High-performance technique for initial velocity field calculation based on PIV images processing

D I Zaripov\(^1\), Renfu Li\(^2\)

\(^1\) School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Luoyu East Road 1037, Wuhan, China
\(^2\) School of Aerospace Engineering, Huazhong University of Science and Technology, Luoyu East Road 1037, Wuhan, China

zaripov.d.i@mail.ru

Abstract. The high-performance technique is proposed in order to calculate the initial velocity field for planar Particle Image Velocimetry. The proposed technique is based on the use of Parallel Projection Correlation technique together with the region of interest that allows acceleration of calculation procedure and significantly increase the maximum value of measurable velocity using small-sized interrogation window at the first iteration. To show the performance of proposed technique, two synthetic test cases are considered in this investigation: one-dimensional uniform flow and linear shear flow. Accuracy analysis is carried out depending on different interrogation window sizes for two velocity components. The results of test cases show good performance of proposed technique in terms of accuracy. Finally, the proposed technique is applied together with the iterative multigrid window-deformation technique based on fractional interrogation window offset.

1. Introduction

Over the last three decades, the Particle Image Velocimetry (PIV) method has become a reliable tool for measurement of kinematic structure of fluid flow [1–3]. Wide variety of approaches is developed and implemented in order to achieve the high accuracy of obtained data such as velocity vectors and their derivatives. The development of PIV techniques that yield the high accuracy of data is the essential problem. At the same time, the problem of computational cost reduction has no less importance.

A Fast Fourier Transform Cross-Correlation (FFT-CC) approach is widely used when processing the PIV images. Its complexity in two-dimensional domain is \(O(N^2\log_2N^2)\) where \(N\) is the interrogation window (IW) size. This makes it much more preferable than the direct cross-correlation (2D-DCC) one which has the complexity of \(O(N^4)\). The drawback of FFT-CC is that the IW in subsequent image cannot be slid over the region of interest (ROI) and its location must be fixed. Thus FFT-CC is commonly used for the predictor displacement field evaluation where particle images displacements are initially unknown and are supposed to be equal to zero [4]. It should be mentioned that 2D-DCC does not suffer from such a drawback and theoretically has no limitation for maximum measurable velocity [5] due to the possibility to use the ROI.
On the other hand FFT-CC imposes constraints on the IW size when considering gradient flow: typical IW size is 32×32 pix². According to [2], the further increase of IW size leads to high errors in determining of particle images displacements even though a low gradient shear flow is considered. According to well-known one-quarter rule [6], the maximum particle images displacement must be less than quarter of IW size, i.e. less than 8 and 16 pix for IW sizes 32×32 and 64×64 pix², respectively. This is a significant limitation of modern PIV techniques that use FFT-CC to calculate the initial velocity field.

Generally speaking, there are two approaches to calculate the initial velocity field: FFT-CC that allows acceleration of calculation but has limitation for maximum measurable velocity and 2D-DCC that has no mentioned limitation and gives more accurate results but is computationally intensive. The present work is dedicated to the novel approach that combines the advantages of both approaches. It is based on a Parallel Projection Correlation technique which has been first proposed in [7] when considering the Tomographic PIV in order to reduce the calculation time.

2. Description of technique for initial velocity field calculation

The technique of initial velocity field calculation consists of two basic operations and is schematically shown in figure 1. In the vicinity of grid nodes preliminarily applied to image \( k \), small IW with sizes \( N \times M \) pix² are selected. Then projections of

\[
I_x(i) = \frac{1}{M} \sum_{j=1}^{M} I(i, j), \quad i = \overline{1, N}, \quad j = \overline{1, M} \tag{1}
\]

and

\[
I_y(j) = \frac{1}{N} \sum_{i=1}^{N} I(i, j), \quad i = \overline{1, N}, \quad j = \overline{1, M} \tag{2}
\]

are considered instead of luminous intensity array of pixels \( I(i, j) \): the elements of those are equal to arithmetic mean value of all elements in columns and rows of the array \( I(i, j) \), respectively. As mentioned before, it is better to use ROI applied to image \( k+1 \) in order to have no limitation for maximum measurable velocity. Thus, the same procedure of calculating of projections \( I_x(i, n, m) \) and \( I_y(i, n, m) \) is conducted for IW within ROI with sizes \( N_{ROI} \times M_{ROI} \) pix², where \( n=\overline{1, N_{ROI}-N+1} \) and \( m=\overline{1, M_{ROI}-M+1} \). Then the horizontal and vertical particle images displacements are determined comparing projections \( I_x \) and \( I_y \) of IW from two consecutive images \( k \) and \( k+1 \) as shown in figure 1. These displacements can be found by cross-correlating as

\[
R_x(n, m) = \sum_{i=1}^{N} I_x(k, i) \cdot I_x(k+1, i, n, m), \quad n = \overline{1, N_{ROI} - N + 1}, \quad m = \overline{1, M_{ROI} - M + 1} \tag{3}
\]

and

\[
R_y(n, m) = \sum_{j=1}^{M} I_y(k, j) \cdot I_y(k+1, j, n, m), \quad n = \overline{1, N_{ROI} - N + 1}, \quad m = \overline{1, M_{ROI} - M + 1} \tag{4}
\]

Then the search for the values \( n \) and \( m \) where the functions \( R_x \) or \( R_y \) have maximum values is conducted according to figure 1. These values correspond to the most probable integer displacements of particle images in pixels. Since the proposed technique deals with one-dimensional projections instead of two-dimensional luminous intensity array, it is proposed to use the abbreviation 1D-DCC for its definition.

Correlating the projections \( I_x \) and \( I_y \) instead of \( I \) yields the computational complexity of the order of \( O(2Nn^2) \) for square IW and ROI. This makes 1D-DCC much more efficient in terms of calculation time than 2D-DCC with complexity of \( O(N^2n^2) \). Thus, the theoretical speed-up of 1D-DCC compared to 2D-DCC is \( N/2 \). In practice, the speed-up is slightly less (see table 1). The computation was
performed on Intel Core i5 with processor speed of 3.0 GHz. Speed-up estimation includes all overheads of cross-correlation map calculation, such as the creation or filling of appropriate arrays. As can be seen from table 1, the speed-up is almost independent of the ROI size that agrees with theoretical estimation. Moreover, the bigger IW size, the higher the speed-up.

| N_{ROI} - N + 1, pix | N_{ROI} = 16 | N_{ROI} = 32 |
|----------------------|--------------|--------------|
| N_{ROI} = 16         | 6.4          | 6.5          |
| N_{ROI} = 32         | 13.6         | 14.0         |

However, it is not obvious what correlation map between $R_x$ and $R_y$ is better to determine $n$ and $m$. In order to understand this, two test cases are further considered: one-dimensional uniform flow and linear shear flow.

### Figure 1. The algorithm of initial velocity field calculation.

#### 3. Synthetic images generation
To quantitatively estimate the accuracy of 1D-DCC, synthetic images are generated using conventional approach to PIV image generation [2,8]. The luminous intensity of particle images is assumed to be continuous and distributed according to the Gaussian function, and the particle images borders are defined using $e^{-2}$ level of maximal value of the Gaussian function. The luminous intensity of each pixel is defined by integration over the area occupied by the pixel [8,9]. Particle images with diameter $d_p = 2$ pix are randomly and uniformly distributed over the whole image area with concentration $C = 0.076$ particles/pix$^2$. To obtain the explicit dependence of the error on particle displacement values, noise is not added to the images.

Two synthetic test cases are considered in this research: one-dimensional uniform flow and linear shear flow. For both cases, a sequence of 35 images (for 34 displacement field) with the sizes 64×560 pix$^2$ is generated. Such image sizes allows the use of IW with sizes 16×16 and 32×32 pix$^2$ that are overlapped by 50%. The total number of IW placed in each image is 102 (3×34). Only the matrix of displacement vectors $1\times32\times32$ placed in the middle windows are statistically analyzed even though 3×34×34 displacement vectors are calculated. The uniform vertical displacement of particles $l_0$ in the range from 0 to 2 pix with the increment of 0.1 pix is simulated. When shear flow is simulated, the particle images are displaced according to linear gradient shift from 0 to 0.3 pix/pix with the increment of 0.02 pix/pix resulting in zero mean displacement for statistically analyzed vectors. In order to make a comparative accuracy estimation between the proposed technique and other ones such as 2D-DCC
and FFT-CC, the ROI sizes are chosen to be equal to $31 \times 31$ and $63 \times 63$ pix$^2$ for IW with sizes $16 \times 16$ and $32 \times 32$ pix$^2$, respectively (the ROI are placed at the center of IW).

The erroneous displacement vectors are detected using a $3 \times 3 \times 3$ local median test proposed in [10] and replaced by the local average of the (accepted) neighbor vectors [11].

4. Results of 1D-DCC accuracy evaluation

Random and bias errors are analyzed for both particle images displacement directions: in flow direction ($y$-component of displacement) and perpendicular to flow direction ($x$-component of displacement). One should note that the particle images are not shifted in perpendicular to flow direction in simulation, i.e. the true values of their $x$-component of displacements are equal to zero. Only one iteration is applied when using 2D-DCC and 1D-DCC for both test cases.

The random and bias errors for uniform flow are plotted in figure 2 depending on IW size and particle images displacement. The comparative error estimation is conducted for data obtained by 2D-DCC and 1D-DCC. Moreover, particle images displacements are determined from both cross-correlation maps $R_x$ and $R_y$ when considering 1D-DCC. The results are compared with appropriate data obtained by the iterative multigrid technique with discrete IW offset [12] for synthetic images with the same particle images concentration and diameters.

Figure 2,a and figure 2,b illustrate the random error patterns which grow with IW size reduction. The difference of approximately one order of magnitude between random errors obtained from $R_x$ and $R_y$ is seen for both particle images displacement components. Besides, the random errors $\sigma_y$ and $\sigma_x$ have lower value if displacements are calculated from $R_y$ and $R_x$, respectively. Such a behavior of random error distribution is explained by the shape of cross-correlation function (see figure 1). For instance, if cross-correlation function is obtained with the help of projection $I_y$, it has prolate shape in the vicinity of its maximum: broad cross-correlation peak in the $y$-direction and narrow peak in the $x$-direction (see top cross-correlation map $R_y$ in figure 1). It is well-known [5] that broad cross-correlation peak reduces the accuracy of peak detection. Thus, it is better to calculate $x$-component of particle images displacement from $R_x$ and, similarly, $y$-component of displacement from $R_y$.

![Figure 2](image-url)

**Figure 2.** Random ($a$, $b$) and bias ($c$, $d$) errors of $x$- ($b$, $d$) and $y$- ($a$, $c$) components of particle images displacement for uniform flow.

Figure 2,c shows the periodic behavior of bias errors with a 1-pixel wave-length that is in accord with [12]. This periodicity appears due to the application of ROI that is similar to discrete IW offset used in [12]. The amplitude of the bias error patterns obtained by different techniques varies not much
with the IW size and their values are in good agreement with [12]. Nevertheless, the bias errors $\beta_v$ calculated from $R_x$ are less than the same one obtained from $R_y$ (see figure 2,c). No explanation is found yet for this result which will be further analyzed. Figure 2,d shows that $x$-component of particle images displacement is better calculated from $R_x$ (note that in $x$-direction the particle images displacements are equal to zero).

In general, it is seen from figure 2 that 2D-DCC yields more accurate results compared to results obtained by 1D-DCC and the iterative multigrid technique with discrete IW offset [12]. However, if particle images displacements are properly obtained from appropriate cross-correlation maps, they are accurate within 0.1 pix that is quite sufficient for initial velocity field calculation.

Figure 3 illustrates the random and bias error patterns for shear flow. It is seen from figure 3,a that the $y$-component of particle images displacement is obtained more accurately from $R_y$ when the displacement gradient is smaller than 0.2 and 0.1 pix/pix for IW sizes 16×16 and 32×32 pix$^2$, respectively. This behavior of random error distribution for low gradient shear flow is similar to the case of uniform flow and has been described above. However, when higher displacement gradient is considered, the cross-correlation map $R_y$ yields worse results compared to $R_x$ (see figure 3,a). This can be explained by the following. According to 1D-DCC, the result of particle images projecting on orthogonal sides of planar image is the one-dimensional field of illumination intensity where particle images are not single but may overlap yielding the homogeneous field. Thus, if particle images are displaced not uniformly in the direction perpendicular to the projecting side of image, the one-dimensional field can become unrecognizable. This effect becomes stronger, the higher the gradient is. Thus, it is better to use $R_y$ when high gradient shear flow is considered (see figure 3,a). The influence of projecting procedure is more obvious when $x$-component of displacement is calculated (see figure 3,b): $\sigma_u$ obtained from $R_x$ is approximately two orders of magnitude less than the same one obtained from $R_y$ and even less than the same one obtained by 2D-DCC. Not big difference in $\beta_u$ is seen in figure 3,c in contrast to $\beta_v$ shown in figure 3,d where $R_x$ provides better results.

In general, the comparison of the random and bias errors for shear flow demonstrates that $R_x$ yields better results in terms of accuracy than $R_y$ especially for the high flow gradient.

![Figure 3](image_url)

**Figure 3.** Random ($a$, $b$) and bias ($c$, $d$) errors of $x$- ($b$, $d$) and $y$- ($a$, $c$) components of particle images displacement for shear flow.

Finally, the application of 1D-DCC together with the iterative multigrid window-deformation (WIDIM) technique [12] is demonstrated. 1D-DCC based on $R_x$ is used to obtain the initial particle images displacement field and then two additional iterations are executed by WIDIM. In order to
interpolate the image deformed according to the predictor displacement field, the \( \text{sinc} \) function with kernel size \( 6 \times 6 \text{ pix}^2 \) is chosen. The results are compared with appropriate data from [12].

The random and bias errors for uniform flow are plotted in figure 4, a and figure 4, c, respectively, depending on IW size and particle images displacement. It is seen from figure 4, a that the random error is one order of magnitude higher than the same one obtained by WIDIM used FFT-CC [12]. Figure 4, c shows the nonmonotonic behavior of the bias errors as well as the same order of magnitude at their peak values. The periodic behavior of bias errors with a 1-pixel wave-length is in good agreement with [13] whereas no explanation has been found in [12] for periodic behavior with the 2-pixels wave-length. The amplitude of the bias error pattern is in accord with [12]. The results of shear flow test case shown in figure 4, b agree well with results from [12]. Nevertheless, further improvement in the proposed technique is needed to reach higher accuracy of data.

Figure 4. Random (a, b) and bias (c, d) errors of \( y \)-component of particle images displacement for uniform flow (a, c) and shear flow (b, d).

5. Conclusions

The high-performance 1D-DCC technique based on the PPC procedure is proposed in order to accelerate the cross-correlation function computation. It allows acceleration of cross-correlation function calculation up to \( N/2 \) times compared to 2D-DIC. It has been shown that, in practice, the speed-up is less. For example, for IW size of 32 pix, the cross-correlation procedure is accelerated about 14 times that is less compared to theoretical estimation of 16 times.

Two synthetic test cases have been considered to estimate the accuracy of the proposed technique: one-dimensional uniform flow and linear shear flow. It has been shown that the use of \( x \)- and \( y \)-projections of IW leads to prolate shapes of cross-correlation peaks in \( y \)- and \( x \)-directions, respectively, which adversely affects the accuracy of appropriate displacement components. In general, the comparison of the random and bias errors for uniform and shear flows demonstrates that the cross-correlation map \( R_y \) yields better results in terms of accuracy than \( R_x \) when particle images are displaced in \( y \)-direction. The results obtained by 1D-DCC together with WIDIM have shown that 1D-DCC can be recommended for the initial velocity field calculation.

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