Choi states, symmetry-based quantum gate teleportation, and stored-program quantum computing

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The stored-program architecture is canonical in classical computing, while its power has not been fully recognized for the quantum case. We study quantum information processing with stored quantum program states, i.e., using qubits instead of bits to encode quantum operations. We develop a stored-program model based on Choi states, following from channel-state duality, and a symmetry-based generalization of deterministic gate teleportation. Our model enriches the family of universal models for quantum computing, and can also be employed for tasks including quantum simulation and communication.

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I. INTRODUCTION

A physical process, or program in information processing, can be encoded and stored as data, whose action can be recovered or simulated at any later stage on demand. In the era of quantum information science, such a stored-program paradigm is highly desirable and turns out to be nontrivial [1–5]. In particular, the seemingly inevitable entanglement between quantum program and processor causes difficulty, which can be simply avoided if the program is entirely stored as classical bits instead of qubits. However, this would sacrifice quantum virtues for saving space and time costs and yield heavy classical side-processing.

In this work, from a symmetry point of view, we show that stored-program quantum computing is appealing and powerful. At the heart of our method are the Choi states [6], which serve as program states, and gate teleportation [7, 8], which assembles the processor. Choi states are based on the channel-state duality [9], which is a fundamental relation in quantum theory yet only has spotted applications such as tomography [10]. We develop a symmetry-based generalization of gate teleportation, which plays central roles in our scheme and reveals the computational power of Choi states.

There are quantum computing models with a certain stored-program features, and they are known as teleportation-based or measurement-based quantum computing [8, 11, 12]. However, the program states are of particular types, and our new model serves as a modification or generalization of them. For instance, the computing on cluster states is by local projective measurements in bases that are determined by previous measurement outcomes. We may pull back the measurement bases as initial states, leading to different program states for different computations. In the scheme based on Bell measurement [8], gates, hence program states, are sorted in the levels of Clifford hierarchy and can be teleported with byproduct correction operators of one-level lower. Here we develop the stored-program model for general program states, namely, states of completely positive and trace-preserving mappings, i.e., quantum channels [9, 10]. We emphasize the central roles of stored programs, and measurements are only part of the processor. In our framework, we treat gates as symmetry operators on tensors (in the language of matrix-product states [13]), instead of operators on states. The symmetry that are involved above are $U(1)$ for the so-called 1-bit teleportation [14] and $Z_2 \times Z_2$ for teleportation.

Given the importance of Choi states, we develop a method for their preparation by quantum circuits based on extreme channels [15, 16], which is more efficient than the standard dilation method. A mixed program state will be stored as a set of extreme program states and random bits, and the later can greatly reduce the circuit costs for preparing program states. With Choi states, our method can also be used for other tasks. With the channel-recovery scheme from Choi states, this directly serves as a quantum simulation method. In the setting of quantum communication, our scheme can also be viewed as an extended form of quantum repeaters that are important to build quantum networks [17, 18].

This work contains the following sections. In section II we highlight the role of symmetry in teleportation. In section III we discuss methods for channel-recovery from Choi states. Then in section IV we develop the symmetry-based gate teleportation and the scheme for composition of Choi states. In section V we present a method for the preparation of Choi program states via generalized-extreme channels. Then we discuss some applications of the model we develop in section VI in particular, the stored-program quantum computing. We conclude with some perspectives in section VII.

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II. PRELIMINARY

A quantum channel \( \mathcal{E} \) acting on \( \mathcal{D}(\mathcal{H}) \) can be represented by a set of Kraus operators \( \{K_i\} \) such that \( \sum_i K_i^\dagger K_i = 1 \), for \( \dim \mathcal{H} = d \). It can be viewed as an isometry \( \sum_i |i\rangle K_i \), or a three-leg tensor: \( i \) as the index for the physical space, and each \( K_i \) acts on the logical space \[19]\.

We say a channel \( \mathcal{E} \) has symmetry group \( G \) when

\[
\sum_j U^*_{ij} K_j = W^g K_i V^g, \quad \forall g \in G, \tag{1}
\]

for unitary representations \( V, W, \) and \( U = (U_{ij}) \). The symmetry is called ‘global’ when \( W = V^\dagger \) since in this case it can be extended to composition of channels; otherwise, the symmetry is called a ‘local’ or ‘gauge’ symmetry. A gauge symmetry can induce a smaller global symmetry under composition.

A simple example is the 1-bit teleportation with tensor \( \{HZ\}, i = 0, 1, H \) as Hadamard gate \[14]\, Note we use \( X, Y, Z \) to denote Pauli matrices. The usual 2-bit teleportation is obtained as a sequence of two 1-bit teleportation \( X, Y, Z \) to denote Pauli matrices. The usual 2-bit teleportation can be shuffled to the 2nd site so that \( \mathcal{E} = (1 \otimes \mathcal{E}')(\omega) \). Note the transpose \( t \) is inevitable. This becomes a spatial reflection symmetry when \( \mathcal{E}' = \mathcal{E} \). For the unitary case, this means \( U \) is a symmetric matrix (not necessarily real).

We are interested in how to use \( \mathcal{C} \) as a program state (or ‘resource’) to realize or simulate \( \mathcal{E} \) on a system, labelled by \( S \). Below we will ignore the labels. Given \( \mathcal{C} \), the channel can be recovered in two ways. The first scheme, named as ‘Bell scheme’, employs Bell measurement and

\[
\mathcal{E}(\rho) = \langle \omega | (\mathcal{C} \otimes \rho) | \omega \rangle \tag{4}
\]

for the projection \( |\omega\rangle \) on the system \( S \) and the site \( B \) as a part of the Bell measurement. Other projections in the Bell measurement will lead to Pauli byproduct operators \( P_i \), which, however, cannot be pulled out after the channel \( \mathcal{E} \). In this scheme, the state \( \rho \) needs to be prepared initially.

By treating \( \rho \) as a measurement element, the channel can also be realized as

\[
\mathcal{E}(\rho) = \text{tr}_B [\mathcal{C}(1 \otimes \rho')], \tag{5}
\]

which can be viewed as a measurement of \( \sqrt{\rho} \) on site \( B \). We name this scheme as ‘POVM scheme’ since here the state \( \rho \) serves as a POVM element. We find this scheme can realize the channel in a heralded way instead of being probabilistic, see Fig. 1 (Left).

Define a recovery channel \( \mathcal{R} \) by two Kraus operators

\[
K_0 = \sqrt{\rho'}, \quad K_1 = \sqrt{1 - \rho'}, \tag{6}
\]

and the two POVM effects are nothing but \( \rho' \) and \( 1 - \rho' \). The channel \( \mathcal{R} \) can be easily realized by a unitary \( U_{\mathcal{R}} \)

![FIG. 1. (Left): Choi state of a channel \( \mathcal{E} \) and the recovery \( \mathcal{R} \) (shaded). The vertical wavy wire is the entangled Bell state \( |\omega\rangle \). The qubit ancilla for \( \mathcal{R} \) is the lower wire, with initial state \( |0\rangle \) and then measured. (Right): Gate teleportation by the indirect Bell measurement \( \mathcal{G} \) and the conditional rotation \( T \) (shaded). The middle wire of \( \mathcal{G} \) is the qubit ancilla, with initial state \( |1\rangle \) and then measured. The symbol \( \bullet \) is classical one-control. The sites for the program states are, from top to bottom, A, B, C, and D. The additional control wires (yellow) are present only for the case of nonunitary channels.]

III. CHANNEL-RECOVERY FROM CHOI STATES

From channel-state duality, a channel \( \mathcal{E} \) is equivalent to the Choi state

\[
\mathcal{C} := (\mathcal{E} \otimes 1)(\omega) \tag{3}
\]
from dilation method, which takes the form

$$U_R = \left( \begin{array}{cc} \sqrt{\rho} & \sqrt{1 - \rho^2} \\ \sqrt{1 - \rho^2} & -\sqrt{\rho} \end{array} \right).$$  \hspace{1cm} (7)$$

As \([K_0, K_1] = 0\), with the eigen-decomposition \(\rho^t = VDV^\dagger\), \(K_0 = V\sqrt{D}V^\dagger\), \(K_1 = V\sqrt{I - D}V^\dagger\). The channel is realized as

$$\mathcal{R}(\sigma) = \text{tr}_a V \circ \mathcal{M} \circ V^\dagger (\sigma \otimes |0\rangle \langle 0|),$$  \hspace{1cm} (8)

with the final trace on the qubit ancilla \(a\), which is initialized at \(|0\rangle\). The unitary \(\mathcal{M}\) is a ‘multiplexer’ that realizes \(\sqrt{D}\) and \(\sqrt{I - D}\), obtained from sine-cosine decomposition \([15]\). Note here \(\sigma\) is not \(\rho\), and \(V(\cdot) := V(\cdot)V^\dagger\) acts on the system.

The ancilla a needs to be measured selectively, i.e., the outcome is recorded. When it is 0 (1), \(K_0\) (\(K_1\)) is realized on site B, leading to \(E(\rho)\) (\(E(I - \rho)\)). We have to correct the case of being 1. The idea is to use another copy of \(C\) and realize \(E(I)\) on site A by tracing out site B. Then we subtract results on \(E(I)\) and \(E(I - \rho)\), which is just \(E(\rho)\). When the channel is unitary \(U\), the correction step can be omitted since the effect \(U(\cdot)\) is trivial. To summarize, in the POVM scheme, two copies of \(C\) are needed in general. When the outcome on the first copy is 0, then the second copy is not needed; when it is 1, then the second copy is needed to make the correction. As such, this scheme is heralded. Compared with the Bell scheme, it is deterministic by using two samples of the Choi states, and we will use it in the stored-program model.

IV. COMPOSITION OF CHOI STATES AND GENERALIZED GATE-TELEPORTATION

In our model, a Choi state serves as a program state. Given two program states, one primary task is to see if there is a composition of them. The composition is important so that programs (‘gates’) can be shuffled around. This turns out to be nontrivial and we need to develop a generalized gate teleportation scheme.

We first study the unitary case, for which the Choi state is pure. Denote two Choi states as \(\ket{\psi_U}\) and \(\ket{\psi_V}\), and the sites as A, B, and C, D, respectively. See Fig. 1 (Right). To simulate \(UV\) or \(VU\), we need to obtain the state \(\ket{\psi_{UV}}\) or \(\ket{\psi_{VU}}\). A standard way is to use the Bell scheme and when the projection is \(\omega\) on sites B and C, the gate \(V\) is teleported from site A to site C, and the state of sites A and D is \(\ket{\psi_{VU}}\). To obtain \(\ket{\psi_{UV}}\), we shall apply the Bell scheme on sites A and D. However, when the Bell measurement yields other outcomes, there will be Pauli byproduct operators in between the two unitary gates, which cannot be corrected in general.

Our method is to replace the Bell scheme by an indirect Bell measurement which requires an extra qubit ancilla. First, we call the Bell state \(\ket{\omega}\) as ‘singlet’, and other threes as ‘triplet’ for the qubit case \([21]\). Recall that in the Bell measurement the circuit to prepare Bell states is firstly run backwards, denoted as \(U_{S}^B\), and then projective measurement is performed in the computational basis. Our method is to use the gate \(U_{S0}\), after \(U_{S}^B\), to couple to the ancilla at state \(|1\rangle\) and measure the ancilla instead. Such an indirect or generalized Bell measurement can be expressed as

$$\mathcal{G}(\sigma) := \text{tr}_a U_{S0} \circ U_{S}^B (\sigma \otimes |1\rangle \langle 1|).$$  \hspace{1cm} (9)$$

The gate \(U_{S0}\) is a Toffoli gate but with one-controls replaced by zero-controls, and with the ancilla as target. This encodes the information of being singlet or triplet to the ancilla: when the outcome on the ancilla is 0 (1), it means the singlet (triplet) is realized. Note that the singlet case flips the ancilla state. Also note that the measurement on the ancilla can also be pushed to the end: the classical control will be replaced by a quantum control. However, we prefer the classical control.

When the outcome is the singlet on sites B and C, the gate \(U\) on site C is teleported to site A, leading to \(\ket{\psi_{UV}}\). When the outcome is the triplet (or adjoint in general), different from the Bell scheme, we need to use the symmetry property of the triplet: it has the full symmetry of \(SU(2)\) (or \(SU(d)\) in general). This is because the channel \(\mathcal{G}\) conditioned on outcome 1 is the set of Pauli operators \(P_t\), i.e., a Pauli channel. We know from above that it has the symmetry \(SU(d)\) in general.

Now if we intend to shuffle the gate \(V\), we can apply the corresponding rotation \(T\) on the triplet, and this is equivalent to the action of \(V^t\) on site C. This finally leads to the state \(\ket{\psi_{V^tU}}\). That is to say, the generalized gate-teleportation scheme can realize the state \(\ket{\psi_{UV}}\) or \(\ket{\psi_{V^tU}}\) in a heralded way. Combined with the POVM scheme on site D for recovery, the gate \(VU\) or \(V^tU\) can be simulated with output on site A.

It seems the transpose is inevitable, which forbids a deterministic gate teleportation (e.g., for \(V\)). It turns out this is only apparent and can be avoided by noticing that any unitary operator can be expressed as a product of two symmetric unitary operators. Namely, given a unitary operator \(U\), from eigen-decomposition we have \(U = U_DDU_D^\dagger\) for unitary \(U_D\) and diagonal \(D\) matrices. It can also be written as

$$U = (U_DU_D^\dagger)(U_D^*DU_D^\dagger),$$  \hspace{1cm} (10)$$

which is a product of two symmetric unitary matrices, denoted as \(U_L\) and \(U_R\), and \(U = U_L^*U_R\). This means that for any unitary \(U\) of arbitrary dimension, it only requires two Choi program states, \(\ket{\psi_{UL}}\) and \(\ket{\psi_{UR}}\), to deterministically teleport \(U\).

In addition, we also believe that using only one Choi state cannot achieve deterministic teleportation. One might intend to replace the Pauli channel and symmetry group \(SU(d)\) by other channels or groups. This shall need a channel that has a symmetry of the form \(U(\cdot)U^*\).
This channel is equivalent to a bipartite symmetric state invariant under $U \otimes U^\dagger$ \cite{22,23}, which turns out to be the trivial identity state. This is nothing but the case when a singlet is established in teleportation. This shows that the composition method above has to be used. A different approach is to allow non-Pauli byproduct, like the Clifford hierarchy \cite{8}, but this is beyond our framework based on symmetry.

The gate teleportation above directly extends to the composition of a sequence of program states. The whole sequence of gates obtained are of the form $P \prod_i U_i$, for $P$ as the final Pauli byproduct, and each gate is $U_i = U_i^L U_i^R$, and corresponding rotations $T_i$ are needed for the adjointor case.

We now study the case for channels whose Choi states are mixed states. The symmetry condition does not generalize directly since now unitary operators are replaced by nonunitary quantum channels. This appears as a nontrivial obstacle, while we find a scheme based on direct-sum dilation shall work. For a channel $E$ with a set of Kraus operators $\{K_i\}$, each of them can be dilated to a unitary

$$U_{K_i} = \left( \begin{array}{cc} K_i & \sqrt{1 - K_i^\dagger K_i} \\ \sqrt{1 - K_i K_i^\dagger} & -K_i \end{array} \right). \tag{11}$$

The gate $U_{K_i}$ acts on a space of dimension $2d$. Denote the original space as $\mathcal{H}_S$, and the additional one as $\mathcal{H}_S^\perp$. Now the channel $E$ can be simulated by a random-unitary channel

$$E'(\sigma) = \text{tr}_e \bar{U}(\sigma \otimes e) \tag{12}$$

for $\bar{U} := \sum_i |i\rangle \langle i| \otimes U_{K_i}$ as a controlled-unitary gate, with the ancilla as control with initial state $e := |e\rangle \langle e|$ and traced out, for $|e\rangle := \sum_i |i\rangle$ as the equal-amplitude superposition state. The state $\sigma = \rho \otimes 0$, for 0 on the dilated subspace $\mathcal{H}_S^\perp$. The action $E(\rho)$ is the restriction of $E'(\sigma)$ to the system subspace $\mathcal{H}_S$.

Now compare with the unitary case, here the task is to teleport controlled-unitary gates instead of unitary gates. This can be done using a slight extension of our scheme above: for each $U_{K_i}$ in $\bar{U}$, there exists a $T_i$ gate that can teleport it. The $T_i$ gates are controlled by the same ancilla for $U_{K_i}$. See Fig. 1 (Right), the yellow control wires. Similar with the unitary case, when we obtain the singlet, the channel $E$ is teleported. To avoid the transpose, we decompose each $U_{K_i}$ as the product of two symmetric unitary matrices $U_{K_i} = U_{K_i}^L U_{K_i}^R$, and then it is not hard to see that, using the same control wire for $U_{K_i}^L$ and $U_{K_i}^R$, and using two program states, the gate $\bar{U}$ can be teleported, i.e., the channel $E$ is teleported.

To realize the action of channel on state, we need to design a POVM and a channel based on state of the form $\rho \otimes 0$. The channel, denoted $\mathcal{R}'$, contains three Kraus operators

$$K_0 = [\sqrt{\rho}, 0], \quad K_1 = [\sqrt{1 - \rho}, 0], \quad K_2 = [0, 1], \tag{13}$$

as an extension of the channel $\mathcal{R}$ \cite{9}. This channel needs a qudit ancilla, and differently from $\mathcal{R}$, when the outcome is 2, the simulated result restricted to the space $\mathcal{H}_S^\perp$ is $\mathcal{E}'(1)$, which equals 1 from the trace-preserving condition. If this occurs, the simulation has to be started again. We observe that the direct-sum dilation suits some models and systems. For instance, in linear optics and quantum walk, each computational basis state can be addressed by encoding in a separate ‘mode’, and a direct-sum dilation is to add more modes to the system. As such, the restriction to a subspace can be easily done by only observing those modes in it.

For special type of channels, our scheme can be greatly simplified. A wide class of channel are the random unitary channels, and it is clear that they can be realized by the controlled-unitary scheme above, and no direct sum dilation is needed. Another kind is known as entanglement-breaking channels \cite{24}, for which the Choi states are bipartite separable states (while the partial traces are $\mathcal{E}(1)$ and $\mathcal{E}'(1)$). These channels and program states would be trivial since there is no entanglement and they can be easily simulated by a measurement-preparation scheme.

V. PREPARATION OF PROGRAM STATES VIA GENERALIZED-EXTREME CHANNELS

A Choi state $\mathcal{C}$ is not easy to prepare on the first hand, namely, this may require the operation of $\mathcal{E}$ on the Bell state $|\omega\rangle$, and realizing $\mathcal{E}$ itself (e.g., by a dilated unitary) is a nontrivial task. From dilation, it requires the form of Kraus operators, which are not easy to find in general given $\mathcal{C}$. Here we develop a framework that relies on Choi states and reduces the quantum circuit complexity by using random bits.

The set of qudit channels forms a convex body. This means that a convex sum of channels still leads to a channel, and there are extreme channels that are not convex sum of others. From Choi \cite{9}, a channel is extreme iff there exists a Kraus representation $\{K_i\}$ such that the set $\{K_i^\dagger K_j\}$ is linearly independent. For a qudit, this means the rank of an extreme channel is at most $d$. Channels of rank $r \leq d$ are termed as generalized-extreme channels \cite{24}, here termed as ‘gen-extreme channels.’ It is clear to see that, a gen-extreme but not extreme channel is a convex sum of extreme channels of lower ranks. It has been conjectured \cite{26} and numerically supported \cite{15,16} that arbitrary channel can be written as a convex sum of at most $d$ gen-extreme channels $\mathcal{E} = \sum_{i=1}^d \mu_i \mathcal{E}_i^g$. This requires a random dit. For the worst case, the upper bound for such a convex sum is $d^4 - d^2$ from Carathéodory theorem on convex sets, which merely costs more random dits.
To simulate the composition $\prod_j E_j$, with each $E_j$ of rank greater than $d$, hence permitting a convex-sum decomposition, one needs to sample the composition of gen-extreme channels. We find there exists a concise form of Choi states for gen-extreme channels, which can be used to find the circuit and also Kraus operators directly.

The Choi state $C$ for a gen-extreme channel $E$ is of rank $r \leq d$ and $\text{tr}_1 C = \mathds{1}$, $\text{tr}_2 C = E(\mathds{1})$. It turns out $C = \sum_{ij} |i \rangle \langle j | \otimes C_{ij}$ for

$$C_{ij} := E((i) \langle j |) = \sqrt{C_i} U_i^\dagger U_j \sqrt{C_j}$$

(14)

for $C_i \equiv C_{ii}$, and $U_i, U_j \in SU(d)$ \[14\]. Observe that

$$E^t(\rho) = \sum_{ij} \rho_{ij} C_{ij} = V^\dagger (\rho \otimes \mathds{1}) V,$$

(15)

for an isometry $V := \sum_i |i \rangle U_i \sqrt{C_i}$. Here $\mathds{1}$ is an ancilla state. Now we show $V$ can be used to find a quantum circuit to realize $E$. Given $V$, we can find a unitary dilation $U$ such that $U|0\rangle = V$, and it relates to the channel by $E^t(\rho) = \langle 0 | U^\dagger (\rho \otimes \mathds{1}) U | 0 \rangle$, while the final projection $|0\rangle \langle 0 |$ is on the system. Define $W := $ swap $:: U^t$ for swap gate between the system and ancilla which are of the same dimension, then we find

$$E(\rho) = \text{tr}_a W^t (\rho \otimes |0\rangle \langle 0 |),$$

(16)

which means $W^t$ is the circuit to realize the channel $E$ as in the standard dilation scheme. The Kraus operators can be obtained from it as $K_i = \langle i | W^t | 0 \rangle$.

Compared with standard (tensor-product) dilation method to simulate a general channel, the method above requires lower circuit cost since it only needs an ancilla as a single qudit instead of two. While the convex-sum decomposition, which is a sort of generalized eigenvalue decomposition since a gen-extreme Choi state can be mapped to a pure state, are difficult for large channels, it shall be comparable with the eigen-decomposition of Choi state to find the set of Kraus operators. Both of the decompositions are solvable for smaller systems.

VI. APPLICATIONS

A. Teleportation of universal gate set

We now discuss applications of our frameworks above. For the unitary case, our method can be used to teleport universal gate sets. Consider the popular Hadamard gate $H$, phase gate $S$, the so-called $Z^{1/4}$ gate $T$, CNOT, CZ, and Toffoli gate. One immediately notice that these gates are all symmetric matrices. We see above that symmetric unitary operators, for which $U = U^t$, can be teleported deterministically, and the byproduct are Pauli operators. Note a product of symmetric matrices are not symmetric in general.

It is easy to check that the affine forms of $H$, $S$, CNOT, and CZ are (generalized) permutations since they are Clifford gates which preserve the Pauli group. A generalized permutation is a permutation which also allows entry of module 1 besides 1 itself. The $T$ gate and Toffoli gate are not Clifford gates, and their affine forms are not permutations. Instead, the affine forms of them contains a Hadamard-like gate as a sub-matrix, which means, in the Heisenberg picture, they are able to generate superpositions of Pauli operators. This fact also generalize to the qudit case, with Hadamard replaced by Fourier transform operators. This serves as an intriguing fact regarding the origin of computational power of quantum computing.

B. Stored-program quantum computing

It is known that the operation $U|d\rangle$ for general $U$ and input data state $|d\rangle$ cannot be simulated by $G(|d\rangle\psi_U)$ in a unitary way, for $G$ independent of $U$ and $|d\rangle$ \[1\]. Our method serves as a modification of it so this is possible: the processor $G$ is the generalized gate teleportation scheme, which instead depends on the input program state. For symmetric matrices $U$, the program state $|\psi_U\rangle$ is sufficient. For general cases, with $U = U^L U^R$, the program states $|\psi_U^L\rangle$ and $|\psi_U^R\rangle$ together are sufficient to obtain deterministic teleportation and composition. The channel-recovery scheme does not depend on the input program, which, however, is destroyed after the computation. One has to either prepare multiple copies of the program states or refresh the program if required. This is similar with the ‘one-way’ character of measurement-based quantum computing \[11\], and indeed, our model can be treated as a generalization of it. By the composition of a sequence of program states, we prepare many-body states in a heralded way, which can be properly treated as modification of graph states \[12\] or valence-bond solid states \[27\].

VII. CONCLUSION

In this work, we have studied computational schemes based on Choi states, which serve as the program states in a stored-program model. With a symmetry point of view of teleportation and computation, we develop a symmetry-based generalization of gate-teleportation, which is shown to be powerful to deterministically build up networks of program states. Our study reveals that quantum stored-program computing or information processing is powerful and shall be pursued further.

A program, as an operator being either unitary or nonunitary, can be stored by one Choi state if it is a symmetric matrix or two Choi states in general. For nonunitary channels, i.e., mixed Choi program states, we first embed each nonunitary Kraus operators into a unitary one based on direct-sum dilation method, which is further treated as a symmetry operator. Whether chan-
nels can be directly treated as a ‘quasi-symmetry’ or not is an interesting open problem.

The framework we develop may also be generalized. We expect that it can be adopted to infinite-dimensional systems for the teleportation of continuous variables. Embedding of fault-tolerance in the stored-program model shall be feasible, which nevertheless is a nontrivial further topic. Our scheme shall also be used in many settings such as universal computing models, quantum simulation, and quantum communication.

VIII. ACKNOWLEDGEMENT

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