Cavitation Influence in 1D Part-load Vortex Models

P K Dörfler
Consulting engineer, Hydro adviser LLC, Zurich (Switzerland)

E-mail: contact@hydroadviser.ch

Abstract. Residual swirl in the draft tube of Francis turbines may cause annoying low-frequency pulsation of pressure and power output, in particular during part-load operation. A 1D analytical model for these dynamic phenomena would enable simulation by some conventional method for computing hydraulic transients. The proper structure of such a model has implications for the prediction of prototype behaviour based on laboratory tests.

The source of excitation as well as the dynamic transmission behaviour of the draft tube flow may both be described either by lumped or distributed parameters. The distributed version contains more information and, due to limited possibilities of identification, some data must be estimated. The distributed cavitation compliance is an example for this dilemma. In recent publications, the customary assumption of a constant wave speed has produced dubious results.

The paper presents a more realistic model for distributed compressibility. The measured influence of the Thoma number is applied with the local cavitation factor. This concept is less sensitive to modelling errors and explains both the Thoma and Froude number influence.

The possible effect of the normally unknown non-condensable gas content in the vortex cavity is shortly commented. Its measurement in future tests is recommended. It is also recommended to check the available analytical vortex models for possible dispersion effects.

1. Introduction
Due to their fixed runner blades and operating speed, Francis turbines generate a load-dependent amount of swirl in their exit flow. Between 50 and 85 per cent of the flow at best efficiency, the flow usually develops into a regular helical vortex, the so-called draft-tube rope. The precession of the rope inside the diffuser is accompanied by undesirable pressure and power fluctuation [1]. The mechanical effects of this phenomenon result in limitations of the useful operating range in many plants.

Aside of the turbine discharge, the draft tube pressure level has the most important influence on the intensity of the pulsation. Due to high circumferential velocities, a lengthy cavitation zone normally resides along the vortex axis; the dimensions of this cavity depend both on the pressure and flow conditions in the draft tube. Already in 1940, Moody [2] realized that the resonance-like increase of amplitudes called draft-tube surge must be due to “the presence of an elastic medium, such as an air or vapor cavity”. In the early 1980s, first attempts were made to quantitatively describe the pulsation phenomena including their interaction with the surrounding system (Dörfler 1982 [3]). Like earlier works of Brennen [4] about pumps, these studies represented the behavior of the cavity by the partial derivatives of its volume, C= -∂Vc/∂h (cavitation compliance) and -∂Vc/∂Q (mass flow gain factor).

Compared to the compressibility C of the gaseous phase, the compressibility of the liquid phase in the draft tube is negligible. In such a lumped-parameter representation, the dynamic characteristic of the draft tube flow therefore supports just a single mode of oscillation.
In the late 1990s, after the discovery of the so-called high-partial-load pulsation, Philibert and Couston [5] proposed to express the compressibility effect by assigning a reduced wave speed to a considerable portion of the draft tube, instead of the lumped compliance C. Their theory comprised the correct relationship between the cavity cross section and wave speed, but then treated the wave speed as constant in the axial direction. This arrangement enabled an analytic representation of higher natural modes, and allowed for an explanation of the high frequencies observed at high partial load.

The constant wave speed was probably just a stopgap for convenient integration of the wave equation. Later investigations by Arpe [6] confirmed that the wave speed is not constant at all, being extremely low in the upper draft tube cone and increasing significantly towards the draft tube elbow. However, the discrepancy between the theoretical advantage of the distributed-parameter model and the practical neglect of the actual distribution of wave speed did not disappear. Recent studies concerned with the transmission behavior of the vortex [8][9] still retained the assumption of a single value of wave-speed along the vortex – with some implausible results, as shown in Section 2.2.

2. 1D model structures compared to test results

Quantitative data is normally retrieved from physical model tests but the goal is to predict the behavior of a prototype machine, with the help of the analytical model. The structure of such an analytical model has implications for the process of prediction and should therefore, as much as possible, be based on the physical phenomena involved. Also, any set of assumptions must consider the practical possibilities to quantify the model parameters. If parameters cannot be measured or computed, more assumptions become necessary.

Synthesis of a model for the pulsation caused by the part-load vortex requires assumptions about the excitation source, the dynamic transmission behavior of the cavitation zone, and the relationship between the asynchronous (i.e. purely local) and the synchronous components of the pulsation.

2.1. Source of excitation

In case of the partial-load vortex, the synchronous pulsation is excited because the draft tube elbow necessitates a periodical deformation of the corkscrew-shaped vortex axis. A momentum source driving a forced oscillation is assumed throughout this paper.

It is, unfortunately, not possible to directly measure the excitation source; measurement of pressure variations can only capture a system response. To determine the source, one must rely on some model for the system response. Uncertainty about the response is relatively low in absence of cavitation. Most reliable measures of the momentum source are therefore obtained in non-cavitating condition, at very high sigma. Few data have been published so far; Figure 1 shows some published amplitude values, in percent of the runner velocity head \( u_2^2 / 2g \).

Depending on the cavitation model (2.2), various assumptions concerning the location and extent of this source are possible. In the simple lumped-parameter model of ref. [3], the source was assumed to be downstream of all the pressure sensors in the draft tube cone and elbow, because location upstream of the cavity could not reproduce the measured dependency between the cavitation number and the pressure amplitudes; as shown in Figure 2 (a).

![Figure 1](image)

Figure 1 Excitation source amplitude at high \( \sigma \): model test results from [3][8][10], expressed as pressure pulsation coefficient [10]
Recent studies with more elaborate models [7][8] considered a distributed cavitation compliance between the draft tube intake and bottom. Alligne’s CFD study based on the FLINDT model [7] found a momentum source to be evenly distributed along the elbow (plus a mass source), while Landry’s analysis [8] of tests with the Mica 1 model (D=0.35m, n_{QE}=0.132, n_{ED}=0.288) ended up with a momentum source concentrated near the draft tube cone exit.

2.2. Dynamic transmission behaviour

Modelling of the cavitating part of the draft tube requires a measure for compressibility, fluid mass, and some representation of positive or negative damping effects. As the void fraction is always small and hence there are no important variations of mass, the main object is compressibility. Damping effects due to mass flow gain or ‘bulk viscosity’ [8] will not be discussed in this article.

Figure 2  Check for location of momentum source, data from [3], for sensor locations see Fig. 3

Figure 3  Influence of $\sigma$ on pulsation test results, including hysteresis, data from [3]
In a lumped-parameter model, compressibility is represented by the cavitation compliance

\[ C \text{ [m}^2] = - \frac{\partial V_c}{\partial h} \tag{1} \]

using an averaged fluid pressure \( h = \frac{p}{\rho g} \) at cavity location. For consistency with the natural frequency, its value depends slightly on its assumed location in the system. But this type of model is deliberately simplifying anyway, thus one may well assume a location in the draft tube entry section.

As the cavitation zone may actually extend well into the draft tube elbow, or even further, the distributed-parameter model for compliance should clearly come closer to reality. The compliance is, in such a model, not a single number \( C \) (expressed in \( \text{m}^2 \)) but a ‘compliance per unit draft tube length’

\[ c(s) \text{ [m]} = \frac{dC}{ds} \tag{2} \]

The usual way to express its effect is, however, by means of a reduced value of wave speed \( a \). The distributed parameter, \( c \) or \( a \), must in either case be some function of the path length \( s \). If this function is not known, or maybe wrong, then the advantage of the more realistic model may vanish. It is therefore advisable to reflect on the customary assumption of a constant wave speed \( (da/ds=0) \).

In experiments concerning the influence of \( \sigma \) on compliance, one always obtains a global effect only. It may be ascribed either to the lumped parameter \( C \), or to some deliberately chosen distribution of \( c(s) \) or \( a(s) \). The simplest assumption, which has survived until today [8], is that the wave speed \( a \) remains constant throughout the vortex length. The relationship between the variables \( a \) and \( c \) is

\[ c = \frac{Ag}{a^2} \tag{3} \]

Between the draft tube intake and elbow, the cross section area \( A(s) \) usually increases by a factor close to 2. In order to maintain constant wave speed \( a \), the compliance \( c \) must double. Note that in a unit with vertical axis, the ambient pressure around the vortex increases considerably with \( s \) while the vorticity remains approximately the same. As a consequence, the distributed compliance \( c \) has to decrease, not increase, with \( s \). The wave speed must therefore increase significantly with \( s \).

**Figure 4** Cavitation compliance \( C \) and cavity volume \( V_C \) proportional to \( \exp(b\sigma) \): examples
An obvious alternative approach would be to regard the distributed compliance $c$ as a function of the local cavitation factor $\chi_E$ [11][8]

$$\chi_E = \frac{(p(s) - p_v)}{\rho \omega E} \quad (4)$$

In Eq. (4), $p(s)$ is the mean pressure in the cross section at path length $s$, considering the elevation, mean velocity and draft tube head loss, and $E$ is the specific energy ($E=gH$).

One may estimate the function $c(\chi_E)$ based on the available function $C=f(\sigma)$. This is facilitated by the fact that an exponential function $a\exp(b\sigma)$ is an acceptable approximation for $C(\sigma)$. Because $C$ and $c$ are related by Eq. (2), and the variation of $\chi_E(s)$ is mainly caused by the gravity term, i.e. approximately linear in $s$, one may use the same coefficient $b$ for both $C(\sigma)$ and $c(\chi_E)$.

Using data from various sources, Figure 4 demonstrates the near-exponential character of $V_C(\sigma)$ and $C(\sigma)$. Figure 5 compares the two compliance and wave speed models described above, for an operating condition at $Q/Q_{opt}=0.8$ examined in [8]. Both distributions produce the same natural frequency but the percentage of compliance $C_1/C$ upstream of the assumed exciter location is quite different.

As already shown in [3] and Figure 2, this percentage is crucial for the $\sigma$-dependency of the resonance.

The natural frequency of 3.8Hz for this test condition corresponds to a lumped cavitation compliance of 8...11 cm$^2$, depending on the model structure.

For frequencies of the order of the vortex frequency, the influence of the cavitation model on the impedance $Z_{DT}$ at the draft tube intake is negligible if the total compliance $C$ is slightly adjusted; see Figure 6.

The relative amount of compliance upstream and downstream of the momentum source plays an important role. This can best be explained by a simplified model with lumped compliance and momentum source. As in reference [8], the source is assumed to be concentrated near the end of the draft tube cone ($s=0.38m$), shown in Figure 5 above.

**Figure 5** Comparison of alternative models for wave speed $a(s)$ and compliance $c(s)$

data corresponding to ref. [8], test PL1 ($Q/Q_{opt}=0.8$), $\sigma=0.11$, Fr=7.66

**Figure 6** Equivalence of compliance models
The compliance $C$ is divided into an upstream part $C_1$ and a downstream part $C_2$, $C = C_1 + C_2$. The relative amount of compliance on either side of the momentum source has little influence on the natural frequency, but very important influence on the forced pulsation.

This influence is visualized in Figure 7. Using the known impedances of the test rig and draft tube, one can calculate the frequency response of the pressure in the draft tube inlet, $\text{abs}(p_{DT}/p_{ex})$. Diagram (a) compares several different scenarios for the partitioning of $C$ to both sides of the source. The cases $C_1/C = 1$ and $C_1/C = 0$ correspond to the two extremes compared in [3], and in Figure 2 (a) and (c). $C_1/C = 0.34$ and $C_1/C = 0.64$ result from the two scenarios for wave speed shown in Figure 5. If the wave speed were independent of $s$ ($C_1/C = 0.34$), then almost two thirds of the compliance would lie downstream of the pressure source $p_{ex}$, resulting in a very low pressure gain. If $a(s)$ actually depends on $\chi_E$ like $C(\sigma)$, and the momentum source does not depend on $\sigma$, then the model with constant wave speed can explain the measured pressure amplitudes only by assuming a strong increase of the source magnitude according to the ratio between the two curves, as shown in diagram (b). Indeed, Landry’s analysis concludes that the source magnitude strongly increases at low $\sigma$, see diagram (c).

Figure 7 (a) Influence of partition of compliance on pressure gain, (b) Source amplitude error due to $da/ds=0$, and (c) Variation of source amplitude according to ref. [8]

2.3. Superimposed asynchronous pulsation

Only the synchronous part of the pulsation propagates throughout the hydraulic system, and is subject to 1D modelling, but it can only be accessed after separating the superimposed asynchronous part. The necessity of separation stems from the fact that pressure recorded in the draft tube always contains components from both phenomena. Two different ways to treat the asynchronous pulsation have been followed so far. In either case one needs some auxiliary assumption.

Dörfler [3] used the pulsation measured upstream of the turbine in cavitation-free condition to synthesize the synchronous part in the draft tube and, by subtracting it from the signals measured in the draft tube, obtained the asynchronous part. Assuming no influence of $\sigma$ on the amplitude of the asynchronous part and its phase against the momentum source, one can approximately identify the location of the source, see Figure 2.

Alternatively, one can use a number $N$ of equally spaced pressure sensors in a cross section of the draft tube, and assume the asynchronous component to be symmetric. In such a case, one obtains the harmonics 1 through $N-1$ of the synchronous pulsation by averaging between the measured signals. The asynchronous part of the pulsation may then be ignored; this concept was applied in Landry’s study [8]. Favrel [9] found remarkably different asynchronous parts, even in the relatively deep Mica draft tube tested by Landry [8].
The example in Figure 2 demonstrates that the asynchronous part could play a meaningful role for analysis. Its phase conveys information concerning the shape of the vortex axis and its changes as a function of cavity size.

2.4. Estimate of Froude number effect

In the following, the development of the local cavitation factor in a model draft tube will be examined. It will be assumed that the same operating condition is tested at different test heads, i.e. Froude numbers. The example will again use data of the operating point PL1 of the Mica 1 model test at LMH [8]. Figure 8 (a) shows how the change of the elevation and flow velocity along the flow path makes the local cavitation factor increase; for small test heads this effect produces large deviations from \( \sigma \).

![Figure 8](image)

(a) Local cavitation factor  
(b) Resulting compliance  
(c) Relative natural frequency

From the cavitation factor, one obtains an estimate for the distributed compliance using

\[
c = a \cdot \exp(b \cdot \sigma) \cdot H_{\text{ref}} / H
\]  

(5)

The exponent \( b = -36 \) was determined to approximate the measured exponential dependency \( C(\sigma) \); the parameter \( a = 0.473 \text{mm} \) provides the total compliance from integration of \( c(s) \) for the reference head of 20.5m, corresponding to \( \text{Fr}=7.66 \). The course of the distributed compliance in diagram (b) suggests that the model with constant cavity length (here 0.95m) may not yet be the best possible solution. The course of the predicted relative natural frequency is compared with the test results, diagram (c). The curves show a surprisingly strong influence of the test head. The lowest Froude number (\( \text{Fr}=5.52 \)) would correspond to the prototype; no test results for this Froude number are included in [8].

2.5. Comparison of different load conditions

It is evident from Figure 4 (b) that the magnitude of parameter \( b \) in the function \( C = a \cdot \exp(b \cdot \sigma) \) is smaller in case of the test point with the larger guide vane opening. Note that both curves (CFD results taken from [12]) belong to points with negative swirl, i.e. conditions with \( Q_{\text{nD}} \) values higher than the swirl-free condition. GVO=0.77 has higher swirl than GVO=0.73.

Table 1 Load dependency of compliance in test results from [8]

| Operating point | \( Q_{\text{nD}} / Q_{\text{nD, opt}} \) | Coefficient b | \( Q_{\text{nD}} - Q_{\text{nD,0}} \) |
|-----------------|------------------|--------------|------------------|
| PL1             | 0.80             | -36          | -0.294           |
| PL2             | 0.64             | -19          | -0.410           |
The compliance estimates in Figure 4 (c), derived from measured natural frequencies, show that the value of \( b \) for the point PL2 with lower part load must be smaller than for PL1. This can be understood from the fact that the reference pressure for \( \sigma \) is the turbine head \( H \), but the compliance is inversely proportional to the circumferential velocity head of the vortex. Therefore the coefficient \( b \) in Eq. (5) is inversely proportional to the square of the deviation from the discharge coefficient in no-rope condition, \( Q_{\text{nd}}=0.85 \), as may be checked from the data in Table 1.

3. Effects of the non-condensable gas content

As suggested by the definition of the cavitation number \( \sigma \), the theoretical pressure at the phase limit is the vapor pressure (usually 2-3 kPa in the hydraulic lab). Depending on the test conditions and the degree of de-aeration of the working fluid, the cavity actually also contains some amount of non-condensable gas. As a consequence, the real pressure inside the cavity is higher than vapor pressure. This affects the dynamics of part-load surge in several ways.

Firstly, the extent of the cavitation zone is a function of the pressure difference between the cavity and the fluid ambient. If two tests are done at equal \( \sigma \) but with different partial pressure of air inside, then the volume of the cavity is not the same. On the other hand, the bulk modulus of the gas content increases together with its pressure, thereby reducing the compliance in case of equal void fraction.

A number of experimental observations suggest that the phenomena described above are actually influencing the test results. In addition, such effects may be approximately quantified by simple calculations. The shift between the curves for increasing and decreasing \( \sigma \) in Figure 3, for example, are certainly due to differences in cavity pressure. One may also suspect that the ‘lock-in’ effect of surge amplitudes observed by Favrel [9] around the resonant condition, in particular at low Froude number, is due to slow variation of the unknown gas content of the cavity.

No data concerning the actual cavity pressure in the draft tube vortex have been published, to the author’s knowledge, during the last 40 years. Before, Nakanishi and Ueda [13] had reported gas pressures in the vortex core of 3 Francis models between 10 and 30 kPa in case of resonant surge. In a model pump turbine, Grein [14] measured a cavity pressure as high as 55 kPa.

A quantitative example for the effect of residual air has already been given in [3]. Another example is given in the following, based on a lumped-parameter model and, once again, on operating point PL1 used in the previous sections. The distribution of compliance \( c(s) \) shown in Figure 8 (b) may be integrated along the path \( s \) to obtain the total compliance \( C \). As \( C \) is linked with the cavity volume by equation (1), the assumption of negligible cavity pressure permits us to estimate the cavity volume by integrating over the cavitation number. As \( c(\sigma) \) is an exponential function, the integration corresponds to a division by the exponent \( b \):

\[
V_C [m^3] \approx - H.C/b \quad (6)
\]

For \( \sigma=0.11 \), the Froude dependency of this cavity volume – without considering any gas content - is shown in Figure 9 (a), together with the mean cavity diameter based on the assumed constant cavity length of 0.95m. If a cavity of given volume \( V_C \) has a substantial content of gas, then the bulk modulus of the gas content reduces the cavitation compliance. The description of the compliance becomes more complex; now there are two terms contributing to the ‘stiffness’ (inverse compliance) of the vortex: one term from the centrifugal action of the vortex and another one from the gas pressure

\[
-1/C = dh/dV_C = dh/dV_C + dh/dV_C = -1/C_V \cdot n.p_C/p_W g V_C \quad (7)
\]

where \( C_V \) is the vortex compliance for equal \( V_C \) at constant cavity pressure, and \( n \) is the polytropic index. Note that in such a case the gas content increases the effective local cavitation number because the cavity pressure is higher than vapor pressure, and a given volume \( V_C \) occurs at a higher \( \sigma \) value.

In the following, it is arbitrarily assumed that during the model tests, the gas in the cavity had a pressure of 2.5 kPa, equal to the vapor pressure.
Figure 9 Influence of Froude number and air content (PL1 [8], Q/Q_{opt}=0.8, σ=0.11)

(a) Cavity volume                          (b) Inverse compliance

Figure 9 (b) compares the effective stiffness 1/C with the term n.p_G/ρ_WgV_C, assuming p_G=p_V, V_C=V_{C0}, n=1.3. This term makes up for a surprisingly high fraction of the total stiffness 1/C. At Fr=6.56 the two terms are already equal. There is no room for the important vortex stiffness due to the peripheral velocity head. Obviously, the actual cavity volume V_C must be bigger than the theoretical value V_{C0} resulting from Eq. (6) – 2 times in Figure 9 (b) - and the discrepancy must increase even more if the effective gas pressure in the cavity is higher than 2.5kPa. Actually it is the ratio of the two unknown parameters - p_G/V_C - that controls the ‘bulk modulus’ term of the compliance. p_G may depend on both Fr and σ; if p_G is not known, then the cavity volume from Eq. (6) has only symbolic meaning. This creates some uncertainty for the interpretation of the model test results and their scale-up.

4. Side note: Wave propagation on a cavitating vortex

In addition to its dependency on cavity pressure, the behaviour of the wave speed on the vortex may hold a pitfall of a more fundamental kind. In hydropower research, it has been tacitly assumed so far that the wave velocity in any part of the vortex is a frequency-independent parameter. It is this assumption that permits us to ‘measure’ the wave velocity using the response at some suitable frequency, and to make a prediction of the behaviour (for instance, resonant draft tube surge) at a different frequency.

But this tacit assumption is probably not justified, taking into account the physics of wave propagation on a cavitating vortex. Until recently, researchers described the cavitation-induced drop of wave speed in analogy to the behavior of a ‘bubbly air-water mixture’ [8]. In such a model, the ‘stiffness’ counteracting compression of the flow would result from increasing gas pressure inside the cavity. In reality however, the main force that resists compression of the cavity is the centrifugal effect. The waves travelling along the vortex are actually inertial waves. To understand this, one may profit from an analogy to ordinary water waves. The centrifugal force in the vortex corresponds to the role of gravity in water waves, and instead of a plane environment, the vortex waves occur in a cylindrical one. Like in the case of water waves, there should be a dispersion relation defining the phase velocity as a function of wave number.

In the shipbuilding industry, wave propagation on a hollow vortex has been extensively studied. For the unbounded case, without a rigid wall, solutions are available for a number of more or less realistic vortex models. For the simplest case – the potential vortex - Bosschers [15] has published the analytical solution. One lesson from this research, and also from the water wave analogy, is that the phase velocity is a function of frequency, at least for the unbounded vortex. The unbounded vortex corresponds to the limit case of deep water waves; in the limit case of very shallow water, the phase velocity is only a function of water depth times gravity g. Between the two limit cases, there is a transition with influence from both water depth and wave number. Analogy suggests this will also be the case if the cavity diameter is very small compared to the draft tube diameter.
5. Conclusions
The customary practice to assume a constant wave speed along the cavitating draft tube vortex should better be replaced by a more realistic assumption. The dependency of the distributed compliance $c$ from the local cavitation factor $\chi_E$ may be approximated by an exponential function $a \exp(b \chi_E)$ proportional to the influence of the Thoma parameter $\sigma$ on the lumped compliance $C$.

It follows that the strong influence of cavitation on the magnitude of the momentum source reported by Landry [8] is a result of the inappropriate assumption of constant wave speed, and that the actual influence of $\sigma$ on the source amplitude - if any - is much smaller.

The unknown partial pressure of air inside the vortex cavity creates some uncertainty of results. Its measurement would not be too difficult, and it should be adopted as an additional test parameter in future research. Data for this parameter in prototype turbines could likewise be useful.

Researchers in the hydropower sector could learn about cavitating vortex dynamics from other sectors, for instance nautical engineering. Strictly speaking, the assumption of a frequency-independent wave speed on the cavitating vortex is not quite correct. The possibility of dispersion should be examined because it could have consequences for the methodology of dynamic testing.

References
[1] Rheingans W F 1940, Power Swings in Hydroelectric Power Plants, Trans. ASME 62, pp 171-177
[2] Moody L F 1940, Discussion of [1], Trans. ASME 62, p 180
[3] Dörfler P K 1982, System oscillations excited by the Francis turbine's part load vortex core: mathematical modeling and experimental verification, PhD thesis (in German) TU Wien (AT), English translation (2013) on researchgate.net
[4] Brennen C E 1978, The unsteady, dynamic characterization of hydraulic systems with emphasis on cavitation and turbomachines, Joint Symp. ASME/ASCE/IAHR, Ft. Collins, pp 97-107
[5] Philibert R, Coustou M, 1998, Francis turbines at part load, matrix simulating the gaseous rope. IAHR Section Hydr Machinery and Cavitation, 19th Symp (Singapore), pp 441-453
[6] Nicolet C, Arpe J, Avellan F, 2004, Identification and modelling of pressure fluctuations of a Francis turbine scale model at part load operation, IAHR Section Hydr Machinery and Cavitation, 22nd Symp (Stockholm)
[7] Alligné S 2011, Forced and Self Oscillations of Hydraulic Systems Induced by Cavitation Vortex Rope of Francis Turbines, PhD thesis no. 5117, EPFL Lausanne (CH)
[8] Landry C 2015, Hydroacoustic Modeling of a Cavitation Vortex Rope for a Francis Turbine, PhD thesis no. 6547, EPFL Lausanne (CH)
[9] Favrel A 2015, Dynamics of the Cavitation Precessing Vortex Rope for Francis Turbines at Part Load operating conditions, PhD thesis no. 6880, EPFL Lausanne (CH)
[10] Dörfler P K, Sick M, Couttu A 2013, Flow-Induced Pulsation and Vibration in Hydroelectric Machinery, London: Springer, chapters 2 and 7
[11] Franc J-P, F. Avellan F et al. 1995, La Cavitation: Mécanismes Physiques et Aspects Industriels. Collection Grenoble Sciences. Presse Universitaires de Grenoble, Grenoble
[12] Koutnik J, Nicolet C, Schohl G A, Avellan F 2006, Overload Surge Event in a Pumped-Storage Power Plant. IAHR Section Hydraulic Machinery and Systems, 23rd Symp (Yokohama)
[13] Nakanishi K, Ueda T 1964, Air Supply into Draft Tube of Francis Turbines. Fuji Electric Review, 10, No.3, pp 81-91
[14] Grein H 1974, Partial pressure and gas content inside a cavitating vortex core and influence on pressure fluctuations in the draft tube of a Francis type pump turbine. Proc. IME 182, pp 313-319
[15] Bosschers J 2008, Analysis of inertial waves on inviscid cavitating vortices in relation to low-frequency radiated noise, WIMRC Cavitation Forum, Warwick University (UK)