Interpolation of partial and full
supersymmetry breakings in $\mathcal{N} = 2$
supergavity

Hiroyuki Abe 1, Shuntaro Aoki 1, Sosuke Imai 1, Yutaka Sakamura 2,3

1 Department of Physics, Waseda University,
Tokyo 169-8555, Japan

2 KEK Theory Center, Institute of Particle and Nuclear Studies, KEK,
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

3 Department of Particles and Nuclear Physics,
SOKENDAI (The Graduate University for Advanced Studies),
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

Abstract

We discuss an $\mathcal{N} = 2$ supergravity model that interpolates the full and
the partial supersymmetry breakings. In particular, we find the conditions for
an $\mathcal{N} = 0$ Minkowski vacuum, which is continuously connected to the partial-
breaking ($\mathcal{N} = 1$ preserving) one. The model contains multiple (Abelian)
vector multiplets and a single hypermultiplet, and is constructed by employ-
ing the embedding tensor technique. We compute the mass spectrum on the
Minkowski vacuum, and find some non-trivial mass relations among the mas-
sive fields. Our model allows us to choose the two supersymmetry-breaking
scales independently, and to discuss the cascade supersymmetry breaking for
the applications to particle phenomenology and cosmology.

*E-mail address: abe@waseda.jp
†E-mail address: shun-soccer@akane.waseda.jp
‡E-mail address: s.i.sosuke@akane.waseda.jp
§E-mail address: sakamura@post.kek.jp
1 Introduction

Extended ($\mathcal{N} \geq 2$) supergravity naturally appears from higher dimensional supergravity and string compactifications (see [1,2] for review). Its interactions are more restricted (predictive) than $\mathcal{N} = 1$ supergravity, which has been intensively investigated from the viewpoint of particle phenomenology and cosmology. Due to the restrictions, it is a non-trivial task to obtain a phenomenologically favorable supersymmetry-breaking vacuum. This fact is related to the no-go theorem for the partial breaking of extended supergravity [3,4]. For example, in $\mathcal{N} = 2$ supergravity, the naive gauging (electric gauging in the frame where...
the prepotential exists) leads to the breaking of the whole \( \mathcal{N} = 2 \) supersymmetries, and some fields acquire masses by (super) higgs mechanism. However, it generally leads to a degenerate mass spectrum due to a single supersymmetry-breaking scale. Therefore, it is impossible to realize the above mentioned \( \mathcal{N} = 1 \) models within this framework. In order to incorporate such phenomenological models, the \( \mathcal{N} = 2 \) supersymmetries need to be broken by two different breaking scales. Then, an approximate \( \mathcal{N} = 1 \) supersymmetry appears between these scales. Furthermore, when they are hierarchical, the situation approaches to the partial breaking of the supersymmetries.

The possibility of the partial breaking in \( \mathcal{N} = 2 \) supergravity was found in Refs. [7–9], and there, it was shown that the partial breaking occurs by gauging a matter (hyper) sector in a specific frame where the prepotential does not exist. The systematic analysis for the partial breaking conditions based on the so-called embedding tensor [10–11] can be found in Ref. [12] (see also [13–15]). These models are studied in connection with the partial breaking in the global \( \mathcal{N} = 2 \) models [16] by taking the rigid supersymmetry limit [8–17–19]. Also, there are various discussions related to the D-brane effective actions, e.g., [20–35]. The partial supersymmetry breaking in non-Abelian gauge theories are discussed in both of the global [36–38] and the local supersymmetry cases [39] (see also [40] for review).

In this paper, we interpolate the full and the partial supersymmetry breaking to obtain the approximate partial breaking of \( \mathcal{N} = 2 \) supersymmetries. Then, the spectrum should be characterized by two independent breaking scales of the \( \mathcal{N} = 2 \) supersymmetries. In Ref. [7], the model realizes the full or the partial supersymmetry breaking, and the parameter spaces for those vacua are continuously connected. For phenomenological applications, we need matter multiplets, in addition to the goldstino multiplets. Therefore, we generalize the model in such a way that it has multiple vector multiplets, more general prepotential, and larger class of the gauging of the isometries, including the model of Ref. [7] as a special case. Then, we investigate the conditions for the supersymmetry breakings at two different scales, and compute the corresponding mass spectrum. As we will see later, there are some non-trivial relations in the spectrum in this class of models.

This paper is organized as follows. In Sec. 2, we briefly review \( \mathcal{N} = 2 \) supergravity and specify our model. In Sec. 3, we derive the stationary conditions of the potential, and discuss the supersymmetry breaking by analyzing the supersymmetry transformations of the fermions. Then, we compute the boson and the fermion masses and find some relations among them. Section 4 is devoted to the summary. In Appendix A, we collect the notations. The technical details of the embedding tensor formalism are summarized in Appendix B. In Appendix C, we list the vacuum expectation values of the gauge kinetic functions. In Appendix D, we show the interaction terms including fermion bilinears on the vacuum, which are necessary to discuss the decay modes in Sec. 3.

---

1 The full supersymmetry breakings in \( \mathcal{N} = 2 \) supergravity are also discussed based on the constrained \( \mathcal{N} = 2 \) superfields, e.g., in Refs. [3,6].

2 A similar approach in the global \( \mathcal{N} = 2 \) supersymmetry can be found in Refs. [41,42].
2 Set up

In this section, we specify the model which is a generalization of Ref. [7]. Here we follow the convention of [46] and use the unit $M_P = 1$, where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

2.1 Vector and hyper sectors

We consider $\mathcal{N} = 2$ gauged supergravity in four dimensions which contains $n_v$ Abelian vector multiplets, one hypermultiplet, in addition to the gravitational multiplet:

- **Vector multiplets** : \{ $z^i, \lambda^{iA}, A^i_{\mu}$ \}, \(i = 1, \cdots, n_v\) (2.1)
- **Hypermultiplet** : \{ $b^u, \zeta_\alpha$ \}, (2.2)
- **Gravitational multiplet** : \{ $g_{\mu\nu}, \psi^A_{\mu}, A^0_{\mu}$ \}. (2.3)

A vector multiplet contains a complex scalar $z^i$, two gauginos $\lambda^{iA}$ (A = 1, 2) and a vector $A^i_{\mu}$. A hypermultiplet contains four real scalars $b^u$ (u = 0, ···, 3)\(^4\) and two hyperinos $\zeta_\alpha$ (\(\alpha = 1, 2\))\(^3\). The gravitational multiplet contains the spacetime metric $g_{\mu\nu}$ ($\mu, \nu = 0, \cdots, 3$), two gravitinos $\psi^A_{\mu}$ (A = 1, 2) and the graviphoton $A^0_{\mu}$. Note that there are $N + 1$ vector fields in the system and they are labeled by $A^\Lambda_{\mu}$ ($\Lambda = 0, 1, \cdots, n_v$).

**Vector sector**

The vector sector is governed by the prepotential $F(X^\Lambda)$, which is a holomorphic function of $n_v + 1$ complex variables $X^\Lambda$ ($\Lambda = 0, 1, \cdots, n_v$), and is homogeneous of degree two. In general, it can be parameterized as

$$F = -i(X^0)^2 f(X^i/X^0),$$

(2.4)

where $f$ is an arbitrary holomorphic function. In $\mathcal{N} = 2$ supergravity, the theory has (on-shell) Sp(2$n_v$ + 2, $\mathbb{R}$) symmetry which acts on the holomorphic section,

$$\Omega^M(z) = \begin{pmatrix} X^\Lambda(z) \\ F_\Sigma(z) \end{pmatrix}, \quad (\Lambda, \Sigma = 0, 1, \cdots, n_v)$$

(2.5)

where $F_\Sigma = \partial F/\partial X^\Sigma$, and the index $M$ specifies $2n_v + 2$ components of the symplectic vector.

Based on the holomorphic section $\Omega$, the Kähler potential $\mathcal{K}$ is given in a manifestly symplectic invariant way as

$$\mathcal{K} = -\log(i\Omega^T \mathcal{C} \Omega) = -\log \left( i\bar{X}^\Lambda F_\Lambda - i\bar{F}_\Lambda X^\Lambda \right),$$

(2.6)

\(^3\)The hyperscalars $b^u$ (u = 0, ···, 3) have the quaternionic structure.

\(^4\)Note that $\alpha$ is not a spinor index. The spinor indices are suppressed throughout this paper.
where $C$ is a symplectic invariant tensor,
\[
C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\] (2.7)

We take a special coordinate as
\[
X^0 = 1, \quad X^i = z^i,
\] (2.8)
where $z^i$ are identified as physical scalars in vector multiplets. Then, $F_\Lambda = \{F_0, F_i\}$ becomes
\[
F_0 = -2if + iz^if, \quad F_i = -if_i,
\] (2.9)
where the subscript $i$ on $f$ denotes the derivative with respect to $z^i$. The Kähler potential is then written by
\[
K = \log K_0, \quad \text{where} \quad K_0 \equiv 2(f + \bar{f}) - (z - \bar{z})^i(f_i - \bar{f}_i).
\] (2.10)
This is the general form of the Kähler potential.

The derivatives of the Kähler potential, the Kähler metric, and the Levi-Civita connection are computed as
\[
\partial_i K = -\frac{\partial_i K_0}{K_0} = -\frac{1}{K_0} \left( f_i + \bar{f}_i - (z - \bar{z})^j f_{ij} \right),
\] (2.11)
\[
g_{ij} = \partial_i K \partial_j K - \frac{1}{K_0} (f_{ij} + \bar{f}_{ij}),
\] (2.12)
\[
\Gamma^k_i{}_{j} = g^{kl} \partial_l g_{ij} - \frac{g^{kl}}{K_0} (\partial_l \partial_j K_0 \partial_k K + \partial_i \partial_j \partial_k K_0).
\] (2.13)
Furthermore, for later convenience, we list several quantities which appear in the Lagrangian:
\[
V^M = \begin{pmatrix} L^\Lambda \\ M_\Sigma \end{pmatrix} \equiv e^\kappa/2\Omega^M = e^\kappa/2 \begin{pmatrix} X^\Lambda (z) \\ F_\Sigma (z) \end{pmatrix},
\] (2.14)
\[
U^M_i = \begin{pmatrix} f^\Lambda_i \\ h^\Lambda_{\Sigma i} \end{pmatrix} \equiv D_i V^M = \left( \partial_i + \frac{1}{2} \partial_i K \right) V^M,
\] (2.15)
\[
D_i U^M_j = \partial_i U^M_j + \frac{1}{2} \partial_i K U^M_j - \Gamma^k_i{}_{j} U^M_k = e^\kappa f_{ijk}g^{kk}\bar{U}^M_k.
\] (2.16)
Finally, the gauge kinetic functions $N_{\Lambda\Sigma}$ are given by
\[
N_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + \frac{2i \text{Im} F_{\Lambda\Gamma} \text{Im} F_{\Sigma\Pi} X^\Pi X^\Gamma}{\text{Im} F_{\Pi\Gamma} X^\Pi X^\Gamma}.
\] (2.17)
Hyper sector

As for the hyper sector, we consider the following quaternion-Kähler metric \[7\],

\[ h_{uv} = \frac{1}{2(b^0)^2} \delta_{uv}, \]  

(2.18)

which describes a nonlinear sigma model on SO(4, 1)/SO(4). As shown recently in Ref. \[15\], this is the unique metric of a single hypermultiplet for the partial breaking in Minkowski space. The vielbein \( U^{\alpha A} = U_u^{\alpha A} db^u \) can be read off as

\[ U^{\alpha A} = \frac{1}{2b^0} \epsilon^{\alpha \beta} \left( db^0 - i \sum_{x=1}^{3} \sigma^x db^x \right)^A, \]  

(2.19)

where \( A = 1, 2 \) and \( \alpha = 1, 2 \) represent the SU(2) and Sp(2) indices respectively (their conventions are shown in Appendix A). \( \sigma^x \) is the standard Pauli matrices. The SU(2) connection is given by

\[ \omega^x_u = \frac{1}{b^0} \delta^x_u. \]  

(2.20)

Note that Eq. (2.18) admits three commuting isometries:

\[ b^m \rightarrow b^m + c^m, \quad (m = 1, 2, 3) \]  

(2.21)

where \( c^m \) are real constants.\(^5\) Then, the Killing vectors \( k^u_m \) which generate these transformations and the moment maps corresponding to \( k^u_m \) are

\[ k^u_m = \delta^u_m, \quad \mathcal{P}^x_m = \frac{1}{b^0} \delta^x_m. \]  

(2.22)

2.2 Gauging

In order to discuss the supersymmetry breaking, we will gauge some of the isometries of the hyper sector. For this purpose, we employ the embedding tensor formalism \[10\][11], which is useful for discussing the general gauging of the extended supergravity (see also \[47\][48] for a review). This formalism formally introduces a double copy of the gauge fields, i.e., the electric gauge fields \( A^{\Lambda}_{\mu} \) and the magnetic gauge fields \( A_{\mu\Sigma} \) (\( \Lambda, \Sigma = 0, 1, \cdots, n_v \)), and gauges some of the global symmetries with the gauge couplings,

\[ \Theta_M^m = \begin{pmatrix} \Theta^m_{\Lambda} \\ \Theta^m_{\Sigma m} \end{pmatrix}, \]  

(2.23)

\(^5\)Although both indices \( m \) and \( x \) run over 1, 2 and 3, \( m \) labels the isometries while \( x \) is used to emphasize the SU(2) structure.
which is called the embedding tensor.

The tensor $\Theta^m_M$ must satisfy several conditions for the self-consistency of the theory \[10,11\]. In our case where no isometry on the vector sector is gauged, the only corresponding constraint is

$$\Theta^m_M \mathcal{C}^{MN} \Theta^n_N = 0. \tag{2.24}$$

Then the covariant derivative is defined by

$$D_\mu \equiv \partial_\mu - A_\mu^\Lambda \Theta^m_m T^\Lambda_m - A_\mu^\Sigma \Theta^\Sigma m T^m_m, \tag{2.25}$$

where $T_m$ are generators of the isometries (2.21), thus $k^u_m = T_m b^u$. We also define

$$k^u_M = \Theta^m_M k^u_m, \quad P^x_M = \Theta^m_M P^x_m. \tag{2.26}$$

Note that the magnetic vectors $A_\mu^\Lambda$ also participate in the gauging.

Our interest is the $\mathcal{N} = 2$ supergravity system with two breaking scales, which can realize the $\mathcal{N} = 1$ (partially broken) vacuum in some limits of the gauge couplings. We need to gauge two of the isometries in Eq. (2.21) to obtain the partial breaking \[7,9\] because the $\mathcal{N} = 1$ massive gravitino multiplet contains two massive vector fields, which come from the gauging of two isometries. Also, we need a magnetic entry in Eq. (2.23) for the partial breaking because it is proven in Ref. [12] that purely electric gauging cannot preserve $\mathcal{N} = 1$ supersymmetry. From these observations, we take the embedding tensor as

$$\Theta^m_M = \begin{pmatrix} 0 & e_2 & e_3 \\ 0 & E_i & 0 \\ 0 & 0 & 0 \\ 0 & M^i & 0 \end{pmatrix}, \tag{2.27}$$

where we choose the directions $b^2$ and $b^3$ to be gauged. All of the parameters in Eq. (2.27) are real constants. The gauge coupling constants $e_2, e_3$ and $E_i$ are the electric ones, and $M^i$ are the magnetic ones. One can check that Eq. (2.27) satisfies Eq. (2.24). This is a generalization of the one discussed in Ref. [15]. Our setup reduces to the model of Ref. [7] when $e_2 = E_i = 0$ and $n_v = 1$.

### 2.3 Action and supersymmetry transformation

Here we show the relevant parts of the Lagrangian [46,49–51]. Due to the existence of the embedding tensor as well as some magnetic vector fields, we have to introduce auxiliary two-form fields $B_{\mu\nu,m}$ for consistency (see Refs. [10,11] for detail).

The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2} R + \mathcal{L}_{\text{kin}} + \mathcal{L}_Y + \mathcal{L}_{\text{Pauli}} + \mathcal{L}_{\text{der}} + \mathcal{L}_{\text{top}} - V, \tag{2.28}$$
where $R$ is the Ricci scalar and we have omitted the four fermi interactions. $\mathcal{L}_{\text{kin}}$ consists of the kinetic terms and we further decompose it for later convenience,

\[
\mathcal{L}_{\text{kin}} = \mathcal{L}_{\text{kin},z} + \mathcal{L}_{\text{kin},b} + \mathcal{L}_{\text{kin},v} + \mathcal{L}_{\text{kin},f} \tag{2.29}
\]

\[
\mathcal{L}_{\text{kin},z} = g_{ij} \partial_{\mu} z^{i} \partial_{\mu} \bar{z}^{j} \tag{2.30}
\]

\[
\mathcal{L}_{\text{kin},b} = h_{uv} D_{\mu} b^{u} D^{\mu} b^{v} \tag{2.31}
\]

\[
\mathcal{L}_{\text{kin},v} = \frac{1}{4} I_{\Lambda \Sigma} H^{\Lambda}_{\mu \nu} H^{\Sigma}_{\mu \nu} + \frac{i}{4} R_{\Lambda \Sigma} H^{\Lambda}_{\mu \nu} \tilde{H}^{\Sigma}_{\mu \nu}, \tag{2.32}
\]

\[
\mathcal{L}_{\text{kin},f} = \frac{1}{\sqrt{-g}} \varepsilon_{\mu \nu \rho \sigma} \bar{\psi}^{A}_{\mu} D_{\rho} \psi_{B}^{B} + \frac{i}{2} g_{ij} \bar{\lambda}^{i A} \gamma_{\mu} D_{\mu} \lambda^{j A} - i \bar{\zeta}^{A} \gamma_{\mu} D_{\mu} \zeta_{A} + \text{h.c.}, \tag{2.33}
\]

where $H^{\Lambda}_{\mu \nu}$ is a gauge invariant combination of the field-strength and the two-form field,

\[
H^{\Lambda}_{\mu \nu} \equiv F^{\Lambda}_{\mu \nu} + \frac{1}{2} \Theta^{\Lambda \mu} B_{\mu \nu}, \tag{2.34}
\]

and $\tilde{H}_{\mu \nu} \equiv -\frac{i}{2} \varepsilon_{\mu \nu \rho \sigma} H^{\rho \sigma}$. $I_{\Lambda \Sigma}$ and $R_{\Lambda \Sigma}$ are imaginary and real parts of $N_{\Lambda \Sigma}$. $D_{\mu} b^{u}$ is defined in Eq. (2.25). The covariant derivatives for fermions are shown in Appendix A.

$\mathcal{L}_{Y}$ denotes the interactions including the fermion bilinears and it is explicitly given by

\[
\mathcal{L}_{Y} = 2 S_{AB} \bar{\psi}_{A}^{\Lambda} \gamma_{\mu} \psi_{B}^{\mu} + ig_{ij} W^{i A B} \bar{\lambda}_{A}^{i} \gamma_{\mu} \psi_{B}^{\mu} + 2i N^{A}_{\alpha} \bar{\zeta}^{\alpha} \gamma_{\mu} \psi_{A}^{\mu} + M^{\alpha \beta} \bar{\zeta}_{\alpha} \zeta_{\beta} + M^{\alpha}_{i B} \bar{\zeta}_{\alpha} \lambda^{i B} + M^{i A j B} \bar{\lambda}_{i A}^{j} \lambda^{j B} + \text{h.c.}, \tag{2.35}
\]

where

\[
S_{AB} \equiv \frac{i}{2} (\sigma_{x})_{AB} P_{M}^{x} V^{M}, \tag{2.36}
\]

\[
W^{i A B} \equiv i (\sigma_{x})_{AB} P_{M}^{x} g^{ij} \bar{U}_{j}^{i}, \tag{2.37}
\]

\[
N^{A}_{\alpha} \equiv -2 U_{u A} k_{M}^{u} \bar{V}^{M}, \tag{2.38}
\]

\[
M^{\alpha \beta} \equiv U_{u}^{\alpha A} U_{v}^{\beta B} \epsilon_{A B} D^{u k_{M}^{v}} V^{M}, \tag{2.39}
\]

\[
M^{\alpha}_{i B} \equiv 4 U_{i u}^{\alpha A} U_{M}^{k_{B} M}, \tag{2.40}
\]

\[
M^{i A j B} \equiv \frac{i}{2} (\sigma_{x})_{A B} P_{M}^{x} D_{i} U_{j}^{M}. \tag{2.41}
\]

In their expressions, $D_{u} k_{M}^{v} = \partial_{u} k_{M}^{v} + \epsilon_{u v}^{x y} \omega_{A}^{x} k_{M}^{y}$ and $D_{u} U_{j}^{M}$ can be found in Eq. (2.16).

The interaction terms $\mathcal{L}_{\text{Pauli}}$ and $\mathcal{L}_{\text{der}}$ are necessary in subsection 3.6 and their explicit expressions are shown in Appendix B.

$\mathcal{L}_{\text{top}}$ is required for consistency of the embedding tensor formalism and it is given by

\[
\mathcal{L}_{\text{top}} = -\frac{i}{4} \Theta^{\Lambda \mu} B_{\mu \nu, \mu} \left( F^{\mu \nu}_{A} - \frac{1}{4} \Theta^{\alpha}_{A} B_{\mu \nu}^{\alpha} \right). \tag{2.42}
\]
Finally, the scalar potential $V$ is given by

$$V = g^{ij} U_i^M U_j^N P_{M}^x P_{N}^x + 4 h_{uv} k_u^i h_v^j \bar{V}^M V^N - 3 \bar{V}^M V^N P_{M}^x P_{N}^x. \quad (2.43)$$

Note that the Lagrangian (2.28) has larger gauge symmetry, in addition to the Abelian gauge symmetry of the vector fields. Indeed, it is invariant under

$$\delta A^i = \frac{1}{2} \Theta^a \xi^a_i, \quad (2.44)$$

$$\delta A_{\mu} = \frac{1}{2} \Theta^a \xi^a_{\mu}, \quad (2.45)$$

$$\delta B_{\mu \nu} = 2 \partial_{\mu} \xi_{\nu}, \quad (2.46)$$

where $\xi^a_{\mu}$ are the gauge transformation parameters of the two-forms.

We need the supersymmetry transformations of the fermions to discuss supersymmetry breaking. The relevant parts are given by

$$\delta \psi_{A \mu} = i S_{AB} \gamma_\mu \epsilon^B + \cdots, \quad (2.47)$$

$$\delta \lambda^i_A = W^i_{AB} \epsilon^B + \cdots, \quad (2.48)$$

$$\delta \zeta^\alpha = N^A \epsilon_A + \cdots, \quad (2.49)$$

where $\epsilon^A$ are the transformation parameters. The ellipses represent other contributions, which vanish in the Minkowski vacuum.

# 3 Spectrum

Here we derive the mass spectrum of our model.

## 3.1 Scalar potential and minimization

First, let us discuss conditions the vacuum satisfies. Under the gauging (2.27), the scalar potential (2.43) is explicitly given by

$$V = \frac{e^K}{(l^0)^2} \left[ g^{ij} D_i^x D_j^x \left( \mathcal{E}^x - i f_j \mathcal{M}^{xj} + N_j^x \bar{z}^j \right) \left( \mathcal{E}^x + i f_k \mathcal{M}^{xk} + N_k^x \bar{z}^k \right) \right], \quad (3.1)$$

where we have defined

$$D_i^x \equiv \mathcal{E}^x \partial_i K - i \mathcal{M}^{xj} (f_j + \partial_i K f_j) + N_j^x (\delta^x_i + \partial_i K z^x), \quad (3.2)$$

$$\mathcal{E}^x \equiv (0, e_2, e_3), \quad \mathcal{M}^{xj} \equiv (0, M^j, 0), \quad N_j^x \equiv (0, E_j, 0). \quad (3.3)$$
Note that the scalar potential (3.1) is independent of $b^m (m = 1, 2, 3)$. Thus, the minimum is obtained by solving

$$
\partial_i V = 0, \quad \partial_{\theta} V = 0 ,
$$
(3.4)

where $\partial_i = \partial/\partial z^i$ and $\partial_{\theta} = \partial/\partial \theta^0$. They give $n_v + 1$ equations in general. By using Eq. (2.16), Eq. (3.4) can be summarized as

$$
f_{ijk} g^{ij} \tilde{D}_j^x \tilde{D}_k^x = 0 .
$$
(3.6)

In general, it is difficult to solve these equations for a general form of $f$. Therefore, for simplicity, we assume that one of $z^i$ has a nonzero expectation value (we choose it the $N$-th direction), and the vacuum satisfies the following conditions:

$$
\langle z^i \rangle = \delta^{iN} \lambda ,
$$
(3.7)

$$
\langle f_i \rangle = \delta_{iN} \langle f_N \rangle ,
$$
(3.8)

$$
\langle f_{IN} \rangle = \delta_{iN} \langle f_{NN} \rangle ,
$$
(3.9)

$$
\langle f_{NNi} \rangle = \delta_{iN} \langle f_{NNN} \rangle .
$$
(3.10)

Then, the derivative of the Kähler potential and the Kähler metric become

$$
\langle \partial_i K \rangle = \delta_{iN} \langle \partial_N K \rangle ,
$$
(3.11)

$$
\langle g_{iN} \rangle = \delta_{iN} \left( | \langle \partial_N K \rangle |^2 - \langle f_{NN} + \bar{f}_{NN} \rangle / \langle K_0 \rangle \right) .
$$
(3.12)

Later, we will check that these assumptions for the vacuum are valid in a concrete choice of $f$. Furthermore, we assume that the gauging is done only for the $N$-th direction, i.e.,

$$
\mathcal{M}^{xj} = \delta^{iN} M^N , \quad \mathcal{N}_j^x = \delta_{jN} E_N ,
$$
(3.13)

which lead to

$$
\langle D_i^x \rangle = \delta_{iN} \langle D_N^x \rangle .
$$
(3.14)

Under these simplifications, Eq. (3.6) is reduced to just one equation,

$$
\langle f_{NNN} (g^{NN})^2 \tilde{D}_{N}^x \tilde{D}_{N}^x \rangle = 0 ,
$$
(3.15)

and the others are trivially satisfied. Therefore, if the vacuum satisfies either

$$
\langle (\tilde{D}_N^x)^2 \rangle = 0 ,
$$
(3.16)

or

$$
\langle f_{NNN} \rangle = 0 ,
$$
(3.17)
then Eq. (3.4) is satisfied at that point.

Next, one can check that Eq. (3.5) is satisfied if

\[ \langle f_{NN} \rangle = -i \frac{E_N}{M_N}. \]  

(3.18)

In this case, we obtain \( \langle \partial_N K \rangle = \langle \partial_S K \rangle \) and \( \langle g_{NN} \rangle = | \langle \partial_N K \rangle |^2 \) since \( \langle f_{NN} \rangle \) is pure imaginary (see Eqs. (2.11) and (2.12)). Also, the condition (3.18) leads to the Minkowski vacuum.

Under the condition (3.18), Eq. (3.16) can be simplified further as,

\[ \langle f_N \rangle = -\frac{1}{M_N} (i\epsilon_2 + iE_N\lambda \pm \epsilon_3). \]  

(3.19)

In the following discussion, we take the plus branch.\(^6\)

In summary, under the assumptions (3.7)-(3.10) and (3.13), the vacuum must satisfy either of the two conditions:

(i) Eqs. (3.18) and (3.17),

(ii) Eqs. (3.18) and (3.19).

These conditions have been derived in Ref. [15] in the case of a single vector multiplet \( (n_v = 1) \). We will investigate how many supersymmetries are (un)broken in the vacuum in the next subsection.

### 3.2 Supersymmetry transformation

Let us begin with the case (ii). Under the conditions (3.18) and (3.19), the supersymmetry transformations (2.47)-(2.49) in the vacuum become

\[ \langle \delta \psi_{A\mu} \rangle = \left( \frac{i\epsilon_3 e^{K/2}}{2b^0} \right) \left( \begin{array}{cc} -1 & 1 \\ 1 & -1 \end{array} \right) \gamma^\mu \left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right), \]  

(3.20)

\[ \langle \delta \lambda^A \rangle = \delta^{iN} \left( \frac{i\epsilon_3 e^{K/2}}{b^0} \right) g^{NS} \partial_S \mathcal{K} \left( \begin{array}{cc} -1 & 1 \\ 1 & -1 \end{array} \right) \left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right), \]  

(3.21)

\[ \langle \delta \zeta_\alpha \rangle = \left( \frac{i\epsilon_3 e^{K/2}}{b^0} \right) \left( \begin{array}{cc} -1 & 1 \\ -1 & 1 \end{array} \right) \left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right). \]  

(3.22)

As can be seen, all of the matrices have a zero eigenvalue. Indeed, defining

\[ \phi_\pm = \frac{1}{\sqrt{2}} (\phi_1 \pm \phi_2), \]  

(3.23)

\(^6\)For the choice of the minus sign, see the comment below Eq. (3.26).
with $\phi = \{\psi, \lambda, \zeta, \epsilon\}$, we can rewrite them as

$$
\begin{align*}
\left( \frac{\langle \delta \psi_+^\mu \rangle}{\langle \delta \psi_-^\mu \rangle} \right) &= \left( \frac{-e_3 e^{K/2}}{b^0} \right) \gamma_\mu \left( \begin{array}{c} 0 \\ \epsilon^- \end{array} \right), \\
\left( \frac{\langle \delta \lambda^{i+} \rangle}{\langle \delta \lambda^{i-} \rangle} \right) &= \delta^i N \left( -\frac{2ie_3 e^{K/2}}{b^0} g^{\bar{N}N} \partial_{\bar{N}} K \right) \left( \begin{array}{c} 0 \\ \epsilon_- \end{array} \right), \\
\left( \frac{\langle \delta \zeta^+ \rangle}{\langle \delta \zeta^- \rangle} \right) &= \left( -\frac{2ie_3 e^{K/2}}{b^0} \right) \left( \begin{array}{c} \epsilon_- \\ 0 \end{array} \right).
\end{align*}
$$

Therefore, one of the two supersymmetries is broken (for $\epsilon^-$ direction), but the other one is still preserved (for $\epsilon^+$ direction). The minus sign in Eq. (3.19) leads to the preservation of $\epsilon^-$ direction. We conclude that for nonzero $e_3$ and $M^N$, we always have $N = 1$ preserving vacuum in the case (ii).\footnote{The global supersymmetry limit of this partial breaking vacuum have been discussed in Refs. [8,17–19].}

In the case (i), on the other hand, the supersymmetry transformations are

$$
\begin{align*}
\langle \delta \psi_{\Lambda^\mu} \rangle &= \left( \frac{e^{K/2}}{2b^0} \right) \left( \begin{array}{cc} \tau & e_3 \\ e_3 & \tau \end{array} \right) \gamma_\mu \left( \begin{array}{c} \epsilon^1 \\ \epsilon^2 \end{array} \right), \\
\langle \delta \lambda^{iA} \rangle &= \delta^i N \left( \frac{i e^{K/2}}{b^0} g^{\bar{N}N} \partial_{\bar{N}} K \right) \left( \begin{array}{cc} \bar{\tau} & e_3 \\ e_3 & \bar{\tau} \end{array} \right) \left( \begin{array}{c} \epsilon^1 \\ \epsilon^2 \end{array} \right), \\
\langle \delta \zeta_\alpha \rangle &= \left( -\frac{i e^{K/2}}{b^0} \right) \left( \begin{array}{cc} e_3 & \bar{\tau} \\ -\bar{\tau} & -e_3 \end{array} \right) \left( \begin{array}{c} \epsilon^1 \\ \epsilon^2 \end{array} \right),
\end{align*}
$$

where

$$
\tau \equiv i e_2 + i E_N \lambda + M^N \langle f_N \rangle.
$$

In this case, the value of $\langle f_N \rangle$ is not determined by the vacuum conditions. The vacuum can be broken to $N = 0$ depending on $\langle f_N \rangle$. If we choose $\langle f_N \rangle$ as $\langle f_N \rangle = -\frac{1}{M^N} (i e_2 + i E_N \lambda + e_3 + \Delta)$, then $N = 1$ supersymmetry is preserved. Thus, we can interpolate the full and the partial breakings in this case.\footnote{In the purely electric gauging, it is possible to break full $N = 2$ supersymmetry, but the supplemental condition for the partial breaking (3.19) is never satisfied [12].}

In order to characterize the deviation from the partial-breaking condition (3.19), we parametrize $\langle f_N \rangle$ as

$$
\langle f_N \rangle = -\frac{1}{M^N} (i e_2 + i E_N \lambda + e_3 + \Delta),
$$

where we have introduced $\Delta$ (complex) as an order parameter of $N = 0$ breaking. $\Delta = 0$ recovers $N = 1$ preserving (partially broken) vacuum. Then, Eqs. (3.27)-(3.29) are expressed
\[ \langle \delta \psi_{A\mu} \rangle = \left\langle -\frac{e^{K/2}}{2b^0} \left( \begin{array}{cc} e_3 + \Delta & -e_3 \\ e_3 & e_3 + \Delta \end{array} \right) \right\rangle \gamma_{\mu} \left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right), \quad (3.32) \]
\[ \langle \delta \lambda^{iA} \rangle = \delta^{iN} \left\langle -\frac{ie^{K/2}}{b^0} g^{N\bar{N}} \partial_{\bar{N}K} \right\rangle \left( \begin{array}{cc} e_3 + \bar{\Delta} & -e_3 \\ e_3 & e_3 + \bar{\Delta} \end{array} \right) \left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right), \quad (3.33) \]
\[ \langle \delta \zeta_\alpha \rangle = \left\langle -\frac{ie^{K/2}}{2b^0} \right\rangle \left( \begin{array}{cc} e_3 & -e_3 - \bar{\Delta} \\ e_3 + \bar{\Delta} & -e_3 \end{array} \right) \left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right), \quad (3.34) \]

In terms of the basis (3.23), these are rewritten as
\[ \left( \begin{array}{c} \langle \delta \psi_{+\mu} \rangle \\ \langle \delta \psi_{-\mu} \rangle \end{array} \right) = \left\langle -\frac{e^{K/2}}{2b^0} \right\rangle \gamma_{\mu} \left( \begin{array}{c} \Delta \epsilon^+ \\ (2e_3 + \Delta)\epsilon^- \end{array} \right), \quad (3.35) \]
\[ \left( \begin{array}{c} \langle \delta \lambda^{i+} \rangle \\ \langle \delta \lambda^{-} \rangle \end{array} \right) = \delta^{iN} \left\langle -\frac{ie^{K/2}}{b^0} g^{N\bar{N}} \partial_{\bar{N}K} \right\rangle \left( \begin{array}{c} \bar{\Delta} \epsilon^+ \\ (2e_3 + \Delta)\epsilon_- \end{array} \right), \quad (3.36) \]
\[ \left( \begin{array}{c} \langle \delta \zeta_+ \rangle \\ \langle \delta \zeta_- \rangle \end{array} \right) = \left\langle -\frac{ie^{K/2}}{b^0} \right\rangle \left( \begin{array}{c} (2e_3 + \bar{\Delta})\epsilon_- \\ -\bar{\Delta}\epsilon_+ \end{array} \right). \quad (3.37) \]

The full supersymmetry breaking occurs unless \( \Delta = 0, -2e_3 \). In the following, we call the supersymmetry transformations caused by \( \epsilon_+ \) and \( \epsilon_- \) as the first and the second supersymmetry transformations, respectively.

### 3.3 Explicit model

Let us specify our model. We assume that \( f \) has the following form,
\[ f = c_0 + c_i z^i + \frac{1}{2} c_{ij} z^i z^j + \frac{1}{6} c_{ijk} z^i z^j z^k, \quad (3.38) \]
where \( c_0, c_i, c_{ij} \) and \( c_{ijk} \) are complex constants and totally symmetric for their indices. This satisfies Eqs. (3.17) and (3.18) if
\[ c_{NNN} = 0, \quad (3.39) \]
\[ c_{NN} = -i \frac{E_N}{2M_N}. \quad (3.40) \]

Also, from the assumptions (3.7)-(3.10), we obtain
\[ f = c_0 + c_N z^N + c_{NN}(z^N)^2 + c_{ab} z^a z^b + 3c_{abc} z^a z^b z^c + c_{a\hat{a}} z^{\hat{a}} z^{\hat{b}} z^{\hat{c}}, \quad (3.41) \]
where we have divided the indices \( i = \{ \hat{a}, N \} \) with \( \hat{a} = 1, \cdots, n_\nu - 1 \). \( ^{10} \)

\(^9\)In Ref. [7], the case \( f = z \) is considered.

\(^{10}\)The convention of the indices is summarized in Appendix \( \Delta \).
3.4 Fermion mass

Let us check the spectrum of the fermion sector. In the vacuum, fermion mass terms from the interactions in Eq. (2.35) are evaluated as

\[ \mathcal{L}_Y = \left\langle \frac{ie^{\mathcal{K}/2}}{b^0} \right\rangle (\Delta \psi^+_\mu \gamma^{\mu\nu} \psi^+_\nu + (\Delta + 2e_3) \psi^-_\mu \gamma^{\mu\nu} \psi^-_\nu) \]

\[ + \left\langle \frac{e^{\mathcal{K}/2}}{b^0} \partial_N \mathcal{K} \right\rangle \left( \Delta \tilde{\lambda}_+^N \gamma_\mu \psi^+_\mu + (\Delta + 2e_3) \tilde{\lambda}_-^N \gamma_\mu \psi^-_\mu \right) \]

\[ + \frac{2e^{\mathcal{K}/2}}{b^0} \left( (\Delta + 2e_3) \tilde{\zeta}_+ \gamma_\mu \psi^+_\mu - \Delta \tilde{\zeta}_- \gamma_\mu \psi^-_\mu \right) \]

\[ + \left\langle \frac{ie^{\mathcal{K}/2}}{b^0} \right\rangle \left( (\Delta + 2e_3) \tilde{\zeta}_+ \lambda^N - \Delta \tilde{\zeta}_- \lambda^{N+} \right) \]

\[ - \left\langle \frac{ie^{3\mathcal{K}/2}}{2b^0} g^{N\bar{N}} \partial_N \mathcal{K} f_{\bar{a}aN} \right\rangle \left( \Delta \tilde{\lambda}_+^a \lambda^\bar{b} - \Delta \tilde{\lambda}_-^a \lambda^\bar{b} + (\Delta + 2e_3) \tilde{\lambda}_+^a \lambda^\bar{b} + \text{h.c.} \right) \]

\[ \equiv m_1 \psi^+_\mu \gamma^{\mu\nu} \psi^+_\nu + im_1 \bar{\lambda} \gamma_\mu \psi^+_\mu + \frac{m_1}{3} \bar{\lambda} \lambda - \frac{m_1}{3} \bar{\eta} \eta \]

\[ + m_2 \psi^-_\mu \gamma^{\mu\nu} \psi^-_\nu + im_2 \bar{\lambda} \gamma_\mu \psi^-_\mu + \frac{m_2}{3} \bar{\lambda} \lambda - \frac{m_2}{3} \bar{\eta} \eta \]

\[ + \sum_{\bar{a} = 1}^{N-1} \left( \frac{1}{2} G m_+ \tilde{\lambda}_+^a \lambda^\bar{b} + \frac{1}{2} G m_- \tilde{\lambda}_-^a \lambda^\bar{b} \right) + \text{h.c.} \quad (3.42) \]

In the second line, we have defined

\[ m_1 = \left\langle \frac{ie^{\mathcal{K}/2}}{b^0} \right\rangle \Delta, \quad m_2 = \left\langle \frac{ie^{\mathcal{K}/2}}{b^0} \right\rangle (\Delta + 2e_3), \quad (3.43) \]

\[ m_- = \left\langle -\frac{ie^{3\mathcal{K}/2}}{b^0} \sqrt{g^{N\bar{N}}} C G \right\rangle \bar{\Delta}, \quad m_+ = \left\langle -\frac{ie^{3\mathcal{K}/2}}{b^0} \sqrt{g^{N\bar{N}}} C G \right\rangle (\Delta + 2e_3), \quad (3.44) \]

\[ \chi \equiv \langle \partial_N \mathcal{K} \rangle \lambda^{N+} - 2\zeta_- \quad \eta \equiv \langle \partial_N \mathcal{K} \rangle \lambda^{N+} + \zeta_-, \quad (3.45) \]

\[ \bar{\chi} \equiv \langle \partial_N \mathcal{K} \rangle \lambda^{N-} + 2\zeta_+ \quad \bar{\eta} \equiv \langle \partial_N \mathcal{K} \rangle \lambda^{N-} + \zeta_. \quad (3.46) \]

Also, we have assumed that

\[ \langle f_{\bar{a}aN} \rangle = C \delta_{\bar{a}a}, \quad \langle g_{\bar{a}a} \rangle = G \delta_{\bar{a}a}, \quad (3.47) \]

where \( C \) and \( G \) are complex and real constants, for simplicity\(^{11}\)

\(^{11}\)These can be achieved by choosing \( c_{\bar{a}a}, c_{\bar{a}aN} \propto \delta_{\bar{a}a} \) in Eq. (3.41).
Thus, in the unitary gauge, χ

In terms of χ(\bar{\chi}) and η(\tilde{\eta}) defined above, we can rewrite them as

In Eqs. (3.42) and (3.49), χ and \bar{\chi} are the goldstinos which correspond to the first and the second supersymmetry breaking, respectively. It can be checked directly by

Thus, in the unitary gauge, χ = \bar{\chi} = 0, they disappear from the spectrum. Then, we obtain

This gives the free parts of the fermion sector. Note that we have two massive pairs, \psi_+, \eta and \psi_-, \tilde{\eta}, whose masses are given by |m_1| and |m_2| respectively. In addition, there are 2n_\nu - 2 massive gauginos. The n_\nu - 1 gauginos (\lambda^\pm) have the mass |m_+|, and the others (\lambda'^\pm) have |m_-|\footnote{\lambda^\pm \rightarrow \frac{1}{\sqrt{2}} \lambda^\pm \text{ and } \eta_\pm (\tilde{\eta}_\pm) \rightarrow \sqrt{\frac{3}{2}} \eta_\pm (\sqrt{\frac{3}{2}} \tilde{\eta}_\pm) lead to the canonical normalization.}. Their mass scales are splitting due to the existence of \epsilon_3.
3.5 Boson mass

First, let us focus on the following terms

\[ \mathcal{L}_B \equiv \mathcal{L}_{\text{kin},v} + \mathcal{L}_{\text{kin},b} + \mathcal{L}_{\text{top}} \]

\[ = \frac{1}{4} \mathcal{I}_{\Lambda \Sigma} \mathcal{H}_{\mu \nu}^\Lambda \mathcal{H}^{\Sigma \mu \nu} + \frac{i}{4} \mathcal{R}_{\Lambda \Sigma} \mathcal{H}_{\mu \nu}^\Lambda \mathcal{H}^{\Sigma \mu \nu} + h_{uv} D_\mu b^u D_\mu b^v \]

\[ - \frac{i}{4} \Theta^{\Lambda m} \bar{B}_{\mu \nu,m} \left( F_{\mu \nu}^\Lambda - \frac{1}{4} \Theta^n_{\Lambda} B_{\mu \nu}^n \right). \] (3.52)

Each term can be decomposed explicitly as

\[ \mathcal{H}_{\mu \nu}^\Lambda = F_{\mu \nu}^\Lambda + \frac{1}{2} \Theta^{\Lambda m} B_{\mu \nu,m} = \begin{pmatrix} F_{\mu \nu}^0 \\ F_{\mu \nu}^N \\ F_{\mu \nu} + \frac{1}{2} M_N B_{\mu \nu,2} \end{pmatrix}, \] (3.53)

\[ - \frac{i}{4} \Theta^{\Lambda m} \bar{B}_{\mu \nu,m} \left( F_{\mu \nu}^\Lambda - \frac{1}{4} \Theta^n_{\Lambda} B_{\mu \nu}^m \right) = - \frac{i}{4} M_N \bar{B}_{\mu \nu,2} \left( F_{\mu \nu}^N - \frac{1}{4} E_N B_{\mu \nu}^N \right), \] (3.54)

and

\[ D_\mu b^2 = \partial_\mu b^2 - (e_2 A_{\mu}^0 + e_N A_{\mu}^N + A_{\mu N} M_N), \]

\[ D_\mu b^3 = \partial_\mu b^3 - A_{\mu}^0 e_3. \] (3.55)

Note that there seems \( n_v + 2 \) vectors \( A_{\mu}^0, A_{\mu}^\hat{a}, A_{\mu}^N, \) and \( A_{\mu N} \) at a glance. However, we can gauge away \( A_{\mu \nu}^N \) by

\[ B_{\mu \nu,2} \rightarrow B_{\mu \nu,2} - \frac{2}{M_N} F_{\mu \nu}^N, \quad A_{\mu N} \rightarrow A_{\mu N} - \frac{E_N}{M_N} A_{\mu}^N, \] (3.57)

and eliminate its degree of freedom. Therefore, we obtain \( n_v + 1 \) vectors as usual. Also, \( A_{\mu N} \) and \( B_{\mu \nu,2} \) do not have kinetic terms, and should be integrated out.\(^{13}\) As shown in Appendix \[3\], eliminating \( B_{\mu \nu,2} \) gives a kinetic term for \( A_{\mu N} \). We only show the result here.

After integrating out the two-form field, Eq. (3.52) at the vacuum is given by

\[ \mathcal{L}_B = \frac{1}{4} \langle \mathcal{I}_{00} \rangle F_0^0 F^0 + \frac{1}{2} \langle \mathcal{I}_{N 0} \rangle F_N F^0 + \frac{1}{4} \langle \mathcal{I}^{NN,0} \rangle F_N F_N + \frac{1}{4} \langle \mathcal{I}_{\hat{a} \hat{b}} \rangle F_{\hat{a}} F_{\hat{b}} \]

\[ + \left\{ \frac{1}{2 (b_0)^2} \right\} \left\{ \sum_{u=0}^1 \partial_\mu b^u \partial^\mu b^u + (\partial_\mu b^3 - e_3 A_{\mu}^0)^2 + (\partial_\mu b^2 - e_2 A_{\mu}^0 - M_N A_{\mu N})^2 \right\}. \] (3.58)

\(^{13}\) \( B_{\mu \nu,1} \) and \( B_{\mu \nu,3} \) are absent under the parametrization \(^{2.27}\).

\(^{14}\) The vacuum expectation values of the gauge kinetic function are shown in Appendix \[C\].
⟨\hat{I}_{00}\rangle = -\frac{\langle e^{-K/2} \rangle}{4} \frac{(e_3 + \text{Re}\Delta)^2 + (e_2 + \text{Im}\Delta)^2}{(e_3 + \text{Re}\Delta)^2}, \quad (3.59)

⟨\hat{I}_{ab}\rangle = \text{Re} \left( 2c_{ab} + 6\lambda c_{abN}\right), \quad (3.60)

⟨\hat{I}_{N0}\rangle = -\frac{\langle e^{-K/2} \rangle}{4} \frac{M^N(e_2 + \text{Im}\Delta)}{(e_3 + \text{Re}\Delta)^2}, \quad (3.61)

⟨\hat{I}_{NN}\rangle = -\frac{\langle e^{-K} \rangle}{4} \frac{(M^N)^2}{(e_3 + \text{Re}\Delta)^2}. \quad (3.62)

For notational simplicity, here we have omitted the spacetime indices of the field-strength \( F_{\mu\nu} \).

It can be found that two hyperscalars, \( b^2 \) and \( b^3 \), can be eliminated by

\[
A^0_\mu \to A^0_\mu + \frac{1}{e_3} \partial_\mu b^3, \\
A'_\mu \to A'_\mu + \frac{1}{M^N} \partial_\mu b^2. 
\]

(3.63) \hspace{1cm} (3.64)

where we have redefined \( A_{\mu N} \) as

\[
A'_\mu = A_{\mu N} + \frac{e_2}{M^N} A^0_\mu. 
\]

(3.65)

Then, the Lagrangian becomes

\[
\mathcal{L}_B = \frac{1}{4} \langle \hat{I}_{00} \rangle F^0 F^0 + \frac{1}{2} \langle \hat{I}_{N0} \rangle F_N F^0 + \frac{1}{4} \langle \hat{I}_{NN} \rangle F_N F_N + \frac{1}{4} \langle \hat{I}_{ab} \rangle F^a F^b \\
+ \left( \frac{1}{2(b^2)^2} \right) \left\{ \sum_{u=0}^1 \partial_\mu b^u \partial^\mu b^u + (e_3 A^0_\mu)^2 + (M^N A'_\mu)^2 \right\}. 
\]

(3.66)

It contains \( n_v - 1 \) massless vector fields and 2 massive ones. There are also two massless hyperscalars, \( b^0 \) and \( b^1 \). For the massive modes, we can diagonalize their kinetic and mass matrices by

\[
\begin{pmatrix}
B_\mu \\
B'_\mu
\end{pmatrix} = \frac{\langle e^{-K/2} \rangle}{2(e_3 + \text{Re}\Delta)} \begin{pmatrix}
\sqrt{E_+ e_3 \cos \theta} & \sqrt{E_+ M^N \sin \theta} \\
-\sqrt{E_- e_3 \sin \theta} & \sqrt{E_- M^N \cos \theta}
\end{pmatrix} \begin{pmatrix}
A^0_\mu \\
A'_\mu
\end{pmatrix},
\]

where

\[
E_\pm = 1 + \frac{\text{Re}\Delta}{e_3} + \frac{|\Delta|^2}{2e_3^2} \pm \frac{|\Delta|}{2e_3^2} \sqrt{|\Delta|^2 + 4e_3 \text{Re}\Delta + 4e_3^2},
\]

\[
\tan 2\theta = \frac{2e_3 \text{Im}\Delta}{|\Delta|^2 + 2e_3 \text{Re}\Delta}.
\]
Finally, we obtain

\[
\mathcal{L}_B = -\frac{1}{4} F(B) F(B) - \frac{1}{4} F(B') F(B') + \frac{1}{4} \langle I_{ab} \rangle F^a F^b \\
+ \left\langle \frac{1}{2 (b^0)^2} \right\rangle \sum_{u=0}^1 \partial_\mu b^u \partial^\mu b^u + \frac{1}{2} m_B^2 B_\mu^2 + \frac{1}{2} m_B'^2 B'_\mu^2,
\]

(3.70)

where \( F(B) \) and \( F(B') \) denote the field-strength of \( B_\mu \) and \( B'_\mu \). Their masses are given by

\[
m_B^2 = \left\langle \frac{e^\kappa}{(b^0)^2} \right\rangle 4 e_3^2 E_-,
\]

(3.71)

\[
m_B'^2 = \left\langle \frac{-e^\kappa}{(b^0)^2} \right\rangle 4 e_3^2 E_+.
\]

(3.72)

Remarkably, \( m_B \) and \( m_B' \) are related to \( m_1 \) and \( m_2 \) by

\[
m_B = \|m_2| - |m_1||,
\]

(3.73)

\[
m_B' = |m_2| + |m_1|.
\]

(3.74)

We see some implications of their hierarchical structures.

Finally, let us check the mass of \( z^i \). Defining the fluctuation around the vacuum,

\[
z^i = \langle z^i \rangle + \tilde{z}^i,
\]

(3.75)

we can expand the scalar potential as

\[
V = \langle V \rangle + \left( \langle \partial_i V \rangle \tilde{z}^i + \text{h.c.} \right) + \left( \frac{1}{2} \langle \partial_i \partial_j V \rangle \tilde{z}^i \tilde{z}^j + \text{h.c.} \right) + \cdots,
\]

(3.76)

where the ellipsis denotes higher order couplings of \( \tilde{z}^i \). The first and the second terms vanish due to the (Minkowski) vacuum conditions.\[15\] The third and fourth terms are expressed as

\[
\langle \partial_a \partial_b V \rangle = \delta_{ab} \left\langle \frac{2 e^{3\kappa}}{(b^0)^2} \frac{C |C|^2}{G} g^{N\bar{N}} \right\rangle (|\Delta|^2 + 2 e_3 \text{Re} \Delta + 2 e_3^2),
\]

(3.77)

\[
\langle \partial_a \partial_b V \rangle = -\delta_{ab} \left\langle \frac{2 e^{3\kappa}}{(b^0)^2} \frac{C^2}{G} g^{N\bar{N}} \right\rangle \Delta (\bar{\Delta} + 2 e_3),
\]

(3.78)

\[
\langle \partial_N \partial_{\bar{N}} V \rangle = \langle \partial_N \partial_{\bar{a}} V \rangle = \langle \partial_N \partial_N V \rangle = \langle \partial_N \partial_{\bar{a}} V \rangle = 0,
\]

(3.79)

by assuming Eq. (3.47). Note that \( \tilde{z}^N \) is massless. Taking into account the canonical normalization (\( \tilde{z}^a \rightarrow \tilde{z}^a / G \)), and diagonalizing the mass matrix, we obtain

\[
V = \frac{1}{2} m_2^2 (x^a)^2 + \frac{1}{2} m_2' (y^a)^2 + \cdots,
\]

(3.80)

\[15\text{In order to avoid the runaway of } b^0, \text{ the vacuum should be the Minkowski one.}\]
where

\[
m_x = |m_+| + |m_-|, \\
m_y = ||m_+| - |m_-||, \\
\left( \begin{array}{c} x^a \\ y^a \end{array} \right) = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{array} \right) \left( \begin{array}{c} \text{Re} \tilde{z}^a \\ \text{Im} \tilde{z}^a \end{array} \right), \\
\tan 2 \varphi = -\frac{\text{Im}(m_+m_-)}{2\text{Re}(m_+m_-)}
\]

(3.81) \quad (3.82) \quad (3.83) \quad (3.84)

The mass of $\tilde{z}^a$ and $\lambda^{a\pm}$ are related by Eqs. (3.81) and (3.82) under the choice of Eq. (3.47). In general cases, this relation does not seem to hold unlike the relations (3.73) and (3.74). For general values of $c_{\hat{a}\hat{b}}$ and $c_{\hat{a}\hat{b}N}$, the gaugino masses (the last line in Eq. (3.42)) are given by

\[
\left\langle -\frac{i}{2b^0} e^{3\kappa/2} \sqrt{g^{NN}} f_{\hat{a}\hat{b}N} \right\rangle \Delta \lambda^{a-} - \lambda^{b-} + \left\langle -\frac{i}{2b^0} e^{3\kappa/2} \sqrt{g^{NN}} f_{\hat{a}\hat{b}N} \right\rangle (\Delta + 2e_3) \lambda^{a+} + \lambda^{b+},
\]

(3.85)

and the gauginos have different masses depending on $c_{\hat{a}\hat{b}}$ and $c_{\hat{a}\hat{b}N}$. The boson masses of $\tilde{z}^a$ are also similar.

### 3.6 Summary of mass spectrum

In Table 1, we summarize the spectrum (at the tree level) obtained in the previous subsections, where $\Delta' \equiv \Delta + 2e_3$.

As we saw before, the first supersymmetry ($\epsilon_+$) recovers in the limit $\Delta \to 0$. In this case, the first gravitino $\psi_{\mu+}$ becomes massless ($|m_1| = 0$), and it belongs to the $\mathcal{N} = 1$
gravitational multiplet with $g_{\mu\nu}$. The second gravitino $\psi_{\mu-}$ and $\tilde{\eta}_{\bullet}$, $B_{\mu}, B'_{\mu}$ have degenerate masses ($|m_2| = m_B = m_{B'}$), and they form an $\mathcal{N} = 1$ massive spin 3/2 multiplet. Also, half of the gauginos $\lambda^{\hat{a}-}$ become massless, and with $A^{\hat{a}}$, they form $n_{v} - 1$ massless $\mathcal{N} = 1$ vector multiplets. The remaining gauginos $\lambda^{\hat{a}+}$ have the mass $|m_+|$, and they form $n_{v} - 1$ massive $\mathcal{N} = 1$ chiral multiplets. Finally, $\{\zeta_{-}, b^{0}, b^{1}\}$ and $\{\lambda^{\hat{N}+}, z^{\hat{N}}\}$ form $\mathcal{N} = 1$ massless chiral multiplets.

On the other hand, $\Delta' = 0$ (or $\Delta = -2\epsilon_3$) corresponds to the case where the second supersymmetry ($\epsilon_{-}$) recovers and the first one ($\epsilon_{+}$) is broken, which can be seen from Eqs. (3.35)-(3.37). The mass spectrum is the same as the case of $\Delta = 0$, but the roles of $\psi_{\mu-}, \tilde{\eta}_{\bullet}$, and $\lambda^{\hat{a}+}$ are just replaced by $\psi_{\mu+}, \eta_{\bullet}$, and $\lambda^{\hat{a}-}$, respectively.

Toward a phenomenological application, let us consider a case of hierarchical supersymmetry breaking $|\Delta'| \gg |\Delta|$. Then, the mass spectrum of the massive vectors $\{B_{\mu}, B'_{\mu}\}$ and the fermions $\Psi_{-} \equiv \{\psi_{\mu-}, \tilde{\eta}_{\bullet}\}$ and $\Psi_{+} \equiv \{\psi_{\mu+}, \eta_{\bullet}\}$ satisfies

$$m_{B'} \gtrsim |m_2| \gtrsim m_B \gg |m_1|,$$

(3.86)

From the results shown in Appendix D, we find that all the interactions among these fields schematically take the following forms:

$$B\Psi_{-}\Psi_{+},$$

(3.87)

$$B'\Psi_{-}\Psi_{+}.$$  
(3.88)

Through these interactions, the possible decay processes of the heavy fields $B'_{\mu}$ and $\Psi_{-}$ are

$$\Psi_{-} \rightarrow \Psi_{+}, B,$$

(3.89)

$$B' \rightarrow \Psi_{-}, \Psi_{+}.$$  
(3.90)

However, due to the mass relations (3.73) and (3.74), their decay rates vanish at least at the tree level.

As for the other massive fields $x^{\hat{a}}, y^{\hat{a}}$, and $\lambda^{\hat{a} \pm}$, there are no specific mass relations since their masses depend on the free parameter $C$ (or $c_{\hat{a}\hat{b}N}$ in general). Thus, their decays to the light states in $\Psi_{+}$ can have nonvanishing rates.

In summary, we found that the direct decays from the heavy particles $B, B', \Psi_{-}$ to the light ones in $\Psi_{+}$ are not allowed (at the tree level), but those from $x^{\hat{a}}, y^{\hat{a}}, \lambda^{\hat{a} \pm}$ to $\Psi_{+}$ can be allowed depending on $C$. We schematically show these allowed/forbidden decay processes in Fig. 1. These restrictions may become important when we discuss the cosmological history based on models of extended supergravity.

4 Conclusion

In this paper, we have investigated the $\mathcal{N} = 2$ supergravity model that interpolates the full and the partial breakings of supersymmetries, which is a generalization of Ref. 7. We
extend the model by introducing additional vector multiplets. As can be inferred from the studies of the partial breaking of extended supergravity [7–9, 12], the magnetic gauging is important and we chose the embedding tensor as Eq. (2.27) with Eq. (3.13). Then we found the conditions for an $\mathcal{N} = 0$ Minkowski vacuum, which is continuously connected to $\mathcal{N} = 1$ preserving ones.

The breaking scales of the two supersymmetries can be chosen independently in our model (see Eqs. (3.35)-(3.37)). Thus, we can discuss the case of the cascade supersymmetry breakings in which the two supersymmetry breaking scales are separated hierarchically. This is phenomenologically interesting when we consider the string compactifications. In such a case, an approximate $\mathcal{N} = 1$ supersymmetry appears at intermediate energies between the two scales. In contrast to other phenomenological supersymmetric models, we can quantitatively discuss the effects of the second supersymmetry breaking on light modes in the approximate $\mathcal{N} = 1$ sector.

We computed the mass spectrum (see table 1), which is indeed characterized by two different scales, $|\Delta|$ and $|\Delta'|$. We found that there are non-trivial relations in the spectrum, (3.73) and (3.74). Even for $|\Delta'| \gg |\Delta|$, there are no direct decay processes from the heavy fields ($B, B', \Psi_-$) whose masses are of $\mathcal{O}(|\Delta'|)$ to the first gravitino ($\Psi_+$) in our model (see figure. 1). This property may be important when models based on extended supergravity are applied to cosmological scenarios.

There are several issues to be addressed: The first one is the generality of the gauging. In this work, we have not discussed the most general gauging, taking the embedding tensor

It is remarkable that supergravity in higher dimensional spacetime (such as the low energy effective theories of superstrings) inevitably becomes an extended supergravity in four dimensions at low energies, if the supersymmetry-breaking scales are lower than the compactification scales.
as Eq. (2.27) with the simplification (3.13). While our model includes that of Ref. [7] as a special case, it is worth investigating the other gauging and checking how general our result is. Also, we may allow other vacuum solutions without assuming the ansatz (3.7)-(3.10), and include higher-order terms in \( z^i \) of the prepotential \( f \). It would be also interesting to extend this analysis to the system with multiple hypermultiplets and non-Abelian gauge symmetries as in Ref. [39]. We leave them for future works.

Acknowledgements

Authors would like to thank Hajime Otsuka and Yusuke Yamada for useful discussion. H. A. is supported in part by JSPS KAKENHI Grant Number JP16K05330 and also supported by Institute for Advanced Theoretical and Experimental Physics, Waseda University. S.A. is supported in part by a Waseda University Grant for Special Research Projects (Project number: 2018S-141).

A Notations

A.1 Index of vector fields

The index of the vector fields \( \Lambda \) is decomposed as in table 2.

| \( \Lambda \) | \( \{0,1\cdots,n_v\} \) |
| --- | --- |
| \( \{0,i\} \) | \( \{0,\hat{a},N\} = \{\hat{\Lambda},N\} \) |
| \( \hat{i} \) | \( 1,2,\cdots,n_v \) |
| \( \hat{a} \) | \( 1,2,\cdots,n_v - 1 \) |
| \( \Lambda \) | \( 0,1\cdots,n_v - 1 \) |

Table 2: Index of vector fields

A.2 Spinor notations

Here, we summarize spinor conventions.

The SU(2) and Sp(2) invariant tensors satisfy

\[
\epsilon^{AB}\epsilon_{BC} = -\delta^A_C, \quad \epsilon^{12} = \epsilon_{12} = 1, \quad (A.1)
\]

\[
C^{\alpha\beta}C_{\beta\gamma} = -\delta^\alpha_\gamma, \quad C^{12} = C_{12} = 1, \quad (A.2)
\]

and the indices of SU(2) and Sp(2) vectors are raised and lowered by

\[
\epsilon_{AB}P^B = P_A, \quad \epsilon^{AB}P_B = -P^A, \quad (A.3)
\]

\[
C_{\alpha\beta}P^{\beta} = P_\alpha, \quad C^{\alpha\beta}P_\beta = -P^\alpha. \quad (A.4)
\]
The Pauli matrices are \((\sigma^x)^A_B(x = 1, 2, 3)\) are
\[
(\sigma^1)^A_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (\sigma^2)^A_B = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad (\sigma^3)^A_B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\] (A.5)
Their indices are raised and lowered by \(\epsilon_{AB}\) and \(\epsilon^{AB}\) defined above.

We denote the chirality of the spinors as
\[
(\gamma^5)^A_B \begin{pmatrix} \psi_A^A \\ \chi_A \end{pmatrix} = \begin{pmatrix} \psi_A^A \\ \chi_A \end{pmatrix},
\]
(A.6)
\[
(\gamma^5)^A_B \begin{pmatrix} \bar{\psi}_A^A \\ \bar{\chi}_A \end{pmatrix} = -i \begin{pmatrix} \bar{\psi}_A^A \\ \bar{\chi}_A \end{pmatrix}.
\] (A.7)

## B Integrating out two-form field

Here we show the process to integrate out the auxiliary two-form field, which is needed for the embedding tensor formalism \[10,11\].

The two-form field appears in Eqs. (2.32), (2.42). Also, it is contained in Eq. (D.6) which is necessary for deriving the interactions in Appendix D. We summarize them as,
\[
L_{\text{kin},v} + L_{\text{top}} + L_{\text{Pauli}} = \frac{1}{4} \mathcal{I}_{\Sigma\Lambda} H_{\mu\nu} H_{\Sigma}^{\mu\nu} + \frac{i}{4} R_{\Lambda\Sigma} H_{\mu\nu} \tilde{H}_{\Sigma}^{\mu\nu} - \frac{i}{4} \Theta^{\lambda n} \tilde{B}_{\mu\nu,m} \left( F_{\lambda n}^{\mu\nu} - \frac{1}{4} \Theta_{\lambda n} B_{\mu\nu}^{\lambda n} \right) + H_{\mu\nu} Q_{\Lambda}^{\mu\nu},
\] (B.1)
where \(Q_{\Lambda}^{\mu\nu}\) comes from Eq. (D.6), and is defined by
\[
Q_{\Lambda}^{\mu\nu} \equiv \text{Re} S_{\Lambda}^{\mu\nu} + i \text{Im} S_{\Lambda}^{\mu\nu},
\] (B.2)
\[
S_{\Lambda}^{\mu\nu} \equiv \mathcal{I}_{\Lambda\Sigma} \left\{ 2 L^{\Sigma} \bar{\psi}_{A\mu} \psi_{B\nu} \epsilon_{AB} - 2 i \tilde{f}_{\lambda}^{\Sigma} \bar{\chi}_{A\lambda} \gamma_{\nu} \psi_{B\mu} \epsilon_{AB} \\
+ \frac{1}{4} D_{\mu} f_{\nu}^{\Sigma} \bar{\chi}_{A\lambda} \gamma_{\mu\nu} \chi_{B\lambda} \epsilon_{AB} - \frac{1}{2} L^{\Sigma} \zeta_{\alpha} \gamma_{\mu\nu} \zeta_{\beta} C^{\alpha\beta} \right\}.
\] (B.3)
Here * denotes a Hodge dual defined by \(*T_{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu\rho\sigma} T^{\rho\sigma}\).

In the gauge (3.57), the E.O.M of \(\tilde{B}_{\mu\nu,2}\) yields
\[
\mathcal{I}_{\Lambda N} H_{\mu\nu}^{\Lambda N} + i \mathcal{R}_{\Lambda N} \tilde{H}_{\mu\nu}^{\Lambda N} - i \tilde{F}_{\mu\nu,N} + \frac{i}{2} E_N \tilde{B}_{\mu\nu,2} + 2 Q_{N\mu\nu} = 0,
\] (B.4)
where we have introduced an index $\hat{\Lambda} = \{0, \hat{a}\}$. This can be solved as

$$B_{\mu \nu} \equiv - \frac{2}{M N} \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \mathcal{J}_{\mu \nu} + \frac{2i}{M N} \frac{\mathcal{P}_N}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \mathcal{J}_{\mu \nu}, \quad (B.5)$$

$$\mathcal{P}_{NN} \equiv \frac{E_N}{M N} + \mathcal{R}_{NN}, \quad (B.6)$$

$$\mathcal{J}_{\mu \nu} \equiv \mathcal{I}_{NN} \mathcal{F}_{\mu \nu}^\Lambda + i \mathcal{R}_{NN} \bar{F}_{\mu \nu}^\Lambda - i \bar{F}_{\mu \nu} + 2 \mathcal{Q}_N. \quad (B.7)$$

By substituting the solution into the Lagrangian (B.1), we obtain

$$\mathcal{L}_{v, \text{kin}} + \mathcal{L}_{\text{top}} + \mathcal{L}_{\text{Pauli}} = \frac{1}{4} \hat{\mathcal{I}}_{\hat{\Lambda} \hat{\Sigma}} \mathcal{F}^\hat{\Lambda} \mathcal{F}^{\hat{\Sigma}} + \frac{1}{2} \hat{\mathcal{I}}_\Sigma \mathcal{F}^{\Sigma} \mathcal{F}^\Sigma + \frac{1}{4} \hat{\mathcal{I}}^N \mathcal{F}_N \mathcal{F}_N$$

$$+ \frac{i}{2} \hat{\mathcal{R}}_{\hat{\Lambda} \hat{\Sigma}} \mathcal{F}^\hat{\Lambda} \bar{F}^{\hat{\Sigma}} + \frac{i}{2} \hat{\mathcal{R}}_\Sigma \mathcal{F}_N \bar{F}^{\Sigma} + \frac{i}{4} \hat{\mathcal{R}}^N \mathcal{F}_N \bar{F}_N$$

$$+ \Sigma^N \mathcal{Q}_N + \hat{\mathcal{H}}^\Lambda \mathcal{Q}_\Lambda + \text{four fermi couplings}, \quad (B.8)$$

where

$$\hat{\mathcal{I}}_{\hat{\Lambda} \hat{\Sigma}} = \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \left[ \mathcal{R}_{\Lambda \Sigma} \mathcal{R}_{\Sigma \Sigma} - \mathcal{I}_{\Lambda \Sigma} \mathcal{I}_{\Sigma \Sigma} - 2 \mathcal{I}_{NN}^{-1} \mathcal{P}_{NN} \mathcal{R}_{(\Lambda \Sigma)} \mathcal{I}_{(\Sigma \Sigma)} \right], \quad (B.9)$$

$$\hat{\mathcal{I}}^{\Lambda}_\Lambda = \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \left[ - \mathcal{R}_{NN} + \mathcal{P}_{NN} \mathcal{I}_{NN}^{-1} \mathcal{I}_{NN} \right], \quad (B.10)$$

$$\hat{\mathcal{I}}^N = \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2}, \quad (B.11)$$

$$\hat{\mathcal{R}}_{\hat{\Lambda} \hat{\Sigma}} = \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \left[ \mathcal{I}_{NN}^{-1} \mathcal{P}_{NN} \left( \mathcal{I}_{\Lambda \Sigma} \mathcal{I}_{\Sigma \Sigma} - \mathcal{R}_{\Lambda \Sigma} \mathcal{R}_{\Sigma \Sigma} \right) - 2 \mathcal{R}_{(\Lambda \Sigma)} \mathcal{I}_{(\Sigma \Sigma)} \right], \quad (B.12)$$

$$\hat{\mathcal{R}}^{\Lambda}_\Lambda = \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \left[ \mathcal{I}_{NN}^{-1} \mathcal{P}_{NN} \mathcal{I}_{NN}^{-1} \mathcal{R}_{NN} \right]; \quad (B.13)$$

$$\hat{\mathcal{R}}^N = - \frac{\mathcal{P}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2}, \quad (B.14)$$

$$\Sigma^N = - \frac{\mathcal{I}_{NN}}{\mathcal{I}_{NN}^2 + \mathcal{P}_{NN}^2} \left[ - i \bar{F}_N + \mathcal{I}_{NN} \bar{F}^\Lambda + i \mathcal{R}_{NN} \bar{F}^\Lambda \right.$$

$$\left. - i \mathcal{I}_{NN}^{-1} \mathcal{P}_{NN} \left( - i F_N + \mathcal{I}_{NN} \bar{F}^\Lambda + i \mathcal{R}_{NN} F^\Lambda \right) \right]. \quad (B.15)$$

The first and the second line in Eq. (B.8) describe the kinetic terms and theta couplings of physical vector fields. The third line shows the interactions including the vector and the fermion biliner couplings (we have neglected four fermi interactions), which are used in Appendix D.
C  Expectation values of gauge kinetic function

Here we list the explicit expressions of the vacuum expectation values of the gauge kinetic function and the couplings appearing in Eq. (B.8).

At the vacuum, the gauge kinetic function (2.17) is evaluated as,

\[
\langle I_{00} \rangle = - \frac{\langle K_0 \rangle}{2} \frac{1 + (\lambda + \bar{\lambda}) \langle \partial N K \rangle + 2|\lambda|^2 \langle \partial N K \rangle^2}{1 - (\lambda - \bar{\lambda})^2 \langle \partial N K \rangle^2},
\]

(C.1)

\[
\langle I_{0N} \rangle = \frac{\langle K_0 \partial_N K \rangle}{2} \frac{1 + (\lambda + \bar{\lambda}) \langle \partial_N K \rangle}{1 - (\lambda - \bar{\lambda})^2 \langle \partial_N K \rangle^2},
\]

(C.2)

\[
\langle I_{NN} \rangle = - \frac{\langle K_0 \rangle}{2} \frac{\langle \partial N K \rangle^2}{1 - (\lambda - \bar{\lambda})^2 \langle \partial_N K \rangle^2},
\]

(C.3)

\[
\langle I_{ab} \rangle = \text{Re} \left( 2c_{ab} + 6\lambda c_{ab}N \right), \quad \langle I_{0a} \rangle = \langle I_{N\bar{a}} \rangle = 0,
\]

(C.4)

\[
\langle R_{00} \rangle = 2\text{Im}c_0 - i\frac{\langle K_0 \rangle(\lambda - \bar{\lambda}) \langle \partial N K \rangle^2}{2} \frac{1 + (\lambda + \bar{\lambda}) \langle \partial N K \rangle}{1 - (\lambda - \bar{\lambda})^2 \langle \partial N K \rangle^2},
\]

(C.5)

\[
\langle R_{0N} \rangle = - \frac{e_2 + \text{Im}\Delta}{M^N} + i\frac{\langle K_0 \rangle(\lambda - \bar{\lambda}) \langle \partial N K \rangle^2}{2} \frac{1 + (\lambda + \bar{\lambda}) \langle \partial N K \rangle}{1 - (\lambda - \bar{\lambda})^2 \langle \partial N K \rangle^2},
\]

(C.6)

\[
\langle R_{NN} \rangle = - \frac{E_N}{M^N} - \frac{i\langle K_0 \rangle(\lambda - \bar{\lambda})^3}{1 - (\lambda - \bar{\lambda})^2 \langle \partial N K \rangle^2},
\]

(C.7)

\[
\langle R_{\bar{a}b} \rangle = \text{Im} \left( 2c_{\bar{a}b} + 6\lambda c_{\bar{a}b}N \right), \quad \langle R_{0\bar{a}} \rangle = \langle R_{N\bar{a}} \rangle = 0.
\]

(C.8)

By substituting these equations into Eqs. (B.9)-(B.15), we obtain

\[
\langle \hat{I}_{00} \rangle = - \frac{e^{-\xi}}{4} \frac{(e_3 + \text{Re}\Delta)^2 + (e_2 + \text{Im}\Delta)^2}{(e_3 + \text{Re}\Delta)^2},
\]

(C.9)

\[
\langle \hat{I}_{\bar{a}b} \rangle = \langle I_{\bar{a}b} \rangle,
\]

(C.10)

\[
\langle \hat{I}_{0a} \rangle = \langle \hat{I}_{\bar{a}N} \rangle = 0,
\]

(C.11)

\[
\langle \hat{I}_{0\bar{a}} \rangle = \langle \hat{I}_{\bar{a}N} \rangle = 0,
\]

(C.12)

\[
\langle \Sigma^N \rangle = e^{-\xi} \frac{(M^N)^2}{4(e_3 + \text{Re}\Delta)^2} \left[ -i\hat{F}_N + 2 \langle \partial N K \rangle \text{Im}F_N - \frac{i(e_2 + \text{Im}\Delta)}{M^N}\hat{F}^0 + \frac{1}{M^N} \left\{ e_3 + \text{Re}\Delta + 2 \langle \partial N K \rangle ((e_3 + \text{Re}\Delta)\text{Re}\lambda + (e_2 + \text{Im}\Delta)\text{Im}\lambda) \right\} \hat{F}^0 \right].
\]

(C.14)

Here we have omitted the expressions of \( \langle \hat{R} \rangle \), which are not necessary in the main text.
D Interaction

Here, we calculate the interaction terms up to three point couplings. We focus on the interactions of the massive fermions including two gravitinos.

Before specifying the interactions on the vacuum, here we enumerate the undefined quantities in subsection 2.3: The covariant derivatives of the fermions, $L_{\text{Pauli}}$, and $L_{\text{der}}$.

The covariant derivatives of the fermions are given by\(^{(46,49–51)}\),

\[
\mathcal{D}_\mu \psi_{A \nu} = \nabla_\mu \psi_{A \nu} + \frac{i}{2} \hat{Q}_\mu \psi_{A \nu} + \hat{\omega}_B^{\text{B}} \psi_{B \nu}, \tag{D.1}
\]

\[
\mathcal{D}_\mu \lambda^{i A} = \nabla_\mu \lambda^{i A} - \frac{i}{2} \hat{Q}_\mu \lambda^{i A} + \hat{\Gamma}^{i}_{j \mu} \lambda^{j A} + \hat{\omega}_B^{A \mu} \lambda^{B}, \tag{D.2}
\]

\[
\mathcal{D}_\mu \zeta_\alpha = \nabla_\mu \zeta_\alpha - \frac{i}{2} \hat{Q}_\mu \zeta_\alpha + \hat{\Delta}^{\beta}_{\alpha \mu} \zeta_\beta, \tag{D.3}
\]

where $\nabla_\mu$ denotes the one including the Levi-Civita connection on spacetime, and $\hat{\Gamma}^{i}_{j \mu} = \Gamma^{i}_{jk} \partial_\mu z^k$. $\hat{Q}_\mu$, $\hat{\omega}_B^{A \mu}$, and $\hat{\Delta}^{\beta}_{\alpha \mu}$ are the $U(1)$, $SU(2)$, and $Sp(2)$ (gauged) connections, which are given by\(^{(46,49–51)}\),

\[
\hat{Q}_\mu = -\frac{i}{2} \left( \partial_\mu K_\mu z^i - \partial_\mu K_\mu z^j \right), \tag{D.4}
\]

\[
\hat{\omega}_B^{A \mu} = \hat{\Delta}^{\beta}_{\alpha \mu} = \frac{i}{2b^0} \left( \begin{array}{ccc}
\partial_\mu b^2 - A^{0 \mu} e_3 & -b_\mu b^1 + iA^\prime_\mu M^N & \partial_\mu b^3 + A^{0 \mu} e_3 \\
-iB_\mu b^2 - iA^\prime_\mu M^N & -\partial_\mu b^3 - A^{0 \mu} e_3 & \partial_\mu b^1 - A^{0 \mu} e_3 \\
\partial_\mu b^3 + A^{0 \mu} e_3 & -\partial_\mu b^1 - A^{0 \mu} e_3 & \partial_\mu b^2 - iA^\prime_\mu M^N \\
\end{array} \right), \tag{D.5}
\]

$L_{\text{Pauli}}$ is given by

\[
L_{\text{Pauli}} = \mathcal{H}^{+ A}_{\mu \nu} \mathcal{I}_{\Lambda \Sigma} \left\{ 2 L^{\Sigma \bar{A} \mu} \psi^{B \nu} \epsilon_{AB} - 2 i j^i_1 \lambda^{\gamma \mu \nu} \psi_{B \mu} \epsilon_{AB} \ight. \\
+ \frac{1}{4} \mathcal{D}_i j^i_1 \lambda^{\gamma \mu \nu} \lambda^{B \mu} \epsilon_{AB} - \frac{1}{2} L^{\gamma \mu \nu} \zeta^{\alpha \beta} C^{\alpha \beta} \right\} + h.c., \tag{D.6}
\]

where $\mathcal{H}^{+ A}_{\mu \nu} \equiv \frac{1}{2} (\mathcal{H}_{\mu \nu} \pm \mathcal{H}_{\mu \nu})$ are (anti) self-dual tensors.

Finally, $L_{\text{der}}$ is given by

\[
L_{\text{der}} = -g_{ij} \partial_\mu z^j \bar{\psi}_A \lambda^{i A} - 2 \mathcal{U}^{A \alpha}_u D_\mu b^\mu \bar{\psi}_A \zeta_\alpha + g_{ij} \partial_\mu z^j \bar{\lambda}^{i A} \gamma^\mu \psi_{A \nu} \\
+ 2 \mathcal{U}^{A \alpha}_u D_\mu b^\mu \bar{\zeta}_\alpha \gamma^\mu \psi_{A \nu} + h.c., \tag{D.7}
\]

Then, let us see the explicit forms of the interactions on the vacuum and under the gauge conditions $\chi = \bar{\chi} = 0$, Eqs. (3.57), (3.63), and (3.63).

From Eq. (D.6), we can obtain the vector and fermion bilinear couplings. It also contains the two-form field $B_{\mu \nu,2}$ and affects the process of integrating out $B_{\mu \nu,2}$ (see Appendix B).
After integrating out $B_{\mu\nu,2}$, $L_B + L_{\text{Pauli}}$ produces

\[
L_{\text{int,1}} = O_1 F_{\mu\nu}^+ (B) \bar{\psi}^\mu \gamma^\mu \psi^- + O_2 F_{\mu\nu}^+ (B) (\bar{\eta}^* \gamma^\nu \psi^\mu_\perp - \bar{\eta}^* \gamma^\nu \psi^\mu_\parallel) + O_3 F_{\mu\nu}^+ (B) \bar{\eta}^* \gamma^\mu \eta^* \\
+ O_{4\bar{a}b} F_{\mu\nu}^+ (B) \left( -\chi^a \gamma^\mu \lambda^\perp + \chi^a \gamma^\mu \lambda^\parallel \right) \\
+ O_5 F_{\mu\nu}^+ (B') \bar{\psi}^\mu \gamma^\mu \psi^- + O_6 F_{\mu\nu}^+ (B') (\bar{\eta}^* \gamma^\nu \psi^\mu_\perp - \bar{\eta}^* \gamma^\nu \psi^\mu_\parallel) + O_7 F_{\mu\nu}^+ (B') \bar{\eta}^* \gamma^\mu \eta^* \\
+ O_{8\bar{a}b} F_{\mu\nu}^+ (B') \left( -\chi^a \gamma^\mu \lambda^\perp + \chi^a \gamma^\mu \lambda^\parallel \right) \\
- 2i \langle e^{K/2} \rangle \langle I_{\bar{a}b} \rangle F_{\mu\nu}^a \left( -\bar{\chi}^a \gamma^\mu \psi^\mu_\perp + \bar{\chi}^a \gamma^\mu \psi^\mu_\parallel \right) \\
- \frac{2}{3} Re\lambda \langle I_{\bar{a}b} \rangle \left| \frac{f_{\bar{a}b}}{\bar{g}^{N\bar{N}}} \right| F_{\mu\nu}^a \left( -\bar{\lambda}^a \gamma\mu \eta^* + \bar{\lambda}^a \gamma\mu \eta^* \right) + \text{h.c.}, \tag{D.8}
\]

in addition to Eq. (3.70). Here we have omitted four fermi interactions which come from the integration of $B_{\mu\nu,2}$. The coefficients are given by

\[
O_1 = \frac{1}{|m_2| + |m_1|} \left( e^i\theta + \Delta \cos\theta \right), \tag{D.9}
\]

\[
O_2 = \frac{1}{|m_2| + |m_1|} \left( e^{-i\theta} + \Delta \cos\theta \right), \tag{D.10}
\]

\[
O_3 = \frac{1}{9b^0} \left( e^{i\theta} \gamma^\mu \gamma^\mu \psi^\perp + \bar{\eta}^* \gamma^\mu \gamma^\mu \psi^\perp \right) \left( e^{-i\theta} + \Delta \cos\theta \right), \tag{D.11}
\]

\[
O_{4\bar{a}b} = \frac{1}{8b^0} \left( e^{K/2} \right) \left( f_{\bar{a}b} \bar{g}^{N\bar{N}} \right) \left( e^{i\theta} + \Delta \cos\theta \right), \tag{D.12}
\]

$O_{5,6,7,8}$ are obtained by the replacements $O_{1,2,3,4}$ with $\cos\theta \rightarrow -\sin\theta, \sin\theta \rightarrow \cos\theta$, and $|m_1| \rightarrow -|m_1|$ in their expressions. In the derivation, we have assumed $Re\lambda = 0$.

From Eq. (D.7), we obtain

\[
L_{\text{int,2}} = \frac{1}{3Re\lambda} \partial_{\mu} \bar{\zeta}^{N} \left( \eta^* \gamma^\nu \gamma^\mu \psi^\perp + \bar{\eta}^* \gamma^\nu \gamma^\mu \psi^\perp \right) - \langle g_{ab} \rangle \partial_{\mu} \bar{\zeta}^{b} \left( \bar{\lambda}^a \gamma^\nu \gamma^\mu \psi^\perp + \bar{\lambda}^a \gamma^\nu \gamma^\mu \psi^\perp \right) \\
+ \left\{ \frac{1}{3b^0} \right\} \partial_{\mu} b^0 \left( \eta^* \gamma^\nu \gamma^\mu \psi^\perp + \bar{\eta}^* \gamma^\nu \gamma^\mu \psi^\perp \right) + \left\{ \frac{i}{3b^0} \right\} \partial_{\mu} b^1 \left( \eta^* \gamma^\nu \gamma^\mu \psi^\perp + \bar{\eta}^* \gamma^\nu \gamma^\mu \psi^\perp \right) \\
+ \left\{ \frac{1}{3(b^0)^2} \right\} \frac{M^N e_3}{Re\lambda(|m_2| + |m_1|)} B_{\mu} \left( e^{-i\theta} \eta^* \gamma^\nu \gamma^\mu \psi^\perp + e^{i\theta} \bar{\eta}^* \gamma^\nu \gamma^\mu \psi^\perp \right) \\
+ \left\{ \frac{1}{3(b^0)^2} \right\} \frac{M^N e_3}{Re\lambda(|m_2| + |m_1|)} B'_{\mu} \left( e^{-i\theta} \eta^* \gamma^\nu \gamma^\mu \psi^\perp + e^{i\theta} \bar{\eta}^* \gamma^\nu \gamma^\mu \psi^\perp \right) + \text{h.c.}, \tag{D.13}
\]

26
Also, we have the scalar and the fermion bilinear couplings from Eq. (2.35),

\[ L_{\text{int},3} = 2 \left( \bar{S}_{11} + \bar{S}_{12} \right) \bar{\psi}_\mu^+ \gamma^{\mu\nu} \psi_\nu^+ + 2 \left( \bar{S}_{11} - \bar{S}_{12} \right) \bar{\psi}_\mu^+ \gamma^{\mu\nu} \psi_\nu^- \\
+ \left\{ i \left( g_{\hat{a}\hat{b}} \right) \left( W^{\hat{a}11} + W^{\hat{a}12} \right) \bar{\lambda}_+ \gamma_+ \psi_\mu^+ + \frac{2i}{3} \left( W_{N11} + W_{N12} \right) \bar{\eta}^* \gamma_+ \psi_\mu^+ \\
+ i \tilde{g}_{N\tilde{a}} \left( g^{N\tilde{N}} \partial_N K \right) \bar{m}_1 \bar{\lambda}_+ \gamma_+ \psi_\mu^+ + \frac{2i}{3} \left( g^{N\tilde{N}} \right) \tilde{g}_{N\tilde{N}} \bar{m}_1 \bar{\eta}^* \gamma_+ \psi_\mu^+ \\
+ \left( \rightarrow -, W^{i12} \rightarrow -W^{i12}, m_1 \rightarrow m_2 \right) \right\} - \frac{2i}{3} \left( \tilde{N}_1^1 + \tilde{N}_2^1 \right) \bar{\eta}^* \gamma_+ \psi_\mu^+ \\
+ \left\{ -\frac{1}{3} \left( \tilde{M}_{a1}^1 + \tilde{M}_{a1}^2 \right) \bar{\eta}^* \lambda_- - \frac{2}{9} \left( \tilde{M}_{N1}^1 + \tilde{M}_{N1}^2 \right) \bar{\eta}^* \eta^* \\
- \left( \rightarrow +, \tilde{M}_{i1}^2 \rightarrow -\tilde{M}_{i1}^2 \right) \right\} + \left\{ \left( \tilde{M}_{12\hat{a}}^1 + \tilde{M}_{12\hat{a}}^2 \right) \bar{\lambda}^+ \eta^* + \frac{4}{9} \left( g^{N\tilde{N}} \right) \left( \tilde{M}_{11NN} + \tilde{M}_{12NN} \right) \bar{\eta}^* \eta^* \\
+ \left( \rightarrow -, \tilde{M}_{12ij} \rightarrow -\tilde{M}_{12ij} \right) \right\} + \text{h.c.,} \right. \]

\[ \text{(D.14)} \]

where we have defined the fluctuations from the expectation values of Eqs. (2.36)-(2.41) and \( g_{ij} \), and distinguished them by adding tilde. They are the functions of \( \tilde{z}^i \) and \( \tilde{b}^0 \). \( +(-) \rightarrow -(+) \) denotes that \( \psi_+(\psi_-) \) and \( \eta(\tilde{\eta}) \) should be replaced by \( \psi_-(\psi_+) \) and \( \tilde{\eta}(\eta) \) respectively.
Finally, the covariant derivatives in Eqs. (D.1)-(D.3) produce the following terms,

\begin{equation}
L_{\text{int.4}} = -\frac{i}{2} \varepsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_{\nu \rho +} \psi_{\sigma +} + \bar{\psi}_\mu \gamma_{\nu \rho -} \psi_{\sigma -} + \frac{i}{2 \lambda_0} \varepsilon^{\mu \nu \rho \sigma} \partial_\rho b_1 \left( \bar{\psi}_\mu \gamma_{\nu \rho +} \psi_{\sigma +} - \bar{\psi}_\mu \gamma_{\nu \rho -} \psi_{\sigma -} \right)
\end{equation}

\begin{align}
&+ \left( \frac{i}{2 (\lambda_0)^2} \right) \varepsilon^{\mu \nu \rho \sigma} e_3 M^N \bar{B}_\rho \left( e^{i \theta} \bar{\psi}_\mu \gamma_{\nu \rho +} \psi_{\sigma +} + e^{-i \theta} \bar{\psi}_\mu \gamma_{\nu \rho -} \psi_{\sigma +} \right) \\
&- \left( \frac{1}{2 (\lambda_0)^2} \right) \varepsilon^{\mu \nu \rho \sigma} e_3 M^N \bar{B}_\rho \left( e^{i \theta} \bar{\psi}_\mu \gamma_{\nu \rho +} \psi_{\sigma +} - e^{-i \theta} \bar{\psi}_\mu \gamma_{\nu \rho -} \psi_{\sigma +} \right) \\
&+ \left\{ -\frac{1}{4} \langle g_{ab} \rangle \bar{\lambda}_+^b \gamma_\mu \tilde{Q}_\mu \lambda^a + \frac{1}{6} \eta \cdot \gamma_\mu \overline{Q}_\mu \eta \cdot - \frac{i}{2} \langle g_{ab} \rangle \bar{\lambda}_+^b \gamma_\mu \tilde{\Gamma}_k \lambda^k + \frac{i}{3} \left( \partial_N K \right) \eta \cdot \gamma_\mu \overline{\Gamma}_k \lambda^k \right\} \\
&+ \left\{ \frac{1}{4 (\lambda_0)^2} g_{ab} \right\} \partial_\mu b^1 \bar{\lambda}_+^b \gamma_\mu \lambda^a + \left\{ \frac{1}{18 (\lambda_0)^2} \right\} \partial_\mu b^1 \bar{\eta} \cdot \eta \cdot - \left\{ \frac{1}{2} \right\} \\
&+ \frac{1}{4 (\lambda_0)^2} g_{ab} \right\} \frac{e_3 M^N}{\text{Re}\lambda(|m_2| + |m_1|)} \bar{B}_\mu \left( e^{-i \theta} \bar{\lambda}_+^b \gamma_\mu \lambda^a + e^{i \theta} \bar{\lambda}_+^b \gamma_\mu \lambda^a \right) \\
&- \frac{1}{4 (\lambda_0)^2} g_{ab} \right\} \frac{e_3 M^N}{\text{Re}\lambda(|m_2| - |m_1|)} \bar{B}_\mu \left( e^{-i \theta} \bar{\lambda}_+^b \gamma_\mu \lambda^a - e^{i \theta} \bar{\lambda}_+^b \gamma_\mu \lambda^a \right) \\
&+ \frac{1}{18 (\lambda_0)^2} \frac{e_3 M^N}{\text{Re}\lambda(|m_2| + |m_1|)} \bar{B}_\mu \left( e^{-i \theta} \bar{\eta} \cdot \gamma_\mu \bar{\eta} \cdot + e^{i \theta} \bar{\eta} \cdot \gamma_\mu \bar{\eta} \cdot \right) \\
&- \frac{1}{18 (\lambda_0)^2} \frac{e_3 M^N}{\text{Re}\lambda(|m_2| - |m_1|)} \bar{B}_\mu \left( e^{-i \theta} \bar{\eta} \cdot \gamma_\mu \bar{\eta} \cdot - e^{i \theta} \bar{\eta} \cdot \gamma_\mu \bar{\eta} \cdot \right) + \text{h.c.}, \\
\end{align}

where we have defined the fluctuations as \( \dot{Q}_\mu = \langle \dot{Q}_\mu \rangle + \tilde{Q}_\mu \) and \( \dot{\Gamma}_k_{ij} = \langle \dot{\Gamma}_k_{ij} \rangle + \tilde{\Gamma}_k_{ij} \). They are the functions of \( \tilde{z}_t \).

References

[1] M. Grana, “Flux compactifications in string theory: A Comprehensive review,” Phys. Rept. 423, 91 (2006) [hep-th/0509003].

[2] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, “Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,” Phys. Rept. 445, 1 (2007) [hep-th/0610327].

[3] S. Cecotti, L. Girardello and M. Porrati, “Two Into One Won’t Go,” Phys. Lett. 145B, 61 (1984).

[4] S. Cecotti, L. Girardello and M. Porrati, “Constraints On Partial Superhiggs,” Nucl. Phys. B 268, 295 (1986).
[5] E. Dudas, S. Ferrara and A. Sagnotti, “A superfield constraint for $\mathcal{N} = 2$ $\mathcal{N} = 0$ breaking,” JHEP 1708, 109 (2017) [arXiv:1707.03414 [hep-th]].

[6] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “New nilpotent $\mathcal{N} = 2$ superfields,” Phys. Rev. D 97, no. 2, 026003 (2018) [arXiv:1707.07390 [hep-th]].

[7] S. Ferrara, L. Girardello and M. Porrati, “Minimal Higgs branch for the breaking of half of the supersymmetries in N=2 supergravity,” Phys. Lett. B 366, 155 (1996) [hep-th/9510074].

[8] S. Ferrara, L. Girardello and M. Porrati, “Spontaneous breaking of N=2 to N=1 in rigid and local supersymmetric theories,” Phys. Lett. B 376, 275 (1996) [hep-th/9512180].

[9] P. Fre, L. Girardello, I. Pesando and M. Trigiante, “Spontaneous N=2 $\rightarrow$ N=1 local supersymmetry breaking with surviving compact gauge group,” Nucl. Phys. B 493, 231 (1997) [hep-th/9607032].

[10] B. de Wit, H. Samtleben and M. Trigiante, “On Lagrangians and gaugings of maximal supergravities,” Nucl. Phys. B 655, 93 (2003) [hep-th/0212239].

[11] B. de Wit, H. Samtleben and M. Trigiante, “Magnetic charges in local field theory,” JHEP 0509, 016 (2005) [hep-th/0507289].

[12] J. Louis, P. Smyth and H. Triendl, “Spontaneous $\text{N}=2$ to $\text{N}=1$ Supersymmetry Breaking in Supergravity and Type II String Theory,” JHEP 1002, 103 (2010) [arXiv:0911.5077 [hep-th]].

[13] J. Louis, P. Smyth and H. Triendl, “The $\text{N}=1$ Low-Energy Effective Action of Spontaneously Broken $\text{N}=2$ Supergravities,” JHEP 1010, 017 (2010) [arXiv:1008.1214 [hep-th]].

[14] T. Hansen and J. Louis, “Examples of $\mathcal{N} = 2$ $\rightarrow$ $\mathcal{N} = 1$ supersymmetry breaking,” JHEP 1311, 075 (2013) [arXiv:1306.5994 [hep-th]].

[15] I. Antoniadis, J. P. Derendinger, P. M. Petropoulos and K. Siampos, “All partial breakings in $\mathcal{N} = 2$ supergravity with a single hypermultiplet,” JHEP 1808, 045 (2018) [arXiv:1806.09639 [hep-th]].

[16] I. Antoniadis, H. Partouche and T. R. Taylor, “Spontaneous breaking of $\text{N}=2$ global supersymmetry,” Phys. Lett. B 372, 83 (1996) [hep-th/9512006].

[17] J. R. David, E. Gava and K. S. Narain, “Partial $\text{N} = 2$ $\rightarrow$ $\text{N} = 1$ supersymmetry breaking and gravity deformed chiral rings,” JHEP 0406, 041 (2004) [hep-th/0311086].
[18] L. Andrianopoli, P. Concha, R. D’Auria, E. Rodriguez and M. Trigiante, “Observations on BI from $\mathcal{N} = 2$ Supergravity and the General Ward Identity,” JHEP 1511, 061 (2015) [arXiv:1508.01474 [hep-th]].

[19] R. A. Laamara, E. H. Saidi and M. Vall, “Partial Breaking in Rigid Limit of $\mathcal{N} = 2$ Gauged Supergravity,” arXiv:1704.05686 [hep-th].

[20] J. Bagger and A. Galperin, “A New Goldstone multiplet for partially broken supersymmetry,” Phys. Rev. D 55, 1091 (1997) [hep-th/9608177].

[21] M. Rocek and A. A. Tseytlin, “Partial breaking of global $D = 4$ supersymmetry, constrained superfields, and three-brane actions,” Phys. Rev. D 59, 106001 (1999) [hep-th/9811232].

[22] F. Gonzalez-Rey, I. Y. Park and M. Rocek, “On dual 3-brane actions with partially broken $\mathcal{N}=2$ supersymmetry,” Nucl. Phys. B 544, 243 (1999) [hep-th/9811130].

[23] A. A. Tseytlin, “Born-Infeld action, supersymmetry and string theory,” In *Shifman, M.A. (ed.): The many faces of the superworld* 417-452 [hep-th/9908105].

[24] C. P. Burgess, E. Filotas, M. Klein and F. Quevedo, “Low-energy brane world effective actions and partial supersymmetry breaking,” JHEP 0310, 041 (2003) [hep-th/0209190].

[25] I. Antoniadis, J.-P. Derendinger and T. Maillard, “Nonlinear $\mathcal{N}=2$ Supersymmetry, Effective Actions and Moduli Stabilization,” Nucl. Phys. B 808, 53 (2009) [arXiv:0804.1738 [hep-th]].

[26] S. M. Kuzenko, “The Fayet-Iliopoulos term and nonlinear self-duality,” Phys. Rev. D 81, 085036 (2010) [arXiv:0911.5190 [hep-th]].

[27] N. Ambrosetti, I. Antoniadis, J.-P. Derendinger and P. Tziveloglou, “Nonlinear Supersymmetry, Brane-bulk Interactions and Super-Higgs without Gravity,” Nucl. Phys. B 835, 75 (2010) [arXiv:0911.5212 [hep-th]].

[28] S. M. Kuzenko and I. N. McArthur, “Goldstino superfields for spontaneously broken $\mathcal{N}=2$ supersymmetry,” JHEP 1106, 133 (2011) [arXiv:1105.3001 [hep-th]].

[29] S. Ferrara, M. Porrati and A. Sagnotti, “$N = 2$ Born-Infeld attractors,” JHEP 1412, 065 (2014) [arXiv:1411.4954 [hep-th]].

[30] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Nilpotent chiral superfield in $\mathcal{N}=2$ supergravity and partial rigid supersymmetry breaking,” JHEP 1603, 092 (2016) [arXiv:1512.01964 [hep-th]].
[31] S. Ferrara, A. Sagnotti and A. Yeranyan, “Two-field Born-Infeld with diverse dualities,” Nucl. Phys. B 912, 305 (2016) [arXiv:1602.04566 [hep-th]].

[32] S. M. Kuzenko, I. N. McArthur and G. Tartaglino-Mazzucchelli, “Goldstino superfields in $\mathcal{N} = 2$ supergravity,” JHEP 1705, 061 (2017) [arXiv:1702.02423 [hep-th]].

[33] I. Antoniadis, J. P. Derendinger and C. Markou, “Nonlinear $\mathcal{N} = 2$ global supersymmetry,” JHEP 1706, 052 (2017) [arXiv:1703.08806 [hep-th]].

[34] F. Farakos, P. Ko, G. Tartaglino-Mazzucchelli and R. von Unge, “Partial $\mathcal{N} = 2$ Supersymmetry Breaking and Deformed Hypermultiplets,” [arXiv:1807.03715 [hep-th]].

[35] N. Cribiori and S. Lanza, “On the dynamical origin of parameters in $\mathcal{N} = 2$ Supersymmetry,” [arXiv:1810.11425 [hep-th]].

[36] K. Fujiwara, H. Itoyama and M. Sakaguchi, “Partial breaking of $\mathcal{N}=2$ supersymmetry and of gauge symmetry in the U(N) gauge model,” Nucl. Phys. B 723, 33 (2005) [hep-th/0503113].

[37] K. Fujiwara, H. Itoyama and M. Sakaguchi, “Partial supersymmetry breaking and $\mathcal{N}=2$ U(N(c)) gauge model with hypermultiplets in harmonic superspace,” Nucl. Phys. B 740, 58 (2006) [hep-th/0510255].

[38] K. Fujiwara, H. Itoyama and M. Sakaguchi, “Supersymmetric U(N) gauge model and partial breaking of $\mathcal{N}=2$ supersymmetry,” Prog. Theor. Phys. Suppl. 164, 125 (2007) [hep-th/0602267].

[39] H. Itoyama and K. Maruyoshi, “U(N) gauged $\mathcal{N}=2$ supergravity and partial breaking of local $\mathcal{N}=2$ supersymmetry,” Int. J. Mod. Phys. A 21, 6191 (2006) [hep-th/0603180].

[40] K. Maruyoshi, “Gauged $\mathcal{N}=2$ Supergravity and Partial Breaking of Extended Supersymmetry,” [hep-th/0607047].

[41] I. Antoniadis, J. P. Derendinger and J. C. Jacot, “$\mathcal{N}=2$ supersymmetry breaking at two different scales,” Nucl. Phys. B 863, 471 (2012) [arXiv:1204.2141 [hep-th]].

[42] R. Ahl Laamara, M. N. El Kinani, E. H. Saidi and M. Vall, “$\mathcal{N}= 2$ Supersymmetry Partial Breaking and Tadpole Anomaly,” Nucl. Phys. B 901, 480 (2015) [arXiv:1512.00704 [hep-th]].

[43] H. Pagels and J. R. Primack, “Supersymmetry, Cosmology and New TeV Physics,” Phys. Rev. Lett. 48, 223 (1982).

[44] T. Moroi, H. Murayama and M. Yamaguchi, “Cosmological constraints on the light stable gravitino,” Phys. Lett. B 303, 289 (1993).
[45] A. de Gouvea, T. Moroi and H. Murayama, “Cosmology of supersymmetric models with low-energy gauge mediation,” Phys. Rev. D 56, 1281 (1997) [hep-ph/9701244].

[46] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fre and T. Magri, “N=2 supergravity and N=2 superYang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map,” J. Geom. Phys. 23, 111 (1997) [hep-th/9605032].

[47] M. Trigiante, “Gauged Supergravities,” Phys. Rept. 680, 1 (2017) [arXiv:1609.09745 [hep-th]].

[48] H. Samtleben, “Lectures on Gauged Supergravity and Flux Compactifications,” Class. Quant. Grav. 25, 214002 (2008) [arXiv:0808.4076 [hep-th]].

[49] G. Dall’Agata, R. D’Auria, L. Sommovigo and S. Vaula, “D = 4, N=2 gauged supergravity in the presence of tensor multiplets,” Nucl. Phys. B 682, 243 (2004) [hep-th/0312210].

[50] R. D’Auria, L. Sommovigo and S. Vaula, “N = 2 supergravity Lagrangian coupled to tensor multiplets with electric and magnetic fluxes,” JHEP 0411, 028 (2004) [hep-th/0409097].

[51] L. Andrianopoli, R. D’Auria, L. Sommovigo and M. Trigiante, “D=4, N=2 Gauged Supergravity coupled to Vector-Tensor Multiplets,” Nucl. Phys. B 851, 1 (2011) [arXiv:1103.4813 [hep-th]].