Neutrino CP violating parameters from nontrivial quark-lepton correlation: a S3 x GUT model

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We investigate the prediction on the lepton phases in theories with a non trivial correlation between quark (CKM) and lepton (PMNS) mixing matrices. We show that the actual evidence, under the only assumption that the correlation matrix $V^M$ product of CKM and PMNS has a zero in the entry (1,3), gives us a prediction for the three CP-violating invariants $J$, $S_1$, and $S_2$. A better determination of the lepton mixing angles will give stronger prediction for the CP-violating invariants in the lepton sector. These will be tested in the next generation experiments.

To clarify how our prediction works, we show how a model based on a Grand Unified Theory and the permutation flavor symmetry $S_3$ predicts $V_{13}^M = 0$.

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I. INTRODUCTION

After the recent experimental evidences about neutrino physics [1-14] we know very well almost all the parameters in the quark [12] and lepton [16-35] sectors. We measured all the quark and charged lepton masses, and the value of the difference between the square of the neutrino masses $\delta m^2_{12} = m_1^2 - m_2^2$ and $\delta m^2_{23} = |m_2^2 - m_3^2|$. We also know the value of the quark mixing angles and phases, and the two mixing angles $\theta_{12}$ and $\theta_{23}$ in the lepton sector. The challenge for the next future [36-53] will be to determine the sign of $\delta m^2_{23}$ (i.e. the hierarchy in the neutrino sector), the absolute scale of the neutrino masses, and the value of the 3rd lepton mixing angle $\theta_{13}$ (in particular if it is zero or not). Finally, if $\theta_{13}$ is not too small, there is a hope to measure the CP violating phases.

Despite the fact that we have all these experimental evidences, still a theory of flavor is missing. The main point is related to the fact that there is a hierarchy in the mixing angles and masses. In particular the quark mixing angles are in general much smaller than the corresponding lepton mixing angles. The small quark mixing angles can be explained with continuous flavor symmetries (see for instance [54]), while the structure of the lepton mixing matrix seems to be better accommodated in models with discrete symmetries (see for example [55-58]). Recently, the disparity of nature indicates between quark and lepton mixing angles has been viewed in terms of a ‘quark-lepton complementarity’ (QLC) [59, 60] which can be expressed in the relations

$$\theta_{12}^{PMNS} + \theta_{12}^{CKM} \simeq 45^\circ; \quad \theta_{23}^{PMNS} + \theta_{23}^{CKM} \simeq 45^\circ. \quad (1)$$

These relations are related to the parametrization used for the CKM and PMNS mixing matrix. From a more general point of view, we can define a correlation matrix $V^M$ as the product of the PMNS [61, 62] and CKM [63, 64] mixing matrices,

$$V^M = U^{CKM} U^{PMNS}. \quad (2)$$

A lot of efforts have been done to obtain the most favorite pattern for the matrix $V^M$ [65-71]. The naive QLC relations in eq. (1) seems to implies $V^M$ to be BiMaximal, i.e. in the standard parametrization it contains two maximal mixing angle, and a third angle to be zero. A BiMaximal $V^M$ implies a nice, in the sense that it is testable at the near future neutrino experiments, prediction on the $\theta_{13}^{PMNS}$ mixing angle [54] and the CP violating parameters for the lepton sector [72]. At first order approximation however $V^M$ BiMaximal seems not to be compatible with the experiments [67]. From our previous work [65] we learn that $V^M$ BiMaximal, although it is not ruled out by the experiments, is excluded at 90% CL in non SUSY models, or in SUSY models with $\tan \beta < 40$ where the RGE correction are negligible [73-76].

Despite the fact that the correlation matrix $V^M$ cannot be BiMaximal, still there are very nice phenomenological consequences from a non trivial $V^M$. All of these consequences seems to be related to the fact that experimental evidences tell us [65] that $V_{13}^M = 0$ is in good agreement with the experimental data, and is even the preferred value. From a theoretical point of view we like very much the fact that experimental data tell us that $V_{13}^M = 0$. First of all the theoretical ingredient of the
quark-lepton complementarity that gives phenomenological predictions is $V^{M}_{13} = 0$. In fact $V^{M}_{13} = 0$, without any other assumptions on the full $V_{M}$ matrix with the exception of the compatibility with the actual experimental evidences, implies a testable prediction for the undetermined lepton mixing angle

$$\theta_{13}^{PMNS} = (9^{+1}_{-2})^\circ \quad \text{(see [65])}$$

Second because $V^{M}_{13} = 0$ can be achieved in a natural way in theoretical models by imposing some symmetry.

In this work first we analyze the consequences of $V^{M}_{13} = 0$ on the CP violating invariants for the lepton sector. Then we show how a toy model, based on $S_{3}$ flavor permutation symmetry [77] and GUT gives $V^{M}_{13} = 0$.

We use a Monte Carlo simulation with two-sided Gaussian distributions around the mean values of the observables to extract the $J$, $S_{1}$, and $S_{2}$ invariants. The input information on $\theta_{13}^{PMNS}$ is taken from the analysis of ref. [65] which uses quark lepton correlation ($V^{M}_{13} = 0$), and neutrino and quark data. We obtain that the main ingredient to obtain a prediction on the CP violating invariants for the lepton sector is the constraint $V^{M}_{13} = 0$.

From the theoretical point of view we show that, if we do not take care about a required fine-tuning in obtaining the masses, it is possible to construct a toy model [77], based on $S_{3}$ flavor permutation symmetry and GUT, that gives us $V^{M}_{13} = 0$. In Grand Unified Theories (GUT), as long as quarks and leptons are inserted in the same representation of the gauge group, we need to include in the definition of $V^{M}$ non trivial phases between the CKM and PMNS mixing matrices. In our model $V^{M}$ is defined by

$$V^{M} = U^{CKM} \Omega U^{PMNS}. \quad (3)$$

where $\Omega$ is a diagonal matrix $\Omega = \text{diag}(e^{i\omega_{i}})$ and the three phases $\omega_{i}$ are free parameters (in the sense that they are not determined by present experimental evidences). The matrix $V_{M}$ is related to the Dirac and Majorana neutrino mass matrix. For this reason its form is given by the symmetries of the model.

The paper is organized as follows: in section II we introduce our notation and the parameterization for CKM and PMNS mixing matrices. With the aid of a Monte Carlo simulation, we study the numerical correlations of the lepton CP violating phases $J$, $S_{1}$, and $S_{2}$ with respect to the mixing angle $\theta_{12}^{PMNS}$. In section III we clarify the relation between the correlation matrix $V^{M}$ and the neutrino mass matrices in GUT model, with a see-saw of type I. After that we show how a $S_{3}$ flavor symmetry can give us $V^{M}_{13} = 0$. Finally in section IV we present a summary and our conclusions.

II. CP VIOLATING INVARIANTS IN THE LEPTON SECTOR

As usually, we parameterize the lepton mixing matrix as

$$U^{PMNS} = U^{23}\Phi U^{13}\Phi^{1}U^{12}\Phi^{m} \quad (4)$$

where $\Phi$ is a diagonal matrix with elements $\{1,1,e^{i\phi}\}$, and $\phi$ is the Dirac CP violating phase. $\Phi^{m}$ contain the Majorana phases and is a diagonal matrix with elements given by $\{e^{i\phi_{1}},e^{i\phi_{2}},1\}$, and $U^{13}$ are rotation in the $(i,j)$ plan. There are two kind of invariants parameterizing CP violating effect. The Jarlskog invariant [28] $J$ that parametrizes the effects related to the Dirac phase, and the two invariants $S_{1}$ and $S_{2}$ that parameterize the effects related to the Majorana phases. The $J$ invariant describes all CP breaking observables in neutrino oscillations [29]. It is the equivalent of the Jarlskog invariant in the quark sector. It is given by

$$J = \text{Im}\{U_{\nu_{e},\nu_{\mu}}U_{\nu_{e},\nu_{\tau}}U_{\nu_{\mu},\nu_{\tau}}^{*}\} \quad (5)$$

In the parametrization of eq. (4) one has

$$J = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \phi. \quad (6)$$

Then we have the two invariants $S_{1}$ and $S_{2}$ that are related to the Majorana phases. They are

$$S_{1} = \text{Im}\{U_{\nu_{e},\nu_{\mu}}U_{\nu_{e},\nu_{\tau}}^{*}\} \quad (7)$$

$$S_{2} = \text{Im}\{U_{\nu_{\mu},\nu_{\tau}}U_{\nu_{\mu},\nu_{\tau}}^{*}\} \quad (8)$$

In the parametrization of eq. (4) we have

$$S_{1} = \frac{1}{2} \cos \theta_{12} \sin 2\theta_{23} \sin(\phi + \phi_{1})$$

$$S_{2} = \frac{1}{2} \sin \theta_{12} \sin 2\theta_{23} \sin(\phi + \phi_{2})$$

The two Majorana phases appear in $S_{1}$ and $S_{2}$ but not in $J$.

A. Prediction for CP violating invariants

In this section we investigate the consequences of a $V^{M}$ correlation matrix with a zero $(1,3)$ entry on the undetermined parameters $J$, $S_{1}$, and $S_{2}$. We remember that $J$ is the Dirac invariant CP-violating phase, and is the only one that can be observed in neutrino oscillations experiments. As shown in a previous paper [65], the data favors a vanishing $(1,3)$ entry in the correlation matrix $V^{M}$. So in the whole analysis we fix $\sin^{2} \theta_{13}^{M} = 0$. Moreover $\tan^{2} \theta_{12}^{M}$ and $\tan^{2} \theta_{23}^{M}$ are allowed to vary respectively within the intervals $[0.3,1.0]$ and $[0.5,1.4]$. We introduce the unitary Wolfenstein parameterization in terms of the variables $\lambda$, $A$, $\rho$, $\eta$ [78]

$$U^{CKM} = U^{23}\Phi U^{13}\Phi^{1}U^{12}, \quad (9)$$
where one has the relations

\[
\sin \theta_{12}^{CKM} = \lambda \\
\sin \theta_{23}^{CKM} = A \lambda^2 \\
\sin \theta_{13}^{CKM} e^{-i \delta_{CKM}} = A \lambda^3 (\rho + i \eta)
\]

to all orders in \( \lambda \). We use the updated values for the CKM mixing matrix, given at 95\%CL by [13]

\[
\lambda = 0.2265_{-0.0041}^{+0.0040}, \quad A = 0.801_{-0.041}^{+0.066}, \\
\eta = 0.189_{-0.114}^{+0.182}, \quad \rho = 0.358_{-0.085}^{+0.086}.
\]

(10)

with

\[
\rho + i \eta = \frac{\sqrt{1 - A^2 \lambda^2 (\eta + i \rho)}}{\sqrt{1 - A^2 \lambda^4 (\eta + i \rho)}}.
\]

(11)

For the lepton mixing angle we impose [16, 17]

\[
\sin^2 \theta_{PMNS}^{12} = 0.44 \times (1_{-0.22}^{+0.41}) \\
\sin^2 \theta_{PMNS}^{13} = 0.314 \times (1_{-0.18}^{+0.15}),
\]

(12)

and [65]

\[
\theta_{PMNS}^{13} = (9_{-2}^{+1})^\circ.
\]

(13)

We allow the \( U_{CKM} \) parameters to vary, with a two-sided Gaussian distribution, within the experimental ranges given in eq. (10). For the \( \Omega \) phases in eq. (3) we take flat distributions in the interval [0, 2\( \pi \)]. We make Monte Carlo simulations for different values of \( \theta_{12}^{V_M} \) and \( \theta_{23}^{V_M} \) mixing angles, allowing \( \tan^2 \theta_{12}^{V_M} \) and \( \tan^2 \theta_{23}^{V_M} \) to vary respectively within their allowed intervals, in consistency with the lepton and quark mixing angles. From eq. (6), by using the fact that \( \theta_{13} \) is small and that \( \theta_{23} \) is maximal, we get

\[
J = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin \phi.
\]

This expression tells us that the \( J \) parameter is within the range \(|J| < 0.042\). However there is a non trivial correlation between \( J \) and \( \theta_{PMNS}^{12} \). Because the \( CKM \) is given by the experimental data, and \( V_{13}^{M} \) is fixed to be zero, the phase \( \phi \) and the \( \theta_{PMNS}^{13} \) angle are almost fixed as a function of \( \theta_{PMNS}^{12} \).

In figs. 1-3 we report the result of our simulation for \( J \). We plot the correlation between the \( J \) invariant and \( \sin^2 \theta_{PMNS}^{12} \) for \( V_M \) BiMaximal (fig. 1), TriBimaximal (fig. 2) and \( V_M \) with \( \tan^2 \theta_{12}^{V_M} = 0.4 \) (fig. 3). First of all, from fig. 1 we see that the solar mixing angle \( \theta_{PMNS}^{12} \) is constrained to have \( \sin^2 \theta_{PMNS}^{12} > 0.36 \) for \( V_M \) Bimaximal. From figs. 1-3 we see the correlation between the structure of \( V_M \) and the CP violating invariant \( J \). In particular, for \( V_M \) BiMaximal \( J \) close to zero. For \( V_M \) TriBimaximal \( J \) is around 0.042. Finally for \( V_M \) such that \( \tan^2 \theta_{12}^{V_M} = 0.4 \) we get that \( J \) can be any value between \(-0.04 \) and 0.04. We also see that a better determination of the \( \sin^2 \theta_{PMNS}^{12} \) could give a stronger prediction for the \( J \) invariant in the case of \( V_M \) TriBimaximal.

In figs. 4-6 we report the result of our simulation for \( S_1 \) (\( S_2 \) plots have similar shapes). The expressions in eqs. (10) give us the range for these invariants:

\[
|S_1| < 0.14 \quad |S_2| < 0.11
\]

(14)

We plot the correlations between the \( S_1 \) invariant with respect \( \sin^2 \theta_{PMNS}^{12} \) for \( V_M \) BiMaximal (fig. 4), TriBimaximal (fig. 5), and such that \( \tan^2 \theta_{12}^{V_M} = 0.4 \) (fig. 6). From the figures we obtain that for \( V_M \) BiMaximal the
correlation between the Majorana CP violating parameter $S_1$ and $\sin^2 \theta_{12}^{PMNS}$. 

**FIG. 5:** Same as fig 2 (V$_M$ TriBimaximal) for the correlation between the Majorana CP violating parameter $S_1$ and $\sin^2 \theta_{12}^{PMNS}$. 

**FIG. 6:** Same as fig 3 (V$_M$ such that $\tan^2 \theta_{12}^{V_M} = 0.4$) for the correlation between the Majorana CP violating parameter $S_1$ and $\sin^2 \theta_{12}^{PMNS}$. 

Majorana CP invariant $S_1$ is close to zero, for $V_M$ TriBimaximal $S_1$ is around 0.13. Finally for $V_M$ such that $\tan^2 \theta_{12}^{V_M} = 0.4$ we obtain that $S_1$ can be any value between $-0.14$ and $0.14$. Similar results hold for the other Majorana CP violating invariant $S_2$. We see that also in this case a better determination of the $\theta_{12}^{PMNS}$ mixing angle will give a stronger constraint for the $S_1$ (and $S_2$) invariant for $V_M$ TriBimaximal. As for $J$, these correlations of $S_1$ (and $S_2$) with respect to $\theta_{12}^{PMNS}$ are predic-

tions of any theoretical GUT model that gives a relation of the type $V^M = U^{CKM} \Omega U^{PMNS}$ with $V_{13}^{PMNS} = 0$. In the next section we will show how to construct an explicit model that predict $V_{13}^{PMNS} = 0$. 

**III. A TOY MODEL**

In this section we will show how to construct a toy model that gives us the relation $V^M = U^{CKM} \Omega U^{PMNS}$ with $V_{13}^{PMNS} = 0$. We will not take care of explicit values for the masses. To obtain them we need an unwanted fine-tuning. However, as long as we refer to the mixing angles only, this model can be seen as a toy model explaining the relationship between the CKM and the PMNS mixing matrix and the appearance of a zero $(1,3)$ entry in the quark-lepton correlation matrix $V^M$. 

### A. $V^M$ in theories with see-saw of type I

Let’s fix the notations in the lepton sector. Let $Y_l$ be the Yukawa matrices for charged leptons. It can be diagonalized by

$$Y_l = U_l Y_l^\Delta V_l^\dagger$$

(15)

Let be $M_R$ the Majorana mass matrix for the right neutrino and $M_D$ the Dirac mass matrix. Under the assumption that the low energy neutrino masses are given by the see-saw of Type I we have that the light neutrino mass matrix is given by

$$M_\nu = M_D \frac{1}{M_R} M_D^\dagger.$$  

(16)

Let us introduce $U_0$ form the diagonalization of the Dirac mass matrix

$$M_D = U_0 M_\nu^\Delta V_0^\dagger$$

(17)

then we define $V^M$ by the diagonalization of the light neutrino mass

$$M_\nu = M_D \frac{1}{M_R} M_D^\dagger = U_\nu^\dagger M_\nu^\Delta U_\nu = U_\nu^\dagger U_\nu V^M$$

(18)

Finally the lepton mixing matrix is

$$U^{PMNS} = U_1^\dagger U_\nu = U_1^\dagger U_0 V^M$$

(19)

From eqs. (17),(18) we see that the mixing matrix $V^M$ diagonalize the following symmetric matrix:

$$C = M_D^\dagger V_0^\dagger \frac{1}{M_R} V_0^\dagger M_D^\dagger$$

(20)

where $V_0$ is the mixing matrix that diagonalize on the right the Dirac neutrino mass matrix in eq. (17).
B. $V^M$ as correlation matrix in GUT

In GUT models, such as generic $SO(10)$ or $E_6$, there are some natural Yukawa unifications. These give up to an interesting relation between the $U^{CKM}$ quark mixing matrix, $U^{PMNS}$ lepton mixing matrix and $V^M$ obtained from eq. (20). In fact $V^M$ turns out to be the correlation matrix defined in eq. (3). In the quark sector we introduce $Y_u$ and $Y_d$ to be the Yukawa matrices for up and down sectors. They can be diagonalized by

$$Y_u = U_u Y_u^a V_u^\dagger$$
$$Y_d = U_d Y_d^a V_d^\dagger$$

where the $Y^a$ are diagonal and the $U$s and $V$s are unitary matrices. Then the quark mixing matrix is given by

$$U^{CKM} = U_u^a U_d$$

In GUT models such as $SO(10)$ or $E_6$ we have intriguing relations between the Yukawa coupling of the quark sector and the one of the lepton sector. For instance, in minimal renormalizable $SO(10)$ with Higgs in the $10$, $126$, and $210$, we have $Y_l \approx Y_\tau$. In fact the flavor symmetry implies the structure of the Yukawa matrices: the equivalent entries of $Y_l$ and $Y_d$ are usually of the same order of magnitude. We conclude that, as long as the flavor symmetry fully constrains the mixing matrices that diagonalize a Yukawa matrices, we have $U_l \approx V_d^\ast$.

From eq. (19) we get

$$U^{PMNS} \simeq V_u^a U_0 V^M$$

where $V^M$ is the mixing matrix that diagonalize the matrix $C$ of eq. (20). If we call $Y_\nu$ the Yukawa coupling that will generate the Dirac neutrino mass matrix $M_D$, we have also the relation

$$Y_\nu \approx Y_\nu^a \rightarrow U_0 \simeq V_u^\ast$$

This relation, together with the previous one, implies

$$U^{PMNS} \simeq V_u^a V_u^\ast V^M$$

If the Yukawa matrices are symmetric, for example in minimal renormalizable $SO(10)$ we have only small contributions from the antisymmetric representation $120$, the previous relationship translates into a relation between $U^{PMNS}$, $U^{CKM}$ and $V^M$. In fact we have

$$Y_u = Y_u^a \rightarrow V_u^a = U_u$$
$$Y_d = Y_d^a \rightarrow V_d^a = U_d.$$  

The first relation tell us that

$$U^{PMNS} = V_u^a U_0 V^M.$$ 

Finally, by using the second relation in eq. (26) and the definition of the CKM mixing matrix of eq. (19) we get that

$$U^{PMNS} = (U^{CKM})^\dagger V^M.$$ 

The form of $V^M$ can be obtained under some assumptions about the flavor structure of the theory. This model will give for example a correlation $V^V$ with $V^M_{13} = 0$. As a consequence of the from of quark-lepton correlation matrix $V^M$ there are some predictions for the model. For example the prediction for $\theta_{13}^{PMNS}$ of [65] and the correlations between CP violating phases and the mixing angle $\theta_{12}$ of sect. [11].

C. $V^M$ from $S_3$ flavor symmetry

As we told $V^M$ diagonalize the matrix $C$ of eq. (20). In this section we will show that a $S_3$ flavor permutation symmetry, softly broken into $S_2$, gives us the prediction of $V^M_{13} = 0$.

The six generators of the $S_3$ flavor symmetry are the elements of the permutation group of three objects. The action of $S_3$ on the fields is to permute the family label of the fields. In the following we will introduce the $S_2$ symmetry with respect the 2nd and 3rd generations. The $S_2$ group is an Abelian one and swap the second family $\{\mu_L, (\nu_\mu)_L, s_L, c_L, \mu_R, (\nu_\mu)_R, s_R, c_R\}$ with the third one $\{\tau_L, (\nu_\tau)_L, b_L, t_L, \tau_R, (\nu_\tau)_R, b_R, t_R\}$.

Let us assume that there is an $S_3$ flavor symmetry at high energy, which is softly broken into $S_2$. In this case, before the $S_3$ breaking all the Yukawa matrices have the following structure:

$$Y = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}$$

where $a$ and $b$ independent. The $S_3$ symmetry implies that $(1/\sqrt{3},1/\sqrt{3},1/\sqrt{3})$ is an eigenvector of our matrix in eq. (20). Moreover these kind of matrices have two equal eigenvalues. This gives us an undetermined mixing angle in the diagonalizing mixing matrices.

When $S_3$ is softly broken into $S_2$, one get

$$Y = \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix}$$

with $c \approx a$ and $d \approx b$. When $S_3$ is broken the degeneracy is removed. In general the $S_2$ symmetry implies that $(0,1/\sqrt{2},-1/\sqrt{2})$ is an exact eigenvector of our matrix $27$. The fact that $S_3$ is only softly broken into $S_2$ allows us to say that $(1/\sqrt{3},1/\sqrt{3},1/\sqrt{3})$ is still in a good approximation an eigenvector of $Y$ in eq. (27). Then the mixing matrix that diagonalize from the right the Yukawa mixing matrix in eq. (27) is given in good approximation by

$$\begin{pmatrix} \approx -\sqrt{2}/\sqrt{3} & \approx 1/\sqrt{3} & 0 \\ \approx 1/\sqrt{6} & \approx 1/\sqrt{3} & 1/\sqrt{2} \\ \approx 1/\sqrt{6} & \approx 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}$$

where we did not prompt the phases.
Let us now investigate the $V^M$ in this model. The mass matrix $M_D$ will have the general structure in eq. (27). To be more defined, let us assumed that there is an extra softly broken $Z_2$ symmetry under which the 1st and the 2nd families are even, while the 3rd family is odd. This extra softly broken $Z_2$ symmetry gives us a hierarchy between the off-diagonal and the diagonal elements of $M_D$, i.e. $b, d \ll a, c$. In fact if $Z_2$ is exact both $b$ and $d$ are zero. For simplicity, we assume also a quasi-degenerate extra softly broken matrix between the off-diagonal and the diagonal elements of $M_D$, i.e. $a, d \ll b, c$. As shown in [77], it is possible to fit the CKM and the correlation in GUT models. In particular we investigated the Majorana right handed neutrino is of the form

$$M_R = \begin{pmatrix} a & b & b' \\ b & c & d \\ b' & d & e \end{pmatrix}$$

(29)

Because $S_3$ is only softly broken into $S_2$ we have that $a \approx c \approx e$, and $b \approx b' \approx d$. In this approximation the $M_R$ matrix is diagonalized by a $U_R$ of the form in eq. (28). In this case we have that $M_\nu$ defined in eq. (10) is near to be $S_3$ and $S_2$ symmetric, then it is diagonalized by a mixing matrix $U_\nu$ near the TriBiMaximal one given in eq. (28). The $C$ matrix is diagonalized by the mixing matrix

$$V_M = U_\nu U_R$$

(30)

We obtain that $V_M$ is a rotation in the $(1,2)$ plan, i.e. it contains a zero in the $(1,3)$ entry.

As shown in [77], it is possible to fit the CKM and the PMNS mixing matrix within this model [81].

IV. SUMMARY AND CONCLUSIONS

In this paper we show the power of the quark lepton correlation in GUT models. In particular we investigated the correlation between the CP violating invariants and the mixing angle $\theta_{12}$ in the lepton sector. To extract these informations we used a Monte Carlo approach to take into full account the presence of unknow unphysical phases in the definition of $V^M$.

We obtain that

$$\theta_{13}^{PMNS} = (9^{-1+1}_0)$$

and are correlated to $\sin^2 \theta_{12}^{PMNS}$

with the theoretical input $V^M_{13} = 0$ only. The results are such that in the next future it will be possible to make cross check from the experimental evidences and discriminate the validity of this approach.

To better clarify the importance of the quark lepton correlation we showed how a toy model, based on the $S_3$ flavor permutation symmetry and GUT, predicts the correlation matrix $V^M$ to have a zero $(1,3)$ entry. In this model $V^M$, defined by $\theta$ is also related to the Dirac and Majorana neutrino mass matrix as in eq. (20). For this reason its form is given by the symmetries of the model.

In this model we have $V^M_{13} = 0$. We can apply all the nice predictions obtained in section [11].

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Notice that in [77] the mass matrix $M_d$ is not symmetric, however the mixing matrix $V_d$ is similar to the mixing matrix $U_e$. [81]