Role of nucleon effective mass and symmetry energy
on the neutrino mean free path in neutron star

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Abstract

The Korea-IBS-Daegu-SKKU energy density functional (KIDS-EDF) models, derived from the universal
Skyrme functional, have been successfully and widely applied in describing the properties of finite nuclei
and infinite nuclear matter. In the present work, we extend the applications of the KIDS-EDF models to
investigate the implications of the nucleon effective mass and nuclear symmetry energy obtained from the
KIDS-EDF models on the properties of neutron star (NS) and neutrino interaction with the NS constituents
matter in the linear response approximation (LRA). We then analyze the total differential cross-section of
neutrino, neutrino mean free path (NMFP), and the NS mass-radius (M-R) relations. We find that the NS
M-R relations predictions for all KIDS-EDF models are in excellent agreement with the recent observations
as well as the NICER result. Remarkable prediction results on the NMFPs are given by the KIDS0-m*77
and KIDS0-m*99 models with $M_n^*/M \leq 1$ which are quite higher in comparison with those obtained for
the KIDS0, KIDS-A, and KIDS-B models with $M_n^*/M \geq 1$. For the KIDS0, KIDS-A, and KIDS-B models,
we obtain the $\lambda \lesssim R_{NS}$, indicating that these models support the slow NS cooling and neutrino trapping in
NS. On the contrary, both KIDS0-m*77 and KIDS0-m*99 models support faster NS cooling and a small
possibility of neutrino trapping within NS, predicting $\lambda \gtrsim R_{NS}$. More interestingly the NMFP decreases as
the density and neutrino energy increase, which is consistent with those obtained in the Brussels-Montreal
Skyrme (BSk17 and BSk18) models at saturation density.

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I. INTRODUCTION

It is widely known that neutron stars (NSs) are compact objects which pose extraordinary laboratories for dense stellar matter physics that cannot be reproduced in terrestrial laboratories, where the neutrino processes in the later stage of stellar evolution play an important role \[1-3\]. However, the internal composition of the core of NSs and the interactions among the NS constituents matter are still poorly understood until now. Also, the most important observable properties of NSs like NS maximum masses and radii are not well-constrained yet \[4, 5\]. The NS maximum mass is the implication of general relativity and it is controlled by the equation of state (EoS) of nuclear matter at a density more than a few times normal density. In addition to the determinations of the NS mass and radius, it is expected that the interior of NS will cool \textit{via} the neutrino emissions process which sensitively depends on the NS composition and the NS constituents’ interactions. The slow or rapid neutrino emissions have a significant implication on the NS cooling process. The neutrinos emission and scattering are sensitive to the nuclear symmetry energy of the EoS and the nucleon effective masses \[6, 7\].

The nucleon effective masses, which are calculated from the effective interaction \[8\], are not only crucial for neutrino scattering and absorption (opacity) in NS \[9, 13\] but also for the structure of rare isotopes, stellar matters, compact stars, and other astrophysical objects, i.e. supernova and NS as well as the dynamics of heavy-ion collision (HIC) \[14\]. Besides the nucleon effective masses, the nuclear symmetry energy also has an essential role in determining the features of the stiffness or softness of EoS in the neutron-rich matter. Generally, the stiffer or softer nuclear symmetry energy can be understood from the enhancement or reduction of pressure gradient in the asymmetric matter. Therefore, the nuclear symmetry energy is a powerful tool for controlling the rate of the NS cooling process in a mixture of nucleons and determining the density for the appearance of hyperon or other exotic particles, the nucleon emissions in the reaction dynamics, and the collective flows in HIC \[15, 16\].

Moreover, the behavior of the density dependence of the nuclear symmetry energy at high densities remains unknown \[17, 18\]. Although the nuclear symmetry energy at saturation density $\rho_0 \approx 0.16 \text{ fm}^{-3}$ was predicted to be around $32 \pm 0.59 \text{ MeV}$ in the recent analysis from the nuclear liquid drop (LD) model \[19\], its values for the baryon densities larger than the saturation densities are poorly known \[17, 18\]. Therefore intensive studies on the density dependence of the nuclear symmetry energy at higher densities are extremely needed to gain a better understanding
of the properties of NS and its matter constituents interactions as well as to constrain the EoS of neutron-rich matter. Several theoretical works using various microscopic theories with realistic nucleon-nucleon forces and phenomenological models [17–21] have been done to observe the features of the nuclear symmetry energy and nucleon effective masses. Besides those theoretical attempts and terrestrial laboratories constraints which allow us to constrain the nuclear symmetry energy at around saturation density, nowadays it is also possible to constrain the nuclear symmetry energy in the higher density via the well-known mass-radius (M-R) relations of NS using the astrophysical or astronomical observations such as the Neutron Star Interior Composition Explorer (NICER) [22, 23], the Laser Interferometer Gravitational-Wave Observatory (LIGO) [24, 25], and other X-ray burst observations [26]. For instance, using the astrophysical observations information of the tidal deformability detected from the binary NS mergers, allows us to constrain the density dependence of nuclear symmetry energy. Motivated by recent progress in theoretical studies, terrestrial laboratories or experiments, and astrophysical observations, in the present theoretical study, we observe the neutrino mean free path (NMFP), which is a crucial quantity for the explanation of NS cooling and an important input for the simulation of the neutrino transport [27], for various Korea-IBS-Daegu-SKKU energy density functional (KIDS-EDF) models and the implications of the nuclear symmetry energies and nucleon effective masses on the NMFP as well as the mass-radii properties of the NS.

In this paper, we investigate the role of the nucleon effective masses and nuclear symmetry energies for various KIDS-EDF models: KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A, and KIDS-B models on the neutrino scattering with NS constituent matters, which consist of the protons (p), neutrons (n), electrons (e) and muons (μ) as the standard matter particles, and properties of the NS. In the neutrino scattering, we perform the linear response approximation (LRA) to describe the interaction between neutrino and NS constituents. Note that in this present work we focus on only the neutrino interactions with the neutrons and protons since their differential cross-sections of neutrino and NMFP are more dominant than those with the leptons. The KIDS-EDF models have been successfully and widely applied in many physics phenomenon such as the finite nuclei [28], the quasi-elastic electron scattering [29], the inclusive electron scattering [30], and nuclear matter [28] as well as NS [31]. The nuclear symmetry energy, the neutron, and proton effective masses, and the neutron and proton fractions of constituents of the β–stable matter are calculated within the KIDS-EDF models. With these parameters obtained in the KIDS-EDF models, we calculate the NMFP, which is an inverse of the differential cross-sections of neutrino, in NS. Furthermore,
we calculate the M-R relations of NS by solving the equations of Tolman-Oppenheimer-Volkoff (TOV) [32, 33] and observe the M-R implications of the neutrino mean free path and vice versa, where it is expected to provide new insight on the rate of NS cooling process and the possibility of neutrino trapping in NS.

This paper is organized as follows. In Sec. II we briefly introduce the formalism of the KIDS-EDF models with various types. We then calculate the nuclear symmetry energy, the neutron, and proton effective masses, and the proton and neutron fractions of constituents of the $\beta$–stable matter for various KIDS-EDF models. In Sec. III we present the M-R relations of NS that are calculated via the TOV equations. In Sec. IV we finally present the differential cross-sections of neutrino and in particular NMFP using the LRA. Section V is devoted to a summary and conclusion.

II. KIDS-EDF MODELS

In this section, we briefly present the formalism of the KIDS-EDF models which were proposed for the first time for homogeneous nuclear matter [34]. It was then applied for describing the properties of finite nuclei. In the finite nuclei, EDF in the nuclear matter was transformed to the form of the Skyrme functional, where the specific values of the effective masses were determined by fitting to nuclear data. Here we first describe the energy per nucleon in the homogeneous nuclear matter by expanding the energy per nucleon or the energy density in terms of the power of the Fermi momentum $k_F$ which is equivalent to the cubic root of the baryon density. An explicit expression of the KIDS-EDF energy per nucleon is given by

$$E(\rho, \delta) = \mathcal{T}(\rho, \delta) + \sum_{j=0}^{3} c_j(\delta) \rho^{(1+a_j)},$$

where $a_j = j/3$ and $\rho = \rho_n + \rho_p$ is the baryon density, where $\rho_n$ and $\rho_p$ are respectively neutron and proton densities. The isospin asymmetry parameter is defined by $\delta = (\rho_n - \rho_p)/\rho$, where $\delta = 0$ for symmetric nuclear matter (SNM) and $\delta = 1$ for pure neutron matter (PNM). The kinetic energy in the first term $\mathcal{T}(\rho, \delta)$ of Eq. (1) is given by

$$\mathcal{T}(\rho, \delta) = \frac{3}{5} \left[ \frac{\hbar^2}{2M_p} \left( \frac{1 - \delta}{2} \right)^{\frac{5}{3}} + \frac{\hbar^2}{2M_n} \left( \frac{1 + \delta}{2} \right)^{\frac{5}{3}} \right] \left( 3\pi^2 \rho \right)^{\frac{2}{3}},$$

where $\hbar = h/2\pi$. The parameters $c_j(\delta)$ are fixed by fitting to the nuclear properties. The first term in Eq. (1) describes the kinetic energy of the nucleon in the homogeneous nuclear matter, while the second term includes the pairing effects. The parameter $c_j(\delta)$ is determined by fitting to the nuclear data, and the specific values of the effective masses are determined by fitting to nuclear data.
where \( M_p \) and \( M_n \) are respectively the proton and neutron masses. The potential energy term of Eq. (1) contains the parameters of \( c_j(\delta) \) which is defined by \( c_j(\delta) = \alpha_j + \beta_j \delta \), where the parameters \( \alpha_j \) are to be fixed by SNM and \( \beta_j \) are to be fixed by the PNM. Note that the expansion parameters \( c_j(\delta) \) can be constrained once we know the empirical properties of nuclear matter, i.e. the EoS.

Thus, after expanding the energy per nucleon in Eq. (1) in terms of the \( \delta \), the nuclear symmetry energy \( S(\rho) \) is straightforwardly determined via the second derivation of the energy density over the \( \delta \). The expressions of the expanding energy per nucleon and the symmetry energy are respectively given by

\[
\mathcal{E}(\rho, \delta) = \mathcal{E}(\rho, \delta = 0) + S(\rho)\delta^2 + O(\delta^4),
\]

\[
S(\rho) = \frac{\hbar^2}{6M} \left( \frac{3\pi^2}{2} \right)^{\frac{3}{2}} \rho^{\frac{3}{2}} + \sum_{j=0}^{3} \beta_j \rho^{(1+a_j)},
\]

(3)

where \( \frac{\hbar^2}{6M} \left( \frac{3\pi^2}{2} \right)^{\frac{3}{2}} \rho^{\frac{3}{2}} \) is the kinetic energy term, and we replace \( M_n \) and \( M_p \) with the average nucleon mass \( M = (M_n + M_p)/2 \). Analogously, at around the nuclear matter saturation density, the energy per nucleon in SNM \( \mathcal{E}(\rho, 0) \) and the nuclear symmetry energy can be respectively expanded by

\[
\mathcal{E}(\rho, 0) = E_0 + \frac{1}{2} K_0 \chi^2 + \frac{1}{6} Q_0 \chi^3 + O(\chi^4),
\]

\[
S(\rho) = J + L \chi + \frac{1}{2} K_{sym} \chi^2 + \frac{1}{6} Q_{sym} \chi^3 + \frac{1}{24} R_{sym} \chi^4 + O(\chi^5),
\]

(4)

where \( \chi = \left( \frac{\rho - \rho_0}{\rho_0} \right)^{3/2} \). Thus, the energy per particle at saturation density \( E_0 \), the compression modulus \( K_0 \), and the skewness coefficient \( Q_0 \) in Eq. (4) are respectively given by

\[
K_0 = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0}, \quad Q_0 = 27\rho_0^3 d^3 \left[ \mathcal{E}(\rho, 0) \right] \bigg|_{\rho = \rho_0}.
\]

(5)

Also, the nuclear symmetry energy can be identified at saturation baryon density by the value of \( J = S(\rho_0) \), the slope \( L \), the curvature \( K_{sym} \), the skewness \( Q_{sym} \), and the kurtosis \( R_{sym} \) which are given respectively by

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
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L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
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L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

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L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

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L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

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L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

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L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]

\[
L = \rho_0 dS(\rho) \bigg|_{\rho = \rho_0}, \quad K_{sym} = 9\rho_0^2 d^2 \left[ \frac{\mathcal{E}(\rho, 0)}{\rho} \right] \bigg|_{\rho = \rho_0},
\]
\[ Q_{\text{sym}} = 27\rho_0^3 \frac{d^3 S(\rho)}{d\rho^3} \bigg|_{\rho = \rho_0}, \quad R_{\text{sym}} = 81\rho_0^4 \frac{d^4 S(\rho)}{d\rho^4} \bigg|_{\rho = \rho_0}. \] (6)

We now turn to determine the KIDS-EDF parameters in terms of the Skyrme force parameters. The conventional Skyrme interaction is defined by [35]

\[
\mathcal{V}_{i,j}(\mathbf{k}, \mathbf{k'}) = t_0(1 + x_0 P_{\sigma}) \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{2} t_1(1 + x_1 P_{\sigma}) \left[ \delta(\mathbf{r}_i - \mathbf{r}_j) \mathbf{k}^2 + \mathbf{k}'^2 \delta(\mathbf{r}_i - \mathbf{r}_j) \right] \\
+ t_2(1 + x_2 P_{\sigma}) \mathbf{k}' \cdot \delta(\mathbf{r}_i - \mathbf{r}_j) \mathbf{k} + \frac{1}{6} t_3(1 + x_3 P_{\sigma}) \rho^\sigma \delta(\mathbf{r}_i - \mathbf{r}_j) \\
+ i W_0 \mathbf{k}' \times \delta(\mathbf{r}_i - \mathbf{r}_j) \mathbf{k} \cdot (\sigma_i - \sigma_j), \tag{7}
\]

where \( P_{\sigma} = (1 + \sigma_1 \cdot \sigma_2) / 2 \) and \( \sigma \) are respectively the spin-exchange operator and Pauli spin matrices. \( \mathbf{k} = (\nabla_i - \nabla_j) / 2i \) and \( \mathbf{k}' = (\nabla'_i - \nabla'_j) / 2i \) are the relative momentum. The strength of the spin-orbit coupling \( W_0 \) is absent in the EDF for the homogeneous matter in the Skyrme force. Thus the energy density functional for infinite nuclear matter can be written in terms of the Skyrme force parameters and it gives

\[
\mathcal{E}(\rho, \delta) = \mathcal{T}(\rho, \delta) + \frac{3}{8} t_0 \rho - \frac{1}{8} (2y_0 + t_0) \rho \delta^2 + \frac{1}{16} t_3 \rho^{(\alpha+1)} \\
- \frac{1}{48} (2y_3 + t_3) \rho^{(\alpha+1)} \delta^2 + \frac{1}{16} (3t_1 + 5t_2 + 4y_2) \tau \\
- \frac{1}{16} [(2y_1 + t_1) - (2y_2 + t_2)] \tau \delta^2. \tag{8}
\]

where \( y_i \equiv t_i x_i \) and \( \tau = \frac{3}{5} \left( \frac{6\pi^2}{\nu} \right)^{\frac{3}{2}} \rho^{\frac{3}{2}} \) where \( \nu \) is degeneracy number. For SNM, \( \nu = 4 \) and for PNM, \( \nu = 2 \). Matching Eq. (1) and Eq. (8) we then determine the relations between \( c_j(\delta) \) and Skyrme coefficients \( (t_i, y_i) \) and one has

\[
c_0(\delta) = \frac{3}{8} t_0 - \frac{1}{8} (2y_0 + t_0) \delta^2, \\
c_1(\delta) = \frac{1}{16} t_{31} - \frac{1}{48} (2y_{31} + t_{31}) \delta^2, \\
c_2(\delta) = \frac{1}{16} t_{32} - \frac{1}{48} (2y_{32} + t_{32}) \delta^2 + \frac{3}{5} \left( \frac{6\pi^2}{\nu} \right)^{\frac{3}{2}} \frac{1}{16} (3t_1 + 5t_2 + 4y_2) \delta^2, \\
c_3(\delta) = \frac{1}{16} t_{33} - \frac{1}{48} (2y_{33} + t_{33}) \delta^2. \tag{9}
\]

where the power of density \( \alpha = 1/3 \) is assigned to \( t_{31} \) and \( y_{31} \), \( \alpha = 2/3 \) to \( t_{32} \) and \( y_{32} \), and \( \alpha = 1 \) to
$t_{33}$ and $y_{33}$. As mentioned before, we expand the EDF as a power series of cubic-root of nuclear density. So in this case we have three extra terms rather than one for the $\rho^{\alpha}$ term. Note that in the KIDS-EDF model we have twelve parameters in total to be determined in the Skyrme force. In the present work we consider the various KIDS-EDF models: KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B. The Skyrme force parameters with their differences and similarities for the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B models are depicted in Tab. I.

In the standard Skyrme force [35], the nucleon effective masses in the asymmetric nuclear matter (ANM) are defined by

$$M_i^* = M_i \left[ 1 + \frac{M_i}{8\hbar^2} \rho \Theta_s - \frac{M_i}{8\hbar^2} \tau_3^i (2\Theta_v - \Theta_s) \rho \delta \right]^{-1},$$

(10)

where $M_i$ is the nucleon mass in free space ($i = n, p$). The Skyrme force parameters are given as $\Theta_s = 3t_1 + t_2(5 + 4x_2)$ and $\Theta_v = t_1(2 + x_1) + t_2(2 + x_2)$, and $\tau_3$ is the third component of the isospin of nucleon with $\tau_3 = +1, -1$ for the neutrons and protons, respectively. With the Skyrme force parameters of $\Theta_s$ and $\Theta_v$, the nucleon isoscalar and isovector masses can be respectively defined by

$$\mu_s^* = M \left( 1 + \frac{M}{8\hbar^2} \rho \Theta_s \right)^{-1}, \quad \mu_v^* = M \left( 1 + \frac{M}{4\hbar^2} \rho \Theta_v \right)^{-1},$$

(11)

where the values for $\mu_s^*$ and $\mu_v^*$ for various KIDS-EDF models are shown in Table I. Note that the nucleon effective masses can be also defined in terms of the nucleon isoscalar and isovector masses.

For the neutrons and protons fractions of the dense matter for all five KIDS-EDF models their fractions can be calculated in the constraint of $\beta$-stability matter that gives the chemical potential relations as

$$\mu_n - \mu_p = \mu_e, \quad \mu_e = \mu_\mu,$$

(12)

and the charge neutrality which is given by

$$\rho_p = \rho_e + \rho_\mu,$$

(13)

where $\mu_{(p,n)} = \frac{\partial E(\rho, \delta)}{\partial \rho_{(p,n)}}$ are respectively protons and neutrons chemical potentials. $\rho_p, \rho_n, \rho_e$ and $\rho_\mu$ are protons, neutrons, electrons and muons densities, respectively. The fractions of protons and
neutrons are respectively defined by $Y_p = \rho_p/\rho$ and $Y_n = \rho_n/\rho$. The chemical potentials for the leptons are defined by $\mu_{(e,\mu)} = \sqrt{k_F^{(e,\mu)} + m_{(e,\mu)}^2}$ with $m_e$ and $m_\mu$ are respectively electrons and muons masses in free space.

Results for the nuclear symmetry energy $S(\rho)$ for various KIDS-EDF models as a function of the baryon density $\rho$ are shown in Fig. 1. The nuclear symmetry energies for the KIDS0, KIDS0-m*77, and KIDS0-m*99 models have the same predictions over the range of the nuclear densities. However, in the low density up to around $\rho = \rho_0 = 0.16$ fm$^{-3}$ all five types of the KIDS-EDF

FIG. 1: Symmetry energy $S(\rho)$ as a function of $\rho$ for various KIDS-EDF models.

FIG. 2: Effective masses of the neutrons (a) and protons (b) for various KIDS-EDF models.
TABLE I: Skyrme force parameters for all five KIDS-EDF models. The units of $t_0, y_0$ are in MeV fm$^3$, the units of $t_{31}, y_{31}$ are in MeV fm$^4$, and the units of $t_1, t_2, t_{32}, y_{32}$ are in MeV fm$^5$. The unit of $y_{33}$ is in MeV fm$^6$.

| Parameters | KIDS0     | KIDS0-m*77 | KIDS0-m*99 | KIDS-A    | KIDS-B    |
|------------|-----------|------------|------------|-----------|-----------|
| $t_0$      | -1772.044 | -1772.044  | -1777.044  | -1855.377 | -1772.044 |
| $y_0$      | -127.524  | -127.524   | -127.524   | 2182.404  | 2057.283  |
| $t_1$      | 275.724   | 441.990    | 318.922    | 276.058   | 271.712   |
| $y_1$      | 0.000     | -109.026   | -361.166   | 0.000     | 0.000     |
| $t_2$      | -161.507  | -295.060   | 26.816     | -167.415  | -161.957  |
| $y_2$      | 0.000     | 259.499    | -215.113   | 0.000     | 0.000     |
| $t_{31}$   | 12216.730 | 12216.730  | 12216.730  | 14058.746 | 12216.730 |
| $y_{31}$   | -11969.990| -11969.990 | -11969.990 | -73482.960| -70716.593|
| $t_{32}$   | 571.074   | -2572.655  | -191.343   | -1022.193 | 622.750   |
| $y_{32}$   | 29485.421 | 37593.402  | 34304.568  | 122670.831| 119258.903|
| $t_{33}$   | 0.000     | 0.000      | 0.000      | 0.000     | 0.000     |
| $y_{33}$   | -22955.280| -22955.280 | -22955.280 | -73105.329| -70290.560|
| $W_0$      | 108.359   | 115.276    | 129.959    | 92.023    | 91.527    |
| $\mu_s$    | 0.991     | 0.700      | 0.900      | 1.004     | 0.997     |
| $\mu_v$    | 0.819     | 0.700      | 0.900      | 0.827     | 0.825     |

models have quite similar values of the nuclear symmetry energies, and their symmetry energy values start to diverge at around $\rho \approx \rho_0$ for the KIDS0, KIDS-A, and KIDS-B models as seen in Fig. 1.

Results for the neutrons $M_n^*/M$ and protons $M_p^*/M$ effective masses in the $\beta$-equilibrium matter for the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B models are shown in Fig. 2. Figure 2(a) shows the effective masses of neutrons for the KIDS0-m*77 and KIDS0-m*99 models are smaller than one, whereas for the KIDS0, KIDS-A, and KIDS-B models the values of $M_n^*/M$ are larger than one. Among all five KIDS-EDF models, the KIDS0 model has the largest value of the $M_n^*/M$ starting at around $\rho \approx 2.0 \rho_0$.

In Fig. 2(b) we show that the values of $M_p^*/M$ for the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A, and KIDS-B models are smaller than one. It is seen that the smallest value of $M_p^*/M$ is given by the KIDS0-m*77 model, whereas the largest value of $M_p^*/M$ is given by the KIDS0-m*99 model up to around $\rho \approx 4.0 \rho_0$. Interestingly it is shown that in the low density up to $\rho \approx 2.0 \rho_0$ the values of the $M_p^*/M$ for the KIDS0, KIDS-A, and KIDS-B models are almost the same and they start to diverge at around $\rho \approx 2.0 \rho_0$. Then the values of $M_p^*/M$ for the KIDS-A and KIDS-B
TABLE II: Effective neutron masses and nuclear symmetry energies for the five KIDS-EDF models.

| Types of the KIDS-EDF models | $M_n^\ast/M$ and features of the symmetry energy |
|------------------------------|-----------------------------------------------|
| KIDS0                        | $M_n^\ast/M \geq 1$ and soft symmetry energy   |
| KIDS0-m*77                   | $M_n^\ast/M \leq 1$ and soft symmetry energy   |
| KIDS0-m*99                   | $M_n^\ast/M \leq 1$ and soft symmetry energy   |
| KIDS-A                       | $M_n^\ast/M \geq 1$ and stiff symmetry energy  |
| KIDS-B                       | $M_n^\ast/M \geq 1$ and stiff symmetry energy  |

start being larger than that for the KIDS0 model. Summary results for the typical effective neutron masses and the nuclear symmetry energies for all five KIDS-EDF models are depicted in Table II.

III. NEUTRON STAR MASS-RADIUS RELATIONS

In this section, we present the NS gravitational mass-radius (M-R) relations before describing the NMPF in NS in Sec. IV. Here we solve the TOV equations for the static (non-rotating) NS, which are respectively given by [32, 33]

\[
\frac{dP(r)}{dr} = -\frac{G[M(r) + 4\pi r^3 P(r)/c^2] [E(r) + P(r)]}{r[r - 2GM(r)/c^2] c^2},
\]

\[
\frac{dM(r)}{dr} = 4\pi r^2 \frac{E(r)}{c^2}, \tag{14}
\]

where the radial distance from the center is symbolized by $r$ and $M(r)$ is the mass profile of neutron star within $r$. $P(r)$ and $E(r)$ are respectively pressure and energy density that are obtained from the KIDS-EDF models. Both equations in Eq. (14) are numerically solved using the Runge-Kutta integration technique by integrating them over the radial distance from the center up to the surfaces of the NS where $P(R_{NS}) = 0$, and the NS mass profile $M(R_{NS}) = M_{NS}$ where $R_{NS}$ is the NS radius and $M_{NS}$ is the NS mass.

Results for the NS mass-radius relations and particle fractions for neutrons and protons for all five KIDS-EDF models are shown in Fig. 3. Figure 3(a) shows the $M_{NS}/M_{sun}$ as a function of the NS radius $R_{NS}$ for the KIDS0, KIDS0-m*77, and KIDS0-m*99 models are quite similar predictions. Comparing with the results of the KIDS0 with $M_n^\ast/M \geq 1$ and soft symmetry energy, KIDS0-m*77 with $M_n^\ast/M \leq 1$ and soft symmetry energy, and KIDS0-m*99 with $M_n^\ast/M \leq 1$ and soft symmetry energy models, the KIDS-A and KIDS-B models with $M_n^\ast/M \geq 1$ and stiff symmetry energy predict larger radius of NS. However the M-R relations results for all five KIDS-EDF
models have in excellent agreement with the recent observations of PSR J0348+0432 [36], PSR J0740+6620 [37] and PSR J1614-2230 [38], predicting the maximum mass of NS $M_{\text{NS}} = 2.0 M_{\text{sun}}$ as well as NICER prediction results for radius $R_{\text{NS}} = 12.35 \pm 0.75$ km for a NS mass $M_{\text{NS}} = 2.08 M_{\text{sun}}$ [39] as clearly shown in Fig. 3(a), where the blue dots denote the data from the low mass X-ray binary (LMXB) data [40].

In Fig. 3(b) we show the results of the particle fractions for the neutrons and protons for all five KIDS-EDF models. The particle fractions of the protons and neutrons obtained for the KIDS0 with $M_{p}^{*}/M \geq 1$ and soft symmetry energy, KIDS0-m*77 with $M_{p}^{*}/M_{N} \leq 1$ and soft symmetry energy, and KIDS0-m*99 with $M_{p}^{*}/M \leq 1$ and soft symmetry energy models have quite similar predictions. This indicates that the particle fraction of protons and neutrons is also affected by the symmetry energy. For example, as shown in Table I, the KIDS0, KIDS0-m*77, and KIDS0-m*99 models have soft symmetry energies that lead to small particle fractions of protons and large particle fractions of neutrons. The KIDS-A and KIDS-B models with $M_{n}^{*}/M \geq 1$ and stiff symmetry energy give different predictions in comparison with those three KIDS-EDF models. However in general all five KIDS-EDF models have similar behavior on the particle fraction of protons, increasing as the nuclear density increases. On the contrary, the neutron fractions decrease with increasing the nuclear density for all five KIDS-EDF models.
In the section, we present the neutrino interaction with constituents of matter of NS. Starting with the Lagrangian density of neutrino interactions with NS constituents matter via current-current interaction and it is then defined by [9–11]

\[ \mathcal{L}^{(n,p)}_{\text{int}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e] \left[ \bar{\psi} \Gamma_{\mu}^{(n,p)} \psi \right], \quad (15) \]

where the nucleon vertex is defined by

\[ \Gamma_{\mu}^{(n,p)} = \gamma_\mu (C_V^{(n,p)} - C_A^{(n,p)} \gamma_5) \] and the weak coupling constant of \( G_F = 1.023 \times 10^{-5}/M^2 \). The values of \( C_V = -0.5 \) and \( C_A = -g_A/2 \) for the neutron, whereas \( C_V = 0.5 - 2 \sin^2 \theta_w \) and \( C_A = g_A/2 \) for the proton, where \( g_A = 1.260 \) is the axial coupling constant and \( \sin^2 \theta_w = 0.223 \). Note that the Lagrangian density for the charged-current absorption reaction is the same as the neutral-current scattering, which leads to the same expressions of the differential cross-section. Only the values of the axial and vector coupling constants are different [10].

From the Lagrangian in Eq. (15) we easily derive the differential cross-section of neutrino and one has

\[ \frac{1}{V} \frac{d^3 \sigma}{dE'd^2d\Omega} = -\frac{G_FE'_\nu}{32\pi^2E_\nu} \text{Im} \left[ L_{\mu\nu}\Pi^{\mu\nu} \right], \quad (16) \]

where \( E_\nu \) and \( E'_\nu \) are respectively the initial and final neutrino energies. The polarization tensors \( \Pi^{\mu\nu} \) for the target neutrons and protons are given by

\[ \Pi_{\mu\nu}^{(n,p)} (q^2) = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ G^{(n,p)}(p) \Gamma_{\mu}^{(n,p)} G^{(n,p)}(p + q) \Gamma_{\nu}^{(n,p)} \right], \quad (17) \]

where \( G^{(n,p)} \) are the propagators of the neutrons and protons targets and \( p = (p_0, \mathbf{p}) \) is the initial four-momentum of the neutrons and protons targets. The propagators of neutrons and protons are explicitly expressed by

\[ G^{(n,p)}(p) = \left[ \frac{p^* + M^*}{p^2 - M^*^2 + i\epsilon} + \frac{i\pi}{E^*} \frac{p^* + M^*}{E^*} \delta (p^*_0 - E^*) \Theta \left( p_F^{(n,p)} - |\mathbf{p}| \right) \right], \quad (18) \]

where \( E^* = E + \Sigma_0 = \sqrt{p^{*2} + M^{*2}} \), being the effective nucleon energy, and \( M^* = M + \Sigma_s \) represents the nucleon effective mass. \( \Sigma_s \) and \( \Sigma_0 \) are the scalar and time-like self-energies, respectively. \( \mathbf{p}^* = \mathbf{p} + \left( \frac{p^*_0}{|\mathbf{p}|} \right) \Sigma_0 \) is the nucleon effective momentum with \( |\mathbf{p}| \) and \( \Sigma_0 \) being the three-component...
momentum of nucleon and the space-like self-energy, respectively.  

\( p_F^{(n,p)} = \sqrt{E_F^{(n,p)^2} - M^{*(n,p)^2}} \)

are the proton and neutron Fermi momentum. The neutrino tensor \( L_{\mu \nu} \) is then given by

\[
L_{\mu \nu} = 8 \left[ 2k_\mu k_\nu + (k \cdot q)g_{\mu \nu} - (k_\mu q_\nu + q_\mu k_\nu) - i\epsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta \right],
\]

where \( q = (q_0, \mathbf{q}) \) is the four-momentum transfer and \( k = (k_0, \mathbf{k}) \) stands for the initial neutrino four-momentum.

Finally, we present the neutrino mean free path for the neutral-current neutrino scattering. Considering a fixed baryon density, the inverse of the neutrino mean free path (opacity) is straightforwardly obtained by integrating the differential cross-section in Eq. (16) over the three-component momentum transfer \( |\mathbf{q}| \) and energy transfer \( q_0 \) and it gives

\[
\lambda^{-1}(E_\nu) = 2\pi \int_{q_0}^{(2E_\nu-q_0)} d|\mathbf{q}| \int_0^{2E_\nu} dq_0 \left| \frac{|\mathbf{q}|}{E_\nu E'_\nu} \left[ \frac{1}{V} \frac{d^3\sigma}{dE'_\nu d^2\Omega} \right] \right|,
\]

where \( E'_\nu = E_\nu - q_0 \).

Results for a differential cross-section of the neutrino with neutrons for all five KIDS-EDF models at fixed \( E_\nu = 5 \text{ MeV} \), which is a typical kinematic for the NS cooling phase, and \( |\mathbf{q}| = 2.5 \text{ MeV} \) are shown in Fig. 4. Figure 4 (a) shows the magnitudes of the differential cross-sections of neutrino for the KIDS0, KIDS-A, and KIDS-B models with \( M^*/M \gtrsim 1 \) at \( \rho = 1.0 \rho_0 \) are bigger than those obtained for the KIDS0-m*77 and KIDS0-m*99 models with \( M^*/M \lesssim 1 \). However, the range of \( q_0 \) for the KIDS0, KIDS-A, and KIDS-B models is smaller than those obtained for the KIDS0-m*77 and KIDS0-m*99 models. It is worth noting the differential cross-section of the neutrino with neutrons concerning the \( M^*/M \). As in Fig. 2 (a), the effective masses of neutrons for the KIDS0, KIDS-A, and KIDS-B models are quite similar up to \( \rho \approx 2.0 \rho_0 \), it then leads to similar results on the differential cross-sections of neutrino for the corresponding densities.

Similar results on the differential cross-sections of neutrino for the KIDS0, KIDS-A, and KIDS-B models can be seen in Fig. 4 (b) at \( \rho = 2.0 \rho_0 \). The magnitude of the differential cross-section of neutrino for the KIDS0 model is larger than those for the KIDS-A and KIDS-B models as the effective masses of neutrons become larger in the high-density regime (\( \rho \gtrsim 2.0 \rho_0 \)). However, the magnitude of the differential cross-sections of neutrino for the KIDS-A and KIDS-B models hold the same as their effective masses of neutrons are quite similar overall density regimes, which can be seen in Fig. 2 (a). Amongst the KIDS0, KIDS-A, and KIDS-B models with \( M^*/M \gtrsim 1 \), the
different magnitudes on the differential cross-section of neutrino are much more pronounced as seen in Fig. 4(d) at $\rho = 4.0 \rho_0$, where the effective mass of neutrons of the KIDS0 model is higher than that for the KIDS-A and KIDS-B models for $\rho \approx 4.0 \rho_0$.

Figure 5 shows the differential cross-sections of neutrino with protons for all five KIDS-EDF models at fixed $E_\nu = 5$ MeV and $|q| = 2.5$ MeV as a function of $q_0$. At $\rho = 1.0 \rho_0$ the magnitude of the differential cross-section of neutrino with protons are quite similar for the KIDS0, KIDS0-m*99, KIDS-A, and KIDS-B models but rather different for the KIDS0-m*77 model with the lowest $M_p^*/M$. This behavior holds not only at $\rho = 1.0 \rho_0$ but also at $\rho = (2 - 4)\rho_0$ as shown clearly in Fig. 5(b)-(d). The ranges of $q_0$ increase for all five KIDS-EDF models as the nuclear density increases. The largest range of $q_0$ is given by the KIDS0-m*77 model with the lowest $M_p^*/M$ as shown in Fig. 5(d).
The results for the total differential cross-section of neutrino, which are obtained by summing the differential cross-sections of the neutrino with protons in Fig. 5 and neutrons in Fig. 4, as a function of $q_0$ for all five KIDS-EDF models at fixed $E_\nu = 5$ MeV and $|q| = 2.5$ MeV are depicted in Fig. 6. Figure 5 clearly shows the dominant contribution to the magnitude size of the total differential cross-sections of neutrino is given by the differential cross-section of the neutrino with neutrons for all five KIDS-EDF models. However, the shape of the total differential cross-sections of neutrino for all five KIDS-EDF models depends on both shapes of differential cross-sections of the neutrino with protons and neutrons. The total cross-section for the KIDS0, KIDS-A, and KIDS-B models increase as the density increases but for the KIDS0-m*77 and KIDS0-m*99 models, the total cross sections of neutrino are almost the same even if the density increases. Amongst the KIDS-EDF models, the highest total cross-section of neutrino is given by the KIDS0 model. Consequently, the NMFP of the KIDS0 model is smaller than other KIDS-EDF models as seen in Fig. 7.

Results for the NMFP for all five KIDS-EDF models at fixed $E_\nu = 5$ MeV and $|q| = 2.5$ MeV...
as a function of the nuclear density is given in Fig. 7 (a). Figure 7 (a) shows the NMPFs for the KIDS-A and KIDS-B are quite similar for all regimes of the baryon densities. This behavior is followed by the KIDS0 model up to $\rho \approx 3.0 \rho_0$ and then the total NMFP for the KIDS0 model decreases faster than those obtained for the KIDS-A and KIDS-B models. Remarkable results on the NMFPs are predicted by the KIDS0-m*77 and KIDS0-m*99 models. The NMFP predictions for the KIDS0-m*77 and KIDS0-m*99 models are rather higher compared with those obtained for the KIDS0, KIDS-A, and KIDS-B models. The magnitude size of the NMPF for the KIDS0-m*77 model is bigger than that for other KIDS-EDF models.

To more clearly and deeply understand the connection between the NMFP and effective masses of nucleon we show, in Fig. 7 (b), the NMFP as a function of $M_n^*/M$. It is clearly shown that the KIDS-EDF models which have smaller values of the nucleon effective masses will give higher magnitudes of the NMFP. For example it can be obviously seen that the KIDS0-m*77 model with $M_n^*/M \leq 1$ predicts $\lambda \approx (22 - 25) \text{ km}$. KIDS0-m*99 model which also has $M *_n /M \leq 1$ predicts
FIG. 7: Neutrino mean free path for various KIDS-EDF models as a function of $\rho$ (a) and neutrino mean free path for various KIDS-EDF models as a function of $M_n^*/M$ (b), that are calculated at fixed $E_\nu = 5$ MeV and $|q| = 2.5$ MeV.

FIG. 8: Neutrino mean free path for various KIDS-EDF models as a function of NS radius $R_{NS}$ and $M_{NS}/M_{sun}$ (a) and neutrino mean free path for various KIDS-EDF models as a function of $M_{NS}/M_{sun}$ (b) that are calculated at fixed $E_\nu = 5$ MeV and $|q| = 2.5$ MeV.

long NMFP $\Lambda \geq 13$ km. It should be noted that even when the $M_n^*/M$ values are similar in the KIDS0-m*77 and KIDS0-m*99 models, NMFP $\lambda$ could have very different values. The highest value of $M_n^*/M$ is given by the KIDS0 model, and it leads to smaller prediction result of the NMFP, i.e. for the $M_n^*/M \approx 5$ it predicts $\lambda \approx 5$ km. For the KIDS-A and KIDS-B models with $M_n^*/M \approx 2$ it predicts the $\lambda \geq 5$ km.

Moreover, results for the NMFP as a function of the NS radius at fixed $E_\nu = 5$ MeV and $|q| = 2.5$ MeV for all five KIDS-EDF models are shown in Fig. 8(a) and the NMFP as a function of...
the $M_{NS}/M_{\odot}$ for all five KIDS-EDF models are shown in Fig. 8 (b). Figure 8 (a) shows for the KIDS0-m*77 model with $R_{NS} = 10$ km it predicts the $\lambda \approx 20$ km and NS mass $M_{NS}/M_{\odot} \approx 2.0$, indicating that the neutrino easily escape from the NS and it leads to fast or rapid NS cooling. For the KIDS0-m*99 model with $R_{NS} = 10$ km one predicts the $\lambda \approx 12$ km with $M_{NS}/M_{\odot} \approx 2.0$, showing that the neutrino can slowly escape from the NS.

On the contrary the KIDS0, KIDS-A and KIDS-B models with $R_{NS} = 10$ km it predicts respectively the $\lambda \approx 4.7$ km with $M_{NS}/M_{\odot} \approx 2.0$ and $\lambda \approx 5.03$ km and 5.04 km with $M_{NS}/M_{\odot} \approx 2.0$. For the $R_{NS} = 12$ km, which is constrained from the NICER experiment, one predicts that $\lambda \approx 23$ km with $M_{NS}/M_{\odot} \approx 1.0$ for the KIDS0-m*77 model and $\lambda \approx 15$ km with $M_{NS}/M_{\odot} \leq 1.0$ for the KIDS0-m*99 model. Also, it indicates that even though with larger radius of NS, neutrino can still easily escape from the NS for the KIDS0-m*77 and KIDS0-m*99 models. For the KIDS0, KIDS-A and KIDS-B models it shows that the $\lambda \leq R_{NS}$. It indicates that the KIDS0, KIDS-A and KIDS-B models support the slow NS cooling and neutrino trapping in NS. The KIDS0-m*99 and KIDS0-m*77 always have $\lambda \geq R_{NS}$ as clearly shown in Fig. 8 (a). This means that both KIDS0-m*77 and KIDS0-m*99 models support the fast NS cooling and small possibility of neutrino trapping within NS.

To more clearly see the relations between NMFP and NS M-R relations, Figure 8 (b) shows the NMFP as a function of the $M_{NS}/M_{\odot}$ for all five KIDS-EDF models. For $M_{NS} = 2.0 M_{\odot}$ the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B models predict respectively $\lambda \approx 5$ km, 24 km, 13 km, 7 km, and 7 km. For $M_{NS} = 1.4 M_{\odot}$, which is well known as a NS canonical mass observed from the binary MS merger GW170817 [24, 25], the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B models predict $\lambda \approx 7$ km, 22 km, 14 km, 8 km, and 8 km, respectively.

Finally, in Fig. 9 we show the NMFP for the KIDS0 model with $M_{n}^{*}/M \gtrsim 1$ and low symmetry energy as a function of the initial neutrino energy $E_{\nu}$ for different densities. Figure 9 clearly shows that the NMFP decreases as the density and initial neutrino energy increase, which is consistent with those obtained in the functionals of the Brussels-Montreal Skyrme (BSk17 and BSk18) models [41] at $\rho = 1.0 \rho_{0}$. However, in the present work, we also predict the NMFP for higher density up to $\rho = 4.0 \rho_{0}$. Note that higher neutrino energy can not only be relevant for the NS but also for supernovae, i.e., $E_{\nu} = 30$ MeV is typical neutrino energy for the core-collapse supernova.
V. SUMMARY AND CONCLUSION

In the present work, we have investigated the nuclear symmetry energy, the proton and neutron effective masses, the NS M-R relations, proton and neutron fractions, and NMFP using various KIDS-EDF models. The KIDS-EDF model has been widely and successfully applied to describe the properties of finite nuclei. In this work, we extend the applications of the KIDS-EDF model to the neutron star matter and the neutrino interaction with the NS matter constituents using the LRA. We then analyze the implications of the nucleon effective masses and symmetry energy of the various KIDS-EDF models on the NS properties through the M-R relations and neutrinos' interaction with the NS matter constituents as well as the implications of the NS M-R relations with the NMFP and vice versa.

We find that the nuclear symmetry energy for the KIDS0, KIDS0-m*77, and KIDS0-m*99 models have quite similar predictions over the range of the nuclear densities. All five KIDS-EDF models predict quite similar results on the symmetry energy in the low density up to $\rho \approx \rho_0$ but it then diverges in the higher density $\rho \gtrsim \rho_0$, where the KIDS0 keeps having similar prediction results with the KIDS0-m*77 and KIDS0-m*99 models, but it is a different prediction with the KIDS-A and KIDS-B models. Similar prediction results on the nuclear symmetry energy for the KIDS0, KIDS0-m*77, and KIDS0-m*99 models lead to a similar result for the NS M-R relations
for the corresponding models as shown in Fig. 3 (a). It is seen that the KIDS-A and KIDS-B models have different prediction results on the M-R relations with the KIDS0, KIDS0-m*77, and KIDS0-m*99 models, because both models have different prediction results on the nuclear symmetry energies with the KIDS0, KIDS0-m*77, and KIDS0-m*99 models, as clearly shown in Table II. The prediction results on the properties of NS (M-R relations) for all five KIDS-EDF models have in excellent agreement with the recent observations of PSR J0348+0432 [36], PSR J0740+6620 [37] and PSR J1614-2230 [38], predicting the maximum mass of NS \( M_{\text{NS}} \approx 2.0 M_{\odot} \) as well as NICER prediction results for radius \( R_{\text{NS}} = 12.35 \pm 0.75 \ km \) for a NS mass \( M_{\text{NS}} = 2.08 M_{\odot} \).

We find that the dominant contribution to the magnitude size of the total differential cross-sections of neutrino is given by the differential cross-section of the neutrino with neutrons for all five KIDS-EDF models. The shape of the total differential cross-sections of neutrino for all five KIDS-EDF models depends on both shapes of differential cross-sections of the neutrino with protons and neutrons. The total cross-section for the KIDS0, KIDS-A, and KIDS-B models increase as the density increases but for KIDS0-m*77 and KIDS0-m*99 models, the total cross sections of neutrino are almost the same as the density increases. Amongst the KIDS-EDF models, the highest total cross-section of neutrino is given by the KIDS0 model. It is clearly understood, since the effective masses of neutrons for the KIDS0, KIDS-A, and KIDS-B models with \( M_{\text{n}}^* / M \approx 1 \) are quite similar up to \( \rho \approx 2.0 \rho_0 \), it then leads to similar prediction results on the differential cross-sections of neutrino.

Interestingly we also find that the differential cross-sections of neutrino for the KIDS0, KIDS-A, and KIDS-B models with \( M_{\text{n}}^* / M \geq 1 \) are bigger than those obtained for the KIDS0-m*77 and KIDS0-m*99 models with \( M_{\text{n}}^* / M \leq 1 \) at \( \rho \approx 1.0 \rho_0 \). The range of \( q_0 \) for the KIDS0, KIDS-A and KIDS-B models with \( M_{\text{n}}^* / M \geq 1 \) are smaller than those obtained for the KIDS0-m*77 and KIDS0-m*99 models with \( M_{\text{n}}^* / M \leq 1 \). At higher densities, \( \rho \approx 4.0 \rho_0 \) differences in the differential cross-sections of neutrino among the KIDS-EDF models are much more pronounced, because the effective mass differences for all five KIDS-EDF models become significant.

In the results for the NMFP for all KIDS-EDF models, we find the NMFPs for the KIDS-A and KIDS-B models are quite similar for all the baryon densities regimes. This behavior is followed by the KIDS0 model up to \( \rho \approx 3.0 \rho_0 \) and the NMFP of the KIDS0 model decreases faster than those obtained for the KIDS-A and KIDS-B models. The NMFP predictions for the KIDS0-m*77 and KIDS0-m*99 models are rather higher compared with those obtained for the KIDS0, KIDS-A, and
KIDS-B models. The magnitude size of the NMPF for the KIDS0-m*77 model is bigger than that for other KIDS-EDF models. It is clearly shown that the KIDS-EDF models which have smaller values of the nucleon effective masses will give higher magnitudes of the NMFP.

From the results for the $\lambda-M_{\text{NS}}$ relations, for $M_{\text{NS}} = 2.0\, M_{\odot}$ the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B models predict respectively $\lambda \approx 5\, \text{km}$, $24\, \text{km}$, $13\, \text{km}$, $7\, \text{km}$, and $7\, \text{km}$. For $M_{\text{NS}} = 1.4\, M_{\odot}$, which is well known as a NS canonical mass observed from the binary MS merger GW170817, the KIDS0, KIDS0-m*77, KIDS0-m*99, KIDS-A and KIDS-B models predict $\lambda \approx 7\, \text{km}$, $22\, \text{km}$, $14\, \text{km}$, $8\, \text{km}$, and $8\, \text{km}$, respectively. Summarizing the result, the KIDS0, KIDS-A and KIDS-B models show that $\lambda \lesssim R_{\text{NS}}$, indicating that they support the slow NS cooling and neutrino trapping in NS. The KIDS0-m*99 and KIDS0-m*77 always have $\lambda \gtrsim R_{\text{NS}}$ as clearly shown in Fig. 8(a), showing that they support the fast NS cooling and a small possibility of neutrino trapping within the NS.

We finally find that the NMFP decreases as the density and initial neutrino energy increase, which is consistent with those obtained in the functionals of the Brussels-Montreal Skyrme (BSk17 and BSk18) models at $\rho = \rho_0$ [41].

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