KONDO EFFECT IN NON-EQUILIBRIUM

Theory of energy relaxation induced by dynamical defects in diffusive nanowires

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Abstract In diffusive Cu and Au quantum wires at finite transport voltage $U$ the non-equilibrium distribution function $f(E,U)$ exhibits scaling behavior, $f(E,U) = f(E/eU)$, indicating anomalous energy relaxation processes in these wires. We show that in nonequilibrium the Kondo effect, generated either by magnetic impurities (single-channel Kondo effect) or possibly by non-magnetic, degenerate two-level systems (two-channel Kondo effect), produces such scaling behavior as a consequence of a Korringa-like (pseudo)spin relaxation rate $\propto U$ and of damped power-law behavior of the impurity spectral density as a remnant of the Kondo strong coupling regime at low temperature but high bias. The theoretical, scaled distribution functions coincide quantitatively with the experimental results, the impurity concentration being the only adjustable parameter. This provides strong evidence for the presence of Kondo defects, either single- or two-channel, in the experimental systems. The relevance of these results for the problem of dephasing in mesoscopic wires is discussed briefly.

1. INTRODUCTION

Recently, it has become possible to determine experimentally [1, 2] the distribution function of quasiparticles, $f_x(E,U)$, in dependence of the particle energy $E$ in a metallic, diffusive nanowire in stationary nonequilibrium with a finite transport voltage $U$ applied between the ends of the wire. The measurements were done by attaching an additional superconducting tunneling electrode at a position $x$ along the wire, so
that \( f_x(E, U) \) could be extracted from the tunneling current

\[
j_t = \frac{e}{h} \gamma N_o \int dE \left[ f_x(E, U) - f^o(E + eU_t) \right] N_{BCS}(E + eU_t).
\]

Here \( N_{BCS} \) is the peaked BCS density of states in the superconducting electrode, \( N_o \) and \( \gamma \) are the approximately constant density of states in the wire and the tunneling matrix element, respectively, and \( U_t \) denotes the voltage between the wire and the tunneling electrode. \( f^o(E) = 1/(e^{-E/k_BT} + 1) \) is the Fermi distribution at temperature \( T \).

As expected for a diffusive wire with elastic impurity scattering [3], \( f_x(E, U) \) has a double-step shape, corresponding to the two chemical potentials at \( E = 0 \) and \( E = eU \) at the ends of the wire. In addition, the steps are rounded, indicating inelastic scattering processes in the wire. In Cu [1] and Au [2] wires, the observed rounding is such that the quasiparticle distribution obeys a remarkable scaling property when \( U \) exceeds a certain low-energy scale, \( eU \sim eU_o \approx 0.1\text{meV} \): \( f_x(E, U) = f_x(E/eU) \) [1]. One may gain phenomenological insight into the nature of the inelastic relaxation processes by observing [1] that the scaling property of \( f_x(E, U) \) implies that the equation of motion of \( f_x(E, U) \), the quantum Boltzmann equation, and consequently the inelastic single-particle collision rate \( 1/\tau \) are scale invariant as well. Assuming a (yet to be determined) two-particle potential \( \tilde{V}(\varepsilon) \) with energy transfer \( \varepsilon \), \( 1/\tau \) is in 2nd order perturbation theory given as

\[
\frac{1}{\tau(E)} = \frac{1}{\tau(E/eU)} \simeq N_o^3 \int d\varepsilon \int d\varepsilon' |\tilde{V}(\varepsilon)|^2 F \left( \frac{E}{eU}, \frac{\varepsilon}{eU}, \frac{\varepsilon'}{eU} \right),
\]

where \( F \) is a combination of distribution functions \( f_x \) ensuring that there is only scattering from an occupied into an unoccupied state. The experimental finding that \( f_x \) and hence \( 1/\tau \) depend only on the dimensionless energies, as indicated in Eq. (1.2), implies that in the term on the right-hand side only dimensionless energies (normalized to \( eU \)) occur, i.e. \( \tilde{V}(\varepsilon) \propto 1/\varepsilon \) in the scaling regime \( |E| \lesssim k_B U_o \lesssim eU \). It follows that within 2nd order perturbation theory (Eq. (1.2)) the energy relaxation rate has a logarithmic energy dependence, \( 1/\tau(E) \propto \ln(E/eU_o) \), non-vanishing at the Fermi energy. This anomalous behavior has raised considerable interest in the energy relaxation measurements [1], especially because of the possible relation of the apparently non-vanishing energy relaxation rate to the problem of dephasing saturation observed at low \( T \) in the magnetoresistance of nanoscopic wires [4, 5, 2].

The infrared singular behavior of \( \tilde{V}(\varepsilon) \), phenomenologically deduced above, means in particular that the interaction has no essential momentum dependence and should be of a local origin, in contrast to an inter-
action mediated by a diffusive or dispersive collective mode. Therefore, it has been proposed [6] that the anomalous energy relaxation may be induced by Kondo impurities, in particular by (nearly) degenerate atomic two-level systems [6] or other dynamical defects in the nanowire which can generate a two-channel Kondo (2CK) effect [7, 8]. The 2CK effect is known to have ideally a non-vanishing zeropoint entropy \( S(0) = k_B \ln \sqrt{2} \) [9, 10] and, hence, can cause dephasing at low energies [11]. In fact, the electron interaction vertex mediated by a Kondo impurity does show a \( 1/\varepsilon \) energy dependence. This was noted for the 2CK effect in the strong coupling region [6] (Fig. 1 a)) and independently for the weak coupling (i.e. high energy) regime of the magnetic, single-channel Kondo (1CK) effect [12], see also [13]. Although this analysis takes elevated single-particle energies into account, it assumes the Kondo defect to be in thermodynamic equilibrium. Since, however, the effective interaction is generated dynamically due to a Fermi edge singularity, it will be substantially modified in the non-equilibrium situation of the energy relaxation measurements considered here.

Although the Kondo effect in stationary non-equilibrium has been treated by various numerical methods before, no complete understanding has been reached up to now. In particular, the scaling properties and to what extent the Kondo effect at finite bias is a strong coupling problem have remained unclear. In this article we, therefore, summarize a predominantly analytical analysis of the 1CK and the 2CK non-equilibrium Kondo effect [14]. It is shown that in nanowires with either 1CK or 2CK defects the quasiparticle distribution function exhibits scaling in terms of the applied bias, \( eU \), when \( eU \) exceeds an intrinsic low-energy scale \( eU_o \) which, up to logarithmic corrections, is proportional to the equilibrium Kondo temperature \( T_K^{(0)} \). The numerical solution of the quantum Boltzmann equation gives quantitative agreement with the experimental results discussed above, where in the scaling regime the impurity concentration is the only adjustable parameter.

2. A SINGLE KONDO IMPURITY IN A NON-EQUILIBRIUM WIRE

The 1CK as well as the 2CK system can be described by a generalized \( \text{SU}(N) \times \text{SU}(M) \) Anderson impurity model in the Kondo limit, i.e. in terms of a quantum degree of freedom in a low-lying level \( \varepsilon_d \), coupled to the continuum of conduction electrons via a hybridization matrix element \( V \), where \( N = 2 \) denotes the (pseudo)spin degeneracy and \( M = 1, 2 \) the number of degenerate, conserved conduction electron channels.
In the auxiliary particle representation [15] the model Hamiltonian reads

$$H = \sum_{p,m,\sigma} \epsilon_p c_{pm\sigma}^\dagger c_{p\sigma} + \epsilon_d \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} + V \sum_{p,m,\sigma} (f_{\sigma}^\dagger b_{m} c_{pm\sigma} + h.c.) , \quad (1.3)$$

where $c_{pm\sigma}^\dagger$ is the creation operator for an electron with momentum $\vec{p}$. The auxiliary fermion and boson operators, $f_{\sigma}^\dagger$, $b_{m}^\dagger$, create the local defect in its quantum state $\sigma$ or in the unoccupied state, respectively. The operator constraint $\hat{Q} = \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} + \sum_{m} b_{m}^\dagger b_{m} \equiv 1$ ensures that the defect is in exactly one of its quantum states at any instant of time. In the 1CK case of a magnetic Anderson impurity $\sigma$ denotes spin, and $m = 1$ has no relevance. For a 2CK defect, $\sigma$ is identified with a pseudospin, e.g. the parity of the local defect wave function, and $m = 1, 2$ is the conduction electron spin acting as the conserved channel degree of freedom. Note that in this case the bosonic operator $b_{m}^\dagger$ transforms according to the conjugate representation of SU(M) in order for the channel degree of freedom to be conserved. The equilibrium Kondo temperature of the model is (to leading exponential order) $T_{K}^{(0)} \simeq W \exp(-1/N_N J)$, with $J = V^2/|\epsilon_d|$ the effective spin exchange coupling and $W$ the half band width.

The auxiliary particle representation allows for the application of standard Green’s function methods and, in particular, is readily generalized [16] to non-equilibrium by means of the Keldysh technique. It is known [17] to describe the universal spectral properties of the multi-channel Kondo effect correctly both in the infrared and in the high energy regime already within the non-crossing approximation (NCA), which is a self-consistent, conserving approximation to leading order in the effective hybridization $\Gamma = V^2 \pi N_\sigma$. Thus, in the multichannel case the NCA constitutes a correct infinite resummation of the logarithmic terms of
perturbation theory. In the 1CK case at finite bias the logarithmic series is the same as for the 2CK effect in equilibrium because of the presence of two Fermi edges in the former case [14]. Thus, the NCA gives a correct description both of the 1CK and of the 2CK effects in a strong non-equilibrium situation \((eU > T_K^{(0)})\) and will be used for the explicit calculations below.

In the remainder of this section we will consider a single Kondo defect at position \(x\) in a diffusive wire of length \(L\) with a bias voltage \(U\) applied between its ends. The generalization to the experimental situation of a dilute ensemble of Kondo impurities will be treated in the following section. The derivations apply both to the single and to the multi-channel case. For purely elastic scattering in the wire the electronic distribution function at the position of the impurity is

\[
f_x(E, U) = \frac{x}{L} f^0(E + eU) + \left(1 - \frac{x}{L}\right) f^0(E). \tag{1.4}
\]

Denoting the momentum integrated greater (+) and lesser (−) Keldysh Green’s functions for the conduction electrons at position \(x\) by

\[
G^\pm_{c,x}(E, p) = 2\pi i f_x(E, U) N_o(E), \quad G^{-c,x}(E, p) = -2\pi i(1 - f_x(E, U)) N_o(E), \quad \text{and similarly the “greater”, “lesser” and retarded auxiliary particle propagators by } G^{\pm}_r, f, b(\omega), \text{ the non-equilibrium NCA equations then read (Fig. 1 b))}
\]

\[
\frac{G^+_f(\omega)}{|G^+_f(\omega)|^2} = -M \Gamma \int \frac{d\epsilon}{2\pi i} G^\pm_{c,x}(-\epsilon) G^\pm_b(\omega + \epsilon) \tag{1.5}
\]

\[
\frac{G^+_b(\omega)}{|G^+_b(\omega)|^2} = +N \Gamma \int \frac{d\epsilon}{2\pi i} G^\mp_{c,x}(\epsilon) G^\mp_f(\omega + \epsilon). \tag{1.6}
\]

This set of non-linear equations is closed by the Kramers-Kroenig relations, \(G^r_{f,b}(\omega) = \int d\epsilon/(2\pi i) G^r_{f,b}(\omega)/(\omega - \epsilon + i0)\), which follow from analyticity and the fact that the auxiliary particle Green’s functions have only forward in time propagating parts. The conduction electron \(t\)-matrix due to the Kondo impurity is

\[
t^\pm_{c,x}(E) = -\frac{\Gamma}{\pi N_o(0)} \int \frac{d\epsilon}{2\pi i} G^\pm_f(E + \epsilon) G^\mp_b(\epsilon). \tag{1.7}
\]

It is crucial to determine the low-energy scales of the problem in non-equilibrium. These may be determined from perturbation theory. At bias \(eU\) the Kondo temperature, defined as the breakdown scale of perturbation theory, is suppressed, e.g. in the middle of the wire \((x = 1/2)\), as

\[
T_K(eU) = \sqrt{T_K^{(0)} + \frac{(eU)^2}{2k_B}} - \frac{eU}{2k_B} \quad eU \gg k_B T_K^{(0)} \simeq \frac{k_BT_K^{(0)}^2}{eU}. \tag{1.8}
\]
In addition, a Korringa-like spin relaxation rate $1/\tau_s \propto eU$ appears, because in non-equilibrium there is finite phase space available for scattering even at $T = 0$,

$$\frac{1}{\tau_s(eU)} = \frac{1}{4\pi} NM (N_oJ)^2 eU < eU .$$  \hspace{1cm} (1.9)

Because of this inelastic relaxation rate, the logarithmic singularities of perturbation theory ($T = 0$) are shifted by $1/\tau_s$ to the complex frequency plane, and the Kondo scale disappears eventually for large bias $eU$ and is replaced by the inelastic rate $1/\tau_s$: The low-temperature scale is $T_o(eU) = \max[T_K(eU), 1/\tau_s(eU)]$. The crossover from the Kondo to the Korringa scale occurs at a bias

$$eU_o = \frac{2}{N_oJ} \sqrt{\frac{\pi}{NM}} k_B T_K^{(0)} = 2N \sqrt{\frac{\pi}{NM}} k_B T_K^{(0)} \ln\left(\frac{W}{T_K^{(0)}}\right) .$$ \hspace{1cm} (1.10)

This means that in the large bias regime, $eU > eU_o$, the low-energy scale of the problem is proportional to the external bias itself.

We now turn to the scaling analysis of the non-equilibrium Kondo equations (1.5)–(1.7). It is well known [18, 19], that at $T = 0$ in equilibrium the exact auxiliary particle propagators exhibit infrared powerlaw divergencies $G_{f,b}^{-}(\omega) \propto \Theta(\omega) \omega^{-\alpha_{f,b}}$, where the exponents $\alpha_{f,b}$ are due to an orthogonality catastrophe between the occupied and the unoccupied impurity state. In the 2CK case the exact exponents are reproduced by NCA [17] and depend in a characteristic way on the spin degeneracy $N$ and the channel number $M$, $\alpha_f = M/(N + M)$, $\alpha_b = N/(N + M)$ with $\alpha_f + \alpha_b = 1$. In the large bias regime ($eU > eU_o$), the NCA equations are valid both in the 1CK and in the 2CK case, as mentioned above. The behavior around the Fermi edges may be determined by defining the complex frequency variables $z = \omega - i/2\tau_s(eU)$ etc. and rewriting the NCA equations for $z \rightarrow 0$ and $z \rightarrow eU$ as coupled differential equations in analogy to the equilibrium case [18]. We obtain damped powerlaw behavior of the auxiliary fermion propagator for energies $|\omega| \lesssim 1/\tau_s$ and of the auxiliary boson propagator for energies around the two Fermi edges $|\omega| \lesssim 1/\tau_s$, $|\omega - eU| \lesssim 1/\tau_s$ where the damping constant is $1/\tau_s$. The exponents are modified in the large bias regime to $\alpha'_f = 2M/(N + 2M)$, $\alpha'_b = N/(N + 2M)$, where the additional factor of 2 originates from the fact that there are two separated Fermi edges. The form of $\alpha'_f$, $\alpha'_b$ is reminiscent of an effective doubling of the channel number $M$ induced by the two Fermi edges. This behavior is confirmed by the numerical evaluation of the problem and is consistent with a recent perturbative renormalization group analysis [20] of the problem. The fact that powerlaw behavior (although damped) instead of logarithmic behavior persists
even at large bias may be seen as a remnant of the strong coupling behavior of the Kondo problem in non-equilibrium. It is crucial for scaling in terms of $eU$ to occur: Putting the powerlaw behavior of the auxiliary propagators back into Eqs. (1.5)–(1.7), using $\alpha_f' + \alpha_b' = 1$ and the important fact that the low-energy scale $T_0$ and the damping rate of the powerlaw behavior themselves scale with the external bias, $T_0 1/\tau_s \propto eU$, it follows immediately that these equations depend only on dimensionless energies $\omega/eU$, $E/eU$ etc., i.e. they are scale invariant in the regime $eU > eU_o = 2N \sqrt{\pi N M k_B T(0) K \ln (W/T_K(0))}$ (Eq. 1.10). It is seen that in experiments one may extract the equilibrium Kondo temperature $T_K^{(0)}$ from the lower breakdown scale of the scaling behavior.

3. SOLVING THE KINETIC EQUATION

The remaining task is to calculate the electronic non-equilibrium distribution function $f_x(E, U)$ explicitly at an arbitrary position $x$ along the wire in the presence of a dilute but finite density of 1CK or 2CK defects $c_{\text{imp}}$.

The Boltzmann quantum kinetic equation for the momentum-dependent “greater” conduction electron Green’s function $G_{c,(x,t)}^+(E, \vec{p})$, which contains information about the distribution function, reads [21]

$$\left[ \frac{\partial}{\partial t} + \nabla_x \cdot \frac{\vec{p}}{m} + e\vec{E}(\vec{x}) \cdot \nabla_p \right] G_{c,(x,t)}^+(E, \vec{p}) = \mathcal{C}(\{G_{c,(x,t)}^+(E, \vec{p})\}, c_{\text{imp}}),$$

(1.11)

where $\mathcal{C}$ is the collision integral induced by inelastic scattering off Kondo defects and $\vec{E}(\vec{x})$ are the external electric fields, including those generated by static, random impurities in the wire. Exploiting the fact that the current in the disordered system is diffusive,

$$\vec{j}_x(E) = \sum_{\vec{p}} \frac{\vec{p}}{m} G_{c,x}^+(E, \vec{p}) = -D \nabla_x \rho_x(E) = -D \nabla_x \sum_{\vec{p}} G_{c,x}^+(E, \vec{p}),$$

(1.12)

with $D$ the diffusion coefficient, and summing in Eq. (1.11) over quasiparticle momenta $\vec{p}$, the electric field term drops out as a surface term, and we obtain in the stationary case a diffusive kinetic equation [3] for the impurity averaged distribution function $\overline{f}_x(E, U) = \sum_{\vec{p}} G_{c,x}^+(E, \vec{p}) / 2\pi i N_o$ as function of the quasiparticle energy $E$ and position $x$,

$$-D \nabla_x^2 \overline{f}_x(E, U) = \mathcal{C}(\{\overline{f}_x(E, U)\}, c_{\text{imp}}).$$

(1.13)

After momentum integration, the collision integral is given in terms of the impurity t-matrix (Eq. 1.7) to arbitrary order in the electron-
Figure 2 Scaled distribution functions $f_x(E,U)$ in nanowires. — Left panel: Two Cu nanowires ($L = 1.5 \mu m$, $D = 65 cm^2/s$, $x/L = 0.5$ and $L = 5.0 \mu m$, $D = 45 cm^2/s$, $x/L = 0.5$). Black curves: Experimental data for $0.05 mV \leq U \leq 0.3 mV$ in steps of $0.05 mV$ [1]. The two curves showing deviations from scaling are at small voltages. Light curves: Theory for 2CK impurities in the scaling regime, $eU > eU_o$. Fitted 2CK impurity concentration: $c_{imp} \approx 8 \cdot 10^{-6}/(lattice\ unit\ cell)$. — Right panel: Two Au nanowires (wire 1 and wire 2 of Ref. [2]), measured at different positions $x/L$. Open circles: Experimental data; Solid lines: Theory for magnetic impurities in the scaling regime. Fitted magnetic impurity concentration: $c_{imp} \approx 2 \cdot 10^{-4}/(lattice\ unit\ cell)$.

impurity coupling $\Gamma$ by

$$C(\{f_x(E,U)\}, c_{imp}) = \frac{c_{imp}}{2\pi N_o} [t_{c,x}^-(E)G_{c,x}^+(E) - t_{c,x}^+(E)G_{c,x}^-(E)]. \quad (1.14)$$

We have solved the set of non-linear integro-differential equations (1.5)–(1.7), (1.13), (1.14) numerically with the boundary conditions $f_{x=0}(E,U) = f^o(E)$, $f_{x=L}(E,U) = f^o(E+eU)$ and that the electron system should be in local equilibrium with the dynamical impurity. The results are shown in Fig. 2. The theoretical $f_{x}(E/eU)$ curves show scaling behavior in the regime $eU > eU_o$ (Eq. (1.10)) for bias voltages $U$ varying within a factor of 4 to 9, depending on the high energy parameters of the model Eq. (1.3). There is excellent quantitative agreement between
theory and experiment for all samples. The experimental data for Cu and Au wires can be fitted equally well by the 1CK or by the 2CK theory, with very similar defect concentrations in the two cases. As an example, Fig. 2 shows the fits of the Cu data to the 2CK theory and of the Au data to the 1CK theory only. The agreement is consistent with the conjecture that the anomalous energy relaxation in those wires is mediated either by 1CK or by 2CK defects.

4. RELATION TO DEPHASING

For an ideal 2CK system in equilibrium, the non-vanishing quasiparticle scattering rate $1/\tau(E)$ crosses over to a pure dephasing rate $1/\tau_\varphi$, as the quasiparticle energy approaches the Fermi energy, $E, T \to 0 \ [11]$. However, because of the finite level splitting $\Delta$ in real systems, one expects an upturn of $\tau_\varphi$ at the lowest $T \lesssim \Delta/k_B$, with an intermediate plateau for $\Delta/k_B \lesssim T \lesssim T_K^{(0)} \ [11]$. Such behavior can also be expected from magnetic (1CK) Kondo impurities in combination with a phonon contribution, because the spin flip rate reaches a maximum at $T \simeq T_K^{(0)} \ [22]$. This has been observed in the Kondo system AuFe [23] and most recently in clean Au wires presumably containing a small concentration of Fe impurities [2]. One might, therefore, conjecture that 1CK or 2CK defects could be the origin of the dephasing time saturation observed in magnetotransport measurements of weak localization [4, 5, 2]. This assumption is indeed supported by several coincidences between the dephasing time measurements and the results on the non-equilibrium distribution function: (1) The dephasing time $\tau_\varphi$ extracted from magnetotransport experiments [5, 4] is strongly material, sample, and preparation dependent. This suggests a non-universal dephasing mechanism, like dynamical defects, which is not inherent to the electron gas. (2) The dephasing time in Au wires is generically shorter than in Cu wires [5]. This is consistent with the fact that the estimates for the dynamical impurity concentration $c_{imp}$, obtained from the fit of the present theory to the experimental distribution functions, is substantially higher in Au than in Cu wires (Fig. 2). (3) In Ag wires one observes neither dephasing saturation nor $E/eU$ scaling of the distribution function [24]. This is consistent with the assumption that dephasing saturation and anomalous energy relaxation in the nanoscopic wires have the same origin and that there are no Kondo defects present in the Ag samples.
5. CONCLUDING REMARKS

We have analysed the single- as well as the two-channel Kondo effect in a stationary non-equilibrium situation. It was found that a Korringa-like, inelastic spin relaxation rate appears which at large bias sets the low-energy scale of the problem. Nevertheless, a remnant of the strong coupling Kondo fixed point persist even at large bias, which manifests itself by damped powerlaw behavior of the local spectral density. The latter leads to scaling of the non-equilibrium distribution function \( f_x(E, U) = \frac{f_x(E/eU)}{eU} \) as an experimentally observable signature. The present theory yields quantitative agreement with the experimental results for all samples measured, the density of Kondo defects in the wire being the only adjustable parameter in the scaling regime. This strongly suggests that the anomalous energy relaxation might be caused by either 1CK or 2CK defects. However, it was shown that the scaling property of the distribution function does not distinguish between 1CK or 2CK impurities. Further experimental tests, like application of a magnetic field, will be required for that purpose.

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References

[1] H. Pothier, S. Guéron, Norman. O. Birge, D. Esteve and M. H. Devoret, Phys. Rev. Lett. 79, 3490 (1997); Z. Phys. B 104, 178 (1997).
[2] F. Pierre, H. Pothier, D. Esteve, M. H. Devoret, A. B. Gougam, N. O. Birge, this volume, cond-mat/0012038.
[3] K. E. Nagaev, Phys. Lett. A 169 103 (1992); Phys. Rev. B 52, 4740 (1995).
[4] P. Mohanty, E.M.Q. Jariwala and R. A. Webb, Phys. Rev. Lett. 78, 3366 (1997).
[5] A. B. Gougam, F. Pierre, H. Pothier, D. Esteve and Norman O. Birge, J. Low Temp. Phys. 118, 447 (2000).
[6] J. Kroha, Adv. Solid. State. Phys. 40, 267 (2000).
[7] P. Nozières and A. Blandin, J. Phys. (Paris) 41, 193 (1980).
For a comprehensive overview and references see D. L. Cox and A. Zawadowski, Adv. Phys. 47 (5), 599–942 (1998).

P. B. Wiegmann and A. M. Tsvelik, Pis’ma Zh. eksp. teor. Fiz. 38, 489 (1983) [JETP Lett. 38, 591 (1983); Adv. Phys. 32, 453 (1983).

N. Andrei and C. Destri, Phys. Rev. Lett. 52, 364 (1984).

A. Zawadowski, J. v. Delft and D. Ralph, Phys. Rev. Lett. 83, 2632 (1999).

A. Kaminski and L. I. Glazman, cond-mat/0010379.

J. Sólyom and A. Zawadowski, Z. Phys. 226, 116 (1969).

J. Kroha and A. Zawadowski, in preparation.

S. E. Barnes, J. Phys. F 6, 1375 (1976); F 7, 2637 (1977).

M. H. Hettler, J. Kroha and S. Hershfield, Phys. Rev. Lett. 73, 1967 (1994); Phys. Rev. B 58, 5649 (1998).

D. L. Cox and A. E. Ruckenstein, Phys. Rev. Lett. 71, 1613 (1993).

E. Müller-Hartmann, Z. Phys. B 57, 281 (1984).

J. Kroha and P. Wölfle, Phys. Rev. Lett., 79, 261 (1997).

P. Coleman, C. Hooley and O. Parcollet, cond-mat/0012005.

E. M. Lifshitz and L. P. Pitaevskii, in Landau and Lifshitz Course of Theoretical Physics, Vol. 10: Physical Kinetics, Chapt. X (Butterworth-Heinemann, Oxford, 1997).

E. Müller-Hartmann and G. T. Zittartz, Phys. Rev. Lett. 26, 428 (1971).

R. P. Peters, G. Bergmann and R. M. Müller, Phys. Rev. Lett. 58, 1964 (1987).

F. Pierre, H. Pothier, D. Esteve and M. H. Devoret, J. Low Temp. Phys. 118, 437 (2000).