Midgap Andreev resonant state affected by superconducting proximity effect of high-$T_c$ cuprate attached to diffusive normal metal

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Abstract. We report how a midgap Andreev resonant state (MARS) is affected by the superconducting proximity effect at an interface of diffusive normal metal/insulator/$d_{x^2-y^2}$-wave superconductor (DN/I/DS) junctions. A zero-bias conductance peak (ZBCP) was observed for the (110)-oriented interface in Ag/SiO/Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi-2212) planar junctions. The experimental ZBCP was analyzed by the extended circuit theory, which was improved by the introduction of spatial variations of the pair potential in a DS electrode near the junction interface. Our experimental results are well explained by the extended circuit theory for DN/I/DS junctions.

1. Introduction

The midgap Andreev resonant state (MARS) is created at an interface of unconventional superconducting junctions [1], and importantly influences on various physical quantities [2]. One of those is a zero-bias conductance peak (ZBCP), which is frequently observed in high-$T_c$ cuprate junctions. The consistency between the MARS theory and experimental results has been studied in details [3]. Recently, by using the generalized boundary condition of the Keldysh-Nambu Green’s function formalism, the circuit theory has been developed to an unconventional superconducting junction attached to a diffusive normal metal (DN) [4]. The circuit theory includes both of the MARS and the coherent Andreev reflection, and the coherent Andreev reflection induces the superconducting proximity effect in the DN conductor near the junction interface. On the other hand, since the pair potential of a $d_{x^2-y^2}$-wave superconductor (DS) is anisotropic, the amplitude of the pair potential near a surface or an interface is significantly reduced. This suppression of the pair potential causes the formation of the MARS at the Fermi energy (zero energy) around nodal directions [5].

In this paper, for the (110)-oriented interface regarded as the nodal direction in high-$T_c$ cuprates, experimental ZBCP’s in Ag/SiO/Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi-2212) planar junctions will be analyzed by the extended circuit theory for diffusive normal metal/insulator/$d_{x^2-y^2}$-wave superconductor (DN/I/DS) junctions. Furthermore, we will focus on analytical results of the ZBCP from the viewpoint of the superconducting proximity effect at a junction interface.
2. Experimental procedure
Bi-2212 single crystals have been grown by the traveling solvent floating-zone (TSFZ) method. The transition temperature $T_c$ of single crystals was 87–90 K, which was decided by the resistivity and magnetic susceptibility measurements. We have prepared planar tunnel junctions by using these single crystals as follows: A single crystal with a size of approximately 10 mm $\times$ 3 mm $\times$ 50 $\mu$m was molded into a block of epoxy resin. The block of epoxy resin was cut to expose the (110)-oriented surface of the single crystals, which was confirmed by the X-ray diffraction (XRD) pattern [6]. The exposed surface was smoothed by diamond pastes with lubricant after roughly polished by sandpaper. At that point, SiO was evaporated on the (110)-oriented surface as an insulating barrier, and the thickness of SiO layer was changed between 0 Å and 300 Å for each planar tunnel junction. Ag was deposited on the SiO thin film through a metal mask patterned with the counterelectrode of strips with 0.5 mm width. The thickness of the Ag layer was controlled to be 1500 Å. The tunneling conductance spectra were measured in a setup depicted schematically in the inset of figure 1, using a lock-in amplifier. The temperature ranged from 4.2 K to 300 K, using a temperature controller with a stability of more than 0.1 K.

3. Results and discussion
3.1. Theoretical model of midgap Andreev resonant state
In order to analyze experimental results, we will construct the theory of tunneling conductance, taking into account of the superconducting proximity effect, i.e., spatial variations of the pair potential $\Delta_{0}(x)$ in the DS electrode and the resistance $R_d$ in the DN conductor. Using $D_\pm(x)$ and $F_+(x)$ obeyed the following Riccati type equations [5]

$$\hbar|v_F|\frac{\partial}{\partial x}D_\pm(x) = \pm \left[2\omega_m D_\pm(x) + \Delta(\phi_\pm, x)D_\pm^2(x) - \Delta^*(\phi_\pm, x)\right], \quad (1)$$

$$\hbar|v_F|\frac{\partial}{\partial x}F_+(x) = -2\omega_m F_+(x) + \Delta^*(\phi_+, x)F_+^2(x) - \Delta(\phi_+, x), \quad (2)$$

we can write the quasiclassical Green’s function in a compact form

$$\hat{g}_{++}(\phi_+, x) = i\left[\frac{2}{1-D_+(x)F_+(x)}\left(\frac{1}{iD_+(x)} - \frac{iF_+(x)}{-D_+(x)F_+(x)}\right) - 1\right]. \quad (3)$$

Here, initial conditions of these equations are

$$D_\pm(\infty) = \frac{\Delta^*(\phi_\pm, \infty)}{\omega_m \pm \sqrt{\omega_m^2 + |\Delta(\phi_\pm, \infty)|^2}}, \quad F_+(0) = \frac{1 - \sigma_N(\phi)}{D_-(0)}, \quad \sigma_N(\phi) = \frac{4\cos^2\phi}{4\cos^2\phi + Z^2}, \quad (4)$$

where $\omega_m$ is the Matsubara frequency and $\sigma_N(\phi)$ stands for tunneling conductance when the system is in the normal state. $Z$ is an insulating barrier height given by $Z = 2mH/(\hbar^2k_F)$ with Fermi momentum $k_F$ and effective mass $m$. $\phi_\pm$ means that $\phi_+ = \phi$ and $\phi_- = \pi - \phi$.

The pair potential in the DS electrode is given by [5]

$$\Delta(\phi, x) = \sum_{0 \leq m < \omega_c/(2\pi T)} \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\phi' \sum_{\alpha=\pm} V(\phi, \phi'_\alpha) \left[\hat{g}_{\alpha\alpha}(\phi'_\alpha, x)\right]_{12}, \quad (5)$$

with $\hat{g}_{-\alpha}(\phi_-, x) = -\hat{g}^{\dagger}_{-\alpha}(\phi_-, x)$, where $\omega_c$ is the cutoff energy and $[\hat{g}_{\alpha\alpha}(\phi_\alpha, x)]_{12}$ means the 12 element of $\hat{g}_{\alpha\alpha}(\phi_\alpha, x)$. Here, $V(\phi, \phi'_\alpha)$ is the effective interelectron potential of the Cooper pair and is given by $V(\phi, \phi'_\alpha) = 2V_d \cos\left[2(\phi - \theta)\right] \cos\left[2(\phi'_\alpha - \theta)\right]$, where $V_d$ denotes the attractive potential of $d_{x^2-y^2}$-wave symmetry and $\theta$ is the angle between normal to the interface and the lobe direction of the $d_{x^2-y^2}$-wave pair potential. The final $\Delta(\phi_\pm, x)$ and $\hat{g}_{\pm\pm}(\phi_\pm, x)$
are obtained using equations (1)–(3) and (5) after we repeat the iteration process until the sufficient convergence is obtained [5]. Therefore, we can calculate the tunneling conductance at an interface based on the self-consistently determined pair potential \( \Delta(\phi, x) \). The resulting tunneling conductance \( \sigma_S(E, \phi) \) in the superconducting state is given by [5]

\[
\sigma_S(E, \phi) = \frac{1 + \sigma_N(\phi)|\Gamma_{S+}(E, \phi_+, 0)|^2 + |\sigma_N(\phi) - 1||\Gamma_{S+}(E, \phi_+, 0)||\Gamma_{S-}(E, \phi_-, 0)||^2}{[1 + |\sigma_N(\phi) - 1||\Gamma_{S+}(E, \phi_+, 0)||\Gamma_{S-}(E, \phi_-, 0)||^2].}
\]

In the above, \( \Gamma_{S\pm}(E, \phi_\pm, x) \) which is obtained by analytic continuation from \( D_{\pm}(x) \) is proportional to the ratio of the wave function of a hole and that of an electron, respectively [5]. The quantity \( \Gamma_{S\pm}(E, \phi_\pm, x) \) satisfies the following equations.

\[
i\hbar v_F x \frac{\partial}{\partial x} \Gamma_{S+}(E, \phi_+, x) = 2E \Gamma_{S+}(E, \phi_+, x) - \Delta(\phi_+, x) \Gamma_{S+}^2(E, \phi_+, x) - \Delta^*(\phi_+, x),
\]

\[
i\hbar v_F x \frac{\partial}{\partial x} \Gamma_{S-}(E, \phi_-, x) = 2E \Gamma_{S-}(E, \phi_-, x) - \Delta^*(\phi_-, x) \Gamma_{S-}^2(E, \phi_-, x) - \Delta(\phi_-, x).
\]

In addition, the circuit theory in DN/I/DS junctions guarantees that the MARS channels sufficiently give influences in line shapes of \( \sigma_T(eV) \) rather than the superconducting proximity effect in the case of our experimental junction geometry [7]. Hence, in analysis for \( \theta = \pi/4 \), such as the nodal direction, the circuit theory is described by a simplified formula, just expressed by the Ohm’s law: an elementary sum of \( R_{R_d=0} \) and \( R_d \) in a superconducting state, where \( R_{R_d=0} \) is the total resistance of the tunnel junction in the condition of \( R_d = 0 \). Then, the normalized tunneling conductance \( \sigma_T(eV) \) can be approximated by the following simplified equations [4]:

\[
\sigma_T(eV) = \frac{R_b + R_d}{R_{R_d=0} + R_d}, \quad R_{R_d=0} = R_b \frac{\langle \sigma_N(eV) \rangle}{\langle \sigma_S(eV) \rangle}, \quad R_b = R_0 \int_{-\pi/2}^{\pi/2} d\phi \frac{2}{\sigma_N(\phi) \cos \phi},
\]

\[
\langle \sigma_S(eV) \rangle = \frac{1}{4k_B T} \int_{-\infty}^{\infty} dE \int_{-\pi/2}^{\pi/2} d\phi e^{-\lambda \phi^2} \sigma_N(\phi) \sigma_S(E, \phi) \text{sech}^2 \left( \frac{E + eV}{2k_B T} \right) \cos \phi,
\]

\[
\langle \sigma_N(eV) \rangle = \frac{1}{4k_B T} \int_{-\infty}^{\infty} dE \int_{-\pi/2}^{\pi/2} d\phi e^{-\lambda \phi^2} \sigma_N(\phi) \text{sech}^2 \left( \frac{E + eV}{2k_B T} \right) \cos \phi,
\]

where we assume \( \Delta(\phi, x) = \Delta_0(x) \cos[2(\phi - \theta)] \) as the spatial dependent pair potential in the DS electrode. Here, \( \Delta_0(x) \) denotes the maximum amplitude of the pair potential of the superconducting state at the position \( x \). In the above equations, \( R_0 \) is the Sharvin resistance at the junction interface and \( k_B \) is the Boltzmann’s constant. Further \( \lambda \) is added in order to introduce the probability of tunneling directions at the junction interface.

3.2. Experimental tunneling conductance and analytical results

We try to analyze experimental ZBCP’s of Ag/SiO/Bi-2212 planar junctions by the circuit theory developed in section 3.1. We anticipate obtaining good fitting results using the extended circuit theory because the calculated height \( \sigma_T(0) \) of the ZBCP is strongly suppressed with increasing the resistance \( R_d \) in the DN conductor. Figure 1 shows the spatial dependence of the pair potential \( \text{Re}[\Delta_0(x)] \) in the DS electrode. \( \text{Re}[\Delta_0(0)]/\Delta_0 \approx 0.11 \), then it is found that \( \text{Re}[\Delta_0(x)] \) sufficiently reduces at the DN/DS interface for the nodal direction. Figure 2 represents the fitting result for an experimental ZBCP at \( T = 5.0 \) K using equations (6) and (9)–(11). Here, the slight discrepancy between the experimental data and theoretical calculation in figure 2 probably comes from the simplification of the insulating barrier in the model [8]. As shown in figure 2, although the Dynes’s broadening parameter \( \Gamma \) [9] does not exist in the analysis, the analytical result indicates a good agreement between the experimental ZBCP and the theoretical curve. The parameter values of the analytical result are also given in table 1.
Table 1. Resulting parameters in fitting procedure for tunneling conductance for the (110)-oriented junction. $Z$, $\lambda$ and $R_d/R_b$ are dimensionless constant.

| $\Delta_0$ (meV) | $Z$ | $\lambda$ | $R_d/R_b$ | $\theta$ (rad) | $T$ (K) |
|------------------|-----|-----------|-----------|----------------|--------|
| 26.4             | 2.6 | 13.0      | 0.44      | $\pi/4$        | 5.0    |

4. Conclusions
We have developed the circuit theory for DN/I/DS junctions into the systems containing spatial variations of the pair potential in the DS electrode near the junction interface. The spatial variation of the pair potential was determined self-consistently on the basis of the quasiclassical theory and the Green’s function method. The pair potential at the junction interface was sufficiently reduced, and the experimental ZBCP was quantitative agreement with the theoretical calculation, due to finite resistance in the DN conductor. Thus, we were able to manifest that the model constructed for the nodal direction is applicable to analysis of actual experimental data of high-\(T_c\) cuprate junctions, rather than the MARS theory for the $d_{x^2−y^2}$-wave symmetry.

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