Improved 2D-MUSIC estimation for low intercept coprime MIMO radar

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Abstract. In electronic warfare, airborne multiple input multiple output (MIMO) radar must not only accurately detect the Angle of moving target, but also ensure the low interception performance of radar. In order to solve this problem, a low interception bistatic MIMO radar with improved coprime array is proposed in this paper. By expanding the coprime array of the bistatic MIMO radar, the array spacing is further expanded to form a sparse array, which reduces the hardware and software cost and improves the anti-jamming capability. In addition, based on the two-dimensional MUSIC algorithm to detect the Angle of the maneuvering target, Taylor expansion method is added to realize the Angle estimation under low SNR. The research results show that, compared with the joint DOD and DOA estimation in a bistatic MIMO radar, the improved method in this paper increases the estimation accuracy, reduces the computational cost of the algorithm, applies to low SNR environment, and ensures low interception performance.

1. Introduction
With the development of information technology, single radar can no longer meet the needs of complex battlefield environment, and the cooperative development of multiple radars has become an inevitable trend[1]. The concept of MIMO radar was formally proposed at the IEEE Radar Conference in 2004[2]. The monostatic radar is located in the same area, it is easy to be threatened by anti-radiation missiles(ARM), active jamming and passive jamming, so the bistatic radar comes into the research field of scholars[3]. The transmitting array is placed on the airborne MIMO radar or far away from the battlefield, and the receiving MIMO radar does not emit electromagnetic signals[4]. Therefore, the bistatic MIMO radar is difficult to be investigated by the enemy, effectively avoiding the precise attack of the enemy, and has a good anti-jamming capability.

The coprime array is not limited by the Nyquist sampling rate and has a larger array element spacing, which effectively reduces the hardware and software overhead cost[5]. Later, in order to make full use of the coprime array, some scholars extended the coprime array to form an extended coprime array[6]. In this paper, the extended coprime array is improved by expanding it and further expanding the array aperture, so that it has better resolution and anti-jamming ability. Moreover, the improved extended coprime array is applied to the bistatic MIMO radar. Combining the virtual array and bistatic characteristics of MIMO radar, the low interception ability of the radar is further guaranteed.

Because the transmitting array and receiving array of bistatic MIMO radar are far apart and have the direction of departure angle (DOD) and the direction of arrival angle (DOA), the positioning research of bistatic MIMO radar adopts the method of joint Angle estimation of DOD and DOA[7]. Usually, two-dimensional MUSIC[8] algorithm is used to estimate the joint Angle of DOD and DOA. However, MUSIC algorithm needs to carry out two-dimensional exhaustive search and traversal angle range is

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large, which makes its algorithm computation heavy and its implementation is small in practical military applications[9,10]. To solve this problem, the 2D MUSIC algorithm is improved in this paper, which avoids the 2D exhaustive search and improves the estimation accuracy of the radar. Moreover, it is combined with the improved extended coprime MIMO radar to ensure the low intercept performance of the radar while having practicability.

2. System model

2.1. Improved coprime bistatic MIMO radar

The bistatic MIMO radar system model adopted in this paper is shown in Figure 1. The transmitting array and receiving array are the same coprime array, which is composed of subarray 1 and subarray 2. Subarray 1 consists of $N$ uniform array elements with spacing of $M\lambda/2$, and subarray 2 consists of $2M$ uniform array elements with spacing of $N\lambda/2$, where $N$ and $M$ are coprime numbers, $\lambda$ is the signal wavelength. The sparse array elements effectively reduce the cost of hardware and software development. We extend the subarray 2 from $M$ to $2M$, which increases the degree of freedom of the coprime array. Then, we expand the extended coprime array to make the first array element of subarray 2 coincide with the last array element of subarray 1, which can effectively improve the resolution and performance in the case of low SNR.

![Figure 1. System model of an improved coprime array bistatic MIMO radar.](image)

In Figure 1, $\phi$ is the launch angle and $\theta$ is the angle of echo reflected. The respective array elements of the transceiver array are $2M+N-1$. The mathematical representation of the transmitting and receiving array element position is as follows, Where $d_0$ is the half wavelength.

$$P_t = P_r = \{Mnd_0|0 \leq n \leq (N-1)| \cup \{M(n-1)d_0 + Nmd_0|0 \leq m \leq (2M-1)\}$$

(1)

2.2. Signal model

Assuming that there are $K$ far-field maneuvering targets, the radar emission signals are orthogonal coded signals as shown in Equation 2. Where, $T$ is the number of phase codes in each repetition period.

$$\frac{1}{T} \sum_{p=1}^{T} s_i(p)s_j^*(p) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

(2)
The received signal $X(t)$ of the radar is shown in Equation 3.

$$X(t) = \sum_{k=1}^{L} a_r(\phi_k)b_r(t)\beta_r(t) + w(t)$$ (3)

Where $t = 1, 2, \ldots, L$ is the number of snapshot. $w(t)$ is the white Gaussian noise with an average value of 0, $b_r(t)$ is the echo scattering signal of the kth target, as shown in Equation 4. Where $\beta_r$ is the complex amplitude of radar cross section (RCS) of the kth target, $f_a$ and $\mu_k$ are the doppler frequency shift and chirp frequency of the kth target. $a_r(\phi_k)$ and $a_r(\theta_k)$ are the guidance vectors of the kth target, as shown in Equation 5.

$$b_r(t) = \beta_r e^{j2\pi(f_a+\frac{\mu_k}{2})t}$$ (4)

$$a_r(\phi_k) = [a_{r1}(\phi_k), a_{r2}(\phi_k)]$$

$$a_r(\theta_k) = [a_{r1}(\theta_k), a_{r2}(\theta_k)]$$ (5)

Where, $a_{r1}(\phi_k)$ and $a_{r2}(\phi_k)$ is the guidance vector of the subarray 1 and 2 of the transmitting array, $a_{r1}(\theta_k)$ and $a_{r2}(\theta_k)$ is the guidance vector of the subarray 1 and 2 of the receiving array. As can be seen from section 2.1, the subarray guidance vectors of the transmitting and receiving arrays are shown in Equation 6 respectively.

$$a_{r1}(\phi_k) = \left[e^{jM(\frac{\phi_k}{\theta_1})}, e^{jM(\frac{\phi_k}{\theta_2})}, \ldots, e^{jM(\frac{\phi_k}{\theta_M})}\right]^T$$

$$a_{r2}(\phi_k) = \left[e^{j\frac{\phi_k}{\theta_1}}, e^{j\frac{\phi_k}{\theta_2}}, \ldots, e^{j\frac{\phi_k}{\theta_M}}\right]^T$$ (6)

The direction matrix of MIMO radar transmitting and receiving arrays are shown in Equation 7.

$$A = [a_r(\phi_1), a_r(\phi_2), \ldots, a_r(\phi_k)]$$

$$A = [a_r(\theta_1), a_r(\theta_2), \ldots, a_r(\theta_k)]$$ (7)

MIMO radar sets $2M+N-1$ matched filters at the receiving array to carry out matched filtering processing on the received signals and form a virtual array. Therefore, the received signal after matched filtering is shown in Equation 8.

$$Y(t) = Ab(t) + n(t)$$ (8)

Where, the echo scattering signal $b(t) = [b_1(t), b_2(t), \ldots, b_L(t)]$. $A$ is the direction matrix of the virtual array of the improved coprime array bistatic MIMO radar, as shown in Equation 9.

$$A = A_r \odot A_l = [a(\phi_1, \theta_1), a(\phi_2, \theta_2), \ldots, a(\phi_k, \theta_k)]$$ (9)

Where, $a(\phi_k, \theta_k) = a_r(\phi_k) \otimes a_l(\theta_k)$. $\odot$ stands for the Khatri-Rao product and $\otimes$ for the Kronecker product.

3. Joint DOD and DOA estimation of MIMO radar

3.1. 2D MUSIC estimation

In practical application, the spatial covariance matrix $R$ can be estimated by $L$ finite discrete sampling snapshot numbers, as shown in Equation 10.
Where $U_s$ is the signal subspace and $U_n$ is the noise subspace. According to the orthogonality of subspace $a^H(\phi, \theta)U_n = 0$, the two-dimensional space spectrum can be expressed as Equation 11.

$$F_{2D,MUSIC} = \frac{1}{a^H(\phi, \theta)U_sU^H_n a(\phi, \theta)}$$

### 3.2. Taylor improved estimation

The traditional 2D MUSIC algorithm needs to traverse the angles within the range, and the exhaustive search method consumes a lot of computation time. Therefore, according to the characteristics of the array structure in this paper, the method of Taylor expansion is adopted to improve the MIMO radar DOD and DOA dimension reduction estimation methods in literature [11].

The guide $a(\phi, \theta)$ vector is expanded by Taylor at $\phi = \phi_0$, as shown in Equation 12. $a(\phi, \theta)$ is decomposed into matrices $V(\phi)$ and $W(\theta)$, as shown in Equation 13 and 14.

$$a(\phi, \theta) = a(\phi_0, \theta) + \sum_{q=1}^{Q} a^{(q)}(\phi_0, \theta) (\phi - \phi_0)^q q!$$

$$V(\phi) = [1, (\phi - \phi_0), \cdots, (\phi - \phi_0)^L]$$

$$W(\theta) = [a(\phi_0, \theta), \cdots, a^{(L)}(\phi_0, \theta), \cdots, a^{(Q)}(\phi_0, \theta)]$$

The denominator of the 2D-MUSIC space spectral function of Equation 11 is expressed by $J$. Substituting the Taylor expansion of the equation into the equation, it can be written as:

$$J(\phi, \theta) = a^H(\phi, \theta)U_sU^H_n a(\phi, \theta) = V^H(\phi)W(\theta)V(\phi) = V^H(\phi)Z(\theta)V(\phi)$$

According to the basic knowledge of MUSIC algorithm, the optimal solution set $(\phi, \theta)$ of $J$ should be obtained. This is an optimization problem, and the constraint condition is set as: $e^H V(\phi) = 1$, in order to eliminate the meaningless solutions in $V(\phi)$ that are all 0. The optimization problem is expressed as follows:

$$\left\{ \begin{array}{l} \min V^H(\phi) Z(\theta) V(\phi) \\ s.t. e^H V(\phi) = 1 \end{array} \right.$$

Where, $e = [1, 0, \cdots, 0]^T$. Using Lagrange multiplier, the corresponding cost function can be obtained as Equation 17. Where, $\lambda_i$ is a constant. Take the partial derivative of Equation 17 and set it to 0, as shown in Equation 18.

$$L(\phi, \theta) = V^H(\phi)Z(\theta)V(\phi) - \lambda_i (e^H V(\phi) - 1)$$

$$\frac{\partial L(\phi, \theta)}{\partial V(\phi)} = 2Z(\theta)V(\phi) + \lambda_i e^H = 0$$

Combined with the constraints, we can get $V(\phi) = \frac{Z^{-1}(\theta) e}{e^H Z^{-1}(\theta) e}$. By substituting it into the optimization problem, we can obtain DOD directly by using Equation 19.

$$\theta = \arg \min_{\phi} \frac{1}{e^H Z^{-1}(\theta) e} = \arg \max e^H Z^{-1}(\theta) e$$
Substitute the obtained $\theta$ into the $V(\phi)$, and use the least square strategy to solve the corresponding DOD, namely $\phi$.

4. Radio frequency stealth technology of MIMO radar

According to the radar distance equation, the maximum intercepted distance of MIMO radar is:

$$R_{\text{D,max}}^2 = \frac{G_T G_R N_v P_t \eta t_s \delta \lambda^2}{(4\pi)^2 N_0 \text{SNR}_{\text{min}}} \tag{20}$$

$G_T$ and $G_R$ are the gain of transmitting and receiving antennas, $N_v$ is the number of virtual array elements, $P_t$ is the radar transmitting power, $\eta$ is the signal duty ratio, $t_s$ is the signal residence time, $\delta$ is the target RCS, $N_0$ is the noise power spectral density at the radar receiver, and $\text{SNR}_{\text{min}}$ is the minimum detectable signal-to-noise ratio between radars.

By combining Schriehl's interception factor with Equation (20), the expression of MIMO radar interception factor shown in Equation (21) can be obtained.

$$\alpha^4 \propto \frac{M \sum (\theta) P_t G_T^2 \lambda^2 N_v \text{SNR}_{\text{r, min}}^2 t_s^2}{4\pi^3 \eta \sigma N_0^2 B_t^2 \text{SNR}_{\text{r, min}} T_p^2} \tag{21}$$

$G_i$ is the antenna gain of the intercepted receiver, $\gamma \in (0,1)$ is the non-phase-coherent accumulation loss. In this paper, $N_0$ is equal to $N_r$, $B$ is the bandwidth of the intercepted receiver, $\text{SNR}_{\text{r, min}}$ is the minimum detectable signal-to-noise ratio of the intercepted receiver, $\text{SNR}_{\text{r, min}}$ is the minimum detectable signal-to-noise ratio of the intercepted receiver, and $\sum(\theta)$ is the normalized pattern factor of the radar in the direction of the intercepted receiver.

According to literature [12], the interception probability of radar power is as follows:

$$p = 1 - e^{-0.5\alpha} \tag{22}$$

The RF stealth capability of MIMO radar is affected by the transmitting power $P_T$ and the resident time $t_s$. First of all, when the environmental parameters are determined, SNR is proportional to the transmitting power $P_T$, that is, the smaller the SNR is, the smaller the $P_T$ is, the smaller the corresponding $\alpha$ is, the smaller the $p$ is, and the better the radar RF stealth capability is. Secondly, according to literature [13], the signal residence time $t_s$ is proportional to the snapshot number $L$, that is, the smaller $L$ is, the smaller $t_s$ is, the smaller $\alpha$ is, the smaller $p$ is, and the better RF stealth capability of the radar is.

5. Simulation experiment

We simulate the improved bistatic MIMO radar model in this paper to detect the DOD and DOA of four moving targets at a time. In the model, $M=7N=5$, SNR is set as 0dB, snap number is set as 200, and the number of sources is set as 57. Assume that the angles of moving targets are $(\phi_1, \theta_1) = (-20', 10')$, $(\phi_2, \theta_2) = (-15', 15')$, $(\phi_3, \theta_3) = (-40', -20')$, $(\phi_4, \theta_4) = (35', 20')$. The starting and stopping frequencies of scattered signals at a certain moment are $f_{s1} = 0.12, f_{s2} = 0.32, f_{s3} = 0.15, f_{s4} = 0.35, f_{s5} = 0.18, f_{s6} = 0.38$.

The 2D-MUSIC algorithm under the traditional coprime array of MIMO radar is searched in the whole area, and the spatial spectrum function graph obtained is shown in Figure 2. The four peaks in Figure 2 are the angle positions corresponding to the four targets. According to the derivation in Section 3, Taylor expansion was selected at $\phi_0 = 15'$ for the extended mutualarray proposed in this paper in the simulation experiment, as shown in Figure 3. Figure 3 shows the clear positioning of the four targets. The effectiveness of the improved algorithm is proved.
In this paper, RMSE is used to measure the estimation accuracy of the two MUSIC algorithms. Monte Carlo experiment is used to make the simulation process close to the real process. RMSE expression based on the Monte Carlo experiment is shown in Equation (23), where, $k$ is the target number, $G$ is the Monte Carlo number, and the Monte Carlo number is 200 in this experiment.

$$RMSE = \frac{1}{K} \sum_{i=1}^{k} \sqrt{\frac{1}{G} \sum_{j=1}^{G} ((\phi_i - \phi_{ij})^2 + (\theta_i - \theta_{ij})^2)}$$

The RMSE of the improved MUSIC algorithm under the improved coprime array MIMO radar in this paper was compared with the RMSE of the traditional coprime array MIMO radar 2D-MUSIC algorithm as a function of SNR, as shown in Figure 4. It is proved that the improved algorithm has better estimation accuracy.

Suppose that the radar and the intercepted receiver have the same noise power spectral density, the noise temperature is 290K, the noise coefficient is 2, the signal duty cycle is 0.01, and the target RCS is 1m$^2$. The parameters of the radar and intercepted receiver in reference [12] make the dwell time 26ms. Figure 5 shows the variation of radar interception probability with SNR drawn according to Equation (22). As can be seen from Figure 4, when SNR is 0, the error of the algorithm is small, while the interception probability at this time is 0.017%, which proves that in the process of improving the radar positioning algorithm in this paper, the low interception performance of the radar is guaranteed, so that it can meet the application in the military field while improving the estimation accuracy.
We compare the running time of two kinds of MIMO radar improved array MUSIC algorithm in Win10 system, the CPU processing model is AMD 2400G, and the software is Matlab2019a. 200 Monte Carlo experiments were carried out, and the calculated average running time was shown in Figure 6. Ten groups of coprime transceiver array elements were selected \{3,5\}, \{4,5\}, \{5,8\}, \{7,9\}, \{9,13\}, \{11,13\}, \{13,14\}, \{13,20\}, \{17,21\}, \{19,21\}.

![Figure 6. Comparison of the running time of the two algorithms](image)

As can be seen from Figure 6, the running time of the improved algorithm in this paper is relatively short, and when the number of array elements increases, the time of the algorithm in this paper is significantly shorter than that of the traditional coprime array 2DMUSIC algorithm of MIMO radar, which proves that the calculation amount of the improved algorithm is reduced.

6. Conclusions
By improved MIMO radar's coprime array and 2D-MUSIC estimation algorithm, the accuracy of locating moving targets is increased, at the same time the low interception performance of radar is satisfied, so that it has better application performance in the military field.

Acknowledgments
This work was supported by National Natural Science Foundation of China (NSFC) (No. 61801212).

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