On Limitations of T Invariance in K Decays

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Abstract

Data from the CPLEAR collaboration coupled with the assumption that the Bell-Steinberger relation holds have provided direct evidence for T violation. In this note we investigate what we can say about T violation without such an assumption.

We show that both the modulus and the phase of $\eta_{+-}$ can be reproduced with T invariant dynamics through finetuning CPT breaking. The large T odd correlation observed by the KTeV collaboration in $K_L \rightarrow \pi^+\pi^-e^+e^-$ thus does not yield direct evidence for T violation. In such a world the phase of $\delta'$ is $\delta_2 - \delta_0 - \phi_{SW} \sim - (85.5 \pm 4)^\circ$. Also, $K^\pm \rightarrow \pi^\pm\pi^0$ could exhibit a CPT asymmetry of up to few$\times 10^{-3}$ without upsetting any known bound.

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CPT symmetry has an impressive theoretical pedigree as an almost inescapable consequence of Lorentz invariant quantum field theories. Observation of a CP asymmetry is therefore usually seen as tantamount to the discovery of a violation of time reversal invariance T. However the experimental verification of CPT invariance is much less impressive. Furthermore the emergence of superstring theories has opened – by their fundamentally non-local structure – a theoretical backdoor through which CPT breaking might slip in. This asks for carefully analysing the empirical basis of CPT invariance and the degree to which an observable can establish T violation directly, i.e. without invoking the CPT theorem[1]. In addressing this issue, we will rely on as few other theoretical principles as possible: since we view the observation of CPT violation as a rather exotic possibility, we believe we should accept other theoretical restrictions very reluctantly only.

Data from the CPLEAR collaboration have provided direct evidence for T violation[2]. In this note we want to address the following questions:

- To which degree and in which sectors of $\Delta S \neq 0$ dynamics is T violated?
- How accurately is the validity of CPT invariance established experimentally?
- Which conclusions can be drawn without invoking the Bell-Steinberger relation.
- Which is the most promising – or the least hopeless – observable for finding CPT violations in kaon decays?

The reader might wonder why we are insisting on analyzing T symmetry without assuming the Bell-Steinberger relation. After all, it is viewed as just a consequence of unitarity. Yet the following has to be kept in mind: when contemplating the possibility of CPT violation – a quite remote and exotic scenario – we should not consider the Bell-Steinberger relation sacrosanct. The latter is based on the assumption that all relevant decay channels are known. Since the major branching
fractions have been measured with at best an error of 1%, some yet undetermined decay mode with a branching fraction of $10^{-3}$ can easily be hidden [3]. We are not arguing that this is a likely scenario – it is certainly not! However we do not view it to be more exotic than CPT violation. Then it does not make a lot of sense to us to allow for the latter while forbidding the former.

The paper will be organized as follows: after briefly reviewing the formalism relevant for $K^0 - \bar{K}^0$ oscillations in Sect. 2 we list the direct evidence for $T$ being violated in Sect. 3; in Sect. 4 we analyse the phases of $\eta_{+}$ and $\eta_{0}$; after evaluating what can be learnt from $K_L \to \pi^+ \pi^- e^+ e^-$ in Sect. 5, we give our conclusions in Sect. 6.

2 Formalism

To introduce our notation and make the paper self-contained we shall record here the standard formalism for the neutral $K$ meson system.

2.1 $\Delta S = 2$ Transitions

The time dependence of the state $\Psi$, which is a linear combination of $K^0$ and $\bar{K}^0$, is given by

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = \mathcal{H} \Psi(t), \quad \Psi(t) = \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix}.$$  (1)

The $2 \times 2$ matrix $\mathcal{H}$ can be expressed through the identity and the Pauli matrices $\sigma_i$

$$\mathcal{H} \equiv M - \frac{i}{2} \Gamma = E_1 \sigma_1 + E_2 \sigma_2 + E_3 \sigma_3 - i D 1.$$  (2)

with

$$E_1 = \text{Re} \, M_{12} - \frac{i}{2} \text{Re} \Gamma_{12}, \quad E_2 = - \text{Im} \, M_{12} + \frac{i}{2} \text{Im} \Gamma_{12},$$

$$E_3 = \frac{1}{2} (M_{11} - M_{22}) - \frac{i}{4} (\Gamma_{11} - \Gamma_{22}), \quad D = \frac{i}{4} (M_{11} + M_{22}) + \frac{1}{4} (\Gamma_{11} + \Gamma_{22}).$$  (3)

It is often convenient to use instead complex numbers $E, \theta$, and $\phi$ defined by

$$E_1 = E \sin \theta \cos \phi, \quad E_2 = E \sin \theta \sin \phi, \quad E_3 = E \cos \theta,$$

$$E = \sqrt{E_1^2 + E_2^2 + E_3^2}.$$  (4)

The mass eigenstates are given by

$$|K_S\rangle = p_1 |K^0\rangle + q_1 |\bar{K}^0\rangle,$$

$$|K_L\rangle = p_2 |K^0\rangle - q_2 |\bar{K}^0\rangle.$$  (5)
with the convention $\text{CP}|K^0⟩ = |\bar{K}^0⟩$ and

$$
\begin{align*}
p_1 &= N_1 \cos \frac{\theta}{2}, \quad q_1 = N_1 e^{i\phi} \sin \frac{\theta}{2}, \\
p_2 &= N_2 \sin \frac{\theta}{2}, \quad q_2 = N_2 e^{i\phi} \cos \frac{\theta}{2}, \\
N_1 &= \frac{1}{\sqrt{|\cos \frac{\theta}{2}|^2 + |e^{i\phi} \sin \frac{\theta}{2}|^2}}, \\
N_2 &= \frac{1}{\sqrt{|\sin \frac{\theta}{2}|^2 + |e^{i\phi} \cos \frac{\theta}{2}|^2}}.
\end{align*}
$$

The discrete symmetries impose the following constraints:

- **CPT or CP invariance** $\implies$ $\cos \theta = 0$, $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$
- **CP or T invariance** $\implies$ $\phi = 0$, $\text{Im} M_{12} = 0 = \text{Im} \Gamma_{12}$

\[ \text{(7)} \]

### 2.2 Nonleptonic Amplitudes

We write for the amplitudes describing decays into final states with isospin $I$:

$$
\begin{align*}
T(K^0 \to [\pi\pi]_I) &= A_I e^{i\delta_I}, \\
T(\bar{K}^0 \to [\pi\pi]_I) &= \overline{A_I} e^{i\delta_I}
\end{align*}
$$

where the strong phases $\delta_I$ have been factored out and find:

- **CPT invariance** $\implies A_I = \overline{A_I}$
- **CP invariance** $\implies A_I = \overline{A_I}$
- **T invariance** $\implies A_I = A_I$

\[ \text{(9)} \]

The expressions for $\eta_{+-}$ and $\eta_{00}$

$$
\begin{align*}
\eta_{+-} &= \frac{1}{2} \left( \Delta_0 - \frac{1}{\sqrt{2}} \omega e^{i(\delta_2 - \delta_0)} (\Delta_0 - \Delta_2) \right), \\
\eta_{00} &= \frac{1}{2} \left( \Delta_0 + \sqrt{2} \omega e^{i(\delta_2 - \delta_0)} (\Delta_0 - \Delta_2) \right), \\
\Delta_I &= \frac{1}{2} \left( 1 - \frac{q_2 A_I}{p_2 A_I} \right), \quad |\omega| \equiv \left| \frac{A_2}{A_0} \right| \lesssim \frac{1}{20},
\end{align*}
$$

\[ \text{(10)} \]

are valid irrespective of CPT symmetry.
2.3 Semileptonic Amplitudes

The general amplitudes for semileptonic $K$ decays can be expressed as follows:

\[
\langle l^+ \nu \pi^- | \mathcal{H}_W | K^0 \rangle = F_l (1 - y_l)
\]

\[
\langle l^+ \nu \pi^- | \mathcal{H}_W | \overline{K}^0 \rangle = x_l F_l (1 - y_l)
\]

\[
\langle l^- \pi^+ | \mathcal{H}_W | K^0 \rangle = \pi_l^* F_l^* (1 + y_l^*)
\]

\[
\langle l^- \pi^+ | \mathcal{H}_W | \overline{K}^0 \rangle = F_l^* (1 + y_l^*)
\]

(11)

with the selection rules

$\Delta S = \Delta Q$ rule: $x_l = \pi_l = 0$

CP invariance: $x_l = \pi_l^*$; $F_l = F_l^*$; $y_l = -y_l^*$

T invariance: $\text{Im } F = \text{Im } y_l = \text{Im } x_l = \text{Im } \pi_l = 0$

CPT invariance: $y_l = 0$, $x_l = \pi_l$.

3 Direct Evidence for T Violation

The so-called Kabir test\cite{5} represents a quantity that probes $T$ violation without reference to CPT symmetry:

\[
A_T \equiv \frac{\Gamma(K^0 \rightarrow \overline{K}^0) - \Gamma(\overline{K}^0 \rightarrow K^0)}{\Gamma(K^0 \rightarrow \overline{K}^0) + \Gamma(\overline{K}^0 \rightarrow K^0)}
\]

(12)

A nonvanishing $A_T$ requires

\[
M_{12} - \frac{i}{2} \Gamma_{12} \neq M_{21} - \frac{i}{2} \Gamma_{21}.
\]

(13)

which constitutes CP as well as $T$ violation. Associated production flavor-tags the initial kaon. The flavor of the final kaon is inferred from semileptonic decays; i.e., we measure the $CP$ asymmetry

\[
A_{CP} \equiv \frac{\Gamma(K \rightarrow l^- \nu \pi^+) - \Gamma(\overline{K} \rightarrow l^+ \nu \pi^-)}{\Gamma(K \rightarrow l^- \nu \pi^+) + \Gamma(\overline{K} \rightarrow l^+ \nu \pi^-)}
\]

(14)

Yet a violation of CPT invariance and/or of the $\Delta S = \Delta Q$ rule can produce an asymmetry in the latter – $A_{CP} \neq 0$ – without one being present in the former – $A_T = 0$. These issues have to be tackled first. There is nothing new in our remarks on this subject; we add them for clarity and completeness.

Analysing the asymmetries in $\Gamma(\overline{K}^0(t) \rightarrow l^+ \nu K^-)$ vs. $\Gamma(K^0(t) \rightarrow l^- \bar{\nu} K^+)$ and $\Gamma(\overline{K}^0(t) \rightarrow l^- \bar{\nu} K^+)$ vs. $\Gamma(K^0(t) \rightarrow l^+ \nu K^-)$ for large times $t$ C Plear has found\cite{6}

\[
\text{Re } \cos \theta = (6.0 \pm 6.6 \pm 1.2) \times 10^{-4}.
\]

(15)
From the decay rate evolution they have inferred

\[ \text{Im} \cos \theta = (-3.0 \pm 4.6 \pm 0.6) \times 10^{-2}, \]
\[ \frac{1}{2} \text{Re} (x_l - \bar{x}_l) = (0.2 \pm 1.3 \pm 0.3) \times 10^{-2}, \]
\[ \frac{1}{2} \text{Im} (x_l + \bar{x}_l) = (1.2 \pm 2.2 \pm 0.3) \times 10^{-2}. \]  

(16)

While there is no sign of CPT violation in any of these observables, the bounds of Eq.(16) are not overly restrictive.

Another input is provided by the charge asymmetry in semileptonic \( K_L \) decays for which the general expression reads as follows:

\[ \delta_{\text{Lept}} = \frac{\Gamma(K_L \rightarrow l^+ \nu \pi^-) - \Gamma(K_L \rightarrow l^- \nu \pi^+)}{\Gamma(K_L \rightarrow l^+ \nu \pi^-) + \Gamma(K_L \rightarrow l^- \nu \pi^+)} = \text{Im} \phi - \text{Re} \cos \theta - \text{Re} (x_l - \bar{x}_l) - 2\text{Re} y_l. \]  

(17)

CPT violation, if it exists, is most likely to surface in \( M_{12} \), which is of second order in the weak interactions. It is then natural to assume \textit{semileptonic} decay amplitudes to conserve CPT, which is fully consistent with Eq.(16), but not confirmed to the required level:

\[ x_l - \bar{x}_l = 0, \quad \text{or} \quad y_l = 0. \]  

(18)

With this assumption, and from the data [7]

\[ \delta_{\text{Lept}} = (3.27 \pm 0.12) \times 10^{-3}. \]  

(19)

one obtains

\[ \text{Im} \phi - \text{Re} \cos \theta = (3.27 \pm 0.12) \times 10^{-3} \]  

(20)

and infers from Eq.(15)

\[ \text{Im} \phi = (3.9 \pm 0.7) \times 10^{-3}. \]  

(21)

showing that \( T \) is violated in kaon dynamics.

This result can be stated more concisely as follows [9]:

\[ A_T \simeq A_{\text{CP}} = (6.6 \pm 1.3 \pm 1.0) \times 10^{-3}. \]  

(22)

In order to get a result independent of the assumption that \textit{direct semileptonic} kaon decays obey CPT symmetry, the CPLEAR collaboration has employed constraints from the Bell-Steinberger relation to deduce the bound [4]

\[ \frac{1}{2} \text{Re} (x_l - \bar{x}_l) - \text{Re} y_l = (-0.4 \pm 0.6) \times 10^{-3}, \]  

(23)

which again is fully consistent with CPT invariance of the semileptonic decays. This results in establishing violation of \( T \) symmetry – provided the assumption mentioned above is valid.
4 Phases of $\eta_{+-}$ & $\eta_{00}$ and CPT

4.1 Basic Expressions

Manipulating Eq.(10) we obtain through $O(\phi)$ and $O(\cos \theta)$

$$|\eta_{+-}| \frac{\Delta \Phi}{\sin \phi_{SW}} = \left( \frac{M_K - M_{\bar{K}}}{2 \Delta M} + R_{direct} \right)$$  \hspace{1cm} (24)$$

$$\Delta \Phi \equiv \frac{2}{3} \phi_{+-} + \frac{1}{3} \phi_{00} - \phi_{SW}$$

$$R_{direct} = \frac{1}{2} \text{Re} \left( r_A - \frac{i e^{-i \phi_{SW}}}{\sin \phi_{SW}} \sum_{f \neq [2\pi]_0} \epsilon(f) \right)$$

$$r_A \equiv \frac{A_0}{A_0} - 1, \quad \phi_{SW} \equiv -\frac{1}{2} \frac{2 \Delta M}{\Delta \Gamma}$$

$$\epsilon(f) = e^{i \phi_{SW}} \cos \phi_{SW} \frac{\text{Im} \Gamma_{12}(f)}{\Delta \Gamma}.$$  \hspace{1cm} (25)$$

Since CPT symmetry predicts $M_K = M_{\bar{K}}$ and Re $r_A = O(\xi_0^2)$, where $\xi_0 = \text{arg} A_0$, it implies $|\Delta \Phi| = 0$ to within the uncertainty given by $|\sum_{f \neq [2\pi]_0} \epsilon(f)|$; the latter sum thus represents the theoretical ‘noise’.

4.2 Estimating $\sum \epsilon(f)$

The major kaon decay modes fall into two classes, namely flavor-non-specific or flavor-specific channels.

- With $A_f = \langle f | H_W | K^0 \rangle$ and $\bar{A}_f = \langle f | H_W | \bar{K}^0 \rangle$, we have, to first order in CP violation,

$$\text{Im} \Gamma_{12}(f) = i \eta_{f} \Gamma(K \to f) \left( 1 - \eta_{f} \frac{\bar{A}_f}{A_f} \right),$$  \hspace{1cm} (26)$$

for CP eigenstates with eigenvalue $\eta_f$. Im $\Gamma_{12}(f) \neq 0$ can hold only if $\bar{A}_f \neq \eta_f A_f$, i.e. if there is direct CP violation in the channel $f$.

Using data on $\epsilon'$, Br$(K_L, S \to 3\pi)$ and

$$\text{Im} \eta_{+-0} = \left( -2 \pm 9 + 2 \right) \times 10^{-3}, \quad \text{Im} \eta_{000} = 0.07 \pm 0.16 \quad [10, \text{[I]}]$$  \hspace{1cm} (27)$$

where

$$\eta_{+-0,000} \equiv \frac{1}{2} \left( 1 + \frac{q_1}{p_1} A(\pi^+ \pi^- \pi^0, 3\pi^0) \right),$$  \hspace{1cm} (28)$$
we obtain

\[ |\epsilon(3\pi^0)| < 1.1 \times 10^{-4} \]
\[ |\epsilon([2\pi]_2)| \simeq 0.28 \times 10^{-6} \]
\[ |\epsilon((\pi^+\pi^-\pi^0)_{CP[-+]|})| < 5 [0.2] \times 10^{-6} , \]

(29)

- Allowing for a violation of the $\Delta Q = \Delta S$ rule in semileptonic decays as expressed by

\[ x_l \equiv \frac{\langle l^+\nu\pi^-|H_{\nu\pi}\rangle_{|K}}{\langle l^+\nu\pi^-|H_{\nu\pi}\rangle_{|K^*}} \]

we find

\[ |\epsilon(\pi l\nu)| \leq 4 \times 10^{-7} . \]

(30)

4.3 Quantifying CPT Tests

With the measured values for the phases $\phi_{+-}$, $\phi_{00} - \phi_{+-}$, and $\phi_{SW}$ we arrive at a result quite consistent with zero [7, 12, 18]:

\[ \Delta \Phi = 0.01^\circ \pm 0.7^\circ|_{exp.} \pm 1.5^\circ|_{theor.} , \]

(31)

i.e., the phases $\phi_{+-}$ and $\phi_{00}$ agree with their CPT prescribed values to within 2°. CPT invariance is thus probed to about the $\delta \phi/\phi_{SW} \sim 5\%$ level. The relationship between $\phi_{+-}$, $\phi_{00}$ on one side and $\phi_{SW}$ on the other is a truly meaningful gauge; yet the numerical accuracy of that test is not overwhelming. The theoretical error can be reduced significantly by making quite reasonable assumptions on CP violation; however, we refrain from doing so based on our belief that assuming observable CPT breaking is not very reasonable to start with.

In Eq.(31), the theoretical uncertainty $\sum_f \epsilon(f)$ provides the limiting factor for this test [13]; it is dominated by $K \rightarrow 3\pi^0$. Future experiments could reduce the uncertainty by a factor of up to two [11]. Alternatively we can state

\[ \frac{M_{\pi^0} - M_K}{2\Delta M} + \frac{1}{2} \Re r_A = (0.06 \pm 4.0|_{exp} \pm 9|_{theor}) \times 10^{-5} . \]

(32)

Yet $\Delta M$ does not provide a meaningful calibrator; for it arises basically unchanged even if CP were conserved while the latter would imply $M_{\pi^0} - M_K = 0$ and $r_A = 0$ irrespective of CPT breaking.

The often quoted truly spectacular bound (for $R_{direct} = 0$)

\[ \frac{M_{\pi^0} - M_K}{M_K} = (0.08 \pm 5.3|_{exp}) \times 10^{-19} \]

(33)

definitely overstates the numerical degree to which CPT invariance has been probed. $M_K$ is not generated by weak interactions and thus cannot serve as a meaningful yardstick.

In summary: while no hint has has found indicating a limitation to CPT symmetry, the experimental evidence for it is far from overwhelming:
• Comparing the phases of $\eta_{+}$ and $\eta_{00}$ with the superweak phase constitutes a meaningful test of CPT symmetry. Yet there is a ‘noise’ level of about $2^{o}$ that cannot be reduced significantly [11].

• Relating the bound on the difference $|M_{K} - M_{K}|$ to the kaon mass itself is extremely impressive numerically – yet meaningless.

• When entertaining the idea of CPT violation, we should not limit our curiosity to a single quantity like $\Delta\Phi$ (or equivalently $M_{K} - M_{K}$).

• Finally, the reader should be reminded that CPT symmetry implies $\Delta\Phi \ll \phi_{SW}$ but the converse does not follow.

5 Consequences in a T Conserving World

5.1 Reproducing $\eta_{+-}$

Assuming nature to conserve $T$, which implies $\phi = 0$, see Eq.(7), we have:

\[
\begin{align*}
\frac{|\eta_{+-}|\Delta\Phi}{\sin\phi_{SW}} &= -\frac{M_{11} - M_{22}}{2\Delta M} + \frac{1}{2}r_{A}, \\
\frac{|\eta_{+-}|}{\cos\phi_{SW}} &= -\frac{\Gamma_{11} - \Gamma_{22}}{4\Delta M}t_{\gamma}\phi_{SW} - \frac{1}{2}r_{A}, \\
\text{Re} \cos \theta &= -\frac{M_{11} - M_{22}}{\Delta M} \sin^{2} \phi_{SW} + \frac{1}{2} \frac{\Gamma_{11} - \Gamma_{22}}{\Delta M} \sin \phi_{SW} \cos \phi_{SW}. 
\end{align*}
\]

Inserting the values of $\eta_{+-}$, $\phi_{SW}$ and Eq.(15) we can solve for the three unknowns:

\[
\begin{align*}
\frac{M_{11} - M_{22}}{\Delta M} &\simeq r_{A} \simeq (-3.9 \pm 0.7) \times 10^{-3} \\
\frac{\Gamma_{11} - \Gamma_{22}}{\Delta M} &\simeq (-5.0 \pm 1.4) \times 10^{-3}. 
\end{align*}
\]

(35)

The solution is very unnatural – Eq.(35), for example, requires cancellation between CPT violating $\Delta S = 1$ and 2 amplitudes. Yet however unnatural they may be, we must entertain this possibility unless we can exclude it empirically.

As a side remark, we mention that if we invoke the Bell-Steinberger relation in its usual form – meaning that kaon decays are effectively saturated by the $K \rightarrow 2\pi, 3\pi, l\nu\pi$ channels, then we have an additional relation [13]:

\[
\frac{1}{2}r_{A} \simeq -\frac{\Gamma_{11} - \Gamma_{22}}{4\Delta M};
\]

(36)

i.e, Eq.(34) then implies $\eta_{+-} \simeq 0$. This is not surprising since these known modes do not exhibit any sign of CPT violation. But, as we have remarked before, in testing CPT we want to stay away from invoking saturation by the known channels.
5.2 \( K \to \pi\pi \)

Where should such a large CPT violation show its face? Imposing \( r_A \neq 0 \) raises the prospects of unacceptably large direct CP violation in \( K_L \to \pi\pi \). Eq. (10) can be reexpressed as follows:

\[
\begin{align*}
\epsilon &\simeq \frac{1}{\sqrt{1 + \left(\frac{\Delta M}{2\Delta M}\right)^2}} e^{i\phi_{SW}} \left( \frac{\text{Im} M_{12}}{\Delta M_K} + \xi_0 \right) \\
\epsilon' &= \frac{1}{2\sqrt{2}}\omega e^{i(\delta_2-\delta_0)} \frac{q_2}{p_2} \left( \frac{A_0}{A_0} - \frac{A_2}{A_2} \right)
\end{align*}
\]

If \( T \) is conserved, \( \frac{q_2}{p_2} \left( \frac{A_0}{A_0} - \frac{A_2}{A_2} \right) \) is real and Eq. (38) then tells us [7]

\[
\arg \left( \frac{\epsilon'}{\epsilon} \right) = \delta_2 - \delta_0 - \phi_{SW} \simeq -(85.5 \pm 4)\degree.
\]

Therefore

\[
\text{Re} \frac{\epsilon'}{\epsilon} \simeq \cos(\delta_2 - \delta_0 - \phi_{SW}) \cdot \frac{\omega}{2\sqrt{2} |\eta_{+1}|} \cdot |\Delta_0 - \Delta_2|
\]

\[
= 0.035 \cdot (0.087^{+0.061}_{-0.078}) \cdot \left| \frac{r'_A}{r_A} - 1 \right| = (3.0^{+2.2}_{-2.7}) \cdot 10^{-3} \cdot \left| \frac{r'_A}{r_A} - 1 \right|
\]

where

\[
r'_A \equiv \frac{A_2}{A_2} - 1
\]

Some remarkable features can be read off from this expression:

- For

\[
\delta_2 - \delta_0 - \phi_{SW} = 90\degree
\]

which is still allowed by the data, one obtains

\[
\text{Re} \frac{\epsilon'}{\epsilon} = 0.
\]

As far as \( K \to \pi\pi \) is concerned this amounts to a superweak scenario!

- The empirical landscape of CP violation has changed qualitatively: KTeV, confirming earlier observations of NA 31, has conclusively established the existence of direct CP violation [8]:

\[
\text{Re} \frac{\epsilon'}{\epsilon} = (2.80 \pm 0.30 \pm 0.28) \cdot 10^{-3}
\]
Including previous data and preliminary results from NA 48 one arrives at a world average of

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) = (2.12 \pm 0.28) \cdot 10^{-3}$$ \hspace{1cm} (45)

This can be reproduced with a ‘canonical’ $r'_A = 0$, but only for a very narrow slice in the phase of $\epsilon'/\epsilon$, namely

$$\delta_2 - \delta_0 - \phi_{SW} \simeq - (86.5 \pm 0.5)^0.$$ \hspace{1cm} (46)

- The dominant uncertainty here enters through the phase shifts $\delta_{0,2}$. If $\delta_2 - \delta_0 - \phi_{SW}$ falls outside the range of Eq.(46), then $r'_A \neq 0$ is needed to reproduce $\text{Re}(\epsilon'/\epsilon)$. As an illustration consider $\delta_2 - \delta_0 - \phi_{SW} = 80^0$. In that case $1/2 \leq r'_A/r_A \leq 5/6$ had to hold to obtain $1 \cdot 10^{-3} \leq \text{Re}(\epsilon'/\epsilon) \leq 3 \cdot 10^{-3}$. Hence $r'_A \sim -(2 \div 4) \cdot 10^{-3}$. More generally if

$$\delta_2 - \delta_0 - \phi_{SW} \leq 83^0$$ \hspace{1cm} (47)

then the observed value of $\text{Re}(\epsilon'/\epsilon)$ would imply

$$r'_A \leq -10^{-3}$$ \hspace{1cm} (48)

if $T$ is conserved.

- This would have a dramatic impact on $K^\pm \rightarrow \pi^\pm \pi^0$ decays. For Eq.(18) implies a sizeable CPT asymmetry there

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \pi^0) - \Gamma(K^- \rightarrow \pi^- \pi^0)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0) + \Gamma(K^- \rightarrow \pi^- \pi^0)} > 10^{-3}$$ \hspace{1cm} (49)

With CPT symmetry we predict here a direct CP asymmetry of at most $O(10^{-6})$ due to electromagnetic corrections. Thirty year old data yield $(0.8 \pm 1.2) \cdot 10^{-2}$. Upcoming experiments will produce a much better measurement.

5.3 $K_L \rightarrow \pi^+\pi^-e^+e^-$

If the photon polarization $\vec{e}_\gamma$ in $K_L \rightarrow \pi^+\pi^-\gamma$ were measured, we could form the CP and T odd correlation $P^\gamma_\perp \equiv \langle \vec{e}_\gamma \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \rangle$. A more practical realization of this idea is to analyze $K_L \rightarrow \pi^+\pi^-e^+e^-$ which proceeds like $K_L \rightarrow \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-e^+e^-$. It allows to determine a CP and T odd moment $\langle A \rangle$ related to $P^\gamma_\perp$ by measuring the correlation between the $\pi^+\pi^-$ and $e^+e^-$ planes. This effect was predicted to be $[14]$:

$$\langle A \rangle = (14.3 \pm 1.3)\%$$ \hspace{1cm} (50)

and observed by KTeV $[15]$:

$$\langle A \rangle = (13.6 \pm 2.5 \pm 1.2)\%$$ \hspace{1cm} (51)
It is mainly due to the interference between the bremsstrahlung process \( K_L \Rightarrow K_{CP+} \Rightarrow \pi^+\pi^- \Rightarrow \pi^+\pi^-\gamma^* \) and a one-step M1 reaction \( K_L \Rightarrow \pi^+\pi^-\gamma^* \). The former is CP violating and described by \( \eta_{+-} \) irrespective of the theory underlying CP violation.

It is a remarkable measurement since it has revealed a huge CP asymmetry in a rare channel that had not been observed before. While T odd correlations have been seen before in production processes and in nonleptonic hyperon decays, those – due to their sheer magnitude – had to be blamed on final state interactions; such an explanation turned out to be consistent with what we know about those. The quantity \( \langle A \rangle \) on the other hand is a T odd correlation sui generis since it has a chance to be generated by microscopic T violation.

Yet the most intriguing question is what does this measurement teach us about T violation without reference to CPT symmetry? The answer is: Nothing really! For we have just shown – by giving a concrete example – that if we are sufficiently determined we can dial CPT violation in such a way that both the modulus and phase of \( \eta_{+-} \) are reproduced even with T invariant dynamics, and it is \( \eta_{+-} \) that controls \( \langle A \rangle \).

### 5.3.1 A Comment on the Intricacies of Final State Interactions

It is well-known that a non-vanishing T-odd correlation does not necessarily establish T violation since final state interactions can induce it even if T is conserved. Yet even so the reader might be surprised by our findings that a value of \( \langle A \rangle \) as large as 10% does not establish T violation. For it would be tempting to argue that in the case at hand final state interactions could not induce an effect even within an order of magnitude of the observed size. The argument might proceed as follows: \( \langle A \rangle \) reflects the correlation between the \( \pi^+ - \pi^- \) and the \( e^+ - e^- \) planes; their relative orientation can be affected by final state interactions – but only of the electromagnetic variety; then \( \langle A \rangle \gg 1\% \) could not arise.

If nothing else, our brute force scenario shows that such an argument is fallacious. This can be seen also more directly. As stated above there are two different contributions to \( K_L \Rightarrow \pi^+\pi^-e^+e^- \), namely the M1 amplitude which is CP neutral, and the bremsstrahlung one due to the presence of CP violation. One should note that the presence of the this second amplitude requires neither T violation nor final state interactions!

Let us assume for the moment that \( \text{arg} \ \eta_{+-} = 0 \) were to hold. Ignoring final state interactions both in the M1 and the bremsstrahlung amplitudes one obtains \( \langle A \rangle = 0 \), since the former is imaginary and the latter real now. When the final state interactions are switched back on, they affect the two amplitudes differently. Interference can take place, and one finds (with \( \text{arg} \ \eta_{+-} = 0 \)) \( \langle A \rangle \sim 8\% \). How can the orientation of the \( \pi^+ - \pi^- \) and the \( e^+ - e^- \) planes get shifted so much by strong final state interactions? The fallacy of the intuitive argument sketched above derives from its purely classical nature. In quantum mechanics it is not surprising at all.
that phase shifts between coherent amplitudes change angular correlations.

6 Summary

In this note we have listed the information we can infer on $T$ and $CPT$ invariance from the data on kaon decays. Our reasoning was guided by the conviction that once we contemplate $CPT$ breaking the notion of a reasonable or natural assumption starts to resemble an oxymoron.

Our findings can be summarized as follows:

- The presence of $T$ violation in $\Delta S \neq 0$ dynamics has been shown without invoking $CPT$ symmetry through the Kabir test performed by CPLEAR. Yet their analysis had to assume semileptonic kaon decays to be $CPT$ symmetric or it had to impose the Bell-Steinberger relation in its conventional form. We do not view either assumption as qualitatively more sacrosanct than $CPT$ symmetry.

- $\phi_{+-,00}$ lie within $2^o$ of what is expected from $CPT$ symmetry.

- A meaningful yardstick for calibrating bounds on limitations to $CPT$ symmetry is provided by $CP$ asymmetries. $CPT$ breaking forces could – empirically – still be as large as few percent of $CP$ violating forces.

- It is grossly misleading to calibrate the bound on $M_{\pi^-} - M_K$ inferred from $\phi_{+-}$, $\phi_{00}$ and $\phi_{SW}$ to the kaon mass.

- The measured values of $\eta_{+-}$ and $\eta_{00}$ provide us with little information on the level of $T$ versus $CPT$ violation. More specifically $\eta_{+-}$ – both its modulus as well as its phase – can be reproduced with $T$ invariant dynamics (unless one imposes the Bell-Steinberger relation):

  - This is achieved by carefully adjusting $CPT$ violation in $\Delta S = 1&2$ transitions.
  - The observed level of direct $CP$ violation – $\eta_{+-} \neq \eta_{00}$ – is not a natural consequence of such a scenario. However it could arise due to a fine-tuning of $\delta_2 - \delta_0 - \phi_{SW}$ – which had to be viewed as completely accidental – or to a compensation of direct $CPT$ violation in $K_L \to [\pi\pi]_0$ and $K_L \to [\pi\pi]_2$.
  - In the latter subscenario one is stuck with a $CPT$ asymmetry in $K^\pm \to \pi^\pm \pi^0$ that could be up to few $\times 10^{-3}$ without upsetting any known empirical bound.

- The KTeV observation of a large $CP$ and $T$ odd correlation in $K_L \to \pi^+\pi^-e^+e^-$ in agreement with theoretical predictions is highly intriguing, yet does not constitute an unequivocal signal for $T$ violation. This has also been noted before [17] using a different line of reasoning.
• We are fully aware that our construction is purely ad-hoc without any redeeming theoretical feature. Nevertheless we do not view it as l’art pour l’art (or more appropriately non-art pour non-art):

- We have shown by constructing an explicit counter-example that the $T$ odd correlation observed in $K_L \rightarrow \pi^+\pi^-e^+e^-$ does not establish $T$ violation without invoking the CPT theorem.
- As a by-product we have found that $K^\pm \rightarrow \pi^\pm\pi^0$ could exhibit a CPT asymmetry large enough to become observable soon.

Finally we would like to add the remark that even negative searches for CPT violation in kaon transitions will not free us from the obligation to probe for such effects in beauty meson decays at the $B$ factories.

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