PUTTING THEORY TO WORK: 
FROM LEARNING BOUNDS TO META-LEARNING ALGORITHMS

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ABSTRACT

Most of existing deep learning models rely on excessive amounts of labeled training data in order to achieve state-of-the-art results, even though these data can be hard or costly to get in practice. One attractive alternative is to learn with little supervision, commonly referred to as few-shot learning (FSL), and, in particular, meta-learning that learns to learn with few data from related tasks. Despite the practical success of meta-learning, many of its algorithmic solutions proposed in the literature are based on sound intuitions, but lack a solid theoretical analysis of the expected performance on the test task. In this paper, we review the recent advances in meta-learning theory and show how they can be used in practice both to better understand the behavior of popular meta-learning algorithms and to improve their generalization capacity. This latter is achieved by integrating the theoretical assumptions ensuring efficient meta-learning in the form of regularization terms into several popular meta-learning algorithms for which we provide a large study of their behavior on classic few-shot classification benchmarks. To the best of our knowledge, this is the first contribution that puts the most recent learning bounds of meta-learning theory into practice for the popular task of few-shot classification.

1 INTRODUCTION

Since the very seeding of the machine learning field, its algorithmic advances were inevitably followed or preceded by the accompanying theoretical analyses establishing the conditions required for the corresponding algorithms to learn well. Such a synergy between theory and practice is reflected in numerous concepts and learning strategies that took their origins in the statistical learning theory: for instance, the famous regularized risk minimization approach is directly related to the minimization of the complexity of the hypothesis space, as suggested by the generalization bounds established for supervised learning (Vapnik 1992), while most of the adversarial algorithms in transfer learning (e.g., DANN from [Ganin & Lempitsky 2015]) follow the theoretical insights provided by the seminal theory of its domain ([Ben-David et al. 2010]).

Even though many machine learning methods now enjoy a solid theoretical justification, some more recent advances in the field are still in their preliminary state which requires the hypotheses put forward by the theoretical studies to be implemented and verified in practice. One such notable example is the emerging field of meta-learning, also called learning to learn (LTL), where the goal is to produce a model on data coming from a set of (meta-train) source tasks to use it as a starting point for learning successfully a new previously unseen (meta-test) target task with little supervision. This kind of approach comes in particularly handy when training deep learning models as their performance crucially depends on the amount of training data that can be difficult and/or expensive to get in some applications. Several theoretical studies (Baxter 2000, Pentina & Lampert 2014)
Maurer et al. [2016], Amit & Meir [2018], Yin et al. [2020]1 provided probabilistic meta-learning bounds that require the amount of data in the meta-train source task and the number of meta-train tasks to tend to infinity for efficient meta-learning. While capturing the underlying general intuition, these bounds do not suggest that all the source data is useful in such learning setup due to the additive relationship between the two terms mentioned above. To tackle this drawback, two very recent studies (Du et al. [2020], Tripuraneni et al. [2020]) aimed at finding deterministic assumptions that lead to faster learning rates allowing meta-learning algorithms to benefit from all the source data. Contrary to probabilistic bounds that have been used to derive novel learning strategies for meta-learning algorithms (Amit & Meir [2018], Yin et al. [2020]), there was no attempt to verify the validity of the assumptions leading to the fastest known learning rates in practice or to enforce them through an appropriate optimization procedure.

In this paper, we bridge the meta-learning theory with practice by harvesting the theoretical results from [Tripuraneni et al. 2020] and [Du et al. 2020], and by showing how they can be implemented algorithmically and integrated, when needed, to popular existing meta-learning algorithms used for few-shot classification (FSC). This latter task consists in classifying new data having seen only few training examples, and represents one of the most prominent examples where meta-learning has shown to be highly efficient. More precisely, our contributions are three-fold:

1. We identify two common assumptions from the theoretical works on meta-learning and show how they can be verified and forced via a novel regularization scheme.
2. We investigate whether these assumptions are satisfied for popular meta-learning algorithms and observe that some of them naturally satisfy them, while others do not.
3. With the proposed regularization strategy, we show that enforcing the assumptions to be valid in practice leads to better generalization of the considered algorithms.

The rest of the paper is organized as follows. After presenting preliminary knowledge on the meta-learning problem in Section 2, we detail the existing meta-learning theoretical results with their corresponding assumptions and show how they can be enforced via a novel regularization technique in Section 3. Then, we provide an experimental evaluation of several state-of-the-art FSL methods in Section 4 and highlight the different advantages brought by the proposed regularization technique in practice. Finally, we conclude and outline the future research perspectives in Section 5.
μ_{T+1}. More precisely, we first use \( \hat{\phi} \) to solve the following problem:

\[
\hat{w}_{T+1} = \arg\min_{w \in \mathbb{R}^k} \frac{1}{n_2} \sum_{i=1}^{n_2} \ell(y_{T+1,i}, \langle w, \hat{\phi}(x_{T+1,i}) \rangle).
\]

Then, we define the true target risk of the learned linear classifier \( \hat{w}_{T+1} \) as:

\[
\mathcal{L}(\hat{\phi}, \hat{w}_{T+1}) = \mathbb{E}_{(x,y) \sim \mu_{T+1}} \left[ \ell(y, \langle \hat{w}_{T+1}, \hat{\phi}(x) \rangle) \right]
\]

and want it to be as low and as close as possible to the ideal true risk \( \mathcal{L}(\phi^*, \hat{w}_{T+1}^*) \) where \( \hat{w}_{T+1}^* \) and \( \phi^* \) satisfy:

\[
\forall t \in [[T + 1]] \text{ and } (x, y) \sim \mu_t, \quad y = \langle w_t^*, \phi^*(x) \rangle + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2). \tag{2}
\]

Equivalently, most of the works found in the literature analyze the excess risk defined as

\[
\text{ER}(\hat{\phi}, \hat{w}_{T+1}) := \mathcal{L}(\hat{\phi}, \hat{w}_{T+1}) - \mathcal{L}(\phi^*, \hat{w}_{T+1}^*)
\]

and seek to upper-bound it in terms of quantities involved in the learning process.

### 3 From Theory to Practice

In this section, we highlight the main theoretical contributions that provably ensure the success of meta-learning in improving the performance on the previously unseen meta-test task with the increasing number of meta-train tasks and the amount of data available for them. We then concentrate our attention on the most recent theoretical advances leading to the fastest learning rates and show how the assumptions used to obtain them can be forced in practice through a novel regularization strategy.

#### 3.1 When Does Meta-Learning Provably Work?

One critical requirement for meta-learning is that a representation learned on meta-train data should be useful for learning a good predictor on the meta-test data set. Such a requirement is reflected by the fact that the excess target risk is bounded by a quantity that involves the number of samples in both meta-train and meta-test samples and the number of available meta-train tasks.

To this end, first studies in the context of meta-learning relied on probabilistic assumption \textbf{[Baxter, 2000, Pentina & Lampert, 2014, Maurer et al., 2016, Amit & Meir, 2018, Yin et al., 2020]} stating that meta-train and meta-test tasks distributions are all sampled i.i.d. from the same random distribution. This assumption leads to the bounds having the following form:

\[
\text{ER}(\hat{\phi}, \hat{w}_{T+1}) \leq O \left( \frac{1}{\sqrt{n_1 T}} + \frac{1}{\sqrt{T}} \right).
\]

Such a guarantee implies that even with the increasing number of source data, one would still have to increase the number of tasks as well, in order to draw the second term to 0. A natural improvement to this bound was then proposed by \textbf{[Du et al., 2020]} and \textbf{[Tripuraneni et al., 2020]} that obtained the bounds on the excess risk behaving as

\[
O \left( \frac{kd}{\sqrt{n_1 T}} + \frac{k}{\sqrt{n_2}} \right) \quad \text{and} \quad \hat{O} \left( \frac{kd}{n_1 T} + \frac{k}{n_2} \right),
\]

respectively, where \( k \ll d \) is the dimensionality of the learned representation and \( \hat{O} (\cdot) \) hides logarithmic factors. Both these results show that all the source and target samples are useful in minimizing the excess risk. Thus, in the FSL regime where target data is scarce, all source data helps to learn well. From a set of assumptions made by the authors in both of these works, we note the following two:

**Assumption 1.** The matrix of optimal predictors \( W^* \) should cover all the directions in \( \mathbb{R}^k \) evenly. More formally, this can be stated as

\[
\frac{\sigma_1(W^*)}{\sigma_k(W^*)} = O(1), \tag{3}
\]

\footnote{For a detailed review of the assumptions, the learning setups and the derived results from these two papers, we refer the interested reader to the Supplementary material.}
where \( \sigma_i(\cdot) \) denotes the \( i^{th} \) singular value of \( W^* \). As pointed out by the authors, such an assumption can be seen as a certain measure of diversity between the source tasks that are expected to be complementary to each other in order to provide a meaningful representation for a previously unseen target task.

**Assumption 2.** The norm of the optimal predictors \( w^* \) should not increase with the number of tasks seen during meta-training\(^3\). This assumption roughly boils down to saying that the classification margin of linear predictors should remain constant thus avoiding over- or under-specialization to the seen tasks.

While being highly insightful, the authors did not provide any experimental evidence suggesting that verifying these assumptions in practice helps to learn more efficiently in the considered learning setting. To bridge this gap, we propose a general regularization scheme that allows to enforce these assumptions when learning the matrix of predictors in several popular meta-learning algorithms.

### 3.2 Putting Theory to Work

Below, we propose a regularization strategy based on the theoretical insights derived from the advances in the meta-learning theory described above.

**Ensuring assumption 1.** For this assumption, we propose to compute the singular values of the matrix of predictors \( W \) during the meta-training stage and follow its evolution during the learning episodes. In practice, this can be done by performing the Singular Value Decomposition (SVD) on the matrix \( W \in \mathbb{R}^{T \times k} \) with a computational cost of \( O(Tk^2) \) floating-point operations (flop). However, as \( T \) is typically quite large, we propose a more computationally efficient solution that is to take into account only the last batch of \( N \) predictors (with \( N \ll T \)) grouped in the matrix \( W_N \in \mathbb{R}^{N \times k} \) that capture the latest dynamics in the learning process. Furthermore, we note that

\[
\sigma_i(W_N W_N^\top) = \sigma_i^2(W_N), \quad \forall i \in [N],
\]

implying that we can calculate the SVD of \( W_N W_N^\top \) (or \( W_N^\top W_N \) for \( k \leq N \)) and retrieve the singular values from it afterwards.

With this computational strategy in hand, we now want to verify whether the optimal linear predictors \( w_i \) are spread out to cover all directions in the embedding space by tracking the evolution of the ratio of singular values during the training process defined as

\[
R_\sigma(W_N) = \frac{\sigma_1(W_N)}{\sigma_N(W_N)}.
\]

For the sake of conciseness, we use \( R_\sigma \) instead of \( R_\sigma(W_N) \) thereafter. According to the theory, we expect \( R_\sigma \) to decrease gradually during the training thus improving the generalization capacity of the learned predictors and preparing them for the target task. When we want to enforce such a behavior in practice, we propose to use \( R_\sigma \) as a regularization term in the training loss of popular meta-learning algorithms.

Alternatively, as the smallest singular value \( \sigma_N(W_N) \) can be arbitrarily close to 0 and lead to numerical errors, we propose a more convenient replacement of \( R_\sigma \) given by the entropy of the vector of singular values defined as follows:

\[
H_\sigma(W_N) = -\sum_{i=1}^{N} \text{softmax}(\sigma(W_N))_i \cdot \log \text{softmax}(\sigma(W_N))_i,
\]

where \( \sigma(W_N) \) is the vector of eigenvalues of \( W_N \) and \( \text{softmax}(\sigma(W_N))_i \) is the \( i^{th} \) output of the softmax function. Analogously to \( R_\sigma \), we write \( H_\sigma \) instead of \( H_\sigma(W_N) \) from now on. Since the distribution with the highest entropy is the uniform distribution, adding \( R_\sigma \) or \( -H_\sigma \) as a regularization term leads to a better coverage of \( \mathbb{R}^k \) by ensuring a nearly identical importance regardless of the direction.

\(^3\)While not stated as a separate assumption in [Du et al., 2020], the authors assume it in their analysis of linear representations and further use it to derive the Assumption 1 mentioned above. See page 5 and the discussion after Assumption 4.3 in their pre-print.
Ensuring assumption 2. In addition to the full coverage of the embedding space by the linear predictors, the meta-learning theory assumes that the norm of the linear predictors does not increase with the number of tasks seen during meta-training, i.e., \( \|w\|_2 = O(1) \) or, equivalently, \( \|W\|_{F}^{2} = O(T) \). If this assumption does not hold in practice, we propose to regularize the norm of linear predictors during training or directly normalize the obtained linear predictors \( \tilde{w} = \frac{w}{\|w\|_2} \).

The final meta-training loss with the theory-inspired regularization terms is given as:

\[
\min_{\phi \in \Phi, W \in \mathbb{R}^{T \times k}} \frac{1}{2Tn_1} \sum_{t=1}^{T} \sum_{i=1}^{n_1} \ell(y_{t,i}, \langle w_t, \phi(x_{t,i}) \rangle) + R_\sigma(W_N) + \|W_N\|_2^2, \tag{4}
\]

and depending on the considered algorithm, we can replace \( R_\sigma \) by \(-H_\sigma \) and/or replace \( w_t \) by \( \tilde{w}_t \) instead of regularizing with \( \|W_N\|_2^2 \).

To the best of our knowledge, such regularization terms based on insights from the advances in meta-learning theory have never been used in the literature before. We also further use the basic quantities involved in the proposed regularization terms as indicators of whether a given meta-learning algorithm naturally satisfies the assumptions ensuring an efficient meta-learning in practice or not.

3.3 RELATED WORK

Below, we discuss several related studies aiming at improving the general understanding of meta-learning, and mention other regularization terms specifically designed for meta-learning.

Understanding meta-learning As mentioned in the introduction, a comprehensive theory for meta-learning is still lacking and is limited to the contributions discussed in Section 3.1. However, several recent works aimed to shed light on phenomena commonly observed in meta-learning by evaluating different intuitive heuristics. For instance, Raghu et al. (2020) investigated whether the popular gradient-based MAML algorithm achieves state-of-the-art performance due to rapid learning with significant changes in the representations when deployed on target task, or due to feature reuse where the learned representation remains almost intact. They establish that the latter factor is dominant and propose a new variation of MAML that freezes all but task-specific layers of the neural network when learning new tasks. In another study provided by Goldblum et al. (2020), the authors explain the success of meta-learning approaches by their capability to either cluster classes more tightly in feature space (task-specific adaptation approach), or to search for meta-parameters that lie close in weight space to many task-specific minima (full fine-tuning approach). Finally, the effect of the number of shots on the classification accuracy was studied theoretically and illustrated empirically in Cao et al. (2020) for the popular metric-based PROTONET algorithm. Our paper is complementary to all other works mentioned above as it investigates a new aspect of meta-learning that has never been studied before, while following a sound theory. Also, we provide a more complete experimental evaluation as the three different approaches of meta-learning (based on gradient, metric or transfer learning), separately presented in Raghu et al. (2020), Cao et al. (2020) and Goldblum et al. (2020), are now compared together.

Other regularization strategies Regularization is a common tool to reduce model complexity during learning for better generalization, and the variations of its two most famous instances given by weight decay (Krog & Hertz, 1992) and dropout (Srivastava et al., 2014) are commonly used as a basis in meta-learning literature as well. In general, regularization in meta-learning is applied to the weights of the whole neural network (Balaji et al., 2018; Yin et al., 2020), the predictions (Jamal & Qi, 2019; Goldblum et al., 2020) or is introduced via a prior hypothesis biased regularized empirical risk minimization (Pentina & Lampert, 2014; Kuzborskij & Orabona, 2017; Denevi et al., 2018a;b; 2019). Our proposal is different from all the approaches mentioned above for the following reasons. First, we do not regularize the whole weight matrix learned by the neural network but the linear predictors of its last layer contrary to what was done in the methods of the first group, and, more specifically, the famous weight decay approach (Krog & Hertz, 1992). Second, we regularize the singular values (through their entropy or their ratio) of the matrix of linear predictors obtained in the last batch of tasks instead of the predictions, as done by the methods of the second group (e.g., using the theoretic-information quantities in Jamal & Qi (2019) and Yin et al. (2020)). Finally, the works of the last group are related to the online setting with convex loss functions only, and, similarly to the algorithms from the second group, do not specifically target the spectral properties
of the learned predictors. Last, but not least, our proposal is built upon the most recent advances in the meta-learning field leading to faster learning rates contrary to previous works on the subject.

Having introduced how to verify the assumptions of interest and our new regularization terms, we set out to implement them for popular meta-learning algorithms on the commonly considered FSL problem.

4 Practical Results

In this section, we use extensive experimental evaluations to answer the following two questions:

Q1) Do popular meta-learning methods naturally satisfy the learning bounds assumptions?

Q2) Does ensuring these assumptions help to (meta-)learn more efficiently?

To this end, we first run the original implementations of popular meta-learning methods to see what their natural behavior is. Then, we observe the impact of forcing them to closely follow the theoretical setup.

4.1 Experimental Setup

Datasets & Baselines For our evaluation, we focus on the few-shot image classification problem on three benchmark datasets, namely: 1) Omniglot (Lake et al., 2015) consisting of 1,623 classes with 20 images of size 28 × 28 per class, 2) miniImageNet (Ravi & Larochelle, 2017) consisting of 100 classes with 600 images of size 84 × 84 per class, and 3) tieredImageNet (Ren et al., 2018) consisting of 779,165 images divided into 608 classes. For each dataset, we follow the commonly adopted experimental protocol used in Finn et al. (2017) and Chen et al. (2019) and use a four-layer convolution backbone (Conv-4) with 64 filters as done by Chen et al. (2019). On Omniglot, we perform 20-way classification with 1 shot and 5 shots, while on miniImageNet and tieredImageNet we perform 5-way classification with 1 shot and 5 shots. Finally, we evaluate four FSL methods: two popular meta-learning strategies, namely, MAML (Finn et al., 2017), a gradient-based method, and Prototypical Networks (PROTONET) (Snell et al., 2017), a metric-based approach; two popular transfer learning baselines, termed as BASELINE and BASELINE++ (Ravi & Larochelle, 2017; Gidaris & Komodakis, 2018; Chen et al., 2019). Even though these baselines are trained with the standard supervised learning framework, such a training can also be seen as learning a single task in the LTL framework.

Implementation details Enforcing Assumptions 1 and 2 for MAML is straightforward as it closely follows the LTL framework of episodic training. For each task, the model learns a batch of linear predictors and we can directly take them as \( W_N \) to compute its SVD. Since the linear predictors are the weights of our model and change slowly, regularizing the norm \( \| W_N \|_F \) and the ratio of singular values \( R_\sigma \) does not cause instabilities during training.

Meanwhile, metric-based methods do not use linear predictors but compute a similarity between features. In the case of PROTONET, the similarity is computed with respect to class prototypes (i.e. the mean features of the images of each class). Since they act as linear predictors, a first idea would be to regularize the norm and ratio of singular values of the prototypes. Unfortunately, this latter strategy hinders the convergence of the network and leads to numerical instabilities. Most likely because prototypes are computed from image features which suffer from rapid changes across batches. Consequently, we regularize the entropy of singular values \( H_\sigma \) instead of the ratio \( R_\sigma \) to avoid instabilities during training to ensure Assumption 1 and we normalize the prototypes to ensure Assumption 2 by replacing \( w_t \) with \( \bar{w}_t \) in Eq. 4.

For transfer learning methods BASELINE and BASELINE++, the last layer of the network is discarded then new linear predictors are learned during meta-testing. Thus, we only regularize the norm \( \| W_N \|_F \) of the new linear predictors learned during the finetuning phase of meta-testing. Similarly to MAML, we compute the ratio of singular values \( R_\sigma \) with the last layer of the network during training and fine-tuning phase.
Accumulating the two methods specifically controls the theoretical quantities of interest, and still, ProtoNet naturally learns to cover the embedding space with the prototypes, while minimizing their norms. This behavior is rather peculiar as neither ProtoNet nor converge toward a constant value when monitoring the last iteration of training, which is in accordance with theory.

Figure 1: (a) Evolution of $\|W_N\|_F$ (left), $R_\sigma$ (middle) and validation accuracy (right) during training of MAML (top) and ProtoNet (bottom) on miniImageNet (1 shot for MAML, 5 shots for ProtoNet). (b) Evolution of $R_\sigma$ (left) and validation accuracy (right) during training of BASELINE (top) and BASELINE++ (bottom) on Omniglot (dashed lines) and tieredImageNet (solid lines). All training curves were reproduced and averaged over 4 different random seeds. For MAML, both $\|W_N\|_F$ and $R_\sigma$ increase during training, which does not fulfill Assumptions 1 and 2. ProtoNet naturally learns to cover the embedding space with the prototypes, while minimizing their norms. $R_\sigma$ converges during training on both datasets for BASELINE++ (similarly to ProtoNet) whereas it diverges for BASELINE on tieredImageNet. With our regularization, $\|W_N\|_F$ and $R_\sigma$ are constant during training, which is in accordance with theory.

4.2 Insights

Q1 – Verifying the assumptions According to theory, $\|W_N\|_F$ and $R_\sigma$ should remain constant or converge toward a constant value when monitoring the last $N$ tasks. From Fig. 1(a), we can see that for MAML (Fig. 1(a) top), both $\|W_N\|_F$ and $R_\sigma$ increase with the number of tasks seen during training, whereas ProtoNet (Fig. 1(a) bottom) naturally learns the prototypes with a good coverage of the embedding space, and minimizes their norm. This behavior is rather peculiar as neither of the two methods specifically controls the theoretical quantities of interest, and still, ProtoNet manages to do it implicitly. As for the transfer learning baselines (Fig. 1(b) top and bottom), we expect them to learn features that cover the embedding space with $R_\sigma$ rapidly converging to a constant value. As can be seen in Fig. 1(b), similarly to ProtoNet, BASELINE++ naturally learns linear predictors that cover the embedding space. As for BASELINE, it learns a good coverage for Omniglot dataset, but fails to do so for the more complicated tieredImageNet dataset. The observed behavior of these different methods leads to a conclusion that some meta-learning algorithms are inherently more explorative of the embedding space.

Q2 – Ensuring the assumptions Armed with our regularization terms, we now aim to force the considered algorithms to verify the assumptions when it is not naturally done. In particular, for
In this paper, we studied the validity of the theoretical assumptions made in recent papers applied to popular meta-learning algorithms and proposed practical ways of enforcing them.

On the one hand, we showed that depending on the problem and algorithm, some models can naturally fulfill the theoretical conditions during training. Some algorithms offer a better covering of the embedding space than others. On the other hand, when the conditions are not verified, learning with our proposed regularization terms allows to learn faster and improve the generalization capabilities of meta-learning methods. The theoretical framework studied in this paper explains the observed performance gain. Notice that no specific hyperparameter tuning was performed as we rather aim

\[ \text{MAML} \]

we regularize both $\|W_N\|_F$ and $R_\sigma$ in order to keep them constant throughout the training. Similarly, we regularize $R_\sigma$ during the training of BASELINE and BASELINE++, and both $\|W_N\|_F$ and $R_\sigma$ during the finetuning phase of meta-testing. For PROTONET, we enforce a normalization of the prototypes. According to our results for Q1, regularizing the singular values of the prototypes through the entropy $H_\sigma$ is not necessary.\(^1\)

Based on the obtained results, we can make the following conclusions. First, from Fig. 1(a) (left, middle) and Fig. 1(b) (left), we note that for all methods considered, our proposed methodology used to enforce the theoretical assumptions works as expected, and leads to a desired behavior during the learning process. This means that the differences in terms of results presented in Table 1 are explained fully by this particular addition to the optimized objective function. Second, from the shape of the accuracy curves provided in Fig. 1(a) (right) and the accuracy gaps when enforcing the assumptions given in Table 1, we can see that respecting the assumptions leads to several significant improvements related to different aspects of learning. On the one hand, we observe that the final validation accuracy improves significantly in all benchmarks for meta-learning methods and in most of experiments for BASELINE (except for Omniglot, where BASELINE already learns to regularize its linear predictors). In accordance with the theory, we attribute the improvements to the fact that we fully utilize the training data which leads to a tighter bound on the excess target risk and, consequently, to a better generalization performance.

On the other hand, we also note that our regularization reduces the sample complexity of learning the target task, as indicated by the faster increase of the validation accuracy from the very beginning of the meta-training. Roughly speaking, less meta-training data is necessary to achieve a performance comparable to that obtained without the proposed regularization using more tasks. Finally, we note that BASELINE++ and PROTONET methods naturally satisfy some assumptions: both learn diverse linear predictors by design, while BASELINE++ also normalizes the weights of its linear predictors. Thus, these methods do not benefit from additional regularization.\(^2\)

### Table 1: Accuracy gap (in p.p.) of the considered algorithms when using the regularization (or normalization in the case of PROTONET) enforcing the theoretical assumptions. All accuracy results are averaged over 2400 test episodes and 4 different seeds. Statistically significant results (out of confidence intervals) are reported with *. Exact performances are on par with those found in the literature and are reported in the Supplementary material.

| Dataset         | Episodes | MAML      | PROTONET  | BASELINE  | BASELINE++ |
|-----------------|----------|-----------|-----------|-----------|------------|
| Omniglot        | 20-way 1-shot | +3.95*  | +0.33*    | −13.2*    | −7.29*     |
|                 | 20-way 5-shot | +1.17*  | +0.01     | +0.66*    | −2.24*     |
| miniImageNet    | 5-way 1-shot | +1.23*  | +0.76*    | +1.52*    | +0.39      |
|                 | 5-way 5-shot | +1.96*  | +2.03*    | +1.66*    | −0.13      |
| tieredImageNet  | 5-way 1-shot | +1.42*  | +2.10*    | +5.43*    | +0.28      |
|                 | 5-way 5-shot | +2.66*  | +0.23     | +1.92*    | −0.72      |

\(^1\)For more details on the effect of entropic regularization on PROTONET, we refer the interested reader to the Supplementary materials.  

\(^2\)See footnote 4 on the previous page.
at showing the effect of ensuring learning bounds assumptions than comparing performance of the methods. Absolute accuracy results are detailed in the Supplementary materials.

While this paper proposes an initial approach to bridging the gap between theory and practice in meta-learning, some questions remain open on the inner workings of these algorithms. In particular, being able to take better advantage of the particularities of the training tasks during meta-training could help improve the effectiveness of these approaches. Self-supervised meta-learning and multiple target tasks prediction are also important future perspectives for the application of meta-learning.

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A  Appendix

A.1  Notations

We introduce here several notations used throughout the main theoretical meta-learning papers (Maurer et al., 2016; [Du et al., 2020]; [Tripuraneni et al., 2020]). We denote by $\mu_{x_t}$, the marginal distribution of $x_t$, and its covariance matrix by $\Sigma_t = E_{x_t \sim \mu_{x_t}}[xx^T]$. We further use $\sigma_t(\cdot)$ to denote the $\mathcal{N}$th singular value of a matrix, let $\hat{R} = \|\hat{\phi}^* W^*\|_{\mathcal{S}} / \sqrt{T}$, where $\|\cdot\|_{\mathcal{S}}$ denotes the nuclear norm and $\hat{\phi}^*$ is the matrix of the transformation applied to the samples from $\mathcal{X}$. The point-wise and uniform covariance convergence refers to the fact that empirical covariance matrices converge to their true counterparts with the increasing number of samples. In [Du et al., 2020], the authors further assume that random vectors have zero mean, i.e., $E_{x \sim \mu_{x_t}}[x] = 0$ for all $t$ and that $x \sim \mu_{x_t}$ can be written as $\Sigma_1^{1/2} x$ with $\bar{x}$ having zero mean and identity covariance matrix. Finally, when considering a two-layer neural network (NN) with Rectifier Linear Unit (ReLU) activation function, the data generating model presented in Eq. 2 is modified by applying the ReLU activation to $\hat{\phi}$. This is denoted as a teacher network assumption. As, for the work of [Tripuraneni et al., 2020], we refer to the method of methods when using SVD to find the top $k$ singular vectors of $\Sigma_1^{1/2} \sum_{t=1}^{T} \sum_{i=1}^{n_t} y_t x_i, x_i^T$, while the linear regression stands for calculating the traditional closed-form solution on the transformed target task given by $\bar{w}_{T+1} = (\sum_{i=1}^{n_2} \hat{\phi}(x_{T+1}, i) \hat{\phi}(x_{T+1}, i)^T)^{-1} \hat{\phi}^T \sum_{i=1}^{n_2} x_{T+1}, y_{T+1}, i$.

A.2  Review of the Meta-Learning Theory

We now formulate the main results of the three main theoretical analyses of meta-learning provided in [Maurer et al., 2016; Du et al., 2020; Tripuraneni et al., 2020] in Table 2.

| Paper                  | Assumptions                                                                 | $\Phi$                           | Bound                        |
|------------------------|-----------------------------------------------------------------------------|----------------------------------|------------------------------|
| Maurer et al. [2016]   | $\mathcal{A}_1$. $\forall t \in [T+1]$, $\mu_t \sim \eta$                 | $-\$                             | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
| Du et al. [2020]       | $\mathcal{A}_{2.1}$. $\forall t \in [T]$, $\exists \nu > 0$, $\Sigma_t \geq \epsilon \Sigma_{\mathcal{S}}$ | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{2.3}$. $|\Sigma_{\mathcal{W}}| \leq O(1)$                     | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{2.4}$. $w_{T+1} \sim \mu_{w_t}$ : $\|E_{w \sim \mu_{w_t}}[ww^T]\|_{\mathcal{S}} \leq O(\frac{1}{T})$ | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{2.5}$. $\mu_t = \mu$, $\Sigma_t = \Sigma$                   | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{2.6}$. Point-wise-unif. cov. convergence                      | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{2.7}$. Teacher network                                        | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
| Tripuraneni et al. [2020]| $\mathcal{A}_{3.1}$. $\forall t \in [T]$, $\mu_t \sim \mu_{x_t}$           | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{3.2}$. $|\Sigma_{\mathcal{W}}| = O(1)$                      | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{3.3}$. $w_t \sim \text{Method of Moments}$                   | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |
|                        | $\mathcal{A}_{3.4}$. $w_{T+1} \sim \text{Linear Regression}$               | $\Phi$                           | $O\left(\frac{\sqrt{C}}{\sqrt{T}} + \frac{1}{T}\right)$ |

Table 2: Overview of main theoretical contributions related to meta-learning with their assumptions, considered classes of representations and the obtained bounds on the excess risk. Here $O(\cdot)$ hides logarithmic factors.

One may note that all the assumptions presented in this table can be roughly categorized into two groups. First one consists of the assumptions related to the data generating process (A1, A2.1, A2.4-7 and A3.1), technical assumptions required for the manipulated empirical quantities to be well-defined (A2.6) and assumptions specifying the learning setting (A3.3-4). We put them together as they are not directly linked to the quantities that we optimize over in order to solve the meta-learning problem. The second group of assumptions include A2.2 and A3.2: both defined as a measure of diversity between source tasks’ predictors that are expected to cover all the directions of $\mathbb{R}^k$ evenly. This assumption is of primary interest as it involves the matrix of predictors optimized in Eq. 1 as thus one can attempt to force it in order for $\mathbf{W}$ to have the desired properties. Finally, we note that assumption A3.2 related to the covariance dominance can be seen as being at the intersection between the two groups. On the one hand, this assumption is related to the population covariance and thus is related to the data generating process that is supposed to be fixed. On the other hand, we can think about a pre-processing step that precedes the meta-train step of the algorithm and transforms the source and target tasks’ data so that their sample covariance matrices satisfy A3.2.
While presenting a potentially interesting research direction, it is not clear how this can be done in practice especially under a constraint of the largest value of $c$ required to minimize the bound.

### A.3 Detailed Experimental Setups

**Omniglot** ([Lake et al., 2015](#)) is a dataset of 20 instances of 1623 characters from 50 different alphabets. Each image was hand-drawn by different people. The images are resized to $28 \times 28$ pixels and the classes are augmented with rotations by multiples of 90 degrees.

**miniImageNet** ([Ravi & Larochelle, 2017](#)) is a dataset made from randomly chosen classes and images taken from the ILSVRC-12 dataset ([Russakovsky et al., 2015](#)). The dataset consists of 100 classes and 600 images for each class. The images are resized to $84 \times 84$ pixels.

**tieredImageNet** ([Ren et al., 2018](#)) is also a subset of ILSVRC-12 dataset. However, unlike miniImageNet, training classes are semantically unrelated to testing classes. The dataset consists of 779,165 images divided into 608 classes. Here again, the images are resized to $84 \times 84$ pixels.

### A.4 Performance Comparisons with According Evaluation Settings

Table 3 shows the performance of our reproduced methods, MAML ([Finn et al., 2017](#)), PROTONET ([Snell et al., 2017](#)), BASELINE ([Ravi & Larochelle, 2017](#)) and BASELINE++ ([Gidaris & Komodakis, 2018](#)), compared to the reported results for the according training and evaluation setting to validate our implementations. We can see that our performance are on par with corresponding reported results. We attribute the differences to minor variations in implementations. Table 4 provides the detailed performance of our reproduced methods with and without our regularization (or normalization for PROTONET). Theses results are summarized in Table 1 of our paper and discussions about them can be found in Section 4.2.

### A.5 Ablative Studies

In the following, we include ablative studies on the effect of each terms in our regularization scheme to complete results given in Section 4.2 of our paper. In Table 5, we compared the performance of our reproduced MAML without regularization, with a regularization on the ratio of singular values, on the norm of the linear predictors, and with both regularization terms on Omniglot. We can see that both regularization terms are important in the training and that using only a single term can be detrimental to the training results.

In Table 6, we report the performance of our reproduced PROTONET without normalization, with normalization and with both normalization and regularization on the entropy. We can see that further enforcing a regularization on the singular values (through the entropy) does not help the training since PROTONET naturally learns to minimize the singular values of the prototypes.

In Table 7 and 8, we show the effect of regularization on different part of the training process of BASELINE and BASELINE++ respectively. The regularization used in training is limited to the ratio of singular values $R_\sigma$, whereas during finetuning, we regularize both the ratio $R_\sigma$ and the norm $\|W_N\|_F$. We can see that for BASELINE, similarly to MAML, both regularization terms are important on miniImageNet and tieredImageNet. For BASELINE++, on the other hand, learning with any of the regularization terms neither improves nor decreases performance in a statistically significant manner.
| Method   | Dataset         | Episodes       | Reported     | Reproduced  |
|----------|-----------------|----------------|--------------|-------------|
|          |                 | 20-way 1-shot  | 93.7* ± 0.7% | 91.72 ± 0.29% |
|          |                 | 20-way 5-shot  | 96.4* ± 0.1% | 97.07 ± 0.14% |
| MAML     | Omniglot        | 5-way 1-shot   | 46.47† ± 0.82% | 47.93 ± 0.83% |
|          |                 | 5-way 5-shot   | 62.71† ± 0.71% | 64.47 ± 0.69% |
|          | tieredImageNet  | 5-way 1-shot   | /             | 50.08 ± 0.91% |
|          |                 | 5-way 5-shot   | /             | 67.5 ± 0.79%  |
|          | miniImageNet    | 20-way 1-shot  | 96.00†        | 95.56 ± 0.10% |
|          |                 | 20-way 5-shot  | 98.90†        | 98.80 ± 0.04% |
|          |                 | 5-way 1-shot   | 44.42† ± 0.84% | 49.53 ± 0.41% |
|          |                 | 5-way 5-shot   | 64.24† ± 0.72% | 65.10 ± 0.35% |
|          | tieredImageNet  | 5-way 1-shot   | /             | 51.95 ± 0.45% |
|          |                 | 5-way 5-shot   | /             | 71.61 ± 0.38% |
|          | miniImageNet    | 20-way 1-shot  | /             | 78.18 ± 0.43% |
|          |                 | 20-way 5-shot  | /             | 95.34 ± 0.15% |
|          | 5-way 1-shot    | 42.11† ± 0.71% | 42.35 ± 0.73% |
|          | 5-way 5-shot    | 62.53† ± 0.69% | 59.58 ± 0.71% |
|          | tieredImageNet  | 5-way 1-shot   | /             | 44.59 ± 0.76% |
|          |                 | 5-way 5-shot   | /             | 66.38 ± 0.75% |
|          | miniImageNet    | 20-way 1-shot  | /             | 77.00 ± 0.49% |
|          |                 | 20-way 5-shot  | /             | 94.18 ± 0.17% |
|          | 5-way 1-shot    | 48.24† ± 0.75% | 48.06 ± 0.76% |
|          | 5-way 5-shot    | 66.43† ± 0.63% | 65.00 ± 0.68% |
|          | tieredImageNet  | 5-way 1-shot   | /             | 52.70 ± 0.87% |
|          |                 | 5-way 5-shot   | /             | 71.58 ± 0.74% |

Table 3: Our reproduced performances compared to reported performances from the according evaluation settings. All accuracy results (in %) are averaged over 2400 test episodes and 4 different random seeds and are reported with 95% confidence interval. *: Results reported from Raghu et al. (2020). †: Results reported from Chen et al. (2019). ‡: Results reported from Snell et al. (2017).
| Method  | Dataset               | Episodes | without Reg./Norm. | with Reg./Norm. |
|---------|-----------------------|----------|-------------------|----------------|
|         |                       | 1-shot   | 5-shot            |                |
| MAML    | Omniglot              | 91.72 ± 0.29% | 97.07 ± 0.14% | 95.67 ± 0.20% | 98.24 ± 0.10% |
|         | miniImageNet          | 47.93 ± 0.83% | 64.47 ± 0.69% | 49.16 ± 0.85% | 66.43 ± 0.69% |
|         | tieredImageNet        | 50.08 ± 0.91% | 67.5 ± 0.79% | 51.5 ± 0.90% | 70.16 ± 0.76% |
| PROTO NET | Omniglot             | 95.56 ± 0.10% | 98.80 ± 0.04% | 95.89 ± 0.10% | 98.80 ± 0.04% |
|         | miniImageNet          | 49.53 ± 0.41% | 65.10 ± 0.35% | 50.29 ± 0.41% | 67.13 ± 0.34% |
|         | tieredImageNet        | 51.95 ± 0.45% | 71.61 ± 0.38% | 54.05 ± 0.45% | 71.84 ± 0.38% |
| BASELINE | Omniglot              | 86.85 ± 0.36% | 96.95 ± 0.12% | 73.65 ± 0.52% | 97.61 ± 0.11% |
|         | miniImageNet          | 42.35 ± 0.73% | 59.58 ± 0.71% | 43.87 ± 0.75% | 61.24 ± 0.71% |
|         | tieredImageNet        | 44.59 ± 0.76% | 66.38 ± 0.75% | 50.02 ± 0.82% | 68.30 ± 0.74% |
| BASELINE++ | Omniglot             | 82.5 ± 0.39% | 95.49 ± 0.15% | 75.21 ± 0.47% | 93.25 ± 0.20% |
|         | miniImageNet          | 48.06 ± 0.76% | 65.00 ± 0.68% | 48.45 ± 0.78% | 64.87 ± 0.68% |
|         | tieredImageNet        | 52.70 ± 0.87% | 71.58 ± 0.74% | 52.98 ± 0.88% | 70.86 ± 0.74% |

Table 4: Performance of several meta-learning algorithms without and with our regularization (or normalization in the case of PROTONET) to enforce the theoretical assumptions. All accuracy results (in %) are averaged over 2400 test episodes and 4 different seeds and are reported with 95% confidence interval. Episodes are 20-way classification for Omniglot and 5-way classification for miniImageNet and tieredImageNet.
The singular values can be detrimental to performance. Similarly to P\textsubscript{ROTO}, further enforcing regularization on both terms is important.

Table 5: Ablative study of the regularization parameter for MAML on Omniglot. All accuracy results (in %) are averaged over 2400 test episodes and 4 different random seeds and are reported with 95% confidence interval. Using both regularization terms is important.

| Dataset         | Episodes     | Reproduced | Ratio | Norm     | Ratio + Norm |
|-----------------|--------------|------------|-------|----------|--------------|
| Omniglot        | 20-way 1-shot| 91.72 ± 0.29% | 89.86 ± 0.31% | 92.80 ± 0.26% | **95.67 ± 0.20%** |
|                 | 20-way 5-shot| 97.07 ± 0.14% | 72.47 ± 0.17% | 96.99 ± 0.14% | **98.24 ± 0.10%** |

Table 6: Performance of PROTO\textsubscript{NET} with and without our regularization on the entropy and/or normalization. All accuracy results (in %) are averaged over 2400 test episodes and 4 different random seeds and are reported with 95% confidence interval. Further enforcing regularization on the singular values can be detrimental to performance.

| Dataset         | Episodes     | Reproduced | Reg. in training | Reg. in finetuning | Reg. in both |
|-----------------|--------------|------------|------------------|--------------------|--------------|
| miniImageNet    | 5-way 1-shot | 49.53 ± 0.41% | 43.12 ± 0.73% | 43.32 ± 0.76% | **43.87 ± 0.75%** |
|                 | 5-way 5-shot | 65.10 ± 0.35% | 60.17 ± 0.71% | 60.72 ± 0.70% | **61.24 ± 0.71%** |
| tieredImageNet  | 5-way 1-shot | 51.95 ± 0.45% | **49.49 ± 0.83%** | 45.78 ± 0.75% | **50.02 ± 0.82%** |
|                 | 5-way 5-shot | **71.61 ± 0.38%** | **68.66 ± 0.74%** | 66.19 ± 0.74% | **68.30 ± 0.74%** |

Table 7: Ablative study on the effect of the regularization on different parts of training process of BASELINE. All accuracy results (in %) are averaged over 2400 test episodes and 4 random seeds and are reported with 95% confidence interval. Similarly to MAML, both regularization terms are important.

| Dataset         | Episodes     | Reproduced | Reg. in training | Reg. in finetuning | Reg. in both |
|-----------------|--------------|------------|------------------|--------------------|--------------|
| miniImageNet    | 5-way 1-shot | 48.06 ± 0.76% | 47.83 ± 0.78% | 48.66 ± 0.79% | 48.45 ± 0.78% |
|                 | 5-way 5-shot | 65.00 ± 0.68% | 64.71 ± 0.68% | 65.35 ± 0.68% | 64.87 ± 0.68% |
| tieredImageNet  | 5-way 1-shot | 52.70 ± 0.87% | 52.75 ± 0.87% | 52.83 ± 0.87% | 52.98 ± 0.88% |
|                 | 5-way 5-shot | 71.58 ± 0.74% | 71.03 ± 0.74% | 71.64 ± 0.74% | 70.86 ± 0.74% |

Table 8: Ablative study on the effect of the regularization on different parts of training process of BASELINE++. All accuracy results (in %) are averaged over 2400 test episodes and 4 random seeds and are reported with 95% confidence interval. Similarly to PROTO\textsubscript{NET}, further enforcing regularization does not improve nor decrease performance.