Entropy generation and momentum transfer in the superconductor-normal and normal-superconductor phase transformations and the consistency of the conventional theory of superconductivity

J. E. Hirsch

Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319

Since the discovery of the Meissner effect the superconductor to normal (S-N) phase transition in the presence of a magnetic field is understood to be a first order phase transformation that is reversible under ideal conditions and obeys the laws of thermodynamics. The reverse (N-S) transition is the Meissner effect. This implies in particular that the kinetic energy of the supercurrent is not dissipated as Joule heat in the process where the superconductor becomes normal and the supercurrent stops. In this paper we analyze the entropy generation and the momentum transfer between the supercurrent and the body in the S-N transition and the N-S transition as described by the conventional theory of superconductivity. We find that it is not possible to explain the transition in a way that is consistent with the laws of thermodynamics unless the momentum transfer between the supercurrent and the body occurs with zero entropy generation, for which the conventional theory of superconductivity provides no mechanism. Instead, we point out that the alternative theory of hole superconductivity does not encounter such difficulties.

I. INTRODUCTION

The conventional theory of superconductivity [1–3] is generally believed to describe both the Meissner effect and its reverse, the superconductor-normal (S-N) transition in the presence of a magnetic field, in a way that is consistent with the laws of thermodynamics that describe reversible first order phase transformations. That the transition is reversible has been established long ago. It was strongly suggested by the discovery of the Meissner effect [4], was first proposed theoretically by Rutgers [5] and by Gorter and Casimir [6], and was confirmed experimentally by extensive experimental work by Keesom and coworkers [7] and Mapother [8]. It is also an implicit assumption of the conventional theory of superconductivity [1–3]. A detailed discussion of this issue is given in the book by Shoenberg [9].

We have recently pointed out [10, 11] that there are key issues regarding basic conservation laws raised by the fact that the transition is reversible that have not been discussed in the theoretical superconductivity literature. We have discussed how these issues are addressed and resolved in an alternative theory of superconductivity, the theory of hole superconductivity [12], and we have raised the possibility that the conventional theory of superconductivity may lack essential physical ingredients that are necessary to describe these transitions in a way that is consistent with the laws of thermodynamics, electrodynamics and mechanics [10, 11, 13–16].

This paper deals with type I superconductors only. A superconductor in a magnetic field has a surface supercurrent that prevents the magnetic field from penetrating its interior. The supercurrent carries mechanical momentum and kinetic energy. When the system undergoes a transition to the normal state, the supercurrent stops since the normal state does not carry any current. In the reverse transition, when the normal metal in a magnetic field becomes superconducting, a supercurrent carrying mechanical momentum and kinetic energy spontaneously starts flowing. In this paper we explore issues associated with entropy generation and momentum transfer associated with these transformations and ask whether the conventional theory of superconductivity has the ability to account for these processes in a consistent way. We compare the superconductor-normal transition to the liquid-vapor transition to clarify some of the issues at play. We find that the conventional theory of superconductivity cannot describe the S-N transition nor the N-S transition in the presence of a magnetic field in a consistent way. In particular, we find that it is required that the momentum of the supercurrent is transferred to the body as a whole without any entropy generation, a process for which the conventional theory provides no mechanism. We find that in the conventional description the transition generates entropy even when it proceeds infinitely slowly, in contradiction with the laws of thermodynamics. We also find that it violates energy conservation. We furthermore find that thermodynamic equilibrium between the normal and superconducting phases is not properly described. We also find that the conventional theory predicts that as the temperature goes to zero the time it takes for the transition to take place diverges, which is inconsistent with observations. We conclude that the conventional theory cannot describe the transition in a consistent way. We briefly sketch the way the unconventional theory of hole superconductivity describes the transition in a way that is consistent with physical laws and with experiment.

In more detail, the outline of this paper is as follows. In Sects. II and III, we discuss the general question of entropy production in a first order phase transition, using the liquid-gas transition as a well-understood model. In
Sect. IV we apply the same reasoning to the superconductor-normal transition in a magnetic field, and show that the entropy production predicted by thermodynamics is entirely accounted for by the Joule heat generated by eddy currents in the transition. The remainder of the paper analyzes whether or not the momentum transfer between the supercurrent and the body, that has to take place to respect momentum conservation, can be described within the conventional theory without additional entropy production. Sect. V quantifies the momentum that needs to be transferred, showing that it is several orders of magnitude larger than would be expected at first sight, as well as its associated kinetic energy, showing that it is a bulk rather than a surface effect. Stringent bounds set by experiment determine that essentially none of this kinetic energy is dissipated as Joule heat. The following sections (VI - XV) explore whether within the physics of the transition as described by the conventional theory of superconductivity it is possible to account for the momentum transfer without introducing irreversibility, and conserving momentum and energy. We find that it is not possible. A key part of the argument is to show that the normal electron distribution that results from pair dissociation is necessarily anisotropic, as shown in Figs. 6 and 11. In Sect. XVI we summarize our findings and discuss their implications. Sect. XVII briefly explains how the theory of hole superconductivity can explain these questions, and we conclude with closing arguments in Sect. XVIII.

The questions discussed in this paper have never been discussed in the superconductivity literature before. Earlier theoretical work on the kinetics of the normal-superconductor transition in a magnetic field [17, 18] used the time-dependent Ginzburg-Landau (TDGL) formalism [19–21]. That formalism is phenomenological and involves a first order differential equation in time with real coefficients for the time evolution of the order parameter. Hence it describes irreversible time evolution, and is therefore irrelevant to the Meissner effect for type I superconductors, which is a reversible process [9], as discussed above. There is also theoretical work in the literature on the resistive transition in a magnetic field [22] describing the onset of resistance in type II superconductors through the creation of phase slips at a finite rate. Such treatments are also not relevant to the physical situation considered in this paper. For our case, a simply connected type I superconductor in the presence of a magnetic field, the phase of the order parameter is uniform in the superconducting phase and the transition to the normal state does not occur through phase slips or phase fluctuations but rather through suppression of the amplitude of the superconducting order parameter.

Superconductivity is undoubtedly a quantum phenomenon. BCS theory undoubtedly describes many of the quantum aspects of superconductivity correctly, for example the existence of macroscopic phase coherence and associated Josephson phenomena. The treatment in this paper is certainly not a fully quantum treatment, it may be characterized as semiclassical. However, whether quantum, semiclassical or classical, a physical description of nature has to satisfy fundamental laws of physics such as momentum and energy conservation. The author can see no way in which a fully quantum description of the phenomena discussed here within the conventional theory would be able to resolve the questions raised here, without introducing new physics such as electron-hole asymmetry, which is not contained in BCS theory and is contained in the theory of hole superconductivity.

II. THERMODYNAMICS OF FIRST ORDER PHASE TRANSFORMATIONS

Consider the generic situation shown in Figure 1. The “universe” that can potentially change its entropy consists of our system (small box) initially at temperature $T$ and a large heat bath at temperature $T + \Delta T$. The heat bath is sufficiently large that its temperature doesn’t change when it gives or takes heat to or from the system. The system is initially in its lower entropy phase (phase 1). We assume its initial temperature $T$ is the temperature of coexistence of phase 1 with another phase (phase 2) of higher entropy for a given other parameter $X$, i.e. $T = T(X)$. Specifically, for the liquid-gas transition $X$ is the pressure $P$, for the S-N transition $X$ is the applied magnetic field $H$.

We will ignore questions related to the initial stages of the transition involving nucleation, surface energy of domains, and superheating. In other words, we assume in the initial state when the system is in phase 1 there is already a coexisting small amount of phase 2 that can grow without having to overcome a barrier.

When thermal contact is established, heat will flow from the heat bath to the system. At the end of the process, the system will be in phase 2 at temperature $T + \Delta T$. Let us call $Q$ the net amount of heat that flowed from the heat bath to the system in this process. The change in entropy of the universe is then

$$\Delta S_{\text{univ}} = -\frac{Q}{T + \Delta T} + \frac{Q}{T} = \frac{Q}{T} \frac{\Delta T}{T}$$

(1)

to lowest order in $\Delta T$.

Let us call $L(T)$ the latent heat of the phase transformation for the given amount of system at temperature $T$, and $C_1$, $C_2$ the heat capacities of phases 1, 2. The latent heat is determined by the difference in entropy of the two coexisting phases

$$L(T) = T(S_2(T) - S_1(T)).$$

(2)
FIG. 1: Schematics of a first order phase transformation of a system (small box) in thermal contact with a heat bath (large box) making a transition from a lower entropy phase (phase 1) to a higher entropy phase (phase 2). We assume an external parameter (pressure or magnetic field) has the value corresponding to phase equilibrium for the system at temperature $T$.

Assume the system absorbs the latent heat while it is at temperature $T$, transforming to phase 2, and then raises its temperature to $T + \Delta T$. The heat transferred from the bath to the system is then

$$Q = L(T) + C_2 T \Delta T$$

and the change in entropy of the universe to first order in $\Delta T$ is from Eq. (1),

$$\Delta S_{\text{univ}} = \frac{L(T) \Delta T}{T}$$

Alternatively, we could assume that the system first raises its temperature to $T + \Delta T$ while remaining in phase 1 and then undergoes the transformation to phase 2. The calculation would be slightly different but the end result for the change in entropy of the universe still has to be equation (4), because entropy is a function of state. We will discuss specific examples in the next sections.

III. THE LIQUID-GAS PHASE TRANSFORMATION

Our system, shown in Fig. 2, consists of 1 mole of a liquid (e.g. water) in a cylinder with a piston of mass $M$ and area $A$ exerting pressure $P=Mg/A$. Assume $P = P(T)$ is the pressure of phase equilibrium between liquid and gas at temperature $T$, which satisfies the Clausius-Clapeyron equation

$$\frac{dP}{dT} = \frac{L(T)}{T \Delta V}$$

with $\Delta V$ the difference in molar volumes of liquid and gas. In the phase transformation at temperature $T$ the system absorbs heat $L(T)$ and performs work $P\Delta V$.

Let us assume there is a small friction force between the piston and the cylinder wall, and we ignore any difference between static and kinetic friction. Because of the friction force the piston will only move if there is additional pressure
\( \Delta P \) in the liquid, and we assume that the point \((P + \Delta P, T + \Delta T)\) is still on the coexistence line. The system will first absorb heat \( Q_1 \) from the bath while still liquid,

\[
Q_1 = C_1 \Delta T
\]

with change in entropy of the universe \( O(\Delta T^2) \). Then, the system undergoes the phase transformation at temperature \( T + \Delta T \) against pressure \( P + \Delta P \), absorbing heat \( L(T + \Delta T) \) from the heat bath. The heat transfer generates no entropy since it happens between the bath and the system at the same temperature. In the process of converting to gas the system does work \( P\Delta V \) and dissipates friction heat

\[
Q_{frict} = \Delta P \Delta V.
\]

We can think of this friction heat equivalently as going back to the heat bath, increasing the bath’s entropy, or as being used as part of the \( L(T + \Delta T) \), so that the heat bath supplies less heat hence decreases its entropy by less. Either way, the change in entropy of the universe to order \( \Delta T \)

\[
\Delta S_{univ} = \frac{Q_{frict}}{T} = \frac{L(T) \Delta T}{T}
\]

where we have used the Clausius-Clapeyron eq. (5). Eq. (8) is in agreement with Eq. (4).

The net heat transferred from the bath to the system in this process was

\[
Q = C_1 \Delta T + L(T + \Delta T) - \Delta P \Delta V
\]

which is the same as Eq. (3), since from Eq. (5), \( \Delta P \Delta V = (L(T)/T) \Delta T \), and from expanding \( L(T + \Delta T) \) and using Eq. (2) for \( dL/dT \),

\[
L(T + \Delta T) = L(T) + L(T)\frac{\Delta T}{T} + T(C_2 - C_1) \Delta T
\]

to first order in \( \Delta T \).

Alternatively, if there is negligible friction between the piston and the cylinder, the piston will accelerate upward and acquire kinetic energy

\[
K_{piston} = \Delta P \Delta V
\]

when all the liquid has evaporated, and will oscillate up and down, eventually dissipating this energy as heat which goes back to the heat bath, giving a change in the entropy of the bath (and the universe)

\[
\Delta S = \frac{K_{piston}}{T} = \frac{L(T) \Delta T}{T}
\]

in agreement with Eq. (4).

In this latter process assuming negligible friction between the piston and the cylinder, the increase in the entropy of the universe will approach zero as the temperature difference \( \Delta T \) goes to zero, in which case the transition will proceed increasingly slower: the speed of the piston goes as \( v \sim \sqrt{(2/M)K_{piston}} \), and as \( \Delta T \to 0 \), \( \Delta P \) and \( K_{piston} \to 0 \). The converse however is not true: the transition may proceed infinitely slowly and yet give rise to a finite increase in the entropy of the universe, as would be the case for finite friction force between the piston of the cylinder \( F_{friction} \) for the case where \( \Delta P \) is infinitesimally larger than \( F_{friction}/A \).

### IV. THE SUPERCONDUCTOR-NORMAL PHASE TRANSFORMATION

Consider a superconducting cylinder at temperature \( T \) in a magnetic field \( H = H_c(T) \), where \( H_c(T) \) is the thermodynamic critical field at temperature \( T \). When the temperature is slightly raised to \( T + \Delta T \) the system will become normal. In many ways this transition, shown in Figs. 3 and 4, is similar to the liquid-vapor transition shown in Fig. 2. In particular, the same thermodynamic laws should apply. The analogous of the Clausius-Clapeyron equation (5) is [23]

\[
\frac{dH_c}{dT} = \frac{L(T)}{T(M_s - M_n)}
\]
FIG. 3: Superconducting cylinder in magnetic field $H = H_c(T)$ becoming normal when $T$ is raised to $T + \Delta T$.

FIG. 4: Superconducting cylinder becoming normal, viewed from the top. The phase boundary moves in with velocity $\dot{r}_0(t)$. The changing magnetic flux generates a Faraday field $E_F$, giving rise to eddy current density $J_F = \sigma E_F$, with $\sigma$ the normal state conductivity, and Joule heat is dissipated.

where $M_n = 0$ and $M_s = -H_c/4\pi$ are the magnetization densities in the normal and superconducting states. Hence from Eq. (13)

$$\frac{L(T)}{T} = -\frac{H_c}{4\pi} \frac{dH_c}{dT}$$

and the change in entropy of the universe predicted by thermodynamics when the superconducting cylinder goes normal is, from Eqs. (4) and (14)

$$\Delta S_{univ} = -\frac{H_c}{4\pi} \frac{dH_c}{dT} \frac{\Delta T}{T}.$$  

Note that the superconductor-normal transition in a magnetic field cannot occur through nucleation of “bubbles” of the normal phase in the interior of the superconducting region, as other first-order phase transformations occur.
This is because in the interior there is no magnetic field and we are assuming that the temperature is well below $T_c$. The transition has to occur through the normal phase moving inward from the surface, as the region carrying the Meissner current becomes normal. This is clearly explained in the theoretical and experimental works of London [24], Pippard [25] and Faber [26]. In particular, Faber [26] argues theoretically and shows experimentally that in a cylindrical geometry the superconducting phase can collapse inwardly retaining near cylindrical symmetry at all times. For simplicity we will assume perfect cylindrical symmetry in this paper except when explicitly stated otherwise (Fig. 10).

What is the physical origin of the entropy increase Eq. (15)? When the system goes normal the magnetic field enters the body, and the changing magnetic flux generates eddy currents in the normal region that decay by dissipation of Joule heat. That is the analogous of the piston friction discussed in the previous section. Here we will show that this physics accounts for the entire entropy increase predicted by thermodynamics Eq. (15). Let us define

$$H_c \equiv H_c(T + \Delta T)$$

$$H_c(1 + p) \equiv H_c(T)$$

hence

$$p = -\frac{1}{H_c(T)} \frac{dH_c(T)}{dT} \Delta T$$

and substituting in Eq. (15)

$$\Delta S_{univ} = \frac{H_c^2}{4\pi T} p$$

Consider the conservation of energy equation that follows from Faraday’s law $\vec{\nabla} \times \vec{E} = (-1/c)\partial \vec{H}/\partial t$ and Ampere’s law $\vec{\nabla} \times \vec{H} = (4\pi/c)\vec{J}$:

$$\frac{d}{dt} \left( \frac{H^2}{8\pi} \right) = -\vec{J} \cdot \vec{E} - \frac{c}{4\pi} \vec{\nabla} \cdot (\vec{E} \times \vec{H}) .$$

The left side represents the change in energy of the electromagnetic field as the magnetic field $H_c(T)$ enters the body, the first term on the right side is the work done by the electromagnetic field in creating currents in this process, and the second term is the inflow of electromagnetic energy. Integrating over the volume of the body $V$ and over time we find for the change in electromagnetic energy per unit volume

$$\frac{1}{V} \int d^3r \int_0^\infty dt \frac{d}{dt} \left( \frac{H^2}{8\pi} \right) = \frac{H_c^2(1 + p)^2}{8\pi} .$$

since initially the magnetic field is completely excluded from the body. From Faraday’s law and assuming cylindrical symmetry we have for the electric field generated by the changing magnetic flux at the surface of the cylinder

$$\vec{E}(R, t) = -\frac{1}{2\pi Re \phi} \frac{d}{dt} \phi(t) \hat{\theta}$$

where $R$ is the radius of the cylinder and $\phi(t)$ is the magnetic flux through the cylinder, with $\phi(t = 0) = 0$, $\phi(t = \infty) = \pi R^2 H_c(1 + p)$. Integration of the second term on the right in Eq. (18), the energy inflow, over space and time, converting the volume integral to an integral over the surface of the cylinder, using that $H = H_c(1 + p)$ at the surface of the cylinder independent of time and Eq. (20) for the electric field at the surface yields

$$\frac{1}{V} \int_0^\infty dt \int (\vec{E} \times \vec{H}) \cdot dS = \frac{H_c^2(1 + p)^2}{4\pi} .$$

for the total electromagnetic energy flowing in through the surface of the sample during the transition.

The current $\vec{J}$ in Eq. (18) flows in the azimuthal direction and is given by the sum of superconducting and normal currents

$$J(r) = J_s(r) + J_n(r)$$
where $J_s(r)$ flows in the region $r \leq r_0(t)$ and is of appreciable magnitude only within $\lambda_L$ of the phase boundary, where $\lambda_L$ is the London penetration depth. $r_0(t)$ is the radius of the phase boundary (see Fig. 4) at time $t$. Integration of the second term in Eq. (18) over the superconducting current yields \cite{24, 27}

$$\frac{1}{V} \int d^3r \int_0^\infty dt (-\vec{J}_s \cdot \vec{E}) = -\frac{H_c^2}{8\pi}. \quad (23)$$

This is because the Faraday field accelerates the supercurrent \cite{27} until its kinetic energy density becomes the difference in free energies between the normal and the superconducting states, at which point the supercurrent stops \cite{24}.

The Joule heat per unit volume generated during the transition is

$$Q_J = \frac{1}{V} \int d^3r \int_0^\infty dt \vec{J}_n \cdot \vec{E} \quad (24)$$

hence from integrating Eq. (18) over space and time using Eqs. (19), (21), (22) and (23) we have

$$\frac{H_c^2(1 + p)^2}{8\pi} = -\frac{H_c^2}{8\pi} - Q_J + \frac{H_c^2(1 + p)^2}{4\pi} \quad (25)$$

which implies

$$Q_J = \frac{H_c^2}{4\pi} p \quad (26)$$

to linear order in $p$. The entropy generated from Joule heat is then

$$\Delta S_{\text{Joule}} = \frac{Q_J}{T} = \frac{H_c^2}{4\pi T} p \quad (27)$$

and comparing Eqs. (26) and (17)

$$\Delta S_{\text{univ}} = \Delta S_{\text{Joule}}. \quad (28)$$

We conclude that the change in entropy of the universe in the superconductor-normal phase transformation results solely from the Joule heat $Q_J$ Eq. (24) produced by normal current $J_n$ generated during the transition.

So far we have not made any assumption about the physical origin of the normal current $J_n$. The Faraday electric field created by the changing magnetic flux will generate a Faraday current

$$\vec{J}_F(r, t) = \sigma \vec{E}_F(r, t) \quad (29)$$

with

$$\sigma = \frac{n e^2 \tau}{m_e} \quad (30)$$

the normal state conductivity, with $n$ the normal state carrier density. Following the calculation in refs. \cite{25, 27} the Faraday electric field is given by

$$\vec{E}_F(r) = \frac{r_0}{cr} \dot{r}_0 \dot{H}_c \hat{\theta} \quad (31)$$

to lowest order in $p$, pointing in the azimuthal direction $\theta$ (counterclockwise). $\dot{r}_0(t)$ is the velocity of motion of the phase boundary. The energy per unit volume dissipated in the transition due to the Faraday current Eq. (29) is given by

$$W = \frac{1}{\pi R^2} \int_0^{t_0} dt \int_{r_0(t)}^{r_0} dr (2\pi r) J_F(r, t) E_F(r, t). \quad (32)$$

$t_0$ is the total time for the transition, given by (for small $p$) \cite{25, 27}

$$t_0 = \frac{\pi \sigma}{pc^2} R^2. \quad (33)$$
Eq. (32) yields using Eqs. (29) and (31)

\[
W = \frac{2\sigma H^2}{R^2 c^2} \int_0^{\ell_0} dt r_0^2 \ln \left( \frac{R}{r_0} \right) r_0^2 = \frac{2\sigma H^2}{R^2 c^2} \int_R^0 dr_0^2 \ln \left( \frac{R}{r_0} \right) r_0
\]

(34)

and using that \[25, 27\]

\[
r_0 \delta_0 \ln \left( \frac{R}{r_0} \right) = -\frac{p c^2}{4\pi \sigma}
\]

(35)
yields

\[
W = \frac{H^2}{4\pi} p = Q_J.
\]

(36)

Therefore, we conclude that the Joule heat Eq. (24) is produced by the Faraday current Eq. (29).

From Eq. (33) we learn that the increase in the entropy of the universe is inversely proportional to the time for the transition to take place. In terms of the average velocity of motion of the phase boundary \[v = R/t_0\] the increase in entropy Eq. (17) can be written, using Eq. (33), as

\[
\Delta S_{\text{Joule}} = \frac{H^2}{16\pi T} \left( \frac{R}{\lambda_L} \right) \left( \frac{\ell}{\lambda_L} \right) \left( \frac{n_s}{n_s} \right) \frac{v}{v_F}
\]

(37)

with \[v_F\] the Fermi velocity, \[\sigma\] given by Eq. (30), \[\ell = v_F\tau\] the mean free path, \[n_s\] the superfluid density, and the London penetration depth \[\lambda_L\] given by the usual expression \[3\]

\[
\frac{1}{\lambda_L} = \frac{4\pi n_s e^2}{m_e c^2}.
\]

(38)

In summary, we find that the increase in entropy predicted by thermodynamics for the S-N transition is completely accounted for by the entropy generated by the Joule heat resulting from the decay of the eddy currents driven by the Faraday electric field. Exactly the same analysis holds for the reverse transition from normal to superconducting state (Meissner effect). In the following sections we analyze the processes of momentum transfer from the supercurrent to the body and vice versa and whether or not they generate additional entropy.

V. MOMENTUM OF THE SUPERCURRENT

Consider the process where the superconductor goes normal. Initially a supercurrent circulates within a London penetration depth of the surface, \[\lambda_L\]. The supercurrent density when the magnetic field is \[H_c\] is

\[
J_s = n_s e v_s = \frac{n_s e^2 \lambda_L}{m_e} H_c
\]

(39a)

with \[v_s\] the superfluid velocity. Associated with it is a mechanical momentum density \[28\]

\[
\mathcal{P}_s = -\frac{m_e}{e} J_s
\]

(39b)

with \[m_e\] the bare electron mass. The total angular momentum of the supercurrent is

\[
L_e = \mathcal{P}_s R (2\pi R \lambda_L h) = -\frac{m_e c}{2e} R^2 h H_c
\]

(40)

where we have used Eq. (38) in the second equality. This mechanical momentum needs to be transferred to the body as a whole in a reversible way when the supercurrent stops.

Alternatively we can derive Eq. (40) as follows. The magnetization that nullifies the magnetic field in the interior is given by

\[
\vec{M} = -\frac{\vec{H}}{4\pi}.
\]

(41)
The associated magnetic moment is
\[ \vec{\mu} = M \pi R^2 \hbar = -\frac{R^2 \hbar}{4} \vec{H}_c. \]  
(42)

Using the relation between magnetic moment and orbital angular momentum \( \vec{L}_c \) of the electrons in the supercurrent
\[ \vec{\mu} = \frac{e}{2m_e c} \vec{L}_c \]  
(43)
we obtain
\[ \vec{L}_c = -\frac{m_e c}{2e} R^2 \hbar \vec{H}_c \]  
(44)
as in Eq. (40).

The kinetic energy associated with the initial supercurrent is
\[ K_c(R) \equiv K_c = \frac{m_e J_0^2}{n_e e^2} (2\pi R \lambda_L \hbar) = \frac{H^2}{8\pi} (2\pi R \lambda_L \hbar) \]  
(45)
where \( \hbar \) is the height of the cylinder. Note that the angular momentum of the supercurrent Eq. (44) is proportional to the volume of the system while its kinetic energy Eq. (45) is proportional to the surface area.

The transfer of the electronic angular momentum to the body also entails transferring some kinetic energy to the body, however the kinetic energy of the body associated with angular momentum Eq. (44) is only a tiny fraction of Eq. (45) because of the large body mass (of order \( (m_c/m_{ion})(\lambda_L/R) \)) and hence can be disregarded.

It may appear that the angular momentum Eq. (44) is all the angular momentum that needs to be transferred to the body when the system becomes normal. Then, it could be argued [29] that when the system becomes normal and finite resistivity sets in, the supercurrent that flows within \( \lambda_L \) of the surface of the body Eq. (39a) will decay by collisions with impurities and/or phonons, its angular momentum Eq. (44) would be transmitted to the body by those collisions making the body rotate, and its kinetic energy Eq. (45) would be dissipated as Joule heat. If so, the Joule heat dissipated per unit volume would be
\[ Q = \frac{K_c}{\pi R^2 \hbar} = \frac{H^2}{8\pi} \frac{2\lambda_L}{R} \]  
(46)
which is a small fraction of the Joule heat dissipated by eddy currents Eq. (26), negligible for macroscopic samples.

However, this is not so. The reason is that the transition cannot occur by the supercurrent near the surface simply stopping. Instead, the phase boundary moves inward as shown in Figs. 4 and 5. For simplicity we assume that the phase boundary has cylindrical symmetry at all times. As the supercurrent near the surface stops, supercurrent further inward is generated. As the magnetic field and the supercurrent move inward, the Faraday field transfers angular momentum to the body in clockwise direction, opposite to the angular momentum Eq. (44) that the body has to acquire. The total angular momentum transferred by the Faraday field to the body in the transition is
\[ \vec{L}_F = e \int_0^\infty dt n_s(2\pi r_0 \lambda_L \hbar) r_0' \times \vec{E}_F(r_0) = -\frac{R}{3\lambda_L} \vec{L}_c, \]  
(47)
which is much larger than \( L_c \). So e.g., for \( \lambda_L = 400 \text{ Å}, R = 1.2 \text{ cm} \), angular momentum \( (10^5+1)\vec{L}_c \) has to be transferred to the body through another mechanism for the body to end up with the correct angular momentum \( \vec{L}_b = \vec{L}_c \).

The total kinetic energy associated with the supercurrents generated during this process is
\[ K_{c,\text{tot}} = \int_0^R \frac{dr_0}{\lambda_L} K_c(r_0) = \frac{H^2}{8\pi} (\pi R^2 \hbar) \]  
(48)
which is the difference in free energies between the normal and superconducting states. This energy cannot be dissipated as Joule heat since the transition is reversible. The latent heat \( L \) associated with the superconductor-normal transition at temperature \( T \) is
\[ L = 4 \frac{(T/T_c)^2}{1 - (T/T_c)^2} K_{c,\text{tot}}. \]  
(49)
Experimentally it is found that less than 1% of the latent heat Eq. (49) is dissipated as Joule heat in the transition [7, 8]. The thermodynamic relations are found to hold even down to temperatures below \( 0.1 T_c \) [30], where the total kinetic energy of the supercurrent Eq. (48) is more than 20 times the latent heat. This confirms that the kinetic energy of the supercurrent is not dissipated when the supercurrent stops, rather it is stored in the electronic degrees of freedom, to be retrieved again in the reverse transition when the system goes superconducting and expels the magnetic field. The question then is: how is the electronic angular momentum Eq. (47), that has associated with it the kinetic energy Eq. (48), transferred to the body without any dissipation?
VI. THE TRANSITION ACCORDING TO THE CONVENTIONAL THEORY

Figure 5 shows schematically how the transition is envisioned to happen within the conventional theory of superconductivity [31, 32]. As the N-S phase boundary moves inward, Cooper pairs at the edge of the S-region dissociate and the resulting normal electrons inherit the center of mass momentum of the Cooper pair, without inheriting the kinetic energy of the Cooper pair. These normal electrons then scatter off impurities in a boundary layer of thickness $\ell=\text{mean free path}$ (hereafter called the “$\ell$-layer”) and in the process transfer this momentum to the body as a whole. This has to happen with no dissipation of energy and no increase in entropy. In the following we will discuss how it may or may not happen and experimental implications. We assume temperature is sufficiently low that impurity scattering dominates over phonon scattering.

Note the role of the Faraday field in Fig. 5. It accelerates the supercurrent in the S region within $\lambda_L$ of the boundary and it transfers momentum to the ions in that region in clockwise direction, opposite to the motion of the body. The total momentum that needs to be transferred to the body by impurity scattering is larger by a factor $R/(3\lambda_L)$ than the net momentum acquired by the body (Eq. (47)) because it has to compensate the momentum in opposite direction transferred by the Faraday field.

A slightly different scenario for the momentum transfer within the conventional theory is the following [3]. The supercurrent is carried by Cooper pairs, all having the same center of mass momentum $(q, 0, 0)$ parallel to the phase boundary. This is equivalent to a rigid shift of the ‘smeared’ Fermi distribution (smeared by the superconducting energy gap $\Delta$). In a free electron approximation, a member of a Cooper pair at the leading edge of the distribution has maximum kinetic energy $(\hbar^2/2m_e)(k_F + q/2)^2$, with $k_F$ the Fermi wavevector. If this electron scatters to the empty state $(-k_F + q/2, 0, 0)$ the energy decrease is $\hbar^2 k_F q/m_e$, and equating this to the energy cost to break a pair, $2\Delta$, gives the criterion for the critical momentum $k_c$, and the critical velocity within BCS theory [3]:

$$v_c = \frac{\hbar q_c}{2m_e} = \frac{\Delta}{\hbar k_F}.$$  

On the other hand, within London theory the supercurrent velocity when a magnetic field $H$ is applied is

$$v_s = \frac{e}{m_e c} \lambda_L H.$$  

FIG. 5: Superconducting cylinder becoming normal, as seen from the top. Magnetic field points out of the page. According to the conventional theory, as electrons in the supercurrent enter the normal region they scatter off impurities and transfer their momentum to the body as a whole, generating the body rotation in direction opposite to the force exerted by the Faraday field $E_F$ on the positive ions.
For $H = H_c$, Eqs. (50a) and (50b) are the same within a numerical factor of order 1 [3]. In this scenario the supercurrent stopping occurs through scattering of Cooper pair members (off impurities or defects or phonons) and in the process breaking up the Cooper pairs with energy gain exceeding $2\Delta$. These scattering processes will also transfer momentum from the supercurrent to the body. Both this scenario and the one discussed previously [31, 32] lead to the same contradictions and we will focus on the one described earlier, depicted in Fig. 5, in what follows.

VII. PHASE SPACE DISTRIBUTION

The mechanical momentum of a Cooper pair is $p_{\text{mech}} = 2m_e v_s = 2e\lambda c H_c \equiv \hbar q$. Consider a Cooper pair with electrons of wavevector $(-k_F + q/2, -k_y), (k_x + q/2, k_y)$. We assume the directions $x$ and $y$ are parallel and perpendicular to the phase boundary respectively. When the pair dissociates, the resulting electrons need to occupy states within $k_B T$ of the Fermi energy. Assuming the temperature is very low, the resulting electrons will be essentially on the Fermi surface. If we assume that momentum is conserved when the pair dissociates, there is essentially a single state on the Fermi surface into which the electrons can dissociate, namely $(q/2, \sqrt{k^2_F - q^2/4}), (q/2, -\sqrt{k^2_F - q^2/4})$, with $k_F$ the Fermi wavevector. It is clear that the transition cannot take place if all Cooper pairs have to dissociate into a single electronic state.

However, it is argued [32] that in this process momentum in the $y$ direction does not have to be conserved, rather the momentum difference in direction perpendicular to the phase boundary can be picked up by the superfluid [32]. If so, the resulting normal states for the electrons will be of the form $(-k_F' + q/2, k_y), (k_x' + q/2, k_y)$ with $(k_x' - q/2)^2 + k_y^2 = (k_x + q/2)^2 + k_y^2 = k_F^2$. Assuming these states are at the Fermi surface, the left panel in Figure 6 shows the region that they occupy. The grey sliver of thickness $\Delta k = (i\tau \ell /k_F$ accommodates the electrons originating from dissociation of Cooper pairs in the time interval $\tau$ assuming they lost their extra kinetic energy in the process. Note that $q >> \Delta k$. States in the arc to the left of the dashed vertical line on the left panel, that have $-k_F \leq k_x \leq -k_F + q$, are excluded, because their ‘partner’ with $k_x' = k_x + q$ would necessarily be outside the Fermi surface. Then, this anisotropic distribution would relax through impurity scattering to the equilibrium distribution shown on the right panel in Fig. 6, transferring the momentum to the body. Here for simplicity we are ignoring the effect of electron-electron interactions; their possible role will be discussed in a later section.

How does finite temperature affect this argument? The products of the dissociating Cooper pair can have energies within $k_B T$ of the Fermi energy. So their maximum wavevector in the $x$ direction is $k_x^{\text{max}} = k_F + \delta k$, with

$$
\delta k = \frac{T}{T_F} k_F
$$

with $T_F$ the Fermi temperature. On the other hand, the center of mass momentum of the Cooper pairs is

$$
q = 2 \frac{e\lambda}{h c} H_c \sim \frac{1}{2\lambda L}.
$$

With a typical $\lambda L \sim 500 \AA$, $k_F \sim 1 \AA^{-1}$, temperature $T \sim 1 K$ or below and $T_F \sim 50,000 K$, it is clear that $\delta k << q$, so we can disregard the effect of temperature in this argument.

It is obvious from Fig. 6 that the distribution on the left panel is more ‘ordered’ than that on the right panel, hence that impurity scattering generates entropy. Let us estimate this entropy increase quantitatively. For simplicity we
assume the system is at a very low temperature \( T_0 \). The solid angle corresponding to the excluded states in Fig. 6 is \( 2\pi q/k_F \). If \( \Delta N \) is the number of electrons resulting from dissociation in time \( \tau \), \( \Delta N_1 = (q/(2k_F)\Delta N \) is the number of single particle states to the left of the dashed line in Fig. 6 that are ‘forbidden’ by momentum conservation. The entropy increase in going from the left to the right panel in Fig. 6 is

\[
\Delta S = k_B\ln\Omega
\]

where \( \Omega \) is the number of ways to distribute \( \Delta N_1 \) indistinguishable particles in \( \Delta N \) boxes. This yields

\[
\Delta S = k_B\Delta N \frac{q}{2k_F} (\ln\left(\frac{2k_F}{q}\right) + 1).
\]

With \( q/(2k_F) \sim 1/1000 \),

\[
\Delta S \sim 0.01k_B\Delta N
\]

or 0.01\( k_B \) per electron that came from dissociation of a Cooper pair. This is an enormous increase in entropy.

The argument above assumes that the system is at a sufficiently low temperature \( T_0 \) that there are much fewer than \( \Delta N \) thermally excited electrons and holes within \( k_B T_0 \) of the Fermi energy. This requires \( k_B T_0 < \left(\dot{r}_0/\ell\right)\epsilon_F \), admittedly a very low temperature. Nevertheless, the important conclusion is that in this regime at least the entropy increase per electron is given by Eq. (54). This violates the principle that in a reversible thermodynamic process the increase in entropy should be arbitrarily small provided the transition occurs sufficiently slowly. Eq. (54) is incompatible with the superconductor-normal transition being reversible.

VIII. REVERSIBILITY AND THE MEISSNER TRANSITION

We next discuss how the conventional theory envisions the Meissner effect, the process where a normal metal in a magnetic field expels the magnetic field and becomes superconducting. Figure 7 shows the schematics, i.e. the inverse process to the one shown in Fig. 5.

In this case, the role of the Faraday field \( E_F \) in the S region close to the phase boundary is to \textit{decelerate} the electrons in the supercurrent as they move deeper into the S region (by way of the phase boundary moving outward), and in the same region (within \( \lambda_L \) of the phase boundary) impart counterclockwise momentum to the ions. The actual rotation...
of the body is clockwise, and the electrons joining the supercurrent acquire counterclockwise momentum in direction opposite to the force exerted on them by the Faraday electric field. How does the conventional theory explain these processes?

According to the conventional theory [31, 32] as normal electrons next to the phase boundary form Cooper pairs and join the supercurrent, they ‘spontaneously’ acquire the momentum of the supercurrent. To the best of this author’s knowledge the dynamics of this process is not explained further. As this process happens, a momentum imbalance is created in the normal region, i.e. the normal electron distribution is left with equal momentum in the opposite direction (dotted arrows in Fig. 7). The normal electron distribution then relaxes through impurity scattering transferring this momentum to the body as a whole, generating the body’s rotation.

However, we point out that the process just described, depicted in Fig. 7, is not the reverse of the process depicted in Fig. 5. Reversibility means reversing the arrow of time. In the S-N transition first the Cooper pair dissociates into normal electrons inheriting the momentum of the supercurrent. Then, these normal electrons lose their extra momentum by impurity scattering, over a time period \( \tau \). So the processes occur in sequence over a time period \( \tau \). Therefore one cannot say that the reverse process is that first electrons join the supercurrent and then normal electron scatter, as depicted in Fig. 7. Rather, the reverse process of Fig. 5 would be that first normal electrons in the \( \ell \)-layer incident on impurities from random directions would scatter preferentially with momentum in the direction of the momentum of the supercurrent, then they would form Cooper pairs and join the supercurrent. The first stage would correspond to reversing the process shown in Fig. 6, shown schematically in Fig. 8. Clearly that cannot physically happen because it would imply lowering the entropy of the universe.

This illustrates again that the physical processes invoked in the conventional theory to explain the S-N and N-S transitions [32] are not reversible.

IX. TEMPERATURE DEPENDENCE OF THE TRANSITION SPEED

Let us examine how the speed of the transition is affected by temperature within the conventional understanding of superconductivity. According to the conventional theory, the processes described in the previous section involve Andreev reflection [32]. In such processes when a normal electron is incident on the N-S phase boundary, it is reflected as a hole and a Cooper pair is created. In order for this to have a non-negligible probability to happen, both the incident electron and the reflected hole have to be within \( k_B T \) of the Fermi energy. This implies that at low temperatures only a small fraction of normal electrons can participate in this process, and this fraction will go to zero as \( T \) approaches zero. In other words, the theory predicts that for temperatures sufficiently close to zero the Meissner effect will not take place, or will take an arbitrarily long time.

Let us estimate the time involved. In the normal layer of thickness \( \ell \) next to the phase boundary (\( \ell \)-layer) a fraction \( k_B T / \epsilon_F \) of electrons can potentially undergo Andreev scattering at the interface and become superconducting. The process also involves relaxing the resulting momentum imbalance of the remaining normal electrons through impurity scattering, which takes time \( \tau \). Therefore at a minimum the time required to convert all the normal electrons in the \( \ell \)-layer to superconducting electrons carrying a current will be

\[
t_{\tau} = \frac{\tau \epsilon_F}{k_B T},
\]  

(56)
This implies that the speed of motion of the phase boundary cannot exceed
\[ \dot{r}_0^{\text{max}} = \frac{\ell}{t_r} = \frac{v_F k_B T}{\epsilon_F} \]
or equivalently
\[ \dot{r}_0^{\text{max}} = \frac{5.1}{\epsilon_F (eV)^{1/2}} T(mK) cm/s. \]

For example, for \( \epsilon_F = 10 eV \) and \( T = 1 mK \), \( \dot{r}_0^{\text{max}} = 1.6 cm/s \). For a cylinder of radius \( R = 1 cm \) at \( T = 10 \mu K \), the transition cannot occur in less than 160 seconds.

The speed of the transition is also limited by Faraday’s law, as discussed in Sect. III. As the temperature decreases the normal state conductivity \( \sigma \) increases and that implies that the time for the transition as described in sect. III will increase as given by Eq.(33). However at very low temperatures \( \sigma \) will reach a constant value and the speed of the transition as limited by Faraday’s law will reach a limiting value. Instead, the physics discussed in this section implies that the time for the transition will increase without bounds as the temperature is lowered further. Reversibility implies that the transition time for the S-N transition should also increase without bounds in this situation.

We know of no experimental evidence suggesting that the Meissner transition (and its reverse) will freeze out as the temperature becomes very low. By increasing the disorder in the sample we can decrease the time of the transition allowed by Faraday’s law Eq. (33) so that the physics discussed here will dominate. We suggest this should be probed experimentally. If it is found that the transition does not slow down significantly as the temperature is lowered it will call into question the validity of the scenario required by the conventional theory.

X. THE QUESTION OF EQUILIBRIUM

How does the conventional theory envision the equilibrium along the normal-superconductor phase boundary in the presence of a magnetic field? The problem was first considered by H. London in 1935 [24], however the issues discussed here were not raised.

According to the conventional theory, “in thermal equilibrium pair recombination and pair breaking processes occur all the time”, “there is a detailed balance of pair decay and pair creation” [33]. “The process and its reverse will be happening all the time in thermal equilibrium and there is clearly no generation of entropy in this situation”, “the process of incident electron being reflected as holes will be balanced by the process of incident holes being reflected as electrons” [32].

However, according to the discussion in Sect. VI, when Cooper pairs become normal their momentum distribution is not isotropic as depicted in Fig. 6, and becomes isotropic through impurity scattering, in the process generating entropy. In the reverse process involving Andreev reflection, relaxation of the normal electron momentum by impurity scattering also generates entropy. Therefore, contrary to the statements above [32, 33] we argue that the conventional scenario predicts that in thermodynamic equilibrium there is in fact continuous generation of entropy from these processes. Clearly this does not make sense and implies that the conventional theory is incompatible with a situation of thermodynamic equilibrium between normal and superconducting phases in a magnetic field. We believe that ample experimental evidence exists that such equilibrium does exist in nature without continuous generation of entropy.

XI. THE ROLE OF ELECTRON-ELECTRON INTERACTION

In the foregoing we have assumed for simplicity that electrons don’t interact with each other. In fact they do. How will this modify the arguments presented earlier?

Electron-electron scattering will tend to make the anisotropic distribution in Fig. 6 isotropic but not relax the momentum, since it conserves total momentum. In the absence of impurity scattering the momentum distribution would change as shown in Fig. 9, giving rise to a slightly shifted isotropic Fermi distribution. In time \( \tau \) the phase boundary moves a distance \( \dot{r}_0 \tau \). Once the distribution becomes isotropic the electrons that became normal in time \( \tau \) will have shared their extra momentum with all the electrons in the \( \ell \)-layer, resulting in a small shift \( \delta q \) in the Fermi surface as shown in the right panel of Fig. 9, given by \( \delta q = (q/2) \dot{r}_0 \tau / \ell = (q/2) \dot{r}_0 / v_F \). Then, this slightly shifted Fermi sea will relax by impurity scattering transferring its momentum to the lattice.

The time scale for electron scattering is \( \tau_{ee} \sim \hbar \epsilon_F / (k_B T)^2 \), which at low temperatures is usually larger than the impurity scattering time \( \tau \), so impurity scattering will dominate the relaxation. Nevertheless, let us assume a very clean sample where a combination of electron-electron scattering and a small amount of impurity scattering will first
FIG. 9: If electron-electron scattering dominates, the distribution will become isotropic conserving total momentum, so the Fermi surface ends up shifted slightly to the right by an amount $\delta q = (\dot{r}_0 \tau / \ell) q/2$ (dashed line).

randomize the momenta as shown in Fig. 9 conserving total momentum, and subsequently impurity scattering will transfer the momentum to the body as a whole. The first process will generate entropy as given by Eq. (54), and the second process will generate additional entropy, as discussed in what follows.

XII. THE CONVENTIONAL ARGUMENT

The conventional argument regarding entropy generation in the process of transferring momentum goes as follows [32]. The extra momentum carried by each dissociating electron $q/2$ is shared by the entire Fermi sea in the $\ell$-layer, giving a very small momentum shift

$$\delta q = (q/2) \dot{r}_0 \tau / \ell = (q/2) \dot{r}_0 / v_F$$

(59)

to the entire Fermi sea. Some entropy will be generated as the momentum is transferred to the body through impurity scattering, but it is argued that it is quantitatively small, goes to zero as the speed of the transition goes to zero, and may not be inconsistent with observations. Let us analyze it quantitatively.

The mechanical momentum density in the $\ell$-layer under this assumption will be

$$P_q = n_s \dot{r}_0 \tau / 2 \ell = n_s \dot{r}_0 \frac{h q}{v_F} = n_s \frac{c \lambda_L}{v_F} \frac{e \lambda_L}{c} H_c$$

(60)

that was acquired from the $(n_s/2) \dot{r}_0 \tau (2\pi r_0 h)$ Cooper pairs of center of mass momentum $h q$ that dissociated in time $\tau$. The mechanical momentum density Eq. (60) is equivalent to an electric current density $J_q$ given by [28]

$$J_q = \frac{e}{m_e} P_q = \frac{e}{m_e} \frac{n_s \dot{r}_0 \tau}{2 \ell} h q = \frac{n_s e^2}{m_e} \frac{\dot{r}_0 \lambda_L}{v_F} H_c.$$  

(61)

When this current decays by collisions with impurities, its mechanical momentum is transferred to the body as a whole. The momentum density Eq. (60) has associated with it a kinetic energy density

$$K_q = \frac{P_q^2}{2 m_e n_s} = \frac{m_e}{2 e^2 n_s} J_q^2 = \frac{H_c^2}{8\pi} \left( \dot{r}_0 / v_F \right)^2$$

(62)

and the decay of this current will generate entropy per unit volume $K_q/T$.

On the other hand the Faraday electric field Eq. (31) that exists in this region also distorts slightly the Fermi distribution, giving rise to a normal current

$$J_F = \sigma E_F = \frac{n e^2}{m_e} \frac{\dot{r}_0}{v_F} c = \frac{\ell}{\lambda_L} n_s J_q$$

(63)

where we have assumed Eq. (30) for the normal state conductivity. We will assume $n = n_s$ for simplicity (in general $n$ can be larger than $n_s$). Note that the currents $J_q$ and $J_F$ are of similar form and similar magnitude for the case $\ell \sim \lambda_L$, $n \sim n_s$, and they both become vanishingly small as the speed of motion of the phase boundary $\dot{r}_0$ becomes vanishingly small. The kinetic energy density associated with $J_F$ will be similar to $K_q$. However, a difference between $J_q$ and $J_F$ is that $J_F$ flows over the entire normal region $r_0(t) \leq r \leq R$ while $J_q$ only flows in the region $r_0(t) \leq r \leq r_0(t) + \ell$. As a consequence, the total entropy generated by the momentum-transfer current $J_q$ will be much smaller than the
entropy generated by the Faraday current $J_F$. Note also that the momentum transferred from $J_F$ to the body by collisions is cancelled by the opposite momentum transferred directly by $E_F$ to the ions.

In the layer of thickness $\ell$ adjacent to the phase boundary the total current density is then

$$J_{\text{tot}} = J_\ell + J_F.$$  

(64)

We have already considered the effect of $J_F$ in producing entropy in section IV, so we need to subtract its contribution here to avoid double counting. The net additional contribution of the current density $J_{\text{tot}}$ is then a kinetic energy density

$$\mathcal{K}_{\text{excess}} = \frac{m_e}{2e^2 n_s} (J_{\text{tot}}^2 - J_F^2) = \frac{H^2_c}{8\pi} \left( \frac{\ell}{v_F} \right)^2 (1 + \frac{2\ell}{\lambda_L})$$  

(65)

where we have used Eq. (38). This excess kinetic energy resides in the region $r_0 \leq r \leq r_0 + \ell$ and were generated by motion of the phase boundary in time $\tau$ and associated conversion of the supercurrent into normal current. As this excess normal current decays through normal scattering processes, the entropy per unit volume generated for the cylinder of radius $R$ in the process of becoming normal due to this physics will be, from Eq. (65)

$$\Delta S_{e} = \frac{H^2_c}{8\pi T} \frac{1}{\pi R^2} \tau (1 + \frac{2\ell}{\lambda_L}) \int_0^{r_0} dt \left( \frac{\ell}{v_F} \right)^2 (2\pi r_0 \ell)$$  

(66)

and changing integration variables from $t$ to $r_0$ and using Eq. (35)

$$\Delta S_{e} = \frac{H^2_c}{4\pi T} p \frac{\lambda_L}{\ell} \frac{\lambda_L}{R} (1 + \frac{2\ell}{\lambda_L}) \int_0^{1-\delta} dx \frac{1}{\ln(\frac{1}{\tau})}.$$  

(67)

In the upper limit of the integral Eq. (67) we have included a small cutoff parameter $\delta > 0$ to render the integral non-divergent. The weak divergence for $\delta \to 0$ originates in the fact that the speed of the phase boundary $\dot{r}_0$, Eq. (35), diverges as $r_0 \to R$. This is of course unphysical and stems from the approximation inherent in Eq. (35) that the only factor limiting the speed of the phase boundary is Faraday’s law [25]. In reality, as the speed becomes large other factors will set in to limit $\dot{r}_0$, e.g. finite thermal conductivity. This has been discussed by Faber [20], who also showed experimentally that Eq. (35) for $\dot{r}_0$ is accurately satisfied over the entire range of $r_0$ except for $(R - r_0)/R \lesssim 0.05$ where it is cut off.

Thus, the integral in Eq. (67) is of order unity, and we have approximately

$$\Delta S_{e} = \frac{H^2_c}{4\pi T} \frac{\lambda_L}{\ell} \frac{\lambda_L}{R} (1 + \frac{2\ell}{\lambda_L}).$$  

(68)

This is smaller than the increase in entropy per unit volume due to Joule heat generation from eddy currents

$$\Delta S_{\text{Joule}} = \frac{H^2_c}{4\pi T} p$$  

(69)

by a factor $\sim (\lambda_L/R)$ if we assume $\lambda_L \sim \ell$.

It should be noted that there is an important difference between the entropy contributions Eqs. (68) and (69). The entropy contribution from eddy currents Eq. (69) does not depend on the particular way the sample went normal, since eddy currents are generated everywhere in the normal parts of the sample. Assume that instead of a process where the boundary moves uniformly inward as in Fig. 4, the sample goes normal through a kind of ‘intermediate state’ where the magnetic field breaks up the superconducting regions into domains that evolve separately, as shown in Fig. 10. Each superconducting region in Fig. 10 has a boundary supercurrent density given by Eq. (39). If the regions shown in Fig. 10 have initial radius $R_0$, the change in entropy per unit volume due to the supercurrent stopping is given by

$$\Delta S_{e} = \frac{H^2_c}{4\pi T} p \frac{\lambda_L}{\ell} \frac{\lambda_L}{R_0} (1 + \frac{2\ell}{\lambda_L})$$  

(70)

instead of by Eq. (68). Instead, the entropy contribution from eddy currents is still given by Eq. (69) that does not depend on the size of the domains. Thus, the extra entropy generation from the supercurrent stopping will be much larger if the transition proceeds through the route shown schematically in Fig. 10 and $R_0$ is small. This is because there is a lot more supercurrent in the situation shown in Fig. 10 than there is in the case shown in Fig. 4.

Nevertheless, it is argued that the increase in entropy obtained here is sufficiently small that it can be ignored, or perhaps that it is cancelled by other small effects that have been neglected [32]. However, in the next section we will argue that the Joule heat and resulting entropy generated by the processes described here is in fact much larger.
FIG. 10: Superconducting cylinder becoming normal through a route where it first breaks up into many superconducting domains of initial radius $R_0$.

XIII. THE FLAW IN THE CONVENTIONAL ARGUMENT

Here we will show that the Joule heat generated by the stopping of the normal current is in fact much larger than calculated in the previous section. In addition to causing problems with the second law of thermodynamics, this violates either the first law of thermodynamics or the essential assumption of the conventional theory of superconductivity that the kinetic energy of the supercurrent equals the difference in free energies in the normal and superconducting states.

The point we wish to make is that the kinetic energy associated with the normal current that originates in the decay of the Cooper pairs is in fact much larger than Eq. (62). The reason is, as discussed in Sect. VII, that a fraction $q/(2k_F)$ of the states available to the dissociating electrons (left panel of Fig. 6) are 'blocked' by momentum conservation, because if one electron of the dissociating pair occupies those states the other electron would have energy higher than the Fermi energy. Quantitatively, this implies that the Fermi level in the $\ell$–layer gets shifted to

$$\bar{k}_F = k_F \left(1 + \frac{1}{3} \frac{q}{2k_F} \frac{\dot{r}_0 \tau}{\ell} \right)$$

as Cooper pairs dissociate into normal electrons. This gives an extra kinetic energy per particle

$$\Delta \epsilon_{\text{kin}} = \frac{\hbar^2}{6m_e} k_F q \frac{\dot{r}_0 \tau}{\ell}$$

for the electrons coming from dissociating Cooper pairs. To obtain the extra kinetic energy in the $\ell$–layer we multiply Eq. (72) by $n_s \dot{r}_0 \tau / \ell$ and obtain

$$\bar{K}_q = \frac{H^2}{8\pi} \left(\frac{\dot{r}_0}{v_F}\right)^2 \frac{4k_F}{3q} = K_q \left(\frac{4k_F}{3q}\right)$$

which is much larger than the earlier result Eq. (62).

The argument just presented is approximate because it assumes that the density of states at the Fermi surface for the dissociating electrons is constant, and this is not the case for $q \neq 0$. Nevertheless, it is a good approximation. We have verified this by numerical calculations in both two- and three-dimensional geometries. In a 2-dimensional geometry, Eq. (72) is modified as

$$\Delta \epsilon_{\text{kin}} = \frac{\hbar^2}{2m_e \pi} \cos^{-1}(1 - \frac{q}{k_F}) \frac{\dot{r}_0 \tau}{\ell} \sim \frac{\hbar^2}{2m_e \pi} \sqrt{\frac{2q \dot{r}_0 \tau}{k_F} / \ell}$$

As an example, Fig. 11 shows numerical results for 2 dimensions, $q/k_F = 0.3$, $\dot{r}_0 \tau / \ell = 0.16$. For these extreme parameters the analytic formula Eq. (74) is not very accurate, since it assumes the distribution is uniform except for the 'hole' in the back end, which clearly is not the case for the left panel of Fig. 11. The ratio of kinetic energies
in the left and right panels of Fig. 11 from the numerical calculation is 8.9 while the analytic formulas Eqs. (74) and (59) yield 17.9 in this case. As somewhat more realistic examples, in 3 dimensions with \( \dot{\tau}/\ell = 0.035 \), with \( q/k_F = 0.2 \) and \( q/k_F = 0.1 \) the ratios of kinetic energies from the numerical calculations are 16.3 and 5, while the analytic formulas Eq. (72) and (59) predict 13.3 and 6.67, i.e. \( 4k_F/3q \). In a realistic situation \( 4k_F/3q \sim 1000 \) or larger.

Returning to Eq. (73), we conclude then that the dissipation of Joule heat from this kinetic energy will far exceed the Joule heat dissipated due to the Faraday field in the \( \ell \)-layer. Ignoring the Faraday field in the \( \ell \)-layer we then obtain by time integration following the steps after Eq. (65) and using \( q = 1/(2\lambda_L) \)

\[
\tilde{K}_q^{\text{total}} = \frac{H^2}{8\pi} \frac{16}{3} \frac{\lambda_L \lambda^2}{\ell} \frac{k_F}{R}
\]

(75)

for the Joule heat per unit volume in addition to the Joule heat Eq. (26) dissipated due to the Faraday field.

The fact that the S-N transition is governed by thermodynamics requires that when the supercurrent stops its entire kinetic energy is used up in paying for the free energy cost of becoming normal, \( H^2/8\pi \) per unit volume. This follows from the fact that the kinetic energy associated with the superfluid current Eq. (39) is

\[
K_s = \frac{m_e}{2e^2n_s} J_0^2 = \frac{m_e}{2e^2n_s} \left( \frac{n_s e^2 \lambda L}{m_e c} H_c \right)^2 = \frac{H^2}{8\pi}
\]

(76)

where we have used Eq. (38). If this was not the case, there could not be phase equilibrium between superconducting and normal phases in the presence of a magnetic field [24]. This argument indicates that in fact the kinetic energy \( K_q^{\text{total}} \) has to be exactly zero in order to not violate energy conservation.

Instead, Eq. (75) is quantitatively very significant. For example, for \( p = 0.1875 \), \( k_F = 1\text{Å}^{-1} \) and \( \lambda_L = 500\text{Å} \), it says that for \( R = 0.1\text{mm} \) this extra Joule heat is 1/4 of the total kinetic energy of the supercurrent. This will also be the case for a larger sample with domains of radius \( R_0 = 0.1\text{mm} \) as shown in Fig. 10. This violates energy conservation because there is no source for this energy. It also violates the second law, since it predicts an entropy generation \( K_q^{\text{total}}/T \) which is comparable to the entropy generated by eddy currents Eq. (27) which we have shown in Sect. IV accounts for the entire entropy generated in the transition according to thermodynamics.

**XIV. Recapitulation and Consistency Check**

Let us rederive the results of the previous section in a simpler way to check their consistency.

The kinetic energy of the supercurrent amounts to each electron having an extra kinetic energy

\[
\epsilon_s = \frac{\hbar^2 q^2}{8m_e}.
\]

All that energy should be paid to dissociate a Cooper pair according to Eq. (76). However, it is impossible that electrons becoming normal keep the momentum of the supercurrent but none of its kinetic energy. The conventional
FIG. 12: Direct and reverse stages in the transition process taking into account the effect of electron-electron interactions. It is clear that these processes cannot occur from right to left.

argument [32] discussed in Sect. XII says that each unpairing electron shares its momentum with all the electrons in the \( \ell \)-layer, and to do that it only needs a tiny amount of kinetic energy, namely

\[
\epsilon_{\text{conv}} = \frac{\hbar^2 q^2}{8 m_e} \left( \frac{\tilde{r}_0 \tau}{\ell} \right) = \epsilon_s \left( \frac{\tilde{r}_0 \tau}{\ell} \right)^2
\]

so the difference between Eqs. (77) and (78) is available to unpair the Cooper pair. This already violates conservation of energy, albeit by a small amount.

However, we have shown in the previous section that in fact Eq. (78) is incorrect. Because momentum in direction parallel to the interface has to be conserved when a Cooper pair dissociates, the kinetic energy of each dissociating electron is

\[
\epsilon_{\text{correct}} = \frac{\hbar^2 q^2}{8 m_e} \left( \frac{4 k_F}{3} q \right) \left( \frac{\tilde{r}_0 \tau}{\ell} \right)^2 = \left( \frac{4}{3} \frac{k_F}{q} \right) \epsilon_{\text{conv}}
\]

which is substantially larger than \( \epsilon_{\text{conv}} \).

From comparing Eqs. (79) and (77) we estimate that the total Joule heat dissipated per unit volume arising from the kinetic energy Eq. (79) will be

\[
Q_{\text{kin}} = \frac{H^2}{8 \pi} \left( \frac{4 k_F}{3} q \right) \left( \frac{\tilde{r}_0 \tau}{\ell} \right)
\]

where \( \tilde{r}_0 \) is the average speed of the phase boundary, which we obtain from

\[
\tilde{r}_0 = \frac{R}{t_0} = \frac{4 \lambda^2}{R \tau} p
\]

where we have used Eqs. (30), (33) and (38). Replacing in Eq. (80) yields

\[
Q_{\text{kin}} = \frac{H^2}{8 \pi} \frac{32}{3} \frac{\lambda_F^2 k_F}{\ell}\frac{R}{R^2}
\]

which differs by a factor of 2 from our earlier result Eq. (75). The discrepancy arises because here we have used an average for the phase boundary speed rather than using the exact expression Eq. (35) and doing the time integration as in Sect. XII. Nevertheless this confirms the consistency of our approach and in particular the validity of introducing a cutoff in the integral in Eq. (67).

XV. REVERSIBILITY AGAIN

Let us consider again the question of reversibility including the role of electron-electron interactions, under the assumption that electron-electron scattering occurs faster than electron-impurity scattering. The processes at play are shown in Fig. 12. If the transition is truly reversible, these processes should be able to happen in both directions shown by the arrows. So starting from the rightmost panel, impurity scattering processes in an isotropic Fermi sea should give rise to an isotropic distribution that is slightly shifted to the right (middle panel), then electron-electron scattering processes should distort this distribution to the anisotropic form shown on the left panel, and finally electrons from this anisotropic distribution should condense into the supercurrent-carrying Cooper pairs. It is obvious that these reverse physical processes are not physically possible because they lower the entropy of the universe and hence do not take place in nature.
We have analyzed the momentum transfer between the supercurrent and the body and the production of entropy when a superconductor in a magnetic field goes normal and in the reverse process within the conventional theory of superconductivity. For comparison, we have also discussed the somewhat analogous process of a liquid evaporating.

For the liquid-gas transition, our understanding suggests no conflicts with known physical laws. Entropy production can occur in a variety of ways: transfer of heat across a temperature gradient, or friction of the piston with the cylinder walls, or internal friction in the gas phase. The entropy that these processes generate is consistent with the laws of thermodynamics. For the total entropy generated to be infinitesimal, the process has to occur infinitely slowly, in which case it becomes exactly reversible, i.e. no change in the entropy of the universe. The reverse is however not true, the process can occur infinitely slowly and still give rise to finite entropy generation.

In the case of the superconductor-normal transition as described by the conventional theory of superconductivity [3, 31, 32], we identified two separate sources for entropy generation: Joule heating due to eddy current decay, generated by the changing magnetic flux, and entropy generation in the process of the supercurrent stopping and transferring its mechanical momentum first to normal electrons and from there to the body, and vice versa.

We showed quantitatively that Joule heating from eddy currents accounts for the entire entropy generated in the transition according to the laws of thermodynamics. This then implies that zero entropy can be generated by the process of the supercurrent stopping and transferring its momentum to the body. However, we analyzed the process of momentum transfer from the supercurrent to the body following the prescriptions of the conventional theory of superconductivity [31, 32] and found that entropy is necessarily generated in this process. We pointed out that there are two different processes that generate entropy. One is the randomization of the anisotropic momentum distribution (Figs. 6, 11) that results from the pair dissociation. This entropy generation is non-zero even in the limit where the transition occurs infinitely slowly. The second is the dissipation of the extra kinetic energy of the distributions (Figs. 6, 11) as Joule heat, as the momentum is transferred to the body. This can be quantitatively comparable to the Joule heat generated by eddy currents and violates both conservation of energy and the second law of thermodynamics.

We have also argued that according to the conventional theory entropy would be continuously generated in a situation of thermodynamic equilibrium between the normal and superconducting phases in the presence of a magnetic field, where pair recombination and pair breaking processes continuously occur, as the resulting transient anisotropic normal electron momentum distribution resulting from these processes (Figs. 6, 11) is randomized by scattering processes. Clearly this does not make physical sense, since in a system in thermodynamic equilibrium no entropy is generated.

It is understandable that no entropy can be generated in the process of the supercurrent stopping for the following reason: initially the current and its associated mechanical momentum is carried by the condensate, all pairs having the same center of mass momentum. That is a single state, carrying no entropy. In the final state, the mechanical momentum is carried by the body as a whole, also a single state carrying no entropy. This implies that there can be no intermediate stage in the process of transferring the mechanical momentum from the supercurrent to the body that does carry entropy, because that entropy cannot be destroyed again to reach the final state. This was already recognized long ago by Keesom [34], who wrote "it is essential that the persistent currents have been annihilated before the material gets resistance, so that no Joule-heat is developed. Within the conventional theory of superconductivity there is no mechanism to transfer the mechanical momentum of the supercurrent directly to the body as a whole without involving the normal electrons.

The unphysicality of the conventional description is particularly striking for the case where the superconducting region is expanding instead of contracting, i.e. the Meissner effect, as discussed in Sect. VIII, where the normal state Fermi sea in the layer of thickness $\ell$ adjacent to the boundary would have to arrange itself, through collisions between electrons and impurities, into the anisotropic momentum distribution shown in the left panel of Fig. 8, to then transfer it to the superconducting region as normal particles bind to form Cooper pairs. There certainly is no physical way to justify this process which necessitates a spontaneous lowering of the entropy of the universe.

In the next section we discuss how the theory of hole superconductivity [12] resolves these issues.

**XVII. EXPLANATION WITHIN THE THEORY OF HOLE SUPERCONDUCTIVITY**

Here we discuss briefly how the theory of hole superconductivity resolves these issues. For more details the reader is referred to references [10, 11, 35] and references therein.

There are two questions that need to be answered. (1) how do electrons acquire or lose the momentum of the supercurrent, and (2) how does the body acquire or loses its momentum. The essence of the answer is that the momentum transfers are mediated by the electromagnetic field rather than by scattering processes, as explained in ref. [11].
First, how do electrons acquire or lose their momentum reversibly? This results from expansion or contraction of electronic orbits, as shown schematically in Fig. 13. The theory of hole superconductivity predicts that when electrons become superconducting their orbits expand from a microscopic radius to radius $2\lambda_L$, driven by lowering of kinetic energy. When an orbit expands from a microscopic radius to radius $r$ in the presence of a magnetic field, the magnetic Lorentz force on the outgoing electron imparts an azimuthal speed

$$v_\phi = -\frac{er}{2m_ec}H,$$

thus when the orbit expands to radius $r = 2\lambda_L$ the azimuthal speed acquired is the speed of the Meissner current Eq. (50b). Similarly when the orbit shrinks from radius $2\lambda_L$ to a microscopic radius the azimuthal Lorentz force acts in the opposite direction and the supercurrent stops.

Why $2\lambda_L$ orbits? The supercurrent flowing in a surface layer of thickness $\lambda_L$ results from superposition of $2\lambda_L$ orbits in the bulk, and electrons in such orbit carry orbital angular momentum $\hbar/2$.

The process just described does not involve any scattering and hence is reversible. The reversible expansion or contraction of the orbits explains how the supercurrent starts and stops in a given external magnetic field.

Second, how does the body as a whole acquire compensating momentum in the opposite direction? In the process just described, momentum conservation holds because as the orbit expands or contracts and the azimuthal momentum of the electron changes, a compensating azimuthal momentum is stored in the electromagnetic field, as explained in [11]. That momentum is retrieved by a radial flow of normal charge to compensate for the radial charge redistribution that occurs when the electronic orbit expands or contracts. Here is where the hole-like nature of the normal state charge carriers plays a key role.

Figure 14 explains the physics of the processes for the S-N and N-S transitions. In the left panel (S-N transition) the superconducting region is shrinking and $2\lambda_L$ orbits at the boundary of the N-S region are contracting, which imparts the electrons with a clockwise azimuthal impulse, stopping the supercurrent. At the same time, to compensate for the radial inflow of negative charge, a backflow of normal charge occurs: normal state hole carriers flow radially inward at speed $\dot{r}_0$, the speed of motion of the phase boundary. The azimuthal forces on this backflow of inflowing hole carriers are $F_H$ and $F_E$ shown in Fig. 14. $F_H$ is the magnetic Lorentz force and $F_E$ is the electric force due to the Faraday field $E_F$, which is given according to Faraday’s law by

$$E_F = \frac{\dot{r}_0}{c}H.$$ 

Thus, $eE_F$ is precisely the magnetic Lorentz force on the hole carriers in opposite direction to the electric force, the azimuthal forces on the holes exactly cancel out and the hole motion is radial. Similarly the right panel shows the process for the N-S transition (Meissner effect). Here the orbits expand as the phase boundary moves out, propelling the supercurrent, and the backflow consists of holes moving radially outward together with the phase boundary. Again, the azimuthal electric and magnetic forces on the holes are balanced.

What we have just described explains the body’s rotation. The radial inflow or outflow of hole carriers creates a torque that makes the body rotate, imparting the same angular momentum gained or lost by the supercurrent in opposite direction, as shown quantitatively in ref. [11]. It is instructive to consider the normal backflow in terms of electrons rather than holes, as shown in Figure 15. Note that now the electric and magnetic forces on these electrons
are not balanced, rather they point in the same direction. To recover force balance we need to add another force \( F_{\text{latt}} \), which is a transverse force that the periodic ionic potential exerts on the charge carriers moving in crossed electric and magnetic field when the charge carriers are holes. By Newton’s third law, there is an equal opposite force exerted by the electrons on the ions, which we denote by \( F_{\text{on-latt}} \). This force acts in the direction of the body rotation, it is in fact the force that makes the body rotate. The transfer of momentum to and from the body is not mediated by impurity scattering. Rather, it is mediated by the coherent interaction of electrons near the top of energy bands, that have a negative effective mass, with the periodic potential of the ions [35].

In summary, the processes just described qualitatively and analyzed quantitatively in the references [11, 35] explain how momentum is transferred between electrons and the body as a whole in a reversible way, without any scattering processes and Joule heating that would create entropy in contradiction with theory and experiment.

XVIII. CLOSING ARGUMENTS

In this paper we have made the simplifying assumption that a sharp boundary between superconducting and normal regions exists, given by \( r_0(t) \), with no supercurrent in the normal region. In other descriptions the superconducting order parameter and the supercurrent may decay to zero over a finite distance. However there is no indication for why in such more complicated descriptions a mechanism would exist that would allow transfer of momentum from the supercurrent to the body without entropy generation and respecting energy conservation, as required by the laws of thermodynamics, within the framework of the conventional theory of superconductivity.

It is important to emphasize that the conclusions reached in this paper about the conventional theory in no way depend on the limitations of BCS theory that relies on weak coupling and long distance averaging. The reason that we are finding a violation of thermodynamics laws within the description given by conventional theory is that conventional theory offers no way to transfer momentum directly from the supercurrent to the body, rather requires an intermediate step where the azimuthal momentum is carried by normal electrons [31], and part of the kinetic energy of the supercurrent is carried by normal electrons. Any theory of superconductivity with that requirement will lead to the same violation. The thermodynamic relations teach us that a correct description of superconductivity in nature cannot involve this intermediate step.
FIG. 15: Figure 14 redrawn replacing the backflowing holes by electrons flowing in the opposite direction. The electric and magnetic forces on electrons $F_E$ and $F_H$ point in the same direction. Since the motion is radial, this implies that another force must exist, $F_{\text{latt}}$, exerted by the periodic potential of the ions on the charge carriers. By Newton’s third law, an equal and opposite force is exerted by the charge carriers on the ions, $F_{\text{on-latt}}$, that makes the body rotate.

In conclusion, in this paper we have argued that the description of the superconductor-to-normal transition in the presence of a magnetic field and its reverse (Meissner effect) provided by the conventional theory of superconductivity [31–33], which involves transfer of mechanical momentum between the supercurrent and the body through an intermediate state involving normal electrons carrying this momentum, is inconsistent with the laws of thermodynamics. We conclude that the conventional theory of superconductivity, at least in its present form, cannot provide a consistent description of the S-N and N-S transitions in a magnetic field exhibited by all type I superconductors [39]. Instead, the alternative theory of hole superconductivity [12] can, as we briefly discussed in Sect. XVII and more extensively in references [10, 11, 35].

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