Nuclear Anapole Moments and the Parity-nonconserving Nuclear Interaction

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Abstract.
The anapole moment is a parity-odd and time-reversal-even electromagnetic moment. Although it was conjectured shortly after the discovery of parity nonconservation, its existence has not been confirmed until recently in heavy nuclear systems, which are known to be the suitable laboratories because of the many-body enhancement. By carefully identifying the nuclear-spin-dependent atomic parity nonconserving effect, the first clear evidence was found in cesium. In this talk, I will discuss how nuclear anapole moments are used to constrain the parity-nonconserving nuclear force, a still less well-known channel among weak interactions.

Introduction
Tests of the unified electroweak theory have long been an important subject in physics. Compared with successes gained in the leptonic, semileptonic, and flavor-changing hadronic sectors, it is fair to say that the flavor-conserving hadronic weak interaction is not well-constrained. Experimentally, this sector could only be studied in nuclear systems, therefore, the major challenge comes from the sensitivity required to separate the parity-nonconserving (PNC) observables from much larger strong and electromagnetic (EM) backgrounds. Despite a number of difficulties, these studies are of fundamental importance because the hadronic neutral weak interaction only appears in the flavor-conserving sector. One also hope that these studies can reveal more information about the hadronic dynamics which can not be probed by parity-conserving (PC) observables.

Several precise and interpretable nuclear PNC measurements have already been made and put constraints on the PNC nucleon-nucleon (NN) interaction. These results along with new developments using polarized proton or neutron beam will be reviewed in the plenary talk by W. D. Ramsay. The focus of this short presentation is the nuclear anapole moment, which provides another window to examine the PNC NN interaction and have to be measured in atomic PNC experiments.

It was first noted independently by Vaks and Zel’dovich [1] that the PNC mechanism allows a parity-odd EM coupling to an elementary particle (actually to a composite system as well) by inducing an exotic electromagnetic moment, called the “anapole moment” (AM). Later on, Flambaum, Khriplovich, and Sushkov [2] pointed out that the AM of a nucleus grows roughly as the nucleon number to the power of two-thirds, and this suggested it might be possible to measure this nuclear AM in heavy atoms.

These theoretical conjectures finally realized when Colorado group [3] announced the first clear evidence of nuclear AM using the polarized atomic cesium beam. By carefully
identifying the hyperfine dependence of atomic PNC effects, a 7σ determination was reported. And the error bar is so small that a very good constraint on the PNC NN interaction could be deduced (if one does the calculation right). In the context of this symposium, it is more than adequate to recognize this discovery by atomic physicists which contributes to the nuclear spin physics at the lowest energy end.

**Nuclear AMs and the PNC NN Interaction**

According to the multipole expansion, the electromagnetic moments are classified as charge $C_J$, transverse electric $E_J$, and transverse magnetic $M_J$, where $J$ denote the angular momentum. Normally, parity ($P$) and time-reversal ($T$) invariance only allow charge moments of even order ($C_0$, total charge; $C_2$, charge quadrupole; ...etc.) and transverse magnetic moments of odd order ($M_1$, magnetic dipole; $M_3$, magnetic octupole; ...etc.). The vector moment $C_1$, which is $P$- and $T$-odd, is the charge dipole moment. The vector moment $E_1$, which is $P$-odd but $T$-even, is exactly the “anapole moment”. Often in the literature, the anapole operator $\vec{a}$, generated by the current density operator $\vec{j}(\vec{r})$, is defined as

$$\vec{a} = -\pi \int d^3r r^2 \vec{j}(\vec{r}) \equiv \frac{G_F}{\sqrt{2}} \kappa_{am} \vec{I}.$$ \(^1\)

It is clear that this operator gives vanishing expectation values unless the wave function is not a parity eigenstate or the current is axial-vector—both are linked to weak interactions at the fundamental level. In the last part of the equation above, a dimensionless quantity $\kappa_{am}$, which characterizes the strength of a nuclear AM, is defined through the Fermi constant (the typical scale of weak interaction) and the nuclear spin vector $\vec{I}$ (the only intrinsic vector of an elementary or composite particle).

An illustrative picture of the AM is the toroidal current winding. Because the $r^2$ weighting factor, the currents on the outer part of the torus give larger moments than the inner part, and this leads to a net AM. Suppose in a system where parity is a good symmetry, the left- and right-handed current windings should be equally probable, therefore no net AM occurs. However, any non-equal mixture of these two by some PNC mechanism will results in a chiral current and thus an AM. Also noteworthy is that the magnetic field generated by the toroidal current winding is confined, therefore, unless a particle is inside the torus, there is no interaction. This contact character of interactions with AMs is the same as the low energy neutral weak interaction, a result anticipated by the unified electroweak theory.

Although one believes the nuclear AMs have their origin in the couplings of quarks and weak bosons, $W^\pm$ and $Z^0$, a hadronic theory from the first principle is still unavailable. Instead, various models are designed to describe the nonperturbative dynamics of hadrons. For the PNC NN interaction, the widely-used framework is a one meson exchange model including $\pi$, $\rho$, and $\omega$, with one of the meson couplings is PC and the

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\(^1\) The current conservation plays a role in defining the form of $E_1$ operator, and the definition for $\kappa$ is different from what Khriplovich et al. adopted. For more details, see Ref. [10].
other PNC. The six PNC meson coupling constants in this model, $h_\pi$, $h_{\rho}^0$, $h_{\rho}^1$, $h_{\rho}^2$, $h_\omega^0$, and $h_\omega^1$, as defined by Desplanques, Donoghue, and Holstein (DDH) [4], undermine the physics of how the fundamental couplings of quark and weak bosons are modified by the strong interaction.  

The theoretical benchmark is given by the so-called DDH “best values” along with some reasonable ranges. It is the hope that experiments could constrain these couplings well and justify the hadronic theories.

Given this PNC NN potential, nuclear AMs arise in three ways: i) one-body contribution, where weak radiative corrections are induced in the form of single nucleon loop or pole diagrams, often called as the nucleonic AM, ii) two-body contribution, where mesons induce extra EM currents by coupling photons to nucleon-antinucleon pairs and mesons in-flight, iii) polarization mixing, where a parity eigenstate state is mixed by opposite-parity states, thus the normally forbidden EM couplings are allowed. While the one-body contribution is incoherent, many-body effects would possibly enhance the two-body and polarization mixing terms.

**Experimental Results and Deduced Constraints**

With the increasing accuracy, atomic PNC experiments have been an important part of the low-energy precision tests of the standard model. The dominant PNC effect comes from the tree-level $Z^0$ exchange between atomic electrons and the nucleus with an axial-vector coupling at electrons and a vector coupling at the nucleus, $A(e)-V(N)$. This is a nuclear-spin-independent (NSI) effect in which every nucleon contributes coherently. On the other hand, the $V(e)-A(N)$ exchange gives a nuclear-spin-dependent (NSD) effect, but is much suppressed because nucleons contribute incoherently and electrons have a weaker vector coupling to $Z^0$.

Although the interaction of electrons with the nuclear AM comes at a higher order, i.e., $G_F \alpha$, it actually dominates the NSD effect in heavy nuclei because the electron coupling is not suppressed by $(1 - 4 \sin^2 \theta_W)$ and the nuclear many-body enhancement grows as $A^{2/3}$. Therefore, the extraction of nuclear AM involves: i) an atomic many-body calculation relating the experiment result to the PNC electron-nucleus interaction, ii) the identification of NSD PNC effect by comparing results from different hyperfine levels, and iii) the subtraction of contributions due to $Z^0$ exchange and hyperfine interaction.

So far, nuclear AMs in cesium and thallium have been reported. The cesium experiment by Colorado group showed a clear evidence, however, the thallium experiments by Seattle [5] and Oxford [6] groups had large error bars so the results are consistent with zero. The extracted AMs in terms of $\kappa_{am}$ are: $\kappa_{am}(Cs) = 0.090 \pm 0.016$ [7] and $\kappa_{am}(Tl) = 0.376 \pm 0.400$ (Seattle’s only).

In order to constrain the PNC meson couplings using these results, one has to perform a model calculation of the nuclear AM and then express $\kappa_{am}$ in terms of these couplings.

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2 $h_\pi$ was named ad $f_\pi$ originally by DDH, however, it is changed in order not confuse with pion decay constant sometimes.

3 The Oxford result is not quoted here, see Ref. [10] for discussion.
FIGURE 1. Constraints on the PNC meson couplings ($\times 10^7$). The error bands are one standard deviation.

Because both Cs and Tl are heavy nuclei, the nuclear structure is the most important issue. There have been quite a few calculations with various treatments, a brief summary and survey could be found in Ref. [8]. Roughly speaking, the calculations based on the single particle approximation, which treat the Cs as a $1g_{7/2}$ proton plus the closed core and Tl as a $3s_{1/2}$ proton hole plus the closed core, tend to predict larger AMs than calculations which consider many-body effects.

The constraints on the PNC meson couplings is presented in Fig. 1. The Cs and Tl bands are plotted based on the shell model results of Ref. [9, 10], a full two-body calculation in which all the exchange currents are included and the polarization mixing is handled by the closure approximation with the aid of nuclear systematics found in light nuclei.

Apparently, the anapole constraints are not in agreement with the existing nuclear PNC results, and also with each other (only a small part of the Tl band is shown here, and the central line of this band has a negative x-intercept). The result for the AM of Tl is rather confusing because the experiment gave a positive value, but all the calculations predict negative. Therefore, it is very possible that the tension between Cs and Tl bands is due to this sign problem. One can also observe that the Cs result tests a similar combination of PNC couplings as $\rho\alpha$ and $^{19}\text{F}$, but favors larger values. By combining Cs and $pp$ bands, the allowed region does fall into the DDH reasonable ranges, with $h_\pi \sim 9$. However, the stringent limit set by the $^{18}\text{F}$ result, $|h_\pi| \leq 1$, which has been performed by five groups, definitely rules out this possibility.

The big discrepancy between the anapole constraints and existing nuclear PNC results is certainly a puzzle to be sorted out. The first criticism of the theory would be on our still
limited knowledge of the structure of heavy nuclei. By the way, the atomic many-body theory, which is the key to the interpretation of experiments, should also be examined.

With only one certain result in Cs, obviously we need more experimental inputs to clarify the current situation. There are several new measurements being in progress or proposal. For example, a new Cs measurement will double-check the existing result, an improved Tl experiment hopefully can solve the sign problem, results of odd-neutron nuclei like Dy, Yb, and Ba would produce constraints roughly perpendicular to what odd-proton nuclei do, and the study of a chain of Fr isotopes should reduce some of the theoretical uncertainties.

However, it ought to be emphasized that, if any of these results, when available, is going be to used for constraining PNC meson couplings reliably, a good nuclear structure calculation is still the top necessity.

Summary

The nuclear anapole moment, a manifestation of nuclear parity-nonconservation which has been conjectured for a long time, is clearly discovered in the atomic PNC experiment of cesium. The precision of this result makes it sensible to constrain the PNC meson couplings. However, a big discrepancy is found by comparing this new constraint with existing nuclear PNC results, most possibly due to the nuclear structure uncertainties. In order to constrain the hadronic theory reliably, this puzzle should be further addressed.

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