A study on discharge gas pulsations in a three stage single acting reciprocating compressor

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Abstract. The growing demand in the market for highly efficient, more reliable and less expensive compressors have activated the manufacturers to develop state-of-the-art analytical tools to predict, evaluate and optimize the existing as well as creating the new designs. Reciprocating action of piston and intermittent flow of gas induced by suction and discharge phases give rise to pressure pulsations and oscillations. Development of an analytical tool for predicting the pressure pulsations and oscillations saves considerable time and reduces the unnecessary number of prototypes as well as the development costs. In the present study, a mathematical model of a W type three stage single acting reciprocating compressor is developed to predict the discharge gas pulsations in the piping system of the compressor. An experimental setup is prepared and the analytical model prepared for simulation of the discharge gas pulsations is initially tested in limited experiments for validation. The model results are compared with the experimental findings and good agreement was observed.

1. Introduction
Reciprocating compressors are widely used in industries. Modelling of the reciprocating system and its simulation is of great importance as it provides an insight into the energy used during the compression process, compressor efficiency and influence of various design parameters on the compressor performance. The steady-state simulations are used in fault diagnosis wherein the simulated pressure profile is compared with the experimental one. Steady state indicator diagram has been used to analyze the importance of phenomena such as valve lift, valve stiffness and leakage on compressor performance. The mass flow rate that occurs through the valves into the suction and discharge plenums leads to gas pulsations due to the finite volume associated with the plenums. These gas pulsations have been identified as a major source of vibrations and noise radiation from reciprocating compressors. In order to predict and hence reduce the vibration and noise generated, a good understanding of the parameters that control the source of discharge gas pulsations is required. Many studies on single cylinder compressors offer a useful approach to solve this problem. However, in order to determine the gas pulsations and PV diagrams of the W type three-stage single acting reciprocating compressors, the discharge gas pulsations, valve and piping systems of each stage should be modeled.

In this study, a mathematical model of a W type three stage single acting reciprocating compressor is developed to predict the discharge gas pulsations in the piping system of the compressor. In order to form the P-V diagram of each stage, cylinder internal pressures, stage valves and piping should be modeled. For the valve model, the mass equation together with equations expressing the position of the pistons must be solved depending on the crankshaft angle. The cylinder internal pressure and pipe models were performed by using the valve model and equations expressing the position of the pistons. Thus, the discharge gas pulsations of the three stage reciprocating compressor is modeled. Simulation of the mathematical model is performed by MATLAB software and parametrical analysis is performed by using model. An experimental setup is prepared and the analytical model prepared for simulation of the discharge gas pulsations is initially tested in limited experiments for validation. In the model verification studies, depending on the crank angle, the measurements of the stage, internal cylinder and stage output pressures for each stage were measured and the model was...
verified. Thus, the mathematical model can be safely used to design more efficient, quiet and reliable reciprocating compressors.

2. Mathematical Model

As shown in Figure 1, an overview of the system that is considered is the W type three stage single acting reciprocating compressor.

![Figure 1. Overview of the compressor](image)

The equations expressing the position of the pistons must be obtained to demonstrate the kinematic properties of the three stage W type reciprocating compressor. The physical model of the crank mechanism is shown in Figure 2.

![Figure 2. General view of crank mechanism](image)

It is assumed that the crankshaft angle is 0° in case the piston (vertical piston) is at the upper dead point before starting to obtain the equations. The direction of rotation of the machine is clockwise. In this case, the position of 1st stage piston is ahead of crankshaft angle as much as angle between cylinders and the position of 2nd stage piston is behind of crankshaft angle as much as angle between cylinders. In this way the positions of pistons can be written as follows [1]:

\[
x_{p1} = r + \frac{r^2}{4l_1} - \left[ r \cos(\omega t + \phi) + \frac{r^2}{4l_1} \cos 2(\omega t + \phi) \right]
\]

\[
x_{p2} = r + \frac{r^2}{4l_2} - \left[ r \cos(\omega t - \phi) + \frac{r^2}{4l_2} \cos 2(\omega t - \phi) \right]
\]

\[
x_{p3} = r + \frac{r^2}{4l_3} - \left[ r \cos \omega t + \frac{r^2}{4l_3} \cos 2\omega t \right]
\]

(1)

In the modeling of cylinder internal pressure, no gas leakage, homogenous system, adiabatic control volume are assumed. Internal pressure of cylinder is divided into two phases as suction and expansion phase and
compression and discharge phase as shown in Figure 3. The change in cylinder pressure depending on the crank angle is obtained due to cylinder volume and mass change [2].

![Figure 3. P-V diagram](image)

The general expression of the ideal gas equation and its application to the cylinder internal pressure are as follows:

$$PV = mRT$$

$$\frac{P_{3-1}(q)V(q)}{P_{3}V_{3}} = \frac{m(q)RT(q)}{m_{3}RT_{3}}$$

The general expression of suction and expansion phase can be written as follows [3]:

$$P_{3-1}(q) = P_{3}\left(\frac{V_{3}}{V(q)}\right)^{n}\frac{m(q)}{m_{3}}$$

$$V_{3-1}(q) = A_{c}x_{p}(q) + V_{1}$$

In the expansion phase, the compressed air (P3) in the dead zone expands and the pressure drops. After the suction valve is opened, mass transfer into the cylinder takes place. In this way, the internal pressure is modeled by Eq. (3). General expression of compression and discharge phase is as follows:

$$P_{1-3}(q) = P_{1}\left(\frac{V_{1}}{V(q)}\right)^{n}\frac{m(q)}{m_{1}}$$

$$V_{1-3}(q) = A_{c}x_{p}(q) + V_{1}$$

$$m(q) = \dot{m}t$$

In the compression phase, the air absorbed into the cylinder (P1) is compressed and its pressure increases. After the discharge valve is opened, mass transfer is carried out through the cylinder. In this way, the internal pressure is modeled by Eq. (4). The valve is modeled as a single-degree of freedom mass-spring system without an inverse flow and with the effect of ideal gas flow as shown in Figure 4.
Figure 4. Valve and flow through the valve

At refers to the minimum cross-sectional area that varies depending on the valve opening. According to the Saint-Venant's equation obtained from the literature, the mass flow through the valve within the minimum cross-section is expressed as follows [4]:

$$\dot{m} = P(q)A_t \sqrt{\frac{2n-1}{n-1} \frac{1}{RT(q)}} \left( \frac{1}{\sigma^2} \right)^{n+1/n} - \left( \frac{1}{\sigma} \right)^{n+1/n}$$

$$A_t = \pi D_p x$$  \hspace{1cm} (5)

Critical pressure ratios (choking condition) constant was taken $\sigma_{cr}$: 1.894.

$$\sigma_{cr} = 1 \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} = 1.894 \ (k = 1.4)$$  \hspace{1cm} (6)

Critical pressure ratios constant $\sigma$ is calculated according to expression and suction phases as written in Eq. (6).

For compression and discharge phase [6];

Figure 5. Cross sectional view of the valve

Gas pressure force ($F_g$), spring force ($F_s$), friction force ($F_f$) and adhesion forces ($F_{adh}$) are effected on the valve plate. When the valve is closed, the cylinder internal pressure exerts pressure on the valve plate from the $A_v$ surface while the outlet pressure exerts a force through the entire plate surface ($A_p$). The force balance equation on the valve can be written as follows.

$$F_g = P(q)A_v - P_d A_p$$  \hspace{1cm} (7)

The spring characteristic is determined linearly when determining spring forces. $k$ refers to the spring constant.

$$F_s = k (x_0 + x)$$  \hspace{1cm} (8)

Assuming that the valve plate is in a random position between the valve seat and the limit it is assumed that it is subject to damping action by the fluid in the opposite direction to the movement of the valve plate. The friction force on the plate is directly proportional to the speed of the plate. $C_f$ refers to the coefficient of friction.

$$F_f = C_f \dot{x}$$  \hspace{1cm} (9)

The oil film consists of the sum of the adhesion force, the viscosity effect ($F_{vis}$), the capillary force ($F_{cap}$) and the surface tension force ($F_{ten}$).
\[ F_{\text{adh}} = F_{\text{vis}} + F_{\text{cap}} + F_{\text{ten}} \]  

The point of Khalifa and Liu model the liquid-gas interfacial tension can be omitted (Ften=0). Since the valve plate makes a vertical movement in the bed, the viscous forces formed in the perpendicular direction to the flow have been neglected (Fig.6) [6].

\[ F_{\text{adh}} \equiv F_{\text{cap}} = \pi \frac{Y_{\text{LG}}}{2} \frac{D_p^2 \cos \beta}{h_{\text{film}}} \left( \frac{D_p}{D_v} \right)^2 - 1 \]  

Simplified adhesion forces are equal to capillary force.

By assuming the piping diameter is constant between the stages, the volumes between the stages are modeled separately as shown in Figure 7. The change in the mass between the stages is expressed below.

Figure 6. Stiction on valve

Figure 7. Schematic view of stages

The pressure balance for each volume can be written as follows:

\[ P_{v_p} = P_d \frac{\Delta m_{\text{st}}}{m_{v_p}} \]  

In general, the operative steps of the reciprocating compressor are indicated in Figure 8.

Figure 8. Scheme of three stage reciprocating compressor
3. Experimental Setup

Up to this point, pressure changes of the cylinder internal pressure, valve and piping systems of each stage have been modeled in order to determine the pressure fluctuations and P-V diagrams of the three-stage reciprocating compressors. An experimental set-up was created to obtain these data experimentally.

In order to obtain the P-V diagrams, pressure measurements were made from the stages inputs and outputs together with the pressure measurements from the bottom of the discharge valves of the cylinder internal pressures of the stages and in the dead volume area. Since the air outlet temperature in the area to be measured is 180°C, the pressure transmitter to be preferred for the test should have high measurement accuracy and high accuracy at this temperature.

Since the pressure measurements will be taken from the dead volume area in the pressure zone of the valve, there is a serious problem for the pressure transducer to be mounted here. For proper connection of the transmitter, the connection size diameter should not exceed M6.

As a result of the research, it was decided to use KELLER PAA-M5 HB type pressure transmitters which meet all these requirements (Figure 9). These transmitters have 10 bar for the first stage, 30 bar for the second stage and 50 bar for the third stage.

| Specifications                  | Description                |
|---------------------------------|----------------------------|
| Operating temperature range     | -40°C - 200°C              |
| Signal measurement range        | 0-50 kHz                   |
| Accuracy                        | ±0.1%FS                    |

The locations of the transmitters are determined according to the valve designs.

![Figure 9. Pressure transmitter assembly of cylinder](image)

![Figure 10. Location of the pressure transmitters](image)

Once the pressure transmitters have been installed, the test device is ready for measurement (Fig.10). Pressure measurements were performed with DEWESOF's DEWE43 model data acquisition device at 20 kHz sampling frequency.

4. Model Validation

The data collected after the completion of the tests were compared with the model outputs. The following shows the change in the internal pressure of the first, second and third stage cylinders. Some basic physical parameters of the piston compressor used in the tests are given in the Table 2.
Table 2. 3 stage piston compressor physical parameters

| Parameters                                      | Description       |
|------------------------------------------------|-------------------|
| First Stage Cylinder Diameter                  | 160 mm            |
| Second Stage Cylinder Diameter                 | 85 mm             |
| Third Stage Cylinder Diameter                  | 52 mm             |
| Suction Capacity                               | 2767 l/min        |
| Inlet Pressure                                 | 1.0 bar.a         |
| Working Pressure                               | 41.0 bar.a        |
| Shaft Power                                    | 27.4 kW           |
| Stroke                                         | 94 mm             |
| Angle Between Cylinders                        | 60 deg            |
| Rotation speed                                 | 1465 rpm          |
| Connecting rod length                          | 252 mm            |
| Valve flow max. cross section are of 1st stage suction | 2079 mm²   |
| Valve flow max. cross section are of 1st stage discharge | 1448 mm²   |
| Valve flow max. cross section are of 2nd stage suction | 670 mm²   |
| Valve flow max. cross section are of 2nd stage discharge | 540 mm²   |
| Valve flow max. cross section are of 3rd stage suction | 352 mm²   |
| Valve flow max. cross section are of 3rd stage discharge | 110 mm²   |

When comparing the internal pressures of the cylinders, a good fit is observed at the peak points in the suction and compression zones. The opening of the discharge valve later than the model is due to the fact that the estimated spring force is actually calculated higher.

The measurements taken from the cylinder head and the comparison of these measurements with the model outputs are shown in Figure 11 and Figure 12.

Good agreement was observed at the minimum and maximum peak values at the outlet pressure. However, pressure pulsations in the system have an impact on differences in some regions. A useful model has been created to reduce the impact of these fluctuations.

![Figure 11. Comparisons of P-V diagrams of each cylinders](image-url)
5. Conclusion
In the study, a mathematical model of a W type three stage single acting reciprocating compressor was developed to predict the discharge gas pulsations in the piping system of the compressor. An experimental setup was prepared and the analytical model prepared for simulation of the discharge gas pulsations was initially tested in limited experiments for validation. The model results are compared with the experimental findings and good agreement was observed.

It is obvious that a validated analytical tool will be useful for designing the piping system of a three stage single acting reciprocating compressor that has reduced sound power levels and improved performance. Conceptual design of the improved compressor has been completed by using the data obtained from analytical tool and prototyping is at the final stage. The development process was completed in a relatively short period of time thanks to the analytical model.

![Figure 12. Comparisons of each cylinder discharge pressures](image)

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Nomenclature

\( x_p \) Position of pistons
\( r \) Radius of crankshaft
\( l \) Connecting rod lengths
\( \omega \) Angular velocity
\( t \) Time
\( \varphi \) Angle between cylinders
\( Q \) Crankshaft angle
\( m(q) \) Mass of pressurized air in cylinder at certain (q) crankshaft angle
\( m_1 \) Mass of pressurized air in cylinder at certain point of 1 in P-V diagram (Figure 3)
\( m_3 \) Mass of pressurized air in cylinder at certain point of 3 in P-V diagram (Figure 3)
\( V(q) \) Volume of cylinder at certain (q) crankshaft angle
\( V_1 \) Volume of cylinder at certain point of 1 in P-V diagram (Figure 3)
\( V_3 \) Volume of cylinder at certain point of 3 in P-V diagram (Figure 3)
\( P \) Pressure of each stages
\( A_c \) Cylinder area
\( \dot{m} \) Mass flow rate through the valves
\( \sigma \) Pressure ratio between upstream and downstream of valves
\( \sigma_{cr} \) Critical pressure ratio at choked point
\( A_f \) Valve flow cross section are
\( D_p \) Valve plate diameter
\( x \) Valve lift
\( P_d \) Discharge pressure of cylinder head
\( Y_{LG} \) Surface tension
\( \beta \) Meniscus contact angle
\( h_{film} \) Oil film thickness

Subscripts

1 1st stage
2 2nd stage
3 3rd stage
3-1 Suction and expansion phase
1-3 Compression and discharge phase
d discharge
s suction