THE MINIMUM TOTAL MASS OF MACHOS AND HALO MODELS OF THE GALAXY

TAKASHI NAKAMURA
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan

YUKITOSHI KAN-YA AND RYOICHI NISHI
Department of Physics, Kyoto University, Kyoto 606, Japan

Received 1996 June 4; accepted 1996 October 9

ABSTRACT

If the density distribution \( \rho(r) \) of MACHOs is spherically symmetric with respect to the Galactic center, it is shown that the minimal total mass \( M_{\text{min}}^{\text{MACHO}} \) of the MACHOs is \( 1.7 \times 10^{10} M_\odot \rho_{LMC}^{-1/2} \), where \( \rho_{LMC} \) is the optical depth \( \rho_{LMC} \) toward the Large Magellanic Cloud (LMC) in units of \( 2 \times 10^{-7} \). If \( \rho(r) \) is a decreasing function of \( r \), it is proved that \( M_{\text{min}}^{\text{MACHO}} \) is \( 5.6 \times 10^9 M_\odot \rho_{LMC}^{1/2} \). Several spherical and axially symmetric halo models of the Galaxy with a few free parameters are also considered. It is found that \( M_{\text{min}}^{\text{MACHO}} \) ranges from \( 5.6 \times 10^9 \) to \( \sim 3 \times 10^{11} M_\odot \rho_{LMC}^{1/2} \). For a general case, the minimal column density \( \Sigma_{\text{min}}^{\text{MACHO}} \) of MACHOs is obtained as \( \Sigma_{\text{min}}^{\text{MACHO}} = 25 M_\odot \text{pc}^{-2} \rho_{LMC}^{1/2} \). If the clump of MACHOs exist only halfway between the LMC and the Sun, \( M_{\text{min}}^{\text{MACHO}} \) is \( 1.5 \times 10^9 M_\odot \rho_{LMC}^{1/2} \). This shows that the total mass of MACHOs is smaller than \( 5 \times 10^{10} M_\odot \), i.e., \( \sim 10\% \) of the mass of the halo inside the LMC, if either the density distribution of MACHOs is unusual or \( \rho_{LMC} \approx 2 \times 10^{-7} \).

Subject headings: dark matter — Galaxy: halo — Galaxy: structure — gravitational lensing

1. INTRODUCTION

Recent second-year analysis of microlensing events toward the Large Magellanic Cloud (LMC) by the MACHO collaboration suggests that the optical depth \( \rho_{LMC} \) is \( \sim 2 \times 10^{-7} \) and the fraction \( f \) of MACHOs is \( \sim 0.4 \), with the typical mass \( \sim 0.34 M_\odot \) in the standard spherical flat rotation halo model (Bennett et al. 1996). The estimated mass of MACHOs is exactly the mass of red dwarfs. However, the contribution of the halo red dwarfs to MACHO events should be small since the observed density of the halo red dwarfs is too low (Bahcall et al. 1994; Graff & Freese 1996a, 1996b).

As for the white dwarf Galaxy halo, Charlot & Silk (1995) combined population synthesis models with constraints from deep galaxy surveys and showed that only a small fraction (\( \leq 10\% \)) of the dark mass in the present-day galaxy halo could be in the form of white dwarf remnants (WDRs) of intermediate-mass stars. Adams & Laughlin (1996) recently argued the implications of white dwarf Galactic halos. From the current limits on the density of red dwarfs (Bahcall et al. 1994; Graff & Freese 1996a, 1996b) and the Galactic metallicity, the initial mass function must be sharply peaked about a characteristic mass scale \( M \sim 2.3 M_\odot \). They concluded that the mass fraction of WDRs in the halo is likely to be less than 25\%, since only a fraction of the initial mass of a star is incorporated into WDRs.

The spatial density distribution function of MACHOs that caused microlensing events is not known in spite of many arguments on the mass distribution of halo dark matter (Paczynski 1986; Griest 1991; Sackett & Gould 1993; Frieman & Scoccimarro 1994; Sahu 1994; Alcock et al. 1995; Gates, Gyuk, & Turner 1996; Turner, Gates, & Gyuk 1996; Evans 1996; Kan-ya, Nishi, & Nakamura 1996). Only the possible value of the optical depth \( \tau_{LMC} \) (Bennett et al. 1996) is known, the fraction of MACHOs in the halo depending on the spatial density distribution function of MACHOs. In this situation, it is important to check the relation between the total mass \( M_{\text{min}}^{\text{MACHO}} \) of MACHOs and the density distribution function of MACHOs for the given optical depth \( \tau_{LMC} \). The results of such a study will be useful in the arguments on the fraction of MACHOs in the halo and on the question of what MACHOs are.

In this Letter, we study \( M_{\text{min}}^{\text{MACHO}} \) for various density distribution functions of MACHOs and discuss the minimal total mass of the MACHO halo. Gates, Gyuk, & Turner (1996) and Turner, Gates, & Gyuk (1996) also discussed the mass of the MACHO halo for various galaxy models. While they separately added unidentified dark, thick disk components, we discuss the total mass responsible for microlensing. In § 2 we obtain the minimal total mass \( M_{\text{min}}^{\text{MACHO}} \) of MACHOs for spherically symmetric density distributions. In § 3 we discuss \( M_{\text{min}}^{\text{MACHO}} \) for various axially symmetric density distributions. Section 4 will be devoted to discussions.

2. SPHERICALLY SYMMETRIC HALO MODELS AND THE MINIMAL TOTAL MASS OF MACHOS

We assume that the density distribution function \( \rho(r) \) of MACHOs is a function of the Galactocentric radius \( r \). The optical depth \( \tau_{LMC} \) toward the LMC is given by (Paczynski 1986; Griest 1991)

\[
\tau_{LMC} = \frac{4 \pi G}{c^2} \int_{D_0}^{D_L} x \left( 1 - \frac{x}{D_i} \right) \rho(r) \, dx ,
\]

and

\[
\eta = \cos b \cos l ,
\]

where \( D_0, l, b, \) and \( R_0 \) are the distance to the LMC (50 kpc), Galactic longitude and latitude of the LMC, and the...
Galactocentric radius of the Sun (8.5 kpc), respectively. In equation (1) we assumed the threshold \( u_r = 1 \) for simplicity.

Equation (1) is rewritten as

\[
\tau^{\text{LMC}} = \int f(x) \, dm, \tag{4}
\]

\[
f(x) = \frac{G \, x \, [1 - (x/D)]}{c^2 \, r^2 \, (dr/dx)}, \tag{5}
\]

and

\[
dm = 4 \pi r^2 \rho(r) \frac{dr}{dx}. \tag{6}
\]

For the LMC, \( f(x) \) is infinite at \( x = x_c = R_c \eta = 0.153 R_c \), so that the minimal total mass \( M_{\text{min}}^{\text{MACHO}} \) of MACHOs is zero for any given \( \tau^{\text{LMC}} \) if MACHOs are distributed in an infinitesimally thin shell at \( r = r_c = (1 - \eta^2)^{1/2} R_c \). However, this is wrong. Since the angular size of the LMC is \( \sim 10^5 \times 10^6 \), \( M_{\text{MACHO}}^{\text{LMC}} \) is minimized if MACHOs are distributed in a shell at \( r = r_c \) with width \( d \) given by

\[
d = \frac{10 \pi}{180} R_c \eta = 227 \text{ pc}. \tag{7}
\]

It is easy to show that \( M_{\text{min}}^{\text{MACHO}} \) is given by

\[
M_{\text{min}}^{\text{MACHO}} = \frac{c^2 \, \tau^{\text{LMC}}}{3 G} \left( r_c + d \right)^3 - r_c^3
\]

\[
= \frac{1.7 \times 10^{10} \, M_\odot \, \tau_{-6.7}^{\text{LMC}}}{}, \tag{8}
\]

where \( \tau_{-6.7}^{\text{LMC}} \) is \( \tau_{-6.7}^{\text{LMC}} \) in the unit of \( 2 \times 10^{-7} \). This shows that, in principle, \( M_{\text{MACHO}}^{\text{LMC}} \) can be only \( \sim 3\% \) of the total mass of the halo inside the LMC. However, the density distribution function of MACHOs in this case is very peculiar, so that we calculate \( M_{\text{MACHO}}^{\text{LMC}} \) for more realistic \( \rho(r) \) in order to get a more realistic \( M_{\text{min}}^{\text{MACHO}} \). We consider two models: (1) the polytropic model, where \( \rho(r) \) is given by a polytropic of index \( N \) and the radius \( R_a \); and (2) the \( \alpha \) model, where \( \rho(r) \) is given by

\[
\rho(r) = \frac{\rho_0}{\left[ 1 + (r^2/R_a^2) \right]^\alpha}. \tag{9}
\]

This model is similar to the beta model of the cluster of galaxies with core radius \( R_a \).

In Figure 1 we show \( M_{\text{MACHO}}^{\text{LMC}} \) as a function of \( R_a \) for several polytropic indices \( N \). \( M_{\text{MACHO}}^{\text{LMC}} \) ranges from \( 5.6 \times 10^{10} \, M_\odot \, \tau_{-6.7}^{\text{LMC}} \) for \( N = 0 \) to \( 7.8 \times 10^{10} \, M_\odot \, \tau_{-6.7}^{\text{LMC}} \) for \( N = 3 \). For \( N = 4 \) and \( 4.5 \), \( M_{\text{MACHO}}^{\text{LMC}} \) is greater than \( 1.0 \times 10^{11} \, M_\odot \, \tau_{-6.7}^{\text{LMC}} \), and the minimum does not exist for \( R_a < R_c \). Under the assumption that \( \rho(r) \) is a decreasing function of \( r \), it is shown in the Appendix that \( M_{\text{MACHO}}^{\text{LMC}} \) is minimized when \( \rho(r) \) is constant. Therefore, \( M_{\text{min}}^{\text{MACHO}} \) is \( 5.6 \times 10^{10} \, M_\odot \, \tau_{-6.7}^{\text{LMC}} \) if \( \rho(r) \) is a decreasing function. In Figure 2 we show the total mass of MACHOs inside the LMC in \( \alpha \) models as a function of \( R_a \) for several values of \( \alpha \). \( M_{\text{min}}^{\text{MACHO}} \) ranges from \( 1.3 \times 10^{11} \, M_\odot \, \tau_{-6.7}^{\text{LMC}} \) for \( \alpha = 1.5 \) to \( 8.7 \times 10^{10} \, M_\odot \, \tau_{-6.7}^{\text{LMC}} \) for \( \alpha = 6 \). For large \( \alpha \), \( M_{\text{min}}^{\text{MACHO}} \) does not change so much but converges, although the value of \( R_a \) at the minimum increases. This behavior can be understood.
analytically by using the asymptotic expression of gamma functions.

3. AXIALLY SYMMETRIC HALO MODELS AND THE MINIMAL TOTAL MASS OF MACHOS

There are several suggestions that the Galactic halo is not spherically symmetric (Aarseth & Binney 1978; Aguilar & Merritt 1990; Binney 1996), so that here we study axially symmetric halo models and calculate $M_{\text{MACHO}}$. We consider two models: (1) the exponential disk model, where the axially symmetric density distribution function $\rho(R, Z)$ in cylindrical coordinates is given by

$$\rho(R, Z) = \rho_0 \exp \left(-\frac{R}{R_d} - \frac{|Z|}{Z_d}\right)$$

(11)

($R_d$ and $Z_d$ are scale heights); and (2) the elliptical model, where $\rho(R, Z)$ is given by

$$\rho(R, Z) = \frac{\rho_0}{[1 + (R^2/a^2) + (Z^2/c^2)]^\alpha}$$

(12)

($a$ and $c$ describe the ellipticity of the equidensity surface). This is an axially symmetric version of the $\alpha$ model.

In Figure 3 we show $M_{\text{MACHO}}$ as a function of $R_d$ in exponential disk models for various aspect ratios ($Z_d/R_d$) of the equidensity surface. We see that $M_{\text{min}}$ is $\sim 1.0 \times 10^{11} M_\odot$, $\tau_{\text{MACHO}}^{l_{\text{MACHO}}}$ for $0.5 < Z_d/R_d < 1.0$, and that it increases with the decrease of $Z_d/R_d$ for $Z_d/R_d < 0.5$.

In Figures 4a and 4b, we show $M_{\text{MACHO}}$ in elliptical models as a function of $a$ and the aspect ratio $c/a$ for $\alpha = 2.5$ and $\alpha = 6.0$, respectively. $M_{\text{min}}$ is $8.9 \times 10^{10} M_\odot$, $\tau_{\text{MACHO}}^{l_{\text{MACHO}}}$ at $a = 10$ kpc and $c = 6$ kpc for $\alpha = 2.5$, and is $7.08 \times 10^{10} M_\odot$, $\tau_{\text{MACHO}}^{l_{\text{MACHO}}}$ at $a = 22$ kpc and $c = 13.2$ kpc for $\alpha = 6.0$. For large $\alpha$, $M_{\text{min}}$ converges, similar to the $\alpha$ models.

4. DISCUSSIONS

A deep north Galactic pole proper-motion survey (Majewski, Munn, & Hawley 1996) suggests that the halo is not dynamically mixed but contains a significant fraction of stars with membership in correlated stellar streams. If MACHOs are also dynamically unmixed, it is possible that the density distribution function is neither spherically nor axially symmetric, but completely inhomogeneous. In such a case, the information we can obtain from the microlensing events toward the LMC is the minimal column density $\sum_{\text{MACHO}}$ of MACHOs. Since, in equation (1), $x(1 - x/D) < D/4$, $\sum_{\text{MACHO}}$ is given by

$$\sum_{\text{MACHO}} = 25 M_\odot \text{pc}^{-2} \tau_{\text{LMC}}^{l_{\text{MACHO}}}.$$  (13)

Similar to equation (7), the linear size of the clump of MACHOs should be larger than $174 \text{pc} (x/kpc)$, where $x$ is the distance to the clump of MACHOs. For $x = D/2$, $M_{\text{MACHO}}$ is $1.5 \times 10^9 M_\odot$. If this is the case, the optical depth toward the Small Magellanic Cloud will be quite different, and the inhomogeneity of the density distribution of MACHOs can be checked.

In conclusion, it is shown that the total mass of MACHOs becomes smaller than $5 \times 10^{10} M_\odot$, i.e., $\sim 10\%$ of the mass of the halo inside the LMC, if either the density distribution of MACHOs is unusual or $\tau_{\text{LMC}}^{l_{\text{MACHO}}} \ll 2 \times 10^{-7}$.

We like to thank A. Hayward for checking the English. This work was supported by a Grant-in-Aid of Scientific Research of the Ministry of Education, Culture, and Sports, Nos. 07640399 (T.N.) and 08740170 (R.N.).

APPENDIX

PROOF THAT $M_{\text{MACHO}}$ IS MINIMIZED IF THE SPHERICALLY SYMMETRIC DENSITY DISTRIBUTION FUNCTION $\rho(R)$ IS A CONSTANT

We first fix the density at $r = r_c$. Since the matter inside $r_c$ does not contribute to $\tau$, $M_{\text{MACHO}}$ is minimized if $\rho(r)$ is constant for $r < r_c$. For $r < r$, from equation (4), it is easy to observe that $d\tau/dm$ is a decreasing function of $r$ irrespective of $\rho(r)$. Now for fixed $m$ (i.e., the same mass but different $r$, depending on the density distribution function), $r$ is smallest and $d\tau/dm$ is largest if $\rho(r) = \rho(r_c) = \text{constant}$. This means that $\tau$ is largest for a constant density distribution for fixed total mass $M_{\text{MACHO}}$. Inversely for fixed $\tau$, $M_{\text{MACHO}}$ is minimized for the constant density distribution. By varying the value $\rho(r_c)$, $M_{\text{min}}$ is obtained.
REFERENCES

Aarseth, S. J., & Binney, J. J. 1978, MNRAS, 185, 227
Adams, F. C., & Laughlin, G. 1996, ApJ, 468, 586
Aguilar, L. A., & Merritt, D. R. 1990, ApJ, 265, 33
Alcock, C., et al. 1995, ApJ, 449, 28
Bahcall, J. N., Flynn, C., Gould, A., & Kirhakos, S. 1994, ApJ, 435, L51
Bennett, D. P., et al. 1996, in Proc. Second International Workshop on Gravitational Microlensing Survey, ed. M. Moniez, 17
Binney, J. J. 1996, in IAU Symp. 169, Unsolved Problems of the Milky Way, ed. L. Blitz (Dordrecht: Kluwer), 1
Charlot, S., & Silk, J. 1995, ApJ, 445, 124
Evans, N. W. 1996, MNRAS, 278, L5
Frieman, J., & Scoccimarro, R. 1994, ApJ, 431, L23
Gates, E., Gyuk, G., & Turner, M. S. 1996, Phys. Rev. D, 53, 4138
Graff, D. S., & Freese, K. 1996a, ApJ, 456, L49
———. 1996b, ApJ, 467, L65
Griest, K. 1991, ApJ, 366, 412
Kan-ya, Y., Nishi, R., & Nakamura, T. 1996, PASJ, 48, 479
Majewski, S. R., Munn, J. A., & Hawley, S. L. 1996, ApJ, 459, L73
Paczynski, B. 1986, ApJ, 304, 1
Sackett, P. D., & Gould, A. 1993, ApJ, 419, 648
Sahu, K. C. 1994, Nature, 370, 275
Turner, M. S., Gates, E. L., & Gyuk, G. 1996, preprint (astro-ph/9601168)