Event-based Consensus Tracking for Nonlinear Multi-Agent Systems under Semi-Markov Jump Topology

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ABSTRACT This paper studies the event-triggering leader-follower consensus with the strictly dissipative performance for nonlinear multi-agent systems (MASs) with semi-Markov changing topologies. First, a polynomial fuzzy model is established to describe the error nonlinear multi-agent system that is formed by one virtual leader and followers. Then, a new event-triggering transmission strategy is proposed to mitigate communication and computational load. By utilizing the event-triggering mechanism and modeling the switching topologies by semi-Markov process, a sampled-data based consensus protocol is designed. Compared with traditional Markov jump topologies, the transition rate is time-varying for semi-Markov switching topologies. By mode-dependent Lyapunov-Krasovskii functional, the sum of square based relaxed stabilization conditions for fuzzy MASs are obtained to guarantee event-triggering consensus with strict dissipativity in an even-square sense, i.e., the derived conditions take into account the joint effects of event-triggering control, semi-Markov jump topologies and external disturbance. An illustrative example is provided to verify the proposed consensus design schemes.

INDEX TERMS Multi-agent system, fuzzy modeling, strict dissipativity, event-triggering control, semi-Markov switching topology.

I. INTRODUCTION

Cooperative consensus of multi-agent systems (MASs) has received considerable attention owing to its wide applications, including flocking [1], formation control [2], [3]. The main purpose of consensus problems is to design a distributed controller (consensus protocol), which can guarantee that all agents can reach a common state by exchanging local information among neighboring agents via communication link. Various control schemes have been utilized, such as finite time control in [4], fault-tolerant control in [5], [6], adaptive control in [7]–[9] and optimal control in [11], [12].

The communication topologies among the agents may not be often fixed due to links interruption and new establishment partly stem from the communication equipment failures and disturbance. To describe the time-varying topology, a common method is that the switching topologies are modeled by the Markov process, which have been attracted a lot of concerns. For example, see [13]–[15]. However, in practice, Markov changing topologies have many limitations because the dwell time obeys exponential distribution and the transition rates are constant. Different from traditional Markov jump topologies, the dwell time of semi-Markov changing topologies obeys more general distribution, including Gaussian distribution and Weibull distribution. For semi-Markov switching topologies, the transition rates are time-varying and depend on the dwell time. Recently, fruitful results have been reported on semi-Markov switching topologies [16]–[18]. Hence, semi-Markov changing topology is one of the issues worth considering here.

Dissipativity theory is introduced in [19], which plays a key role in the analysis and synthesis of control systems. In practice, it is necessary to guarantee the dissipativity to reach the purpose of interference attenuation. The dissipativity is regarded as a generalization of the $H_{\infty}$ performance, the passivity theory, and the bounded real lemma. The dissipative performance is discussed for a variety of dynamic systems [20], [21]. For instance, [21] studies the observed-based event-triggering sliding mode control with the strict
dissipativity of the switched stochastic discrete system.

Event-triggering control (ETC), as an effective scheme in saving communication resources and alleviating control updates, has gain remarkable attention. Different from the time-triggering scheme, data transmission and control updates are decided by an event-triggering condition. When the triggering condition is met, the event occurs. The central idea and challenge of ETC are to establish the time sequence of data transmission through a predefined event-triggering strategy which is different from the time series of traditional periodic control. For example, see [22] and the references therein. Recently, event-triggering consensus problems for MASs have attracted extensive attention. Rich results have been obtained [23]–[28]. For instance, the control problem of event-triggering consensus is discussed for linear MASs with changing topologies in [26].

Recently, the polynomial fuzzy model in [29] is introduced for modeling a nonlinear system by polynomial expression. The new fuzzy model can be viewed as a generalization of the T-S fuzzy model [30]–[32], which has attracted extensive attention. For example, see [22] and the references therein. The main contributions of this paper are summarized as follows:

(i) Most existing results deal with the consensus problems of the nonlinear MASs by using the Lipschitz conditions, such as [9], [10]. A polynomial fuzzy model is established to describe the error nonlinear multi-agent system in this paper. Compared with [34], the fuzzy model here is simpler and without extra assumptions.

(ii) In [35], [36], the consensus problems of continuous-time communication are investigated for nonlinear MASs under changing topologies. [13] addresses the time-triggering consensus problems for nonlinear MASs under Markov switching topologies. Unlike [13], [35], [36], a sampled-data mode-dependent event-triggering transmission strategy is presented here to reduce communication and computational load. By using the event-triggering scheme and modeling the switching topologies by semi-Markov process, the mode-dependent event-triggering consensus protocols are designed.

(iii) Using mode-dependent Lyapunov-Krasovskii functional, relaxed stabilization conditions based on sum of square (SOS) [37] are obtained to assure event-triggering consensus with strict dissipativity in an even-square sense, i.e., the presented conditions take into account the joint effects of event-triggering communication, semi-Markov jump topologies and external disturbance.

The remainder of this paper is organized as follows: In Section 2, the related knowledge of graph theory is introduced and the problem formulation is given. In Section 3, the polynomial fuzzy model is built and the sample-data mode-dependent event-triggering transmission scheme is designed. In Section 4, event-triggering consensus protocols and the main results are presented. In Section 5, an illustrative example is provided. We conclude this paper in Section 6.

**Notation:** The symbol ⊗ denotes the Kronecker product. \| \cdot \| is the Euclidean norm. \( I \) represents the identity matrix with appropriate dimensions. \( E \{ \cdot \} \) is the expectation operator. \((Ω, F, P)\) denotes a probability space. \( Q > 0 \) means that the matrix \( Q \) is positive definite. The superscript T for matrix \( Q \) denotes transpose of matrix \( Q \). \( \text{Sym}(A) \) means \( A + A^T \). \( Σ^2 \) represents SOS.

II. PRELIMINARIES AND PROBLEM FORMULATION

Here, we introduce the related knowledge of graph theory and the problem formulation is presented.

A. GRAPH THEORY

Let \( \mathcal{G} = (V, E, A) \) be a digraph generated by \( N \) follower agents, in which \( V = \{1, \ldots, N\} \) is a nonempty node set, \( E = \{(i, j) : i, j \in V\} \) denotes an edge set, and \( A = [a_{ij}] \in \mathbb{R}^{N \times N} \) represents a weighted adjacency matrix. \( a_{ij} = 1 \) if \((i, j) \in E\), otherwise \( a_{ij} = 0 \). Define Laplacian matrix \( L = \{L_{ij}\} \in \mathbb{R}^{N \times N} \) as \( L_{ij} = \sum_{k=1, k \neq i}^{N} a_{ik} \) if \( i = j \), otherwise \( L_{ij} = -a_{ij} \). We denote \( \mathcal{G} \) as a digraph formed by one virtual leader labeled 0 and \( N \) follower agents marked \( 1 \sim N \).

Define a switching signal by \( γ(t) : [0, +∞) \rightarrow S = \{1, \ldots, s\} \), whose value is the index of network topology. Denote \( L(γ(t)) \) as the Laplacian matrix of the topology \( \mathcal{G}(γ(t)) \) at \( γ(t) \).

B. SEMI-MARKOV SWITCHING TOPOLOGIES

The semi-Markov switching signal \( γ(t) \) is determined by the probability transitions:

\[
P_r\{γ(t + φ) = ν|γ(t) = μ\} = \begin{cases} \lambda_{μν}(φ) + o(φ), & μ ≠ ν, \\ 1 + \lambda_{μμ}(φ) + o(φ), & μ = ν \end{cases} \tag{1}
\]

where \( φ > 0 \) denotes the sojourn time and \( \text{lim}_{φ \rightarrow 0} o(φ)/φ = 0 \). \( λ_{μν}(φ) \geq 0 \) denotes the transition rate with \( μ ≠ ν \) from mode \( μ \) at time \( t \) to mode \( ν \) at time \( t + φ \) and \( λ_{μμ}(φ) = -Σ_{ν=1, ν≠μ}^{s} λ_{μν}(φ) \). In this paper, we consider time-varying transition rate \( λ_{μν}(φ) \) depending on \( φ \). When \( λ_{μν}(φ) = λ_{μν}(φ) \) constant, the semi-Markov process reduces to a traditional Markov process.

C. PROBLEM FORMULATION

Here, we consider a leader-follower nonlinear multi-agent system formed by \( N \) followers and one virtual leader. Each agent’s dynamics is described by

\[
\begin{align*}
\dot{x}_0 &= f(x_0), \\
\dot{x}_i &= f(x_i) + u_i + d_p w_i,
\end{align*} \tag{2}
\]

where \( x_0 \in \mathbb{R}^n \) is the state of virtual leader, \( x_i \in \mathbb{R}^n \) is the state of agent \( i \), and \( i = 1, \ldots, N \). \( f(x_i) \in \mathbb{R}^n \) is
a polynomial vector in \( x_i, u_i \in \mathbb{R}^n \) is the control input. \( d_p \in \mathbb{R}^{n \times q} \), and \( w_i \in \mathbb{R}^q \) is the external disturbance.

The error state is \( e_i = x_i - x_0 \). Then, the error dynamics can be expressed as

\[
\dot{e}_i = f(x_i) - f(x_0) + u_i + d_p w_i, \quad (3)
\]

Before going further, the following assumptions and concepts are given to obtain the main results.

Assumption 1: Each \( G^{\gamma(t)} \), \( \gamma(t) \in S \), contains a directed spanning tree with the root of the virtual leader.

Assumption 2: States of each agent are periodically sampled. The sampling period is synchronized by a clock.

**Definition 1**: [36]: Given matrices \( Y \in \mathbb{R}^{r \times q} \), \( X \in \mathbb{R}^{r \times t} \), and \( Z = Z^T \in \mathbb{R}^{q \times q} \) with \( X' \leq 0 \) and \( Z > 0 \), if for \( T^* \geq 0 \) and \( \delta > 0 \),

\[
\int_0^{T^*} \left[ \begin{array}{c} z \\ w \end{array} \right]^T \left[ \begin{array}{cc} I_N \otimes X & I_N \otimes Y \\ I_N \otimes Y & I_N \otimes Z \end{array} \right] \left[ \begin{array}{c} z \\ w \end{array} \right] \, dt \geq \delta \int_0^{T^*} w^T w \, dt
\]

then (3) is called strictly \((X, Y, Z)\)-\(\delta\)-dissipative.

**Definition 2**: Under the consensus protocol \( u_i, (2) \) is called mean-square consensus if

\[
\lim_{t \to \infty} E\{\|x_i - x_0\|^2\} = 0, \quad i = 1, \ldots, N
\]

for any initial conditions \( x_i(0), x_0(0) \in \mathbb{R}^n \).

**Remark 1**: Inspired by [35], the definition of mean-square consensus in (5) for event-based MASs under semi-Markov jump topologies is presented.

**III. POLYNOMIAL FUZZY MODEL AND EVENT-TRIGGERING MECHANISM**

**A. POLYNOMIAL FUZZY MODEL**

To describe system (3), a polynomial fuzzy model is established below:

\[
\mathcal{R}^p: \quad \text{If } \theta_{i1} = \zeta_{j1}, \ldots, \theta_{iu} = \zeta_{ju}, \text{ then } \ 
\dot{e}_i = a_p(e_i)e_i + u_i + d_p w_i, \quad (6)
\]

where \( \theta_i = [\theta_{i1}, \ldots, \theta_{iu}]^T \) denotes the premise variable vector. \( \zeta_{j1}, \ldots, \zeta_{ju} \) refer to the fuzzy sets. \( r \) is IF-THEN rules’ number. \( a_p(e_i) \) denotes the polynomial matrix in \( e_i \).

The compact form of (6) is

\[
\dot{e}_i = \sum_{p=1}^{r} h_p(\theta_i)\{a_p(e_i)e_i + u_i + d_p w_i\}, \quad (7)
\]

where

\[
h_p(\theta_i) = \frac{\omega_p(\theta_i)}{\sum_{p=1}^{r} \omega_p(\theta_i)} = \prod_{j=1}^{u} C_{j}^p(\theta_{i,j}).
\]

The function \( h_p(\theta_i) \) has the properties of

\[
h_p(\theta_i) \geq 0, \quad \sum_{p=1}^{r} h_p(\theta_i) = 1.
\]

**B. EVENT-TRIGGERING MECHANISM**

To save communication resources, an event-triggering control strategy is presented for the system (2). To determine whether the sampled data is transmitted or not, the mode-dependent event-triggering condition for the \( i \)th agent is defined as

\[
\begin{align*}
\{ \varepsilon_i(t_k^i + lh) \}^T \Phi^{(i)}(t_k^i + lh) &> \rho_i \{ \varepsilon_i(t_k^i + lh) \}^T \Phi^{(i)}(t_k^i + lh), \quad (8)
\end{align*}
\]

where \( \rho_i > 0 \) denotes the threshold, and \( \Phi^{(i)} > 0 \) is the weighting matrices to be designed later. \( \varepsilon_i(t_k^i + lh) = \sum_{j \in N_i} a_{ij} |x_i(t_k^i) - x_j(t_k^i)| + d_i |x_i(t_k^i) - x_0(t_k^i + lh)| \), where \( i = 1, 2, \ldots, \) and \( h \) is the sampling period. Define

\[
\varepsilon_i(t_k^i + lh) = \alpha(x_i(t_k^i) - x_i(t_k^i + lh)), \quad (9)
\]

where \( \alpha \) is a scalar in \((0, 1]\). \( \varepsilon_i(t_k^i + lh) \) denotes the measurement error formed by the last released state \( x_i(t_k^i) \) and the current state \( x_i(t_k^i + lh) \). Let \( m_h = t_k^i + lh, \varepsilon_i(m_h) = \alpha(x_i(t_k^i) - x_i(t_k^i + lh)) \), and \( e_i(m_h) = \alpha(x_i(m_h) - x_0(m_h)) \), where \( m \) is an integer.

**Remark 2**: If the condition (8) holds, the event is trig- erred, and then the sampled data is sent to its neighbors and controller. The event-triggering time sequence represents \( \{t_k^i, t_{k+1}^i, \ldots\} \), in which \( t_k^i \) is the initial time. Since the minimum inter-event time \( \min\{t_{k+1}^i - t_k^i\} \geq \bar{h} \), Zeno behavior does not happen.

**Remark 3**: In (9), motivated by [32], \( \alpha \) is introduced to smooth the input signal. If \( \alpha = 1 \), the event-triggering mechanism will reduced to the conventional one as in [16], [38]. Compared with the conventional event-triggering mechanism, the event-triggering mechanism in (9) will reduce erroneuous events induced by the abrupt changing of the output measurement.

**IV. EVENT-TRIGGERING CONSENSUS DESIGN AND CONSENSUS CONDITIONS**

Now, we consider the event-triggering dissipative consensus conditions for system (7) under semi-Markov jump topologies. Then, the derived results can be extended to a fixed topological case.

**A. EVENT-TRIGGERING CONSENSUS PROTOCOL**

Here, we first design a distributed event-triggering consensus protocol for system (7) under semi-Markov changing topologies.

Considering the controlled output \( z_i \), the augmented system of agent \( i \) is

\[
\dot{e}_i = \sum_{p=1}^{r} h_p(\theta_i)\{c_{zp}e_i + d_{zp} w_i\}, \quad (10)
\]

where \( c_{zp} \in \mathbb{R}^{r \times n} \), \( d_{zp} \in \mathbb{R}^{r \times q} \), and \( z_i \in \mathbb{R}^i \).
An event-triggering consensus protocol for agent $i$ is designed as follows:

$$u_i(t) = - \sum_{j \in N_i(t)} a_{ij} \gamma_i(t) [x_i(t_k^j) - x_j(t_k^i)] - d_i \gamma_i(t) [x_i(t_k^i) - x_0(mh)],$$

where $t \in [mh, (m+1)h)$, $N_i = \{ j \in V : (j, i) \in E \}$ is the $i$th agent’s neighboring set, $\gamma_i(t) \in \mathbb{R}^{n \times n}$ denotes positive definite control gain matrix, $d_i$ is the weight of information flow. If agent $i$ can get the leader’s information at $\gamma(t)$, then $d_i = 1$.

**Remark 4:** Our purpose is to design the consensus protocol (11) to ensure that all agents can achieve agreement and alleviate the consumption of communication resources.

The sampled-data based protocol (11) is expressed as

$$u_i(t) = - \frac{1}{\alpha} \sum_{j \in N_i(t)} a_{ij} \gamma_i(t) \left[ e_i(mh) - e_j(mh) \right] + \frac{1}{\alpha} \gamma_i(t) [e_i(mh) + E_i(mh)],$$

where $t \in [mh, (m+1)h)$.

For notation simplicity, $\gamma(t) = \mu, \mu \in S$. The compact form of (12) is

$$u(t) = - \frac{1}{\alpha} (\tilde{E} \gamma \Gamma + \mu) e(mh) + \frac{1}{\alpha} (\tilde{E} \mu + \Gamma \mu) e(mh),$$

where

$$\tilde{E} \mu = - E \mu - D \mu, \quad \Gamma \mu = \text{diag}(d_1, \ldots, d_N),$$

$$e(mh) = [e_1^T(mh), \ldots, e_N^T(mh)]^T,$$

$$\tilde{E}(mh) = [\tilde{E}_1^T(mh), \ldots, \tilde{E}_N^T(mh)]^T.$$

Substituting (13) into (10), the closed-loop system at $\gamma(t)$ is

$$\dot{e}(t) = \sum_{p=1}^{r} h_p(\theta) \left\{ A_p(e) e + D_p w \right\} + \frac{1}{\alpha} \gamma_i(t) [e_i(mh) - e_j(mh)],$$

$$\dot{z}(t) = \sum_{p=1}^{r} h_p(\theta) (C_{zp} e + D_{zp} w),$$

$$e(\theta) = \varphi(\theta), \quad \theta \in [-h, 0],$$

where $t \in [mh, (m+1)h)$,

$$h_p(\theta) = \text{diag}(h_p(\theta_1), \ldots, h_p(\theta_N)),$$

$$A_p(e) = \text{diag}(a_p(e_1), \ldots, a_p(e_N)),$$

$$D_p = I_N \otimes d_p, C_{zp} = I_N \otimes c_{zp},$$

$$D_{zp} = I_N \otimes d_{zp} e = [e_1^T, \ldots, e_N^T]^T,$$

$$z = [z_1^T, \ldots, z_N^T]^T, w = [w_1^T, \ldots, w_N^T]^T.$$

Define artificial delay by $mh = t - \tau(t)$, $0 \leq \tau(t) \leq h$, $\hat{\tau}(t) = 1$ at $t \neq mh$.

**B. CONSENSUS CONDITIONS UNDER SEMI-MARKOV SWITCHING TOPOLOGIES**

**Theorem 1:** Given matrices $Y \in \mathbb{R}^{n \times q}, X = X^T \in \mathbb{R}^{t \times t}$, and $Z = Z^T \in \mathbb{R}^{q \times q}$ with $X \leq 0$ and $Z > 0$, and scalars $\delta > 0$, $h > 0$, and $0 < \alpha \leq 1$, under Assumptions 1 and 2 by semi-Markov process and the consensus protocol (13), there exist positive matrices $P^\mu \in \mathbb{R}^{n \times n}, \Gamma^\mu \in \mathbb{R}^{n \times n}, Q \in \mathbb{R}^{n \times n}, R \in \mathbb{R}^{n \times n}$, and $\Phi^\mu \in \mathbb{R}^{n \times n}$, nonnegative $\varepsilon_1, \varepsilon_2$ and $\varepsilon_3$, such that, for $\mu = 1, \ldots, s, p = 1, \ldots, r$,

$$\eta_1(p^\mu - \varepsilon_1 I) \eta_1 \leq \Sigma^2,$$

$$\eta_2(\Gamma^\mu - \varepsilon_2 I) \eta_2 \leq \Sigma^2,$$

$$-\eta_3(\Pi^\mu + \varepsilon_3 I) \eta_3 \leq \Sigma^2.$$  \(\text{(15)}\) \(\text{(16)}\) \(\text{(17)}\)

Then, (14) is asymptotically stable, i.e. all agents can reach an event-triggering consensus with a strictly dissipative performance in mean-square sense, where $\eta_1, \eta_2$ and $\eta_3$ are arbitrary vectors,

$$\Pi^\mu \triangleq \sum_{i=1}^{s} \lambda_{i\mu} (I_N \otimes \mu^\varepsilon) + \text{Sym}((I_N \otimes \mu^\varepsilon) A_p(e)),$$

$$\Xi^\mu \triangleq -C_{zp}^T (I_N \otimes \varphi) C_{zp},$$

$$\Sigma^\mu \triangleq -C_{zp}^T (I_N \otimes \varphi) D_{zp} - C_{zp}^T (I_N \otimes \varphi) + (I_N \otimes \mu^\varepsilon) D_p,$$

$$\Lambda^\mu \triangleq \text{diag}(\rho_1, \ldots, \rho_N),$$

$$\Delta^\mu \triangleq \mu^\varepsilon A_p(e).$$

**Proof 1:** Choose Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$

Where

$$V_1(t) = e^T(I_N \otimes \mu^\varepsilon(e)) e,$$

$$V_2(t) = \int_{\hat{\tau}(t)}^{t} e^T(t) (I_N \otimes Q) e(t) dt,$$

$$V_3(t) = h \int_{\hat{\tau}(t)}^{t} \hat{e}^T(t) (I_N \otimes R) \hat{e}(t) d\sigma d\beta.$$
From (15) and (16), we can get \( V(t) > 0 \) and \( \Gamma^\mu > 0 \), respectively.

The weak infinitesimal operator \( \mathcal{F} \) of \( V(t) \) is defined as
\[
\mathcal{F}V(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} \{ E \{ V(t + \Delta)|e(t), \gamma(t) \} - V(t) \}. 
\]
(19)

Therefore, one has
\[
\mathcal{F}V_1(e) = \sum_{\nu=1}^s \lambda_{\nu}(\phi)e^T(t)(I_N \otimes p^{\nu})e(t) + 2e^T(t)(I_N \otimes p^{\nu})\dot{e}(t),
\]
(20)
\[
\mathcal{F}V_2(e) = e^T(I_N \otimes Q)e,
\]
(21)
\[
\mathcal{F}V_3(e) = h^2e^T(I_N \otimes R)e - h \int_{t-h}^t \dot{e}(\sigma)(I_N \otimes R)e(\sigma)d\sigma,
\]
(22)

Employing Jensen’s inequality, the second item in (22) implies that
\[
- \int_{t-h}^t \dot{e}(\sigma)(I_N \otimes R)e(\sigma)d\sigma \leq - (e^T(t) - e^T(t-h))(I_N \otimes R) \times (e(t) - e(t-h)).
\]
(23)
Furthermore, if event-triggering condition (8) is broken, the event is not triggering. We obtain
\[
(\mathcal{E}(mh))^T(I_N \otimes \Phi^\mu)\mathcal{E}(mh) = \sum_{i=1}^N (\mathcal{E}_i(mh))^T\Phi^\mu\mathcal{E}_i(mh) \leq \Lambda(M(mh))^T(I_N \otimes \Phi^\mu)Y(mh)
\]
(24)
where \( Y(mh) = -(\hat{\mathcal{E}}^\mu \otimes I_n)e(mh) - (\hat{\mathcal{E}}^\mu \otimes I_n)\mathcal{E}(mh) \).

Therefore, (24) can be expressed as
\[
(\mathcal{E}(t-\tau(t)))^T(I_N \otimes \Phi^\mu)\mathcal{E}((t-\tau(t))) \leq [e(t-\tau(t)) + \mathcal{E}(t-\tau(t))]^T \times (\hat{L}^\mu)^T \Lambda(\hat{L}^\mu \otimes \Phi^\mu) [e(t-\tau(t)) + \mathcal{E}(t-\tau(t))].
\]
(25)
Combining (20)-(23) and (25), and assuming that \( w(t) = 0 \), one obtains
\[
\mathcal{F}V(t) = \xi^T(t) \left( \sum_{p=1}^r L_p(\theta)\Sigma \right) \xi(t),
\]
where
\[
\xi(t) = \begin{bmatrix} e^T & e^T(t-\tau(t)) & e^T(t-h) & \mathcal{E}^T(t-\tau(t)) \end{bmatrix}^T,
\]
\[
\Sigma = \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_2 \\
* & \Pi_5 & 0 & \Pi_5 \\
* & * & \Pi_7 & 0 \\
+ h^2 & 0 & \Pi_6 & \Pi_6 \\
\end{bmatrix},
\]
(26)
From (17) and Proposition 2 in [39], it follows that \( \Sigma < 0 \). Hence, \( \mathcal{F}V(t) < 0 \).

Under zero initial condition, an index is defined as
\[
\mathcal{J}(T^*) = \int_0^{T^*} \begin{bmatrix} z \ w \end{bmatrix}^T \begin{bmatrix} I_N \otimes X & I_N \otimes Y \\ \ast & I_N \otimes Z \end{bmatrix} \cdot \begin{bmatrix} z \ w \end{bmatrix} dt - \delta \int_{T^*}^{\infty} w^T w dt.
\]
(27)
For any nonzero \( w(t) \in I_2(0, \infty) \), one shows
\[
\mathcal{J}(T^*) = \int_0^{T^*} \xi^T(t) \left( \sum_{p=1}^r h_p(\theta)\Psi \right) \xi dt,
\]
(28)
where
\[
\xi = \begin{bmatrix} e^T & e^T(t-\tau(t)) & e^T(t-h) & \mathcal{E}^T(t-\tau(t)) & w^T \end{bmatrix}^T,
\]
\[
\Psi = \begin{bmatrix} \Pi_1 + \Xi_1 & \Pi_2 & \Pi_3 & \Pi_2 & \Xi_2 \\
* & \Pi_5 & 0 & \Pi_5 & 0 \\
* & * & \Pi_7 & 0 & 0 \\
* & * & * & \Pi_8 & 0 \\
* & * & * & * & \Xi_3 \end{bmatrix}
\]
Employing Schur complement, if
\[
\begin{bmatrix} \Pi_1 + \Xi_1 & \Pi_2 & \Pi_3 & \Pi_2 & \Xi_2 & \Pi_4 \\
* & \Pi_5 & 0 & \Pi_5 & 0 & \Omega_6 \\
* & * & \Pi_7 & 0 & 0 & 0 \\
* & * & * & \Pi_8 & 0 & \Omega_6 \\
* & * & * & * & \Xi_3 & hD_P^T \\
* & * & * & * & * & \Pi_9 \end{bmatrix} < 0,
\]
(29)
then \( \Psi < 0 \). From (17), it follows that (28) holds. One gets
\[
\int_0^{T^*} \xi^T(t)\Psi\xi dt < 0.
\]
(30)
From (27) and \( \mathcal{V}(T^*) > 0 \), one obtains
\[
\mathcal{J}(T^*) > 0.
\]
By Definition 1, (14) is strictly \((X, Y, Z)\)-\(\delta\)-dissipative. Based on (17), one obtains
\[
\mathcal{F}V(t) < 0, \quad t \in [mh, (m+1)h).
\]
(31)
Therefore, there exists a scalar \( \epsilon > 0 \), such that
\[
\mathcal{F}V(t) < -\epsilon e^T(t)e(t), \quad t \in [mh, (m+1)h),
\]
(32)
By using Dynkin’s formula, one has
\[
E\{V((m+1)h-o(h)) \} - V\{mh\} < -\epsilon E\int_{mh}^{(m+1)h-o(h)} \|e(\sigma)\|^2 d\sigma.
\]
Similarly, one obtains
\[
E\{V mh-o(h)\} - V\{(m-1)h\} < -\epsilon E \left\{ \int_{(m-1)h}^{mh-o(h)} \|e(\sigma)\|^2 d\sigma \right\},
\]
\[
E\{V mh-o(h)\} - E\{V(0)\} < -\epsilon E \left\{ \int_{mh}^{mh-o(h)} \|e(\sigma)\|^2 d\sigma \right\},
\] (32)

Since \( V_2(t) \geq 0 \) and \( \int_{t=0}^{t} e(t)(I N \otimes Q)e(t)d\sigma = 0 \) for \( t=mh \), so \( V_2((mh-o(h)) \geq V_2(mh) \), which implies that
\[
E\{V mh-o(h)\} \geq V\{(mh)\}.
\] (33)

Therefore, it follows from (31)-(33) that
\[
E\{V((m+1)h-o(h))\} - E\{V(0)\} \leq -\epsilon \sum_{m=0}^{\infty} E \left\{ \int_{mh}^{(m+1)h-o(h)} \|e(\sigma)\|^2 d\sigma \right\}.
\] (34)

By (34), one obtains
\[
\sum_{m=0}^{\infty} E \left\{ \int_{mh}^{(m+1)h-o(h)} \|e(\sigma)\|^2 d\sigma \right\} \leq e^{-1} E\{V(0)\},
\]

That is
\[
\lim_{T \to \infty} E \left\{ \int_{0}^{T} \|e(\sigma)\|^2 d\sigma \right\} \leq \infty,
\]

which indicates that \( \lim_{T \to \infty} E\{e(\sigma)\|^2 = 0 \). According to Definition 2, all agents achieve consensus. This proof is completed.

**Remark 5:** In Theorem 1, by polynomial Lyapunov-Krasovskii functional technique, SOS-based relaxed sufficient condition is presented to ensure that the polynomial fuzzy MASs can achieve even-square agreement with strictly dissipative performance under event-triggering control and semi-Markov switching topologies. Based on Theorem 1, we present the following approach to design the control gains.

**Theorem 2:** Given matrices \( X = X^T \in \mathbb{R}^{r \times r}, Z = Z^T \in \mathbb{R}^{q \times q} \), and \( Y^{\mu} \times q \) with \( X \leq 0 \) and \( Z > 0 \), and scalars \( \delta > 0, h > 0 \), and \( 0 < \alpha \leq 1 \), under Assumptions 1 and 2 by semi-Markov process and the consensus protocol (13), there exist positive matrices \( \vec{p}^\mu, \vec{r}^\mu, \vec{q}^\mu \in \mathbb{R}^{n \times n} \), \( \vec{p}^\mu, \vec{r}^\mu \in \mathbb{R}^{n \times n} \) and \( \vec{q}^\mu \in \mathbb{R}^{n \times n} \), nonnegative polynomials \( \varepsilon_1, \varepsilon_2 \), \( \varepsilon_3 \) such that, for \( \mu = 1, \ldots, s, p = 1, \ldots, r, \)
\[
[\eta_1^T (\vec{p}^\mu - \varepsilon_1 I) \eta_1] \leq \Sigma^2, \quad (35)
\]
\[
[\eta_2^T (\vec{q}^\mu - \varepsilon_2 I) \eta_2] \leq \Sigma^2, \quad (36)
\]
\[
-\eta_3^T (\vec{q}^\mu + \varepsilon_3 I) \eta_3 \leq \Sigma^2. \quad (37)
\]

Then, (14) is asymptotically stable with strict dissipativity, where \( \eta_1, \eta_2 \) and \( \eta_3 \) are arbitrary vectors.
we get
\[
\begin{bmatrix}
\Omega_1 - (I_N \otimes \bar{p}^\mu) \\
\times \Xi_1 (I_N \otimes \bar{p}^\mu) \\
-\Omega_{10} (\Omega_{11})^{-1} \Omega_{10}
\end{bmatrix}
\begin{bmatrix}
\Omega_2 & \Omega_3 & \Omega_2 & \hat{\Xi}_2 & \Omega_4 \\
\ast & \Omega_7 & 0 & 0 & 0 \\
\ast & \ast & \Omega_8 & 0 & 0 \\
\ast & \ast & \ast & \Xi_3 & hD_pT \\
\ast & \ast & \ast & \ast & \Omega_9 \\
\ast & \ast & \ast & \ast & \hat{\Xi}_1
\end{bmatrix}
\begin{bmatrix}
\Omega_1 \\
\Omega_2 \\
\Omega_3 \\
\Omega_2 \\
\hat{\Xi}_2 \\
\Omega_4 \\
\gamma \\
\Omega_{10}
\end{bmatrix}
\triangleq \Omega^\mu < 0,
\]
where \( \bar{p}^\mu = (p^\mu)^{-1}, \bar{R}^\mu = \bar{p}^\mu R \bar{p}^\mu, Q^\mu = \bar{p}^\mu Q \bar{p}^\mu, \Phi^\mu = \bar{p}^\mu \Phi \bar{p}^\mu \).

Since \((I_N \otimes \bar{p}^\mu) - (I_N \otimes \bar{R}^\mu)^T (I_N \otimes \bar{R}^\mu)^{-1} (I_N \otimes \bar{p}^\mu) \geq 0\), it follows that
\[
- (I_N \otimes R)^{-1} = -(I_N \otimes \bar{p}^\mu) (I_N \otimes \bar{R}^\mu)^{-1} (I_N \otimes \bar{p}^\mu) \leq I_N \otimes \bar{R}^\mu - 2(I_N \otimes \bar{p}^\mu).
\]

By Schur complement, (39) can be expressed by SOS conditions (37). The rest part of the proof is similar to those of Theorem 1.

Remark 6: By contragredient transformation of (38), then SOS conditions (37) is derived. From (35)-(37), we obtain the solutions of \( \hat{\Gamma}^\mu, \bar{p}^\mu \) and \( \Phi^\mu \). Then, the control gain and event-triggering parameters are calculated by \( \Gamma^\mu = \hat{\Gamma}^\mu (\bar{p}^\mu)^{-1} \) and \( \Phi^\mu = (\bar{p}^\mu)^{-1} \Phi \bar{p}^\mu \). When \( \lambda_{\mu\nu}(\phi) = \lambda_{\mu\nu} \), where \( \lambda_{\mu\nu} \) is the constant transition rate, the semi-Markov process is reduced to the traditional Markov process. In Theorems 1 and 2, since the transition rate \( \lambda_{\mu\nu}(\phi) \) depending on \( \phi \) is time-varying, the SOS condition (37) is not linear, which is difficult to solve. To overcome this drawback, we present the following result.

C. CONSENSUS CONDITIONS UNDER SEMI-MARKOV SWITCHING TOPOLOGIES

Here, based on the obtained result in Theorem 2, consensus conditions for MASs under semi-Markov switching topologies is given.

Theorem 3: Given matrices \( X = X^T \in \mathbb{R}^{n \times 1}, Z = Z^T \in \mathbb{R}^{q \times q} \), and \( Y^{i \times q} \) with \( X \leq 0 \) and \( Z > 0 \), and scalars \( \delta > 0, h > 0 \), and \( 0 < \alpha \leq 1 \), under Assumptions 1 and 2 by semi-Markov process and the protocol (13) depending on mode \( \mu \), there exist positive matrices \( \mu, \mu \), \( \bar{p}^\mu \in \mathbb{R}^{n \times n}, \bar{R}^\mu \in \mathbb{R}^{n \times n} \), \( \bar{Q}^\mu \in \mathbb{R}^{n \times n}, \bar{R}^\mu \in \mathbb{R}^{n \times n} \), and \( \bar{p}^\mu \in \mathbb{R}^{n \times n} \), nonnegative polynomials \( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) and \( \varepsilon_4 \), such that, for \( m = 1, \ldots, s, \)
\begin{align*}
\eta_1^T \mu \bar{p}^\mu - \varepsilon_1 I & \leq \Sigma^2, \\
\eta_2^T \mu \bar{p}^\mu - \varepsilon_2 I & \leq \Sigma^2, \\
\eta_3^T \mu \bar{p}^\mu + \varepsilon_3 I & \leq \Sigma^2, \\
\eta_4^T \mu \bar{p}^\mu + \varepsilon_4 I & \leq \Sigma^2.
\end{align*}

Then, (14) is asymptotically stable with strict dissipativity, where \( \eta_1, \eta_2, \eta_3 \) and \( \eta_4 \) are arbitrary vectors.

Proof 3: For a specific \( \phi \), the transition rate \( \lambda_{\mu\nu}(\phi) \) can be written as convex combination \( \lambda_{\mu\nu}(\phi) = \kappa_1 \lambda_{\mu\nu} + \kappa_2 \lambda_{\mu\nu} \), where \( \kappa_1 + \kappa_2 = 1 \) and \( \kappa_1, \kappa_2 > 0 \). Multiplying \( \hat{\Omega}^\mu \) by \( \kappa_1 \) and \( \hat{\Omega}^\mu \) by \( \kappa_2 \), it follows that
\[
\begin{bmatrix}
M_1 & \Omega_2 & \Omega_3 & \Omega_2 & \hat{\Xi}_2 & \Omega_4 & \gamma & M_2 \\
\ast & \Omega_5 & 0 & \Omega_5 & 0 & \Omega_6 & 0 & 0 \\
\ast & \ast & \Omega_7 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \Omega_8 & 0 & \Omega_6 & 0 & 0 \\
\ast & \ast & \ast & \ast & \Xi_3 & hD_pT & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \Omega_9 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \Omega_{11} & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \hat{\Xi}_1 & 0
\end{bmatrix} < 0.
\]
D. Consensus Conditions under a Fixed Topology

Design event-triggering consensus protocol for the $i$th agent under a fixed topology

$$u_i(t) = - \sum_{j \in N_i} \alpha_{ij} \Gamma [x_i(t_k^j) - x_j(t_k^j)] - d_i \Gamma [x_i(t_k^j) - x_0(mh)],$$

where $t \in [mh, (m + 1)h]$. $\Gamma \in \mathbb{R}^{n \times n}$ denotes positive definite control gain matrix. The other parameters are similar to those in (11).

The sampled-data consensus protocol (44) can be represented as

$$u_i(t) = - \sum_{j \in N_i} \alpha_{ij} \Gamma \left[ e_i(mh) - e_j(mh) \right] + \varepsilon_i(mh) - \varepsilon_j(mh),$$

where $t \in [mh, (m + 1)h]$.

Rewrite (45) in compact form as

$$u = \frac{1}{\alpha} (\tilde{\mathcal{L}} \otimes \Gamma) e(mh) + \frac{1}{\alpha} (\tilde{\mathcal{L}} \otimes \Gamma) \mathcal{E}(mh),$$

where

$$\tilde{\mathcal{L}} = - \mathcal{L} - \mathcal{D}, \mathcal{D} = \text{diag} \{d_1, \ldots, d_N\},$$

$$e(mh) = [e_{11}^T(mh), \ldots, e_{N1}^T(mh)]^T,$$

$$\mathcal{E}(mh) = [\mathcal{E}_{11}(mh), \ldots, \mathcal{E}_{N1}(mh)]^T.$$  

Substituting (46) into (10), the closed-loop system is

$$\dot{e}(t) = \sum_{p=1}^{r} h_p(\theta) \left\{ A_p(e) e + D_p w + \frac{1}{\alpha} (\tilde{\mathcal{L}} \otimes \Gamma) e(t - \tau(t)) + \frac{\varepsilon_i}{\alpha} (\tilde{\mathcal{L}} \otimes \Gamma) \mathcal{E}(t - \tau(t)) \right\},$$

$$z(t) = \sum_{p=1}^{r} h_p(\theta) (C_{zp} e + D_{zp} w),$$

$$e(\vartheta) = \varphi(\vartheta), \vartheta \in [-h, 0],$$

where $t \in [mh, (m + 1)h]$.

From Theorems 1 and 2, the derived results can be extended to event-triggering dissipative consensus under a fixed topology.

\textbf{Theorem 4:} Given matrices $\mathcal{X} = \mathcal{X}^T \in \mathbb{R}^{n \times n}$, $\mathcal{Z} = \mathcal{Z}^T \in \mathbb{R}^{n \times q}$, and $\mathcal{Y}^{n \times q}$ with $\mathcal{X} \leq 0$ and $\mathcal{Z} > 0$, and scalars $\delta > 0$, $h > 0$, and $0 < \alpha \leq 1$, under Assumption 1 and the consensus protocol (46), there exist positive matrices $\bar{\rho} \in \mathbb{R}^{n \times n}$, $\Gamma \in \mathbb{R}^{n \times n}$, $\bar{Q} \in \mathbb{R}^{n \times n}$, $\bar{R} \in \mathbb{R}^{n \times n}$, and $\bar{\Phi} \in \mathbb{R}^{n \times n}$, nonnegative polynomials $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$, such that for $p = 1, \ldots, r$,

$$\eta_1^T(\bar{\rho} - \varepsilon_1 \Gamma) \eta_1 \leq \Sigma^2,$$

$$\eta_2^T(\bar{\Gamma} - \varepsilon_2 \Gamma) \eta_2 \leq \Sigma^2,$$

$$-\eta_3^T(\bar{\Omega} + \varepsilon_3 \Gamma) \eta_3 \leq \Sigma^2.$$  

Then, (14) is asymptotically stable with strict dissipativity, where $\eta_1$, $\eta_2$ and $\eta_3$ are arbitrary vectors, $\Sigma^2$ is a positive definite matrix, and $\bar{\mathcal{L}} \leq \tilde{\mathcal{L}} \otimes \Gamma$.

\textbf{Proof 4:} This proof is similar to those of Theorems 1 and 2.

V. Illustrative Example

Consider a nonlinear multi-agent network, which the switching topologies are shown in Figure 1. Each agent’s dynamics is described from [40]

$$\begin{align*}
\dot{x}_0 &= f(x_0) \\
\dot{x}_i &= f(x_i) + u_i + d_p w_i,
\end{align*}$$

where

$$f(x_i) = \begin{bmatrix} 10(x_{i2} - x_{i1}) \\ 28x_{i1} - x_{i1}x_{i3} - x_{i2} \\ x_{i1}x_{i2} - \frac{2}{3}x_{i3} \end{bmatrix}.$$  

The polynomial fuzzy model is established as follows:

$$R^{n \times \psi} : \begin{cases} x_{i1}(t) = \xi_1(t), x_{i2}(t) = \xi_2(t), x_{i3}(t) = \xi_3(t), \end{cases}$$

Then, (14) is asymptotically stable with strict dissipativity, where $\eta_1$, $\eta_2$ and $\eta_3$ are arbitrary vectors, $\Sigma^2$ is a positive definite matrix, and $\bar{\mathcal{L}} \leq \tilde{\mathcal{L}} \otimes \Gamma$. 

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\textbf{Proof 4:} This proof is similar to those of Theorems 1 and 2.
The augmented fuzzy error system is
\[
\dot{e} = \sum_{p=1}^{8} h_p(\theta) \left( A_p(e)e + D_p w(t) + \frac{1}{\alpha}(L^\mu \otimes \Gamma^\mu) e(t-\tau(t)) + \frac{1}{\alpha}(L^\mu \otimes \Gamma^\mu) E(t-\tau(t)) \right),
\]
\[
z = \sum_{p=1}^{8} h_p(\theta) (C_{zp} e + D_{zp} w(t)),
\]
where
\[
C_{zp} = I_4 \otimes \begin{bmatrix} 1.75 & 0 & 0 \\ 1 & 0.01 & 0.06 \\ 0 & 0.08 & 0 \end{bmatrix},
\]
\[
D_{zp} = I_4 \otimes \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}.
\]

Without loss of generality, assume that the edges' weights of all communication topologies are 1. Figure 2 depicts the semi-Markov switching signal. Laplacian matrices are expressed as
\[
L^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
\]
\[
L^2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\]

The external disturbance is \( w(t) = 1.5e^{-0.25t} |\cos t| \). Let \( X = -0.8, Y = -0.65, Z = 1.25, \delta = 0.9 \). The sampling period is \( h = 0.1 \). In (9), \( \alpha = 0.5 \). The transition rates are given by \( \lambda_{11}(\phi) = (-2.4, -1.6), \lambda_{12}(\phi) = (1.6, 2.4), \lambda_{21}(\phi) = (1.4, 2.6), \lambda_{22}(\phi) = (-2.6, -1.4) \). The event-triggering parameters \( \rho = (0.0625, 0.025, 0.0625, 0.05) \). From Theorem 3, using the SOSTOOLS [37] to solve SOS conditions, we obtain
\[
\Gamma^1 = \begin{bmatrix} 3.4766 & 0 & 0 \\ * & 2.6436 & 0 \\ * & * & 2.6436 \end{bmatrix},
\]
\[
\Gamma^2 = \begin{bmatrix} 3.3807 & 0 & 0 \\ * & 2.5661 & 0 \\ * & * & 2.5661 \end{bmatrix}.
\]

The event-triggering instants of each agent are depicted in Figure 3, which indicates the amounts of sampling data transmitted are reduced. The state trajectories of each agent are shown in Figure 4. The error states are given in Figure 5. The simulation results show that all agents achieve consensus, which demonstrates the effectiveness of the presented design schemes.

**VI. CONCLUSION**

In this paper, the event-triggering consensus with strict dissipativity have been studied for fuzzy MASs with semi-Markov jump topologies and external disturbance. A new
event-triggering consensus protocol has been designed to guarantee that MASs can achieve consensus and save communication resources. With the mode-dependent Lyapunov-Krasovskii functional, SOS-based relaxed conditions have been obtained. The simulation results demonstrate the effectiveness of the proposed design techniques. Future work will consider the event-based containment control problems for MASs with switching topologies and cyber attacks.

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