An Axion-Like Particle from an $SO(10)$ Seesaw with $U(1)_X$

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Abstract

We investigate the decoupling of heavy right handed neutrinos in the context of an SO(10) GUT model, where a remnant anomalous symmetry is $U(1)_X$. In this model the see-saw mechanism which generates the neutrino masses is intertwined with the Stueckelberg mechanism, which leaves the CP-odd phase of a very heavy Higgs in the low energy spectrum as an axion-like particle. Such pseudoscalar is predicted to be ultralight, in the $10^{-20}$ eV mass range. In this scenario, the remnant anomalous $X$ symmetry of the particles of the Standard Model is interpreted as due to the incomplete decoupling of the right handed neutrino sector. We illustrate this scenario including its realisation in the context of SO(10).
1 Introduction

Recently there has been considerable interest in the occurrence of axion-like particles [1–3] including the appearance in model building of anomalous $U(1)$ symmetries with a Stueckelberg field [4–19]. In this paper we examine the simplest GUT example where this phenomenon is closely related to the see-saw mechanism [20] for generating the neutrino masses and may provide a link between axions and right-handed neutrinos.

At the same time our scenario establishes a possible link between leptogenesis and dark matter [21–23] in a generalized setting, due to the prediction of an axion in the low energy spectrum. Stueckelberg axions ($b(x)$) appear in the field theory realization of the Green-Schwarz mechanism of anomaly cancelation of string theory, in the dualization of a 3-form, and correspond to pseudoscalar gauge degrees of freedom (see also the discussion in [18]). As ordinary Nambu-Goldstone modes they undergo a local shift

$$b(x) \rightarrow b(x) + M\theta(x)$$

under an abelian gauge transformation and are coupled to the anomaly via a dimension-5 operators of the form $b(x)/MF \wedge F$ where $F$ is, generically, the field strength of the gauge fields which share a mixed anomaly with the $U(1)$ symmetry, and $M$ is the Stueckelberg scale.

In these scenarios, pseudoscalar gauge degrees of freedom may develop physical components only after the breaking of the shift symmetry by some extra potential. This is expected to occur in the case of phase transitions in a non-abelian gauge theory, when instanton interactions naturally arise and induce a mixing between the Stueckelberg field and the Higgs sector of the theory, with the generation of a periodic potential, after spontaneous symmetry breaking.

This scenario in which the CP odd phases of the scalar sector mix and generate such a potential, has provided the basic template for the emergence of a physical CP odd state, in a way which is very close to what was conjectured to occur in the case of the electroweak or DFSZ version of the Peccei-Quinn [24] axion (see the review [25]), where the anomalous symmetry is a global rather than a local one.

Indeed, we recall that in the DFSZ case one writes down a general potential, function of three scalar fields, which is $SU(2) \times U(1)$ invariant. The simplest realization of this scenario is in the two-Higgs doublet model, where the Higgs fields $H_u$ and $H_d$ are assigned the global symmetry

$$H_u \rightarrow e^{i\alpha X_u} H_u, \quad H_d \rightarrow e^{i\alpha X_d} H_d$$

under $U(1)_{PQ}$ and are accompanied by an additional scalar $\Phi$, which is singlet under the Standard Model (SM) symmetry

$$\Phi \rightarrow e^{i\alpha X_\Phi} \Phi$$

with $X_u + X_d = -2X_\Phi$. The potential is given by a combination of terms of the form

$$V = V(H_u^2, H_d^2, |\Phi|^2, |H_uH_d^\dagger|^2, |H_u \cdot H_d^\dagger|^2, |H_u \cdot H_d|^2, \Phi^2)$$
(with $H_u \cdot H_d \equiv H_u^\alpha H_d^\beta \epsilon_{\alpha\beta}$) which is invariant under the Standard Model gauge symmetry and is in addition invariant under the global $U(1)_{PQ}$.

As pointed out in [19] a similar effective theory can be obtained in the case of a gauge symmetry, in a scenario that leaves most of the intermediate steps in the generation of Stueckelberg-like Lagrangian unchanged. In this realization of the Stueckelberg Lagrangian, the Stueckelberg pseudoscalar emerges from the phase of the complex scalar field which is responsible for the breaking of the gauged $U(1)$ symmetry. The breaking takes place at the GUT (Grand Unified Theory) scale, which takes the role of the Stueckelberg mass for the low energy effective theory.

In our case such abelian symmetry is contained within $SO(10)$ and it is identified with $U(1)_X$. This provides the basic observation which motivates our work, which connects the decoupling of a gauge boson corresponding to an $U(1)_X$ symmetry within $SO(10)$ and of a right-handed neutrino to the appearance of an axion in the spectrum of the low energy theory. Being the construction sequential in each of the three generations, this scenario predicts three axions in the spectrum. Building on a similar analysis by two of us in [4] based on a $E_6 \times U(1)_X$, such axions are expected to be ultralight, in the $10^{-20}$ eV mass range.

1.1 Incomplete decoupling of a chiral fermion and global anomalous $U(1)_X$

We believe it is useful to scrutinise this within a transparent model where two examples of physics beyond the standard model, the non-zero neutrino masses and the Stueckelberg axion are closely related. Since we know from experiment [26] that the first extension exists in Nature, it increases our expectation that the second should be realised. We shall review the group theory of $SO(10)$ including the available irreducible representations for the matter particles and the symmetry breaking. $SO(10)$ naturally provides three right-handed neutrinos which can participate in the see-saw. Because of the decoupling of these additional neutrino states at high masses, the resultant effective theory possesses an anomalous $U(1)_X$ symmetry. Since we shall discuss neutrino masses it is worth recalling the various possibilities for introducing them into the minimal SM. We shall mention four of these, one being the see-saw mechanism, and reveal why the other three are less attractive. One of them, introduced in [27], once appeared to be compelling when based only on the SuperKamiokande experiment [26] but it predicted maximal solar neutrino mixing which unfortunately was subsequently excluded by the SNO experiment [28]. This left as the most popular possibility the see-saw mechanism which we shall employ in the present model. When neutrino masses were established experimentally in 1998 there was confusion about to whom priority for the see-saw idea belonged and it was temporarily assigned to a number of theory papers published in 1979. Further scholarship revealed, however, that priority belonged to a 1977 paper by Minkowski [20].
2 SO(10) Grand Unification

The SO(10) model for unifying quarks and leptons was invented over forty years ago in 29,30. After non-zero masses for neutrinos were discovered, it became the most popular GUT superseding the otherwise more economical SU(5) GUT 31. A recent discussion of an SO(10) GUT can be found in 32. In the minimal Standard Model (SM), as in the minimal SU(5) GUT, the neutrinos were assumed to be massless. In the SO(10) GUT, each family in a 16 contains, in addition to the fifteen helicity states of the minimal SM, a right- handed neutrino $N$. This gives rise to several additional features, beyond the most obvious one that the neutrinos can acquire mass through the see-saw mechanism. An SU(5) GUT subsumes the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ but an SO(10) GUT with one additional rank includes also a $U(1)_X$. It is this gauged $(X)$ symmetry and its breaking which will play a central role in our present discussion.

The group theory underlying the SO(10) GUT is well-known and reviewed in many papers; one reliable such reference is 33. For the purposes of establishing notation we shall briefly discuss this with special emphasis on the role of $(X)$ symmetry which will be treated further in subsequent subsections.

2.1 Breaking patterns

The gauge group $SO(10)$ has the dimension 45 of its adjoint. An adjoint of scalars can break the symmetry while preserving rank-5 to

$$SO(10) \rightarrow [SU(3)_C \times SU(2)_L \times U(1)_Y]_{SM} \times U(1)_X$$

with $3X = 12Y - 15(B - L)$. We shall need more scalars to give mass to the fermions. Each family is in a 16 irreducible representation. For example the first family is

$$16 \equiv (u^r, u^g, u^b, d^r, d^g, d^b; u^r, u^g, u^b, d^r, d^g, d^b; \nu_e, e^-, N, e^+)_{L}$$

where we have designated the colours as $r, g, b$ (= red, green, blue). The Yukawa couplings which can provide fermion masses require scalar fields which are included in

$$16 \times 16 = 10_s + 120_a + 126_s$$

where the subscripts $s,a$ specify symmetric, antisymmetric. The 10 is the vector representation of SO(10), although the spinor representation 16 is really the defining representation, because one can make 10 from 16, as in Eq.(7), but not vice versa. We first consider the decomposition of SU(5) into $SU(3)_c \times SU(2)_L \times U(1)_Y$, adopting the notation $(SU(3)_C, SU(2)_L)_Y$ with the result that
\[ 5 = (3, 1)_{+2/3} + (1, 2)_{-1} \]
\[ 10 = (3, 2)_{+1/3} + (3, 1)_{-4/3} + (1, 1)_{+2} \]
\[ 15 = (6, 1)_{-4/3} + (3, 2)_{+1/3} + (1, 3)_{+2} \]
\[ 24 = (8, 1)_0 + (3, 2)_{-5/3} + (3, 2)_{+5/3} + (1, 1)_0 + (1, 1)_0 \]
\[ 45 = (8, 2)_{+1} + (6, 1)_{-2/3} + (3, 2)_{-7/3} + (3, 1)_{-4/3} + (3, 3)_{-2/3} + (3, 1)_{-2/3} + (1, 2)_+ \]
\[ 50 = (8, 2)_{+1} + (6, 1)_{+8/3} + (6, 3)_{-2/3} + (3, 2)_{-7/3} + (3, 3)_{-2/3} + (1, 1)_{-4} \] (8)

The states in the first two lines of Eq. (8) are the familiar ones of one SM family, without a right-handed neutrino, which is why \((10 + \bar{5})\) is used in an SU(5) GUT. The scalars in the SU(5) Yukawa couplings must be among

\[ 5 \times 5 = 10_a + 15_s \]
\[ 10 \times \bar{5} = 5 + 45 \]
\[ 10 \times 10 = \bar{5}_s + 45_a + 50_s \] (9)

and we note that the usual Higgs boson, which in this notation is the complex doublet \((1, 2)_{+1}\), appears uniquely in the \(5\) and \(45\) of SU(5), as can be seen from Eq. (8). Armed with these preliminaries about SU(5), it is rendered almost trivial to extend the analysis to SO(10), but the \((X)\) symmetry means we must tread carefully. We return to Eq. (7) and adopt a new notation in the SO(10) decompositions of \((SU(5))_X\). From [33], we are able to decompose the scalar SO(10) irreducible representations into their SU(5) components:

\[ 10 = 5_2 + 5_{-2} \]
\[ 120 = 5_2 + 5_{-2} + 10_{-6} + \bar{10}_6 + 45_2 + 45_{-2} \]
\[ 126 = 1_{-10} + 5_{-2} + 10_{-6} + \bar{15}_6 + 45_2 + 45_{-2} \]
\[ 45 = 24_0 + 10_4 + \bar{10}_{-4} + 1_0 \] (10)

All of 10, 120 and 126 necessarily contain a candidate for the SM complex Higgs doublet. From Eq. (8), we can, if needed, translate the SU(5) representations in Eq. (10) into SM representations. This provides all the group theory we shall need in the present article. In the following we shall focus on the breaking of \(U(1)_X\) which is intimately related to the mass of the right-handed neutrinos N in Eq. (8) and hence to the see-saw mechanism.
2.2 The two complex singlet scalars in the effective potential

If we introduce a scalar field \( \Phi \), singlet under SU(5) with lepton number \( L = +2 \), we can write the Majorana mass \( M \) of the right-handed neutrino \( N_R^i \) (\( i,j = 1,2,3 \)) of the three generations as

\[
\lambda_{ij} N_R^i N_R^j \Phi. \tag{11}
\]

The masses \( \lambda_{ij} \langle \Phi \rangle \) may be taken to be \( \sim 10^{10} \) GeV, far above the weak scale, whereupon we may integrate out the right-handed neutrino \( N \) to derive an effective field theory with interesting properties. In particular, the gauged \( U(1)_X \) of the SO(10) GUT has become anomalous, because in the \( (X)^3 \) triangle diagram \( N \) has been removed from the internal states.

We note that the 126 of scalars in Eq. (10) contains an SU(5) singlet, charged under \( (B-L) \), in addition to the SU(5) singlet in the 45 of Eq. 10 \( \Phi \). The presence of two such states in our model will be relevant in our subsequent analysis.

Let us step back to a purely bottom-up approach. Consider the original minimal standard model (MSM) with massless neutrinos. In perturbation theory, it conserves baryon number (B) and lepton number (L) so there is a global \( U(1)_{B-L} \) which, without a right-handed neutrino, is anomalous. Such a statement is obviously not connected to grand unification. Of course, this model is ruled out because neutrinos have non-zero masses so some modification is necessary to the MSM and there is a number of possibilities \[34]. The most popular is the addition of right-handed neutrinos which permit the see-saw mechanism for generating neutrino masses. This is achieved most naturally in SO(10) unification.

Now we carefully discuss a top-down analysis of SO(10) spontaneous symmetry breaking. At the GUT scale \( (10^{15-16} \text{ GeV}) \) the adjoint 45 is used to break the symmetry in a necessarily rank-preserving manner according to

\[
SO(10) \rightarrow SU(5) \times U(1)_X \rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)_X \tag{12}
\]

so that the \( U(1)_X \), with \( 3X = 12Y - 15(B-L) \), is still unbroken and its gauge boson is massless. At an intermediate scale \( M_I \sim 10^{10-11} \) GeV the complex 126 is used spontaneously to break \( U(1)_X \) and to give Majorana masses to the three right-handed neutrinos. This arises from a VEV of the \( SU(5) \)-singlet complex component in Eq. (33) which has the Mexican-hat type of potential required for the Higgs mechanism.

3 See-Saw Mechanism

In the MSM neutrinos are massless. The minimal standard model involves three chiral neutrino states, but it does not admit renormalizable interactions that can generate neutrino masses. Nevertheless, experimental evidence suggests that both solar and atmospheric neutrinos display flavor oscillations, and hence that neutrinos do have mass. Two very different neutrino squared-mass differences are required to fit the data:
\[ 6.9 \times 10^{-5} \text{eV}^2 \leq \Delta_s \leq 7.9 \times 10^{-5} \text{eV}^2 \quad \text{and} \quad \Delta_a \sim (2.4 - 2.7) \times 10^{-3} \text{eV}^2, \]

where the neutrino masses \( m_i \) are ordered such that:

\[ \Delta_s = |m_2^2 - m_1^2| \quad \text{and} \quad \Delta_a = |m_3^2 - m_2^2| \simeq |m_3^2 - m_1^2| \]

and the subscripts \( s \) and \( a \) pertain to solar (\( s \)) and atmospheric (\( a \)) oscillations respectively. The large uncertainty in \( \Delta_s \) reflects the several potential explanations of the observed solar neutrino flux: in terms of vacuum oscillations or large-angle or small-angle MSW solutions, but in every case the two independent squared-mass differences must be widely spaced with

\[ r = \Delta_s / \Delta_a \sim 3 \times 10^{-2}. \]

In a three-family scenario, four neutrino mixing parameters suffice to describe neutrino oscillations, akin to the four Kobayashi-Maskawa parameters in the quark sector. Solar neutrinos may exhibit an energy-independent time-averaged suppression due to \( \Delta_a \), as well as energy-dependent oscillations depending on \( \Delta_s / E \). Atmospheric neutrinos may exhibit oscillations due to \( \Delta_a \), but they are almost entirely unaffected by \( \Delta_s \). It is convenient to define neutrino mixing angles as follows:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
=
\begin{pmatrix}
c_{2c_3} & c_{2s_3} & s_{2e}^{i\delta} \\
+c_{1s_3} + s_{1s_2c_3}e^{i\delta} & -c_{1c_3} - s_{1s_2s_3}e^{i\delta} & -s_{1c_2} \\
+s_{1s_3} - c_{1s_2c_3}e^{i\delta} & -s_{1c_3} - c_{1s_2s_3}e^{i\delta} & +c_{1c_2}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

with \( s_i \) and \( c_i \) standing for sines and cosines of \( \theta_i \). For neutrino masses satisfying (13), the vacuum survival probability of solar neutrinos is:

\[ P(\nu_e \rightarrow \nu_e)|_s \simeq 1 - \frac{\sin^2 2\theta_2}{2} - \cos^4 \theta_2 \sin^2 2\theta_3 \sin^2 (\Delta_s R_s / 4E) \]

whereas the transition probabilities of atmospheric neutrinos are:

\[
\begin{align*}
P(\nu_\mu \rightarrow \nu_\tau)|_a & \simeq \sin^2 2\theta_1 \cos^4 \theta_2 \sin^2 (\Delta_a R_a / 4E) \\
P(\nu_e \rightarrow \nu_\mu)|_a & \simeq \sin^2 2\theta_2 \sin^2 \theta_1 \sin^2 (\Delta_a R_a / 4E) \\
P(\nu_e \rightarrow \nu_\tau)|_a & \simeq \sin^2 2\theta_2 \cos^2 \theta_1 \sin^2 (\Delta_a R_a / 4E)
\end{align*}
\]

None of these probabilities depend on \( \delta \), the measure of CP violation. Let us turn to the origin of neutrino masses. Among the many renormalizable and gauge-invariant extensions of the standard model that can do the trick are (i) The introduction of a complex triplet of mesons \((T^{++}, T^+, T^0)\) coupled bilinearly to pairs of lepton doublets. They must also couple bilinearly to the Higgs doublet(s) so as to avoid spontaneous \((X)\) violation and the appearance of a massless and experimentally excluded majoron. This mechanism can generate an arbitrary complex symmetric Majorana mass matrix for
neutrinos. (ii) The introduction of singlet counterparts to the neutrinos with very large Majorana masses. The interplay between these mass terms and those generated by the Higgs boson, the so-called see-saw mechanism, yields an arbitrary but naturally small Majorana neutrino mass matrix. (iii) The introduction of a charged singlet meson $f^+$ coupled antisymmetrically to pairs of lepton doublets, and a doubly-charged singlet meson $g^{++}$ coupled bilinearly both to pairs of lepton singlets and to pairs of $f$-mesons. An arbitrary Majorana neutrino mass matrix is generated in two loops. (iv) The introduction of a charged singlet meson $f^+$ coupled antisymmetrically to pairs of lepton doublets and (also antisymmetrically) to a pair of Higgs doublets. This simple mechanism was first proposed in [27] and results at one loop in a Majorana mass matrix in the flavor basis $(e, \mu, \tau)$ of a special form:

$$
\begin{pmatrix}
0 & m_{e\mu} & m_{e\tau} \\
m_{e\mu} & 0 & m_{\mu\tau} \\
m_{e\tau} & m_{\mu\tau} & 0
\end{pmatrix}
$$

This Zee model is attractive as an simple extension of the SM. It predicts maximal solar neutrino mixing, $\theta_{12} = \frac{\pi}{4}$, a value which was strongly disfavoured by SNO data [28, 35]. Of all the models preserving only the three chiral left-handed neutrinos of the SM - models (i), (iii) and (iv) above - model (iv) is surely the most appealing and it fails. Therefore one is led to additional neutrino states, typically two or more massive right-handed neutrinos which we denote $N_i$ ($i = 1, 2, \ldots, p$).

In the model we shall discuss $p$ is necessarily $p = 3$ because each of the three quark-lepton families is in a 16 of $SO(10)$ and each contains one $N$ state. There has been considerable interest in more minimal models with $p = 2$ as introduced in the so-called FGY model of [36]. This choice has the property of reducing the number of free parameters such that the CP-violating phase in $N_i$ mixing matrix is simply related to the CP-violating phase, $\delta$, in Eq. (16). This means that the measurement of $\delta$ in long-baseline neutrino oscillation experiment would shine light on the origin of matter-antimatter asymmetry arising from leptogenesis [37] where it arises from $N_i$ decay. In general, this connection does not exist so that an optimistic logic could argue that the FGY model, sometimes called the minimal see-saw, is possibly correct.

For the present case of $p = 3$ we introduce a mass basis

$$(\nu_e, \nu_\mu, \nu_\tau, N_1, N_2, N_3)$$

so that there is a $6 \times 6$ mass matrix in four $3 \times 3$ blocks with the top-left block vanishing and the bottom-right being the large Majorana masses for the $N_i$. The two off-diagonal blocks are Dirac masses coupling the $\nu_{iL}$ to the $N_{iR}$.

The effective mass matrix of the light Majorana neutrinos is given by

$$M = M_D (M_R)^{-1} M_D^T$$
where \( M_D \) and \( M_R \) are the 3 × 3 mass matrices for the Dirac and right-handed Majorana neutrinos, respectively. \( M_D^T \) designates the transpose.

The see-saw strategy is immediately evident from Eq. (21). Denoting the mean values of the 3 × 3 blocks by \( m \) and \( M \)

\[
\begin{pmatrix}
0 & m \\
\end{pmatrix}
\]

the eigenvalues for \( m \ll M \) are close to \( m^2/M \) and \( M \). This shows how large the \( N_i \) masses are expected to be. Taking the first family, with a typical quark mass 10 MeV and electron neutrino mass \( 10^{-5} \) eV, we find \( M \sim 10^{10} \) GeV. Coincidentally, and suggestively, such a mass fits well with the mass required for successful leptogenesis [37].

This discussion exhibits the great advantage of the see-saw mechanism compared to the alternative models discussed above: the smallness of the neutrino masses relative to those of the quarks and leptons occurs naturally. That being said, the other side of the coin is that experimental observation of the very massive \( N_i \) is challenging.

The crucial observation for our present purposes is to consider the \( U(1)_X \) triangle anomalies. If we keep all the states in Eq. (23) for one family

\[
16 \equiv (u^r, u^d, u^b, d^r, d^d, d^b; u^r, u^d, u^b, d^r, d^d, d^b; \nu_e, e^-, N, e^+)_{L},
\]

then we can examine this question.

The pure gauge anomaly \( U(1)_X^3 \) has cancelling contributions from the states in Eq. (23) as follows

\[
6 \left( \frac{1}{27} \right) + 6 \left( -\frac{1}{27} \right) + 2(+1) + 2(-1) = 0
\]

For the gravitational triangle anomaly which has only one \( U(1)_X \) vertex the respective cancelling contributions are

\[
6 \left( \frac{1}{3} \right) + 6 \left( -\frac{1}{3} \right) + 2(+1) + 2(-1) = 0.
\]

When we decouple the \( N \) state in Eq. (23) by taking it to very high mass, the right hand sides of Eq. (24) and Eq. (25) both change from zero to \( -1 \), the anomalies do not cancel, and therefore there exists in the effective theory an anomalous \( U(1) \) symmetry of the sort considered in different contexts in e.g. [38–41].

4 Anomalous \( U(1)_X \)

Let us introduce the matter fields in our model. The fermions are in three \( 16 \)’s, \( \Psi_i \) \( (i = 1, 2, 3) \). Each \( 16 \) contains a right-handed neutrino \( N^i_R \) with \( (X) = +1 \).

\( SO(10) \) contains the usual \( SU(5) \) subgroup [31] which plays a rôle in containing the minimal standard
model (MSM) as if without neutrino mass. To provide mass to $N_R$ without breaking $SU(5)$ we introduce a complex scalar $\Phi$ in the $126$ of $SO(10)$ which under $SU(5)$ contains

$$126 \subset 1 + 5 + 10 + 15 + 45 + 50$$

and the $N_R^i$ acquire mass as in Eq. [11] when the $SU(5)$-singlet component of $\Phi$ in Eq.[34] gains an intermediate mass scale VEV

$$< \Phi > = M_I$$

where for the see-saw mechanism the intermediate mass scale $M_I$ is typically $\sim 10^{10}$ GeV.

To break the symmetry $SU(5)$ to that of the standard model we introduce more scalars in the representations of $SO(10)$ which are the adjoint $A$ in a $45$, the vector $V$ in a $10$ and finally a spinor $B(16)$. The adjoint $45$ decomposes under $SU(5)$ as

$$45 \supset 1 + 10 + 1\overline{0} + 24$$

so that the $24$ can provide the rank-preserving $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. We recall that in $SO(10)$, $45$ decomposes as in Eq. [10] within which the $24$ can provide the rank-preserving $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$ symmetry breaking. The fermion masses arise from the Yukawa couplings

$$L_{Yukawa} = \Psi (Y_V V + Y_\Phi \Phi) \Psi$$

which may be understood to contain the coupling of Eq.[11].

We adopt the convention that Latin indices $a, b, c, \ldots$ run from 1 to 10 and Greek indices $\alpha, \beta, \gamma, \ldots$ run from 1 to 16. The vector field $V$ is $V_a$ and the adjoint $A$ is $A_{ab} = -A_{ba}$ so that all the $V$ and $A$ couplings up to quartic in the Higgs potential can be written, bearing in mind that

$$10 \times 10 \supset 1 + 45 + 50$$

$$10 \times 45 \supset 10 + 120 + 320$$

$$45 \times 45 \supset 1 + 45 + 54 + 210 + 770 + 945.$$  

in the form

$$V(V, A) = V_a V_a + (V_a V_a)^2 + A_{ab} A_{ab} + (A_{ab} A_{ab})^2 + (V_a V_a) (A_{bc} A_{bc}) + \ldots$$

among other terms.

To deal with the $126$ it is essential to introduce the $\Gamma$ matrices

$$\Gamma^a_{\alpha\beta}$$

which are ten $16 \times 16$ matrices which roughly generalise the four $4 \times 4$ Dirac matrices $\gamma^\mu$ pertinent to $O(4)$, and likewise satisfy a Clifford algebra. The $\Phi$ field of the $126$ is a symmetric scalar field satisfying the trace condition

$$\Gamma^a_{ij} \Phi_{ji} = Tr(\Gamma^a \Phi) = 0$$
Now, in addition to Eq. (30), we shall need
\begin{align*}
126 \times 10 & \supset 210 + 1050, \\
126 \times 45 & \supset 120 + 126 + 1728 + 3696, \\
126 \times 126 & \supset 54_{S} + 945_{A} + 1050_{S} + 2772_{S} + 4125_{S} + 6930_{A}.
\end{align*}
(34)

to write the Higgs potential terms involving $\Phi$ such as
\[
V(\Phi) = \Phi_{ij}\Phi_{ij} + (\Phi_{ij}\Phi_{ij})^2 + \Phi_{ij}\Phi_{jk}\Phi_{kl}\Phi_{li} \\
+ \Gamma^a_{ij}\Phi_{jk}\Phi_{kl}\Phi_{ln}\Phi_{mn}\Gamma^a_{ni} + \ldots
\]
(35)
among other terms including mixed $\Phi - A$ terms possible under $SO(10)$ symmetry, as can be seen from Eqs. (30) and (34). We take note of the cubic scalar coupling $16.16.126$ which may be written
\[
B_\alpha B_\beta B^*_{\alpha\beta}
\]
(36)
and which we shall use in the next section.

5 Stueckelberg Axion

In order to illustrate how the mixing of the CP-odd phases takes place in the breaking of $SO(10) \rightarrow SU(5) \times U(1)$ we consider specific terms in the potential, describing the conditions which need to be satisfied in order to generate a periodic potential function of a single gauge invariant field. The latter takes the role of a physical axion and will be denoted by $\chi$.

The periodic potential is generated at the scale at which $SU(5) \times U(1)$ is broken necessarily at $> 10^{15}$ GeV to avoid too-fast proton decay. At this GUT scale, instanton effects are present. In order to understand why this happens, we consider an $SO(10)$ invariant term in the original theory such as
\[
16 \times 16 \times 126
\]
(37)
which is built out of the spinorial (16) of $SO(10)$ and the complex conjugate of the 126. The $SO(10)$ singlet is obtained from
\[
16 \times 16 = 10_s + 120_a + 126_s
\]
(38)
by combining the $126_s$ taken from the symmetric part of the product $(16 \times 16)_s = 126_s + 10_s$ with the $126$. We can specialize (17) by indicating the $X$ content of the decomposition using
\[
16 = 1_{-5} + 5_{+3} + 10_{-1}
\]
(39)
from which gives for their antisymmetric product
\[
120_a = (16 \times 16)_a \\
= 5_{-2} + 10_{-6} + (5 + 45)_{+2} + 10_{+6} + \overline{10}_{-2}
\]
(40)
while the symmetric component can be specialized in the form

\[(16 \times 16)_s = 126_s + 10_s\]

\[= (1_{-10} + 5_{-2} + 10_{-6} + \overline{15}_{+6} + 45_{+2} + \overline{50}_{-2}) + (5_{+2} + 5_{-2}) \quad (41)\]

where the two contributions in brackets refer respectively to the $126_s$ and to the $10_s$ of $SO(10)$.

A periodic potential can be extracted from the decomposition above starting from the $126_s \times \overline{126}$, $SO(10)$ singlet, by combining the $1_{-10}$ in Eq. (11) with the $1_{+10}$ in the $\overline{126}$, the latter obtained by conjugation of (34) - with the inclusion of its complete $SU(5) \times U(1)_X$ content -

\[\overline{126} = 1_{+10} + 5_{+2} + \overline{10}_{+6} + 15_{-6} + \overline{35}_{-2} + 50_{+2}. \quad (42)\]

A term in this form in the potential allows to induce a mixing of the CP-odd phases of the two $SU(5)$ singlet representations in such a way that one linear combination of these will correspond to a physical axion while the second one will be part of the Nambu-Goldstone mode generated by the breaking of $U(1)_X$.

We will be denoting with $\sigma$ and $\phi$ the two fields corresponding to the $1_{-10}$ and $1_{10}$ respectively, denoting their vevs with $v_\sigma$ and $v_\phi$ respectively. We will assume that $v_\phi$ will be large in such a way to provide a mass term for the right-handed neutrino, as specified in (11) using the Majorana operator $N_R N_R \phi$.

In order to characterise the structure of the Stueckelberg Lagrangian at classical level we focus our attention on the extra (periodic) potential related to $\sigma$ and $\phi$

\[V_p = \lambda M_G^2 \sigma \phi + \text{h.c.} \quad (43)\]

Since there must be an $SU(5)$ singlet it is important to realise that the other parts of Eq.(42) do not contribute. The coupling $\lambda$ is instanton generated at the scale $M_{GUT}$, a fact which provides a drastic suppression in $V_p$. We parameterize both fields around their vevs as

\[
\sigma = \frac{v_\sigma + \sigma_1 + i \sigma_2}{\sqrt{2}} \\
= \frac{v_\sigma + \rho_\sigma e^{iF_\sigma(x)/(g_B v_\sigma)}}{\sqrt{2}} \\
\phi = \frac{v_\phi + \rho_\phi e^{ib(x)/v_\phi}}{\sqrt{2}}
\]

and $v_\phi$ is at the GUT scale $M_{GUT} \sim 10^{15}$ GeV. The parameterization of $V_p$ in a broken phase is made possible by the remaining - non periodic - general scalar potential which will assume a typical mexican-hat shape as for an ordinary $U(1)$ symmetry. Both $\sigma$ and $\phi$ are charged under $U(1)_X$ and therefore their vevs break the gauged $(X)$ which as we have discussed survives as an anomalous $U(1)$
in the effective theory at low energies. We denote with \( g_B \) the gauge coupling of the \( U(1)_X \) gauge boson \((B_\mu)\), while \( \pm q_B \) will denote the corresponding \( X \) charges of the scalars. Their normalization, equal to \( \pm 10 \) in the normalization of \[33\], is indeed arbitrary. The role of the Stueckelberg field is taken by \( b(x) \) in the polar parameterization of \( \phi \), which is normalized to 1 in mass dimension, while \( F_\sigma \) is massless.

The two covariant derivatives of the scalars take the form

\[
D_\mu \sigma = (\partial_\mu + iq_B g_B B_\mu) \sigma \\
D_\mu \phi = (\partial_\mu + iq_B g_B B_\mu) \phi
\]  

with the typical Stueckelberg kinetic term generated from the decoupling of the radial fluctuations of the \( \phi \) field

\[
|D_\mu \phi|^2 = \frac{1}{2} \partial_\mu \rho_\phi \partial^\mu \rho_\phi + \frac{1}{2} (\partial_\mu b - MB_\mu)^2
\]  

with \( M = q_B g_B v_\phi \sim M_I \) takes the role of the Stueckelberg scale. In general it is natural to assume that both \( v_\phi \) and \( v_\sigma \) are of the same order, and the mass of \( B_\mu \), the \( X \) gauge boson, will be given as a mean of both vevs

\[
M_B = \sqrt{(q_B g_B v_\sigma)^2 + M^2}
\]  

The quadratic action, neglecting the contribution of the radial excitations of \( \sigma \) and \( \phi \), can be easily written down for such \( \sigma - \phi \) combination

\[
\mathcal{L}_q = \frac{1}{2} (\partial_\mu \sigma_2)^2 + \frac{1}{2} (\partial_\mu b)^2 + \frac{1}{2} M_B^2 B_\mu B^\mu + B_\mu \partial^\mu (M_1 b + v_\sigma g_B q_B \sigma_2),
\]  

from which, after diagonalization of the mass terms we obtain

\[
\mathcal{L}_q = \frac{1}{2} (\partial_\mu \chi_B)^2 + \frac{1}{2} (\partial_\mu G_B)^2 + \frac{1}{2} (\partial_\mu h_1)^2 + \frac{1}{2} M_B^2 B_\mu B^\mu - \frac{1}{2} m_1^2 h_1^2 + M_B B^\mu \partial_\mu G_B.
\]  

where we are neglecting all the other terms generated from the decomposition which will not contribute to the breaking. We can identify the linear combinations

\[
\chi_B = \frac{1}{M_B} (-M \sigma_2 + q_B g_B v_\sigma b), \\
G_B = \frac{1}{M_B} (q_B g_B v_\sigma \sigma_2 + M b),
\]  

(50)

corresponding to the physical axion \( \chi_B \), and to a massless Nambu-Goldstone mode \( G_B \). The rotation matrix that allows the change of variables \((\sigma_2, b) \rightarrow (\chi, G_B)\) is given by
\[
U = \begin{pmatrix}
-\cos \theta_B & \sin \theta_B \\
\sin \theta_B & \cos \theta_B
\end{pmatrix}
\]  
(51)

with

\[
\theta_B = \arcsin(q_B g_B v_\sigma / M_B).
\]  
(52)

The potential, as shown in similar analysis \[5\], is periodic in \(\chi/f\chi\) where \(f_\chi \sim M_I\) takes the role of the axion decay constant. As already stressed before, the origin of this potential is nonperturbative and linked to the presence of instantons at the SO(10) GUT phase transition. For such reason, the size of the constants \(\lambda\) in such potential are exponentially suppressed with \(\lambda_i \sim e^{-2\pi/\alpha_{GUT}}\), with the value of the coupling \(\alpha_{GUT}\) fixed at the scale \(M_{GUT}\) when the SO(10) instantons are exact. The value of \(\alpha_{GUT}\) here is in the range \(1/33 \leq \alpha_I \leq 1/32\), giving \(10^{-91} \leq \lambda_{ij} \leq 10^{-88}\), determining an axion mass given by \(m_\chi^2 \sim \lambda M_I^2\) in the range

\[10^{-22} \text{eV} < m_\chi < 10^{-20} \text{eV}\]  
(53)

corresponding to an ultralight axion, which has been invoked for the resolution of several astrophysical constraints \[42\].

## 6 Conclusions

We have investigated the possibility that the decoupling of a right-handed neutrino in the context of an SO(10) GUT can be accompanied by an axion-like particle. Such a particle shares many of the properties already considered for a similar model discussed by two of us in the context of an \(E_6 \times U(1)_X\) unification, interpreted as low-energy GUT theory derived from string theory \[4\].

While, in the previous construction, the Stueckelberg Lagrangian was generated by the dualisation of a 3-form and required an anomalous \(U(1)\) gauge symmetry, in this construction we have simply considered the possibility that the \(U(1)_X\) symmetry of the Standard Model has an interesting implication.

Starting from an SO(10) symmetry, broken to an \(SU(5) \times U(1)_X\) GUT symmetry, the decoupling of a right-handed neutrino leaves at low energy an action which is Stueckelberg like, with a global anomaly which couples to a CP-odd phase, \(\chi(x)\). We have invoked the generation of a periodic potential in the \(SU(5) \times U(1)_X\) effective theory in order to extract such gauge invariant degree of freedom in the pseudoscalar sector which couples to a global anomaly. Such Stueckelberg-like pseudoscalars are expected to be ultralight, around \(10^{-20}\) eV and to decouple at the scale corresponding to the mass of the right-handed neutrino. An earlier paper which relates the lightness of the axion to neutrino mass is \[43\].

We have illustrated, by analysing the representation content of the scalar sector of the SO(10) and \(SU(5) \times U(1)_X\) theories how this could be achieved.
We believe that we have merely identified the general tracts of this mechanism to which we hope to return in the near future in a more extensive analysis.

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