Pythagorean Fuzzy Based AHP-VIKOR Integration to Assess Rail Transportation Systems in Turkey

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Abstract Rail transit systems, one of the important public transportation (PT) systems, are widespread in large cities to satisfy passengers or commuters for their daily trips. However, rail transportation (RT) investments are costly, and the budgets are minimal. Thus assessing RT systems in big cities play a crucial role in deciding on the best RT investment. In this study, for the first time, a two-stage fuzzy set is used among the multi-criteria decision-making (MCDM) studies in the area of transportation. The proposed method is a unique technique integrating the Analytic Hierarchy Process (AHP) and Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) techniques with an extension of Pythagorean fuzzy sets. In this manner, datasets from the observations of three rail lines in Antalya, one of the largest cities in Turkey, are applied to the proposed method. Fuzzy Analytic Hierarchy Process (FAHP), a commonly used MCDM method, is used to weighting four main criteria and thirteen sub-factors. AHP is strengthened by interval-valued Pythagorean fuzzy numbers (IVPFNs). The Fuzzy VIKOR (FVIKOR) approach is then applied for the prioritization of three rail line alternatives. Results are achieved by analyzing not only real RT service observations, also actual RT expenditures or costs. Consequently, the uncertainty in results is thus minimized. Outcomes of the case study reveal the most serviceable RT line in the city evaluating the main criteria, namely economy, comfort, environment, and safety. Therefore, the proposed approach methodologically contributes to adopting a new insight into the integration of MCDM methods with fuzzy set.

Keywords Public transportation · Rail transit · Multi-criteria decision-making · Pythagorean fuzzy sets

1 Introduction

Urban rail transportation (RT) systems are essential in public transportation (PT) systems in large and crowded cities, either in developed or developing countries [1–5]. Factors such as a faster and safer journey, comfort, and accessibility are among people’s preferences for rail systems. In addition, rail systems are preferred to help solve the traffic problem due to the high demand for travel in large metropolises. However, decision-makers require a very high budget to implement rail system projects. The evaluation process and making the right decision are of great importance in accomplishing these PT plans. Although there are various studies on the evaluation of rail systems in the literature, it is evident that the number of multi-criteria decision-making (MCDM) studies on RT is few and insufficient [6–15]. This study will examine methodologically diverse MCDM methods: Pythagorean AHP [16, 17] and VIKOR with fuzzy logic integration. Thanks to this study, comparisons of light rail investments made in Antalya, Turkey were examined according to MCDM methods. According to the analysis results, the most appropriate and efficient light rail investment in Antalya was determined.
The energy demands of societies rise in lockstep with their living standards and degrees of development. Furthermore, environmental issues, pollution levels, and waste indicate that some practices should be avoided. Transportation has become one of the most pressing issues due to rapid urbanization, dense population expansion, air pollution, and energy scarcity. Many issues have been resolved such as designing quiet and comfortable PT vehicles to meet user needs. They started to become more environmentally friendly and have higher energy efficiency. Those attributes promoted the RT systems, one of the most popular modes of PT [12, 18–20]. Rail transit has higher investment expenses but lower operational costs than road transport. Furthermore, compared to road transportation, the chances of accidents, energy consumption, traffic congestion, and personnel employment are lower. On the other hand, the rail system has a significantly higher transit capacity than road transport. All of these factors have hastened the growth of RT today.

Assessing PT is a very noteworthy subject because it includes advanced systems of transportation modes. The outcomes could have an impact on changing the use of PT sharply if the PT system has defects. Hence, there is a precedent for investigating PT systems and looking for the most convenient and reliable networks for PT users. Although several studies have been conducted to evaluate transportation infrastructure including all areas of transportation, in recent years [21–23], many studies have been conducted on MCDM methods to assess the transportation modes in PT systems [24]. They prepared a review paper for presenting studies using MCDM techniques in transportation systems between 1993 and 2015. For example, the study of Celik et al. [25] was presented because they introduced an approach, namely an integrated novel interval type-2 fuzzy TOPSIS and GRA method. In addition to those studies, Keyvan Ekbatani and Cats [26], Zak and Kruszyński [27], Nassereddine and Eskandari [28], Awasthi et al. [29], Hamurcu and Eren [30], Kumar et al. [31] and Seker and Aydin [32] evaluated PT using MCDM modeling frameworks. While Keyvan Ekbatani and Cats [26] included lexicographic, Simple Additive Weighting (SAW), a technique for order preference by similarity to the ideal solution (TOPSIS), and Concordance Analysis; Nassereddine and Eskandari [28] included the Delphi method, Group Analytic Hierarchy Process (GAHP), and preference ranking organization method for enrichment of evaluations (PROMETHEE). Ghorbanzadeh et al. [33] performed an Analytical Hierarchical Process (AHP) and Interval Analytic Hierarchy Process (IAHP) based methodology. They processed surveys completed in large cities to assess passenger satisfaction. They evaluated the PT systems including metro, taxi, regular bus (RB), bus rapid transit (BRT), rail rapid transit (RRT), and van. Thus, they made suggestions to improve PT service quality. Another study was performed by Kiciński and Solecka [34] to evaluate PT using several criteria. These were travel time, fleet use effectiveness, effects on the environment, compatibility of PT, safety, and security, making profits, and costs such as investment. According to their MCDM approach with AHP and Electre III, seven PT solutions compared to the current state were created. Pérez-Domínguez et al. [35] evaluated a large city PT network performance by Pythagorean Fuzzy combinative distance-based assessment (CODAS). Moreover, fuzzy sets are used in the evaluation of transportation systems. For instance, Wang et al. [36] included polygonal fuzzy sets to assess logistics transportation. Additionally, Öztürk [37] applied type-2 fuzzy set MCDM to find performance of PT services’ applications by assessing customer satisfaction surveys. They found that the performance criteria such as punctuality and providing traveler information are well enough to satisfy passenger happiness.

However, PT has many subgroups. Most of the time, only one transportation mode is considered for an assessment.

**Bus Rapid Transit (BRT):** For example, there are many recently conducted studies on assessing public bus networks using MCDM approaches. Nassereddine and Eskandari [28] and de Aquino et al. [38] evaluated PT systems, including city bus and BRT lines in a large city. A study was performed by Mavi et al. [39] in order to introduce a new method, namely Grey Step-wise Weight Assessment Ratio Analysis (SWARA-G). They proposed to add new buses to the lines to improve the bus service. Moreover, Erdoğan and Kaya [40], Güner [41], and Büyüközköy et al. [42] assessed public bus technologies alternating other PT modes in urban areas.

**Airway:** Locations of airports were evaluated by using MCDM approaches such as PROMETHEE, VIKOR, COPRAS, MAIRCRA, and MABAC methods [43]. Such studies looked for the solution of the most convenient areas for increasing the airports’ usability. New airline routes were created to increase the number of passengers for specific airlines [44]. Also, the services in airways were evaluated in a couple of studies. For instance, Chen [45] and Ghorabaee et al. [46] used DEMATEL, ANP, TOPSIS, WASPAS, COPRAS, and EDAS methods to assess the service quality of airline companies.

**Railway:** For instance, Aydin et al. [47] inspected the level of customer satisfaction at RT lines in Istanbul, a populated city. They proposed a framework combining statistical analysis, fuzzy analytic hierarchy process, trapezoidal fuzzy sets, and Choquet integral. Suggestions were made according to the results to improve the RT lines. Moreover, Azadeh et al. [48] proposed a framework to
evaluate an RT system, a kind of PT. Their model considered a couple of criteria such as health, safety, ergonomics, and resilience engineering. For assessing the decision-making units (DMUs), they used a data enveloped analysis (DEA) approach. Özgür [49] also assessed the performance of many railway projects and the lines in large cities, considering cost and ridership forecasts.

In addition, MCDM methods were also used for ranking railway networks or lines. For example, Mandic et al. [50], Hamurcu and Eren [30] and Görgün [51] used the AHP method for evaluating major railway networks in their countries. In addition, Kilic and Çerçioğlu [52] criticized applying Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and Vise Kriterijumska Optimizacija i Kompromisno Resenje (VIKOR) methods for prioritization. In the study of Yücel and Taşbat [53], AHP was used to evaluate PT problems, and alternative weights were defined by applying Best Worst Method (BWM). Among the scheduled PT projects, as a result, the most suitable ones for the city of Istanbul were determined. Li et al. [54] introduced a technique synthesizing fuzzy theory, statistical analysis, and TOPSIS approach for appraising a populated city’s RT lines. Correspondingly a new approach was proposed by Stoilova et al. [55] to evaluate a railway network performance using the Sequential Interactive Modelling for Urban Systems (SIMUS) integrated MCDM. Such studies suggested applicable alternatives or improvements for RT lines.

Additionally, MCDM approaches emphasized the importance of risk management in RT systems. For instance, an integrated MCDM approach was introduced to opt for a strategy of optimum train derailment risk response [56]. They claimed the best risk response strategy could be used as their simple and easy-to-use approach was applied.

In the literature, fuzzy set theory accounts for the vagueness and uncertainty of subjective evaluations [57]. Pythagorean fuzzy sets are widely used in the calculation of uncertainty. Pythagorean fuzzy sets give experts more freedom to express their views on the uncertainty and MCDM problem’s uncertainty under consideration [58, 59]. In another study, spherical fuzzy sets in the AHP-VIKOR methodology have been used to determine the selection of gas stations serving during the COVID-19 quarantine [60]. Another study with innovative generalized distance measurement formula is proposed to extend the classic VIKOR technique for MCDM problems within the Pythagorean fuzzy scenario [61]. When the studies in the literature are examined, it is a major drawback that the uncertainty and its effect are not taken into account sufficiently in the applications of the MCDM methods. At the same time, the lack of integrated work based on the Pythagorean fuzzy set in criteria weighting and alternative prioritization can be a deficiency.

This article is prearranged as follows. First, Sect. 2 presents the methodology with a concise description of the proposed approach. Then, while Sect. 3 delivers the application using our proposed technique and its results, Sect. 4 discusses the outcomes, and the paper is deduced.

### 2 Methodology

People, businesses, and organizations face multidimensional decision problems in all areas of life. Managers often make decisions in situations where more than one factor and conflicting goals (criteria) are desired for achievement. MCDM consists of approaches and methods that try to reach a possible “best or appropriate” solution that meets more than one conflicting criterion. As a result, decision-makers can make more scientific and successful decisions using MCDM techniques to overcome such problems.

In MCDM approaches, many candidates are graded by comparing the alternatives of plans, policies, strategies, and action styles, and the best among them is tried to be selected. MCDM methods use weight information about the criteria to solve complex problems with conflicting qualities [62]. First, alternatives and qualifications are defined. Afterward, measurements of each alternative are obtained separately according to each criterion. Then, their weights are assigned according to the criteria. Next, the overall values of the alternatives are determined by combining the assigned criteria weights and the single-criteria value measurements of the alternatives with an integration model. Finally, a sensitivity analysis is performed, and results suggestions and evaluations are presented.

Although the use of Pythagorean fuzzy sets is not new when the literature is reviewed, it is pretty challenging to find a study that focuses on both the calculation of Pythagorean fuzzy numbers and the weights of the criteria using AHP and their prioritization with FVIKOR. Moreover, unlike the mentioned literature, an integrated method including PFAHP and FVIKOR is proposed in this study. The main difference between the proposed approach is integrating both methods to enable more accurate decision-making. In addition, the proposed approach gives decision-makers more independence in their evaluation.

#### 2.1 Proposed Integrated Approach

The theoretical background of the methods used in the proposed integrated approach is described in this section. The PFAHP steps are provided in the first subsection. Then, in the second subsection, the Fuzzy VIKOR (FVIKOR) method is used to assess RT line alternatives. Finally, the results of the proposed approach of PFAHP and
FVIKOR methods are demonstrated. The proposed approach consists of both the PFAHP and FVIKOR methods. The following lists the key steps of the suggested approach as in Fig. 1.

2.2 Pythagorean Fuzzy Analytic Hierarchy Process (PFAHP)

This subsection includes the proposed fuzzy analytic hierarchy methodology. The stages of the PFAHP method are shown in the following steps:

- **Step 1** Compromised pairwise comparison matrix \( R = (r_{ji})_{mn} \) is structured according to experts’ evaluations. The linguistic scale used for decision matrices is presented in Table 1 adopted from the study of Ilbahar et al. [63].

- **Step 2** The difference matrices \( D = (d_{ij})_{mn} \) between the lower and upper values of the membership and non-membership functions are calculated using Eqs. (1) and (2):

  \[
  d_{ijL} = \mu_{ij}^2 - \nu_{ij}^2
  \]

  \[
  d_{ijU} = \mu_{ij}^2 - \nu_{ij}^2
  \]

- **Step 3** The interval multiplicative matrix \( S = (s_{ij})_{mn} \) is calculated via Eqs. (3) and (4):

  \[
  S_{ijL} = \sqrt{1000d_{ijL}}
  \]

  \[
  S_{ijU} = \sqrt{1000d_{ijU}}
  \]

- **Step 4** The determinacy value \( H = (h_{ij})_{mn} \) of the \( r_{ji} \) is calculated using Eq. (5):

  \[
  h_{ij} = 1 - \left( \mu_{ijU}^2 - \mu_{ijL}^2 \right) - \left( \nu_{ijU}^2 - \nu_{ijL}^2 \right)
  \]

- **Step 5** The indeterminacy degrees are multiplied with \( S = (s_{ij})_{mn} \) matrix to calculate the matrix before normalization weights \( T = (\tau_{ij})_{mn} \) via Eq. (6):

  \[
  \tau_{ij} = \left( \frac{S_{ijL}}{S_{ijU}} \right) h_{ij}
  \]

- **Step 6** The normalized priority weights \( w_i \) are obtained via Eq. (7):

  \[
  w_i = \frac{\sum_{j=1}^{m} w_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} W_{ij}}
  \]

2.3 FVIKOR (Fuzzy Vise Kriterijumska Optimizacija I Kompromisno Resenje)

In 2004, in addressing MCDM issues, Opricovic and Tzeng used the VIKOR process, first articulated as Opricovic [64]. The definition of VIKOR, which is the abbreviation of the word Vise Kriterijumska Optimizacija I Kompromisno Resenje, is an optimization of multiple parameters and a solution to compromise. Within the context of alternatives and the scope of assessment criteria, the basis of the approach is to establish a compromise solution. This approach to compromise is the nearest to the optimal solution. In this approach, constructing a multi-criteria rating index for alternatives makes it possible to take the closest decision to the optimal solution under some conditions. Compromise rating is accomplished by comparing proximity values to the optimal alternative.

2.4 Steps of Fuzzy VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje)

This subsection includes the proposed FVIKOR methodology. The stages of the FVIKOR method are shown in the following steps:

- **Step 1** For each assessment criterion, the best \( (f_{ij}^*) \) and worst \( (f_{ij}^-) \) values are calculated. If criterion \( i \) is a criterion in terms of evaluation in the sense of “utility” for \( I = 1, 2, \ldots, n; f_{ij}^* \) and \( f_{ij}^- \) can be expressed as follows.
Table 1 Scale for weighting of interval-valued PFAHP

| Linguistic term                  | Interval-valued Pythagorean fuzzy numbers |
|----------------------------------|-------------------------------------------|
|                                 | $\mu_L$ | $\mu_U$ | $\nu_L$ | $\nu_U$ |
| Certainly low important (CLI)   | 0.00    | 0.00    | 0.90    | 1.00    |
| Very low important (VLI)        | 0.10    | 0.20    | 0.80    | 0.90    |
| Low important (LI)              | 0.20    | 0.35    | 0.65    | 0.80    |
| Below average important (BAI)   | 0.35    | 0.45    | 0.55    | 0.65    |
| Average important (AI)          | 0.45    | 0.55    | 0.45    | 0.55    |
| Above average important (AAI)   | 0.55    | 0.65    | 0.35    | 0.45    |
| High important (HI)             | 0.65    | 0.80    | 0.20    | 0.35    |
| Very high important (VHI)       | 0.80    | 0.90    | 0.10    | 0.20    |
| Certainly high important (CHI)  | 0.90    | 1.00    | 0.00    | 0.00    |
| Exactly equal (EE)              | 0.197   | 0.197   | 0.197   | 0.197   |

$\begin{align*}
\text{Step 2} & \quad \text{For each assessment unit } S_j \text{ and } R_j \text{ values are determined. The criteria weights are expressed by } w_i. \\
S_j &= \sum_{i=1}^{n} w_i (f_i^+ f_i) / (f_i^+ f_i) \\
R_j &= \max_{i} \left[ w_i (f_i^+ f_i) / (f_i^+ f_i) \right] \\
\text{Step 3} & \quad \text{For each evaluation unit, } Q_j \text{ values are computed.} \\
Q_j &= v \left( S_j - S^* \right) + (1 - v) \left( R_j - R^* \right) \\
& \text{where } S^* = \min_{j} S_j, S^* = \max_{j} S_j, R^* = \min_{j} R_j, R^* = \max_{j} R_j, \quad v \in [0, 1] \text{ is the weight of the decision making strategy of the “majority of attributes” (or “maximum group utility”).} \\
\text{Step 4} & \quad \text{It lists the measured values of } S, R, \text{ and } Q. \text{ Then, the evaluation unit with the smallest } Q \text{ value is expressed as the best option in the alternative category. The last step is to propose a compromise solution that was the best ranked by the measure } Q. \\
\text{Step 5} & \quad \text{Two conditions must be met for the results obtained and considered true. In this way only, the alternative can be considered better or the most fitting with the minimum } Q \text{ value. The first condition relates to acceptable advantage, and the second is acceptable stability in decision making. The best alternative must be best ranked by } S \text{ or } R \text{ parameters too.} \\
\end{align*}$

3 Application of the Proposed Approach

Four main criteria evaluate the rail lines in our analysis, shown in Table 2. These are specifically economy, comfort, environment, and safety. Expert opinions were achieved for these main criteria, and the values depending on their opinions were recorded. Thus, the Pythagorean fuzzy set for four main criteria can be constructed as in Table 2.

Each criterion has sub-factors to understand a deep investigation of rail lines and their viable and exclusive comparison. For example, the leading criterion economy has four sub-factors: personnel cost, energy cost, passenger cost, and operation cost per km (Table 3). While personnel cost relies on the expenditure on the staff, such as their salaries, energy cost infers the expenses paid to the energy companies. Moreover, passenger cost signifies the charges taken from the commuters, and operation cost per km stands for the operation cost of the line.

The other main criterion, comfort, is divided into five subgroups (Table 4). These are specifically named service rate, the number of service delays, service delay times, stations’ access, and the rate of sedentary journeys per service. Service rate indicates the proportion of trams in service according to the schedule, which is critical for the journey planning comfort of passengers. On the other hand, delays may occur, which may also affect the passengers’ comfort. For this, the number of service delays and service delay times are considered the sub-factors of comfort for the analysis. Further, access to the stations is crucial for the commuters’ comfort to attain the tram services because stations’ or stops’ low accessibility may discourage the PT users from selecting the railway mode. Likewise, high accessibility to the stations may increase ridership. Therefore, access to the service is vital. However, taking the trip seated generally increases the passengers’ comfort and satisfaction. Thus, the rate of sedentary journeys per service is also a sub-factor in our analysis. According to the
experts’ opinions, Table 4 is established as the Pythagorean fuzzy set for the main criterion of comfort’s sub-factors.

The main criterion environment is essential to evaluate the PT rail lines because most lines pass through urban or residential areas where many people live. It comprises three sub-groups: greenhouse gas emission, noise level, and upkeep (Table 5). Greenhouse gas emission signifies the amount of emission or exhaust gases due to the rail line causing air pollution. At the same time, noise level indicates the amount of noise pollution mainly caused by the railway services, such as the friction noise between the rail cars and the railway. Since air pollution causes respiratory disorders or diseases, the amount of emission gas generated by the transit lines must be considered to evaluate the lines. Likewise, noise pollution must be included in the evaluation because it decreases the quality of life and may result in many illnesses. Moreover, upkeep is another sub-measure for the environment criterion because cleaning cars and stops encourages passengers to use PT.

According to the expert opinions, four main criteria are prioritized as in Fig. 2. The criterion economy has the highest priority with 47.2%, while safety is the second highest with 25.6%. Later, comfort and environment come as the third and the fourth, respectively. The weight of the economy criterion is the most because the budgets are very limited in the construction sector of PT. Decision-makers, most of the time, seek to gain the highest benefit with a minimum amount of budget and scarce resources. Thus, establishing the rail lines can also be included in this side of the recent projects. Expert opinions produce that safety has the second order for the tram lines. In many developing countries, this outcome can be understandable. One reason for this consequence is that many commuters somehow adopt using non-prioritized safety public transit lines. Comfort has a higher priority weight than environment since public transits can be more attractive to PT users if the journey proposes comfort. Ridership can be increased if delays are few or none and the services’ schedule is reliable. Increasing ridership at the end escalates the income of the tram line companies and service providers.

Table 2 Pythagorean fuzzy set for main criteria criticizing the rail lines and the values according to the expert opinions

| Main criterion | Economy | Comfort | Environment | Safety |
|----------------|---------|---------|-------------|--------|
| Economy        | 0.197   | 0.197   | 0.197       | 0.197  |
| Comfort        | 0.200   | 0.350   | 0.650       | 0.800  |
| Environment    | 0.200   | 0.350   | 0.650       | 0.800  |
| Safety         | 0.350   | 0.450   | 0.550       | 0.650  |

Table 3 Pythagorean fuzzy set for the main criterion of economy’s sub-factors

| Personnel cost | Energy cost | Passenger cost | Operation cost per km |
|----------------|-------------|----------------|-----------------------|
| Personnel cost| 0.197       | 0.197          | 0.197                 | 0.197 |
| Energy cost   | 0.450       | 0.550          | 0.450                 | 0.550 |
| Passenger cost| 0.350       | 0.450          | 0.550                 | 0.650 |
| Operation cost| 0.450       | 0.550          | 0.650                 | 0.800 |

Expert opinions lead to the assessment of priority weight calculation of the leading criterion economy. As a result, the highest priority weight corresponds to the energy cost sub-factor (Fig. 3). The rest of the sub-factors weights are similar; however, personnel cost weight is slightly more than the passenger cost weight and the operation cost per km weight.

Depending on the expert opinions, the sub-factors of the main criterion of comfort are ordered as follows: service rate, service delay times, the number of service delays, stations’ access, rate of sedentary journeys per service starting from the highest priority weight to the lowest priority weight (Fig. 4). The reliability of the service rate makes a significant contribution to the quality of the line. Therefore, the highest weights are seen for the service rate, service delay times, and the number of service delays. The total proportion of them is almost 80%. This result explains that reliability and punctuality play a significant role in rail services. Stations’ access and the rate of sedentary journeys per service are the other necessary measures for customer satisfaction. However, according to the experts, timely
Table 4 Pythagorean fuzzy set for the main criterion of comfort’s sub-factors

|                       | Service rate | The number of service delays | Service delay times | Stations’ access | Rate of sedentary journeys per service |
|-----------------------|--------------|------------------------------|---------------------|------------------|---------------------------------------|
| Service rate          | 0.197        | 0.197                        | 0.197               | 0.550            | 0.650                                 |
| The number of service delays | 0.200        | 0.350                        | 0.650               | 0.800            | 0.197                                 |
| Service delay times   | 0.200        | 0.350                        | 0.650               | 0.800            | 0.200                                 |
| Stations’ access      | 0.350        | 0.450                        | 0.550               | 0.650            | 0.350                                 |
| Rate of sedentary journeys per service | 0.200        | 0.350                        | 0.650               | 0.800            | 0.200                                 |
services make passengers more comfortable than the station’s access level and seated while commuting.

According to the expert opinions, the main criterion environment’s sub-factors are also ordered in priority weights (Fig. 5). The sub-factor greenhouse gas emission achieves the highest priority weight of 62.5%. More than half of the environment weights are occupied by the priority weight of air pollution, demonstrating the importance of that sub-factor compared to the other sub-factors such as noise level and upkeep. The weights of them are almost similar depending on the expert opinions. However, noise level weight has slightly more than upkeep weight.

In our analysis, the expert opinions are first recorded to evaluate the main criteria and their sub-factors, which is necessary for PFAHP. Since expert opinions are used to gain sub-factors priority weights, a vital transportation information background is prepared for the following analysis phase, which FVIKOR does. In the next phase by FVIKOR, numerical data achieved from actual cases observed on the tram lines are taken into account. Therefore, PFAHP results strongly support the outcome of the FVIKOR analysis. The priority orders of hazards

| Greenhouse gas emission | Noise level | Upkeep |
|-------------------------|-------------|--------|
| 0.197 | 0.197 | 0.197 |
| 0.200 | 0.350 | 0.650 |
| 0.200 | 0.350 | 0.650 |

Table 5 Pythagorean Fuzzy Set for the main criterion of environment’s sub-factors

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Fig. 2 The normalized priority weights of main criteria by PFAHP

Fig. 3 Priority weights of the main criterion economy’s sub-factors

Fig. 4 Priority weights of the main criterion comfort’s sub-factors

Fig. 5 Priority weights of the main criterion environment’s sub-factors

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concerning these parameters are then determined by using
FVIKOR. The evaluation scale of experts is given in
Table 6 [65]. Experts expressed their fuzzy rates through it.

According to the actual case data of the economy’s sub-
factors, a gradation is made among the alternative rail
transit lines (Table 7), such as Line-1, Line-2, and Line-3.
After processing the economy data using the VIKOR
analysis, the consequences demonstrate that Line-3 is the
best option considering the personnel cost, energy cost,
passenger cost, and operation cost per km. On the other
hand, the worst alternative is Line-2 in light of personnel
cost, energy cost, and operation cost per km.

The results of the FVIKOR analysis for the comfort
criterion are presented in Table 8. All rail lines are similar
according to the service rate. However, Line-3 is the best
option regarding punctuality issues because it shows the
best performance regarding the number of service delays
and service delay times. Moreover, Line-3 is again the best
alternative in terms of its stations’ access, but it shares the
leading position with Line-2 in the category of providing
sedentary journeys.

Table 9 displays the results of the FVIKOR decision
matrix for the environment criterion. In this case, the best
alternative is Line-3 under greenhouse gas emission and
noise level criteria. Nevertheless, Line-2 is the best in
achieving cleaning services, in other words, upkeep. One
reason for this outcome may be that it serves the airport
area, where many tourists and locals use the line.

Regarding safety, FVIKOR analysis offers the outcome
that Line-3 is the best option, where the number of acci-
dents or crashes is less than the other alternatives
(Table 10).

To sum up, according to the FVIKOR parameters dis-
cussed, Table 11 and Fig. 6 illustrate the gradation of
alternatives. The crisp value of the alternatives for $Q_i$
is ranked from the smallest to the highest value. The best
option is Line-3, and the second-best alternative is Line-1
based on the FVIKOR analysis. Thus, the third option is
Line-2.

4 Conclusion

This study proposed a two-stage fuzzy multi-criteria
framework that provides more consistency in the decision-
making process and gives an appropriate final rank of
alternatives. Additionally, a unique integration of system-
atically various MCDM methods was assessed, which
synthesized Pythagorean fuzzy AHP and FVIKOR. The

Table 6 Linguistic terms and equivalent Pythagorean fuzzy numbers
for assessing line alternatives

| Linguistic expression | Corresponding Pythagorean fuzzy number $(u, v)$ |
|-----------------------|----------------------------------------------|
| Very low              | $0.15$ $0.85$                               |
| Low                   | $0.25$ $0.75$                               |
| Moderately low        | $0.35$ $0.65$                               |
| Medium                | $0.50$ $0.45$                               |
| Moderately high       | $0.65$ $0.35$                               |
| High                  | $0.75$ $0.25$                               |
| Very high             | $0.85$ $0.15$                               |

Table 7 Aggregated Pythagorean fuzzy decision in matrix form for economy criterion

| Alternative | Personnel cost $(u, v)$ | Energy cost $(u, v)$ | Passenger cost $(u, v)$ | Operation cost per km $(u, v)$ |
|-------------|-------------------------|---------------------|------------------------|-------------------------------|
| Line-1      | $0.40$                  | $0.58$              | $0.50$                 | $0.48$ $0.72$                |
| Line-2      | $0.55$                  | $0.42$              | $0.63$                 | $0.35$ $0.40$                |
| Line-3      | $0.25$                  | $0.75$              | $0.22$                 | $0.78$ $0.18$                |

Table 8 Aggregated Pythagorean fuzzy decision in matrix form for comfort criterion

| Alt   | Service rate $(u, v)$ | The number of service delays $(u, v)$ | Service delay times $(u, v)$ | Stations’ access $(u, v)$ | Rate of sedentary journeys per service $(u, v)$ |
|-------|-----------------------|--------------------------------------|-----------------------------|--------------------------|-------------------------------------------------|
| Ln-1  | $0.50$                | $0.48$ $0.40$                        | $0.58$ $0.39$               | $0.59$ $0.46$            | $0.52$ $0.30$                                   |
| Ln-2  | $0.50$                | $0.48$ $0.54$                        | $0.45$ $0.53$               | $0.45$ $0.44$            | $0.54$ $0.46$                                   |
| Ln-3  | $0.50$                | $0.48$ $0.15$                        | $0.85$ $0.18$               | $0.83$ $0.49$            | $0.51$ $0.46$                                   |
The proposed method was applied through a case study in the RT system of Antalya/Turkey. The new methodology used and analyzed all three rail lines in service. First, Pythagorean fuzzy AHP was used in four main criteria: economy, comfort, environment, and safety. Then, the prioritization of rail line alternatives was determined by using the FVIKOR method. Results of the case study showed that the most crucial and serviceable line is Line-3 according to the transport-related parameters applied in the proposed method in this study. Furthermore, analysis results revealed that Line-3 is the most appropriate rail investment in the city compared to the other rail lines, Line-1 and -2.

The contribution of this study is the following: (1) It is the first MCDM study in the area of transportation that used a two-stage Fuzzy set. (2) Performing applications especially with datasets from real-life rail transit trips and experiences contribute to the literature. (3) Since the dataset is from the observations, this study minimizes uncertainty. This uncertainty minimization leads to a decrease in future transportation investment costs, which are recently very high. (4) This article can serve as an example for future studies because the proposed approach can develop a new insight into the integration of MCDM methods with a fuzzy set.

It is important to remember that other multi-criteria decision methods such as fuzzy PROMETHEE and fuzzy TOPSIS and their variations may be used for more efficient approaches in the second stage of the methodology presented in this study. The disadvantage of Pythagorean fuzzy aggregation operators is that they produce undetermined results in some special situations where the membership value or non-membership value receives a weight vector value of 0 or 1. In future work, various versions of fuzzy sets theory could be considered for resolving this problem. Furthermore, a couple of work developments might be done, such as including more experts among the survey group and weighting more criteria.

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Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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