Microscopic origin of the effective spin-spin interaction in a semiconductor quantum dot ensemble

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We present a microscopic model for a singly charged quantum dot (QD) ensemble to reveal the origin of the long-range effective interaction between the electron spins in the QDs. Wilson’s numerical renormalization group (NRG) is used to calculate the magnitude and the spatial dependency of the effective spin-spin interaction mediated by the growth induced wetting layer. Surprisingly, we found an antiferromagnetic Heisenberg coupling for very short inter-QD distances that is caused by the significant particle-hole asymmetry of the wetting layer band at very low filling. Using the NRG results obtained from realistic parameters as input for a semiclassical simulation for a large QD ensemble, we demonstrate that the experimentally reported phase shifts in the coherent spin dynamics between single and two color laser pumping can be reproduced by our model, solving a longstanding mystery of the microscopic origin of the inter QD electron spin-spin interaction.

Spins confined in semiconductor QDs have been discussed as candidates for implementation of quantum bits (qubits) in quantum information technologies [1, 2] since it allows integration into conventional semiconductor logic elements. For information processing, qubit initialization and readout [3, 4] are as important as manipulations of the spins. Optical control experiments in QD ensembles [5, 6] as well as the measurements of the dephasing time as function of the laser spectral width [7] in such samples provided strong evidence for a long-range electron spin-spin interactions between the different QDs in the ensemble of unknown microscopic origin. Initially, it was speculated [8] that it might be caused by an optically induced Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction [9, 10]. Since the laser pulse duration is of the order of ps such an optically induced interaction, however, would decay rather rapidly and is not compatible with the observed spin coherence on a scale of several ns.

In order to make use of the intrinsic long range spin-spin interaction between the localized spins in QDs by spin manipulation protocols, its origin needs to be understood. In this letter, we propose a microscopic mechanism based on the analysis of a multi-impurity Anderson model [12, 13]. We start from a localized electron bound state in each QD that weakly hybridize with the conduction band (CB) of the thin wetting layer (WL) [13–17] that is left below the QDs in the Stranski-Krastanow growth protocol. The basic setup is sketched in Fig. 1. By including all virtual charge fluctuations in the leading order, significant corrections to the conventional textbook expression starting from an effective local moment picture have been reported [12, 18] for short distances. We believe that implementing backgates below the WL would allow to manipulate the properties of the low concentration WL electron gas and hence to control the effective spin-spin interaction.

Recently a growth procedure for InGaAs QD ensembles was proposed [19] to eliminate the WL. In such samples, the effective spin-spin interaction between the QD electron spins should be absent. While those types of QD ensembles are produced to decrease the energy loss when used as photon emitters, a detailed investigation of their spin properties in electron doped samples is still missing to the best of our knowledge.

Model.— The WL is treated as a free two-dimensional CB

\[ H_{\text{WL}} = \sum_{\vec{k}} \frac{\hbar^2 \vec{k}^2}{2m^*} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} \]  

in the isotropic effective mass approximation with \(m^*/m_0 = 0.023 [20]\). The minimal model for the \(N_{\text{QD}}\) QDs in the ensemble is given by

\[ H_{\text{QD}} = \sum_{i} \sum_{\sigma} \varepsilon_i^d d_{i\sigma}^\dagger d_{i\sigma} + U_i n_{i\uparrow} n_{i\downarrow} \]

where \(\varepsilon_i^d\) denotes the single particle energy of the bound electron state with spin \(\sigma\) in the \(i\)-th QD and \(U_i\) the corresponding Coulomb repulsion preventing the bound state to be doubly occupied. The creation (annihilation) operator \(d_{i\sigma}^\dagger(d_{i\sigma})\) adds (removes) an electron to (from) the \(i\)-th QD. The electronic wave function is localized but covers the whole diameter of the QD. The parameters \(U_i\) and \(\varepsilon_i^d\) depend on the individual shape of the QD.

![FIG. 1. Sketch of two quantum dots that are linked by the InAs wetting layer.](image-url)
Using the experimental estimates \([21]\), we approximate \(U_i \approx 4\text{ meV}\) for a QD with a diameter \(D = 25\text{ nm}\).

The concentration of the donors in the QD ensemble used in Ref. \([3]\) was selected to match the QD density in the sample. Each QD can capture at least one electron if the bound state \(\varepsilon_i^d\) is below the CB of the WL. It remains empty when the chemical potential \(\mu \leq \varepsilon_i^d\). Spin-spectroscopy experiments \([22]\), however, indicate that not all QDs are filled with one electron. This might be due to local imperfections that shift individual \(\varepsilon_i^d\) to higher energies. Also doubly occupied QD states are possible when \(\varepsilon_i^d < \mu - U_i\). Excluding this doubly occupied QD ground state configuration puts a lower bound on \(\varepsilon_i^d\). We assume that a fraction \(q_{\text{WL}}\) of the total donor excess electrons is filling up the WL such that the chemical potential \(\mu > \varepsilon_F\) and \(q_{\text{WL}} = 1\) yielding \(\varepsilon_F \approx 1\text{ meV}\) for a dot density of \(10^{10}\text{cm}^{-2}\) \([5]\).

The bound state of the \(i\)-th QD can tunnel with a finite tunneling amplitude \(V_i^m\) into the Wannier orbital \(m\) of the WL. The resulting hybridization term between the bound state of each QD and the WL takes the form

\[
H_{\text{hyb}} = \sum_{i=1}^{N_{\text{QD}}} \sum_{\sigma} \sum_{m} \left( V_i^m d_i^\sigma \epsilon_{m\sigma} + h.c. \right),
\]

where \(\epsilon_{m\sigma}\) is the annihilation operator of the Wannier orbital at site \(\vec{R}_m\) of the 2D WL and whose spatial Fourier transform is \(c_{\vec{k}\sigma}\). The total Hamiltonian of the coupled QD problem is given by \(H = H_{\text{WL}} + H_{\text{QD}} + H_{\text{hyb}}\).

The effect of the WL onto the dynamics of the localized QD states is determined by the hybridization function matrix \([12, 13]\)

\[
\Delta_{ij}(z) = \sum_{lm} \frac{1}{\pi} \sum_{\vec{k}} \frac{[V_i^0]^* V_j^0 e^{i\vec{k}(\vec{R}_m - \vec{R}_l)}}{z - \varepsilon_{\vec{k}}}.
\]

In the wide band limit, i.e., \(V_0/D \ll 1\) where \(D\) is the band width of the WL CB, \(t_{ij} = \Re \Delta_{ij}(\mu)\) generates an effective hopping between the QDs \(i\) and \(j\). The average distance between the QDs is of the same order of magnitude as the Fermi wave length \(\lambda_F\). The distance variations between the WL sites, however, are small compared to \(\lambda_F\). Consequently, we can replace \(\vec{R}_m - \vec{R}_l\) by the distance between the two centers of the QDs, i.e., \(\vec{R}_i - \vec{R}_j\), and include the spatial extension of the QD \([23]\) by defining \(V_i = \sum_m V_i^m\). We introduce the average hybridization matrix element \(V_0^2 = \langle V_i^0 \rangle\) and define a reference energy scale \(\Gamma_0 = \pi V_0^2 \rho_0\), \(\rho_0\) being the constant density of state of the 2D WL CB. The charge fluctuation scale \(\Gamma_0\) determines the order of magnitude of \(\Delta_{ij}(z)\). The antiferromagnetic (AF) part of the RKKY interaction can be estimated \([12, 13]\) as \(J_{\text{AF}}^{\text{RKKY}} \approx 4t_0^2/U \times 4\Delta_{ij}^2/\Gamma_0\) serving as a first estimate for \(\Gamma_0 \approx 10 - 100\text{ meV}\).

In a conventional model, the AF part of the RKKY interaction is compensated by the FM contribution for \(|\vec{R}| \rightarrow 0\) \([18, 24]\). The relevant parameter regime of our model, however, does not fit the conventional regime discussed in the literature where \(D, U > J_K\), \(J_K\) being the averaged local Heisenberg coupling \(J_K\) between the QD electron spin and the local CB spin density \([25]\). The energy \(\varepsilon_{ij}^d\) is located below the lower band edge of the CB. Many Wannier orbitals contribute to \(\vec{V}_i\), and virtual charge fluctuations in the localized orbitals below the CB continuum are allowed. We expect deviations between the correct effective spin-spin interaction within our model and the prediction from the conventional two stage mapping of the QD model: first onto a Kondo ensemble model by applying a Schrieffer-Wolff transformation (SWT) \([20]\) that is \(O(V_0^2)\) followed by a perturbative estimation of the \(J_{\text{RKKY}}^{\text{AF}}\) in \(O(V_0^4)\). Significant corrections to \(J_{\text{RKKY}}^{\text{AF}}\) have been already reported \([12, 18]\) at short distances since additional terms in \(O(V_0^4)\) neglected in the two stage mapping become important.

**Numerical renormalization group approach.**— To circumvent the inconsistency problem of the two stage perturbative approach, we apply the numerical renormalization group (NRG) \([27, 28]\) to the Hamiltonian with \(N_{\text{QD}} = 2\) to determine the distance dependent effective Heisenberg interaction between the localized electron spins. For each distance \(R = |\vec{R}_i - \vec{R}_j|\), the two QD problem is mapped onto an effective two band model \([12, 24, 26]\), where we used \(\Gamma_0, \mu = \varepsilon_F\) and \(\varepsilon_i^d = \varepsilon_{ij}^d\) as the adjustable parameters such that each QD remains singly charged. Note that the NRG mapping of the strong asymmetric bands onto a Wilson chain \([27, 28]\) leads to modification of the hopping parameters \([30]\) and of the NRG fixed point (FP) spectrum \([31]\). This adjustment only depends on the support of the CB density of states and, therefore, is independent of the QD distance \(R\).

Using the NRG, we calculated the temperature dependent spin-spin correlation function \(\langle \hat{S}_i \hat{S}_j(T) \rangle\) as shown for two distances in Fig. (2a). The sign of \(\langle \hat{S}_i \hat{S}_j(T) \rangle\) determines the sign of \(J_{\text{RKKY}}^{\text{AF}}(R)\), and \(|J_{\text{RKKY}}^{\text{AF}}(R)|\) is obtained from the fit to the universal functions of the spin-spin correlation function of a \(s = 1/2\) toy model, \(S_{12}(T) = \mu_0 e^{-\beta J} - 1)/(1 + 3e^{-\beta J})\), where the parameter \(\mu_0^2\) includes possible Kondo screening effects of the localized spins \([32]\). We added the fit functions as solid lines to Fig. (2a). The extracted \(J_{\text{RKKY}}^{\text{AF}}(R) = J\) are shown in Fig. (2b) for different fixed set of parameters.

For two localized moments coupled by a Heisenberg interaction \(J_K\) to the local spin-density of 2D CB with a quadratic dispersion in 2D, Klein and Fischer \([25]\) derived the analytic expression

\[
J_{ij}^{\text{RKKY}} = -\rho_0 J_K^2 \frac{\delta k_F^2}{4\pi} \times [J_0(k_F R)N_0(k_F R) + J_1(k_F R)N_1(k_F R)]
\]

for the effective RKKY interaction, where \(J_i(x) = N_i(x)\) is the Bessel (Neumann) function of order \(l\) and \(\delta\) the area
of the 2D unit cell. We added a fit to the expression Eq. \[5\] to our NRG data as solid magenta line in Fig. 2(b). The NRG results follow excellently the analytic predictions for larger distances. The magnitude of \(J_{\text{RKKY}}(R)\) is proportional to \(\Gamma_0^{\text{F}}\) as expected, but the absolute value differs for those predicted by Eq. \[5\]. \(J_{\text{RKKY}}(R)\) remains invariant under rescaling \(\alpha_{\text{F}}, \alpha_{\text{U}}, \alpha_{\text{e}}^{\text{d}}\) for \(R_{\text{K}}/\pi > 0.5\). For short distances, \(R_{\text{K}}/\pi < 0.25\), however, we observe significant deviations: The NRG reveals an AF RKKY interaction in contrary to the conventional RKKY result. The origin of this surprising effect can be linked to the large effective single-particle hopping between the QD orbitals induced by the very strong particle-hole asymmetry of the CB for small WL fillings. The conventional two stages perturbation theory is inconsistent and requires additional corrections in \(O(V_0)\) \[12, 18\].

\[\text{Spin polarization in laser pulsed QD ensembles. — After establishing the distance dependent effective interaction between the electron spins in the different QDs, we investigate its influence on an ensemble of singly charged QD in an external magnetic field subject to a two color laser pumping with circularly polarized light}\ [5]. We used a semiclassical simulation \[33, 34\] for the spin dynamics of a QD ensemble model \[7\]

\[H_{\text{array}} = \sum_{i}^{N_{\text{QD}}} H_{1}^{(i)} + \sum_{i<j}^{N_{\text{QD}}} J_{ij} \bar{S}_{1}^{(i)} \bar{S}_{1}^{(j)}, \quad (6)\]

where the central spin model (CSM) \(H_{1}^{(i)}\),

\[H_{1}^{(i)} = g_{\text{e}}^{(i)} \mu_{\text{B}} \bar{B}_{\text{ext}} \bar{S}_{1}^{(i)} + \sum_{k=1}^{N_{i}} g_{\text{e}}^{(i)} \mu_{\text{N}} B_{\text{ext}} \mathbf{I}_{k}^{(i)} + \sum_{k=1}^{N_{i}} A_{k}^{(i)} \mathbf{I}_{k}^{(i)} \bar{S}_{1}^{(i)}, \quad (7)\]

includes the electron and nuclear Zeeman term in the external magnetic field \(\bar{B}_{\text{ext}}\) as well as the hyperfine coupling between the \(N_{i}\) nuclei spins denoted by \(\mathbf{I}_{k}^{(i)}\) and the electron spin \(\bar{S}_{1}^{(i)}\) \[32, 33\]. We generated random 2D ensembles of QD QDs with a dot density of \(10^{10}\) \(\text{cm}^{-2}\) \[5\] and assigned \(J_{ij} = x_{i} x_{j} J_{\text{RKKY}}(R_{\text{K}}, \epsilon_{\text{F}}, \Gamma_{0})\) from the NRG data depicted in Fig. 2(b) where \(x_{i}\) accounts for the variations of \(\bar{V}_{i}\). We attributed \(N_{QD}/2\) randomly selected QDs to be resonant to one of the two different laser frequencies and, therefore, also assigned the corresponding \(g_{\text{e}}^{(i)}\). We set the average \(g_{\text{e}} = 0.55\), the ratios between the two subsets \(g_{e}^{1}/g_{e}^{2} = 1.03\), and \(z = \mu_{\text{B}} / (\mu_{\text{N}} N_{i}) = 1/800\).

We run three different types of semiclassical simulations for the spin dynamics with \(I_{k} = 3/2\) nuclear spins: (i) a box model with \(A_{k} = A_{0} = \text{const.}, N_{\text{QD}} = 10000\), and \(N_{i} = 10^{6}\) nuclear spins, (ii) a simulation for a frozen nuclear spin dynamics (FOA) \[36\] for \(N_{\text{QD}} = 10000\) and \(N_{i}\) as in (i) and (iii) an \(A_{k}\) distribution with \(N_{\text{QD}} = 1000\) and \(N_{i} = 1000\) including the full nuclear spin dynamics. In all cases, we kept the characteristic energy scale \(T^{*} = (\sum (A_{k}^{(i)})^2 (I_{k}^{(i)})^2)^{-1/2} = 1 \text{ ns}\) fixed. The technical aspects of the simulation including the treatment of the laser pulse using a trion excitation cycle can be found in the literature \[7, 34\] which is summarized in the Supplemental Material.

The experiments reported in Ref. \[5\] were performed in a transversal magnetic field of \(1\) T, and laser pulses of two different colors were applied pumping two different resonant QD subsets. In order to reproduce one of the experimental key results, we first determined a reference curve for \(\langle S_{1}(t)\rangle\) of subset 1 subjected to a circular polarized laser pulse at \(t = 0\) that shows damped coherent oscillations characterized by the Larmor frequency. Then we run the same setup but apply a second circular polarized laser pulse with a color resonating with subset 2 at a moment when \(\langle S_{z}(t)\rangle\) of subset 1 reached a minimum, i. e. at a delay of \(\Delta t \approx 0.111 \text{ ns}\).

We tracked the relative phase shifts of the coherent oscillations between the reference curve \(\langle S_{1}(t)\rangle\) and those in the presence of a laser pulse onto subset 2. The sign of the phase shift depends only on the relative sign of the circular polarization. The result is plotted in Fig. \[5\].
As in the experiment, the phase shift increases roughly linearly in time. The slope is determined by overall magnitude of \( J_{ij}^{\text{RKKY}}(R) \). The results that resembles the experiments the best were obtained for \( \Gamma_0 = 140 \mu eV \) and \( \varepsilon_F = 0.5 \text{meV} \) corresponding to \( q_{\text{WL}} = 0.5 \) and \( U/\Gamma_0 = 29 \). Leaving the ratio \( U/\Gamma_0 = 29 \) fixed, we can also reproduce the experimental phase shifts with \( \Gamma_0 = 1.25 \text{meV}, \varepsilon_d = -15 \text{meV}, U = 40 \text{meV} \) indicating the physics is driven by an effective local Kondo coupling \( 2\hbar \) plus corrections in \( O(V_{0}^{\prime 4}) \) for short distances. Note that the individual hybridization strength between the bound electron QD state and the local Wannier orbital remains small and is of the order \( V_{m}^{\prime} = 10 - 40 \text{meV} \). Only by the summation to \( \bar{V}_{i} \), a significant contribution arises.

We run the simulation for fixed \( x_0 = 1 \) and for the FOA additional with a Gaussian normal distribution with \( \delta x = 0.2 \) to reveal the influence of the local derivation of the QD hybridization matrix element from its average value. The phase shifts are nearly independent of the distribution of local hybridization matrix elements \( \bar{V}_{i} \). The major disorder contributions stems from the randomly distribution of distances to the neighbouring QDs. As a result, each QD senses a slightly different effective coupling converting the quadratic increase of the phase shift in a two QD model to a linear increase for short times. Tracking the full dynamic of the Overhauser field is not required on the time scale of 3 ns: The frozen Overhauser approximation and the case \( A_k = \text{const.} \) yields the almost same results as the full nuclear spin dynamics but with a reduced \( N_{\text{QD}} \). We also run the simulation for a \( J_{ij}^{\text{RKKY}}(R) \) after rescaling \( \Gamma_0, U, \varepsilon_d \) with a factor of 10 to accommodate for errors in the rough parameter estimates. The slope of the phase shift \( \phi \) came out about 50% to large requiring a reduction of \( \Gamma_0 \). The origin of the increase is caused by contribution of the first minimum in \( J_{ij}^{\text{RKKY}}(R) \) at around \( RK_{F}/\pi \approx 0.2 \) where we observe significant deviations from the analytic prediction. Overall, the simulation data agree remarkably well with the experimental findings even over a large range of parameters.

**Conclusion.**— In this letter, we present a QD WL model to explain the microscopic origin of the inter QD electron–spin–spin interaction conjectured in Ref. \[5\]. We used Wilson’s NRG \[25\] to extract the strength and distance dependency of this spin–spin interaction mediated by the WL via an RKKY based mechanism. While the long distance behavior is in agreement with the low-filling perturbative RKKY result \[25\], we found significant deviations for shorter distances. The unconventional AF RKKY interaction at short distances can be connected to the large effective inter QD hopping between the QD orbitals \[12\, \[13\] as a consequence of the very low band filling. The conventional RKKY approach \[25\] starts already from an effective local moment picture and does not include all corrections in \( O(V_{0}^{\prime 4}) \).

We used our NRG data as input for a semiclassical simulation of the spin dynamics in a coupled QD ensemble which is able to reproduce the experimental phase shifts reported for two-color pumping setup \[3\] very accurately for a variety of different realistic parameters. Although the interaction in the effective model might appear substantial, one has to bare in mind that all WL Wannier orbital below a QD contribute in Eq. \[5\] only by a very weak coupling.

Our theory solves the long standing mystery about the microscopic origin of the inter QD spin–spin interaction. We predict that this interaction can be eliminated by a QD growth process that excludes an InAs WL \[19\]. By gating the 2D WL, it might be possible to modulate the 2D electron gas and, therefore, control the effective Heisenberg interactions \( J_{ij}^{\text{RKKY}}(R) \) by a change of gate voltage.

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[1] F. P. G. de Arquer, D. V. Talapin, V. I. Klimov, Y. Arakawa, M. Bayer, and E. H. Sargent, Semiconductor quantum dots: Technological progress and future challenges, Science 373, 10.1126/science.aaz8541 (2021).
[2] R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen, Spins in few-electron quantum dots, Rev. Mod. Phys. 79, 1217 (2007).
[3] J. M. Elzerman, R. Hanson, L. H. W. van Beveeren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouven-
hoven, Single-shot read-out of an individual electron spin in a quantum dot, Nature 430, 431 (2004).

[4] M. Atatüre, J. Dreiser, A. Badolato, A. Högele, K. Karrai, and A. Imamoglu, Quantum-dot spin-state preparation with near-unity fidelity, Science 312, 551 (2006).

[5] S. Spatzek, A. Greilich, S. E. Economou, S. Warvig, A. Schwan, D. R. Yakovlev, D. Reuter, A. D. Wieck, T. L. Reinecke, and M. Bayer, Optical Control of Coherent Interactions between Electron Spins in InGaAs Quantum Dots, Phys. Rev. Lett. 107, 137402 (2011).

[6] S. Warvig, A. René, S. E. Economou, A. Greilich, D. R. Yakovlev, D. Reuter, A. D. Wieck, T. L. Reinecke, and M. Bayer, All-optical tomography of electron spins in (In, Ga)As quantum dots, Phys. Rev. B 89, 081310 (2014).

[7] A. Fischer, E. Evers, S. Warvig, A. Greilich, M. Bayer, and F. B. Anders, Signatures of long-range spin-spin interactions in an (In, Ga)As quantum dot ensemble, Phys. Rev. B 98, 205308 (2018).

[8] C. Piermarocchi, F. Chen, L. J. Sham, and D. G. Steel, Optical RKKY Interaction between Charged Semiconductor Quantum Dots, Phys. Rev. Lett. 89, 167402 (2002).

[9] M. A. Ruderman and C. Kittel, Indirect exchange coupling of nuclear magnetic moments by conduction electrons, Phys. Rev. 96, 99 (1954).

[10] T. Kasuya, A theory of metallic ferromagnetism and antiferromagnetism on Zener’s model, Progress of Theoretical Physics 16, 45 (1956).

[11] K. Yosida, Magnetic properties of Cu-Mn alloys, Phys. Rev. 106, 893 (1957).

[12] F. Eickhoff, B. Lechtenberg, and F. B. Anders, Effective low-energy description of the two-impurity Anderson model: RKKY interaction and quantum criticality, Phys. Rev. B 98, 115103 (2018).

[13] F. Eickhoff and F. B. Anders, Strongly correlated multi-impurity models: The crossover from a single-impurity problem to lattice models, Phys. Rev. B 102, 205132 (2020).

[14] F. Eickhoff and F. B. Anders, Kondo holes in strongly correlated impurity arrays: RKKY-driven Kondo screening and hole-hole interactions, Phys. Rev. B 104, 045115 (2021).

[15] S. Sanguinetti, K. Watanabe, T. Tateno, M. Wakaki, N. Koguchi, T. Kuroda, F. Minami, and M. Gurioli, Role of the wetting layer in the carrier relaxation in quantum dots, Applied Physics Letters 81, 613 (2002).

[16] R. V. N. Melnik and M. Willatzen, Bandstructures of conical quantum dots with wetting layers, Nanotechnology 15, 1 (2003).

[17] S. Lee, O. L. Lazarenkova, P. von Allmen, F. Oyafuso, and G. Klimeck, Effect of wetting layers on the strain and electronic structure of InAs self-assembled quantum dots, Phys. Rev. B 70, 125307 (2004).

[18] R. Žitko and J. Bonča, Multiple-impurity anderson model for quantum dots coupled in parallel, Phys. Rev. B 74, 045312 (2006).

[19] M. C. Löbl, S. Scholz, I. Söllner, J. Ritzmann, T. Denneulin, A. Kovács, B. E. Kardynal, A. D. Wieck, A. Ludwig, and R. J. Warburton, Excitons in inaas quantum dots without electron wetting layer states, Communications Physics 2, 93 (2019).

[20] M. O. (eds), ed., Semiconductors — basic data (Springer, Berlin, Heidelberg, 1996).

[21] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav, and M. Kastner, Kondo effect in a single-electron transistor, Nature 391, 156 (1998).

[22] P. Glasenapp, D. S. Smirnov, A. Greilich, J. Hackmann, M. M. Glazov, F. B. Anders, and M. Bayer, Spin noise of electrons and holes in (In, Ga)As quantum dots: Experiment and theory, Phys. Rev. B 93, 205429 (2016).

[23] We checked that by an explicit calculation assuming a Gaussian distribution of $V_a$, centered at $\bar{R}$, with a width given by the QD radius of $r_{QD} \approx 15 \text{ nm}$.

[24] B. A. Jones and C. M. Varma, Study of two magnetic impurities in a ferrimag, Phys. Rev. Lett. 58, 843 (1987).

[25] B. Fischer and M. W. Klein, Magnetic and nonmagnetic impurities in two-dimensional metals, Phys. Rev. B 11, 2025 (1975).

[26] J. R. Schrieffer and P. A. Wolff, Relation between the Anderson and Kondo Hamiltonians, Phys. Rev. 149, 491 (1966).

[27] H. R. Krishna-murthy, J. W. Wilkins, and K. G. Wilson, Renormalization-group approach to the Anderson model of dilute magnetic alloys. I. Static properties for the symmetric case, Phys. Rev. B 21, 1003 (1980).

[28] R. Bulla, T. A. Costi, and T. Pruschke, The numerical renormalization group method for quantum impurity systems, Rev. Mod. Phys. 80, 395 (2008).

[29] I. Affleck, A. W. W. Ludwig, and B. A. Jones, Conformal-field-theory approach to the two-impurity Kondo problem: Comparison with numerical renormalization-group results, Phys. Rev. B 52, 9528 (1995).

[30] R. Hager, Kondo-Effekt in Systemen mit niedriger Ladungsträgerkonzentration, Master’s thesis, Department of Physics, University of Augsburg (2007).

[31] See Supplemental Material for additional technical details.

[32] $\mu_{eff} = s(s+1)$ for unscreened spins.

[33] M. M. Glazov and E. L. Ivchenko, Spin noise in quantum dot ensembles, Phys. Rev. B 86, 115308 (2012).

[34] N. Jäschke, A. Fischer, E. Evers, V. V. Belykh, A. Greilich, M. Bayer, and F. B. Anders, Nonequilibrium nuclear spin distribution function in quantum dots subject to periodic pulses, Phys. Rev. B 96, 205419 (2017).

[35] M. Gaudin, J. Physique 37, 1087 (1976).

[36] I. A. Merkulov, A. L. Efros, and M. Rosen, Electron spin relaxation by nuclei in semiconductor quantum dots, Phys. Rev. B 65, 205309 (2002).

[37] W. A. Cossh and D. Loss, Hyperfine interaction in a quantum dot: Non-markovian electron spin dynamics, Phys. Rev. B 70, 195340 (2004).

[38] W. Zhang, V. V. Dobrovitski, K. A. Al-Hassanieh, E. Dagotto, and B. N. Harmon, Hyperfine interaction induced decoherence of electron spins in quantum dots, Phys. Rev. B 74, 205313 (2006).

[39] G. Chen, D. L. Bergman, and L. Balents, Semi-classical dynamics and long-time asymptotics of the central-spin problem in a quantum dot, Phys. Rev. B 76, 045312 (2007).