Logical Team Q-learning: An approach towards factored policies in cooperative MARL

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Abstract

We address the challenge of learning factored policies in cooperative MARL scenarios. In particular, we consider the situation in which a team of agents collaborates to optimize a common cost. Our goal is to obtain factored policies that determine the individual behavior of each agent so that the resulting joint policy is optimal. In this work we make contributions to both the dynamic programming and reinforcement learning settings. In the dynamic programming case we provide a number of lemmas that prove the existence of such factored policies and we introduce an algorithm (along with proof of convergence) that provably leads to them. Then we introduce tabular and deep versions of Logical Team Q-learning, which is a stochastic version of the algorithm for the RL case. We conclude the paper by providing experiments that illustrate the claims.

1 Introduction

In recent times Reinforcement Learning (RL) has seen great success in many domains. In particular, Q-learning [1] and its deep learning extension DQN [2] have shown great performance in challenging domains such as the Atari Learning Environment [3]. At the core of DQN lie two important features: the ability to use expressive function approximators (in particular, neural networks) which allow it to estimate complex Q-functions; and the ability to learn off-policy and use replay buffers [4], which allows DQN to be very sample efficient. Traditional RL focuses on the interaction between one agent and an environment. However, in many cases of interest, a multiplicity of agents will need to interact with a unique environment and with each other. This is the object of study of Multi-agent RL (MARL), which goes back to the early work of [5] and has seen renewed interest of late (for an updated survey see [6]). In this paper we consider the particular case of cooperative MARL in which the agents form a team and have a shared unique goal. We are interested in tasks where collaboration is fundamental and a high degree of coordination is necessary to achieve good performance. In particular, we consider two scenarios.

In the first scenario, the global state and all actions are visible to all agents (one example of this situation could be a team of robots that collaborate to move a big and heavy object). It is well known that in this scenario the team can be regarded as one single agent where the aggregate action consists of the joint actions by all agents [7]. The fundamental drawback of this approach is that the joint action space grows exponentially in the number of agents and the problem quickly becomes intractable [8, 9]. Another important inconvenience with this approach is that it cannot cope with a changing number of agents (for example if the system is trained with 4 agents, it cannot be executed by a team of 5 agents; we expand on this point in a later section). One well-known and popular approach to solve these issues, is to consider each agent as an independent learner (IL) [5]. However, this approach has

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a number of issues. First, from the point of view of each IL, the environment is non-stationary (due to the changing policies of the other agents), which jeopardizes convergence. And second, replay buffers cannot be used due to the changing nature of the environment and therefore even in cases where this approach might work, the data efficiency of the algorithm is negatively affected. Ideally, it is desirable to derive an algorithm with the following features: i) it learns individual policies (and is therefore scalable), ii) local actions chosen greedily with respect to these individual policies result in an optimal team action iii) can be combined with NNs, iv) works off-policy and can leverage replay buffers (for data efficiency), v) and enjoys theoretical guarantees to team optimal policies at least in the dynamic programming scenario. Indeed, the main contribution of this work is the introduction of Logical Team Q-learning (LTQL), an algorithm that has all these properties. We start in the dynamic programming setting and derive equations that characterize the desired solution. We use these equations to define the Factored Team Optimality Bellman Operator and provide a Theorem that characterizes the convergence properties of this operator. A stochastic approximation of the dynamic programming setting is used to obtain the tabular and deep versions of our algorithm. For the single agent setting, these steps reduce to: the Bellman optimality equation, the Bellman optimality operator (and the theorem which states the linear convergence of repeated application of this operator) and Q-learning (in its tabular form and DQN).

In the second scenario, we consider the centralized training and decentralized execution paradigm. During execution, agents only have access to observations which we assume provide enough information to play an optimal team policy. An example of this case would be a soccer team in which the attackers have the ball and see each other but do not see the goalkeeper or the defenders of their own team (arguably this information is enough to play optimally and score a goal). The techniques we develop for the previous scenario can be applied to this case without modification.

1.1 Relation to prior work

Some of the earliest works on MARL are [5, 10]. Tan [5] studied Independent Q-learning (IQL) and identified that IQL learners in a MARL setting may fail to converge due to the non-stationarity of the perceived environment. Claus and Boutilier [10] compared the performance of IQL and joint action learners (JAL) where all agents learn the Q-values for all the joint actions, and identified the problem of coordination during decentralized execution when multiple optimal policies are available. Littman [7] later provided a proof of convergence for JALs. Recently, Tampuu et al. [11] did an experimental study of ILs using DQNs in the Atari game Pong. All these mentioned approaches cannot use experience replay due to the non-stationarity of the perceived environment. Following Hyper Q-learning [12], Foerster et al. [13] addressed this issue to some extent using fingerprints as proxies to model other agents’ strategies.

Lauer and Riedmiller [14] introduced Distributed Q-learning (DistQ), which in the tabular setting has guaranteed convergence to an optimal policy for deterministic MDPs. However, this algorithm performs very poorly in stochastic scenarios and becomes divergent when combined with function approximation. Later Hysteretic Q-learning (HystQ) was introduced in [15] to improve these two limitations. HystQ is based on a heuristic and can be thought of as a generalization of DistQ. These works also consider the scenario where agents cannot perceive the actions of other agents. They are related to LTQL (from this work) in that they can be considered approximations to our algorithm in the scenario where agents do not have information about other agents’ actions. Recently Omidshafiei et al. [16] introduced Dec-HDRQNs for multi-task MARL, which combines HystQ with Recurrent NNs and experience replay (which they recognize is important to achieve high sample efficiency) through the use of Concurrent Experience Replay Trajectories.

Wang and Sandholm [17] introduced OAB, the first algorithm that converges to an optimal Nash equilibrium with probability one in any team Markov game. OAB considers the team scenario where agents observe the full state and joint actions. The main disadvantage of this algorithm is that it requires estimation of the transition kernel and rewards for the joint action state space and also relies on keeping count of state-action visitation, which makes it impractical for MDPs of even moderate size and cannot be combined with function approximators.

Guestrin et al. [18, 9], Kok and Vlassis [8] introduced the idea of factoring the joint Q-function to handle the scalability issue. These papers have the disadvantage that they require coordination graphs that specify how agents affect each other (the graphs require significant domain knowledge). The main shortcoming of these papers is the factoring model they use, in particular they model the optimal
We consider a situation where multiple agents form a team and interact with an environment and with each other. We model this interaction as a Team Markov Decision Process (TMDP), which we define by the tuple \((S,T,K,o^\tau,A^\tau,P,r)\). Here, \(S\) is a set of global states shared by all agents; \(T\) is the set of types of agents; \(K\) is the total amount of agents, each of type \(\tau_k \in T\); \(o^\tau: S \to O^\tau\) is the observation function for agents of type \(\tau \in T\), whose output lies in some set of observations \(O^\tau\); \(A^\tau\) is the set of actions available to agents of type \(\tau\); \(P(s'|s,a_1^K,\ldots,a^K)\) specifies the probability of transitioning to state \(s'\) from state \(s\) having taken joint actions \(a_1^K \in A^\tau_1; \ldots; a^K \in A^\tau_K\); and \(r: S \times A^\tau_1 \times \cdots \times A^\tau_K \times S \to \mathbb{R}\) is a global reward function. Specifically, \(r(s,a_1^K,\ldots,a^K,s')\) can be a random variable following some distribution \(f_{s,a_1^K,\ldots,a^K}(r)\). We clarify that from now on we will refer to the collection of all individual actions as the team’s action, denoted as \(\bar{a}\). Furthermore we will use \(a^{-k}\) to refer to the actions of all agents except for action \(a^K\). Therefore we can write \(P(s'|s,a_1^K,\ldots,a^K) = P(s'|s,a^K,a^{-K}) = P(s'|s,\bar{a})\), \(r(s,a_1^K,\ldots,a^K,s') = r(s,a^K,a^{-K},s') = r(s,\bar{a},s')\) and \(f_{s,a_1^K,\ldots,a^K}(r) = f_{s,\bar{a},s}(r)\). The goal of the team is to maximize the team’s return:

\[
J(\pi) = \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{\pi,\mathcal{P},d,f} \left[ r(s_t, \bar{a}_t, s_{t+1}) \right]
\]

(1)

where \(s_t\) and \(\bar{a}_t\) are the state and actions at time \(t\), respectively, \(\pi(\bar{a}|s)\) is the team’s policy, \(d\) is the distribution of initial states, and \(\gamma \in [0,1)\) is the discount factor. We clarify that we use bold font to denote random variables and the notation \(\mathbb{E}_\ell\) makes explicit that the expectation is taken with respect to distribution \(\ell\). From now on, we will only make the distributions explicit in cases where doing so makes the equations more clear. Accordingly, the team’s optimal state-action value function \(Q(\pi)\) and \(Q(\bar{a})\) are defined as

\[
Q(\pi)(s,a) = \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, \bar{a}_t, s_{t+1}) \right] \\
Q(\bar{a})(s) = \mathbb{E}_{\bar{a}} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, \bar{a}_t, s_{t+1}) \right]
\]

The approach we introduce in this paper also considers learning factored Q-functions. In particular, TMDPs include the notion of Games \([17]\). However, these definitions are different from ours, which is why we opted for the alternative name VDN. The main issue with this factorization model is that the optimal Q-function cannot always be factored in this way, in fact, the tasks for which this model does not hold are typically the ones that require a high degree of coordination, which happen to be the tasks where one is most interested in applying specific MARL approaches as opposed to ILs. Moreover, even if the Q-function can be accurately modeled in this way, there is no guarantee that if individual agents select their optimum strategies by maximizing their local Q-functions the resulting joint action maximizes the global Q-function. The approach we introduce in this paper also considers learning factored Q-functions. However, the fundamental difference is that the factored relationships we estimate always exist and the joint action that results from maximizing these individual Q-functions is optimal. VDN \([19]\) and QMIX \([20]\) are two recent deep methods that also factorize the optimal Q-function assuming additivity and monotonicity, respectively. This factoring is their main limitation since many MARL problems of interest do not satisfy any of these two assumptions. Indeed, Son et al. \([21]\) showed that these methods are unable to solve a simple matrix game. Furthermore, the individual policies cannot be used for prediction, since the individual Q-values are not estimates of the return. To improve on the representation limitation due to the factoring assumption, Son et al. \([21]\) introduced QTAN which factors the Q-function in a more general manner and therefore allows for a wider applicability. The main issue with QTAN is that although it can approximate a wider class of Q-functions than VDN and QMIX, the algorithm resorts to other approximations, which degrade its performance in complex environments (see \([22]\)).

Recently, actor-critic strategies have been explored \([23,24]\). However, these methods have the inconvenience that they are on-policy and therefore do not enjoy the data efficiency that off-policy methods can achieve. This is of significant importance in practical MARL settings since the state-action space is very large.

### 2 Problem formulation

We consider a situation where multiple agents form a team and interact with an environment and with each other. We model this interaction as a Team Markov Decision Process (TMDP) which we define by the tuple \((S,T,K,o^\tau,A^\tau,P,r)\). Here, \(S\) is a set of global states shared by all agents; \(T\) is the set of types of agents; \(K\) is the total amount of agents, each of type \(\tau_k \in T\); \(o^\tau: S \to O^\tau\) is the observation function for agents of type \(\tau \in T\), whose output lies in some set of observations \(O^\tau\); \(A^\tau\) is the set of actions available to agents of type \(\tau\); \(P(s'|s,a_1^K,\ldots,a^K)\) specifies the probability of transitioning to state \(s'\) from state \(s\) having taken joint actions \(a_1^K \in A^\tau_1; \ldots; a^K \in A^\tau_K\); and \(r: S \times A^\tau_1 \times \cdots \times A^\tau_K \times S \to \mathbb{R}\) is a global reward function. Specifically, \(r(s,a^K)\) is the reward when the team transitions to state \(s'\) from state \(s\) having taken actions \(a^K\). The reward \(r(s,a^K)\) can be a random variable following some distribution \(f_{s,a^K}(r)\). We clarify that from now on we will refer to the collection of all individual actions as the team’s action, denoted as \(\bar{a}\). Furthermore we will use \(a^{-K}\) to refer to the actions of all agents except for action \(a^K\). Therefore we can write \(P(s'|s,a^{-K}) = P(s'|s,a^K,a^{-K}) = P(s'|s,\bar{a})\), \(r(s,a^{-K}) = r(s,a^K,a^{-K},s') = r(s,\bar{a},s')\) and \(f_{s,a^{-K}}(r) = f_{s,\bar{a},s}(r)\). The goal of the team is to maximize the team’s return:

\[
J(\pi) = \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{\pi,\mathcal{P},d,f} \left[ r(s_t, \bar{a}_t, s_{t+1}) \right]
\]

(1)

where \(s_t\) and \(\bar{a}_t\) are the state and actions at time \(t\), respectively, \(\pi(\bar{a}|s)\) is the team’s policy, \(d\) is the distribution of initial states, and \(\gamma \in [0,1)\) is the discount factor. We clarify that we use bold font to denote random variables and the notation \(\mathbb{E}_\ell\) makes explicit that the expectation is taken with respect to distribution \(\ell\). From now on, we will only make the distributions explicit in cases where doing so makes the equations more clear. Accordingly, the team’s optimal state-action value function \(Q(\pi)\) and \(Q(\bar{a})\) are defined as

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Q(\pi)(s,a) = \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, \bar{a}_t, s_{t+1}) \right] \\
Q(\bar{a})(s) = \mathbb{E}_{\bar{a}} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, \bar{a}_t, s_{t+1}) \right]
\]
optimal policy \((\pi^1)\) are given by [26]:

\[
\pi^1(\bar{a}|s) = \arg\max_{\pi(a|s)} E \sum_{s',R} \gamma q^1(s',\bar{a}'), q^1(s',\bar{a}')
\]

(2a)

\[
q^1(s,\bar{a}) = E \sum_{s',R} \gamma q^1(s',\bar{a}'), q^1(s',\bar{a}')
\]

(2b)

As already mentioned, a team problem of this form can be addressed with any single-agent algorithm. The fundamental inconvenience with this approach is that the joint action space scales exponentially with the number of agents, more specifically \(|\mathcal{A}| = \prod_{k=1}^{K} |A^{\tau_k}|\) (where \(\mathcal{A}\) is the joint action space).

Another problem with this approach is that the learned Q-function cannot be executed in a decentralized manner using the agents’ observations. Furthermore, the learned quantities (value functions or policies) are useless if the number of agents changes. However, if factored policies are learned, then these could be executed by teams with different number of agents (as long as the extra agents are of the same "type" as the agents used for learning. In section 5.3 we provide one example of this scenario). For these reasons, in the next sections we concern ourselves with learning factored quantities.

**Assumption 1.** We assume that if for two states \(s_1\) and \(s_2\) we have \(o^t_{s_1} = o^t_{s_2}\), then \(q^1(s_1,a^t_k,a^{-k})|a^{-k} = \pi^t_{a^{-k}}(a^{-k})|a^{-k} = b\) if and only if \(q^1(s_2,a^t_k,a^{-k})|a^{-k} = \pi^t_{a^{-k}}(a^{-k})|a^{-k} = b\).

**Assumption 2.** Agents of the same type are assumed to be homogeneous. Mathematically, if two agents \(n\) and \(k\) are homogeneous, then for every state \(s_1\) there is another equivalent state \(s_2\) such that:

\[
(o^t_{s_1} = o^t_{s_2} \forall \ell \neq (n,k)) \wedge (o^t_{s_2} = o^t_{s_1}) \wedge (o^t_{s_1} = o^t_{s_2}) \rightarrow q^1(s_1,\bar{a})|a^n = b = q^1(s_2,\bar{a})|a^n = b.
\]

(3)

In simple terms, assumption 1 means that even though observations are not full descriptions of the state, they provide enough information to know the effect of individual actions assuming everybody else in the team acts optimally (intuitively this is a reasonable requirement if the agents are expected to be able to play a team optimum strategy using only their partial observations). Assumption 2 means that if two agents of the same type are swapped (while other agents remain unchanged), then the value functions of the corresponding states are equal independently of the policy being executed by the team (as long as the agents swap their corresponding policies as well).

### 3 Factored Bellman relations and dynamic programming

Similarly to the way that relations 2 are used to derive Q-learning in the single agent setting, the goal of this section is to derive relations in the dynamic programming setting from which we can derive a MARL algorithm. The following two lemmas take the first steps in this direction.

**Lemma 1.** All TMDPs that satisfy assumptions 1 and 2 have, for each deterministic team optimal policy \(\pi^*\) a \(\mathcal{T}\) factored functions \(q^{\pi^*} : \mathcal{O}^* \times \mathcal{A}^* \rightarrow \mathbb{R}\) such that:

\[
\max_{\bar{a}} q^1(s,\bar{a}) = \max_{a^K} q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^K) = \cdots = \max_{a^K} q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^K)
\]

(4a)

\[
\max_{\bar{a}} q^1(s,\bar{a}) = q^1(s,a^K,\bar{a}^{\pi},a^{\pi})\max_{\bar{a}} q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^K)
\]

(4b)

\[
q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^K) = \sum_{s',R} \gamma q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^K)|a^n = \arg\max_{a^n} q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^{\ast}) \forall n \neq k
\]

(4c)

**Proof.** See Appendix 6.1.

A simple interpretation of equation 4c is that \(q^{\pi^{K\cdot \ast}}(o^{K\cdot \ast},a^K)\) is the expected return starting from state \(s\) when agent \(k\) takes action \(a^K\) while the rest of the team acts in an optimal manner.

**Lemma 2.** All TMDPs that satisfy assumptions 1 and 2 have at least one deterministic team optimal policy that can be factored into \(\mathcal{T}\) deterministic policies \(\pi^{\pi^{\ast}}(a|o)\), where \(a \in \mathcal{A}^* \) and \(o \in \mathcal{O}^*\). Such factored deterministic policies can be obtained as follows:

\[
\pi^{\pi^{\ast}}(a|o) = \begin{cases} 
1, & \text{if } a = \arg\max_a q^{\pi^{\ast}}(o,a) \\
0, & \text{else}
\end{cases}
\]

(5)
why this kind of stubborn rationale cannot escape Nash equilibria (i.e., agents do not learn when A simple interpretation of operator $\beta$ The answer is no, since the poor outcome was player $k$. View of some player $k$, what the first term of (7) means is "I will only learn from experiences in which my teammates acted according to what I think is the optimal team strategy". It is easy to see why this kind of stubborn rationale cannot escape Nash equilibria (i.e., agents do not learn when the team deviates from its current best strategy, which obviously is a necessary condition to learn better strategies). The interpretation of the full operator $B^\psi$ is "I will learn from experiences in which: a) my teammates acted according to what I think is the optimal team strategy; or b) my teammates deviated from what I believe is the optimal strategy and the outcome of such deviation was better than I expected if they had acted according to what I thought was optimal", which arguably is a logical player would do (this is the origin of the algorithm’s name).

\footnote{This problem arises in situations in which the TMDF has multiple deterministic team optimal policies and the agents learn factored functions of the form $\max_{a-k} q^\tau((s,a_k,a^{-k}))$ (we remark that these are not the same as $q^\tau((s,a_k,a_k))$.)}
The mean convergence rate is exponential with constant lower bounded by $\gamma$. As a sanity check, notice that in the single agent case operator $B^\psi$ reduces to the Bellman optimality operator and Theorem 1 reduces to the well-known result that repeated application of the Bellman optimality operator to any initial $Q$-function converges at an exponential rate (with constant $\gamma$) to $q^\dagger$.

**Theorem 1.** Repeated application of the operator $B^\psi$ to any initial $|T|$ $q^\tau$-functions converge to set $S$ with probability one. Mathematically:

$$
\mathbb{P}\left( \lim_{N \to \infty} (B^\psi)^N q^\tau(\omega^\tau_a, a^k) \in S \right) = 1 \\
S = \left\{ q^\tau \mid q^\tau(\omega^\tau_a, a^k) \leq q^\tau(\omega^\tau_o, a^k) \leq \max_{a^k} q^1(s, a^k, a^{-k}) \forall (\tau_k, o^\tau_k, a^k) \in (T, O^\tau_k, A^\tau_k) \right\}
$$

The mean convergence rate is exponential with constant lower bounded by $\gamma^p$, where $p$ is the lowest probability assigned to any $a^{-k}$ by $\psi$ (i.e. $p = \arg \min_{a^{-k}} \mathbb{P}_\psi(a^{-k})$).

**Proof.** See appendix 6.3. ■

As a sanity check, notice that in the single agent case operator $B^\psi$ reduces to the Bellman optimality operator and Theorem 1 reduces to the well-known result that repeated application of the Bellman optimality operator to any initial $Q$-function converges at an exponential rate (with constant $\gamma$) to $q^\dagger$.

### 4 Reinforcement Learning

In this section we present LTQL (see algorithm 1), which we obtain as a stochastic approximation to operator $B^\psi$. Note that the algorithm utilizes two $q$ estimates for each type $\tau$, a biased one parameterized by $\theta^\tau$ (which we denote $q_{\theta^\tau}$) and an unbiased one parameterized by $\omega^\tau$ (which we denote $q_{\omega^\tau}$). We clarify that in the listing of algorithm 1 we used a constant step-size, however this can be replaced with decaying step-sizes or other schemes such as AdaGrad [28] and Adam [29]. Note that the target of the unbiased network is used to calculate the target values for both functions; this prevents the bias in the estimates $q_{\theta^\tau}$ (which arises due to the $c_2$ condition) from propagating through bootstrapping. The target parameters of the biased estimates ($\theta^\tau$) are used solely to evaluate condition $c_1$. We have found that this stabilizes the training of the networks, as opposed to just using $\theta^\tau$. Hyperparameter $\alpha$ weights samples that satisfy condition $c_2 ((r + \max_a q_{\theta^\tau} (o^{\tau_k}_a, a) > q_{\theta^\tau} (o^{\tau_k}_a, a^{\kappa}))$ differently from those who satisfy $c_1$. Intuitively, since the purpose of condition $c_2$ is to escape Nash equilibria, $\alpha$ should be chosen as small as possible as long as the algorithm doesn’t get stuck in such equilibria. As we remarked in the introduction, LTQL reduces to DQN for the case where there is a unique agent. In appendix 6.6 we include the tabular version of the algorithm along with a brief discussion.

**Algorithm 1: Logical Team Q-Learning**

**Initialize:** an empty replay buffer $R$, parameters $\theta^\tau$ and $\omega^\tau$ and their corresponding targets $\theta^\tau_T$ and $\omega^\tau_T$ for all types $\tau \in T$.

**for** iterations $i = 0, \ldots, E$ **do**

Sample $T$ transitions $(o^{\tau_1}_a, \ldots, o^{\tau_K}_a, \bar{a}, r, o^{\tau_1}_a, \ldots, o^{\tau_K}_a)$ by following some behavior policy which guarantees all joint actions are sampled with non-zero probability and store them in $R$.

**for** iterations $i = 0, \ldots, E$ **do**

Sample a mini-batch of $B$ transitions $(o^{\tau_1}_a, \ldots, o^{\tau_K}_a, \bar{a}, r, o^{\tau_1}_a, \ldots, o^{\tau_K}_a)$ from $R$.

Set $\Delta_{\theta^\tau} = 0$ and $\Delta_{\omega^\tau} = 0$ for all types $\tau$.

**for** each transition of the mini-batch $b = 1, \ldots, B$ and each agent $k = 1, \ldots, K$ **do**

if $a^k = \arg \max_a q_{\theta^\tau} (o^{\tau_k}_a, a^k)$ then $\Delta_{\theta^\tau} = \Delta_{\theta^\tau} + (r + \max_a q_{\theta^\tau} (o^{\tau_k}_a, a) - q_{\theta^\tau} (o^{\tau_k}_a, a^k))$ else if $\alpha \max_a q_{\theta^\tau} (o^{\tau_k}_a, a) > q_{\theta^\tau} (o^{\tau_k}_a, a^k)$ then $\Delta_{\theta^\tau} = \Delta_{\theta^\tau} + \alpha (r + \max_a q_{\theta^\tau} (o^{\tau_k}_a, a) - q_{\theta^\tau} (o^{\tau_k}_a, a^k))$

**end if**

**end for**

$\theta^\tau = \theta^\tau + \mu \Delta_{\theta^\tau}$  \hspace{1cm} $\omega^\tau = \omega^\tau + \mu \Delta_{\omega^\tau}$

**end for**

Update targets $\theta^\tau_T = \theta^\tau$ and $\omega^\tau_T = \omega^\tau$.  

**end for**
Note that LTQL works off-policy and there is no necessity of synchronization for exploration. Therefore, it can be implemented in a fully decentralized manner as long as all agents have access to all observations (and therefore to the full state) and actions of other agents (so that they can evaluate $c_1$). Interestingly, if condition $c_1 (a^n = \arg \max_{a^n} q \tau_k (o^n, a^n) \forall n \neq k)$ was omitted (to eliminate the requirement that agents have access to all this information), the resulting algorithm is exactly DistQ \cite{14}. However, as the proof of theorem\cite{13} indicates, the resulting algorithm would only converge in situations where it could be guaranteed that during learning overestimation of the $q$ values is not possible (i.e., the tabular setting applied to deterministic MDPs; this remark was already made in \cite{14}). In the case where this condition could not be guaranteed (i.e., when using function approximation and/or stochastic MDPs), some mechanism to decrease overestimated $q$ values would be necessary, as this is the main task of updates due to $c_1$. One possible way to do this would be to use all transitions to update the $q$ estimates but use a smaller step-size for the ones that do not satisfy $c_2$. Notice that the resulting algorithm would be exactly HystQ \cite{15}.

5 Experiments

5.1 Matrix game

The first experiment is a simple matrix game (figure \ref{fig:1a} shows the payoff structure) with multiple team optimum policies to evaluate the resilience of the algorithm to the coordination issue mentioned in section\cite{3}. In this case, we implemented LTQL and DistQ in tabular form (we do not include HystQ because in deterministic environments with tabular representation this algorithm is dominated by DistQ) and we also implemented Qmix (note that this algorithm cannot be implemented in tabular form due to the use of the mixing network). In all cases we used uniform exploratory policies (c = 1) and we did not use replay buffer. DistQ converges to \ref{12}, which clearly shows why DistQ has a coordination issue. However, LTQL converges to either of the two possible solutions shown in \ref{13} (depending on the seed) for which individual greedy policies result in team optimal policies. Qmix converges to \ref{14}. Note that Qmix fails at identifying an optimum team policy and the resulting joint $Q$-function $q_{jt}$ obtained using the mixing network also fails at predicting the rewards. The full $q_{jt}$ is shown in appendix 6.7, where we also include the learning curves of all algorithms for the readers reference along with a brief discussion.

\begin{equation}
q^1(a^1) = \max_{a^2} q^1(a^1, a^2) = [2, 2] \quad q^2(a^2) = \max_{a^1} q^1(a^1, a^2) = [0, 2, 2] \quad \text{(12)}
\end{equation}

\begin{equation}
q^1(a^1) = [2, 1] \quad q^2(a^2) = [0, 2, 0] \quad \text{or} \quad q^1(a^1) = [0, 2] \quad q^2(a^2) = [0, 1, 2] \quad \text{(13)}
\end{equation}

The average test return is the return following a greedy policy averaged over 50 games.

5.2 Stochastic finite TMDP

In this experiment we use a tabular representation in a stochastic episodic TMDP. The environment is a linear grid with 4 positions and 2 agents. At the beginning of the episode, the agents are initialized in the far right. Agent 1 cannot move and has 2 actions (\textit{push button} or \textit{not push}), while agent 2 has 3 actions (\textit{stay}, \textit{move left} or \textit{move right}). If agent 2 is located in the far left and chooses to \textit{stay} while agent 2 chooses \textit{push}, the team receives a +10 reward. If the button is pushed while agent 2 is moving left the team receives a −30 reward. This negative reward is also obtained if agent 2 stays still in the leftmost position and agent 1 does not push the button. All rewards are subject to additive Gaussian noise with mean 0 and standard deviation equal to 1. Furthermore if agent 2 tries to move beyond an edge (left or right), it stays in place and the team receives a Gaussian reward with mean 0 and standard deviation equal to 3. The TMDP finishes after 5 timesteps or if the team gets the +10 reward (whichever happens first). We ran the simulation 5 times with different seeds. Figure \ref{fig:1b} shows the average test return\cite{5} (without the added noise) of LTQL, HystQ, DistQ and Qmix. As can be seen, LTQL is the only algorithm capable of learning the optimal team policy. In appendix 6.8 we specify the hyperparameters and include the learning curves of the $Q$-functions along with a discussion on the performance of each algorithm.

\begin{equation}
q^1 = [-0.75, 1.09] \quad q^2 = [-3.49, 1.83, 0.62] \quad \text{(14)}
\end{equation}
5.3 Cowboy bull game

In this experiment we use a more complex environment. The TMDP is a challenging predator-prey type game, in which 4 (homogeneous) cowboys try to catch a bull (see figure 1c). The position of all players is a continuous variable (and hence the state space is continuous). The space is unbounded and the bull can move 20% faster than the cowboys. The bull follows a fixed stochastic policy, which is handcrafted to mimic natural behavior and evade capture. Due to the unbounded space and the fact that the bull moves faster than the cowboys, it cannot be captured unless all agents develop a coordinated strategy (the bull can only be caught if the agents first surround it and then close in evenly). The task is episodic and ends after 75 timesteps or when the bull is caught. Each agent has 5 actions (the four moves plus stay). When the bull is caught a +1 reward is obtained and the team also receives a small penalty (−1/(4 × 75)) for every agent that moves. Note that due to the reward structure there is a very easily attainable Nash equilibrium, which is for every agent to stay still (since in this way they do not incur in the penalties associated with movement). In this game, since all agents are homogeneous, only one Q-function is learned whose input is the agent’s observation and the output are the Q-values corresponding to the 5 possible actions. Figure 1d shows the test win percentage and figure 1e shows the average test return for LTQL, HystQ and Qmix. The best performing algorithm is LTQL. HystQ learns a policy that catches the bull 80% of the times, although it fails at obtaining returns higher than zero. We believe that the poor performance of Qmix in this task is a consequence of its limited representation capacity due to its monotonic factoring model. As we mentioned in the introduction, we can test the learned policy on teams with different number of agents, figure 1f shows the results. The policy scores above 70% for teams of all sizes bigger than 4. Note that the policy can be improved for any particular team size by further training if necessary. In the appendix we provide all hyperparameters and implementation details, we detail the bull’s policy and the observation function. All code, a pre-trained model and a video of the policy learned by LTQL are included as supplementary material.

6 Conclusions

In this article we have introduced theoretical groundwork for cooperative MARL. We also introduced LTQL, which has the 5 desirable properties mentioned in the introduction. Furthermore, it does not

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Figure 1: (c) through (f) correspond to the cowboy bull game. In (d) and (e) the blue, green and red curves correspond to LTQL, HystQ and Qmix, respectively. In (b), (d) and (e) the dark curves show the mean over all seeds while the shaded region show the min and max limits over the seeds.
impose constraints on the learned individual $Q$-functions and hence it can solve environments where previous algorithms, which are considered to be state of the art such as $Qmix$ [22], fail. The algorithm fits in the centralized training and decentralized execution paradigm. It can also be implemented in a fully distributed manner in situations where all agents have access to each others’ observations and actions.

**Broader Impact**

This paper introduces novel concepts and algorithms to MARL theory. We believe the material we present does not introduce any societal or ethical considerations worth mentioning in this section.

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Appendix

6.1 Proof of Lemma 1

Assumption 3. We assume that if for two states \(s_1\) and \(s_2\) we have \(o^{a_k}_{s_1} = o^{a_k}_{s_2}\), then
\[
q^1(s_1, a^k, a^{-k})|_{a^{-k} \sim \pi^1(a^k, a^{-k}|s_1)} = q^1(s_2, a^k, a^{-k})|_{a^{-k} \sim \pi^1(a^k, a^{-k}|s_2)}.
\]

We start by rewriting equation (2b) for convenience:
\[
q^1(s, \bar{a}) = E [r(s, \bar{a}, s') + \gamma \max_{\bar{a}'} q^1(s', \bar{a}')] \tag{15}
\]

Now assume that we have some team optimal policy \(\pi^\dagger(\bar{a}|s)\). We define \(q^{k,\dagger}(s, a^k)\) as follows:
\[
q^{k,\dagger}(s, a^k) = q^1(s, a^k, a^{-k})|_{a^{-k} \sim \pi^1(a^k, a^{-k}|s)} \tag{16}
\]

In simple terms \(q^{k,\dagger}(s, a^k)\) is the \(q\)-value if agent \(k\) takes action \(a^k\) while the rest of the agents act optimally (note that this is not the same as \(\max_{q^{-k}_s} q^1(s, a^k, a^{-k})\)). Due to assumption 1 \(q^{k,\dagger}(s, a^k)\) can be written as a function of the observations. Therefore, we define \(q^{k,\bullet}\) as:
\[
q^{k,\bullet}(o^{a_k}_s, a^k) = q^{k,\dagger}(s, a^k) = q^1(s, a^k, a^{-k})|_{a^{-k} \sim \pi^1(a^k, a^{-k}|s)} \tag{17}
\]

Note that by construction we get:
\[
\max_{a^k} q^{k,\bullet}(o^{a_k}_s, a^k) = \max_{\bar{a}} q^1(s, \bar{a}) \forall k \tag{18}
\]
\[
\arg \max_{a^k} q^{k,\bullet}(o^{a_k}_s, a^k) = a^k \sim \pi^1(a^k, a^{-k}|s) \forall k \tag{19}
\]
\[
\max_{\bar{a}} q^1(s, \bar{a}) = q^1(s, \arg \max_{a^k} \max_{\bar{a}} q^{k,\bullet}(o^{a_k}_s, a^k), \cdots, \arg \max_{a^K} \max_{\bar{a}} q^{k,\bullet}(o^{a_k}_s, a^K)) \forall k \tag{20}
\]

Combining (15), (17) and (19) we get the following relation for \(q^{k,\bullet}\):
\[
q^{k,\bullet}(o^{a_k}_s, a^k) = q^1(s, a^k, a^{-k})|_{a^{-k} \sim \pi^1(a^k, a^{-k}|s)} \forall k \tag{21}
\]
\[
q^{k,\bullet}(o^{a_k}_s, a^k) = q^k\dagger(s, a^k, a^{-k})|_{a^{-k} = \arg \max_{a^{-k}} q^{n,\bullet}(o^{a_k}_s, a^{-k})} \forall k \tag{22}
\]
\[
= E [r(s, a^k, a^{-k}, s') + \gamma \max_{a_{\bar{k},k}} q^{k,\bullet}(o^{a_k}_s, a^k), a^{\bar{k},k})|_{a^{-k} = \arg \max_{a^{-k}} q^{n,\bullet}(o^{a_k}_s, a^{-k})} \forall k \tag{23}
\]

Note that (18), (20) and (23) satisfy equations (4a), (4b) and (4c), respectively. However, functions \(q^{k,\bullet}\) are defined on a per agent basis (which means that there are \(K\) such functions) while functions \(q^{n,\bullet}\) depend on the type (and hence there are \(|T|\) such functions). Therefore, still we have to prove that functions \(q^{k,\bullet}\) corresponding to agents of the same type are equal. From assumption 2 it follows that choosing \(q^\tau\) in (3) to be \(q^1(s, a^k, a^{-k})|_{a^{-k} \sim \pi^1(a^k, a^{-k}|s)}\) we get:
\[
q^1(s_1, a^k, a^{-k})|_{a^{-k} = b} = q^1(s_2, a^n, a^{-n})|_{a^{-n} = b} \forall b \in A^\tau \tag{24}
\]
\[
o^{k}_s = o^{n}_s \tag{25}
\]

where \(s_1\) and \(s_2\) are two equivalent states that swap agents \(k\) and \(n\) such that \(\tau_k = \tau_n = \tau\) for some type \(\tau\). Since (24) holds for any \(b \in A^\tau\) and for all equivalent states, combining (24), (25) and (17) and setting \(o = o^{k}_s = o^{n}_s\) we get that:
\[
q^{k,\bullet}(o, a) = q^{n,\bullet}(o, a) = q^{\tau,\bullet}(o, a) \forall (k, o, a)|_{\tau_k = \tau, o \in O^\tau, a \in A^\tau} \tag{26}
\]

which completes the proof.

6.2 Proof of remark 1

Consider the matrix game with two homogeneous agents, each of which has two actions \((A = \{\alpha, \beta\})\) and the following reward structure:
For this case $q^l, q^*, \pi^l$ and $\pi^*$ are given by:

\[
\begin{array}{c|cc}
 q^l(a^1, a^2) & \alpha & \beta \\
\hline
\alpha & 0 & -1 \\
\beta & -1 & 1 \\
\end{array}
\]

\[
\begin{array}{c|cc}
 \pi^l(a^1, a^2) & \alpha & \beta \\
\hline
\alpha & 0 & 0 \\
\beta & 0 & 1 \\
\end{array}
\]

\[
\begin{array}{c|cc}
 q^*(a) & \alpha & \beta \\
\hline
\alpha & -1 & 1 \\
\beta & 0 & 1 \\
\end{array}
\]

Notice that as expected, $q^*$ satisfies (4c). However, note that (4c) is also satisfied by the following $q$ function which is different from $q^*$.

\[
q(a = \alpha) = 0, \quad q(a = \beta) = -1
\]  

(27)

Notice further that the team policy obtained by choosing actions in a greedy fashion with respect to $q$ constitutes a Nash equilibrium.

6.3 Proof of Theorem[I]

We start defining the following auxiliary constants and operators:

\[
q_{\max} = r_{\max}(1 - \gamma)^{-1}
\]

\[
q_{\min} = r_{\min}(1 - \gamma)^{-1}
\]

\[
B_E q^r_k(o^k_s, a^k) = \mathbb{E}_{\pi, \gamma}(r(s, a, k, \bar{s}')) + \gamma \max_{a'} \{ q^r_k((o^k_s, a^k)) \}_{a_n = \arg \max_{a_n} q^r_k(o^k_s, a_n)} \forall \gamma \neq k
\]

(30)

\[
B_I q^r_k(o^k_s, a^k) = \max \{ q^r_k(o^k_s, a^k), \max_{a_n} \mathbb{E}_{\pi, \gamma}(r(s, a, k, \bar{s}')) + \gamma \max_{a'} q^r_k((o^k_s, a')) \}
\]

(31)

\[
B_U \psi q^r_k(o^k_s, a^k) = \begin{cases} B_E q^r_k(o^k_s, a^k) & \text{with probability } p \\ B_I q^r_k(o^k_s, a^k) & \text{else} \end{cases}
\]

(32)

\[
B_L \psi q^r_k(o^k_s, a^k) = [\mathbb{E}(c_1) B_E q^r_k(o^k_s, a^k) + \mathbb{E}(c_2) B_I q^r_k(o^k_s, a^k)] + \mathbb{E}(c_1 \land c_2) B_I q^r_k(o^k_s, a^k)
\]

(33)

\[
c_1 = \Delta \left( a^n = \arg \max_{a^n} q^n(o^k_s, a^n) \quad \forall n \neq k \right)
\]

(34)

\[
c_2 = \Delta \left( a^{-k} = \arg \max_{a^{-k}} B_{a^{-k}} q^r_k(o^k_s, a^k) \right)
\]

(35)

where $r_{\max}$ is the maximum mean reward, $r_{\min}$ is the minimum mean reward and $p$ is the minimum probability assigned to any $a^{-k}$ by the discrete distribution $\psi$.

Lemma 3. Repeated application of operator $B_U \psi$ to any $q^r_k(o^k_s, a^k) \geq q_{\max}$ converges to set $S_U = \{ q^r_k|q^r_k(o^k_s, a^k) \leq \max_{a^{-k}} q^l(s, a, k, a^{-k}) \forall o^k_s \in \mathcal{O}^k_s \land a^k \in \mathcal{A}^k \}$ with probability one. The mean rate of convergence is exponential with Lipschitz constant $\gamma^p$.

Proof. See appendix 6.4.

Lemma 4. Repeated application of operator $B_L \psi$ to any $q^r_k(o^k_s, a^k) \leq q_{\min}$ converges to set $S_L = \{ q^r_k|q^r_k(o^k_s, a^k) \geq q^{r-k}(o^k_s, a^k) \forall o^k_s \in \mathcal{O}^k_s \land a^k \in \mathcal{A}^k \}$ with probability one. The mean rate of convergence is exponential with Lipschitz constant $\gamma^p$. 

1
Proof. See appendix 6.5.

Now assume we have an initial Q-function and define \( q_L(o_s^n, a^k) = \min \{ q_{\min}, q(o_s^n, a^k) \} \) and \( q_U(o_s^n, a^k) = \max \{ q_{\max}, q(o_s^n, a^k) \} \). We conclude the proof by noting that \( B_L^q q_L(o_s^n, a^k) \leq B^q q(o_s^n, a^k) \leq B^q q_U(o_s^n, a^k) \) and making use of lemmas (3) and (4) and the sandwich theorem.

6.4 Proof of Lemma 3

We start defining \( q_I^q(o_s^n, a^k) = q_U, \forall \tau_k, o_s^n, a^k \), where \( q_U \geq q_{\max} \). The first part of the proof consists in showing that any sequence of the form \( B^{K_0} B_E \cdots B^{K_1} B_E B^{K_0} q_I^q(o_s^n, a^k) \) is equal to \( B^{K_0} q_I^q(o_s^n, a^k) \) where \( K_0 \in \mathbb{N} \) and \( n \in \mathbb{N} \) is the number of times that operator \( B_I \) is applied in the aforementioned sequence. Applying operator \( B_I \) to \( q_I^q(o_s^n, a^k) \) we get:

\[
B_I q_I^q(o_s^n, a^k) = \max \left\{ q_U, \max_{a \in \mathbb{A}} E_{P,I} \left( r(s, a^k, a^{-k}, s') + \gamma q_U \right) \right\} = q_U
\]

Therefore, \( B^{K_0} q_I^q(o_s^n, a^k) = q_I^q(o_s^n, a^k) \) for any \( K_0 \in \mathbb{N} \). Applying operator \( B_E \) we get:

\[
B_E q_I^q(o_s^n, a^k) = \mathbb{E}_{x' \sim P} \left( r(s, a^k, a^{-k}) + \gamma q_U \right) \left| a^n = \arg \max_{a^n} q_U \right. \forall n \neq k
\]

\[
\leq \max_{a \in \mathbb{A}} r(s, a^k, a^{-k}) + \gamma q_U
\]

\[
B_E q_I^q(o_s^n, a^k) \leq \mathbb{E}_{x' \sim P} \left( r(s, a^k, a^{-k}) + \gamma \max_{a'} r(s', a') + \gamma^2 q_U \right) | a^n = \arg \max_{a^n} B_E q_I^q(o_s^n, a^n) \forall n \neq k
\]

\[
\leq \max_{a \in \mathbb{A}} \mathbb{E}_{x' \sim P} \left( r(s, a^k, a^{-k}) + \gamma \max_{a'} r(s', a') + \gamma^2 q_U \right)
\]

\[
B^{K_1} q_I^q(o_s^n, a^k) \leq \max_{a_0^k, a_1^k, \cdots, a_{K_1-1}^k} \mathbb{E} \left( \sum_{i=0}^{K_1-1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_1} q_U
\]

where to simplify notation we defined \( \mathbb{E}_{P,I} r(s, a^k, a^{-k}, s') = r(s, a^k, a^{-k}) \). Further application of \( B_I \) we get:

\[
B_I B^{K_1} q_I^q(o_s^n, a^k) \leq \max \left\{ \max_{a_0^k, a_1^k, \cdots, a_{K_1-1}^k} \mathbb{E} \left( \sum_{i=0}^{K_1-1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_1} q_U, \right\}
\]

\[
\leq \max_{a_0^k, a_1^k, \cdots, a_{K_1-1}^k} \mathbb{E} \left( \sum_{i=0}^{K_1-1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_1+1} q_U
\]

Therefore, we conclude that \( B^{K_2} B^{K_1} B_I q_I^q(o_s^n, a^k) = B^{K_1} q_I^q(o_s^n, a^k) \). More generally, we can write:

\[
B^{K_n} \cdots B^{K_1} B_I q_I^q(o_s^n, a^k) = B^{q_U} (o_s^n, a^k)
\]

\[
\leq \max_{a_0^k, a_1^k, \cdots, a_{n-1}^k} \mathbb{E} \left( \sum_{i=0}^{n-1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^n q_U
\]

\[
\leq \max_{a \in \mathbb{A}} q^l(s, a^k, a^{-k}) + \gamma^n \left( q_U - \min_{a \in \mathbb{A}} q^l(s, a) \right)
\]

where \( n \) is the number of times that operator \( B_E \) is applied. Therefore, we get:

\[
(B^{q_U})^n q_I^q(o_s^n, a^k) \leq \max_{a \in \mathbb{A}} q^l(s, a^k, a^{-k}) + \gamma^n \left( q_U - \min_{a \in \mathbb{A}} q^l(s, a) \right)
\]

where \( n \in \mathbb{N} \) is defined as before (i.e., it is the amount of times that \( B_E \) was selected out of \( N \). Now we proceed to show that the probability that \( n \) is finite in any infinite sequence is zero. The
We conclude the proof by lower bounding the mean convergence rate as follows:

\[
\mathbb{P}(n; N) \leq \left( \frac{N}{n} \right)^n (1 - P)^{N - n} = \left( \frac{1}{n!} \prod_{k=N-n+1}^{N} k \right) P^n (1 - P)^{N - n} \leq \frac{N^n}{n!} P^n (1 - P)^{N - n} \tag{43}
\]

\[
\lim_{n \to \infty} \mathbb{P}(n; N) = 0 \tag{44}
\]

where in (b) we upper bounded the probability of \( B_E \) with \( P \), which is the highest probability assigned to any \( a^{-k} \) by \( \psi \) (i.e. \( P = \max_{a^{-k}} \mathbb{P}_\psi(a^{-k}) \)) and then calculated the probability using the binomial distribution. Since \( \lim_{n \to \infty} \mathbb{P}(n; N) = 0 \), it follows that the probability of \( n \) being finite in an infinite sequence is also 0 (because it is the finite sum of probabilities which tend to 0).

Therefore we can conclude:

\[
\lim_{N \to \infty} (B_E^\psi)^N q_L^\gamma (a_{s_i}^{x_i}, a^k) \overset{w.p.1}{\leq} \max_{a^{-k}} q^+ (s, a^k, a^{-k}) \tag{45}
\]

We conclude the proof by lower bounding the mean convergence rate as follows:

\[
\mathbb{E} (\gamma^n) \geq \gamma \mathbb{E} n \geq (\gamma P)^N \tag{46}
\]

where in (c) we used Jensen’s inequality and in (d) we lower bounded the probability \( \mathbb{P}(n; N) \) using a binomial distribution with probability \( p \), where \( p \) is the lowest probability assigned to any \( a^{-k} \) by \( \psi \) (i.e. \( p = \min_{a^{-k}} \mathbb{P}_\psi(a^{-k}) \)).

### 6.5 Proof of Lemma 4

The first part of the proof consists in showing that any sequence of the form \( B_I, B_E^{K_1} B_I, B_E^{K_2} B_I, \ldots \) \( B_E^{K_n} q_L^\gamma (a_{s_i}^{x_i}, a^k) \) is lower bounded by the sequence \( B_I^n q_L^\gamma (a_{s_i}^{x_i}, a^k) \) where \( K_n \in \mathbb{N} \) and \( n \in \mathbb{N} \) is the number of times that operator \( B_I \) is applied in the aforementioned sequence. We start defining \( q_L^\gamma (a_{s_i}^{x_i}, a^k) = q_L \) where \( q_L \leq q_{\min} \). If we apply \( B_E \) to \( q_L (a_{s_i}^{x_i}, a^k) \) we get:

\[
B_E q_L^\gamma (a_{s_i}^{x_i}, a^k) \geq q_L = q_L^\gamma (a_{s_i}^{x_i}, a^k) \tag{47}
\]

\[
B_E^{K_n} q_L^\gamma (a_{s_i}^{x_i}, a^k) \geq q_L^\gamma (a_{s_i}^{x_i}, a^k) \tag{48}
\]

Therefore, we get \( B_I \) \( B_E^{K_n} q_L^\gamma (a_{s_i}^{x_i}, a^k) \geq B_I q_L^\gamma (a_{s_i}^{x_i}, a^k) \) more specifically, we can write:

\[
B_I q_L^\gamma (a_{s_i}^{x_i}, a^k) = \max \left\{ q_L, \max_{a^{-k}} r(s, a^k, a^{-k}) + \gamma q_L \right\} = \max \{ r(s, a^k, a^{-k}) + \gamma q_L \} \tag{49}
\]

\[
B_I^{K_n} q_L^\gamma (a_{s_i}^{x_i}, a^k) = \max_{a_{s_i}^{-k}} \mathbb{E} \left( \sum_{i=0}^{K_n-1} \gamma^i r(s_i, a_{s_i}^k, a_{s_i}^{-k}) | s_0 = s \right) + \gamma^{K_n} q_L \tag{50}
\]

where like in the previous section we defined \( \mathbb{E}_{P, f} r(s, a^k, a^{-k}, s') = r(s, a^k, a^{-k}) \). Applying \( B_E \) we get:

\[
B_E B_I^{K_1} q_L^\gamma (a_{s_i}^{x_i}, a^k) = \max_{a_{s_1}^{-k}, \ldots, a_{s_{K_1}}^{-k}} \mathbb{E} \left( \sum_{i=0}^{K_1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_1+1} q_L
\]

\[
B_E B_I^{K_1} q_L^\gamma (a_{s_i}^{x_i}, a^k) = \max_{a_{s_1}^{-k}, \ldots, a_{s_{K_1}}^{-k}} \mathbb{E} \left( \sum_{i=0}^{K_1+1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_1+2} q_L
\]

\[
B_E B_I^{K_1} q_L^\gamma (a_{s_i}^{x_i}, a^k) = \max_{a_{s_1}^{-k}, \ldots, a_{s_{K_1+1}}^{-k}} \mathbb{E} \left( \sum_{i=0}^{K_1+1} \gamma^i r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_1+2} q_L
\]

\[
\text{\ where } a_{s_n}^{-k} \sim \text{binomial distribution because the sampled } a^{-k} \sim \psi \text{ in successive applications of operator } B_E \text{ are independent of each other.}
\]
Expanding \( \arg \max_a B_E B_I^{K_I} q_L^n(o_s^n, a^n) \) we get:

\[
\arg \max_a B_E B_I^{K_I} q_L^n(o_s^n, a^n) = \\
\arg \max_{a_0} \max_{a_1, \ldots, a_{K_I-1}} \max_{s_0} \left( \sum_{i=0}^{K_I} \gamma^n r(s_i, a_i^n, a_i^{-n}) | s_0 = s \right) \text{ for } a_0 = \arg \max B_I^{q_L^n} \forall n \neq k
\]

where in (a) we used equation (50). Combining (53) with (52) we get:

\[
B_E^2 B_I^{K_I} q_L^n(o_s^n, a^k) = \max_{a_0, \ldots, a_{K_I-I}} \max_{s_0} \left( \sum_{i=0}^{K_I-1} \gamma^n r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_I+2} q_L \]

Therefore, it follows:

\[
B_E^{K_I} B_I^{K_I} q_L^n(o_s^n, a^k) \geq B_E B_I^{K_I} q_L^n(o_s^n, a^k)
\]

Applying \( B_I \) to (50) we get:

\[
B_I B_E B_I^{K_I} q_L^n(o_s^n, a^k) \geq \max_{a_0, \ldots, a_{K_I-I}} \left( \sum_{i=0}^{K_I-1} \gamma^n r(s_i, a_i^k, a_i^{-k}) | s_0 = s \right) + \gamma^{K_I+4} q_L = B_I^{K_I+1} q_L^n(o_s^n, a^k)
\]

Therefore, we conclude that any sequence of the form \( B_I \cdots B_E^{K_I} B_I^{K_I} q_L^n(o_s^n, a^k) \) is lower bounded by the sequence \( B_I^{K_I} q_L^n(o_s^n, a^k) \) where \( n \) is the number of times that operator \( B_I \) is applied in the aforementioned sequence. Furthermore, from equation (50) we see that as \( n \to \infty \), \( B_I^{q_L^n}(o_s^n, a^k) \) converges to \( \max_{a^k} q^i(s, a^k, a^{-k}) \) at an exponential rate with Lipschitz constant \( \gamma \):

\[
\lim_{n \to \infty} B_I^{q_L^n}(o_s^n, a^k) = \lim_{n \to \infty} \max_{a^k} q^i(s, a^k, a^{-k}) + \gamma^n (q_L - \max_{a} q^i(s_{K_I+1}, a)) = \max_{a^k} q^i(s, a^k, a^{-k})
\]

A corollary of the previous statement is that any sequence of the form \( B_E \cdots B_E B_I^{K_I} B_E^{K_I} q_L^n(o_s^n, a^k) \) is lower bounded by the sequence \( B_E B_I^{q_L^n}(o_s^n, a^k) \) (note that this sequence ends with the application of \( B_E \) as opposed to \( B_I \), which is the case for the sequence studied in the previous paragraph). In this case the sequence still converges at the same rate to \( q^* (o_s^n, a^k) \) as opposed to \( \max_{a^k} q^i(s, a^k, a^{-k}) \). This can easily be seen as follows:

\[
\lim_{n \to \infty} B_E B_I^{q_L^n}(o_s^n, a^k) = \lim_{n \to \infty} B_E \left[ \max_{a^{-k}} q^i(s, a^k, a^{-k}) + \gamma^n (q_L - \max_{a} q^i(s_{K_I+1}, a)) \right] = B_E \max_{a^k} q^i(s, a^k, a^{-k}) = q^{k*}(o_s^n, a^k)
\]

Since by definition \( q^{k*}(o_s^n, a^k) \leq \max_{a^k} q^i(s, a^k, a^{-k}) \) and operator \( B_I^\psi \) applies \( B_I \) or \( B_E \) at random (and therefore a sequence of composed of several applications of \( B_L^\psi \) can end with either \( B_I \) or \( B_E \)) we get:

\[
(B_L^\psi)^N q_L^n(o_s^n, a^k) \geq B_E \left[ \max_{a^{-k}} q^i(s, a^k, a^{-k}) + \gamma^n (q_L - \max_{a} q^i(s_{K_I+1}, a)) \right] = q^{k*}(o_s^n, a^k) + O(\gamma^n)
\]
where \( n \in \mathbb{N} \) is defined as before (i.e., it is the amount of times that \( B_i \) was selected out of \( N \)). The same arguments we used in the previous section to prove the convergence \( w.p.1 \) and lower bound the rate of convergence apply here without modification. Therefore, we conclude:

\[
\lim_{N \to \infty} (B^L)_{\pi_n} \leq q^\pi_n(o^k, a^k) \quad \text{w.p.} \quad 1 \quad \text{(62)}
\]

\[
\mathbb{E}(\tau^n) \geq \gamma \mathbb{E} n \geq \gamma \rho N \quad \text{(63)}
\]

6.6 Tabular Logical Team Q-Learning

In the particular case where the MDP is deterministic (and hence the same arguments we used in the previous section to prove the convergence \( w.p.1 \) and lower bound the rate of convergence apply here without modification. Therefore, we conclude:

\[
\lim_{N \to \infty} (B^L)_{\pi_n} \leq q^\pi_n(o^k, a^k) \quad \text{w.p.} \quad 1 \quad \text{(62)}
\]

\[
\mathbb{E}(\tau^n) \geq \gamma \mathbb{E} n \geq \gamma \rho N \quad \text{(63)}
\]

Algorithm 2 Tabular Logical Team Q-Learning for deterministic MDPs

**Initialize:** an empty replay buffer \( \mathcal{R} \) and estimates \( \hat{q}_B^\pi \) and \( \hat{q}_U^\pi \).

**for** iterations \( e = 0, \ldots, E \) **do**

Sample \( T \) transitions \((o^0_i, \ldots, o^K_i, a_i, r_i, o^0_{i+1}, \ldots, o^K_{i+1})\) by following some behavior policy which guarantees all joint actions are sampled with non-zero probability and store them in \( \mathcal{R} \).

**for** iterations \( i = 0, \ldots, I \) **do**

Sample a transition \((o^0_i, \ldots, o^K_i, a_i, r_i, o^0_{i+1}, \ldots, o^K_{i+1})\) from \( \mathcal{R} \).

**for** agent \( k = 1, \ldots, K \) **do**

**if** \( a^n = \arg \max_a \hat{q}_B(o^{n,k}, a^n) \) \( \forall n \neq k \) **then**

\[
\hat{q}_B(o^{n,k}, a^k) = \hat{q}_B(o^{n,k}, a^k) + \mu (r + \max_a \hat{q}_U(o^{n,k}, a) - \hat{q}_B(o^{n,k}, a^k))
\]

**end if**

**else** if \( (r + \max_a \hat{q}_U(o^{n,k}, a) > \hat{q}_B(o^{n,k}, a^k)) \) **then**

\[
\hat{q}_U(o^{n,k}, a^k) = \hat{q}_U(o^{n,k}, a^k) + \mu (r + \max_a \hat{q}_U(o^{n,k}, a) - \hat{q}_B(o^{n,k}, a^k))
\]

**end if**

**end for**

**end for**

**end for**

Algorithm 3 Tabular Logical Team Q-Learning

**Initialize:** an empty replay buffer \( \mathcal{R} \) and estimates \( \hat{q}_B^\pi \) and \( \hat{q}_U^\pi \).

**for** iterations \( e = 0, \ldots, E \) **do**

Sample \( T \) transitions \((o^0_i, \ldots, o^K_i, a_i, r_i, o^0_{i+1}, \ldots, o^K_{i+1})\) by following some behavior policy and store them in \( \mathcal{R} \).

**for** iterations \( i = 0, \ldots, I \) **do**

Sample a transition \((o^0_i, \ldots, o^K_i, a_i, r_i, o^0_{i+1}, \ldots, o^K_{i+1})\) from \( \mathcal{R} \).

**for** agent \( k = 1, \ldots, K \) **do**

**if** \( a^n = \arg \max_a \hat{q}_B(o^{n,k}, a^n) \) \( \forall n \neq k \) **then**

\[
\hat{q}_B(o^{n,k}, a^k) = \hat{q}_B(o^{n,k}, a^k) + \mu (r + \max_a \hat{q}_U(o^{n,k}, a) - \hat{q}_B(o^{n,k}, a^k))
\]

**end if**

**else** if \( (r + \max_a \hat{q}_U(o^{n,k}, a) > \hat{q}_B(o^{n,k}, a^k)) \) **then**

\[
\hat{q}_U(o^{n,k}, a^k) = \hat{q}_U(o^{n,k}, a^k) + \mu (r + \max_a \hat{q}_U(o^{n,k}, a) - \hat{q}_B(o^{n,k}, a^k))
\]

**end if**

**end for**

**end for**

**end for**

If algorithm 2 were applied to a stochastic MDP, due to condition \( c_2 ((r + \max_a \hat{q}_U(o^{n,k}, a) > \hat{q}_B(o^{n,k}, a^k))) \), it would be subject to bias, which would propagate through bootstrapping and hence
could compromise its ability to find optimal team policies. This can be solved by having a second unbiased estimate $q_U$ that is updated only when $c_1$ is satisfied and use this unbiased estimate to bootstrap. The resulting algorithm is shown in algorithm 3.

### 6.7 Matrix game additional results

We start specifying the hyperparameters. For *Logical Team Q-learning* and *DistQ* we used a step-size equal to $0.1$. The mixing network in *Qmix* has 2 hidden layers with 5 units each, the nonlinearity used was the ELu and the step-size used was $0.05$ (we had to make it smaller than the others to make the SGD optimizer converge). We finally remark that due to the more complex structure of *Qmix* (compared to the other two algorithms) we had to train this algorithm with 100 times more games (notice the x-axis in figure 2).

In figure 2 we show the convergence curves for *Qmix*, *DistQ* and *Logical Team Q-learning*. Figures 2a and 2c correspond to the deterministic (algorithm 2) and general version (algorithm 3), respectively. Figures 2d and 2e also show curves for *Logical Team Q-learning* but use different seeds and provide evidence to our claim that this algorithm can converge to either of the two following options:

$$q^{1,*}(a^1) = [2, 1] \quad q^{2,*}(a^2) = [0, 2, 0] \quad \text{or} \quad q^{1,*}(a^1) = [0, 2] \quad q^{2,*}(a^2) = [0, 1, 2]$$  \hspace{1cm} (64)

One interesting fact to note is that the suboptimal values of $q^*_B$ (in figures 2b and 2d) do not converge while the same values do converge in the case of $q^*_U$. This is a direct consequence of theorem 1. As the theorem suggests, only the optimal values of the estimates generated by *Logical Team Q-learning* converge to $q^{**,*}$, and suboptimal values lie in the region between $q^{**,*}(o^*, a^k)$ and $\max_{a^k} q^1(s, a^*, a^k)$. Due to the bias generated by condition $c_2$, the values corresponding to sub-optimal actions converge to values in between $q^{**,*}$ and $q^1$. All the values $q^*_U$ converge because these estimates are not affected by this bias (because $q^U$ is only updated through $c_1$, see algorithm 3).

![Figure 2: Matrix game. In all figures the red curves correspond to the three actions of agent 2, while the blue curves correspond to the two actions from agent 1.](image)

Below we show the joint $q$ values generated by *Qmix*’s mixing network.
6.8 Stochastic TMDP additional results

The observation corresponding to agent 1 is a vector with two binary elements: the first one indicates whether or not agent 2 is in the leftmost position, and the second element indicates whether or not there is enough time for agent 2 to reach the leftmost position. The observation corresponding to agent 2 is a vector with two elements: the first one is the number of the position it occupies and the second one is the same as agent 1 (whether there is enough time to reach the leftmost position). Note that these observations are not full descriptions of the state, however they do satisfy assumption 1.

All algorithms are implemented in an on-line manner with no replay buffer. $\epsilon$-greedy exploration with a decaying schedule is used in all cases ($\epsilon = \max[0.05, 1 - \text{epoch}/5 \times 10^4]$). The learning rate used is $\mu = 10^{-1}$ and the smaller second rate for $HystQ$ is $\mu_{\text{small}} = 5 \times 10^{-2}$, in the case of $Qmix$ we used $\mu = 10^{-3}$ to guarantee stability.

Figures show the estimated $q$-values for Logical Team $Q$-learning corresponding to 4 different observations at the 4 positions. Note that in all figures the optimum action has the highest value and correctly estimates the return corresponding to the optimal team policy (+10).

Figures show the learning curves for $DistQ$. Note that the reason that this algorithm cannot solve this environment is that it severely overestimates the value of choosing to move to the right whilst on the rightmost position. It is well known that this is a consequence of the fact that $DistQ$ only performs
updates that increase the estimates of the $Q$-values combined with the stochastic reward received when agent 2 "stumbles" against the right edge.

Figures 4 show the learning curves for agent 2 of DistQ for a random seed. This algorithm cannot solve this environment because it has two issues and the way to solve one makes the other worse. More specifically, one can be solved by increasing the smaller step-size, while the other needs to decrease it. The first issue is the same one that affects DistQ, i.e., the overestimation of the move right action in the rightmost position. Note that this can be ameliorated by increasing the small step-size. The second issue is the penalty incurred due to moving to the left when agent 1 presses the button. This can be ameliorated by decreasing the small step-size. The fact that there is no intermediate value for the small step-size to solve both issues is the reason that this algorithm cannot solve this environment.

Figures 5 show the learning curves for HystQ. This algorithm cannot solve this environment because it has two issues and the way to solve one makes the other worse. More specifically, one can be solved by increasing the smaller step-size, while the other needs to decrease it. The first issue is the same one that affects DistQ, i.e., the overestimation of the move right action in the rightmost position. Note that this can be ameliorated by increasing the small step-size. The second issue is the penalty incurred due to moving to the left when agent 1 presses the button. This can be ameliorated by decreasing the small step-size. The fact that there is no intermediate value for the small step-size to solve both issues is the reason that this algorithm cannot solve this environment.

Figures 6 show the learning curves for Qmix. The architecture used is as follows: we used tabular representation for the individual $q$ functions, and for the mixing and hypernetworks we used the architecture specified in [22]. More specifically, the mixing network is composed of two hidden layers (with 10 units each) with ELu nonlinearities in the first layer while the second layer is linear. The hypernetworks that output the weights of the mixing network consist of two layers with ReLU nonlinearities followed by an activation function that takes the absolute value to ensure that the mixing network weights are non-negative. The bias of the first mixing layer is produced by a network with a unique linear layer and the other bias is produced by a two layer hypernetwork with a ReLU nonlinearity. All hypernetwork layers are fully connected and have 5 units.
Figure 5: Learning curves for agent 2 of HystQ for a random seed.

(a) Leftmost position and $t = 3$
(b) Slot adjacent to leftmost and $t = 2$
(c) Slot adjacent to rightmost and $t = 1$
(d) Rightmost position and $t = 0$

Figure 6: Learning curves for agent 2 of Qmix for a random seed.

(a) Leftmost position and $t = 3$
(b) Slot adjacent to leftmost and $t = 2$
(c) Slot adjacent to rightmost and $t = 1$
(d) Rightmost position and $t = 0$

All code is available as supplementary material. This experiment takes approximately 8 minutes to run in our hardware (2017 iMac with 3.8 GHz Intel Core i5 and 16GB of RAM).
6.9 Cowboy bull game additional results

The bull’s policy is given by the pseudocode shown in algorithm 4.

**Algorithm 4** Bull’s policy.

```plaintext
if distance to all predators > 10 then
    Natural foraging behavior: Stay still with 90% probability, otherwise make a small move in a random direction.
else
    if the maximum angle formed by two predators is > 108° then
        There’s a hole to escape: Escape through the direction in between these two predators.
    else if distance to farthest predator - distance to closest predator > 5 then
        There’s no hole, but one predator is much closer than the others so run in the direction opposite to this predator.
    else
        No way out (scared): Stay still with 70% probability, otherwise make a fast move in a random direction.
end if
end if
```

We now specify the hyperparameters for *Logical Team Q-learning*. All NN’s have two hidden layers with 50 units and ReLu nonlinearities. However, for each Q-network, instead of having one network with 5 outputs (one for each action), we have 5 networks each with 1 output. At every epoch the agent collects data data by playing 32 full games and then performs 50 gradient backpropagation steps. Half of the 32 games are played greedily and the other half use a Boltzmann policy with temperature $b_T$ with decays according to the following schedule $b_T = \max[0.05, 0.5 \times (1 - \text{epoch}/15 \times 10^3)]$. We use this behavior policy to ensure that there are sufficient transitions that satisfy condition $c_1$ and that also there are transitions that satisfy $c_2$. The target networks are updated every 50 backprop steps. The capacity of the replay buffer is $2 \times 10^5$ transitions, the mini-batch size is 1024, we use a discount factor equal to 0.99 and optimize the networks using the Adam optimizer with initial step-size $10^{-5}$.

The hyperparameters of the HystQ implementation are the same as those of LTQL, the ratio of the two step-sizes used by HystQ is 0.1.

The architecture used by Qmix is the one suggested in [22] with the exception that, for fairness, the individual Q-networks used the same architecture as the ones used by the other algorithms (i.e., 5 networks with a unique output as opposed to 1 network with 5 outputs). All hidden layers of the hypernetworks as well the mixing network have 10 units. In this case we did 5 backprop iterations per epoch and the target network update period is 15. We use a batch size of 256, a discount factor equal to 0.98 and optimize the networks with the Adam optimizer with initial step-size $10^{-6}$. In this case the behavior policy is always Boltzmann with the following annealing schedule for the temperature parameter $b_T = \max[0.005, 0.05 \times (1 - \text{epoch}/25 \times 10^3)]$. The batch size, Boltzmann temperature value, learning step-size and target update period were chosen by grid search.

All implementations use TensorFlow 2. The code is available as supplementary material. Running one seed for one agent takes approximately 12 hours in our hardware (2017 iMac with 3.8 GHz Intel Core i5 and 16GB of RAM).