Photonic and Leptonic Rare B Decays

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Abstract

Some selected topics involving photonic and leptonic rare $B$ decays are reviewed. The interest in their measurement for the CKM phenomenology is underlined. They are also potentially interesting in searching for physics beyond the standard model. This is illustrated on the examples of the decays $B \rightarrow (K, K^*)\ell^+\ell^-$ and $B \rightarrow (\rho, \omega)\gamma$ by contrasting their anticipated phenomenological profiles in the standard model and some variants of supersymmetric models.

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1 Introduction

Rare $B$ Decays (induced in the quark language by transitions such as $b \rightarrow s\gamma$, $b \rightarrow d\gamma$, $b \rightarrow s\ell^+\ell^-$, $b \rightarrow d\ell^+\ell^-$, ...) and particle-antiparticle mixings ($B^0 - \overline{B}^0$, $B^0_s - \overline{B}^0_s$, ...) represent flavour-changing-neutral-current (FCNC) processes. In standard model (SM), FCNC processes are not allowed in the Born approximation and are induced by loops which impart them a sensitivity to higher scales. These transitions are either dominated by the top quark in the SM, or else the contribution of the lighter ($u,d,s,c$)-quarks can be estimated using QCD and the knowledge of the entries in the first two rows of the CKM (Cabibbo-Kobayashi-Maskawa) matrix [1]. In either case, FCNC $B$ decays can be used to determine the CKM matrix elements involving the top quark, $V_{td}$, $V_{ts}$, and $V_{tb}$. Since direct determination of only $|V_{tb}|$ in the decay $t \rightarrow Wb$ is presently available [2], and there is no credible method to measure the other two directly, FCNC $B$ (and $K$)-decays are the only feasible alternatives in quantifying our knowledge about $V_{td}$ and $V_{ts}$.

With the advent of the $B$-factories, the era of precision $B$-physics is already upon us. With definite plans to accumulate $O(500)$ fb$^{-1}$ luminosity at the BABAR detector within the next five years (and a similar projection for BELLE), a large number of rare decays and CP-violating asymmetries in partial decay rates of the $B^\pm$ or $B^0/\overline{B}^0$ mesons will be measured. Hadron machines will push this frontier even beyond, in particular in the $B_s$ and $\Lambda_b$ sectors, as discussed at this meeting [3] and elsewhere [4]. In anticipation of this, a lot of theoretical effort has gone in consolidating the $B$-physics profile in the SM and in suggesting search strategies for physics beyond the SM. The prime candidate in new physics searches is supersymmetry. While not expected to provide a "smoking gun" proof of supersymmetry, for which high energy colliders are arguably more suited, yet precise measurements in low energy processes may also reveal effects anticipated in supersymmetric theories. Typical of these are incremental contributions to the mass differences $\Delta M_d$ and $\Delta M_s$ in the $B - \overline{B}$ system, the effective FCNC vertices $bs\gamma$, $bs\ell^+\ell^-$, $bs\nu\bar{\nu}$, inducing the $b \rightarrow s$ transitions (likewise, in $b \rightarrow d$ transitions), and CP-violating phases. We review here some selected topics in the photonic and leptonic rare $B$-meson decays from the point of view of their impact on the CKM phenomenology and/or their potential for the discovery of induced supersymmetric effects.
2 Radiative Decays $B \to (X_s, X_d) + \gamma$ and the CKM Phenomenology

2.1 $\mathcal{B}(B \to X_s \gamma)$ and determination of $|V_{ts}|$

We start with the inclusive radiative decays $B \to X_s \gamma$. The lowest order (one-loop) contribution to the decay $b \to s + \gamma$ is described by the amplitude

$$
\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \sum_{i=u,c,t} \lambda_i F_2(x_i) q^\mu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b ,
$$

(1)

where $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$, and $x_i = m_i^2/m_W^2$, with $m_i$ and $m_W$ being the quark and $W^\pm$-boson masses, respectively; $G_F$ is the Fermi coupling constant and $e$ is the electromagnetic coupling, with $e^2/4\pi = \alpha_{em} \simeq 1/137$. The quantity $F_2(x_i)$ is the Inami-Lim Function

$$
F_2(x) = \frac{x}{24(x-1)^4} \left[ 6x(3x - 2) \ln x - (x - 1)(8x^2 + 5x - 7) \right] ,
$$

(2)

and $\lambda_i$ are the CKM factors, $\lambda_i \equiv V_{ib}^{\ast} V_{ts}$. Since $\lambda_u/\lambda_c \ll 1$, CKM Unitarity implies $\lambda_c \simeq -\lambda_t$, yielding the GIM-amplitude

$$
\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \lambda_t
\times (F_2(x_t) - F_2(x_c)) q^\mu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b .
$$

(3)

With $m_t$ already measured, this amplitude depends on $\lambda_t$.

To quantify this, one has to incorporate QCD corrections to the decay rate. A truly cooperative effort by several theoretical groups has led to a determination of the branching ratio $\mathcal{B}(B \to X_s \gamma)$ up to and including the next-to-leading order (NLO) accuracy. The result is the following NLO (in $O(\alpha_s)$) and power (in $1/m_b^2$ and $1/m_c^2$) corrected branching ratio in the SM:

$$
\mathcal{B}(B \to X_s \gamma) = [(3.29 \pm 0.33) \times 10^{-4}] \cdot |V_{ts}^{\ast} V_{tb}/V_{cb}|/0.976] .
$$

(4)

Explicit dependence on the CKM factor $|V_{ts}^{\ast} V_{tb}/V_{cb}|$ and the default value following from the (indirect) unitarity fit of this quantity are displayed. The present world average for the branching ratio, based on the CLEO,
ALEPH [11] and the more recent BELLE [12] measurements, is \( \mathcal{B}(B \to X_s \gamma) = (3.21 \pm 0.39) \times 10^{-4} \). This yields the CKM ratio:

\[
\left| \frac{V_{ts}^*V_{tb}}{V_{cb}} \right| = 0.964 \pm 0.075,
\]

to be compared with the unitarity fits of this quantity: \( \left| \frac{V_{ts}^*V_{tb}}{V_{cb}} \right| = 0.976 \pm 0.010 \). Using the present measurements of the matrix element \( |V_{cb}| = 0.04 \pm 0.002 \) [7] and assuming \( |V_{tb}| \simeq 1.0 \), yields \( |V_{ts}| = 0.038 \pm 0.003 \). Using, instead, the value \( |V_{tb}| = 0.97^{+0.16}_{-0.12} \), measured by the CDF collaboration [2], gives \( |V_{ts}| = 0.04 \pm 0.007 \), which corresponds to a measurement with an error of \( \pm 18\% \). This error will be greatly reduced in future.

The determination of \( |V_{ts}| \) from the \( B \to X_s \gamma \) is occasionally questioned [9]. The issue raised is the following: The dominance of the top-quark contribution, manifest in the lowest order GIM-amplitude in Eq. (3), is no longer present after including the QCD corrections in the decay rate. This deserves a closer look. Internal book keeping of the decay rate for \( B \to X_s \gamma \) shows that the QCD corrections do not significantly change the intermediate \( uu \) contribution which remains small and at the level of 2\% and can be neglected. However, the QCD-corrected amplitude \( \mathcal{M}(b \to s \gamma) \) receives a very large contribution from the matrix element of the four-quark operator \( O_2 = (s \Gamma_i c)(\bar{c} \Gamma_i \bar{b}) \), involving the intermediate \( cc \) loop, proportional to \( \alpha_s(m_b)C_2(m_b)\lambda_c \), where \( \lambda_c = V_{cs}^*V_{cb} \). The CKM dependence of the \( cc \) contribution to the branching ratio is hence proportional to \( (\lambda_c^2/V_{cb})^2 = V_{cs}^2 \). Fortunately, this CKM matrix element is known very precisely thanks to LEP \( |V_{cs}| = 0.989 \pm 0.016 \) [13]. Putting in the other relevant dynamical quantities, the \( cc \) contribution by itself gives a branching ratio which is twice the present experimental value of \( \mathcal{B}(B \to X_s \gamma) \) [8, 9]. Were it not for the top quark, SM would have been ruled out by data on \( \mathcal{B}(B \to X_s \gamma) \)! Hence, the top quark contribution proportional to \( \lambda_i \) is absolutely crucial in bringing the SM prediction for this branching ratio in accord with data. This evidently constrains the CKM ratio \( |V_{ts}^*V_{tb}/V_{cb}| \) whose present estimate is given above.

2.2 \( \mathcal{B}(B \to X_d \gamma) \) and \( a_{CP}(B \to X_d \gamma) \) in the SM

Measuring the CKM-suppressed inclusive radiative decay \( B \to X_d + \gamma \) will be an experimental \textit{Tour de Force}! The CKM-allowed \( (B \to X_s \gamma) \) and the
CKM-suppressed \((B \to X_d \gamma)\) decays are anticipated to have rather similar photon energy spectra \([14]\). However, the excellent \(K/\pi\)-separation at the B-factory experiments (and at CLEO) could be used in effectively separating the two radiative branches. With an estimated branching ratio \(\mathcal{B}(B \to X_d + \gamma) = \mathcal{O}(10^{-5})\) \([15]\), one would need \(\mathcal{O}(10^8)\) \(B \bar{B}\) events, which are well within reach of the \(B\)-factories in the next several years. The experimental effort will yield high dividends, as one also expects direct CP violation in the decay rates for \(B \to X_d \gamma\) and its charge conjugate \(\bar{B} \to X_d \gamma\) which is measurably large in the SM, with typical estimates being \(\mathcal{O}(20\%)\) \([15]\).

The effective Hamiltonian for the \(B \to X_d \gamma\) decays can be written in the form \([15]\)

\[
\mathcal{H}_{\text{eff}}(b \to d) = -\frac{4G_F}{\sqrt{2}} \xi_t \sum_{j=1}^{8} C_j(\mu) \hat{O}_j(\mu). \tag{6}
\]

The CKM factors are given by \(\xi_j \equiv V_{jb}V_{jd}^*\), \(j = u, c, t\). The Wilson coefficients \(C_j(\mu)\) are the same as in the decays \(B \to X_s \gamma\), but the four-quark operators have now an implicit CKM-parametric dependence

\[
\hat{O}_1 = -\frac{\xi_c}{\xi_t} (\bar{c}_{L\beta} \gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha} \gamma^\mu c_{L\beta}) - \frac{\xi_u}{\xi_t} (\bar{u}_{L\beta} \gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha} \gamma^\mu u_{L\beta})
\]

\[
\hat{O}_2 = -\frac{\xi_c}{\xi_t} (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{d}_{L\beta} \gamma^\mu c_{L\beta}) - \frac{\xi_u}{\xi_t} (\bar{u}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{d}_{L\beta} \gamma^\mu u_{L\beta}). \tag{7}
\]

Using the Wolfenstein parametrization \([16]\), one has:

\[
\xi_u = A \lambda^3 (\bar{\rho} - i\bar{\eta}), \quad \xi_c = -A \lambda^3, \quad \xi_t = -\xi_u - \xi_c. \tag{8}
\]

Here, \(A \simeq 0.84\), \(\lambda = 0.22\), but \(\bar{\rho}\) and the phase \(\bar{\eta}\) are only poorly determined, typically lying in the correlated range \(0.05 \leq \bar{\rho} \leq 0.40\) and \(0.25 \leq \bar{\eta} \leq 0.50\) (at 95\% C.L.) \([17]\). The quantities \(\bar{\rho}\) and \(\bar{\eta}\) are the \(O(\lambda^2)\) corrected Wolfenstein parameters \(\bar{\rho} = \rho(1 - \lambda^2/2)\) and \(\bar{\eta} = \eta(1 - \lambda^2/2)\) \([18]\). Thus, all three CKM factors in Eq. \((8)\) are comparable and of order \(\lambda^3\). This yields the following expression for the branching ratio in the SM:

\[
\mathcal{B}(B \to X_d + \gamma) = \lambda^2[(1 - \bar{\rho})^2 + \bar{\eta}^2]D_t
\]

\[
\times \left[1 + \frac{\bar{\rho}^2 + \bar{\eta}^2}{(1 - \bar{\rho})^2 + \bar{\eta}^2} \hat{D}_u + \frac{\bar{\rho}(1 - \bar{\rho}) - \bar{\eta}^2}{(1 - \bar{\rho})^2 + \bar{\eta}^2} \hat{D}_r - \frac{\bar{\eta}}{(1 - \bar{\rho})^2 + \bar{\eta}^2} \hat{D}_i \right], \tag{9}
\]
where, typically, $D_t = 3.6 \times 10^{-4}$, $\hat{D}_u = 0.078$, $\hat{D}_r = -0.136$, and $\hat{D}_i = 0.27$. Using the range of the parameters $(\rho, \eta)$ given above yields

$$6.0 \times 10^{-6} \leq B(B \to X_d + \gamma) \leq 2.0 \times 10^{-5}, \quad (10)$$

The direct CP-asymmetry in the decay rates defined as

$$a_{CP}(B \to X_d + \gamma) \equiv \frac{\Gamma(B \to X_d + \gamma) - \Gamma(B \to X_d + \bar{\gamma})}{\Gamma(B \to X_d + \gamma) + \Gamma(B \to X_d + \bar{\gamma})}, \quad (11)$$

has not so far been calculated in the NLL precision. We recall that, as opposed to the decay rate $\Gamma(B \to X_d + \gamma)$, which receives contributions starting from the lowest order, i.e., terms of the form $(\alpha_n s(m_b) \ln n(m_W/m_b))$, the CP-odd numerator in Eq. (11) is suppressed by an extra factor $\alpha_s$, i.e., it starts with terms of the form $\alpha_s(m_b) (\alpha_s \log(m_W/m_b))$. This results in a moderate scale dependence of $a_{CP}$.

Using the LL expression for the denominator in eq. (11), the CP rate asymmetry can be written as \[15\]

$$a_{CP}(B \to X_d + \gamma) = -\frac{Im(\xi_t^* \xi_u) D_i}{|\xi_t|^2 D_t^{(0)}} = \frac{D_i \bar{\eta}}{D_t^{(0)}|(1 - \bar{\rho})^2 - \bar{\eta}^2|}, \quad (12)$$

where $D_t^{(0)}$ stands for the LL part of $D_t$, and its typical value is $D_t^{(0)} \simeq 2.7 \times 10^{-4}$. Varying the CKM parameters in the range given above yields:

$$0.10 \leq a_{CP}(B \to X_d + \gamma) \leq 0.35 \quad (13)$$

Hence, a measurement of $B(B \to X_d + \gamma)$ and $a_{CP}(B \to X_d + \gamma)$ will have a big impact on the CKM Phenomenology.

The corresponding CP-asymmetry in the CKM-allowed decay $B \to X_s + \gamma$ in the SM is small, $a_{CP}(B \to X_s + \gamma) \propto \lambda^2 \eta$ yielding $a_{CP}(B \to X_s + \gamma) \leq 1\%$, reflecting the fact that the complex phases in the matrix elements $V_{ts}$ and $V_{tb}$ are absent in leading order. Hence, a measurement of the CP-asymmetry $a_{CP}(B \to X_s + \gamma)$ will be a sure signal of new physics \[19\]. Present bounds from CLEO imply that $a_{CP}(B \to X_s + \gamma)$ lies between $-0.27$ and $+0.10$ at 90% C.L. \[10\], which is consistent with being zero and with the SM. It should be stressed that in the experimental analysis of the CP-asymmetries in inclusive radiative decays $B \to X \gamma$, it is imperative to separate the hadrons $X$ into $X_s$ (emerging from $b \to s$ transitions) and $X_d$ (emerging from $b \to d$ transitions). Else, as pointed out by Soares \[20\], the sum of the rate difference $\Delta \Gamma(B \to X_d + \gamma) + \Delta \Gamma(B \to X_s + \gamma)$ vanishes in the SM in the limit $m_d = m_s = 0$. 

\[5\]
3 Exclusive Decays $B \rightarrow V + \gamma$

Inclusive decay $B \rightarrow X_d \gamma$ is theoretically more robust but experimentally very challenging to measure. The exclusive decays $B \rightarrow V \gamma$, with $V = \rho^0, \rho^\pm$ and $\omega$, should be relatively easier to measure, given the large statistics at the B factories. In anticipation of this, considerable effort has gone in studying the theoretical profile of the exclusive radiative $B$-decays \cite{21}. We shall first review some of these estimates in terms of the predicted branching ratios for $B \rightarrow V \gamma (V = K^*, \rho, \omega)$, which are being used to constrain the CKM ratio $|V_{td}|/|V_{ts}|$ from the current experiments.

Using the effective Hamiltonian

$$H(b \rightarrow d \gamma) = \frac{G_F}{\sqrt{2}} \left[ \xi_u (C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)) - \xi_t C_{\gamma}^{eff}(\mu)O_7(\mu) + \ldots \right],$$

where the Four-quark operators are now defined as:

$$O_1 = (\bar{d}_\alpha \Gamma_\mu u_\beta)(\bar{u}_\beta \Gamma_\nu b_\alpha), \quad O_2 = (\bar{d}_\alpha \Gamma_\mu u_\alpha)(\bar{u}_\beta \Gamma_\nu b_\beta),$$

with $\Gamma_\mu = \gamma_\mu (1 - \gamma_5)$, and $O_7 = \frac{m_B}{8\pi^2} \bar{d}\sigma_{\mu\nu} (1 - \gamma_5) F_{\mu\nu} b$, the crucial step is to calculate the form factors in $B \rightarrow V$ transitions involving these operators. Defining the form factor in $B \rightarrow V + \gamma$ from $O_7$ ($\psi = d, s$)

$$\langle V, \lambda | \frac{1}{2} \bar{\psi} \sigma_{\mu\nu} q^\nu b | B \rangle = i \epsilon_{\mu \nu \rho \sigma} e^{(\lambda)} p_{\rho \nu} F^B_{\sigma} F^{B \rightarrow V} (0),$$

the form factor $B_S^{B \rightarrow K^*}(0)$ has been measured in the decays $B \rightarrow K^* \gamma$. There are several estimates of this quantity based on the light cone QCD-sum rules \cite{22, 23} and lattice-QCD \cite{24}, yielding branching ratios in agreement with the CLEO \cite{23}, BELLE \cite{12} and BABAR \cite{26} data.

Assuming only the $O_7$ contribution (SD-Dominance) leads to simple relations among the decay rates for $B \rightarrow \rho \gamma$ and $B \rightarrow K^* \gamma$. For example, one has

$$R(\rho/K^*) \equiv \frac{\Gamma(B \rightarrow \rho + \gamma)}{\Gamma(B \rightarrow K^* + \gamma)} \simeq \kappa_{u,d} \left[ \frac{|V_{td}|}{|V_{ts}|} \right]^2,$$

where $\kappa_i \equiv [F_S(B_i \rightarrow \rho \gamma)/F_S(B_i \rightarrow K^* \gamma)]^2$ is the ratio of the form factors entering in the SD-part of the amplitudes. (In the limit of $SU(3)$ symmetry $\kappa_i = 1$.) Isospin symmetry implies $B(B^+ \rightarrow \rho^+ \gamma) = 2B(B^0 \rightarrow \rho^0 \gamma)$ and
\[ \mathcal{B}(B^+ \to K^{*+}\gamma) = \mathcal{B}(B^0 \to K^{0*}\gamma) \]. In addition using SU(3) symmetry, one has \[ \mathcal{B}(B^0 \to \omega\gamma) = \mathcal{B}(B^0 \to \rho^0\gamma) \]. These relations are frequently used in the current experimental analyses. Present upper limits on \( R(\rho/K^{*+}) \) are: \( R(\rho/K^{*+}) < 0.32 \) \([25]\) and \( R(\rho/K^{*+}) < 0.28 \) \([12]\), yielding \( |V_{td}/V_{ts}| < 0.75 \) and \( |V_{td}/V_{ts}| < 0.70 \), respectively, at 90% C.L., using \( |\kappa_i| = 0.58 \) \([27]\). This is not yet competitive with the bound from the ratio on the mass difference \( \Delta M_d/\Delta M_s \), which currently yields \( |V_{td}/V_{ts}| < 0.24 \) at 95% C.L. \([17]\).

Experiments at \( B \) factories will be able to reach the sensitivity of the SM, \( |V_{td}/V_{ts}| \approx 0.2 \).

Next, we discuss the isospin-violating ratio \( \Delta \), defined as follows:

\[ \Delta = \frac{1}{2}[\Delta^+ + \Delta^-] \],

\[ \Delta = \frac{1}{2}[\Delta^0 + \Delta^+] \].

The charge-conjugate averaged ratio

\[ \Delta^0 = \frac{\Gamma(B^- \to \rho^-\gamma)}{2\Gamma(B^0 \to \rho^0\gamma)} - 1 \],

\[ \Delta^+ = \frac{\Gamma(B^+ \to \rho^+\gamma)}{2\Gamma(B^0 \to \rho^0\gamma)} - 1 \].

(18)

The long-distance contributions in the ratio \( \Delta^+ \) and \( \Delta^- \) due to the long-distance contributions in the decays \( B^\pm \to \rho^\pm\gamma \) may turn out to be sizable. Hence, given enough data, one should analyze the ratios \( \Delta^0 \) and \( \Delta^- \) for anticipated deviations from the isospin value of unity. In fact, as argued in Ref. \([28]\), the interference of the LD and SD-amplitudes in \( \Delta \) and \( \mathcal{A}_{CP}(B^\pm \to \rho^\pm\gamma) \) may provide a sensitive probe for physics beyond the SM.

The long-distance contributions arise from the matrix elements of the operators \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \). There are two form factors in the matrix elements \( \langle V,\lambda|\mathcal{O}_{1,2}|B \rangle \), which we denote as \( F_1^L(q^2) \) and \( F_2^L(q^2) \). QCD sum rule-based estimates put them almost equal \([23, 34]\), \( F_{1,2}^{B\to V}(q^2) = F_{L}^{B\to V}(q^2) \). Taking into account the dominant LD-contributions arising from \( W^\pm \)-annihilation and \( W^\pm \)-exchange, one has

\[ \mathcal{M}(B^- \to \rho^-\gamma) = \xi_t a_{P}^{(-)}(1 - \frac{|\xi_u|}{|\xi_t|} R_{L}^{(-)} e^{i\alpha}) \],

\[ \mathcal{M}(B^0 \to \rho^0\gamma) = \xi_t a_{P}^{(0)}(1 - \frac{|\xi_u|}{|\xi_t|} R_{L}^{(0)} e^{i\alpha}) \].

(19)

(20)
where $\alpha$ is one of the inner angles of the unitarity triangle; $a_P^{(-)}$ and $a_P^{(0)}$ are SD (Penguin)-amplitudes and $R_L^{(\pm)}$ and $R_L^{(0)}$ are model-dependent quantities involving the form factors and other dynamical variables. Typical estimates obtained using factorization of the matrix elements of $O_1$ and $O_2$ and the Light-cone QCD sum rules are $[29, 30]$:

$$a_P^{(-)} \approx a_P^{(0)}, \quad R_L^{(-)} \approx -0.3 \pm 0.07, \quad R_L^{(0)} \approx 0.03 \pm 0.01. \quad (22)$$

The quantity $R_L^{(0)}$ entering in $\overline{B^0} \rightarrow \rho^0\gamma$ decay is suppressed due to the electric charge of the down quark ($e_d = +1/3$) in $\overline{B^0} = b\bar{d}$, as opposed to the up quark charge ($e_u = -2/3$) in $B^- = b\bar{u}$, as radiation from the light quarks dominates, and the color-suppression factor, which phenomenologically has a relative weight of about 0.25.

Dropping $R_L^{(0)}$ and rewriting $R_L^{(-)} = \epsilon_A e^{i\phi_A}$, where $\phi_A$ is a strong interaction phase, on has in the lowest order

$$\frac{B(B^- \rightarrow \rho^0\gamma)}{2B(\overline{B^0} \rightarrow \rho^0\gamma)} \approx \left| 1 - \epsilon_A e^{i\phi_A} \frac{\xi_u}{\xi_t} e^{i\alpha} \right|^2. \quad (23)$$

The isospin ratios can now be expressed as:

$$\Delta_{\pm 0} = 2\epsilon_A \left[ \cos \phi_A F_1 \mp \sin \phi_A F_2 + \frac{1}{2} \epsilon_A (F_1^2 + F_2^2) \right]. \quad (24)$$

Here, $F_{1,2}$ are functions of the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$, with

$$F_1 = - \left| \frac{\lambda_u^{(d)}}{\lambda_t^{(d)}} \right| \cos \alpha, \quad F_2 = - \left| \frac{\lambda_u^{(d)}}{\lambda_t^{(d)}} \right| \sin \alpha, \quad (F_1^2 + F_2^2) = \left| \frac{\lambda_u^{(d)}}{\lambda_t^{(d)}} \right|^2. \quad (25)$$

The charge-conjugated ratio $\Delta$ in the leading order is:

$$\Delta_{LO} = 2\epsilon_A \left[ \cos \phi_A F_1 + \frac{1}{2} \epsilon_A (F_1^2 + F_2^2) \right] \approx 2\epsilon_A \left[ F_1 + \frac{1}{2} \epsilon_A (F_1^2 + F_2^2) \right], \quad (26)$$

the second equality follows from assuming $\phi_A = 0$, obtaining in the factorization approximation.

Recently, the leading-twist non-factorizable contribution to the weak annihilation amplitude has been computed in perturbation theory by Grinstein and Pirjol, using heavy quark expansion techniques $[31$]. In the chiral limit
(m_u, m_d \to 0), these authors show that the one-loop non-factorizable corrections in weak annihilation amplitude vanish in the leading twist and one recovers the factorization result. Interestingly, the weak annihilation contribution, which is the dominant long-distance amplitude, can be determined model independently from the radiative decays \( B^\pm \to \gamma \ell^\pm \nu \ell \) [32]. To generate non-zero \( A_{CP} \), it is necessary to compute the NLO corrections [20, 33], which generates \( \phi \) perturbatively in the penguin amplitude. Of course, \( \Delta_{LO} \) is also renormalized by perturbative QCD corrections.

### 3.1 NLO Effects in \( \Delta \) and \( A_{CP}(B^\pm \to \rho^\pm \gamma) \)

Ignoring contributions which are formally power \((1/m_b)\)-suppressed, the NLO corrections to the decay rate \( \Gamma(B \to \rho \gamma) \) can be retrieved from the corresponding corrections to the inclusive radiative decays \( \Gamma(B \to X_s \gamma) \) and \( \Gamma(B \to X_d \gamma) \). The isospin-violating ratio \( \Delta_{NLO} \) in the NLO approximation and the CP-violating decay rate asymmetry \( A_{CP}(B^\pm \to \rho^\pm \gamma) \) have been calculated to \( O(\alpha_s) \) in Ref. [28]: The isospin-violating ratio \( \Delta_{NLO} \) and \( \Delta_{LO} \) are shown as functions of the angle \( \alpha \) in Fig. 1. As can be seen in this figure, the NLO corrections to \( \Delta_{LO} \) are small for all values of \( \alpha \). The anticipated range of \( \alpha \) from the CKM-unitarity fits with 95% C.L. [17] is also indicated. Note, \( \Delta_{LO} \) (but numerically also \( \Delta_{NLO} \)) is essentially proportional to \( F_1 \), while \( A_{CP}(B^\pm \to \rho^\pm \gamma) \) is proportional to \( F_2 \). Thus, for the central value of the CKM-unitarity fits, yielding \( \alpha \simeq \pi/2 \), \( \Delta_{NLO} \) is very small, implying that isospin symmetry in \( \Delta \) holds to a very high precision, while for this value of \( \alpha \), \( A_{CP}(B^\pm \to \rho^\pm \gamma) \) takes its maximum value. Conversely, if the value of \( \alpha \) is sufficiently different from \( \pi/2 \), the CP asymmetry \( A_{CP}(B^\pm \to \rho^\pm \gamma) \) decreases while \( \Delta_{NLO} \) increases, reaching \( \Delta_{NLO} \simeq \pm 0.2 \) for extreme allowed values of \( \alpha \). Thus, measurements of \( \Delta \) and \( A_{CP}(B^\pm \to \rho^\pm \gamma) \) determine the angle \( \alpha \) in the SM [28], providing complementary information on this angle.

As a possible candidate for new physics, we discuss supersymmetric effects on \( \Delta \) and \( A_{CP} \) which enter through the modification of the Wilson coefficient \( C_7^{(0)_{\text{eff}}}(m_b) \) and some \( O(\alpha_s) \) functions (called \( A_R^{(1)\ell} \) and \( A_I^{(1)\ell} \) in Ref. [28]). In addition, \( F_1 \) and \( F_2 \) are also in general modified. We shall restrict ourselves to the minimal supersymmetric model (MSSM) case, in which one expects additional contributions to \( \Delta M_d \), \( \Delta M_s \), and \( \epsilon_K \), but no new phases, leading to a shift in the apex of the CKM-unitarity triangle. The branching ratio \( \mathcal{B}(B \to X_s \gamma) \) constrains the real and imaginary parts
of \( C_{7}^{\text{eff}} \). Including upper bounds from the electric dipole moment of the neutron leads to \( \text{Im}(C_{7}/C_{7}^{\text{SM}}) \ll 1 \) \cite{36,37}. Depending on the supersymmetric parameters, one has typically three different situations: (i) Small \( \tan \beta \)-solution (say, \( \tan \beta = 3 \)): \( \text{Re}(C_{7}^{\text{eff}}/C_{7}^{\text{eff,SM}}) = 1 \pm 15\% \); (ii) Medium \( \tan \beta \)-solution (say, \( \tan \beta = 10 \)): \( \text{Re}(C_{7}^{\text{eff}}/C_{7}^{\text{eff,SM}}) = 0.7–1.2 \); (iii) Large \( \tan \beta \)-solution (say, \( \tan \beta = 30 \)). In the last case, two possible branches are: either \( \text{Re}(C_{7}^{\text{eff}}/C_{7}^{\text{eff,SM}}) = 0 \).\( \frac{7–1}{2} \), or \( \text{Re}(C_{7}^{\text{eff}}/C_{7}^{\text{eff,SM}}) = (−0.8)–(−1.5) \). It is this second possibility involving a flip of the sign of \( C_{7}^{\text{eff}} \), compared to the SM, which is of particular interest in the context of the isospin-violating ratio \( \Delta_{\text{NLO}} \) and the CP asymmetry \( A_{\text{CP}}(B^\pm \rightarrow \rho^\pm \gamma) \). The interference of the SD- and LD-contribution in the amplitude, impacting on \( \Delta \) and \( A_{\text{CP}}(B^\pm \rightarrow \rho^\pm \gamma) \), is indeed sensitive to the sign of \( C_{7}^{\text{eff}} \). Possible supersymmetric effects on \( \Delta \) and \( A_{\text{CP}}(B^\pm \rightarrow \rho^\pm \gamma) \) are shown in Figs. 2, where the assumed values of \( \text{Re}(C_{7}^{\text{eff}}/C_{7}^{\text{eff,SM}}) \) are also indicated.

4 The decays \( B \rightarrow (K,K^*)\ell^+\ell^- \) in the SM

4.1 The effective Hamiltonian approach

The decays \( B \rightarrow (X_s,X_d)\ell^+\ell^- \), as well as their exclusive counterparts such as the decays \( B \rightarrow (K,K^*,\pi,\rho)\ell^+\ell^- \), provide the possibility of measuring Dalitz-distributions in a number of variables from which the effective vertices in the underlying theory can be extracted \cite{38}. Inclusive decays are theoretically more robust than exclusive decays which require additionally the knowledge of the relevant form factors. However, despite comparatively larger branching ratios, inclusive decays are more difficult to measure, in particular, in experiments operating at hadron machines. Exclusive semileptonic decays are accessible to a wider variety of experiments \cite{39,40}. We review here the anticipated profiles for the decays \( B \rightarrow (K,K^*)\ell^+\ell^- \) in the SM and some variants of supersymmetry.

At the quark level, the semileptonic decay \( b \rightarrow s\ell^+\ell^- \) can be described in terms of the effective Hamiltonian

\[
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^*V_{tb}\sum_{i=1}^{10} C_i(\mu)O_i(\mu),
\]

where unitarity of the CKM matrix and the numerical hierarchy \( V_{us}^*V_{ub} \ll V_{ts}^*V_{tb} \) are implicit. The definitions of the operators and the expressions for the
Wilson coefficients in the SM can be seen in Ref. [41]. Restricting ourselves to
the SM and SUSY, the short-distance contributions in the decays $B \to X_s \gamma$
and $B \to X_s \ell^+ \ell^-$, and the exclusive decays of interest to us, are determined
by three coefficients, called $C_7^{eff} \equiv C_7 - C_5/3 - C_6$, $C_9$ and $C_{10}$ [41].

The above Hamiltonian leads to the following free quark decay amplitude
for $b \to s \ell^+ \ell^-$:

$$
\mathcal{M}(b \to s \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{tb} \left\{ C_9^{eff} \left[ \bar{s} \gamma_\mu L b \right] \left[ \bar{\lambda} \gamma^\mu \lambda \right] + C_{10} \left[ \bar{s} \gamma_\mu L b \right] \left[ \bar{\lambda} \gamma_\mu \gamma_5 \lambda \right] 
- 2 \hat{m}_b C_7^{eff} \left[ \bar{s} i \sigma_\mu \nu \frac{q^\nu}{\hat{s}} R b \right] \left[ \bar{\lambda} \gamma^\mu \lambda \right] \right\} .
$$

(28)

Here, $s = q^2$, $q = p_+ + p_-$, $p_\pm$ are the four-momenta of the leptons, respectively,
and the hat on a variable denotes its normalized value with respect to the
$B$-meson mass, $m_B$, e.g., $\hat{s} = s/m_B^2$, $\hat{m}_b = m_b/m_B$, and we denote by
$m_b \equiv m_b(\mu)$ the $\overline{\text{MS}}$ mass evaluated at a scale $\mu$. Note that $\mathcal{M}(b \to s \ell^+ \ell^-)$,
although a free quark decay amplitude, contains certain long-distance effects
from the matrix elements of four-quark operators, $\langle \ell^+ \ell^- s | O_i | b \rangle$, $1 \leq i \leq 6$,
which usually are absorbed into a redefinition of the short-distance Wilson-
coefficients. Thus, the effective coefficient of the operator $O_9$ is defined as

$$
C_9^{eff}(\hat{s}) = C_9 + Y(\hat{s}) ,
$$

(29)

where $Y(\hat{s})$ stands for the above-mentioned matrix elements of the four-quark
operators. A perturbative calculation yields [41]:

$$
Y_{\text{pert}}(\hat{s}) = g(m_c, \hat{s}) C^{(0)} - \frac{1}{2} g(1, \hat{s}) \left( 4 C_3 + 4 C_4 + 3 C_5 + C_6 \right)
- \frac{1}{2} g(0, \hat{s}) \left( C_3 + 3 C_4 \right) + \frac{2}{9} \left( 3 C_3 + C_4 + 3 C_5 + C_6 \right) ,
$$

(30)

with $C^{(0)} \equiv 3 C_1 + C_2 + 3 C_3 + C_4 + 3 C_5 + C_6$. The functions $g(\hat{m}, \hat{s})$ are
given in Ref. [41]. Thus, whereas the coefficients $C_7^{eff}$ and $C_{10}$ are constants,
the effective coefficients $C_9^{eff}$ is a $\hat{s}$-dependent function and has a non-local
character.

Nonperturbative effects originate in particular from resonance contributions.
It is essentially only the $c\bar{c}$-resonant contribution that matters.

---

†In general, more operators are present in supersymmetric theories and their possible
effects are discussed in Ref. [23].
Ref. [42] suggests to add the contributions from $J/\Psi, \Psi', \ldots$ to the perturbative result, with the former parametrized in the form of Breit-Wigner functions with known widths. The function $Y(\hat{s})$ is then replaced by

$$Y_{\text{ann}}(\hat{s}) = Y_{\text{pert}}(\hat{s}) + \frac{3\pi}{\alpha^2 C(0)} \sum_{V_i=\psi(1s), \ldots, \psi(6s)} \Omega_i \frac{\Gamma(V_i \to \ell^+\ell^-) m_{V_i}}{m_{V_i}^2 - \hat{s} m_B^2 - im_{V_i} \Gamma_{V_i}}. \quad (31)$$

The phenomenological factors $\Omega_i$ can be fixed from the relation

$$B(B \to K^{(*)}V_i \to K^{(*)}\ell^+\ell^-) = B(B \to K^{(*)}V_i) B(V_i \to \ell^+\ell^-), \quad (32)$$

where the right-hand side is given by data [4].

In Ref. [43], Krüger and Sehgal have argued that the perturbative/non-perturbative dichotomy in $Y(\hat{s})$ can be avoided by using the measured cross-section $\sigma(e^+e^- \to \text{hadrons})$ together with the assumption of the quark-hadron duality for large $\hat{s}$ to reconstruct $Y(\hat{s})$ from its imaginary part by a dispersion relation. In principle, this approach has a merit. However, the quark-hadron duality argument is not completely quantitative either, as perturbative contributions in $\sigma(e^+e^- \to \text{hadrons})$ and $B(B \to X_s\ell^+\ell^-)$ are not identical. In particular, the (non-local) perturbative part of $Y(\hat{s})$ has genuine hard contributions proportional to $m_b^2$, which can neither be ignored nor taken care of by the $e^+e^-$ data. The issue remains as to how much of the genuine perturbative contribution in $B \to X_s\ell^+\ell^-$ arising from the $c\bar{c}$-continuum should be kept and there is at present no unique solution to this problem. Luckily, the inherent theoretical uncertainty in the two approaches is not overwhelming and the SD-contribution can be meaningfully extracted from data.

### 4.2 Dilepton invariant mass and the forward-backward asymmetry

Exclusive decays $B \to (K, K^*)\ell^+\ell^-$ are described in terms of matrix elements of the quark operators in Eq. (28) over meson states, which can be parametrized in terms of form factors. For $B \to K$ transition, there are three of them, called $f_+(s), f_0(s)$, involving the vector current, and $f_T(s)$, entering in the matrix element of the tensor current. For the vector meson $K^*$, there are five form factors in the $V - A$ current, but only four are independent which we call as $A_1, A_2, A_0$ and $V$. The tensor part involves three form factors, called $T_1, T_2$ and $T_3$ [28], with $T_1(0) = T_2(0)$. 12
The dilepton mass spectrum for the decays $B \to (K, K^*)\ell^+\ell^-$ and the forward backward asymmetry (FBA), defined as \[12\]

$$\frac{dA_{FB}}{d\hat{s}} = -\int_{0}^{\hat{u}(\hat{s})} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}}, \tag{33}$$

where the variable $\hat{u}$ corresponds to $\theta$, the angle between the momentum of the $B$ meson and the positively charged lepton $\ell^+$ in the dilepton center of mass system, are worked out in a number of papers (see, for example, Ref. [23]). Assuming CP-symmetry, the forward-backward asymmetry in the decays of the $B$ and $\bar{B}$ mesons are equal and opposite \[44\].

For $B \to K$ transitions, the dilepton invariant mass spectrum is simplified

$$\frac{d\Gamma}{d\hat{s}} \sim \left| V_{ts}^* V_{tb} \right|^2 \left( |C_{9}^{eff} f_+(s)|^2 + \frac{2m_b}{1 + \hat{m}_K} C_{7}^{eff} f_T(s)^2 + |C_{10} f_+(s)|^2 \right). \tag{34}$$

Note, there is no dependence on the form factor $f_-$ for $m_\ell = 0$. Also, since $|C_{7}^{eff}| \ll |C_{9}^{eff}|, |C_{10}|$, to a good approximation $d\Gamma/d\hat{s} \propto |f_+|^2$, with the effect from the $C_{7}^{eff} f_T$ term of order $-10\%$. Hence, the $B \to K\ell^+\ell^-$ decay is dominated by the matrix element of the vector current, just like the charged current induced transition $B \to \pi\ell\nu_\ell$. This observation could be put to good use to determine the CKM ratio $|V_{ub}/V_{ts}^* V_{tb}|$ \[13\]. The FBA vanishes in $B \to K\ell^+\ell^-$ decays.

For the $B \to K^*\ell^+\ell^-$ transition, there is no dependence on the form factor $A_0$ for $m_\ell = 0$. However, in general, the dilepton invariant mass distribution depends on all the three effective Wilson coefficients and the form factors discussed above \[23\]. The expression for the FBA for the decay $B \to K^*\ell^+\ell^-$ is rather simple and instructive

$$\frac{dA_{FB}}{d\hat{s}} = \frac{G_F^2 \alpha^2 m_B^5}{2^8 \pi^5} \left| V_{ts}^* V_{tb} \right|^2 \hat{s}\hat{u}(\hat{s})^2$$

$$\times C_{10} \left[ \text{Re}(C_{9}^{eff}) V A_1 + \frac{\hat{m}_b}{\hat{s}} C_{7}^{eff} (VT_2(1 - \hat{m}_{K^*}) + A_1 T_1(1 + \hat{m}_{K^*})) \right]. \tag{35}$$

The position of the zero $\hat{s}_0$ of the FBA is given by

$$\text{Re}(C_{9}^{eff}(\hat{s}_0)) = -\frac{\hat{m}_b}{\hat{s}_0} C_{7}^{eff} \left\{ \frac{T_2(\hat{s}_0)}{A_1(\hat{s}_0)} (1 - \hat{m}_{K^*}) + \frac{T_1(\hat{s}_0)}{V(\hat{s}_0)} (1 + \hat{m}_{K^*}) \right\}, \tag{36}$$
which depends on the value of $m_b$, the ratio of the effective coefficients
$C_7^{\text{eff}} / \text{Re}(C_9^{\text{eff}}(\hat{s}_0))$, and the ratio of the form factors shown above. It is interesting to observe that using the heavy quark symmetry, formulated in the form of a Large Energy Effective Theory (LEET) \[46\], both the ratios of the form factors appearing in Eq. (36) can be shown to have essentially no hadronic uncertainty, i.e., all dependence on the form factors cancels:

$$
\frac{T_2}{A_1} = \frac{1 + \hat{m}_{K^*}}{1 + \hat{m}_{K^*}^2 - \hat{s}} \left( 1 - \frac{\hat{s}}{1 - \hat{m}_{K^*}^2} \right), \quad \frac{T_1}{V} = \frac{1}{1 + \hat{m}_{K^*}}. \tag{37}
$$

With these relations, one has a particularly simple form for the right hand side in Eq. (36) determining $\hat{s}_0$, namely

$$
\text{Re}(C_9^{\text{eff}}(\hat{s}_0)) = -2 \frac{\hat{m}_b}{\hat{s}_0} C_7^{\text{eff}} \frac{1 - \hat{s}_0}{1 + \hat{m}_{K^*}^2 - \hat{s}_0}. \tag{38}
$$

Thus, the zero-point of the FB-asymmetry in $B \to K^*\ell^+\ell^-$ is determined essentially by the precision of the ratio of the effective coefficients and $m_b$, putting it on almost same footing as the corresponding quantity in the inclusive decays $B \to X_s\ell^+\ell^-$, for which the zero-point of the FBA is given by the solution of the equation $\text{Re}(C_9^{\text{eff}}(\hat{s}_0)) = -\frac{2}{\hat{s}_0} C_7^{\text{eff}}$. The insensitivity of $\hat{s}_0$ to the decay form factors in $B \to K^*\ell^+\ell^-$ has also been pointed out in Ref. \[47\] based on a comparative study of a number of form factor models. Perturbative stability of the LEET results given in Eqs. (37) has recently been studied by Beneke and Feldmann \[48\], who find that the $O(\alpha_s)$ corrections, modifying the above relations, are within 10%. Thus, the LEET-based result in Eq. (38) stands theoretically on quite rigorous grounds, making the zero-point of the FBA a precision test of the SM. Typically, $\hat{s}_0 = 0.10$ (i.e. $s_0 = 2.9 \text{ GeV}^2$) in the SM.

While none of the experiments has so far reached the SM-sensitivity in $B \to (K, K^*)\ell^+\ell^-$ decays, some do provide interesting upper limits on the parameter space of models with new physics. This has been worked out in the context of the SUSY models in Ref. \[23\]. The exclusive decay $B \to K^*\mu^+\mu^-$ provides at present the most stringent bounds on the effective coefficients in the SM. The current upper limit \[39\] and the SM-expectations \[23\] are:

$$
\mathcal{B}_{nr}(B \to K^*\mu^+\mu^-)_{\text{SM}} = 2.0 \pm 0.5 \times 10^{-6}, \quad \mathcal{B}_{nr}(B \to K^*\mu^+\mu^-)_{\text{CDF}} < 4.0 \times 10^{-6} \text{ (at 90\% C.L.)}. \tag{39}
$$
5 The decays $B \to (K, K^*)\ell^+\ell^-$ in SUSY

Rare $B$-decays $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$ have been extensively studied in the context of supersymmetric theories. Recent developments can be seen in Refs. [36, 37, 49, 50, 51, 52]. We do not consider models with broken R-parity and assume that there are no new phases from *new physics* beyond the SM. This covers an important part of the supersymmetric parameter space, but not all.

Possible contributions from new physics (NP) in the relevant Wilson coefficients can be taken into account by the (correlated) ratios, $(i = 7, 9, 10)$:

$$R_i(\mu) \equiv \frac{C_i^{NP} + C_i^{SM}}{C_i^{SM}} = \frac{C_i}{C_i^{SM}},$$

which depend on the renormalization scale (except for $C_{10}$). The experimental constraint from $B \to X_s\gamma$ translates into the bound

$$0.80 < |R_7(\mu = 4.8 \text{ GeV})| < 1.20,$$

where the coefficients are understood to be calculated in the LLA precision. Some representative cases are discussed below.

5.1 $B \to (K, K^*)\ell^+\ell^-$ in SUGRA models

The parameter space of these models may be decomposed into two qualitatively different regions, which can be characterized by tan $\beta$ values. For small tan $\beta$, say tan $\beta \sim 3$, the sign of $C_{7}^{\text{eff}}$ is the same as in the SM. Here, no spectacular deviations from the SM can be expected in the decays $B \to (K, K^*)\ell^+\ell^-$. For large tan $\beta$, the situation is more interesting due to correlations involving the branching ratio for $B \to X_s\gamma$, the mass of the lightest CP-even Higgs boson, $m_h$, and sign($\mu_{\text{susy}}$), appearing in the Higgs superpotential. The interesting scenario for SUSY searches in $B \to (K, K^*)\ell^+\ell^-$ is the one in which sign($\mu_{\text{susy}}$) and $m_h$ admit $C_{7}^{\text{eff}}$ to be positive [36]. In this case one expects a constructive interference of the terms depending on $C_{7}^{\text{eff}}$ and $C_9$ in the dilepton invariant mass spectra. For the sake of illustration, we use

$$R_7 = -1.2, \quad R_9 = 1.03, \quad R_{10} = 1.0,$$

obtained for tan $\beta = 30$ [50], as a representative large-tan $\beta$ solution.
In Fig. 3, we show the dilepton invariant mass spectrum for the decay $B \to K\ell^+\ell^-$. Below the $J/\psi$ mass, the decay rate is enhanced in SUSY by about 30% compared to the SM. This enhancement is difficult to disentangle from the non-perturbative uncertainties attendant with the SM-distributions (shown as the shaded band in this figure). The dilepton mass distribution for $B \to K^*\mu^+\mu^-$ is more promising, as in this case the SUSY enhancement is around 100%, see Fig. 4, and this is distinguishable from the SM-related theoretical uncertainties (shown as the shaded band in this figure). The supersymmetric effects in $B \to K^*\ell^+\ell^-$ are very similar to the ones worked out for the inclusive decays $B \to X_s\ell^+\ell^-$ [50], where enhancements of (50–100)% were predicted in the low-$q^2$ branching ratios. The effect of $R_7$ being negative is striking in the FB asymmetry as shown in Fig. 5, in which the two SUGRA curves are plotted using Eq. (42) (for $R_7 < 0$) and by flipping the sign of $R_7$ but keeping the magnitudes of $R_i$ to their values given in this equation. Summarizing for the SUSY theories, large $\tan\beta$ solutions lead to $C_{7\text{eff}}$ being positive, which implies that FB-asymmetry below the $J/\psi$-resonant region remains negative (hence, no zero in the FB-asymmetry in this region) and one expects an enhancement up to a factor two in the dilepton mass distribution in $B \to K^*e^+e^-$ and $B \to K^*\mu^+\mu^-$.

**5.2 $B \to (K, K^*)\ell^+\ell^-$ in the MIA Approach**

The minimal insertion approach aims at including all possible squark mixing effects in a model independent way. Choosing a $q, \tilde{q}$ basis where the $q-\tilde{q}-\tilde{\chi}^0$ and $q-\tilde{q}-\tilde{g}$ couplings are flavor diagonal, flavor changes are incorporated by a non-diagonal mass insertion in the $\tilde{q}$ propagator, which can be parametrized as ($A, B =$Left, Right) [53]

$$
(\delta_{ij}^{up,down})_{A,B} = \frac{(m_{ij}^{up,down})^2_{A,B}}{m_{\tilde{q}}^2},
$$

where $(m_{ij}^{up,down})_{A,B}$ are the off-diagonal elements of the up(down) squark mass squared matrices that mix flavor $i$ and $j$, for both the right- and left-handed scalars, and $m_{\tilde{q}}^2$ is the average squark mass squared. The sfermion propagators are expanded in terms of the $\delta$s. The Wilson coefficients have the following structure ($k = 7, 9, 10$):

$$
C_k = C_k^{SM} + C_k^{diag} + C_k^{MIA},
$$
where $C^{MIA}$ is given in terms of $(\delta_{ij}^{\text{up, down}})^{1,2}$ up to two mass insertions [49], and $C^{\text{diag}}_k$ being the SUSY contribution in the basis where only flavor-diagonal contributions are allowed.

The MIA-SUSY approach has been recently used in the analysis of the decays $B \to X_s \ell^+ \ell^-$ [49], taking into account the present bounds on the coefficient $C^{\text{eff}}_7(m_B)$ following from the decay $B \to X_s \gamma$. The other two coefficients $C^{MIA}_9$ and $C^{MIA}_{10}$ are calculated by scanning over the allowed supersymmetric parameter space [49]. Illustrative examples of the dilepton invariant mass spectrum in the decays $B \to K \mu^+ \mu^-$ and $B \to K^* \mu^+ \mu^-$ in the MIA approach are shown in Figs. 3 and 4, respectively. They have been calculated for the following values:

$$R_7 = \pm 0.83, \quad R_9 = 0.92, \quad R_{10} = 1.61 , \quad (45)$$

which are allowed by the present experimental bounds. The characteristic difference in this case, as compared to the SUGRA models, lies in the significantly enhanced value of $C_{10}$.

A characteristic of the MIA approach is that the sign of $C_{10}$ ($C^{SM}_{10} < 0$) depends on the quantities $(\delta_{23}^u)^{\text{LR}}$ and $(\delta_{23}^d)^{\text{LL}}$. In particular, in this scenario, SUSY effects could change the sign of the Wilson coefficient $C_{10}$. This has no effect on the dilepton invariant mass distributions, as they depend quadratically on $C_{10}$, but it would change the sign of $A_{FB}$ in $B \to K^* \ell^+ \ell^-$. To illustrate this, we use the following parameters

$$R_7 = \pm 0.83, \quad R_9 = 0.79, \quad R_{10} = -0.38 , \quad (46)$$

and plot the resulting normalized FB asymmetry in Fig. 5. The positive FB-asymmetry in $B \to K^* \ell^+ \ell^-$ (as well as in $B \to X_s \ell^+ \ell^-$ shown in [49]) for the dilepton invariant mass below the resonant $J/\psi$ region is rather unique to the MIA approach.

With $O(10^9)$ $B\bar{B}$ events anticipated at the B-factories, and much higher yields at the Tevatron and LHC experiments, measurements of the differential rates and the forward-backward asymmetry in $B \to (K,K^*)\ell^+ \ell^-$ will allow precision tests of the SM. If we are lucky, some of these measurements may lead to the discovery of new physics. We illustrated the case for supersymmetry.
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Figure 1: $\Delta_{\text{LO}}$ and $\Delta_{\text{NLO}}$ in SM with $\epsilon_A = -0.3$ and $|V_{ub}/V_{td}| = 0.48$. (From Ref. [28].)

Figure 2: $\Delta_{\text{LO}}$ (left) and $A_{\text{CP}}(B^\pm \to \rho^\pm \gamma)$ (right) with $\epsilon_A = -0.3$ in the SM (solid line), and in the MSSM with $C_7^{(0)\text{eff}}/C_7^{(0)\text{eff(SM)}} = 0.95$ (dot-dashed line), $C_7^{(0)\text{eff}}/C_7^{(0)\text{eff(SM)}} = 0.8$ (dashed line) and $C_7^{(0)\text{eff}}/C_7^{(0)\text{eff(SM)}} = -1.2$ (dotted line). The SM and MSSM curves correspond respectively to $|V_{ub}/V_{td}| = 0.48$ and $|V_{ub}/V_{td}| = 0.63$. (From Ref. [28].)
Figure 3: The dilepton invariant mass distribution in $B \rightarrow K \mu^+ \mu^-$ decays. The solid line represents the SM and the shaded area depicts the form factor-related uncertainties. The dotted line corresponds to the SUGRA model with $R_7 = -1.2$, $R_9 = 1.03$ and $R_{10} = 1$. The long-short dashed lines correspond to the MIA-SUSY model, given by $R_7 = -0.83$, $R_9 = 0.92$ and $R_{10} = 1.61$. The corresponding pure SD spectra are shown in the lower part of the plot. (From Ref. [23].)

Figure 4: The dilepton invariant mass distribution in $B \rightarrow K^* \mu^+ \mu^-$ decays. Legends are the same as in Fig. 3. (From Ref. [23].)
Figure 5: The normalized forward-backward asymmetry in $B \to K^* \mu^+ \mu^-$ decay as a function of $s$. The solid line denotes the SM prediction. The dotted (long-short dashed) lines correspond to the SUGRA (the MIA-SUSY) model, using the parameters given in Eq. (42) (Eq. (45)) with the upper and lower curves representing the $C_7^{\text{eff}} < 0$ and $C_7^{\text{eff}} > 0$ case, respectively. The dashed curves indicating a positive asymmetry for large $s$ correspond to the MIA-SUSY models using the parameters given in Eq. (46). (From Ref. [23].)