Field-induced axion decay $a \rightarrow e^+e^-$ via plasmon

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The axion decay $a \rightarrow e^+e^-$ via a plasmon is investigated in an external magnetic field. The results we have obtained demonstrate a strong catalyzing influence of the field as the axion lifetime in the magnetic field of order $10^{15}$ G and at temperature of order 10 MeV is reduced to $10^3$ sec.

1. INTRODUCTION

The pseudo-Goldstone boson associated with Peccei-Quinn symmetry $U_{PQ}(1)$, the axion, is of interest not only in theoretical aspects of elementary particle physics, but in some astrophysical and cosmological applications as well. It is also known that astrophysical and cosmological considerations leave a narrow window on the axion mass:

$$10^{-5} \text{eV} \lesssim m_a \lesssim 10^{-2} \text{eV},$$

where axions could exist and provide a significant fraction or all of the cosmic dark matter.

At present the interest in axions as a possible dark matter candidate stimulates fullscale searches for galactic axions in experiments. Negative results of experiments are naturally explained by the fact that axions are very weakly coupling and very long living. The axion lifetime in vacuum is gigantic one:

$$\tau \sim 6.3 \cdot 10^{42} \text{s} \left(\frac{10^{-2} \text{eV}}{m_a}\right) ^6 \left(\frac{E_a}{1 \text{MeV}}\right).$$

On the other hand, in some astrophysical considerations where axions effects could be substantial, it is important to take into account the influence of plasma and the magnetic fields. One of the most physically realistic situations presented in many astrophysical objects is that when from both these components of the active medium the plasma dominates. For the physical circumstances of interest to us, the temperature $T$ appears to be the largest physical parameter. So, we will use a well describing by a crossed field limit ($E \perp B$, $E = B$) case, $T^2 \gg eB \gg m_e^2$, when a great number of the Landau levels are excited. At the same time the condition $T^2 \gg eB$ is fulfilled, the magnetic field is strong enough, $eB \gg m_e^2$, in comparison with the known Schwinger value $B_e = m_e^2/e \approx 4.41 \cdot 10^{13}$ G. Possible mechanisms of a generation of such strong fields as $B \sim 10^{15} - 10^{17}$ G in astrophysics were discussed in a number of papers.

In this paper we investigate the influence of the magnetic field and plasma on the axion decay into electron-positron pair via a photon intermediate state $a \rightarrow \gamma \rightarrow e^+e^-$ in KSVZ model in which axions have not direct coupling to leptons. The reason for which this forbidden in vacuum and plasma channel is opened in the magnetic field is that $e^+e^-$ pair can have both time-like and space-like total momentum as it occurs in photon splitting $\gamma \rightarrow e^+e^-$.

2. MATRIX ELEMENT

A diagram describing $a \rightarrow e^+e^-$ decay via the plasmon intermediate state is shown in Fig. 1.
the magnetic field in the electron wave functions and undulating double lines imply the influence of medium in the photon propagator.

The Lagrangian describing the axion-photon coupling can be presented in the form:
\[ \mathcal{L}_{a\gamma} = g_{a\gamma} \partial_\mu A_\mu \tilde{F}_{\nu\mu} a, \]  
(3)
where \( A_\mu \) is the four potential of the quantized electromagnetic field, \( \tilde{F} \) is the dual external field tensor, \( a \) is the axion field. Here \( g_{a\gamma} \) is the known axion-photon coupling constant with the dimension in the magnetic field. The condition of the internal field;

\[ q_\perp = \sqrt{q^2 - |q|^2}. \]
(4)
in terms of the currents
\[ J_\alpha = \int d^4x \bar{\psi}(p, x) \gamma_\alpha \psi(-p', x) e^{-iqx}, \]
\[ \hbar \alpha = -ie(q\tilde{F}G(q))_\alpha = -ieq_\perp \tilde{F}_{\mu\nu} G_{\nu\alpha}(q). \]
Here, \( e > 0 \) is the elementary charge, \( p = (E, p) \) and \( p' = (E', p') \) are the quasi-momenta of final electron and positron (\( p^2 = p'^2 = m_e^2 \)) in an external field; \( q = (E_a, q) \) is the axion momentum; \( \psi(p, x) \) is the exact solution of the Dirac equation in the magnetic field. The condition of the relative weakness of the magnetic field, \( eB \ll T^2 \), means that the plasma influence determines basically the properties of the photon propagator \( G_{\alpha\beta} \) which can be presented as a sum of transverse and longitudinal parts:
\[ G_{\alpha\beta} = -i \left( \frac{\mathcal{P}^{(T)}_{a\beta}}{g^2 - \Pi^{(T)}} + \frac{\mathcal{P}^{(L)}_{a\beta}}{g^2 - \Pi^{(L)}} \right), \]
(5)
\[ \mathcal{P}^{(T)}_{a\beta} = -\sum_{\lambda=1}^{2} t^\lambda_{\alpha} t^\lambda_{\beta}, \]
\[ \mathcal{P}^{(L)}_{a\beta} = - l_{\alpha} l_{\beta}. \]
Here, \( \Pi^{(T)} \) and \( \Pi^{(L)} \) are the transverse and longitudinal eigenvalues of the polarization operator; \( t^\lambda_{\alpha} = (0, t^\lambda) \) and \( l_{\alpha} \) denote transverse and longitudinal photon polarization vectors:
\[ \mathbf{t}^{(1)} = \frac{\mathbf{q} \times \mathbf{B}}{|q|B \sin \theta}, \quad \mathbf{t}^{(2)} = \frac{\mathbf{q} \times \mathbf{t}^{(1)}}{|q|}, \]

\[ \ell_{\alpha} = \sqrt{-\frac{q^2}{(uq)^2 - q^2} \left( u_{\alpha} - \frac{uq}{q^2} q_\alpha \right)}, \]

where \( u_\alpha \) the four-velocity of the medium; \( \theta \) is the angle between the external magnetic field \( \mathbf{B} \) and the axion momentum \( \mathbf{q} \).

Figure 1. The diagram describing the axion decay into electron-positron pair via virtual photon.

\[ a(q) \rightarrow \gamma \rightarrow e(p) \rightarrow e(-p'), \]
\[ e \rightarrow e' \]
where \( \alpha E = (E, \mathbf{p}) \) and \( e' = (E', \mathbf{p}') \) are the quasi-momenta of final electron and positron (\( p^2 = p'^2 = m_e^2 \)) in an external field; \( q = (E_a, q) \) is the axion momentum; \( \psi(p, x) \) is the exact solution of the Dirac equation in the magnetic field. The condition of the relative weakness of the magnetic field, \( eB \ll T^2 \), means that the plasma influence determines basically the properties of the photon propagator \( G_{\alpha\beta} \) which can be presented as a sum of transverse and longitudinal parts:

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where \( u_\alpha \) the four-velocity of the medium; \( \theta \) is the angle between the external magnetic field \( \mathbf{B} \) and the axion momentum \( \mathbf{q} \).

Being integrated over the variable \( x \) the expression (6) can be presented in the form:

\[ S = \frac{(2\pi)^4 \delta^2(\mathbf{Q}_\perp)}{\sqrt{2E_a V - 2E V - 2E'} V} \frac{g_{a\gamma}}{\pi(4\beta)^{1/3}} \]

\[ \times \int \frac{d^4p}{(2\pi)^4} \hbar \Phi(\mathbf{q}) + i\epsilon_{\alpha\beta\gamma\delta} \frac{\gamma_5}{2m_e^2} (\gamma Fh) \Phi'(\mathbf{q}) \]

\[ - \frac{e\mathbf{q} \cdot \mathbf{B}}{2m_e^2} \eta \Phi(\mathbf{q}) \]

\[ + \frac{m_e^2}{2z^2} (k(kh))(k(kp')) \eta \Phi(\mathbf{q}) \]

\[ \zeta \phi \]

\[ x_{\pm} = \pm \frac{1}{\lambda} \sqrt{\lambda}, \]

\[ z = \left( \frac{\lambda}{2\sqrt{\lambda}} \right)^{1/3}, \]

\[ \beta = \frac{1}{4} u^2 z^3, \quad u^2 = -e^2 a^2, \]

\[ \chi^2 = \frac{1}{2z^2} (k(kh))(k(kp')) \eta \Phi(\mathbf{q}) \]

\[ \chi_{\alpha}^2 = \frac{e^2(qFq)}{m_e^2}, \]

where \( Q = q - p - p' \), \( Q_{\perp} \) is the perpendicular to \( \mathbf{k} \) component (\( Q_{\perp} \mathbf{k} = 0 \)). With the four-potential
\( A_\mu = (k x) a_\mu \) the external field tensor is \( F_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu \). Finally, \( \Phi(\eta) \) is the Airy function:

\[
\Phi(\eta) = \int_0^\infty dt \cos \left( \eta t + \frac{t^3}{3} \right),
\]

\[
\eta = z^2 (1 + \tau^2), \quad \tau = -\frac{e(\rho F q)}{m_e^4 \lambda a},
\]

and \( \Phi'(\eta) = \partial \Phi(\eta)/\partial \eta \).

3. DECAY PROBABILITY

To obtain the decay probability one has to carry out a non-trivial integration over the phase space of the \( e^+ e^- \) pair taking their specific kinematics in the magnetic field into account.

As the analysis shows the contribution of the transverse photon mode to \( a \rightarrow e^+ e^- \) decay probability in the ultrarelativistic case is negligibly small. The main contribution due to the longitudinal plasmon intermediate state has a form:

\[
W = \frac{g^2 n_s (eB)^2}{36 \pi} \frac{E_3^2 \cos^2 \theta}{(E_a^2 - \xi^2)^2 + \gamma^2 \xi^4} \rho,
\]

\[
\rho = 6 \int_0^1 dx x(1-x)(1-n)(1-\bar{n}),
\]

\[
n = \left( \exp \frac{x E_a - \mu}{T} + 1 \right)^{-1},
\]

\[
\bar{n} = \left( \exp \frac{(1-x) E_a + \mu}{T} + 1 \right)^{-1},
\]

where \( n \) and \( \bar{n} \) are the Fermi-Dirac distributions of electrons and positrons at a temperature \( T \) and a chemical potential \( \mu \), respectively. The function \( \rho(E_a, T, \mu) \) has a meaning of the average value of supressing statistical factors and is, in general case, inside the interval \( 0 < \rho < 1 \).

Eq. (8) has a resonant behaviour at the point \( (E_a^2)_{\text{res}} = \xi^2 \) where axion and longitudinal plasmon dispersion curves cross (Fig. 3). The dimensionless resonance width \( \gamma \) of the \( a \rightarrow e^+ e^- \) process in Eq. (8) is

\[
\gamma = \frac{\xi^2 \Gamma_L(\xi)}{q^2 Z_L},
\]

where \( \Gamma_L(\xi) \) is the total width of the longitudinal plasmon; \( Z_L \) is the renormalization factor of longitudinal plasmon wave-function:

\[
Z_L^{-1} = 1 - \frac{\partial \Pi^L}{\partial q^2}. \tag{10}
\]

Notice that without the external field the plasmon decay into neutrino pair takes place only. In the presence of the magnetic field which, from one hand, is weak, \( eB \ll E_a \), and, from the other hand, strong enough, \( eB \gg \alpha^3 E_a^2 \), novel channel of the longitudinal plasmon decay is opened \( \gamma_L \rightarrow e^+ e^- \). However the main contribution to the width \( \Gamma_L(\xi) \) is determined by the process of the longitudinal plasmon absorption \( \gamma_L e^- \rightarrow e^- \) which becomes possible in this kinematical region in the magnetic field.

Below we give the expressions for \( \xi^2 \) and \( \gamma \) in two limits:

i) degenerate plasma

\[
\xi^2 \approx \frac{4 \alpha}{\pi} \mu^2 \left( \ln \frac{2 \mu}{m_e} - 1 \right), \tag{11}
\]

\[
\gamma \approx \frac{2 \alpha \mu^2}{3 \xi^2},
\]

Figure 2. Dispersion relations \( \omega^2 = \omega^2_a(k) \) for longitudinal plasmons (solid line), axions \( E_a^2 = k^2 + m_a^2 \) (short dashes), and vacuum photons \( \omega = k \) (long dashes).
ii) nondegenerate hot plasma

\[ \mathcal{E}^2 \simeq \frac{4\pi \alpha}{3} T^2 \left( \ln \frac{4T}{m_e} - 0.647 \right), \quad (12) \]
\[ \gamma \simeq \frac{2\alpha \mu^2}{3 \mathcal{E}^2}. \]

Considering possible applications of the result we have obtained to cosmology it is necessary to take an influence of a hot plasma into account. Under the early Universe conditions the hot plasma is nondegenerate one (\( \mu \ll T \)) and the medium parameter \( \rho \) is inside the interval \( 1/4 < \rho < 1 \). With \( \mathcal{E}^2 \) and \( \gamma \) from (13) we obtain the following estimation for the axion lifetime in the resonance region:

\[ \tau(a \to \gamma pl \to e^+ e^-) \simeq 2.5 \cdot 10^4 \text{s} \quad (13) \]
\[ \times \left( \frac{10^{-10}}{g_{ae} \text{ GeV}} \right)^2 \left( \frac{T}{10 \text{ MeV}} \right) \left( \frac{10^{15} \text{G}}{B} \right)^2. \]

It is interesting to compare (13) with the field-induced axion lifetime (14) in the model where axions couple with electrons on the tree level:

\[ \tau(a \to e^+ e^-) \simeq 3.4 \cdot 10^6 \text{s} \quad (14) \]
\[ \times \left( \frac{10^{-13}}{g_{ae}} \right)^2 \left( \frac{T}{10 \text{ MeV}} \right)^{1/3} \left( \frac{10^{15} \text{G}}{B} \right)^{2/3}. \]

The expressions (13) and (14) we have presented here demonstrate the strong catalyzing influence of the medium, plasma and the magnetic field, on the axion lifetime in comparison with the vacuum one (2). Due to the resonance behaviour of \( a \to \gamma pl \to e^+ e^- \) via the longitudinal plasmon the axion lifetime in KSVZ model with induced axion-electron interaction can be smaller than in DFSZ model with the direct coupling.

ACKNOWLEDGEMENTS

N. Mikheev and L. Vassilevskaya thank the organizers of the 5-th IFT Workshop on Axions for their warm hospitality during the visit. This research was partially supported by INTAS under grant No. 96-0659 and by the Russian Foundation for Basic Research under grant No. 98-02-16694.

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