Simple Pulses for Universal Quantum Computation with a Heisenberg ABAB Chain

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Recently Levy [1] has shown that quantum computation can be performed using an ABAB...
pairs being concatenated to produce $\hat{R}_z(\pi)$.

![Diagram](image)

**FIG. 2.** (a) A sequence of four steps to synthesize $\hat{R}_z(\pi)$.
(b) Two steps suffice for a certain two-qubit gate.

Given that we can achieve pure z-rotations and pure y-rotations, we can use the sequence $\hat{R}_z(\alpha)\hat{R}_z(\beta)\hat{R}_z(\gamma)$ to synthesize (up to a meaningless global phase) the general single-qubit transform

$$\hat{G} = \begin{pmatrix} e^{i(\alpha/2+\gamma/2)} \cos \frac{\beta}{2} & -e^{i(\alpha/2-\gamma/2)} \sin \frac{\beta}{2} \\ e^{i(\alpha/2-\gamma/2)} \sin \frac{\beta}{2} & e^{i(\alpha/2+\gamma/2)} \cos \frac{\beta}{2} \end{pmatrix}.$$  

This formal construction therefore corresponds to a maximum of 7 steps for any single qubit gate (1 + 6 + 1 = 8, but we may amalgamate the last two, since both are z-rotations). In practice, there will be shorter sequences for any given operation. For example, the important Hadamard transform corresponds to just a single step (e.g. applying $J = 2\omega_0$ for time $t = \pi/(2\sqrt{2}\omega_0)$). The time requirement for the $\hat{R}_z(\pi)$ rotation shown in Fig. 2(a) is probably quite typical - it is $\pi(1 + \sqrt{2})/(2\omega)$.

One might object that since the other qubits in the computer are also (presumably) represented by an AB pair, these qubits will have performed a z-axis rotation $\hat{R}_z(2\omega\tau)$ whilst we were performing $\hat{G}$ on our target qubit. We should take these rotations into account, i.e. we should really be working in the rotating frame of a passive qubit. A naive method (not the most efficient) for achieving this is to supplement our $\hat{G}$ sequence with a rotation $\hat{R}_z(2\pi)$, which has no net effect in the lab frame but takes time $\tau' = \pi/\omega'$. With an appropriate choice of $\theta$ ($\Rightarrow \omega'$) the total gate time $\tau$ is then such that $\omega\tau = 2\pi\tau'$, so that the ‘other’ qubits have experienced zero net rotation. More efficiently, one would incorporate this consideration into the process of deriving the optimal short rotation sequence for $\hat{G}$.

The above analysis therefore demonstrates that any single qubit-gate can be efficiently performed on the logical qubit via a short sequence of fixed $J$ values. It is straightforward to extend this approach to produce a particular two-qubit gate which, together with our universal single-qubit gate, will form a complete set of gates for computation. Consider an BABA section of a quantum computer, and suppose that two logical qubits are represented in this section, one in the first BA pair and one in the second (see Fig 2(b)). Now suppose that the interaction is “off” between all spins except the middle AB pair (which spans the two logical qubits). With an appropriate short sequence of non-zero $J$ values, we can produce the net effect

$$\hat{U}_\text{gate} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ of the central two spins. Here $\hat{U}_0$ denotes the time evolution that would have occurred if the interaction had simply been off for the whole period. Thus the effect (up to a meaningless global phase of $i$) is to introduce a phase of $-1$ conditional on central spin-pair AB being in state $|10\rangle$. Remembering that the logical qubits on the two BA pairs are represented as $|01\rangle \equiv |0\rangle_L$ and $|10\rangle \equiv |1\rangle_L$, this condition translates to both logical qubits being in state $|0\rangle_L$. Our transformation is therefore a two-qubit gate comparable to the “nAND” gate, except that it singles-out $|0\rangle_L|0\rangle_L$ rather than $|1\rangle_L|1\rangle_L$. We might therefore describe this gate as a “nNOR”.

As a final remark, it is worth noting that although the above approach does not require the $\omega \equiv (A - B)/2$ parameter to be varied, never-the-less such an ability would be advantageous. In particular, it would be useful if $\omega$ could be switched to zero, because this would then allow the SWAP operation to be performed with a single pulse, and on a time scale limited only by the maximum strength of $J$. Any one-dimensional computer based on nearest-neighbor interactions must spend much of its time simply moving qubits around, therefore efficient performance of SWAP is very desirable. One might imagine a quantum dot implementation where the B-field has a cycle involving being ‘off’ for a period of the time (during which qubits are moved around), before being pulsed to a large value in order to allow general one and two-qubit gates as described above.

To conclude, we have explicitly shown that one can perform universal computation in the system described by Levy using only simple fixed values of $J$. This scheme, with its relatively modest set of physical requirements, is a strong candidate architecture for solid state quantum computing.

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[1] J. Levy, preprint quant-ph/0101057 at xxx.lanl.gov.

[2] Certain physical systems do have this property, which would allow somewhat shorter pulse sequences in our analysis.
[3] See for example Nielsen and Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press 2000.

[4] One such sequence for the given matrix consists of just two steps as follows: for time $t = 2\pi a_+ / \omega_0$ apply $J = 2\omega_0(1/a_+^2 + 1)^{1/2}$, then for time $t = 2\pi a_- / \omega_0$ apply $J = 2\omega_0(1/a_-^2 - 1)^{1/2}$. Here $a_+ = (5 + \sqrt{7})/8$. Here $a_- = (5 - \sqrt{7})/8$. 