The Design Theory of the Shaving Cutter of Conjugate Curved Gears

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Abstract: Based on the existing research on the conjugate curve gear, the meshing equation of the shaving process is deduced according to the gear meshing principle and the working principle of the disc shaving cutter, so as to obtain the tooth surface equation of the shaving cutter corresponding to the conjugate curve gear, which provides a theoretical basis for the design of the shaving cutter of the conjugate curve gear.

1. Introduction

Gear transmission is one of the most widely used mechanical transmission methods in modern equipment. Different from the common involute gear drive, worm gear drive, and bevel gear drive, the principle of the conjugate curve of gear drive has been proposed in recent years \cite{1,2}. Studies have shown that the suitable curved conjugate meshing pair has the characteristics of approximately pure rolling, a low sliding rate of the tooth surface, and high transmission efficiency \cite{3}.

In the preliminary study, gear hobbing was widely used in rough processes \cite{4}, and the theory of form grinding was put forward in finishing processes \cite{5}.

Compared with tooth grinding, tooth shaving, as one of the finishing processes of gears, can not only steadily improve the precision and the surface quality of the gears, but also has the characteristics of high processing efficiency and long tool life \cite{6}. In addition, due to the relatively simple equipment of the shaving process, the high quality of the shaving gear, the durable shaving cutter, and the simple structure of the machine tool, the shaving process is widely used as a gear finishing process in the production industries such as automobiles, tractors, and machine tools.

Finishing the conjugate curve gear with the shaving process is the main way to further improve the transmission characteristics of the conjugate curve gear.

2. Tooth Surface Equation of Conjugate Curved Gear

The coordinate axis is established, as shown in Figure 1. The coordinate system $S_p(O_p - x_p, y_p, z_p)$ is a fixed coordinate system in space, and the coordinate system $S_z(O_z - x_z, y_z, z_z)$ is fixedly connected with the gear, which is changed by the fixed coordinate system $S_p$ with the rotation of the gear. The coordinate system $S_n(O_n - x_n, y_n, z_n)$ and the coordinate system $S_s(O_s - x_s, y_s, z_s)$ are fixedly connected with the rack tool, respectively representing the normal surface and the end surface of the
rack. The angle between the coordinate axes \( z_n \) and \( z_s \) is the helix angle \( \beta \) of the gear, and the distance between the coordinate origins \( O_n \) and \( z_s \) is \( u \). \( r_2 \) refers to the radius of the pitch cylinder of the gear.

![Figure 1 Rack tool generation tooth surface](image)

During the generating motion, the pitch plane of the rack cutter is tangent to the pitch cylinder face of the gear. The rack tool moves at a constant speed at the linear speed of \( V_s \) along the opposite direction of the coordinate axis \( y_s \), and at the same time, the gear rotates counterclockwise along the coordinate axis \( z_p \) at the angular speed of \( \omega_0 \). When the angle that the gear rotates is \( 2\theta_2 \), the distance that the rack cutter moves will be \( r_2\theta_2 \).

The following coordinate transformations can be obtained from the above figure:

1. The transformation matrix from coordinate system \( S_n \) to \( S_s \) is:

\[
M_{sn} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \beta & \sin \beta & u \sin \beta \\ 0 & -\sin \beta & \cos \beta & u \cos \beta \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

2. The transformation matrices of coordinate systems \( S_s \) and \( S_2 \) are:

\[
M_{2s} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & r_2(\cos \theta_2 + \theta_2 \sin \theta_2) \\ \sin \theta_2 & \cos \theta_2 & 0 & r_2(\sin \theta_2 - \theta_2 \cos \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

There is \( z_n = 0 \) in the normal tooth profile equation of the rack, which is transformed into the coordinate system \( S_s \) according to the formula to obtain the rack tooth surface \( r_s \):

\[
\begin{align*}
(x_s &= x_n \\
y_s &= y_n \cos \beta + u \sin \beta \\
z_s &= -y_n \sin \beta + u \cos \beta
\end{align*}
\]

The normal vector of any point on the rack tooth surface is:

\[
n_s = \frac{\partial r_s}{\partial \xi} \times \frac{\partial r_s}{\partial \eta}
\]

Where \( \xi \) represents the parameter in the normal tooth profile equation of the rack.

In the coordinate system \( S_s \), the motion velocity vectors of the rack and gear are as follows.
Where $\omega$ is the angular velocity vector, whose direction is the negative direction of the $p_z$ axis, and the magnitude is $\omega_0$. $r_p$ is the position vector of any point on the tooth surface in the coordinate system $S_p$.

The conjugate mesh of the rack and pinion satisfies the equation:

$$n_s \cdot V = 0$$

In the formula, $V = V_p - V_s$ is the relative movement speed of the rack and pinion.

The tooth surface equation of the gear can be solved by equations (2), (3), and (6):

$$\begin{align*}
x_2 &= (r_2 + x_n) \cos \theta_2 + \frac{x_n x_n \cos \beta \sin \theta_2}{y_n} \\
y_2 &= (r_2 + x_n) \sin \theta_2 - \frac{x_n x_n \cos \beta \cos \theta_2}{y_n} \\
z_2 &= (r_2 \theta_2 - \frac{(x_n x_n + y_n y_n) \cos \beta}{y_n}) \cot \beta - y_n \sin \beta
\end{align*}$$

3. Design of Conjugate Curve Gear Razor

When the shaving cutter processes the gear, it meshes with the staggered axis helical gear. The working principle of axial shaving is that the shaving cutter rotates to drive the gear to rotate and shave it, and the shaving cutter feeds slowly along the axis of the gear to ensure that the full length of the shaving gear is processed. When the axial feed is reversed, the rotation direction of the shaving cutter will also be reversed, and the feed will be perpendicular to the axial direction of the shaving cutter. When the shaving allowance of the shaved gear is all removed, 2 ~ 3 idle strokes will be performed without vertical feed to improve the tooth surface quality of the shaved gear. Knowing the gear tooth surface equation, according to the calculation method of the point contact helical tooth surface [7], the above working principle is equivalent to the rotation of the shaving cutter to drive the gear to rotate. At the same time, it makes a uniform linear motion along the axial direction of the shaving cutter.

3.1 Coordinate Systems and Coordinate Transformations

The coordinate system is constructed as shown in Figure 2. The coordinate systems $S(O - x, y, z)$ and $S_p(O_p - x_p, y_p, z_p)$ are fixed coordinate systems. The axis $z$ and the $p_z$ represent the rotation axes of the shaving cutter and the gear, and the angle $\Sigma$ between the two axes is the staggered angle of the shaving cutter and the gear. Axis $x$ coincides with the axis $x_p$, and the distance $a$ between them is the center distance.

![Figure 2. Shaving meshing coordinate system](image-url)
Coordinate systems $S_1(O_1-x_1,y_1,z_1)$ and $S_2(O_2-x_2,y_2,z_2)$ are respectively fixed to the shaving cutter and gear, and initially coincide with coordinate systems $S$ and $S_p$. The shaving cutter rotates uniformly around the axis $z$ at an angular velocity $\omega_1$, and the gear rotates uniformly around the axis $z_p$ at angular velocity $\omega_2$. Meanwhile, the shaving cutter moves in a straight line at a constant speed along the $z$ axis at speed $v_0$. When the razor turns through the angle $\varphi_1$, the gear turns through the angle $\varphi_2$. At this time, $OO_2 = l_1$.

1. The transformation matrix from coordinate system $S_2$ to $S$ is

$$M_{02} = \begin{bmatrix}
\cos \varphi_2 & \sin \varphi_2 & 0 & -a \\
-cos \varphi_2 \sin \varphi_2 & cos \varphi_2 \cos \varphi_2 & \sin \varphi_2 \sin \varphi_2 & \cos \varphi_2 \cos \varphi_2 & \cos \varphi_2 \\
sin \varphi_2 \cos \varphi_2 & -sin \varphi_2 \cos \varphi_2 & \cos \varphi_2 \sin \varphi_2 & \cos \varphi_2 \cos \varphi_2 & \cos \varphi_2 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

(8)

2. The transformation matrix from coordinate system $S$ to $S_1$ is

$$M_{10} = \begin{bmatrix}
\cos \varphi_1 & \sin \varphi_1 & 0 & 0 \\
-sin \varphi_1 \cos \varphi_1 & \cos \varphi_1 \cos \varphi_1 & \sin \varphi_1 \sin \varphi_1 & \cos \varphi_1 \cos \varphi_1 & \cos \varphi_1 \\
0 & 0 & 1 & -l_1 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(9)

3.2 Normal Vector and Relative Motion Velocity

As shown in Figure 3, in the coordinate system $S(O-x,y,z)$, $i$, $j$, and $k$ are the unit vectors of the coordinate axes $x$, $y$, and $z$, respectively. The position vector of any point on the tooth surface of the shaved gear is $r(x,y,z)$, which is obtained by combining equation 7 and the transformation matrix equation 8.

![Figure 3 The relative motion speed of the gear meshing with the razor](image_url)

The angular velocity of the shaving cutter 1 and the velocity of the axial movement can be represented by a vector as:

$$\begin{align*}
\omega^{(1)} &= \omega_1 k \\
v_0^{(1)} &= v_{01} k
\end{align*}$$

(10)

Similarly, the angular velocity of the shaved gear 2 and the velocity of the axial movement can be represented by a vector as:
\[
\begin{align*}
\omega^{(2)} &= -\omega_2 \sin \Sigma j - \omega_2 \cos \Sigma k \\
\mathbf{v}_0^{(2)} &= 0
\end{align*}
\]  

(11)

At a point \( M \) in space, its coordinate position in the coordinate system \( S(O-x, y, z) \) is \( (x, y, z) \), and the following relational expression can be obtained:

\[
\begin{align*}
\mathbf{r}^{(1)} &= \overrightarrow{OM} = xi + yj + zk \\
\mathbf{r}^{(2)} &= \overrightarrow{O_pM} = (a + x)i + yj + zk
\end{align*}
\]  

(12)

The speed of point \( M \) when moving with the shaving cutter 1 and the shaving gear 2 is as follows.

\[
\begin{align*}
\mathbf{v}^{(1)} &= \omega^{(1)} \times \mathbf{r}^{(1)} + \mathbf{v}_0^{(1)} \\
\mathbf{v}^{(2)} &= \omega^{(2)} \times \mathbf{r}^{(2)} + \mathbf{v}_0^{(2)}
\end{align*}
\]  

(13)

The relative velocity of the two gears at the point \( M \) is:

\[
\mathbf{v}^{(12)} = \mathbf{v}^{(1)} - \mathbf{v}^{(2)}
\]  

(14)

The normal vector is:

\[
\mathbf{n} = (n_x, n_y, n_z) = \frac{\partial \mathbf{r}(x,y,z)}{\partial \theta} \times \frac{\partial \mathbf{r}(x,y,z)}{\partial \theta_2}
\]  

(15)

### 3.3 Tooth Surface Equation of Shaving Cutter

During the shaving process, since the shaving cutter and the shaved gear are always in contact, the meshing equation is satisfied when in contact:

\[
\mathbf{n} \cdot \mathbf{v}^{(12)} = 0
\]  

(16)

The tooth surface equation of the shaved gear 2 is converted into the coordinate system \( S_i(O_1-x_1, y_1, z_1) \) fixed with the shaving cutter through the transformation matrix, which satisfies the meshing equation. The tooth surface equation of the shaving cutter 1 can be obtained as follows.

\[
\begin{align*}
\varphi_1 &= \varphi_2 + \lambda_1 \\
\varphi_1 &= \lambda_1 \omega_2 + \frac{v_{01}}{P_1} \\
\omega_1 &= \lambda_1 \omega_2 + \frac{v_{01}}{P_1}
\end{align*}
\]  

(17)

Among them,

\[
\begin{align*}
\varphi_1 &= \lambda_1 \varphi_2 + \lambda_1 \\
\omega_1 &= \lambda_1 \omega_2 + \frac{v_{01}}{P_1}
\end{align*}
\]  

### 4. Conclusion

As a new transmission mechanism, the conjugate curve gear pair is worthy of in-depth understanding and research due to its advantages of large bearing capacity and long service life. In terms of finishing, the feasibility of shaving technology on the conjugate curve gear pair is studied in this paper. Meanwhile, the corresponding tooth surface equation of the axial shaving cutter is designed and derived, providing the theoretical basis for the shaving finishing of conjugate curve gears.

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