MAGNETIC ENERGY SPECTRA IN SOLAR ACTIVE REGIONS

VALENTyna ABRAMenko and VASYL YURCHYSHYN

Big Bear Solar Observatory, 40386 N. Shore Lane, Big Bear City, CA 92314, USA

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ABSTRACT

Line-of-sight magnetograms for 217 active regions (ARs) with different flare rates observed at the solar disk center from 1997 January until 2006 December are utilized to study the turbulence regime and its relationship to flare productivity. Data from the SOHO/MDI instrument recorded in the high-resolution mode and data from the BBSO magnetograph were used. The turbulence regime was probed via magnetic energy spectra and magnetic dissipation spectra. We found steeper energy spectra for ARs with higher flare productivity. We also report that both the power index, $\alpha$, of the energy spectrum, $E(k) \sim k^{-\alpha}$, and the total spectral energy, $W = \int E(k)dk$, are comparably correlated with the flare index, $A$, of an AR. The correlations are found to be stronger than those found between the flare index and the total unsigned flux. The flare index for an AR can be estimated based on measurements of $\alpha$ and $W$ as $A = 10^b(\alpha W)^c$, with $b = -7.92 \pm 0.58$ and $c = 1.85 \pm 0.13$. We found that the regime of the fully developed turbulence occurs in decaying ARs and in emerging ARs (at the very early stage of emergence). Well-developed ARs display underdeveloped turbulence with strong magnetic dissipation at all scales.

Key words: Sun: flares – Sun: photosphere – Sun: surface magnetism – turbulence

Online-only material: color figures

1. INTRODUCTION

Existing methods for predicting the flare activity of active regions (ARs) are not numerous (e.g., Abramenko et al. 2002; Falconer et al. 2002, 2003, 2006; Leka & Barnes 2003, 2007; McAteer et al. 2005; Schrijver 2007; Georgoulis & Rust 2007; Barnes & Leka 2008; see review by McAteer et al. 2009) and the majority of them are based on first-order statistical moments of the magnetic field (test parameters are derived from $B$). At the same time, Leka & Barnes (2007) noted that the exploration of higher order statistical moments seems beneficial given the nonlinear nature of a flaring process.

Earlier we presented results of a small statistical study (Abramenko 2005) focused on the relationship between the flare productivity of solar ARs and the power-law index, $\alpha$, of the magnetic energy spectrum, $E(k) \sim k^{-\alpha}$. This magnetic energy spectrum technique is based on the second-order statistical moment of a two-dimensional field and shows the distribution of energy over spatial scales. The study was based on 16 ARs observed predominantly during 2000–2003 with the SOHO/MDI instrument (Scherrer et al. 1995) performing in the high-resolution (HR) mode. The findings were promising: flaring ARs were reported to display steeper power spectra with the power index exceeding a magnitude of 2. Flare-quiet ARs exhibited a Kolmogorov-type spectrum with $\alpha$ close to 5/3 (Kolmogorov 1991, hereafter K41).

Power spectra calculations, based on the technique described in Abramenko (2005), will be a part of the pipeline system designed to process real-time data flowing from the Helioseismic and Magnetic Imager$^1$ that operates on board the Solar Dynamics Observatory.$^2$ Here, we evaluate the performance of the method based on a large data set. We report that both the magnetic energy stored in large-scale structures and the magnetic energy cascaded by turbulence at small-scale structures (below 10 Mm) are comparably correlated with flare productivity (Section 3). From the magnetic energy spectra, we also determined at what spatial scales the bulk of the magnetic energy dissipation occurs (Section 4). This allowed us to make an inference about the characteristics of the turbulent regime in ARs, which may be useful as a constraint criterion for the MHD modeling of ARs.

2. DATA

We selected 217 ARs measured near the solar disk center (no further than 20' away from the central meridian), so that the projection effect can be neglected. The set covers the period between 1997 January and 2006 December. All the ARs displayed nonzero flare activity, i.e., at least one GOES flare was produced by an AR during its passage across the solar disk. Given the typical average flux densities in ARs, smooth power spectra are not usually obtained for ARs with unsigned magnetic flux less than $10^{22}$ Mx. Therefore, we required that each AR should possess unsigned magnetic flux that exceeds this threshold value.

For the majority of ARs (215), MDI/HR magnetograms (pixel size of 0.6") were utilized. For two extremely flare-productive ARs only Big Bear Solar Observatory (BBSO) data, obtained at very good seeing conditions with the Digital Magnetograph (DMG; Spirock 2005; pixel size of 0.6"), were available. As we have shown earlier (Abramenko et al. 2001), magnetic energy spectra calculated for the same AR from MDI/HR and BBSO/DMG magnetograms agree very well at scales larger than 3 Mm. This allows us to use different data without the risk of skewing the correlation. From MDI full disk magnetograms, we determined the trend in the total unsigned flux in each AR during a three-day time interval centered at the time of the MDI/HR magnetogram acquisition. When AR flux variations did not exceed $\pm 10\%$ of the mean value, we classified the AR as a stable, well-developed AR. ARs displaying larger monotonous changes in the flux were classified as emerging or decaying. Unipolar sunspots were detected by visual assessment.
The flare productivity of an AR was measured by the flare index, $A$, introduced in Abramenko (2005). Since the X-ray classification of solar flares (X, M, C, and B) is based on a denary logarithmic scale, we can define the flare index as

$$A = (100S^{(X)} + 10S^{(M)} + S^{(C)} + 0.1S^{(B)})/1.$$

Here, $S^{(j)}$ is the sum of all GOES flare magnitudes in the $j$th X-ray class:

$$S^{(j)} = \sum_{i=1}^{N_j} I_i^{(j)},$$

where $N_j = N_X, N_M, N_C,$ and $N_B$ are the numbers of flares of X, M, C, and B classes, respectively, that occurred in a given AR during its passage across the solar disk, which is represented by the time interval $t$ measured in days. $I_i^{(X)}, I_i^{(M)}, I_i^{(C)},$ and $I_i^{(B)}$ are the GOES magnitudes of X, M, C, and B flares. The interval $t$ was taken to be 27/2 days for the majority of the ARs with the exception of the emerging ones. In general, those ARs that produced only C-class flares have flare indices smaller than 2, whereas several X-class flares will result in the flare index exceeding 100. The highest flare index of 584 was registered during its passage across the solar disk, which is represented by a time interval $t$ measured in days. A comparison of the spectra in Figure 1 also suggests that not only the slope but also the amplitudes of a spectrum may be related to flare productivity. Thus, we calculated two additional parameters from the spectra. The first parameter, $\langle E(k_{int}) \rangle$, characterizes the amplitude of a spectrum at low wavenumbers and is derived as $E(k)$ averaged over the interval ranging from the smallest $k$ to $k = 2\pi/10$ Mm$^{-1}$. The correlation coefficient between this parameter and the flare index $A$ is 0.65 with a 95% confidence interval of 0.57–0.72. The second parameter is the total spectral energy, $W = \int E(k)dk$, where the integration is performed over all the wavenumbers where $E(k)$ is nonzero. On average, 85% ± 5% of the total spectral energy is concentrated at scales larger than 10 Mm, so that $W$ may be considered a measure of the energy accumulated in large-scale structures of an AR. The correlation between $W$ and the flare index $A$ (see Figure 2(b)) was found to be somewhat higher: $\rho = 0.68$ with a 95% confidence interval of 0.60–0.75.

Note that $W$ can also be computed directly from $B^2$ according to Parseval’s theorem (e.g., Kammler 2000, p. 74). However, computing $W$ via integration of a Fourier transform has an advantage because the integration over an annulus also acts as a noise filter by leaving out the high-frequency corners in a two-dimensional wavenumber box. When analyzed data are obtained from MDI/HR, the difference in derived values is less than 0.5%. However, the difference increases when ground-based data are analyzed. When a mixed data set is used (space- and ground-based observations), the values of $W$ derived via integration of the Fourier transform are more consistent. Therefore, the total spectral energy used here was calculated using the integration approach.

So, the power index of the spectrum, the total spectral energy, and the averaged large-scale amplitude are comparably correlated with the flare index. We may conclude that both (green circles) form a separate low-flaring/shallow-spectrum subset. Emerging, decaying, and stable ARs are distributed more or less uniformly over the diagram.

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the turbulent energy cascade and the presence of large-scale structures in an AR are relevant to flare activity.

For the purpose of prediction of an AR’s flare productivity, we weighted the total spectral energy, $W$, and the power index, $\alpha$, to capture the contribution of both small- and large-scale effects (Figure 2(c)). The correlation coefficient in this case increased to 0.71 with a 95% confidence interval of 0.63–0.77. The flare index can then be fitted assuming

$$A = 10^{b}(\alpha W)^{c},$$

(3)

where $b = -7.92 \pm 0.58$ and $c = 1.85 \pm 0.13$, with the reduced $\chi^2 = 0.29$.

In general, the most powerful flares occur in strong, large ARs with a considerable amount of magnetic flux (e.g., Barnes & Leka 2008). And yet, the total unsigned flux is only very weakly correlated with the flare index (see Figure 2(d), the correlation coefficient is 0.37 with a 95% confidence interval of 0.25–0.49). This is understandable when we recall previous studies showing that not only the magnetic flux but rather the complexity of the magnetic structure is relevant to the flaring rate (Sammis et al. 2000; Abramenko et al. 2002; Falconer et al. 2002, 2003, 2006; Leka & Barnes 2003, 2007; McAteer et al. 2005; Schrijver 2007; Georgoulis & Rust 2007).

4. MAGNETIC DISSIPATION SPECTRA

As it follows from Figure 2(a), the majority of ARs (especially those of high flare activity) display energy spectra steeper than the Kolmogorov-type spectrum. To infer a physical meaning of this result, we will analyze the magnetic energy dissipation rates and magnetic dissipation spectra.

The magnetic energy dissipation rate is related to the presence of electric currents, i.e., $(\varepsilon) \sim \eta j^2$ (e.g., Biskamp 1993). In MHD models of turbulence, dissipative structures are visualized via (squared) currents (e.g., Biskamp & Welter 1989; Biskamp 1996; Schaffenberger et al. 2006; Pietarila Graham et al. 2009; Servidio et al. 2009), which in two-dimensional images appear to be predominantly located along magnetic field discontinuities, frequently referred to as current sheets. From two-dimensional MHD modeling, Biskamp & Welter (1989) found that when the magnetic Reynolds number (which is the ratio of the characteristic values of the advection terms to the magnetic diffusivity, and quantifies the strength of advection relative to magnetic diffusion) is low, these current sheets are extended and rare, and they become shorter and more numerous as the Reynolds number increases. Thus, the magnetic dissipation spectrum represents the distribution of dissipative structures over many spatial scales, and is a reasonable proxy for statistics of current structures in an AR.

The magnetic dissipation spectrum allows us to probe the state of the turbulence. For fully developed turbulence (K41; high Reynolds number), the bulk of the magnetic energy dissipation occurs at small scales, $k_d$, whereas the energy input occurs at large scales, $k_e$ (Figure 3), and the energy cascades from large to small scales without any losses. When the energy input interval and the dissipation interval overlap, dissipation occurs at intermediate scales along the cascade. This condition occurs in the state of underdeveloped turbulence (low Reynolds number), when large-scale structures might interfere with the turbulent
cascade at small scales. It is a challenge to model such a field because no K41 simplifications are applicable.

The magnetic energy dissipation spectra are defined as (Monin & Yaglom 1975; Biskamp 1993)

\[ E_{\text{dis}}(k) = 2\eta k^2 E(k), \]

where \( \eta \) is the magnetic diffusivity coefficient. (Note that \( E \) and \( E_{\text{dis}} \) in Equation (4) have different dimensions.) Then the rate of magnetic energy dissipation normalized by the magnetic diffusivity can be derived as (Biskamp 1993)

\[ \langle \varepsilon \rangle / \eta = 2 \int_{0}^{\infty} k^2 E(k) dk. \]

From observations we can derive the function \( k^2 E(k) \), which is proportional to the dissipation spectrum under the assumption that \( \eta \) is uniform over the AR area. In our case, both \( k^2 E(k) \) and \( \langle \varepsilon \rangle / \eta \) are associated with the dissipation of the \( B_z \) component only.

We calculated \( k^2 E(k) \) spectra for all ARs in our data set. Typical examples are shown in Figure 4. At the early stages of development of the emerging ARs, the separation distance \( (k_d - k_c) \) is the largest, which is similar to the fully developed turbulence conditions seen in the quiet Sun (see Figure 3). Later on this distance decreases as \( k_d \) shifts toward smaller wavenumbers (larger scales), so that the intervals of energy and dissipation become exceedingly overlapping. This implies the formation of large-scale dissipative structures. Decaying magnetic complexes show quite opposite behavior (Figure 4, middle row). Well-developed ARs (bottom row in Figure 4) show a significant overlap of the energy and dissipation intervals suggesting that, to the contrary of the fully developed turbulence phenomenology, significant dissipation takes place at all spatial scales. Thus, for the majority of well-developed ARs, one should expect a state of underdeveloped turbulence in the photosphere with the dissipation of the magnetic energy at all observable spatial scales.

We then compared the magnitudes of \( \langle \varepsilon \rangle / \eta \) to the flare index, \( A \). Their correlation turned out to be positive with \( \rho = 0.53 \) with a 95% confidence interval of 0.43–0.62. This indicates that the rate of magnetic energy dissipation in the photosphere is relevant to the flare activity.

5. CONCLUSIONS AND DISCUSSION

In this study, we analyzed second-order statistical moments of solar magnetic fields of 217 ARs observed with the MDI instrument in the HR mode during the 23rd solar cycle. The angle-integrated magnetic energy spectra of solar ARs display a well-defined power-law region, which indicates the presence of a turbulent nonlinear energy cascade. The power index, \( \alpha \), measured at 3–10 Mm scale range was found to be correlated well with the flare index, \( A \) (correlation coefficient \( \rho = 0.57 \)). This result further supports our previous findings based on only 16 ARs (Abramenko 2005). The power indices range between 1.3 and 3.0, with the majority of ARs having the power index in the range of 1.6–2.3. No particular preference for the classical 5/3 index was found. These values are surprisingly in agreement with recent numerical simulations of decaying MHD turbulence (Lee et al. 2010). The model results showed that in equivalent initial magnetic configurations different types of spectra (from \( k^{-3/2} \) to \( k^{-2} \)) may emerge depending on the intrinsic nonlinear dynamics of the flow.

The total spectral energy, \( W = \int E(k) dk \), is found to be correlated well with the flare index (\( \rho = 0.68 \)), while spectral energy weighted by the power index shows the strongest correlation with the flare index (\( \rho = 0.71 \)), which allowed us to determine an empirical description of this relationship: \( A = 10^{b(\alpha W)_c} \), where \( b = -7.92 \pm 0.58 \) and \( C = 1.85 \pm 0.13 \).

Combined analysis of magnetic energy and magnetic dissipation spectra showed that in the majority of well-developed ARs, the turbulent energy cascade is augmented by magnetic energy dissipation at all scales. We thus argue that a state of underdeveloped turbulence exists in the photosphere of mature ARs.

The magnetic energy dissipation rate, \( \langle \varepsilon \rangle / \eta \), correlates with flare productivity in the same degree as the power index does (\( \rho = 0.53 \)). As long as the energy dissipation rate is proportional to the square of the electric currents, we argue that the presence of the currents is relevant to flare productivity. Also, good correlation between the energy dissipation rate and the flaring rate is in agreement with earlier reports by Schrijver et al. (2005).

It is known from direct calculations based on vector magnetograms that electric currents are ubiquitous in ARs (see, e.g., Abramenko et al. 1991, 1996; Leka et al. 1993, 1996; Pevtsov et al. 1994; Wheatland 2000; Zhang 2002; Schrijver et al. 2005, Schrijver et al. 2008; Leka & Barnes 2007; Schrijver 2009). We found here that photospheric magnetic fields are in a state of underdeveloped turbulence when both the energy cascade and the energy dissipation at all scales are present in the system. We, therefore, arrive at the conclusion that both large- and small-scale dissipative structures (currents) are relevant to flaring.

On the other hand, Fisher et al. (1998) found no correlation between photospheric currents and the soft X-ray luminosity of magnetic complexes.
Figure 4. Top: energy spectra, $E(k)$ (blue lines), and dissipation spectra, $k^2 E(k)$ (double red lines), plotted for an emerging AR. Panels (a), (b), and (c) correspond to three consecutive moments during the emergence. Vertical blue (red) bars mark the maximum of the energy (dissipation) spectrum. The blue bars correspond to $k_e$ and the red bars correspond to $k_d$. As the AR emerges, $k_d$ shifts toward the smaller wavenumbers. Middle: energy and dissipation spectra for a decaying magnetic complex NOAA ARs 9682 and 9712. The right panel shows a superposition of the dissipation spectra at the well-developed (double red line) and decaying (solid green line) state of the magnetic complex. As the magnetic fields decay, $k_d$ shifts toward small scales (larger wavenumbers). Bottom: energy spectra and dissipation spectra for three well-developed ARs. The spectra are overlapped for each case. 

(A color version of this figure is available in the online journal.)

ARs. This was also noted, but not discussed, by Schrijver et al. (2005). We suggest that this apparent controversy is due to the difference in the nature of flares and the soft X-ray emission. We may consider flares as sporadic explosive events caused by strongly nonlinear dynamics relevant to large- and small-scale magnetic discontinuities (Falconer et al. 2002, 2003, 2006; see also Schrijver 2009), whereas soft X-ray emission reflects a more stationary and homogeneous process of coronal heating rather related to ubiquitous small-scale discontinuities formed in situ (Klimchuk 2006).

We would also like to note that the power law of the magnetic energy spectra, explored in this paper, should not be confused with the power law found in the distribution of the magnetic flux in flux concentrations recently reported by Parnell et al. (2009). At first sight, both of them characterize the structure of the magnetic field. However, they address different physical consequences of the magnetic field structuring. The power law of the magnetic energy spectrum represents the distribution of magnetic energy, $B^2$, over spatial scales and quantifies turbulence in an AR. Here, the smallest magnetic elements are represented by the tail of the spectrum usually associated with low spectrum amplitudes. The power law found in the distribution of magnetic flux represents the frequency (abundance) of magnetic elements of different sizes and implies a unique mechanism of the formation of magnetic flux concentrations (say, fragmentation process; see Abramenko & Longcope 2005 for more discussion). The smallest magnetic elements are the most frequent and are represented by the highest amplitudes of the distribution.

Modern computational capabilities allow us to develop MHD models that take into account the turbulent regime and turbulent dissipation (e.g., Lionello et al. 2010; Klimchuk et al. 2010). Therefore, diagnostics of turbulence derived from a large uniform data set is essential for constructing and restraining these models.

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