A strongly-coupled Λ-type micromechanical system

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We study a classical Λ-type three-level system based on three high-Q micromechanical beam resonators embedded in a gradient electric field. By modulating the strength of the field at the difference frequency between adjacent beam modes, we realize strong dynamic two-mode coupling, via the dielectric force. Driving adjacent pairs simultaneously, we observe the formation of a purely mechanical ‘dark’ state and an all-phononic analog of coherent population trapping — signatures of strong three-mode coupling. The Λ-type micromechanical system is a natural extension of previously demonstrated ‘two-level’ micromechanical systems and offers new perspectives on the architecture of all-phononic micromechanical circuits and arrays.

The ability to control phonon transport in micromechanical systems has important consequences for applications such as low-power mechanical filters, switches, routers, memories, and logic gates [1–9]. As a basic starting point, considerable effort has been aimed towards achieving tunable coupling between two modes of a micromechanical resonator, using a combination of piezoelectric [10, 11], photothermal [12, 13], and dielectric [14] forces. Realization of strong coupling — that is, coherent energy exchange between two modes at a rate (g) larger than their mechanical dissipation (γ) — has recently enabled classical analogs of ‘two-level system’ coherent control, including Rabi flops, Ramsey fringes and Hahn echo [10, 15]. These demonstrations provide a dress rehearsal for phonon transport in future micromechanical circuits and arrays [16, 17].

In this Letter, we demonstrate strong coupling between three radiofrequency micromechanical resonators forming a classical analog of a Λ-type three-level system (two nearly degenerate, low energy levels and single high energy level). The inclusion of a third ‘level’ opens the door to a rich variety of physics not accessible to two-level systems, such as analogs of EIT (electromagnetically induced transparency [18]) and CPT (coherent population trapping [19]). To our knowledge, a fully micromechanical Λ-type system has not been previously implemented. By contrast, V-type three-level systems formed by a pair of optical cavity modes coupled to a common mechanical resonator mode have been extensively explored in the context of cavity optomechanics [20–22], enabling demonstrations of optomechanical state transfer [20] and optomechanical dark states [21]. In analogy to the latter, we show that by driving both pairs of the Λ-system micromechanical system simultaneously, destructive interference leads to the formation of a mechanical dark state and a phononic analog of CPT.

The micromechanical system, shown in Fig. 1, consists of a planar stack of three high-stress Si3N4 microbeams. The outer left and right beams possess fundamental out-of-plane flexural modes with frequencies ωL ∼ 2π · 1.67 MHz and ωR ∼ 2π · 1.72 MHz, respectively. The middle beam has a fundamental mode frequency of ωM ∼ 2π · 4.50 MHz. To couple these modes, a voltage is applied between two electrodes patterned on the outer beams. The resulting electric field produces a dielectric potential, which, owing to the spatial dependence of the field, gives rise to a static intermodal coupling [15], analogous to the strain-mediated coupling of two beams sharing a non-rigid anchor [10]. We wish to emphasize that, contrary to the latter case, dielectric intermode coupling can be switched completely off by grounding the electrodes. In this sense, the mechanical modes may be considered to reside on three physically isolated mechanical resonators. Though conceptually the same as three modes of a single resonator, physical separation is advantageous for building micromechanical circuits/arrays, as it enables the resonators...
FIG. 2: Dielectric tuning of mechanical eigenfrequencies and optical readout. (a-c) Thermal motion of the middle beam (mode M), right beam (mode R), and left beam (mode L), readout using a lensed-fiber interferometer. For these measurements no DC voltage is applied across the metal electrodes. Displacement noise spectral densities, $S_x$, are expressed in units scaled to the twice the peak zero point spectral density, $S_{zp x}(\omega_m)$ (i.e., the displacement noise produced by single thermal phonon). In these units, the peak spectral density is equal to with the thermal occupation, $n_{th} \approx k_B T / \hbar \omega_m$, where $\omega_m$ is the mechanical frequency. (d) Dependence of mode frequencies on the DC voltage applied across the electronics. Mode shapes computed by the finite element method are shown on the left.

Experimental characterization of the device is shown in Fig. 2. Mechanical spectra are recorded used a lensed fiber-based interferometer [24] with a spot size (~ 4 μm) large enough to simultaneously record the thermal displacement of each beam. The optical field, supplied by a 780 nm Ti:sapphire laser, is attenuated until photothermal effects are negligible. Sample and fiber are embedded in a vacuum chamber at $10^{-4}$ mbar. When no voltage is applied across the electrodes, the fundamental out-of-plane modes of the middle (M), right (R), and left (L) beams reside at their natural frequencies with energy dissipation rates of $\gamma_M = 2\pi \cdot 100$ Hz, $\gamma_R = 2\pi \cdot 170$ Hz, and $\gamma_L = 2\pi \cdot 140$ Hz, respectively, shown in descending order in Fig. 2a-c.

To realize dynamic intermode coupling, an AC voltage is applied across the electrodes at one of the two difference frequencies, $\Delta \omega_L(R) = \omega_M - \omega_L(R)$, exhibiting normal mode splitting of the left (right) and middle beam modes. (d) Energy level diagram of the three-level system in the presence of simultaneous pump and probe tones. All three levels exhibit mode splitting. Notably, one of the dressed states is ‘dark’ in the sense that involves no motion of the center beam.

FIG. 3: Energy level diagram. (a) Energy level diagram of the A-type micromechanical system in the absence of dynamic coupling. (b,c) Energy level diagram in the presence of pump (probe) tone $\Delta \omega_L(R) = \omega_M - \omega_L(R)$, exhibiting normal mode splitting of the left (right) and middle beam modes. (d) Energy level diagram of the three-level system in the presence of simultaneous pump and probe tones. All three levels exhibit mode splitting. Notably, one of the dressed states is ‘dark’ in the sense that involves no motion of the center beam.

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portional to the AC voltage amplitude, and corresponds to twice the energy coupling rate \( g_{L(R)} \) between the left(right) and middle beam modes. Strong coupling, \( g_{L(R)} > \{\gamma_{L(R)}, \gamma_{M}\} \), is achieved for \( V_{L(R)} > 1 \ V_{PP} \) in both cases. At the largest AC voltage amplitudes, the cooperativities achieved are \( C_{L(R)} = 4g_{L(R)}^2/(\gamma_{L(R)}\gamma_{M}) > 1 \times 10^4 \).

When the pump and probe tones are applied simultaneously, strong coupling is induced among all three beams. This coupling is akin to a resonant Raman interaction, and features the formation of a dynamically decoupled ‘dark’ state due to destructive interference (see Fig. 3d). To study this effect, the displacement noise spectrum is recorded versus pump amplitude \( V_{L(R)} \) with pump frequency tuned to \( \Delta_{L} = \omega_{L} - \omega_{R}, V_{DC} = 10 \ V, \) and no probe voltage applied. Right column: Noise spectrum of mode M (d), mode R (e), and mode L (f) versus probe amplitude \( V_{P} \) with probe frequency tuned to \( \Delta_{R} = \omega_{M} - \omega_{R}, V_{DC} = 10 \ V, \) and no pump voltage applied.

![FIG. 4: Dynamic intermode coupling by dielectric frequency modulation.](image)

Left column: Noise spectrum of mode M (a), mode R (b), and mode L (c) versus pump amplitude \( V_{L(R)} \) with pump frequency tuned to \( \Delta_{L} = \omega_{L} - \omega_{R}, V_{DC} = 10 \ V, \) and no probe voltage applied.

Here, \( x_{L(M)} \) is the generalized displacement of the left(right, middle) beam mode, \( g_{L(R)} = \Omega_{L(R)} \cos(\Delta_{L(R)} t) \) is the modulated intermode coupling rate with strength \( \Omega_{L(R)} \) and \( F_{L(M)} \) is a generalized external force. The normal modes of Eq. 1 are:

\[
x_{\pm} = (g_{L}x_{L} + g_{R}x_{R} \pm g_{0}x_{M})/(\sqrt{2}g_{0}) \quad \text{and} \quad x_{D} = (-g_{R}x_{L} + g_{R}x_{R})/g_{0}, \quad \text{where} \quad g_{0} = \sqrt{\frac{g_{L}}{\gamma_{L}} + \frac{g_{R}}{\gamma_{R}}}.
\]

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The experimental results shown in Fig. 5a-c are reproduced in Fig. 5d-f using a classical coupled-resonator model:

\[
\dot{x}_{L} + \gamma_{L}\dot{x}_{L} + \omega_{L}^{2}x_{L} = F_{L} - g_{L}x_{M} \quad (1a)
\]

\[
\dot{x}_{M} + \gamma_{M}\dot{x}_{M} + \omega_{M}^{2}x_{M} = F_{M} = -g_{L}x_{L} - g_{R}x_{R} \quad (1b)
\]

\[
\dot{x}_{R} + \gamma_{R}\dot{x}_{R} + \omega_{R}^{2}x_{R} = F_{R} - g_{R}x_{M}. \quad (1c)
\]

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FIG. 5: Analog of coherent population trapping among three dynamically coupled micromechanical beams. Probe voltage ($V_R$) dependence of mode M (a), mode R (b), and mode L (c) for $V_{DC} = 10$ V when the pump frequency is tuned to $\Delta_L = \omega_M - \omega_L$ and the probe frequency is tuned to $\Delta_R = \omega_M - \omega_R$. The pump voltage is here fixed at $V_L = 10\, V_{pp}$. Simulated results for (a), (b), and (c) are shown in (d), (e), (f), respectively. Cross-sections of (a), (b), and (c) at $V_R = 10\, V_{pp}$ are shown in (g), (h), and (i).

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