A POSSIBLE BIFURCATION IN ATMOSPHERES OF STRONGLY IRRADIATED STARS AND PLANETS

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ABSTRACT

We show that under certain circumstances the differences between the absorption mean and Planck mean opacities can lead to multiple solutions for an LTE atmospheric structure. Since the absorption and Planck mean opacities are not expected to differ significantly in the usual case of radiative equilibrium, nonirradiated atmospheres, the most interesting situations in which the effect may play a role are strongly irradiated stars and planets, and also possibly structures in which there is a significant deposition of mechanical energy, such as stellar chromospheres and accretion disks. We have presented an illustrative example of a strongly irradiated giant planet in which the bifurcation effect is predicted to occur for a certain range of distances from the star.

Subject headings: binaries: general — planetary systems — planets and satellites: general — radiative transfer — stars: atmospheres — stars: low-mass, brown dwarfs

1. INTRODUCTION

There are many situations in which the atmosphere of an object, a star or a planet, is irradiated by a companion star in such a way that this irradiation significantly influences its atmospheric structure. In the case of classical close binary stars, the effect exists, but is rarely dramatic because the effective temperatures differ by a factor of a few. On the other hand, the effect may be quite dramatic in the case of substellar mass objects such as giant planets and brown dwarfs, irradiated by a solar-type star, in which case the ratio of their effective temperatures may reach a factor of 100 or more.

We stress that we use the term “effective temperature” as used in the theory of stellar atmospheres, namely, as a measure of the total energy flux coming from the interior of the object.

In the case of extrasolar giant planets (EGPs), we recently have witnessed a significant increase in interest in predicting EGP spectra. This was motivated in part by two detected planetary transits, which are in principle able to provide direct information about the planet’s atmosphere. Another motivation is the need to predict spectra of extrasolar planets to guide the design of future missions that aim at recording EGP spectra.

We have recently computed a large set of model atmospheres and spectra of EGPs (Sudarsky, Burrows, & Hubeny 2003, hereafter SBH03). We have used a modification of our universal stellar atmosphere program TLUSTY (Hubeny 1988; Hubeny & Lanz 1995), called COOLTLUSTY. This variant does not compute opacities on the fly; instead it uses a pretabulated opacity as a function of wavelength, temperature, and density. The tables are computed using an updated version of the chemical equilibrium code of Burrows & Sharp (1999), which includes a prescription to account for the rainout of species in a gravitational field.

Motivated by the recent discovery of the second transiting planet OGLE-TR-56 (Konacki et al. 2003; Sasselov 2003), which is believed to have a separation of a mere 0.0225 AU from its parent star, we tried to extend the SBH03 models to higher irradiations. However, COOLTLUSTY faced significant convergence problems. After various attempts and using various strategies, we have discovered that the convergence behavior is not the result of a bug in the program or of an insufficiency in our numerical scheme, but is in fact a consequence of an existence of multiple solutions that may lead to disastrous effects for convergence. When studying the effect, we found that its roots are quite general and are, in fact, applicable to other cases in the theory of stellar atmospheres. Therefore, we devote the present paper to explaining the effect in general terms.

2. BASIC EQUATIONS

The atmospheric structure is obtained by solving simultaneously the radiative transfer equation, the radiative equilibrium equation, and the hydrostatic equilibrium equation. For simplicity, we assume LTE. Since our basic aim here is to study atmospheres of irradiated giant planets, this approximation is a reasonable one, although it should be relaxed in the future, as the LTE assumption was eventually relaxed in modern studies of stellar atmospheres. Here we present a brief overview of the basic equations.

The radiative transfer equation is written as

\[
\frac{\mu}{dn} \frac{dI_{\nu}}{dm} = \chi_{\nu} (I_{\nu} - S_{\nu}) ,
\]

where \(I_{\nu}\) is the specific intensity of radiation as a function of frequency \(\nu\), angle (described through the cosine of the angle of propagation with respect to the normal to the surface) \(\mu\), and the geometrical coordinate, taken here as the column mass \(m\). The latter is defined as \(dm = -\rho dz\), where \(z\) is a geometrical distance (measured outward) and \(\rho\) the mass
density. The monochromatic optical depth is defined as \( \tau_{\ell} = \chi_{\ell} \, dm \). Finally, \( S_\ell \) is the source function, given in LTE by

\[
S_\ell = \frac{\kappa_\ell}{\chi_\ell} B_\ell + \frac{\sigma_\ell}{\chi_\ell} J_\ell .
\]

(2)

Here, \( \kappa_\ell \) is the true absorption coefficient, \( \sigma_\ell \) the scattering coefficient, and \( \chi_\ell = \kappa_\ell + \sigma_\ell \) the total extinction coefficient. All coefficients are per unit mass.

The boundary conditions are provided through the diffusion approximation at the deepest point,

\[
I_{\ell}(\tau_{\text{max}}) = B_\ell [T(\tau_{\text{max}})] + \mu \frac{dB_\ell}{dT_\ell} |_{\tau_{\text{max}}} , \quad \mu > 0 ,
\]

(3)

and the upper boundary condition is

\[
I_{\ell}(0) = I_{\ell}^{\text{ext}} , \quad \mu < 0 ,
\]

(4)

where \( I_{\ell}^{\text{ext}} \) is the specific intensity of the external irradiation. For simplicity, we assume isotropic irradiation, \( I_{\ell}^{\text{ext}} = J_{\ell}^{\text{ext}} \).

It is convenient to express the frequency-integrated irradiation intensity as

\[
J_{\ell}^{\text{ext}} \equiv \int_0^\infty J_{\ell}^{\text{ext}} \, d\nu = WB(T_\star) ,
\]

(5)

where \( T_\star \) is an effective temperature of the irradiating star (in case the irradiation source is not a star, \( T_\star \) is merely the characteristic temperature of the incoming radiation), and \( W \) is a dilution factor.

The moments of the specific intensity are defined as

\[
(J_\ell, H_\ell, K_\ell) \equiv \frac{1}{2} \int_{-1}^1 I_{\ell\mu}(1, \mu, \mu^2) \, d\mu .
\]

(6)

The first moment of the transfer equation is written

\[
\frac{dH_\ell}{dm} = \chi_\ell (J_\ell - S_\ell) = \kappa_\ell (J_\ell - B_\ell) ,
\]

(7)

where the second equality follows from equation (2).

Integrating over frequency we obtain

\[
\frac{dH_\ell}{dm} = \kappa_J J - \kappa_B B ,
\]

(8)

where \( \kappa_J \) and \( \kappa_B \) are the absorption and Planck mean opacities, respectively, defined by

\[
\kappa_J = \int_0^\infty \kappa_{\ell J} J_\ell \, d\nu / \int_0^\infty J_\ell \, d\nu ,
\]

(9)

and

\[
\kappa_B = \int_0^\infty \kappa_{\ell B} B_\ell \, d\nu / \int_0^\infty B_\ell \, d\nu .
\]

(10)

These two opacities are usually assumed to be equal. This is an excellent approximation in the case of nonirradiated atmospheres, because \( J_\ell \approx B_\ell \) in optically thick regions, and \( J_\ell(\tau < 1) \propto 1/2 B_\ell(\tau = 1) \) (the Eddington-Barbier relation), so \( J \) is proportional to \( B \) and the averaging over \( J \) and \( B \) leads to very similar results. However, here we maintain the distinction because the difference between \( \kappa_J \) and \( \kappa_B \) turns out to be crucial in the case of strongly irradiated atmospheres.

The second moment of the transfer equation is

\[
\frac{dK_\ell}{dm} = \chi_\ell H_\ell ,
\]

(11)

and integrating over frequency we obtain

\[
\frac{dK}{dm} = \chi H H ,
\]

(12)

where

\[
\chi H = \int_0^\infty \chi_\ell H_\ell \, d\nu / \int_0^\infty H_\ell \, d\nu ,
\]

(13)

which is called the flux mean opacity. Notice that unlike the two previous opacities, which were averages of the true absorption coefficient (without the scattering term), the flux mean opacity contains the total absorption coefficient.

Finally, the radiative equilibrium equation is written

\[
\int_0^\infty \kappa_\ell(J_\ell - B_\ell) \, d\nu = 0 ,
\]

(14)

which can be rewritten, using the above defined mean opacities, as

\[
\kappa_J J - \kappa_B B = 0 .
\]

(15)

Substituting equation (15) into equation (8), we obtain another form of the radiative equilibrium equation:

\[
\frac{dH}{dm} = 0 , \quad \text{or} \quad H = \text{const} = \frac{\sigma}{4\pi} T_{\ell \ell}^4 ,
\]

(16)

where \( \sigma \) is the Stefan-Boltzmann constant.

3. TEMPERATURE STRUCTURE

3.1. General

The above equations are exact, given LTE, but are of course only formal because the mean opacities \( \kappa_J \) and \( \chi H \) are not known in advance; only \( \kappa_B \) is a known function of temperature and density. Nevertheless, assuming that they are known and equal to the Rosseland mean opacity, we can write a solution for temperature, following the classical textbook procedure, known as “LTE-gray model atmospheres” (e.g., Mihalas 1978). A generalization of a classical model for the case of external irradiation is given by Hummer (1982), and for the case of accretion disks and unequal mean opacities by Hubeny (1990).

The procedure is as follows. From equation (15) we have \( B = (\kappa_J/\kappa_B)J \), which allows us to express \( T \) through \( J \) using the well-known relation \( B = (\sigma/\pi) T^4 \). To determine \( J \), we use the solution for the second moment of the transfer equation \( K(\tau_H) = H \tau_H + K(0) = (\sigma/4\pi) T_{\ell \ell}^4 \tau_H + K(0) \), where \( \tau_H \) is the optical depth associated with the flux mean opacity, and express the moment \( K \) in terms \( J \) using the Eddington factor, \( f_K \equiv K/J \). Similarly, we express the surface flux through the second Eddington factor, \( f_H \equiv H(0)/J(0) \), and we end up with (see also Hubeny 1990)

\[
T^4 = \frac{3}{4} T_{\ell \ell}^4 \frac{\kappa_J}{\kappa_B} \left( \frac{1}{3f_K} \tau_H + \frac{1}{3f_H} \right) + \frac{\kappa_J}{\kappa_B} W T_{\ell \ell}^4 .
\]

(17)

Again, this solution is exact within LTE. The usual LTE-gray model consists of assuming that all the mean opacities
are equal to the Rosseland mean opacity. Moreover, if one adopts the Eddington approximation \((f_k = 1/3; f_H = 1/\sqrt{3})\), then one obtains a simple expression
\[
T^4 = \frac{1}{4} T_{\text{eff}}^4 \left( \tau + 1/\sqrt{3} \right) + WT^4. \tag{18}
\]

We consider the most interesting case, namely that of strong irradiation, defined by \(WT^4 >> T_{\text{eff}}^4\). In this case, the second term in brackets is negligible, and we can define a penetration depth as the optical depth where the usual thermal part \((\propto T_{\text{ef}}^4)\) and the irradiation part \((\propto WT^4)\) are nearly equal, viz.,
\[
\tau_{\text{pen}} \approx W \left( \frac{T_{\text{eff}}}{T} \right)^4. \tag{19}
\]

The behavior of the local temperature in the case of the strictly gray model is very simple—it is essentially constant, \(T = T_0 \equiv W^{1/4} T_*\) for \(\tau < \tau_{\text{pen}}\), and follows the usual distribution \(T \propto \tau^{1/4} T_{\text{eff}}\) in deep layers, \(\tau > \tau_{\text{pen}}\). We stress that while the strictly gray model exhibits an essentially isothermal structure down to \(\tau \approx \tau_{\text{pen}}\), such will not be the case for a nongray model, as we show in detail in the next sections.

### 3.2. Surface Layers

In the general case, we have to retain the ratio of the absorption and Planck mean opacities. In the irradiation-dominated layers \((\tau_H < \tau_{\text{pen}})\), the temperature is given by
\[
T = \gamma W^{1/4} T_, \tag{20}
\]
where
\[
\gamma \equiv (\kappa_J/\kappa_B)^{1/4}. \tag{21}
\]
As stated before, \(\gamma \approx 1\) in the case of no or weakly irradiated atmospheres. However, in the case of strong irradiation, \(\gamma\) may differ significantly from unity, and, moreover, may be a strong function of temperature, and to a lesser extent density. This is easily seen by noticing that in optically thin regions, the local mean intensity is essentially equal to twice the irradiation intensity, because the incoming intensity is equal to the irradiation intensity, and the outgoing intensity is roughly equal to it as well. The reason is that in order to conserve the total flux when it is much smaller than the partial flux in the inward or the outward direction, both fluxes should be almost equal, as are the individual specific and mean intensities. More specifically,
\[
H = H_{\text{out}} - H_{\text{ext}} = \int_0^\infty \int_0^1 I_{\nu,\mu} d\mu d\nu - \int_0^\infty \int_0^1 I_{\nu,-\mu} d\mu d\nu. \tag{22}
\]
If \(H < H_{\text{ext}}\), then we must have \(\int_0^\infty I_{\nu,\mu} d\nu \approx \int_0^\infty I_{\nu,-\mu} d\nu\) for all angles, and thus \(J_\nu \approx 2J_{\text{ext}} \approx 2 W^{1/4} B_\nu(T_*)\). We can then write the absorption mean opacity as a function of \(T\) and \(T_*\) as
\[
\kappa_J(T, T_*) \approx \frac{\int \kappa_J(T) B_\nu(T_*) d\nu}{\int B_\nu(T_*) d\nu} \approx \frac{\int \kappa_J(T) B_\nu(T_*) d\nu}{\int B_\nu(T_*) d\nu}. \tag{23}
\]
The dilution factor cancels out, and only the spectral distribution of the irradiation intensity matters.

In other words, the mean intensity is an average of the opacity weighted by the Planck function corresponding to the effective temperature of the source of irradiation, while the Planck mean is an average of the opacity weighted by the Planck function corresponding to the local temperature. Obviously, they can be quite different. If the monochromatic opacity differs significantly in the region where \(B_\nu(T)\) and \(B_\nu(T_*)\) have their local maxima, the resulting \(\kappa_J\) and \(\kappa_B\) will differ substantially. If, moreover (and this is the crucial point), the monochromatic opacity depends sensitively on temperature, the opacity ratio \(\gamma\) may have a complex, and generally nonmonotonic, dependence on temperature.

The local temperature in the upper layers is given, in view of equation (20), by the expression
\[
T/T_0 = \gamma(T). \tag{24}
\]
It is now clear that if \(\gamma\) exhibits a strongly nonmonotonic behavior in the vicinity of \(T_0\), for instance if it has a pronounced minimum or maximum there, equation (24) may have two or even more solutions!

### 3.3. Deep Layers

The bifurcation behavior is not limited to the surface layers, but may continue to large optical depths. This might seem surprising at first sight because from equation (17) one might expect that once \(\tau > 1\), the mean opacities \(\kappa_J\) and \(\kappa_B\) become roughly equal, and the Eddington factor is \(f_k \approx 1/3\), so that \(T = T_0\) for all the way till \(\tau_{\text{pen}}\). We recall that the penetration depth may be quite large, for instance, for the case \(T_{\text{eff}} = 75,000\) K, \(T_* = 6000\) K, and \(W = 2.2 \times 10^{-3}\) (the case studied in detail in the next section), \(\tau_{\text{pen}} = 9 \times 10^4\).

However, the behavior of the local temperature may be more complicated. This is linked to another interesting inequality of mean opacities that is usually taken for granted, namely, the flux mean opacity \(\chi_H\) and the Rosseland mean opacity \(\chi_{\text{Ross}}\). The Rosseland mean opacity is, in fact, defined in such a way that it is equal to the flux opacity in the diffusion approximation. Indeed, in this approximation,
\[
H_\nu \approx \frac{1}{3} \frac{dB_\nu}{d\tau_\nu} = \frac{1}{3} \frac{1}{\chi_\nu} \frac{d\chi_\nu}{d\tau_\nu} = \frac{1}{3} \frac{1}{\chi_\nu} \frac{dT}{d\tau_\nu} \frac{d\tau_\nu}{d\tau}. \tag{25}
\]
and therefore
\[
\chi_H \approx \frac{\int_0^\tau dB_\nu/d\tau_\nu d\tau_\nu}{\int_0^\tau (1/\chi_\nu)(dB_\nu/d\tau_\nu) d\tau_\nu} = \chi_{\text{Ross}},
\]
where the latter equality represents the definition of the Rosseland mean opacity.

In the case of strong irradiation, an interesting, and fundamentally different, situation appears. Since the net flux is very small, the total flux in the \(\mu > 0\) (outgoing) hemisphere is roughly equal to that in the \(\mu < 0\) (incoming) hemisphere (see above). Because the monochromatic opacity varies strongly with frequency, there are frequency regions where the net monochromatic flux is positive, and regions where it is negative. The flux mean opacity close to the surface may attain large values, either positive or negative, depending on whether \(\chi_\nu\) weighs the positive or negative net flux regions more.

Going to deeper layers, the net monochromatic flux decreases because the radiation field becomes more...
isotropic for all frequencies, so that the flux mean opacity decreases. Consequently, the corresponding flux mean optical depth increments $\Delta \tau_H \approx \chi_H \Delta m$ become very small, and $\tau_H$ will exhibit a plateau where it remains essentially constant with $m$. Finally, in the regions where all the influence of the external irradiation dies out, the usual diffusion approximation sets in, and the flux mean optical depth becomes essentially equal to the Rosseland optical depth.

This means that for strong irradiation, the temperature in the deep layers should exhibit a plateau with

$$T_{\text{plateau}} \approx \frac{4}{3} T_{\text{eff}}^4 \tau_H,$$

where $\tau_H$ is the plateau value of the flux mean optical depth. The value of $\tau_H$ and thus $T_{\text{plateau}}$ depends on the exact form of the monochromatic opacity; a rough estimate is provided by setting $\tau_H \approx \tau_{\text{pen}}$, in which case $T_{\text{plateau}} \approx (7/4)^{1/4} W^{1/4} T_\gamma \approx 1.15T_0$.

4. EXAMPLE OF AN EXTRASOLAR GIANT PLANET IRRADIATED BY A SOLAR-TYPE STAR

4.1. Opacities

Using partition functions, LTE level densities, stimulated emission corrections, and broadening algorithms, we generated opacity tables from numerous available line lists. For gaseous H$_2$O, Partridge & Schwenke (1997) have calculated the strengths of more than 3 $\times 10^8$ lines. We used a subset of their 4 $\times 10^7$ strongest lines. For various other molecular species (e.g., NH$_3$, PH$_3$, H$_2$S, and CO), we used the HITRAN (Rothman et al. 1992, 1998) and GEISA (Husson et al. 1994) databases, augmented with additional lines from theoretical calculations and measurements (Tyuterev et al. 1994; Goorvitch 1994; Tipping 1990; Wattson & Rothman 1993). For methane, shortward of $\sim 1.0$ $\mu$m we used the Karkoschka (1994) opacities, and between $\sim 1.0$ and $1.58$ $\mu$m we used the Strong et al. (1993) opacities. Longward of $1.58$ $\mu$m, we used the Dijon methane database (Borysow et al. 2003) that includes the hot bands. Opacity due to collision-induced absorption (CIA) by H$_2$ and helium is taken from Borysow & Fromhold (1990), Zheng & Borysow (1995), and Borysow, Jørgensen, & Zheng (1997), as updated in Borysow (2002).

FeH and CrH opacities were taken from Dulick et al. (2003) and Burrows et al. (2002), respectively. The TiO line lists for its nine major electronic systems were taken from the Schwenke ab initio calculations, as modified by Allard, Hauschildt, & Schwenke (2000). These data include lines due to isotopically substituted molecules $^{46}$Ti$^{16}$O, $^{48}$Ti$^{16}$O, $^{49}$Ti$^{16}$O, and $^{50}$Ti$^{16}$O relative to the most abundant isotopic form $^{48}$Ti$^{16}$O. For VO, we used the line list provided by B. Plez (1999, private communication). Because $^{51}$V is by far vanadium’s most abundant isotope, the lines of isotopically substituted molecules are not necessary. The line lists and strengths for the neutral alkali elements (Li, Na, K, Rb, and Cs) were obtained from the Vienna Atomic Line Data Base (Piskunov et al. 1995). The general line shape theory of Burrows, Marley, & Sharp (2000) was used for Na and K, while those of Nefedov, Sinel’shchikov, & Usachev (1999) and Dimitrijević & Peach (1990) were used for the other alkali metals.

In practice, we pre-calculate a large opacity table in $T/\rho$ frequency space in which we interpolate during the iteration of the atmosphere/spectral model to convergence. The table contains 30,000 frequency points from 0.3 to 300 $\mu$m, uniformly spaced in steps of 1 cm$^{-1}$. Generally, 5000 geometrically spaced frequency points are used in the radiative transfer model. In other words, our approach belongs to the category of “opacity sampling” methods for treating line blanketing.

We consider two different opacity tables; one that includes opacity due to TiO and VO, and the other one without these molecules. In the case without TiO/VO, these species were removed both from the opacity table as well as from the state equation, so these species are considered completely absent everywhere in the atmosphere. These two different opacity tables suggest themselves because for an irradiated atmosphere we obtain high temperatures in the low-pressure outer atmosphere. Importantly, temperature inversions are very possible. However, thermochemical equilibrium calculations with rainout (Burrows & Sharp 1999; Lodders & Fegley 2002) suggest that TiO and VO would exist at such high-$T$/low-$P$ points. Since the presence of TiO and VO is quite natural at high-$T$/high-$P$ points, one confronts a situation in which there are two regions rich in TiO/VO, separated by a middle region that is TiO/VO free. In a gravitational field with a monotonically pressure profile, this gap could act like a cold trap in which the TiO/VO that is transported by molecular or eddy diffusion from the upper low-$P$ region into the intermediate cooler region would condense and settle out, thereby depleting the upper low-$P$ TiO/VO-rich region. This would eventually leave no TiO/VO at an altitude to provide the significant absorption that could lead to a bifurcation. However, the cold-trap effect can not be anticipated in abundance tables that are functions of temperature and pressure alone. These tables are not cognizant of gravity, nor are they aware of the global $T/P$ profile. Whether this cold-trap effect in fact happens is not yet clear. Furthermore, the same cold-trap effect might obtain for other EGP species (e.g., Fe). Hence, we use the presence or absence of TiO/VO at high-$T$/low-$P$ points and the associated ambiguity as a means to explore the real mathematical bifurcation effect we identify in this paper and leave to a future work the study of the true viability of cold traps in irradiated EGP atmospheres.

4.2. Results

We consider an intrinsically cold giant planet, with $T_{\text{eff}} = 75$ K and $\log g = 3$, irradiated by a G0 V star with a spectral energy distribution taken for simplicity to be the corresponding Kurucz (1994) model atmosphere. We consider the class V, “roaster” situation, as defined by SBH03.

The opacity ratio index $\gamma$ is plotted in Figure 1 as a function of temperature for three densities characteristic to the outer layers of an atmosphere: $\rho = 10^{-12}, 10^{-11}$, and $10^{-10}$ g cm$^{-3}$. The upper panel shows the results for the opacities including TiO/VO, while the lower panel shows those without TiO/VO. It is clearly seen that in the former case the ratio $\kappa_J/\kappa_B$, and thus $\gamma$, exhibits a sharp peak around 1500 K; the dependence on density is not strong. The ratios $T/T_0$ for $T_0 = 450$ K (weak irradiation), 1250 K (medium strong irradiation), and 2600 K (very strong irradiation) are over-plotted with dotted lines. The latter roughly corresponds to the case of the recently discovered transiting planet OGLE-TR-56. According to equation (24), the temperature at the surface layers is found at the intersection of the curves of $\gamma(T)$ and $T/T_0$; thus, we see that for both low and high
irradiation there are unique solutions (corresponding to $T \approx 250$ K and $T \approx 2600$–3000 K), but that in the intermediate cases there are indeed multiple solutions.

The opacity table without TiO/VO does not exhibit a peak around 1500 K, so one obtains unique solutions for the temperature for all irradiations.

To explain the behavior of the $\kappa_j$ over $\kappa_B$ ratio, we plot in Figure 2 two examples of the monochromatic opacity, for $T = 1600$ K (close to the maximum of $\gamma$) and $T = 520$ K (the region near the minimum of $\gamma$). In both cases $\rho = 10^{-11}$ g cm$^{-3}$. We also plot normalized Planck functions for the two nominal temperatures, 1600 and 520 K (dotted lines), and for $T = T_0 = 6000$ K (dashed line). The latter weighs most heavily the optical region around $\nu \approx (4-5) \times 10^{14}$ Hz, while the local Planck functions put the weight at much lower frequencies (around $10^{14}$ and $3 \times 10^{13}$ Hz, respectively). In the case of the higher temperature, $T = 1600$ K, the monochromatic opacity exhibits a maximum just in the region where the Planck function corresponding to $T_0$ is largest (because of TiO and VO); therefore, $\kappa_j > \kappa_B$. For the lower temperature, $T = 520$ K, the opacity in the optical range is lower by orders of magnitude, so $\kappa_j$ is now significantly lower. Since the Planck function for $T = 520$ K emphasizes frequencies below $7 \times 10^{13}$ Hz, where the monochromatic opacity is very large, we obtain $\kappa_j < \kappa_B$.

There are two other features seen in Figure 2 that are worth noticing. First, there is a large increase in $\gamma$ for temperatures around and below 100 K. This is a consequence of low opacity at low frequencies. However, the opacity sources are quite uncertain in this regime. For studies of extremely cool objects the opacity table should be checked and possibly upgraded, but studying these objects is not our objective here. Second, in the case of no TiO/VO opacity (lower panel), it appears that $\gamma$ nearly coincides with $T/T_0$. Taken at face value, this would imply that the ratio $\int \kappa_j B_\nu(T_0) d\nu / \int \kappa_j B_\nu(T) d\nu$ is very close to unity. However, we do not see any compelling reason why this should be generally valid, so we conclude that such a behavior is just a coincidence.

In Figure 3 we demonstrate the behavior of the flux mean opacity discussed in § 3.3. The high–surface-$T$ branch exhibits a negative flux mean optical depth at the surface layers because the opacity is high in the optical region where the next flux is negative. Consequently, the flux mean opacity is negative there, and thus, the flux mean opacity (and the corresponding optical depth) on the plateau is lower than in the case of the low–surface-$T$ branch.

We have also computed exact LTE models as described in SBH03. We have used the stellar atmosphere code COOLTLUSTY, which is a variant of the universal program TLUSTY (Hubeny 1988; Hubeny & Lanz 1995). We stress that the emergent flux is not computed by a separate program; instead it is computed by COOLTLUSTY. This also means that we do not use different opacity tables for computing the atmospheric structure and for computing the spectra. We have also tested the sensitivity of the computed

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4 We are indebted to the referee for pointing out this possibility to us.

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model on different opacity samplings; we have resampled our original frequency grid of 5000 points to lower numbers of frequencies, and even for 300 points we found very small differences in the temperature structure (of the order of a few K). In some cases it was found advantageous to first converge a model with 300 frequencies, then using this as an input, to reconverge a model with the full grid of 5000 frequencies in the next step.

The temperature structure is displayed in Figure 4, which demonstrates that the simplified description put forward above faithfully reflects the true behavior of temperature. Solid lines represent models computed for the opacity table without TiO/VO, while the dashed and dot-dashed lines represent models with TiO/VO. The models without TiO/VO always converged to a unique solution, and with temperature monotonically decreasing outward, as we might have expected from the above discussion.

The behavior of the models with TiO/VO is more interesting. The high-irradiation model (Fig. 4, top dashed line) indeed converged to a unique solution. The surface temperature is 2600 K, in excellent agreement with the value expected from Figure 1 (the density at the uppermost point is about $3 \times 10^{-12}$ g cm$^{-3}$). Even the initial increase of temperature monotonically decreasing outward, as we might have expected from the above discussion.

The low-irradiation model also converged to a unique solution; moreover, its surface temperature of about 250 K is in excellent agreement with the value one may expect from Figure 1. Again, both models, without and with TiO/VO, agree very well.

The two middle curves of Figure 4, the dashed and dot-dashed curves, correspond to the same irradiation, and, thus, represent the bifurcation predicted by our analysis. The model with high surface temperature was converged starting from the extremely irradiated model (through several intermediate steps), while the model with the low surface temperature was converged starting from scratch, i.e., from a usual LTE-gray model in which all mean opacities are created equal. The high surface temperature at about 2200 K essentially coincides with the high-temperature intersection of about 2200 K in Figure 1. The low–surface–$T$ solution exhibits a somewhat higher temperature than would follow from the low-$T$ intersection in Figure 1—650 versus 550 K, but in view of all the approximations involved such an agreement is still very good. We did not numerically find the third possible solution with a surface $T$ of about 1300–1400 K.

We stress that the numerical solutions were not contrived in any way to lead to the predicted bifurcation. In fact, we first discovered the effect purely numerically—we obtained two different, yet perfectly well converged solutions, depending on the input model. We have continued the convergence of both solution until the maximum relative change of temperature and density decreased below $10^{-5}$ in all depth points.

Several features of the overall accuracy of the model are worth mentioning. In fact, the low value of maximum relative change of temperature and density is not a satisfactory criterion of the convergence. One should also check the
The accuracy of the computed flux gradient and the value of the net flux. That is, for the former case we have to examine the quantity $e_{\text{grad}} = (\kappa_J - \kappa_B) / \kappa_B$, and for the latter case the quantity $e_{\text{flux}} = (H - H_0) / H_0$, where $H_0 = (\sigma/4\pi)T_{\text{eff}}^4$ is the net flux. For the models presented in Figure 4, $e_{\text{grad}}$ is very small, typically $10^{-10}$ or less. The conservation of the total net flux is much harder to achieve; the bifurcated models have typically $e_{\text{flux}} \approx \text{few} \times 10^{-4}$ near the surface, while $e_{\text{grad}}$ reaches a few $\times 10^{-2}$ for column mass around $m \approx 10^2 \, \text{g cm}^{-2}$ (that is, a few percent), and drops to less than $10^{-3}$ at $m \approx 10^3 \, \text{g cm}^{-2}$ and deeper. The error in the net total flux of several percent might seem large, but one has to bear in mind that the net flux is a subtraction of two large quantities, $H_{\text{out}} - H_{m}$, and the actual numerical error $(H_{\text{out}} - H_{m} - H_0) / H_{\text{out}}$ is of the order of $10^{-5}$ or less. A dramatic demonstration of this fact is that a model, which is almost converged (maximum relative change in $T$ and $\rho$ was less than $10^{-3}$), exhibits a seemingly ridiculous value of $e_{\text{flux}} \approx 10^5$ at upper layers $m < 10^2 \, \text{g cm}^{-2}$ (i.e., an error in the total net flux of some $10^{-5}$), yet the actual temperature difference between this and the fully converged model is it most about 2–3 K, and the predicted flux in both cases is completely indistinguishable on a plot. This again shows that in the case of strong irradiation it is the value of $(H_{\text{out}} - H_{m} - H_0) / H_{\text{out}}$ (or $e_{\text{grad}}$) that is critical for practical convergence, not a much more stringent criterion of the error in the total net flux, $e_{\text{flux}}$.

The difference in the temperature structure is, of course, reflected in the emergent spectrum. In Figure 5 we show the emergent flux corresponding to the two solutions of the intermediate-irradiation (distance 0.08 AU) model, computed with TiO/VO. The models differ dramatically in the optical and near-IR region because of the dramatically different surface temperature and the corresponding increase of the TiO/VO opacity. Since the low–surface-$T$ branch exhibits a higher temperature at the plateau (see Fig. 4), the flux at the low-opacity regions is higher.

Finally, Figure 6 displays the emergent flux for the two models for the highest irradiation, namely for the distance 0.0225 AU, which corresponds to OGLE-TR-56. The thick line is a model without TiO/VO, and the thin line with TiO/VO. There is no bifurcation here, since both opacity tables led to unique solutions, although quite different ones. The corresponding spectra are thus significantly different as well.

5. DISCUSSION AND CONCLUSIONS

We have demonstrated that under certain circumstances the differences between the absorption mean and the Planck mean opacities can lead to multiple solutions for the atmospheric structure. This result is quite robust, and in fact represents a so-far overlooked general result of elementary stellar atmospheres theory. Since we do not expect that the absorption and Planck mean opacities will differ significantly in the usual case of radiative equilibrium of non-irradiated atmospheres, the most interesting situations in which the effect may play role are strongly irradiated stars and planets, and also, possibly, structures in which there is a significant deposition of mechanical energy, such as stellar chromospheres and accretion disks. We are not concerned with these objects here, but would like to stimulate further study of possible bifurcations for these situations. We note that Nolan & Lunine (1988) also found a bimodal behavior of the atmosphere of Triton that is linked to external irradiation.

From the physical point of view we may interpret the absorption mean opacity as the global absorption efficiency, and the Planck mean opacity as the global emission efficiency, of the medium. The integrated mean intensity $J$ acts as a total absorption pool, while the integrated Planck function $B$ as a total thermal pool. The radiative equilibrium, which stipulates that the total radiation energy absorbed on the spot is equal to the total energy emitted on the same spot, therefore acts as a thermostat: the radiative equilibrium sets the local temperature in such a way that $\kappa_J = \kappa_B$. Here $J$ is determined by the radiative transfer, and the local temperature follows as $B \equiv (\sigma/\pi)T_4^4 = \ldots$
In the case of strong irradiation, the weighting that determines $\kappa_J$ is dominated by the incoming radiation. If the monochromatic opacity in the region around the peak of the external irradiation is very sensitive to temperature (for instance if it is very high for high temperatures and very low for low temperatures), the “radiative-equilibrium thermostat” may find two solutions—either a high $T$ together with high $\kappa_J$, or a low $T$ with low $\kappa_J$; in both cases the radiative equilibrium $\kappa_J J = \kappa_B B$ is satisfied.

However, the applicability of this effect to real objects depends critically on a degree of reliability of adopted opacities. For high-temperature conditions in which the gaseous opacities dominate, the situation is relatively well under control, thanks to recent enormous progress in computing atomic data (Opacity Project, Iron Project, OPAL, and others; for a recent review see, e.g., Nahar 2003).

In the case of irradiated giant planets or other substellar-mass objects like L and T dwarfs, the opacity for low density ($\rho \approx 10^{-12}$ to $10^{-9}$ g cm$^{-3}$) and high temperature ($T > 2000$ K) is still relatively uncertain. At the present stage, we cannot be sure that the bifurcation effect really occurs, but we have shown that this is certainly a very real possibility that should be taken into account when constructing atmospheric models.

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