Bias from gas inhomogeneities in the pressure profiles as measured from X-ray and Sunyaev–Zeldovich observations

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ABSTRACT
X-ray observations of galaxy clusters provide emission measure weighted spectra, arising from a range of density and temperature fluctuations in the intracluster medium (ICM). This is fitted to a single temperature plasma emission model to provide an estimate of the gas density and temperature, which are sensitive to the gas inhomogeneities. Therefore, X-ray observations yield a potentially biased estimate of the thermal gas pressure, $P_X$. At the same time Sunyaev–Zeldovich (SZ) observations directly measure the integrated gas pressure, $P_{SZ}$. If the X-ray pressure profiles are strongly biased with respect to the SZ, then one has the possibility to probe the gas inhomogeneities (their amplitude and physical nature), even at scales unresolved by the current generation of telescopes. At the same time, a weak bias has implications for the interchangeable use of mass proxies like $Y_{SZ}$ and $Y_X$ as cosmological probes. In this paper, we investigate the dependence of the bias, defined as $b_P(r) \equiv P_X(r)/P_{SZ}(r) - 1$, on the characteristics of fluctuations in the ICM taking into account the correlation between temperature and density fluctuations. We made a simple prediction of the irreducible bias in idealized X-ray versus SZ observations using multitemperature plasma emission model. We also provide a simple fitting form to estimate the bias given the distribution of fluctuations. In real observations, there can be additional complications arising from instrumental background, insufficient photon statistics, asphericity, method of deprojection, etc. Analysing a sample of 16 clusters extracted from hydrodynamical simulations, we find that the median value of bias is within $\pm 3$ per cent within $R_{500}$, it decreases to $-5$ per cent at $R_{500} < r < 1.5R_{500}$ and then rises back to $\sim 0$ per cent at $r \gtrsim 2R_{500}$. The scatter of $b_P(r)$ between individual relaxed clusters is small at the level of $< 0.03$ within $R_{500}$, but turns significantly larger (0.25) and highly skewed ($\overline{b_P(r)} > 0$) at $r \gtrsim 1.5R_{500}$. For any relaxed cluster, we find $|b_P(r)| < 15$ per cent within $R_{500}$, across different implementations of input physics in the simulations. Unrelaxed clusters exhibit a larger scatter in $b_P(r)$ (both from radius to radius and from cluster to cluster).

Key words: galaxies: clusters: intracluster medium – cosmic background radiation – X-rays: galaxies: clusters.

1 INTRODUCTION
Galaxy clusters offer an interesting possibility of probing physics relevant on both cosmological as well as galactic scales (see recent review by Kravtsov & Borgani 2012). On the one hand, they are useful for constraining cosmological parameters (Haiman, Mohr &...
performed (in various versions; see e.g. Kriss, Cioffi & Canizares 1983; McLaughlin 1999; Vikhlinin et al. 2006) to yield the threedimensional (3D) radial profiles of both temperature and density. Henceforth we shall use the term ‘deprojection’ to imply the particular method as implemented in Churazov et al. (2003) (see also Russell, Sanders & Fabian 2008). Briefly, assuming spherical symmetry, a set of measured values of the surface brightness in concentric rings is converted into a set of emissivities in spherical shells, using the inverse of a simple geometrical projection matrix. The procedure is repeated for many energies, yielding for each shell an energy-dependent volume emissivity — i.e. the spectrum, which is then fitted in XSPEC with the standard model(s).

Thanks to rapid technological progress in SZ observations it has been possible to apply similar methods to well-resolved SZ images obtained with the Planck (Planck Collaboration et al. 2012b), SPT (Plagge et al. 2010), ACT (Reese et al. 2012), Combined Array for Research in Millimeter-wave Astronomy (CARMA; Plagge et al. 2012), Multiplexed SQUID/TES Array at Ninety Gigahertz (MUSTANG; Korngut et al. 2011) and BOLOCAM (Sayers et al. 2012). From these measurements one may then obtain the volume integrated thermal pressure $P = \int n_e T dV$, in each concentric shell both in X-rays and SZ, following the deprojection method as outlined in the above paragraph.

X-ray observations reveal the presence of substructures, even in relaxed clusters, with the fluctuations in density and temperature having amplitudes of $\sim 10$ per cent (Zhang et al. 2009) at $r \lesssim 0.5R_{500}$ (see also Churazov et al. 2012). Zhuravleva et al. (2012b), hereafter Z12 investigated properties of the ICM fluctuations in simulated clusters and found their distribution to be near-lognormal along with a high-density tail contributing $\sim 1$ per cent by volume (see also Roncarelli et al. 2006; Vazza et al. 2011). The width of the density fluctuations is small (0.1 dex) at the centre of relaxed clusters but grows by a factor of 6 at $2R_{500}$; with unrelaxed clusters found to have a broader distribution than relaxed clusters. Z12 propose a simple method for identifying the high-density clumps using a sample of simulated clusters and show that the median values of the ICM pressure remain robust to the extraction of such clumps.

If we assume that the X-ray emissivity is independent of temperature,\footnote{The X-ray emissivity in the energy band $E_1$–$E_2$ is $\varepsilon_X \propto n_e^2 \Lambda(E_1, E_2, T)$. For $T \lesssim 3$ keV, the soft band (0.5–2 keV) emissivity is almost independent of the temperature.} and the width of the lognormal distribution of gas density fluctuations $\sigma_{n_e}$ is small ($\sigma_{n_e} \ll 1$), the bias, $b_p$, between the pressures measured from X-ray ($P_X$) and SZ ($P_{SZ}$) observations, as a function of $\sigma_{n_e}$, in the following three different hypotheses (isothermal, adiabatic and isobaric gas) may be written as

$$\begin{align*}
b_p = \frac{P_X}{P_{SZ}} - 1 = \frac{\sqrt{(n_e^2)}(T)}{n_e^2} - 1 \\
= \exp\left[\frac{\sigma_{n_e}^2}{2}\right] - 1 \approx \frac{\sigma_{n_e}^2}{2} \quad \ldots \text{isothermal} \\
= \exp\left[\frac{3}{2} - \gamma\right]\sigma_{n_e}^2 - 1 \approx -\frac{\sigma_{n_e}^2}{6} \quad \ldots \text{adiabatic} \\
= \exp\left[\frac{3\sigma_{n_e}^2}{2}\right] - 1 \approx \frac{3}{2}\sigma_{n_e}^2 \quad \ldots \text{isobaric}
\end{align*}$$

for the adiabatic index $\gamma = 5/3$. It is interesting to note here that the bias takes positive values for isobaric fluctuations but is negative for adiabatic fluctuations.
Thus, for such fluctuations one expects the bias to be simply related to the gas clumping factor, 
\[ C \equiv \langle n_e^2/n_e \rangle^2 = \exp(\sigma_c^2) \approx 1 + \sigma_c^2. \]
Note also that the magnitude of this bias is small as long as the fluctuations are not large. Mathiesen, Evrard & Mohr (1999) showed using simulations that the average bias in gas mass within \( R_{500} \) due to clumpiness in the ICM is about \(+16\) per cent, while Nagai, Vikhlinin & Kravtsov (2007a) found the bias to be lower at \( \lesssim 6\) per cent. Nagai & Lau (2011) showed that clumping may be even larger \((C \sim 2–6)\) at the cluster outskirts \((r \gtrsim 1.5R_{500})\;\text{see also Vazza et al. 2012}.\)

X-ray and SZ observations provide complimentary information for studying clumpiness and fluctuations in the ICM. However, in reality the actual bias between \( P_X \) and \( P_{SZ} \) may well be different from the simple estimate presented in equation (1) due to the effects of X-ray instrumental response and, more importantly, the bias resulting from fitting a single temperature model to a spectrum produced by gas with a range of temperatures (e.g. Mazzotta et al. 2004; Kawahara et al. 2007). For example, in a uniform gas having only temperature fluctuations, the measured temperature is weighted down by \( \Lambda(E1,E2,T) \) due to the relative importance of line spectra at lower temperatures, producing a negatively biased \( P_X \) with respect to \( P_{SZ} \). In addition, there might also be a bias arising from the application of a simple \( \beta \) model (Rasia et al. 2006) or from the assumption of spherical symmetry for clusters having more complicated morphology (Ameglio et al. 2007), however, we shall not discuss these issues here.

It is therefore necessary to make a realistic estimate of the irreducible bias between X-ray and SZ measurements of pressure profiles. By irreducible, we mean the bias that would persist even for the most favourable or idealized observations. In this work, we estimate this bias in two ways: (i) from an idealized hot gas described by a lognormal distribution of density and temperature values; (ii) and then from hydrodynamical simulations (with different implementations of input physics) of clusters in a broad mass range. Mock X-ray spectra generated from the given distributions of gas density and temperature are used to estimate this bias. Our computation of this bias takes into account both the density and temperature fluctuations along with their correlation, as well as the effects of the X-ray instrumental response function (for the Chandra telescope) involved in the spectral fitting. We wish to emphasize that in real observations, the bias is likely to depend on the method of deprojection, X-ray instrumental background, ability to remove dense clumps from images (which may be limited by photon statistics), etc. The actual bias inherent in a given X-ray/SZ analysis method should be estimated via detailed tests.

2 THE BIAS FROM AN IDEALIZED HOT GAS WITH A LOGNORMAL DISTRIBUTION

Hydrodynamical simulations of clusters show that the density, \( n_e \), and temperature, \( T \), of the ICM follow a nearly lognormal distribution (Kawahara et al. 2007; Z12) with an extended tail of high density fluctuations (HDF), see Fig. 1. We model an idealized hot gas \((T \sim \text{few keV})\) as a weighted sum of \(30 \times 30\) components (in densities and temperature) with the weights (here, the emission measures) correctly normalized to follow a bivariate lognormal distribution in \( n_e \) and \( T \), defined by the following probability density function:

\[ f(n_e, T) = \frac{1}{2\pi\sigma_{n_e}\sigma_T} \exp \left( -\frac{1}{2} \frac{X}{C^{-1} X^T} \right), \]

where \( X = [\ln(n_e) - \mu_{n_e}, \ln(T) - \mu_T] \) and \( C \) is the covariance matrix,

\[ C = \begin{bmatrix} \sigma_{n_e}^2 & \sigma_{n_e}\sigma_T\xi \xi^-1 \\ \sigma_{n_e}\sigma_T\xi^-1 & \sigma_T^2 \end{bmatrix}. \]

Such a distribution is characterized by the means of the lognormal distribution, \( \mu_{n_e} = \ln(n_e) \times 10^{-3} \) cm\(^{-3} \) and \( \mu_T = \ln(T/1 \text{ keV}) \) (or \( T_{\text{median}} = \exp(\mu_T) \text{ keV} \)), the variances, \( \sigma_{n_e}^2 \) and \( \sigma_T^2 \), along with the coefficient of correlation, \( \xi \), between \( \ln(n_e) \) and \( \ln(T) \). Of these, one may always set the density parameter, \( \mu_{n_e} \), to an arbitrary value since we are only interested in computing the ratio \( P_X/P_{SZ} \); henceforth we shall not mention the value of this parameter.

2.1 Method: mock X-ray spectrum

We use the MEKAL code in XSPEC (Mewe, Gronenschild & van den Oord 1985; Mewe, Lemen & van den Oord 1986; Kaatza 1992; Liedahl, Osterheld & Goldstein 1995) to generate the X-ray model spectra from a hot diffuse gas, including the line emissions from the astrophysically abundant elements, for each of the components defined by the values of \( n_e \) and \( T \). The choice of the MEKAL model was motivated by its ability to calculate anew the spectrum at any given temperature, rather than interpolate from tabulated values as in the APEC model.\(^3\) These spectra are then added together to produce a composite spectrum of X-ray emission from an ICM having a distribution of temperatures and densities. This composite spectrum is next fitted by a single component MEKAL model (in the energy range 0.5–8.0 keV) to obtain the best-fitting values \( n_{e,\text{fit}} \) and \( T_{\text{fit}} \) using the Chandra instrumental response file. The X-ray electron pressure, \( P_X = n_{e,\text{fit}} T_{\text{fit}} \), for the ICM is then computed from these best-fitting values of density and temperature.\(^4\)

The bias in the gas density is defined with respect to the volume-weighted density, and the bias in temperature is defined with respect to the mass-weighted temperature:

\[ n_{e,\text{ref}} = \frac{\int n_e \, dV}{\int dV}, \quad T_{\text{ref}} = \frac{\int T n_e \, dV}{\int n_e \, dV}. \]

\(^3\) We find that the APEC model (Smith et al. 2001) produces small spurious jumps in the bias caused by the interpolation of the tabulated spectra. These jumps are absent in the MEKAL model if the emission spectra are calculated anew for each model.

\(^4\) Note that when fitting a MEKAL model to the composite spectrum, we allow only the temperature and normalization to be free parameters, while the redshift, metal abundances and the hydrogen column density are kept fixed.
Bias in the X-ray versus SZ pressure profiles

Figure 1. Distribution of density, temperature and pressure in simulated clusters (see Z12, for details). The solid lines show the actual distribution, while the dashed lines indicate the lognormal distribution having the same full width at half maxima. The distributions shown in blue/red/black colours are measured for the (highly relaxed) cluster CL7 in the CSF simulation (see Section 3) in concentric shells at the radii \( r/R_{500} \): 0.9–1.0/1.1–1.2/1.6–1.8. Note the presence of an extended high-density tail in the distribution of density fluctuations.

Figure 2. Single temperature fit to a composite spectrum for the Chandra response. Upper panel: the red curve shows the composite spectrum generated from a bivariate lognormal distribution of gas components with \( \sigma_T = 0.45, \sigma_{ne} = 0.55 \) and correlation \( \xi = -0.55 \), while the blue curve is the best-fitting single temperature MEKAL spectrum. Lower panel: difference between the data and model. For this spectrum, the best-fitting values of density and temperature are \( 1.38 \times 10^{-3} \text{ cm}^{-3} \) and \( 3.91 \text{ keV} \), giving biases of \( +16.8 \) and \( -23.3 \) per cent with respect to \( n_{\text{ref}} = 1.16 \times 10^{-3} \text{ cm}^{-3} \) and \( T_{\text{ref}} = 3.91 \text{ keV} \), respectively (see equation 3). Using equation (4) we compute the pressure bias \( b_P = -10.4 \) per cent.

2.2 Results: bias from a lognormal distribution of gas density and temperature fluctuations

As a preliminary check we verified that our scheme reproduces the simple estimate of the ratio \( P_X/P_{\text{SZ}} = n_{\text{fit}}/n_{\text{ref}} = \sqrt{C} \) when the temperature fluctuations are set to zero, while retaining only the density perturbations (see also equation 1). This is done by generating a mock spectrum from a lognormal distribution of density fluctuations, convolving it with the instrument response (Chandra response in our case) and then fitting it to a single temperature model, as explained before in Section 2.1, to compute the bias using equation (4). We then go on to systematically explore the dependence of bias in an idealized hot gas described by a bivariate lognormal distribution of density and temperature fluctuations (see equation 2). For this we compute the bias (as described in Section 2.1) as a function of the parameters \( \{ \sigma_T, \sigma_{ne}, \xi, T_{\text{median}} \} \) for values in the range \(-0.01 \leq \sigma_T, \sigma_{ne} \leq 0.6 \) and \(-0.6 \leq \xi \leq 0.6 \) at various temperatures, \( T_{\text{median}} \geq 0.5 \text{ keV} \), for an assumed value of metallicity. We find that the bias is most sensitive to the value of \( \xi \), the coefficient of correlation between density and temperature fluctuations (see equation 2). Keeping the other parameters fixed, the bias always increases almost linearly with \( \xi \) for \( |\xi| < 0.55 \). The rate of increase, however, depends on the widths of the distribution of temperature and density fluctuations, i.e. \( \sigma_T \) and \( \sigma_{ne} \). The simple linear dependence of the bias on \( \xi \) implies the possibility of probing the nature of perturbations in the ICM: \( \xi < 0 \) would indicate the relative importance of isobaric or entropy perturbations, while \( \xi > 0 \).

\[ b_P \equiv \frac{P_X}{P_{\text{SZ}}} - 1 = \frac{n_{\text{fit}}}{n_{\text{ref}}} \frac{T_{\text{fit}}}{T_{\text{ref}}} - 1. \]  \hspace{1cm} (4)

The top panel of Fig. 2 shows the mock X-ray spectrum generated from a hot diffuse gas that has a bivariate lognormal distribution (red line) with a median temperature \( T_{\text{median}} = 4 \text{ keV} \) and median density \( n_{\text{e, median}} = 1 \times 10^{-3} \text{ cm}^{-3} \). The blue line shows the best-fitting spectrum specified by a single value of temperature and density using equation (4) for pressure bias \( b_P = -10.4 \) per cent.

\[ S \equiv k_B T/n_e^{2/3}, \]  \hspace{1cm} (5)

Here entropy is defined as \( S \equiv k_B T/n_e^{2/3} \), where \( n_e \) and \( T \) are electron density and temperature of the ICM, respectively.
0 would imply that adiabatic or pressure perturbations (caused by sound waves or weak shocks) are dominant.

We now try to describe the dependence of the bias in pressure in terms of the parameters of the bivariate lognormal distribution using a fitting form. For values of the median gas temperature, \( T_{\text{median}} \geq 3 \text{ keV} \), the bias may be parametrized using three free parameters as follows:

\[
b_P = \frac{1}{2} a_{ne}^2 - p_1 a_T^2 + p_2 \ln(1 + p_3 a_T) a_{ne} \xi.
\]

(5)

The best-fitting values of the parameters are indicated in Table 1 for a range of assumed metallicities. This fitting formula is accurate (in absolute terms) to within 0.01 for \( \sigma_{ne/T} \leq 0.3 \) with \( |\xi| \leq 0.6 \) and within 0.05 for \( \sigma_{ne/T} \leq 0.6 \) with the same range for \( \xi \).

We verified that these conditions hold for the ICM within \( R_{500} \) in simulated clusters (see Section 4.3 and the discussion on Fig. 7). For median temperatures greater than 3 keV, the X-ray emissivity in the 0.5–8 keV band is only a weak function of temperature; at lower temperatures, however, there is a significant dependence due to relative importance of discrete spectral line emission, causing the measured bias to depart significantly from this parametrization. We find that a decrease in the median temperature below \( \sim 1 \text{ keV} \) causes the bias to change steeply in a non-trivial manner, which is difficult to parametrize through a simple form.

In general the bias in pressure for an assumed value of metal abundances depends on four parameters – \( \sigma_T, \sigma_{ne}, \xi \) and \( T_{\text{median}} \). However, we may simplify this by eliminating a parameter using an approximate relation \( \sigma_T = 0.73 \sigma_T - 0.02 \) that is seen in hydrodynamical simulations of clusters (see Fig. 5). With this simplification, in Fig. 3 we plot the bias contours for various values of median temperatures as a function of \( \xi \) and \( \sigma_{ne} \) assuming two values of gas metallicities, \( Z/Z_\odot = 0.2 \) and 0.5; the former being the typical value beyond 0.2 \( R_{180} \), while the latter is indicative of the ICM metallicities at the centre of the cluster (e.g. Leccardi & Molendi...
2008). We see that for ICM temperatures, $T_{\text{median}} > 3$ keV, the bias is small (from $-10$ to $+5$ per cent) for most of the parameter space in the $(\xi, \sigma_{\text{nz}})$ plane. Only for higher values of both the fluctuations and the negative correlation $\xi$, the bias goes beyond $-15$ per cent.

For lower temperatures ($1 < T_{\text{median}} < 3$ keV), this bias can be between $-25$ and $+15$ per cent, depending on the metal abundance; while for even lower temperatures the bias turns positive, mostly lying at the $0$--$10$ per cent level. Note that especially for low temperatures ($T_{\text{median}} < 3$ keV), this bias can be be-

\[ \sigma_{\text{nz}} \approx 1 \text{ keV}, \]

we find that our results on bias are more significant.

In the range of $3$ keV, the bias increases with metallicity.

Two types of simulations were used and

\[ n_{\text{med}} \approx 500 \]

These clusters were selected from low-

\[ T_{\text{M}} > 0.35, \]

as described before in Section 2.1. The fake spec-

\[ g_{\text{art}} < 0.9. \]

Two runs were

\[ n_{\text{med}} = 0.3, \]

and NR run without radiative cooling and star formation. The last column

\[ n_{\text{med}} = 0.3, \]

for density

\[ T_{\text{M}} = 0.35, \]

properties otherwise.

3 SIMULATIONS AND SAMPLE OF GALAXY CLUSTERS

We use a sample of 16 simulated clusters at $z = 0$ taken from Nagai et al. (2007a,b). The simulations were done using the Adaptive Refinement Tree (ART) N-body + gas-dynamics code (Krausov, Klypin & Khokhlov 1997; Kravtsov, Klypin & Hoffman 2002) assuming a flat $\Lambda$ cold dark matter ($\Lambda$CDM) cosmology with the following values for the cosmological parameters: $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.04286$, $h = 0.7$ and $\sigma_8 = 0.9$. Two types of simulations were used with different physics involved, but with the same initial conditions:

(i) non-radiative (NR) runs without any radiative cooling and (ii) cooling plus star formation (CSF) runs, which included metallicity-dependent radiative cooling, star formation, supernova feedback and ultraviolet (UV) background. The clusters in the sample were selected randomly to uniformly sample the mass range, $7 \times 10^{13} < M_{500} < 2 \times 10^{15} M_{\odot}$. These clusters were selected from low-resolution simulations and resimulated at a higher resolution.

The division of the sample into relaxed and unrelaxed subsamples (see Table 2) was done in Nagai et al. (2007a) by visually examining the morphology of mock X-ray images in three different projections. This was done by producing mock images of the simulated clusters using the Chandra response function with a 100 ks exposure. From

Table 2. Properties of simulated clusters in the sample of Nagai et al. (2007a) and Nagai, Kravtsov & Vikhlinin (2007b) at $z = 0$. Two runs were performed with different physics: CSF run with cooling+star formation and NR run without radiative cooling and star formation. The last column indicates if the cluster is visually identified as relaxed (R) or unrelaxed (U).

| Cluster ID | $R_{500}$ ($h^{-1}$ Mpc) | $M_{500}^{\text{CSF}}$ ($10^{14} h^{-1} M_{\odot}$) | Relaxed (R) or unrelaxed (U) |
|------------|--------------------------|---------------------------------|-------------------------------|
| CSF/NR     | CSF/NR                   | CSF/NR                          | CSF/NR                        |
| CL101      | 1.16/1.14                | 9.08/8.62                       | U/U                           |
| CL102      | 0.98/0.95                | 5.45/4.63                       | U/U                           |
| CL103      | 0.99/0.99                | 5.71/5.71                       | U/U                           |
| CL104      | 0.97/0.97                | 5.39/5.31                       | R/R                           |
| CL105      | 0.94/0.92                | 4.86/4.50                       | U/U                           |
| CL106      | 0.84/0.84                | 3.47/3.40                       | U/U                           |
| CL107      | 0.76/0.78                | 2.57/2.74                       | U/U                           |
| CL3        | 0.71/0.70                | 2.09/1.98                       | R/R                           |
| CL5        | 0.61/0.61                | 1.31/1.34                       | R/U                           |
| CL6        | 0.66/0.61                | 1.68/1.32                       | U/R                           |
| CL7        | 0.62/0.60                | 1.41/1.25                       | R/R                           |
| CL9        | 0.52/0.51                | 0.823/0.775                     | U/U                           |
| CL10       | 0.49/0.47                | 0.672/0.621                     | R/R                           |
| CL11       | 0.54/0.44                | 0.899/0.482                     | U/R                           |
| CL14       | 0.51/0.48                | 0.769/0.652                     | R/R                           |
| CL24       | 0.39/0.39                | 0.347/0.347                     | U/U                           |

In this section we shall compute the bias between ’measured’ pressure profiles from X-ray and SZ observations using the sample of simulated clusters introduced in the previous section. We wish to emphasize that the term ‘measured’ (here and henceforth) refers to the value of pressure as would be inferred from observations in X-ray/SZ through a deprojection analysis of real data. Note that this may well be different from the actual values of thermal pressure of the ICM.

Each cluster is divided into 32 concentric shells (spaced in equal logarithmic intervals) between the radii $0.1 < r/R_{500} < 2.3$, in which we compute the histogram of density and temperature values. This is then used to generate fake spectra assuming a metallicity of $Z/Z_{\odot} = 0.35$, as described before in Section 2.1. The fake spectra are then analysed using the Chandra response matrix to give the best-fitting values of density, $n_{\text{med}}$, and temperature, $T_{\text{med}}$, for each shell. The ’measured’ X-ray pressure is then simply $P_X = n_{\text{med}} T_{\text{med}}$. On the other hand the SZ pressure for each shell is obtained by computing the mean thermal pressure in the shell $P_{\text{SZ}} = \sum n_e T_e w_i / \sum w_i$, where $w_i$ is the weight, i.e. the fractional volume occupied by the ICM specified by the values $n_e$ and $T_e$ for density and temperature, respectively. These are used now to compute the

\[ \text{Bias in the X-ray versus SZ pressure profiles} \]

4 THE BIAS FROM SIMULATED GALAXY CLUSTERS

These images the ‘relaxed’ clusters were identified as those with a regular X-ray morphology and without any significant deviations from elliptical symmetry. On the other hand, clusters were classified as ‘unrelaxed’ when departures from elliptical symmetry, filamentary X-ray structures or large isophotal centroid shifts were seen in the produced images. This procedure is similar to that used by observers. Z12 made another classification based on the widths of the density distribution, $\sigma_{\text{nz}}$, at $r = R_{500}$. This classification agrees very well with that done in Nagai et al. (2007a), and the difference in $\sigma_{\text{nz}}(R_{500})$ between relaxed and unrelaxed clusters was at the level of $\pm 2.5 \sigma$. There also exist other schemes of classification e.g. using the two-dimensional (2D) multipole expansion of the projected gravitational potential (Buote & Tsai 1995) or computing the centroid shifts (Poole et al. 2006); however, we did not compare our classification with these.

The ART code uses a non-uniform and dynamic mesh adjusting to the evolving particle distribution, thus allowing for a large dynamic range. To simplify our analysis, the code output (gas densities, temperatures, velocities and metallicities) was interpolated on to a uniform grid with grid cells of size comparable to the effective resolution of the cluster simulations. Following Z12, we randomly sample $4 \times 10^4$ data points within the sphere of radius of $5 h^{-1}$ Mpc with a weight $\propto 1/r^2$, where $r$ is the distance from the cluster centre. This sampling is uniform in azimuthal and polar directions and provides equal number of points per spherical shell of a given thickness. Such a scheme is useful for dealing with averaged quantities computed over spherical shells as discussed further in the following sections.

6 Strictly speaking, assuming a fixed metallicity value for different shells is reasonable only within $\pm 0.5 R_{500}$ where $T_{\text{median}} > 1$ keV for most of the simulated clusters, and the bias depends only weakly on the assumed metal abundance. For $T_{\text{median}} \lesssim 1$ keV, we find that our results on bias are more influenced by metallicity; when higher accuracy is required, especially for lower temperatures, one has to do the direct analysis on a case by case basis.
bias $b_p(r) = P_X(r)/P_{SZ} - 1$ between the ‘measured’ X-ray and SZ pressures in each shell.

### 4.1 Properties of the ICM fluctuations

Z12 used the same set of simulations, as described in Section 3, to study the properties of the inhomogeneities in the ICM. They developed a simple and robust method to separate the HDF from the nearly hydrostatic bulk component. This was implemented by imposing a cut within each shell to exclude all particles having densities, $n_e$, such that $\log(n_e) > \log(n_e,\text{median}) + 3.5\sigma_{n_e}$. Henceforth we shall use the term ‘HDF’ to exclusively denote the particles in the simulation lying above this cut, and the term ‘bulk’ to describe the particles lying within this cut.

Here we review some of the results presented in Z12. Fig. 4 shows the radial profiles indicating the properties of ICM fluctuations for the sample of relaxed clusters in the NR simulations. In Fig. 4(a) the solid lines denote the median values (over the sample of relaxed clusters in the NR run) of the parameters obtained by fitting a bivariate lognormal distribution (see equation 2) within each shell. The dashed lines are the same quantities computed after the removal of the HDF components. The thin lines indicate the maximum and minimum outliers of the sample that shows the scatter across the sample. Fig. 5 shows the relation between the parameters $\sigma_{n_e}$ and $\sigma_T$ in each shell; the straight line is the best-fitting linear relation, $\sigma_T = 0.73\sigma_{n_e} - 0.02$, which is also the assumption used in producing the contours in Fig. 3. The data from NR and CSF simulations indicate an intrinsic scatter of 0.07 in $\sigma_T$ for a given value of $\sigma_{n_e}$ in this relation.

Figs 4(a) and (b) show that the values of the parameters, $\sigma_{n_e}$ and $\xi$, are fairly robust to the exclusion of the HDF components from the bulk.

In both the CSF and NR runs, the unrelaxed clusters have not only higher values of temperature and density fluctuations, but also show significantly larger scatter from cluster to cluster. Both relaxed and unrelaxed clusters show a steady increase in the fluctuation amplitude with radius, which steepens in the outskirts ($r > R_{500}$). As shown in Fig. 4(a), the fluctuations in gas density, $\sigma_{n_e}$, are large beyond $R_{500}$, and these fluctuations are reduced if the HDF components are removed. Fig. 4(b) shows that the correlation between temperature and density fluctuations, $\xi$, is mostly negative, but increases towards zero at larger radii. The increase in $\xi$ on removal of high-density clumps is in line with our expectations, as the clumpy regions contribute to an anticorrelation (or $\xi < 0$) between density and temperature fluctuations.

### 4.2 Removing the contribution from the brightest clumpy regions in the cluster outskirts

We expect the ‘measured’ quantities related to the ICM (e.g. gas pressure, density and temperature) to be less biased when the HDF components are removed from the simulation data. Clearly the removal of HDF according to the criterion described in the previous section is only possible in simulations. In real observations, the complete removal of HDF seems unlikely, due to the presence of...
the X-ray background noise and limited photon statistics, especially relevant at the cluster outskirts or for the high-redshift clusters. Here we shall attempt to check this by dealing directly with the 2D projected X-ray images obtained from the simulated clusters, only in the limit of infinite photon statistics (i.e. without any Poisson noise) and in the absence of instrumental background. However, we shall argue in Section 4.3 that even a partial removal of the clumps, especially the brightest ones (as permitted by the photon statistics), helps to reduce the bias to a large extent.

For each of the clusters, we divide the X-ray images into six radially equal annular regions starting from $r = R_{200}$ as the innermost ring and up to $r = 2.3 R_{200}$. We choose $R_{200}$ as the inner radius for the purpose of removing clumps since we find that the bias outliers become large only beyond this radius, suggesting that the effect of clumps is dominant in the bias profiles only in the outskirts ($r > R_{200}$) of clusters (see also Mathiesen et al. 1999; Nagai & Lau 2011; Battaglia et al. 2012b; Vazza et al. 2012, and Fig. 7). Using the pixels lying within each annular region we then create histograms of the X-ray brightness (in the soft band from 0.5 to 2.0 keV using the Chandra response) and mask all pixels with brightness, $B$, such that $\log(B) > \log(B_{\text{median}}) + f_{\text{cut}} \sigma_B$, to zero values, where $\sigma_B$ is the standard deviation of the logarithm of pixel brightness in each annulus. Using three different values of $f_{\text{cut}}$, 2, 2.5 and 3, we then eliminate all the particles from the simulation data which lie exactly along the line of sight of the masked pixels, some of which are responsible for producing the very bright regions in the outskirts of the X-ray image. We shall henceforth refer to this method of removing the high-density contributions from the X-ray images as the 2D cut in contrast with the 3D cut performed directly on the radial shells and as outlined in Section 4.1 and detailed in section 4 of Z12.

We note that the 2D cuts do not remove all of the clumpy contributions from the 3D radial shells, partly because clumps lying along the line of sight through the bright central region of the projected image of the cluster would not show up with high enough contrast in the brightness images, and would remain in the filtered (i.e. after a 2D cut) simulation data. For a fair comparison of the results from 2D and 3D cuts we attempt to correct this by applying a projection in another plane and repeating the masking procedure in this new plane. Fig. 6 shows a comparison of projected X-ray images of the simulated cluster CL101 under 2D (using $f_{\text{cut}} = 2.5$) and 3D cuts. On the left is the original image produced without any cuts, while the central and the rightmost images are after application of 3D and 2D cuts, respectively. Note that the 3D cut is applied everywhere while the 2D cut is only applied within $R_{200} < r < 2.3 R_{200}$ (indicated by the annulus which is bounded by green circles). We see that the bright clumps in this annulus, visible prominently on the leftmost image, are absent after applying either the 3D or 2D cuts. The fact that this annulus appears very similar after 2D and 3D cuts indicate that both of these cuts remove nearly the same points from the 3D shells.

We now try to examine if a similar 2D cut (or masking) is possible in real observations, where the ability to mask the clumps would depend on the signal-to-noise ratio at $r \gtrsim R_{200}$. To be able to identify and mask such regions it is necessary that the intrinsic fluctuations of surface brightness should dominate over the Poisson fluctuations of the photon counts. We now estimate the minimum number of counts required in order to be able to mask such clumps, provided that they are resolved by the instrument. Let $N$ be the total number of counts, $n_{\text{reg}}$ be the number of independent regions (or pixels used for masking); then, we have $n = N/n_{\text{reg}}$ as the number of counts per region. To reliably remove the region with the surface brightness $X$ times the mean value (i.e. $X \sigma$ counts) one needs (case with no background) as a conservative estimate: $(X - 1)n > 5\sqrt{n}$ for $n \gg 1$, or $(X - 1)n > 5$ for $n \ll 1$. For the case shown in Fig. 6 we use $63 \times 63$ kpc pixels, yielding $n_{\text{reg}} = 9048$ in the annulus from $R_{200}$ to $2.5 R_{200}$. Assuming 100 000 counts for the whole image, and 7.0 per cent flux coming from this annulus, the typical number of counts per pixel $N/n_{\text{reg}} = 0.78$. Thus, pixels with the surface brightness $X > 6.4$ are easily detectable as more than 5σ deviations on top of the mean level. To get 100 000 photons in the 0.5–2.0 keV band the Chandra Advanced CCD Imaging Spectrometer-I (ACIS-I) instrument with an effective area of $\sim 370 \text{cm}^2$ would require an exposure time

\footnote{This is the weighted average effective area over the energy range 0.5–2.0 keV for a hot gas with $T \sim$ few keV. The photon rate used to estimate the exposure was also computed for the same energy range.}
of $\sim 10$ ks for the simulated cluster CL101 with a mass of $\sim 1.2 \times 10^{15} M_\odot$ and $\sim 150$ ks for CL24 with a mass of $\sim 5.0 \times 10^{13} M_\odot$, assuming a redshift of $z = 0.1$ for both objects. Further, we see that the clumps in the image will still be resolved if the size of the pixels is increased by a factor of $\sim 2.5$. This reduces the required number of photons, we now find that pixels with surface brightness $X > 10.25$ can be used for masking, using $f_{\text{cut}} = 2.5$ and $3.0$, even if there are 10000 counts for the whole image.

4.3 Results: bias between X-ray and SZ pressure profiles

Fig. 7(a) shows the median bias profile of the relaxed clusters in the NR run. The profiles with and without HDF components (the 3D cut) are shown by the thick solid blue and thick dashed magenta lines, respectively. Thick dot-dashed black line shows the median profile with the exclusion of the bright clumps beyond $R_{500}$ in the projected map (the 2D cut with $f_{\text{cut}} = 2.5$). The sample outliers are indicated by the thin lines of similar type. Note that this bias is computed from the composite spectra corresponding to the actual distribution of temperature and density values within each shell (see Section 2.1), and not from the idealized lognormal distribution. We find the median bias computed from the relaxed clusters to be within 0 to $+2$ per cent at $r < 0.5 R_{500}$. This then decreases to negative values $\sim -6$ per cent at $r = R_{500}$. The bias profiles remain unaffected by the HDF component (due to small fluctuations) within $R_{500}$, producing almost identical profiles with and without HDF. Beyond $R_{500}$, the median bias profiles from ‘only bulk’ follow the same trend as ‘bulk+HDF’, but are smoother and are limited to within $-2.5$ to $+2$ per cent. With the 2D cuts, the median bias profile is also smoother and agrees with the bias profile after the 3D cut (see the lower panel of Fig. 8).

We find that the unrelaxed clusters display a larger negative bias and larger scatter, both from shell-to-shell and across the sample at a given value of $r/R_{500}$. The cluster-to-cluster scatter increases with radius. At $r = R_{500}$, the scatter is less than 3 and 13 per cent for relaxed and unrelaxed clusters, respectively, for the NR run. At larger radii ($r \sim 2 R_{500}$), the scatter is highly skewed in the positive direction, with some shells showing $b_P \gg 0$ (see Fig. 8), from the presence of dense clumps. After the 3D cut, the scatter in the pressure bias drops to less than 7 per cent within $R_{500}$ even in unrelaxed clusters.

Fig. 7(b) shows the ‘measured’ values of the X-ray and SZ pressures with respect to the median pressure in concentric shells at various radii. Since the value of $P_{\text{SZ}}$ actually corresponds to the
of the original value after the 2D and 3D cuts (as the very dense clumps are cold).

In Fig. 8 we compare the bias profiles in pressure, in the cluster outskirts ($r > R_{500}$), for the various cuts in surface brightness applied in concentric annuli on the projected images of the NR simulations. We also plot for comparison the results from the ‘HDF+bulk’ (no cut) and ‘only bulk’ (3D cut).

We find that application of more stringent cuts (specified by lower values of $f_{\text{cut}}$) reduces the scatter in the bias profiles. Exclusion of the bright clumps in the cluster outskirts ($r > R_{500}$) reduces the outliers (on the positive side) in the bias ($b_p$) profiles, by more than an order of magnitude, from +8.2 to +1.8, +1.2 and +0.7, respectively, at $2.2 R_{500}$, for $f_{\text{cut}} = 3.0, 2.5$ and 2.0. For $f_{\text{cut}} = 2.0$ the excluded clumps have surface brightness between $\sim$9 and 13 times that of the average value (across the inner to the outer annuli). For $f_{\text{cut}} = 2.5$ this is 13 and 18, while for $f_{\text{cut}} = 3.0$ these numbers are 20 and 28. It is clear that such exclusions would remove only the brightest regions associated with high-density clumps, as shown in Fig. 6 for the unrelaxed cluster CL101 in the NR run. On the negative side (i.e. outliers with $b_p < 0$), the various cuts do not affect the outliers as much as for the positive side. We see from the bias profiles plotted in the upper panel of Fig. 8 that the 2D cut specified by the value $f_{\text{cut}} = 2.5$ produces similar results to the 3D cut. A more stringent cut ($f_{\text{cut}} = 2.0$) may further eliminate the intrinsic brightness fluctuations associated with inhomogeneities in the diffuse ICM, while a more relaxed cut ($f_{\text{cut}} = 3.0$) does not remove all the visibly bright clumps. In the lower panel of Fig. 8 we indicate the median values of the bias for the 16 clusters; these are again quite robust to the application of the various 2D and 3D cuts.

Finally, we compare the bias in the median pressure profiles, $b_p = P_X/P_{SZ} - 1$, for both relaxed and unrelaxed clusters in simulations with very different input physics. As expected, we find more clumping in the CSF simulations when compared to the NR simulations; the bias profiles in the CSF runs also display significantly larger scatter from shell to shell (over the same radial range) due to the presence of some very dense clumps. Fig. 9 shows the median bias profiles to be within $-4$ to $+2$ per cent for $r < 0.5 R_{500}$ and within $-6$ to $+2$ per cent for $0.5 R_{500} < r < R_{500}$ for relaxed and unrelaxed clusters in both NR and CSF simulations. At $r \approx 1.2 R_{500}$, the bias reaches $-13$ per cent, arising from the presence of infalling clumps, but reduces to less than $-5$ per cent when the HDF component is removed. In both the NR and CSF simulations the bias increases beyond $r \gtrsim 2 R_{500}$.

More importantly, we find that the pressure bias is limited to within $\pm 15$ per cent for any of the relaxed simulated clusters in both CSF and NR runs of the simulation inside $R_{500}$. Also, the results on median bias profiles are quite similar (see Fig. 9) across very different input physics used in the NR and CSF runs. This suggests that our results are robust to possible uncertainties in the implementation of physical processes in the hydro codes.

5 COMPARISON WITH CURRENT AND FUTURE MEASUREMENTS

High-resolution observations of galaxy clusters jointly in X-rays and millimetre/submillimetre bands provide independent measurements of the thermal pressure profiles through a deprojection analysis. In this work we provide an estimate of the expected bias in the pressure profiles derived from the X-ray and microwave bands taking into account both temperature and density fluctuations in the ICM, as well as the correlation between them. We also predict biases in 3D
density and temperature structure derived from X-ray observations. The bias in density depends on the ability (determined by the photon statistics and X-ray background) to mask out X-ray-emitting gas clumps in the outskirts of galaxy clusters. Our estimates from Section 4.2 seem to suggest that X-ray observations would be able to identify these clumps (if resolved) in clusters up to moderate redshifts (z \sim 0.1).

Accurate measurements of the bias as a function of cluster radius will allow us to probe the inhomogeneity and the nature of perturbations (pressure versus entropy) in the ICM. Our work is also relevant for the use of galaxy clusters in cosmological studies. A robust characterization of biases between X-ray and SZ observations is critical for constraining cosmological parameters through measurements of the angular diameter distances, d_A, at various redshifts (Bonamente et al. 2006; Khedekar & Majumdar 2010). The prediction of the bias in density profiles is also important for the determination of the baryon budget in clusters (e.g. Simionescu et al. 2011; Vazza et al. 2012).

Our results appear to be consistent with the bias (−10 to −40 per cent, from the cluster centre to r = 0.8 R_{500}) in the pressure profiles seen in the Coma cluster (Planck Collaboration et al. 2012b), using data from XMM–Newton and Planck observations. Coma is an unrelaxed cluster and the bias outliers seen in the small sample of unrelaxed simulated clusters range from −15 to +20 per cent and −60 to +20 per cent for the NR and CSF simulations, respectively, in the same radial range. The current results on the ratio Y_{SZ}/Y_X appear to be mixed and inconsistent with each other. On the one hand, recent observations from the CARMA (Bonamente et al. 2012) indicate that this ratio is consistent with unity, while other measurements (e.g. Andersson et al. 2011; Rozo et al. 2012) are quite different from the biases derived from our simulations. In our view this is likely due to some underlying systematic uncertainties associated with the interpretation of the current observations (see Bonamente et al. 2006, for a list of possible systematic errors).

Recently, Battaglia et al. (2012b) estimated biases in the gas mass fraction, f_{gas}, derived from X-ray observations, using a set of cosmological cluster simulations. Our results on the bias in the derived gas density from X-ray data due to clumpiness are consistent with their findings. They estimate the contribution of clumping to M_{gas}(r < R_{500}) to be 10–20 per cent while our results indicate a median bias of 16 per cent in the density at the same radius.

6 CONCLUSIONS

The main conclusions of this paper are summarized below.

(i) We first investigate the dependence of bias between the X-ray andSZ pressures by fitting the X-ray emission from an idealized hot gas having a lognormal distribution of density and temperature values. Our results demonstrate that the bias is expected to be small (within ±10 per cent), as long as the magnitude of density and temperature fluctuations is sufficiently small (σ_r/σ_{mg} < 0.45) and their correlation does not take large negative values (ξ ≈ −0.3), for median temperatures, T_{median} \geq 3 keV. However, the bias may be somewhat larger (≈15 per cent) for clusters with lower temperatures (T_{median} \leq 1 keV). We provide a fitting form to predict the bias as a function of the properties of fluctuations of the ICM for T_{median} \geq 3 keV and plot bias contours in the (ξ, σ) plane that may be used to probe the properties of fluctuations from a given observed ratio of P_X/P_{SZ}. At lower temperatures (T_{median} < 3 keV), the bias is also found to be a function of the metal abundance in the gas.

(ii) We then use a sample of 16 simulated clusters to study properties of the ICM fluctuations. The clusters in the simulations show an increase in the amplitude of both density and temperature fluctuations with radius. The fluctuations of temperature and density are mostly negatively correlated within r < R_{500}. This correlation is close to zero at higher radii. We compute P_X and P_{SZ} profiles using the ICM from simulated clusters as would be measured from X-ray and SZ observations. Both P_{SZ} and P_X are biased with respect to the median pressures values. For P_{SZ} this bias is purely due to the asymmetric (near lognormal) distribution of pressure fluctuations; for P_X, in addition to the bias from asymmetry, there is also a bias arising from fitting a single temperature model to the multicomponent emission spectrum.

(iii) We show that the median bias, b_p(r) ≡ P_X(r)/P_{SZ}(r) − 1, between the pressure estimated from X-ray and SZ observations is small and lies within −6 to +2 per cent, up to R_{500}, even for unrelaxed clusters. The scatter in the pressure bias is significantly smaller for relaxed clusters (<0.03) than unrelaxed clusters (<0.13) at r < R_{500}; however, it becomes large and positively skewed at r ~ 2 R_{500}. For non-relaxed clusters, there is a noticeably higher scatter in the bias values from shell to shell. The outliers responsible for the scatter bias can be reduced by almost an order of magnitude just by using a simple method of excluding the brightest clumps (as permitted by reasonably good photon statistics) from the cluster outskirts (r > R_{500}), for both relaxed and unrelaxed clusters.

(iv) Finally, our results on the median values of pressure bias between X-ray and SZ observations are not significantly different in the two types of simulations with different input physics: (i) NR runs and (ii) CSF runs. Therefore these results (i.e. a small bias...
within $R_{200}$ may be considered to be robust with respect to possible uncertainties in the physical implementations in the hydro codes.

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