SMEFT and the Drell-Yan Process at High Energy

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Abstract

The Drell-Yan process is a copious source of lepton pairs at high energy and is measured with great precision at the Large Hadron Collider (LHC). Barring any new light particles, beyond the Standard Model effects can be studied in Drell-Yan production using an effective field theory. At tree level, new 4-fermion interactions dominate, while at one loop operators modifying 3-gauge boson couplings contribute effects that are enhanced at high energy. We study the sensitivity of the neutral Drell-Yan process to these dimension-6 operators and compare the sensitivity to that of $W^+W^-$ pair production at the LHC.
I. INTRODUCTION

The exploration of the electroweak sector is a major task for the LHC. Without new low scale particles, the only tools available for studying deviations from the SM predictions are effective field theories (EFT). In this approach, low scale physics is assumed to be sensitive to the presence of higher dimension operators. When a complete basis of these operators is constructed, they will affect predictions for many observables, including in Higgs physics [1, 2], gauge boson pair production [3–7], top quark production [8–10], and many other processes. The measurements from different processes provide complementary information about the parameters of the EFT and potential insights into the underlying UV physics.

In this work, we consider the effects of a consistent EFT analysis on neutral Drell-Yan production. The Drell-Yan process is extremely precisely measured at numerous energies, while the Standard Model theoretical predictions exist at NNLO QCD [11–15], along with the resummation of the logarithms [16, 17]. The QCD corrections have been combined with NLO electroweak effects [18, 19] and implemented in the FEWZ code [20–22]. We study neutral Drell Yan production in the context of the SMEFT [23], where the Higgs boson is assumed to be part of an SU(2) doublet. The effects of the dimension-6 SMEFT operators can potentially be of the same magnitude as the higher order Standard Model corrections and both need to be considered in precision studies. New 4-fermion operators can contribute to $q\bar{q} \rightarrow l^+l^-$ production at tree level and have been extensively studied in the literature [24, 25]. Precision measurements at the $Z$-pole and other low energy measurements place stringent bounds on the strengths of the non-Standard Model 4-fermion operators.

At the LHC, new information can be gained by looking at high $p_T$ (or $m_{ll}$) events where the new physics effects are potentially enhanced by contributions of $O(\frac{p_T^2}{\Lambda^2})$. In an EFT approach, the NLO corrections to the EFT contributions are also necessary, and new operators that do not contribute at tree level can have measurable effects. The QCD corrections to Drell-Yan production in the SMEFT are known [26]. The program of electroweak corrections to the SMEFT is in its infancy, however, with results for $H \rightarrow VV$ [27–30], $H \rightarrow b\bar{b}$ [31, 32] and $Z \rightarrow f\bar{f}$ [33, 34] known. Here we begin the program of one-loop EFT contributions to Drell-Yan production. We consider the one-loop contributions from anomalous 3-gauge boson interactions, and compare with the sensitivity to these interactions in $W^+W^-$ pair.
production. The sensitivity of Drell Yan production to oblique corrections at high energy has also been studied in Ref. [35].

In Section II, we review the SMEFT and write the leading order amplitude for Drell-Yan production. Section III shows the results of our NLO calculation involving SMEFT operators. Then, in Section IV we demonstrate the impact of SMEFT operators on kinematic distributions in Drell-Yan production, and estimate the reach of the LHC in probing these operators. Section V contains our conclusions.

II. BASICS

In the SMEFT, new physics is described by a tower of operators,

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{k=5}^{\infty} \sum_{i=1}^{n} \frac{C^k_i}{\Lambda^{k-4}} O^k_i. \quad (1)$$

The dimension-\(k\) operators are constructed from SM fields and the new beyond the SM (BSM) physics effects reside in the coefficient functions, \(C^k_i\). For large \(\Lambda\), it is sufficient to retain only the lowest dimensional operators. The operators have been classified in several different bases, which are related by the equations of motion [1, 23, 36, 37]. In this paper we will use the Warsaw basis of Ref. [23] and the convenient implementation of Ref. [38].

Only a few operators contribute to the neutral Drell Yan process, \(f \bar{f} \rightarrow e^+_p e^-_\mu\), at tree level (\(p\) is a generation index). There are new 4-fermion operators, along with operators that shift the tree level relationships among the parameters [39, 40]. The operators relevant for Drell-Yan production at tree level, along with the operator \(O_W\) that is the focus of the next section, are given in Table I. We define \(q\) and \(l\) to be the \(SU(2)_L\) quark and lepton doublets, respectively.

The operators in Table I change the form of the kinetic terms in the gauge sector,

$$\mathcal{L} = -\frac{1}{4} W^{I,\mu\nu} W^{I}_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

$$+ \frac{1}{\Lambda^2} \left( C_{\phi W} (\phi^\dagger \phi) W^{I,\mu\nu} W^{I}_{\mu\nu} + C_{\phi B} (\phi^\dagger \phi) B^{\mu\nu} B_{\mu\nu} + C_{\phi W B} (\phi^\dagger \tau^I \phi) W^{I,\mu\nu} B_{\mu\nu} \right). \quad (2)$$

We define “barred” fields, \(\overline{W}_\mu \equiv (1 - C_{\phi W} v^2/\Lambda^2) W_\mu\) and \(\overline{B}_\mu \equiv (1 - C_{\phi B} v^2/\Lambda^2) B_\mu\) and “barred” gauge couplings, \(\overline{g}_2 \equiv (1 + C_{\phi W} v^2/\Lambda^2) g_2\) and \(\overline{g}_1 \equiv (1 + C_{\phi B} v^2/\Lambda^2) g_1\) so that \(\overline{W}_\mu \overline{g}_2 = W_\mu g_2\) and \(\overline{B}_\mu \overline{g}_1 = B_\mu g_1\). The “barred” fields defined in this way have their kinetic
\[ \mathcal{O}_W = \epsilon^{IJK} W^I_{\mu \nu} W^J_{\nu \rho} W^K_{\rho \mu} \]

\[ \mathcal{O}_{\phi D} = (\phi^I D^\mu \phi)^* (\phi^I D_{\mu} \phi) \]

\[ \mathcal{O}_{\phi WB} = (\phi^I \gamma^I \phi)^* W^I_{\mu \nu} B^{\mu \nu} \]

\[ \mathcal{O}^{(3)}_{\phi l p, r} = (\phi^I_{\mu} D^I_{\mu} \phi)(\bar{l}^I_{p} \gamma^I_{\mu} l^I_{r}) \]

\[ \mathcal{O}^{(1)}_{l q p, r s, t} = (\bar{l}^I_{p} \gamma^I_{\mu} l^I_{r})(\bar{u}^I_{s} \gamma^I_{\mu} u^I_{t}) \]

\[ \mathcal{O}^{(3)}_{q e p, r s, t} = (\bar{l}^I_{p} \gamma^I_{\mu} l^I_{r})(\bar{e}^I_{s} \gamma^I_{\mu} e^I_{t}) \]

\[ \mathcal{O}^{(3)}_{u d p, r s, t} = (\bar{l}^I_{p} \gamma^I_{\mu} l^I_{r})(\bar{d}^I_{s} \gamma^I_{\mu} d^I_{t}) \]

\[ \mathcal{O}^{(3)}_{l l p, r s, t} = (\bar{l}^I_{p} \gamma^I_{\mu} l^I_{r})(\bar{l}^I_{s} \gamma^I_{\mu} l^I_{t}) \]

**TABLE I:** Dimension-6 operators relevant for our study (from [23]). For brevity we suppress fermion chiral indices \( L, R \). \( I = 1, 2, 3 \) is an \( SU(2) \) index. \( p, r, s, t = 1, 2, 3 \) are generation indices.

Terms properly normalized and preserve the form of the covariant derivative. The masses of the \( W \) and \( Z \) fields are then, \([38, 41]\):

\[
\begin{align*}
M^2_W &= \frac{g_2^2 v^2}{4}, \\
M^2_Z &= \frac{(g_1^2 + g_2^2) v^2}{4} + \frac{v^4}{\Lambda^2} \left( \frac{1}{8} (g_1^2 + g_2^2) C_{\phi D} + \frac{1}{2} g_1 g_2 C_{\phi WB} \right). \tag{3}
\end{align*}
\]

Dimension-6 operators contribute to the decay of the \( \mu \) lepton at tree level, changing the relation between the vev, \( v \), and the Fermi constant \( G_\mu \) obtained from the measurement of the \( \mu \) lifetime,

\[
G_\mu = \frac{1}{\sqrt{2} v^2} - \frac{1}{2 \sqrt{2} \Lambda^2} ( C_{U_{1,2,2,1}^U} + C_{U_{1,1,2,2}^U} ) + \frac{\sqrt{2}}{2 \Lambda^2} ( C_{\phi l_{1,1}^{(3)}} + C_{\phi l_{2,2}^{(3)}} ) \]

\[
\equiv \frac{1}{\sqrt{2} v^2} - \frac{1}{2 \Lambda^2} C_{U} + \frac{\sqrt{2}}{\Lambda^2} C_{\phi l_{1,1}^{(3)}}, \tag{4}
\]

where we assume flavor universality of the coefficients in the last line above.

We choose the \( G_\mu \) scheme, where we take the physical input parameters to be,

\[
G_\mu = 1.1663787(6) \times 10^{-5} \text{GeV}^{-2}
\]

\[
M_Z = 91.1876 \pm .0021 \text{GeV}
\]

\[
M_W = 80.385 \pm .015 \text{ GeV} \tag{5}
\]
Using an input basis of $M_W$, $M_Z$ and $G_\mu$\cite{40}, the effective fermion-$Z/\gamma$ interactions are,

\[
L = \frac{2M_W}{v} \sqrt{1 - \frac{M_W^2}{M_Z^2}} Q_f \left\{ 1 - \frac{c_W}{s_W} v^2 C_{\phi WB} - \frac{1}{4} \frac{c_W^2}{s_W^2} v^2 C_{\phi D} \right\} \bar{f} \gamma^\mu f A_\mu \\
+ \frac{2M_Z}{v} \left\{ T_3^f - Q_f \left( \frac{1 - M_W^2}{M_Z^2} \right) \left( 1 - \frac{c_W}{s_W} v^2 C_{\phi WB} \right) \right\} \bar{f} \gamma^\mu P_L f Z_\mu \\
- \frac{v^2}{4} \left( T_3^f - Q_f \left( 1 - \frac{M_W^2}{M_Z^2} \right) C_{\phi D} - \frac{v^2}{2} \left( C_{\phi f}^{(1)} - 2T_3^f C_{\phi f}^{(3)} \right) \right) \bar{f} \gamma^\mu P_L f Z_\mu \\
+ \frac{2M_Z}{v} \left\{ -Q_f \left( 1 - \frac{M_W^2}{M_Z^2} \right) \left( 1 - \frac{c_W}{s_W} v^2 C_{\phi WB} \right) \right\} \bar{f} \gamma^\mu P_R f Z_\mu \\
+ \frac{v^2}{4} Q_f \left( 1 - \frac{M_W^2}{M_Z^2} \right) C_{\phi D} - \frac{v^2}{2} C_{\phi f} \right\} \bar{f} \gamma^\mu P_R f Z_\mu , \tag{6}
\]

where $c_W = M_W/M_Z$, $s_W = \sqrt{1 - c_W^2}$, $T_3^f = \pm \frac{1}{2}$, $P_{L,R} = \frac{(1 + \gamma_5)}{2}$, and $f = q, l$ for left-handed fermions and $f = u, d, e$ for right-handed fermions. (We omit the generation indices for simplicity).

The tree level SM result for $q_f \bar{q}_i \rightarrow e^+_p e^-_p$ receives corrections from s-channel $Z/\gamma$ exchange,

\[
A_{SM}^{XY} = M_{XY} \left\{ \frac{4M_W^2}{v^2} \frac{Q_l Q_q}{s} + \frac{g_{qX} g_{lY}}{s - M_Z^2} \right\} , \tag{7}
\]

where $X,Y = L,R$,

\[
g_{fL} = \frac{2M_Z}{v} \left[ T_3^f - Q_f \left( \frac{1 - M_W^2}{M_Z^2} \right) \right] \\
g_{fR} = \frac{2M_Z}{v} \left[ -Q_f \left( 1 - \frac{M_W^2}{M_Z^2} \right) \right] , \tag{8}
\]

and,

\[
M_{XY} \equiv \left( \bar{f}_r \gamma^\mu P_X f_r \right) \left( \bar{c}_p \gamma^\mu P_Y e_p \right) . \tag{9}
\]

The tree level SMEFT amplitudes instead read,

\[
A_{XY}^{SMEFT} = A_{XY}^{SM} - \frac{v^2}{\Lambda^2} M_{XY} \left\{ \frac{8M_W^2}{v^2} \left( s_W c_W C_{\phi WB} + \frac{c_W^2}{4} C_{\phi D} \right) \frac{Q_l Q_q}{s} + \frac{1}{2} C_{\phi D} \frac{g_{qX} g_{lY}}{s - M_Z^2} \right\} \\
- \frac{2M_Z}{v} \left( s_W c_W C_{\phi WB} + \frac{c_W^2}{2} C_{\phi D} \right) \frac{g_{qX} Q_l + g_{lY} Q_q}{s - M_Z^2} + A_{XY,q}^{4-fermions} . \tag{10}
\]
where

\[ A_{4-fermions}^{LL,q} = \frac{M_{XY}}{\Lambda^2} \left( C_{lq}^{(1)} - 2 T_3^q C_{lq}^{(3)} \right) \]

\[ A_{4-fermions}^{LR,q} = \frac{M_{XY}}{\Lambda^2} \left( C_{qe} \right) \]

\[ A_{4-fermions}^{RL,u(d)} = \frac{M_{XY}}{\Lambda^2} \left( C_{lu(d)} \right) \]

\[ A_{4-fermions}^{RR,u(d)} = \frac{M_{XY}}{\Lambda^2} \left( C_{eu(d)} \right). \]  \hspace{1cm} (11)

The 4-fermion operators give contributions that grow with energy relative to the SM contributions and the phenomenological effects have been examined in Ref. \[25\].

III. NLO AMPLITUDES

At one loop, there are contributions to Drell-Yan from new operators not contributing at tree level. We focus on \( O_W \). This operator is particularly interesting because it is strongly restricted from \( f \bar{f} \rightarrow W^+W^- \) both at LEP and at the LHC as its effects grow with energy. The sensitivity to \( C_W \) from LHC measurements has been found in Ref. \[7\], and is roughly,

\[-0.17 < C_W \left( \frac{1 \text{ TeV}}{\Lambda^2} \right)^2 < 0.18. \]  \hspace{1cm} (12)

It has been speculated \[10\] that because of the large cross section and precision of the measurements that Drell Yan could also yield a precise determination of \( C_W \).

The diagrams of Fig. 1 give contributions to the left-hand amplitudes. The complete amplitudes are in the supplemental material and the energy enhanced (relative to the SM)
FIG. 2: The ratio of the differential cross section as a function of dilepton mass in the SMEFT to that in the SM. In each curve, one operator coefficient is turned on at a time, while the others are set to zero. The sizes of the operators are taken to be at their current bounds. The left (right) plot shows the distribution for 14 (100) TeV.

The contributions are,

\[
A_{LL,u}^{NLO} = A_{LL,u}^{SMEFT} \left( 1 - \frac{3s v^2}{\Lambda^2 M_Z^2 (1 + 2c_w)} \left[ g^3 C_W \right] \frac{32 \pi^2}{2 \pi^2} \right)
\]

\[
A_{LL,d}^{NLO} = A_{LL,d}^{SMEFT} \left( 1 + \frac{3s v^2}{\Lambda^2 M_Z^2 (1 - 4c_w)} \left[ g^3 C_W \right] \frac{32 \pi^2}{2 \pi^2} \right). \tag{13}
\]

IV. RESULTS

We now use the amplitudes of Sections II and III to calculate the effects of SMEFT operators on kinematic distributions in Drell-Yan production. Current and future precision measurements of \(pp \rightarrow e^+e^-p\) will constrain not only 4-fermion operators involving quarks and leptons, but also purely bosonic operators that contribute at loop level. We concentrate on the effect of the latter, that would dominate in a universal theory [42] or one in which the sizes of the coefficients of BSM 4-fermion operators involving 1st and 2nd generation quarks and leptons are small.

In Figure 2, we show the fractional modification of the dilepton mass distribution due to different bosonic operators. We include not only the operators \(O_{\phi WB}\) and \(O_W\), which cause loop effects and are the focus of our calculation, but also one of the 4-fermion operators that affects Drell-Yan production at tree level, for comparison. Each operator is taken to be at its current maximum allowed size from LHC data. \(C_{\phi WB}\) corresponds to the \(S\) parameter.
$S$ \cite{43, 44} and is constrained to be

$$-0.004 < C_{\phi WB} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 < 0.006 ,$$

where we take our limit from the Gfitter collaboration \cite{45}. $C_W$ is bounded by $W^+W^-$ production, as described in Sec. III. The 4-fermion operators are limited by existing Drell-Yan measurements \cite{46}, and of the many potential operators which contribute, the best constrained operator is $O_{lq}^{(3)}_{2,2,1,1}$, whose coefficient is limited to be

$$-0.012 < C_{lq}^{(3)}_{2,2,1,1} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 < 0.0047 .$$

The left panel shows the effects of these operators at the LHC, while the right panel shows them at a future 100 TeV collider. For the 4-fermion operator, the EFT loses validity at high invariant mass because the neglected dimension-8 operators become important. Nevertheless, it is clear that in a non-universal theory, new 4-fermion operators can change Drell-Yan production much more than purely bosonic operators, even those such as $C_{\phi WB}$ which contribute at tree level. At 100 TeV, where Drell-Yan could be potentially measured up to $m_{\ell\ell} = 20$ TeV \cite{47}, the operator $C_W$ at its current $2\sigma$ limit could provide up to a 50% deviation in the number of events at high energy. The effects are similarly sized for the $p_T$ distribution, as shown in Figure 3.

From our derived effects of new operators on the Drell-Yan $m_{\ell\ell}$ distribution, we can place limits on the sizes of these operators from existing measurements of Drell-Yan production. We use the 8 TeV CMS measurement \cite{48}, which goes up to 2 TeV in the dilepton invariant mass. Using the CMS data above 240 GeV and neglecting correlated uncertainties
FIG. 4: Contours in the $C_W - C_{\phi WB}$ plane resulting from a fit to the CMS measurement of Drell-Yan. The contours indicate 68.3%, 95.4%, and 99.7% confidence contours around the best-fit point. Solid contours are for the current 8 TeV measurement [48], while dashed contours are for the HL-LHC. The Standard Model is indicated at (0, 0). The region between the blue (red) lines is allowed by the current limits of Eq. 14 (Eq. 12).

among different bins\(^1\), we construct a $\chi^2$ function expressing the goodness of fit between the observed data and the prediction for arbitrary sizes of the SMEFT operators $O_W$ and $O_{\phi WB}$. We consider only how the new operators affect the ratio of the data to theory, and are not sensitive to the overall normalization of the Drell-Yan invariant mass distribution.

In Figure 4, we show the allowed region in the plane of the sizes of these two operators. The 13 TeV measurement [49] using 2.8 fb\(^{-1}\) of data already has comparable uncertainties in bins going out to 3 TeV in $m_{\ell\ell}$, and we also show a projection for the high luminosity upgrade of the LHC assuming that statistical uncertainties scale as $1/\sqrt{L}$ such that the uncertainties in each bin would be limited only by systematics, which are currently around 5%. The currently allowed region in the plane is significantly larger than that allowed by constraints from electroweak precision and $WW$ production on the operator coefficients $C_{\phi WB}$ and $C_W$, respectively. However, as both $O_{\phi WB}$ and $O_W$ contribute to Drell-Yan, an

\(^1\) At high invariant mass, in the region where the SMEFT operators are expected to have the greatest effect, the uncertainty is dominantly statistical.
external constraint on one of the operator coefficients in conjunction with a measurement of the Drell-Yan differential distribution can constrain the other better.

V. CONCLUSION

In the absence of new light physics, the EFT approach provides a parametrization of BSM effects in terms of higher dimension operators. Given the minute precision with which many processes can be measured at the LHC, especially at high luminosity, it is of use to know the loop contributions of EFT operators to SM physics. In this work, we have evaluated some of these corrections in the SMEFT for Drell-Yan production.

Notably, operators which do not contribute at tree level to Drell-Yan can have sizable contributions at one loop. We have shown that at the upper end of the energy range that can be probed at the LHC, the effect of the operator $O_W$ can be several percent, or even up to 50% at a future 100 TeV collider. Over the lifetime of the LHC, statistical uncertainties will go down significantly as more data is collected, significantly increasing the sensitivity of precision Drell-Yan measurements in the region where new physics operators have the greatest effect. As a consequence, future measurements of Drell-Yan offer the possibility to constrain operators such as $O_W$ even though their contributions are only at one loop. While gauge boson production is still a more sensitive probe of $O_W$, the effects which we have computed here must be taken into account to ensure consistency in a full NLO fit to the coefficients of SMEFT operators.

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