On-demand Entanglement Source with Spatial Phase Modulation

Xiang-Bin Wang, Cheng-Xi Yang, and Yan-Bing Liu

Department of Physics and the Key Laboratory of Atomic and Nanosciences, Ministry of Education, Tsinghua University, Beijing 100084, China

(Dated: November 27, 2009)

Abstract

The polarization entanglement photon pairs generated from the biexciton cascade decay in a single semiconductor quantum dot is corrupted by the position-dependent (time-dependent) phase difference of the two polarization mode due to the fine structure splitting. We show that, by taking voltage ramping to an electro-optic modulator, such phase-difference can be removed. In our first proposed set-up, two photons are sent to two separate Pockels cell under reverse voltage ramping, as a result, the position-dependent phase difference between the two polarization mode is removed in the outcome state. In our second proposed set-up, the polarization of the first photon is flipped and then both photons fly into the same Pockels cell. Since we only need to separate the two photons rather than separate the two polarization modes, our schemes are robust with respect to fluctuations of the optical paths.

*Electronic address: xbwang@mail.tsinghua.edu.cn
I. INTRODUCTION

Quantum entanglement plays an important role in the study of fundamental principles of quantum mechanics[1]. It is also the most important resource in quantum information processing[2, 3]. Among all types of quantum entanglement, polarization entangled photon-pairs are particularly useful because of easy manipulation and transmission. There are many mature techniques to produce such entangled pairs probabilistically[4, 5, 6, 7], while an on-demand entangled photon pair is essential in many tasks in quantum information processing.

Recently, an on-demand entangled photon-pair source was proposed[8] and realized in a semiconductor quantum dot system[9, 10, 11]. However, because of the fine-structure splitting (FSS) there, the relative phase of the entangled state is randomized so that only classical correlation can be detected by traditional time-integrated measurement[12, 13, 14, 15]. So far, there are many methods proposed to explore this “hidden entanglement”[12, 13, 14, 15, 16, 17, 18, 19, 20], for example, reducing FSS[16, 17, 18, 19], spectral filtering[10], time resolving post-selection[15], and so on. Up to now, the smallest FSS realized in experiment is about 0.3 $\mu$eV and non-classical nature of the radiation field is verified by directly observing violation of the Bell inequality[19]. However, the entanglement quality is considerably decreased even by very small FSS, and further reducing FSS is very difficult in experiment. Furthermore, the severe restriction on FSS greatly limits the selection range of quantum dot systems. Certain quantum dots with large FSS cannot be used even if they have distinct advantage, such as emitting photons of frequencies in the easy transmission frequency window in free space or optical fiber. Also, the post-selection method in frequency domain or time domain will significantly decrease the photon collection efficiency. To overcome all these drawbacks, Stace et al proposed to use cavities to control the frequencies. Latter, Jones and Stace proposed a more simplified, downstream solution of polarization-dependent frequency shift to the photons by an acousto-optical modulator (AOM). Basically, there are 3 steps in the circuit: 1) Split the polarization modes of each photons; 2) Shift the frequency of vertical polarization modes by AOM; 3)Combine split beams by a polarization beam splitter. However, the efficiency of a normal commercially available AOM is fairly low. The efficiency of a very good AOM for a single photon is about 80%. The joint efficiency of two photons in the scheme in Ref[20] is not larger than 64%. Moreover, in the proposed set-up[20], special
FIG. 1: Energy levels of the semiconductor quantum dot used to generate polarization entangled photons. The biexciton state (XX) is a zero-spin state formed by two electrons and two heavy holes. When the dot decays, two photons are emitted sequentially, and their polarization is determined by the “decay path”. Usually an FSS $S$ exists between the two excitons ($X_H$ and $X_V$).

care has to be taken to make the two optical paths stable. Say, a fluctuation of half a micrometer in any optical path will entirely destroy the result.

Here we propose another solution through using an electro-optical modulator (EOM). EOM is a mature technique which has been demonstrated in many experiments. In particular, two-photon interference has been experimentally observed very recently. In our proposed set-up, we use a dichroic mirror to separate the two photons, and then remove the position-dependent phase difference by using Pockels cells under a ramping voltage. Since a Pockels cell itself makes the phase shift differently to different polarization mode, we don’t have to separate the different polarization modes as was proposed in Ref. 20. Instead, we only need to separate the two photons. In this way, compared with the existing proposal, our method has an advantage in its robustness to fluctuations of optical paths. As calculated later, a fluctuation of 1 mm will cause $10^{-3}$ fluctuation in the phase. Moreover, compared with Ref. 20, our scheme seem to have a significantly higher efficiency. A commercially available Pockels cell almost has no loss.

II. THE PROBLEM

The energy levels of the quantum dot used for photon-pair generation are shown in Fig. 1. After exciting a single quantum dot into biexciton state (XX), two photons are emitted sequentially as the dot decays in a cascade process. Because the two exciton states ($X_H$ and $X_V$).
and $X_V$) are not degenerate\cite{22, 23}, the two photons are actually entangled in the complex space of both polarization and frequency

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left[ \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_H(\omega_1, \omega_2) |H_1 H_2; \omega_1, \omega_2\rangle 
+ \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_V(\omega_1, \omega_2) |V_1 V_2; \omega_1, \omega_2\rangle \right].$$

(1)

The spectral functions for the two decay path of the quantum dot system can be written as\cite{10, 24}

$$\Phi_H(\omega_1, \omega_2) = \sqrt{2\Gamma} \frac{1}{2\pi} \frac{1}{\omega_1 + \omega_2 - \omega_0 + i\Gamma} \times \frac{1}{\omega_2 - \omega_{H_2} + i\Gamma/2},$$

(2a)

$$\Phi_V(\omega_1, \omega_2) = \sqrt{2\Gamma} \frac{1}{2\pi} \frac{1}{\omega_1 + \omega_2 - \omega_0 + i\Gamma} \times \frac{1}{\omega_2 - \omega_{V_2} + i\Gamma/2}.$$

(2b)

Here, as shown in Fig. 1, $\omega_{H_2} = \omega_{X_H} - \omega_{GS}$, $\omega_{V_2} = \omega_{X_V} - \omega_{GS}$, and $\omega_0 = \omega_{XX} - \omega_{GS}$, where $\hbar \omega_{XX}$, $\hbar \omega_{X_H}$, $\hbar \omega_{X_V}$, $\hbar \omega_{GS}$ are the eigenenergy of levels $XX$, $X_H$, $X_V$, and $GS$, respectively, and $\Gamma$ is the decay rate of the four transitions $XX \rightarrow X_H$, $XX \rightarrow X_V$, $X_H \rightarrow GS$, and $X_V \rightarrow GS$\cite{10}.

Therefore, the state is actually inseparable in the composite space of both polarization and frequency. This hides the entanglement in polarization space only.

III. PHASE MODULATION WITH POCKELS CELLS

A Pockels cell contains a crystal whose refraction index of a certain optical axis changes linearly with the external voltage, due to the so called electro-optical effects. Consider a Pockels cell with a time-dependent voltage $V(t)$ and a wave packet passing through it, as shown in Fig. 2. For simplicity, we assume that any non-trivial phase modulation only happens to the vertical polarization of the incident light.

Suppose initially the wave function of a wave train in vertical polarization is $e^{ik_{V}x}$, with a certain reference original point $O(0)$, at the left side of the crystal. We shall always use the reference framework of the flying wave train itself, i.e., the reference point $O(t)$ propagates with the wave train, in the same speed. Suppose at time $t_0$, the distance between the
reference point $O(t_0)$ and the left side surface of the crystal is $L$. The ramping voltage $V(t)$ applied to the crystal is a linear function of time $t$, $V(t) = a + bt$. Suppose the position of any point $X(t_0)$ is $x(t_0)$ at this reference framework. At a later time $t = \tau$, the wave train is at the right side of the crystal and the phase of original points $O(t_0), X(t_0)$ have now propagated to points $O(\tau), X(\tau)$, respectively. At time $\tau$, we take $O(\tau)$ as the reference original point and denote $x(\tau)$ as the position of $X(\tau)$ in the new reference framework of $O(\tau)$. To see the phase modulation after the wave train passes the crystal, we study the relation between $x(t_0)$ and $x(\tau)$.

The refraction index of the crystal is linearly dependent on the applied voltage. At any time $t$, the vertical-polarization-mode light speed inside the crystal is

$$v(t) = \frac{v_0}{1 + \eta V(t)} \quad (3)$$

and $v_0$ is the light speed inside the crystal when there is no applied voltage, $\eta$ is a constant parameter which is dependent on the crystal property itself. Suppose the crystal thickness is $s$. At time $t_0$, the original phase at point $O(t_0)$ is $\varphi_{OV}$ or $\varphi_{OH}$, for vertical polarization wave train or horizontal polarization wave train, respectively.

**Frequency shift.** Consider the vertical polarization case first. Suppose it takes time $\Delta t(X)$ for point $X$ to pass through the crystal. Explicitly,

$$\int_{t_{in}(X)}^{t_{in}(X)+\Delta t(X)} v(t) dt = s \quad (4)$$

where $t_{in}(X) = \frac{-x}{c} + L/c$ is the time point that point $X$ in the original wave train reaches the left side of the crystal. For a linearly rising voltage $V(t) = a + bt$, Eq. (4) gives rise to

$$\Delta t(X) = \frac{1 + \eta(a + bL/c - bx/c)}{\eta b} (e^{\eta bs/v_0} - 1). \quad (5)$$

At time point $\tau$, the phases of points $O(t_0), X(t_0)$ have propagated to points $O(\tau), X(\tau)$, respectively. Using the formula above we find that the position of $X(\tau)$ in the new reference framework $O(\tau)$ is

$$x(\tau) = e^{\eta bs/v_0} x. \quad (6)$$

This is to say, after passing through the crystal, the spatial phase function (with reference original point $O(\tau)$) is changed into

$$\varphi_{out}(x(\tau)) = \varphi_{out}(e^{\eta bs/v_0} x(t_0)) = \varphi_{in}(x(t_0)) \quad (7)$$
where $\varphi_{in}(x) = k_V x$ is the spatial phase function of the wave train before passing through the crystal, with reference original point $O(t_0)$. Therefore, the spatial phase for the wave train after passing through the crystal is

$$\varphi_{out}(x) = e^{-\eta bs/v_0} k x,$$

where the reference original point is $O(\tau)$. (For simplicity, here we set all initial phases at reference point to be 0.) This is actually a frequency shift to the wave train. The crystal under voltage ramping of $V(t) = a + bt$ will transform the original frequency $\omega_V$ (or wave vector $k_V$) of the wave train at the left side of the crystal into the new frequency $\omega'_V$ (or wavevector $k'_V$) of the wave train at the right side of the crystal by the following formula:

$$\frac{\omega'_V}{\omega_V} = \frac{k'_V}{k_V} = e^{-\eta bs/v_0} = f(b).$$

Phase change. Eq.(9) is the spatial phase modulation of vertical polarization only. If the original wave train is in horizontal polarization mode, it takes time

$$\Delta t_H = s/v_0$$

for any point $X(t_0)$ in the original wave train to pass through the crystal. At time $\tau$, the original reference point $O(t_0)$ propagates to the new reference point $O_H(\tau)$. In this new reference point, the spatial phase function is

$$\varphi_H(x_H(\tau)) = k_H x_H(\tau),$$

where $k_H$ is the wave vector of the horizontal polarized mode. At the reference framework of $O(\tau)$ (reference point of vertical polarization), the position of $O_H(\tau)$ is

$$d(a, b, L) = c(\tau - \frac{s}{v_0}) - c(\tau - \Delta t(O))$$

$$= \left( \frac{c}{\eta b} + \frac{ac}{b} + L \right) (e^{\eta bs/v_0} - 1) - \frac{cs}{v_0}$$

where $\Delta t(O)$ is given by Eq.(5) with $x = 0$. Therefore, in the same reference framework $O(\tau)$, the spatial phase of horizontal polarization at time $\tau$ is

$$\varphi_H(x) = k_H(x - d(a, b))$$

where $k_H$ is the wave vector of the horizontal polarization mode and $d(a, b)$ is given by \text{Eq.}(30). The spatial phase difference of two polarizations in reference framework $O(\tau)$ is

$$\Delta \varphi(x) = (e^{-\eta bs/v_0} k_V - k_H) x + k_H d(a, b).$$
FIG. 2: Phase modulation. After the wave train passed the crystal (the square box in the middle), the point $X(t_0)$ propagated to $X(\tau)$. The relationship of the positions of $X$ and $X'$ is given in Eq. (6).

From this we can see that the crystal under a time dependent voltage not only changes the frequency, but also offers a position-independent phase difference between the two polarization modes of the outcome wave train. This phase is dependent on the parameters $a, b$ in the linear function $V(t)$.

In the derivation above, we have ignored the possible small dispersion of the crystal. Obviously, our result can be directly extended to this case that the crystal’s refraction index is dependent on the frequency of the incident light by setting $v_0$ and $\eta$ frequency dependent.

For simplicity, we shall only consider the case without dispersion hereafter. In such a case, taking a very similar derivation, we have the following wavefunction transform formulas for arbitrary wavefunction by its polarization mode:

$$
\psi_V(x) \rightarrow e^{-\eta bs/(2v_0)} \psi_V(e^{-\eta bs/v_0} x);
$$

$$
\psi_H(x) \rightarrow \psi_H(x - d(a, b)).
$$

(15)

IV. OUR SCHEME

We propose two schemes here.

As shown in Fig. 3, scheme 1 contains two EOM phase modulators with ramping voltage of $V_1(t) = a_1 + b_1 t$ and $V_2(t) = a_2 + b_2 t$. The two photons are separated by their frequency, and then pass through each modulator. Each voltage ramping covers the time that each photon pass through its modulator, as shown in Fig. 4. Scheme 1 has no limit to the frequencies of the two photons, say, no matter they are quite close or quite different. However, as shown
FIG. 3: Proposed scheme 1. The first photon and second photon are separated by a dichroic mirror (DM). The two Pockels cells start to run before the photons arrive. They make reverse phase modulation.

FIG. 4: Voltage ramping in scheme 1.

below, we need the starting time difference of two ramping voltages be controlled much smaller than 1 ns.

If the frequency difference of two photons is rather small compared with the frequencies of each photons, we can use Scheme 2. In scheme 2, the problem of starting time difference control is circumvented. Scheme 2 contains only one EOM phase modulator under voltage ramping of $V(t) = a + bt$, as shown in Fig.(5). Two photons are separated and they pass through the modulator in different path. Polarization of photon one is flipped before it reaches the modulator. Also, the voltage ramping covers the time of both photon passing through the modulator.
FIG. 5: Proposed scheme 2. The first photon and second photon are separated by a dichroic mirror (D). Polarization of photon 1 is flipped by a flipper (F) before enters the Pockels cell P. Here both photons enter the same Pockels cell.

A. Voltage ramping of scheme 1

There are two photons emitted from the quantum dot. If we transform Eq. (1) from frequency space to position space, the state of the field can be rewritten as

$$|\Psi_{in}\rangle = \frac{\Gamma}{c} \int \int_{0>x_1>x_2} dx_1 dx_2 e^{\frac{\Gamma}{c} (x_2+x_1)}$$

$$\times (e^{i(k_{H_1}x_1+k_{H_2}x_2)}|H_1H_2\rangle + e^{i(k_{V_1}x_1+k_{V_2}x_2)}|V_1V_2\rangle),$$

(16)

where $x_1$ and $x_2$ refer to the position of the first photon and second photon, respectively, with a common reference point, the right end of the wave packet. Eq. (16) is equivalent to the result given in Ref. [14]. Suppose at a certain time $t_0$, the reference point $O(t_0)$ arrives at the dichroic mirror. According to Eq. (15), the outcome state is

$$|\Psi_{out}\rangle = \frac{\Gamma}{c} \int \int_{0>x_1-d_1>x_2-d_2} dx_1 dx_2 A_H |H_1H_2\rangle$$

$$+ \frac{\Gamma}{c} \sqrt{f_1 f_2} \int \int_{0>f_1 x_1>f_2 x_2} dx_1 dx_2$$

$$\times A_V e^{i\Delta \varphi_1 + i\Delta \varphi_2} |V_1V_2\rangle,$$

(17)

where $f_i = e^{-\eta_{bs}/m_0}$, $\Delta \varphi_i$ is given by Eq. (14), with parameters $a = a_i$, $b = b_i$, $L = L_i$ there;

$$A_H = e^{\frac{\Delta}{c} (x_1+x_2-d_1-d_2)}$$

$$A_V = e^{\frac{\Delta}{c} (f_2 x_2+f_1 x_1)}$$

(18)
and $d_i$ is given by Eq. (30) with parameters $a = a_i, b = b_i$. If we set
\[
\begin{align*}
  b_1 &= \frac{v_0}{\eta s} \ln \left( \frac{k v_1}{k H_1} \right), \\
  b_2 &= \frac{v_0}{\eta s} \ln \left( \frac{k v_2}{k H_2} \right), \tag{19}
\end{align*}
\]
we find that
\[
\begin{align*}
  \Delta \varphi_1 + \Delta \varphi_2 &= k H_1 \left( \frac{c}{\eta b_1} + \frac{ca_1}{b_1} + L_1 \right) \left( e^{\eta b_1/s/v_0} - 1 \right) \\
  &\quad + k H_2 \left( \frac{c}{\eta b_2} + \frac{ca_2}{b_2} + L_2 \right) \left( e^{\eta b_2/s/v_0} - 1 \right) - k_0 cs/v_0. \tag{20}
\end{align*}
\]
where $L_1, L_2$ are the optical path from $O(t_0)$ to the two separate Pockels cells, $k_0 = k H_1 + k H_2$ is a constant (i.e., position independent). Therefore the value above is a constant phase independent of positions $x_1, x_2$. Also, we find that
\[
\sqrt{f_1 f_2 A_V/A_H} = \sqrt{f_1 f_2} \exp \left[ \frac{\Gamma}{2c} (f_2 x_2 + f_1 x_1) - \frac{\Gamma}{2c} (x_1 + x_2 - d_1 - d_2) \right] = 1 + \epsilon \tag{21}
\]
where $\epsilon$ is in the magnitude order of $10^{-6}$, given the coherence length of the wave train $L \approx 0.3$ m and $\Gamma \approx 10^9$. Therefore, with the setting of Eq. (19), our scheme 1 can produce high quality polarization entangled photon pairs.

In our schemes, we only split the two photons by frequency difference instead of splitting the polarization mode. Even though the optical paths of each photons may fluctuate significantly, the result only changes negligibly. This is different from the AOM based scheme in Ref. [20] which separates two polarization modes.

A fluctuation of amount $\delta l_i$ in the optical path of the $i$'th photon will cause a fluctuation of amount
\[
\delta \varphi_i = k H_i \delta l_i (e^{\eta b_i/s/v_0} - 1) \tag{22}
\]
This means, even a fluctuation of 1 mm in one of the optical path will cause only a phase difference fluctuation of in the magnitude order of $10^{-3}$ in our scheme.

Since $a_1, a_2$ plays no role in our scheme, we can ramp the voltage from 0, i.e., setting $a_1 = a_2 = 0$. We can also consider the consequence of non-exact simultaneous voltage ramping of the two modulators. Suppose the starting times of ramping are $t_1$ and $t_2$, respectively. To be sure that the voltage ramping covers the incident wave train, we need $t_1 \leq t_0$ and $t_2 \leq t_0$. This is equivalent to set $a_1 = b_1(t_0 - t_1)$ and $a_2 = b_2(t_0 - t_2)$ and start
the voltage ramping exactly at \( t = t_0 \). The fluctuation in the value of Eq.(20) is now

\[
\delta(\varphi) \approx \frac{cnS}{v_0} [k_{H_1} b_1(t_0 - t_1) + k_{H_2} b_2(t_0 - t_2)] \approx c k_S \delta t
\]

(23)

where \( k_S = k_{V_1} - k_{H_1} \) which correspond to FSS, and \( \delta t = t_2 - t_1 \). We see that our result only dependent on the time difference \( \delta t \), independent of the absolute time. Therefore, the trigger time uncertainty of the quantum dot does not affect the result here. To obtain high quality entanglement, we need

\[
\delta t << \frac{1}{ck_S}
\]

(24)

Given an FSS of 1 GHz, we need the ramping time difference to be much smaller than 1 ns. The consequence of time difference \( \delta t \) here is equivalent to the time window post-selection scheme with time resolving detection[15]. However, here in our scheme there is almost no photon loss.

In scheme 1, we need to control the time difference of two voltage ramping in a rather small range (much less than 1 ns). As shown below, our scheme 2 has an intrinsic fault tolerance property, where the technical problem of simultaneous ramping is bypassed.

**B. Robustness of scheme 2**

In our scheme 2 as shown in Fig.(5), we can set \( V(t) = bt \) and

\[
b = \frac{v_0}{\eta S} \ln \left( \frac{k_{H_1}}{k_{V_1}} \right),
\]

(25)

Before the photons pass through each crystal, the state is

\[
|\Psi_{in}\rangle = \frac{\Gamma}{c} \int \int_{0 > x_1 > x_2} dx_1 dx_2 e^{\frac{\Gamma}{2c}(x_2 + x_1)}
\]

\[
\times \left( e^{i(k_{H_1} x_1 + k_{H_2} x_2)}|V_1 H_2\rangle + e^{i(k_{v_1} x_1 + k_{v_2} x_2)}|H_1 V_2\rangle \right),
\]

(26)

After the two photons pass through the same Pockels cell under voltage ramping, the state is

\[
|\Psi_{out}\rangle = \frac{\Gamma}{c} \sqrt{f} \int \int_{0 > f x_1 > x_2 - d_2} dx_1 dx_2 A_H |V_1 H_2\rangle
\]

\[
+ \frac{\Gamma}{c} \sqrt{f} \int \int_{0 > x_1 - d_1 > f x_2} dx_1 dx_2
\]

\[
\times A_V e^{-i\Delta \varphi_1 + i\Delta \varphi_2} |H_1 V_2\rangle,
\]

(27)
where

$$\Delta \varphi_1 = (f k_{H_1} - k_{V_1}) x_1 + k_{V_1} d_1,$$

$$\Delta \varphi_2 = (f k_{V_2} - k_{H_2}) x_2 + k_{H_2} d_2,$$

(28)

with $f = e^{-\eta bs/v_0}$;

$$A_H = e^{\frac{c}{2\pi} (f x_1 + x_2 - d_2)}$$

$$A_V = e^{\frac{c}{2\pi} (f x_2 + x_1 - d_1)}$$

(29)

As defined in Eq.(25), here $b_1 = b_2 = b$ and $a_1 = a_2 = 0$. According to Eq.(30), $d_i$ here is

$$d(a = 0, b, L_i) = \left( \frac{c}{\eta b} + L_i \right) \left( e^{\eta bs/v_0} - 1 \right) - \frac{cs}{v_0}$$

(30)

Direct calculations show that $|A_V/A_H| - 1$ is in the magnitude order of $10^{-6}$. As shown earlier, the consequence to the final outcome due to the optical path fluctuation is negligible, therefore we assume zero fluctuation in the optical paths. Also, since both photons enter the same Pockels cell, there is no ramping voltage starting time difference. After calculation, we find that the only non-constant term in $\Delta \varphi_2 - \Delta \varphi_1$ is

$$\epsilon_2 \approx \frac{k_S \Delta k}{k_{H_1}} x_2$$

(31)

where $\Delta k = k_{V_2} - k_{H_1}$, which is position dependent. In the set-up of Ref.[9], the coherence length of the whole wave train is only about 0.3m, so the maximal value of $k_s x_2$ is around 1, also, the magnitude order of $\Delta k/k_{H_1}$ is $10^{-3}$, therefore the position-dependent term $\epsilon_2$ is around $10^{-3}$ and hence negligible.

C. Feasibility

Similar EOM phase modulation has been used in laser spectroscopy, such as Pound-Drever-Hall laser frequency stabilization[25]. Technically, such phase modulation can be accomplished by a commercially available optical device, such as a Pockels cell, which introduces a phase shift to vertically polarized mode. In passing through the Pockels cell, a photon will acquire an additional phase shift $\alpha V$ given the applied voltage $V$. Here $\alpha$ is the phase sensitivity of the Pockels cell. Obviously, $\alpha$ is related to the parameters used in our earlier calculations by

$$n_0 \eta S = \frac{\alpha \lambda}{2\pi}$$

(32)
where $\lambda$ is the wave length of the incident light and $n_0$ equals to $c/v_0$. As far as we have known, the phase sensitivity $\alpha$ of commercially available Pockels cells can be up to 52 mrad/volt @ 830nm [26]. In order to compensate an FSS of $1 \mu eV = 2\pi \times 254.6\, MHz$, we only need to set $b$ and $b'$ around 30 V/ns according to Eq. (19). This is obviously doable by the existing technology. Because the duration of the field radiation is about several nanoseconds [15], the scan voltage needs only last several nanoseconds. Therefore, the maximal voltage requested is only a few hundred volts according to Eq. (19), and this is easily accessible. Moreover, we can also choose to arrange several Pockels cells in series along one photon’s path to compensate larger FSS.

V. CONCLUDING REMARK

We have shown how to compensate the position dependent phase in the entangled photon pair generated by the biexciton cascade decay in a single semiconductor quantum dot with FSS. The EOM phase modulation is done by voltage ramping on a Pockels cell. It is shown that our proposed schemes are robust with respect to imperfections such as optical path fluctuation. With our scheme, the quality of entangled photon pairs can be improved to almost perfect level.

Acknowledgments

Acknowledgments— We would like to thank H. P. Zeng, C.Z. Peng, S. Jiang, Jia-Zhong Hu and Ming Gao for helpful discussions. This work was supported in part by the National Basic Research Program of China grant nos 2007CB907900 and 2007CB807901, NSFC grant number 60725416, and China Hi-Tech program grant no. 2006AA01Z420.

[1] J. J. Sakurai, Modern Quantum Mechanics, (Addison-Wesley Publishing Company, 1994).
[2] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, (Cambridge University Press, Cambridge, 2000).
X. B. Wang, T. Hiroshima, A. Tomita, and M. Hayashi, Physics Reports 448, 1 (2007);
N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. 74, 145 (2002); S. L. Braunstein, and P. van Loock, Rev. Mod. Phys. 77, 513 (2005).

T. E. Kiess, Y. H. Shih, A. V. Sergienko, and C. O. Alley, Phys. Rev. Lett. 71, 3893 (1993).
P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, Phys. Rev. Lett. 75, 4337 (1995).

K. Edamatsu, G. Oohata, R. Shimizu, and T. Itoh, Nature 431, 167 (2004).

D. Fattal, K. Inoue, J. Vuckovic, C. Santori, G. S. Solomon, and Y. Yamamoto, Phys. Rev. Lett. 92, 037903 (2004).

O. Benson, C. Santori, M. Pelton, and Y. Yamamoto, Phys. Rev. Lett. 84, 2513 (2000).

R. M. Stevenson, R. J. Young, P. Atkinson, K. Cooper, D. A. Ritchie, and A. J. Shields, Nature 439, 179 (2006).

N. Akopian, N. H. Lindner, E. Poem, Y. Berlatzky, J. Avron, D. Gershoni, B. D. Gerardot, and P. M. Petroff, Phys. Rev. Lett. 96, 130501 (2006).

A. J. Shields, Nature Photonics 1, 215 (2007).

T. M. Stace, G. J. Milburn, C. H. W. Baenes, Phys. Rev. B 67, 085317 (2007).

Z. Q. Zhou, C. F. Li, G. Chen, J. S. Tang, Y. Zou, M. Gong, and G. C. Guo, arXiv:0909.0078 (2009).

A. J. Hudson, R. M. Stevenson, A. J. Bennett, R. J. Young, C. A. Nicoll, P. Atkinson, K. Cooper, D. A. Ritchie, and A. J. Shields, Phys. Rev. Lett. 99, 266802 (2007).

R. M. Stevenson, A. J. Hudson, A. J. Bennett, R. J. Young, C. A. Nicoll, D. A. Ritchie, and A. J. Shields, Phys. Rev. Lett. 101, 170501 (2008).

R. J. Young, R. M. Stevenson, P. Atkinson, K. Cooper, D. A. Ritchie, and A. J. Shields, New Journal of Physics 8, 29 (2006).

R. Hafenbrak, S. M. Ulrich, P. Michler, L. Wang, A. Rastelli, and O. G. Schmidt, New Journal of Physics 9, 315 (2007).

L. He, M. Gong, C.-F. Li, G.-C. Guo, and A. Zunger, Phys. Rev. Lett. 101, 157405 (2008).

R. J. Young, R. M. Stevenson, A. J. Hudson, C. A. Nicoll, D. A. Ritchie, and A. J. Shields, Phys. Rev. Lett. 102, 030406 (2009).

Nick S. Jones and T. M. Stace, Physical Review A 73, 033813 (2006)

S. Barrett, Nature Photonics, 3, 430 (2009)
[22] D. Gammon, E. S. Snow, B. V. Shanabrook, D. S. Katzer, and D. Park, Phys. Rev. Lett. 76, 3005 (1996).

[23] M. Bayer, G. Ortner, O. Stern, A. Kuther, A. A. Gorbunov, A. Forchel, P. Hawrylak, S. Fafard, K. Hinzer, T. L. Reinecke, S. N. Walck, J. P. Reithmaier, F. Klopf, and F. Schafer, Phys. Rev. B 65, 195315 (2002).

[24] M. O. Scully and M. S. Zubairy, Quantum optics (Cambridge University Press, Cambridge, 1997).

[25] E. D. Black, American Journal of Physics 69, 79 (2001).

[26] Http://www.conoptics.com/Modulation-Systems-Products.html