Experimentally Accessible Quantum Phase Transition in a non-Hermitian Tavis-Cummings Model Engineered with Two Drive Fields

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We study the quantum phase transition (QPT) in a non-Hermitian Tavis-Cummings (TC) model of experimentally accessible parameters, which is engineered with two drive fields applied to an ensemble of two-level systems (TLSs) and a cavity, respectively. When the two drive fields satisfy a given parameter-matching condition, the coupled cavity-TLS ensemble system can be described by an effective standard TC Hamiltonian in the rotating frame. In this ideal Hermitian case, the engineered TC model can exhibit the super-radiant QPT with spin conservation at an experimentally-accessible critical coupling strength, but the QPT is however spoiled by the decoherence. We find that in this non-Hermitian case, the QPT can be recovered by introducing a gain in the cavity to balance the loss of the TLS ensemble. Also, the spin-conservation law is found to be violated due to the decoherence of the system. Our study offers an experimentally realizable approach to implementing QPT in the non-Hermitian TC model.

Introduction.—Quantum phase transition (QPT) has been widely studied in various quantum systems (see, e.g., [1–12]), because of its fundamental importance in many-body physics and potential applications in quantum technologies [13]. Among them, the super-radiant QPT was predicted [14–17] in, e.g., the Dicke model [18], which involves the collective interaction between an ensemble of two-level systems (TLSs) and the quantized field in a cavity. In the thermodynamic limit of a large number of TLSs, the ground-state properties of the Dicke model, such as the excitations in the TLS ensemble, can display an abrupt change related to the QPT in the system [19–21], when continuously varying the collective coupling strength around a critical value. As required to be comparable to the frequencies of the TLS ensemble and the cavity mode, this critical coupling strength is difficult to achieve experimentally. Moreover, the no-go theorem due to the squared electromagnetic vector potential also hinders the presence of the super-radiant QPT in the Dicke model [22–24]. Due to these difficulties, the nonequilibrium QPT (i.e., the simulated QPT) has been proposed [25–27] and observed [28–30] in cavity quantum electrodynamics (QED) systems by engineering an effective Dicke Hamiltonian.

In the experiment, most quantum systems can only reach the strong-coupling regime [31, 32], where the Dicke model can be reasonably reduced to the Tavis-Cummings (TC) model [33] via the rotating-wave approximation (RWA) [34], i.e., ignoring the counter-rotating coupling terms between the TLS ensemble and the cavity mode. Similar to the Dicke model, the TC model is another archetypal model known to exhibit the super-radiant QPT [35]. Nevertheless, QPT in the TC model can occur only in the regime of extremely strong coupling, which is unaccessible for a realistic system and in which the RWA actually breaks down [34]. In addition, the decoherence inevitably occurring in the realistic system can spoil the QPT in the TC model [36–39]. Therefore, it is of great importance to explore the nonequilibrium QPT by engineering an effective experimentally-accessible TC Hamiltonian [39–41].

In this Letter, we study the QPT in a driven quantum system consisting of a TLS ensemble coupled to a cavity mode. To reduce the critical coupling strength of the TC model, we use two external fields with the same frequency to drive the TLS ensemble and the cavity, respectively. By choosing proper drive-field parameters, an effective TC model can be rebuilt in the rotating frame, with the effective frequencies of the TLS ensemble and the cavity mode being their respective frequency detunings from the drive fields. In the ideal Hermitian case (i.e., without decoherence in the system), we can demonstrate the QPT in the driven system, where the corresponding critical coupling strength is tunable (via varying the drive-field frequency) and experimentally accessible. However, decoherence inevitably occurs in the system and the QPT of the TC model is spoiled in this non-Hermitian case [36–39]. We find that the QPT in the non-Hermitian TC model can be recovered by using a cavity with gain to balance the loss of the TLS ensemble. In sharp contrast to the spin conservation in the ideal Hermitian case, the spin-conservation law is found to be violated in the non-Hermitian case [42, 43]. Moreover, we propose to implement this non-Hermitian TC model with a hybrid circuit-QED system. To the best of our knowledge, this is the first proposal to engineer an experimentally accessible non-Hermitian TC model that can have QPT in the presence of decoherence in the system.

QPT in the ideal Hermitian case.—The proposed system consists of $N$ TLSs (e.g., spins) in a cavity, each with the same transition frequency $\omega_s$ and coupled to a cavity mode with coupling strength $\lambda_s$ [see Fig. 1(a)]. This system can be described in a RWA by the TC model (we set $\hbar = 1$)

$$H_{\text{TC}} = \omega_s a^\dagger a + \omega_c J_z + \frac{\lambda}{\sqrt{N}}(a^\dagger J_- + aJ_+),$$

(1)
Hamiltonian of the system can be converted to the drive field and each TLS. In a rotating reference frame with \( \Omega \) between the drive field and the cavity mode, and a waveguide resonator, where the gain of the resonator results from an ensemble of NV centers in diamond interacting with an active coplanar-TC model using a hybrid circuit-QED system composed of an ensemble of NV centers in diamond interacting with an active coplanar-TC model. Two drive fields of the same frequency \( \omega \) are applied to the ensemble and the cavity mode, respectively. Two fields with the same frequency drive the ensemble and the resonator, respectively.

where \( a \) (\( a^\dagger \)) is the annihilation (creation) operator of the cavity mode with resonant frequency \( \omega_c \). \( J_x, J_y, \) and \( J_z \) are the collective spin operators of the TLS ensemble with raising and lowering operators \( J_x = J_z = 0 \), and \( \lambda = \lambda_s \sqrt{\omega_c/\Omega} \) is the collective coupling strength between the TLS ensemble and the cavity mode. This Hamiltonian has a conserved parity \([35], [H_{TC}, \Pi] = 0, \) with \( \Pi = \exp[i\pi(a^\dagger a + J_z + N/2)] \). In the ideal case without decoherence in the system, the TC model exhibits a QPT at the critical coupling strength \( \lambda = \lambda_c \equiv \sqrt{\omega_c/\Omega} \) in the thermodynamic limit \( N \to +\infty \) [35]. However, this QPT is spoiled by the decoherence of the system [36–39]. Also, it is extremely difficult to demonstrate the QPT due to the inaccessibility of the very large critical coupling strength \( \lambda_c \) in a realistic system.

To solve these problems, we first manage to reduce the critical coupling strength by applying two drive fields of the same frequency \( \omega_d \) to the cavity and the TLS ensemble, respectively. This corresponds to adding the drive Hamiltonian \( H_d = \Omega_d a^\dagger e^{-i\omega_d t} + a e^{i\omega_d t} + (\Omega_d / \sqrt{N})(J_x e^{-i\omega_d t} + J_y e^{i\omega_d t}) \) to the Hamiltonian in Eq. (1). Here \( \omega_d \) is the Rabi frequency between the drive field and the cavity mode, and \( \Omega_d = \sqrt{N} \) is the collective Rabi frequency between the drive field and the TLS ensemble, where \( \Omega_d \) is the Rabi frequency between the drive field and each TLS. In a rotating reference frame with respect to the frequency \( \omega_d \) of the two drive fields, the total Hamiltonian of the system can be converted to

\[
H_{TC}^{(d)} = \Delta_c a^\dagger a + \Delta_s J_z + \frac{\lambda}{\sqrt{N}}(aJ_x + a^\dagger J_-) + (\Omega_d a^\dagger + \Omega_d^\dagger a)
\]

where \( \Delta_{c(s)} \equiv \omega_{c(s)} - \omega (\geq 0) \) is the frequency detuning of the cavity mode (TLS ensemble) relative to the corresponding drive field. By introducing a displacement \( a = A + \alpha \) and \( a^\dagger = A^\dagger + \alpha^* \) with \( \alpha = -\Omega_d/\Delta_c \), i.e., a translation transform, the above Hamiltonian becomes

\[
H_{TC}^{(d)} = \Delta_c A^\dagger A + \Delta_s J_z + \frac{\lambda}{\sqrt{N}}(A J_x + A^\dagger J_-) + \frac{1}{\sqrt{N}}[(\Omega_d + i\lambda\alpha)J_+ + (\Omega_d^\dagger + i\lambda\alpha^*)J_-],
\]

which has the same form as the standard TC model in Eq. (1), but the critical coupling strength \( \lambda = \lambda_c \equiv \sqrt{\Delta_c/\Delta_s} \) becomes experimentally accessible by reducing \( \Delta_c / \Delta_s \) via tuning the drive-field frequency \( \omega_d \). Like the standard TC model in Eq. (1), the effective TC model in Eq. (4) also exhibits a normal (super-radiant) phase with unbroken (broken) parity symmetry when \( \lambda < \lambda_c \) (\( \lambda > \lambda_c \)), and the TLS ensemble in the thermodynamic limit has the critical behaviors [41]

\[
\frac{\langle J_x \rangle}{(N/2)} = \begin{cases} -1, & \lambda < \lambda_c; \\ -\kappa_1 \lambda^2, & \lambda \geq \lambda_c, \end{cases}
\]

and

\[
\frac{\langle J_+ \rangle}{(N/2)} = \begin{cases} 0, & \lambda < \lambda_c; \\ (1 - \lambda_0^2/\lambda_1^2)^{1/2}, & \lambda \geq \lambda_c. \end{cases}
\]

This gives \( \langle J_x \rangle \sim |\lambda - \lambda_0|^2 \) and \( \langle J_+ \rangle \sim |\lambda - \lambda_0|^2 \) around \( \lambda = \lambda_c \), with critical exponents \( \nu_1 = 1 \) and \( \nu_2 = 1/2 \). The displaced cavity field can also display a QPT because \( \langle A \rangle = -\lambda J_x/(\sqrt{\Delta_c} \Delta_s) \), i.e., \( \langle A^\dagger \rangle \) behaves as \( \langle A^\dagger \rangle = \langle A^\dagger \rangle /|\lambda - \lambda_0|^2 \), with \( \nu_2 = 1 \). Therefore, \( \langle a^\dagger a \rangle \sim |\lambda - \lambda_0|^2 \), with \( \nu_2 = 1/2 \) [44], owing to \( \langle a \rangle \equiv \langle A \rangle + \alpha = -\lambda(J_x)/(\sqrt{\Delta_c} \Delta_s) \). Note that two suitable drive fields are required to demonstrate the QPT. If only one finite drive field is applied (i.e., either \( \Omega_d \neq 0 \) but \( \Omega_d = 0 \) or \( \Omega_d = 0 \) but \( \Omega_d \neq 0 \)), the effective Hamiltonian of the driven system does not preserve the parity symmetry [45]. In this case, the system can only tend to exhibit the critical behavior when the sole drive field becomes extremely weak [41].

QPT in the non-Hermitian case with gain.—When the decoherence of the system is considered, the system becomes non-Hermitian. We use a Langevin equation approach [34] to study the critical behavior of the non-Hermitian system. With the Hamiltonian in Eq. (2), the dynamics of the driven system is governed by the following Langevin equations:

\[
\dot{a} = -i(\Delta_c - i\kappa_c) a - i\frac{\lambda}{\sqrt{N}} J_x - i\Omega_d.
\]
\begin{align}
J_z &= -i(\Delta_r - i\gamma_z)J_r - i\frac{2\lambda}{\sqrt{N}}J_r a + i2J_z \frac{\Omega_J}{\sqrt{N}}, \\
J_r &= -i\frac{\lambda}{\sqrt{N}}(a J_r - a^\dagger J_r) - i\frac{1}{\sqrt{N}}(\Omega_J J_r - \Omega_J^* J_r - \gamma_\perp(N/2 + J_z)),
\end{align}

where \(\Gamma_r\) is the decay rate of the cavity, and \(\gamma_{\perp}/(\gamma_{\parallel})\) is the transversal (longitudinal) relaxation rate of the TLS ensemble. The operator \(\hat{O} = \{a, J_r, J_r^\dagger\}\) can be written as a sum of its expectation value \(\langle \hat{O} \rangle\) and fluctuation \(\delta \hat{O}\), i.e., \(\hat{O} = \langle \hat{O} \rangle + \delta \hat{O}\). At the steady state \(\langle \hat{O} \rangle = 0\), it follows from Eq. (7) that

\begin{align}
(\Delta_r - i\kappa_r)\langle a \rangle + \frac{\lambda}{\sqrt{N}}\langle J_r \rangle + \Omega_r = 0, \\
(\Delta_r - i\gamma_z)\langle J_r \rangle - \frac{2\lambda}{\sqrt{N}}\langle J_r \rangle a - i\langle J_r \rangle \frac{\Omega_J}{\sqrt{N}} = 0, \\
\frac{\lambda}{\sqrt{N}}(\langle a^\dagger \rangle\langle J_r \rangle - \langle a \rangle\langle J_r \rangle) + \gamma_\perp(N/2 + \langle J_z \rangle) \\
- i\gamma_\parallel(N/2 + \langle J_z \rangle) = 0,
\end{align}

where a mean-field approximation is applied to the two-operator terms [46]. Substituting the first equation in Eq. (8) into the second and third equations in Eq. (8) to eliminate \(\langle a \rangle\), we have

\begin{align}
\left[ 1 + \frac{\lambda^2}{(\Delta_r \gamma_z - \kappa_r \gamma_\perp)^2} \right] \langle J_r \rangle \langle J_r \rangle = 0, \\
\kappa_r \lambda^2 \frac{(\langle J_r \rangle)^2}{(N/2)^2} + \gamma_\parallel \frac{\left(1 + \langle J_z \rangle \right)^2}{(N/2)} = 0
\end{align}

when the parameter-matching condition in the non-Hermitian case, \(\Omega_r/\Omega_J = (\Delta_r - i\kappa_r)/\lambda\), is used. Obviously, Eq. (9) has only a set of trivial solutions \(\langle J_r \rangle/(N/2) = 1\) and \(\langle J_z \rangle = 0\). This verifies that the decoherence of the system ruins the QPT occurring in the standard Hermitian TC model [36–39].

To recover the QPT, we manage to introduce a gain medium in the dissipative cavity. With the gain included, we obtain the same equations as Eqs. (7)-(9), but \(\kappa_r\) is replaced by \(\kappa = \kappa_r - \kappa_g\), where \(\kappa_g\) is the gain rate of the cavity owing to the gain medium. The parameter-matching condition becomes

\begin{align}
\Omega_r/\Omega_J = (\Delta_r - i\kappa)/\lambda,\quad \text{and Eq. (9) is converted to}
\end{align}

\begin{align}
\left[ 1 + \frac{\lambda^2}{(\Delta_r \gamma_z - \kappa_r \gamma_\perp)^2} \right] \langle J_r \rangle \langle J_r \rangle = 0, \\
\kappa_r \lambda^2 \frac{(\langle J_r \rangle)^2}{(N/2)^2} + \gamma_\parallel \frac{\left(1 + \langle J_z \rangle \right)^2}{(N/2)} = 0.
\end{align}

To have the QPT, we need \(\Delta_r \gamma_z + \Delta_r \kappa \equiv \Delta_r \gamma_z - \Delta_r (\kappa_g - \kappa_r) = 0\), which gives the required gain rate \(\kappa_g(0) = \kappa_r + \gamma_z \Delta_r/\kappa_r\). Now, the parameter-matching condition \(\Omega_r/\Omega_J = (\Delta_r - i\kappa)/\lambda\) is reduced to \(\Omega_r/\Omega_J = (1 + i\gamma_z/\Delta_r)/\lambda\). When \(\kappa_g = \kappa_g(0)\), besides the set of trivial solutions \(\langle J_r \rangle/(N/2) = 1\) and \(\langle J_z \rangle = 0\), Eq. (10) has also another set of nontrivial solutions

\begin{align}
\frac{(\langle J_z \rangle)}{(N/2)} = \lambda \frac{\langle J_r \rangle}{\langle J_r \rangle}, \\
\frac{(\langle J_r \rangle)}{(N/2)} = \frac{\lambda}{\gamma_\parallel \left(1 - \frac{\lambda^2}{\Delta_r^2} \right)} \frac{\gamma_\parallel}{(N/2)^{1/2}}
\end{align}

with \(\lambda_r\) modified as

\begin{align}
\lambda_r \equiv \sqrt{\Delta_r \gamma_z (1 + \gamma^2_\parallel/\Delta_r^2)}.
\end{align}

Here \(\langle J_z \rangle\) is assumed to be real for simplicity. As shown in Fig. 2, when varying the critical coupling strength \(\lambda_r\) from \(\lambda < \lambda_r\) to \(\lambda > \lambda_r\), the proposed driven system exhibits a QPT from the normal phase with \(\langle J_r \rangle/(N/2) = 1\) and \(\langle J_z \rangle = 0\) to the super-radiant phase with \(\langle J_r \rangle/(N/2) = -1\) and \(\langle J_z \rangle = 0\). In the non-Hermitian case with gain, \(\langle J_r \rangle/(N/2)\) has the same behavior as in the Hermitian case [see Fig. 2(a)]. In fact, around \(\lambda = \lambda_r\), \(\langle J_r \rangle \sim |\lambda - \lambda_r|^\nu\), with \(\nu = 1\). Figure 2(b) shows that in the non-Hermitian case with gain, \(\langle J_r \rangle/(N/2)\) is much reduced in the regime of super-radiant phase, but it can be analytically derived from Eq. (11) that around \(\lambda = \lambda_r\), \(\langle J_r \rangle/(N/2)\) still exhibits the same critical behavior as in the Hermitian case: \(\langle J_z \rangle \sim |\lambda - \lambda_r|^\nu\), with \(\nu = 1/2\).

From the first equation in Eq. (8), but with \(\kappa_r\) replaced by \(\kappa = \kappa_r - \kappa_g(0) = -\gamma_z \Delta_r/\kappa_r\), we have

\begin{align}
\langle a \rangle = -\frac{\lambda}{\sqrt{N} \Delta_r (1 + i\gamma_z/\Delta_r)} \langle J_r \rangle - \frac{\Omega_r}{\Delta_r (1 + i\gamma_z/\Delta_r)}
\end{align}

With \(\gamma_z = 0\), it reduces to the result in the ideal Hermitian case. In Fig. 3(a), we plot \(\langle a^\dagger a \rangle/N\) versus the reduced coupling strength \(\lambda/\Delta_r\) in both the ideal Hermitian case and the
monotonically decreases when \( \frac{\lambda}{\text{of the auxiliary qubit}, \text{ the e}} \)  microwave fields \([49]\). After eliminating the degree of freedom

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the auxiliary flux qubit transversely coupled to the resonator with

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with gain, \( \text{F} \). Possible implementation.—In the experiment, the proposed model with gain can be implemented using a hybrid circuit-QED system composed of a dissipative TLS ensemble, such as the nitrogen-vacancy (NV) centers in diamond, coupled to an active coplanar-waveguide resonator [see Fig. 1(b)]. The transition frequency of NV centers can be tuned by an external magnetic field and the number of NV centers in the sample can be \( N \approx 10^{12} \) [47, 48], approaching the thermodynamic limit of the proposed system. The hybrid system is usually in the strong-coupling regime and can be described by a standard TC Hamiltonian in Eq. (1).

To engineer an active resonator, one can harness an auxiliary flux qubit transversely coupled to the resonator with a coupling strength \( g_0 \) and longitudinally driven by two microwave fields [49]. After eliminating the degree of freedom of the auxiliary qubit, the effective gain rate \( k_g \) of the resonator can be tuned from 0 to, e.g., \( 0.2g_0 \) (i.e., \( 2\pi \times 6 \text{ MHz} \) for \( g_0/2\pi = 30 \text{ MHz} \)) by varying the amplitudes and frequencies of the two microwave fields [49]. Moreover, as shown in Fig. 1(b), two additional drive fields with Rabi frequencies \( \Omega_d \) and \( \Omega_f \) pump the NV-center ensemble and the resonator, respectively. When the parameter-matching condition \( \Omega_d/\Omega_f = \Delta_0/(1 + i\gamma_\perp/\Delta_0)/\lambda \) is achieved for these two drive fields, an experimentally accessible TC model is then engineered and the relevant quantities for demonstrating the QPT are governed by Eqs. (10) and (13).

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\text{Discussions and Conclusions}.
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The special case with only one drive field was studied in Refs. [41, 45], which is related to the cavity or the TLS ensemble pumped by a drive field, i.e., Eq. (2) with \( \Omega_c \neq 0 \) but \( \Omega_f = 0 \) [41] or \( \Omega_f \neq 0 \) but \( \Omega_c = 0 \) [45]. At a finite strength of this drive field, the biased TC model does not preserve the parity symmetry due to the bias term and the QPT disappears in the system [45]. Only in the weak drive-field limit [i.e., either \( \Omega_c \rightarrow 0 \) when \( \Omega_f = 0 \) or \( \Omega_f \rightarrow 0 \) when \( \Omega_c = 0 \) in Eq. (2)] can the model tend to have the QPT [41]. For a realistic system, the cup-like QPT behavior is however much smoothed by the inevitable cavity loss and the dissipation of the TLS ensemble even in the weak drive-field limit [39]. In our proposal, when the two drive fields satisfy the parameter-matching condition, we can obtain an effective TC Hamiltonian with parity symmetry [i.e., Eq. (4)]. This TC Hamiltonian can exhibit QPT at an experimentally-accessible critical coupling strength and the weak drive-field limit is not necessary. Even with the decoherence of the system, the QPT behavior is still achievable by harnessing an active cavity (instead of a passive cavity). Very recently, the QPT in a TC model induced by a single squeezed drive field was investigate [40], where the effective Hamiltonian is a biased TC model with a two-photon drive term, corresponding to Eq. (2) with \( \Omega_c a^\dagger + \Omega_f a \) replaced by \( \Omega_c a^2 + \Omega_f a^\dagger a^\dagger \) and \( \Omega_f = 0 \). Using a squeezing transformation, this biased TC Hamiltonian can be transformed to an anisotropic Dicke Hamiltonian [50]. Experimentally, if a pure squeezed drive field, \( a^2 + a^\dagger \), cannot be achieved, as discussed above, any finite unsqueezed part of the single drive field can ruin the QPT. In our study, the obtained effective Hamiltonian in Eq. (4) is a standard TC model without the bias term and no squeezed drive field is required. Also, the spin-conservation law is used in Ref. [40]. It is different from our non-Hermitian case in which the spin-conservation law is found to be violated [42, 43].

In conclusion, we have studied the QPT in a TC model engineered with two drive fields applied to the TLS ensemble and the cavity, respectively. In the ideal Hermitian case without decoherence, the QPT can occur at an experimentally-accessible critical coupling strength, but it is spoiled by the decoherence of the system. In this non-Hermitian case, we find that the QPT can be recovered by harnessing a gain in the cavity to balance the loss of the TLS ensemble. In sharp contrast to the spin conservation in the ideal Hermitian case, the spin-conservation law is however found to be violated in our non-Hermitian case. Moreover, we propose to implement this non-Hermitian TC model using a hybrid circuit-QED system. Our work provides an experimentally realizable approach to achieving QPT in the non-Hermitian TC model.

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