Feynman Rules in $N=2$ projective superspace III: Yang-Mills multiplet

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Abstract

The kinetic action of the $N=2$ Yang-Mills vector multiplet can be written in projective $N=2$ superspace using projective multiplets. It is possible to perform a simple $N=2$ gauge fixing, which translated to $N=1$ component language makes the kinetic terms of gauge potentials invertible. After coupling the Yang-Mills multiplet to unconstrained sources it is very simple to integrate out the gauge fixed vector multiplet from the path integral of the free theory and obtain the $N=2$ propagator. Its reduction to $N=1$ components agrees with the propagators of the gauge fixed $N=1$ component superfields. The coupling of Yang-Mills multiplets and hypermultiplets in $N=2$ projective superspace allows us to define Feynman rules in $N=2$ superspace for these two fields.

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1 Introduction

Quantization of $N = 2$ superfields in $N = 2$ superspace can be achieved for multiplets living in certain subspaces of $N = 2$ superspace. One such subspace is projective superspace \[1\]. Recently we have presented the Feynman rules for the quantization of massless hypermultiplets \[2\] and massive hypermultiplets \[3\] living in this subspace (for an alternative description of $N = 2$ supersymmetric systems see \[4\]). We consider now the $N = 2$ Yang-Mills vector multiplet.

We briefly review the form of this multiplet in projective superspace (the real tropical multiplet) as an infinite power series on the projective complex coordinate. In the abelian case the $N = 1$ components of the tropical multiplet can be simply related to the prepotentials of the $N = 1$ chiral spinor and chiral scalar contained in the $N = 2$ gauge field strength, plus pure gauge degrees of freedom \[5\]. In the nonabelian case the prepotentials we mentioned are the ones corresponding to the kinetic action only, and the field strengths have a highly nonlinear dependence on them.

We propose a $N = 2$ supersymmetric kinetic action for the projective Yang-Mills multiplet. In the abelian case it corresponds to the holomorphic $N = 2$ gauge superpotential at tree level (in the literature often called the $N = 2$ prepotential). As usual, gauge fixing in the path integral is needed to be able to invert such kinetic terms. In $N = 1$ superspace the prepotentials of the chiral spinor and chiral scalar appearing in the kinetic action each requires its own gauge fixing terms \[1\]. We show that gauge fixing of the enlarged $N = 2$ gauge symmetry \[5\] reproduces the $N = 1$ gauge fixing and we conjecture that it also introduces invertible kinetic terms for the pure gauge superfields. Following the same procedure as in \[2\], we use the $N = 1$ propagators of component superfields in the tropical multiplet to try and guess the form of a $N = 2$ gauge propagator that contains them all.

To justify our conjecture we quantize the action in $N = 2$ superspace by gauge fixing the $N = 2$ real tropical multiplet as a whole. Once we have an invertible kinetic term for this multiplet in projective superspace, we can add the coupling to an unconstrained source and integrate out the gauge potential from the free theory path integral to find the $N = 2$ propagator we guessed. In the nonabelian case we have in addition interacting ghosts whose kinetic action is of the same type as that of the hypermultiplet.

Finally, we introduce suitable vertex factors describing the interaction of the vector multiplet with the charged hypermultiplet. This multiplet is described by a complex superfield analytic in the projective complex coordinate \[2\]. Self-interaction vertices for the nonabelian gauge multiplet are still under investigation at the present time. We can give diagram construction rules in $N = 2$ superspace using the first type of vertices.

2 Projective Superspace

We briefly review the basic ideas of $N = 2$ projective superspace. For a more complete review of $N = 2$ projective superspace we refer the reader to \[1\],\[2\].

The algebra of $N = 2$ supercovariant derivatives in four dimensions is\[1\]

\[ D^2 = \frac{1}{2}D^\alpha D_\alpha, \]

\[ \Box = \frac{1}{2}\partial^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}. \]
\{D_{a\alpha}, D_{b\beta}\} = 0 \, , \quad \{D_{a\alpha}, D_{b}^{\dot\beta}\} = i\delta_{a}^{b}\partial_{a\dot\beta} \, . \tag{1}

The projective subspace of $N = 2$ superspace is parameterized by a complex coordinate $\zeta$, and it is spanned by the following projective supercovariant derivatives

\[
\nabla_{\alpha}(\zeta) = D_{1\alpha} + \zeta D_{2\alpha} \tag{2}
\]

\[
\bar{\nabla}_{\dot{\alpha}}(\zeta) = \bar{D}_{\dot{\alpha}}^{2} - \zeta \bar{D}_{\dot{\alpha}}^{1} \tag{3}
\]

The conjugate of any object is constructed in this subspace by applying the antipodal map to the complex coordinate stereo-graphically projected onto the Riemann sphere, and composing it with complex conjugation back on the complex plane. To obtain the barred supercovariant derivative we conjugate the unbarred derivative and we multiply by an additional factor $-\zeta$

\[
-\zeta \bar{\nabla}_{\alpha}(\zeta) = \bar{\nabla}_{\dot{\alpha}}(\zeta) \tag{4}
\]

The projective supercovariant derivatives and the orthogonal combinations

\[
\Delta_{\alpha}(\zeta) = -D_{2\alpha} + \frac{1}{\zeta}D_{1\alpha} \, , \quad \bar{\Delta}_{\dot{\alpha}}(\zeta) = \bar{D}_{\dot{\alpha}}^{1} + \frac{1}{\zeta}\bar{D}_{\dot{\alpha}}^{2} \tag{5}
\]

constitute an alternative basis of spinor derivatives. They give the following algebra and identities

\[
\{\nabla(\zeta), \nabla(\zeta)\} = \{\nabla(\zeta), \bar{\nabla}(\zeta)\} = \{\Delta(\zeta), \Delta(\zeta)\} = \{\Delta(\zeta), \bar{\Delta}(\zeta)\} = \{\nabla(\zeta), \bar{\Delta}(\zeta)\} = 0
\]

\[
\{\nabla_{\alpha}(\zeta), \bar{\nabla}_{\dot{\alpha}}(\zeta)\} = -\{\bar{\nabla}_{\dot{\alpha}}(\zeta), \Delta_{\alpha}(\zeta)\} = 2i\partial_{\alpha\dot{\alpha}} \\
\{\nabla_{\alpha}(\zeta_{1}), \bar{\nabla}_{\dot{\alpha}}(\zeta_{2})\} = i(\zeta_{1} - \zeta_{2})\partial_{\alpha\dot{\alpha}} \\
\nabla^{2}(\zeta_{1})\nabla^{2}(\zeta_{2}) = (\zeta_{1} - \zeta_{2})^{2}(D_{1})^{2}(D_{2})^{2} \\
\nabla^{2}(\zeta)\Delta^{2}(\zeta) = 4(D_{1})^{2}(D_{2})^{2} \tag{6}
\]

For notational simplicity we will denote from now on $D_{1\alpha} = D_{\alpha}, D_{2\alpha} = Q_{\alpha}$. Superfields living in $N = 2$ projective superspace are annihilated by the projective supercovariant derivatives (3). This constraints can be rewritten as follows

\[
D_{\alpha}Y = -\zeta Q_{\alpha}Y \, , \quad \bar{Q}_{\dot{\alpha}}Y = \zeta \bar{D}_{\dot{\alpha}}Y \tag{7}
\]

Manifestly $N = 2$ supersymmetric actions have the form

\[
\frac{1}{2\pi i} \oint_{C} \frac{d\zeta}{\zeta} \, dx \, D^{2}\bar{D}^{2}f(Y, \bar{Y}, \zeta) \tag{8}
\]

where $C$ is a contour around some point of the complex plane that generically depends on the function $f(Y, \bar{Y}, \zeta)$. 

2
The superfields obeying (7) may be classified [1] as i) $O(k)$ multiplets, ii) rational multiplets iii) analytic multiplets. The $O(k)$ multiplet can be expressed as a polynomial in $\zeta$ with powers ranging from 0 to $k$. Rational multiplets are projective quotients of $O(k)$ superfields, and analytic multiplets are analytic in the coordinate $\zeta$ on some region of the Riemann sphere.

For even $k$ we can impose a reality condition on the $O(k)$ multiplet. We refer to it as the real $O(2p)$ multiplet and we reserve the name $\eta$ for this field. The reality condition can be written

$$\frac{\eta}{\zeta^p} = \frac{\bar{\eta}}{\zeta^p} ,$$

(9)

or equivalently in terms of coefficient superfields

$$\eta_{2p-n} = (-)^{p-n}\bar{\eta}_n .$$

(10)

The arctic multiplet is the limit $k \to \infty$ of the complex $O(k)$ multiplet. It is therefore analytic in $\zeta$ around the north pole of the Riemann sphere

$$\Upsilon = \sum_{n=0}^{\infty} \Upsilon_n \zeta^n .$$

(11)

Its conjugate (antarctic) superfield

$$\bar{\Upsilon} = \sum_{n=0}^{\infty} \bar{\Upsilon}_n (-\frac{1}{\zeta})^n$$

(12)

is analytic around the south pole of the Riemann sphere. Similarly if we consider the self-conjugate superfield $\eta/\zeta^p$ the real tropical multiplet is the limit $p \to \infty$ of this multiplet

$$V(\zeta, \bar{\zeta}) = \sum_{n=-\infty}^{+\infty} v_n \zeta^n .$$

(13)

It is analytic away from the polar regions and it contains a piece analytic around the north pole of the Riemann sphere (though not projective) and a piece analytic around the south pole. The reality condition in terms of its coefficient superfields is the following

$$v_{-n} = (-)^n \bar{v}_n .$$

(14)

The constraints obeyed by multiplets living in projective superspace [7] can be written in terms of their coefficients

$$D_\alpha \Upsilon_{n+1} = -Q_\alpha \Upsilon_n , \quad \bar{D}_\alpha \Upsilon_n = \bar{Q}_\alpha \Upsilon_{n+1} .$$

(15)

Such constraints imply that the lowest order coefficient superfield of any multiplet is antichiral in $N = 1$ superspace, and the next to lowest order is antilinear. The same constraints are referred to as constraints (15).

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2 Throughout this paper we reserve the term multiplet to describe constrained superfields, while unconstrained superfields with similar complex coordinate dependence are simply called $O(k)$, rational, and analytic superfields.
imply that the highest order coefficient superfield is chiral in $N = 1$ superspace and the next to highest order is linear.

\begin{align*}
D_\alpha \Upsilon_0 &= 0, \quad D^2 \Upsilon_1 = 0 \\
\bar{D}_\bar{\alpha} \bar{\Upsilon}_0 &= 0, \quad D^2 \bar{\Upsilon}_1 = 0.
\end{align*}

(16)

In the case of a complex $O(k)$ hypermultiplets its highest and lowest order superfields are not conjugate to each other, and the complex multiplet describes twice as many physical degrees of freedom as the real one \[2\].

In the case of the real projective multiplet there is no lowest or highest order coefficient, and therefore none of the coefficient superfields is constrained in $N = 1$ superspace.

3 Kinetic Yang-Mills action in $N = 2$ superspace

The minimal action of the $N = 2$ Yang-Mills multiplet is well known

\begin{align*}
S &= \frac{1}{2} Tr \left( \int dx D^2 Q^2 W W + \int dx \bar{D}^2 \bar{Q}^2 \bar{W} \bar{W} \right),
\end{align*}

(17)

where the $N = 2$ superfield strength $W$ is a covariantly chiral scalar proportional to the anticommutator of gauge covariantized $N = 2$ spinor derivatives

\begin{align*}
\{ D_\alpha, Q_\beta \} &= i C_{\alpha\beta} W.
\end{align*}

(18)

Its expansion in the Grassmann coordinate of the second supersymmetry gives the N=1 covariantly chiral field strengths

\begin{align*}
W|_{\theta_2 = 0} &= \Phi, \quad Q_\alpha W|_{\theta_2 = 0} = -W_\alpha.
\end{align*}

(19)

The action expressed in terms of this fields is

\begin{align*}
S &= \int dx D^2 \bar{D}^2 Tr \bar{\Phi} \Phi + \frac{1}{2} \left( \int dx D^2 Tr \frac{W^\alpha W_\alpha}{2} + \int dx \bar{D}^2 Tr \frac{W^{\bar{\alpha}} W_{\bar{\alpha}}}{2} \right).
\end{align*}

(20)

The term quadratic in the $N = 1$ covariantly chiral field $\Phi$ and its conjugate contains the interactions of an ordinary chiral scalar with the other degrees of freedom in the gauge multiplet. This is made manifest by using the gauge chiral representation of the gauge covariantized $N = 1$ spinor derivatives \[4\]. The barred derivatives annihilate ordinary chiral fields while the unbarred ones annihilate covariantly antichiral fields defined in terms of ordinary antichiral superfields and a real gauge prepotential $v$

\begin{align*}
\int dx D^2 \bar{D}^2 Tr \bar{\Phi} \Phi &= \int dx D^2 \bar{D}^2 Tr e^{v \bar{\phi}} e^{-v \phi}.
\end{align*}

(21)

The gauge superfield $v$ is a prepotential for the $N = 1$ chiral spinor field strength

\begin{align*}
W_\alpha &= i \bar{D}^2 (e^{-v} D_\alpha e^v),
\end{align*}

(22)

and similarly the chiral field $\phi$ can also be defined by a complex prepotential.
\[ \phi = D^2 \bar{\psi} . \] (23)

The kinetic part of the action (20) written in terms of \( N = 1 \) prepotentials is then

\[ S_0 = \int d^4x D^2 \bar{D}^2 \text{Tr} \left( \psi D^2 \bar{D}^2 \bar{\psi} + \frac{1}{2} v D^a \bar{D}^2 D_a v \right) . \] (24)

In the abelian theory this action is also the full gauge action because there are no self-interactions.

This well known description of the gauge multiplet can be related to a real tropical multiplet \( V(\zeta) \) that we will call the projective vector multiplet. For the abelian multiplet the relation among component fields of both descriptions is very simple and direct. For the nonabelian multiplet the relation is very nonlinear, but we can still formulate the theory in terms of projective vector multiplets.

To understand the projective superspace description of the gauge multiplet and write the kinetic action (24) using real tropical multiplets, we consider the \( N = 2 \) supersymmetric interaction of a real tropical multiplet and a complex (ant)arctic hypermultiplet \([5]\) in the (anti)fundamental representation of the gauge group

\[ S_V = \int dx d^4 \theta \oint d\zeta \frac{2}{\pi i \zeta} \bar{\Upsilon} e^V \Upsilon . \] (25)

This action is invariant under gauge transformations

\[ \bar{\Upsilon}' = (\bar{\Upsilon})' = \bar{\Upsilon} e^{-i\bar{\Lambda}} , \quad (e^V)' = e^{i\bar{\Lambda}} e^V e^{-i\Lambda} , \quad \Upsilon' = e^{i\Lambda} \Upsilon , \] (26)

where the gauge parameter is an (ant)arctic multiplet. This guarantees that the transformed hypermultiplet is also (ant)arctic. The infinitesimal abelian transformation of the real tropical multiplet in terms of \( \zeta \)-coefficient superfields \([5]\) is

\[ \delta V = i(\bar{\Lambda} - \Lambda) \longrightarrow \delta v_0 = i(\bar{\lambda}_0 - \lambda_0) , \quad \delta v_n = -i\lambda_n . \] (27)

Since \( \lambda_0 \) is antichiral and \( \lambda_1 \) is antilinear, while higher order coefficients are unconstrained, the gauge transformation can be used to identify the physical degrees of freedom in the real tropical multiplet \([5]\). First we put the real tropical multiplet in a gauge where it becomes a real \( O(2) \) multiplet by setting the components \( v_n = 0 \ \forall \ n \neq -1, 0, 1 \). We can further gauge away all of \( v_1 \) except for the antichiral piece \( D^2 v_1 \) and correspondingly keep the chiral piece in \( v_{-1} \), taking this \( O(2) \) gauge multiplet to a Lindström-Roček gauge. Finally, we can put the coefficient \( v_0 \) in a Wess-Zumino gauge, and then we have isolated the physical degrees of freedom contained in \( V(\zeta) \). This suggests \([5]\) that \( i v_{-1} = \bar{\psi} \) is a prepotential for the chiral scalar gauge field strength, \( i v_1 = \psi \) is a prepotential for the antichiral scalar, and \( v_0 = v \) is the usual \( N = 1 \) prepotential of the chiral spinor gauge field strength \( W_a = -iQ_a D^2 v_{-1} = i\bar{D}^2 D_a v_0 \). All other coefficient superfields in \( V \) are gauge degrees of freedom.

If the real tropical multiplet is Lie algebra valued, the corresponding nonabelian infinitesimal transformation is highly nonlinear

\[ \delta V = L_{\frac{1}{2}} \left[-i \left( \bar{\Lambda} + \Lambda \right) + c o t h L_{\frac{1}{2}} i \left( \bar{\Lambda} - \Lambda \right) \right] , \] (28)
where the Lie derivative is defined as the commutator \[ L_Y X = [Y, X] . \] (29)
The individual components in \( V \) transform in a complicated way, but we can see that it is possible to put the nonabelian real tropical multiplet in an \( O(2) \) gauge by noticing that the most general real tropical multiplet can be written as a gauge transformed \( O(2) \) multiplet

\[
V = V^{O(2)} + L_{V^{O(2)}} \left[ -i (\bar{\Lambda} + \Lambda) + cothL_{V^{O(2)}} i (\bar{\Lambda} - \Lambda) \right] .
\] (30)
The linearized transformation is of the form (27), giving a most general real tropical multiplet. The nonlinear corrections do not change this condition.

The projective superspace description of the gauge multiplet can be used to construct an explicit representation of gauge covariantized spinor derivatives [5]. We split the exponential of the real tropical multiplet into a part analytic around the north pole of the Riemann sphere and a part analytic around the south pole

\[
e^V = e^{V^+} e^{-V^+} , \quad e^{-V^+} = (e^{V^+}) .
\] (31)
Using the fact that the vector multiplet is a projective multiplet \( (\nabla_\alpha e^V) = 0 \), we can see that the projective spinor derivatives

\[
\tilde{\nabla}_\alpha = e^{V^+} \nabla_\alpha e^{-V^+} = e^{-V^+} \nabla_\alpha e^{V^+}.
\] (32)
and

\[
\tilde{\tilde{\nabla}}_{\dot{\alpha}} = e^{V^+} \tilde{\nabla}_{\dot{\alpha}} e^{-V^+} = e^{-V^+} \tilde{\nabla}_{\dot{\alpha}} e^{V^+}.
\] (33)
annihilate a covariantly projective (ant)arctic multiplet

\[
\tilde{\Upsilon}^i = (e^{V^+})^i_j \Upsilon^j , \quad \tilde{\tilde{\Upsilon}}_i = \tilde{\Upsilon}_j (e^{V^+})^j_i .
\] (34)
In the abelian case it is very easy to evaluate the anticommutator of such gauge covariantized spinor derivatives

\[
\{D_\alpha, Q_\beta\} = C_{\alpha\beta} D^2 v^1 , \quad \{\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = C_{\dot{\alpha}\dot{\beta}} \bar{D}^2 v^{-1} .
\] (35)
They are proportional to the \( N = 2 \) abelian gauge field strengths \( \bar{W} = i D^2 v^1 \) and its conjugate. In the nonabelian case the anticommutator also defines the gauge field strength, although its explicit form is highly nonlinear in the \( \zeta \)-coefficient superfields and (35) gives only the lowest order terms.

To see that the action (25) describes hypermultiplets interacting with a gauge multiplet we rewrite this action using covariantly projective hypermultiplets

\[
S_\Upsilon = \int dx d^4 \theta \oint \frac{d\zeta}{2 \pi i \zeta} \tilde{\Upsilon} e^V \Upsilon = \int dx d^4 \theta \oint \frac{d\zeta}{2 \pi i \zeta} \tilde{\tilde{\Upsilon}} \tilde{\Upsilon} ,
\] (36)
\footnote{In the abelian case the exponents correspond to the polar pieces of the tropical multiplet \( V = V^+ + V^- \), but for the nonabelian multiplet this is not true.}
and we perform the duality that (in the hypermultiplet free theory) exchanges the complex linear field of the (ant)arctic multiplet by a chiral one \[.\] This duality relates the off-shell $N = 2$ description of the hypermultiplet to the traditional on-shell realization. The algebra (35) induces a modified $N = 1$ linearity constraint on $\bar{\tilde{\Upsilon}}_1$. We can impose this constraint using a Lagrange multiplier in the conjugate fundamental representation of the gauge group

$$S_T = \int dx \; d^4\theta \; \left( \hat{\bar{\tilde{\Upsilon}}}_0 \bar{\tilde{\Upsilon}}_0 - \hat{\bar{\tilde{\Upsilon}}}_1 \bar{\tilde{\Upsilon}}_1 + \hat{\bar{\tilde{\Upsilon}}}_2 \bar{\tilde{\Upsilon}}_2 + \ldots + \hat{\bar{\tilde{\Upsilon}}} (\bar{D}^2 \bar{\tilde{\Upsilon}}_1 - i W \bar{\tilde{\Upsilon}}_0) + \bar{\tilde{\Upsilon}} (D^2 S + i \bar{W} \tilde{\Upsilon}_0) \right).$$

(37)

We can integrate out the unconstrained field $\bar{\tilde{\Upsilon}}_1$ in the path integral of the theory. The dualization gives the action of two $N = 1$ covariantly chiral scalars $\bar{\tilde{\Upsilon}}_0$ and $\bar{D}^2 \bar{\tilde{\Upsilon}}$ (in the fundamental and antifundamental representation respectively) interacting with a chiral gauge scalar $W \mid = \Phi$. In addition we have auxiliary fields that decouple

$$S_{\text{dual}} = \int dx \; D^2 \bar{D}^2 \left( \bar{\tilde{\Upsilon}}_0 e^v \bar{\tilde{\Upsilon}}_0 + D^2 \bar{\tilde{\Upsilon}} e^{-v} \bar{D}^2 \bar{\tilde{\Upsilon}} + \ldots \right)$$

$$-i \int dx \; D^2 (\bar{D}^2 \Phi \bar{\tilde{\Upsilon}}_0) + i \int dx \; \bar{D}^2 (D^2 \bar{\tilde{\Upsilon}} \Phi).$$

(38)

It is possible to rewrite this dual action in terms of ordinary chiral fields by going to the gauge chiral representation of the gauge covariantized derivatives

$$S_{\text{dual}} = \int dx \; d^4 \theta \; (\Upsilon_0 e^v \bar{\Upsilon}_0 + D^2 \bar{\Upsilon} e^{-v} \bar{D}^2 \bar{\Upsilon} + \ldots)$$

$$-i \int dx \; d^2 \theta (\bar{D}^2 \phi \bar{\Upsilon}_0) + i \int dx \; d^2 \theta (D^2 \bar{\Upsilon} \phi \bar{\Upsilon}_0).$$

(39)

This well known $N = 1$ formulation of the charged hypermultiplet gives 1-loop logarithmic divergences \[7\] proportional to the gauge kinetic action (24). In our formulation we can compute analogous $N = 1$ diagrams \[8\] with the component field interactions in (25). Combining all the diagrams with external potentials $v_i$ and hypermultiplets running in the loop, we find that the final logarithmic divergence is indeed proportional to (24).

We also have the tools to compute these 1-loop divergences directly in $N = 2$ superspace \[8\]: we use the hypermultiplet $N = 2$ propagator given in ref. \[4\] and the projective superspace interactions in (23). Just as in the $N = 1$ calculation, the logarithmic divergences must be proportional to the gauge kinetic action and induce a wave function renormalization.

The only nonvanishing logarithmic divergence we find comes from the two point function $\langle V(1) V(2) \rangle$. It is an integral of a function local in $N = 2$ superspace but nonlocal in the complex coordinate $\zeta$: it involves two complex contour integrals on overlapping contours around the origin of the complex plane. Its projection to the chiral and antichiral $N = 2$ subspaces can be trivially integrated on the complex coordinates

$$S_0 = -\frac{Tr}{2} \int dx d^8 \theta \int \frac{d\zeta_1}{2\pi i} \frac{d\zeta_2}{2\pi i} \frac{V(\zeta_1) V(\zeta_2)}{\zeta_1 - \zeta_2}$$

(40)
\[
\begin{align*}
&= - \frac{Tr}{2} \int dx D^2 Q^2 \oint \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} \frac{\Delta_2^2 V(1) \bar{\Delta}_2^2 V(2)}{4(\zeta_1 - \zeta_2)^2} \\
&= - \frac{Tr}{2} \int dx D^2 Q^2 \oint \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} \bar{D}^2 V(1) \bar{D}^2 V(2) \\
&= - \frac{Tr}{2} \int dx D^2 Q^2 \left( \bar{D}^2 v_{-1} \bar{D}^2 v_{-1} \right).
\end{align*}
\]

In the abelian case this $N = 2$ superpotential is the tree level gauge action (17). We can write this expression in $N = 1$ superspace

\[
S_0 = -\frac{1}{2} \int dx D^2 Q^2 \left( \bar{D}^2 v_{-1} \bar{D}^2 v_{-1} \right) 
\]

(41)

\[
= -\frac{1}{2} \int dx D^2 Q^2 \left( 2\bar{D}^2 Q^2 v_{-1} \bar{D}^2 v_{-1} + \bar{D}^2 Q^\alpha v_{-1} \bar{D}^2 Q_{\alpha} v_{-1} \right) 
\]

\[
= - \int dx D^2 \bar{D}^2 \left( D^2 v_{1} \bar{D}^2 v_{-1} - \frac{1}{2} v_0 D \bar{D}^2 D v_0 \right),
\]

and we find the kinetic action (24) after identifying again $iv_1 = \psi$, $v_0 = \nu$.

4 \hspace{1cm} $N = 1$ gauge fixing

It is well known that the $N = 1$ action (24) does not have invertible kinetic terms and the system needs gauge fixing to remove the gauge group volume from the path integral. Suitable gauge fixing conditions for the $N = 1$ prepotentials are [3]

\[
D^2 v = 0 = \bar{D}^2 v, \quad D_\alpha \bar{\psi} = 0 = \bar{D}_\alpha \psi.
\]

(42)

This gauge fixing is imposed by inserting unity in the path integral as the product of a functional Dirac delta times the inverse Faddeev-Popov determinant. The determinant can be written as a functional integral of an exponential. The exponent is just the gauge fixing function evaluated on anticommuting unconstrained scalar and spinorial ghosts [3]. These unconstrained prepotential ghosts can be traded for anticommuting chiral and complex linear field strength ghosts, and their kinetic term is

\[
S_{FP} = \int dx d^4 \theta \left( b D^2 \bar{D}^2 \bar{c} + b^\alpha D_\alpha \bar{D}^\alpha \bar{c}_\alpha + c.c. \right) = \int dx d^4 \theta \left( D^2 b \bar{D}^2 \bar{c} - D^\alpha b_\alpha \bar{D}^\alpha \bar{c}_\alpha + c.c. \right).
\]

(43)

With convenient gauge fix averaging, the gauge kinetic terms are supplemented with the pieces needed to invert the kinetic operators [8], and we get the well known gauge fixed action

\[
S_0 + S_{fix} = \frac{Tr}{2} \int dx d^4 \theta \left( \psi \left[ -D^2 \bar{D}^2 - \frac{1}{\alpha}(\bar{D}^2 D^2 - D \bar{D}^2 D) \right] \bar{\psi} \right. \\
\left. + \bar{\psi} \left[ -\bar{D}^2 D^2 - \frac{1}{\alpha}(D^2 \bar{D}^2 - D \bar{D}^2 D) \right] \psi + v \left[ D \bar{D}^2 D - \frac{1}{\alpha}(D^2 \bar{D}^2 + \bar{D}^2 D^2) \right] \nu \right).
\]

(44)
In $N = 1$ superspace the physical prepotentials of the gauge multiplet can be gauge fixed with apparently different $\alpha$ parameters. That is possible because in the usual formulation of the gauge multiplet the interactions of the potentials $\psi$ and $\bar{\psi}$ always involve the corresponding field strengths $\bar{\phi}$ and $\phi$. The propagator $\langle \psi(1) \bar{\psi}(2) \rangle$ is always acted upon with spinor derivatives from the interaction vertices to give

$$\langle D^2 \psi(1) \bar{D}^2 \bar{\psi}(2) \rangle = \langle \bar{\phi}(1) \phi(2) \rangle$$

or the conjugate expression. No matter what the value of $\alpha$ is, the derivatives in (45) select the antichiral-chiral projector in the inverted kinetic operator

$$- \frac{\bar{D}^2 D^2}{\Box^2} - \alpha \left( \frac{D^2 \bar{D}^2}{\Box^2} - \frac{D \bar{D}^2 D}{\Box^2} \right),$$

and this is precisely the $\alpha$-independent piece. In the projective superspace formulation of the gauge multiplet the interactions (25) of the superfield $v_1$ do not involve the field strength $\bar{\phi}$ and we have to be more careful. In $N = 2$ superspace we use the gauge transformation (27) to fix the gauge for the whole real tropical multiplet containing $v_0$, $v_{-1}$ and $v_1$. Their gauge transformations are not independent and we must use the same gauge fixing parameter for both. Indeed, the $N = 2$ relations among coefficient superfields (7) guarantee that a single condition $v_2 = 0 = v_{-2}$ automatically reproduces the $N = 1$ gauge fixing conditions (42) on the prepotentials $v_0$, $v_1$ and $v_{-1}$. However, it also gives unwanted gauge fixing conditions on the prepotentials $v_3$, $v_{-3}$ and $v_{-4}$ which are otherwise absent from the kinetic action. We need additional gauge fixings for the gauge fixing, i.e. it is not enough to set $v_2 = 0 = v_{-2}$, but we have to fix $v_n = 0 \forall n \neq -1, 0, 1$. This is precisely the $O(2)$ gauge discussed before.

Our ultimate goal is to produce an invertible kinetic term for the whole real vector multiplet with the full $N = 2$ superspace measure, so that we can compute the propagator with its full $N = 2$ Grassmann coordinate dependence. When we reduce this $N = 2$ invertible kinetic term to $N = 1$ components we expect to find also a kinetic term for the prepotentials $v_n$, $|n| > 1$. These are unphysical degrees of freedom and it maybe surprising to introduce kinetic operators for them. This is however analogous to what happens with the unphysical fields of the $N = 1$ vector prepotential: although they can be set equal to zero in Wess-Zumino gauge, the standard gauge fixing $D^2 v = 0 = \bar{D}^2 v$ gives an invertible kinetic term for the whole $v$.

With this argument in mind we tentatively propose the following $N = 1$ gauge fixed action

$$S_0 + S_{fix} = \frac{1}{2} \int dxd^4 \theta \ Tr \left( -\frac{1}{\alpha} \sum_{n \neq -1, 0, 1} v_n \Box v_{-n} + v_0 \left[ D \bar{D}^2 D - \frac{1}{\alpha} (D^2 \bar{D}^2 + \bar{D}^2 D^2) \right] v_0 \right)$$

$$-v_1 \left[ D^2 \bar{D}^2 + \frac{1}{\alpha} (\bar{D}^2 D^2 - D \bar{D}^2 D) \right] v_{-1} - v_{-1} \left[ \bar{D}^2 D^2 + \frac{1}{\alpha} (D^2 \bar{D}^2 - D \bar{D}^2 D) \right] v_1 \right).$$

Of course this ansatz will only be justified once we have found a gauge fixing in $N = 2$ superspace that reproduces this expression.
5 \textbf{N = 2 propagator obtained from N = 1 component propagators}

We can now proceed to reconstruct the propagator of the vector multiplet in N = 2 superspace the same way it has been done for the hypermultiplet [2]. We add to the action a source term

\[ S_j = \int dx^4 \theta \oint \frac{d\zeta}{2\pi i} \; Tr \; j V = \int dx^4 \theta \sum_{n=-\infty}^{+\infty} Tr \; j_{-n}v_n, \quad (48) \]

where the source is a real tropical multiplet itself, making the whole expression N = 2 supersymmetric. The \( \zeta \)-coefficient sources are unconstrained in N = 1 superspace as we mentioned when we defined the real tropical multiplet. This allows us to complete squares trivially on the N = 1 component action of the vector multiplet. Integrating out the N = 1 superfields in the free theory path integral, we are left with the following terms quadratic in sources

\[
\ln Z_0[j, \bar{j}] = Tr \int dx^4 \theta \left( -\frac{1}{2} j_0 \left[ \frac{D \bar{D}^2 D}{\Box^2} - \alpha \frac{D^2 D^2 + \bar{D}^2 D^2}{\Box^2} \right] j_0 \right. \\
+ \left. j_1 \left[ \frac{D^2 \bar{D}^2}{\Box^2} + \alpha \frac{D^2 D^2 - \bar{D} \bar{D}^2 \bar{D}}{\Box^2} \right] j_1 + \alpha \sum_{n=2}^{+\infty} j_n \Box^{-1} j_{-n} \right). 
\]

The vector multiplet propagator will have the following projection into N = 1 superspace

\[
\langle V^a(1)V^b(2)\rangle_{\theta_2=0} = \sum_{n=2}^{+\infty} \langle v^a_n(1) v^b_n(2) \rangle \left( \frac{\zeta_2}{\zeta_1} \right)^n + \langle v^a_{-1}(1) v^b_1(2) \rangle \frac{\zeta_2}{\zeta_1} + \langle v^a_0(1) v^b_0(2) \rangle \\
+ \langle v^a_1(1) v^b_{-1}(2) \rangle \frac{\zeta_1}{\zeta_2} + \sum_{n=2}^{+\infty} \langle v^a_n(1) v^b_{-n}(2) \rangle \left( \frac{\zeta_1}{\zeta_2} \right)^n 
\]

\[
= \delta^{ab} \left( \alpha \sum_{n=2}^{+\infty} \left[ \left( \frac{\zeta_2}{\zeta_1} \right)^n + \left( \frac{\zeta_1}{\zeta_2} \right)^n \right] \left[ D^2 \bar{D}^2 + \bar{D}^2 D^2 - D \bar{D}^2 \bar{D} \right] \\
+ \frac{\zeta_2}{\zeta_1} \left[ (1-\alpha)D^2 \bar{D}^2 + \alpha(D^2 \bar{D}^2 + \bar{D}^2 D^2 - D \bar{D}^2 \bar{D}) \right] \\
+ \frac{\zeta_1}{\zeta_2} \left[ (1-\alpha)\bar{D}^2 D^2 + \alpha(D^2 \bar{D}^2 + \bar{D}^2 D^2 - D \bar{D}^2 \bar{D}) \right] \right) \frac{\delta^4(x_{12})\delta^4(\theta_{12})}{\Box^2}.
\]

Rewriting the term proportional to \( \alpha \) we obtain

\[
\langle V^a(1)V^b(2)\rangle_{\theta_2=0} = \delta^{ab} \left( \alpha \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n + (1-\alpha) \right) \times \\
\times \left[ \frac{\zeta_2}{\zeta_1} D^2 \bar{D}^2 - D \bar{D}^2 \bar{D} + \frac{\zeta_1}{\zeta_2} \bar{D}^2 D^2 \right] \frac{\delta^4(x_{12})\delta^4(\theta_{12})}{\Box^2}. 
\]

10
When we quantized the hypermultiplet \([2]\) we made the ansatz that the \(N = 2\) propagator of a projective multiplet should contain the projective spinor derivatives \(\nabla^4_1 \nabla^4_2 \delta^8(\theta_1 - \theta_2)\) \((\nabla^4_1 = \nabla^2(\zeta_1) \nabla^2(\zeta_1))\). The projection of this expression into \(N = 1\) superspace is

\[
\nabla^4_1 \nabla^4_2 \delta^8(\theta_1 - \theta_2) \mid_{\theta_2 = 0} = (\zeta_1 - \zeta_2)^2 (\zeta_2^2 D^2 D^2 + \zeta_2^2 D^2 D^2 - \zeta_1 \zeta_2 D D^2 D) \delta^4 (1) \delta^4 (\theta_1 - \theta_2). \tag{52}
\]

Making the same ansatz for the tropical vector multiplet it is straightforward to realize that the \(N = 2\) propagator we look for is

\[
\langle V^a(1) V^b(2) \rangle = \delta^{ab} \left( \alpha \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n + (1 - \alpha) \right) \frac{\nabla^4_1 \nabla^4_2}{\zeta_1 \zeta_2 (\zeta_2 - \zeta_1)^2} \delta^8 (\theta_{12}) \delta^4 (x_{12}). \tag{53}
\]

The first term is the only one present in Fermi-Feynman gauge \(\alpha = 1\). The infinite series defines the full (tropical) Dirac delta distribution \([2]\) on the Riemann sphere for any function with a power series expansion in \(\zeta\)

\[
\int \frac{d\zeta_1}{2\pi i \zeta_1} F(\zeta_1) \delta^{+\infty} (\zeta_2, \zeta_1) = \int \frac{d\zeta_1}{2\pi i \zeta_1} F(\zeta_1) \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n = F(\zeta_2). \tag{54}
\]

Using the following identities

\[
\frac{\nabla^4_1 \nabla^4_2}{\zeta_1 \zeta_2 (\zeta_2 - \zeta_1)^2} = \frac{\nabla^4_1 \nabla^4_2 \Delta^2 \nabla^2_2}{4 \zeta_1 \zeta_2} = \frac{\nabla^4_1}{\zeta_1^3} \left[ \frac{\zeta_2}{\zeta_1} D^2 D^2 - D D^2 D + \frac{\zeta_1}{\zeta_2} \bar{D}^2 \bar{D}^2 \right]
\]

\[
= \frac{\nabla^4_1 \Delta^2 \nabla^2_2}{4 \zeta_1 \zeta_2} = \left[ \frac{\zeta_2}{\zeta_1} D^2 \bar{D}^2 - D \bar{D}^2 D + \frac{\zeta_1}{\zeta_2} \bar{D}^2 \bar{D}^2 \right] \nabla^4_2 \tag{55}
\]

and reordering

\[
\sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \left[ \frac{\zeta_2}{\zeta_1} D^2 D^2 - D \bar{D}^2 D + \frac{\zeta_1}{\zeta_2} \bar{D}^2 \bar{D}^2 \right] = \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \Box \tag{56}
\]

we can rewrite the \(N = 2\) propagator in Fermi-Feynman gauge \(\alpha = 1\) in two equivalent forms

\[
\langle V^a(1) V^b(2) \rangle = \delta^{ab} \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \delta^8 (\theta_{1 - \theta_2}) \delta^4 (x_{1 - x_2}) \tag{57}
\]

\[
= \delta^{ab} \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \delta^8 (\theta_{2 - \theta_1}) \delta^4 (x_{1 - x_2}). \tag{58}
\]

In Landau gauge \(\alpha = 0\) the gauge fixing is given an infinite weight in the path integral. Not surprisingly, the propagator is the one corresponding to an \(O(2)\) multiplet \([2]\) \(V^{O(2)} = \eta / \zeta\) with kinetic term

\[
S_0 = - \int dxd^4 \theta \int \frac{d\zeta}{2\pi i} \eta \frac{\partial}{\partial \zeta} \eta \frac{\partial}{\partial \zeta}. \tag{59}
\]
6 Gauge fixing in $N = 2$ superspace

The $N = 2$ vector multiplet has a well defined kinetic action in $N = 2$ superspace as we have seen, and the $\zeta$-coefficient superfields can be suitably gauge fixed to produce invertible kinetic terms in $N = 1$ superspace. It is therefore natural to expect that a $N = 2$ gauge fixing term exists, allowing us to invert the $N = 2$ kinetic operator.

We want to gauge fix the vector multiplet in the $N = 2$ free theory path integral

$$Z_0 = \int D[V] D[f] \Delta_{FP}^{-1}(V_G - f) e^{iS_0(V)} e^{iS_{avg}(f)},$$

where we define the truncated tropical fields $V_G$ and $f$ using the polar Dirac delta distributions on the Riemann sphere [4]

$$V_G(\zeta) = \oint \frac{d\zeta'}{2\pi i \zeta'} \left( \delta^{(-2)}(\zeta, \zeta') + \delta^{(2)}(\zeta, \zeta') \right) V(\zeta') = \sum_{n=-\infty}^{-2} \zeta^n v_n + \sum_{n=2}^{+\infty} \bar{\zeta}^n v_n.$$  

In the abelian case the Faddeev-Popov ghosts only have a quadratic kinetic term and since they decouple from the other fields we will not be concerned with them anymore. In the nonabelian case we obtain the Faddeev-Popov determinant as usual from the functional derivative of the gauge fixing function

$$\frac{\delta V_G(1)}{\delta (\Lambda(2), \bar{\Lambda}(2))} = \oint \frac{d\zeta_0}{2\pi i \zeta_0} \left( \delta^{(-2)}_0(\zeta_1, \zeta_0) + \delta^{(2)}_0(\zeta_1, \zeta_0) \right) \times$$

$$\times L_{V_0} \left[ \left( \delta^{(+\infty)}_0(\zeta_0, \zeta_2) \bar{\nabla}^4_0 + \delta^{(0)}_0(\zeta_0, \zeta_2) \frac{\bar{\nabla}^4_0}{\zeta_0^2} \right) + \frac{\text{coth}L}{V_0} \left( \delta^{(+\infty)}_0(\zeta_0, \zeta_2) \bar{\nabla}^4_0 - \delta^{(0)}_0(\zeta_0, \zeta_2) \frac{\bar{\nabla}^4_0}{\zeta_0^2} \right) \right] \delta^8(\theta_2) \delta^4(x_{12}),$$

where we have used the functional derivatives with respect to (ant)arctic multiplets $\Lambda = \nabla^4 \Psi$ and $\bar{\Lambda} = \nabla^4 \bar{\Psi}/\zeta^4$ [4]. The inverse Faddeev-Popov determinant is obtained by taking the matrix elements of this operator evaluated on the infinite basis of unconstrained anticommuting real tropical superfield points $b(\zeta_1, \theta_1, x_1), c(\zeta_2, \theta_2, x_2)$, and integrating out its exponential in the path integral

$$\Delta_{FP}^{-1} = \int D[b] D[c] \text{exp} \left( Tr \int dx_1 dx_2 d\theta_1 d\theta_2 \oint d\zeta_0 d\zeta_1 d\zeta_2 b(\zeta_1, \theta_1, x_1) \delta^{(-2)}_0(\zeta_1, \zeta_0) + \delta^{(+\infty)}_0(\zeta_1, \zeta_0) \delta^8(\theta_2) \delta(\Lambda(2), \bar{\Lambda}(2)) c(2) \right).$$

Defining the (ant)arctic ghost multiplets

$$C = \nabla^4_0 \sum_{n=0}^{+\infty} \bar{\zeta}_0^n c_n, \quad \bar{C} = \nabla^4_0 \sum_{n=0}^{+\infty} \frac{\bar{\zeta}_0^n}{\zeta_0^n} c_n,$$

$$B = \nabla^4_0 \sum_{n=2}^{+\infty} \bar{\zeta}_0^n b_n, \quad \bar{B} = \nabla^4_0 \sum_{n=2}^{+\infty} \frac{\bar{\zeta}_0^n}{\zeta_0^n} b_n.$$
the resulting $N = 2$ ghost action is

$$S_{FP} = \int dx d^4\theta \int \frac{d\zeta_0}{2\pi i \zeta_0} (B + \bar{B})L_{\frac{1}{2}}[(C + \bar{C}) + \coth L_{\frac{1}{2}}(C - \bar{C})] .$$

The kinetic piece in this action is

$$S^{FP}_0 = \int dx d^4\theta \int \frac{d\zeta_0}{2\pi i \zeta_0} Tr (B + \bar{B})(C - \bar{C}) = \int dx d^4\theta \int \frac{d\zeta_0}{2\pi i \zeta_0} Tr (BC - \bar{B}\bar{C}) ,$$

and the ghost quantization is very similar to that of two mixed (ant)arctic hypermultiplets in the adjoint representation of the gauge group, with the peculiarity that this superfields are anticommuting. Performing the contour integral we obtain

$$N = 1$$

the adjoint representation of the gauge group, with the peculiarity that this superfields are

$$N = 2$$

linear kinetic ghost terms of the form \([43]\).

The gauge fixing weight we propose is the following

$$S_{avg} = -\frac{Tr}{2\alpha} \int dx d^8\theta \left( \oint_{|\zeta_1|<|\zeta_2|} \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} \frac{\zeta_1 f(\zeta_1)f(\zeta_2)}{(\zeta_2 - \zeta_1)^3} + \oint_{|\zeta_2|<|\zeta_1|} \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} \frac{\zeta_2 f(\zeta_1)f(\zeta_2)}{(\zeta_1 - \zeta_2)^3} \right) .$$

Integrating the weighted function $f$ in the path integral we obtain the gauge fixing action

$$S_{fix} = S_{avg}(f \rightarrow V_G) .$$

Performing the contour integrals we notice that we may replace $V_G \rightarrow V$ because the integration automatically projects out the components $v_{-1}, v_0$ and $v_1$

$$S_{fix} = -\frac{Tr}{2\alpha} \int dx d^8\theta \left( \oint_{|\zeta_1|<|\zeta_2|} \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} \frac{\zeta_1 V(\zeta_1)V(\zeta_2)}{(\zeta_2 - \zeta_1)^3} + \oint_{|\zeta_2|<|\zeta_1|} \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} \frac{\zeta_2 V(\zeta_1)V(\zeta_2)}{(\zeta_1 - \zeta_2)^3} \right)$$

$$= -\frac{1}{\alpha} \int dx d^8\theta \sum_{n=2}^{\infty} \frac{n(n-1)}{2} \, \text{Tr} \, v_{-n} v_n .$$

When we project this expression into $N = 1$ superspace we find the extra pieces needed to invert the kinetic terms. A convenient way to perform such projection is to replace the $N = 2$ superspace measure by $D^2 \bar{D}^2 Q^2 \bar{Q}^2 = D^2 \bar{D}^2 \nabla^4 / \zeta^2$ and act with the projective derivatives on the integrand \([13]\). To simplify the contour integrals we use the identity

$$\frac{\nabla^4_1}{(\zeta_2 - \zeta_1)^2} V(\zeta_2) = \nabla^2_1 \bar{D}^2 V(\zeta_2) ,$$

and we rewrite the $N = 1$ projection of the gauge fixing action as follows

$$S_{fix} = -\frac{Tr}{2\alpha} \int dx d^2 \bar{D}^2 \left( \oint_{|\zeta_1|<|\zeta_2|} \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} V(\zeta_1) \frac{\nabla^2_1}{\zeta_1^2} \bar{D}^2 \frac{\zeta_1}{(\zeta_2 - \zeta_1)} V(\zeta_2) \right.$$

$$+ \oint_{|\zeta_2|<|\zeta_1|} \frac{d\zeta_1 d\zeta_2}{2\pi i 2\pi i} V(\zeta_1) \frac{\nabla^2_1}{\zeta_1^2} \bar{D}^2 \frac{\zeta_2}{(\zeta_1 - \zeta_2)} V(\zeta_2) \right) .$$

We have transformed a triple pole into a simple one. The remaining pole can be written as a convergent geometric series
\[ S_{\text{fix}} = -\frac{Tr}{2\alpha} \int dx d\theta \left( \oint_{|\zeta_1|<|\zeta_2|} \frac{d\zeta_1}{2\pi i} \frac{d\zeta_2}{2\pi i} V(\zeta_1) \frac{\nabla^2_1}{\zeta_1^2} D^2 \frac{\zeta_1}{\zeta_2} \sum_{n=0}^{+\infty} \left( \frac{\zeta_1}{\zeta_2} \right)^n \frac{\zeta_1}{\zeta_2} \right) V(\zeta_2) \]

+ \int_{|\zeta_2|<|\zeta_1|} \frac{d\zeta_1}{2\pi i} \frac{d\zeta_2}{2\pi i} V(\zeta_1) \frac{\nabla^2_1}{\zeta_1^2} D^2 \sum_{n=0}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \frac{\zeta_1}{\zeta_2} \right) . \quad (71)

In this form the only poles are in the origin of the complex plane and the radial ordering is irrelevant for the contour integration. We can deform the contours so that they become overlapping and combine both terms in the same double integral. We also use the identity (69) in the \( N = 1 \) projection of the kinetic action (40), and we find the following \( N = 1 \) gauge fixed action

\[ S_0 + S_{\text{fix}} = -\frac{Tr}{2} \int dx d\theta \int \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} V(\zeta_1) \frac{\nabla^2_1}{\zeta_1^2} D^2 \zeta_1 \zeta_2 \left[ 1 + \frac{1}{\alpha} \sum_{n\neq 0} \left( \frac{\zeta_2}{\zeta_1} \right)^n \right] V(\zeta_2) . \quad (72)\]

Finally, we rewrite the differential operator acting on \( V(\zeta_2) \)

\[ \left( \frac{\zeta_2}{\zeta_1} \right)^{n+1} \nabla^2_1 D^2 V(\zeta_2) = \left[ \left( \frac{\zeta_2}{\zeta_1} \right)^{n+1} D^2 \bar{D}^2 - \left( \frac{\zeta_2}{\zeta_1} \right)^n \bar{D}^2 D + \left( \frac{\zeta_2}{\zeta_1} \right)^{n-1} \bar{D}^2 D^2 \right] V(\zeta_2) , \quad (73) \]

and we recover the form of the \( N = 1 \) gauge fixed action (47) we proposed

\[ S_0 + S_{\text{fix}} \]

\[ = -\frac{Tr}{2} \int dx d\theta \int \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} V(\zeta_1) \left( \frac{\zeta_1}{\zeta_2} \left[ \bar{D}^2 D^2 + \frac{1}{\alpha} (D^2 \bar{D}^2 - D\bar{D}^2 D) \right] \right) \]

\[ + \left[ \frac{1}{\alpha} (D^2 \bar{D}^2 + \bar{D}^2 D^2) \right] V(\zeta_2) \]

\[ + \frac{1}{\alpha} \sum_{n\neq -1,0,1} \left( \frac{\zeta_1}{\zeta_2} \right)^{n} \left[ \bar{D}^2 D^2 + D^2 \bar{D}^2 - D\bar{D}^2 D \right] V(\zeta_2) . \quad (74) \]

7 Computation of the \( N = 2 \) propagator in \( N = 2 \) superspace

We have successfully found a gauge-fixing in \( N = 2 \) superspace that reproduces the ansatz (47) for a gauge fixed action in \( N = 1 \) components. To invert the kinetic term in \( N = 2 \) superspace we use an unconstrained real tropical superfield \( X(\zeta) \) which defines a prepotential for the projective gauge multiplet

\[ V(\zeta) = \frac{\nabla^4}{\zeta^2} X(\zeta) . \quad (75) \]
We rewrite the gauge fixed action in $N = 2$ superspace as

$$S_0 + S_{fix} = -\frac{Tr}{2} \int dx d\theta \oint \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} X(\zeta_1) K(\zeta_1, \zeta_2) X(\zeta_2)$$

$$= -\frac{Tr}{2} \int dx d\theta \oint \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} X(\zeta_1) \frac{\nabla_1^4}{\zeta_1^2} \frac{\nabla_2^4}{\zeta_2^2} X(\zeta_2) \left[ 1 + \frac{1}{\alpha} \sum_{n \neq 0} \left( \frac{\zeta_2}{\zeta_1} \right)^n \right] \frac{\nabla_2^4(\zeta_2)}{\zeta_2^2} X(\zeta_2).$$

Now we must add a source term in $N = 2$ superspace that reproduces the $N = 1$ sources (8). The source term we want must involve and unconstrained real tropical source $J$, so that the projection into $N = 1$ superspace contains the projective multiplets $V$ and $j = \nabla^4 J/\zeta^2$

$$S_J = \int d^4 x d^8 \theta \oint \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} J(\zeta) V(\zeta) = \int d^4 x \bar{D}^2 \bar{D}^2 \oint \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} J(\zeta) \frac{\nabla_1^4(\zeta_1)}{\zeta_1^2} J(\zeta_2),$$

where the shift superfield

$$J(\zeta_1) = \oint \frac{d\zeta_0}{2\pi i \zeta_0} P(\zeta_1, \zeta_0) J(\zeta_0)$$

obeys

$$S_J = \int d^4 x d^8 \theta \oint \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} J(\zeta_2) \frac{\nabla_1^4(\zeta_1)}{\zeta_1^2} X(\zeta_2)$$

$$= \int d^4 x d^8 \theta \oint \frac{d\zeta_1}{2\pi i \zeta_1} \frac{d\zeta_2}{2\pi i \zeta_2} \left[ \oint \frac{d\zeta_0}{2\pi i \zeta_0} P(\zeta_1, \zeta_0) J(\zeta_0) \right] K(\zeta_1, \zeta_2) X(\zeta_2).$$

The last term in (78) gives the source dependence of the free theory path integral. We expect it to contain the differential operators present in the propagator (53), and therefore $P \propto \nabla^4$. Indeed acting with additional projective derivatives on the kinetic operator we obtain

$$\nabla_0^4 \left( \nabla_1^2 \bar{D}^2 \bar{V}_1^2 \nabla_2^2 \bar{V}_2^2 \right) = \nabla_0^4 \left( \nabla_1^2 \bar{D}^2 \bar{Q}^2 \nabla_2^2 \bar{Q}^2 \right) = (\zeta_0 - \zeta_1)^2 \nabla_0^2 \bar{D}^2 \bar{Q}^2 \nabla_2^2,$$

and the operator we are looking for is

$$P(\zeta_0, \zeta_1) = \left( 1 + \alpha \sum_{n \neq 0} \left( \frac{\zeta_1}{\zeta_0} \right)^n \right) \frac{\zeta_1 \nabla_0^4}{\zeta_0 (\zeta_0 - \zeta_1)^2 \Box^2} = \mathcal{P}(\zeta_0, \zeta_1).$$
It is very straightforward to check that it obeys the condition we imposed in (81)

$$\oint d\zeta_1 \frac{d\zeta_0}{2\pi i \zeta_0} J(\zeta_0) P(\zeta_0, \zeta_1) K(\zeta_1, \zeta_2) = \oint d\zeta_0 \frac{d\zeta_1}{2\pi i \zeta_1} \sum_{n=-\infty}^{\infty} \left( \frac{\zeta_2}{\zeta_0} \right)^n J(\zeta_0) \frac{\nabla^4_0 D^2 \bar{Q}^2 \nabla^2_2}{\zeta_0^2 \Box} = J(\zeta_2) \frac{\nabla^4_2}{\zeta_2^2}. \quad (83)$$

Thus the free theory path integral is

$$\ln Z_0[J] = \frac{T}{2} \int d^8 \theta \oint d\zeta' \frac{d\zeta}{2\pi i \zeta' \zeta} J(\zeta') \frac{\nabla^4(\zeta')}{\zeta'^2} P(\zeta', \zeta) J(\zeta), \quad (84)$$

$$= \frac{T}{2} \int d^8 \theta \oint d\zeta' \frac{d\zeta}{2\pi i \zeta' \zeta} J(\zeta') \left( \alpha \sum_{n=-\infty}^{\infty} \left( \frac{\zeta}{\zeta'} \right)^n + (1 - \alpha) \right) \frac{\nabla^4(\zeta') \nabla^4(\zeta)}{\zeta' \zeta (\zeta - \zeta')^2 \Box^2} J(\zeta).$$

Now the propagator in $N = 2$ superspace can be simply obtained by functionally differentiating the path integral with respect to the sources. Since the source is unconstrained, the functional derivative with respect to it is just the product of Dirac delta distributions in $N = 2$ superspace and in the Riemann sphere. The propagator we find reproduces correctly our ansatz (83)

$$\langle V^a(1) V^b(2) \rangle = \frac{\delta}{\delta J_a(1) \delta J_b(2)} Z_0[J] \quad (85)$$

$$= \delta^{ab} \left( \alpha \sum_{n=-\infty}^{\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n + (1 - \alpha) \right) \frac{\nabla^4_1 \nabla^4_2}{\zeta_1 \zeta_2 (\zeta_2 - \zeta_1)^2 \Box^2} \delta^8(\theta_{12}) \delta^4(x_{12}).$$

### 8 Vertices

The abelian vector multiplet can only interact with charged fields. The best example we have of such interactions is the charged (ant)arctic hypermultiplet coupling (25). This couplings can be generalized to nonabelian vector multiplets interacting with hypermultiplets in the fundamental or adjoint representation of the gauge group. In addition the $N = 2$ superpotential of the nonabelian vector multiplet contains self-interactions. At the time of writing this manuscript we have not yet been able to rewrite them as functionals of the real tropical multiplet, as we did with the kinetic term. We can only give at this moment the vertex factors of the first type.

As we explained in [2], when the propagators of projective multiplets contain the maximum number of projective derivatives we can formally put such derivatives in the vertex factors. The factor corresponding to an interaction (25) where all fields give internal lines will be

$$\int d^8 \theta \oint d\zeta \frac{\nabla^4 e^\text{exp} (\nabla^4) \nabla^4}{\zeta}. \quad (86)$$

whereas in interactions with external and internal fields we only replace the latter with projective derivatives.
9 Feynman rules for the interacting vector multiplet and hypermultiplet

The Feynman rules for construction of diagrams in $N = 2$ superspace are a simple generalization of those given in [2]. As we mentioned above we can choose to put the projective derivatives of the propagators in the corresponding lines of the interaction vertices, working formally with propagators

$$\langle \Upsilon(1) \bar{\Upsilon}(2) \rangle = (-) \delta^{(+\infty)}(\zeta_1, \zeta_2) \frac{1}{\zeta_2^2(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) , \quad (87)$$

and

$$\langle V^a(1)V^b(2) \rangle = \delta^{ab} \left( \alpha \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n + (1 - \alpha) \right) \frac{1}{\zeta_1 \zeta_2(\zeta_2 - \zeta_1)^2} \delta^8(\theta_{12}) \delta^4(x_{12}) . \quad (88)$$

However, if we use the simplified form of the gauge propagator in Fermi-Feynman gauge (57) such a choice is not possible because it does not have enough projective derivatives. In many cases it is much simpler to use this propagator, and to treat all multiplets the same way we keep the spinor derivatives in the propagators. In that case we work with hypermultiplet propagators

$$\langle \Upsilon(1) \bar{\Upsilon}(2) \rangle = (-) \delta^{(+\infty)}(\zeta_1, \zeta_2) \frac{\nabla^4_1 \nabla^4_2}{\zeta_2^2(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) , \quad (89)$$

and vector multiplet propagators

$$\langle V^a(1)V^b(2) \rangle = \delta^{ab} \frac{\nabla^4_1}{\zeta_1^2} \delta^8(\theta_1 - \theta_2) \delta^4(x_1 - x_2) \sum_{n=-\infty}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n , \quad (90)$$

To construct a given diagram we expand the exponential of the interacting action and we let the functional derivatives with respect to internal line sources act on the free theory path integral. This standard procedure gives the combinatorial factors associated with each diagram.

After this we follow the usual strategy in the computation of superspace diagrams: we extract a total derivative $\nabla^4_i$ for each vertex $i$ and complete the restricted measure $d^4 \theta_i$ to a full $N = 2$ superspace measure $\zeta_i^2 d^8 \theta_i$. Then we reduce the Grassmann coordinate dependence of all propagators except the last one to bare Dirac delta functions.

In Fermi-Feynman gauge the most efficient way to perform these two steps is to extract the total derivative $\nabla^4_i$, used to complete the measure of the vertex $i$, from a gauge internal line connecting it to a vertex $j$. This manipulation reduces the gauge propagator to a Grassmann delta function $\delta^8(\theta_{ij})$ and a Riemann sphere delta distribution $\delta^{(+\infty)}(\zeta_i, \zeta_j)$. Integrating with the $N = 2$ superspace measure and the complex contour measure of the vertex $i$, we bring
the vertices $i$ and $j$ to the same point in $N = 2$ Grassmann space and in the Riemann sphere. After reducing all the gauge propagators in this fashion, we complete the superspace measure in all other vertices by extracting total derivatives from the hypermultiplet propagators. The final steps are the same as those described in [2]: we perform the “D”-algebra using transfer rules and integration by parts to reduce the hypermultiplet propagators to bare Grassmann delta functions. Integrating the corresponding coordinates, at the end of the process we are left with one bare Grassmann delta function multiplying certain number of spinor derivatives that act on a similar delta function

$$
\delta^8(\theta_n - \theta_m) \nabla_p \ldots \nabla_q \delta^8(\theta_n - \theta_m) .
$$

(91)

Any number of spinor derivatives larger than 8 must be reduced using the anticommutation relations (6) of projective derivatives. Any number of spinor derivatives smaller than 8 makes the product vanish, while 8 derivatives completely eliminate the Dirac delta function on the right [2]. The last bare delta function can be integrated over one of the Grassmann coordinates and the final amplitude is local Grassmann functional integrated with the full $N = 2$ superspace measure.

Finally, we must perform the complex contour integrals. As we mentioned in references [2] and [8] this step usually involves using a radial ordering prescription on the complex contours. Some examples of 1-loop and 2-loop calculations using these Feynman rules will be presented in a future publication [9].

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