A drop bouncing on a vertically-vibrated surface may self-propel forward by Faraday waves and travels along a fluid interface. This system called walking drop forms a non-quantum wave-particle association at a macroscopic scale. The dynamics of one particle has triggered many investigations and has resulted in spectacular experimental results in the last decade. We investigate numerically the dynamics of a large number of walking drops evolving on an unbounded fluid interface in a presence of a confining potential acting on the particles. We show that even if the individual trajectories are erratic the system presents well-defined ordered internal structure that remains invariant to many parameter variations. We rationalize such non-stationary self-organization thanks to the symmetry of the waves and show that oscillatory pair potentials form a wavy collective state of active matter.

I. INTRODUCTION

In a seminal article, Y. Couder and E. Fort [1] have shown that a submillimetric drop may bounce on a vertically-vibrated fluidic interface. Above a certain fluid acceleration threshold the drop may self-propel and be guided by standing waves [2,3] which are the footprint of Faraday waves [14,16]. The system has triggered a flurry of thought-provoking experiments mimicking effects which were previously thought to be peculiar to the quantum scale [2,3]. Specifically, the dynamics of one drop has been investigated in many configurations, like moving through a slit and double slits [17], in cavities [18,21], with a Coriolis force [22,24], in a harmonic potential [13,23,25,27], exhibiting Friedel-like oscillations [28], enabling statistical projections [29] and analog of superradiance [30]. In the last decade, these studies have achieved a converging understanding of both the emergence of wave-like statistics and classical quantization at a macroscopic scale [2,3].

In a parallel stream of research, it has also shown complex dynamical behaviors such as memory-induced diffusion, and run and tumbling [31,39] for a single droplet system, in which the degrees of freedom in the wave field play the role of a tailored thermal bath [39]. The dynamics of two drops has also been investigated and promenade modes [5,13,40,43], as well as quantized orbiting states have been observed and rationalized [13,43,45]. As for the many drops’ dynamics, crystalline structures have been obtained in 2D [40,47], and the collective dynamics in toroidal channel [48,49] as well as the spatially periodic potentials [50] have been investigated in the linear and non-linear regime [51,55]. Recently 2D arrays of interacting circular cavities have shown the spontaneous emergence of long distance motion synchronisation in analogy with ferromagnetism and antiferromagnetism [50]. In all these investigations, and because of the complexity of preventing the coalescence during potential collisions, the drops are carefully placed in either particular positions or specific relative distances to each other. As a result of this constraint, the dynamical behaviors of many "freely"-moving interacting walking drops, which would be a remarkable playground for statistical physics, is mainly an experimental uncharted territory. A numerical investigation would be a crucial first step to trigger and initiate further experimental realisations. In this article, we investigate numerically the dynamics of a large number of walking drops evolving on an unbounded fluid interface in the presence of a confining potential acting on the particles.

The article is organised as follows. In Sec. II we introduce the numerical model. Then in Sec. III we characterize and rationalize the properties of the dynamical phases and propose a mechanism to predict the main properties of this statistical quantized self-organization. Finally, we conclude in Sec. IV.

II. NUMERICAL MODEL AND METHODS

We perform a numerical simulation of many walking drops in two dimensions. Several models [11,12,22,57,58], varying in their strategies to solve the equations and the dynamics of the drops, have been proposed. They are in fairly good qualitative agreement as they share the same main essential features. We implement the walking drops dynamics by considering a discrete time evolution [22], which can be summarized as follows.

First, we model the wave created by a single impact on an unbounded fluid interface by a Bessel function of order 0, J₀, centered at the point of impact. The standing Faraday wave frequency \( f_F \) is directly linked to the vibration frequency of the bath \( f_0 \) by the relation \( f_F = f_0/2 \). \( \lambda_F \) is the wavelength, in our case \( \lambda_F = 6.1 \) mm. As the drop impacts the bath at constant relative phase every period, \( T_F = 1/f_F = 40 \) ms, the wave field is a strobed quantity. Thus a single impact at the surface position \( r_0 \) gives rise to a surface standing wave field with a spatial structure proportional to
Fig. 1. Schematics of the system. (a) Example of the system with \( N = 3 \) drops, \( M_e = 30 \), \( R_{\text{max}} = 3\lambda_F \), seen from a top view. The coloured surface illustrates the wave field (in arbitrary units) created by the bouncing drops dotted in grey. The waves are not restrained by the outer wall, only the drops are. (b) A sketch of the confining force: it is equivalent to a step of force of 1 N acting beyond a distance to the center larger than \( R_{\text{max}} \).

\[ J_0(K_F \| r - r_0 \|), \]

where \( r \) is a two-dimensional vector and \( K_F = 2\pi/\lambda_F \). Experimentally, taking into account the fluid bath viscosity is known to lead to a spatial decay more pronounced than a Bessel function \([12, 58, 59]\).

However considering a simplified wave field is a widely used pragmatical approximation \([3, 65]\) as it is one of the very few ways to get an analytically-tractable model that can provide valuable predictions. As a consequence, we keep this simplified Bessel approximation for the wave field expression.

Secondly, the persistence time of these waves is defined by a parameter called \( M_e \), the memory time. The higher the value, the longer the waves remain on the surface and influence the dynamics of the drops. Experimentally, the memory time is tuned by varying the acceleration amplitude of the bath relative to the critical Faraday acceleration threshold. In the numerical model, the memory is a scalar parameter \( M_e \) which we can vary at will. \( M_e \approx 10 \) is considered as a low memory regime, and \( M_e \approx 50 \) as a moderately small memory regime \([59]\). We only investigate \( M_e \) values ranging from 10 to 50. In practice, this memory range corresponds to an acceleration amplitude of the fluid bath \( \gamma_m/\gamma_F = (1 - 1/M_e) \in [0.9, 0.98] \), with \( \gamma_F \) the fluid-dependent Faraday acceleration threshold.

Thirdly, we consider the system as sketched in Fig. 1. \( N \) walking drops are confined by a very steep external potential acting beyond a radius \( R_{\text{max}} \). The wave field is considered as not influenced by the presence of its confining potential and as a consequence, the waves are generated as if they were feeling an unbounded fluid interface. We integrate the position of the \( i \)-th drop (\( i \in \{1, \ldots, N\} \)) from time \( t_p \) at position \( r_i(t_p) \) to time \( t_{p+1} = t_p + T_F \), i.e. one bounce further, at position \( r_i(t_{p+1}) = r_i(t_p) + v_i(t_p)T_F \) and by computing the evolution of the velocity of the \( i \)-th drop, \( v_i \), as

\[ v_i(t_{p+1}) = v_i(t_p) + T_F \left( -\eta v_i(t_p) + F_{\text{wave},i}(t_p) + F_{\text{wall},i} + F_{\text{drops} \rightarrow \text{drop} i} \right). \]

wherein \( \eta \) is the dissipation coefficient is set to 4.72 s\(^{-1}\) \([23, 32]\), which corresponds to a silicone oil with a kinematic viscosity of 20 cSt. It takes into account both the friction due to the lubrication layer between the drop and the bath, and the transfer of momentum to the fluid bath. \( F_{\text{wave},i} \) is the force propelling the \( i \)-th drop and is proportional to the local value of the two-dimensional gradient of the wave field. Specifically, it writes

\[ F_{\text{wave},i}(t_p) = -\frac{C T_F}{k_F} \nabla r_i(t_p) \sum_{j=1}^{N} \sum_{k=p-3M_e}^{p} J_0(k_F \| r_i(t_p) - r_j(t_k) \|) e^{-\frac{(p-k)}{M_e}} \]

The first sum accounts for the linear superposition of the waves generated by the drops \( j (j \in \{1 \ldots N\}) \). The second sum is the memory term which indicates that the amplitude of the waves generated by the \( j \)-th drop at time \( t_k < t_p \) are exponentially damped. \( C = 1.1 \) m s\(^{-1}\) is a coupling coefficient \([23, 32]\) expected from a drop of diameter 0.9 mm. Rigorously, the sum over \( k \) should start from \( k = 0 \) but thanks to the exponential term, we start the summation from \( p - 3M_e \), neglecting the other residual terms. We checked that this significant truncation does not affect the results. \( F_{\text{wall}} \) is the force accounting for the presence of the outer wall. It is phenomenologically modelled by a step of force at \( R_{\text{max}} \). It is only acting for a radius larger than \( R_{\text{max}} \) as sketched in Fig. 1. It writes

\[ F_{\text{wall}} = -F_0 e_r H (r_i(t_p) - R_{\text{max}}) \]

with \( e_r \) the radial unit vector pointing outward the center, \( H \) the Heaviside step function, \( r_i = \| r_i \| \) the distance of the \( i \)-th drop to the center and \( F_0 = 1 \) N the wall force magnitude. In practice, \( F_{\text{wall}} \) is a step of force which value is at least one order of magnitude larger than all the other forces in the external region. Finally, \( F_{\text{drops} \rightarrow \text{drop} i} \) accounts for the short distance drop-drop repulsion which we model as an elastic repulsion, with a spring constant (per unit mass) \( K \), provided the distance between a pair of drops is smaller than twice the drop radius and zero otherwise. In what follows, we consider, \( K = 0 \), i.e. \( F_{\text{drops} \rightarrow \text{drop} i} = 0 \), unless explicitly stated otherwise. This sum of forces determines the evolution of velocity of a given drop (Eq. 2) hence its position, period by period.

The initial position of drops is defined randomly.
on a circular surface of radius equal to half the radius of the circular accessible domain. We initialize the memory kernel by generating randomly and uniformly the $3 \times \text{Me}$ last past positions around each drop within a distance $0.01 \times \lambda_F$ from the drop. The drops evolve in the bath for at least $10^4$ Faraday periods, which we find to be a good duration to make statistics for most simulations. For experiments with 5 drops or less, $10^5$ Faraday periods are considered to decrease the measurements uncertainties. We discard the first $10^3$ Faraday periods to avoid any transient effects.

### III. RESULTS : DYNAMICAL SELF-ORGANIZATION

We investigate the dynamics by changing the number of drops $N$, the domain radius $R_{\text{max}}$, and the memory parameter $\text{Me}$. Figures 2a and 2b illustrate two characteristic snapshots of the positions of 30 drops confined after $10^3$ time steps in a domain of radius $R_{\text{max}} = 3 \lambda_F$ and $5 \lambda_F$, respectively (see also Supplemental Movie 1 and 2). The magnitude of the memory parameter is indicated by a blue tail which visually links a drop to its $\text{Me}$ past positions. The mean density of drops, $\rho = N/(\pi R_{\text{max}}^2)$ is low in both Fig. 2a and b, specifically $\rho_{\text{drop}} \simeq 1.1 \lambda_F^{-2}$ and $0.4 \lambda_F^{-2}$, respectively, and we observe erratic trajectories, regardless of the bath size. We remark that sometimes, the drops walk together by forming small groups during a few tens of periods but we did not observe any reproducible patterns nor any obviously-identifiable long-standing stable emerging structure. Very small excursions outside $R_{\text{max}}$ can be observed in Figures 2 and are dictated by the precise details of the modelling of the wall repulsion. It is very close to a hard wall repulsion and we check that, in practice, the dynamics is hardly influenced by the details of the wall modelling. We also observe that the drops are overlaying and crossing each other sometimes. In this low drop density regime, we check that the dynamics is statistically not influenced by this effect by switching on the force $F_{\text{drops-\rightarrow drop \; i}}$, a investigation which we shall discuss further.

As we observe no apparent structure by looking at the sole dynamics, we proceed by investigating the statistical properties on the $N$ drops system. We first compute the probability density function, $F(r)$. We define the radial probability function, $f(r)$, by integrating $F$ over all possible angular components, specifically, $f(r) = \int_0^{2\pi} F(r, \theta) d\theta$. By construction, we get the normalization $\int f(r) r dr = \int \int F(r, \theta) r dr d\theta = 1$. As a consequence, Figure 3a shows the evolution of $rf(r)$, with the length expressed in $\lambda_F$ units, for $N = 5$ and 30, confining radius $R_{\text{max}} = 3 \lambda_F$ and $5 \lambda_F$, and for a common memory parameter $\text{Me} = 30$. For all parameters configurations, we observe a linear dependency of $rf(r)$ with $r$ up to a radius $R^* \simeq R_{\text{max}} - \lambda_F$, where $rf(r)$ is maximum. For $r < R^*$, the slopes are independent of the drop-drop density, at least in this low density regime, and we find that they scale as $1/R_{\text{max}}^2$. For distances from the center $r > R^*$, the drops feel the influence of the boundary and $rf(r)$ sharply decreases at the vicinity of $R_{\text{max}}$. For radial distance to the center $r < R^*$, a linear behavior of $rf(r)$ means that $f$ is constant with $r$ and the distribution of drops is radially homogeneous. Then we analyse the angular structure of the system by computing the angular probability distribution $h(\theta)$, specifically $h(\theta) = \int F(r, \theta) \theta dr$. Figure 3b shows that the drops angular distribution remains isotropic for all drop densities and all values of $R_{\text{max}}$, investigated.
We rationalize the evolution of the speed density by taking inspiration from the active statistical theory developed in [39]. Essentially, in the large memory regime ($\text{Me} \gg 10^2$) investigated by the authors, the motion of one single drop is equivalent to a self-propelled particle coupled to an effective white noise thermal bath. Following [39], we fit in Fig. 4a the numerical speed densities probability function with $h(v) = a v \exp(-\beta ((v - v_0)^2))$ wherein $v_0$ originates from a constraint on the speed arising from the self-propulsion of the drops and $\beta$ the effective temperature of the system. $a$ is a normalisation factor such that $\int h(v) dv = 1$. In the absence of the speed constraint term $v_0$, $h$ identifies to a two-dimensional Maxwell-Boltzmann distribution as one would expect from a classical ideal gas in two dimensions. We observe some systematic quantitative differences between the numerical results and the fitting model especially with the tail of the distribution, suggesting strong remaining correlations.

To reveal these correlations, we measure in Fig. 4b and 4c, the evolution of the mean speed and of its standard deviation as a function of the number of drops along with their fits with 95% confidence interval. We observe a small and steady increase of the mean speed with the number of drops in the system. It is a difference with the case of promenade modes in which two correlated drops move slower than one single drop. We conclude that we do not reach a statistical limit by increasing the number of drops, an important difference with an ideal gas and with the behavior of one single drop in the large memory regime [39]. We note that the increase of the mean speed should obviously reach an upper limit in experiments as non-linear wave effects are expected to start playing a saturation role.

The speed fluctuations also increase with the number of drops (see Fig. 4c) and we rationalize its evolution as follows. We expect that the squared speed fluctuation of a particle $i$, $\sigma_i^2$, should be the sum of two contributions: one intrinsic, denoted $\sigma_{i,0}^2$, and one due to the presence of all the other drops $j \neq i$, denoted $\Sigma_i^2$:

$$\sigma_i^2 = \sigma_{i,0}^2 + \Sigma_i^2. \quad (4)$$

In principle, $\Sigma_i^2$ is a function of (i) the number of drops, $N$, and (ii) all possible degrees of freedom of all the other drops $j \neq i$. All drops being indistinguishable, we assume that $\Sigma_i^2$ can be factorized as $\Sigma_i^2 = \Sigma_{m,i}^2 F(N)$ where $F$ is a function independent on $i$ and only dependent on the number of drops, while $\Sigma_{m,i}^2$ is a function which depends possibly on all possible degrees of freedom of all the other drops $j \neq i$. This factorization ansatz is equivalent to a mean-field approach, which we expect to hold for large $N$.

The number of drops is a scale-free parameter, so $F(N)$ is expected to be an algebraic function of $N$, specifically $F(N) = N^{\nu}$. $\nu$ is an exponent to be determined from the fitting of the numerical results. By combining Eq. [4]
with the factorization ansatz, and then averaging on both time and on all drops \(i = 1 \ldots N\), we infer that the speed standard deviation \(\delta v\) takes the form

\[
\delta v^2 = \delta v_0^2 + \delta v_m^2 N^\nu \tag{5}
\]

Fig. 4c indicates a good agreement with the theoretical predictions (Eq. 5).

To investigate in more details the correlations, we aim at spotting the signature of any internal structure. To do so, we compute the drops’ pair correlation function

\[
g(r) = \left\langle \sum_{i,j,i\neq j} \delta (r - (r_i - r_j)) \right\rangle \tag{6}
\]

where \(\langle \rangle\) denotes an average over time. As there is no preferred angle we assume \(g(r) = g(|r|)\). The corresponding results are presented sequentially in Fig. 5, each panel corresponding to the variations of one single parameter. Fig. 5a indicates that the larger the radius is, the more peaks appear. The pair correlation function decreases with the distance up to twice \(R_{\text{max}}\) with the presence of well-pronounced equally-spaced peaks. The global decreasing trend is expected as a situation for which the drops would be homogeneously distributed in a circular domain of radius \(R_{\text{max}}\), would exhibit the exact same decreasing behaviour. Fig. 5b indicates that varying memory in the range \(\lambda_{\text{Me}} \in [10;50]\) does not change the pair correlation function profile, and moderately alters the magnitude of the peak-to-peak amplitude. Finally Fig. 5c shows that varying the number of drops only affects the signal-to-noise ratio, which is very low for 5 drops or less. We observe on Figure 5 local maxima on the curves. It means that drops tend to locate statistically from one to each other at a preferred set of quantized distances. Some of them are well pronounced with a well-measurable position and some of them are emerging which make their position difficult to estimate accurately. Among the peaks clearly formed, we measure, for example at \(R_{\text{max}} = 7\lambda_{\text{F}}\), the position of maxima \(r^* = 1.09, 2.07, 3.08, 4.06, 5.04, 6.06, 6.97, 7.91 (\pm 0.04)\).

This quantized distance originates from the force due to the wave field (Eq. 2), which we rationalize now by considering the dominant terms in Eq. 2. We introduce the notations \(r_{ij}(t_p) = r_i(t_p) - r_j(t_p)\) between the pair \(i\) and \(j\), and \(\Delta_j(t_{p},t_k) = r_j(t_{p}) - r_j(t_k)\). Additionally, we denote \(r_{ij}(t_p) = |r_{ij}(t_p)|\), \(\Delta_j(t_{p},t_k) = |\Delta_j(t_{p},t_k)|\), \(J_n\) the Bessel function of first kind of order \(n\) and \(\theta_{ij}(t_{p},t_k)\) the relative angle between \(r_{ij}(t_p)\) and \(\Delta_j(t_{p},t_k)\). Note that the symbol \(i\) in the exponential denotes the imaginary number and differs from the italicized subscript \(i\).

We identify the dominant terms in Eq. 2 by proceeding as follows. As the memory lets the position of maxima unchanged, we consider the regime of large \(\lambda_{\text{Me}}\) which means that we neglect the exponential decay hereafter in Eq. 2. Then, we note that the wave \(J_0\) term in Eq. 2 can be written

\[
J_0(k_F||r_i(t_p) - r_j(t_k)||) = J_0(k_F||r_i(t_p) - r_j(t_p)||) - J_0(k_F||r_i(t_p) - r_j(t_k)||) = J_0(k_F||r_i(t_p) - \Delta_j(t_{p},t_k)||) = \sum_{n=-\infty}^{\infty} J_n(k_F r_{ij}(t_p) J_n(k_F \Delta_j(t_{p},t_k))) e^{i n \theta_{ij}(t_{p},t_k)} \tag{7}
\]
each drop is very complex we expect that, after the double summation in Eq. (2) over both the past positions and drops, the corresponding terms with complex phases in Eq. (7) rapidly self-average and give rise to small contributions. As a consequence, we expect the term \( n = 0 \) in Eq. (7), namely \( U_{0,j}(r_{ij}) = J_0(k_F r_{ij}(t_p)) J_0(k_F \Delta_j(t_p, t_k)) \) to be the leading term. As the wave-induced force, \( F_{\text{wave},i} \) in Eq. (2) derives from a gradient, the term inside this gradient corresponds to an effective wave potential. Thus, we expect the equilibrium solution to be given by the extrema of \( U_{0,j}(r_{ij}) \), apart from stability. This can be found by looking for the zeros of \( r \mapsto J_1(2\pi r/\lambda_F) \) which are

\[
r^*/\lambda_F \approx 0, 0.63, 1.13, 1.63, 2.13, 2.63, 3.13, \ldots \quad (8)
\]

The even zeros correspond to those found numerically up to the uncertainty measures (see vertical dashed line in Fig. 5b). We recall that the situation is unstable and does not correspond to a crystalline structure. We conjecture that the odd zeros correspond to pair modes that are linearly unstable similarly to that observed in [43]. The peaks in the pair correlation function are the signature of a dynamically evolving internal structure and we have established its statistical wave origin.

Finally, we numerically predict that the pair correlation function exhibits a maximum at the origin. Experimentally, it is obviously impossible as short distance interactions must be taken into account. We present briefly in this last paragraph the influence of a short-distance elastic repulsion interaction. In Fig. 6, we observe that the systems with repulsion have pair correlation function \( g(r) \) converging towards 0 when \( r \) tends to 0, contrary to the system in the absence of repulsion. The stronger the repulsion, the more efficient the convergence towards 0 at \( r = 0 \), is. We also observe that including short distance interactions let the positions of the peaks, outside the origin, unchanged while their aspects are mainly unaltered.

### IV. CONCLUSION

We have investigated numerically the dynamics of a large number of walking drops that are confined by an external potential well while the surface Faraday waves are created as if they were in an unbounded fluid domain. The drops interact between each other by the waves they generated and evolve in a complex chaotic dynamics which explores the whole accessible domain. A statistical analysis of the dynamics shows that the drops are homogeneously distributed below critical radius with an angular isotropy. A fine analysis of the pairwise density function shows that while being dynamic, time-evolving and presenting many indications of a good mixing in the phase space, the system adopts, in average, preferred distances whose origin has been rationalized by analysing

![FIG. 5. Evolution of the pair correlation function \( g \) for various (a) confining potential radii \( R_{\text{max}} \), (b) memory parameters \( M \), and (c) numbers of drops \( N \). Color code is on the legend of each figure. The dashed lines in (a) represent the theoretical situation for which all the drops are homogeneously distributed in a circular domain of radius \( R_{\text{max}} \). Vertical black lines in (b) represent the theoretical predictions for the position of the stable maxima (even solutions in Eq. 8).](image-url)
FIG. 6. Evolution of pairwise density function $g$ for the system $N = 30$, $Me = 30$, $R_{\text{max}} = 5\lambda_F$ for various repulsion strengths in levels of grey. Color code: from light grey to black, the spring constant per unit mass is $K = 500, 1000, 1500, 2000, 2500\,\text{s}^{-1}\cdot\text{kg}^{-1}$. Darker curves correspond to higher repulsions. The salmon curve is the case in the absence of repulsion. Inset: zoom at low distance. The blue vertical dashed line corresponds to the drop size in $\lambda_F$ units.

the internal symmetry of the waves. Thus, this numerical investigation sheds light on a statistical many-body wave self-organization in an apparent erratic dynamics. We hope these exploratory findings will motivate further experimental realisations in the field.

V. STATEMENTS AND DECLARATIONS

A. Competing Interests

The authors declare no competing interests.

B. Author contributions

AH performed the numerical simulation. ML designed the numerical schemes. AH and ML analysed the numerical results. AH and ML wrote the manuscript. All authors agree with the latest revisions.

VI. APPENDIX

The C++ numerical code to adapt to various platforms with a Readme file is available at the link repository https://mycore.corecloud.net/index.php/s/qlFPCxcMHDejQr6. Supplementary Movie 1: movie associated to Fig. 2a. Supplementary Movie 2: movie associated to Fig. 2b.

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