Modeling of the stress-strain state of reinforced concrete beams under prolonged load action

Z Holovata¹,², S Neutov¹, M Surianinov¹

¹ Odesa State Academy of Civil Engineering and Architecture, 4, Didrikhsona st., Odesa 65029, Ukraine
² Address all correspondence to this author.
E-mail: zlataholovataya@gmail.com

Abstract. The paper considers the use of a layered deformation model of the stress-strain state of reinforced concrete beams experiencing transverse bending taking into account the long-term action of the load. The results of experimental studies of twelve beams made of concrete of three classes for short-term and long-term acting load are given. It has been established that long-term strength of inclined cross-sections follows the general laws of deformation and fracture of reinforced concrete. Comparison of calculation results of the proposed deformation model with experimental data has shown their satisfactory convergence. A tendency of some increase (within 10...15%) in the long-term strength of the reinforced concrete elements under study in the inclined sections has been established experimentally.

1. Introduction
There are many works devoted to the study of reinforced concrete beams for strength, stiffness and cracking resistance under prolonged exposure to load. Much fewer publications are connected with the consideration of this problem in relation to fiber reinforced concrete beams.

Long-term exposure to loading that causes microcracks in the material during initial loading shows that the failure of the specimen proceeds slowly over a long period of time. Experiments with concrete (and to an even greater extent with fiber concrete) show that the long-term strength of concrete lies above the limit that determines the strength under long-term loading. There is still no consensus on the factors influencing the long-term strength of concrete and fiber concrete beams when they are bent. The problem requires further research and continues to be relevant. Nowadays in Ukraine the design of concrete and reinforced concrete structures is carried out according to the limit states of the first and the second groups taking into account as a rule the buildings and structures responsibility classes and responsibility categories of structural elements set by corresponding normative documents, variability of properties, loads and influences, geometry characteristics, operation term and working conditions of structures [1]. Calculation of fiber concrete structures when reinforced with steel fiber is regulated by the normative document DSTU-N B B.2.6-218:2016 [2].

2. Literature review
The existing methods of calculation of reinforced concrete elements under the action of bending moments, longitudinal and transverse forces can be divided into two main groups: methods based on consideration of the stress-strain state of the section [3-6] and methods not using strain relations. The first group usually includes the methods of calculation of elements based on normal sections, contained
in the design standards of most developed countries, including the European standards [7-9]. These methods are based on calculation models based on unified principles, the main of which are the hypothesis of plane sections and material deformation diagrams. This approach makes it possible to calculate strength, crack formation and crack opening, as well as deformations from a unified position. Characteristic representatives of the second group are methods of calculation, which are in good agreement with experiments, but do not have a common basis in the calculations for various types of limit states. These methods are full of numerous empirical relations.

The study of bendable reinforced concrete elements under long-term loads was carried out by many well-known scientists. However, in our country in recent years, such studies were carried out little. The work [10] is devoted to the study of cracking in reinforced concrete beams under long-term loads on the basis of the energy approach. Development of calculation methods for steel reinforced concrete beams as elements of slabs under different types and modes of loading is devoted to the article [11].

In foreign publications, the emphasis has shifted to the study of beams with dispersed reinforcement. Thus, the paper [12] presents the results of studying the long-term behavior of composite reinforced concrete beams under long-term loads. It is noted that the deflections in the middle of the span after 3 years under load increased by 2.5 times compared to the initial deflections. Work [13] is aimed at studying, both analytically and experimentally, the deflections of beams reinforced with fiberglass that have been subjected to prolonged loads for 2 years. Experiments with beams made of high-strength fiber reinforced concrete (SFRC) subjected to long-term bending loads are described in [14].

3. The purpose of this work is to use a layered deformation model of the stress-strain state of reinforced concrete beams experiencing transverse bending, taking into account the long-term action of the load.

4. Materials and Methods
Mathematical methods of experiment planning with obtaining and analysis of appropriate experimental and statistical models have been used. Determination of physical, physical-mechanical and construction-technical properties of concrete and fiber concrete has been carried out in accordance with the current regulatory documents and generally accepted methods.

5. Results and Discussion
We assume the deformation model of the beam in discrete form: we divide the beam into \( i \) sections (multiples of the step of the transverse reinforcement) along the length and \( j \) layers (1 cm each) along the height of the section. The stress-strain state of the formed sections is assumed to be the same.

The reinforcement of the tensile and compressed zones of the beam is "tied" to the faces of the section at the level of their centers of gravity with the values of the protective layers \( a \) and \( a' \) (figure 1).
Figure 1. Design cross-section.

The modeling of the stress-strain state of the beam, experiencing transverse bending under prolonged load action, is based on the following: the relations between the stresses and strains of concrete and reinforcement are represented by real diagrams $\sigma - \varepsilon$; distribution of deformations along the height of sections occurs in accordance with the hypothesis of plane sections; the hypothesis of joint deformations of concrete and reinforcement is accepted; the loading of the beam is carried out by two symmetrically arranged forces $F$, applied at a distance $c$ from the supports (figure 2); all characteristics of the stress-strain state of the beam are determined by the method of successive approximations.

To describe diagram $\sigma_b - \varepsilon_b$ (figure 3), we use relation:

$$\frac{\sigma_b(t)}{R_b} = \frac{k \cdot \eta - \eta^2}{1 + (k - 2) \cdot \eta^k},$$

where $\eta = \varepsilon_b(t)/\varepsilon_{bu}$ – value, which characterizes the intensity of the increase in deformation in concrete $\varepsilon_b(t)$; $\varepsilon_{bu}$ – the deformation value that corresponds to the greatest value of prism strength $R_b$; $k$ – a coefficient that takes into account the influence of the inelastic component of the deformation.

![Diagram Q(x), kN](image)

![Diagram M(x), kN/m](image)

Angles of rotation diagram

![Deflection diagram](image)

Figure 2. Diagrams of internal forces, rotation angles and deflections of the beam from the action of the transverse load without taking into account its own weight

The prism strength of concrete at any point in time $t$ from the day of concreting is determined by the known formula:

$$R_b = \frac{1}{\alpha_{bt}} \cdot \frac{\varepsilon_{bt}}{\varepsilon_{bu}}$$
\[ R_b = R_{b28} \frac{\lg t}{\lg 28}, \]  
where \( R_{b28} \) – prism strength of concrete at 28 days of age.

\[ \sigma_b = 3,6R_b - \left( \frac{3R_b}{36} \right)^5 + 51 \cdot 10^{-5}. \]  

In order to bring the ascending branch of the curve (1) closer to the experimental data, an additional condition was introduced when determining the coefficient \( k \). Such a condition is the point of the diagram, which corresponds to the initial deformation modulus of concrete \( E_{b0} \). In the current norms in Ukraine [1], the initial modulus \( E_{b0} \) is determined at stresses \( \sigma_b = 0,3R_b \). For this stress level, the deformation of the concrete is

\[ \varepsilon_b = \frac{\sigma_b}{E_{b0}} = \frac{0,3R_b}{E_{b0}}, \]  
then

\[ \eta_0 = \frac{0,3R_b}{\varepsilon_{bu}E_{b0}}. \]  

After substitution into (1) \( \sigma_b/R_b = 0,3 \) and expression (4) we have:

\[ 0,3 = \frac{k \cdot \eta_0 - \eta_0^2}{1 + (k - 2)\eta_0}, \]  

After transforming (5), we get the expression for the coefficient of elasticity of concrete \( k \) on the ascending branch of the diagram:

\[ k = \frac{\eta_0^2 + 0,6\eta_0 + 0,3}{0,7\eta_0}. \]
To describe the descending branch of the concrete compression diagram, the point corresponding to the final value of the concrete strain $\varepsilon_{\text{bmax}}$ is introduced:

$$\varepsilon_{\text{bmax}} = \frac{R_b \cdot 10^{-2}}{10 + 2.75R_b}. \quad (7)$$

On the assumption that $\varepsilon_{\text{bmax}}$ is observed at stresses $\sigma_b = 0.8R_b$, coefficient of elasticity $k_1$ on the downward branch of the diagram $\sigma - \varepsilon$ of the concrete takes the form of

$$k_1 = \frac{\eta_0^2 + 1.6\eta_0 + 0.8}{0.2\eta_0}. \quad (8)$$

To describe diagram $\sigma - \varepsilon$ for tensile concrete we use the same dependence (1) as for compression. Values $\varepsilon_{\text{tmax}}$, $\varepsilon_{\text{tmax}}$, we obtain by multiplying $\varepsilon_{\text{bmax}}$, $\varepsilon_{\text{bmax}}$ on the ratio $R_{\text{t}}/R_b$. In the final form, the tensile stresses in the concrete are determined using the dependence

$$\frac{\sigma_{\text{b}}(t)}{R_{\text{t}}} = \frac{k \cdot \eta + \eta^2}{1 + (k - 2)\eta}, \quad (9)$$

where $\eta = \varepsilon_{\text{b}}(t)/\varepsilon_{\text{tmax}}$ – value that characterizes the intensity of tensile deformation $\varepsilon_{\text{b}}$ in concrete; $\varepsilon_{\text{tmax}}$ – deformation value of concrete at maximum tensile stresses.

The tensile strength of concrete is determined by the formula

$$R_{m} = R_{m28} \frac{\lg t}{\lg 28}. \quad (10)$$

The secular modulus of deformation of concrete at any age and level of loading at some point $i$ of the diagram is defined by the dependence

$$E_{\text{b}}(t) = \frac{\sigma_{\text{b}}(t)}{\varepsilon_{\text{b}}(t)} = \tan \alpha_t. \quad (11)$$

Creep deformations $\varepsilon_{cr}$ can be taken into account by using the known dependences proposed in [5, 6] for concretes without damage and in [15, 16] for concretes damaged by corrosive processes. The determination of creep deformations [15] is based on the law of change in the value of concrete creep characteristic suggested by A. B. Golyshev [5], which can be represented in the following form:

$$\varphi(t) = \varphi_k (1 - e^{-\lambda(t)}), \quad (12)$$

where $\lambda(t) = 1.5 \sqrt{t/365}$; $\varphi_k$ – the final value of the creep characteristic, which can be determined taking into account the corrosion damage of concrete by the formula:

$$\varphi_k = \xi_0 \cdot \xi_{\text{cr}1} \cdot \xi_{\text{cr}2} \cdot \xi_{\text{cr}3} \cdot C_m \cdot E_b. \quad (13)$$

The normative value of the creep measure can be determined by the formula

$$C_m = 12.5 \cdot 10^{-6} \frac{B}{R_b}. \quad (14)$$

Conclusively, given that $\varphi(t) = \varepsilon_{cr}(t)/\varepsilon_b(t)$, after some transformations, the formula for determining the creep deformation of concrete with regard to corrosion damage will look like

$$\varepsilon_{cr}(t) = \frac{\varphi_e \cdot \varphi_{\text{cr}1} \cdot \xi_{\text{cr}2} \cdot \xi_{\text{cr}3} \cdot 12.5 \cdot 10^{-6} \frac{B \cdot \sigma_b(t)}{R_b}(1 - e^{-\lambda(t)}), \quad (15)$$
where  $\xi^c_b$ – coefficient, which takes into account the presence of corrosion damage and which is recommended to take equal to 1.05 ... 1.15, depending on the type of concrete;  $\xi_{c1}$, $\xi_{c2}$, $\xi_{c3}$ – coefficients that take into account the humidity of the environment, the size of the structure, the time of their loading.

According to the recommendations of O. Y. Berg [17], they can be determined by the formulas:

$$
\xi_{c1} = \frac{1.54(135 - W)}{100};
$$

$$
\xi_{c2} = 0.90\left(0.70\frac{1}{r}\right);
$$

$$
\xi_{c3} = 0.36\frac{26.4}{(13 + t)};
$$

where  $t$ – loading time;  $r$ – exposed concrete specific surface;  $W$ – ambient humidity.

Swelling deformations of concrete  $\varepsilon_{sh}(t)$ can be determined using the dependencies [5], as well as taking into account the effect of corrosion [15].

The value of shrinkage deformations of concrete according to [5] is determined by the formula

$$
\varepsilon_{sh}(t) = \varepsilon_{shb}(1 - e^{-\lambda(t)}),
$$

where  $\varepsilon_{shb}$ – the final value of concrete shrinkage deformations, which, taking into account corrosion damage, can be determined from the condition:

$$
\varepsilon_{shb} = \frac{\xi^c_b \cdot \varepsilon_{sh1} \cdot \xi_{sh2} \cdot \varepsilon_{shu}}{\xi^c_b};
$$

At the same time

$$
\varepsilon_{shu} = 0.14 \cdot 10^{-6} \sqrt{B};
$$

$$
\varepsilon_{sh}(t) = \varepsilon_{shb} \cdot \varepsilon_{sh1} \cdot \varepsilon_{sh2} \cdot 0.14 \cdot 10^{-6} \sqrt{B}(1 - e^{-\lambda(t)});
$$

The final formula for determining the deformations takes the form

$$
\varepsilon_{sh}^0(t) = \gamma_{sh}^0 \varepsilon_{sh}(t),
$$

where  $\gamma_{sh}^0$ – transition coefficient from shrinkage deformation to swelling deformation;  $\varepsilon_{sh1}$,  $\varepsilon_{sh2}$ – coefficients, which take into account the effect of humidity and size of structures and which [18] are recommended to determine from the conditions:

$$
\varepsilon_{sh1}(t) = 1.52\left[1 - \frac{W}{100}\right]^3,
$$

$$
\varepsilon_{sh2}(t) = 0.035(31 - r).
$$

The total design deformations of concrete, taking into account long-term processes and corrosion, are:

$$
\varepsilon_{bij}(t) = \varepsilon_{bijb} + \varepsilon_b(\sigma_b, t) + \varepsilon_{ct}(t) - \varepsilon_{sh}(t)_d,
$$

where  $\varepsilon_{bijb}$ – deformations of concrete under short-term loading;  $\varepsilon_b(\sigma_b, t)$ – deformations of concrete due to stress growth in the specimens due to a decrease in the cross-section due to corrosion stresses.

In [19] it was proposed to take into account the transformation of the parameters of the deformation diagram of concrete depending on the time and level of long-term loading, based on the values of these parameters during short-term loading using the following dependencies:
\[
\begin{align*}
\dot{\xi}_{b_{\text{max}}}(t) &= \dot{\xi}_b [1 + \eta_c (0.1800 + 0.2734 \ln(t))]; \\
\dot{\xi}_{bu}(t) &= \dot{\xi}_b [1 + \eta_c (0.206 + 0.239 \ln(t))]; \\
\beta(t) &= \beta + \left( \frac{\dot{\xi}_{bu}(t) - \dot{\xi}_{bu}}{\dot{\xi}_{b_{\text{max}}}} \right) \left( \frac{0.932 - \beta}{2} \right); \\
\sigma_{b_{\text{max}}}(t) &= R_b [0.9488 - 0.0166 \ln(t)] k (\eta_c, t); \\
k (\eta_c, t) &= 1 - 0.152 \ln(t) [1 - 1.4 \eta_c + 0.491 \eta_c^2].
\end{align*}
\]

where \( \eta \) – long-term stress level; \( R_b (\tau - \tau_c) \) – prism strength of concrete after prolonged stress action; \( R_b (\tau) \) – is the prism strength of the unloaded concrete specimen at the considered moment of time; \( \tau \) – stress action duration.

The relationship between the stresses and deformations of concrete under prolonged load action, as recommended in [19], can be taken as a diagram described by a polynomial

\[
\sigma_b(t) = \sigma_{b_{\text{max}}}(t) \sum_{i=1}^{5} a_i(t) \left( \frac{\dot{\xi}_b(t)}{\dot{\xi}_{b_{\text{max}}}} \right)^i,
\]

where \( \sigma_b(t) \) – stress in the concrete at an arbitrary moment of time; \( \dot{\xi}_b(t) \) – total deformations of concrete at an arbitrary moment of time; \( a_i(t) \) – coefficients of the polynomial describing the deformation diagram of concrete under long-term load action.

The relationship between stresses and strains is represented in the form of two linear diagrams (Figure 4).

![Figure 4. Diagram \( \sigma - \varepsilon \) for reinforcing steel](image)

The bending moment and shear force along the length of the element are variables:

\[
\begin{align*}
- &at \ 0 < x \leq c \quad M(x) = Fx; \quad Q(x) = F; \\
- &at \ c < x < l - c \quad M(x) = Fc; \quad Q(x) = 0; \\
- &at \ l - c < x \leq l \quad M(x) = F(l - x); \quad Q(x) = -F.
\end{align*}
\]

Experimental studies have been carried out for the long-term action load of three series of specimens of 4 beams in each with the shear span \( c = 35 cm \), made of concrete class C12/15, C20/25, C28/35, respectively. The destruction of all the beams, both under short-term and long-term action of the load, occurred along the inclined crack. In each series, one beam was subjected to short-term loading until failure, and the remaining 3 beams were subjected to long-term loading, respectively, with load levels of 0.85; 0.9 and 0.95 of the destructive load. The experiments showed that the beams with a load level of 0.95 failed, depending on the concrete class, in the interval from 1 to 65 hours. The higher the concrete class, the longer the interval to failure. Beams loaded to the 0.9 level of breaking load stood under load for 5 to 70 days, after which failure occurred. When tested at this load level, a steady increase in deflections and crack opening widths was observed. By the time of destruction, the width of inclined
cracks opening under prolonged action of load exceeded the similar indicators under short-term load in 1.6–2.1 times, and the deflections - in 1.7–2 times.

In beams of all three classes of concrete at a load of 0.85Fu, stabilization of deflections and crack opening width were observed. After an eight-month observation period, these beams were brought to failure. As a result of studies, it was found that the failure load for beams loaded with 0.85Fu long-term load increased by 13-15% compared to short-term loading.

6. Conclusions
Thus, it has been established that the long-term strength of inclined sections obeys the general laws of deformation and fracture of reinforced concrete.

Comparison of the results of calculations using the proposed deformation model with experimental data showed their satisfactory convergence.

The tendency of some growth (in the range of 10...15%) of the long-term strength of the studied reinforced concrete elements in inclined sections has been established experimentally.

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