Extended state observer-based integral line-of-sight guidance law for path following of underactuated unmanned surface vehicles with uncertainties and ocean currents

Mingcong Li*, Chen Guo and Haomiao Yu

Abstract
This article focuses on the problem of path following for underactuated unmanned surface vehicles (USVs) considering model uncertainties and time-varying ocean currents. An extended state observer (ESO)-based integral line-of-sight (ILOS) with an integral sliding mode adaptive fuzzy control scheme is proposed as the main control framework. First, a novel ESO is employed to estimate the surge and sway velocities based on the kinetic model, which are difficult to measure directly. Then, the adaptive ILOS guidance law is proposed, in which the integral vector is incorporated into the adaptive method to estimate the current velocities. Meanwhile, an improved fuzzy algorithm is introduced to optimize the look-ahead distance. Second, the controller is extended to deal with the USV yaw and surge velocity signal tracking using the integral sliding mode technique. The uncertainties of the USV are approximated via the adaptive fuzzy method, and an auxiliary dynamic system is presented to solve the problem of actuator saturation. Then, it is proved that all of the error signals in the closed-loop control system are uniformly ultimately bounded. Finally, a comparative simulation substantiates the availability and superiority of the proposed method for ESO-based ILOS path following of USV.

Keywords
Unmanned surface vehicles, ocean currents, line-of-sight, path following, actuator saturation

Date received: 19 March 2020; accepted: 31 March 2021

Topic Area: Robot Manipulation and Control
Topic Editor: Yangquan Chen
Associate Editor: Yangquan Chen

Introduction
In recent decades, intelligent control of unmanned surface vehicles (USVs) has become one of the most challenging topics in the nonlinear control community and has attracted great attention in the marine, military, and commerce fields for applications, such as path following, collision avoidance, and formation control.1–5 The problem of USV path following has expanded over the past decade and is a thorny aspect of USV intelligent control because of its complicated mathematical model. Additionally, a difficult problem is that only the surge and yaw direction can be directly controlled, while the sway velocity is passive for most USVs.

Most USVs are underactuated in that the number of actuators in the mechanism is less than its degree of freedom. 

College of Marine Electrical Engineering, Dalian Maritime University, Dalian, China

Corresponding author:
Chen Guo, College of Marine Electrical Engineering, Dalian Maritime University, Dalian 116026, China. Email: dregexpoc1126@com
freedom. Although an underactuated system is more complex than a full-drive system, the former possesses several advantages including conservation of energy, material, and space.\(^6\)\(^7\) The authors proposed the underactuated spherical parallel mechanism-based robotic ankle exoskeleton, and the lightweight mechanism in low-carbon design was verified. In the literature,\(^8\) a detailed calculation model for each stage of the sustainable supply chain was proposed, and findings revealed that the underactuated system can be used to achieve lightweight and energy saving, thereby leading to a low carbon footprint. The underactuated system has its special values, but it needs to achieve breakthroughs or unprecedented innovations in both theoretical and practical techniques. At present, research on this type of system has become popular in USVs and robotics,\(^9\)\(^10\) and the control of underactuated systems has been researched mainly by state stabilization, trajectory tracking, and path following.\(^6\)\(^11\) High nonlinearity renders the problem of control design in the execution module represents another indispensable aspect of the path-following cascade control system. Essentially, the execution module control system should be designed to force the USV state to track the reference signals of the proposed LOS guidance law. When controlling an underactuated system, the first problem is controllability; this system is complex and nonlinear, such that linear control theories cannot be directly applied. Underactuated systems must be analyzed using nonlinear controllability theory based on their own characteristics. Extensive research has presented controllability analysis of underactuated systems. From the mathematical standpoint, the authors\(^20\)\(^21\)\(^22\) provided a theoretical basis for the controllability analysis in underactuated systems. The control of the underactuated system is always realized using motion coupling or dynamic coupling.\(^23\)\(^24\)\(^25\) In a controllable system, an effective control technique is backstepping control, which has been widely adopted given its systematic calculated amount.\(^17\)\(^26\)\(^27\) The backstepping technique can eliminate the constraint that the relative degree must be 1 in classical passive systems. However, the heavy calculation burden of backstepping makes some control strategies impractical. From this point of view, the active-disturbance-rejection controller (ADRC), trajectory linearization controller (TLC), and sliding mode controller (SMC) have been proposed by many researchers. In the literature,\(^28\) a control scheme combining the LOS guidance law with the ADRC technique was proposed to make the USV follow a reference parameterized curved path. Liu et al.\(^29\) introduced the TLC scheme in relation to USV path following and illuminated a new direction in TLC technology. Considering the robustness to external disturbances, parameter perturbations, and unmodeled dynamics, SMC is an effective and powerful advanced controllers that have been developed considerably in USV and robot areas.\(^30\)\(^31\)\(^32\)\(^33\)\(^34\)\(^35\) In practical applications, SMC has successfully applied to underactuated biped robot,\(^36\) satellites,\(^37\) and overhead crane.\(^38\) For example, sliding mode observer was designed by Van et al.\(^39\) to estimate the robot velocities in the presence of model uncertainties and external disturbances. A backstepping sliding mode AUV path-following control algorithm was proposed by Liang et al.\(^40\) However, the single SMC usually cannot satisfy the system requirement, such as high efficiency and strong robustness. In this case, a hybrid
control scheme that switched between proportional-derivative (PD) control and SMC was proposed by Ouyang et al.\textsuperscript{31} for tracking control of robot manipulators, where PD control was used to stabilize the controlled system, while SMC was used to compensate the disturbance and uncertainty and reduce tracking errors. In the literature,\textsuperscript{30} the integral SMC (ISMC) was first employed for USV trajectory tracking. Moreover, the ISMC was introduced to the USV path following control\textsuperscript{26} and applied in the attitude loop and surge velocity loop, respectively, findings were fairly encouraging. In the literature,\textsuperscript{41} an adaptive fuzzy control method was proposed to estimate the model uncertainty and achieve remarkable tracking performance in terms of both tracking and unknown estimation. Actuator saturation can strongly influence system stability, such as undershooting, lag, and performance degradation. To solve this physical problem, researchers have thus proposed many approaches to preventing this issue, such as by using the continuous sigmoid function instead of the sign function,\textsuperscript{46} adding a filter,\textsuperscript{46} introducing a fuzzy/neural network to approximate the sign function,\textsuperscript{47} or applying mathematical optimization to the switching function.\textsuperscript{48}

System uncertainty and disturbance are common in practical control systems. The robustness against them is critical for motion control of USV. A variety of methods were proposed to deal with the uncertainty, ranging from Fourier series expansion,\textsuperscript{49} observers,\textsuperscript{50} and neural networks\textsuperscript{51–53} to fuzzy techniques. Fuzzy control is an early form of intelligent control and it imitates the ambiguity of human’s thought and controls objects using the control experience of human experts.\textsuperscript{6} A weakness of fuzzy techniques is that approximator accuracy relies on the number of nodes. An effective approach involves estimating the norm of the ideal weighting vector by replacing the vector elements. From this point of view, in the literature,\textsuperscript{34} an adaptive fuzzy control method was proposed to estimate model uncertainty and achieve remarkable tracking performance in terms of both tracking and unknown estimation. In the literature,\textsuperscript{54} the fuzzy techniques were used to estimate the model uncertainty and external disturbance simultaneously. Considering the structure of approximator, Wang and Er\textsuperscript{55} proposed a self-constructing fuzzy control USV trajectory tracking scheme, which contained self-learning membership functions and parameter adaptation.

Every input into real systems should be bounded by actuators’ physical restrictions. The actuator saturation (i.e. input saturation) tends to be ignored when designing control systems. Actuator saturation can strongly influence the stability of systems, such as undershooting, lag, and performance degradation. To solve this physical problem, Chen et al.\textsuperscript{56} proposed an auxiliary dynamic system to compensate for the input constraints. The system states were applied for the adaptive tracking control design in uncertain MIMO nonlinear systems. In the literature,\textsuperscript{37} a finite-time trajectory tracking scheme was proposed based on PD plus dynamics compensation in the presence of input saturation, where the Sat function was introduced to deal with the saturation problem. In the literature,\textsuperscript{19} an auxiliary design scheme was presented to compensate for the surge and yaw controller in an underactuated USV, and the uniformly ultimately bounded (UUB) stability was confirmed for the cascade path following system.

In this article, an ESO-based ILOS (EILOS) guidance law and adaptive fuzzy SMC (EIAFSM) with actuator saturation are proposed for USV path following in the presence of ocean current velocities and external model uncertainties. The ESO is developed to identify surge and sway velocities considering their immeasurability, and the ILOS guidance law is designed to produce the reference heading angle. In addition, an improved algorithm is proposed for look-ahead distance. Then, the SMC is designed to maintain the USV surge velocity and heading angle tracking the reference signals generated by the LOS guidance law. Meanwhile, the USV model uncertainty and sign functions in the control law are estimated using the fuzzy logic system (FLS), and an auxiliary system is provided to compensate for the part exceeding the actuator limit.

The remainder of this article is structured as follows. Several necessary preliminaries and explanations about the USV model are detailed in the second section. The EILOS guidance scheme is introduced in the third section. The fourth section outlines the actuator control method for USVs. The system convergence analysis is presented in the fifth section. The sixth section provides an example to illustrate the feasibility of the proposed method, and the seventh section offers our conclusion and directions for future work.

**Preliminaries**

**Lemmas**

**Definition 1.** $\mathbb{R}^n$ is the $n$-dimensional Euclidean space. The solution of the differential equation is $x$ and $x(t_0) = x_0$.\textsuperscript{58}

For a set containing the origin $W \subset \mathbb{R}^n$, the system is UUB if there is a non-negative constant $T(x_0, W) < \infty$, so that the following equation holds for all $t \geq t_0 + T(x_0, W)$:

$$\|x(t_0)\| < \delta \Rightarrow x(t) \in W$$

(1)

**Lemma 1.** If $x = 0$ is an equilibrium point of the system $\dot{x} = f(x, t)$, and the function $f$ is Lipschitz, there exists a positive Lyapunov function $V$ satisfying\textsuperscript{58,59}

$$\dot{V} = \frac{\partial V}{\partial x} f(x, t) \leq -CV + \rho \leq 0, \forall t \geq 0, \forall x \in \mathbb{R}^n$$

(2)

where $C$ is a non-negative parameter and $\rho < \infty$. The system $\dot{x} = f(x, t)$ is UUB.
Lemma 2. The USV input signal \( \tau_i \) is limited by \(-\tau_{imax}\) and \(\tau_{imax}\) such that \(-\tau_{imax} \leq \tau_i \leq \tau_{imax}\). The relational expression between the real \( \tau_i \) and the command \( \tau_{0i} \) is

\[
\tau_i = \begin{cases} 
\tau_{imax}, & \tau_{0i} > \tau_{imax} \\
\tau_{0i}, & -\tau_{imax} \leq \tau_{0i} \leq \tau_{imax} \\
-\tau_{imax}, & \tau_{0i} < -\tau_{imax} 
\end{cases}
\]

(3)

Lemma 3. For \( a, b \geq 0 \), the Young’s inequality holds

\[
ab \leq \frac{\delta^m}{m} |a|^m + \frac{1}{m^\delta n} |b|^n
\]

(4)

where \( \delta \) is positive, \( m, n > 1 \) and \((m-1)(n-1) = 1\). When \( m = n = 2, \delta = 1 \), inequality (4) becomes

\[
ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2.
\]

In this case, the right side of the inequality sign is non-negative. Thus, if \( a \leq 0 \) or \( b \leq 0 \), the inequality also holds.

Unmanned surface vehicle models

This subsection describes the USVs’ kinematic and dynamic models with ocean currents. The mathematical model of a USV on a horizontal plane can be described as follows

\[
\begin{align*}
\dot{x} &= u_r \cos \psi - v_r \sin \psi + V_x \\
\dot{y} &= u_r \sin \psi + v_r \cos \psi + V_y \\
\dot{\psi} &= \nu_r \\
\dot{u}_r &= H_u(v_r, r) - \frac{d_{11}}{m_{11}} u_r + \tau_u + \delta_u \\
\dot{v}_r &= E(u_r)r + F(u_r)v_r + \delta_v \\
\dot{r} &= H_r(u_r, v_r, r) + \tau_r + \delta_r
\end{align*}
\]

(5)

where \((x, y)\) provide the positional information and \(\psi\) denotes the heading angle. \((u_r, v_r, r)\) represent the USV relative surge velocity, the sway velocity, and the yaw rate within the body-fixed frame, respectively. \((V_x, V_y)\) describe the \(x, y\) directions of ocean current velocities within the inertial frame. Define

\[
R(\psi) = \begin{bmatrix} 
\cos \psi & -\sin \psi \\
\sin \psi & \cos \psi
\end{bmatrix}
\]

and

\[
[u_r, v_r]^T = R(\psi)[V_x, V_y]^T,
\]

where \((u_r, v_r)\) describe the velocities of ocean current within the body-fixed frame. \([u, v]^T = [u_r, v_r]^T + [u_c, v_c]^T\), where \((u, v)\) are the absolute USV velocities. \((\tau_u, \tau_r)\) represent the input signals that directly control the actuator. The external disturbances can be expressed as \((\delta_u, \delta_v, \delta_r)\).

The definitions of \(H_u(v_r, r), E(u_r), F(u_r), H_r(u_r, v_r, r)\) are given in Appendix.

Assumption 1. The absolute USV resultant velocity \(U\), heading angle \(\psi\), and yaw rate \(r\) are measurable, but the relative velocities \(u_r\) and \(v_r\) are not.

Assumption 2. \(\delta_u, \delta_v,\) and \(\delta_r\) are bounded, their upper bounds can be described as \((\eta_1, \eta_2, \eta_3)\), which are unknown.

Assumption 3. The ocean currents are assumed to be slow changing such that \(V_x \approx V_y \approx 0\) and bounded by \(V_{max}\). The magnitudes of ocean currents are much lower than the USV velocities.

Assumption 4. The time derivatives of \(u_r\) and \(v_r\) are bounded.

Remark 1. In Assumption 1, we can easily determine the state vectors \(U, \psi,\) and \(r\) from common navigational instruments. The relative velocities are difficult to measure with common apparatuses.\(^6\) Assumptions 2 and 3 are reasonable due to the finite energy of external disturbances including ocean currents,\(^62\) and similar assumptions appear in the literature.\(^1\) For Assumption 4, similar theories can be found in Proposition 1\(^63\) and Assumption 1.\(^6\) This assumption is justified given that the energy of the USV actuator is finite and abrupt signal change is not allowed, thereby leading to \(|\dot{u}(\psi)| \leq \omega_{max}\), where \(\omega_{max}\) is a positive constant.

Control objective

This article aims to propose a control algorithm to keep the USV following a prescribed path parameterized by \((x_k(s), y_k(s))\), such that the velocities and position tracking errors of USV converge to a small range ultimately, that is, \(\lim_{s \rightarrow \infty} (x - x_k) \leq \epsilon_x, \lim_{s \rightarrow \infty} (y - y_k) \leq \epsilon_y,\) and \(\lim_{s \rightarrow \infty} (u_r - u_{rd}) \leq \epsilon_u\), where \(u_{rd}\) denotes the desired relative surge velocity and \(\epsilon_x, \epsilon_y, \epsilon_u\) are bounded constants.

Remark 2. More precisely, the objective of the velocity is to realize \(\lim_{s \rightarrow \infty} U = U_{rd}\), where \(U_{rd}\) represents the desired absolute resultant velocity. If \(V_x, V_y, u_r\) and \(v_r\) are known or accurately estimated, then, we can easily calculate the desired relative sway velocity \(u_{rd}\). Therefore, it is reasonable for \(u_{rd}\) to be the velocity objective.

Guidance subsystem design

The schematic diagram of USV path following and an ELOS guidance law are presented in this section. The LOS geometry is shown in Figure 1.

Let \(c\) describe the desired path \((x_k(s), y_k(s))\) and the tangential angle of the path is defined as \(\gamma_k(s) = \arctan(\gamma_k(s))\), satisfying \(\gamma_k(s) = [\pi, \pi]\), where \(\gamma_k(s)\) and \(\gamma_k(s)\) denote the partial derivatives of \(y_k(s)\) and \(x_k(s)\), respectively. The sideslip angle that is always ignored by some researchers can be expressed as \(\beta = \arctan(\nu, u)\). Note that the USV velocities in system (5) are relative due to the ocean currents. To facilitate calculations, define \(\beta = \arctan(\nu, u)\) as the relative sideslip angle that can only be a dummy variable.

Assumption 5. The guidance signal \(\psi_d\) can be completely tracked by the actual heading angle in the guidance part regardless of the control effects of \(\tau_u\) and \(\tau_r\), that is, \(\psi = \psi_d\).
As shown in Figure 1, the along- and cross-tracking errors \((x_e, y_e)\) of USV can be expressed as

\[
\begin{bmatrix}
  x_e \\
  y_e
\end{bmatrix} =
\begin{bmatrix}
  \cos \gamma_k & -\sin \gamma_k \\
  \sin \gamma_k & \cos \gamma_k
\end{bmatrix}^T \begin{bmatrix}
  x - x_k(s) \\
  y - y_k(s)
\end{bmatrix}
\]

(6)

Similar to equation (6), we have

\[
\begin{bmatrix}
  \dot{x}_k(s) \\
  \dot{y}_k(s)
\end{bmatrix} =
\begin{bmatrix}
  \cos \gamma_k & -\sin \gamma_k \\
  \sin \gamma_k & \cos \gamma_k
\end{bmatrix} \begin{bmatrix}
  u_m \\
  0
\end{bmatrix}
\]

(7)

where \(u_m\) represents the virtual speed of the desired path and \(u_m = \xi \sqrt{x_k^2 + y_k^2}\).

The time derivative of the along-tracking error is expressed by

\[
\dot{x}_e = x \cos \gamma_k + y \sin \gamma_k - x_k(s) \cos \gamma_k - y_k(s) \sin \gamma_k + \dot{\gamma}_k \left( (x - x_k(s)) \sin \gamma_k + (y - y_k(s)) \cos \gamma_k \right)
\]

(8)

Similarly, we have

\[
\dot{y}_e = -x \sin \gamma_k + y \cos \gamma_k + x_k(s) \sin \gamma_k - y_k(s) \cos \gamma_k + \dot{\gamma}_k \left( (x - x_k(s)) \cos \gamma_k + (y - y_k(s)) \sin \gamma_k \right)
\]

(9)

Substituting equations (5) to (7) into equations (8) and (9) results in

\[
\begin{align*}
\dot{x}_e &= u_e \cos (\psi_d - \gamma_k) - v_e \sin (\psi_d - \gamma_k) + y_c \dot{\gamma}_k - u_m + \theta_x \\
\dot{y}_e &= u_e \sin (\psi_d - \gamma_k) + v_e \cos (\psi_d - \gamma_k) - x_e \dot{\gamma}_k + \theta_y
\end{align*}
\]

(10)

where \(\theta_x = V_e \cos (\beta_e - \gamma_k)\) and \(\theta_y = V_e \sin (\beta_e - \gamma_k)\), in which \(V_e = \sqrt{V_x^2 + V_y^2}\) and \(\beta_e = \text{atan2}(V_y, V_x)\). \(\theta_x\) and \(\theta_y\) are bounded under Assumption 3.

To estimate the relative velocities of the USV, two novel ESOs are proposed as follows

\[
\begin{align*}
\dot{u}_e &= p_1 + k_1 x_e \\
\dot{p}_1 &= -k_1(\dot{u}_e \cos (\psi_d - \gamma_k) - \dot{v}_e \sin (\psi_d - \gamma_k) + \dot{\gamma}_e y_c - u_m) + y_c \sin (\psi_d - \gamma_k) + x_e \cos (\psi_d - \gamma_k)
\end{align*}
\]

(11)

and

\[
\begin{align*}
\dot{v}_e &= p_2 + k_1 y_e \\
\dot{p}_2 &= -k_1(\dot{u}_e \sin (\psi_d - \gamma_k) + \dot{v}_e \cos (\psi_d - \gamma_k) + \dot{\gamma}_e x_e) + y_c \cos (\psi_d - \gamma_k) - x_e \sin (\psi_d - \gamma_k)
\end{align*}
\]

(12)

where \(\dot{u}_e\) and \(\dot{v}_e\) are the estimations of \(u_e\) and \(v_e\) respectively, and \(k_1\) is a positive parameter. Define the estimation errors \(\bar{u}_e = \hat{u}_e - u_e\) and \(\bar{v}_e = \hat{v}_e - v_e\). By combining equations (10) to (12), the corresponding error dynamics of the velocity estimations can be written as

\[
\begin{align*}
\dot{\bar{u}}_e &= \bar{u}_e - \bar{u}_e = p_1 + k_1 \bar{x}_e - \bar{u}_e \\
\dot{\bar{p}}_1 &= -k_1(\bar{u}_e \cos (\psi_d - \gamma_k) - \bar{v}_e \sin (\psi_d - \gamma_k) + k_1 \theta_x) \\
&\quad + y_c \sin (\psi_d - \gamma_k) + x_e \cos (\psi_d - \gamma_k) - \bar{u}_e \\
\dot{\bar{v}}_e &= \bar{v}_e - \bar{v}_e = p_2 + k_1 \bar{v}_e - \bar{v}_e \\
\dot{\bar{p}}_2 &= -k_1(\bar{u}_e \sin (\psi_d - \gamma_k) + \bar{v}_e \cos (\psi_d - \gamma_k) + k_1 \theta_y) \\
&\quad + y_c \cos (\psi_d - \gamma_k) - x_e \sin (\psi_d - \gamma_k) - \bar{v}_e
\end{align*}
\]

(13)

Remark 3: The ESO system is significantly different from the ESOs provided in the literature.\(^{63}\) We take ocean currents into account, two compulsory terms \(\theta_x\) and \(\theta_y\) are added into the error system (13). In addition, to achieve the stability of a more complicated guidance system, there are two additional terms \(y_c \cos (\psi_d - \gamma_k)\) and \(x_e \sin (\psi_d - \gamma_k)\) of the ESO system.

Note that \(u_e = \hat{u}_e - \bar{u}_e\) and \(v_e = \hat{v}_e - \bar{v}_e\), rewrite equation (10) as

\[
\begin{align*}
\dot{x}_e &= u_e \cos (\psi_d - \gamma_k) - \bar{u}_e \cos (\psi_d - \gamma_k) - \bar{v}_e \sin (\psi_d - \gamma_k) \\quad + V_e \dot{\gamma}_k - u_m + \theta_x \\
\dot{y}_e &= u_e \sin (\psi_d - \gamma_k) - \bar{u}_e \sin (\psi_d - \gamma_k) + V_e \cos (\psi_d - \gamma_k) \\
&\quad - V_e \dot{\gamma}_k - x_e \dot{\gamma}_k - \theta_y
\end{align*}
\]

(14)

The guidance law is presented as

\[
\psi_d = \gamma_k - \hat{\theta}_x + \arctan \left( \frac{-V_y - \alpha_x}{\Delta} \right)
\]

(15)

where \(\hat{\theta}_x = \text{atan2}(\bar{v}_e, \bar{u}_e)\), \(\Delta > 0\) represents the look-ahead distance, and the integral term \(\alpha_x\) is the virtual input that is used to shape the dynamics of the system.

Define \((\hat{\theta}_x, \hat{\theta}_y)\) is the estimation of \((\theta_x, \theta_y)\), \((\bar{\theta}_x, \bar{\theta}_y)\) are the estimation errors, and \(\hat{\theta}_x = \theta_x - \bar{\theta}_x, \hat{\theta}_y = \theta_y - \bar{\theta}_y\).
In equation (14), $u_m$ can be treated as the moving speed of the reference path and is proposed as

$$u_m = \hat{u}_r \cos(\psi_d - \gamma_k) - \hat{v}_r \sin(\psi_d - \gamma_k) + k_2 x_r + \hat{\theta}_x$$  \hspace{1cm} (16)

where $k_2$ is a positive constant. Therefore, the update law of the path variable $\varsigma$ can be expressed as

$$\dot{\varsigma} = \frac{\hat{u}_r \cos(\psi_d - \gamma_k) - \hat{v}_r \sin(\psi_d - \gamma_k) + k_2 x_r + \hat{\theta}_x}{\sqrt{x_r^2 + y_r^2}}$$  \hspace{1cm} (17)

Remark 4. Note that the physical meaning of virtual variable $u_m$ is that the speed of the reference path and its value depend on some USV state variables, such as $u_r$, $v_r$, $x_r$, and $\psi_d$. One can adjust the tracking speed between the real and reference path, that is, the desired path also tracks the real path to some extent, dramatically reducing the computational burden.

The derivative of $y_c$ can be rewritten as

$$\dot{y}_c = \hat{U}_r \sin(\psi_d + \beta_r - \gamma_k) - \hat{u}_r \sin(\psi_d - \gamma_k) - \hat{v}_r \cos(\psi_d - \gamma_k) - x_r \gamma_k + \hat{\theta}_y$$

where $\hat{U}_r$ is the estimate of resultant resultant velocity and $\hat{U}_r = \sqrt{\hat{u}_r^2 + \hat{v}_r^2}$.

Substituting equation (15) into equation (18) yields

$$\dot{y}_c = - \frac{\hat{U}_r}{\sqrt{(y_c + \alpha_r)^2 + \Delta^2}} (y_c + \alpha_r) - \hat{u}_r \sin(\psi_d - \gamma_k) - \hat{v}_r \cos(\psi_d - \gamma_k) - x_r \gamma_k + \hat{\theta}_y$$

Note that the ocean current parameter $\theta_y$ can be eliminated by designing the virtual control input $\alpha_r$ in asymptotically as follows

$$\frac{\hat{U}_r \alpha_r}{\sqrt{(y_c + \alpha_r)^2 + \Delta^2}} = \hat{\theta}_y$$

(20)

Then, the position errors dynamic system (14) becomes

$$\begin{cases}
\dot{x}_c = - \hat{u}_r \cos(\psi_d - \gamma_k) + \hat{v}_r \sin(\psi_d - \gamma_k) + y_c \gamma_k + \hat{\theta}_x \\
\dot{y}_c = - \frac{\hat{U}_r}{\sqrt{(y_c + \alpha_r)^2 + \Delta^2}} y_c - \hat{u}_r \sin(\psi_d - \gamma_k) - \hat{v}_r \cos(\psi_d - \gamma_k) - x_c \gamma_k + \hat{\theta}_y
\end{cases}$$

(21)

Solving for $\alpha_r$ given one feasible solution (the positive root) given by

$$\alpha_r = \frac{\hat{U}_r \frac{\hat{v}_r}{\hat{U}_v} \sqrt{1 - \frac{\hat{U}_v^2}{\hat{U}_r^2}} - \frac{\hat{U}_v}{\hat{U}_r} \sqrt{\Delta^2 \left(1 - \frac{\hat{U}_v^2}{\hat{U}_r^2}\right) + y_c^2}}{1 - \frac{\hat{U}_v^2}{\hat{U}_r^2}}$$

(22)

The condition $\left|\frac{\dot{\theta}_y}{\hat{U}_r}\right| < 1$ must be satisfied to guarantee that $\alpha_r$ is bounded.

Design the adaptive law for ocean currents parameters as follows

$$\begin{cases}
\dot{\hat{\theta}}_x = - \frac{1}{\hat{U}_1}(x_c - \hat{\theta}_x \hat{\theta}_y) \\
\dot{\hat{\theta}}_y = - \frac{1}{\hat{U}_2}(y_c - \hat{\theta}_x \hat{\theta}_y)
\end{cases}$$

(23)

Remark 5. The magnitudes of the ocean current velocities can be a far cry from USV. Therefore, if the initial conditions of $(\hat{\theta}_x, \hat{u}_r, \hat{v}_r)$ and the control parameters of the ESO and the adaptive law for ocean currents are set appropriately, $\left|\frac{\dot{\theta}_y}{\hat{U}_r}\right| < 1$ can be easily enforced.

Theorem 1. The subsystems (10) and (13), viewed as a guidance system containing position tracking errors $(x_c, y_c)$ and estimation errors of the USV and ocean current velocities $(\hat{u}_r, \hat{v}_r, \hat{\theta}_x, \hat{\theta}_y)$, are UUB under Assumptions 3 to 5.

Proof. Considering the following Lyapunov function candidate (LFC) $V_1 = \frac{1}{2} x_c^2 + \frac{1}{2} y_c^2 + \frac{1}{2} \hat{u}_r^2 + \frac{1}{2} \hat{v}_r^2 + \frac{1}{2} \hat{\theta}_x^2 + \frac{1}{2} \hat{\theta}_y^2$. Taking the derivative of $V_1$ with respect to time, we obtain

$$\dot{V}_1 = x_c \dot{x}_c + y_c \dot{y}_c + \hat{u}_r \dot{\hat{u}}_r + \hat{v}_r \dot{\hat{v}}_r + \frac{1}{\hat{U}_1} \hat{\theta}_x \dot{\hat{\theta}}_x + \frac{1}{\hat{U}_2} \hat{\theta}_y \dot{\hat{\theta}}_y$$

$$= -k_2 x_c^2 - \phi_1 y_c^2 - k_1 \hat{u}_r^2 \cos(\psi_d - \gamma_k) - \hat{u}_r \hat{\theta}_x - \hat{v}_r \hat{\theta}_y - k_1 \hat{v}_r^2 \cos(\psi_d - \gamma_k) + x_c \hat{\theta}_x + y_c \hat{\theta}_y + k_1 \hat{u}_r \hat{\theta}_x + k_1 \hat{v}_r \hat{\theta}_y - x_c \hat{\theta}_x + y_c \hat{\theta}_y + \hat{\theta}_x \dot{\hat{\theta}}_x + \hat{\theta}_y \dot{\hat{\theta}}_y$$

(24)

where $\phi_1 = \frac{\hat{U}_r}{\sqrt{(y_c + \alpha_r)^2 + \Delta^2}} > 0$. According to Lemma 3, we can obtain

$$\begin{cases}
\dot{\hat{u}}_r \leq - \frac{1}{2} \hat{u}_r^2 + \frac{1}{2} \hat{v}_r^2 \\
\dot{\hat{v}}_r \leq - \frac{1}{2} \hat{v}_r^2 + \frac{1}{2} \hat{\theta}_y^2 \\
k_1 \dot{\hat{u}}_r \leq - \frac{1}{2} \hat{u}_r^2 + \frac{1}{2} k_1^2 \hat{\theta}_x^2 \\
k_1 \dot{\hat{v}}_r \leq - \frac{1}{2} \hat{v}_r^2 + \frac{1}{2} k_1^2 \hat{\theta}_y^2 \\
k_1 \dot{\hat{\theta}}_x \leq \frac{1}{2} \hat{\theta}_x^2 - \frac{1}{2} k_1^2 \hat{\theta}_x^2 \\
k_1 \dot{\hat{\theta}}_y \leq \frac{1}{2} \hat{\theta}_y^2 - \frac{1}{2} k_1^2 \hat{\theta}_y^2
\end{cases}$$

(25)
In this section, the control laws \(\tau_u\) and \(\tau_r\) to achieve the desired velocity and heading angle are calculated.

It yields that
\[
\dot{V}_1 \leq -(k_2 - 1)x_r^2 - \dot{\phi}_1x_r^2 - (k_1\cos(\psi_d - \gamma_k) - 1)\dot{u}_r^2 - (k_1\cos(\psi_d - \gamma_k) - 1)v_r^2 \leq \frac{\vartheta_1}{2} - \frac{k_1}{2} \dot{h}_s^2 - \left(\frac{\vartheta_2}{2} \frac{k_2}{2}\right)^2 \dot{h}_s^2 + \frac{1}{2} \dot{u}_r^2 + \frac{1}{2} v_r^2 + \frac{\vartheta_1}{2} \vartheta_2 + k_2^2 \vartheta_{\max}^2 + \frac{\vartheta_2}{2} + k_1^2 \vartheta_{\max}^2 \\
\leq -2\mu_1 V_1 + C_1
\]  

(26)

where \(\mu_1 = \min\{k_2 - 1, \vartheta_1, k_1\cos(\psi_d - \gamma_k) - 1, (\vartheta_1 - k_1)\vartheta_2_1/2, (\vartheta_2 - k_1)\vartheta_2_2/2\} \) and \(C_1 = (\vartheta_1 + k_2^2)\vartheta_{\max}/2 + (\vartheta_2 + k_1^2)\vartheta_{\max}/2 + \dot{u}_r^2 + v_r^2/2\).

Thus, \(V_1\) is a monotonically decreasing function outside the range \(\omega_1 = \{V_1 \leq \frac{C_1}{2\mu_1}\} \) if \(k_2 - 1 > 0, k_1\cos(\psi_d - \gamma_k) > 1\) and gives
\[
\dot{V}_1 \leq (V_1(0) - \frac{C_1}{2\mu_1})e^{-2\mu_1 t} + C_1 \frac{1}{2\mu_1}
\]  

(27)

and it follows that the errors \(\dot{\dot{u}}_r, \ddot{\dot{v}}_r, \dot{x}_e, \dot{y}_e, \dot{\theta}_x, \) and \(\dot{\theta}_y\) are UUB from Lemma 1.

The look-ahead distance, \(\Delta\), which impacts the tracking performance in the guidance system, has been deemed time invariant by most researchers.1,12,17,26 This phenomenon leads to slow convergence of position-tracking errors. Actually, if the distance between the USV and the reference path is long, we should choose a smaller value for \(\Delta\) to make the absolute value of \(y_e\) decrease more quickly; conversely, a larger \(\Delta\) is corresponding to the close range between USV and reference path. In this context, Mu et al.15 proposed a fuzzy algorithm of \(\Delta\) according to this principle but did not consider the changing trend of \(y_e\). Therefore, an improved FLS with the inputs being \(y_e\) and \(\dot{y}_e\), the output being \(\dot{\lambda}\) is introduced to optimize the value of \(\Delta\), where \(\dot{\lambda}\) represents the gain. Then, the look-ahead distance \(\Delta\) can be expressed as \(\Delta = \Delta_{\min} + \dot{\lambda}(\Delta_{\max} - \Delta_{\min})\). \(y_e, \dot{y}_e, \) and \(\dot{\lambda}\) are equally divided into five parts. The fuzzy rules are given in Table 1, and the fuzzy surface of \(y_e, \dot{y}_e, \) and \(\dot{\lambda}\) is shown in Figure 2.

As shown in Table 1, the EILOS guidance law is a part of the whole path following the scheme of USV. We can employ the guidance law together with the actuator control system, which will be designed later using the sliding mode technique, FLS, and an auxiliary dynamic system.

### Control subsystem design

In this section, the control laws \(\tau_u\) and \(\tau_r\) to achieve the desired velocity and heading angle are calculated.

**Table 1. Fuzzy rules of \(\Delta\)**

| \(\dot{y}_e\) | NB | NS | Z | PS | PB |
|-------------|----|----|---|----|----|
| y_e         |    |    |   |    |    |
| NB          | VS | VS | VS| VS | S  |
| NS          | S  | S  | M | B  | B  |
| Z           | B  | VB | VB| VB | B  |
| PS          | B  | B  | M | S  | S  |
| PB          | S  | VS | VS| VS | VS |

**Figure 2. The fuzzy surface.**

**Velocity tracking control**

\(u_d\) denotes the desired absolute surge velocity, as shown in Figure 3. To facilitate calculation, we generally choose the desired relative surge velocity as our control objective. The rationality is analyzed in Remark 2. For simplicity, we assume \(u_r = \dot{u}_r\) in this section. Define \(u_c = \dot{u}_r - u_{dr}\) and the sliding surface \(s_1 = u_c + c_1 \int_0^t u_{dr} d\tau\), where \(c_1\) is a positive parameter to be designed. The derivative of \(s_1\) is
\[
\dot{s}_1 = H_u(v_r, r) - \frac{d_1}{m_1} u_c + \tau_u + \vartheta_{\dot{u}_d} + c_1 u_e
\]

(28)

Subsequently, by virtue of FLS to approximate the external model uncertainties \(\tilde{g}_u = \tilde{\theta}_u \xi_u(s_1)\).

To solve the problem of input saturation, an auxiliary dynamic system is proposed as
\[
\dot{\sigma}_u = \begin{cases} 
-k_{\sigma u} \sigma_u - \frac{|s_1|\Delta \tau_u + 0.5\kappa^2 \Delta \tau_u^2}{\sigma_u} + \kappa \Delta \tau_{\dot{u}_d}, |\sigma_u| \geq \sigma_k \\
0, |\sigma_u| < \sigma_k
\end{cases}
\]

(29)

where \(k_{\sigma u}, \kappa,\) and \(\sigma_k\) are positive constants, and \(\Delta \tau_u = \tau_u - \tau_{u0}\). The corresponding nominal surge control law is proposed as
\[ \tau_{u0} = -\hat{\theta}_e \xi_u(s_1) + \dot{u}_d - \eta_1 \text{sgn}(s_1) - c_1 u_e - k_u s_1 + k_o \sigma_u \]

where \( \eta_1 \) is the upper bound of \( \delta_u \) and is unknown. In addition, the sign function \( \text{sgn}(s_1) \) will cause system input chattering. Therefore, we define that \( \hat{h}_1 = \hat{\theta}_h \xi_b(s_1) \) represents the approximation to \( \eta_1 \text{sgn}(s_1) \). The control law can be rewritten as

\[ \tau_{u0} = -\hat{\theta}_e \xi_u(s_1) + \dot{u}_d - \hat{h}_1 - c_1 u_e - k_u s_1 + k_o \sigma_u \]  

(31)

The update laws of FLS are proposed as follows

\[
\begin{align*}
\dot{\hat{\theta}}_e &= \gamma_1 (s_1 \xi_u(s_1) - \rho_1 \hat{\theta}_e) \\
\dot{\hat{\theta}}_{h1} &= p_1 (s_1 \xi_b(s_1) - q_d \hat{\theta}_{h1})
\end{align*}
\]

(32)

where \( \gamma_1, \rho_1, p_1 \) and \( q_d \) are positive parameters.

**Theorem 2.** All of the errors of the USV path-following velocity control system are UUB with the control law (31) and the update law (32).

**Proof.** Assign the following LFC \( V_u = \frac{1}{2} \xi_u^T \dot{\xi}_u + \frac{1}{2\rho_1} \hat{\theta}_e^T \dot{\hat{\theta}}_e + \frac{1}{2p_1} \hat{\theta}_{h1}^T \dot{\hat{\theta}}_{h1} + \frac{1}{2} \sigma_u^2 \), where \( \dot{\hat{\theta}}_e = \theta_e - \hat{\theta}_e \) and \( \hat{\theta}_{h1} = \theta_{h1} - \hat{\theta}_{h1} \).

When \( |\sigma_u| \geq \sigma_k \), the time derivative of \( V_u \) is

\[
\dot{V}_u = s_1 (\hat{\theta}_e^T \xi_u(s_1) + \dot{e}_u - k_u s_1 - \hat{\theta}_{h1} \xi_b(s_1) + \hat{\theta}_{h1} \xi_b(s_1))
\]

\[
- \theta_{h1}^T \xi_b(s_1) + \dot{e}_u + k_o \sigma_u + \Delta \tau_u + \sigma_u (-k_o \sigma_u)
\]

\[
- \frac{|s_1 \Delta \tau_u + 0.5 \kappa^2 \Delta \tau_u^2|}{\sigma_u} + \frac{1}{\gamma_1} \hat{\theta}_e^T \dot{\hat{\theta}}_e
\]

\[
- \frac{1}{p_1} \hat{\theta}_{h1}^T \dot{\hat{\theta}}_{h1}
\]

(33)

In view of equation (32) and Lemma 3, we have

\[
\dot{V}_u \leq -\left( k_u - \frac{1}{2} k_o \right) s_1^2 - \left( k_u - \frac{k_o}{2} \right) \frac{1}{2} \sigma_u^2
\]

\[
- \rho_1 \hat{\theta}_e^T \dot{\hat{\theta}}_e - q_u \hat{\theta}_{h1}^T \dot{\hat{\theta}}_{h1} + s_1 \dot{e}_u - \eta_1 |s_1| + \frac{1}{2} (\varepsilon_u^2 + \varepsilon_{h1}^2)
\]

\[
+ \frac{\rho_1}{2} \hat{\theta}_{h1}^T \dot{\hat{\theta}}_{h1} + \frac{\rho_1}{2} \hat{\theta}_{h1}^T \dot{\hat{\theta}}_{h1} + \frac{k_o}{2} \sigma_u^2
\]

\[
+ s_1 \dot{e}_u - \eta_1 |s_1| + \frac{1}{2} \Delta \tau_u^2 + \frac{k_o}{2} \sigma_u^2
\]

\[
\leq -2 \mu_2 V_u + C_{u2}
\]

(34)
where \( \mu_{u_2} = \min \{ k_u - \frac{1}{2} - \frac{1}{2} k_{r_0} \beta_{\sigma}^2 + \frac{1}{2} \sigma_{\tau}^2 \} \) and \( C_{u_2} = \frac{1}{2} (\varepsilon_2^2 + \varepsilon_3^2) + \frac{1}{2} \theta_{\max}^2 \theta_{\max} + \frac{1}{2} \theta_{\max}^2 \theta_{\max} + \frac{1}{2} \Delta \tau_r^2 + \frac{1}{2} k_{r_0} \sigma_{\tau}^2. \)

Synthesizing equations (34) and (35), we have

\[
\dot{V}_u \leq -2 \mu_u V_u + C_u
\]

where \( \mu_u = \min \{ \mu_{u_1}, \mu_{u_2} \} \) and \( C_u = \max \{ C_{u_1}, C_{u_2} \}. \)

Thus, \( V_u \) in a monotone decreasing function out of the range \( \omega_2 = \{ V_u \leq \frac{C_u}{2 \mu_u} \} \) and that gives

\[
V_u \leq (V_u(0) - \frac{C_u}{2 \mu_u}) e^{-2 \mu_u t} + \frac{C_u}{2 \mu_u}
\]

It follows that all of the errors signals of the velocity tracking subsystem are UUB from Lemma 1, and the sub-system is stable. Without the assumption \( u_t = \bar{u}_t \), the stability of the velocity tracking system can also be guaranteed (see the literature65).

**Attitude tracking control**

Define attitude tracking error \( \phi = \psi - \psi_d \) and the sliding surface \( s_2 = c_2 \phi + \dot{\phi} \), where \( c_2 \) is a positive constant. Differentiating both sides of \( s_2 \) with respect to time results in

\[
s_2 = c_2 \dot{\phi} - \dot{\psi}_d + H_r(u_t, v_r, r) + \tau_r + \delta_r
\]

Similar to the last subsection, the auxiliary system is given by

\[
\dot{\sigma}_r = \begin{cases} 
- k_{\sigma} \sigma_r - |s_2 \Delta \tau_r + 0.5 \kappa^2 \Delta \tau_r^2|/\sigma_r + \kappa \Delta \tau_r, |\sigma_r| \geq \sigma_k \\
0, |\sigma_r| \leq \sigma_k 
\end{cases}
\]

where \( \Delta \tau_r = \tau_r - \tau_{r_0} \) and \( k_{\sigma} \) is a positive parameter. In view of the unascertained bound of \( \delta_r \), the nominal heading control law \( \tau_{r_0} \) is designed as

\[
\tau_{r_0} = -\dot{\theta}_r \xi_u(s_2) - c_2 \dot{\phi} + \dot{\psi}_d - \hat{h}_2 - k_r s_2 - \psi_e + k_{r_0} \sigma_r
\]

where \( \hat{g}_r = \dot{\theta}_r \xi_u(s_2) \) represents the approximation of \( g_r \), and \( \hat{h}_2 = \hat{h}_2 \xi_b(s_2) \). The update laws are presented as

\[
\begin{align*}
\dot{\theta}_r &= \gamma_2 (s_2 \xi_u(s_2) - q_r \dot{\theta}_r) \\
\dot{\hat{h}}_2 &= p_2 (s_2 \xi_b(s_2) - q_r \hat{h}_2)
\end{align*}
\]

where \( \gamma_2, q_r, p_2 \), and \( q_r \) are positive parameters.

**Theorem 3.** All of the tracking errors of the USV attitude control system are UUB with the control law (40) and the update law (41).

**Proof.** Considering the following LFC

\[
V_r = \frac{1}{2} \dot{\psi}_e^2 + \frac{1}{2} s_2^2 + \frac{1}{2} \dot{\sigma}_r^2 + \frac{1}{2} \dot{\hat{h}}_2^2 + \frac{1}{2} \Delta \tau_r^2
\]

When \( |\sigma_r| \geq \sigma_k \), the derivative of \( V_r \) yields

\[
\dot{V}_r = -c_2 \dot{\psi}_e^2 + s_2^2 \left( \dot{\theta}_r \xi_u(s_2) + \dot{\psi}_e - k_r s - \psi_e + \Delta \tau_r \\
+ \hat{h}_2 \xi_b(s_2) \right) + s_2^2 \left( \dot{\theta}_r \xi_u(s_2) - \dot{\psi}_e + k_r \sigma_r \right)
\]

\[
+ \frac{1}{2} \dot{\sigma}_r^2 + \frac{1}{2} \dot{\hat{h}}_2^2 + \frac{1}{2} \Delta \tau_r^2
\]

then, substituting equation (41) into (42) and using Lemma 3 yields

\[
\dot{V}_r \leq \begin{cases} 
-c_2 \dot{\psi}_e^2 - \frac{1}{2} \dot{\theta}_r \xi_u(s_2) + s_2^2 \left( \dot{\theta}_r \xi_u(s_2) + \dot{\psi}_e - k_r s - \psi_e + \Delta \tau_r \\
+ \hat{h}_2 \xi_b(s_2) \right) + s_2^2 \left( \dot{\theta}_r \xi_u(s_2) - \dot{\psi}_e + k_r \sigma_r \right)
\end{cases}
\]

Substituting equation (41) into (42) and using Lemma 3 yields

\[
\dot{V}_r \leq -k_r \left( r_k - \frac{1}{2} k_{r_0} \right) s_2^2 - \frac{1}{2} \left( k_{\sigma r} - \frac{1}{2} k_{r_0} \right) \sigma_r^2
\]

\[
- \frac{1}{2} q_r \xi_u(s_2) - \frac{1}{2} \xi_b(s_2) + \frac{1}{2} \xi_b(s_2) + \frac{1}{2} \xi_b(s_2)
\]

\[
+ \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2)
\]

\[
+ \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2)
\]

\[
+ \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2)
\]

\[
- \eta_3 |s_2| \leq -2 \mu_r V_r + C_r
\]

where \( \mu_r = \min \{ c_2, k_r - \frac{1}{2} k_{r_0}, \frac{1}{2} k_{r_0} \} \) and \( C_r = \frac{1}{2} \Delta \tau_r^2 + \frac{1}{2} k_r \sigma_{\tau}^2 + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2) + \frac{1}{2} \xi_u(s_2)
\]

From the above, we could reach

\[
\dot{V}_r \leq -2 \mu_r V_r + C_r
\]
where \( \mu_r = \min\{\mu_1, \mu_2\} \) and \( C_r = \max\{C_1, C_2\} \). Thus, \( V_r \) is a monotone decreasing function out of the range \( \omega_3 = \{V_r \leq \frac{C_r}{2\mu_r}\} \), and then, we have

\[
V_r \leq \left(V_r(0) - \frac{C_r}{2\mu_r}\right) e^{-2\mu_r t} + \frac{C_r}{2\mu_r} \quad (46)
\]

We can conclude that all of the error signals of the attitude tracking subsystem are UUB from Lemma 1, and the subsystem is stable.

Closed-loop system stability analysis

Theorem 4. Define the tracking errors \( \zeta_e = [X_e, Y_e, Z_e]^T \), where \( X_e = [x_e, y_e]^T \), \( Y_e = [s_1, \psi_e, s_2]^T \), and \( Z_e = [\dot{u}_r, v_r, \theta_1, \theta_2, \theta_3, \theta_4, \theta_{01}, \theta_{02}]^T \), in the presence of model uncertainties, ocean currents, and other unknown disturbances. If the mathematical model of USV is defined as equation (5), the guidance law is calculated by equation (15), the controllers are designed by equations (31) and (40), based on Assumptions 1 to 5, we have the following conclusions:

1. All of the tracking errors and estimation errors of the closed-loop system are UUB, and the system is stable.
2. The sway velocity is passively bounded.

Proof. For the closed-loop system of USV.

1. Assign the complete LFC \( V = V_1 + V_u + V_r \). The derivative of \( V \) with respect to time satisfies

\[
\dot{V} \leq \mu_1 V_1 + \mu_2 V_u + \mu_3 V_r + C_1 + C_2 + C_r \leq \mu V + C_r
\]

where \( \mu = \min\{\mu_1, \mu_2, \mu_3\} \) and \( C = C_1 + C_2 + C_r \) such that

\[
V \leq (V(0) - \frac{C}{2\mu}) e^{-2\mu t} + \frac{C}{2\mu} \quad (47)
\]

It follows that all of the errors of the closed-loop system are UUB, thus, the USV path following system is stable. It is indicated that \( \zeta_e \) ultimately converges to the range \( \{\zeta_e \in \mathbb{R}^5, ||\zeta_e|| \leq C_0\} \). We can see from equation (50) that the ultimate compact set can be adjusted by tuning control parameters \( k_1, k_2, k_{u0}, k_{r0}, k_{r1} \), and so on.

2. For the sway velocity \( v_r \), consider a Lyapunov function \( V_r = \frac{1}{2}v_r^2 \), differentiating it with respect to time, we have

\[
\dot{V}_r = F(u_r)v_r^2 + E(u_r)v_r + \delta_v v_r \leq F(u_r)v_r^2 + E(u_r)v_r + \delta_v ||v_r||
\]

where \( F(u_r) < 0 \) and \( E(u_r)v_r + \delta_v \) are bounded.\(^{19}\)

Therefore, \( v_r \) is bounded referring to Chapter 4.8 of the literature.\(^{66}\)

| Table 2. Control parameters of USV simulation. |
|-----------------------------------------------|
| Notation Value Notation Value Notation Value |
|-----------------------------------------------|
| \( k_1 \) | 1.5 | \( \kappa \) | 1 | \( p_1 \) | 5 |
| \( k_2 \) | 3 | \( \sigma_k \) | 0.001 | \( p_2 \) | 5 |
| \( f_1 \) | 1 | \( k_{u0} \) | 4 | \( q_u \) | 0.1 |
| \( f_2 \) | 1 | \( k_{r0} \) | 2 | \( q_r \) | 0.2 |
| \( \phi_1 \) | 0.1 | \( \gamma_1 \) | 10 | \( c_1 \) | 1 |
| \( \phi_2 \) | 0.05 | \( \gamma_2 \) | 30 | \( c_2 \) | 1 |
| \( k_{ru} \) | 1.5 | \( \rho_u \) | 0.2 | \( k_u \) | 1 |
| \( k_{rr} \) | 2.5 | \( \rho_r \) | 0.3 | \( k_r \) | 2 |

USV: unmanned surface vehicle.

Simulation studies

To illustrate the availability of the proposed path following scheme, some simulation studies are conducted in this section with USV, whose parameters can be found in the literature.\(^{19}\) The look-ahead distance is defined as \( d_{\text{min}} = 6, d_{\text{max}} = 12 \). The absolute value of \( (v_u, \tau_r) \) is restricted to \( (2 \text{ N}, 1.5 \text{ Nm}) \).\(^{67}\) The control parameters are given in Table 2. The time-varying ocean currents within the inertial frame are set as \( V_s = 0.03 \sin(t/20) \text{ m/s} \) and \( V_r = 0.02 \sin(t/20) \text{ m/s} \). The other disturbances are assumed to be \([v_u, \delta_v, \delta_r]^T = [0.15 \sin(0.1t), 0.1 \sin(0.1t), 0.15 \sin(0.1t)]^T\) to emphasize the superiority of the method proposed in this article, we take the PLOS\(^{12}\) and CLOS\(^{18}\) methods as position comparisons and the ELOS method as USV velocity estimation comparisons, where ELOS represents the proposed EILOS scheme without dealing with the ocean currents. Note that the ocean currents were not taken into account in the literature.\(^{12}\) therefore, we consider the same ocean currents as in the EILOS scheme and employ the same adaptive strategy to treat them. Specifically, we consider the USV’s relative resultant velocities as measurable states in the PLOS and CLOS schemes because they calculate the sideslip angles instead of USV velocities. In addition, the backstepping method is contrasted for \( \psi_e \) and \( u_{ed} \) in the control part, where \( u_{ed} = u_{rd} - u_r \).

The velocity \( u_{ed} \) is set to 0.6 m/s, and the initial USV states are given by \([x(0), y(0), u_r(0), v_r(0), r(0), \psi(0)] = [0, 15, 0.5, 0.01, 0, 0]\). The initial values of \( \dot{u}_r \) and \( \dot{v}_r \) are \((0.3, 0.15)\). The reference path is

\[
\begin{aligned}
x_h(t) &= 30 \sin(0.1t) + \zeta \\
y_h &= 3\zeta
\end{aligned}
\]

Results are depicted in Figures 4 to 14. Figures 4 and 5 show that the USV can follow the reference path, and the proposed EILOS scheme performs best because it converges to the reference path in minimal time. In addition, the PLOS and CLOS schemes exhibit obvious fluctuations in cross-tracking error, \( v_r \), at the steady period. Figure 7 indicates that the unmeasured surge and sway velocities can be precisely and quickly extracted using the proposed
ESO. Furthermore, it is reasonable to compare the relative USV resultant velocities as measurable states in the PLOS and CLOS schemes. If ocean currents are not compensated in the ELOS scheme, as shown in Figure 8, then obvious fluctuations exist in the velocity estimate errors. Attitude and surge velocity tracking errors are displayed in Figure 9,
in which the SMC and backstepping methods can each cause \( (\psi, u_r) \) to converge to \( (\psi_d, u_{rd}) \), and the SMC method exhibits a faster response. Figure 10 illustrates that the virtual control input \( \alpha_r \) canceling the drift term is bound, and the bound is small. Figure 11 shows the value of \( \Delta \), as the aforementioned theory, \( \Delta \) increases as \( y_e \) decreases.

Ocean current parameters and their estimates appear in Figure 12, in which the estimates \( (\hat{\theta}_e, \hat{\theta}_r) \) can be identified by the adaptive method. Figure 13 describes the approximation errors of model uncertainties (i.e. \( g_u \) and \( g_r \)), and the FLS has an excellent approximation effect. Last, the performance of input signals is shown in Figure 14, revealing that the control inputs are within the allowable range once auxiliary dynamic systems are added. In addition, a comparison analysis between the control input with sign functions and with the estimations of sign functions \( \text{esgn} \) by FLS is also depicted in Figure 14, in which we can see that the problem of chattering is solved by FLS and control inputs satisfy engineering applications.

**Conclusions**

In this article, an ISMC is proposed based on a novel ESO, an ILOS guidance law, an auxiliary dynamic system, and the FLS. The salient features of the proposed algorithm
are as follows. First, the unmeasured velocity can be precisely estimated by ESO. Second, the ILOS guidance law is able to provide the reference heading angle as well as estimating the ocean current velocities simultaneously. Third, the ISMC with FLS is able to force the state tracking errors converge to a neighborhood of zero. It is verified that the closed-loop system of the USV is UUB. The simulation results show the availability and superiority of the EIAFSM scheme.

Many problems warrant closer investigation, and some methods must be enhanced for USV path following (e.g. neglecting the hysteresis characteristic of the actuator and the lack of an accurate adaptive method for fast time-varying ocean currents). Therefore, disturbance observers are powerful tools that will be the focus of our subsequent work.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by “the National Natural Science Foundation of China” [Grant numbers 51879027, 51579024, and 51809028] and “the Fundamental Research Funds for the Central Universities” [Grant numbers 3132019318 and 3132019109].

ORCID iD
Mingcong Li https://orcid.org/0000-0002-5687-8870

References
1. Fossen TI, Pettersen KY, and Galeazzi R. Line-of-sight path following for Dubins paths with adaptive sideslip compensation of drift forces. IEEE Trans Control Syst Technol 2015; 23(2): 820–827.
2. Zhang J, Yu S, and Yan Y. Fixed-time extended state observer-based trajectory tracking and point stabilization control for marine surface vessels with uncertainties and disturbances. Ocean Eng 2019; 186(15): 106109.
3. Shojaei K. Neural adaptive robust control of underactuated marine surface vehicles with input saturation. Appl Ocean Res 2015; 53: 267–278.
4. Sun T, Zhang J, and Pan Y. Active disturbance rejection control of surface vessels using composite error updated extended state observer. Asian J Control 2017; 19(5): 1802–1811.
5. Miao B and Li T. A novel neural network-based adaptive control for a class of uncertain nonlinear systems in strict feedback form. Nonlin Dynam 2015; 79(2): 1005–1013.
6. He B, Wang S, and Liu Y. Underactuated robotics: a review. Int J Adv Rob Syst 2019; 16(4): 1–29.
7. He B, Cao X, and Gu Z. Kinematics of underactuated robotics for product carbon footprint. J Clean Prod 2020; 257: 120491.
8. He B, Liu Y, Zeng L, et al. Product carbon footprint across sustainable supply chain. J Clean Prod 2019; 241: 118320.
9. Zhang A, Lai X, Wu M, et al. Nonlinear stabilizing control for a class of underactuated mechanical systems with multi degree of freedoms. Nonlin Dynam 2017; 89: 2241–2253.
10. Chen Y and Tian Y. Formation tracking and attitude synchronization control of underactuated ships along closed orbits. Int J Robust Nonlin 2015; 25: 3023–3044.
11. Healey A and Lienard D. Multivariable sliding mode control for autonomous diving and steering of unmanned underwater vehicles. IEEE J Ocean Eng 1993; 18(3): 327–339.
12. Liu L, Wang D, Peng Z, et al. Predictor-based los guidance law for path following of underactuated marine surface vehicles with sideslip compensation. Ocean Eng 2016; 124: 340–348.
13. Kelasidi E, Liljebak P, Pettersen KY, et al. Integral line-of-sight guidance for path following control of underwater snake robots: theory and experiments. IEEE Trans Rob 2017; 33(3): 1–19.
14. Bevly DM, Sheridan R, and Pettersen KY. Integrating INS sensors with GPS velocity measurements for continuous estimation of vehicle sideslip and tire cornering stiffness. In: American automatic control conference, AACC, Washington, USA, 25–27 June 2001, pp. 483–493.
15. Mu D, Wang G, Fan Y, et al. Adaptive LOS path following for a podded propulsion unmanned surface vehicle with uncertainty of model and actuator saturation. Appl Sci 2017; 7(12): 1232–1251.
16. Liu L, Wang D, and Peng Z. ESO-based line-of-sight guidance law for path following of underactuated marine surface vehicles with exact sideslip compensation. IEEE J Ocean Eng 2017; 42(2): 477–487.
17. Wang N, Sun Z, Yin J, et al. Finite-time observer based guidance and control of underactuated surface vehicles with unknown sideslip angles and disturbances. IEEE Access 2018; 6: 14059–14070.
18. Miao J, Wang S, Tomovic M, et al. Compound line-of-sight nonlinear path following control of underactuated marine vehicles exposed to wind, waves, and ocean currents. Nonlin Dynam 2017; 89(4): 2441–2459.
19. Zheng Z and Sun L. Path following control for marine surface vessel with uncertainties and input saturation. Neurocomputing 2016; 177(C): 158–167.
20. Sussmann HJ. A general theorem on local controllability. SIAM J Control Optim 1987; 25(1): 158–194.
21. Bianchini RM and Stefani G. Controllability along a trajectory: a variational approach. SIAM J Control Optim 1993; 31(4): 900–927.
22. Hermann R and Krener A. Nonlinear controllability and observability. IEEE Trans Autom Control 1977; 22(5): 728–740.
23. Arai H, Tanie K, and Tachi S. Dynamic control of a manipulator with passive joints in operational space. IEEE Trans Robot Autom 1993; 9: 85–93.
24. Nakamura Y, Suzuki T, and Koinuma M. Nonlinear behavior and control of a nonholonomic free-joint manipulator. IEEE Trans Robot Autom 1997; 13: 853–862.

25. Levine WS. Open-loop control using oscillatory inputs. In: The control systems handbook. Boca Raton: Chemical Rubber Company Press, 2010, pp. 1191–1211.

26. Yu Y, Guo C, and Yu H. Finite-time predictor line-of-sight based adaptive neural network path following for unmanned surface vessels with unknown dynamics and input saturation. Int J Adv Rob Syst 2018; 15(6): 172988141881469.

27. Do KD, Jiang ZP, and Pan J. Robust adaptive path following of underactuated ships. Automatica 2004; 40(6): 929–944.

28. Wang R, Wang S, Wang Y, et al. Path following for a biomimetic underwater vehicle based on ADRC. In: IEEE international conference on robotics and automation, ICRA, Singapore, 29 May–3 June 2017, pp. 3519–3524. IEEE.

29. Liu Y, Zhu J, Li R, et al. Omni-directional mobile robot controller based on trajectory linearization. Rob Autom Syst 2008; 56(5): 461–479.

30. Ashrafiuon H, Muske KR, McNinch LC, et al. Sliding mode tracking control of surface vessels. IEEE Trans Ind Electron 2008; 55(11): 4004–4012.

31. Ouyang P, Acob J, and Pano V. PD with sliding mode control for trajectory tracking of robotic system. Rob Comput Integr Manuf 2014; 30(2): 189–200.

32. Matveev A, Wang C, and Savkin A. Real-time navigation of mobile robots in problems of border patrolling and avoiding collisions with moving and deforming obstacles. Rob Autom Syst 2012; 60(6): 769–788.

33. Bessa W, Dutra M, and Kreuzer E. Depth control of remotely operated underwater vehicles using an adaptive fuzzy sliding mode controller. Rob Autom Syst 2008; 56(8): 670–677.

34. Veysi M, Soltanpour M, and Khooban M. A novel self-adaptive modified bat fuzzy sliding mode control of robot manipulator in presence of uncertainties in task space. Robotica 2015; 33(10): 2045–2064.

35. Chen Y, Yan Y, Wang K, et al. An adaptive fuzzy sliding mode controller for the depth control of an underactuated underwater vehicle. Int J Adv Rob Syst 2019; 16(2): 1–10.

36. Nikkhah M, Ashrafiuon H, and Fahimi F. Robust control of underactuated bipeds using sliding modes. Robotica 2007; 25: 367–374.

37. Ashrafiuon H and Erwin RS. Shape change maneuvers for attitude control of underactuated satellites. In: Proceedings of the 2005 american control conference, Portland, OR, USA, 8–10 June 2005, pp. 895–900. IEEE.

38. Almutairi NB and Zribi M. Sliding mode control of a three-dimensional overhead crane. J Vib Control 2009; 15: 1679–1730.

39. Van M, Kang H J, Suh Y S, et al. Output feedback tracking control of uncertain robot manipulators via higher-order sliding-mode observer and fuzzy compensator. J Mech Sci Technol 2013; 27(8): 2487–2496.

40. Liang X, Wan L, Blake J, et al. Path following of an underactuated AUV based on fuzzy backstepping sliding mode control. Int J Adv Rob Syst 2016; 13(3): 1.

41. Meng Q, Zhang T, Gao X, et al. Adaptive sliding mode fault-tolerant control of the uncertain Stewart platform based on offline multibody dynamics. IEEE ASME Trans Mechatron 2014; 19(3): 882–894.

42. Yu H, Guo C, and Yan Z. Globally finite-time stable three-dimensional trajectory-tracking control of underactuated UUVs. Ocean Eng 2019; 189: 106329.

43. Jin M, Jin Y, Hun P, et al. High-accuracy tracking control of robot manipulators using time delay estimation and terminal sliding mode. Int J Adv Rob Syst 2011; 8(4): 65–78.

44. Jia H, Zhang L, Cheng X, et al. Three-dimensional path following control for an underactuated UUV based on nonlinear iterative sliding mode. Zidonghua Xuebao Acta Auto Sin 2012; 38(2): 308–314.

45. Cheng C, Liu S, and Wu H. A transformed Lure problem for sliding mode control and chattering reduction. IEEE Trans Autom Control 1999; 44(3): 563–568.

46. Su W, Drakunov S, Ozguner U., et al. Sliding mode with chattering reduction in sampled data systems. In: IEEE conference on decision and control, San Antonio, TX, USA, 15–17 December 1993, pp. 2452–2457. Piscataway, NJ, United States: IEEE.

47. Yoo B and Ham W. Adaptive fuzzy sliding mode control of nonlinear system. IEEE Trans Fuzzy Syst 1998; 6(2): 315–321.

48. Konno Y and Hashimoto H. Design of sliding mode dynamics in the frequency domain. Adv Rob 1992; 7(6): 587–598.

49. Khorashadizadeh S and Fateh MM. Uncertainty estimation in robust tracking control of robot manipulators using the Fourier series expansion. Robotica 2017; 35(2): 310–336.

50. Coelho P and Nunes U. Path-following control of mobile robots in presence of uncertainties. IEEE Trans Rob 2005; 21(2): 252–261.

51. Liu H and Zhang T. Neural network-based robust finite-time control for robotic manipulators considering actuator dynamics. Rob Comput Integr Manuf 2013; 29(2): 301–308.

52. Xiang X, Yu C, and Zhang Q. Robust fuzzy 3D path following for autonomous underwater vehicle subject to uncertainties. Comput Oper Res 2017; 84: 165–177.

53. Tao Y, Zheng J, and Lin Y. A sliding mode control-based on a RBF neural network for deburring industry robotic systems. Int J Adv Rob Syst 2016; 13(1): 1.

54. Wang N, Sun Z, Zheng Z, et al. Finite-time sideslip observer-based adaptive fuzzy path-following control of underactuated marine vehicles with time-varying large sideslip. Int J Fuzzy Syst 2018; 20(6): 1767–1778.

55. Wang N and Er MJ. Self-construction of adaptive robust fuzzy neural tracking control of surface vehicles with uncertainties and unknown disturbances. IEEE Trans Control Syst Technol 2015; 23(3): 991–1002.

56. Chen M, Ge SS, and Ren B. Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints. Automatica 2011; 47(3): 452–465.

57. Su Y and Swevers J. Finite-time tracking control for robot manipulators with actuator saturation. Rob Comput Integr Manuf 2014; 30(2): 91–98.
58. Loria A and Panteley E. Cascaded nonlinear time-varying systems: analysis and design. *Lect Notes Control Inf Sci* 2005; 311: 23–64.

59. Ge SS and Wang C. Adaptive neural control of uncertain MIMO nonlinear systems. *IEEE Trans Neural Netw* 2004; 15(3): 674–692.

60. Hardy GH, Littlewood JE, and Polya G. *Inequalities*. Reprint of the 1952 edition. Cambridge: Cambridge Mathematical Library, 1988.

61. Lekkas AM and Fossen TI. Integral LOS path following for curved paths based on a monotone cubic hermite spline parametrization. *IEEE Trans Control Syst Technol* 2014; 22(6): 2287–2301.

62. Fossen TI. *Handbook of marine craft hydrodynamics and motion control*. Trondheim: John Wiley & Sons, 2011.

63. Caoyang Y, Chunhu L, Lian L, et al. ELOS-based path following control for underactuated surface vehicles with actuator dynamics. *Ocean Eng* 2019; 187(1): 106139.

64. Fossen TI and Lekkas AM. Direct and indirect adaptive integral line-of-sight path-following controllers for marine craft exposed to ocean currents. *Int J Adapt Control Signal Process* 2017; 31(4): 445–463.

65. Wang Y, Tong H, and Wang C. High-gain observer-based line-of-sight guidance for adaptive neural path following control of underactuated marine surface vessels. *IEEE Access* 2019; 7: 26088–26101.

66. Khalil HK. Boundedness and ultimate boundedness. In: Khalil HK (ed) *Nonlinear systems*. 3rd ed. New Jersey: Prentice Hall, 2002, pp. 168–174.

67. Fredriksen E and Pettersen KY. Global-exponential waypoint maneuvering of ships: theory and experiments. *Automatica* 2006; 42(4): 677–687.

**Appendix**

\[
H_u(v_r, r) = \frac{(m_{22}r + m_{23}r)r}{m_{11}}
\]

\[
H_r(u_r, v_r, r) = \frac{m_{23}d_{22} - m_{22}(d_{32} + (m_{22} - m_{11})u_r)}{m_{22}m_{33} - m_{23}^2} v_r
\]

\[
+ \frac{m_{23}(d_{23} + m_{11}u_r) - m_{22}(d_{33} + m_{23}u_r)}{m_{22}m_{33} - m_{23}^2} r
\]

\[
E(u_r) = \frac{m_{23}^2 - m_{11}m_{33}}{m_{22}m_{33} - m_{23}^2} u_r + \frac{d_{33}m_{23} - d_{23}m_{33}}{m_{22}m_{33} - m_{23}^2} u_r
\]

\[
F(u_r) = \frac{(m_{22} - m_{11})m_{23}}{m_{22}m_{33} - m_{23}^2} u_r - \frac{d_{22}m_{33} - d_{32}m_{23}}{m_{22}m_{33} - m_{23}^2} u_r
\]