The modified dynamics is conducive to galactic warp formation

Rafael Brada and Mordehai Milgrom
Condensed Matter Department, Weizmann Institute of Science, Rehovot Israel

ABSTRACT

There is an effect in the modified dynamics (MOND) that is conducive to formation of warps. Because of the nonlinearity of the theory the internal dynamics of a galaxy is affected by a perturber over and above possible tidal effects. For example, a relatively distant and light companion or the mean influence of a parent cluster, with negligible tidal effects, could still produce a significant warp in the outer part of a galactic disk. We present results of numerical calculations for simplified models that show, for instance, that a satellite with the (baryonic) mass and distance of the Magellanic clouds can distort the axisymmetric field of the Milky Way enough to produce a warp of the magnitude (and position) observed. Details of the warp geometry remain to be explained: we use a static configuration that can produce only warps with a straight line of nodes. In more realistic simulations one must reckon with the motion of the perturbing body, which sometimes occurs on time scales not much longer than the response time of the disk.

Subject headings: galaxies: kinematics and dynamics

1. introduction

It is not yet known what exactly induces and maintains galactic warps. It may well be that a number of mechanisms–from among those proposed already, or some new ones–act together or alone to produce this ubiquitous phenomenon. Some proposed mechanisms rely on a dark galactic halo as a direct actuator or as a mediator of perturbations (for reviews with extensive references see Briggs 1990, Binney 1992, and Binney & Merrifield 1998). The modified dynamics (MOND) repudiates dark halos, but it offers a new mechanism that increases the warping efficacy of external perturbers over and above their possible tidal effects, which, notoriously, are too weak. This results from the nonlinearity of MOND and is most clearly demonstrated in the case where a system (a galaxy) falls in an external field
that by itself is approximately constant in space. In a linear theory, such as Newtonian
gravity, the constant external field has no effect on the internal dynamics of the system
(motions with respect to its center of mass); in MOND it very much does. When the
external field dominates the internal field of the system it is easy to deduce what its effects
are, as discussed e.g. in [Bekenstein & Milgrom (1984)] and [Milgrom (1986)]. In the present
context the external acceleration is small compared with the internal ones at the position
of the warp, which necessitates numerical studies.

For our mechanism to work in field galaxies, one or more perturbers must be present.
There is, indeed, growing evidence that the appearance of a warp in a galaxy is strongly
correlated with the presence of nearby perturbers (see e.g. [Reshetnikov & Combes 1998]).
Even galaxies that had been thought to be isolated might, in fact, not be so ([Shang & al
1998]). Of course, perturber companions have always been suspected, but their direct tidal
effects on disks seem to be too small.

The purpose of this letter is to demonstrate, by numerical solutions of simplified
galaxy-perturber systems, that, with reasonable parameter values, this MOND effect
produces galactic warps of the magnitude observed. We have not included effects due to
variations in the external field (due to the motion of the perturber, or to the motion of
the galaxy in a parent cluster). From the symmetry of our model problem the warps we
produce have a straight line of nodes.

The method is described in section 2; the results are detailed in sections 3 and 4;
conclusions are drawn in section 5.

2. Method

We use the nonrelativistic, modified-gravity formulation of MOND suggested by
[Bekenstein & Milgrom (1984)]. The acceleration field \( \vec{g} = -\vec{\nabla}\phi \) produced by a mass
distribution \( \rho \) is derived from a potential \( \phi \) that satisfies

\[
\vec{\nabla} \cdot [\mu(|\vec{\nabla}\phi|/a_0)\vec{\nabla}\phi] = 4\pi G\rho
\]

instead of the usual Poisson equation \( \vec{\nabla} \cdot \vec{\nabla}\phi = 4\pi G\rho \), where \( \mu(x) \approx x \) for \( x \ll 1 \),
and \( \mu(x) \approx 1 \) for \( x \gg 1 \), and \( a_0 \) is the acceleration constant of MOND. The form
\( \mu(x) = x/\sqrt{1 + x^2} \) has been used in all rotation curve analyses, and we also use it here.
This nonlinear potential equation is solved numerically using multi-grid methods as detailed
in [Brada (1996)], and adumbrated in [Brada & Milgrom (1999)].
We consider two classes of models. To simulate a far away companion, or the effect of the mean field of a cluster on a member galaxy, we solve for the field of a rigid disk in the presence of a given external acceleration field $\vec{g}_{ex}$. In this case the field equation is solved subject to the boundary condition at infinity $\phi_\infty(\vec{r}) = -\vec{r} \cdot \vec{g}_{ex}$. Then $\vec{g}_{ex}$ is subtracted from $-\vec{\nabla}\phi$ to get the field relative to the galaxy. This latter determines the galaxy’s internal dynamics, warps, etc. To simulate the effect of a nearby companion, exemplified here by the effect of the Magellanic clouds (MC) on the Milky-Way (MW), we solve fully for a disk-plus-perturber system (in which case $\vec{\nabla}\phi \to 0$ at infinity). Then, the center-of-mass acceleration of the galaxy is computed using the surface-integral method given by eq.(14) of Bekenstein & Milgrom (1984), and is subtracted from the acceleration field to get the internal dynamics.

In each case, after the acceleration field relative to the center of mass of the galaxy is found, we find closed, nearly circular, nearly planar, test-particle orbits. The orbits are integrated for many periods to insure that, within our accuracy and patience, they are closed. Thus, inasmuch as adiabaticity is a good approximation, these are non-precessing orbits. They are also found to be stable under small changes in their initial conditions. These are taken to trace a warp, in the spirit of the tilted-ring model (Rogstad, Lockhart, & Wright 1974).

3. An exponential disk in a constant external field

We take the model galaxy to be an exponential disk smoothly truncated at a radius that we use as our unit length, $R_{cut} = 1$, and with a scale length of $h = 0.2$ in these units. The surface density is of the form $\Sigma_0 \exp(-R/h)(1 - R^4)$ for $0 \leq R \leq 1$. The disk lies in the $x-y$ plane. To optimize the warping effect we take the external field to lie $45^\circ$ from the $x$ axis in the third quadrant of the $x-z$ plane. Its absolute value is taken as $g_{ex} = 0.01$ in units of $a_0$. (We work in units where $a_0 = 1$, $G = 1$, so masses are given in units of $a_0 R_{cut}^2/G$.) In a more extensive study we plan to calculate the effect as a function of the field direction (relative to the disk axis) and also to follow the test particle orbits as the external field changes with time to mimic the relative motion of the galaxy and perturber. For the present, pilot study we ran models with two values of the disk mass $m = 0.01$ and $m = 0.04$. The (MOND) accelerations of the isolated disk models at $R = 1$ are $\approx m^{1/2} = 0.1, 0.2$; i.e. respectively, ten and twenty times larger than the external field. Both accelerations are small compared with 1 ($a_0$) so we are rather deep in the MOND regime in the warp region. In the deep-MOND regime the theory has obvious scaling.
properties so the above parameters represent a family of models spanned e.g. by scaling by the same factor $m^{1/2}$ and $g_{ex}$, or $m$ and $R_{cut}^2$ (with $h/R_{cut}$ fixed).

The results are summarized in Figures 1 and 2. We first show for each model a plot of the absolute value of the torque $T \equiv |\vec{r} \times \vec{\nabla}\phi_c|$ in the $x-z$ plane containing the disk axis and the external field ($\vec{\nabla}\phi_c$ is the field in the center-of-mass frame). This is a quantity that brings out clearly the departure of the field from both axisymmetry and left-right symmetry. In the spherical case $T = 0$ everywhere; in the isolated-disk case the $T = 0$ line is the $x$ axis (and the $z$ axis). This torque plot is also useful for homing in on closed orbits of the potential whose center is near the galactic center, because these should cross the $x-z$ plane near the zero-torque line. The orbits are found by actual integration, starting from a set of initial conditions. The projections of some such orbits are then shown. We surmise that in the spirit of the tilted-ring model they delineate the shape of the warp.

![Fig. 1.](image)

**Fig. 1.**—The field geometry and some closed orbits for the $m = 0.01$ case. The upper panel shows the lines of constant $|\vec{r} \times \vec{\nabla}\phi_c|$ in the $x-z$ plane. The lower panels show the projections of some closed orbits on the $x-z$ and $x-y$ planes. The three symbols □, △, and ◇ show their crossing points of the $x-z$ plane.
4. A disk-plus-companion system—the effect of the Magellanic Clouds on the Milky Way.

The disk of the Milky Way is known to be warped beyond the solar circle (Burke 1957, Kerr 1957, Henderson, Jackson, & Kerr 1982, and for a recent description and references Binney & Merrifield 1998). At galactic longitude \((l \approx 90^\circ)\) the HI disk curls steadily away from the plane. At \((l \approx 270^\circ)\) the disk curves southward before turning back towards the plane (see Binney & Merrifield 1998 for an analytic expression that approximates the warp shape beyond 11 kpc).

Fig. 2.— The same as Figure 1 with \(m = 0.04\) for 4 orbits.

The line of nodes in the tilted-ring picture is straight within the uncertainties, and is nearly perpendicular to the plane spanned by the inner-disk axis and the radius vector to the Magellanic clouds. This makes the cloud system a prime suspect in producing the warp. It was, however, appreciated long ago that the tidal field of the Clouds, in their present position, is too small to distort the disk to the extent observed. For example, Hunter & Toomre (1969) estimated that a cloud mass \(\approx 10^{10}M_\odot\) would generate a warp of amplitude \(\leq 70\) pc at a radius of 16 kpc. It has, however, been suggested by Weinberg (1995) that

[Diagram of warp in Milky Way disk with text details]

[Another diagram showing line of nodes and cloud system]

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a “live” halo that actively responds to the perturbation of the clouds might augment the
effect to produce a warp of the observed magnitude and geometry.

MOND, as we said, excludes a dynamically important halo, but might lead to a large
enough warp due to the non-linear effect discussed above. We model the system as follows.
The MW is taken as a pure disk in the $x - y$ plane, centered at the origin, with the cutoff,
exponential surface-density law described in section 3 (with $h = 0.2$); its dimensionless
mass is $M_{\text{disk}} = 0.04$. The Magellanic clouds are represented by one point mass $M_{\text{sat}}$ at a
position whose dimensionless coordinates are $(2.52, 0, -1.63)$. This is at $15h$ from the center
of the Galaxy, and at the correct galactic latitude of the LMC. This ratio would correspond
for example to $h = 3$ kpc (Binney & Merrifield 1998) and an LMC distance of 45 Mpc
(Mould & al 1999). The uncertainties in these parameters are still large. Two mass ratios
were considered: $M_{\text{sat}}/M_{\text{disk}} = 0.1, 0.2$. (The B luminosity ratio of the clouds to the MW is
about 0.2. Since the baryonic $M/L$ values of the two might be different, reasonable values
of the mass ratio lie between 0.1 and 0.4.) Other nearby galaxies are expected to have a
smaller effect than the LMC; for example, M31, despite its higher mass, causes a rather
smaller acceleration near the MW.

In Figure 3 we show, for each mass ratio, two closed, quasi-circular, stable orbits
beyond the cutoff radius of the disk—presumed to delineate our calculated warp—together
with a representation of the observed warp (as given by the formula in Binney & Merrifield
1998).

![Fig. 3.— The calculated Milky Way warp. Shown are the crossing points of two stable, test-
particle orbits for two satellite-to-disk mass ratios: 0.1 (circles), and 0.2 (crosses). The central bar
marks one disk scale length (=3 kpc) on each side. The heavy line shows the observed warp of the
galaxy. The arrow marks the direction to the perturber representing the MC. The vertical scale is
stretched by a factor of 2 relative to the horizontal scale.](image-url)
5. Conclusions and discussion

We see that for a constant external field whose ratio to the field of the isolated disk at \(5h\) is only (5-10)%, a noticeable warp beyond this radius is indicated by test-particle orbits. This acceleration ratio increases in proportion to the galactocentric radius, but, still, rotational velocities will be affected only little even up to \((10 - 15)h\). Note that the warp is not symmetric but is less pronounced on the attracting side of the field.

We used an external-field inclination that is favorable for an S-shape warp. When the field is in the disk plane, the axisymmetry is broken but the up-down one is not, so no warp will be induced. If the field is perpendicular to the disk, axisymmetry is preserved but not the up-down one; this might induce bowl-shaped warps. (In the limit of a highly dominant, perpendicular, external field, the analytic results in Milgrom (1986) show that the geometry remains up-down symmetric, but for a weak perturbing field this is not so.)

Regarding the results for the MW-MC system, we see that even for a mass ratio of 0.1 a satellite at the position of the MC produces enough field distortion to accommodate inclined, quasi-circular orbit that rise to 0.25\(h\) at \(R = 6h\) on one side, and to a height of 0.2\(h\) at \(R = 5.5h\) on the other. With a mass ratio of 0.2 the amplitude of the warp is close to that observed for the MW.

Our analysis requires various improvements, which we hope to include in a future, more extensive analysis.

1. A larger volume of the parameter space has to be surveyed. This includes more values of the relative strength of the perturbation, different disk-perturber alignments (leading perhaps to a wider varieties of warp shapes), more complex perturbations such as two or more satellites, which would bend the line of nodes at larger radii, where we cannot approximate the combined effect by a constant field. We would also have to study other galaxy mass distributions. For example, we expect that if a considerable fraction of the galaxy mass is put in a round bulge, a warp will form more easily. For the same reason, if \(h\) is smaller (but the MC distance remains the same) the warp will be stronger at the same position.

2. Viewing the warp as an envelope of test-particle orbits may be too naive. Certainly in more complicated geometries we expect orbits to cross and gas dynamics must be considered.

3. We must reckon with the fact that in many relevant cases the geometry of the perturbation changes considerably during the response time of the disk (say the orbital period at the position of the warp). This is true of galaxies moving in or near the core of
galaxy clusters; it is also true for the MW-MC system. The warp geometry will thus not just follow the perturbation adiabatically but, at larger radii, the geometry may reflect the past history of the perturbation (leading, among other things, to curvature of the line of nodes). According to proper-motion observations and models of the MC and the Magellanic stream motion (see e.g. Lin, Jones, & Klemola 1995), the MC binary is moving on a nearly polar orbit around the galaxy with a tangential velocity that is now comparable with the rotational velocity of the galaxy. This means that the radius vector to the MC changes its angle with the Galaxy’s axis by 90° during the Galaxy’s orbital period at about 15 kpc. This means that the adiabaticity assumption we have made might be broken, and more and more so at larger radii. Our subtraction of a constant center-of-mass acceleration is then also not valid. This could lead to a more complicated warp geometry than the integral-sign shape that we get with adiabaticity. Because the MC orbit is nearly polar we expect the line of nodes to remain straight.

4. We think self-gravity of the mass in the warped region is not so important. This is because the relative contribution of the warped mass to the acceleration field is small everywhere, even within the warp itself. (The surface density in the warp is smaller than the integrated surface density there.) This contribution can then be treated as a perturbation–linearizing in it the MOND field equation—and in MOND such density perturbations produce an even weaker effect than in Newtonian dynamics (hence the added stability in MOND). So we can at least expect that the nonlinearity of MOND will not beget some peculiar amplification of self gravity. But we do not really know what these effects might be—a point that has to be checked numerically.

We plan to perform $(N+1)$-body simulations whereby the $N$-body warped disk and the point-mass perturber orbit each other. This will account for non-adiabaticity and for self gravity in the disk, and also partly for point 2 above.

We thank James Binney for helpful suggestions and for comments on the manuscript, and the referee for improving suggestions.

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