Formulating E- & T-Model Inflation in Supergravity

C Pallis
Laboratory of Physics, Faculty of Engineering,
Aristotle University of Thessaloniki, Thessaloniki
GR-541 24, Greece
E-mail: kpallis@gen.auth.gr

Abstract. We present novel realizations of E- and T-model inflation within Supergravity which are largely associated with the existence of a pole of order one and two respectively in the kinetic term of the inflaton superfield. This pole arises due to the selected logarithmic Kahler potentials $K$, which parameterize hyperbolic manifolds with scalar curvature related to the coefficient $(-N) < 0$ of a logarithmic term. The associated superpotential $W$ exhibits the same $R$ charge with the inflaton-accompanying superfield and includes all the allowed terms. The role of the inflaton can be played by a gauge singlet or non-singlet superfield. Models with one logarithmic term in $K$ for the inflaton, require $N = 2$, some tuning -- of the order of $10^{-5}$ -- between the terms of $W$ and predict a tensor-to-scalar ratio $r$ at the level of $0.001$. The tuning can be totally eluded for more structured $K$’s, with $N$ values increasing with $r$ and spectral index close or even equal to its present central observational value.

1. Introduction

It is well-known [1–3] that the presence of a pole in the kinetic term of the inflaton gives rise to inflationary models collectively named $\alpha$-attractors [4, 5]. Confining ourselves to poles of order one and two, the lagrangian of the homogenous inflaton field $\phi = \phi(t)$ can be written as

$$L = \sqrt{-g} \left( \frac{N_p}{2 f_p^2} \dot{\phi}^2 - V_I(\phi) \right) \quad \text{with} \quad f_p = 1 - \phi^p, \quad p = 1, 2 \quad \text{and} \quad N_p > 0. \quad (1)$$

Also, $g$ is the determinant of the background Friedmann-Robertson-Walker metric $g^{\mu\nu}$ with signature $(+,-,-,-)$, dot stands for derivation with respect to (w.r.t) the cosmic time and $V_I = V_I(\phi)$ is a polynomial function of $\phi$. Expressing the canonically normalized field, $\hat{\phi}$, in terms of $\phi$ as follows

$$\frac{d\hat{\phi}}{d\phi} = J = \sqrt{\frac{N_p}{f_p}} \Rightarrow \phi = \begin{cases} 1 - e^{-\frac{\hat{\phi}}{\sqrt{N_1}}} & \text{for} \quad p = 1, \\ \tanh \left( \frac{\hat{\phi}}{\sqrt{N_2}} \right) & \text{for} \quad p = 2, \end{cases} \quad (2)$$

we easily infer that $V_I(\hat{\phi})$ develops a plateau for $\hat{\phi} \gg 1$, and so it becomes suitable for driving inflation of chaotic type. It is called $E$-Model Inflation (EMI) [5, 6] (or $\alpha$-Starobinsky model [7]) and T-Model Inflation (TMI) [6, 8] for $p = 1$ and $2$ respectively.

In this talk, based on Ref. [2, 3], we present novel realizations of E and/or T-Model Inflation (ETI) in the context of Supergravity (SUGRA). Namely, in Sec. 2 we describe our general strategy...
in establishing those models and then – in Sec. 3 – we specify three models ($\delta EM, EM2, EM4$) employing a gauge singlet inflaton and three ($\delta TM, TM4, TM8$) using a gauge non-singlet inflaton. We end up with our numerical results in Sec. 4 and our conclusions in Sec. 5.

Throughout the text, the subscript, $\chi$ denotes derivation w.r.t the field $\chi$, charge conjugation is denoted by a star ($^*$) and we use units where the reduced Planck scale $m_P = 2.44 \cdot 10^{18}$ GeV is set equal to unity.

2. Strategy of the SUGRA Embedding

We start our investigation presenting the basic formulation of scalar theory within SUGRA in Sec. 2.1 and then – in Sec. 2.2 and 2.3 – we outline our methodology in constructing viable ETI.

2.1. General Formalism

The part of the SUGRA lagrangian including the (complex) scalar fields $z^\alpha$ can be written as

$$\mathcal{L} = \sqrt{-g} \left( K_{\alpha \beta} D_\mu z^{\alpha} D^\mu z^{\beta} - V_{\text{SUGRA}} \right), \quad (3)$$

where the kinetic mixing is controlled by the Kähler potential $K$ and the relevant metric defined as

$$K_{\alpha \beta} = K_{z^\alpha z^{\beta}} > 0 \quad \text{with} \quad K^{\beta \alpha} K_{\alpha \gamma} = \delta_\gamma^\beta. \quad (4a)$$

Also, the covariant derivatives for the scalar fields $z^\alpha$ are given by

$$D_\mu z^\alpha = \partial_\mu z^\alpha + ig A_\mu^a T_{\alpha \beta} z^\beta \quad (4b)$$

with $A_\mu^a$ being the vector gauge fields, $g$ the (unified) gauge coupling constant and $T_{\alpha \beta}$ with $a = 1, ..., \dim G_{\text{GUT}}$ the generators of a Grand Unified Theory (GUT) gauge group $G_{\text{GUT}}$. Here and henceforth, the scalar components of the various superfields are denoted by the same superfield symbol.

The SUGRA scalar potential, $V_{\text{SUGRA}}$, is given in terms of $K$, and the superpotential, $W$, by

$$V_{\text{SUGRA}} = V_F + V_D \quad \text{with} \quad V_F = e^K \left( K^{\alpha \beta} D_\alpha W D_\beta W^* - 3|W|^2 \right) \quad \text{and} \quad V_D = g^2 \sum_a D_\alpha^2 / 2. \quad (5)$$

The Kähler covariant derivative and the D-terms read

$$D_\alpha W = W, z^\alpha + K_{z^\alpha W} \quad \text{and} \quad D_\alpha = z_\alpha (T_a), \quad K_{z^\alpha} = K_{z^\alpha}. \quad (6)$$

Therefore, ETI introduced in Sec. 1 can be supersymmetrized, if we select conveniently the functions $K$ and $W$ so that Eq. (3) reproduces Eq. (1).

2.2. Inflaton’s Sector

We concentrate on ETI driven by $V_F$ assuring that $V_D = 0$ during it. This condition may be attained identifying the inflaton either with (the radial part of) a gauge singlet superfield $z^2 := \Phi$ or with the radial part of a conjugate pair of Higgs superfields, $z^2 := \Phi$ and $z^3 := \bar{\Phi}$.

To achieve a kinetic term in Eq. (3) similar to that in Eq. (1) for $p = 1$ and 2, we need to establish suitable $K$’s so that (during ETI)

$$\langle K \rangle_1 = -N \ln f_p \quad \text{and} \quad \langle K_{\alpha \beta} \rangle_1 = N / f_p^2 \quad (7)$$

with $N$ related to $N_p$. However, from the F-term contribution to Eq. (5), we remark that $K$ affects – besides the kinetic mixing – $V_{\text{SUGRA}}$, which, in turn, depends on $W$ too. Therefore, $f_p$ is generically expected to emerge also in the denominator of $V_{\text{SUGRA}}$ making difficult the establishment of an inflationary era. This problem can be addressed [2, 3] either by tuning the terms of $W$ so that the pole is removed from $V_{\text{SUGRA}}$ thanks to cancellations or adopting a more structured $K$ which yields the desired kinetic terms in Eq. (1) but lets $V_{\text{SUGRA}}$ immune.
2.3. Stabilizer’s Sector
We reserved $\alpha = 1$ for a gauge singlet superfield, $z^1 = S$ called stabilizer or goldstino, which assists [10] us to formulate ETI of chaotic type in SUGRA. We assume that $S$ and $W$ are equally charged under a global $R$ symmetry and so it appears linearly in $W$ multiplying its other terms. Also, it generates for $\langle S \rangle_I = 0$ the inflationary potential via the only term of $V_{\text{SUGRA}}$ which remains alive

$$V_I = \langle V_F \rangle_I = \langle e^K K^{SS^*} |W,S|^2 \rangle_I,$$

where the symbol “$\langle Q \rangle_I$” denotes the value of a quantity $Q$ during ETI. Such an adjustment assures the boundedness of $V_I$, whereas $S$ can be stabilized at $\langle S \rangle_I = 0$, if we select [11]

$$K_2 = N_S \ln \left(1 + \left|S/N_S\right|^2\right) \Rightarrow \langle K_2^{SS^*} \rangle_I = 1 \text{ with } 0 < N_S < 6. \quad (9)$$

$K_2$ parameterizes the compact manifold $SU(2)/U(1)$. Note that for $\langle S \rangle_I = 0$, $S$ is canonically normalized and so we do not mention it again henceforth.

3. Inflationary Settings
We here explain the construction of the EMI and TMI in Sec. 3.1 and 3.2 respectively.

3.1. E-Model Inflation
This setting is realized in presence of two gauge singlet superfields $S$ and $\Phi$. The relevant SUGRA setup is presented in Sec. 3.1.1 whereas the resulting models are derived in Sec. 3.1.2.

3.1.1. SUGRA Set-up
We adopt the most general renormalizable $W$ consistent with the $R$ symmetry mentioned in Sec. 2.3, i.e.,

$$W = S \left(\lambda_1 \Phi + \lambda_2 \Phi^2 - M^2\right) \quad (10)$$

where $\lambda_1, \lambda_2$ and $M$ are free parameters. As regards $K$, this includes, besides $K_2$ in Eq. (9), one of the following $K$’s:

$$K_{1s} = -N \ln \left(1 - \Phi/2 - \Phi^*/2\right) \text{ or } \tilde{K}_{1s} = -N \ln \left(\frac{1 - \Phi/2 - \Phi^*/2}{1 - \Phi}^{1/2}\right), \quad (11)$$

with $\text{Re}(\Phi) < 1$ and $N > 0$. For the considered versions of EMI, $\delta\text{EM}$ and $\text{EM}2–\text{EM}4$, the total $K$ respectively is

(a) $K_{21s} = K_2 + K_{1s}$ and (b) $\tilde{K}_{21s} = K_2 + \tilde{K}_{1s}. \quad (12)$$

The elimination of pole in $V_I$ for $\delta\text{EM}$ can be achieved if we set

$$N = 2 \text{ and } r_{21} = -\lambda_2/\lambda_1 \simeq 1 + \delta_{21} \text{ with } \delta_{21} \sim 0 \text{ and } M \ll 1. \quad (13)$$

No denominator exists in $V_I$ for $\text{EM}2$ and $\text{EM}4$ and so $N, \lambda_1, \lambda_2$ and $M$ are free parameters.

3.1.2. Inflationary Configuration
The inflationary potential can be derived from Eq. (8) specifying the inflationary trajectory as follows

$$\langle S \rangle_I = 0 \text{ and } \langle \theta \rangle_I := \arg \langle \Phi \rangle_I = 0. \quad (14)$$

Inserting the quantities above into Eq. (8) and taking into account Eq. (9) and

$$\langle e^K \rangle_I = \begin{cases} f_{1}^{-N} & \text{for } K = K_{21s}, \\ 1 & \text{for } K = \tilde{K}_{21s}, \end{cases} \quad (15)$$
we arrive at the following master equation

\[ V_1 = \lambda^2 \left\{ \left( \phi - r_{21}\phi^2 - M_1^2 \right)^2 / f_1^2 \right\} \quad \text{for } \delta \text{EM}, \]
\[ \left( \phi - r_{12}\phi^2 - M_2^2 \right)^2 \quad \text{for EM2}, \]
\[ \left( \phi^2 - r_{12}\phi - M_2^2 \right)^2 \quad \text{for EM4}, \]

where \( \phi = \text{Re}(\Phi), r_{ij} = -\lambda_i/\lambda_j \) with \( i, j = 1, 2 \) and \( \lambda_i \) and \( M_i \) are identified as follows

\[ \lambda = \begin{cases} \lambda_1 & \text{and } M_1 = M/\sqrt{\lambda_1} \text{ for } \delta \text{EM and EM2}, \\ \lambda_2 & \text{and } M_2 = M/\sqrt{\lambda_2} \text{ for EM4}. \end{cases} \]

As advertised in Sec. 3.1.1, the pole in \( f_1 \) is presumably present in \( V_1 \) for \( \delta \text{EM} \), but it disappears for EM2 and EM4. The arrangement of Eq. (13), though, renders the pole harmless for \( \delta \text{EM} \).

We introduce the canonically normalized fields, \( \hat{\phi} \) and \( \hat{\theta} \), as follows

\[ \langle K_{\Phi\Phi}\rangle_1 |\hat{\Phi}|^2 \simeq \frac{1}{2} \left( \hat{\phi}^2 + \hat{\theta}^2 \right) \Rightarrow \frac{d\hat{\phi}}{d\phi} = J = \frac{\sqrt{N/2}}{f_1} \text{ and } \hat{\theta} \simeq J\phi\theta \text{ with } \langle K_{\Phi\Phi}\rangle_1 = \frac{N}{4f_1^2}. \]

We see that the relation between \( \phi \) and \( \hat{\phi} \) is identical with Eq. (2) for \( p = 1 \), if we do the replacement \( N_1 = N/2 \).

To check the stability of \( V_{\text{SUGRA}} \) in Eq. (5) along the trajectory in Eq. (14) w.r.t the fluctuations of \( z^\alpha \)'s, we construct the mass spectrum of the theory. Our results are summarized in Table 1. Taking the limit \( \delta_{21} = M_1 = 0 \) for \( \delta \text{EM}, r_{21} = M_1 = 0 \) for EM2 and \( r_{12} = M_2 = 0 \) for EM4, we find the expressions of the masses squared \( \hat{m}^2_{\phi,\theta} \) (with \( \chi^\alpha = \theta \) and \( s \)) arranged in Table 1. We there display the masses \( \hat{m}^2_{\phi,\theta} \) of the corresponding fermions too – we define \( \hat{\psi}_\Phi = J\psi_\Phi \) where \( \psi_\Phi \) and \( \psi_S \) are the Weyl spinors associated with \( S \) and \( \Phi \) respectively. We notice that the relevant expressions can take a unified form for all models – recall that we use \( N = 2 \) in \( \delta \text{EM} \) – and approach, close to \( \phi = \phi_* \approx 1 \), rather well the quite lengthy, exact ones employed in our numerical computation. From them we can appreciate the role of \( N_S < 6 \) in retaining positive \( \hat{m}^2_{\phi,\theta} \). Also, we confirm that \( \hat{m}^2_{\phi,\theta} \gg H^2_1 \simeq V_{f_0}/3 \) for \( \phi_i < \phi \lessgtr \phi_* \).

### 3.2. T-Model Inflation

Although TMI is mostly realized by a gauge-singlet inflaton [5], we here propose an alternative scheme where the inflaton is included in a conjugate pair of Higgs superfields, \( \Phi \) and \( \Phi \) with charges \( B - L = -1 \) and 1 respectively. These fields break the GUT symmetry \( G_{\text{GUT}} = G_{\text{SM}} \times U(1)_{B-L} \) down to the Standard Model (SM) gauge group \( G_{\text{SM}} \) through their vacuum expectation value (v.e.v). We below outline the SUGRA setting in Sec. 3.2.1 and its inflationary outcome in Sec. 3.2.2.
3.2.1. SUGRA Set-up  Consistently with the imposed symmetries we adopt the following $W$

$$W = S \left( \frac{1}{2} \lambda_2 \dot{\Phi}^2 + \lambda_4 (\dot{\Phi})^2 - \frac{1}{4} M^2 \right),$$  \hspace{1cm} (19)

where $\lambda_2, \lambda_4$ and $M$ are free parameters. This type of $W$ is well-known from the models of F-term hybrid inflation – for reviews see, e.g., Ref. [12]. The invariance of $K$ under $G_{\text{GUT}}$ enforces us to introduce a pole of order two in the $\Phi^2 - \Phi^2$ kinetic terms. One possible option – for another equivalent one see Ref. [2] – is

$$K_{(11)}^2 = - \frac{N}{2} \ln \left(1 - 2|\Phi|^2\right) \left(1 - 2|\Phi|^2\right) \text{ or } \tilde{K}_{(11)}^2 = - \frac{N}{2} \ln \left(1 - 2|\Phi|^2\right) \left(1 - 2|\Phi|^2\right),$$  \hspace{1cm} (20)

which parameterizes the manifold $M_{(11)}^2 = SU(1,1)/U(1)\Phi \times SU(1,1)/U(1)\bar{\Phi}$ [2] with scalar curvature $\mathcal{R}_{(11)}^2 = -8/N$. From the selected above $W$ and $K$’s, we obtain the models $\delta \text{TM}$ and $\text{TM4} - \text{TM8}$, which employ the following total $K$’s correspondingly:

$$(a) \quad K_{(11)}^2 = K_2 + K_{(11)}^2 \quad \text{and} \quad (b) \quad \tilde{K}_{(11)}^2 = K_2 + \tilde{K}_{(11)}^2.$$  \hspace{1cm} (21)

Within $\delta \text{TM}$, an elimination of the singular denominator appearing in $V_1$ is obtained setting

$$N = 2 \quad \text{and} \quad r_{42} = - \lambda_4/\lambda_2 \simeq 1 + \delta_{42} \quad \text{with} \quad \delta_{42} \sim 0 \quad \text{and} \quad M \ll 1.$$  \hspace{1cm} (22)

On the other hand, no singularity exists in $V_1$, within $\text{TM4}$ and $\text{TM8}$ and so $N, \lambda_2, \lambda_4$ and $M$ remain free parameters.

3.2.2. Inflationary Configuration  Adopting for the relevant fields the parameterization

$$\Phi = \phi e^{i\theta} \cos \theta_\Phi \quad \text{and} \quad \bar{\Phi} = \phi e^{i\theta} \sin \theta_\Phi \quad \text{with} \quad 0 \leq \theta_\Phi \leq \pi/2 \quad \text{and} \quad S = (s + is)/\sqrt{2},$$  \hspace{1cm} (23)

we can easily verify that a D-flat direction is

$$\langle \theta \rangle_1 = \langle \bar{\theta} \rangle_1 = 0, \quad \langle \theta_\Phi \rangle_1 = \pi/4 \quad \text{and} \quad \langle S \rangle_1 = 0,$$  \hspace{1cm} (24)

which can be qualified as inflationary path. Regarding the exponential prefactor of $V_1$ in Eq. (5) we obtain

$$\langle e^{K_1} \rangle_1 = \begin{cases} f_2^{-N} & \text{for} \quad K = K_{(11)}^2, \\ 1 & \text{for} \quad K = \tilde{K}_{(11)}^2, \end{cases}$$  \hspace{1cm} (25)

Substituting it and Eqs. (9) and (19) into Eq. (8), the inflationary potential $V_1$ takes its master form

$$V_1 = \frac{\lambda^2}{16} \begin{cases} \left(\phi^2 - r_{42}\phi^4 - M_2^2 \right)^2 / f_2^N & \text{for} \delta \text{TM}, \\ \left(\phi^2 - r_{42}\phi^4 - M_2^2 \right)^2 & \text{for} \text{TM4}, \\ \left(\phi^4 - r_{42}\phi^2 - M_2^2 \right)^2 & \text{for} \text{TM8}, \end{cases}$$  \hspace{1cm} (26)

where $r_{ij} = -\lambda_i/\lambda_j$ with $i, j = 1, 2$ and $\lambda$ and $M_i$ are identified as follows

$$\lambda = \begin{cases} \lambda_2 & \text{and} \quad M_2 = M/\sqrt{\lambda_2} \quad \text{for} \delta \text{TM and TM4}, \\ \lambda_4 & \text{and} \quad M_4 = M/\sqrt{\lambda_4} \quad \text{for} \text{TM8}. \end{cases}$$  \hspace{1cm} (27)

From Eq. (26), we infer that the pole in $f_2$ is presumably present in $V_1$ of $\delta \text{TM}$ but it disappears in $V_1$ of $\text{TM4}$ and $\text{TM8}$ and so no $N$ dependence in $V_1$ arises. The elimination of the pole in the regime of Eq. (22) lets open the realization of $\delta \text{TM}$, though.
The canonically normalized (hatted) fields of the $\bar{\phi} - \Phi$ system during TMI, are defined as follows

$$\langle K_{\alpha\beta} \rangle = \frac{1}{\kappa} \text{diag}(1, 1)$$

(28b)

where, via Eq. (20), we find

$$\langle (K_{\alpha\beta}) \rangle = \langle M_{\Phi\Phi} \rangle_1 \text{ with } \langle M_{\Phi\Phi} \rangle_1 = \frac{\kappa}{f_{2}}.$$  

Inserting the expressions above in Eq. (28a) we obtain the hatted fields

$$d\hat{\phi}/d\phi = J = \sqrt{2N}/f_2, \quad \hat{\theta}_\pm = \sqrt{\kappa} \phi \theta_\pm \quad \text{and} \quad \hat{\theta}_\Phi = \sqrt{2\kappa} \phi (\theta_\Phi - \pi/4),$$

(29)

where $\theta_\pm = (\bar{\theta} \pm \theta) / \sqrt{2}$. From the first equation above we conclude that Eq. (2) for $p = 2$ is reproduced for $N_2 = 2N$.

We can also verify that the direction of Eq. (24) is stable w.r.t the fluctuations of the non-inflaton fields. Approximate, quite precise though, expressions for $\phi \sim 1$ are arranged in Table 2. We confine ourselves to the limits $\delta_{r_{42}} = M_2 = 0$ for $\delta T M$, $r_{42} = M_2 = 0$ for TM4 and $r_{24} = M_4 = 0$ for TM8. As in the case of the spectrum in Table 1, $N_S < 6$ plays a crucial role in retaining positive and heavy enough $\hat{m}_{\phi}^2$. Here, however, we also display the masses, $M_{BL}$, of the gauge boson $A_{BL}$ (which signals the fact that $U(1)_B-L$ is broken during TMI) and the masses of the corresponding fermions. It is also evident that $A_{BL}$ becomes massive absorbing the massless Goldstone boson associated with $\bar{\theta}_-$.

### Table 2. Mass spectrum for TMI along the inflationary trajectory of Eq. (24).

| Fields       | Eigenstates | Masses Squared |
|--------------|-------------|----------------|
| 2 real scalars | $\hat{\theta}_+$ $\hat{\theta}_\Phi$ | $m_{\hat{\theta}+}^2$ $3H_1^2$ |
|              | $\hat{\theta}_\Phi$ | $\hat{m}_{\hat{\theta}+}^2$ |
| 1 complex scalar | $s, \bar{s}$ | $\hat{m}_{s}^2$ |
|              | $H_1^2(1/N_S - 8(1 - \phi^2)/N + N\phi^2/2)$ |
|              | $+2(1 - 2\phi^2) + 8\phi^2/N)$ |
| 1 gauge boson | $A_{BL}$ | $M_{BL}^2$ |
|              | $M_{BL}^2$ |
|              | $2Ng_2^2\phi_2^2/f_2^2$ |
| 4 Weyl spinors | $\hat{\psi}_\pm$ $\hat{\psi}_\Phi^-$ | $\hat{m}_{\hat{\psi}+}^2$ |
|              | $M_{BL}^2$ |
|              | $12f_2^2H_1^2/N^2\phi_2^2$ |
|              | $2Ng_2^2\phi_2^2/f_2^2$ |

4. Inflationary Observables

We here constrain the parameters of our EMI and TMI – in Sec. 4.2 and 4.3 respectively – taking into account a number of observational and theoretical requirements described in Sec. 4.1.

4.1. Constraints

We impose to ETI the following observational and theoretical requirements:

- ...
(a) The number of e-foldings \( N_* \) that the scale \( k_* = 0.05/\text{Mpc} \) experiences during ETI must be enough for the resolution of the problems of standard Big Bang, i.e., [13]

\[
N_* = \int_{\phi_I}^{\phi_*} d\phi J \frac{V_{1*}}{V_{1,\phi}} \simeq 61.3 + \frac{1 - 3w_{rh}}{12(1 + w_{rh})} \ln \frac{\pi^2 g_{*h} T_{rh}^4}{30 V_I(\phi_I)} + \frac{1}{4} \ln \frac{V_I(\phi_*)^2}{g_{*h}^{1/2} V_I(\phi_I)},
\]

where \( \hat{\phi}_* \) is the value of \( \phi \) when \( k_* \) crosses the inflationary horizon whereas \( \hat{\phi}_I \) is the value of \( \phi \) at the end of ETI, which can be found, in the slow-roll approximation, from the condition

\[
\max\{\epsilon(\phi_I), |\eta(\phi_I)|\} = 1, \quad \text{where } \epsilon = \frac{1}{2} \left( \frac{V_{I,\phi}}{V_I} \right)^2 \quad \text{and} \quad \eta = \frac{V_{I,\phi}\phi}{V_I}.
\]

Also we assume that ETI is followed in turn by an oscillatory phase with mean equation-of-state parameter \( w_{rh} \), radiation and matter domination. We determine it applying the formula [3]

\[
w_{rh} = 2 \frac{\int_{\phi_{mn}}^{\phi_{mx}} d\phi J (1 - V_I/V_I(\phi_{mx}))^{1/2}}{\int_{\phi_{mn}}^{\phi_{mx}} d\phi J (1 - V_I/V_I(\phi_{mx}))^{-1/2}} - 1,
\]

where \( \phi_{mn} = \langle \phi \rangle \) is the v.e.v of \( \phi \) after ETI. Motivated by implementations [14] of non-thermal leptogenesis, which may follow ETI, we set \( T_{rh} \simeq 10^9 \text{ GeV} \) for the reheat temperature. Finally, \( g_{*h} = 228.75 \) is the energy-density effective number of degrees of freedom include corresponding to the Minimal SUSY SM (MSSM) spectrum.

(b) The amplitude \( A_* \) of the power spectrum of the curvature perturbations generated by \( \phi \) at \( k_* \) has to be consistent with data [13], i.e.,

\[
A_* = V_I(\hat{\phi}_*)^3/12 \pi^2 V_{1,\phi}(\hat{\phi}_*)^2 \simeq 2.105 \cdot 10^{-9}.
\]

(c) The remaining inflationary observables \( (n_s, \alpha_s, r) \) have to be consistent with the latest Planck release 4 (PR4), Baryon Acoustic Oscillations (BAO), CMB-lensing and BICEP/Keck (BK18) data [15,16], i.e.,

\[
(i) \ n_s = 0.965 \pm 0.009 \quad \text{and} \quad (ii) \ r \leq 0.032,
\]

at 95\% confidence level (c.l.) – pertaining to the \( \Lambda \text{CDM}+r \) framework with \( |\alpha_s| \ll 0.01 \). These observables are estimated through the relations

\[
(i) \ n_s = 1 - 6\hat{\epsilon}_* + 2\hat{\eta}_* \quad \text{and} \quad (ii) \ \alpha_s = \frac{2}{3} \left( 4\hat{\eta}_s^2 - (n_s - 1)^2 \right) - 2\hat{\xi}_* \quad \text{and} \quad (iii) \ r = 16\hat{\epsilon}_*,
\]

with \( \xi = V_{I,\phi} V_{1,\phi\phi}/V_I^2 \) – the variables with subscript * are evaluated at \( \phi = \phi_* \).

(d) The effective theory describing ETI has to remain valid up to a Ultra Violet (UV) cutoff scale \( \Lambda_{UV} \simeq m_p \) to ensure the stability of our inflationary solutions, i.e.,

\[
(i) \ V_I(\phi_*)^{1/4} \leq \Lambda_{UV} \quad \text{and} \quad (ii) \ \phi_* \leq \Lambda_{UV}.
\]

Finally, we quantify the tuning needed for the attainment of efficient ETI defining

\[
\Delta_* = 1 - \phi_*.
\]

The naturalness of the achievement of ETI increases with \( \Delta_* \).
After imposing Eqs. (30) and (32) the free parameters of

\[ \delta EM, \text{ EM2, EM4 are} \quad (\delta_{21}, M_1), \ (N, r_{21}, M_1) \text{ and} \quad (N, r_{12}, M_2), \]

respectively. Recall that we use \( N = 2 \) exclusively for \( \delta EM \). Fixing \( M_1 = 0.001 \) for \( \delta EM \), \( M_1 = 0.01 \) and \( r_{21} = 0.001 \) for EM2 and \( M_2 = 0.01 \) and \( r_{12} = 0.001 \) for EM4, we obtain the curves plotted and compared to the observational data in Fig. 1. We observe that:

(a) For \( \delta EM \) the resulting \( n_s \) and \( r \) increase with \( |\delta_{21}| \) – see solid line in Fig. 1. From the considered data we collect the results

\[ 0 \lesssim \delta_{21}/10^{-5} \lesssim 3.3, \quad 3.5 \lesssim r/10^{-3} \lesssim 5.3 \quad \text{and} \quad 9 \cdot 10^{-3} \lesssim \Delta_* \lesssim 0.01. \quad (37) \]

In all cases we obtain \( N_* \simeq 44 \) consistently with Eq. (30) and the resulting \( w_{rh} \simeq -0.237 \) from Eq. (31b). Fixing \( n_s = 0.965 \), we find \( \delta_{21} = -1.7 \cdot 10^{-5} \) and \( r = 0.0044 \) – see the leftmost column of the Table in Fig. 1.

(b) For EM2 and EM4, \( n_s \) and \( r \) increase with \( N \) and \( \Delta_* \), which increases w.r.t its value in \( \delta EM \). Namely, for EM2 – see dashed line in Fig. 1 – we obtain

\[ 0.96 \lesssim n_s \lesssim 0.9654, \quad 0.1 \lesssim N \lesssim 65, \quad 0.05 \lesssim \Delta_*/10^{-2} \lesssim 16.7 \quad \text{and} \quad 0.0025 \lesssim r \lesssim 0.039 \quad (38a) \]

with \( w_{rh} \simeq -0.05 \) and \( N_* \simeq 50 \). On the other hand, for EM4 – see dot-dashed line in Fig. 1 – we obtain

\[ 0.963 \lesssim n_s \lesssim 0.965, \quad 0.1 \lesssim N \lesssim 55, \quad 0.23 \lesssim \Delta_*/10^{-2} \lesssim 8.5 \quad \text{and} \quad 0.0001 \lesssim r \lesssim 0.04 \quad (38b) \]

with \( w_{rh} \simeq (0.25 – 0.39) \) and \( N_* \simeq 54 – 56 \). In both equations above the lower bound on \( N \) is just artificial. For \( N = 10 \), specific values of parameters and observables are arranged in the rightmost columns of the Table in Fig. 1.
indicated on the solid line or (ii) TM4

After enforcing Eqs. (30) and (32) – which yield $\delta_{42}/r_{42}/r_{24}$, $\phi_{i}/0.1$, $\Delta_{s}(%)$, and $\phi_{t}/0.1$ – we obtain results similar to those obtained for TM8 and $r_{24} = 10^{-6}$ and various $N$'s indicated on the dashed and dot-dashed line respectively. The shaded corridors are identified as in Fig. 1. The relevant field values, parameters and observables corresponding to points shown in the plot are listed in the Table.

4.3. T-Model Inflation

After enforcing Eqs. (30) and (32) – which yield $\lambda$ together with $\phi_{s}$ – the free parameters of the models

$\delta\text{TM}$, TM4, TM8 are $(\delta_{42}, M_{2})$, $(N, r_{42}, M_{2})$ and $(N, r_{24}, M_{4})$,

respectively. Recall that we use $N = 2$ exclusively for $\delta\text{TM}$. Also, we determine $M_{2}$ and $M_{4}$ demanding that the GUT scale within MSSM, $M_{\text{GUT}} \simeq 2.433 \times 10^{-2}$, is identified with the value of $M_{BL}$ – see Table 2 – at the vacuum value of $\phi_{s}$, $\langle \phi \rangle$. We approximately obtain $M_{2}$ and $M_{4} \leq 0.001$. By varying the remaining parameters for each model we obtain the allowed curves in the $n_{s} - r$ plane – see Fig. 2. A comparison with the observational data is also displayed there. We observe that:

(a) For $\delta\text{TM}$ – see the solid line in Fig. 2 – we obtain results similar to those obtained for $\delta\text{EM}$ in Sec. 4.2. Namely, the resulting $n_{s}$ and $r$ increase with $|\delta_{42}|$ with $n_{s}$ covering the whole allowed range in Eq. (33). From the considered data we collect the results

$$2 \lesssim -\delta_{42}/10^{-5} \lesssim 5.5, \quad 2 \lesssim r/10^{-3} \lesssim 3.6 \quad \text{and} \quad 4 \lesssim \Delta_{s}/10^{-3} \lesssim 4.75. \tag{39}$$

Also, Eq. (30) yields $N_{s} \simeq (54.8 - 55.7)$ and Eq. (31b) results to $w_{rh} \simeq 0.3$. Fixing $n_{s} = 0.965$ we find $\delta_{42} = -3.6 \cdot 10^{-5}$ and $r = 0.0026$ – see the leftmost column of the Table in Fig. 2.

(b) For TM4 and TM8, $n_{s}$ and $r$ increase with $N$ and $\Delta_{s}$ which is larger than that obtained in $\delta\text{TM}$. More specifically, for TM4 – see dashed line in Fig. 2 – we obtain

$$0.963 \lesssim n_{s} \lesssim 0.964, \quad 0.1 \lesssim N \lesssim 36, \quad 0.09 \lesssim \Delta_{s}/10^{-2} \lesssim 7.6 \quad \text{and} \quad 0.0005 \lesssim r \lesssim 0.039, \tag{40a}$$

with $w_{rh} \simeq 0.3$ and $N_{s} \simeq 56$. On the other hand, for TM8 – see dot-dashed line in Fig. 2 – we obtain

$$0.963 \lesssim n_{s} \lesssim 0.965, \quad 0.1 \lesssim N \lesssim 40, \quad 0.45 \lesssim \Delta_{s}/10^{-2} \lesssim 3.8 \quad \text{and} \quad 0.0001 \lesssim r \lesssim 0.039, \tag{40b}$$

with $w_{rh} \simeq (0.25 - 0.6)$ and $N_{s} \simeq (54.6 - 60)$. For $N = 12$, specific values of parameters and observables are arranged in the rightmost columns of the Table in Fig. 2.

| Model: | $\delta\text{TM}$ | TM4 | TM8 |
|-------|------------------|-----|-----|
| $\delta_{42}/r_{42}/r_{24}$ | $-3.6 \cdot 10^{-5}$ | 0.01 | $10^{-6}$ |
| $N$ | 2 | 12 | 12 |
| $\phi_{s}/0.1$ | 9.9555 | 9.75 | 9.877 |
| $\Delta_{s}(%)$ | 0.445 | 2.5 | 1.23 |
| $\phi_{t}/0.1$ | 5.9 | 3.9 | 6.5 |
| $w_{rh}$ | 0.33 | 0.266 | 0.58 |
| $N_{s}$ | 55.2 | 56.4 | 58 |
| $\lambda/10^{-5}$ | 3.6 | 8.6 | 8.5 |
| $n_{s}/0.1$ | 9.65 | 9.64 | 9.65 |
| $-\alpha_{s}/10^{-4}$ | 6.6 | 6.4 | 5.98 |
| $r/10^{-2}$ | 0.26 | 1.4 | 1.3 |

Figure 2. Allowed curves in the $n_{s} - r$ plane fixing $M_{BL} = M_{\text{GUT}}$ for (i) $\delta\text{TM}$ and various $\delta_{42}$'s indicated on the solid line or (ii) TM4 and $r_{42} = 0.01$ or TM8 and $r_{24} = 10^{-6}$ and various $N$'s indicated on the dashed and dot-dashed line respectively. The shaded corridors are identified as in Fig. 1. The relevant field values, parameters and observables corresponding to points shown in the plot are listed in the Table.
5. Conclusions
We reviewed the implementation of ETI in the context of SUGRA. We employed as inflaton a gauge singlet or non-singlet superfield for EMI or TMI respectively. The models are relied on the superpotentials in Eqs. (10) and (19) which respect an $R$ symmetry and include, besides inflaton, an inflaton-accompanying field which facilitates the establishment of ETI. In each model we singled out three subclasses of models ($\delta EM$, $EM2$ and $EM4$) and ($\delta TM$, $TM4$ and $TM8$). The models $\delta EM$ and $\delta TM$ are based on the Kähler potentials in Eq. (12a) and (21a) whereas ($EM2$, $EM4$) and ($TM4$, $TM8$) in those shown in Eq. (12b) and (21b). Within $\delta EM$ and $\delta TM$ any observationally acceptable $n_s$ is attainable by tuning $\delta_{21}$ and $\delta_{42}$ respectively to values of the order $10^{-5}$, whereas $r$ is kept at the level of $10^{-3}$ – see Eqs. (37) and (39). On the other hand, $EM2$, $EM4$, $TM4$ and $TM8$ avoid any tuning, larger $r$’s are achievable as $N$ increases beyond 2, while $n_s$ lies close to its central observational value – see Eqs. (38a) and (38b) for EMI and Eqs. (40a) and (40b) for TMI.

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