Near-Field Wideband Beamforming for Extremely Large Antenna Arrays

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Abstract—The natural integration of extremely large antenna arrays (ELAAs) and terahertz (THz) communications can potentially establish Tb/s data links for 6G networks. However, due to the extremely large array aperture and wide bandwidth, a new phenomenon termed as “near-field beam split” emerges. This phenomenon causes beams at different frequencies to focus on distinct physical locations, leading to a significant loss of the beamforming gain. To address this challenging problem, we first harness a piecewise-far-field channel model to approximate the complicated near-field wideband channel. In this model, the entire large array is partitioned into several small sub-arrays. While the wireless channel’s phase discrepancy across the entire array is modeled as near-field spherical, the phase discrepancy within each sub-array is approximated as far-field planar. Built on this approximation, a phase-delay focusing (PDF) method employing delay phase precoding (DPP) architecture is proposed. Our PDF method could compensate for the intra-array far-field phase discrepancy and the inter-array near-field phase discrepancy via the joint control of phase shifters and time delayers, respectively. Theoretical and numerical results are provided to demonstrate the efficiency of the proposed PDF method in mitigating the near-field beam split effect. Finally, we define and derive a novel metric termed as the “effective Rayleigh distance” by the evaluation of beamforming gain loss. Compared to classical Rayleigh distance, the effective Rayleigh distance is more accurate in determining the near-field range for practical communications.

Index Terms—Extremely large antenna array, wideband, nearfield beam split, beamforming, Rayleigh distance.

I. INTRODUCTION

As a key technology for 5G communication systems, large antenna arrays (LAAs) could improve the transmission rate by orders of magnitude via efficient beamforming/precoding [1]. To further reap the benefits of massive antennas, LAAs are evolving to extremely large antenna arrays (ELAAs) for 6G communications [2], where the array aperture is dramatically increased to support ultra-high-speed communications. There are abundant possible implementations of ELAA. For instance, ELAA could be employed in distributed multiple-input-multiple-output (MIMO) systems relying on radio stripes [3] or in reconfigurable intelligent surface (RIS) systems [4] to improve the network capacity. It is also envisioned to coat ELAAs on entire walls to enhance the coverage of wireless signals [5].

In addition, ELAAs are usually combined with high-frequency communications. Benefiting from the abundant spectrum resources, terahertz (THz) communications can provide a very large bandwidth of several GHz, allowing for Tb/s data rates for 6G networks [6]. Besides, the extremely small size of THz antennas also favorably facilitates the deployment of ELAAs [7]. As a consequence, the natural integration of THz wideband communications and ELAAs has been regarded as a pivotal candidate for next-generation wireless networks [7].

A. Prior Works

The evolution from LAA to ELAA not only implies a sharp increment in array aperture, but also leads to a fundamental change in the characteristics of the electromagnetic (EM) field [8]. The electromagnetic radiation field can generally be divided into the far-field and radiation near-field regions [9], [10]. In the far-field region, the wireless channel could be modeled under the planar wave assumption, where the phase of the antenna response vector is a linear function of the antenna index [11], [12]. In contrast, the wavefront of near-field channel has to be modeled accurately as spherical, where the phase of near-field array response vector is a non-linear function of the antenna index [9]. The boundary between near-field and far-field is typically quantified by the Rayleigh distance [9], also known as the Fraunhofer distance [13], which is proportional to the square of array aperture normalized by wavelength. Since the array aperture is typically not very large in the current 5G systems, the near-field range of 5G LAA is negligible. That is why classical beamforming techniques usually direct a beam with planar wavefront in a specific direction [1]. In contrast, as the number of ELAA’s antennas increases dramatically, the Rayleigh distance will be expanded by orders of magnitude. The near-field range of an ELAA could be up to several hundreds of meters [8], covering a large part of typical cells. In this scenario, it is necessary to perform near-field beamforming to focus the energy of a beam on a desired user location [14] by exploiting the spherical wavefront property. Given this non-negligible near-field range,
near-field communications will be of pivotal significance in next-generation communications.

Moreover, when it comes to wideband systems, another critical change in EM waves known as beam split is induced, which also has the terminology “beam squint” \cite{15}, \cite{16}. Classically, LAA relies on phased shifter (PS) based analog beamforming architecture, allowing only frequency-independent phase shifting for narrowband beamforming \cite{17}. However, the array response vectors of wireless channels are frequency-dependent, especially for THz wideband networks, causing the wavefront of a beam at different frequencies to deviate from that at the center frequency. To elaborate, in far-field scenarios, the beam split effect makes beams of different frequency components propagate in distinct directions \cite{15}. On the other hand, in near-field scenarios, the beam split effect results in a new phenomenon where beams at different frequencies are focused at varying directions and distances. Consequently, the signal energy fails to converge on the desired receiver’s location. Only signals around the center frequency can be captured by the receiver, while most beams with frequencies far away from the center frequency suffer from an unacceptable beamforming gain loss.

Over the past few years, intensive research has been devoted to studying advanced beamforming technologies to address the far-field beam split effect \cite{16}, \cite{18}, \cite{19}, \cite{20}, \cite{21}, \cite{22}. Relevant methods fall into two primary categories, i.e., algorithmic methods and hardware-based mitigation methods. In the first category, researchers have endeavored to generate wide beams by carefully optimizing the PSs to achieve flattened beamforming gain across the entire bandwidth \cite{16}, \cite{18}, \cite{19}. While these algorithms are relatively straightforward to implement in practice, their beamforming performance is severely hindered by the presence of beam split as well. This limitation arises because they still rely on PS-based analog beamforming. The second category of solutions employs true-time-delay (TTD) circuits instead of PSs to generate frequency-dependent beams, which offers the potential to eliminate far-field beam split \cite{20}, \cite{21}, \cite{22}. Inspired by this idea, several array structures, such as true-time-delay (TTD) arrays \cite{20} and delay-phase precoding (DPP) arrays \cite{21}, \cite{22}, have been envisioned and developed to counteract the far-field beam split effect. Despite the rapid development of solutions to far-field beam split, it is essential to note that the aforementioned methods are all customized for far-field communications. They are not applicable to tackle the challenges posed by near-field beam split, because the models of far-field channels and near-field channels differ remarkably. To the best of our knowledge, the near-field beam split effect has not been studied in the literature.

**B. Our Contributions**

To fill in this gap, a phase-delay focusing (PDF) method is proposed to tackle the near-field beam split problem. Our key contributions are summarized as follows.

1. First, we introduce the near-field beam split effect of ELAA by comparing the loss of beamforming gain resulting from both far-field and near-field beam split effects. We formulate the model of near-field beam split effect and reveal that this effect causes beams at different frequencies to focus on distinct locations.

2. Second, a piecewise-far-field wideband channel model is proposed to approximate the near-field wideband channel model with high accuracy. In this model, the entire ELAA is partitioned into multiple small sub-arrays. In this way, we could reasonably assume that the receiver is located in the far-field region of each small sub-array while being in the near-field region of the entire ELAA. This partition allows us to decompose the complicated phase discrepancy of a near-field channel into two distinct components: the inter-array near-field phase discrepancy and the intra-array far-field phase discrepancy. Leveraging this decomposition, a phase-delay focusing (PDF) method is proposed based on the DPP array architecture, where the inter-array phase and the intra-array phase are compensated by the PSs and TTDs of DPP, respectively. Simulation results validate the efficacy of the proposed PDF method in mitigating the near-field beam split effect.

3. Finally, by evaluating the gain loss of far-field beamforming in the near-field region, a new metric called effective Rayleigh distance is derived to distinguish the far-field and near-field regions. Classical Rayleigh distance, which is defined by evaluating the phase error between planar wave and spherical wave, is not precise enough to capture the near-field region where far-field beamforming methods are not applicable. To tackle this problem, we conduct a theoretical evaluation on the gain loss of far-field beamforming in the physical space. Subsequently, the close-form expression of effective Rayleigh distance is derived, which defines the region where the gain loss of far-field beamforming exceeds a threshold. Since beamforming gain directly affects the received signal power, our proposed effective Rayleigh distance is a more accurate metric for measuring the near-field range for practical communications.

**C. Organization and Notation**

The rest of the paper is organized as follows. In section II, the wideband ELAA channel model is introduced and the near-field beam split effect is discussed. In section III, the proposed piecewise-far-field channel model and the proposed PDF method are explained in detail. Theoretical analysis on the beamforming gain of the PDF method is also offered. Section IV elaborates on the effective Rayleigh distance. Numerical results are provided in section V. Finally, conclusions are drawn in section VI.

**Notation:** Lower-case boldface letters \( x \) denote vectors; \((\cdot)^T, (\cdot)^H, (\cdot)^*\) and \( \| \cdot \|_k \) denote the transpose, conjugate transpose, conjugate, and \( k \)-norm of a vector or matrix respectively; \( |x| \) denotes the the amplitude of scalar \( x \); \( \arg(x) \) denotes the phase of \( x \); \( |x|_n \) represents the \( n \)th element of vector \( x \); \( |X|_{ij} \) represents the \((i,j)\)th entry of matrix \( X \); \( CN(\mu;\Sigma) \) and \( U(a; b) \) denote the Gaussian distribution with mean \( \mu \) and
As shown in Fig. 1, the center of the BS array is
\[ \rho_n = n - \frac{N - 1}{2} \] with \( n \in \{0, 1, \ldots, N - 1\} \). Therefore, the array aperture is given as \( \delta = (N - 1)d \approx Nd \). The \( u \)th user is located at \( (x_u, y_u) \), where its polar coordinate is
\[ (r_u, \theta_u) = \left( \sqrt{x_u^2 + y_u^2}, \arctan \frac{y_u}{x_u} \right) \]. Then, adopting the free space Maxwell equation, the line-of-sight near-field channel \( h_{u,m} \) can be modeled [9] as
\[ h_{u,m} = g_{u,m} \left[ e^{-jk_m r_u^{(0)}}, e^{-jk_m r_u^{(1)}}, \ldots, e^{-jk_m r_u^{(N-1)}} \right]^T = g_{u,m} \sqrt{N} a_m(r_u, \theta_u), \quad (2) \]
where \( a_m(r_u, \theta_u) \) represents the near-field array response vector, \( k_m = \frac{2\pi f_m}{c} \) denotes the wavenumber at frequency \( f_m \), and \( g_m \) denotes the complex path loss. Let \( r_u^{(n)} \) be the distance from the \( n \)th BS antenna to the \( u \)th user expressed as
\[ r_u^{(n)} = (x_u^2 + (y_u - \delta_N d)^2)^{1/2} = (r_u^2 + (\delta_N d)^2 - 2\delta_N dr_u \sin \theta_u)^{1/2}. \quad (3) \]
Since the PS-based analog beamformer \( \mathbf{F} \) is frequency-independent, each column of \( \mathbf{F} \) is generally set to align with the array response vector of the wireless channel at the center frequency \( f_c \) [11, 23]. Thereafter, let \( \mathbf{F} = [\mathbf{f}_0, \mathbf{f}_1, \ldots, \mathbf{f}_{U-1}] \), then \( \mathbf{f}_u \) can be obtained by
\[ \mathbf{f}_u = \mathbf{a}_c^*(r_u, \theta_u) = \frac{1}{\sqrt{N}} [e^{jk c r_u^{(0)}}, \ldots, e^{jk c r_u^{(N-1)}}]^T, \quad (4) \]
where \( k_c = \frac{2\pi f_c}{c} \) and \( \mathbf{a}_c^*(r_u, \theta_u) \) represent the wavenumber and array response vector on the center frequency \( f_c \). The fundamental model difference from the near-field array response vector \( \mathbf{a}_m(r_u, \theta_u) \) to the analog beamformer \( \mathbf{f}_u = \mathbf{a}_c^*(r_u, \theta_u) \) causes the near-field beam split effect.

### B. Discussion on Near-Field Beam Split

To delve into the near-field beam split effect, we would like to compare the beamforming properties under far-field/near-field and narrowband/wideband conditions. In the upcoming discussions, we will omit the subscript \( u \) for ease of expression. Notice that the phase \( k_m r_u^{(n)} \) in (2) and \( k_c r_u^{(n)} \) in (4) are non-linear functions with respect to (w.r.t) the antenna index \( n \). Traditionally, since the array aperture is not very large, the far-field model under the planar wave assumption [11] is widely adopted to simplify this non-linear distance as
\[ r^{(n)} = r \left( 1 + \frac{(\delta_n d)^2}{r^2} - \frac{2\delta_n d \sin \theta}{r} \right)^{1/2}, \quad (5) \]
where (a) arises because of the first-order Taylor expansion \((1 + x)^{1/2} \approx 1 + \frac{1}{2} x \) and the ignorance of the second-order term \( (\delta_n d x)^2 \). It is clear from (5) that in the far-field region, the phase becomes \( k_m r^{(n)} \approx k_m r - k_m \delta_n d \sin \theta \), which is a linear function of the antenna index \( n \). Then the far-field beamforming vector becomes \( |\mathbf{f}_n| = \frac{1}{\sqrt{N}} e^{jk \delta_n d \sin \theta} \). Because the term \( e^{jk \delta_n d \sin \theta} \) is independent of the antenna index \( n \),

**Fig. 1.** The system layout of ELAA.
Clearly, the maximum value of $G$ for rowband systems where $\theta = 0$, we have

$$G(\hat{r}, \hat{\theta}, r, \theta, f_r) = \text{max}$$

the lowest, the center, and the highest frequencies are plotted (e.g., the three lines in the sub-figures (c) and (d)).

(b) the near-field narrowband scenario, (c) the far-field wideband scenario, and (d) the near-field wideband scenario. In each sub-figure, the beam energy of the lowest, the center, and the highest frequencies are plotted (e.g., the three lines in the sub-figures (c) and (d)).

$[f]_n$ can be rewritten as $[f]_n = \frac{1}{\sqrt{N}} e^{-jk\hat{r}e^{(n)} d \sin \theta} = [f_{\text{far}}(\theta)]_n$, depending only on direction $\theta$. As shown in Fig. 2(a), the beam at the center frequency generated by $[f_{\text{far}}(\theta)]_n$ is transmitting towards a specific direction $\theta$.

However, since the linear approximation in (5) is not accurate when $n$ is very large, the above far-field assumption does not hold anymore for ELAAs. The typical near-field range is determined by the Rayleigh distance $R = \frac{\lambda c}{4 \pi f}$, where $\lambda$ is the wavelength and $f$ is the frequency. In this case, the beamforming gain $G_{\text{near}}(\theta)$ can be approximated as $G(\hat{r}, \hat{\theta}, r, \theta, f_r) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-j(km - km)\hat{r}e^{(n)}}$, rendering it much lower than 1. Consequently, the beam energy at $f_r$ is split from the desired location $(r, \theta)$.

In the far-field scenario where the distance is considerable, as shown in Fig. 2(c), this beam split effect causes beams at different frequencies to transmit towards different directions. However, in the near-field setting, as shown in Fig. 2(d), the near-field beam split effect causes beams at different frequencies to focus on different locations. These distinct beamforming properties distinguish the far-field and near-field beam split effects.

Furthermore, since beams over large bandwidth are split to different locations/directions, the user can only access to signals close to the center frequency. For example, considering $f_c = 100$ GHz, $B = 5$ GHz, and $N = 512$, the far-field beam-split effect results in over 50% of the sub-carriers experiencing a beamforming gain loss of at least 60%. Recent works mainly concentrate on mitigating the far-field beam-split effect. This is accomplished by either deploying a large number of high power consumption time-delay elements or heavily relying on the linear phase property of the far-field channel, which is not applicable in the near field. To the best of our knowledge, the near-field beam-split effect has not been studied in the literature.

Notice that the sub-figures (c) and (d) of Fig. 2 also appear in our earlier work [26], where [26] employs near-field beam split to perform fast beam training, while the objective of this work is to mitigate this phenomenon via energy-efficient beamforming.
III. PROPOSED METHODS

In this section, we commence by introducing a piecewise-far-field channel characterized by piecewise-linear phase properties to approximate the intricate near-field channel. Subsequently, a PDF method is built on this approximation to mitigate the near-field beam split effect.

A. Piecewise-Far-Field Channel Model

The non-linear phase \(-k_{m}r_{n}^{(n)}\) w.r.t the antenna index \(n\) makes it intractable to directly devise near-field wideband beamforming techniques. In order to get a manageable simplification of this non-linear phase while maintaining acceptable accuracy, we observe that the Rayleigh distance \(\frac{1}{2} N^{2} \lambda_{c}\) scales proportionally with the square of the number of antennas, signifying that fewer antennas corresponds to a better accuracy of the far-field assumption in (5).

Inspired by this observation, as depicted in Fig. 3(a)-(c), a piecewise-far-field channel model is harnessed to approximate the intricate near-field channel. In this model, the entire large array is partitioned into multiple sub-arrays, each equipped with much fewer antennas compared to the entire array. This partition leads to a notable reduction of the near-field range for each sub-array. Consequently, even if the receiver is inside the near-field region of the entire array, we can reasonably assume that the receiver is situated in the far-field region of each sub-array.

To elaborate, as shown in Fig. 3(d), we divide the entire large array into \(K\) sub-arrays. For each sub-array, there are \(P\) adjacent antennas, satisfying \(N = KP\). Then, the near-field channel from the BS to the \(u\)th user is rearranged as follows

\[
h_{u,m} = \left[ h_{u,m}^{(0)}, h_{u,m}^{(1)}, \ldots, h_{u,m}^{(K-1)} \right]^T,
\]

where \(h_{u,m}^{(k)} \in \mathbb{C}^{P \times 1}\) represents the sub-channel between the \(k\)th sub-array and the \(u\)th user. We define \(\delta_{K}^{(k)} = k - \frac{K-1}{2}\) for \(k \in \{0, 1, \ldots, K-1\}\). Then, the distance and direction from the center of the \(k\)th sub-array to the \(u\)th user are expressed as

\[
r_{u,k}^{(p)} = \sqrt{x_{u}^{2} + (y_{u} - \delta_{K}^{(k)} P d)^{2}} = \sqrt{r_{u,k}^{2} + (\delta_{K}^{(k)} P d)^{2} - 2\delta_{K}^{(k)} P d r_{u,k} \sin \theta_{u,k}}
\]

and \(\sin \theta_{u,k} = \frac{y_{u} - \delta_{K}^{(k)} P d}{r_{u,k}}\), respectively. We further define \(\delta_{P}^{(p)} = p - \frac{P-1}{2}\) for \(p \in \{0, 1, \ldots, P-1\}\). Consequently, according to (5), the distance \(r_{u,k}^{(p)}\) from the \(p\)th antenna in the \(k\)th sub-array to the \(u\)th user is expressed as

\[
r_{u,k}^{(p)} \approx r_{u,k} - \delta_{P}^{(p)} d \sin \theta_{u,k}.
\]
There, the approximation (a) holds because each sub-array is small enough. Accordingly, the near-field channel $h_{u,m}^{(k)}$ of the $k$th sub-array is approximated by a far-field channel $\hat{h}_{u,m}^{(k)}$:

$$
\hat{h}_{u,m}^{(k)}|_p \approx g_{u,m} e^{-jk_{m} r_{u,k} \cos \theta_{u,k}} \sin \theta_{u,k} = [\hat{h}_{u,m}^{(k)}]_p. \tag{9}
$$

By introducing the parameter $\eta_m = \frac{L_m}{c}$ and plugging $d = \frac{\lambda}{2}$ and $k_m = \frac{2\pi f_m}{c}$ into (9), we arrive at $[\hat{h}_{u,m}^{(k)}]_p = e^{-jk_{m} r_{u,k} \cos \theta_{u,k}} \sin \theta_{u,k}$. Consequently, the intricate near-field channel is approximated by a piecewise-linear far-field channel:

$$
h_{u,m} \approx \hat{h}_{u,m} = \left[\hat{h}_{u,m}^{(0)}, \hat{h}_{u,m}^{(1)}, \cdots, \hat{h}_{u,m}^{(k-1)}\right]. \tag{10}
$$

It is notable from (9) that the phase of $[\hat{h}_{u,m}^{(k)}]_p$ is a linear function of $p$, the antenna index of the $k$th sub-array. This linear phase property suggests that $[\hat{h}_{u,m}^{(k)}]_p$ can be regarded as a far-field channel. Furthermore, considering the different $r_k$ and $\theta_k$ in each sub-array, the planar waves impinging on different sub-arrays come from different directions. This is why we call the entire array’s channel the piecewise-linear far-field channel. To illustrate the fidelity of the proposed model, Fig. 3(e) depicts the channel phase as a function of the antenna index for the near-field, far-field, and piecewise-linear far-field channel models. The phase profile of the piecewise-linear far-field channel model closely approaches that of the true near-field channel.

In essence, our proposed channel model can be recognized as a piecewise-linearization of the intricate near-field channel model, wherein the phase exhibits local linear behavior within each sub-array. Harnessing this piecewise-linear phase characteristics, we proceed to devise a near-field wideband beamforming method referred to as phase-delay focusing to alleviate the near-field beam-split effect in the subsequent subsection.

### B. Proposed Phase-Delay Focusing Method

We first elaborate on overcoming the near-field beam-split for an arbitrary user by analog beamforming, while the extension to multi-user hybrid beamforming is studied in Section III-D. For ease of expression, the subscript $u$ is omitted in Section III-B to III-C. Accordingly, the variables $h_{u,m}$, $\hat{h}_{u,m}$, $f_{u}$, $r_{u}$, $\theta_{u}$, $r_{u,k}$, and $\theta_{u,k}$ become $h_{m}$, $\hat{h}_{m}$, $f$, $r$, $\theta$, $r_k$, and $\theta_k$.

Specifically, the introduced piecewise-linear far-field channel model makes it straightforward to decouple the phase in (9) into two components: the inter-array phase discrepancy $-k_{m} r_{k}$ across different sub-arrays, and the intra-array phase discrepancy $\pi \eta_m \delta_p \sin \theta_k$ within each sub-array. It is notable that $-k_{m} r_{k}$ is a non-linear function of $k$, giving rise to a near-field channel phase property, whereas $\pi \eta_m \delta_p \sin \theta_k$ follows a linear function in relation to $p$, as the same to a far-field model. Both of these two phase components contribute to the near-field beam split effect. The following fact inspires us to neglect the influence of intra-array phase $\pi \eta_m \delta_p \sin \theta_k$ on near-field beam split. As suggested in [21], [28], the degree of beam-split effect is proportional to the physical antenna aperture. A larger antenna aperture results in a severe beam-split effect. Although this conclusion is derived in the far-field region, it is valid to near-field as well, because the physical propagation delay always increases with the antenna aperture, no matter far-field or near-field. Following this intuition, we can find that the intra-array phase discrepancy $k_{m} \delta_p \sin \theta_k$ corresponds to a sub-array’s aperture $Pd$, while the inter-array phase discrepancy $k_{m} r_{k}$ is related to the entire array’s aperture $Nd$. Since $P \ll N$, it is reasonable to deduce the near-field beam split is dominated by the inter-array phase discrepancy. Consequently, our target is converted to compensating for the inter-array phase $-k_{m} r_{k}$.

Note that the channel phase $-k_{m} r_{k} = -2\pi f_{m} r_{k}$ is equivalent to the frequency response of a time delay of $\frac{c}{\lambda} r_{k}$. Therefore, the delay-phase precoding architecture [21] employing TTD circuits can be used to compensate for the inter-array phase $-k_{m} r_{k}$. As illustrated in Fig. 4, compared to conventional hybrid precoding architecture, one additional TTD circuit is inserted in each sub-array to connect the RF chain and the PS-based sub-array. The frequency response of a TTD at $f_{m}$ is $e^{-j2\pi f_{m} \tau}$, where $\tau$ represents the adjustable time delay parameter. For brevity, let $\tau' = c \tau$ be the adjustable distance parameter. Then, the corresponding frequency response is transformed into $e^{-j2\pi f_{m} \tau'}$. Thereby, a TTD is able to compensate for the frequency-dependent phase $-k_{m} r_{k}$ if $\tau' = -r_{k}$. Moreover, as shown in Fig. 4, the main function of the PS-based sub-array is to generate far-field planar waves to match the intra-array far-field phase $\pi \eta_m \delta_p \sin \theta_k$. Through the joint manipulation of PSs and TTDs, the beam energy across the entire bandwidth can be focused on the receiver location $(r, \theta)$. We henceforth refer to this method as phase-delay focusing.

Recall that the analog beamformer $\mathbf{f}$ realized by PS is frequency-independent. In contrast, the introduction of TTD makes the corresponding analog beamformer $\mathbf{f}_{m}$ frequency-dependent. To be specific, similar to the decomposition of wireless channel presented in (7), the beamforming vector $\mathbf{f}_{m}$ realized by the PDF method at $f_{m}$ is composed of $K$ sub-vectors, i.e.,

$$
\mathbf{f}_{m} = \left[\mathbf{f}^{(0)}_{m}, \mathbf{f}^{(1)}_{m}, \cdots, \mathbf{f}^{(K-1)}_{m}\right]^T, \tag{11}
$$

Fig. 4. A single-user example of the delay-phase precoding architecture [21] for performing the PDF method.
where $f_m^{(k)} \in \mathbb{C}^{P \times 1}$ represents the beamforming vector of the $k$th sub-array. As illustrated in Fig. 4, $f_m^{(k)}$ is generated by one TTD element and $P$ PSs, which can be expressed as follows:

$$f_m^{(k)} = \frac{1}{\sqrt{N}} e^{-jk_m r'_k} [e^{jk_p \beta_k}, \cdots, e^{jk_p (f - 1) \beta_k}]^T. \quad (12)$$

Here, $r'_k$ denotes the adjustable distance parameter of the $k$th TTD element and $\beta_k$ denotes the adjustable phase parameter associated with the $k$th PS-based sub-array. It is evident from (12) that $f_m^{(k)}$ includes two distinct components. The first one is the frequency-independent phase $\pi \rho_m^{(k)} \beta_k'$ generated by the $P$ PSs within the $k$th sub-array. The second one involves the frequency-dependent phase $-k_m r'_k$ realized by the $k$th TTD element. As discussed earlier, the purpose of $\pi \rho_m^{(k)} \beta_k'$ is to produce planar waves that align with the far-field phase discrepancy $\pi \rho_m \delta^{(p)} \sin \theta_k$, while the introduction of $t'_k = \frac{r'_k}{r}$ serves to compensate for the near-field phase discrepancy $-k_m r'_k$.

To elaborate, the normalized beamforming gain achieved by the proposed PDF method on the user location is expressed as

$$\min_{\{r_k\}} \frac{1}{\sqrt{|g_m|}} |\tilde{h}_m^T f_m| = \frac{1}{\sqrt{|g_m|}} \sum_{k=0}^{K-1} \tilde{h}^{(k)}_m^T f^{(k)}_m \quad (13),$$

where $\Xi_p(x) = \frac{\sin(\frac{\pi}{P} x)}{P \sin(\frac{\pi}{P} x)}$. To generate planar waves aligning with the sub-array channel, $\beta_k'$ is typically devised according to the spatial direction $\sin \theta_k$ at the center frequency [29], i.e.,

$$\beta_k' = -\sin \theta_k. \quad (14)$$

By substituting (14) into (13), we obtain

$$\tilde{h}^{(k)}_m^T f^{(k)}_m = e^{-jk_m (r'_k + r_k)} \Xi_p(\epsilon_m \sin \theta_k), \quad (15)$$

where $\epsilon_m = \rho_m - 1 = \frac{B}{f} \left(\frac{2}{M-1} m - 1\right)$. The subsequent objective of our PDF method is to find proper $\{r'_k\}$ to maximize the beamforming gain on the user location $(r, \theta)$ over the entire bandwidth. Hence, the corresponding optimization problem can be formulated as

$$\max_{\{r_k\}} \frac{1}{MK} \left(\sum_{m=1}^{M-1} \sum_{k=0}^{K-1} e^{-jk_m (r'_k + r_k)} \Xi_p(\epsilon_m \sin \theta_k)\right) \quad (16)$$

s.t. $r'_k \geq 0$ for $k \in \{1, 2, \cdots, K\}$.

We provide the following Lemma 1 to solve problem (16).

**Lemma 1:** If $|\epsilon_m| \leq \frac{2}{P}$ for $\forall m \in \{0, 1, \cdots, M - 1\}$, then the optimal solution to problem (16) is

$$r'_k = L - r_k, \quad (17)$$

where $L$ is a global distance parameter chosen to ensure $\min \{r'_k\} \geq 0$.

**Proof:** By substituting (17) into (15), the beamforming gain of the $k$th sub-array can rewritten as

$$\tilde{h}^{(k)}_m^T f^{(k)}_m = P e^{-jk_m k} \Xi_p(\epsilon_m \sin \theta_k). \quad (18)$$

The condition $|\epsilon_m| \leq \frac{2}{P}$ means that the parameters $|\epsilon_m \sin \theta_k| \leq |\epsilon_m|$, are within the main lobe of $\Xi_p(\cdot)$ for all subcarriers $f_m$. Hence, it is obvious that $\Xi_p(\epsilon_m \sin \theta_k) > 0$, and the beamforming gain can be presented as

$$\frac{1}{K} \sum_k \Xi_p(\epsilon_m \sin \theta_k), \quad (19)$$

In addition, according to the Cauchy-Schwarz inequality, the beamforming gain $G_m$ has an upper bound:

$$G_m \leq \frac{1}{K} \sum_k \Xi_p(\epsilon_m \sin \theta_k). \quad (20)$$

It is clear from (19) and (20) that $r'_k$ is the optimal solution to maximize $G_m$ at frequency $f_m$. Moreover, since $r'_k$ is frequency independent, it is the optimal solution to all subcarriers and thus optimal to problem (16), which completes the proof.

To sum up, (14) and (17) complete the beamforming design for the proposed PDF method. In the next subsection, theoretical analysis will be provided to validate the efficiency of the proposed PDF method in alleviating the near-field beam split effect.

**C. Analysis of Beamforming Gain Performance**

This subsection presents an analysis of the performance of the proposed PDF method for large numbers $M$ and $K$. We begin by introducing Lemma 2, which provides the average beamforming gain over all sub-carriers achieved by the PDF method.

**Lemma 2:** Suppose the user is located in the far-field region of each sub-array and $|\epsilon_m| \leq \frac{2}{P}$ for $\forall m$, then the average beamforming gain over all sub-carriers achieved by (11) can be approximated as

$$\frac{1}{M} \sum_{m=0}^{M-1} G_m \approx 1 - \frac{1}{MK} \sum_{m=0}^{M-1} \frac{e_m}{P} \sum_{k=0}^{K-1} \sin^2 \theta_k. \quad (21)$$

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Proof: Plugging (14) and (17) into (13), the average beamforming gain could be expressed as
\[
\frac{1}{M} \sum_{m=0}^{M-1} G_m = \frac{1}{MR} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} \Xi_P(\epsilon_m \sin \theta_k). \tag{22}
\]

It is intractable to compute (22) as the variables \(\epsilon_m\) and \(\sin \theta_k\) are included in Dirichlet sinc functions \(\Xi_P(\epsilon_m \sin \theta_k)\). To deal with this problem, we use a two-variable quadratic function to fit function \(\Xi_P(ab)\). To be specific, because \(|\epsilon_m| \leq \epsilon_{M-1} = \frac{B}{2\pi r}\) and \(|\sin \theta_k| \leq 1\), the following five points on \(\Xi_P(ab)\) are used for function fitting: \((0, 0, 1), \left(\frac{B}{2\pi r}, 1, \Xi_P(\frac{B}{2\pi r})\right), \left(-\frac{B}{2\pi r}, -1, \Xi_P(-\frac{B}{2\pi r})\right), \left(-\frac{B}{2\pi r}, 1, \Xi_P(-\frac{B}{2\pi r})\right)).\) Therefore, we have
\[
\Xi_P(ab) \approx 1 - (1 - \Xi_P(\frac{B}{2\pi r})) \frac{a^2}{\left(\frac{B}{2\pi r}\right)^2} b^2. \tag{23}
\]

The graph of function \(1 - (1 - \Xi_P(\frac{B}{2\pi r})) x^2\) is depicted in Fig. 5 for parameters \(P = 32, B = 5\) GHz, and \(f_c = 100\) GHz, which is quite close to the Dirichlet sinc function \(\Xi_P(\frac{B}{2\pi r})\).

Finally, substituting (23) into (22) and replacing \((a, b)\) with \((\epsilon_m, \sin \theta_k)\), we could arrive at the conclusion (21).

Lemma 2 allows us to separately compute the factors affecting the average beamforming gain: the factor \(\sum_m \frac{\epsilon_m^2}{(2\pi r)^2}\) arising from the wideband effect and the factor \(\sum_k \sin^2 \theta_k\) resulting from the near-field effect.

Specifically, the following Corollary 1 presents a more analytical form of (21).

**Corollary 1:** For large numbers \(K\) and \(M\), the average beamforming gain \(G = \frac{1}{M} \sum_{m=0}^{M-1} G_m\) in (21) could be represented as
\[
G = \frac{1}{M} \sum_{m=0}^{M-1} G_m \approx 1 - \gamma(B, f_c, P) \times \xi(r, \theta, D), \tag{24}
\]

where
\[
\gamma(B, f_c, P) = \frac{1 - \Xi_P(\frac{B}{2\pi r})}{3}, \tag{25}
\]
\[
\xi(r, \theta, D) = 1 - \frac{r \cos \theta}{D} \left(\pi \mathbb{1}_{2r \leq D} + \arctan \frac{Dr \cos \theta}{r^2 - \frac{1}{4} D^2}\right). \tag{26}
\]

Here, \(D = (KP - 1)d \approx KPD, \mathbb{1}_{2r \leq D} = 1\) if \(2r \leq D\), and \(\mathbb{1}_{2r \leq D} = 0\) otherwise.

**Proof:** The key to this proof lies in calculating the closeform expressions of \(\sum_m \frac{\epsilon_m^2}{(2\pi r)^2}\) and \(\sum \sin^2 \theta_k\). Firstly, due to the fact that \(1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\), we can represent \(\sum_m \frac{\epsilon_m^2}{(2\pi r)^2}\) as
\[
\sum_m \frac{\epsilon_m^2}{(2\pi r)^2} = \sum_{m=0}^{M-1} \left(\frac{2}{M-1} m - 1\right)^2 = \frac{M(M+1)}{3(M-1)} \tag{27}.
\]

Then for a large number \(M\), \(\sum_m \frac{\epsilon_m^2}{(2\pi r)^2}\) could be further approximated as \(\frac{M}{3}\).

As for the second summation \(\sum_k \sin^2 \theta_k\), it could be rewritten in an integral form for a large number \(K\):
\[
\sum_{k=0}^{K-1} \sin^2 \theta_k = \sum_{k=0}^{K-1} \frac{(r \sin \theta - \delta_k^P) Pd^2}{r^2 + (\delta_k^P Pd)^2 - 2 \delta_k^P Pd \sin \theta} \approx \int_{-\frac{K}{2}}^{\frac{K}{2}} \frac{(r \sin \theta - kPd)^2}{r^2 + (kPd)^2 - 2 kPd \sin \theta} \, dk \tag{28}
\]
\[
= K \frac{r \cos \theta}{PD} \arctan \left(\frac{Dk \cos \theta}{r^2 - \frac{1}{4} D^2}\right) - \frac{K^2}{2} \tag{28}.
\]

where \(D = (KP - 1)d \approx KPD\) for large \(KP\). Here, (a) comes from the indefinite integral \(\frac{1}{\sqrt{A^2 + B^2} + C^2} = \frac{d}{d^{1/2} + d} \arctan \left(\frac{AC + B}{\sqrt{A^2 + B^2} + C^2}\right)\), and (b) arises from a characteristic of the inverse tangent function:
\[
\arctan A + \arctan B = \begin{cases} \arctan \frac{A + B}{1 - AB} & (AB < 1) \\ \pi + \arctan \frac{A + B}{1 - AB} & (AB > 1) \end{cases}
\]
for \(A > 0\). Finally, by combining (21), (27), and (28), the conclusion of **Corollary 1** could be obtained.

**Corollary 1** yields several crucial conclusions. The beamforming gain loss of our PDF method arises from two factors: the loss caused by wideband effect \(\gamma(B, f_c, P)\) and the loss posed by the geometry \(\xi(r, \theta, D)\).

First, the geometry loss \(\xi(r, \theta, D)\) captures the degree of beam split effect with varying user locations. Due to the non-decreasing property of function \(\arctan()\), it is straightforward to prove that \(\xi(r, \theta, D)\) monotonically increases w.r.t \(\theta\) when \(r > \frac{1}{2} D\). This fact implies that a larger angle of arrival leads to a more significant beamforming gain loss caused by nearfield beam split, making it harder for our PDF method to compensate for this loss. Similar conclusions also appear in existing far-field beam split solutions. Moreover, \(\xi(r, \theta, D)\) could account for the geometry loss in both far-field and near-field regions, as it incorporates the distance parameter \(r\). A simple evidence is that when \(r \rightarrow +\infty\), \(\xi(r, \theta, D)\) tends to \(\sin^2 \theta\), which is exactly the geometry loss in far-field conditions provided in (21).

Second, \(\gamma(B, f_c, P)\) is induced by the loss of beam split within each sub-array. This is due to the employment of PS based frequency-independent beamforming for each sub-array. Fortunately, since the number \(P\) of a sub-array’s antennas is much less than the number \(N\) of the entire array’s antennas, this loss \(\gamma(B, f_c, P)\) can approach 0 by choosing an appropriate value of \(P\). To be specific, it can be easily proven that \(\gamma(B, f_c, P)\) is an increasing function w.r.t \(P\) when \(P \leq \frac{4Kd}{B}\). With a smaller \(P\), the impact of intra-array beam split is reduced. For instance, when \(P = 32, B = 5\) GHz, \(f_c = 100\) GHz, \(r = 10\) m, \(D = 0.5\) m, and \(\theta = \frac{\pi}{2}\), we have \(\gamma(B, f_c, P) \approx 0.081, \xi(r, \theta, D) \approx 0.7496\), and \(\gamma(B, f_c, P) \times \xi(r, \theta, D) \approx 0.9393\). This implies that more than 93% average beamforming gain is achievable by the proposed PDF method.
Algorithm 1 The Proposed PDF Algorithm

Require:
Channel matrix \( H_m \); the user locations \( \{(r_u, \theta_u)\}_{i=0}^{U-1} \); the total transmit power \( \rho \).

Ensure:
The digital beamformer \( D_m \) and the analog beamformer \( F_m \).

1: for \( u \in \{0, 1, \ldots, U - 1\} \) do
2: for \( k \in \{0, 1, \ldots, K - 1\} \) do
3: Determine the distance parameter of the \( k^{th} \) TTD element: \( r_{u,k}^{'} = -r_{u,k} = -\sqrt{r_u^2 + (\delta_k^j P d)^2 - 2\delta_k^j P d r_u \sin \theta_u} \)
4: Determine the phase shift of the \( k^{th} \) sub-array: \( \beta_{u,k}^{'} = -\sin \theta_{u,k} = -(-y_u \sin \theta_u - \delta_k^j P d) \)
5: Shift \( r_{u,k}^{'} \) by \( L = \min_k \{r_{u,k}\} \) to make them positive: \( r_{u,k}^{'} = L + r_{u,k}^{'} \)
6: end for
7: Build the beamforming vector connected to the \( u^{th} \) RF chain using (29)
8: end for
9: Build the analog beamformer: \( F_m \leftarrow [f_0, f_1, \ldots, f_{U-1}] \).
10: Calculate the digital beamformer by ZF: \( D_m \leftarrow F_m H_m^H(\sum_u F_m H_m^H)^{-1} \).
11: Normalize the digital beamformer to power \( \rho \): \( D_m \leftarrow \frac{1}{\sqrt{\|D_m\|^2}} D_m \).
12: return \( F_m \) and \( D_m \).

D. Extension to Multi-User Hybrid Beamforming

In this sub-section, we extend the proposed PDF method to multi-user hybrid beamforming systems in Section II. The same DPP architecture depicted in Fig. 4 is employed for each RF circuit. Specifically, recall that the introduction of TTD circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent. Specifically, recall that the introduction of TTD circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent. Therefore, we denote this new analog beamformer as circuits makes the analog beamformer frequency-dependent.

The Proposed Algorithm

1: Require:
2: Channel matrix \( H_m \); the user locations \( \{(r_u, \theta_u)\}_{i=0}^{U-1} \); the total transmit power \( \rho \).
3: Ensure:
The digital beamformer \( D_m \) and the analog beamformer \( F_m \).

4: 1: for \( u \in \{0, 1, \ldots, U - 1\} \) do
5: 2: for \( k \in \{0, 1, \ldots, K - 1\} \) do
6: 3: Determine the distance parameter of the \( k^{th} \) TTD element: \( r_{u,k}^{'} = -r_{u,k} = -\sqrt{r_u^2 + (\delta_k^j P d)^2 - 2\delta_k^j P d r_u \sin \theta_u} \)
7: 4: Determine the phase shift of the \( k^{th} \) sub-array: \( \beta_{u,k}^{'} = -\sin \theta_{u,k} = -(-y_u \sin \theta_u - \delta_k^j P d) \)
8: 5: Shift \( r_{u,k}^{'} \) by \( L = \min_k \{r_{u,k}\} \) to make them positive: \( r_{u,k}^{'} = L + r_{u,k}^{'} \)
9: 6: end for
10: Build the beamforming vector connected to the \( u^{th} \) RF chain using (29)
11: 7: end for
12: Build the analog beamformer: \( F_m \leftarrow [f_0, f_1, \ldots, f_{U-1}] \).
13: Calculate the digital beamformer by ZF: \( D_m \leftarrow F_m H_m^H(\sum_u F_m H_m^H)^{-1} \).
14: Normalize the digital beamformer to power \( \rho \): \( D_m \leftarrow \frac{1}{\sqrt{\|D_m\|^2}} D_m \).
15: return \( F_m \) and \( D_m \).


IV. EFFECTIVE RAYLEIGH DISTANCE

In the previous section, we assume that the user is positioned within the near-field range of the entire array while being in the far-field range of each sub-array. Therefore, it is essential to accurately identify the near-field ranges of a sub-array and the entire array. Traditionally, the classical Rayleigh distance is employed as a standard for quantifying the near-field range. However, our experiments show that the Rayleigh distance overestimates the actual near-field range. For example, when the array aperture is \( D = 0.384 \) m and the carrier is \( f_c = 100 \) GHz, the Rayleigh distance is around 98 m. Yet, the far-field wideband beamforming method [21] only exhibits a noticeable beamforming gain loss when the distance is less than 30 m. This fact implies that classical Rayleigh distance is derived by evaluating the largest phase error between planar wave and spherical wave, which does not directly affect the transmission rate. By contrast, the near-field effect has a direct impact on beamforming gain, which in turn plays a pivotal role in determining transmission rates. Therefore, it becomes apparent that a more precise metric for defining the near-field range in terms of beamforming gain is required.

Specifically, we first introduce the derivation of the classical Rayleigh distance by the evaluation of phase error. For ease of discussion, we only consider an arbitrary frequency \( f_m \) and an arbitrary user. Therefore, the subscript \( m \) and \( u \) is omitted in this section. Denote \( h(r, \theta) \) as the near-field channel as a function of user location \((r, \theta)\), of which the \( n^{th} \) entry is given by.

\[
|h(r, \theta)_n| = \left| ge^{-jkr_0^n} \right| = \left| ge^{-jkr_0^n} \right| = \left| ge^{-jkr_0^n} \right| = \left| ge^{-jkr_0^n} \right| = \left| ge^{-jkr_0^n} \right|
\]

where \( g \) denotes the path loss and \( k = \frac{2\pi}{\lambda} \) is the wavenumber. By utilizing the far-field approximation in (5), the far-field channel is expressed as \( H_{far}(r, \theta)_n = h(\infty, \theta)_n = h(\infty, \theta)_n = h(\infty, \theta)_n = h(\infty, \theta)_n = h(\infty, \theta)_n \).
Subsequently, the definition of Rayleigh distance is as follows [13]: if the distance \( r \) from user to BS exceeds the Rayleigh distance \( R \), then the largest phase error \( E(r) = \max_{n} E_n(r, \theta) \) is no more than \( \frac{\pi}{8} \). That is to say, once the largest phase error \( E(r) \) surpasses \( \frac{\pi}{8} \), the user is situated in the near-field region. To derive the close-form expression of \( E(r) \), the second-order Taylor expansion \((1 + x)^{\frac{1}{2}} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2\) is commonly used [31] to approximate the distance \( r^{(n)} \) as

\[
E_n(r, \theta) = |\angle[h(r, \theta)]_n - \angle[h_{\text{far}}(r, \theta)]_n| = |kr^{(n)} - k(r - \delta_N d \sin \theta)|. \tag{30}
\]

In the field of microwave and antenna, the approximation (31) is known as the Fresnel approximation [24]. Next, the phase error can be approximated as

\[
E_n(r, \theta) \approx k \frac{(\delta^{(n)} d)^2 \cos^2 \theta}{2r} \tag{31}
\]

In order for \( E(r) \leq \frac{\pi}{8} \), it is necessary to make \( r \geq \frac{2D^2}{\lambda} \). Consequently, the Rayleigh distance is given by

\[
R \approx \frac{2D^2}{\lambda}. \tag{33}
\]

On the other hand, we define a new effective Rayleigh distance via the evaluation of beamforming gain loss. To elaborate, the normalized coherence between the channel \( h(r, \theta) \) and its far-field approximation \( h_{\text{far}}(r, \theta) \) is characterized by

\[
\mu(r, \theta) = \frac{1}{|g|^2 N} |h^H(r, \theta) h_{\text{far}}(r, \theta)|. \tag{34}
\]

The coherence \( \mu(r, \theta) \) equivalents to the achievable beamforming gain at frequency \( f \) when the BS utilizes the far-field beamforming vector \( f = \frac{1}{|g| \sqrt{N}} h_{\text{far}}(r, \theta) \) to serve a user located at \( (r, \theta) \). Clearly, this beamforming gain would gradually decline when the user is moving close to BS and the near-field effect becomes remarkable. When the beamforming gain loss, denoted as \( 1 - \mu(r, \theta) \), exceeds a predefined threshold \( \Delta \), it indicates that the user has entered the near-field region.

Consequently, the boundary \( R_{\text{eff}} \), where \( 1 - \mu(R_{\text{eff}}, \theta) \) exactly equals to \( \Delta \), is defined as the effective Rayleigh distance. Notably, the direct influence of beamforming gain \( \mu(r, \theta) \) on the received signal power makes \( R_{\text{eff}} \) a more accurate metric for characterizing the near-field range in communication systems. **Lemma 3** gives out the close-form expression of effective Rayleigh distance \( R_{\text{eff}} \).

**Lemma 3**: We define the effective Rayleigh distance \( R_{\text{eff}} \) such that the inequality \( 1 - \mu(r, \theta) \geq \Delta \) always holds for \( 0 < r \leq R_{\text{eff}} \). Then, the value of \( R_{\text{eff}} \) is given by

\[
R_{\text{eff}} \approx C \Delta \cos^2 \theta \frac{2D^2}{\lambda}, \tag{35}
\]

where \( C = \frac{1}{4 \Delta} \) and \( \beta \Delta \) is the solution of the equation

\[
\frac{1}{\beta} \int_0^{\beta} e^{-j \pi t^2} dt = \Delta. \tag{36}
\]

**Proof**: To obtain the value of the effective Rayleigh distance, we need to derive the close-form expression of \( \mu(r, \theta) \). Based on the second-order Taylor expansion in (31), \( \mu(r, \theta) \) can be expressed as

\[
\mu(r, \theta) \approx \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{-j \pi (\alpha^{(n)} d)^2 \cos^2 \theta} \right| = \frac{1}{N} \left| \sum_{m=-\frac{1}{4} + \frac{1}{4N}}^{\frac{1}{4}} e^{-j \pi m^2 (N d)^2 \cos^2 \theta} \right|. \tag{36}
\]

Notice that the operator \( \sum_{m=-\frac{1}{4} + \frac{1}{4N}}^{\frac{1}{4}} \) performs the summation over \( m = -\frac{1}{2} + \frac{1}{2N}, -\frac{1}{2} + \frac{3}{2N}, \ldots, \frac{1}{2} - \frac{1}{2N} \).

Let’s define \( \zeta = \frac{N^2 \Delta \cos \theta}{\lambda} \) for brevity. Since the number of antennas \( N \) is quite large, (36) can be represented in an integral form as

\[
\mu(r, \theta) \approx \frac{1}{\sqrt{2\pi\zeta}} \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j \pi \zeta m^2 \cos \theta} dm = 2 \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j \pi \zeta m^2 \cos \theta} dm. \tag{37}
\]

Additionally, we introduce the variable transformation: \( \frac{1}{2} t^2 = m^2 \zeta \). Then, \( \mu(r, \theta) \) can be rewritten as

\[
\mu(r, \theta) = \frac{2}{\sqrt{2\pi\zeta}} \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j \sqrt{2\pi \zeta} t^2} dt = G(\beta), \tag{38}
\]

where \( G(\beta) = \int_0^\beta e^{-j \sqrt{2\pi} t^2} dt / \beta \) and \( \beta = \frac{\sqrt{2\pi\zeta}}{\sqrt{2\pi\zeta}} = \sqrt{\frac{\lambda^2}{N^2 \Delta \cos^2 \theta}} = \sqrt{\frac{D^2 \cos^2 \theta}{2\lambda N^2 \Delta}} \). It is clear from (38) that the coherence heavily relies on the characteristics of the function \( G(\beta) \). Fortunately, \( G(\beta) \) does not contain any parameters, allowing us to obtain its numerical result via offline integration.
As illustrated in Fig. 6, the function $G(\beta)$ shows a significant downward trend w.r.t $\beta$. Therefore, to make the beamforming gain loss $1 - \mu(r, \theta) = 1 - G(\beta)$ larger than the threshold $\Delta$, we need $\beta \geq \beta_\Delta$, where $G(\beta_\Delta) = 1 - \Delta$.

In the end, due to the relationship $\beta = \sqrt{\frac{2\Delta \cos \theta}{2\lambda r^2}}$, the near-field region is determined by $r < \frac{1}{2\lambda} \sqrt{\frac{D \cos \theta}{2\Delta}}$, giving rise to the result $R_{\text{eff}} = \frac{1}{2\lambda} \sqrt{\frac{D \cos \theta}{2\Delta}}$.

**Lemma 3** offers a comprehensive approach to compute $R_{\text{eff}}$. To illustrate, let’s consider a simple example where $D = 5\%$. We could solve the equation $\frac{1}{2\lambda} \int_{\theta(\Delta)} \sqrt{\frac{D \cos \theta}{2\Delta}} dt = 0.05$ via the Newton method and obtain $\beta_\Delta = 0.8257$. Hence, the effective Rayleigh distance is evaluated as $R_{\text{eff}} = 0.367 \cos \theta \frac{2D^2}{\lambda}$.

It is evident from (33) and (35) that effective Rayleigh distance needs two more variables compared to Rayleigh distance, i.e., the constant $C_\Delta$ related to beamforming gain loss and the angle of arrival $\theta$. These two variables enable effective Rayleigh distance to accurately capture where far-field beamforming are not applicable, and thus make it a more accurate metric for quantifying near-field region. In Section V, the accuracy of effective Rayleigh distance will be verified through simulation.

### A. Discussion on the Fresnel Approximation

In deriving the effective Rayleigh distance, the Fresnel approximation (31) is employed. As indicated in [13], the approximation (31) is accurate when the distance $r$ is larger than the “Fresnel distance” $0.5 \sqrt{\frac{D \cos \theta}{\lambda}}$. To validate the rationality of the Fresnel approximation, we would like to show that the effective Rayleigh distance $0.367 \cos \theta \frac{2D^2}{\lambda}$, with $\Delta = 5\%$, is much larger than the Fresnel distance. Take it into account that the section range of a typical cell is around $2\pi$, thus $\theta$ is restricted between $-\frac{\pi}{3}$ and $\frac{\pi}{3}$. Accordingly, $0.367 \cos \theta \frac{2D^2}{\lambda} \geq 0.5 \sqrt{\frac{D \cos \theta}{\lambda}}$ is equivalent to $N > 15.8490$. Given that an extremely large antenna array have hundreds or thousands of antennas, which is greater largely than 15.8490, the effective Rayleigh distance is much longer than the Fresnel distance, resulting in the accuracy of approximation (31).

### B. Discussion on the Piecewise-Far-Field Approximation

The effective Rayleigh distance is capable of verifying the accuracy of the piecewise-far-field approximation as well. Recall that the user should locate in the far-field region of each sub-array, i.e., $r_k$ must be larger than $0.367 \cos \theta \frac{2(P\ell - 1)^2}{\lambda l}$ with $\Delta = 5\%$. Take a small sub-array configuration as an example: $P = 32$ and $f_c = 100$ GHz. The effective Rayleigh distance per sub-array is upper bounded by $0.367 \cos \theta \frac{2(P\ell - 1)^2}{\lambda l} \leq 0.367 \frac{2(P\ell - 1)^2}{\lambda l} = 0.5286m$. In this context, as long as the user-to-sub-array distance $r_k$ is larger than 0.5286m, a common situation in mobile communications, each sub-array’s channel can be precisely modeled as far-field. Therefore, under a small sub-array configuration, the piecewise-far-field approximation is accurate.

3The function $G(\beta)$ is applied in our another paper [32] as well to evaluate the quasi-orthogonality of near-field channels, while $G(\beta)$ is used to derive the effective Rayleigh distance in this paper.

### C. Discussion on the Number of Antennas per Sub-Array

In this sub-section, by combining **Lemma 1**, **Lemma 2**, and **Lemma 3**, the value of the essential parameter $P$, the number of antennas per sub-array, is designed.

The value of $P$ needs to meet three key requirements. First, as stated in **Lemma 1**, $|\epsilon_m| \leq \frac{B}{2\lambda}$ holds for $\forall m$. Owing to the fact that $\max|\epsilon_m| = \frac{B}{2\lambda}$, we have $P \leq \frac{4\lambda}{B}$.

Next, we made an assumption that the user is located in the far-field region of each sub-array. We evaluate the effective Rayleigh distance at the center frequency $f_c$ with a wavelength $\lambda_c$. Suppose the user’s activity range is $r \in [\rho_l, \rho_u]$ and $\theta \in [-\theta_h, \theta_h]$, where $\rho_l$ and $\rho_u$ can be regarded as the least allowable distance from user to BS and the cell radius, and $\theta_h$ refers to the sector range of a cell. Applying **Lemma 3**, the effective Rayleigh distance of a sub-array is

$$C_\Delta \cos \theta \frac{2(P\ell)^2}{\lambda_c} = \frac{1}{2} C_\Delta \cos \theta \frac{2P^2}{\lambda_c} \leq \frac{1}{2} C_\Delta \cos \theta \frac{2P^2}{\lambda_c}$$

Then, to keep the user consistently outside the region bounded by $C_\Delta \cos \theta \frac{2P^2}{\lambda_c}$, we can let $\rho_l \geq \frac{1}{2} C_\Delta \cos \theta \frac{P^2}{\lambda_c}$ and arrive at the condition $P \leq \frac{2\rho_l}{C_\Delta \cos \theta \frac{P^2}{\lambda_c}}$.

Finally, in order to guarantee the performance of PDF method, we would like to design $P$ such that the least average beamforming gain in **Corollary 1** is greater than a predefined threshold $\delta$. This requirement can be formulated as follows:

$$\min_{r,\theta} G(r, f_c, P) \max_{r,\theta} \xi(r, \theta, D) \geq \delta.$$  

$$\Rightarrow \Xi_P \left(\frac{B}{2f_c}\right) \geq 1 - \frac{3(1 - \delta)}{\max_{r,\theta} \xi(r, \theta, D)}$$

Notice that $\max_{r,\theta} \xi(r, \theta, D) = \max_{r,\theta} \xi(r, \theta, D)$ given that $\xi(r, \theta, D)$ is a decreasing function w.r.t $|\theta|$. Furthermore, we can utilize the gradient ascend method to solve $\max_{r,\theta} \xi(r, \theta, D)$, and thereafter employ the Newton method to attain $P_3$ from the equation $\Xi_P \left(\frac{B}{2f_c}\right) = 1 - \frac{3(1 - \delta)}{\max_{r,\theta} \xi(r, \theta, D)}$.

Finally, taking into consideration the monotonic decreasing property of function $\Xi_P \left(\frac{B}{2f_c}\right)$ w.r.t $P$, we can draw the conclusion $P \leq P_3$.

As a result, applying the three requirements above, the number of antennas per sub-array $P$ should satisfy

$$P \leq \min \left\{ \frac{4f_c}{B}, \sqrt{\frac{2\rho_l}{C_\Delta \cos \theta \frac{P^2}{\lambda_c}, P_3}} \right\}.$$  

For instance, considering the following parameters: $N = 400$, $B = 5 \text{ GHz}$, $f_c = 100 \text{ GHz}$, $\rho_l = 1 \text{ m}$, $\theta_h = \frac{\pi}{3}$, $\Delta = 5\%$, and $\delta = 90\%$, we have $\frac{4f_c}{B} = 80$, $\sqrt{\frac{2\rho_l}{C_\Delta \cos \theta \frac{P^2}{\lambda_c}}} \approx 43$ and $P_3 \approx 42$. Therefore, the number of antennas per sub-array can be $P = 40$, meaning that a 400-antenna array just needs $K = \frac{400}{40} = 10$ TTDs to alleviate the near-field beam split effect. Moreover, the lower bound of $P$ is exactly 1, because a reduced $P$ leads to an increased deployment of TTD units, enabling the PDF method to achieve more flexible frequency-dependent beamforming. In this context, the beamforming gain is increasingly improved with the reduction of $P$.

### V. Simulation Results

In this section, numerical results are provided to demonstrate the performance of the proposed PDF method and the
TABLE I
SYSTEM CONFIGURATIONS

| Configuration                  | Value         |
|-------------------------------|---------------|
| The number of the BS antennas $N$ | 256           |
| The number of Users $U$        | 1, 4          |
| The center frequency $f_c$     | 100 GHz       |
| The bandwidth $B$              | 5 GHz         |
| The number of subcarriers $M$  | 256           |
| The user’s activity range $[\rho_L, \rho_H]$ | [1 m, 100 m] |
| The sector range of a cell $\theta_b$ | $\pi/3$      |
| Threshold parameter $\Delta$   | 5%            |
| Threshold parameter $\delta$   | 90%           |
| The number of a sub-array’s antennas $P$ | 32           |
| The number of TTDs $K$         | 8             |

Fig. 7. Beamforming gain per sub-carrier w.r.t direction.

accuracy of effective Rayleigh distance. The default simulation parameters are presented in Table I unless particularly specified.

A. Beamforming Gain

We begin with comparing the beamforming gain performance for single-user scenarios, i.e., $U = 1$.

Fig. 7 presents the beamforming gain performance for different sub-carriers as a function of direction. Here, the user is located at $(r, \theta) = (2 \text{ m}, \frac{\pi}{8})$, and we evaluate the beamforming gain for various frequencies and physical directions. Fig. 7(a) showcases the beamforming gain achieved by traditional near-field narrowband beamfocusing method [23], while Fig. 7(b) presents the results of the proposed PDF method. Let $f_L$, $f_c$, and $f_H$ be the lowest, the center, and the highest frequency, respectively. For the traditional narrowband beamfocusing method, the near-field beam split effect causes the beams at $f_L$, $f_c$, and $f_H$ to be focused on different locations, leading to a significant beamforming gain loss at $f_L$ and $f_H$.

However, as illustrated in Fig. 7 (b), the proposed PDF method can effectively focus the energy of the beams at $f_L$, $f_c$, and $f_H$ on the desired user location. Besides, more than 95% beamforming gain on the user location is achieved for $f_L$ and $f_H$. Therefore, the proposed PDF method is able to effectively mitigate the near-field beam split effect.

Fig. 8 illustrates the average beamforming gain performance w.r.t direction $\theta$. In this context, the distance $r$ is fixed as 10 m and the direction $\theta$ spans from $-\theta_b$ to $\theta_b$. We can observe that the analysis result (24) for average beamforming gain is quite close to the real average beamforming gain achieved by the PDF method. With the increment of $|\theta|$, the average beamforming gain achieved by both the PDF method and the narrowband beamfocusing method declines. This is attributed to the fact that the beam split effect becomes more significant with larger $|\theta|$. Nevertheless, our PDF method could remain more than $\delta = 90\%$ average beamforming gain over $\theta \in [-\theta_b, \theta_b]$, which is consistent with our discussion on the number of a sub-array’s numbers in section IV-C.

Moreover, Fig. 9 shows the average beamforming gain performance w.r.t bandwidth $B$. In this simulation, the user is located at $(r, \theta) = (10 \text{ m}, \frac{\pi}{4})$, and the bandwidth increases from 100 MHz to 10 GHz. It is clear from Fig. 9 that our PDF method could remarkably enhance the near-field beamforming capability by offering near optimal average beamforming gain. For instance, when $B = 5 \text{ GHz}$, around 3 times higher average beamforming gain is reaped by the PDF method than narrowband beamfocusing. Besides, it is notable that when the bandwidth is around 10 GHz, our analysis results slightly differ from the real performance of PDF. This is because, with larger bandwidth $B$, the precision of quadratic fitting in (23) gets reduced, leading to an error of the analytical beamforming gain.
as the performance upper bound. All phase shifters in Fig. 1 are replaced with TTD circuits and the optimal beamforming achieved by fully TTD arrays, where an algorithm tailored for solving far-field beam split in [21]. Last, hybrid beamforming designed in [19], and the TTD-DPP focusing method in [23], the PE-AltMin algorithm for wideband large-scale fading is compensated by transmit power control.

To explicitly illustrate the impact of the near-field effect, we keep the SNR as 10 dB for different distances, where the number of users is set as $N = 31$.

\begin{align}
\text{SE} &= \frac{1}{M} \sum_{m=0}^{M-1} \sum_{u=0}^{U-1} \log_2 \left( 1 + \frac{\| h_{u,m}^T F_m d_{u,m} \|^2}{\sum_{v \neq u} \| h_{u,m}^T F_m d_{v,m} \|^2 + \sigma^2} \right) \tag{41}
\end{align}

is evaluated to compare different beamforming algorithms, where $d_{u,m}$ represents the $u$th column of $D_m$. The path gains $g_m$ are generated from the Complex Gaussian distribution $CN(0, 1)$. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \frac{P}{\sigma^2}$.

The compared algorithms include the narrowband beamforming method in [23], the PE-AltMin algorithm for wideband hybrid beamforming designed in [19], and the TTD-DPP algorithm tailored for solving far-field beam split in [21]. Last, the optimal beamforming achieved by fully TTD arrays, where all phase shifters in Fig. 1 are replaced with TTD circuits and $f_{u,m}$ is constructed as $a_m^*(r_u, \theta_u)$ in Algorithm 1, is employed as the performance upper bound.

To begin with, the spectral efficiency w.r.t the distance $r$ is depicted in Fig. 10. The number of users is set as $U = 4$. The signal-to-noise ratio (SNR) is set as follows: $U = 4$; $K = 8$; $\text{SNR} = 10$ dB; the distances $\{u\}_{u=0}^{U-1}$ and directions $\{\theta_u\}_{u=0}^{U-1}$ are sampled from the uniform distribution $U(1, 30)$ and $U(-\frac{\pi}{2}, \frac{\pi}{2})$, respectively. The spectral efficiency is obtained through 10000 times Monte-Carlo simulations. To elaborate, Fig. 11 can be divided into three regions according to the number of BS antennas. When $N < 50$, the array aperture is small, so the user is located in the far-field area and the beam split effect is negligible. In this context, all algorithms could achieve good performance. Next, when $50 < N < 100$, the far-field beam split effect appears, leading to a severe degradation to the narrowband beamforming and PE-AltMin methods. On the other hand, since TTD-DPP and PDF can effectively alleviate far-field beam split, both of them could remain near-optimal average rate performance. Finally, when the number of BS antennas is further increased such that $N > 100$, the array aperture is quite large so that the near-field beam split effect is negligible.
the optimal beamforming based on the fully TTD (FTTD) based on conventional hybrid beamforming (HB) architecture, PE-AltMin [19] and narrowband beamfocusing [23] methods and analog circuits. The compared benchmarks include the total power consumed by the baseband digital processing defined as the ratio between the spectral efficiency and the

\[ \frac{\text{Spectral Efficiency}}{\text{Power Consumption}} = \frac{C}{P} \]

Energy Efficiency

priority of the PDF method. To address this challenge, we first propose a piecewise-far-field model to approximate the near-field model with high accuracy. Applying this model, a PDF method is proposed to efficiently alleviate the near-field beam split effect through the joint manipulation of PSs and TDs. Moreover, we define a new metric called as “effective Rayleigh distance” by evaluating the beamforming gain, which is more accurate in quantifying the near-field range than the classical Rayleigh distance for practical communications. Finally, numerical results are provided to demonstrate the effectiveness of our work.

The discussion on the near-field beam split effect and our PDF method provide new vision to ELAA beamforming. Besides, our proposed effective Rayleigh distance offers a new way to evaluate the near-field range. For future works, people could investigate the near-field beam split effect in more general situations, such as multi-antenna users, uniform planar arrays, reconfigurable intelligent surfaces [34], and so forth. In addition, extending the effective Rayleigh distance to more applications deserves in-depth study as well.

C. Energy Efficiency

Fig. 13 provides an energy efficiency comparison when \( U = N_{RF} \) varies from 1 to 8. The energy efficiency is defined as the ratio between the spectral efficiency and the total power consumed by the baseband digital processing and analog circuits. The compared benchmarks include the PE-AltMin [19] and narrowband beamfocusing [23] methods based on conventional hybrid beamforming (HB) architecture, the optimal beamforming based on the fully TTD (FTTD) arrays, and the TTD-DPP and PDF methods built on the DPP architecture. The power consumption of these three architectures, denoted by \( P_{HB} \), \( P_{FTTD} \), and \( P_{DPP} \) are given by [21]

\[
P_{HB} = P_t + P_B + N_{RF}P_{RF} + N_{RF}N_{PS},
\]

\[
P_{FTTD} = P_t + P_B + N_{RF}P_{RF} + N_{RF}N_{PS},
\]

\[
P_{DPP} = P_t + P_B + N_{RF}P_{RF} + N_{RF}N_{PS} + N_{RF}KP_{TTD},
\]

where \( P_t \) and \( P_B \) represent the transmission power and digital processing power, and \( P_{RF} \), \( P_{PS} \), and \( P_{TTD} \) denote the power consumption of each RF chain, phase shifter, and TTD element. The following typical values are adopted: \( P_t = 30 \text{ mW} \) [21], \( P_B = 200 \text{ mW} \) [21], \( P_{RF} = 250 \text{ mW} \) [33], \( P_{PS} = 30 \text{ mW} \) [33], and \( P_{TTD} = 100 \text{ mW} \) [21]. The other settings are as follows: \( SNR = 5 \text{ dB} \); \( K = 8 \); \( N = 256 \); \( r_u \sim U(1, 30, 30) \); \( \theta_u \sim U(-\frac{\pi}{3}, \frac{\pi}{3}) \). It is clear from Fig. 13 that even though the proposed beamforming has the highest spectral efficiency, its energy efficiency is pretty low because of the large number of high-power TTDs used, i.e., \( UN \). In contrast, we can observe that the proposed PDF achieves much higher energy efficiency than all compared benchmarks. This observation is attributed to the fact that the PDF method can efficiently overcome the near-field beam-split effect with a quite small number of expensive TTDs, i.e., \( UK \ll UN \), which further demonstrates the efficacy of the proposed PDF method.



**VI. Conclusion**

In this paper, we reveal an important challenge for future ELAA communications, i.e., the near-field beam split effect. To address this challenge, we first propose a piecewise-far-field model to approximate the near-field model with high accuracy. Applying this model, a PDF method is proposed to efficiently alleviate the near-field beam split effect through the joint manipulation of PSs and TDs. Moreover, we define a new metric called as “effective Rayleigh distance” by evaluating the beamforming gain, which is more accurate in quantifying the near-field range than the classical Rayleigh distance for practical communications. Finally, numerical results are provided to demonstrate the effectiveness of our work.

The discussion on the near-field beam split effect and our PDF method provide new vision to ELAA beamforming. Besides, our proposed effective Rayleigh distance offers a new way to evaluate the near-field range. For future works, people could investigate the near-field beam split effect in more general situations, such as multi-antenna users, uniform planar arrays, reconfigurable intelligent surfaces [34], and so forth. In addition, extending the effective Rayleigh distance to more applications deserves in-depth study as well.

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