HIGGS PARTICLES AT FUTURE
HADRON AND ELECTRON–POSITRON COLLIDERS

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ABSTRACT

The prospects for discovering Higgs particles and studying their fundamental properties at future high–energy electron–positron and hadron colliders are reviewed. Both the Standard Model Higgs boson and the Higgs particles of its minimal supersymmetric extension are discussed. We update various results by taking into account the new value of the top quark mass obtained by the CDF Collaboration and by including radiative corrections some of which have been calculated only recently.

1. Introduction

1.1. Standard Model Higgs Sector

The Higgs mechanism is a cornerstone in the electroweak sector of the Standard Model (SM). The fundamental particles, leptons, quarks and weak gauge bosons, acquire their masses through the interaction with a scalar field. The self–interaction of the scalar field leads to a non–zero field strength in the ground state, inducing the spontaneous breaking of the electroweak SU(2) × U(1) symmetry down to the electromagnetic U(1) symmetry.

In order to accomodate the well–established electromagnetic and weak phenomena, this mechanism for generating particle masses requires the existence of at least one weak iso–doublet scalar field. The three Goldstone bosons among the four degrees of freedom are absorbed to build up the longitudinal polarization states of the massive W±, Z gauge bosons. One degree of freedom is left over, corresponding to a real physical scalar particle. The discovery of this Higgs particle is the experimentum crucis for the standard formulation of the electroweak interactions.

The only unknown parameter in the SM Higgs sector is the mass of the Higgs particle. Even though the value of the Higgs mass cannot be predicted in the Standard Model, interesting constraints can be derived from assumptions on the energy range within which the model be valid before perturbation theory breaks down at a scale Λ and new dynamical phenomena would emerge.

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(i) If the Higgs mass is larger than $\sim 1$ TeV, the interactions between longitudinal $W$ and $Z$ bosons would become so strong that non-perturbative effects are needed to ensure unitarity at high energies. In the 500 GeV energy range, residual final state interactions in $e^+e^- \rightarrow W^+W^-$ would still be too small to be observable; their detection would require much higher energy $e^+e^-$ machines. However, this scenario can eventually be studied at proton colliders in an exploratory way.

(ii) The strength of the Higgs self-interaction is determined by the Higgs mass itself at the scale $M_H$. Increasing the scale, the quartic self-coupling of the Higgs field grows logarithmically with the energy scale, similarly to the electromagnetic coupling in QED. If the Higgs mass is small, the energy cut-off $\Lambda$ is large at which the coupling grows beyond any bound and new phenomena may be observed; conversely, if the Higgs mass is large, the cut-off $\Lambda$ is small. The condition $M_H < \Lambda$ sets an upper limit on the Higgs mass in the Standard Model. Thorough analyses of the non-perturbative regime near the cut-off lead to an estimate of about 630 GeV for the upper limit of $M_H$. However, if the Higgs mass is less than 180 to 200 GeV, the Standard Model can be extended up to the GUT scale $\Lambda_{\text{GUT}} \sim 10^{15}$ GeV with weakly interacting particles. Including the effect of $t$-quark loops on the running coupling, a detailed analysis predicts the area of the allowed $(m_t, M_H)$ values shown in Fig. 1 for several values of the cut-off parameter $\Lambda$.

On quite general grounds, the hypothesis that interactions of $W, Z$ bosons and Higgs particles remain weak up to the GUT scale is an attractive idea and may play a key rôle in the explanation of the experimental value of the electroweak mixing parameter $\sin^2 \theta_W$. Based on the SM particle spectrum, the mixing parameter evolves from the symmetry value $3/8$ at the GUT scale down to $\sim 0.2$ at $O(100 \text{ GeV})$. Even though additional degrees of freedom are needed to account for the small discrepancy to the experimentally observed value 0.23, the hypothesis that the particle interactions remain weak up to the GUT scale is nevertheless qualitatively supported by this result.

(iii) Top-quark quantum corrections to the quartic Higgs coupling are negative, driving the coupling to negative values for which the vacuum becomes unstable. The boundary on the right hand side of the allowed domain in the $(m_t, M_H)$ plane corresponds to the values where the quartic coupling vanishes. For top masses larger than about 100 GeV, this leads to a lower limit on the Higgs mass. The $(m_t, M_H)$ mass pairs are attracted by the line connecting the tips of the allowed areas if the theory evolves from the cut-off energies $\Lambda$ down to the $O(100 \text{ GeV})$ range.

From the requirement of vacuum stability and from the assumption that the Standard Model can be continued up to the GUT scale, upper and lower bounds on the Higgs mass can be derived. Based on these arguments, the Higgs mass could well be expected in the window $100 < M_H < 180 \text{ GeV}$ for a top mass value of 150 GeV. It must however be stressed again that the upper limit is based on assumptions which are backed only qualitatively by the measured value of $\sin^2 \theta_W$.

The important range of $M_H$ between $M_Z$ and $2M_Z \approx 180 \text{ GeV}$ is generally referred to as the intermediate Higgs mass range.
Once the Higgs mass is fixed, the triple and quartic self–couplings of the SM Higgs particle are uniquely determined [at the tree level]. The size of the Higgs couplings to massive gauge bosons and quarks/leptons is set by the masses of these particles. Hence, the profile of the Higgs particle can be predicted completely for a given value of the Higgs mass: the decay properties are fixed and the production mechanisms and the production rates can be determined.

So far, the most comprehensive search for Higgs particles has been carried out in Z decays at LEP based on the Bjorken process $Z \rightarrow Z^* H$. By now, a lower bound on the Higgs mass of about $M_H \geq 63.8$ GeV could be established. This limit can be raised by a few GeV by accumulating more statistics. In the second phase of LEP with a total energy close to 200 GeV, Higgs particles can be searched for up to masses of 85 to 90 GeV in Higgs bremsstrahlung off the Z line. Higher energy colliders are required to sweep the entire mass range for the Higgs particle.

1.2. Supersymmetric Extension

Supersymmetric theories (SUSY) are very attractive extensions of the Standard Model, incorporating the most general symmetry of the $S$ matrix in field theory. At low energies they provide a theoretical framework in which the problem of naturalness and hierarchy in the Higgs sector is solved by retaining Higgs bosons with moderate masses as elementary particles in the presence of high mass scales demanded by grand unification. The Minimal Supersymmetric extension of the Standard Model ($\text{MSSM}$) may serve as a useful guideline into this domain. This point is underlined by the fact that the model led to a prediction of the electroweak mixing angle $\theta_W$ that is in very nice agreement with present high–precision measurements of $\sin^2 \theta_W$. Although some of the phenomena will be specific to this
minimal version, the general pattern will nevertheless be characteristic to more general extensions so that the MSSM can be considered as representative for a wide class of SUSY models.

Supersymmetry requires the existence of at least two isodoublet fields $\Phi_1$ and $\Phi_2$, thus extending the physical spectrum of scalar particles to five. The MSSM is restricted to this minimal extension. The field $\Phi_2$ [with vacuum expectation value $v_2$] couples only to up–type quarks while $\Phi_1$ [with vacuum expectation value $v_1$] couples to down–type quarks and charged leptons. The physical Higgs bosons introduced by this extension are of the following type: two $CP$–even neutral bosons $h$ and $H$ [where $h$ will be the lighter particle], a $CP$–odd neutral boson $A$ [usually called pseudoscalar] and two charged Higgs bosons $H^\pm$. Besides the four masses $M_h$, $M_H$, $M_A$ and $M_{H^\pm}$, two additional parameters define the properties of the scalar particles and their interactions with gauge bosons and fermions: the ratio of the two vacuum expectation values $\tan\beta = v_2/v_1$ and a mixing angle $\alpha$ in the neutral $CP$–even sector. Supersymmetry leads to several relations among these parameters and, in fact, only two of them are independent at tree level. These relations impose a strong hierarchical structure on the mass spectrum [$M_h < M_Z$, $M_A < M_H$ and $M_W < M_{H^\pm}$] which however is broken by radiative corrections since the top quark mass is large. The parameter $\tan\beta$ will in general be assumed in the range $1 < \tan\beta < m_t/m_b$ [$\pi/4 < \beta < \pi/2$], consistent with restrictions that follow from interpreting the MSSM as low energy limit of a supergravity model.

Since the lightest $CP$–even scalar boson $h$ is likely to be the Higgs particle which will be discovered first, an attractive choice of the two input parameters is the set [$M_h, \tan\beta$], with $\tan\beta$ to be determined by the production cross sections. Once these two parameters [as well as the top quark mass and the associated squark masses which enter through radiative corrections] are specified, all other masses and the mixing angle $\alpha$ can be predicted. To incorporate radiative corrections we first shall neglect, for the sake of simplicity, non–leading effects due to non–zero values of the supersymmetric Higgs mass parameter $\mu$ and of the parameters $A_t$ and $A_b$ in the soft symmetry breaking interaction. The radiative corrections are then determined by the parameter $\epsilon$ which grows as the fourth power of the top quark mass $m_t$ and logarithmically with the squark mass $M_S$:

$$\epsilon = \frac{3\alpha}{2\pi} \frac{1}{s_W^2 c_W^2} \frac{1}{\sin^2 \beta} \frac{m_t^4}{M_Z^2} \log \left(1 + \frac{M_Z^2}{m_t^2}\right) \quad (1.1)$$

$s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W$. These corrections are positive and they shift the mass of the light neutral Higgs boson $h$ upward with increasing top mass. The variation of the upper limit on $M_h$ with the top quark mass is shown in Fig. 2 for $M_S = 1$ TeV and two representative values of $\tan\beta = 2.5$ and 20; and update of Ref. While the dashed lines correspond to the leading radiative corrections in eq. (1.1) [$\mu = A_t = A_b = 0$], the solid lines correspond to $\mu = -200, 0, +200$ GeV and $A_t = A_b = 1$ TeV. The upper bound on $M_h$ is shifted from the tree level value $M_Z$ up to $\sim 130$ GeV for $m_t = 175$ GeV and $\sim 140$ GeV for $m_t = 200$ GeV.
Fig. 2 The masses of the Higgs particles in the MSSM including radiative corrections; the squark masses are fixed to 1 TeV. The dashed curve shows the leading correction \( A_t = A_b = \mu = 0 \) while the solid curves include the full corrections \( A_t = A_b = 1 \text{ TeV} \) and \( \mu = -200, 0, 200 \text{ GeV} \) (a) Upper limit on \( M_h \) as a function of \( m_t \); (b)–(d) masses of the \( H, A \) and \( H^\pm \) Higgs bosons as functions of \( M_h \). The top mass is fixed to 175 GeV.
Taking $M_h$ and $\tan \beta$ as the input parameters, the mass of the pseudoscalar $A$ is given by

$$M_A^2 = \frac{M_h^2(M_Z^2 - M_h^2 + \epsilon) - \epsilon M_Z^2 \cos^2 \beta}{M_Z^2 \cos^2 2\beta - M_h^2 + \epsilon \sin^2 \beta}$$ (1.2)

and the masses of the heavy neutral and charged Higgs bosons follow from the sum rules

$$M_H^2 = M_A^2 + M_Z^2 - M_h^2 + \epsilon$$ (1.3)
$$M_{H^\pm}^2 = M_A^2 + M_W^2$$ (1.4)

In the subsequent discussion, we will use for definiteness the two values $m_t = 175$ GeV and $M_S = 1$ TeV. For the two representative values of $\tan \beta$ introduced above, the masses $M_A, M_H$ and $M_{H^\pm}$ are displayed in Fig. 2b–d as a function of the light neutral Higgs mass $M_h$. Apart from the range near the upper limit of $M_h$ for a given value of $\tan \beta$, the masses cluster in characteristic bands of 100 to 200 GeV for $M_H$ and $M_{H^\pm}$, and up to $\sim 150$ GeV for $M_A$ [similarly to $M_h$]. On general grounds, the masses of the heavy neutral and charged Higgs bosons are expected to be of the order of the electroweak symmetry breaking scale.

The mixing parameter $\alpha$ is determined by $\tan \beta$ and the Higgs masses,

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_h^2 - M_Z^2 + \epsilon \cos 2\beta} \quad \left[ -\frac{\pi}{2} \leq \alpha \leq 0 \right]$$ (1.5)

The couplings of the various neutral Higgs bosons to fermions and gauge bosons will in general depend on the angles $\alpha$ and $\beta$. Normalized to the SM Higgs couplings, they are summarized in Table 1. The pseudoscalar particle $A$ has no tree level couplings to gauge bosons, and its couplings to down (up) type fermions are (inversely) proportional to $\tan \beta$.

Tab. 1: Higgs bosons couplings in the MSSM to fermions and gauge bosons relative to the SM Higgs boson couplings.

| $\Phi$ | $g_{\Phi uu}$ | $g_{\Phi dd}$ | $g_{\Phi VV}$ |
|-------|-------------|-------------|-------------|
| $H_{SM}$ | | | |
| $h$ | $\cos \alpha / \sin \beta$ | $- \sin \alpha / \cos \beta$ | $\sin (\beta - \alpha)$ |
| $H$ | $\sin \alpha / \sin \beta$ | $\cos \alpha / \cos \beta$ | $\cos (\beta - \alpha)$ |
| $A$ | $1 / \tan \beta$ | $\tan \beta$ | $0$ |
Fig. 3 Coupling parameters of SUSY Higgs bosons as functions of the lightest Higgs boson mass $M_h$. The couplings are normalized to the SM Higgs boson couplings as in Tab. 1. The parameters are the same as in Fig. 2.
Typical numerical values of these couplings are shown in Fig. 3 as a function of the lightest neutral Higgs boson mass and for two values of $\tan \beta$; the same set of $\mu$ and $A_t, A_b$ values as in Fig. 2 has been chosen. The figure demonstrates that the dependence on the parameters $\mu$ and $A_t, A_b$ is very weak and the leading radiative corrections provide a very good approximation. [This is also the case for the radiative corrections to the Higgs boson masses, Fig. 2, although the dependence on $A_t, A_b$ is slightly stronger in this case, leading to a shift of a few GeV]. There is in general a strong dependence on the input parameters $\tan \beta$ and $M_h$. The couplings to down (up) type fermions are enhanced (suppressed) compared to the SM Higgs couplings. If $M_h$ is very close to its upper limit for a given value of $\tan \beta$, the couplings of $h$ to fermions and gauge bosons are SM like. It is therefore very difficult to distinguish the Higgs sector of the MSSM from the SM, if all other Higgs bosons are very heavy.

Negative searches for SUSY Higgs particles at LEP in the processes $Z \to Z^* H$ and $Z \to AH$ exclude $h$ and $A$ bosons with masses smaller than $M_{h,A} \simeq 45$ GeV for $m_t = 140$ GeV, $M_S = 1$ TeV and $\tan \beta > 1$ [if the parameters of the model are allowed to vary arbitrarily and if one includes all possible decay modes, this bound becomes $M_h > 44$ GeV and $M_A > 21$ GeV].

Charginos and neutralinos are expected to be the lightest supersymmetric particles. In general, they are mixtures of the [non–colored] gauginos and Higgsinos, the spin 1/2 supersymmetric partners of the gauge bosons and Higgs bosons. There are two chargino $\tilde{\chi}^{\pm}_i \ [i = 1, 2]$ and four neutralino $\tilde{\chi}^{0}_i \ [i = 1, ..., 4]$ states, where $\tilde{\chi}^{0}_1$ will be assumed to be the stable lightest supersymmetric particle. The masses and the couplings of these particles are obtained by diagonalizing the charged and neutral mass matrices; the unitary matrices which diagonalize the mass matrices can be found in Ref. Interpreting the MSSM as the low–energy limit of a supergravity model, the matrix elements will depend on four parameters, one Higgs mass $[M_h$ or $M_A]$, $\tan \beta$, $\mu$ and the SU(2) gaugino mass $M$ which, without loss of generality, can be taken to be positive. [The fifth SUSY parameter of the model is the universal mass parameter $m_0$ of the scalar particles at the unification scale.]

From the negative search of supersymmetric particles in $Z$ decays, the lightest neutralino $[\tilde{\chi}^{0}_1]$ mass is restricted to be larger than 20 GeV for $\tan \beta = 2.5$ and larger than 22 GeV for $\tan \beta > 4$; the second lightest neutralino $[\tilde{\chi}^{0}_2]$ and the charginos are excluded if their masses are less than $\sim M_Z/2$. If the search at LEP200 with a c.m. energy of 180 GeV is negative, charginos with masses $m_{\tilde{\chi}^\pm_i} < 90$ GeV will also be excluded. The present LEP data also exclude sleptons with masses below $\sim M_Z/2$; if sleptons will not be observed at LEP200 these limits can be improved by roughly a factor of two. On the other hand, CDF data restrict the squarks masses to be larger than $\sim 150$ GeV if cascade decays are suppressed. Since the sfermion masses depend on the universal mass parameter $m_0$, they are not fixed by the mass scales which appear in the chargino/neutralino mass matrices. We shall assume in this discussion that squarks and sleptons are heavy so that they will not affect Higgs boson decays and production.
1.3. Organization of the paper

The discovery of Higgs particles is the most fundamental test of the modern formulation of the electroweak theory, and therefore is the major goal of future accelerators. In this review, we will discuss the prospects for producing Higgs particles and once produced, for studying their fundamental properties in the Standard Model as well as in its minimal supersymmetric extension at future high-energy colliders: the CERN proton–proton collider LHC with a center of mass energy of \( \sim 14 \) TeV and a future \( e^+e^- \) linear collider [which can also be turned into a high-energy \( \gamma\gamma \) collider] with an energy in the range 300 to 500 GeV.

We will update various results for Higgs masses and couplings in the supersymmetric extension of the Standard Model, partial decay widths and production cross sections, to take into account the value of the top quark mass recently published by the CDF collaboration:

\[
m_t = 174^{+10}_{-11} \pm 10 \text{ GeV} \tag{1.6}
\]

with a central value which coincides with the favored value obtained from a global fit of electroweak precision measurements at LEP and SLC:

\[
m_t = 174^{+11}_{-12}^{+17}_{-19} \text{ GeV} \tag{1.7}
\]

Whenever possible we will use the value \( m_t = 175 \) GeV, although sometimes we will also use the values 150 or 200 GeV which can be viewed as conservative lower and upper bounds on the top mass, respectively.

In section 2, we will summarize the various decay modes of the Higgs particles including a discussion of the main QCD and electroweak radiative corrections, some of which have been calculated only recently. Partial and total decay widths as well as the most important decay branching fractions will be given. In the case of supersymmetric Higgs bosons, both the standard and the supersymmetric decay modes will be discussed.

In section 3, we will discuss Higgs production at the CERN “Large Hadron Collider” (LHC). We will summarize the various production processes, with a special emphasis on the dominant one, gluon–gluon fusion via a heavy quark loop, for which we will discuss the QCD corrections [which turned out to be very important] and the leading electroweak correction. The main detection channels for standard and supersymmetric Higgs bosons will be summarized.

In section 4, we will analyze the potential of a future \( e^+e^- \) linear collider with a center of mass energy of 500 GeV. We will discuss in some detail the main production processes, especially the bremsstrahlung process, and the corresponding backgrounds. We then analyze the possibility of measuring some fundamental properties of the Higgs particles, such as their couplings to the other elementary particles and their spin–parity quantum number assignments. A short discussion about the complementary potential of \( \gamma\gamma \) colliders will be given.

Section 5 contains our conclusions.
2. Decays Modes

2.1. Standard Model

As previously discussed, the profile of the Higgs particle in the Standard Model is uniquely determined at tree level if the Higgs mass is fixed. The strength of the Yukawa couplings of the Higgs boson to fermions is set by the fermion masses $m_f$, and the coupling to the electroweak gauge bosons $V = W, Z$ by their masses $M_V$:

\[
g_{ffH} = \left(\frac{\sqrt{2} G_F}{m_f}\right)^{1/2} \quad (2.1)
\]

\[
g_{V VH} = 2 \left(\frac{\sqrt{2} G_F}{M_V}\right)^{1/2} \quad (2.2)
\]

The decay width, the branching ratios and the production cross sections are given by these parameters. The partial widths for the main decay channels, including the dominant QCD and electroweak radiative corrections, and their characteristics are summarized below; an update of Ref. 32

In the Born approximation the width of the Higgs decay into lepton pairs is

\[
\Gamma(H \rightarrow l^+l^-) = \frac{G_F m_l^2}{4\sqrt{2}\pi} M_H \beta^3 \quad (2.3)
\]

with $\beta = (1 - 4m_l^2/M_H^2)^{1/2}$ being the velocity of the leptons in the final state. For the decay widths into quark pairs, eq. (2.3) has to be supplemented by a color factor $N_c = 3$ in the Born approximation. However, the QCD corrections turn out to be quite large and therefore must be included, and the partial decay width reads

\[
\Gamma(H \rightarrow q\bar{q}) = \frac{3G_F m_q^2}{4\sqrt{2}\pi} M_H \beta^3 \left[1 + \frac{4\alpha_s}{3\pi} \Delta_{H}^{QCD}\right] \quad (2.4)
\]

where the QCD correction factor is given by

\[
\Delta_{H}^{QCD} = \frac{1}{\beta} A(\beta) + \frac{1}{16 \beta^3} (3 + 34\beta^2 - 13\beta^4) \log \frac{1 + \beta}{1 - \beta} + \frac{3}{8\beta^2} (7\beta^2 - 1) \quad (2.5)
\]

with $[\text{Li}_2$ is the Spence function defined by, $\text{Li}_2(x) = - \int_0^x dy \log(1 - y)]$

\[
A(\beta) = (1 + \beta^2) \left[4\text{Li}_2 \left(\frac{1 - \beta}{1 + \beta}\right) + 2\text{Li}_2 \left(\frac{1 - \beta}{1 + \beta}\right) - 3\log \frac{1 + \beta}{1 - \beta} \log \frac{2}{1 + \beta} - 2\log \frac{1 + \beta}{1 - \beta} \log \beta \right] - 3\beta \log \frac{4}{1 - \beta^2} - 4\beta \log \beta \quad (2.6)
\]

In the limit $M_H \gg m_q$, the decay width receives logarithmic contributions which in the case of the $b$ quark and for Higgs masses around 100 GeV, decrease the $H \rightarrow b\bar{b}$ decay width by more than 50%. The bulk of these QCD corrections can be absorbed into running quark masses evaluated at the scale $\mu = M_H$. In the
limit $M_H \gg m_t$, one can also include the order $\alpha_s^2$ corrections which are known \cite{4} and the decay width reads

$$\Gamma(H \to q\bar{q}) = \frac{3G_F}{4\sqrt{2}\pi}m_q^2(M_H^2)M_H \left[ 1 + 5.67 \left( \frac{\alpha_s}{\pi} \right) + (35.94 - 1.36N_F) \left( \frac{\alpha_s}{\pi} \right)^2 \right]$$ \hspace{1cm} (2.7)

$\alpha_s \equiv \alpha_s(M_H^2)$ and $N_F = 5$ is the number of active quark flavors; all quantities are defined in the $\overline{\text{MS}}$ scheme with $\Lambda_{\overline{\text{MS}}} \sim 150$ MeV for $N_F = 5$. For $M_H = 120$ GeV, the $b$ and $c$ quark masses $m_b(m_b^2) = 4.2$ GeV and $m_c(m_c^2) = 1.35$ GeV have dropped to the effective values $m_b(M_H^2) = 3$ GeV and $m_c(M_H^2) = 0.77$ GeV, respectively. For Higgs decays to top quarks, the QCD corrections are not large (except near threshold) since $m_t$ is of the same order as $M_H$ and the decay width is approximately given by eq. (2.3) apart from the additional color factor $N_c = 3$.

The electroweak radiative corrections to fermionic Higgs decays are well under control \cite{10, 23}. Parametrizing the Born formulae (2.3) in terms of the Fermi coupling constant $G_F$, the corrections are free of large logarithms associated with light fermion loops. Below the $W$ threshold, the total correction $\delta = \delta_{\text{EW}} + \delta_{\text{m}}$ can be approximated by \cite{10, 23} [similarly to the QCD corrections, the additional logarithmic term $\delta_m$ for photonic corrections can be mapped into the running mass]

$$\delta_{\text{EW}} = \alpha \left[ \frac{9}{4} Q_f^2 + \frac{1}{16\hat{s}_W} \left( k_f \frac{m_t^2}{\hat{M}_W^2} - 5 + \frac{3}{s_W^2} \log c_W^2 \right) - 3\hat{\delta}_f^2 + \frac{1}{2} \hat{\alpha}_f^2 \right]$$ \hspace{1cm} (2.8)

with $\hat{\delta}_f = (2I_1^f - 4s_W^2 Q_f)/(4s_W c_W)$ and $\hat{\alpha}_f = 2I_3^f/(4s_W c_W)$. The coefficient $k_f = 7$ for $\tau$, $c$ is reduced to $k_f = 1$ for $b$ quarks and the total correction for these fermions is of the order of a few percent.

Above the $H \to WW$ and $ZZ$ decay thresholds, the partial width into massive gauge boson pairs may be written as \cite{10, 23}

$$\Gamma(H \to VV) = \delta_V \frac{\sqrt{2}G_F}{32\pi} M_H^3 \delta' \left( 1 - 4x + 12x^2 \right)^2$$ \hspace{1cm} (2.9)

where $x = M_V^2/M_H^2$, $\beta = \sqrt{1 - 4x}$ and $\delta_V = 2(1) \quad V = W(Z)$. For large Higgs masses, the vector bosons are longitudinally polarized. Since the longitudinal wave functions are linear in the energy, the width grows as the third power of the Higgs mass. The electroweak radiative corrections \cite{10, 23} are positive and amount to a few percent above the threshold.

Below the threshold for two real bosons, the Higgs particle can decay into real and virtual $VV^*$ pairs, primarily $WW^*$ pairs above $M_H \sim 110$ GeV. The partial decay width, $W$ charges summed over, is given by \cite{12}

$$\Gamma(H \to VV^*) = \frac{3G_F^2}{16\pi^3} M_H^4 \delta_W X_R(x)$$ \hspace{1cm} (2.10)

where $\delta_W = 1$ and $\delta_Z = 7/12 - 10\sin^2\theta_W/9 + 40\sin^4\theta_W/27$ and

$$X_R(x) = \frac{3(1 - 8x + 20x^2)}{(4x - 1)^{3/2}} \arccos \left( \frac{3x - 1}{2x^{3/2}} \right) - \frac{1 - x}{2x} (2 - 13x + 47x^2)$$

$$- \frac{3}{2} (1 - 6x + 4x^2) \log x$$ \hspace{1cm} (2.11)
The $\gamma\gamma$, $\gamma Z$, and $gg$ couplings to Higgs bosons are mediated by heavy particle triangular loops. These loop decays are important only for Higgs masses below $\sim 140$ GeV where the total decay width is rather small. However, they are very interesting since their strength is sensitive to scales far beyond the Higgs mass and can be used as a possible telescope for new particles whose masses are generated by the Higgs mechanism.

In the Standard Model only the $W$ and top quark loops contribute significantly to the $H\gamma\gamma$ coupling. For a Higgs boson in the intermediate mass range, $M_W \leq M_H \leq 2M_W$, the $W$ loop contribution is always dominating and interferes destructively with the fermion amplitude. The decay width varies from $\sim 5$ to $\sim 50$ keV in this mass range. The QCD corrections to the quark amplitude have been calculated in Refs.\textsuperscript{46,47,48} in the intermediate mass range, the correction factor is well under control being of $\mathcal{O}(\alpha_s/\pi)$. Thus, contrary to the $H \to \bar{q}q$ case, the QCD corrections to $H \to \gamma\gamma$ do not generate large logarithms. A more detailed discussion of these corrections will be given in section 4.3.

Similar to the $\gamma\gamma$ case, the $H \to Z\gamma$ coupling is built up by top quark and $W$ loops; the $W$ loop being by far dominating. The decay occurs for $M_H > M_Z$ and the decay width varies from a few keV for $M_H \sim 120$ to $\sim 100$ keV for $M_H \sim 2M_W$. The QCD corrections to the quark loop, calculated in Ref.\textsuperscript{44}, are rather small in the previous range for $M_H$, being of $\mathcal{O}(\alpha_s/\pi)$.

Gluonic Higgs decays, $H \to gg$, are mediated by top quark loops\textsuperscript{45} in the Standard Model and the decay width is of significance only for Higgs masses below the top threshold. Incorporating the QCD radiative corrections which include $ggg$ and $gq\bar{q}$ final states and which are very important since they increase the partial width by $\sim 65\%$, the partial width can be cast in the approximate form\textsuperscript{46}

$$
\Gamma(H \to gg) = \frac{G_F \alpha_s^2(M_H^2)}{30\sqrt{2}\pi^3} M_H^3 \left[ 1 + \frac{215 \alpha_s(M_H^2)}{12 \pi} \right]
$$

By adding up all possible decay channels, we obtain the total width shown in Fig. 4a for $m_t = 175$ GeV. Up to masses of 140 GeV, the Higgs particle is very narrow, $\Gamma(H) \leq 10$ MeV. After opening the [virtual] gauge boson channels, the state becomes rapidly wider, reaching $\sim 1$ GeV at the $ZZ$ threshold. The width cannot be measured directly in the intermediate mass range. Only above $M_H \geq 250$ GeV it becomes wide enough to be resolved experimentally.

The branching ratios of the main decay modes are displayed in Fig. 4b; an update of Ref.\textsuperscript{51} A large variety of channels will be accessible for Higgs masses below 140 GeV. The by far dominant mode are $b\bar{b}$ decays, yet $c\bar{c}$, $\tau^+\tau^-$ and $gg$ still occur at a level of several percent. [At $M_H = 120$ GeV for instance, the branching ratios are $68\%$ for $b\bar{b}$, $4.6\%$ for $c\bar{c}$, $6.6\%$ for $\tau^+\tau^-$ and $6\%$ for $gg$.] The branching ratios for the $H \to \gamma\gamma$ and $\gamma Z$ are small, being of $\mathcal{O}(10^{-3})$. Above the mass value $M_H = 140$ GeV, the Higgs boson decay into $W$’s becomes dominant, overwhelming all other channels once the decay mode into two real $W$’s is kinematically possible.
Fig. 4 Total decay width (a) and decay branching ratios (b) of the SM Higgs boson; the top quark mass is fixed to $m_t = 175$ GeV. The QCD corrections to the hadronic decay modes are included.
2.2. Supersymmetric Extension

The lightest Higgs boson will decay mainly into fermion pairs since its mass is smaller than $\sim 130$ GeV. This is also the dominant decay mode of the pseudoscalar boson $A$ which has no tree level couplings to gauge bosons. The partial decay width of a neutral Higgs boson $\Phi$ into fermion pairs is given by

$$
\Gamma(\Phi \to \bar{f}f) = \frac{N_c G_F m_f^2}{4\sqrt{2}\pi} g_{\Phi ff}^2 M_\Phi \beta^p
$$

(2.13)

where $\beta = (1 - 4m_f^2/M_\Phi^2)^{1/2}$ and $p = 3(1)$ for the $\mathcal{CP}$–even (odd) Higgs boson; the couplings $g_{\Phi ff}$ are listed in Tab.1. For final state quarks one has to include QCD corrections; in the case of $\mathcal{CP}$–even neutral Higgs bosons the correction factor is given by eq. (2.5) while in the case of the pseudoscalar boson one has

$$
\Delta_{A}^{QCD} = \frac{1}{\beta} A(\beta) + \frac{1}{16\beta} (19 + 2\beta^2 + 3\beta^4) \log \frac{1 + \beta}{1 - \beta} + \frac{3}{8} (7 - \beta^2)
$$

(2.14)

with $A(\beta)$ given by eq. (2.6). Here, again one has to use the running masses which take into account the bulk of these QCD corrections, which in the limit $M_H \gg m_q$ are the same for $\mathcal{CP}$–odd and $\mathcal{CP}$–even Higgs bosons; as discussed previously in the case of $\bar{b}$ quarks and for $M_\Phi \sim 100$ GeV, this results in a decrease of the decay width by roughly a factor of two. For values of $\tan\beta$ larger than unity and for masses less than $\sim 130$ GeV, the main decay modes of the neutral Higgs bosons will be decays into $b\bar{b}$ and $\tau^+\tau^-$ pairs; the branching ratios being always larger than $\sim 90\%$ and $5\%$, respectively. The decays into $c\bar{c}$ and gluons are in general strongly suppressed especially for large values of $\tan\beta$. For large masses, the top decay channels $H, A \to t\bar{t}$ open up; yet this mode remains suppressed for large $\tan\beta$.

If the mass is high enough, the heavy $\mathcal{CP}$–even Higgs boson can in principle decay into weak gauge bosons $H \to VV$, $V = W$ or $Z$. Below the threshold for two real bosons, the $\mathcal{CP}$–even neutral Higgs bosons $H$ and $h$ can decay into $VV^*$ pairs, one of the vector bosons being virtual. The partial decay widths are given by

$$
\Gamma(\Phi \to VV^{(*)}) = g_{\Phi VV}^2 \Gamma(H_{SM} \to VV^{(*)})
$$

(2.15)

with $\Phi = h$ or $H$ and $\Gamma(H_{SM} \to VV^{(*)})$ given by eqs. (2.9–2.11). Since $h$ is light and the $H$ partial width is proportional to $\cos^2(\beta - \alpha)$, they are strongly suppressed and the width of the $ZZ$ signal is very small in the supersymmetric extension. [If $M_H$ is large enough for these decay modes to be kinematically allowed, $M_h$ is very close to its maximum so that $\cos^2(\beta - \alpha) \to 0$.] For the same reason, the cascade decay of the $\mathcal{CP}$–odd Higgs boson, $A \to Zh$, is suppressed in general

$$
\Gamma(A \to Zh) = \frac{G_F}{8\sqrt{2}\pi} \cos^2(\beta - \alpha) \frac{M_A^2}{M_A} \lambda^{1/2}(M_Z^2, M_h^2, M_A^2) \lambda(M_A^2, M_h^2, M_Z^2)
$$

(2.16)

with $\lambda(x, y; z) = (1 - x/z - y/z)^2 - 4xy/z^2$ being the usual two–body phase space function.
The heavy neutral Higgs boson $H$ can also decay into two lighter Higgs bosons:\[ \Gamma(H \to hh) = \frac{G_F}{16\sqrt{2}\pi} \frac{M_H^4}{M_H} \left(1 - 4 \frac{M_h^2}{M_H^4}\right)^{1/2} \left[ \cos 2\alpha \cos(\beta + \alpha) - 2 \sin 2\alpha \sin(\beta + \alpha) \right]^2 \] (2.17)

These modes, however, are restricted to small domains in the parameter space.

Gluonic Higgs decays $\Phi \to gg$ are mediated by top and bottom quark loops [the squarks decouple from the effective $\Phi gg$ vertex for high masses]. For the light Higgs particle this decay mode is significant only for $h$ masses close to the maximal value where $h$ has $S\mathcal{M}$ like couplings, and for $H$ masses only below 140 GeV and small values of $\tan\beta$ where the coupling to top quarks is sufficiently large. Therefore, one can neglect the $b$ loop contribution, and the decay width $\Gamma(\Phi \to gg)$ with $\Phi = h$ or $H$ is given by eq. (2.12) up to the factor $g^2_{\Phi h}$. For the pseudoscalar Higgs particle, the gluonic decay mode is marginal.

Decays into of the $CP$–even and $CP$–odd Higgs bosons into $\gamma\gamma$ and $Z\gamma$ final states are very rare with branching ratios of order $\mathcal{O}(10^{-3})$ or below. The QCD corrections to the two photon decay widths have been calculated in Ref.\cite{ref2} and found to be small, $\sim \mathcal{O}(\alpha_s/\pi)$ across the physically interesting mass ranges if the running of the quark masses is properly taken into account; for details see section 4.3 where Higgs production in $\gamma\gamma$ fusion will be discussed.

The coupling of the charged Higgs particle to fermions is a $P$ violating mixture of scalar and pseudoscalar couplings

$$g_{H^{\pm} ud} = \left( \frac{G_F}{\sqrt{2}} \right)^{1/2} \left[ (1 - \gamma_5) \frac{m_u}{\tan\beta} + (1 + \gamma_5) m_d \tan\beta \right]$$

(2.18)

The charged Higgs particles decay into fermions with a partial decay width

$$\Gamma(H^+ \to ud) = N_c \frac{G_F}{4\sqrt{2}\pi} \frac{\lambda^{1/2}(m_u^2, m_d^2, M_{H^\pm})}{M_{H^\pm}} \left[ (M_{H^\pm}^2 - m_u^2 - m_d^2) \right]
\left( m_u^2 \tan^2\beta + \frac{m_d^2}{\tan^2\beta} \right) - 4m_u^2 m_d^2\right]$$

(2.19)

Here again one has to include the QCD corrections in the limit $m_d = 0$, corresponding to the approximate width in the case of the top–bottom decays where the $m_b$ effects can be neglected, the QCD factor is given by $\left[ \alpha = m_t^2/M_{H^\pm}^2 \right]$\[ \Delta_{H^\pm}^{QCD} = \frac{9}{4} + \frac{3 - \alpha + 2\alpha^2}{2(1 - \alpha)} \log \frac{\alpha}{1 - \alpha} - \left[ 2\text{Li}_2 \left( \frac{\alpha}{\alpha - 1} \right) \right]
+ \log(1 - \alpha) - \log(1 - \alpha) \log \frac{\alpha}{1 - \alpha}\right]

(2.20)

If allowed kinematically, charged Higgs bosons also decay into the lightest neutral Higgs plus a $W$ boson,

$$\Gamma(H^+ \to Wh) = \frac{G_F \cos^2(\beta - \alpha)}{8\sqrt{2}\pi c_W} \frac{M_W}{M_H} \lambda^4(M_W, M_H; M_{H^\pm}) \lambda(M_{H^\pm}, M_H; M_W)$$

(2.21)
Below the $tb$ and $Wh$ thresholds, the charged Higgs particles will decay mostly into $\tau\nu_\tau$ and $c\bar{s}$ pairs, the former being dominant for $\text{tg}\beta > 1$. For large $M_{H^\pm}$ values, the top–bottom decay $H^+ \rightarrow t\bar{b}$ becomes dominant.

Adding up the various decay modes, the width of all five Higgs bosons remains very small, even for large masses. This is shown for the two representative values $\text{tg}\beta = 2.5$ and 20 in Fig. 5. Apart from the $CP$–even heavy neutral Higgs boson $H$ and small $\text{tg}\beta$, the pattern of branching ratios is in general quite simple. The neutral Higgs bosons decay preferentially to $b\bar{b}$, and to a lesser extent to $\tau^+\tau^-$ pairs; the charged Higgs bosons to $\tau\nu_\tau$ and, preferentially, $t\bar{b}$ pairs above this threshold; see Fig. 6.
Fig. 6 Decay branching ratios of the Higgs bosons [without SUSY decays] as functions of their masses for two values of $\tan \beta = 2.5$ and 20; $m_t = 175$ GeV and $M_S = 1$ TeV.
In most studies of supersymmetric Higgs bosons at future colliders it is assumed that they do not decay into supersymmetric particles. However, while sfermions are probably too heavy to affect Higgs decays, the Higgs boson decays into charginos and neutralinos could eventually play a significant role since some of these particles are expected to be lighter or of the same order as $M_Z$. These new channels could open up at least for the heavy Higgs bosons $H, A$ and $H^\pm$. They could be so large that they reduce the branching fractions for the standard decays in a sizable way, therefore altering the signals, and as a result, the search strategies for these particles.

The decay widths of the neutral Higgs bosons $H_k$, with $H_1 = H$, $H_2 = h$ and $H_3 = A$, into chargino or neutralino pairs are given by

$$\Gamma(H_k \to \tilde{\chi}_i \tilde{\chi}_j) = K \frac{M_H}{\delta(i,j)} \left[ (F^2_{ijk} + F^2_{ji}) \left( 1 - \frac{M_i^2}{M_H^2} - \frac{M_j^2}{M_H^2} \right) - 4F_{ijk}F_{jik}\eta_k \frac{|M_iM_j|}{M_H^2} \right]$$

where $\eta_{1,2} = +1$, $\eta_3 = -1$ and $\delta(i,j) = 1$ unless the final state consists of two identical Majorana neutralinos in which case $\delta(i,j) = 2$; $K = G_F/(2\sqrt{2}\pi)\lambda^1/2M_W^2$. The coefficients $F_{ijk}$ can be expressed in terms of the elements of the matrices $U, V$ which diagonalize the chargino mass matrices, and of the neutralino matrix $Z$, given in Ref.\[55\] For the charged Higgs boson decays into neutralino/chargino pairs, the partial widths read [Ref.\[55\] $F_{L,R}$ can be found in Ref.\[55\]]

$$\Gamma(H^\pm \to \tilde{\chi}_i^\pm \tilde{\chi}_j^0) = KM_{H^\pm} \left[ (F^2_L + F^2_R) \left( 1 - \frac{M_i^2}{M_{H^\pm}^2} - \frac{M_j^2}{M_{H^\pm}^2} \right) - 4F_LF_R\frac{|M_iM_j|}{M_{H^\pm}^2} \right]$$

The decay branching ratios of the lightest $CP$–even neutral Higgs boson $h$ into the lightest chargino pair and the lightest and next-to-lightest neutralino pairs, are displayed in Fig. 7, an update of Ref.\[21,57\]. The contours are shown in the $[\mu, M]$ plane where the sum of these branching ratios exceeds 5% [dashed lines] and 50%...
[solid lines]; the dotted lines are the contours which are excluded by LEP100 data and which can be probed at LEP200. These decays can be sizable in the area of the $[\mu, M]$ plane between the two LEP contours. They are particularly important for $h$ masses close to the maximum allowed values [see Fig. 2]; in this case the lightest Higgs boson has $S_M$ couplings and the dominant $b\bar{b}$ decay mode is not enhanced anymore for large $\tan\beta$ values, so that other decay modes can become significant. For these masses and for large $\tan\beta$ values, the branching ratios for neutralino/chargino decays are sizable even outside the regions which can be probed at LEP200.

The branching ratios of the heavy $CP$–even, the $CP$–odd and the charged Higgs boson decays into chargino and neutralino pairs can be very large and they can even be dominant in some areas of the MSSM parameter space, exceeding 50% for positive values of $\mu$ and/or $M$ values below 200 GeV. This is mainly due to the fact that the couplings of the Higgs bosons to charginos and neutralinos are gauge couplings which can be larger than the Yukawa couplings to standard fermions and the couplings to the gauge bosons [the latter being zero at tree level for the $CP$–odd and being suppressed for the heavy $CP$–even Higgs particles]. Finally, the branching fractions of the invisible neutral Higgs decays can be important, Fig. 8, and they could jeopardize the search for the Higgs particles at hadron colliders.

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Fig. 7. Contour lines in the $[\mu, M]$ plane where the sum of the branching ratios of the lightest Higgs boson $h$ into charginos and neutralinos exceeds 5% (dashed lines) and 50% (full lines) for two values of $M_h$ and $\tan\beta$; the shaded areas are the regions which are (can be) excluded at LEP100 (LEP200). The top mass was taken to be $m_t = 140$ GeV.
Fig. 8. Branching ratios of the decays of the three neutral Higgs bosons into the lightest neutralino pair [invisible decays] as a function of their masses for $\tan \beta = 2.5$ and 20. The top mass is fixed to 175 GeV and $M_S = 1$ TeV.
3. Production at pp Colliders

3.1. Standard Model Higgs

3.1.1. Production mechanisms

The major goal of the CERN future “Large Hadron Collider” [and the one of the late “Superconducting Super Collider” planned in Texas] is the search for Higgs particles. There are four main production mechanisms of neutral Higgs bosons at hadron colliders, and all of them make use of the fact that the Higgs boson couples preferentially to heavy particles. These four processes, see Fig. 9, are the following:

\begin{align*}
\text{gluon – gluon fusion mechanism} : & \quad gg \rightarrow H \\
\text{massive vector boson fusion} : & \quad W W/ZZ \rightarrow H \\
\text{associate production with } W/Z : & \quad q\bar{q} \rightarrow V + H \\
\text{associate production with } \bar{t}t : & \quad gg, q\bar{q} \rightarrow \bar{t}t + H
\end{align*}

Fig. 9 Main production mechanisms of Higgs bosons at proton colliders.

The cross sections are shown in Fig. 10 for a center of mass energy of 16 TeV, typical of LHC. [Note that for a luminosity of \( \mathcal{L} = 10^{33}(10^{34}) \) cm\(^{-2}\)s\(^{-1}\), \( \sigma = 1 \) pb would correspond to \( 10^4(10^5) \) events per year.]

All the way up to Higgs masses of the order of 1 TeV, the dominant production process is the gluon–gluon fusion mechanism, which proceeds through a triangular top quark loop in the Standard Model. In the intermediate Higgs mass range \( M_W < M_H < 2M_Z \), the cross section is of the order of a few tens of pb [in Fig. 10 the two extreme values of the top mass, \( m_t = 150 \) and \( m_t = 200 \) GeV, are used]. The cross section is enhanced for Higgs masses near the \( 2m_t \) threshold and drops down to values of the order of 0.1 pb for \( M_H \sim 1 \) TeV.
The next most important production mechanism is the $WW/ZZ$ fusion process, $qq \rightarrow V^*V^* \rightarrow qqH$. For intermediate mass Higgs bosons, the cross section is of the order of a few pb, i.e. one order of magnitude below the $gg$ fusion mechanism cross section, but it drops less rapidly than the $gg \rightarrow H$ cross section and becomes competitive with the latter for Higgs masses of the order of 1 TeV. In fact, when the top quark was believed to be rather light, this process has received much attention since it was the dominant production mechanism in a wide Higgs mass range.

Fig. 10 Cross sections [in pb] for $S_M$ Higgs production at the LHC as a function of the Higgs mass. The c.m. energy was taken to be $\sqrt{s} = 16$ TeV; for the processes $gg \rightarrow H$ and $pp \rightarrow t\bar{t}H$, the top quark mass was chosen to be $m_t = 150$ and 200 GeV.

The process where the Higgs particle is produced in association with $W/Z$ gauge bosons is important only for intermediate mass Higgs bosons where the cross sections are another order of magnitude smaller than the one for $WW/ZZ$ fusion. However, if the luminosity is high enough, this process can be useful since one can trigger on the $W$ or $Z$ bosons through their leptonic decay modes. Note that the $WH$ process dominates over the $ZH$ process, a consequence of the fact that the charged current couplings are larger than the neutral couplings.

Finally, the Higgs production in association with $t\bar{t}$ pairs has a cross section of the same order as the $q\bar{q} \rightarrow VH$ cross section in the intermediate mass range, i.e. $O (1$ pb $)[for$ $m_t = 175$ GeV, the cross section would be slightly smaller than
shown in the figure]. The process proceeds through gluon–gluon fusion as well as $q\bar{q}$ annihilation, the contribution of the former being much larger than the one of the latter. As for the $q\bar{q} \rightarrow VH$ process, it could be useful in the intermediate mass range since one can also trigger on the $W$ boson produced in the $t \rightarrow bW$ decay.

Note that for the SSC with a planned c.m. energy of 40 TeV, the cross section for the main production mechanism, $gg \rightarrow VH$, has been found to be rather large; they will be discussed in some detail in the following.

For completeness, we will give expressions for the matrix element squared, once the sum/average over spin/color is performed, for the two processes $q\bar{q} \rightarrow VH$ and $qq \rightarrow V^*V^* \rightarrow qqH$ where $V = W, Z$ [note that there are a few misprints in the corresponding formulae in the compilation of the first paper in Ref. 25].

For the $q\bar{q} \rightarrow VH$ process, one has

$$|\mathcal{M}(q\bar{q} \rightarrow VH)|^2 = \frac{G_F^2 M_V^4}{18s} \left(v_q^2 + a_q^2\right) \lambda(\hat{s}, M_W^2/\hat{s}) + \frac{12 M_Z^2/\hat{s}}{(1 - M_V^2/\hat{s})^2}$$

(5)

where $\lambda$ is the usual two–body phase space function $\lambda(x, y) = (1 - x - y)^2 - 4xy$ and $v_q/a_q$ the vector/axial–vector couplings of the quarks to the vector bosons:

$v_q = \sqrt{2}, a_q = -\sqrt{2}$ for the $W$ and $v_q = 2I_{3q} - 4e_eq_W$ and $a_q = 2I_{3q}$ for the $Z$ [$I_{3q}$ is the third component of weak isospin and $e_q$ the electric charge of the quark].

In the case of the $qq \rightarrow q\bar{q}H$ production mechanism, one has in terms of the scalar products of the quark momenta

$$|\mathcal{M}|^2 (q(p_1)q(p_2) \rightarrow q(p_1)q(p_2)H) = \frac{32\sqrt{2}G_F^3 M_V^8}{[(p_1 - p_1)^2 - M_V^2]^2[(p_2 - p_2)^2 - M_V^2]^2} \times \left[(g_{LL}g_{LR}^2 + g_{RR}g_{RL}^2)(p_1 \cdot p_2 p_1 \cdot p_2) + (g_{LL}g_{RR}^2 + g_{RR}g_{LL}^2)(p_1 \cdot p_2 p_1 \cdot p_2)\right]$$

(6)

where $g_{LL, R} = (v_q \mp a_q)/2$ with $v_q$ and $a_q$ defined as previously. To obtain the total cross–sections, one has to sum over the contributions of quarks and antiquarks [with the appropriate couplings] and convolute with the appropriate quark distribution functions. The cross section for the $pp \rightarrow t\bar{t}H$ is rather involved, the $gg$ fusion mechanism will be discussed separately in the next subsection.

Besides the errors due to the poor knowledge of the gluon distribution at small $x$ [this will be improved by future measurements at the $ep$ collider HERA], the lowest order cross sections are affected by large uncertainties due to higher order corrections. Including the next to leading QCD corrections, the total cross sections can be defined properly: the scale at which one defines the strong coupling constant is fixed and the [generally non–negligible] corrections are taken into account. The “K–factors” for $VH$ production [which can be inferred from the Drell–Yan production of weak bosons] and the $VV$ fusion mechanisms [are small, increasing the total cross sections by $\sim 20\%$ and $10\%$ respectively; the corrections to the associate $ttH$ production process are still not known. The QCD radiative corrections to the main production mechanism, $gg \rightarrow H$, have been computed in Refs. 64 and have been found to be rather large; they will be discussed in some detail in the following.
3.1.2. gg fusion mechanism

As previously mentioned, the dominant production mechanism for the Standard Higgs boson at proton–proton colliders is the gluon–gluon fusion process. The Higgs boson couple to gluons primarily through a heavy top quark triangle loop. To lowest order (LO), the cross section is given by

$$\sigma_{\text{LO}}(pp \rightarrow H + X) = \sigma_0^H \frac{d\mathcal{L}^{gg}}{d\tau_H}$$

where $d\mathcal{L}^{gg}/d\tau_H$ denotes the gluon luminosity at $\tau_H = M^2_H/s$ with $\sqrt{s}$ being the c.m. energy of the proton collider. The parton cross sections can be expressed in terms of a form factor derived from the quark triangle diagrams in Fig. 9,

$$\sigma_H = \frac{G_F^2}{288\sqrt{2}\pi} \left| \sum_Q F^H_{HQ}(\tau_Q) \right|^2$$

The form factor is related to the scalar triangle integral $f$,

$$f(\tau_Q) = \begin{cases} \arcsin^2 \sqrt{\tau_Q} & \tau_Q < 1 \\ -\frac{1}{4} \left[ \log \frac{1+\sqrt{1-\tau_Q}}{1-\sqrt{1-\tau_Q}} - i\pi \right]^2 & \tau_Q > 1 \end{cases}$$

in the following way

$$F^H_{HQ}(\tau_Q) = \frac{3}{2}\tau_Q^{-1} \left[ 1 + (1 - \tau_Q^{-1}) f(\tau_Q) \right]$$

with $\tau_Q = M^2_H/4m^2_Q$. The form factor is normalized such that for $m_Q \gg M_H$, $F^H_Q \rightarrow 1$ and it approaches zero in the chiral limit $m_Q \rightarrow 0$. To incorporate the QCD corrections to $\sigma(pp \rightarrow H + X)$, one has to consider the processes,

$$gg \rightarrow H(g) \quad \text{and} \quad gq \rightarrow Hq, \quad q\bar{q} \rightarrow Hg$$

Characteristic diagrams of the QCD radiative corrections are shown in Fig. 11. They consist of two–loop gluon–quark and Higgs–quark vertex corrections, rescattering corrections and non–planar diagrams. The renormalization program has been carried out in the $\overline{\text{MS}}$ scheme. The “physical” quark mass $m_Q$ is defined at the pole of the propagator; this assures the correct [perturbative] threshold behavior of the triangle amplitude for $M_H \approx 2m_Q$. The renormalization of the scalar $HQ\bar{Q}$ vertex is connected with the renormalization of the quark mass and the quark wave–function, $Z_{HQ\bar{Q}} = (Z_Q - 1) - \delta m_Q/m_Q$. In addition to these virtual corrections, the gluon radiation off the initial state gluons and the heavy quark lines must be taken into account. After adding up all these contributions, ultraviolet and infrared singularities cancel. Leftover collinear singularities are absorbed into the renormalized parton densities, which we define in the $\overline{\text{MS}}$ scheme. Finally the subprocesses $gq \rightarrow Hq$ and $q\bar{q} \rightarrow Hg$ must be added.
The results for the cross sections can be summarized in the following form:

\[
\sigma(pp \rightarrow H + X) = \sigma_0^H \left[ 1 + C^H \frac{dL_{gg}}{d\tau_H} \right] \tau_H + \Delta \sigma_{gg}^H + \Delta \sigma_{gq}^H + \Delta \sigma_{qq}^H \tag{12}
\]

The coefficient \(C^H\) denotes the contributions from the virtual two–loop quark corrections regularized by the infrared singular part of the cross section for real gluon emission,

\[
C^H = \tau^2 + c^H + \frac{33 - 2N_f}{6} \log \frac{\mu^2}{M_H^2}
\]

\[
c^H = \text{Re} \sum_Q F_Q^H c_Q^H(\tau_Q) / \sum_Q F_Q^H \tag{13}
\]

The coefficient \(C^H\) splits into the infrared term \(\tau^2\), a term depending on the renormalization scale \(\mu\) of the coupling constant, and a piece \(c^H\) which depends on the mass ratio \(\tau_Q\); it has been reduced from 5–dimensional Feynman parameter integrals to 1–dimensional integrals analytically and the remaining integration has been performed numerically. In the limit of large quark masses, the coefficient can be calculated analytically and one finds\[6,13\]

\[
m_Q \gg M_H: \quad c_Q^H \rightarrow \frac{11}{2}
\]

For large Higgs masses but moderate quark masses the real and imaginary parts of \(c_Q^H\) remain small, \(\leq \pm 5\) for \(\tau_Q \leq 10^4\). However, the numerical results clearly indicate
the onset of the asymptotic behavior $I_m c_Q^H \sim \log \tau_Q$ and suggest $Re c_Q^H \sim \log^2 \tau_Q$ for large $\log \tau_Q$. The (non–singular) contributions from gluon radiation in $gg$ scattering, from $gq$ scattering and $q\bar{q}$ annihilation, Figs. 11, depend on the renormalization scale $\mu$ and the factorization scale $M$ of the parton densities,

\begin{align*}
\Delta \sigma_{gg}^H &= \int_{\tau_H}^1 d\tau \frac{d\mathcal{L}_{gg}}{d\tau} \frac{\alpha_s}{\pi} \sigma_0^H \left\{ -zP_{gg}(z) \log \frac{M^2}{\tau_s} + d_{gg}^H(z, \tau_Q) \right. \\
&\left. +12 \left[ \frac{\log(1-z)}{1-z} - z [2-z(1-z)] \log(1-z) \right] \right\} \\
\Delta \sigma_{gq}^H &= \int_{\tau_H}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}_{gq}}{d\tau} \frac{\alpha_s}{\pi} \sigma_0^H \left\{ -\frac{1}{2} \log \frac{M^2}{\tau_s} + \log(1-z) \right\} zP_{gq}(z) + d_{gq}^H(z, \tau_Q) \\
\Delta \sigma_{q\bar{q}}^H &= \int_{\tau_H}^1 d\tau \sum_{q} \frac{d\mathcal{L}_{q\bar{q}}}{d\tau} \frac{\alpha_s}{\pi} \sigma_0^H \sigma_{qq}^H(z, \tau_Q)
\end{align*}

(15)

with $z = \tau_H/\tau$ and $P_{ij}$ being the standard Altarelli–Parisi splitting functions. Again, the coefficients $d_{gg}^H, d_{gq}^H$ and $d_{q\bar{q}}^H$ have been reduced to 1–dimensional integrals which have been evaluated numerically. In the limit of large quark masses, the coefficients can be determined analytically,

\begin{align*}
m_Q \gg M_H: \quad d_{gg}^H &\to -\frac{11}{2} (1-z)^3 \\
d_{gq}^H &\to -1 + 2z - \frac{1}{3} z^2 \\
d_{q\bar{q}}^H &\to \frac{32}{27} (1-z)^3
\end{align*}

(16)

In Fig. 12 we present the final QCD corrected cross sections for a value of the top mass of 200 GeV, assuming that the Higgs mass is sufficiently below the $2m_t$ threshold. The cross sections are shown for the DFLM parametrization of the parton densities and of the coupling constant in the next–to–leading order at $\mu^2 = M^2 = \hat{s}$ and also $\mu^2 = M^2 = M_H^2$ with $A_{MS}^2 = 170$ MeV. They are compared with the Born terms in which all quantities are consistently used in the leading order expansion $\Lambda_{\overline{MS}}^5 = 130$ MeV. The QCD corrections are found to be at the level of 50 to 80%. To demonstrate the uncertainty of the cross section due to the presently unknown small $x$ behavior of the gluon density and the magnitude of the coupling constant, we compare the predictions of the DFLM and GRV parametrizations; even for these two extremes choices, the difference remains at a level less than 20%.

The QCD corrections to $gg \to H$ for arbitrary values of $M^2_H/m_t^2$ have been calculated in Refs. and the result is shown in Fig. 13. As one can see, the $K$ factor which characterizes the size of the QCD corrections, $K_H = \sigma_{H\overline{O}}^H/\sigma_{H\overline{O}}^L$, is rather insensitive to the mass ratio and is of order 1.8. Therefore the result in the limit $m_t \gg M_H$ is a good approximation of the complete result.
Fig. 12 Production cross sections with and without the QCD corrections at the LHC in the approximation $m_t \gg M_H/2$; the top mass is fixed to 200 GeV.

Fig. 13 Individual and total contributions to the “K–factor” of the $gg \rightarrow H$ process at the LHC as a function of the Higgs mass; the top mass is fixed to 175 GeV. The GRV parametrization for the parton densities has been used.
The next important radiative correction to the $Hgg$ amplitude, that is proportional to the square of the mass of the quark in the loop and is therefore potentially very large, is the two–loop $\mathcal{O}(G_F m_Q^2)$ electroweak correction. We will summarize below the result of a very recent evaluation of this correction. 

As already discussed, in the minimal Standard Model with three families, only the top quark significantly contributes to the $Hgg$ coupling. The $\mathcal{O}(\alpha_S G_F m_t^2)$ correction to the top quark loop amplitude in the limit $m_t \gg m_b$ is given by

$$F_t^H \to F_t^H \left[ 1 + \frac{G_F \sqrt{2}}{32\pi^2} m_t^2 \right]$$

Due to a large cancellation among the various contributions, the total correction amounts to a positive contribution of a mere 0.2% for a top mass value $m_t \sim 200$ GeV. Therefore, contrary to the very large QCD corrections, the leading electroweak correction to the top quark loop mediated Higgs–gluons coupling turns out to be very small and well under control.

Since the measurement of the $Hgg$ coupling is a very powerful tool to count the number of heavy quarks which couple to the Higgs boson, it is interesting to evaluate the effect of this correction in the case of a fourth generation of heavy fermions. Because the mass splitting between the members of the extra weak isodoublet of quarks is highly constrained by present electroweak precision measurements, one can work in the approximation where the two quarks are degenerate in mass. In this case, the $\mathcal{O}(G_F m_Q^2)$ correction to one of the quarks amplitude is given by

$$F_Q^H \to F_Q^H \left[ 1 - \frac{G_F \sqrt{2}}{8\pi^2} m_Q^2 \right]$$

This negative correction will therefore screen the value of the one–loop generated $Hgg$ coupling. However, the correction is rather small since for realistic values of the quark masses, $m_Q < 500$ GeV [an upper bound obtained from the requirement that weak interactions do not become strong and perturbation theory is reliable], it does not exceed the 5% level. It is only for quark masses larger than $\sim 2$ TeV, for which perturbation theory breaks down already at the tree level, that this radiative correction will exceed the one–loop result.

Note that in the previous equation only the contribution of the heavy quarks of the fourth generation has been taken into account. Additional contributions will be induced by the extra weak isodoublet of leptons [from the renormalization of the Higgs boson wave–function and vacuum expectation value]. If one assumes that the masses of the heavy leptons are approximately equal to those of the quarks, the total correction in eq. (3.18) will be smaller by a factor of three.

Therefore, the $\mathcal{O}(G_F m_Q^2)$ correction to the $Hgg$ amplitude is well under control for quark masses in the range interesting for perturbation theory, and the counting of new heavy quarks via the $Hgg$ coupling will not be jeopardized by these corrections.
3.1.3. *Higgs Detection*

The signals which are best suited to identify the Higgs boson at proton colliders have been studied in great detail; see for instance Refs. 58, 62. Here, we will briefly summarize the main conclusions of these studies for LHC energies. For the sake of convenience we will divide the Higgs mass into two ranges: the “high mass” range $M_H > 140$ GeV and the “low mass” range $M_H < 140$ GeV.

**High mass range:**

For a Higgs mass above the $ZZ$ threshold, $M_H > 2M_Z$, one can exploit the $H \rightarrow ZZ$ decays which have clean signatures and are affected by rather small backgrounds. The best signal is the so called “gold–plated” signal $H \rightarrow ZZ \rightarrow 4l^\pm$ with two pairs of charged electrons or muons with invariant masses equal to $M_Z$. The branching ratio is of the order of $\sim 0.14\%$ and is therefore somewhat small. To improve the signal cross sections, one can also use the $H \rightarrow l^+l^-\bar{\nu}\bar{\nu}$ decay mode for which the branching ratio is 6 times larger; however in this latter mode the Higgs mass cannot be measured directly.

The backgrounds, which come mainly from $Z$ boson pair production in $q\bar{q} \rightarrow ZZ$ and in the loop mediated process $gg \rightarrow ZZ$ [for the $H \rightarrow l^+l^-\bar{\nu}\bar{\nu}$ mode one has to consider in addition the process $pp \rightarrow Z + $ jets, where some of the jets are undetected or their energy not well measured to give a fake missing energy] are manageable: they have softer transverse momentum distributions than to the $H \rightarrow ZZ$ signal for which $P_T(Z) \sim M_H/2$. For a total integrated luminosity of $10^5$ pb$^{-1}$, one can detect the Standard Model Higgs boson up to masses of $M_H \sim 800$ GeV with the $4l^\pm$ signal, but for a luminosity 10 times smaller only masses $M_H \sim 500$ GeV can be reached. The complementary signal $H \rightarrow l^+l^-\bar{\nu}\bar{\nu}$ as well as the process $H \rightarrow WW \rightarrow l\nu jj$, where the Higgs boson is produced in the $VV$ fusion processes and where one uses forward jet tagging, can extend the discovery limit up to Higgs masses of 1 TeV [only $M_H \sim 700$ GeV for an integrated luminosity of $10^4$ pb$^{-1}$].

Below the $2M_Z$ threshold, $H \rightarrow ZZ^* \rightarrow 4l^\pm$, where one of the $Z$ bosons is off–shell, has a still appreciable cross section times branching ratio. The signal is very clean and after appropriate transverse momentum cuts on the charged leptons and with a good lepton isolation and identification, one can suppress the backgrounds [which mainly come from $ZZ^*, Zb\bar{b}, Zt\bar{t}$ and $t\bar{t}$ production, the heavy quarks decaying semi–leptonically] down to a manageable level. Higgs bosons with masses down to $M_H \simeq 140$ GeV can be detected in this process.

One can therefore conclude that with the $H \rightarrow ZZ^{(*)} \rightarrow 4$ leptons signal, the SM Higgs boson with a mass in the “high range”, $140$ GeV $< M_H < 1$ TeV can be relatively easily detected at the LHC, provided that the luminosity is high enough, i.e. $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$. The high luminosity is demanding on the detector, though.

**Low mass range:**

For Higgs bosons in the “low mass” range, the situation is a bit more complicated.
The branching ratio for $H \rightarrow ZZ^*$ becomes too small to be useful and because of the huge QCD jet background, the dominant Higgs decay mode $H \rightarrow bb$ is useless in the main production process $gg \rightarrow H$; one has then to rely on the very rare $\gamma\gamma$ decay mode with a branching ratio of $\mathcal{O}(10^{-3})$.

At the LHC with a luminosity of $\int L = 100 \text{ fb}^{-1}$, the cross section times branching ratio leads to $\mathcal{O}(0.5 - 1 \times 10^3)$ events in the mass range $80 < M_H < 140 \text{ GeV}$ but one has to fight against formidable backgrounds. Jets faking photons need a rejection factor larger than $10^4$ per jet to be reduced to the level of the physical background $q\bar{q}, gg \rightarrow \gamma\gamma$ which is still very large. However, if very good geometric resolution and stringent isolation criteria, combined with excellent electromagnetic energy resolution to detect the narrow $\gamma\gamma$ peak of the Higgs boson are available [one also needs a high luminosity $L \simeq 10^{34} \text{ cm}^{-2}\text{s}^{-1}$], this channel, although very difficult, is feasible.

A complementary channel in this mass range would be the $q\bar{q} \rightarrow WH, t\bar{t}H \rightarrow \gamma\gamma l\nu$ using the charged lepton from the $W$ decay as a tag. At LHC, the signal for the associate production of the Higgs with top quark pairs is two times larger than the one for associate production with a $W$. The background cross sections are rather small but the signal cross sections are also small: only $\sim 20$ events for a luminosity of $10^5 \text{ pb}^{-1}$ are expected [when one adds up the two channels], making this process also difficult to use.

Two alternatives of detecting the Higgs boson through decays which are not the two-photon decays have been proposed. The first one is the $H \rightarrow \tau^+\tau^-$ channel with the Higgs boson produced in the gluon–gluon fusion mechanism. Unfortunately, because of overwhelming physical backgrounds [coming mainly from $tt$ pair production and also from the Drell–Yan production of $\tau^+\tau^-$ pairs for $M_H$ in the vicinity of $M_Z$], this method has been found hopeless for the Standard Model Higgs. However, it could be used in the supersymmetric extension for $H$ and $A$ in some range of the SUSY parameter space.

A second method which has been proposed very recently, is to use the dominant $H \rightarrow bb$ decay mode with the Higgs particles produced in association with $tt$ pairs leading to $ttbb$ final states. Requiring that at least one of the top quarks decays into an isolated $e$ or $\mu$ lepton and with the help of a micro–vertex detector with very good efficiency and purity for tagging the $b$ quarks, it would be possible to use this channel at the LHC for a top quark heavier than 150 GeV. However, because of the many overlapping events expected at the LHC with a very high–luminosity option [which is needed to have a reasonable number of events], the efficiencies for $b$–quark tagging might be difficult to achieve at the required level. A careful analysis including the simulation of the experimental environment might be required to assess firmly the viability of this mode.

In conclusion, while it is relatively easy to detect Higgs bosons in the “high mass” range up to $\sim 1 \text{ TeV}$ at LHC, it is rather difficult to detect Higgs bosons with masses below 140 GeV: a very high luminosity $\simeq 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ and a dedicated detector are required.
3.2. Supersymmetric Extension

3.2.1. Production mechanisms

In the minimal supersymmetric extension of the Standard Model, the mechanisms for producing the neutral Higgs bosons are practically the same as in the SM, one simply has to take into account the contributions of the b quark whose couplings are in general strongly enhanced for large $\tan \beta$ values, and for the pseudoscalar Higgs boson, discard the $WW/ZZ$ fusion processes and the associate production process with a W or Z boson because there is no $AVV$ coupling at the tree level. The main features of these production mechanisms are summarized below:

(a) $gg \to h/H/A$: in the MSSM this process proceeds through top and bottom quark loops. Because the top (bottom) couplings to the SUSY Higgs bosons are suppressed (enhanced) compared to those of the SM Higgs, the $b$ quark loop contribution can compete with the one of the top quark and becomes even dominant for large values of $\tan \beta$. Extra contributions may come from squark loops, but contrary to quarks, these particles will decouple from the Higgs–gluon vertex if their masses are very large. The rather large QCD corrections to this process will be discussed in the next subsection.

(b) $WW/ZZ$ fusion mechanisms: in the MSSM, the couplings of the CP–even Higgs bosons to $VV$ pairs are suppressed by $\sin(\alpha-\beta)$ or $\cos(\alpha-\beta)$ factors compared to the SM couplings and the production cross sections are therefore always smaller than in the SM. In fact, this process is not important in general since for large Higgs masses where it dominates over the $gg$ fusion mechanism in the SM, the $HVV$ coupling is extremely small; for small Higgs masses, the $hVV$ couplings can approach the SM value but the cross section is much below the one for the $gg$ fusion process. In the case of the pseudoscalar Higgs boson, because there is no $AVV$ coupling at the tree level, these processes are absent.

(c) Associate production with $W/Z$: this process is important only for the lightest Higgs boson $h$ and for the heavier $H$ with a mass in the “low mass” range [again, because there is no $AVV$ coupling this process is absent for the pseudoscalar]. Because of the suppressed $h/HVV$ couplings, the cross sections are always smaller than in the case of the SM Higgs.

(d) Associate production with a heavy quark pair: because the top quark couplings to Higgs bosons are smaller than in the SM for $\tan \beta > 1$, the cross section for the process $q\bar{q}, gg \to Q\bar{Q} + h/H/A$ where $Q = t$ is smaller than the SM Higgs cross section. However, for $Q = b$ this process can become the dominant production mode in some range of the SUSY parameter space; this is because the $b$ couplings to Higgs bosons are extremely enhanced for large values of $\tan \beta$.

(e) For charged Higgs bosons, the most interesting production processes are: the top decay $t \to H^+ b$ with the top quarks mainly produced through gluon fusion, $gg \to tt$ and possibly $gb \to tH \to tbb$ in a limited range of the SUSY parameter space. Other production processes seem to be hopeless at hadron colliders.
3.2.2. \textit{gg} fusion mechanism

In this subsection we discuss the gluon–gluon fusion production mode for supersymmetric neutral Higgs bosons and present the QCD corrections to the production rates. This demands the extension of the SM calculation in two ways: (i) since the Higgs–gg coupling is mediated, in part of the MSSM parameter space, dominantly by bottom quark loops one has to deal with a properly weighted superposition of \( t \) and \( b \) loop contributions, (ii) the QCD corrections have to be analyzed for arbitrary ratios of Higgs and quark masses, and (iii) the QCD corrections are to be determined for pseudoscalar Higgs bosons.

To lowest order (LO), the cross sections for the production of Higgs particles in \( gg \) fusion are given by eq. (3.7) where the Standard Model Higgs boson \( H \) has to be replaced by \( \Phi = h/H \) or \( A \). For the \( CP \)-even Higgs bosons \( h \) and \( H \) the parton cross section is given by eq. (3.8), while for the pseudoscalar one has

\[
\sigma_0^A = \frac{G_F \alpha_s^2}{128 \sqrt{2} \pi} \left| \sum_Q F_Q^A(\tau_Q) \right|^2
\]  

With the help of the scalar triangle integral \( f \) the form factors \( F_{Q}^{h/H} \) and \( F_{Q}^{A} \) are

\[
F_{Q}^{h/H}(\tau_Q) = \frac{3}{2} \tau_Q^{-1} \left[ 1 + (1 - \tau_Q^{-1})f(\tau_Q) \right] g_{QQQ}^{h/H}
\]

\[
F_{Q}^{A}(\tau_Q) = \tau_Q^{-1} f(\tau_Q) g_{QQQ}^A
\]

where the coefficients \( g_{QQQ} \) denote the couplings of the Higgs bosons normalized to the SM Higgs couplings to top and bottom quarks and are given in Tab. 1.

The leading electroweak radiative corrections are taken into account for the Higgs masses and couplings. For small values \( tg_\beta \sim 1 \), the \( t \) couplings are dominant while \( b \) couplings are large for large \( tg_\beta \). The form factors are normalized such that for \( m_Q \gg M_\Phi \):

\[
F_Q^{h/H} \rightarrow g_{QQQ}^{h/H}
\]

\[
F_Q^{A} \rightarrow \frac{m_Q^2}{M_A^2} \left( \log \frac{M_A^2}{m_Q^2} - i\pi \right)^2 g_{QQQ}^{A}
\]

Both form factors approach zero in the chiral limit \( m_Q \rightarrow 0 \). The leading logarithmic terms also give rise to the same cross section in the scalar and pseudoscalar case in the approach to the chiral limit.

The diagrams for the QCD radiative corrections are the same shown in Figs. 11. As in the SM case, the renormalization program has been carried out in the \( \overline{MS} \) scheme and the “physical” quark mass is defined at the pole of the propagator. The renormalization is also the same; however for the pseudoscalar case one has
to deal with the problem of $\gamma_5$ in the dimensional regularization scheme and we have adopted the ’t Hooft–Veltman scheme. In this scheme, the renormalization $Z_{AQ\bar{Q}}$ of the pseudoscalar $AQ\bar{Q}$ vertex must be supplemented by an additional term $8\alpha_s/(3\pi)$ to restore chiral invariance in the limit $m_Q \to 0$ and the correct form of the Adler–Bell–Jackiw anomaly. As usual, after adding up all contributions, ultraviolet and infrared singularities cancel and the leftover collinear singularities are absorbed into the renormalized (in the $\overline{MS}$) parton densities.

The calculation has been performed following the same lines as in the Standard Model case and the results for the cross sections are given by eqs. (3.12–3.15) with $H$ replaced by $\Phi = h/H/A$. In the limit of large quark masses, the corrections can be calculated analytically, and one obtains:

$$m_Q \gg M_\Phi : \quad c_{h/H}^Q \to 11/2, \quad c_A^Q \to 6$$

and the coefficients $d_{q\bar{q}}^\Phi, d_{gg}^\Phi$ and $d_{gq}^\Phi$ coincide for scalar and pseudoscalar Higgs particles and are given in eq. (3.16). Also in the chiral limit of small quark but large Higgs masses, these coefficients approach a common limit for scalar and pseudoscalar Higgs particles.

The $K$ factors which characterize the size of the QCD radiative corrections properly, are defined by the ratios $K_{\text{tot}}^\Phi = \sigma_{\text{HO}}^\Phi/\sigma_{\text{LO}}^\Phi$. The cross sections $\sigma_{\text{HO}}^\Phi$ in next–to–leading order are normalized to $\sigma_{\text{LO}}^\Phi$, evaluated for parton densities and $\alpha_s$ in leading order. $K_{\text{tot}}^\Phi$ split into contributions from the (regularized) virtual corrections $K_{\text{virt}}^\Phi$ plus the real corrections $K_{ij}^\Phi = \Delta \sigma_{ij}^\Phi/\sigma_{ij}^\Phi$. For both the renormalization and the factorization scale $\mu = M = M_\Phi$ has been chosen.

The $K$ factors can be determined in the way defined above, by adopting the GRV parametrizations of the parton densities for which separate $LO$ and $HO$ analyses have been performed. As shown in Fig. 14, $K_{\text{virt}}^\Phi$ and $K_{gg}^\Phi$ are of similar size and of order 50%, in general, while $K_{gq}^\Phi$ and $K_{q\bar{q}}^\Phi$ are quite small. Apart from the $t\bar{t}$ threshold region, the $K$ factors $K_{\text{tot}}^\Phi$ are rather insensitive to the values of the Higgs masses. Near the threshold the present perturbative analysis, based on one–gluon exchange, cannot be applied anymore for the pseudoscalar particle $A$. In particular, since $t\bar{t}$ pairs at rest can form $0^{-+}$ bound states, the $Agg$ coupling develops a Coulombic singularity for $M_A \approx 2m_t$. The range within a few GeV of the threshold mass must therefore be excluded from the analysis.

Outside this singular range, the QCD corrections are in general large and positive with values up to $\sim 2$, except for the light neutral Higgs boson $h$ and small values of $M_A$ for which the $K$ factor is close to unity for large $\tan \beta$. As expected, the $K$ factor for $h$ coincides with the corresponding $SM$ value if $M_h$ approaches its maximum for a given $\tan \beta$. [In this limit the MSSM reduces effectively to the $SM$ with one light Higgs boson and $SM$ type couplings.] The rapid change of the $K$ factors near this limit corresponds to a rather gradual change if the $SUSY$ space is parametrized by $M_A$ instead of $M_h$. The $K$ factors do not vary dramatically with the Higgs masses. The analytical results in the limit $\tau_Q \to 0$ provide in general a useful guideline for processes mediated by top loops.
3.2.3 Higgs Detection

As for the Standard Model Higgs boson, the various signals which can be used for the detection of supersymmetric neutral Higgs bosons are: two isolated photons from $gg \rightarrow h/H/A \rightarrow \gamma\gamma$; a tagged lepton and two isolated photons $pp \rightarrow l\nu\gamma\gamma$; the “gold–plated” signal $pp \rightarrow H \rightarrow ZZ \rightarrow 4l^{\pm}$; in addition one can use $H, A \rightarrow \tau^{+}\tau^{-}$ mode which is hopeless in the case of the SM Higgs boson. The charged Higgs boson can be detected in top quark decays. The situation at hadron colliders can be summarized as follows:

i) Since the lightest Higgs boson mass is always smaller than $\sim 140$ GeV, the $ZZ$ signal where one of the $Z$ bosons is off–shell cannot be used for it. Furthermore, the $hWW(hbb)$ coupling is suppressed (enhanced) leading to a smaller $\gamma\gamma$ branching ratio than in the $SM$ [additional contributions from chargino and sfermion loops can also alter the two–photon decay width] making the search for the $h$ boson more difficult than for the Standard Model Higgs boson.
ii) Since the pseudoscalar $A$ has no tree level couplings to gauge bosons and since the couplings of the heavy $CP$–even $H$ boson are strongly suppressed, the gold–plated $ZZ \rightarrow 4l^\pm$ signal is lost [for $H$ it survives only for small $\tan\beta$ and $M_H$ values, provided that $M_H < 2m_t$]. One has therefore to rely on the $A, H \rightarrow \tau^+\tau^-$ channels for large $\tan\beta$ values; this mode, which is hopeless for the SM Higgs boson, seems to be feasible in the supersymmetric case. The channel where the neutral Higgses are produced with $t\bar{t}$ pairs and decay to $b$ quarks seems also promising.

iii) As discussed previously, charged Higgs particles, if lighter than the top quark, can be accessible in top decays $t \rightarrow H^+b$. This results in a surplus of $\tau$ lepton final states [in this mass range, the main decay mode of the charged Higgs is $H^- \rightarrow \tau\nu_\tau$] over $\mu, e$ final states, an apparent breaking of $\tau$ vs. $e, \mu$ universality. At LHC, $H^\pm$ masses up to $\sim 100$ GeV can be probed for $m_t \approx 150$ GeV.

Note that, if the mass of the lightest Higgs boson $h$ is close to its maximal allowed value for a given value of $\tan\beta$, all the other Higgs bosons are very heavy [and degenerate] and $h$ has exactly the couplings of the SM Higgs boson. In this case, the situation is similar to the SM case with $M_H = 100–140$ GeV, unless squark or chargino loops alter the $gg$ production and the $\gamma\gamma$ decay processes. Note also that if the neutralinos and charginos are light enough, Higgs decays into these particles are possible and would significantly change the search strategies. In particular, if the neutral Higgs bosons can decay into lightest neutralino pairs with large branching ratios, the search for $SU(3)$ Higgs bosons could be jeopardized at hadron colliders.

In conclusion, the search for $SU(3)$ Higgs bosons is more difficult than the search for the SM Higgs, unless $M_h$ is close to its maximum for a given $\tan\beta$. Detailed analyses have shown that there is a substantial area in the $SU(3)$ parameter space where no Higgs particle can be found at the $pp$ collider LHC; this is illustrated in [the hatched area of] Fig. 15 from Ref. 78.

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Fig. 15 $SU(3)$ parameter space where Higgs bosons can be produced at the LHC, the top mass was fixed to 150 GeV; from Ref. 78.
4. Production at $e^+e^-$ Colliders

$e^+e^-$ linear colliders operating in the energy range between 300 to 500 GeV have a very high physics potential, especially for the discovery of Higgs particles and the study of their fundamental properties. In this section, we will survey Higgs physics at these future colliders, and for illustration we will assume in most of the cases a centre of mass energy of $\sqrt{s} = 500$ GeV.

4.1. Standard Model

At $e^+e^-$ linear colliders operating in the 300–500 GeV energy range, the main production mechanisms for Higgs particles, Fig. 16, are the following processes:

| Process Type         | Process Description                                                                 | Equation |
|----------------------|--------------------------------------------------------------------------------------|----------|
| Bremsstrahlung       | $e^+e^- \rightarrow Z \rightarrow Z + H$                                            | (4.1)    |
| WW Fusion            | $e^+e^- \rightarrow \bar{\nu} \nu (WW) \rightarrow \bar{\nu} \nu + H$               | (4.2)    |
| ZZ Fusion            | $e^+e^- \rightarrow e^+e^- (ZZ) \rightarrow e^+e^- + H$                              | (4.3)    |
| Radiation off Tops   | $e^+e^- \rightarrow (Z, \gamma) \rightarrow t\bar{t} + H$                          | (4.4)    |

Fig. 16 Main production mechanisms for SM Higgs particles in high-energy $e^+e^-$ colliders.

The bremsstrahl process is dominant for moderate values of the ratio $M_H/\sqrt{s}$, but falls off according to the scaling law $\sim s^{-1}$ at high energies. The fusion processes, on the other hand, are most important for small values of the ratio $M_H/\sqrt{s}$, i.e. high energies where the cross sections grow $\sim M_W^2 \log s$. For Higgs masses in the intermediate mass range, the cross sections for the bremsstrahl process and the $WW$ fusion process are of comparable size at $\sqrt{s} = 500$ GeV, while the $ZZ$ fusion cross section is smaller by an order of magnitude [Fig. 17]. With $\sigma \sim 100$ fb, a total of $\sim 1000$ Higgs particles can be created at an integrated luminosity of $\int \mathcal{L} = 10$ fb$^{-1}$, which corresponds to a running time of $10^7$ s per year at $\mathcal{L} = 10^{33}$ cm$^{-2}$s$^{-1}$. For Higgs masses below 100 GeV, the cross section for Higgs radiation off top quarks, $e^+e^- \rightarrow t\bar{t}H$, is of the order of a few fb; this process can be used only to measure the $ttH$ Yukawa coupling once the Higgs boson is detected in the previous processes.
Additional production mechanisms are provided by $\gamma\gamma$ and $e\gamma$ collisions, the high–energy photons generated by Compton back–scattering of laser light. Higgs particles can then be created in $\gamma\gamma$ fusion

$$\gamma\gamma \rightarrow H \quad (4.5)$$

and through bremsstrahlung off the $W$ line

$$e\gamma \rightarrow \nu WH \quad (4.6)$$

The most important production processes in $e^+e^-$ collisions will be discussed in some detail below.

4.1.1. Higgs Bremsstrahlung

The cross section for the bremsstrahl process eq. (4.1) can be presented in a compact form

$$\sigma(e^+e^- \rightarrow ZH) = \frac{G_F^2 M_Z^4}{96\pi s} \left( v_e^2 + a_e^2 \right) \frac{\lambda^{1/2} \lambda + 12 M_Z^2/s}{(1 - M_Z^2/s)^2} \quad (4.7)$$

where $a_e = -1$ and $v_e = -1 + 4s^2_W$ are the $Z$ charges of the electron and $\lambda = (1 - M_H^2/s - M_Z^2/s)^2 - 4M_H^2M_Z^2/s^2$ is the usual two–particle phase space function. With $\sigma \sim 200$ fb, a rate of $\sim 2000$ Higgs particles in the intermediate mass range is produced at an energy $\sqrt{s} = 300$ GeV and an integrated luminosity of $\int L = 10$ fb$^{-1}$; see Fig. 18. Asymptotically, the cross section scales $\sim 1/s$. 

Fig. 17 Production cross sections for SM Higgs particles at $\sqrt{s} = 500$ GeV.
The angular distribution of the \(Z/H\) bosons in the bremsstrahl process is sensitive to the spin of the Higgs particle as will be discussed later. For high energies, the \(Z\) boson is produced in a state of longitudinal polarization so that, according to the equivalence theorem, the production amplitude becomes equal to the amplitude \(A(e^+e^- \rightarrow \phi^0H)\), with \(\phi^0\) being the neutral Goldstone boson which is absorbed to give mass to the vector boson. The angular distribution, \(d\sigma/d\cos\theta \sim \lambda\sin^2\theta + 8M_Z^2/s\), therefore approaches the spin–zero angular distribution asymptotically

\[
\frac{d\sigma}{d\cos\theta} \to \frac{3}{4}\sin^2\theta
\]  

The radiative corrections to the angular distribution and the total cross section are well under control. The bulk of these corrections is due to photon radiation from the incoming electrons and positrons. The weak corrections are relatively modest being of the order of a few percent; see Fig. 18.

The recoiling \(Z\) boson in the two–body reaction \(e^+e^- \rightarrow ZH\) is mono–energetic, \(E_Z = (s - M_H^2 + M_Z^2)/(2\sqrt{s})\), and the mass of the Higgs boson can be derived from the energy of the \(Z\) boson, \(M_H^2 = s - 2\sqrt{s}E_Z + M_Z^2\), if the initial \(e^+\) and \(e^-\) beam energies are sharp. However, beamstrahlung smears out the c.m. energy and the system moves along the beam axes [initial state photon radiation and the beam energy spread have also to be taken into account]. The intensity of the beamstrahlung depends on the machine design. It must be suppressed as strongly as possible for a given luminosity in order to allow for the high quality experimental analyses which are based on the powerful kinematical constraints familiar from low–energy \(e^+e^-\) physics. Good examples in this context are the DESY–Darmstadt and the TESLA design studies; see Fig. 19. For these designs the smearing of the missing

Fig. 18 Total cross sections for the bremsstrahl process for three energy values \(\sqrt{s} = 300, 400\) and 500 GeV with and without electroweak radiative corrections.

The recoiling \(Z\) boson in the two–body reaction \(e^+e^- \rightarrow ZH\) is mono–energetic, \(E_Z = (s - M_H^2 + M_Z^2)/(2\sqrt{s})\), and the mass of the Higgs boson can be derived from the energy of the \(Z\) boson, \(M_H^2 = s - 2\sqrt{s}E_Z + M_Z^2\), if the initial \(e^+\) and \(e^-\) beam energies are sharp. However, beamstrahlung smears out the c.m. energy and the system moves along the beam axes [initial state photon radiation and the beam energy spread have also to be taken into account]. The intensity of the beamstrahlung depends on the machine design. It must be suppressed as strongly as possible for a given luminosity in order to allow for the high quality experimental analyses which are based on the powerful kinematical constraints familiar from low–energy \(e^+e^-\) physics. Good examples in this context are the DESY–Darmstadt and the TESLA design studies; see Fig. 19. For these designs the smearing of the missing
mass is of the same magnitude as the experimental uncertainties in the reconstruction of the $Z$ boson in the leptonic decay channels. This is shown in Fig. 20a where the missing mass in the signal $e^+e^- \rightarrow ZX$, $X = H$, is compared for $M_H = 130$ GeV with the background from $e^+e^- \rightarrow ZZ$ final states.

Fig. 19 Beamstrahlung corrections for $e^+e^- \rightarrow ZH$ total cross sections.

Since the recoiling $Z$ boson remains approximately mono–energetic, even if beamstrahlung is taken into account, it is easy to separate the signal from the background. Four mass regions must be considered:

(i) For Higgs masses close to the $Z$ mass, double $Z$–production $e^+e^- \rightarrow ZZ$ is the main background source. The cross section is large but can be reduced by cutting out the forward production and by selecting $b\bar{b}$ final states by means of flavor tagging through vertex detection. While the Higgs particle decays almost exclusively to $b\bar{b}$ final states, the branching ratio of the decay $Z \rightarrow b\bar{b}$ is small, $\sim 15\%$.

(ii) For masses between 100 and 140 GeV, the background comes from single $Z$–production in $e^+e^- \rightarrow ZZ^*(\rightarrow q\bar{q})$. The cross section is suppressed by one order of the electroweak coupling compared to the signal. Further reduction of the background can be achieved through flavor tagging.

(iii) In the mass range above $\sim 140$ GeV where gauge boson decays become dominant, the most important background is due to $e^+e^- \rightarrow Z + WW^*(\rightarrow q\bar{q}')$. The cross section of this reaction is suppressed by two powers of the electroweak coupling relative to the signal.

(iv) Beyond 160 GeV and 180 GeV, the reactions with three gauge bosons in the final state, $e^+e^- \rightarrow Z + WW$ and $e^+e^- \rightarrow Z + ZZ$ are the main background channels. In the background the invariant mass of the $WW$ or $ZZ$ final states is broad as opposed to the resonance structure of the signal. The background under the signal is therefore small as demonstrated in Fig. 20b.
Fig. 20 (a) Distribution of the missing mass $M^2 = (p_{e^+} + p_{e^-} - p_{\mu^+} - p_{\mu^-})^2$ for the $e^+ e^- \rightarrow \mu^+ \mu^- H$ signal assuming $M_H = 130 \text{ GeV}$ and the $e^+ e^- \rightarrow \mu^+ \mu^- Z$ background, including bremsstrahlung, beamstrahlung and the beam-energy spread [angular cut $|\cos \theta_{\mu\mu}| < 0.6$]. Distributions $d\sigma/dM_{VV}$ (in pb/GeV) for heavy–Higgs signals plus backgrounds in the (b) bremsstrahl and (c) $WW/ZZ$-fusion channels for a selection of $M_H$ values; acceptance cuts are applied and bremsstrahlung, beamstrahlung and the beam-energy spread are taken into account. Everywhere the DESY-Darmstadt narrow-band design with $\sqrt{s} = 500 \text{ GeV}$ has been assumed; the results for the TESLA design are very similar.
4.1.2 Fusion Processes

The cross section for the fusion processes can be cast into a compact form

$$\sigma = \frac{G_F^2 M_V^4}{64\sqrt{2}\pi^3} \int_{\kappa_H}^{1} dx \int_{x}^{1} dy \frac{dy}{[1 + (y - x)/\kappa_V]^{2}} \left[(v^2 + a^2)^2 f(x, y) + 4v^2 a^2 g(x, y)\right] \tag{4.9}$$

$$f(x, y) = \left(\frac{2x}{y^2} - \frac{1 + 2x}{y^2} + \frac{2x}{2y} - \frac{1}{2}\right) \left[\frac{z}{1 + z} - \log(1 + z)\right] + \frac{x}{y^3} \frac{z^2(1 - y)}{1 + z}$$

$$g(x, y) = \left(-\frac{x}{y^2} + \frac{2 + x}{2y} - \frac{1}{2}\right) \left[\frac{z}{1 + z} - \log(1 + z)\right]$$

with $\kappa_H = M_H^2/s$, $\kappa_V = M_V^2/s$, $z = y(x - \kappa_H)/(\kappa_V x)$ and $v, a$ the electron couplings to the massive gauge bosons [$v = -1 + 4s_W^2, a = -1$ for the $Z$ boson and $v = a = \sqrt{2}$ for the $W$ boson].

Fig. 21 Total cross sections for the fusion mechanisms for three energy values $\sqrt{s} = 300$, 400 and 500 GeV as a function of the Higgs boson mass.
For a total energy $\sqrt{s} = 500$ GeV and a Higgs mass in the intermediate range, the $WW$ fusion cross section is of about the same magnitude as the bremsstrahl cross section, for lower energies it is smaller and for higher energies larger [see Fig. 21]. Asymptotically it keeps growing logarithmically $M_W^{-2} \log s/M_H^2$ in contrast to the bremsstrahl cross section which falls $\sim s^{-1}$. The cross section for $ZZ$ fusion is about an order of magnitude smaller than the cross section for $WW$ fusion; this is a mere consequence of the fact that the NC couplings are smaller than the CC couplings. The lower rate however is, at least partly, compensated by the clean signature of the $e^+e^-$ final state in (4.3) that allows for a missing–mass analysis to tag the Higgs particle. The smearing of the Higgs peak due to beamstrahlung is similar to the case of Higgs bremsstrahlung in Fig. 20b.

The production of the Higgs particle in the fusion processes is central with a spread in rapidity of about one unit; the energy distribution peaks at about 30 GeV above the mass value. The transverse momentum of the Higgs particle has a broad maximum at $P_T \sim 60$ GeV. For a light Higgs mass, the by far dominant background is the process $e^+e^- \rightarrow e^+W^-\nu_e$; the cross section exceeds the signal for jet–jet final states by about a factor of 60. Another background arises from $WW$ fusion into a $Z$ boson which is three times larger than the signal. All other backgrounds can be efficiently reduced.

The single $W(Z)$ boson production shows a behavior similar to the Higgs boson in the signal process such that cuts enhance the signal/background ratio very little except for three distinctive features: the resonance structure, the spin of the resonance and the flavor composition of the decays. As the Higgs mass enters the $Z$ and $W$ resonance region, flavor tagging is indispensable. Its application would lead to an event sample of about 240 events composed of 60 $W \rightarrow jj$, 80 $Z \rightarrow jj$ and 100 $H \rightarrow jj$ tagged as $b\bar{b}$-jets [for realistic tagging efficiencies and a luminosity of 20 fb$^{-1}$]. For a Higgs mass around $2M_W$, the background process with $W^+W^-$ and $WZ$ final states can be reduced to a negligible level.

The process $e^+e^- \rightarrow e^+e^-H$ is, at $M_H \sim M_Z$, contaminated by $e^+e^- \rightarrow e^+e^-Z$ which is about 10 times stronger than the signal after requiring both $e^+$ and $e^-$ to be detected with $P_T > 30$ GeV but before tagging the $b$'s in the Higgs decay. The signal to background ratio improves rapidly as $M_H$ moves away from the $Z$ mass.

4.1.3. Measurement of Higgs Couplings

The fundamental particles acquire masses through the interaction with the Higgs field. The size of the Higgs couplings to fermions and gauge bosons is therefore set by the masses of these particles. This is a necessary requirement to unitarize the theory of electroweak interactions. Once the Higgs is found, it will be of great importance to measure its couplings to the fundamental particles, which are uniquely predicted by the Higgs mechanism.

The Higgs couplings to massive gauge bosons can directly be determined from the measurement of the cross sections: the $HZZ$ coupling in the bremsstrahl and
in the $ZZ$ fusion processes; the $HWW$ coupling in the $WW$ fusion process. For sufficiently large Higgs masses above $\sim 250$ GeV, these couplings can also be determined experimentally from the decay widths $H \to VV$, $V = Z,W$. Higgs couplings to fermions are not easy to measure directly. For Higgs bosons in the intermediate mass range where the decays into $b\bar{b},c\bar{c}$ and $\tau^{+}\tau^{-}$ are important, the decay width is so narrow that it cannot be resolved experimentally. Nevertheless, the branching ratios into $\tau$ leptons and charm quarks reveal the couplings of these fermions relative to the coupling of the $b$ quarks into which the Higgs boson decays predominantly. In the upper part of the intermediate mass range but below the threshold for real $WW$ decays, the branching ratios $\text{BR}(H \to VV^{*})$ are sizeable and can be determined experimentally. In this case, the absolute values of the $b$ and eventually of the $c,\tau$ couplings can be derived once the $HVV$ couplings are fixed by the cross sections.

The decays $H \to gg$ and $\gamma\gamma, Z\gamma$ and the process $\gamma\gamma \to H$ are mediated by loop diagrams and are proportional to the couplings of the Higgs boson to heavy particles, not only the top quark but also particles beyond the $SM$. Indeed, the number of heavy particles can be counted in these processes if their masses are generated by the Higgs mechanism, so that their coupling to the Higgs grow with the mass; one therefore needs a more direct way to measure the $ttH$ coupling.

A direct way to determine the Yukawa coupling of the intermediate mass Higgs boson to the top quark in the range $m_H \leq 120$ GeV is provided by the bremsstrahlung process $e^{+}e^{-} \to t\bar{t}H$. For large Higgs masses above the $t\bar{t}$ threshold, the decay channel $H \to t\bar{t}$ increases the cross section of $e^{+}e^{-} \to t\bar{t}Z$ through the reaction $e^{+}e^{-} \to ZH(\to t\bar{t})$; without the Higgs decay this final state is produced mainly through virtual $\gamma$ and $Z$ bosons.

The $ttH$ final state is generated almost exclusively through Higgs bremsstrahlung off the top quarks. Additional contributions from Higgs particles emitted by the $Z$ line, are very small. In general, the top and Higgs masses must be kept non–zero so that the cross section for Higgs bremsstrahlung is quite involved. However, neglecting these mass effects and the Higgs emission from the $Z$ line at a level of $O(10\%)$, the Dalitz plot density can be written in a simple form:

\[
\frac{d\sigma}{dx_t dx_\tau} = \frac{\alpha^2 g_{ttH}^2}{s} \left\{ \frac{Q_e^2 Q_t^2}{1 - M_Z^2/s} + \frac{\hat{v}_e \hat{v}_t (\hat{v}_e^2 + \hat{a}_e^2)(\hat{v}_t^2 + \hat{a}_t^2)}{(1 - M_Z^2/s)^2} \right\} \times \frac{x_H^2}{(1 - x_t)(1 - x_\tau)} - \frac{(\hat{v}_e^2 + \hat{a}_e^2)\hat{a}_t^2}{(1 - M_Z^2/s)^2} 2(1 + x_H) \right\} \tag{4.10}
\]

where $\hat{v}_e, \hat{a}_e$ are the $Z$ charges of the electron introduced earlier, $x_H = 2E_H/\sqrt{s}$ is the reduced energy of the Higgs boson and $g_{ttH}$ is the coupling of the Higgs particle to the top quark. The integrated cross section is shown for various c.m. energy values in Fig. 22 as a function of $M_H$. At $\sqrt{s} = 500$ GeV, while for small $M_H$ the cross sections increase with $m_t$ as a result of the rising Yukawa coupling, this trend is reversed for heavy Higgses by the reduction of the available phase space. For an integrated luminosity of $\int L = 20$ fb$^{-1}$, some 100 events can be expected at Higgs masses of order 60 GeV, falling to less than 20 events at 100 GeV.
Fig. 22 The cross sections $\sigma(e^+e^- \to t\bar{t}H)$ at $\sqrt{s} = 0.5, 1$ and 1.5 TeV as a function of the Higgs mass; the top quark mass is fixed to 175 GeV.

Taking acceptance losses into account, this appears to be the upper limit at which the $ttH$ coupling in the intermediate mass range can be measured directly in the course of a few years. Since the signature of the process $e^+e^- \to t\bar{t}H \to WWb\bar{b}b\bar{b}$ is spectacular, there is reasonable hope to isolate these events experimentally despite the low rates. The large number of $b$ quarks together with the mass constraints $m(bb) = m(H)$ and $m(Wb) = m(t)$ will be crucial in rejecting background events. QCD initiated $t\bar{t}b\bar{b}$ final states are suppressed strongly, $\sigma_{QCD} < 0.1$ fb, if the $b\bar{b}$ invariant mass is restricted to values larger than 50 GeV.

The reaction $e^+e^- \to t\bar{t}Z$ is generally mediated by the subprocess $e^+e^- \to (\gamma^*, Z^*) + Z$ with the top quark pair coupled to the virtual photon and $Z$ boson. The Higgs contribution is important only if the Higgs boson is produced as a real particle, with a mass large enough to allow for the decay into a top quark pair; the effect on the cross section is most pronounced for Higgs masses not far above the top decay threshold. At a 500 GeV collider and because the top mass is large, there is not enough available phase space and the cross section $\sigma(e^+e^- \to Zt\bar{t})$ is extremely small being of the order of 1 fb; the effect of $H \to t\bar{t}$ is also marginal; therefore, higher energy colliders will be required for this process to be useful. Note that for large Higgs masses the partial width into top quarks rises linearly with $M_H$ while the decay widths to $W$ and $Z$ bosons grow with the third power, thus suppressing the branching ratio to top quark pairs.

Even though both reactions, Higgs bremsstrahlung off top quarks and Higgs decay to top quark pairs, are not easy to handle experimentally in view of the small cross sections, they nevertheless deserve attention as they may provide the only opportunity to measure the Higgs–fermion coupling directly.
4.1.4 Measurement of spin and parity

The scalar character of the Higgs particle can be tested at $e^+e^-$ colliders in several ways. The angular distribution of the $e^+e^- \rightarrow ZH$ final state depends on the spin and parity of the Higgs particle. The same is true of the angular correlations in the Higgs decay to fermion and gauge boson pairs.

Since the production of the final state $e^+e^- \rightarrow Z^* \rightarrow ZH$ is mediated by a virtual Z boson [transversely polarized to the $e^\pm$ beam axis], the production amplitude could be a monomial in $\cos/\sin \theta$. In the SM, however, the $ZZH$ coupling, $L(ZZH) = (\sqrt{2} G_F)^{1/2} M_Z^2 HZ^\mu Z^\mu$, is an S–wave coupling $\sim \vec{\epsilon}_1 \cdot \vec{\epsilon}_2$ in the laboratory frame, linear in $\sin \theta$ and even under parity and charge conjugation, corresponding to the $0^{++}$ assignment of the Higgs quantum numbers. At high energies the outgoing Z boson is longitudinally polarized and the angular distribution follows the $\sin^2 \theta$ law, eq. (4.8). Nevertheless, it is interesting to confront these properties of the Higgs boson to those of a pseudoscalar state $A(0^{-+})$ in order to underline the uniqueness of the SM prediction. The pseudoscalar case is realized in two–doublet Higgs models, in which the $ZZA$ couplings are induced by loop effects. The effective point-like coupling, $L(ZZA) = (\eta/4) (\sqrt{2} G_F)^{1/2} M_Z^2 AZ^{\mu \nu} \tilde{Z}_{\mu \nu}$, with $\eta$ being a dimensionless factor and $\tilde{Z}^{\mu \nu} = \epsilon^{\mu \nu \rho \sigma} Z_{\rho \sigma}$, is a P–wave coupling, odd under parity and even under charge conjugation. It reduces to $(\vec{\epsilon}_1 \times \vec{\epsilon}_2) \cdot (\vec{p}_1 - \vec{p}_2)$ in the laboratory frame. Since the Z spins are coupled to a vector, the angular distribution is again a binomial in $\sin \theta$, $d\sigma/d\cos \theta \sim 1 - \frac{1}{2} \sin^2 \theta$, independent of the energy. The total cross section is proportionally to $\beta^3$ characteristically different from ZH production near threshold.

The angular correlations specific to the S–wave $0^{++}$ Higgs production in $e^+e^- \rightarrow ZH$ can directly be confronted experimentally with the process $e^+e^- \rightarrow ZZ$. This process has an angular momentum structure that is distinctly different from the Higgs process. Mediated by electron exchange in the $t$–channel, the amplitude is built up by many partial waves, peaking in the forward/backward direction. The two distributions are compared with each other in Fig. 23, which demonstrates the specific character of the Higgs production process.

Since the longitudinal wave function of a vector boson grows with the energy of the particle [in contrast to the energy independent transverse wave function] the Z boson in the S–wave Higgs production process (4.1) must asymptotically be polarized longitudinally. By contrast, the Z bosons from ZA associate production or ZZ pair production are transversally polarized [at high energies in the second case]. This pattern can be checked experimentally. While the distribution of the light fermions in the $Z \rightarrow f \bar{f}$ rest frame with respect to the flight direction of the Z is given by $\sin^2 \theta_\phi$ for longitudinally polarized Z bosons, it behaves as $(1 \pm \cos \theta_\phi)^2$ for transversally polarized states, after averaging over the azimuthal angles. Including the azimuthal angles, and integrating out the polar angles, the angular correlations for $e^+e^- \rightarrow ZH$ [Z $\rightarrow f \bar{f}$] are the familiar $\cos \phi_\ast$ and $\cos 2\phi_\ast$ dependence associated with $P$–odd and even amplitudes, respectively,
\[
\frac{d\sigma(ZH)}{d\phi_*} \sim 1 - \frac{9\pi^2}{32} \frac{\gamma v_e a_e}{\gamma^2 + 2 v_e^2} \frac{2 v_f a_f}{v_e^2 + a_e^2} \cos \phi_* + \frac{1}{2} \frac{1}{\gamma^2 + 2} \cos 2\phi_*. \tag{4.11}
\]

The azimuthal angular dependence disappears for high energies \( \sim 1/\gamma \) as a result of the dominating longitudinal polarization of the Z boson. Note again the characteristic difference to the pseudoscalar \( 0^- \) case \( e^+e^- \rightarrow ZA \ [Z \rightarrow f\bar{f}] \), where the azimuthal dependence is \( \mathcal{P} \)-even and independent of the energy in contrast to the \( 0^{++} \) case; after integrating out the polar angles,

\[
\frac{d\sigma(ZA)}{d\phi_*} \sim 1 - \frac{1}{4} \cos 2\phi_*. \tag{4.12}
\]

We can thus conclude that the angular analysis of the Higgs production in \( e^+e^- \rightarrow Z^* \rightarrow ZH \ [Z \rightarrow f\bar{f}] \) allows stringent tests of the \( J^{PC} = 0^{++} \) quantum numbers of the Higgs boson. This is a direct consequence of the \( 0^{++} \) coupling \( \epsilon_1 \cdot \epsilon_2 \) of the \( ZZH \) vertex in the production amplitude.

The spin and parity quantum numbers of Higgs particles can also be checked in the decay processes

\[
H \rightarrow V V^* \rightarrow (f_1\bar{f}_2) (f_3\bar{f}_4) \quad [V = W, Z] \tag{4.13}
\]

The angular distributions of the fermions in the decay process are quite similar to the rules found for the production process; a complete discussion can be found in Refs.\[14\]. Note that the angular correlations in Higgs decays can also be studied at hadron colliders, although background problems require a more sophisticated discussion.

Fig. 23 Angular distributions of the processes \( e^+e^- \rightarrow ZH, ZA \) and \( ZZ \) at \( \sqrt{s} = 500 \) GeV. The Higgs mass is fixed to \( M_H = 120 \) GeV.
4.2. Supersymmetric Extension

4.2.1 Neutral Higgs Bosons

The main production mechanisms of neutral Higgs bosons at $e^+e^-$ colliders are the bremsstrahlung process and pair production,

(a) bremsstrahlung $e^+e^- \rightarrow (Z) \rightarrow Z + h/H$ \hspace{1cm} (4.14)
(b) pair production $e^+e^- \rightarrow (Z) \rightarrow A + h/H$ \hspace{1cm} (4.15)

as well as the fusion processes, similarly to the SM Higgs boson,

(c) fusion processes $e^+e^- \rightarrow \nu\bar{\nu} (WW) \rightarrow \nu\bar{\nu} + h/H$
$e^+e^- \rightarrow e^+e^- (ZZ) \rightarrow e^+e^- + h/H$ \hspace{1cm} (4.16)

The $\mathcal{CP}$–odd Higgs boson $A$ cannot be produced in fusion processes to leading order.

The cross sections for the four bremsstrahl and pair production processes can be expressed as:

$$\sigma(e^+e^- \rightarrow Zh) = \sin^2(\beta - \alpha) \sigma_{SM}$$
$$\sigma(e^+e^- \rightarrow ZH) = \cos^2(\beta - \alpha) \sigma_{SM}$$
$$\sigma(e^+e^- \rightarrow Ah) = \cos^2(\beta - \alpha) \sigma_{SM} \bar{\lambda}$$
$$\sigma(e^+e^- \rightarrow AH) = \sin^2(\beta - \alpha) \sigma_{SM} \bar{\lambda}$$ \hspace{1cm} (4.17)

where

$$\sigma_{SM} = \frac{G_F^2 M_Z^4}{96\pi s} (v_e^2 + a_e^2) \lambda^{1/2}(M_{h,H}, M_Z^2 ; s) \lambda(M_{h,H}, M_Z^2 ; s) + 12 M_Z^2 / s \right) (1 - M_Z^2 / s)^2$$ \hspace{1cm} (4.18)

is the SM cross section for Higgs bremsstrahlung and the factor $\bar{\lambda}$ accounts for the correct suppression of the $P$–wave cross sections near the threshold

$$\bar{\lambda} = \frac{\lambda^{3/2}(M_j^2, M_Z^2 ; s)}{\lambda^{1/2}(M_j^2, M_Z^2 ; s) \left[ \lambda(M_j^2, M_A^2 ; s) + 12 M_Z^2 / s \right]} \right) \left[ j = h, H \right]$$ \hspace{1cm} (4.19)

The cross sections for the bremsstrahlung and the pair production as well as the cross sections for the production of the light and the heavy neutral Higgs bosons $h$ and $H$ are mutually complementary to each other, coming either with a coefficient $\sin^2(\beta - \alpha)$ or $\cos^2(\beta - \alpha)$. Since $\sigma_{SM}$ is large, at least one of the $\mathcal{CP}$–even Higgs bosons should be detected. From the mass and $\sin^2/\cos^2(\beta - \alpha)$ plots, we conclude that the following final states will be observed [Fig. 24], depending on the values of the parameters $M_h$ and $\tan\beta$:

- $M_h$ “small”, $\tan\beta$ small: $hZ, HZ, hA, HA$
- $M_h$ “large”, $\tan\beta$ large: $H, hA$

- $M_h$ “large”, $\tan\beta$ small: $hZ, [HA]$
- $M_h$ “large”, $\tan\beta$ large: $hZ, [HA]$
where “$M_h$ small” and “large” are synonymous for “considerably below” and “close to the upper limit of the light $\mathcal{CP}$–even Higgs boson” for a given value of $\tan \beta$. If $M_h$ is “large” the $H, A$ masses can exceed the kinematical limit for $HA$ pair production.

Fig. 24 Regions of the $[M_h, \tan \beta]$ plane where the four cross sections $e^+e^- \to hZ, hA, HZ$ and $HA$ are observable. The contours are defined such that the cross sections are larger than 2.5 fb [corresponding to 25 events for an integrated luminosity of $\int \mathcal{L} = 10 \text{ fb}^{-1}$]. The dashed area corresponds to the theoretically forbidden region $[M_h^2 \not> M_Z^2 \cos^2 2\beta + \epsilon \sin^2 \beta]$.

The region which can be probed at LEP200 [defined such that for $\sqrt{s} = 180$ GeV and $\int \mathcal{L} = 500 \text{ pb}^{-1}$, the number of events in one of the two processes $e^+e^- \to hZ$ or $hA$ is larger than 25] is the area to the left of the thin line. We have taken $m_t = 175$ GeV and $M_S = 1$ TeV. The process $e^+e^- \to hZ$ is accessible in the entire area below the full line, $hA$ in the entire area above the broken line and $HZ$ in the entire area above the full line; $HA$ final states can be detected in the area between the two dashed lines.

The cross sections for the production of the $\mathcal{CP}$–even light and heavy Higgs bosons $h$ and $H$ via bremsstrahlung are shown as functions of the Higgs mass in Fig. 25a. The cross section for $h$ is large for small values of $\tan \beta$ and/or large values of $M_h$ where $\sin^2(\beta - \alpha)$ approaches its maximal value. In these two cases the cross section is of the order of $\sim 50 \text{ fb}$, which for an integrated luminosity of 10 \text{ fb}^{-1}$ corresponds
to \sim 500\) events. By contrast, the cross section for \(H\) is large for large \(\text{tg}\beta\) and light \(h\) [implying small \(M_H\)]. In the case of \(h\) [and also for \(H\) in most of the parameter space] the signal consists of a \(Z\) boson accompanied by a \(b\bar{b}\) or a \(\tau^+\tau^-\) pair. The signal is easy to separate from the background which comes mainly from \(ZZ\) production if the Higgs mass is close to \(M_Z\).

Fig. 25 Production cross sections of the \(CP\)–even neutral Higgs bosons at \(\sqrt{s} = 500\) GeV as functions of their masses for three values of \(\text{tg}\beta = 2.5, 5\) and 20; (a) Bremsstrahl processes \(e^+e^- \rightarrow Z + h/H\), and (b) in association with the pseudoscalar Higgs boson \(e^+e^- \rightarrow A + h/H\).
The cross sections for the associate channels $e^+e^- \rightarrow Ah$ and $AH$ are displayed in Fig. 25b. As anticipated, the situation is opposite to the previous case: the cross section for $Ah$ is large for light $h$ and/or large values of $\tan \beta$ whereas $AH$ production is preferred in the complementary region. The sum of the two cross sections decreases from $\sim 50$ to $10$ fb if $M_A$ increases from $\sim 50$ to $200$ GeV. In major parts of the parameter space, the signals consist of four $b$ quarks in the final state, requiring facilities for efficient $b$ quark tagging. Mass constraints will help to eliminate the backgrounds from QCD jets as well as $ZZ$ final states.

Fig. 26 Production cross sections of the $\mathcal{CP}$–even neutral Higgs bosons at $\sqrt{s} = 500$ GeV as functions of their masses for three values of $\tan \beta = 2.5, 5$ and $20$: (a) $WW$ fusion process $e^+e^- \rightarrow \nu\bar{\nu} + h/H$ (b) and $ZZ$ fusion process $e^+e^- \rightarrow e^+e^- + h/H$. 
and ZZ fusion provide additional mechanisms for the production of the CP–
even neutral Higgs bosons. They lead to Higgs bosons in association with a ν¯ν or
e+e– pair in the final state. The cross sections can again be expressed in terms of
the corresponding SM cross sections, given in eq. (4.9)

\[
\sigma(e^+e^- \rightarrow (VV) \rightarrow h) = \sin^2(\beta - \alpha)\sigma_{SM}^{VV} \\
\sigma(e^+e^- \rightarrow (VV) \rightarrow H) = \cos^2(\beta - \alpha)\sigma_{SM}^{VV}
\] (4.20)

For the WW fusion mechanism, the cross sections are larger than for the bremsstrahl
process if the Higgs mass is moderately small – less than 160 GeV at \(\sqrt{s} = 500\)
GeV. However, since the final state cannot be fully reconstructed, the signal is more
difficult to extract. As in the case of the bremsstrahl process, the production of light
h and heavy H bosons are complementary. The cross sections for the ZZ fusion
mechanism are about an order of magnitude smaller than for the one for WW. ZZ
fusion will nevertheless be useful as the final state can be fully reconstructed. The
cross sections are displayed for representative values of \(\text{tg} \beta\) in Fig. 26.

The preceding discussion on the neutral MSSM Higgs sector in \(e^+e^–\) linear colliders can be summarized in the following points:

(i) The lightest CP–even Higgs particle \(h\) can be detected in the entire range of
the MSSM parameter space, either through the bremsstrahlung process \(e^+e^- \rightarrow
hZ\) or through pair production \(e^+e^- \rightarrow hA\). In fact, this conclusion holds true even
at a c.m. energy of 300 GeV, independently of the top and squark mass values, and
also if invisible neutralino decays are allowed for.

(ii) There is a substantial area of the \([M_h, \text{tg} \beta]\) parameter space where all neutral
SUSY Higgs bosons can be discovered at a 500 GeV collider. This is possible if the
masses of the scalar \(H\) and the pseudoscalar \(A\) boson are less than \(\sim 230\) GeV.

(iii) In some part of the MSSM parameter space, the lightest Higgs particle \(h\) can be detected, but it cannot be distinguished from the SM Higgs boson [if SUSY
decays are not allowed]. This happens if, for a given value of \(\text{tg} \beta\), \(M_h\) is very close
to its maximum value and \(H\) and \(A\) are too heavy to be produced in association. In
this case, the couplings of \(h\) to gauge bosons and fermions are SM like. A way to
distinguish between the two is provided by \(\gamma\gamma\) collisions as will be discussed shortly.

(iv) Higgs boson decays into charginos and neutralinos can be very important
in some areas of the MSSM parameter space; in particular, invisible decays of the
neutral Higgs bosons can be larger than the decays into standard particles. At \(e^+e^–\)
colliders, missing mass techniques allow to isolate these events in the bremsstrahlung
process for the CP–even Higgs bosons or in a mixture of visible and invisible decay
modes of \(Ah\) and \(AH\) in the pair production processes.

Some of these features are not specific to the minimal extension but they are
expected to be realized also in more general SUSY models. For example, a light
Higgs boson with a mass in the intermediate range is quite generally predicted in su-
persymmetric theories [27,28] [The detection of this particle however is not necessarily
guaranteed as the production rate may be suppressed.]
4.2.2 Charged Higgs Bosons

An unambiguous signal of an extended Higgs sector would be the discovery of a charged Higgs boson. In a general two–Higgs doublet model, charged Higgs bosons can be as light as $\sim 45$ GeV, the lower limit derived from the negative search at LEP100. In the MSSM however, $H^\pm$ is constrained to be heavier than the $W$ boson. More precisely, the lower limit $M_A > 45$ GeV obtained at LEP100 implies $M_{H^\pm} > 90$ GeV.

In $e^+e^-$ collisions, the production of a pair of charged Higgs bosons proceeds through virtual photon and $Z$ boson exchange. The cross section depends only on the charged Higgs mass and does not depend on any extra parameter,

$$\sigma(e^+e^- \rightarrow H^+H^-) = \frac{\pi\alpha^2}{3s} \left[ 1 - \frac{2\hat{v}_e\hat{v}_H}{1 - M_Z^2/s} + \frac{(\hat{a}_e^2 + \hat{a}_H^2)\hat{v}_H^2}{(1 - M_Z^2/s)^2} \right] \beta^3 \quad (4.21)$$

with the standard $Z$ charges $\hat{v}_e = (-1 + 4s_W^2) / 4c_W s_W$, $\hat{a}_e = -1 / 4c_W s_W$ and $\hat{v}_H = (-1 + 2s_W^2) / 2c_W s_W$, and $\beta = (1 - 4M^2_{H^\pm}/s)^{1/2}$ the velocity of the Higgs particles. The cross section is shown in Fig. 27a as a function of the charged Higgs mass for a c.m. energy $\sqrt{s} = 500$ GeV. For small Higgs masses the cross section is of order 100 fb, but it drops very quickly due to the $P^-$–wave suppression factor $\beta^3$ near the threshold. For $M_{H^\pm} = 220$ GeV, the cross section has fallen to a level of $\sim 5$ fb, which for an integrated luminosity of 10 fb$^{-1}$ corresponds to 50 events. The angular distribution of the charged Higgs bosons follows the $\sin^2 \theta$ law typical for spin–zero particle production.

Charged Higgs particles can also be created in $\gamma\gamma$ collisions. Generating the $\gamma$ beams through back–scattering of laser light, the total energy of the $\gamma\gamma$ collider can go up to $\sim 80\%$ of the original $e^+e^-$ energy, which corresponds to $\sqrt{s_{\gamma\gamma}} \sim 400$ GeV for a 500 GeV $e^+e^-$ collider. The $\gamma\gamma$ luminosity is expected to be of the same magnitude as the original $e^+e^-$ luminosity. The cross section is given by

$$\sigma(\gamma\gamma \rightarrow H^+H^-) = \frac{2\pi\alpha^2}{s} \beta \left[ 2 - \beta^2 - \frac{1 - \beta^4}{2\beta \log 1 - \beta} \right] \quad (4.22)$$

where $\beta$ is the velocity of the Higgs particles. The numerical result is displayed in Fig. 27a for the $\gamma\gamma$ luminosity without beam polarization. Due to the reduced energy, the maximum Higgs mass which can be probed in $\gamma\gamma$ collisions is smaller than in the original $e^+e^-$ collisions; the cross section however is enhanced by a factor $\sim 3$ in the low mass range.

The charged Higgs boson, if lighter than the top quark, can also be produced in top decays. The ratio of the decay widths $t \rightarrow bH^+$ and the standard mode $t \rightarrow bW^+$ is given by

$$\frac{\Gamma(t \rightarrow bH^+)}{\Gamma(t \rightarrow bW^+)} = \frac{(m_t^2 + m_b^2 - M_{H^\pm}^2)(m_t^2\csc^2 \beta + m_b^2\csc^2 \beta) + 4m_t^2m_b^2}{M_{W^\pm}^2(m_t^2 + m_b^2 - 2M_{W^\pm}^2) + (m_t^2 - m_b^2)^2} \times \frac{\lambda(M_{H^\pm}^2, m_t^2; m_b^2)^{1/2}}{\lambda(M_{W^\pm}^2, m_t^2; m_b^2)^{1/2}} \quad (4.23)$$
In the range $1 < \tan \beta < m_t/m_b$ favored by SUSY models, the branching ratio $\text{BR}(t \to bH^\pm)$ varies between $\sim 2\%$ and $20\%$. Since the cross section for top pair production is of order of $0.5$ pb at $\sqrt{s} = 500$ GeV, this corresponds to $200$ and $2000$ charged Higgs bosons at a luminosity $\int \mathcal{L} = 10$ fb$^{-1}$; Fig. 27b.

The signature for $H^\pm$ production can be read off the graphs displaying the branching ratios in section 2. If $M_{H^\pm} < m_t + m_b$, the charged Higgs boson will decay mainly into $\tau \nu_\tau$ and $c \bar{s}$ pairs, the $\tau \nu_\tau$ mode dominating for $\tan \beta$ larger than unity. This results in a surplus of $\tau$ final states over $e, \mu$ final states, an apparent breaking of $\tau$ vs. $e, \mu$ universality. For large Higgs masses the dominant decay mode is the top decay $H^+ \to tb$. In some part of the parameter space also the decay $H^+ \to W^+ h$ is allowed, leading to cascades with heavy $\tau$ and $b$ particles in the final state.

Fig. 27 Production cross sections of the charged Higgs boson (a) at $e^+e^-$ colliders with $\sqrt{s} = 500$ GeV and in $\gamma\gamma$ collisions with $\sqrt{s} = 400$ and (b) in top decays with $m_t = 175$ GeV and for three values of $\tan \beta = 2.5, 5$ and 20.
4.3. $\gamma\gamma$ Collisions

Future high–energy $e^+e^-$ linear colliders can be made to run in the $e\gamma$ or $\gamma\gamma$ mode by using Compton back scattering of laser light. One then obtains $e\gamma$ colliders by converting only one of the initial electron or positron beams to a very energetic photon, or $\gamma\gamma$ colliders by converting both the electron and positron to photons. These colliders will have practically the same energy [\$\sim 90\%$ for $e\gamma$ and $\sim 80\%$ for $\gamma\gamma$ machines] and luminosity as the original $e^+e^-$ collider [although they will depend in a sensitive way on various machine parameters]; for details see Refs. 106, 107.

One of the best motivations for turning single–pass future high–energy $e^+e^-$ linear colliders into $\gamma\gamma$ colliders is undoubtedly the study of the properties of neutral Higgs particles which can be produced as resonances in the $s$–channel. In the context of the supersymmetric and Standard Model Higgs sector, two main features which are difficult to study in the $e^+e^-$ mode, can be investigated at such colliders:

i) As discussed previously, since the photons couple to Higgs bosons via heavy particle loops, the $H\gamma\gamma$ amplitudes are sensitive to particle masses, standard and also supersymmetric, well above the Higgs masses themselves. We have seen in the preceding section that in the $e^+e^-$ mode, the lightest supersymmetric Higgs particle $h$ can be detected but it cannot be distinguished from the $SM$ Higgs boson in some part of the $MSSM$ parameter space if $SU(5)$ decays are not allowed. As long as this ambiguity cannot be resolved by proceeding to higher $e^+e^-$ collider energies, the only way to distinguish $h$ from the $SM$ Higgs particle is provided by Higgs production in $\gamma\gamma$ fusion. While in the Standard Model this process is built up by $W$ and top quark loops, additional contributions in $SU(5)$ models are provided by supersymmetric particle loops [chargino, sfermion and charged Higgs boson loops] which alter the $SM$ production rates.

ii) In the $e^+e^-$ mode, since the standard Higgs boson and the scalar $MSSM$ Higgs particles $h, H$ couple to vector bosons directly, the positive parity can be checked by analyzing the $Z$ final states in $e^+e^- \rightarrow Z^* \rightarrow ZH$ [$Z \rightarrow f\bar{f}$] as discussed above; this method is equivalent to the analysis of the $h, H \rightarrow VV$ decays. However, in the $MSSM$ the pseudoscalar $A$ boson has no tree level couplings to the vector bosons and the latter methods cannot be used. To study the $CP$ properties of scalar and pseudoscalar Higgs particles on the same footing [and to check whether $CP$ is violated in the Higgs sector, as it would be the case in a general two–Higgs doublet model where the three neutral Higgs bosons could correspond to arbitrary mixed $CP$ states], one can run in the $\gamma\gamma$ mode where both type of particles are produced through loop diagrams with similar rates. Indeed, the fusion of Higgs particles by linearly polarized photon beams depends on the angle between the polarization vectors. For scalar particles the production amplitude $\sim \hat{\epsilon}_1 \cdot \hat{\epsilon}_2$ is maximal only for parallel vectors while pseudoscalar particles with amplitudes $\sim \hat{\epsilon}_1 \times \hat{\epsilon}_2$ prefer perpendicular polarization vectors. However, for typical experimental set–ups for Compton back–scattering of laser light, the maximum degree of linear polarization of the generated hard photon beams is less than about $30\%$ so that the efficiency for two polarized beams is reduced to less than $10\%$. \[11\]
To study both features, very high luminosities and a very good control on the photon beam polarization are required. Moreover, in case where the Higgs bosons decay into $b$–quarks, one needs a careful analysis of background rejection due to the enormous number of $b\bar{b}$ pairs [and also other jets] produced in $\gamma\gamma$ collisions. For heavy neutral $CP$–even Higgs bosons which decay mostly into massive vector bosons pairs, one also needs to deal with the large backgrounds from $WW$ and $ZZ$ pair production [although the latter proceeds only through loop diagrams as for Higgs production].

Furthermore, it is mandatory to have a precise prediction for the value of the Higgs–$\gamma\gamma$ coupling, especially if one attempts to exploit this coupling to look for new particles whose masses are not entirely generated through the Higgs mechanism and therefore have small effects [as is the case for charginos, sfermions and charged Higgs bosons]. Therefore, one needs to control properly the radiative corrections to the coupling mediated by the standard particles and to include the QCD corrections to the quark amplitude. [Note that a precise prediction of the $\Phi \rightarrow \gamma\gamma$ decay widths is also important because they play a crucial rôle for the search of these particles in the lower part of the intermediate mass range at the LHC, as discussed in the previous section]. For completeness, the QCD corrected $\Phi \rightarrow \gamma\gamma$ decay widths will be briefly discussed below.

Denoting the quark amplitudes by $A_Q$ [where $Q = t, b$ in the $MSSM$ etc.], the $H = H_{SM}/h/H \rightarrow \gamma\gamma$ and $A \rightarrow \gamma\gamma$ decay rates are given by

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128 \sqrt{2} \pi^3} \left| \sum_Q N_{C e}^2 g_{Q Q H} A_Q^H + g_W A_W^H + \cdots \right|^2$$ (4.24)

$$\Gamma(A \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_A^3}{32 \sqrt{2} \pi^3} \left| \sum_Q N_{C e}^2 g_{Q Q A} A_Q^A + \cdots \right|^2$$ (4.25)

where the quark amplitudes at lowest order are given in eq. (3.18) while the $W$ amplitude is given by

$$A_{WWH} = -\frac{3}{4} \tau^{-1} \left[ 3 + 2\tau + 3(2 - \tau^{-1}) f(\tau) \right]$$ (4.26)

where, as usual, the scaling variable is defined as $\tau = M^2_{\Phi}/s_{\gamma\gamma}$ with $m_i$ denoting the loop mass, and the scalar triangle integral $f$ is given by eq. (3.9).

The cross section for the $\gamma\gamma$ fusion of Higgs bosons is found by folding the parton luminosity, see e.g. Ref. and the parton cross section is determined by the $\gamma\gamma$ width [$s_{\gamma\gamma}$ is the photon c.m. energy],

$$\sigma(\gamma\gamma \rightarrow \Phi) = \frac{8\pi^2}{M_{\Phi}^2} \Gamma(\Phi \rightarrow \gamma\gamma) \delta(1 - M_{\Phi}^2/s_{\gamma\gamma})$$ (4.27)

The QCD corrections to the quark amplitudes can be parametrized as

$$A_{QQ\Phi} = A_{QQ\Phi}^{LO} + C \frac{\alpha_s}{\pi}$$ (4.28)
The coefficient $C$ depends on $\tau = M^2_\Phi/4m_Q^2(\mu^2)$ where the running quark mass $m_Q(\mu^2)$ is defined at the renormalization point $\mu$ which is taken to be $\mu = M_\Phi/2$ in our analysis; this value is related to the pole mass $m_Q(m^2_Q) = m_Q$ in the on–shell renormalization scheme by

$$m_Q(M^2_\Phi/4) = m_Q \left[ \frac{\alpha_s([M_\Phi/2]^2)}{\alpha_s(m^2_Q)} \right]^{12/(33-2N_f)} \{ 1 + O(\alpha_s^2) \}$$

The lowest order amplitude $A^{LO}_{QQ\Phi}$ is to be evaluated for the same mass value $m_Q([M_\Phi/2]^2)$. The choice $\mu = M_\Phi/2$ of the renormalization point avoids large logarithms $\log M^2_\Phi/m^2_Q$ in the final results for Higgs masses much larger than the quark mass. $\alpha_s$ is taken at $\mu$ for $\Lambda = 200$ MeV.

The two–loop diagrams contributing to the decay widths are a subset of the diagrams which appear in the case of the $gg \to$ Higgs fusion processes. We have evaluated these diagrams in a way similar to what has been discussed in the $gg \to H$ case, but for the the running quark mass at $\mu = M_\Phi/2$. The output of the calculation is shown in Fig. 28 where the amplitudes $C_H$ for scalar loops and $C_A$ for pseudoscalar loops are shown as functions of $\tau$.

The coefficients are real below the quark threshold $\tau < 1$, and complex above. Very close to the threshold, within a margin of a few GeV, the present perturbative analysis cannot be applied anymore. [It may account to some extent for resonance effects in a global way.] Since $Q\bar{Q}$ pairs cannot form $0^{++}$ states at the threshold, $\Im C_H$ vanishes there; $\Re C_H$ develops a maximum very close to the threshold. By contrast, since $Q\bar{Q}$ pairs do form $0^{-+}$ states, the imaginary part $\Im C_A$ develops a step which is built up by the Coulombic gluon exchange [familiar from the Sommerfeld singularity of the QCD correction to $Q\bar{Q}$ production in $e^+e^-$ annihilation]; $\Re C_A$ is singular at the threshold. The threshold effects have been discussed in detail recently. For large $\tau$, both coefficients approach a common numerical value, as expected from chiral invariance in this limit. In the opposite limit, the QCD corrections can be evaluated analytically,

$$m_Q \gg M_\Phi : \quad C_H \to -1 \quad \text{and} \quad C_A \to 0$$

These results can easily be traced back to the form of the $\gamma\gamma$ anomaly in the trace of the energy–momentum tensor and to the non-renormalization of the axial–vector anomaly. In Fig. 29, the QCD corrected $\gamma\gamma$ widths for $h, H, A$ Higgs bosons are displayed in the MSSM for two values $\tan\beta = 2.5$ and $\tan\beta = 20$, taking into account only quark and $W$ boson loops. While in the first case top loops give a significant contribution, bottom loops are the dominant component for large $\tan\beta$. The overall QCD corrections are also shown: since we have used the running quark masses, the corrections to the widths are small, $\sim O(\alpha_s/\pi)$ everywhere. However, artificially large $\delta$ values occur for specific large Higgs masses when the lowest order amplitudes vanish accidentally as a consequence of the destructive interference between $W$ and quark–loop amplitudes, see also Ref.
Fig. 28 Real and imaginary parts of the QCD correction to the quark amplitudes of the scalar and pseudoscalar couplings normalized to the Born terms.

Fig. 29 QCD corrected two–photon decay widths [in keV] and size of the QCD corrections for the MSSM neutral Higgs bosons with $\tan\beta = 2.5$ and 20; the top mass was taken to be $m_t = 140 \text{ GeV}$. 
5. Summary

Probing electroweak symmetry breaking will be the most important mission of future high–energy colliders. In this review, we have discussed the properties of the Higgs particles of the Standard Model and of its Minimal Supersymmetric Extension. We have updated various results for couplings, decay widths and production cross sections to take into account the new value of the top quark obtained recently by the CDF and LEP/SLC Collaborations, and to include radiative corrections, some of which have been calculated only recently.

We have then discussed the potential of future high–energy colliders for discovering and studying the properties of these Higgs particles, and considered the case of the CERN hadron collider LHC with a center of mass energy of \( \sim 14 \) TeV and of a future \( e^+e^- \) collider in the energy range 300–500 GeV.

At the hadron collider LHC, Higgs particles can be produced with very high rates. The main production mechanism is the fusion mechanism \( gg \rightarrow H \) which proceeds through heavy quark loops, followed by the \( WW/ZZ \) fusion process and the associate production with \( W/Z \) bosons or heavy quark pairs. The QCD corrections are important for the \( gg \) fusion mechanism, their inclusion enhances the production rate by practically a factor of two.

For Standard Model Higgs bosons in the “high mass” region, \( M_H > 140 \) GeV, the signal consists of the so–called “gold–plated” events \( H \rightarrow ZZ^{(*)} \rightarrow 4l^\pm \) with \( l = e, \mu \), with which one can probe Higgs masses up to \( \sim 800 \) GeV with a luminosity \( \int \mathcal{L} = 100 \text{ fb}^{-1} \) at LHC. The \( H \rightarrow l^+l^-\bar{\nu}\bar{\nu} \) and \( H \rightarrow WW \rightarrow l\nu jj \) decay channels can push this limit up to \( \sim 1 \) TeV. For the “low mass” range, the situation is more complicated and one has to rely on the rare \( H \rightarrow \gamma\gamma \) decay mode. At the LHC, the event rate is rather large, \( \mathcal{O}(10^3) \) events for \( \int \mathcal{L} = 100 \text{ fb}^{-1} \), but the backgrounds pose a formidable task. With a dedicated detector and a high–luminosity option, this channel although very difficult, is feasible. A complementary channel in this low mass range would be the \( pp \rightarrow WH, t\bar{t}H \rightarrow \gamma\gamma\nu\bar{\nu} \) but unfortunately the rates are small; a promising process would be \( pp \rightarrow t\bar{t}H \) with the Higgs decay \( H \rightarrow b\bar{b} \), if very high efficiency and purity for \( b \)–quark tagging in the LHC environment [especially with the high–luminosity option] could be made available.

In the Minimal Supersymmetric Standard Model, the situation is even more complicated. The production mechanisms are the same as for the \( SM \) Higgs but one has to take the \( b \) quark contributions into account. Since the lightest Higgs boson mass is always smaller than \( \sim 140 \) GeV, only the two–photon decay channel can be used for this particle; the branching ratio is smaller than in the \( SM \) making the detection of this Higgs boson more difficult, except in the case where \( M_h \) is close to its maximal value where the situation is similar to the \( SM \) case. Since the pseudoscalar \( A \) has no tree level couplings to gauge bosons and since the couplings of the heavy \( \mathcal{CP} \)–even \( H \) are strongly suppressed, the gold–plated \( ZZ \) signal is lost in a large part of the parameter space; a promising alternative is to use the \( A, H \rightarrow \tau^+\tau^- \) channels for large \( \tan \beta \) values. Another possibility would be neutral Higgs production in association with \( t\bar{t} \) pairs with the Higgs particle decaying into...
$b$–quarks if very good micro–vertex detectors are available. Charged Higgs particles, if lighter than the top quark, can be accessible in top decays. Up to now, there is still a substantial area in the SUSY parameter space where no Higgs particle can be found at the LHC [although, these analyses have to be updated to take into account the high value of the top quark mass and the new possibilities offered by $b$–tagging at hadron colliders].

$e^+e^-$ linear colliders with energies in the range 300 – 500 GeV and a luminosity of $\mathcal{L} = a$ few times $10^{33} \text{cm}^{-2}\text{s}^{-1}$ are ideal instruments to search for Higgs bosons in the mass range below the scale of electroweak symmetry breaking [in fact, standard Higgs bosons can be discovered up to masses of the order of 350 GeV at a 500 GeV $e^+e^-$ collider]. This range is the most likely region for the mass of the Standard Model Higgs boson since in this case, the particle remains weakly interacting up to the GUT scale.

In the intermediate mass range, Standard Higgs particles can be observed irrespectively of their decay modes, in three independent production channels: the bremsstrahlung process $e^+e^- \rightarrow ZH$ and the fusion processes $e^+e^- \rightarrow \bar{\nu}\nu H$ and $e^+e^- \rightarrow e^+e^-H$. The particle is relatively easy to detect especially in the $ZH$ channel, where the main background from $ZZ$ pair production can be suppressed efficiently by using micro–vertex detectors since Higgs bosons with masses below 140 GeV decay mainly into $b\bar{b}$ pairs. Once the Higgs boson is found, its fundamental properties can be investigated. The Higgs spin can be measured by analyzing the angular dependence of the $ZH$ production process and of the Higgs decays into massive gauge bosons. The Higgs couplings to the massive gauge bosons can be determined through the production rates, the coupling to heavy fermions through the Higgs decay branching fractions, and in some mass window, Higgs radiation off top quarks.

An even stronger case for $e^+e^-$ colliders operating in the 500 GeV range is made by supersymmetric extensions of the Standard Model. Since in the minimal extension, the lightest Higgs particle has a mass below 140 GeV and decays mainly into $b\bar{b}$ and $\tau^+\tau^-$ pairs, it cannot be missed at an $e^+e^-$ collider with an energy $\sqrt{s} > 300$ GeV, independently of its decay modes and in the entire SUSY parameter space. The heavy neutral Higgs particles can be produced in the bremsstrahlung and fusion processes or pairwise, these processes being complementary. At least one neutral Higgs boson must be detected, and in a large part of the SUSY parameter space, all neutral Higgs bosons can be observed. Charged Higgs particles can be detected up to practically the kinematical limit.

Future $e^+e^-$ can be turned to very high–energy $\gamma\gamma$ or $e\gamma$ colliders by using back–scattering of laser light. The $\gamma\gamma$ mode of the $e^+e^-$ collider could be useful to measure accurately the Higgs–photons coupling to which new particles might contribute, and to study the $\mathcal{CP}$ properties of the Higgs particles.

$e^+e^-$ linear colliders operating in the 300–500 GeV energy range and hadron colliders operating in the multi–TeV range have a complementary potential for addressing the key issue of the mechanism of electroweak symmetry breaking.
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