Neutrinoless $\beta\beta$ decay transition matrix elements within mechanisms involving light Majorana neutrinos, classical Majorons and sterile neutrinos

P. K. Rath\textsuperscript{1}, R. Chandra\textsuperscript{2}, K. Chaturvedi\textsuperscript{3}, P. Lohani\textsuperscript{1}, P. K. Raina\textsuperscript{4}, and J. G. Hirsch\textsuperscript{5}

\textsuperscript{1}Department of Physics, University of Lucknow, Lucknow-226007, India
\textsuperscript{2}Department of Applied Physics, Babasaheb Bhimrao Ambedkar University, Lucknow-226025, India
\textsuperscript{3}Department of Physics, Bandelkhanda University, Jhansi-284128, India
\textsuperscript{4}Department of Physics, Indian Institute of Technology, Ropar, Rupnagar - 140001, India and
\textsuperscript{5}Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, 04510 México, D.F., México

(Dated: May 11, 2014)

In the PHFB model, uncertainties in the nuclear transition matrix elements for the neutrinoless double-$\beta$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes within mechanisms involving light Majorana neutrinos, classical Majorons and sterile neutrinos are statistically estimated by considering sets of sixteen (twenty-four) matrix elements calculated with four different parametrization of the pairing plus multipolar type of effective two-body interaction, two sets of form factors and two (three) different parameterizations of Jastrow type of short range correlations. In the mechanisms involving the light Majorana neutrinos and classical Majorons, the maximum uncertainty is about 15% and in the scenario of sterile neutrinos, it varies in between approximately 4 (9)%–20 (36)% without (with) Jastrow short range correlations with Miller-Spencer parametrization, depending on the considered mass of the sterile neutrinos.

PACS numbers: 21.60.Jz, 23.20.-g, 23.40.Hc

I. INTRODUCTION

In the last decade, the confirmation of neutrino flavor oscillations at atmospheric, solar, reactor and accelerator neutrino sources [1, 2] and the reported observation of neutrinoless double beta ($\beta\beta$) decay [3-8] have together played a great inspirational role in the advancement of a vast amount of experimental as well as theoretical studies on nuclear double-$\beta$ decay in general and ($\beta\beta$) decay in particular [9, 10]. The former has provided information on the neutrino mass square differences ($\Delta m^2_{31}$ and $\Delta m^2_{41}$), mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ and possible hierarchies in the neutrino mass spectrum [11]. In addition to hinting on the Majorana nature of neutrinos, the latter has also ascertainment the role of various mechanism in different gauge theoretical models [12]. Presently, a number of projects for observing the ($\beta\beta$) decay of $^{48}$Ca (CANDLES), $^{76}$Ge (GERDA, MAJORANA), $^{82}$Se (SuperNEMO, Lucifer), $^{100}$Mo (MOON, Amore), $^{116}$Cd (COBRA), $^{130}$Te (CUORE), $^{136}$Xe (XMASS, EXO, KAMLand-Zen, NEXT), $^{150}$Nd (SNO++, SuperNEMO, DCBA/MTD) [13-17] have been designed and hopefully, the reported observation of ($\beta\beta$) decay [3-8] would be confirmed in the near future.

In the left-right symmetric model [16, 17], the three possible mechanisms of ($\beta^-\beta^-$)$_{0\nu}$ decay are the exchange of left handed light as well as heavy Majorana neutrinos and the exchange of right handed heavy Majorana neutrinos. Alternatively, the occurrence of lepton number violating Majoron accompanied ($\beta\beta$)$_{0\nu}$ decay is also a possibility. Based on the most recent experimental evidences [18-21], regarding the observability of all the nine Majoron models [22-24], it has been concluded that the study of classical Majoron models is the most preferred one.

In the short base line experiments [24, 25], the indication of $\tau_{\mu} \rightarrow \tau_e$ conversion was explained with 0.2 eV < $\Delta m^2$ < 2 eV and 10^{-3} < sin$^2$2$\theta$ < 4.10^{-2}. New results of the reactor fluxes favor short base line oscillation [26, 27]. The confirmation of all these observations would imply the existence of more than three massive neutrinos [28]. In Ref. [22], it was shown that the mixing of a light sterile neutrino (mass ≪ 1 eV) with a much heavier sterile neutrino (mass ≫ 1 GeV) would result in observable signals in current $\beta\beta$ decay experiments, as is the case in other interesting alternative scenarios [31, 32].

The study of ($\beta^-\beta^-$)$_{0\nu}$ decay within mechanisms involving light Majorana neutrinos, classical Majorons and sterile neutrinos can be performed under a common theoretical formalism [32-34]. In the mass mechanism, the contributions of the pseudoscalar and weak magnetism terms of the recoil current can change the NTMEs $M_{0\nu}$ up to 30% in the QRPA [35, 36], about 20% in the interacting shell model (ISM) [37] and 15% in the interacting Boson model (IBM) [38].

In the evaluation of NTMEs, the most desirable approach is to employ the successful large scale shell-model calculations [37, 39, 41], if feasible. However, the QRPA [42, 43] and its extensions [44, 45] have emerged as the most employed models for explaining the observed suppression of $M_{2\nu}$ in addition to correlating the single-$\beta$ GT strengths and half-lives of ($\beta^-\beta^-$)$_{2\nu}$ decay by including a large number of basis states in the model space. The necessity for the inclusion of nuclear deformation has resulted in the employment of deformed QRPA [46-48], projected-Hartree-Fock-Bogoliubov (PHFB) [13, 52],
pseudo-SU(3) \cite{53}, IBM \cite{38, 54, 56} and energy density functional (EDF) \cite{47} approaches in the calculation of NTMEs. Additionally, there are many possibilities for the inclusion of the model dependent form factors for the finite size of nucleons (FNS), short range correlations (SRC) \cite{53, 58, 62}, and the value of axial vector current coupling constant \( g_A \) \cite{63}. Each model has a different truncation scheme for the unmanageable Hilbert space, and employs a variety of residual interactions, resulting in NTMEs \( M^{(0\nu)} \), which are of the same order of magnitude but not identical.

In the analysis of uncertainties in NTMEs for \((\beta^{-}\beta^{-})_{0\nu} \) decay, the spread between the available calculated results \cite{64} was translated in to an average of all the available NTMEs, and the standard deviation was treated as the measure of the theoretical uncertainty \cite{65, 66}. Bilenky and Grifols \cite{67} have suggested that the possible observation of \((\beta^+\beta^-)_{0\nu} \) decay in several nuclei could be employed to check the calculated NTMEs in a model independent way by comparing the ratios of the NTMEs-squared with the ratios of observed half-lives \( T_{1/2}^{0\nu} \). Model specific theoretical uncertainties have been analyzed in the QRPA approach \cite{68, 70}. Further, studies on uncertainties in NTMEs due to the SRC have also been preformed in Refs. \cite{60, 72, 71}.

The main objective of the present work is to study the effects of pseudoscalar and weak magnetism terms on the Fermi, Gamow-Teller (GT) and tensorial NTMEs for the \((\beta^{-}\beta^{-})_{0\nu} \) decay of \( ^{94}_{\alpha}Zr, ^{98}_{\alpha}Mo, ^{104}_{\alpha}Ru, ^{110}_{\alpha}Pd, ^{128}_{\alpha}Te \) and \( ^{150}_{\alpha}Nd \) isotypes in the light Majorana neutrino mass mechanism. In addition, we investigate the effects due to deformation, FNS and the SRC vis-a-vis the radial evolution of NTMEs. Uncertainties in NTMEs are calculated statistically by employing four different parametrizations of effective two-body interaction, form factors with two different parametrizations and three different parametrizations of the SRC. In the same theoretical formalism, the \((\beta^{-}\beta^{-})_{0\nu} \) decay involving classical Majorons and sterile neutrinos is also studied. The theoretical formalisms to calculate the half-lives of the \((\beta^{-}\beta^{-})_{0\nu} \) decay with induced currents \cite{35, 36}, classical Majorons \cite{33, 35} and sterile neutrinos \cite{31} have already been reported. Hence, we briefly outline the steps of the above derivations in Sec. II. In Sec. III, we present the results and discuss them vis-a-vis the existing calculations done in other nuclear models. Finally, the conclusions are given in Sec. IV.

## II. THEORETICAL FRAMEWORK

The detailed theoretical formalism required for the study of \((\beta^-\beta^-)_{0\nu} \) decay due to the exchange of light Majorana neutrinos has been given by Simkovic et al. \cite{32} as well as Vergados \cite{36}. The observability of Majoron accompanied \((\beta^-\beta^-)_{0\nu} \) decay in nine Majoron models \cite{22} has already been discussed by Hirsch et al. \cite{23}. Further, the \((\beta^-\beta^-)_{0\nu} \) decay within the mechanism involving sterile neutrinos has been given by Benes et al. \cite{31}. In the following, we present a brief outline of the required theoretical formalism for the clarity in notations used in the present paper.

### A. Light Majoron neutrino mass mechanism

In the Majorana neutrino mass mechanism, the half-life \( T_{1/2}^{0\nu} \) for the \( 0^+ \rightarrow 0^+ \) transition of \((\beta^-\beta^-)_{0\nu} \) decay due to the exchange of light Majorana neutrinos between nucleons having finite size is given by \cite{32, 36}

\[
\left[ T_{1/2}^{0\nu} (0^+ \rightarrow 0^+) \right]^{-1} = G_{01} \left| \frac{\langle m_\nu \rangle}{m_e} M^{(0\nu)} \right|^2 ,
\]

(1)

where

\[
\langle m_\nu \rangle = \sum_i U_{e_i}^2 m_i, \quad m_i < 10 \text{ eV},
\]

(2)

and in the closure approximation, the NTME \( M^{(0\nu)} \) is defined as

\[
M^{(0\nu)} = \sum_{n,m} \left\langle 0^+_F \left| -\frac{H_F(r_{nm})}{g_A} + \sigma_n \cdot \sigma_m H_{GT}(r_{nm}) + S_{nm} H_T(r_{nm}) \right| 0^+_F \right\rangle \tau_n^+ \tau_m^+ ,
\]

(4)

with

\[
S_{nm} = 3 (\sigma_n \cdot \mathbf{r}_{nm}) (\sigma_m \cdot \mathbf{r}_{nm}) - \sigma_n \cdot \sigma_m.
\]

(5)

The neutrino potentials associated with Fermi, Gamow-Teller (GT) and tensor operators are given by

\[
H_\alpha(r_{nm}) = \frac{2 R}{\pi} \int \frac{f_\alpha(q r_{nm})}{(q + A)} h_\alpha(q) dq ,
\]

(6)

where \( f_\alpha(q r_{nm}) = j_0(q r_{nm}) \) and \( f_\alpha(q r_{nm}) = j_2(q r_{nm}) \) for \( \alpha = \text{Fermi/GT} \) and tensor potentials, respectively.
The effects due to the FNS are incorporated through the dipole form factors and the form factor related functions $h_F(q)$, $h_{GT}(q)$ and $h_T(q)$ are written as

\begin{align}
    h_F(q) &= g_V^2 \left( q^2 \right), \\
    h_{GT}(q) &= \frac{g_A^2(q^2)}{g_A^2} \left[ 1 - \frac{2}{3} \frac{g_P(q^2)q^2}{g_A^2} + \frac{1}{3} \frac{g_M(q^2)q^4}{g_A^2(2q^2)} \right] + \frac{2}{3} \frac{g_M^2(q^2)q^2}{g_A^24M_F^2}, \\
    h_T(q) &= \frac{g_A^2(q^2)}{g_A^2} \left[ 1 - \frac{2}{3} \frac{g_P(q^2)q^2}{g_A^2} + \frac{1}{3} \frac{g_M(q^2)q^4}{g_A^2(2q^2)} \right] + \frac{1}{3} \frac{g_M^2(q^2)q^2}{g_A^24M_F^2},
\end{align}

where

\begin{align}
    g_V(q^2) &= g_V \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^2, \\
    g_A(q^2) &= g_A \left( \frac{\Lambda_A^2}{q^2 + \Lambda_A^2} \right)^2, \\
    g_P(q^2) &= \frac{2M_Pg_A(q^2)}{(q^2 + m_n^2)} \left( \frac{\Lambda_A^2 - m_n^2}{\Lambda_A^2} \right), \\
    g_M(q^2) &= \kappa g_V \left( q^2 \right)
\end{align}

with $g_V = 1.0$, $g_A = 1.254$, $\kappa = \mu_p - \mu_n = 3.70$, $\Lambda_V = 0.850$ GeV and $\Lambda_A = 1.086$ GeV. The presence of pseudoscalar and weak magnetism terms of the higher order currents (HOC) \[35\], results as seen by the consideration of Eqs. \[4\]–Eq. \[13\], in no change of the Fermi matrix element $M^{(0v)}_{F} = -g_A^2M^{(0v)}_{F-V^2}$, the addition of three new terms $M^{(0v)}_{GT-AP}$, $M^{(0v)}_{GT-PP}$, $M^{(0v)}_{GT-MM}$ to the conventional GT $M^{(0v)}_{GT} = M^{(0v)}_{GT-AP}$ matrix element and the addition of three new terms $M^{(0v)}_{T-AP}$, $M^{(0v)}_{T-PP}$, $M^{(0v)}_{T-MM}$ as the tensor matrix element $M^{(0v)}_{T}$. Consideration of internal structure of protons and neutrons suggests an alternative parametrization of $g_V(q^2)$ given by \[72\]

\begin{align}
    g_V(q^2) &= F_1^v(q^2) - F_1^n(q^2)
\end{align}

where

\begin{align}
    F_1^v(q^2) &= \frac{1}{1 + \frac{q^2}{4M^2}} \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^2 \left[ 1 + \left( 1 + \mu_p \right) \frac{q^2}{4M_F^2} \right], \\
    F_1^n(q^2) &= \frac{\mu_n}{1 + \frac{q^2}{4M^2}} \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^2 \left( 1 - \xi_n \right) \frac{q^2}{4M^2}
\end{align}

with $\mu_p = 1.79$ nm, $\mu_n = -1.91$ nm and $\Lambda_V = 0.84$ GeV.

![FIG. 1: Distribution of $g_V(q^2)$ and $g_M(q^2)$ for FNS1 and FNS2.](image-url)

In addition,

\[ F_2^v(q^2) = \frac{\mu_p}{1 + \frac{q^2}{4M^2}} \left( \frac{\Lambda_V^2}{q^2 + \Lambda_V^2} \right)^2 \left( 1 + \frac{q^2}{4M^2} \xi_n \right) \]

with

\[ \xi_n = \frac{1}{1 + \lambda_n \frac{q^2}{4M^2}} \]

and $\lambda_n = 5.6$. We get two sets of form factors by considering Eq. \[10\]–Eq. \[13\] and Eqs. \[11\]–\[17\].
are denoted by FNS1 and FNS2, respectively. In Fig. 1, we present the plot of \( g_V(q^2) \) and \( g_M(q^2) \) of FNS1 and FNS2, the shapes of which have definite relation with the magnitudes of NTMEs \( M^{(0\nu)} \).

### B. Majoron accompanied \((\beta^-\beta^-)_{0\nu}\) decay

In the classical Majoron model, the inverse half-life \( T_{1/2}^{(0\nu)} \) for the \( 0^+ \rightarrow 0^+ \) transition of Majoron emitting \( \nu \) has been derived by considering the exchange of a Majorana neutrino between two nucleons and is given by \([33, 35]\)

\[
[T_{1/2}^{(0\nu)} (0^+ \rightarrow 0^+)]^{-1} = |(g_M)|^2 G_{0\nu} M^{(0\nu)} ,
\]

where \( (g_M) \) is the effective Majoron–neutrino coupling constant and the NTME \( M^{(0\nu)} \) is same as the \( M^{(0\nu)} \) for the exchange of light Majorana neutrinos. The phase space factors \( G_{0\nu} \) are evaluated by using

\[
G_{0\nu} = \left( \frac{2 \lambda e^4}{256 \pi^2 (m_e R)^4 \ln 2} \right) \int_1^{T+1} F_0 (Z_f, \varepsilon_1) p_1 \varepsilon_1 d\varepsilon_2 \int_1^{T+2-\varepsilon_1} (T + 2 - \varepsilon_1 - \varepsilon_2) F_0 (Z_f, \varepsilon_2) p_2 \varepsilon_2 d\varepsilon_2,
\]

and have been calculated for all nuclei of general interest \([23, 73]\).

### C. Mechanism involving Sterile neutrinos

The contribution of the sterile \( \nu_h \) neutrino to the half-life \( T_{1/2}^{(0\nu)} \) for the \( 0^+ \rightarrow 0^+ \) transition of \((\beta^-\beta^-)_{0\nu}\) decay has been derived by considering the exchange of a Majorana neutrino between two nucleons and is given by \([33, 35]\),

\[
[T_{1/2}^{(0\nu)} (0^+ \rightarrow 0^+)]^{-1} = |U_{e\nu}|^2 \frac{m_h}{m_e} M^{(0\nu)} (m_h) ,
\]

where the phase space factor \( G_{0\nu} \) is the same as Eq. \([33]\), \( U_{e\nu} \) is the \( \nu_e \) mixing matrix element and the NTME \( M^{(0\nu)} (m_h) \) is written as

\[
M^{(0\nu)} (m_h) = \left\langle 0_F \left| \left[ -\frac{H_F (m_h, r)}{g_A} + \sigma_n \cdot \sigma_m H_{GT} (m_h, r) + S_{nm} H_T (m_h, r) \right] \right| 0_I \right\rangle .
\]

In Eq. \([24]\), the neutrino potentials are of the form

\[
H_\alpha (m_h, r) = \frac{2 \lambda R}{\pi} \int_0^\infty \frac{f_\alpha (qr) h_\alpha (q^2) q^2 dq}{\sqrt{q^2 + m_h^2} \left( \sqrt{q^2 + m_h^2} + x \right)} ,
\]

with the same \( h_\alpha (q^2) \) as given in Eqs. \([7]\)–Eq. \([9]\).

### D. Uncertainties in NTMEs within PHFB Model

In the PHFB model, the calculation of the NTMEs \( M_k^{(0\nu)} \) of the \((\beta^-\beta^-)_{0\nu}\) decay has already been discussed in Ref. \([54]\). Employing the HFB wave functions, one obtains the following expression for the NTME \( M_k^{(0\nu)} \) of the \((\beta^-\beta^-)_{0\nu}\) decay corresponding to an operator \( O_k \)

\[
[\beta^-\beta^-]_{0\nu} \) decay is given by \([33, 35]\)

\[
M_k^{(0\nu)} (m_h) = \left[ \frac{n_{J_f = 0} n_{J_i = 0}}{\pi} \sum_{\alpha\gamma\delta} (\alpha\beta | O_k | \gamma\delta) \right.
\]

\[
\times \int_0^\pi \frac{\left( F_{Z+2,N-2}^{(\nu)} (\theta) F_{Z+2,N-2}^{(\nu)*} \right)}{\left( 1 + F_{Z+2,N-2}^{(\nu)} (\theta) F_{Z+2,N-2}^{(\nu)*} \right)} \left( F_{Z,N}^{(\nu)} \right) \sin \theta d\theta .
\]

The required amplitudes \( (u_{im}, v_{im}) \) and expansion coefficients \( C_{O,m} \) of axially symmetric HFB intrinsic state \( |\Phi_0 \rangle \) with \( K = 0 \) to evaluate the expressions \( n_{J_f} (Z,N),(Z+2,N-2) (\theta) \), \( f_{Z,N} \) and \( F_{Z,N} (\theta) \) \([54]\), are obtained by minimizing the expectation value of the effec-
in a basis consisting of a set of deformed states. The details about the single particle Hamiltonian $H_{sp}$ as well as the pairing $V(P)$, quadrupole-quadrupole $V(QQ)$ and hexadecapole-hexadecapole $V(HH)$ parts of the effective two-body interaction have been given in Ref. 49. To perform a statistical analysis, sets of twenty-four NTMEs $M^{(0v)}_{\nu}$ for $(\beta^–\beta^-)_{0\nu}$ decay are evaluated using Eq. (20) in conjunction with four different parametrization of the two body effective interaction, two sets of form factors and three different parametrizations of the SRC. The details about the four different parametrizations have already been given in Refs. [49, 50]. However, a brief discussion about them is presented here for completeness.

The strengths of the proton-proton, the neutron-neutron and the proton-neutron parts of the $V(QQ)$ are denoted by $\chi_{2pp}$, $\chi_{2nn}$ and $\chi_{pn}$, respectively. In Refs. 74, 75, it has been shown that the experimental excitation energy of the $2^+$ state, $E_2^+$, could be fitted by taking equal strengths of the like particle components of the QQ interaction i.e. $\chi_{2pp} = \chi_{2nn} = 0.0105$ MeV $b^{-4}$ and by varying the strength of the proton-neutron component of the QQ interaction $\chi_{pn}$. In Ref. [49], it was also feasible to employ an alternative isoscalar parametrization of the quadrupole-quadrupole interaction, by taking $\chi_{2pp} = \chi_{2nn} = \chi_{pn}/2$ and the three parameters were varied together to fit $E_2^+$. These two alternative parametrizations of the quadrupole-quadrupole interaction were referred to as $PQQ\text{H1}$ and $PQQ\text{H2}$. Two additional parametrizations, namely $PQQ\text{HH1}$ and $PQQ\text{HH2}$ were obtained with the inclusion of the hexadecapolar $HH$ part of the effective interaction. Presently, we consider a form of Jastrow short range correlations simulating the effects of Argonne V18, CD-Bonn potentials in the self-consistent CCM [52], given by

$$f(r) = 1 - ce^{-ar^2}(1 - br^2),$$

where $a = 1.1 fm^{-2}$, $1.59 fm^{-2}$, $1.52 fm^{-2}$, $b = 0.68 fm^{-2}$, $1.45 fm^{-2}$, $1.88 fm^{-2}$ and $c = 1.0$, $0.92$, $0.46$ for Miller and Spencer parametrization, Argonne NN, CD-Bonn potentials, which are denoted by SRC1, SRC2 and SRC3, respectively. Finally, the uncertainties associated with the NTMEs $M^{(0v)}_{\nu}$ for $(\beta^–\beta^-)_{0\nu}$ decay are evaluated by calculating the mean and standard deviation given by

$$\overline{M}^{(0v)} = \frac{\sum_{i=1}^{N} M^{(0v)}_{i}}{N},$$

and

$$\Delta \overline{M}^{(0v)} = \frac{1}{\sqrt{N-1}} \left[ \sum_{i=1}^{N} \left( M^{(0v)}_{i} - \overline{M}^{(0v)} \right)^2 \right]^{1/2}.$$

III. RESULTS AND DISCUSSIONS

In the present work, we use the same wave functions as used in the earlier works [50, 51, 61, 73]. It has been already shown that [50], the experimental excitation energies of $2^+$ state $E_{2^+}$ can be reproduced to about 98% accuracy by adjusting the proton-neutron quadrupole correlation strength parameter $\chi_{3pn}$ or $\chi_{3pp}$. The maximum change in $E_{2^+}$ and $E_{0^+}$ energies with respect to $PQQ1$ interaction [74, 75] is about 8% and 31%, respectively. Further, the reduced $B(E2:0^+ \rightarrow 2^+)$ transition probabilities, deformation parameters $\beta_2$, static quadrupole moments $Q(2^+)$ and gyromagnetic factors $g(2^+)$ change by about 20%, 10% (except for $^{94}\text{Zr}$ and $PQQ2$ interaction), 27% and 12% (except for $^{94,96}\text{Zr}$ and $PQQ2$ interaction), respectively, and are usually in an overall agreement with the experimental data [73, 78]. The experimental result for the quadrupole moment $Q(2^+)$ of $^{96}\text{Zr}$ is not available and the theoretical results exhibit a sign change for the $PQQ2$ interaction. The theoretically calculated NTMEs $M_{2\nu}$ for the $0^+ \rightarrow 0^+$ transition also change up to approximately 11% except $^{94,128,130}\text{Te}$, and $^{150}\text{Nd}$ isotopes, for which the changes are approximately 42%, 21%, 24% and 18% respectively. The change in the ratio for deformation effect $D_{2\nu}$ is approximately 18% but for $^{94}\text{Zr}$ in which case the change is approximately 30%.

A. Exchange of light Majorana neutrinos

Employing the HFB wave functions generated with four different parametrizations of effective two-body interaction, the required NTMEs $M^{(0v)}_{\nu}$ for $^{94,96,98,100}\text{Zr}$, $^{98,100}\text{Mo}$, $^{104}\text{Ru}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ isotopes are calculated with the consideration of two sets of form factors and three different parametrizations of the Jastrow type of SRC. In Table II we present sets of twenty-four NTMEs $M^{(0v)}_{\nu}$ for $^{100}\text{Mo}$ are presented without (HOC - 3rd column) and with the SRC (HOC+SRC -4th to 6th columns) to exhibit the role of HOC as well as the SRC explicitly. To test the validity of closure approximation used in the present work, the NTMEs $M^{(0v)}_{\nu}$ are also calculated for $\overline{A}/2$ in the energy denominator in the case of HOC+SRC given in 7th to 9th columns of Table II. It is remarkable that the variation in the NTMEs calculated with FNS1 and FNS2 are almost negligible.

In Table II we present sets of twenty-four NTMEs $M^{(0v)}_{\nu}$ for $^{94,96,98,100}\text{Zr}$, $^{98,100}\text{Mo}$, $^{104}\text{Ru}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ isotopes calculated in the same approximations as mentioned above. The sets of NTMEs calculated with FNS1 and FNS2 are donated by HOC1 and HOC2, respectively. It is noticed in general but for $^{128}\text{Te}$ isotope that the NTMEs evaluated for both $PQQ1$ and $PQQ2$ parameterizations are quite close. The inclusion of the hexadecapolar term tends to reduce them by magnitudes, specifically depending on the structure of nuclei.
TABLE I: Decomposition of NTMEs $M^{(0\nu)}$ for the $\beta^{-\beta^{-}}_{0\nu}$ decay of $^{100}$Mo including higher order currents (HOC) with (a) FNS1, (b) FNS2 and SRC (HOC+SRC) for the PQQI1 parameterization.

| NTMEs | FNS | HOC | HOC+SRC | HOuC+SRC (A/2) |
|-------|-----|-----|---------|----------------|
|       |     | SRC1 | SRC2 | SRC3 | SRC1 | SRC2 | SRC3 |
| $M^0_{F}(^{0\nu})$ | (a) | 2.1484 | 1.8911 | 2.1492 | 2.2216 | 2.0691 | 2.3412 | 2.4168 |
|       | (b) | 2.2034 | 1.9152 | 2.1883 | 2.2707 | 2.0943 | 2.3817 | 2.4673 |
| $M^0_{GT-\AA}$ | (a) | -6.3815 | -5.4584 | -6.3022 | -6.5663 | -5.9813 | -6.8682 | -7.1424 |
|       | (b) | -0.4503 | -0.2962 | -0.4054 | -0.4510 | -0.3060 | -0.4177 | -0.4640 |
| $M^0_{GT-PP}$ | (a) | 1.5521 | 1.1518 | 1.4644 | 1.5810 | 1.2013 | 1.5222 | 1.6143 |
|       | (b) | -0.2370 | -0.1192 | -0.1832 | -0.2201 | -0.1239 | -0.1892 | -0.2266 |
| $M^0_{GT-MM}$ | (a) | -5.5167 | -4.7220 | -5.4265 | -5.6564 | -5.2098 | -5.9529 | -6.1918 |
|       | (b) | -5.5107 | -4.7250 | -5.4283 | -5.6549 | -5.2128 | -5.9546 | -6.1902 |
| $M^0_{GT-P}$ | (a) | -0.0227 | -0.0230 | -0.0235 | -0.0234 | -0.0236 | -0.0241 | -0.0240 |
|       | (b) | -0.0092 | 0.0700 | 0.0710 | 0.0709 | 0.0718 | 0.0729 | 0.0728 |
| $M^0_{GT-AP}$ | (a) | 0.0059 | 0.0060 | 0.0062 | 0.0062 | 0.0061 | 0.0064 | 0.0064 |
|       | (b) | 0.0058 | 0.0058 | 0.0060 | 0.0060 | 0.0059 | 0.0062 | 0.0062 |
| $M^0_{GT-MM}$ | (a) | 0.0524 | 0.0529 | 0.0538 | 0.0537 | 0.0543 | 0.0552 | 0.0551 |
|       | (b) | 0.0522 | 0.0528 | 0.0536 | 0.0535 | 0.0542 | 0.0550 | 0.0549 |
| $|M^{(0\nu)}|$ | (a) | 6.8305 | 5.8716 | 6.7394 | 7.0154 | 6.4712 | 7.3865 | 7.6736 |
|       | (b) | 6.8597 | 5.8902 | 6.7664 | 7.0454 | 6.4904 | 7.4142 | 7.7044 |

The maximum variation in $M^{(0\nu)}$ due to the PQQHH1, PQQ2 and PQQH2 parameterizations with respect to PQQ1 one lies between 20–25%. The relative change in NTMEs $M^{(0\nu)}$, by changing the energy denominator to $\bar{A}/2$ instead of $\bar{A}$ is in between 8.7%–12.7%, which confirms that the dependence of NTMEs on average excitation energy $\bar{A}$ is small and thus, the validity of the closure approximation for $(\beta^{-\beta^{-}})_{0\nu}$ decay is supported.

The study of the radial evolution of NTMEs defined by

$$M^{(0\nu)} = \int C^{(0\nu)}(r) \, dr, \quad (31)$$

is the best possible tool to display the role of the HOC as well as the SRC. By the study of radial evolution of NTMEs $M^{(0\nu)}$, Šimkovic et al. in the QRPA [71] and Menéndez et al. in the ISM [72] have shown that the magnitude of $M^{(0\nu)}$ for all nuclei undergoing $(\beta^{-\beta^{-}})_{0\nu}$ decay exhibit a maximum at about the internucleon distance $r \approx 1$ fm, and that the contributions of decaying pairs coupled to $J = 0$ and $J > 0$ almost cancel out beyond $r \approx 3$ fm. Similar observations on the radial evolution of NTMEs $M^{(0\nu)}$ and $M^{(0N)}$ due to the exchange of light [50] and heavy Majorana neutrinos [51], respectively have also been reported within the PHFB approach.

In Fig. 2, the effects due to the HOC and SRC are made more transparent by plotting the radial dependence of $C^{(0\nu)}$ for $^{100}$Mo isotope with the PQQ1 parameterization of the effective two body interaction in four cases, namely HOC1, HOC1+SRC1, HOC1+SRC2 and HOC1+SRC3. In Fig. 3, the radial dependence of $C^{(0\nu)}$ for the same four combinations of HOC1 and SRC are plotted for $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes. It is noticed that the $C^{(0\nu)}$ are peaked at $r = 1.0$ fm for the HOC1 and the addition of SRC1 and SRC2 shifts the peak to 1.25 fm. However, the position of the peak remains unchanged at $r = 1.0$ fm with the inclusion of SRC3. Although, the radial distributions of $C^{(0\nu)}$ extends up to about 10 fm, the maximum contribution to the radial evolution of $M^{(0\nu)}$ results from the distribution of $C^{(0\nu)}$ up to 3 fm. In addition, the above observations also remain valid with the other three parameterizations of the effective two-body interaction. The observed variation in the areas of curves in different cases is intimately related with the large changes in NTMEs $M^{(0\nu)}$. 
presented in Table II.

In Table III the relative changes in NTMEs $M^{(0\nu)}$ (in \%) due to the different approximations are presented. It is noticed that the consideration of FNS1 induces changes about 9.0\%–11.0\% in the NTMEs $M^{(0\nu)}_{VV} + M^{(0\nu)}_{AA}$ of point nucleon case. The inclusion of HOCl reduces the NTMEs further by 11.0\%–13.0\%. The NTMEs $M^{(0\nu)}$ are approximately reduced by 13.0\%–17.0\%, 1.0\%–2.0\% and 2.5\%–3.0\% with the addition of SRC1, SRC2 and SRC3, respectively, relative to the HOCl case.

The effect of deformation on $M^{(0\nu)}$ is quantified by the quantity $D^{(0\nu)}$ defined as the ratio of $M^{(0\nu)}$ at zero deformation ($\zeta_{qq} = 0$) and full deformation ($\zeta_{qq} = 1$) [80].

$$D^{(0\nu)} = \frac{M^{(0\nu)}(\zeta_{qq} = 0)}{M^{(0\nu)}(\zeta_{qq} = 1)}.$$  (32)
Table III: Changes (in %) of the NTMEs $M^{(0\nu)}$ due to exchange of light Majorana neutrinos, for the $(\beta^-\beta^-)^{0\nu}$ decay with the inclusion of FNS1, HOC1 and HOC1+SRC (HOC1+SRC1, HOC1+SRC2 and HOC1+SRC3) for the four different parameterizations of the effective two-body interaction, namely (a) $PQQ_1$, (b) $PQQHH_1$, (c) $PQQ_2$ and (d) $PQQHH_2$.

| Nuclei     | FNS1  | HOC1  | HOC1+SRC |
|------------|-------|-------|----------|
| $(^{96}\text{Zr})$ | 8.94–10.63 | 11.17–12.96 | (i) 12.38–15.61 |
|            |       |       | (ii) 0.99–1.96 |
|            |       |       | (iii) 2.48–2.97 |
| $(^{96}\text{Zr})$ | 9.40–11.07 | 11.43–13.03 | (i) 13.17–16.40 |
|            |       |       | (ii) 1.11–2.11 |
|            |       |       | (iii) 2.50–3.04 |
| $(^{98}\text{Mo})$ | 8.95–10.68 | 10.13–13.00 | (i) 12.40–15.71 |
|            |       |       | (ii) 0.99–1.99 |
|            |       |       | (iii) 2.39–2.99 |
| $(^{100}\text{Mo})$ | 9.25–11.15 | 11.38–12.73 | (i) 12.91–16.56 |
|            |       |       | (ii) 1.07–2.15 |
|            |       |       | (iii) 2.51–3.06 |

Table IV: Deformation ratios $D^{(0\nu)}$ of $(\beta^-\beta^-)^{0\nu}$ decay for the $PQQ_1$ parameterization.

| Nuclei | HOC1 | HOC1+SRC |
|--------|------|----------|
| $(^{94}\text{Zr})$ | 2.48 | 2.52 |
| $(^{96}\text{Zr})$ | 4.40 | 4.53 |
| $(^{98}\text{Mo})$ | 1.94 | 1.95 |
| $(^{100}\text{Mo})$ | 2.15 | 2.17 |
| $(^{104}\text{Ru})$ | 3.80 | 3.91 |
| $(^{110}\text{Pd})$ | 2.61 | 2.66 |
| $(^{128}\text{Te})$ | 4.38 | 4.50 |
| $(^{130}\text{Te})$ | 2.94 | 2.95 |
| $(^{150}\text{Nd})$ | 6.18 | 6.17 |

In Table IV, we tabulate the values of $D^{(0\nu)}$ for $^{94,96}\text{Zr}$, $^{98,100}\text{Mo}$, $^{104}\text{Ru}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ nuclei. It is observed that owing to deformation effects, the NTMEs $M^{(0\nu)}$ are suppressed by factor of about 2–6 in the mass range $A = 90–150$. Thus, the deformation plays a crucial role in the nuclear structure aspects of $(\beta^-\beta^-)^{0\nu}$ decay.

In the present statistical analysis, we employ only the twenty-four NTMEs listed in the columns 4–6 (HOC1+SRC) and columns 11–13 of Table III employing the bare and quenched values of axial vector coupling constant $g_A = 1.254$ and $g_A = 1.0$, respectively. In Table IV we display the calculated averages $\bar{M}^{(0\nu)}$ and their variances $\Delta M^{(0\nu)}$ for $^{94,96}\text{Zr}$, $^{98,100}\text{Mo}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ isotopes along with all the available theoretical results in other models. It turns out that the uncertainties $\Delta M^{(0\nu)}$ are of the order of 10%, but for $^{130}\text{Te}$ and $^{150}\text{Nd}$ isotopes for which $\Delta M^{(0\nu)}$ are approximately 12% and 15%, respectively. Further, the effect due to the Miller-Spenser parameterization of Jastrow type of SRC is estimated by evaluating the same mean...
TABLE V: Average NTMEs $\overline{M}^{(0\nu)}$ and uncertainties $\Delta \overline{M}^{(0\nu)}$ for the $\beta^-\beta^-$ decay of $^{94,96,98}_{\text{Mo}},^{100}_{\text{Mo}},^{104}_{\text{Ru}},^{110}_{\text{Pd}},^{128,130}_{\text{Te}}$ and $^{150}_{\text{Nd}}$ isotopes. Both bare and quenched values of $g_A$ are considered. Case I and Case II denote calculations with and without SRC1, respectively. In the 5th column, (a), (b), (c), (d) and (e) correspond to Jastrow, FHCh, UCOM and CCM SRC with Argonne V18 and CD-Bonn potentials, respectively. The range of values in the QRPA and RQRPA for $g_A=1.0$ are tabulated.

| Nuclei | $g_A$ | Case I $\overline{M}^{(0\nu)}$ | Case II $\overline{M}^{(0\nu)}$ | SRC | QRPA | RQRPA | QRPA | ISM | EDI* | IBM |
|--------|-------|---------------------|---------------------|------|------|------|------|-----|-----|-----|
| $^{94}_{\text{Zr}}$ | 1.254 | 3.88±0.37 | 4.08±0.24 | (c) | 1.22±0.03 | 1.31±0.15 | 5.32±0.39 |
|       | 1.0   | 2.76±0.26 | 2.90±0.17 | (c) | 1.23±0.04 | 1.33±0.15 | 3.90±0.13 |
| $^{96}_{\text{Zr}}$ | 1.254 | 2.86±0.26 | 3.03±0.12 | (a) | 1.22±0.03 | 1.31±0.15 | 2.53 |
|       |       |          |          | (b) | 1.23±0.04 | 1.33±0.15 | 3.17 |
|       |       |          |          | (c) | 1.77±0.02 | 1.77±0.02 | 5.65 |
|       |       |          |          | (d) | 2.07±0.10 | 2.01±0.17 | 2.89 |
|       |       |          |          | (e) | 2.28±0.03 | 2.19±0.22 | 3.00 |
|       | 1.0   | 2.04±0.19 | 2.16±0.09 | (a) | 1.32±0.08 | 1.22±0.12 | 2.764 |
|       |       |          |          | (c) | 1.22±0.03 | 1.31±0.15 | 2.53 |
| $^{98}_{\text{Mo}}$ | 1.254 | 6.49±0.55 | 6.81±0.32 | (a) | 3.64±0.21 | 3.03±0.21 | 3.73 |
|       | 1.0   | 4.64±0.41 | 4.88±0.25 | (c) | 3.73±0.21 | 3.12±0.21 | 3.62 |
| $^{100}_{\text{Mo}}$ | 1.254 | 6.26±0.63 | 6.59±0.44 | (a) | 4.71±0.28 | 3.88±0.26 | 5.08 |
|       |       |          |          | (b) | 5.18±0.36 | 4.20±0.24 | 4.31 |
|       |       |          |          | (c) | 5.73±0.34 | 4.67±0.31 | 4.50 |
|       | 1.0   | 4.49±0.47 | 4.73±0.33 | (a) | 2.96±0.15 | 2.55±0.13 | 3.103 |
|       |       |          |          | (c) | 2.55±0.13 | 2.10±0.16 | 2.764 |
| $^{104}_{\text{Ru}}$ | 1.254 | 4.36±0.44 | 4.61±0.28 | (a) | 4.44±0.24 | 3.75±0.21 | 4.10±1.40 |
|       | 1.0   | 3.15±0.33 | 3.33±0.22 | (c) | 5.63±0.49 |           | 3.62 |
| $^{110}_{\text{Pd}}$ | 1.254 | 7.16±0.74 | 7.53±0.54 | (a) | 5.99±0.17 | 5.26±0.16 | 4.15 |
|       |       |          |          | (c) | 5.14±0.69 |           | 4.31 |
|       | 1.0   | 5.12±0.55 | 5.39±0.41 | (a) | 3.97±0.14 | 3.52±0.13 | 2.26 |
|       |       |          |          | (b) | 4.15±0.15 | 3.68±0.14 | 4.48 |
|       |       |          |          | (c) | 5.04±0.15 | 4.45±0.15 | 5.62 |
|       |       |          |          | (d) | 5.38±0.17 | 4.71±0.17 | 5.62 |
|       |       |          |          | (e) | 5.99±0.17 | 5.26±0.16 | 5.13 |
|       | 1.0   | 2.61±0.28 | 2.75±0.20 | (a) | 3.11±0.09 | 2.77±0.09 | 5.74 |
|       |       |          |          | (b) | 4.54±0.13 | 4.0±0.12 | 4.03 |
|       |       |          |          | (c) | 5.04±0.15 | 4.45±0.15 | 5.62 |
|       |       |          |          | (d) | 5.38±0.17 | 4.71±0.17 | 5.62 |
|       |       |          |          | (e) | 5.99±0.17 | 5.26±0.16 | 5.13 |
| $^{128}_{\text{Te}}$ | 1.254 | 3.62±0.39 | 3.82±0.28 | (a) | 3.56±0.13 | 3.22±0.13 | 2.04 |
|       |       |          |          | (b) | 3.72±0.14 | 3.36±0.15 | 4.03 |
|       |       |          |          | (c) | 4.53±0.12 | 4.07±0.13 | 5.12 |
|       |       |          |          | (d) | 4.77±0.15 | 4.27±0.15 | 5.12 |
|       |       |          |          | (e) | 5.37±0.13 | 4.80±0.14 | 4.47 |
|       | 1.0   | 2.91±0.35 | 3.07±0.28 | (a) | 2.95±0.04 | 2.15±0.14 | 3.48 |
|       |       |          |          | (b) | 2.15±0.14 | 1.71 | 2.88 |
| $^{150}_{\text{Nd}}$ | 1.254 | 2.83±0.42 | 2.96±0.39 | (a) | 2.95±0.04 | 2.15±0.14 | 3.48 |
|       |       |          |          | (c) | 2.95±0.04 | 2.15±0.14 | 3.48 |
|       |       |          |          | (d) | 2.95±0.04 | 2.15±0.14 | 3.48 |
|       |       |          |          | (e) | 2.95±0.04 | 2.15±0.14 | 3.48 |
| 1.0   | 2.04±0.31 | 2.13±0.29 |          |          |          |          |          |          |          |          |

$\overline{M}^{(0\nu)}$ and their standard deviations $\Delta \overline{M}^{(0\nu)}$ for sixteen NTMEs calculated using SRC2 and SRC3 parameterizations. By excluding the NTMEs due to SRC1 in the statistical analysis, the uncertainties reduce by 1.5%-5%.

The figure-of-merit defined by

$$C_{mn}^{(0\nu)} = \frac{10^{24}}{m_e} \left| M^{(0\nu)} \right|^2,$$

is another NTME related convenient quantity and is usually used in the analysis of experimental data. The arbitrary scaling factor $10^{24}$ is used so that the $C_{mn}^{(0\nu)}$ are of the order of unity. Recently, the phase space factors have been calculated by Kotila and Lachello with screening correction. However, we evaluate the twenty-four $C_{mn}^{(0\nu)}$ using rescaled phase space factors of Boehm and Vogel.
for $g_A = 1.254$ and perform a statistical analysis for estimating the averages $C_{mm}^{(0\nu)}$ and uncertainties $\Delta C_{mm}^{(0\nu)}$. In the 3rd and 4th columns of Table VI, the averages $C_{mm}^{(0\nu)}$ and uncertainties $\Delta C_{mm}^{(0\nu)}$ are displayed. The uncertainties in $\Delta C_{mm}^{(0\nu)}$ are about twice of those of $\Delta M^{(0\nu)}$.

The limits on the effective mass of light neutrino $\langle m_\nu \rangle$ are extracted from the largest observed limits on half-lives $T_{1/2}^{(0\nu)}$ of $\nu^\beta \rightarrow \nu^\beta$ decay using the average $\overline{C}_{mm}^{(0\nu)}$. It is observed that the extracted limits on $\langle m_\nu \rangle$ for $^{100}$Mo and $^{130}$Te nuclei are 0.36 eV–0.50 eV and 0.32 eV–0.45 eV, respectively. In the last but one column of Table VI, the predicted half-lives of $\nu^\beta \rightarrow \nu^\beta$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes are given for $\langle m_\nu \rangle = 50$ meV. In the last column of Table VI we present the nuclear sensitivities, which are related to mass sensitivities, defined by [33]

$$\xi^{(0\nu)} = 10^8 \sqrt{G_{01}} \left| M^{(0\nu)} \right|,$$  

with an arbitrary normalization factor $10^8$ so that the nuclear sensitivities turn out to be order of unity. It is observed that the nuclear sensitivities for $\nu^\beta \rightarrow \nu^\beta$ decay of $^{100}$Mo, $^{150}$Nd, $^{130}$Te, $^{104}$Pd, $^{96}$Zr, $^{94}$Zr $^{128}$Te, and $^{98}$Mo isotopes are in the decreasing order of their magnitudes.

### B. Majoron emission

In the classical Majoron model, the NTMEs $M^{(0\nu)}$ for the $\nu^\beta \rightarrow \nu^\beta$ decay are same as the $M^{(0\nu)}$ for $\nu^\beta \rightarrow \nu^\beta$ decay. Hence, the average NTMEs $\overline{M}^{(0\nu)}$ and uncertainties $\Delta \overline{M}^{(0\nu)}$ are same as the $M^{(0\nu)}$ and $\Delta M^{(0\nu)}$, respectively. The phase space factors $G_{0\nu}$ for the $0^+ \rightarrow 0^+$ transition of $\nu^\beta \rightarrow \nu^\beta$ decay mode have been calculated by Hirsch et al. [23] using $g_A = 1.254$. We calculate the phase space factors given in Table VII for the $0^+ \rightarrow 0^+$ transition of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru and $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes with $g_A = 1.254$. The maximum difference between the two sets of phase space factors is less than 5%.

The extracted limits on the effective Majoron-neutrino coupling parameter ($g_M$) form the largest limits on the observed half-lives $T_{1/2}^{(0\nu)}$ are given in Table VII. The most stringent extracted limit on $\langle g_M \rangle < (2.22 \pm 3.09) \times 10^{-5}$. In the column 6 of the same Table VII, the predicted half-lives $T_{1/2}^{(0\nu)}$ for $\langle g_M \rangle = 10^{-6}$ are displayed. The sensitivities $\xi^{(0\nu)}$ defined similar to $\xi^{(0\nu)}$ are presented in the last column of the Table VII. It is noticed that the sensitivities for $\nu^\beta \rightarrow \nu^\beta$ decay mode of $^{98}$Mo, $^{128}$Te, $^{94}$Zr, $^{104}$Ru, $^{110}$Pd, $^{130}$Te, $^{96}$Zr, $^{100}$Mo and $^{150}$Nd isotopes are in the increasing order of their magnitudes.

### C. Sterile neutrinos

To estimate the uncertainties in NTMEs $M^{(0\nu)}(m_\nu)$, sets of twenty-four NTMEs are calculated for five potential $(\beta^\nu \beta^\nu)$ candidates, namely $^{96}$Zr, $^{100}$Mo, $^{128,130}$Te and $^{150}$Nd isotopes using HFB wavefunctions generated with four different parametrizations of effective two-body interaction, form factors with two different parametrizations and three different parameter-
TABLE VII: Phase space factors, extracted effective Majoron-neutrino coupling \(\langle g_M \rangle\), predicted half-lives \(T_{1/2}^{(0\nu)}\) and sensitivities for the \((\beta^–\beta^–)_{0\nu}\) decay of \(^{94,96}\)Zr, \(^{98,100}\)Mo, \(^{104}\)Ru, \(^{110}\)Pd, \(^{128,130}\)Te and \(^{150}\)Nd isotopes.

| Nuclei    | \(G_{OM}\)       | Expt. \(T_{1/2}^{(0\nu)}\) (yr) | Ref. | \(g_A\) | \(\langle g_M \rangle\) | \(\langle g_M \rangle = 10^{-6}\) | \(\zeta^{(0\nu)}\) |
|-----------|------------------|---------------------------------|------|---------|-----------------|---------------------------------|---------|
| \(^{94}\)Zr | \(1.299 \times 10^{-17}\) | \(2.3 \times 10^{18}\) | \([85]\) | (a) | \(4.49 \times 10^{-2}\) | \(4.63 \times 10^{-2}\) | \(1.47\) |
| \(^{96}\)Zr | \(2.603 \times 10^{-15}\) | \(1.9 \times 10^{21}\) | \([21]\) | (a) | \(1.49 \times 10^{-4}\) | \(4.19 \times 10^{-2}\) | \(15.45\) |
| \(^{98}\)Mo | \(2.926 \times 10^{-21}\) | \(a\) | \(|\langle g_M \rangle|\) | \(2.08 \times 10^{-4}\) | \(8.20 \times 10^{-2}\) | \(11.04\) |
| \(^{100}\)Mo | \(1.736 \times 10^{-15}\) | \(2.7 \times 10^{22}\) | \([18]\) | (a) | \(2.22 \times 10^{-5}\) | \(1.33 \times 10^{-2}\) | \(27.45\) |
| \(^{104}\)Ru | \(3.037 \times 10^{-17}\) | \(a\) | \(|\langle g_M \rangle|\) | \(3.09 \times 10^{-5}\) | \(2.58 \times 10^{-2}\) | \(19.69\) |
| \(^{110}\)Pd | \(2.758 \times 10^{-16}\) | \(a\) | \(|\langle g_M \rangle|\) | \(1.55 \times 10^{-3}\) | \(1.55 \times 10^{-2}\) | \(2.54\) |
| \(^{128}\)Te | \(9.669 \times 10^{-18}\) | \(1.5 \times 10^{24}\) | \([91, 92]\) | (a) | \(6.88 \times 10^{-5}\) | \(7.0 \times 10^{-2}\) | \(1.19\) |
| \(^{130}\)Te | \(1.300 \times 10^{-15}\) | \(a\) | \(|\langle g_M \rangle|\) | \(9.54 \times 10^{-5}\) | \(1.36 \times 10^{-2}\) | \(0.86\) |
| \(^{150}\)Nd | \(1.038 \times 10^{-14}\) | \(1.52 \times 10^{21}\) | \([20]\) | (a) | \(8.50 \times 10^{-5}\) | \(1.10 \times 10^{-2}\) | \(30.19\) |
|           |                  | (b) | \(|\langle g_M \rangle|\) | \(1.18 \times 10^{-4}\) | \(2.12 \times 10^{-2}\) | \(21.73\) |

IV. CONCLUSIONS

In the PHFB model, the required NTMEs to study the \((\beta^-\beta^-)_{0\nu}\) decay of \(^{94,96}\)Zr, \(^{98,100}\)Mo, \(^{104}\)Ru, \(^{110}\)Pd, \(^{128,130}\)Te and \(^{150}\)Nd isotopes within mechanisms involving the light Majorana neutrino, classical Majorons and sterile neutrinos are calculated for four different parametrization of pairing plus multipolar type of effective two body interaction, two sets of form factors and three different parametrizations of SRC. It is observed that the closure approximation is quite valid as expected. The effect due to FNS is about 10% and inclusion of the HOC further reduces the NTMEs by approximately 12%.

With the consideration of the SRCs, the NTMEs are in addition reduced by 16.0% (1.0%) for SRC1 (SRC2). The effects due to deformation are in between a factor of 2–6.

The sets of twenty-four NTMEs \(M^{(0\nu)}\) have been employed for estimating the uncertainties therein for the bare axial vector coupling constant \(g_A = 1.254\) and quenched value of \(g_A = 1.0\). In the mechanisms involving light Majorana neutrino and classical Majorons, the uncertainties in NTMEs are in about 4.0% (9.0%)–13.5% (15.0%) without (with) the SRC1. In the case of sterile neutrinos, the uncertainties in NTMEs are in between 4% (9%)–20% (36%) depending on the considered mass of the sterile neutrinos without (with) SRC1.
TABLE VIII: Calculated average NTMEs $\Delta M^{(0\nu)}(m_\nu)$ and uncertainties $\Delta M^{(0\nu)}(m_\nu)$ for the $\beta^{-}\beta^{-}$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes. The (a) Case I and (b) Case II denote calculations with and without SRC1, respectively.

| Nuclei | $m_\nu = (MeV)$ | $10^{-4} - 1$ | $10^{-2}$ | $10^{-3}$ | $10^{-5} - 10^{-7}$ $10^{5}/m_\nu^2$ |
|--------|----------------|--------------|-----------|-----------|-----------------------------|
| $^{90}$Zr | (a) 2.863±0.260 | 2.692±0.252 | 1.273±0.170 | 0.50±0.015 | 0.486±0.175 |
| (b) 3.028±0.116 | 2.853±0.109 | 1.384±0.067 | 0.60±0.008 | 0.593±0.102 |
| $^{100}$Mo | (a) 6.262±0.626 | 5.910±0.597 | 2.808±0.360 | 0.105±0.030 | 0.998±0.347 |
| (b) 6.589±0.438 | 6.229±0.405 | 3.028±0.188 | 0.124±0.016 | 1.210±0.204 |
| $^{128}$Te | (a) 3.619±0.388 | 3.412±0.374 | 1.640±0.239 | 0.064±0.019 | 0.613±0.220 |
| (b) 3.819±0.278 | 3.607±0.263 | 1.776±0.153 | 0.076±0.011 | 0.744±0.138 |
| $^{130}$Te | (a) 4.054±0.488 | 3.815±0.450 | 1.808±0.225 | 0.069±0.019 | 0.659±0.223 |
| (b) 4.262±0.392 | 4.018±0.349 | 1.949±0.104 | 0.081±0.010 | 0.796±0.127 |
| $^{150}$Nd | (a) 2.831±0.422 | 2.659±0.396 | 1.213±0.198 | 0.043±0.013 | 0.413±0.149 |
| (b) 2.963±0.395 | 2.788±0.368 | 1.303±0.161 | 0.051±0.008 | 0.500±0.098 |

We have also extracted limits on the effective neutrino mass $m_\nu$ from the available limits on experimental half-lives $T^{0\nu}_{1/2}$ using average NTMEs $\bar{M}^{(0\nu)}$ calculated in the PHFB model. In the case of $^{130}$Te isotope, one obtains the best limit on the effective neutrino mass $m_\nu < 0.32$ eV. From the observed limit on the half-life $T^{0\nu}_{1/2} > 3.0 \times 10^{24}$ yr of $(\beta^{-}\beta^{-})_{0\nu}$ decay [10]. The best limit on the Majoron-neutrino coupling constant $(g_{\nu M})$ turns out to be $< 2.22 \times 10^{-5}$ from the observed half-life of $^{100}$Mo isotope. The study of sensitivities of nuclei suggest that to extract the effective mass of light Majorana neutrino $m_\nu$, $^{100}$Mo is the preferred isotope and $^{150}$Nd is the favorable isotope to extract the $|g_{\nu M}|$. Thus, the sensitivities of different nuclei are mode dependent. It has been observed that the extracted limits on the sterile neutrino $\nu_s$, mixing matrix element $U_{e s}$ extend over a wider region of mass $m_\nu$ than those of laboratory experiments, astrophysical and cosmological observations.

Acknowledgments

This work is partially supported by the Council of Scientific and Industrial Research (CSIR), India vide sanction No. 03(1216)/12/EMR-II, Indo-Italian Collaboration DST-MAE project via grant no. INT/Italy/P-7/2012 (ER), Consejo Nacional de Ciencia y Tecnología (Conacyt)-México, and Dirección General de Asuntos del Personal Académico, Universidad Nacional Autónoma de México (DGAPA-UNAM) project IN103212. One of us (PKR) thanks Prof. S. K. Singh, HNBG University, India for useful discussions.

[1] M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rep., 460, 1 (2008).
[2] S. M. Bilenky and C. Giunti, arXiv:1203.5250v3 [hep-ph].
[3] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney, and I. V. Krivosheina, Mod. Phys. Lett. A 16, 2409 (2001); H. V. Klapdor-Kleingrothaus et al., Eur. Phys. J. A 12, 147 (2001).
[4] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, A. Dietz, and O. Chklovets, Phys. Lett. B 586, 198 (2004).
[5] F. Feruglio, A. Strumia and F. Vissani, Nucl. Phys. B 637, 345 (2002).
[6] C. E. Aalseth et al., Phys. Rev. D 65, 092007 (2002); Mod. Phys. Lett. A 17, 1475 (2002).
[7] Yu. G. Zdesenko, F. A. Danevich, and V. I. Tretyak, Phys. Lett. B 546, 206 (2002).
[8] A. Ianni, Nucl. Instrum. Meth. A 516, 184 (2004).
[9] E. T. Avignone III, S. R. Elliott, and J. Engel, Rev. Mod. Phys. 80, 481 (2008).
[10] J. D. Vergados, H. Ejiri, and F. Šimkovic, Rept. Prog. Phys. 75, 106301 (2012).
[11] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, Phys. Rev. D 84, 053007 (2011).
[12] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, and I. V. Titkov, Int. J. Mod. Phys. A 21, 1159 (2006); H.V. Klapdor-Kleingrothaus and I. V. Krivosheina, Mod. Phys. Lett. A 21, 1547 (2006).
[13] A. S. Barabash, Phys. At. Nucl., 73, 162, (2010); IV International Pontecorvo Neutrino Physics School, Alushta, Ukraine September 26-October 6, 2010, arXiv:1107.5663 [nucl-ex].
[14] A. Giuliani, Acta Physica Polonica B 41, 1447 (2010).
[15] B. Schwingerheuer, arXiv:1201.4916v1 [hep-ex].
[16] M. Doi and T. Kotani, Prog. Theor. Phys. 89, 139 (1993).
[17] M. Hirsch, H. V. Klapdor-Kleingrothaus, and O. Panella, Phys. Lett. B 374, 7 (1996).
[18] R. Arnold et al., Phys. Rev. Lett. 107, 062504 (2011); Nucl. Phys. A 765, 483 (2006); Nucl. Phys. A 678, 340 (2000).
[19] A. Barabash and V. Brudanin (NEMO Collaboration), Phys. At. Nucl. 74, 312 (2011).
[20] J. Argyriades et al., Phys. Rev. C 80, 032501(R) (2009).
[21] J. Argyriades et al., Nucl. Phys. A 847, 168 (2010).
[22] P. Bamert, C. P. Burgess and R. N. Mohapatra, Nucl. Phys. B 440, 25 (1995).
[89] C. Arnaboldi et al., Phys. Lett. B 557, 167 (2003).
[90] C. Arnaboldi et al., Phys. Rev. C 78, 035502 (2008).
[91] O. K. Manuel, J. Phys. G 17, 221 (1991).
[92] A. S. Barabash, Phys. Rev. C 81, 035501 (2010).
[93] A. D. Dolgov, S. H. Hansen, G. Raffelt, and D. V. Semikoz, Nucl. Phys. B 590, 562 (2000); Nucl. Phys. B 580, 331 (2000).
[94] B. Blennow, E. F. Martinez, J. L. Pavon and J. Menéndez, JHEP07, 096 (2010).