Strongly screening on electron capture for Nuclides $^{52,53,59,60}$Fe by the Shell-Model Monte Carlo method in pre-supernova

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Abstract The death of the massive stars due to supernova explosion is a key ingredient in stellar evolution, stellar population synthesis. The electron capture (EC) plays a vital role in supernovae explosions. According to the Shell-Model Monte Carlo (SMMC) method, basing on the Random Phase Approximation (RPA) and Linear Response Theory Model (LRTM), we study the strongly screening EC rates of nuclides $^{52,53,59,60}$Fe in presupernova. The results show that the screening rates can decrease about 18.66%. We compare our results with those of Fuller et al. (FFN), Aufderheide et al. (AUFD), and Nabi et al. (NKK) in the case with and without strong electron screening (SES). For the case without SES, our calculations are in very good agreement with those of AUFD in relatively high density surroundings and the maximum error is within 0.35% at $\rho r = 100$ (e.g., even-even nuclei $^{60}$Fe). However, for odd-A nuclei $^{59}$Fe, our rates are close to one, one, two order magnitude smaller than those of FFN, AUFD, and NKK. For the case with SES, our screening results are about three, two orders magnitude, and 7.27% lower than those of FFN, AUFD, NKK for $^{60}$Fe, respectively, and it is lower about two, two orders magnitude, and 12.42% than those of FFN, AUFD, NKK for odd-A nuclide $^{59}$Fe.

Keywords stars: supernovae, stars: evolution, Physical Date and Processes: nuclear reactions.

1 Introduction

It is well known that supernovae not only plays a critical role in the universe, but also supernovae are major sources of nucleosynthesis of stellar evolution and galactic chemical evolution. However, the driving mechanisms are still not well understood for two typical supernova (i.e. the core-collapse (type II) and thermonuclear (type Ia) supernovae). Some researches show that electron captures (hereafter EC) on medium-heavy nuclei play an important role in both types of supernovae. The weak interaction (e.g., EC and Beta decay) leads up the unstable nuclear burning and iron nucleus collapse in the process of the supernova explosions [1,2,3]. Therefore, some pioneer works on EC are investigated by Fuller, et al. [4,5] (FFN); Aufderheide, et al. [6,7] (AUFD); According to the Shell-Model Monte Carlo method [8,9,10,11], Langanke, et al. [12,13]; and Juodagalvis, et al. [14] in supernova. Liu, et al. [15,16,17,18,19,20,21,22,23,24,25,26] (NKK) also discussed the weak interaction reactions in explosive stellar environments due to the importance of EC.

Nonetheless, the problem on supernova explosion has always been the interesting and challenging issue in the fields of astrophysics. On the other hand, the strong electron screening (SES) has been raised a strong interesting and challenging problem among nuclear astrophysicist in pre-supernova stellar evolution and nucleosynthesis.

In the process of supernova explosion, what role on earth should the EC play in stars? How do the temperature and the density affect on the EC rates? what role will the SES play in stars? How does SES affect on the EC rates? These problems show that it is extremely important and useful to calculate accurately the EC rates for the research of supernova explosion and numerical-simulation. It is also extremely necessary for us to understand, solve and calculate accurately the SES and screening corrections in stellar interior for the relativistic degenerate electron liquid.
Some works \(6,7,27,28,18,20,4,5\) show that nuclides \(52,53,59,60\)Fe are very important and dominated during the process of supernova explosions. Thus the rates of them are widely investigated by some eminent scholars (e.g. \(6,7,27,28\)) in supernova. In the same environment, Liu, et al. and Gutierrez, et al. \(22,29\) also discussed the weak interaction reactions on \(52,53,59,60\)Fe. However, their works seem not to consider the influence of SES on EC. The problems about SES has already been discussed by Bravo et al. \(30\), and Liu et al. \(31\). The works mentioned above show that it is extremely important and necessary to calculate accurately the screening corrections to the EC rates in dense stars.

The effects of the charge screening on the nuclear physics (e.g. electron capture and beta decay) Come at least from three factors. Firstly, the screening potential changes the electron Coulomb wave function in the process nuclear reaction. Secondly, the electron screening potential decreases the energy of incident electron joining the capture reaction, which generally also decreases the electron capture rates of the nuclei. Thirdly, the electron screening increases the energy of atomic nucleus (i.e. the single particle energy will increases) in the process of nuclear reaction, thus increases the nuclear reaction rates. Finally, the electron screening evidently and effectively decreases the number of the higher-energy electrons, of which the energy is more than the threshold of the capture reaction. The screening relatively increases the threshold of the reaction and decreases the capture rates, but increases for the beta decay rates.

Due to the importance of SES about the EC of \(52,53,59,60\)Fe in astrophysical environments, in this paper, we focus on these nuclei and reinvestigate their EC rates according to the Shell-Model Monte Carlo (hereafter RPA) \(27\). For the case without SES, we analyse the error factor \(C\) and compared our results \((\lambda_{ec}^{0}(LJ))\) with those of AUFD \((\lambda_{ec}^{0}(AUFD))\), which is based on the theory of Brink Hypothesis \(6,7\). Furthermore, we discuss the electron capture cross section (hereafter ECCS) and the rates of the change of electron fraction (hereafter RCEF) in process of EC by using the theory of RPA. The comparisons of our rates with those of FFN \(12\), AUFD \(6\), and NKK \(26\) are presented in table format in the case without SES. On the other hand, basing on the linear response theory \(32\), we investigate the strongly screening rates and the screening factors \(C_1\). In order to understand the influence of SES on EC, we also compare our screening rates with those of FFN, AUFD, and NKK. We find the influence of SES on the rates is very significant.

Our work differs from previous works \(6,7,4,5,26\) about the discussion of EC. Firstly, the works of FFN and AUFD are based on the theory of Brink Hypothesis (BH). Basing on quasi-particle random phase approximation, NKK also discussed the EC reaction in the case without SES. We analyze the EC process for iron group nuclei and derive new screening results by using the Shell-Model Monte Carlo (SMMC) method \(27\) and basing on LRTM \(32\) and RPA theory \(12,13\). We also make detailed comparison of the results for the strong screening rates and no-screening rates among the calculations by FFN, AUFD, and NKK.

The present paper is organized as follows: in the next section, we analyses the EC rates in stellar interiors in the case with and without strong electron screening. Some numerical results and discussion are given in Section 3. And some conclusions are summarized in Section 4.

2 The EC in stellar interiors

2.1 The EC process in the case without SES

The stellar electron capture rates for the \(k\) th nucleus \((Z, A)\) in thermal equilibrium at temperature \(T\) is given by a sum over the initial parent states \(i\) and the final daughter states \(f\) \(4,5,6,7\)

\[
\lambda_k = \lambda_{ec}^0 = \sum_i \frac{2(J_i + 1) \exp(-E_i/kT)}{G(Z, A, T)} \sum_f \lambda_{if} \tag{1}
\]

The EC rate from one of the initial states to all possible final states is \(\lambda_{if}\). The \(J_i\) and \(E_i\) are the spin and excitation energies of the parent states, \(G(Z, A, T)\) is the nuclear partition function and given by

\[
G(Z, A, T) = \sum_i (2J_i + 1) \exp(-E_i/kT) \tag{2}
\]

Using the level density formula, \(\vartheta(E, J, \pi)\), the contribution from the excite states is discussed. Thus the
nuclear particle function approximately becomes [7]

\[ G(Z, A, T) \approx (2J_f + 1) + \int_0^{\infty} dE \int_{-\pi}^{\pi} dJ d\pi (2J_i + 1) \times \vartheta(E, J, \pi) \exp(-\frac{E_i}{kT}) \] (3)

where the level density is given by [16,38]

\[ \vartheta(E, J, \pi) = \frac{1}{\sqrt{2\pi a^{2}}} \times \frac{1}{2^{\frac{3}{2}}(E - \delta)} f(E, J, \pi) \] (4)

where

\[ a = \sqrt{2w^2} \]

\[ \delta = \frac{\sqrt{2a}}{} \]

\[ f(E, J, \pi) = \frac{1}{2^{2\frac{3}{2}}} \exp[-\frac{J(J + 1)}{2w^2}] \] (5)

and where

\[ w = \text{the radius and } m_u = \frac{1}{NA} \text{is the atomic mass unit}. \]

Based on the RPA theory with a global parameterization of the single particle numbers, the EC rates in the case without SES is related to the electron capture cross-section by [13]

\[ \lambda_{cc}^0(LJ) = \frac{1}{2^{2\frac{3}{2}}} \sum_{if} \int_{-\infty}^{\infty} \psi_i^{2} \sigma_{cc}(\varepsilon_e, \varepsilon_i, \chi)(\varepsilon_e, U_F, T) d\varepsilon_e \] (7)

where \( \varepsilon_0 = \text{max}(Q_{if}, 1) \), \( p_e = \sqrt{\varepsilon_e - 1} \) is the momenta of the incoming electron, and \( \varepsilon_e \) is the total rest mass and kinetic energies of the incoming electron, \( U_F \) is the electron chemical potential, \( T \) is the electron temperature. Note that in this paper all of the energies and the momenta are respectively in units of \( m_e c^2 \) and \( m_e c \), where \( m_e \) is the electron mass and \( c \) is the light speed in vacuum. The electron Fermi-Dirac distribution is defined as

\[ f = f(\varepsilon_e, U_F, T) = [1 + \exp(\frac{\varepsilon_e - U_F}{kT})]^{-1} \] (8)

Due to the energy conservation, the electron, proton and neutron energies are related to the neutrino energy, and Q-value for the capture reaction [36]

\[ Q_{if} = \varepsilon_e - \varepsilon_\nu = \varepsilon_n - \varepsilon_\nu = \varepsilon_f^p - \varepsilon_i^p \] (9)

and we have

\[ \varepsilon_f^n - \varepsilon_i^p = \varepsilon_{f, if} + \mu + \Delta_{np} \] (10)

where \( \mu = m_n - m_p \), the difference between neutron and proton chemical potentials in the nucleus and \( \Delta_{np} = M_n c^2 - M_p c^2 = 1.293 MeV \), the neutron and the proton mass difference. \( Q_{i0} = M_f c^2 - M_i c^2 = \mu + \Delta_{np} \), with \( M_i \) and \( M_f \) being the masses of the parent nucleus and the daughter nucleus respectively; \( \varepsilon_{f, if} \) corresponds to the excitation energies in the daughter nucleus at the states of the zero temperature.

The electron chemical potential is found by inverting the expression for the lepton number density

\[ n_e = \frac{8\pi}{(2\pi)^3} \int_0^{\infty} p_e^2 (f_{-e} - f_{+e}) dp_e \] (11)

where \( f_{-e} = [1 + \exp(\frac{\varepsilon_e}{kT})]^{-1} \) and \( f_{+e} = [1 + \exp(\frac{\varepsilon_e}{kT})]^{-1} \) are the electron and positron distribution functions respectively, \( k \) is the Boltzmann constant.

According to the Shell-Model Monte-Carlo method, which discussed the Gamow-Teller strength distributions, the total cross section by EC is given by [27]

\[ \sigma_{cc} = \sigma_{cc}(E_e) = \sum_{if}(2J_i + 1) \exp(-\beta E_i) \sigma_{fi}(E_e) \]

\[ = 6g_{wek}^2 \int d\xi (E_e - \xi)^2 \frac{G_A^2}{12\pi} S_{GT}(\xi) F(Z, E_e) \] (12)

where \( \beta = 1/T_N \) is the inverse temperature, \( T_N \) is the nuclear temperature and in unit of Mev, and \( E_e = \varepsilon_e \) is the electron energy. \( S_{GT} \) is the Gamow-teller(GT) strength distribution, which is as a function of the transition energy \( \xi \). The \( g_{wek} = 1.1661 \times 10^{-5} \text{Gev}^{-2} \) is the weak coupling constant and \( A \) is the axial vector form-factor which at zero momentum is \( G_A = 1.25 \). \( F(Z, \varepsilon_e) \) is the Coulomb wave correction which is the ratio of the square of the electron wave function distorted by the coulomb scattering potential to the square of wave function of the free electron.

The SMMC method is also used to calculate the response function \( R_A(\tau) \) of an operator \( A \) at an imaginary time \( \tau \). By using a spectral distribution of initial and final states \( |i \rangle \) and \( |f \rangle \) with energies \( E_i \) and \( E_f \). \( R_A(\tau) \) is given by [12]

\[ R_A(\tau) = \frac{\sum_{if}(2J_i + 1) \exp(-\beta E_i) \exp(-\tau(E_f - E_i))|\langle f|\hat{A}|i \rangle|^2}{\sum_{if}(2J_i + 1) \exp(-\beta E_i)} \] (13)

Note that the total strength for the operator is given by \( R(\tau = 0) \). The strength distribution is given by

\[ S_{GT}(E) = \frac{\sum_{if} \delta(E - E_f + E_i)(2J_i + 1) \exp(-\beta E_i) |\langle f|\hat{A}|i \rangle|^2}{\sum_{if}(2J_i + 1) \exp(-\beta E_i)} \] (14)

which is related to \( R_A(\tau) \) by a Laplace Transform, \( R_A(\tau) = \int_{-\infty}^{\infty} S_{GT}(E) \exp(-\tau E) dE \). Note that here \( E \) is the energy transfer within the parent nucleus, and that the strength distribution \( S_{GT}(E) \) has units of Mev\(^{-1}\).

The presupernova EC rates in the case without SES is given by folding the total cross section with the flux of a degenerate relativistic electron gas [12]

\[ \lambda_{cc}^0(LJ) = \frac{\ln 2}{6163} \int_0^{\infty} \frac{d\xi S_{GT}(E)}{c^3 (mn e)^2} \]

\[ \int_{\mu_0}^{\infty} dp e^{2p^2} (-\xi + \varepsilon_e)^2 F(Z, \varepsilon_e) f(\varepsilon_e, U_F, T) \text{ (s}^{-1}) \] (15)
where the $\xi$ is the transition energy of the nucleus, and $f(\varepsilon_n, U_F, T)$ is the electron distribution function. The $p_0$ is defined as

$$
\begin{align*}
  p_0 &= \begin{cases} 
    \sqrt{\Delta Q_{if}^2 - m_e^2c^4} & (Q_{if} < -m_e c^2) \\
    0 & \text{(otherwise)}.
  \end{cases}
\end{align*}
$$

(16)

In the case without SES, we define the error factors $C$, which compare our results of $\lambda^e_{ec}(LJ)$, which discussed by method of SMMC with those of $\lambda^e_{ec}(AFUD)$, which calculated basing on the method of Brink Hypothesis by AUFD.

$$
C = \frac{(\lambda^e_{ec}(LJ) - \lambda^e_{ec}(AFUD))}{\lambda^e_{ec}(LJ)}
$$

(17)

On the other hand, the RCEF plays a key role in stellar evolution and presupernova outburst. In order to understand how would the SES effect on RCEF, the RCEF due to EC reaction on the $k$ th nucleus in SES is defined as

$$
Y_{ec}(k) = -\frac{X_k}{A_k} \lambda_k
$$

(18)

where $\lambda_k$ is the EC rates; $X_k$ is the mass fraction of the $k$ th nucleus and $A_k$ is the mass number of the $k$ th nucleus.

2.2 The EC process in the case with SES

Using the linear response theory, Itoh et al. [32] calculated the screening potential for relativistic degenerate electrons. We name this the linear response theory model (hereafter LRTM) with SES. A more precise screening potential in LRTM is given by

$$
D = 7.525 \times 10^{-3} Z \frac{10^5 \rho_T}{A} \frac{1}{(m_e c^2)^5} J(\varepsilon_s, R) \text{(MeV)}
$$

(21)

where $J(\varepsilon_s, R)$, $\varepsilon_s$ and $R$ can be found in Ref. [17]. The formula (21) is valid for $10^{-5} \leq \varepsilon_s \leq 10^{-1}$, $0 \leq R \leq 50$ conditions, which are usually fulfilled in the pre-supernova environment.

If the electron is strongly screened and the screening energy is high enough in order not to be neglected in high density plasma, its energy will decrease from $\varepsilon$ to $\varepsilon' = \varepsilon - D$ in the decay reaction due to electron screening. At the same time, the screening relatively decreases the number of high energy electrons with energies higher than the threshold energy for electron capture. The threshold energy increases from $\varepsilon_0$ to $\varepsilon_s = \varepsilon_0 + D$. Thus the EC rates with SES becomes

$$
\lambda^e_{ec}(LJ) = \ln \frac{2}{6163} \int_0^\infty d\xi S_{GT}(\xi) \frac{e^3}{(m_e c^2)^5} \int_{\varepsilon_0}^\infty d\varepsilon' \left( (\varepsilon'^2 - 1) \frac{1}{2} (\varepsilon'^2 - 1) \frac{1}{2} \right) F(Z, \varepsilon') f(\varepsilon', U_F, T)
$$

(22)

where $C_1$ is the screening factors $C_1$ in the care with and without SES in order to understand the effect of SES on the EC process as follows:

$$
C_1 = \frac{\lambda^e_{ec}(LJ)}{\lambda^e_{ec}(LJ)}
$$

(23)

3 The results and discussion

Figure 1 shows the ECCS of nuclides $^{52,53,59,60}\text{Fe}$ as a function of electron energy at temperature $T_0 = 9, 11$. We find with increasing of electron energy, the ECCS increases greatly. The higher the temperature, the faster the changes of ECCS becomes. It is because that the higher the temperature, the larger the electron energy and electron chemical potential are. So even more electrons will join in the EC process due to their energy is greater than the $Q$-values. Furthermore, the Gamow-Teller transition would be dominated in this process at high temperature surroundings.

As we all know, the trigger of the electron capture requires a minimum electron energy given by the mass splitting between parent and daughter (i.e. $Q_{\beta}$). This threshold is lowered by the internal excitation energy at finite temperature. For even-even parent nuclei the Gamow-Teller strength centered at daughter.
energies in comparison to even-even parent nuclei by electron energies above threshold. But for odd-A nuclei the Gamow-Teller distribution will peak at noticeably higher daughter excitation energies at low temperatures. So the ECCS are shifted to higher electron energies in comparison to even-even parent nuclei by about 3 MeV.

The EC rates as function of $\rho T$ at some typical astrophysics condition are shown in Figure 2 and 3. We detailed discuss the EC process according to SMMC method, especially for the contribution for EC due to the GT transition base on RPA theory. We find the EC rates are increased greatly and may be in excess of six orders of magnitude with increasing of density (e. g. for $^{60}$Fe at $T_0 = 3.40, Y_e = 0.47$). On the other hand, the density make the different effect on the EC rates for different nuclides due to different Q-values and transition orbits.

According to our calculations, we find that the GT transition in EC reaction may not be dominant at a relative low temperature. This process can be dominated by low energy transition. On the contrary, the distribution of the electron gas should satisfy the Fermi-Dirac distribution under the condition of high temperature and density. The GT transition strength of nuclei is distributed in the form of the centrosymmetric Gaussian function about the GT resonance point. So the energies of the electrons, which can participate in the GT resonance transitions are not symmetric in relative high energy range.

The Gamow-Teller strength distributions from shell model Monte Carlo studies of fp-shell nuclei play an important role in the pre-collapse evolution of supernovae. The GT$^+$ transitions, which change protons into neutrons, have so far been addressed only qualitatively in presupernova simulations because of insufficient experimental information, assuming the GT$^+$ strength to reside in a single resonance whose energy relative to the daughter ground state has been parametrized phenomenologically $[15]$. (n, p) experiments show that the GT$^+$ strength is fragmented over many states, while the total strength is significantly quenched compared to the single particle model.

As an example, we plot the strength distributions $S_{\text{GT}^+}$ as a function of daughter state excitation energy for nuclei $^{59}$Fe. We show the calculated strength functions for GT$^+$ to the two parent states, the ground state ($0^+$) and first excited state ($2^+$) of $^{60}$Fe in Figure 4. We consider and reproduce the first few low-lying levels in $^{60}$Fe, which are 0, 1.1, 2.2, 2.4MeV corresponding to the spin parity of 0$^+$, 2$^+$, 0$^+$, 2$^+$. We find the peak of $S_{\text{GT}^+}$ will get to 1.562, 0.223MeV$^{-1}$ at 0.5MeV, 3.40MeV of daughter nuclei $^{60}$Mn for ground state and 1st excited state, respectively. The total GT strength distribution $B(\text{GT})_{\text{tot}}$ for the ground state ($0^+$) and first excited state ($2^+$) is 9.47, 8.19 MeV, respectively. From above discussion, by simply displacing the ground state strength distribution by the excitation energy, one can see that the GT distribution for the excited state may not be qualitatively inferred from the information of the ground state. In fact we think an average value of the excited state distributions may be the most standard distribution, which would appear to be the one pertaining to the excited states.

The RCEF is very sensitivity parameter in the electron capture process. We find the RCEF decreases and even more than by four orders of magnitude for $^{60}$Fe at $T_0 = 7.33$ in Figures 5. With increasing of the density and temperature, the electron chemical potential becomes so high that large numbers of electrons join to the EC process. Thus the RCEF reduce greatly.

It is well known that the electron capture (EC) plays a vital role in supernovae explosions. AUFD expended FFN’s work based on shell model. Their works are based on the theory of Brink Hypothesis, which detailed discussed by FFN in the case without SES. We analyze the EC process and derive new results according to the Shell-Model Monte Carlo (SMMC) method and the Random Phase Approximation (RPA) theory. In this paper, we define the error factors C in order to compare our results ($\lambda_{\nu}^\alpha$ (LJ)) with those of AUFD ($\lambda_{\nu}^\alpha$ (AUFD)). We hope to find some difference between these two methods at different typical stellar conditions.

Figure 6 shows the results of the error factors C as a function of density $\rho T$. One can see that with increasing of density, the factor C reduces greatly. According to our calculations, we find that our results is agreed reasonably well with AUFD in a high density environment (e. g. $\rho T = 100$) and the maximum error is within 0.35%. On the other hand, in a relative low density surroundings the maximum error is within 3.982% (e. g. $\rho T = 10, Y_e = 0.41, T_0 = 12.6$).

The EC rates on these iron group nuclei are important from the oxygen shell burning phase up to the end of convective core silicon burning phase of massive stars. Some pioneer works on EC rates have been done by FFN, AUFD, and NKK. As example, the comparisons of several EC rates (i.e. FFN’s, AUFD’s, NKK’s, and ours) for nuclides $^{59}$Fe and $^{60}$Fe are presented in a tabular form at $\rho T = 4010, Y_e = 0.41, T_0 = 7.33$ in the case without SES. We find it is well agreement between ours and AUFD’s for even-even nuclide $^{60}$Fe. The factor C is about 0.832, 3.848, and 1.7267 corresponding to those of AUFD, FFN and NKK. The comparisons for
odd-A nuclide $^{59}$Fe show that the rates of FFN, AUFD, and NKK are close to by one, one, and two order magnitude bigger than ours.

Table 2 presents the comparison of our strongly screening results with those of FFN, AUFD, NKK. From the results of $s_i$ (i=1, 2, 3, 4), one can conclude that the strongly screening rates are about three and two orders magnitude lower than those of FFN and AUFD for even-even nuclide $^{60}$Fe, respectively, but is about two orders magnitude for odd-A nuclide $^{59}$Fe. On the other hand, due to SES, our screening rates decreases about 12.42%, 7.27% comparing to those of NKK for odd-A nuclide $^{59}$Fe and even-even nuclide $^{60}$Fe, respectively.

The screening factors $C_1$ is plotted as a function of $\rho_T$ from figure 7 to 9. Due to SES, we find the rates decrease greatly and even more than by $\sim 18.66\%$ and $\sim 17.80\%$ in Figure 6. The lower the temperature, the larger the effect on EC rates becomes. This is due to the fact that the SES mainly decreased the number of higher energy electrons, which can actively join in the
EC reaction. On the other hand, the lower the temperature (e.g. in Figure 7), the larger the effect on $C_1$ is. However, the higher temperature (e.g., in Figure 8), the higher the average electron energy becomes, thus the smaller the effect on $C_1$ is, due to the relatively low screening potential. One can also find from Figure 7 to 9 that the screening factor is nearly the same at higher density and independent on the temperature and density. The reason is that at higher density surroundings the electron energy is mainly determined by its Fermi energy, which is strongly decided by density.

Because of relative low electron screening potential at the low density, we find that the lower the density, the smaller the effect on EC becomes. As the density increases, the $C_1$ increases gradually due to the increases of the screening potential. As the density further increases, the factor $C_1$ will close to identical at relative high density. This is because the electron energy is mainly determined by Fermi energy at higher density.
and the effect is relatively weakened by temperature. As the density increases, the electronic Fermi and shielding potential increases. The ratio between shielding potential and Fermi energy has nothing to do with density approximately.

Of course, we know the screening of nuclear electric charges with a high electron density means a short screening length, which means a lower enhancement factor from Coulomb wave correction. However, even a relatively short electric charge screening length will not have much effect on the overall rate due to the weak interaction is effectively a contact potential. A bigger effect is that electrons are bound in the plasma. Table 3 in detail shows the numerical calculations about the relationship by the minimum value of screening factor $C_{1 min}$ between the rates in the case with and without SES. For example, the EC rates of nuclei $^{52,53,59,60}$Fe are decreased about $\sim 1.40\%$, $\sim 2.12\%$, $\sim 17.80\%$, $\sim 18.66\%$ at $T_9 = 0.133, Y_e = 0.485$, respectively.
The Q-value of electron capture for some neutron rich nuclei (e.g. \(^{60}\text{Fe}\)) has not been measured, so that the EC Q-value has to be estimated with a mass formal by FFN. FFN used the Semiempirical atomic mass formula (see Ref. [38]), thus the Q-value used in the effective rates are quite different. On the other hand, For odd-A nuclei (e.g. \(^{59}\text{Fe}\)), FFN places the centroid of the GT strength at too low excitation energies (see the discussions in Ref. [5]). Their rates are thus somewhat overestimated. Using the nuclear shell model, AUFD expanded the FFN’s works. AUFD analyzed the nuclear excited level by a simple calculation on the nuclear excitation level transitions. The capture rates are made up of the lower energy transition rates between the ground states and the higher energy transition rates between GT resonance states. The works of FFN and AUFD may be an oversimplification and therefore the accuracy is limited. they adopt the so-called Brinks hypothesis in their calculations. This hypothesis assumes that the Gamow-Teller strength distribution on excited
states is the same as for the ground state, only shifted by the excitation energy of the state. This hypothesis is used because no experimental data is available for the Gamow-Teller strength distributions from excited states and they did not employ any microscopic theory to calculate the Gamow-Teller strength functions from excited states.

Using the pn-QRPA theory, NKK expanded the FFN’s works and analyzed nuclear excitation energy distribution. They have taken into consideration the particle emission processes, which constrain the parent excitation energies. The pn-QRPA theory calculates stronger Gamow-Teller strength distribution from these excited states compared to those assumed using Brinks hypothesis. However in the GT transitions considered process in their works, only low angular momentum states are considered.

The method of SMMC is actually adopts an average of GT intensity distribution of electron capture and the calculated results are in good agreement with experiments, but the results for most nuclei are generally smaller than other methods, especially for some odd-A nuclides(e.g. $^{59}$Fe). The charge exchange reactions (p, n) and (n, p) make it possible to observe in the process of weak interaction, especially for the information of the total GT strength distribution in nuclei. The experimental information is particularly rich for some iron nuclei and it is the availability of both GT$^{+}$ and GT$^{-}$, which makes it possible to study the problem of renormalization of $\sigma_T$ operators in detail. We have calculated the total GT strength in a full p-f shell calculation, resulting in $B_{GT} = g_A^2 |\langle \sigma_T \rangle|^2$, where $g_A^2$ is axial-vector coupling constant.

For example, in presupernova the electron capture reaction on $^{59}$Fe is dominated by the wave functions of the parent and daughter states. The total GT strength for $^{59}$Fe in a full p-f shell calculation, is resulting in $B_{GT} = 10.1 g_A^2 \rho_7^2$. An average of the GT strength distribution is in fact obtained by SMMC method. A reliable replication of the GT distribution in the nucleus is carried out and detailed analysis by using an amplification of the electronic shell model. Thus the method is relative accuracy.

Summing up the above discussion, basing on RPA and linear response theory, by using the method of SMMC, we have discussed the EC rates in SES. One can see that the SES has an evident effect on EC rates for different nuclei, particularly for heavier nuclides whose threshold is negative (e.g. $^{59,60}$Fe) at relative lower temperature and higher density environment. According to above calculations and discussion, one can conclude that the strongly screening rates are decreased greatly and may be in excess of $\sim 18.66\%$ based on the RPA theory and SMMC method.

4 Conclusions

In this paper, basing on RPA theory and LRTM, by using SMMC method, we have carried out an estimation for the EC rates of $^{52,53,59,60}$Fe in the case with and without SES. Meanwhile, the ECCS and the RCEF are discussed in SES. We also detailed compare our results
is the rates for FFN, AUFD, and NKK respectively in the case without SES. Even more than by four orders of magnitude (e.g. for ete in the EC process and the RCEF decreases and increases greatly. The RCEF is very sensitivity parameter of SES. With increasing of electron energy, the ECCS case without SES. with those of FFN, AUFD, and NKK, which are in the case without SES.

Table 1 The comparisons of our calculations of EC rates in the case without SES for nuclides $^{59}$Fe and $^{60}$Fe with those of FFN [5], AUFD [7] and NKK [26] at $\rho_T = 4010, Y_e = 0.41, T_0 = 7.33$. The ratio computes as $k_i = \frac{\lambda_i^{(LJ)}}{\lambda_i^{(FFN)}}, \lambda_i^{(AUFD)} (i = 1, 2, 3)$ is the rates for FFN, AUFD, and NKK respectively in the case without SES.

| Nuclide | $\lambda_i^{(FFN)}$ | $\lambda_i^{(AUFD)}$ | $\lambda_i^{(NKK)}$ | $\lambda_i^{(LJ)}$ | $k_1$ | $k_2$ | $k_3$ |
|---------|----------------|----------------|----------------|----------------|-----|-----|-----|
| $^{59}$Fe | 7.20e+02 | 7.43e+02 | 2.7e+02 | 7.789e+01 | 9.244 | 9.539 | 34.704 |
| $^{60}$Fe | 6.73e+01 | 1.44e+01 | 3.02e+01 | 1.749e+01 | 3.848 | 0.823 | 1.7267 |

Table 2 The comparisons of our calculations of EC rates for nuclides $^{59}$Fe and $^{60}$Fe with those of FFN [5], AUFD [7] and NKK [26] at $\rho_T = 33, Y_e = 0.45, T_0 = 4.24$. The ratio computes as $s_j = \frac{\lambda_s^{(LJ)}}{\lambda_s^{(FFN)}}, \lambda_s^{(AUFD)} (j = 1, 2, 3, 4)$ is the rates for FFN, AUFD, NKK, and ours respectively in the care without SES.

| Nuclide | $\lambda_s^{(FFN)}$ | $\lambda_s^{(AUFD)}$ | $\lambda_s^{(NKK)}$ | $\lambda_s^{(LJ)}$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|---------|----------------|----------------|----------------|----------------|-----|-----|-----|-----|
| $^{59}$Fe | 6.30e–03 | 5.30e–03 | 6.20e–05 | 5.43e–05 | 8.6190e–03 | 1.0245e–02 | 0.8758 | 0.9644 |
| $^{60}$Fe | 4.60e–03 | 1.00e–03 | 1.10e–05 | 1.02e–05 | 2.2174e–03 | 1.0200e–02 | 0.9273 | 0.9444 |

Table 3 The minimums value of strong screening factor $C_1$, which is comparisons of the screening rates with those of no-screening rate for some typical astronomical condition when $1 \leq \rho_T \leq 200$.

| Nuclide | $\rho_T$ | $C_{min}$ | $\rho_T$ | $C_{min}$ | $\rho_T$ | $C_{min}$ | $\rho_T$ | $C_{min}$ |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|
| $^{52}$Fe | 25 | 0.9986 | 18 | 0.9997 | 19 | 0.9998 | 41 | 0.9999 |
| $^{59}$Fe | 10 | 0.9788 | 10 | 0.9854 | 9 | 0.9960 | 8 | 0.9984 |
| $^{59}$Fe | 15 | 0.8220 | 15 | 0.9670 | 13 | 0.9944 | 12 | 0.9978 |
| $^{60}$Fe | 26 | 0.8134 | 26 | 0.9641 | 14 | 0.9937 | 21 | 0.9971 |

with those of FFN, AUFD, and NKK, which are in the case without SES.

Firstly, We find the influence on ECCS is very obvious and significant by temperature under the condition of SES. With increasing of electron energy, the ECCS increases greatly. The RCEF is very sensitivity parameter in the EC process and the RCEF decreases and even more than by four orders of magnitude (e.g. for $^{60}$Fe at $T_0 = 7.33$).

Secondly, for the case without SES, the EC rates increase greatly and ever exceed by six orders of magnitude (e.g. for $^{60}$Fe at $T_0 = 3.40, Y_e = 0.47$). We compare our results with those of AFUD due to different methods for calculating the EC rates. One can find our calculations are in very good agreement with those of AUFD in relative high density surroundings (e. g. $\rho_T = 100$) and the maximum error is within 0.35%. However, it is within 3.982% in a relative low density surroundings (e. g. $\rho_T = 10, Y_e = 0.41, T_0 = 15.6$). On the other hand, as examples, we also discuss the comparisions of our calculated rates with those of FFN, AUFD and NKK of $^{59}$Fe and $^{60}$Fe. We find it is well agreement between our results and AUFD’s for even-even nuclide $^{60}$Fe (i.e. the factor C is about 0.832, but is 3.848, 1.7267 corresponding to FFN and NKK, respectively). The comparisons for odd-A nuclide $^{59}$Fe show that the rates of FFN, AUFD, and NKK are close to by one, one, and two order magnitude bigger than ours.

Finally, for the case with SES, by using SMNC method, we discuss the strongly screening rates in supernovae explosive stellar environments basing on RPA and linear response theory. We compare our strongly screening results with those of FFN, AUFD, and NKK in the case without SES. One can find that the strongly screening rates are about three and two orders magnitude lower than those of FFN and AUFD for even-even nuclei $^{60}$Fe, respectively, but it is lower about two orders magnitude for odd-A nuclei $^{59}$Fe. Our screening rates are decreased about 12.42%, 7.27% comparing to those of NKK for odd-A nuclei $^{59}$Fe and even-even nuclide $^{60}$Fe, respectively. However, according to our calculations, our strongly screening rates ($\lambda_s^{(LJ)}$) are decreased greatly and even exceed $\sim 18.66\%$ corresponding to those of $\lambda_s^{(LJ)}$ in the case without SES.

As we all know, the EC by SES play an important role in the dynamics process of the collapsing core of
a massive star. It is the main parameter which leads to a supernova explosion and stellar collapse. It also is quite relevant for simulations in the process of collapse and explosion for massive star. The SES also strongly influence on the cooling rate and evolutionary timescale in EC and beta decay process. Our calculations may be helpful for study of the stellar and galactic evolution and nucleosynthesis calculations. The results we derived, may become a good foundation for the future investigation of the evolution of late-type stars, the nature of mechanism of supernova explosions and the numerical simulation of supernovas.

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