The longitudinal spin relaxation of 2d electrons in Si/SiGe quantum wells in a magnetic field

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The longitudinal spin relaxation time, $T_1$, in a Si/SiGe quantum well is determined from the saturation of the ESR signal. We find values of a few microseconds. Investigations of $T_1$ as a function of Fermi energy, concentration of scattering centers and of the momentum scattering time, $\tau_k$, lead to the conclusion that for high electron mobility the spin relaxation is caused by the Dyakonov-Perel (DP) mechanism while for low mobility the Elliott-Yaffet mechanism dominates. The DP relaxation is caused by Bychkov-Rashba coupling. Evaluation of the DP mechanism shows that $T_1^{-1}$ for high electron mobility can be effectively reduced by an external magnetic field. The effect of the degenerate Fermi-Dirac statistics on the DP process is discussed.

I. INTRODUCTION

Looking for spin systems suitable for spintronics or quantum computing devices, the longitudinal spin relaxation time, $T_1$, is of basic importance. $T_1$ is ruled by spin-flip processes and it corresponds to the characteristic spin memory time. In this paper we investigate $T_1$ in the high mobility 2d electron gas in a Si/SiGe quantum well, where electrons can be easily manipulated by illumination with light and by an electric field.\textsuperscript{[1,2]} We show that for this material system, which magnetically is one of the cleanest, $T_1$ is of the order of microseconds whereas the time needed for a spin manipulation by a microwave magnetic field is by more than two orders of magnitude shorter.

We also investigate the mechanism for spin relaxation. Analyzing the spin relaxation rate as a function of the momentum scattering rate, $\tau_k^{-1}$, allows to distinguish the Elliott-Yaffet (EY) mechanism\textsuperscript{[3,4]} and the D’yakonov-Perel (DP) mechanism\textsuperscript{[5,6]}. The EY mechanism describes the probability of a spin-flip in a momentum scattering event. This probability is ruled by spin-orbit coupling and the resulting admixture of a state with opposite spin projection\textsuperscript{[7,8]}:

\begin{equation}
(T_1^{-1})_{EY} = \alpha_{EY} \tau_k^{-1}
\end{equation}

The DP relaxation, in contrast, originates from a zero field spin splitting of the conduction band states\textsuperscript{[9,10,11,12]}. For a Si quantum well the zero field splitting is described by the Bychkov-Rashba (BR) term\textsuperscript{[10,11,12]}:

$$\mathcal{H}_{BR} = \alpha_{BR} (\mathbf{k} \times \sigma) \cdot \hat{e}_z$$

which was shown to exist also for single sided modulation doped Si quantum wells\textsuperscript{[11,12]}.

Here $\sigma$ stands for the vector of a Pauli spin-matrix of a conduction electron\textsuperscript{[10,11,12]}, $\mathbf{k}$ is the k-vector proportional to the electron momentum, $\hat{e}_z$ is a unit vector perpendicular to the 2d layer and $\alpha_{BR}$ is the Rashba parameter that depends on the spin-orbit coupling and details of the interface\textsuperscript{[10,12]}. Momentum scattering causes also a time dependent modulation of the BR interaction. As a consequence, the probability for spin-flips becomes finite. For non-quantizing magnetic field, the DP mechanism\textsuperscript{[10,12]} is expected to be proportional to the momentum scattering time:

$$\langle T_1^{-1} \rangle_{DP} = \Omega_{BR}^2 \alpha_{BR} \tau_k$$

where the frequency $\Omega_{BR}$ is proportional to the k-vector and the BR parameter $\alpha_{BR}$:

$$\Omega_{BR}^2 = \alpha_{BR} k / 2 \hbar$$

In this paper we present results obtained from conduction electron spin resonance (CESR) spectroscopy. Simultaneous measurements of CESR, which allows to evaluate $T_1$, and cyclotron resonance (CR), which allows to estimate $\tau_k$, permit the evaluation of $T_1$ as a function of $\tau_k$. Such data are obtained from samples with different donor and electron concentrations.

II. SAMPLES AND EXPERIMENTAL RESULTS

Samples were grown by molecular beam epitaxy on 1000 $\Omega$cm Si(001) substrates, which show complete carrier freeze-out below 30 K. A 20 nm thick Si channel with tensile in-plane strain was deposited on a strain-relaxed Si$_{0.75}$Ge$_{0.25}$ buffer layer, which consists of a 0.5 $\mu$m thick Si$_{0.75}$Ge$_{0.25}$ layer on top of a 2 $\mu$m thick Si$_{1-x}$Ge$_x$ layer with compositional grading. The upper Si$_{0.75}$Ge$_{0.25}$ barrier was modulation doped with a 12.5 nm thick, nominally undoped spacer layer, and capped with 5 nm of Si. Three modulation doped Si/SiGe structures with different donor concentrations were examined. The electron concentration was changed by the light illumination.

All measurements were performed with a the standard X-band ESR spectrometer, at a microwave frequency 9.4 GHz. The sample were situated in the center of the rectangular TM$_{001}$ cavity, at the maximum of the magnetic component of microwave field (which is perpendicular to
the applied magnetic field). The sample layer was oriented to be perpendicular to the applied magnetic field (and to electric component of microwave field).

The spin resonance has an exceedingly narrow linewidth in the range 3 ± 10 μT. In spite of the fact that the sample was situated in the minimum of the electric microwave field and perpendicular to it, the strong absorption due to CR was well observed allowing for to monitor the carrier density from the integral absorption and the momentum scattering rate from the CR linewid th 2.

In Fig. 1 the longitudinal spin relaxation time, $T_1$, and the momentum relaxation rate, $\tau_k^{-1}$, are plotted as a function of the electron concentration, $n_s$. The parameter $T_1$ has been evaluated from the saturation of the ESR signal amplitude and the ESR line broadening at high microwave power 11, 12. Estimating the quality factor of the loaded cavity from the resonance dip width, we obtain the amplitude of the magnetic component of the microwave field of 1.1 G at a microwave power of 200 mW. The data for different samples are marked by different symbols. The results for the spin relaxation time, $T_1$, vary in the range of 1 to 5 μs. For different samples $T_1$ is different and it depends on the electron concentration. The momentum scattering rate varies with $n_s$ by an order of magnitude. The increase of the momentum scattering is related to the screening breakdown and an increase of the potential fluctuations at low Fermi energy 8. Samples with a higher doping level show also a higher $\tau_k^{-1}$.

The dependence of $T_1$ on $n_s$ is governed by the complex dependence of the relaxation rate on the Fermi k-vector and of the dependence of $\tau_k^{-1}$ on the electron concentration. In order to follow the dependence of the spin relaxation rate, $T_1^{-1}$, on the momentum relaxation, $\tau_k^{-1}$, our data are plotted in Figs. 2a and 2b in two different ways. In Fig. 2a the spin relaxation rate, $T_1^{-1}$, is given as a function of the momentum scattering rate, $\tau_k^{-1}$. In Fig. 2b, the spin relaxation rate, $T_1^{-1}$, is normalized by the electron concentration, $n_s$. This normalization allows to account for the dependence of $T_1^{-1}$ on the BR parameter, $\alpha BR$, and to study the dependence of the DP rate $T_1^{-1}$ on $\tau_k^{-1}$.
III. THE SPIN RELAXATION CAUSED BY THE ELLIOTT-YAFET MECHANISM

Comparison of the data in the two figures demonstrates the existence of two different ranges with different spin relaxation behavior. For high scattering rate, \( \tau_k^{-1} > 3 \cdot 10^{11} \, \text{s}^{-1} \), the spin relaxation is simply proportional to the momentum scattering rate indicating the EY process as the dominating one. The EY coefficient is independent both of the electron concentration and the doping level within the experimental error. The solid line in Fig. 2a corresponds to \( \alpha_{\text{EY}} = 1.0 \cdot 10^{-6} \).

For low momentum scattering rate, \( \tau_k^{-1} < 3 \cdot 10^{11} \, \text{s}^{-1} \), the spin relaxation rate is bigger than expected from the EY mechanism. Moreover, \( T_1^{-1} \) depends on the electron concentration in the high electron mobility range. On the other hand, the normalized spin relaxation (see Fig. 2b) is characterized by a systematic dependence, common for all investigated samples. As we argue below the observed dependence in the high mobility range is well described by the DP mechanism.

IV. THE D’YAKONOV-PEREL MECHANISM OF THE RELAXATION

The prediction of the DP scattering rate, as described by Eqs. (3) and (4) is marked by the dashed line in Fig. 2a. For the BR parameter we took the value \( \alpha_{\text{BR}} = 0.55 \cdot 10^{-12} \, \text{eV cm} \) evaluated earlier from the analysis of the linewidth and g-factor anisotropy in the same samples. No correlation between the dashed line and the experimental data is recognizable. Eq. (3) stands, however, for the case of a weak external magnetic field, when the momentum scattering rate is much smaller as compared to the Zeeman frequency, and it does not consider cyclotron motion and Landau quantization. In an external magnetic field, because of the cyclotron motion, the electron velocity changes its direction all the time. The time correlation function of the k-vector, and consequently the correlation function of the effective BR field seen by an electron, is described by:

\[ \langle k \cdot k(t) \rangle \propto (\Omega_{\text{BR}} \cdot \Omega_{\text{BR}}(t)) = \Omega_{\text{BR}}^2 e^{i \omega_t - \tau_k^{-1} t} \]  

The corresponding probabilities of spin-up and down flips are obtained by the Fourier component of the correlation function (Eq.(5)) at the Zeeman frequency \( \omega_z \):

\[ W_\pm = \frac{\Omega_{\text{BR}}^2}{2} \frac{\tau_k}{1 + (\omega_z \pm \omega_o)^2 \tau_k^2} \]  

The longitudinal spin relaxation time for a single electron is equal to:

\[ (T_1^{-1})_{\text{DP}} = W_+ + W_- \]  

Eqs. (6) and (7) describe the DP relaxation in an external magnetic field. For a short momentum relaxation time, Eq.(7) becomes equivalent to Eq.(3). But for quantizing magnetic field, where \( \omega_z \tau_k > 1 \), the DP relaxation rate is expected to be reduced by the denominator in Eq. (6). The dependence corresponding to Eqs. (6-7) is shown in Fig. 2 by the dash-dotted line. The reduction of the spin relaxation caused by the external magnetic field is well visible. Moreover, for low scattering rate, \( \omega_z \tau_k \gg 1 \), the DP relaxation rate is expected to be proportional to the momentum scattering.

For an electron gas, in which the final states to which electron can be scattered, are partially occupied the evaluation of the mean spin relaxation rate of the whole electron system requires thermodynamic averaging. The scattering probability and the momentum relaxation rate depend on energy. These quantities are proportional to the population of empty states: \( \tau_k^{-1}(\varepsilon) = \tau_{k \omega}^{-1}[1 - f_{\text{FD}}(\varepsilon)] \) where \( \tau_{k \omega}^{-1} \) is the momentum relaxation rate (as used in the Boltzmann equation approach) and \( f_{\text{FD}}(\varepsilon) \) is the Fermi-Dirac distribution function. For moderate magnetic field, where \( h \omega_c < k_B T < E_F \) the dependence of the BR frequency on energy can be neglected and the mean value of the transition probability, \( \langle W_\pm \rangle \), weighted by the derivative of the Fermi-Dirac distribution function, is described by:

\[ \langle W_\pm \rangle = \frac{\Omega_{\text{BR}}^2}{2} \int f_{\text{FD}}(\varepsilon) \frac{\tau_k(\varepsilon)}{1 + (\omega_c \pm \omega_o)^2 \tau_k^2(\varepsilon)} \, d\varepsilon \]  

The solid line in Fig. 2b corresponds to the DP scattering rate as described by Eqs. (7) and (8). For the BR parameter again a value of \( 0.55 \cdot 10^{-12} \, \text{eV cm} \) has been taken. The theoretical curve, without any other fitting parameter, fits well to the experimental points for high electron mobility. For the highest mobility the effect of a moderate magnetic field (B=0.34 T) is the reduction of the DP mechanism by about two orders of magnitude. In the limit of very high electron mobility, the DP relaxation rate for degenerate statistics (Eq. (7-8)) is by a factor 2 smaller as compared to the non-degenerate case described by Eqs. (6-7)

For the high momentum scattering rate the DP relaxation rate is expected to tend to the solution for weak magnetic field. But for degenerate statistics (solid line), where the final states are partially occupied, the spin relaxation rate is by a factor 2 bigger as compared to the non-degenerate case (dashed and dash-dotted lines) for a given momentum relaxation rate.

V. CONCLUSIONS

In conclusion, we have shown that:

- the DP mechanism dominates for high mobility structures but the quantization due to the applied magnetic field leads to a considerable reduction of the DP relaxation rate. In Eq. (3) a reduction factor of about \( 1 + \frac{\tau_k^2 \omega_c^2}{2} \) must be introduced, where \( \omega_c \) is the cyclotron frequency (compare Eq. (3) and
Eqs. (6-7)). As a consequence, for weak momentum scattering the reduced DP spin relaxation rate is proportional to $\tau_k^{-1}$, in contrast to Eq. (3).

- the value of the BR parameter, $\alpha_{BR}$, as determined from $T_1$ turns out to be the same within the experimental accuracy as previously evaluated from the anisotropy of the CESR linewidth (dephasing time, $T_2$) and the g-factor [8].

- for low mobility samples the spin relaxation is dominated by the EY mechanism. We find an EY coefficient of: $\alpha_{EY} = 1.0 \cdot 10^{-6}$, which is common for all samples and, for the investigated range of parameters (the Fermi energy does not exceed 2.5 meV), $\alpha_{EY}$ does not depend on the electron concentration.

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