Tachyons and the preferred frames

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Abstract
Quantum field theory of space-like particles is investigated in the framework of absolute causality scheme preserving Lorentz symmetry. It is related to an appropriate choice of the synchronization procedure (definition of time). In this formulation existence of field excitations (tachyons) distinguishes an inertial frame (privileged frame of reference) via spontaneous breaking of the so called synchronization group. In this scheme relativity principle is broken but Lorentz symmetry is exactly preserved in agreement with local properties of the observed world. It is shown that tachyons are associated with unitary orbits of Poincaré mappings induced from $SO(2)$ little group instead of $SO(2,1)$ one. Therefore the corresponding elementary states are labelled by helicity. The cases of the helicity $\lambda = 0$ and $\lambda = \pm \frac{1}{2}$ are investigated in detail and a corresponding consistent field theory is proposed. In particular, it is shown that the Dirac-like equation proposed by Chodos et al. [1], inconsistent in the standard formulation of QFT, can be consistently quantized in the presented framework. This allows us to treat more seriously possibility that neutrinos might be fermionic tachyons as it is suggested by experimental data about neutrino masses [2, 3, 4].

1 Introduction
Almost all recent experiments, measuring directly or indirectly the electron and muon neutrino masses, have yielded negative values for the mass square. It suggests that these particles might be fermionic tachyons. This intriguing possibility was written down some years ago by Chodos et al. [1] and Recami et al. [6].

On the other hand, in the current opinion, there is no satisfactory theory of superluminal particles; especially it is commonly believed that there is no respectable tachyonic quantum field theory at present [1]. This persuasion
creates a psychological barrier to take such possibility seriously. Even if we con-
sider eventuality that neutrinos are tachyons, the next problem arises; namely a
modification of the theory of electro-weak interaction will be necessary in such
a case. But, as we known, in the standard formulation of special relativity, the
unitary representations of the Poincaré group, describing fermionic tachyons, are
induced from infinite dimensional unitary representations of the non-compact
$SO(2,1)$ little group. Consequently, in the conventional approach, the neutrino
field should be infinite-component one so a construction of an acceptable local
interaction is extremely difficult.

In this paper we suggest a solution to the above dilemma. To do this we
use the ideas developed in the papers \cite{8, 9} based on the earlier works \cite{10, 11},
where it was proposed a consistent description of tachyons on both classical and
quantum level. The basic idea is to extend the notion of causality without a
serious change of special relativity. This can be done by means of a freedom
in the determination of the notion of the one-way light velocity, known as the
“conventionality thesis” \cite{12, 13}.

The main results of this paper can be summarized as follows:

- The relativity principle and the Lorentz covariance are formulated in
  the framework of a nonstandard synchronization scheme (the Chang–
  Thangherlini (CT) scheme). The absolute causality holds for all kinds
  of events (time-like, light-like, space-like).

- For bradyons and luxons our scheme is fully equivalent to the stand-
  ard formulation of special relativity.

- For tachyons it is possible to formulate covariantly canonical formalism,
  proper initial conditions and the time development.

- There exists a (covariant) lower bound of energy for tachyons; in terms of
  the contravariant zero-component of the four-momentum this lower bound
  is simply zero.

- The paradox of “transcendental” tachyons, apparent in the standard ap-
  proach, disappears.

- Tachyonic field can be consistently quantized using the CT synchroni-
  zation scheme.

- Tachyons distinguish a preferred frame via mechanism of the spontaneous
  symmetry breaking \cite{8, 11}; consequently the relativity principle is broken,
  but the Lorentz covariance (and symmetry) is preserved. The preferred
  frame can be identified with the cosmic background radiation frame.

- Classification of all possible unitary Poincaré mappings for space-like
  momenta is given. The important and unexpected result is that unitary or-
  bits for space-like momenta are induced from the $SO(2)$ little group. This
  holds because we have a bundle of Hilbert spaces rather than a single
  Hilbert space of states. Therefore unitary operators representing Poincaré
  group act in irreducible orbits in this bundle. Each orbit is generated from
  subspace with $SO(2)$ stability group. Consequently, elementary states are
  labelled by helicity, in an analogy with the light-like case. This fact is ex-
  tremely important because we have no problem with infinite component
  fields.
• A consistent quantum field theory for tachyons with helicity $\lambda = 0$ and $\lambda = \pm \frac{1}{2}$ is formulated.

In the paper [14] the $\beta$-decay amplitude is calculated under assumption that neutrino is a tachyon.

2 Preliminaries

As is well known, in the standard framework of the special relativity, space-like geodesics do not have their physical counterparts. This is an immediate consequence of the assumed causality principle which admits time-like and light-like trajectories only.

In the papers by Terletsky [15], Tanaka [16], Sudarshan et al. [17], Recami et al. [18, 19, 20] and Feinberg [21] the causality problem has been reexamined and a physical interpretation of space-like trajectories was introduced. However, every proposed solution raised new unanswered questions of the physical or mathematical nature [22]. The difficulties are specially frustrating on the quantum level [23, 24].

It is rather evident that a consistent description of tachyons lies in a proper extension of the causality principle. Notice that interpretation of the space-like world lines as physically admissible tachyonic trajectories favour the constant-time initial hyperplanes. This follows from the fact that only such surfaces intersect each world line with locally nonvanishing slope once and only once. Unfortunately, the instant-time hyperplane is not a Lorentz-covariant notion in the standard formalism, which is just the source of many troubles with causality.

The first step toward a solution of this problem can be found in the papers by Chang [25, 26, 27], who introduced four-dimensional version of the Tangherlini transformations [28], termed the Generalized Galilean Transformations (GGT). In [10] it was shown that GGT, extended to form a group, are hidden (non-linear) form of the Lorentz group transformations with $SO(3)$ as a stability subgroup. Moreover, a difference with the standard formalism lies in a non-standard choice of the synchronization procedure. As a consequence a constant-time hyperplane is a covariant notion. In the following we will call this procedure of synchronization the Chang–Tangherlini synchronization scheme.

It is important to stress the following two well known facts: (a) the definition of a coordinate time depends on the synchronization scheme [12, 29, 30], (b) synchronization scheme is a convention, because no experimental procedure exists which makes it possible to determine the one-way velocity of light without use of superluminal signals [13]. Notice that a choice of a synchronization scheme, different that the standard one, does not affect seriously the assumptions of special relativity but evidently it can change the causality notion, depending on the definition of the coordinate time.

As it is well known, intrasystemic synchronization of clocks in their “setting” (zero) requires a definitional or conventional stipulation—for discussion see Jammer [13], Sjödin [31] (see also [32]). Really, to determine one-way light speed it is necessary to use synchronized clocks (at rest) in their “setting” (zero) [3]. On the other hand to synchronize clocks we should know the one-way light velocity.

2 Evidently, without knowledge of the one-way light speed, it is possible to synchronize clocks in their rate only [3].
Thus we have a logical loophole. In other words no experimental procedure exists (if we exclude superluminal signals) which makes possible to determine unambiguously and without any convention the one-way velocity of light (for analysis of some experiments see Will [34]). Consequently, an operational meaning has the average value of the light velocity around closed paths only. This statement is known as the conventionality thesis [13]. Following Reichenbach [12], two clocks $A$ and $B$ stationary in the points $A$ and $B$ of an inertial frame are defined as being synchronous with help of light signals if

$$t_B = t_A + \epsilon_{AB}(t_A' - t_A).$$

Here $t_A$ is the emission time of light signal at point $A$ as measured by clock $A$, $t_B$ is the reception-reflection time at point $B$ as measured by clock $B$ and $t_A'$ is the reception time of this light signal at point $A$ as measured by clock $A$. The so called synchronization coefficient $\epsilon_{AB}$ is an arbitrary number from the open interval $(0, 1)$. In principle it can vary from point to point. The only conditions for $\epsilon_{AB}$ follow from the requirements of symmetry and transitivity of the synchronization relation. Note that $\epsilon_{AB} = 1 - \epsilon_{BA}$. The one-way velocities of light from $A$ to $B$ ($c_{AB}$) and from $B$ to $A$ ($c_{BA}$) are given by

$$c_{AB} = \frac{c}{2\epsilon_{AB}}, \quad c_{BA} = \frac{c}{2\epsilon_{BA}}.$$

Here $c$ is the round-trip average value of the light velocity. In standard synchronization $\epsilon_{AB} = \frac{1}{2}$ and consequently $c = c_{AB}$ for each pair $A, B$.

The conventionality thesis states that from the operational point of view the choice of a fixed set of the coefficients $\epsilon$ is a convention. However, the explicit form of the Lorentz transformations will be $\epsilon$-dependent in general. The question arises: Are equivalent notions of causality connected with different synchronization schemes? As we shall see throughout this work the answer is negative if we admit tachyonic world lines. In other words, the causality requirement, logically independent of the requirement of the Lorentz covariance, can contradict the conventionality thesis and consequently it can prefer a definite synchronization scheme, namely CT scheme if an absolute causality is assumed.

It is very interesting, that in the framework of CT synchronization two old-standing theoretical problems [35], completely unconnected with tachyons, have solutions; namely: (a) the manifestly covariant canonical formalism for relativistic particle can be found and (b) it is possible to construct a covariant position operator and set of covariant relativistic localizable states [36].

### 3 The Chang–Tangherlini synchronization

As was mentioned in Section 2 in the paper by Tangherlini [28] a family of inertial frames in $1+1$ dimensional space of events was introduced with the help of transformations which connect the time coordinates by a simple (velocity dependent) rescaling. This construction was generalized to the $1+3$ dimensions by Chang [25, 26]. As was shown in the paper [10], the Chang–Tangherlini inertial frames can be related by a group of transformations isomorphic to the orthochronous Lorentz group. Moreover, the coordinate transformations should be supplemented by transformations of a vector-parameter interpreted as the velocity of a privileged frame. It was also shown that the above family of frames is equivalent to the Einstein–Lorentz one; (in a contrast to the interpretation in [25, 26]). A difference lies in another synchronization procedure for clocks [11].
In the Appendix we derive realization of the Lorentz group given in \[10\] in a systematic way \[11\].

Let us start with a simple observation that the description of a family of inertial frames in the Minkowski space-time is not so natural. Instead, it is obvious that the geometrical notion of bundle of frames is more natural. Base space is identified with the space of velocities; each velocity marks out a coordinate frame. Indeed, from the point of view of an observer (in a fixed inertial frame) all inertial frames are labelled by their velocities with respect to him. Therefore, in principle, to define the transformation rules between frames, we should use, except of coordinates, also this vector-parameter, related to velocities of frames with respect to a distinguished observer.

Notice that a distinguishing of a preferred inertial frame is in full agreement with local properties of the observed expanding world. Indeed, we can fix a local frame in which the Universe appears spherically; it can be done, in principle, by investigation of the isotropy of the Hubble constant \[37\]. It coincides with the cosmic background radiation frame. Thus it is natural to ask for a formalism incorporating locally Lorentz symmetry and the existence of a preferred frame.\[1\]

Below we list our basic requirements:

1. Coordinate frames are related by a set of transformations isomorphic to the Lorentz group (Lorentz covariance).

2. The average value of the light speed over closed paths is constant \((c)\) for all inertial observers (constancy of the round-trip light velocity).

3. With respect to the rotations \(x^0\) and \(\vec{x}\) transform as \(SO(3)\) singlet and triplet respectively (isotropy).

4. Transformations are linear with respect to the coordinates (affinity).

5. We admit an additional set of parameters \(u\) (velocity space—the base space for the bundle of inertial frames).

We see that assumptions 1–4 are the standard ones. In the following we consider also two distinguished cases corresponding to the relativity principle and absolute causality requirements respectively. Hereafter we shall use the natural units \(c = \hbar = 1\).

### 3.1 Poincaré group transformation rules in the CT synchronization

According to our assumptions, transformations between two coordinate frames \(x^\mu\) and \(x'^\mu\) have the following form

\[
x'(u') = D(\Lambda, u)(x(u) + a).
\]

(1)

Here \(D(\Lambda, u)\) is a real (invertible) \(4 \times 4\) matrix, \(\Lambda\) belongs to the Lorentz group and \(u^\mu\) is assumed to be four-velocity of distinguished frame, i.e., it transforms like \(dx^\mu\)

\[
u' = D(\Lambda, u)u.
\]

(2)

\[^{a}\text{Frequently, the notion of preferred frame is associated with a violation of Lorentz invariance [38] but in our case the Lorentz invariance is assumed to be exact.}\]
The \( a^\mu \) are translations in the frame \( x^\mu(u) \). It is easy to verify that the transformations (1)–(2) constitute a realization of the Lorentz group if the following composition law holds

\[
D(\Lambda_2, D(\Lambda_1, u)) D(\Lambda_1, u) = D(\Lambda_2 \Lambda_1, u).
\]

(3)

Now, because \( u^\mu \) is assumed to be four-velocity of an inertial frame, it must be related to a time-like Lorentzian four-velocity \( u_E \) \((u_E^2 = 1)\); subscript \( E \) means Einstein–Poincaré synchronization (EP synchronization), where \( u_E \) has the standard transformation law

\[
u'_E = \Lambda u_E.
\]

(4)

Let us denote the intertwined matrix by \( T(u) \), i.e.

\[
u_E = T^{-1}(u)u.
\]

(5)

Therefore the explicit form of \( D(\Lambda, u) \) satisfying the assumptions 1–5 (see also the Appendix) is

\[
D(\Lambda, u) = T(u') \Lambda T^{-1}(u),
\]

(6)

where

\[
T(u) = \begin{pmatrix}
1 & b(u) \bar{u}^T \\
0 & I
\end{pmatrix}.
\]

(7)

Here \( b(u) \) is rotationally invariant function of \( u \); the superscript \( T \) denotes transposition. Furthermore, the transformed four-velocity \( u' \) is determined from the relation (5) and the transformation law (4). Thus the square of the line element

\[
ds^2 = g_{\mu\nu}(u) dx^\mu dx^\nu
\]

(8)

with

\[
g(u) = (T(u) \eta T^T(u))^{-1},
\]

(9)

where the Minkowski tensor \( \eta = \text{diag}(+, -, -, -) \), is invariant under the transformations (1)–(2). Now, by means of (8) for null geodesics, it is easy to calculate the light velocity. To do this let us notice that

\[
u^2 = g_{\mu\nu}(u)u^\mu u^\nu = 1.
\]

(10)

The velocity of light propagation in a direction \( \vec{n} \) \((\vec{n}^2 = 1)\) reads

\[
\vec{c} = \frac{\vec{n}}{1 + \vec{n} \bar{u}b(u)}.
\]

(11)

\footnote{Both \( u \) and \( u_E \) must be related to the quotient space \( SO(3, 1)/SO(3) \) — the base space of the frame bundle under consideration.}

\footnote{In the papers by Chang [25, 26, 27] it was used some kinematical objects with an improper physical interpretation [39, 40]. For this reason we should be precise in the nomenclature related to different synchronizations.}
so the Reichenbach synchronization coefficient takes the form

$$\varepsilon(\vec{n}, \vec{u}) = \frac{1}{2} (1 + \vec{n} \vec{u} b(u)).$$

(12)

Therefore the function $b(u)$ distinguishes between different synchronizations. The most interesting choices of $b(u)$ correspond to the Einstein–Poincaré synchronization and to the Chang–Tangherlini one.

In the first case (EP), $b(u) = 0$, i.e. $T(u) = I$, so $g(u) = \eta$, $\vec{c} = \vec{c}_E = \vec{n}$ and $x = x_E$, $u = u_E$ with the standard transformation law (4).

Now, the CT case is obtained under condition that the instant-time hyper-plane $x^0 = \text{constant}$ is an invariant notion, i.e. that $x^0 = D(\Lambda, u)^0_0 x^0$ so $D(\Lambda, u)^0_k = 0$. Thus from eqs. (6), (7) we have in this case

$$b(u) = -u^0.$$

(13)

In the following we assume the CT synchronization defined by eq. (13). In this case

$$T(u) = \begin{pmatrix} 1 & -\vec{u}^T u^0 \\ 0 & I \end{pmatrix}.$$  

(14)

Thus the interrelation between coordinates in EP and CT synchronizations is given by

$$x^0_E = x^0 + u^0 \vec{u} \vec{x}, \quad \vec{x}_E = \vec{x},$$  

(15)

i.e. the essential difference is in the coordinate time definition.

Now, by means of (6) and (13), we have determined the form of the matrix $D(\Lambda, u)$ (i.e. the transformation law (1)–(2)); namely

for rotations $R \in SO(3) \subset SO(3, 1)$

$$D(R, u) = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix};$$  

(16)

for boosts

$$D(W, u) = \begin{pmatrix} 1 \frac{1}{W^0} & \frac{0}{W \otimes W^T} \\ -W \frac{1}{1+\sqrt{1+u^0 u^0}} & W \otimes \vec{u}^T u^0 \end{pmatrix}.$$  

(17)

Here $W^\mu$ is the four-velocity of the primed frame ($x'$) with respect to the initial one ($x$). We see that absolute causality can be introduced in our framework because the coordinate time is rescaled only by the positive factor $1/W^0$, i.e.

$$x'^0 = \frac{1}{W^0} x^0.$$  

(18)

\(^6\text{I.e. } W^\mu = \frac{dX^\mu}{ds}, \text{ where } dX \text{ is the displacement of the origin of the primed frame in the time } dX^0 \text{ and can be expressed by } u \text{ and } u' = D(W, u)u \text{ as follows} \)

$$W^0 = \frac{u^0}{u^0'}, \quad W = \frac{(u^0 + u'^0)(\vec{u} - \vec{u}')}{1 + u^0 u'^0(1 + \vec{u}' \vec{u})}.$$
Moreover, taking $W^\mu = u^\mu$, we can check from (17) that $u^\mu$ is the four-velocity of a distinguished (privileged) inertial frame (fixed by $\bar{u} = (1, 0, 0, 0)$) as seen from the unprimed frame ($x$).

Now, the covariant metric tensor $g(u)$ (9) takes the form
\[
[g_{\mu\nu}(u)] = \begin{pmatrix} 1 & u^0 \bar{u}^T \\ u^0 \bar{u} & -I + \bar{u} \otimes \bar{u}^T (u^0)^2 \end{pmatrix},
\] (19)
while the contravariant one reads
\[
g^{-1}(u) = \begin{pmatrix} (u^0)^2 & u^0 \bar{u}^T \\ u^0 \bar{u} & -I \end{pmatrix},
\] (20)
so the square of the length has the Euclidean form
\[
dl^2 = -g^{ik} dx_i dx_k = d\bar{x}^2.
\]
Furthermore, the light velocity $\bar{c}$ is given by
\[
\bar{c} = \frac{\bar{u}}{1 - \bar{u} \bar{u}^0},
\] (21)
and we can check that $\langle |\bar{c}| \rangle_{\text{closed path}} = 1$.

Now, by means of relations between differentials (see (15))
\[
dx^0_E = dx^0 + u^0 \bar{u} d\bar{x}, \quad d\bar{x}_E = d\bar{x},
\] (22)
we obtain interrelations between velocities in both synchronizations; namely
\[
\bar{v} = \frac{\bar{v}_E}{1 - \frac{\bar{v} \bar{u}}{\bar{c}}}, \quad \bar{v}_E = \frac{\bar{v}}{1 + \bar{v} \bar{u} u^0}.
\] (23)
(24)
Notice, that for $|\bar{v}| > |\bar{c}|$ the above formulas have a singularity.

We can also express the transformation matrix (13) by means of the velocities $\bar{\sigma} = \frac{\bar{v}}{\bar{c}}$ and $\bar{V} = \frac{\bar{v}}{W}$ via the relation
\[
\frac{1}{W} = \sqrt{\left(1 + \bar{\sigma} \bar{V} \gamma_0^{-2}\right)^2 - (\bar{V})^2}
\] (25)
with
\[
\gamma_0 = \left[\frac{1}{2} \left(1 + \sqrt{1 + (2\bar{\sigma})^2}\right)\right]^{1/2}.
\] (26)
Finally, let us notice that a second rank tensor, say $\theta^\mu_{\nu}$, transforms under the transformation law (1)–(2) according to
\[
\theta'(u') = D(\Lambda, u)\theta(u)D^{-1}(\Lambda, u).
\] (27)
Therefore, taking into account the triangular form (13)–(17) of $D(\Lambda, u)$, it is easy to see that the following conditions are invariant under (27)
1. $\theta^0_k = 0$ (this implies $\theta^0_0 = \text{const}$);
2. $\theta^0_k = 0$, $\theta^0_0 = \text{const}$, $\theta^i_j = \alpha \delta^i_j$, where $\alpha$ is a scalar function.

The second condition in the form $\theta^0_\mu = 0$, $\theta^i_j = -\delta^i_j$ is crucial for construction of a covariant canonical formalism for tachyons. This will be done in the Sec. 3.3.
3.2 Causality and kinematics in the CT synchronization

In CT scheme *causality has an absolute meaning*. This follows from the transformation law (18) for the coordinate time: \( x^0 \) is rescaled by a positive, velocity dependent, factor \( \frac{1}{W_0} \). Thus this formalism extends the EP causality by allowing faster than light propagation. It can be made transparent if we consider the relation derived from eq. (22)

\[
\frac{dx^0}{d\tau^E} = 1 - \frac{\vec{v}_E \cdot \vec{v}_E}{u^0_E^2}.
\]  

(28)

For \( |\vec{v}_E| \leq 1 \) we have \( \frac{dx^0}{d\tau^E} > 0 \), so the EP and CT causality coincide in this case. Nevertheless for \( |\vec{v}_E| > 1 \), the sign of \( \frac{dx^0}{d\tau^E} \) is indefinite which is a consequence of an inadequacy of the EP synchronization to description of faster than light propagation. On the other hand in the CT synchronization framework, by means of the eq. (18), each time interval \( \Delta x^0 \) is observer-independent, so there are no causal problems for tachyons.

Let us consider in detail a free particle case associated with a space-like geodesics. The corresponding action \( S \) is of the form

\[
S_{12} = -\kappa \int_{\lambda_1}^{\lambda_2} \sqrt{-ds^2}
\]  

(29)

where the square of the space-like line element

\[
ds^2 = g_{\mu\nu}(u) \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda^2 < 0
\]  

(30)

and the continuous affine paramenter \( \lambda \) is defined along the trajectory as monotonically increasing as one proceeds along the curve in a fixed direction.

The equations of motion are obtained by means of the variational principle and reads

\[
\frac{d}{d\lambda} \left( \frac{\dot{x}^\mu}{\sqrt{-g_{\mu\nu}(u)\dot{x}^\mu\dot{x}^\nu}} \right) = 0
\]  

(31)

with \( \dot{x}^\mu = \frac{dx^\mu}{d\lambda} \equiv w^\mu \). Now, we are free to take the path parameter as \( d\lambda = \sqrt{-ds^2} \), so the four-velocity \( w^\mu \) satisfies

\[
w^2 = g_{\mu\nu}(u)w^\mu(u)w^\nu(u) = -1
\]  

(32)

and consequently

\[
\ddot{w}^\mu = \dddot{w}^\mu = 0.
\]  

(33)

Let us focus our attention on the constraint (32). Obviously it defines an one-sheet hyperboloid; in particular in the preferred frame (for \( u = \tilde{u} = (1, 0, 0, 0) \)) \( g_{\mu\nu}(\tilde{u}) = \eta_{\mu\nu} \), so \( \eta_{\mu\nu}w^\mu(\tilde{u})w^\nu(\tilde{u}) = -1 \), like in the Einstein synchronization. However, there is an important difference; namely under Lorentz boosts the zeroth component \( w^0(u) \) of \( w^\mu \) is rescaled by a positive factor only (see eq. (18)) i.e. \( w^0(u') = \frac{1}{W_0}w^0(u) \). Therefore, contrary to the Einstein–Poincaré synchronization, in this case points of the upper part of the hyperboloid

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(satisfying \( w^0(u) > 0 \)) transforms again into points of the upper part. This allows us to define consistently the velocity of a tachyon:

\[
v = \frac{dx}{dx^0} = \frac{\vec{w}}{w^0}
\]

because now, for each observer, the tachyon speed is finite (i.e. \(|v| < \infty, w^0 > 0\)). We see that the infinite velocity is a limiting velocity, like in the non-relativistic case (it corresponds to \( w^0 = 0 \) which is an invariant condition). Notice that the constraint relation \( (32) \) implies that velocity of a tachyon moving in a direction \( \vec{n} \) is restricted by the inequality

\[
|\vec{c}| = \frac{1}{1 - \vec{n} \vec{u} u^0} < |\vec{v}| < \infty.
\]

Furthermore, the transformation law for velocities in the EP synchronization, derived from \( (1), (2), (17) \) reads

\[
v' = W_0 \left[ \vec{v} + \vec{W} \left( \frac{(\vec{W} \vec{v})}{1 + \sqrt{1 + (\vec{W})^2}} - w^0(\vec{u} \vec{v}) - 1 \right) \right]
\]

We see that the transformation law \( (36) \) is well defined for all velocities (sub- and superluminal). Recall that in the EP synchronization the corresponding transformation rule reads

\[
v'_E = \frac{\vec{v}_E - W_E \left( 1 - \frac{W_E \vec{v}_E}{(1 + W_E^2)} \right)}{W_E^0 - W_E \vec{v}_E}
\]

We observe that the denominator in the first part of the above transformation rule can vanish for \( |\vec{v}_E| > 1 \). Thus a tachyon moving with \( 1 < |\vec{v}_E| < \infty \) can be converted by a finite Lorentz map into a “transcendental” tachyon with \( |\vec{v}'_E| = \infty \). This discontinuity is an apparent inconsistency of this transformation law; namely in the EP scheme tachyonic velocity space does not constitute a representation space for the Lorentz group. A technical point is that the space-like four-velocity cannot be related to a three-velocity in this case by the relation \( \vec{v}_E = \frac{\vec{w}_E}{w^0_E} \), because \( w^0_E \) can take the value zero for a finite Lorentz transformation.

Concluding, in the CT synchronization the problem of “transcendental” tachyons does not appear—contrary to the eq. \( (37) \), the transformation law \( (36) \) is continuous, does not “produce” “transcendental” tachyons and completed by rotations, forms (together with the mapping \( u \rightarrow u' \)) a realization of the Lorentz group and the relation of \( \vec{v} \) to the four-velocity is nonsingular.

### 3.3 Canonical formalism

Let us identify the Lagrangian of a free tachyon related to the action \( (29) \); by means of the formulas \( (19), (8), (34) \) we have

\[
L = \kappa \sqrt{(\vec{v})^2 - (1 + w^0 \vec{u} \vec{v})^2}
\]
Thus the canonical momenta read
\[ \pi_k = \frac{\partial L}{\partial v^k} = \frac{\kappa [v^k - u^k u^0 (1 + u^0 \vec{u} \vec{v})]}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} = -\kappa \omega_k \] (39)
where we have used eq. (32). The Hamiltonian
\[ H = \pi_k v^k - L = \frac{\kappa (1 + u^0 \vec{u} \vec{v})}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} = +\kappa \omega_0 \] (40)
Therefore the covariant four-momentum \( k_\mu \) of tachyon can be defined as
\[ k_0 = H = \kappa \omega_0, \quad \vec{k} = -\pi_\vec{v} = \kappa \omega \] (41)
i.e. \( k_\mu = \kappa \omega_\mu \).
Notice that
\[ k^2 = g^{\mu\nu}(u) k_\mu k_\nu = -\kappa^2 \] (42)
and the energy \( H = \kappa \omega_0 \) has in each inertial frame a finite lower bound corresponding to \( |\vec{v}| \to \infty \), i.e.
\[ E > \frac{\kappa \sqrt{1 - (u^0)^2} \cos \phi}{\sqrt{1 - (\sqrt{1 - (u^0)^2} \cos \phi)^2}} \equiv E(u^0, \phi) \] (43)
where \( \cos \phi = \frac{\vec{u} \vec{v}}{|\vec{u}||\vec{v}|} \).
Therefore, contrary to the standard case, the energy of tachyon is always restricted from below by \( E(u^0, \phi) > -\infty \). Moreover, if we calculate the contravariant four-momentum \( k^\mu = g^{\mu\nu}(u) k_\nu = \kappa \omega^\mu \) we obtain that
\[ k^0 = \frac{\kappa}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} > 0 \] (44)
which confirm our statement that the sign of \( k^0 \) is Lorentz invariant also for tachyons (recall the transformation law (18)). In terms of \( \vec{k}_\mu \) the zeroth components of \( k \) read
\[ k^0 = u^0 \sqrt{\left(\vec{k} \vec{k}\right)^2 + (|\vec{k}|^2 - \kappa^2)}, \quad k_0 = \frac{1}{u^0} \left(-\vec{k}_\omega + \sqrt{\left(\vec{k} \vec{k}\right)^2 + (|\vec{k}|^2 - \kappa^2)} \right) \] (45) (46)
and the range of the covariant momentum \( \vec{k}_\mu \) is determined by the inequality
\[ |\vec{k}| > \kappa \left(1 + \left(\frac{\vec{k} \vec{k}}{|\vec{k}|^2}\right)^2 \right)^{-\frac{1}{2}} \] (47)
i.e. values of \( |\vec{k}| \) lie outside of the oblate spheroid with half-axes \( \kappa \) and \( \kappa u^0 \) and with the symmetry axis parallel to \( \vec{u} \).
Now, the Hamilton equations have the form
\[ \frac{d\vec{x}}{dt} = \frac{\partial H}{\partial \pi_{\vec{v}}}, \quad -\frac{\partial k_0}{\partial \vec{k}} = \vec{k} = \vec{v}, \quad \frac{dk_0}{dt} = -\frac{\partial H}{\partial \vec{x}} = 0. \] (48) (49)
From the second equation it follows that $\frac{dv}{dt} = 0$.

Furthermore, the most general Poincaré covariant Poisson bracket reads
\begin{equation}
\{A, B\} = C^{\mu \nu} \left( \frac{\partial A}{\partial x^\mu} \frac{\partial B}{\partial k_\nu} - \frac{\partial A}{\partial k_\mu} \frac{\partial B}{\partial x^\nu} \right) \tag{50}
\end{equation}
where $C^{\mu \nu}$ is a second rank tensor. Because we assume parity and translational invariance, its most general form is
\begin{equation}
C^{\mu \nu} = a \delta^{\mu \nu} + bk_\mu u_\nu + du_\mu k_\nu + ek_\mu k_\nu + f u_\mu u_\nu
\end{equation}
where $a$, $b$, $d$, $e$, $f$ are Lorentz scalars.

Notice that (50) imply
\begin{equation}
\{x^\mu, k_\nu\} = C^{\mu \nu}.
\end{equation}
However, we should demand
\begin{equation}
\{x^0, k_\nu\} = 0 \quad \text{i.e. } C^{0 \nu} = 0, \tag{51}
\end{equation}
\begin{equation}
\{x^i, k_k\} = -\delta^i_k \quad \text{i.e. } C^{i k} = -\delta^i_k, \tag{52}
\end{equation}
as well as
\begin{equation}
\{x^i, k_0\} = \frac{\vec{k}}{k^0} \quad \text{i.e. } C^{i 0} = \frac{\vec{k}}{k^0}. \tag{53}
\end{equation}
The first condition tells that the coordinate time $t = x^0$ is not any dynamical variable, but a dynamical parameter. The second one is evidently the statement that $\vec{x}$ and $\pi_\vec{x} = -\vec{k}$ are canonically conjugated. Finally the last requirement follows from the Hamilton equations (48)–(49). The above conditions determine the Poisson bracket (50) which takes the form
\begin{equation}
\{A, B\} = -\left( \delta^{\mu \nu} \frac{k^\mu u_\nu}{uk} \right) \left( \frac{\partial A}{\partial x^\mu} \frac{\partial B}{\partial k_\nu} - \frac{\partial A}{\partial k_\mu} \frac{\partial B}{\partial x^\nu} \right) \tag{54}
\end{equation}
with $uk = u_\mu k^\nu = u_0 k^0$; this last equality follows from the fact that $u_k = g_{k \mu}(u) u^\mu = 0$.

It is easy to see that the Poisson bracket defined by the relation (54) satisfies all necessary conditions:

- It is linear with respect to the both factors, antisymmetric, satisfying the Leibniz rule and fulfill the Jacobi identity.
- It is manifestly Poincaré covariant in the CT synchronization (recall the comment after the eq. (27)).
- It is consistent with all canonical conditions (51)–(53).
- It is easy to check that the tachyonic dispersion relation (42), $k^2 = -\kappa^2$, is consistent with this bracket. i.e. $\{k^2, k_\nu\} = \{k^2, x^\nu\} = 0$; therefore we do not need to introduce a Dirac bracket.

It is clear that in analogous way we can construct canonical formalism for bradyons ($k^2 = m^2$) in the CT synchronization. We also are able to introduce in this scheme a covariant position operator on the quantum ground [36].
3.4 Synchronization group and the relativity principle

From the foregoing discussion we see that the CT synchronization prefers a privileged frame corresponding to the value $\vec{u} = 0$. It is clear that if we forget about tachyons such a preference is only formal; namely we can choose each inertial frame as a preferred one.

Let us consider two CT synchronization schemes, say $A$ and $B$, under two different choices of privileged inertial frames, say $\Sigma_A$ and $\Sigma_B$. Now, in each inertial frame $\Sigma$ two coordinate charts $x_A$ and $x_B$ can be introduced, according to both schemes $A$ and $B$ respectively. The interrelation is given by the almost obvious relations

$$x_B = T(u_B)T^{-1}(u_A)x_A,$$  \hspace{1cm} (55)

$$u^B = D(\Lambda_{BA}, u^A)u^A,$$  \hspace{1cm} (56)

where $u^A$ ($u^B$) is the four-velocity of $\Sigma_A$ ($\Sigma_B$) with respect to $\Sigma$. $T(u)$ is given by the eq. (14). We observe that a set of all possible four-velocities $u$ must be related by Lorentz group transformations too, i.e. $\{\Lambda_{BA}\} = L_S$. Of course it does not coincide with our intersystemic Lorentz group $L$. We call the group $L_S$ a synchronization group \cite{8, 11}.

Now, if we compose the transformations (1), (2) of $L$ and (55)–(56) of $L_S$ we obtain

$$x' = T(u')\Lambda T^{-1}(u)x,$$ \hspace{1cm} (57)

$$u' = D(\Lambda_S\Lambda, u)u$$ \hspace{1cm} (58)

with $\Lambda_S \in L_S, \Lambda \in L$.

Therefore (57)–(58) can be obtained as a composition of two mutually commuting transformations

$$L_0 \ni (I, \Lambda_0): \begin{cases} x' = T(u)\Lambda_0 T^{-1}(u)x \\ u' = u \end{cases}$$ \hspace{1cm} (59)

$$L_S \ni (\Lambda_S, I): \begin{cases} x' = T(u')\Lambda T^{-1}(u)x \\ u' = D(\Lambda, u)u \end{cases}$$ \hspace{1cm} (60)

Thus the composition law for $(\Lambda_S, \Lambda_0)$ reads

$$(\hat{\Lambda}_S, \hat{\Lambda}_0)(\Lambda_S, \Lambda_0) = (\hat{\Lambda}_S \Lambda_S, \hat{\Lambda}_0 \Lambda_0).$$ \hspace{1cm} (61)

It is evident that both $L_S$ and $L_0$ are isomorphic to the Lorentz group. Therefore the set $\{(\Lambda_S, \Lambda)\}$ is the direct product of two Lorentz groups $L_0 \otimes L_S$. The intersystemic Lorentz group $L$ is the diagonal subgroup in this direct product, i.e. elements of $L$ are of the form $(\Lambda, \Lambda)$. Thus $L$ acts as an authomorphism group of $L_S$.

Now, the synchronization group $L_S$ realizes in fact the relativity principle. In our language the relativity principle can be formulated as follows: Any inertial frame can be choosen as a preferred frame. What happens, however, when the tachyons do exist? In that case the relativity principle is obviously broken: If tachyons exist then one and only one inertial frame must be a preferred frame to preserve an absolute causality. Moreover, the one-way light velocity becomes
a real, measured physical quantity because conventionality thesis breaks down. It means that the synchronization group $L_S$ is broken to the $SO(3)_u$ subgroup (stability group of $u$); indeed, transformations from the $L_S/SO(3)_u$ do not leave the causality notion invariant. As we show later, on the quantum level we have to deal with spontaneous breaking of $L_S$ to $SO(3)$.

Notice, that in the real world a preferred inertial frames are distinguished locally as the frames related to the cosmic background radiation. Only in such frames the Hubble constant is direction-independent.

4 Quantization

The following two facts, true only in CT synchronization, are extremely important for quantization of tachyons:

• Invariance of the sign of the time component of the space-like four-momentum i.e. $\varepsilon(k^0) = \text{inv}$,

• Existence of a covariant lower energy bound; in terms of the contravariant space-like four-momentum $k^\mu$, $k^2 < 0$, this lower bound is exactly zero, i.e. $k^0 \geq 0$ as in the time-like and light-like case.

This is the reason why an invariant Fock construction can be done in our case.

4.1 Tachyonic representations

Our fundamental object will be a bundle of Hilbert spaces $\mathcal{H}_u$ corresponding to the bundle of the inertial frames. Here we classify unitary Poincaré mappings in this bundle of Hilbert spaces for a space-like four-momentum. Furthermore we find the corresponding canonical commutation relations. As result we obtain that tachyons correspond to unitary mappings which are induced from $SO(2)$ group rather than $SO(2,1)$ one. Of course, a classification of unitary orbits for time-like and light-like four-momentum is standard, i.e., it is the same as in EP synchronization; this holds because the relativity principle is working in these cases (synchronization group is unbroken).

As usually, we assume that a basis in a Hilbert space $\mathcal{H}_u$ (fibre) of one-particle states consists of the eigenvectors $|k, u; \ldots\rangle$ of the four-momentum operators namely

$$P^\mu|k, u; \ldots\rangle = k^\mu|k, u; \ldots\rangle$$

where

$$(k', u; \ldots|k, u; \ldots) = 2k^0 \delta^3(k'_\perp - k_\perp)$$

i.e. we adopt a covariant normalization. The $k^0 = g^{0\mu}k_\mu$ is positive and the energy $k_0$ is the corresponding solution of the dispersion relation (42); both $k^0$ and $k_0$ are given by (45)–(46). In the following we will use the abbreviation $\omega_k \equiv k^0$ also.

The covariant normalization in (63) is possible because in CT synchronization the sign of $k^0$ is an invariant. Thus we have no problem with an indefinite norm in $\mathcal{H}_u$.  

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Now, \( ku \equiv k_\mu u^\mu \) is an additional Poincaré invariant. Summarizing, irreducible family of unitary operators \( U(\Lambda, a) \) in the bundle of Hilbert spaces \( H_u \) acts on an orbit defined by the following covariant conditions

- \( k^2 = -\kappa^2 \);
- \( \varepsilon(k^0) = \text{inv} \), for physical representations \( k^0 > 0 \) so \( \varepsilon(k^0) = 1 \) which guarantee a covariant lower bound of energy;
- \( q \equiv uk = \text{inv} \), it is easy to see that \( q \) is the energy of tachyon measured in the privileged frame.

As a consequence there exists an invariant, positive definite measure

\[
d\mu(k, \kappa, q) = d^4k \theta(k^0) \delta(k^2 + \kappa^2) \delta(q - uk).
\]

(64)

Let us return to the problem of classification of irreducible unitary mappings \( U(\Lambda, a) \) from \( H_u \) to \( H_u' \):

\[
U(\Lambda, a)|k, u; \ldots \rangle = |k', u'; \ldots \rangle;
\]

here the pair \( (k, u) \) is transported along trajectories belonging to an orbit fixed by the above mentioned invariant conditions. To follow the familiar Wigner procedure of induction, one should find a stability group of the double \( (k, u) \). To do this, let us transform \( (k, u) \) to the preferred frame by the Lorentz boost \( L_u^{-1} \). Next, in the privileged frame, we rotate the spatial part of the four-momentum to the \( z \)-axis by an appropriate rotation \( R_\hat{n} \). As a result, we obtain the pair \( (\hat{k}, \hat{u}) \) transformed to the pair \( (\tilde{k}, \tilde{u}) \) with

\[
\begin{pmatrix}
q \\
0 \\
0 \\
\sqrt{\kappa^2 + q^2}
\end{pmatrix}, \quad
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix}.
\]

(65)

It is easy to see that the stability group of \( (\tilde{k}, \tilde{u}) \) is the \( SO(2) = SO(2, 1) \cap SO(3) \) group. Thus tachyonic unitary representations should be induced from the \( SO(2) \) instead of \( SO(2, 1) \) group! Recall that unitary representations of the \( SO(2, 1) \) non-compact group are infinite dimensional (except of the trivial one). As a consequence, local fields in the standard case are necessarily infinite component ones (except of the scalar one). On the other hand, in the CT synchronization case unitary representations for space-like four-momenta in our bundle of Hilbert spaces are induced from irreducible, one dimensional representations of \( SO(2) \) in a close analogy with a light-like four-momentum case. They are labelled by helicity \( \lambda \), by \( \kappa \) and by \( q \) (\( \varepsilon(k^0) = \varepsilon(q) \) is determined by \( q \); of course a physical choice is \( \varepsilon(q) = 1 \)).

Now, by means of the familiar Wigner procedure we determine the Lorentz group action on the base vectors; namely

\[
U(\Lambda)|k, u; \kappa, \lambda, q \rangle = e^{i\lambda \phi(\Lambda, k, u)}|k', u'; \kappa, \lambda, q \rangle
\]

(66)

where

\[
e^{i\lambda \phi(\Lambda, k, u)} = U(R_{\hat{n}}^{-1} \Omega R_{\hat{n}})
\]

(67)
with
\[ \Omega = L_u^{-1} \Lambda L_u. \] (68)

Here \( k \) and \( u \) transform according to the law (1), (2), (16), (17). The rotation \( R_{\tilde{n}} \) connects \( \tilde{k} \) with \( D(L_u^{-1}, u)k \), i.e.
\[ R_{\tilde{n}} \tilde{k} = D(L_u^{-1}, u)k = \tilde{k}. \] (69)

Taking into account (65) we can derive the explicit form of \( R_{\tilde{n}} \)
\[ R_{\tilde{n}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 + \frac{(n^3)^2}{n^3 - 1} & \frac{n^1 n^2}{n^3 - 1} & n^1 \\ 0 & \frac{n^1 n^2}{n^3 - 1} & 1 + \frac{(n^2)^2}{n^3 - 1} & n^2 \\ 0 & n^1 & n^2 & n^3 \end{pmatrix} \] (70)
where \( \tilde{n} = \frac{\tilde{k}}{|\tilde{k}|} \). Notice that for rotations \( S \) form the stability group \( SO(2) \)
\[ R_S R_{\tilde{n}} = S R_{\tilde{n}} S^{-1}. \]

It is easy to check that \( R_{\tilde{n}}^{-1} \Omega R_{\tilde{n}} \) is a Wigner-like rotation belonging to the stability group \( SO(2) \) of \( (\tilde{k}, \tilde{u}) \) and determines the phase \( \varphi \). By means of standard topological arguments \( \lambda \) can take integer or half-integer values only i.e. \( \lambda = 0, \pm 1/2, \pm 1, \ldots \).

Now, the orthogonality relation (63) reads
\[ \langle k', u; \kappa', \lambda', q' | k, u; \kappa, \lambda, q \rangle = 2 \omega_k \delta^3(k' - k) \delta_{\lambda', \lambda}. \] (71)

\subsection*{4.2 Canonical quantization}
Following the Fock procedure, we define canonical commutation relations
\[ [a_\lambda(k, u), a_\tau(p, u)]_{\pm} = [a_\lambda^\dagger(k, u), a_\tau^\dagger(p, u)]_{\pm} = 0, \] (72)
\[ [a_\lambda(k, u), a_\tau^\dagger(p, u)]_{\pm} = 2 \omega_k \delta(k - p) \delta_{\lambda, \tau}, \] (73)
where - or + means the commutator or anticommutator and corresponds to the bosonic (\( \lambda \) integer) or fermionic (\( \lambda \) half-integer) case respectively. Furthermore, we introduce a Poincaré invariant vacuum \( |0\rangle \) defined by
\[ \langle 0 | 0 \rangle = 1 \quad \text{and} \quad a_\lambda(k, u) |0\rangle = 0. \] (74)

Therefore the one particle states
\[ a_\lambda^\dagger(k, u) |0\rangle \] (75)
are the base vectors belonging to an orbit in our bundle of Hilbert spaces iff
\[ U(\Lambda) a_\lambda^\dagger(k, u) U(\Lambda^{-1}) = e^{i \lambda \varphi(\Lambda, k, u)} a_\lambda^\dagger(k', u'), \] (76)
\[ U(\Lambda) a_\lambda(k, u) U(\Lambda^{-1}) = e^{-i \lambda \varphi(\Lambda, k, u)} a_\lambda(k', u'), \] (77)
and
\[ [P_\mu, a_\lambda^\dagger(k, u)]_{\pm} = k_\mu^\pm a_\lambda^\dagger(k, u). \] (78)
Notice that
\[ P_\mu = \int d^4k \theta(k^0) \delta(k^2 + \kappa^2) k_\mu \left( \sum_\lambda a^\dagger_\lambda(k, u) a_\lambda(k, u) \right) \]  
(79)
is a solution of (78).

Let us determine the action of the discrete transformations, space and time inversions, \( P \) and \( T \) and the charge conjugation \( C \) on the states \( |k, u; \kappa, \lambda, q\rangle \).

\[ P |k, u; \kappa, \lambda, q\rangle = \eta_s |k_\pi, u_\pi; \kappa, -\lambda, q\rangle, \]  
(80)
\[ T |k, u; \kappa, \lambda, q\rangle = \eta_t |k_\pi, u_\pi; \kappa, \lambda, q\rangle, \]  
(81)
\[ C |k, u; \kappa, \lambda, q\rangle = \eta_c |k, u; \kappa, \lambda, q\rangle_c, \]  
(82)
where \( |\eta_s| = |\eta_t| = |\eta_c| = 1, k_\pi = (k^0, -\vec{k}), u_\pi = (u^0, -\vec{u}) \), the subscript \( c \) means the antiparticle state and \( P, C \) are unitary, while \( T \) is antiunitary.

Consequently the actions of \( P, T \) and \( C \) in the ring of the field operators read
\[ P a^\dagger_\lambda(k, u) P^{-1} = \eta_s a^\dagger_{-\lambda}(k_\pi, u_\pi), \]  
(83)
\[ T a^\dagger_\lambda(k, u) T^{-1} = \eta_t a^\dagger_{\lambda}(k_\pi, u_\pi), \]  
(84)
\[ C a^\dagger_\lambda(k, u) C^{-1} = \eta_c b^\dagger_{\lambda}(k_\pi, u_\pi), \]  
(85)
where \( b_\lambda \equiv a^c_\lambda \) —antiparticle operators.

Finally we can deduce also the form of the helicity operator:
\[ \hat{\lambda}(u) = - \frac{\hat{W}_\mu u_\mu}{(Pu)^2 - P^2} \]  
(86)
where
\[ \hat{W}_\mu = \frac{1}{2} \varepsilon^{\sigma\lambda\tau} J_{\sigma\lambda} P_\tau \]
is the Pauli-Lubanski four-vector.

Notice that
\[ P \hat{\lambda}(u) P^{-1} = -\hat{\lambda}(u^\pi), \]  
(87)
\[ T \hat{\lambda}(u) T^{-1} = \hat{\lambda}(u^\pi), \]  
(88)
\[ C \hat{\lambda}(u) C^{-1} = \hat{\lambda}(u), \]  
(89)
as well as
\[ [\hat{\lambda}(u), a^\dagger_{\lambda}(u, k)] = \lambda a^\dagger_{\lambda}(u, k). \]  
(90)

### 4.3 Local fields

As usually we define local tachyonic fields as covariant Fourier transforms of the creation–annihilation operators. Namely
\[ \varphi_\alpha(x, u) = \frac{1}{(2\pi)^3} \int dq \int d\mu(k, \kappa, q) \sum_\lambda \left[ w_{\alpha\lambda}(k, u) e^{ikx} b^\dagger_{\lambda}(k, u) + v_{\alpha\lambda}(k, u) e^{-ikx} a_\lambda(k, u) \right], \]  
(91)
where the amplitudes $w_{\alpha\lambda}$ and $v_{\alpha\lambda}$ satisfy the set of corresponding consistency conditions (the Weinberg conditions). Here we sum irreducible Poincaré orbits labelled by selected helicities and over the invariant $q$. Thus the integration in (91) reduces to the integration with the measure

$$d\mu(k, \kappa) = d^4k \theta(k^0) \delta(k^2 + \kappa^2)$$

i.e. to the integration over the space of all initial conditions.

## 5 Scalar tachyonic field and its plane-wave decomposition

Let us consider a hermitian, scalar field $\varphi(x, u)$ satisfying the Klein–Gordon equation with imaginary “mass” $i\kappa$, i.e.

$$(g^\mu\nu(u) \partial_\mu \partial_\nu - \kappa^2) \varphi(x, u) = 0.$$  

(93)

The Fourier decomposition of the field $\varphi$ reads

$$\varphi(x, u) = \frac{1}{(2\pi)^{3/2}} \int d\mu(k, u) \left( e^{ikx} a^\dagger(k, u) + e^{-ikx} a(k, u) \right).$$

(94)

Integrating with respect to $k_0$ we obtain

$$\varphi(x, u) = \frac{1}{(2\pi)^{3/2}} \int_\Gamma \frac{d^3k}{2\omega_k} \left( e^{ikx} a^\dagger(k, u) + e^{-ikx} a(k+, u) \right)$$

(95)

where the integration range $\Gamma$ is determined by the eq. (47).

The canonical commutation rules (72), (73) take the form

$$[a(k, u), a(p, u)] = [a^\dagger(k, u), a^\dagger(p, u)] = 0,$$

(96)

$$[a(k, u), a^\dagger(p, u)] = 2\omega_k \delta(k - p_\nu).$$

(97)

By the standard procedure we obtain the commutation rule for $\varphi(x, u)$ valid for an arbitrary separation

$$[\varphi(x, u), \varphi(y, u)] = -i \Delta(x - y, u),$$

(98)

where the analogon of the Schwinger function reads

$$\Delta(x, u) = \frac{-i}{(2\pi)^3} \int d^4k \delta(k^2 + \kappa^2) e^{ikx}.$$  

(99)

It is remarkable that $\Delta$ does not vanish for a space-like separation which is a direct consequence of the faster-than-light propagation of the tachyonic quanta. Moreover $\Delta(x, u)|_{x^\nu = 0} = 0$ and therefore no interference occurs between two measurements of $\varphi$ at an instant time. This property is consistent with our interpretation of instant-time hyperplanes as the initial ones.

Now, because of the absolute meaning of the arrow of time in the CT synchronization we can introduce an invariant notion of the time-ordered product of field operators. In particular the tachyonic propagator

$$\Delta_F(x - y, u) = -i \langle 0 | T(\varphi(x, u) \varphi(y, u)) | 0 \rangle$$

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The singular functions are well defined as distributions on the space of "well behaved" solutions of the Klein–Gordon equation \((93)\). The role of the Dirac delta plays the generalized function
\[
\phi^4_\Gamma(x - y) = \frac{1}{(2\pi)^3} \int d^3 k \theta(\pm k^0) \delta(k^2 + \kappa^2) e^{ikx}. 
\]
(102)

The above form of \(\phi^4_\Gamma(x)\) express impossibility of the localization of tachyonic quanta. In fact, the tachyonic field does not contain modes with momentum \(\vec{k}\) inside the spheroid defined in eq. \((47)\). Consequently, by the Heisenberg uncertainty relation, an exact localization of tachyons is impossible.

Note also that
\[
\partial^0 \Delta(x - y, u) \delta(x^0 - y^0) = \phi^4_\Gamma(x - y)
\]
so the equal-time canonical commutation relations for \(\phi(x, u)\) and its conjugate momentum \(\pi(x, u) = \partial^0 \phi(x, u)\) have the correct form
\[
[\phi(x, u), \phi(y, u)] = \delta(x^0 - y^0) [\pi(x, u), \pi(y, u)] = 0, 
\]
(103)\

\[
[\phi(x, u), \pi(y, u)] \delta(x^0 - y^0) = i\phi^4_\Gamma(x - y)
\]
(104)
as the operator equations in the space of states.

To do the above quantization procedure mathematically more precise, we can use wave packets rather than the plane waves. Indeed, with a help of the measure we can define the Hilbert space \(\mathcal{H}_u\) of one particle states with the scalar product
\[
(f, g)_u = \int d\mu(k, u) f^*(k, u) g(k, u) < \infty. 
\]
(105)

Now, using standard properties of the Dirac delta we deduce
\[
(f, g)_u = \int d^3 k \frac{\delta^4(k)}{2\omega_k} f^*(k, u) g(k, u) .
\]
(106)

It is remarkable that for \(\omega_k \rightarrow 0\) to preserve inequality \(\|f\|^2_u < \infty\) the wave packets \(f(k, u)\) rapidly decrease to zero. This means that probability of "momentum localization" of a tachyon in the infinite velocity limit is going to zero in agreement with our intuition. As usually we introduce the smeared operators
\[
a(f, u) = (2\pi)^{-3/2} \int d\mu(k, u) a(k, u) f^*(k, u)
\]
(107)

and the conjugate ones. The canonical commutation rules \((96)–(97)\) take the form
\[
[a(f, u), a(g, u)] = [a^\dagger(f, u), a^\dagger(g, u)] = 0,
\]
(108)\

\[
[a(f, u), a^\dagger(g, u)] = (f, g)_u.
\]
(109)
We have also \( a(f, u) |0\rangle = 0 \) and \( \langle f, u | g, u \rangle = (f, g)_u \), where \( |f, u\rangle = a^\dagger(f, u) |0\rangle \).

According to our assumption of scalarity of \( \varphi(x, u) \)

\[
L \ni \Lambda : \ varphi'(x', u') = \varphi(x, u). \quad (110)
\]

The transformation law (76)–(77) is realized as follows

\[
U(\Lambda) \varphi(x, u) U^{-1}(\Lambda) = \varphi(x', u'). \quad (111)
\]

Therefore the wave packets must satisfy the scalarity condition

\[
f'(k', u') = f(k, u). \quad (112)
\]

It follows that the family \( \{ U(\Lambda) \} \) forms an unitary orbit of the intersystemic Lorentz group \( L \) in the bundle of the Hilbert spaces \( \mathcal{H}_u \); indeed we see that

\[
(f', g')_u = (f, g)_u. \quad (113)
\]

Now we introduce wave-packet solutions of the Klein–Gordon equation via the Fourier transformation

\[
F(x, u) = (2\pi)^{-3/2} \int d\mu(k, u) f(k, u) e^{-ikx}. \quad (114)
\]

In terms of these solutions the scalar product (105) reads

\[
(F, G)_u = -i \int d^3x F^*(x, u) \mathcal{O}^0 G(x, u). \quad (115)
\]

It is easy to see that for an orthonormal basis \( \{ \Phi_\alpha(x, u) \} \) in \( \mathcal{H}_u \) the completeness relation holds

\[
\sum_\alpha \Phi_\alpha^*(x, u) \Phi_\alpha(y, u) = i\Delta^+_\varphi(x - y, u), \quad (116)
\]

where \( \Delta^+ \) has the form (101) and it is the reproducing kernel in \( \mathcal{H}_u \) i.e.

\[
(i\Delta^+_\varphi(x, u), \Phi)_u = \Phi(x, u).
\]

Finally, the four-momentum operator has the form

\[
P_\mu = \int d\mu(k, u) k_\mu a^\dagger(k, u) a(k, u). \quad (117)
\]

Thus we have constructed a consistent quantum field theory for the hermitian, scalar tachyon field \( \varphi(x, u) \). We conclude, that a proper framework to do this is the CT synchronization scheme.

### 5.1 Spontaneous breaking of the synchronization group

As we have seen in the foregoing section, the intersystemic Lorentz group \( L \) is realized unitarily on the quantum level. In this section we will analyse the role of the synchronization group \( L_S \) in our scheme.

As was stressed in the Sec. 3.4, if tachyons exist then one and only one inertial frame is the preferred frame. In other words the relativity principle is
broken in this case: tachyons distinguish a fixed synchronization scheme from the family of possible CT synchronizations. Consequently, because all admissible synchronizations are related by the group \( L_S \), this group should be broken. To see this let us consider transformations belonging to the subgroup \( L_0 \) (see Sec. 3.4). They are composed from the transformations of intersystemic Lorentz group \( L \) and the synchronization group \( L_S \); namely they have the following form (see eq. 53) and the definition of \( L_0 \),

\[
    u' = u, \quad x' = T(u)\Lambda_0 T^{-1}(u)x \equiv \Lambda_0(u)x.
\]

We search an operator \( W(\Lambda_0) \) implementing (118) on the quantum level; namely

\[
    \phi'(x, u) = W(\Lambda_0^{-1})\phi(x, u)W^\dagger(\Lambda_0^{-1}) = \phi(x', u). \tag{119}
\]

This means that we should compare both sides of (119) i.e.

\[
    \int d\mu(k, u) \left[ e^{ikx} a'(k, u) + e^{-ikx} a'(k, u) \right] = \int d\mu(p, u) \left[ e^{ipx'} a'(p, u) + e^{-ipx'} a'(p, u) \right], \tag{120}
\]

where \( x' \) is given by eq. (118), while, formally

\[
    a' = WaW^\dagger, \quad a'^\dagger = Wa^\dagger W^\dagger. \tag{121}
\]

Taking into account the form of the measure \( d\mu \) (eq. (122)) and the fact that \( \Lambda_0(u) \) does not leave invariant the sign of \( k^0 \), after some calculations, we deduce the following form of \( W \):

\[
    a'(k, u) = \theta(k^0) a(k', u) + \theta(-k^0) a^\dagger(-k', u), \tag{122}
\]

\[
    a'^\dagger(k, u) = \theta(k^0) a^\dagger(k', u) + \theta(-k^0) a(-k', u), \tag{123}
\]

where \( k' = \Lambda_0^{-1}(u)k \).

We see that formally unitary operator \( W(\Lambda_0) \) is realized by the Bogolubov-like transformations; the Heaviside \( \theta \)-step functions are the Bogolubov coefficients. The form (122)–(123) of the transformations of the group \( L_0 \) reflects the fact, that a possible change of the sign of \( k^0 \) causes a different decomposition of the field \( \phi \) on the positive and negative frequencies. Furthermore it is easy to check that the transformation (122)–(123) preserves the canonical commutation relations (96)–(97).

However, the formal operator \( W(\Lambda_0) \) realized in the ring of the field operators, cannot be unitarily implemented in the space of states in general; only if \( \Lambda_0 = \Lambda_u \) is an element of the stability group \( SO(3)_u \) of \( u \) in \( L_S \), it can be realized unitarily. This is related to the fact that \( \Lambda_u \) does not change the sign of \( k^0 \) for any \( k \). Indeed, notice firstly (see (122)–(123)) that \( W(\Lambda_0) \) does not annihilate the vacuum \( |0\rangle \). Moreover, the particle number operator

\[
    N = \int d\mu(k, u) a^\dagger(k, u)a(k, u) \tag{124}
\]

applied to the “new” vacuum

\[
    |0\rangle' = W^{-1} |0\rangle \tag{125}
\]
gives
\[ N |0\rangle' = \delta^3(0) \int d^3 k \ \theta(-\Lambda_0(u)k^0) |0\rangle'. \quad (126) \]

The right side of the above expression diverges like \( \delta^6(0) \) for any \( \Lambda_0(u) \in L_0/SO(3)_u \). Only for the stability subgroup \( SO(3)_u \) vacuum remains invariant. Thus, a “new” vacuum \(|0\rangle'\), related to an essentially new synchronization, contains an infinite number of “old” particles. As is well known, in such a case, two Fock spaces \( H \) and \( H' \), generated by creation operators from \(|0\rangle\) and \(|0\rangle'\) respectively, cannot be related by an unitary transformation \( W(\Lambda_0) \) in our case. Therefore, we have deal with the so called spontaneous symmetry breaking of \( L_S \) to the stability subgroup \( SO(3) \) (recall that \( L \) is realised unitarily). This means that physically privileged is only one realization of the canonical commutation relations \( (96)–(97) \) corresponding to the vacuum \(|0\rangle\). Such a realization is related to a definite choice of the privileged inertial frame and consequently to a definite CT synchronization scheme. Thus we can conclude that, on the quantum level, tachyons distinguish a preferred frame via spontaneous breaking of the synchronization group.

To complete discussion, let us apply the four-momentum operator \( P_\mu \) to the new vacuum \(|0\rangle'\). As the result we obtain
\[ P_\mu |0\rangle' = -\delta^3(0) \Lambda_0^{\nu \mu} \theta(-\Lambda_0(u)k^0) |0\rangle'. \quad (127) \]

This expression diverges again like \( \delta^7(0) \) for \( \Lambda_0 \in L_0/SO(3)_u \). Therefore a transition to a new vacuum (≡ change of the privileged frame) demands an infinite momentum transfer, i.e. it is physically inadmissible. This last phenomenon supports our claim that existence of tachyons is associated with spontaneous breaking of the the synchronization group. On the other hand it can be shown \([11]\) that a free field theory for standard particles (bradyons or luxons), formulated in CT synchronization, is unitarily equivalent to the standard field theory in the EP synchronization; we do not repeat the corresponding proof because of its simplicity.

6  Fermionic tachyons with helicity \( \lambda = \pm \frac{1}{2} \)

To construct tachyonic field theory describing field excitations with the helicity \( \pm \frac{1}{2} \), we assume that our field transforms under Poincaré group like bispinor (for discussion of transformation rules for local fields in the CT synchronization see \([10]\)); namely
\[ U(\Lambda) \psi(x, u) U(\Lambda^{-1}) = S(\Lambda^{-1}) \psi(x', u'), \quad (128) \]

where \( S(\Lambda) \) belongs to the representation \( D^{\pm 0} \oplus D^{0 \pm} \) of the Lorentz group. Because we are working in the CT synchronization, it is convenient to introduce

\[ \text{We can treat } (128) \text{ in some sense, as a quantum version of the familiar reinterpretation principle } [21]. \text{ We find that the reinterpretation principle cannot be unitarily implemented—this is just the source of inconsistencies in approaches incorporating this principle.} \]
an appropriate (CT-covariant) base in the algebra of Dirac matrices as

\[ \gamma^\mu = T(u)^\mu_\nu \gamma^\nu_E, \] (129)

where \( \gamma^\nu_E \) are standard \( \gamma \)-matrices, while \( T(u) \) is given by the eq. (14). Therefore

\[ \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}(u)I. \] (130)

However, notice that the Dirac conjugate bispinor \( \bar{\psi} = \psi^\dagger \gamma^0_E \). Furthermore

\[ \gamma^5 = -\frac{i}{4!} \epsilon_{\mu\nu\sigma\lambda} \gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\lambda = \gamma^5_E. \]

Now, we look for covariant field equations which are of degree one with respect to the derivatives \( \partial_\mu \) and imply the Klein–Gordon equation

\[ (g^{\mu\nu}(u)\partial_\mu \partial_\nu - \kappa^2) \psi = 0, \] (131)

related to the space-like dispersion relation \( k^2 = -\kappa^2 \). We also require the \( T \)-invariance of these equations.

As the result we obtain the following family of the Dirac-like equations

\[ \{(u \gamma \sin \alpha - 1) ((iu\partial) \cos \beta - \kappa \sin \beta) - \gamma^5 \left[ (-i\gamma \partial) + \frac{i}{2}[\gamma \partial, u\gamma] \right] \sin \alpha 
\quad +uv ((iu\partial)(1 + \cos \alpha \sin \beta) + \kappa \cos \alpha \cos \beta) \} \psi(x,u) = 0, \] (132)

derivable from an appropriate hermitian Lagrangian density. Here \( u\gamma = u_\mu \gamma^\mu \), \( u\partial = u_\mu \partial_\mu \), \( \gamma \partial = \gamma^\mu \partial_\mu \) and \( \alpha, \beta \) — real parameters, \( \alpha \neq (2n+1)\frac{\pi}{2} \). To guarantee the irreducibility of the elementary system described by (132), the equation (132) must be accomanied by the covariant helicity condition

\[ \hat{\lambda}(u)\psi(u,k) = \lambda \psi(u,k) \] (133)

where \( \hat{\lambda} \) is given by (13) taken in the coordinate representation (see below) and \( \lambda \) is fixed (\( \lambda = \frac{1}{2} \) or \( -\frac{1}{2} \) in our case). This condition is quite analogous to the condition for the left (right) bispinor in the Weyl’s theory of the massless field. It implies that particles described by \( \psi \) have helicity \( -\lambda \), while antiparticles have helicity \( \lambda \). For the obvious reason in the following we will concentrate on the case \( \lambda = \frac{1}{2} \).

Notice that the pair of equations (132,133) is not invariant under the \( P \) or \( C \) inversions separately for every choice of \( \alpha \) and \( \beta \).

Now, in the bispinor realization the helicity operator \( \hat{\lambda} \) has the following explicit form

\[ \hat{\lambda}(u) = \frac{\gamma^5 [i\gamma \partial, u\gamma]}{4\sqrt{(iu\partial)^2 + \Box}} \] (134)

where the integral operator \( ((iu\partial)^2 + \Box)^{-\frac{1}{2}} \) in the coordinate representation is given by the well behaving distribution

\[ \frac{1}{\sqrt{(-iu\partial)^2 + \Box}} = \frac{1}{(2\pi)^4} \int \frac{d^4p \varepsilon(\nu)p^\nu e^{ipx}}{\sqrt{(|u|^2 - p^2).} \] (135)

\[ \text{In the Ref. \cite{9} we found a class of the second order equations under condition of the \( P \)-invariance.} \]
Now, let us notice that the equation (132), supplemented by the helicity condition (133), are noninvariant under the composition of the $P$ and $C$ inversions (see eqs. (83)–(85) and the Appendix), except of the case $\sin \alpha = \cos \beta = 0$. Because (132)–(133) are $T$-invariant, therefore only for $\sin \alpha = \cos \beta = 0$ they are $CPT$-invariant. Taking $\sin \beta = \cos \alpha = 1$ we obtain from (132)

$$\left\{ \kappa + \gamma^5 [i \gamma \partial - 2u \gamma(i u \partial)] \right\} \psi = 0,$$

supplemented by (133). On the other hand, for $\cos \alpha = -\sin \beta = 1$ we obtain

$$(\gamma^5 (i \gamma \partial) - \kappa) \psi = 0. \quad (137)$$

The last equation is exactly the Chodos et al. [1] Dirac-like equation for tachyonic fermion. However, contrary to the standard EP approach, it can be consistently quantized in our scheme (if it is supplemented by the helicity condition (133)). In the following we will analyze the eqs. (137) and (133) by means of the Fourier decomposition

$$\psi(x, u) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^4k \delta(\kappa^2 + \kappa^2) \theta(k^0) \left[ w_+ (k) e^{i k x} b^{\dagger}_+ (k) + v_- (k) e^{-i k x} a_- (k) \right]$$

of the field $\psi$. The creation and annihilation operators $a$ and $b$ satisfy the corresponding canonical anticommutation relations (72)–(73), i.e., the nonzero ones are

$$[a_- (k), a^\dagger_- (p)]_+ = 2\omega_k \delta(k^- - p^-) \quad (139)$$

$$[b_+ (k), b^\dagger_+ (p)]_+ = 2\omega_k \delta(k^- - p^-) \quad (140)$$

In (138) $b_- \frac{1}{2}$ and $a_+ \frac{1}{2}$ do not appear because we decided to fix $\lambda = \frac{1}{2}$ in (133) (compare with (90)). As the consequence of (133) the corresponding amplitudes $w_- \frac{1}{2}$ and $v_+ \frac{1}{2}$ vanish. The nonvanishing amplitudes $w_+ \frac{1}{2}$ and $v_- \frac{1}{2}$ satisfy

$$(\kappa + \gamma^5 k \gamma) w_+ \frac{1}{2} (k, u) = 0, \quad (141)$$

$$\left( 1 + \frac{\gamma^5 [k \gamma, u \gamma]}{2 \sqrt{q^2 + \kappa^2}} \right) w_+ \frac{1}{2} (k, u) = 0, \quad (142)$$

$$(\kappa - \gamma^5 k \gamma) v_- \frac{1}{2} (k, u) = 0, \quad (143)$$

$$\left( 1 + \frac{\gamma^5 [k \gamma, u \gamma]}{2 \sqrt{q^2 + \kappa^2}} \right) v_- \frac{1}{2} (k, u) = 0. \quad (144)$$

Here $k^0 = \omega_k$, $q = uk$. The solution of (141)–(144) reads

$$w_+ \frac{1}{2} (k, u) = \left( \frac{\kappa - \gamma^5 k \gamma}{2\kappa} \right) \frac{1}{2} \left( 1 - \frac{\gamma^5 [k \gamma, u \gamma]}{2 \sqrt{q^2 + \kappa^2}} \right) w_+ \frac{1}{2} (\tilde{k}, \tilde{u}), \quad (145)$$

$$v_- \frac{1}{2} (k, u) = \left( \frac{\kappa + \gamma^5 k \gamma}{2\kappa} \right) \frac{1}{2} \left( 1 - \frac{\gamma^5 [k \gamma, u \gamma]}{2 \sqrt{q^2 + \kappa^2}} \right) v_- \frac{1}{2} (\tilde{k}, \tilde{u}). \quad (146)$$
Furthermore, the projections $w\bar{w}$ and $v\bar{v}$ read

$$
w_{\frac{1}{2}}(k,u)\bar{w}_{\frac{1}{2}}(k,u) = (\kappa - \gamma^5 k\gamma) \frac{1}{2} \left( 1 - \frac{\gamma^5[k\gamma,u\gamma]}{2\sqrt{q^2 + \kappa^2}} \right), \tag{147}$$

$$
v_{-\frac{1}{2}}(k,u)\bar{v}_{-\frac{1}{2}}(k,u) = -(\kappa + \gamma^5 k\gamma) \frac{1}{2} \left( 1 - \frac{\gamma^5[k\gamma,u\gamma]}{2\sqrt{q^2 + \kappa^2}} \right). \tag{148}$$

The above amplitudes fulfil the covariant normalization conditions

$$
\bar{w}_{\frac{1}{2}}(k,u)\gamma^5 u\gamma w_{\frac{1}{2}}(k,u) = \bar{v}_{-\frac{1}{2}}(k,u)\gamma^5 u\gamma v_{-\frac{1}{2}}(k,u) = 2q, \tag{149}
$$

$$
\bar{w}_{\frac{1}{2}}(k^\pi, u)\gamma^5 u\gamma w_{\frac{1}{2}}(k, u) = 0. \tag{150}
$$

The amplitudes $w_{\frac{1}{2}}(\tilde{k}, \tilde{u})$ and $v_{-\frac{1}{2}}(\tilde{k}, \tilde{u})$, taken for the values $\tilde{k}$ and $\tilde{u}$ given in the eq. (63), have the following explicit form (for $\gamma_\mu$ matrix convention—see Appendix)

$$
w_{\frac{1}{2}}(\tilde{k}, \tilde{u}) = \begin{pmatrix} 0 \\ \sqrt{-q + \sqrt{q^2 + \kappa^2}} \\ 0 \\ \sqrt{q + \sqrt{q^2 + \kappa^2}} \end{pmatrix}, \quad v_{-\frac{1}{2}}(\tilde{k}, \tilde{u}) = \begin{pmatrix} 0 \\ -\sqrt{-q + \sqrt{q^2 + \kappa^2}} \\ 0 \\ \sqrt{q + \sqrt{q^2 + \kappa^2}} \end{pmatrix}. \tag{151}
$$

It is easy to see that in the massless limit $\kappa \to 0$ the eqs. (141)–(144) give the Weyl equations

$$
k\gamma w_{\frac{1}{2}} = k\gamma v_{-\frac{1}{2}} = 0, \quad \gamma^5 w_{\frac{1}{2}} = -w_{\frac{1}{2}}, \quad \gamma^5 v_{-\frac{1}{2}} = -v_{-\frac{1}{2}},$$

as well as the amplitudes (143), (146) have a smooth $\kappa \to 0$ limit (it is enough to verify (151)).

Now, the normalization conditions (149)–(150) generate the proper work of the canonical formalism. In particular, starting from the Lagrangian density $\mathcal{L} = \bar{\psi} \left( \gamma^5 (i\gamma\partial - \kappa) \right) \psi$ we can derive the translation generators; with help of (138,139,140) and (149,150) we obtain

$$
P_\mu = \int \frac{d^4k}{2\omega_k} k_\mu (a_{\frac{1}{2}}^\dagger a_{-\frac{1}{2}} + b_{\frac{1}{2}}^\dagger b_{-\frac{1}{2}}) \tag{152}$$

In agreement with (18). Thus we have constructed fully consistent Poincaré covariant free field theory for a fermionic tachyon with helicity $-\frac{1}{2}$, quite analogous to the Weyl’s theory for a left spinor which is obtained as the $\kappa \to 0$ limit.

7 The stability of vacuum

One of the serious defects of the standard approach to the tachyon field quantization is apparent instability of the vacuum. The reason is that relativistic
kinematics admits in this case many-particle states with vanishing total four-momentum. It is related directly to the fact that for each (space-like) four-momentum, say $k^\mu_E$, the four-momentum $-k^\mu_E$ with the opposite sign is kinematically admissible, because there is no covariant spectral condition $k^0_E > 0$ for space-like $k^\mu_E$.

Notwithstanding, such a situation does not take place in the presented scheme, because space-like four-momentum $k$ satisfies the invariant spectral condition, $k^0 > 0$ in each inertial frame. Thus the sum of $k$ and $k'$ satisfies the same spectral condition. In brief, we have exactly the same situation as in the case of the time-like (or light-like) four-momenta under the invariant spectral condition, $k^0 > 0$. This means that in our scheme multiparticle states with vanishing total four-momentum do not appear, so the vacuum $|0\rangle$ cannot decay.

For example, for two particle state $|q = k + p\rangle \equiv |k\rangle \otimes |p\rangle$, where $k$ and $p$ satisfy spectral condition, i.e., $k^0 > 0$, $p^0 > 0$, we have the inequality $q^0 > 0$ (i.e., $q \neq 0$), so there is no vacuum-like state with the four-momentum $q = 0$. Concluding, this theory is stable.

8 Conclusions

The main result of this work is demonstration that it is possible to construct Poincaré-covariant theory for tachyons on both classical and quantum level. The only price is necessity of existence of a preferred frame. Tachyons are classified according to the unitary representations of $SO(2)$ rather than $SO(2, 1)$ group; so they are labelled by the eigenvectors of the helicity operator. In particular for the helicity $\lambda = \pm \frac{1}{2}$ we have constructed family of $T$-invariant equations (132). Under condition of $PCT$ invariance we selected two equations (136) and (137). The equation (137) coincide with the one proposed by Chodos et al.

We show by explicit construction that, in our scheme, theory described by this equation, supplemented by the helicity condition (133) can be consistently quantized. This theory describe fermionic tachyon with helicity $-\frac{1}{2}$. It has a smooth massless limit to the Weyl’s left-handed spinor theory. These results show that there are no theoretical obstructions to interpret the experimental data about square of mass of neutrinos as a signal that they can be fermionic tachyons. A more detailed discussion of this problem is given in the paper [14].

We can conclude that, contrary to the current opinion, it is possible to agree the Lorentz covariance with universal causality and existence of a privileged frame. Moreover, a consistent quantization of the tachyonic field in this framework is possible. From this point of view the Einstein–Poincaré synchronization is useless in the tachyonic case—the proper choice is the CT synchronization. On the other hand, in a description of bradyons and luxons only, we are free in the choice of a synchronization procedure. For this reason we can use in this case CT-synchronization as well as the EP one.

The CT-synchronization, a natural one for a description of tachyons, favours a reference frame (privileged frame). This preference is only formal (it is a convention) if tachyons do not exist. However, if they exist, then an inertial reference frame is really (physically) preferred, what in fact holds in the real

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9Recall that in the asymptotics $k^0 \to 0$ the wave packets decrease to zero (see remark below the eq. (106)).
world. As a consequence, the one-way light velocity can be measured in this case and, in general, it will be direction-dependent for a moving observer. Light velocity is isotropic only in the privileged frame. In the observed world we have a serious candidate to such a frame; namely frame related to the cosmic background radiation.

A Derivation of the Lorentz group transformation rules

Let us derive the form of transformations between two coordinate frames \( x^\mu \) and \( x'^\mu \); for simplicity we denote \( D(\Lambda, u_E) \equiv D(\Lambda, u_E) \)

\[
x'(u'_E) = D(\Lambda, u_E)x(u_E),
\]

(153)

where \( D(\Lambda, u_E) \) is a real (invertible) \( 4 \times 4 \) matrix, \( \Lambda \) belongs to the Lorentz group and \( u_E^{\mu} \) is assumed to be a Lorentz four-vector, i.e.,

\[
u'_E = \Lambda u_E, \quad u_E^2 = 1 > 0.
\]

(154)

The transformations (153)–(154) constitute a realization of the Lorentz group if the following composition law holds

\[
D(\Lambda_2, \Lambda_1 u_E)D(\Lambda_1, u_E) = D(\Lambda_2 \Lambda_1, u_E).
\]

(155)

Now we demand that \( (x^\mu) \equiv (x^0, \vec{x}) \) transform under subgroup of rotations as singlet + triplet (isotropy condition), i.e. for \( R \in SO(3) \)

\[
\Omega \equiv D(R, u_E) = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}.
\]

(156)

From eqs. (153)–(155) we see that the identity and the inverse element have the form

\[
I = D(I, u_E),
\]

(157)

\[
D^{-1}(\Lambda, u_E) = D(\Lambda^{-1}, \Lambda u_E).
\]

(158)

Using the familiar Wigner’s trick we obtain that

\[
D(\Lambda, u_E) = T(\Lambda u_E)\Lambda T^{-1}(u_E),
\]

(159)

where the real matrix \( T(u_E) \) is given by

\[
T(u_E) = D(L_{u_E}, \hat{u}_E)L_{u_E}^{-1}.
\]

(160)

Here \( \hat{u}_E = (1, 0, 0, 0) \) and \( L_{u_E} \) is the boost matrix: \( u_E = L_{u_E} \hat{u}_E \). We use the following parametrization of the matrix \( L_{u_E} \)

\[
L_{u_E} = \begin{pmatrix}
u^0_E & \hat{u}^T_E \\
\vec{u}_E & I + \frac{\hat{u}_E \vec{u}_E}{1+u_E^2} \end{pmatrix}.
\]
Note that the transformations (153)–(154) leave the bilinear form $x^T(u_E) \times g(u_E)x(u_E)$, where the symmetric tensor $g(u_E)$ reads

$$g(u_E) = (T(u_E)\eta T^T(u_E))^{-1},$$

invariant. Here $\eta$ is the Minkowski tensor and the superscript $^T$ means transposition.

Now we determine the matrix $T(u_E)$. To do this we note that under rotations $T(\Omega u_E) = \Omega T(u_E)\Omega^{-1}$, so the most general form of $T(u_E)$ reads

$$T(u_E) = \left( \begin{array}{cc}
a(u_E^0) & b(u_E^0)u_E^T \\
d(u_E^0)u_E^T & e(u_E^0)I + (u_E^0 \otimes u_E^T)f(u_E^0) \end{array} \right),$$

(162)

where $a, b, d, e$ and $f$ are some functions of $u_E^0$. Inserting eq. (162) into eq. (161) we can express the metric tensor $g(u_E)$ by $a, b, d, e$ and $f$. In a three dimensional flat subspace we can use an orthogonal frame (i.e. $(g^{-1})_{ik} = -\delta_{ik}$; $i, k = 1, 2, 3$), so we obtain

$$e(u_E^0) = 1, \quad d^2 = (2 - f \hat{u}_E^2)f.$$

(163)

Furthermore, from the equation of null geodesics, $dx^Tgdx = 0$, we deduce that the light velocity $\hat{c}$ in the direction $\hat{n}$ ($\hat{n}^2 = 1$) is of the form

$$\hat{c} = \hat{n}\left(\frac{\sqrt{\alpha + \beta^2\hat{u}_E^2} - \beta\hat{u}_E\hat{n}}{\alpha}\right)^{-1},$$

(164)

where $\alpha = a^2 - b^2\hat{u}_E^2$, $\beta = ad - b(1 + f\hat{u}_E^2)$. From eq. (164) we see that the synchronization convention depends on the functions $\alpha$ and $\beta$ only. Now, because $a, b$ and $d$ can be expressed as functions of $\alpha$, $\beta$ and $f$ and we are interested in essentially different synchronizations only, we can choose

$$f = 0,$$

(165)

so

$$d = 0, \quad \beta = -b, \quad \alpha = a^2 - b^2\hat{u}_E^2.$$  

(166)

Finally, from (164)–(166) the average value of $|\hat{c}|$ over a closed path is equal to

$$\langle |\hat{c}| \rangle_{\text{cl. path}} = \frac{1}{a}.$$

Because we demand that the round-trip light velocity ($\langle |\hat{c}| \rangle_{\text{cl. path}} = c = 1$) be constant, we obtain

$$a = 1.$$

(167)

Summarizing, $T(u_E)$ has the form

$$T(u_E) = \left( \begin{array}{cc}1 & b(u_E^0)u_E^T \\0 & I \end{array} \right),$$

(168)
while the light velocity

\[ \vec{c} = \vec{n} \left( 1 + b\vec{u}_E \vec{n} \right)^{-1}, \]  

so the Reichenbach coefficient reads

\[ \varepsilon(\vec{n}, \vec{u}_E) = \frac{1}{2} \left( 1 + b\vec{u}_E \vec{n} \right). \]  

In the special relativity the function \( b(u_0^E) \) distinguishes between different synchronizations. Choosing \( b(u_0^E) = 0 \) we obtain \( \vec{c} = \vec{n}, \ v = \frac{1}{2} \) and the standard transformation rules for coordinates: \( x^i_E = \Lambda x_E \), where, as before the subscript \( E \) denotes EP-synchronization. On the other hand, if we demand that the instant-time hyperplane \( x^0 = \text{constant} \) be an invariant notion, i.e. that \( x^0 = D(\Lambda, u^0_E) \) so \( D(\Lambda, u^0_E)_k = 0 \), then from eqs. (159, 168) we have

\[ b(u^0_E) = -\frac{1}{u_0^E}, \]  
i.e. finally

\[ T(u_E) = \begin{pmatrix} 1 & -\vec{u}_E^T \\ 0 & I \end{pmatrix}. \]  

Thus we have determined the form of the transformation law (153) in this case in terms of the EP four-velocity \( u^0_E \). Notice that in terms of \( u = T(u_E)u_E \), the matrix \( T = T(u) \), by means of eq. (168) has the form

\[ \begin{pmatrix} 1 & b(u)\vec{n}^T \\ 0 & I \end{pmatrix} \]  

so for \( b(u) \) determined by (171), \( b = -u_0 \), i.e.

\[ T^\dagger = T \]  

in this case.

### B Representation of the discrete symmetries

The discrete transformations \( P, T \) and \( C \), defined by the eqs. (80)–(82) are realised in the bispinor space standardly, i.e. \( P \) by \( \gamma_0^E \), while \( T \) and \( C \) by \( T \) and \( C \) satisfying the conditions

\[ T^\dagger T = I, \quad T^* T = -I, \quad T^{-1}\gamma^\mu T = \gamma^\mu, \]  

\[ C^\dagger C = I, \quad C^* C = -I, \quad C^T = -C, \quad C^{-1}\gamma^\mu C = -\gamma^\mu T. \]  

Notice that the last condition in (175) and (176) can be formulated in terms of the standard \( \gamma_E^\mu \) exactly in the same form.
In explicit calculations of the amplitudes (151) we have used the following representations of the $\gamma_E$ matrices: $\vec{\gamma}_E = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$, $\gamma_0^0_E = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$. In this representation the parity, charge conjugation and time inversion are given, up to a phase factor by

$$P = \gamma^0_E, \quad C = i\gamma^0_E \gamma_5^E, \quad T = -i\gamma^0_E \gamma^2_E \gamma^1_E \gamma^5_E.$$

Finally, we use $\epsilon^{\mu\nu\sigma\lambda}$ with $\epsilon^{0123} = 1$; consequently $\gamma_5^E = i\gamma^0_E \gamma_1^E \gamma_2^E \gamma_3^E$.

References

[1] A. Chodos, A. I. Hauser, and V. A. Kostelecky. *Phys. Lett.*, B150:431, 1985.

[2] L. Montanet et al. *Phys. Rev.*, D50:1173, 1994. And 1995 off-year partial update for the 1996 edition available on the PDG WWW pages (URL: [http://pdg.lbl.gov/](http://pdg.lbl.gov/)).

[3] K. Assamagan et al. *Phys. Lett.*, B335:231, 1994.

[4] G. Gelmini and E. Roulet. Neutrino masses. Preprint CERN-TH.7541/94, CERN, Geneva, 1994.

[5] L. S. Brown and C. Zhai. Atomic effects in Tritium beta decay. Preprint UW/PT-95-16, University of Washington, 1995.

[6] E. Giannetto, G. D. Maccarrone, R. Mignani, and E. Recami. *Phys. Lett.*, B178:115, 1986.

[7] K. Kamoi and S. Kamefuchi. In E. Recami, editor, *Tachyons, Monopoles and Related Topics*. North-Holland, New York, 1978.

[8] J. Rembieliński. Quantization of the tachyonic field. Preprint KFT UL 2/94, Łódź Univ., 1994. ([hep-th/9410079](http://arxiv.org/abs/hep-th/9410079)).

[9] J. Rembieliński. Tachyonic neutrinos? Preprint KFT UL 5/94, Łódź Univ., 1994. ([hep-th/9411230](http://arxiv.org/abs/hep-th/9411230)).

[10] J. Rembieliński. *Phys. Lett.*, A78:33, 1980.

[11] J. Rembieliński. A quantum description of tachyons. Preprint IF UL/1/1980, Łódź Univ., 1980.

[12] H. Reichenbach. *Axiomatization of the Theory of Relativity*. University of California Press, Berkeley, CA, 1969.

[13] M. Jammer. In *Problems in the Foundations of Physics*. North-Holland, Bologne, 1979.

[14] J. Ciborowski and J. Rembieliński. Experimental results and the hypothesis of the tachyonic neutrinos. The talk presented at 28th International Conference on High Energy Physics, Warsaw, July 1996.
[15] Y. P. Terletsky. *Sov. Phys. Dokl.*, 5:782, 1960.
[16] S. Tanaka. *Prog. Theor. Phys.*, 24:171, 1960.
[17] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan. *Am. Journ. Phys.*, 30:718, 1962.
[18] E. Recami. *Giornale di Fisica*, 10:195, 1968.
[19] V. Olkhovsky and E. Recami. *Nuov. Cim.*, A63:814, 1969.
[20] E. Recami. *Rev. del Nuov. Cim.*, 9:1, 1986.
[21] G. Feinberg. *Phys. Rev.*, 159:1089, 1967.
[22] E. Recami. In E. Recami, editor, *Tachyons, Monopoles and Related Topics*. North-Holland, New York, 1978.
[23] K. Kamoi and S. Kamefuchi. *Prog. Theor. Phys.*, 45:1646, 1971.
[24] N. Nakanishi. *Prog. Theor. Phys. Suppl.*, 51:1, 1972.
[25] T. Chang. *Phys. Lett.*, A70:1, 1979.
[26] T. Chang. *J. Phys.*, A13:L207, 1980.
[27] T. Chang. *J. Phys.*, A12:L203, 1979.
[28] F. R. Tangherlini. *Nuov. Cim. Suppl.*, 20:1, 1961.
[29] D. G. Torr and J. G. Vargas. *Found. of Phys.*, 16:1089, 1986.
[30] J. G. Vargas. *Found. of Phys.*, 16:1003, 1986.
[31] T. Sjödin. *Nuov. Cim.*, B51:229, 1979.
[32] R. Mansouri and R. U. Sexl. *Gen. Relativ. Grav.*, 8:479, 515, 809, 1977.
[33] J. L. Anderson. *Principles of Relativity Physics*. Academic Press, New York, 1967.
[34] C. M. Will. *Phys. Rev.*, D45:403, 1992.
[35] H. Bacry. *Localizability and Space in Quantum Physics*. Springer-Verlag, Berlin—Heidelberg.
[36] P. Caban and J. Rembieliński. Localization of quantum states and the preferred frame. Preprint, Lódź University, 1996.
[37] S. Weinberg. *Gravitation and Cosmology*, page 474. J. Wiley & Sons, New York, 1972.
[38] C. M. Will. *Theory and Experiment in Gravitational Physics*. Cambridge University Press, 1993.
[39] T. Sjödin and M. F. Podlaha. *Lett. al Nuov. Cim.*, 31:433, 1981.
[40] A. Flidrzyński and A. Nowicki. *J. Phys.*, A15:1051, 1982.