How Emergent is Gravity?

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Abstract

General theory of relativity (or Lovelock extensions) is a dynamical theory; given an initial configuration on a space-like hypersurface, it makes a definite prediction of the final configuration. Recent developments suggest that gravity may be described in terms of macroscopic parameters. It finds a concrete manifestation in the fluid-gravity correspondence. Most of the efforts till date has been to relate equilibrium configurations in gravity with fluid variables. In order for the emergent paradigm to be truly successful, it has to provide a statistical mechanical derivation of how a given initial static configuration evolves into another. In this essay, we show that the energy transport equation governed by the fluctuations of the horizon-fluid is similar to Raychaudhuri equation and, hence gravity is truly emergent.

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Emergent phenomena occur when simple interactions working cooperatively create more complex interaction [1]. Physically, simple interactions occur at smaller length scales (microscopic level), and collective behaviour manifests at much larger length scales. For example, the Coloumb force \(1/r^2\) experienced by two charges separated by a distance \(r\) is understood to be a fundamental force while the interaction force \(1/r^4\) between two moving bubbles in a superfluid is understood as an emergent law. In the same spirit, one is tempted to ask whether the General theory of relativity is the low-energy limit of a strongly correlated system and gravitons their collective excitations [2].

Due to the long range and attractive nature of gravity, gravitational systems are far-from-equilibrium. Over the last three decades, it has been noticed that the condensed matter systems that are far-from-equilibrium exhibit a convenient separation of length and time scales [3]. One of the key physical ingredient is that the dynamics of a system with many degrees of freedom can be described by the interaction of only a few (such as those at long length and time scales). This so-called hydrodynamic approach provides a successful basis for describing systems far from equilibrium [3, 4].

The thermodynamics of black-holes [5], seem to suggest the hydrodynamic approach. Due to Hawking radiation, the black-holes are out-of-equilibrium systems [6, 7] and the black-hole entropy is dominated by the degrees of freedom close to horizon [8]. Over the last few years, a growing body of evidence suggests that gravitational dynamics near the black-hole horizon, is analogous to the dynamics of fluids [9, 10, 11]. (See also Refs. [2, 12, 13, 14, 15].)

Recently, the present authors have taken an alternative route and shown that the fluid-gravity correspondence is more physical and can be used to derive physical quantities from the horizon-fluid [20]. Identifying the long wavelength limit as that used in the Mean Field Theory description of Phase Transitions, we showed that the entropy of the ordered phase is same as the Bekenstein-Hawking entropy. The flow chart below provides a birds eye view of the fluid-gravity programme and the right side describes the programme undertaken by the authors.

Mean Field Theory may yield correct critical exponents for certain phase transitions, however, it ignores fluctuations that drive system from one physical state to another. In the same spirit, while we were able to recover Bekenstein-Hawking entropy by modelling the horizon-fluid using Mean Field Theory, our previous analysis is incomplete, or for that matter, any emergent gravity approach [10]. More specifically, the question that needs to be addressed in any emergent gravity approach is whether the fluctuations of the horizon-
fluid provide necessary information about how a given horizon-fluid configuration evolves into another. In this essay, we show that the transport equation for the order parameter of the horizon-fluid system is identical to the Raychaudhuri equation.

We start by assuming that the Horizon-fluid forms a condensate at a critical temperature. The justification comes from two lines of arguments. First, the evidence provided by Carlip\([18]\) that the black-hole horizon has some properties that exhibit universality. This indicates that the physics near the horizon is that of a system near a critical point. Second, recently, Skakala and Shankaranarayanan\([19]\) modelled the fluid as a Bose gas with \(N\) particles and found that all the particles stayed in the ground state for large horizon radius. This suggests that horizon-fluid forms a BEC at some critical temperature \(T_c\). The conditions underlying this are \([20]\):

1. There is a temperature \(T_c\) (critical temperature), at which, all the \(N\) fluid degrees of freedom on the horizon form a condensate.

2. The system remains close to the critical point.

One can describe this critical system, a homogeneous fluid, using Mean Field Theory. The order parameter is

\[
\eta = \sqrt{kN},
\]

and the thermodynamic Potential \(\Phi\) is

\[
\Phi = \Phi_0 + a(P)(T - T_c)\eta^2 + B(P)\eta^4.
\]

where \(k\) is a constant, coefficients \(a\) and \(B\) are determined by the relation \(P = -TA\). One can determine the entropy of the fluid in \(\eta \neq 0\) phase and show that it corresponds to the
Bekenstein-Hawking entropy, $S = \frac{A}{4}$ \cite{20}. The horizon-fluid is near a critical point and when the system goes over to the ordered phase, its entropy is the same as the black-hole entropy. We showed that the formalism can be extended to include black-holes in AdS background\cite{20}. The negative cosmological constant can be treated as an external field analogous to an external magnetic field. This causes first order phase transition and the existence of a tri-critical point.

What has been described so far, concerns only static variables in the gravity theory (in this case, the parameters which fully describe a black-hole), correspond to the equation of state of the horizon-fluid. Our aim now is to investigate, within the emergent paradigm, how a given static configuration with a given equation of state is driven to another static configuration. We do this in two steps. First, we consider fluctuations from the equilibrium that are adiabatic; fluctuations whose wavelengths are of the same order as the horizon-fluid and occur over a longer time scale. Second, we consider fluctuations that are non-adiabatic; wavelength of these fluctuations are usually smaller than the fluid scale.

**Adiabatic Process:** Fluctuations take the fluid from one equilibrium configuration to another. Since these fluctuations are adiabatic, the new equilibrium configuration can also be described by Mean Field Theory. The equilibrium position is one of the two minima of the double well potential on which the system has settled, i.e. $\eta_{min}^2 = \frac{a(T_c - T)}{2B} \delta N_s$. Let, $\delta \eta_s(\delta N_s)$ denote the change in the value of the order parameter (number) due to fluctuations, then the change in the potential is

$$\delta \Phi_s = \frac{1}{4} \alpha (T - T_c) \frac{\delta N_s^2}{N_0}. \tag{3}$$

The change in entropy during this process can be determined in two ways, $\delta S = -\frac{\partial \delta \Phi_s}{\partial T}$ and $\delta S = \frac{\delta S}{\delta N_s}$. Comparing them, we get, $\delta E = T \delta S$. This is the statement of the First Law of Thermodynamics for the fluid system. It is of the same form as the mathematical statement of the process 1st law for event horizons,

$$\delta M = \frac{1}{4} T \delta A = T \delta S, \tag{4}$$

which relates the increase in the mass of a black-hole due to matter-energy falling through the horizon to the increase in the horizon area \cite{16, 14, 21}. It shows us that the physical process 1st law can be thought of as the adiabatic restoration of equilibrium after a fluctuation in the fluid system.

**Non-Adiabatic Process:** In this general case, the fluid system is away from equilibrium and some amount of energy is being transferred to it from the external source. Due to the
fluctuations, energy is gained by the horizon-fluid, correspondingly, the number of fluid d.o.f and order parameter $\eta$ change. Applying Onsager’s hypothesis$^{22}$, and assuming $k$ does not change in Eq. (1), we can describe the change in the order parameter $\eta$ by the Langevin equation. We note that the Horizon fluid has negative Bulk viscosity. Since the fluid is taken to be homogeneous, only the bulk viscosity needs be considered.

The change in the order parameter $\eta$ is given by the Brownian motion (See, for instance, Ref. $^{23}$):

$$\ddot{\eta} = -\beta \dot{\eta} + F(t),$$  

(5)

where, $F(t)$ is the random term and $\beta \dot{\eta}$ is the damping term due to the bulk viscosity of the fluid. Using (1), we get,

$$\frac{dx}{dt} = -\beta x - \frac{1}{2}x^2 + 2\frac{F(t)}{\eta},$$  

(6)

where, $x = \frac{\dot{\eta}}{N}$. $\eta \neq 0$ here, as it fluctuates around $\eta = \eta_{\text{min}} \neq 0$. Taking the ensemble average on both sides of Eq. (6) and using $\langle \frac{F(t)}{\eta} \rangle = 0$, we get,

$$\frac{d\langle x \rangle}{dt} = -\beta \langle x \rangle - \frac{1}{2}\langle x \rangle^2 - \frac{1}{2}\langle \Delta x \rangle^2,$$  

(7)

where, $\langle \Delta x \rangle^2 (= \langle x^2 \rangle - \langle x \rangle^2)$ is the mean squared fluctuation in $x$.

To determine $\langle x^2 \rangle$, we note, $K.E. = \frac{1}{2}\langle \dot{\eta}^2 \rangle$ and $P.E. = \frac{\alpha}{k}(T - T_c)\delta \eta^2$, where, $X$ denotes the time average of a quantity. Using Virial theorem$^{23}$, we have, $K.E. = P.E.$.

The fluctuations in $\eta$ are related to the dissipated energy density of the fluid $\rho_d$ implying that $\langle \dot{\eta}^2 \rangle$ and $\langle x^2 \rangle$ are related to $\rho_d$. Thus, we get,

$$\frac{d\langle x \rangle}{dt} = -\beta \langle x \rangle - \frac{1}{2}\langle x \rangle^2 + 8k\alpha \rho_d.$$  

(8)

The damping coefficient $\beta$ has dimensions of length-inverse. For the horizon-fluid, this corresponds to the horizon radius, which is inversely proportional to $T$. This leads to:

$$\frac{d\langle x \rangle}{dt} = C T \langle x \rangle - \frac{1}{2}\langle x \rangle^2 + 8k\alpha \rho_d$$  

(9)

where, $C$ is a constant. This is the key result of this essay regarding which we would like to stress the following points:

1. The above equation is similar to the Raychaudhuri equation sans the shear term,

$$\frac{d\theta}{dt} = 2\pi T \theta - \frac{1}{2}\theta^2 - 8\pi T_{\alpha\beta} \xi^\alpha \xi^\beta.$$  

(10)
To relate (9) to the gravity side, we shift to the geometric variable, $A$, which is the cross-sectional area of the null congruence in addition to being the area of the $2 + 1$ dimensional fluid. Then $x = \frac{\dot{A}}{A} = \theta$. Here $t$ denotes the affine parameter along the null congruence as used in [12]. Comparing (9) with (10), one can write, $C = -2\pi$ and $ho_d = \frac{1}{8k\alpha}(8\pi T_{\alpha\beta}\xi^\alpha\xi^\beta + \langle \dot{\eta} \rangle^2)$. It is to be noted that except for the values of the coefficients, the signatures are the same. The Bulk viscosity is negative for the fluid on the horizon. Hence, what is normally the damping term in a standard Brownian motion, here reinforces the transport of energy. It is reasonable to assume that the coefficient of such a term is proportional to the Bulk Viscosity, $\eta_B$ (Stokes’ law is one example of this), i.e. $\beta = -CT$.

2. The dissipated energy density is of the form, $T_{\alpha\beta}\xi^\alpha\xi^\beta + \frac{1}{2}\langle \dot{\eta} \rangle^2$. So there is an extra term apart from the amount of the matter-energy that falls across the horizon. In the gravity picture, the dissipated energy then consists of two parts, one matter-part and one geometry-part. Thus, our analysis provides an alternative view of the Raychaudhuri equation i.e. governing the transport of energy on the fluid side.

General Theory of Relativity is a dynamical theory and makes a clear and precise prediction about how an initial configuration would evolve to a final configuration. Raychaudhuri equation is a crucial ingredient in describing the dynamics of gravity. It governs the dynamics of null geodesic congruences. The emergent gravity paradigm has been with us for close to fifty years. Most of the efforts within the emergent gravity approach has been directed towards relating the equilibrium configurations in gravity with the fluid variables. In this essay, we have shown that the differential equation governing the transport of energy into the fluid is similar to the Raychaudhuri equation. This clearly demonstrates that the Horizon-Fluid description provides dynamical information about gravity and gravity is truly emergent. One might compare this with the long wavelength or the hydrodynamic limit of AdS-CFT, where the dynamics of the fluctuations on the fluid side can be mapped to the macroscopic variables on the gravity side[24, 9]. Such a programme, however, has not been carried out for the gravity theory for a more general class of space-time. The results reported here constitute a first step towards that direction.

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