Dynamical Constraints in the Nonsymmetric Gravitational Theory

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Abstract

We impose in the nonsymmetric gravitational theory, by means of Lagrange multiplier fields in the action, a set of covariant constraints on the antisymmetric tensor field. The canonical Hamiltonian constraints in the weak field approximation for the antisymmetric sector yield a Hamiltonian energy bounded from below. An analysis of the Cauchy evolution, in terms of an expansion of the antisymmetric sector about a symmetric Einstein background, shows that arbitrarily small antisymmetric Cauchy data can lead to smooth evolution.

1 Introduction

Recently, a modified version of the nonsymmetric gravitational theory (NGT) field equations has been published \[1, 2, 3, 4, 5\], which leads to a linear approximation for weak fields that incorporates Einstein’s linear field equations and the massive Kalb-Ramond field equations for the antisymmetric field \(g_{[\mu\nu]}\) \([6, 7, 8]\). These equations do not have any ghost poles, tachyons or unphysical asymptotic behaviour, removing the inconsistencies discovered by Damour, Deser and McCarthy, in an earlier version of NGT \([9]\). Clayton \([10, 11, 12]\) has developed a Hamiltonian formalism for NGT that reveals a local instability of the NGT field equations associated with three of the six possible propagating degrees of freedom in the skew sector. Thus, the three components \(g_{[0]} (i, j = 1, 2, 3)\) of the six components \(g_{[\mu\nu]}\) become unphysical propagating degrees of freedom. Since NGT does not possess a rigorous gauge invariance associated with the skew sector of the form:

\[ g_{[\mu\nu]} \rightarrow g_{[\mu\nu]} + \xi_{\mu,\nu} - \xi_{\nu,\mu}, \]

where \(\xi_{\mu}\) is an arbitrary vector field, we must impose dynamical constraints on the action through covariant Lagrangian constraint equations, which guarantee that the antisymmetric transverse-longitudinal degrees of freedom vanish throughout the evolution of the dynamical NGT field equations. We shall show in the following how this can be implemented and how the new field degrees of freedom associated
with the Lagrange multiplier fields produce consistent field equations in the linear approximation and a Hamiltonian bounded from below.

We also show that the Cauchy instability deduced by Clayton [10, 11, 12] for the massive NGT field equations is no longer present in the evolution of the equations for an expansion of the antisymmetric sector about an arbitrary Einstein background.

2 The NGT Action and the Constrained Field Equations

We shall decompose the nonsymmetric \( g_{\mu\nu} \) and \( \Gamma^{\lambda}_{\mu\nu} \) as

\[
g_{\mu\nu} = \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu}), \quad g_{[\mu\nu]} = \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu}),
\]

and

\[
\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{(\mu\nu)} + \Gamma^{\lambda}_{[\mu\nu]}.
\]

The contravariant tensor \( g^{\mu\nu} \) is defined in terms of the equation:

\[
g^{\mu\nu} g_{\sigma\nu} = g^{\nu\mu} g_{\nu\sigma} = \delta^{\mu}_{\sigma}.
\]

The notation follows that of earlier work on NGT [1, 2, 3, 4]. The Lagrangian density is given by

\[
\mathcal{L}_{ngt} = \mathcal{L} + \mathcal{L}_M,
\]

where

\[
\mathcal{L} = g^{\mu\nu} R_{\mu\nu}(W) - 2\Lambda\sqrt{-g} - \frac{1}{4} \mu^2 g^{\mu\nu} g_{[\nu\mu]} - \frac{1}{6} g^{\mu\nu} W_{\mu} W_{\nu} + g^{\mu\nu} J_{[\mu} \phi_{\nu]},
\]

and \( \mathcal{L}_M \) is the matter Lagrangian density (\( G = c = 1 \)):

\[
\mathcal{L}_M = -8\pi g^{\mu\nu} T_{\mu\nu}.
\]

Here, \( g^{\mu\nu} = \sqrt{-g} g^{\mu\nu} \), \( g = \text{Det}(g_{\mu\nu}) \), \( \Lambda \) is the cosmological constant and \( R_{\mu\nu}(W) \) is the NGT contracted curvature tensor:

\[
R_{\mu\nu}(W) = W_{\mu\nu,\beta}^\beta - \frac{1}{2}(W_{\mu\beta,\nu}^\beta + W_{\nu\beta,\mu}^\beta) - W_{\alpha\nu} W_{\mu\beta}^\alpha + W_{\alpha\beta} W_{\mu\nu}^\alpha,
\]

defined in terms of the unconstrained nonsymmetric connection:

\[
W_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda - \frac{2}{3} \delta_{\mu\nu}^\lambda W_{\nu},
\]

where

\[
W_{\mu} = \frac{1}{2}(W_{\mu\lambda}^\lambda - W_{\lambda\mu}^\lambda).
\]
Eq. (9) leads to the result:

\[ \Gamma_\mu = \Gamma^\lambda_{[\mu,\lambda]} = 0. \]

The contracted tensor \( R_{\mu\nu}(W) \) can be written as

\[ R_{\mu\nu}(W) = R_{\mu\nu}(\Gamma) + \frac{2}{3} W_{[\mu,\nu]}, \]

where

\[ R_{\mu\nu}(\Gamma) = \Gamma^\beta_{\mu\nu,\beta} - \frac{1}{2} \left( \Gamma^\beta_{(\mu,\beta),\nu} + \Gamma^\beta_{(\nu,\beta),\mu} \right) - \Gamma^\alpha_{\alpha\nu} \Gamma^\beta_{\mu\beta} + \Gamma_{(\alpha\beta)}^\beta \Gamma^\alpha_{\mu\nu}. \]

The term in Eq. (9):

\[ g^{\mu\nu} J_{[\mu,\nu]}, \]  \hspace{1cm} (10)

contains the Lagrange multiplier fields \( \phi_\mu \) and the source vector \( J_\mu \).

A variation of the action

\[ S = \int d^4x L_{ngt} \]

yields the field equations in the presence of matter sources:

\[ G_{\mu\nu}(W) + \Lambda g_{\mu\nu} + S_{\mu\nu} = 8\pi (T_{\mu\nu} + K_{\mu\nu}), \]  \hspace{1cm} (11)

\[ g_{[\mu\nu]} W_{\nu} = -\frac{1}{2} g^{(\mu\alpha)} W_{\alpha}, \]  \hspace{1cm} (12)

\[ g^{\mu\nu,\sigma\rho} + g^{\mu\nu} W^\mu_{\rho\sigma} + g^{\mu\nu \rho} W^\nu_{\sigma\rho} - g^{\mu\nu} W^\rho_{\sigma\rho} + \frac{2}{3} \delta^\nu_{\sigma}\delta^\mu_{\rho} W_{[\rho\beta]} \]

\[ + \frac{1}{6} (g^{(\mu\beta)} W_{\rho\delta} \delta^\nu_{\sigma} - g^{(\nu\beta)} W_{\rho\delta} \delta^\mu_{\sigma}) = 0. \]  \hspace{1cm} (13)

Here, we have \( G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \) and

\[ S_{\mu\nu} = \frac{1}{4} \mu^2 (g_{[\mu\nu]} + \frac{1}{2} g_{\mu\nu} g^{[\sigma\rho]} g_{[\rho\sigma]} + g^{[\sigma\rho]} g_{\mu\sigma} g_{\rho\nu}) \]

\[ - \frac{1}{6} (W_\mu W_\nu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} W_\alpha W_\beta). \]  \hspace{1cm} (14)

Moreover, the contribution from the variation of (10) with respect to \( g^{\mu\nu} \) and \( \sqrt{-g} \) is given by

\[ K_{\mu\nu} = -\frac{1}{8\pi} [J_{[\mu,\nu]} - \frac{1}{2} g_{\mu\nu} (g^{[\alpha\beta]} J_{(\alpha,\beta)})]. \]  \hspace{1cm} (15)

The variation of \( \phi_\mu \) yields the constraint equations

\[ g^{[\mu\nu]} J_\nu = 0. \]  \hspace{1cm} (16)

These equations hold globally for the evolution of the field equations. We have not varied the source vector \( J_\mu \).
If we use (13), then (15) becomes

$$K_{[\mu\nu]} = -\frac{1}{8\pi} J_{[\mu\phi\nu]}.$$  \hspace{1cm} (17)

We can choose the vector $J_\mu$ to be $J_\mu = (0, 0, 0, J_0)$, so that (14) corresponds to the three constraint equations

$$g^{[10]} = 0.$$  \hspace{1cm} (18)

The generalized Bianchi identities

$$[g^{\alpha\nu} G_{\rho\nu}(\Gamma) + g^{\nu\rho} G_{\nu\rho}(\Gamma)]_{,\alpha} + g^{\mu\nu,\rho} G_{\mu\nu} = 0,$$  \hspace{1cm} (19)

give rise to the matter response equations [4]:

$$g_{\mu\rho} T^{\mu\nu,\nu} + g_{\rho\mu} T^{\nu\mu,\nu} + (g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho}) T^{\mu\nu} = 0.$$  \hspace{1cm} (20)

3 Linear Approximation

Let us assume that $\Lambda = 0$ and expand $g_{\mu\nu}$ about Minkowski spacetime:

$$g_{\mu\nu} = \eta_{\mu\nu} + (1) h_{\mu\nu} + ...,$$  \hspace{1cm} (21)

where $\eta_{\mu\nu}$ is the Minkowski metric tensor: $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, +1)$. We shall also expand $\Gamma^\lambda_{\mu\nu}, W^\lambda_{\mu\nu}$ and $K_{[\mu\nu]}$:

$$\Gamma^\lambda_{\mu\nu} = (1) \Gamma^\lambda_{\mu\nu} + (2) \Gamma^\lambda_{\mu\nu} + ...,$$  \hspace{1cm} (22)

$$W^\lambda_{\mu\nu} = (1) W^\lambda_{\mu\nu} + (2) W^\lambda_{\mu\nu} + ...,$$  \hspace{1cm} (23)

$$K_{[\mu\nu]} = (1) K_{[\mu\nu]} + (2) K_{[\mu\nu]} + ....$$  \hspace{1cm} (24)

We adopt the notation, $\psi_{\mu\nu} = (1) h_{[\mu\nu]}$, and using (4), we find that $\psi^{\mu\nu} = \eta^{\mu\lambda} \eta^{\nu\sigma} \psi_{\sigma\lambda}$. To first order of approximation, Eq.(12) gives

$$\psi_\mu = -\frac{1}{2} W_\mu,$$  \hspace{1cm} (25)

where for convenience $W_\mu$ denotes $(1) W_\mu$. Moreover,

$$\psi_\mu = \psi_{\mu\beta}^\beta = \eta^{\beta\sigma} \psi_{\mu\beta,\sigma}.$$

The antisymmetric and symmetric field equations derived from Eq.(11) decouple to lowest order and the symmetric equations are the usual Einstein field equations in the linear approximation. The skew equations are given by [2]

$$\psi_{\mu\nu,\sigma}^\sigma + \mu^2 \psi_{\mu\nu} = J_{\mu\nu},$$  \hspace{1cm} (26)
where
\[ J_{\mu\nu} = 16\pi(T_{[\mu\nu]} + \frac{2}{\mu^2}T_{[\mu\sigma],\nu}^\sigma + K_{[\mu\nu]} + \frac{2}{\mu^2}K_{[\mu\sigma],\nu}^\sigma). \] (27)

We can write the Lagrangian density for the \( \psi_{\mu\nu} \) in the form:
\[ L_{\text{skew}} = \frac{1}{12}F_{\mu\nu\lambda}F^{\mu\nu\lambda} - \frac{1}{4}\mu^2\psi_{\mu\nu,\lambda} - 16\pi\psi^{\mu\nu}(T_{[\mu\nu]} + K_{[\mu\nu]}), \] (28)
where
\[ F_{\mu\nu\lambda} = \psi_{\mu\nu,\lambda} + \psi_{\nu\lambda,\mu} + \psi_{\lambda\mu,\nu}. \]

and to linear order we have from Eq. (17):
\[ K_{[\mu\nu]} = -\frac{1}{8\pi}J_{[\mu\phi_i]}, \] (29)
where for convenience, we have used \( K_{[\mu\nu]} \) in place of \( (1)^{1}K_{[\mu\nu]} \) and \( \phi_{\mu} \) in place of \( \phi_{\mu} \). From Eq. (25) we get
\[ \psi_{\mu} = 16\pi \frac{\mu}{\mu^2}(T_{[\mu\nu]}^{\nu} + K_{[\mu\nu]}^{\nu}). \] (30)

The source current density \( T_{[\mu\nu]} \) is identified at the microscopic level with a bosonic open string current density \( [4, 8] \), and for open strings the \( T_{[\mu\nu]} \) is not conserved.

In the wave-zone, \( T_{\mu\nu} = 0 \), and Eqs. (28) and (30) become
\[ \psi_{\mu\nu,\sigma}^{\sigma} - \psi_{\mu\sigma,\nu}^{\nu} + \psi_{\nu\sigma,\mu}^{\sigma} + \mu^2\psi_{\mu\nu} = 16\pi K_{[\mu\nu]}, \] (31)
\[ \psi_{\mu} = \frac{16\pi}{\mu^2}K_{[\mu\nu]}^{\nu}. \] (32)

Eq. (31) can be written as
\[ F_{\mu\nu,\sigma}^{\sigma} + \mu^2\psi_{\mu\nu} = 16\pi K_{[\mu\nu]}. \] (33)

By using the frame in which \( J_\mu = (0, 0, 0, J_0) \) and substituting the weak field constraint, \( \psi_{i0} = 0 \), into (32), we obtain
\[ \phi_{i,i} + J_{0,i}\phi_i = 0, \] (34)
and
\[ \dot{\phi}_i = -\mu^2(\psi_{ij,j} + \frac{1}{\mu^2}J_0\phi), \] (35)
where \( \dot{\phi}_i = \partial\phi_i/\partial t. \)

Choosing \( \mu = 0 \) and \( \nu = i \) in (31), we get
\[ \dot{\psi}_{ij,j} = J_0\phi_i, \] (36)
while choosing $\mu = i$ and $\nu = k$ yields an equation for the transverse-transverse components of $\psi_{\mu\nu}$:

$$\psi_{ik,\sigma}{}^\sigma - \psi_{ij,k}{}^j + \psi_{kij}{}^j + \mu^2 \psi_{ik} = 0,$$

(37)

where we have used

$$K_{[0]} = -\frac{1}{16\pi} J_0 \phi_i, \quad K_{[ik]} = 0$$

(38)

obtained from (29).

By using (35) and the time derivative of (36), we get

$$\ddot{\psi}_{ij,j} = -\mu^2 \psi_{ij,j}.$$

(39)

This equation has the solution:

$$\psi_{ij,j}(\vec{x}, t) = c_i(\vec{x}) \cos(\mu t + \theta),$$

(40)

where $\theta$ is a constant. By taking the divergence of (37), we obtain

$$\ddot{\psi}_{ij,j} - 2\nabla^2 \psi_{ij,j} + \mu^2 \psi_{ij,j} = 0.$$

(41)

Substituting the solution (40) into (41) gives

$$\nabla^2 c_i(\vec{x}) = 0.$$

(42)

Let us substitute

$$\psi_{ik}(\vec{x}, t) = b_{ik}(\vec{x}) \cos(\mu t + \theta)$$

(43)

into (37) and use (40) to give

$$\nabla^2 b_{ik}(\vec{x}) = 2 c_{i,k}(\vec{x}).$$

(44)

By employing the solution of the Laplace equation (42), we can obtain the $b_{ik}$ from (44) which in turn determines the transverse-transverse components $\psi_{ik}$.

For the static case, we obtain from (35):

$$\psi_{ij,j} = 0$$

and (37) becomes

$$\nabla^2 \psi_{ik}(\vec{x}) - \mu^2 \psi_{ik}(\vec{x}) = 0,$$

(45)

which is the Helmholtz equation with the point source, spherically symmetric Yukawa solution:

$$\psi_{ik}(r) = \lambda_{ik} \frac{\exp(-\mu r)}{r},$$

(46)

where $\lambda_{ik}$ and $\mu$ are constants.
4 Canonical Form of the Hamiltonian in the Weak Field Antisymmetric Sector

By decomposing the Lagrangian \( \tilde{\mathcal{L}} \) for \( T_{\mu
u} = 0 \) into space and time components and using \( \gamma_\mu = (0, 0, 0, 1) \), we get

\[
\mathcal{L}_{\text{skew}} = \frac{1}{4} \left[ (\dot{\psi}_{ik})^2 + 4\psi_{k0,i}\dot{\psi}_{ik} + 2(\psi_{k0,i})^2 - 2\psi_{k0,k}\psi_{i0,i} + 2\psi_{kj,k}\psi_{ij,i} - (\psi_{jk,i})^2 - \mu^2((\psi_{ik})^2 - 2(\psi_{0i})^2) \right] + J_0 \phi_i \psi_{0i}. \tag{47}
\]

The conjugate momenta \( \pi_{\mu\nu} \) are given by

\[
\pi_{i0} = \frac{\partial \mathcal{L}_{\text{skew}}}{\partial \dot{\psi}_{i0}} = 0, \tag{48}
\]

\[
\pi_{ij} = \frac{\partial \mathcal{L}_{\text{skew}}}{\partial \dot{\psi}_{ij}} = \dot{\psi}_{ij} + \psi_{0i,j} - \psi_{0j,i}. \tag{49}
\]

We can now obtain the following form of the Lagrangian:

\[
\mathcal{L}_{\text{skew}} = \frac{1}{4} \left[ (\pi_{ij})^2 + 2\psi_{jk,k}\psi_{ji,i} - (\psi_{jk,i})^2 - \mu^2((\psi_{ik})^2 - 2(\psi_{0i})^2) \right] + J_0 \phi_i \psi_{0i}. \tag{50}
\]

The Hamiltonian is given by

\[
\mathcal{H}_{\text{skew}} = \frac{1}{2} \pi_{ik}\dot{\psi}_{ik} - \mathcal{L}_{\text{skew}} + \theta_i \pi_{i0}, \tag{51}
\]

where \( \theta_i \) acts as a Lagrange multiplier for the constraint \( \pi_{i0} = 0 \). Using (50) we get

\[
\mathcal{H}_{\text{skew}} = \frac{1}{4}(\pi_{ik})^2 + \pi_{ij}\psi_{i0,j} - \frac{1}{2}\psi_{jk,k}\psi_{ji,i} + \frac{1}{4}(\psi_{jk,i})^2 + \frac{1}{4}\mu^2((\psi_{ik})^2 - 2(\psi_{0i})^2) + J_0 \phi_i \psi_{i0} + \theta_i \pi_{i0}. \tag{52}
\]

By employing the Poisson bracket relation:

\[
\{\psi_{i0}, \pi_{j0}\} = \delta_{ij}, \tag{53}
\]

we get the evolution equation:

\[
\dot{\pi}_{i0} = \{\pi_{i0}, \mathcal{H}_{\text{skew}}\} \approx -\pi_{ik,k} - \mu^2\psi_{i0} + J_0 \phi_i \approx 0, \tag{54}
\]

where

\[
\mathcal{H}_{\text{skew}} = \int d^3x \mathcal{H}_{\text{skew}}. \]

Varying with respect to \( \phi_i \) yields the constraint

\[
\psi_{i0} \approx 0. \]

7
Taking the divergence of (54) leads to Eq.(34). Moreover, we have
\[ \dot{\psi}_i = \{\psi_i, H_{\text{skew}}\} \approx \dot{\theta}_i \approx 0. \] (55)

We have the set of constraints:
\[ \pi_{i0} \approx 0, \quad \psi_{i0} \approx 0, \] (56)
which constitute six second class constraints. Thus, there are \((6 \text{ degrees of freedom}) - \frac{1}{2}(6 \text{ second class constraints}) = 3\) independent degrees of freedom.

Imposing the constraints, we obtain the Hamiltonian energy of the system:
\[ H_{\text{skew}} \approx \int d^3x \frac{1}{2}[\pi_i^2 + (\psi_{i,i})^2 + \mu^2 \psi_i^2], \] (57)
where \(\pi_i = \frac{1}{2} \epsilon_{ijk} \pi_{jk}\) and \(\psi_i = \frac{1}{2} \epsilon_{ijk} \psi_{jk}\). Thus, the energy is bounded from below and the weak field antisymmetric sector leads to a physically consistent Hamiltonian, free of ghost poles and tachyons.

5 Linearization about a Fixed Einstein Background

In recent papers, Clayton has developed a Hamiltonian constraint formalism for NGT [10, 11, 12]. The field equations are decomposed into a (3+1) form by foliating spacetime into spacelike hypersurfaces. Configuration space has therefore been chosen to consist of \(g^\perp_{ij}\) and \(g^{ik}\), where the index \(\perp\) denotes a component normal to a hypersurface \(\Sigma\). The Cauchy data is chosen so that the antisymmetric sector is an arbitrarily small perturbation of the symmetric sector. Thus, the Cauchy data is expanded in powers of the antisymmetric components \(g^{[\mu \nu]} = (B_i, \gamma^{[ik]}\) about an arbitrary Einstein background described by the symmetric metric \(\gamma^{(ik)}\).

From the Hamiltonian constraint formalism, equations for the velocity components \(\dot{B}^i\) and the acceleration components \(\ddot{B}^i\) are derived. These are given by (see ref. [11, 12] for details):
\[ \dot{B}^i \approx Y^i + \frac{3}{4} N \gamma^{(ik)} O_{2kj}^{-1} Z^j, \] (58)
\[ \ddot{B}^i \approx \frac{3}{4} N \sqrt{\mathcal{S}} O_2^{-1} k X_k, \] (59)
where
\[ O_{2ik}^{-1} \approx -\frac{1}{\gamma \cdot \tilde{\gamma}} \left[ \gamma^{(ik)} - \frac{1}{\gamma \cdot \tilde{B}} (B_i \gamma_k + \gamma_k B_i) + \frac{\tilde{B} \cdot \tilde{B} - \tilde{\gamma} \cdot \tilde{\gamma}}{(\gamma \cdot \tilde{B})^2} \gamma_i \gamma_k \right]. \] (60)
The \(X_i, Y_i\) and \(Z_i\) are quantities that are well behaved as the antisymmetric components become vanishingly small and \(B_i\) is the densitised \(B_i\). Moreover, \(\gamma_i = \frac{1}{4} \sqrt{S} \epsilon_{ijk} \gamma^{[jk]}\), \(S = \text{Det}(S_{ik})\) with \(S_{ik}\) defined by \(\gamma^{(ik)} S_{kj} = \delta^i_j\), \(N\) is the lapse function.
and we have used the notation: \( \vec{A} \cdot \vec{B} = A^i B_i \). The operator \( O^{-1}_{2ik} \sim O(\text{skew}^{-2}) \) is singular in the limit of arbitrarily small antisymmetric fields.

In the version of NGT presented here, the global constraints (16) and (18), obtained from the action principle, guarantee that the velocity components \( B^i \) and the acceleration components \( \dot{B}^i \), determined by Eqs. (58) and (59), are absent from the evolution equations. Thus, this version of NGT can yield smooth solutions arbitrarily close to the symmetric Einstein background solutions, allowing for stable Cauchy evolution and linearization. However, further work is required to demonstrate that there exists a complete, rigorous solution to the Cauchy evolution problem.

6 Conclusions

We have formulated a version of NGT with a covariant action principle, including constraints on the antisymmetric field variables \( g^{[\mu \nu]} \), implemented by the Lagrange multiplier fields \( \phi_\mu \), which guarantees a system of consistent field equations. By expanding the antisymmetric field sector about Minkowski spacetime and performing a Hamiltonian constraint analysis, it was shown that the Hamiltonian for the weak antisymmetric fields was bounded from below, ensuring a physically consistent weak field approximation in conjunction with the weak field symmetric Einstein sector.

The field equations were then expressed in the \((3+1)\) Hamiltonian formalism developed by Clayton [10, 11, 12], and the antisymmetric sector was expanded about an arbitrary symmetric Einstein background. It was then shown that the new field equations with the dynamical constraints, \( g^{[\mu \nu]} J_\nu = 0 \), led to Cauchy stable evolution equations and linearization stable solutions for arbitrarily small \( g^{[\mu \nu]} \) fields. Therefore, both the massless and the massive NGT field equations yield physically consistent solutions with asymptotically flat boundary conditions with well-defined Newtonian and Einstein gravity limits.

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