Creating Teams of Simple Agents for Specified Tasks:  
A Computational Complexity Perspective

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Abstract: Teams of interacting and co-operating agents have been proposed as an efficient and robust alternative to monolithic centralized control for carrying out specified tasks in a variety of applications. A number of different team and agent architectures have been investigated, e.g., teams based on single vs multiple behaviorally-distinct types of agents (homogeneous vs heterogeneous teams), simple vs complex agents, direct vs indirect agent-to-agent communication. A consensus is emerging that (1) heterogeneous teams composed of simple agents that communicate indirectly are preferable and (2) automated methods for verifying and designing such teams are necessary. In this paper, we use computational complexity analysis to assess viable algorithmic options for such automated methods for various types of teams. Building on recent complexity analyses addressing related questions in swarm robotics, we prove that automated team verification and design are by large both exact and approximate polynomial-time intractable in general for the most basic types of homogeneous and heterogeneous teams consisting of simple agents that communicate indirectly. Our results suggest that tractability for these problems must be sought relative to additional restrictions on teams, agents, operating environments, and tasks.

1 Introduction

Teams of interacting and co-operating agents have been proposed as an efficient and robust alternative to monolithic centralized control for carrying out specified tasks in a variety of applications. A number of different team and agent architectures have been investigated. Three dimensions of these architectures are of particular importance:

1. Should teams consist of a single type of agent or multiple types of agents, i.e., should teams be homogeneous or heterogeneous?
2. Should individual agents have simple or complex control mechanisms, i.e., simple reflex or model- / goal- / utility-based agents [1, Section 2.4]?

3. Should individual agents communicate directly with each other by agent-to-agent messages or indirectly via their sensed presences and/or environmental modifications, i.e., via stigmergy [2]?

Based on the experience gained with various proof-of-concept experiments and implementations, the consensus is emerging that (1) heterogeneous teams composed of relatively simple agents that communicate indirectly are preferable [3, 4, 5] and (2) automated methods for verifying and designing such teams are necessary [6, 7].

A natural question at this point is what algorithmic options are and are not available for the efficient verification and design of teams relative to the three dimensions listed above. In this paper, we give some initial answers to this question, building on recent work [8, 9, 10, 11, 12, 13] addressing related questions in swarm robotics for distributed construction. In particular, we give proofs (modified from several given previously in [9, 10, 11]) which demonstrate that the problems of team verification and design are by large both exact and approximate polynomial-time intractable in general relative to the most basic types of homogeneous and heterogeneous teams consisting of simple reflex agents that do not use stigmergy. This in turn suggests that tractability must be sought relative to additional restrictions on teams, agents, operating environments, and tasks.

2 Methods

In this section, we first review the basic entities in our model of task performance by robot teams — namely, environments, individual robots, robot teams, and tasks (with the last of these being new to this paper). Though this is a basic model in which robots sense and move without uncertainty in a discrete and synchronous manner in a 2D grid-based environment, it is flexible enough to allow investigations along the three team and agent architectural dimensions listed in the introduction. In the interests of concision, most of this review is the “short form” given in [8, 10]; readers wishing more details should consult [9, 11]. This will be followed by formalization of computational problems within this model corresponding to various types of robot team verification and design.

The basic entities in our model are as follows:

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1 Though there has been other complexity-theoretic work on individual agent verification and design [14, 15, 16], the models of agents and environments used were too abstract to allow examination of agent teams, simple reflex agents, or stigmergy.

2 These modifications are described along with the proofs of our results in the appendix.
• **Environments:** Our robots operate in a finite 2D square-based environment $E$ in which each square is either a freespace (which a robot can occupy or move through) or an obstacle (which a robot cannot occupy or move through), and has a square-type, e.g., grass, gravel, wall, drawn from a set $E_T$. Let $E_{i,j}$ denote the square that is in the $i$th column and $j$th row of $E$ such that $E_{1,1}$ is the square in the southwest-most corner of $E$.

• **Robots:** Each robot occupies a square in $E$ and in a basic movement-action can either move exactly one square to the north, south, east or west of its current position or elect to stay at its current position. Each robot has a sensing-distance bound $r$ such that the robot can sense the type of the square at any position within Manhattan distance $r \geq 0$ of the robot’s current position (with $r = 0$ corresponding to the square on which the robot is standing). These square-types are accessible via predicates of the form $enval(e, pos)$ which returns True if the square at position $pos$ has type $e \in E_T \cup \{e_{\text{robot}}\}$ (with the sensor returning $e_{\text{robot}}$ if a robot is occupying square $pos$) and False otherwise, where a position $pos$ is specified in terms of a pair $(x, y)$ specifying an environment-square $E_{i+x,j+y}$ if the robot is currently occupying $E_{i,j}$. Each robot can change the type of the square at any position within Manhattan distance one of the robot’s current position to type $e$ via predicates of the form $enmod(e, pos)$ where $pos$ is specified as for $enval()$.

Each robot has a finite-state controller and is hence known as a Finite-State Robot (FSR). Each such controller consists of a set $Q$ of states linked by transitions, where each transition $(q, f, x, dir, q')$ between states $q$ and $q'$ has a propositional logic trigger-formula $f$, an environment modification specification $x$, and a movement-direction $dir \in \{\text{goNorth}, \text{goSouth}, \text{goEast}, \text{goWest}, \text{stay}\}$. Trigger-formulas and modification specifications are typically stated in terms of predicates $enval()$ and $enmod()$, respectively. Both of these specifications can also be stated as a special symbol $\ast$, which is interpreted as follows: If $f \neq x \neq \ast$ and the transition’s trigger-formula evaluates to True, i.e., the transition is enabled, this causes the environment-modification specified by $x$ to occur, the robot to move one square in direction $dir$, and the robot’s state to change from $q$ to $q'$. If $f = \ast$, the transition executes if no other transition executes (making this in effect the default transition); if $x = \ast$, no environment-modification is made.

The transitions in the FSR described above can be viewed as condition-action rules within the agent framework given in [1, Section 2.4] such that the $\ast$ trigger formula on a transition leaving state $q$ can be viewed as the negation of the disjunction of the trigger formulas of all other transitions leaving $q$. Given this,
single- and multi-state FSR correspond to simple reflex and model-based reflex agents, where the model and UPDATE-STATE() function in [1, Section 2.4.3] are implicit in the transitions between states. Note that actions correspond to FSR movements and environmental modifications as required, and are flexible enough to allow situations in which FSR movements or environment modifications do not occur.

- **Robot teams**: A team $T$ consists of a set of the robots described above, where there may be more than one robot with the same controller on a team; as such, we allow both homogeneous and heterogeneous teams. Let $T_i$ denote the $i$th robot on the team. Each square in $E$ can hold at most one member of $T$; if at any point in the execution of a task two robots in a team attempt to occupy or modify the same free space or a robot attempts to occupy the same space as an obstacle, the execution terminates and is considered unsuccessful. A **positioning** of $T$ in $E$ is an assignment of the robots in $T$ to a set of $|T|$ squares in $E$. For simplicity, team members move synchronously, and once movement is triggered, it is atomic in the sense that the specified movement is completed.

Note that robots in our teams do not communicate with each other directly — rather, they can communicate with each other indirectly through their sensed presences in and changes they make to the environment, i.e., via stigmergy [2]. In the remainder of the paper, we will find it useful to distinguish these two types of communication, which will be denoted as **agent-** and **environment-based stigmergy**, respectively.

- **Tasks**: Tasks are specified in terms of a desired set of environment square-values, robot positions, and/or robot internal states, e.g., a $3 \times 3$ square of square-type $e_X$ has been created at a particular location in the environment, all robots in a team are in state $q_F$ and located on the eastmost edge of the environment. Such is a specification will be denoted as a task’s **target configuration**. We will assume that for each task $T_{sk}$ and an environment $E$ in which a robot team $T$ is operating, $E$ can be checked for the target configuration associated with $T_{sk}$ in time polynomial in the size of $E$.

We use the notions of deterministic robot and team operation proposed in [8][11] (i.e., requiring that at any time as the team operates in an environment, all transitions enabled in a robot relative to the current state of that robot perform the same environment modifications and progress to the same next state). Given this, an individual FSR is not itself deterministic but rather the operation of that FSR is deterministic in the context of a particular FSR team operating in a particular environment.
Let us now consider the team verification design problems that we will analyze in this paper, starting with verification.

**Team / Environment Verification for Task $T_{sk}$ (TeamEnvVer)**

*Input*: An environment $E$ based on square-type set $E_T$, an FSR team $T$, an initial positioning $p_I$ of $T$ in $E$, and an integer $#_{ec} \geq 0$.

*Question*: Does $T$ started at $p_I$ perform task $T_{sk}$ while making at most $#_{ec}$ square-type changes in $E$?

We will consider two types of robot team design. Both types of design are done relative to a given design library. In the first case, $L$ consists of transition templates of the form $(q, f, x, move, q')$ which are used to construct FSR controllers from a specified set of states by instantiating transition templates relative to those states. Note it may be the case that $q = q'$ in such a construction, i.e., a transition may loop back on the same state.

**Controller Design by Library Selection for Task $T_{sk}$ (ContDesLS)**

*Input*: An environment $E$ based on square-type set $E_T$, a requested team-size $|T|$, an initial positioning $p_I$ of $T$ in $E$, a transition template library $L$, and integers $r \geq 0$, $|Q| \geq 1$, $d \geq 1$, $h \geq 1$, and $#_{ec} \geq 0$.

*Output*: A controller-set $C$ in which each controller has sensory radius $r$, at most $|Q|$ states, and at most $d$ transitions chosen from $L$ out of any state such that an FSR team $T$ based on $h$ controllers from $C$ started at $p_I$ performs $T_{sk}$ while making at most $#_{ec}$ square-type changes in $E$, if such a $C$ and $T$ exists, and special symbol $\bot$ otherwise.

In the second case, $L$ consists of complete FSR controllers.

**Team Design by Library Selection for Task $T_{sk}$ (TeamDesLS)**

*Input*: An environment $E$ based on square-type set $E_T$, a requested team-size $|T|$, an FSR library $L$, an initial region $E_I$ of size $T$ in $E$, and integers $h \geq 1$ and $#_{ec} \geq 0$.

*Output*: An FSR team $T$ based on $h$ robots selected from $L$ such that $T$ started in $E_I$ performs $T_{sk}$ while making at most $#_{ec}$ square-type changes in $E$, if such a a $T$ exists, and special symbol $\bot$ otherwise.

We will subsequently analyze these problems relative to three parameters:

1. The number of different types of FSR controllers in a team ($h$);
2. The maximum number of states in the robots in a team ($|Q|$); and
3. The maximum allowable number of environmental changes made by a team in performing specified tasks ($#_{ec}$).
Different values of these parameters will allow us to investigate the question posed in the introduction — namely, the effects on the computational difficulty of team verification and design when using (1) teams based on single \((h = 1)\) and multiple \((h > 1)\) types of FSR controllers (i.e., homogeneous and heterogeneous teams), (2) simple reflex \(|Q| = 1\) and model-based reflex \(|Q| > 1\) agents, and (3) agents that do \(#_{ec} > 0\) and do not \(#_{ec} = 0\) use environment-based stigmergy in the performance of tasks.

## 3 Results

In this section, we analyze the computational difficulty of our team verification and design problems relative to the three restrictions proposed at the end of the previous section. We evaluate this difficulty relative to several criteria of efficient algorithm operation using two standard techniques — namely, proving tractability by giving algorithms and intractability by giving reductions from known intractable problems (see [17][18] for details of these techniques). All proofs of results are relegated to the appendix.

Let us first consider exact polynomial-time solvability. An exact polynomial-time algorithm is an algorithm which always produces the correct output for a given input and whose runtime is asymptotically upper-bounded, i.e., upper-bounded when \(|x|\) goes to infinity, by \(c|x|^c'\), where \(|x|\) is the size of the input \(x\) and \(c\) and \(c'\) are constants. A problem that has a polynomial-time algorithm is said to be polynomial-time tractable. Polynomial-time tractability is desirable because runtimes increase slowly as input size increases, and hence allow the solution of larger inputs. We start with team verification.

**Result A** (Modified from Lemma 4 in the Supplementary Materials of [11]): TeamEnvVer is not exact polynomial-time tractable when \(|Q| = 1, h = 1, \) and \(#_{ec} = 0\). Moreover, this intractability holds for any version of TeamEnvVer when \(h > 1, |Q| > 1, \) and \(#_{ec} > 0\).

This result demonstrates that verification is polynomial-time intractable in general for homogeneous teams of simple reflex agents that do not use environment-based stigmergy. This in turn motivates the notion (introduced in [11]) of \((c_1, c_2)\)-completabillity, which requires that each robot team complete its task within \(c_1(|E| + |Q|)c_2\) timesteps for constants \(c_1\) and \(c_2\). Let the versions of ContDesLS and

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3Results A, B, D, and E hold relative to some combination of the conjectures \(P \neq NP\) and \(P = BPP\), which though unproven are widely believed to be true within computer science [19][20].

4For technical reasons that are described in detail in [10], \(c_1\) and \(c_2\) are not part of the problem input but are specified beforehand. To ensure generous but still low-order polynomial team runtime bounds, we will assume that \(c_1 = 10\) and \(c_2 = 3\).
TeamDesLS with this completability restriction be denoted by ContDesLS$_{res}$ and TeamDesLS$_{res}$. It turns out that this restriction does not always help in general.

**Result B** (Modified from Lemma 5 in [11]): ContDesLS$_{res}$ is not exact polynomial-time tractable when $|Q| = 1$, $h = 1$, and $\#_{ec} = 0$. Moreover, this intractability holds for any version of ContDesLS$_{res}$ when $h > 1$, $|Q| > 1$, and $\#_{ec} > 0$.

**Result C** (Modified from Result A in [9]): TeamDesLS$_{res}$ is exact polynomial-time tractable when $|Q| \geq 1$, $h = 1$, and $\#_{ec} \geq 0$.

**Result D** (Modified from Lemma A.7 in [10]): TeamDesLS$_{res}$ is not exact polynomial-time tractable when $|Q| = 1$, $h = 2$, and $\#_{ec} = 0$. Moreover, this intractability holds for any version of TeamDesLS$_{res}$ when $h > 2$, $|Q| \geq 1$, and $\#_{ec} \geq 0$.

The above demonstrates that (1) restricted controller design (like verification) is polynomial-time intractable in general for homogeneous teams of simple reflex agents that do not use environment-based stigmergy (Result B) and (2) though restricted team design is polynomial-time tractable for any type of homogeneous team (Result C), it is polynomial-time intractable in general for the simplest heterogeneous teams based on simple reflex agents that do not use environment-based stigmergy (Result D).

Let us now consider polynomial-time approximate solvability. This type of solvability may be acceptable in situations where always getting the correct output for an input is not required. Three popular types of polynomial-time approximation algorithms are:

1. algorithms that always run in polynomial time but are frequently correct in that they produce the correct output for a given input in all but a small number of cases (i.e., the number of errors for input size $n$ is bounded by function $err(n)$) [21];

2. algorithms that always run in polynomial time but are frequently correct in that they produce the correct output for a given input with high probability [22]; and

3. algorithms that run in polynomial time with high probability but are always correct [23].

Algorithms of type (2) are of particular interest as they include evolutionary algorithms. Unfortunately, none of these options are in general open to us either courtesy of the following result.

**Result E** (Modified from Results A.4 and A.5 in [10]): None of the intractable versions of TeamEnvVer, ContDesLS$_{res}$, or TeamDesLS$_{res}$ described in Results A, B,
and D are polynomial-time approximable in senses (1–3).

Note that all intractability results above hold relative to the simplest types of tasks (do some subset of the robots in team $T$ reach particular positions in $E$?) and (in the case of ContDesLS$_{res}$ and TeamDesLS$_{res}$) the most restrictive type of completability, i.e., $(1,1)$-completabaility.

4 Discussion

In the previous section, we demonstrated that agent team verification and two types of design by library selection (agent controller and team) are by large both exact (Results A, B, and D) and approximate (Result E) polynomial-time intractable in general relative to the simplest possible types of teams and agents, i.e., homogeneous teams consisting of simple reflex agents that do not use environment-based stigmergy. Even in the one case where we have polynomial-time tractability (Result C; homogeneous team design by library selection), intractability asserts itself when as few as two types of agents co-exist on a team (Result D). That the various intractabilities we have demonstrated cannot be vanquished by invoking verification and design relative to teams with higher values of $h$, $|Q|$, and $\#_{ec}$ suggests that we are seeing a tight frontier of tractability [18, Section 4.1] relative to the lowest possible values of these parameters.

As all of our results are derived relative to a simplified team operation model in which deterministic agents operate in a synchronous and discrete manner within a 2D grid-based environment, these results are not immediately applicable to probabilistic agents that operate in an asynchronous and continuous manner in the real world. That being said, our results do for now offer some reasons for real-world roboticists to be cautious. In particular, the fact that team verification and design are polynomial-time intractable even when agent motion and sensing are error-free and occur in

5It is tempting to think that polynomial-time intractability of team verification and design follows from the well-known combinatorial explosion in the number of possible team states and designs in multi-agent systems. However, there are many examples of problems with such exponential-size search spaces that are nonetheless solvable in polynomial time by algorithms that exploit structure in those spaces, e.g., Minimum Spanning Tree [24, Chapter 23]. Hence, definitive proof of polynomial-time intractability requires proofs such as those given here.

6It is also intriguing that agent-based stigmergy is critical to some (Results A and D) but not all (Result B) of our intractability results. It is all too often assumed in discussions about stigmergy that environmental modifications (including “smart” materials [25]) are key. Our proofs suggest that in certain situations, environmental modifications are unnecessary given sufficiently large and appropriately-structured groups of mobile agents. As large groups of agents are of increasing interest in certain applications, this warrants further investigation.

7Results like ours do nonetheless have a surprising generality; the interested reader is referred to Sections 5 and 6.2 of [9] and Section 5 of [11] for details.
completely-observable environments hints that there may be additional sources of computational difficulty in these problems that are not associated with agent motion and sensing under partial observability and uncertainty [26]. These sources should be acknowledged and investigated, particularly if team verification and design must behave both efficiently and correctly when fully automated without human oversight.

This last point highlights a crucial caveat when interpreting our intractability results – namely, these results hold relative to a simplified model of agent team operation and, perhaps more importantly, restrictions on the values of \( h, |Q|, \) and \( \#ec \). Our frequent proviso that intractability results apply “in general” was not mere rhetoric — tractability may well hold for the cases we considered when additional restrictions are in place; then again, it may not. In either case, this must be determined by future complexity analyses. Hence, our results should be seen not as final statements on the intractability of team verification and design but rather as interim guidelines suggesting where in the universe of restriction possibilities tractability does and does not hold.

Given the above, future research into team verification and design should perhaps more closely incorporate computational complexity analyses like those given here. Such research could initially focus on more fully characterizing those combinations of restrictions that do and do not render team verification and design tractable. Such work has already been started [8, 9, 10, 12, 11, 13] for team verification and design in swarm robotics relative a variety of restrictions using more advanced analysis techniques (e.g., parameterized complexity analysis [27]). Additional restrictions of particular interest here are those that “break” the reductions underlying our intractability results, e.g., restrictions on the degree and type of structure encountered by agents in their environments (including the presences of other agents). Once these initial intractability maps have been derived for simplified team operation models, they should be extended to more realistic models incorporating stochasticity and uncertainty. Part of this can be done by using complexity analysis techniques that explicitly incorporate stochasticity [22, 28, 29]. Complexity-based frameworks that incrementally build on simplified operation models in a systematic and principled manner to create results applicable to more realistic models (analogous to those developed in linguistics [30] and cognitive science [31]) may also be of use in this endeavour.

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A Proofs of Results

All of our intractability results will be derived using polynomial-time reductions from the following problems:

**3-Satisfiability (3SAT)** [18, Problem LO2]
*Input:* A set $U$ of variables, a set $C$ of disjunctive clauses over $U$ such that each clause $c \in C$ has $|c| = 3$.
*Question:* Is there a satisfying truth assignment for $C$?

**Dominating set** [18, Problem GT2]
*Input:* An undirected graph $G = (V, E)$ and a positive integer $k$.
*Question:* Does $G$ contain a dominating set of size $k$, i.e., is there a subset $V' \subseteq V$, $|V'| = k$, such that for all $v \in V$, either $v \in V'$ or there is at least one $v' \in V'$ such that $(v, v') \in E$?

For each vertex $v \in V$, let the complete neighbourhood $N_C(v)$ of $v$ be the set composed of $v$ and the set of all vertices in $G$ that are adjacent to $v$ by a single edge, i.e., $v \cup \{u \mid u \in V \text{ and } (u, v) \in E\}$. We assume below for each instance of Dominating set an arbitrary ordering on the vertices of $V$ such that $V = \{v_1, v_2, \ldots, v_{|V|}\}$; we also assume analogous orderings for the variables and clauses of each instance of 3-Satisfiability such that $U = \{u_1, u_2, \ldots, u_{|U|}\}$ and $C = \{c_1, c_2, \ldots, c_{|C|}\}$.

For technical reasons [17, 18], our intractability results are initially derived relative to decision versions of our problems, i.e., problems whose answers are either “Yes” or “No”. Problem TeamEnvVer is already phrased as a decision problem. The decision versions of ContDesSres and TeamDesLSres (denoted by ContDesLSres,D and TeamDesLSres,D, respectively) ask if the structures requested in each problem ($(C, T)$ and $T$, respectively) exist or not. The following lemma (based on the observation that any algorithm for non-decision problem $X$ can be used to solve $X_D$) will be useful below in transferring results from decision problems to their associated non-decision problems.

**Lemma 1** If $X_D$ is not solvable in polynomial time relative to conjecture $C$ then $X$ is not solvable in polynomial time relative to conjecture $C$.

All proofs of results given here are modifications of proofs given previously in [9, 10, 11]. It is thus appropriate to describe the nature of these modifications, starting with an overview. All previous proofs had $|Q| = 1$, so modifications were made to ensure

1. the initial values of $h$ and $\#_{ec}$ stated in the first part of each result, and
2. the higher values of $|Q|$, $h$, and $\#_{ec}$ stated in the second part of each result.
As previous work was done with respect to distributed constructions tasks, $\#_{ec} = 0$ was easy to ensure in the modifications once we introduced our task model (which was new to the current paper) and focused on tasks that involve achieving specified environment / agent / state configurations — we just needed robots to get to specified environmental positions where we previously had them also building structures in those positions. Other result-specific modifications are summarized below in text prior to each result’s proof.

We now present the proofs of results in our paper, starting with Result A. Part (1) of this result requires relatively straightforward modification of environments and FSR transitions to convert the previous $h = 5$ construction in [11] to the needed $h = 1$ construction while preserving $|Q| = 1$. Part (2) requires relatively straightforward modification to allow $|Q| > 1$ and $h > 1$.

**Result A** (Modified from Lemma 4 in the Supplementary Materials of [11]): TeamEnvVer is not exact polynomial-time tractable when $|Q| = 1$, $h = 1$, and $\#_{ec} = 0$ unless $P = NP$. Moreover, this intractability holds for any version of TeamEnvVer when $h > 1$, $|Q| > 1$, and $\#_{ec} > 0$ for fixed values of $h$, $|Q|$, and $\#_{ec}$.

**Proof:** Lemma 4 in [11] gives a reduction from 3SAT to problem ContEnvVer$_{syn}$, which is our problem TeamEnvVer without the restriction on $\#_{ec}$. In the instance of ContEnvVer$_{syn}$ created by this reduction, all possible truth assignments to the variables in $U$ in the given instance of 3SAT are generated one at a time by the movements of a team of $|U| + 3$ single-state FSR of 4 types (Variable, Carry, CarryDetect, and CarrySignal) in an environment $E$ such that the values in the truth assignment are encoded in the positions of the Variable FSRs. Each of these assignments is in turn checked by a single-state FSR of a fifth type (Evaluate) against the collection of clauses $C$ in the given instance of 3SAT, and if the truth assignment satisfies the conjunction of the clauses in $C$, the Evaluate FSR moves one square to the east and modifies the type of the square it is now placed on.

We modify this reduction to create a reduction from 3SAT to TeamEnvVer when $h = 1$ and $\#_{ec} = 0$ as follows:

1. As each of the five types of FSR can only occupy very specific non-overlapping areas in $E$, change the types of the squares in the areas occupied by each type of FSR to new FSR-type-specific square-types $e_{Var}$, $e_{Car}$, $e_{CD}$, $e_{CS}$, and $e_{Evl}$.

2. Create a single-state universal FSR that can simulate all five types of FSR by combining modified versions of all transitions in the five types of FSR, where a transition of the form $(q, f, m, q)$ in FSR-type $t$ is changed to $(q, eval(e_t, (0, 0)) \land f, m, q)$, i.e., FSR-type-specific transitions can only trigger if the FSR is in the environmental area associated with FSR of that type.
3. Replace the environmental modification made by the \textit{Evaluate} FSR with $\ast$.

Note that this reduction creates an FSR team in which $h = 1$, $|Q| = 1$, and $\#_{ec} = 0$. If we then make the target configuration of the task associated with this instance of TeamEnvVer be that an FSR is positioned immediately to the west of the initial position of the \textit{Evaluate} FSR, the proof of reduction correctness in Lemma 4 also establishes that this reduction is correct. As 3SAT is \textit{NP}-hard [18, Problem L02], this reduction establishes that TeamEnvVer is also \textit{NP}-hard when $h = 1$, $|Q| = 1$, and $\#_{ec} = 0$ and hence not polynomial-time solvable under these restrictions unless $P = \text{NP}$.

We now need to establish the \textit{NP}-hardness of TeamEnvVer for fixed values of $h > 1$, $|Q| > 1$, and $\#_{ec} > 0$. Observe that the instance of TeamEnvVer constructed above makes no environmental modifications and hence trivially makes at most $\#_{ec}$ environmental modifications for any fixed value of $\#_{ec}$. In the case of $h$ and $|Q|$, construct a modified instance of TeamEnvVer above in which there is an additional “holding area” in $E$ consisting of $h-1$ squares enclosed by obstacle-squares such that no FSR inside this area can leave it. Populate this holding area with arbitrary FSR based on $|Q|$ states such that none of these FSR makes an environmental modification. As $h$ and $|Q|$ are of fixed value, this modified instance of TeamEnvVer can still be constructed in time polynomial in the size of the given instance of 3SAT. Moreover, the reduction from 3SAT to this modified instance of TeamEnvVer shows the \textit{NP}-hardness of TeamEnvVer for the specified values of $h$ and $|Q|$.

Part (1) of Result B requires relatively straightforward modification to use transition-template library $L$, which actually ends up simplifying the original proof in [11]. As the team consisted of a single FSR, this trivially gives $h = 1$. Part (2) requires complex and decidedly non-trivial modification to allow $|Q| > 1$ and relatively straightforward modification of the construction used in the proof of part (2) of Result A to allow $h > 1$.

Result B (Modified from Lemma 5 in [11]): \text{ContDesLS}_{\text{res}}$ is not exact polynomial-time tractable unless $P = \text{NP}$ when $|Q| = 1$, $h = 1$, and $\#_{ec} = 0$. Moreover, this intractability holds for any version of \text{ContDesLS}_{\text{res}}$ when $h > 1$, $|Q| > 1$, and $\#_{ec} > 0$ for any fixed values of $h$, $|Q|$, and $\#_{ec}$.

Proof: Lemma 5 in [11] gives a reduction (based on a reduction in [12]) from \textit{Dominating Set} to a problem \text{ContDes}_{D,syn}^{fast}$ which is essentially \text{ContDesLS}_{\text{res},D}$ in which selection from a library $L$ of transition-templates is simulated by specifying bounds in the problem input on $|f|$, the maximum length of any transition trigger-formula. This reduction creates a somewhat complex environment for a team composed of a single-state FSR. [12] Figure 2(b)]. In order to force the transitions in such a robot to encode a candidate dominating set of size $k$ in the graph $G$ in the given instance
of Dominating set, the robot has to navigate from the southwestmost corner of the environment to the top of the \((k+1)\)st column in subgrid \(SG1\) [12, Figure 2(c)]. From there, the robot navigates the \(|V|\) columns of subgrid \(SG2\) [12, Figure 2(d)], where each column represents the vertex neighbourhood of a particular vertex in \(G\) and the robot progresses eastward from one column to the next if and only if that robot has a transition corresponding to a vertex in the neighbourhood encoded in the first column. Subgrid \(SG2\) thus checks if the robot encodes an actual dominating set of size \(k\) in \(G\), such that the robot enters the northeastmost square of the environment and builds the requested structure there if and only if the \(k\) east-moving transitions in the robot encode a dominating set of size \(k\) in \(G\).

Given the above, consider the following reduction from Dominating set to \(\text{ContDesLS}_{\text{res},D}\). Given an instance \(\langle G = (V,E), k \rangle\) of Dominating set, construct an instance \(\langle E', E'_T, |T|, p_I, |L|, r, |Q|, d, h, \#_{ec} \rangle\) of \(\text{ContDesLS}_{\text{res},D}\) as follows: Let \(E'\) be the environment constructed in Lemma 5 of [11] with the northwest \(e_N\)-based and \(SG1\) subgrids removed, \(E'_T\) be the version of \(E_T\) in that same lemma, \(p'_I = E'_{1,1}\), \(L = \{\langle q, \text{enval}(y, (0,0)), *, \text{goEast}, q' \rangle \mid y \in \{e_1, \ldots, e_{|V|}\}\} \cup \{\langle q, *, *, \text{goNorth}, q' \rangle \}\}, |T| = |Q| = h = 1, r = 0, d = k + 1, and \#_{ec} = 0. Finally, let the target configuration of the task associated with this instance be an FSR positioned in the northeastmost square in \(E'\); let us call this position \(p_F\). This instance of \(\text{ContDesLS}_{\text{res},D}\) can be constructed in time polynomial in the size of the given instance of Dominating Set.

Observe that the use of \(L\) means that we no longer need subgrid \(SG1\) and the restrictions on \(|f|\) posited in Lemma 5 mentioned above to force the created FSR to have \(k\) east-moving transitions corresponding to a candidate dominating set of \(k\) distinct vertices in \(G\). Hence, by slight simplifications and modifications of the proof of correctness of the reduction in Lemma 5 mentioned above, it can be shown that there is a dominating set of size \(k\) in graph \(G\) in the given instance of Dominating set if and only if there is an FSR with the structure specified in the constructed instance of \(\text{ContDesLS}_{\text{res},D}\) such (1) the lone FSR in \(T\) can progress to \(p_F\) if this FSR starts at \(p_I\) and (2) the \(k+1\) transitions in this FSR are \(k\) east-moving transitions from \(L\) whose activation-formula predicates correspond to the vertices in a dominating set of size \(k\) in \(G\), and the final transition in \(L\). As each transition in this FSR has an activation-formula consisting of either \(*\) or a single predicate evaluating if that square has a particular square-type, there can be at most one transition enabled at a time and the operation of this FSR in \(E'\) is deterministic. As the single robot in \(T\) can only move north or east and does one of either in each move, the number of transitions executed in this construction task is the Manhattan distance from \(p_I\) to \(p_F\) in \(E\). This distance is \(|V| + 1 + |V|^2 + 1 < |E'| = c_1|E'|^{c_2} < c_1(|E'| + |Q|)^{c_2}\) when \(c_1 = c_2 = 1\), which means that this navigation task is \((1,1)\)-completable. As Dominating set is \(NP\)-complete [18, Problem GT2], the reduction above establishes that \(\text{ContDesLS}_{\text{res},D}\) is
Graph $G$ to modify eastward-moving transitions that correspond to a dominating set of size $t$. Hence, an FSR that can successfully navigate the first the rules of deterministic FSR operation require that there cannot be more than one To then enter and traverse SG2, the state $q_{i-1}$ is to include an additional “holding area” consisting of $h-1$ squares enclosed...
by obstacle-squares such that no FSR inside this area can leave it, and populate this holding area with arbitrary FSR based on \(|Q|\) states such that none of these FSR makes an environmental modification. As \(h\) and \(|Q|\) are of fixed value, the modified instance of ContDesLS\(_{res,D}\) can still be constructed in time polynomial in the size of the given instance of DOMINATING SET. Moreover, the reduction from DOMINATING SET to this modified instance of ContDes:S\(_{res,D}\) shows the \(NP\)-hardness of ContDesLS\(_{res,D}\) for the specified values of \(h\) and \(|Q|\).

Result C requires a very straightforward modification of the algorithm presented previously in [9] to incorporate our new task model.

**Result C** (Modified from Result A in [9]): TeamDesLS\(_{res}\) is exact polynomial-time tractable when \(|Q| \geq 1\), \(h = 1\), and \(#ec \geq 0\).

**Proof:** The algorithm in the proof of Result A in [9], which tests for each controller \(c\) in \(L\) whether a team based entirely on \(c\) can construct \(X\) at \(p_X\) in at most \(c_1(|E|+|Q|)^{c_2}\) timesteps, operates in polynomial time. We need only modify that algorithm such that after each timestep of robot team operation we check if the target configuration associated with the task is in \(E\), which can also be done in polynomial time.

Part (1) of Result D requires relatively straightforward modification to environment and FSR transitions to force \(h\) to be exactly 2 (as the proof presented previously in [10] only required that \(h \leq 2\)). Part (2) requires relatively straightforward modification of the construction used in the proof of part (2) of Result A to allow \(h > 2\) and \(|Q| > 1\).

**Result D** (Modified from Lemma A.7 in [10]): TeamDesLS\(_{res}\) is not exact polynomial-time tractable when \(|Q| = 1\), \(h = 2\), and \(#ec = 0\). Moreover, this intractability holds for any version of TeamDesLS\(_{res}\) when \(h > 2\), \(|Q| \geq 1\), and \(#ec \geq 0\).

**Proof:** Lemma A.7 in [10] gives a reduction from 3SAT to a version of problem SelAlg\(_D\) which is for all intents and purposes our problem TeamDesLS\(_{res,D}\) without the restriction on \(#ec\). In the instance of SelAlg\(_D\) created by this reduction, a team \(T\) of \(|U|\) single-state FSR chosen from an FSR library of size 2 encodes a truth assignment to the variables in \(U\) in the given instance of 3SAT and each of the southmost \(|C|\) rows in environment \(E\) encodes a clause in \(C\) in this same instance. All robots in the team can reach the northmost row of \(E\) and deposit a strip of \(|T|\) squares of type \(e_X\) if and only the truth-assignment to the variables encoded in \(T\) satisfies all clauses in \(C\); moreover, as at least one robot moves north in each timestep and strictly less than \(|E|\) such moves can be made if \(T\) encodes such a valid truth-assignment, the task is \((1,1)\)-completable.

We modify this reduction to create a reduction from 3SAT to TeamDesLS\(_{res,D}\) when \(h = 2\) and \(#ec = 0\) as follows:
1. Add new clauses \(c_T\) and \(c_F\) to \(C\) and new variables \(u_T\) and \(u_F\) to \(U\) such that clause \(c_T\) (\(c_F\)) is satisfied if and only if variable \(u_T\) (\(u_F\)) is assigned value \(True\) (\(False\)).

2. Add two extra columns to \(E\) corresponding to new clauses \(c_T\) and \(c_F\) and add two to the size of the robot team.

3. In each of two robots in \(L\), replace the transition that modifies the type \(e_B\) of a square in the northmost row of \(E\) to \(e_X\) with a transition that stays on a square of type \(e_B\).

Note that this reduction creates an FSR team in which \(h = 2\), \(|Q| = 1\), and \(#_{ec} = 0\). If we then make the target configuration of the task associated with this instance of TeamDesLS\(_{res,D}\) be \(|T|\) FSRs positioned in the central squares of the northmost row of \(E\), the proof of reduction correctness in Lemma A.7 also establishes that this reduction is correct. As 3SAT is \(NP\)-hard [18, Problem L02], this reduction establishes that TeamDesLS\(_{res,D}\) is also \(NP\)-hard when \(h = 1\), \(|Q| = 1\), and \(#_{ec} = 0\); our main result then follows from Lemma 1.

To complete the proof, observe that to establish the \(NP\)-hardness of TeamDesLS\(_{res,D}\) and polynomial-time unsolvability unless \(P = NP\) of TeamDesLS\(_{res}\) for fixed values of \(h > 2\), \(|Q| > 1\), and \(#_{ec} > 0\), we can use appropriately modified versions of the constructions and logic that we used to show similar results for TeamEnvVer in the proof of Result A above.

Finally, in Result E, inapproximability in senses (1) and (2) follows from the previously presented proofs in [10] and the \(NP\)-hardness of decision versions of our verification and design problems shown in Results A, B, and D above. Inapproximability in sense (3) is new to the current paper but follows in a very straightforward manner from a known class inclusion result in computational complexity theory and the inapproximability proof for sense (2).

**Result E** (Modified from Results A.4 and A.5 in [10]): None of the intractable versions of TeamEnvVer, ContDesLS\(_{res}\), or TeamDesLS\(_{res}\) described in Results A, B, and D are polynomial-time approximable in senses (1–3) unless \(P \neq BPP\) and \(P = NP\).

**Proof:** That approximate polynomial-time solvability in sense (1) for any of the listed problems implies \(P = NP\) follows from the \(NP\)-hardness of the decision versions of each of these problems (which is established in the proofs of Result A, B, and D) and Corollary 2.2 in [21].

With respect to approximate polynomial-time solvability in sense (2), it is widely believed that \(P = BPP\) [20, Section 5.2] where \(BPP\) is considered the most inclusive
class of decision problems that can be efficiently solved using probabilistic methods (in particular, methods whose probability of correctness is \( \geq \frac{2}{3} \) and can thus be efficiently boosted to be arbitrarily close to one). Hence, if any of the listed problems has a probabilistic polynomial-time algorithm which operates correctly with probability \( \geq \frac{2}{3} \) then the decision version of that problem is by definition in \( BPP \). However, if \( BPP = P \) and we know that each of the decision versions of the listed problems is \( NP \)-hard by the proofs of Results A, B, and D, this would then imply by the definition of \( NP \)-hardness that \( P = NP \).

With respect to approximate polynomial-time solvability in sense (3), it is known that \( ZPP \subseteq BPP \), where \( ZPP \) is the class of decision problems that can be always be solved correctly by algorithms with expected polynomial runtime \cite{23}. Hence, if any of the listed problems is approximately solvable in sense (3) then the decision version of that problem is by definition in \( ZPP \) as well as \( BPP \). However, as each of the decision versions of the listed problems is \( NP \)-hard by the proofs of Results A, B, and D, this would then imply by the definition of \( NP \)-hardness and the widely-believed conjecture \( P = BPP \) that \( P = NP \).