Scaling Behavior at High $p_T$ and the $p/\pi$ Ratio

Rudolph C. Hwa$^1$ and C. B. Yang$^{1,2}$

$^1$Institute of Theoretical Science and Department of Physics
University of Oregon, Eugene, OR 97403-5203, USA

$^2$Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, P. R. China

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We first show that the pions produced at high $p_T$ in heavy-ion collisions over a wide range of high energies exhibit a scaling behavior when the distributions are plotted in terms of a scaling variable. We then use the recombination model to calculate the scaling quark distribution just before hadronization. From the quark distribution it is then possible to calculate the proton distribution at high $p_T$, also in the framework of the recombination model. The resultant $p/\pi$ ratio exceeds one in the intermediate $p_T$ region where data exist, but the scaling result for the proton distribution is not reliable unless $p_T$ is high enough to be insensitive to the scale-breaking mass effects.

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I. INTRODUCTION

There are three separate and independent aspects about the hadrons produced at large transverse momentum ($p_T$) in heavy-ion collisions at high energies that collectively contribute to a coherent picture to be addressed in this paper. One is the existence of a scaling behavior at large $p_T$ that we have found by presenting the data in terms of a new variable. Another is the issue about the surprisingly large proton-to-pion ratio at moderate $p_T$ ($\sim 2 - 3$ GeV/c) discovered by PHENIX \cite{1} in central $AuAu$ reactions at $\sqrt{s} = 130$ and 200 GeV. The third issue concerns the hadronization process relevant for the formation of hadrons at large $p_T$ and the applicability of the recombination model \cite{2}. It is our goal to show that, in light of the scaling behavior of the $\pi^0$ produced, the recombination mechanism naturally gives rise to a $p/\pi$ ratio that exceeds 1 in the $2 < p_T < 3$ GeV/c range.

Particle production in heavy-ion collisions at very high energies is usually described in terms of hydrodynamical flow \cite{3}, jet production at high $p_T$ \cite{4}, thermal statistical model \cite{5}, or a combination of various hadronization mechanisms \cite{6}. In none of the conventional approaches does one expect protons to be produced at nearly the same rate as the pions. If all hadrons with $p_T > 2$ GeV/c are regarded as products of jet fragmentation, then the known fragmentation functions of quark or gluon jets would suppress proton relative to pion by the sheer weight of the proton mass. Such a discrepancy from the observed data led some to regard the situation as an anomaly and proposed the gluonic baryon junction as a mechanism to enhance the proton production rate \cite{7}. Their predictions remain to be checked by experiments.

The parton fragmentation functions have been used even at low $p_T$ in string models where the production of particles in hadronic collisions is treated as the fragmentation of diquarks, as done in the dual parton model \cite{8}. There has been a long-standing dichotomy on whether particle production in the fragmentation region can better be described by fragmentation \cite{8,9} or recombination \cite{10}. It is possible that the two pictures might be unified in a more comprehensive treatment of hadronization in the future. Here we extend the recombination model to the central region at large $p_T$. It should be recognized that an essential part of the recombination model is the determination of the distribution functions of the quarks and antiquarks that are to recombine. In the case of large-$p_T$ hadrons the underlying physics is undoubtedly hard collisions of partons and the associated radiation of gluons. If the parton distributions can be calculated just before hadronization, then the final step of recombination can readily be completed. If those distributions cannot be determined in pQCD, then the step between the initiating large-$p_T$ parton and the resultant hadrons may efficiently be described by a fragmentation function, determined phenomenologically from experiments. Thus in that sense the two approaches, recombination and fragmentation, are not contradictory, but complementary.

We state from the outset that no attempt will be made here to perform a first-principle calculation of the parton distributions at large $p_T$ before recombination. However, from the observed data on pion production in central $AuAu$ collisions at the Relativistic Heavy-Ion Collider (RHIC), it is possible to work backwards in the recombination model to determine the quark (and antiquark) distribution at large $p_T$. On the basis of the quark distributions inferred, it is then possible to calculate the proton distribution in the recombination model. The basic idea is that if there is a dense system of quarks and antiquarks produced in a heavy-ion collision whatever the dynamics responsible for them may have been (gluons having been converted to $q\bar{q}$ pairs before hadronization), then the formation of pions and protons (and whatever else) is prescribed by the recombination model without any arbitrariness in normalization and momentum dependence.

One limitation of the recombination model as it stands at present is that it is formulated in a frame-independent way in terms of momentum fractions and is therefore inapplicable to a system where the particle momenta are low and the mass effects are large. The physics of recomb.
bination is still valid at low momentum, but the details of the wave functions of the constituent quarks become important; they have not been built into the recombination function that takes the simplest form in the infinite momentum frame. Thus our calculation of particles produced at midrapidity is not reliable when $p_T$ is of the order of the masses of the hadrons under consideration. For protons we can trust the results only for $p_T > 3$ GeV/c. For pions the lower limit of validity can be pushed much lower.

Since our approach makes crucial use of the experimental data on the pion spectrum as the input, it is essential to relate the spectra determined at different energies to an invariant distribution so that the scale-invariant recombination model can be applied. To discover the existence of an invariant distribution with no theoretical prejudices is a problem worthy in its own right. Fortunately, that turns out to be possible. The analysis for that part of the study will be presented below first to emphasize its independence from the theoretical modeling of hadronization. It should be mentioned that the scaling of transverse mass spectra has been investigated recently [11]. The emphasis there has been on the dependence of hadronization. It should be mentioned that the scaling property in Fig. 1 is also unchanged, the only modifications being the scales of the horizontal and vertical axes. Thus the absolute magnitude of the dimensionless variable $\sqrt{s}/K$, which will be referred to as the scaling function $\Phi(z)$, which can be parametrized by

$$\Phi(z,K) = K^2 f(p_T,s) = \frac{1}{2\pi z} \frac{dN}{dz}.$$

(3)

We adjust $K$ for each $s$ and check whether all three data sets coalesce into one universal dependence on $z$, which we would simply label as $\Phi(z)$, if it is possible.

In Fig. 1 we show $\Phi(z)$, where the three symbols represent the three data sets for the three energies. Evidently, the universality exists and is striking. While this behavior needs to be confirmed by more data, and the theoretical implication remains to be explored, the existence of this scaling behavior is a significant phenomenological property of the $p_T$ distributions that suggests some underlying simplicity. It is like the KNO scaling of the multiplicity distributions $P(n,s)$ in $pp$ collisions, where for $\sqrt{s} < 200$ GeV they can be expressed by one universal scaling function $\psi(z)$, with $z = n/(n)$ [12,16].

The values of $K$ that are used for the plot in Fig. 1 are in units of GeV: $K = 1(200), 0.9(130)$ and $0.717(17)$, the quantities in the parentheses being the values of $\sqrt{s}$. The $\sqrt{s}$ dependence of $K$ forms nearly a straight line, as shown in Fig. 2. Since the high and low energy data differ both in colliding nuclei and in centrality, one does not expect strict regularity in how $K$ depends on $\sqrt{s}$. Nevertheless, an approximate linear dependence is a simple behavior expected on dimensional grounds. The straight line in Fig. 2 corresponds to the best fit

$$K(s) = 0.69 + 1.55 \times 10^{-3} \sqrt{s},$$

(4)

where $\sqrt{s}$ is in units of GeV. It should be recognized that the normalization of $K(s)$ is arbitrary; it is chosen to be 1 at $\sqrt{s} = 200$ GeV for simplicity. If it is normalized to some other value at that point, the linear behavior in Fig. 2 is unchanged, only the scale of the vertical axis is shifted accordingly. The scaling property in Fig. 1 is also unchanged, the only modifications being the scales of the horizontal and vertical axes. Thus the absolute magnitude of the dimensionless variable $z$ has no significance.

If the $z$ dependence of $\Phi(z)$ in Fig. 1 were strictly linear, so that it is a power-law dependence

$$\Phi(z) \propto z^\alpha,$$

(5)

then there would be no relevant scale in the problem. The fact that it is not a straight line implies that there is an intrinsic scale in the $p_T$ problem, which is hardly surprising. What is significant is that while there is no strict scaling in $z$, there is no explicit dependence on $s$. That is, at any energy we have the same universal function $\Phi(z)$, which will be referred to as the scaling behavior in $s$. That function can be parametrized by

$$\Phi(z) = 1500 \left(z^2 + 2\right)^{-4.9},$$

(6)

which is represented by the smooth curve in Fig. 1. For large enough $z$ Eq. (6) does have the form of the power
law given in Eq. (5) with \( \alpha = 9.8 \). It is a succinct statement of the universal properties at high \( p_T \). The departure from Eq. (5) at small \( z \) reflects the physics at low \( p_T \). Since there is no data on \( \pi^0 \) for \( p_T < 1 \) GeV/c, the extrapolation of \( \Phi(z) \) to \( z < 1 \) is not reliable. However, there is a more accurate determination of \( \Phi(z) \) that includes the low \( z \) region when the charge \( \pi^+\) data are considered; it is given in [17], and is not needed here.

![FIG. 1: Scaled transverse momentum distribution of produced \( \pi^0 \). Data are from Ref. [12, 13]. The solid line is a fit of the data by Eq. (6).](image)

Note that there is no fixed scale in \( p_T \) that separates the high- and low-\( p_T \) physics. Equation (6) gives a smooth transition from one to the other in the variable \( z \), thus implying different ranges of values of the transition \( p_T \) at different \( s \).

While Eq. (6) gives a good parametrization of the scaling function \( \Phi(z) \) throughout the whole range of \( z \), one notices, however, that the WA98 data at 17 GeV shows a slight departure from \( \Phi(z) \) at the high \( z \) end of that data set. It should be recognized that those data points have \( p_T > 3 \) GeV/c, which represents a huge fraction of the available energy at \( \sqrt{s} = 17 \) GeV. In fact, one expects the violation of universality to be more severe at higher \( z \) at that \( \sqrt{s} \), since energy conservation would suppress the inclusive cross section at higher \( p_T \). What is amazing is that most of the WA98 data points are well described by \( \Phi(z) \), even though the corresponding \( p_T \) values take up a much larger fraction of the available energy than the other data points from RHIC. It demonstrates the significance of the variable \( z \) in revealing the scaling property.

![FIG. 2: The dependence of \( K(s) \) on \( \sqrt{s} \). The line is a linear fit.](image)

**III. PION AND QUARK DISTRIBUTIONS IN THE RECOMBINATION MODEL**

Having found a scaling distribution for the produced \( \pi^0 \) independent of \( s \), we now consider the hadronization process in the recombination model in search for an origin of such a scaling behavior. In previous investigations the recombination model has been applied only to the fragmentation region where the longitudinal momenta are large and the transverse momenta are either held fixed at low \( p_T \) or integrated over [2, 17, 18]. We now consider the creation of pions in the central region of \( AA \) collisions and study the \( p_T \) dependence. Unlike the former case where the longitudinal momentum fractions of the partons are essentially known (from the structure functions), the \( p_T \) distributions of the partons in the latter case are essentially unknown. Indeed, it is the aim of this section to determine the parton \( p_T \) distributions from the \( \pi^0 \) distribution found in the previous section.

Let us start by writing down the basic equation for recombination in the 3-space

\[
E \frac{d^3 N_\pi}{dp} = \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \mathcal{F}(\vec{p}_1, \vec{p}_2) R_\pi(\vec{p}_1, \vec{p}_2, \vec{p})
\]  

where the left-hand side (LHS) is the inclusive distribution of pion with energy-momentum \((E, \vec{p})\). \( \mathcal{F}(\vec{p}_1, \vec{p}_2) \) is
the probability of having a quark at $p_i^y$ and an antiquark at $p_{\bar{i}}^y$ just before hadronization. $R_\pi(p_1, \bar{p}_2, \bar{\rho})$ is the invariant distribution, $Ed^3N_\pi^y/d^3p$, of producing a pion at $p^y$ given a $q$ at $p_i^y$ and a $\bar{q}$ at $p_{\bar{i}}^y$. Note that $R_\pi$ has the dimension (momentum)$^{-2}$, same as the LHS.

Writing the phase-space density in the form

$$\frac{d^3p}{E} = dy d\phi p_T dp_T,$$  \hspace{1cm} (8)

we define the inclusive distribution in $p_T$, averaged over $y$ and $\phi$, as follows,

$$\frac{d^3N_\pi}{p_T dp_T} = \frac{1}{\Delta y} \int_{\Delta y} dy \frac{1}{2\pi} \int_0^{2\pi} d\phi E \frac{d^3N_\pi}{dp^3},$$  \hspace{1cm} (9)

where $\Delta y$ is limited to one unit of rapidity in the central region. Our focus will be on the transverse momentum fractions in the hadron can vary between 0 and 1, but the deviation in the momentum components of the partons transverse to the hadron $\bar{\rho}$ must be severely limited because of the limited transverse size of the hadron. Thus we write

$$R_\pi(p_1, \bar{p}_2, \bar{\rho}) = R_\pi^0 \delta(y_1 - y_2) \delta(\phi_1 - \phi_2) \delta(\frac{y_1 + y_2}{2} - y) \delta^2(p_{1T} + p_{2T} - p_T),$$  \hspace{1cm} (10)

where $R_\pi^0$ is dimensionless, since $\delta^2(p_{1T} + p_{2T} - p_T)$ has the dimension of $R_\pi(p_1, \bar{p}_2, \bar{\rho})$. If this $\delta$-function is further written in the collinear form due to the $\delta(\phi_1 - \phi_2)$ in Eq. (10)

$$\delta^2(p_{1T} + p_{2T} - p_T) = \delta(\frac{\phi_1 + \phi_2}{2} - \phi) \frac{1}{p_T} \delta(p_{1T} + p_{2T} - p_T),$$  \hspace{1cm} (11)

then Eq. (10) can be reduced to the 1D form

$$\frac{dN_\pi}{p_T dp_T} = \int dp_{1T} dp_{2T} F(p_{1T}, p_{2T}) R_\pi^0 \frac{p_T^{-2}}{p_T} \delta\left(p_{1T} + p_{2T} - 1\right),$$  \hspace{1cm} (12)

where $F(p_{1T}, p_{2T})$ is the $q\bar{q}$ distribution in $p_{1T}$ averaged over $y$ and $\phi$.

We can reexpress this equation in terms of the scaling variable $z = p_T/K$, introduced in Eq. (2), and obtain

$$\frac{dN_\pi}{z dz} = \int dz_1 dz_2 F(z_1, z_2) R_\pi(z_1, z_2, z)$$  \hspace{1cm} (13)

where

$$F(z_1, z_2) = K^4 F(p_{1T}, p_{2T})$$  \hspace{1cm} (14)

$$R_\pi(z_1, z_2) = R_\pi^0 z^{-2} \delta\left(\frac{z_1 + z_2}{z} - 1\right).$$  \hspace{1cm} (15)

Since $F(p_{1T}, p_{2T})$ is the parton density in $p_{1T}, p_{2T}$, $F(z_1, z_2)$ is the corresponding dimensionless density in $z_1 z_2$. Equation (13) is now our basic formula for recombination in the scaled transverse-momentum variable. The total number of $q$ and $\bar{q}$ is $\int dz_1 dz_2 F(z_1, z_2, z)$, which should be invariant under a change of scale

$$z = \lambda x$$  \hspace{1cm} (16)

so that

$$x = p_T/K', \quad K' = \lambda K.$$  \hspace{1cm} (17)

The corresponding change on $F(z_1, z_2)$ is that it becomes

$$F'(x_1, x_2) = \lambda^4 F(z_1, z_2).$$  \hspace{1cm} (18)

Thus the normalization of $F(z_1, z_2)$ is scale dependent, as it should in view of Eq. (14).

So far the recombination function $R_\pi(z_1, z_2, z)$ is not fully specified because $R_\pi^0$ has not been. In Eq. (16) the factor $z^{-2}$ is associated with the dimension of the pion density, and the $\delta$-function with momentum conservation. To introduce the pion wave function in terms of the constituent quarks, we rewrite Eq. (15) as

$$R_\pi(z_1, z_2, z) = R_\pi^0 z^{-2} G_\pi(\xi_1, \xi_2),$$  \hspace{1cm} (19)

where $R_\pi^0$ is a normalization constant to be determined and $G_\pi(\xi_1, \xi_2)$ is the valon distribution of the pion [14]. Since the recombination of a $q$ and $\bar{q}$ into a pion is the time-reversed process of displaying the pion structure, the dependence of $R_\pi(z_1, z_2, z)$ on the pion structure is expected. During hadronization the initiating $q$ and $\bar{q}$ dress themselves and become the valons of the produced hadron without significant change in their momenta. The variable $\xi_i$ in Eq. (19) denotes the momentum fraction of the $i$th valon, i.e.,

$$\xi_i = z_i/z,$$  \hspace{1cm} (20)

which is denoted by $y_i$ in the valon model [2, 14], a notation that cannot be repeated here on account of the rapidity variables already used in Eq. (10). In general, the valon distribution of a hadron $h$ has a part specifying the wave-function squared, $G_h$, and a part specifying momentum conservation

$$G_h(\xi_1, \cdots) = \tilde{G}_h(\xi_1, \cdots) \delta\left(\sum_i \xi_i - 1\right),$$  \hspace{1cm} (21)

where the functional form of $\tilde{G}_h$ is determined phenomenologically. Although for proton $G_p$ is found to be
highly nontrivial [19], for pion $\tilde{\gamma}_\pi$ turns out to be very simple [10]

$$\tilde{\gamma}_\pi(\xi_1, \xi_2) = 1,$$

(22)

which is a reflection of the fact that the pion mass is much lower than the constituent quark masses, so tight binding results in large uncertainty in the momentum fractions of the valons. Equation (22) implies that the valon momenta of the pion is uniformly distributed in the range $0 < \xi < 1$.

What remains in Eq. (19) for us to determine is $R^\eta_z$. At this point we need to be more specific about the quark and antiquark that recombine. If the colors of $q$ and $\bar{q}$ are considered, then the probability of forming a color singlet pion is 1/9 in $3 \times 3$. Similarly, for three quarks forming a proton the probability is 1/27 in $3 \times 3 \times 3$. In the parton distributions, $F_{q\bar{q}}$ for pion production involves two color triplets and $F_{qq\bar{q}}$ for proton production involves three color triplets so the color factors work out just right in that the factors of 9 for $qq \bar{q}$ and 27 for $qqq \bar{q}$ are cancelled by the corresponding inverse factors in the recombination probabilities. In other words, for the $p/\pi$ ratio to be considered later, we can ignore the factors associated with the color degrees of freedom and proceed with the determination of $F_{q\bar{q}}$ without specifying the quark colors and summing over them.

The situation with flavor is not the same. For a $u\bar{u}$ pair and a $d\bar{d}$ pair, they can form $\pi^0$ and $\eta$, respectively. The branching ratio of $\eta$ to $3\pi^0$ is 32.5% and to $\pi^+\pi^-\pi^0$ is 22.6%. Thus for every $\eta$ produced there is on average 1.2$\pi^0$. Due to the higher mass of $\eta$ we make the approximation that the rate of indirect production of $\pi^0$ via $\eta$ is roughly the same as the direct production from $u\bar{u}$ and $d\bar{d}$. If we now use $q\bar{q}$ to denote either $u\bar{u}$ or $d\bar{d}$, but not both $u\bar{u}$ and $d\bar{d}$, then each pair of $q\bar{q}$ leads to one $\pi^0$.

Since in a heavy-ion collision there are many quarks and antiquarks produced in the central region, it is reasonable to assume that the $q$ distribution is independent of the $\bar{q}$ distribution so we can write $F_{q\bar{q}}$ in the factorizable form

$$F_{q\bar{q}}(z_1, z_2) = F_q(z_1) F_{\bar{q}}(z_2),$$

(23)

where $F_q$ stands for either $u$ or $d$ distributions, and similarly for $F_{\bar{q}}$, but for $\pi^0$ production $\bar{q}$ should be the antiquark partner of $q$.

The fact that we consider $\eta$ production above, but not the vector meson $\rho$ requires an explanation. We defer that discussion until the next section, after we have presented the formalism for the production of protons.

Returning now to the normalization of $R_\pi(z_1, z_2, z)$, we note that, using Eqs. (19), (21) and (22),

$$\int dzzR_\pi(z_1, z_2, z) = \int_0^z \frac{dz_1}{Z} R^\pi_{z_1} \delta \left( \frac{z_1 + z_2 - z}{z} - 1 \right)$$

$$= R^\pi_z$$

(24)

is the probability that a $q$ at $z_1$ and a $\bar{q}$ at $z_2$ recombine to form a pion at any $z$. According to our counting in the second paragraph above, the total probability for $q\bar{q} \rightarrow \pi^0$ integrated over all momenta is

$$\int_0^Z \frac{dz_1}{Z} \int_0^Z \frac{dz_2}{Z} \int dz z R_\pi(z_1, z_2, z) = 1,$$

(25)

where $Z$ is the maximum $z_i$, whatever it is. This normalization condition is scale invariant, and we find, using Eq. (24), that

$$R_\pi^0 = 1.$$  

(26)

Putting Eqs. (19) - (23) and (25) in (15) we obtain

$$\frac{dN_\pi}{zdz} = \int dzz_2 \frac{z_1z_2}{z} F_q(z_1) F_{\bar{q}}(z_2) \delta(z_1 + z_2 - z).$$

(27)

This is obtained from Eq. (9) where $y$ and $\phi$ are both explicitly averaged over. The LHS is to be identified with $\Phi(z)$. Note that the $1/2\pi$ factors in Eqs. (11) and (13), where $\Phi(z)$ is defined, are there to render $f(p_T, s)$ an average distribution in $\phi$; that is the notation for the experimental distribution, defined in [12]. The distribution defined by us in Eq. (9) already includes the $1/2\pi$ factor, so our $dN_\pi/zdz$ is just the experimental $\Phi(z)$. As we have mentioned earlier, the normalization of $z$ has no significance. By means of a scale change in Eq. (16) we can move from $z$ to $x$, or vice-versa, without changing the scale invariant form of Eq. (24). In Eq. (9) we found $\Phi(z)$ to have the form

$$\Phi(z) = A \left( z^2 + c \right)^{-n}.$$  

(28)

If we change $z$ to $x$ according to Eq. (16), then by keeping the total number of pions invariant, i.e.,

$$\int dz z \Phi(z, K) = \int dx x \Phi'(x, K'),$$

(29)

we have

$$\Phi'(x, \lambda K) = \lambda^2 \Phi(\lambda x, K).$$

(30)

It thus follows that

$$\Phi'(x) = \lambda^{2(1-n)} A \left( x^2 + c/\lambda^2 \right)^{-n}.$$  

(31)

Similarly, in the $x$ variable the transformed quark distributions is

$$F_q'(x_1, K') = \lambda^2 F_q(z_1, K).$$  

(32)

Without having to specify the arbitrary scale factor $\lambda$, let us work with the $z$ variable and rewrite Eq. (27) as

$$\Phi(z) = \int_0^z dzz_1 \left( 1 - \frac{z_1}{z} \right) F_q(z_1) F_{\bar{q}}(z - z_1).$$  

(33)

We must now consider how the $q$ and $\bar{q}$ distributions differ. Unlike the structure functions of the nucleon, where $q$ and $\bar{q}$ have widely different distributions, we
are here dealing with the partons at high \( p_T \) in heavy-ion collisions just before recombination. The dynamics underlying their \( p_T \) dependences is complicated. Many subprocesses are involved, which include hard scattering, gluon radiation, jet quenching, gluon conversion to quark pairs, thermalization, hydrodynamical expansion, to name a few familiar ones. At very large \( p_T \) there are far more quark jets than antiquark jets, since the valence quarks have larger longitudinal momentum fractions than the sea quarks. By hard scattering the quarks therefore can acquire larger \( p_T \) than the antiquarks. Thus in that way one would expect the \( p_T \) distribution of the quarks to be very different from that of the antiquarks. However, that view does not apply to our problem. Those are the \( q \) and \( \bar{q} \) that initiate jets, along with jets initiated by gluons. The conventional approach is to follow the jet production by jet fragmentation, which can be modified by gluons. The conventional approach is to follow the jet production by jet fragmentation, which can be modified by the dense matter that the initiating partons traverse. As discussed earlier, our approach is not to delve into the dynamical origins of the \( q \) and \( \bar{q} \) distributions, but to consider the recombination of \( q \) and \( \bar{q} \) just at the point of hadronization. Such \( q \) and \( \bar{q} \) are not the partons that initiate jets, but are the parton remnants after the hard partons radiate gluons which subsequently convert to \( q\bar{q} \) pairs. Those parton remnants have similar momentum distribution for \( q \) and \( \bar{q} \), since gluon conversion creates \( q \) and \( \bar{q} \) on equal basis; those partons are the ones that recombine to form hadrons. They are not to be confused with the jet-initiating hard partons that fragment into hadrons in the fragmentation model. In the recombination picture those hard partons that acquire large \( p_T \) immediately after hard scattering are not ready for recombination; they lose momenta and virtuality through gluon radiation until a large body of low-virtuality quarks and antiquarks are assembled for recombination—a view that is complementary to the fragmentation picture. Of course, there are more quarks than antiquarks, since the number of valence quarks of the participating nucleons cannot diminish. For that reason we allow \( F_q(z) \) to differ in normalization from \( F_{\bar{q}}(z) \). However, as a first approximation we assume that their \( z \) dependences are the same.

There is some indirect experimental evidence in support of our assumption. In Ref. [1] the \( \bar{p}/p \) ratio for central collisions is reported to be essentially constant within errors; more precisely, it ranges between 0.6 and 0.8 for \( p_T \) in the range 0.5 < \( p_T \) < 3.8 GeV/c. Since \( \bar{p} \) is formed by the recombination of three \( \bar{q} \), while \( p \) is formed from three \( q \), a quick estimate of the \( \bar{q}/q \) ratio is that it varies between 0.63/3 and 0.80/3, i.e., from 0.843 to 0.928. Such a narrow range of variation is sufficient for us to assume that \( F_q(z) \) has the same \( z \) dependence as \( F_{\bar{q}}(z) \). For their relative normalization we take the mean \( \bar{p}/p \) ratio to be 0.7. Thus we adopt the \( \bar{q}/q \) ratio to be

\[
F_q(z)/F_{\bar{q}}(z) = F_q^0(z)/F_{\bar{q}}^0(z) = 0.7^{1/3}. \tag{34}
\]

With this input we are finally ready to infer the quark distribution from the pion distribution.

![Diagram](https://example.com/diagram.png)

**FIG. 3:** The solid line is the fit of the data as shown in Fig. 1 (in a different scale), and the dashed line is the theoretical calculation of \( \Phi(z) \) using the quark distribution in Fig. 4.

We parameterize \( F_q(z) \) by

\[
F_q(z) = a \left(z^2 + z + z_0 \right)^{-m} \tag{35}
\]

and adjust the three parameters \( a \), \( z_0 \) and \( m \) to fit \( \Phi(z) \) by using Eq. (35). We obtain an excellent fit with the values

\[
a = 90, \quad z_0 = 1, \quad m = 4.65. \tag{36}
\]

In Fig. 3 we show in solid line the data represented by the formula in Eq. 30 and in dashed line the result of the theoretical calculation using Eqs. (35)-(36). They coalesce nearly completely in the interval \( 1 < z < 8 \). The quark distribution \( F_q(z) \) is shown in Fig. 4. To appreciate the \( p_T \) range corresponding to \( z \) in Fig. 4, recall Eq. (2), \( p_T = zK \), and Fig. 2 for \( K \). Thus at \( \sqrt{s} = 200 \) GeV, \( p_T \) is \( z \) in GeV. Equations (35) and (36) represent a main result of this study. What is important is that we have found a scaling quark distribution that is independent of \( s \) from SPS to RHIC, and perhaps to LHC. It is a succinct summary of the effects of all the dynamical subprocesses in heavy-ion collisions. The non-trivial \( z \) dependence in Eq. (35) indicates that there are intrinsic scales in the low-\( p_T \) problem.
As in Eq. (19) we relate the recombination function $R_p$ where $F$ therefore calculate the proton distribution at high test, since hadronization occurs near the end. We shall stages. Proton production provides the most appropriate tion at the end of its evolution, massive dileptons would not be sensitive to it due to their production at the early stages. Proton production provides the most appropriate test, since hadronization occurs near the end. We shall therefore calculate the proton distribution at high $p_T$ and compare with the data on the $p/\pi$ ratio. This is not a completely satisfactory venture, since the proton mass is large, so only at very high $p_T$ can our scale invariant calculation be valid without explicit consideration of the mass effect. Present data on the $p/\pi$ ratio do not extend beyond $p_T \sim 3.8$ GeV/c \cite{1}. Nevertheless, our calculation should provide some sense on the magnitude of the rate of proton production at the high $p_T$ end.

The inclusive distributions in the scaled $p_T$ variable can be obtained in the recombination model by generalizing Eq. (13) to the recombination of three quarks

$$\frac{dN_p}{dz_1} = \int dz_1 dz_2 dz_3 \ F(z_1, z_2, z_3) \ R_p(z_1, z_2, z_3, z)$$

where $F(z_1, z_2, z_3)$ is given the factorizable form

$$F(z_1, z_2, z_3) = F_u(z_1) F_u(z_2) F_d(z_3).$$

As in Eq. (19) we relate the recombination function $R_p$ to the valon distribution, $G_p$, of the proton

$$R_p(z_1, z_2, z_3, z) = R_p^{(0)} f z^{-2} G_p(\xi_1, \xi_2, \xi_3),$$

where $G_p$ has the general form given in Eq. (21), and $R_p^{(0)}$ remains to be determined. In Ref. \cite{19}, a detailed study of the proton structure functions has been carried out in deriving the valon distribution from the parton distributions that fit the deep inelastic scattering data. It is

$$\tilde{G}_p(\xi_1, \xi_2, \xi_3) = g \ (\xi_1 \xi_2) \ \xi_3,$$

where

$$\alpha = 1.755, \quad \beta = 1.05,$$

$$g = [B(\alpha + 1, \beta + 1) B(\alpha + 1, \alpha + \beta + 2)]^{-1}.$$ 

Single-valon distributions $G_p(\xi_i)$ can be obtained from the three-valon distribution by integration and are peaked around $\xi = 1/3$, indicating that each of the three valons carries on average roughly $1/3$ the momentum of the proton, their sum being strictly 1. Details of the valon model, described in \cite{18}, are not needed for the following. It is only necessary to recognize that the recombination of two $u$ quarks with a $d$ quark to form a proton has a probability proportional to the proton’s valon distribution that accounts for the proton structure. The other point to bear in mind is that the valon distribution in the proton is obtained in the frame where the proton momentum is infinitely large so the finite masses of the proton and valons are unimportant. However, the validity of that result when the proton momentum is only two or three times larger than its mass is questionable. With that caveat we proceed with our scale invariant calculation and see what can emerge.

As discussed in the preceding section, there is no need to consider the color factors for either pion or proton formation since hadrons are color singlets, but the flavor octets for these hadrons do introduce some factors. The $|uud\rangle$ state appears in $10 + 8 + 8'$ of $3 \times 3 \times 3$; among them the first two contain $\Delta^+$ and $\rho$. Thus the flavor parts of $|\langle \Delta^+ |uud\rangle|^2$ and $|\langle \rho |uud\rangle|^2$ are $1/3$ for each. Since $\Delta^+$ decays to $p + \pi^0$ and $n + \pi^+$, $|\langle \rho |\Delta^+\rangle|^2$ gives another factor $1/2$. The spin decomposition of $2 \times 2 \times 2$ is $4 + 2 + 2$, among which the $\Delta^+$ component is $4/8$ and $p$ is $2/8$. Putting the flavor and spin factors together, we have

$$|\langle \rho |uud\rangle|^2 + |\langle \rho |\Delta^+\rangle|^2 \langle \Delta^+ |uud\rangle|^2 = \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} - \frac{1}{6}.$$ 

We thus normalize $R_p$, as we have done in Eq. (20), by

$$\int_0^\infty \prod_{i=1}^3 dz_i z^3 \int dz \ R_p(z_1, z_2, z_3, z) = \frac{1}{6}.$$
In view of Eqs. (21) and (39) we have
\[ R_p^0 \int_0^Z \prod_{i=1}^3 \frac{dz_i}{z} \tilde{G}_p \left( \frac{z_1}{z_{i_1}}, \frac{z_2}{z_{i_2}}, \frac{z_3}{z_{i_3}} \right) = \frac{1}{6}, \] (45)
where \( z_i = \sum_{i'} z_{i'} \). Using Eq. (40), the above integral can be transformed to
\[ g \int_0^1 \prod_{i=1}^3 d\zeta_i \left( \frac{\zeta_i \zeta_2}{\zeta_1 \zeta_3} \right)^{\alpha} \left( \frac{\zeta_3}{\zeta_i} \right)^{\beta} = 2.924 \] (46)
with \( \zeta_i = z_i/Z \) and \( \zeta = \sum \zeta_i \). There is no explicit dependence on \( Z \), and Eqs. (41) and (42) have been used in getting the numerical value in Eq. (40). It thus follows that
\[ R_p^0 = 0.057. \] (47)

At this point we should address the question why we consider \( \Delta^+ \) production above, but not \( \rho \) production in the preceding section. For the production of \( \pi^0 \), if we are to consider the contribution from \( \rho^\pm \) (since \( \rho^0 \) does not decay strongly into \( \pi^0 \)), we would be extending our scope to other flavored states besides \( uu \) and \( dd \). Then other vector mesons and higher resonances, such as \( K^* \), that can decay into \( \pi^0 \) must also be included. Similarly, for \( p \) production the consideration of other states beside \( uud \) would involve many resonances that can decay into \( p \). The system is not closed without more phenomenological input beside \( \pi^0 \). Thus for a closed system in which a prediction can be made, we limit ourselves to only the \( uu \) and \( dd \) in the meson states and \( uud \) in the baryon states; hence, only \( \pi^0, \eta, p \) and \( \Delta^+ \) are considered. To include \( ud \) and \( du \), we must also include \( uu \) and \( udd \), and so on. We surmise that if more resonances are included in both the meson and baryon sectors, the \( p/\pi \) ratio to be determined below would change somewhat; however, the result is not likely to differ by a factor greater than 2.

With the recombination function \( R_p \) completely determined, and the quark distribution \( F_q(z_i) \) given by Eqs. (38) and (39), we can now use Eq. (47) to calculate the proton distribution in \( z \). The result is shown by the solid line in Fig. 5, where only the portion \( z > 2 \) is exhibited. We have stated at the outset that the scale invariant form of \( dN_p/zdz \) cannot be expected to be valid when the mass effect is important. The relevant value of \( z \) corresponding to the proton mass (let alone the \( \Delta^+ \) mass) is
\[ z_m = m_p/K, \] (48)
which ranges from 1.3 at \( \sqrt{s} = 17 \) GeV to 0.94 at 200 GeV. As expected, the scaling violating effects are energy dependent. Thus we should not regard the calculated result to be reliable for \( z < 3 \). At very low \( p_T \) the distributions of all hadrons can be given exponential fits in the transverse mass. The STAR data for most central collisions at \( \sqrt{s} = 130 \) GeV give for \( \bar{p} \) production for \( p_T < 0.6 \) GeV
\[ \frac{1}{2\pi m_T} \frac{d^2N_{\bar{p}}}{dm_T dz} = 4 \exp \left\{ -(m_T - m_p)/T_p \right\} \] (49)
where \( m_T = (m_T^2 + p_T^2)^{1/2} \) and \( T_p = 565 \) MeV. To convert this distribution to that for \( p \) we assume that only the normalization at \( p_T = 0 \) needs to be adjusted. The \( \bar{p}/p \) ratio at low \( p_T \) is 0.6. Since \( m_T dm_T = p_T dp_T \) and the distribution in \( p_T \) changes by a scale factor \( K^2 \) given in Eq. (2), where \( K = 0.9 \) for \( \sqrt{s} = 130 \) GeV, the factor 4 in Eq. (45) should therefore be changed to \( 4 \times 0.81/0.6 = 5.4 \). Expressing \( m_T \) in terms of \( z \) by use of Eq. (2) with \( K = 0.9 \), we show the \( z \) dependence of the distribution for \( p \) in Fig. 5 by the short dashed line. The region \( 0.5 < z < 2 \) is left blank because our scaling result cannot be reliably extended into that region. Nevertheless, it is gratifying to observe that the theoretical calculation without any free parameters produces a proton distribution at large \( z \) that is reasonable in normalization and shape and can smoothly be connected with the low-\( p_T \) distribution by interpolation.

![Proton distribution](image)

**FIG. 5:** Proton distribution in \( z \). Solid line is the theoretical result; the dashed line is the fit of data at low-\( p_T \).

With the proton distribution now at hand, we can calculate the \( p/\pi \) ratio. For the pion distribution we use \( \Phi(z) \) given in Eq. (4). For proton we use the calculated result based on Eq. (47). Their ratio, defined by
\[ R_{p/\pi}(z) = \frac{dN_p}{zdz} / \Phi(z) \] (50)
is shown by the solid line in Fig. 6. The preliminary data on the \( p/\pi \) ratio were reported in Ref. 40, which we show also in Fig. 6 for both \( \sqrt{s} = 130 \) and 200 GeV. Note that because it is a ratio there is no change in the normalizations of \( R_{p/\pi} \) for the two energies, but in transforming
from $p_T$ to $z$ the factor $K$ in Eq. (2) must be taken into account. Unlike the pion case the effects of the proton mass are not negligible for $p_T \lesssim 3$ GeV/$c$, and one sees no scaling in $s$ or $z$ in Fig. 6. Our scale invariant calculation is unreliable for $z < 3$ and shows a result that is obviously too high at $z \sim 2$. There seems to be a good chance that the theory and experiment can agree well for $z > 4$. In Fig. 6 we show two curves that can connect our scaling result with the data. The dotted curve is an eyeball fit of the 130 GeV data with a connection at $z = 3.5$, while the dashed curve fits the 200 GeV data with a connection at $z = 4$. In the absence of a theoretical study that takes the mass-dependent effects into account in the intermediate $p_T$ region, the only point we can make here is that it is not hard to produce a $p/\pi$ ratio that exceeds 1 in the scale invariant calculation in the recombination model, but it does so in a region where both theory and experiment need refinement. Judging by what is self-evident in Fig. 6, we see no strong need for any exotic mechanism for proton production (as proposed in [6]) beyond the conventional subprocess where three quarks recombine to form a proton.

![Graph](image)

FIG. 6: Proton-to-pion ratio: solid line is the scaling distribution from calculation; data (preliminary) are from Ref. [1]. The dotted and dashed lines are eyeball fits of the data as extrapolations from the scaling result.

V. CONCLUSION

The discovery of a scale invariant distribution $\Phi(z)$ for pion production at intermediate and high values of $p_T$ in heavy-ion collisions ranging over energies in excess of an order of magnitude of variation is an important phenomenology observation that should be checked experimentally in great detail. Additional energy points should be added not only to strengthen the validity of the scaling behavior, but also to find the onset of scaling violation, if it exists.

The phenomenological properties of hadron production provide useful insights into the hadronization process and into the nature of the quark system just before they turn into hadrons. The usual approach to the study of heavy-ion collisions is from inside out, following the evolution of the dense matter, either in terms of hydrodynamical flow or of hard parton scattering and subsequent hadronization by fragmentation [21]. Our approach pursued here is from outside in, by starting from the observed scaling behavior of the pions produced and deriving the momentum distribution of the quarks that can give rise to such a behavior. That is accomplished by use of the recombination model. There is no direct way to check the validity of the quark distribution thus obtained. However, we have used it to determine the proton distribution at high $p_T$ where the mass effects are unimportant. The data on proton production have not yet reached that regime where the predicted scaling distribution can be checked. In the region where data exist on the $p/\pi$ ratio we find that our calculated result, though not reliable, is in rough agreement with the imprecise data to the extent that the ratio exceeds 1, a feature that is notable.

While the recombination model needs further work to take the proton mass into account at intermediate $p_T$, its formulation in the invariant form has been developed here to treat the very high $p_T$ region. We have made the assumption that the quark and antiquark distributions are the same, apart from normalization, just before recombination. That assumption is supported by the constancy of the $\bar{p}/p$ ratio in the PHENIX data in the central region. That experimental fact can also be used to lend credibility to our general approach to hadronization that is treated as a recombination process, for which we have given arguments why the distributions of quarks and antiquarks should be similar before they recombine. In contrast, the fragmentation model would suggest a decreasing function of $\bar{p}/p$ in $p_T$ because of the dominance of quark jets over antiquark jets at large $p_T$ [1].

In this paper we have only considered the energy dependence of the $p_T$ spectrum at fixed maximum centrality. It is natural to ask what the dependence is on centrality. We have investigated that problem by making a phenomenological analysis of the data on centrality dependence without using any hadronization model, and found a scaling behavior very similar to what is reported here. The scaling distribution found there [14] includes the very small $p_T$ region in the fit, and is therefore more
accurate. But the fits in the intermediate and large $p_T$ region are the same. The implication on the centrality dependence of the $p/\pi$ ratio in the recombination model is still under study.

To have an invariant quark distribution independent of $s$ just before hadronization provides an unexpected picture of the quark system. It suggests that the evolution of the system proceeds toward a universal form whatever the collision energy may be. We expect that universal form to depend on rapidity. The origin of such a scaling distribution in $z$ is not known at this point and can form the focus of a program of future theoretical investigations.

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