Vortex glass transitions in disordered three-dimensional XY models: Simulations for several different sets of parameters

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The anisotropic frustrated 3D XY model with strong disorder in the coupling constants is studied as a model of a disordered superconductor in an applied magnetic field. Simulations with the exchange Monte Carlo method are performed for frustrations $f = 1/5$ and $f = 1/4$, corresponding to two different values of the magnetic field along the $z$ direction. The anisotropy is also varied. The determination of the helicity modulus from twist histograms is discussed in some detail and the helicity modulus is used in finite size scaling analyses of the vortex glass transition. The general picture is that the behavior in [Phys. Rev. Lett. 91, 077002 (2003)] is confirmed. For strong (e.g. isotropic) coupling in the $z$ direction the helicity modulus fails to scale and it is argued that this is due to a too small effective randomness of such systems for the accessible system sizes.

I. INTRODUCTION

An applied magnetic field in a type-II superconductor will give rise to vortex lines that penetrate the sample. A current applied perpendicular to these vortex lines will give rise to a force perpendicular to both the current and the magnetic field. In a pure system there is nothing that hinders the motion of the vortex lines and their motion leads to flux-flow resistivity and therefore a loss of superconductivity. The presence of point disorder could mean a substantial reduction of the mobility of the vortex lines, but the resistivity would, in the conventional picture, nevertheless always be non-zero.

A vortex glass phase is an alternative possibility that was suggested to restore the true superconducting state. The idea is that the finite disorder strength together with the vortex line interaction leads to diverging energy barriers against the vortex motion, and thereby a vanishing resistivity. This was suggested to take place through a continuous transition with universal exponents and certain scaling properties. Experimental results in support of this picture have been reported but the conclusion of a vortex glass phase has also often been questioned.

There has also been much work on simulations of vortex glass models. The simplest three-dimensional (3D) vortex glass model, that was also the first to be studied, is the 3D gauge glass model that includes the disorder through a random vector potential added to the phase difference of the superconducting order parameter. Already the early simulations found strong evidence for a transition, and with the exchange Monte Carlo (MC) technique it has been possible both to give more convincing evidence for a transition and to determine the value of the correlation length exponent to $\nu = 1.39 \pm 0.04$.

A problem with using the 3D gauge glass as a model of a disordered superconductor in an applied magnetic field, is the generally recognized fact that the model lacks some of the properties and symmetries of the physical system. The applied field both breaks the spatial symmetry of the system and introduces an additional length scale. In a model that properly includes these features one would e.g. have the possibility of anisotropic scaling, i.e. different divergences of the correlations parallel and perpendicular to the applied field.

Several attempts have recently been done to simulate systems with the correct symmetry. The first published results are from simulations of a frustrated 3D XY model with filling $f = 1/4$ and disorder in the coupling constants. The correlation length exponent was there determined to be $\nu = 2.2$ even though the quality of the data did not allow for any firm conclusions. In a second paper by the same author the open boundary conditions employed in the first study were changed to standard periodic boundary conditions. The data now rather suggested $\nu \approx 1.1$, but some quantities still failed to provide good scaling. Some aspects of these simulations are discussed in Sec. VI.

Simulations have also been performed with vortex lines instead of the phase variables of the XY model. A simulation study of such a vortex line model with strong point disorder gives the value $\nu = 0.7$, indistinguishable from the 3D XY exponent. In that study the pinning energy was quite strong which presumably means that most plaquettes are either always occupied or always empty. One possible reason for the 3D XY-like exponents could then be that the model supports vortex loop excitations (as in the 3D XY model) against a background of frozen-in field lines from the applied field.

The present paper is a sequel of Ref. 15 which gave the first numerical support for 3D gauge glass exponents in vortex glass simulations with the correct symmetry. The approach was there to study an anisotropic model with much weaker couplings in the field direction than in the directions perpendicular to the field. This was a natural choice due to experiences from the first order transition between the Abrikosov lattice and the vortex line liquid. In these simulations it has been found that the correct behavior required a great flexibility of the field induced vortex lines, which could be obtained either with a very large size of the system along the direction of the applied field or with weaker couplings between the...
phase angles in the same direction. As we will see below the choice of an anisotropic model turns out to be crucial for obtaining convincing scaling collapses.

Another recent study of the vortex glass transition has been done on a model that extends the elastic description of a vortex lattice to include dislocations. The correlation length exponent of the transition was found to be $\nu \approx 1.3$, which within reasonable error bars also is consistent with 3D gauge glass universality.

In the present paper we present detailed analyses of the frustrated 3D XY model with strong disorder in the coupling constants. The paper is an extension of Ref. 17 in two respects: (i) The determination of the helicity modulus from simulations with twist fluctuations as well as analyses of the thermalization and the exchange steps in the Monte Carlo simulations are described in considerably more detail. (ii) Simulations and analyses have been done for several different sets of parameters.

The organization of the paper is as follows: In Sec. II we discuss the determination of the helicity modulus from twist histograms. Section III deals with the vortex glass model and the different sets of parameters used in the simulations. In Sec. IV the simulation methods are discussed with emphasis on some aspects of the exchange Monte Carlo technique, and Sec. V gives the simulation results. Section VI, finally, contains a discussion together with a short summary.

II. DETERMINATION OF THE HELICITY MODULUS

The XY model is defined by the Hamiltonian

$$H = \sum_{\langle ij \rangle} U(\theta_i - \theta_j),$$

where a common choice for the spin interaction is $U(\phi) = -J \cos(\phi)$. The helicity modulus, which is the standard probe of phase coherence in XY models, is defined through the response to an applied twist. One way to define the twist is to generalize the standard periodic boundary conditions $\theta_{(L,0,0)} = \theta_{(0,0,0)}$ to

$$\theta_{(L,0,0)} = \theta_{(0,0,0)} + \Delta_x,$$

and similarly in the other directions. Here $\Delta_x$ is the phase mismatch or the total twist in the $x$ direction. One may alternatively think about the twist as being spread out across the whole system and introduce the twist per link, $\delta_\mu = \Delta_\mu/L_\mu$. The Hamiltonian may then be written

$$H = \sum_i \sum_\mu U(\theta_{i+\mu} - \theta_i - \delta_\mu).$$

The helicity modulus is defined through the change in the free energy $F(\Delta_\mu)$, or the free energy per site $f = F/V$, as

$$\Upsilon_\mu = \frac{1}{V} \frac{\partial^2 F}{\partial \delta_\mu^2} \bigg|_{\delta_\mu = 0} = \frac{L_\mu^2}{V} \frac{\partial^2 F}{\partial \Delta_\mu^2} \bigg|_{\Delta_\mu = 0},$$

which gives the correlation function

$$\Upsilon_\mu = \frac{1}{V} \left\langle \sum_i U''(\phi_i) \right\rangle - \frac{1}{V^2} \left\langle \left( \sum_i U'(\phi_i) \right)^2 \right\rangle.$$  \hspace{1cm} (2)

With this correlation function the determination of the helicity modulus is done in simulations performed with zero twist. Note that the derivative in Eq. (3) is evaluated at the minimum of the free energy which typically is $\Delta = 0$. However, in some disordered models there is nothing that guarantees that the minimum of the free energy is at zero twist. The approach taken here is to study such systems with simulations that include the twist fluctuations as additional dynamical variables.

A. Twist fluctuations

There is a well-known duality relation between an XY model in the Villain representation and a gas of interacting charges. In two dimensions this is a Coulomb gas with logarithmic interactions and in three dimensions a gas of interacting loops. As observed by several authors the XY model that is dual to a Coulomb gas with periodic boundary conditions also includes twist fluctuations. Physically, the twist fluctuations are necessary for the process when a pair of vortices separate, cross the boundary and recombine. In the absence of twist fluctuations such a process gives a configuration where the phase rotates by $2\pi$ across the system in the direction perpendicular to the vortex separation, as illustrated in Fig. 1. The effect is that recombinations of vortices effectively is prohibited. Figure 2 illustrates the vortex separation in the presence of twist fluctuations in the $y$ direction.

FIG. 1: The separation of a vortex pair in a system with periodic boundary conditions gives a configuration with the phase rotating by $2\pi$.

FIG. 2: The separation of a vortex pair in a system with fluctuating twist boundary conditions. The twist variable is here applied between the top and the bottom rows of spins that are connected through the boundary conditions. The four panels are for $\Delta_y = 0, \pi, 0.85 \times 2\pi,$ and $2\pi$, respectively.
B. Basic relations

An alternative means to obtain $\Upsilon_\mu$ is by first determining the free energy, $F(\Delta_\mu)$. The simulations are then performed with fluctuating twists in the $\mu$ direction and periodic boundary conditions in the other two directions,

$$H = \sum_{i, \lambda \neq \mu} U(\theta_i + \lambda - \theta_i) + \sum_i U(\theta_{i+\mu} - \theta_i + \Delta_\mu/L_\mu).$$

The free energy is obtained from the histogram $P(\Delta_\mu)$ through

$$F(\Delta_\mu) = -T \ln P(\Delta_\mu),$$

and the helicity modulus may be determined from a fit of the free energy in a narrow range $r$ of $\Delta$ around zero. Dropping the index $\mu$ we write,

$$F(\Delta) = F_0 + \frac{1}{2} \Upsilon\Delta^2, \quad |\Delta| < r. \quad (3)$$

This is trivial in principle, but some complications arise when this is applied to simulation data with limited accuracy. The following sections will discuss this question in some detail.

C. Range of $\Delta$

Since $\Upsilon$ is defined as a derivative of the free energy, the range of $\Delta$ used for the fit to Eq. (3) should be chosen as small as possible. To check for the dependence of $\Upsilon$ on the range $r$ we made use of a twist histogram $P(\Delta)$ for an ordinary 3D XY model, with $L = 8$ and $T = 2.2$ close to $T_c$. It is then found that there is a strong dependence on $r$ which to a good approximation is $\Upsilon(r) - \Upsilon(0) \sim -r^2$, due to the presence of a $\Delta^4$ term in $F(\Delta)$. From $\Upsilon(r)$ for small $r$ an extrapolation to $r = 0$ gives $\Upsilon = 0.1389(3)$ in excellent agreement with the more precise value, $\Upsilon = 0.13899(8)$ obtained with Eq. (2) from a MC simulation with the Wolff cluster algorithm.

D. Disordered systems: unknown $\Delta^0$

We have so far only been concerned with models with the known minimizing twist $\Delta^0 = 0$. The presence of disorder may however mean that the minimizing twist becomes different for different disorder realizations and is not known at the outset, and this turns out to add an unexpected complication to the analysis.

In the case when $\Delta^0$ is unknown the analysis consists of two steps: (i) take some data from a certain range around the maximum of $P(\Delta)$ (the minimum of the free energy) (ii) fit the free energy from this data to a second order polynomial in $\Delta$ to determine $\Upsilon$. For this second step Eq. (3) has to be changed to

$$F(\Delta) = F_0 + \frac{1}{2} \Upsilon(\Delta - \Delta^0)^2, \quad |\Delta - \Delta^0| < r. \quad (4)$$

When used on simulation data, where statistical fluctuations are always present, this method happens to give values of the helicity modulus that are biased towards too large values.

To illustrate this fact we have again made use of twist histograms for the 3D XY model. Even though the minimizing twist is still zero we now take $\Delta^0$ to be a free variable in the analysis. For the complete run which consists of about 7000 bins there is no discernible effect of the randomness, but by constructing twist histograms from $\tau_{\text{aver}}$ (say 2–40) consecutive bins the effect becomes significant and may be systematically examined. The bin size is $2^{18} = 262144$ sweeps across the system. Figure 3 shows $\Upsilon(\tau_{\text{aver}})$ versus $1/\tau_{\text{aver}}$. The values of these run-length dependent values $\Upsilon(\tau_{\text{aver}})$ are based on close to 7000 different twist histograms constructed from $\tau_{\text{aver}}$ consecutive bins:

$$\Upsilon(\Delta; \tau_{\text{aver}}) = \frac{1}{\tau_{\text{aver}}} \sum_{\tau=1}^{\tau_{\text{aver}}} P(\Delta; i + \tau). \quad (5)$$

The message from this figure is that there is a bias in determinations of $\Upsilon$ that are based on too short runs. It is also clear that there is a $1/\tau_{\text{aver}}$-dependence and the data may be extrapolated to $\Upsilon(\tau_{\text{aver}} \rightarrow \infty) \approx 0.1381$. Since this data is obtained with a finite range, $r = 0.0625$, this value should be compared to, and agrees well with, the corresponding value obtained in Sec. II C.

![FIG. 3: When $\Upsilon$ is determined from Eq. (4) with $\Delta^0$ as a free parameter, the obtained $\Upsilon$ becomes biased towards too large values. This bias decays with $1/\tau_{\text{aver}}$ where $\tau_{\text{aver}}$ is the number of bins used for collecting data, cf. Eq. (5). The data is obtained from an isotropic lattice with $L = 8$ and $T = 2.2$.](image-url)
width of the distribution. For \( N \) samples \( x_i \), the width of the distribution is characterized by the variance and from elementary texts in statistics it is well known that 
\[
\sigma^2_{\text{naive}} = \frac{1}{N} \sum (x_i - \overline{x})^2
\]
gives a biased estimate which may be corrected by
\[
\sigma^2 = \frac{N}{N - 1} \sigma^2_{\text{naive}}.
\]
The well known reason for this correction is the use of \( \overline{x} \) instead of the true average of the distribution. In the analysis of the histogram data the location of \( \Delta^0 \) is a similar source of error and it is natural to expect the same kind of effect in the analysis of the histogram data. With \( \overline{\Upsilon}(\tau_{\text{aver}}) \) and \( \overline{\Upsilon}(\infty) \) inversely related to \( \sigma^2_{\text{naive}} \) and \( \sigma^2 \), respectively, and the number of independent samples given by \( N = \tau_{\text{aver}}/b \), we obtain
\[
\overline{\Upsilon}(\tau_{\text{aver}}) = \frac{1}{1 - b/\tau_{\text{aver}}} \overline{\Upsilon}(\infty).
\]
Here \( b \) is a constant with dimension of time. The expression above may also be written
\[
\frac{1}{\overline{\Upsilon}(\tau_{\text{aver}})} = \frac{1}{\overline{\Upsilon}(\infty)} - \frac{b}{\tau_{\text{aver}}},
\]
and for small values of \( b/\tau_{\text{aver}} \) Eq. (6) becomes
\[
\overline{\Upsilon}(\tau_{\text{aver}}) = \overline{\Upsilon}(\infty) + b/\tau_{\text{aver}},
\]
which explains the rectilinear behavior of \( \overline{\Upsilon} \) in Fig. 3. To determine the unbiased quantity \( \overline{\Upsilon}(\infty) \) we need to obtain \( \overline{\Upsilon}(\tau_{\text{aver}}) \) for a few values of \( \tau_{\text{aver}} \) and fit that data to one of the equations above.

E. Twist fluctuations in several directions

We have now discussed the use of twist fluctuations in a single direction and ordinary PBC in the two other. The simulations of Ref. 12 were however done in a somewhat different way with twist fluctuations in all three directions. For this discussion we introduce the generalization \( P^{(3)}(\Delta_x, \Delta_y, \Delta_z) \). With the phase angles of the XY model discretized to 256 different values, the computer memory needed to store such histograms rapidly becomes enormous. The collected histograms were therefore instead
\[
P_x(\Delta_x) = \sum_{\Delta_y} \sum_{\Delta_z} P^{(3)}(\Delta_x, \Delta_y, \Delta_z),
\]
and the analogous \( P_y(\Delta_y) \) and \( P_z(\Delta_z) \). For the quantity defined earlier with twist fluctuations in one dimension only we write,
\[
P^{(1)}(\Delta) \equiv P^{(3)}(\Delta, 0, 0).
\]
Figure 4 shows \( P^{(1)}(\Delta_x) \) together with \( P_x(\Delta_x) \). These two curves are very different and it becomes clear that the "helicity modulus" determined from \( P_x \) is not the same as the proper helicity modulus from the fluctuation formula or from \( P^{(1)} \). However, from the universality hypothesis one would expect scaling of all kinds of quantities based on the free energy, and the similar behavior of \( \Upsilon_x \) and \( \Upsilon^{(1)} \) in Fig. 4 suggests that that actually is the case. Here we use the standard scaling assumption,
\[
L \overline{\Upsilon} = f_T(tL^{1/\nu}),
\]
with the reduced temperature \( t = (T/T_c - 1) \).

FIG. 4: Comparison of the twist histograms from simulations of the pure 3D XY model with twist fluctuations in one and three directions. The solid line is the distribution of \( \Delta_x \) in simulations with \( \Delta_y = \Delta_z = 0 \). The dashed line is the same quantity obtained with fluctuations in all three directions. The data is obtained from a cubic isotropic lattice with \( L = 8 \) and \( T = 2.2 \).

FIG. 5: Scaling collapse of the helicity moduli obtained in the pure 3D XY model. The upper symbols (solid) are the proper helicity modulus, \( \Upsilon^{(1)} \) obtained from Eq. (2) and the low symbols are for \( \Upsilon_x \) obtained from \( P_x(\Delta_x) \). The good collapse of the latter quantity confirms the expectation that it equally well may be used for examining the critical properties.
F. Use in vortex glass simulations

As discussed above there are some complications in the determination of the helicity modulus from twist histograms. However, when our interest is only to determine the critical properties of a model, two of the above discussed complications may be disregarded. If the scaling hypothesis is phrased such that the properties of the free energy is a function only of the combination \( tL^{1/\nu} \) it is clear that the precise method to examine these properties is not important as long as it is the same for all system sizes. Among other things this means that the choice of the parameter \( b \) in Eq. \( (10) \) has the dimension of time and may be interpreted as the time between two independent measurements. In a disordered system one expects the characteristic time to be different for different disorder realizations and one would need an average of a number of functions with different time constants. However, since the correction is linear in \( b \), c.f. Eq. \( (8) \), such an average has the same functional form, but now with \( b \) as an average characteristic time.

III. THE VORTEX GLASS MODEL

The model we simulate is given by the Hamiltonian \(^{15,24} \)

\[
\mathcal{H} = - \sum_{\text{bonds } i\mu} J_{i\mu} \cos(\theta_i - \theta_{i+\mu} - A_{i\mu} + \delta(\mathbf{r}_i, \mathbf{r}_\mu)), \quad (10)
\]

where \( \theta_i \) is the phase of the superconducting wave function at site \( i \) with position \( \mathbf{r}_i \) of a periodic \( L_x \times L_y \times L_z \) lattice, and the sum is over all bonds in directions \( \mu = x, y, z \). The size in the \( x \) and \( y \) directions are the same; \( L_x = L_y = L \). An applied magnetic field in the \( z \) direction is obtained through the quenched vector potential with the choice \( A_{ix} = 2\pi f_y i \), and \( A_{iy} = A_{iz} = 0 \). The simulations are performed with fluctuating twist boundary conditions\(^{25} \) which in the duality relation corresponds to a vortex line model with periodic boundary conditions. We make use of \( L_{\mu} \) twist variables \( \delta_{\mu}^{(r;i)} \) in each direction and the total twists in the respective directions are \( \Delta_{\mu} = \sum_{j=1}^{L_{\mu}} \delta_{\mu}^{(j)} \). These variables are updated with the usual Metropolis method. We have run simulations for four different sets of parameters, summarized in Tab. \[ II \]. The disorder is put in the coupling constants which are chosen as

\[
J_{i\mu} = J_L(1 + \varepsilon_{i\mu}), \quad \mu = x, y, \]

\[
J_{iz} = J_L(1 + \varepsilon_{i\mu}), \quad \text{only sets C and D}.
\]

For set A the \( \varepsilon_{i\mu} \) are independent variables from a Gaussian distribution with \( \langle \varepsilon_{i\mu} \rangle = 0 \) and \( p = \sqrt{\langle \varepsilon_{i\mu} \rangle^2} = 0.40 \). For sets B through D \( \varepsilon_{i\mu} \) were instead from a uniform distribution between \(-1\) and \(1\). Another difference (as indicated in the Table) is that the disorder for sets A and B is only put on the couplings in the \( x \) and \( y \) directions whereas the sets C and D are also disordered along \( z \). The reason for this choice is to facilitate a direct comparison with the simulations in Ref. \[ 12 \].

| Set | \( f \) | \( J_{ij}/J_L \) | disorder directions |
|-----|------|-----------------|------------------|
| A   | 1/5  | 1/40 Gaussian    | \( x, y \)       |
| B   | 1/5  | 1/10 rectangular | \( x, y \)       |
| C   | 1/4  | 1/10 rectangular | \( x, y, z \)    |
| D   | 1/4  | 1 rectangular   | \( x, y, z \)    |

TABLE I: Four different parameter sets have been simulated. Information about the runs are given in Tab. \[ II \].

IV. SIMULATION METHODS

A. The exchange steps

The exchange MC method—also called parallel tempering—is an elegant method that makes it possible to calculate the correct statistical averages in disordered systems where the usual MC methods would only be stuck in a local minimum. The idea is to simulate many different configuration in parallel and, beside the ordinary Metropolis MC steps, let the configurations perform a kind of constrained random walk in temperature space. These occasional changes in temperature means that the configurations sometimes are at higher temperatures where the energy barriers between various local minima are low and easily may be overcome.

Our simulations were done with \( N_T \) temperatures, \( T_0 \) through \( T_{N_T-1} \), chosen according to

\[
T_m = T_{\min} \left( \frac{T_{\max}}{T_{\min}} \right)^{m/N_T}, \quad m = 0, \ldots, N_T - 1. \quad (11)
\]

The values of \( N_T \), \( T_{\min} \), and \( T_{\max} \) as well as the number of disorder realizations and the length of the runs are detailed in Tab. \[ II \].

B. Check for equilibration

In spite of its beauty the exchange MC method does not alleviate the need for thermalizing the system and
it is therefore necessary to in some way monitor the approach to equilibrium. Since our main quantities from the simulations are the histograms $P_\mu(\Delta \mu, \tau)$ we use these quantities in the analysis of the approach to equilibrium. The idea is to quantify the similarity of each histogram $P_\mu(\Delta \mu, \tau)$ to the last histogram $P_\mu(\Delta \mu, \tau_{\text{max}})$ which is assumed to be typical of a thermalized system. The disorder averaged histogram difference is then defined as

$$Q_\mu(\tau) = \left[ \sum_{\Delta \mu} \left| P_\mu(\Delta \mu, \tau) - P_\mu(\Delta \mu, \tau_{\text{max}}) \right| \right]_{\text{av}} \tag{12}$$

The notation $[\cdots]_{\text{av}}$ denotes the disorder averaging. $Q = \frac{1}{2}(Q_x + Q_y)$ is shown in Fig. 6 for $T = 0.125$ (close to $T_c$) and our four system sizes. The decrease of $Q$ at small $\tau$ down to constant levels is an effect of the thermalization and one can read off the number of bins needed for thermalization ($\tau_{\text{eq}}$ in Tab. 11) from Fig. 6. The decrease of $Q$ as $\tau \to \tau_{\text{max}}$ is due to similarities between $P_\mu(\Delta \mu, \tau)$ and $P_\mu(\Delta \mu, \tau_{\text{max}})$ that are present because of the slow dynamics of the MC simulations. The equilibration time $\tau_{\text{eq}}$ is chosen from the time needed to reach the constant value of $Q(\tau)$ whereas $\tau_{\text{max}}$, the total length of the simulations, was chosen to get enough data for the extrapolation shown in Fig. 8.

The constant level of $Q(\tau)$ is, especially for the larger systems, surprisingly large. To interpret the data correctly it should however be kept in mind that the figure shows the difference between two histograms that both deviate from the true histogram of a hypothetical run of infinite length; the difference between a single histogram and the true one would give values that are roughly a factor of two smaller. The values nevertheless signal large fluctuations and conveys a message about a complicated phase space with many different local minima.

### C. Efficiency of the exchange steps

A common way to monitor the efficiency of the exchange MC steps is to measure the exchange acceptance. This is however only a measure of the local mobility of the configurations and doesn’t answer the more relevant question about the efficiency of the algorithm to move configurations across a larger temperature range. To keep track of all the exchange steps would mean producing an enormous amount of data and is therefore usually not desirable. A simple method has therefore been devised that gives the most relevant information with very little overhead. The idea is to, for each configuration, keep track of all the exchange steps would mean producing a vector of integers, $v_m$, with information about how long it was since the temperature $T_m$ was last visited by that very configuration.

One way to use that information is to examine the vector $v_m$ for all configurations that were at the lowest temperature at the end of the run. A measure of the disorder averaged time since the last visit at temperature $T_m$ is shown in Fig. 6. The same figure also shows the results from a simple simulation of an unconstrained random walk with the same properties and acceptance probability as in the exchange MC. As seen in the figure the difference is an order of magnitude, which indicates that conclusions about the efficiency of the exchange steps

| Data set | $L$ | $N_d$ | $N_T$ | $T_{\text{min}}$ | $T_{\text{min}}$ | $\tau_{\text{eq}}$ | $\tau_{\text{max}}$ |
|----------|-----|------|------|----------------|----------------|----------------|----------------|
| A        | 10  | 600  | 12   | 0.09          | 0.24           | 1              | 16             |
|          | 15  | 600  | 24   | 0.09          | 0.24           | 4              | 16             |
|          | 20  | 600  | 36   | 0.09          | 0.24           | 11             | 32             |
|          | 25  | 200  | 36   | 0.115         | 0.24           | 17             | 48             |
| B        | 10  | 500  | 12   | 0.18          | 0.40           | 5              | 13             |
|          | 15  | 500  | 24   | 0.18          | 0.40           | 5              | 21             |
|          | 20  | 300  | 36   | 0.18          | 0.40           | 7              | 21             |
| C        | 8   | 400  | 8    | 0.16         | 0.38           | 1              | 12             |
|          | 12  | 700  | 16   | 0.16         | 0.38           | 2              | 15             |
|          | 16  | 400  | 24   | 0.16         | 0.38           | 3              | 17             |
| D        | 8   | 400  | 8    | 0.55         | 1.10           | 1              | 13             |
|          | 12  | 600  | 16   | 0.55         | 1.10           | 2              | 15             |
|          | 16  | 600  | 24   | 0.55         | 1.10           | 3              | 17             |
|          | 20  | 400  | 32   | 0.55         | 1.10           | 4              | 13             |

![FIG. 6: The quantity $Q_\mu(\tau)$ is a disorder averaged measure of the difference between the histograms $P_\mu(\Delta \mu, \tau)$ and $P_\mu(\Delta \mu, \tau_{\text{max}})$. The initial decrease down to a constant level shows the thermalization of the collection of $N_T$ configurations. The decrease as $\tau \to \tau_{\text{max}}$ is there because of correlations between $P_\mu(\Delta \mu, \tau)$ for consecutive $\tau$. This data is for set A with $L_z/L = 3/5$; the shown quantity is $Q = \frac{1}{2}(Q_x + Q_y)$.](image)
cannot be safely determined from the acceptance ratio alone. The reason for the long times needed for a configuration to travel from the highest to the lowest temperature is presumably that most high-temperature states are far away from the phase space regions typical of the lowest temperatures, which means that the configuration will usually have to undergo many thorough and time-consuming reorganizations before it can reach an energy compatible with the lowest temperatures.

The error bars are the statistical errors associated with the extrapolation to zero 1

The disorder average of the square root of temperature is presumably that most high-temperature states are far away from the phase space regions typical of the lowest temperatures, which means that the configuration to travel from the highest to the lowest temperature is usually many thorough and time-consuming reorganizations before it can reach an energy compatible with the lowest temperatures.

D. Eliminating the bias

For the following discussion we introduce a notation for the disorder averaged helicity moduli in the transverse and the parallel directions, respectively,

\[
\tau_{\perp} = \frac{1}{2} [\tau_x + \tau_y]_{\text{av}}, \quad (13a)
\]

\[
\tau_{\parallel} = [\tau_z]_{\text{av}}. \quad (13b)
\]

The procedure used to determine the disorder averaged helicity modulus consists of three steps: (i) Determine \(\tau_{\mu}(\tau_{\text{aver}})\) for each disorder configuration and several values of \(\tau_{\text{aver}}\) by fitting histogram \(P_{\mu}(\Delta\mu; \tau_{\text{aver}})\) based on \(\tau_{\text{aver}}\) consecutive bins, \(P_{\mu}(\Delta\mu, \tau)\) to Eq. (3). (ii) Calculate the disorder averaged quantities \(\tau_{\perp}(\tau_{\text{aver}})\) and \(\tau_{\parallel}(\tau_{\text{aver}})\), cf. Eqs. (13a, 13b). (iii) Fit this data to Eq. (7) to obtain the unbiased estimates \(\tau_{\perp} \equiv \tau_{\perp}(\infty)\) and \(\tau_{\parallel} \equiv \tau_{\parallel}(\infty)\). The last step is illustrated in Fig. 8.

The error bars are the statistical errors associated with the disorder average for each size. It seems that the errors associated with the extrapolation to zero \(1/\tau_{\text{aver}}\) are smaller than the errors due to the limited number of disorder realizations.

V. RESULTS

In this section we report the results from the analysis described above, with a number of different sets of parameters, cf. Tab. I. The purpose is to check that the proposed behavior is a generic feature and is not limited to the parameters of Ref. 15 but as we have been doing simulations with several different sets of parameters it has not been possible to achieve very high precision in the estimates of the critical exponents. The emphasis is therefore rather on checking for scaling that is consistent with 3D gauge glass universality. Generally speaking that picture is confirmed, but the new simulations also give information about failure of finite size scaling for certain sizes and parameters.

A. High anisotropy, \(J_{\parallel}/J_{\perp} = 1/40\)

The results in Ref. 15 were obtained with a rather high anisotropy, \(J_{\parallel}/J_{\perp} = 1/40\) and the aspect ratio \(L_{z}/L = 3/5\). We have now also performed simulations with a smaller aspect ratio, \(L_{z}/L = 2/5\), which is a good consistency test since not only the critical exponents but also the critical temperature should be independent of the aspect ratio. We have also performed additional simulations with several other aspect ratios to determine the anisotropy exponent.

1. Varying the aspect ratio

Figure 8 shows the helicity moduli for the same parameters as in Ref. 15 but with the aspect ratio \(L_{z}/L = 2/5\). We find a nice crossing for \(L\tau_{\perp}\) at the expected value.
and 3 sizes. However, somewhat unexpectedly, the scaling only should work for rather large system sizes. The deviation from the scaling behavior is another example of the well-known fact that finite size scaling often fails for very small system sizes.

$T_c = 0.123^{15}$ The results for the perpendicular quantity $L\gamma_{\perp}$ also agree with this behavior for the two larger sizes but the data for the smallest system, $10 \times 10 \times 4$, is significantly off. This is in line with the general expectation that the scaling only should work for rather larger system sizes. However, somewhat unexpectedly, the scaling in $L\gamma_{\perp}$ prevails even though it fails in the direction parallel to the applied field.

Scaling collapses for the two aspect ratios $L_z/L = 2/5$ and $3/5$ are shown in Fig. 10. When discarding $\gamma_{\parallel}$ for $10 \times 10 \times 4$ the collapses with $T_c = 0.123$ and $\nu = 1.5$ (from Ref. 15) are excellent for both quantities. Note the similar shapes of the scaling functions for the two aspect ratios. For panel (b) this requires the use of $L_z$ instead of $L$ on the $x$ axis. Also note that the dependency on the aspect ratio is the opposite for $L_z\gamma_{\parallel}$ compared to $L\gamma_{\perp}$.

2. The anisotropy exponent

The above data is consistent with isotropic scaling, the anisotropy exponent $\xi = 1$, but to estimate the error bars we need a determination of the exponent. The idea behind finite size scaling is that certain quantities only should depend on the fraction $\xi/L$, and to generalize this concept to anisotropic scaling one has to allow for the possibility of two different correlation lengths, $\xi$ and $\xi_z$, that grow in different ways as $T_c$ is approached, $\xi_z \sim \xi^\xi$. To do finite size scaling one needs sizes such that $\xi/L \propto \xi_z/L_z$ and with the above relation between $\xi$ and $\xi_z$ we need to determine the behavior of systems with $L_z \propto L^\xi$. For general values of $\xi$ this gives non-integral $L_z$ and the common practice is to obtain the appropriate data through interpolation of data for neighboring $L_z$-values. To make that possible we have thus simulated with several different $L_z$: for $L = 10$ we have used $L_z = 5$, 6, and 7, and for $L = 15$ simulations have been done with $L_z = 8$, 9, and 10.

To determine limits on $\xi$ the most straightforward test would be to repeat the scaling analysis with different values of $\xi$ and check how the quality of the scaling collapse depends on $\xi$. Because of the statistical errors in the raw data that is however not a very useful technique. A more sensitive test is obtained by combining results from analyses of both $\gamma_{\perp}$ and $\gamma_{\parallel}$. To do that we focus on how the crossing temperatures of $L^\xi\gamma_{\perp}$ and $L^{2-\xi}\gamma_{\parallel}$ depend on $\xi$. To make the test clean and simple we only make...
use of two sizes at the time. Fig. 11 shows the dependency of the crossing temperatures on $\zeta$ for sizes $L = 15$ and 25. The two different crossing temperatures coincide at $T \approx 0.12$ and $\zeta \approx 1$. Note that the two quantities have the opposite dependency on $\zeta$. This is the key to this more precise determination of $\zeta$, and together with a rough error estimate Fig. 11 gives $\zeta = 1 \pm 0.1$.

**FIG. 11:** The figure shows how the crossing temperature of $L^2 \gamma_\perp$ and $L^2 \gamma_\parallel$ with $L = 15$ and $L = 25$ depend on the assumed value of $\zeta$. One set of data is for $(L, L_z) = (25, 15)$ and the other for $(L, L_z) = (15, 9 \cdot (15/25)^{\zeta - 1})$. The second set is obtained by interpolating the results from simulations with $L_z = 8, 9$, and 10. Since a crossing of the data in both directions should occur at $T_c$, the correct value of $\zeta$ is obtained at the crossing of these two sets of data points. This gives $\zeta = 1 \pm 0.1$, strongly suggestive of isotropic scaling.

**B. Less anisotropic, $J_\parallel/J_\perp = 1/10$**

The simulations discussed in the previous section are for a rather strong anisotropy, $J_\parallel/J_\perp = 1/40$. It is generally expected that the critical behavior should be independent of details as the anisotropy, and we now check this expectation with simulations for $J_\parallel/J_\perp = 1/10$; data sets B and C in Tab. I. Figure 12 shows scaling collapses of the helicity moduli for data set B. Beside the weaker anisotropy the simulations also differ in that the disorder $\xi_{ij}$ is stronger and is now chosen from a uniform rectangular distribution between $-1$ and 1, corresponding to $p = \sqrt{\langle \xi_{ij}^2 \rangle} = 1/\sqrt{3} \approx 0.577$. In a fit with $T_c$ and $\nu$ as adjustable parameters a collapse of $LT_\perp$ gives $T_c = 0.239$ and $\nu = 1.56$ whereas a collapse of $LT_\parallel$ gives $T_c = 0.241$ and $\nu = 1.97$. The different values of $\nu$ is an indication of the rather low precision in these determinations. In Fig. 12 we show that it is possible to collapse both sets of data with the same parameters, $T_c = 0.24$ and $\nu = 1.6$. The collapse of $LT_\perp$ is very nice whereas the collapse of $LT_\parallel$, especially in a region around $T_c$, is somewhat worse. However, considering the statistical errors, we believe this to be just a statistical fluctuation. The fact that several points around $T_c$ all deviate in the same way is an artifact of the exchange Monte Carlo method since the exchange steps have the effect to give correlations between results at neighboring temperatures.

We have also simulated the same model but with filling factor $f = 1/4$; data set C in Tab. I. The collapse which is found in Fig. 13 is excellent and we obtain $\nu = 1.35$ and $\nu = 1.48$ from the scaling collapses of $LT_\perp$ and $LT_\parallel$, respectively.

**C. Isotropic system**

The values for $\nu$ given above, obtained from simulations with different values of anisotropy and filling factor, are within reasonable error bars consistent with 3D gauge glass universality, $\nu \approx 1.39$. This seems to rule out the possibility that the nice scaling in Ref. [13] was only a coincidence. Still, the results presented in the present section show that scaling fails when the analysis is applied to an isotropic model. This finding is of some importance since isotropic couplings have been used in several investigations of vortex glass models. These papers reach differing conclusions and yet other investigations fail to find acceptable finite size scaling (private discussion). We believe that an understanding of the problem to scale our data from isotropic couplings may shed light on problems in these other investigations.
clearly different from the anisotropic systems with
had case is exceptionally large. For all the other cases we
quantity is considerably larger in the temperature region

FIG. 13: Data collapse of $L\gamma_\perp$ and $L\gamma_\parallel$ for $f = 1/4$ and $J_\parallel/J_\perp = 0.1$. The parameters used in the data collapses are $\nu = 1.4$ and $T_c = 0.225$.

1. Failure to scale the data

For simulations of an isotropic system we use the same parameters as Kawamura in Ref. 12 but the analysis differs from theirs in that we focus on the behavior of the helicity moduli instead of the rms-current.

Figure 14(a) shows $L\gamma_\perp$ for the four system sizes $L = 8, 12, 16$, and $20$. The data for $L\gamma_\perp$ weakly suggests the possibility of scaling and panel (b) shows the attempted scaling collapse with $T_c \approx 0.63$ and $\nu \approx 1.50$. Even though the value of $\nu$ is in good agreement with our earlier findings, the poor quality of the collapse makes it impossible to draw any more definite conclusions. Turning to $L\gamma_\parallel/J_\parallel$ shown in Fig. 15 we find that it is impossible to collapse the data since the crossing points for two successive system sizes shift systematically to lower temperatures for increasing $L$. Beside the failure to scale the data it should be noted that $L\gamma_\parallel/J_\parallel$ for the isotropic case is exceptionally large. For all the other cases we had $L\gamma_\parallel/J_\parallel \approx 1.0$ at $T_c$, but in the isotropic model, this quantity is considerably larger in the temperature region of interest.

2. The reason for the failure to scale

The behavior of $\gamma_\parallel$ in the isotropic system is thus clearly different from the anisotropic systems with $J_\parallel/J_\perp = 1/40$ or $1/10$. We will now argue that this is because the disorder in the coupling constants is not effective in fully disordering the system for the accessible system sizes.

As a probe of the loss of order we use $\Delta^0_\mu$ which is the position of the minimum of the free energy, $F_\mu(\Delta_\mu)$. This quantity has been used before as a measure of the effective strength of the disorder. The disorder fixed point was there characterized by $\langle |\Delta^0_\mu| \rangle = \pi/2$ which corresponds to a uniform distribution between $-\pi$ and $\pi$. Figure 14 shows histograms of $\Delta^0_\mu$ and $\Delta^0_\pi$ from our data and it is clear that the histograms are very different from a uniform distribution. Especially the histograms of $\Delta^0_\mu$ are very narrow with $|\Delta^0_\mu/\pi| < 0.1$ for almost 99% of the disorder realizations. For $\Delta^0_\pi$ the distributions are considerably wider but are still clearly peaked around zero. In both cases there is some finite size dependence, with a wider distribution for larger system sizes. For comparison we also show the corresponding histograms for the anisotropic model with $J_\parallel/J_\perp = 1/10$ in Fig. 17.

For the anisotropic case the histograms of $\Delta^0_\mu$ are close to a uniform distribution; only the data for $L = 8$ have somewhat more weight around zero. This shows that the data that exhibits good scaling are from strongly disordered systems. In contrast, the isotropic model appears to be far from the disorder fixed point and we believe
that this is at the root of the failure to find a convincing data collapse.

To discuss the physical meaning of $\Delta_0^0$ we return to Fig. 10 which illustrates the relation between the size of a vortex pair and the value of the twist variable in the direction perpendicular to the separation. As the pair separates in the $x$ direction the twist $\Delta_y$ gradually increases. At zero temperature the twist is to a good approximation proportional to the distance, $d$, between the vortices, $\Delta_y = 2\pi d/L$. For the more general situation with several vortices the vortex separation generalizes to the total dipole moment of the system of vortices, $p_x = \sum x_i q_i$, where $i$ enumerates the vortices, $x_i$ is the $x$-coordinate of vortex $i$, and $q_i$ is the vorticity (charge). At non-zero $T$ the distribution of $\Delta_y$ at constant $p_x$ will be wider; the relevant expressions are given in Ref. 27. For the three-dimensional case the dipole moment generalizes to the projection of the vortex loops on a certain plane, $C_{xy}$. The corresponding relation is then $\Delta_y = 2\pi C_{xy}/L^2$.

In a pure system the twist histogram will always be symmetric around zero, $\Delta_y^0 = 0$, but the effect of the disorder is to favor certain vortex loops between the layers and suppress others. The net effect may be a non-zero $C_{xy}$ and accordingly a shift of $\Delta_y^0$ away from zero. Our interpretation of the results in Fig. 11 is therefore that the disorder is not strong enough to introduce loops between the layers. Note that field-induced vortex lines that have a non-vanishing projection on the $x$-$y$ plane also contribute to $C_{xy}$. The absence of large disorder-induced vortex loops between the layers (or the equivalent deflection of the field-induced vortex lines) means that $\Delta_y^0$ is always close to zero.

The analyses above suggests that a strong coupling in the field direction has the effect to reduce the amount of disorder-induced vortex loops between the planes. The effect is to get $\Delta_0^0$ close to zero which means that the effect is to get $\Delta_0^0$ close to zero which means that the effect is to get $\Delta_0^0$ close to zero which means that the effect of disorder is not strong enough to introduce loops between the layers. Note that field-induced vortex lines that have a non-vanishing projection on the $x$-$y$ plane also contribute to $C_{xy}$. The absence of large disorder-induced vortex loops between the layers (or the equivalent deflection of the field-induced vortex lines) means that $\Delta_y^0$ is always close to zero.

The figures show histograms of $\Delta_x^0$ and $\Delta_y^0$ for anisotropy $J_1/J_\perp = 1/10$ and $f = 1/4$ in panel (a) and panel (b), respectively. The results for the larger sizes, $L = 12$ and 16 (circles and squares), are consistent with a uniform distribution whereas the distributions for $L = 8$ (crosses) have somewhat more weight around zero. However, it seems that such small deviations from perfect disorder (a flat histogram) have no discernible effects on the scaling shown in Fig. 13. The histograms are calculated on the basis of data for all the simulated temperatures.
fective disorder is small in the system and we believe that this is responsible for the failure of the helicity moduli to scale. Considering the broadening of the histograms with increasing \( L \) in Fig. 14, we expect this to be a finite size effect, but are presently unable to estimate the size where scaling could be expected to set in.

VI. DISCUSSION

The use of an anisotropic model in the study of critical phenomena with finite size scaling deserves some comments. To get data with high precision for finite size scaling from Monte Carlo simulations, the correlation volume should ideally have the same shape as the simulation cell. In an isotropic model a cubic simulation cell is therefore the best choice and in the general case one wants a common value of the fraction \( \xi_\mu / L_\mu \) in all directions. For the model in the present paper with a symmetry breaking field there is nothing that guarantees that isotropic couplings are best. It is however possible to extract some information about the correlations from the helicity moduli. With \( \Upsilon_\mu \) as the measure of the phase coherence, a larger \( \Upsilon_\mu \) implies stronger correlations and thereby a larger correlation length in the \( \mu \) direction. By comparing data for the isotropic model in Figs. 14 and 15, the fact that \( \Upsilon_\parallel \) is considerably larger than \( \Upsilon_\perp \) leads us to conclude that the correlations are considerably stronger in the field direction compared to the perpendicular direction. One way to reach the goal of a simulation cell with the same shape as the correlation volume would then be to increase the aspect ratio \( L_z / L \), but a different and more efficient way is to instead decrease \( J_\parallel \), the coupling strength in the field direction.

To get a better understanding of the effect of anisotropic couplings on the helicity moduli we have made some additional simulations on the ordinary 3D XY model (zero field and no disorder) with \( J_z / J = 1/4 \). Since one expects \( \xi_\mu \propto \sqrt{J_\mu} \) the aspect ratio was then chosen to be \( L_z / L = 1/2 \) which gives a simulation cell with the same shape as the correlation volume. With this value of the aspect ratio the simulations give \( \Upsilon / J = \Upsilon_z / J_z \) at \( T_c \) to a good approximation. As shown in Fig. 12 the same relation holds to a good approximation at and close to \( T_c \) in the simulations of the vortex glass with \( J_\parallel / J_\perp = 1/10 \) and \( L_z / L = 1 \). This suggests that the correlations in the different directions are about equally strong when the anisotropy is set to \( J_\parallel / J_\perp = 1/10 \) and that this value therefore is close to optimal for the anisotropy in the vortex glass simulations with \( f = 1/5 \).

Even though it thus seems that our model is best examined with a rather large anisotropy we now turn to the results obtained with isotropic couplings. These simulations were performed with the parameters of Ref. 12 to make it possible to directly compare the results. It is however clear that the results are significantly different. Whereas our \( LT_\perp \) almost collapse at \( T = 0.63 \) with \( \nu \approx 1.5 \) their corresponding quantity, \( I_T \), collapses for the three largest sizes at \( T_g = 0.81 \) with \( \nu = 1.0 \). Especially the different values of the critical temperature points to a systematic difference.

We believe that the reason for this difference is their calculation of \( I_{\text{rms}} \) as the derivative of \( F(\Delta) \) evaluated at \( \Delta = 0 \), rather than at random values of \( \Delta \). From our direct determinations of \( F(\Delta_\mu) \) we have found that the typical structure of this quantity (obtained with parameter set D) is a single minimum of the free energy with a shape that in most cases to a very good approximation is parabolic, \( F(\Delta_\mu) = \text{const} + \Upsilon_\mu (\Delta_\mu - \Delta_0^\mu)^2 / 2 \), where both \( \Upsilon_\mu \) and \( \Delta_0^\mu \) depend on the disorder realization. When the derivative is evaluated at \( \Delta_\mu = 0 \) one gets \( I_\mu = -\Upsilon_\mu \Delta_0^\mu \). This means that the obtained rms-current is not only a measure of the amount of structure in \( F(\Delta_\mu) \) but also depends on the location of this structure. Against that background the size-dependence of the distribution of \( \Delta_0^\mu \) shown in Fig. 16 is problematic and we believe this to be the reason for the different critical behavior in Ref. 12 compared to the results in this paper. This undesired finite size will affect the determination of the rms-current and we therefore believe that the scaling behavior of \( I_T \) in Ref. 12 is only accidental.

The failure of the helicity modulus to scale in the isotropic model is a related but different question. As discussed in Sec. V C 2 a message from Fig. 10 is that the isotropic model is not sufficiently disordered for the simulated system sizes and it seems possible that this remaining order destroys the transition. The broadening of the histograms in Fig. 10 with increasing system size would lead to a flat distribution in the limit of large \( L \) and one would then expect scaling with the 3D gauge glass exponents. However, considering the slow widening of the histograms as \( L \) increases, scaling would presumably only be seen for very large systems.

It should finally be noted that the problem with determining the rms-current from the derivative at \( \Delta = 0 \) is not present in the 3D gauge glass model. The reason is that the randomness there is put into the vector potential and that a random \( \Delta \) then may be absorbed in the similarly random \( A_{ij} \). There is then no need to make use of a random \( \Delta \) and the standard way to evaluate \( I_{\text{rms}} \) at \( \Delta = 0 \) is acceptable.

To summarize: the main conclusion of the present investigation is that the vortex glass model is in the same universality class as the 3D gauge glass model. This is a confirmation of the behavior found in Ref. 13. Still, it is found that simulations with isotropic couplings do not give any convincing scaling collapse and we argue that the reason is that the effective randomness for the accessible system sizes is too small to give the correct behavior of the vortex glass transition.
VII. ACKNOWLEDGEMENTS

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