Induced-gravity inflation in no-scale supergravity and beyond

C. Pallis

Departament de Física Teòrica and IFIC, Universitat de València-CSIC, E-46100 Burjassot, Spain
E-mail: kpallis@gen.auth.gr

Received March 24, 2014
Accepted July 6, 2014
Published August 27, 2014

Abstract. Supersymmetric versions of induced-gravity inflation are formulated within Supergravity (SUGRA) employing two gauge singlet chiral superfields. The proposed superpotential is uniquely determined by applying a continuous $R$ and a discrete $\mathbb{Z}_n$ symmetry. We select two types of logarithmic Kähler potentials, one associated with a no-scale-type $SU(2,1)/SU(2) \times U(1)_R \times \mathbb{Z}_n$ Kähler manifold and one more generic. In both cases, imposing a lower bound on the parameter $c_R$ involved in the coupling between the inflaton and the Ricci scalar curvature — e.g. $c_R \gtrsim 76, 105, 310$ for $n = 2, 3$ and 6 respectively —, inflation can be attained even for subplanckian values of the inflaton while the corresponding effective theory respects the perturbative unitarity. In the case of no-scale SUGRA we show that, for every $n$, the inflationary observables remain unchanged and in agreement with the current data while the inflaton mass is predicted to be $3 \cdot 10^{13}$ GeV. Beyond no-scale SUGRA the inflationary observables depend mildly on $n$ and crucially on the coefficient involved in the fourth order term of the Kähler potential which mixes the inflaton with the accompanying non-inflaton field.

Keywords: inflation, supersymmetry and cosmology

ArXiv ePrint: 1403.5486
### Contents

1 Introduction 1

2 Embedding IG inflation in SUGRA 3
   2.1 The general set-up 4
   2.2 Modeling IG inflation in SUGRA 5
   2.3 Framework of inflationary analysis 6

3 The inflationary scenario 6
   3.1 Inflationary observables — constraints 7
   3.2 No-scale SUGRA 8
      3.2.1 The inflationary potential 9
      3.2.2 Analytical results 10
      3.2.3 Numerical results 11
   3.3 Beyond no-scale SUGRA 12
      3.3.1 The inflationary potential 12
      3.3.2 Analytical results 13
      3.3.3 Numerical results 14

4 The effective cut-off scale 15
   4.1 Jordan frame computation 15
   4.2 Einstein frame computation 16

5 Conclusions 17

### 1 Introduction

The announcement of the recent PLANCK results [1, 2] fuelled increasing interest in inflationary models implemented thanks to a strong enough non-minimal coupling between the inflaton field, $\phi$, and the Ricci scalar curvature, $\mathcal{R}$. Indeed, these models predict [2, 3] a (scalar) spectral index $n_s$, tantalizingly close to the value favored by observational data. The existing non-minimally coupled to Gravity inflationary models can be classified into two categories depending whether the non-minimal coupling to $\mathcal{R}$ is added into the conventional one, $m_P^2 \mathcal{R}/2$ — where $m_P = 2.44 \cdot 10^{18}$ GeV is the reduced Planck scale — or it replaces the latter. In the first case the vacuum expectation value (v.e.v) of the inflaton after inflation assumes sufficiently low values after inflation, such that a transition to Einstein gravity at low energy to be guarantied. In the second case, however, the term $m_P^2 \mathcal{R}/2$ is dynamically generated via the v.e.v of the inflaton; these models are, thus, named [4–9] Induced-Gravity (IG) inflationary models. Despite the fact that both models of non-Minimal Inflation are quite similar during inflation and may be collectively classified into universal “attractor” models [10], they exhibit two crucial differences. Namely, in the second category, (i) the *Einstein frame* (EF) inflationary potential develops a singularity at $\phi = 0$ and so, inflation is of Starobinsky-type [11] actually; (ii) The *ultraviolet* (UV) cut-off scale [12–16] of the theory, as it is recently realized [17, 18], can be identified with $m_P$ and, thereby, concerns regarding
the naturalness of inflation can be safely eluded. On the other hand, only some [16] of the
remaining models of nonminimal inflation can be characterized as unitarity safe.

In a recent paper [17] a supersymmetric (SUSY) version of IG inflation was, for first time,
presented within no-scale [19–22] Supergravity (SUGRA). A Higgs-like modulus plays there
the role of inflaton, in sharp contrast to [21] where the inflaton is matter-like. For this reason
we call in [17] the inflationary model no-scale modular inflation. Although any connection
with the no-scale SUSY breaking [19, 20, 23, 24] is lost in that setting, we show that the model
provides a robust cosmological scenario linking together non-thermal leptogenesis, neutrino
physics and a resolution to the µ problem of the Minimal SUSY SM (MSSM). Namely, in [17],
we employ a Kähler potential, \( K \), corresponding to a \( SU(N,1)/SU(N) \times U(1)_R \times Z_2 \)
symmetric Kähler manifold. This symmetry fixes beautifully the form of \( K \) up to an holomorphic
function \( \Omega_H \) which exclusively depends on the inflaton, \( \phi \), and its form \( \Omega_H \sim \phi^2 \) is fixed by
imposing a \( Z_2 \) discrete symmetry which is also respected by the superpotential \( W \). Moreover,
the model possesses a continuous \( R \) symmetry, which reduces to the well-known \( R \)-parity of
MSSM. Thanks to the strong enough coupling between \( \phi \) and \( \mathcal{R} \), inflation can be attained
even for subplanckian values of \( \phi \), contrary to other SUSY realizations [22, 25, 26] of the
Starobinsky-type inflation.

Most recently a more generic form of \( \Omega_H \) has been proposed [18] at the non-SUSY
level. In particular, \( \Omega_H \) is specified as \( \Omega_H \sim \phi^n \) and it was pointed out that the resulting
IG inflationary models exhibit an attractor behavior since the inflationary observables and
the mass of the inflaton at the vacuum are independent of the choice of \( n \). It would be,
thereby, interesting to investigate if this nice feature insists also in the SUSY realizations
of these models. This aim gives us the opportunity to generalize our previous analysis [17]
and investigate the inflationary predictions independently of the post-inflationary cosmolog-
cal evolution. Namely, we here impose on \( \Omega_H \) a discrete \( Z_n \) symmetry with \( n \geq 2 \), and
investigate its possible embedding in standard Poincaré SUGRA, without invoking the su-
perconformal formulation — cf. [27]. We discriminate two possible embeddings, one based
on a no-scale-type symmetry and one more generic, with the first of these being much more
predictive. Namely, while the embedding of IG models in generic SUGRA gives adjustable
results as regards the inflationary observables, — see also [29] —, no-scale SUGRA predicts
independently of \( n \) results identical to those obtained in the non-SUSY case. Therefore,
no-scale SUGRA consists a natural framework in which such models can be implemented.

Below, in section 2, we describe the generic formulation of IG models within SUGRA. In
section 3 we present the basic ingredients of our IG inflationary models, derive the inflationary
observables and confront them with observations. We also provide a detailed analysis of the
UV behavior of these models in section 4. Our conclusions are summarized in section 5.
Throughout the text, the subscript of type \( \chi \) denotes derivation with respect to (w.r.t) the
field \( \chi \) (e.g., \( \chi\chi = \partial^2/\partial\chi^2 \)) and charge conjugation is denoted by a star.

## 2 Embedding IG inflation in SUGRA

In section 2.1 we present the basic formulation of a theory which exhibits non-minimal cou-
pling of scalar fields to \( \mathcal{R} \) within SUGRA and in section 2.2 we outline our strategy in
constructing viable models of IG inflation. The general framework for the analysis of the
emerged models is given in section 2.3.
2.1 The general set-up

Our starting point is the EF action for $N$ gauge singlet scalar fields $z^a$ within SUGRA [30–33] which can be written as

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} m_P^2 \hat{R} + K_{\alpha\beta} \hat{g}^{\mu\nu} \partial_\mu z^\alpha \partial_\nu z^{\ast \beta} - \hat{V} \right), \tag{2.1a}$$

where summation is taken over the scalar fields $z^a$, $K_{\alpha\beta} = \hat{K}_{z^a z^{\ast \beta}}$ with $K_{\alpha\beta} = \delta_\gamma^{\beta} \delta_\gamma^\alpha$, $\hat{g}$ is the determinant of the EF metric $\hat{g}_{\mu\nu}$, $\hat{R}$ is the EF Ricci curvature, $\hat{V}$ is the EF F-term SUGRA scalar potential which can be extracted once the superpotential $W$ and the Kähler potential $K$ have been selected, by applying the standard formula

$$\hat{V} = e^{K/3m_P^2} \left( K^{\alpha\beta} F_\alpha F^{\beta} - \frac{3}{4} |W|^2 / m_P^4 \right), \tag{2.1b}$$

where we take into account that the phase of $\Phi$, $\text{arg}\Phi$ is stabilized to zero; we thus get

$$-\Omega/3 = e^{-K/3m_P^2} \Rightarrow K = -3m_P^2 \ln (-\Omega/3), \tag{2.2}$$

we arrive at the following action

$$S = \int d^4x \sqrt{-g} \left( -\frac{m_P^2}{2} \left( -\frac{\Omega}{3} \right) \hat{R} + m_P^2 \partial_\mu z^a \partial^\mu z^{\ast \alpha} - K_{\alpha\beta} \partial_\mu z^\alpha \partial_\nu z^{\ast \beta} - \Omega A_\mu A^\mu / m_P^2 - V \right), \tag{2.3}$$

where $g_{\mu\nu} = -(3/\Omega) \hat{g}_{\mu\nu}$ and $V = \Omega^2 \hat{V} / 9$ are the JF metric and potential respectively, we use the shorthand notation $\Omega_\alpha = \Omega_{z^\alpha}$ and $\Omega_\beta = \Omega_{z^{\ast \beta}}$ and $A_\mu$ is the purely bosonic part of the on-shell value of an auxiliary field given by

$$A_\mu = -im_P^2 \left( \Omega_\alpha \partial_\mu z^\alpha - \Omega_\beta \partial_\mu z^{\ast \beta} \right) / 2\Omega. \tag{2.4}$$

It is clear from eq. (2.3) that $S$ exhibits non-minimal couplings of the $z^a$’s to $\hat{R}$. However, $\Omega$ enters the kinetic terms of the $z^a$’s too. In general, $\Omega$ can be written as [30–32]

$$-\Omega/3 = \Omega_H(z^a) + \Omega^{\ast}_H(z^{\ast \alpha}) - \Omega_K (z^a z^{\ast \alpha}) / 3, \tag{2.5}$$

where $\Omega_K$ is a dimensionless real function while $\Omega_H$ is a dimensionless, holomorphic function. For $\Omega_H > \Omega_K$, $\Omega_K$ expresses mainly the kinetic terms of the $z^a$’s whereas $\Omega_H$ represents the non-minimal coupling to gravity — note that $\Omega_{\alpha\beta}$ is independent of $\Omega_H$ since $\Omega_{H z^a z^{\ast \beta}} = 0$.

To realize the idea of IG, we have to assume that $\Omega_H$ depends on a Higgs-like modulus, $z^1 := \Phi$ whose the v.e.v generates the conventional term of the Einstein gravity at the SUSY vacuum, i.e.

$$\langle \Omega_H \rangle + \langle \Omega^{\ast}_H \rangle = 1 \Rightarrow \langle \Omega_H \rangle = 1/2 \text{ for } \langle \Omega_K \rangle \sim 0 \tag{2.6}$$

where we take into account that the phase of $\Phi$, $\text{arg}\Phi$ is stabilized to zero; we thus get $\langle \Omega_H \rangle = \langle \Omega^{\ast}_H \rangle$.

In order to get canonical kinetic terms, we need [30–32] $A_\mu = 0$ and $\Omega_{K\alpha\beta} \simeq 0$ or $\delta_{\alpha\beta}$. The first condition is attained when the dynamics of the $z^a$’s is dominated only by the real moduli $|z^a|$. The second condition is satisfied by the choice

$$\Omega_K (|z^a|^2) = k_\alpha |z^a|^2 / m_P^2 - k_{\alpha\beta} |z^\beta|^2 / m_P^4 \tag{2.7}$$
with sufficiently small coefficients $k_\alpha$ and $k_{\alpha \beta} \simeq 1$. Here we assume that the $z^\alpha$’s are charged under a global symmetry, so as mixed terms of the form $z^\alpha z^\beta$ are disallowed. The inclusion of the fourth order term for the accompanying non-inflaton field, $z^2 := S$ is obligatory in order to evade \cite{30–32} a tachyonic instability occurring along this direction during IG inflation. As a consequence, all the allowed terms are to be considered in the analysis for consistency. Let us here note that such a consistency is not observed in the SUGRA incarnations of similar models \cite{10, 30–32}. On the other hand, if we assume that $k_1 = 0$ and $k_{1 \alpha} = 0$, $\forall \alpha = 1, \ldots, N - 1$ (2.8)

the emergent Kähler manifold associated with $K$ can be identified with $SU(N, 1)/SU(N) \times U(1)_R \times \mathbb{Z}_n$ — where the symmetries $U(1)_R$ and $\mathbb{Z}_n$ are specified in section 2.2 — and highly simplifies the realization of IG inflation. The option in eq. (2.8) is inspired by the early models of soft SUSY breaking \cite{19, 20} and defines \cite{22} no-scale SUGRA. We below show details of these two realizations of IG inflation.

2.2 Modeling IG inflation in SUGRA

As we anticipated above, the realization of the idea of IG in SUGRA requires at least two singlet superfields, i.e., $z^\alpha = S, \Phi$; $\Phi$ is a Higgs-like superfield whose the v.e.v generates $m_P$ and $S$ is an accompanying superfield, whose the stabilization at the origin assists us to isolate the contribution of $\Phi$ into $\hat{V}$, eq. (2.1b). To see how this structure works, let us below specify the form of $\Omega_H$ and $W$.

Inspired by \cite{18}, we here determine $\Omega_H$ by postulating its invariance under the action of a global $\mathbb{Z}_n$ discrete symmetry. Therefore it can be written as

$$\Omega_H(\Phi) = c_R \frac{\Phi^n}{m^2_P} + \sum_{k=1}^{\infty} \lambda_k \frac{\Phi^{2kn}}{m^{2kn}_P}$$

with $k$ being a positive integer. Restricting ourselves to subplanckian values of $\Phi$ and assuming relatively low $\lambda_k$’s, we can say that $\mathbb{Z}_n$ uniquely determines the form of $\Omega_H$. Confining ourselves to a such situation we ignore henceforth the $k$-dependent terms in eq. (2.9). On the other hand, $W$ has to be selected so as to achieve the arrangement of eq. (2.6). The simplest choice is that used in the models of F-term hybrid inflation \cite{34}. As a consequence $\Omega_H(\Phi)$ has to be involved also in the superpotential $W$ of our model which has the form

$$W = \lambda m^2_P S (\Omega_H - 1/2) / c_R$$

and can be uniquely determined if we impose, besides $\mathbb{Z}_n$, a nonanomalous $R$ symmetry $U(1)_R$ under which

$$S \rightarrow e^{i\varphi} S, \quad \Omega_H \rightarrow \Omega_H, \quad W \rightarrow e^{i\varphi} W.$$  

Indeed, $U(1)_R$ symmetry ensures the linearity of $W$ w.r.t. $S$ which is crucial for the success of our construction. To verify that $W$ leads to the desired $\langle \Omega_H \rangle$ we minimize the SUSY limit, $V_{SUSY}$, of $\hat{V}$, obtained from the latter, when $m_P$ tends to infinity. This is

$$V_{SUSY} = \frac{\lambda^2 m^4_P |\Omega_H - 1/2|^2}{c^2_R} + \frac{\lambda^2 m^4_P |S\Omega_{H,\Phi}|^2}{c^2_R},$$

where the complex scalar components of $\Phi$ and $S$ are denoted by the same symbol. From eq. (2.12a), we find that the SUSY vacuum lies at

$$\langle S \rangle = 0 \quad \text{and} \quad \langle \Omega_H \rangle = 1/2,$$
as required by eq. (2.6). Let us emphasize that soft SUSY breaking effects explicitly break \( U(1)_R \) to a discrete subgroup. Usually [17] combining the latter with the \( \mathbb{Z}_2^f \) fermion parity, yields the well-known \( R \)-parity of MSSM, which guarantees the stability of the lightest SUSY particle and therefore it provides a well-motivated CDM candidate.

The selected \( W \) and \( K \) by construction give also rise to a stage of IG inflation. Indeed, placing \( S \) at the origin, the only surviving term of \( \hat{V} \) in eq. (2.1b) is

\[
\hat{V}_{IG0} = e^{K/m_P^2} K^{SS^*} |W_S|^2 = \frac{\lambda^2 m_P^4 |2\Omega_H - 1|^2}{4e^2 f_{SF} f_R} \quad \text{since} \quad e^{K/m_P^2} = \frac{1}{f_R} \quad \text{and} \quad K^{SS^*} = \frac{f_R}{f_{SF}}, \tag{2.13a}
\]

where the functions \( f_R \) and \( f_{SF} \) are computed along the inflationary track, i.e.,

\[
f_R = -\Omega/3 \quad \text{and} \quad f_{SF} = m_P^2 \Omega_{SS^*} \quad \text{for} \quad S = \arg\Phi = 0. \tag{2.13b}
\]

Given that \( f_{SF} \ll f_R \approx 2\Omega_H \) with \( c_R \gg 1 \), an inflationary plateau emerges since the resulting \( \hat{V}_{IG0} \) in eq. (2.13a) is almost constant. Therefore, \( \Phi \) involved in the definition of \( \Omega_H \), eq. (2.9), arises naturally as an inflaton candidate. Note that the non-vanishing values of \( \Phi \) during IG inflation break spontaneously the imposed \( \mathbb{Z}_n \); no domain walls are thus produced due to the spontaneous breaking of \( \mathbb{Z}_n \) at the SUSY vacuum, eq. (2.12b).

### 2.3 Framework of inflationary analysis

To consolidate the validity of the inflationary proposal we have to check the stability of the inflationary direction

\[ \theta = s = \bar{s} = 0, \tag{2.14} \]

w.r.t. the fluctuations of the various fields, which are expanded in real and imaginary parts as follows

\[ \Phi = \frac{\phi}{\sqrt{2}} e^{i\theta/m_P} \quad \text{and} \quad S = \frac{s + i\bar{s}}{\sqrt{2}}. \tag{2.15} \]

To this end we examine the validity of the extremum and minimum conditions, i.e.,

\[
\frac{\partial \hat{V}_{IG0}}{\partial \chi^\alpha} \bigg|_{\text{eq. (2.14)}} = 0 \quad \text{and} \quad \hat{m}_{\chi^\alpha}^2 > 0 \quad \text{with} \quad \chi^\alpha = \theta, s, \bar{s}. \tag{2.16a}
\]

Here \( \hat{m}_{\chi^\alpha}^2 \) are the eigenvalues of the mass matrix with elements

\[
\hat{M}_{\alpha\beta} = \frac{\partial^2 \hat{V}_{IG0}}{\partial \chi^\alpha \partial \chi^\beta} \bigg|_{\text{eq. (2.14)}} \quad \text{with} \quad \chi^\alpha = \theta, s, \bar{s} \tag{2.16b}
\]

and hat denotes the EF canonically normalized fields. The kinetic terms of the various scalars in eq. (2.1a) can be brought into the following form

\[
K_{\alpha\beta} \dot{z}^\alpha \dot{z}^\beta = \frac{1}{2} \left( \dot{\phi}^2 + \dot{\theta}^2 \right) + \frac{1}{2} \left( \dot{s}^2 + \dot{\bar{s}}^2 \right), \tag{2.17a}
\]

where the dot denotes derivation w.r.t. the JF cosmic time and the hatted fields are defined as follows

\[
\frac{d \hat{\phi}}{d\phi} = J = \sqrt{K_{\Phi\Phi}}, \quad \hat{\theta} = m_P \sqrt{K_{\Phi\Phi^*}} \theta/\phi, \quad \text{and} \quad (\hat{s}, \hat{\bar{s}}) = \sqrt{K_{SS^*}}(s, \bar{s}). \tag{2.17b}
\]
Note, in passing, that the spinors $\psi_Φ$ and $\psi_S$ associated with the superfields $S$ and $Φ$ are normalized similarly, i.e., $\hat{ψ}_S = \sqrt{K_{SS}}\psi_S$ and $\hat{ψ}_Φ = \sqrt{K_{ΦΦ}}\psi_Φ$.

Upon diagonalization of $\hat{M}^2_{αβ}$, eq. (2.16b), we can construct the scalar mass spectrum of the theory along the direction in eq. (2.14) — see section 3.2.1 and 3.3.1. Besides the stability requirement in eq. (2.16a), from the derived spectrum we can numerically verify that the various masses remain greater than $\hat{H}_{IG}$ during the last 50 e-foldings of inflation, and so any inflationary perturbations of the fields other than the inflaton are safely eliminated. Due to the large effective masses that $θ, s$ and $\bar{s}$ in eq. (2.16b) acquire during inflation, they enter a phase of oscillations about zero with reducing amplitude. As a consequence, the $φ$ dependence in their normalization — see eq. (2.17b) — does not affect their dynamics. Moreover, we can observe that the fermionic (4) and bosonic (4) degrees of freedom are equal — here we take into account that $\hat{φ}$ is not perturbed. Employing the well-known Coleman-Weinberg formula [35], we find that the one-loop corrected inflationary potential is

$$\hat{V}_{IG} = \hat{V}_{IG0} + \frac{1}{64\pi^2} \left( \frac{\hat{m}_θ^4}{\Lambda^2} \ln \frac{\hat{m}_θ^2}{\Lambda^2} + 2\hat{m}_s^4 \ln \frac{\hat{m}_s^2}{\Lambda^2} - 4\hat{m}_ψ^4 \ln \frac{m_ψ^2}{\Lambda^2} \right),$$

(2.18)

where $Λ$ is a renormalization group mass scale, $\hat{m}_θ$ and $\hat{m}_s = \hat{m}_\bar{s}$ are defined in eq. (2.16a) and $\hat{m}_ψ$ are the mass eigenvalues which correspond to eigenstates $\hat{ψ}_± ≃ (\hat{ψ}_S ± \hat{ψ}_Φ)/\sqrt{2}$. As we numerically verify, the one-loop corrections have no impact on our results, since the slope of the inflationary path is generated at the classical level and the various masses are proportional to the weak coupling $λ$.

3 The inflationary scenaria

In this section we outline the salient features and the predictions of our inflationary scenaria in sections 3.2 and 3.3 respectively, testing them against a number of criteria introduced in section 3.1.

3.1 Inflationary observables — constraints

A successful inflationary scenario has to be compatible with a number of observational requirements which are outlined in the following.

3.1.1. The number of e-folds, $\hat{N}_s$, that the scale $k_s = 0.05$/Mpc suffers during IG inflation,

$$\hat{N}_s = \int_{\hat{φ}_\text{f}}^{\hat{φ}_istar} \frac{d\hat{φ}}{m_ϕ^2} \frac{\hat{V}_{IG}}{\hat{V}_{IG,\hat{φ}}} = \frac{\phi_{\text{f}}}{\phi_{\text{istar}}} \int_{\hat{φ}_\text{f}}^{\hat{φ}_istar} j^2 \frac{\hat{V}_{IG}}{\hat{V}_{IG,\hat{φ}}} d\hat{φ},$$

has to be at least enough to resolve the horizon and flatness problems of standard big bang, i.e., [2]

$$\hat{N}_s ≃ 19.4 + 2 \ln \frac{\hat{V}_{IG}(\phi_istar)^{1/4}}{1 \text{ GeV}} - \frac{4}{3} \ln \frac{\hat{V}_{IG}(\phi_istar)^{1/4}}{1 \text{ GeV}} + \frac{1}{3} \ln \frac{T_{\text{rh}}}{1 \text{ GeV}} + \frac{1}{2} \ln \frac{f_{R}(\phi_istar)^{1/3}}{f_{R}(\phi_istar)^{1/3}},$$

(3.2)

where we assumed that IG inflation is followed in turn by a decaying-inflaton, radiation and matter domination, $T_{\text{rh}}$ is the reheat temperature after IG inflation, $φ_istar$ is the value of $φ$ when $k_s$ crosses outside the inflationary horizon, and $φ_istar$ is the value of $φ$ at the
achieved adopting a Kähler potential which depends at least on two gauge singlet superfields. According to our analysis in section 2.2, IG inflation in the context of no-scale SUGRA can be valid if the effective theory arises.

3.2 No-scale SUGRA

As we show in section 4, the UV cutoff of our model is \( m_P \) and so no concerns regarding the validity of the effective theory arise.

3.1.3. The (scalar) spectral index, \( n_s \), its running, \( a_s \), and the scalar-to-tensor ratio \( r \) — estimated through the relations:

\[
\begin{align*}
n_s &= 1 - 6\tilde{\epsilon}_s + 2\tilde{\eta}_s, \quad a_s = 2 \left( 4\tilde{\epsilon}_s^2 - (n_s - 1)^2 \right) / 3 - 2\hat{\xi}_s \quad \text{and} \quad r = 16\tilde{\epsilon}_s, \\
& \quad \text{where} \quad \hat{\xi} = m_P^4 \hat{V}_{IG,\phi} \hat{V}_{IG,\phi\phi\phi} / \hat{V}^2 = m_P^2 \hat{V}_{IG,\phi} \hat{\eta}_s / \hat{V}_{IG} J^2 + 2\tilde{\eta} \quad \text{and the variables with subscript * are evaluated at} \quad \phi = \phi_s — \text{must be in agreement with the fitting of the data} \quad [2] \quad \text{with LCDM model, i.e.,} \\
& \quad (a) \quad n_s = 0.9603 \pm 0.0146, \quad (b) \quad -0.0314 \leq a_s \leq 0.0046 \quad \text{and} \quad (c) \quad r < 0.135, \\
& \text{at 95\% confidence level (c.l.)}
\end{align*}
\]

3.1.4. To avoid corrections from quantum gravity and any destabilize of our inflationary scenario due to higher order non-renormalizable terms — see eq. (2.9) —, we impose two additional theoretical constraints on our models — keeping in mind that \( \hat{V}(\phi_t) \leq \hat{V}(\phi_s) \):

\[
\begin{align*}(a) \quad \hat{V}(\phi_s)^{1/4} &\leq m_P \quad \text{and} \quad (b) \quad \phi_s \leq m_P. \\
&\text{As we show in section 4, the UV cutoff of our model is} \quad m_P \quad \text{and so no concerns regarding the validit}\y\text{ of the effective theory arise.}
\end{align*}
\]

3.2 No-scale SUGRA

According to our analysis in section 2.2, IG inflation in the context of no-scale SUGRA can be achieved adopting a Kähler potential which depends at least on two gauge singlet superfields — the inflaton \( \Phi \) and an accompanying one \( S \) — and has the form

\[
K = -3m_P^2 \ln \left( \Omega_H(\Phi) + \Omega_H(\Phi^*) - \frac{|S|^2}{3m_P^2} + k_S |S|^4 / 3m_P^2 \right),
\]

as inferred by inserting eqs. (2.8), (2.7) and (2.5) into eq. (2.2). Consequently, the Kähler manifold which corresponds to \( K \) is \( SU(2,1) / SU(2) \times U(1)_R \times \mathbb{Z}_n \) globally symmetric. The underlying symmetry of Kähler manifold allows us to avoid any mixing of inflaton \( \Phi \) with \( S \) which fixes \( f_{SK} = 1 \) — see eq. (2.13b). We below extract the inflationary potential in section 3.2.1 and present our analytical and numerical results in section 3.2.2 and 3.2.3 respectively.
### Fields, Eigensates, Masses Squared

| Fields       | Eigensates | Masses Squared                                      |
|--------------|------------|-----------------------------------------------------|
| 1 real scalar| $\hat{\theta}$ | $\hat{m}_\theta^2 = \lambda^2 m_P^2 (2^{n-2} - c_R x_\phi^n f_\phi)/3c_R x_\phi^{2n} \simeq 4\hat{H}_{IG}^2$ |
| 2 real scalars| $\hat{s}, \hat{\bar{s}}$ | $\hat{m}_s^2 = \lambda^2 m_P^2 (2^{3n/2} + 4c_R x_\phi^n (2n - 2^{n/2}c_R x_\phi^n + 12k_S f_\Phi))/(3 \cdot 2^{3+n/2}c_R x_\phi^n)$ |
| 2 Weyl spinors| $\hat{\psi}_\pm = \hat{\psi}_\pm \sqrt{2}$ | $\hat{m}_{\psi_\pm}^2 \simeq 2^{n-2}\lambda^2 m_P^2 /3c_R x_\phi^{2n}$ |

Table 1. The mass spectrum along the trajectory of eq. (2.14) during IG inflation.

### 3.2.1 The inflationary potential

Taking into account the form of $\Omega_H$, $f_R$ and $f_S\Phi$ from eqs. (2.9) and (2.13b), eq. (2.13a) reads

$$\hat{V}_{IG0} = \left[ \Omega_H f_R \right] = \frac{\lambda^2 m_P^4 f_\Phi^2}{4f_R^2}$$

since $f_S\Phi = 1$ and $f_R = 2c_R x_\phi^n /2^{n/2}$ where we introduce the dimensionless quantities

$$x_\phi = \phi / m_P \quad \text{and} \quad f_\Phi = 2^{n/2} - 1 - c_R x_\phi^n.$$ (3.10)

Obviously $\hat{V}_{IG0}$ in eq. (3.9) develops a plateau with almost constant potential energy density corresponding to the Hubble parameter

$$\hat{H}_{IG} = \frac{\hat{V}_{IG0}^{1/2}}{\sqrt{3} m_P} \simeq \frac{\lambda m_P}{2\sqrt{3} c_R} \quad \text{with} \quad \hat{V}_{IG0} \simeq \frac{\lambda^2 m_P^4}{4c_R^2}. \tag{3.11}$$

Along the configuration of eq. (2.14) $K_{\alpha\bar{\beta}}$ defined in eq. (2.17a) takes the form

$$K_{\alpha\bar{\beta}} = \frac{1}{f_R} \text{diag} \left( \frac{3m_P^2 |\Omega_H|}{f_R}, 1 \right) = \text{diag} \left( \frac{3n^2}{2x_\phi^2} \frac{2^{n/2}}{2c_R x_\phi^n} \right), \tag{3.12}$$

where the explicit form of $\Omega_H$ in eq. (2.9) is taken into account. Integrating the first equation in eq. (2.17b) we can identify the EF field:

$$\hat{\phi} = \hat{\phi}_c + \frac{3}{2} m_P \ln \frac{\phi}{\langle \phi \rangle} \quad \text{with} \quad \langle \phi \rangle = \frac{\sqrt{2} m_P}{\sqrt{2} c_R}. \tag{3.13}$$

where we take into account eqs. (2.9) and (2.12b). Also $\hat{\phi}_c$ is a constant of integration.

Following the general analysis in section 2.3 we derive the mass spectrum along the configuration of eq. (2.14). Our results are arranged in table 1. We see there that $k_S \gtrsim 1$ assists us to achieve $\hat{m}_s^2 > 0$ — in accordance with [22, 25, 26]. inserting the extracted masses in eq. (2.18) we can proceed to the numerical analysis of IG inflation in the EF [4–6], employing the standard slow-roll approximation [36–39] — see section 3.2.3. For the sake of the presentation, however, we first — see section 3.2.2 — present analytic results based on eq. (3.11), which are quite close to the numerical ones.
3.2.2 Analytical results

The duration of the slow-roll IG inflation is controlled by the slow-roll parameters which, according to their definition in eq. (3.3b), are calculated to be

$$\hat{\epsilon} \simeq \frac{2^n}{3f_\phi^2} \text{ and } \hat{\eta} \simeq \frac{2^{1+n/2}(2^{n/2} - c_R x_n^c)}{3f_\phi^2}. \quad (3.14)$$

The termination of IG inflation is triggered by the violation of the $\hat{\epsilon}$ criterion at $\phi = \phi_\ell$ given by

$$\hat{\epsilon}(\phi_\ell) = 1 \Rightarrow \phi_\ell = \sqrt{2} m_P \left( (\sqrt{3} + 2)/2\sqrt{3} c_R \right)^{1/n}, \quad (3.15a)$$

since the violation of the $\hat{\eta}$ criterion occurs at $\phi = \phi_c$ such that

$$\hat{\eta}(\phi_c) = 1 \Rightarrow \phi_c = \sqrt{2} m_P \left( \frac{5}{6c_R} \right)^{1/n} = \left( 3 + 2\sqrt{3}/5 \right)^{-1/n} \phi_\ell < \phi_\ell. \quad (3.15b)$$

In the EF, $\hat{\phi}_\ell$ remains independent of $c_R$ and $n$, so substituting eq. (3.15a) into eq. (3.13) we obtain

$$\hat{\phi}_\ell - \hat{\phi}_c \simeq \sqrt{3/2m_P} \ln(1 + 2/\sqrt{3}). \quad (3.16)$$

E.g., setting $\hat{\phi}_c = 0$, we obtain $\hat{\phi}_\ell = 0.94m_P$.

Given that $\phi_\ell < \phi_\star$, we can find a relation between $\phi_\star$ and $\hat{N}_s$ as follows

$$\hat{N}_s \simeq \frac{3c_R}{2^{1+n/2}m_P^2} (\phi_\star^c - \phi_\star^p) \Rightarrow \phi_\star \simeq m_P \sqrt{2^{1+n/2}\hat{N}_s}/3c_R. \quad (3.17a)$$

Obviously, IG inflation consistent with eq. (3.7b) can be achieved if

$$x_\star \leq 1 \Rightarrow c_R \geq 2^{1+n/2}\hat{N}_s/3 \text{ with } x_\star = \phi_\star/m_P. \quad (3.17b)$$

Therefore, we need relatively large $c_R$’s which increase with $n$. On the other hand, $\hat{\phi}_c$ remains transplanckian, since plugging eq. (3.17a) into eq. (3.13) we find

$$\hat{\phi}_c \simeq \hat{\phi}_c + \sqrt{3/2m_P} \ln(4\hat{N}_s/3), \quad (3.18)$$

which gives $\hat{\phi}_c = 5.3m_P$ for $\hat{\phi}_c = 0$. Despite this fact, our construction remains stable under possible corrections from non-renormalizable terms in $\Omega_R$ since these are expressed in terms of initial field $\Phi$, and can be harmless for $|\Phi| \leq m_P$.

Upon substitution of eqs. (3.11), (3.12) and (3.17a) into eq. (3.4) we find $A_s$ as follows

$$A_s^{1/2} = \frac{\lambda m_P^4 f_\Phi(\phi_\star)^2}{2^{n/2+2}\sqrt{2\pi c_R} \phi_n^p} = \frac{\lambda(3 - 4\hat{N}_s)^2}{96\sqrt{2}\pi c_R\hat{N}_s} \Rightarrow \lambda \simeq 6\pi \sqrt{2A_s c_R/\hat{N}_s} \Rightarrow c_R \simeq 41637\lambda, \quad (3.19)$$

for $\hat{N}_s \simeq 52$. Therefore, enforcing eq. (3.4) we obtain a relation between $\lambda$ and $c_R$ which turns out to be independent of $n$. Replacing $\phi_\star$ by eq. (3.17a) into eq. (3.5) we estimate, finally, the inflationary observable through the relations:

$$n_s = \frac{(1 + 4\hat{N}_s)(4\hat{N}_s - 15)}{(3 - 4\hat{N}_s)^2} \simeq 1 - 2/\hat{N}_s - 9/2\hat{N}_s^2 = 0.960, \quad (3.20a)$$

$$a_s \simeq -2\xi_s = \frac{128(3 - \hat{N}_s)}{(4\hat{N}_s - 3)^3} \simeq -2/\hat{N}_s^2 + 3/2\hat{N}_s^3 = -0.0007, \quad (3.20b)$$

$$r = \frac{192}{(3 - 4\hat{N}_s)^2} \simeq 12/\hat{N}_s^2 = 0.0045 \quad (3.20c)$$

for $\hat{N}_s \simeq 52$. These outputs are fully consistent with the observational data, eq. (3.6).
Figure 1. The inflationary potential \( \hat{V}_{IG} \) as a function of \( \phi \) for \( n = 2, \lambda = 1.7 \cdot 10^{-3} \) and \( c_R = 76 \) or \( n = 6, \lambda = 6.8 \cdot 10^{-3} \) and \( c_R = 310 \). The values corresponding to \( \phi_* \) and \( \phi_f \) are also depicted.

### 3.2.3 Numerical results

The inflationary scenario under consideration depends on the parameters:

\[
\lambda, \ c_R, \ k_S \ \text{and} \ T_{ih}.
\]

Our results are essentially independent of \( k_S \)'s, provided that we choose them so as \( \hat{m}_s^2 > 0 \) for every allowed \( \lambda \) and \( c_R \) — see table 1. We therefore set \( k_S = 1 \) throughout our calculation. We also choose \( \Lambda \approx 10^{13} \) GeV so as the one-loop corrections in eq. (2.18) vanish at the SUSY vacuum, eqs. (2.12b) and (2.6). Finally we choose \( T_{ih} = 10^9 \) GeV as suggested by reliable post-inflationary scenaria — see [17]. Upon substitution of \( \hat{V}_{IG} \) from eqs. (2.18) and (3.11) in eqs. (3.3b), (3.1) and (3.4) we extract the inflationary observables as functions of \( c_R, \lambda \) and \( \phi_* \). The two latter parameters can be determined by enforcing the fulfilment of eq. (3.2) and (3.4), for every chosen \( c_R \). Our numerical findings are quite close to the analytic ones listed in section 3.2.2 for presentational purposes.

The variation of \( \hat{V}_{IG} \) as a function of \( \phi \) for two different values of \( n \) can be easily inferred from figure 1, where we depict \( \hat{V}_{IG} \) versus \( \phi \) for \( \phi_* = m_P \) and \( n = 2 \) or \( n = 6 \). The imposition \( \phi_* = m_P \) correspond to \( \lambda = 0.0017 \) and \( c_R = 76 \) for \( n = 2 \) and \( \lambda = 0.0068 \) and \( c_R = 310 \) for \( n = 6 \). In accordance with our findings in eqs. (3.13) and (3.17b) we conclude that increasing \( n \) (i) larger \( c_R \)'s and therefore lower \( \hat{V}_{IG0} \)'s are required to obtain \( \phi < m_P \); (ii) larger \( \phi_f \) and \( \langle \phi \rangle \) are obtained. Combining eqs. (3.15a) and (3.19) with eq. (3.11) we can convince ourselves that \( \hat{V}_{IG0}(\phi_f) \) is independent of \( c_R \) and to a considerable degree of \( n \).

By varying \( \lambda \) we can delineate the region of the parameters allowed by a simultaneous imposition of eqs. (3.4), (3.2) and (3.7). Our results are displayed in figure 2, where we draw as functions of \( \lambda \) the allowed values of \( c_R \) and \( \langle \phi \rangle \) — see figure 2-(a) — \( \phi_* \) (solid line) and \( \phi_f \) (dashed line) — see figure 2-(b). We use black, gray and light gray lines for \( n = 2, 3 \) and 6 respectively. As anticipated in eq. (3.19) the relation between \( c_R \) and \( \lambda \) is independent of \( n \); the various lines, thus, coincide. However, eq. (3.7) is fulfilled to the right of the thin line. Indeed, the lower bound of the depicted lines comes from the saturation of eq. (3.17b) whereas the upper bound originates from the perturbative bound on \( \lambda, \lambda \leq \sqrt{4\pi} \approx 3.54 \).

Moreover, the variation of \( \phi_f \) and \( \phi_* \) as a function of \( \lambda \) — drawn in figure 2-(b) — is consistent with eqs. (3.15a) and (3.17a).
Figure 2. The allowed by eqs. (3.2), (3.4) and (3.7) values of \(c_R\) and the resulting \(\langle \phi \rangle\) [\(\phi_*\) (solid line) and \(\phi_f\) (dashed line)] versus \(\lambda\) [(a) [(b)]. We use black, gray and light gray lines for \(n = 2, 3\) and 6 respectively, \(k_S = 1\) and \(T_{rh} = 10^9\) GeV. eq. (3.7) is fulfilled to the right of the thin line.

The overall allowed parameter space of the model for \(n = 2, 3\) and 6 is correspondingly
\[
76, 105, 310 \lesssim c_R \lesssim 1.5 \cdot 10^5 \quad \text{and} \quad (1.7, 2.4, 6.8) \cdot 10^{-3} \lesssim \lambda \lesssim 3.54 \quad \text{for} \quad \tilde{N}_* \simeq 52 \quad (3.21a)
\]
with \(\langle \phi \rangle\) being confined in the ranges \((0.0026 - 0.1), (0.021 - 0.24)\) and \((0.17 - 0.48)\). Moreover, the masses of the various scalars in table 1 remain well above \(\tilde{H}_{IG}\) both during and after IG inflation for the selected \(k_S\). E.g., for \(n = 3\) and \(c_R = 495\) (corresponding to \(\lambda = 0.01\)) we obtain
\[
(\tilde{m}_2^2(\phi_*), \tilde{m}_2^2(\phi_*)) / \tilde{H}_{IG}^2(\phi_*) \simeq (4, 905) \quad \text{and} \quad (\tilde{m}_2^2(\phi_f), \tilde{m}_2^2(\phi_f)) / \tilde{H}_{IG}^2(\phi_f) \simeq (10.5, 26.8). \quad (3.21b)
\]

Letting \(\lambda\) or \(c_R\) vary within its allowed region in eq. (3.21a), independently of \(n\), we obtain
\[
0.961 \lesssim n_s \lesssim 0.963, \quad -7 \lesssim a_s / 10^{-4} \lesssim -6.4 \quad \text{and} \quad 4.2 \gtrsim r / 10^{-3} \gtrsim 3.6, \quad (3.22)
\]
which lie close to the analytic results in eqs. (3.20a), (3.20b) and (3.20c) and within the allowed ranges of eq. (3.6), with \(n_s\) being impressively spot on its central observationally favored value — see eq. (3.6a). Therefore, the inclusion of the variant exponent \(n \geq 2\), compared to the initial model of [17], does not affect the successful predictions on the inflationary observables.

3.3 Beyond no-scale SUGRA

If we lift the assumption of no-scale SUGRA in eq. (2.8), \(\Omega\) takes its more general form, obtained by inserting eqs. (2.7) and (2.9) into eq. (2.5); the resulting through eq. (2.2) Kähler potential is
\[
K = -3m_T^2 \ln \left( \Omega_H(\Phi) + \Omega_H^*(\Phi^*) - \frac{|S|^2}{3m_T^2} - \frac{|\Phi|^2}{3m_T^2} + k_S \frac{|S|^4}{3m_T^4} + 2k_\Phi \frac{|\Phi|^4}{3m_T^4} + 2k_{S\Phi} \frac{|S|^2|\Phi|^2}{3m_T^4} \right), \quad (3.23)
\]
where the factors of 2 are added just for convenience. The description of the inflationary potential, our analytical and numerical results are exhibited below in sections 3.3.1, 3.3.2 and 3.3.3 correspondingly.
3.3.1 The inflationary potential

The tree-level scalar potential in this case has its general form in eq. (2.13a) where \( f_R \) and \( f_{S\Phi} \) are calculated by employing their definitions in eq. (2.13b) as follows

\[
\begin{align*}
f_R &= 2c_R x_\phi^2 + x_\phi^2 + k_\Phi x_\phi^4/6 \quad \text{and} \quad f_{S\Phi} = 1 - k_{S\Phi} x_\phi^2. 
\end{align*}
\]  

(3.24)

Taking into account the form of \( f_R \) above, \( \hat{V}_{IG0} \) can be cast as follows

\[
\hat{V}_{IG0} = \frac{m_I^4\lambda^2f_\Phi^2}{4c_R^2f_{S\Phi}x_\phi^4(c_R x_\phi^{-2} - 2n/2 - 2f_{x_\phi}/3)^2} 
\] 

(3.25a)

where \( f_{x_\phi} = 1 - k_{x_\phi} x_\phi^2 \), while \( x_\phi \) and \( f_{x_\phi} \) are defined in eq. (3.10). Similarly to section 3.2, \( \hat{V}_{IG0} \) in eq. (3.25a) develops a plateau with almost constant potential energy density corresponding to the Hubble parameter

\[
\hat{H}_{IG} = \frac{\hat{V}_{IG0}^{1/2}}{\sqrt{3m_p}} \simeq \frac{\lambda m_p}{2\sqrt{3f_{S\Phi}c_R}} \quad \text{with} \quad \hat{V}_{IG0} \simeq \frac{\lambda^2m_I^4}{4f_{S\Phi}c_R^2}. 
\]  

(3.25b)

Moreover, the EF canonically normalized inflaton, \( \hat{\phi} \), is found via eq. (2.17b) with \( J^2 \) given by

\[
J^2 = \frac{3n^2c_R^2x_\phi^2 + 2^{14+n/2}c_R x_\phi^{2+n}(1 - n + 2k_{x_\phi}(n - 2)x_\phi^2)}{(c_R x_\phi^{1-n} - 2n/2 - 2f_{x_\phi}/3)^2} \simeq \frac{3n^2}{2x_\phi^2} + \frac{2n/2(1 - n)}{2c_R x_\phi^n}. 
\]  

(3.26)

Consequently, \( J \) turns out to be close to that obtained in section 3.2.1.

Following the standard procedure of section 2.3 we construct the mass spectrum of the theory along the path of eq. (2.14). The precise expressions of the relevant masses squared, taken into account in our numerical computation, are rather lengthy due to the numerous contributions to \( \hat{V}_{IG0} \), eq. (3.25a). Our findings, though, can be considerably simplified, if we perform an expansion for small \( x_\phi \)'s — retaining \( f_{x_\phi} \) intact —, consistently with our restriction, eq. (3.7). If we keep the lowest order terms, the masses squared for the scalars reduce to those displayed in table 1, whereas the mass squared of the chiral fermions shown in table 1 has to be multiplied by the factor

\[
1 + k_{S\Phi}c_R x_\phi^{2-n}/2^{n/2-1}n. 
\]  

(3.27)

As in the case of section 3.2, employing the mass spectrum along the direction of eq. (2.14), we can calculate \( \hat{V}_{IG} \) in eq. (2.18) to further analyze the model.

3.3.2 Analytical results

Upon substitution of eq. (3.25b) into eq. (3.3b), we can extract the slow-roll parameters which determine the strength of the inflationary stage. Performing expansions about \( x_\phi \simeq 0 \), as above, we can extract approximate expressions which assist us to interpret the numerical results presented in section 3.3.3. Namely, we find

\[
\hat{\epsilon} = \frac{(2^{n/2}n + 2k_{S\Phi}c_R x_\phi^{2+n})^2}{3n^2f_{x_\phi}^2}, 
\]  

(3.28a)

\[
\hat{\eta} = \left( 2^n n^2 + 4k_{S\Phi}c_R^2 x_\phi^{2(1+n)} + 2^{n/2}c_R x_\phi^{n} \left( \left( (n - 2)^2/6 + 4k_{S\Phi}(n - 1) \right) x_\phi^2 - n^2 \right) / 3n^2f_{x_\phi}^2. 
\]  

(3.28b)
As it may be numerically verified, \( \phi_s = x_s m_P \) and \( \phi_t \) do not decline a lot from their values in eqs. (3.17a) and (3.15a), which can be served for our estimations below. In particular, replacing \( \hat{V}_{IG0} \) from eq. (3.25b) in eq. (3.4) we obtain

\[
A_s^{1/2} = \frac{n \lambda f_\phi^2(x_s)}{4 \sqrt{2 \pi} c_R^2 x_s^2 (2^{n/2} n + 2 k_{S\Phi} c_R x_s^2 n)} \Rightarrow \lambda \simeq 2 \pi \sqrt{2 A_s c_R} \left( \frac{3}{N_*} + 8 k_{S\Phi} n \left( \frac{2 \hat{N}_s}{3 c_R} \right)^{2/n} \right).
\]

Comparing this expression with the one obtained in the case of no-scale SUGRA, eq. (3.19), we remark that \( \lambda \) acquires a mild dependence on both \( k_{S\Phi} \) and \( n \). Inserting, eq. (3.17a) into eq. (3.5) we can similarly provide an expression for \( n_s \). This is

\[
n_s \simeq 1 - 2/\hat{N}_s + \left( \frac{4}{9} \right)^{1/n} \left( \frac{\hat{N}_s}{c_R} \right)^{2/n} \frac{128 k_{S\Phi} + 27 n^2 / \hat{N}_s^3}{12 n^2}.
\]

Therefore, a clear dependence of \( n_s \) on \( n \) and \( k_{S\Phi} \) arises, with the second one being much more efficient. On the other hand, \( a_s \) and \( r \) remain pretty close to those obtained in the absence of \( k_{S\Phi} \) — see section 3.2.2. In particular, the dependence of \( r \) on \( n \) and \( k_{S\Phi} \) can be encoded as follows

\[
r \simeq \frac{12}{\hat{N}_s^2} + 32 \frac{2^{2/n+1} k_{S\Phi}}{3^{3/n} \hat{N}_s^{1-2/n} c_R^{2/n}} + 64 \frac{2^{4/n+2} k_{S\Phi} \hat{N}_s^{4/n}}{3^{2(n+1)/n} n^2 c_R^{4/n}}.
\]

It is clear from the results above that \( k_{S\Phi} \neq 0 \) has minor impact on \( r \) since its presence is accompanied by large denominators where \( c_R \gg 1 \) is involved.

### 3.3.3 Numerical results

This inflationary scenario depends on the following parameters:

\( \lambda, c_R, k_S, k_{S\Phi}, k_\Phi \) and \( T_{th} \).

As in the case of section 3.2.3 our results are independent of \( k_S \), provided that \( \hat{m}_s^2 > 0 \) — see in table 1. The same is also valid for \( k_\Phi \) since the contribution from the second term in \( f_{R'}, \)

eq. (3.24), is overshadowed by the strong enough first term including \( c_R \gg 1 \). We therefore set \( k_S = 1 \) and \( k_\Phi = 0.5 \). We also choose \( T_{th} = 10^9 \text{ GeV} \). Besides these values, in our numerical code, we use as input parameters \( c_R, k_{S\Phi} \) and \( \phi_s \). For every chosen \( c_R \geq 1 \), we restrict \( \lambda \) and \( \phi_s \) so that the conditions eqs. (3.1), (3.4) and (3.7) are satisfied. By adjusting \( k_{S\Phi} \) we can achieve \( n_s \)'s in the range of eq. (3.6). Our results are displayed in figure 3-(a1) and (a2) [figure 3-(b1) and (b2)], where we delineate the hatched regions allowed by Eqs. (3.1), (3.4), (3.6) and (3.7) in the \( \lambda - c_R \) [\( \lambda - k_{S\Phi} \)] plane. We take \( n = 2 \) in figure 3-(a1) and (b1) and \( n = 3 \) in figure 3-(a2) and (b2). The conventions adopted for the various lines are also shown. In particular, the dashed [dot-dashed] lines correspond to \( n_s = 0.975 \) \( [n_s = 0.946] \), whereas the solid (thick) lines are obtained by fixing \( n_s = 0.96 \) — see eq. (3.6). Along the thin line, which provides the lower bound for the regions presented in figure 3, the constraint of eq. (3.7b) is saturated. At the other end, the perturbative bound on \( \lambda \) bounds the various regions.

From figure 3-(a1) and (a2) we see that \( c_R \) remains almost proportional to \( \lambda \) and for constant \( \lambda, c_R \) increases as \( n_s \) decreases. From figure 3-(b1) we remark that \( k_{S\Phi} \) is confined
Figure 3. The (hatched) regions allowed by eqs. (3.2), (3.4), (3.6) and (3.7) in the $\lambda - c_R$ plane ($a_1$, $a_2$) and $\lambda - k_{S\Phi}$ plane ($b_1$, $b_2$) for $k_S = 1$, $k_\Phi = 0.5$ and $n = 2$ ($a_1$, $b_1$) or $n = 3$ ($a_2$, $b_2$). The conventions adopted for the various lines are also shown.

close to zero for $n_s = 0.96$ and $\lambda < 0.16$ or $\phi_* > 0.1 m_P$ — see eq. (3.17a). Therefore, a degree of tuning (of the order of $10^{-2}$) is needed in order to reproduce the experimental data of eq. (3.6a). On the other hand, for $\lambda > 0.16$ (or $\phi_* < 0.1 m_P$), $k_{S\Phi}$ takes quite natural (of order one) negative values — consistently with eq. (3.30). This feature, however, does not insist for $n = 3$ — see figure 3-(b2) —, where the allowed (hatched) region is considerably shrunk and so, $k_{S\Phi}$ remains constantly below unity for any $\lambda$. As we explicitly verified, for $n = 6$ the results turn out to be even more concentrated about $k_{S\Phi} \simeq 0$. Therefore, we can conclude that this embedding of IG inflation in SUGRA favors low $n$ values.

More explicitly, for $n_s = 0.96$ and $\hat{N}_* \simeq 52$ we find:

$$71 \lesssim c_R \lesssim 1.5 \cdot 10^5 \quad \text{with} \quad 1.6 \cdot 10^{-3} \lesssim \lambda \lesssim 3.5 \quad \text{and} \quad 0 \lesssim -k_{S\Phi} \lesssim 2.4 \quad (n = 2); \quad (3.32a)$$
$$100 \lesssim c_R \lesssim 1.4 \cdot 10^5 \quad \text{with} \quad 2.1 \cdot 10^{-3} \lesssim \lambda \lesssim 3.5 \quad \text{and} \quad 0.002 \lesssim -k_{S\Phi} \lesssim 0.3 \quad (n = 3); \quad (3.32b)$$
$$270 \lesssim c_R \lesssim 1.65 \cdot 10^5 \quad \text{with} \quad 5.6 \cdot 10^{-3} \lesssim \lambda \lesssim 3.5 \quad \text{and} \quad 0.01 \lesssim -k_{S\Phi} \lesssim 0.1 \quad (n = 6). \quad (3.32c)$$

Note that the lower bounds on $c_R$ and $\lambda$ are quite close to those obtained in eq. (3.21a). In both cases $6.8 \lesssim |a_s|/10^{-4} \lesssim 8.2$ and $r \simeq 3.8 \cdot 10^{-3}$ which lie within the allowed ranges of eq. (3.6). Needless to say that, as in section 3.2.3, we here also obtain $\hat{m}_{\chi^0}^2/H_{IG}^2 \gg 1$ with $\hat{m}_{\chi^0}^2$ being defined in eq. (2.16a).
4 The effective cut-off scale

An outstanding trademark of IG inflation is that it is unitarity-safe, despite the fact that its implementation with subplanckian \( \phi \)'s — see eq. (3.17b) — requires relatively large \( c_R \)'s. To show this we below extract the UV cut-off scale, \( \Lambda_{\text{UV}} \), of the effective theory first in the JF — section 4.1 — and then in the EF — see section 4.2. Although the expansions about \( \langle \phi \rangle \) presented below are not valid [15] during IG inflation, we consider the extracted this way \( \Lambda_{\text{UV}} \) as the overall cut-off scale of the theory, since reheating is an unavoidable stage of the inflationary dynamics [16].

4.1 Jordan frame computation

The possible problematic process in the JF, which causes [12–14] concerns about the unitarity-violation, is the \( \delta \phi - \delta \phi \) scattering process via s-channel graviton, \( h^{\mu \nu} \), exchange — \( \hat{\phi} \) represents an excitation of \( \phi \) about \( \langle \phi \rangle \), see below. The relevant vertex is \( c_R \delta \phi \Box h/m_P \) — with \( h = h^\mu_\mu \) — can be derived from the first term in the right-hand side of eq. (2.3) expanding the JF metric \( g_{\mu \nu} \) about the flat spacetime metric \( \eta_{\mu \nu} \) and the inflaton \( \phi \) about its v.e.v as follows:

\[
\delta L = -\frac{(\Omega_H)}{4} F_{EH}(h^{\mu \nu}) + \frac{1}{2} (F_K)\partial_\mu \delta \phi \partial^\mu \delta \phi + \left( m_P \langle \Omega_{H,\phi} \rangle + \delta_R c_R^{2/n} \frac{\delta \phi}{m_P} \right) F_R \delta \phi + \cdots
\]

\[
= -\frac{1}{8} F_{EH}(h^{\mu \nu}) + \frac{1}{2} \partial_\mu \overline{\delta \phi} \partial^\mu \overline{\delta \phi} + \frac{2}{\sqrt{m_P}} \frac{c_R^{2/n} \sqrt{(\Omega_H)}}{(\Omega_H)} \overline{\delta \phi}^2 \Box h + \cdots, \tag{4.2a}
\]

where \( \delta_R = 1/2 \left[ \delta_R = 2^{2/n}(n-1)/8 \right] \) for \( n = 2 \) \([n > 2]\) and the functions \( F_{EH}, F_R \) and \( F_K \) read

\[
F_{EH}(h^{\mu \nu}) = h^{\mu \nu} \Box h_{\mu \nu} - h \Box h + 2 \partial_\rho h^{\mu \rho} \partial^\nu h_{\mu \nu} - 2 \partial_\rho h^{\mu \nu} \partial_\mu h, \tag{4.2b}
\]

\[
F_R(h^{\mu \nu}) = \Box h - \partial_\rho \partial^\rho h^{\mu \nu}. \tag{4.2c}
\]

and

\[
F_K = \begin{cases} 
0, & \text{for no-scale SUGRA;} \\
1, & \text{beyond no-scale SUGRA.} 
\end{cases} \tag{4.2d}
\]

The JF canonically normalized fields \( \overline{h}_{\mu \nu} \) and \( \overline{\delta \phi} \) are defined by the relations

\[
\overline{\delta \phi} = \sqrt{\frac{(\Omega_H)}{(\Omega_H)}} \overline{\delta \phi} \quad \text{and} \quad \overline{h}_{\mu \nu} \frac{\sqrt{2}}{ \sqrt{2} } = \sqrt{\frac{(\Omega_H)}{(\Omega_H)}} h_{\mu \nu} + \frac{m_P \langle \Omega_{H,\phi} \rangle}{\sqrt{(\Omega_H)}} \eta_{\mu \nu} \delta \phi \tag{4.2e}
\]

with

\[
\overline{\Omega_H} = F_K \Omega_H + 3m_P^2 \Omega_{H,\phi}^2. \tag{4.2f}
\]

The interaction originating from the last term in the right-hand side of eq. (4.2a) gives rise to a scattering amplitude which is written in terms of the center-of-mass energy \( E \) as follows

\[
\mathcal{A} \sim \left( \frac{E}{\Lambda_{\text{UV}}} \right)^2 \quad \text{with} \quad \Lambda_{\text{UV}} = \frac{m_P}{\delta_R c_R^{2/n} \sqrt{(\Omega_H)}} = \frac{m_P}{\delta_R c_R^{2/n}} \left( \frac{(F_K)}{(\Omega_H)} + 3 \sqrt{2} m_P^2 \langle \Omega_{H,\phi} \rangle^2 \right) \sim m_P \tag{4.3}
\]
(up to irrelevant numerical prefactors) since \( \Omega_H = 1/2 \ll m_P^2 \Omega_{H, \phi}^2 \approx 2^{2/n^2} c_R^2/n^2/8 \). Here \( \Lambda_{UV} \) is identified as the UV cut-off scale in the JF, since \( \mathcal{A} \) remains within the validity of the perturbation theory provided that \( E < \Lambda_{UV} \). Obviously, the argument above can be equally well applied to both implementations of IG inflation in SUGRA — see section 3.2 and 3.3 — since the extra terms included in eq. (3.23) — compared to eq. (3.8) — are small enough and do not generate any problem with the perturbative unitarity.

### 4.2 Einstein frame computation

Alternatively, \( \Lambda_{UV} \) can be determined in EF, following the systematic approach of [16]. Note, in passing, that the EF (canonically normalized) inflaton,

\[ \tilde{\phi} = (J) \delta \phi \quad \text{with} \quad \langle J \rangle = \sqrt{\frac{3}{2}} \frac{n}{\langle x_\phi \rangle} = \sqrt{\frac{3}{2}} n \sqrt{2 c_R} \]  

(4.4)

acquires mass which is given by

\[ \hat{m}_{\delta \phi} = \left( \tilde{V}_{IG0, \tilde{\phi}} \right)^{1/2} = \left( \tilde{V}_{IG0, \phi \phi} / J^2 \right)^{1/2} = \lambda m_P / \sqrt{3 c_R} . \]  

(4.5)

Making use of eq. (3.19) we find \( \hat{m}_{\delta \phi} = 3 \cdot 10^{13} \text{ GeV} \) for the case of no-scale SUGRA independently of the value of \( n \) — in accordance with the findings in [18]. Beyond no-scale SUGRA, replacing \( \lambda \) in eq. (4.5) from eq. (3.29), we find that \( \hat{m}_{\delta \phi} \) inherits from \( \lambda \) a mild dependence on both \( n \) and \( k_{\phi \phi} \). E.g., for \( n = 0.5 m_P \), \( n = 2 - 6 \) and \( n_s \) in the range of eq. (3.6) we find \( 2.2 < \hat{m}_{\delta \phi} / 10^{13} \text{ GeV} \lesssim 3.8 \) with the lower [upper] value corresponding to the lower [upper] bound on \( n_s \) in eq. (3.6) — see figure 3-(a1) and (a2).

The fact that \( \tilde{\phi} \) does not coincide with \( \delta \phi \) — contrary to the standard Higgs inflation [12–15] — ensures that the IG models are valid up to \( m_P \). To show it, we write the EF action \( S \) in eq. (2.1a) along the path of eq. (2.14) as follows

\[ S = \int d^4 x \sqrt{-g} \left( - \frac{1}{2} m_P^2 \hat{R} + \frac{1}{2} J^2 \phi^2 - \tilde{V}_{IG0} + \cdots \right) , \]  

(4.6a)

where the dot denotes derivation w.r.t. the JF cosmic time and the ellipsis represents terms irrelevant for our analysis. Also \( \tilde{V}_{IG0} \) and \( V_{IG0} \) are respectively given by eqs. (2.17b) and (3.11) [eqs. (3.26) and (3.25b)] for the model of section 3.2 [section 3.3]. For both models, \( J^2 \) is accurately enough estimated by eq. (3.12) — cf. eq. (3.26). Expanding \( J^2 \hat{\phi}^2 \) about \( \langle \phi \rangle \) — see eq. (3.13) — in terms of \( \tilde{\phi} \) in eq. (4.4) we arrive at the following result

\[ J^2 \hat{\phi}^2 = \left( 1 - \frac{2}{n} \sqrt{\frac{2}{3}} \hat{\phi}^2 \right) \left( 2 + \frac{2}{n} \hat{\phi}^2 \right) - \frac{1}{4} \frac{2}{n} \hat{\phi}^4 \frac{8}{9 \sqrt{3} c_R} + \frac{2}{9 n^4} \hat{\phi}^4 \hat{\phi}^2 \hat{\phi}^2 - \cdots \right) \]  

(4.6b)

On the other hand, \( \tilde{V}_{IG0} \) in eq. (3.11) can be expanded about \( \langle \phi \rangle \) as follows

\[ \tilde{V}_{IG0} = \frac{\lambda^2 m_P^2}{6 \sqrt{3} c_R} \tilde{\phi} \left( 1 - \sqrt{\frac{2}{3}} \left( 1 + \frac{1}{n} \right) \hat{\phi} \hat{m}_P + \left( \frac{7}{18} + \frac{1}{n} + \frac{11}{18 n^2} \right) \hat{\phi}^4 m_P^4 \hat{\phi}^2 \hat{\phi}^2 \hat{\phi}^2 \hat{\phi}^2 \cdots \right) . \]  

(4.6c)

From the expressions above, eqs. (4.6b) and (4.6c), — which reduce to the ones presented in [17] for \( n = 2 \) — we can easily infer that \( \Lambda_{UV} = m_P \) even for \( n > 2 \). The same expansion is also valid for the model of section 3.3. In any case, therefore, we obtain \( \Lambda_{UV} = m_P \), in agreement with our findings in section 4.1.
5 Conclusions

In this work we showed that a wide class of IG inflationary models can be naturally embedded in standard SUGRA. Namely, we considered a superpotential which realize easily the IG idea and can be uniquely determined by imposing two global symmetries — a continuous $R$ and a discrete $\mathbb{Z}_n$ symmetry — in conjunction with the requirement that inflation has to occur for subplanckian values of the inflaton. On the other hand, we adopted two forms of Kähler potentials, one corresponding to the Kähler manifold $SU(2,1)/SU(2) \times U(1)_R \times \mathbb{Z}_n$, inspired by no-scale SUGRA, and one more generic. In both cases, the tachyonic instability, occurring along the direction of the accompanying non-inflaton field, can be remedied by considering terms up to the fourth order in the Kähler potential. Thanks to the underlying symmetries the inflaton, $\phi$, appears predominantly as $\phi^n$ in both the super- and Kähler potentials.

In the case of no-scale SUGRA, the inflaton is not mixed with the accompanying non-inflaton field in Kähler potential. As a consequence, the model predicts results identical to the non-SUSY case independently of the exponent $n$. In particular, we found $n_s \simeq 0.963$, $a_n \simeq -0.00068$ and $r \simeq 0.0038$, which are in excellent agreement with the current data, and $\hat{m}_{\delta\phi} = 3 \times 10^{13}$ GeV. Beyond no-scale SUGRA, all the possible terms up to the forth order in powers of the various fields are included in the Kähler potential. In this case, we can achieve $n_s$ precisely equal to its central observationally favored value, mildly tuning the coefficient $k_{3\Phi}$. Furthermore, a weak dependence of the results on $n$ arises with the lower $n$‘s being more favored, since the required tuning on $k_{3\Phi}$ is softer. In both cases a $n$-dependent lower bound on $c_R$ assists us to obtain inflation for subplanckian values of the inflaton, stabilizing thereby our proposal against possible corrections from higher order terms in $\Omega_H$. Furthermore we showed that the one-loop radiative corrections remain subdominant during inflation and the corresponding effective theory is trustable up to $m_p$. Indeed, these models possess a built-in solution into long-standing naturalness problem [12–14, 16] which plagued similar models. As demonstrated both in the EF and the JF, this solution relies on the dynamical generation of $m_p$ at the vacuum of the theory.

As a bottom line we could say that although no-scale SUGRA has been initially coined as a solution to the problem of SUSY breaking [19, 20, 23, 24] ensuring a vanishing cosmological constant, it is by now recognized — see also [17, 22, 26] — that it provides a flexible framework for inflationary model building. In fact, no-scale SUGRA is tailor-made for IG (and nonminimal, in general) inflation since the predictive power of this inflationary model in more generic SUGRA incarnations is lost.

Note added. When this work was under completion, the Bicep2 experiment [40] announced the detection of B-mode polarization in the cosmic microwave background radiation at large angular scales. If this mode is attributed to the primordial gravity waves predicted by inflation, it implies [40] $r = 0.16^{+0.06}_{-0.05}$ — after subtraction of a dust model. Combining this result with eq. 3.6c we find — cf. [41] — a simultaneously compatible region $0.06 \lesssim r \lesssim 0.135$ (at 95% c.l.) which, obviously, is not fulfilled by the models presented here, since the predicted $r$ lies one order of magnitude lower — see eq. 3.22 and comments below eq. 3.32c. However, it is still premature to exclude any inflationary model with $r$ lower than the above limit since the current data are subject to considerable foreground uncertainty — see e.g. [42–44].

Acknowledgments

This research was supported by the Generalitat Valenciana under contract PROMETEOII/2013/017.
References

[1] WMAP collaboration, G. Hinshaw et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, Astrophys. J. Suppl. 208 (2013) 19 [arXiv:1212.5226] [SPIRE].

[2] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XXII. Constraints on inflation*, arXiv:1303.5082 [SPIRE].

[3] S. Tsujikawa, J. Ohashi, S. Kuroyanagi and A. De Felice, *Planck constraints on single-field inflation*, Phys. Rev. D 88 (2013) 023529 [arXiv:1305.3044] [SPIRE].

[4] A. Zee, *A Broken Symmetric Theory of Gravity*, Phys. Rev. Lett. 42 (1979) 417 [SPIRE].

[5] D.S. Salopek, J.R. Bond and J.M. Bardeen, *Designing Density Fluctuation Spectra in Inflation*, Phys. Rev. D 40 (1989) 1753 [SPIRE].

[6] R. Fakir and W.G. Unruh, *Induced gravity inflation*, Phys. Rev. D 41 (1990) 1792 [SPIRE].

[7] J.L. Cervantes-Cota and H. Dehnen, *Induced gravity inflation in the SU(5) GUT*, Phys. Rev. D 51 (1995) 395 [astro-ph/9412032] [SPIRE].

[8] N. Kaloper, L. Sorbo and J. Yokoyama, *Inflation at the GUT scale in a Higgsless universe*, Phys. Rev. D 78 (2008) 043527 [arXiv:0803.3809] [SPIRE].

[9] A. Cerioni, F. Finelli, A. Tronconi and G. Venturi, *Inflation and Reheating in Spontaneously Generated Gravity*, Phys. Rev. D 81 (2010) 123505 [arXiv:1005.0935] [SPIRE].

[10] R. Kallosh, A. Linde and D. Roest, *Universal Attractor for Inflation at Strong Coupling*, Phys. Rev. Lett. 112 (2014) 011303 [arXiv:1310.3950] [SPIRE].

[11] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, Phys. Lett. B 91 (1980) 99 [SPIRE].

[12] J.L.F. Barbon and J.R. Espinosa, *On the Naturalness of Higgs Inflation*, Phys. Rev. D 79 (2009) 081302 [arXiv:0903.0355] [SPIRE].

[13] C.P. Burgess, H.M. Lee and M. Trott, *Comment on Higgs Inflation and Naturalness*, JHEP 07 (2010) 007 [arXiv:1002.2730] [SPIRE].

[14] M.P. Hertzberg, *On Inflation with Non-minimal Coupling*, JHEP 11 (2010) 023 [arXiv:1002.2995] [SPIRE].

[15] F. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, *Higgs inflation: consistency and generalisations*, JHEP 01 (2011) 016 [arXiv:1008.5157] [SPIRE].

[16] A. Kehagias, A.M. Dizgah and A. Riotto, *Comments on the Starobinsky Model of Inflation and its Descendants*, Phys. Rev. D 89 (2014) 043527 [arXiv:1312.1155] [SPIRE].

[17] C. Pallis, *Linking Starobinsky-Type Inflation in no-Scale Supergravity to MSSM*, JCAP 04 (2014) 024 [arXiv:1312.3623] [SPIRE].

[18] G.F. Giudice and H.M. Lee, *Starobinsky-like inflation from induced gravity*, Phys. Lett. B 733 (2014) 58 [arXiv:1402.2129] [SPIRE].

[19] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, *Naturally Vanishing Cosmological Constant in N = 1 Supergravity*, Phys. Lett. B 133 (1983) 61 [SPIRE].

[20] J.R. Ellis, C. Kounnas and D.V. Nanopoulos, *No Scale Supersymmetric Guts*, Nucl. Phys. B 247 (1984) 373 [SPIRE].

[21] J. Ellis, D.V. Nanopoulos and K.A. Olive, *No-Scale Supergravity Realization of the Starobinsky Model of Inflation*, Phys. Rev. Lett. 111 (2013) 111301 [arXiv:1305.1247] [SPIRE].

[22] J. Ellis, D.V. Nanopoulos and K.A. Olive, *Starobinsky-like Inflationary Models as Avatars of No-Scale Supergravity*, JCAP 10 (2013) 009 [arXiv:1307.3537] [SPIRE].
[23] J. Ellis, D.V. Nanopoulos and K.A. Olive, A no-scale supergravity framework for sub-Planckian physics, Phys. Rev. D 89 (2014) 043502 [arXiv:1310.4770] [SPIRE].

[24] T. Leggett, T. Li, J.A. Maxin, D.V. Nanopoulos and J.W. Walker, No Naturalness or Fine-tuning Problems from No-Scale Supergravity, arXiv:1403.3099 [SPIRE].

[25] R. Kallosh and A. Linde, Superconformal generalizations of the Starobinsky model, JCAP 06 (2013) 028 [arXiv:1306.3214] [SPIRE].

[26] D. Roest, M. Scalisi and I. Zavala, Kähler potentials for Planck inflation, JCAP 11 (2013) 007 [arXiv:1307.4343] [SPIRE].

[27] R. Kallosh, Planck 2013 and Superconformal Symmetry, arXiv:1402.0527 [SPIRE].

[28] R. Kallosh, More on Universal Superconformal Attractors, arXiv:1306.3214 [SPIRE].

[29] C. Pallis, Models of Non-Minimal Chaotic Inflation in Supergravity, PoS(Corfu2012)061 [arXiv:1307.7815] [SPIRE].

[30] M.B. Einhorn and D.R.T. Jones, Inflation with Non-minimal Gravitational Couplings in Supergravity, JHEP 03 (2010) 026 [arXiv:0912.2718] [SPIRE].

[31] H.M. Lee, Chaotic inflation in Jordan frame supergravity, JCAP 08 (2010) 003 [arXiv:1005.2735] [SPIRE].

[32] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Superconformal Symmetry, NMSSM and Inflation, Phys. Rev. D 83 (2011) 025008 [arXiv:1008.2942] [SPIRE].

[33] G.R. Dvali, Q. Shafi and R.K. Schaefer, Large scale structure and supersymmetric inflation without fine tuning, Phys. Rev. Lett. 73 (1994) 1886 [hep-ph/9406319] [SPIRE].

[34] S.R. Coleman and E.J. Weinberg, Radiative Corrections as the Origin of Spontaneous Symmetry Breaking, Phys. Rev. D 7 (1973) 1888 [SPIRE].

[35] G. Lazarides, Basics of inflationary cosmology, J. Phys. Conf. Ser. 53 (2006) 528 [hep-ph/0607032] [SPIRE].

[36] B. Audren, D.G. Figueroa and T. Tram, A note of clarification: BICEP2 and Planck are not in tension, arXiv:1405.1390 [SPIRE].

[37] H. Liu, P. Mertsch and S. Sarkar, Fingerprints of Galactic Loop I on the Cosmic Microwave Background, Astrophys. J. 789 (2014) L29 [arXiv:1404.1899] [SPIRE].

[38] R. Flauger, J.C. Hill and D.N. Spergel, Toward an Understanding of Foreground Emission in the BICEP2 Region, arXiv:1405.7351 [SPIRE].