Fuzzy Logic Models for Detection of Critical Processes in Manufacturing

Pavel Moor\textsuperscript{1,a}, Igor Gluhih\textsuperscript{1,b}, Anton Moor\textsuperscript{1,c}, Svetlana Moor\textsuperscript{2,d}

\textsuperscript{1}Tyumen State University, Tyumen, Russia, 625000
\textsuperscript{2}Industrial University of Tyumen, Tyumen, Russia, 625000

E-mail: \textsuperscript{a}moorpk@gmail.com, \textsuperscript{b}igluhih@utmn.ru, \textsuperscript{c}anton.moor@gmail.com, \textsuperscript{d}moorsm@mail.ru

Abstract. This article describes how methods of fuzzy logic apply to detection of critical production processes for the purposes of optimization and control of production and automated processes. We propose a mathematical model based on fuzzy logic methods to identify critical processes. The system of fuzzy logic inference is formulated. The existence of task solution is proven. Criteria for task solving efficiency are formulated and described.

Introduction

In the last decades the development of the systems for monitoring and detection of the processes that require reengineering became one of the important areas of production activities automation [1, 6, 7, 8]. Reengineering is a very costly complex process because it involves the process itself being changed as well as its structure and content modified. Therefore, the decision making about the necessity of reengineering of one process or several processes is one of the serious issues of production process management.

In order to deal with this issue, it is necessary to identify the processes that have critical parameters. Due to a lack of regulations and norms, the data being ambiguous or uncertain the links between the borderline parameters of that critical area are conditional and fuzzy. In this work we call the processes that need significant changes to be introduced into as critical processes. We suggest identifying these processes by using fuzzy logic methods. Numerous researches, studies and practical implementations have demonstrated that usage of fuzzy logic methods in planning and production management has its benefits [2,3,4,5]. The reason for it is that with the help of Fuzzy Logic System [12] it becomes possible to develop managing effects in the conditions of uncertainty, inaccuracy, incompleteness of information by using qualitative, not formalized effectiveness criteria.

This allows to imitate a way of expert thinking during decision making in management systems without introducing strict formalized mathematical links between processes parameters. At the same time, such automated systems can process and match a number of parameters that is much greater than a person can do as well as it can find hidden regularities and tendencies that are not obvious from the first glance.

In this article we propose to use fuzzy logic methods to identify critical manufacturing processes. The general idea is to develop a set of rules using terminology of fuzzy logic that will allow to make conclusion about a process that is studied - whether it is critical or not. The values of the parameters that are received during the monitoring are formulated with the use of linguistic variables and can be used being incomplete and (or) not exact. Using the processing of uncertainty and combination of numerous rules, the conclusion about the level of criticality of studied processes is made with a high level of accuracy and validity. Further in the article we offer a mathematical model for describing processes in
the terms of fuzzy logic as well as we propose a general algorithm for finding a task solution in detecting critical processes that need reengineering.

Method and models

The basis of a processes space is a set of the manufacturing processes – $T$.

Each process $t \in T$ is characterized by a set of its attributes $H = \{h_1, h_2, \ldots, h_n\}$. (1)

The following attributes can be evaluated:

1. quality – a ratio of defect-free products against total number of goods produced in a manufacturing process;
2. performance – an amount of production manufactured in a certain period of time;
3. duration – a length of the process’ manufacturing cycle;
4. cost – the total cost of the manufacturing process.

Each $h_i$ attribute gets a single-valued correspondence with a set of its possible values – $S_i$, belonging to the collection of sets $S = \{S_1, S_2, \ldots, S_n\}$. (2)

In general case $S_i \subset \mathbb{R}$, $S_i \neq \mathbb{R}$.

Representation $\varphi: T \rightarrow S$ puts the attributes’ values $\{s_1, s_2, \ldots, s_n\}$ to a correspondence with each process $t$.

Out of the range of the process’ attributes the following sets can be selected

$K \subseteq H$ – a set of the base attributes and

$J \subseteq H$ – a set of the derived attributes.

It is obvious that

$K \cup J = H, \ K \cap J = \emptyset$. (3)

We will consider that each $h_i \in J$ has a representation

$\psi_i: (S_i - S_j) \rightarrow S_i$, (4)

which defines the rules of the attribute calculation, and there is at least one sequence

$\psi_{i_1}, \psi_{i_2}, \ldots, \psi_{i_m}$, (5)

where $m = |J|$ - is the cardinal number $J$, which makes it possible to sequentially calculate the values of each of the derived attributes.

In practice, often

$\psi_i: S_j \rightarrow S_i, \ h_j \in K$, (6)

i.e. there is a direct dependency of derived attribute from the base attribute.

So, the processes’ space is a system

$T_S = \langle T, H, S, \varphi, \psi \rangle$. (7)

Fuzzy inference system

Definition 1. Fuzzy set of the critical processes will be the further definition of a fuzzy subset $A \subseteq T$, which is characterized by $\mu_A$ membership function of a set $A$, $\mu_A(t) \in [0, 1]$.

Fuzzy set $B$ of non-critical processes with $\mu_B$ membership function will be defined in a similar way.

Definition 2. A set of definitely critical processes will be the name of a defined subset $A_p \subseteq T$: it will be possible to define each $t \in A_p$ within the model as critical.
Similarly, fuzzy set \( B_p \) of non-critical processes is identified. Sets \( A_p \) and \( B_p \) do not intersect, \( A_p \cap B_p = T \).

We have proved the following theorem.

**Theorem 1. Regarding the existence of the task solution.**

\[ \forall \varepsilon \in (0; 1) \exists \mu A: \forall t_1 \in A_p, t_2 \in B_p \mu A(t_1) > \varepsilon, \mu A(t_2) < \varepsilon. \]

**Definition 3.** Fuzzy attribute value of the process will be the further definition of a fuzzy variable \( D \), which is a set \( D = \{\alpha, S_i, A_{S_i}\} \),

where:
- \( \alpha \) - a value name; \( S_i \) - a (clear) set of the attribute’s values, where the fuzzy value \( D \) is defined; \( A_{S_i} \) - a fuzzy set, determined on \( S_i \) with corresponding membership function \( \mu_{S_i} \).

**Definition 4.** By a linguistic attribute we will understand a linguistic variable

\[ \gamma = \{\beta, L, S_i, G, M\}, \]

where:
- \( \beta \) - a linguistic attribute name; \( L \) - a set of linguistic attribute’s terms; \( S_i \) - a definition scope of the linguistic attribute, a set of (exact) possible values of attribute; \( G \) - a syntax procedure, which allows us to operate with values from term-set \( L \); \( M \) - a semantic procedure, which allows us to match each element of an extended term-set \( L \cup G(L) \) with an attribute’s fuzzy value.

**Definition 5.** Statement \( \tau = \{\gamma \mid D\} \) will be called a local criterion of process’ criticality.

Parts of the local criterion - linguistic attribute \( \gamma \) and fuzzy value of attribute \( D \) are related by a formula:

\[ D \in M(L \cup G(L)), \gamma = \{\beta, L, S_i, G, M\}, \]

meaning that fuzzy value \( D \) is a result of applying the semantic procedure \( M \) of a linguistic attribute \( \gamma \) to its extended term set.

**Definition 6.** Antecedent (premise) of a critical process’s rule detection is a set of a local criteria of process’ criticalities \( \{\tau_1, \tau_2, \ldots, \tau_n\} \).

Consequent (consequence) of a critical process rule detection is a statement \( \{\mu_A \mid D_\rho\} \), where \( D_\rho \) is a special linguistic variable, determined on \( \mu_A \).

Antecedent together with consequent comprise a rule of critical process detection \( \omega \).

We can apply methods of fuzzy logic inference to a system of rules \( \Omega = \{\omega_1, \omega_2, \ldots, \omega_k\} \). As an input value for the algorithms of fuzzy logic inference we will use manufacturing process \( t \) together with its attributes values.

Algorithm’s appliance results in a computed value

\[ \mu_A^* = F_x(t), \]

where \( F_x \) is a procedure of fuzzy logic inference.

Resulting system of fuzzy logic inference is determined by three components:

\[ F_S = \{A, B, \Omega\}. \]

**Base model**

of critical processes detection \( \{T_S, F_S\} \) is comprised of processes set and system of fuzzy logic inference.

**Effectiveness criteria**

Having set a certain \( \varepsilon \), we will consider exact set

\[ A^* = \{t: \mu_A^* \geq \varepsilon\} \]

as a set of critical processes, and similarly,
B*=\{t: \mu_A^*<\varepsilon\} \text{ as a set of non-critical processes.} \quad (14)

We will use probability of type I errors and type II errors as effectiveness criteria of the problem solution.

Type I error (false positive) \( E_{R1} \) – is a probability of a critical process’ acceptance as a non-critical (erroneous rejection of a correct zero hypothesis):

\[
E_{R1} = \frac{|B^* \cap A_p|}{|A_p|},
\]

where \(|X|\) is a power of corresponding set.

Type II error is a probability of acceptance of a non-critical process as a critical (erroneous acceptance of a false zero hypothesis):

\[
E_{R2} = \frac{|A^* \cap B_p|}{|B_p|}.
\]

The following statement can be easily proved:

Theorem 2. If \( E_{R1}=0 \) and \( E_{R2}=0 \), then \( A^*=A_p \) and \( B^*=B_p \).

Sequence 1. For \( E_{R1}=0, E_{R2}=0 \) it is necessary to have \(|A^*|=|A_p|\).

Thus, we can use condition \(|A^*|=|A_p|\) as a criterion of fuzzy system’s effectiveness (as a necessary condition).

Practically it is more convenient to check the proximity of \( \frac{|A^*|}{|T|} \) and \( \frac{|A_p|}{|T|} \).

If

\[
\frac{|A^*|}{|T|} \ll \frac{|A_p|}{|T|},
\]

then we can speak with a fair degree of certainty about the high probability of a system’s type I error.

If

\[
\frac{|A^*|}{|T|} \gg \frac{|A_p|}{|T|},
\]

then we can consider high probability of a type II error.

If value

\[
\frac{|A_p|}{|T|}
\]

can be known based on statistical and historical data, then value

\[
\frac{|A^*|}{|T|},
\]

and particularly, the value \(|A^*|\) should be calculated individually for a given fuzzy rule set and chosen \( \varepsilon \).
Based on a nature of the task, attributes \( \{ S_1, S_2, ..., S_n \} \) are usually defined on discrete sets. Having enough historical data on the processes we can compose spectral probabilities distribution function \( f_i(s_j) \).

From function \( f_i(s_j) \) we can get distribution function of a local criterion of a critical process:

\[
f_z(\mu_D) = \sum_{\mu_D(s_j)=\mu_D} f_i(s_j),
\]

(21)

where \( \mu_D(s_j) \) – is a membership function of a corresponding fuzzy attribute.

After that we can consequently get distribution functions of rule \( \omega \) antecedents and distribution of a membership function \( \mu_\omega \).

The simplest way to get a distribution of an antecedent of a rule \( \omega \) on a basis of local criteria is applying the fuzzy logic inference algorithm to all elements of distribution definition set of every criterion \( \tau \) with cumulative sum of production of their distributions.

Using the same method, we can get a distribution of a membership \( \mu_\omega \) from distribution of antecedents of rules \( \omega \).

Set power correlation we intend to get can be calculated using a formula:

\[
\frac{|\Lambda^*|}{|\Lambda|} = 1 - F(\mu_\Lambda) = \sum_{\mu_\omega \in \omega} f(\mu_\Lambda).
\]

(22)

These rules and criteria can be used to build a system of critical processes detection. Further on we will analyze the algorithmic implementation of a base mathematic model. It uses Mamdani’s algorithm of fuzzy inference [13]. The scheme of the algorithm’s work is shown at the Fig. 1.

**Conclusion**

Thus, in fuzzy logic terms we have formulated and built the mathematic model of critical processes in manufacturing detection, as well as we have proved the theorem of existence of the task solution, have identified the process of fuzzy inference. Based on this approach, we have built the base algorithm of critical processes detection. An important feature of the proposed model is its ability to manage processes when the information received is uncertain, and when there are no strictly formalized criteria of efficiency.

This model has universal properties and can be utilized to detect critical processes in various enterprises. Apart from this, the model helps to develop an automated system which is focused not only on managing separate production parameters, but also on managing processes. This includes editing, replacement or full reengineering of these processes.
1. **The beginning of the algorithm**
   Input to the algorithm: information about a manufacturing process \(t\) with values of its attributes \(\{s_1, s_2, \ldots, s_n\}\).

2. **Calculation of the derived attributes**
   For all \(i=1\ldots n\) perform: calculate \(s_i \leftarrow \psi(s_1, s_2, \ldots, s_n)\).

3. **Calculation of local criteria grade of membership**
   For all \(i=1\ldots m, j=1\ldots n\) (\(m\) – count of rules, \(n\) – count of criteria in the rule)
   perform \(\mu_{ij} \leftarrow \mu_D(s_k)\), where \(s_k\) is a process’ attribute at which a fuzzy attribute \(D\), belonging to the criterion \(\tau_{ij}\), is defined.

4. **Calculation of antecedents’ grade of membership**
   For all \(i=1\ldots m\) calculate \(\alpha_i \leftarrow \min(\mu_{i1}, \mu_{i2}, \ldots, \mu_{in})\).

5. **Fuzzy implication**
   For all \(i=1\ldots m\) find functions \(\chi_i(\mu_A) \leftarrow \min(\alpha_i, \mu_D(\mu_A))\).

6. **Composition**
   Define function \(\chi(\mu_A) \leftarrow \max(\chi_1(\mu_A), \chi_2(\mu_A), \ldots, \chi_m(\mu_A))\).

7. **Defuzzyfication.**
   Calculate \(\mu^*_A \leftarrow \frac{\int_0^1 \mu_A \chi(\mu_A) d\mu_A}{\int_0^1 \chi(\mu_A) d\mu_A}\).

**Figure 1.** The scheme of the base model algorithm
References

[1] Francois Yves Simon, Continuous Process Improvement (CPI) Algorithm, https://www.automation.com/library/articles-white-papers/process-control-process-monitoring/continuous-process-improvement-cpi-algorithm

[2] Miss. Ashwini. A. Mate, Dr. I. P. Keswani, Scheduling By Using Fuzzy Logic in Manufacturing, Journal of Engineering Research and Applications, Vol. 4, Issue 7(Version 6), July 2014, pp.104-111

[3] Jiping Niu, John Dartell, Application Of Fuzzy Mrp-ii In Fast Moving Consumer Goods Manufacturing Industry, Simulation Conference, 2008. WSC 2008. Winter, Miami, FL., USA, USA

[4] E. De Gentili, A. De Cicco, J.-F. Santucci, Devs and Fuzzy logic to model and simulate a manufacturing process, International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'06), 2005, Vienna, Austria

[5] M. A. S. Monfared, J. B. Yang, Design of an intelligent manufacturing scheduling and control system using fuzzy logic: Sensitivity analysis and parameter optimization, Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology, Volume 15 Issue 2, April 2004, Pages 89 – 104

[6] Panagis Foteinopoulos, Alexios Papacharalampopoulos, Panagiotis Stavropoulos, On thermal modeling of Additive Manufacturing processes, CIRP Journal of Manufacturing Science and Technology, Volume 20, January 2018, Pages 66-83

[7] M. Abbas, H. El Maraghy, Synthesis and optimization of manufacturing systems configuration using co-platforming, CIRP Journal of Manufacturing Science and Technology, Volume 20, January 2018, Pages 51-65

[8] Lihui Wang, Machine availability monitoring and machining process planning towards Cloud manufacturing, CIRP Journal of Manufacturing Science and Technology, Volume 6, Issue 4, 2013, Pages 263-273

[9] P. Stavropoulosa, D. Chantzisa, C. Doukasa, A. Papacharalampopoulosa, G.Chryssoulouris, Monitoring and control of manufacturing processes: A review, 14th CIRP Conference on Modeling of Machining Operations (CIRP CMMO)

[10] Jonathan Downey, Denis O’Sullivan, Miroslaw Nejmen, Sebastian Bombinski, Paul O’Leary, Ramesh Raghavendra, Krzysztof Jemieliaki, Real Time Monitoring of the CNC Process in a Production Environment- the Data Collection & Analysis Phase, Procedia CIRP Volume 41, 2016, Pages 920-926

[11] Siler W., Buckley J. J, Fuzzy expert systems and fuzzy reasoning, John Wiley & Sons, Inc, 2005. 422 P.

[12] Zadeh L. A., Fuzzy Sets //Information and Control. 1965. Vol. 8. №3. P. 338 - 353.

[13] Leonenkov A.V., Fuzzy modeling in MATLAB and fuzzyTECH. Spb.: BHV-Peterburg, 2003. 736 P.