The Impact of Lithium-Ion Battery Polarising Impedance Modelling on End-of-Discharge Prognosis Accuracy

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Abstract: In Lithium-Ion batteries, the polarising impedance is an important characteristic that has been shown to be a complex function of, among others, both the state of charge and the demanded current. Therefore within a prognostic framework, which typically solely relies on the a priori modelling of the hidden state evolution, the correct characterisation of the functional surface with respect to state of charge and current impacts the accuracy of the predicted end-of-discharge probability density function. This is important in critical systems that rely solely on the Lithium-Ion as a power source and require an unbiased prediction of the end-of-discharge time. This paper demonstrates how a correctly modelled polarising surface can improve prediction accuracy over the state of the art models found in literature.

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1. INTRODUCTION

In Lithium-Ion battery monitoring and the subsequent estimation of the State of Charge (SoC), the polarising impedance is an important characteristic that has been shown to be a complex function of, among others, both the SoC and the demanded current. Therefore within a prognostic framework, which typically solely relies on the hidden state evolution model, the correct characterisation of the functional surface with respect to state of charge and current, impacts the accuracy of the predicted end-of-discharge probability mass function (pmf). This is important in critical systems that rely solely on the Lithium-Ion as a power source and require an unbiased prediction of the end-of-discharge time.

The typical solution to this problem in literature, such as in Quintero et al. (2017): Acuña and Orchard (2017a), is augment the state evolution of the SoC to include an additional state that represents the polarising impedance. This state evolves through a combination of outer-feedback loops, simulated annealing and the re-sampling of the Particle Filter observer to converge on the current value of the polarising impedance.

The problem with this approach arises when you consider a scenario where the usage profile constantly changes (a very real scenario), which, inhibits the convergence of this second artificial evolution (AE) state. This is due to, as demonstrated for example in Burgos-Mellado et al. (2016), that in fact the polarising impedance is a multidimensional surface which is both a function of current and SoC.

In the following sections, it is shown, how the standard state of the art model struggles in this real world scenario to produce consistent & accurate predictions, regardless of hyper-parameter tuning and how by replacing the polarising impedance AE state with a reproducing kernel significantly improves prognosis accuracy, as well as algorithmic stability.

2. POLARISING IMPEDANCE IN LI-ION BATTERY MODELLING

The state space modelling (SSM) used in battery monitoring are often based on a simplified equivalent circuit model (ECM)Waag et al. (2013), such as the one depicted in Fig. 1. ECMs are used as their name suggests as an equivalence, as well as a simplification of, the underlying mechanism of ion transfer, kinetics and subsequent Red-Ox reactions within the Lithium-ion battery. ECMs are effective due to their significantly reduced computational complexity compared to a complete model of Lithium-ion kinetics as well as reducing the dependency of the model on parameterisation. The dependence of the ECM performance on correct parameterisation is reduced, but not eliminated, in fact it obscures one of the main difficulties in Lithium-ion battery inference, state varying parameters, specifically the polarising impedance.

In order to understand the problem we will focus on the equivalent model shown in Fig. 1, under this equivalence the resistance $R(I_R)$ can be described by the Butler-Volmer equation (1) Waag et al. (2013):

$$R(I_R) = R_1 \cdot \left( \frac{\ln(k_1 \cdot I_R + \sqrt{(k_1 \cdot I_R)^2 + 1})}{k_1 \cdot I_R} \right)$$

(1)

where $R_1$ is the resistance at $I_R = 0A$ and $k_1$ is a parameter that describes the current dependency of the resistance. It appears as though the resistance varies...
depending on the users demand of the battery, but the problem becomes more complex when it can be shown that both parameters \((R_1 \text{ and } R_f)\) vary with different temperatures \((T)\), States of Health \((\text{SoH})\) and States of Charge \((\text{SoC})\) Burgos-Mellado et al. (2016). So if we were to express generally a lumped parameter model for the impedance \((R_0, R(I_R) \& C)\) we would be considering a highly non-linear model which can be expressed as

\[
Z_p = f(\text{SoC}, I, \text{SoH}, T).
\]

which we will call the general polarising impedance model.

3. GENERAL STATE SPACE MODELLING OF LITHIUM-ION BATTERY

The State of Charge \((\text{SoC})\) is an abstract construct that represent an important indicator for estimating the energy of our battery, with this in mind we desire a function that

\[
\begin{align*}
\text{sup } \text{SoC} &\rightarrow E_{\text{crit}} \quad &\text{inf } \text{SoC} &\rightarrow 0
\end{align*}
\]

where \(E_{\text{crit}}\) is the maximum amount of energy \((\text{Watt} \cdot \text{hour})\) the battery is currently capable of containing. Therefore if we consider our ECM (Fig. 1) we can express the rate of change of the \(\text{SoC}\) as the rate of change in power, the product of the demanded battery current \(I\) and observed voltage \(V\), with respect to time as some fraction of our total battery energy \(E_{\text{crit}}\):

\[
\frac{d\text{SoC}}{dt} = \frac{d}{dt} \left( \frac{I \cdot V}{E_{\text{crit}}} \right).
\]

For the sake of simplicity we will adopt a positive power convention where a positive current \(I > 0\) represents a decrease in energy of our battery, with this in mind we can further discretise (3) with respect to time \(t\), which leaves us with a SSM for \(\text{SoC}\) in the form:

\[
\text{SoC}_{k+1} = \text{SoC}_k + \frac{\Delta t \cdot I_k \cdot V_k}{E_{\text{crit}}}.
\]

where \(\Delta t\) is unit of discretisation in this context called our “sample time”. Now we have defined how the \(\text{SoC}\) evolves as a function of our control variable \(I_k\) and the internal state \(\text{SoC}\), we need to define how those two states affect the output voltage, which is included in our model.

In order to do so we must consider our equivalent circuit Fig. 1 or its functional equivalency

\[
V_k = V_{\text{OCV}}(\text{SoC}_k) - I_k \cdot Z_p
\]

where \(V_{\text{OCV}}\) is the open circuit voltage \((\text{OCV})\) and is modelled as

\[
V_{\text{OCV}}(\text{SoC}) = v_L + (v_0 - v_L) \cdot e^{\gamma(\text{SoC}(k) - 1)} \\
+ \alpha \cdot v_L \cdot (\text{SoC}(k) - 1) \\
+ (1 - \alpha) \cdot v_L \cdot e^{-\beta} - e^{-\beta \sqrt{\text{SoC}(k)}}
\]

where \((\alpha, \beta, \gamma)\) are fixed parameters that are determined initially from battery tests (see Pola et al. (2015) for a rigorous explanation) and form an integral part of the characteristic curve. It is apparent the implication that the accuracy of \(Z_p\) is of significance as it directly impacts the accuracy of estimation of the \(\text{SoC}\) and subsequently its prediction.

4. THE PARTICLE FILTER

All estimation within this paper is performed via the Particle Filter (PF) observer framework, the PF is defined as follows.

Consider a sequence of probability distributions \(\pi_t(x) \geq 1\), where it is assumed that \(\pi_t(x_{0:t})\) can be evaluated point-wise up to a normalising constant. Particle Filters (PF) are a class of algorithms designed to approximately obtain samples sequentially from \(\{\pi_t\}\) to generate a collection of \(N \gg 1\) weighted random samples \(\{w_{t}^{(i)}, x_{t-1}^{(i)}\}_{i=1, \ldots, N}, w_{t}^{(i)} \geq 0, \forall t \geq 1\), satisfying Andrieu et al. (2001):

\[
\sum_{i=1}^{N} w_{t}^{(i)} \varphi_t \left( x_{t-1}^{(i)} \right) \sim \frac{1}{N} \rightarrow \int \varphi_t(x_{t-1}) \pi_t(x_{t-1}) dx_{t-1},
\]

where \(\varphi_t\) is any \(\pi_t\)-integrable function. In the particular case of the Bayesian Filtering problem, the target distribution \(\pi_t(x_{0:t}) = p(x_{0:t} | y_{1:t})\) is the posterior pdf of \(X_{0:t}\), given a realisation of noisy observations \(Y_{1:t} = y_{1:t}\). In a Bayesian filtering observer framework we define the general state evolution

\[
p(x_k|x_{k-1}) \approx f(x_k|x_{k-1})
\]

and the general observation

\[
p(y_k|x_k) \approx g(y_k|x_k)
\]

where \(x_k\) is the hidden state at instance \(k\), such as the \(\text{SoC}_k\) in (4), and \(y_k\) is the observation at instance \(k\), e.g. \(V_k\) in (5).

Using (8) and (9), \(\pi_t(x_{0:t})\) may be written as Doucet et al. (2001)

\[
\pi_t(x_{0:t}) = p(x_0) \prod_{k=1}^{t} f_k(x_k|x_{k-1}) g_k(y_k|x_k)
\]

Let a set of \(N\) paths \(\{x_{t}^{(i)}\}_{i=1, \ldots, N}\) be available at time \(t-1\). Furthermore, let these paths distribute according to \(q_{t-1}(x_{0:t-1})\), also referred to as the importance density function at time \(t-1\). Then, the objective is to efficiently obtain a set of \(N\) new paths \((\text{particles})\) \(\{x_{t}^{(i)}\}_{i=1, \ldots, N}\) approximately distributed according to \(\pi(x_{0:t})\) Andrieu et al. (2001).
For this purpose, the current paths \( x_{i:t-1} \) are extended by using the kernel \( q_i(x_{0:t-1} | x_{0:t-1}) = \delta(x_{0:t-1} - x_{0:t-1}) \cdot q_i(\tilde{x}_i | x_{0:t-1}) \), i.e., \( \tilde{x}_{0:t} = (x_{0:t-1}, \tilde{x}_i) \). The importance sampling procedure generates consistent estimates for the expectation by approximating (10) with the empirical distribution Andrieu et al. (2001):

\[
\hat{x}_t^N(x_{0:t}) = \sum_{i=1}^N w_{0:t}^{(i)} \delta(x_{0:t} - \hat{x}_t^{(i)})
\]

where \( w_{0:t}^{(i)} \propto w_{0,t}^{(i)}(x_{0:t}) \) and \( \sum_{i=1}^N w_{0:t}^{(i)} = 1 \).

### 4.1 Sequential Importance Sampling/Resampling Particle Filter (SIR-PF)

The simplest PF implementation - the SIS particle filter - computes the value of the particle weights \( w_{0:t}^{(i)} \), by setting the importance density function equal to a priori pdf for the state, i.e., \( q_i(x_{0:t} | x_{0:t-1}) = p(\tilde{x}_i | x_{t-1}) = f_i(\tilde{x}_i | x_{t-1}) \). In that manner, the weights for the newly generated particles are evaluated from the likelihood of new observations. The efficiency of the procedure improves as the variance of the importance weights is minimized. The choice of the importance density function is critical for the performance of the particle filter scheme and hence, it should be considered in the filter design.

One of the main difficulties that must be addressed in the implementation of SIS particle filters is the problem of particle degeneracy Doucet et al. (2001) since, after a few iterations, all but one particle could have a negligible weight Andrieu et al. (2001); Doucet et al. (2001). Several methods have been proposed to overcome this problem, one of which is to perform the resampling procedure conditionally on a test of the Effective Sampling Size (\( N_{eff} \)) criterion, given by Li et al. (2014):

\[
N_{eff} = \left( \sum_{i=1}^N \left( \frac{\omega_i}{\bar{\omega}} \right)^2 \right)^{-1}
\]

s.t. \( \sum_{i=1}^N \omega_i^2 = 1 \) (12)

The re-sampling is only implemented when the \( N_{eff} \) is below a threshold \( N_r \), e.g. \( N_r = 0.85 \cdot N \), where \( N \) is the size of the particle population. This process avoids sample impoverishment and permits the Particle Filter to converge to the optimal filter, in the weak sense, with the rate of \( 1/\sqrt{N} \). The SIS in combination with the re-sampling step is referred to as the Sequential Importance Sampling Resampling (SIR) Particle Filter, full details of which can be found in Doucet and Johansen (2011).

### 5. PARTICLE-FILTERING BASED PROGNOSIS

It is possible to describe the evolution in time of the degradation of the non-linear state equations used in the estimation phase (Sec. 4). By using the state equation (8), it is possible to generate long-term predictions using kernel functions to reconstruct the estimate of the state probability density function (pdf) in future time instants.

The goal is to produce a predicted conditional state pdf \( \hat{p}(x_{1:t+k} | \tilde{x}_{1:t+k-1}) \), which describes the state distribution at the future time instant \( t + k, (k = 1, \ldots, \rho) \) when the particle \( \tilde{x}_{t+k}^{(i)} \) is used as an initial condition. Assuming that the current weights \( \{w_{t+k}^{(i)}\}_{i=1}^N \) are a good representation of the state pdf at time \( t+k \), by using the law of total probabilities and the particle weights at time \( t+k-1 \), as shown in (13)

\[
\hat{p}(x_{t+k} | \tilde{x}_{1:t+k-1}) \approx \sum_{i=1}^N w_{t+k-1}^{(i)} \cdot \hat{p}(x_{t+k} | \tilde{x}_{1}^{(i)}),
\]

where \( \tilde{x}_{t+k}^{(i)} = \tilde{x}_t^{(i)} \), \( k = 1, \ldots, \rho \).

To evaluate (13), the weight of every particle should be modified (at each prediction step) to take into account the fact that noise and process non-linearities could change the shape of the state pdf as time passes. However, since the weight update procedure is needed as part of a prediction problem, it cannot depend on the acquisition of new measurements. Additionally, before proceeding with the next prediction step, it is necessary to allocate a new set of particles within the domain of the probability distribution (13). To overcome most of these difficulties Orchard and Vachtsevanos (2009) proposed a method of using a kernel to represent the uncertainty growth of each prediction step.

Consider a discrete approximation (14) for the predicted state pdf (13), where \( K(\cdot) \) is a kernel density function, which may correspond to the process noise pdf, a Gaussian Kernel or a re-scaled Epamechnikov Kernel Orchard and Vachtsevanos (2009).

\[
\hat{p}(x_{t+k} | \tilde{x}_{1:t+k-1}) \approx \sum_{i=1}^N w_{t+k-1}^{(i)} K \left( x_{t+k} - E \left[ x_{t+k} | \tilde{x}_{1}^{(i)} \right] \right)
\]

The kernel method utilised is a modified version of the Regularised Particle Filter presented by Musso et al. (2001), where they present an optimum re-scaled Epamechnikov kernel, given by:

\[
A = (8c_{n_x}^{-1}(n_x + 4)(2\sqrt{\pi})^{n_x})^{1/(n_x+4)}
\]

\[
h_{opt} = A \cdot N^{-1/(n_x+4)}
\]

\[
K_h = \frac{1}{h_{opt}^{n_x}} K \left( \frac{x}{h} \right)
\]

\[
K(x) = \begin{cases} 
  \frac{n_x + 2}{2c_{n_x}} (1 - \|x\|) & \text{if } \|x\| < 1 \\
  0 & \text{otherwise}
\end{cases}
\]

where \( c_{n_x} \) is the volume of the unit sphere in \( \mathbb{R}^{n_x} \). Rather than simply projecting the expectation of the state variables into the future, the uncertainty of (14) can be represented by generating a population of equally weighted particles at time instant \( t + k, 1 \leq k \leq \rho \), and performing an inverse transform re-sampling procedure for the particle population. The focus is to represent the growth of uncertainty presented in (14). For that, samples distributed according to (14) are obtained by selecting \( u^{(i)} = i \cdot (N + 1)^{-1} \) for \( i = 1, \ldots, N \) and interpolating a value for \( \tilde{x}_{t+k}^{(i)} \) from the cumulative state distribution

\[
F(X_{t+k} \leq x_{t+k}) = \int_{-\infty}^{x_{t+k}} \hat{p}(x_{t+k} | \tilde{x}_{1:t+k-1}) dx_{t+k}
\]
in accordance with \( \hat{x}_{t+k} = F^{-1}(u^{(i)}) \). Considering all of the above, the regularisation algorithm Musso et al. (2001) is used to perform prognosis by Orchard and Vachtsevanos (2009):

1. Apply modified inverse transform re-sampling procedure for \( i = 1, \cdots, N \), where \( \omega_{t+k} = N^{-1} \)
2. Calculate \( \hat{S}_{t+k} \), the empirical co-variance matrix of
   \[
   \left\{ E \left[ \hat{x}_{t+k}^{(i)} | \hat{x}_{t+k-1}^{(i)} \right] , \hat{x}_{t+k}^{(i)} \right\}_{i=1}^{N}
   \]
3. Compute \( \hat{D}_{t+k} \) such that \( \hat{D}_{t+k} \hat{S}_{t+k}^{-1} = \hat{S}_{t+k} \)
4. For \( i = 1, \cdots, N \), draw \( \epsilon_i \sim K \), the Epanechnikov kernel and assign \( \hat{x}_{t+k}^{(i)*} = \hat{x}_{t+k}^{(i)} + h_{t+k}^{opt} \hat{D}_{t+k} \epsilon_i \) where \( h_{t+k}^{opt} \) is calculated as in (15) and \( \hat{x}_{t+k}^{(i)*} \) is the regularised particle population.
5. For the next prediction step \( \hat{x}_{t+k-1}^{(i)*} = \hat{x}_{t+k-1}^{(i)} \)

It is assumed that the state co-variance matrix \( \hat{S}_{t+k} \) is equal to the empirical co-variance of \( \hat{x}_{t+k} \) and that a set of equally weighted samples for \( \hat{x}_{t+k-1} \) is available. This serves to maximise the efficiency of the Epanechnikov Kernel.

6. CALCULATING THE PROBABILITY MASS FUNCTION OF THE END OF DISCHARGE

Using the Particle Filter based Prognosis framework outline in Sec. 5, a projection of the evolution of the state probability density function is produced for all time \( k > k_p \), where \( k_p \) is the prognosis start point. In the case of battery State of Charge (SoC) prediction this is only informative in limited sense. Often the desired prediction in practice is not the probability of state with respect to time but, given a desired limit (of SoC), what is the probability we will reach the limit with respect to time.

In battery prognosis often the limit is referred to as the End-of-Discharge (EoD) point, in the case of this paper it is set as \( EoD \triangleq SoC \leq 0.3 \) or \( EoD = 0.3 \) is the out limiting point. Acuña and Orchard (2017a) outlines how to calculate the probability mass function with respect to time, described as

\[
P(EoD = 0.3) = \sum_{i=1}^{N_k} W_k^{(i)} p(EoD|SoC_k^{(i)})
\]

which involves summing the projected state trajectories with respect to time at the instance the particle \( SoC_{k_{p,k}} \leq EoD \) in order to accumulate samples from the projected PMF, the process is thoroughly described in Acuña and Orchard (2017a).

7. TESTING PROFILE

With the observation and subsequent prognosis framework defined in Sec. 4 & 5, we will focus on the application case. In this paper we will be using a profile developed from the battery data found in Burgos-Mellado et al. (2016), whose specifications can be found in Appendix A and whose profile is demonstrated in Fig. 2.

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**Fig. 2. Lithium-Ion Battery Validation Profile**

8. STATE OF THE ART STANDARD MODEL

As it was alluded to in Sec. 1 the state of the art state space model (SSM) in Lithium-Ion battery State of Charge estimation and prognosis, such as the one found in Quintero et al. (2017), consists of defining an additional artificial evolution state that represents a constantly evolving \( Z_{pol} \) in order to resolve the observation and state equations dependence on (5), which includes \( Z_{pol} \). This is defined by the random walk model

\[
Z_{pol}(k) = Z_{pol}(k-1) + \eta_k
\]

where \( \eta_k \) is artificially augmented noise model that is modified through simulated annealing and outer-feedback correction loops (see Orchard et al. (2009) for more details) in order to produce convergence, essentially learning the polarising impedance through observations. The state of charge evolution is generally defined by

\[
SoC_{k+1} = SoC_k - \frac{\Delta t \cdot I_k \cdot V_k}{E_{crit}} + \omega_k
\]

where \( \omega_k \) represents the state evolution uncertainty generally modelled as zero mean i.i.d Gaussian noise modelled as \( N(0, \sigma_{\omega}^2) \) for simplicity. The observations are modelled by the equation

\[
V_k = V_{OCV}(SoC_k) - I_k \cdot Z_p + \nu_k
\]

where \( \nu_k \) is zero mean i.i.d Gaussian noise generally modelled as \( N(0, \sigma_{\nu}^2) \).

8.1 Estimation and Prediction Results

As it has already been mentioned this convergence can be hindered by a profile such as the one used in Fig. 2. The result is a model that is constantly correcting during the estimation phase as we can see in Fig. 3 & Fig. 4.

It is often left unsaid that the performance of the particle filter (PF) in the case of the standard model (19), (20), and (21) is often heavily dependent on the hyper-parameter tuning, specifically \( \sigma_{\omega}, \sigma_{\nu}, \eta \) and their relative sizes. Fig. 3 & Fig. 4 are examples of two extreme cases of this reality.

Fig. 3 represents the situation where the user wishes force the algorithm to actually converge, which can be achieved by decreasing \( \sigma_{\omega} \) and increasing \( \sigma_{\nu} \), effectively biasing the PF to favour the a prior state model (20) over any observed differences from measurements. This results in a case where the model is artificially confident in it’s estimation of the state and subsequently the polarising impedance
true (PMF, described in Sec. 6) is artificially narrow which (2017b), is that the predicted probability mass function ground truth and has failed to correct. The result of this is it can be seen that the model is biased away from the (19) converges. This can be easily observed in Fig 3 as 

Fig. 3. Artificially high precision in prognosis



(1) for estimation accuracy a large range of values is required for the polarising impedance

The second situation, shown in Fig. 4, is a pessimistic model that understands the poor modelling capability of a single dimension polarising impedance. This is represented by a hyper-parameter tuning that increases σ_ω and uses the measured σ_p (or the true measurement uncertainty). This results in significantly improved estimation accuracy, as shown in Fig. 4 by the estimation constantly correcting to close to the true SoC. The consequence of this tuning is that, due to the constantly changing profile, that the AE state never converges. This, in turn, during the prognosis stage creates a large uncertainty (from the large variance in Z_{pol}). It can be seen in Fig. 6 that this is almost invariant of the starting point (up to the point the estimation is very close to the prediction point). Despite a large prediction uncertainty it is mostly stable, although not very accurate.

In a broad sense the competing philosophies of these two models highlight two important factors that should be incorporated in an accurate model,

(2) for accurate prognosis low state uncertainty is required
these two competing ideals in the standard model can be incorporated into one model if we instead consider a polarising surface.

9. KERNEL STATE SPACE MODEL

By refactoring (21) to include \( Z_{\text{pol}} \approx f(\text{SoC}_k, I_k) \) as
\[
V_k = V_{OC_k}(\text{SoC}_k) - I_k \cdot Z_{\text{pol}}(\text{SoC}_k, I_k) + \nu_k
\]
we can consider \( f(\text{SoC}_k, I_k) \) as a non-linear surface, this suggests that we could approximate said surface in the reproducing kernel Hilbert space (RKHS) \( \mathcal{H} \) with reproducing kernel \( K \).

According to the representer theorem Tobar et al. (2015), the optimal estimate with respect to an arbitrary loss function can be expressed in terms of the kernel function \( K \), input training samples \( x_i \) and a vector of mixing parameters \( a = [a_1, ..., a_N] \) in the form
\[
Z_{\text{pol}} = \sum_{i=1}^{N} a_i K((\text{SoC}_k, I_k), s_i)
\]
where the centres of the kernel evaluations are referred to as support vectors. The Gaussian Kernel is defined
\[
K(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{\sigma_K^2}\right)
\]
where \( x_i, x_j \) are the input vectors and the parameter \( \sigma_K > 0 \) is the kernel width.

9.1 Application to estimation and Prognosis

The kernel was learnt from a previous set of discharge tests of the same battery using a Particle Markov Chain Monte Carlo algorithm to infer the joint probability \( p(\text{SoC}_0:T, \theta|V_0:T) \), where \( \theta = [a_0, ..., a_N, \sigma_\omega] \) (details of which will be outlined in a future publication). The surface can be seen to be highly nonlinear with respect to both SoC and I as can be seen from the plot of level curves of the surface in Fig. 7.

The result of incorporating the learnt surface into the estimation and prognosis stage can be seen firstly in Fig. 8. It can be seen that the estimation incorporates the state uncertainty correctly (as it was part of the learning paradigm) and performs consistently regardless of the usage profile. Fig. 5 and Fig. 6 demonstrate the unbiased correctly confident prediction results (shown in red). It is important to note that the model correctly incorporates in its prediction results correct entropy growth, i.e. the further the prediction is from the estimated PMF, the greater the uncertainty, accurately representing reality.

Further more if we plot the results of one of the commonly used precision metric, such as the Online Precision Index Ley and Orchard (2017)
\[
I_k = \exp\left(-\frac{\text{\inf}(EoD) - \text{\sup}(EoD)}{|E[EoD] - t|}\right)
\]
where \( EoD \) is the PMF of the desired End-of-Discharge estimate (e.g \( EoD = 0.3 \) in the case of this paper), we can see that in Fig. 9 that the Kernel Model is almost always an improvement on even the most precise standard model.

10. CONCLUSION

It can be seen that in all presented cases the correct modelling of the polarising impedance surface is always an improvement on the performance of the standard model that is used in the state of the art. This is due to the fact that the kernel model leads to a more informed estimation
model which, leads to a smaller estimation uncertainty and subsequently a reduced initial uncertainty (for the prediction evolution). This can easily be reasoned theoretically due to an increase in the information contained in the model, reduces the Shannon entropy contributed by the state model.

This leads naturally to an emphasis on the importance of the proposed future work of demonstrating a Particle Markov Chain Monte Carlo method for learning the polarising impedance kernel weights.

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Appendix A. BATTERY MODEL PARAMETRISATION

| Battery Parameter | \( I \geq 0 \) | \( I < 0 \) |
|-------------------|-----------|-----------|
| Nominal Capacity  | 40        | A · h     |
| Nominal Voltage   | 25.6      | V        |
| \( E_{\text{crit}} \) | 313.3440  | 0        |
| \( v_L \)         | 1.0       | 0.98345  |
| \( v_0 \)         | 26.792    | 26.750    |
| \( \gamma \)      | 0.556     |          |
| \( \alpha \)      | −2.631    | −2.733    |
| \( \beta \)       | 0.567     |          |
| \( \omega_k \)    | \( \mathcal{N}(0,0.001^2) \) |          |
| \( \mu_k \)       | \( \mathcal{N}(0,0.1^2) \) |          |

Table A.1. Battery Specifications