Production and decay of spinning black holes at colliders and tests of black hole dynamics

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Abstract

We analyse the angular momentum distribution of black holes produced in high energy collisions in space-times with extra spatial dimensions. We show that the black hole spin significantly affects the energy and angular spectra of Hawking radiation. Our results show the experimental sensitivity to the angular momentum distribution and provide tests of black hole production dynamics.
The possibility of extra spatial dimensions (ESDs) has been raised to explain the hierarchy problem, either through the volume of the ESDs \([1]\) or an exponential warp factor \([2]\). Given a number \(n\) of ESDs, the hypothesis is that gravity propagates in \(4+n\) dimensions (the bulk), while the standard model (SM) fields are confined to the 4 observed dimensions (the brane). At the fundamental scale \(M_F\) in \(4+n\) dimensions, which is close to the electroweak scale \(\mathcal{O}(1 \text{ TeV})\), all interactions including gravity have the same strength.

In a parton-parton collision with energy exceeding \(M_F\), the strong gravitational field can induce the formation of a black hole if the energy is confined to a region smaller than the corresponding event horizon \([3]\). The production and decay of black holes of mass \(M_{bh} \sim M_F\) will be heavily influenced by quantum gravitational effects. However, for \(M_{bh} \gg M_F\), the classical picture based on Einstein’s equations of general relativity should be valid. The analysis of the behavior of quantum fields in the classical metric results in the Hawking effect \([4]\): the black hole radiates energy with a spectrum characteristic of a black body. The resulting thermodynamic description \([5]\) identifies the black hole surface gravity and the surface area with the temperature and the entropy, respectively, of the black body.

The Hawking radiation from black holes provides information about their properties \([6,7]\). In this Letter we show that the angular momentum is an important parameter in the production and decay of collider-produced black holes. In a purely geometrical picture, partial wave analysis indicates that black holes are preferentially produced with large angular momentum. We study the properties of Hawking radiation from spinning black holes, and propose two measurements that test various production models. These measurements are relevant for understanding the role played by the black hole entropy and the Gibbons-Hawking action in the production dynamics.

In \(4+n\) dimensions, the radius of the black hole event horizon \(R_{bh}\) is given by \([8]\)

\[
R_{bh}^{n+1}(1 + a_s^2) = \frac{16\pi M_{bh}}{(n+2)A_{n+2}M_F^{n+2}},
\]

where \(A_{n+2} = 2\pi^{(3+n)/2}/\Gamma\left(\frac{n+3}{2}\right)\) is the area of the unit \(n+2\) sphere, and \(a_s = (n + 2)J/(2M_{bh}R_{bh})\) is a dimensionless rotation parameter for the angular momentum \(J\) of the black hole. We use the convention \(\hbar = c = k_B = 1\), where \(k_B\) is the Boltzmann constant, and the gravitational constant \(G = M_F^{-(n+2)}\).

The size \(L\) of the ESDs is related to \(M_F\) by \(L \sim (M_{Pl}^2/M_F^{n+2})^{1/n}\) \([1]\), where \(M_{Pl}\) (\(\sim 10^{16} \text{ TeV}\) ) is the 4-dimensional Planck scale. The horizon radius \(R_{bh} \sim (M_{bh}/M_F^{n+2})^{1/(n+1)}\) is much smaller than \(L\) when

\[
M_{bh}^{n+1} M_F^{n+2} \ll M_{Pl}^2.
\]

This is the case for the collider-produced black holes we consider, so these black holes “live” in \(4+n\) dimensions.

In the center-of-mass (CM) frame of a two-parton collision, the energy of each parton is \(M_{bh}/2\). The geometrical transverse phase space is maximized for parton impact parameter near \(R_{bh}\), implying that \(J \sim R_{bh}M_{bh} \sim (M_{bh}/M_F)^{n+2}\). In 4 dimensions \((n = 0)\), the initial state of the black hole tends to be extremal, i.e. \(J\) approaches the upper limit \(M_{bh}^2/M_F^2\).

The temperature of a spinning black hole is given by \([8]\)

\[
T_{bh} = \frac{n + 1 + (n - 1)a_s^2}{4\pi R_{bh}(1 + a_s^2)}.
\]
The energy ($\omega$) spectrum of the radiated particles from a spinless black hole is given by the Planck spectrum of thermal black body radiation \cite{3}, for which the mean energy $\omega_s$ is $O(T_{bh})$. The decay angular distribution is affected by structure of size $r$, where $r \sim \omega^{-1} \sim T_{bh}^{-1} \sim R_{bh}$. Thus, for a typical spinning black hole produced in a high energy collision, we expect the angular distribution of the Hawking radiation to deviate from an isotropic distribution.

We discuss below the predicted black hole $J$ distribution, and the radiation $\omega$ and angular distributions, based on a detailed Monte Carlo model of black hole production and decay.

**Black hole production:** We compute the probability of producing a black hole with given $M_{bh}$ and $J$ by using the partial wave expansion of the initial state, the interaction Hamiltonian and the black hole phase space. We make two assumptions: (i) the Hamiltonian conserves angular momentum, and (ii) the Hamiltonian is not spin-dependent. We take the Hamiltonian to be non-zero when $|\vec{r}_1 - \vec{r}_2| \leq 2R_{bh}$, i.e. $\hat{H} \sim \Theta(2R_{bh} - |\vec{r}_1 - \vec{r}_2|)$, where $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the colliding partons.

We approximate the wave functions of the colliding partons by plane waves. We define the CM and relative position vectors to be $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$ respectively, and we take $\vec{k}_{1,2} = \pm k \hat{z}$ to be the momenta of the incoming partons ($k = \omega$). Using the CM frame, we denote the two-parton initial state by $|\Psi, \psi, \chi >$, where $<\vec{R} | \Psi > = A e^{i(k_1 + k_2) \cdot \vec{R}} = A$, $<\vec{r} | \psi > = A' e^{i(k_1 - k_2) \cdot \vec{r}/2}$ and $\chi$ is the two-parton intrinsic spin state. $A$ and $A'$ are normalization factors that include a cut-off for the volume containing the plane waves. We denote the black hole state by $|\hat{H} | \Psi \psi \chi >$. Using the plane wave expansion in spherical harmonics \cite{8}, the matrix element $M = < J J_z | \hat{H} | \Psi \psi \chi >$ reduces to

$$M = \int d^3 \vec{R} d^3 \vec{r} < J J_z | \hat{H} | \vec{R} \vec{r} \chi > < \vec{R} \vec{r} | \Psi \psi > = A'' \sum_{lsms} \left[ < J J_z | l 0 s m_s > \sqrt{2l+1} \right. \\
\left. \times \int_{2R_{min}}^{2R_{bh}} r^2 dr j^3_l (\frac{M_{bh}}{r} 2r) \right] , \quad (4)$$

where $< J J_z | l 0 s m_s >$ are the Clebsch-Gordon coefficients and $s$ ($m_s$) is the total ($z$ component of the) two-parton intrinsic spin. We assume that space-time is quantized in the underlying theory of quantum gravity, setting the minimum black hole radius ($R_{min}$) to $M_F^{-1}$.

Another method \cite{11} of calculating the matrix element using the path-integral approach has introduced the Gibbons-Hawking action \cite{11}. The results of this approach have been debated \cite{12,13}. We comment on this model at the end of our paper.

One may take the black hole phase space $\mathcal{F}$ as the number of available black hole states $N_{bh}$ \cite{14}, which is related to the entropy $S_{bh} = \ln(N_{bh})$, given by \cite{8}

$$S_{bh} = \frac{4\pi R_{bh} M_{bh}}{n+2} \quad . \quad (5)$$

However, this choice of $\mathcal{F}$ contradicts the purely geometrical picture, where $\mathcal{F} = 1$. These two choices of $\mathcal{F}$ result in dramatically different $J$ distributions. Figure \cite{4} shows the normalized distributions of $|\mathcal{M}|^2$, $N_{bh}$, and $|\mathcal{M}|^2 N_{bh}$ as a function of $J$, for a black hole of mass...
$M_{bh} = 50M_F$ produced by two colliding fermions in 4+3 dimensions. As we will show, the different $J$ distributions result in different radiation spectra.

![Diagram](image)

FIG. 1. The normalized distributions of black hole angular momentum $J$, for $M_{bh} = 50M_F$ in 4+3 dimensions. We show the separate contributions of $|M|^2$ and $N_{bh}$, and their product.

**Black hole decay:** The decay by Hawking radiation is prescribed by thermodynamics. In our Monte Carlo simulation, we model the black hole decay as the sequential emission of individual partons. We derive the $\omega$ and angular momentum ($\vec{j}$) spectra using the micro-canonical ensemble:

$$P(\omega, \vec{j}) = \frac{g e^{\Delta S(\omega, \vec{j})}}{\sum_i g_i e^{\Delta S(\omega_i, \vec{j}_i)}},$$

where $g$ is the degeneracy of parton states, $\Delta S$ is the change in entropy of the black hole, and the sum in the denominator is over accessible parton ($\omega, \vec{j}$) values. The use of the micro-canonical ensemble incorporates finite mass effects such as the variation of the temperature and spin of the black hole during the decay. Since the black hole entropy always increases when matter falls into the black hole, time reversal invariance requires $\Delta S$ to be negative in the black hole decay. We omit in Eqn. 6 a multiplicative grey-body factor $\Gamma_{\omega \vec{j}}$, which would represent the absorption probability of a parton of given $\omega$ and $\vec{j}$. A differential equation for $\Gamma_{\omega \vec{j}}$ has been derived, which suggests that high $j$ decays are suppressed for low energy radiation due to the grey body factor. Including this general behavior of $\Gamma_{\omega \vec{j}}$ in Eqn. 6 would further enhance the effects of black hole spin on the decay distributions that we discuss here.
The exponential term $e^{\Delta S}$ in $P$ prevents large changes in black hole entropy in the emission of each parton. Since the entropy decreases with decreasing mass, but increases with decreasing angular momentum, radiation of high energy partons is not suppressed if the partons also carry high angular momentum. Therefore a spinning black hole emits more high energy partons compared to a spinless black hole of the same mass. This property can be used to distinguish between production models with different choices of the phase space factor $F$. The distributions of angular momentum versus energy of the radiated partons are shown in Fig. 4. When $F = 1$, the black hole radiates partons with higher $j$, and thus higher $\omega$, compared to $F = N_{bh}$.

**Measurement:** The partons radiated with high energy provide two tests of the black hole production dynamics. Figure 3 shows the mean multiplicity $N$ of high energy partons per black hole for $F = N_{bh}$ and $F = 1$ as a function of $M_{bh}$. For the mass ranges of 4-10 $M_F$ (the potential LHC range) and 5-50 $M_F$ (the potential VLHC range), we choose partons with $\omega > 8T_i$ and $\omega > 10T_i$ respectively, where $T_i$ is the initial temperature for $J = 0$. We find a detectable difference in the multiplicity when $M_{bh} > 8M_F$, and a substantial difference when $M_{bh} > 15M_F$. The sensitivity can be increased beyond the results of this counting experiment by analysing the shape of the $\omega$ spectrum at high $\omega$.

The second test of black hole production dynamics is provided by the angular distribution of the high energy partons. Figure 4 shows the $\cos \theta$ distributions of the high energy partons predicted by the two production models. The anisotropy when $F = 1$ is due to the larger fraction of black holes produced with high $J$ values, resulting in Hawking radiation with higher angular momentum.

Figures 3 and 4 show that one can experimentally distinguish between the production of high and low angular momentum black holes. We expect this ability to be a powerful test of any theory of quantum gravity. For example, Voloshin [10] has used the path-integral approach to introduce in the production cross section the exponential factor $e^{-I_E}$, where $I_E(M_{bh}, J)$ is the Gibbons-Hawking action [11]. Our results can be used to test this prediction and the $J$ dependence of $I_E$.

In four dimensions, $I_E = M_{bh}/(2T_{bh})$ [11]. If we assume that this dependence continues to hold in higher dimensions (up to numerical prefactors), we may use the dependence of $T_{bh}$ on $J$ (Eqn. 3) to extract the $J$ dependence of $I_E$ in higher dimensions. For $n > 1$, $T_{bh}$ is non-zero for any $J$ and has a weak $J$ dependence. For $n > 3$, $T_{bh}$ increases with $J$ for large $J$ so that $I_E$ is smaller at the typical $J \sim R_{bh}M_{bh}$ than at $J = 0$. Therefore in this scenario the exponential factor $e^{I_E}$ increases the fraction of black holes produced with large $J$ values, leading to an enhancement of the multiplicity and anisotropy of high energy Hawking radiation. It would be useful to calculate the $4+n$ dimensional Gibbons-Hawking action as a function of $M_{bh}$ and $J$, so that these predictions can be tested.

In conclusion, we have shown that angular momentum $J$ is a salient property of collider-produced black holes in space-times with extra spatial dimensions. In a purely geometrical picture, we have used partial wave analysis to calculate the $J$ distribution, showing that the typical value of $J$ is approximately $R_{bh}M_{bh}$. The large black hole spin is manifest in the increased multiplicity and anisotropy of the Hawking radiation at high energy. We have shown that black hole phase space considerations can significantly alter the $J$ distribution if the phase space is taken as the number of black hole states calculated from the entropy. In a path-integral approach to black hole production, the role of the $4+n$ dimensional
FIG. 2. The distribution of angular momentum versus energy of the decay partons for the two choices of $\mathcal{F}$ in the black hole production model. The distributions are shown for $n = 3$ and $M_{bh} = 50M_F$. The density of points is indicated on a logarithmic scale.
FIG. 3. The mean multiplicity $N$ of high energy partons per black hole as a function of black hole mass for the two choices of $F$. We use the thresholds $\omega > 8T_i$ (top) and $\omega > 10T_i$ (bottom), where $T_i$ is the initial black hole temperature for $J = 0$. The distributions are shown for $n = 3$. 
FIG. 4. The $\cos \theta$ distribution of the decay partons with $\omega > 10T_i$, for $n=3$ and $M_{bh}=50M_F$. The distributions obtained from the two choices of $F$ are compared (solid: $F = 1$, dashed: $F = N_{bh}$).
Gibbons-Hawking action can be tested using the observables we have proposed.

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\[
\psi_{\vec{k}\cdot\vec{r}} = 4\pi \sum_{lm} i^l j_l(kr) Y_{l^m}(\hat{r}) Y_{l^m}^*(\hat{k}) ,
\]

where \( l(l+1) \) is the eigenvalue of \( \hat{L}^2 \) (\( \hat{L} \) is the two-parton relative orbital angular momentum operator), \( m \) is the eigenvalue of \( \hat{L}_z \), and \( j_l \) are the spherical Bessel functions

\[
j_l(x) = (-x)^l \left( \frac{1}{x} \frac{d}{dx} \right)^l \left( \frac{\sin x}{x} \right) .
\]

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