Associated $J/\psi + \gamma$ production as a probe of the polarized gluon distribution

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Abstract

Associated production of $J/\psi$ and a $\gamma$ has recently been proposed as a clean probe of the gluon distribution. The same mechanism can be used to probe the polarized gluon content of the proton in polarized proton-proton collisions. We study $J/\psi + \gamma$ production at both polarized fixed target and polarized collider energies.
Interest in high energy spin physics has been recently revived with the result from (and interpretations thereof) the EMC collaboration\cite{1} on polarized $\mu - p$ scattering. Processes in polarized $pp$ collisions (such as achievable at an upgraded Fermilab fixed target facility or at a polarized collider \cite{2}) sensitive to the polarized gluon content of the proton, such as jets\cite{3, 4, 5}, direct photons\cite{5, 6, 7}, and heavy quark production\cite{8}, have been discussed. Another intriguing suggestion, due to Cortes and Pire\cite{9}, is to consider $\chi_2(c\bar{c})$ production where the dominant lowest-order subprocess would be $gg \rightarrow \chi_2$. The partonic level asymmetries for $\chi_2/\chi_0$ production have been calculated in the context of potential models\cite{11} and are large. Low transverse momentum quarkonium production in polarized $pp$ collisions using other methods has also been considered\cite{8, 12} as has high $p_T \psi$ production\cite{13}.

In all cases of charmonium production, the experimental signal is $\ell^+\ell^-$ ($\ell = e$ or $\mu$) with the lepton-lepton invariant mass giving the $J/\psi$ mass, since $\chi_J$ can decay radiatively to $J/\psi + \gamma$, and the $J/\psi$ signature is quite clean. As has been noted\cite{14}, the question of extracting the gluon distribution is made less clean by the multitude of contributing processes, \textit{e.g.}:

\begin{align}
g + g & \rightarrow \chi_{0,2} \\
g + g & \rightarrow \chi_J + g \\
q + g & \rightarrow \chi_{0,2} + q \\
q + \bar{q} & \rightarrow \chi_{0,2} + g \\
g + g & \rightarrow J/\psi + g \\
g + g & \rightarrow b(\rightarrow J/\psi + X) + \bar{b} \\
q + \bar{q} & \rightarrow b(\rightarrow J/\psi + X) + \bar{b}.
\end{align} \hfill (1)
The simplicity of the Cortes and Pire idea is now gone. A full $\mathcal{O}(\alpha_s^3)$ calculation of
the spin-dependent production of $\chi_J$ is necessary. At low $p_T$, $\chi_J$ production will also
involve $q + g$ and $q + \bar{q}$ initial states, while at high $p_T$ in addition the $2 \rightarrow 2$ kinematics
make the extraction of parton distribution functions less direct. Furthermore, a very
careful calculation is required because even processes with small cross section can have
a large effect on the asymmetry. The extraction of $\Delta g(x, Q^2)$ using inclusive $J/\psi$ will
be a challenge.

Recently, $J/\psi$ produced in association with a $\gamma$ has been proposed as a clean
channel to study the gluon distribution at hadron colliders\textsuperscript{[15]}. The radiative $\chi_J$
decays can produce $J/\psi$ at both low and high $p_T$, but the photon produced will be
soft ($E \sim O(400 \text{ MeV})$). If we insist that the experimental signature consist of a $J/\psi$
and $\gamma$, with large but equal and opposite $p_T$ there is only one production mechanism

$$ g + g \rightarrow J/\psi + \gamma. $$

(2)

Following Ref. \textsuperscript{[15]}, this mechanism has been proposed in Ref. \textsuperscript{[16]} to study the po-
larized gluon distribution in polarized fixed target experiments; we perform a more
detailed analysis, including the analysis of this mechanism at the Brookhaven Rela-
tivistic Heavy Ion Collider (RHIC) at both 50 GeV and 500 GeV center of mass energy
and at the Superconducting Super Collider (SSC). Polarized proton-proton operation
is being considered for RHIC, for at least several months data collection, while the
tunnel design of the SSC has been modified for the possible future inclusion of the
Siberian Snakes needed for polarized proton-proton mode. Also, we list the full set of
helicity amplitudes for this process, explicitly stating the Lorentz frame in which the
$J/\psi$ helicities are given.
The full helicity amplitudes for $g + g \rightarrow J/\psi + \gamma$ can be calculated following the approach of Gastmans, Troost and Wu\cite{17}, with the addition of explicit helicity polarization vectors for the $J/\psi$. A convenient set of polarization vectors can be found in Böhm and Sack\cite{18}. These polarization vectors reduce to the usual massive vector boson $(+, -, 0)$ polarization vectors in the parton center of mass frame, and so, although the expressions for the helicity amplitudes have Lorentz invariant form, the $(+, -, 0)$ only refer to the $J/\psi$ helicity in this one particular frame. We find only one independent helicity amplitude ($M(++, ++)$, where the ‘++,++’ refer to the helicity of $g_1g_2, \gamma J/\psi$ respectively), and the remaining 5 non-zero helicity amplitudes can be found by crossing and parity symmetries:

$$M(++, ++) = M(−−, −−) = C\frac{\hat{s}(\hat{s} − M^2)}{(\hat{s} − M^2)(\hat{t} − M^2)(\hat{u} − M^2)}$$

$$M(−+, −+) = M(−+, ++) = C\frac{\hat{u}(\hat{u} − M^2)}{(\hat{s} − M^2)(\hat{t} − M^2)(\hat{u} − M^2)}$$

$$M(−+, −−) = M(−+, −+) = C\frac{\hat{t}(\hat{t} − M^2)}{(\hat{s} − M^2)(\hat{t} − M^2)(\hat{u} − M^2)}$$

where $C = \frac{4e_q^2g_s^2R(0)M\delta^{ab}}{\sqrt{3\pi}M}$. Here, $M$ is the $J/\psi$ mass, $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables, $R(0)$ is the radial wavefunction at the origin of the $c\bar{c}$ in the $J/\psi$ and $a,b$ are the color indices of the incident gluons. Thus, the (spin and color) summed and averaged matrix element squared can be found\cite{15}:}

$$|M(g + g \rightarrow J/\psi + \gamma)|^2 = \frac{(16\pi)^2\alpha\alpha_s^2M|R(0)|^2}{27}\left[\frac{\hat{s}^2}{(\hat{t} − M^2)^2(\hat{u} − M^2)^2} + \frac{\hat{t}^2}{(\hat{u} − M^2)^2(\hat{s} − M^2)^2} + \frac{\hat{u}^2}{(\hat{s} − M^2)^2(\hat{t} − M^2)^2}\right].$$

$|R(0)|^2$ can be related to the leptonic width of the $J/\psi$:

$$\Gamma(J/\psi \rightarrow e^+e^-) = \frac{16\alpha^2}{9M^2}|R(0)|^2 = 4.72 \text{ keV}$$

$$|R(0)|^2 = 0.48 \text{ GeV}^3.$$
We are interested in the longitudinal spin-spin asymmetry, defined as:

\[ A_{LL} = \frac{\sigma(++) - \sigma(+-)}{\sigma(++) + \sigma(+-)} \]  

(6)

where \( \sigma(++) \) (\( \sigma(+-) \)) is the cross section for the collision of 2 protons with the same (opposite) helicities. This can be calculated in the parton model,

\[ A_{LL} \sigma = \int dx_1 \, dx_2 \, \hat{a}_{LL} \, \hat{\sigma} \, \Delta g(x_1, Q^2) \, \Delta g(x_2, Q^2) \]  

(7)

where \( \hat{\sigma} \) is the parton level cross section (related to \( |M|^2 \) given earlier), \( \Delta g(x, Q^2) \) is the polarized gluon distribution in the proton (\( = (g^+(x, Q^2) - g^-(x, Q^2)) \)) where \( g^+(x, Q^2) \) \( (g^-(x, Q^2)) \) is the distribution for gluons with the same (opposite) helicity as that of the proton) and \( \hat{a}_{LL} \) is the parton level asymmetry

\[ \hat{a}_{LL} = \frac{\hat{\sigma}(++) - \hat{\sigma}(+-)}{\hat{\sigma}(++) + \hat{\sigma}(+-)}. \]  

(8)

Given the known helicity amplitudes for this process, the parton level asymmetry is simply

\[ \hat{a}_{LL} = \frac{s^2(\hat{s} - M^2)^2 - t^2(\hat{t} - M^2)^2 - u^2(\hat{u} - M^2)^2}{s^2(\hat{s} - M^2)^2 + t^2(\hat{t} - M^2)^2 + u^2(\hat{u} - M^2)^2}. \]  

(9)

Measurable quantities of interest are the \( p_T \) distribution and the joint \( p_T-y_1-y_2 \) distribution with \( y_1 = y_2 = 0 \), where \( y_{1(2)} \) is the rapidity of the \( \gamma \) \( (J/\psi) \). In the latter case, both partons have the same Bjorken-\( x \) (which is a function of \( p_T \) only). The corresponding asymmetries are given by:

\[ A_{LL}^1 = \frac{\sigma(++) - \sigma(+-)}{\sigma(++) + \sigma(+-)} \]  

\[ A_{LL}^2 = \frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)} \]  

\[ A_{LL}^3 = \frac{\left. \frac{d\sigma(++)}{dp_T \, dy_1 \, dy_2} \right|_{y_1 = y_2 = 0} - \left. \frac{d\sigma(+-)}{dp_T \, dy_1 \, dy_2} \right|_{y_1 = y_2 = 0}}{\left. \frac{d\sigma(++)}{dp_T \, dy_1 \, dy_2} \right|_{y_1 = y_2 = 0} + \left. \frac{d\sigma(+-)}{dp_T \, dy_1 \, dy_2} \right|_{y_1 = y_2 = 0}}. \]  

(10)
Note that $A_{LL}^3$ is proportional to $[\Delta g(x(p_T), Q^2)]^2$. Another interesting theoretical concept (though not measurable experimentally) is the average $\hat{a}_{LL}$, or ‘resolving power’. It is defined in the following way

$$\langle \hat{a}_{LL} \rangle \sigma = \int dx_1 \int dx_2 \hat{a}_{LL} \hat{\sigma} g(x_1, Q^2) g(x_2, Q^2).$$

(11)

As we wish to determine if a given experimental scenario can shed light on the size of the polarized gluon in the proton, we need, in addition to calculating the asymmetry, to estimate the experimental uncertainty in the asymmetry. We will approximate the uncertainty by the statistical uncertainty, since ratios of cross sections should be relatively free of systematic uncertainties. The statistical uncertainty in the measurement of an asymmetry is given by $\delta A$, where

$$\delta A = \frac{\sqrt{1 - A^2}}{\sqrt{N}}$$

and $N$ is the number of events.

We examine this process in several different experimental settings. First, we consider an hypothetical fixed target experiment and to be specific, take the proton beam energy to be 800 GeV (such as would exist at the upgraded Fermilab fixed target facility). In order to estimate the luminosity possible at such an experiment, we must make some assumptions. First, the Main Injector at Fermilab can provide $\sim 10^{14}$ (unpolarized protons)/sec, with a 65% duty cycle[19]. We’ll assume a one month run, at a much reduced proton rate (say, a factor of 100), combined with a small polarized gas ($H_2$) jet target (approximately 1 cm long). This will give, we think, a very conservative estimate of $\int \mathcal{L} dt = 50 \text{ pb}^{-1}$. We place no cuts on the rapidity of the photon or $J/\psi$, nor on the $p_T$ of the photon or leptons. We find a cross section of approximately
200 pb, most of which is at low $p_T$. The resolving power (or average $\hat{a}_{LL}$) is found to be about 28%. We use the polarized distributions of Bourrely, Guillet and Chiappetta[20]. They provide 2 sets of distributions, one with a large polarized gluon distribution and small polarized strange quark distribution (we’ll refer to it as the set BGC0) and one with a moderately large polarized gluon and moderately large polarized strange quark distribution (we’ll refer to this set as BGC1). The $p_T$ distribution is shown in Figure 1a (in cross section) and in Figure 1b (in $A_{LL}^2$). We were also interested the asymmetry $A_{LL}^3$, (technically, instead of taking $y_1$ and $y_2$ derivatives, we bin the events in the usual way, displaying the contents of the bin with $-0.1 \leq y_1, y_2 \leq 0.1$). The results are shown in Figure 3a (distribution in cross section) and 3b ($A_{LL}^2$ vs. $p_T$). We present in Table 1 the total number of events expected (at all $p_T$ and $y_{1,2}$ consistent with our cuts) as well as the ‘resolving power’ and asymmetry $A_{LL}^1$ and an estimate of the statistical uncertainty, $\delta A_{LL}^1$. We also list the number of events in a single $p_T$ bin ($p_T$ given in the table caption), and $A_{LL}^2$ and $\delta A_{LL}^2$ for that particular $p_T$ bin. Finally, we present the number of events in the same $p_T$ bin, further restricting the events to lie within $|y_{1,2}| \leq 0.1$, and the value of $A_{LL}^3$ and $\delta A_{LL}^3$ in the particular $p_T$ bin. These are representative results. Higher statistics can be obtained by the inclusion of all $p_T$ bins.

At this point, we would like to further address the work of Ref. [10]. The large asymmetries shown are surprising, and in our opinion not correct. The parton level asymmetry, making the following replacements for $\hat{t}$ and $\hat{u}$ (i.e. working in the parton center of mass frame):

\[
\hat{t} = -\frac{1}{2}(\hat{s} - M^2)(1 - \cos \theta)
\]
\[
\hat{u} = -\frac{1}{2}(\hat{s} - M^2)(1 + \cos \theta)
\]
reduces to
\[
\hat{a}_{LL} = \frac{1 - \frac{1}{8}[(1 + 6 \cos^2 \theta + \cos^4 \theta) + \frac{2M^2}{s}(1 - \cos^4 \theta) + \frac{M^4}{s^2}(1 - \cos^2 \theta)^2]}{1 + \frac{1}{8}[(1 + 6 \cos^2 \theta + \cos^4 \theta) + \frac{2M^2}{s}(1 - \cos^4 \theta) + \frac{M^4}{s^2}(1 - \cos^2 \theta)^2]}.
\]  

(14)

Here \(\cos \theta\) is measured in the parton center of mass frame. It is obvious that for \(\cos \theta = \pm 1\), \(\hat{a}_{LL}\) is a minimum (actually zero), and so, for any \(\hat{s}\), the maximum of \(\hat{a}_{LL}\) should be at \(\cos \theta = 0\). In this limit, the asymmetry reduces to
\[
\hat{a}_{LL}(\cos \theta = 0) = \frac{1 - \frac{1}{8} \left( \frac{\hat{s} + M^2}{\hat{s}} \right)^2}{1 + \frac{1}{8} \left( \frac{\hat{s} + M^2}{\hat{s}} \right)^2}.
\]

(15)

Two further limiting cases are possible, namely production at threshold (\(\hat{s} = M^2\)) which gives \(\hat{a}_{LL} = \frac{1}{3}\) and production at very high energy (\(\hat{s} \rightarrow \infty\)) which gives \(\hat{a}_{LL} = \frac{7}{9}\). For \(\sqrt{\hat{s}} = \sqrt{s} = 38.75\) GeV (the fixed target energy considered both here and in Ref. [16]), the parton level asymmetry is near its maximum value. Since \(\Delta g(x, Q^2)/g(x, Q^2) \leq 1\) generally, the maximum observable asymmetry is bounded by the maximum parton level asymmetry. Thus we are unable to understand the prediction, in Ref. [16], that the observable asymmetry can be as large as 85%.

Next, we consider collider experiments at RHIC. RHIC is a high luminosity (\(\mathcal{L} = 2 \times 10^{32}\) cm\(^{-2}\) sec\(^{-1}\) = 6000 pb\(^{-1}\)/yr) collider capable of producing proton on proton collisions for center of mass energies between 50 and 500 GeV. A program of polarized proton on proton collisions, at full energy and luminosity, is being discussed[21]. We will assume a nominal running time of 2 months, at full luminosity, for 50 GeV and 500 GeV each. In order to be somewhat conservative, we will estimate event numbers based on 300 pb\(^{-1}\) integrated luminosity. We will assume a generic collider type detector, and in order to simulate the acceptance we will require the photon and electrons observed to lie in the rapidity range \(|y| \leq 2\) (this simulates the acceptance of
the proposed STAR detector at RHIC\cite{22}, level 2 for photons and electrons. We will not consider the possibility of the detection of the $\mu^+\mu^-$ final state at RHIC. Furthermore, we will (rather arbitrarily) require the $p_T$ of the photon larger than 1 GeV in the following discussion. We present our results for the $p_T$ distribution in Figure 3a, and $A^2_{LL}$ in Figures 3b ($\sqrt{s} = 50$ GeV) and 3c ($\sqrt{s} = 500$ GeV). See Figure 4a for $\frac{d\sigma}{dp_T dy_1 dy_2}$ vs. $p_T$ and Figures 4b ($\sqrt{s} = 50$ GeV) and 4c ($\sqrt{s} = 500$ GeV) for $A^3_{LL}$ vs. $p_T$. The ‘resolving power’ increases with energy (actually $p_T$), even though the observed asymmetry decreases. This is simply a consequence of the behavior of the polarized gluon distribution. Please refer to Table 1 for some representative results.

Finally, we consider a collider experiment at the SSC. The luminosity of the SSC is $\mathcal{L} = 10^{33} \text{ cm}^{-2}\text{sec}^{-1} = 30000 \text{ pb}^{-1}/\text{yr}$. We will again assume a running time of 2 months at full luminosity and energy, and conservatively calculate event numbers based on 1500 pb$^{-1}$ integrated luminosity. We require the photons and leptons to have $p_T \geq 10$ GeV and lie in the range $|y| \leq 2.5$ (these approximate the acceptances of the SDC detector\cite{23}). In this case, the resolving power is quite high, $\langle \hat{a}_{LL} \rangle = 60\%$, although because of the extremely small-$x$ probed the observed asymmetry $A^1_{LL}$ is tiny. Similarly, $A^2_{LL}$ and $A^3_{LL}$ are both smaller than 1% for all $p_T < 125$ GeV, while there will only be a handful of events at (or beyond) $p_T \sim 25$ GeV, so there is no observable asymmetry. Again, see Table 1 for some representative results.

In conclusion, we have studied the process $p + p \rightarrow J/\psi + \gamma + X$ in polarized proton-proton collisions. We first presented the necessary helicity amplitudes and discussed the calculation. Then we studied this process at polarized fixed target and in colliders, at polarized RHIC (50 and 500 GeV center of mass energy) and at polarized SSC. Our results indicate that a polarized (double spin) fixed target program can be
very useful in the determination of the polarized gluon distribution. It is unfortunate that no such experiment is planned. RHIC (especially at lower energies) is an excellent probe of the polarized gluon distribution. Since \( A_{LL}^3 \) is directly proportional to \( [\Delta g(x(p_T), Q^2)/g(x(p_T), Q^2)]^2 \), this distribution provides an easy determination of the polarized gluon distribution at various \( x \) values. It will prove especially useful to measure this distribution at several center of mass energies. Even a measurement of \( A_{LL}^2 \) can provide much useful information (though it is not clear whether the higher statistics involved in this measurement will outweigh the cleanliness of the extraction of the polarized gluon distribution in a measurement of \( A_{LL}^3 \)). The SSC probes a much lower \( x \) in this process, and since \( \Delta g(x, Q^2)/g(x, Q^2) \ll 1 \) there is no measurable asymmetry. However, the ‘resolving power’ at SSC is still very large, so the smallness of the asymmetry is purely a consequence of the small-\( x \) behavior of \( \Delta g(x, Q^2) \). Polarized SSC can still be a useful tool for the study of high energy spin properties of the proton by utilizing a subprocess that will probe larger \( x \) (e.g. heavy Higgs production). We should also point out that we have considered only the color singlet model of heavy quarkonium production in this paper. A similar analysis can be performed using local duality, if it is determined at HERA that this mechanism contributes to \( J/\psi + \gamma \) production\(^2\). Some slight modifications will be required, namely the inclusion of charm in the proton (this effect should be small) and light \( q\bar{q} \) fusion, and in addition the modification of the parton level asymmetries. As a final related comment, we plan to study \( J/\psi + \gamma \) production at HERA using a polarized lepton beam and angular distributions of the final leptons to learn something of the polarized gluon distribution of the photon.

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|                  | \(N_{TOT}\) | \(\langle \hat{a}_{LL} \rangle\) | \(A_{LL}^1(\delta A_{LL}^1)\) | \(N_{p_T}\) | \(A_{LL}^2(\delta A_{LL}^2)\) | \(N_{p_T}\) | \(A_{LL}^3(\delta A_{LL}^3)\) |
|------------------|-------------|-----------------|-----------------|-------------|-----------------|-------------|-----------------|
| Fixed Target     | 10500       | 28.4%           | 12.5% (1%)      | 5000        | 16% (1.4%)      | 200         | 22% (6%)        |
|                  |             |                 | 3.2% (1%)       |             | 4% (1.4%)       |             | 5% (6%)         |
| RHIC 50 GeV      | 11430       | 43.3%           | 19.1% (1%)      | 4500        | 26% (1.5%)      | 1080        | 32% (3%)        |
|                  |             |                 | 4.6% (1%)       |             | 8% (1.5%)       |             | 8% (3%)         |
| RHIC 500 GeV     | 86400       | 44.7%           | .4% (0.3%)      | 4500        | 1.7% (1.5%)     | 840         | 1.8% (3%)       |
|                  |             |                 | .05% (0.3%)     |             | .2% (1.5%)      |             | .3% (3%)        |
| SSC              | 8835        | 60.2%           | .005% (1%)      | 3000        | .008% (2%)      | 540         | .01% (4%)       |
|                  |             |                 | .0006% (1%)     |             | .001% (2%)      |             | .001% (4%)      |

Table 1: Summary of representative predictions for \(J/\psi + \gamma\) production in polarized proton-proton interactions. \(N_{TOT}\) is the total number of events above some minimum \(p_T\) (= 0 GeV for fixed target, 1 GeV for RHIC and 10 GeV for SSC). \(\langle \hat{a}_{LL} \rangle\) is the ‘resolving power’ as defined in the text (this is independent of the polarized parton distributions). \(A_{LL}^i\) and \(\delta A_{LL}^i\) are defined in the text; the upper entry corresponds to the large \(\Delta g(x, Q^2)\) (set BGC0) and the lower entry corresponds to the moderately large \(\Delta g(x, Q^2)\) (set BGC1). \(N_{p_T}\) is the number of events in the particular \(p_T\) bin (0.5-1.5 GeV for fixed target, 1-2 GeV for RHIC at 50 GeV, 3-5 GeV for RHIC at 500 GeV and 10-20 GeV for SSC).
Figure Captions

Figure 1 - $p_T$ distribution, $\frac{d\sigma}{dp_T}$ vs. $p_T$ (1a) and $A_{LL}^2$ vs. $p_T$ (1b) for large $\Delta g(x, Q^2)$ (solid line) and for moderately large $\Delta g(x, Q^2)$ (dashed line) at fixed target.

Figure 2 - $\frac{d\sigma}{dp_T}dy_1dy_2|_{y_1=y_2=0}$ vs. $p_T$ (2a) and $A_{LL}^3$ vs. $p_T$ (2b) for large $\Delta g(x, Q^2)$ (solid line) and moderately large $\Delta g(x, Q^2)$ (dashed line) at fixed target.

Figure 3 - $p_T$ distribution, $\frac{d\sigma}{dp_T}$ vs. $p_T$ (3a) for RHIC at $\sqrt{s} = 500$ GeV (solid line) and at $\sqrt{s} = 50$ GeV (dot-dashed line), and $A_{LL}^2$ vs. $p_T$ for RHIC at $\sqrt{s} = 50$ GeV (3b) and at $\sqrt{s} = 500$ GeV (3c) for large $\Delta g(x, Q^2)$ (solid line) and for moderately large $\Delta g(x, Q^2)$ (dashed line).

Figure 4 - $\frac{d\sigma}{dp_T}dy_1dy_2|_{y_1=y_2=0}$ vs. $p_T$ (4a) for RHIC at $\sqrt{s} = 500$ GeV (solid line) and at $\sqrt{s} = 50$ GeV (dot-dashed line), and $A_{LL}^3$ vs. $p_T$ for RHIC at $\sqrt{s} = 50$ GeV (4b) and at $\sqrt{s} = 500$ GeV (4c) for large $\Delta g(x, Q^2)$ (solid line) and moderately large $\Delta g(x, Q^2)$ (dashed line).