Numerical investigation of flow structures with an oblique detonation wave in a hypersonic annular cylindrical chamber

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Abstract. A new supersonic flow-type annular detonation combustor is designed in which steady oblique detonation waves in the channel are generated using a compression body in the form of a solid single-wound spiral with a constant pitch angle. A two-dimensional unsteady mathematical model of the reacting flow in this device is formulated. The flow dynamics at the start of the chamber operation and steady supersonic flow structures for a stoichiometric hydrogen-air flow with an inlet Mach number $M_0=5$ are numerically investigated. Two-dimensional numerical simulation is carried out for different spiral angles and geometrical dimensions of the chamber. A bifurcation of steady flow structures with respect to the initial condition of the problem is observed.

1. Introduction
The detonation combustion of mixtures in supersonic flows is currently considered to be a more effective method of obtaining the release of the chemical energy of fuel in scramjet engines than the widely used method based on turbulent diffusion combustion. At present, there are two different concepts of flow-type detonation chambers (DC). In the first concept, unsteady combustion of the mixture occurs in rotating transverse detonation waves (TDWs) in an annular cylindrical chamber [1, 2]. The second version uses steady oblique detonation waves (ODW) formed over the surface of the compression body [3]. In the simplest case of this concept, the DC is a rectangular channel in which at least one of the walls is a flat wedge. Each of these DC concepts has its own advantages and disadvantages.

In this paper, we propose a new design of a flow-type DC, which, in a sense, is a combination of the DC geometries described above. The chamber is an annular channel located between inner and outer cylindrical walls. The compression surface is an infinitely thin, solid, single-wound spiral that covers the entire cross-section of the annular channel from the inner to the outer wall of the DC, see Figure 1. The spiral has a constant pitch angle $\alpha$ with respect to the axis of the chamber. The width of the annular channel is considered to be much smaller than the radius of the chamber, so that the gas flow can be considered to be uniform along the radius of the DC. This makes it possible to model the flow in the DC in a two-dimensional formulation in the same manner as is usually done for a simulation of rotating DWs in annular chambers [1,2]. At the DC inlet, there is a hypersonic flow ($M_0 > M_{CJ}$) of a uniformly mixed fuel-air mixture. Here $M_0$ is the freestream Mach number and $M_{CJ}$ is the Mach number of the Chapman-Jouguet DW.
This paper presents a numerical study of the possible flow regimes with a steady ODW in the proposed annular DC with varying geometrical dimensions of the DC, spiral pitch angle $\alpha$, and initial flow parameters. Two-dimensional modeling makes it possible to determine the main structures constituting the reacting flow in the DC considered, the degree of steadiness of the flow, the mechanism of interaction of these structures with each other, and their variations with changes in the above-mentioned parameters of the problem.

2. **Governing equations and model of chemical kinetics**

The dynamics of the compressible chemically reactive medium is described by the two-dimensional Euler equations.

The chemical reaction in the DW is described according to the two-stage model of the detonation kinetics (induction period and main heat release stage) first proposed in [4]. The duration of the induction period is determined by known empirical formula [5] for hydrogen mixtures. The main heat release takes place after the induction period. The changes in internal energy and all thermodynamic parameters of the gas are described at this stage by the equations of the approximate kinetic model [6, 7]. The model is highly accurate and consistent with the second law of thermodynamics. The constants of the model have a clear physical meaning and are calculated from the tabulated thermochemical parameters of the mixture before the two-dimensional numerical calculation. This two-stage kinetic model has been used for numerical simulation of the two-dimensional cellular DW structure in hydrogen–oxygen mixtures in our previous study [8]. A good agreement has been obtained between the numerical results and experimental data on the size $a_0$ over a wide range of initial pressures and degrees of dilution with argon.

The system of governing equations was closed by the well-known thermal equation of state for an ideal gas.

3. **Numerical method**

The resultant systems of equations were solved numerically using the code based on the Godunov-type finite-volume scheme [9] with the fourth-order MUSCL TVD reconstruction [10] and the advanced HLLC algorithm [11] for an approximate solution of the Riemann problem. In implementation of HLLC algorithm for the case of a chemically reacting mixture, the “energy relaxation method” [12] was used. This method eliminates the problem of numerical solution of the Riemann problem for a medium with a complicated nonlinear equation of state (including that with a variable ratio of specific heats). Integration in time was performed with second-order accuracy by using additive semi-implicit Runge-Kutta methods [13]. The time step was determined at each time layer of the solution from the stability condition [9]. In the present simulations, the values of the Courant number were $\text{CFL}=0.3–0.4$. A uniform fixed grid was used in both directions.

At the inlet boundary $x=0$ of the solution domain, incoming unperturbed supersonic flow parameters were specified. In the external peripheral grid cells at the outlet boundary $x=L$, low counterpressure for the guaranteed critical gas flow from the DC outlet into the surrounding virtual

![Figure 1. Circuit design of the annular cylindrical chamber with single solid spiral.](image)
space were specified. At the upper and lower boundaries, periodic boundary conditions were defined at \( x \leq L_1 \) and the conditions of an impervious solid wall at \( x > L_1 \).

The code is parallelized with MPI library using the domain decomposition technique.

4. Results and Discussion
Numerical simulation was performed for a stoichiometric hydrogen-air mixture. At the DC inlet (\( x = 0 \)), the flow of the mixture was assumed to be uniform at a static pressure \( p_0 = 0.166 \text{ atm} \) and a static temperature \( T_0 = 850 \text{ K} \), and the values of \( \alpha \) and \( M_0 \) were varied. For these initial parameters of the mixtures, the Mach number \( M_{CJ} = 2.73 \). The study was started with a strongly overdriven DW as a simpler type of DW with a smooth front, since the front in this case does not have a cellular structure.

We used a sufficiently large Mach number \( M_0 = 5 \) and a large spiral angle \( \alpha = 30^\circ \). The length of the DC was chosen to be \( L = 1.5 \text{ cm} \) during preliminary calculations, and the height \( H \) of the two-dimensional

![Figure 2](image1)

**Figure 2.** Stationary structure of flow field in a detonation chamber for \( H = 1 \text{ cm} \) and \( \alpha = 30^\circ \): (a) normalized density; (b) temperature (K). Start from a uniform supersonic flow.

![Figure 3](image2)

**Figure 3.** Stationary structure of flow field in a detonation chamber for \( H = 1 \text{ cm} \) and \( \alpha = 30^\circ \): (a) normalized density; (b) temperature (K). Start from a quiescent mixture.

The solution domain was varied: \( H = 2 \pi r_c \), where \( r_c \) is the radius of the annular layer of the mixture. The beginning of the spiral was set at \( x = L_1 = 0.1 \text{ cm} \).

Figures 2 and 3 show the normalized density and temperature fields of the mixture for \( H = 1 \text{ cm} \). As can be seen from the figures, the main flow components in the DC are an overdriven oblique DW, formed over the spiral compression surface, and a rarefaction region formed on the back side of the
spiral (i.e., behind the rarefaction surface). The crimson solid line in the figures shows the boundary of the induction zone, behind which a heat release zone begins.

It is seen from Figs. 2 and 3 that for the given value of \( H \), there is a strong interaction between the ODW and the flow region behind the rarefaction surface. Investigation of the flow dynamics shows that the emerging oblique DW interacting with the classical two-dimensional rarefaction wave (RW) paradoxically initiates induction combustion in the latter. That is, detonation combustion is initiated in a flow zone with decreasing gas parameters (temperature, density, etc.). A particularly interesting finding in this study is the existence of two steady-state (on average) solutions depending on the initial conditions of the problem, which occur under the same boundary conditions, i.e., a bifurcation effect with respect to the initial solution vector. Figure 1 shows the obtained steady flow at the start \((t=0)\) from a uniform flow of the mixture \( u=(M_0c_0, 0) \), where \( u \) is the two-dimensional gas particle velocity vector and \( c_0 \) is the initial speed of sound. Physically, this start corresponds to the instantaneous placement of the compression body under study in the initial uniform supersonic flow. At subsequent times, an unsteady shock-wave pattern is formed around the investigated body in the solution domain; the final configuration is shown in Figure 2. When starting from a quiescent mixture, \( u=(0,0) \), steady flow occurs shown on Fig. 3. The start from this state corresponds to the removal of the virtual wall at the left boundary \( x=0 \) and the propagation of a plane shock wave (SW) from left to right in the solution domain and its interaction with the compression body.

It can be seen that the obtained steady-state solutions differ greatly from each other in the size, shape, and structure of the flow and the values of the RW parameters. In the case of Fig. 2, the first characteristic of the two-dimensional RW is almost immediately followed by an oblique shock wave with a jump in the gas parameters which then decrease toward the DC outlet. Penetration of this jump into the detonation products behind the ODW causes a jump in the parameters in the energy release region \( 1 \leq x \leq 1.5 \) cm (see Fig. 1). When starting from the state of rest, we have a 2D rarefaction wave behind the spiral surface, which is perturbed by compression waves, and the gas parameters in this flow region continuously (though not monotonically) decrease toward the DC outlet. In the detonation products behind the ODW, we have a region of rapidly decreasing flow parameters (see Fig. 3).

There is also a significant difference in gas temperature in the reverse flow region, cf. Figs. 2 and 3 at \( y \approx 1.3 \). The more uniform and low-temperature region behind the rarefaction surface is obtained in the case of starting from \( u=(0,0) \). Before the interaction with the RW, the flow in the ODW has no differences.

Thus, the flow region behind the rarefaction surface of the spiral has a complex structure with unsteady turbulent shear layers, reverse flow zones, and combustion regions. These flow elements are essentially unsteady, but, on the average over time, this region is fairly stable in terms of its shape and the average values of the gas parameters in it. The differences in the obtained steady-state (on average)
solutions are due to the difference between the SW and the RW formed during the transient process, which interact differently with each other at the start from different initial conditions. Investigation of the time stability of the obtained two solutions and the dependence of the 2D modeling results on the difference grid resolutions used shows that these structures are steady (on average) and do not transform into one another or degenerate into any third similar flow configuration.

The flow structure is found to change suddenly as the parameter \( H \) is changed. Figure 4 gives the steady field of the normalized flow density for \( H=1.5 \) cm and the same values of the other parameters as previously. Figure 4a corresponds to the start from a uniform flow, and Fig. 4b to the start from the state of rest. It is seen that, in both cases, there is no interaction between the RW and the smooth oblique DW. In the first case, we have a classical smooth two-dimensional RW behind the back side of the spiral and combustion is completely absent. In the case of starting from \( u=(0,0) \), the RW has small perturbations and a small stagnation zone with detonation products remaining from the preceding nonstationary transient process. It can be said that with decreasing interaction between the flow components, the bifurcation effect on the initial solution vector is extremely weak.

Next, the flow structure was studied at smaller pitch angles of the spiral. The dimensions of the solution domain were considerably increased due to a decrease in the temperature of the gas in the induction zone with decreasing angle \( \alpha \) and due to the exponential dependence of the length of the induction zone on temperature. Figures 5 and 6 show the numerical Schlieren visualization of the obtained steady flow fields for \( \alpha=13.4^\circ \) and \( M_0=5 \). The calculation for \( H=20 \) cm and \( L=60 \) cm is presented in Fig. 5, and that for \( H=35 \) cm and \( L=50 \) cm in Fig. 6; \( L_1=0.5 \) cm in both cases. In these figures, we have the classical \( \lambda \)-structure of the formation of an oblique DW previously observed in the calculation of DWs on a wedge and cone [3]. Digits 1 and 2 denote two jumps in the induction zone, which, sequentially interact with the leading shock front to form a steady oblique DW over the compression surface of the annular spiral. In both cases, we have a classical two-dimensional RW behind the rarefaction surface of the spiral and there are no perturbations or combustion zones in the RW. As can be seen in Fig. 5, the geometry of the solution domain allows a strong interaction between the ODW and the RW. However, careful calculations show that, for this geometry, there is no bifurcation of the steady-state solution for the initial state. Thus, the bifurcation effect can be completely eliminated by decreasing the spiral angle and hence the intensity of the oblique SW and the RW during the transient process. In the case of Fig. 6, the geometry does not allow interaction between the flow components, and we have an isolated ODW and an unperturbed two-dimensional RW. In this case, the bifurcation effect was also not observed.

Figure 5. Stationary structure of flow field in a detonation chamber for \( H=20 \) cm. Figure 6. Stationary structure of flow field in a detonation chamber for \( H=35 \).
In the case of Fig. 5, we have an almost ideal situation in which the geometrical parameters of the proposed annular DC and the spiral angle are selected so that for an inlet flow Mach number $M_i=5$, almost all of the reacting mixture supplied to the DC is burned in the ODW.

5. Concluding Remarks
A two-dimensional unsteady mathematical model was formulated for the flow in a supersonic flow-type annular detonation chamber with a compression body in the form of a solid single-wound spiral with a constant pitch angle $\alpha$ with respect to the axis of the chamber.

The flow dynamics at the start of the chamber was studied numerically, and steady-state solutions were obtained for the supersonic flow of a stoichiometric hydrogen-air mixture with an inlet Mach number $M_i=5$ in the chamber. The angle of the spiral and the geometrical dimensions of the chamber were varied.

It is found that the main components of the steady flow in the chamber are an oblique overdriven detonation wave and a two-dimensional rarefaction wave. For $\alpha=30^\circ$, a bifurcation of the solution with respect to the initial condition problem was observed. The existence of this effect was shown to depend on both the intensity of these two main flow components and the geometry of their interaction with each other.

With decreasing spiral angle, an oblique detonation with the classical $\lambda$-structure was obtained. The calculations for $\alpha=13.4^\circ$, $H=20$ cm, and $L=60$ cm resulted in a quite efficient detonation combustor configuration of the proposed design.

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