Large electric dipole moment of charged leptons in the standard model

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The electric dipole moment (EDM) of a system against electric field \( H = -\mathbf{d} \cdot \mathbf{E} \), often quoted as the most sensitive observable to the CP violation beyond the standard model (SM), was successively updated since the 60’s \[6, 10, 12, 14, 19–25\], and currently shows a record of \( d_e < 1.1 \times 10^{-29} \text{e cm} \[24\]. There is also much effort to push it down using paramagnetic atoms trapped in three-dimensional optical lattice \[26, 27\], polyatomic molecules \[28, 29\], or polar molecules and inert gas matrix \[30\], etc. The experimental EDM studies of the neutron and the \( \tau \) lepton are also established fields, with the former one measured using storage rings \[15\] and the latter one extracted from the precision analysis of collider experimental data \[16\]. The measurability of the muon EDM using storage rings was also recently deeply discussed in the context of the general relativity \[31\].

One of the most attractive advantage of the EDM is that the effect of the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix \[32\], which is the representative CP violation of the SM, is extremely small, at least for all known systems. There, the EDMs of light quarks \[33–39\] and electrons \[40–42\] appear from the three- and four-loop levels, with the estimated values \( d_u,d_d \sim O(10^{-35}) \text{e cm} \) and \( d_e \sim O(10^{-45}) \text{e cm} \), respectively (for the electron, an example of the four-loop level diagram is displayed in Fig. 1). The Weinberg operator (chromo-EDM of gluons) is also very small, yielding an EDM to the neutron of \( O(10^{-46}) \text{e cm} \[43\].

This extreme suppression is due to the antisymmetry of the Jarlskog invariant \[44\] in the exchange of flavor, which is an important consequence of the Glashow-Iliopoulos-Maiani (GIM) mechanism \[12, 45, 46\], leading to the cancellation of almost equal terms and thus bringing additional factors of quark masses. It can actually be proven that, under the GIM mechanism, the EDM of the charged lepton evaluated at the elementary level suffers from the suppression factor \( m_e^2 / m_t^2 \) at all orders of perturbation, yielding at most \( d_e \sim 10^{-48} \text{e cm} \[47\].

On the other hand, the CKM contributions to the EDM of composite systems are believed to be more enhanced thanks to the long distance effect, where the Jarlskog combination is realized with two distinct \( |\Delta S| = 1 \) hadron level interactions. As for the nucleon EDM, this contribution is larger than the quark EDM and chromo-EDM contributions by two or three orders of magnitude \[48–50\].

Then what about the lepton EDM? We actually found that similar long distance mechanism also happens for the case of the EDM of charged leptons. At the hadronic
level, one-loop level diagrams (see Fig. 2) give the leading contribution and the cancellation is much milder because the loop momenta, given by the hadron masses, are sufficiently different between diagrams, so that we expect a much larger EDM. In this letter, we report on the evaluation of this new contribution in hadronic effective model.

The leading order contribution of the CKM matrix to the lepton EDM is constructed with at least two $W$ boson exchanges. To avoid severe GIM cancellation, we have to split the short distance flavor changing process at least into two parts, while keeping the Jarlskog combination split the short distance flavor changing process at least into two parts, while keeping the Jarlskog combination chosen so as to form the Jarlskog invariant.

Let us now introduce the interactions to calculate the one-loop level diagrams of the hadron masses, as shown in Fig. 2. It is convenient to describe the $|\Delta S| = 0$ vector meson interactions with the hidden local symmetry (HLS) formulation. The HLS is a framework introduced to extend the domain of applicability of chiral perturbation to include vector meson resonances, and it is successful in phenomenology. It generates three-vector meson interactions as follows:

$$\mathcal{L}_{\text{HLS}}^{VVV} = igtr[(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu],$$

where $g \equiv \frac{m_\nu}{2\pi} \sim 4.2$, and

$$V_\mu \equiv \left(\frac{(\rho^0 + \omega)/\sqrt{2}}{\rho^0} - \frac{(\rho^- + \omega)/\sqrt{2}}{K^{*0}}\right).$$

We note that this lagrangian is renormalizable if we assume the vector meson mass has been generated by the Higgs mechanism.

Let us now model the weak interaction at the hadron level. From Fig. 2 the $|\Delta S| = 1$ semi-leptonic interaction appears in the semi-leptonic creation of $K^*$ and in the transition between $K^*$ and chargeless vector mesons $\rho, \omega, \phi$. In Fig. 2 the $K^*$-lepton vertex does not change the charge of the lepton, so this interaction must effectively be generated by a loop involving $W$ boson so as to change twice the quark flavor at short distance, as shown in Fig. 3. Then it is best to also include heavy flavored quarks in this loop as well to derive benefit from the large loop momentum if we wish to maximize the effective coupling. Therefore this $|\Delta S| = 1$ effective interaction is attributed the CKM matrix elements $V_{cs}V_{ds}^*$ or $V_{ts}V_{ds}^*$. It also has to be parity violating, otherwise the $|\Delta S| = 1$ meson transition has to create axial vector mesons which are heavier.

The parity violating interaction between $K^*$ and the charged lepton is given by

$$\mathcal{L}_{K^*\ell l} = g_{K^*\ell l}K^*_\mu I^\mu_{\gamma\gamma}\gamma\ell l + (\text{h.c.}),$$

where $K^*_\mu$ is the field operators of the $K^*$ meson. In the limit of zero momentum exchange, the coupling constant is given by

$$\text{Im}(g_{K^*\ell l})\epsilon_{\mu}\epsilon^*_\mu = \text{Im}(V_{cs}V_{td})(0|\bar{s}\gamma_\mu d|K^*)I_{d\ell l},$$

where we fixed the complex phases of $V_{ud}V_{ts}^*$ to be real. The $K^*$ meson matrix element is defined by $m_{K^*}, f_{K^*}\epsilon_\mu K^*_{\mu}$ where $\epsilon_\mu K^*$, $m_{K^*} = 890$ MeV and $f_{K^*} = 204$ MeV are the polarization vector, the mass, and the phenomenologically derived decay constant of $K^*$, respectively. The quark level amplitude $I_{d\ell l}$ can be obtained by calculating the one-loop level diagrams of Fig. 3. By neglecting all external momenta $I_{d\ell l}$ can be calculated as follows:

$$I_{d\ell l} = -7.7 \times 10^{-8} \text{GeV}^{-2}.$$
This value is quite consistent in absolute value with that
of the naive dimensional analysis \( I_{d_{\text{eff}}} \sim \frac{\alpha_{\text{QED}}}{\sin^2\theta_W m_{W}^2} \sim 1.7 \times 10^{-7} \text{ GeV}^{-2} \).

We now model the \( K^*V \) transition \((V = \rho, \omega, \phi)\), which is either a two-point vertex or a three-point one. It is generated by the \(|\Delta S| = 1\) four-quark effective hamiltonian

\[
H_{\text{eff}}(\mu) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{6} z_i(\mu) Q_i(\mu) + \text{h.c.,} \quad (6)
\]

where the Fermi constant \( G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \).

Here \( Q_i \) (\( i = 1 \to 6 \)) are defined as \([36]\)

\[
Q_1 \equiv \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) u_\beta \cdot \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\alpha,
\]

\[
Q_2 \equiv \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha \cdot \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta,
\]

\[
Q_3 \equiv \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\alpha \cdot \sum_{q=u,d,s} \bar{q}_\beta \gamma_\mu (1 - \gamma_5) q_\beta,
\]

\[
Q_4 \equiv \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\beta \cdot \sum_{q=u,d,s} \bar{q}_\beta \gamma_\mu (1 - \gamma_5) q_\alpha,
\]

\[
Q_5 \equiv \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\alpha \cdot \sum_{q=u,d,s} \bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\beta,
\]

\[
Q_6 \equiv \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\beta \cdot \sum_{q=u,d,s} \bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha,
\]

where \( \alpha \) and \( \beta \) are the color indices. For the case of the \(|\Delta S| = 1\) four-quark interaction, the renormalization from the electroweak scale \( \mu = m_W \) to the hadronic scale \( \mu = 1 \) GeV changes the Wilson coefficients \( z_i \). From the numerical calculation of the running in the next-to-next leading logarithmic approximation \([51, 56]\), we obtain

\[
z(\mu = 1 \text{ GeV}) = \begin{pmatrix} -0.107 & 1.02 \\ 1.76 \times 10^{-5} & -1.39 \times 10^{-2} \\ 6.37 \times 10^{-3} & -3.45 \times 10^{-3} \end{pmatrix}. \quad (13)
\]

We use the standard factorization to derive the \(|\Delta S| = 1\) vector meson transition from the \(|\Delta S| = 1\) four-quark interaction of Eq. \([4]\). We first construct the \(|\Delta S| = 1\) meson transition in the factorization with vacuum saturation approximation \([37, 58]\). The lagrangian of the weak \(|\Delta S| = 1\) vector meson transition is given by

\[
\mathcal{L}_{V'K^*} = V_{ud} V_{us}^* \sum_{V=\rho,\omega,\phi} g_{V'K^*} V'_\mu V'^\nu K^{\ast}_\nu + \text{h.c.,} \quad (14)
\]

where \( V' \) is the field operator of the \( \rho_1, \omega, \phi \) or \( \phi \) mesons. The \(|\Delta S| = 1\) four-quark interaction has two distinct contributions, as shown in Fig. \([4]\) (a) and (b). The former one is generated by all \( Q_i \) ’s, while the latter is only possible through the Fierz transform of \( Q_3 \) and \( Q_6 \). The couplings can be calculated in the factorization as

\[
\langle \rho|\bar{s}_\gamma^\mu d \bar{q}_\gamma_\mu q| K^* \rangle \approx \langle 0|\bar{s}_\gamma^\mu d | K^* \rangle \langle \rho|\bar{q}_\gamma^\mu q | 0 \rangle, \quad (15)
\]

\[
\langle \rho|\bar{s} d d| K^* \rangle \approx \langle \rho|\bar{s} d | K^* \rangle \langle 0|\bar{d} d | 0 \rangle, \quad (16)
\]

where \( q = u, d, s \). We note that the vacuum saturation approximation gives the leading contribution in the large \( N_c \) expansion for mesonic processes. The vector meson matrix elements are related to the decay constants like Eq. \([4]\) \([55]\). The chiral condensate \( \langle 0|\bar{q}q |0 \rangle \) is derived from the Gell-Mann-Oakes-Renner relation, and the vector meson scalar density \( \langle K^*|\bar{s}d |\rho \rangle \approx 1.3 \text{ GeV} \) is extracted from the lattice QCD result of the quark mass dependence of the vector meson mass \([53, 59]\) and flavor \( SU(3) \) symmetry. The appropriate scale of the factorization procedure was chosen as \( \mu = 1 \) GeV. Using these input parameters, the coupling constants are given by \( g_{\rho K^*} = 4.2 \times 10^{-8} \text{ GeV}^2 \), \( g_{\omega K^*} = 3.4 \times 10^{-8} \text{ GeV}^2 \), and \( g_{\phi K^*} = -5.8 \times 10^{-9} \text{ GeV}^2 \).

Let us also construct the weak three-meson interactions. Its Lagrangian is given by

\[
\mathcal{L}_{V'K'K^*} = V_{ud} V_{us}^* \sum_{V=\rho,\omega,\phi} g_{V'K'K^*} V'_\mu V'^\nu V'^\nu K^{\ast}_\nu + \text{h.c.,} \quad (17)
\]

where \( A \cdot \partial^\mu B \equiv A(\partial^\mu B) - (\partial^\mu A)B \). Again by using the vacuum saturation approximation, we have

\[
\langle \rho|\bar{s}_\gamma^\mu q \bar{s}_\gamma^\nu d| K^{\ast} \rangle \approx \langle 0|\bar{s}_\gamma^\mu q| \rho \rangle \langle \rho|\bar{s}_\gamma^\nu d | K^{\ast} \rangle, \quad (18)
\]

where \( \langle V(p')|\bar{s}_\gamma^\nu d | K^{\ast}(p) \rangle \approx (p'^\mu + p'^\nu) \epsilon_\nu^{(V')} \epsilon^{(K'^*)} \mu \) \((V = \rho, \omega, \phi)\). The coupling constants are then given by 

\[
\delta_{\rho K^*} = -g^4_{\rho K^*} = -g^4_{\phi K^*} = 1.7 \times 10^{-7}, \quad g^4_{\omega K^*} = -g^4_{\phi K^*} = 1.4 \times 10^{-7}, \quad g^4_{\phi K^*} = -g^4_{\phi K^*} = -1.8 \times 10^{-8}.
\]

After evaluation, we obtain the following lepton EDM:

\[
d_e^{\text{(SM)}} = 1.8 \times 10^{-39} \text{ cm}, \quad (19)
\]

\[
d_\mu^{\text{(SM)}} = 6.2 \times 10^{-38} \text{ cm}, \quad (20)
\]

\[
d_\tau^{\text{(SM)}} = -8.9 \times 10^{-38} \text{ cm}. \quad (21)
\]
These values are much larger than the estimation at the four-loop level \(d_e^{(SM)} \sim 10^{-48} \text{e cm}\). As anticipated in the beginning of this letter, the enhancement is due to the absence of severe GIM cancelation (antisymmetry in the interchange of quark flavor \[12\]) thanks to the scatter of the typical loop momentum according to the hadron masses, whereas in the short-distance case the momenta were made almost equal by the heavy top quark or W boson. This is a general feature of the long distance effect, and similar enhancement could be seen in the case of the neutron EDM compared to the short distance quark EDM \[39, 42, 49, 50\].

We also analyze the potential systematics of our study. The first large source of uncertainty is that due to the renormalization of the \(|\Delta S| = 1\) four-quark operators. It may be estimated by looking at the variation of Wilson coefficients between \(\mu = 0.6\) GeV and \(\mu = m_e = 1.27\) GeV, yielding about 10%. The second important source of systematics is the factorization used to calculate the vector meson interactions. As we saw above, the vacuum saturation approximation gives the leading order effect in the large \(N_c\), so we expect the error bar to be about \(1/N_c \sim O(40%)\). Finally, we have to discuss the hadronic uncertainty, associated with the neglect of heavier hadrons. In the dimensional analysis, the most important omitted contribution should be that of the axial vector mesons, the CP-odd electron-nucleon interaction may also contribute at the same order \[42\]. To truly distinguish the effect of the electron EDM, other independent tests are also required.

We also point that this long distance effect is also generated by other semileptonic and quark flavor violating processes. The most interesting target should be the physics of the \(B\) meson decay anomaly, recently suggested by several \(B\) factory experiments \[61, 62\] or the recent result of \(K\) meson decay of KOTO experiment \[63, 44\].

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**FIG. 5.** Plot of the SM predictions (quark and hadron levels) of the electron EDM compared with the experimental upper limits updated from the 60’s \[6, 10, 12, 14, 16, 20, 22, 25\]. The prospective experimental sensitivity \[30\] is also shown.

In conclusion, we evaluated for the first time the hadronic level contribution to the EDM of charged leptons. As a result, we found that this long distance effect is much larger than the previously known one, which was estimated at the elementary level. The main reason of the enhancement at the hadronic level is because we could avoid severe suppression due to the GIM mechanism. In Fig. 5 we plot the EDM of the electron in the SM compared with the progress of the experimental accuracy. The electron EDM obtained in this work is \(d_e = O(10^{-39}) \text{e cm}\), which is still well below the current sensitivity of the molecular beam experiments \[14\]. The EDM experiments are however improving very fast, and we have to be very sensitive to their progress and to proposals with new ideas, with some of them claiming to be able to ideally reach the level of \(O(10^{-35} - 10^{-37}) \text{e cm}\) \[39\]. This potential breakthrough, combined with our result, is maybe cautioning us that we have to be careful with the statement that the window left for the electron EDM to new physics beyond the SM is almost infinite. We also note that in EDM experiments using atoms and molecules, the CP-odd electron-nucleon interaction may also contribute at the same order \[42\]. To truly distinguish the effect of the electron EDM, other independent tests are also required.

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