Magnetic fields generated by r-modes in accreting millisecond pulsars

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Abstract. In millisecond pulsars the existence of the Coriolis force allows the development of the so-called Rossby oscillations (r-modes) which are known to be unstable to emission of gravitational waves. These instabilities are mainly damped by the viscosity of the star or by the existence of a strong magnetic field. A fraction of the observed millisecond pulsars are known to be inside Low Mass X-ray Binaries (LMXBs), systems in which a neutron star (or a black hole) is accreting from a donor whose mass is smaller than 1 $M_{\odot}$. Here we show that the r-mode instabilities can generate strong toroidal magnetic fields by inducing differential rotation. In this way we also provide an alternative scenario for the origin of the magnetars.

1. Introduction
The r-mode oscillations in all rotating stars are unstable for emission of gravitational waves [1]. These modes play therefore a very important role in the astrophysics of compact stars and in the search for gravitational waves. On the other hand the existence of millisecond pulsars implies the presence of damping mechanisms of the r-modes. Damping mechanisms are associated with bulk and shear viscosity and with the possible existence of the so called Ekman layer. The latter is located at the interface between the solid crust and the fluid of the inner core and in this region friction is significantly enhanced respect to friction in a purely fluid component. All these mechanisms are strongly temperature dependent.

An important class of rapidly rotating neutron stars are the accreting millisecond pulsars associated with Low Mass X-ray Binaries (LMXBs). For these objects the internal temperature is estimated to be in the range $10^8$-$10^{8.5}$ K [2, 3] and their frequencies can be as large as $\sim 650$ Hz. In this range of temperatures and in the case of a purely nucleonic star, bulk and shear viscosities alone cannot stabilize stars whose frequency exceeds $\sim 100$ Hz. A possible explanation of the stability of stars rotating at higher frequencies is based on the Ekman layer, but recent calculations show that this explanation holds only for rather extreme values of the parameters [4]. In this contribution we propose a new damping mechanism based on the generation inside the star of strong magnetic fields produced by r-mode instabilities. This same mechanism has been proposed in the case of rapidly rotating, isolated and newly born neutron stars in [5, 6]. In that paper the mechanism which generates the magnetic field is investigated only during the relatively short period in which the star remains always in the instability region. In our work we consider accreting stars and we investigate the interplay between r-modes and magnetic field on an extremely long period and we show that in this scenario a very strong magnetic field can...
be produced.

2. R-mode equations in the presence of magnetic field

R-mode instabilities are associated to kinematical secular effects which generate differential rotation in the star and large scale mass drifts, particularly in the azimuthal direction. Differential rotation in turn can produce very strong toroidal magnetic fields in the nucleus and these fields damp the instabilities extracting angular momentum from the modes. In order to derive the equations regulating the evolution of r-modes in the presence of a pre-existent poloidal magnetic field we have modified the equations derived in [7], taking into account also the magnetic damping.

We use the estimate given in [1] for the gravitational radiation reaction rate due to the $l = m = 2$ current multipole

$$F_g = \frac{1}{47} M_{1.4} R_{10}^4 P_{-3}^{-6} \text{ s}$$

as well as the bulk and shear viscosity damping rates

$$F_b = \frac{1}{2.7 \times 10^{11}} M_{1.4}^{-1} R_{10}^{-2} T_g^6 \text{ s}$$

$$F_s = \frac{1}{6.7 \times 10^7} M_{1.4}^{5/4} R_{10}^{-23/4} T_g^{-2} \text{ s}$$

where we have used the notation $M_{1.4} = M/1.4M_\odot$, $R_{10} = R/10$ Km, $P_{-3} = P/1$ ms and $T_g = T/10^9$ K.

The total angular momentum $J$ of a star can be decomposed into an equilibrium angular momentum $J_\epsilon$ and a perturbation proportional to the canonical angular momentum of a r-mode $J_c$:

$$J = J_\epsilon(M, \Omega) + (1 - K_j)J_c, \quad J_c = -K_c \alpha^2 J_\epsilon$$

where $K_{(j, \epsilon)}$ are dimensionless constants and $J_\epsilon \equiv I_\epsilon \Omega$.

Following Ref. [8] the canonical angular momentum obeys the following equation:

$$\frac{dJ_c}{dt} = 2J_\epsilon \{ F_g(M, \Omega) - [F_v(M, \Omega, T_v) + F_m_i(M, \Omega, B_p)] \}$$

where $F_v = F_b + F_s$ is the viscous damping rate and we have introduced the magnetic damping rate $F_{m_i}$ that we discuss in the next section.

The total angular momentum satisfies instead the equation:

$$\frac{dJ}{dt} = 2J_\epsilon F_g + \dot{J}_a(t) - I_\epsilon \Omega F_{m_e}$$

where $\dot{J}_a$ is the rate of accretion of angular momentum and we have assumed it to be $\dot{J}_a = M(GMR)^{1/2}$, and $F_{m_e}$ is the magnetic braking rate associated to the poloidal magnetic field.

Combining the equations (4) and (5) than we give the dynamical evolution relation

$$\frac{d\alpha}{dt} = \alpha(F_g - F_v - F_{m_i}) + \alpha[K_j F_g + (1 - K_j)(F_v + F_{m_i})]K_c \alpha^2 - \frac{\alpha M}{2\Omega} \left( \frac{G}{MR^3} \right)^{1/2} + \frac{\alpha F_{m_e}}{2}$$

$$\frac{d\Omega}{dt} = -2K_c \alpha^2[K_j F_g + (1 - K_j)(F_v + F_{m_i})] - \frac{\dot{M} \Omega}{M} + \frac{\dot{M}}{I} \left( \frac{G}{MR^2} \right)^{1/2} - \Omega F_{m_e}$$

where $I = \tilde{I}MR^2$ with $\tilde{I} = 0.261$ for an n=1 polytrope. The previous equations can be simplified when the star is close to the instability region identified by the condition $F_g - F_v - F_m \approx 0$ and
we obtain:
\[
\frac{d\Omega}{dt} = -2\Omega \alpha^2 K_c (F_v + F_m) - \frac{\dot{M}}{M} \Omega + \frac{\dot{M}}{I} \left( \frac{G}{MR^3} \right)^{1/2}
\]
\[
\frac{d\alpha}{dt} = \alpha F_g - \alpha (F_v + F_m) (1 - \alpha^2 K_c) - \frac{\dot{M}}{2I \Omega} \left( \frac{G}{MR^3} \right)^{1/2}
\]
(7)

We note that \( K_c = 9.4 \times 10^{-2} \) and that in this approximation the value of \( K_j \) is unimportant. Moreover the magnetic breaking is negligible for magnetic fields typical of the LMXBs (10^8-10^9 G) and for accretion rates considered (10^{-8}-10^{-9} M_\odot yr^{-1}).

Viscosity depends critically on temperature. We include three factors in modelling the temperature evolution: modified URCA cooling, shear viscosity reheating, and accretion heating. The cooling rate due to the modified URCA reactions, \( \dot{\varepsilon}_u \), is given in [9]

\[
\dot{\varepsilon}_u = 7.5 \times 10^{39} M_{1.4}^{2/3} T_9^8 \text{ erg s}^{-1}
\]
(8)
The neutron star will be heated by the action of shear viscosity on the r-mode oscillations. The heating rate due to shear viscosity, \( \dot{\varepsilon}_s \), is given by [1]

\[
\dot{\varepsilon}_s = 2\alpha^2 \Omega^2 R^2 \tilde{J} F_s = 8.3 \times 10^{37} \alpha^2 \Omega^2 \tilde{J} M_{1.4}^{10/3} R_9^{15/4} T_9^{-2} \text{ erg s}^{-1}
\]
(9)

where \( \tilde{J} = 1.635 \times 10^{-2} \).

Accretion heating have two components. We use the estimates given in [10]. The first contribution arises when accreting matter undergoes nuclear burning at the surface of the star

\[
\dot{\varepsilon}_n = \frac{\dot{M}}{m_B} \times 1.5\text{MeV} = 4 \times 10^{51} \dot{M}_{1.4} \text{ erg s}^{-1}
\]
(10)

where \( m_B \) is the mass of a baryon.

The second contribution arise because the flow is assumed to be advection dominated. Matter falling in towards a star liberate \( \sim GM/R \) of potential energy per unit mass. In a non-advection dominated flow most of this would be dissipated as heat in accretion disk, while in an advection dominated flow this energy is carried in with the flow of matter. The heating rate is

\[
\dot{\varepsilon}_h \sim \frac{R G M \dot{M}}{\lambda} = 8 \times 10^{51} M_{1.4}^{13/6} \dot{M}_{1.4} \text{ erg s}^{-1}
\]
(11)

Finally we use the estimate of the heat capacity \( C_v \) given in [10]

\[
C_v = 1.6 \times 10^{39} M_{1.4}^{1/3} T_9 \text{ erg K}^{-1}
\]
(12)

3. Magnetic damping

The crucial ingredient introduced in the previous Section is the magnetic damping rate, which we have inserted in the evolution of the r-modes. This specific modification of the r-mode equations is at the basis of the phenomenology we are going to discuss and was never considered in previous calculations. The expression of the magnetic damping rate has been derived in [5, 6], where it has been shown that while the star remains in the instability region, the r-modes generate a differential rotation which can greatly amplify a pre-existent magnetic field. More specifically, if a poloidal magnetic field was originally present, a strong toroidal field is generated inside the star. The energy of the modes is therefore transferred to the magnetic field and the instability is damped.
The expression of the magnetic damping rate reads:

$$F_m \equiv \frac{1}{\tau_m} \approx \frac{4(1-p)}{9 \pi p \cdot (8.2 \times 10^{-3})} \frac{B_p^2 R \Lambda' \int_0^t \alpha^2(t') \Omega(t') dt'}{M \Omega},$$

(13)

where $p$ and $\Lambda'$ are dimensionless parameters of order unit. The time integral over the r-mode amplitude $\alpha$ takes contribution from the period during which the star is inside the instability region. The crucial point is if the toroidal magnetic field generated during an instability phase can be stabilized or unwind on an Alfvén timescale.

4. Results

We consider a scenario in which the mass accretion spin up a initially slowly rotating neutron star and we investigate the case in which the star cannot find a state of thermal equilibrium, then it enters a regime of thermo-gravitational runaway within a few hundred years of crossing the r-mode stability curve. In figure 1 and figure 2 we show the evolution of temperature, spin frequency and r-mode amplitude obtained without magnetic field. At the instant $A$ the accreting neutron star enters the r-modes instability window. Until the star is close to equilibrium the instability grows slowly and the star moves to higher temperatures mainly because of non thermal equilibrium. After the instant $B$ the r-mode instability grows exponentially due to the decrease of the shear viscosity with increasing temperature. As a consequence the r-modes amplitude rapidly reaches the saturation value and the viscosity heats significantly the star. At this stage the star loses angular momentum by emission of gravitational waves and within few hundred years goes out of the instability region (instant $C$).

Taking into account the magnetic field, the evolutionary scenario for the star is quite different. After the instant $B$, the secular effects are extremely large and the toroidal magnetic field is either produced or amplified by the wrapping of the poloidal magnetic field produced by the (mostly) toroidal secular velocity field. Our results obtained solving Eqs.(7) and reported in figure 3 show how the new magnetic field so generated reaches a value $B_{tor} \sim 10^{14}$ G in about one hundred years and stabilizes the star when it is still in the instability region.

The subsequent evolution of the system is difficult to control because it strongly depends on the evolution of the magnetic field. Analytical and numerical simulations show that strong purely toroidal fields are unstable. We will limit ourselves to outline a possible scenario that we discuss in a forthcoming paper. Following the results of some numerical simulations, the magnetic field can evolve on an Alfvén timescale into a stable configuration of a mixed poloidal-toroidal twisted-torus shape [11]. In this scenario the newly formed magnetic field can provide a effective damping of the r-modes and rapidly rotating stars can be stabilized also in the absence of the Ekman layer.

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Figure 1. Path followed by the accreting neutron star, without toroidal magnetic field, in the Temperature vs Frequency plane. Here $\dot{M} = 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$ and poloidal magnetic field $B_{\text{pol}} = 10^8 \text{ G}$.

Figure 2. Temporal evolution, without toroidal magnetic field, of frequency, r-mode amplitude and temperature of the accreting neutron star.
Figure 3. Temporal evolution, with toroidal magnetic field, of frequency, r-mode amplitude and temperature of the accreting neutron star.

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