain the ground-based interferometers, such as LIGO [9], Advanced LIGO [10, 11], VIRGO [12, 13], GEO [14], KAGRA [15] and ET [16, 17] aiming at the frequency range $10^3$–$10^8$ Hz; the space interferometers, such as the future eLISA/NGO [18, 19] which is sensitive in the frequency range $10^{-4}$–$10^{-1}$ Hz, BBO [20, 21] and DECIGO [22, 23] which both are sensitive in the frequency range 0.1–10 Hz; and the pulsar timing array, such as PPTA [24, 25] and the planned SKA [26] with the frequency window $10^{-9}$–$10^{-6}$ Hz. Besides, there are some potential very high frequency GW detectors, such as the waveguide detector [27], the proposed Gaussian maser beam detector around GHz [28] and the 100 MHz detector with a pair of 75 cm baseline synchronous recycling interferometers [29]. Furthermore, the very low frequency portion of RGWs also contribute to the anisotropies and polarizations of CMB [30], yielding a magnetic type polarization of CMB as a distinguished signal of RGWs. WMAP [31–34], Planck [35], the ground-based ACTPol [36] and the proposed CMBpol [37] are of this type.

The reheating temperature, $T_{RH}$, carries rich information of the early Universe, and relates to the decay rate of the inflation as $T_{RH} \propto \sqrt{\Gamma}$ [38, 39]. Recently, the temperature of the reheating, $T_{RH}$, was evaluated [40] according to the CMB observations by WMAP 7 [34], combining with the slow-roll inflation scenarios. Furthermore, the resultant RGWs was studied in [41]. However, these pieces of work underwent the assumption of a fixed form of the potential of the scalar field driving the slow-roll inflation, $V(\phi) = \frac{1}{2}m^2\phi^2$. In this paper, we study a more general case of $V(\phi) = \lambda\phi^n$ [42], where $\lambda$ is constant. Moreover, we adopt the limitation $n < 2.1$ obtained from the spectrum of CMB [43]. For a non-fixed value of $n$, it is hard to evaluate the temperature of the reheating process, $T_{RH}$, using the method employed in [40]. Thus, we choose several values of $T_{RH}$ lying in the range of $\sim 10^4$–$10^8$ GeV, where the lower limit and the upper limit of $T_{RH}$ are obtained from the constraints in [43] and [44], respectively. In the text, one will see that $n$ and $T_{RH}$ affect the increases of the scale factor in the early stages of the Universe. Once all the expansion histories of different stages are determined, the evolutions of the RGWs at various phases can be determined subsequently. For the present time, the solutions of RGWs can be obtained, whose different frequency bands correspond to the $k$ modes re-entered the horizon at different phases. On the other hand, the anisotropies due to the tensor metric perturbations (GWs) can be scaled to those due to the observations of the scalar perturbations by introducing a parameter $r$ called the tensor-to-scalar ratio. Under the frame of the slow-roll inflation scenario, $r$ will be constrained in a narrow range due to the constraints from $n$ for a given value of the scalar spectral index $n_s$. Similarly, the inflation expansion index $\beta$ will also be constrained in a narrow range. Besides, there is a simple relation between $n$ and the preheating expansion index $\beta$, describing the expansion behavior of the Universe from the end of inflation to the reheating process. As will be shown below, the RGWs in the very high frequencies are sensitively dependent on the parameters $n_s$, $\beta$, and $T_{RH}$. Furthermore, the spectra of RGWs also depends on the condition of the quantum normalization. To this end, the spectra of RGWs given by different parameters and different conditions will confront the various current and planed GW detectors. The future detectors BBO and DECIGO are quite promising not only to determine various parameters but also to examine the validity of the quantum normalization.
Throughout this paper, we use the units \( c = h = k_B = 1 \). Indices \( \lambda, \mu, \nu, \ldots \) run from 0 to 3, and \( i, j, k, \ldots \) run from 1 to 3.

## 2. RGWs in the accelerating universe

In a spatially flat universe, the existence of perturbations modifies the Friedmann–Robertson–Walker metric to be

\[
dx^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})\, dx^i\, dx^j],
\]

(1)

where \( a(\tau) \) is the scale factor, \( \tau \) is the conformal time and \( h_{ij} \) stands for the perturbations to the homogenous and isotropic spacetime background. In general, there are three kinds of perturbations: scalar perturbation, vectorial perturbation and tensorial perturbation. In this paper we only consider the tensorial perturbation, that is, GWs. In the transverse-traceless (TT) gauge, \( h_{ij} \) satisfies: \( \frac{\partial h_{ij}}{\partial \tau} = 0 \) and \( h_{ij} = 0 \), where we used the Einstein summation convention. In the Fourier \( k \)-modes space, it can be written as

\[
h_{ij}(\tau, x) = \sum_{\sigma} \int \frac{d^3k}{(2\pi)^3/2} \epsilon_{ij}^{(\sigma)}(k) e^{ikx},
\]

(2)

where \( \sigma = +, \times \) stands for the two polarization states, the comoving wave number \( k \) is related with the wave vector \( k \) by \( k = (\delta_{ij} k^i k^j)^{1/2} \), \( h_{ij}^{(\sigma)}(\tau) = h_k^{(\sigma)}(\tau) \) ensuring \( h_{ij} \) be real, and the polarization tensor \( \epsilon_{ij}^{(\sigma)} \) satisfies [2]

\[
\epsilon_{ij}^{(\sigma)} = 2\delta_{\sigma\sigma} \epsilon_{ij}^{(s)}, \quad \epsilon_{ij}^{(s)} \delta_{ij} = 0, \quad \epsilon_{ij}^{(s)} n^i = 0, \quad \epsilon_{ij}^{(s)} (-k) = \epsilon_{ij}^{(s)}(k).
\]

(3)

In terms of the mode \( h_k^{(\sigma)} \), the wave equation is

\[
h_k^{(\sigma)}(\tau) + 2\frac{a'(\tau)}{a(\tau)} h_k^{(\sigma)}(\tau) + k^2 h_k^{(\sigma)}(\tau) = 0,
\]

(4)

where a prime means taking derivative with respect to \( \tau \). The two polarizations of \( h_k^{(\sigma)}(\tau) \) have the same statistical properties and give equal contributions to the unpolarized RGWs background, so the super index \((\sigma)\) can be dropped. The approximate solutions of equation (4) are well analyzed in [2, 3, 7], and are detailed in [41] given an accelerating universe at present. Furthermore, the analytic solutions were also studied by many authors [8, 45–48]. For a power-law form of the scale factor \( a(\tau) \propto \tau^\alpha \), the analytic solution to equation (4) is a linear combination of Bessel and Neumann functions

\[
h_k(\tau) = \tau^{1/\alpha - 1} \left[ C_1 J_{\nu - 1}(k \tau) + C_2 N_{\nu - 1}(k \tau) \right],
\]

(5)

where the constants \( C_1 \) and \( C_2 \) for each stage are determined by the continuities of \( h_1(\tau) \) and \( h_2(\tau) \) at the joining points \( \tau_1, \tau_2, \tau_3 \) and \( \tau_4 \) [8, 45]. Therefore, all the constants in the solutions of RGWs can be completely fixed, once the initial condition is given. In a spatially flat \( (k = 0) \) universe, the scale factor indeed has a power-law form in various stages [2, 41, 45, 47]. It is described by the following successive stages.

The inflationary stage:

\[
a(\tau) = l_0 |\tau|^{1+\beta}, \quad -\infty < \tau \leq \tau_1,
\]

(6)

where the inflation index \( \beta \) is a model parameter describing the expansion history during inflation. The special case of \( \beta = -2 \) corresponds to the exact de Sitter expansion. However, both the model-predicted and the observed results indicate that the value of \( \beta \) could differ slightly from \(-2\).

The preheating stage:

\[
a(\tau) = a_\tau |\tau - \tau_p|^{1+\beta_\tau}, \quad \tau_1 \leq \tau \leq \tau_2,
\]

(7)

where the parameter \( \beta_\tau \) describes the expansion behavior of the preheating stage from the end of inflation to the occurrence of the reheating process followed by the radiation-dominant
stage. In some literary works [41, 49], $\beta_5$ is set to be 1; however, we take $\beta_5$ as a free parameter in the paper.

The radiation-dominant stage:
\[
a(\tau) = a_0(\tau - \tau_1), \quad \tau_1 \leq \tau \leq \tau_2.
\]
(8)

The matter-dominant stage:
\[
a(\tau) = a_m(\tau - \tau_m)^2, \quad \tau_2 \leq \tau \leq \tau_E.
\]
(9)

The accelerating stage up to the present time $\tau_0$ [7]:
\[
a(\tau) = l_H|\tau - \tau_0|^{-\gamma}, \quad \tau_E \leq \tau \leq \tau_0,
\]
(10)

where $\gamma$ is a $\Omega_\Lambda$-dependent parameter, and $\Omega_0$ is the energy density contrast. To be specific, we take $\gamma \simeq 1.97$ [50] for $\Omega_0 = 0.73$ [34] in this paper. It is convenient to choose the normalization $|\tau_0 - \tau_m| = 1$, i.e. the present scale factor $a(\tau_0) = l_H$. From the definition of the Hubble constant, one has $l_H = \gamma / H_0$, where $H_0 = 100 h$ km s$^{-1}$Mpc$^{-1}$ is the present Hubble constant. We take $h \simeq 0.704$ [34] throughout this paper. Supposing $\beta$ and $\beta_5$ are model parameters, all the constants included through equation (6) to equation (10) can be fixed by the continuity of $a(\tau)$ and $a'(\tau)$ at the four given joining points $\tau_1$, $\tau_2$, $\tau_3$ and $\tau_4$, if one knows the increases of the scale factor of various stages, i.e. the definite values of $\xi_1 \equiv a(\tau_1)/a(\tau)$, $\xi_2 \equiv a(\tau_2)/a(\tau_1)$, $\xi_3 \equiv a(\tau_3)/a(\tau_2)$ and $\xi_4 \equiv a(\tau_4)/a(\tau_3)$.

The spectrum of RGWs $h(k, \tau)$ is defined by
\[
\langle h^2(\tau) \rangle \equiv \int_0^\infty h^2(k, \tau) \frac{dk}{k},
\]
(11)

where the angle brackets mean ensemble average. The dimensionless spectrum $h(k, \tau)$ relates to the mode $h_k(\tau)$ as [47]
\[
h(k, \tau) = \frac{\sqrt{2}}{\pi} k^{3/2} h_k(\tau).
\]
(12)

The one that we are interested in is the present RGWs spectrum $h(k, \tau_0)$. The characteristic comoving wave number at a certain joining time $\tau_0$ is give by [41]
\[
k_0 \equiv k(\tau_0) = \frac{2\pi a(\tau_0)}{1/H(\tau_0)}.
\]
(13)

After a long but simple calculation, it is easy to obtain $k_H = 2\pi \gamma$ and the following relations:
\[
\frac{k_E}{k_H} = \frac{\xi_1^{-\frac{1}{2}}}{\xi_2}, \quad \frac{k_2}{k_1} = \xi_2, \quad \frac{k_3}{k_2} = \xi_4, \quad \frac{k_4}{k_3} = \xi_5^{1/\gamma}.
\]
(14)

In the present universe, the physical frequency relates to a comoving wave number $k$ as
\[
f = \frac{k}{2\pi a(\tau_0)} = \frac{k}{2\pi l_H}.
\]
(15)

The present energy density contrast of RGWs defined by $\Omega_{GW} = \langle \rho_e \rangle / \rho_c$, where $\rho_e = \frac{1}{8\pi G} h_{ij} h_{ij}$ is the energy density of RGWs and $\rho_c = 3H_0^2/8\pi G$ is the critical energy density, is given by [3, 5]
\[
\Omega_{GW} = \frac{\int_{f_{\text{low}}}^{f_{\text{upper}}} \Omega_\phi(f) \frac{df}{f}}{f},
\]
(16)

with
\[
\Omega_\phi(f) = \frac{2\pi^2}{3} \frac{f_{\text{upper}}^2}{f^2} \left( \frac{f}{H_0} \right)^2
\]
(17)

being the dimensionless energy density spectrum. We have used a new notation, $h_k(f) = h(f, \tau_0)/\sqrt{2}$, called the characteristic strain spectrum [5] or chirp amplitude [51]. The lower and upper limit of integration in equation (16) can be taken to be $f_{\text{low}} \simeq f_E$ and $f_{\text{upper}} \simeq f_1$, respectively, since only the wavelength of the modes inside the horizon contribute to the total energy density.
3. The increases of the scale factor

For the simple $\Lambda$CDM model, the late-time acceleration of the universe is well know. One easily has $\xi_F = 1 + z_E \equiv (\Omega_\Lambda / \Omega_m)^{1/3} \simeq 1.4$, where $z_E$ is the redshift when the accelerating expansion begins. The increase of the scale factor duration of the matter-dominated stage can also be obtained straightforwardly, $\xi_M = a(t_M) / a(t_E) = (1 + z_E)^{-1}$ with $z_E = 3240$ [34]. However, the histories of the radiation-dominated stage and the preheating stage are not known well. Recently, Mielczarek [40] proposed a method to evaluate the reheating temperature, $T_{RH}$, under the frame of the slow-roll inflation model with a quadratic potential $V(\phi) = \frac{1}{2} m^2 \phi^2$ combing the observations from WMAP. Using this method, $\xi_\phi$ and $\xi_M$ can be determined subsequently with the evaluation of $T_{RH}$ [41]. In this paper, we consider a more general power-law form of the potential, $V(\phi) = \lambda \phi^n$, where $\lambda$ is a constant. For this general form of $V(\phi)$, it is hard to obtain the analytic expression of the energy density of the universe at the end of inflation, and in turn, it is hard to obtain the temperature of reheating analytically. Hence, we will take some reasonable values of $T_{RH}$ constrained by CMB observations [43].

Firstly, we discuss the value of $\xi_\phi$. After reheating, the universe is filled with the relativistic plasma, which undergoes an adiabatic expansion as long as the entropy transfer between the radiation and other components can be neglected. The adiabatic approximation leads to the conservation of the entropy, i.e. $dS = 0$. It implies $s a^3 = \text{const}$, where the entropy density $s$ of radiation is given by

$$s = \frac{2\pi^2}{45} g_s T^3.$$  \hfill (18)

Here, $g_s$ counts the effective number of relativistic species contributing to the radiation entropy. Another similar quantity $g$, counting the effective number of relativistic species contributing to the energy density of radiation, relates to energy density:

$$\rho = \frac{\pi^2}{30} g T^4.$$  \hfill (19)

The behavior of $g$ and $g_s$ with different energy scale were demonstrated in [46]. At the energy above $\sim 0.1$ MeV, one has $g = g_s$. Moreover, at the energy scales above $\sim 1$ TeV, $g = 106.75$ in the standard model, and $g \sim 220$ in the minimal extension of supersymmetric standard model, respectively. On the other hand, at the energy scales below $\sim 0.1$ MeV, $g = 3.36$ and $g_s = 3.91$, respectively. According to the conservation of entropy, one easily gets the increase of the scale factor from the reheating till the recombination [40],

$$\frac{a_{rec}}{a(\tau_*)} = \frac{T_{RH}}{T_{rec}} \left( \frac{g_{es}}{g_{s*}} \right)^{1/3},$$  \hfill (20)

where $a_{rec}$ and $T_{rec}$ stand for the scale factor and the temperature at the recombination, respectively. $g_{es}$ and $g_{s*}$ count the effective number of relativistic species contributing to the entropy during the reheating and that during recombination, respectively. As discussed in [43], the lower band of the reheating energy scale is 17.3 TeV constrained by the observed scalar power spectrum of CMB at 95$\%$ of the confidence limit. Thus, in this paper we assume $g_{es} \simeq 200$ eclectically, which was also employed in [43]. On the other hand, one has $g_{s*} = 3.91$ including the contributions of the effective number from photons and three species of massless neutrinos to the radiation entropy during the recombination, since the energy scale at the recombination $T_{rec} = T_{CMB}(1 + z_{rec}) \sim 10^{-7}$ MeV. Under the assumption of $g_{es} \simeq 200$, the lower bound of $T_{RH} \gtrsim 6 \times 10^3$ GeV was obtained [43]. On the other hand, gravitinos production gives an upper bound [52]. For instance, in the framework of the constrained minimal supersymmetric standard model [44], the upper bound of $T_{RH}$ was found that $T_{RH} \lesssim a$ few $\times 10^7$ GeV from over-production of $^6\text{Li}$ from bound state effects, and moreover, $T_{RH}$
can be relaxed to \( \lesssim 3 \times 10^{15} \) GeV coming from the energy scale at the end of inflation [43]. Based on equation (20), one easily obtain

\[
\zeta_s = a(\tau_2) \frac{a_{rec}}{a(\tau_2)} = \frac{T_{RH}}{T_{CMB} (1 + z_{eq}) (g_{*s})^{1/3}},
\]

where we have used \( T_{rec} = T_{CMB} (1 + z_{rec}) \). With \( T_{CMB} = 2.725 K = 2.348 \times 10^{-13} \) GeV [34], one has \( \zeta_s \approx 5 \times 10^{16} \) for example. Secondly, we discuss the evaluation of \( \zeta_1 \). First of all, we briefly recall the slow-roll inflation model. For slow-roll inflation, the evolution is described by the usual slow-roll parameters [53]:

\[
\epsilon \equiv \frac{m^2_{Pl}}{16\pi} \left( \frac{V'}{V} \right)^2,
\]

\[
\eta \equiv \frac{m^2_{Pl}}{8\pi} \left( \frac{V''}{V} \right),
\]

which are required to be much small than unity for the slow-roll approximation to be valid. \( \epsilon \) approaches unity at the end of inflation. When the slow-roll conditions are satisfied, inflation continues keeping the Hubble rate nearly constant, and the primordial tensor power spectrum and the scalar power spectrum are respectively given as [48, 51]:

\[
\Delta_\mathcal{H}^2(k, \tau_*) \approx \frac{16}{\pi} \left( \frac{H_*}{m_{Pl}} \right)^2,
\]

\[
\Delta_\mathcal{R}^2(k, \tau_*) \approx \frac{1}{\pi \epsilon} \left( \frac{H_*}{m_{Pl}} \right)^2,
\]

where \( H_* \) is the Hubble rate during inflation, and \( \tau_* \) stands for the moment when the \( k \) mode exits the horizon. On the other hand, based on the observations of CMB, the present scalar power spectrum can be expanded in power laws,

\[
\Delta_\mathcal{H}^2(k) = \Delta_\mathcal{H}^2(k_0) \left( \frac{k}{k_0} \right)^{n_t},
\]

\[
\Delta_\mathcal{R}^2(k) = \Delta_\mathcal{R}^2(k_0) \left( \frac{k}{k_0} \right)^{n_s-1},
\]

where \( \Delta_\mathcal{H}^2(k_0) \) and \( \Delta_\mathcal{R}^2(k_0) \) are the power spectrum of the tensor perturbations and curvature perturbations evaluated at the pivot wave number \( k_0 = a(\tau_0) = 0.002 \) Mpc\(^{-1} \) [34], respectively. Furthermore, under the slow-roll approximation, at the pivot wave number \( k_0 \) the spectral parameters are given by [53]

\[
n_t \simeq -2\epsilon,
\]

\[
n_s \simeq 1 - 6\epsilon + 2\eta.
\]

In general, the spectral indices \( n_t \) and \( n_s \) are \( k \)-dependent, described by the running parameters \( \alpha_t \equiv \delta n_t / \delta \ln k \) and \( \alpha_s \equiv \delta n_s / \delta \ln k \), respectively [47, 53–55]. However, \( \alpha_t \) and \( \alpha_s \) are only second-order small quantities. Moreover, if one uses the quantum normalization (see below) as the initial condition for the generation of RGWs, \( \alpha_t \) should be exactly zero. On the other hand, as will be seen below, non-zero \( \alpha_t \) would induce an \( n_s \) greater than 1, which make it difficult to evaluate the increase of the scale factor from the \( k_0 \) mode exiting the horizon during inflation to the end of inflation. Hence, in this paper we will simply set \( \alpha_t = \alpha_s = 0 \). Note that Even though the value of \( n_t \) is quite uncertain, \( n_s \) can be well constrained by CMB [34] or
BAO [56]. The ratio of the primordial tensor power spectrum to the scalar power spectrum is defined as [48, 51]

\[ r \equiv \frac{\Delta_\gamma^2(k, \tau_i)}{\Delta_\zeta^2(k, \tau_i)} = 16\epsilon, \]

based on equations (23) and (24). Therefore, at the pivot number \( k_0 \), one has

\[ r = \frac{\Delta_\gamma^2(k_0, \tau_i)}{\Delta_\zeta^2(k_0, \tau_i)} \approx \frac{\Delta_\gamma^2(k_0)}{\Delta_\zeta^2(k_0)} = 16\epsilon, \]

(30)

where \( \tau_i \) is the \( k_0 \) mode which exits the horizon during inflation. The approximation of the second equation in equation (30) accounts for the fact that the pivot \( k_0 \) wave mode re-entered the horizon a little earlier than the present time, and then has suffered a decay. Therefore, the ratio \( \Delta_\gamma^2(k_0)/\Delta_\zeta^2(k_0) \) cannot exactly reflect the true value of \( r \) given by its definition; however, the deviation would be expected to be less than \( \sim 0.8\% \) [41]. Hence, we will use this approximation when confronted with the CMB observations. Furthermore, under this approximation, one has a simple relation

\[ n_t = 2\beta + 4, \]

(31)

since the primordial spectrum of RGWs has a power-law form \( \Delta(k_0) \approx \Delta(k_0, \tau_i) \propto k^{2\beta+4} \) [41]. WMAP 7 Mean [34] fixed \( \Delta_\zeta^2(k_0) = \Lambda_i = (2.43 \pm 0.11) \times 10^{-9} \). Thus, the non-zero value of \( r \) implies the existence of GW background, which induced uniquely the B-mode polarization of CMB [57]. At present only observational constraints on \( r \) have been given [33, 34]. The upper bounds of \( r \) are recently constrained [34] as \( r < 0.24 \) by WMAP+BAO+H\(_0\) and \( r < 0.36 \) by WMAP 7 only for \( \alpha_s = 0 \), and \( r < 0.49 \) for \( \alpha_s \neq 0 \) by both the combination of WMAP+BAO+H\(_0\) and the WMAP 7 only, respectively. Furthermore, using a discrete, model-independent measure of the degree of fine-tuning required, if \( 0.95 \lesssim n_r < 0.98 \), in accord with current measurements, the tensor-to-scalar ratio satisfies \( r \approx 10^{-2} \) [58]. Therefore, one can normalize the RGWs at \( k = k_0 \) using equation (30), if \( r \) can be determined definitely.

As analyzed by Mielczarek [40], for the pivot wave number \( k_0^p \), the total increase of the scale factor from the mode exiting the horizon during inflation up to the present time can be evaluated as

\[ \xi_{\text{tot}} \simeq \frac{H_*}{k_0^p}. \]

(32)

Due to equations (24) and (26), one has

\[ \frac{1}{\pi \epsilon} \left( \frac{H_*}{m_{\text{pl}}^2} \right)^2 \approx \Delta_\zeta^2(k_0), \]

(33)

where the approximation \( \Delta_\zeta^2(k_0) \approx \Delta_\zeta^2(k_0, \tau_i) \) was used. Taking the form \( V(\phi) = \lambda \phi^n \), one can easily have a relation

\[ \epsilon = \frac{n(1 - n_s)}{2(n + 2)}, \]

(34)

from equations (22) and (28). Plugging equations (33) and (34) into equation (32) gives

\[ \xi_{\text{tot}} \simeq \frac{m_{\text{pl}}^2}{k_0^p} \sqrt{\frac{\pi n}{2(n + 2)}} \left( 1 - n_s \right) \Delta_\zeta^2(k_0). \]

(35)

On the other hand, if we assume that the universe underwent a quasi-de Sitter expansion (\( \beta \approx -2 \)), then the increase of a scalar factor from the moment of \( k_0 \) mode exiting the horizon during inflation to the end of inflation is give by

\[ \xi_s = \epsilon^N, \]

(36)
where $N$ is the e-folding number, which can be estimated as
\[
N \simeq - \frac{8\pi}{m_{\text{Pl}}^2} \int_{\phi_0}^{\phi_f} V(\phi) \, d\phi.
\] (37)

Concretely, for $V(\phi) = \lambda \phi^n$, one can obtains
\[
N \simeq \frac{n+2}{2(1-n_\phi)},
\] (38)

with the help of equations (22) and (28). So, if $n = 2$, equation (38) reduces to the result shown in [40]. Plugging equation (38) into equation (36), and using the identity
\[
\zeta_{\text{tot}} = \zeta_{\xi_1} \zeta_{\xi_2} \zeta_{\xi_3} \zeta_{\xi_4},
\] (39)

one can easily obtain the complete expression of $\zeta_1$,
\[
\zeta_1 = \frac{m_{\text{Pl}}}{k_0^3} \pi \Delta_R^2(k_0)(1-n_\phi) \frac{n}{2(n+2)} \left[ \frac{T_{\text{CMB}}}{T_{\text{RH}}} \left( \frac{S_{\text{et}}}{S_{\text{et}}} \right) \right]^{1/3} \exp \left[ - \frac{n+2}{2(1-n_\phi)} \right].
\] (40)

One can examine that, for $n = 2$, the above expression reduces to equation (11) in [41] after using equation (7) in the same reference. In the following, let us see the reasonable range of the index $n$ constrained by both theories and observations. As well known, at the end of inflation, the scalar field $\phi$ oscillates quickly around some point where $V(\phi)$ has a minimum. In the limit that the oscillation rate is much greater than Hubble expansion rate $H$, and ignoring the coupling between the scalar field $\phi$ and other components, it is found that [42] the scalar field oscillations behave like a fluid with $p = \bar{\rho} \rho$, where the average equation of state $\bar{\rho}$ depends on the form of the potential $V(\phi)$. For $V(\phi) = \lambda \phi^n$, one has
\[
\bar{\rho} = \frac{n-2}{n+2}
\] (41)

and $\rho$ decreases as $a^{-6n/(n+2)}$. In particular, $n = 2$, one has $\bar{\rho} = 0$ and $\rho \propto a^{-3}$, which imply a matter-dominant-like expansion of the preheating stage [49]. Adding the consideration of the coupling between the scalar field and the resulting relativistic particle creation, Martin and Ringeval [43] verified the relation (41) using a numerical method, and it was found that the average $\bar{\rho}$ never deviates from zero exceeding 8%. From theoretical consideration, one should have $\bar{\rho} < 1$ to satisfy the positivity energy conditions; while $\bar{\rho} > -1/3$ to make sure that the inflation must stop and the preheating stage begins. Due to equation (41), the condition $-1/3 < \bar{\rho} < 1$ leads to $n > 1$. On the other hand, Martin and Ringeval [43] firstly gave a constraint on $n$ based on the CMB observations, $n < 2.1$. Therefore, based on both the theories and observations, the index $n$ is constrained to be
\[
1 < n < 2.1.
\] (42)

Note that there is a relation between $n$ and $\beta_\nu$. According to the energy conservation equation and the Friedmann equation,
\[
\dot{\rho} + 3\frac{a}{a} \rho (1 + \bar{\rho}) = 0,
\] (43)

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho,
\] (44)

one can easily obtain $a \propto t^{2/(3+3\bar{\rho})} \propto t^{2/(1+3\bar{\rho})}$. Using equations (7) and (41), and allowing for $\rho \propto a^{-6n/(n+2)}$, one has
\[
\beta_\nu = \frac{4 - n}{2(n - 1)}.
\] (45)

Then, in principle, the expression of $\zeta_1$ in equation (40) can be rewritten as a function of $\beta_\nu$. From the combination of equations (42) and (45), one finds that, $n > 1$ leads to $\beta_\nu > -0.5$ and $n < 2.1$ leads to $\beta_\nu > 0.86$, respectively. Based on the range of $n$ (or $\beta_\nu$) discussed above, we try to constrain some parameters combining with CMB observations.
4. Parameter constraints from observations

As shown in the previous section, many parameters are dependent on the value of $n_s$. Seven-year WMAP Mean [34] gives $n_s = 0.967 \pm 0.014$ and $n_s = 0.982^{+0.020}_{-0.019}$ when one also considers the tensor mode contributions to the anisotropies of CMB. Moreover, the combination WMAP+BAO+$H_0$ Mean gives $n_s = 0.968 \pm 0.012$ and $n_s = 0.973 \pm 0.014$ when the tensor mode contributions are included. Independently, SDSS III predicts $n_s = 0.96 \pm 0.009$ [56]. As can be seen in equation (40), $\zeta$ is sensitively dependent on $n_s$, and in turn one can expect that the spectrum of RGWs also depend sensitively on $n_s$ in the very high frequencies. Therefore, for a general demonstration, we consider the cases: $n_s = 0.96, 0.97$ and $0.98$, respectively.

Firstly, let us constrain the tensor-to-scalar ratio $r$. According to equations (22) and (29), it is straightforward to obtain

$$r = \frac{8n}{n+2}(1-n_s).$$ (46)

We show this relation in figure 1. One can see that $r$ increases slowly with $n$. $r$ lies in $(0.11, 0.16)$, $(0.08, 0.12)$, and $(0.05, 0.08)$ for $n_s = 0.96$, $n_s = 0.97$ and $n_s = 0.98$, respectively. Similarly, from equations (27) and (31), one has

$$\beta = -2 - \frac{n}{2(n+2)}(1-n_s).$$ (47)

which is shown in figure 2. The parameter $\beta$ is constrained in the range of $(-2.007, -2.010)$, $(-2.005, -2.008)$ and $(-2.003, -2.005)$ for $n_s = 0.96$, $n_s = 0.97$ and $n_s = 0.98$, respectively. Therefore, the range of $n$ in equation (42) leads to very tight constraints on $r$ and $\beta$, which are limited in very narrow ranges with definite value of $n_s$.

Now, let us see the increase of the scale factor during preheating stage $\zeta_1$, which is expressed in equation (40). We plot it in figure 3 as a function of $n$ with definite values of $T_{RH}$. Allowing for the expansion of the universe, one would expect that $\xi_1 > 1$. As can be seen in figure 3, the cases of $n_s = 0.96$ and 0.97 can make sure that the resultant $\zeta_1$ is much larger than 1; however, the case of $n_s = 0.98$ cannot be compatible with the fact that $\zeta_1 > 1$ in the whole range of $n$ as shown in equation (42). If $n_s$ is determined well to be as high as 0.98, it will give very tight constraints on $n$. Concretely, $n \lesssim 1.7$ and $n \lesssim 1.3$ for $T_{RH} = 10^4$ GeV.
and $T_{RH} = 10^8$ GeV, respectively. What we are more interested in are the characteristic frequencies given by equation (15). With the help of equation (14), one can easily obtain the characteristic frequencies: $f_H = H_0 \simeq 2.28 \times 10^{-18}$ Hz, $f_H = H_0 \simeq 2.28 \times 10^{-18}$ Hz, $f_L \simeq 1.93 \times 10^{-18}$ Hz, $f_L \simeq 9.3 \times 10^{-17}$ Hz. The value of $f_s$ depends linearly on $T_{RH}$. For instance, $f_s \simeq 4.54 \times 10^{-3}$ Hz for $T_{RH} = 10^4$ GeV and $f_s \simeq 45.4$ Hz for $T_{RH} = 10^8$ GeV, respectively. Since $f_1 = f_s \frac{\zeta_1}{\zeta_1 - 1}$, it depends on the values of $n$ (or $\beta_s$), $n_s$ and $T_{RH}$. It is worthwhile to be studied in detail, since the characteristic frequency $f_1$ is approximately the highest frequency of RGWs. The modes whose frequency higher than $f_1$ decay with the expansion of the Universe [2, 3]. We plot $f_1$ as a function of $n$ with definite $n_s$ and $T_{RH}$ in figure 4. One can see that the behavior of $f_1$ along with the increasing $n$ is quite different for different values of $n_s$. For the case of $n_s = 0.98$, we plotted $f_1$ using dotted lines for the values of $n$ constrained by $\zeta_1 > 1$ which are shown in figure 3. In the part of large $n$, $f_1$ is
Figure 4. The upper limit frequency $f_1$ as a function of $n$ for different combinations of $n_s$ and $T_{\text{RH}}$. The dotted parts for $n_s = 0.98$ are constrained by the condition $\zeta_1 > 1$ shown in figure 3.

larger for smaller values of $n_s$. On the other hand, in the limit of $n \to 1$, $f_1$ becomes a fixed value independent of $n_s$, and moreover, the asymptotic fixed $f_1$ is larger for a larger value of $T_{\text{RH}}$. It is easy to understand if one has found that $\beta_s \to +\infty$ as $n \to 1$ from equation (45) which leads to $f_1 \to f_s$. The value of $f_1$ should be below the constraint from the rate of the primordial nucleosynthesis, $f_1 \lesssim 3 \times 10^{10}$ Hz [2]. When the acceleration epoch is considered, the constraint becomes $f_1 \simeq 4 \times 10^{10}$ Hz. This will in turn give some constraints on $n$, $n_s$, and $T_{\text{RH}}$.

As analyzed in our previous work [41], when the quantum normalization for the generation of RGWs during inflation is employed, one has

$$\Delta_{\text{RG}}(k_0)r^{1/2} = 8\sqrt{\pi} l_P l_0 T_{\text{RH}}^{-\frac{3}{2}} \left( \frac{\zeta_1}{\zeta_2} \right)^{\beta_s - \gamma_E} \left( \frac{k_0}{k_H} \right)^{\beta},$$

(48)

where $l_P = \sqrt{G}$ is the Planck length. In equation (48), there are totally six parameters: $r$, $\beta_s$, $n_s$, $n$, $T_{\text{RH}}$, and $l_P$. However, among them only three are independent, due to equations (45), (46) and (47). We show the $T_{\text{RH}} - \beta$ relation with definite values of $n_s$ in figure 5. First of all, we define the range of $6 \times 10^3 - 10^8$ GeV as region I, while the range of $6 \times 10^3 - 3 \times 10^{15}$ GeV as region II. It is found that, under the condition of quantum normalization, $n_s = 0.96$ and $n_s = 0.98$ can be ruled out, since the resultant $T_{\text{RH}}$ is outside region II. If one considers the gravitinos production problem, the case $n_s = 0.97$ would also be ruled out, since the resultant $T_{\text{RH}}$ lies well inside region I for the whole range of $\beta$ given by equation (47). Moreover, as shown in figure 5, the quantum normalization will give little tighter constraints on the range of $\beta$ for $n_s = 0.967$ and $n_s = 0.968$. Note that these results are based on the validity of quantum normalization; however, it is not the unique initial condition. Let us make a comparison with the previous results in [41]. Taking $n_s = 0.966$ for example, $n = 2$ leads to $T_{\text{RH}} \simeq 3.4 \times 10^6$ GeV; while $T_{\text{RH}} \simeq 2.8 \times 10^{12}$ GeV shown in figure 1 in [41]. Hence, the discrepancy of $T_{\text{RH}}$ at six orders of magnitude, indicates that the quantum normalization may not be a good initial condition. However, one should also keep in mind that we have used many approximations, which would also contribute a lot to the discrepancy of $T_{\text{RH}}$ discussed above. Note that, if one does not consider quantum normalization, the zero point energy should be removed or
Figure 5. The relation between $T_{\text{RH}}$ and $\beta$ based on equation (48) due to the condition of quantum normalization.

else the cosmological constant would be 120 orders of magnitude larger than observed. Some effective methods have been proposed, such as the decaying vacuum model [59]. In following section, we will demonstrate the spectra of RGWs with and without quantum normalization, respectively.

5. The spectra of RGWs and their detection

In this section, we demonstrate the energy density spectra of RGWs with reasonable values of the parameters and discuss the detection due to the current running and planned GW detectors.

As discussed in the previous sections, there are many parameters involved in the spectrum of RGWs. They are $n_s$, $n$, $r$, $\beta$, $\beta_s$ and $T_{\text{RH}}$. However, among them only three are independent due to equations (45)–(47). Furthermore, $n_s$ has been constrained well from observations of CMB, BAO and $H_0$. Since the spectrum of RGWs in high frequencies sensitively depends on $n_s$, we discuss two cases of $n_s = 0.968$ and $n_s = 0.973$, respectively, based on the combination of WAMP+BAO+$H_0$ [34]. In the following, we regard $\beta_s$ and $T_{\text{RH}}$ as parameters, and choose some representative values of them since they have large uncertainties. In order to give a complete discussion, we will consider the spectra of RGWs both with and without quantum normalization. As analyzed in section 3, $T_{\text{RH}}$ is constrained to be $T_{\text{RH}} \sim 10^4 - 10^8$ GeV, and $\beta_s$ is limited to be larger than 0.86. Firstly, let us see the case of no quantum normalization.

Setting $\beta_s = 1$, figure 6 shows the energy density spectra of RGWs, $\Omega_{\text{GW}}(f)$, with different values of $n_s$ and $T_{\text{RH}}$. One can see that all the $\Omega_{\text{GW}}(f)$ nearly overlap each other in the low frequencies. This is because the spectrum for $f \leq f_s$ is only related to $r$ and $\beta$ [41], and, moreover, the differences of $r$ and $\beta$ are very small between the case $n_s = 0.968$ and the case $n_s = 0.973$ due to equations (46) and (47). Explicitly, one has $r = 0.128$, $\beta = -2.008$ for $n_s = 0.968$ and $r = 0.108$, $\beta = -2.007$ for $n_s = 0.973$, respectively. However, in the part of high frequencies, $\Omega_{\text{GW}}(f)$ exhibits different properties for different combinations of $n_s$ and $T_{\text{RH}}$. On one hand, for the same value of $T_{\text{RH}}$, the spectrum $\Omega_{\text{GW}}(f)$ with $n_s = 0.968$ and that with $n_s = 0.973$ have the same ‘turning point’ from which $\Omega_{\text{GW}}(f)$ decreases rapidly with the increasing frequency, and the ‘turning point’ is just $f_s$ which is only dependent on $T_{\text{RH}}$. Moreover, the decreasing slope of the logarithm of the two spectra for $f \geq f_s$ is nearly
Figure 6. The energy density spectra of RGWs for the fixed $\beta_s = 1$ with different combinations of $n_s$ and $T_{RH}$, without considering the quantum normalization.

Figure 7. The energy density spectra of RGWs for the fixed $T_{RH} = 10^6$ GeV with different combinations of $n_s$ and $\beta_s$, without considering the quantum normalization.

the same since $\Omega_{GW}(f) \propto f^{4+2\beta-2\beta_s}$ [41] which is reduced to $\Omega_{GW}(f) \propto f^{-2}$ for $\beta \approx -2$ and $\beta_s = 1$. However, the $\Omega_{GW}(f)$ with a smaller $n_s$ has a larger upper limit frequency $f_1$ which responds to a lower amplitude of $\Omega_{GW}(f)$. On the other hand, for the same value of $n_s$, the $\Omega_{GW}(f)$ with a higher $T_{RH}$ leads to not only a larger $f_s$ but also to a larger $f_1$ since $f_1 \propto T_{RH}^{1/2}$ for $\beta_s = 1$ which can seen from the combination of equations (14), (21) and (40). Figure 7 shows the energy density spectra of RGWs for the fixed value $T_{RH} = 10^6$ GeV. One can see that a larger $\beta_s$ leads to a steeper slope of the logarithm of $\Omega_{GW}(f)$ and a smaller $f_1$ for the same values of $n_s$. In words, $T_{RH}$ determines the value of $f_s$, $\beta_s$ determines the slope of the logarithm of $\Omega_{GW}(f)$ for the fact that $\beta \approx -2$, and $f_1$ depends on all the three parameters especially $n_s$. Secondly, let us consider the case of the quantum normalization. Due
Figure 8. The energy density spectra of RGWs under the condition of the quantum normalization.

Table 1. The definitions of $h_c$ with different parameters. 'N' stands for 'No' meaning that the condition of the quantum normalization is not considered; while 'Y' stands for 'Yes' meaning that the condition of the quantum normalization is considered.

| $h_c$ | $n_s$ | $\beta_s$ | $T_{RH}$ | Quantum normalization |
|------|------|---------|---------|---------------------|
| $h_{c1}$ | 0.968 | 1 | $10^4$ GeV | N |
| $h_{c2}$ | 0.968 | 1 | $10^6$ GeV | N |
| $h_{c3}$ | 0.968 | 4 | $10^6$ GeV | N |
| $h_{c4}$ | 0.973 | 1 | $10^6$ GeV | N |
| $h_{c5}$ | 0.968 | 1 | $2 \times 10^{11}$ GeV | Y |
Figure 9. The strain of RGWs with different parameters for $n_s = 0.968$, $\beta_s = 1$ and $T_{RH} = 10^6$ GeV confronts the current and planned GW detectors. The sensitivity curves of PPTA and SKA using the pulsar timing technique are taken from [25] and [60], respectively. The curve of BBO is generated using the online ‘sensitivity curve generator’ [19] with the parameters in table II of [20] and table I of [21]. The curve of DECIGO is taken from [22].

Figure 10. The characteristic amplitude of RGWs in high frequencies confronting the instrument noise $\sqrt{\mathcal{H}(f)}$ of BBO, ultimate DECIGO [23], AdvLIGO and ET.

even though AdvLIGO and ET are hard to catch the signals of RGWs, BBO has the potential to distinguish RGWs with different parameters or different conditions in the frequency band $10^{-2}$–$10^0$ Hz. Furthermore, the ultimate DECIGO has the capability to distinguish them more easily. Thus, the future BBO and DECIGO detections provide an important tool not only for determining the parameters but also for examining the validity of the quantum normalization when RGWs were generated during inflation. It is worthwhile to point out that, at frequencies...
lower than $10^{-2}$ Hz the signals of RGWs are contaminated by the confusion noise produced by galactic binaries [61]. Hence, we should focus on the frequencies higher than $10^{-2}$ in order to distinguish various spectra of RGWs.

6. Conclusions and discussions

In the frame of the slow-roll inflation with a power-law form $V = \lambda \phi^n$, we calculated the analytic solutions of RGWs. In the narrow range $1 < n < 2.1$, the tensor-to-scalar ratio $r$ and the inflation expansion index $\beta$ are both tight limited to lie in narrow ranges for a given value of the scalar spectral index $n_s$. Concretely, $r$ lies in (0.11, 0.16), (0.08, 0.12) and (0.05, 0.08) for $n_s = 0.96$, $n_s = 0.97$ and $n_s = 0.98$, respectively; while $\beta$ lies in the range of $(-2.007, -2.010)$, $(-2.005, -2.008)$ and $(-2.003, -2.005)$ for $n_s = 0.96$, $n_s = 0.97$ and $n_s = 0.98$, respectively. Moreover, the preheating expansion index $\beta_s$ is constrained to be $\beta_s > 0.86$. We found that the spectrum of RGWs in high frequencies depends on the parameters $n_s$, $\beta_s$ and $T_{\text{RH}}$. Explicitly, $T_{\text{RH}}$ determines $f_s$ where the flat RGWs spectrum decreases suddenly. $\beta_s$ determines the decreasing slope of the logarithm of the spectrum. Whereas, the upper limit frequency $f_1$ is dependent on all the three parameters $n_s$, $\beta_s$ and $T_{\text{RH}}$. Besides, the quantum normalization for the generation of RGWs also affect the spectrum of RGWs in high frequencies.

Among the current and planed GW detectors, SKA using the pulsar timing technique and the space-based interferometers eLISA, BBO and DECIGO are promising to catch the signals of RGWs. Furthermore, BBO and DECIGO have the potential not only to distinguish the spectra with different parameters but also to examine the validity of the quantum normalization. Therefore, RGWs could become the most important tool to know the physics of the very early Universe such as the inflation and reheating process. Even though we chose a series power-law form potential of the scalar field, as shown in [42], a polynomial form of the potential will be dominated by the lowest power of $\phi$ in $V$. In this case, the conclusion is not substantially modified. In our previous work [41], we obtained the $r - \beta$ relation for a particular potential $V = \frac{1}{2} m^2 \phi^2$. However, for the more general case $V = \lambda \phi^n$, it is hard to obtain a complete analytic solution of $T_{\text{RH}}$ and in turn the increases of the scale factor $\zeta_s$ and $\zeta_1$. Therefore, in this paper, we set a series reasonable values of $T_{\text{RH}}$ as additional parameters. The determination of $n_s$ is very important, since it sensitively affects our results. The future CMB experiments such as the Plank satellite [35], the ground-based ACTPol [36] and the planned CMBPol [37] will help us to determine the more convincible value of $n_s$. Therefore, one can expect accordingly that the spectrum of RGWs would be known better.

In principle, our analysis is valid for the slow-roll inflation with other forms of the potential $V(\phi)$. However, for some particular forms of $V(\phi)$, it would be difficult to obtain the analytic result of $\zeta_1$ as a function of the parameters included in $V(\phi)$. Moreover, one could not effectively constrain the parameters in $V(\phi)$. However, one can still calculate $\zeta_1$ numerically according to the whole calculating processes presented in [40], and then calculate the spectra of RGWs accordingly. More general inflationary models other than the slow-roll inflation and the slow-roll inflation with other forms of $V(\phi)$ would be studied in our future work.

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