Hybrid Adaptive Control for the
DC-DC Boost Converter *

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Abstract: In this paper, we consider the problem of practically asymptotically stabilizing the DC-DC boost converter under parameter uncertainty. In particular, we propose an estimation algorithm that identifies the input voltage and output load of the converter in finite time. Using these estimates, we design a control algorithm that “unites” global and local control schemes. The global control scheme induces practical asymptotic stability of a desired output voltage and corresponding current, and the local control scheme maintains industry-standard PWM behavior during steady state. Stability properties for the resulting hybrid closed-loop system are established and simulation results illustrating the main results are provided.

Keywords: Electrical circuits, energy and power networks, cyber-physical systems, switched systems, learning, identification, stabilization, modeling.

1. INTRODUCTION

The DC-DC boost converter is widely used in the power systems of electric vehicles [Bellur and Kazimierzczuk 2007]. These systems operate under constantly changing demands such as supplying energy during acceleration and storing it during braking, necessitating power conversion technology that is capable of adapting to these changes. The industry standard control scheme for the boost converter is pulse-width modulation (PWM). However, since PWM controllers typically utilize a linearized model of the converter dynamics, the stability properties only hold locally near the set-point [Kassakian et al. 1991]. Recently, a renewed interest in power converters has originated from the rise of hybrid modeling paradigms and new perspectives on their control, including time-based switching, state-event triggered control, and optimization-based control, were proposed (Vasca and Iannelli, 2012).

In this paper, motivated by the prevalence of PWM control implementations in industry and the widespread familiarity with its operation, we propose a control algorithm that stabilizes the boost converter even under uncertainty in the input voltage and load resistance and maintains PWM behavior during steady-state operation. We utilize the modeling approach first proposed by Themis et al. (2015) that captures the transient behavior and every possible state of the converter system. Using hybrid systems tools presented in Section 2, we study the properties of a modular “uniting” control framework, discussed in Section 3, that switches between global and local control schemes. In Section 4, we propose an estimator that permits finite-time estimation of the converter input voltage and load resistance. We show in Section 5 that the closed-loop unifying framework and finite-time estimator induce global practical asymptotic stability of a desired voltage value. The hybrid systems tools recently developed in [Goebel et al. 2012, Sanfelice 2021] form the enabling techniques to achieve these results. Due to space constraints, some proofs are sketched or omitted, and will be published elsewhere.

Notation. We denote the real, nonnegative, positive, and natural numbers as $\mathbb{R}, \mathbb{R}_0^+, \mathbb{R}^+$, and $\mathbb{N}$, respectively.

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The solution jump set on which jumps are permitted. The vector \( \mathbf{C} \) differential inclusion capturing the continuous dynamics, and the corresponding constraint sets associated with each \( \mathbf{F} \) modeling the discrete behavior, and \( \mathbf{M} \) is the number of jumps that have occurred. The domain \( x \in \mathbb{R}^{n} \), \( n \) is the amount of ordinary time that has passed and \( j \) is the number of jumps that have occurred. The domain of \( x \), denoted \( \mathbb{D} \), \( x \in \mathbb{R}^{n} \times \mathbb{R}^{m} \), is a hybrid time domain, in the sense that for every \((T, J) \in \mathbb{D} \), there exists a nondecreasing sequence \( \{t_{j} \}^{j+1} \) with \( t_{0} = 0 \) such that \( \mathbb{D} \cap ([0, T] \times \{0, 1, \ldots , J\}) = \bigcup_{j=0}^{J} ([t_{j}, t_{j+1}), \{j\}) \). A solution \( x \) to \( \mathbb{H} \) is called maximal if it cannot be extended. A solution is called complete if its domain is unbounded.}

\[ x(\mathbb{H}) \subset \mathbb{R}^{n} \]

\[ \text{A solution is called complete if its domain is unbounded.} \]

2.2 Boost converter model

The boost converter is a class of switched-mode power supply that utilizes a switch \( S \), inductor \( L \), diode \( d \), and capacitor \( C \) to raise the voltage at the output load \( R \) compared to the input voltage \( E \). The state of the switch \( S \) (open or closed) represents the control input to the boost converter plant. When the switch is closed, current flows through the inductor and generates a magnetic field. When the switch is opened, the inductor magnetic field decays to maintain the current towards the load, causing a polarity reversal within the inductor. The primary voltage source in series with the inductor then charges the capacitor through the diode to a higher voltage than is attainable using the voltage source alone. If the switch is cycled fast enough, the inductor does not fully discharge between cycles and the load voltage remains higher than that of the source.

The boost converter dynamics may be expressed as a (continuous time) plant, \( \mathbb{H} \), with discrete-valued data denoting the position of the switch \( S \). We model it as \( \mathbb{H} \) in (1) but with no jumps. That is, \( \mathbb{H} \) with state \( x := (v_{c}, i_{L}) \), \( x \in \mathbb{X} \), \( \mathbb{M} \), \( \mathbb{F} \), \( \mathbb{C} \), and \( \mathbb{D} \) are given below, input \( q \in \{0, 1\} \), and output given by \( x \). Following Theunisse et al. (2015), its dynamics reduce to the differential inclusion with constraints

\[ \dot{x} \in F_{P}(x, q) \quad (x, q) \in C_{P} \]  

where \( C_{P} := (\mathbb{M} \cup \{0\}) \cup (\mathbb{M} \times \{1\}) \), \( \mathbb{M} \), \( \mathbb{F} \), \( \mathbb{C} \), and \( \mathbb{D} \) are given by

\[ \mathbb{M} = \{x \in \mathbb{R}^{2} : i_{L} \geq 0\} \cup \{x \in \mathbb{R}^{2} : v_{c} \leq 0\} \cup \{x \in \mathbb{R}^{2} : v_{c} \leq E, i_{L} = 0\} \]

\[ \mathbb{F} = \{x \in \mathbb{R}^{2} : v_{c} \geq 0\} \]

\[ \mathbb{C} = \{x \in \mathbb{R}^{2} : v_{c} \leq E, i_{L} = 0\} \]

\[ \text{(3) for details) } \]

\[ F_{P}(x, 0) := \begin{cases} \frac{-\pi}{2} v_{c} + \frac{1}{2} i_{L} & \text{if } x \in \mathbb{M} \setminus \mathbb{F} \\ \frac{-\pi}{2} v_{c} + \frac{1}{2} i_{L} & \text{if } x \in \mathbb{F} \]  

\[ \text{4. HYBRID PARAMETER ESTIMATION AND UNITING CONTROL} \]

4.1 Parameter estimation

We begin by studying Problem 1 from Section 3. For the purpose of estimating \( R \) and \( E \), we establish the
following lemma, which allows us to express the dynamics of solutions to $H_P$ in a convenient form.

**Lemma 4.1.** Each maximal solution $t \mapsto x(t)$ to $H_P$ in (2) with input $t \mapsto q(t)$ satisfies

$$
\dot{x}(t) = f_1(x(t), q(t)) + f_2(x(t), q(t))\vartheta
$$

for all $t \in \text{dom}(x, q)$, where $\vartheta = (\vartheta_1, \vartheta_2) := (R^{-1}, E)$ and

$$
f_1(x, q) := \begin{cases} 
\left( \begin{array}{c}
\frac{q}{\omega} \\
0
\end{array} \right) & \text{if } q = 0, \ x \in \mathbb{M}_1 \ \setminus \mathbb{M}_3 \\
0 & \text{if } (q = 1, \ x \in \mathbb{M}_1) \ \text{or} \ (q = 0, \ x \in \mathbb{M}_5)
\end{cases}
$$

$$
f_2(x, q) := \begin{cases} 
\left( \begin{array}{c}
\frac{-q}{\omega} \\
0 \\
0
\end{array} \right) & \text{if } (q = 0, \ x \in \mathbb{M}_1) \ \text{or} \ (q = 1, \ x \in \mathbb{M}_5) \\
\left( \begin{array}{c}
0 \\
\vartheta \\
0
\end{array} \right) & \text{if } q = 0, \ x \in \mathbb{M}_5.
\end{cases}
$$

Estimating the parameters $R$ and $E$ is equivalent to estimating the parameter vector $\vartheta$ in (4). For this purpose, we extend the finite-time parameter estimator proposed in [Hartman et al., 2012] to classes of hybrid systems whose solutions satisfy (4). The algorithm is expressed as a hybrid system, denoted $H_E$, and operates as follows. Let $(t, j) \mapsto z_E(t, j)$ be a solution to $H_E$, hence, defined on a hybrid time domain $t^0 := [t_0, t_1] \times \{0\}$ for the solution $z_E$ with constant $\vartheta$ and initial conditions $\omega(0, 0) = 0, Q(0, 0) = 0, \eta(0, 0) = 0, \Gamma(0, 0) = 0$, and $\dot{\vartheta}(0, 0)$ arbitrary. Omitting the $(t, j)$ for solutions, for the sake of making an argument, suppose that over this interval of flow, $Q$ and $\Gamma$ satisfy

$$
\dot{Q} = \omega^\top \omega, \quad \dot{\Gamma} = \omega^\top \omega \vartheta.
$$

(5)

Then, if there exists a positive time $t_1 \in t^0$ such that $Q(t_1, 0)$ is invertible, $\dot{\vartheta}$ can be reset to $Q^{-1} \Gamma$, leading to

$$
\dot{\vartheta}(t_1, 1) = Q^{-1}(t_1, 0) \Gamma(t_1, 0)
$$

$$
\left( \int_0^{t_1} \omega(t, 0) \omega(t, 0) \right) \int_0^{t_1} \omega(t, 0) \omega(t, 0) \vartheta \vartheta(t, 0) dt \vartheta = \vartheta.
$$

(6)

However, since $\vartheta$ is unknown prior to hybrid time $(t_1, 1)$, a trajectory for $\Gamma$ satisfying (5) cannot be generated. Due to this, we rewrite the dynamics of $\Gamma$ as

$$
\dot{\vartheta} = \omega \vartheta (\omega^\top x - \dot{x} - \eta),
$$

where $\eta = x - \dot{x} - \omega (\theta - \dot{\theta})$. Note that since $\omega(0, 0) = 0$, the initial condition $\eta(0, 0) = 0$ implies $\dot{x}(0, 0) = 0$. Differentiating $\eta$ yields

$$
\dot{\eta} = \dot{x} - \dot{x} - \omega (\theta - \dot{\theta}) + \omega \dot{\theta}
$$

(7)

Next, we define a matrix function $(x, q) \mapsto K(x, q) = R^\top(x, q) > 0$ to be designed. The arguments of $K$ are omitted below for simplicity. Then, let $\dot{x}, \omega$, and $\dot{\theta}$ satisfy

$$
\dot{x} = f_1(x, q) + f_2(x, q) \dot{\theta} + K(x - \dot{x}) + \omega \dot{\theta}
$$

$$
\dot{\omega} = f_2(x, q) - K \omega.
$$

(8)

Plugging the above expressions into (7) yields $\dot{\eta} = -K \eta$. Hence, $\omega, Q, \eta$, and $\Gamma$ are now expressed in terms of known quantities and we can compute $\dot{\vartheta}$ as in (6).

Following [Hartman et al., 2012], we implement the estimation scheme outlined above as a hybrid algorithm whose jump map imposes the initial conditions specified just above (5) and computes $\dot{\vartheta}$ as in (6). Then, the hybrid system $H_E = (C_E, F_E, D_E, G_E, \theta)$ has state $z_E := (x, \dot{x}, \omega, Q, \eta, \Gamma) \in X_E := \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$, inputs $(x, \omega, Q, \eta, \Gamma)$ \in $CP$, output $\dot{\vartheta} \in \mathbb{R}^2$, and dynamics

$$
\dot{z}_E = F_E(x, Q, \eta, \Gamma)
$$

where $G_E(x, Q, \eta, \Gamma) \in D_E$ and $\theta := (\vartheta_1, \vartheta_2) = (\dot{R}, \dot{E})$. Given $G_E(x, Q, \eta, \Gamma) := (x, Q^{-1} \Gamma, 0, 0, 0, 0)$,

$$
F_E(x, Q, \eta, \Gamma) := \begin{bmatrix}
f_1(x, Q, \eta, \Gamma) + K \omega + \omega \dot{\theta} \\
\omega \omega \dot{\theta}
\end{bmatrix}
$$

where $h(x, Q, \eta, \Gamma) = \Omega(t^0 + f_2(x, Q, \eta, \Gamma)) (x - \dot{x})$, and

$$
C_E := \{(x, Q, \eta, \Gamma) \in CP \times XC : \det(Q) \leq \mu \}
$$

$$
D_E := \{(x, Q, \eta, \Gamma) \in CP \times XC : \det(Q) \geq \mu \}.
$$

The matrix function $K$ and the parameter $\Omega = \Omega^> 0$ modify the convergence rate of $x$ and $\dot{\vartheta}$ during flows, and $\mu > 0$ ensures that $Q^{-1}$ is well-defined in the jump map.

The dynamics of $H_E$ in (8) are similar to the estimator proposed in [Hartman et al., 2012]. However, in [Hartman et al., 2012], $f_1$ and $f_2$ are continuous functions of the state and input, compared to piecewise continuous in (4).

Similarly to [Hartman et al., 2012], each maximal solution to $H_E$ is guaranteed to jump if the following holds.

**Assumption 4.2.** Given a compact set $\Lambda \subset \mathbb{R}^2 \times \{0, 1\}$, there exist $a, b > 0$ such that, for any maximal solution $t \mapsto x(t)$ to $H_P$ with input $t \mapsto q(t)$ satisfying $rge(x, q) \subset \Lambda$ and any $\tilde{t} > 0$ such that $[\tilde{t}, \tilde{t} + a] \subset \text{dom}(x, q)$,

$$
\int_\tilde{t}^{\tilde{t} + a} f_2(x(s), q(s)) \dot{f}_2(x(s), q(s)) ds \geq b I
$$

(9)

Next, we establish the following proposition, which states the stability properties of the interconnection of the plant $H_P$ and estimator $H_E$. The proof follows similarly to [Li and Sanfelice, 2019, Proposition 4.4].

**Proposition 4.3.** Consider the interconnection of $H_P$ in (2) and $H_E$ in (8) with $K(x, q) = k + k_1 f_2(x, q) \Omega f_2(x, q)$ where $k > \frac{1}{2} I$ and $\Omega = \Omega^> 0$, with input $t \mapsto q(t) \rightarrow q(t, j) \in \{0, 1\}$. Given a compact set $\Lambda \subset \mathbb{R}^2 \times \{0, 1\}$ satisfying Assumption 4.2, there exists $\mu > 0$ in (8) such that, for each maximal solution $\phi = (x, Q, \eta, \Gamma) \rightarrow q(t, j) \in \{0, 1\}$, the interconnection satisfying $rge(x, q) \subset \Lambda$, there exists a hybrid time $(\tilde{t}, \tilde{j}) \in dom \phi$ such that $\phi(\tilde{t}, \tilde{j}) \in \Lambda \times A_E$ with

$$
A_E := \{z_E \in XC : \dot{x} = x, \dot{\theta} = \theta, \eta = 0\}.
$$

4.2 Global control algorithm

Next we study Problem 2 from Section 3 in the context of the uniting control framework described therein, beginning with the global control algorithm. The hybrid control

\textsuperscript{1} Since $H_P$ in (2) is a continuous-time system, its solutions are parameterized using only $t$.

\textsuperscript{2} Since the interconnection of $H_P$ and $H_E$ is a hybrid system, the input and state of $H_P$ are now parameterized by $(t, j)$.}
algorithm proposed in Theunisse et al. (2015) represents an ideal candidate for the global controller. Given a desired output voltage \( v^*_c \), this algorithm renders the set
\[
\mathcal{A}_P := \{ x \in \mathbb{R}^2 : v_c = \hat{v}_c^* , i_L = \frac{v^*_c}{\pi L} \}
\]
globally asymptotically stable for the boost converter when the converter parameters \( c, L, R, E > 0 \) are known.

However, in contrast to Theunisse et al. (2015), the parameters \( R \) and \( E \) are unknown in this paper. Hence, we apply the certainty equivalence principle and substitute the parameter estimates \( \hat{R} \) and \( \hat{E} \) from \( \mathcal{H}_E \) in (8) for \( R \) and \( E \), respectively. Then, following the derivation in Theunisse et al. (2015), given a desired output voltage \( \hat{v}_c^* \), the set-point \( x^*(\hat{\theta}) := (v^*_c, \hat{i}_L^*) \) with \( \hat{i}_L^*: = \frac{v^*_c}{\pi \hat{L}} \) is stabilized using the control Lyapunov function \( V(x, \hat{\theta}) = (x - x^*(\hat{\theta}))^\top P(x - x^*(\hat{\theta})) \), where \( P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} > 0 \) with \( 0 < \frac{p_{11}}{p_{22}} \). We define a hybrid system \( \mathcal{H}_1 \) with state \( z_i := q \in X_i := \{0, 1\} \), inputs \( x \in \mathcal{X} \) and \( \hat{\theta} \in \mathbb{R}_{>0} \), and dynamics
\[
\begin{align*}
\dot{q} &= 0 =: F_1(z_1), \\
q^+ &= 1 - q =: G_1(z_1)
\end{align*}
\]
where \( k_1 \) represents the input of \( H_P \),
\[
C_1 := \{(x, z_1, \hat{\theta}) \in \mathcal{X}_P \times \mathcal{X}_1 \times \mathbb{R}_{>0} : \hat{\gamma}_0(x, \hat{\theta}) \leq \rho, q = 0 \}
\]
\[
\cup \{(x, z_1, \hat{\theta}) \in \mathcal{X}_P \times \mathcal{X}_1 \times \mathbb{R}_{>0} : \hat{\gamma}_1(x, \hat{\theta}) \leq \rho, q = 1 \}
\]
\[
D_1 := \{(x, z_1, \hat{\theta}) \in \mathcal{X}_P \times \mathcal{X}_1 \times \mathbb{R}_{>0} : \hat{\gamma}_0(x, \hat{\theta}) \geq \rho, q = 0 \}
\]
\[
\cup \{(x, z_1, \hat{\theta}) \in \mathcal{X}_P \times \mathcal{X}_1 \times \mathbb{R}_{>0} : \hat{\gamma}_1(x, \hat{\theta}) \geq \rho, q = 1 \}
\]
and \( \rho \in \mathbb{R} \) is a design parameter that, as in Theunisse et al. (2015), spatially regularizes the closed-loop global controller by modifying the separation between the functions \( \hat{\gamma}_0 \) and \( \hat{\gamma}_1 \) to avoid Zeno behavior. The functions \( \hat{\gamma}_0 \), \( \hat{\gamma}_1 \) given by
\[
\begin{align*}
\hat{\gamma}_0(x, \hat{\theta}) &= \gamma_0(x, \hat{\theta}) \quad \text{where} \quad \gamma_0(x, \hat{\theta}) := 2(a_0 (v_c - \hat{v}_c)^2 + b_0 x^2 + c_0 \hat{L} + d_0 x) \\
\hat{\gamma}_1(x, \hat{\theta}) &= \gamma_1(x, \hat{\theta}) \quad \text{where} \quad \gamma_1(x, \hat{\theta}) := 2(a_1 (v_c - \hat{v}_c)^2 + b_1 x^2 + c_0 \hat{L} + d_1 x)
\end{align*}
\]
where \( a_0 = -\frac{p_{11}}{\pi \hat{L}} \), \( a_1 = -\frac{p_{11}}{\pi \hat{L}} \), \( b_0 = \frac{p_{21}}{\pi \hat{L}} + \frac{p_{22} \hat{i}_L}{\pi \hat{L}} \), \( b_1 = -\frac{p_{21}}{\pi \hat{L}} \), \( c_0 = \frac{p_{22} \hat{i}_L}{\pi \hat{L}} \), \( c_1 = \frac{p_{22} \hat{i}_L}{\pi \hat{L}} \), \( d_0 = -\frac{p_{22} \hat{i}_L}{\pi \hat{L}} \), \( d_1 = -\frac{p_{22} \hat{i}_L}{\pi \hat{L}} \), and \( \rho_0 = k_0 \frac{p_{21}}{\pi \hat{L}}, \quad k_1 = \frac{p_{21}}{\pi \hat{L}} \) where \( k_0, k_1 \in (0, 1) \) are design parameters that ensure \( \mathcal{H}_1 \in (0, 2p_{11}/(\pi \hat{L})) \).

Given \( (t, j) \mapsto x(t, j) \) and \( (t, j) \mapsto \hat{\theta}(t, j) \), each solution \( (t, j) \mapsto q(t, j) \) to \( \mathcal{H}_1 \) maintains a constant switch state until \( x(t, j) \) intersects with the \( \rho \) level-set of \( \hat{\gamma}_0 \), at which point the value of \( q \) is toggled. Note that the jump set \( D_1 \) below (12) has been modified compared to the model in Theunisse et al. (2015). In particular, the conditions \( \hat{\gamma}_0(x, \hat{\theta}) = \rho \) and \( \hat{\gamma}_1(x, \hat{\theta}) = \rho \) in Theunisse et al. (2015) are instead \( \hat{\gamma}_0(x, \hat{\theta}) \geq \rho \) and \( \hat{\gamma}_1(x, \hat{\theta}) \geq \rho \), respectively. This change ensures completeness of maximal solutions for the closed-loop unifying control algorithm discussed in Section 4.4.

To ensure that Assumption 4.2 is satisfied for the closed-loop global controller, we define the set
\[
\Pi := \{ x \in \mathcal{X}_P : v_c > 0, i_L > 0 \}.
\]
yielding the closed-loop dynamics
\[
\dot{x} = A_{cl}(\hat{\theta})\hat{x}
\]  
(19)

where \( \hat{K} \) is chosen such that \( A_{cl}(\hat{\theta}) := A_{avg}(\hat{\theta}) - B_{avg}(\hat{\theta})\hat{K}(\hat{\theta}) \) is Hurwitz for each \( \hat{\theta} \). Then, the PWM duty cycle is computed as \( d(x, \hat{\theta}) := \psi(d^*(\hat{\theta}) - \hat{K}(\hat{\theta})\hat{x}) \), where \( \psi(s) := \min\{\max\{0, s\}, 1\} \) is a saturation function.

Then, we define the hybrid system \( \mathcal{H}_0 \) with state \( z_0 := \tau \in \mathcal{X}_0 := [0, 1] \), inputs \( x \in \mathcal{X}_P \) and \( \theta \in \mathbb{R}^2_+ \), and dynamics
\[
\begin{align*}
\dot{\tau} &= \frac{1}{\varepsilon}; F_0(z_0) \quad (x, z_0, \hat{\theta}) \in C_0 \\
\tau^+ &= 0; G_0(z_0) \quad (x, z_0, \hat{\theta}) \in D_0 \\
\kappa_0(x, z_0, \hat{\theta}) := \begin{cases} 
1 & \text{if } \tau < d(x, \hat{\theta}) \\
0 & \text{if } \tau > d(x, \hat{\theta})
\end{cases} (20)
\end{align*}
\]

where \( C_0 := \mathcal{X}_P \times [0, 1] \times \mathbb{R}^2_+ \) and \( D_0 := \mathcal{X}_P \times \{1\} \times \mathbb{R}^2_+ \).

Each solution \((t, j) \mapsto (\tau(t), j) \) to \( \mathcal{H}_0 \) represents a timer that counts continuously with a rate of \( 1/\varepsilon \) and resets to zero each time \( \tau = 1 \). The output \( \kappa_0 \) is a square wave representing the PWM signal that determines the converter switch state. The parameter \( \varepsilon > 0 \) represents the PWM period.

To ensure validity of the linearization in (17), and that the converter operates only in the continuous conduction mode under the local controller, we define the set \( \mathcal{X}_C := \mathcal{L} \cap \Pi \). Then, since the matrix \( A_{cl}(\hat{\theta}) \) in (19) is Hurwitz for each \( \hat{\theta} \), there exists an open set \( \mathcal{B}_{AP} \subset \mathcal{X}_C \) containing a neighborhood of \( \mathcal{A}_P \) that is forward invariant for (19).

Next, we establish the following proposition, which states the stability properties of the closed-loop local controller.

**Proposition 4.5.** Consider the interconnection of the plant \( \mathcal{H}_P \) in (2) with \( c, L, R, E > 0 \), local controller \( \mathcal{H}_0 \) in (20) with \( \varepsilon > 0 \), and parameter estimator \( \mathcal{H}_E \) with \( K(x, q) := k + \frac{1}{4}f_2(x, q)\Omega f_2(x, q) \), where \( k > \frac{1}{4}I \) and \( \Omega = \Omega^\top > 0 \). Given a desired set-point voltage \( \psi_0 \in E \) and a compact set \( \Delta \subset \Pi \times \mathcal{X}_E \times \mathcal{X}_P \), with \( \Pi \) given in (14), that is forward invariant for the interconnection, there exists \( \mu > 0 \) in (8) such that, for each maximal solution \( \phi = (x, z_0, \varphi, \phi) \) to the interconnection with \( \phi(0, 0) \in \Delta \), there exists a hybrid time \( (t^*, j^*) \in \Delta \) \phi(\varphi, \phi) \) in \( (x, z_0, \varphi, \phi) \) to the interconnection with \( \phi(0, 0) \in \mathcal{X}_E \times \mathcal{X}_P \). And, \( \mathcal{B}_{AP} \) such that, for each compact set \( \mathcal{T} \subset \mathcal{B}_{AP} \) and each \( \varepsilon > 0 \), there exists \( \varepsilon > 0 \) guaranteeing the following property: for each \( \varepsilon \in (0, \varepsilon^*) \) defining \( D_0 \) in (20), every solution \( \phi \) to the interconnection with \( \phi(0) \in \mathcal{X}_E \times \mathcal{X}_P \) is such that, for all \( (t, j) \in \mathcal{T} \), \( \phi(\varphi, \phi) \) is a solution satisfies
\[
|x(t, j)|_{\mathcal{B}_{AP}} \leq \beta(|x(0, 0)|_{\mathcal{B}_{AP}}, t + j) + \nu
\]  
(21)

**Sketch of Proof:** Assumption 4.2 holds for every maximal solution with \( \phi(0, 0) \in \Delta \). Then, from Proposition 4.3 we have \( \hat{\theta} = \theta \) in finite time, and the stability result follows from (Teel and Nesic, 2010, Theorem 2).

4.4 Unitifying control algorithm

To implement the uniting control framework, the supervisor logic outlined in Section 3 is applied to the interconnection of the boost converter plant \( \mathcal{H}_P \) using the global and local control algorithms \( \mathcal{H}_1 \) and \( \mathcal{H}_0 \), respectively. Recall that \( z_0 \) is the state of \( \mathcal{H}_0 \), \( z_1 \) is the state of \( \mathcal{H}_1 \), and the output \( \kappa \) of the selected controller is mapped to the input \( q \) of \( \mathcal{H}_P \). Then, we define the hybrid system \( \mathcal{H} \) with state \( \xi = (x, z_0, z_1, p) \in X := \mathcal{X}_P \times \mathcal{X}_0 \times \mathcal{X}_1 \times \{0, 1\} \), input \( \hat{\theta} \in \mathbb{R}^2_+ \), and dynamics
\[
\begin{align*}
\dot{\xi} &\in F(\xi, \hat{\theta}) \quad (\xi, \hat{\theta}) \in C \\
\xi^+ &\in G(\xi) \quad (\xi, \hat{\theta}) \in D.
\end{align*}
\]  
(22)

The logic variable \( p \in \{0, 1\} \) is set to 0 when the global controller is selected and to 1 when the local controller is selected. The flow map \( F \) is equal to \( (F_p, F_0, 0, 0) \) when \( p = 0 \) and to \( (F_p, 0, F_1, 0) \) when \( p = 1 \). It is written concisely as
\[
F(\xi, \hat{\theta}) := \begin{cases} 
F_p(\xi, \kappa_p(x, z_0, \hat{\theta})) & (1 - p) F_0(z_0) \\
p F_1(z_1) & 0
\end{cases}.
\]

The flow set \( C \) is given by
\[
C := \{(\xi, \hat{\theta}) \in \mathcal{X} \times \mathbb{R}^2_+ : (x, \kappa_p(x, z_0, \hat{\theta})) \in C_p, (x, z_0, \hat{\theta}) \in C_0, (x, z_1, \hat{\theta}) \in C_1, (x, p) \in (\mathcal{M} \times \{0\}) \cup (\mathbb{R}^2 \setminus \mathcal{N} \times \{1\}) \}
\]
where the sets \( \mathcal{N} \) and \( \mathcal{M} \) are to be designed.

The jump map \( G \) permits jumps by \( G_0 \) when \( p = 0 \) and \( G_1 \) when \( p = 1 \), and toggles the value of \( p \) based on the converter state \( x \) in relation to the sets \( \mathcal{N} \) and \( \mathcal{M} \). This is expressed as
\[
\begin{align*}
G_0(\xi) := \{G^0(\xi), G^2(\xi)\} & \quad \xi \in D^0 \cap D^2 \\
G^1(\xi) & \quad \xi \in D^1 \cap D^2 \\
G^2(\xi) & \quad \xi \in D^2 \cap (D^0 \cup D^1)
\end{align*}
\]

Next, we will design the uniting control sets \( \mathcal{N} \) and \( \mathcal{M} \).

5. UNITING CONTROL SETS AND MAIN RESULT

Any sets \( \mathcal{N} \) and \( \mathcal{M} \) that satisfy Assumption 3.1 are acceptable for the uniting control framework in (22). We provide one example of how these sets can be designed for the boost converter. We define the closed set \( \mathcal{N} \) as a ball given by
\[
\mathcal{N} := x^*(\hat{\theta}) + r_N \mathcal{B}
\]  
(23)

where \( r_N \in \mathbb{R}^2 \) is chosen such that \( \mathcal{N} \subset \mathcal{B}_{AP} \). Then, we choose \( p \) in (12) such that each maximal solution to the closed-loop global controller converges to \( \mathcal{N} \). The choice of a ball for \( \mathcal{N} \) is arbitrary and was done for simplicity.

4 The jump maps associated with the sets \( D^0 \cap D^2 \) and \( D^1 \cap D^2 \) are necessary to satisfy outer semicontinuity of \( G \) in (Goebel et al., 2012, Assumption 6.5).
The reachable set from $\mathcal{N}$ may be computed, for example, via Poisson analysis as in [Almer et al., 2007]. However, since this technique is computationally intensive for real-time implementation, we approximate $\mathcal{M}$ using the linearized model (19). A rigorous analysis of this approximation is beyond the scope of the paper. Using the Lyapunov function $\hat{V}(\bar{x}) := \hat{x}^T P \hat{x}$, where $P = P^T > 0$ solves $A^T_\mathcal{F}(\hat{\theta}) P + PA_\mathcal{F}(\hat{\theta}) = -Q$ and $Q = Q^T > 0$, we choose a parameter $r_0 \in \mathbb{R}_{>0}$ such that $L^*_\mathcal{F}(r_0) \supset \mathcal{N}$. Then, solutions to (19) from $\mathcal{N}$ remain inside $\mathcal{M} := L^*_\mathcal{F}(r_0)$.

To bound the trajectories of the closed-loop local controller, points on the boundary of $\mathcal{M}$ are parameterized in a grid such that the variation in the vector field $F_P$ between adjacent points is small. Since the converter switch remains in one state for at most $\varepsilon$ seconds during each PWM period, we compute the finite-time reachable set from each point on the boundary of $\mathcal{M}$ by integrating $F_P$ for $\varepsilon$ seconds for each $q \in \{0, 1\}$. Then, $\mathcal{M}$ is defined as

$$\mathcal{M} := \text{int}(L^*_\mathcal{F}(r_\mathcal{M}))$$

where $r_\mathcal{M} \in \mathbb{R}_{>0}$ is chosen such that $\mathcal{M}$ bounds the set reachable in $\varepsilon$ seconds from $\mathcal{M}$ for each switch state $q \in \{0, 1\}$. Finally, we choose the matrix function $K$ in (18) and the parameter $\varepsilon > 0$ in (20) so that $\mathcal{M} \subset X_2$. Next, we establish our main result, which states the stability properties of the closed-loop uniting controller.

**Theorem 5.1.** Consider the interconnection of the hybrid system $\mathcal{H}$ in (22) with $c, L, R, E > 0$, $k_0, k_1 \in (0, 1)$, $\rho > 0$, $\varepsilon > 0$, and parameter estimator $\hat{\theta}_E$ in (8) with $K(x, q) = k + \frac{1}{2} f_2(x, q) f_2^T(x, q)$, where $k > \frac{1}{2} I$ and $\Omega = \Omega^T > 0$. Given a desired set-point voltage $v^*_\mathcal{P}$, uniting control sets $\mathcal{N}$ and $\mathcal{M}$ satisfying Assumption 3.1, and a compact set $\Delta \subset X_0 \times X_1 \times \{0, 1\} \times X_2$, with $\Pi$ given in (14), that is forward invariant for the interconnection, there exists $\mu > 0$ in (8) such that, for each maximal solution $\phi = (x, z_0, z_1, p, z_2)$ to the interconnection with $\phi(0, 0) \in \Delta$, there exists a hybrid time $(t', j') \in dom \phi$ such that $\phi(t', j') \in \Pi \times X_0 \times X_1 \times \{0, 1\} \times X_2$, with $\hat{A}_E$ given in (10). Furthermore, there exists $\beta \in K\mathcal{C}$ such that, for each compact set $\Upsilon \subset \mathbb{R}^2$ and each $n > 0$, there exist $\rho^*, \varepsilon^* > 0$ guaranteeing the following property: for each $n$ in $(\rho, \rho^*)$ defining $C_1$ and $D_1$ in (12) and each $n$ in $(0, \varepsilon^*)$ defining $F_0$ in (20), every solution $\phi$ to the interconnection with $\phi(0, 0) \in \Upsilon \times X_0 \times X_1 \times \{0, 1\} \times X_2$ is such that, for all $(t, j) \in dom \phi$, its $x$ component satisfies

$$|x(t, j)|_{A_P} \leq \beta(|x(0, 0)|_{A_P}, t + j) + \nu$$

and such solutions exhibit no more than two toggles in the value of the solution component $p$.

**Sketch of Proof:** The stability result follows from Propositions 4.4 and 4.5. The value of $p$ can be shown to toggle at most twice by analysis of the trajectories from the sets $\mathcal{N}$ and $\mathcal{M} \setminus \mathcal{N}$ (see Sanfelice [2021, Theorem 4.6]).

### 6. SIMULATION RESULTS

In this section, we present simulation results for the interconnection of $\mathcal{H}$ and $\mathcal{H}_E$. Simulations are performed using the Hybrid Equations Toolbox [Sanfelice et al., 2013] with $c = 0.1F$, $L = 0.2H$, $P = \begin{bmatrix} 0 & \varepsilon^2/2 \\ \varepsilon^2/2 & 0 \end{bmatrix}$, $\varepsilon = 0.0001$, $\rho = 0.001$, $\mu = 0.001$, and $A_P = (7, 3.27)$. The set $\mathcal{N}$ in (23) is defined with $r_\mathcal{N} = 0.05v^*_\mathcal{P}$, and a grid of 10 points is used to compute $\mathcal{M}$ in (24) from $\mathcal{M}$. Initial conditions are

$$x_0 = (3, 6), E_0 = 6, R_0 = 3.6, \dot{R}_0 = 0, \dot{E}_0 = 5,$$

as shown in Figure 1. Both parameter estimates converge at 0.5 seconds. They converge again when $E$ changes at $t = 3$ and when $R$ changes at $t = 5$. The plant state converges to a neighborhood of $A_F$ following each convergence of the parameter estimate to the true value.

![Fig. 1. Left: trajectories under $\mathcal{H}_1$ shown in blue and $\mathcal{H}_0$ in magenta. Right: parameter estimates $\hat{R}$ and $\hat{E}$.

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[5] Code at [https://github.com/HybridSystemsLab/UnitingBoost](https://github.com/HybridSystemsLab/UnitingBoost)