A Reinforcement Learning Based Approach for Joint Multi-Agent Decision Making

Mridul Agarwal and Vaneet Aggarwal

Purdue University, West Lafayette, IN, email: {agarw180,vaneet}@purdue.edu

Abstract

Reinforcement Learning (RL) is being increasingly applied to optimize complex functions that may have a stochastic component. RL is extended to multi-agent systems to find policies to optimize systems that require agents to coordinate or to compete under the umbrella of Multi-Agent RL (MARL). A crucial factor in the success of RL is that the optimization problem is represented as the expected sum of rewards, which allows the use of backward induction for the solution. However, many real-world problems require a joint objective that is non-linear and dynamic programming cannot be applied directly. For example, in a resource allocation problem, one of the objective is to maximize long-term fairness among the users. This paper addresses and formalizes the problem of joint objective optimization, where not only the sum of rewards of each agent but a function of the sum of rewards of each agent needs to be optimized. The proposed algorithms at the centralized controller aims to learn the policy to dictate the actions for each agent such that the joint objective function based on average per step rewards of each agent is maximized. We propose both model-based and model-free algorithms, where the model-based algorithm is shown to achieve $\tilde{O}(\sqrt{T})$ regret bound for $K$ agents over a time-horizon $T$, and the model-free algorithm can be implemented using deep neural networks. Further, using fairness in cellular base-station scheduling as an example, the proposed algorithms are shown to significantly outperform the state-of-the-art approaches.

1 Introduction

Many fields are increasingly adopting reinforcement learning for optimizing sequential decision making. These fields range from the operation of cyber-physical systems (e.g., cooperative robots in the production factories (Nguyen, Nguyen, and Nahavandi 2019), to optimizing communication networks (Mao, Netravali, and Alizadeh 2017), and neural networks (Li and Malik 2016)). These problems may include more than one agent, and the environment interlink the decisions of the different agents. This mandates the use of multi-agent learning (Busoniu, Babuska, and De Schutter 2010) to yield the best results. (Zhang and Shah 2014a) considered maximin fairness in multi-agent systems, which is an example of a joint function optimization based on the individual rewards of the agents. This paper aims to provide a novel formulation for joint decision making among multiple agents using reinforcement learning approaches and to analyze the performance of the proposed algorithms.

We consider a setup where we want to optimize a possibly nonlinear joint objective function of long-term rewards of all the agents. Some examples include fairness optimization and optimizing neural networks. For fairness optimization, agents jointly want to optimize a fairness objective among the different agents, e.g., proportional fairness, $\alpha$-fairness, or improve the worst-case average reward of the users (Altman, Avrachenkov, and Garnaev 2008). In such situations, the overall joint objective function cannot be written as sum utility at each time instant. This prohibits the application of the standard single-agent reinforcement learning based policies as the backward induction step update cannot be directly applied here. For example, if a process has 2 agents and $T > 1$ steps, and all the resource was allocated to the first agent till $T - 1$ steps. Then, at $T^{th}$ step the resource should be allocated to the second agent to ensure fairness. This requires the need to track past allocation of all the resources and not just the current state of the system. We define a novel multi-agent formulation, making several practical assumptions, which optimizes the joint function of the average per-step rewards of the different agents to alleviate the need for maintaining history.

We further note that even though multi-agent reinforcement learning algorithms have been widely studied, (Tan 1993) Shoham, Powers, and Grenager 2003, Busoniu, Babuska, and De Schutter 2010, Ono and Fukumoto 1996, there are no convergence proofs to the optimal joint objective function without the knowledge of the transition probability, to the best of our knowledge. This paper assumes no knowledge of the state transition probability of the agents and aims to provide algorithms for the decision making of the different agents. We provide two algorithms; The first is a model-based algorithm that learns the transition probability of the next state given the current state and action. The second algorithm is model-free, which uses policy gradients to find the optimal policy.

The proposed model-based algorithm uses posterior sampling with Dirichlet distribution. We show that the proposed algorithm converges to an optimal point when the joint objective function is Lipschitz continuous. In addition, we show that the proposed algorithm achieves a regret bound sub-linear in the number of time-steps and number of agents. This regret bound characterizes the gap between the optimal objective and the objective achieved by the algorithm in $T$ time-steps. We show a regret bound of $\tilde{O}(\sqrt{KT})$, where $K, T$ denotes the number of agents, and time steps, respectively.

The proposed model-free algorithm can be easily imple-
mented using deep neural networks for any differentiable objective function. Further, we note that the reward functions of the different agents can be very different, and can optimize different metrics for the agents. As long as there is a joint objective function, the different agents can make decisions to optimize this function and achieve the optimal decision at convergence.

We also present evaluation results for both the algorithms for optimizing proportional fairness of multiple agents connecting to a cellular base station. We compare the obtained policies with existing heuristics of optimizing proportional fairness for wireless networks (Margolies et al. 2016) and SARSA based RL solution proposed by (Perez et al. 2009). We developed a simulation environment for wireless network for multiple number of agents and states for each agent. Our results beat the existing heuristics by a significant margin.

Key contributions of our paper are:

- A structure for joint function optimization with multiple agents based on average per-step rewards.
- A model-based algorithm using posterior sampling with Dirichlet distribution, and its regret bounds.
- A model-free policy gradient algorithm which can be efficiently implemented using neural networks.
- Evaluation results and comparison with existing heuristics for optimizing proportional fairness in cellular networks.

The rest of the paper is organized as follows. Section 2 describes related works in the field of RL and MARL. Section 3 describes the problem formulation. The proposed model based algorithm and model free algorithm are described in Sections 4 and 5 respectively. In Section 6 the proposed algorithms are evaluated for cellular scheduling problem. Section 7 concludes the paper with some future work directions.

2 Related Work

Reinforcement learning for single agent has been extensively studied in past (Sutton and Barto 2018). Dynamic Programming was used in many problems by finding cost to go at each stage (Puterman 1994; Bertsekas 1995). These models optimize linear additive utility and utilize the power of Backward Induction.

Following the success of Deep Q Networks (Mnih et al. 2015), many new algorithms have been developed for reinforcement learning (Schulman et al. 2015; Lillicrap et al. 2015; Wang et al. 2015; Schulman et al. 2017). These papers focus on single agent control, and provide a framework for implementing scalable algorithms. Sample efficient algorithms based on rate of convergence analysis have also been studied for model based RL algorithms (Agrawal and Jia 2017; Osband, Russo, and Van Roy 2013), and for model free Q learning (Jin et al. 2018). However, sample efficient algorithms use tabular implementation instead of a deep learning based implementation.

In most applications such as financial markets, swarm robotics, wireless channel access, etc., there are multiple agents that make a decision (Bloembergen et al. 2015), and the decision of any agent affects the others agents. This requires extending single agent reinforcement learning to multi-agent learning, where the joint action of the agents must be optimized. In early work on multi-agent reinforcement learning (MARL) for stochastic games (Littman 1994), it was recognized that no agent works in a vacuum. In his seminal paper, Littman (Littman 1994) focused on only two agents that had opposite and opposing goals. This means that they could use a single reward function which one tried to maximize and the other tried to minimize. The agent had to work with a competing agent and had to behave to maximize their reward in the worst possible case. In MARL, the agents select actions simultaneously at the current state and receive rewards at the next state. Different from the algorithm that can solve for a Nash equilibrium in a stochastic game, the goal of a reinforcement learning algorithm is to learn equilibrium strategies through interaction with the environment (Tan 1993; Shohami, Powers, and Grenager 2003; Busoniu, Babuška, and De Schutter 2010; Ono and Fukumoto 1996; Shalev-Shwartz, Shammah, and Shashua 2016).}

3 Problem Formulation

We consider an infinite horizon discounted Markov decision process (MDP) $\mathcal{M}$ defined by the tuple $(S, A, P, K, r_1, r_2, \ldots, r_K, \gamma, \rho_0, D, f)$. $S$ denotes a finite set of state space, and $A$ denotes a finite set of actions. $P : S \times A \rightarrow S$ denotes the probability transition distribution. $K$ denotes the number agents and $[K] = \{1, 2, \ldots, K\}$ is the set of $K$ agents. $r^k : S \times A \rightarrow [0, 1]$ denotes reward generated by agent $k \in [K]$. $\gamma$ is the discount factor and $\rho_0 : S \rightarrow [0, 1]$ is the distribution of initial state. $f : \mathbb{R}^K \rightarrow \mathbb{R}$ denotes the joint objective function. $D$ is the diameter of the MDP, which is the maximum expected number of steps needed to reach any state $s' \in S$ from some state $s \in S$. We assume that the diameter $D$ of $\mathcal{M}$ is bounded.

If each agent $k$ maintains state space $S_k$, then the joint state space will become $S = S_1 \times \cdots \times S_K$. The same also holds true when each agent $k$ maintains action space $A_k$. The joint action space here is $A = A_1 \times \cdots \times A_K$. We use a joint stochastic policy $\pi : S \times A \rightarrow [0, 1]$ which returns the probability of selecting action $a \in A$ for any given state $s \in S$. The expected long term reward and expected per step reward of the agent $k$ are given by $J^k_\pi$ and $J^k_\pi$, respectively.
respectively, when the joint policy $\pi$ is followed. Formally, $J^k_\pi$ and $\lambda^k_\pi$ are defined as

$$J^k_\pi = \mathbb{E}_{s_0, a_0, s_1, a_1, \ldots} \left[ \lim_{T \to \infty} \sum_{t=0}^{\infty} \gamma^t r^k(s_t, a_t) \right]$$  \hspace{1cm} (1)$$

$$s_0 \sim \rho_0(s_0), a_t \sim \pi(a_t | s_t), s_{t+1} \sim P(s_{t+1} | s_t, a_t)$$  \hspace{1cm} (2)$$

$$\lambda^k_\pi = \mathbb{E}_{s_0, a_0, s_1, a_1, \ldots} \left[ \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} r^k(s_t, a_t) \right]$$  \hspace{1cm} (3)$$

$$\lambda^k_\pi = \lim_{\gamma \to 1} (1 - \gamma) J^k_\pi$$  \hspace{1cm} (4)$$

Equation (4) follows from the Laurent series expansion of $J^k_\pi$ (Puterman 1994). For brevity, in the rest of the paper, $\mathbb{E}_{s_t, a_t, s_{t+1}, \ldots} [f] \geq 0$ will be denoted as $\mathbb{E}_{\rho, \pi, P} [\cdot]$, where $s_0 \sim \rho_0(s_0)$, $a_t \sim \pi(s_t | a_t)$, $s_{t+1} \sim P(s_{t+1} | s_t, a_t)$.

The agents aim to collaboratively optimize the joint objective function $f$, which is defined over the long-term rewards of the individual agents. We make certain practical assumptions on this joint objective function $f$, which are listed as follows:

**Assumption 1.** The objective function $f$ is concave. Hence for any arbitrary distribution $D$, the following holds.

$$f(\mathbb{E}_{\infty \sim D} [x]) \geq \mathbb{E}_{\infty \sim D} [f(x)] ; \quad x \in \mathbb{R}^K$$  \hspace{1cm} (5)$$

The objective function $f$ represents the utility obtained from the expected per step reward of each agent. These utility functions are often concave to reduce variance when the agents are risk averse (Pratt 1964). To model this concave utility function, we assume the above form of Jensen’s inequality.

**Assumption 2.** The function $f$ is assumed to be differentiable everywhere. Further, we let $f$ be a $L-$Lipschitz function, or

$$\|f(x) - f(y)\| \leq L \|x - y\|_2 ; \quad x, y \in \mathbb{R}^K$$  \hspace{1cm} (6)$$

Assumption 2 ensures that for a small change in long run rewards of any agent does not cause drastic changes in the objective.

In Appendix C, we provide an additional assumption using which we can show that the proposed algorithms lead to Pareto optimal strategy when applied for fairness optimization problems.

Based on Assumption 1, we maximize the function of expected sum of rewards of agents. Further to keep the formulation independent of time horizon or $\gamma$, we maximize the function over expected per step rewards of each agent. Hence, our goal is to find the optimal policy as the solution for the following optimization problem.

$$\pi^* = \arg \max_{\pi} f(\lambda^1_\pi, \ldots, \lambda^K_\pi)$$  \hspace{1cm} (7)$$

Any online algorithm $\mathcal{A}$ starting with no prior knowledge will require to obtain estimates of transition probabilities $P$ and obtain rewards $r^k_s$, $\forall k \in [K]$ for each state action pair. Initially, when algorithm $\mathcal{A}$ does not have good estimates of the model, it accumulates a regret for not working as per optimal policy. We define a time dependent regret $R_T$ to achieve an optimal solution defined as the difference between the optimal value of the function and the value of the function at time $T$, or

$$R_T = \left| f(\lambda^1_\pi^*, \ldots, \lambda^K_\pi^*) - f\left(\frac{1}{T} \sum_{t=0}^{T} r^1(s_t, a_t), \ldots, \frac{1}{T} \sum_{t=0}^{T} r^K(s_t, a_t)\right)\right|$$  \hspace{1cm} (8)$$

In the following section, we present a model-based algorithm to obtain this policy $\pi^*$, and regret accumulated by the algorithm. We will present a model-free algorithm in Section 5 which can be efficiently implemented using Deep Neural Networks.

## 4 Model-based Algorithm

Single-agent problem can be optimally solved using backward induction on the expected rewards due to additive function over time (Puterman 1994). However, since our controller is optimizing a joint non-linear function of the individual long-term rewards of the agents, only optimizing what is ahead does not yield an optimal algorithm. Our goal is to find the optimal policy as solution of Equation (7). Using average per-step reward and infinite horizon allows to use Markov policies as there is always infinite time available to make the system fair.

The individual long-term reward for each agent is still linearly additive ($\frac{1}{T} \sum_{t=0}^{T} \gamma^t r^k(s_t, a_t)$). Thus, the value function for individual agents can be obtained using backward induction. For infinite horizon optimization problems (or $\tau \to \infty$), we can use steady state distribution of the state to obtain expected cumulative rewards. For all $k \in [K]$, we define

$$V^k_\pi(s) = \mathbb{E}_{\pi, P} \left[ \sum_{j=0}^{\infty} \gamma^{j-t} r^k(s_j, a_j) | s_t = s \right]$$  \hspace{1cm} (9)$$

$$V^k_\pi = (I - \gamma P_\pi)^{-1} \pi^k \gamma P_\pi$$  \hspace{1cm} (10)$$

where $P_\pi$ is the state transition probability matrix for the policy $\pi$, $\pi^k = [\pi^k(1), \ldots, \pi^k(|S|)]^T$, $V^k_\pi = [V^k_\pi(1), \ldots, V^k_\pi(|S|)]^T$, and superscript $(\cdot)^T$ denotes transpose. Thus, we have

$$J^k_\pi = \sum_{s \in S} d^s(s) V^k_\pi(s)$$  \hspace{1cm} (11)$$

$$\lambda^k_\pi = (1 - \gamma) J^k_\pi$$  \hspace{1cm} (12)$$

where $d^s_\pi$ is the steady state distribution of policy $\pi$. The optimal policy is solution of the Equation (7) with $\lambda^k_\pi$ defined according to Equation (12). Equation (11) and Equation (12) form the key structure for our model based algorithm.

The proposed model-based algorithm estimates the transition probabilities by interacting with the environment. We need the steady state distribution of each policy $\pi$. We ensure this by sampling transition probability from Dirichlet distribution. Proposition 1 formalizes the result of the existence of a steady state distribution when the transition probability is sampled from a Dirichlet distribution.
Proposition 1. For MDP \( \tilde{M} \) with state space \( S \) and action space \( A \), let the transition probabilities \( \tilde{P} \) come from Dirichlet distribution. Then, any policy \( \pi \) for \( \tilde{M} \) will have a steady state distribution \( \tilde{d}_{\pi} \) given as

\[
\tilde{d}_{\pi}(s') = \sum_{s \in S} \tilde{d}_{\pi}(s) \left( \sum_{a \in \tilde{A}} \pi(a|s)P(s, a, s') \right) \quad \forall s' \in \tilde{S}
\]  

(13)

Proof. Transition probabilities \( P(s, a, \cdot) \) follow Dirichlet distribution, and hence they are strictly positive. Further, as the the policy \( \pi(a|s) \) is a probability distribution on actions conditioned on state, \( \pi(a|s) \geq 0 \), and \( \sum_{a} \pi(a|s) = 1 \). So, there is no zero transition probability to reach from state \( s \in \tilde{S} \) to state \( s' \in \tilde{S} \). Since the single step transition probability matrix is strictly positive for any policy \( \pi \), a steady state distribution exists for any policy \( \pi \). \( \square \)

Algorithm 1 Policy search algorithm

1: procedure POLICY SEARCH \( P, N, r^k, [K], \gamma, f \) \( \triangleright \) Current estimate of the model parameters
2: for \( \pi \in \Pi \) do \( \triangleright \) search from the set of all policies \( \Pi \)
3: \( P_\pi(s, s') = \sum_{a} \pi(a|s)P(s, a, s') \forall (s, s') \)
4: for \( k \in [K] \) do
5: \( r^k(s) = \sum_{a} \pi(a|s)\sum_{s'} N(s, a, s') \triangleright \) Vector of expected rewards for all states
6: \( V_k^\pi = (I - \gamma P_\pi)^{-1} r^k \)
7: \( \lambda_k^\pi = (1 - \gamma) \sum_s d_\pi(s) V_k^\pi(s) \)
8: end for
9: Calculate steady state distribution \( d_\pi(s) \) from \( P_\pi \)
10: Evaluate Policy
\[
\hat{f}_\pi = f \left( \lambda_{\pi}^1, \ldots, \lambda_{\pi}^K \right)
\]  

(14)

11: end for
12: Return \( \pi^* = \arg \max_{\pi} \hat{f}_\pi \)
13: end procedure

4.1 Algorithm Description

The proposed algorithm to search for optimal policy is described in Algorithm 1. Policy Search algorithm uses the state transition probabilities \( P \), and reward function \( r_k \) for each agent \( k \in [K] \) of the infinite horizon MDP, \( M \) as input. The algorithm then evaluates the state transition probability matrix for policy \( \pi \) (Line 3). In Line 5-6, for each agent, we calculate the expected reward for each state and the value function vector using Equation (10) for policy \( \pi \). Further, the steady state distribution \( d_{\pi} \) is calculated from the estimated state transition distribution matrix, and the value of the joint objective function is obtained for the policy using Equation (14). The steps in Line 3-9 are repeated for all the policies (outer loop in Line 2), and the policy that maximizes the joint objective function is returned.

Algorithm 1 uses an estimate of the MDP, which is not available apriori. Algorithm 3 describes the overall procedure that estimates the transition probabilities and the reward functions. The algorithm takes as input the state space \( S \), action space \( A \), set of agents \( [K] \), discount factor \( \gamma \), and the objective function \( f \). It initializes the next state visit count for each state-action pair \( N(s, a, s') \) by one for Dirichlet sampling. The reward estimate for each agent \( k \), \( r_k(s, a) \), is initialized by zero. For initial exploration, the policy uses a uniform distribution over all the actions. The algorithm proceeds in epochs, we assume that the controller is optimizing for the infinite horizon and thus there is no stopping condition for epoch loop in Line 3. For each time index \( t \) in epoch, the controller observes the state, samples and plays the action according to \( \pi(a|s) \), and observes the rewards for each agent and next state (Line 5-7). It then updates the state visit count for the observed state and played action pair. Also, the reward for each agent is updated in Line 7. In Line 10, we use sampling using the updated posterior and update the policy using Algorithm 1 in Line 11. The updates in Line 10-11 are made after completing an epoch; the condition after which an epoch is marked as complete (line 9) is described in Section 4.2.

4.2 Regret

We first state and prove a regret bound for Algorithm 3, which depends on the epoch complete condition in Line 9. Let the regret till time \( T \) of Algorithm 2 for \( N = 1 \), objective function \( f \) as identity (i.e., \( f(x) = x \)), and some particular epoch update condition be defined as,

\[
R_T = \lambda_{\pi^*} - \frac{1}{T} \sum_{t=1}^{T} r_t
\]  

(15)

Then, the regret of Algorithm 2 for \( N > 1 \), and a L-Lipschitz function \( f \) satisfying Equation 6 is given in following theorem

Theorem 1. Let the regret \( R_T \) of Algorithm 2 for \( N = 1 \) and \( f(x) = x \), be bounded by \( \mathcal{R} \) with probability \( 1 - \delta \). Then implementing Algorithm 2 for multiagent MDP \( M \) to solve for policy \( \pi^* \), given in Equation 7, will incur a regret \( R_T \) given as:

\[
R_T \leq \sqrt{KR} \mathcal{R} \text{ w.p. } 1 - K\delta
\]  

(16)

Proof. The definition of regret \( R_T \) in Equation 6 gives

\[
R_T = \left| f \left( \ldots, \lambda_k^{h, *}, \ldots \right) - f \left( \ldots, \frac{1}{T} \sum_{t=0}^{T} r^k(s_t, a_t), \ldots \right) \right|
\]  

(17)

\[
\leq L \left( \sum_{k=1}^{K} \left( \lambda_k^{h, *} - \frac{1}{T} \sum_{t=0}^{T} r^k(s_t, a_t) \right)^2 \right)^{\frac{1}{2}}
\]  

(18)

\[
\leq L \sqrt{K} \max_k \left| \lambda_k^{h, *} - \frac{1}{T} \sum_{t=0}^{T} r^k(s_t, a_t) \right|
\]  

(19)

Equation (18) follows from Assumption 2 stated in Equation 6. Equation (19) follows from the fact that the mean of \( K \) values is bounded by the maximum of the \( K \) values. Now, regret for Algorithm 2 for any single agent \( k \in [K] \) is defined as,

\[
R_k = \lambda_k^{h, *} - \frac{1}{T} \sum_{t=0}^{T} r^k(s_t, a_t)
\]  

(20)
Using union bound on Equation \ref{eq:bound}, we obtain Equation \ref{eq:thm1} with probability at least $1 - K\delta$.

This analysis shows that the regret of joint function optimization only scales as square root of the number of agents for any online algorithm.

We now discuss the effect of epoch update condition (line 9, Algorithm \ref{alg:alg3}). One approach for epoch update condition is to choose fixed effective horizon $\tau$ of $O\left((1 - \gamma)^{-1}\right)$ and update policy after every $\tau$ steps, as mentioned in \cite{osband2013optimistic}. They obtain a regret bound of $\tilde{O}\left(rS\sqrt{A/T}\right)$ for $N = 1$ and $f(x) = x$.

Another common approach, as mentioned in \cite{agrawal2017thompson}, is to use dynamic effective horizon and update policy after every horizon. The epoch is updated only when the total visits to any state-action pair is doubled. That is, the number of visits to state-action pair $(s, a)$ in epoch $c$, $N_c(s, a)$ satisfies $N_c(s, a) \geq \sum_{c' < c} N_{c'}(s, a)$ for some state-action pair $(s, a)$. Their algorithm also uses an Extended MDP approach that generates $O(|S|)$ independent samples from the posterior. The policy is then optimized for the extended MDP. Using these tricks, they obtain a regret bound of $\tilde{O}\left(D\sqrt{SA/T}\right)$ for $N = 1$ and $f(x) = x$.

Thus, a corollary of Theorem 1 is obtained when we modify epoch update condition in Line 9 of Algorithm \ref{alg:alg3} and use an extended MDP formulation from Optimistic Posterior Sampling (OPS) algorithm from \cite{agrawal2017thompson}. This corollary is given as follows.

**Corollary 1.** Choosing the update condition based on doubling epoch trick in Line 9 of Algorithm \ref{alg:alg3} and using an Extended MDP formulation from Optimistic Posterior Sampling algorithm \cite{agrawal2017thompson} on $M$ achieves the following regret bound to find a policy satisfying Equation \ref{eq:thm1} with probability $1 - K\delta$

$$R_T \leq \tilde{O}\left(LD\sqrt{KA\sqrt{T}}\right) \text{ for } T > S^6A$$

where $\tilde{O}$ hides the logarithmic factors in $S, A, T, \delta$, and absolute constants.

### 5 Model Free Algorithm

In the previous section, we developed a model based tabular algorithm for joint function optimization. However, as the state space, action space, or number of agents increase the tabular algorithm becomes infeasible to implement. In this section, we consider a policy gradient based algorithm which can be efficiently implemented using (deep) neural networks thus alleviating the requirement of a tabular solution for large MDPs.

For the model free policy gradient algorithm, we will use finite time horizon MDP, or $T < \infty$ in our MDP $M$. This is a practical scenario where communication networks optimize fairness among users for finite duration \cite{margolies2016}. We now describe a model free construction to obtain the optimal policy. We use a neural network parameterized by $\theta$. The objective thus becomes to find optimal parameters $\theta^*$, which maximizes,

$$\arg \max_{\theta} f \left( (1 - \gamma) J^1_{\theta}, \cdots, (1 - \gamma) J^K_{\theta} \right)$$

\begin{algorithm}[h]
\caption{Model-based Algorithm for Joint Decision Making with Multiple Agents}
\begin{algorithmic}[1]
\Procedure{Model Based Online}{$S, A, [K], \gamma, f$}
\State Initialize $N(s, a, s') = 1 \forall (s, a, s') \in S \times A \times S$
\State $\tau^k(s, a) = 0 \forall (s, a, k) \in S \times A \times [K], \pi(a|s) = \frac{P(s, a, s')}{\sum_{a'} P(s, a', s')}$
\For {epoch $e = 1, 2, \cdots$}
\State $t_e = 1, 2, \cdots$
\For {epoch update condition met then}
\State $P(s, a, s') \sim \text{Dir}(N(s, a, s')) \forall (s, a, s')$
\State $\pi = \text{Policy Search}(P_N, \tau^k, [K], \gamma, f)$
\State break epoch
\EndIf
\EndFor
\EndFor
\EndProcedure
\end{algorithmic}
\end{algorithm}

Gradient estimation for Equation \ref{eq:grad} can be obtained using chain rule:

$$\nabla_\theta f = \sum_{k \in [K]} \frac{\partial f}{\partial J^k_\theta} \nabla_\theta J^k_\theta$$

\begin{equation}
= (\nabla_j \pi J^k) (\nabla_\theta \pi) \left( \frac{\partial f}{\partial J^k_\theta} \right) \tag{23}
\end{equation}

Note that, $\tilde{J}_\pi$ is the expected cumulative reward. $\tilde{J}_\pi$ can be replaced with averaged cumulative rewards over $N$ trajectories for the policy at $i$th step, where a trajectory $\tau$ is defined as the tuple of observations and $\tau = (s_0, a_0, r_0^1, \cdots, r^K_0, s_1, a_1, r_1^1, \cdots, r^K_1, \cdots)$. Further, $\nabla_\theta \tilde{J}_\pi$ can be estimated using REINFORCE algorithm proposed in \cite{williams1992simple, sutton2000policy}, and is given as

$$\tilde{\nabla}_\theta \tilde{J}_\pi = \frac{1}{N} \sum_{j=0}^{N} \sum_{t=0}^{T} \log \pi(a_{t,j} | a_{t-1,j}) \cdot \tilde{J}(s_{t,j}, a_{t,j}).$$

Further, $\tilde{J}_\pi$ is estimated as

$$\hat{J} = \frac{1}{N} \sum_{t=0}^{T} \tilde{J}(s_{t,j}, a_{t,j})$$

For a learning rate $\eta$, parameter update step to optimize the parameters becomes

$$\theta_{t+1} = \theta_t + \eta \left( \nabla_\theta \hat{J} \right)$$

The proposed Model Free Policy Gradient algorithm for joint function optimization is described in Algorithm \ref{alg:alg4}. The algorithm takes as input the parameters $S, A, [K], T, \gamma, f$ of MDP $M$, number of sample trajectories $N$, and learning rate $\eta$ as input. The policy neural network is initialized with weights $\theta$ randomly. It then collects $N$ sample trajectories using the policy with current weights in Line 4. In Line 5, the gradient is calculated using Equations \ref{eq:grad}, \ref{eq:reinforce}, and \ref{eq:thm3} on the $N$ trajectories. In optimization step of Line 6, the weights are updated using gradient ascent.
Algorithm 3 Model Free Joint Policy Gradient

1: procedure JOINT POLICY GRADIENT
2: \((S, A, [K], T, \gamma, f, N, \eta)\)
3: Initialize \(\pi_{\theta_0}(a, s)\)  
4: for \(i = 0, 1, \ldots, \) until convergence do
5: Collect \(N\) trajectories using policy \(\pi_{\theta}\)
6: Estimate gradient using Equation (24), (25), (26)
7: Perform Gradient Ascent using Equation (27)
8: Return \(\pi_{\theta}\)
9: end for

6 Evaluations

We evaluate both the proposed model-based algorithm (Algorithm 2) and the proposed model-free algorithm (Algorithm 3) for maximizing fairness among agents in a cellular network where multiple users are connected to a base station. The fairness maximization has also been at the heart of many other resource allocation problems such as cloud resource management, manufacturing optimization, etc. (Perez et al. 2009; Zhang and Shah 2015). The problem of maximizing wireless network fairness has been extensively studied in the past by (Margolies et al. 2016; Kwan, Leung, and Zhang 2009; Bu, Li, and Ramjee 2018). With increasing number of devices that need to access wireless network and ever upgrading network architectures, this problem still remains of practical interest.

6.1 Problem Setup

Proportional Fair (PF) scheduling algorithms are the de facto standard for opportunistic schedulers in cellular networks (Holma and Toskala 2005). They aim to provide high throughput while maintaining fairness among the users. The problem of maximizing finite horizon proportional fairness for multiple agents attached to a base station is defined as

\[
C = \max_{(\alpha, k, t) \in K \times T} \sum_{k=1}^{K} \log \left( \frac{1}{T} \sum_{t=1}^{T} \alpha_k r_{k,t} \right) \tag{28}
\]

\[
\text{s.t.} \sum_{k=1}^{K} \alpha_k = 1 \forall t \in \{1, 2, \ldots, T\} \tag{29}
\]

\[
\alpha_{k,t} \in [0, 1] \tag{30}
\]

where, \(\alpha_{k,t} = 1\) if the agent \(k\) obtains the network resource at time \(t\), and \(0\) otherwise. Further, \(r_{k,t}\) denotes the rate at which agent \(k\) can transmit at time \(t\) if allocated network resource. We use \(T = 1000\) for the simulations and \(K \in \{2, 4, 6\}\). We note that \(r_{k,t}\) is only known causally limiting the use of offline optimization techniques and making the use of learning-based strategies for the problem important.

The state space of each agent comes from its channel conditions. We assume that the channel for a channel can only be in two conditions \{good, bad\}, where the good and bad conditions for each agent could be different. At each time, the scheduler gives all the resources to a single agent. Thus, the action at each time is a one-hot vector with the entry corresponding to the agent receiving the resources set to one. This gives \(|S| = 2^K\) (corresponding to the joint channel state of all agents), and \(|A| = K\) (actions correspond to the agent that is selected in a time slot). Based on the channel state of agent, the scheduling decision determines the agent that must be picked in the time-slot. Rate \(r_{k,t}\), for agent \(k\) at time \(t\), is dependent on the state of the agent \(s_{k,t}\) and is mentioned in Table 1. Each agent remain in the same state with probability \(0.8\), and move to a different state \(s \sim U(S)\) with probability \(0.2\). The state transition model becomes:

\[
\forall k, s_{k,t+1} = \begin{cases} s_{k,t}, & \text{w.p.} \ 0.8 \\ s \sim U(\{\text{good, bad}\}) & \text{w.p.} \ 0.2 \end{cases} \tag{31}
\]

Table 1: Agent rate \(r_{k,t}\) (in Mbps) based on agent state \(s_{k,t}\). Rate values are practically observable data rates over a wireless network such as 4G-LTE.

6.2 Evaluated Algorithms

We compare our model-based and model-free algorithms with practically implemented algorithm of Blind Gradient Estimation (Margolies et al. 2016; Bu, Li, and Ramjee 2018) in network schedulers, SARSA based algorithm devised by (Perez et al. 2009). We first describe the algorithms used in evaluations.

- **Blind Gradient Estimation Algorithm (BGE):** This heuristic allocates the resources based on the previously allocated resources to the agents. Starting from \(t = 1\), this policy allocates resource to agent \(k^*_t\) at time \(t\), where

\[
k^*_t = \arg \max_{k \in [K]} \frac{r_{k,t}}{\sum_{k'=0}^{K} \alpha_{k,t} r_{k,t}} \tag{32}
\]

In this policy, an agent gets the resource if it can utilize to maximum potential when all the other agents have been allocated sufficient resources in past, or if it is currently the most starved agent. BGE is used as de facto standard for scheduling in cellular systems (Holma and Toskala 2005).

- **SARSA Algorithm:** This algorithm based on SARSA (Sutton and others). The reward at each time \(t\) is the fairness of the system at time \(t\), or

\[
f_t = \sum_{k=1}^{K} \log \left( \frac{1}{t} \sum_{t'=1}^{t} \alpha_{k,t'} r_{k,t'} \right) \tag{33}
\]

We use \(\gamma = 0.9, \epsilon = 0.05\), and learning rate \(\eta = 0.01\) for SARSA implementation.

- **Proposed Model Based Algorithm:** We describe the algorithm for infinite horizon, so we maximize the policy
for infinite horizon proportional fairness problem by discounting the rewards as
\[
\lim_{T \to \infty} \sum_{k=1}^{K} \log \left( \frac{1}{T} \sum_{t=1}^{T} \gamma^{t} \alpha_{k,t} r_{k,t} \right)
\]  
(34)

The learned policy is evaluated on finite horizon environment of \( T = 1000 \). We keep \( \gamma = 0.99 \) for implementation of Algorithm 2. We use a fixed episode length of \( O \left( \frac{1}{1-\gamma} \right) = O(100) \) as mentioned in (Osband, Russo, and Van Roy 2013) and update policy after every \( \tau = 100 \) steps.

In Algorithm 1 we use gradient ascent to find a locally optimal policy \( \pi(\cdot|s) \) with step size of \( \eta = 0.1 \) for each state \( s \in S \). For small chance in the stochastic policy, change in average per-step rewards of agents are small. From Assumption 2, the change in function value is also small, hence the direction for steepest ascent exists. The policy obtained after applying gradient ascent is projected back to the probability simplex to satisfy the probability axioms.

- **Proposed Model Free Algorithm:** The proposed algorithm uses the reward metric \( C \) from Equation (35). Since \( \log(\cdot) \) is differentiable, the gradient in Equation (24) is evaluated using Equation (35).

\[
\nabla_{\theta} f = \sum_{k \in [K]} \sum_{j=1}^{N} \nabla \log \pi(\tau_{t,j}|s_{t,j}) \sum_{t=0}^{T} \gamma^{t} r_{k}(s_{t,j}, a_{t,j})
\]

(35)

The neural network consists of a single hidden layer with 200 neurons, each having ReLU activation function. We use Adam optimizer with \( \beta_1 = 0.9 \), and \( \beta_2 = 0.999 \) to train the network. The value of other hyperparameters are \( \gamma = 0.99, \eta = 1 \times 10^{-3} \), and batch size \( N = 100 \).

### 6.3 Simulation Results

We trained the SARSA algorithm and the model based Algorithm 2 for 5000 time steps for each value of \( K \). To train model free Algorithm 3, we used \( 2 \times 10^{6} \) epochs, and each epoch consists of 1000 time steps. Note that the Blind Gradient Estimation algorithm doesn’t need training as it selects the agent based on observed rewards.

We show the performance of policies implemented by each of the algorithm. Each policy is run 100 times and median and interquartile range is shown in Figure 1 for each policy. The policy performance for \( K = 2, K = 4, \) and \( K = 6 \) is shown in Figure 1(a), Figure 1(b) and Figure 1(c) respectively.

We note that SARSA based solution performs significantly worse than other algorithms. The performance of SARSA significantly decrease for \( K = 4 \) and \( K = 6 \), hence we exclude the plots for the same to keep the plots comprehensible.

We note that the performance of model-based algorithm (Algorithm 2) and that of the model-free algorithm (Algorithm 3) are almost overlapping. For \( K = 2 \), the model-based algorithm outperforms the model-free algorithm. However, for \( K \in \{4, 6\} \), the model-free algorithm performs marginally better. We believe that this difference in the performance of model-based algorithm is because we used convex approximation in Algorithm 1. The proposed algorithms (model-based as well as model-free) consistently outperform BGE algorithm for \( K \in \{2, 4, 6\} \) which is the de-facto standard in cellular networks. Further, the gap between the performance of BGE algorithm, and the proposed algorithms increase considerably as the number of agents increase.

We present the percentage improvements of proportional fairness obtained by the proposed model-based and the model-free algorithms as well as SARSA algorithm over BGE algorithm in Appendix A where we see that the proposed algorithms are about 17% better than BGE for \( K = 6 \). We also show the convergence rate of model based and model free algorithms in Appendix B where we see that the model-based approach converges faster.

### 7 Conclusion

This paper presents a novel average per step reward based formulation for optimizing joint objective function of long-term rewards of each agent for infinite horizon multi-agent systems. In case of finite horizon, Markov policies may not be able to optimize the joint objective function, hence an average reward per step formulation is considered. A tabular model based algorithm which uses Dirichlet sampling to obtain regret bound of \( \tilde{O} \left( \sqrt{\frac{K}{T}} \right) \) for \( K \) agents over a time horizon \( T \) is provided. Further, a model free algo-
rithm which can be efficiently implemented using neural networks is also proposed. The proposed algorithms outperform standard heuristic currently in use by a significant margin for maximizing proportional fairness in cellular scheduling problem.

Possible future works include modifying the framework to obtain actions from policies instead of probability values for infinite action space, and obtaining decentralized policies by introducing a message passing architecture.

References

[Agrawal and Jia 2017] Agrawal, S., and Jia, R. 2017. Optimistic posterior sampling for reinforcement learning: worst-case regret bounds. In Advances in Neural Information Processing Systems, 1184–1194.

[Altman, Avrachenkov, and Garnaev 2008] Altman, E.; Avrachenkov, K.; and Garnaev, A. 2008. Generalized α-fair resource allocation in wireless networks. In 2008 47th IEEE Conference on Decision and Control, 2414–2419. IEEE.

[Bertsekas 1995] Bertsekas, D. P. 1995. Dynamic programming and optimal control, volume 1. Athena scientific Belmont, MA.

[Bloembergen et al. 2015] Bloembergen, D.; Tuyls, K.; Hennes, D.; and Kaisers, M. 2015. Evolutionary dynamics of multi-agent learning: A survey. Journal of Artificial Intelligence Research 53:659–697.

[Bu, Li, and Ramjee 2010] Bu, T.; Li, L.; and Ramjee, R. Generalized proportional fair scheduling in third generation data networks. In Proceedings IEEE INFOCOM 2006. 25TH IEEE International Conference on Computer Communications.

[Busoniu, Babuška, and De Schutter 2010] Bušoniu, L.; Babuška, R.; and De Schutter, B. 2010. Multi-agent reinforcement learning: An overview. In Innovations in multi-agent systems and applications-I. Springer. 183–221.

[Holma and Toskala 2005] Holma, H., and Toskala, A. 2005. WCDMA for UMTS: Radio Access for Third Generation Mobile Communications. John wiley & sons.

[Jin et al. 2018] Jin, C.; Allen-Zhu, Z.; Bubeck, S.; and Jordan, M. I. 2018. Is q-learning provably efficient? In Advances in Neural Information Processing Systems, 4863–4873.

[Kwan, Leung, and Zhang 2009] Kwan, R.; Leung, C.; and Zhang, J. 2009. Proportional fair multiuser scheduling in lte. IEEE Signal Processing Letters 16(6):461–464.

[Li and Malik 2016] Li, K., and Malik, J. 2016. Learning to optimize. arXiv preprint arXiv:1606.01885.

[Li et al. 2018] Li, X.; Shankaran, R.; Orgun, M. A.; Fang, G.; and Xu, Y. 2018. Resource allocation for underlay d2d communication with proportional fairness. IEEE Transactions on Vehicular Technology 67(7):6244–6258.

[Lillicrap et al. 2015] Lillicrap, T. P.; Hunt, J. J.; Pritzel, A.; Heess, N.; Erez, T.; Tassa, Y.; Silver, D.; and Wierstra, D. 2015. Continuous control with deep reinforcement learning. arXiv preprint arXiv:1509.02971.

[Littman 1994] Littman, M. L. 1994. Markov games as a framework for multi-agent reinforcement learning. In Machine learning proceedings 1994. Elsevier. 157–163.

[Mao, Netravali, and Alizadeh 2017] Mao, H.; Netravali, R.; and Alizadeh, M. 2017. Neural adaptive video streaming with pensieve. In Proceedings of the Conference of the ACM Special Interest Group on Data Communication, 197–210. ACM.

[Margolis et al. 2016] Margolis, R.; Sridharan, A.; Aggarwal, V.; Jana, R.; Shankaranarayanan, N.; Vaishampayan, V. A.; and Zussman, G. 2016. Exploiting mobility in proportional fair cellular scheduling: Measurements and algorithms. IEEE/ACM Transactions on Networking (TON) 24(1):355–367.

[Mnih et al. 2015] Mnih, V.; Kavukcuoglu, K.; Silver, D.; Rusu, A. A.; Veness, J.; Bellemare, M. G.; Graves, A.; Riedmiller, M.; Fidjeland, A. K.; Ostrovski, G.; et al. 2015. Human-level control through deep reinforcement learning. Nature 518(7540):529.

[Nguyen, Nguyen, and Nahavandi 2019] Nguyen, T. T.; Nguyen, N. D.; and Nahavandi, S. 2019. Deep reinforcement learning for multi-agent systems: A review of challenges, solutions and applications.

[Ono and Fukumoto 1996] Ono, N., and Fukumoto, K. 1996. Multi-agent reinforcement learning: A modular approach. In Second International Conference on Multiagent Systems, 252–258.

[Osband, Russo, and Van Roy 2013] Osband, I.; Russo, D.; and Van Roy, B. 2013. (more) efficient reinforcement learning via posterior sampling. In Advances in Neural Information Processing Systems, 3003–3011.

[Perez et al. 2009] Perez, J.; Germain-Renaud, C.; Kégl, B.; and Loomis, C. 2009. Responsive elastic computing. In Proceedings of the 6th international conference industry session on Grids meets autonomic computing, 55–64. ACM.

[Pratt 1964] Pratt, J. W. 1964. Risk aversion in the small and in the large. Econometrica 32(1/2):122–136.

[Puterman 1994] Puterman, M. L. 1994. Markov decision processes: Discrete stochastic dynamic programming.

[Schulman et al. 2015] Schulman, J.; Levine, S.; Abbeel, P.; Jordan, M.; and Moritz, P. 2015. Trust region policy optimization. In International conference on machine learning, 1889–1897.

[Schulman et al. 2017] Schulman, J.; Wolski, F.; Dhariwal, P.; Radford, A.; and Klimov, O. 2017. Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347.

[Shalev-Shwartz, Shammah, and Shashua 2016] Shalev-Shwartz, S.; Shammah, S.; and Shashua, A. 2016. Safe, multi-agent, reinforcement learning for autonomous driving. arXiv preprint arXiv:1610.03295.

[Shoham, Powers, and Grenager 2003] Shoham, Y.; Powers, R.; and Grenager, T. 2003. Multi-agent reinforcement learning: a critical survey. Web manuscript.

[Sutton and Barto 2018] Sutton, R. S., and Barto, A. G. 2018. Reinforcement learning: An introduction. MIT press.
A Numerical Comparison of Performance

The performance improvement at $t = 1000$ for Algorithm 2 and Algorithm 3 over the Blind Gradient Estimation based algorithm is presented in Table 2. Table 2 also shows the decrease in performance when using the policy obtained by SARSA algorithm. We calculate the performance improvement of any algorithm as per the following equation,

$$\frac{C_{\text{algorithm}} - C_{\text{BGE}}}{|C_{\text{BGE}}|} \times 100$$

(36)

| Algorithm | $K = 2$ | $K = 4$ | $K = 4$ |
|-----------|---------|---------|---------|
| Algorithm 2 | 10.382% | 6.859%  | 16.885% |
| Algorithm 3 | 2.745%  | 7.96%   | 17.626% |
| SARSA     | -576.17%| -208.79%| -119.40%|

Table 2: Performance gaps in median fairness of RL algorithms over the BGE algorithm

B Convergence Rate Results

Figure 2 shows the convergence rates of the proposed model-based and model-free algorithms for $K = 2, K = 4$, and $K = 6$ respectively. We ran 20 independent instances of Algorithm 2 and show the median and interquartile range. We observe that the model based algorithm converges much faster than the model free algorithm.

C Applications to Fairness maximization

Many multi-agent reinforcement learning problems focus on optimizing fairness in the system. Many such examples can be found in (Zhang and Shah 2014b). We now provide an extra assumption on the fairness function, and show that the optimal policy satisfying Equation (7) is Pareto optimal.

Assumption 3. If $f$ is an element-wise monotonically strictly increasing function. Or, $\forall k \in [K]$, the function satisfies,

$$x^k > y^k \implies f(\cdots, x^k, \cdots) > f(\cdots, y^k, \cdots)$$

(37)

Element wise increasing property motivates the agents to be strategic as by increasing its per-step average reward, agent can increase the joint objective. Based on Equation (37), we notice that the solution for Equation (7) is Pareto optimal.

Definition 1. A policy $\pi^*$ is said to be Pareto optimal if and only if there exists no other policy $\pi$ such that the average per-step reward is at least as high for all agents, and strictly higher for at least one agent. Or

$$\forall k \in [K], \lambda^k_{\pi^*} \geq \lambda^k_{\pi} \text{ and } \exists k, \lambda^k_{\pi^*} > \lambda^k_{\pi}$$

(38)

Theorem 2. Solution of Equation (7), or the optimal policy $\pi^*$ is Pareto Optimal

Proof. We will prove the result using contradiction. Let $\pi^*$ be the solution of Equation (7) and not be Pareto optimal. Then there exists some policy $\pi$ for which the following equation holds,

$$\forall k \in [K], \lambda^k_{\pi^*} \geq \lambda^k_{\pi} \text{ and } \exists k, \lambda^k_{\pi^*} > \lambda^k_{\pi}$$

(39)

From element-wise monotone increasing property in Equation (37), we obtain

$$f(\cdots, \lambda^k_{\pi^*}, \cdots) > f(\cdots, \lambda^k_{\pi}, \cdots)$$

(40)

$$= \arg\max \pi f(\lambda^k_{\pi}, \cdots, \lambda^k_{\pi})$$

(41)

This is a contradiction. Hence, $\pi^*$ is a Pareto optimal solution. □

This result shows that algorithms presented in this paper can be used to optimally allocate resources among multiple agents using average per step allocations.
Figure 2: Rate of Convergence of Model Based algorithm and Model Free algorithm (Best viewed in color)