Golden fraction in the theory of nucleation

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Abstract

The problem of the universal form of the size spectrum is analyzed. The half-widths of two wings of spectrum is introduced and it is shown that their ratio is very close to the golden fraction. In appendix it is shown that behind the golden fraction of an image one can find the information basis, i.e. the proportion of the golden fraction corresponds to some method to find extremum. The method to find extrema associated with Fibonacci numbers also leads to proportions which can be seen in nature or can be introduced artificially. The information origin of proportions is proved theoretically and confirmed by examples in nature and human life.

1 Universal proportion in the form of the size spectrum

It is well known that the phenomenon of a golden fraction is widely spread in nature. This fact is proven by numerous measurements during hundreds of years. The most striking feature is that some fundamental proportions in nature satisfy the golden fraction.

It is worth seeking the golden fraction in the process of nucleation. The most natural conditions are the dynamic ones. Under these conditions there is a universal form of size spectrum derived in [1]. The form of the universal spectrum is given by the following formula

\[ f = \exp(x - \exp(x)) \]

in the special coordinates (see [1]) after the special renormalization.
The spectrum has the amplitude 

\[ f_{am} = \exp(-1) \]

which is attained at \( x = 0 \).

The relaxation length is ordinary defined as the length where the function is diminished in \( \exp(1) \) times. So, here appeared two lengths - one corresponding to the right wing and that corresponding to the left wing. We shall denote them as \( -x_1 \) and \( x_2 \). They can be expressed through the W-Lambert function and have the following values

\[ x_1 = 1.84 \]
\[ x_2 = 1.14 \]

The ratio \( x_2/x_1 \) is very close to the golden fraction

\[ x_2/x_1 = 0.622 \]

This value is very close to the precise value of the golden fraction 0.618. The relative error is less than one percent.

The situation is clarified by fig.1.

There is no yet any clear interpretation of such good coincidence of this result with the golden fraction. It is quite possible that this is explained by the information origin of the golden fraction which is derived in Appendix.

2 Appendix: The role of the information interaction in the golden fraction

The proportions of a human body satisfy the golden fraction rule as it was stated many times, for example, by Pythagoras, Leonardo da Vinci, etc. But investigations of Adolf Zeising [2] showed that only the main proportion of a male body satisfies in the global proportions the rule of the golden fraction. The global proportion of a female body slightly differs from the golden fraction 1.618 and it is 1.60. Why this slight deviation takes place? Below the answer on this question will be given. This answer is based on the information origin of the golden fraction which will be analyzed below together with incomplete fractions appeared as ratios of Fibonacci numbers.
2.1 Information origin of the golden fraction

We admit that there is some natural process behind the phenomena of the golden fraction. What process can it be? At least this is the process of observation. Certainly, the process of observation is the information interaction between the observer and the environment. What purposes are attained in this interaction? We suppose that the observer wants to reconstruct the shape and the content of the image. The points which produces the maximal information are certainly the bifurcation points. The second important class of points are the points of extrema. Bifurcation points compose the shape of the object and form the information background to find all other characteristics of the object. At this shape the points of extrema have to be found. So, the primary task is to find extrema.

It is known that practically all methods to find exremum different from the simple comparison of function in different points contain the one dimensional procedure as a elementary step in the global procedure \[5\]. So, it is worth to consider namely the one dimensional procedure of the extremum seeking.
Consider an elementary interval \([0, 1]\). This interval will be the initial interval where the extremum (maximum) of the known function exists. Our task is to determine the position of this maximum.

To state that there is a maximum inside the given interval it is necessary to have at least three points at the interval. Two points will be at the ends of interval. This is clear because during the sequential procedure we diminish the initial interval and automatically the boundary points of a new interval will be already measured. When we have three points we can only state that there is a maximum in the inner point when the function in the inner point is greater than at the boundary points. But we can not diminish the interval without the forth point. Let the inner points be \(x_1\) and \(x_2\). We have to determine the positions of these points. Let \(x_1\) be less than \(x_2\).

When \(f(x_1) > f(x_2)\) then we can reduce \([0, 1]\) to \([0, x_2]\). When \(f(x_1) < f(x_2)\) then \([0, 1]\) can be reduced to \([x_1, 1]\).

The symmetry requires that

\[x_1 = 1 - x_2\]

Then to determine the position of \(x_1\) one can note that at interval \([0, x_2]\) it will be necessary to put two points and it would be very profitable when one of these points coincides with \(x_1\). This point will be the left point in the interval \([0, x_2]\). Then

\[x_1 = x_2 \times x_2\]

or

\[x_1 = (1 - x_1) \times (1 - x_1)\]

with a root

\[x_1 = \frac{3 - \sqrt{5}}{2}\]

which belongs to \([0, 1]\). The value

\[1 - x_1 = \frac{-1 + \sqrt{5}}{2} = 0.618\]

is called the golden fraction.

This value is namely the golden fraction mentioned at the beginning. So, there appeared a hypothesis that the golden fraction in nature is associated with a process of observation and with the procedure of seeking the extremum. This is the main idea of this publication. But it is necessary to confirm this observation. This will be done below.
2.2 Example of inapplicability of the pure golden fraction

We shall check the method of the golden fraction on example of the seeking for the approximate extremum by two measurements in the inner points of interval. After only two measurements it is necessary to take the final decision. This is the minimal number of measurements because one measurement can not specify the interval smaller than the initial one.

The simple analysis shows that the smallest interval will be when two points are \( x_1 = 1/3, \ x_2 = 2/3 \). It does not correspond to the golden fraction, but \( x_2 = 0.6666 \) is rather close to 0.618. Here

\[
x_1 = 1 - x_2 = x_2 - x_1
\]

The reason of the discrepancy is the finite number of measurements. So, it is necessary to analyze the optimal procedures to find extremum with finite number of points.

2.3 The method to find extremum in the finite number of measurements

One of the oldest methods to find extrema is the Fibonacci method described already by Euclid [3].

Let the process be the one dimensional searching of an extremum restricted by \( N \) measurements. The process is supposed to be a sequential one, i.e. the observer makes conclusions about the interval for the possible values of an argument at every step of the measurements. We shall call this interval as the uncertainty interval \( I_N \).

Now we consider the last measurement \( X_N \). It has to be made in interval \( I_{N-1} \). This interval contains the point of extremum and also the point \( E_{N-1} \) at which the extremum between all taken measurements is attained.

If we take the new point of measurement \( X_N \) equal or very close to \( E_{N-1} \) then we get no new information about the behavior of the function and such measurement is useless. So, it is necessary to have a distance between \( X_N \) and \( E_{N-1} \). Certainly, we do not know this distance and and speak only about the lowest boundary for this distance \( \delta \).

The best estimate for the \( |I_N| \) will be when we put \( X_N \) symmetrically to \( E_{N-1} \) with respect to the middle of interval \( I_{N-1} \). Then

\[
I_{N-1} = 2I_N - \delta
\]
This completes the step in the recurrent procedure.

Now we come to the previous pair of experiments. The interval $I_{N-1}$ contains $E_{N-2}$. In this interval two experiments will be made. The best experiment in this pair will be $E_{N-1}$. Another experiment will be denoted as $D_{N-1}$. This point will be the boundary between two parts of $I_{N-2}$: one part will be included in the further investigations and the other part will be thrown out.

But at the beginning of experiment it is not known what value from the pair will be the best and what will be thrown out. So, these values have to be symmetric with respect to the ends of interval. So, the distances from these points to the ends of interval have to be equal.

Since both points are symmetric with respect to the middle of interval and one of the points will be the optimal $E_{N-1}$ then every point has to be at the distance $L_{N-1}$ from the end of interval. Then

$$L_{N-2} = L_{N-1} + L_N$$

These recurrent relations are typical for the Fibonacci numbers. It is necessary to check the initial numbers with $N = 1$ and $N = 2$ but according to (1) these numbers are equal and after the renormalization of $L_1$ and $L_2$ we come to

$$F_1 = 1, F_2 = 1$$

Then $L_N = F_N$ are the Fibonacci numbers.

The sequential necessary proportions will be

$$F_2/F_3 = 2/3, \quad F_3/F_4 = 3/5, \quad F_4/F_5 = 5/8$$

Already $F_4/F_5$ is very close to the golden fraction and later all sequential fractions will approach to the golden fraction. So, it is worth to consider only the first fractions.

One can see that this method is optimal in the case of $N$ measurements.

### 2.4 Examples of proportions

When human bodies or some other objects in nature have the mentioned proportions it allows to grasp their image rather fast. So, one can speak about the increase of the interaction speed. The time necessary to get the approximate image is smaller when the main extreme points of an image
coincide with proportions prescribed by the golden fraction or Fibonacci fractions.

If our hypothesis is true then there will be numerous examples of the Fibonacci fractions $F_2/F_3$ and especially $F_3/F_4$. The higher fractions cannot be observed because they are too close to the golden fraction. Really, in many cases it is necessary to get the extremum after two or three measurements. As an example one can consider professions of drivers, hunters, etc., where it is very important to take decisions immediately.

So, one can come to conclusion that there exist some observers who have the habits to estimate the extrema in several first steps. The object under such observation will correspond to their habits.

As it is known from statistical mechanics the additional time spending for a fixed job corresponds to some surplus energy (because the small time corresponds to the nonequilibrium process which requires the surplus energy). So, the construction of the image with ideal proportions is energetically profitable.

Now it is clear that the proportion of a female body 0.60 corresponds to $F_3/F_4$ and it is explained by historical role of a hunter in a pre-historical period. Since it was necessary to take decisions immediately the hunters used to estimate the image in two or three basic points, Contrary to men the women have enough time for observations in their silent life and, thus, the male body have a proportion of a golden fraction corresponding to the infinite number of observations.

Certainly, women can not immediately transform their body to the golden fraction proportion in our society where professions of men are now rather calm. But later the evolution choice will inevitably bring this proportion to a golden fraction. The women with long feet are sexually attractive now and have more chances to get children, So, earlier or later this proportion will come to the golden fraction. But it takes thousands of years and now we have the proportion $F_3/F_4$ which is the trace of men’s professions in the pre-historical times.

One can see the following interesting example confirming this theory. The Kuroi in Greece created before the classic period have proportions (see fig.2) corresponding to the female fraction 0.60.

An explanation is very simple since the sculptor and spectators were mainly the men who found the sexually attractive proportions as the female ones. Only in classic period these proportions were reconsidered and brought to the real proportions of a male body.
Is it possible to view the first proportion \( F_2/F_3 \) in a human body? In nature it does not exist. But it can be seen in artificial images of women clothes in a fashion industry images (see fig.3).

The mentioned main ratio here is close to \( F_2/F_3 \). One can continue this type of examples. The different heights of heels help women to modify the main ratio of a body. One and a half or two centimeters of a heel give approximately one percent in ratio. So, the heels in three-four centimeters transforms the ratio 0.60 to the golden fraction. This corresponds to the English heel. The high heels in 10 centimeters transforms the ratio into the fraction \( F_3/F_4 \). This is a French heel. Two types of heels give a clear answer on applicability of the Fibonacci ratios. Women evidently vote by their heels for the information basis of the harmonic proportions in nature.

As the result of the given considerations one can state that now the information origin of appearance of the golden fraction is clarified. If we start from the principle of the minimal energy we can derive the golden fraction analytically since every observation requires some time and, thus, some additional energy. The facts appeared from the incomplete golden fractions, i.e. from the Fibonacci numbers, show experimentally that behind the golden
proportions there is the Fibonacci method of the extrema determination.

One can also mention that now it is clear why ordinary in the women fashion the waist line is outlined. Really, the waist line goes approximately three centimeters higher the umbilicus point which brings the ratio $0.60 = \frac{F_3}{F_4}$ to the golden fraction.

One can also see that the spatial sequence of different Fibonacci proportions introduces the sequence of different times for observation of these proportions. So, there appeared the connection between the space image and the sequence of times (or the melody) of observation. This allows to speak about the space-time connection and about the melody of paintings.

References

[1] Kurasov V.B. Theoretical and mathematical physics, 2002, 131:3, 503528

[2] Adolf Zeising, Neue Lehre von den Proportionen des menschlichen Körpers aus einem bisher unerkannt gebliebenen, die ganze Natur
und Kunst durchdringenden morphologischen Grundgesetze entwickelt, Leipzig, 1854

[3] Kusin L.T. Foundations of cybernetics, vol.1, Moscow, Energy, 1973, 504 p.