Non-factorizable long distance contributions
in color suppressed decays of $B$ mesons *

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Abstract

$\bar{B} \to D\pi$, $D^*\pi$, $J/\psi K$ and $J/\psi \pi$ decays are studied. Their amplitude is
given by a sum of factorized and non-factorizable ones. The latter which is
estimated by using a hard pion approximation is rather small in color favored
$\bar{B} \to D\pi$ and $D^*\pi$ decays but still can efficiently interfere with the main
amplitude given by the factorization. In the color suppressed $\bar{B} \to J/\psi K$
and $J/\psi \pi$ decays, the non-factorizable contribution is very important. The
sum of the factorized and non-factorizable amplitudes can reproduce well
the existing experimental data on the branching ratios for the color favored
$\bar{B} \to D\pi$ and $D^*\pi$ and the color suppressed $\bar{B} \to J/\psi K$ and $J/\psi \pi$ decays by
taking reasonable values of unknown parameters involved.

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Nonleptonic weak decays of charm and $B$ mesons have been studied extensively by using the so-called factorization (or vacuum insertion) prescription. It has been supported by two different arguments. One is that the factorization is applicable in the large $N_c$ (color degree of freedom) limit and the other is that it can be a good approximation under a certain kinematical condition, i.e., a heavy quark decays into another heavy quark plus a pair of light quark and anti-quark which are emitted colinearly with sufficiently high energies, for example, like $b \to c + (\bar{u}d)_1$, where $(\bar{u}d)_1$ denotes a color singlet $(\bar{u}d)$ pair.

The factorization predicts the so-called color suppression [suppression of color mismatched decays like $\bar{B}^0 \to D^0\pi^0$, $D^{*0}\pi^0$, etc., described by $b \to (c\bar{u})_8 + d$, where $(c\bar{u})_8$ means that the $(c\bar{u})$ pair is of color octet]. According to recent measurements of $\bar{B} \to D\pi$ decays, the $\bar{B}^0 \to D^0\pi^0$ decay is actually suppressed in comparison with color favored decays like $B^- \to D^0\pi^-$, etc., described by $b \to c + (\bar{u}d)_1$. However, semi-phenomenological analyses in two-body decays of $B$ mesons within the framework of the factorization suggest that the value of $a_2$ to reproduce the observed branching ratios for these decays should be larger by about a factor 2 than the one with the leading order (LO) QCD corrections, where $N_c = 3$ and that its sign should be opposite to the one in the large $N_c$ limit although the phenomenological value of $a_1$ is very close to the one expected in the same approximation. [$a_1$ and $a_2$ are the coefficients of four quark operators in the effective weak Hamiltonian in the Bauer-Stech-Wirbel (BSW) scheme which will be reviewed briefly in the next section.] The above fact implies that the large $N_c$ argument fails, at least, in hadronic weak decays of $B$ mesons. Since the large $N_c$ argument is independent of flavors, it also does not work in nonleptonic weak decays of charm mesons. The kinematics of charm meson decays is far from the condition, i.e., like $b \to c + (\bar{u}d)_1$ mentioned before, under which the factorization is applicable. Therefore the factorization of charm decay amplitudes has no theoretical support. In fact, a naive application of the factorization to charm decay amplitudes again leads to the color suppression [suppression of color mismatched decays,
$D^0 \to K^0\pi^0, \bar{K}^*\pi^0$, etc., described by $c \to (s\bar{d})_8 + u$] and therefore the amplitudes for two body decays of charm mesons must be approximately real. However the observed decay rates for these decays are not always suppressed and the amplitudes for $D \to \bar{K}\pi$ and $\bar{K}^*\pi$ decays have large phases [3]. In this way, it will be understood that the factorized amplitudes might be dominant only in some specific decays like the color favored $\bar{B} \to D\pi$ and $D^*\pi$ decays [4]. The above result on the $B$ decays that the phenomenological value of $a_2$ is much larger than the one with the LO corrections seems to suggest that, even in the decays of $B$ mesons with high mass, factorization is not complete but non-factorizable long distance hadron dynamics still cannot be neglected. Buras [10] calculated next-to-leading order (NLO) QCD corrections to the effective weak Hamiltonian in three different renormalization schemes and observed that $a_2$ can have large NLO corrections which are strongly dependent on choice of renormalization scheme while the corrections to $a_1$ are very small and are stable with respect to change of renormalization scheme. Then he discussed that the above instability of the NLO corrections to $a_2$ may imply importance of non-factorizable effects on $B$ decays. Soares [11] tried to estimate phenomenologically non-factorizable contributions to $\bar{B} \to D\pi$ and $\bar{B} \to J/\psi K$ decays and found several possible solutions which indicate large non-factorizable contributions.

In this article, we study $\bar{B} \to D\pi, D^*\pi, J/\psi K$ and $J/\psi\pi$ decays describing the amplitude for these decays by a sum of factorizable and non-factorizable ones. The latter amplitude is estimated by using a hard pion (or kaon) approximation. The $\bar{B} \to D\pi$ and $D^*\pi$ decays will be studied in the next section. It will be seen that, in the color suppressed $\bar{B}^0 \to D^0\pi^0$ and $D^{*0}\pi^0$ decays, the hard pion amplitudes as the non-factorizable long distance contributions are important. In the section 3, the decays, $\bar{B} \to J/\psi K$ and $B^- \to J/\psi\pi^-$, both of which are color suppressed, will be investigated in the same way. It will be demonstrated that, in these decays, non-factorizable long distance amplitudes are again important. A brief summary will be given in the final section.
II. \( \bar{B} \to D\pi \) AND \( D^*\pi \) DECAYS

Our starting point is to describe the amplitude for two body decay of \( B \) meson by a sum of factorizable and non-factorizable ones [12],

\[
M_{\text{total}} = M_{\text{fact}} + M_{\text{non-fact}}.
\]  

The factorizable amplitude \( M_{\text{fact}} \) is evaluated by using the factorization in the BSW scheme [1, 2] in which the relevant part of the effective weak Hamiltonian is given by

\[
H_{\text{BSW}}^w = \frac{G_F}{\sqrt{2}} U_{cb} U_{ud} \{ a_1 O_1^H + a_2 O_2^H + \text{h.c.} \}.
\]

It can be obtained by applying the Fierz reordering to the usual effective Hamiltonian

\[
H_w = \frac{G_F}{\sqrt{2}} U_{cb} U_{ud} \{ c_1 O_1 + c_2 O_2 + \text{h.c.} \},
\]

where \( c_1 \) and \( c_2 \) are the Wilson coefficients of the four quark operators,

\[
O_1 = [ (\bar{d}u)_L + (\bar{s}c)_L ](\bar{c}b)_L, \quad O_2 = (\bar{c}u)_L(\bar{d}b)_L + (\bar{c}c)_L(\bar{s}b)_L
\]

with \((\bar{q}'q)_L = \bar{q}'\gamma_\mu(1 - \gamma_5)q\). The quark bilinears in \( O_1^H \) and \( O_2^H \) are treated as interpolating fields for the mesons and therefore should be no longer Fierz reordered. The coefficients, \( a_1 \) and \( a_2 \), in Eq.(2) are given by

\[
a_1 = c_1 + \frac{c_2}{N_c}, \quad a_2 = c_2 + \frac{c_1}{N_c}.
\]

The LO QCD corrections lead to \( a_1 \simeq 1.03 \) and \( a_2 \simeq 0.11 \) for \( N_c = 3 \) [4]. \( U_{ij} \) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element [13] which is taken to be real in this article since \( CP \) invariance is always assumed.

The factorization prescription in the BSW scheme leads to the following factorized amplitude, for example, for the \( B^-\to D^0(p')\pi^-(q) \) decay,

\[
M_{\text{fact}}(B^-\to D^0(p')\pi^-(q)) = \frac{G_F}{\sqrt{2}} U_{cb} U_{ud} \{ a_1 \langle \pi^-(q) | (\bar{d}u)_L | 0 \rangle \langle D^0(p') | (\bar{c}b)_L | B^- (p) \rangle + \}
\]

\[
+ a_2 \langle D^0(p') | (\bar{c}u)_L | 0 \rangle \langle \pi^-(q) | (\bar{d}b)_L | B^- (p) \rangle \}.
\]
Table I. Factorized amplitudes for $B \to D\pi$ and $D^*\pi$ decays ($m^2_\pi = 0$).

| Decay          | $M_{\text{fact}}$                                                                 |
|----------------|------------------------------------------------------------------------------------|
| $\bar{B}^0 \to D^+\pi^-$ | $iU_{cb}U_{ud}\sqrt{2}\alpha f_\pi(m^2_B - m^2_D)F^D_{0B}(0)[1 - \left(\frac{a_2}{a_1}\right)\left(\frac{f_{D*}}{f_D}\right)\left(\frac{m^2_D}{m^2_B - m^2_D}\right)\frac{F^{D*}_{0B}(m^2_\pi)}{F^{DB}_{0}(0)}]$ |
| $B^0 \to D^0\pi^0$ | $-iU_{cb}U_{ud}\sqrt{2}\alpha f_Dm_B^2F^{*B}_{0B}(m^2_D)[1 + \left(\frac{f_{D*}}{f_D}\right)\left(\frac{m^2_D}{m^2_B - m^2_D}\right)\frac{F^{D*}_{0B}(m^2_\pi)}{F^{DB}_{0}(0)}]$ |
| $B^- \to D^0\pi^-$  | $iU_{cb}U_{ud}\sqrt{2}\alpha f_\pi(m^2_B - m^2_D)F^D_{0B}(0)[1 + \left(\frac{a_2}{a_1}\right)\left(\frac{f_{D*}}{f_D}\right)\left(\frac{m^2_D}{m^2_B - m^2_D}\right)\frac{F^{D*}_{0B}(m^2_\pi)}{F^{DB}_{0}(0)}]$ |
| $\bar{B}^0 \to D^{*-}\pi^-$ | $-iU_{cb}U_{ud}\sqrt{2}\alpha f_DA_{0}^{D*}(0)[1 - \left(\frac{a_2}{a_1}\right)\left(\frac{f_{D*}}{f_D}\right)\frac{A_{0}^{D*}(m^2_\pi)}{A_{0}^{DB}(0)}]2m_{D*}\epsilon^*(p') \cdot p$ |
| $B^0 \to D^{*0}\pi^0$ | $iU_{cb}U_{ud}\sqrt{2}\alpha f_DF^{*B}_{1B}(m^2_D)[1 + \left(\frac{f_{D*}}{f_D}\right)\frac{A_{0}^{D*}(m^2_\pi)}{A_{0}^{DB}(0)}]2m_{D*}\epsilon^*(p') \cdot p$ |
| $B^- \to D^{*0}\pi^-$  | $-iU_{cb}U_{ud}\sqrt{2}\alpha f_\pi A_{0}^{D*}(0)[1 + \left(\frac{a_2}{a_1}\right)\left(\frac{f_{D*}}{f_D}\right)\frac{F^{*B}_{0}(m^2_\pi)}{F^{*B}_{0}(0)}]2m_{D*}\epsilon^*(p') \cdot p$ |

Factorizable amplitudes for the other $B \to D\pi$ and $D^*\pi$ decays also can be calculated in the same way. To evaluate these amplitudes, we use the parameterization of matrix elements of currents in Ref. [2],

$$\langle \pi(q)|A_\mu|0\rangle = -if_\pi q_\mu, \quad \langle 0|A_\mu|\pi(q)\rangle = if_\pi q_\mu,$$

(7)

$$\langle D(p')|V_\mu|\bar{B}(p)\rangle = \left\{(p + p')_\mu - \frac{m^2_B - m^2_D}{q^2}q_\mu\right\}F_1(q^2) + \frac{m^2_B - m^2_D}{q^2}q_\mu F_0(q^2),$$

(8)

$$\langle D^*(p')|A_\mu|\bar{B}(p)\rangle = \left\{(m_B + m_{D^*})\epsilon^*_\mu(p')A_1(q^2) - \frac{\epsilon^*(p') \cdot q}{m_B + m_{D^*}}(p + p')_\mu A_2(q^2)
-2m_{D^*}\frac{\epsilon^* \cdot q}{q^2}q_\mu A_3(q^2)\right\} + 2m_{D^*}\frac{\epsilon^* \cdot q}{q^2}q_\mu A_0(q^2),$$

(9)

where $q = p - p'$ and the form factors satisfy

$$A_3(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}}A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}}A_2(q^2),$$

(10)

$$F_1(0) = F_0(0), \quad A_3(0) = A_0(0).$$

(11)

To get rid of useless imaginary unit except for the overall phase in the amplitude, however, we adopt the following parameterization of matrix element of vector current [14]

$$\langle V(p')|V_\mu|0\rangle = -if_{V}\cdot m_{V}\epsilon^*_\mu(p').$$

(12)

As stressed in Ref. [14], the above matrix element of vector current can be treated in parallel to those of axial vector currents in Eq. (7) in the infinite momentum frame (IMF). Using these
expressions of current matrix elements, we obtain the factorized amplitudes for $\bar{B} \to D\pi$ and $D^*\pi$ decays in Table I, where we have put $m^2_\pi = 0$.

Before we evaluate numerically the factorized amplitudes, we study non-factorizable amplitudes for $\bar{B} \to D\pi$ and $D^*\pi$ decays using a hard pion technique in the IMF; $p \to \infty$ \cite{15,16}. In our hard pion approximation, the non-factorizable amplitude for the $\bar{B}(p) \to D(p')\pi(q)$ decay is given by

$$M_{\text{non-f}}(\bar{B} \to D\pi) \simeq M_{\text{ETC}}(\bar{B} \to D\pi) + M_S(\bar{B} \to D\pi). \quad (13)$$

The equal-time commutator term ($M_{\text{ETC}}$) and the surface term ($M_S$) are given by

$$M_{\text{ETC}}(\bar{B} \to D\pi) = \frac{i}{f_\pi} \langle D| [V_\pi, H_w]|\bar{B}\rangle \quad (14)$$

and

$$M_S(\bar{B} \to D\pi) = \frac{i}{f_\pi} \left\{ \sum_n \left( \frac{m^2_D - m^2_B}{m^2_n - m^2_B} \right) \langle D| A_\pi|n\rangle \langle n|H_w|\bar{B}\rangle 
+ \sum_\ell \left( \frac{m^2_D - m^2_B}{m^2_\ell - m^2_D} \right) \langle D| H_w|\ell\rangle \langle \ell|A_\pi|\bar{B}\rangle \right\}, \quad (15)$$

respectively, where $[V_\pi + A_\pi, H_w] = 0$ has been used. (See Refs. \cite{15} and \cite{16} for notations.) $M_{\text{ETC}}$ and $M_S$ have to be evaluated in the IMF. The surface term has been given by a sum of all possible pole amplitudes, \textit{i.e.}, $n$ and $\ell$ run over all possible single mesons, not only ordinary \{$q\bar{q}$\}, but also hybrid \{$q\bar{q}g$\} and exotic \{$qq\bar{q}\bar{q}$\} mesons. Since the $B$ meson mass $m_B$ is much higher than those of charm mesons and since wave function overlaps between the ground-state \{$q\bar{q}$\}_0 and excited-state-meson states are expected to be small, however, excited meson contributions will be small in these decays and can be safely neglected. Therefore the hard pion amplitudes as the non-factorizable long distance contributions are approximately described in terms of \textit{asymptotic ground-state-meson matrix elements} (matrix elements taken between single ground-state-meson states with infinite momentum) of $V_\pi$, $A_\pi$ and $H_w$. Hard pion amplitudes for $\bar{B} \to D^*\pi$ decays can be obtained by exchanging $D$ for $D^*$.

Asymptotic matrix elements of $V_\pi$ and $A_\pi$ are parameterized as
\begin{align}
\langle \pi^0 | V_\pi^+ | \pi^- \rangle &= \sqrt{2} \langle K^+ | V_\pi^+ | K^0 \rangle = -\sqrt{2} \langle D^+ | V_\pi^+ | D^0 \rangle = \sqrt{2} \langle B^+ | V_\pi^+ | B^0 \rangle = \cdots = \sqrt{2}, \\
\langle \rho^0 | A_\pi^+ | \pi^- \rangle &= \sqrt{2} \langle K^{*+} | A_\pi^+ | K^0 \rangle = -\sqrt{2} \langle D^{*+} | A_\pi^+ | D^0 \rangle = \sqrt{2} \langle B^{*+} | A_\pi^+ | B^0 \rangle = \cdots = h, 
\end{align}
\tag{16}

\begin{align}
\langle \rho^0 | A_\pi^+ | \pi^- \rangle &= \sqrt{2} \langle K^{*+} | A_\pi^+ | K^0 \rangle = -\sqrt{2} \langle D^{*+} | A_\pi^+ | D^0 \rangle = \sqrt{2} \langle B^{*+} | A_\pi^+ | B^0 \rangle = \cdots = h, 
\end{align}
\tag{17}

where \( V_\pi \)'s and \( A_\pi \)'s are isospin charges and their axial counterpart, respectively. The above parameterization can be obtained by using asymptotic \( SU_f(5) \) symmetry \cite{17}, or \( SU_f(5) \) extension of the nonet symmetry in \( SU_f(3) \). Asymptotic matrix elements of \( V_\pi \) between vector meson states can be obtained by exchanging pseudo scalar mesons for vector mesons with corresponding flavors in Eq.(16), for example, as \( \pi^0, - \rightarrow \rho^0, - \), etc. The \( SU_f(4) \) part of the above parameterization reproduces well \cite{16,18} the observed values of decay rates, \( \Gamma(D^* \rightarrow D\pi) \) and \( \Gamma(D^* \rightarrow D\gamma) \).

Amplitudes for dynamical hadronic processes can be decomposed into \((\text{continuum contribution}) + (\text{Born term})\). Since \( M_S \) is given by a sum of pole amplitudes, \( M_{\text{ETC}} \) corresponds to the continuum contribution \cite{19} which can develop a phase relative to the Born term. Therefore we parameterize the ETC terms using isospin eigen amplitudes and their phases. Since the \( D\pi \) final states can have isospin \( I = \frac{1}{2} \) and \( \frac{3}{2} \), we decompose \( M_{\text{ETC}} \)'s as

\begin{align}
M_{\text{ETC}}(\bar{B}^0 \rightarrow D^+ \pi^-) &= \sqrt{\frac{1}{3}} M_{\text{ETC}}^{(3)} e^{i\delta_3} + \sqrt{\frac{2}{3}} M_{\text{ETC}}^{(1)} e^{i\delta_1}, \\
M_{\text{ETC}}(\bar{B}^0 \rightarrow D^0 \pi^0) &= -\sqrt{\frac{2}{3}} M_{\text{ETC}}^{(3)} e^{i\delta_3} + \sqrt{\frac{1}{3}} M_{\text{ETC}}^{(1)} e^{i\delta_1}, \\
M_{\text{ETC}}(B^- \rightarrow D^0 \pi^-) &= \sqrt{3} M_{\text{ETC}}^{(3)} e^{i\delta_3},
\end{align}
\tag{18, 19, 20}

where \( M_{\text{ETC}}^{(2I)} \)'s are the isospin eigen amplitudes with isospin \( I \) and \( \delta_{2I} \)'s are the corresponding phase shifts introduced. In the present approach, therefore, the final state interactions are included in the non-factorizable amplitudes.

In this way we can describe the non-factorizable amplitudes for the \( \bar{B} \rightarrow D\pi \) decays as

\begin{align}
M_{\text{non-f}}(\bar{B}^0 \rightarrow D^+ \pi^-) &\simeq -i \frac{\langle D^0 | H_w | \bar{B}^0 \rangle}{f_\pi} \left\{ \frac{4}{3} e^{i\delta_1} - \frac{1}{3} e^{i\delta_3} + \frac{\langle D^{*0} | H_w | \bar{B}^0 \rangle}{\langle D^0 | H_w | \bar{B}^0 \rangle} \left( \frac{m_B^2 - m_D^2}{m_B^2 - m_{D^*}^2} \right) \sqrt{\frac{1}{2}} h + \cdots \right\},
\end{align}
\tag{21}
\( M_{\text{non-f}}(\bar{B}^0 \rightarrow D^0 \pi^0) \simeq -i \frac{\langle D^0 | H_w | \bar{B}^0 \rangle}{f_\pi} \left\{ \sqrt{2} \frac{2}{3} \left[ 2e^{i\delta_1} + e^{i\delta_3} \right] + \sqrt{\frac{1}{2}} \left[ \frac{\langle D^0 | H_w | \bar{B}^0 \rangle}{\langle D^0 | H_w | B^0 \rangle} \left( \frac{m_{B}^2 - m_{D}^2}{m_{B}^2 - m_{D}^2} \right) \right] + \sqrt{\frac{1}{2}} \left[ \frac{\langle D^0 | H_w | \bar{B}^0 \rangle}{\langle D^0 | H_w | B^0 \rangle} \left( \frac{m_{B}^2 - m_{D}^2}{m_{B}^2 - m_{D}^2} \right) \right] \right\} \right\} \). 

\( M_{\text{non-f}}(B^- \rightarrow D^0 \pi^-) \simeq -i \frac{\langle D^0 | H_w | \bar{B}^0 \rangle}{f_\pi} \left\{ e^{i\delta_3} + \frac{\langle D^0 | H_w | \bar{B}^0 \rangle}{\langle D^0 | H_w | B^0 \rangle} \left( \frac{m_{B}^2 - m_{D}^2}{m_{B}^2 - m_{D}^2} \right) \right\} \sqrt{\frac{1}{2}} \). 

where the ellipses denote the neglected excited meson contributions. The corresponding amplitudes for the \( \bar{B} \rightarrow D^* \pi \) decays can be obtained by replacing \( D^0 \leftrightarrow D^{*0} \) and \( \delta_{2I} \rightarrow \delta_{2I}^* \) in Eqs. (21) – (23). Therefore the non-factorizable amplitudes in the hard pion approximation are controlled by the asymptotic ground-state-meson matrix elements of \( H_w \) and the phases.

Now we evaluate the amplitudes given above. The factorized amplitudes in Table I contain many parameters which have not been measured by experiments, \( i.e., \) form factors, \( F_0(q^2), A_0(q^2), F_1(q^2), \) decay constants, \( f_D, f^*_D, f_B, \) etc. The form factors \( F_{0DB}(0) \) and \( A_{0D^*B}(0) \) can be calculated by using the heavy quark effective theory (HQET) \[20\] but the other form factors are concerned with light meson states and therefore have to be estimated by some other models. In color favored decays, main parts of the factorized amplitudes depend on the form factor, \( F_{0DB}(0) \) or \( A_{0D^*B}(0) \), and the other form factors are included in minor terms proportional to \( a_2 \). Therefore our result may not be lead to serious uncertainties although we here take specific values of the form factors given in Ref. \[7\]. In the color suppressed \( \bar{B}^0 \rightarrow D \pi^0 \) and \( D^{*0} \pi^0 \) decays, the factorized amplitudes contain the form factors, respectively, \( F_{0DB}(m_{D}^2) \) and \( F_{1D^*B}(m_{D^*}^2) \), whose values are model dependent. However, we need not seriously worry about ambiguities arising from these form factors as long as the non-factorizable amplitudes which do not involve them are dominant. [If the non-factorizable contribution is not dominant, the results on these decays will be not very much different from those of the usual factorization.] For the decay constants of heavy mesons, we assume \( f_D \simeq f_{D^*} \) (and \( f_B \simeq f_{B^*} \)) since \( D \) and \( D^* \) (\( B \) and \( B^* \)) are expected to be degenerate because of heavy quark symmetry \[20\] and are approximately degenerate in reality. Here we take
Table II. Factorized and non-factorizable amplitudes for the $B \to D\pi$ and $D^*\pi$ decays. The CKM matrix elements are factored out.

| Decay          | $A_{\text{fact}} \times 10^{-5}$ GeV | $A_{\text{non-f}} \times 10^{-5}$ GeV |
|----------------|-------------------------------------|--------------------------------------|
| $\bar{B}^0 \to D^+\pi^-$ | $1.54a_1\{1 - 0.11\left(\frac{a_2}{a_1}\right)\}$ | $-3.52a_2B_H\left\{\left[\frac{4}{3}e^{i\delta_1} - \frac{1}{3}e^{i\delta_3}\right] - 0.55\right\}$ |
| $\bar{B}^0 \to D^0\pi^0$  | $-1.23a_2\left\{\frac{f_D}{0.205\text{ GeV}}\right\}$ | $-3.52a_2B_H\left\{\frac{\sqrt{2}}{3}\left[2e^{i\delta_1} + e^{i\delta_3}\right] - 0.05\right\}$ |
| $B^- \to D^0\pi^-$      | $1.54a_1\{1 + 1.14\left(\frac{a_2}{a_1}\right)\}$ | $3.52a_2B_H\left\{e^{i\delta_3} + 0.48\right\}$ |
| $B^0 \to D^{*+}\pi^-$  | $-1.53a_1\{1 - 0.39\left(\frac{a_2}{a_1}\right)\}$ | $2.13a_2B_H\left\{\left[\frac{4}{3}e^{i\delta_1} - \frac{1}{3}e^{i\delta_3}\right] - 0.91\right\}$ |
| $\bar{B}^0 \to D^{*0}\pi^0$ | $1.25a_2\left\{\frac{f_D}{0.205\text{ GeV}}\right\}$ | $2.13a_2B_H\left\{\frac{\sqrt{2}}{3}\left[2e^{i\delta_1} + e^{i\delta_3}\right] + 0.02\right\}$ |
| $B^- \to D^{*0}\pi^-$ | $-1.53a_1\{1 + 1.20\left(\frac{a_2}{a_1}\right)\}$ | $-2.13a_2B_H\left\{e^{i\delta_3} + 0.95\right\}$ |

$f_{D^*} \simeq f_D \simeq 205$ MeV and $f_{B^*} \simeq f_B \simeq 175$ MeV from a recent result of lattice QCD [21], $f_D = 203 \pm 7 \pm 20$ MeV and $f_B = 178 \pm 9 \pm 18$ MeV. In this way, we can obtain the factorized amplitudes in the second column of Table II, where we have neglected very small annihilation terms in the $\bar{B}^0 \to D^0\pi^0$ and $D^{*0}\pi^0$ decay amplitudes.

To evaluate the non-factorizable amplitudes, we need to know the size of the asymptotic matrix elements of $H_w$ and $A_x$ taken between heavy meson states. The latter which was parameterized in Eq.(17) is estimated to be $|h| \simeq 1.0$ [13,14] by using partially conserved axial-vector current (PCAC) and the observed rate $\Gamma(\rho \to \pi\pi)_{\text{expt}} \simeq 150$ MeV. To estimate the asymptotic matrix elements,

$$\langle D^0|H_w|\bar{B}^0\rangle, \langle D^{*0}|H_w|\bar{B}^0\rangle, \langle D^0|H_w|\bar{B}^{*0}\rangle \quad \text{and} \quad \langle D^{*0}|H_w|\bar{B}^{*0}\rangle,$$  \hspace{1cm} (24)

included in the non-factorizable amplitudes, we apply the factorization to them since the heavy mesons annihilate at the weak vertex in the weak boson mass $m_W \to \infty$ limit; for example,

$$\langle D^0|H_w|\bar{B}^0\rangle = \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}\left(\frac{m_{\pi}^2 + m_{B}^2}{2}\right)B_H f_D f_B a_2,$$  \hspace{1cm} (25)

in the IMF, where $H_w = H_w^{BSW}$ and $B_H$ is an analogue to the $B$ parameter in the matrix element of $O_{\Delta B=2}$ providing the $B - \bar{B}$ mixing. Since the other asymptotic matrix elements
of $H_w$ in Eq.(24) can be estimated in the same manner as in Eq.(25), we obtain the hard pion amplitudes as the non-factorizable contributions listed in the third column of Table II where we have used $f_{D^*} \simeq f_D \simeq 205$ MeV and $f_{B^*} \simeq f_B \simeq 175$ MeV as before and used the same $B_H$ parameter for all the asymptotic matrix elements in Eq.(24). The CKM matrix elements have been factored out.

We now estimate branching ratios, $B(\bar{B} \to D\pi)$ and $B(\bar{B} \to D^*\pi)$, by taking a sum of the factorized amplitude (the second column in the Table II) and the non-factorizable amplitude (the third column in Table II) as the total amplitude [22]. To this, we have to give values of remaining parameters. We take $U_{cb} = 0.038$ from the updated value $|U_{cb}| = 0.0388 \pm 0.0036$ [23]. For the coefficients $a_1$ and $a_2$ in the effective weak Hamiltonian $H_w (= H_w^{BSW})$, we do not know their true values. According to Ref. [10], NLO corrections to $a_1$ are small while corresponding corrections to $a_2$ may be large (approximately of the same size as the LO corrections, which seems to be unnatural from the view point of the perturbation theory) and very unstable with respect to change of renormalization scheme as mentioned before. We expect that the value of $a_1$ with the LO corrections is not far from the true value and that higher order corrections should be small as is expected formally in the perturbation theory. Therefore we take conservatively $a_1 = 1.03$ and $a_2 = 0.11$ which are given in the LO approximation [2]. For the phases $\delta_1$ and $\delta_3$ ($\delta_1^*$ and $\delta_3^*$) arising from contributions of non-resonant multi-hadron intermediate states with isospin $I = \frac{1}{2}$ and $\frac{3}{2}$ which have not been measured by experiments, they will be restricted in the region $|\delta_{2I}|, |\delta_{2I}^*| < 90^\circ$. (Resonant contributions have already been extracted as pole amplitudes in $M_S$, although they were not very important and were neglected as discussed before.) We here assume $\delta_1 \simeq \delta_1^*$ and $\delta_3 \simeq \delta_3^*$ because of the heavy quark symmetry. For the value of the $B_H$ parameter, we here consider two cases, $B_H = 1$ and 0.5 as typical examples, and list the branching ratios for $U_{cb} = 0.038$, $U_{ud} = 0.98$, $\delta_1 = \delta_1^* = 60^\circ$, $\delta_3 = \delta_3^* = -60^\circ$ in Table III, where we have used the observed lifetime [3], $\tau(B^-) \simeq \tau(\bar{B}^0) \simeq 1.5 \times 10^{-12}$ s. $B_{\text{fact}}$, $B_{\text{non-f}}$ and $B_{\text{total}}$ are estimated branching ratios which include only the factorized amplitude, only the non-factorizable one and the sum of them, respectively. It is seen that the non-factorizable contributions to the
Table III. Branching ratios (%) for $\bar{B} \to D\pi$ and $D^*\pi$ decays where $a_1 = 1.03$, $a_2 = 0.11$, $U_{cb} = 0.038$, $U_{ud} = 0.98$ and $\tau(B^-) = \tau(\bar{B}^0) = 1.5 \times 10^{-12}$ s. The values $\delta_1 = \delta_1^* = 60^\circ$ and $\delta_3 = \delta_3^* = -60^\circ$ of phases are taken tentatively. $B_{\text{fact}}$, $B_{\text{non-f}}$ and $B_{\text{total}}$ include only the factorized amplitude, only the non-factorizable one and the sum of them, respectively. The data values are taken from Ref. [5].

| Decay       | $B_{\text{fact}}$ | $B_{\text{non-f}}$ | $B_{\text{total}}$ | $B_{\text{expt}}$ |
|-------------|-------------------|---------------------|---------------------|-------------------|
|             | $B_H = 1$         | $B_H = 0.5$         | $B_H = 1$           | $B_H = 0.5$       |
| $\bar{B}^0 \to D^+\pi^-$ | 0.26              | 0.033               | 0.008               | 0.30              | 0.27              | 0.31 ± 0.04       |
| $\bar{B}^0 \to D^0\pi^0$ | 0.002             | 0.010               | 0.002               | 0.019             | 0.008             | $< 0.048$         |
| $B^- \to D^0\pi^-$   | 0.34              | 0.027               | 0.007               | 0.51              | 0.42              | 0.50 ± 0.07       |
| $\bar{B}^0 \to D^{*+}\pi^-$ | 0.24             | 0.013               | 0.003               | 0.29              | 0.26              | 0.28 ± 0.05       |
| $\bar{B}^0 \to D^{*0}\pi^0$ | 0.002            | 0.004               | 0.001               | 0.011             | 0.006             | $< 0.097$         |
| $B^- \to D^{*0}\pi^-$ | 0.34              | 0.024               | 0.006               | 0.49              | 0.41              | 0.52 ± 0.10       |

Color favored $\bar{B} \to D\pi$ and $D^*\pi$ decays are small but still can interfere efficiently with the main amplitude given by the naive factorization and that the long distance hadron dynamics can improve remarkably $B(\bar{B} \to D\pi)_{\text{fact}}$ and $B(\bar{B} \to D^*\pi)_{\text{fact}}$. The predicted branching ratios $B(\bar{B} \to D\pi)_{\text{total}}$ and $B(\bar{B} \to D^*\pi)_{\text{total}}$ for the color favored $\bar{B} \to D\pi$ and $D^*\pi$ decays reproduce well the observed ones.

In the color suppressed $\bar{B}^0 \to D^0\pi^0$ and $D^{*0}\pi^0$ decays, the non-factorizable contributions are considerably larger than the factorized ones in the case of $B_H \simeq 1$ while the former is comparable with the latter in the case of $B_H \simeq 0.5$. Therefore, the ambiguities arising from the uncertainties of the values of $F^{\pi B}_1(m_D^2)$ and $F^{\pi B}_1(m_{D^*}^2)$ which are involved in the factorized amplitudes for these decays are not very serious as long as $B_H \simeq 1$. Within the present approximation, the predicted branching ratio $B(\bar{B}^0 \to D^0\pi^0)_{\text{total}}$ will be not much lower than the present experimental upper limit if $B_H \simeq 1$ while the former will be much less than the latter if $B_H \lesssim 0.5$. 


III. $\bar{B} \to J/\psi \bar{K}$ AND $J/\psi \pi$ DECAYS

Now we study Cabibbo-angle favored $\bar{B} \to J/\psi \bar{K}$ and suppressed $B^- \to J/\psi \pi^-$ decays in the same way as in the previous section. Both of them are color suppressed and their kinematical condition is much different from the color favored $\bar{B} \to D \pi$ and $D^* \pi$ decays at the level of underlying quarks, i.e., $b \to (c\bar{c})_s + s$ in the former but $b \to c + (\bar{u}d)_1$ in the latter. Therefore, dominance of factorized amplitudes in the $\bar{B} \to J/\psi \bar{K}$ and $B^- \to J/\psi \pi^-$ decays has no theoretical support and hence it is expected that non-factorizable long distance contribution is important in these decays.

The factorized amplitude for the $\bar{B} \to J/\psi \bar{K}$ decays is given by

$$M_{\text{fact}}(\bar{B} \to J/\psi \bar{K}) = -iU_{cb}U_{cs}\left\{ \frac{G_F}{\sqrt{2}}a_2f_{\psi}F_{1}^{KB}(m_{\psi}^2) \right\}2m_{\psi}e^{i(p' \cdot p)}.$$  \hspace{1cm} (26)

The value of the decay constant of $J/\psi$ is estimated to be $f_{\psi} \simeq 0.38$ GeV from the observed rate \cite{8} for the $J/\psi \to \ell^+\ell^-$. The value of the CKM matrix element $U_{cs}$ is given by $U_{cs} \simeq U_{ud} \simeq 0.98$. The value of the form factor $F_{1}^{KB}(m_{\psi}^2)$ has not been measured and its theoretical estimates are model dependent. We pick out tentatively the values of $F_{1}^{KB}(m_{\psi}^2)$ based on the following five models, i.e., BSW \cite{1}, GKP \cite{24}, CDDFGN \cite{25}, AW \cite{26} and ISGW \cite{27}, and list the corresponding $B(\bar{B} \to J/\psi \bar{K})_{\text{fact}}$ in Table IV, where we have used $U_{cb} = 0.038$, $\tau_B = 1.5 \times 10^{-12}$ s and $a_2 = 0.11$ as before. The results $(B_{\text{fact}})$ from the factorized amplitudes for all the values of $F_{1}^{KB}(m_{\psi}^2)$ listed in Table IV are much smaller than the observations \cite{3},

$$B(B^- \to J/\psi \bar{K}^-)_{\text{expt}} = (0.102 \pm 0.014) \%$$

$$B(\bar{B}^0 \to J/\psi \bar{K}^0)_{\text{expt}} = (0.075 \pm 0.021) \%.$$  \hspace{1cm} (27)

Non-factorizable contributions to these decays are estimated by using a hard kaon approximation which is a simple extension of the hard pion technique in the previous section. With this approximation and isospin symmetry, non-factorizable amplitude for the $\bar{B} \to J/\psi \bar{K}$ decays is given by

$$M_{\text{non-f}}(\bar{B} \to J/\psi \bar{K}) = \frac{i}{f_{\bar{K}}}\langle \psi |H_{w}|\bar{B}_{s0}^0 \rangle \left\{ e^{i\delta_{\psi \bar{K}}} + \frac{\langle \psi |H_{w}|\bar{B}_{s0}^0 \rangle}{\langle \psi |H_{w}|\bar{B}_{s0}^0 \rangle} m_{B}^2 - m_{\psi}^2 \right\} m_{B}^2 - m_{\psi}^2 \sqrt{\frac{1}{2}} h + \cdots.$$  \hspace{1cm} (28)
Table IV. Branching ratios (%) for the $B \to J/\psi K$ decays where the values of $F_1^{KB}(m_{\psi}^2)$ estimated in the five models, BSW, GKP, CDDFGN, AW and ISGW, in Refs. [1], [24], [25], [26] and [27], respectively, and tentatively chosen $|\delta_{\psi K}| = 60^\circ$ are used. Values of the other parameters involved are the same as in Table III. The data values are taken from Ref. [5].

| Models | BSW | GKP | CDDFGN | AW | ISGW |
|--------|-----|-----|--------|----|------|
| $F_1^{KB}(m_{\psi}^2)$ | 0.565 | 0.837 | 0.726 | 0.542 | 0.548 |
| $B_{fact}$ | 0.010 | 0.022 | 0.016 | 0.009 | 0.009 |
| $B'_{non-f}$ | | | | | |
| $B'_{H} = 1$ | | | | | |
| $B'_{H} = 0.5$ | | | | | |
| $B_{total}$ | | | | | |
| $B'_{H} = 1$ | 0.11 | 0.14 | 0.13 | 0.11 | 0.11 |
| $B'_{H} = 0.5$ | 0.045 | 0.067 | 0.058 | 0.044 | 0.044 |
| Experiment | | | | | |
| $B(B^- \to J/\psi K^-) = (0.102 \pm 0.014)\%$ | | | | | |
| $B(\bar{B}^0 \to J/\psi \bar{K}^0) = (0.075 \pm 0.021)\%$ | | | | | |

where $\delta_{\psi K}$ is the phase from contributions of multi-hadron intermediate states into the $\psi \bar{K}$ final state and the ellipsis denotes neglected contributions of excited mesons [28]. We have used $\langle \bar{B}_s^0 | V_{K+} | B^- \rangle = -1$ and $\sqrt{2} \langle \bar{B}_s^0 | A_{K+} | B^- \rangle = -h$ which are flavor $SU_f(3)$ extensions of Eqs.(16) and (17). Values of asymptotic matrix elements, $\langle \psi | H_w | \bar{B}_s^0 \rangle$, etc., can be estimated by using the factorization as before. Then the total amplitude for the $B \to J/\psi K$ decays is approximately given by

$$M(\bar{B} \to J/\psi \bar{K})_{total} 
\simeq -iU_{cb}U_{cs}\{5.73F_1^{KB}(m_{\psi}^2) + B'_{H}\{3.82e^{i\delta_{\psi K}} + 5.11\}\}a_2 \times 10^{-5} \text{ GeV}$$

(29)

where $f_K \simeq 0.16$ GeV and $f_{B_s} \simeq f_{B_s} = 202 \pm 26$ MeV from the updated lattice QCD result [21], $f_{B_s} = 202 \pm 26$ MeV, have been taken. $B'_{H}$ is a $B$ parameter corresponding to $B_H$ in Eq.(25).
From the above amplitude, we see that \( |M_{\text{fact}}(B \to J/\psi K)| \lesssim |M_{\text{non-f}}(B \to J/\psi K)| \) unless \( B'_{H} \lesssim 0.5 \). If \( B'_{H} \simeq 1 \), the non-factorizable contribution will be dominant and \( B(\bar{B} \to J/\psi \bar{K})_{\text{total}} \) from the amplitude in Eq.(29) can reproduce the observed values in Eq.(27) when reasonable values of \( |\delta_{\psi\bar{K}}| \) are taken. [Since the \( \psi\bar{K} \) state is exotic, \( \delta_{\psi\bar{K}} \) will be not very far from \( \delta_{3}^{*} \) in the \( \bar{B} \to D^{*}\pi \) decays.] We list our results for \( |\delta_{\psi\bar{K}}| = 60^\circ \) in Table IV as an example. It is seen that ambiguities arising from uncertainty of \( F_{KB}1(m_{\psi}^{2}) \) which is included in the factorized amplitude will be considerably diluted because of the non-factorizable contribution if \( B_{H} \simeq 1 \).

For the Cabibbo-angle suppressed \( B^{-} \to J/\psi\pi^{-} \), the same technique and values of parameters as the above lead to

\[
M(B^{-} \to J/\psi\pi^{-})_{\text{total}} \approx -iU_{cb}U_{cd}\{5.73F_{1}^{\pi B}(m_{\psi}^{2}) + B'_{H}[3.79e^{i\delta_{\psi\pi}} + 5.52]\}a_{2} \times 10^{-5} \text{ GeV.} \tag{30}
\]

Using \( F_{1}^{\pi B}(m_{\psi}^{2}) \approx F_{1}^{KB}(m_{\psi}^{2}) \) and \( \delta_{\psi\pi} \approx \delta_{\psi\bar{K}} \) expected from \( SU_{f}(3) \) symmetry, we obtain

\[
B(B^{-} \to J/\psi\pi^{-})_{\text{total}} \approx \left| \frac{U_{cd}}{U_{cs}} \right|^{2} B(B^{-} \to J/\psi K^{-})_{\text{total}} \tag{31}
\]

which is well satisfied by experiments \([29,30]\). From Eq.(30), it is seen that the non-factorizable long distance contribution is again dominant in this decay when \( B'_{H} \simeq 1 \). \( B(B^{-} \to J/\psi\pi^{-})_{\text{total}} \) from the amplitude Eq.(30) which includes both of the factorized amplitude and the non-factorizable one can reproduce the existing experimental data,

\[
B(B^{-} \to J/\psi\pi^{-})_{\text{expt}} = (4.7 \pm 2.6) \times 10^{-5} \text{ (CLEO [29])},
\]

\[
= (5.0 \pm \frac{2.1}{1.9}) \times 10^{-5} \text{ (CDF [30])}, \tag{32}
\]

by taking \( a_{2} \approx 0.11 \) and reasonable values of \( |\delta_{\psi\pi}| \) and \( B'_{H} \).

**IV. SUMMARY**

In summary, we have investigated the \( \bar{B} \to D\pi, D^{*}\pi, J/\psi \bar{K} \) and \( J/\psi\pi^{-} \) decays describing the amplitude for these decays by a sum of factorizable and non-factorizable contribu-
tions. The former amplitude has been estimated by using the naive factorization while the latter has been calculated by using a hard pion (or kaon) approximation. The so-called final state interactions have been included in the non-factorizable long distance contributions. The non-factorizable contribution to the color favored $B \to D\pi$ and $D^*\pi$ decays is rather small and therefore the final state interactions seem to be not very important in these decays although still not negligible. By taking reasonable values of the phase shifts arising from contributions of multi-hadron intermediate states to the non-factorizable amplitudes, the observed branching ratios for these decays can be well reproduced in terms of a sum of the hard pion amplitude and the factorized one when the values of the coefficients $a_1 \simeq 1.03$ and $a_2 \simeq 0.11$ of four quark operators in the BSW weak Hamiltonian are taken. Namely, the factorized amplitudes are dominant but not complete and long distance hadron dynamics should be carefully taken into account in hadronic weak interactions of $B$ mesons. If $a_2 \simeq 0.2$, which may be given by the next-to-leading order corrections but very strongly dependent on the choice of renormalization scheme and unstable [10], were taken, however, these decays might be saturated by the factorizable contributions [3,7], and then the sum of factorized and non-factorizable amplitudes would provide too large rates for these decays unless $B_H \ll 1$.

In color suppressed $B \to D^0\pi^0, D^{*0}\pi^0, J/\psi\bar{K}$ and $J/\psi\pi^-$ decays, non-factorizable long distance contributions are important. In particular, in the $B^0 \to J/\psi\bar{K}^0$ decay, long distance physics should be treated carefully since it will play an important role to determine the $CP$ violating CKM matrix element. When $a_2 \simeq 0.11$ are taken, a sum of factorized and non-factorizable amplitudes with reasonable values of the phases $|\delta_{\psi\bar{K}}|$ and $|\delta_{\psi\pi}|$ can reproduce the observed values of $B(B \to J/\psi\bar{K})$ and $B(B^- \to J/\psi\pi^-)$, respectively, although both of the present theory and the existing data still contain large ambiguities.

The non-factorizable amplitudes are proportional to asymptotic ground-state-meson matrix elements of $H_w$, i.e., the $B$ parameter, $B_H$ or $B'_H$. If $B_H \simeq B'_H \simeq 1$, the non-factorizable amplitudes will be dominant in the amplitudes for the color suppressed decays and the value of $B(\bar{B}^0 \to D^0\pi^0)$ is predicted to be not much lower than the present experimental upper
limit. If $B(B^0 \to D^0\pi^0)$ is measured to be much less than the present upper limit, $B_H$ will be considerably smaller than unity. If $a_2 \simeq 0.2$ were taken, the sum of the factorized and non-factorizable amplitudes would give too large branching ratios for the color suppressed decays unless $B_H$ and $B'_H$ are much less than unity.

Therefore more precise measurements of branching ratios for the color suppressed decays, in particular, $B(\bar{B} \to D^0\pi^0)$ are useful to determine the non-factorizable long distance contributions in hadronic weak decays of $B$ mesons.

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