Price Impact of Order Flow Imbalance:
Multi-level, Cross-sectional and Forecasting

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December 2021

Abstract
We quantify the impact of order flow imbalances (OFIs) on price movements in equity markets. First, we examine the contemporaneous price impact of multi-level OFIs and reveal their additional explanatory power. We propose an integrated OFI by applying PCA to OFIs across different levels, leading to superior results in both in-sample and out-of-sample. Second, we introduce a method to examine the existence of cross-impact. Unlike previous works, we leverage LASSO to highlight the sparsity of the cross-impact terms. Compared with the best-level OFI price-impact model, the cross-impact model with the entire cross-sectional best-level OFIs provides additional explanatory power. However, once multi-level OFIs have been incorporated, cross-impact terms cannot provide additional explanatory power. Last, we apply price-impact and cross-impact models to predict future returns, and provide evidence that cross-sectional OFIs significantly increase both in-sample and out-of-sample $R^2$.

Keywords: Order flow imbalance; Price impact; Cross impact; LASSO; PCA; Return prediction.

JEL Codes: C31, C52, C53, G10, G17

1 Introduction
Nowadays, more than half of the world’s stock exchanges, such as the New York Stock Exchange, Nasdaq Stock Exchange, London Stock Exchange, Hong Kong Stock Exchange and etc., use the limit order book (LOB) mechanism [25] to record the outstanding limit orders. A limit order is a type of order to buy or sell a security at a specific price or better, as shown in Figure 1. For example, a buy limit order is an order to buy at a preset price or lower, while a sell limit order is an order to sell a security at a preset price or higher. Owing to the availability of high-frequency LOB data, there has been an increasing number of investigations concerning various aspects of LOB data, including statistical properties of limit order books [8, 46].
Motivated by optimal execution and market-making strategies, Moallemi and Yuan proposed a model of queue position valuation in a limit order book [43]. Yet another example of a valuable use case of such high-frequency data was recently put forth by Rahimikia and Poon, who examined the performance of machine learning (ML) models in realized volatility forecasting using big data sets such as LOBSTER limit order book and news sentiment variables [47], and reported superior performance of such ML models over traditional volatility forecasting models.

Order Flow Imbalance (OFI) is the net order flow resulting from the imbalance between the supply and demand sides [17, 11, 56]. Most studies in the field of quantitative finance (e.g. [12]) have only focused on analyzing trade imbalances, that arise from the arrival of market orders. To date, several studies [17, 56, 11, 53] have used LOB data to examine the price impact of OFIs. For example, the works of [56, 17] examine the effect of multi-level OFIs. A number of studies [11, 53] explore the relationship between cross-sectional OFIs and returns. To the best of our knowledge, this paper is the first study to systematically examine effects of order flow imbalances from multiple perspectives, including price impact on contemporaneous returns, cross-impact on contemporaneous returns, and efficacy in forecasting future returns. Throughout this paper, contemporaneous returns denote the returns that materialize over the same bucket of time as the OFI.

**Price impact - on contemporaneous returns.** Our first study concerns the price impact model using multi-level OFIs, as opposed to only using the best-level OFIs. Our empirical evidence shows that, as OFIs at deeper order book levels are included as predictors, the explained percentage of variance steadily increases in the in-sample tests. Nonetheless, the performance on out-of-sample data gets worse after a certain level of OFIs. We further study the relationships across multi-level OFIs, and observe strong multicollinearity among such OFIs within the top 10 levels of the book. Through principal component analysis (PCA), we observe that the first principal component of multi-level OFIs can explain more than 80% of the total variance. Therefore, we propose to use the first principal component to transform multi-level OFIs to an integrated...
variable. Our results demonstrate that such integrated OFIs are superior to both best-level OFIs and multi-level OFIs, in both in-sample and out-of-sample tests. It is important to emphasize that we use both in-sample and out-of-sample tests to assess the model’s performance, while previous works [17, 11] only focus on in-sample tests. Interestingly, we find that the explained percentage of variance in the price dynamics is associated with stock properties such as the bid-ask spread, volatility, and trading volume.

**Cross impact - on contemporaneous returns.** Another important question concerns the existence of cross-impact, i.e. assessing whether one stock’s prices and returns are influenced by the order flow imbalances of other stocks (see [6, 11, 45]). We provide a new approach to examining cross-impact, which leverages the sparsity of cross-impact terms and employs the Least Absolute Shrinkage and Selection Operator (LASSO) to fit a linear regression that uses cross-sectional OFIs as predictors. By comparing with the price-impact model using best-level OFIs, we find that the cross-impact model using the entire cross-sectional best-level OFIs as candidate features can provide *small but significant* additional explanatory power to price movements. Moreover, we observe that once the multi-level OFIs has been taken into account, the cross-impact model cannot provide additional explanatory power to the price-impact model. These findings suggest that consolidating multi-level OFIs, such as our integrated OFIs, is a more effective method to model price dynamics than introducing cross-impact terms. To the best of our knowledge, this is the first study to comprehensively analyze the relations between contemporaneous individual returns and multi-level order flow imbalances of individual or cross-sectional stocks.

**Forecasting future returns.** Finally, we investigate the performance of forward-looking price-impact and cross-impact models, i.e. using OFIs to forecast future returns, which has received a lot less attention in the literature, as it is an inherently much more challenging problem than explaining contemporaneous returns. For example, the forward-looking price-impact model uses the lagged OFIs of a stock to predict its own returns, while the forward-looking cross-impact model considers all cross-sectional OFIs as candidate predictors and applies LASSO to solve the regression. Our results show that even though both models achieve negative $R^2$ values, the forward-looking cross-impact model can significantly increase $R^2$ values compared to the forward-looking price-impact model. To testify the profitability of these forecasts, we employ two strategies based on the forecast returns. Our results indicate that the forward-looking cross-impact model yields more consistent and higher profits compared to the forward-looking price-impact model. To the best of our knowledge, this is the first study that demonstrates existence of the cross-impact phenomenon when analyzing order flow, in the forward-looking model; in contrast, no evidence of cross-impact was identified when considering contemporaneous returns.
The main contributions of our work can be summarized as follows:

1. We show that order flow at deeper levels of the limit order books provides additional explanatory power for price impact, when compared to the order flow imbalance at the best-level.

2. We provide a systematic approach for combining OFIs at the top levels of the book into an integrated OFI variable which best explains price impact.

3. We show evidence that there is no need to introduce cross-impact terms for modeling the contemporaneous returns, as long as the OFI incorporates information from multiple levels, not just the best level.

4. Cross-asset OFIs improve the forecast of future returns, thus providing evidence of cross-impact in the forward-looking model.

Paper outline. The remainder of this paper is structured as follows. In Section 2, we establish the notations and terminology employed throughout the paper. Section 3 presents studies about the price-impact model on contemporaneous returns. Section 4 turns to the cross-impact model on contemporaneous returns. In Section 5, we first discuss the forward-looking versions of the price-impact and cross-impact models, and then examine the economic gains from forecast returns. Finally, we conclude our analysis in Section 6 and highlight potential future research directions.

2 Notation

In this short section, we establish notations for various quantities used throughout the paper.

Best-level OFI. We use the following definition of OFI (proposed by Cont et al. [17]) in our analysis

\[ \text{OFI}^1_{i,t} := \sum_{t - h < \tau \leq t} q_{i,\tau}^{b,1} \{ p_{i,\tau}^{b} \geq p_{i,\tau}^{1,b} \} - q_{i,\tau}^{b,1} \{ p_{i,\tau}^{b} \leq p_{i,\tau}^{1,b} \} - q_{i,\tau}^{a,1} \{ p_{i,\tau}^{a} \geq p_{i,\tau}^{1,a} \} + q_{i,\tau}^{a,1} \{ p_{i,\tau}^{a} \leq p_{i,\tau}^{1,a} \}. \]  

Here, \( p_{i,\tau}^{b} \) and \( q_{i,\tau}^{b,1} \) denote the best bid price and size (in number of shares) of stock \( i \), respectively. Similarly, \( p_{i,\tau}^{a} \) and \( q_{i,\tau}^{a,1} \) denote the ask price and ask size at the best level, respectively. \( \tau \) represents the arriving time of events and \( \tau' \) is the last event time before \( \tau \). Thus \( \text{OFI}^1_{i,t} \) calculates the accumulative order flow imbalances at the best bid/ask side during the time interval \([t - h, t]\), where \( h \) denotes the lookback horizon. Note that the variable \( \text{OFI}^1_{i,t} \) increases when

- the bid size increases and the best bid price remains the same;
• the ask size decreases and the best ask price remains the same;

• the best bid/ask prices increase.

One can perform an analogous analysis for the decreasing case.

**Deeper-level OFI.** A natural extension of the best-level OFI defined in Eqn (1) is deeper-level OFI. We define OFI at level \( m \) \((m \geq 1)\) as follows

\[
\text{OFI}_{m,i,t} := \sum_{t-h<\tau \leq t} q_{m,b}^{i,\tau} 1\{P_{m,b}^{i,\tau} \geq P_{m,b}^{i,\tau'}\} - q_{m,b}^{i,\tau'} 1\{P_{m,b}^{i,\tau} \leq P_{m,b}^{i,\tau'}\} - q_{m,s}^{i,\tau} 1\{P_{m,s}^{i,\tau} \leq P_{m,s}^{i,\tau'}\} + q_{m,s}^{i,\tau'} 1\{P_{m,s}^{i,\tau} \geq P_{m,s}^{i,\tau'}\}.
\]

(2)

Here, \( P_{m,b}^{i,\tau} \) and \( q_{m,b}^{i,\tau} \) denote, respectively, the bid price and bid size (in number of shares) at level \( m \) of the order book, respectively. Similarly, \( P_{m,s}^{i,\tau} \) and \( q_{m,s}^{i,\tau} \) denote the ask price and ask size at level \( m \).

**Normalized OFI.** Due to the intraday pattern in limit order depth [2], we use the average size [28] to scale OFI at the corresponding level.

\[
\text{ofi}_{m,i,t} = \frac{\text{OFI}_{m,i,t}}{Q_{M,i,t}},
\]

(3)

where \( Q_{M,i,t}^{M} = \sum_{m=1}^{M} \frac{1}{n(t)} \sum_{t-h<\tau \leq t} \left[ q_{m,b}^{i,\tau} + q_{m,b}^{i,\tau'} \right] \) is the sum order book depth across the first \( M \) levels and \( n(t) \) is the number of events during \((t-h,t]\). Throughout the paper, we use the first \( M = 10 \) levels.

**Logarithmic return.** We denote the logarithmic return of asset \( i \) during \((t-h,t]\) by

\[
R_{i,t} = \log \left( \frac{P_{i,t}}{P_{i,t-h}} \right),
\]

(4)

where \( P_{i,t} \) is the mid-price at time \( t \), i.e. \( P_{i,t} = \frac{P_{i,t}^1 + P_{i,t}^1}{2} \), and \( h \) is the time interval.

**Normalized return.** Taking into account the fact that the intraday pattern in price volatility is has been extensively demonstrated [41, 17, 24], we normalize \( R_{i,t} \) by the historical volatility of asset \( i \) and denote \( r_{i,t} \) as the normalized return.

\[
r_{i,t} = \frac{R_{i,t}}{\sigma_{i,t} \sqrt{h}},
\]

(5)

where \( \sigma_{i,t} \) is the average value of standard deviations of minutely returns during \([t-30,t]\), across the previous five trading days.
3 Price Impact

In this section, we study the price-impact model using LOB data. We first compare the effects of best-level OFIs with multi-level OFIs. Due to the redundancy of information across multi-level OFIs, we propose an integrated version of the OFIs, and evaluate its efficacy on modeling price changes.

3.1 Related Works

The relation between contemporaneous order flow imbalance and price moves, also known as price-impact [7, 17, 40], has drawn substantial attention in recent decades. It is an important constituent of studying price dynamics and relevant to practical problems, e.g. trading costs [22] and optimal execution models [27].

A series of previous studies mainly focus on the impact of imbalances between buy and sell market orders. Taranto et al. [50] propose a framework aiming to build a precise linear model of price dynamics based on two types of market orders: (1) those that leave the price unchanged, and (2) those which lead to an immediate price change. Brown et al. [9] adopt a bivariate vector auto-regression (VAR) to study the interaction between order imbalance and stock return. Using data from the Australian Stock Exchange, the authors conduct causality tests and observe a strong bi-directional contemporaneous relationship. Instead of modeling the temporary impact, Eisler et al. [20] empirically analyze the impact of different order events on future price changes.

The literature regarding the impact of limit orders includes the following works, that are more relevant to our present study. Cont et al. [17] use a linear model to describe the contemporaneous impact of the order flow imbalance on price dynamics. Their proposed measure, order flow imbalance, is built on the following order book events: limit orders, market orders, and cancellations. Ample evidence presents a significant relationship between the order flow imbalance and price changes. However, Cont et al. [17] only investigate the in-sample $R^2$ and ignore the generalization error of their regressions. Xu et al. [56] extend the model of [17] to multi-level order flow imbalance. They argue that even though strong correlations exist between the net order flow at different price levels, multi-level limit orders are still statistically informative. Furthermore, Xu et al. [56] adopt a cross-validation method to estimate the out-of-sample root-mean-square error; however, they do not preserve the chronological order of the data, and their experiments are performed on only six stocks for one year of data. In contrast, in our study, we obey the temporal ordering of the data, and our study is based on around 100 stocks over three years of data.
3.2 Models for Price-impact

Price-impact of best-level OFIs. We first pay attention to the impact of best-level OFIs, i.e. $\text{ofi}^1_{i,t}$ on contemporaneous returns\(^1\), which materialize over the same time bucket as the OFI.

$$\text{PI}^{[1]} : \quad r_{i,t} = \alpha_i^{[1]} + \beta_i^{[1]} \text{ofi}^1_{i,t} + \epsilon_i^{[1]}_t.$$  \hfill (6)

Here, $\epsilon_i^{[1]}_t$ is a noise term summarizing the influences of other factors, such as the order flow imbalances at multiple levels, and potentially the trading behaviours of other stocks. For the sake of simplicity, we denote the above regression model as PI\(^{[1]}\), and use ordinary least squares (OLS) to estimate it.

Price-impact of multi-level OFIs. We then perform an extended version of PI\(^{[1]}\) using multi-level OFIs as predictors in the model,

$$\text{PI}^{[m]} : \quad r_{i,t} = \alpha_i^{[m]} + \sum_{k=1}^{m} \beta_i^{[m],k} \text{ofi}^k_{i,t} + \epsilon_i^{[m]}_t,$$  \hfill (7)

where $\text{ofi}^k_{i,t}$ is the normalized OFIs at level $k$. We refer to this model as PI\(^{[m]}\), and use ordinary least squares (OLS) to estimate it.

Principal Components Analysis (PCA) is a widely-used statistical procedure that applies an orthogonal transformation to convert a number of correlated variables into a smaller number of uncorrelated variables, i.e. principal components [35, 54]. The main objective of PCA is to determine the important directions which can explain most of the original variability in the data. PCA projects data onto these important directions, thus reducing dimensionality. For example, we can drop the eigenvectors with the lowest eigenvalues due to their low information content. As a result, PCA can be employed to capture dominant factors driving the market (see [34]).

In general, PCA entails the following steps: computing the covariance/correlation matrix, computing eigenvalues and eigenvectors, sorting eigenvalues and identifying principal components, and finally, interpreting the results. More specifically, assume the data is given by the random vector $\mathbf{x} = (x_1, \cdots, x_p)^\top \in \mathbb{R}^p$. If we have $n$ observations $\{\mathbf{x}^{(i)}\}_{i=1}^n$ of $\mathbf{x}$, we can construct the sample matrix, denoted as $\mathbf{X} = [\mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(n)}]^\top \in \mathbb{R}^{n \times p}$. Next, we compute the empirical covariance matrix of $\mathbf{x}$ by

$$\mathbf{S} = \frac{1}{n-1} (\mathbf{X} - \bar{\mathbf{x}})^\top (\mathbf{X} - \bar{\mathbf{x}}),$$  \hfill (8)

where $\bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{x}^{(i)}$ is the empirical mean vector.

\(^1\)We also conduct experiments on price changes. The results are similar to the ones associated with returns.
In order to maximize the variance of the projected variable, the principal vectors \( \{w_k\}_{k=1}^p \) and their associated eigenvalues \( \{\lambda_k\}_{k=1}^p \) have to satisfy

\[
\begin{align*}
    w_k &= \arg \max_{w \in \mathbb{R}^p} \{w^\top Sw : w^\top w_j = 0, \forall 1 \leq j < k, ||w||_2 = 1\}, \\
    \lambda_k &= \max_{w \in \mathbb{R}^p} \{w^\top Sw : w^\top w_j = 0, \forall 1 \leq j < k, ||w||_2 = 1\}.
\end{align*}
\]

(9)

The Lagrange multipliers method (see [51] for more details) can help transform the problem (9) into the eigen-decomposition of the sample covariance matrix \( S \). Suppose the eigenvalues \( \{\lambda_k\}_{k=1}^p \) and eigenvectors of \( S \) are sorted in decreasing order of the eigenvalue, i.e. \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0 \). In order to obtain lower-dimensional structures, while preserving as much information in the data as possible, one can project data points onto only the first \( k \) principal components, \( (w_1^\top x, \cdots, w_k^\top x) \).

**Price-impact of Integrated OFIs.** In order to make use of the information embedded in multiple LOB levels and avoid overfitting, we propose an integrated version of OFIs, as shown in Eqn (10), which only preserves the first principal component. We further normalize the first principal component by dividing by its \( l_1 \) norm so that the weights of multi-level OFIs in constructing integrated OFIs sum to 1. Specifically, denoting the multi-level OFI vector as \( o_{i,t} = (o_{i,1,t}, \cdots, o_{i,10,t})^\top \), we consider

\[
    o_{i,t}^I = \frac{w_1^\top o_{i,t}}{||w_1||_1},
\]

(10)

where \( w_1 \) is the first principal vector computed from historical data. To the best of our knowledge, this is the first work to *aggregate multi-level OFIs into a single variable*. 

In practice, we use samples of \( o_{i,s} \) during the previous 30 minutes, \( s \in [t-30 \text{ minutes}, t) \), to compute the principal components. Then we apply the first principal vector to transform the multi-level OFI vector \( o_{i,t} \) to \( o_{i,t}^I \). For example, if using minutely buckets, we perform PCA on a matrix of size \( 30 \times 10 \). For 10-second buckets, the PCA input matrix is of size \( 180 \times 10 \), since there are 180 buckets of 10-second length in a 30-minute interval.

Finally, we estimate the impact of \( o_{i,t}^I \) on the returns, and denote this model as \( PI^I \).

\[
    PI^I : \quad r_{i,t} = \alpha_{i,t} + \beta_{i} o_{i,t}^I + \epsilon_{i,t}^I.
\]

(11)

### 3.3 Empirical Results

**Data.** We use the Nasdaq ITCH data from LOBSTER\(^{2}\) to compute OFIs and returns during the intraday time interval 10:00AM-3:30PM. The reason for excluding the first and last 30 minutes of the trading day is

\(^{2}\text{https://lobsterdata.com/} \)
due to the increased volatility near the opening and closing session \[29, 14, 15\]. Our data includes the top 100 components of S&P500 index, for the period 2017-01-01 to 2019-12-31.

**Implementation.** For a more representative and fair comparison with previous studies \[17, 56\], we apply the same procedure described in \[17\] for our experiments. Specifically, we use a 30-minute estimation window and within each window, data associated with returns and OFIs are computed for every 10 seconds, i.e. \( h = 10 \) seconds. Altogether, this amounts to \( 30 \times 60 / 10 = 180 \) observations for each regression.

**Performance evaluation.** We use the past 30-minute data to fit the above regressions, namely Eqns (6), (7), and (11). We then calculate their adjusted-\( R^2 \), denoted as the in-sample \( R^2 \) (or IS \( R^2 \)). In order to assess how well the model generalizes to unseen data, we propose to perform the following out-of-sample tests. We use the fitted model to estimate returns on the following 30-minute data and compute the corresponding \( R^2 \), denoted as the out-of-sample \( R^2 \) (or OOS \( R^2 \)). Because we drop the first and last half an hour of each trading day from our samples and perform out-of-sample tests, this leads to 10 linear regressions contained in each trading day for a given stock.

**Price-impact of multi-level OFIs.** From Table 1, we observe that \( \pi^1 \) can explain 71.16% of the in-sample variation of a stock’s contemporaneous returns. In terms of out-of-sample data, \( \pi^1 \) can explain 64.64% of returns’ variation. The top panel of Table 1 shows that the in-sample \( R^2 \) values increase as more multi-level OFIs are included as features, which is not surprising given that \( \pi^m \) is a nested model of \( \pi^{m+1} \). However the increments of the in-sample \( R^2 \) are descending, indicating that much deeper LOB data might be unable to provide additional information. This argument is confirmed by the models’ performance on out-of-sample data, as shown at the bottom panel of Table 1. Note that out-of-sample \( R^2 \) reaches a peak at \( \pi^8 \).

|          | \( \pi^1 \) | \( \pi^2 \) | \( \pi^3 \) | \( \pi^4 \) | \( \pi^5 \) | \( \pi^6 \) | \( \pi^7 \) | \( \pi^8 \) | \( \pi^9 \) | \( \pi^{10} \) |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| In-Sample| \( R^2 \) (%) | 71.16       | 81.61       | 85.07       | 86.69       | 87.66       | 88.30       | 88.74       | 89.04       | 89.24       | **89.38**   |
|          | \( \Delta R^2 \) (%) | 10.45       | 3.46        | 1.62        | 0.97        | 0.64        | 0.44        | 0.30        | 0.20        | 0.14        |
|          | p-Value     | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        |
| Out-Of-Sample| \( R^2 \) (%) | 64.64       | 75.81       | 79.47       | 81.13       | 82.05       | 82.65       | 83.01       | **83.16**   | 83.15       | 83.11       |
|          | \( \Delta R^2 \) (%) | 11.17       | 3.66        | 1.66        | 0.92        | 0.60        | 0.36        | 0.15        | -0.01       | -0.04       |
|          | p-Value     | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.00        | 0.51        | 0.93        |             |

Table 1: Average adjusted-\( R^2 \) (in percentage points) of \( \pi^1 \) (Eqn (6)) and \( \pi^m \) (Eqn (7)). The top and bottom panels report the results in in-sample and out-of-sample tests, respectively. \( \Delta R^2 \) is the increase in adjusted \( R^2 \) between \( \pi^m \) and \( \pi^{m+1} \) \( (m = 1, \ldots, 9) \). The p-value represents the probability of observing the increase in adjusted \( R^2 \) under the null hypothesis of no increase from \( \pi^m \) to \( \pi^{m+1} \) \( (m = 1, \ldots, 9) \).
Impact comparison between multi-level OFIs. An interesting question is whether the OFIs at different price levels contribute evenly in terms of price impact. Based on Figure 2 (a), we conclude that multi-level OFIs have different contributions to price movements. Particularly, ofi\textsuperscript{2} (OFIs at the second-best level) reflects greater influence than ofi\textsuperscript{1} (OFIs at the best level) in model PI\textsuperscript{10}, which is perhaps counter-intuitive, at first sight.

We further investigate how the coefficients vary across stocks with different characteristics, such as volume, volatility, and bid-ask spread. Figure 2 (b)-(d) reveals that for stocks with high-volume and small-spread, order flow posted deeper in the LOB has more influence on price movements. The results regarding spread are in line with [56], where it is observed that for large-spread stocks (AMZN, TSLA, and NFLX), the coefficients of ofi\textsuperscript{m} tend to get smaller as m increases, while for small-spread stocks (ORCL, CSCO, and MU), the coefficients of ofi\textsuperscript{m} may become larger as m increases.

Nonetheless, Cont et al. [17] conclude that the effect of ofi\textsuperscript{m}(m ≥ 2) on price changes is only second-order or null. There are two likely causes for the differences between the findings of [17] and ours. First, the data used in [17] includes 50 stocks (randomly picked from S&P500 constituents) for a single month in 2010, while we use the top 100 large-cap stocks for 36 months during 2017-2019. Second, Cont et al. [17] consider the average of the coefficients across 50 stocks. In our work, we first group 100 stocks by firm characteristics, and then study the average coefficients of each subset. Therefore, our results are based on a more granular analysis, across a significantly longer period of time.

Next, we investigate the correlations between the multi-level OFIs. Figure 3 reveals that even though the correlation structure of multi-level OFIs may vary across stocks, they all show strong relationships. It is worth pointing out that the best-level OFI exhibits the smallest correlation with any of the remaining nine levels, a pattern which persists across the different stocks.

Principal component of multi-level OFIs. Table 2 shows the average percentage of variance attributed to each principal component, and reveals that the first principal component can explain more than 80% of the total variance. As a result, we create an integrated form of OFIs by projecting multi-level OFIs along the first principal component, as summarized in Eqn (10).

| Principal Component | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|---------------------|----|----|----|----|----|----|----|----|----|----|
| Explained Variance Ratio (%) | 83.71 | 6.74 | 3.25 | 1.98 | 1.33 | 0.97 | 0.73 | 0.56 | 0.42 | 0.31 |

Table 2: Average percentage of variance attributed to each principal component, i.e. the ratio between the variance of each principal component and the total variance.
Figure 2: Coefficients of the model PI\textsuperscript{10}. (a) Average coefficients and one standard deviation (error bars), (b)-(d) coefficients sorted by stock characteristics. Volume: trading volume on the previous trading day. Volatility: volatility of one-minute returns during the previous trading day. Spread: average bid-ask spread during the previous trading day. [0\%, 25\%), respectively [75\%, 100\%], denote the subset of stocks with the lowest, respectively highest, 25\% values for a given stock characteristic.

Figure 3: Correlation matrix of multi-level OFIs. (a) averaged across stocks, (b)-(d): correlation matrix of Apple (AAPL), JPMorgan Chase (JPM), and Johnson & Johnson (JNJ). The x-axis and y-axis represent different levels of OFIs.

Figure 4: First principal component of multi-level OFIs. (a) averaged across stocks, (b)-(d): 1st principal component of Apple (AAPL), JPMorgan Chase (JPM), and Johnson & Johnson (JNJ). The x-axis represents different levels of OFIs and y-axis represents the weights of OFIs in the first principal component.

In Figure 4 we show statistics pertaining to the weights attributed to the 10 levels in the first principal component. Plot (a) shows the average weights, and the one standard deviation bars, across all stocks in
the universe. In Figures (b-d), we plot the average principal component for three different stocks, where the average and one standard deviation bars are computed across buckets of time for a given stock. Figure 4 (a) reveals that the best-level OFI has the smallest weight in the first principal component, but the highest standard deviation, hinting that it fluctuates significantly across stocks. Altogether, Figures 3 and 4 provide evidence that stocks with different correlation structures of multi-level OFIs have different first principal component patterns. For example, for AAPL, its weight in the first component is increasing as the OFI goes deeper, whereas the weight in the first component of JPM peaks at the $6^{th}$ level. Furthermore, Figure 5 shows various patterns for the first principal component of multi-level OFIs, for each quantile bucket of various stock characteristics, in particular, for volume, volatility and spread. For example, in Figure 5 (a), the red curve shows the average weights in the first principal component for each of the 10 levels, where the average is taken over all the top 25% largest volume stocks. A striking pattern that emerges from this figure is that for high-volume, and low-volatility stocks, OFIs deeper in the LOB receive more weight in the first component. However, for low-volume, and large-spread stocks, the best-level OFIs account more than the deeper-level OFIs.

![Graphs](image)

Figure 5: First principal component of multi-level OFIs, in quantile buckets for various stock characteristics. The $x$-axis indexes the top 10 levels of the OFIs. **Volume**: trading volume on the previous trading day. **Volatility**: volatility of one-minute returns during the previous trading day. **Spread**: average bid-ask spread during the previous trading day. [0%, 25%), respectively [75%, 100%], denote the subset of stocks with the lowest, respectively highest, 25% values for a given stock characteristic.

**Performance of Integrated OFIs.** Table 3 summarizes the relevant statistics of the price-impact model using integrated OFIs. Compared with Table 1, we observe that even though the in-sample $R^2$ of $PI^I$ is lower than $PI^m$ (when $m \geq 5$), its performance in out-of-sample tests is better than $PI^m$ ($\forall 1 \leq m \leq 10$). It demonstrates the ability of PCA to extract information from data and reduce the chance of overfitting.
|                  | PI$^I$  |
|------------------|---------|
| In-Sample $R^2$ (%) | 87.14   |
| Out-Of-Sample $R^2$ (%) | 83.83   |

Table 3: This table presents the average results of PI$^I$ (Eqn (11)). The first and second row report the adjusted $R^2$ in the in-sample and out-of-sample tests, respectively.

**Time-series variation.** Furthermore, we compare PI$^I$ with PI$^{[m]}$ by month, as shown in Figure 6. For better readability, we only report the results when $m = \{1, 8\}$, that correspond to the simplest model and the model with highest out-of-sample $R^2$, respectively. To arrive at this figure, we average $R^2$ values in each month. Figure 6 reveals that the improvement of PI$^I$ compared to PI$^{[1]}$ in the in-sample tests is consistent. In terms of out-of-sample tests, it is typically the case that the univariate regression PI$^I$ can explain slightly more about price movements than the best multivariate regression, PI$^{[8]}$.

![Figure 6: Time-series variation of $R^2$ by month, for different price-impact models. Panels (a) and (b) are based on the average results in the in-sample tests and out-of-sample tests, respectively.](image)

**Cross-sectional variation.** Next, we ask the question whether the integrated OFIs affect stocks differently, depending on their characteristics. To this end, we report in Table 4 the results, both for in-sample and out-of-sample, for the best-level OFI and the integrated OFI, for each quartile bucket of the distribution obtained from the volume, volatility, and spread of the stocks in our universe. We conclude from Table 4 that the integrated OFIs can improve $R^2$ in price impact consistently across different subsets grouped by firm characteristics. Table 4 also shows that price-impact models can better explain *high-volume, low-volatility, and small-spread stocks*. These results shed new light on the modeling of price dynamics. Take spread as an example; for large-spread stocks, new orders are very likely to arrive inside the bid-ask spread [56]. Assume
there are two potential scenarios when a limit buy order $B_1$ ($B_2$), as depicted in Figure 7, arrives within the bid-ask spread. $B_1$ and $B_2$ have the same size but different prices; they will lead to the same multi-level OFI vectors but different mid-prices. Hence, it is more difficult to explain the impact of OFIs on prices for large-spread stocks. In terms of volatility and volume, a possible explanation may be due to their correlations with bid-ask spread. Previous studies [1, 55] found that there is a strong positive correlation between spread and volatility, while trading volume is negatively correlated with spread. Therefore, for further studies on the price impact model, it is recommended to take these findings into account.

Table 4: Results showing the $R^2$ (in percentage) of the price-impact model, sorted by stock characteristics. $R^2$ is measured for both in-sample (IS) and out-of-sample (OOS) tests. The top panel is based on experiments using OFIs at best level ofi$^1$. The bottom panel is based on experiments using integrated OFIs ofi$^I$. Volume: trading volume on the previous trading day. Volatility: volatility of one-minute returns during the previous trading day. Spread: average bid-ask spread during the previous trading day. $[0\%, 25\%)$, respectively $[75\%, 100\%)$, denote the subset of stocks with the lowest, respectively highest, 25% values for a given stock characteristic.
Figure 7: Two potential scenarios when a limit buy order, $B_1$ ($B_2$), arrives within the bid-ask spread with a specific target price. Assuming $B_1$ and $B_2$ have the same size but different prices, then they will lead to the same multi-level OFI vectors but different mid-prices.

4 Cross Impact

In this section, we study the cross-impact model, i.e. how the stock prices are related to the order flow imbalances of other stocks. We provide a new method, applying Least Absolute Shrinkage and Selection Operator (LASSO), to examine the existence of contemporary cross-impact.

4.1 Related Works

The cross-impact phenomenon has attracted considerable interest in the recent literature [30, 6, 11, 45, 52]. Most studies focus on the cross-correlation structure of returns and order flows. Hasbrouck and Seppi [30] find that commonality in returns among Dow 30 stocks is mostly attributed to order flow commonality. Wang et al. [52] empirically analyze the price responses of a given stock influenced by the trades of other stocks, which they denote as cross-response. They show that the average cross-responses are weaker than the self-response. Furthermore, Benzaquen et al. [6] adopt the multivariate linear propagator model to explain the facts regarding cross-response.

Other empirical studies use the multi-asset extension of the linear model used in [39] and investigate the off-diagonal terms, i.e. cross-impact coefficients. Pasquariello and Vega [45] provide empirical evidence of cross-asset informational effects in NYSE and NASDAQ stocks between 1993 and 2004. They show that the daily order flow imbalance in one stock, or across one industry, has a significant and persistent impact on daily returns of other stocks or industries.
One closely related work is [11], where the authors render a different conclusion about cross-impact from a viewpoint of causality. They first show that the positive covariance between returns of a specific stock and order flow imbalances of other stocks cannot constitute evidence of cross-impact. They decompose the order flow imbalances into a common factor and idiosyncratic components, in order to verify that, as long as the common factor is involved in the model, adding cross-impact terms can improve the explained proportion of the variance only by 0.5%. Our present study differs from [11] in the following several aspects.

- (1) Capponi and Cont [11] only focus on the in-sample performance; the results in Appendix C reveal that the model proposed by them has a tendency to overfit the training data and leads to poor out-of-sample performance.

- (2) Our model takes into account the potential sparsity of the cross-impact terms, while Capponi and Cont [11] ignore this aspect; to this end, one can check more experimental results in Appendix C.

- (3) In addition to examining the cross-impact of best-level OFIs, we also consider the cross-impact from multi-level OFIs, in order to gauge a comprehensive understanding of the relations between cross-sectional multi-level OFIs and individual returns. To the best of our knowledge, this is the first study to investigate such relations.

### 4.2 Models for Cross-impact

We use the entire cross-sectional OFIs (including a stock’s own OFI) as candidate features to fit the returns \(r_{i,t}\) of the \(i\)-th stock, as shown in Eqn (12). In this model, \(\eta_{i,t}\) is the noise term with \(E[\eta_{i,t}^{[1]}] = 0\) and \(\text{Var}[\eta_{i,t}^{[1]}] < \infty\). Thus, \(\beta_{i,j}^{[1]} (j \neq i)\) represents the influence of the \(j\)-th stock’s OFI on the return of stock \(i\), after excluding the own OFI impact of stock \(i\). In addition, we also study the cross-impact model using the integrated OFIs, denoted as CI.

\[
\text{CI}^{[1]}: \quad r_{i,t} = \alpha_{i}^{[1]} + \beta_{i,i}^{[1]} \text{ofi}_{i,t}^{1} + \sum_{j \neq i} \beta_{i,j}^{[1]} \text{ofi}_{j,t}^{1} + \eta_{i,t}^{[1]}.
\]

\[
\text{CI}^{I}: \quad r_{i,t} = \alpha_{i}^{I} + \beta_{i,i}^{I} \text{ofi}_{i,t}^{I} + \sum_{j \neq i} \beta_{i,j}^{I} \text{ofi}_{j,t}^{I} + \eta_{i,t}^{I}.
\]

**Sparsity of Cross-impact.** On one hand, ordinary least squares (OLS) regression becomes ill-posed when there are fewer observations than parameters to estimate. We are considering \(N\) predictors in Eqns (12) and (13), i.e., the contemporaneous OFIs of each stock in our universe. Assuming the stock universe consists of the components of the S&P500 index, i.e., \(N \approx 500\) and the time interval is one minute, then each linear regression requires at least 500 observations, accounting for 500 minutes of historical data, i.e., more than one trading day. Therefore, it seems inappropriate to estimate Eqns (12) and (13) for intraday scenarios.
using OLS regression. On the other hand, Capponi and Cont \[11\] find that a certain number of cross-impact coefficients $\beta_{i,j}(j \neq i)$ from their OLS regressions\(^3\) are not statistically significant at the 1% significance level.

Therefore, our approach assumes that there is a small number of assets $j \neq i$ having significant cross-impact on stock $i$, as opposed to the entire universe. To this end, we apply the Least Absolute Shrinkage and Selection Operator (LASSO) to solve Eqns (12) and (13). Note that even though the sparsity of the cross-impact terms is not theoretically guaranteed, our empirical evidences confirm this modeling assumption.

**LASSO** is a regression method that performs both variable selection and regularization, in order to enhance the prediction accuracy and interpretability of regression models \[31\]. It can be formulated as linear regression models. Take Eqn (12) as an example, the objective function of LASSO consists of two parts, i.e. the sum of squared residuals, and the $l_1$ constraint on the regression coefficients, as shown in (14).

$$
\min_{\alpha^{[1]}_i, \beta^{[1]}_{i,j}} \mathcal{L}_i = \sum_t \left[ r_{i,t} - \alpha^{[1]}_i - \beta^{[1]}_{i,i} \text{ofi}_{i,t} - \sum_{j \neq i} \beta^{[1]}_{i,j} \text{ofi}_{j,t} \right]^2 + \lambda \sum_{j=1}^N \left\| \beta^{[1]}_{i,j} \right\|_1.
$$

There is a hyperparameter $\lambda$ in the loss function that controls the penalty weight. In practice, we use cross-validation \[49\], a standard procedure in machine learning for choosing the penalty weight for each regression. Several techniques \[23, 19\] from convex analysis and optimization theory have been developed to compute the solutions of LASSO, including coordinate descent, subgradient methods, least-angle regression (LARS), and proximal gradient methods. In this work, we use coordinate descent algorithm to fit the model.

### 4.3 Empirical Results

**Implementation.** We follow the same setting as in Section 3.3 to implement the cross-impact model, and compare its in-sample and out-of-sample performance with the price-impact model. Table 5 reports the average $R^2$ values of the PI and CI models with normalized OFIs at the best level (in the top panel), and the results with the integrated OFIs (in the bottom panel). We observe improvements for in-sample (2.71%) and out-of-sample tests (1.39%) when using the OFIs at the best level. For integrated OFIs, the improvement for in-sample tests is 0.71%, statistically significant. However, there is no increase in out-of-sample $R^2$ when using the integrated OFIs. We refer the reader to Appendix B for more experimental results of cross-impact models with multi-level OFIs, which further support our observations.

\(^3\)The reason why Capponi and Cont \[11\] are able to adopt OLS regressions is that they use an entire trading day to estimate cross-impact on the basis of 67 stocks.
|        | PI (%) | CI (%) | $\Delta R^2$ (%) | p-Value |
|--------|--------|--------|------------------|---------|
| ofi\(^1\) |        |        |                  |         |
| In-Sample | 71.16  | 73.87  | 2.71             | 0.00    |
| Out-Of-Sample | 64.64  | 66.03  | 1.39             | 0.00    |
| ofi\(^I\) |        |        |                  |         |
| In-Sample | 87.14  | 87.85  | 0.71             | 0.00    |
| Out-Of-Sample | 83.83  | 83.62  | -0.21            | 1.00    |

Table 5: Average statistics of the models PI\(^1\) (Eqn (6)), CI\(^1\) (Eqn (12)), PI\(^I\) (Eqn (11)) and CI\(^I\) (Eqn (13)), for the contemporaneous returns. The first and second column report the adjusted-$R^2$ of the price-impact and cross-impact models, respectively. $\Delta R^2$ is the increase in adjusted-$R^2$. p-Value represents the probability of observing the increase in adjusted-$R^2$ under the null hypothesis of no increase. Note the results of PI\(^1\) (PI\(^I\)) model in the first column are the same as those in Table 1 (Table 3), respectively.

**Sparsity of the Cross-impact coefficients.** The results in Figure 8 reveal that the average number of statistically significant cross-impact coefficients ($\beta_{i,j}^{[1]} \neq 0$, when $j \neq i$) for each stock is around 10, supporting the sparsity assumption. Returns of Facebook (FB) are influenced by the smallest number (fewer than 5) of cross-sectional OFIs, while almost 27 other stocks’ OFIs can affect the price movements of General Electric (GE).

![Figure 8](image_url)

Figure 8: Bar plot showing the number of non-zero cross-impact coefficients for individual stocks averaged across days; the dashed blue line represents the average number across assets in our universe. Stocks are sorted according to their market capitalization at the end of 2019.

Next, we take a closer look at the cross-impact coefficients of each stock based on best-level and integrated OFIs, i.e. $\beta_{i,j}^{[1]}, \beta_{i,j}^{I}$ ($j \neq i$). Figure 9 shows a comparison of the top singular values of the coefficient matrices given by the best-level and integrated OFIs. The relatively large singular values of the best-level OFI is a consequence of the higher edge density and thus average degree of the network. Note that both networks exhibit a large top singular value (akin to the usual *market mode*), and the integrated OFI network has a
faster decay of the spectrum, thus revealing its low rank structure.

Figure 9: Barplot of singular values in descending order for the average coefficient matrix in **contemporaneous cross-impact** models. The coefficients are averaged over 2017–2019. We perform Singular Value Decomposition (SVD) on the coefficient matrix to obtain the singular values. The $x$-axis represents the singular value rank, and the $y$-axis represents the singular values.

We construct a network for each coefficient matrix, which only preserves the edges larger than the a given threshold, as shown in Figure 10. We color stocks according to the GICS sector division\(^4\), and sort them by their market capitalization within each sector. As one can see from Figure 10(a), the cross-impact coefficient matrix $\beta^{(1)}_{i,j}$ displays a sectorial structure, in accordance with previous studies [6]. This demonstrates that the price of a specific stock is more influenced by the OFIs of stocks in the same industry, as expected. This behaviour could be fueled by index arbitrage strategies, where traders may, for example, trade an entire basket of stocks coming from the same sector against an index.

Figure 10(b) presents the average cross-impact coefficients based on integrated OFIs, i.e. $\beta^{I}_{i,j}(j \neq i)$. Comparing with Figure 10(a), the connections in Figure 10(b) are much weaker, implying that the cross-impact from stocks can be potentially explained by a stock’s own multi-level OFIs, to a large extent. Note that there is only one connection from GOOGL to GOOG, as pointed out at the top of Figure 10(b). This stems from the fact that both stock ticker symbols pertain to Alphabet (Google), therefore there exists a strong relation between them. Our study also revealed that OFIs of GOOGL have more influence on the returns of GOOG, not the other way around. The main reason might be that GOOGL shares have voting rights, while GOOG shares do not.

In Figures 10(c) and 10(d), we set lower threshold values (75-th / 25-th percentile of coefficients) in order to promote more edges in the networks based on the integrated OFIs. More interestingly, we observed 4 connections in Figure 10(c). Except from bidirectional links between GOOGL and GOOG, there exists a

\(^4\)The Global Industry Classification Standard (GICS) is an industry taxonomy developed in 1999 by MSCI and Standard & Poor’s (S&P) for use by the global financial community.
one-way link from Cigna (CI) to Anthem (ANTM), and one-way link from Duke Energy (DUK) to NextEra Energy (NEE). Anthem announced to acquire Cigna in 2015. After a prolonged breakup, this merger finally failed in 2020. Therefore, it is unsurprising that the OFIs of Cigna can affect the price movements of Anthem. In terms of the second pair, NextEra was interested in acquiring Duke Energy, but Duke rebuffed the takeover in 2020. Note that 2020 is not in our sample period. This indicates that some participants in the market had noticed the special relationship between Duke Energy and NextEra Energy before the mega-merger was proposed.

Discussion about Cross-Impact. We consider the following scenario, also depicted in Figure 11, to explain the above findings. For simplicity, we denote the order from trading strategy $A$ on stock $i$ (resp. $j$) as $A_i$ (resp. $A_j$). Analogously, we define orders from strategy $B$ and $S$. Let us next consider the OFIs of stock $i$. There are three orders from different portfolios, given by $A_i$, $B_i$ and $S_i$. $A_i$ is at the third bid level of stock $i$ and linked to an order at the best ask level of stock $j$, i.e. $A_j$. Also, $B_i$ is at the best ask level of stock $i$ and linked to an order at the best bid level of stock $j$, i.e. $B_j$. Finally, $S_i$ is an individual bid order at the best level of stock $i$.

Now assume that the best-level limit orders from stock $j$ are linked to price movements of stock $i$ through paths $B_j \rightarrow B_i \rightarrow \text{ofi}_i^1 \rightarrow r_i$ and $A_j \rightarrow A_i \rightarrow \text{ofi}_i^3 \rightarrow r_i$. Thus the price-impact model which only utilizes its own best-level orders of stock $i$ will ignore the information along the second path $A_j \rightarrow A_i \rightarrow \text{ofi}_i^3 \rightarrow r_i$. This might illustrate why the cross-sectional best-level OFIs can provide additional explanatory power to the price-impact model using only the best-level OFIs.

Nonetheless, if we can integrate multi-level OFIs in an efficient way (in our example, aggregate order imbalances caused by orders $A_i$, $B_i$ and $S_i$), then there is no need to consider OFIs from other stocks for modeling price dynamics. In other words, information hidden in the path $A_j \rightarrow A_i \rightarrow \text{ofi}_i^3 \rightarrow r_i$ can be leveraged as long as $A_i$ is well absorbed into new integrated OFIs. We put forward this mechanism which potentially explains why the cross-impact model with integrated OFIs cannot provide extra explanatory power compared to the price-impact model with integrated OFIs, which also highlights the advantage of our integrated OFIs.

Another possible explanation for this is that the duration of the cross-impact terms might be shorter than the current time interval (30 minutes) used in our experiments, rendering the cross-impact terms to disappear in out-of-sample tests. To verify this assertion, we implement additional experiments where models are updated more frequently. The results (deferred to Appendix A) reveal that even under higher-frequency updates of the models, there is no benefit from introducing cross-impact terms to the price-impact model with integrated OFIs.
Figure 10: Illustrations of the coefficient networks constructed from contemporaneous cross-impact models. The coefficients are averaged over 2017–2019. To render the networks more interpretable and for ease of visualization, we only plot the top 5% largest (a–b), or top 25% largest (c), or top 75% largest (d), in magnitude coefficients [37, 18]. Nodes are coloured by the GICS structure and sorted by market capitalization. Green links represent positive values while black links represent negative values. The width of edges is proportional to the absolute values of their respective coefficients.
5 Forecasting Future Returns

In previous sections, the definitions of price-impact and cross-impact are based on contemporaneous OFIs and returns, meaning that both quantities pertain to the same bucket of time. In this section, we extend the above studies to future returns, and probe into the forward-looking price-impact and cross-impact models.

5.1 Related Works

In recent years, machine learning models, including deep neural networks, have achieved substantial developments, leading to their applications in financial markets, especially for the task of predicting stock returns. Huck [33] utilizes state-of-the-art techniques, such as Random Forests, to construct a portfolio over a period of 22 years, and the results demonstrate the power of machine learning models to produce profitable trading signals. Krauss et al. [38] apply a series of machine learning methods to forecast the probability of a stock to outperform the market index, and then construct long-short portfolios from the predicted one-day-ahead trading signals. Gu et al. [26] employ machine learning methods, such as LASSO, random forest, neural networks etc., to make one-month-ahead return forecasts, and demonstrate the potential of machine learning approaches in empirical asset pricing, due to their ability to handle nonlinear interactions.

In addition, there is a large body of literature researching the lead-lag effect in equity returns [32, 13, 48, 42, 5] and crypto returns [3]. Hou [32] shows that within the same industry, returns of large firms help
predict returns of small firms. Chinco et al. [13] apply LASSO to produce rolling one-minute-ahead return forecasts using the entire cross-section lagged returns as candidate predictors, and observe that cross-asset returns can improve Sharpe Ratios for a given trading strategy. Rapach et al. [48] investigate the role of the United States in international markets, and show that lagged U.S. returns can significantly predict returns in numerous non-U.S. industrialized countries. Finally, Menzly and Ozbas [42] find that stocks that are in economically related industries can cross-predict each other’s returns.

A few studies have begun to examine the relation between order imbalances and future daily returns [14, 15, 10]. Chordia et al. [14] reveal that daily stock market returns are strongly related to contemporaneous and lagged order imbalances. Chordia and Subrahmanyam [15] further find that there exists a positive relation between lagged order imbalances and daily individual stock returns. They also show that imbalance-based trading strategies, i.e. buy if the previous day’s imbalance is positive, and sell if the previous day’s imbalance is negative, are able to yield statistically significant profits. Cao et al. [10] show that the order imbalances behind the best level are informative, and have additional power in forecasting future short-term returns. Nonetheless, to the best of our knowledge, cross-sectional order flow imbalances have not been considered as predictors for predicting future returns in the literature, which is a direction we explore in the remainder of this section.

5.2 Predictive Models

We propose the following forward-looking price-impact and cross-impact models, denoted as F-PI[1] and F-CI[1], respectively. F-PI[1] uses the lagged OFIs of stock \(i\) to predict its own future return \(R_{i,t+1}\), while F-CI[1] involves the entire lagged cross-sectional OFIs. For the impact of lagged OFIs on multi-horizon future returns, we refer to Appendix E for more details.

\[
\text{F-PI[1]} : \quad R_{i,t+1} = \alpha_i^F + \beta_i^F \text{ofi}_{i,t}^1 + \gamma_i^F \text{ofi}_{i,t-1}^1 + \theta_i^F \text{ofi}_{i,t-2}^1 + \epsilon_i^F. \tag{15}
\]

\[
\text{F-CI[1]} : \quad R_{i,t+1} = \alpha_i^F + \sum_{j=1}^{N} \left[ \beta_{i,j}^F \text{ofi}_{j,t}^1 + \gamma_{i,j}^F \text{ofi}_{j,t-1}^1 + \theta_{i,j}^F \text{ofi}_{j,t-2}^1 \right] + \eta_i^F. \tag{16}
\]

We also study models with integrated OFIs (ofi\(^I\), denoted as F-PI\(^I\) and F-CI\(^I\), respectively.

\[
\text{F-PI}^I : \quad R_{i,t+1} = \alpha_i^{FI} + \beta_i^{FI} \text{ofi}_{i,t}^I + \gamma_i^{FI} \text{ofi}_{i,t-1}^I + \theta_i^{FI} \text{ofi}_{i,t-2}^I + \epsilon_i^{FI}. \tag{17}
\]

\[
\text{F-CI}^I : \quad R_{i,t+1} = \alpha_i^{FI} + \sum_{j=1}^{N} \left[ \beta_{i,j}^{FI} \text{ofi}_{j,t}^I + \gamma_{i,j}^{FI} \text{ofi}_{j,t-1}^I + \theta_{i,j}^{FI} \text{ofi}_{j,t-2}^I \right] + \eta_i^{FI}. \tag{18}
\]

Furthermore, we compare OFI-based models with those return-based models proposed in previous works

\(^5\)For ease of interpretation in the following strategy profitability analysis, here we only report the results for forecasting the raw log-returns. In an unreported experiment, we predict the normalized future returns by lagged OFIs. We also find significant improvements in out-of-sample \(R^2\) by introducing cross-asset terms.
(see Chinco et al. [13]), where the lagged returns are involved as predictors. F-AR is an autoregressive (AR) model of order 3, as shown in (19). F-CR uses the entire cross-section lagged returns as candidate predictors, as detailed in (20).

\[
\text{F-AR : } R_{i,t+1} = \alpha_i^R + \beta_i^R R_{i,t} + \gamma_i^F R_{i,t-1} + \theta_i^R R_{i,t-2} + \epsilon_i^R. \tag{19}
\]

\[
\text{F-CR : } R_{i,t+1} = \alpha_i^R + \sum_{j=1}^{N} \left[ \beta_{i,j}^R R_{j,t} + \gamma_{i,j}^R R_{j,t-1} + \theta_{i,j}^R R_{j,t-2} \right] + \eta_i^R. \tag{20}
\]

5.3 Empirical Results

Implementation. Observations associated with returns and OFIs are computed minutely, i.e. \( h = 1 \) minute. We use data from the previous 30 minutes to estimate the model parameters and apply the fitted model to forecast the one-minute-ahead return. We repeat this procedure for each trading day, to compute rolling one-minute-ahead return forecasts.

|       | F-PI (%) | F-CI (%) | \( \Delta R^2 \) (%) | p-Value |
|-------|----------|----------|----------------------|---------|
| ofi   | In-Sample | 0.24     | 12.24                | 12.00   | 0.00       |
|       | Out-Of-Sample | -19.07   | -11.98               | 7.09    | 0.00       |
| ofiI  | In-Sample | 0.26     | 12.19                | 11.93   | 0.00       |
|       | Out-Of-Sample | -17.62   | -11.81               | 5.81    | 0.00       |

|       | F-AR (%) | F-CR (%) | \( \Delta R^2 \) (%) | p-Value |
|-------|----------|----------|----------------------|---------|
| \( R \) | In-Sample | 0.38     | 13.66                | 13.28   | 0.00       |
|       | Out-Of-Sample | -36.62   | -11.19               | 25.43   | 0.16       |

Table 6: Statistics of models using lagged OFIs (in the top panel), including F-PI\(^I\) (Eqn (15)), F-CI\(^I\) (Eqn (16)), along with their integrated OFI-based counterparts denoted as F-PI\(^I\) and F-CI\(^I\), and models using lagged returns (in the bottom panel), including F-AR (Eqn (19)) and F-CR (Eqn (20)), for predicting future returns. The first and second column report the adjusted-\( R^2 \) of price-impact and cross-impact model, respectively. \( \Delta R^2 \) is the increase in adjusted-\( R^2 \). p-Value represents the probability of observing the increase in adjusted-\( R^2 \) under the null hypothesis of no increase.

Table 6 summarizes the in-sample and out-of-sample performance of the aforementioned predictive models. It is not surprising that the cross-impact models F-CI\(^I\) (resp. F-CI\(^I\)) attain higher adjusted-\( R^2 \) statistics compared to the price-impact models F-PI\(^I\) (resp. F-PI\(^I\)), for the in-sample tests. In terms of out-of-sample \( R^2 \) performance, F-CI\(^I\) and F-CI\(^I\) show an improvement of 7.0% and 5.8%, respectively. For models using lagged returns, we reach a similar conclusion as in [13], that lagged cross-asset returns can improve both in-sample and out-of-sample \( R^2 \).

We remark that negative \( R^2 \) values do not imply that the forecasts are economically meaningless. To
emphasize this point, we will incorporate these return forecasts into two common forecast-based trading strategies, and showcase their profitability in the following subsection.

**Coefficient network.** Considering the different magnitudes of the OFIs and returns, we first normalize the coefficient matrix of each model by dividing by the average of the absolute coefficients. We construct a network for each normalized coefficient matrix, which only preserves the edges larger than the 95-th percentile of coefficients. Figure 12 illustrates some of the main characteristics of the coefficient networks for various forward-looking models, including F-PF\textsuperscript{[1]} (16), F-PF\textsuperscript{I}, F-CR (20).

To gain a better understanding of the structural properties of the resulting network, we aggregate node centrality measures at the sector level, and also perform a spectral analysis of the adjacency matrix [36, 44]. From Table 8, we observe that on average, the network based on best-level OFIs contains more inner-sector connections than the other two counterparts, thus implying a sectorial structure. Table 7 presents the top five most significant stocks in terms of out-degree centrality in each network. For example, in Figure 12(a), Amazon (AMZN), Google (GOOGL, GOOG), Nvidia (NVDA), NFLX (Netflix) are five hubs that possess the highest predictive power for other stocks, thus hinting at the possibility of building cross-asset models. Interestingly, Figure 12(b) shows that even though Amazon, Netflix and Nvidia are still hubs, Google is replaced by Goldman Sachs (GS) and Facebook (FB). In Figure 12(c), Intuitive Surgical (ISRG), Broadcom (AVGO) and General Electric (GE) are new hubs, while Amazon (AMZN) is not.

Figure 13 shows a barplot with the average value for each of the top 20 largest singular values of the network adjacency matrix, for best-level OFIs, integrated OFI, and returns, where the average if performed over all constructed networks. For ease of visualization and comparison, we first normalize the adjacency matrix before computing the top singular values, which exhibit a fast decay. In addition to the significantly large top singular value revealing that the networks have a strong rank-1 structure, the next 6-8 singular values are likely to correspond to the more prominent industry sectors.
Figure 12: Network structure of the coefficient matrix (β_{ij}) constructed from forward-looking cross-impact models. The coefficients are averaged over 2017–2019. To render the networks more interpretable and for ease of visualization, we only plot the top 5% largest in magnitude coefficients. Nodes are coloured by the GICS structure and sorted by market capitalization. Green links represent positive values while black links represent negative values. The width of edges is proportional to the absolute values of their respective coefficients.

Table 7: Top 5 stocks according to node out-degree centrality in threshold networks shown in Figure 12.

The out-degree centrality for a specific node is the fraction of nodes its outgoing edges are connected to.

Table 8: Group degree centrality [21] for each GICS sector. According to the threshold networks as shown in Figure 12, we compute the fraction of stocks outside of a specific sector connected to stocks in this specific sector. The color of each sector in this table corresponds to the color in Figure 12.
Figure 13: Barplot of normalized singular values in descending order for the average coefficient matrix in *forward-looking cross-impact* models. The coefficients are averaged over 2017–2019. We perform Singular Value Decomposition (SVD) on the coefficient matrix and obtain the singular values. The $x$-axis represents the singular value rank, and the $y$-axis shows the normalized singular values.

### 5.4 Strategy Profitability Analysis

On the basis of one-minute-ahead return forecasts, we employ two common portfolio construction methods to evaluate the economic gains of the aforementioned predictive models.

**Forecast-implied portfolio.** Chinco et al. [13] propose a method for portfolio construction based on predicted returns from a specific forecasting model $F$, denoted as forecast-implied portfolio. The motivations can be summarized as follows.

- It only executes an order when the one-minute-ahead return forecast exceeds the bid-ask spread.
- It buys/sells more shares of the $i$-th stock when the absolute value of one-minute-ahead return forecast for $i$-th stock is higher.
- It buys/sells more shares of the $i$-th stock when the one-minute-ahead return forecasts for the $i$-th stock tend to be less volatile throughout the trading day.

This strategy allocates a fraction $w_{i,t}$ of its capital to the $i$-th stock, based on

$$w_{i,t} \overset{\text{def}}{=} \frac{1\{\lvert f_{i,t}^F \rvert > \text{sprd}_{i,t} \} \cdot f_{i,t}^F / \sigma_{i,t}^F}{\sum_{n=1}^{N} 1\{\lvert f_{n,t}^F \rvert > \text{sprd}_{n,t} \} \cdot \lvert f_{n,t}^F \rvert / \sigma_{n,t}^F}.$$  

(21)

Here $f_{i,t}^F$ represents the one-minute-ahead return forecast according to model $F$ for minute $(t + 1)$, $\text{sprd}_{i,t}$ represents the relative bid-ask spread, $\sigma_{i,t}^F$ represents the standard deviation of the model’s one-minute-ahead return forecasts for the $i$-th stock during the previous 30 minutes of trading, i.e. the standard deviation of
in-sample fits. The denominator is the total investment, so that the strategy is self-financed. If there are no stocks with forecasts that exceed the spread in a given minute, then we set \( w_{i,t} = 0 \), \( \forall i \).

**Long-short portfolio.** In this strategy, we sort stocks into deciles according to forecast returns of model \( F \). We then construct a portfolio that longs stocks with the highest forecast return (in decile 10), and shorts stocks with the lowest ones (in decile 1). We refer to this portfolio as the long-short portfolio [38]. Its weights are calculated as follows

\[
\begin{align*}
w_{i,t} & \overset{\text{def}}{=} \frac{1 \{ f_{t+1}^F > d_t^{(9)} \} - 1 \{ f_{t+1}^F < d_t^{(1)} \}}{\sum_{n=1}^N \left[ 1 \{ f_{t+1}^F > d_t^{(9)} \} + 1 \{ f_{t+1}^F < d_t^{(1)} \} \right]}.
\end{align*}
\]

(22)

Here \( f_{t+1}^F \) is the one-minute-ahead return forecast according to model \( F \) for minute \((t + 1)\). \( d_t^{(1)} \) and \( d_t^{(9)} \) represent the lowest and highest one tenth predicted returns of stocks, respectively. Alternatively, \( d_t^{(1)} = \inf \{ x \in \mathbb{R} : \frac{1}{N} \sum_{n=1}^N 1 \{ f_{t+1}^F \leq x \} \geq 10\% \} \) and \( d_t^{(9)} = \inf \{ x \in \mathbb{R} : \frac{1}{N} \sum_{n=1}^N 1 \{ f_{t+1}^F \leq x \} \geq 90\% \} \). The denominator is used to normalize the weights, so that the sum of the absolute weights equals 1.

Finally, we compute the profit and loss (PnL) of the resulting portfolios at each minute. Recall that \( R_{i,t} \) is the raw log return of stock \( i \) at time \( t \), thus \( (e^{R_{i,t}} - 1) \) is the raw return.

\[
\begin{align*}
PnL_{t+1} & = \sum_{i=1}^N w_{i,t} \times (e^{R_{i,t+1}} - 1).
\end{align*}
\]

(23)

**Performance.** Table 9 compares the performance of the above two strategies, based on forecast returns from various predictive models. It is worth noting that in the following analysis, the strategy ignores trading costs, as this is not the focus of our paper; furthermore, we report the PnL in basis points, for ease of comparison with typical transaction fees, often also reported in basis points. Table 9 reveals that portfolios based on forecasts of the forward-looking cross-impact model outperform those based on forecasts of the forward-looking price-impact model. We also note that compared with models using the lagged returns as predictors, the models incorporating lagged OFIs yield higher PnL, implying that the OFIs contain more predictive information than the returns. We deem this to be an interesting finding, and are not aware of previous works in the literature that provided empirical evidence in this direction.

The results in Table 10 indicate that the profitability of F-CI is relatively stable during 2017-2019, but F-PI experienced a steep fall in the last quarter of 2017. Future research should be undertaken to investigate this phenomenon, preferably over a more extended period of time.
Table 9: Results of the PnLs of two trading strategies based on different forward-looking models. The top, middle and bottom panels report results of models using lagged best-level OFIs ofi \( (F-\text{PI}^{[1]}, F-\text{CI}^{[1]}), \) lagged integrated OFIs ofi \( (F-\text{PI}^{I}, F-\text{CI}^{I}) \), and lagged returns \( (F-\text{AR}, F-\text{CR}) \), respectively.

Table 10: Average statistics of the forward-looking models \( F-\text{PI}^{[1]} \) (Eqn (15)), and \( F-\text{CI}^{[1]} \) (Eqn (16)). Q1, respectively Q4, denote the first, respectively last, quarter in a year. OOS \( R^2 \) is the out-of-sample \( R^2 \). PnL is average PnL in each minute for each quarter.

Discussion on the forward-looking models. Tables 5 and 6 reveal that, in contrast to the price-impact model, cross-sectional OFIs can provide considerably more additional explanatory power for future returns compared to contemporaneous returns. A possible explanation for this asymmetric phenomenon is that there exists a time lag between when the OFIs of a given stock are formed (a so-called flow formation period) and the actual time when traders notice this change of flow and incorporate it into their trading models. For example, assume a trader submitted an unexpectedly large amount of buy limit orders of Apple (AAPL) at 10:00, at either the first level or potentially deeper in the book. Other traders (most likely, not high-frequency players) may notice this anomaly and adjust their portfolios (including Apple) at a much later time, for example 10:01. In this case, the OFIs of Apple may indicate future price changes of other stocks.

Consistent with our explanation, Hou [32] argues that the gradual diffusion of industry information is a leading cause of the lead-lag effect in stock returns. Cohen and Frazzini [16] find that certain stock prices do not promptly incorporate news pertaining to economically related firms, due to the presence of investors subject to attention constraints. Further research should be undertaken to investigate the origins of success of cross-sectional OFIs in predicting future returns.

It is also interesting to note that forward-looking models using integrated OFIs (the bottom panel in Table 6) cannot significantly outperform models using the best-level OFIs (the top panel in Table 6). This
phenomenon might stem from the fact that the integrated OFIs do not explicitly take into account the level information (distance of a given level to the best bid/ask) of multi-level OFIs, and are agnostic to different sizes resting at different levels on the bid and ask sides of the book. To verify this claim, we conduct experiments of forward-looking price-impact models with multi-level OFIs in Appendix D. The results support our earlier intuition that certain level-specific information, useful for predicting future returns, is lost in the integration process of the multi-level OFIs. In accordance with our findings, previous studies [29, 10] have demonstrated that limit orders at different price levels may have different information content with respect to predicting future returns.

6 Conclusion

In this work, we systematically examine price impacts of order flow imbalances on returns from multiple perspectives. Our main contributions can be summarized as follows.

First, we verify the effects of multi-level OFIs on price dynamics. We find that, as more multi-level OFIs are included as predictors, the explained percentage of total variance steadily increases in in-sample tests, but the increments become smaller as we go deeper in the book. Meanwhile, the model’s performance in out-of-sample tests gets worse after a certain level of OFIs. This is a sign of overfitting and implies the redundancy of information in multi-level OFIs. We also observe a strong multi-collinearity across multi-level OFIs, leading us to perform principal component analysis on them. The results show that the first principal component can explain a large proportion (over 80%) of the total variance. In order to avoid overfitting, we propose to transform multi-level OFIs into an integrated variation according to the first principal component. Our integrated OFIs demonstrate a higher $R^2$ in spite of time-series variations and cross-sectional variations in out-of-sample tests. Interestingly, $R^2$ values of the price-impact model are found to be linked to bid-ask spread, volatility, and volume.

Second, we introduce a new procedure to examine the cross-impact on contemporaneous returns. Under the sparsity assumption of cross-impact coefficients, we use LASSO to describe such a structure and compare the performances with the price-impact model, which only utilizes a stock’s own OFIs. We implement models with the best-level OFIs and integrated OFIs, respectively. The results demonstrate that in comparison with the price-impact model using best-level OFIs, the cross-impact model can provide additional explanatory power, small but significant, in both in-sample and out-of-sample tests. However, the cross-impact model with integrated OFIs cannot provide extra explanatory power to the price-impact model with integrated OFIs, indicating the effectiveness of our integrated OFIs.

At last, we apply the price-impact and cross-impact models to the challenging task of predicting future
returns. The results reveal that involving cross-sectional OFIs can increase both in-sample and out-of-sample $R^2$. We subsequently demonstrate that this increase in out-of-sample $R^2$ leads to additional profits, when incorporated in common trading strategies.

**Future research directions.** There are a number of interesting avenues to explore in future research. One such direction pertains to the assessment of whether cross-asset **multi-level** OFIs can improve the forecast of the future returns (in the present work, we only considered the best-level OFI and integrated OFI in this context). Another interesting direction pertains to performing a similar analysis as in the present paper, but for the last 15-30 minutes of the trading day, where a significant fraction of the total daily trading volume occurs. For example, according to [4], for the first few months of 2020 in the US equity market, about 23% of trading volume in the 3,000 largest stocks by market value has taken place after 3:30 p.m, compared with about 4% from 12:30 p.m. to 1 p.m. It would be an interesting study to explore the interplay between the OFI dynamics and this surge of volume at the end of US market hours.

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In this experiment, we use a 30-minute window to estimate models using LASSO. We then apply the estimated coefficients to fit data in the following one minute, and repeat this procedure every minute. Results are summarized in Table 11, and reveals similar conclusion as in Section 4.

|                  | PI (%) | CI (%) | $\Delta R^2$ (%) | p-Value |
|------------------|--------|--------|------------------|---------|
| **PI**           |        |        |                  |         |
| In-Sample        | 70.80  | 73.55  | 2.75             | 0.00    |
| Out-Of-Sample    | 59.67  | 61.46  | 1.79             | 0.00    |
| **CI**           |        |        |                  |         |
| In-Sample        | 86.10  | 86.84  | 0.74             | 0.00    |
| Out-Of-Sample    | 78.88  | 78.91  | 0.03             | 0.46    |

Table 11: Average statistics of the models $\text{PI}^{[1]}$ (Eqn (6)), $\text{CI}^{[1]}$ (Eqn (12)), $\text{PI}^{[I]}$ (Eqn (11)) and $\text{CI}^{[I]}$ (Eqn (13)) in one-minute update frequency. The first and second column report the adjusted-$R^2$ of price-impact and cross-impact model, respectively. $\Delta R^2$ is the increase in adjusted-$R^2$. p-Value represents the probability of observing the increase in adjusted-$R^2$ under the null hypothesis of no increase.

Cross-Impact of Multi-level OFIs on Contemporaneous Returns

We use the entire cross-sectional multi-level OFIs as candidate predictors to fit return $r_{i,t}$, as shown in Eqn (24). In this experiment, there are approximately 1,000 candidate features (100 stocks $\times$ 10 levels) in each
regression. We follow the same setting described in Section 3.3 and use LASSO to train this regression. Specifically, we use a 30-minute estimation window to fit models and then apply the estimated coefficients to fit data in the next 30 minutes. The time interval is 10 seconds, and thus the number of observations in each regression is 180.

\[ r_{i,t}^{[10]} = \alpha_i^{[10]} + \sum_{m=1}^{10} \beta_{i,i}^{[10],m} \text{ofi}_{i,t}^m + \sum_{m=1}^{10} \sum_{j \neq i} \beta_{i,j}^{[10],m} \text{ofi}_{j,t}^m + \eta_{i,t}^{[10]}. \]  

(24)

The results are summarized in Table 12 and reveal a similar conclusion as in Section 4. Additionally, compared with Table 5, we can conclude that CI and CI\[10\] have similar performances. Given the fact that the number of predictors used in the former is 1/10 of the number in the latter, the advantages of integrated OFIs are once again reflected.

|          | PI (%) | CI (%) | \(\Delta R^2\) (%) | \(p\)-Value |
|----------|--------|--------|---------------------|------------|
| [ofi\[1\],..., ofi\[10\]]  |        |        |                     |            |
| In-Sample| 89.28  | 89.61  | 0.33                | 0.00       |
| Out-Of-Sample | 84.21  | 83.35  | -0.86               | 1.00       |

Table 12: Average statistics of the models PI\[10\], and CI\[10\] (Eqn (24)). The first and second column report the adjusted-\(R^2\) of price-impact and cross-impact model, respectively. \(\Delta R^2\) is the increase in adjusted-\(R^2\). \(p\)-Value represents the probability of observing the increase in adjusted-\(R^2\) under the null hypothesis of no increase. Note that here we use LASSO to fit PI\[10\] considering the multi-collinearity among multi-level OFIs; therefore, the results in first column are different from those reported in Table 1.

## C Common Factors of OFIs

Capponi and Cont [11] propose a two-step procedure to justify the significance of cross-impact terms. In the first step, they use ordinary least squares (OLS) to decompose each stock’s OFIs into the common factor and the idiosyncratic components. In the second step, they regress returns of stock \(i\) on the common factor, its own idiosyncratic component and the idiosyncratic component of all other stocks. Nonetheless, Capponi and Cont [11] only consider the in-sample performance of their proposed model, and ignore the out-of-sample performance in the experiments. In addition, they employ OLS regression in both steps of their proposed pipeline, a technique unable to capture the potential sparsity of the cross-impact terms.

Motivated by this line of work, we follow their first step to obtain the market factor and residuals of each stock. However, we employ LASSO in the second step to testify the intraday cross-impact of the idiosyncratic OFIs. More specifically, as shown in Eqn (25), we first regress a stock’s OFI (ofi\[1\]_{i,t}) on the common factor of OFIs (\(F_{ofi,t}\)), that is the first principal component of the cross-sectional order flow imbalances, and obtain
the idiosyncratic components \((\tau_{i,t})\) of the OFIs, for each individual stock.

\[
ofi_{i,t}^1 = \mu_i + \gamma_i F_{ofi,t} + \tau_{i,t}. \tag{25}
\]

Subsequently, we regress returns \((r_{i,t})\) of stock \(i\) against (i) the common factor of OFIs \((F_{ofi,t})\), (ii) the idiosyncratic components of its own OFIs \((\tau_{i,t})\), and (iii) the idiosyncratic components of the OFIs of other stocks \((\tau_{j,t}, j \neq i)\) via LASSO. In other words, we use the common factor of OFIs and the entire cross-section of OFI residuals as candidate features to fit returns. Eventually, we arrive at the cross-impact model in Eqn (26), denoted as \(CI^M\).

\[
CI^M : \quad r_{i,t} = \alpha_i^M + \beta_{i0}^M F_{ofi,t} + \beta_{ii}^M \tau_{i,t} + \sum_{j \neq i} \beta_{ij}^M \tau_{j,t} + \eta_{i,t}. \tag{26}
\]

We compare \(CI^M\) with a parsimonious model \(PI^M\) (Eqn (27)), in which only a stock’s own OFI residual and the common order flow factor are utilized.

\[
PI^M : \quad r_{i,t} = \alpha_i^M + \beta_{i0}^M F_{ofi,t} + \beta_{ii}^M \tau_{i,t} + \epsilon_{i,t}. \tag{27}
\]

We then train the \(PI^M\) and \(CI^M\) models on historical data, under the same setting as in Section 4.3. Similarly, we present the average \(R^2\) values of \(PI^M\) and \(CI^M\) in Table 13. We observe small improvements (of only 1.13%) from \(PI^M\) to \(CI^M\) in in-sample tests. From considering Tables 5 and 13, we also observe that introducing the common factor leads to small changes in the model’s explanatory power of price dynamics in the in-sample tests. However, both the price-impact and cross-impact models perform much worse in out-of-sample data, highlighting once again the importance of out-of-sample testing.

| \(ofi^1\) | \(PI^M\) (%) | \(CI^M\) (%) | \(\Delta R^2\) (%) | \(p\)-Value |
|---------|-------------|-------------|-----------------|--------|
| In-Sample | 71.38 | 72.51 | 1.13 | 0.0 |
| Out-Of-Sample | 31.87 | 33.52 | 1.65 | 0.0 |
| \(ofi^I\) | \(PI^M\) (%) | \(CI^M\) (%) | \(\Delta R^2\) (%) | \(p\)-Value |
| In-Sample | 85.71 | 86.09 | 0.38 | 0.0 |
| Out-Of-Sample | 46.27 | 46.82 | 0.55 | 0.0 |

Table 13: Average statistics of the \(PI^M\) (Eqn (27)) and \(CI^M\) (Eqn (26)) models. The first and second column report the adjusted-\(R^2\) of the price-impact and cross-impact models, respectively. \(\Delta R^2\) is the increase in adjusted-\(R^2\). \(p\)-Value represents the probability of observing the increase in adjusted-\(R^2\), under the null hypothesis of no increase.
D Price-Impact of Multi-level OFIs on Future Returns

We use multi-level OFIs as candidate predictors to fit future returns $R_{i,t+1}$, as shown in Eqn (28). Given the fact that multi-level OFIs have a strong multicollinearity, we use LASSO to fit this regression. We follow the same setting described in Section 5. Specifically, we use a 30-minute estimation window to fit models, and then apply the estimated coefficients for making one-minute-ahead return forecasts.

\[
F\text{-PI}^{[10]} : \quad R_{i,t+1} = \alpha_i^F + \sum_{m=1}^{10} \left[ \beta_i^{F,m} \text{ofi}_{i,t}^m + \gamma_i^{F,m} \text{ofi}_{i,t-1}^m + \delta_i^{F,m} \text{ofi}_{i,t-2}^m \right] + \eta_i^F. \tag{28}
\]

The results in Table 14 provide evidence that the forward-looking price-impact model using multi-level OFIs significantly outperforms the forward-looking price-impact model with best-level OFIs or with integrated OFIs. These numerical results indicate that the integrated OFIs potentially lose valuable information in multi-level OFIs, which is important to forecast future returns.

|                  | F-PI$^{[1]}$ | F-PI$^{I}$ | F-PI$^{[10]}$ |
|------------------|-------------|------------|--------------|
| $R^2$ (%)        | In-Sample   | 0.24       | 0.26         | 5.68         |
|                  | Out-Of-Sample| -19.07     | -17.62       | -10.42       |
| PnL (bps)        | Forecast-implied | 0.19       | 0.20         | 0.46         |
|                  | Long-Short  | 0.10       | 0.12         | 0.51         |

Table 14: Average statistics of the models F-PI$^{[1]}$ (Eqn (15)), F-PI$^{I}$ and F-PI$^{[10]}$ (Eqn (28)). The top panel reports the adjusted-$R^2$ on in-sample and out-of-sample data, respectively. The bottom panel reports the PnLs of forecast-implied portfolio and long-short portfolio, respectively. Note that the results with respect to F-PI$^{[1]}$ and F-PI$^{I}$ (the first two columns in this table) are the same as results in Tables 6 and 9.

E Price-Impact with Best-level OFIs over Multiple Future Horizons

To study the impact of lagged OFIs on multi-horizon future returns, we provide the following two approaches.

**PnL-based.** In this final experiment, we turn to study the profitability of a simple OFI-based trading strategy [15], i.e. to long a stock if its current OFI is positive, and short if the current OFI is negative, then hold for a specific period. We compute the profit and loss (PnL) as follows, and consider PnL$_h$ as the daily PnL in a holding period $h$.

\[
PnL_{t,h} = \text{sign} \left( \text{ofi}_i^1 \right) \times \sum_{t=1}^{h} R_{t+t}.
\]

\[
PnL_h = \frac{1}{N_h} \sum_{t<t_{\text{max}}} \text{PnL}_{t,h}, \tag{29}
\]

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where \( N_h \) is the number of time buckets and \( t_{\text{max}} \) is the time of the last OFI. For example, if we study the minutely PnLs over the next 1 hour, then \( N_h = 330 \) and \( t_{\text{max}} = 15:00 \). An example of the PnL calculation is depicted in Figure 14. Figure 15 shows the PnL profile curve, as a function of the holding period. We find that the OFI over a given bucket, attains the highest PnL over the following 2-3 minutes; this PnL remains positive for up to approx 12 minutes, beyond which the PnL becomes slightly negative.

**Impact-based.** In this method, we first estimate the following model for each stock. To facilitate notation in this section, we ignore the subscript for stocks. In F-PI\(^{MF}\), the coefficient \( \beta_s \) represents the impact of \( \text{ofi}_1^{t-s+1} \) (i.e. OFIs in \([t-s, t-s+1]\)) on \( R_{t+1} \) (i.e. return in \([t, t+1]\)) after excluding the influences from other order flows during \([t-p, t)\),

\[
\text{F-PI}^{MF} : \quad r_{t+1} = \alpha^{MF} + \sum_{s=1}^{P} \beta_s \text{ofi}_{t-s+1} + \eta_t^{MF}.
\]  

Figure 16 provides an example to illustrate the connection between the model (30) and the impact of a specific \( \text{ofi}_1 \) on multi-horizon future returns. In this example, we focus on the impact of order flow imbalance in \([11:59, 12:00]\) (the blue line in each panel). At \( t = 12:00 \), we see that the impact of one unit of \( \text{ofi}_1 \) in \([11:59, 12:00]\) on the return in \([12:00, 12:01]\) is \( \beta_1 \) (the red line in the top panel). Similarly, at \( t = 12:01 \), the impact of one unit of \( \text{ofi}_1 \) in \([11:59, 12:00]\) on the return in \([12:01, 12:02]\) is \( \beta_2 \) (the red line in the middle
Thus we conclude that the impact of a specific ofi\textsuperscript{1} on future returns in \([t, t + 1) \ (t \geq t_0)\) can be considered as \(\beta_{t+1-t_0}\). Furthermore, the cumulative future price changes or returns up to time \(t + 1 \ (t \geq t_0)\) due to ofi\textsuperscript{1} are the cumulative sum of coefficients \(\sum_{s=1}^{t+1-t_0} \beta_s\).

\[
\begin{align*}
\text{Coefficients } \{\beta_1, \ldots, \beta_p\} \text{ are estimated from model (30).}
\end{align*}
\]

In order to perform a granular analysis of the multi-horizon impact, while still maintaining the computations feasible, we set the computation frequency of OFIs and returns at 10 seconds, while updating the forecasting model F-PI\textsuperscript{MF} every day rather than every 1-min or 30-min. Figure 17 reveals that in the first bucket after the creation of a specific OFI, on average, the price is increasing because the first coefficient is positive. However, after the first phase, the coefficients are mostly negative, implying prices are decreasing. Finally, the OFI loses its impact on price dynamics at around 55 minutes after its arrival.

\[
\begin{align*}
\text{Figure 17: Cumulative sum of the coefficients in model (30). The x-axis represents the future horizons (in minutes); the y-axis represents the cumulative sum of coefficients (in basis points). Coefficients are averaged across stocks and days.}
\end{align*}
\]