Exact Solutions of Interference Fit of a High-speed Coupling for Micro Gas Turbine

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**Abstract.** Interference fit is widely used in the high-speed coupling of micro gas turbine since it can transfer large torque, are easy to produce and offer significant cost advantages. In this paper, a interference fit model was developed to analyse the conterction strength of coupling and shaft for micro gas turbine. Then using elastic theory, the radial stress, tangential stress and radial displacement of high-speed coupling are derived with taking account of contact pressure, angular velocity and the thickness of coupling. Finally taking a 100kW micro gas turbine as an example, the above solutions were calculated by using the numerical method and results show that the contact pressure, angular velocity and thickness of the coupling, should be considered for the design of the interference fit between the coupling and shaft in micro gas turbine. The direction of the radial stress of the coupling is determined by the value of the contact pressure. The tangential stresses of the coupling are identically greater than zero. The radial displacement of the coupling is a monotony decrease function of the radius and positively correlated with the contact pressure, angular velocity and thickness of the coupling.

1. **Introduction**

As key parts in micro gas turbine, couplings are widely used in high-speed or ultra-high-speed shafting for the connection and coaxial driving of generator, compressor and gas turbine. Many methods can connect the coupling and the shaft together at low rotational speed, such as bolts, a pair of mating spline teeth or interference fit.

However, the bolts could introduce certain unavoidable unbalances at high rotational speed, and the mating spline with an internal tooth and an external tooth has hysteretic forces and moments [1]. The interference fit, which can transfer large torque, are easy to produce and offer significant cost advantages, are used in the design for fixation of pinions, couplings and the like on shafts in order to overcome the disadvantages of the bolts or the mating spline [2].

There is a vast literature [3-8], most of which is concerned with a laminated composite tube which is interference fitted onto other structure like a bearing bush or a hub. Such a clearance between coupling and shaft is given, that the stresses and displacements of all regions of the fit can be calculated by Lame’s solution commonly. However, the centrifugal force of the coupling and shaft
should be considered at high-speed or ultra-high-speed shafting to guarantee the connection strength and safe operating. Accordingly, the aim of this paper is to develop a better calculation method used in the design of the interference fit for the couplings which transfer torques at the high speed, due to higher operating speeds and lighter weights for micro gas turbines.

2. Analytical model

2.1. Structure

In this study, the structure analyzed here is only the part of the interference fit of a high-speed coupling-shaft system. The inner radius of the shaft and the outer radius of the coupling are defined as \( r_i \) and \( r_o \) before assembly. The outer radius of the shaft is equal to the inner radius of the coupling which is indicated as \( R \) after assembly. Then \( u_r \) and \( u_\theta \) are the radial displacements of the coupling and shaft respectively. Further we stipulate that the elastic moduli \( E \) and Poisson’s ratio \( \mu \) as well as the density \( \rho \) of the coupling and shaft are equal.

![Figure 1. The interference fit between the coupling and shaft.](image)

In order to study the connection characteristics for the interference fit of the coupling and shaft, the influences of contact pressure of the interface of the interference fit \( p \) and angular velocity \( \omega \) as well as the outer radius of the coupling \( r_o \) on the stresses and radial displacement of the coupling are investigated as follow.

2.2. Modeling

Suppose \( \sigma_r \), \( \sigma_\theta \) and \( \tau_{r\theta} \) are the radial stress, tangential stress and the shear stress. Using the classic elastic plane stress theory, the equilibrium differential equation can be expressed as

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + \rho \omega^2 r = 0
\] (1)

Suppose \( \varepsilon_r \) and \( \varepsilon_\theta \) are the radial strain and tangential strain, then the strain-displacement relation and constitutive equation can be given as

\[
\begin{bmatrix}
\varepsilon_r \\
\varepsilon_\theta
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
u \\
\frac{du}{dr}
\end{bmatrix}
\] (2)
\[ \left[ \varepsilon_r, \varepsilon_\theta \right] = \begin{bmatrix} \frac{1}{E} & -\frac{1}{E} \mu \\ -\frac{1}{E} \mu & \frac{1}{E} \end{bmatrix} \begin{bmatrix} \sigma_r \\ \sigma_\theta \end{bmatrix} \] \hspace{1cm} (3)

Then combine equations above, the radial stress \( \sigma_{rc} \), tangential stress \( \sigma_{\theta c} \) and radial displacement \( u_{rc} \) of the interference fit of the coupling are obtained as

\[ \sigma_{rc} = A + \frac{B}{r^2} - \frac{3 + \mu}{8} \rho r^2 \omega^2 \] \hspace{1cm} (4)

\[ \sigma_{\theta c} = A - \frac{B}{r^2} - \frac{1 + 3\mu}{8} \rho r^2 \omega^2 \] \hspace{1cm} (5)

\[ u_{rc} = \frac{1}{E} \left[ (1 - \mu) A r - (1 + \mu) B - \frac{1 - \mu^2}{8} \rho \omega^2 r^3 \right] \] \hspace{1cm} (6)

Where \( a \) and \( B \) are the integration constants and determined by the boundary conditions of the interference fit.

2.3. Stresses and Displacement

The boundary conditions of the interference fit of the coupling can be introduced as follow to guarantee the equations above have a unique solution.

\[ \sigma_{rc}(r) \bigg|_{r=R} = -p \] \hspace{1cm} (7)

\[ \sigma_{rc}(r) \bigg|_{r=r_0} = 0 \] \hspace{1cm} (8)

Where contact pressure \( p \) can be given by

\[ p = \frac{M}{2\pi f L R^2} \] \hspace{1cm} (9)

Where \( M \), \( f \) and \( L \) are torque, static friction coefficient of the interface of the interference and length of the interference fit respectively.

Then the radial stress \( \sigma_{rc} \) and tangential stress \( \sigma_{\theta c} \) of the interference fit of the coupling can be expressed as

\[ \sigma_{rc} = \frac{p R^2}{r_0^2 - R^2} \left( 1 - \frac{r_0^2}{r^2} \right) + \frac{(3 + \mu) \rho \omega^2}{8} \left( R^2 + r_0^2 - \frac{R^2 r_0^2}{r^2} - r^2 \right) \] \hspace{1cm} (10)

\[ \sigma_{\theta c} = \frac{p R^2}{r_0^2 - R^2} \left( 1 + \frac{r_0^2}{r^2} \right) + \frac{(3 + \mu) \rho \omega^2}{8} \left( R^2 + r_0^2 + \frac{R^2 r_0^2}{r^2} - \frac{1 + 3\mu}{3} r^2 \right) \] \hspace{1cm} (11)

And the radial displacement \( u_{rc} \) of the interference fit of the coupling can be written as

\[ u_{rc} = \frac{p R^2 \left( (1 - \mu) r^2 + (1 + \mu) r_0^2 \right)}{E \left( r_0^2 - R^2 \right) r} + \frac{(1 + \mu)(3 - 2\mu) \rho \omega^2 r}{8E(1 - \mu)} \left[ (1 - 2\mu)(R^2 + r_0^2) + \frac{R^2 r_0^2}{r^2} + \frac{2\mu^2 - 1}{3 - 2\mu} r^2 \right] \] \hspace{1cm} (12)

3. Numerical analysis
This section studies the influences of the contact pressure $p$ and angular velocity $\omega$ as well as the outer radius of the coupling $r_o$ on the radial stress, tangential stress and displacement of the coupling. Extensive numerical computations have been performed with the following data in Table 1.

**Table 1.** Invariable parameters of the micro gas turbine.

| Parameter | Value |
|-----------|-------|
| $E$ (pa)         | $2.1 \times 10^{11}$ |
| $\rho$ (kg m$^{-3}$) | $7.8 \times 10^3$ |
| $\mu$ | $0.3$ |
| $P$ (kW) | $100$ |
| $L$ (m) | $0.0346$ |
| $f$ | $0.13$ |
| $R$ (m) | $0.0198$ |
| $r_i$ (m) | $0.0126$ |

The radial stress of the coupling arc is shown in Figure 2 for the contact pressure $p=20$ MPa and the angular velocity $\omega=3000$ rad/s. Figure 3 shows the radial stress of the coupling arc for the outer radius of the coupling $r_o=0.04$ m and the angular velocity $\omega=5000$ rad/s, and Figure 4 shows the radial stress of the coupling arc for the outer radius of the coupling $r_o=0.04$ m and the contact pressure $p=20$ Map. The radial stress of the coupling arc increases firstly and then decreases as the value of the radius grows. The radial stress of the coupling arc is equal to the contact pressures $-p$ for $r = R$, and The radial stress of the coupling arc is zero for $r = r_o$. The radial stress of the coupling arc increases scientifically when the thickness of the coupling is growing.

**Figure 2.** The radial stress of the coupling $\sigma_{rc}$ for the contact pressure $p=20$ Mpa and the angular velocity $\omega=3000$ rad/s.

**Figure 3.** The radial stress of the coupling $\sigma_{rc}$ for the outer radius of the coupling $r_o=0.04$ m and the angular velocity $\omega=5000$ rad/s.

**Figure 4.** The radial stress of the coupling $\sigma_{rc}$ for the outer radius of the coupling $r_o=0.04$ m and the contact pressure $p=20$ Mpa.

**Figure 5.** The tangential stress of the coupling $\sigma_{\theta c}$ for the contact pressure $p=20$ Mpa and the angular velocity $\omega=1000$ rad/s.
The tangential stresses of the coupling $\sigma_{\theta c}$ are shown in Figure 5, Figure 6 and Figure 7 for the different contact pressures $p$, angular velocities $\omega$ and outer radiuses of the coupling $ro$. The tangential stresses of the coupling $\sigma_{\theta c}$ are identically greater than zero as the growing of the contact pressures $p$, angular velocities $\omega$ and outer radiuses of the coupling $ro$.

Figure 8 shows the radial displacement of the coupling $urc$ for the contact pressure $p=40$ Mpa and the angular velocity $\omega=3000$ rad/s. Figure 9 shows the radial displacement of the coupling $urc$ for the outer radius of the coupling $ro=0.04$ m and the angular velocity $\omega=3000$ rad/s.

### 4. Conclusion

Three determine parameters: the contact pressure, angular velocity and thickness of the coupling, should be considered for the design of the interference fit between the coupling and shaft in micro gas turbine. The direction of the radial stress of the coupling is determined by the value of the contact pressure. The tangential stresses of the coupling are identically greater than zero. The radial displacement of the coupling is a monotony decrease function of the radius and positively correlated with the contact pressure, angular velocity and thickness of the coupling.

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