Extra dimensions and possible modifications of Newton’s law

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Abstract. The recent understanding of string theory opens the possibility that the string scale can be as low as a few TeV. The apparent weakness of gravitational interactions can then be accounted by the existence of large internal dimensions, in the submillimeter region. Furthermore, our world must be confined to live on a brane transverse to these large dimensions, with which it interacts only gravitationally. In my lecture, I describe briefly this scenario which gives a new theoretical framework for solving the gauge hierarchy problem and the unification of all interactions. I also discuss its main properties and implications for observations at both future particle colliders, and in non-accelerator gravity experiments. Such effects are for instance the production of Kaluza-Klein resonances, graviton emission in the bulk of extra dimensions, and a radical change of gravitational forces in the submillimeter range.

1. Introduction

In all physical theories, the number of dimensions is a free parameter fixed to three by observation, with one exception: string theory, which predicts the existence of six new spatial dimensions. This is the only known theory today that unifies the two great discoveries of 20th century: quantum mechanics, describing the behavior of elementary particles, and Einstein’s General Relativity, describing gravitational phenomena in our Universe.

String theory replaces all elementary point-particles that form matter and its interactions with a single extended object of vanishing width: a tiny string. Thus, every known elementary particle, such as the electron, quark, photon or neutrino, corresponds to a particular vibration mode of the string (see Fig. 1). The diversity of these particles is due to the different properties of the corresponding string vibrations.

Until now, there is no experimental confirmation of string theory. String physicists believe today in string theory, mainly for theoretical reasons, because it provides a framework for unification of all interactions including gravity. However, some precise experimental tests are necessary to decide whether this theory describes the physical reality.

2. Small dimensions

How can it be tested? If our universe has really six additional dimensions, we should observe new phenomena related to the existence of these dimensions. Why nobody has detected them until

1 On leave from CPHT (UMR 7644 of CNRS), Ecole Polytechnique, F-91128 Palaiseau.
Figure 1. In string theory, the elementary constituent of matter is a miniscule string, having vanishing width but finite size. It can be open with free ends (upper part), or closed (lower part). Its vibration modes, like the ones shown above in two dimensions, correspond to various elementary particles.

now? String theorists had an answer for a long time: because the size of the new dimensions is very small, in contrast to the size of the other three that we know, which is infinitely large.

An infinite and narrow cylinder for example is a two-dimensional space, with one dimension forming a very small cycle: one can move infinitely far away along the axis, while one returns back at the same point when moving along the orthogonal direction (see Fig. 2). If one of the three known dimensions of space was small, say of millimeter size, we would be flat and, while

Figure 2. Possible forms of small extra dimensions of space. Far away they are unobservable, but at distances comparable to their size we start feeling their existence and exploring their shapes.
we could move freely towards left or right, forward or backward, it would be impossible to do more than a few millimeters up or down where space ends.

For a long time, string physicists thought that the six extra dimensions were extremely small, having the smallest possible size of physics, associated to the Planck length $\sim 10^{-35}$ meters. In fact, strings were introduced to describe gravitation whose strength becomes important and comparable to the strength of the other three fundamental interactions (electromagnetic, nuclear strong and weak) at very short distances, of the order of the Planck length. It was then natural to assume that the size of the extra dimensions should be of the same order. In this case, the manifestation of new phenomena associated to the extra dimensions are by far out of experimental reach, at least in particle accelerators. Indeed, the Large Hadron Collider (LHC) which is the biggest accelerator under construction at CERN will explore short distances, only up to $10^{-19}$ meters.

The situation changed drastically recently. During the last three years, more and more theorists examine the possibility that the new dimensions of string theory may be much larger than we thought in the past [1, 2]. These ideas lead in particular to experimental tests of string theory that can be performed at TEVATRON and LHC, or at future colliders.

3. Supersymmetry breaking and TeV dimensions
The first indication of large extra dimensions in string theory came in 1988 from studies of the problem of supersymmetry breaking [3, 4]. Supersymmetry is believed to be a new fundamental symmetry of matter which renders the masses of elementary particles compatible with gravitation. In fact, in a quantum theory without supersymmetry, the presence of gravity, which is much weaker than the other interactions, introduces a new high energy scale, the Planck mass $\sim 10^{19}$ GeV, which attracts all masses of elementary particles to become $10^{16}$ times heavier than their observed values: this is the so-called mass hierarchy problem.

One of the main predictions of supersymmetry is that every known elementary particle has a partner, called superparticle. However, since none of these superparticles have ever been produced in accelerators, they should be heavier than the observed particles. Supersymmetry should therefore be broken. On the other hand, protection of the mass hierarchy requires that its breaking scale, i.e. the mass splitting between the masses of ordinary particles and their partners, cannot be larger than a few TeV. They can therefore be produced for instance at LHC, which will test the idea of supersymmetry.

Assuming that supersymmetry breaking in string theory arises by the process of compactification of the extra dimensions (i.e. from their intrinsic geometry and topology), one can show that its energy breaking scale is tied to the size of these dimensions [3, 4]. Thus, a breaking scale in the TeV region would imply the existence of an extra dimension of size $\sim 10^{-18}$ meters. This was one of the few general predictions of string theory which has the chance to be testable in the next generation of particle experiments in the near future.

At that time however, this result was not taken seriously. The reason was rather due to a theoretical prejudice to evade invalidating the only computations we could do. Even now, string theory can be studied in most cases only approximately, namely in perturbation theory. More precisely, computations can be performed if strings interact weakly. The strength of string interactions is controlled by a parameter, called string coupling, which increases when extra dimensions become large. Therefore, when their size is larger than $10^{-35}$ meters the approximations of perturbative computations do not hold. Thus, the above result has been interpreted negatively. Namely, that supersymmetry breaking could not arise by compactification and remained as an open question.

Two years later specific models were proposed, where perturbative computations were possible in part of the theory, even in the presence of large extra dimensions of size of $10^{-18}$ meters [1]. In this class of models, the new dimensions form small intervals with our world localized at their
ends. The mediators of interactions on the other hand can propagate in the bulk (along the intervals). The study of the physical consequences of these models was performed subsequently in Refs. [5, 6, 7]. Their main characteristics is the production of the Kaluza-Klein states in particle accelerators (see Fig. 3).

![Graph](image)

**Figure 3.** If there is an extra dimension of size $10^{-18}$ meters, felt by the electroweak interactions, LHC should produce the first Kaluza-Klein states of the photon and of the $Z$ boson. We can then detect the electron-positron pairs produced by the disintegration of these states. The number of the expected events is computed as a function of the energy of the pair in GeV. From highest to lowest: excitation of photon+$Z$, photon and $Z$ boson.

If for instance the photon propagates along an extra dimension, one should observe a tower of massive particles with the same properties as the photon but with a mass that becomes higher as the size of the extra dimension is getting smaller. It follows, that for a size of order $10^{-18}$ meters, an energy of order of a few TeV would be sufficient to produce them. Thus, the existence of such dimensions is testable in LHC. Moreover, these models contain a very light particle that mediates new attractive forces at short distances of the order of a fraction of millimeter, which can be tested in table-top experiments that measure the Newton’s law [8] (see Fig. 4 below).

4. **String dualities**

In 1996, it was realized that the string size $l_s = M_s^{-1}$ is a free parameter of the theory, with a priori no relation to the Planck length [9]. In particular, it could be as large as $10^{-18}$ meters which is just below the limiting distance that can be probed by present experiments [10]. In order to understand the change of situation, let us return a couple of years earlier. All the works discussed until now were in the context of the so-called heterotic string theory. On the other hand, there were in total five consistent string theories! Four of them contain only closed strings that form closed loops; the other contains also open strings with ends that move freely with the speed of light; besides, all these theories do not have the same amount of supersymmetry.

This multiplicity of theories was creating a problem, since string theory was supposed to provide a unified framework of all physical theories. We now know that every known string
theory describes a particular limit of an underlying more general fundamental theory that can be defined in eleven dimensions of spacetime, called M-theory [11]. This discovery made an important progress but did not solve all problems. The main achievement was the connection of the five string theories due to the existence of duality symmetries. One type of these symmetries relates two theories with mutually inverse string couplings. Thus, to solve a problem in the context of some theory with large coupling, it is sufficient to perform an appropriate duality transformation. One obtains then a new problem in the context of a dual theory which has a small coupling, the inverse of the former. The new problem can be solved in perturbation theory of the small coupling. Finally, the resulting solution can be transformed back using the inverse duality transformation that takes us in the first theory. Since computations with large coupling became effectively possible, the road was open to study models with extra dimensions much larger than the Planck length.

Figure 4. Present limits on non-Newtonian forces at short distances (yellow regions), as a function of their range $\lambda$ (horizontal axis) and their strength relative to gravity $\alpha$ (vertical axis). The limits are compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

5. The universe on a membrane
A particularly attractive scenario is when the string scale is in the TeV region, which stabilizes the mass hierarchy problem without need of supersymmetry [2]. A possible realization of this idea without experimental conflict is in models possessing large extra dimensions along which only gravity propagates: gravity appears to us very weak at macroscopic scales because its intensity is spread in the “hidden” extra dimensions. On the other hand, at TeV energies, it becomes comparable in strength with the other interactions, i.e. $10^{32}$ times stronger than what we believed in the past. In order to increase the gravitational force without contradicting present observations, one has to introduce at least two such extra dimensions of size that can be as large
as a fraction of a millimeter. At these distances, gravity should start deviate from Newton’s law, which may be possible to explore in laboratory experiments [12] (see Fig. 4).

A convenient perturbative framework realizing this idea is one of the five string theories, called type I, that contains simultaneously closed and open strings [2, 13, 14]. Our universe should be localized on a hypersurface, i.e. a membrane extended in \( p \) spatial dimensions with \( p < 7 \), called \( p \)-brane (see Fig. 5). Closed strings describe gravity and propagate in all nine dimensions of space: in those extended along the \( p \)-brane, as well as in the transverse ones. On the contrary, the endpoints of open strings describing the other (gauge) interactions are confined on the \( p \)-brane.

![Diagram showing a type I string framework](image)

**Figure 5.** In the type I string framework, our Universe contains, besides the three known spatial dimensions (denoted by a single blue line), some extra dimensions \((d_{\parallel} = p - 3)\) parallel to our world \( p \)-brane (green plane) along which the light described by open strings propagates, as well as some transverse dimensions (yellow space) where only gravity described by closed strings can propagate. The longitudinal extra dimensions have the string size of the order of \(10^{-18}\) meters, while the size of the transverse dimensions varies in the range of \(10^{-14}\) meters to a fraction of a millimeter.

Obviously, our \( p \)-braneworld must have at least the three known dimensions of space. But it may contain more: as opposed to the transverse dimensions that interact with us only gravitationally, the “longitudinal” to the brane extra dimensions can be “seen” by the light at sufficiently high energies, giving rise to the production of massive Kaluza-Klein particles in accelerators (see Fig. 3). On the other hand, the existence of the extra large (sub)millimeter dimensions, transverse to our \( p \)-brane universe, guarantee that gravitational interactions appear to us very weak at macroscopic distances, larger that a millimeter. The size of these transverse dimensions varies from a fraction of millimeter (in the case of two) to a Fermi (\(10^{-14}\) meters, in the case of six). Their characteristic signal in particle colliders is graviton emission into the bulk, leading to missing energy that escapes detection [2, 15] (see Fig. 6).

### 6. Gravity modification and sub-millimeter forces

Besides the spectacular experimental predictions in particle accelerators, string theories with large volume compactifications and/or low string scale predict also possible modifications of gravitation in the sub-millimeter range, which can be tested in “table-top” experiments that measure gravity at short distances. There are three categories of such predictions:
Figure 6. Missing energy due to graviton emission in the LHC experiment, as a function of the fundamental scale $M_{(4+n)}$ of quantum gravity that propagates in $n$ large transverse dimensions. It is produced together with a hadronic jet that one detects in the collision of the two proton beams. The figure shows the expected cross-section for $n = 2$ and $n = 4$ extra dimensions, together with the background (horizontal dotted-dashed line) coming from other known sources.

(i) Deviations from the Newton’s law $1/r^2$ behavior to $1/r^{2+n}$, for $n$ extra large transverse dimensions, which can be observable for $n = 2$ dimensions of (sub)-millimeter size. This case is particularly attractive on theoretical grounds because of the logarithmic sensitivity of Standard Model couplings on the size of transverse space [14], which allows to determine the desired hierarchy [16], but also for phenomenological reasons since the effects in particle colliders are maximally enhanced [15]. Notice also the coincidence of this scale with the possible value of the cosmological constant in the universe that recent observations seem to support.

(ii) New scalar forces in the sub-millimeter range, motivated by the problem of supersymmetry breaking, and mediated by light scalar fields $\varphi$ with masses [17, 8, 2, 18]:

$$m_\varphi \simeq \frac{m^2_{\text{susy}}}{M_P} \simeq 10^{-4} - 10^{-6} \text{ eV},$$  

for a supersymmetry breaking scale $m_{\text{susy}} \simeq 1 - 10$ TeV. These correspond to Compton wavelengths in the range of 1 mm to 10 $\mu$m. $m_{\text{susy}}$ can be either the KK scale $1/R$ if supersymmetry is broken by compactification [8, 17], or the string scale if it is broken “maximally” on our world-brane [2, 18]. A model independent and universal attractive scalar force is mediated by the radius modulus (in Planck units)

$$\varphi \equiv \ln R,$$

with $R$ the radius of the longitudinal ($||$) or transverse ($\perp$) dimension(s), respectively. In the former case, the result (1) follows from the behavior of the vacuum energy density $\Lambda \sim 1/R^4_||$ for large $R_||$ (up to logarithmic corrections). In the latter case, supersymmetry is broken primarily on the brane only, and thus its transmission to the bulk is gravitationally suppressed, leading to masses (1). Note that in the case of two-dimensional bulk, there may be an enhancement
factor of the radion mass by \( \ln R_\perp M_s \simeq 30 \) which decreases its wavelength by roughly an order of magnitude [16].

The coupling of the radius modulus (2) to matter relative to gravity can be easily computed and is given by:

\[
\sqrt{\alpha_\varphi} = \frac{1}{m} \frac{\partial m}{\partial \varphi} ; \quad \alpha_\varphi = \begin{cases} 
\frac{\partial \ln \Lambda_{QCD}}{\partial \ln R} \simeq \frac{1}{3} & \text{for } R_\parallel \\
\frac{2n}{n+2} = 1 - 1.5 & \text{for } R_\perp 
\end{cases}
\]

(3)

where \( m \) denotes a generic physical mass. In the upper case of a longitudinal radius, the coupling arises dominantly through the radius dependence of the QCD gauge coupling [8], while in the lower case of transverse radius, it can be deduced from the rescaling of the metric which changes the string to the Einstein frame and depends on the dimensionality of the bulk \( n \) (varying from \( \alpha = 1 \) for \( n = 2 \) to \( \alpha = 1.5 \) for \( n = 6 \)) [16]. Moreover, in the case of \( n = 2 \), there may be again model dependent logarithmic corrections of the order of \( (g_s/4\pi) \ln R M_s \simeq O(1) \). Such a force can be tested in microgravity experiments and should be contrasted with the change of Newton’s law due the presence of extra dimensions that is observable only for \( n = 2 \) [12]. In principle there can be other light moduli which couple with even larger strengths. For example the dilaton, whose VEV determines the (logarithm of the) string coupling constant, if it does not acquire large mass from some dynamical supersymmetric mechanism, can lead to a force of strength 2000 times bigger than gravity [19].

(iii) Non universal repulsive forces much stronger than gravity, mediated by possible abelian gauge fields in the bulk [20, 21]. Such gauge fields may acquire tiny masses of the order of \( M_s^2/M_P \), as in (1), due to brane localized anomalies [21]. Although the corresponding gauge coupling is infinitesimally small, \( g_A \sim M_s/M_P \simeq 10^{-16} \), it is still bigger that the gravitational coupling \( \sim E/M_P \) for typical energies \( E \) of the order of the proton mass, and the strength of the new force would be \( 10^6 - 10^8 \) stronger than gravity. This an interesting region which will be soon explored in micro-gravity experiments (see Fig. 4). Note that in this case the supernova constraints impose that there should be at least four large extra dimensions in the bulk [20].

In Fig. 4 we depict the actual information from previous, present and upcoming experiments [16]. The solid lines indicate the present limits from the experiments indicated. The excluded regions lie above these solid lines. Measuring gravitational strength forces at such short distances is quite challenging. The most important background is the Van der Walls force which becomes equal to the gravitational force between two atoms when they are about 100 microns apart. Since the Van der Walls force falls off as the 7th power of the distance, it rapidly becomes negligible compared to gravity at distances exceeding 100 \( \mu \)m. The dashed thick lines give the expected sensitivity of the present and upcoming experiments, which will improve the actual limits by roughly two orders of magnitude, while the horizontal dashed lines correspond to the theoretical predictions for the graviton in the case of two large extra dimensions and for the radion in the case of transverse radius. These limits are compared to those obtained from particle accelerator experiments in Table 1.

7. Conclusions

Clearly, today, these theories exist only in our imagination. However, we look forward at the next generation of high energy experiments and in particular at the most powerful machine, the LHC at CERN. I am convinced, as the majority of my colleagues, that LHC will play a very important role for the future of high-energy physics of fundamental interactions. In fact, it is designed since last decade to explore the origin of mass of elementary particles and to test, in particular, the idea of supersymmetry, looking for the production of superparticles. We now hope that this accelerator may discover more spectacular and “exotic” phenomena, such as the existence of large extra dimensions of space and of fundamental strings.
Table 1. Limits on $R_\perp$ in mm from missing-energy processes.

| Experiment | $R_\perp (n = 2)$ | $R_\perp (n = 4)$ | $R_\perp (n = 6)$ |
|------------|------------------|------------------|------------------|
| **Collider bounds** |                  |                  |                  |
| LEP 2      | $4.8 \times 10^{-4}$ | $1.9 \times 10^{-8}$ | $6.8 \times 10^{-11}$ |
| Tevatron   | $5.5 \times 10^{-4}$ | $1.4 \times 10^{-8}$ | $4.1 \times 10^{-11}$ |
| LHC        | $4.5 \times 10^{-3}$ | $5.6 \times 10^{-10}$ | $2.7 \times 10^{-12}$ |
| NLC        | $1.2 \times 10^{-2}$ | $1.2 \times 10^{-9}$  | $6.5 \times 10^{-12}$ |
| **Present non-collider bounds** |                  |                  |                  |
| SN1987A    | $3 \times 10^{-4}$  | $1 \times 10^{-8}$ | $6 \times 10^{-10}$ |
| COMPTEL    | $5 \times 10^{-8}$  | -                 | -                |

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