Scattering unitarity with effective dimension-6 operators

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Abstract

The effects of physics beyond the Standard Model may be parametrized by a set of higher-dimensional operators leading to an effective theory. The introduction of these operators makes the theory nonrenormalizable, and one may reasonably expect a violation of unitarity in $2 \to 2$ scattering processes, depending on the values of the Wilson coefficients of the higher dimensional operators. Bounds on these coefficients may be obtained from demanding that there be no such unitarity violation below the scale of the effective theory. We show, at the lowest level, how the new operators affect the scattering amplitudes with longitudinal gauge bosons, scalars, and $t\bar{t}$ in the final state, and find that one may expect a violation of unitarity even at the LHC energies with small values of some of the new Wilson coefficients. For most of the others, such a violation needs large coefficients, indicating nonperturbative physics for the ultraviolet-complete theory, although a proper treatment necessitates the inclusion of even higher-dimensional operators. However, deviations from the Standard Model expectations may be observed with even smaller values for these coefficients. We find that $WW \to WW$, $WW \to ZZ$, and $ZZ \to hh$ scatterings are the best possible channels to probe unitarity violations.

1 Introduction

Even after a few years of running at the energy frontier, the Large Hadron Collider (LHC) is yet to give us any direct evidence of new physics (NP) beyond the Standard Model (SM), except for occasional will-o’-the-wisp signals drifting in and flickering out of existence. On the other hand, we have more than enough reasons to believe that NP exists in some form or other. This leads us to parametrize the NP in a model-independent way in terms of effective higher-dimensional operators, assuming all new degrees of freedom to be sufficiently heavy. A well-used tool for low-energy physics, like Chiral Perturbation Theory or Heavy Quark Effective Theory, this has also become a powerful weapon at the LHC energies. Use of such effective field theories (EFT) to parametrize NP effects was first demonstrated in the seminal work of Ref. [1]. While such theories are in general not renormalizable and hence cannot possibly be the ultimate ultraviolet-complete theory, higher dimensional operators can generate new tree-level interactions, which might include the effects of some hitherto unknown heavy degrees of freedom.

It is important to have a complete basis of gauge and Lorentz-invariant operators of any given dimension; fortunately, this is well-known for the SM (i.e., when all NP fields are integrated out) \(^2\). A minimal set of 59 dimension-6 operators was given in Ref. [6] and later confirmed in other works [7, 8]. The importance of identifying and choosing a proper basis for the effective operators has recently been emphasized in Ref. [9]. Refs. [10, 11] show a practical way to construct and use the SM EFTs. However, only a subset of the higher-dimensional operators is relevant to study a particular process. For example, effects of the new operators on Higgs physics were studied, with a subset of all the dimension-6 operators, in Refs. [12, 13, 14, 15, 16, 17]. Experiments can constrain the Wilson coefficients of the higher dimensional operators, either directly or indirectly [18, 19, 20, 21, 22, 23, 24]. Among other...
interesting uses of the EFT formalism, one can mention the attempt to address the naturalness issue of the Higgs boson mass [25].

There are many equivalent bases to write a complete set of operators (e.g., see [26]). We will use the Strongly Interacting Light Higgs (SILH) basis, as shown in [27, 28], but work with only those operators that are relevant for the scattering processes. As our aim is to look at possible unitarity violations in $2 \rightarrow 2$ scattering processes when such higher dimensional operators are present, we would prefer a basis that couples the Higgs sector strongly with the NP [3]. Let $\Lambda$ be the scale where the new degrees of freedom show up, so this will act as the cutoff for the effective theory. We will keep $\Lambda$ a free parameter, as the Wilson coefficients (WC) scale trivially with $\Lambda^2$. We emphasize that setting $\Lambda = 1$ TeV as a fiducial mark does not necessarily mean that one must observe NP effects beyond that, say at the LHC.

The four-point vertices coming from the dimension-6 operators generally contain a prefactor of $c_i v^2/\Lambda^2$, where $c_i$ is the respective WC, and $v$ is the vacuum expectation value (VEV) of the Higgs field. Thus, the cancellation of the bad high-energy behaviour is affected. It is affected even more if the prefactor contains momentum dependence. However, we will always keep terms of the order of $1/\Lambda^2$. Going beyond that would necessitate the consideration of even higher-dimensional operators. This is to be kept in mind as the new operators will manifest themselves in two different ways, which we discuss below.

The scattering of longitudinally polarized gauge bosons to two-particle final states gave, perhaps, the strongest motivation to have a Higgs sector in the SM [29], because without the Higgs field the scattering amplitudes tend to violate unitarity at high enough energies. In the SM, the bad high-energy behaviour of the scattering amplitudes is completely tamed because of the precise gauge structure and the Higgs mechanism. With effective operators, one may re-introduce the bad behaviour because such cancellations no longer hold, unless the cutoff scale $\Lambda$ is extremely large. This puts a bound on the effective WCs, originating from the fact that the magnitude of real part of any partial wave amplitude must be less than $\frac{1}{2}$. A study somewhat similar in approach, assuming that the SM vertices may allow some deviations from their predicted values, through possibly higher-order interactions, was undertaken in Refs. [30, 31].

The new dim-6 operators contribute in two ways to $2 \rightarrow 2$ scattering processes. First, they contribute to the scattering vertices in a non-trivial way, whose examples we will give later. They can change the vertex factors by terms typically going as $v^2/\Lambda^2$. They can also introduce momentum dependence to the erstwhile momentum-independent vertex factors. The second point is not very obvious and is often neglected in the literature. The new operators can also modify the kinetic terms in the Lagrangian; for example, an operator of the form $(c/\Lambda^2)W^{\mu\nu}W_{\mu\nu}\Phi^\dagger\Phi$, where $\Phi$ is the SM scalar doublet, produces an extra contribution of $cv^2/2\Lambda^2$ to the kinetic term. If $v \ll \Lambda$, this correction is negligible, but in principle this affects the scattering amplitudes by redefining the fields, as well as modifying the corresponding vertex factors for the relevant Feynman diagrams. The Higgs VEV gets modified, and so do the masses of the gauge bosons and fermions where the Higgs VEV is fed. The WCs of some of the field-redefining operators, as we will see, can be tightly constrained from electroweak precision observables. A similar study in another choice of basis was performed in Ref. [32], but without the proper normalization of the kinetic terms.

The effects of NP on the partial wave amplitudes are expected to be small, suppressed typically by $v^2/\Lambda^2$ unless there is some momentum-dependent enhancement. However, $\Lambda$ may very well be within a few TeV, as predicted by most of the NP theories. In fact, if one takes the cutoff scale to be $\Lambda \sim \mathcal{O}(1)$ TeV, even at the LHC energies one may observe violations of the unitarity bound with small to moderate WC. To get a meaningful estimate, one must not go beyond the small-WC limit, if $d > 6$ operators are not taken into account.

In this paper, we will try to see how the scattering amplitudes (the zero-th partial wave, to be more precise) behave with the introduction of these new operators, and when we may expect a violation of

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3 Any other basis could have been used too. The choice of the most effective basis will also be governed by the observables the experiments measure. However, one has to be careful of not introducing redundant operators.
unitarity. As expected, if $\Lambda \to \infty$, all the amplitudes will be well-behaved. We will show that if the NP is indeed at the electroweak scale, one may expect, at the LHC, a violation of unitarity even with perturbative new couplings. A similar study will be applicable to the future $e^+e^-$ colliders too, and in a much cleaner environment. Effects of the dimension-6 operators in hadronic [33, 34] and leptonic colliders [35, 36] have already been discussed in the literature, and we will comment on their bounds later. Another interesting point from the collider perspective that we will not discuss is the introduction of new three- and four-point interactions from the effective operators, as shown in the Appendix. They can in principle affect processes like Higgs production from gluon fusion, or its decay to a photon and a $Z$.

In Section II, we enlist the set of operators that might be interesting to study unitarity violation. We also illustrate how one properly normalizes the fields. We discuss two sets of operators, bosonic and fermionic; the former involves only bosonic fields and the latter involves fermionic fields too. While the bosonic operators can contribute even to $V_L V_L \to t\bar{t}$ scattering (where $V_L$ is any generic longitudinally polarized gauge boson) amplitudes, fermionic operators can never contribute to bosonic final states. The bounds on the corresponding WCs are shown in Section III. As expected, $W_L W_L \to W_L W_L$, $W_L W_L \to Z_L Z_L$, and $Z_L Z_L \to hh$ scatterings almost invariably put the strongest bound to whichever operators it gets a contribution from. We summarize and conclude in Section IV, and relegate some detailed calculation, including that of the modified vertex factors, to the Appendix.

2 Formalism

2.1 The effective Lagrangian

For any scattering we can decompose the amplitude into partial waves

$$A = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\cos \theta) a_\ell,$$  (1)

and by virtue of the optical theorem which relates the cross-section with the imaginary part of the amplitude for zero scattering angle, one gets \(^4\)

$$|a_\ell|^2 = \text{Im} a_\ell \Rightarrow \text{Re} a_\ell \leq \frac{1}{2}.$$  (2)

We will be interested in $\ell = 0$ partial waves only.

We start with the set of SILH operators as given in Ref. [28], and follow their notation and convention. In particular, $\tau_i$ denotes any generic WC, and the operator that comes with $\tau_i/\Lambda^2$ is denoted by $O_i$. Let us first pick up only those operators that lead to $2 \to 2$ bosonic scatterings, and set the generic cut-off scale at some high momentum $\Lambda$. Our fiducial marker is at $\Lambda = 1$ TeV but the WCs scale with $\Lambda^2$, so the actual bound on any generic WC should be read as $\tau_i (\Lambda/1 \text{ TeV})^2$. We will show our results for $\sqrt{s} = 2$ TeV, the typical parton-level energy at the LHC, but this does not mean that we are necessarily in the new physics regime, as already emphasized. The values of $\tau_i$ for $\Lambda = 1$ TeV are denoted by $C_i$. We confine ourselves to $|C_i| < 1$ so that $d > 6$ operators can be neglected.

\(^4\)To treat all possible spins of incoming and outgoing particles, one should use Wigner's $D$-functions, but $D_{00}^\ell$ is directly related with $a_0$. See Ref. [31] for details.
For bosonic scatterings, the relevant terms are

$$\mathcal{L}_{\text{boson}} = \frac{1}{\Lambda^2} \sum_i \bar{c}_i O_i = \frac{\bar{c}_H}{\Lambda^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{\bar{c}_T}{\Lambda^2} (\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi)(\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi) - \frac{\bar{c}_6}{\Lambda^2} (\Phi^\dagger \Phi)^3$$

$$+ \frac{ig\bar{c}_W}{2\Lambda^2} (\Phi^\dagger \tau^i \stackrel{\leftrightarrow}{D}_\mu \Phi)(\Phi^\dagger D^\nu W_{\mu\nu})^i + \frac{ig'\bar{c}_B}{2\Lambda^2} (\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi)(\partial^\nu B_{\mu\nu}) + \frac{ig\bar{c}_HW}{2\Lambda^2} (D^\mu \Phi)^\dagger \tau^i (D^\nu \Phi) W^i_{\mu\nu} + \frac{ig'\bar{c}_HB}{2\Lambda^2} (D^\mu \Phi)^\dagger (D^\nu \Phi) B_{\mu\nu}$$

$$+ \frac{g^2\bar{c}_3}{\Lambda^2} (\Phi^\dagger \Phi) B_{\mu\nu} B^{\mu\nu} + \frac{g_3^2\bar{c}_g}{\Lambda^2} (\Phi^\dagger \Phi) G^a_{\mu\nu} G^{a\mu\nu} + \frac{g^3\bar{c}_{3W}}{\Lambda^2} \epsilon_{ijk} W^{i\mu} W^{j\alpha} W^{k\mu\alpha},$$

(3)

where $\tau_i$ are the Pauli matrices, $g'$, $g$, and $g_3$ are the $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ gauge couplings respectively, and $\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi = \Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi$. The gluon operator $O_g$ will not be relevant for us as we do not consider final states involving gluons, but this will lead to a direct production of the Higgs boson from gluon fusion. These operators can contribute to $W_L W_L (Z_L Z_L) \to t\bar{t}$ too, with the fermionic vertex being SM-like and the bosonic vertex involving the contributions from the dimension-6 operators.\footnote{Only top quark, because of its mass, can contribute to the $\ell = 0$ channel. The other fermionic final states occur at higher $\ell$ and therefore give a much weaker constraint [32]. That is why we do not consider $qf \to VV$ scattering where the $q$ and $f$ come from the initial protons. This is also true for the fermionic dimension-6 operators.}

Note that there are other operators involving three gauge tensors but the last one, $O_{3W}$, is the only one that contributes [32]. At the same time, $O_{3W}$ is not produced at the tree-level [12] and hence $\bar{c}_{3W}$ is expected to have a further loop suppression.

The relevant fermionic operators are

$$\mathcal{L}_{\text{fermion}} = \frac{1}{\Lambda^2} \sum_j \bar{c}_j O_j = \left( \frac{\bar{c}_6}{\Lambda^2} y_u Q_L u_R \Phi^c \Phi^\dagger \Phi + \frac{\bar{c}_H}{\Lambda^2} (\bar{u}_R \gamma^\mu d_R)(\Phi^c \stackrel{\leftrightarrow}{D}_\mu \Phi) + \text{h.c.} \right)$$

$$+ \frac{i\bar{c}_H q}{\Lambda^2} (\bar{Q}_L \gamma^\mu Q_L)(\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi) + \frac{i\bar{c}_H d}{\Lambda^2} (\bar{Q}_L \tau^i \gamma^\mu Q_L)(\Phi^\dagger \tau^i \stackrel{\leftrightarrow}{D}_\mu \Phi)$$

$$+ \frac{i\bar{c}_H u}{\Lambda^2} (\bar{u}_R \gamma^\mu u_R)(\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi) + \frac{i\bar{c}_H d}{\Lambda^2} (\bar{d}_R \tau^i \gamma^\mu d_R)(\Phi^\dagger \tau^i \stackrel{\leftrightarrow}{D}_\mu \Phi)$$

$$+ \frac{g\bar{c}_W}{\Lambda^2} y_u \bar{Q}_L \Phi^c \sigma_{\mu\nu} u_R B^{\mu\nu} + \frac{g\bar{c}_W}{\Lambda^2} y_d \bar{Q}_L \Phi^c \sigma_{\mu\nu} d_R B^{\mu\nu} + \frac{g\bar{c}_W}{\Lambda^2} y_d \bar{Q}_L \Phi^c \sigma_{\mu\nu} d_R W^{i\mu\nu},$$

(4)

where we have neglected operators involving lepton and gluon fields. The Feynman rules involving the new operators are shown in Appendix A, while the detailed structure of one of the operators is shown in Appendix B.

2.2 Field redefinition

Let us spend some time on the field redefinition here, to bring the kinetic terms to their canonical form. From the expression of $O_W$ in Eq. (B.1), one finds that the following terms

$$O_W \supset \frac{g^2}{4} v^2 \left[ (\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu})(\partial_\mu W^{-\nu} - \partial_\nu W^{-\mu}) \right]$$

$$+ \frac{g^2}{4 \cos \theta_W} v^2 \left[ - \cos \theta_W \partial^\nu Z^\mu (\partial_\mu Z_\nu - \partial_\nu Z_\mu) - \sin \theta_W \partial^\nu Z^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) \right]$$

(5)
contribute to the 2-point functions. Similar contributions come from other operators also. All the contributions to the 2-point functions from the operators can be summed up as

\[
\mathcal{L}_{\text{2point}} = \frac{\bar{\tau}_H v^2}{\Lambda^2} (\partial_\mu h)(\partial^\mu h) + g^2 v^2 \frac{\bar{c}_G}{2\Lambda^2} G^\mu_\nu G^{\mu\nu} + g^2 v^2 \frac{\bar{c}_W}{4\Lambda^2} W^-_{\mu\nu} W^{+\mu\nu} + g^2 \sin^2 \theta_W v^2 \frac{\bar{c}_\gamma}{2\Lambda^2} Z^-_{\mu\nu} Z^{\mu\nu} \\
+ g^2 v^2 \frac{\bar{c}_B}{8\Lambda^2} Z^-_{\mu\nu} Z^{\mu\nu} + g^2 v^2 \frac{\bar{c}_W}{8\Lambda^2} Z_{\mu\nu} Z^{\mu\nu} + g^2 \sin^2 \theta_W v^2 \frac{\bar{c}_B}{2\Lambda^2} A^-_{\mu\nu} A^{\mu\nu} \\
- gg' \sin^2 \theta_W v^2 \frac{\bar{c}_W}{8\Lambda^2} A_{\mu\nu} Z^{\mu\nu} - gg' v^2 \frac{\bar{c}_B}{8\Lambda^2} A_{\mu\nu} Z^{\mu\nu}. \tag{6}
\]

With this, one should add the canonical kinetic terms, which gives

\[
\mathcal{L}_{\text{kinetic}} = \frac{1}{2} (\partial_\mu \bar{h})(\partial^\mu h) - \frac{1}{4} G^a_\mu \bar{G}^{a\mu\nu} - \frac{1}{2} W^-_{\mu\nu} W^{+\mu\nu} - \frac{1}{4} Z^-_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} \bar{A}_{\mu\nu} A^{\mu\nu} \tag{7}
\]

where

\[
\bar{h} = \sqrt{1 + 2 \frac{\bar{\tau}_H v^2}{\Lambda^2}} h \equiv \sqrt{N_h} h, \\
\bar{c}^a_\mu = \sqrt{1 - 2 g_s^2 v^2 \bar{c}_G} G^a_\mu \equiv \sqrt{N_G} G^a_\mu, \\
W^\pm_\mu = \sqrt{1 - g^2 v^2 \bar{c}_W} W^\pm_\mu \equiv \sqrt{N_W} W^\pm_\mu, \\
Z_\mu = \sqrt{1 - 2 g^2 \sin^2 \theta_W v^2 \bar{c}_\gamma} Z_\mu \equiv \sqrt{N_Z} Z_\mu, \\
\bar{A}_\mu = \left[ 1 - g^2 \sin^2 \theta_W v^2 \bar{c}_W \right] A_\mu + \left[ 2 g g' \sin^2 \theta_W v^2 \bar{c}_B + gg' v^2 \bar{c}_W \right] Z_\mu \\
\equiv N_A A_\mu + N_{AZ} Z_\mu. \tag{8}
\]

This gives the field redefinitions; also, this shows that one may not extend the \( \bar{\tau}_i \)'s beyond their range of validity, given by

\[
\bar{\tau}_W < 83 \left( \frac{\Lambda}{1 \text{ TeV}} \right)^2, \quad \bar{\tau}_B < 276 \left( \frac{\Lambda}{1 \text{ TeV}} \right)^2, \quad \bar{\tau}_\gamma < 179 \left( \frac{\Lambda}{1 \text{ TeV}} \right)^2. \tag{9}
\]

This, again, naively assumes that there are no other operators of mass dimension greater than 6, and therefore one does not need to take these constraints too seriously. Stronger constraints come from electroweak precision observables, like the \( \rho \)-parameter, as we will soon show.

If \( \bar{\tau}_i v^2 / \Lambda^2 \ll 1 \), one can invert these relations by a binomial expansion and obtain

\[
\begin{align*}
    h & \rightarrow h\left[ 1 - \frac{\bar{\tau}_H v^2}{\Lambda^2} \right], \\
    G^a_\mu & \rightarrow G^a_\mu\left[ 1 + \frac{\bar{c}_G}{\Lambda^2} g_s^2 v^2 \right], \\
    W^\pm_\mu & \rightarrow W^\pm_\mu\left[ 1 + \frac{\bar{c}_W}{\Lambda^2} g^2 v^2 \right], \\
    Z_\mu & \rightarrow Z_\mu\left[ 1 + \frac{\bar{c}_\gamma}{\Lambda^2} g^2 \sin^2 \theta_W v^2 + \frac{\bar{c}_W}{\Lambda^2} g^2 v^2 + \frac{\bar{c}_B}{4\Lambda^2} g^2 v^2 \right], \\
    A_\mu & \rightarrow A_\mu\left[ 1 + \frac{\bar{c}_\gamma}{\Lambda^2} g^2 \sin^2 \theta_W v^2 \right] + Z_\mu \frac{\bar{c}_W - \bar{c}_B - 8\bar{c}_\gamma}{\Lambda^2} \sin^2 \theta_W g g' v^2. \tag{10}
\end{align*}
\]

Note that the binomial expansion is valid only in the proper limit. For numerical evaluations, we work with the exact definitions. However, the vertex factors of the effective theory depend on the WCs,
and in the limit when binomial expansion fails, they become non-perturbative and so large as to make the higher-order effects more important than the tree-level ones. Fortunately, the possible unitarity violations, in at least one channel, occur much before that range.

The expressions for the particle masses, except for the photon which remains massless because of unbroken electromagnetism, are also modified and can be read off from the bilinear term. The couplings now depend on the higher-dimensional WCs, so that the particle masses are tuned to their experimental values:

\[
\begin{align*}
m_h^2 &= \left[1 - 2v^2\equiv H^2 \Lambda^2\right]\left(3\lambda v^2 - \mu^2\right) + \frac{15}{4} \lambda v^4 \tau_6 \Lambda^2, \\
m_W^2 &= \frac{g^2v'^2}{4} \left[1 + g^2v'^2 \tau_W \frac{2\Lambda^2}{2\Lambda^2}\right], \\
m_Z^2 &= \frac{g^2v'^2}{4 \cos^2\theta_W} \left[1 + g^2v'^2 \tau_W \frac{2\Lambda^2}{2\Lambda^2} + g^2v'^2 \tau_B \frac{2\Lambda^2}{2\Lambda^2} + 2g^2v'^2 \sin^2\theta_W \frac{\tau_1 \Lambda^2}{2\Lambda^2} - 2v'^2 \tau_T \frac{\Lambda^2}{2\Lambda^2}\right].
\end{align*}
\]

Here, \(v'\) is the modified Higgs VEV which follows from the redefinition \(m_h^2 = 3\lambda v'^2 - \mu'^2\), where \(v'\) and \(\mu'\) contain the effects of the operators \(O_6\) and \(O_H\). Note that \(O_W\) alone does not affect the tree-level \(\rho\) parameter. The other operators do, and from \(T - T_{SM} = 0.08 \pm 0.12\) where \(\rho = 1 + \alpha T\), one gets

\[-0.44 < c_B < 0.08, \quad -0.48 < c_\gamma < 0.09, \quad -2.5 \times 10^{-3} < c_T < 0.013,\]

where \(c_i = \bar{c}_i(A/1 \text{ TeV})^2\). While these bounds are somewhat stronger than the Froissart bounds for the WCs, the deviation of the cross-section from the SM expectations should be observable at about these values, or even less, as we will see later. This is why we do not talk about the precision observable bounds any further.

One now has to write down the scattering amplitudes not only involving the dimension-6 terms of the effective Lagrangian, but also in terms of the normalized fields. This in turn means that even the SM vertices as well as the propagators are modified and become functions of the WCs. These vertex factors are enlisted in Appendix A. At the same time, we keep ourselves confined to such small values of \(c_i/\Lambda^2\) that only the term linear in \(c_i/\Lambda^2\) is sufficient. One may ask whether we need to take into account the field normalizations in that case. The answer is yes, as such lowest-order corrections appear even when the SM amplitude is calculated with the normalized fields. Sometimes the corrections coming from the vertices are cancelled or enhanced by a similar correction coming from the fields.

We will discuss only about those scatterings that can be observed either by the Large Hadron Collider (LHC) or the next generation International Linear Collider (ILC). They include \(WW \to WW\), \(WW \to ZZ\), \(ZZ \to ZZ\), \(WW \to hh\), \(ZZ \to hh\), \(WW \to t\bar{t}\), and \(ZZ \to t\bar{t}\), with crossed channels included wherever necessary, and the longitudinal mode is implied for the gauge bosons. As we will see, only bosonic scatterings produce any useful constraints.

### 3 Bounds on the Wilson coefficients

To get the bounds on the WCs, we fix \(\sqrt{s} = 2\) TeV, which is a typical parton-level value for the proton-proton collision at the LHC with \(\sqrt{s} = 13\) or 14 TeV. We use FeynArts/FormCalc [37, 38] to calculate the helicity amplitudes using the FeynArts model files for the effective Lagrangian generated by FeynRules [39]. This gives us all the field normalizations as well as the vertex factors. We then observe how the zero-th partial wave amplitude, \(a_0\), varies with the WCs; the bound comes from \(|a_0| \leq \frac{1}{2}\). For \(WW(ZZ) \to t\bar{t}\), we use the helicity amplitude for \(00 \rightarrow ++\) as this gives the tightest constraints.

In general, such operators also contribute to the higher-\(\ell\) states. However, as has been shown in Ref. [32, 31], such constraints are always weaker than those coming from \(\ell = 0\). An intuitive way to
Table 1: Dimension-6 operators affecting the bosonic and fermionic scatterings. The entries marked with √ are affected by the modification of the SM vertices. The entries marked with ⊗ are affected by the wavefunction normalization.

| PROCESS     | $O_6$ | $O_T$ | $O_H$ | $O_γ$ | $O_{HW}$ | $O_B$ | $O_{HB}$ | $O_{3W}$ |
|-------------|-------|-------|-------|-------|----------|-------|----------|----------|
| $WW \rightarrow WW$ | ⊗     | ⊗     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |
| $WW \rightarrow ZZ$ | √     | ⊗     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |
| $ZZ \rightarrow ZZ$ | √     | ⊗     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |
| $WW \rightarrow hh$ | √     | √     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |
| $ZZ \rightarrow hh$ | √     | √     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |
| $WW \rightarrow t \bar{t}$ | ⊗     | ⊗     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |
| $ZZ \rightarrow t \bar{t}$ | √     | √     | ⊚     | ⊚     | ⊚        | ⊚     | ⊚        | ⊚        |

Figure 1: The unitarity limits on the effective Wilson coefficients, where they violate the bound $|\text{Re}a_0| \leq \frac{1}{2}$. We have taken $\sqrt{s} = 2$ TeV and $\Lambda = 1$ TeV, the coefficients scale with $\Lambda^2$. 

Figure 7
understand this is that the worst high-energy behaviour of $\ell = 0$ partial wave goes as $s/m^2$ whereas it is $\sqrt{s}/m^2$ for $\ell = 1$. We have found no $\ell = 1$ constraints that are either stronger than the same coming from $\ell = 0$, or lie in the perturbative domain.

Figure 2: The unitarity limits on the WCs, assuming three of the $d = 6$ operators are generated at the same time, with $C_{HB}$ and $C_{HW}$. Only the lower left white portion is allowed.

Note that only one operator is taken to be nonzero at a time. One may ask whether this is a reasonable assumption, given that almost all the NP models necessarily generate more than one effective operators, if not the (almost) full set. Even with two such operators, the deviation of the scattering cross-section from the SM expectation can set in either before or after the single-operator mark, depending on the signs of the WCs. As a toy example, we show, in Fig. 2, the unitarity bound on $\bar{c}_W$ and $\bar{c}_{HB} = \bar{c}_{HW}$, where all the three operators are present, but the WCs for $O_{HB}$ and $O_{HW}$ are taken to be the same for simplicity. Thus, (i) one either has to know the ultraviolet complete theory, construct the effective operators, and then study the scattering sensitivities, or (ii) perform a complete scanning over the entire 16-dimensional parameter space. None of them is a viable option. In other words, this study may be useful to find the pattern of nonzero WCs if deviations are seen in several channels and are quantified.

Only the worst high-energy behaviour is important; thus, if there are terms going as $s^2$ and $s$ in the amplitude, we consider only the $s^2$ term. Again, note that the cutoff scale $\Lambda$ has been fixed at 1 TeV just as a fiducial mark and has nothing to do with the actual onset of NP.

In Table 1, we show which operators affect which $2 \rightarrow 2$ scattering processes. The notation is self-explanatory; the WC, $\bar{c}_i$, accompanies the operator $O_i$ in Eqs. (3) and (4). In Fig. 1, we show the bounds on the corresponding Wilson coefficients of the bosonic operators. We show only those operators for which one gets an interesting bound that can be probed at the LHC; thus, $O_6$, $O_H$, and $O_3W$ have been dropped, as they do not violate the unitarity bound for $\sqrt{s} = 2$ TeV. They would do so if we considered $2 \rightarrow n$ scattering processes but the chances of observing them at the LHC is negligible. None of the fermionic operators turn out to be interesting; this is also corroborated by Eq. (28) of Ref. [32]. The corresponding Table 2 shows the point where the Froissart bound is reached. We emphasize again that anomalous behaviour of the scattering cross-section should be observable way before this bound is reached.

One may note that some of the coefficients, like $\bar{c}_W$, $\bar{c}_\gamma$, $\bar{c}_{HB}$, $\bar{c}_{HW}$, and $\bar{c}_B$, violate the unitarity bound even for relatively smaller values, and increasing $\Lambda$ by a factor of 5 or 10 will still keep them in
| WC   | Bound | Process  | WC   | Bound | Process |
|------|-------|----------|------|-------|---------|
| $\tau_W$ | 0.06  | $WW \rightarrow hh$ | $\tau_B$ | 0.27  | $ZZ \rightarrow hh$ |
| $\tau_{HB}$ | 0.27  | $ZZ \rightarrow hh$ | $\tau_\gamma$ | 0.14  | $ZZ \rightarrow ZZ$ |
| $\tau_{HW}$ | 0.06  | $WW \rightarrow hh$ | $\tau_T$ | 1.2   | $ZZ \rightarrow hh$ |

Table 2: The limit on the Wilson coefficients, with $\Lambda = 1$ TeV. They scale with $\Lambda^2$. Only bounds below $\tau_i \leq 1.2$ are shown. Gauge boson polarizations are longitudinal.

The perturbative domain \(^6\). At the same time, we would like to mention that the anomalous behaviour of the scattering amplitudes should be observable much before the unitarity bound is reached, and therefore one may surmise the presence of NP for even lower values of the coefficients. Also note that the actual coefficient of $O_{3W}$, apart from the loop suppression mentioned before, should be much larger than that quoted in the Table.

The values of the WCs where unitarity bound is reached decrease with increasing $\sqrt{s}$. While our results are displayed for $\sqrt{s} = 2$ TeV keeping in mind the maximum partonic centre-of-mass energy to be had at 14 TeV LHC, we also show how the limits change with $\sqrt{s}$ in Fig. 3. Among the six couplings shown in Table 2, the variation of $\tau_{HW}$ and $\tau_{HB}$ are identical to that of $\tau_W$ and $\tau_B$ respectively.

![Figure 3](image-url)  

Figure 3: The variation of $|C_i|_{\text{max}}$, the value where the unitarity limit is reached, as a function of $\sqrt{s}$. The variation of $C_{HW}$ and $C_{HB}$ are identical to that of $C_W$ and $C_B$ respectively.

As we have spent some time on the issue of wavefunction normalization, one may ask how significant the corrections are. As can be seen from Table 1 and Fig. 1, such corrections can indeed put a strong constraint, \(e.g.,\) on $\tau_B$ and $\tau_\gamma$ from $WW \rightarrow WW$ scattering. This is because in the amplitude, the field normalization factors appear at the same order of $1/\Lambda$ as the vertex corrections.

There are three points that we would like to mention here.

1. These bounds are valid if only one operator is present at a time. This may not be the case for the particular NP model at hand and there is always the chance of a cancellation, either numerical accident or motivated by the theory, in which case the bounds will be strengthened. They can, in principle, also be relaxed. One may, in principle, also get stronger bounds if coupled-channel final states are considered.

\(^6\)By this, we mean that an ultraviolet-complete theory with perturbative couplings can generate such WCs at the low scale. However, we do not consider any possible running of these coefficients.
considered.

2. The amplitudes start deviating from their SM values much before the $|a_0| = \frac{1}{2}$ bound is reached. Roughly speaking, the amplitudes increase by a factor of 2 over the SM values (which means a fourfold increase in the number of final state particles) when the WCs are about one order below their unitarity bound. Therefore, a precision measurement can unveil any such new physics much before the unitarity bound is reached. In other words, even those operators whose WCs have to be large to hit the unitarity bound (like $O_T$ or $O_{3W}$) — or in other words, whose effects are going to be small if the couplings are perturbative — may still be probed in a precision machine. This feature is important if one wants to work with perturbative couplings with an increased $\Lambda$. At the same time, a detailed study in the moderate-WC region needs the consideration of $d > 6$ operators.

3. This is where the future leptonic colliders like the International Linear Collider may have the advantage over the LHC, where polarization measurement is going to pose a tough challenge. However, they will lose on the $\sqrt{s}$ factor.

3.1 Bounds from collider studies

Effects of the dimension-6 operators in hadronic and leptonic colliders have been studied recently [33, 34, 35, 36] with respect to the SILH Lagrangian [27, 28], using event generators like MadGraph-aMC@NLO [40]. The LHC Run-1 data can provide limits only on the following couplings $^{7}$, where we have normalised the mass scale to $\Lambda = 1$ TeV instead of $m_W^2$ or $v^2$ [33]:

$$\bar{c}_\gamma \in [-0.12 : 0.067], \quad \bar{c}_{HW} \in [-7.3 : 2.2].$$

However, this has been obtained with a fit to the Higgs branching ratios, and the numbers can substantially change if the Higgs sector is extended. Thus, while the $\bar{c}_\gamma$ bound is apparently compatible to the unitarity limit, a direct search is always preferred. Such studies have been performed, and the reach of the LHC [34] and the ILC [36] are as follows:

$$\bar{c}_\gamma : [-2.0 : 3.5] \text{ (LHC300)}, \quad [-0.65 : 1.16] \text{ (LHC3000)}, \quad [-3.0 : 1.0] \text{ (ILC)},$$

$$\bar{c}_{HB} : [-5.9 : 7.8] \text{ (LHC300)}, \quad [-1.9 : 2.5] \text{ (LHC3000)}, \quad [-2.6 : 1.0] \text{ (ILC)},$$

$$\bar{c}_{HW} : [-8.2 : 5.9] \text{ (LHC300)}, \quad [-2.6 : 1.9] \text{ (LHC3000)}, \quad [-0.29 : 0.27] \text{ (ILC)},$$

$$\bar{c}_W : [-0.21 : 0.20] \text{ (ILC)},$$

$$\bar{c}_H : [-0.65 : 0.67] \text{ (ILC)}.$$ 

(14)

where the LHC numbers are for $\sqrt{s} = 14$ TeV with integrated luminosity of 300 fb$^{-1}$ (LHC300) and 3000 fb$^{-1}$ (LHC3000). The ILC numbers are for $\sqrt{s} = 350$ GeV and integrated luminosity of 3 ab$^{-1}$. Thus, one may hopefully expect some deviation from the SM expectations at the LHC. A study on CLIC has also been performed in the second reference of [35], with a slightly different operator basis, and the sensitivity to the new operators is a bit higher compared to the ILC.

4 Summary

If all the NP fields are heavy (and possibly outside the reach of the LHC), their effects on the SM dynamics can be parametrized by a set of higher-dimensional operators. These operators spoil the renormalizability of the effective theory and in turn can make some scattering amplitudes violate unitarity. Significant constraints on the NP parameter space can be obtained if the unitarity violation occurs below the cutoff scale $\Lambda$.

In this paper, we work with a particular basis for the dimension-6 effective operators that are especially helpful for scattering studies. However, one can use any such basis for this study, as long as the

$^{7}$The gluon coupling, $c_g$, is most tightly constrained: $c_g \in [-0.01 : 0.007]$. 
basis is a complete one and does not contain redundant operators. The new operators generate several three-point and four-point interactions that contribute to $2 \rightarrow 2$ scattering processes by modifying the SM vertex factors. This in turn spoils the unitarity as bad high-energy behaviours are not cancelled out.

Some of the new operators also modify the canonical kinetic terms. To bring them back to the canonical forms, one has to redefine the fields by some multiplicative normalization. Such a redefinition indirectly affects the vertices, as can be seen in the list of vertex factors given in Appendix A. Both the effects are important, and as can be seen from the plots, a single operator may affect a number of scattering processes, and a single process may get affected by several operators. We have followed the approach of minimality and assumed the presence of only one operators at a time while discussing the bounds. This need not be the actual case.

The bounds depend on the cutoff $\Lambda$ but always scale as $\Lambda^2$, so it is easy to set a fiducial mark at $\Lambda = 1$ TeV and show the bounds. They also depend on $\sqrt{s}$ and get stronger as $\sqrt{s}$ increases. We have shown all the bounds for $\sqrt{s} = 2$ TeV, a typical parton-level energy at the LHC.

As can be seen, even with $\Lambda = 10$ TeV, there are some WCs $\tilde{c}_i$ that remain $\sim O(1)$ when the unitarity bound $|a_0| = 1/2$ is reached. This is what we can expect very reasonably if the NP interaction that generates the effective operators is tree-level and with perturbative couplings. At the same time, deviations from SM values can be observed for much smaller values of the WCs. However, precise measurement of polarization at the LHC environment remains a challenge to the experimentalists.

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A Feynman Rules

We list here all the relevant Feynman rules with bosons in external legs. All momenta are taken to be going in to the vertex. The symmetry factors are also included. The SM vertices can be obtained by putting all $\tilde{c}_i = 0$ or $C_i = 0$, where we use $C_i = \tilde{c}_i/\Lambda^2$ for brevity. We also use the following shorthand notations already defined in Eq. (8), with $e = g s_W = g' c_W$:

\begin{align}
N_h &= 1 + 2 v^2 C_H , \\
N_Z &= 1 - \frac{1}{2} g^2 v^2 (C_B + 4 s_W^2 C_\gamma) - \frac{1}{2} g^2 v^2 C_W , \\
N_W &= 1 - \frac{1}{2} g^2 v^2 C_W , \\
N_A &= 1 - g^2 v^2 s_W^2 C_\gamma , \\
N_{AZ} &= \frac{1}{4} gg' v^2 (8 s_W^2 C_\gamma + C_B - C_W) .
\end{align}

(A.1)

We will show here only the unnormalized vertices, i.e., vertices obtained with $h, W, Z$ and $A$ and not their barred (normalized) counterparts. For physical processes, the normalized vertices are relevant. To get them, this is what one should do.

- If the number of external $h, W, A$ legs in a vertex be $n_h, n_W, n_A$ respectively, divide the vertex factor by $(N_h)^{n_h/2}(N_W)^{n_W/2}(N_A)^{n_A}$. Note the difference in the exponent of the photon legs.

- External $Z$ legs are slightly more complicated. The major contribution comes from the unnormalized vertex involving $Z$, and should be divided, in the same vein, by $(N_Z)^{n_Z/2}$ where $n_Z$ is
the number of Z-legs. However, there will also be contributions coming from the normalization of the photon fields. For example, suppose we have the following terms in the Lagrangian:

\[ \mathcal{L} \supset P_{\alpha\beta\mu\nu} W^{\alpha} W^{\beta} A^\mu A^\nu + Q_{\alpha\beta\mu\nu} W^{\alpha} W^{\beta} A^\mu Z^\nu + R_{\alpha\beta\mu\nu} W^{\alpha} W^{\beta} Z^\mu Z^\nu \]  
(A.2)

where \(\{P, Q, R\}_{\alpha\beta\mu\nu}\) contain all the momenta and other constants. In terms of the normalized fields, this becomes

\[
\mathcal{L} \supset \frac{1}{N_W N_A^2} P_{\alpha\beta\mu\nu} \bar{W}^\alpha \bar{W}^\beta \bar{A}^\mu \bar{A}^\nu + \frac{1}{N_W N_A \sqrt{N_Z}} \left( Q_{\alpha\beta\mu\nu} - \frac{2N_A Z}{N_A} P_{\alpha\beta\mu\nu} \right) \bar{W}^\alpha \bar{W}^\beta \bar{A}^\mu Z^\nu + \frac{1}{N_W N_Z} \left( R_{\alpha\beta\mu\nu} - \frac{N_A Z}{N_A} Q_{\alpha\beta\mu\nu} + \frac{N_A^2}{N_A^2} P_{\alpha\beta\mu\nu} \right) \bar{W}^\alpha \bar{W}^\beta Z^\mu Z^\nu. \]  
(A.3)

- It is trivial to reproduce Table 1 from the vertex factors and relevant field normalizations. One needs to draw all the possible tree-level diagrams with three- or four-point interactions. If a field appears either as an external leg or as an internal propagator in any of these diagrams, the WCs involved in its normalization will be affected. For example, if there is any diagram involving \(Z\), \(\bar{\tau}_\gamma\), \(\bar{\tau}_W\), and \(\bar{\tau}_B\) appear in Table 1. The WCs coming from vertex factors can easily be picked out from the following list.

Note also that we have never used the on-shell condition \(p_i^2 = m_i^2\). If some of the legs are on-shell, suitable modifications can be easily performed. The symmetrization over the external momenta has also not been done.

- **Four-scalar vertex:**

\[
h(p_1)h(p_2)h(p_3)h(p_4) : -i \left[ 6\lambda + 45\lambda v^2 C_6 + 4C_H \sum_{i \neq j} p_i.p_j \right] \]  
(A.4)

where \(i, j\) run from 1 to 4. This has to be divided by \(N_h^2\) for the normalized vertex.

- **Three-scalar vertex:**

\[
h(p_1)h(p_2)h(p_3) : -iv \left[ 6\lambda + 15\lambda v^2 C_6 + 4C_H \sum_{i \neq j} p_i.p_j \right] \]  
(A.5)

with \(i, j = 1, 2, 3\). This has to be divided by \(N_h^{3/2}\) for the normalized vertex.

- **Four-gauge vertices:**

\[
W^{\mu}(p_1)W^{\nu}(p_2)W^{-\alpha}(p_3)W^{-\beta}(p_4) : \frac{ig^2}{4} \left[ -4 + 4g^2 v^2 C_W + g^2 v^2 C_{HW} \right] \Gamma^{\mu\nu\alpha\beta} + 6ig^4 C_{3W} F^{\mu\nu\alpha\beta} \\
W^{\mu}(p_1)W^{-\nu}(p_2)Z^\alpha(p_3)Z^\beta(p_4) : -\frac{ig^2}{4} \left[ 4g^2 v^2 C_{HW} + 2g^2 v^2 C_W (1 + c_W^2) - 4c_W^2 \right] \Gamma^{\mu\nu\alpha\beta} + 6ig^4 c_W^2 C_{3W} F^{\mu\nu\alpha\beta} \\
W^{\mu}(p_1)W^{-\nu}(p_2)A^\alpha(p_3)Z^\beta(p_4) : \frac{ig^2}{8} \left[ gg^2 v^2 C_{HW} + 2gg^2 v^2 C_W (1 + 2c_W^2) - s_W c_W \right] \Gamma^{\mu\nu\alpha\beta} - 6ig^4 s_W C_{3W} F^{\mu\nu\alpha\beta}, \\
W^{\mu}(p_1)W^{-\nu}(p_2)A^\alpha(p_3)A^\beta(p_4) : \frac{ie^2}{2} \left( 2 - g^2 v^2 C_W \right) \Gamma^{\mu\nu\alpha\beta} - 6ig^4 s_W^2 C_{3W} F^{\mu\nu\alpha\beta}, \]  
(A.6)
where

$$\Gamma^{\mu\nu\alpha\beta} = \left( \eta^{\mu\beta} \eta^{\nu\alpha} + \eta^{\mu\alpha} \eta^{\nu\beta} - 2 \eta^{\mu\nu} \eta^{\alpha\beta} \right), \quad (A.7)$$

and

$$F^{\mu\nu\alpha\beta} = \left( p_1 \cdot p_3 + p_2 \cdot p_4 \right) \eta^{\mu\beta} \eta^{\nu\alpha} + \left( p_1 \cdot p_4 + p_2 \cdot p_3 \right) \eta^{\mu\alpha} \eta^{\nu\beta} - \left( p_1 + p_2 \right) \left( p_3 + p_4 \right) \eta^{\mu\nu} \eta^{\alpha\beta}$$

$$+ \eta^{\mu\nu} \left( p_1 + p_2 \right) \eta^{\mu\beta} \eta^{\nu\alpha} + \left( p_1 + p_2 \right) \eta^{\mu\alpha} \eta^{\nu\beta} - \eta^{\mu\nu} \left( p_3 + p_4 \right) \eta^{\mu\beta} \eta^{\nu\alpha}$$

$$+ \eta^{\mu\nu} \left( p_3 + p_4 \right) \eta^{\mu\beta} \eta^{\nu\alpha} + \eta^{\mu\alpha} \eta^{\nu\beta} \left( p_4 \cdot (p_2 - p_1) \right) - \eta^{\mu\nu} \left( p_2 \cdot p_4 - p_2 \cdot p_1 \right)$$

$$\left( \eta^{\mu\beta} (p_1 \cdot p_3) - \eta^{\mu\alpha} (p_2 \cdot p_4) - \eta^{\nu\beta} (p_1 \cdot p_3) + \eta^{\nu\alpha} (p_2 \cdot p_4) \right). \quad (A.8)$$

Note that the $WW\gamma\gamma$ four-point vertex contributes to $WWZZ$ and $WWAZ$ vertices as shown in Eq. (A.3).

- Three-gauge vertices:

$$W^{\pm \mu}(p_1) W^{- \nu}(p_2) Z^{\alpha}(p_3) : \quad \frac{ig}{8c_W} \left[ -8c_W \eta^{\mu\nu} + (p_2 - p_3)^{\mu} \eta^{\nu\alpha} + (p_3 - p_1)^{\nu} \eta^{\mu\alpha} \right]$$

$$+ 4g^2 \eta^{\mu\nu} (-p_1^{\mu} p_3^{\nu} + p_1^{\nu} p_3^{\mu}) + p_3 \cdot p_4 \left( p_1^{\mu} \eta^{\nu\alpha} + p_1^{\nu} \eta^{\mu\alpha} \right)$$

$$+ \frac{g^2}{2} \left( p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha} \right) + p_2 \cdot p_4 \left( p_1^{\nu} \eta^{\mu\alpha} + p_1^{\mu} \eta^{\nu\alpha} \right)$$

$$+ 2g^2 \eta^{\mu\nu} (p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha} + c_W (p_1^{\mu} \eta^{\nu\alpha} - p_1^{\nu} \eta^{\mu\alpha}))$$

$$+ 2g^2 \eta^{\mu\nu} C_{HW} (p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha})$$

$$\left( \eta^{\mu\nu} (p_1 - p_2) - p_2 \eta^{\mu\nu} + (p_2 - p_3)^{\mu} \eta^{\nu\alpha} + (p_3 - p_1)^{\nu} \eta^{\mu\alpha} \right)$$

$$+ \frac{g^2}{8} \left( p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha} \right) + 2g^2 \eta^{\mu\nu} (C_{HW} + C_{HB}) (p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha}) \quad (A.9)$$

- Three-point mixed vertices:

$$h(p_1) A^{\mu}(p_2) Z^{\nu}(p_3) : \quad \frac{ig^2}{4} \left[ 2g^2 \eta^{\mu\nu} + 2C_B \eta^{\mu\nu} - 2p_1^{\mu} p_2^{\nu} + p_1^{\mu} p_2^{\nu} \right]$$

$$+ C_{HW} (p_1^{\nu} p_2^{\mu} - (p_1 \cdot p_2) \eta^{\mu\nu})$$

$$\left( \eta^{\mu\nu} (p_1 - p_2) - p_2 \eta^{\mu\nu} + (p_2 - p_3)^{\mu} \eta^{\nu\alpha} + (p_3 - p_1)^{\nu} \eta^{\mu\alpha} \right)$$

$$+ \frac{g^2}{4} \left( p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha} \right) + 2g^2 \eta^{\mu\nu} (C_{HW} + C_{HB}) (p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha})$$

$$\left( \eta^{\mu\nu} (p_1 - p_2) - p_2 \eta^{\mu\nu} + (p_2 - p_3)^{\mu} \eta^{\nu\alpha} + (p_3 - p_1)^{\nu} \eta^{\mu\alpha} \right)$$

$$+ \frac{g^2}{4} \left( p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha} \right) + 2g^2 \eta^{\mu\nu} (C_{HW} + C_{HB}) (p_1^{\nu} \eta^{\mu\alpha} - p_1^{\mu} \eta^{\nu\alpha}) \quad (A.10)$$

Note that the $hAZ$ and $hAA$ vertices are generated by the new operators only.
Four-point mixed vertices:

\[
\begin{align*}
    h(p_1)h(p_2)W^{+\mu}(p_3)W^{-\nu}(p_4) & : \quad \frac{ig^2}{4} \left[ 2\eta^{\mu\nu} + 2C_W \left( \frac{2}{3} \eta^{\mu\nu} - \eta^{p_3} p_3^{\mu} + \eta^{p_4} p_4^{\nu} - \eta^{s_1} p_1^{\nu} p_4^{\mu} \right) \\
    & \quad \quad + C_{HW} \left( (p_1 + p_2)^{\mu} p_3^{\nu} + (p_1 + p_2)^{\nu} p_3^{\mu} - (p_1 + p_2)(p_3 + p_4) \right) \right], \\
    h(p_1)h(p_2)Z^{\mu}(p_3)Z^{\nu}(p_4) & : \quad \frac{ig^2}{4c_W^2} \left[ 2\eta^{\mu\nu} - 24v^2C_T \eta^{\mu\nu} + 16s_W^2 C_\gamma \left( \eta^{p_3} p_3^{\mu} - \eta^{p_4} p_4^{\nu} \right) \\
    & \quad \quad - 2c_W^2 C_W \left( \eta^{p_3} p_3^{\mu} - \eta^{p_4} p_4^{\nu} \right) \\
    & \quad \quad - 2s_W^2 C_B \left( \eta^{p_3} p_3^{\mu} - \eta^{p_4} p_4^{\nu} \right) \\
    & \quad \quad + (c_W^2 C_{HW} + s_W^2 C_{HB}) \times \\
    & \quad \quad \left( (p_1 + p_2)^{\mu} p_3^{\nu} + (p_1 + p_2)^{\nu} p_3^{\mu} - (p_1 + p_2)(p_3 + p_4) \right) \right], \\
    h(p_1)W^{+\mu}(p_2)W^{-\nu}(p_3)Z^{\alpha}(p_4) & : \quad \frac{ig^3}{v} \left[ C_W \left( (2 + 4c_W^2)(p_2 - p_3) \right)^{\alpha \eta^{\mu\nu}} \\
    & \quad \quad + (6p_4^{\alpha} c_W^2 - 2p_3^2 s_W^2 - p_2^2 (4 + 2c_W^2)) \eta^{\alpha \eta^{\mu\nu}} \\
    & \quad \quad - (6p_4^{\alpha} c_W^2 - 2p_3^2 s_W^2 - p_2^2 (4 + 2c_W^2)) \eta^{\alpha \eta^{\mu\nu}} \\
    & \quad \quad + C_{HW} \left( (p_2 - p_3)^{\alpha \eta^{\mu\nu}} + (p_4^{\alpha} c_W + p_2^2 s_W - p_2^2) \eta^{\alpha \eta^{\mu\nu}} \\
    & \quad \quad - (p_4^{\alpha} c_W + p_2^2 s_W - p_2^2) \eta^{\alpha \eta^{\mu\nu}} \\
    & \quad \quad + C_{HB} \left( p_4^{\alpha \eta^{\mu\nu}} - p_2^4 \eta^{\alpha \eta^{\mu\nu}} \right) \right], \\
    h(p_1)A^{\mu}(p_2)W^{+\nu}(p_3)W^{-\alpha}(p_4) & : \quad \frac{ie^2}{4} \left[ C_W \left( 6(p_2^2 \eta^{\mu\nu} - p_2^2 \eta^{\alpha \eta^{\mu\nu}}) \\
    & \quad \quad - \left( (p_3 - p_4)^{\alpha \eta^{\mu\nu}} + (p_3 - p_4)^{\nu \eta^{\mu\alpha}} - 2(p_3 - p_4) \right) \eta^{\alpha \eta^{\mu\nu}} \\
    & \quad \quad + C_{HW} \left( (p_1 - p_2) \eta^{\alpha \eta^{\mu\nu}} + (p_1 - p_2)^{\nu \eta^{\mu\alpha}} \right) \\
    & \quad \quad + C_{HB} \left( p_3^2 \eta^{\mu\nu} - p_2^4 \eta^{\alpha \eta^{\mu\nu}} \right) \right], \\
    h(p_1)h(p_2)A^{\mu}(p_3)A^{\nu}(p_4) & : \quad 4ie^2 C_\gamma \left( p_5^{\nu} p_4^{\mu} - p_3 \eta^{\mu\nu} \right), \\
    h(p_1)h(p_2)A^{\mu}(p_3)Z^{\nu}(p_4) & : \quad \frac{ie^2}{4} \left[ C_B - C_W \right] \left( p_3^2 \eta^{\mu\nu} - p_3^2 \eta^{\nu \mu} \right) \\
    & \quad \quad + 16s_W^2 C_\gamma \left( p_5^{\nu} p_4^{\mu} - p_3 \eta^{\mu\nu} \right) \\
    & \quad \quad + \left( C_{HB} - C_{HW} \right) \left( p_5^{\nu} (p_1 + p_2)^{\mu} - p_3 \right), \quad (\text{A.11})
\end{align*}
\]

Last four vertices have been generated only through the effective operators.
• Three-point fermionic vertices:

\[
\tilde{b}(p_1)b(p_2)h(p_3) : -\frac{im_b}{v} \left[ 1 + \frac{3}{2} C_d v^2 \right], \\
\tilde{t}(p_1)t(p_2)h(p_3) : -\frac{im_t}{v} \left[ 1 + \frac{3}{2} C_u v^2 \right], \\
\tilde{b}(p_1)b(p_2)A^\mu(p_3) : -\frac{ie}{3} \gamma^\mu + e (C_{dB} - C_{dW}) m_b \sigma^{\mu\nu} p_{3\nu}, \\
\tilde{t}(p_1)t(p_2)A^\mu(p_3) : 2\frac{ie}{3} \gamma^\mu + e (C_{uB} + C_{uW}) m_t \sigma^{\mu\nu} p_{3\nu}, \\
\tilde{b}(p_1)b(p_2)Z^\mu(p_3) : \frac{i g}{2c_W} \gamma^\mu \left[ \frac{2}{3} s_W^2 - P_L - (C_{Hq} + C'_{Hq}) v^2 P_L - C_{Hd} v^2 P_R \right] \\
\quad - \frac{g}{c_W} \left[ (C_{dB}s_W^2 + C_{dW}c_W^2) m_b \sigma^{\mu\nu} p_{3\nu} \right], \\
\tilde{t}(p_1)t(p_2)Z^\mu(p_3) : \frac{i g}{2c_W} \gamma^\mu \left[ P_L - \frac{4}{3} s_W^2 - (C_{Hq} - C'_{Hq}) v^2 P_L - C_{Hd} v^2 P_R \right] \\
\quad - \frac{g}{c_W} \left[ (C_{uB}s_W^2 - C_{uW}c_W^2) m_t \sigma^{\mu\nu} p_{3\nu} \right], \\
\tilde{t}(p_1)b(p_2)W^{-\mu}(p_3) : \frac{i g}{\sqrt{2}} V_{tb} \left[ 1 + C'_{Hq} v^2 \right] \gamma^\mu P_L + \frac{i g}{\sqrt{2}} C_{Hud} v^2 \gamma^\mu P_R \\
\quad + \sqrt{2} g V_{tb} [m_t C_{uW} \sigma^{\mu\nu} p_{3\nu} P_L + m_b C_{dW} \sigma^{\mu\nu} p_{3\nu} P_R]. \quad \text{(A.12)}
\]

• Four-point fermionic vertices

\[
\tilde{b}(p_1)b(p_2)W^{-\mu}(p_3)W^{+\nu}(p_4) : g^2 m_b C_{dW} \sigma^{\mu\nu}, \\
\tilde{t}(p_1)t(p_2)W^{-\mu}(p_3)W^{+\nu}(p_4) : -g^2 m_t C_{uW} \sigma^{\mu\nu}, \\
\tilde{t}(p_1)b(p_2)W^{-\mu}(p_3)Z^\nu(p_4) : \sqrt{2} g c_W V_{tb} [m_t C_{uW} \sigma^{\mu\nu} P_L + m_b C_{dW} \sigma^{\mu\nu} P_R]. \quad \text{(A.13)}
\]

There is no field normalization for the fermions as their kinetic terms are not affected but the bosonic fields need to be normalized, and as before, \(\tilde{t}\tilde{b}bZ\) vertices get a contribution from \(\tilde{t}\tilde{b}bA\).

**B  The operator in detail**

It is instructive to write out at least one of the operators in detail to show the rich structure. For example, the operator \(O_W \equiv \left( \Phi^\dagger \tau^\dagger \tilde{D}^\mu \Phi \right) \left( D^\nu \Phi \right)^\dagger\) looks like (note that \(\tau_W\) is the WC while \(c_W\) is the cosine of the Weinberg angle):
\[ O_W = \frac{g^2}{4} \left[ v^2 \left[ (\partial^\mu W^{\nu -} - \partial^\nu W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) ight] + igc_W (W^{\mu +} \partial^\nu W^{\nu -} - W^{\nu -} \partial^\nu W^{\mu +}) Z_\mu Z_\nu - igc_W (W^{\mu +} W^{\nu -} - W^{\nu -} W^{\mu +}) (\partial_\mu Z_\nu - 2 \partial_\nu Z_\mu) + igc_W W^{\mu +} (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) Z^\nu - igc_W W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) Z^\nu + ig_{SW} (W^{\mu +} \partial^\nu W^{\nu -} - W^{\nu -} \partial^\nu W^{\mu +}) A_\mu + ig_{SW} (W^{\mu +} W^{\nu -} - W^{\nu -} W^{\mu +}) (\partial_\mu A_\nu - 2 \partial_\nu A_\mu) + ig_{SW} W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) A^\nu - ig_{SW} W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) A^\nu 
+ g^2 (W^{\mu +} W^{\nu -} - 2 W^{\nu -} W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) - g^2 (W^{\mu +} W^{\nu -} + 2 W^{\nu -} W^{\mu +}) Z_\mu Z_\nu + 2 g^2 (W^{\mu +} W^{\nu -} + 2 W^{\nu -} W^{\mu +}) Z_\mu Z_\nu 
+ g^2 (W^{\mu +} W^{\nu -} + 2 W^{\nu -} W^{\mu +}) (\partial_\mu A_\nu - 2 \partial_\nu A_\mu) + 4 g^2 (W^{\mu +} W^{\nu -} + 2 W^{\nu -} W^{\mu +}) A_\mu A_\nu + 2 g^2 (W^{\mu +} W^{\nu -} + 2 W^{\nu -} W^{\mu +}) A_\mu A_\nu + 2 c_W (\partial^\mu Z^\nu (\partial_\mu A_\nu - \partial_\nu A_\mu) - s_W \partial^\nu Z^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) 
+ ig Z^\mu (W^{\mu +} W^{\nu -} - W^{\nu -} W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) + ig Z^\mu W^{\nu -} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) + ig Z^\mu W^{\nu -} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) Z^\nu - igc_W W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) Z^\nu + ig_{SW} (W^{\mu +} W^{\nu -} - 2 W^{\nu -} W^{\mu +}) (\partial_\mu A_\nu - 2 \partial_\nu A_\mu) + ig_{SW} W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) A^\nu - ig_{SW} W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) A^\nu + 2 c_W (\partial^\mu W^{\nu -} - \partial^\nu W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) 
+ 2 c_W (\partial^\mu W^{\nu -} - \partial^\nu W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) - s_W \partial^\nu Z^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) + 2 c_W (\partial^\mu Z^\nu (\partial_\mu A_\nu - \partial_\mu A_\mu) - s_W \partial^\nu Z^\mu (\partial_\mu A_\nu - \partial_\mu A_\mu) 
+ ig Z^\mu (W^{\mu +} W^{\nu -} - W^{\nu -} W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) + igc_W (W^{\mu +} W^{\nu -} - W^{\nu -} W^{\mu +}) (\partial_\mu Z_\nu - 2 \partial_\nu Z_\mu) + igc_W W^{\mu +} (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) Z^\nu - igc_W W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) Z^\nu + igc_W (W^{\mu +} W^{\nu -} - W^{\nu -} W^{\mu +}) (\partial_\mu A_\nu - 2 \partial_\nu A_\mu) + igc_W W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) A^\nu - igc_W W^{\mu +} (\partial_\mu W^{\nu -} - 2 \partial_\nu W^{\mu +}) A^\nu + 2 c_W (\partial^\mu W^{\nu -} - \partial^\nu W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) 
+ h^2 (\partial^\mu W^{\nu -} - \partial^\nu W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) - h^2 (\partial^\mu W^{\nu -} - \partial^\nu W^{\mu +}) (\partial_\mu W^{\nu -} - \partial_\nu W^{\mu +}) - h^2 (\partial^\mu Z^\nu (\partial_\mu Z_\nu - \partial_\nu Z_\mu) - s_W c_W h^2 (\partial^\nu Z^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) 
\] (B.1)

References

[1] W. Buchmuller and D. Wyler, “Effective Lagrangian Analysis of New Interactions and Flavor Conservation,” Nucl. Phys. B 268, 621 (1986).

[2] E. E. Jenkins, A. V. Manohar and M. Trott, “On Gauge Invariance and Minimal Coupling,” JHEP 1309, 063 (2013) [arXiv:1305.0017 [hep-ph]].

[3] J. Brehmer, A. Freitas, D. Lopez-Val and T. Plehn, “Pushing Higgs Effective Theory to its Limits,” Phys. Rev. D 93, no. 7, 075014 (2016) [arXiv:1510.03443 [hep-ph]].

[4] B. Gripaios and D. Sutherland, “An operator basis for the Standard Model with an added scalar singlet,” JHEP 1608, 103 (2016) [arXiv:1604.07365 [hep-ph]].

[5] H. Bélusca-Maito and A. Falkowski, “On the exotic Higgs decays in effective field theory,” Eur. Phys. J. C76(2016) no. 9, 514 [arXiv:1602.02645 [hep-ph]].
[6] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, “Dimension-Six Terms in the Standard Model Lagrangian,” JHEP 1010, 085 (2010) [arXiv:1008.4884 [hep-ph]].

[7] M. B. Einhorn and J. Wudka, “The Bases of Effective Field Theories,” Nucl. Phys. B 876, 556 (2013) [arXiv:1307.0478 [hep-ph]].

[8] E. Masso, “An Effective Guide to Beyond the Standard Model Physics,” JHEP 1410, 128 (2014) [arXiv:1406.6376 [hep-ph]].

[9] G. Passarino, “Field reparametrization in effective field theories,” Eur. Phys. J. Plus 132, 16 (2017) [arXiv:1610.09618 [hep-ph]].

[10] B. Henning, X. Lu and H. Murayama, “How to use the Standard Model effective field theory,” JHEP 1601, 023 (2016) [arXiv:1412.1837 [hep-ph]].

[11] M. Boggia, R. Gomez-Ambrosio and G. Passarino, “Low energy behaviour of standard model extensions,” JHEP 1605, 162 (2016) [arXiv:1603.03660 [hep-ph]].

[12] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, “The Strongly-Interacting Light Higgs,” JHEP 0706, 045 (2007) [hep-ph/0703164].

[13] I. Low, R. Rattazzi and A. Vichi, “Theoretical Constraints on the Higgs Effective Couplings,” JHEP 1004, 126 (2010) [arXiv:0907.5413 [hep-ph]].

[14] J. Elias-Mir, J. R. Espinosa, E. Masso and A. Pomarol, “Renormalization of dimension-six operators relevant for the Higgs decays $h \to \gamma \gamma, \gamma Z$,” JHEP 1308, 033 (2013) [arXiv:1302.5661 [hep-ph]].

[15] A. Falkowski, F. Riva and A. Urbano, “Higgs at last,” JHEP 1311, 111 (2013) [arXiv:1303.1812 [hep-ph]].

[16] R. Contino, M. Ghezzi, C. Grojean, M. Muhlleitner and M. Spira, “Effective Lagrangian for a light Higgs-like scalar,” JHEP 1307, 035 (2013) [arXiv:1303.3876 [hep-ph]].

[17] M. B. Einhorn and J. Wudka, “Higgs-Boson Couplings Beyond the Standard Model,” Nucl. Phys. B 877, 792 (2013) [arXiv:1308.2255 [hep-ph]].

[18] J. Ellis, V. Sanz and T. You, “Complete Higgs Sector Constraints on Dimension-6 Operators,” JHEP 1407, 036 (2014) [arXiv:1404.3667 [hep-ph]].

[19] H. Belusca-Maito, “Effective Higgs Lagrangian and Constraints on Higgs Couplings,” arXiv:1404.5343 [hep-ph].

[20] A. Biektter, A. Knochel, M. Krmer, D. Liu and F. Riva, “Vices and virtues of Higgs effective field theories at large energy,” Phys. Rev. D 91, 055029 (2015) [arXiv:1406.7320 [hep-ph]].

[21] C. Y. Chen, S. Dawson and C. Zhang, “Electroweak Effective Operators and Higgs Physics,” Phys. Rev. D 89, no. 1, 015016 (2014) [arXiv:1311.3107 [hep-ph]].

[22] A. Falkowski and F. Riva, “Model-independent precision constraints on dimension-6 operators,” JHEP 1502, 039 (2015) [arXiv:1411.0669 [hep-ph]].

[23] L. Berthier and M. Trott, “Consistent constraints on the Standard Model Effective Field Theory,” JHEP 1602, 069 (2016) [arXiv:1508.05060 [hep-ph]].
[24] A. Alboteanu, W. Kilian and J. Reuter, “Resonances and Unitarity in Weak Boson Scattering at the LHC,” JHEP 0811, 010 (2008) [arXiv:0806.4145 [hep-ph]];
W. Kilian, T. Ohl, J. Reuter and M. Sekulla, “High-Energy Vector Boson Scattering after the Higgs Discovery,” Phys. Rev. D 91, 096007 (2015) [arXiv:1408.6207 [hep-ph]].

[25] S. Bar-Shalom, A. Soni and J. Wudka, “Effective field theory analysis of Higgs naturalness,” Phys. Rev. D 92, no. 1, 015018 (2015) [arXiv:1405.2924 [hep-ph]].

[26] G. Buchalla, O. Cata, A. Celis and C. Krause, “Note on Anomalous Higgs-Boson Couplings in Effective Field Theory,” Phys. Lett. B 750, 298 (2015) [arXiv:1504.01707 [hep-ph]].

[27] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, “The Strongly-Interacting Light Higgs,” JHEP 0706, 045 (2007) [hep-ph/0703164].

[28] R. Contino, M. Ghezzi, C. Grojean, M. Muhlleitner and M. Spira, “Effective Lagrangian for a light Higgs-like scalar,” JHEP 1307, 035 (2013) [arXiv:1303.3876 [hep-ph]].

[29] B. W. Lee, C. Quigg and H. B. Thacker, “Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass,” Phys. Rev. D 16, 1519 (1977).

[30] D. Choudhury, R. Islam and A. Kundu, “Anomalous Higgs Couplings as a Window to New Physics,” Phys. Rev. D 88, 013014 (2013) [arXiv:1212.4652 [hep-ph]].

[31] M. Dahiya, S. Dutta and R. Islam, “Investigating perturbative unitarity in the presence of anomalous couplings,” Phys. Rev. D 93, 055013 (2016) [arXiv:1311.4523 [hep-ph]].

[32] T. Corbett, O. J. P. Éboli and M. C. Gonzalez-Garcia, “Unitarity Constraints on Dimension-Six Operators,” Phys. Rev. D 91, 035014 (2015) [arXiv:1411.5026 [hep-ph]].

[33] C. Englert, R. Kogler, H. Schulz and M. Spannowsky, “Higgs coupling measurements at the LHC,” Eur. Phys. J. C 76, no. 7, 393 (2016) [arXiv:1511.05170 [hep-ph]].

[34] H. Khanpour, S. Khatibi and M. Mohammadi Najafabadi, “Probing Higgs boson couplings in $H + \gamma$ production at the LHC,” Phys. Lett. B 773, 462 (2017) [arXiv:1702.05753 [hep-ph]].

[35] H. Abramowicz et al., “Higgs Physics at the CLIC Electron-Positron Linear Collider,” arXiv:1608.07538 [hep-ex];
J. Ellis, P. Roloff, V. Sanz and T. You, “Dimension-6 Operator Analysis of the CLIC Sensitivity to New Physics,” JHEP 1705 096, (2017) [arXiv:1701.04804 [hep-ph]].

[36] H. Khanpour and M. Mohammadi Najafabadi, “Constraining Higgs boson effective couplings at electron-positron colliders,” Phys. Rev. D 95, 055026 (2017) [arXiv:1702.00951 [hep-ph]].

[37] T. Hahn, “Generating Feynman diagrams and amplitudes with FeynArts 3,” Comput. Phys. Commun. 140, 418 (2001) [hep-ph/0012260].

[38] T. Hahn and M. Perez-Victoria, “Automatized one loop calculations in four-dimensions and D-dimensions,” Comput. Phys. Commun. 118, 153 (1999) [hep-ph/9807565];
C. Groß, T. Hahn, S. Heinemeyer, F. von der Pahlen, H. Rzehak and C. Schappacher, “New Developments in FormCalc 8.4,” PoS LL 2014, 035 (2014) [arXiv:1407.0235 [hep-ph]].

[39] A. Alloul, B. Fuks and V. Sanz, “Phenomenology of the Higgs Effective Lagrangian via FEYNRULES,” JHEP 1404, 110 (2014) [arXiv:1310.5150 [hep-ph]].
[40] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer and T. Stelzer, “MadGraph 5 : Going Beyond,” JHEP 1106, 128 (2011) [arXiv:1106.0522 [hep-ph]];
J. Alwall, C. Duhr, B. Fuks, O. Mattelaer, D. G. ztrk and C. H. Shen, “Computing decay rates for new physics theories with FeynRules and MadGraph 5 _aMC@NLO,” Comput. Phys. Commun. 197, 312 (2015) [arXiv:1402.1178 [hep-ph]];
J. Alwall et al., “The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations,” JHEP 1407, 079 (2014) [arXiv:1405.0301 [hep-ph]].