Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Personal protective equipment market coordination using subsidy

Hamid R. Sayarshad

School of Civil and Environmental Engineering, Cornell University, Ithaca, NY 14853, USA

Abstract
During a pandemic, various resources, including personal protective equipment (PPE), are required to protect people and healthcare workers from getting infected. Due to the high demand and limited supply chain, countries experience a shortage in PPE products. This global crisis imposes a decline in the international trade of PPE supplies. In fact, most governments implement a localization strategy motivating domestic manufacturers to pivot their operations to respond to PPE demands. An oligopolistic market cannot reach the socially optimal coverage without government subsidies. On the other hand, the government subsidy pays the proportion of production costs to reach the socially optimal coverage, while the government's budget is limited. Therefore, the government collaborates with manufacturers via procurement contracts to increase the supply of PPE products. We propose the first supply chain model of PPE products that investigates manufacturer costs and government expenditure. We consider how different behavioral aspects of manufacturers and government can self-organize towards a system optimum. Additionally, we integrate the consumer surplus, producer surplus, and societal surplus into the game model to maximize social benefit. A cost-sharing contract under the system optimum between government and manufacturers is designed to increase the production of PPEs and hence, helps in reducing the number of infected individuals. We conducted our computational study on real data generated from the mask usage during the Covid-19 pandemic in Los Angeles (LA) County to respond to the reported PPE shortage. Under the socially optimal strategy, the PPE coverage increases by up to 33%, and the number of infected individuals reduces by up to 30% compared to other strategies.

1. Introduction
A pandemic such as Covid-19 can deeply impact the global trade of medical products. For instance, the U.S. trade deficit (that is the gap between exports and imports) expanded by about 20%, reaching a value of $8.65 billion (Federal Reserve Bank of St. Louis on the Economy Blog, Peterson Institute for International Economics). On the other hand, suppliers experience a sharp demand for essential medical items because of the speedy consumption of medical products during the pandemic. One of the most common challenges that people and healthcare workers face was limited access to personal protective equipment (PPE). PPE includes N95 respirators, surgical masks, gloves, eye and foot protection, protective hearing devices, hard hats, respirators and full-body suits, ventilators, test kits, swabs, sanitizer, and disinfectant.

Before the Covid-19 pandemic PPE used to be produced by only a few countries and the rest of the world were dependent on them for PPE production (Loey et al., 2021). During the Covid-19 pandemic, governments issue policies to support domestic production and distribution of PPE by providing economic subsidies to manufacturers for pivoting their operations to produce PPE. In the short time, multiple agencies, including government and private companies attempt to increase PPE supplies and productive capacities. However, in the long time, a balance between trade surplus and budget deficit should be considered.

The localization of critical medical industries builds a resilient supply chain strategy that not only responds to the PPE shortage during a pandemic, but also provides economic development opportunities for domestic manufacturers, specifically for small and medium-sized businesses. The employment growth increases with the economic fundamentals conducive to advanced manufacturing. This localization strategy can help manufacturers to increase the benefits by potentially reducing the production costs of PPE items. This strategy can also develop a skilled workforce that increases capabilities to create product innovations for the PPE market.

It is impossible to encourage manufacturers to produce PPE products when there are high costs to pivot their operations. An oligopolistic market is unable to achieve the socially optimal coverage without government subsidy (Geoffard & Philipson, 1997). Governments look at PPE not in the context of how much fund is needed to support the firms, but in the context of what is the impact if they do not have PPE.
The government’s budget is limited and needs to be assigned more effectively to reach the socially optimal coverage of PPE items. The government collaborates with manufacturers via procurement contracts to pivot their operations to produce PPE items. Government subsidies help domestic companies pay part of the cost of production. However, an optimal strategy is needed to efficiently allocate subsidies to manufacturers to achieve a social equilibrium coverage. Hence, the lack of a centralized coordination system for the PPE market may make these actions only partially successful.

Manufacturers also need to invest in pivoting their operations based on their business strategy. For example, firms need to have access to the source of prototypes for designing their products. Therefore, the government needs to assign subsidies to firms more effectively in order to reach an equilibrium coverage (Liu et al., 2021). In this study, we consider an equilibrium system between government and manufacturers that reduces the sum of expected operational and health costs for both players that are higher than if each player acts individually. The investment cost of companies on links is also considered to increase the production capacity of PPE products. The government allocates subsidies to firms based on firms’ performance in order to achieve an equilibrium coverage.

The lack of a centralized coordination strategy was experienced during the Covid-19 pandemic. While healthcare workers and people were concerned about the PPE supply, a response strategy to increase the PPE supply was absent. For example, only a few raw materials are needed to manufacture N95 masks. While these materials are readily available, the PPE shortage was reported during the Covid-19 pandemic by healthcare sectors (Shullih & LeMay, 2021). Therefore, a proactive contingency strategy needs to be prepared at the pre-disruption stage of the pandemic and can be implemented at the post-disruption stage (Elavarasan et al., 2021). In this paper, we propose a coordination system to respond to the Covid-19 types of pandemics that provides the socially optimal coverage in the PPE market.

An SIR model is an epidemiological model that estimates the number of infected individuals with an infectious disease in a closed population over time. There are three categories of population during any epidemic: “susceptible”, “infected”, and “removed/recovered” population. Jahedi and Yorke (2020) introduced a simple SIR model to estimate the number of infected people and the number of susceptible people using satellite equations. Most epidemiological models do not address manufacturing concerns. On the other hand, supply chain models often address logistical issues but do not consider the key characteristics of epidemiology. For instance, the number of infected individuals increases due to the shortage of PPE supply. Hence, this study incorporates the SIR model into the supply chain coordination problem to estimate the expected system costs based on epidemiological dynamics.
We use fear-sentiment information to estimate the percentage of individuals who have switching behaviors that is equivalent to the PPE burn rate for the entire population. Then, a supply-chain resilience model for PPE products is proposed that includes different stakeholders in terms of suppliers, manufacturers, consumers, and government. Furthermore, the impact of different strategies on the proposed game model is studied. Our contributions include the following epidemic management features:

• We propose a medical supply chain model with the epidemiology of the disease that coordinates manufacturers and the government to respond to medical items shortage.
• We investigate an actual demand price function by modeling the elasticity of demand and individuals' willingness to pay to avoid risks.
• We study a system problem between government and manufacturers that considers expected production, purchasing, and health costs due to the shortage of PPE products.
• We design a cost-sharing contract to allocate government subsidy to manufacturers that it provides incentives to both parties.
• A socially optimal strategy with the objective of maximizing social welfare is proposed to achieve the largest market coverage.

The rest of this paper is organized as follows: The literature is reviewed in Section 2. Section 3 presents a formal problem definition. Section 4 introduces the proposed coordination system for the PPE market during the Covid-19 types of pandemics. The numerical results of a real-world case study are given in Section 5. Finally, the concluding remarks are presented in Section 6.

2. Literature review

After the Covid-19 pandemic, domestic companies, including small, medium, and large sizes, are motivated by the government for pivoting their operations to produce essential medical items. Government and private companies put in their best efforts to make a balance between the supply and the demand of PPE products. Manufacturers need to have access to actual demands for PPE products during the pandemic. Companies are supported by the emergency response team in the duration of pandemic by estimating the demand projection of PPE items. However, small and medium companies face excess inventory buildup due to trading again with the global supply chain. Therefore, an equilibrium status between the demand and supply can solve the shortage of PPE products.

One way to estimate PPE demand for the entire population during a pandemic is to conduct a survey with a large number of interviews or questionnaires. However, the survey can be conducted at a single time, and it is expensive to conduct the survey again. For instance, the New York Times estimated mask usage by conducting a single survey New York Times Coronavirus Face Mask Map. Additionally, the estimation of PPE usage by multiplying the entire population and the rate of PPE usage per person is not an efficient method because the number of individuals who use PPE over time cannot be estimated by this method. Therefore, such these methods are not effective ways to show PPE usage per person. It is needed for a fraction of the population. In this study, we propose an equilibrium status between the demand and supply can solve the shortage of PPE products.

Several models have focused on combining SIR models and logistics models. For instance, Savachkin and Uribe (2012) proposed an SIR model to distribute the resources that minimizes the impact of pandemics on public healthcare sectors. Rachaniotis et al. (2012) investigated a scheduling model using an SIR model to allocate the limited medical resources during the pandemics. A dynamic logistics model for the allocation of medical resources is studied by Liu and Zhang (2016) that can be used to control the pandemics. Li et al. (2021) proposed an optimization formulation that incorporates an epidemiology model to consider the interactive effects of infectious disease dynamics and supply chain disruption. Rozhkov et al. (2022) investigated a simulation model that combines supply chain design, pandemic dynamics, and operational dynamics to consider the impacts of the Covid-19 pandemic on supply chains. In this study, we propose a game model between government and manufacturers that considers expected production cost, purchasing cost, and health cost due to the shortage of PPE items. Moreover, an optimization model based on the social welfare function is considered to achieve the largest coverage of PPE items during the pandemics.

Several studies address the vaccine supply chain; however, the supply chain of medical and protective equipment has different characteristics on the demand and supply sides. For example, Chandra and Vipin (2021) proposed centralized and decentralized systems for the vaccine supply chain where the government is a buyer, and the manufacturer is a supplier. Sinha et al. (2021) predicted demands to reduce vaccine wastage where a cost-effective multi-echelon inventory model is studied to protect the customer service level. Chick et al. (2008) proposed a game theory model to design a contract of influenza...
vaccination by considering the transmission rate and the rate of infected people. Bauch et al. (2003) considered individuals’ behavior into a game theory to evaluate the rate of individuals who will be vaccinated. Arifoglu et al. (2012) considered the impact of uncertainty on the demand side and supply side in the influenza vaccine supply chain by solving a game model. Mamani et al. (2012) studied the socially efficient coverage of vaccines by incorporating the government subsidy and vaccine producer under an oligopoly market. Alam et al. (2021) considered fifteen challenges to COVID-19 vaccine supply chain using fuzzy set method.

In this research work, for the first time, we propose a coordination system to respond to the PPE shortage that incorporates the special characteristics of epidemiology. We focus on the supply chain network model that considers manufacturer costs and government expenditure. We propose an equilibrium system that minimizes the system cost in terms of the expected production and health costs. Using the proposed model, we design a cost-sharing contract between manufacturers and the government that satisfies the goals of both parties. Hence, a cost-sharing contract under the system optimum condition is studied that increases the production of PPEs and hence, helps in reducing the number of infected individuals. Lastly, we integrate the consumer surplus, producer surplus, and societal surplus (the cost that may accrue to society, such as the burden of infection on the healthcare sector) into the model to maximize social welfare.

3. Problem definition

We denote our medical supply chain network by \( Q = [A, C] \) where \( A \) is the set of all the links and \( C \) is the set of nodes, see Fig. 1 for an illustration. The set \( C \) is a union of the three sets \( G \) the set of manufacturing companies, \( z \) the set of distribution centers, \( E \) the set of demand markets. A link in the network either connects a manufacturer company to a distribution center, or a distribution center to a storage center, or a storage center to a demand node. A path of links corresponds to the possible shipment of products from a firm to a demand node. Note that the index \( j \) is dropped for convenience since the proposed model can be solved for each PPE item \( j \in \{1, \ldots, J\} \). In this study, we consider our case study for face masks \( j = 1 \).

The set of links \( A = A_1 \cup A_2 \) is divided into two sets where \( A_1 \) is the set of links in which firms have capabilities to increase the production capacity of PPE items by investing in machines, designs, and prototypes. The second set \( A_2 \) contains links that have fixed production capacity of PPE items. The set of demand nodes \( E \) can be written as \( E = E_1 \cup E_2 \) where \( E_1 \) denotes the PPE demands of the entire population and \( E_2 \) represents the PPE demands of large facilities and public sectors such as hospitals, police and fire stations.

This study investigates a resilient supply chain model to coordinate the PPE market. We combine supply chain design, disease dynamics, and operational dynamics in the proposed game model. In this model it is assumed manufacturers and the government work together to respond to the PPE shortage. The proposed supply chain of PPE products is described in Fig. 2, which contains several modules. We consider a non-homogeneous susceptible individuals to estimate the consumption vector of PPE products for the entire population. Social media such as Twitter provides fear-sentiment information utilized by the epidemiological model to determine the ratio of people who have switching behaviors during the pandemics (Sayarshad, 2022). Based on the fear-sentiment information, we consider an epidemiological model with a heterogeneous susceptible population. We estimate the number of susceptible individuals who have low-risk behaviors, i.e., the number of individuals who use PPE products to protect themselves from the infection. This is equivalent to the consumption vector of PPE items for the entire population.

The proposed demand price function is calculated by multiplying the number of individuals with switching behaviors and the probability that individuals have the willingness to pay for PPEs (or masks) to avoid risks of the disease. Government and manufacturers collaborate during the pandemic to respond to the shortage. We study how different behavioral aspects of manufacturers and government can self-organize towards a system optimum. We investigate the proposed supply chain model to minimize the expected total system cost.

We consider the expected health cost due to the PPE shortage during the pandemics. When an individual gets infection, there are costs borne by the society. During the pandemics, individuals might change their behaviors by getting fear-sentiment information about the disease. We called them the individuals who have low-risk behaviors. This is equivalent to the consumption vector of PPE products for the entire population. Otherwise, if the individuals do not change their behaviors and act as usual, they have high-risk behaviors. Hence, the associated reduction in infections only depends on PPE coverage in the market. Lastly, a social welfare function is established by considering the producer surplus, consumer surplus in terms of the cost of PPE products for individuals and the societal surplus in terms of the burden of infection on healthcare sectors. The proposed supply chain model is considered in a way that maximizes social welfare. The proposed model has the following main applications:

• Using the proposed model, we can evaluate manufacturers' capabilities to pivot their operations to reach the most extensive equilibrium coverage of medical items after a pandemic. Thus, the proposed supply-chain resilience reduces the risk of excess inventory buildup where small and medium manufacturers can compete with large manufacturers to produce medical products.

• Using the proposed model, we can analyze how different behavioral aspects of manufacturers and government can self-organize towards a system optimum. Hence, the manufacturing emergency response team (MERT) can provide response and recovery plans for pandemics may cause for the society in the future.

• Using the proposed model, we can estimate the consumption vector for PPE products that helps manufacturers to predict the actual demands for PPE items over time.

• Using the proposed model, we can estimate the net demand for each traded PPE item by determining the total consumption vector and the total production quantity by domestic firms. This can be a great strategy for future pandemics where domestic
After the Covid-19 pandemic, many national governments have to rethink their offshoring strategies for the PPE products. A coordination system for the supply of PPE items can be developed using the proposed model that combines the domestic supply networks and the global trade networks. Hence, offshoring and onshoring strategies for the PPE market can be evaluated using the proposed model.

4. The proposed model

We investigate an equilibrium model between manufacturers and the government. The system problem (called Problem 1) is studied by the expected health cost and production cost between the government and manufacturers. A cost-sharing contract between manufacturers and government is considered to allocate government subsidies to manufacturers in order to achieve the socially optimal coverage. Hence, the aim of maximizing social welfare is more efficient. A socially optimal coverage to reach the largest market coverage under the objective of maximizing social benefit (called Problem 2) is also proposed. A connection between supply and demand functions is provided by implementing the proposed supply chain model, which includes an epidemiology model of the disease. Each of these elements is explained in the following sections.

4.1. Expected net manufacturer costs

The utility function of manufacturers is to minimize the total expected costs. The manufacturer picks a production quantity that minimizes the expected costs, as shown in Eq. (1). The first expression of Eq. (1) is the operational costs of links in the supply chain network. The second term of Eq. (1) is the total investment cost on the set of links that can increase the production capacity of PPE products. The last expression of Eq. (1) is the total revenue of manufacturer. Therefore, the expected net manufacturer costs is determined by

\[
\text{Min MC} = \sum_{n \in A} \sum_{p \in P} c_p(f_{n,p}) + \sum_{n \in A_1} \sum_{n \in A} \hat{c}_{n,p}(\Gamma_{n,p}) - \sum_{n} \hat{\delta}(Y_n)
\]

The optimal coverage of PPE products is calculated by minimum value between the production quantity on paths and a fraction of the aggregate demand for PPE products during each time period.

\[
Y_n = \min \left\{ \sum_{p \in P} x_{n,p} \delta_{n,p} \delta_p \sum_{e=1}^E d_{n,e} \right\} \forall n \in N, e \in E
\]

We convert Eq. (2) by the following constraints, which is an equivalent linear constraint. Let \( M \) be a big constant and define a new binary decision variable \( y \), which is either 1 or 0. Thus, we have

\[
y_n \leq \sum_{p \in P} x_{n,p} \delta_{n,p} \delta_p \sum_{e=1}^E d_{n,e}
\]

\[
y_n \leq \sigma_n d_n
\]

\[
y_n \geq \sum_{p \in P} x_{n,p} - M(1 - y)
\]

\[
y_n \geq \sigma_n d_n - M y
\]

The link flows of each manufacturer is equal to the sum of PPE products on paths that contain that link, thus we have

\[
f_{n,a} = \sum_{p \in P} x_{n,p} \delta_{a,p} \forall n \in N; \forall a \in A
\]

where \( \delta_{a,p} \) is equal 1 if link \( a \) is contained in path \( p \), and 0 otherwise.

Manufacturers increase their manufacturing capacity by investing in automation and technical technologies, including designing and testing prototypes. For instance, investment cost of adding capacity on a link can be estimated by the types of machines. A basic machine can produce 50 face masks per minute (or 32400 3-ply masks per day), while it is 400 face masks per minute (or 273600 3-ply masks per day) with a high-speed machine (Allan, 2022; Ilyukhina, 2022). Therefore, the link capacity can be increased as (Daniele & Sciaccia, 2021):

\[
\Gamma_{n,a} \leq \tilde{\Gamma}_{n,a} \forall n \in N; \forall a \in A_1
\]

where \( \tilde{\Gamma}_{n,a} \) is the upper bound on the added capacity by manufacturers on link \( a \). We now discuss how production capacity is related to product flow. Specifically, we assume that the production capacity at each link is a linear production function. Hence, we have that (Daniele & Sciaccia, 2021):

\[
f_{n,a} \leq \Gamma_{n,a} \forall n \in N; \forall a \in A_1
\]

Moreover, manufacturers have fixed existing production capacity on links. We consider that there is a bound on production capacity on the links, such that (Daniele & Sciaccia, 2021):

\[
f_{n,a} \leq \tilde{\Gamma}_{n,a} \forall n \in N; \forall \tau \in A_2
\]

4.2. Incorporation of an epidemiology model

This section combines supply chain design, disease dynamics, and operational dynamics to consider the impacts of pandemics on supply chains. The SIR model is one of the most basic models of disease transmission within a population. For instance, Jahedi and Yorde (2020) investigated an SIR model as shown in Eqs. (7a) and (7b) to estimate the ratio of individuals who are infectious \( I_n \) per week \( n \) and the ratio of individuals who are susceptible \( S_n \) per week \( n \). Susceptible individuals on average make \( g \) infectious contact which is the intensity function of Poisson distribution. According to Poisson distribution, the probability that no new infectious case happens in
week \( n \) is \( \exp(-\beta_n I_n) \). Hence, the ratio of new infectious cases and the ratio of susceptible individuals in week \( n \) is estimated by

\[
I_n = S_{n-1}(1 - e^{-\beta_n I_n}) \quad (7a)
\]

\[
S_n = S_{n-1} - I_n \quad (7b)
\]

Model E+ (7) suggests that we can calculate the contact rate in week \( n \) by

\[
\beta_{n-1} = \frac{1}{I_{n-1}} \ln(1 - \frac{I_n}{1 - \sum_{i=1}^{n-1} I_i}) \quad (8)
\]

The susceptible individuals on average \( \beta_n I_n \) infectious contact will vary over time, so it makes more sense to predict the intensity by an online prediction method based on a non-homogeneous Poisson process (NHPP) that can be considered in the future work (Sayarshad, 2015; Sayarshad & Chow, 2016).

Sayarshad (2022) studied a heterogeneous susceptible individuals where the population is divided into two groups. One group is the number of susceptible individuals with low-risk behaviors \( S^L_n \), while another group is the number of susceptible individuals with high-risk behaviors \( S^H_n \). The ratio of PPE (or mask) usage is equivalent of the ratio of susceptible individuals who have switching behaviors during the pandemics, thus we have (Sayarshad, 2022):

\[
S^L_n = \frac{S_n \exp(F_n \theta)}{1 + \exp(F_n \theta)} \quad \forall n \in N \quad (9a)
\]

\[
S_n = S^L_n + S^H_n \quad \forall n \in N \quad (9b)
\]

where \( \theta \) is a coefficient to amplify the fear-sentiment since it is between \([-1, 0]\). During the pandemics, the final sentiment information \( F_n \) contains the new sentiment information and the prior sentiment information that is affected by the memory performance. Hence, an individual is able to learn new information by \( \alpha \beta_{n-1} \), while the past information is being forgotten by \((1 - \alpha_0)F_n \). Lastly, the final sentiment, \( F_n \), is (Sayarshad, 2022):

\[
F_n = \alpha_0 \beta_n + (1 - \alpha_0)F_{n-1} \quad \forall n \in N \quad (10)
\]

Let \( c_1 \) represents the degree of learning and \( c_2 \) denotes the rate of forgetting of information, then after \( \eta_F \) the longest memory epoch, memory performance can be measured by \( a_0 = c_1 \cdot c_2^{\eta_F} \). More details about the fear-sentiment function can be found in Sayarshad (2022).

An epidemiology model only focuses on the epidemics of the disease and does not address manufacturing concerns. On the other hand, a resilient supply chain model needs to address logistical concerns that considers the key characteristics of epidemiology. For example, the number of infected individuals increases due to the PPE shortage. We incorporate the number of PPE coverage in the epidemiology model of the disease to estimate the infectious population due to the supply shortage. Hence, the number of infected individuals who have low-risk behaviors during the pandemics is formulated by

\[
I^L_n \geq \left[ S^L_{n-1} - Y_{n-1} \right] \left[ 1 - e^{-\beta_n I^L_{n-1}} \right] \quad \forall n \in N \quad (11)
\]

Similarly, the additional constraint that estimates the number of infected individuals who have high-risk behaviors is:

\[
I^H_n \geq \left[ S^H_{n-1} + \left[ S^L_{n-1} - Y_{n-1} \right] \left[ 1 - e^{-\beta_n I^H_{n-1}} \right] \right] \quad \forall n \in N \quad (12)
\]

Moreover, the total number of infected individuals is the sum of the number of infected individuals with low-risk and high-risk behaviors. Hence, we have

\[
I_n \geq I^L_n + I^H_n \quad \forall n \in N \quad (13)
\]

4.3. Demand price function

After the Covid-19 pandemic, the demand for medical items such as PPE items goes up, and the PPE market experiences a shortage of supply. Domestic companies, including small, medium, and large sizes, are invited by the government to pivot their operations to produce the medical items. After the shortage of PPE supply, small and medium companies have the demand issue and excess inventory buildup (Freight-Waves). Hence, the actual demand for PPE was absent in the supply chain resilience models. This section estimates the demand projection for PPE products under the assumption of elastic demand. The demand price function is determined by multiplying the number of individuals with switching behaviors \( S^L_n \) and the probability that individuals are willing to pay for PPEs (or masks) to avoid the risk of the disease.

\[
d_{e,e} = S^L_n \left[ e^{\gamma \nu-e^{-\lambda-1}} \right] e^{\gamma \nu-e^{-\lambda-1}} \quad \forall n \in N, \; e = 1 \quad (14)
\]

where the probability that individuals are willing to pay and use PPE products is determined by Theorem 1.

The PPE is also a critical item for large facilities such as hospitals, police stations, and fire stations. Hence, PPE demand for health-care sectors can be computed by the PPE burn rate at hospitals as follow:

\[
d_{e,e} = k_1 e_0 I_n + k_2 H_n \quad \forall n \in N, \; e = 2 \quad (15)
\]

where \( e_0 \) is the ratio of hospitalization, \( k_1 \) is the burn rate of PPE items for infected individuals, and \( k_2 \) is the burn rate of PPE items by staff at healthcare sectors, including hospitals and medical health centers.

We consider the infection loss of individuals to estimate the probability that individuals are willing to pay for PPE products to avoid the risk of disease. For example, we investigate mask usage by individuals in society as shown in Fig. 3. Suppose that a population consisting of \( Q \) individuals is considered. Let \( \sigma_{\eta} \) denote the fraction of mask coverage in the society. The infection loss of an individual is \( \delta \) and \( \sigma \) is consumer price of mask (or PPE) for an individual, which is calculated by multiplying the number of mask usage per week and the mark price.

The infection result of every individual can be one of the four categories: (a) individuals who wear a mask and get the infection, (b) individuals who wear a mask and are healthy, (c) individuals who do not wear a mask and get the infection, and (d) individuals who do not wear a mask and are healthy. The ratio of infected individuals who have low-risk behaviors is \( I^L_n \) and the ratio of infected individuals who have high-risk behaviors is \( I^H_n \). Hence, the expected infection cost for individuals who wear a mask or do not wear a mask is calculated by \( I^L_n (\delta + \sigma) + \sigma (1 - I^L_n) \) and \( \delta I^H_n \), respectively.

\[
I^L_n (\delta + \sigma) + \sigma (1 - I^L_n) = \delta I^H_n
\]

\[
\sigma + \delta I^L_n = \delta I^H_n
\]
But the infection loss of the marginal individual is determined by Eq. (16).

\[ \delta_n = \frac{C}{I^u_n - I^L_n} \]  

(16)

where \( \delta_n \) is the infection loss of the marginal individual in period \( n \).

The following theorem is considered to calculate the probability that individuals want to join the system and use the PPE items.

**Theorem 1.** If the infection loss of marginal individual is higher than the previous time \( t_n - 1 \), \( \delta_n > \delta_{n-1} \), the individual selects to wear a mask (or use PPE). Hence, the probability that individuals are willing to use the PPE products to avoid the risk of infection is

\[ P(\delta_n > \delta_{n-1}) \leq e^{\lambda_n(e^{\alpha-1} - 1)} e^{\lambda_n(e^{\alpha-1} - 1)} \]  

(17)

**Proof.** Let \( X, Y \) be random variables with Poisson distributions where the intensity function of infection is defined by \( \lambda_{n-1} = \beta_{n-1} I_{n-1}^u \) and \( \lambda_n = \beta_n I_n^u \), respectively. We define \( X \equiv I_n^u - I_{n-1}^u = S^u_{n-1} [1 - e^{-\beta_{n-1} I_{n-1}^u}] \) and \( Y \equiv I_n^u - I_{n-1}^u = S^u_{n-1} [1 - e^{-\beta_n I_{n-1}^u}] \) based on Eqs. (16) and (7a), for any \( \gamma \geq 0 \), we have

\[ P(\delta_n > \delta_{n-1}) = P(Y \geq X) = P(e^{-\gamma X} \geq 1) \leq E[e^{-\gamma X}] = e^{-\gamma (X, Y)} \]

Using Markov inequality and independence, we have \( P(X \geq a) = \frac{E[X]}{a} \). \( E[e^{\gamma X}] \) is the moment generating function of \( Y \) and the same for \( X \) (Ross, 1998). By substituting the moments of the general function, we get a bound that is equal to \( e^{\lambda_{n-1}(e^{\alpha-1} - 1)} e^{\lambda_n(e^{\alpha-1} - 1)} \). Thus, we finally have \( P(\delta_n > \delta_{n-1}) \leq e^{\lambda_{n-1}(e^{\alpha-1} - 1)} e^{\lambda_n(e^{\alpha-1} - 1)} \).

### 4.4. Expected government costs

In response to the pandemics, the government allocates subsidies to help manufacturers to pivot their operations. The government expenditure increases due to collaboration with manufacturers. Hence, the goal of the government problem is to minimize the expected production cost and the expected health cost (in terms of the burden of infection on the health-care sector due to PPE shortage).

After a pandemic, the new infection imposes a burden on the health care system. The health care sector is not adequately prepared to deal with it first, but their knowledge will change over time. Hence, the burden on the health care system due to PPE shortage is determined by:

\[ \phi(I_n^u) = u_t e^{\lambda_n (S^L_n - Y_n) I_n^u} \]  

(18)

Based on Eq. (19) given by Gonzalez-Eiras and Niepelt (2020) and Sayarshad (2022), \( u_t \) is calibrated by the damage of the pandemic on the economic of a country. For example, in the USA, it was about 13 billion dollars which is corresponding to 61% of the annual U.S. GDP (\( \rho = 0.61 \)). It is assumed that the health care sector can be prepared to deal with the pandemic during a half of year or \( (365/2) = 182 \) days.

\[ u_t \int_0^{182} e^{-\lambda_n (S^L_n - Y_n) I_n^u} \, dn = 365 \rho \]  

(19)

The second term of (21) is the expected purchasing cost for PPE products by government, where the PPE coverage per week by manufacturers is computed by Eq. (2). Moreover, manufacturers keep the optimal production level when the expected revenue exceeds the cost per unit, because the PPE production is profitable for firms. The revenue ratio is a measure of efficiency that compares a manufacturer’s expenses to its earnings. Hence, we consider constraint (20) which is based on the cost revenue ratio to ensure that manufacturers act optimally (Chick et al., 2008).

\[ \sum_{n=1}^N \sum_{a \in A} \gamma_a \delta_n \sum_{e \in E} f_a(e) + \sum_{n=1}^N \sum_{a \in A} \bar{\gamma}_a \delta_n (\Gamma_n)^a \sum_{e \in E} \gamma_n Y_n \]  

(20)

Formally, the government problem is

\[ \operatorname{Min} \left\{ \sum_{n=1}^N u_t e^{\lambda_n (S^L_n - Y_n) I_n^u} + \sum_{n=1}^N \delta_n (\Gamma_n)^a \right\} \forall n \in N \]  

(21)

Subject to:

\[ Y_n = \min \left\{ \sum_{n=1}^N \sum_{a \in A} x_a e^\lambda \delta_n \sum_{e \in E} f_a(e) + \sum_{n=1}^N \delta_n (\Gamma_n)^a \right\} \forall n \in N \]  

\[ f_a(e) = \sum_{n=1}^N x_a e^\lambda \delta_n \sum_{e \in E} f_a(e) \forall n \in N \]  

\[ \Gamma_n = \sum_{n=1}^N \sum_{a \in A} \bar{\gamma}_a \delta_n (\Gamma_n)^a \sum_{e \in E} \gamma_n Y_n \]  

(22)

\[ \sum_{n=1}^N \sum_{a \in A} \bar{\gamma}_a (\Gamma_n)^a \sum_{e \in E} \gamma_n Y_n \]  

4.5. Total system cost

This section considers an equilibrium model between manufacturers and the government. As shown in Eq. (22), the system problem between government and manufacturers is determined by the purchasing cost, the expected health cost, and production cost. The first expression of Eq. (22) is the expected purchasing cost by the government. The second term of Eq. (22) is the burden of infection on the healthcare sector due to the shortage of PPE supply. The third expression of Eq. (22) is the operation costs of links to produce PPE products. The last term of Eq. (22) is the investment cost of links to increase the capacity. Thus, the system problem is:

**Problem 1 (System Problem).**

\[ \operatorname{Min} \left\{ \sum_{n=1}^N \delta_n (\Gamma_n)^a \sum_{e \in E} f_a(e) + \sum_{n=1}^N \delta_n (\Gamma_n)^a \right\} \forall n \in N \]  

(22)

Subject to:

\[ Y_n = \min \left\{ \sum_{n=1}^N \sum_{a \in A} x_a e^\lambda \delta_n \sum_{e \in E} f_a(e) + \sum_{n=1}^N \delta_n (\Gamma_n)^a \right\} \forall n \in N \]  

\[ f_a(e) = \sum_{n=1}^N x_a e^\lambda \delta_n \sum_{e \in E} f_a(e) \forall n \in N \]  

\[ \Gamma_n = \sum_{n=1}^N \sum_{a \in A} \bar{\gamma}_a \delta_n (\Gamma_n)^a \sum_{e \in E} \gamma_n Y_n \]  

(22)

Subject to:

\[ Y_n = \min \left\{ \sum_{n=1}^N \sum_{a \in A} x_a e^\lambda \delta_n \sum_{e \in E} f_a(e) + \sum_{n=1}^N \delta_n (\Gamma_n)^a \right\} \forall n \in N \]  

\[ f_a(e) = \sum_{n=1}^N x_a e^\lambda \delta_n \sum_{e \in E} f_a(e) \forall n \in N \]  

\[ \Gamma_n = \sum_{n=1}^N \sum_{a \in A} \bar{\gamma}_a \delta_n (\Gamma_n)^a \sum_{e \in E} \gamma_n Y_n \]  

(22)
H.R. Sayarshad

Fig. 4. The proposed algorithm.

\[ d_{n,e} = S_L^e \left[ e^{\lambda_n (\tau - 1)} e^{\lambda_n (\tau - 1)} \right], \forall n \in N, \quad e = 1 \]
\[ d_{n,e} = k_1 e_{1n} L + k_2 e_{2n}, \quad \forall n \in N, \quad e = 2 \]
\[ I_n^L \geq [S_{n-1} - Y_{n-1}] [1 - e^{\lambda_n (\tau - 1)}] \quad \forall n \in N \]
\[ I_n^H \geq [S_{n-1} + [S_{n-1} - Y_{n-1}]] [1 - e^{\lambda_n (\tau - 1)}] \quad \forall n \in N \]
\[ S_n^L \geq Y_n, \forall n \in N \]
\[ I_n \geq I_n^L + I_n^H, \forall n \in N \]
\[ S_n = S_{n-1} - I_n, \forall n \in N \]
\[ S_n^L = S_n \frac{\exp(F_n \theta)}{1 + \exp(F_n \theta)}, \forall n \in N \]

Cost-sharing contracts

An oligopolistic market without government subsidies cannot reach the socially optimal coverage of PPE products. The government can offer subsidy to manufacturers that helps to achieve the demand-supply equilibrium. For instance, the government pays the proportion of total production costs in terms of the operation costs and the investment costs on links. Hence, the government cost function increases under cost-sharing contracts as follows:

\[ SC = \sum_n \alpha_n e^{-\lambda_n} (S_n^L - Y_n) I_n^L + \sum_n \hat{\delta}(Y_n) + \theta \left( \sum_{a \in A} \hat{c}_a(f_{a,n}) + \sum_{a \in A_1} \hat{\pi}_a(\Gamma_{n,a}) \right) \]

\[ (23) \]

where \( \theta \) is the percentage of coverage costs, \( 0 \leq \theta \leq 1 \). The proportion of production costs can be shared by the government under the cost-sharing contract. Moreover, manufacturers keep the optimal production level under cost-sharing contracts as Eq. (24). Other constraints of Problem 1 remain unchanged.

\[ \sum_{n \in N} d_{n,e} = \left( 1 - \theta \right) \frac{\sum_{a \in A} \hat{c}_a(f_{a,n}) + \sum_{a \in A_1} \hat{\pi}_a(\Gamma_{n,a})}{\theta Y_n}, \forall n \in N \]

\[ (24) \]

Therefore, a mechanism to allocate government subsidies is proposed that satisfies manufacturers and government and it helps to achieve a higher coverage of PPE during pandemics.
4.6. Social welfare function

An optimal social coverage of PPE products is considered in this section that integrates producer surplus, consumer surplus, and societal surplus under the objective of maximizing social welfare.

4.6.1. Producer surplus

In order to determine producer surplus, we consider the difference between the operational costs of links and the revenue of firm. Hence, we have

\[ \text{Producer surplus} = \left( \sum_{n} S_{n}^{L} \eta_{n} Y_{n} - \sum_{a \in A} \sum_{L} \hat{c}_{a}^{L}(f_{a,n}) - \sum_{n} \sum_{a \in A} \hat{a}_{n,a}(f_{n,a}) \right) \]

(25)

Let \( I_{n}^{L} \) be the ratio of infected individuals with low-risk behaviors and use PPE products to avoid the risk of infection. \( I_{n}^{H} \) the ratio of infected individuals with high-risk behaviors. The ratio of infection depends jointly on the ratio of PPE coverage in the market and the ratio of susceptible individuals who have high-risk and low-risk behaviors (Bauch & Earn, 2004). Under the PPE coverage for the population \( \sigma_{n} \), the ratio of infected individuals for the entire population is defined by:

\[ I_{n} = \sigma_{n} I_{n}^{L} + (1 - \sigma_{n}) I_{n}^{H} \quad \forall n \in N \]

After simplification, we get

\[ I_{n}^{H} - I_{n}^{L} = \frac{I_{n} - I_{n}^{L}}{1 - \sigma_{n}} \quad \forall n \in N \]

(26)

The willingness to pay for any consumer product depends on Eq. (26). Substituting these and letting \( \eta_{n} = (1 - \sigma_{n}) \left( I_{n}^{L} - I_{n}^{H} \right) \), the equilibrium price of PPE products is

\[ \eta_{n} = I_{n} - I_{n}^{H} \quad \forall n \in N \]

Note that the equilibrium price is associated with the optimal market coverage where the ratio of infection depends on PPE coverage \( \sigma_{n} \).

In other words, the equilibrium price of PPE products is the marginal reduction of infection due to use of PPE products by individuals.

4.6.2. Consumer surplus

We consider heterogeneous susceptible individuals for PPE consumers during pandemics. The consumer surplus is defined by the total expected cost of individuals willing to use PPE products. Thus, we have

\[ \text{Consumer surplus} = \sum_{n} \eta_{n} \left( S_{n}^{L} - I_{n}^{L} \right) \]

(28)

4.6.3. Government surplus

In addition to the costs incurred by individuals and the manufacturers for producing the PPE items, we also consider the expected health cost that may accrue on the society due to the shortage of PPE supply during the pandemics. The expression in the right-hand side of Eq. (29) is the burden of infection on the health care system where it is proportional to the impact of PPE shortage on infectious population \( S_{n}^{L} - Y_{n} \) \( I_{n}^{L} \) and a factor \( u_{i} \) that shows the health-care performance during the pandemic.

\[ \text{Government surplus} = \sum_{n} u_{i} e^{-u_{i} Y_{n}} \left( S_{n}^{L} - Y_{n} \right) I_{n}^{L} \]

(29)

where \( u_{i} \) shows the health care cost due to the stress of health care and \( u_{i} \) denotes the efficiency level or the speed of learning related to the healthcare sector. Combining all these together, the social welfare is defined as follows:

**Problem 2 (Social Welfare Problem).**

\[ \text{MaxSW} = \sum_{n} S_{n}^{L} \left( I_{n} - I_{n}^{L} \right) Y_{n} - \sum_{n} \sum_{a \in A} \hat{c}_{a}^{L}(f_{a,n}) - \sum_{n} \sum_{a \in A} \hat{a}_{n,a}(f_{n,a}) \]
\[ - \sum_n f_n e^{-\alpha_n} \left( S_n^L - Y_n \right) I_n^L - \sum_n \sigma \left( S_n^L - I_n^L \right) \]  

Subject to:
\[ Y_n = \min \left\{ \sum_{e\in E} x_{n,e} \gamma_{e,n} + \sum_{e\in E} d_{n,e} \right\} \forall n \in N, \forall e \in E \]
\[ \sigma \sum_{e\in E} d_{n,e} = \sum_{e\in E} \gamma_{e,n} + \sum_{e\in E} \delta_{e,n} \left( I_{n,e} \right) \forall n \in N \]
\[ f_{n,e} = \sum_{n\in N} x_{n,e} \delta_{e,n} \forall n \in N; \forall e \in A \]
\[ \Gamma_{n,e} \leq T_{n,e} \forall n \in N; \forall e \in A \]
\[ f_{n,e} \leq T_{n,e} \forall n \in N; \forall e \in A \]
\[ f_{n,e} \leq T_{n,e} \forall n \in N; \forall e \in A \]
\[ d_{n,e} = S_{n}^L \left[ e^{-\lambda_{n,e}} - 1 \right] \delta_{n,e} \left( I_{n,e} \right) \forall n \in N, \forall e = 1 \]
\[ d_{n,e} = k_1 \delta_{n,e} I_{n,e} + k_2 H_{n,e} \in N, \forall e = 2 \]
\[ I_{n,e} \geq \left[ S_{n,1,e}^L - Y_{n,1,e} \right] \left[ 1 - e^{-\gamma_{n,1,e} - 1} \right] \forall n \in N \]
\[ I_{n,e} \geq \left[ S_{n,1,e}^L + \left( S_{n,1,e}^L - Y_{n,1,e} \right) \right] \left[ 1 - e^{-\gamma_{n,1,e} - 1} \right] \forall n \in N \]
\[ S_{n,e} \geq Y_{n,e} \forall n \in N \]
\[ I_{n,e} \geq I_{n,e}^L \forall n \in N \]
\[ S_{n,e} = S_{n,e} - I_{n,e} \forall n \in N \]
\[ S_{n,e} = \exp \left( F_{\theta} e \right) \frac{1}{1 + \exp \left( F_{\theta} e \right)} \forall n \in N \]

4.7. Formulating the proposed model in the form of a stage-by-stage algorithm

The proposed coordination system for PPE market including the epidemiology model can be summarized as the following stage-by-stage algorithm:

- **Stage 1:** Input parameters including \( c_1, c_2, n_F, u_1, u_2, \theta_1 \), and time step \( n \in N \) should be specified. Detailed information about other input parameters is shown in Table 1.
- **Stage 2:** We estimate the number of infected individuals \( I_n \) and the number of susceptible individuals \( S_n \) using Eqs. (7a) and (7b), respectively.
- **Stage 3:** We estimate the temporary fear-sentiment information \( \rho_n \) using Twitter data, which its details can be found in Sayarshad (2022).
- **Stage 4:** We compute the final fear-sentiment information using Eq. (10) where memory performance of individuals during the pandemic for learning and forgetting information about the disease is considered.
- **Stage 5:** The number of susceptible individuals who have low-risk behaviors \( S_n^L \) is estimated by Eqs. (9a). This is equivalent to the PPE burn rate for the entire population.
- **Stage 6:** We determine the demand price function for the entire population and large facilities such as hospitals and medical centers using Eqs. (14) and (15), respectively.
- **Stage 7:** We solve the proposed game model under different strategies, as shown in Section 4.
- **Stage 8:** If \( n \) has reached the last step of the study period, go to the next stage. Otherwise, increment \( n (n = n + 1) \) and go back to stage 2.
- **Stage 9:** Report results: the link flow of PPE item on link \( f_{n,e} \), the equilibrium path flow \( x_{n,e} \), the investment cost of adding capacity on a link \( I_{n,e} \), the quantity coverage of PPE item \( Y_n \), infected individuals with low-risk behaviors \( I_n^L \), infected individuals with high-risk behaviors \( I_n^H \), the fraction of coverage \( \sigma_n \).

By implementing the above process for all time periods, we can determine the equilibrium coverage of PPE products based on different strategies. A flowchart of the proposed model is shown in Fig. 4.

5. Experimental results

Experimental tests are performed to study the efficiency of the proposed model. We investigate a case study in LA County under different strategies to show the efficiency of the coordination supply chain model in a real-world case. We used a laptop computer with a 2.50 GHz processor, Intel Core i7-2450, 12 GB of RAM, and 64 bit Windows 10 Professional operating system. The model was coded in Julia 1.7.1 and we solved the mathematical models using the Juniper package (Kröger et al., 2018), Jump package (Lubin & Dunning, 2015), and Ipopt package in Julia.

5.1. Data

We carried out an experimental study of the Covid-19 pandemic using the proposed model for mask coordination based on real-world data. The list of our data used for the case study is presented below:

- Individuals change their behaviors by following social distancing and wearing a mask. Therefore, actual mask usage in LA County can be estimated by the rate of individuals who have low-risk behavior. Similar to Sayarshad (2022), we apply the fear-sentiment information to calculate the percentage of individuals who have switching behaviors. Fig. 5 shows mask usage during Covid-19 in LA County for 36 weeks.
- The demand for the mask in large facilities such as the hospitals and medical centers in LA County is calculated by Eq. (15) which is based on the mask burn rate per shift (Centers for Disease Control and Prevention: PPE burn rate) and the number of staff at hospitals and medical centers in LA County (Department of Public Health, Hospitals and Medical Centers). The burn rate of mask per week for each individual is assumed to be \( k = 3 \) and the consumer price for face mask is assumed to be $1.5. The burn rate of mask for infected individuals per week is assumed to be \( k_1 = 7 \). We assume that the burn rate of masks per week for staff in hospitals and medical centers is \( k_2 = 14 \) (Centers for Disease Control and Prevention: PPE burn rate).
- The number of Covid-19 confirmed positive cases in LA County that is used in the SIR model, can also be found in the USAFacts dataset (USAFACTS).
- The health care cost due to the stress of health care is \( u_1 = 265 \), and the speed of learning for the health care service is assumed to be \( u_2 = -\ln(0.5)/182 \) (Sayarshad, 2022). Coefficient to amplify the final sentiment is assumed to be \( \theta = 10 \) (Sayarshad, 2022).
5.2. Illustrative example

Our supply chain network consists of one firm, six manufacturers, two distribution centers, two distribution storage centers, and two demands market, see Fig. 6 for an illustration. The input parameters of our case study are shown in Table 1. The information contains the set of links, the set of paths from firm until the demand nodes and the production capacities on links under different capacity. We assume that the cost function for operation cost is given:

\[ c(f, n) = c_a + 1.1f_{a,n} \]  

The cost function for the investment capacity is assumed as:

\[ \pi(n, \alpha)(G_{n, \alpha}) = \pi_a + 1.2G_{n, \alpha} \]  

Moreover, the total revenue of production is assumed to be:

\[ \delta(Y_n) = \delta Y_n \]  

The parameters related to the cost functions are also presented in Table 1.

In this study, we proposed a demand price function that considers the elasticity of demand. The demand function is determined by multiplying the number of individuals with switching behaviors \( S_L \) and the probability that individuals have the willingness to pay for masks to avoid getting infected. The aggregate demand for masks in hospitals and the whole population is shown in Fig. 7. We investigate our game setting using three strategies as follow:

- **Strategy 1 (MC):** minimizing net manufacturer costs.
- **Strategy 2 (SC):** minimizing total system costs (Problem 1).
- **Strategy 3 (SW):** maximizing social welfare (Problem 2).

We solve our case study under the first strategy that minimizes net manufacturer costs. The CPU run-time in seconds of the proposed model for the test case is about 6 s. The equilibrium links flow of mask production for one week is shown in Table 2. We experienced a higher number of infection cases in LA County during the selected week. The equilibrium path flows of mask products is presented in Table 2. According to the results of strategy 1, the production capacity on links that have capabilities to increase the capacity is zero. The results show that a monopoly market is unable to solve the PPE shortage during a pandemic when manufacturers act individually. Hence, the government needs to allocate subsidies to firms to achieve the largest coverage of PPE products.

We find an equilibrium coverage of mask products where the government and firms collaborate together to respond to mask shortages during the pandemic. The equilibrium links flow of mask production for one week is shown in Table 3. The equilibrium path flows of mask products is presented in Table 3. The production capacity on links is demonstrated in Table 3. The CPU run-time in seconds of the proposed model for the test case is about 10 s.

Lastly, the socially optimal coverage for coordinating masks to maximize social welfare is solved. The equilibrium links flow of mask production for one week is shown in Table 4. The equilibrium path flows of mask products are presented in Table 4. The production capacity on links is demonstrated in Table 4. The CPU run-time in seconds of the proposed model for the test case is about 20 s.

5.3. Equilibrium behavior evaluation

In this section, we evaluate the proposed game model by comparing three strategies that were mentioned in Section 5.2. We compare the optimal number of coverage \( Y_n \) when different strategies are investigated. As shown in Fig. 8, the maximum mask coverage is found
when social welfare is maximized. On the other hand, the minimum mask coverage is observed when net manufacturing costs is minimized. The number of coverage under system optimum is also presented in Fig. 8 where the government collaborates in mask production ramp up to combat pandemics. It is interesting to note that the manufacturers produce a fixed production quantity per week when manufacturers act individually. Hence, the capacity on links that can be increased through investments is zero, as shown in Table 2. Under socially optimal level, the number of coverage increases by 34% and 2% compared to strategy 1 and strategy 2, respectively. Moreover, we compare the fraction of mask coverage $\sigma_m$ where three strategies are studied in strategy one and strategy two, respectively. Interestingly, we can respond to mask shortages more efficiently when the government collaborates with firms under the system problem and social welfare maximization.

Finally, three strategies are evaluated by comparing the number of infections. Under the socially optimal level, the minimum number of infections is found, as shown in Fig. 10. On the other hand, the number of infection cases increases under the objective of minimizing the net manufacturing costs. Hence, the number of infections reduces under government intervention due to increased mask supply. As shown in Table 5, the number of infected individuals under the goal of maximizing social welfare was reduced by 30% and 10% compared to strategy one and strategy two, respectively.

### Table 2

| $f_{m1}$ | $f_{m2}$ | $f_{m3}$ |
|----------|----------|----------|
| $1814176.3$ | $2002103.9$ | $1613716.5$ |
| $702104.9$ | $1299999$ | $613717.5$ |
| $2499999.0$ | $2299999.2$ | $120335.6$ |
| $1146469.5$ | $1185329.6$ | $221911.5$ |

### Table 3

| $f_{m1}$ | $f_{m2}$ | $f_{m3}$ |
|----------|----------|----------|
| $1085450.9$ | $2030485.8$ | $1679560.3$ |
| $1213849.2$ | $1085450.9$ | $730485.8$ |
| $679560.3$ | $1000000.0$ | $2500000.0$ |
| $1219525.4$ | $1284074.6$ | $1792171.5$ |

### Table 4

| $f_{m1}$ | $f_{m2}$ | $f_{m3}$ |
|----------|----------|----------|
| $2190072.4$ | $1045077.0$ | $190088.9$ |
| $1935267.4$ | $1512636.6$ | $1034907.8$ |
| $98818.9$ | $946269.1$ | $53869.8$ |
| $53914.5$ | $137951.3$ | $655194.5$ |
| $576546.9$ | $974176.7$ | $2473254.2$ |
| $1218345.1$ | $1254907.3$ | $2216352.9$ |

### Table 5

| $f_{m1}$ | $f_{m2}$ | $f_{m3}$ |
|----------|----------|----------|
| $50923.8$ | $528984.0$ | $551212.4$ |
| $49343.3$ | $49434.7$ | $455159.9$ |
| $26934.3$ | $26935.6$ | $67927.1$ |
| $26956.2$ | $33145.2$ | $68824.1$ |
| $32377.2$ | $32377.3$ | $604949.9$ |
| $285365.3$ | $291181.6$ | $606437.2$ |

### 5.4. Cost-sharing contract and evaluation

In this section, we consider a cost-sharing contract that increases the production of PPEs and hence, helps in reducing the number of infected individuals. The government needs to pay the proportion of production costs to firms. Otherwise, the cost of infections and the burden of infection on the healthcare sector can be increased. Therefore, the government collaborates with manufacturers via a cost-sharing contract to
Fig. 8. Optimal PPE coverage using different strategies.

Fig. 9. Optimal fraction of coverage in the market.

Fig. 10. The number of infection under different strategies.
increase the supply of mask (or PPE) products. In order to compare the cost-sharing contract between manufacturers and the government, we evaluate the proposed game model based on the following strategies:

- **Before contract**: manufacturers act individually to produce PPE products during the pandemic.
- **After contact, 70% costs**: manufacturers collaborate with the government and 70% of production costs can be paid by the government.
- **After contact, 100% costs**: manufacturers collaborate with the government and 100% of production costs can be paid by the government.

The results of coverage under different actions between government and manufacturers are shown in Fig. 11. Manufacturers collaborate with the government to produce masks where 70% and 100% of production costs are paid by the government. Note that the system optimum cannot be fully achieved when costs coverage by government is less than 70%. As shown in Table 6, the fraction of mask coverage when manufacturers act individually to produce masks decreases by 32%, while the optimal fraction of coverage increases by 32% and 27% when the government pays 70% and 100% of production costs, respectively. The proposed model can estimate the optimal government subsidy that needs to achieve an optimal equilibrium coverage of PPE products. Moreover, a balance between trade surplus and budget deficit can be determined by the proposed strategy in order to deal with the global supply chain.

We evaluate the optimal number of mask coverage where manufacturers act individually or collaborate with the government via a cost-sharing contract. The results of actions between both parties are shown in Fig. 12. As shown in Table 6, the optimal number of coverage with government involvement increases by up to 32% compared to a strategy without government intervention. Therefore, without government subsidies, an oligopolistic market cannot reach the socially optimal coverage.

Furthermore, we compare the number of infected individuals that is related to the mask coverage in the market. The number of infected individuals under different actions between government and manufacturers is shown in Fig. 13. As shown in Table 6, when the government intervenes, the number of infected individuals decreases by up to 19% compared to a strategy when manufacturers act individually. The Covid-19 infection wave has caused 13 trillion dollars costs in the U.S. which this amount corresponds to 61% of the annual U.S. GDP (Scherbina, 2020). The economic burden of healthcare associated with new infections that the government pays for is extremely high. Hence, the cost-sharing contract can optimize the coordination between manufacturers and the government.

### 6. Conclusion

The first coordination model of the PPE supply chain was proposed that investigates the socially optimal coverage using the subsidy. We proposed a medical supply chain model with the epidemiology of the pandemic that considers expected production cost, purchasing cost, and health cost due to the shortage of PPE products. We evaluated different strategies between government and manufacturers where the elasticity of demand and willingness to pay to avoid risks were investigated. A socially optimal coverage model with the objective of maximizing social welfare was also proposed to achieve the largest market coverage. Compared to other strategies, the fraction of coverage under the socially optimal level increased by up to 33%. Additionally, the number of infected individuals under social welfare maximization was reduced by 30%.

An oligopolistic market cannot achieve socially optimal coverage without government subsidies. Therefore, a cost-sharing contract to allocate government subsidies to manufacturers was investigated that increased production, leading to a reduction in infection cases. The results showed that the optimal number of coverage with government involvement increased by up to 32% compared to a strategy without government intervention. Additionally, the number of infected individuals with government intervention decreased by up to 19% compared to when manufacturers acted individually.

The proposed model can be extended in the future research works, as below:

- After the Covid-19 pandemic, many countries have to rethink their offshoring strategies for the PPE market. The net demand for each traded PPE item can be estimated by the total consumption vector for the PPE item and the total production quantity of PPE items that should be produced by the domestic manufacturing firms (in terms of private sectors and public sectors). Hence, different response plans in terms of trading with the global supply chain and considering a localization strategy by domestic firms would be evaluated in future research.

### Table 5

| Strategies | Relative difference (%) | Relative difference (%) | Relative difference (%) |
|------------|--------------------------|--------------------------|--------------------------|
| MC         | 0.577 -33%               | 4799998.2 -34%           | 8247.1 +30%              |
| SC         | 0.844 -2%                | 7175440.8 -2%            | 6952.6 +10%              |
| SW         | 0.857 -                     | 7285514.4 -              | 6325.6 -                    |

Fig. 11. The fraction of mask coverage.
An international trade resilience with the epidemiology of the disease can be investigated in future research where countries compete for limited supplies of PPE items to maximize social benefits (Li et al., 2021; Salarpour & Nagurney, 2021). The balance of trade (in terms of trade surplus and trade deficit) with the goal of social benefit could be investigated in the future study in order to find the best strategies for future pandemics (Nader et al., 2022).

A blockchain technology that provides smart contracts and improves the transparency of information between stakeholders, including suppliers, market demand, and government, would be considered in future research (Bhushan et al., 2020; Kumar et al., 2020).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All dataset sources are cited in the article.

Acknowledgments

This research did not receive any grant from funding agencies in the public, commercial, or not-for-profit sectors. The author is grateful to the Editor in Chief of the journal, Guest Editor, and two anonymous reviewers, for their valuable comments.

References

Alam, S. T., Ahmed, S., Ali, S. M., Sarker, S., Kabir, G., & ul Islam, A. (2021). Challenges to COVID-19 vaccine supply chain: Implications for sustainable development goals. *International Journal of Production Economics*, 239, Article 108193.

Allan, D. (2022). *Know your face mask costs*. Naukateks AS, [https://www.fastmarkets.com/insights/face-masks-how-material-costs-can-impact-your-bottom-line](https://www.fastmarkets.com/insights/face-masks-how-material-costs-can-impact-your-bottom-line).
Arifoglu, K., Deo, S., & Iravani, S. M. R. (2012). Consumption externality and yield uncertainty in the influenza vaccine supply chain: Interventions in demand and supply sides. Management Science, 58, 1072–1091.

Bauch, C. T., & Earn, D. J. D. (2004). Vaccination and the theory of games. Proceedings of the National Academy of Sciences, 101, 13291–13294.

Bauch, C. T., Galvani, A. P., & Earn, D. J. D. (2003). Group interest versus self-interest in smallpox vaccination policy. Proceedings of the National Academy of Sciences, 100, 10564–10567.

Bernstein, F., & Federgruen, A. (2005). Decentralized supply chains with competing retailers under demand uncertainty. Management Science, 51, 18–29.

Bhushan, B., Khampari, A., Sagayam, K. M., Sharma, S. K., Ahad, M. A., & Debnath, N. C. (2020). Blockchain for smart cities: A review of architectures, integration trends and future research directions. Sustainable Cities and Society, 61, Article 102360.

Centers for Disease Control and Prevention. (2020). PPE burn rate (2018). Estimated personal protective equipment (PPE) needed for healthcare facilities. https://www.cdc.gov/vh/cdc/ebola/healthcare-us/ppe/calculate.html.

Chandra, D., & Vipin, B. (2021). On the vaccine supply chain coordination under subsidy contract. Vaccine, 39, 4039–4045.

Chick, S. E., Mannani, H., & Simchi-Levi, D. (2008). Supply chain coordination and influenza vaccination. Operations Research, 56, 1493–1506.

Danielle, P., & Siciaca, D. (2021). A dynamic supply chain network for ppe during the COVID-19 pandemic. Journal of Applied and Numerical Optimization, 3, 403–424, http://dx.doi.org/10.23952/jano.3.2021.2.09.

Department of Public Health (2020). COVID-19 dashboard for skilled nursing facilities. http://publichealth.lacounty.gov/snfdashboard.htm.

Elavarasan, R. M., Pugazhendhi, R., Shafiullah, G. M., Irfan, M., & Anvari-Moghaddam, A. (2021). A hover view over effectual approaches on pandemic management for sustainable cities – The endowment of prospective technologies with revitalization strategies. Sustainable Cities and Society, 68, Article 102789.

Federal Reserve Bank of St. Louis on the Economy Blog (2020). https://www.stlouisfed.org/on-the-economy/2020/april/covid-19-protectionism-imports-essential-medical-equipment.

FreightWaves (2021). https://www.freightwaves.com/news/excess-ppp-cuts-into-profits-for-distributors-manufacturers.

Fu, J., Chen, X., & Hu, Q. (2018). Subsidizing strategies in a sustainable supply chain. Journal of the Operational Research Society, 69, 283–295.

Geoffard, P. Y., & Philippon, T. (1997). Disease eradication: Private versus public vaccination. The American Economic Review, 87, 223–230.

Gonzalez-Eiras, M., & Niepelt, D. (2020). On the optimal “lockdown” during an epidemic: Technical report 20.01, Swiss National Bank, Study Center Gerzensee, Publication title: Working papers.

Hospitals and Medical Centers (2020). https://geohub.lacity.org/datasets/c10c3e6f9b9f46c97e354816fa464a/explore.

Ilyushkina, I. (2022). Know your face mask cost. Naukateks AS, https://www.fastmarkets.com/insights/facemask-cost-modeling.

Jahedi, S., & Yorke, J. A. (2020). When the best pandemic models are the simplest. Management Science, 66, 13391–13394.

Kumar, G., Saha, R., Buchanan, W. J., Geetha, G., Thomas, R., Rai, M. K., Kim, T.-H., Jahedi, S., & Yorke, J. A. (2020). When the best pandemic models are the simplest. Know your face mask costs

Lubin, M., & Dunning, I. (2015). Computing in operations research using Julia. In Computing in Operations Research using Julia. Springer International Publishing, (pp. 377–386). Springer International Publishing.

Liu, T., Wang, J., Zhu, Y., & Qu, Z. (2021). Linking economic performance and sustainable operations of China’s manufacturing firms: What role does the government involvement play? Sustainable Cities and Society, 67, Article 102771.

Mamani, H., Adida, E., & Dey, D. (2012). Vaccine market coordination using subsidy. IEEE Transactions on Healthcare Systems Engineering, 2, 78–96.

Nader, J., El-Khalil, R., Nassar, E., & Hong, P. (2022). Pandemic planning, sustainability practices, and organizational performance: An empirical investigation of global manufacturing firms. International Journal of Production Economics, 246, Article 108419.

Nagurney, A. (2021). Supply chain game theory network modeling under labor constraints: Applications to the Covid-19 pandemic. European Journal of Operational Research, 293, 880–891.

New York Times Coronavirus Face Mask Map (2020). https://www.nytimes.com/interactive/2020/07/17/upshot/coronavirus-face-mask-map.html.

Peterson Institute for International Economics (2020). https://www.piie.com/blogs/trade-and-investment/policy-watch/covid-19-trumps-carbs-expports-medical-gear-americans-and.

Rachaniotis, N. P., Danakis, T. K., & Pappis, C. P. (2012). A deterministic resource scheduling model in epidemic control: A case study. European Journal of Operational Research, 216, 225–231.

Raz, G., & Ovchinikov, A. (2015). Coordinating pricing and supply of public interest goods using government rebates and subsidies. IEEE Transactions on Engineering Management, 62, 65–79.

Roxs, S. M. (1998). A first course in probability (5th ed.). Upper Saddle River, N.J.: Prentice Hall.

Rozhkov, M., Ivanov, D., Blackhurst, J., & Nair, A. (2022). Adapting supply chain operations in anticipation of and during the COVID-19 pandemic. Omega, 110, Article 102635.

Salarpour, M., & Nagurney, A. (2021). A multicountry, multicommodity stochastic game theory network model of competition for medical supplies inspired by the Covid-19 pandemic. International Journal of Production Economics, 236, Article 108074.

Savachkin, A., & Uribe, A. (2012). Dynamic redistribution of mitigation resources during influenza pandemics [Special issue: Disaster planning and logistics: Part 1]. Socio-Economic Planning Sciences, 46, 33–45.

Sayarshad, H. R. (2015). Smart transit dynamic optimization and informatics (PhD dissertation), Toronto: Ryerson University.

Sayarshad, H. R. (2022). An optimal control policy in fighting Covid-19 and infectious diseases. Applied Soft Computing, Article 109289.

Sayarshad, H., & Chow, J. (2016). Survey and empirical evaluation of nonhomogeneous arrival process models with taxi data. Journal of Advanced Transportation, 110, Article 102674.

Scherbina, A. (2020). Determining the optimal duration of the COVID-19 suppression policy: A cost-benefit analysis. Technical report 20.03, American Enterprise Institute, Washington, Publication title: Working papers.

Shullih, J., & LeMay, S. (2021). Adapting to slow-moving crises: The personal protection equipment supply chain in the time of COVID19. The American Economic Review, 28.

Sinha, P., Kumar, S., & Chandra, C. (2021). Strategies for ensuring required service level for COVID-19 herd immunity in Indian vaccine supply chain. European Journal of Operational Research.

Taylor, T. A., & Xiao, W. (2014). Subsidizing the distribution channel: Donor funding of the National Academy of Sciences.

Wang, J., Zhao, Z. (2019). Game analysis of charging service fee based on benefit of multi-party participants: A case study analysis in China. Sustainable Cities and Society, 48, Article 101528.