Coulomb blockade in two island systems with highly conductive junctions

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We report measurements on single-electron pumps, consisting of two metallic islands formed by three tunnel junctions in series. We focus on the linear-response conductance as a function of gate voltage and temperature of three samples with varying system parameters. In all cases, strong quantum fluctuation phenomena are observed by a \( \log (k_B T / (2E_{\text{co}})) \) reduction of the maximal conductance, where \( E_{\text{co}} \) measures the coupling strength between the islands. The samples display a rich phenomenology, culminating in a non-monotonic behavior of the maximal conductance as a function of temperature.

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The transport properties of single electron devices are well described by the so called sequential tunneling model as long as the conductances of the underlying tunneling contacts are small compared to the conductance quantum \( G_K = e^2 / h \). In most practical cases, however, one has to include higher-order corrections for a quantitative description due to quantum fluctuations of the charge number on the single electron islands. This has been demonstrated clearly for the single electron transistor (SET) \( \text{[3, 4, 8]} \). Excellent agreement between experimental investigations and theoretical studies has been established at low conductances with the aid of perturbation expansion including the correct description of a logarithmic contribution to the linear-response conductance of the multichannel-Kondo type. At higher conductances however, perturbation theory breaks down. Nevertheless, by applying Monte-Carlo methods, the experimental results can be described with amazingly high accuracy \( \text{[9, 10, 11]} \). The findings on the SET indicate that a close match between the model Hamiltonian and its experimental realization exists for single electron devices. It is not clear a priori whether the good agreement between perturbation expansion and experiment as well as Quantum-Monte-Carlo numerics found for the SET survives if more complex arrangements of single electron islands are investigated. E. g. the low temperature conductance of the SET involves finite occupation of only two states and the mapping onto the multichannel-Kondo model is based on identifying those two states with a pseudo spin. This procedure has no simple analogy in general arrangements. It is worthwhile to check the range of validity of perturbation theory in more general cases. Two island systems, readily accessible both by experiment and theory, serve as a good starting point.

In this Letter we present an experimental study of the linear-response conductance of two single electron islands in series, an arrangement nicknamed single electron pump (SEP) \( \text{[3, 4, 8]} \) and sketched in Fig. 1. The linear-response conductance varies with the gate voltages and is bound between temperature dependent values \( G_{\text{min}}(T) \) and \( G_{\text{max}}(T) \). For samples in an interesting and accessible parameter range, the maximal conductance \( G_{\text{max}} \) obeys a non-monotonic behavior as a function of temperature, which is reported here for the first time. This behavior, astonishing at first sight, is naturally explained by the sequential tunneling model in this Letter, giving evidence that non trivial behavior is to be expected in complex arrangements of single-electron islands. Beyond the validity range of the sequential model, we find a logarithmic correction of the conductance at low temperature due to quantum fluctuations. Our high quality data make a sensitive test of perturbation theory as developed e. g. in Ref. 12 feasible. This however, requires an elaborate calculation along the line of formulae in Ref. 12 which is out of the scope of the work presented here.

The linear-response conductance of the SEP can be modeled in various coordinate systems spanning the \((V_1, V_2)\) plane. In this letter we choose dimensionless coordinates \( n_x \) and \( n_y \) defined such that the charging energy \( E_{\text{ch}} = E_{\text{cx}}(n_x - n_s)^2 + E_{\text{cy}}(n_y + \Delta n + \kappa n_s)^2 \). Here
The coordinate capacitances in Fig. 1 (i.e. all capacitances except $E_{c_h}$) possess the lowest energy $E_{c_h}(n_l, n_r)$. b) Measurement of the linear-response conductance of sample 2 as a function of both gate voltages $V_1$ and $V_2$ at 150 mK. The outermost contour line indicates a conductance of 0.01 pS. The following lines range from 0.05 pS to 0.4 pS with a 0.05 pS spacing. The stability diagram (thick lines) is deformed in the coordinates of this figure.

$E_{c_x} = e^2/(2C_s)$ with $C_s$ being the sum of all external capacitances in Fig. 1 (i.e. all capacitances except $C_m$), $E_m = (e^2/2)C_s/(C_m C_s + C_r C_s)$ with $C_r$ and $C_s$ denoting the sum of the external capacitances on the left and the right island separately, and $\kappa = (C_r - C_m)/C_s$. The coordinate $n_x$ is associated with the change of the total charge number $n_x = n_l + n_r$, while $n_y$ redistributes the charge between both islands ($\Delta n = n_l - n_r$).

Each point in the plane spanned by $n_x$ and $n_y$ can be mapped onto a charge ground state $(n_l, n_r)$ which gives the lowest possible electrostatic charging energy $E_{c_h}(n_l, n_r)$. This procedure divides the $(n_x, n_y)$ plane into the grid of hexagonal cells depicted in Fig. 2. The length $E_{co}/E_{c_x}$ of the horizontal cell boundaries is a measure of the coupling strength, where $E_{co} = E_{c_x} + E_m(\kappa^2 - 1)/4 \propto C_m$. Due to the periodicity it suffices to study the linear-response conductance in a small exemplary portion of the $(n_x, n_y)$ plane. Within the sequential model, the linear-response conductance of the system vanishes exponentially at low temperatures except close to the triple points in the $(n_x, n_y)$ plane, where the ground-state energy of all three adjacent states is degenerate. The conductance peaks near these points. It is worthwhile to mention a peculiar behavior of the SEP: Exactly at the triple points the low temperature conductance is constant and given by $G_0/3$ where $G_0^{-1} = G_l^{-1} + G_r^{-1} + G_m^{-1}$. Although for $T \ll E_{co}/k_B$ the maximal conductance $G_{\text{max}}$ is temperature independent as well, the system assumes $G_{\text{max}}$ at a slightly different position in the $n_x$ direction [12]. Taking only three states into account (the occupation probability of all other states is exponentially small for $T < E_{co}/k_B$) we get

$$G(n_x)|_{n_y = -\kappa} = \frac{G_s}{2 + e^{-\beta \Delta E}} \left( \frac{g_s}{g_m} + \frac{e^{\beta \Delta E} - 1}{\beta \Delta E} \right)^{-1}, \quad (1)$$

with $\beta = 1/(k_B T)$, $g_s = g_l g_r/(g_l + g_r) = G_s/G_K$, and $\Delta E = 2E_{c_x}(n_x - 1) + E_{co}$. Depending on the ratio $g_s/g_m$ the peak position deviates from the triple point at $\Delta E = 0$ for $g_m \neq 2g_s$.

In a measurement as a function of $n_x$ and $n_y$ the conductance displays a periodic grid of peaks (grouped as pairs), marking the endpoints of the horizontal boundaries in Fig. 2. With rising temperature the peaks broaden and shift towards the center of the horizontal boundaries. Fig. 2, gives an example at $T = 150$ mK$\sim 0.09E_{co}/k_B$, where thermal broadening already is effective in merging the two separate peaks. Finally the two peaks merge completely and the conductance takes its maximal value at the mid-points, e.g. at $n_x = 1$ and $n_y = -\kappa$.

We study three samples in two different layouts. All samples have been produced by standard shadow-evaporation technique from aluminum with aluminum-oxide barriers. The barriers for the middle and outer contacts are fabricated in different oxidation steps, making different barrier thicknesses for internal and external contacts possible. Sample 1 has a simple layout (not shown) with three contacts in a row, which is the most natural arrangement to fabricate a SEP. In this layout the serial conductance is accessible, but the conductance of the individual contacts remains unknown. In the slightly more complex structure of sample 2 and 3 (see Fig. 3) each island is connected to independent leads via two contacts. This permits measuring the conductance of different serial combinations of the contacts, and thus the individual contact conductances can be determined. In the final experiments both external contacts of each island are operated in parallel (connected to the same voltage source), and the two contacts then act exactly as a single contact. We could not detect any degradation (considering noise performance or sensitivity to external disturbances) for the latter samples compared to sample 1.

FIG. 2: a) Stability diagram for the SEP sketched in Fig. 1. The hexagonal cells mark regions where the indicated charge state $(n_l, n_r)$ possesses the lowest energy $E_{c_h}(n_l, n_r)$. b) Measurement of the linear-response conductance of sample 2 as a function of both gate voltages $V_1$ and $V_2$ at 150 mK. The outermost contour line indicates a conductance of 0.01 pS. The following lines range from 0.05 pS to 0.4 pS with a 0.05 pS spacing. The stability diagram (thick lines) is deformed in the coordinates of this figure.

FIG. 3: Scanning electron microscope picture of sample 2. The two T-shaped structures in the middle are the islands. The inner tunneling contact is visible in the center of the picture. The outer contacts are turned by 90° with respect to the former. The gate lines are visible at top and bottom pointing up and down towards the inner contact. In the final measurement the top and bottom leads are biased in parallel.
The (n_x, n_y) plane at low temperatures (see Fig. 2) we obtain the parameters κ and E_{co}/E_{cx}. In addition we measure the conductance in the high temperature region where it does not depend on either n_x or n_y. In close analogy to the high temperature expansion for the SET, it behaves as G(T) ≈ G_0 \((1 - E_c/(3k_B T)) \). It can be shown 13, 14 that the relation
\[
E_{cx} = \frac{E_c}{G_0} \left( \frac{e_+}{G_I} + \frac{e_-}{G_r} + \frac{e_+ + e_- - 2E_{co}/E_{cx}}{G_m} \right)^{-1}
\]
holds, where e_{±} = \((E_{co}/E_{cx}) (κ ± 1)/2 \)/(κ ± 1) and E_c is an experimentally determined fitting parameter. Tab. I gives all relevant sample parameters.

To simplify the analysis of our data and facilitate the comparison with theoretical considerations we focus on the temperature dependence of three special values of the conductance in the (n_x, n_y) plane, namely G_{min}, G_{max}, and G_m, the latter defined as the conductance at the center of the horizontal boundaries in Fig. 2b, (i.e. at n_x = 1, n_y = −κ). In Fig. 4 we display our main findings. In addition to our experimental results we present the outcome of a calculation in the framework of the sequential tunneling model using the parameters from Tab. I. The techniques for such a calculation are well documented in the literature (e.g. 2) so we do not comment on this calculation in detail. It requires the solution of a master equation which one may set up using golden rule rates for the inelastic tunneling events. For T ∝ E_c/k_B at most four (n_1, n_r) states are occupied with reasonable probability. Restricting the master equation to these four states allows for an analytical solution. At higher temperatures numerical relaxation methods are used.

The sequential tunneling model gives good agreement with our experimental data with the exception of G_{max} at low temperature. The latter deviations are discussed at the end of the letter. The overall behavior (see Fig. 4) is governed by two scales \((E_{co}/k_B\) and \(E_c/k_B\) at which \(G_m(T)\) and \(G_{min}(T)\), respectively, start to increase and finally merge with \(G_{max}(T)\). To get finite conductance through the SEP at least three charge states have to be occupied in thermal equilibrium. At T ∝ E_{co}/k_B this is only possible near the triple points where three adjacent states are occupied. Here e.g. the sequence (0, 0) → (1, 0) → (0, 1) → (0, 0), corresponding to a charge transfer from left to right, occurs with finite probability. The inverse process is equiprobable, but under voltage bias a net current occurs. At n_x = 1, n_y = −κ

where \(G_m\) is measured, the charging energy of states \((0, 0)\) and \((1, 1)\) lie \(ΔE = E_{co}\) above the two fold degenerate ground state \((0, 1), (1, 0)\). As a result the linear response conductance at T ∝ E_{co}/k_B as calculated from the sequential model, is exponentially suppressed since no charge transfer is possible using only two states. At T ∝ E_{co}/k_B the states \((0, 0)\) and \((1, 1)\) are thermally occupied with increasing probability leading first to an exponential increase of \(G_m\) and finally to the merging of \(G_m\) and \(G_{max}\). For n_x = n_y = 0 states besides (0, 0) are occupied for T ∝ E_c/k_B only. Thus \(G_{min}\) is exponentially small for T ∝ E_{co}/k_B.

The most striking feature of our measurements is the non-monotonic dependence of G_{max} on the temperature found for sample 3. It is clear from Fig. 4 that the phenomenology is correctly described by the sequential model. To get more insight into the nature of the drop of G_{max} at T ∝ E_{co}/k_B we analyze G_m(T), which coincides with G_{max} in the relevant temperature range. The

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\begin{tabular}{cccccccc}
samp. & E_{cx} & E_{co} & E_c & κ & g_l & g_r & g_m & G_0 \\
& (K) & (K) & (K) & (K) & (K) & (K) & (K) & (K) \\
1 & 2.8 & 5.8 & 1.3 & 5.6 & -0.018 & 0.44 & 0.44 & 0.04 & 1.42 \\
2 & 1.6 & 3.0 & 0.9 & 2.5 & 0.10 & 0.52 & 0.83 & 1.32 & 10.0 \\
3 & 1.5 & 4.7 & 0.3 & 4.5 & 0.0013 & 0.73 & 0.57 & 0.03 & 0.95 \\
\end{tabular}
\]

FIG. 4: Linear response conductance of the three samples as a function of temperature. Shown are G_{min}(○), G_{max}(●), and G_m (△, the conductance at n_x = 0.5), the latter defined as the conductance at the center of the horizontal boundaries in Fig. 2b. (i.e. at n_x = 1, n_y = −κ). As a result the linear response conductance at T ∝ E_{co}/k_B as calculated from the sequential model, is exponentially suppressed since no charge transfer is possible using only two states. At T ∝ E_{co}/k_B the states (0, 0) and (1, 1) are thermally occupied with increasing probability leading first to an exponential increase of \(G_m\) and finally to the merging of \(G_m\) and \(G_{max}\). For n_x = n_y = 0 states besides (0, 0) are occupied for T ∝ E_c/k_B only. Thus \(G_{min}\) is exponentially small for T ∝ E_{co}/k_B.

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amp. & E_{cx} & E_{co} & E_c & κ & g_l & g_r & g_m & G_0 \\
& (K) & (K) & (K) & (K) & (K) & (K) & (K) & (K) \\
1 & 2.8 & 5.8 & 1.3 & 5.6 & -0.018 & 0.44 & 0.44 & 0.04 & 1.42 \\
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3 & 1.5 & 4.7 & 0.3 & 4.5 & 0.0013 & 0.73 & 0.57 & 0.03 & 0.95 \\
\end{tabular}
\]
The four-state approximation mentioned above yields an analytical solution:

$$G_m^{(4)} = \frac{G_s}{2} \frac{\beta E_{co}}{\sinh(\beta E_{co})} \left(1 - \frac{2g_s}{g_m} \frac{\beta E_{co}}{1 - e^{\beta E_{co}}} \right)^{-1}. \quad (2)$$

We have drawn this function for our sample parameters in Fig. 4 as dashed lines. It has a distinct maximum for $g_m < g_s$, the position and strength of which depends on the ratio $g_s/g_m$. The above approximation breaks down at $T \sim E_c/k_B$ where more than four states are occupied significantly resulting in a rapid rise of the conductance. For an experimental observation of a local minimum the relation $E_{co} < E_c$ has to be fulfilled in addition to $g_m < g_s$.

In Fig. 5 we plot the $n_s$ coordinate of the conductance maximum $n_{max}$ as a function of temperature. Again, reasonable agreement with the sequential model is found. For $T \to 0$ the position of the maximum approaches the location of the triple point. At low temperature the conductance peak is described by Eq. 1. The maximum can shift in either direction depending on the ratio $g_s/g_m$. This can be used to determine $g_s/g_m$ for sample 1 ($g_s/g_m = 5$) where this ratio can not be measured directly due to the simple layout (see above).

The sequential model fails to predict the temperature dependence of $G_{max}$ at low temperature. This deviation can be described phenomenologically by adding a term of the form $\Delta G_{qf} = \alpha \log(k_BT/(2E_{co}))$. The thin solid lines in Fig. 4 display the outcome of a fitting procedure in $\alpha$ that minimizes the mean square deviation between the measured values of $G_{max}$ and $G_{seq} + \Delta G_{qf}(\alpha)$, showing very good agreement. We find $\alpha_1 = 45$ nS, $\alpha_2 = 200$ nS, and $\alpha_3 = 33$ nS for sample 1, 2, and 3, respectively. For the SET much the same behavior is observed and attributed to quantum fluctuations of the charge states. Pohjola et al. [12] analyzed the linear response of the SEP by renormalization group methods. They also found a logarithmic behavior of the low temperature conductance, in qualitative agreement with our experimental result. However, quantitative results were obtained for certain limiting cases of special interest only. For a detailed comparison with our experiment a calculation using our sample parameters is highly desirable.

In summary we have presented an experimental study of the linear response conductance of the SEP in a regime where quantum fluctuations of the charge eigenstates can not be ignored. Depending on the ratio $g_s/g_m$ the SEP shows a remarkably rich phenomenology even within the framework of lowest-order perturbation theory, the so-called sequential tunneling model. Most strikingly, in the easily accessible regime $E_{co} < E_{cx}$ and $g_m \ll g_s$ a pronounced non-monotonic temperature dependence of the conductance has been observed. Phenomena of this kind are to be expected in all single electron devices which are more complex than the SET. They can uncover internal characteristics unaccessible by other means – here e.g. $g_s/g_m$ which is not directly measurable in the most natural SEP layout (sample 1). At low temperature deviations from the sequential behavior due to quantum fluctuations become clearly visible. They are described in close analogy to the SET by a logarithmic correction term of the form $\Delta G_{qf} = \alpha \log(k_BT/(2E_{co}))$, in qualitative agreement with the findings of Ref. [12]. We propose a reevaluation of the formulae of Ref. [12] with parameters in accordance with our experiment so as to check the applicability of perturbation expansion for devices more complex than the SET.

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