Efficacy of crustal superfluid neutrons in pulsar glitch models

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ABSTRACT

Within the framework of recent hydrodynamic models of pulsar glitches, we explore systematically the dependence on the stiffness of the nuclear symmetry energy at saturation density $L$, of the fractional moment of inertia of the pinned neutron superfluid in the crust $G$ and the initial post-glitch relative acceleration of the crust $K$, both of which are confronted with observational constraints from the Vela pulsar. We allow for a variable fraction of core superfluid neutrons coupled to the crust on glitch rise timescales, $Y_g$. We assess whether the crustal superfluid neutrons are still a tenable angular momentum source to explain the Vela glitches when crustal entrainment is included. The observed values $G$ and $K$ are found to provide nearly orthogonal constraints on the slope of the symmetry energy, and thus taken together offer potentially tight constraints on the equation of state. However, when entrainment is included at the level suggested by recent microscopic calculations, the model is unable to reproduce the observational constraints on $G$ and $K$ simultaneously, and is limited to $L > 100$ MeV and $Y_g \approx 0$ when $G$ is considered alone. One solution is to allow the pinned superfluid vortices to penetrate the outer crust, which leads to a constraint of $L \lesssim 45$ MeV and $Y_g \lesssim 0.04$ when $G$ and $K$ are required to match observations simultaneously. When one allows the pinned vortices to penetrate into the crust by densities of up to $0.082$ fm$^{-3}$ above crust-core transition density (a total density of $0.176$ fm$^{-3}$) for $L = 30$ MeV, and $0.048$ fm$^{-3}$ above crust-core transition density (a total density of $0.126$ fm$^{-3}$) for $L = 60$ MeV, the constraint on $G$ is satisfied for any value of $Y_g$. We discuss the implications of these results for crust-initiated glitch models.

Key words: dense matter - equation of state - (stars:) pulsars: individual - glitch - stars:neutron

1 INTRODUCTION

The rotational evolution of young pulsars ($\lesssim 10^7$ years old) is often observed to be interrupted by glitches - sudden increases in spin frequency $\nu$. The range of glitch sizes $\Delta \nu / \nu_0 \sim 10^{-11}$ to $10^{-5}$ and modes of post-glitch recovery is observed to be quite diverse across the pulsar population (Espinoza et al. 2011). One of the most studied pulsars, Vela, quasi-periodically produces large glitches $\Delta \nu / \nu_0 \sim 10^{-6}$ (Melatos et al. 2008; the relatively large number of glitches observed from Vela (21) has made it a test-bed for proposed glitch mechanisms. Much effort has been devoted to examining two-component glitch models, in which one component of the neutron star (most commonly the superfluid neutron component of the crust) is for most of the time decoupled from that component to which the magnetic field lines are anchored and whose rotational evolution we directly observe. This decoupled component acts as an angular momentum reservoir, and occasionally re-couples to the rest of the star, transferring some of its angular momentum and spinning the star up (Anderson & Itoh 1975; Alpar 1977). In most models, the neutron superfluid vortices in the crust interact with the lattice of nuclei to ‘pin’ to the crust (Pizzochero et al. 1997; Avogadro et al. 2008), therefore preventing them from moving radially outwards from the rotation axis in response to the secular spin-down of the star under the action of magnetic torque (Alpar 1977; Pines et al. 1980; Anderson et al. 1982; Alpar et al. 1984). They thus become decoupled from the rotational evolution of the charged component of the crust and that part of the core that couples strongly to it. As a lag builds up between the angular frequency of the charged component (that which we observe) and the frequency of the crustal superfluid, the Magnus force acting radially outwards on the vortices grows, until eventually it overcomes the pinning force, and the vortices unpin en masse and couple to the charged component, transferring angular momentum.

The observed glitch activity of the Vela pulsar can be
used to infer the fractional moment of inertia of the angular momentum reservoir $\Delta I$ compared to that of the portion of the star it couples to at the time of glitch, $I$ (which we shall referred to as the charged component of the star), $\Delta I/I \gtrsim 1.6\%$ (Link et al. 1999). Assuming an upper limit to the moment of inertia of the angular momentum reservoir to be that of the entire crustal neutron superfluid $\Delta = I_{\text{csf}}$, and the charged component it couples to is essentially the whole star $I = I_{\text{tot}}$, many realistic neutron star (NS) equations of state (EoSs) can satisfy the condition $\Delta I/I \gtrsim 1.6\%$, and constraints placed on the NS EoS (Lorenz et al. 1993 Link et al. 1999). However, recent calculations of the strength of the entrainment of superfluid neutrons by the crustal lattice via Bragg scattering suggest that only a fraction of the crustal neutrons are decoupled from the crust, and consequently the upper limit to the moment of inertia of the angular momentum reservoir is reduced: $\Delta I \sim 0.2 I_{\text{csf}}$. This makes $\Delta I/I$ too small to explain the observed glitch sizes at a first glance (Chamel et al. 2013, 2012 Andersson et al. 2012), and suggests one must at least involve other components of the star in addition to the crust superfluid as the store of angular momentum. These studies assume a strong coupling between crust and core so that $I \sim I_{\text{tot}}$. However, estimates of the crust-core coupling timescales due to interactions of neutron vortices and type-I or type-II superconducting protons ([Alpar & Sauls 1988 Sedrakian 2005 Andersson et al. 2006 Jones 2006 Babayan 2009 Link 2012]) raise the possibility of only a small fraction of the core neutrons being coupled to the crust on the glitch rise timescale $\sim 40 s$ (Dodson et al. 2002). Therefore it is possible that $I \ll I_{\text{tot}}$, allowing the ratio $\Delta I/I$ to satisfy the lower bound of $1.6\%$ again with only crustal superfluid neutrons involved.

There has been substantial recent progress in going beyond phenomenological glitch models and simulating the neutron star hydrodynamically through the glitch event (e.g. Sidery et al. 2010 van Eysden & Melatos 2010). Recently, a detailed glitch model incorporating microscopic calculations of the pinning force throughout the crust and a hydrodynamic evolution of the vortices, was shown to explain qualitatively the Vela glitch sizes and post-glitch rotational evolution, and to potentially constrain the EoS (Haskell et al. 2012 Pizzochero 2011 Seveso et al. 2012), despite remaining uncertainties in aspects of the glitch model such as the unpinning trigger mechanism and details of how the vortices subsequently unpin (Glampedakis & Andersson 2009 Warszawski & Melatos 2008 Melatos & Warszawski 2009 Warszawski & Melatos 2013 Warszawski et al. 2012). In this model the dynamics of the vortices are such that, on timescales of $\sim 5 s$, crustal superfluid neutrons unpin in a front which moves radially outwards from the base of the crust until it reaches densities at which the pinning force is a maximum. At this point, the accumulated angular momentum of the pinning front is transferred to the charged component of the star suddenly, and the glitch occurs. Due to the analogy of pushing snow slowly up a hill before releasing it down the other side, it is referred to as the “snowplough” model by the authors. One feature of the model is that the vortices are pinned only in the region in which they are totally immersed in the crust, an equatorial ring which accounts for $\sim 10\%$ of the mass of the whole crust. This implies the moment of inertia of the angular momentum reservoir $\Delta I$ will be reduced by a factor of $\sim 10$. In this model, the crust-core coupling is such that $I \ll I_{\text{tot}}$, and so the ratio $\Delta I/I$ is still able to account for the observed Vela glitch activity for selected EoSs (Seveso et al. 2012). In addition, it can also account for the initial relative post-glitch acceleration of the crust inferred from the 2000 Vela glitch timing data (Dodson et al. 2002). However, crustal entrainment is yet to be taken into account in the model, so it remains an open question as to its efficacy.

The aim of this paper is to examine the range of predictions for $\Delta I/I$ and for the initial post-glitch acceleration within the framework of the “snowplough” model by varying the most uncertain nuclear matter parameters over their experimentally and theoretically constrained ranges, and taking into account entrainment consistently throughout the crust using the recently calculated values (Chamel 2012). To do this, we shall apply systematically and consistently generated sequences of crust and core EOSs together with the relevant crust compositions (Newton et al. 2013) to modeling glitches for the first time. The consistent modeling of crust and core properties when exploring the dependence of neutron star observables has been presented before (Gearheart et al. 2011 Wen et al. 2012 Newton et al. 2013a), and here extends to modeling the crust thickness, density of superfluid neutrons throughout the crust, core EOS and core proton fraction using the same underlying nuclear matter EOSs.

Much effort has been devoted to constraining the EOS of nuclear and neutron star matter, particularly through constraining the density dependence of the symmetry energy at nuclear saturation density $n_0$, parameterized by $L = 3n_0p_0$ where $p_0$ is the pressure of pure neutron matter at saturation density, which is strongly correlated with the pressure in neutron stars at that density. Nuclear experimental probes (for a recent review see Tsang et al. 2012) give a conservative range of $L = 25–105 \text{ MeV}$, although some more recent results on the nuclear experimental side (Lattimer & Lim 2013), as well as tentative constraints from neutron star observation (Newton & Li 2009, Gearheart et al. 2011, Wen et al. 2012, Steiner & Gandolfi 2012) and from ab-initio pure neutron matter calculations (Gezerlis & Carlson 2010, Hebeler & Schwenk 2010, Gandolfi et al. 2012) favor the lower half of that range (although, for a counter-example, see e.g. Sotani et al. 2012). The high-density behavior of the EOS is even more uncertain both theoretically and experimentally (Xiao et al. 2009, Russotto et al. 2011), even if one restricts the composition to purely nucleonic matter, with some of the only constraints coming from analysis of heavy-ion collisions (Danielewicz et al. 2002) and the recent observations of $\sim 2M_\odot$ neutron stars (Demorest et al. 2010, Antoniadis et al. 2013). In this paper we shall explore the impact of systematically varying the density dependence of the symmetry energy $L$ at saturation density on the glitch model.

In Section 2 we describe our glitch modeling and series of EOSs and how we compare with observational quantities. In Section 3 we present and discuss our results and in Section 4 we discuss our conclusions.
THE GLITCH MODEL

The observed angular frequency, $\Omega$, of a pulsar is presumed to be that of its ionic crustal lattice in which the magnetic field lines are anchored. When considering glitch sizes and immediate post-glitch evolution, it is important to define that component of the star strongly coupled to the lattice on timescales comparable with the glitch rise time, which is observationally constrained to be $\lesssim 40s$ (Dodson et al. 2002). In our minimal model of the core which contains purely nucleonic matter, this component contains the core protons and some fraction of the core neutrons, and we shall refer to it as the charged component of the star.

We will outline the glitch mechanism according to the recent “snowplough” model, a full hydrodynamical simulation of the star through a glitch with the inclusion of microscopically-based pinning forces in the crust (Pizochero 2011; Haskell et al. 2012; Seveso et al. 2012). The neutron superfluid vortices are assumed to continuously thread the interior of the star parallel with the rotation axis. As the pulsar spins down, the vortices move outwards from the rotation axis toward the equatorial crust regions under the action of the Magnus force. The vortices that thread through the core permeate the inner crust only at each end. The Magnus force, when integrated over the vortex length, is sufficient to unpin the ends of the vortices almost as soon as they become pinned, so the vortices are free to creep outwards and core superfluid can follow the rotational evolution of the star with very little lag. Only in the equatorial region of the inner crust is the whole length of a vortex contained within the inner crust, and therefore capable of being pinned on timescales of order the inter-glitch time. This is referred to as the strong pinning region of the crust, and is shown shaded in Fig. 1. The pinning force varies throughout the crust and depends on, among other microphysics, the magnitude of the $^4\text{He}$ superfluid gap. Calculations using the most recent estimates of the gap strength indicate the pinning force rises from the crust-core boundary to a maximum somewhere between densities of $n_{\text{max}} = 0.14-0.32n_0$ (Gandolfi et al. 2009; Seveso et al. 2012). However, these calculations neglect crustal entrainment, which might have a significant effect. As the lag between the stellar angular frequency and that of the superfluid develops, an unpinning front moves outwards from the crust-core boundary to the density of maximum pinning force. When that density is reached, the vortices move outwards rapidly, and recouple to the crust via interactions between vortex Kelvin-waves and lattice phonons (Epstein & Baym 1992; Jones 1992), thus transferring their angular momentum to the charged component of the star. The charged component of the star spins up, and then the post-glitch recovery phase is entered as the rest of the core superfluid neutrons re-couple to the charged component on longer timescales, thus exerting a torque on the charged component and accelerating the star towards its pre-glitch spin-down rate. Simulations suggest the unpinning front is finite in thickness, and crust neutrons at densities above that of maximum pinning force could participate in the angular momentum transfer responsible for the glitch. Given the remaining uncertainties in the model, in this paper we shall take the maximum density of the strong pinning region at the time of glitch $n_{\text{max}}$ to be at least the crust core transition density $n_{cc}$. This effectively constitutes an upper limit to the amount of angular momentum transferred between crust and core. We shall also explore the effect of that pinning region extending into the outer core to densities of $n_{\text{max}} = n_{cc} + n_+$. We shall examine values of $n_+$ up to 0.05 fm$^{-3}$.

We shall use the above framework to estimate the average glitch activity of the Vela pulsar, as well as its immediate post-glitch relative acceleration. To do so we need to compute the moment of inertia ($\text{MoI}$) of the crustal superfluid neutrons in the strong pinning region of the crust $I_{\text{csf}}^{(n)}$, the MoI of the charged component of the star $I_c$ and the total MoI of the star $I_{\text{tot}}$.

The moment of inertia of a star of radius $R$ in the limit of small angular frequency $\Omega$ (Hartle & Thorne 1968) is given by

$$I_{\text{tot}} = \frac{8\pi}{3} \int_0^R r^4 e^{-\nu(r)} \tilde{\omega}(r) \frac{\varepsilon(r) + P(r) - G}{\sqrt{1 - 2GM(r)/r^2}} dr,$$  \hspace{1cm} (1)

where $\varepsilon(r)$ is energy density of matter in the star, $P(r)$ is the pressure and $M(r)$ is the mass contained in radius $r$. $\nu(r)$ is a radially-dependent metric function given by

$$\nu(r) = \frac{1}{2} \ln \left(1 - \frac{2GM}{R}\right) - G \int_{r}^{R} \frac{M(x) + 4\pi x^3 P(x)}{x^2 (1 - 2GM(x)/x)} dx,$$ \hspace{1cm} (2)

and $\tilde{\omega}$ is the frame dragging angular velocity

$$\frac{1}{r^2} \frac{d}{dr} \left(r^4 j(r) \frac{d\tilde{\omega}(r)}{dr}\right) + \frac{4j(r)}{dr} \tilde{\omega}(r) = 0,$$ \hspace{1cm} (3)

where

$$j(r) = e^{-\nu(r) - \lambda(r)} = \sqrt{1 - 2GM(r)/re^{-\nu(r)}}$$ \hspace{1cm} (4)

for $r \lesssim R$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Neutron star cross-section in plane of rotation axis ($\Omega$) depicting the geometry of the strong pinning region in inner crust (shaded area). The inner boundary of the strong pinning region is given by the tangent to the crust-core boundary between the crust-core boundary $R(\theta) = R(\theta_{\text{inner}})$ and the outer crust-inner crust boundary $R(\theta) = R(\theta_{\text{outer}})$. The outer boundary of the strong pinning region is taken to be the outer-crust - inner-crust boundary.}
\end{figure}
The charged component of the star includes the crust lattice plus the protons in the core and some fraction of core superfluid neutrons. In any regions in which the protons form a Type I superconductor, neutron vortices can entrain protons and become magnetized, allowing electrons to scatter off them and therefore coupling to the charged component on timescales of $\sim 10 - 1000s$ for the Vela pulsar (Alpar et al. 1983; Alpar & Sanaul 1988). In regions where protons are Type II superconductors, they form fluxtubes to which neutron vortices can become pinned, coupling them to the charged component on timescales of days (Babaev 2009; Link 2012). Indeed, the number of recovery timescales required to fit post-glitch timing data suggests multiple coupling timescales involving different physical mechanisms in the core. By comparing the above estimates of coupling timescales with the upper limit of the glitch rise time $\lesssim 40s$, one may infer that only some fraction of the core neutron superfluid will contribute to the charged component of the star at the time of glitch. That fraction is quite uncertain, and enters into the model as a free parameter $Y_g$, but above estimates indicate that it is possible to have $Y_g \ll 1$ (Haskell et al. 2012; Link 2012). We also denote the total neutron fraction of the core at a given radius $r$ by $Q(r)$. Then the MoI of the charged component can be expressed (Seveso et al. 2012)

$$I_c = \frac{8\pi}{3} \int_0^R r^4 [1 - Q(r)(1 - Y)] e^{-\nu(r)} \frac{\bar{\omega}(r)}{\Omega} \frac{(\bar{\varepsilon}(r) + P_n(r))}{\sqrt{1 - 2GM(r)/r}} dr,$$

The total moment of inertia of superfluid neutrons in the inner crust of the star is given by

$$I_{\text{csf}}^{(\text{tot})} = \frac{8\pi}{3} \int_{R_{\text{inner}}}^{R_{\text{outer}}} r^4 e^{-\nu(r)} \frac{\bar{\omega}(r)}{\Omega} \frac{(\varepsilon_n(r) + P_n(r))}{\sqrt{1 - 2GM(r)/r}} dr$$

where $\varepsilon_n(r)$ is the energy density of crustal superfluid neutrons, $P_n(r)$ is the pressure of the crustal superfluid neutrons and $R_{\text{inner}}$ and $R_{\text{outer}}$ are the radius boundaries for the inner crust. In the model we consider, only the crustal superfluid neutrons within the strong pinning region of the crust, defined as the region within which vortices are totally immersed in the inner crust, contribute to the glitch itself. Defining

$$r^2 I = \frac{8\pi}{3} \int_0^R r^4 e^{-\nu(r)} \left( \bar{\omega}(r) \frac{(\bar{\varepsilon}(r) + P_n(r))}{\Omega} \sqrt{1 - 2GM(r)/r} \right) dr$$

we can estimate the moment of inertia of the strong pinning region of inner crust superfluid neutrons as

$$I_{\text{csf}}^{(\text{up})} = \int_{\theta_{\text{inner}}}^{\pi/2} \left[ \int_{\theta_{\text{inner}}}^{R(\theta_{\text{outer}})} r^2 I dr \right] \sin \theta d\theta$$

where $R(\theta)$ is the distance from the core of the star to the inner boundary of the strong pinning region at an angle $\theta$ to the rotation axis, $R(\theta_{\text{inner}}) \equiv R_{\text{inner}}$ and $R(\theta_{\text{outer}}) \equiv R_{\text{outer}}$ (see Fig. 1).

Entrainment of superfluid neutrons by the crust’s lattice reduces the mobility of the neutrons with respect to that lattice. It can be shown that this effect is encoded by introducing an effective “mesoscopic” neutron mass $m^*_n$ (Chamel 2005; Chamel & Carter 2006; Chamel 2012); larger values correspond to stronger coupling between the neutron superfluid and the crust, and a reduction in the fraction of superfluid neutrons able to store angular momentum for the glitch event. One can include this effect by modifying the integrand Eq. 7:

$$r^2 I \rightarrow r^2 I^* = \frac{m^*_n}{m_n(r)} r^2 I$$

where $m^*_n(r)$ is the effective mass at radius $r$ in the crust. We obtain $m^*_n(r)$ from the results of Chamel (Chamel 2012) by interpolating between the values calculated at specific densities to find the effective mass at arbitrary locations in the inner crust.

The work of Chamel (Chamel 2012) ignores the spin-orbit interaction, which the author notes might weaken the entrainment effect. In order to account for this and other uncertainties, we introduce a parameter $e$ which we use to control the strength of the entrainment:

$$m^*_n \rightarrow 1 + (m^*_n - 1)e$$

where $e = 0$ corresponds to no entrainment and $e = 1$ corresponds to full strength entrainment.

The analysis of (Link et al. 1999) identifies the minimum amount of angular momentum stored in the crustal superfluid reservoir $I_{\text{csf}}$ relative to that of the charged component of the star $I_c$ with the parameter $G$ defined

$$G \equiv \frac{I_{\text{csf}}^{(sp)}}{I_c} \geq \frac{\bar{\Omega}}{|\bar{\Omega}|} A = 0.016$$

where $A$ is the glitch activity parameter of the pulsar, that is the slope of the straight line fit to a plot of the cumulative relative glitch size over time (Link et al. 1999). The observational constraint $G \gtrsim 0.016$ comes from analysis of the Vela glitches (Link et al. 1999; Espinoza et al. 2011). In addition, the 2001 Vela glitch yielded the first measurement of the relative angular acceleration of the charged component immediately after the glitch $\Delta \dot{\Omega}_{\text{gl}}/\dot{\Omega}_0 = 18 \pm 6$ (Dodson et al. 2002; Pizzochero 2011). Assuming that this relative acceleration is the result of the initial re-coupling of the remaining uncoupled component of the core to the charged component (i.e. the change in the effective moment of inertia of the star acted upon by the magnetic torque), it can be calculated within the model as (Pizzochero 2011)

$$\frac{\Delta \dot{\Omega}_{\text{gl}}}{\dot{\Omega}_0} = \frac{(I_{\text{csf}} - I_c)}{I_c} \equiv K$$

We shall confront our calculations of the $G$ and $K$ as defined above with the observed constraints $G \geq 0.016, K = 18 \pm 6$.

2.1 Nuclear matter parameters and crust and core equations of state

The microphysical ingredients in the glitch model include the total pressure and energy density $P(\varepsilon)$ and those of the superfluid neutrons $P_n(\varepsilon_n)$, $\varepsilon_n(\varepsilon_n)$ as a function of baryon density throughout the core and crust, as well
as the crust-core transition baryon density \( n_{cc} \), the effective mass of neutrons in the crust \( m^*_n(r) \).

In order to calculate the crust and core EOSs a model for uniform nuclear matter is required. Nuclear matter models can be characterized by their behavior around nuclear saturation density \( n_0 = 0.16 \text{ fm}^{-3} \), the density region from which much of our experimental information is extracted. We can denote the energy per particle of nuclear matter around saturation density by \( E(n, \delta) \), where \( n \) is the baryon density and \( \delta = 1 - 2x \) the isospin asymmetry; where \( x \) is the proton fraction. \( x = 0.5, \delta = 0 \) corresponds to symmetric nuclear matter (SNM), and \( x = 0, \delta = 1 \) to pure neutron matter (PNM). By expanding \( E(n, x) \) about \( \delta = 0 \) we can define the symmetry energy \( S(n) \),

\[
E(n, \delta) = E_0(\chi) + S(n)\delta^2 + \ldots, \tag{13}
\]

which encodes the energy cost of decreasing the proton fraction of matter. Expanding the symmetry energy about \( \chi = 0 \) where the density parameter \( \chi = \frac{n - n_0}{3n_0} \), we obtain

\[
S(n) = J + L\chi + \frac{1}{2}K_{sym}\chi^2 + \ldots, \tag{14}
\]

where \( J, L \) and \( K_{sym} \) are the symmetry energy, its slope and its curvature at saturation density. At present, the energy of SNM around saturation density is well constrained by experiment. Much experimental effort has been focused on determining the symmetry energy \( J \) and its density dependence \( J \) around nuclear saturation density \( \text{[Li et al. 2008 Tsang et al. 2012]} \), and from these results we take as a conservative range \( 25 < L < 105 \text{ MeV} \) in this work. We will also pay attention, however, to the fact that the congruence of the experimental results \( \text{[Hebeler et al. 2013]} \) favors a range \( 30 < L < 60 \text{ MeV} \).

We calculate the crust and core EOSs and the transition density consistently using the widely used Skyrme nuclear matter model. As the baseline Skyrme parameterization, we choose the SkIUFSU model used in previous work \( \text{[Fattoyev et al. 2012 2013]} \), which shares the same saturation symmetric nuclear matter (SNM) properties as the relativistic mean field (RMF) UFSU model \( \text{[Fattoyev et al. 2010]} \), has isovector nuclear matter parameters obtained from a fit to state-of-the-art PNM calculations, and describes well the binding energies and charge radii of doubly magic nuclei \( \text{[Fattoyev et al. 2012]} \). Two parameters in the Skyrme model are purely isovector - they can be systematically adjusted to vary the symmetry energy \( J \) and its density slope \( L \) at saturation density while leaving SNM properties unchanged \( \text{[Chen et al. 2009]} \). The constraint from PNM at low densities induces correlation between the magnitude and slope of the symmetry energy at saturation density described by \( J = 0.167L + 23.33 \text{ MeV} \). In this work we will create EOSs by varying \( L \) between 25 MeV and 105 MeV under this constraint. For softer symmetry energies at high densities, the resulting EOS is matched onto two successive polytropic equations of state as described in \( \text{[Steiner et al. 2010 Wen et al. 2012]} \) in order to match the constraint on the maximum mass of \( M \geq 2M_\odot \) \( \text{[Demorest et al. 2010 2013]} \).

The crust EOS and crust-core transition densities used in this work are obtained from a simple compressible liquid drop model (CLDM) for the crust \( \text{[Newton et al. 2013b]} \). This model gives the composition of the crust (including the free neutron density) and the extent of the so-called ‘pasta’ phases, in which nuclei become deformed into exotic shapes. Full tables of our crust and core EOSs are available from our website \( \text{http://williamnewton.wordpress.com/ns-eos/} \).

To illustrate the relevant correlations between nuclear matter properties and neutron star properties, we plot in Fig. 2 as a function of \( L \) the crust-core transition densities \( n_{cc} \) and pressures \( P_{cc} \) (left plot) and the radii \( R \) and inner crust thicknesses \( \Delta R_{inner} \) of a \( 1.4M_\odot \) neutron star for the same sequence of models (b).
density $h = P + \varepsilon$ and that of the neutrons only. In the inner crust, we include the free neutrons and those bound in nuclei, and thus our results represent an upper limit to those that would be obtained using free neutrons alone. In Fig. 3 (a-c) we illustrate the effect of varying the saturation density symmetry energy stiffness $L$ from 25 to 105 MeV. The previously mentioned trends of $R$ and $\Delta R_{\text{max}}$ with $L$ are evident once more. A higher value of $L$ gives a higher value of the symmetry energy at super-saturation densities; therefore the proton fraction in the core $1 - Q(r)$ increases with $L$, increasing the moment of inertia of the charged stellar component coupled to the crust at the time of glitch.

### 3 RESULTS AND DISCUSSION

We begin by calculating the quantities $G$ (Fig. 4) and $K$ (Fig. 5) as a function of mass for a representative range of the model parameters. To recap, the model parameters are: the fraction of core neutrons coupled to the crust at the time of glitch $Y_{\text{c}}$, the strength of the crustal neutron entrainment $e$, the inner boundary of the strong pinning region $n_{\text{cc}}$, which is set equal to the crust-core transition density $n_{\text{cc}}$ if we choose to consider the crust only as an angular momentum reservoir, and the slope of the symmetry energy at saturation density $L$.

Before we examine the dependence on the model parameters, we first note the trend of $G$ as a function of stellar mass $M$ that is clear in all plots in Fig. 4, that $G$ decreases with increasing $M$. As mass decreases, the compactness of the neutron star $(M/R)$ decreases and the crust gets relatively thicker, giving a larger contribution to moment of inertia of the star. $G$ thus generically increases as mass decreases.

In Fig. 4, the inferred lower limit on $G$, 1.6%, from the Vela pulsar is shown as the horizontal black line. In Fig. 4 (a-c) we show $G$ as a function of mass $M$ taking $n_{\text{max}} = n_{\text{cc}}$: results are then shown for $Y_{\text{c}} = 0.0$ (a), 0.1 (b) and 0.5 (c), and in each plot we show results for $L = 25$, 65 and 105 MeV (softest to stiffest symmetry energy), with $(e = 1)$ and without $(e = 0)$ entrainment.

As $L$ increases (i.e. the symmetry energy at saturation density becomes stiffer), $G$ becomes larger except for the highest mass stars $M \sim 2M_{\odot}$. This relation is the result of the folding together of competing trends. As $L$ increases, the radius of the star and the core proton fraction increases and thus so does the moment of inertia of the charged component of the star $I_c$: if the crust thickness were held constant, this would result in a smaller value of $G$. In addition, though, as $L$ increases, the crust thickness increases (Fig. 2b) (as a results of the increasing radius and the increasing crust-core transition pressure) thus increasing $I_{\text{op}}^{\text{cr}}$. The overall trend requires a consistent calculation such as those performed here, for which the result is the increasing of $G$ with $L$ for most of the mass range.

Increasing $Y_{\text{c}}$ for a fixed mass star increases the fraction of the core superfluid that couples to the crust at the time of glitch and thus $I_{c}$, therefore decreasing the glitch magnitudes and $G$. Taking these effects into account, we can still satisfy the Vela constraint for a 1.4 $M_{\odot}$ star without entrainment for $Y_{\text{c}}$ as high as 0.5 for $L \geq 85$ MeV; for lower values of $Y_{\text{c}}$ we can satisfy $G \geq 1.6\%$ for lower values of $L$. When entrainment is included at the full strength predicted by (Chamel 2012) $(e = 1)$, the Vela constraint is only satisfied for a 1.4 $M_{\odot}$ when very little of the neutron superfluid is coupled to the crust at the time of glitch $(Y_{\text{c}} \lesssim 0.01)$ and when $L$ attains its highest values $L$ (e.g. $L \gtrsim 100$ MeV).

In Fig. 4 (d-f) we compare the results with no entrainment and full entrainment to those with weaker entrainment (two-thirds strength $e = 0.67$ and one-third strength $(e = 0.33)$). Results are displayed for $L = 65$ MeV and $n_{\text{max}} = n_{\text{cc}}$ over the same range of $Y_{\text{c}}$. One can see that at $L=65$ MeV, it barely becomes possible to satisfy $G \geq 1.6\%$ in a 1.4 $M_{\odot}$ star for $Y_{\text{c}}=0.0$; for $e = 0.33$, it can be satisfied at $L=65$ MeV up to $Y_{\text{c}} \approx 0.05$.

In Fig. 4 (g-i) we allow for the participation of outer core neutrons in the strong pinning region up to densities of 0.01 fm$^{-3}$, 0.03 fm$^{-3}$ and 0.05 fm$^{-3}$ above the crust-core transition density. In each of these plots, crustal entrainment is at full strength $e = 1$, and $L = 65$ MeV. $Y_{\text{c}}$ takes the same values as in the previous sets of 3 plots. The inclusion of pinned core neutrons has a large effect; inclusion of core neutrons up to 0.03 fm$^{-3}$ above the crust-core transition (corresponding to a density of 0.11 fm$^{-3}$ at $L = 65$ MeV) allows a 1.4 $M_{\odot}$ star to satisfy the Vela constraint with
L = 65 MeV with up to half of the remaining core neutrons coupled to the crust at the time of glitch Y_{g} = 0.5.

Fig. 5 (a-c) plots the relative crust acceleration post-glitch \( K \) as a function of mass \( M \). \( K \) is equal to the ratio of the moment of inertia of the core of the star that remains uncoupled to the crust immediately after the glitch to that which is coupled and is therefore insensitive to crust properties such as strength of crustal entrainment. It depends on the stiffness of the symmetry energy at saturation density \( L \) (plots a-c) and while in principle it depends also on \( n_{\text{max}} \) (because increasing \( n_{\text{max}} \) decreases the region of the core decoupled from the crust), in practice the dependence is negligible.

We display results for \( Y_{g} = 0.0 \) (a), \( Y_{g} = 0.1 \) (b), and \( Y_{g} = 0.5 \) (c). As \( L \) increases or \( M \) decreases, the star becomes less compact and correspondingly harder to accelerate under a given torque, and therefore \( K \) will be smaller.

We show the constraint on \( K = 18 \pm 6 \) from the 2000 Vela glitch by the horizontal black lines. It can be seen that the constraint is very difficult to satisfy, accommodating only the smallest values of \( L \) and relatively low values of \( Y_{g} \).

We shall express the above trends more succinctly...
shortly, but let us pause to make explicit two important points: (1) The measured values $G \geq 1.6\%$ and $K = 18 \pm 6$ are both relatively high compared to predictions within the framework of the ‘snowplough’ glitch model from NS models over a realistic range of EOS parameters. A high $G$ is typically satisfied by lower mass stars and higher values of $L$, while a high $K$ is typically satisfied by lower values of $L$ and higher masses; therefore, taken simultaneously, they have the potential to give stringent constraints on $L$ and the neutron star mass. These trends persist when one extends the region of pinned vortices into the core. (2) The inclusion of crustal superfluid entrainment at the level predicted by (Chamel 2012) provides a significant challenge to the model, as expected ((Chamel 2013, Andersson et al. 2012), if one only accepts the presence of pinned vortices in the crust.

Figure 5. (Color online) The initial acceleration of the pulsar spin frequency toward its pre-glitch spin-down rate, $K$, versus mass $M_\odot$ for coupled core superfluid fractions of $Y_g = 0.0$ (a), 0.1 (b) and 0.5 (c). Results are shown for $L = 25$ MeV (solid line), 65 MeV (dashed line), and 105 MeV (dotted line). The observational bounds from the 2001 Vela glitch are shown by the horizontal lines.

Figure 6. (Color online) Upper limit on the mass for which the Vela constraint $G \geq 1.6\%$ is satisfied for a given value of $L$ (colored lines), and upper and lower values of $L$ for which the inferred value of $K$ from the 2001 Vela glitch is satisfied for a given mass (thick black lines marked $K_{\text{Vela}}^+$ and $K_{\text{Vela}}^-$ respectively). The region that satisfies $G \geq 1.6\%$ lies below each line, while the region that satisfies the $K$ constraint lies between the thick solid lines. In the case of the constraint $G > 1.6\%$, results are shown for $e = 0$ (solid colored line), 0.35 (dashed line), 0.67 (dotted line) and 1 (dash-dotted line). The top row shows results considering only the crust to be involved in the angular momentum transfer $n_{\text{max}} = n_{cc}$ and for $Y_g = 0$ (a), $Y_g = 0.1$ (b), $Y_g = 0.5$ (c). The bottom row shows results for $Y_g = 0.5$ and an extension of the strong pinning region into the outer core by a density of 0.01fm$^{-3}$ (d), 0.03fm$^{-3}$ (e) and 0.05fm$^{-3}$ (f) above the crust-core transition density.
alone. (3) If one can justify the pinning region extends only a small way into the outer core, that problem can be solved.

For a given set of model parameters, we can identify a mass $M$ below which the $G \geq 1.6\%$ is satisfied, and bounding masses within which $K = 18 \pm 6$ is satisfied. These limiting masses are plotted as a function of $L$ in Fig. 6.

In Fig. 6 (a-c), results are shown for $n_{\text{max}} = n_{\text{cc}}$ and $Y_g = 0.0$ (a), 0.1 (b) and 0.5 (c). In each plot, the thin solid and dashed curves representing, as a function of $L$, lower limits to the masses that satisfy $G \geq 1.6\%$ are shown for $e = 0$, 0.33, 0.67 and 1.0. Masses which satisfy $G \geq 1.6\%$ lie below the curve. Note that in Fig. 6a, the thin solid line below which are masses which satisfy $G \geq 1.6\%$ for $e = 0$ lies above 2.0$M_{\odot}$ for all $L$. The range of masses that satisfy $K = 18 \pm 6$ lie between the solid thick black lines. When entrainment of crust neutrons is neglected $e = 0$, $G \geq 1.6\%$ is satisfied for all $L$ for masses below 2.0$M_{\odot}$ and $Y_g \approx 0$, and for $L \geq 80$ MeV for a 1.4$M_{\odot}$ star with $Y_g = 0.5$. When entrainment is included (dashed lines), it becomes very difficult to satisfy the constraint on $G$ for reasonable neutron star masses ($\sim 1.4M_{\odot}$ and above) unless $Y_g$ is very low or $L$ is very high. One can immediately see that the $K$ constraint is satisfied only by very low values of $Y_g \approx 0.0$, and low values of $L (37 \lesssim L \lesssim 50$ MeV for a star of mass up to 2.0$M_{\odot}$, and $L \lesssim 40$ MeV for a neutron star of mass $1.4M_{\odot}$).

We shall delineate more precisely the range of masses that simultaneously satisfy $G \geq 1.6\%$ and $K$ shortly.

In Fig. 6 (d-f), results are shown for $Y_g = 0.5$, $e=0$, 0.33, 0.67 and 1.0 and a pinning region that extends into the core by a density of 0.01 fm$^{-3}$ (d), 0.03 fm$^{-3}$ (e), and 0.05 fm$^{-3}$ (f). One can now see that to satisfy the constraint on $G$ for stars above 1.4$M_{\odot}$ with full strength entrainment $e = 1$ and $L$ in a range 30 – 60 MeV requires the extension of the pinning region to be $\approx 0.03$ fm$^{-3}$ - 0.05 fm$^{-3}$ above crust-core transition density (i.e. the pinning region extends up to densities of 0.1 fm$^{-3}$ - 0.14 fm$^{-3}$).

The range of values of $L$ and $Y_g$ that simultaneously satisfy $G \geq 1.6\%$ and $K = 18 \pm 6$ are plotted in Fig. 7. Considering crustal pinning alone: neglecting entrainment, $G \geq 1.6\%$ and $K = 18 \pm 6$ constrain $L \leq 40$ MeV and $Y_g < 0.027$; crustal

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**Table 1.** Extension of the strong pinning region into the outer core required to satisfy the Vela constraint on $G$ for a 1.4$M_{\odot}$ neutron star for $L = 30$ MeV and different values of $Y_g$. The extension is given as the baryon number density above the crust core transition density, $n_{cc}$.

| $Y_g$ | $n_+(e=0)$ | $n_+(e=0.33)$ | $n_+(e=0.67)$ | $n_+(e=1)$ |
|-------|------------|---------------|---------------|------------|
| 0.0   | 0.000      | 0.002         | 0.007         | 0.010      |
| 0.1   | 0.006      | 0.022         | 0.025         | 0.027      |
| 0.5   | 0.048      | 0.056         | 0.058         | 0.059      |
| 1.0   | 0.073      | 0.089         | 0.082         | 0.082      |
of L suggested by the congruence of the experimental and observational constraints $30 \lesssim L \lesssim 60 \text{ MeV}$.

We calculate (i) the ratio of the moment of inertia of the crustal superfluid neutrons strongly pinned in the crust to that of the components of the star coupled to the crust on glitch rise times, $G$, and (ii) the relative angular acceleration of the crust immediately post-glitch $K$ under the assumption that the post-glitch acceleration is caused entirely by the increase in the effective moment of inertia of the star. The average giant glitch activity of the Vela pulsar predicts $G \geq 1.6\%$ and timing of the 2000 Vela glitch gives $K = 18 \pm 6$. Our particular aim is to examine claims that the inclusion of crustal neutron entrainment by the lattice renders the crustal neutrons an insufficient angular momentum reservoir to explain the Vela glitch activity.

In addition to $L$, we have examined the dependence of predictions on the fraction of core neutrons coupled to the crust at the time of glitch $Y_g$, the strength of the crustal neutron entrainment $\epsilon$ resulting from Bragg scattering of neutrons off the crustal nuclear lattice, and the penetration off pinned neutron vortices into the outer core up to a baryon density of $n_{\text{max}}$, and found the region of model space consistent with the observational constraints. We emphasize that, within the particular EOS model we use, our results give an upper limit on the glitch activity that can be produced by “snowplough”-like models.

Our main findings can be summarized as follows:

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Figure 8. (Color online) Fraction of the core superfluid coupled to the crust at the time of glitch below which $G \geq 1.6\%$ is satisfied for a given value of $L$. (a-c): Results are shown for a $1.4M_\odot$ star, crustal entrainment strengths of $\epsilon = 0$ (solid colored line), 0.33 (dashed line), 0.67 (dotted line) and 1 (dash-dotted line) and for a strong pinning region that is contained only within the inner crust $n_{\text{max}} = n_{\text{cc}}$ (a), and that extends into the outer core by a density of 0.03 fm$^{-3}$ (b) and 0.05 fm$^{-3}$ (c). (d-f): Results are shown for $e = 1$, neutron star masses of 1.0 (solid line), 1.2 (dashed line), 1.4 (dotted line) and 1.6M$\odot$ (dash-dotted line) and for a strong pinning region that is contained only within the inner crust $n_{\text{max}} = n_{\text{cc}}$ (d), and that extends into the outer core by a density of 0.03 fm$^{-3}$ (e) and 0.05 fm$^{-3}$ (f). Regions of parameter space consistent with $G \geq 1.6\%$ lie to the left of a given curve.

Table 2. Same quantities at Table 1 with $L = 60 \text{ MeV}$.

| $Y_g$ | $n_+ (\epsilon=0)$ | $n_+ (\epsilon=0.33)$ | $n_+ (\epsilon=0.67)$ | $n_+ (\epsilon=1)$ |
|------|------------------|------------------|------------------|------------------|
| 0.0  | 0.000            | 0.000            | 0.002            | 0.005            |
| 0.1  | 0.000            | 0.007            | 0.011            | 0.013            |
| 0.5  | 0.015            | 0.028            | 0.031            | 0.033            |
| 1.0  | 0.034            | 0.044            | 0.047            | 0.048            |

4 CONCLUSIONS

Using neutron star crust and core EOSs and compositions calculated using the same Skyrme nuclear matter model we have explored the ability of the recent “snowplough” glitch model to reproduce the observed glitch characteristics of the Vela pulsar. The Skyrme model contains two parameters that allow for systematic variation of the symmetry energy magnitude and slope at saturation density without changing symmetric nuclear matter properties, which we exploit to examine predictions spanning the conservative estimate of the range of uncertainty in the density-slope of the symmetry energy $25 < L < 105 \text{ MeV}$ resulting from a wide variety of experimental and observational constraints, while simultaneously being constrained to fit the results of state-of-the-art PNM calculations at low density. In addition to the conservative range of $L$, we pay attention to the range
• The general trends found are as follows: the observed value of $K$ is relatively high compared to model predictions, and favors a soft symmetry energy at saturation density (low values of $L$) and low core superfluid fraction coupled to the crust at the time of the glitch, $Y_g$. A softer symmetry energy results in more compact neutron stars that undergo larger accelerations for a given applied torque. The observational constraint on $G$, on the other hand, favors a stiffer symmetry energy at saturation density (higher values of $L$), which produces larger neutron stars of a given mass and thicker crusts, and therefore an increased MoI of crustal superfluid neutrons participating in the glitch. Thus observational constraints on $G$ and $K$ together are potentially quite constraining on the EOS around saturation density. We note also that the trends with $L$ persist even if the strong pinning region is extended into the core.

• $G \geq 1.65\%$ and $K = 18 \pm 6$ together constrain $L \leq 45$ MeV and $Y_g \lesssim 0.04$. These values are obtained considering strong pinning solely in the crust only when no crustal entrainment is present; when crustal entrainment is at full strength $e = 1$, these values are obtained when the strong pinning region penetrates into the outer core by up to 0.05fm$^{-3}$ above the crust core transition density. Thus, together, the observational constraints require a relatively soft symmetry energy (that is still consistent with experiment) and a relatively weak coupling between core superfluid neutrons and crust.

• Without entrainment the constraint on $G$ alone is satisfied below a maximum value of $Y_g$ that increases with $L$ up to $Y_g < 0.65$ at $L = 105$ MeV. In the range $L = 30 - 60$ MeV the upper limit to $Y_g$ ranges over $0.07 - 0.25$, implying weak to intermediate coupling between core neutrons and crust.

• With entrainment at full strength $e = 1$ and pinned vortices confined to the crust, observational constraints on $G$ imply $L > 100$ MeV and $Y_g \approx 0$. For $e = 0.67$, observational constraints on $G$ imply $L > 70$ MeV and $Y_g < 0.06$, and at $e = 0.33$, observational constraints on $G$ imply $L > 40$ MeV and $Y_g < 0.2$.

• When one allows the pinned vortices to penetrate into the crust by densities of up to 0.082 fm$^{-3}$ above crust-core transition density (a total density of 0.176 fm$^{-3}$) for $L = 30$ MeV, and 0.048fm$^{-3}$ above crust-core transition density (a total density of 0.126 fm$^{-3}$) for $L = 60$ MeV, the constraint on $G$ is satisfied for any value of $Y_g$, thus consistent with strong core neutron-crust coupling.

Hydrodynamical simulations suggest a strong contribution from mutual friction to the post-glitch spin-down rate (Haskell & Pizzochero 2013) which is not considered here. In such a scenario, the immediate post-glitch crust decelerates arises from drag exerted by already-coupled core neutrons, with a weaker deceleration arising later as the rest of the core neutrons couple to the crust on longer timescales. In this case, comparison of the relative moments of inertia of the uncoupled and coupled core components $K$ should be compared with the measured post-glitch acceleration at later times ($\sim$ minutes) after the glitch, when the acceleration is smaller. This would potentially widen the region of model space consistent with both the glitch magnitudes and the post-glitch rotational evolution.

The high-density behavior of the EOS is even more uncertain both theoretically and experimentally (Xiao et al. 2009; Russotto et al. 2011), even if one restricts the composition to purely nucleonic matter, with some of the only constraints coming from analysis of heavy-ion collisions (Danielewicz et al. 2002) and the observations of $\sim 2M_{\odot}$ neutron stars (Demorest et al. 2010). A stiffer high density EOS will cause an increase in the total moment of inertia of the core as well as the moment of inertia of the crust, and a calculation using consistent crust-core EOSs needs to be performed to evaluate the effect on predicted values of $G$ and $K$.

$Y_g$ can be related to the strength of mutual friction in the core via the crust-core coupling timescale (Haskell et al. 2012; Haskell & Antonopoulou 2013) and therefore, in principle, calculated consistently with the EOS, thereby removing it as a free model parameter. This will be the next step in the microphysically-consistent analysis of glitch models, to be reported in an upcoming paper. Note that the strength of mutual friction and hence $Y_g$ will vary throughout the core as the composition and overall density changes.

The current results, given our present knowledge of nuclear matter parameters and crustal microphysics, strongly support the conclusions of Chan (2013; Andersson et al. 2012) that a region of superfluid larger than that contained within the crust is involved in the glitch process. We argue that one possible solution is to extend the region in which neutron vortices are pinned into the core up to, at most, around nuclear saturation density. Extending the pinning region into the outer core still results in predictions that are strongly dependent on the symmetry energy around saturation density, however, and taken together with the predictions of $K$, the model retains the potential to constrain the nuclear equation of state and other microscopic parameters. One possible mechanism for pinning in the outer core is that of neutron vortices being pinned to superconducting proton flux-tubes if protons are type-II superconductors (Ruderman et al. 1998; Link 2003). The protons in the outer core immediately below the crust are indeed predicted to form a type-II superconductor, transitioning to a type-I superconductor at densities about twice nuclear saturation (Sedrakian 2005). It has yet to be addressed how such a strong pinning region in the outer crust couples to the crustal superfluid through the crust-core interface, with the possible mediation of the nuclear “pasta” phases.

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