Accuracy of a teleported trapped field state inside a single bimodal cavity

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We propose a simplified scheme to teleport a superposition of coherent states from one mode to another of the same bimodal lossy cavity. Based on current experimental capabilities, we present a calculation of the fidelity that can be achieved, demonstrating accurate teleportation if the mean photon number of each mode is at most 1.5. Our scheme applies as well for teleportation of coherent states from one mode of a cavity to another mode of a second cavity, both cavities embedded in a common reservoir.

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The teleportation phenomenon has received increasing attention, and a number of protocols have been suggested for its implementation in various contexts, for example running waves and cavity-QED. Experimentally, teleportation has been demonstrated for discrete variables and for a single mode of the electromagnetic field with continuous variables. More recently, teleportation between matter and light was announced. Our scheme to teleport a trapped field state with continuous spectra, the "Schrödinger cat"-like state (SCS). The experiments involving teleportation - the cornerstone of universal quantum computation - are expected to be reported soon in the context of high-Q cavity. Aiming at this goal, our group recently proposed a simplified scheme to teleport a superposition of zero- and one-photon states, which makes use of only one single bimodal high-Q cavity, the teleportation occurring from one mode to another inside the high-Q cavity. Pursuing this idea, here we propose an oversimplified scheme in which the Hamiltonian including the required dispersive interaction between an atom and the dissipating cavity field is

\[ H = H_0 + H_I, \]

where

\[ H_0 = \sum_{i=1}^{2} \hbar \omega_i a_i^\dagger a_i + \sum_{k} \hbar \omega_k b_k^\dagger b_k + \frac{\hbar \omega_0}{2} \sigma_z + \sum_{i=1}^{2} \hbar a_i^\dagger a_i \chi_i \sigma_{ee}, \]

\[ H_I = \sum_{k} \hbar \left( \lambda_{1k} a_1^\dagger b_k + \lambda^*_{1k} a_1 b_k^\dagger \right) + \sum_{k} \hbar \left( \lambda_{2k} a_2^\dagger b_k + \lambda^*_{2k} a_2 b_k^\dagger \right). \] 

Here, \( \sigma_{ee} = |e\rangle \langle e| \), and \( a_i \) and \( a_i^\dagger \) are, respectively, the creation and annihilation operators for the \( i \)th cavity mode of frequency \( \omega_i \), and \( b_k \) and \( b_k^\dagger \) are the analogous operators for the \( k \)th reservoir oscillator mode, whose corresponding frequency and coupling with the mode \( \omega_i \) and \( \lambda_i \) are.

The atom-field coupling parameter \( \chi_i = \frac{g^2}{2} \) will always be adjusted to ensure \( g^2 \tau / \delta_i = \pi \), where \( g \) is the Rabi frequency, \( \tau \) is the atom-field interaction time, and \( \delta_i = (\omega_i - \omega_0) \) is the detuning between the field frequency \( \omega_i \) and the atomic frequency \( \omega_0 \). The evolution outside the cavity occurs with \( \chi_i = 0 \). It is important to note that the last term in Eq. involving \( \chi_i \) will be effective only with one mode at a time.

Thus, while the interaction of an atom with mode 1 (2) of the cavity field is taking place, the relative phase due to dispersive interaction of this atom with mode 2 (1) of the cavity field will be negligible. This is true provided that the difference between the two modes be large enough. In addition, to simplify our estimation of the fidelity of the teleported SCS, we will assume that the atom-field coupling is turned on (off) suddenly at the instant the atom enters (leaves) the cavity.

The evolution of coherent states governed by the Heisenberg equations corresponding to Eqs. (1) are given in detail in Ref. [21]. Here, for brevity, we collect only the main results, assuming a reservoir at absolute zero temperature, which is an excellent approximation. The results of interest for modes \( j = 1, 2 \) are

\[ a_1(t) = \sum_{j=1}^{2} u_{1j}(t)a_j(0) + \sum_{k} \vartheta_{1k}(t)b_k(0), \]

\[ a_2(t) = \sum_{j=1}^{2} u_{2j}(t)a_j(0) + \sum_{k} \vartheta_{2k}(t)b_k(0), \]

where
\[
\begin{align*}
  u_{11}(t) &= \exp \left[ -\frac{(A + B)}{2} t \right] \frac{(B - A)}{\sqrt{(B - A)^2 + 4CD}} \sinh \left( \frac{\sqrt{(B - A)^2 + 4CD}}{2} t \right) + \cosh \left( \frac{\sqrt{(B - A)^2 + 4CD}}{2} t \right) \\
  u_{12}(t) &= -\exp \left[ -\frac{(A + B)}{2} t \right] \frac{2C}{\sqrt{(B - A)^2 + 4CD}} \sinh \left( \frac{\sqrt{(B - A)^2 + 4CD}}{2} t \right),
\end{align*}
\]

and
\[
A = i (\omega_1 + \chi + \Delta \omega_1) + \gamma_{11}/2 \\
B = i (\omega_2 + \chi + \Delta \omega_2) + \gamma_{22}/2 \\
C = i \Delta \omega_{12} + \gamma_{12}/2 \\
D = i \Delta \omega_{21} + \gamma_{21}/2.
\]

The \(\gamma_{jj'}\), and \(\Delta \omega_{jj'}\), \(j, j' = 1, 2\), as explained in Ref. [21], are the damping rates and the Lamb-shifts for the two modes, obtained through the Wigner-Weisskopf approximation [22]
\[
\sum_{j'} \frac{\gamma_{jj'}}{\omega_j + \omega_{j'}} \rightarrow i \Delta \omega_{jj'} + \frac{\gamma_{jj'}}{2}, \quad \omega_{jj'} = \Delta \omega_{jj'} + \gamma_{jj'}, u_{11}(t) \text{ and } u_{22}(t) \text{ can be obtained from } u_{12}(t) \text{ and } u_{11}(t), \text{ respectively, by simply swapping } A \text{ and } B; \text{ and } \partial_{jk}(t) \text{ is an unimportant function when the reservoir is kept at zero temperature.}
\]

Eqs. (3)–(6) can be further simplified by assuming the following experimental parameters, in the microwave domain. For the field mode damping times, \(\gamma_{11}^{-1} = 10^{-8} \text{s} \text{ and } \gamma_{22}^{-1} = 0.9 \times 10^{-3} \text{s} \), corresponding respectively to modes 1 and 2, whose frequencies obey the relation \(\omega_2 = \omega_1 + \Delta\), where \(\Delta/2\pi \) can be adjusted in the range 100 kHz to 2 MHz [24].

The two-level atom must be prepared in such a way that the frequency \(\omega_0\) of the atomic transition \(|e\rangle \to |g\rangle\), when the atom enters the cavity, be detuned from mode 1 by \(\delta = \omega_s - \omega_0\) and fulfilling the condition \(g\pi \ll \delta + \kappa\), where \(\kappa\) is the rate of spontaneous emission and \(\pi\) is the mean photon number in mode 1. This condition thus implies, for the detuning with mode 2, \(g\pi \ll \Delta + \delta + \kappa\). Experimentally, the atomic frequency can be Stark shifted using a time-varying electric field to detune the atomic frequency with each mode [24] by the large amount \(\Delta\). As an example, let us consider an experiment setup prepared obeying \(\delta \sim 10^6 \text{Hz}, \ g \sim 10^4 \text{Hz}, \ \Delta \sim 10^5 \text{Hz}\). Then, the interaction with mode 2 (which we are assuming as possessing higher frequency), will be also dispersive, and when the atom – mode 1 interaction produces a \(\pi\) pulse, the coherent state in mode two will evolve according to \(|\beta\rangle \rightarrow |e^{i\phi} \beta\rangle \sim |\beta\rangle\), with \(\phi = g^2 t / (\Delta + \delta) \sim 0.03\), which we take into account when calculating the fidelity. Further, we can assume the cross-damping rates \(\gamma_{12}\) and \(\gamma_{21}\) taking as maximum values those of each mode separately, i.e., \(\gamma_{12}, \gamma_{21} \sim 10^8 s^{-1}\) [26]. With these assumptions and taking into account the dispersive interaction in Eq. (1), Eqs. (5)–(6) are simplified as following.

\[
\begin{align*}
  u_{12}(t) &= u_{21}(t) \cong 0, \quad \text{When the atom is out of or enters the cavity in the ground state, } u_{11}(t) = \exp \left[ -\frac{\pi}{2} - i \omega_1 \right] t \text{ and } u_{22}(t) = \exp \left[ \left( -\frac{\pi}{2} - i \omega_2 \right) t \right].
\end{align*}
\]

When the atom is in the cavity in the excited state, \(u_{11}(t) = \exp \left[ \left( -\frac{\pi}{2} - i (\omega_1 + \chi) \right) t \right]\) and \(u_{22}(t) = \exp \left[ \left( -\frac{\pi}{2} - i (\omega_2 + \chi) \right) t \right]\) when the atom is interacting with mode 1; or \(u_{11}(t) = \left[ \left( -\frac{\pi}{2} - i \omega_1 \right) t \right]\) and \(u_{22}(t) = \exp \left[ \left( -\frac{\pi}{2} - i (\omega_2 + \chi) \right) t \right]\) when the atom is interacting with mode 2. Here \(\chi = (\gamma_{11} + \gamma_{22})/2\), and therefore we have the important result that the damping rate for each of the two modes is simply the mean damping rate of the two modes.

**Ideal process.** The ideal SCS to be teleported is prepared by injecting a coherent state \(|\beta\rangle_2\) into mode 2, assuming \(\lambda_{1k} = 0\) in Hamiltonian (2). Then a two-level atom 1 is laser-excited and rotated in \(R_1\) to an arbitrary superposition \(C_+ |e\rangle_1 + C_- |g\rangle_1\). After that, the atom 1 crosses the cavity, having being velocity-selected to interact off-resonantly with mode 2 such that \(\chi \tau = \pi\), where \(\tau\) is the atom-field interaction time. The atom 1 then crosses \(R_2\), undergoing a \(\pi/2\) pulse, and is detected, inducing a collapse of the cavity field to the even (+) or odd (−) SCS, \(C_+ |\beta\rangle_2 \pm C_- |\beta\rangle_2\), where \(C_+\) and \(C_-\) are unknown coefficients obeying \(|C_+|^2 + |C_-|^2 = 1\). The + (−) sign occurs if the atom 1 is detected in the state \(|g\rangle_1\) \((|e\rangle_1\) ). From now on let us suppose that the even SCS has been prepared.

The procedure to teleport the SCS is as follows. Firstly, the atom 2 crosses the Ramsey zone \(R_1'\), undergoing a \(\pi/2\) pulse, as shown in Fig. 1, being rotated to the superposition

![Diagram of the experimental setup for engineering and teleporting a Schrödinger cat state inside a bimodal cavity.](image-url)

In this diagram, the atom is shown interacting with two cavities labeled as 1 and 2, and detectors 1 and 2 are positioned to monitor the state of the system.
\[ \sqrt{\frac{1}{2}} (|e\rangle_2 + |g\rangle_2). \]
Assuming mode 1 has previously been prepared in the coherent state \( |\alpha\rangle_1 \), the whole state of the system is
\[ |\varphi\rangle = \frac{1}{\sqrt{2}} (|e\rangle_2 + |g\rangle_2) |\alpha\rangle_1 (C_+ |\beta\rangle_2 + C_- |\beta\rangle_2). \] (11)

Next, atom 2 interacts off-resonantly with mode 1, such that \( \chi = \pi \), resulting in:
\[ |\psi\rangle = \frac{1}{\sqrt{2}} [C_+ |e\rangle_2 |\beta\rangle_2 + C_+ |g\rangle_2 |\alpha\rangle_1 |\beta\rangle_2 + C_- |e\rangle_2 |\beta\rangle_2 + C_- |g\rangle_2 |\alpha\rangle_1 |\beta\rangle_2]. \] (12)

Soon after the atom 2 and mode 1 interaction, which leads to Eq. (12), the Stark shift is switched to a large detuned \( \delta = (\omega_a - \omega_b) \), thus freezing the evolution corresponding to mode 1 and, at the same time, initiating the atom 2 and mode 2 interaction. The result, after this interaction, is
\[ |\chi\rangle = \frac{1}{\sqrt{2}} [C_+ |e\rangle_2 |\beta\rangle_2 - |\beta\rangle_2 + C_+ |g\rangle_2 |\alpha\rangle_1 |\beta\rangle_2 + C_- |e\rangle_2 |\beta\rangle_2 + C_- |g\rangle_2 |\alpha\rangle_1 |\beta\rangle_2]. \] (13)

After crossing the bimodal cavity, atom 2 crosses the Ramsey zone \( \mathbb{R}^2 \) undergoing a \( \pi / 2 \) pulse, such that Eq. (13) evolves to
\[ |\varphi\rangle_{2ab} = \frac{1}{2} \left[ |e\rangle_2 |\beta\rangle_2 (C_+ |\beta\rangle_2 - |\beta\rangle_2 + C_+ |g\rangle_2 |\alpha\rangle_1 + C_- |\alpha\rangle_1 + C_- |\alpha\rangle_1 + C_- |\alpha\rangle_1) \right] + |g\rangle_2 |\beta\rangle_2 (C_+ |\alpha\rangle_1 + C_- |\alpha\rangle_1 + C_- |\alpha\rangle_1) \] (14)

Therefore, by detecting the atom 2 and measuring the phase of the field in mode 2, the field state in mode 1 is projected on to one of the four possibilities allowed by Eq. (14). Assuming atom 2 being detected in its ground state, the phase of the field in mode 2 can be measured by injecting a reference field of known amplitude \( \beta \) into mode 2, which makes the field states \( |\beta\rangle_2 \) and \( |\beta\rangle_2 \) in Eq. (14) evolve respectively to the states \( |\beta\rangle_2 \) and \( |0\rangle_2 \). Such states can then easily be distinguished by sending a stream of two-level atoms, all of them in the ground state \( |g\rangle_2 \), to interact resonantly with mode 2 of the cavity field. Thus, if at least one of these atoms are detected in their excited state \( |e\rangle_2 \), indicating the result \( |g\rangle_2 |\beta\rangle_2 \) in Eq. (14), then mode 1 is projected exactly on the desired state \( |\Psi\rangle_1 = C_+ |\alpha\rangle_1 + C_- |\alpha\rangle_1 \), thus completing successfully the teleportation process. On the other hand, if the measurement result is always \( |g\rangle_2 \), indicating the result \( |g\rangle_2 |\beta\rangle_2 \) in Eq. (14), a second atom interacting off-resonantly with mode 1 leads to \( C_+ |\alpha\rangle_1 + C_- |\alpha\rangle_1 \rightarrow C_+ |\alpha\rangle_1 + C_- |\alpha\rangle_1 \), For measurements revealing the states \( |e\rangle_2 |\beta\rangle_2 \) and \( |e\rangle_2 |\beta\rangle_2 \) in Eq. (14), the teleportation process cannot be completed unless additional cavities and/or atoms be introduced, thus overcomplicating the scheme. The teleportation is accomplished provided we let \( \alpha = \beta \), and the probability of success for the ideal case is then limited to 50%.

**Real process.** In real processes, the state \( |\Psi\rangle_1 \) to be teleported will evolve under the influence of the reservoir, becoming a mixture \( \rho(t) \) after traced out the reservoir. To estimate losses in teleportation, we have to compute i) the known value of the reference field \( \beta(t) \) we have to inject in the cavity in order to obtain \( D [\beta(t)] \langle \beta(t) | = 2 |\beta(t)\rangle_2 \), as remarked after Eq. (14), and ii) the fidelity \( F = 1 \langle \Psi | \rho(t) | \Psi \rangle \) of the teleported SCS. To answer question i), we have to compute the evolution \( |\alpha(0)\rangle_1 \langle \beta(0)|_2 (|0\rangle_2) \rightarrow |\Psi(t)\rangle_{12R} \) and then to trace out mode 1 and the infinite modes of the reservoir, denoted by \( |0\rangle_1 \), in order to obtain \( |\beta(t)\rangle_2 \). Again we quote the result in [21]: starting from the initial state \( |\alpha(0)\rangle_1 \langle \beta(0)|_2 (|0\rangle_2) \), we obtain \( |\beta(t)\rangle_2 \rightarrow |\beta(t)\rangle_2 \), where \( \beta(t) \approx u_{22}(t) \beta(0) \), thus answering question i). Note that the remarkable result that at zero temperature a coherent state loses excitation coherently remains valid, even when more than one mode is considered. To answer question ii), we need the evolution of the teleported ideal state \( |\Psi\rangle_1 \) in the presence of mode 2 and the reservoir, i.e., we have to calculate the evolution of the combined state \( |\Psi\rangle_1 |\beta(0)\rangle_2 (|0\rangle_2) \rightarrow (C_+ |\alpha\rangle_1 |\beta(0)\rangle_2 (|0\rangle_2 + C_- |\alpha\rangle_1 |\beta(0)\rangle_2 (|0\rangle_2) \) and, after that, to trace out mode 2 and the reservoir. This calculation only differs from that in i) by the second term. The result is the mixed SCS

\[
\rho_1(t) = N \left\{ \begin{array}{l}
|u_{11}(t)\alpha_0\rangle_{11} \langle u_{11}(t)\alpha_0| + |u_{11}(t)\alpha_0\rangle_{11} \langle u_{11}(t)\alpha_0| + \\
+Z(t) [\langle u_{11}(t)\alpha_0|_{11} \langle u_{11}(t)\alpha_0| + h.c.]
\end{array} \right\}
\] (15)

where \( h.c. \) means Hermitian conjugate, \( \alpha_0 = \alpha(0) \), and \( Z(t) = \exp[-2 \alpha(0)^2 (1 - |u_{11}(t)|^2)] \) is the term responsible for decoherence. It is important to note that while \( t \) in step i) is the time spent preparing SCS in mode 2, in step ii) \( t \) is the time after the SCS is teleported to mode 1. Restricting ourselves to the joint measurement corresponding to \( |g\rangle_2 \langle \beta|_2 \), which is the only result promptly leading to teleportation without requiring additional unitary operations (see Eq. (14)), in about 25% of the trials the final teleported state will be exactly the original SCS provided we let \( \beta(0) = \alpha(0) \). According to [24], the time the atom spends inside the cavity is 40 – 50 µs, while the total flight time in the experiment is in
However, while the portation process is obtained for all times, for the difference reservoir. In this last case our scheme will work irrespective of a second cavity, if both cavities are placed in the same mode of a second cavity, if both cavities are placed in the same mode 

For teleportation of SCS from one mode of a cavity to another in the second mode [25]. Also, our scheme applies as well dependent of the second reservoir as well as of the excitation level Rydberg atoms, the value of the Rabi frequency, i.e., the fidelity resulting from 25% of the trials, it would be possible to recover, from any of the measurement results, the original SCS to be teleported, at the expense of introducing additional cavities and/or atoms. However, this procedure would demand considerable effort, itself decreasing the fidelity of the teleportation process and overcomplicating the present protocol.

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[1] C. H. Bennett et al., Phys. Rev. Lett. 70 (1993) 1895.
[2] S. L. Braunstein, H. J. Kimble, Phys. Rev. Lett. 80, 869 (1998).
[3] C. J. Villas-Bôas, N. G. de Almeida and M. H. Y. Moussa, Phys. Rev. A 60, 2759 (1999).
[4] L. Davidovich et al., Phys. Rev. A 50, R895 (1994).
[5] D. Bowmeester et al., Nature 390, 575 (1997).
[6] D. Boschi et al., Phys. Rev. Lett. 80, 1121 (1998).
[7] Y. H. Kim et al. Phys. Rev. Lett. 86, 1370 (2001).
[8] M. Riebe et al., Nature, 429, 734 (2004).
[9] M. D. Barrett et al., Nature, 429, 737 (2004).
[10] A. Furusawa et al., Science, 282, 706 (1998).
[11] N. Takei et al., Phys. Rev. Lett. 94, 220502 (2005).
[12] J. F. Sherson et al., Nature 443, 557 (2006).
[13] M. H. Y. Moussa, Phys. Rev. A 55 R3287 (1997).
[14] M. Ikrar, Y. Z. Zhu and M. S. Zubairy, Phys. Rev. A 62, 022307 (2000).
[15] Geisa Pires et al., Phys. Rev. A 70, 025803 (2004).
[16] Sh-Biao Zheng, Phys. Rev. A 69, 064302 (2004).
[17] J. M. Raimond, M. Brune, and S. Haroche, Phys. Rev. Lett. 79, 1964 (1996); M. Weidinger et al., Phys. Rev. Lett. 82, 3795 (1999).
[18] A. Rauschenbeutel et al., Phys. Rev. A 64, 050301(R) (2001).
[19] D. Gottesman, and I. L. Chuang, Nature 402, 399 (1999).
[20] G. Pires et al., Phys. Rev. A 71, 060301(R) (2005).
[21] Iara P. de Queirós, Wesley B. Cardoso, and N. G. de Almeida, J. Phys. B: At. Mol. Opt. Phys. 40, 1 21-27 (2007).
[22] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge Univ. Press,1995); M. O. Scully, and M. S. Zubairy, Quantum Optics (Cambridge Univ. Press, 1997).
[23] S. Osnaghi et al., Phys. Rev. Lett. 87, 037902 (2001).
[24] P. Domokos et al., Eur. Phys. J. D 1, 1-4 (1998).
[25] Iara P. de Queirós and N. G. de Almeida, unpublished.
[26] A. R. Bosco de Magalhaes et al., Physica A 341, 234 (2004). For an investigation of the cross decay rates see also quant-ph/0410200 v1.