Re-examination of the Hubble Constant from the Sunyaev-Zel’dovich Effect: Implication for Cosmological Parameters

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Abstract

We performed a systematic and comprehensive estimate of the Hubble constant $H_0$ via the Sunyaev-Zel’dovich temperature decrement and the X-ray measurements for clusters up to the redshift $z = 0.541$, with particular attention to its dependence on the density parameter $\Omega_0$, and the cosmological constant $\lambda_0$. The resulting values of $H_0$ are largely consistent with that derived from the Cepheid distance for nearby galaxies. We discuss how the angular diameter distance $d_\Lambda$ vs. $z$ diagram from the accurate SZ measurements of high-$z$ clusters can distinguish the values of $\Omega_0$ and $\lambda_0$.

Key words: cosmology: theory — large-scale structure of the universe — methods: statistical

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1. Introduction

There have been a lot of attempts to determine the Hubble constant \( H_0 \) (\( \equiv 100h \) km s\(^{-1}\) Mpc\(^{-1}\)) from the Sunyaev-Zel’dovich (SZ) temperature decrement and the X-ray measurements (Sunyaev & Zel’dovich 1972; see Rephaeli 1995 for a review). The current results are subject to several uncertainties due to simplified model assumptions in describing the realistic cluster gas distribution as well as the limited spatial resolution of the X-ray and radio observations (Birkinshaw, Hughes & Arnaud 1991; Inagaki, Suginoara & Suto 1995). Nevertheless this method is very important for two reasons; it does not rely on any local (or empirical) calibrator, and it can be used even at high redshift \( z \gtrsim 1 \) in principle where no other reliable distance indicator is known, perhaps except for the SNe Ia (Perlmutter et al. 1996).

As discussed by Silk & White (1978) and Birkinshaw et al. (1991), the observation of the SZ effect determines the angular diameter distance \( d_A(z) \) to the redshift \( z \) of the cluster. For \( z \ll 1 \), \( d_A(z) \) is basically given only in terms of \( H_0 (\sim cz/H_0, \) with \( c \) being the light velocity). If \( z \gtrsim 0.1 \), however, the density parameter \( \Omega_0 \), the dimensionless cosmological constant \( \lambda_0 \), and possibly the degree of the inhomogeneities in the light path (see §3 below; Dyer & Roeder 1972) make significant contribution to the value of \( d_A(z) \) as well.

In addition, the current estimate of \( H_0 \) from optical observations is converging with fractional uncertainty of about 10 percent level, \( h = 0.7 \pm 0.1 \) (Freedman et al. 1994; Tanvir et al. 1995). This implies that one may place useful limits on \( \Omega_0 \) and \( \lambda_0 \) if accurate and reliable SZ observations are carried out even for a single high-\( z \) cluster. Unfortunately the non-spherical distribution and clumpiness of the cluster gas among others would inevitably contaminate any SZ observation (Birkinshaw et al. 1991; Inagaki et al. 1995), and this is why we have to proceed statistically. In this Letter, we present estimates of \( H_0 \) for seven SZ clusters (Table 1) by taking proper account of the effect of \( \Omega_0 \) and \( \lambda_0 \) for the first time.

2. Angular Diameter Distance from the SZ Effect

The temperature decrement in the cosmic microwave background (CMB) by the SZ effect observed at an angle \( \theta \) from the center of a cluster is given by the integral over the
line of sight:

\[
\frac{\Delta T}{T_0}(\theta) = -2\xi(x) \int_{-\infty}^{\infty} \frac{k_B T_{\text{gas}}}{m_e c^2} \sigma_T n_e dl, \tag{1}
\]

\[
\xi(x) = \frac{x^2 e^x}{2(e^x - 1)} \left( 4 - x \coth \frac{x}{2} \right), \tag{2}
\]

where \(T_{\text{gas}}\) and \(n_e\) are the temperature and the number density of the electron gas in the cluster, \(T_0\) is the temperature of the CMB (= 2.726K) and \(x\) is the dimensionless frequency \(h_p \tilde{\nu}/k_B T_0\) (\(\tilde{\nu}\) is the frequency of the radio observation, \(h_p\) is the Planck constant), and the other symbols have their usual meanings. The relativistic correction to the above formula is negligible for \(T_{\text{gas}} \lesssim 15\text{keV}\) and \(x \lesssim 1\) (e.g., Rephaeli 1995).

Similarly the X-ray surface brightness \(S_X\) of the cluster observed at \(\theta\) in the X-ray detector bandwidth \(\nu_1 \sim \nu_2\) is written as

\[
S_X(\theta) = \frac{1}{4\pi(1+z)} \int_{-\infty}^{\infty} dl \int_{\nu_1(1+z)}^{\nu_2(1+z)} \frac{d^2 L_X}{d\nu dV} d\nu, \tag{3}
\]

where \(z\) is the redshift of the cluster, and \(d^2 L_X/d\nu dV\) is the X-ray emissivity per unit frequency. If \(T_{\text{gas}}\) is greater than several keV, the line contribution is negligible\(^1\) and \(S_X\) is well approximated by that of the thermal bremsstrahlung:

\[
\frac{d^2 L_X}{d\nu dV} = \alpha(T_{\text{gas}}) n_e^2 \overline{g}_{ff}(T_{\text{gas}}, \nu) \exp \left( -\frac{h_p \nu}{k_B T_{\text{gas}}} \right), \tag{4}
\]

\[
\alpha(T_{\text{gas}}) \equiv \frac{2^5 \pi e^6}{3 m_e c^2} \left( \frac{2\pi}{3 m_e c^2 k_B T_{\text{gas}}} \right)^{1/2} \frac{2}{1 + X}. \tag{5}
\]

We assume the primordial abundances of hydrogen and helium with the hydrogen mass fraction \(X = 0.755\). The velocity averaged Gaunt factor of the thermal bremsstrahlung, \(\overline{g}_{ff}(T_{\text{gas}}, \nu)\), is computed according to Kellogg, Baldwin & Koch (1975).

The conventional assumption is that the cluster gas follows a spherically symmetric isothermal \(\beta\)-model: \(n_e(r) = n_{e0}/(1 + r^2/d_\lambda^2(z) \theta_c^2)^{3\beta/2}\), where \(\theta_c\) and \(n_{e0}\) are the angular core radius (fitted from the radio or X-ray imaging) and the central density of the cluster gas,

\(^1\) The X-ray flux due to line emission for a solar abundance plasma at \(T_{\text{gas}} = 6.8\text{keV}\), for example, is about 6 percent of that of bremsstrahlung in the ROSAT band.
and $d_\Lambda(z)$ denotes the angular diameter distance to the redshift $z$ of the cluster. Then one can integrate equations (1) and (3) over the line of sight, and their central values ($\theta = 0$) reduce to

$$\frac{\Delta T(0)}{T_0} = -2\xi(\nu)\sqrt{\pi}\sigma_T \frac{k_B T_{\text{gas}}}{m_e c^2} \frac{\Gamma(3\beta_r/2 - 1/2)}{\Gamma(3\beta_r/2)} n_e d_\Lambda(z) \theta_r,$$

$$S_X(0) = \frac{\alpha(T_{\text{gas}})k_B T_{\text{gas}}}{h_p} \frac{n_e^2 d_\Lambda(z) \theta_X}{4\sqrt{\pi}(1+z)^4} \frac{\Gamma(3\beta_X - 1/2)}{\Gamma(3\beta_X)} \int_{h_p\nu(1+z)/k_B T_{\text{gas}}} h_p \nu e^{-x} \frac{\xi^2(\nu) S_X(0)}{h_p \nu(1+z)/k_B T_{\text{gas}}} \sigma_f e^{-x} dx,$$

where $\Gamma(x)$ denotes the gamma function. Although the values of $\beta$ and $\theta_c$ obtained by the radio ($\beta_r$ and $\theta_r$) and the X-ray observations ($\beta_X$ and $\theta_X$) should be identical as long as the spherical $\beta$-model is exact, different values are often reported in the literature. While we distinguish them in the above equations following the literature, the difference due to the different choices of the parameters should be regarded as an additional uncertainty in the present context (see §3 below).

Finally equations (6) and (7) can be solved for $d_\Lambda$:

$$d_\Lambda = \frac{\alpha(T_{\text{gas}})k_B T_{\text{gas}}}{16\pi^{3/2}h_p \sigma_f^2 (1+z)^4} \left( \frac{m_e c^2}{k_B T_{\text{gas}}} \right)^2 \frac{\Gamma(3\beta_X - 1/2)}{\Gamma(3\beta_X)} \left( \frac{\Gamma(3\beta_r/2)}{\Gamma(3\beta_r/2 - 1/2)} \right)^2 \frac{\Delta T/T(0)^2 \theta_X}{\xi^2(\nu) S_X(0)} \frac{\theta_r^2}{h_p \nu(1+z)/k_B T_{\text{gas}}} \int_{h_p \nu(1+z)/k_B T_{\text{gas}}} h_p \nu e^{-x} dx.$$ 

Comparison of the above estimate with the theoretical formula for $d_\Lambda$ in the Friedman-Robertson-Walker model, yields a relation among $H_0$, $\Omega_0$ and $\lambda_0$ (e.g., Silk & White 1978; Inagaki et al. 1995) as we discuss extensively below.

3. $H_0$, $\Omega_0$ AND $\lambda_0$ FROM A SAMPLE OF SZ CLUSTERS

In order to estimate $d_\Lambda$ and see the constraints on $H_0$, $\Omega_0$ and $\lambda_0$, we use seven clusters of which we can collect the necessary data in radio and X-rays for the analysis from the literature and the private communications\[^2\]. In the case of CL0016+16, we adopt the value of $S_X(0)$ given in White, Silk & Henry (1981) and Birkinshaw, Gull, & Moffet

\[^2\]We did not include A665 (Birkinshaw et al. 1991) since we could not find the proper X-ray data from the literature.
Otherwise we first compute $S_X(0)$ in equation (8) from the value of $n_{e0}$ given in the literature of the X-ray observation, and then estimate $d_A$ (eq.[8]) so as not to include the line emission contribution (even though it is small). Our estimates of $H_0$ listed in Table 1 are for the $\Omega_0 = 1$ and $\lambda_0 = 0$ model, just for example; they are in good agreement with the previous work as long as we adopt the same parameters. In what follows, we assign $1\sigma$ errors simply by taking account of those of $n_{e0}$, $T_{\text{gas}}$ and $\Delta T(0)$ in quadrature. The errors are dominated by the uncertainty in the SZ temperature decrement. In reality those values would be strongly dependent on the fitted parameters $\beta$ and $\theta_c$, and thus it is likely that our quoted error bars underestimate the actual ones.

Figure 1 illustrates the dependence of the estimated $H_0$ on $\Omega_0$ for the two highest-$z$ clusters in Table 1. More specifically, for CL0016+16 ($z = 0.541$) and A2163 ($z = 0.201$), $h = 1.03^{+0.59}_{-0.28}$ and $h = 0.82^{+0.56}_{-0.24}$ in the $\Omega_0 = 1$ and $\lambda_0 = 0$ model, and $h = 1.43^{+0.82}_{-0.38}$ and $h = 0.94^{+0.64}_{-0.27}$ in the $\Omega_0 = 0$ and $\lambda_0 = 1$ model, respectively. Figure 2 summarizes the estimated $H_0$ for all clusters in models with four different sets of $\Omega_0$ and $\lambda_0$. The derived values except for A478 are consistent with $h = 0.7 \pm 0.1$ from the HST Cepheid distance (Tanvir et al. 1995). Note that if one adopts incorrect cosmological models, the result would show an artificial $z$-dependence of $H_0$ (Suto, Suginoara & Inagaki 1995). Since CL0016+16 is at the highest $z$ in our sample, we plot three different estimates of $H_0$ using different sets of $\beta$ and $\theta_c$ from radio and X-ray observations (in open pentagon, open triangle and filled triangle). The results are quite sensitive to the choice indicating that the higher resolution image analysis both in radio and X-ray observations is highly desired (see aslo Neumann & Böhringer 1996)\textsuperscript{3}.

Finally Figure 3 compares the angular diameter distances $d_A$ estimated from the SZ effect with several theoretical ones. Short dash – long dash line plots the approximation of $d_A = czH_0^{-1}(1 - 3z/2)$ valid only for $z \ll 1$ and $q_0 = 0$, simply to illustrate how it deviates from the correct one; clearly at $z \gtrsim 0.1$ one should treat $\Omega_0$ and $\lambda_0$ differently and

\textsuperscript{3}In fact, a recent detailed analysis using the ROSAT image for CL0016+16, especially taking account of the ellipsoidal shape and its temperature structure, indicated a substantially smaller value for $H_0$ than we derived here (Birkinshaw 1996).
the constraints cannot be given only in terms of $q_0$. Also plotted (thin solid line) is the Dyer-Roeder (or empty beam) distance in the $\Omega_0 = 1$ and $\lambda_0 = 0$ model (Dyer & Roeder 1972) so as to indicate the possible effect of the inhomogeneity in the light propagation (compare with the filled beam case in thick solid line); in the light of the current systematic and statistical errors of the SZ measurement, the latter effect is safely neglected.

4. Discussion and Conclusions

We have estimated the Hubble constant $H_0$ from seven SZ clusters by taking proper account of the effect of the cosmological parameters, which has been neglected previously. The estimated $H_0$ varies around 30 percent depending on the assumed $\Omega_0$ and $\lambda_0$ for the two highest-$z$ clusters. Considering the uncertainties of the SZ effect (Birkinshaw et al. 1991; Inagaki et al. 1995) and the observed scatters in the data, however, it is definitely premature to draw any strong conclusion at this point. Nevertheless it is reassuring that our estimates are consistent with Cepheid-based estimate $h = 0.7 \pm 0.1$ (Tanvir et al. 1995) within $2\sigma$ error-bars. In addition, the $d_A$ vs. $z$ diagram from the future SZ measurements may provide a promising insight into the values of cosmological parameters as the Hubble diagram of distant SNe Ia (Perlmutter et al. 1996).

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Table 1: Data for the SZ clusters

|      | $T_{gas}$ | $-\Delta T(0)$ | $\beta_x$ | $\theta_x$ | $n_{e0}$ | $H_0$     | ref. |
|------|-----------|----------------|-----------|-----------|----------|-----------|------|
|      | keV       | mK             | arcmin    |           |          | km/s/Mpc  |      |
| CL 0016 | 8.4 ± 1.2 | 0.687 ± 0.062  | 2/3       | 0.7       | e)       | 103$^{+59}_{-28}$ | *  |
| $z = 0.541$ | 8.4$^{+1.2}_{-0.6}$ | 1.584 ± 0.256 | 0.5       | 0.5       | e)       | 41$^{+15}_{-12}$  | 1   |
|        |           |                |           |           |          | 0.5 ± 0.3  | e)  |
|        |           |                |           |           |          |           | 2   |
|        |           |                |           |           |          | 0.687 ± 0.062$^a$ | 2/3 |
|        |           |                |           |           |          | 0.7       |      |
| A2163 | 14.6 ± 0.5 | 0.735 ± 0.144  | 0.62      | 1.2       | 10.9 ± 0.4$^f$ | 82$^{+56}_{-24}$ | *  |
| $z = 0.201$ | 14.6 ± 0.55 | 0.62 ± 0.01   | 1.20 ± 0.05 | 82$^{+35}_{-22}$ | 4 |
|        | 14.6 ± 0.5 | 0.62$^{+0.009}_{-0.012}$ | 1.2 ± 0.046 | 10.9 ± 0.4$^f$ | 5 |
|        | 0.735$^{+0.144}_{-0.120}$ | 0.59$^{+0.07}_{-0.05}$ | 1.15$^{+0.33}_{-0.27}$ |       | 6 |
| A2218 | 6.7 ± 0.45 | 0.62 ± 0.08   | 0.65      | 1.0       | 8.9      | 60$^{+24}_{-13}$ | *  |
| $z = 0.171$ | 6.7 ± 0.45 | 0.62 ± 0.08$^c$ | 0.65$^{+0.05}_{-0.03}$ | 1.0 ± 0.2 | 8.9     | 65 ± 25   | 7   |
| A2142 | 8.68 ± 0.12 | 0.878 ± 0.050 | 1.0       | 3.69      | 6.97 ± 0.41 | 51$^{10}_{-7}$ | *  |
| $z = 0.0899$ | 8.68 ± 0.12 | 0.878 ± 0.050$^d$ | 1.0 ± 0.3 | 3.69 ± 0.14 | 6.97 ± 0.41 | 48$^{43}_{-29}$ | 8   |
| A478  | 6.56 ± 0.09 | 0.906 ± 0.068 | 0.667     | 1.93      | 9.55 ± 1.75 | 33$^{+22}_{-9}$ | *  |
| $z = 0.0881$ | 6.56 ± 0.09 | 0.906 ± 0.068$^d$ | 0.667 ± 0.029 | 1.93 ± 0.30 | 9.55 ± 1.75 | 32$^{+18}_{-14}$ | 8   |
| A2256 | 7.51 ± 0.11 | 0.397 ± 0.047 | 0.795     | 5.33      | 3.55 ± 0.18 | 74$^{+26}_{-15}$ | *  |
| $z = 0.0581$ | 7.51 ± 0.11 | 0.397 ± 0.047$^d$ | 0.795 ± 0.020 | 5.33 ± 0.20 | 3.55 ± 0.18 | 72$^{+22}_{-19}$ | 8   |
| Coma  | 9.10 ± 0.40 | 0.505 ± 0.092 | 0.75      | 10.5      | 4.09 ± 0.06 | 63$^{+28}_{-15}$ | *  |
| $z = 0.0235$ | 9.10 ± 0.40 | 0.505 ± 0.092$^d$ | 0.75 ± 0.03 | 10.5 ± 0.6 | 4.09 ± 0.06 | 67$^{+26}_{-22}$ | 8   |

*) our adopted values. Our $H_0$ is estimated for the $\Omega_0 = 1$ and $\lambda_0 = 0$ model. 1) Yamashita (1994), 2) Birkinshaw et al. (1981), 3) Carlstrom, Joy, & Grego (1996), 4) Rephaeli (1995), 5) Elbaz, Arnaud & Böhringer (1995), 6) Arnaud et al. (1992), Wilbanks et al. (1994), 7) Birkinshaw & Hughes (1994), 8) Myers et al. (1996).

a) the observed radio frequency $\nu = 28.7$GHz, b) $\nu = 136$GHz, c) $\nu = 20.3$GHz, d) $\nu = 32$GHz, e) in this cluster we use $S_X(0) = (5.2 \pm 0.83) \times 10^{-6} \text{erg cm}^{-2} \text{s}^{-1} \text{arcmin}^{-2}$ directly in eq.(9) to compute $H_0$ and $d_\lambda$, f) estimated assuming $\Omega_0 = 1$ and $\lambda_0 = 0$. 
Fig. 1.— \(H_0\) estimated from the SZ effect as a function of \(\Omega_0\) in the case of \(\lambda_0 = 0\) (left panels) and \(\lambda_0 = 1 - \Omega_0\) (right panels) assuming the parameters in Table 1. Upper panels are for CL0016+16, while lower panels for A2163. The solid and dotted lines represent the central value and \(\pm 1\sigma\) error ranges, respectively.
Fig. 2.— \( H_0 \) estimated from the SZ effect for seven clusters with 1\( \sigma \) error bars in models with \((\Omega_0, \lambda_0) = (1.0, 0.0), (1.0, 1.0), (0.0, 0.0) \) and \((0.0, 1.0)\). For CL0016+16, we plot values with different \( \beta \) and \( \theta_c \): \( \beta_X = \beta_r = 0.5, \theta_X = \theta_r = 0.5' \) (open pentagon), \( \beta_X = 0.5, \beta_r = 2/3, \theta_X = 0.5', \theta_r = 0.7' \) (open triangle), and \( \beta_X = \beta_r = 2/3, \theta_X = \theta_r = 0.7' \) (filled triangle). The error bars for the latter two are artificially tilted to avoid confusion.
Fig. 3.— The angular diameter distance $d_A$ estimated from the SZ effect as a function of $z$ for seven clusters (symbols). Lines indicate theoretical curves for several sets of cosmological parameters ($h, \Omega_0, \lambda_0$). The meanings of the symbols are the same as Fig. 2.