Observation of a dynamical sliding phase superfluid with P-band bosons

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Sliding phases have been long sought-after in the context of coupled XY-models, of relevance to various many-body systems such as layered superconductors, free-standing liquid-crystal films, and cationic lipid-DNA complexes. Here we report an observation of a dynamical sliding-phase superfluid that emerges in a nonequilibrium setting from the quantum dynamics of a three-dimensional ultracold atomic gas loaded into the P-band of a one-dimensional optical lattice. A shortcut loading method is used to transfer atoms into the P-band at zero quasi-momentum within a very short time duration. The system can be viewed as a series of “pancake”-shaped atomic samples. For this far-out-of-equilibrium system, we find an intermediate time window with lifetime around tens of milliseconds, where the atomic ensemble...
exhibits robust superfluid phase coherence in the pancake directions, but no coherence in the lattice direction, which implies a dynamical sliding-phase superfluid. This experiment potentially opens up a novel venue to search for exotic dynamical phases by creating high-band excitations in optical lattices.

A sliding phase mechanism has been proposed in the study of weakly coupled stacks of XY models, which was introduced to characterize intricate phase transitions in a broad range of many-body systems such as layered superconductors, free-standing liquid-crystal films, and even biological molecules. In the sliding phase, the system behaves like a stack of decoupled layers. With field theory analysis, it has been shown that the sliding phase typically appears under extreme conditions for thermal equilibrium systems or quantum ground states, causing a grievous challenge in experimental implementation.

Recent experimental progress in synthetic quantum systems has achieved unprecedented approaches to investigate fascinating collective phenomena in controllable quantum dynamics, such as light-induced non-equilibrium superconductivity, time-crystals in trapped ion-qubits, correlated quantum kinematics in reduced-dimensional systems, and many-body localization with ultracold atoms in artificial light-crystals. While a complete theoretical framework to describe nonequilibrium phase-transition is still lacking, a formal analogy between temperature and time by comparing partition-function in the thermal-ensemble and unitary-evolution operator in quantum dynamics allows such concepts in statistical physics as many-body phases and condensation, to generalize to the time-domain.
Here, we report on an observation of a sliding-phase superfluid in a dynamical system of ultracold atoms loaded into the P-band of an optical lattice. Our work goes beyond previous studies in P-band optical lattices focused on static phases, by considering nonequilibrium quantum aspects. We have a three-dimensional quantum gas confined with a one-dimensional lattice, which can be thought as a series of “pancake”-shaped subsystems (Fig. 1). Using an adiabatic short-passage to load atoms into the zero quasi-momentum state in the P-band of the lattice, the system is driven far-out-of-equilibrium. During the process of rethermalization, a metastable region is observed, where the atomic sample shows strong phase-coherence in the pancake directions, but no coherence in the lattice direction. These observations imply the first experimental discovery of the sliding-phase superfluid in the time-domain. This work would also shed light on the understanding of high-Tc mechanism in light-probed cooperates.

**Experimental procedure.** The experiment is performed with a BEC of $^{87}$Rb prepared in a hybrid trap with the harmonic trapping frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (28\text{Hz}, 55\text{Hz}, 60\text{Hz})$. A one-dimensional optical-lattice is produced by a standing wave with the lattice constant $d = \pi/k = 426 \text{ nm}$ along $x$ axis with $k$ the wave-number. As shown in Fig. 1(a), atoms are confined in more than 50 discrete pancakes in the $yz$-plane, and the sizes of the condensate in the $\hat{y}$ and $\hat{z}$-directions are about $L_y = 15.6 \mu\text{m}$ and $L_z = 14.9 \mu\text{m}$, respectively.

A shortcut method with the designed pulse sequences is applied to load atoms into the P-band of the optical-lattice (see Fig. 1 and Methods). The loading pulse sequence consist of two sets of pulses whose nodes are shifted in the $\hat{x}$-axis by half of the lattice constant. We stress here
that after loading to the zero quasi-momentum state of the P-band, the quantum system is driven to a far-out-of-equilibrium but at the same time phase-coherent state. We hold the condensate in the P-band for certain amount of time \( t \) and let the system evolve, then the TOF images are taken in two probe directions—Probe1 and Probe2 to be described below.

For probe from \( \hat{z} \)-direction (Probe1), the lattice potential is switched off adiabatically, this band mapping (BM) procedure enables measurements of the atomic population in each band. From such images at different time \( t \), we can quantitatively determine the time-dependent proportion of atoms between S-band and P-band, as shown in Fig. 1(d). We investigate the dynamical phase during the decay process from the excited P-band to the S-band. The probe from \( \hat{x} \)-direction is performed by an abrupt non-adiabatic switch-off (NAS) of the lattice as shown in Fig. 1, which is referred to as Probe2. From a bimodal fitting, we extract the coherent fraction so that the phase coherence of the dynamical many-body state can be inferred (see Methods).

**Observation of the sliding-phase superfluid.** In order to characterize the real-time dynamics after P-band gets occupied, we measure momentum distributions in the lattice (Probe1) and pancake (Probe2) directions at the different holding-times (Fig. 2). Since the system is approximately rotation-symmetric in the \( yz \)-plane, the momentum distribution in the \( \hat{z} \)-direction is equivalent to that in the \( \hat{y} \)-direction, and is thus not shown here. From the time evolution, we identify three distinct dynamical regions. At early time—the first stage, the system has superfluid phase coherence in all three directions, which is clearly demonstrated through the sharp peaks observed in the momentum distribution shown in Fig. 2 at \( t = 1 \) ms. A bimodal fitting shows the
system is coherent in all directions. At late time—the final stage, the quantum gas has rethermalized with a complete loss of phase-coherence, owing to the excessive energy in the atomic-gas. There is yet an intermediate time region with significant time-duration where the phase-coherence of the quantum system survives partially. In this region, the phase-coherence in the lattice direction already disappears, whereas the coherence in the pancake directions still persists as revealed by momentum distributions. The evolution of phase-coherence in the three stages is described by a time-dependent correlation-function,

\[
\langle \hat{\phi}(\mathbf{r}, t) \hat{\phi}^\dagger(\mathbf{r}', t) \rangle \propto \exp\left(\frac{|\mathbf{r}_x - \mathbf{r}'_x|}{\xi_x} + \frac{|\mathbf{r}_y - \mathbf{r}'_y|}{\xi_y} + \frac{|\mathbf{r}_z - \mathbf{r}'_z|}{\xi_z}\right),
\]

where \( \hat{\phi}(\mathbf{r}, t) \) is the bosonic field operator with spatial coordinate \( \mathbf{r} \), \( \xi_{x,y,z} \) the superfluid correlation length in the three directions. At the first stage, the three correlation lengths diverge, or equivalently are comparable with the system-size, whereas at final stage the correlation lengths are all finite. In the intermediate time region, we have divergent correlation lengths in the \( yz \) plane, i.e., \( \xi_{y,z} \sim L \) with \( L \) the system-size, but finite correlation length in the \( x \) direction, i.e., \( \xi_x/L \to 0 \). The peculiar dynamical phase in the intermediate time region represents the long sought-after sliding-phase superfluid—each pancake is phase coherent, but the relative phase across different pancakes is sliding.

The emergence of the sliding phase in the time domain implies two thermalization timescales, associated with the discrete and continuous degrees of freedom in the lattice and pancake directions, respectively. Here we provide an intuitive physical picture for the underlying mechanism. As atoms decay from P- to S-band, topological defects are generated in the superfluid field. But at early time of the dynamics, the fraction of thermal atoms that cause defect-generation is in-
sufficient, so that these defects can further nucleate through quantum kinematics to maintain the superfluid coherence across the whole sample, where the nucleation rate should be attributed to kinetic couplings. It is worth mentioning that signatures of defect-nucleation also exist in previous experiments on P-band bosons in a bipartite optical-lattice. The dynamical evolution of the superfluid coherence is then a competition between thermal-atom powered defect-generation and kinetic-coupling driven defect-nucleation. Note that in our system the kinetic coupling in the lattice direction is much weaker compared to the pancake directions, which means the defect-nucleation rate is much less efficient in the lattice direction. Then in the nonequilibrium process, the phase-coherence in the lattice direction should disappear earlier than in the pancake directions, which then leads to the intermediate time window that supports sliding-phase superfluidity. This physical picture is consistent with our measurements of the dependence of the sliding-phase-lifetime on the lattice depth and temperature, to be presented below. Our results demand theoretical modelling of correlated dynamics in weakly-coupled XY-models beyond Gross-Pitaevskii treatment. A quantitative description is expected to be nontrivial, for example, whether thermal atoms could act as effective-disorder previously proposed to stabilize the sliding-phase is worth consideration.

**Phase lifetime.** To test the robustness of the dynamical sliding-phase superfluid, we measure its lifetime in dynamics. In the experiment, the lifetime is defined from time-dependent phase-coherent fractions in pancake and lattice directions, which are extracted from the momentum distributions (see Methods). Experimental results are shown in Fig. 3. The overall decrease in the lifetime of the phase-coherence with increasing lattice depth can be attributed to the confinement-induced increase of effective interactions which would accelerate the band-decay process. The in-
crease of the sliding-phase-lifetime is consistent with the defect-nucleation picture provided above. Increasing lattice depth would cause decrease of the kinetic coupling in the lattice direction, but not affect much the coupling in the pancake directions. As the sliding-phase emerges from the difference in these kinetic couplings, it is then natural to have an increase in the lifetime when the difference becomes larger.

Finally, we examine finite temperature effects on the dynamical sliding-phase (see Fig. 4). The temperature dependence of its lifetime can be attributed to the competition of thermal-atom powered defect-generation and kinetic-coupling driven defect-nucleation. Note that the sliding-phase appears due to the difference in the kinetic couplings in lattice and pancake directions. Upon temperature increase, the defect-generation is enhanced, and the difference in kinetic couplings is effectively reduced, which then gives a shorter lifetime for the sliding-phase.

In conclusion, through quantum dynamics of ultracold atoms loaded in the excited band, our measurements unveil a sliding-phase superfluid. This sliding-phase appears due to thermalization time-scale separation for discrete and continuous degrees of freedom. The robustness of the dynamical phase has been tested by increasing lattice-depth and temperature. This potentially opens up a novel route to search for metastable correlated phases with ultracold atoms driven far-out-of-equilibrium. The intricate exotic phases challenging to achieve in thermal-equilibrium or the quantum groundstate might find their natural realization in nonequilibrium settings. This work is also expected to shed light on understanding the high-Tc mystery in the nonequilibrium layered-cooperates.
Methods

**Shortcut method to load atoms into P-band.** In the experiment, the condensate is quickly loaded into the P-band (first excited band) of the optical-lattice by a series of pulsed laser potentials. In the first series of pulses the atom experiences spatial potential \( V_{\text{even}}(x) = V_0 \cos^2(kx) \). In the second series, a shifted potential \( V_{\text{odd}}(x) = V_0 \cos^2(kx + \pi/4) \) is applied. The relative spatial shift of the two pulses is introduced to efficiently change the parity from even to odd. The pulse sequence is designed to optimize the eventual P-band population.

In the experiment, two acousto-optic-modulators (AOM) with a frequency difference are used to form our designed pulse sequence with the frequency difference \( \delta\omega = 182.5 \text{MHz} \) which leads to a phase shift of \( 5\pi/4 \) between two pulses. We measure the P-band loading fidelity by the oscillations of the relative population in different momenta from the images by NAS\textsuperscript{33}. By comparing the experimental data with the peripheral contour of the beating signal, the corresponding fidelity is found to be about 95%.

**Finding coherent fraction.** To extract time dependence of coherent fractions in the lattice and pancake directions, we perform five experimental runs under each condition, and take the mean atomic distributions. In the pancake directions (Probe2), the atomic distribution in the \( \hat{y} \)-direction is analyzed by a standard bimodal fitting according to

\[ f(y) = GF_{th}(e^{-(y-y_0)^2/\sigma^2}) + H(\max[1 - (y - y_0)^2/\chi^2, 0])^2, \]

\[ y \leq \chi^2, \]

\[ y > \chi^2, \]
with the thermal distribution $F_{th}$ and a Thomas Fermi distribution for the condensate. The parameters $G'$ and $H$ are amplitudes of two components, $y_0$ is center of the atom cloud, and $\sigma$, $\chi$ represent widths of corresponding distributions. The coherent fraction is given by the atom number in the condensate component divided by the total atom number. In the lattice direction (Probe1), we have coexistence of S- and P-band contributions in the atomic distributions which causes difficulty in extracting the coherent fraction in the system. In analyzing the measured atomic distribution, we take a general form of

$$f(x) = G'f_T(x) + f_C(x),$$

(3)

with $f_T(x)$ the thermal atom distribution, $f_C(x)$ the contribution from the condensed phase-coherent component. The form of thermal-atom distribution $f_T(x)$ is fixed by using the atomic distribution in a parallel line with a 10-pixel offset from the central line, where only thermal atoms exists. The coherent component is then determined using the atomic distribution along the central line in the $\hat{x}$-direction (lattice direction).

**Determination of sliding-phase-lifetime.** In order to determine the lifetime of the sliding phase in the intermediate time-window, we need to find out the time $t_0$ where the coherent fractions in lattice and pancake directions vanish. For both directions, we find that the decay of coherent fractions fit well to an exponential form $Ae^{-t/\tau}$ with different initial amplitude $A$ and decay time constant $\tau$. The vanishing time $t_0$ is determined when the coherent fraction $Ae^{-t/\tau}$ drops to its value of 1%, with the fraction 1% the noise level in condensate fraction measured for incoherent atoms.
Figure 1 a, Experimental configuration for a 1d P-orbital lattice in the $\hat{x}$-direction, where atoms form discrete pancakes confined by the lattice potential. b, The system is probed in two ways. Probe1: probe with a laser beam along $\hat{z}$-direction after the band mapping. Probe2: with the probe beam along the $\hat{x}$-direction, the image is taken by switching off the potential abruptly within 30 ns. Atoms are loaded to the zero quasi-momentum state of the P-band of an optical-lattice through a designed pulse sequence, an example of which is shown in c, with the lattice-depth $V_0 = 5E_r$ ($E_r$ is one-photon recoil energy). After loading into the P-band, atoms are hold in the lattice potential for a time $t$, and the absorption images after the time of flight (TOF) are taken in two directions. Before the measurement for Probe1, the lattice potential is exponentially ramped down in the form $e^{-t/\eta}$ within 500 $\mu$s, the characteristic time being $\eta = 100$ $\mu$s. d, the proportion...
of atoms in the S- and P-bands. For each data point, we perform five experimental runs and take the average. All experimental results in this figure are taken with lattice-depth $V_0 = 5E_r$ at temperature $T = 120$ nK. We note here that the temperature is defined out of the system at equilibrium before the dynamical evolution—temperature is not a well-defined quantity for the dynamical phase itself.

**Figure 2** Momentum distributions measured at three different holding times $t = 1$ ms, 60 ms, 100 ms, along different directions. The images shown in the first row represent experimental TOF measurements in the $xy$-plane by Probe1 (see main text), and the corresponding momentum distribution in the central line along the $\hat{x}$-direction is given in the second row with blue dots. The red solid line gives a full fitting line while the green dashed line gives the distribution of thermal component. Atomic distribution in the S-band (P-band) is revealed in the first (second) Brillouin zone. The third row shows the atomic distribution along the central line in $\hat{y}$-direction for the
experimental images in the $yz$-plane measured by Probe2. In our experiment each pixel has a size of 6.8 µm. Experimental results shown here are taken at temperature 120 nK.

**Figure 3 a**. The coherent fraction measured by Probe1, 2 with the different holding times, for $V_0 = 3E_r$, $5E_r$, $8E_r$, and $11E_r$, respectively. **b**. The time $t_0$ for the atoms to lose coherence in lattice and pancake directions with different optical-lattice depths. The blue diamonds are for $\hat{x}$-direction (lattice) by Probe1 and the dotted points for the $\hat{y}$-direction (pancake). Errorbars are from the fitting with 95% confidence interval. We find the time-dependence of the coherent fractions fits to a form of $Ae^{-t/\tau}$ with $A$ the amplitude and $\tau$ the characteristic time. The starting (ending) point of the sliding-phase superfluid is defined by the time $t_0$ when the coherent fraction in the lattice (pancake) direction vanishes. With a relatively shallow lattice, say lattice depth $V_0 = 3E_r$, there is essentially no difference in the time-dependence of the coherent fractions in lattice and pancake directions, which means a shallow lattice does not support the intermediate sliding-phase superfluid. With a deeper lattice, we find a significant difference in coherent fractions for lattice
and pancake directions, leading to two dynamical timescales and an intermediate time window supporting the sliding-phase superfluid. The sliding-phase-lifetime can be systematically improved upon increasing lattice depth. Experimental results shown here are taken at temperature 120 nK.

**Figure 4 a.** The coherent fraction with different holding times for a lower atomic temperature $T=90 \text{nK}$ at $V_0 = 5E_r$. **b.** The time for atoms to lose coherence in lattice and pancake directions at different temperatures. By increasing the temperature, we find the intermediate time window supporting the sliding-phase gets smaller with larger thermal fluctuations. The temperature dependence indicates the sliding-phase-lifetime can be further improved by cooling down to lower temperature.
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