Nonconservation of Lepton Current and Asymmetry of Relic Neutrinos

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Abstract—The neutrino asymmetry, \(n_\nu - n_\bar{\nu}\), in the plasma of the early Universe generated both before and after the electroweak phase transition (EWPT) is calculated. It is well known that in the Standard Model the leptogenesis before the EWPT, in particular, for neutrinos, owes to the Abelian anomaly in a massless hypercharge field. At the same time, the generation of neutrino asymmetry in the Higgs phase after the EWPT has not been considered previously due to the absence of any quantum anomaly in an external electromagnetic field for such electroneutral particles as neutrinos, in contrast to the Adler anomaly for charged left- and right-handed massless electrons in the same electromagnetic field. Using the Boltzmann equation for neutrinos modified to include the Berry curvature term in momentum space, we establish a violation of the macroscopic neutrino current in the plasma after the EWPT and exactly reproduce the non-conservation of the lepton current in the symmetric phase before the EWPT that owes to the contribution of the triangle anomaly in an external hypercharge field but already without computing the corresponding Feynman diagrams. We apply the new kinetic equation to calculate the neutrino asymmetry by taking into account the Berry curvature and the electroweak interaction with plasma particles in the Higgs phase, including that after the neutrino decoupling in the absence of their collisions in the plasma. We find that this asymmetry is too small for observations. Thus, a difference between the relic neutrino and antineutrino densities, if it exists, must appear already in the symmetric phase of the early Universe before the EWPT.

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1. INTRODUCTION

The non-conservation of the baryon and lepton numbers in the Standard Model (SM) is known to owe to the quantum non-Abelian and (or) Abelian anomalies that correspond to global charge conservation (t Hooft’s rule). \(B/3 - L_\alpha = \text{const}\), \(a = e, \mu, \tau\). As one possibility to generate the baryon asymmetry of the Universe (BAU) through leptogenesis with global charge conservation before the electroweak phase transition (EWPT) at temperatures \(T \geq T_{\text{EWPT}}\) \(\approx 100\) GeV, in [1, 2] we considered the well-known baryogenesis scenario with an initial right-handed electron asymmetry [3]. In this approach we took into account the Abelian triangle anomaly in a massless hypercharge field \(Y\) [4]. We established in [1, 2] that such an Abelian anomaly for the left-handed doublet of leptons \(L = (\nu_e, e_\nu)\),

\[
\frac{\partial j^\mu_L}{\partial x^\mu} = -\frac{g'}{16\pi^2} (E_Y \cdot B_Y),
\]

where \(g' = e/\cos\theta_W\) is the SM gauge coupling constant, \(\sin^2\theta_W = 0.23\) is the Weinberg parameter, and \(E_Y = -\partial Y - \nabla Y_0\) and \(B_Y = (\nabla \times Y)\) are the hyperelectric and hypermagnetic fields, respectively, more likely plays a secondary role in baryogenesis for the chosen initial conditions, because the left-handed lepton asymmetry has no time to develop before the EWPT in such a way as to level off the BAU through sphaleron transitions due to the interaction of sphalerons with left-handed primordial plasma components.

In any case, one might expect an initial neutrino asymmetry \(n_\nu - n_\bar{\nu} = \xi_\nu S^T/6\), where \(\xi_\nu = \mu_\nu/T \neq 0\) is the neutrino asymmetry parameter and \(\mu_\nu\) is the chemical potential of the neutrino–antineutrino Fermi gas,\textsuperscript{1} to appear after the EWPT at \(T < T_{\text{EWPT}}\). The specific (possibly larger) initial neutrino asymmetry in a hot plasma at temperatures \(\tilde{C}(\text{MeV}) < T < \)

\(1\) For the initial conditions chosen in [1] the left-handed lepton asymmetry \(\xi_{\nu e L} = \xi_{\nu e L0}\) reaches a value comparable to the BAU, \(\xi_{\nu e L} \sim 10^{-10}\), at the EWPT time (see Fig. 2 in [1]).
$T_{EWPT}$ remains unknown, except for the Big Bang nucleosynthesis constraint on this parameter $|\xi_{\nu_\tau}| < 0.07$ [5] at $T_{BBN} = 0.1$ MeV when the neutrino oscillations with equivalent asymmetries, $\xi_{\nu_A} \sim \xi_{\nu_\tau} \sim \xi_{\nu_\tau}$, already at $T = O$(MeV) are taken into account. Below we attempt to find other ways to obtain constraints on the relic neutrino asymmetry both before and after the EWPT.

Since the neutrino has zero electric charge, no triangle anomaly exists for it after the EWPT, in contrast to charged fermions in QED. Recall that the non-conservation of the current that owes to the Adler triangle anomaly for chiral (massless) charged fermions,

$$\partial_t n_{R,L} + (\nabla \cdot j_{R,L}) = \pm \frac{Q}{\pi} (E \cdot B),$$

leads to magnetic field instability in a relativistic plasma, for example, for the hot plasma of the early Universe in the presence of a seed difference between the chemical potentials (densities) of right and left-handed particles, $\mu_\ell (t) = (\mu_R - \mu_L)/2 \neq 0$, which decreases due to a change in helicity when the fermion mass is taken into account [6].

Note that the anomaly in (2) has recently been deduced independently from the Boltzmann kinetic equation by taking into account the Berry curvature in momentum space (see Eq. (20) in [7]). This equation is extensively studied and widely used in condensed matter physics and in studying the collisions of heavy ions to produce a quark–gluon plasma [8]. Below we consider a chiral medium for massless neutrinos before and after the EWPT described by the Boltzmann kinetic equation in both cases by taking into account the Berry curvature and obtain analogs of the anomaly in (2) with the 4-current non-conservation for a $v\nu\bar{\nu}$ gas.

In our paper we consider the following questions. In Section 2 we recall the concept of the Berry curvature for neutrinos (or the induced gauge field associated with the topological Berry phase [9, 10]). Then, in Section 3, beginning with the Boltzmann equations without this curvature, we generalize the latter by adding the terms that owe to this topological effect. The generalization is made in two cases: (i) in the presence of massless hypercharge fields before the EWPT; and mainly (ii) in a hot plasma after the EWPT at temperatures $T \ll T_{EWPT}$, when the Fermi approximation is applicable for the interactions of neutrinos with background plasma particles. In case (i) we reproduce the well-known Abelian anomaly for right- and left-handed leptons, in particular, the non-conservation of the current $j^\mu_L$ in Eq. (1). In case (ii) we predict an anomalous violation of the neutrino current, $\partial_t j^\mu_L (x, t) \neq 0$, hitherto unknown in the literature. Then, in Section 4 we calculate the time evolution of the neutrino asymmetry in an expanding Universe using this new anomaly. In Section 5 we summarize our results.

2. THE BERRY CURVATURE

We consider massless neutrinos, $m_{\nu_a}$, $a = e, \mu, \tau$, i.e., neglect the neutrino oscillations. In electroweak interactions in the SM, these particles are chiral, i.e., their fields enter into the interaction Lagrangian as $\psi_{\nu_a} \equiv \psi_{\nu_a} = (1 - \gamma^5)\psi_{\nu_a}/2$, which corresponds to left-handed neutrinos, $(\sigma \cdot k)u_-(k) = -ku_-(k)$, and right-handed antineutrinos, $(\sigma \cdot k)u_+(k) = ku_+(k)$. Here,

$$u_-(k) = \left\{ \begin{array}{ll} e^{-i\phi} \sin(\theta/2) & \\ \cos(\theta/2) & \\ \sin(\theta/2) & \end{array} \right\}$$

are the corresponding two-component spinors and $\sigma$ are the Pauli matrices. These spinors define a nonzero Berry connection (the components of the induced gauge field in momentum space, see [10]):

$$a^\pm = (a^+_k, a^+_\ell, a^\mp_\ell) = iu^\dagger k \nabla_k u_k (k).$$

Using the spinors in (3), we can easily find the components of this field in spherical coordinates $a^\pm = a^\mp = 0$, $a^\ell = \tan(\theta/2)/2k$, and $a^\ell = \cot(\theta/2)/2k$. This allows the Berry curvature $\Omega_k^\pm$ to be calculated:

$$\Omega_k^\pm = \nabla_k \times a_k^\pm = \pm \frac{k}{2\ell^2}, \quad n \equiv \hat{k} = \frac{k}{k}, \quad n^2 = 1,$$

where the upper and lower signs correspond to $\nu_\tau$ and $\nu_\nu$, respectively. The Berry connection in (4) enters as an additional term into the action for chiral right- and left-handed charged particles in the presence of an electromagnetic field $A^\mu = (A_\mu, A) [7]:$

$$S = \int d\tau (p - eA) \cdot \dot{x} - (e_\ell - e_\ell - e_\ell) - a^\pm \cdot \dot{p}.$$  

A similar relation also holds for a lepton (in particular, a left-handed neutrino) interacting with the hypercharge field before the EWPT via the SM coupling constant $g_{R,L} = g'Y_{R,L}/2$, where $Y_L = -1$ is the hypercharge for a left-handed doublet and $Y_R = 2$ is the hypercharge for a right-handed singlet (right-handed electron):

$$S = \int d\tau (k - g_{R,L} Y) \cdot \dot{x} - (e_k - e_{R,L} Y_0) - a_k^\pm \cdot \dot{k}.$$  

In the latter case, we neglected the contribution of the pseudo-vector current\(^2\) in the lepton–massless hypercharge field interaction Lagrangian

\(^2\)As a result, we omitted the contribution of the Chern–Simons term in the particle–hypercharge field interaction Lagrangian proportional to $\mu_c \sigma (Y \cdot B_\mu)$ in the model [4] that is related to the SM parity non-conservation and owes to the polarization effect of the currents of electrons and positrons at the ground Landau level moving along the external hypermagnetic field [11]. This term leads to hypermagnetic field instability, which is not considered here.
\[ \mathcal{L}_{\text{int}} = \sum_a \left[ g_L T_a \gamma_\mu L_a + g_R T_a \gamma_\mu R_a \right] Y^\mu, \] (8)

taking into account only the vector interaction in (7) by analogy with the vector electromagnetic interaction, \( \mathcal{L}_{\text{em}} = e \bar{\psi} \gamma_\mu \gamma^\mu \psi. \) This simplifies the derivation of the Boltzmann kinetic equation

\[ \frac{\partial f_{R,L}}{\partial t} + \mathbf{x} \cdot \nabla f_{R,L} + \mathbf{k} \cdot \nabla f_{R,L} = J_{\text{coll}}, \] (9)

see below.

Finally, to derive the kinetic equation from the action for neutrinos in an unpolarized medium after the EWPT, in the Fermi approximation at temperatures \( T \ll T_{\text{EWPT}} \),

\[ S = \int d^4l \left( (k - G_V \sqrt{2} \nu_e \delta j^\nu) \cdot \mathbf{x} ight) \]

\[ = - (\epsilon_k - G_V \sqrt{2} \nu_e \delta n^\nu) - \mathbf{a}_k \cdot \mathbf{k}, \] (10)

we will give the corresponding equations of motion. Given the Berry curvature in (5), the latter are (see the case of charged particles in [7, 12])

\[ \mathbf{x} = \frac{\partial \mathbf{x}}{\partial \mathbf{k}}, \]

\[ \mathbf{k} = G_V \sqrt{2} \nu_e, \]

\[ \times \left[ - \frac{\partial \delta j^\nu}{\partial t} - \nabla \delta n^\nu + \mathbf{x} \cdot \left[ \nabla \times \delta j^\nu \right] \right]. \] (11)

For nonlinear \( \nu \nu \) interactions the coefficient \( 2G_V \sqrt{2} \nu_e \) with the substitution of the superscript \( e \rightarrow \nu \) for the particle number and 3-current densities should be used instead of \( 2G_V \sqrt{2}. \) Here, \( G_V = 1.17 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi constant, \( \nu_e = 2 \sin^2 \theta_W \) is the vector coupling constant for \( \nu_e \) interactions (the upper sign for electron neutrinos), \( \delta n^{(\nu)} (x, t) = n_{\nu_e} (x, t) - n_{\bar{\nu}_e} (x, t) \) is the electron–positron density asymmetry in an \( e^- e^+ \) plasma, and \( \delta j^{(\nu)} (x, t) = \mathbf{j}_{\nu_e} (x, t) - \mathbf{j}_{\bar{\nu}_e} (x, t) \) is the 3-current density asymmetry. Note the complexity of the coupled equations of motion in (11) with the Berry curvature, when after some algebraic transformations their expressions and the phase volume \( d^3x d^3k \) change as the velocities \( \mathbf{x} \) and forces \( \mathbf{k} \) are decoupled, see below.

3. THE KINETIC EQUATIONS FOR NEUTRINOS WITH THE BERRY CURVATURE

(i) Without the Berry curvature. It is not surprising that if we restrict ourselves to the vector interaction with the hypercharge field without the Berry curvature, in complete analogy with the ordinary Boltzmann equation for charged particles, the kinetic equation for neutrinos and antineutrinos in the symmetric phase of the early Universe is

\[ \frac{\partial f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t)}{\partial t} + \mathbf{n} \cdot \frac{\partial f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t)}{\partial x} \]

\[ \pm g_L \left[ E_V (x, t) + \mathbf{n} \times \mathbf{B}_V (x, t) \right] \cdot \frac{\partial f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t)}{\partial k} \]

\[ = f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t) \] (12)

where \( f^{(\nu_{\nu}, \nu_{\bar{\nu}})} \) are the collision integrals and \( f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t) \) are the distribution functions for neutrinos (antineutrinos) with the substitution of the upper (lower) sign in the force term.

Recall that the Boltzmann equation for neutrinos (antineutrinos) in an unpolarized medium at \( T \ll T_{\text{EWPT}} \), also without the Berry curvature has a form [13–16] that follows from the action (10), when for \( \Omega_k = 0 \) the velocity becomes the ordinary unit velocity of a massless particle, \( \mathbf{x} = \partial \mathbf{x}/\partial \mathbf{k} = \mathbf{n}: \)

\[ \frac{\partial f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t)}{\partial t} + \mathbf{n} \cdot \frac{\partial f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t)}{\partial x} \]

\[ \pm \left[ E_V (x, t) + \mathbf{n} \times \mathbf{B}_V (x, t) \right] \cdot \frac{\partial f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t)}{\partial k} \]

\[ = f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t), \] (13)

where for massless \( \nu_e (\bar{\nu}_e) \) we substitute the upper (lower) sign in the third (force) term defined by the weak \( \nu \nu \) interactions in the Fermi approximation:

\[ E_V (x, t) = G_V \sqrt{2} \nu_e \]

\[ \left[ \mathbf{n} \cdot \nabla \delta n^{(\nu)} (x, t) \right], \]

\[ \mathbf{B}_V (x, t) = G_V \sqrt{2} \nu_e \nabla \times \delta j^{(\nu)} (x, t). \] (14)

Interestingly, since the force term in (13) has the form of a Lorentz force, the corresponding electromagnetic fields (14) obey the standard Maxwell equations: \( \nabla \cdot \mathbf{B}_e = 0 \) and \( \partial_t \mathbf{B}_e = - \nabla \times \mathbf{E}_e. \)

We will emphasize that, neglecting the Berry curvature, we have the spectrum \( \epsilon_k = k \) for which the neutrino 4-current

\[ j^{(\nu_{\nu}, \nu_{\bar{\nu}})} (x, t) = (n_{\nu_{\nu}} (x, t), \mathbf{j}_{\nu_{\nu}} (x, t)) \]

\[ = \int \frac{d^3k}{(2\pi)^3} \epsilon_k \]

\[ f^{(\nu_{\nu}, \nu_{\bar{\nu}})} (k, x, t) \] (15)

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is conserved, as can be seen from the kinetic equations (12) and (13) integrated over \(d^3k\):

\[
\frac{\partial}{\partial t} f^{(\nu_e, \nu_e)}_{\mu}(x,t) = \frac{\partial n^{(\nu_e, \nu_e)}(x,t)}{\partial t} + \frac{\partial [V^{(\nu_e, \nu_e)}(x,t) n^{(\nu_e, \nu_e)}(x,t)]}{\partial x} = 0,
\]

where the macroscopic velocity of the \(\nu_e \nu_e\) gas

\[
V^{(\nu_e, \nu_e)}(x,t) = \frac{1}{n^{(\nu_e, \nu_e)}(x,t)} \int d^3k \frac{f^{(\nu_e, \nu_e)}(k, x, t)}{(2\pi)^3},
\]

(17)
can be nonrelativistic, \(|V| \ll 1\), in contrast to the microscopic velocity in the kinetic equation: \(|n| = 1\).

(ii) **With the Berry curvature.** Now let us turn to the case where the Berry curvature in (5) is taken into account and consider, for example, a generalization of the Boltzmann kinetic equation for neutrinos in the broken phase of the early Universe (13). In complete analogy with the approach in [7, 12], for chiral fermions with a modified spectrum

\[
\varepsilon_k = k[1 - (\Omega_k \cdot B_e(x,t))],
\]

(18)
we can write a modified Boltzmann equation for the neutrino distribution function \(f^{(\nu_e)}_k(k, x, t)\) (for simplicity, only for neutrinos):

\[
\frac{\partial f^{(\nu_e)}_k}{\partial t} + \frac{1}{\sqrt{\omega}} \left( \vec{\nu} \times (\vec{E}_e + \vec{\nu} \times \vec{B}_e) + (\vec{\nu} \cdot \Omega_k) B_e \right) \frac{\partial f^{(\nu_e)}_k}{\partial k} + \frac{1}{\sqrt{\omega}} \left( \vec{E}_e + \vec{\nu} \times \vec{B}_e \right) \frac{\partial f^{(\nu_e)}_k}{\partial x} = j^{(\nu_e)}(f^{(\nu_e)}_k).
\]

(19)
Here, \(\omega = [1 + (B_e \cdot \Omega_k)]^2\) is the coefficient modifying the phase volume, \(d^3k d^3x \rightarrow \sqrt{\omega} d^3k d^3x\), due to the Berry curvature (5) and the effective magnetic field (14), \(\vec{\nu} = \partial \varepsilon_k / \partial k\) is the effective neutrino velocity, and \(\vec{E}_e = E_e - \partial \varepsilon_k / \partial x\) is the effective electric field in the modified Boltzmann equation (19), both with allowance made for the spectrum in (18).

The neutrino number density and the neutrino 3-current density,

\[
n^{(\nu_e)}(x,t) = \int \frac{d^3k}{(2\pi)^3} \sqrt{\omega} f^{(\nu_e)}_k,
\]

(20)
\[
j^{(\nu_e)}(x,t) = \int \frac{d^3k}{(2\pi)^3} \left[ \vec{E}_e + \vec{\nu} \times \Omega_k \right] f^{(\nu_e)}_k.
\]

(21)
satisfy the quantum anomaly (the non-conservation of the macroscopic neutrino 4-current, \(\partial^\mu j^{(\nu_e)}_\mu \neq 0\)) owing to the weak interactions in the SM in the presence of Berry curvature (see [7]):

\[
\partial_t n^{(\nu_e)} + \nabla \cdot j^{(\nu_e)} = - \left( E_e \cdot B_e \right) \int \frac{d^3k}{(2\pi)^3}
\]

(22)
\[
\times \left( \Omega_k \cdot \frac{\partial f^{(\nu_e)}_k}{\partial k} \right) = - C^{(\nu_e)}(E_e \cdot B_e) \neq 0.
\]

The integral in Eq. (22) can be easily calculated for neutrinos with a Fermi distribution

\[
f^{(\nu_e)}(k) = \left[ \exp \left( \frac{k - \mu_{\nu_e}}{T} \right) / T \right]^{-1},
\]

by substituting \(\Omega_k = -k/2k^3\) from the definition of curvature (5) for neutrinos:

\[
C^{(\nu_e)} = \int \frac{dk}{4\pi^2} \frac{\exp \left[ \frac{(k - \mu_{\nu_e})}{T} \right]}{[\exp \left[ \frac{(k - \mu_{\nu_e})}{T} \right] + 1]^2} = \frac{1}{4\pi^2(1 + \exp(-\mu_{\nu_e}/T))}.
\]

(23)
Owing to the opposite sign of this parameter for antineutrinos due to the different sign of the Berry curvature \(\Omega_k\) and given the opposite sign of the chemical potential for \(\nu_e, \mu_{\nu_e} \rightarrow -\mu_{\nu_e}\), we obtain (after the volume integration) the main result of this paper for the evolution of the neutrino asymmetry from the non-conservation of the neutrino current (22) and the expression for the antineutrino current opposite to it in sign:

\[
\frac{d}{dt} (n_{\nu_e} - n_{\nu_e}) = - \frac{1}{4\pi^2} \int \frac{d^3k}{V} (E_e \cdot B_e),
\]

(24)
where the effective electromagnetic fields \(E_e\) and \(B_e\) are specified by Eq. (14).

3.1. *Lepton Current Anomalies in the Symmetric Phase of the Early Universe*

Now we are able to obtain the quantum effect of the non-conservation of lepton currents in a hypercharge field \(Y\) from the Boltzmann equation with the Berry curvature. Similarly to the spectrum (18) and the neutrino...
trino anomaly after the EWPT (24), introducing the
neutrino spectrum $\varepsilon_k = k[1 - g_k(\Omega \cdot B)]$, from the
Boltzmann equation (12), by analogy with the modifi-
cation (13) leading to (19) and then to the 4-current
(20) and (21), we derive a similar anomaly for the left-
handed lepton current before the EWPT:

$$\frac{d}{dt}(n_{e_L} - n_{\mu_L}) = -\frac{g_R^2}{4\pi^2} \int \frac{d^3x}{V} (E_Y \cdot B_Y)$$

(25)

Obviously, absolutely the same Abelian anomaly exists
for left-handed electrons $e_L$ or their density asymme-
try $(n_e - n_\mu)$ obeys Eq. (25) due to the same initial
form of the Boltzmann equation (12).

However, in the case of a right-handed electron $e_R$, we
should, first, substitute $g_L \rightarrow g_R$ in the kinetic equa-
tion (12) and, second, take into account the fact that the
massless $e_R$ is right-handed like antineutrinos, i.e.,
the Berry curvature $\Omega_\mu$ for it leads to the opposite sign
in Eq. (23):

$$C^{(e_R)} = \frac{g_R^2}{4\pi^2 T} \int_0^\infty dk \frac{\exp[(k - \mu_{e_R})/T]}{[\exp[(k - \mu_{e_R})/T] + 1]^2}$$

(26)

On the contrary, the massless right-handed positron is
left-handed like $\nu_{e_L}$ and $e_L$, or the same sign as that in
Eq. (23) obtained for it with the substitution of the
chemical potential $\mu_{e_L} \rightarrow -\mu_{e_R}$, so that

$$C^{(\nu_e)} = (\frac{g_R^2}{4\pi^2})(1 + \exp(\mu_{e_R}/T))^{-1}.$$

Finally, it is easy to reconstruct the Abelian anomaly
for right-handed electrons with an effective charge
$g_R = g_Y^e/2$:

$$\frac{d}{dt}(n_{e_R} - n_{\mu_R}) = \frac{g_R^2}{4\pi^2} \int \frac{d^3x}{V} (E_Y \cdot B_Y)$$

(27)

Thus, we exactly deduced the well-known Abelian anoma-
lies for lepton currents in hypercharge fields without resenting to the Feynman diagram technique.

To conclude this section, we will emphasize that to
get some analog of the Abelian anomaly in the Higgs
(broken) phase after the EWPT in form (24) for neu-
trinos, we needed to consider the electroweak neu-
trino–plasma interaction using the effective electro-
magnetic fields (14) and the Berry curvature in Eq. (22).

4. NEUTRINO ASYMMETRY GENERATION
IN A HOT PLASMA

In this section we will use the neutrino anomaly
(24) to study the generation of neutrino asymmetry in the
early Universe. We will consider a hot plasma at
relativistic temperatures much lower than the EWPT
temperature, $m_e \ll T \ll T_{EWPT}$, when we can apply the
Fermi approximation for weak neutrino—matter interac-
tions. Note that, for simplicity, our calculations are
limited to the neutrino interactions in a leptonic $e^+e^-$
plasma, although allowance for other plasma compo-
ents for $T > T_{QCD} = 100$ MeV causes no fundamental
difficulties.

The effective electromagnetic fields $E_e$ and $B_e$ are
specified by Eqs. (14). Taking into account the stan-
dard Maxwell equations for ordinary electromagnetic
fields $E$ and $B$ in the MHD approximation,

$$\dot{B} = -\nabla \times E, \quad \nabla \cdot B = 0,$$

(28)

$$\nabla \cdot E = -e\delta n^{(e)}, \quad \nabla \times B = j_{em} = -e\delta j^{(e)},$$

where the charge and 3-current density asymmetries
$\delta n^{(e)} = n_e - n_\mu$ and $\delta j^{(e)} = j_e - j_\mu$ are defined in the
equations of motion (11), we find the effective fields
(14) that owe to the weak interaction to be directly
expressed via the Maxwellian fields:

$$E_e(x,t) = A\nabla^2 E(x,t),$$

(29)

$$B_e(x,t) = A\nabla^2 B(x,t).$$

Here, $A = G_F 2\sqrt{2}/e$, $e = \sqrt{\frac{4}{3}\pi\alpha_{em}} \approx 0.3 > 0$ is the abso-
lute value of the electron charge, and $\alpha_{em} = 1/137$ is the
fine-structure constant.

We then use the Fourier representation of the elec-
 tromagnetic field,

$$E(x,t) = \int \frac{d^3k}{(2\pi)^3} e^{ikx} E_k(t),$$

(30)

$$B(x,t) = \int \frac{d^3k}{(2\pi)^3} e^{ikx} B_k(t),$$

where we take into account the fact that $B(x,t) = B^*(x,t)$. In this case, the evolution of the neutrino
asymmetry in the basic equation (24),

$$\frac{d}{dt}(n_{e_R} - n_{\mu_R}) = -\frac{A^2}{8\pi^2 V}$$

$$\times \int \frac{d^3k}{(2\pi)^3} k^4 |E_k(t) \cdot B^*_k(t) + c.c.\|$$

(31)

$$= \frac{A^2}{8\pi^2} \int k^4 \frac{d}{dt} h(k,t)dk,$$

where we used the constraint on the dependence of
$h(k,t)$. The above equation has the same form as the
Boltzmann equation for the left-handed electrons:

$$\frac{d}{dt}(n_{e_L} - n_{\mu_L}) = -\frac{g_R^2}{4\pi^2} \int \frac{d^3x}{V} (E_Y \cdot B_Y).$$
is defined by an isotropic spectrum \( h(k, \tau) \) of the magnetic helicity density \( h(t) = \int dk h(k, \tau) \), where \( h(k, \tau) = k^2 A_k(t) \cdot B_\tau^*(t) + \text{c.c.} / 4 \pi^2 V \) and
\[
\frac{\partial}{\partial t} h(k, \tau) = -\frac{k^2}{2 \pi^2 V} [E_k(t) \cdot B_\tau^*(t) + \text{c.c.}] \tag{32}
\]
Substituting the neutrino density asymmetry \( n_{\nu e} - n_{\bar{\nu} e} = T^2 \xi_{\nu e}(T)/6 \), introducing a variable \( \xi_{\nu e} = \mu_{\nu e}(T)/T \), where \( \mu_{\nu e} \) is the neutrino chemical potential, and using the conformal dimensionless variables \( t \rightarrow \eta = M_{\nu}/T, a = T^{-1} \), and \( h(\kappa, \eta) = a^2 h(k, \tau) \), where \( \kappa = ak \) is a quantity conserved in time, we can rewrite Eq. (31) in the comoving volume as
\[
d\xi_{\nu e}(\eta) = \frac{3 A_s^2}{4 \pi^2 a^2} \int d^4 \kappa \frac{\partial}{\partial \eta} \left[ \frac{h(\kappa, \eta)}{a^2} \right] d\kappa. \tag{33}
\]

The evolution of the magnetic helicity and magnetic energy density spectra obeys the system of equations [6]
\[
\frac{\partial}{\partial \eta} \tilde{h}(\kappa, \eta) = -2 \frac{\xi_{\nu e}^2}{\sigma_c} \tilde{h}(\kappa, \eta) + 4 \tilde{\Pi} \rho_\mu(\kappa, \eta), \tag{34}
\]
\[
\frac{\partial}{\partial \eta} \rho_\mu(\kappa, \eta) = -2 \frac{\xi_{\nu e}}{\sigma_c} \rho_\mu(\kappa, \eta) + \tilde{\Pi} k^2 \tilde{h}(\kappa, \eta),
\]
where \( \tilde{\Pi} = 2 \alpha_{em} a_5 / \pi \) and \( a_5 = a_\mu_5 \). In Eq. (34) we assume the electric conductivity to be \( \sigma_{\text{cond}} = \sigma_c T \), where \( \sigma_c \approx 100 \) in a hot QED plasma. In what follows, we will assume that the magnetic field has maximum helicity: \( \tilde{h}(\kappa, \eta_0) = 2 \tilde{\rho}_\mu(\kappa, \eta_0)/\kappa \).

For the system of equations (34) to be closed, we must describe the evolution of the chiral imbalance \( \mu_5 = (\mu_{eR} - \mu_{eL})/2 \neq 0 \). This can be done using the conservation law (see, e.g., [18])
\[
\frac{d}{dt} \left[ n_{eR} - n_{eL} + \frac{\alpha_{em}}{\pi} h(t) \right] = 0. \tag{35}
\]
Using Eq. (35), the kinetic equation for the chiral imbalance \( \mu_5 = (\xi_{eR} - \xi_{eL})/2 \) takes the form
\[
\frac{d\mu_5}{d\eta} + 6 \frac{\alpha_{em}}{\pi} \int d\kappa \frac{\partial}{\partial \eta} h(\kappa, \eta) = -\Gamma_{\mu_5}, \tag{36}
\]
where we took into account the chirality flipping rate \( \Gamma_{\mu_5} = a T F_{\mu_5} \) due to the nonzero electron mass \( m_e \neq 0 \). This rate was estimated in [6], which will be used in our calculations in Section 4.2.

### 4.1. Monochromatic Magnetic Helicity Spectrum: A Toy Model

To show the possibility of the generation of neutrino asymmetry due to the electroweak interaction with an \( e^- e^+ \) plasma, let us consider a monochromatic magnetic helicity spectrum \( h(k, \tau) = h(t) \delta(k - k_0) \), where \( k_0 = r_D^{-1} \), \( r_D = \nu_F / \omega_p \) is the Debye length, and \( \omega_p = \sqrt{4 \pi e n_\nu / (e^2)} \) is the plasma frequency. Note that \( \omega_p \) coincides with the transverse plasmon mass in the dispersion relation \( \omega = \sqrt{k^2 + \omega_p^2} \), where \( \langle E \rangle \approx 3 T \) is the mean energy in a hot ultrarelativistic plasma for which the thermal velocity \( v_T = 1 \). If a nonrelativistic plasma (after the annihilation of positrons) at \( T \ll m_e \) is considered, then \( \langle E \rangle \approx m_e \) and \( v_T = \sqrt{T/m_e} \). From Eq. (31) we then derive a conservation law similar to the law (35) deduced for charged particles from the Adler anomaly:
\[
\frac{d}{dt} \left[ n_{\nu e} - n_{\bar{\nu} e} - \frac{\alpha_{ind}^a}{2 \pi} h(t) \right] = 0, \tag{37}
\]
where \( \alpha_{ind}^a = \left| e_{\text{ind}}^{(\nu_e)} \right|^2 / 4 \pi \) is the effective electromagnetic constant and \( e_{\text{ind}}^{(\nu_e)} \) is the induced neutrino charge in the plasma.\(^6\) The charge \( e_{\text{ind}}^{(\nu_e)} \) for a Dirac neutrino was found in [19, 20]:
\[
e_{\text{ind}}^{(\nu_e)} = -G_F c_F^e (1 - \lambda) \frac{1}{\sqrt{2} r_D^2}, \tag{38}
\]
where \( \lambda = \mp 1 \) is the neutrino helicity and the lower sign corresponds to a sterile neutrino.

Using Eq. (38), we find that
\[
\left| e_{\text{ind}}^{(\nu_e)} \right|^2 = 7 (e_F^e)^2 \times 10^{-14} (T/m_p)^4
\]
in a hot plasma at \( T \gg m_e \) and \( r_D^{-1} = 0.075 T \). Finally, the effective coupling in the conservation law (37) changes with expansion of the Universe as
\[
\alpha_{ind}^a(T) = 5.6 \left( e_F^e \right)^2 \times 10^{-15} \left( \frac{T}{m_p} \right)^4, \tag{39}
\]
where the electron density \( n_e = 0.183 T^3 \) is substituted.

Taking into account the expressions for the Debye length, \( r_D = \omega_p^{-1} \), and the maximum monochromatic magnetic helicity density, \( \dot{h}(t) = 2 \rho_\mu(t)/k_0 = B^2(T)/\omega_p(T) \), from the conservation law (37) we obtain the electron neutrino asymmetry at \( T \gg m_e \):
\[
\xi_{\nu e}(T) = \frac{3}{\pi T^3} \left[ \frac{\alpha_{ind}^a(T) B^2(T)}{\omega_p(T)} - \frac{\alpha_{ind}^a(T_0) B^2_0}{\omega_p(T_0)} \right], \tag{40}
\]
\[^6\] The expression for the induced charge for a Dirac neutrino \( e_{ind}^{(\nu_e)} = -G_F c_F^e / 2 \pi \sqrt{2} e_{\text{ind}}^{(\nu_e)} \) found in [19] coincides here with formula (38) taken from Eq. (5.16) in the more recent paper [20].
where in the broken phase at $m_e \ll T \ll T_0 \ll T_{\text{EWPT}}$ we will assume zero initial asymmetry, $n_{\nu_e} - n_{\bar{\nu}_e} = \xi_{\nu_e}(t_0)T_0^3/6 = \xi_{\nu_e}(t_0) = 0$.

Substituting $\alpha_{\text{ind}}^2(T)$ from Eq. (39) and the seed magnetic field $B_0 = 0.1T_0^2$, which for a magnetic field frozen into a plasma as $B = 0.1T^2$ successfully satisfies the limit $B < 10^3$ G [11] at the Big Bang nucleosynthesis temperature $T = T_{\text{BBN}} = 0.1$ MeV, from (40) we obtain

$$\xi_{\nu_e}(T) = -0.712(c_p^2)^2 \times 10^{-15}\left(\frac{T_0}{m_p}\right)^4\left(\frac{T}{T_0}\right)^3.$$  \hspace{1cm} (41)

For the initial temperature $T_0 = 1$ GeV this gives a negative asymmetry at $T = O(\text{MeV})$, $\xi_{\nu_e} = -0.912(c_p^2)^2 \times 10^{-6}$. By requiring that the asymmetry (41) be no greater than the upper limit $|\xi_{\nu_e}| < 0.07$ [5] determined by Big Bang nucleosynthesis when taking into account the equalization of the asymmetries at $T = O(\text{MeV})$ owing to the neutrino oscillations, $\xi_{\nu_e} \sim \xi_{\nu_\tau} \sim -\xi_{\bar{\nu}_e}$, we obtain a constraint for the initial temperature, $T_0 < 5$ GeV. Note that at higher initial temperatures the conservation law (37) must be supplemented by the evolution of the magnetic helicity and magnetic energy spectra (34), in which the magnetic diffusion, being explicitly disregarded when deriving the result (41), increases in importance at higher temperatures. Given the diffusion, the generated helicity must decay as

$$h(\eta) \sim h_0(\eta_0)\exp[-2k_0^2(t - t_0)/\sigma_{\text{cond}}],$$

where $k_0^2/\sigma_{\text{cond}} \sim T$ in the exponential.

Thus, we have demonstrated that a small-scale (corresponding to $k_0 = r_D^{-1}$) magnetic field with maximum helicity amplified by dynamo in an $e^-e^+$ plasma can generate a neutrino asymmetry through the electroweak interaction with the plasma. To make the toy model more realistic, in Section 4.2 we will consider large-scale fields with a continuous spectrum of the initial magnetic energy density.

### 4.2. Continuous Kolmogorov Spectrum of the Magnetic Energy Density

We will consider an initial Kolmogorov spectrum of the magnetic energy density $\rho_B(\vec{k}, \eta_0) = \mathcal{C}k^\gamma \tilde{k}$ with

$$\mathcal{C} = -\frac{(v_B + 1)B_0^5}{2k_{\text{max}}^5 - k_{\text{min}}^5},$$  \hspace{1cm} (42)

In this case, the evolution of the spectra in Eq. (34) is determined by the seed spectrum with the maximal helicity $\tilde{h}(\vec{k}, \eta_0) = 2\tilde{\rho}_B(\vec{k}, \eta_0)/k$.

The neutrino asymmetry derived from the basic equation (33),

$$\xi_{\nu_e}(\eta) = \xi_{\nu_e}(\eta_0) + \frac{3A^2M_\eta^4}{4\pi^2} \int k^4\tilde{k}d\tilde{k} \left[ \frac{\tilde{h}(k, \eta)}{\eta} - \frac{\tilde{h}(k, \eta_0)}{\eta_0} + 2\int_0^\eta d\eta' \frac{\tilde{h}(k, \eta')}{\eta'} \right],$$  \hspace{1cm} (43)

is determined by the solution of the self-consistent equations (34) and (36) for the magnetic helicity spectrum and chiral imbalance $\tilde{\mu}_s$.

We solve Eqs. (34) and (36) with the following initial conditions: $B_0 = 0.1$ (see the motivation in Section 3.1) and $\tilde{\mu}_s(\eta_0) = 4 \times 10^{-5}$. We will assume that $k_{\text{max}}$ is the smallest scale of the magnetic field determined by the Debye length $r_D = \omega_p^{-1}$ in a hot plasma: $k_{\text{max}} = \omega_p/T = 0.1$. We will also consider the evolution of the Universe at $T < T_0 = 10$ GeV. The Fermi approximation remains applicable for these temperatures, because $T_0 \ll M_\eta \ll 10^2$ GeV. Moreover, as in Section 3.1, we assume zero initial neutrino asymmetry, $\xi_{\nu_e}(\eta_0) = 0$. Note also that the initial chiral imbalance in our numerical simulations is consistent with the results from [6].

The figure shows the electron neutrino asymmetry in the broken phase as a function of $T$ for various $\tilde{k}_{\text{min}}$. It can be seen that $\tilde{\xi}_\nu$ becomes negative and at $T \sim 1$ GeV reaches saturation dependent on $\tilde{k}_{\text{min}}$. The larger the value of $\tilde{k}_{\text{min}}$, the greater the saturation of $|\tilde{\xi}_\nu|$. The largest $\tilde{k}_{\text{min}}$ used in our analysis is $\tilde{k}_{\text{min}} = 10^{-2} \ll \tilde{k}_{\text{max}}$. Since $\tilde{\xi}_\nu$ becomes constant, it makes no sense to study the dependences at $T < 1$ GeV.

It can be seen from the figure that $|\tilde{\xi}_\nu|$ reaches a considerably smaller value compared to the results of our calculations in Section 4.1 performed in our toy model for a monochromatic spectrum. The predicted $|\tilde{\xi}_\nu|$ generated in the broken phase at $T \sim O(\text{MeV})$ turns out to be many orders of magnitude smaller than that for the same left-handed neutrinos in the symmetric phase, $|\xi_{\nu_e}| \sim 10^{-10}$, as was obtained in [2] in the leptogenesis scenario by taking into account the influence of sphaleron transitions.
5. CONCLUSIONS

In this paper we derived new kinetic equations for massless neutrinos in the hot plasma of the early Universe by taking into account both the Berry curvature and the electroweak interaction of neutrinos with background fermions. Based on these equations, we deduced a new neutrino anomaly in the plasma after the EWPT related to the non-conservation of the lepton number when the Berry curvature was taken into account and obtained the well-known Abelian anomaly for lepton currents in a hypercharge field before the EWPT without using the Feynman diagram technique. We then applied the inferred anomaly to generate the neutrino asymmetry in the broken phase of the early Universe.

Based on our results in Section 4.2, we can conclude that the new neutrino anomaly describing the neutrino current non-conservation in Eq. (24) at temperature much lower than the EWPT temperature, \( T < T_{\text{EWPT}} \), for a real continuous spectrum of the magnetic energy density leads to the neutrino asymmetry in Eq. (43). As can be seen from the figure, this asymmetry turns out to be too small for any observations of relic neutrinos. On the contrary, in the case of the Abelian anomaly in Eq. (1) obtained independently in Eq. (25) by generalizing the Boltzmann equation for neutrinos in (12) using the Berry curvature (5), the generation of neutrino asymmetry turned out to be more efficient. By considering the neutrino asymmetry before the EWPT at \( T > T_{\text{EWPT}} \), for example, in [1], it turns out that \( \xi_{\nu,\mu} \) can be as high as \( 10^{-10} \) for some spatial scales of the hypermagnetic field \( k_0^{-1} \) at \( T_{\text{EWPT}} \approx 100 \text{ GeV} \). This value is close to the observed baryon asymmetry \( B \sim 10^{-10} \).

Nevertheless, our results are important, because we showed that a neutrino asymmetry could be generated due to allowance for two factors: the additional Berry curvature terms in the Boltzmann equation in momentum space as part of the full phase volume for the neutrino distribution functions and the electroweak interaction of neutrinos with the background plasma. The predicted effect can also manifest itself at zero seed neutrino asymmetry owing to the non-conservation of the neutrino lepton number associated with the non-conservation of the neutrino 4-current in a medium with an external magnetic (hypermagnetic) field.

It is generally agreed that after the neutrino decoupling at \( T < T_{\text{dec}} \approx (2–3) \text{ MeV} \) in a nonrelativistic plasma \( (T \ll m_n) \) during the radiation era (at redshifts \( z > 10^4 \)) the neutrino decoupling with matter means that the neutrino asymmetries freeze out at the original level before the neutrino decoupling. Actually, however, the neutrino current still is not conserved due to the Berry curvature in the Vlasov approximation of the Boltzmann equation without collision integrals, \( \partial_j j^j_{\nu} \neq 0 \), as follows from Eq. (24). We studied this case separately and found that, although the growth of the neutrino asymmetry is nonzero, it is too small even compared to the result in an ultrarelativistic plasma; see Eq. (43). Of course, the plasma electric conductivity in this situation is different: \( \sigma_{\text{cond}} \sim T^{3/2} \) instead of \( \sigma_{\text{cond}} \sim T \) in Eq. (43) based on the evolution of magnetic helicity in Eq. (34).

To summarize, we will emphasize that the bulk of the neutrino asymmetry could be acquired mainly before the EWPT through the quantum (Abelian) anomaly in Eq. (1) in an external hypermagnetic field (see, e.g., [1]). In this paper we deduced this anomaly by a new method using the Berry curvature in the Boltzmann equation (12).

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