Definition of positive displacement pump output regulation mechanism parameters with providing of constant flow capacity in output

S Barmin\(^1\) and O Nikitin\(^1,2\)

\(^1\)Bauman Moscow State Technical University, 5 Second Baumanskaya Street, Moscow, 105005, Russian Federation

\(^2\)E-mail: nof1936@yandex.ru

Abstract
This paper surveys mechanism of supply regulation for axial-piston pump with adjustable-angle swash plate operating with constant power. The mechanism includes a piston, whose working space is connected to delivery port, regulation unit and two springs. Least square method and successive approximations method are used to calculate parameters of the mechanism: cut-in pressure level for the second spring and springs stiffness. Regulation effectiveness is estimated by calculation of relative error of power delivered.

Introduction
Pump units allowing hydraulic servo to operate at constant power are widely spread in manufacturing industry. [1, 2] While designing this type of pump units, it is necessary to make pump output regulation unit provide constant power of pressure fluid flow (multiplication of pump output and pressure in pressure pipe values) in all of modes, so that \( N = Qp = \text{const} \). In \( Q — p \) coordinates the plot of the relation is hyperbolic. Hyperbolic law of supply variation can hardly be sustained mechanically or hydromechanically, which causes necessity to approximate the law by two linear functions. Constructionally it’s relatively easy to provide linear relation between supply and pressure. Connection of pressure pipe and stock cavity of shiftable ram combined with availability of spring group enables us to provide flow control depending on pumping pressure. It can be provided by a mechanism including a piston, whose working space is connected to delivery port, regulation unit and two springs. Practically applied method of spring selection doesn’t provide necessary effectiveness of energy transfer. [3,4]

Mathematical description of approximation process
While conducting approximating of hyperbolic law of supply conversion in conditions of supply pressure conversion, the problem of implementation of two sequential linear laws which provide the fullest power supply. In other words, \( N_{hyp} = N_{app} \) so that the energy is fully supplied and the moment of the second spring \( p_a \) switching is defined. Approximating straight lines are drawn through edge points \( p_0 \) and \( p_{\text{max}} \) of pressure change range. [5,6]

Supply regulation process is shown on the example of axial-piston pump with adjustable-angle swash plate. On the base of it preparative prepression of first spring \( z_0 \), stiffness of springs \( c_1 \) and \( c_2 \), diameter of rod \( d_r \), shoulders a and b are defined. On fig.1 construction scheme of mechanism of
supply regulation for axial-piston pump with adjustable-angle swash plate is shown. The swash plate changes pump operating volume from maximum to zero while changing angle in range of 20…0°, and it corresponds to linear law in this range.

Constant power \( N=Q \cdot p = \text{const} \) in coordinates \( Q - p \) theoretically defined by hyperbolic law \( Q_T(p) = N/p \). If the pump is equipped with a leverage mechanism providing operating volume change, supply change law will be of the form of \( Q(p_i) \), where \( i \) corresponds to a certain part of approximation (here \( i = 1 \) and 2). In the considered case supply regulation mechanism includes two springs of different stiffness and a rod. Control signal passed to rod side is pressure level of hydraulic fluid from delivery line defined by external load of hydraulic fluid discharged flow consumer. First spring operates in range of \( p_0 \leq p_1 \leq p_{\text{st}} \) of discharge pressure level, and the second one operates in the range of \( p_{\text{st}} \leq p_2 \leq p_{\text{max}} \) (fig. 2).

Initial pre-pressure of the spring holds the swash plate at position of maximal angle to initial pressure of supply regulation \( p_0 \). The other spring starts operating when hydraulic fluid discharge pressure \( p_{\text{st}} \) is achieved (it corresponds to a certain angle of the swash plate), whose level is defined by law \( Q(p) \) shown on fig. 2. Further both springs operate together in the range of \( p_{\text{st}} \leq p_2 \leq p_{\text{max}} \) of discharge pressure. The piston and springs 1, 2 of supply regulation mechanism may have different shoulders relative to the swash plate rotation axis. This type of construction allows to provide constant power of the pump with lower error. Operating parameters corresponding to each pressure change range get indexes 1 and 2. [7, 8]

Fig. 1. Construction scheme of mechanism of pump supply regulation

In the theoretical variant of regulation pump supply \( Q \) and pressure \( p \) are connected by relation \( Q_T(p) = N = \text{const} \) in operating range of \( p_0 \leq p \leq p_{\text{max}} \). In case of approximation we get the following system:

\[
\begin{align*}
Q_1(p_1) &= A_1 - B_1 p_1 \quad \text{at} \quad p_0 \leq p \leq p_{\text{st}} \\
Q_2(p_2) &= A_2 - B_2 p_2 \quad \text{at} \quad p_{\text{st}} \leq p \leq p_{\text{max}}
\end{align*}
\]  

(1) (2)

In order to increase regulation parameters calculation effectiveness we will derive a formula for \( p_{\text{st}} \), which would allow to deliver pressure with the lowest possible error while taking into account pressure change range between values of \( p_0 \) and \( p_{\text{max}} \). For that we will use non-dimensional values.[9, 10]

Maximum pump supply \( Q_{\text{max}} = A_1 - B_1 p_0 \) is provided in the beginning of regulation process \( p_0 \). Dividing values for pump supply \( Q_1(p_1) \) and \( Q_2(p_2) \) by \( Q_{\text{max}} \), we obtain the following system used to calculate pump supply in a non-dimensional form:

\[
\begin{align*}
\bar{Q}_1(p) &= \frac{A_1 - B_1 p}{A_1 - B_1 p_0} \quad \text{at} \quad 1 \leq \bar{p} \leq \bar{p}_{\text{st}} \\
\bar{Q}_2(p) &= \frac{A_2 - B_2 p}{A_2 - B_2 p_0} \quad \text{at} \quad \bar{p}_{\text{st}} \leq \bar{p} \leq \bar{p}_{\text{max}} (2) \quad \text{and} \quad \bar{Q}_T(p) = 1 / \bar{p}.
\end{align*}
\]  

(3)
At first approximation $p_{st}$ is defined as follows. Tangentials to $Q_T$ are drawn through edge points of regulation range with coordinates of $p_0 = 1$, $Q_0 = 1$ and $p_{max}$, $Q(p_{max})$. Point of crossing of the tangentials has coordinates of $p_{st}$ and $(\tau, \sigma)$, which are defined as follows:

1. \( B_1 = Q_T(l) - \text{derivative of function } Q_T(p) \) at the moment of start of the pump regulation start;
2. \( B_2 = Q_T(p_{max}) - \text{derivative of function } Q_T(p) \) at a point with coordinates of $p = p_{max}$ (final moment of pump supply regulation process);
3. \( A_1 = Q_T(l) - B_1 \cdot l \); \( A_2 = Q_T(p_{max}) - B_2 \cdot p_{max} \);
4. \( A_1 - B_1 \cdot p_{st} = A_2 - B_2 \cdot p_{st} \)

and we obtain: $p_{st} = (A_1 - A_2) / (B_1 - B_2)$. (4)

In order to calculate $p_0 \leq p_1 \leq p_{st}$ more precisely approximation by least square method is used. [9, 10]

In order to do that range of $1...p_{max}$ is broken into $n$ random parts. Number of parts should be as big as possible. Calculation of differential derivatives of the function for variables $A_1$ and $B_1$ and equating them to zero allows to calculate values of coefficients $A_1$ and $B_1$ for the first part.

Coefficients $A_2$ and $B_2$ for the second part are obtained the same way by breaking the range of $p_{st} \leq p_2 \leq p_{max}$ into $m$ random parts.

\[
\begin{align*}
A_1 &= \sum_{i=1}^{n} Q_T(p_1) \cdot \sum_{i=1}^{n} p_1^2 - \sum_{i=1}^{n} Q_T(p_i) \cdot p_1 \cdot \sum_{i=1}^{n} p_1 \\
&\quad - n \cdot \sum_{i=1}^{n} p_1^2 - (\sum_{i=1}^{n} p_1)^2 \\
B_1 &= \frac{\sum_{i=1}^{n} Q_T(p_1) \cdot p_1 - \sum_{i=1}^{n} p_1 \cdot \sum_{i=1}^{n} Q_T(p_i)}{(\sum_{i=1}^{n} p_1)^2 - n \cdot \sum_{i=1}^{n} p_1^2}
\end{align*}
\] (5)

Coefficients $A_2$ and $B_2$ for the second part are obtained the same way. For that the range of $p_{st} ... p_{max}$ is broken into $m$ random parts. And then the following system is obtained:

\[
\begin{align*}
A_2 &= \sum_{i=1}^{m} Q_T(p_{max}) \cdot \sum_{i=1}^{m} p_{max}^2 - \sum_{i=1}^{m} Q_T(p_{st}) \cdot p_{max} \cdot \sum_{i=1}^{m} p_{max} \\
&\quad - m \cdot \sum_{i=1}^{m} p_{max}^2 - (\sum_{i=1}^{m} p_{max})^2 \\
B_2 &= \frac{\sum_{i=1}^{m} Q_T(p_{max}) \cdot p_{max} - \sum_{i=1}^{m} p_{max} \cdot \sum_{i=1}^{m} Q_T(p_{max})}{(\sum_{i=1}^{m} p_{max})^2 - m \cdot \sum_{i=1}^{m} p_{max}^2}
\end{align*}
\] (6)

In case of approximation by least square method and successive approximation method correspondences of start pressure level $p_{st}$ in range of $p_0 = 1$ and $p_{max} = p_{max} / p_0$ define coefficients $A_1$, $B_1$, $A_2$, and $B_2$, $A_1$, $B_1$, $A_2$ and $B_2$ by formulas (5) and (6). The results are shown in table №1.

The analysis shows that definition of second spring starting moment by least square method and successive approximation methods takes quite long time because a sum of large number of components is required, and also because change in intimal data (regulation start moment $p_0$, final regulation moment $p_{max}$) leads to repeated calculation of second spring start moment $p_{st}$. 

3
Table 1. Values of approximation parameters for the corresponding values.

| Values | $\bar{p}_{\text{st}}$ | $\Delta$, % | $A_1$ | $B_1$ | $A_2$ | $B_2$ |
|--------|------------------|-----------|------|------|------|------|
| 1,5    | 1,116            | 0,008     | 2,075| 1,067| 1,549| 0,595|
| 1,75   | 1,275            | 0,008     | 1,803| 0,809| 1,34 | 0,445|
| 2      | 1,363            | 0,007     | 1,725| 0,738| 1,217| 0,366|
| $\bar{p}_{\text{max}}$ | 2,25       | 1,44      | 0,011| 1,677| 0,696| 1,116| 0,306|
| 2,5    | 1,516            | 0,012     | 1,632| 0,657| 1,03 | 0,259|
| 2,75   | 1,538            | 0,037     | 1,636| 0,66 | 0,98 | 0,233|
| 3      | 1,615            | 0,027     | 1,591| 0,621| 0,913| 0,201|

Equality of areas (power) under plots $\bar{Q}_1(p)$, $\bar{Q}_2(p)$ and $\bar{Q}_T(p)$ (fig.2) is used as approximation accuracy criteria, which means $S_T = S_1 + S_2 = S$,

where $S_1 = \int_{\bar{p}_0}^{\bar{p}_{\text{st}}} \bar{Q}_1(p) d\bar{p}$ – area under plot $\bar{Q}_1(p)$; $S_2 = \int_{\bar{p}_{\text{st}}}^{\bar{p}_{\text{max}}} \bar{Q}_2(p) d\bar{p}$ – area under plot $\bar{Q}_2(p)$;

$S_T = \int_{\bar{p}_0}^{\bar{p}_{\text{max}}} \bar{Q}_T(p) d\bar{p}$ – area under plot $\bar{Q}_T(p)$.

Relative error in this case is: $\Delta = \frac{S_T - (S_1 + S_2)}{S_T} \cdot 100$. (7)

Correspondence $\bar{p}_{\text{st}} = f(\bar{p}_{\text{max}})$ in form of $\bar{p}_{\text{st}} = 0,71 + 0,311 \bar{p}_{\text{max}} = 1,021 + 0,311(\bar{p}_{\text{max}} - 1)$, (8)

is obtained taking into account $\bar{p}_0 = 1$ and parameters used for calculating errors $\Delta$ by formula (7).

On fig. 3 non-dimensional operating characteristics are shown: $T$ – theoretical and 1 and 2 - approximated for pump with supply regulation mechanism and providing constant power at any pressure level in accepted change range. Approximating non-dimensional lines are not obligated to be drawn through points at which non-dimensional values of pump supply are equal to 1 and $\bar{Q}_T(\bar{p}_{\text{max}})$. Approximating lines don’t have to pass through points in which non-dimensional value of pump output is equal to 1 and $\bar{Q}_T(\bar{p}_{\text{max}})$. It is connected with the specificity of least square method.[11,12]

Fig.3. Dimensionless operating characteristics of pump with supply regulation mechanism: $T$ – theoretical and 1 and 2 - approximated

Checking calculation of chosen mechanism parameters

Taking into account calculations conducted to obtain $p_{\text{st}} = f(p_{\text{max}})$, calculations were performed in order to check parameters values of mechanism, which regulates supply providing constant capacity of hydraulic fluid flow for a pump with a capacity of 15kW and operating in the pressure range of 18MPa-40MPa. Structural scheme of pump supply regulation mechanism and operating characteristic of a pump with regulation mechanism are similar to ones shown on fig.1, while functional diagrams are shown on fig.2. Checking calculations were performed taking into account geometrical parameters ($A$, $b$, $V_0$, $\gamma$) and using system (1). [13,14]
On the first part \( p_0 \leq p_1 \leq p_{st} \) force applied to the swash plate by rod, acts against the force applied by first spring to the swash plate and defined by initial pre-pressure of spring \( z_0 \), its stiffness \( c_1 \), diameter of \( d_r \), shoulders \( a \) and \( b \) (fig.1).

Equations of forces balance relative to the swash plate rotation axis are as follows:

\[
F_{01} \cdot b = p_0 \alpha \pi d_r^2 / 4, \tag{9}
\]

where \( F_{01} = c_1 z_0 \) – force applied to the swash plate by first spring and provided by its initial pre-pressure \( z_0 \), \( c_1 \) – first spring stiffness, \( p_0 \) – discharge line pressure before movement start, or start of regulation process.

Further pressure growth \( p_1 > p_0 \) causes spring deformation and increase of force applied to the swash plate by it.

\[
F_{sp1} = C_1 [z_0 + z_1(p)] \tag{10}
\]

where \( z_1(p) \) - first spring deformation, caused by change of pressure in the range of \( p_0 < p_1 < p_{st} \), and it can be derived through swash plate angle as follows:

\[
z_1(p) = b [\tan \gamma_{max} - \tan \gamma(p)] \tag{11}
\]

where \( \gamma_{max} \) and \( \gamma(p) \) – maximal value and current value of swash plate angle defined by pressure \( p \).

Pump supply \( Q \) is in direct proportion to swash plate angle tangent:

\[
Q = \frac{V_0}{2 \pi} \frac{\tan \gamma(p)}{\tan \gamma_{max}} \tag{12}
\]

where \( \omega \) - rotational speed of pump shaft \( V_0 \) - pump operating volume.

Using equations (9), (10), (11) in equation (12) and taking into account construction parameters of operating volume regulator we obtain relation between pump supply \( Q \) and supply pressure in the range of \( p_0 \leq p_1 \leq p_{st} \) with only first spring operating as follows:

\[
Q_1(p_1) = A_1 - B_1 p_1 \tag{13}
\]

where \( A_1 = \frac{V_0}{\omega} \left( \frac{z_0}{\tan \gamma_{max}} + 1 \right), B_1 = \frac{V_0}{\omega} \frac{\pi d_r^2}{4c_1 \tan \gamma_{max}} \cdot \frac{a}{b^2} \).

When supply pressure level equal to \( p_{st} \) and corresponding angle of swash plate is achieved, the second spring with a stiffness \( c_2 \) of starts deforming. Therefore, in the range of \( p_{st} < p_2 < p_{max} \) two springs operate simultaneously. Equations of forces balance relative to the swash plate rotation axis can be described in the following form:

\[
F_{s1} \cdot b + F_{s2} \cdot b = p \frac{\pi d_r^2}{4} a, \tag{14}
\]

where \( F_{s2} = c_2 b [\tan \gamma_{st} - \tan \gamma(p)] \) – force applied by second spring to the swash plate, \( c_2 \) - second spring stiffness, \( \gamma_{st} \) - swash plate angle at the pressure of \( p_{st} \), which is pressure level for second spring to start operate. It depends on second spring size and defined during pump design.

After transforming equation (12) taking into account equations (9), (10), (11), and (14), we will obtain relation between pump supply \( Q_2 \) and supply pressure when two springs operate simultaneously in the following form:

\[
Q_2(p_2) = A_2 - B_2 p_2 \tag{15}
\]

where \( A_2 = \frac{V_0}{\omega} \frac{c_1 (z_0 + b \cdot \tan \gamma_{max}) + c_2 b \cdot \tan \gamma_{st}}{b (c_1 + c_2) \tan \gamma_{max}} \) and \( B_2 = \frac{V_0}{\omega} \frac{\pi d_r^2}{4} \cdot \frac{a}{b^2 (c_1 + c_2) \tan \gamma_{max}} \).

After calculating coefficients \( A_i, B_1, A_2 \) and \( B_2 \) by formulas (5) and (6) in dimensional form \( (p_{st} \) for first approximation defined same as for non-dimensional \( \dot{p}_{st} \)) all parameters of regulating unit are
defined \((c_1\) and \(c_2\) — spring stiffness, \(z_0\) — initial pre-pressure of first spring, \(\gamma\) — swash plate angle causing the second spring to work, \(d_r\) — rod diameter, shoulders \(a\) and \(b\) diameter). Number of parts \(n\) and \(m\) is chosen to make pressure value change with a step of 0.001 MPa.

After defining coefficients \(A_1, B_1, A_2\) and \(B_2 p_{st}\) is calculated by formula (4) for the second approximation and coefficients \(A_1, B_1, A_2\) and \(B_2\) are defined repeatedly by formulae (13) and (15). After each approximation relative error \(\Delta\) is calculated by formula (7).

Results of calculation of \(p_{st}\) and \(\Delta\) for successive approximations for the pressure change range of 18..40Mpa are shown in table 2:

| Approximation number | 1   | 2   | 3   | 4   | 5   | 6   |
|----------------------|-----|-----|-----|-----|-----|-----|
| \(p_{st}\), MPa      | 24.828 | 25.082 | 25.268 | 25.403 | 25.5 | 25.569 |
| \(\Delta\), %        | 4.9952 | 0.0255 | 0.0238 | 0.0217 | 0.0245 | 0.0205 |

Values of relative error and cut-in pressure \(p_{st}\) show high convergence with the results of theoretical calculations. After 4-5 approximations \(\lim\Delta \to 0\) for concerned variant of output regulation design we have received \(p_{st} = 25.27...25.4\) MPa (\(\Delta_{\text{min}} = 0.0217\) %) in terms of transmission of energy \(\Delta = N_{\text{app}} - N_{\text{hyp}}\).

On the basis of table 2 data we conclude that second spring start moment \(p_{st}\) corresponds to one third of range of regulated pressure from the moment of regulation start \(p_0\). In this case we obtain the following:

\[
p_{st} = p_0 + \frac{1}{3}(p_{max} - p_0) \quad (16)
\]

In a non-dimensional form this relation can be described as follows:

\[
\bar{p}_{st} = 1 + \frac{1}{3}(\bar{p}_{max} - 1). \quad (17)
\]

**Discussion and analysis of the results**

Expressions (16) and (17) describe characteristics and values of parameters quite accurate in correspondence to formula (8). It allows to conclude that expressions (16) and (17) can be used for defining second spring start moment with high precision and applied for relative pressure change range of any length in a relative range of 1...3,0.

Relative error of power transfer in separate specific points of supply change characteristic (for example, two edge points and one middle point for each of 3 ranges obtained after approximation of hyperbolic function by two lines) aren’t higher than \(\pm 1\) %.

Therefore, possibility of usage of least square method for defining supply regulation mechanism for axial-piston pump providing operating flow supply at constant capacity at the output with low loss (almost full usage of available input power)

Practical checking of chosen parameters of supply regulation mechanism and providing constant hydraulic fluid flow intensity for pump with capacity of 15 kW operating in the pressure range of 18..40Mpa, verified the results of mathematical calculations.

**Conclusion**

1. Proposed method of calculation of second spring start pressure in consideration of pressure change range allows to create and define parameters and size of highly effective pump supply regulation mechanism.

2. Results of research and approximation with the help of least square method and successive square method showed the following:
   - second spring start pressure level in the range of obtained errors depends on value of supply pressure change range;
the following expression can be practically used to define second spring start pressure $p_{st} = p_0 + \frac{1}{3} (p_{\text{max}} - p_0)$.

3. Other parameters of the mechanism (spring stiffness, initial deformation of first spring, swash plate angle providing second spring to start, rod diameter, shoulders a and b diameter) should be defined on the base of obtained expressions by formulae (13) and (15), taking into account construction size of regulation mechanism.

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