Flow regimes and phase diagram of inertial particle suspensions

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The aim of this Letter is to characterize the flow regimes of suspensions of finite-size solid particles in a viscous fluid at finite inertia. We explore the system behavior as function of the particle volume fraction and the Reynolds number (the ratio of flow and particle inertia to viscous forces). Unlike single phase flows where a clear distinction exists between the laminar and the turbulent regime, three different states can be identified in the presence of a particulate phase, with smooth transitions between them. At low volume fractions, the flow becomes turbulent when increasing the Reynolds number, transitioning from the laminar regime dominated by viscous forces to the turbulent regime characterized by enhanced momentum transport by turbulent eddies. At larger volume fractions, we identify a new state characterized by an even larger increase of the wall friction. The wall friction increases with the Reynolds number (inertial effects) while the turbulent transport is unaffected, as in a state of intense inertial shear-thickening.

Understanding transition from laminar to turbulent flow has puzzled many scientists since the seminal work by Reynolds [1]. Despite the vast number of investigations, this phenomenon is not well understood even in the simplest configurations as plane channel flow. To be able to predict, and possibly control, the onset of turbulence is crucial in numerous applications as this is associated with a sharp increase of the wall friction and of the total drag. Transition in channel flows is of subcritical nature, it occurs if the flow is forced by strong enough perturbations, and takes place at Reynolds numbers, Re, significantly lower than that predicted by linear stability [2]. (Re quantifies the ratio of inertial and viscous forces)

When a suspension of rigid particles is considered instead of a pure fluid, peculiar and unexplained effects appear in the transitional regime. These aspects, important in several environmental and industrial applications, constitute the subject of the present Letter.

Matas et al. [3] performed experiments on the laminar-turbulent transition in particle-laden pipe flows. For sufficiently large particles, they found a non-monotonic variation of the transitional Reynolds number when increasing the particle volume fraction Φ (the bulk volume fraction). They were not able to collapse the transitional Reynolds numbers to the one of the single-phase fluid by rescaling with the suspension viscosity for the larger particles used, and thus suggested the presence of some additional physical mechanisms, still to be explored. They also underlined the difficulties found to unambiguously define the flow regime (laminar or turbulent) since suspensions manifest large fluctuations also at low speeds.

Similar findings were recently reproduced numerically in [4]. These authors also note how arbitrary it is to distinguish between laminar and turbulent states in suspensions where the value of the transitional Reynolds number becomes very sensitive to the velocity fluctuation threshold chosen to discern the flow state. The characteristics of the different flow regimes in particle-laden flows and the mechanisms behind the transition still need to be understood.

In this Letter we show the existence of three different flow states in the presence of a particulate phase with smooth transitions between them. First, a laminar-like regime (low Re and Φ), dominated by viscous forces, and, second, a turbulent-like regime (high Re and sufficiently low Φ), characterized by increased wall friction due to the turbulent transport, classically quantified by significant Reynolds stresses [5]. The third state appears in the dense regimes (high Φ), where we observe a significant increase of the wall friction, larger than in the turbulent-like regime. This is not associated with the increase of the Reynolds stresses, but to an increase of the particle-induced stress indicating a transport mechanism different than that of turbulence. Following rheology literature, we recognize this state as an intense inertial shear-thickening: it corresponds to an increase of the wall friction (effective viscosity) with the flow shear rate.

FIG. 1: Sketch of the coordinate system and instantaneous flow field at Re = 2500 & Φ = 0.3. The rigid particles are displayed only on half of the domain for clarity.
(Reynolds number) due to increasing particle stresses with inertia; thus the additional dissipation mechanism suggested in [3].

The analysis is based on data from fully resolved direct numerical simulations of a channel flow laden with rigid naturally-buoyant spherical particles. The code used, the Immersed Boundary solver developed by Breugem [6], couples an uniform Eulerian mesh for the fluid phase with a Lagrangian mesh for the solid phase. Lubrication forces and a soft-sphere collision model have been implemented to address the near field interactions (below a grid cell size). The code has been validated against several test cases [6] and recently used to study the rheology of dense and active suspensions [7, 8].

We simulate a channel flow with periodic boundary conditions in the streamwise and spanwise directions (see Fig. 1). Here, the streamwise, wall-normal and spanwise coordinates and velocities are denoted by $y, z, x$ and $v, w, u$, respectively. The size of the computational domain is $6h \times 2h \times 3h$ where $h$ is the half-channel width. The diameter of the particles is set to a fixed value, $2a = 2h/10$. The number of uniform Eulerian grid points in the domain is $480 \times 160 \times 240$ in the streamwise, wall-normal and spanwise directions; 746 Lagrangian grid points have been used on the surface of each particle to resolve the interactions between the two phases. We define the Reynolds number by the bulk velocity $U_b$, channel height and the fluid kinematic viscosity $\nu$: $Re = U_b h/\nu$. The dataset explores a wide range of Reynolds numbers, $500 \leq Re \leq 5000$, and particle volume fractions $0 < \Phi \leq 0.3$.

The initial condition for the simulations is chosen to be a high-amplitude localized disturbance in the form of a pair of streamwise vortices [3]. The critical value for transition of the single phase flow is found to be $Re \approx 2300$ in the present setup; the velocity fluctuations vanish in time for Reynolds number $Re < 2300$ while they reach a finite amplitude for $Re > 2300$.

To document the difficulty of clearly defining a transition threshold at high volume fractions, time histories of the root-mean-square of the streamwise velocity fluctuations, $\tilde{v}_{rms}$, are displayed in figure 2 for low and high volume fractions (velocity fluctuations are normalized by $U_b$ and time by $h/U_b$). For $\Phi = 0.001$, the transition to turbulence follows classic results in single phase fluids. For $Re \geq 2000$ the time histories exhibit a transient peak after which the level of fluctuation saturates to the values of the turbulent regime, $0.1 < \tilde{v}_{rms} < 0.12$, independent of the Reynolds number in the range considered here. Interestingly, the presence of few finite-size particles in the flow create continuous background disturbances such that the threshold at which turbulence can be sustained decreases from $Re \approx 2300$ for the unladen flow to $1750 < Re < 2000$. Figure 2(b) shows that the level of streamwise velocity fluctuations saturate to $0.05 < \tilde{v}_{rms} < 0.07$ for all the Reynolds numbers under investigation when $\Phi = 0.3$. The velocity fluctuations in the flow smoothly increase with the Reynolds number and approach a regime value lower than that of a turbulent flow at small $\Phi$. As shown in the plot and discussed recently in [4], the definition of a transitional Reynolds number becomes very sensitive to the threshold values chosen, and probably not completely meaningful as we show below.

A contour map of the time-averaged streamwise velocity fluctuation intensity $v_{rms} = \langle v'^2 \rangle$ in the $(\Phi, Re)$ plane is presented in figure 3(a) ($\langle \rangle$ denote ensemble average, $v'$ the streamwise velocity fluctuation). The samples for the ensemble average are collected after the transient phase of the disturbance evolution. The blue dashed-line in the figure represent the threshold value 0.07 that can be reasonably used to identify the chaotic regime. This shows a non-monotonic behavior of the transitional Reynolds number, in line with the experimental observation in [3] and the simulations in [4]. We have marked three points on the map corresponding to the (a) laminar-like, (b) inertial shear-thickening and (c) turbulent-like regimes for future reference. Figure 3(b) displays the average wall
friction $\tau_w(\Phi, Re)$ normalized by the friction of the laminar single-phase Poiseuille flow $\tau^0_w(Re)$. We thus define a relative viscosity $\nu = \tau_w / \tau^0_w$, similarly to what usually adopted in rheology [10]. A jump in the value of the relative viscosity, to about the double, is evident at the critical conditions at low volume fractions. Increasing the particle concentration, the relative viscosity smoothly increases with the Reynolds number (inertial effects) while the level of fluctuations, yet significant, remains almost unaffected. This behavior points toward an new dynamic state that we call intense inertial shear-thickening. It is important to note that the values of $\nu$, for $Re = 500$ (weak inertia) well follow classic semi-empirical fits for viscous laminar suspensions, e.g., Eilers fit [10].

To characterize the different regimes of the particulate flow, we study the stress budget in the flow. Based on the formulation proposed in [11], we write the momentum balance in the wall-normal direction assuming streamwise and spanwise homogeneity,

$$\tau(z/h) = -(1 - \varphi)\langle w^f v^f \rangle - \varphi \langle w^p v^p \rangle + \nu \frac{\partial \langle v^f \rangle}{\partial z} \left(1 - \frac{z}{h}\right),$$

where $\varphi(z)$ denotes the mean local volume fraction, the superscripts $f$ and $p$ the fluid and particle phase, $\rho$ the density and $\sigma_{yz}$ the particle stress. The first two terms represent the contribution of both phases to the Reynolds shear stress: $\tau_R(z)$. The third and forth term are the viscous, $\tau_V(z)$, and particle stresses, $\tau_P(z)$. The total stress balances the external pressure gradient that gives a linear profile across the channel with maximum at the wall, $\tau_w = \nu \frac{\partial \langle v^f \rangle}{\partial z} |_{w}$, and zero at the centerline.

We evaluate the contribution of each term in the stress budget for the cases marked by a,b and c in figure 3(a) and display the wall-normal stress profiles, normalized by $\tau_w$, in fig. 4. For the laminar-like regime, the Reynolds shear stress is negligible and the viscous stress dominates (see panel a); the particle stress has a weak contribution, maximum around $z \approx 0.2h = 2a$. Figure 4(c) displays the stress budget of the turbulent-like regime when the Reynolds stress is the dominant term and both viscous and particle stresses are relevant only in the near-wall region. This is the typical behavior of turbulent flows [2]: the increase of the wall friction $\tau_w$ with respect to a laminar state is caused by the formation of coherent velocity fluctuations and increased transport. For the inertial shear-thickening regime, figure 4(b), the particle stress is larger than the Reynolds shear stress, and accounts for the majority of the momentum transfer (68%) when integrated across the channel. In the middle of the channel, $z \geq 0.6h$, the particle stress is the only term to transfer stress exceeding by more than one order of magnitude the viscous and the Reynolds stresses. This is due to the particle migration that leads to a mean local volume fraction $\varphi \approx 0.5$ in the channel centre (not shown) [12].

Using the momentum transfer budget we are therefore in the position to suggest an unambiguous classification of the flow. In particular, dividing equation 4 with $\tau = \tau_w(1-z/h)$ and integrating across the channel

$$1 = \frac{\nu}{\tau} + \frac{\tau_R}{\tau} + \frac{\tau_P}{\tau} = \Sigma_V + \Sigma_R + \Sigma_P. \hspace{1cm} (2)$$

The flow is denoted as laminar-like if the viscous stress gives the largest contribution to the total momentum transfer across the channel (relative majority), i.e. $\Sigma_V > \Sigma_R$ and $\Sigma_V > \Sigma_P$, and similarly, turbulent-like if the Reynolds shear stress provides the largest contribution. We denote as intense inertial shear-thickening the regime where the momentum transfer is dominated by the particle stress, $\Sigma_P > \Sigma_V$ and $\Sigma_P > \Sigma_R$.

The region of existence of these three regimes is depicted in the $(Re, \Phi)$ plane in figure 3(a). The point of intersection between the different regimes occurs at $Re \approx 2000$ and $\Phi \approx 0.1$ for our setup. The solid black lines show the boundary of the regions where each term in the stress budget is over 50% of the total stress (absolute majority), displaying a relatively smooth transition among the three state, yet clearly distinguishable.

At vanishing volume fractions, $\Phi \to 0$, the transition between the laminar and turbulent regime is, as expected, sharp. Indeed, criteria based either on fluctuation levels or on the stress budget determine the same critical Reynolds number. At finite $\Phi$, the transition among the states become smooth. In this case, the definition of the transitional Reynolds number would depend on the threshold value chosen for the velocity fluctuations and on the choice of the velocity component. In this scenario,
the stress balance can quantitatively identify the dominant interactions.

The flows determined by the particle stress correspond to an intense form of continuous shear-thickening. Inertial shear-thickening amounts to an increase of the relative viscosity $\nu_r = \tau_w/\tau_0^w$ with the imposed shear rate $\dot{\gamma}$ (Reynolds number) at fixed $\Phi$ without an increase of the Reynolds stresses (turbulent activity) \cite{1, 10}. Note that the shear rate $\dot{\gamma} = U_b/h$ is proportional to the Reynolds number $Re = \dot{\gamma} h^2/\nu$. Defining the relative viscosity as $\nu_r = \tau_w/\tau_0^w$, we can extract the contributions to the total wall friction due to the viscous $\nu_r^V = \Sigma v \nu_r$, particle $\nu_r^P = \Sigma p \nu_r$ and Reynolds stresses $\nu_r^R = \Sigma R \nu_r$. These quantities are depicted vs $Re$ in figure 5(b) for $\Phi = 0.025$ and $\Phi = 0.3$. At low $\Phi$, the relative viscosity $\nu_r$ is constant up to $Re \approx 1500$ and determined mainly by the viscous contribution; its value is slightly higher than unity following the increase of viscosity due to the solid phase, in agreement with rheological values in viscous flows. The increase of wall friction at higher $Re$ is due to the action of the Reynolds stresses and to the onset of a turbulent flow.

The situation dramatically changes at $\Phi = 0.3$ where we observe a smooth and continuous increase of $\nu_r$ that can be closely correlated with the increase of the particle-induced stress, and only in minor part to the increase of the Reynolds stress contribution. At sufficiently high $\Phi$, where $\Sigma P$ dominates, an intense shear-thickening occurs causing a large increase of the wall friction. If inertial effects were not relevant, one would expect a constant contribution from the particle stress and an almost constant relative viscosity. Inertial shear-thickening takes place only if inertial effects are present at the particle size (finite particle Reynolds number $Re_a = Re(a/h)^2$ \cite{7, 10}). This behavior is therefore expected only for relatively large particles (the particle Reynolds number $\sim a^2$); small enough particles are not expected to show these peculiar inertial effects. Indeed, Matas et al. \cite{3} are able to scale the experimental data for small particles only with the suspension viscosity measured in the laminar regime, providing indirect evidence for our interpretation. The boundary between the shear-thickening and the turbulent-like regime is not sharp, and one can observe a coexistence of the two states with different relevance (or probability) when varying the number of particles or the importance of inertia.

In summary, we observe in semi-dilute particulate flows an increase of the normalized wall friction (relative viscosity) with increasing inertial effects not associated with a corresponding increase of the Reynolds stresses. We identify this regime as an intense form of continuous inertial shear thickening, induced by inertial effects at particle size at large enough nominal volume fractions, see e.g. \cite{3}. This leads to a phase diagram, function of $\Phi$ and $Re$, with three different regimes. In the limit of vanishing volume fraction, the transition between laminar and turbulent regime is sharp, while this is not the case for the different states at finite $\Phi$. This implies that at moderate and high $\Phi$ inertial shear-thickening and turbulence coexist with different relevance.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{(a) Phase diagram for particle laden channel flow in the $Re - \Phi$ plane (blue, red and green colors indicate laminar-like, turbulent-like and inertial shear-thickening regime). The solid black lines show the boundary of the regions where each term in the stress budget is over 50% of the total stress. (b) Contributions to the total wall friction from the viscous stresses, Reynolds stresses and particle stresses versus the Reynolds number $Re$ for $\Phi = 0.025$ and $\Phi = 0.3$.}
\end{figure}

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