Mass spectra in $\mathcal{N} = 1$ SQCD with additional fields. III

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Abstract

This paper continues arXiV: 1205.0410, 1211.1487 [hep-th]. We also consider here the $\mathcal{N} = 1$ SQCD - like theories (and their Seiberg’s dual) with $N_c$ colors and $N_F$ flavors of light quarks and with $N_F^2$ additional colorless fields $\Phi$, but now with $N_F$ in the range $2N_c < N_F < 3N_c$. The multiplicities of various vacua and quark condensates in these vacua are found. The mass spectra of the direct and dual theories in various vacua are calculated within the dynamical scenario #2 which assumes that quarks in such $\mathcal{N} = 1$ SQCD - like theories can be in two standard phases only. These are either the heavy quark phase where they are confined or the Higgs phase. Besides, this scenario implied that, unlike e.g. $\mathcal{N} = 2$ SQCD, no additional parametrically lighter particles (like magnetic monopoles or dyons) appear in these $\mathcal{N} = 1$ SQCD - like theories without adjoint colored scalar fields. The calculated mass spectra of these direct and dual theories were found to be different, in general.

Besides, the mass spectrum of the dual theory with $\bar{N}_c = N_F - N_c$ colors and $N_c + 1 < N_F < 3N_c/2$ dual quark flavors was calculated.

And finally, the mass spectrum in the direct $\mathcal{N} = 2$ SQCD with $N_c$ colors, $N_c + 1 < N_F < 3N_c/2$ flavors of quarks with the mass term $m\text{Tr}(\bar{Q}Q)$ in the superpotential, broken down to $\mathcal{N} = 1$ by the mass term $\mu_x X^2$ of the colored adjoint scalar field $X$, $m \ll \mu_x \ll \Lambda_2$, was calculated in vacua of the baryonic branch, and compared with those in the two different Seiberg’s dual theories. Our conclusions for this case disagree with those in the recent paper arXiv:1304.0822 of Shifman and Yung.
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1 Introduction

This article continues [1, 2]. We calculate below in sections 3-7 the mass spectra in \( \mathcal{N} = 1 \) SQCD-like theories which include, in addition to standard quarks and gluons, also \( N_F^2 \) colorless fion fields \( \Phi_i^F \) with the large mass parameter \( \mu_\Phi \gg \Lambda_Q \) in the superpotential. The Lagrangian of the direct \( \Phi \)-theory at the scale \( \mu = \Lambda_Q \) is taken in the form

\[
K = \text{Tr} \left( \Phi^\dagger \Phi \right) + \text{Tr} \left( Q^\dagger Q + (Q \rightarrow \overline{Q}) \right), \quad W = -\frac{2\pi}{\alpha(\mu, \Lambda_Q)} S + \Phi W + W_Q, \\
W_\Phi = \frac{\mu_\Phi}{2} \left[ \text{Tr} (\Phi^2) - \frac{1}{N_c} (\text{Tr} \Phi)^2 \right], \quad W_Q = \text{Tr} \overline{Q} (m_Q - \Phi) Q, \quad N_c = N_F - N_c. \tag{1.1}
\]

Here: the gauge group is \( SU(N_c) \), \( \mu_\Phi \) and \( m_Q = m_Q(\mu = \Lambda_Q) \) are the mass parameters, \( S = -W^a_\beta W^{a, \beta}/32\pi^2 \), where \( W^a_\beta \) is the gauge field strength, \( a = 1...N_c^2 - 1 \), \( \beta = 1, 2 \), \( a(\mu) = N_c \alpha(\mu)/2\pi = N_c g^2(\mu)/8\pi^2 \) is the gauge coupling with its scale factor \( \Lambda_Q \).

Besides, in addition to the direct \( \Phi \)-theory in (1.1), we calculate also the mass spectra in the Seiberg dual variant [3]. The Lagrangian of the dual \( d\Phi \)-theory at the scale \( \mu = \Lambda_Q \) looks as

\[
K = \text{Tr} \Phi^\dagger \Phi + \text{Tr} \left( q^\dagger q + (q \rightarrow \overline{q}) \right) + \text{Tr} \frac{M^i_M}{\Lambda_Q^2}, \quad W = -\frac{2\pi}{\alpha(\mu = \Lambda_Q)} \overline{s} + W_\Phi + W_q, \\
W_q = \text{Tr} M (m_Q - \Phi) - \frac{1}{\Lambda_Q} \text{Tr} \left( \overline{q} M q \right). \tag{1.2}
\]

Here: the number of dual colors is \( \overline{N}_c = (N_F - N_c) \) and \( M^i_j \) are \( N_F^2 \) elementary fion fields, \( \overline{\pi}(\mu) = \overline{N}_c \alpha(\mu)/2\pi = \overline{N}_c \overline{g}^2(\mu)/8\pi^2 \) is the dual gauge coupling (with its scale parameter \( \Lambda_\overline{q} \sim \Lambda_Q \), \( \overline{s} = -w^b_\beta w^{b, \beta}/32\pi^2 \), \( w^b_\beta \) is the dual gluon field strength. The gluino condensates of the direct and dual theories are matched as well as \( \langle M^i_j \rangle \) and \( \langle \overline{Q}_j Q^i \rangle \)

\[
\langle - \overline{s} \rangle = \langle S \rangle = \Lambda^3_{YM}, \quad \langle M^i_j \rangle \equiv \langle M^i_j(\mu = \Lambda_Q) \rangle = \langle \overline{Q}_j Q^i \rangle \equiv \langle \overline{Q}_j Q^i(\mu = \Lambda_Q) \rangle.
\]

In sections 3-7 the hierarchies of parameters entering (1.1),(1.2) will be: \( m_Q \ll \Lambda_Q \ll \mu_\Phi, \mu_\Phi \) will be varied while \( m_Q \) and \( \Lambda_Q \) will stay intact.

In those cases when the fields \( \Phi \) are too heavy and dynamically irrelevant, they can be integrated out and the Lagrangian of the dual theory takes the form

\[
K = \text{Tr} \left( q^\dagger q + (q \rightarrow \overline{q}) \right) + \text{Tr} \frac{M^i_M}{\Lambda_Q^2}, \tag{1.3}
\]

\[
W_{\text{matter}} = W_M - \frac{1}{\Lambda_Q} \text{Tr} \left( \overline{q} M q \right), \quad W_M = m_Q \text{Tr} M - \frac{1}{2 \mu_\Phi} \left[ \text{Tr} (M^2) - \frac{1}{\overline{N}_c} (\text{Tr} M)^2 \right].
\]

The mass spectra of the direct and dual theories will be calculated below within the dynamical scenario #2 introduced in [4]. Recall that this scenario assumes that, when such \( \mathcal{N} = 1 \) SQCD-like theories are in the strong coupling regime \( a(\mu) \gtrsim 1 \), the quarks can be in the two standard

\footnote{The gluon exponents are always implied in the Kahler terms. Besides, here and everywhere below in the text we neglect for simplicity all RG-evolution effects if they are logarithmic only.}
phases only. These are either the HQ (heavy quark) phase where they are confined or the Higgs phase. The word standard also implies here that, unlike e.g. the very special \( \mathcal{N} = 2 \) SQCD theories with colored adjoint scalar superfields, no additional parametrically lighter particles (like magnetic monopoles or dyons) appear in these \( \mathcal{N} = 1 \) SQCD-like theories without colored adjoint scalars.\(^2\)

The mass spectra were calculated before in [4] within this scenario \#2 in the standard \( \mathcal{N} = 1 \) SQCD and its dual variant (i.e. without the fion fields \( \Phi \) in (1.1) and (1.2)), and in the direct and dual \( \Phi \) - theories (1.1),(1.2) for the case \( 3N_c/2 < N_F < 2N_c \) in [2]. The purpose of this paper is to extent the results [2] to the region \( 2N_c < N_F < 3N_c \). This is the content of sections 2 - 6. As will be shown therein, the main properties of both \( \Phi \) and \( d\Phi \) - theories with \( 2N_c < N_F < 3N_c \), i.e. multiplicities of vacua and mass spectra differ, in general, from those in [2] with \( 3N_c/2 < N_F < 2N_c \).

Besides, we calculate in section 7 the mass spectrum of the dual \( d\Phi \) - theory for the case \( N_c + 1 < N_F < 3N_c/2 \). This is of interest by itself, but is also useful for the next section 8.

In this last section we consider \( \mathcal{N} = 2 \) SQCD with \( N_c \) colors (with the scale factor \( \Lambda_2 \) of the gauge coupling), \( N_c + 1 < N_F < 3N_c/2 \) flavors of original ”electric” quarks \( Q^i, \bar{Q}_i \) with the mass term \( m_{\text{tot}}Q^i\bar{Q}_i \) in the superpotential, broken down to \( \mathcal{N} = 1 \) by the mass term \( \mu_x \text{Tr}X^2 \) of the colored adjoint scalar superfield \( X \). The considered hierarchies of parameters look as: \( m \ll \mu_x \ll \Lambda_2 \). The purpose here was to calculate the mass spectra of this theory and its various Seiberg’s dual variants in vacua of the baryonic branch [11, 12] at scales \( \mu < \mu_x \) and to compare the results with those appeared recently in [10].

2 Quark condensates and multiplicity of vacua at \( N_F > 2N_c \)

For the reader convenience, we reproduce here first some useful formulas from [1] for the theories (1.1),(1.2).

The Konishi anomalies [5] for the \( i \)-th flavor look in the \( \Phi \) - theory (1.1) as \( (i = 1 \ldots N_F) \)

\[
\langle \Phi_i \rangle \langle \partial W_\Phi \rangle = 0, \quad \langle m_{\text{tot}, i}^{\Phi} \rangle \langle \bar{Q}_i Q^i \rangle = \langle S \rangle, \quad \langle m_{\text{tot}, i}^{\text{tot}} \rangle = m_Q - \langle \Phi_i \rangle,
\]

\[
\langle \Phi_i \rangle = \frac{1}{\mu_\Phi} \left( \frac{\bar{Q}_i Q^i}{\text{Tr} \bar{Q}Q} - \delta_j^i \frac{1}{N_c} \text{Tr} \bar{Q}Q \right), \quad \langle \bar{Q}_i Q^i \rangle = \delta_j^i \langle \bar{Q}_j Q^j \rangle \quad (2.1)
\]

and \( \langle m_{\text{tot}, i}^{\text{tot}} \rangle \) is the value of the quark running mass at the scale \( \mu = \Lambda_Q \).

At all scales until the field \( \Phi \) remains too heavy and non-dynamical, i.e. until its perturbative running mass \( \mu_\Phi^{\text{pert}}(\mu) > \mu \), it can be integrated out and the superpotential takes the form

\[
W_Q = m_Q \text{Tr}(\bar{Q}Q) - \frac{1}{2\mu_\Phi} \left( \text{Tr} (\bar{Q}Q)^2 - \frac{1}{N_c} \left( \text{Tr} \bar{Q}Q \right)^2 \right). \quad (2.2)
\]

The values of the quark condensates for the \( i \)-th flavor, \( \langle \bar{Q}_i Q^i \rangle \), in various vacua can be obtained from the effective superpotential [1]

\[
W_{\text{eff}} = -N_c S + W_Q, \quad S = \left( \frac{\det \bar{Q}Q}{\Lambda_{b_0}^{b_0}} \right)^{1/N_c}, \quad b_0 = 3N_c - N_F. \quad (2.3)
\]

\(^2\) In those cases when the coupling is logarithmically small this scenario \#2 is literally standard.
For the vacua with the spontaneously broken flavor symmetry, \(U(N_F) \to U(n_1) \times U(n_2)\), the most convenient way to find the quark condensates is to use \([1]\)

\[
\langle (\overline{Q}Q)_1 + (\overline{Q}Q)_2 - \frac{1}{N_c} \text{Tr} \overline{Q}Q \rangle_{\text{br}} = m_Q \mu_\Phi, \quad \langle S \rangle_{\text{br}} = \left( \frac{\text{det}(\overline{Q}Q)_{\text{br}}}{\Lambda_Q^{b_0}} \right)^{1/N_c} = \frac{\langle (\overline{Q}Q)_1 \rangle_{\text{br}} \langle (\overline{Q}Q)_2 \rangle_{\text{br}}}{\mu_\Phi},
\]

\[
det(\overline{Q}Q)_{\text{br}} = \langle (\overline{Q}Q)_1 \rangle_{\text{br}}^{n_1} \langle (\overline{Q}Q)_2 \rangle_{\text{br}}^{n_2}, \quad \langle \overline{Q}Q \rangle_1 = \overline{Q}_1 Q^1, \quad \langle \overline{Q}Q \rangle_2 = \overline{Q}_2 Q^2,
\]

\[
\langle m_{Q,1}^{\text{tot}} \rangle_{\text{br}} = m_Q - \langle \Phi_1 \rangle_{\text{br}} = \frac{\langle (\overline{Q}Q)_2 \rangle_{\text{br}}}{\mu_\Phi}, \quad \langle m_{Q,2}^{\text{tot}} \rangle_{\text{br}} = m_Q - \langle \Phi_2 \rangle_{\text{br}} = \frac{\langle (\overline{Q}Q)_1 \rangle_{\text{br}}}{\mu_\Phi}.
\]

The Konishi anomalies for the \(i\)-th flavor look in the dual \(d\Phi\) - theory (1.2) as \((i = 1 \ldots N_F)\)

\[
\langle M_i \rangle_{\overline{\gamma} q_i} = \langle S \rangle_{\Lambda_Q}, \quad \frac{\langle \overline{\tau} q_i \rangle_{\Lambda_Q}}{\langle \overline{\tau} q_i \rangle_{\Lambda_Q}} = \langle m_{Q,i}^{\text{tot}} \rangle_{\text{br}} = m_Q - \frac{1}{\mu_\Phi} \left( \langle M_i \rangle_{\overline{\gamma} q_i} \right).
\]

In vacua with the broken flavor symmetry these can be rewritten as (recall that \(\langle M_j^i \rangle = \delta^i_j \langle M_i \rangle, \langle M_i \rangle = \langle (\overline{Q}Q)_1 \rangle, \langle M_2 \rangle = \langle (\overline{Q}Q)_2 \rangle\))

\[
\langle M_1 + M_2 - \frac{1}{N_c} \text{Tr} M \rangle_{\text{br}} = m_Q \mu_\Phi, \quad \langle S \rangle_{\text{br}} = \left( \frac{\text{det}(M)_{\text{br}}}{\Lambda_Q^{b_0}} \right)^{1/N_c} = \frac{1}{\mu_\Phi} \langle M_1 \rangle_{\text{br}} \langle M_2 \rangle_{\text{br}},
\]

\[
\frac{\langle (\overline{q}q)_1 \rangle_{\text{br}}}{\langle \overline{q}q \rangle_{\text{br}}} = \frac{\langle S \rangle_{\text{br}}}{\langle M_1 \rangle_{\text{br}}} = \frac{\langle M_2 \rangle_{\text{br}}}{\mu_\Phi} = \langle m_{Q,1}^{\text{tot}} \rangle_{\text{br}}, \quad \frac{\langle (\overline{q}q)_2 \rangle_{\text{br}}}{\langle (\overline{q}q) \rangle_{\text{br}}} = \frac{\langle S \rangle_{\text{br}}}{\langle M_2 \rangle_{\text{br}}} = \frac{\langle M_1 \rangle_{\text{br}}}{\mu_\Phi} = \langle m_{Q,2}^{\text{tot}} \rangle_{\text{br}},
\]

\[
\frac{\langle (\overline{q}q)_2 \rangle_{\text{br}}}{\langle (\overline{q}q) \rangle_{\text{br}}} = \frac{\langle (\overline{q}q)_1 \rangle_{\text{br}}}{\langle (\overline{q}q) \rangle_{\text{br}}} = \overline{\tau} q_1, \quad \langle q_2 \rangle = \overline{\tau} q_2.
\]

### 2.1 Vacua with the unbroken flavor symmetry

One obtains from (2.3) at \(\mu_\Phi \leq \mu_{\Phi,o}\) and with \((\overline{q}q_i) = \delta^i_j \langle \overline{Q}Q \rangle\).

a) There are only \(N_c = (N_F - N_c)\) classical \(S\)-vacua at \(\mu_\Phi \ll \mu_{\Phi,o}\) with

\[
\langle \overline{Q}Q \rangle_{S} \equiv \langle \overline{Q}Q(\mu = \Lambda_Q) \rangle_{S} \simeq -\frac{N_c}{N_c} m_Q \mu_\Phi, \quad \mu_{\Phi,o} = \Lambda_Q (m_Q/\Lambda_Q)^{(N_F - 2N_c)/N_c} \ll \Lambda_Q.
\]

b) There are \((N_F - 2N_c)\) quantum \(L\)-vacua at \(\mu_\Phi \gg \mu_{\Phi,o}\) with

\[
\langle \overline{Q}Q \rangle_{L} \equiv \langle \overline{Q}Q(\mu = \Lambda_Q) \rangle_{L} \sim \Lambda_Q^2 \left( \frac{\mu_\Phi}{\Lambda_Q} \right)^{\frac{N_c}{N_F - 2N_c}}.
\]

c) There are \(N_c\) quantum SQCD - vacua at \(\mu_\Phi \gg \mu_{\Phi,o}\) with

\[
\langle \overline{Q}Q \rangle_{SQCD} = \langle \overline{Q}Q(\mu = \Lambda_Q) \rangle_{SQCD} \simeq \langle S \rangle_{SQCD} = \left( \frac{\Lambda_{SQCD}^{y_M}}{m_Q} \right)^3 = \frac{1}{m_Q} \left( \Lambda_Q^{b_0} m_F^{N_F} \right)^{1/N_c}.
\]

The total number of vacua with the unbroken flavor symmetry is

\[
N_{\text{unbr}}^{\text{tot}} = N_c = (N_F - 2N_c) + N_c.
\]
2.2 Vacua with the broken flavor symmetry

\[ U(N_F) \rightarrow U(n_1) \times U(n_2), \quad 1 \leq n_1 \leq [N_F/2] \]

a) There are \((n_1 - N_c)Nc_{N_F}\) br1-vacua\(^3\) at \(N_c < n_1 \leq [N_F/2]\) and \(\mu_\Phi \ll \mu_{\Phi,0}\) with

\[
\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{br1}} \simeq \frac{N_c}{N_c - n_1} m_Q \mu_\Phi, \quad \left\langle (\overline{Q}Q)_2 \right\rangle_{\text{br1}} \sim \Lambda_Q^2 \left( \frac{\mu_\Phi}{\Lambda_Q} \right) \frac{n_1}{N_c - n_1} \left( \frac{m_Q}{\Lambda_Q} \right)^{\frac{n_1 - N_c}{n_1}}.
\]

\[
\frac{\left\langle (\overline{Q}Q)_2 \right\rangle_{\text{br1}}}{\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{br1}}} \sim \left( \frac{\mu_\Phi}{\mu_{\Phi,0}} \right) \frac{N_c}{n_1} \ll 1.
\]

b) There are \((n_2 - N_c)Nc_{N_F}\) br2-vacua at all values \(1 \leq n_1 \leq [N_F/2]\) and \(\mu_\Phi \ll \mu_{\Phi,0}\) with

\[
\left\langle (\overline{Q}Q)_2 \right\rangle_{\text{br2}} \simeq \frac{N_c}{N_c - n_2} m_Q \mu_\Phi, \quad \left\langle (\overline{Q}Q)_1 \right\rangle_{\text{br2}} \sim \Lambda_Q^2 \left( \frac{\mu_\Phi}{\Lambda_Q} \right) \frac{n_2}{N_c - n_2} \left( \frac{m_Q}{\Lambda_Q} \right)^{\frac{n_2 - N_c}{n_2}}.
\]

\[
\frac{\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{br2}}}{\left\langle (\overline{Q}Q)_2 \right\rangle_{\text{br2}}} \sim \left( \frac{\mu_\Phi}{\mu_{\Phi,0}} \right) \frac{N_c}{n_2} \ll 1.
\]

On the whole, there are \((\theta(z)\) is the step function)

\[ N_{\text{br}}^{\text{tot}}(n_1) = \left[ (n_2 - N_c) + \theta(n_1 - N_c)(n_1 - N_c) \right] Nc_{N_F}, \quad N_{\text{br}}^{\text{tot}} = \sum_{n_1=1}^{[N_F/2]} N_{\text{br}}^{\text{tot}}(n_1) \]

vacua with the broken flavor symmetry at \(\mu_\Phi \ll \mu_{\Phi,0}\).

c) There are \((N_c - n_1)Nc_{N_F}\) br1-vacua at \(1 \leq n_1 < N_c\) and \(\mu_\Phi \gg \mu_{\Phi,0}\) with

\[
\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{br1}} \simeq \frac{N_c}{N_c - n_1} m_Q \mu_\Phi, \quad \left\langle (\overline{Q}Q)_2 \right\rangle_{\text{br1}} \sim \Lambda_Q^2 \left( \frac{\Lambda_Q}{\mu_\Phi} \right) \frac{n_1}{N_c - n_1} \left( \frac{m_Q}{\Lambda_Q} \right)^{\frac{n_1 - N_c}{n_1}}.
\]

\[
\frac{\left\langle (\overline{Q}Q)_2 \right\rangle_{\text{br1}}}{\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{br1}}} \sim \left( \frac{\mu_{\Phi,0}}{\mu_\Phi} \right) \frac{N_c}{n_1} \ll 1.
\]

d) There are \((N_F - 2N_c)Nc_{N_F}\) Lt (L - type) vacua at \(n_1 \neq N_c\) and \(\mu_\Phi \gg \mu_{\Phi,0}\) with

\[
(1 - \frac{n_1}{N_c})\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{Lt}} \simeq - (1 - \frac{n_2}{N_c})\left\langle (\overline{Q}Q)_2 \right\rangle_{\text{Lt}} \sim \Lambda_Q^2 \left( \frac{\mu_\Phi}{\Lambda_Q} \right) \frac{N_c}{N_F - 2N_c},
\]

i.e. as in the L - vacua above but \(\left\langle (\overline{Q}Q)_1 \right\rangle_{\text{Lt}} \neq \left\langle (\overline{Q}Q)_2 \right\rangle_{\text{Lt}}\) here.

---

\(^3\) \(Nc_{N_F}\) differ from the standard \(C_{N_F}^{n_1} = N_F! / n_1! n_2!\) only for \(C_{N_F}^{n_1=k} = C_{N_F}^{n_1=k}/2\).

Besides, by convention, we ignore the continuous multiplicity of vacua due to the spontaneous flavor symmetry breaking. Another way, one can separate slightly all quark masses, so that all Nambu-Goldstone particles will acquire small masses \(O(\delta m_Q) \ll m_Q\).
e) There are \((N_F - 2N_c)C_{N_F}^{n_1}\) special vacua at \(n_1 = N_c\) and \(\mu_\Phi \gg \mu_{\Phi,0}\) with

\[
\langle (\overline{Q}Q)_2 \rangle_{\text{spec}} = \frac{N_c}{2N_c - N_F} m_Q \mu_\Phi , \quad \langle (\overline{Q}Q)_1 \rangle_{\text{spec}} \sim \Lambda_Q^2 \left( \frac{\mu_\Phi}{\Lambda_Q} \right)^{\frac{N_c}{N_F - 2N_c}} ,
\]

\[
\frac{\langle (\overline{Q}Q)_2 \rangle_{\text{spec}}}{\langle (\overline{Q}Q)_1 \rangle_{\text{spec}}} \sim \left( \frac{\mu_{\Phi,0}}{\mu_\Phi} \right)^{\frac{N_c}{N_F - 2N_c}} \ll 1 .
\]

As one can see from the above, similarly to [14] with \(N_c < N_F < 2N_c\), all quark condensates become parametrically the same at \(\mu_\Phi \sim \mu_{\Phi,0}\), see (2.7). Clearly, just this region \(\mu_\Phi \sim \mu_{\Phi,0}\) and not \(\mu_\Phi \sim \Lambda_Q\) is very special and most of the quark condensates change their parametric behavior and hierarchies at \(\mu_\Phi \lesssim \mu_{\Phi,0}\). For example, the br1 - vacua with \(1 \leq n_1 < N_c\), \(\langle (\overline{Q}Q)_1 \rangle \sim m_Q \mu_\Phi \gg \langle (\overline{Q}Q)_2 \rangle\) at \(\mu_\Phi \gg \mu_{\Phi,0}\) evolve into br2 - vacua with \(\langle (\overline{Q}Q)_2 \rangle \sim m_Q \mu_\Phi \gg \langle (\overline{Q}Q)_1 \rangle\) at \(\mu_\Phi \ll \mu_{\Phi,0}\), while the br1 - vacua with \(n_1 > N_c\), \(\langle (\overline{Q}Q)_1 \rangle \sim m_Q \mu_\Phi \gg \langle (\overline{Q}Q)_2 \rangle\) at \(\mu_\Phi \ll \mu_{\Phi,0}\) evolve into the L-type vacua with \(\langle (\overline{Q}Q)_1 \rangle \sim \langle (\overline{Q}Q)_2 \rangle \sim \Lambda_Q^2 (\mu_\Phi/\Lambda_Q)^{N_c/(N_F - 2N_c)}\) at \(\mu_\Phi \gg \mu_{\Phi,0}\), etc.

The exception is the special vacua with \(n_1 = N_c\), \(n_2 = \overline{N_c}\). In these, the parametric behavior \(\langle (\overline{Q}Q)_2 \rangle \sim m_Q \mu_\Phi\), \(\langle (\overline{Q}Q)_1 \rangle \sim \Lambda_Q^2 (\mu_\Phi/\Lambda_Q)^{N_c/(N_F - 2N_c)}\) remains the same but the hierarchy is reversed at \(\mu_\Phi \lesssim \mu_{\Phi,0}\): \(\langle (\overline{Q}Q)_1 \rangle/\langle (\overline{Q}Q)_2 \rangle \sim (\mu_\Phi/\mu_{\Phi,0})^{N_c/(N_F - 2N_c)}\).

On the whole, there are

\[
N_{\text{br}}^{\text{tot}}(n_1) = \left[ (N_F - 2N_c) + \theta(N_c - n_1)(N_c - n_1) \right] C_{N_F}^{n_1}, \quad N_{\text{br}}^{\text{tot}} = \sum_{n_1=1}^{[N_F/2]} N_{\text{br}}^{\text{tot}}(n_1)
\]

vacua with the broken flavor symmetry at \(\mu_\Phi \gg \mu_{\Phi,0}\). The total number of vacua is the same at \(\mu_\Phi \lesssim \mu_{\Phi,0}\), as it should be.

We point out finally that the multiplicities of vacua at \(N_F > 2N_c\) are not the analytic continuations of those at \(N_c < N_F < 2N_c\), see [1].

3 Direct theory. Unbroken flavor symmetry.

\(2N_c < N_F < 3N_c\), \(\mu_\Phi \gg \Lambda_Q\)

It is worth noting first that in all calculations below in the text we use the NSVZ \(\beta\) - function [6] (recall also the footnote 1).

Because \(\mu_{\Phi,0} = \Lambda_Q(m_Q/\Lambda_Q)^{(N_F - 2N_c)/N_c} \ll \Lambda_Q\) and \(\mu_\Phi \gg \Lambda_Q\), we are always here in the region \(\mu_\Phi \gg \mu_{\Phi,0}\). There are \(N_c\) SQCD-vacua (these were considered previously in [1]) and \((N_F - 2N_c)\) L - vacua (2.8) with

\[
\frac{\langle \overline{Q}Q \rangle_L}{\Lambda_Q^2} \sim \left( \frac{\mu_\Phi}{\Lambda_Q} \right)^{\frac{N_c}{N_F - 2N_c}} \gg 1 .
\]

So, the gluon masses due to possible higgsing of some quark flavors are large, \(\mu_{\text{gl}} \sim \langle \overline{Q}Q \rangle_L^{1/2} \gg \Lambda_Q\).

But the quark masses are even larger, see (2.4)

\[
\langle m_Q^{\text{tot}} \rangle_L = \langle m_Q - \Phi \rangle_L \sim \langle \Phi \rangle_L \sim \langle \overline{Q}Q \rangle_L \frac{\mu_\Phi}{\mu_\Phi} \sim \Lambda_Q \left( \frac{\mu_\Phi}{\Lambda_Q} \right)^{\frac{N_c}{N_F - 2N_c}} , \quad \frac{\mu_{\text{gl}}}{\langle m_Q^{\text{tot}} \rangle_L} \sim \left( \frac{\Lambda_Q}{\mu_\Phi} \right)^{\frac{3N_c - N_F}{2(N_F - 2N_c)}} \ll 1 .
\]
Therefore, all quarks are in the HQ (heavy quark) phase and are confined. After integrating out all quarks as heavy ones at scales $\mu < m_Q^{\text{pole}} \sim \langle m_Q^{\text{tot}} \rangle$ there remain $N_F^2$ fions $\Phi$ and the $SU(N_c)$ SYM with the scale factor of its gauge coupling

$$\Lambda_{YM}^{(L)} = \left( \Lambda_Q^{bo} \det m_Q^{\text{tot}} \right)^{1/3N_c}, \quad \frac{\langle \Lambda_{YM}^{(L)} \rangle}{\Lambda_Q} = \left( \frac{\mu_{\Phi}}{\Lambda_Q} \right)^{\frac{N_F}{2(N_F - 2N_c)}} \gg \frac{\mu_{\Phi}}{\Lambda_Q}, \quad b_0 = 3N_c - N_F.$$  (3.3)

After integrating out all gluons at scales $\mu < \langle \Lambda_{YM}^{(L)} \rangle$ through the Veneziano-Yankielowics (VY) procedure $[8, 9]$, the Lagrangian of $N_F^2$ fions looks as, see (1.1)

$$K_\Phi = \Phi^\dagger \Phi, \quad W = N_c \left( \Lambda_Q^{bo} \det m_Q^{\text{tot}} \right)^{1/N_c} + W_\Phi, \quad m_Q^{\text{tot}} = m_Q - \Phi.$$  (3.4)

From (3.4), the fion masses are $\mu_{\Phi} \sim \mu_{\Phi}$.

On the whole, the mass spectrum looks in these $(N_F - 2N_c) L$ vacua as follows: a) there is a large number of hadrons made of the non-relativistic (and weakly confined, the string tension is $\sqrt{\sigma} \sim \Lambda_{YM}^{(L)} \ll m_Q^{\text{pole}}$) quarks with masses $m_Q^{\text{pole}} \sim \Lambda_Q (\mu_{\Phi}/\Lambda_Q)^{N_c/(N_F - 2N_c)}$; b) a large number of gluonia with the mass scale $\sim \Lambda_{YM}^{(L)} \sim \Lambda_Q (\mu_{\Phi}/\Lambda_Q)^{N_F/3(N_F - 2N_c)}$; c) the lightest are $N_F^2$ fions with the masses $\mu_{\Phi}(\Phi) \sim \mu_{\Phi} \gg \Lambda_Q$, the hierarchies look as $\Lambda_Q \ll \mu_{\Phi} \ll \Lambda_{YM}^{(L)} \ll m_Q^{\text{pole}}$.

4 Dual theory. Unbroken flavor symmetry.

$$2N_c < N_F < 3N_c, \quad \mu_{\Phi} \gg \Lambda_Q$$

The dual theory in the UV region $\mu > \Lambda_Q$ and $2N_c < N_F < 3N_c$ is taken as UV - free. The largest mass in the case considered is that of dual quarks

$$\mu_q^{\text{pole}} \sim \langle M \rangle_L = \langle Q \rangle_L \sim \Lambda_Q \left( \frac{\mu_{\Phi}}{\Lambda_Q} \right)^{\frac{N_F}{2(N_F - 2N_c)}} \gg \Lambda_Q,$$  (4.1)

while the gluon mass due to possible higgsing of dual quarks is smaller

$$\bar{\mu}_{gl}^2 \sim \langle \bar{q} q \rangle_L = \langle m_Q^{\text{tot}} \rangle \Lambda_Q \sim \langle \Phi \rangle \Lambda_Q \sim \frac{\Lambda_Q \langle Q \rangle_L}{\mu_{\Phi}} \sim \Lambda_Q^2 \left( \frac{\mu_{\Phi}}{\Lambda_Q} \right)^{\frac{N_F}{2(N_F - 2N_c)}}.$$  (4.2)

$$\frac{\bar{\mu}_{gl}}{\mu_q^{\text{pole}}} \sim \left( \frac{\Lambda_Q}{\mu_{\Phi}} \right)^{\frac{N_F}{2(N_F - 2N_c)}} \ll 1.$$  (4.3)

Therefore, the dual quarks are in the Hq (heavy quark) phase and are confined. After they are integrated out at scales $\mu < \mu_q^{\text{pole}}$, there remain $N_F^2$ mions $M$ and the $SU(N_c)$ SYM with the scale factor of its gauge coupling

$$\Lambda_{YM}^{(L)} = \left( \Lambda_Q^{bo} \det M \right)^{1/3N_c}, \quad \frac{\langle \Lambda_{YM}^{(L)} \rangle}{\Lambda_Q} = \left( \frac{\mu_{\Phi}}{\Lambda_Q} \right)^{\frac{N_F}{3(N_F - 2N_c)}} \gg \frac{\mu_{\Phi}}{\Lambda_Q}, \quad b_0 = 3N_c - N_F.$$  (4.3)

After integrating out all dual gluons at scales $\mu < \langle \Lambda_{YM}^{(L)} \rangle$ the Lagrangian of mions looks as

$$K_M = \frac{M^\dagger M}{\Lambda_Q^2}, \quad W = -N_c \left( \frac{\det M}{\Lambda_Q^{bo}} \right)^{1/N_c} + W_M.$$  (4.4)
From (4.4),(4.5) the mion masses are

\[ \mu_{\text{pole}}(M) \sim \frac{\Lambda_Q^2}{\mu_\Phi} \ll \Lambda_Q. \] (4.6)

On the whole, the mass spectrum looks in these \((N_F - 2N_c)\) dual L - vacua as follows:

a) there is a large number of hadrons made of the non-relativistic (and weakly confined, the string tension is \(\sqrt{\sigma} \sim \langle \Lambda_{YM}^{(L)} \rangle \ll \mu_{\text{pole}}\)) dual quarks with the masses \(\mu_{\text{pole}} \sim \Lambda_Q(\mu_\Phi/\Lambda_Q)^{N_c/(N_F - 2N_c)}\);

b) a large number of gluonia with the mass scale \(\sim \langle \Lambda_{YM}^{(L)} \rangle \sim \Lambda_Q(\mu_\Phi/\Lambda_Q)^{N_F/3(N_F - 2N_c)}\);

c) there are \(N_F^2\) fions with the pole masses \(\mu(\Phi) \sim \mu_\Phi\); d) the lightest are \(N_F^2\) mions with the masses \(\mu_{\text{pole}}(M) \sim (\Lambda_Q^2/\mu_\Phi) \ll \Lambda_Q\). The hierarchies look as: \(\mu_{\text{pole}}(M) \ll \Lambda_Q \ll \mu(\Phi) \ll \langle \Lambda_{YM}^{(L)} \rangle \ll \mu_{\text{pole}}\).

5 Direct theory. Broken flavor symmetry.

\[ 2N_c < N_F < 3N_c, \quad \mu_\Phi \gg \Lambda_Q \]

5.1 Lt - vacua

The main difference with the L - vacua in section 3 is that the flavor symmetry is broken spontaneously in these Lt - vacua, \(\langle (Q\bar{Q})_1 \rangle \neq \langle (Q\bar{Q})_2 \rangle\) and, from (3.4), the fions \(\Phi_{12}\) and \(\Phi_{21}\) are the Nambu-Goldstone particles here and are exactly massless.

5.2 br1 - vacua

In these vacua with \(n_1 < N_c\) and at \(\Lambda_Q \ll \mu_\Phi \ll \Lambda_Q^2/m_Q\) the regime is conformal at scales \(m_{Q,1}^{\text{pole}} \ll \mu \ll \Lambda_Q\) (see below) and potentially most important masses look as follows.

The gluon masses due to possible higgsing of quarks are

\[ \mu_{\text{gl},1}^2 \sim z_Q(\Lambda_Q, \mu_{\text{gl},1}) \langle (Q\bar{Q})_1 \rangle_{\text{br1}} \sim \Lambda_Q^2 \left( \frac{m_Q \mu_\Phi}{\Lambda_Q^2} \right)^{2N_F/3N_c} \gg \mu_{\text{gl},1}^2, \quad z_Q(\Lambda_Q, \mu_{\text{gl},1}) \sim \left( \frac{\mu_{\text{gl},1}}{\Lambda_Q} \right)^{b_0/N_F}, \]

while the quark masses are

\[ \langle m_{Q,2}^{\text{tot}} \rangle = \left( \frac{\langle (Q\bar{Q})_1 \rangle_{\text{br1}}}{\mu_\Phi} \right) \sim m_Q, \quad \bar{m}_{Q,2}^{\text{pole}} \sim \frac{\langle m_{Q,2}^{\text{tot}} \rangle_{\text{br1}}}{z_Q(\Lambda_Q, \bar{m}_{Q,2}^{\text{pole}})} \sim \Lambda_Q^2 \left( \frac{m_Q}{\Lambda_Q} \right)^{N_F/3N_c} \gg \bar{m}_{Q,1}^{\text{pole}}, \]

\[ \bar{m}_{Q,2}^{\text{pole}} \sim \left( \frac{\mu_{\text{gl},2}}{\mu_\Phi} \right)^{N_F/3N_c} \sim \left( \frac{\Lambda_Q}{\mu_\Phi} \right)^{N_F/3N_c} \left( \frac{m_Q}{\Lambda_Q} \right)^{N_F(N_F - 2N_c)/3N_c} \ll 1. \]

Therefore, the quarks \(Q^1, \bar{Q}_1\) are higgsed and the overall phase is \(Higgs_1 - HQ_2\). If we take \(2n_1 < b_0\), then \(b'_0 = (b_0 - 2n_1) > 0\) and the lower energy theory with \(SU(N_c - n_1)\) colors and \(n_2\) flavors will be in the conformal regime at scales \(m_{Q,2}^{\text{pole}} \sim \Lambda_{YM}^{(br1)} \ll \mu \ll \mu_{\text{gl},1}\). Then all results for the mass spectra will be the same as in section 6.1 of [2].
6 Dual theory. Broken flavor symmetry.

\[ 2N_c < N_F < 3N_c, \quad \mu_\Phi \gg \Lambda_Q \]

6.1 \( \text{Lt - vacua} \)

The main difference with the \( \text{L - vacua} \) in section 4 is that the flavor symmetry is broken spontaneously in these dual \( \text{Lt - vacua} \), \( \langle M_1 \rangle \neq \langle M_2 \rangle \). The fions have masses \( \mu_{\text{pole}}(\Phi) \sim \mu_\Phi \gg \Lambda_Q \) and are dynamically irrelevant at scales \( \mu < \mu_\Phi \). The low energy Lagrangian of mions is (4.4), but the mion masses look now as

\[ \mu_{\text{pole}}(M_{11}) \sim \mu_{\text{pole}}(M_{22}) \sim \frac{\Lambda_Q^2}{\mu_\Phi} \ll \Lambda_Q, \quad \mu_{\text{pole}}(M_{12}) = \mu_{\text{pole}}(M_{21}) = 0. \] (6.1)

6.2 \( \text{br1 - vacua} \)

Not going into any details we give here the results only. The overall phase is \( H_{q_1} - H_{q_2} \) and the massless Nambu-Goldstone particles here are the mions \( M_{12} \) and \( M_{21} \). The mass spectra are as in section 7.1 of \[2\], the only difference is that \( Z_q \rightarrow 1 \) here because \( b_0/N_F = O(1) \) now.

7 Dual theory. \( N_c + 1 < N_F < 3N_c/2, \quad \Lambda_Q \ll \mu_\Phi \ll \mu_\Phi,0 \)

7.1 \( \text{L and Lt - vacua} \)

The condensates in \( \text{L - vacua} \) look as \[1\]

\[ \langle M \rangle_L = \langle \overline{Q}Q \rangle_L \sim \Lambda_Q^2 \frac{L}{\Sigma \nu_N}, \quad \langle \overline{q}q \rangle_L = \frac{\langle S \rangle_L \Lambda_Q^2}{\langle M \rangle_L} \sim \Lambda_Q^2 \frac{\Lambda_Q}{\mu_\Phi} \frac{N_c^2 - N_F}{N_c}, \quad S = \left( \frac{\det M}{\Lambda_Q b_0} \right)^{1/N_c}, \] (7.1)

\[ \Lambda_Q \ll \mu_\Phi \ll \mu_\Phi,0 = \Lambda_Q(\Lambda_Q/m_Q)^{(2N_c-N_F)/N_c}. \]

Therefore, the pole masses \( \mu_{\text{pole}}^q \) of dual quarks and the possible gluon masses \( \overline{M}_{gl} \) due to their higgsing look as

\[ \mu_{\text{pole}}^q \sim \frac{\langle M \rangle_L}{\Lambda_Q}, \quad \overline{M}_{gl} \sim \langle \overline{q}q \rangle_L^{1/2}, \quad \overline{M}_{gl} \sim \frac{\Lambda_Q}{\mu_\Phi} \frac{N_c^2 - N_F}{N_c} \ll 1 \] (7.2)

and the overall phase is \( H_q \).

After integrating out all quarks as heavy ones at \( \mu < \mu_{\text{pole}}^q \) and then all \( SU(N_c) \) gluons at \( \mu < \Lambda_{YM} \ll \mu_{\text{pole}}^q \) through the VY-procedure \[3\] \[4\], the Lagrangian of \( N_F^2 \) mions looks as, see (4.4),(4.5)

\[ K = \frac{M^\dagger M}{\Lambda_Q^2}, \quad W = -\overline{N}_c \left( \frac{\det M}{\Lambda_Q b_0} \right)^{1/N_c} + W_M. \] (7.3)

From (7.3), the masses of all \( N_F^2 \) mions are \( \mu(M) \sim \Lambda_Q^2/\mu_\Phi \).

On the whole, the mass spectrum in these \( \text{L - vacua} \) includes. -
1) A large number of hadrons made of weakly interacting non-relativistic and weakly confined dual quarks, the scale of their masses is $\mu_q^{\text{pole}} \sim \langle M \rangle_{L} / \Lambda_Q$ (7.1) (the tension of the confining string is much smaller, $\sqrt{\sigma} \sim \Lambda_{Y-M}^{(L)} \ll \mu_q^{\text{pole}}$).

2) A large number of gluonia made of $SU(N_c)$ gluons with their mass scale $\Lambda_{Y-M}^{(L)} = \langle S \rangle_{L}^{1/3} \sim \Lambda_Q (\Lambda_Q / \mu_\Phi)^{N_F / 3(2N_c - N_F)}$.

3) $N_F^2$ mions with masses $\mu(M) \sim \Lambda_Q^2 / \mu_\Phi$.

The mass hierarchies look as $\mu(M) \ll \Lambda_{Y-M}^{(L)} \ll \mu_q^{\text{pole}} \ll \Lambda_Q$.

In comparison with these $L$ - vacua, the only qualitative difference in $L_t$ - vacua with the broken flavor symmetry is that the hybrid mions $M_{12}$ and $M_{21}$ are the Nambu-Goldstone particles there and are exactly massless.

### 7.2 S - vacua

The condensates look here as \[1\]

$$\langle \bar{Q}Q \rangle_S = \langle M \rangle_S \sim m_Q \mu_\Phi,$$  
$$\langle \bar{q}q \rangle_S = \langle M \rangle_S \frac{\langle S \rangle_{S} \Lambda_Q}{\langle M \rangle_S} \sim \Lambda_Q^2 \left( \frac{m_Q \mu_\Phi}{\Lambda_Q^2} \right)^{N_c / N_c},$$

$$\langle S \rangle_S = \left( \frac{\det \langle M \rangle_S}{\Lambda_Q^{3N_c}} \right)^{1 / N_c}, (7.4)$$

where

$$\mu_q^{\text{pole}} \sim \frac{\langle M \rangle_S}{\Lambda_Q}, \quad \bar{p}_{gl}^{\mu_\Phi} \sim \frac{\left( m_Q \mu_\Phi / \Lambda_Q^2 \right)^{(3N_c - 2N_F) / 2N_c}}{\mu_q^{\text{pole}}}$$  

and so the overall phase is also $Hq$. Proceeding as before we obtain the Lagrangian (7.3) (but now in $S$ - vacua). From this, the masses of all $N_F^2$ mions are $\mu(M) \sim \Lambda_Q^2 / \mu_\Phi$, while the scale of gluonia masses is $\Lambda_{Y-M}^{(S)} = \langle S \rangle_S^{1/3} \sim \Lambda_Q (m_Q \mu_\Phi / \Lambda_Q^2)^{N_F / 3N_c} \ll \mu_q^{\text{pole}}$.

As a result, the hierarchies of masses (except for $\mu(\Phi) \sim \mu_\Phi \gg \Lambda_Q$) look as:

a) $\Lambda_Q \gg \mu(M) \gg \mu_q^{\text{pole}} \gg \Lambda_{Y-M}^{(S)}$, at $\Lambda_Q \ll \mu_\Phi \ll \mu'_\Phi = \Lambda_Q (\Lambda_Q / m_Q)^{1/2}$;

b) $\Lambda_Q \gg \mu_q^{\text{pole}} \gg \mu(M) \gg \Lambda_{Y-M}^{(S)}$, at $\mu'_\Phi \ll \mu_\Phi \ll \tilde{\mu}_\Phi = \Lambda_Q (\Lambda_Q / m_Q)^{N_F / (4N_F - 3N_c)}$;

c) $\Lambda_Q \gg \mu_q^{\text{pole}} \gg \Lambda_{Y-M}^{(S)} \gg \mu(M)$ at $\tilde{\mu}_\Phi \ll \mu_\Phi \ll \mu_{\Phi,0} = \Lambda_Q (\Lambda_Q / m_Q)^{(2N_c - N_F) / N_c}$.

### 7.3 br2 - vacua

The condensates look in these vacua as \[1\],

$$\langle \bar{Q}Q \rangle_{br2} = \langle M_2 \rangle_{br2} \sim m_Q \mu_\Phi \gg \langle \bar{q}q \rangle_{br2} = \langle M_1 \rangle_{br2} \sim \Lambda_Q^2 \left( \frac{m_Q \mu_\Phi}{\Lambda_Q} \right)^{N_c - n_1 / n_2 - N_c},$$

$$\langle \bar{q}q \rangle_{br2} = \frac{\langle M_2 \rangle_{br2} \Lambda_Q}{\mu_\Phi} \sim m_Q \Lambda_Q \gg \langle \bar{q}q \rangle_{br2} = \frac{\langle M_1 \rangle_{br2} \Lambda_Q}{\mu_\Phi}, (7.6)$$

$$\bar{p}^{\mu_\Phi}_{gl,1} \sim \langle \bar{q}q \rangle_1 \sim m_Q \Lambda_Q, \quad \mu_{q,2}^{\text{pole}} \sim \frac{\langle M_2 \rangle_{br2} \Lambda_Q}{\Lambda_Q} \sim m_Q \mu_\Phi \gg \mu_{q,1}^{\text{pole}}, \quad \frac{\mu_{q,2}^{\text{pole}}}{\mu_{gl,1}^{\mu_\Phi}} \sim \left( \frac{m_Q \mu_\Phi^2}{\Lambda_Q^3} \right)^{1/2}, \quad n_2 > N_c.$$

11
A) The range $\Lambda_Q \ll \mu_\Phi \ll \Lambda_Q (\Lambda_Q / m_Q)^{1/2}$

The largest mass $\mathcal{M}_{q_{1,1}}$ have in this case gluons (and their superpartners) due to higgsing of $q_1, \overline{q}_1$ dual quarks, $SU(N_c) \rightarrow SU(N_c - n_1)$. The new scale factor of the gauge coupling is

$$
(\Lambda')_{0-2n_1} = \Lambda_Q^{\mu_0} / \det N_{11}, \quad \mu_{q,2}^\text{pole} \ll \langle \Lambda' \rangle \ll \Lambda_Q, \quad b_o = 3N_c - N_F < 0, \quad \langle N_{11} \rangle = \langle \overline{q}_1 q_1 \rangle
$$

(7.7)

and the overall phase is $Higgs_1 - Hq_2$.

After integrating out higgsed quarks and higgsed gluons the Lagrangian of the matter at the scale $\mu = \langle \Lambda' \rangle$ looks as

$$
K = \text{Tr} \left( \frac{M^\dagger M}{\Lambda_Q^2} + 2\sqrt{N_{11}^\dagger N_{11} + K_{\text{hybrid}}} + \left( (q_2')^\dagger q_2' + (\overline{q}^{t, 2})^\dagger \overline{q}^{t, 2} \right) \right),
$$

(7.8)

$$
K_{\text{hybrid}} = N_{21} \frac{1}{\sqrt{N_{11}^\dagger N_{11}}} N_{21}^\dagger + N_{12}^\dagger \frac{1}{\sqrt{N_{11}^\dagger N_{11}}} N_{12},
$$

where $N_{11} = (\overline{q}_1 q_1)$ are $n_1^2$ nion fields remained from the higgsed quarks $q_1, \overline{q}_1$, while the hybrids $N_{12}$ and $N_{21}$ are in essence the quarks $q_2, \overline{q}^2$ with higgsed colors,

$$
W = W_M - W_{MN} - W_q, \quad W_{MN} = \frac{1}{\Lambda_Q} \text{Tr} \left( M_{11} N_{11} + (M_{12} N_{21} + M_{21} N_{12}) + M_{22} (N_{21} 1/N_{11} N_{12}) \right),
$$

$$
W_M = m_Q \text{Tr} \left( M - \frac{1}{2\mu_\Phi} \left( \text{Tr} (M^2) - \frac{1}{N_c} (\text{Tr} M)^2 \right) \right), \quad W_q = \text{Tr} \left( q_2' \frac{M_{22}}{\Lambda_Q} q_2' \right),
$$

(7.9)

where $q_2'$ and $\overline{q}^{t, 2}$ are quarks with unhiggsed colors.

Next, at scales $\mu < \mu_{q,2}^\text{pole} \sim m_Q \mu_\Phi / \Lambda_Q$, we integrate out $q_2', \overline{q}^{t, 2}$ quarks as heavy ones and then $SU(N_c - n_1)$ gluons at $\mu < \Lambda_{Y_M}^{(b2)} \ll \mu_{q,2}^\text{pole}$. The Lagrangian of remaining mions and nions is

$$
K = \text{Tr} \left( \frac{M^\dagger M}{\Lambda_Q^2} + 2\sqrt{N_{11}^\dagger N_{11} + K_{\text{hybrid}}} \right),
$$

$$
W = -(N_c - n_1) \left( \Lambda_Q^{b_o} \frac{\det (M_{22} / \Lambda_Q)}{\det N_{11}} \right)^{1/(N_c - n_1)} + W_M - W_{MN}.
$$

(7.10)

We obtain from (7.10) at $m_Q \mu_\Phi^2 / \Lambda_Q^3 \ll 1$:

a) the mixing of mions and nions is parametrically small and masses of all $N^2_F$ mions are $\mu(M) \sim \Lambda_Q^2 / \mu_\Phi$;

b) the masses of $N_{11}$ nions are much smaller, $\mu(N_{11}) \sim m_Q \mu_\Phi / \Lambda_Q$, $\mu(N_{11}) / \mu(M) \sim m_Q \mu_\Phi^2 / \Lambda_Q^2 \ll 1$;

c) the hybrid nions $N_{12}$ and $N_{21}$ are massless.

On the whole for this case the mass spectrum looks as follows.

1) The heaviest (among the masses $< \Lambda_Q$) are $N^2_F$ mions $M$ with masses $\mu(M) \sim \Lambda_Q^2 / \mu_\Phi$. 

2) There are \( n_1(2N_c - n_1) \) massive gluons (and their superpartners) with masses \( \frac{1}{\mu_{q,1}} \sim \langle N_1 \rangle^{1/2} \sim (m_Q \Lambda_Q)^{1/2} \).

3) There is a large number of hadrons made of weakly interacting non-relativistic and weakly confined quarks \( q' \) and \( \bar{q}' \) with unhiggsed colors, the scale of their masses is \( \mu^{\text{pole}}_{q,2} \sim m_Q \mu_\Phi/\Lambda_Q \) (the tension of confining string is much smaller, \( \sqrt{\sigma} \sim \Lambda_{YM}^{(br2)} \ll \mu^{\text{pole}}_{q,2} \)).

4) The masses of \( n_2^1 \) nions \( N_{11} \) (dual pions) are also \( \mu(N_{11}) \sim m_Q \mu_\Phi/\Lambda_Q \).

5) There is a large number of \( SU(\overline{N}_c - n_1) \) SYM gluons, the scale of their masses is \( \sim \Lambda_{YM}^{(br2)} \sim (m_Q \langle M_1 \rangle^{br2})^{1/3} \), see (7.6).

6) Finally, \( 2n_1n_2 \) Nambu-Goldstone hybrid nions \( N_{12}, N_{21} \) are massless.

The overall hierarchy of masses looks as:

\[
\mu(N_{12}) \ll \Lambda_{YM}^{(br2)} \ll \mu(N_{11}) \sim \mu^{\text{pole}}_{q,2} \ll \mu_{gl,1} \ll \mu(M) \ll \Lambda_Q.
\]

B) The range \( \Lambda_Q (\Lambda_Q/m_Q)^{1/2} \ll \mu_\Phi \ll \mu_{\Phi,o} = \Lambda_Q (\Lambda_Q/m_Q)^{(2N_c-N_F)/N_c} \)

The largest mass have in this case \( q_2, \bar{q}^2 \) quarks, see (7.6). After integrating them out the new scale factor of the gauge coupling is

\[
\left( \Lambda'' \right)^{3N_c-n_1} = \Lambda^{\text{po}}_{q} \left( \frac{m_Q \mu_\Phi}{\Lambda_Q} \right)^{n_2}, \quad \Lambda'' \ll \mu^{\text{pole}}_{q,2}, \quad n_1 < \overline{N}_c.
\] (7.11)

The ratio of the pole masses \( \mu^{\text{pole}}_{q,1} \) of \( q_1, \bar{q}^1 \) quarks to the possible gluon masses \( \mu_{gl,1} \) due to their higgsing looks as

\[
\mu^{\text{pole}}_{q,1} \sim \frac{\langle M_1 \rangle^{br2}}{\Lambda_Q}, \quad \mu_{gl,1} \sim (m_Q \Lambda_Q)^{1/2}, \quad \frac{\mu^{\text{pole}}_{q,1}}{\mu_{gl,1}} \sim \left( \frac{\mu_\Phi}{\mu_\Phi} \right)^{n_2/(3N_c-n_1)}, \quad (7.12)
\]

\[
\frac{\Lambda''}{\mu_{gl,1}} \sim \left( \frac{\mu_\Phi}{\mu_\Phi} \right)^{n_2/(3N_c-n_1)}, \quad \hat{\mu}_\Phi \sim \Lambda_Q \left( \frac{\Lambda_Q}{m_Q} \right)^{b_2-n_1 > 0}.
\]

Therefore, in the range \( (\Lambda_Q^3/m_Q)^{1/2} \ll \mu_\Phi \ll \hat{\mu}_\Phi \) the hierarchies look as: \( \mu_{gl,1} \gg \Lambda'' \gg \mu^{\text{pole}}_{q,1} \) and the quarks \( q_1, \bar{q}^1 \) are higgsed in the weak coupling region at \( \mu = \mu_{gl,1} \gg \Lambda'' \), the overall phase is also \( H_{q_2} - H_{\text{iggs}} \). But \( \mu^{\text{pole}}_{q,1} \gg \Lambda'' \gg \mu_{gl,1} \) at \( \hat{\mu}_\Phi \ll \mu_\Phi \ll \mu_{\Phi,o} \), the quarks \( q_1, \bar{q}^1 \) are too heavy and not higgsed, the overall phase is \( H_{q_2} - H_{q_1} \).

1) \( (\Lambda_Q^3/m_Q)^{1/2} \ll \mu_\Phi \ll \hat{\mu}_\Phi \)

After integrating out the heaviest quarks \( q_2, \bar{q}^2 \) and then higgsed quarks \( q_1, \bar{q}^1 \) and higgsed gluons at \( \mu < \mu_{gl,1} \ll \mu^{\text{pole}}_{q,2} \), and finally \( SU(\overline{N}_c - n_1) \) gluons at \( \mu < \Lambda_{YM}^{(br2)} \ll \mu_{gl,1} \), there remain only \( N_F^1 \) mions \( M_j^i \) and \( n_1^2 \) nions \( N_{11} \) (dual pions). The Lagrangian looks as, see (4.5)

\[
K = \text{Tr} \left( \frac{1}{\Lambda_Q^2} M^\dagger M + 2\sqrt{N_{11}^1 N_{11}} \right),
\] (7.13)

\[
W = -(\overline{N}_c - n_1) \left( \frac{\Lambda^{\text{po}}_Q \det(M_{22}/\Lambda_Q)}{\det N_{11}} \right)^{1/(\overline{N}_c-n_1)} + \frac{1}{\Lambda_Q} \text{Tr} N_{11} \left( M_{12} \frac{1}{M_{22}} M_{21} - M_{11} \right) + W_M.
\]
We obtain from (7.13):
\[ \mu(N_{11}) \sim \mu(M_{11}) \sim (m_Q \Lambda_Q)^{1/2}, \quad \mu(M_{22}) \sim \Lambda_Q^2/\mu_\Phi, \quad \mu(M_{12}) = \mu(M_{12}) = 0. \] (7.14)

2) \( \mu_\Phi \ll \mu_\Phi \ll \mu_{\Phi, o} \)

After integrating out the quarks \( q_2, \Phi^2 \) and \( q_1, \Phi^1 \) as heavy ones and then \( SU(N_c) \) gluons at \( \mu < \Lambda_{Y M}^{(br2)} \ll \mu_{q,1}^{pole} \ll \mu_{q,2}^{pole} \), the Lagrangian of \( N_F^2 \) mions makes as
\[ K = \text{Tr} \left( \frac{1}{\Lambda_Q^2} M^\dagger M \right), \quad W = -\overline{\Phi} \left( \frac{\text{det} M}{\Lambda_{br2}^2} \right)^{1/N_c} + W_M. \] (7.15)

From (7.15), see (4.5) and (7.6),
\[ \mu(M_{22}) \sim \frac{\Lambda_Q^2}{\mu_\Phi}, \quad \mu(M_{11}) \sim \frac{\langle M_2 \rangle \Lambda_Q^2}{\langle M_1 \rangle \mu_\Phi} \gg \mu(M_{22}), \quad \mu(M_{12}) = \mu(M_{12}) = 0. \] (7.16)

8 Broken \( \mathcal{N} = 2 \) SQCD and exercises with Seiberg’s duality

It was considered recently in \([10]\), \( \mathcal{N} = 2 \) SQCD theory broken down to \( \mathcal{N} = 1 \) with single trace perturbation \( \sim \mu_x X^2 \), with \( U(N_c) \) gauge group and \( N_c + 1 < N_F < 3N_c/2 \) flavors of "electric" fundamental quarks \( Q^a_i, a = 1 \ldots N_c, i = 1 \ldots N_F \) and anti-fundamental quarks \( \overline{Q}^a_{j} \). The matter Lagrangian at the scale \( \mu = (\text{several})\Lambda_2 \) looks as (the gluon exponents are implied here and below in the Kahler terms, \( \Lambda_2 \) is the scale factor of the non-abelian gauge coupling \( g^2 \))
\[ K = \frac{1}{g_A^2(\mu)} X^A X^A + Q^a_i Q^a_i + Q^a_{j} \overline{Q}^a_{j}, \quad g_A^2 = g^2 \text{ for } A = 1 \ldots N_c^2 - 1 \text{ and } g_1^2 \text{ for } A = N_c^2, \]
\[ W_{\text{matter}} = \frac{\mu_x}{2} X^A X^A + \text{Tr} \left( m \overline{Q} Q - \sqrt{2} \overline{Q} X Q \right), \quad A = 1 \ldots N_c^2. \] (8.1)

8.1 Broken \( U(N_F) \rightarrow U(n_1) \times U(n_2) \) and unbroken \( Z_{2N_c-N_F} \)

A) There are br2 - vacua in this theory at \( 1 \leq n_1 < N_c = (N_F - N_c) \), \( \mu_x \ll \mu_{x, o} \), see e.g. eq.(3.9) in \([11]\) (these are the vacua of the baryonic branch in the language of \([11]\) \([12]\) or zero vacua in the language of \([10]\), with the condensates of "electric" quarks \( Q^a_i, \overline{Q}^a_{j} \) \( m \ll \Lambda_2, \quad \Lambda_Q^2 N_F = \Lambda_2^{2N_c-N_F} \mu_x^{N_c}, \quad N_c + 1 < N_F < 3N_c/2 \)
\[ \langle \overline{Q} Q \rangle_{br2} \simeq m \mu_x, \quad \mu_{x, o} \sim \Lambda_Q \left( \frac{\Lambda_Q}{m} \right)^{2N_c-N_F} N_c \sim \mu_x \left( \frac{\Lambda_2}{m} \right)^{2N_c-N_F} N_c \sim \mu_x \mu_{x, o} \sim \left( \frac{m}{\Lambda_2} \right)^{2N_c-N_F} N_c \ll 1, \] (8.2)
\[ \langle \overline{Q} Q \rangle_{br2} \simeq \mu_x \Lambda_2 \left( \frac{m}{\Lambda_2} \right)^{N_c-n_1}, \quad \langle \overline{Q} Q \rangle_{br2} \simeq \left( \frac{m}{\Lambda_2} \right)^{2N_c-N_F} N_c \ll 1, \quad 1 \leq n_1 < N_c. \]

If \( \mu_x \ll \Lambda_2 \), the lower energy gauge group at scales \( \mu = \Lambda_2/(\text{several}) \) in these br2 - vacua is \( U(N_c^2) \times U(1)^{2N_c-N_F} \) \([11]\). \( \text{\textsuperscript{4}} \)

\[ \text{\textsuperscript{4}} \] The perturbative NSVZ \( \beta \)-function of \( \mathcal{N} = 2 \) UV-free SQCD is exactly one loop \([6, 7]\) and \( g^2 \) has a pole at \( \mu = \Lambda_2 \). Therefore, to avoid a singularity, the field \( X \) is necessarily higgsed breaking the \( SU(N_c) \) group, with (at least some) components \( (X^A) \sim \Lambda_2 \). Moreover, for the same reasons, if there remains the non-abelian "electric" subgroup unbroken at the scale \( \Lambda_2 \), it has to be IR-free (or conformal).

There is the residual discrete symmetry \( Z_{2N_c-N_F} \) in the theory \((8.1)\) \([11]\). It is broken spontaneously in Lt-vacua (resulting in the multiplicity \( 2N_c - N_F \)) and unbroken in br2-vacua. From all this it follows that in br2-vacua \( \langle X \rangle \sim \text{diag}(a_1, \ldots, a_{N_q^c}, \omega \Lambda_2, \omega^2 \Lambda_2, \ldots, \omega^{2N_c-N_F} \Lambda_2) \) \([11]\), where \( \omega = \exp(2\pi i/(2N_c - N_F)) \) is a \((2N_c - N_F)\)-th root of unity, while \( a_i \) are either \( m \) or much smaller \( \sim mf(z), \quad f(z \rightarrow 0) \rightarrow 0, \quad z = (m/\Lambda_2)^{2N_c-N_F} \ll 1. \)
According to [10] (see the footnote 4), the lower energy $\mathcal{N} = 2$ $U(\mathcal{N}_c)$ theory at scales $\mu < \Lambda_2/(\text{several})$ contains the light adjoint fields $x^B$, $B = 1 \ldots \mathcal{N}_c^2$ (scalar partners of remained light $U(\mathcal{N}_c)$ ”electric” gluons) and $N_F$ flavors of remained light (i.e. with masses $\ll \Lambda_2$) original ”electric” quarks $Q^i_b$ and $\overline{Q}^i_b$, $i = 1 \ldots N_F$, $b = 1 \ldots \mathcal{N}_c$ (they will be denoted below as $Q^i_b$ to distinguish them from $Q^i_a$ with $\mathcal{N}_c$ colors).\footnote{ignoring here and below the abelian $U(1)^{2N_c-N_F}$ ”magnetic” part and its magnetic monopoles with their mass scale $\sim \sqrt{\mu_2}$} This theory is IR free in the range of scales $m \ll \mu \ll \Lambda_2$ and the scale factor of the $SU(\mathcal{N}_c)$ gauge coupling at scales $\mu_x \ll \mu \ll \Lambda_2$ is $\sim \Lambda_2$. Therefore the Lagrangian of light fields at the scale $\mu = \Lambda_2/(\text{several})$ looks as

$$K = \frac{1}{g_A^2(\mu)} x^B x^B + Q^i Q + \overline{Q}^i \overline{Q}, \quad g_A^2 = g^2 \quad \text{for} \quad B = 1 \ldots \mathcal{N}_c - 1 \quad \text{and} \quad g_1^2 \quad \text{for} \quad B = \mathcal{N}_c,$$

$$W_{\text{matter}} = \frac{\mu_x}{2} x^B x^B + \text{Tr} \left( m \overline{Q} Q - \sqrt{2} \overline{Q} x Q \right), \quad m \ll \mu_x \ll \Lambda_2. \quad (8.3)$$

After integrating out all fields $x^B$ as heavy ones at $\mu < \mu_{\text{pole}}(x) \sim \mu_x \ll \Lambda_2$ (all logarithmic effects of the RG evolution are ignored here and below for simplicity), the new scale factor of the $SU(\mathcal{N}_c)$ gauge coupling is

$$\tilde{\Lambda} = \Lambda_2 b_2/\mu_x, \quad \tilde{\Lambda} \gg \Lambda_2, \quad b_2 = (2 \mathcal{N}_c - N_F) < 0, \quad \tilde{b}_o = (3 \mathcal{N}_c - N_F) < 0, \quad (8.4)$$

and the Lagrangian at the scale $\mu = \mu_x$ looks as [10]

$$K = \text{Tr} \left( Q^i Q + \overline{Q}^i \overline{Q} \right), \quad W_{\text{matter}} = m \left( \overline{Q} Q \right) - \frac{1}{2 \mu_x} \left( \overline{Q} Q \right)^i \left( \overline{Q} Q \right)^i. \quad (8.5)$$

The above $\text{br}_2$ -vacua (8.2) of the theory (8.1) with $\mathcal{N}_c$ colors, $\mathcal{N}_c + 1 < N_F < 3 \mathcal{N}_c/2$ and with $

\langle (\overline{Q} Q)_{2}\rangle_{\text{br}_2} \sim m \mu_x \gg \langle (\overline{Q} Q)_{1}\rangle_{\text{br}_2}, \quad 1 \leq n_1 < \mathcal{N}_c, \quad \mu_x \ll \mu_{x, o}, \quad \text{will correspond now to the \text{br}_1} - \text{vacua of the theory (8.5) with \mathcal{N}_c colors, \mathcal{N}_F > 3 \mathcal{N}_c, \mu_x \gg \mu_{x, o}} \quad \text{(see (2.14) in section 2 and (8.8) below)}, \quad \text{and with}$

$$\langle (\overline{Q} Q)_{1}\rangle_{\text{br}_1} \sim m \mu_x \gg \langle (\overline{Q} Q)_{2}\rangle_{\text{br}_1}, \quad \tilde{\mu}_{x, o} \sim \tilde{\Lambda} \left( \frac{\mathcal{N}_c - N_F}{m} \right)^{\frac{N_F}{N_c}} \sim \mu_x \left( \frac{m}{\Lambda_2} \right)^{(2 \mathcal{N}_c - N_F)/\mathcal{N}_c}, \quad \frac{\mu_x}{\mu_{x, o}} \gg 1. \quad (8.6)$$

B) To calculate the spectrum of masses smaller than $\mu_x$ in the theory (8.5) it will be convenient to introduce the additional colorless but flavored fion fields $\Phi^i_f$ (these will be dynamically irrelevant finally at $\mu < \mu_x$, see below). Therefore, the direct $\Phi$ - theory is defined as follows. It has the same quark and $U(\mathcal{N}_c)$ gluon fields as in (8.5), the scale factor of the $SU(\mathcal{N}_c)$ gauge coupling is $\tilde{\Lambda} \gg \Lambda_2$ (8.4), and there are $N_F^2$ fion fields $\Phi$ in addition. It is IR free in the range of scales $m \ll \mu \ll \tilde{\Lambda}$, and we will deal with its $\text{br}_1$ - vacua, see (8.8) below. Its Lagrangian at the scale $\mu = \tilde{\Lambda}$ looks as

$$K = \text{Tr} \left( \Phi^i \Phi \right) + \text{Tr} \left( Q^i Q + \overline{Q}^i \overline{Q} \right), \quad W_{\text{matter}} = \frac{\mu_x}{2} \text{Tr} \left( \Phi^2 \right) + \text{Tr} \left( \overline{Q} (m - \Phi) Q \right). \quad (8.7)$$

In what follows the parameter $\mu_x$ can be varied in the range $m \ll \mu_x \ll \Lambda_2$ while $m$ and $\Lambda_2$ will stay intact. Then, with the change of notations: $\mathcal{N}_c \rightarrow \mathcal{N}_c, \mu_\Phi \rightarrow \mu_x, \Lambda_Q \rightarrow \tilde{\Lambda}, m_Q \rightarrow m$, we can use the results from (2.14) for the quark condensates, $1 \leq n_1 < \mathcal{N}_c, n_2 > \mathcal{N}_c$ (the Konishi anomalies

\footnotesize{11, 12}
(2.4) for $SU(N_c)$ theory (1.1) are replaced now with $(\langle \overline{Q} Q \rangle_1 + \langle \overline{Q} Q \rangle_2)_{br} = m_{\mu x}$ for $U(N_c)$ theory (8.7)

$$
\langle (\overline{Q} Q)_1 \rangle_{br1} \simeq m_{\mu x}, \quad \langle (\overline{Q} Q)_2 \rangle_{br1} \sim \frac{\tilde{A}^2 (\frac{\Lambda}{\mu_x})^{n_1 (N_c-n_1)}}{(N_c-n_1)} \rho^2 (\frac{m}{\Lambda})^{(N_c-n_1)/(N_c-n_1)} / m_{\mu x} \Lambda_2 (\frac{(N_c-n_1)}{(N_c-n_1)}),
$$

$$
\frac{\langle (\overline{Q} Q)_2 \rangle_{br1}}{\langle (\overline{Q} Q)_1 \rangle_{br1}} \sim \left( \frac{m}{\Lambda_2} \right)^{2(N_c-N_F)/(N_c-n_1)} \ll 1, \quad 1 \leq n_1 < N_c.
$$

We obtain from (8.7),(8.8) for the potentially most important masses

$$
\mu_{gl,2} \ll \mu_{gl,1} \sim \langle (\overline{Q} Q)_1 \rangle_{br1}^{1/2} \sim (m_{\mu x})^{1/2} \ll \Lambda_2,
$$

$$
m_{Q,1}^{pole} \ll m_{Q,2}^{pole} \sim \langle m_{Q,2}^{tot} \rangle = \langle m - \Phi_2 \rangle_{br1} = \frac{\langle (\overline{Q} Q) \rangle_{br1}}{\mu_x} \sim m \ll \mu_{gl,1},
$$

so that the overall phase is Higgs$_1 = HQ_2$ (HQ=heavy quark). Proceeding now as in previous sections, we can calculate the mass spectrum. Not going into any details we give here the results only.

a) The mixing of the pions $\bar{\Pi}_{11}$ (originating from higgsing of $Q^1$, $\overline{Q}_1$ quarks) and $\bar{\Pi}_{12}$, $\bar{\Pi}_{21}$ (these are in essence the quarks $Q^2$, $\overline{Q}_2$ with higgsed colors) with fions $\Phi$ is parametrically small and neglected. The masses of all $N_c^2$ fions are the largest ones, $\mu(\Phi) \sim m_{\mu x}$. Therefore, they are irrelevant at scales $\mu < m_{\mu x}$ we are interested in.

b) There are $n_1(2N_c - n_1)$ higgsed ”electric” gluons (and their superpartners) with masses $\mu_{gl,1} \sim (m_{\mu x})^{1/2}$.

c) There is a large number of hadrons made of the weakly interacting and weakly confined quarks $\overline{Q}_2$, ($Q^2$)’ with unhiggsed colors and with masses $m_{Q,2}^{pole} \sim m$ (the tension of the confining string originating from the unbroken non-abelian group $SU(N_c - n_1)$ is much smaller, $\sqrt{\sigma} \sim \Lambda^{(br1)}_{YM} \ll m$).

d) The masses of $n_1^2$ pions $\bar{\Pi}_{11}$ are $\sim m$.

e) There is a large number of gluonia made of unhiggsed non-abelian $SU(N_c - n_1)$ gluons with the mass scale $\Lambda^{(br1)}_{YM} = \langle S \rangle^{1/3} \sim (m_{\overline{Q} Q})_{br1}^{1/3}$, see (8.8).

f) $2n_1 n_2$ hybrid pions $\bar{\Pi}_{12}$, $\bar{\Pi}_{21}$ are the Nambu-Goldstone particles and are massless.

g) There remains one extra massless photon.

The overall mass hierarchies look as

$$
\mu(\bar{\Pi}_{12}) = \mu(\bar{\Pi}_{21}) \ll \Lambda^{(br1)}_{YM} \ll \mu(\bar{\Pi}_{11}) \sim m_{Q,2}^{pole} \ll \mu_{gl,1} \ll \mu(\Phi) \ll \Lambda_2 \ll \tilde{\Lambda}.
$$

C) Let us consider now the $d\Phi$ - theory which is the literal Seiberg dual to the direct $\Phi$ theory (8.7). According to the Seiberg rules $[3, 13]$, this is the theory with $U(N_c = N_F - N_c)$ dual colors, the scale factor of the dual gauge coupling is $\Lambda_q = C_o \tilde{\Lambda}$, $C_o = O(1)$, with $N_c + 1 < N_F < 3N_c/2$ flavors of the quarks $q_a$ and $\overline{q}_a$ (dual to $Q$, $\overline{Q}$), $N_F^2$ fions $\Phi_i^j$ and with $N_F^2$ elementary mions $M_j^j \rightarrow (\overline{Q}, Q^{'})$.

The case $n_1 = N_c - 1$ is different. The whole $SU(N_c)$ group is higgsed, there is no confinement and no gluonia with masses $\sim \Lambda^{(br1)}_{YM}$, the non-perturbative contribution to the low-energy superpotential originates not from the unhiggsed non-abelian SYM, but directly from the instanton.

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6 The case $n_1 = N_c - 1$ is different. The whole $SU(N_c)$ group is higgsed, there is no confinement and no gluonia with masses $\sim \Lambda^{(br1)}_{YM}$, the non-perturbative contribution to the low-energy superpotential originates not from the unhiggsed non-abelian SYM, but directly from the instanton.
$SU(N_c)$ gauge coupling $\sigma(\mu \sim \Lambda) \sim 1$ and $\sigma(\mu \ll \Lambda) \sim (\Lambda/\mu)^{\nu>0} \gg 1$. The dual Lagrangian at the scale $\mu = \Lambda$ is

$$K = \text{Tr} (\Phi^\dagger \Phi) + \frac{1}{\Lambda^2} \text{Tr} (\bar{M}^\dagger M) + \text{Tr} (q^\dagger q + \bar{q}^\dagger \bar{q}),$$

$$W_{\text{matter}} = \frac{\mu_x}{2} \text{Tr} (\Phi^2) + \text{Tr} (m - \Phi) M - \frac{1}{\Lambda} \text{Tr} (\bar{q} M q). \quad (8.11)$$

Proceeding as in [4] and not going into any details we also give below the results only. The overall phase in the br1 - vacua considered, see (8.8), is $H q_1 - H q_2$ and the masses of dual quarks look as

$$\mu_{q,1} \equiv \mu_{q,1}(\mu = \Lambda) = \frac{\langle M_1 \rangle_{\text{br1}} = \langle (\bar{Q} Q) \rangle_{\text{br1}}}{\Lambda}, \quad \mu_{q,1}^{\text{pole}} = \frac{\mu_{q,1}}{z_q(\Lambda, \mu_{q,1})} \sim \Lambda_2 \left( \frac{m}{\Lambda_2} \right)^{N_c/N_c}, \quad (8.12)$$

$$\bar{m}_{q,2}^{\text{pole}} \sim \rho^{(+)} \rho^{(-)} \left[ \langle (Q Q) \rangle_{\text{br1}} \right] = \frac{\rho^{(+)} \rho^{(-)}}{\rho^{(+) \rho^{(-)}}} \simeq m \Lambda, \quad \bar{m}_{gl,2} \sim m \ll \mu_{q,1}^{\text{pole}}, \quad (8.13)$$

$$\mu_{q,2}^{\text{pole}} \sim \frac{\langle M_2 \rangle_{\text{br1}} = \langle (\bar{Q} Q) \rangle_{\text{br1}}}{\Lambda} \frac{1}{z_q(\Lambda, \mu_{q,1}) z_q(\mu_{q,1}, \mu_{q,2})} \sim m \ll \mu_{q,1}^{\text{pole}}, \quad (8.14)$$

where the anomalous dimensions are (see section 7 in [14] and section 4 in [1])

$$\gamma_q^{(+)M} = -1 + \gamma_q^{(+)}, \quad \nu^{(+)M} = -\frac{N_c}{N_c}, \quad \nu^{(+)} = \frac{3N_c - 2N_F}{N_c}, \quad (8.15)$$

$$\gamma_q^{(-)M} = -(1 + \gamma_q^{(-)}), \quad \nu^{(-)} = \frac{3N_c - 2N_F}{N_c}. \quad (8.16)$$

The mass spectrum of this dual $d\Phi$ - theory looks as follows.

a) The mixing of all $N_F^2$ quarks $\bar{M}$ and $N_F^2$ fions $\Phi$ is small at scales $\mu \ll \mu_x$ and is neglected. The masses of all $N_F^2$ fions, $\mu(\Phi) \sim \mu_x$, will be the largest ones provided that $\mu_x \gg \mu_{q,1}^{\text{pole}}$ (8.12) which is always possible to adjust. So, all fions are dynamically irrelevant at scales $\mu \ll \mu_x$ and all other masses will be smaller than $\mu_x$. Therefore, all $N_F^2$ fion fields $\Phi$ can be integrated out as heavy ones at the scale $\mu \ll \mu_x$ we are interested in, and the Lagrangian at the scale $\mu = \mu_x$ will be

$$K = z_q(\Lambda, \mu_x) \text{Tr} \left( \frac{\bar{M}^\dagger M}{\Lambda^2} \right) + z_q(\Lambda, \mu_x) \text{Tr} (q^\dagger q + \bar{q}^\dagger \bar{q}), \quad (8.16)$$
\[ z_M(\tilde{\Lambda}, \mu_x) = \left( \frac{\tilde{\Lambda}}{\mu_x} \right)^{1+\gamma_q} \gg 1 \quad \text{and} \quad z_q(\tilde{\Lambda}, \mu_x) = \left( \frac{\mu_x}{\tilde{\Lambda}} \right)^{\gamma_q} \ll 1, \]

\[ W_{\text{matter}} = -\frac{1}{2\mu_x} \text{Tr}(\tilde{M}^2) + m \text{Tr} \tilde{M} - \frac{1}{\Lambda} \text{Tr}(\overline{q} \tilde{M} q). \]

b) There is a large number of hadrons made of confined \( q_1, \overline{q}_1 \) quarks (and heavy hybrids made of \( q_2, \overline{q}_2 \) quarks), the quark masses are \( \mu_{q,1}^{\text{pole}} \sim \Lambda_2(m/\Lambda_2)^{N_c/N_c}, \) see (8.12).

c) There is a large number of hadrons made of confined \( q_2, \overline{q}_2 \) quarks, the quark masses are \( \mu_{q,2}^{\text{pole}} \sim m \ll \mu_{q,1}^{\text{pole}} \) (the tension of the confining string is much smaller, \( \sqrt{\sigma} \sim \Lambda_{YM}^{(bri)} \ll m \)).

d) The masses of \( n_1^2 \) mions \( \tilde{M}_{11} \) and \( n_2^2 \) mions \( \tilde{M}_{22} \) are \( \sim m \).

e) There is a large number of gluonia made of \( SU(N_c) \) dual gluons with the mass scale \( \Lambda_{YM}^{(bri)} = \langle S \rangle_{br1}^{1/3} \sim (m\langle \tilde{M}_2 \rangle_{br1} = m\langle (\overline{Q}Q)_2 \rangle_{br1})^{1/3} \ll m, \) see (8.8).

f) \( 2n_1n_2 \) hybrid mions \( \tilde{M}_{12}, \tilde{M}_{21} \) are the Nambu-Goldstone particles and are massless.

g) There remains one extra massless photon.

The overall hierarchies look as

\[ \mu(\tilde{M}_{12}) = \mu(\tilde{M}_{21}) \ll \Lambda_{YM}^{(bri)} \ll \mu(\tilde{M}_{11}) \sim \mu(\tilde{M}_{22}) \sim \mu_{q,2}^{\text{pole}} \ll \mu_{q,1}^{\text{pole}} \ll \mu(\Phi) \ll \Lambda_2 \ll \tilde{\Lambda}. \]

Comparing the mass spectra of the direct \( \Phi \) (8.5) and dual \( d\Phi \) (8.16) theories it is seen that they are different.

Therefore, the next question is as follows. - Is it possible at all to start from the direct \( \mathcal{N} = 1 \) theory with the original "electric" quark fields \( Q^a_i, \overline{Q}^a_j \) with \( N_c \) colors, i.e. those in (8.1), and to adjust the parameters so that its Seiberg’s dual variant with \( \overline{N}_c \) colors and dual quarks \( q_i, \overline{q}_j \) will coincide at scales \( \mu < \mu_x \) with the theory (8.5)? As we will show below, the answer is "nearly yes" (the meaning of "nearly" will become clear below).

The Lagrangian of this desired direct \( \mathcal{N} = 1 \) theory at the scale \( \mu = \Lambda_Q \) looks as

\[ K = \text{Tr}(\Phi^\dagger \Phi) + \text{Tr}(Q^i Q + \overline{Q}^j \overline{Q}), \quad W_{\text{matter}} = \frac{\mu_{\Phi}}{2} \text{Tr}(\Phi^2) + \text{Tr}(\overline{Q}(m_Q - \Phi)Q), \quad (8.17) \]

the gauge group is \( U(N_c) \), the scale factor \( \Lambda_Q \) of the gauge coupling is chosen as \( \Lambda_Q = \tilde{\Lambda} \) (8.4) from the beginning, and hierarchies of parameters in (8.17) are \( m_Q \ll \Lambda_2 \ll \tilde{\Lambda} \ll \mu_{\Phi} \).

Then the corresponding Seiberg dual theory has the gauge group \( U(\overline{N}_c) \), the dual quarks \( q_i, \overline{q}_j \), and \( N_f^2 \) additional elementary mions \( \tilde{M}_j \rightarrow (\overline{Q}^j Q^j) \), the scale factor of the dual gauge coupling is \( \Lambda_q \sim \Lambda_Q \sim \tilde{\Lambda} \) and the dual Lagrangian at the scale \( \tilde{\Lambda} \) looks as

\[ K = \text{Tr}(\Phi^\dagger \Phi) + \frac{1}{\Lambda^2} \text{Tr}(M^\dagger M) + \text{Tr}(q^i \overline{q} + \overline{q}^j q); \]

\[ W_{\text{matter}} = \frac{\mu_{\Phi}}{2} \text{Tr}(\Phi^2) + \text{Tr}(m_Q - \Phi)M - \frac{1}{\Lambda} \text{Tr}(\overline{q} M q). \quad (8.18) \]

The masses of \( N_f^2 \) fion fields \( \Phi \) are very large, \( \mu(\Phi) \sim \mu_{\Phi} \), and they all can be integrated out once and forever,

\[ K = \frac{\text{Tr}(M^\dagger M)}{\Lambda^2} + \text{Tr}(q^i \overline{q} + \overline{q}^j q), \quad (8.19) \]
The Lagrangian (8.22) will have the same form as (8.5), with the same mass parameters \( q, \Phi \), with the same gauge group \( U \) above and (8.23) below logarithmic RG evolution (which is neglected). Let us choose now \( \mu > \Lambda \) than \( \tilde{\Lambda} = \Lambda = \Lambda \)

\[ m \]smaller than \( \mu \)

\[ m \]will be the largest ones among the masses smaller than \( \Lambda \) and they all can be integrated out at the scales \( \mu < \mu \) we are interested in. As a result, the Lagrangian at \( \mu = \mu \) looks as

\[ K = \text{Tr} \left( q^\dagger q + \bar{\tau}^\dagger \tau \right), \quad W_{\text{matter}} = m_Q \text{Tr} M - \frac{1}{2\mu_\Phi} \text{Tr} (M^2) - \frac{1}{\Lambda} \text{Tr} (\bar{\tau} M q). \]  (8.22)

Therefore, with the choice of parameters in (8.17)

\[ \Lambda_Q = \Lambda \left( \frac{\Lambda_2}{\mu_\chi} \right)^{3N_c - 2N_F}, \quad m_Q = \frac{\mu_\chi}{\Lambda} = m \left( \frac{\mu_\chi}{\Lambda_2} \right)^{3N_c - 2N_F}, \quad \mu_\Phi = \frac{\Lambda^2}{\mu_\chi} = -\Lambda \left( \frac{\Lambda_2}{\mu_\chi} \right)^{3N_c - 2N_F} \]  (8.23)

the Lagrangian (8.22) will have the same form as (8.5), with the same mass parameters \( m \) and \( \mu_\chi \), with the same gauge group \( U(N_c) \) and the same scale factor \( \Lambda \) of its non-abelian gauge coupling, and all hierarchies will be as needed

\[ m_Q \ll m \ll \mu_\chi \ll \Lambda_2 \ll \Lambda \ll \mu_\Phi. \]  (8.24)

And the mass spectrum at scales smaller than \( \mu_\chi \) will be the same in this theory with the quark fields \( q, \bar{\tau} \) as those described above in the theory (8.5) with the quark fields \( Q, \bar{Q} \).
Does it mean that these two theories, (8.5) and (8.22), will be completely equivalent? The answer is negative. The reason is as follows. Let the quarks $Q^i_a$ in (8.1),(8.17) to have the positive baryon charge $B = 1$ and to be in the fundamental representation $N_F$ of the $SU(N_F)_L$ flavor group, so that the baryon field with the baryon charge $N_c$ looks as $\sim Q^{N_c}$. The same baryon field looks then as $\sim q^{\overline{N_c}}$ in terms of dual quarks, so that the quarks $q_i$ in (8.19),(8.22) will have the positive baryon charge $N_c/\overline{N_c}$ and will be in the conjugate fundamental representation $\overline{N}_F$ of $SU(N_F)_L$ (up to $q \leftrightarrow \overline{7}$ which is simply the change of notations). But the quarks $Q^i_a$ in (8.5) are in the same fundamental representation $N_F$ of the $SU(N_F)_L$ flavor group as the quarks $Q^i_a$ in (8.1) because $Q$ and $Q$ differ only by a number of colors.

Now, in the interval of scales $(m\mu_x)^{1/2} \ll \mu \ll \mu_x$, all terms in the superpotentials (8.5) and (8.22) are dynamically irrelevant and all quark and gluon fields are effectively massless in both theories, so that the non-abelian flavor symmetry is enhanced and is $SU(N_F)_L \times SU(N_F)_R$. Therefore, the quarks $Q^i$ from (8.5) give the positive contribution, $(+\overline{N_c}d_{\text{fund}})$, to the flavor 't Hooft triangle $SU^3(N_F)_L$, while the contribution of quarks $q_i$ from (8.22) to this triangle is negative, $(-\overline{N_c}d_{\text{fund}})$. Clearly, this is a manifestation of the fact that the quarks $Q^i$ and $q_i$ behave differently under $SU(N_F)_L$ transformations. This conclusion differs from those in [10] where it was stated that the quarks $Q$ and $q$ are the same.\[7\]

### 8.2 Unbroken $U(N_F)$ and unbroken $Z_{2N_c-N_F}$

It is interesting that the vacua with these properties in the theory (8.7) with $N_c$ colors and $\mu_x \gg \mu_x$, (8.2) are the $N = 1$ SQCD-vacua with the multiplicity $N_c$ (see (2.9), with replacements $N_c \rightarrow \overline{N_c}$, $\Lambda_Q \rightarrow \overline{\Lambda}$, $\mu_\Phi \rightarrow \mu_x$, $m_Q \rightarrow m$, and (8.4))

$$
\langle S \rangle_{\text{SQCD}} \simeq \left( \Lambda_{\overline{N_c}} \det m \right)^{1/\overline{N_c}} \simeq \mu_x \Lambda_2^2 (m/\Lambda_2)^{N_F/N_c},
$$

$$
\langle \overline{Q}Q \rangle_{\text{SQCD}} \simeq \frac{\langle S \rangle_{\text{SQCD}}}{m} \simeq \frac{1}{m} \left( \Lambda_{\overline{N_c}} \det m \right)^{1/\overline{N_c}} \simeq \mu_x \Lambda_2 (m/\Lambda_2)^{N_c/\overline{N_c}}.
$$

The theory (8.7) in the SQCD-vacua is IR-free in the range of scales $m \ll \mu \ll \overline{\Lambda}$ and the RG-evolution is only logarithmic (and ignored). Therefore, the potentially competing masses look as

$$
m_Q^{\text{pole}} \sim m_Q^{\text{tot}} = (m - \langle \Phi \rangle) = \frac{\langle S \rangle_{\text{SQCD}}}{\langle \overline{Q}Q \rangle_{\text{SQCD}}} \sim m,
$$

$$
\mu_{gl}^2 \sim \langle \overline{Q}Q \rangle_{\text{SQCD}}, \quad \frac{\mu_{gl}}{m_Q^{\text{pole}}} \sim \left( \frac{m}{\overline{\Lambda}} \right)^{(3N_c-2N_F)/2\overline{N_c}} \ll 1
$$

and so the overall phase is $H\overline{Q}$.

The mass spectrum looks as follows.
1) The masses of all fions $\Phi$ are $\sim \mu_x$ and they are irrelevant at scales $\mu < \mu_x$ we are interested in.
2) There is a large number of hadrons made of weakly interacting and weakly confined non-relativistic quarks $Q$ and $\overline{Q}$ with the pole masses $m_Q^{\text{pole}} \sim m$ (the tension of the confining string is

---

7 Recall also that (at least within the dynamical scenario #2 used in this paper) the mass spectra in the two theories (8.17) and (8.18) are different.
\[ \sqrt{\sigma} \sim \Lambda_M^{(\text{SQCD})} \ll m. \]

3) There is a large number of gluonia made of \( SU(N_c) \) gluons, the scale of their masses is \( \sim \Lambda_{YM}^{(\text{SQCD})} = \langle S \rangle_{\text{SQCD}}^{1/3} \), see (8.25).

4) There remains one extra massless photon.

Consider now the direct theory (8.17) and its dual (8.18), (8.19). For this case, see (8.23)
\[ \mu_{\Phi, o} = \Lambda_Q (\frac{\Lambda_Q}{m_Q})^{\frac{2N_c-N_F}{N_c}} = \mu_{\Phi} (\frac{\Lambda_2}{m})^{\frac{2N_c-N_F}{N_c}} \gg \mu_{\Phi} \] (8.27)
and so for these two theories (8.17) and (8.19) the corresponding vacuum with the unbroken \( U(N_F) \) and \( Z_{2N_c-N_F} \) symmetries is the S - vacuum with the multiplicity \( N_c \), see (2.7) and (8.23). In this vacuum
\[ \langle \overline{Q}Q \rangle_S \sim m_Q \mu_{\Phi} = m\Lambda = m\Lambda_2 (\frac{\Lambda_2}{\mu_x})^{\frac{N_F}{2N_c}} \]
\[ \langle S \rangle_S = \left( \frac{\det(\overline{Q}Q)_S}{\Lambda_Q^{3N_c-N_F}} \right)^{1/N_c} \sim \mu_x \Lambda_2^2 \left( \frac{m}{\Lambda_2} \right)^{N_F/N_c} \]
\[ \langle M \rangle_S = \langle \overline{Q}Q \rangle_S , \quad \langle \overline{q}q \rangle_S = \frac{\langle S \rangle_S \Lambda_Q}{\langle M \rangle_S} \simeq \mu_x \Lambda_2 \left( \frac{m}{\Lambda_2} \right)^{N_c/N_c} \] (8.28)

The dual theory (8.19) is in the IR-free regime at scales \( m \ll \mu \ll \Lambda_Q = \tilde{\Lambda} \) and the RG-evolution is only logarithmic (and ignored). From (8.28), the potentially important masses look as
\[ \mu_q^{\text{pole}} \sim \mu_q = \frac{\langle M \rangle_S}{\Lambda} \simeq m , \quad \overline{\mu}_{\text{gl}} \sim \langle \overline{q}q \rangle_S^{1/2} , \quad \frac{\mu_q^{\text{pole}}}{\mu_q} \sim \frac{\mu_x}{\Lambda_2} \left( \frac{m}{\Lambda_2} \right)^{(3N_c-2N_F)/2N_c} \ll 1 \] (8.29)
so that the overall phase is \( Hq \). The masses of all \( N_F^2 \) mions \( M \) are the largest ones, \( \mu(M) \sim \tilde{\Lambda}^2/\mu_{\Phi} = \mu_x \), and they are dynamically irrelevant at scales \( \mu \ll \mu_x \). As one can expect with the choice (8.23), the spectrum of masses smaller than \( \mu_x \) in the S - vacua of the theory (8.19) with the dual quarks \( q \), \( \overline{q} \) is the same as those in the SQCD - vacua of the theory (8.5) with the quarks \( Q, \overline{Q} \). But, clearly, the values of 't Hooft triangles \( SU(N_F)^2 \) in the range of scales \( m \ll \mu \ll \mu_x \) where all particles in both theories (8.5) and (8.19) are effectively massless are also different in these vacua with unbroken symmetries.

On the whole, our results in this section on comparing the properties of direct theories and their Seiberg’s dual differ from those in [10].

9 Conclusions

This paper continues [1, 2] studying the direct \( \mathcal{N} = 1 \) SQCD - like theories and their Seiberg’s dual, with various numbers of colors and quark flavors and with additional colorless but flavored fields \( \Phi_j \). Unlike [1, 2] where only the region \( 1 < N_F < 2N_c \) was considered, we calculated here the mass spectra of the direct and dual theories in the region \( 2N_c < N_F < 3N_c \).

The calculations in this article were performed within the dynamical scenario #2 introduced in [4]. This scenario assumes that, when such \( \mathcal{N} = 1 \) SQCD-like theories are in the strong coupling regime \( a(\mu) \gtrsim 1 \), the quarks can be in the two \emph{standard} phases only. - These are either the HQ (heavy quark) phase where they are confined or the Higgs phase where they are condensed with \( \langle Q \rangle = \langle \overline{Q} \rangle \neq 0 \). The word \emph{standard} also implies here that, unlike e.g. the very special \( \mathcal{N} = 2 \)
SQCD theories with colored adjoint scalar fields, no additional "unexpected" parametrically lighter particles (like magnetic monopoles or dyons) appear in these $\mathcal{N} = 1$ SQCD-like theories without colored adjoint scalars (see also footnote 2).

The calculations of mass spectra are based on finding first the quark and gluino condensates in various vacua. This was done for $N_F > 2N_c$ in section 2, where the multiplicities of various vacua with the unbroken or spontaneously broken flavor symmetry were also found. It is worth noting that the explicit expressions for the total number and multiplicities of various vacua in the three regions: $1 < N_F < N_c$, $N_c < N_F < 2N_c$ and $N_F > 2N_c$ are different and are not analytic continuations of each other, see [11] and section 2. And the hierarchies among the quark condensates are also different.

The mass spectra of the direct theories and their Seiberg’s dual at $2N_c < N_F < 3N_c$, $\mu_\Phi \gg \Lambda_Q$, were calculated in various vacua and compared with each other in sections 3-6. These two mass spectra are different, in general.

We calculated also in section 7 the mass spectra of the dual theory with $\mathcal{N}_c = N_F - N_c$ colors and $N_c < N_F < 3N_c/2$ flavors. Besides of interest by itself, the results were useful for the next section 8.

In this last section $\mathcal{N} = 2$ SQCD (8.1) was considered in vacua of the baryonic branch, with $N_c$ colors (with the scale factor $\Lambda_2$ of the $SU(N_c)$ gauge coupling) and $N_c + 1 < N_F < 3N_c/2$ flavors of the original ”electric” quarks $Q$ with the mass term $m\text{Tr}(Q^2)$ in the superpotential, broken down to $\mathcal{N} = 1$ by the mass term $\sim \mu_x X^2$, $m \ll \mu_x \ll \Lambda_2$, of the adjoint scalar field $X$. This theory was considered previously e.g. in [11] (for the case $m = 0$) and in [12], and recently in [10].

It was stated in [10] that, in these vacua of the baryonic branch with spontaneously broken flavor symmetry $U(N_F) \rightarrow U(n_1) \times U(n_2)$, the quarks $q$ with $\mathcal{N}_c$ colors which are the standard Seiberg’s quarks dual to the original ”electric” quarks $Q$ with $N_c$ colors in (8.1), and the quarks $Q$ with $\mathcal{N}_c$ colors in the theory (8.1) at scales $\mu \ll \Lambda_2$ (these are the light quarks of this theory with masses $\ll \Lambda_2$) are the same.

We calculated in section 8.1 the mass spectra of the theory (8.5) with $\mathcal{N}_c$ colors and $N_F$ flavors of quarks $Q$ in these vacua, and in the two variants of Seiberg’s dual. The first variant was the literal Seiberg’s dual to (8.5) strongly coupled theory (8.11),(8.16) with $N_c = N_F - \mathcal{N}_c$ colors and $N_F$ flavors of quarks $q$ (dual to $Q$). It was found that the mass spectra of the direct theory (8.5) with the quarks $Q$ and the dual one (8.16) with the quarks $q$ are different. The second variant was the direct theory (8.17) with the $N_c$ colors and $N_F$ flavors of original ”electric” quarks $Q$ and its Seiberg’s dual theory (8.19),(8.22) with $\mathcal{N}_c$ colors and $N_F$ flavors of quarks $q$ (dual to $Q$), and with especially chosen values of parameters $m_Q, \mu_\Phi$ and $\Lambda_Q$. It was shown that it is possible to adjust the values of $m_Q, \mu_\Phi, \Lambda_Q$ so that the spectra of masses smaller than $\mu_\Phi$ become the same in these two theories (8.5) and (8.22). Nevertheless, in distinction with the statement in [10], even in this case the flavor quantum numbers of the quarks $Q$ from (8.5) and $q$ from (8.22) are different and this reveals itself in different values of the ’t Hooft triangles.

And finally, we calculated in section 8.2 the mass spectra in the theories (8.5) and (8.19),(8.22) in vacua of the baryonic branch with unbroken $U(N_F)$. It is interesting that these are the $\mathcal{N} = 1$ SQCD vacua for the theory (8.5). As in section (8.1), the mass spectra of these two theories can be made the same at scales $\mu < \mu_\Phi$ in these vacua also by adjusting the values of $m_Q, \mu_\Phi, \Lambda_Q$ as in (8.23) but, clearly, the values of the ’t Hooft triangles $SU(N_F)^c_L$ also remain different.

This work was supported in part by Ministry of Education and Science of the Russian Federation and RFBR grant 12-02-00106-a.
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