A Globally Convergent State Observer for Multimachine Power Systems with Lossy Lines

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Abstract: We present the first solution to the problem of estimation of the state of multimachine power systems with lossy transmission lines. We consider the classical three-dimensional “flux-decay” model of the power system and assume that the active and reactive power as well as the rotor angle and excitation voltage at each generator is available for measurement—a scenario that is feasible with current technology. The design of the observer relies on two recent developments proposed by the authors: a parameter estimation based approach to the problem of state estimation and the use of the dynamic regressor extension and mixing technique to estimate these parameters. Thanks to the combination of these techniques it is possible to overcome the problem of lack of persistent excitation that stymies the application of standard observer designs. Simulation results illustrate the performance of the proposed observer.

Keywords: Electric power systems, Observers, Nonlinear systems, Dynamic state estimation

1. INTRODUCTION

Power systems are experiencing major changes and challenges, such as an increasing amount of power-electronics-interfaced equipment, growing transit power flows and fluctuating (renewable) generation, see (Winter et al., 2015). Therefore power systems are operated under more and more stressed conditions and, thus, closer to their stability limits as ever before, see (Winter et al., 2015). In addition, as detailed in (Milano et al., 2018), their dynamics become faster, more uncertain and also more volatile. Hence, fast and accurate monitoring of the system states is crucial in order to ensure a stable and reliable system operation, see (Zhao et al., 2019). This, however, implies that the conventional monitoring approaches based on steady-state assumptions are no longer appropriate and instead novel dynamic state estimation (DSE) tools have to be developed, see (Zhao et al., 2019; Singh and Pal, 2018).

By recognizing this need, DSE has become a very active research area in the past years, see (Zhao et al., 2019; Singh and Pal, 2018). The interest in DSE has been further accelerated by the growing deployment of phasor measurement units (PMUs), see (Terzija et al., 2010). As of today, the prevalent algorithms in the literature for DSE are Kalman Filter techniques, including Extended Kalman Filters, see (Ghahremani and Kamwa, 2011; Paul et al., 2018) and Unscented Kalman Filters, see (Valverde and Terzija, 2011; Wang et al., 2011; Anagnostou and Pal, 2017). The main reasoning behind using these techniques is that they are in principle applicable to nonlinear systems, see (Julier and Uhlmann, 1997; Wan and Van Der Merwe, 2000), such as power system models. However, a major drawback of the aforementioned results is that the estimator convergence is only shown empirically, e.g., via simulations, but no rigorous convergence guarantees are provided.

An important application of DSE is the design and practical implementation of excitation controllers for transient stability improvement, e.g., via power system stabilizers, see (Zhao et al., 2019). To the best of our knowledge, all existing solutions to this problem with rigorous stability claims rely on the availability of the full state of the system. The assumption that all generating units dispose of full state measurement severely stymies the practical applicability of these controllers. Besides its potential application in excitation controllers, the present result may be utilized by the wide-area control system to control the transient and oscillatory dynamics of system voltage and frequency (Singh and Pal, 2018). See the recent books by (Sauer et al., 2017; Singh and Pal, 2018) for a detailed treatment on this new technological developments.
Recently, power system state observers have been proposed in (Rinaldi et al., 2017, 2018, 2019) based on sliding mode techniques and their convergence has been established under certain assumptions. Yet, all these results neglect the voltage dynamics and assume purely inductive transmission lines. In (Anagnostou et al., 2018) an observer-based anomaly scheme has been derived using a detailed linearized power system model in combination with a linear time-varying observer.

The abovementioned facts and developments motivate the observer design conducted in the present paper. For its derivation, we consider a large-scale power system consisting of $n$ synchronous generators represented by the standard three-dimensional "flux-decay" model, see (Kundur, 1994; Sauer et al., 2017), and interconnected through a transmission network, which we assume to be lossless. That is, we explicitly take into account the presence of transfer conductances. It is well known that the dynamics of power systems with lossy transmission lines is considerably more complicated than the lossless one—see (Anderson and Fouad, 2003; Ortega et al., 2005) for a discussion on this issue.

The main contribution of the present work is the proof that using the measurements of active and reactive power as well as rotor angle and excitation voltage at each generator—a reasonable assumption given the latest technology, see (Yang et al., 2007)—it is possible to recover the full state of a multimachine power system, even in the presence of lossy lines. An important observation that is made in the paper is that, under the assumed measurement scenario, the problem of estimation of the generator voltages can be recast as estimation of the state of a linear time-varying (LTV) system, to which classical state estimation techniques, e.g., Kalman-Bucy filters (Anderson, 1971), can be applied. Unfortunately, we prove in the paper that the associated LTV system is not uniformly completely observable (UCO), a necessary condition to ensure convergence of the estimated states, see (Rueda-Escobedo et al., 2019) for a recent discussion of this issue and (Zhao et al., 2019) for a general discussion on the challenges of observability analysis in DSE.

To overcome this problem we invoke two recent results reported by the authors.

1. Following the parameter estimation based observer (PEBO) design proposed in (Ortega et al., 2015), we reformulate the problem of state observation into one of parameter estimation.
2. The unknown parameters are reconstructed using the dynamic regression extension and mixing (DREM) procedure (Aranovskiy et al., 2017), which is a novel, powerful, parameter estimation technique particularly suitable for PEBO design problem.

See (Ortega et al., 2019; Pyrkin et al., 2019) for two recent applications of these combined techniques. Two outstanding features of DREM estimators that are exploited in the paper are that, on one hand, no appeal is made to persistent excitation (PE) arguments to prove its convergence. Instead, a condition of non-square integrability of a scalar signal is imposed in DREM, see (Aranovskiy et al., 2017) for further details. On the other hand, as shown in (Gerasimov et al., 2018), a simple modification to DREM allows to achieve the very important feature of finite-time convergence (FTC) under the weakest sufficient excitation assumption (Kreisselmeier and Rietze-Augst, 1990). The convergence conditions mentioned above are, of course, necessary since as is well-known some excitation assumptions are needed to ensure the success of state observers (Besançon, 2007; Bernard, 2019) and parameter estimators (Ljung, 1987; Sastry and Bodson, 1989).

The remainder of the paper is organized as follows. The mathematical model of the power system and the problem formulation is given in Section 2. The proposed observers and their stability proof are given in Section 3. Simulation results with a two machine example, which illustrate the performance of the proposed observer, are presented in Section 4. The paper is wrapped-up with concluding remarks in Section 5.

2. SYSTEM MODEL AND STATE OBSERVER PROBLEM FORMULATION

As standard in centralized DSE, see (Zhao et al., 2019), we consider a Kron-reduced power system consisting of $n > 1$ interconnected machines and with the dynamics of the $i$-th generator described by the classical third order model
defined as

$\delta_i = \omega_i$, $M_i\dot{\omega}_i = -D_{mi}\omega_i + \omega_i(P_{mi} - P_{ci})$, $\tau_iE_i = -E_i - (x_{di} - x_{qi})I_{di} + E_{f1} + \nu_i$, $i \in \mathbb{N} := \{1, ..., n\}$,

where the state variables are the rotor angle $\delta_i \in [0, 2\pi)$, rad, the speed deviation $\omega_i \in \mathbb{R}$ in rad/sec and the generator quadrature internal voltage $E_i \in \mathbb{R}_+$, $I_{di}$ is the d axis current, $P_{ci}$ is the electromagnetic power, the voltages $E_{f1}$ and $\nu_i$ are the constant voltage component applied to the field winding, and the control voltage input, respectively. Furthermore, $D_{mi}$, $M_i$, $P_{mi}$, $\tau_i$, $\omega_0$, $x_{di}$ and $x_{qi}$ are positive parameters.

The active $P_{ci}$ and reactive $Q_{ci}$ power are defined as

$P_{ci} = E_iI_{qi}$, $Q_{ci} = E_iI_{di}$, (2)

where $I_{qi}$ is the q axis current.

These currents establish the connections between the machines and are given by

$I_{qi} = G_{mi}E_i + \sum_{j=1, j \neq i}^{n} E_jY_{ij}\sin(\delta_{ij} + \alpha_{ij})$

$I_{di} = -B_{mi}E_i - \sum_{j=1, j \neq i}^{n} E_jY_{ij}\cos(\delta_{ij} + \alpha_{ij})$, (3)

where we defined $\delta_{ij} := \delta_i - \delta_j$ and the constants $Y_{ij} = Y_{ji}$ and $\alpha_{ij} = \alpha_{ji}$ are the admittance magnitude and admittance, respectively.

1 We recall that PE of the classical linear regression model is equivalent to non UCO of the associated LTV system (Sastry and Bodson, 1989).
2 To simplify the notation, whenever clear from the context, the qualifier "$i \in \mathbb{N}$" will be omitted in the sequel.
admittance angle of the power line connecting nodes $i$ and $j$, respectively. Furthermore, $G_{mi}$ is the shunt conductance and $B_{mi}$ the shunt susceptance at node $i$. Finally, combining (1), (2) and (3) results in the well-known compact form (Ortega et al., 2005; Langarica et al., 2015)

$$\dot{\delta}_i = \omega_i$$

$$\dot{\omega}_i = -d_i \omega_i + P_i - d_i \sum_{j=1, j \neq i}^n E_{ji} Y_{ij} \sin(\delta_{ij} + \alpha_{ij})$$

$$\dot{E}_i = -a_i E_i + b_i \sum_{j=1, j \neq i}^n E_{ji} Y_{ij} \cos(\delta_{ij} + \alpha_{ij}) + u_i,$$

where we have defined the signals

$$u_i := \frac{1}{\tau_i} (E_{fi} + v_i)$$

and the positive constants

$$D_i := \frac{M_{mi}^2}{M_i}, \quad P_i := d_i M_{pi}, \quad d_i := \frac{\omega_i}{M_i}$$

$$a_i := \frac{1}{\tau_i} [1 - (x_{di} - x_{di}')] B_{mi}, \quad b_i := \frac{1}{\tau_i} (x_{di} - x_{di}').$$

To formulate the observer problem we consider that all parameters are known, and make the following assumption on the available measurements.

**Assumption 1.** The signals $\delta_i$, $u_i$, $P_{ei}$ and $Q_{ei}$ of all generating units are measurable.

As usual in observer problems (Besançon, 2007; Bernard, 2019) we assume that $u_i$ is bounded and such that all state trajectories are **bounded**.

**Problem Formulation:** Consider the multimachine power system (4), verifying Assumption 1. Design an observer 3

$$\dot{\hat{X}} = F(\chi, \delta, P_e, Q_e)$$

$$\dot{\hat{E}} = H(\chi, \delta, P_e, Q_e, \hat{\omega})$$

such that

$$\lim_{t \to \infty} \begin{bmatrix} \dot{\hat{E}}(t) \\ \dot{\hat{\omega}}(t) \end{bmatrix} = 0,$$

where we, generically, define the estimation errors $(\cdot) := (\cdot) - (\cdot)$.

Although the centralized measurement of the rotor angles $\delta_i$ is unlikely in applications, the observer problem without this assumption is a daunting task. Even if this transfer of information is possible, another practical consideration that is neglected in the problem formulation is the difficulty of synchronizing these measurements (Singh and Pal, 2018; Sauer et al., 2017).

3. MAIN RESULT

Before presenting the proposed observer we first give in this section an LTV systems perspective of the voltage dynamics and show its fundamental lack of observability limitation. To overcome the latter, we then adopt a PEBO perspective for the state observation problem and apply the DREM procedure to estimate the unknown parameters.

3.1 An LTV description of the voltage dynamics

To simplify the notation we find convenient to introduce the following notation

$$\Delta_{ij}(t) := \delta_i(t) - \delta_j(t) + \alpha_{ij}$$

$$A(t) := \begin{bmatrix} -a_1 & b_1 Y_{12} \cos \Delta_{12}(t) & \cdots & b_1 Y_{1n} \cos \Delta_{1n}(t) \\ b_2 Y_{21} \cos \Delta_{21}(t) & -a_2 & \cdots & b_2 Y_{2n} \cos \Delta_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ b_n Y_{n1} \cos \Delta_{n1}(t) & b_n Y_{n2} \cos \Delta_{n2}(t) & \cdots & -a_n \end{bmatrix}$$

Given this notation we observe that we can write the voltage equations of (4) as an LTV system

$$\dot{E} = A(t) E + u.$$

We underscore that, under the standing assumptions, $A(t)$ is measurable and obviously bounded.

The following simple lemma is instrumental for our further developments.

**Lemma 2.** There exists a measurable matrix $C(t) := C(P_e(t), Q_e(t), \delta(t)) \in \mathbb{R}^{n \times n}$ such that

$$C(t) E = 0$$

(8)

**Proof.** From (2) we have that

$$P_e I_d - Q_e I_q = 0.$$

Clearly, the equations (3)—which are linearly dependent on $E$—may be written in the compact form

$$I_q = S(\delta) E, \quad I_d = T(\delta) E,$$

(9)

for some suitably defined $n \times n$ matrices $S(\delta)$ and $D(\delta)$. The proof is completed replacing (9) in the identity above and defining

$$C(P_e, Q_e, \delta) := \begin{bmatrix} P_{e1} T_{1}^T (\delta) - Q_{e1} S_{1}^T (\delta) \\ \vdots \\ P_{en} T_{n}^T (\delta) - Q_{en} S_{n}^T (\delta) \end{bmatrix},$$

(10)

where $T_{i}^T (\delta)$, $S_{i}^T (\delta)$ are the rows of the matrices $D(\delta)$ and $S(\delta)$, respectively.

One might be tempted to view the system (7) and (8) as an LTV system with output $y_E := C(t) E$ that—in view of (8)—turns out to be equal to zero, and design a classical Kalman-Bucy filter for it (Anderson, 1971). That is, to propose the Kalman-Bucy observer

$$\dot{\hat{E}} = [A(t) - H(t) C^T(t) C(t)] \hat{E} + u$$

$$H = H A^T(t) + A(t) H - H C^T(t) C(t) H + S, \quad H(0) > 0, \quad S > 0,$$

which ensures $\lim_{t \to \infty} \hat{E}(t) = 0$ (exponentially), provided the pair $(A(t), C(t))$ is UCO—see (Anderson, 1971) and the derivations in Section 5 of (Ruiza-Escobedo et al., 2019). Alas, this system turns out to be non UCO. Indeed, we recall the following (Sastry and Bodson, 1989)

4 Actually, it is shown in (Anderson, 1971) that the UCO condition ensures the Riccati equation above has a bounded, invertible solution $H$, which is necessary for the analysis of the associated observer error dynamics.
An LTV system $\dot{x} = A(t)x$, $y = C(t)x$, with bounded matrices $A(t), C(t)$ is uniformly completely observable (UCO) if there exists positive constants $c_1, c_2$ and $T$ such that
\[ c_1 I_n \geq \int_{t_0}^{t_0 + T} \Phi^T(r, t_0) C^T(r) C(r) \Phi(r, t_0) dr \geq c_2 I_n, \forall t_0 \geq 0, \]
where $\Phi(t, t_0)$ is the system’s state transition matrix.

Now, we recall that the state trajectories of the system $\dot{x} = A(t)x$ with initial condition $x(t_0) \in \mathbb{R}^n$ satisfy (Demidovich, 1967; Rugh, 1996)
\[ x(t) = \Phi(t, t_0)x(t_0), \forall t \geq t_0. \] (11)
In our scenario $C(t)x(t) = 0$, hence, it is clear that the lower bound on the inequality of Definition 3 cannot be satisfied.

### 3.2 An PEBO approach to the estimation of the voltage

The lemma below illustrates how, applying the PEBO approach proposed in (Ortega et al., 2015), we can generate a linear regression equation (LRE) where estimation of the unknown parameters leads to estimation of the unknown state $E$.

**Lemma 4.** Consider the dynamic equations of the voltage (6)-(8) and the dynamic extension
\[ \dot{\xi}_E = A(t)\xi_E + u, \quad \dot{\phi} = A(t)\phi, \phi(0) = I_n. \] (12)
There exists a constant vector $\theta \in \mathbb{R}^n$, and measurable signals $y \in \mathbb{R}^n$ and $\psi \in \mathbb{R}^{n \times n}$ such that
\[ E = \xi_E - \phi \theta \]
\[ y = \psi \theta. \] (13) (14)
**Proof.** Define the signal $e := \xi_E - E$. From (7) and (12) we see that $e$ satisfies
\[ \dot{e} = A(t)e. \]
From the equation above and the properties of the state transition matrix $\Phi$ of $A(t)$ given in (11), we conclude that there exists a constant vector $\theta \in \mathbb{R}^n$ such that $e = \phi \theta$ and, consequently, (13) holds.

To establish (14) we multiply (13) by $C(t)$ and, recalling (8), obtain the equation
\[ C(t)\xi_E = C(t)\phi \theta. \]
The proof is completed defining
\[ y := C(t)\xi_E \]
\[ \psi := C(t)\phi. \] (15)

### 3.3 Generation via DREM of n scalar LRE for the unknown parameters $\theta_i$

In view of Lemma 4 the only remaining step to construct the observer for $E$ is to, proceeding from the linear regression (14), propose an estimator for $\theta$. Here, again, we are confronted with the lack of UCO mentioned in Subsection 3.1. Indeed, the UCO condition of Definition 3 is equivalent to the PE of the regressor $\Psi$. Hence, standard gradient or least squares estimators will not ensure (exponential) convergence (Sastry and Bodson, 1989). To bypass this difficulty we invoke the DREM parameter estimation procedure proposed in (Aranovskiy et al., 2017).

Although the construction of DREM allows for the use of general, LTV, $L_\infty$-stable, operators, for the sake of simplicity we consider here the use of simple LTI filters. Towards this end, we propose a stable transfer matrix $F(s) \in \mathbb{R}^{n \times n}(s)$, and define the signals
\[ Y := F(p)y \in \mathbb{R}^n \]
\[ \Psi := F(p)\psi \in \mathbb{R}^{n \times n} \]
\[ \gamma := \text{adj(} \Psi \text{)} \in \mathbb{R}^n \]
\[ \Delta := \text{det}(\Psi) \in \mathbb{R}, \] (16)
where $p := \frac{d}{dt}$ and $\text{adj(} \cdot \text{)}$ is the adjugate operator.

**Lemma 5.** Consider the LRE (14) and the signals (16). Then, the following scalar LREs hold
\[ \gamma_i = \Delta \theta_i \] (17)
**Proof.** Applying the filter $F(p)$ to the LRE (14) we obtain
\[ Y = (\Psi + \eta) \gamma \]
where $\eta$ is an exponentially decaying term that, following the standard procedure, is neglected in the sequel. The proof is completed by multiplying the equation above by $\text{adj(} \Psi \text{)}$ and recalling that for any, possibly singular, $n \times n$ matrix $B$ we have $\text{adj(} B \text{)} B = \text{det(} B \text{)} I_n$. □ □ □

### 3.4 Proposed state observers

We are now in position to present our main result: two globally convergent observers for the state of the multimachine power system (4). The first one ensures asymptotic convergence while the latter guarantees FTC. In both cases, the required excitation conditions are rather weak.

**Proposition 6.** Consider the multimachine power system (4), verifying Assumption 1. Fix an $n \times n$ stable transfer matrix $F(s)$ and $2n$ positive numbers $\gamma_i$ and $k_{\omega_i}$. The voltage state observer defined by (6), (10), (12), (15), (16), together with
\[ \dot{\gamma}_i = -\gamma_i \Delta(\Delta \theta_i - \gamma_i) \]
\[ \dot{E} = \xi_E - \phi \theta, \] (18)
and the speed observer given by
\[ \dot{\omega}_i = -D_i \omega_i + P_i - d_i P_{ei} - k_{\omega_i} \omega_i \]
\[ \tilde{\omega}_i = \xi_{\omega_i} + k_{\omega_i} \delta_i, \] (19)
ensure (5), with all signals bounded, provided $\Delta \notin \mathcal{L}_2$.

**Proof.** Replacing (14) in the parameter estimator equation yields
\[ \dot{\theta}_i = -\gamma_i \Delta^2 \theta_i. \] (20)
Given the standing assumption on $\Delta$ we have that $\dot{\theta}(t) \to 0$. Hence, invoking (13) and (18), this implies that $\dot{E}(t) \to 0$.

To prove the convergence of the speed estimator notice that, using (2) and (3), the rotor speed dynamics (4) may be written as
\[ \dot{\omega}_i = -(D_i + k_{\omega_i}) \omega_i. \]
Then, compute from (4) and (19) the error dynamics
\[ \dot{\tilde{\omega}}_i = -(D_i + k_{\omega_i}) \tilde{\omega}_i. \]
This completes the proof of the proposition. □□□

As shown in (Gerasimov et al., 2018), a simple modification to DREM allows to achieve FTC for the voltage observer, under the weakest sufficient excitation assumption (Kreisselmeier and Rietze-Augst, 1990), which is articulated in the assumption below. For ease of presentation, and without loss of generality, we assume that all the adaptation gains $\gamma_i$ in the parameter estimator (18) are equal to $\gamma > 0$.

Assumption 7. Fix a small constant $\mu \in (0, 1)$. There exists a time $t_c > 0$ such that

$$\int_0^{t_c} \Delta^2(\tau) d\tau \geq \frac{1}{\gamma} \ln(1 - \mu).$$

Proposition 8. Consider the multimachine power system (4), verifying Assumption 1. Fix an $n \times n$ stable transfer matrix $F(s)$, a positive numbers $\gamma$ and $\mu \in (0, 1)$. The voltage state observer defined by (6), (10), (12), (15), (16), together with

$$\dot{\hat{\theta}} = -\gamma \Delta (\hat{\theta} - \gamma),$$

$$\dot{w} = -\gamma \Delta^2 w, \quad w(0) = 1$$

ensures

$$\tilde{E}(t) = 0, \quad \forall t \geq t_c$$

with all signals bounded, provided $\Delta(t)$ verifies Assumption 7.

Proof. First, notice that the definition of $w_c$ ensures that $\tilde{E}$ is well-defined. Now, from (20) and the definition of $w$ we have that

$$\tilde{\theta} = w \hat{\theta}(0).$$

Clearly, this is equivalent to

$$\int_0^t (1 - w) \theta = \hat{\theta} - w \hat{\theta}(0).$$

On the other hand, under Assumption 7, we have that $w_c(t) = w(t), \quad \forall t \geq t_c$. Consequently, we conclude that

$$\int_0^t 1 - w_c \theta(0) \theta = \theta, \quad \forall t \geq t_c.$$

Replacing this identity in (22) completes the proof. □□□

The following remarks are in order.

(R1) The stability properties established in Propositions 6 and 8 are “trajectory-dependent”, in the sense that they pertain only to the trajectory generated for the given initial conditions. This means that the flow of the observer dynamics may contain unbounded trajectories, and the appearance of a perturbation may drive our “good” trajectory towards a “bad” one. This is, of course, a robustness problem that needs to be further investigated.

(R2) It is interesting to note that the “excitation” condition imposed by Assumption 7 can be weakened increasing the adaptation gain $\gamma$. However, as is well-known, the use of fast adaptation entails a series of undesirable phenomena. On the other hand, if the assumption of $\Delta \notin L_2$ is strengthened to $\Delta$ being PE, then the convergence is exponential.

4. SIMULATION RESULTS

For simulation we consider a two-machine system, see for instance (Ortega et al., 2005). The dynamics of the system result in the sixth-order model

$$\begin{align*}
\dot{\delta}_1 &= \omega_1, \\
\dot{\omega}_1 &= -D_1 \omega_1 + P_1 - G_{11} E_1^2 - Y_{12} E_1 E_2 \sin(\delta_{12} + \alpha_{12}) \\
E_1 &= -a_1 E_1 + b_1 E_2 \cos(\delta_{12} + \alpha_{12}) + E_f + \nu_1; \\
\dot{\delta}_2 &= \omega_2, \\
\dot{\omega}_2 &= -D_2 \omega_2 + P_2 - G_{22} E_2^2 + Y_{21} E_1 E_2 \sin(\delta_{12} + \alpha_{12}) \\
E_2 &= -a_2 E_2 + b_2 E_1 \cos(\delta_{21} + \alpha_{21}) + E_f + \nu_2,
\end{align*}$$

with the current equations defined as

$$I_{q1} = G_{11} E_1 + E_2 Y_{12} \sin(\delta_{12} + \alpha_{12})$$

$$I_{q1} = -B_{11} E_1 - E_2 Y_{12} \cos(\delta_{12} + \alpha_{12})$$

$$I_{q2} = G_{22} E_2 + E_1 Y_{21} \sin(\delta_{21} + \alpha_{21})$$

$$I_{d2} = -B_{22} E_2 - E_1 Y_{21} \cos(\delta_{21} + \alpha_{21}).$$

In this case we have that

$$A(t) = \begin{bmatrix} -a_1 & b_1 \cos(\delta_{12}(t) + \alpha_{12}) \\ b_2 \cos(\delta_{21}(t) + \alpha_{21}) & -a_2 \end{bmatrix}$$

$$S(\delta) = \begin{bmatrix} G_{11} & Y_{12} \sin(\delta_{12} + \alpha_{12}) \\ Y_{21} \sin(\delta_{21} + \alpha_{21}) & G_{22} \end{bmatrix}$$

$$T(\delta) = \begin{bmatrix} -B_{11} & Y_{12} \cos(\delta_{12} + \alpha_{12}) \\ -B_{22} & -Y_{21} \cos(\delta_{21} + \alpha_{21}) \end{bmatrix}.$$}

For the observer design we selected the simplest filter

$$F(p) = \begin{bmatrix} 1 & 0 \\ k & p + k \end{bmatrix},$$

with $k > 0$. The parameters of the model (24) are given in Table 1.

| Parameter | Initial values | After load change |
|-----------|----------------|-------------------|
| $Y_{12}$  | 0.1032         | 0.1032            |
| $Y_{21}$  | 0.1032         | 0.1032            |
| $b_1$     | 0.0223         | 0.0223            |
| $b_2$     | 0.0205         | 0.0205            |
| $D_1$     | 1              | 1                 |
| $D_2$     | 0.2            | 0.2               |
| $\nu_1$   | 1              | 1                 |
| $\nu_2$   | 1              | 1                 |
| $B_{11}$  | -0.4373        | -0.5685           |
| $B_{22}$  | -0.4294        | -0.3582           |
| $G_{11}$  | 0.0966         | 0.1256            |
| $G_{22}$  | 0.0626         | 0.1204            |
| $a_1$     | 0.2614         | 0.2808            |
| $a_2$     | 0.2532         | 0.2804            |
Fig. 1. Transients of the first voltage observation error $E_1 - \hat{E}_1$ for DREM and FTC observers with a 30% load change at $t = 10$ sec

Fig. 2. Transients of the second voltage observation error $E_2 - \hat{E}_2$ for DREM and FTC observers with a 30% load change at $t = 10$ sec

for the system $E(0) = \text{col}(7,6)$, and zero for all remaining states. All initial conditions for the observers were also set equal to zero. For the DREM observer two different adaptation gains $\gamma_i$ were tried. For the FTC observer we set $\mu = 0.1$ of Assumption 7. For the speed observer we selected different values of the coefficient $k_{\omega i}$. For the sake of comparison we also present the behavior of the signals $\xi_{Ei}$ that—as since the matrix $A(t)$ corresponds to an asymptotically stable system—provides also an estimate of the voltage $E$. The results of the simulations are shown in Fig. 1-Fig. 4. As expected by the theory, the speed of convergence of the DREM estimator for the voltage increases for large values of $\gamma_i$, with a similar behavior of the speed error with respect to $k_{\omega i}$. Also, it is seen that the FTC estimator has the fastest convergence, and this happens in finite-time. It is interesting to remark that the behavior of the observers is highly insensitive to the load change, as it is hardly visible in $\hat{E}_2$.

5. CONCLUDING REMARKS AND FUTURE RESEARCH

We have given in this paper a solution to the problem of state observation of multimachine power systems with lossy lines assuming measurable the rotor angles, the excitation voltage and the active and reactive power of all generators. To the best of the authors’ knowledge this is the first time a solution like this is presented. It is shown that, under a suitable observability assumption, global convergence of the voltage estimator is achieved.

By imposing the classical PE condition, the convergence is exponential ensuring good robustness properties. The speed observation is ensured without imposing any excitation condition, and it is exponential with tunable and, possibly, arbitrarily fast rate of convergence.

Besides its direct application in (unrealistic) state-feedback excitation controllers for transient stability improvement, e.g. (Ortega et al., 2005; Shen et al., 2003; Langarica et al., 2015; Yan et al., 2010), the present result may be utilized by the wide-area control system to control the transient and oscillatory dynamics of system voltage and frequency (Singh and Pal, 2018).

Several open topics remain to be investigated.

- A better understanding of the key excitation assumption $\Delta \notin L_2$ is required. In particular, to know if it is verified in the normal (or fault) conditions of the power system.
- As indicated in the introduction a major practical issue is the lack of synchronism in the measurements of the rotor angles. A possible scenario to capture this fact is to include time delays in the transmission information—where the recent results of (Rueda-Escobedo et al., 2019) might be useful.
- The parameters of the network are highly uncertain. In particular, the line impedances, are poorly known. An adaptive implementation of the proposed observer is currently been explored.
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