Dendritic flux avalanches in superconducting films of different thickness

J I Vestgården, Y M Galperin, T H Johansen

1 Department of Physics, University of Oslo, PO Box 1048 Blindern, 0316 Oslo Norway
3 Ioffe Physical Technical Institute, 26 Polytekhnicheskaya, St Petersburg 194021, Russian Federation
2 Institute for Superconducting and Electronic Materials, University of Wollongong, Northfields Avenue, Wollongong, NSW 2522, Australia

E-mail: j.i.vestgarden@fys.uio.no

Abstract. At low temperatures the critical state in superconducting films can be unstable with respect to thermomagnetic dendritic avalanches. By numerical simulations of disk-shaped superconductors, we consider how the dynamics and morphology of the avalanches depend on the disk thickness. We find that as the disks get thicker, the jumps in magnetic moment caused by the avalanches get larger and the threshold magnetic field for the appearance of the first avalanche increases. At the same time, the branches are straighter and the number of branches decreases. Comparison with theory suggests that strong spatial disorder to some extent cancels the stabilizing effects of the substrate kept at constant temperature.

PACS numbers: 74.25.Ha, 68.60.Dv, 74.78.-w
1. Introduction

The critical state in type-II superconducting films subjected to transverse applied magnetic field or current can be susceptible to intermittent dynamics, where magnetic flux rushes in from the edges, forming large branching flux structures. The patterns remaining after such dendritic flux avalanches have been imaged in many materials, e.g., Pb [1], Sn [2], Nb [3], YBa$_2$Cu$_3$O$_{7-x}$ [4], MgB$_2$ [5], Nb$_3$Sn [6], YNi$_2$B$_2$C [7], NbN [8], and α-MoGe [9]. Because the rapid motion of magnetic flux also implies a major redistribution of currents in the samples, the flux avalanches are associated with sudden drops in the magnetic moment values [10, 11, 12, 13].

Dendritic flux avalanches are caused by a thermomagnetic instability initiated when a temperature fluctuation facilitates uncontrolled penetration of magnetic flux and rise in temperature [14]. A model based on continuum electrodynamics and flow of heat has explained the phenomenon with great success, as numerical solutions have produced avalanche dynamics and patterns in striking resemblance with the experiments [16, 17], and linear stability analysis of the model has explained many features, such as the existence of a threshold temperature and magnetic field [18, 19, 20] and a threshold electric field [21] for onset of avalanche activity. Experimentally it has been demonstrated that the threshold magnetic field increases with shrinking lateral size [18, 22] and simulations have shown that the properties of avalanches can be described by a small number of dimensionless parameters [23]. The velocity of avalanches triggered by a laser-pulse has been shown to be inversely proportional to the sample thickness [24]. At the same time, it is not clear from previous works how the threshold magnetic field and avalanche morphology depend on the sample’s thickness.

In this work, we consider how the properties of dendritic flux avalanches depend on the sample thickness. We perform numerical simulations for various thicknesses, but with otherwise identical parameters, and consider the effect on the magnetic moment, the threshold field, and the morphology of the dendritic flux patterns.

2. Model

Let us consider a superconducting sample in gradually increasing transverse applied magnetic field $H_a$, as depicted in figure 1. The film is shaped as a disk with radius $R$ and thickness $d$, where $d \ll R$. In the flux creep regime, the resistivity is very non-

![Figure 1](image-url) **Figure 1.** The sample is a disk of radius $R$ and thickness $d \ll R$. The magnetic field is applied transverse to the plane, causing penetration of magnetic flux from the edges.
linear as the sheet current \( J \) approaches the sheet critical current, which we assume scales with sample thickness as \( dj_c \), where \( j_c \) is the critical current density. We use the conventional power law relation \[25\]

\[
E = \rho J/d,
\]

\[
\rho = \rho_0 \begin{cases} 
1, & T > T_c \text{ or } J > dj_c, \\
(J/dj_c)^{n-1}, & \text{otherwise},
\end{cases}
\]

where \( n \) is the creep exponent and \( \rho_0 \) is the normal resistivity. The parameters depend on local temperature \( T \) as

\[
j_c = j_{c0} (1 - T/T_c), \quad n = n_1 T_c/T + n_0,
\]

for \( T < T_c \).

The fields must satisfy the Maxwell equations

\[
\nabla \times \mathbf{H} = J \delta(z), \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\dot{\mathbf{B}},
\]

with \( \mathbf{B} = \mu_0 \mathbf{H} \) and \( \nabla \cdot \mathbf{J} = 0 \). Here \( \delta(z) \) is the Dirac delta function.

The local magnetization \( g = g(x,y,t) \) is defined by

\[
\mathbf{J} = \nabla \times \hat{z} g = \nabla g \times \hat{z}.
\]

By performing the calculations using \( g \) one assures that \( \nabla \cdot \mathbf{J} = 0 \) holds at any time.

The contour lines of \( g \) are the current stream lines. Integrated over the sample area, \( g \) gives the total magnetic moment,

\[
m = \frac{1}{2} \int \mathbf{r} \times \mathbf{J} dxdy = \hat{z} \int g dxdy.
\]

From the Maxwell equations, one gets the time evolution of \( g \) as \[26\]

\[
\dot{g} = \mathcal{F}^{-1} \left[ \frac{2}{k} \mathcal{F} \left[ \frac{1}{\mu_0} \dot{\mathbf{B}}_z - \dot{H}_a \right] \right],
\]

where \( \mathcal{F} \) and \( \mathcal{F}^{-1} \) are Fourier and inverse Fourier transforms, respectively, and \( k = |\mathbf{k}| \) is the wave-vector.

Inside the sample, \( \dot{\mathbf{B}}_z \) is calculated from the Faraday law and the material law as

\[
\dot{\mathbf{B}}_z = \nabla \cdot (\rho \nabla g)/d.
\]

Outside the sample \( \dot{\mathbf{B}}_z \), is fixed by the condition \( \dot{g} = 0 \), which is implemented using the iterative procedure described in \[26\].

The propagation of heat in the sample is governed by \[18\]

\[
c \dot{T} = \kappa \nabla^2 T - h(T - T_0)/d + JE/d,
\]

with specific heat \( c \), thermal conductivity \( \kappa \), coefficient of heat transfer to substrate \( h \). The substrate is kept at constant temperature \( T_0 \).

The simulations are initiated with parameter values typical for films of MgB\(_2\) \[18, 17\]: \( T_c = 39 \) K, \( \rho_0 = 7 \cdot 10^{-8} \Omega \text{m} \), \( j_{c0} = 1.2 \cdot 10^{11} \) A/m\(^2\), \( n_1 = 20 \), \( n_0 = -10 \). The thermal parameters were \( \kappa = [0.17 \text{ kW/Km}] (T/T_c)^3 \), \( c = [35 \text{ kJ/Km}^3] (T/T_c)^3 \), and \( h = [200 \text{ kW/Km}^2] (T/T_c)^3 \). In all runs the applied magnetic field is driven with the same rate \( \mu_0 \dot{H}_a = 10 \) T/s.

Let us write the equations on dimensionless form, in order to identify the effective parameters of the problem, and how they scale with \( d \). The dimensionless quantities are \( \tilde{g} = g/Rdj_{c0} \), \( \tilde{J} = J/dj_{c0} \), \( \tilde{j}_c = j_c/j_{c0} \), \( \tilde{H} = H/dj_{c0} \), \( \tilde{r} = \mathbf{r}/R \), \( t = t\rho_0/\mu_0 dR \),
Figure 2. The magnetic moment as function of applied field, with \( d = 0.2 - 0.6 \, \mu \text{m} \), at \( T = 0.15 \) (top) and \( 0.2T_c \) (bottom). Each jump in the curves corresponds to a dendritic flux avalanche.

\[ \tilde{E} = E/\rho_0 j_0, \quad \tilde{T} = T/T_c, \quad d\tilde{H}_n/d\tilde{t} = \dot{H}_n\mu_0 R/j_0\rho_0. \]  
In these units, the material law becomes

\[ \tilde{E} = \tilde{\rho} \tilde{J}, \quad \tilde{\rho} = \begin{cases} 1, & \tilde{T} > 1 \text{ or } \tilde{J} > 1 - \tilde{T}, \\ \left(\tilde{J}/(1 - \tilde{T})\right)^{n-1}, & \text{otherwise}. \end{cases} \]  

(9)

The Maxwell equations become

\[ \nabla \times \tilde{H} = \tilde{J}\delta(\tilde{z}), \quad \nabla \cdot \tilde{H} = 0, \quad \nabla \times \tilde{E} = -d\tilde{H}/d\tilde{t}, \]  

(10)

and finally the heat propagation equation becomes

\[ \frac{d\tilde{T}}{dt} = \alpha \nabla^2 \tilde{T} - \beta(\tilde{T} - \tilde{T}_0) + \gamma \tilde{T}^{-3} \tilde{J} \tilde{E}, \]  

(11)

where

\[ \alpha \equiv \frac{d}{R \rho_n c}, \quad \beta \equiv R\frac{\mu_0}{\rho_n c}, \quad \gamma \equiv Rd\frac{\mu_0 \gamma_0^2}{c T_c}. \]  

(12)

Here all parameters are evaluated at the critical temperature.

The state is found by discrete integration in time of (10) and (11), as described in [26]. The independent parameters of the dimensionless problem are: \( \alpha, \beta, \gamma, n_1, \) and \( d\tilde{H}/d\tilde{t} \). Since both \( \alpha \) and \( \gamma \) depend on \( d \), it is not possible to predict the thickness-dependency of the results just from inspection of the dimensionless parameters.

The disk-shaped specimen was embedded in a \( \times \) square, with \( a = 1.2R \), where the extra space was used for implementation of the boundary conditions. The embedding square was discretized on a 512 \( 512 \) equidistant grid. Spatial disorder
was included into the formalism by randomly changing the value of $dj_{c0}$ in each grid point by ±5%. All runs have the same realization of spatial disorder.

3. Results

Starting from zero-field-cooled state, an applied magnetic field is gradually increased with constant rate $\dot{H}_a$. Then a critical state is formed from the edges, with $J = dj_c$ and nonuniform $B_z$, which is highly peaked at the edge and falls to zero at the flux front. Inside the flux front the superconductor is in the flux-free Meissner state, where $B_z = 0$, but $J \neq 0$ as a consequence of the nonlocal electrodynamics.

Figure 2 shows the magnetic moment $m$ as a function of applied field, extracted from the simulations during the field-ramp using (5). Each curve corresponds to a different thickness $d = 0.2 - 0.6 \mu m$, and the two panels are at substrate temperatures $T_0 = 0.15$ and $0.2T_c$. Qualitatively, all curves have the same behaviour. Initially, the magnetic moment increases smoothly as predicted by the critical state model, until the first avalanches appears as a jump in the curve at the threshold field $H_{th}$. At $T_0 = 0.15T_c$ the first jumps are quite small, but except from that, most jumps are of comparable sizes, and each one of them is clearly visible. The figure shows that the size of jumps in magnetization are larger for increasing $d$. Unlike in bulks, where the thermomagnetic instability typically causes a global breakdown in superconductivity, with consequent magnetization drop to zero [27, 28], the magnetization values of the figure are always nonzero, fluctuating around a more or less constant value, c.f. [22, 29].

Let us see to what extent the thresholds for onset of instability in the simulations agree with the criteria from linear stability analysis. In disks, the magnetic flux penetration depth as a function of applied field is [30]

$$l = R - R/\cosh(H_a/H_c),$$

(13)

where $H_c = dj_c/2$. Since there is only a numerical factor difference compared to strips, where $H_c = dj_c/\pi$ [31], we can reuse the formulas for the threshold field previously

![Figure 3.](image)

*Figure 3.* The threshold field as a function of sample thickness. The points are extracted from the simulations, the lines are plots of the analytical prediction, $H_{th}$. 

derived for strips, only with a change of numerical constants. By assuming that the most unstable mode is spatially constant one gets \[20\]

\[H_{th} = \frac{d j_c}{2} \left[ \frac{\pi^2 \kappa T^*}{n R^3 j_c \mu_0 H_a} \right]^{\frac{1}{5}},\]

where \(1/T^* \equiv |\partial \log j_c / \partial T|\).

Figure 3 shows the threshold field as a function of sample thickness, with \(T_0 = 0.15\) and \(0.2 T_c\). The discrete points in the figure are extracted from the simulations (the magnetic moment curves of figure 2). The figure shows that \(H_{th}\) increases with \(d\), and that the increase is close to linear. At the same time \(H_{th}\) increases with \(T_0\), as expected from previous theory and experiments \[18, 20\]. To make an interpretation of the simulation results, figure 3 also plots the analytical \(H_{th}\), equation \(14\), for the two temperatures. We see that the analytical predictions are matching the simulation results quite well. This indicates that the instability is prevented mainly by the lateral heat diffusion, which is the only mechanism included in \(14\). In particular, the \(H_{th} \propto d\) dependency seen both in the analytical curve and the numerical results is typical for avalanches being prevented by the lateral heat diffusion. A deviation from proportionality would on the contrary indicate the presence of surface effects, such as the heat removal to the substrate. The absence of such effects and comparison with previous works \[20\] indicate that the stabilizing effect of the substrate is being neutralized by the presence of the spatial disorder used in the simulation.

Figure 4 shows the flux distributions for \(d = 0.2, 0.4\) and \(0.6 \mu m\), and \(T_0 = 0.15\)
Dendritic flux avalanches

and $0.2T_c$. All panels are at different times, as the applied fields are $H_a = 0.15d_j c_0$ and $0.19d_j c_0$ for $T_0 = 0.15T_c$ and $0.2T_c$, respectively. We see that the avalanches are significantly larger at the highest temperature, and that they have got more branches and more complex morphology. Also the sample thickness seems to affect the morphology of the avalanches, in the sense that increasing $d$ gives straighter avalanches, with fewer and thicker branches. This dependency is partly a consequence of the changing depth of penetration prior to the avalanches, partly a consequence of changing heat removal to substrate during the avalanches.

Because all runs are with identical realization of the spatial disorder, there is some overlap between the nucleation spots of the avalanches. This implies that the avalanches are not nucleated by uniform instabilities, which is the case for the thermomagnetic instability in spatially uniform samples [20], but instead they appear on places selected by an interplay between the randomly distributed disorder and fluctuations in the electric field values.

4. Summary

The critical state in superconducting films can, at low temperatures, be unstable with respect to thermomagnetic avalanches. We have considered how the properties of avalanches depend on the sample’s thickness by performing numerical simulations on disks of various thicknesses, but otherwise identical parameters.

We have shown that a ticker sample gives a larger jump in the magnetic moment and higher threshold field for the appearance of the first avalanche. At the same time the branches get straighter and the number of branches decreases. The threshold field in the simulation grows linearly with sample thickness and matches the theoretical prediction where the only mechanism included was the lateral heat transport. Due to the strong spatial disorder, the substrate kept at constant temperature has only minor effect on the stability.

Acknowledgments

This work was financially supported by the Research Council of Norway.

[1] W. DeSorbo and V. L. Newhouse. Optical detection of domain strucures and current flow in superconducting lead films. *J. Appl. Phys.*, 33:1004, 1962.

[2] G. J. Dolan. Direct observation of the magnetic structure in thin films of Pb, Sn, and In. *J. Low temp. Phys.*, 15:111, 1973.

[3] C. A. Durán, P. L. Gammel, R. E. Miller, and D. J. Bishop. Observation og magnetic-field penetration via dendritic growth in superconducting niobium films. *Phys. Rev. B*, 52:75, 1995.

[4] P. Brüll, D. Kirchgässner, P. Leiderer, P. Berberich, and H. Kinder. Magnetic-field-induced damage in a superconducting YBa$_2$Cu$_3$O$_{7-\delta}$. *Annalen der Physic*, 504:243, 1992.

[5] T. H. Johansen, M. Baziljevich, D. V. Shantsev, P. E. Goa, Y. M. Galperin, W. N. Kang, H. J. Kim, E. M. Choi, M.-S. Kim, and I. Lee. Dendritic magnetic instability in superconducting MgB$_2$ films. *EPL*, 59:599, 2002.

[6] I. A. Rudnev, S. V. Antonenko, D. V. Shantsev, T. H. Johansen, and A. E. Primenko. Dendritic flux avalanches in superconducting Nb$_3$Sn films. *Cryogenics*, 43:663, 2003.

[7] S. C. Wimbush, B. Holzapfel, and Ch. Jooss. Observation of dendritic flux instabilities in YNi$_2$B$_2$C thin films. *J. App. Phys.*, 96:3589, 2004.

[8] I. A. Rudnev, D. V. Shantsev, T. H. Johansen, and A. E. Primenko. Avalanche-driven fractal flux distributions in NbN superconducting films. *Appl. Phys. Lett.*, 87:042502, 2005.

[9] M. Motta, F. Colauto, W. A. Ortiz, J. Fritzche, J. Cuppens, W. Gillijns, V.V. Moshchalkov, T. H. Johansen, A. Sanchez, and A. V. Silhanek. Enhanced pinning in superconducting thin films with graded pinning landscapes. *Appl. Phys. Lett.*, 102:212601, 2013.
Dendritic flux avalanches

[10] Z. W. Zhao, S. L. Li, Y. M. Ni, H. P. Yang, Z. Y. Liu, H. H. Wen, W. N. Kang, H. J. Kim, E. M. Choi, and S. I. Lee. Suppression of superconducting critical current density by small flux jumps in MgB$_2$ thin films. Phys. Rev. B, 65:064512, 2002.

[11] E.-M. Choi, H.-S. Lee, H.-J. Kim, S.-I. Lee, H.-J. Kim, and W. N. Kang. Enhancement at low temperatures of the critical current density for Au-coated MgB$_2$ thin films. Appl. Phys. Lett., 84:82, 2004.

[12] F. Colauto, E. Choi, J. Y. Lee, S. I. Lee, E. J. Patiño, M. G. Blamire, T. H. Johansen, and W. A. Ortiz. Suppression of flux avalanches in superconducting films by electromagnetic breaking. Appl. Phys. Lett., 96:092512, 2010.

[13] Jae-Yeap Lee, Hu-Jong Lee, Myung-Hwa Jung, Sung-Ik Lee, Eun-Mi Choi, and W. N. Kang. Saw-tooth pattern from flux jumps observed by high resolution M-H curves in MgB$_2$ thin films. J. Appl. Phys., 108:033909, 2010.

[14] R. G. Mints and A. L. Rakhmanov. Critical state stability in type-II superconductors and superconducting-normal-metal composites. Rev. Mod. Phys., 53:551, 1981.

[15] J. I. Vestgård, D. V. Shantsev, Y. M. Galperin, and T. H. Johansen. Lightning in superconductors. Sci. Rep., 2:886, 2012.

[16] I. S. Aranson, A. Gorevich, M. S. Wellin, R. J. Wijngaarden, V. K. Vlasko-Vlasov, V. M. Vinokur, and U. Welp. Dendritic flux avalanches and nonlocal electrodynamics in thin superconducting films. Phys. Rev. Lett., 94(3):037002, Jan 2005.

[17] J. I. Vestgård, D. V. Shantsev, Y. M. Galperin, and T. H. Johansen. Dynamics and morphology of dendritic flux avalanches in superconducting films. Phys. Rev. B, 84:054537, 2011.

[18] D. V. Denisov, D. V. Shantsev, Y. M. Galperin, Eun-Mi Choi, Hyun-Sook Lee, Sung-Ik Lee, A. V. Bobyl, P. E. Goa, A. A. F. Olsen, and T. H. Johansen. Onset of dendritic flux avalanches in superconducting films. Phys. Rev. Lett., 97:077002, 2006.

[19] J. I. Vestgård, D. V. Shantsev, Y. M. Galperin, and T. H. Johansen. The thermomagnetic instability in superconducting films: Threshold magnetic field and temperature. arXiv:1309.4591, 2013.

[20] J. I. Vestgård, Y. M. Galperin, and T. H. Johansen. The thermomagnetic instability in superconducting films with adjacent metal layer. arXiv:1304.5405, 2013.

[21] E.-M. Choi, H.-S. Lee, J. Y. Lee, S.-I. Lee, Å. A. Olsen, V. V. Yurchenko, D. V. Shantsev, T. H. Johansen, H.-J. Kim, and M.-H. Cho. Width-dependent upper threshold field for flux noise in MgB$_2$ films. Phys. Rev. Lett., 98(11):117001, Mar 2007.

[22] J. I. Vestgård, D. V. Shantsev, Y. M. Galperin, and T. H. Johansen. The diversity of flux avalanche patterns in superconducting films. Supercond. Sci. Technol., 26:055012, 2013.

[23] U. Bolz, B. Biehler, D. Schmidt, B.-U. Runge, and P. Leiderer. Dynamics of the dendritic flux instability in YBa$_2$Cu$_3$O$_{7-δ}$. EPL, 64:517, 2003.

[24] E. H. Brandt. Susceptibility of superconductor disks and rings with and without flux creep. Phys. Rev. B, 55(21):14513, 1997.

[25] J. I. Vestgård, P. Mikheenko, Yu. E. Kuzovlev. Nonlocal electrodynamics of normal and superconducting films. New J. Phys., 15:093001, 2013.

[26] Y. B. Kim, C. F. Hempstead, and A. R. Strnad. Magnetization and critical supercurrents. Phys. Rev., 129:528–535, Jan 1963.

[27] Y.-H. Zhou and X. Yang. Numerical simulations of thermomagnetic instability in high-$T_c$ superconductors: Dependence on sweep rate and ambient temperature. Phys. Rev. B, 74:054507, Aug 2006.

[28] F. Colauto, E. J. Patiño, M. G. Blamire, and W. A. Ortiz. Boundaries of the instability region of the $H T$ diagram of Nb thin films. Supercond. Sci. Technol., 21:045018, 2008.

[29] P. N. Mikheenko and Yu. E. Kuzovlev. Inducance measurements of HTSC films with high critical currents. Physica C, (204):229, 1993.

[30] E. H. Brandt, M. V. Indenbom, and A. Forkl. Type-II superconducting strip in perpendicular magnetic field. EPL, 22:735, 1993.