Hybrid Seesaw Neutrino Masses
with $A_4$ Family Symmetry

Shao-Long Chen, Michele Frigerio, and Ernest Ma

Physics Department, University of California, Riverside, California 92521

Abstract

We consider the scenario in which neutrino data are explained by the interplay of type I and II seesaw terms in the Majorana neutrino mass matrix $M_\nu = M_L - M_D M_R^{-1} M_D^T$. We construct a predictive model with $M_L$ proportional to the unit matrix, 3 diagonal texture zeros in $M_R$, and $M_D$ diagonal. We show how this pattern can be maintained by the non-Abelian discrete symmetry $A_4$, and discuss its phenomenological consequences. It turns out that the two types of seesaw give contributions of the same order to $M_\nu$. In the CP conserving case, we find $\sin \theta_{13} \approx 2/(\tan^2 \theta_{23} \tan \theta_{12})$ and we predict inverted (normal) ordering of the mass spectrum for $\tan^2 \theta_{12} < 0.5$ ($> 0.5$).

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In many well-motivated extensions of the Standard Model of particle interactions, the Majorana neutrino mass matrix is in general given by

\[ M_\nu = M_L - M_D M_R^{-1} M_D^T, \]  

(1)

where the first term comes from the coupling of two left-handed neutrinos to a heavy Higgs triplet with a naturally small vacuum expectation value (type II seesaw [1]) and the second term comes from the canonical seesaw mechanism [2] assuming the existence of heavy singlet right-handed neutrinos. In the past, perhaps for reasons of simplicity or economy, the common practice was to assume the dominance of one or the other of these two terms. After all, if both terms were considered, predictability would be largely lost. However, if there exists a symmetry which limits the forms of both terms, a simple and realistic Hybrid Seesaw model may still emerge.

An early discussion of the role of the two contributions in the generation of a large mixing angle can be found in [3]. One common strategy (see e.g. [4] and references therein) was to assume a symmetry such as \( SO(3) \) which requires \( M_L \) to be proportional to the unit matrix, but allow it to be broken arbitrarily in the second term. Another recent paper [5] applies an \( S_3 \times S_3 \) symmetry to both terms, but a number of symmetry-breaking parameters are needed to fit data. Here we propose that the structure of both terms is fixed by the same family symmetry and thus obtain the first example of a predictive Hybrid Seesaw model with a well-defined symmetry (the discrete group \( A_4 \)) for the complete Lagrangian.

Consider first the type I contribution. If \( M_D \) is diagonal (which may be maintained by the \( A_4 \) symmetry), then the texture zeros of \( M_R \) are reflected in \( M_\nu \) as zero sub-determinants [6]. In fact, in the perspective of the type I seesaw formula, instead of the texture zeros of \( M_\nu \) [7], those of \( M_R \) [8] are expected to have a deeper theoretical meaning (the two types
of zeros may be related \[9\]). In particular, consider the case

\[
\mathcal{M}_R = \begin{pmatrix}
0 & \times & \times \\
\times & 0 & \times \\
\times & \times & 0 \\
\end{pmatrix},
\]

where \(\times\) denotes a nonzero entry. This structure is rather unique, as it is the only possibility to have more than 2 zeros in \(\mathcal{M}_R\) (and therefore less than 4 free parameters) without inducing 2 or more zeros in \(\mathcal{M}_R^{-1}\). Assuming \(\mathcal{M}_D\) diagonal, one then obtains

\[
\mathcal{M}_\nu^I = \frac{1}{a} \begin{pmatrix}
a^2 & ab & ac \\
ab & b^2 & -bc \\
ac & -bc & c^2 \\
\end{pmatrix},
\]

which has no texture zero but 3 zero sub-determinants. This three-parameter structure, as we will show, cannot reproduce all present neutrino data \[10\]. On the other hand, it is possible that a significant contribution comes from \(\mathcal{M}_L\) and, if it is proportional to the unit matrix, Eq. (1) becomes

\[
\mathcal{M}_\nu = \begin{pmatrix}
d + a & b & c \\
b & d + b^2/a & -bc/a \\
c & -bc/a & d + c^2/a \\
\end{pmatrix},
\]

which (i) turns out to fit all present data and (ii) may be stabilized by a simple family symmetry, as shown below. This model of Hybrid Seesaw, depending on 4 parameters, is the most minimal constructed so far.

To maintain the pattern of \(\mathcal{M}_\nu\) in Eq. (4), a suitable family symmetry is \(A_4\), the discrete group of the even permutation of four objects. It is also the symmetry group of the regular tetrahedron (Plato’s “fire” \[12\]), and has been applied to the neutrino mass matrix in a number of ways \[13\] \[14\]. The irreducible representations of \(A_4\) are \(1, 1', 1'', 3\). The group multiplication rule \[13\] is

\[
3 \times 3 = 1 + 1' + 1'' + 3_1 + 3_2 ,
\]

where \(\psi_i, \varphi_j \sim 3\) implies

\[
1 = \psi_1 \varphi_1 + \psi_2 \varphi_2 + \psi_3 \varphi_3 ,
\]
\[ 1' = \psi_1 \varphi_1 + \omega^2 \psi_2 \varphi_2 + \omega \psi_3 \varphi_3 , \]  
(7)  
\[ 1'' = \psi_1 \varphi_1 + \omega \psi_2 \varphi_2 + \omega^2 \psi_3 \varphi_3 , \]  
(8)  
\[ 3_1 = (\psi_2 \varphi_3, \psi_3 \varphi_1, \psi_1 \varphi_2) , \]  
(9)  
\[ 3_2 = (\psi_3 \varphi_2, \psi_1 \varphi_3, \psi_2 \varphi_1) , \]  
(10)  

with \( \omega = e^{2\pi i/3} \).

Here we make the following assignment: the 3 families of leptons transform as a triplet,

\[ (\nu_i, l_i), \quad l_i^c, \quad \nu_i^c \sim 3 , \]  
(11)  

and the scalar sector consists of three Higgs doublets \( \Phi_i \sim 1, 1', 1'' \), one Higgs triplet \( \xi \sim 1 \), and three Higgs singlets \( \Sigma_i \sim 3 \). This implies that the Dirac mass matrices linking \( l_i \) to \( l_j^c \) (\( \mathcal{M}_l \)) as well as \( \nu_i \) to \( \nu_j^c \) (\( \mathcal{M}_D \)) are both diagonal, with 3 independent entries each. Explicitly,

\[
\begin{pmatrix}
  m_e \\
  m_\mu \\
  m_\tau \\
\end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix}
  1 & 1 & 1 \\
  1 & \omega & \omega^2 \\
  1 & \omega^2 & \omega \\
\end{pmatrix} \begin{pmatrix}
  y_{l1}(\Phi_1) \\
  y_{l2}(\Phi_2) \\
  y_{l3}(\Phi_3) \\
\end{pmatrix},
\]  
(12)  

where the Yukawa couplings \( y_{li} \) should be tuned to fit the charged lepton masses, as in the Standard Model. Our assignment also implies that \( \mathcal{M}_L \) in Eq.(1), which is generated by \( \langle \xi \rangle \), is proportional to the unit matrix and \( \mathcal{M}_R \) has nonzero off-diagonal entries, as in Eq.(2):

\[ (\mathcal{M}_R)_{ij} = f_R \langle \Sigma_k \rangle \]  

with \( i \neq j \neq k \). Notice that, even if \( \langle \Sigma_k \rangle \) were related among each other by the symmetry of the scalar potential, the parameters \( a, b, c \) in Eq.(3) would be completely independent, since they are determined by the 3 unknown diagonal entries of \( \mathcal{M}_D \). This is what is needed to obtain Eq.(4).

However, the bare Majorana mass term \( \nu_i^c \nu_i^c \) is invariant under \( A_4 \) and it cannot be removed by hand. This is a generic issue in models with texture zeros in the mass matrix of gauge singlets. Thus one is naturally led to consider a left-right gauge extension of the Standard Model with \( (l^c, \nu^c) \) transforming as a doublet under \( SU(2)_R \). In that case, these bare mass terms are forbidden by the gauge symmetry and \( \Sigma_i \) should now be considered as
triplets under $SU(2)_R$, i.e. the counterpart of $\xi$ which is a triplet under $SU(2)_L$. In this way our initial assumption in Eq.(2) is justified and the pattern of our proposed Hybrid Seesaw model in Eq.(4) is completely stabilized.

Let us briefly consider the phenomenology associated with the three $SU(2)_L$ doublets $\Phi_i$. Since charged lepton Yukawa couplings are diagonal, Lepton Flavor Violation processes are suppressed by the smallness of neutrino masses and therefore negligible. The Standard Model like Higgs doublet is given by $\Phi = (v_1 \Phi_1 + v_2 \Phi_2 + v_3 \Phi_3)/v$, where $v^2 = v_1^2 + v_2^2 + v_3^2 = (174 \text{ GeV})^2$. The orthogonal combinations $\Phi'$ and $\Phi''$ decay into $e^+ e^-$, $\mu^+ \mu^-$ and $\tau^+ \tau^-$ with similar rates (the couplings are of the order $m_\tau/v$, the exact values depending on the scalar potential parameters). One loop contributions to the anomalous magnetic moment of the muon, $g_\mu - 2$, are induced, but their size is generically negligible even for Higgs masses as light as 100 GeV (for a precise estimation in a similar model, see [15]).

Notice also that, if the 3 families of quarks transform as an $A_4$ triplet in the same way as leptons, up and down quark mass matrices are both diagonal, thus describing in first approximation the smallness of CKM mixing angles. Then, since all fermions transform in the same way under $A_4$, they may be embedded in multiplets of a Grand Unified gauge group. However, the construction of an appropriate scalar sector is highly non-trivial and beyond the purposes of the present paper.

Let us study the phenomenological implications for neutrino masses and mixing angles. Data on neutrino oscillations [10] indicate that $\theta_{23}$ is close to maximal and $\theta_{13}$ is small. One can check that the matrix structure [11] may accommodate $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ if and only if $b^2 = c^2$. It is useful to discuss first this limiting case and, in the following, possible deviations from it.

**Case $b = c$:** The matrix $\mathcal{M}_\nu$ has a form [11] such that $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. In the basis
spanning $\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2}$, and $(\nu_\tau - \nu_\mu)/\sqrt{2}$, it becomes

$$\mathcal{M}_\nu = \begin{pmatrix} d + a & \sqrt{2}b & 0 \\ \sqrt{2}b & d & 0 \\ 0 & 0 & d + 2b^2/a \end{pmatrix}. \quad (13)$$

The parameters $a, b, d$ are in general complex. Diagonalizing $\mathcal{M}_\nu \mathcal{M}_\nu^\dagger$, we obtain

$$\tan 2\theta_{12} = \frac{2|B|}{|d|^2 - |d + a|^2}, \quad (14)$$

where $B = \sqrt{2}[2Re(b^*d) + ab^*]$. Notice that $\theta_{12} < \pi/4$ implies $|a|^2 + 2Re(ad^*) < 0$. The mass squared differences are given by

$$\Delta m^2_{\text{sol}} \equiv |m_3|^2 - |m_1|^2 = \sqrt{|d|^2 - |d + a|^2}^2 + 4|B|^2 = \frac{|d|^2 - |d + a|^2}{\cos 2\theta_{12}}, \quad (15)$$

$$\pm \Delta m^2_{\text{atm}} \equiv |m_3|^2 - \frac{1}{2}(|m_2|^2 + |m_1|^2) = 4 \left| \frac{b^2}{a} \right|^2 + 4Re \left( \frac{d^*b}{a} \right) - 2|b|^2 + \frac{1}{2}(|d|^2 - |d + a|^2). \quad (16)$$

**Subcase (1):** If the parameters $a, b$ and $d$ are real, they are uniquely determined by the experimental values of $\theta_{12}$, $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$. In particular $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}$ implies $2d \approx -a$, so that $d = 0$ (pure type I seesaw) is not a solution, as already mentioned. Since

$$|m_3|^2 - \frac{1}{2}(|m_2|^2 + |m_1|^2) = -\frac{a^2 \tan^2 2\theta_{12}}{2} \left( 1 - \frac{1}{8} \tan^2 2\theta_{12} \right) + \frac{1}{2} \Delta m^2_{\text{sol}} \left( 1 - \frac{1}{2} \tan^2 2\theta_{12} \right), \quad (17)$$

the ordering of the mass spectrum is inverted for $\tan^2 2\theta_{12} < 0.5$, as favored (but only at about $1\sigma$ level) by present data. For the best fit values of oscillation parameters ($\tan^2 2\theta_{12} = 0.45$, $\Delta m^2_{\text{sol}} = 8.0 \times 10^{-5} \text{ eV}^2$, $\Delta m^2_{\text{atm}} = 2.5 \times 10^{-3} \text{ eV}^2$, $[16]$), one finds $a = \pm 0.057 \text{ eV}$, $|b| = 0.049 \text{ eV}$, $d = \mp 0.029 \text{ eV}$ and $|m_{1,2,3}|$ are respectively 0.0748, 0.0753, 0.0560 eV, i.e. a mild inverted ordering. In this case the effective mass parameter relevant for neutrinoless $2\beta$-decay takes the value $m_{ee} \equiv |d + a| = 0.028 \text{ eV}$. However, for values of $\tan^2 2\theta_{12}$ closer to 0.5, the absolute mass scale increases and the spectrum becomes quasi-degenerate, as shown in Fig[1]. Correspondingly, $m_{ee} \approx \cos 2\theta_{12} |m_1|$ becomes larger.
Figure 1: The mass eigenvalue $m_3$ as a function of $\tan^2 \theta_{12}$, in the limit $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and no complex phases (subcase (1)). The displayed interval is the 99% C.L. allowed range for $\tan^2 \theta_{12}$ [16]. The solid line corresponds to the best fit values for $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$, whereas the shaded region accounts for the 99% C.L. allowed ranges of the mass squared differences. The star indicates the best fit. The ordering of the mass spectrum is inverted for $\tan^2 \theta_{12} < 0.5$ (left branch) and normal for $\tan^2 \theta_{12} > 0.5$ (right branch).

Subcase (2): If the parameters $a$ and $d$ are real and $b = i|b|$ is imaginary, then Eq. (17) is replaced by

$$|m_3|^2 - \frac{1}{2}(|m_2|^2 + |m_1|^2) = \frac{(\Delta m^2_{\text{sol}})^4 \sin^4 2\theta_{12}}{16a^6} + \frac{(\Delta m^2_{\text{sol}})^3 \sin^2 2\theta_{12} \cos 2\theta_{12}}{4a^4} + \frac{1}{2} \Delta m^2_{\text{sol}} \cos 2\theta_{12}.$$  \hspace{1cm} (18)

This is a solution with normal ordering and again the 3 experimental conditions (best fit values) determine $a$, $|b|$, and $d$, i.e. $\pm 0.0032$ eV, $0.0084$ eV, and $\mp 0.0064$ eV, with $|m_{1,2,3}| = 0.011, 0.014, 0.052$ eV respectively. Differently from subcase (1), the absolute mass scale depends weakly on $\tan^2 \theta_{12}$ within the experimental range.

More in general, when $b = c$ there are two complex free parameters given by the relative phases among $a$, $b$ and $d$. For illustration, let us consider the following extensions of subcases (1) and (2), with only one additional degree of freedom. Subcase (1'): Let $d$ be real with
Figure 2: The dependence of the neutrino mass eigenvalues $m_i$ on complex phases, in the limit $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ (Eq.(13)). In the left panel we assume $d$ real and $a, b$ having the same phase $\phi$ (subcase (1')). In the right panel we assume $d$ real, $a = |a|e^{i\phi}$ and $b = i|b|$ purely imaginary (subcase (2')). We take the best fit values of mass squared differences and $\tan^2 \theta_{12} = 0.45$.

$a = |a|e^{i\phi}$ and $b = |b|e^{i\phi}$, i.e. $a$ and $b$ have the same phase. Then the 3 relations for $\tan 2\theta_{12}$, $\Delta m^2_{\text{sol}}$, and $\Delta m^2_{\text{atm}}$ are exactly the same as in subcase (1), with the replacements $a \rightarrow |a|$, $b \rightarrow |b|$ and $d \rightarrow d \cos \phi$. This means that we again have an inverted ordering for $\tan^2 \theta_{12} < 0.5$. Since only the combination $d \cos \phi$ is determined by phenomenology and

$$|m_3|^2 = \left| d + \frac{2b^2}{a} \right|^2 = \left( d \cos \phi + 2 \left| \frac{b^2}{a} \right| \right)^2 + (d \cos \phi)^2 \tan^2 \phi,$$

(19)

the overall scale of neutrino masses increases with increasing values of $\tan^2 \phi$. This means that the mass spectrum can be quasi-degenerate independently from the value of $\tan^2 \theta_{12}$.

**Subcase (2'):** Let $d$ be real with $a = |a|e^{i\phi}$ and $b = i|b|$. The 3 conditions are the same as in subcase (2), with $a$ replaced by $|a|$ and $d$ by $d \cos \phi$. We now have

$$|m_3|^2 = \left( d \cos \phi - 2 \left| \frac{b^2}{a} \right| \right)^2 + (d \cos \phi)^2 \tan^2 \phi,$$

(20)

so that, as in subcase (1'), the overall mass scale increases with $\tan^2 \phi$. The dependence of neutrino masses on $\phi$ is shown in Fig.2 for both subcases (1') and (2').
Case $b \neq c$: In this general case $\theta_{13}$ may be non-zero and $\theta_{23}$ may deviate from the maximal value $\pi/4$. If one neglects complex phases, the type II term $M_L = d \mathbb{I}$, being proportional to the identity, does not affect the mixing angles but only the mass spectrum: calling $\lambda_i$ the eigenvalues of $M'_\nu$ in Eq.(3), one has simply $m_i = d + \lambda_i$. In order to extract the constraints on the mixing angles and $\lambda_i$, one should notice that $(M'_\nu)^{-1}$ has 3 texture zeros on the diagonal, by construction. It then follows that

$$0 = \frac{1}{\lambda_3} + \frac{1}{\lambda_2} + \frac{1}{\lambda_1},$$

$$\tan^2 \theta_{13} = \frac{\lambda_2 \cos^2 \theta_{12} + \lambda_1 \sin^2 \theta_{12}}{\lambda_1 + \lambda_2},$$

$$\tan 2\theta_{23} = \frac{\lambda_2 \sin^2 \theta_{12} + \lambda_1 \cos^2 \theta_{12}}{(\lambda_1 - \lambda_2) \cos \theta_{12} \sin \theta_{12}} \frac{1}{\sin \theta_{13}}.$$

Therefore, given the values of $\theta_{12}$ and $\theta_{23}$ ($\theta_{13}$), the ratio $\lambda_1/\lambda_2$ and $\theta_{13}$ ($\theta_{23}$) are predicted. In particular, taking into account that $\tan^2 \theta_{13} < 0.05 \ll 1$, one finds

$$\sin \theta_{13} \approx \frac{1}{\tan 2\theta_{23} \tan 2\theta_{12}},$$

so that the size of the 1–3 mixing angle is proportional to the deviation from maximal atmospheric mixing. This result is illustrated in Fig.3 which shows that the present upper bound $\sin \theta_{13} < 0.2$ can be saturated, given the experimental uncertainty on $\theta_{23}$ and $\theta_{12}$.

Since $m_i = d + \lambda_i$, the parameters $d$ and $\lambda_{1,2}$ are uniquely determined once $\Delta m^2_{sol}$ and $\Delta m^2_{atm}$ are given, so that the mass spectrum is predicted too. After some algebra, one finds that the ordering is inverted for

$$\tan^2 \theta_{12} < \frac{1 + \tan^2 \theta_{13}}{2 - \tan^2 \theta_{13}} = 0.5 \div 0.54$$

and normal for $\tan^2 \theta_{12}$ larger. The absolute neutrino mass scale $|m_3|$ is shown, as a function of $\theta_{13}$ ($\theta_{23}$), in Fig.4 (Fig.5). The dependence of $|m_3|$ on $\theta_{12}$ is strong, analogously to the case $b = c$: quasi-degeneracy of the spectrum (and accordingly sizable $m_{ee}$) is obtained for $\tan^2 \theta_{12}$ close to the right-hand side of Eq.(23).
Figure 3: The correlation between $\theta_{13}$ and $\theta_{23}$ in the CP conserving case (no complex phases). The displayed interval is the 99% C.L. allowed range for $\tan^2 \theta_{23}$ \cite{16}. The curves depend only on the value of the solar mixing angle $\theta_{12}$ (they are independent of the neutrino mass spectrum).

Figure 4: The correlation between $m_3$ and $\theta_{13}$, for different values of the solar mixing angle $\theta_{12}$. The value of $\theta_{12}$ determines if the ordering of the mass spectrum is inverted (left panel) or normal (right panel). The lines correspond to the best fit values for $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$, whereas the shaded regions account for the 99% C.L. allowed ranges of the mass squared differences. Complex phases are put to zero.
Figure 5: The same as Fig. 4 but as a function of $\theta_{23}$. The displayed interval is the 99% C.L. allowed range for $\tan^2 \theta_{23}$.

If complex phases are introduced, the type II contribution will affect not only the mass spectrum but also the mixing angles. In general there will be more freedom to fit data and we do not elaborate further in this direction. Just notice that $\theta_{13} \neq 0$ can be possibly associated with Dirac type CP violation.

In conclusion, we have considered the Hybrid Seesaw scenario, where light neutrino masses receive comparable contributions from super-heavy right-handed neutrinos and scalar isoriplets. We have shown that a family symmetry (the discrete group $A_4$) is able (i) to control the structure of type I and type II seesaw terms at the same time and (ii) to restrict the number of free parameters so that predictions are possible and experimentally testable.

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