First proton-pair breaking in semi-magic nuclei beyond $^{132}$Sn and $^{208}$Pb: Configuration of the long-lived isomer of $^{217}$Pa

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The close similarity between the shell structures in the $^{132}$Sn and $^{208}$Pb regions is a well known phenomenon. Thus, using the correspondence between the high-\(j\) orbits located above the $Z = 50$ and $Z = 82$ shell gaps, we discuss the evolutions of the fully-aligned states with one broken proton pair in the $N = 82$ and $N = 126$ isotones. A long-lived isomeric state was discovered in $^{217}$Pa more than thirty years ago and despite two other experiments giving new experimental results, the discussions on its main properties (spin, parity, configuration) remained inconclusive. Then, using the comparison with the $I^+ = 17/2^+$ isomeric state recently measured in $^{139}$La, the isomeric state of $^{217}$Pa is assigned as the fully-aligned state of the $(\pi f_{7/2})^2 (\pi f_{7/2})^2$ configuration.

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Isomeric states occupy a prime position in nuclear structure study. As a general rule, their configuration is nearly pure, so their discovery enables the identification of single-particle states among a great number of other excited states of the nucleus. Then their properties can be compared to theoretical predictions. For instance, in odd-\(A\) nuclei close to magic numbers, the first long-lived isomers that had been discovered are due to the M4 character of the isomeric transitions $^{[1]} \text{[2]}$, which is explained by the difference in angular momentum of two orbits close in energy $^{[3]}$, such as $p_{1/2} - g_{9/2}$ for $N(Z) < 50$, $d_{3/2} - h_{11/2}$ for $N(Z) < 82$, and $f_{5/2} - i_{13/2}$ for $N < 126$. Another mechanism is known to produce isomeric states in spherical nuclei close to magic numbers, the breaking of a high-\(j\) nucleon pair. When the angular momenta of the two nucleons are fully aligned, the overlap of their wavefunctions is maximized and their residual interaction is large. Thus the excitation energy of the state having the maximum spin ($I_{\text{max}} = 2j - 1$) is lowered and its $E2$ decay towards the next state of the multiplet is slowed down. Textbook examples are the $8^+$ isomeric state ($T_{1/2} = 201$ ns) in $^{210}$Pb and the $8^+$ isomeric state ($T_{1/2} = 99$ ns) in $^{210}$Po, where the two nucleons outside the doubly-magic core are located in the $\pi g_{9/2}$ and $\pi h_{9/2}$ orbits, respectively. Such isomeric states due to breaking of a high-\(j\) nucleon pair are also found in semi-magic nuclei having much more than two valence nucleons, for instance in the $N = 82$ isotones with $54 < Z < 60$ and in the $N = 126$ isotones with $85 < Z < 92$ $^{[4]}$.

In the nuclear chart, the comparison of the regions lying above the two doubly-magic nuclei, $^{132}$Sn and $^{208}$Pb, is very interesting since each single-particle state in the $^{132}$Sn region can be related to one particular state in the $^{208}$Pb region which only differs by one unit of orbital angular momentum $^{[5]}$. These corresponding states have approximately the same ordering and spacing, so one may expect similarities in nuclei belonging to these two regions. In this paper, we discuss the properties of the fully-aligned states with one broken proton pair involving the high-\(j\) orbits located above the $Z = 50$ gap in the $N = 82$ isotones with $52 \leq Z \leq 58$. The isomeric state recently identified in $^{139}$La is particularly emphasized. Then the comparison with the fully-aligned states measured in the $N = 126$ isotones having $84 \leq Z \leq 92$ allows us to assign the configuration of the long-lived isomeric state of $^{217}$Pa, for which no firm conclusion was provided in the previous papers.

The high-spin structures of five $N = 82$ isotones ($54 \leq Z \leq 58$) have been recently studied $^{[6]}$. Positive-parity states dominate the low-energy part of the level schemes, since the two proton orbits located just above the $Z = 50$ gap are $\pi g_{9/2}$ and $\pi d_{5/2}$. In $^{133}$Sb, the latter is located 962 keV above the former. Even though the pairing correlations dilute the occupancy of the two subshells when $Z$ is increasing, the crossing of the two orbits occurs at the ‘right’ place, the ground state of $^{139}$La having $I^+ = 7/2^+$ and the one of $^{141}$Pm having $I^+ = 5/2^+$.

All the even-\(Z\) isotones having $52 \leq Z \leq 58$ exhibit a low-lying isomeric state with $I^+ = 6^+$. It is due to the first breaking of a proton pair which has either the $(\pi g_{7/2})^2$ configuration or the $(\pi g_{7/2})^1 (\pi d_{5/2})^1$ one, since both configurations lead to $I_{\text{max}} = 6^+$ and display a small gap in energy between the $6^+$ and the $4^+_1$ states. Indeed two $6^+$ states have been identified in these isotones $^{[7]}$, their configurations depend on the location of the Fermi level within the two orbits. For $Z = 52$ and $54$, the $6^+_1$ state has most likely the first configuration and the $6^+_2$ state the second one, given that the $\pi g_{7/2}$ orbit is filling first. On the other hand, the results of $(^3\text{He}, d)$ reactions $^{[4]}$ clearly indicate that (i) the $6^+_1$ state of $^{144}$Ce comes from the $(\pi g_{7/2})^1 (\pi d_{5/2})^1$ configuration and its $6^+_2$ state from the $(\pi g_{7/2})^2$ one, and (ii) the two $6^+$ states of $^{136}$Ba are strongly mixed. It is worth noting that, while the $6^+_1$ state of these four isotones is isomeric, whatever its configuration, the decay of their $6^+_2$ state is never isomeric, as the $M1$ transition to the $6^+_1$ state is never delayed.

As to the odd-\(Z\) isotones, the result of the first breaking of a proton pair varies with the occupation of the two orbits. If the odd proton is located in the $\pi g_{7/2}$ orbit, this leads to the $(\pi g_{7/2})^{\pm 3}$ configuration or to
The $(\pi g_{7/2})^{\pm 1}(\pi d_{5/2})^2$ one, depending on the total number of protons. It is important to note that both configurations give $I_{\text{max}}^{\pi} = 15/2^+$. When the odd proton is promoted to the $\pi d_{5/2}$ orbit, the configuration is $(\pi g_{7/2})^2(\pi d_{5/2})^1$, with $I_{\text{max}}^{\pi} = 17/2^+$. As expected, the $(\pi g_{7/2})^{-2,3}$ configuration gives rise to the first yrast states of $^{135}_{53}$Cs, with $I^\pi = 11/2^+$ and $15/2^+$. The $(\pi g_{7/2})^2(\pi d_{5/2})^1$ configuration lies at higher excitation energy and only the state with $I_{\text{max}}^{\pi} = 17/2^+$ belongs to the yrast sequence \[6\rightarrow 7\]. Unlike the even-Z isotones, neither $^{135}_{53}$I nor $^{137}_{53}$Cs exhibit a low-lying isomeric state, as the distance in energy between the $15/2^+$ and $11/2^+$ states of the $(\pi g_{7/2})^3$ configuration is well higher than the one between the $6^+$ and $4^+$ states of the $(\pi g_{7/2})^2$ configuration (compare Fig. 12 and Fig. 13(a) of Ref. \[6\]).

$^{139}_{57}$La behaves differently from $^{135}_{53}$I and $^{137}_{53}$Cs \[6\]: (i) two coexisting structures have been identified, one built on the $7/2^+$ ground state and another one built on the first-excited state at 166 keV having $I^\pi = 5/2^+$; (ii) the $17/2^+$ state is isomeric, with $T_{1/2} = 315$ ns. The first structure contains four states with spins $7/2^+$, $11/2^+$, $13/2^+$ and $15/2^+$. The second structure also contains four states, with spins $5/2^+$, $9/2^+$, $13/2^+$ and $17/2^+$. Results of the shell model calculations done in Ref. \[6\] indicate that the states of the two groups have not the same configuration. The main configuration of all the states of the first structure is $(\pi g_{7/2})^5(\pi d_{5/2})^2$, i.e., the odd proton occupies the $\pi g_{7/2}$ orbit. On the other hand, the main configuration of all the states of the second structure is $(\pi g_{7/2})^6(\pi d_{5/2})^1$, i.e., the odd proton occupies the $\pi d_{5/2}$ orbit. The relative energies of the states of these two structures explain why the $17/2^+_1$ state is isomeric in $^{139}_{57}$La. Unlike $^{135}_{53}$I and $^{137}_{53}$Cs, it cannot be linked to the $15/2^+$ state which is located above it, therefore it decays towards the $13/2^+_1$ state by a low-energy $E2$ transition.

Fig. (11) illustrates the close similarity between the shell structures in the $^{132}$Sn and $^{208}$Pb regions. As mentioned above, the orbital angular momenta differ by one unit, the orientation of the intrinsic spin remaining unchanged, namely, $\pi g_{7/2} \rightarrow \pi h_{9/2}$, $\pi d_{5/2} \rightarrow \pi f_{7/2}$, and $\pi h_{11/2} \rightarrow \pi i_{13/2}$. In Fig. (11a), we have gathered the results discussed above, i.e., the evolution of the excitation energy of the fully-aligned states with one broken proton pair in the $N = 82$ isotones with $52 \leq Z \leq 58$. For the sake of completeness, the states involving the third high-j orbit located above the $Z = 50$ gap, $\pi h_{11/2}$, are also shown (the experimental data are from Refs. \[4,7\]).

Fig. (11b) shows the analogous excited states measured in the $N = 126$ isotones with $84 \leq Z \leq 92$ (the low-energy parts of their level schemes are shown in Fig. (2)). The two $8^+$ states of the $N = 126$ isotones display the same evolution as the two $6^+$ states of the $N = 82$ isotones. Similarly the behaviors of the $21/2^-$ and $23/2^-$ states of the $N = 126$ isotones are the same as the ones of the $15/2^+$ and $17/2^+$ states of the $N = 82$ isotones. Thus
the long-lived isomeric state of $^{217}$Pa, which was located at $\sim 1854(7)$ keV by means of its $\alpha$ decay, is very likely the counterpart of the isomeric state of $^{139}$La.

Before going further, it is important to recall the experimental status of this isomeric state. The alpha decay of an isomeric state of $^{217}$Pa was observed many years ago from the $^{40}$Ar+$^{181}$Ta reaction at 176 MeV bombarding energy [11]. Compared with the information known at that time in the lighter $N = 126$ isotones, this isomer was associated with a fully-aligned three-proton state, either with $I^\pi = 29/2^+$ from the ($\pi h_9/2$)$^2(\pi i_{13/2})^1$ configuration or with $I^\pi = 23/2^-$ from the ($\pi h_9/2$)$^2(\pi f_{7/2})^1$ one. Its very long half-life ($T_{1/2} = 1.6$ ms) was proposed to be due to a large hindrance from the angular momentum of the emitted alpha particle. Many years later, this isomeric state was produced in another reaction, $^{28}$Si+$^{194}$Pt at a beam energy of 163 MeV [12], and the large hindrance of the transition was entirely attributed to the centrifugal barrier, a value of $\ell \sim 9h$ being estimated from standard formula. Given that the ground state of the daughter nucleus, $^{213}$Ac, has $I^\pi = 9/2^-$, the isomeric state of $^{217}$Pa was thus assigned as the fully-aligned state of the ($\pi h_9/2$)$^2(\pi i_{13/2})^1$ configuration, with $I^\pi = 29/2^+$. Later on, a more detailed study of the decay scheme of $^{217}$Pa was performed by means of $\alpha - \gamma$ spectroscopy [9]. Fine structures in the $\alpha$ decay of the isomeric state as well as of the ground state of $^{217}$Pa were observed, leading to an unambiguous identification of several excited states of $^{213}$Ac. Moreover, the measured decay curve of the main $\alpha$ transition decaying the ground state of $^{217}$Pa was found to be composite, indicating that the isomeric state ($T_{1/2} = 1.2(2)$ ms) partly decays to the ground state, $\%IT=27(4)$, giving the partial half-life, $T_{1/2} \sim 4.4$ ms. This last result was used to provide support to the $I^\pi = 29/2^+$ choice [9]: The isomeric transition was attributed to a $29/2^+ \rightarrow 23/2^-$ decay, i.e., an $E3$ transition with an energy low enough to be in agreement with a long half-life.

Nevertheless, such an interpretation is clearly at variance with the behavior of the neighboring isotones, where the excitation energies of the fully-aligned states involving the $\pi i_{13/2}$ subshell decrease slowly as a function of $Z$ [see the evolution of the 11$^-$ and 29/2$^+$ states drawn in green in Fig. [1b] and Fig. [2]. A drop of about 650 keV of the 29/2$^+$ state from $^{215}$Ac to $^{217}$Pa is very unlikely, especially since the evolution of the 11$^-$ state of the even-$Z$ remains smooth up to $Z = 90$. On the other hand, the excitation energy of the long-lived isomeric state of $^{217}$Pa

FIG. 2. (Color online) Systematics of the first excited state $s$ of the $N = 126$ isotones with $84 \leq Z \leq 92$, the colored states are the fully-aligned states with one broken proton pair shown in Fig. [1b]. The $g$ factors of most of the 8$^+$, 11$^-$, 21/2$^-$, and 29/2$^+$ states have been measured, confirming their proton configurations (see for instance, the figures 4(a) and 4(b) of Ref. [8]). The $\alpha$-decaying states of $^{216}$Th, $^{217}$Pa and $^{218}$U are drawn with a thicker line. The experimental data come from Refs. [8–10] for $Z = 89, 91$ and 92, respectively and from the 'Evaluated Nuclear Structure Data File' [3] for the others.
fits very well the evolution of the fully-aligned $23/2^-$ state, which is expected at lower energy than the $21/2^-$ state from the $(\pi h_9/2)^3$ configuration at $Z = 91$, given their opposite evolution as a function of $Z$ [see Fig. 1(b)].

Thus the isomeric states of $^{139}$La and $^{217}$Pa share several similar properties, (i) their excitation energies are close to each other, (ii) their configurations involve the two first orbitals located just above the proton shell gaps, differing by one unit of orbital angular momentum, namely $(\pi g_{7/2})^2(\pi d_{5/2})^1$ for the former and $(\pi h_9/2)^2(\pi f_{7/2})^1$ for the latter.

On the other hand, the half-life of their gamma decays are very different. The half-life of the isomeric state of $^{139}$La is 315 ns, which is due to the emission of an $E2$ transition of 89 keV with $B(E2) = 1.9$ W.u. [9], value in line with the fact that both the emitting and populated states have the same configuration, $(\pi g_{7/2})^0(\pi d_{5/2})^1$. The partial half-life of the isomeric state of $^{217}$Pa is $T_{1/2} \approx 4.4$ ms. Such a long half-life cannot be due to a low-energy $E2$ transition between two states having the same configuration, implying that the $19/2^-$ state belonging to the $(\pi h_9/2)^2(\pi f_{7/2})^1$ multiplet has to be located above the $23/2^-$ state. This is in agreement with results of large-scale shell model calculations which were performed for the $N = 126$ isotones, from $Z = 84$ to $Z = 94$ [13].

As said above, an $E3$ transition with a low energy can lead to a partial half-life in the ms range. For instance, for $E_\gamma = 20$ keV and $T_{1/2} \approx 4.4$ ms, the value of the reduced transition probability is $B(E3) \sim 30$ W.u.. Then the $E3$ isomeric transition of $^{217}$Pa would link the $23/2^-$ fully-aligned state of the $(\pi h_9/2)^2(\pi f_{7/2})^1$ configuration to a $17/2^+$ state. Given that in $^{215}$Pa, the $\pi i_{13/2}$ subshell is expected at an energy about 600 keV by extrapolating the values known in the lighter isotones [4], the $17/2^+$ state belonging to the $(\pi i_{13/2})^1(\pi h_9/2)^2$ multiplet could be located below $\sim 1850$ keV, the energy of the long-lived isomeric state. Moreover it is worth recalling that the same configurations are involved in the $E3$ transitions measured in the lighter isotones $^{211}$At, $^{213}$Fr and $^{215}$Ac [4]) and their $29/2^+ \rightarrow 23/2^-$ transitions have $B(E3) \sim 25$ W.u., close to the value mentioned above for $^{217}$Pa.

As to the large hindrance of the $\alpha$ decay, $^{217}$Pa$^m \rightarrow ^{213}$Ac$^{\alpha}$, it is likely due to (i) the centrifugal barrier (the angular momentum of the emitted $\alpha$ has to be at least 8 h for a $23/2^- \rightarrow 9/2^-$ transition) and (ii) the change in proton configuration (the proton pair of the $\alpha$ particle has to be formed from two protons lying in two different orbits and having their angular momentum almost aligned, $[(\pi h_9/2)^2(\pi f_{7/2})^1]_{23/2} \rightarrow [(\pi h_9/2)^1]_{9/2^-}$).

In summary, using the correspondence between the high-$j$ orbits located above the $Z = 50$ and $Z = 82$ shell gaps, as well as the $I^\pi = 17/2^+$ isomeric state recently measured in $^{139}$La, we propose that the long-lived isomeric state of $^{217}$Pa, which was discovered more than thirty years ago, is the fully-aligned state with $I^\pi = 23/2^-$ of the $(\pi h_9/2)^2(\pi f_{7/2})^1$ configuration, its gamma partial half-life being in agreement with an $E3$ transition having an energy of about 20 keV. It is worth recalling that the $g$ factors of all the high-spin isomeric states of the $N = 126$ isotones [drawn with filled symbols in Fig. 1(b)] have been measured for $Z \leq 89$, confirming the identification of their two-proton configurations. Unfortunately the measurement of the $g$ factor of $^{217}$Pa$^m$ seems out of reach by now, both its very low production rate in reactions induced by heavy ions and its long lifetime prevent use of the standard methods available in the present experimental facilities.

Finally, one may wonder whether such a mechanism giving rise to long-lived isomeric states may exist elsewhere in the nuclear chart. Starting with the $^{78}$Ni region, we find the $\pi f_{5/2}$ and $\pi p_{3/2}$ orbits lying above the $Z = 28$ shell gap. Thus the three-proton isomeric state would be the fully-aligned state of the $(\pi f_{5/2})^2(\pi p_{3/2})^1$ configuration with $I^\pi_{\text{max}} = 11/2^-$, provided that it is located below the $9/2^-$ state of the $(\pi f_{5/2})^3$ configuration. We know that it is not the case, since the yrast states of $^{83}$As have been recently identified and the $11/2^-$ state, higher in energy, decays to the $9/2^-$ state by means of a $M1$ transition of 323 keV [14].

On the other hand, long-lived isomeric states from three-neutron configurations could be found in heavy-A Pb isotopes. Indeed high-spin states were observed in $^{211}$Pb having three valence neutrons in the $\nu g_{9/2}$ and $\nu i_{11/2}$ orbits which are lying just above the $N = 126$ shell closure [15]. The fully-aligned states of the $(\nu g_{9/2})^3$ and $(\nu g_{9/2})^2(\nu i_{11/2})^1$ configurations (with $I^\pi = 21/2^+$ and $27/2^+$, respectively) are isomeric, because of the low energy of their decaying $E2$ transition. As stated in the conclusion of Ref. [15], predictions can be made for the expected behavior of more neutron-rich Pb isotopes: "The $27/2^+$ level will continue to be isomeric, either an $E2$-decaying isomer as in $^{211}$Pb or, should it move below the $23/2^+$ state, very long lived." Thus, even though the order of the two valence orbits is reversed as compared to the $N = 82$ and $N = 126$ isotones discussed above (here, the low-$j$ orbit lies above the high-$j$ one), the heavy-A Pb isotopes could display an yrast trap with $I^\pi = 27/2^+$, from the $(\nu g_{9/2})^2(\nu i_{11/2})^1$ configuration.

When moving to the super-heavy part of the chart, another interesting case may be also found. Different spherical doubly-magic super-heavy nuclei are predicted, depending on the parametrization of the mean field [16]. Nevertheless the existence of a spherical gap at $N = 184$ seems to be almost 'universal'. The two orbits lying just above this gap are $\nu i_{13/2}$ and $\nu h_{11/2}$, i.e., with $\Delta\ell = 2$ and $\Delta j = 1$, as all the couples of orbits discussed above. Thus an isomeric state with $I^\pi = 35/2^-$ from the $(\nu i_{13/2})^2(\nu h_{11/2})^1$ configuration would be expected in the $Z = 114(120)$ isotopes having $N > 187$, provided that such nuclei remain spherical when adding neutrons in the two orbits. A very large hindrance of the $\alpha$ decay of such a $35/2^-$ state is expected because of the centrifugal barrier ($\ell > 11h$ when considering its decay
to the ground state of the daughter nucleus). Therefore the half-life of the isomeric state would be well longer than the one of the ground state (the same situation as the 18\(^+\) isomeric state of \(^{212}\)Po, with \(T_{1/2}^{\text{is}} = 45\) s, as compared to \(T_{1/2}^{\text{gs}} = 0.299\) \(\mu\)s [4]).

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