Testing General Relativity with the Doppler magnification effect

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ABSTRACT
The apparent sizes and brightnesses of galaxies are correlated in a dipolar pattern around matter overdensities in redshift space, appearing larger on their near side and smaller on their far side. The opposite effect occurs for galaxies around an underdense region. These patterns of apparent magnification induce dipole and higher multipole terms in the cross-correlation of galaxy number density fluctuations with galaxy size/brightness (which is sensitive to the convergence field). This provides a means of directly measuring peculiar velocity statistics at low and intermediate redshift, with several advantages for performing cosmological tests of GR. In particular, it does not depend on empirically-calibrated scaling relations like the Tully-Fisher and Fundamental Plane methods. We show that the next generation of spectroscopic galaxy redshift surveys will be able to measure the Doppler magnification effect with sufficient signal-to-noise to test GR on large scales. We illustrate this with forecasts for the constraints that can be achieved on parametrised deviations from GR for forthcoming low-redshift galaxy surveys with DESI and SKA2.

Key words: Large-scale structure of Universe – techniques: radial velocities

1 INTRODUCTION
Until recently, the application of General Relativity (GR) to cosmology has represented a tremendous extrapolation of the theory to distance scales far exceeding those over which it has been subjected to precision tests. The most stringent tests of GR remain those involving experiments in the Solar System (e.g. with the Cassini probe, lunar laser ranging, and Earth-orbit frame dragging and equivalence principle experiments; Bertotti et al. 2003; Williams et al. 2004; Will 2006; Everitt et al. 2011; Touboul et al. 2017), and observations of binary pulsar systems (Taylor & Weisberg 1982; Taylor et al. 1992; Esposito-Farese 1996; Weisberg & Taylor 2005; Kramer et al. 2006; Wex 2014), all covering distances substantially less than a parsec.

A host of new precision tests are starting to become feasible that can greatly extend the range over which GR has been validated, however (Damour & Taylor 1992; Baker et al. 2015; Berti et al. 2015; Sakstein 2018). A notable example is the recent detection of gravitational waves from binary black hole and neutron star coalescences by the LIGO and VIRGO detectors (Abbott et al. 2016a, 2017b). All of the events that have been observed so far appear to be consistent with GR (Abbott et al. 2017a), thus extending a subset of precision tests out to a comoving distance of $\sim 800$ Mpc ($z \approx 0.2$) for the most distant event seen so far (Abbott et al. 2017a). This is sufficient to extend the envelope of precision tests out into the Hubble flow, beyond the gravitational environment dominated by our local cluster.

On larger scales and at higher redshifts, an extremely wide variety of tests have been proposed, involving such diverse objects and observables as galaxy clusters and their mass function; supermassive black holes embedded in galaxies; weak lensing distortions of galaxies, clusters, and the CMB; and the redshift-space clustering of galaxies, including relativistic effects (Bonvin & Fleury 2018); see Berti et al. (2015) for a review. These cover a broad range of distance scales and gravitational environments, and can be quite precise in some instances (e.g. for Chameleon mod-
A number of technical issues stand in the way of attaining the comprehensive, high-precision constraints that have been achieved in the Solar System however. First, most of these tests depend on accurately modelling complex astrophysical phenomena, which introduces significant systematic uncertainties. Second, astronomical measurements are inherently noisier, and require considerably more data to reach similar levels of precision to Solar System tests. Finally, possible deviations from GR are more diverse and harder to parametrise in the cosmological regime (c.f. the Parametrized Post-Newtonian framework on Solar System scales; Will 2011), leaving many tests effectively model-dependent.

To overcome these difficulties, cosmological tests of GR are needed that are sufficiently sensitive and general while being less susceptible to astrophysical systematics. A particularly promising class of observables involve direct measurements of the peculiar velocity field (e.g. Koyama et al. 2009; Hudson & Turnbull 2012; Hellwing et al. 2014; Mueller et al. 2015; Gronke et al. 2015; Johnson et al. 2016; Ivarsen et al. 2016). Velocities are a sensitive probe of gravitational physics, as they respond to changes in the effective strength of gravity over long periods of time. Since most non-GR theories are expected to modify the growth rate of structure, they should therefore leave an imprint in the cosmic peculiar velocity distribution. The equivalence principle implies that freely-falling galaxies should all respond to gravitational potentials in the same way, regardless of their mass or type, so galaxy peculiar velocities should also be unbiased with respect to the underlying dark matter distribution (at least to linear order on large scales; see Zheng et al. 2015; Desjacques et al. 2016). This means that velocities do not depend on tracer-dependent bias terms, which are an important source of uncertainty for other galaxy clustering observables – particularly as they can be degenerate with signatures of modified gravity (e.g. Baldi et al. 2014; Barreira et al. 2014). Combinations of observables that have similar bias-independent properties can be constructed in principle, such as the $E_G$ statistic that combines galaxy density and lensing measurements (Zhang et al. 2007; Reyes et al. 2010), but tracer-dependent quantities tend to enter the resulting quantity in practice (Leonard et al. 2015; Moradinezhad Dizgah & Durrer 2016).

While this makes direct velocity-based observables cleaner in principle, most practical methods of measurement reintroduce dependences on hard-to-model astrophysical phenomena. The Tully-Fisher method (Tully & Fisher 1977) commonly used at low redshifts relies on an empirically-calibrated scaling relation between the luminosity and circular velocity of a galaxy, for example. Similarly, the kinetic Sunyaev-Zel’dovich effect can be used to measure the velocities of galaxy clusters at higher redshifts, but only in a way that is degenerate with the integrated optical depth of the cluster, which must be modelled based on other measurements (Bhattacharya & Kosowsky 2008; Mueller et al. 2015; Alonso et al. 2016; Battaglia 2016). Nevertheless, these methods have all been successfully used to make cosmological measurements in the past, including some that even appear to show deviations from $\Lambda$CDM+GR (e.g. Kashlinsky et al. 2009; Watkins et al. 2009; Macaulay et al. 2011). Concerns about astrophysical systematics have contributed to scepticism of these anomalous results, however, which remain contentious.

In this paper, we describe how tests of GR can be performed using a different direct peculiar velocity observable called Doppler magnification (Bonvin 2008; Bacon et al. 2014; Bonvin et al. 2017). The Doppler magnification is an effect on the apparent sizes of objects in redshift-space: galaxies with a component of their peculiar velocity directed away from us appear to be closer than they actually are (due to a Doppler shift), while their surface brightness remains unchanged. This causes them to look slightly bigger than average for galaxies at that observed redshift, and is therefore akin to an apparent magnification. By cross-correlating size estimates for a set of galaxies with the galaxy density field, it is possible to isolate multipoles of the resulting correlation function that measures the density-velocity and velocity-velocity power spectra (Bonvin et al. 2017), which can then be compared to the GR predictions.

This is similar in spirit to methods like the Tully-Fisher effect, except we use a ‘statistical ruler’ (galaxy sizes) rather than standard candles (see Kaiser & Hudson (2015) for a discussion). The need to model the properties of the target galaxy population is also much reduced compared to these methods. There is no need to separately calibrate an empirical scaling relation for example, as the mean galaxy size as a function of redshift can be measured from the survey itself. Also, the galaxy bias does not enter into the velocity-velocity term, which can be measured directly from the octupole of the correlation function. Measurement systematics do of course remain (the size estimates can depend on sensor characteristics, for example), but can at least be mitigated through experimental design or high-fidelity instrumental simulations and calibration strategies. This goes some way to achieving what the $E_G$ statistic originally sets out to do – removing the bias dependence whilst also being sensitive to modified gravitational physics. Finally we note that Doppler magnification differs from redshift-space distortions (RSDs), since it is due to the effect of peculiar velocities on the size of galaxies, rather than on their distribution. As a consequence, Doppler magnification is sensitive to the velocity field itself, whereas RSDs probe the gradient of this field. The sensitivity of these two probes to modified gravity models is therefore expected to differ, especially in models of gravity where the growth of structure depends on scale.

The paper is organised as follows. In Section 2, we review the Doppler magnification effect and how it can be extracted from the cross-correlation of the galaxy density and convergence fields, and show how the relevant quantities are affected by deviations from GR. We present forecasts for the detectability of deviations from GR with DESI and SKA HI spectroscopic galaxy surveys in Section 3, and then conclude in Section 4.

We use a flat $\Lambda$CDM+GR cosmology with cosmological parameters $\Omega_\text{CDM} = 0.048$, $\Omega_\text{b} = 0.02548$, $\Omega_\Lambda = 0.96$, and $\sigma_8 = 0.83$ as the fiducial model.

## 2 Doppler Magnification in Modified Gravity

In this section we briefly review the Doppler magnification effect, and a method to extract it from the dipole of the number count-convergence correlation function. We then discuss how modifications to GR affect the Doppler dipole, and study some illustrative examples of modified gravity theories, and their effects on the number count-convergence correlation.
2.1 Number count-convergence correlation

The Doppler magnification effect is observed in the cross-correlation of the galaxy number count fluctuation, $\Delta$, and a suitable proxy for the convergence field, $\kappa$. We begin by calculating the cross-correlation as a function of redshift and angle,

$$\xi^{\Delta\kappa} = \langle \Delta(z,n) \kappa(z',n') \rangle,$$

(1)

where $n$ denotes the direction of observation. This quantity can be expanded in a hierarchy of multipoles around an observed overdensity, with various correlations between the density, velocity, and lensing terms contributing differently to each multipole as a function of redshift.

At linear order, the galaxy number count fluctuation is given by

$$\Delta(z,n) = b \delta - \frac{1}{2H} \partial_r (V \cdot n),$$

(2)

where $b$ is the local bias, $H \equiv aH$ is the conformal Hubble rate, $r$ is the comoving distance of the galaxy and $V \cdot n$ is the line-of-sight peculiar velocity of the galaxy. The second term is the redshift-space distortion. We have neglected here the lensing and relativistic corrections to $\Delta$ (Yoo et al. 2009; Bonvin & Durrer 2011; Challinor & Lewis 2011) since their contribution to the cross-correlation is subdominant at low redshift and on sub-horizon scales.\(^1\)

The convergence contains two dominant contributions: the standard weak lensing convergence, $\kappa_l$, and a Doppler contribution (Bonvin 2008; Bacon et al. 2014)

$$\kappa_v = \left( \frac{1}{rH} - 1 \right) V \cdot n.$$  

(3)

The Doppler term dominates at low redshift $z \lesssim 0.5$, whereas the lensing term dominates a high redshift.

As shown in Bonvin et al. (2017), the correlation of $\kappa_v$ with $\Delta$ has a distinctive dipolar structure. For a realistic survey, this can be projected out and measured using an estimator of the form (Bonvin et al. 2017)

$$\xi_{\text{Dop}}(d) = a_\kappa \sum_{ij} \Delta_i \kappa_j \cos \beta_{ij} \delta_K (d_i - d),$$

(4)

where $a_\kappa$ is a normalisation factor, and the sum is over pairs of pixels in the survey separated by a physical comoving distance $d_i$. The angle $\beta_{ij}$ is the angle formed at $\Delta_i$ between the line-of-sight angle $n$ and the direction vector to $\kappa_j$ (see Fig. 1). In what follows, we will refer to the quantity in Eq. (4) as the ‘Doppler magnification dipole’. This estimator provides an efficient way of isolating the $\kappa_v$ contribution. Projecting $\Delta \kappa$ onto a dipole does indeed strongly suppress the gravitational lensing contribution $\kappa_l$ up to redshift $\sim 1$, see Bonvin et al. (2017). Note that the cross-correlation between $\Delta$ and $\kappa$ also contains an octupole modulation, which can be isolated by weighting the two-point function by the Legendre polynomial $P_3(\cos \beta_{ij})$. As shown in Bonvin et al. (2017), this octupole is however significantly smaller than the dipole, and we therefore concentrate only on the latter in the following.

As implied by Eqs. (3) and (4), the Doppler dipole can be used as a probe of the line-of-sight peculiar velocity field. This is of particular interest in studies of modified gravity theories, which generically alter the growth rate of structure, $f$. Since at linear order we have $V \propto f$, this suggests that Doppler magnification can be used as an independent probe of gravitational physics on cosmological scales. Note that the correlation $\langle \Delta \kappa \rangle$ depends on $f$ in a different way than the standard redshift-space disturbance terms in the $\langle \Delta \Delta \rangle$ correlation, since $\kappa \propto V \cdot n$, whereas $\Delta$ depends on the velocity gradient $\partial_r (V \cdot n)$. As such the sensitivity of these two probes to modified gravity may be different, especially in the case where the growth rate is scale-dependent.

In the following sections, we show how deviations from General Relativity enter into the calculation of the Doppler magnification dipole, and derive expressions for the dipole (and higher multipoles) of the number count-convergence correlation function in the presence of such effects.

2.2 Velocity potential and growth factor

While alternative theories of gravity can be extremely complex in general, the vast majority can be described by a handful of new functional degrees of freedom to linear order in perturbations on an assumed FLRW background with an effective dark energy equation of state $w(z)$ (Amendola et al. 2013b; Baker et al. 2013; Gleyzes et al. 2013; Lagos et al. 2018). A further simplification can be made by applying the quasi-static approximation, which neglects time derivatives of any new gravitational degrees of freedom, and by restricting our attention to scales much smaller than the horizon ($k \gg H$). Under these assumptions, the Poisson equation relating the density and time-time gravitational potential in Fourier space is\(^2\)

$$-k^2 \Psi = 4\pi G a^2 \bar{\rho}(a,k) \delta,$$

(5)

where $\bar{\rho}(a,k)$ is an arbitrary function of scale factor and wavenumber that encodes the modified gravitational

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\(^1\) The contribution from the relativistic corrections to the dipole and octupole of $\langle \Delta \kappa \rangle$ is suppressed by $(d/r)^2$ ($d$ being the pixels’ separation) with respect to the contribution from the standard Newtonian terms in Eq. (2). The lensing contribution is strongly subdominant at small redshift, similar to what was found in Bonvin et al. (2014).

\(^2\) We assume the following metric convention throughout this paper: $ds^2 = a^2 [-(1 + 2\Psi)d\tau^2 + (1 - 2WY_{ij} J^{ij}d\tau^2)]$, and we use the Fourier convention $f(x, \tau) = (2\pi)^{-3} \int d^3k e^{-ik \cdot \mathbf{x}} f(k, \tau)$. 

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**Figure 1.** The geometrical set-up and definitions of various angles used in the calculations.
physics, and has the value $\mu = 1$ in GR. A second modification also arises, in the form of a non-trivial ‘gravitational slip’ relation relating the two potentials in the metric, $\Phi = \eta(a,k)\Psi$. The slip parameter $\eta$ is again an arbitrary function of time and scale, and $\eta = 1$ in GR. As shown in e.g. Hall et al. (2013), $\eta$ does not enter into the growth equations for $\delta$ and $V$ at subhorizon scales. Since those fields are the only ones contributing to the Doppler magnification dipole in the regime we are interested in (where the lensing contribution is negligible), our observable will be insensitive to $\eta$. Finally, theories of modified gravity can break Einstein’s equivalence principle, generating modifications to Euler’s equation (see Gleyzes et al. 2015). We do not consider this possibility here and we assume that Euler’s and the continuity equations are the same as in GR.

The growth equation for $\delta$ is determined by solving the modified Bardeen equation on sub-Hubble scales (Pogosian et al. 2010; Amendola et al. 2013a),

$$\frac{\partial^2 \delta}{\partial (ln a)^2} + \left(2 + \frac{\partial \ln H}{\partial \ln a}\right) \frac{\partial \delta}{\partial \ln a} = \frac{3}{2} \Omega_m \mu \delta.$$  \hspace{1cm} (6)

We write the solution to this equation in Fourier space in terms of the usual $\Lambda$CDM transfer function $T(k)$ and the primordial scalar potential $\Psi_p(k)$, such that

$$\delta(k, z) = \frac{2}{3} \left(\frac{k}{H_0}\right)^2 T(k) \Omega_m \mu(z) \Psi_p(k).$$  \hspace{1cm} (7)

The growth factor $D$ is found by solving Eq. (6) with initial condition $D(k,a_0) = 1$ at early times, $a_s < a_{ini} \ll 1$, where $a_s$ is the scale factor at recombination. Inserting (7) into the continuity equation on sub-Hubble scales

$$V(k, z) = -\frac{3}{k} \delta(k, z),$$  \hspace{1cm} (8)

where a dot denotes a conformal time derivative, the velocity potential becomes

$$V(k, z) = \mathcal{G}(k, z) T(k) \Psi_p(k).$$  \hspace{1cm} (9)

The ‘velocity growth factor’ $\mathcal{G}$ is defined by

$$\mathcal{G}(a, k) \equiv \frac{2a k}{3\Omega_m H_0} \left(\frac{D}{a} - \frac{\mu}{\mu} \frac{D}{a}\right).$$  \hspace{1cm} (10)

### 2.3 Multipoles of the correlation function

Combining $\Delta$ with the Doppler convergence $\kappa_v$ in Eq. (3) we obtain

$$\xi^{\Delta \kappa_v}(z', \theta) = \frac{1}{\mathcal{H}(z')(z')} - 1$$  \hspace{1cm} (11)

$$\int \frac{d^3k}{(2\pi)^3} 4\pi \delta^{(3)}(\mathbf{x}' - \mathbf{x}) \mathcal{G}(z, k) T^2(2)\mathcal{P}(k) \langle \mathbf{k} \cdot \mathbf{n} \rangle \times \left[\frac{2b}{3\Omega_m H_0} \frac{k^2}{\mu(z)} + (\mathbf{k} \cdot \mathbf{n}) \mathcal{G}(z, k)\right],$$

where $\mathcal{P}(k)$ denotes the primordial power spectrum ($\Psi_p(k)\Psi_p(k') = (2\pi)^3 P(k) \delta^{(3)}(k + k')$). Note that the primes on redshift and direction in Eq. (11) refer to the pixel where $\kappa$ is estimated. The cross-correlation (11) is a function of $\theta$ which is the angle between $n$ and $n'$. We can re-express this cross-correlation in terms of $(z, d, \beta)$, where $d$ is the comoving distance between the galaxies and $\beta$ is the orientation of the pair with respect to the line-of-sight (see Fig. 1). Following Szalay et al. (1998); Szapudi (2004); Papai & Szapudi (2008); Montanari & Durrer (2012); Bonvin et al. (2017), we expand the exponential and factors of $\mathbf{k} \cdot n$ in terms of spherical harmonics, which allows us to integrate over the direction of $\mathbf{k}$. The cross-correlation then takes the simple form

$$\xi^{\Delta \kappa_v}(z', \theta) = \frac{1}{2\pi^2} \left[1 - \frac{1}{\mathcal{H}(z')\mathcal{H}(z)}\right] \times \left\{\frac{2r_1 - 3r_3}{10} \left(1 + \cos 2\beta\right) + I_1 \sin a + \frac{r_1 + r_3}{5} \sin a \sin 2\beta\right\},$$  \hspace{1cm} (12)

The angles $\alpha$ and $\beta$ are defined in Fig. 1, and the functions $\lambda_\ell$ and $\nu_\ell$, with $\ell = 1, 3$, are given by

$$\lambda_\ell(d, r, \beta) = \int dk k^2 j_2(kd) G(z', k) T^2(k) \mathcal{P}(k)$$  \hspace{1cm} (13)

$$\times \frac{2b}{3\Omega_m H_0} \frac{k^2}{\mu(z)} + \frac{1}{3} \frac{k}{\mathcal{H}(z)} G(z, k) \xi^\mathcal{G}(z', k).$$  \hspace{1cm} (14)

Note that the functions $\lambda_\ell$ and $\nu_\ell$ depend not only on the pixels’ separation $d$, but also on $r$ and $\beta$ through the evolution of $\mathcal{G}(z', k)$ with redshift.

To extract the dipole signal from Eq. (12), we write $z', r(z')$ and $\alpha$ explicitly as a function of $(d, \beta, r(z))$ and weight the cross-correlation $\xi$ by the Legendre polynomial $P_1(\cos \beta)$ (i.e multiplying Eq. (12) by $\cos \beta$ and integrate it over $\beta$).

In the flat-sky approximation and neglecting evolution between $z$ and $z'$, Eq. (12) can be simplified. Using $\cos \alpha \cos 2\beta = -\cos \beta + 2 \cos^3 \beta$ and $\sin \alpha \sin 2\beta = 2 \cos \beta - 2 \cos^3 \beta$, we find, in terms of orthogonal Legendre polynomials, $P_1(x) = x$, $P_2(x) = 5x^3 - 3x$,

$$\xi^{\Delta \kappa_v} \approx \frac{1}{2\pi^2} \left(1 - \frac{1}{\mathcal{H}(z')}\right) \left[I_1 + \frac{4}{15}\right] P_1(\cos \beta) - \frac{2}{5} r_3 P_3(\cos \beta).$$  \hspace{1cm} (15)

Since we have neglected the evolution of $\mathcal{G}$ between $z$ and $z'$, the functions $\lambda_\ell$ and $\nu_\ell$ are now independent of the orientation $\beta$, so that in the flat-sky approximation, the dipole signal is just given by the $P_1(\cos \beta)$ term in Eq. (15). As shown in Bonvin et al. (2017), the octupole, $P_3(\cos \beta)$, is also detectable in future surveys, but we do not consider it further here since its signal-to-noise is significantly smaller than that of the dipole.

The dependence of the dipole on the theory of gravity is encoded in the functions $\lambda_\ell$ and $\nu_\ell$, which depend on $D$, $\mathcal{G}$ and $\mu$. Deviations from GR have two impacts on the dipole. First, they change the evolution of the dipole with redshift, via the redshift dependence of $D$, $\mathcal{G}$ and $\mu$. And second, they change the shape of the dipole as a function of $d$. The $k$-dependences of $D$, $\mathcal{G}$ and $\mu$ do indeed modify the integrals over $\mathbf{k}$ in Eqs. (13) and (14), leading to a different scaling with $d$.

### 2.4 Modified gravity models

We now consider two parameterisations of $\mu$ that are representative of some alternative theories of gravity. First we consider a specialisation of the model presented in Planck Collaboration XIV (2016)

$$\mu(a, k) = 1 + E_1 \Omega_{DE} a.$$  \hspace{1cm} (16)
In this model, the growth is scale-independent, making this also an effective dark energy parametrisation. The second model we consider is an \( f(R) \) model studied in Gianantonio et al. (2010); Hu et al. (2013)

\[
\mu(a, k) = \frac{1}{1 - 1.4 \times 10^{-8} (\Lambda/\text{Mpc})^2 a^3} \frac{1 + \frac{2}{3} \kappa^2 k^2 a^2}{1 + \frac{2}{3} \kappa^2 k^2 a^2}.
\] (17)

The resulting equation of motion from varying the modified Einstein-Hilbert action with respect to the metric introduces a scalar degree of freedom \( fR = df/dR \), the *scalaron*, whose Compton wavelength \( \lambda \) (in Eq. 17) at present can be expressed in terms of its dimensionless counterpart \( B_0 \) as \( \lambda^2 = B_0/(2H_0^2) \). The general expression of the dimensionless Compton wavelength is given by (Song et al. 2007b)

\[
B = \frac{f_{RR}}{1 + f_R \frac{dR}{d\ln a}} \left( \frac{d\ln H}{d\ln a} \right)^{-1},
\]

where \( f_{RR} \) is the second derivative of \( f(R) \) with respect to the Ricci scalar \( R \). As highlighted in Song et al. (2007a), a one parameter family of \( f(R) \) models labelled by \( B_0 \) exists for any given background expansion history.

In each of these two models we only have one additional parameter to constrain, \( E_{11} \) or \( B_0 \). In Fig. 2 (left panel), we show the dipole at \( z = 0.15 \) for these two models – solid green for the scale-independent model with \( E_{11} = 0.06 \); and solid orange for the \( f(R) \) model with \( B_0 = 0.1 \) – compared to ACDM in solid black. The shaded regions show the error bars on the dipole, calculated with the specifications of a survey like SKA phase 2 (see Section 3 for details). The light grey corresponds to an error on the convergence of \( \sigma_8 \), whereas the dark grey is for \( \sigma_8 = 0.3 \). We see that in the range 80 – 150 Mpc/h, deviations from the GR prediction of the order of 15% for the scale-independent model and \( \geq 20\% \) for the \( f(R) \) model are clearly visible in the Doppler magnification dipole.

In this figure we also compare the flat-sky approximation (dotted lines) to the full-sky calculation, finding rea-
sonable agreement on relevant scales. In particular, the departure from the flat-sky approximation happens at the same scale in the three models, despite the different $k$-dependence of their growth rate. Note that in the following we will use the full-sky expression for the dipole, since the surveys we are interested in will cover large areas of the sky, allowing a measurement of the dipole up to large separations, where the flat-sky approximation breaks down.

In the right panel of Fig. 2, we plot the percentage difference between $\Lambda$CDM and the two models. In the scale-independent model, the deviation does not depend on separation $d$. As expected, the deviation decreases towards higher redshift, due to the function $\Omega_{\text{DE}}(a)$ in Eq. (16), which suppresses deviations from GR at high redshift. The $f(R)$ model, on the other hand, has a distinct scale dependence. The function $\mu$ in Eq. (17) deviates from GR at both large scales, where $\mu(a, k) \to [1 - 1.4 \times 10^{-3} (4/Mpc)^2 a^3]^{-1}$, and small scales, where $\mu(a, k) \to 4/3 [1 - 1.4 \times 10^{-3} (4/Mpc)^2 a^3]^{-1}$. As a consequence, the dipole exhibits departure from $\Lambda$CDM at both small and large separations. The scale at which $\mu$ transitions from one asymptotic value to the other is governed by the parameter $B_0$. In particular, decreasing $B_0$ tends to shift this transition to smaller scales. This would in turn shift the deviations in the dipole to smaller scales.

For comparison, we plot in Fig. 3 the percentage difference in the monopole and the quadrupole of redshift-space distortions (i.e. the monopole and quadrupole of $\langle \delta \Delta \rangle$), induced by the two models. We see that, for the $f(R)$ model, the deviations in the monopole of RSD are significantly larger than those in the dipole. The deviations in the quadrupole on the other hand are quite similar to those in the dipole. This suggests that the $f(R)$ model generates larger deviations in the density than in the velocity. However, since modifications in the density are degenerated with the bias, the constraining power on $B_0$ is expected to be governed by deviations in the quadrupole.

Comparing Fig. 3 with Fig. 2, we also see that the deviations in the dipole clearly increase with separation, whereas those in the monopole and quadrupole have no clear scale-dependence. This behaviour is related to the fact that Doppler magnification is directly sensitive to peculiar velocities, whereas RSD are sensitive to their gradient. As a consequence the dipole contains a factor $k/H$ less than RSD, which gives more weight to larger scales. The dipole is therefore particularly well adapted to test modifications of gravity in the linear regime.

For the scale-independent model, we see that the deviations in the RSD monopole and quadrupole are of the same order of magnitude as those in the Doppler magnification dipole. This is not surprising, since in this case the functions $D_1$, $\mu$ and $\mathcal{G}$ can be taken out of the integrals over $k$. The different weighting in $k$ has consequently no impact on the amplitude of the deviations, leading to similar results for the three multipoles.

In the next section, we study forecasts for the overall sensitivity to the $B_0$ and $E_{11}$ parameters.

### 3 FORECASTS FOR FUTURE GALAXY SURVEYS

We now present predicted constraints on cosmological parameters in each model to show how deviations from GR can be constrained with the Doppler magnification dipole. We consider the set of parameters $h, \Omega_m, \Omega_b$, together with $E_{11}$ for the scale-independent model and $B_0$ for the $f(R)$ model. The fiducial values we choose are those of $\Lambda$CDM+GR with $h = 0.68$, $\Omega_m = 0.3028$, $\Omega_b = 0.048$ and the MG parameters zero. We fix the other cosmological parameters to their fiducial value: $n_s = 0.96$, and $\sigma_8 = 0.83$.

#### 3.1 Galaxy survey specifications

We assess the ability of two spectroscopic galaxy redshift surveys to constrain GR with Doppler magnification observations. The first, DESI (Wechsler & DESI Collaboration 2015), is expected to begin in 2018, and will yield multiple spectroscopic galaxy samples from a 5-year survey over a 14,000 deg$^2$ footprint. The sample of most relevance to Doppler magnification is the Bright Galaxy Sample (BGS), which covers the redshift range $0.05 \leq z \lesssim 0.4$, with a median redshift of $z \simeq 0.2$. BGS galaxies will be selected from existing r-band imaging from DECam and Bok 90Prime, but g, z, and 3 – 4µm band imaging will also be available.
(Wechsler & DESI Collaboration 2015). This provides multiple avenues for measuring the galaxy sizes to estimate $\kappa$, while DESI itself will provide high-resolution spectroscopic redshifts.

The second survey we consider is a HI galaxy survey on Phase 2 of the Square Kilometre Array (SKA), which is expected to enter operation in the late 2020s, potentially yielding a deep hemispherical ($\sim 20,000$ deg$^2$) survey on the southern sky around 2030. Spectroscopic redshifts are estimated from detections of the 21cm emission line of neutral hydrogen, while sizes can be estimated from imaging of either resolved radio continuum emission, or cross-matched optical counterparts of the radio galaxies. While an SKA2 galaxy survey is expected to be sample variance limited over $0 \leq z \leq 1.5$, we will focus on the $z \leq 0.5$ range here, where the contamination from the lensing convergence is negligible (as shown in Bonvin et al. (2017), in this regime it reaches at most 7% at large separations $\geq 180$ Mpc/h).

For both surveys, we bin the expected galaxy number density into tophat redshift bins of width $\Delta z = 0.1$, covering the range $0.1 \leq z \leq 0.5$. This quantity is shown in Fig. 4. The DESI values are taken from DESI Collaboration et al. (2016), while the SKA2 values are taken from Bull (2016). For both surveys, we fix the bias in each redshift bin to its fiducial value. Constraints for other upcoming surveys such as Euclid and LSST are broadly similar to SKA2.

We use a Planck CMB prior in all our analyses, which we construct by Gaussianising the Planck Collaboration XIII (2016) MCMC chains, forming an effective Fisher matrix by inverting the resulting covariance matrix. This is used to constrain the standard $\Lambda$CDM parameters only; constraints on the modified gravity model parameters from the CMB are not included (although see Planck Collaboration XIV (2016); Planck Collaboration (2018) for Planck analyses that do include them). The main reason for choosing to ignore information from the CMB in this way is that we wish to focus on how the Doppler magnification effect is able to directly constrain modified gravity scenarios, rather than studying its role in (e.g.) breaking degeneracies within the CMB-derived parameters to yield better constraints. Some information from the CMB is nevertheless necessary to help fix the various background parameters that would otherwise be poorly constrained by Doppler magnification alone. A more holistic analysis that includes information from contemporary surveys (such as CMB lensing and redshift-space distortions) is left for future work.

An expression for the covariance matrix was calculated in Bonvin et al. (2017). It is dominated by three contributions: the shot noise in the galaxy number count, the cosmic variance of the galaxy number count, and the intrinsic error on the size measurement $\text{cov} \kappa$, for which we choose two representative values, $\text{cov} \kappa = 0.3, 0.8$. We refer the interested reader to Alsing et al. (2015); Bonvin et al. (2017) for a discussion on how the value of $\text{cov} \kappa$ may change depending on the type of galaxies in a given survey. Note that while the covariance matrix calculation in Bonvin et al. (2017) does account for correlations between different pixel’s separations, it does not account for correlations between different redshift bins. However since the size of the bins that we use is relatively large, we can safely neglect the covariance between them.

3 To calculate the impact of non-linearities on the dipole we use the following approximation: we use the linear continuity equation to relate the velocity to the density, and then we calculate the non-linear density power spectrum with HaloFit. This procedure is not correct, since in the non-linear regime the continuity equation is modified. However, it allows us to evaluate at which scales non-linearities become relevant.

3.2 Results

In this section, we present forecasts for how well Doppler dipole measurements with DESI and SKA2 galaxy survey will be able to constrain the scale-independent and $f(R)$ modified gravity parametrisations discussed above.

Since our expression for the dipole is based on linear perturbation theory, we restrict our analysis to separations $d = 40-180$ Mpc/h. In Fig. 5 we estimate the impact of non-linearities on the dipole and we show that $d \geq 40$ Mpc/h is a conservative minimum separation, for which the effect of non-linearities is less than 2%. The left panel of Fig. 6 shows the joint constraints in the $f(R)$ model on $\Omega_m$ and the parameter $B_0$ marginalised over the other parameters. The constraints are sensitive to the value of the error on the size measurement: the constraints for $\sigma_\kappa = 0.3$ are better than those corresponding to $\sigma_\kappa = 0.8$ by a factor of $\sim 2$. It is worth pointing out that, without a Planck prior, the value of $\sigma_\kappa$ predominantly affects the diagonal of the forecast parameter covariance matrix, while it can affect both the resulting constraint and the correlation between parameters when a prior is included. The marginalised constraints on $B_0$ (95% CL), obtained by combining the Doppler magnification dipole with Planck, are $B_0 < 1.1 \times 10^{-5}$ with DESI and $B_0 < 5.2 \times 10^{-6}$ with SKA2, assuming $\sigma_\kappa = 0.3$. Including scales down to $d = 20$ Mpc/h tightens the constraints by a factor of $\sim 4$: $B_0 < 2.71 \times 10^{-6}$ with DESI and $B_0 < 1.46 \times 10^{-6}$ with SKA2. This shows that the constraining power of the dipole is not strongly degraded by limiting the analysis to linear scales.

For comparison, the current constraints on $B_0$ from Planck Collaboration XIV (2016) are $B_0 < 8.6 \times 10^{-5}$ (95% CL), obtained from a combination of Planck CMB temperature, polarisation, weak lensing, BAO, and RSD. The Doppler magnification dipole is therefore expected to improve the current constraints by one order of magnitude with SKA2.

In the right panel of Fig. 6, we show the joint constraints for the scale-independent model on $\Omega_m$ and the parameter $E_{11}$, marginalised over the other parameters. The marginalised constraints on $E_{11}$ (68% CL), obtained by combining the Doppler magnification dipole with Planck, are $E_{11} < 0.03$ with DESI and $E_{11} < 0.01$ with SKA2, assuming $\sigma_\kappa = 0.3$. Comparing with current constraints from Planck Collaboration XIV (2016): $E_{11} = -0.3^{+0.20}_{-0.18}$ (68% CL), we see that the Doppler magnification dipole is again expected to improve the constraints by one order of magnitude.

To understand the different constraining power of the Doppler magnification dipole versus RSD, it is first informative to compare the cumulative signal-to-noise of these two probes. The signal-to-noise for the Doppler magnification dipole within SKA2, for $0.1 \leq z \leq 0.5$ and $40 \leq d \leq 180$ Mpc/h is of 70 for $\sigma_\kappa = 0.3$. For RSD, we can calculate the signal-to-noise of the monopole and quadrupole, using the specifications of the CMASS DR11 sample (Samushia et al. 2014) used for the Planck constraints (Planck Collaboration XIV 2016). Using the publicly available code
Figure 6. Joint marginalised constraints $B_0 - \Omega_m$ for the $f(R)$ model (left) and $E_{11} - \Omega_m$ for the scale-independent model (right). Dashed blue and solid blue ellipses are 68% CL for the DESI survey, considering $\sigma_8 = 0.8$ and $\sigma_8 = 0.3$ respectively. Dashed red and solid red ellipses are 68% CL for the SKA2 survey, using $\sigma_8 = 0.8$ and $\sigma_8 = 0.3$ respectively.

Figure 7. Constraints on all the parameters in the $f(R)$ model (left) and the scale-independent model (right). All the ellipses are 68% CL. Dashed blue corresponds to DESI with $\sigma_8 = 0.8$, solid blue to DESI with $\sigma_8 = 0.3$, dashed red corresponds to SKA2 with $\sigma_8 = 0.8$ and solid blue to SKA with $\sigma_8 = 0.3$.

COFFE (Tansella et al. 2018) to calculate the signal and covariance matrices, we obtain a cumulative signal-to-noise of 80 for $24 \leq d \leq 152 \text{Mpc}/h$. This shows that the precision with which the Doppler magnification dipole will be measured with SKA2 is similar to the precision of current RSD measurements.

Since the deviations in the Doppler magnification dipole are similar to those in the multipoles of RSD (see Figs. 2 and 3), we would then expect similar constraints on $B_0$ and $E_{11}$ from these two probes. The fact that we find instead an order of magnitude improvement with the Doppler magnification dipole is due to the fact that in our analysis we fix the value of the bias. This automatically breaks the degeneracy between modifications of gravity and bias evolution. On the other hand, RSD analyses consider the bias as a free parameter, which can be measured from the combination of the monopole and quadrupole. However, due to measurement uncertainty, the degeneracy between modifications in the growth rate and bias is only partly broken, leading to a degradation of the constraints compared to our analysis.

Finally, let us mention that the Planck constraints on $B_0$ are highly sensitive to how a degeneracy between $\tau$ and $B_0$ is broken. With Planck CMB measurements alone, the upper limit is $B_0 < 0.79$ (95% CL), which is reduced to $< 0.69$ when BAO, Type Ia supernovae, and $H_0$ (‘BSH’) measurements are added. Further adding RSD measurements reduces the upper limit to $< 0.90 \times 10^{-4}$ however – an improvement of around four orders of magnitude! The explanation for this dramatic improvement is that even relatively weak constraints on structure formation (i.e. those providing measurements of $\sigma_8(z)$) are sufficient to break the degeneracy with $\tau$ and therefore tightly constrain $B_0$. This effect is not captured by our forecasts, which only use the Planck constraints as a prior on the standard cosmological parameters; if $B_0$ were included in our Planck prior Fisher matrix, a similar effect would be observed when combined with the Doppler dipole Fisher matrix, as this also constrains the growth rate of structure.


| $\sigma_\kappa$ | $z$ | SKA2 (95% CL) | DESI (95% CL) |
|----------------|-----|---------------|---------------|
| 0.3            | 0.15| $< 6.12 \times 10^{-6}$ | $< 1.32 \times 10^{-5}$ |
|                | 0.25| $< 9.70 \times 10^{-6}$ | $< 2.14 \times 10^{-5}$ |
|                | 0.35| $< 1.54 \times 10^{-5}$ | $< 3.40 \times 10^{-5}$ |
|                | 0.45| $< 2.44 \times 10^{-5}$ | $< 5.30 \times 10^{-5}$ |
| 0.8            | 0.15| $< 1.49 \times 10^{-5}$ | $< 3.44 \times 10^{-5}$ |
|                | 0.25| $< 2.43 \times 10^{-5}$ | $< 5.62 \times 10^{-5}$ |
|                | 0.35| $< 3.91 \times 10^{-5}$ | $< 8.93 \times 10^{-5}$ |
|                | 0.45| $< 6.21 \times 10^{-5}$ | $< 1.39 \times 10^{-4}$ |

For completeness, forecast constraints on all parameters of the scale-independent and $f(R)$ models that we considered are shown in the left and right panels of Fig. 7 respectively.

The marginalised constraints on the $B_0$ and $E_{11}$ parameters are shown as a function of redshift in Tables 1 and 2 respectively. For both models, and both SKA and DESI, the constraints are best in the lowest redshift bin, and get gradually worse with increasing redshift. This behaviour is expected, given how both the overall amplitude of the Doppler dipole and number density of detected galaxies decrease with redshift. As a consequence, the regime $0.1 \leq z \leq 0.5$ is well adapted to the analysis. This regime has also the advantage that the gravitational lensing contribution is always strongly suppressed and can therefore be safely neglected. It is notable that the predicted constraints from SKA2 are always around a factor of two better than for DESI, irrespective of the value of $\sigma_\kappa$ that is assumed. This is primarily due to the significantly larger survey area of SKA2, despite it having a lower number density than the DESI sample in the lowest redshift bins.

In all of the cases we have considered so far, the Doppler dipole measurement is dominated by the intrinsic error on the galaxy size, $\sigma_\kappa$. Fig. 8 shows results for several different values of $\sigma_\kappa$ from the more optimistic value of 0.3 that was used in the forecasts above, to a highly optimistic value of 0.01. The latter value is quite extreme, and we make no claim that it can be achieved in practice – galaxies are complex objects formed by messy nonlinear processes, and so there will always be a significant amount of scatter in the size distribution of any population (c.f. Albing et al. 2015). There is some hope that galaxy samples (or proxy observables) can be selected to reduce the scatter however; for example, the relationship between HI mass and disk radius has a particularly low scatter of $\approx 0.06$ dex ($\sigma_\kappa \approx 0.14$) (Wang et al. 2016). As shown in Fig. 8, the gain from reducing $\sigma_\kappa$ from 0.3 to 0.1 is a factor of $\sim 2$, with $B_0 < 2.28 \times 10^{-6}$ and $E_{11} < 1.14 \times 10^{-2}$ (95% CL) when $\sigma_\kappa = 0.1$. If $\sigma_\kappa$ can be reduced further (perhaps by using galaxy size measurement methods that do not depend on statistical galaxy size distributions), the constraining power of the Doppler dipole method can be enhanced even further.

### 4 CONCLUSIONS

We have shown the potential of using the Doppler magnification dipole, prescribed by Bonvin et al. (2017), to constrain departures from GR. To illustrate the sensitivity of Doppler magnification to modified gravity, we have chosen two toy models in the Parametrised Post-Friedmann formalism, one with a scale-independent growth rate, and one $f(R)$ model with a scale-dependent growth rate.

In the quasi-static regime within the scales of interest, we have derived an expression for the peculiar velocity in the two models. We have shown that the velocity is sensitive to the function $\mu$, which modifies Poisson’s equation, but that it is insensitive to the slip (which modifies the relation between the two Bardeen potentials).

We have then derived the cross-correlation between the convergence and the galaxy number counts $\xi^{\Delta \sigma^2}$, in the two models, and we have compared it with that of GR. As expected, the difference between the scale-independent model and GR is constant at all separations $\delta$. It is however redshift dependent, decreasing at higher redshift. On the other hand, the scale-dependence of the $f(R)$ model modifies the shape of the dipole. The departure from GR exhibits then a minimum at small separation and then constantly increases towards large separation.

Since the Doppler magnification dipole should be detected with a high signal-to-noise in SKA2 and DESI, we have used these surveys to forecast constraints on the parameters $E_{11}$ and $B_0$ that encode deviations from GR in the two models. We have found that for DESI the constraints on $B_0$ and $E_{11}$ are expected to be similar to current RSD constraints. The constraints from SKA2 are expected to be one order of magnitude better. This improvement is however mainly due to the fact that we fix the bias in our analysis.

We have used four tomographic bins to get the constraints on both $B_0$ and $E_{11}$. To investigate which tomographic bin provides the constraining power, we have computed constraints as a function of redshift bin for the two surveys and found that the resulting constraints decrease with redshift, in other words the constraining power mainly comes from the bin at low redshift ($z = 0.15$). Overall, constraints from SKA2 are approximately twice as tight as those from DESI at all redshift bins.

To get an idea of how sensitive to the errors on size measurement the constraints are, we have chosen optimistic and pessimistic cases with $\sigma_\kappa = 0.3, 0.8$ respectively. We have found that decreasing $\sigma_\kappa$ from 0.8 to 0.3 improves the constraints by a factor of 2. We have also explored how the constraints vary if we decrease $\sigma_\kappa$ from 0.3 to the (unrealistic) value of 0.01, finding an improvement by a factor of $\sim 20$ for the $f(R)$ model and by a factor of $\sim 10$ for the scale-independent model. This shows that the error on size

Table 2. Marginalised constraints on the $E_{11}$ parameter, obtained at each redshift bin with two different values of $\sigma_\kappa$.

| $\sigma_\kappa$ | $z$ | SKA2 (95% CL) | DESI (95% CL) |
|----------------|-----|---------------|---------------|
| 0.3            | 0.15| $< 2.82 \times 10^{-2}$ | $< 6.05 \times 10^{-2}$ |
|                | 0.25| $< 3.48 \times 10^{-2}$ | $< 7.72 \times 10^{-2}$ |
|                | 0.35| $< 4.43 \times 10^{-2}$ | $< 8.80 \times 10^{-2}$ |
|                | 0.45| $< 5.85 \times 10^{-2}$ | $< 1.12 \times 10^{-1}$ |
| 0.8            | 0.15| $< 6.80 \times 10^{-2}$ | $< 1.54 \times 10^{-1}$ |
|                | 0.25| $< 8.12 \times 10^{-2}$ | $< 1.80 \times 10^{-1}$ |
|                | 0.35| $< 9.88 \times 10^{-2}$ | $< 2.14 \times 10^{-1}$ |
|                | 0.45| $< 1.24 \times 10^{-1}$ | $< 2.66 \times 10^{-1}$ |
Figure 8. Constraints on $\Omega_m$ and the $B_0$ parameter of the $f(R)$ model (left) and $E_{11}$ parameter of the scale-independent model (right) for SKA2 + Planck (68% CL), for several different values of $\sigma_6$.

measurement is the dominant source of uncertainty in the dipole.

Finally, let us mention that in our analysis we have considered only two specific models: an $f(R)$ model which modifies the growth rate at both small and large scales, and a scale-independent model. If on the other hand, we would have modifications of gravity that are significant only at large scales, then we would expect the Doppler magnification dipole to be more sensitive to these modifications than RSDs. As discussed above, the Doppler magnification dipole has one factor of $k/H$ less than RSD, making it especially sensitive to modifications at large scales.

We conclude that the Doppler magnification dipole, considering future surveys like SKA, has good prospects for investigating modification of gravity on sub-horizon scales. In the event that RSD measures a departure from GR in the future, it will be crucial to check this result with an independent probe. Our analysis shows that the Doppler magnification dipole does provide an alternative way of testing GR with peculiar velocities and that it is complementary to RSD.

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REFERENCES

Abbott B. P., et al., 2016a, Phys. Rev. Lett., 116, 061102
Abbott B. P., et al., 2016b, Phys. Rev. Lett., 116, 221101
Abbott B. P., et al., 2017a, Phys. Rev. Lett., 118, 221101
Abbott B. P., et al., 2017b, Astrophys. J., 845, L13
Alonso D., Louis T., Bull P., Ferreira P. G., 2016, Phys. Rev., D94, 043522
Alsing J., Kirk D., Heavens A., Jaffe A. H., 2015, Monthly Notices of the Royal Astronomical Society, 452, 1202

Amendola L., et al., 2013a, Living Rev. Rel., 16, 6
Amendola L., Kunz M., Motta M., Saltas I. D., Sawicki I., 2013b, Phys. Rev., D87, 023501
Bacon D. J., Andrianomena S., Clarkson C., Bolejko K., Maartens R., 2014, Mon. Not. Roy. Astron. Soc., 443, 1900
Baker T., Ferreira P. G., Skordis C., 2013, Phys. Rev., D87, 024015
Baker T., Psaltis D., Skordis C., 2015, Astrophys. J., 802, 63
Baldi M., Villaescusa-Navarro F., Viel M., Puchwein E., Springel V., Moscardini L., 2014, MNRAS, 440, 440
Barreira A., Li B., Hellwing W. A., Lombriser L., Baugh C. M., Pascoli S., 2014, J. Cosmology Astropart. Phys., 4, 029
Battaglia N., 2016, JCAP, 1608, 058
Bert E., et al., 2015, Class. Quant. Grav., 32, 243001
Bertotti B., Iess L., Tortora P., 2003, Nature, 425, 374
Bhattacharya S., Kosowsky A., 2008, J. Cosmology Astropart. Phys., 8, 030
Bonvin C., 2008, Phys. Rev., D78, 123530
Bonvin C., Durrer R., 2011, Phys. Rev., D84, 063505
Bonvin C., Fleury P., 2018, JCAP, 1805, 061
Bonvin C., Hui L., Gaztanaga E., 2014, Phys. Rev., D89, 083535
Bonvin C., Andrianomena S., Bacon D., Clarkson C., Maartens R., Moloi T., Bull P., 2017, Mon. Not. Roy. Astron. Soc., 472, 3906
Bull P., 2016, Astrophys. J., 817, 26
Burrage C., Sakstein J., 2016, JCAP, 1611, 045
Challinor A., Lewis A., 2011, Phys.Rev., D84, 043516
DESI Collaboration et al., 2016, preprint, (arXiv:1611.00036)
Damour T., Taylor J. H., 1992, Phys. Rev., D45, 1840
Desjacques V., Jeong D., Schmidt F., 2013, JCAP, 1308, 025
Desjacques V., Jeong D., Schmidt F., 2013, JCAP, 1308, 025
Esposito-Farese G., 1996, in Colloquium on Pulsar Timing, General Relativity, and the Internal Structure of Neutron Stars Amsterdam, Netherlands, September 24-28, 1996. pp 90–6984 (arXiv:gr-qc/9612039)
Everitt C. W. F., et al., 2011, Phys. Rev. Lett., 106, 221101
Giannantonio T., Martinelli M., Silvestri A., Melchiorri A., 2010, Journal of Cosmology and Astroparticle Physics, 2010, 030
Gleyzes J., Langlois D., Piazza F., Vernizzi F., 2013, JCAP, 1308, 025
Gleyzes J., Langlois D., Piazza F., Vernizzi F., 2013, JCAP, 1308, 025
Gleyzes J., Langlois D., Piazza F., Vernizzi F., 2013, JCAP, 1308, 025
Gronke M., Llinares C., Mota D. F., Winther H. A., 2015, MNRAS, 450, 2837
Hall A., Bonvin C., Challinor A., 2013, Phys. Rev., D87, 064026
Hellwing W. A., Barreira A., Frenk C. S., Li B., Cole S., 2014, Physical Review Letters, 112, 221102

MNRAS 000, 1–11 (2018)
