Health states assessment and prediction of manufacturing system based on brittleness

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Abstract. Accurate recognition of system health states is the key to ensure the safe operation of the system. In view of the shortcomings of the existing methods, a new method of manufacturing system health states assessment and prediction based on brittleness is proposed. Firstly, according to the real-time effective performance parameters, the brittle risk entropy model of equipment is constructed, and the brittleness of corresponding equipment on each station is calculated; Secondly, based on the structural characteristics of manufacturing system, the calculation model of system brittleness is constructed by analysing from equipment to system step by step, and the mapping relationship between system brittleness and health states is established to complete the assessment of system health states; Thirdly, based on the historical data samples, the quintic polynomial regression model of system brittleness and time is constructed to predict the future health states of system and the time nodes that need to be maintained. Finally, an assembly manufacturing system is taken as an example to verify the correctness and effectiveness of the proposed method.

1. Introduction

With the rapid development of industrial technology, the characteristics of manufacturing system such as large scale, complex structure and diversified tasks are becoming more and more prominent. In the current environment of multi tasks and variable load, the interference and uncertainty faced by manufacturing system will continue to increase. If a key component in the system is abnormal, it will cause chain reaction within the system, and lead to fluctuations of the system health states during operation, which may result in system failure or shutdown, and even endanger personal safety. Therefore, the accurate assessment of the health states of manufacturing system is not only the guarantee to improve the health management level and ensure the safe operation of the system, but also the theoretical basis for the system maintenance decision-making, which has important research value and practical significance.

With the concept of Prognostics and Health Management (PHM) first proposed in the F-35 Joint attack aircraft project in western countries [1], health states assessment has been paid more and more attention by researchers in recent years. The existing health states assessment methods of system can be divided into the following three categories: 1) The methods based on failure physical model [2-4]. This kind of methods mainly evaluate and predict the health states by establishing an accurate mathematical model that reflecting the deterioration physical process of the system, so it is not very
suitable for complex systems which are difficult to build accurate mathematical models. 2) The methods based on knowledge rules [5-6]. This kind of methods is based on the existing complete knowledge base. The accuracy of this kind of evaluation method depends on whether the knowledge covered by the base is comprehensive and whether the reasoning mechanism is systematic and reasonable, so its application has some limitations. 3) The methods based on the data-driven [7-9]. This kind of method is based on the effective data collected by experiments or sensors, through mathematical mining and other advanced methods to evaluate the health states of the system. This kind of method does not need to establish an accurate physical model, but only needs to mine and describe the system health information hidden behind the data. Therefore, this method has wide applicability under the support of the current big data environment.

In conclusion, different experts and scholars have used different methods to evaluate and predict the health states of system, but most of them lack to explore their health states from the inherent attributes of the system themselves. Brittleness, as an inherent property of manufacturing system, is used to measure the possibility of system damage and its anti-interference capability [10]. In the actual operation of manufacturing system, brittleness, health state and maintenance strategy are closely related. Brittleness can reflect the health states of the system in essence, and the accurate health states can make the maintenance strategy more reasonable. Therefore, in view of the shortcomings of the existing research, this paper proposes a new method of health states assessment and prediction of manufacturing system from the perspective of brittleness, which is based on the real-time effective performance parameters and the related brittleness theory. Through this method, the system health states can be accurately evaluated and the maintenance time node can be accurately predicted, which provides a new idea for the research of system health states.

2. Related theories

2.1. Brittle risk entropy

The brittle risk refers to the risk that the brittleness of the equipment will be activated and collapse due to the interference of internal and external uncertain factors [11]. Brittle risk is not only related to the uncertainty degree of the equipment states, but also related to the consequences of the state on the equipment.

Assuming that equipment E has n performance states (x₁, x₂, ⋯, xₙ), and the probability of each performance state occurrence is pᵢ (i=1, 2, ⋯, n), then the expression of equipment brittle risk entropy H(E) can be defined as follows based on the definition of brittle risk and the principle of traditional information entropy [11]:

$$H(E) = H(C, p) = -\sum_{i=1}^{n} q_i \log p_i$$

$$= -\sum_{i=1}^{n} \left[ \frac{C_i p_i}{\sum C_i p_i} \right] \log p_i,$$

$$0 \leq p_i \leq 1; \sum_{i=1}^{n} p_i = 1$$ (1)

In Equation (1), Ci (1 ≥ Ci ≥ 0) is the collapse coefficient of the equipment, which indicates the probability of equipment collapse when equipment is in performance state xi; where qᵢ is the utility coefficient of performance state xi, and its expression is shown in Equation (2).
\[ q_i = \frac{C_i p_{id}}{\sum_{i=1}^{n} C_i p_i}, \quad i = 1, 2, \ldots, n \]  

(2)

\[ 0 \leq q_i \leq 1, \quad \sum_{i=1}^{n} q_i = 1, i = 1, 2, \ldots, n \]

2.2. Brittleness

Brittleness is a quantitative evaluation index to measure the damage intensity and evaluate the current health states of the system or equipment. The greater the value of brittleness is, the faster the system will collapse [12]. The calculation of the value of brittleness is shown in Equation (3) as below [13].

\[
B = \begin{cases} 
0, & H < H' \\
\frac{H - H'}{H'' - H'}, & H' \leq H \leq H'' \\
1, & H > H'' 
\end{cases}
\]

(3)

In Equation (3), \(B\) represents the degree of brittleness, \(H'\) represents the value of brittle risk entropy under the initial normal working condition during the operation period of the equipment, \(H''\) represents the maximum value of brittle risk entropy within the operation period of the equipment, and \(H\) represents the value of brittle risk entropy which is actually calculated when the equipment is in any intermediate states. It can be seen from Equation (3): \(0 \leq B \leq 1\). When \(B = 0\), it means the system is in the initial normal working state. When \(0 < B < 1\), it means the system is in the degraded operating states. When \(B = 1\), it means the brittleness of the system is activated and the system is in the collapse or fault state.

3. Assessment of system health states

3.1. Calculation model of brittle risk entropy for single equipment

In order to effectively measure the brittleness of the equipment, firstly, the brittle risk entropy is calculated by constructing the brittleness risk entropy model, and then the brittleness of the equipment in each state is calculated by Equation (3) in Section 2.2.

Based on the theory of brittle risk entropy in Section 2.1, the calculation model of brittle risk entropy for equipment which have single performance parameter is established as follows:

\[ H(x_d) = H(C, p) = -\sum_{i=1}^{n} q_i \log_2 p_{id} \]

\[ = -\sum_{i=1}^{n} \left[ \frac{C_{id} p_{id}}{\sum_{i=1}^{n} C_{id} p_{id}} \right] \log_2 p_{id}, \]  

(4)

\[ i = 1, 2, \ldots, n \]

In Equation (4), \(H(x_d)\) represents the value of brittle risk entropy of the equipment on the day \(d\) during operation; \(C_{id}\) represents the collapse coefficient of the equipment in the performance state \(i\) on day \(d\); \(p_{id}\) represents the state probability of the equipment in the performance state \(i\) on day \(d\); \(q_{id}\) represents the utility coefficient; \(n\) is the total number of performance states of the equipment.

In the actual operation process of equipment, due to the external interference of multi tasks and complex environment, it is often difficult to use a single performance parameter to characterize the performance state of equipment. Therefore, in order to more accurately and comprehensively evaluate the performance state and brittleness of equipment, multiple performance parameters are usually used...
to analyse and calculate the brittle risk entropy of equipment. Based on the calculation model of brittle risk entropy for single performance parameter of equipment above, the calculation model of brittle risk entropy for multi performance parameter of equipment is defined as follows:

\[
H_s(x_d) = \sum_{j=1}^{k} \rho_j H_j(x_d)
\]  

(5)

In Equation (5), \(\rho_j\) represents the weight of each performance parameter of the equipment, and it can be calculated based on the entropy weight method by Equation (6); \(k\) is the total number of performance parameters; \(H_j(x_d)\) represents the brittle risk entropy that measuring the brittleness of the equipment by using the performance parameter \(j\); \(H_s(x_d)\) represents the comprehensive brittle risk entropy for multiple performance parameters of equipment.

\[
\rho_j = \begin{cases} 
\frac{1 - H_j(x_d)}{\sum_{j=1}^{k} (1 - H_j(x_d))}, & H_j(x_d) \leq 1 \\
\frac{1 - 1 - H_j(x_d)}{\sum_{j=1}^{k} (1 - H_j(x_d))}, & H_j(x_d) > 1 
\end{cases}
\]

(6)

3.2. Calculation model of system brittleness

According to the structural characteristics of manufacturing system, the system can be divided into several subsystems in series, and each subsystem is composed of several equipment in series or parallel. Based on the principle of equivalent partition, the calculation of the brittleness of manufacturing system can be transformed into the hybrid calculation of the brittleness of multiple equipment. Taking the system shown in Figure 1 as an example, the calculation model of each subsystem and system brittleness can be defined as follows:

1) In Figure 1, subsystem \(a\) is composed of \(m\) equipment in series. When the brittleness of any equipment in the series system is activated, the system will collapse. Therefore, the brittleness of series subsystem \(a\) is defined as the maximum brittleness of all equipment in the subsystem, which is expressed by Equation (7):

\[
B_a = \max(B_{ae}), \quad e = 1, 2, \ldots, m
\]

(7)

In Equation (7), \(B_a\) is the brittleness of series subsystem \(a\); \(B_{ae}\) is the brittleness of each equipment in series subsystem \(a\); \(m\) is the number of equipment in series subsystem \(a\).

2) In Figure 1, subsystem \(b\) is composed of \(q\) equipment in parallel. When the brittleness of all equipment in the parallel system is activated, the system will collapse. Therefore, the brittleness of the parallel subsystem \(b\) is defined as the minimum brittleness of all equipment in the subsystem, which is expressed by Equation (8):

\[
B_b = \min(B_{be}), \quad e' = 1, 2, \ldots, q
\]

(8)

In Equation (8), \(B_b\) is the brittleness of parallel subsystem \(b\); \(B_{be}\) is the brittleness of each equipment in parallel subsystem \(b\); \(q\) is the number of equipment in parallel subsystem \(b\).

3) System \(S\) is composed of subsystem \(a\) and subsystem \(b\) in series. Combined with the above brittleness calculation model of series and parallel subsystem, the brittleness of system \(S\) can be calculated by Equation (9) as follows:

\[
B_S = \max \{B_a, B_b\} = \max \{\max(B_{ae}), \min(B_{be})\}
\]

(9)

In Equation (9), \(B_S\) is the brittleness of system \(S\).
3.3. Mapping relationship between brittleness and system health states

The health states of the system describe the deviation degree of its current performance state from the normal initial state. In order to make the classification of the health states of system more accurately and objectively, this paper starts from the brittleness, using the value of brittleness to describe the health states of the system. The system brittleness at any time in operation can be calculated based on Section 3.2, and the health states of the system is accurately divided by building the mapping model of the value of brittleness and health states of the system. Through the mapping relationship model in Figure 2, the degradation rule of system performance, the process of brittleness being activated and health states of the system can be organically combined. According to the method in reference [14], the system is divided into four health states based on the value of brittleness, it can be seen from Figure 2 that different values of brittleness correspond to different health states of the system. Health states can not only clearly reflect the actual situation of system, but also reflect the internal laws of performance degradation and brittleness effect accumulation of system, which can provide theoretical basis for the research of maintenance strategy.

**Figure 1.** Flow chart of the system configuration.

**Figure 2.** Mapping relationship between the brittleness and health states of system.

### 4. Prediction of system health states

According to the calculation method of system brittleness in Section 3.2, the value of system brittleness BS(t) at any time t (t=\{t1, t2, t3, ⋯, tx\}) in operation cycle can be obtained, which is expressed by Equation (10):
In the above Equation (10), the health states of the system is optimal at t1. With the development of operation time, the performance of the system will decline irreversibly under the influence of multi tasks and external dynamic uncertainty, and the brittleness effect will accumulate gradually. When the cumulative amount of brittleness effect is within the controllable range, the brittleness of the system remains recessive because it is not activated, but it may not only lead to the uncertainty of the system increases, but also lead to the performance parameters of the system fluctuate; When at time tx, the cumulative amount of brittleness effect exceeds the threshold, the brittleness of the system will be activated and dominant, which will lead to the system failure and need to be shut down for maintenance.

In order to accurately express the relationship between the value of brittleness BS(t) and time during the operation of the system, and make full use of historical data, this paper uses regression analysis method to establish the polynomial regression model of brittleness and time, which can be expressed as follows:

\[ B_s(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \cdots + \alpha_{i-1} t^{i-1} + \alpha_i t^i \]  \hspace{1cm} (11)

In Equation (11), t is the monitoring time; \( \alpha \) is the fitting coefficient; \( i \) is the order of a polynomial, which is a positive integer.

5. Case study

Taking an engine cylinder head assembly manufacturing system as an example to verify the proposed method in the paper. Figure 3 in below shows the simple configuration diagram of the system. According to different processes, the system is divided into five subsystems in series, and each subsystem is composed of several equipment in parallel. Subsystem 1 includes an automatic turnover equipment M1; Subsystem 2 consists of two automatic gluing equipment M2 and M3, which have the same performance and are in parallel with each other; Subsystem 3 consists of three automatic tightening equipment M4, M5 and M6, which have the same performance and are in parallel with each other; Subsystem 4 is composed of two automatic press mounting equipment M7 and M8 with the same performance in parallel; Subsystem 5 consists of only one detection equipment M9. Each subsystem completes the corresponding installation task in sequence, and finally completes the assembly task of cylinder head together.

![Figure 3. Configuration diagram of cylinder head assembly manufacturing system.](image)

Based on the historical experience data and the material performance of the assembly object, and combined with the real-time monitoring data during operation, the standard values and thresholds of performance parameters that characterizing the performance states of each equipment in the system are extracted as shown in Table 1.
5.1. Calculation of brittleness of single equipment

In the assembly process of engine cylinder head, most parts are connected and fastened by bolts, and tightening equipment is an important guarantee of product assembly quality. Therefore, in this case, the calculation process of single equipment brittleness is described in detail by taking the automatic tightening equipment $M_4$ on the key station of the assembly system as an example.

1) Calculation the brittle risk entropy of equipment

Firstly, based on the mapping relationship between the performance parameter intervals and performance states of tightening equipment $M_4$, the probability distribution of the tightening equipment $M_4$ in different performance states is obtained by real-time statistics; Secondly, combined with the definition of collapse coefficient above and the method in reference [11], the collapse coefficients in Equation (4) are obtained as follows: $C_{ld} = 0.1, C_{sd} = 0.2, C_{d4} = 0.3, C_{d4} = 0.4$; Finally, the comprehensive brittle risk entropy of tightening equipment $M_4$ in sampling period is calculated quantitatively by using the calculation model of brittle risk entropy for single equipment in Section 3.1.

2) Calculation the brittleness of equipment

Based on the concept and calculation Equation (3) of brittleness in Section 2.2, the brittleness value $B_{M4}(t)$ of $M_4$ at each time in the sampling period can be calculated, as shown in Equation (12).

$$
B_{M4}(t) = \begin{cases} 0, & H(t) < 0.671 \\ \frac{H(t) - 0.671}{1.298}, & 0.671 \leq H(t) \leq 1.969 \\ 1, & H(t) > 1.969 \end{cases}
$$

(12)

In Equation (12), $H(t)$ represents the comprehensive brittle risk entropy corresponding to each moment of $M_4$ in the sampling period.

5.2. Calculation of system brittleness and assessment of system health states

According to the calculation model of system brittleness in Section 3.2 and the configuration diagram of the system in this case, the brittleness $B_5$ of the system can be calculated by Equation (13) as follows:

$$
B_5 = \max \{B_{S1}, B_{S5}, B_{S4}, B_{S6}, B_{S8}\} = \max \{B_{M4}, \min(B_{M1}, B_{M5}), \min(B_{M1}, B_{M5}, B_{M4}), \min(B_{M1}, B_{M5}, B_{M4}, B_{M4})\}
$$

(13)

In Equation (13), $B_{S1}$-$B_{S8}$ respectively represent the corresponding brittleness values of each subsystem in the sampling period; $B_{M1}$-$B_{M4}$ respectively represent the brittleness values of each equipment in each subsystem during the sampling period.

Based on the calculation Equation (13), the brittleness value of the system at each time in the sampling period can be obtained, and then the health state of the system can be accurately evaluated through the mapping relationship between the brittleness values and the system health states in Section 3.3. The detailed calculation and evaluation results are shown in Table 2.

5.3. Prediction of system health states

The three months performance parameters of the system before maintenance are collected continuously in days. Based on the above calculation steps, the daily brittleness values (100 groups) of the system are measured in real time. Using these historical measured data as samples, the health states of the system in the future and the accurate time nodes that need to be maintained are predicted. Based on the sample data, the relationship curve between system brittleness and time is obtained, as shown in Figure 4.

It can be seen from Figure 4 that during the sampling period, the brittleness values of the system increase with the operation time. The growth rate of brittleness in the early 30 days is relatively slow, but it increases significantly in the middle 30-70 days, which tends to be stable in the later period. This
is because the continuous manufacturing tasks and the external disturbance of dynamic uncertainty will inevitably lead to an irreversible decline trend of the performance state of the system. The gradual accumulation of the performance degradation effect will destroy the original order state and form a new disorder state. This phenomenon is called the accumulation process of the system brittleness effect.

In order to ensure the accuracy of polynomial regression model of system brittleness and time, this paper uses the quintic polynomial to fit the relationship between the system brittleness and time, which not only ensures the accuracy of the model, but also avoids the occurrence of over fitting phenomenon. Based on the real-time historical sample data, the fitting coefficients in the quintic polynomial are solved, and the quintic polynomial regression model of system brittleness and time is finally obtained, as shown in the following Equation (14).

### Table 1. The performance parameter values of each equipment in the system.

| Equipment               | Performance parameters | Standard values | Thresholds |
|-------------------------|------------------------|-----------------|------------|
| automatic turnover      | Turnover speed / (r·min⁻¹) | 35              | [20,50]    |
| equipment               |                         |                 |            |
| automatic gluing        | Gluing time /s          | 10              | [6,14]     |
| equipment               | Working pressure/MPa    | 2               | [1,4]      |
|                         | Operating speed / (m·min⁻¹) | 15              | [10,20]    |
| automatic tightening    | Torque/ (N·M)           | 60              | [55,65]    |
| equipment               | Angle/(°)              | 55              | [40,70]    |
|                         | Press mounting force/9.8N | 4000            | [2000,5000] |
| automatic press         | Working speed / (mm·s⁻¹) | 20              | [10,30]    |
| mounting equipment      | Return speed / (mm·s⁻¹) | 130             | [110,140]  |
| automatic detection     | Detection cycle/s      | 8               | [6,10]     |
| equipment               | Detection pressure/Pa  | 50              | [30,80]    |

The values of system brittleness at various times in the future can be obtained by Equation (14), and combined with the mapping relationship between system brittleness and health states, the health states of the system at various time in the future and the time node of maintenance can be predicted.
Figure 4. The relationship curve between system brittleness and time.

Table 2. Britteness values and health state of the system at each monitoring time.

| Date (day) | System brittleness B_s(t) | Health states |
|------------|---------------------------|---------------|
| 1          | 0.0012                    | healthy       |
| 8          | 0.0714                    | healthy       |
| 15         | 0.1418                    | healthy       |
| 22         | 0.2297                    | healthy       |
| 29         | 0.3092                    | favorable     |
| 36         | 0.3894                    | favorable     |
| 43         | 0.3495                    | favorable     |
| 50         | 0.5228                    | favorable     |
| 57         | 0.6998                    | favorable     |
| 64         | 0.8271                    | sub-healthy   |
| 71         | 0.8549                    | sub-healthy   |
| 78         | 0.8995                    | sub-healthy   |
| 85         | 0.9437                    | sub-healthy   |
| 92         | 0.9790                    | sub-healthy   |

\[ B_s(t) = -0.0215 + 1.6828t - 7.6131t^2 + 25.8799t^3 - 31.4619t^4 + 12.5622t^5 \]  \hspace{1cm} (14)

6. Conclusion
Based on the brittleness theory, this paper proposes a new method to effectively evaluate and predict the health states of manufacturing system by using brittleness. The main conclusions are as follows:

1) The brittle risk entropy model of single equipment with multiple performance parameters is constructed to calculate the brittleness of equipment. Based on the configuration of manufacturing
system, the "bottom-to-top" method is adopted to analyse from equipment to system step by step, and then the calculation model of system brittleness is established to calculate the system brittleness.

2) By establishing the mapping relationship between the brittleness and the health states of the system, the health state of the system is evaluated; According to the data-based health state prediction method, the quintic polynomial regression model of system brittleness and time is constructed, and the health states of the system in future and the time nodes that need to be maintained are predicted.

3) An actual assembly manufacturing system is taken as an example to verify the effectiveness of the model and method.

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