Abstract: A hybrid technique combining the multi-level fast multipole algorithm (MLFMA) and the modified adaptive division beam tracing (MADBT) is presented to analyze the radiation patterns of the antennas mounted on large-scale complex platforms. In this technique, the MLFMA is used to characterize the antenna and the transition region that cannot be analyzed accurately by high-frequency asymptotic methods. The MADBT method is used to analyze the contribution of the platforms to the entire radiation pattern by tracing all beams effectively. By applying the beam-based MADBT method instead of the conventional current-based physical optics (PO) method to the platforms, the multi-bounce effects inside the platforms are considered, which enhances the accuracy of the radiation patterns, especially for the complex platforms with corner reflector. An iteration method is proposed to model the interaction between the antennas and the platforms strictly. The proposed iterative MLFMA-MADBT method is mesh-independent and can avoid the matrix-vector production (MVP) of the iterative MLFMA-PO method in each iteration. These characters significantly reduce the memory and time consumption in computation while keeping high accuracy. Numerical results are presented to demonstrate the accuracy and efficiency of the proposed hybrid technique.

Keywords: Iterative MLFMA-MADBT; antenna radiation analysis; multi-bounce effects; mesh-independent

1. Introduction

The analysis of the antennas mounted on large complex platforms is of growing importance in the area of computational electromagnetics (CEM). For analysis of large electromagnetic problems in frequency domain, there are two main kinds of method: the low-frequency numerical method and the high-frequency asymptotic method. Among the low-frequency methods, the method of moments (MoM) [1], the multi-level fast multipole algorithm (MLFMA) [2,3], the precorrected-FFT (P-FFT) [4] and the adaptive integral method (AIM) [5] are developed to provide reliable accuracy. However, the expensive computational resources cost of the low-frequency methods makes them unsuitable for the analysis of the radiation problem including large platforms. In the high-frequency asymptotic area, the physical optics (PO) and the iterative PO based on ray tracing (RT) has been widely used in the analysis of electromagnetic scattering and radiation. The shooting and bouncing ray (SBR) technique [6] is one of the most efficient ways to predict the scattering field of electrically large and complex targets with great accuracy and efficiency. The divergence problem in the BT leads to the beam tracing method (BT) in which the ray tube is replaced by the adaptive division beam [7,8].

A promising approach to solve this problem is to combine the fast but approximate high-frequency asymptotic method and the slow but accurate low-frequency method to produce a hybrid technique. The method of moments-physical optics (MoM-PO) [9–12] and the multi-level fast multipole algorithm-physical optics(MLFMA-PO) [13] are the most
classic hybrid technique with high accuracy and high efficiency. The high order MoM-PO [14,15] method was introduced to decrease the number of unknowns. The method of moments-iterative physical optics (MoM-IPO) method is developed to model the multi-bounce effects outside the MoM region [16]. To consider the interaction between the MoM region and the PO region, the matrix-vector production (MVP) is applied in the iterative MLFMA-PO method [13] at the expense of computation time.

In this paper, a modified adaptive division beam tracing (MADBT) is proposed to model the multi-bounce effects outside the MLFMA region. In contrast to the previous work, which only uses the BT to solve the scattering problem under the plane wave [17], we use the BT for analyzing the complex radiation problem. To the best of our knowledge, it is the first time that the beam-based method is used to analyze the antennas mounted on large platforms. A point source in the MLFMA region is used as the original point of the MADBT when considering the coupling from the MLFMA region to BT region. And the radiation pattern of the point source is modeled by the FFAFA-FMLMA [18] after a quantitative estimation of the size of the MLFMA region. Thus, the proposed MLFMA-MADBT hybrid method can also be used to analyze complex antenna structure such as patch antenna array.

In detail, the analysis of the antennas mounted on large platforms is solved in several steps: First, the strong coupling problem, namely the connection between the antenna and the platform, is solved and decoupled by partitioning a transition region surrounding the junction. Second, the coupling interaction between the MLFMA region and the BT region is calculated with the induced field from one region to another region. The field from the MLFMA region acts as the incident field of the initial beams launched toward the BT region, while the PO field from the BT region impacts on the MLFMA region as an additional excitation voltage vector. Third, by applying the MADBT method, the multi-bounce effects within the platform can be obtained. The MADBT method is based on beams rather than the PO current, which means the MVP can be avoided and memory usage can be greatly reduced. Furthermore, the BT process of the MADBT method only depends on the geometry of the platform, so the adaptive division of BT is mesh-independent. The BT region can be modeled with an extremely coarse mesh and scarcely affect the accuracy.

The remainder of the paper is organized as follows. The outline of the iterative MLFMA-MADBT method is introduced in Section 2. The key points and the details of the proposed method are described in Section 3. The numerical results of the proposed method and two conventional methods are compared in Section 4, while conclusions are drawn in Section 5.

2. Method Overview

Consider an antenna mounted on a large perfect electric conducting (PEC) platform. The structure can be divided into two parts: the MLFMA region and the BT region. As shown in Figure 1, to solve the strong coupling problem, namely the antenna connected with the platform, a part of platform surrounding the junction is allocated as the transition region. The transition region and the antenna are together classified as the MLFMA region, and the remaining part of the platform is allocated to the BT region. Usually, the accuracy and computational complexity improves with the increment of the area of transition region. The rule to estimate the size of the transition region is presented in Section 3.1.

A general flowchart in Figure 2 is presented to illustrate the iterative MLFMA-MADBT method for the radiation analysis of an antenna mounted on a large-scale platform. The method mainly consists of four parts:

1. The MLFMA region (antenna and transition region) is analyzed by the MLFMA and regarded as a point source.
2. The initial beams are generated from the point source and the MADBT method is performed on the BT region.
3. The iterative process is used to handle the coupling interaction between the MLFMA region and BT region.
4. The far field is obtained as the superposition of the MLFMA region and BT region. Details of each part are described in the following sections.

![Diagram showing the division of the MLFMA region and BT region for the whole structure.](image)

**Figure 1.** The division of the MLFMA region and BT region for the whole structure.

![Flowchart showing the iterative MLFMA-MADBT method.](image)

**Figure 2.** A general flowchart of the iterative MLFMA-MADBT method.
3. Formulation

3.1. Point Source Modeling and Size Estimation of the MLFMA Region

The first problem to be solved is how to build the mutual relationship between the MLFMA region and BT region. To analyze the radiation field from the MLFMA region to the BT region, a general way is calculating the integral from all RWG basis functions in the MLFMA region. When the platform is large, this solution requires large memory and long computational time. Thus, a novel method is proposed in this paper to reduce the computational complexity.

In this paper, considering the mechanism of the MADBT method, the radiation field from the MLFMA region to the BT region is regarded as transmitted from a point source at the geometrical center of the MLFMA region. In addition, the electric field radiated by the MLFMA region can be calculated by

$$E(r_i) = -jw\mu \int_S G(r_i, r_j) [J(r_j) + \frac{1}{k^2} \nabla' \nabla' \cdot J(r_j)] dr_j$$  \hspace{1cm} (1)$$

where \(w\), \(\mu\) and \(k\) are the angular velocity, permeability and wave number in free space, \(r_i\) is the observation point located in the BT region, \(r_j\) is the source point in the MLFMA region, \(G(r_i, r_j)\) is the Green’s function in free space, \(J(r_j)\) is the electric current at \(r_j\).

When the source point and observation point is relatively far, the fast far-field approximation technique combined with MLFMA (FAFFA-MLFMA) can be used to alleviate the burden of field calculations \[18,19\]. In this paper, the oct-tree boxes in the MLFMA region define the source groups, and each triangle in the BT region becomes a single far-field observation group. Suppose the \(r_m\) and \(r_n\) are the centers of the observation and source groups. For the far-field BT region, the spherical-wave function can be approximated as

$$e^{-jk|r_i-r_j|} \approx -\frac{jk}{4\pi} e^{-jk_0 (r_m+r_n)/\alpha_{mn}}$$  \hspace{1cm} (2)$$

where \(k_0 = \hat{k}k_0\), \(\hat{k}_0\) is unit spatial vector from \(r_n\) to \(r_m\), \(r_{mn} = r_i - r_m\), \(r_{nj} = r_n - r_j\) and \(\alpha_{mn}\) is a translator which is defined as

$$\alpha_{mn} = 4\pi \frac{e^{-jkr_{mn}}}{-jkr_{mn}}$$  \hspace{1cm} (3)$$

where \(r_{mn}\) is the distance between \(r_m\) and \(r_n\).

Therefore, the MLFMA region can be simply treated as a point source with the newly defined coupling formula to reduce the number of radiation sources dramatically, while the original is the number of RWG basis functions in the MLFMA region.

The size of the MLFMA region must be determined before calculating the radiated field from the MLFMA region. As mentioned in \[18\], the quantitative criterion for the far-field approximation in the BT region can be presented as

$$r_{mn} \geq 3\gamma D$$  \hspace{1cm} (4)$$

where \(\gamma \geq 1\) is a threshold set as 1.5 in this paper and \(D\) is the maximum value of \(|r_{im} + r_{nj}|\).

In the BT region, \(r_{mn} = 0\) because of the observation point in each triangle (group) is exactly the center of the group. Then we have

$$D = \max|r_{nj}| = \frac{1}{2} \sqrt{D_x^2 + D_y^2 + D_z^2}$$  \hspace{1cm} (5)$$

where \(D_x\), \(D_y\) and \(D_z\) are the side lengths of the box (group) in the oct-tree of the MLFMA.

Suppose there is no transition region and only an antenna is included in the MLFMA region. According to (4), for a large platform, only the triangles whose distances from the center point to any part the antenna is bigger than \(3\gamma D\) can be identified as the far-field BT region. The remaining triangles in the platform must be treated as the transition region.
and calculated as a part of the MLFMA region. In actual applications, we tend to extend the transition region from (4) to make sure that the far-field approximation is correct.

3.2. Initial Beams Generation and Visibility Determination

A novel and efficient way proposed in this paper is creating the initial beams according to the geometry of the BT region. As illustrated in Figure 3, the initial radiation beams are generated by directly connecting the point source with mesh triangles in the BT region. Thus, the initial beams in this paper are pyramids emitted from a point source.

![Figure 3. The generation of initial beams. P is the point source, M and $\hat{n}$ are the center and normal of the triangle.](image)

Please note that only the mesh triangles visible to the point source can be used to create the initial beams. The visibility determination is implemented by progressive processing to minimize the calculation.

First, a sketchy filtering with a back-face culling based on the normal [20] is presented in (6).

$$\hat{n} \cdot (r_M - r_P) < 0$$  \hspace{1cm} (6)

where $\hat{n}$ denotes the normal of the triangle, $r_P$ and $r_M$ are the locations of the point source and the center of the triangle.

Then, the triangles satisfying (6) will be verified more precisely by sending a test ray from the point source to the center of the triangle. The test ray of a visible triangle should not intersect with other triangles before arriving the destination. The searching of the triangles near the test ray can be accelerated by kd-tree algorithm [21].

3.3. MADBT Method

In this paper, a modified adaptive beam division algorithm is introduced to analyze radiation problem where the initial beams are pyramids with different corner ray directions. Beams in this method are dynamically split when they are divergent, and more details are described as follows.

After the generation of initial beams, beams are launched toward the BT region and the recursive beam tracing is performed using the MADBT. During the tracing process, several beam-triangle intersections are performed. Please note that the kd-tree algorithm can be used to accelerate the time-consuming beam-triangle intersections when the platform is complex.
As shown in Figure 4, there are three basic cases of the beam-triangle intersections, i.e., non-intersection, partial intersection and full intersection. Different cases can be determined by checking the relative position between the beam and the projected mesh triangle on the beam base plane.

![Figure 4](image)

**Figure 4.** Three cases of intersection. Project the mesh triangle on the beam base plane, we can get the relative position between beam and mesh triangle.

Among the three intersection cases, the non-intersection and full intersection are easy to handle. For the non-intersection case, the beam-triangle intersection will come to an end and other beam-triangle intersections will be done for the beam until it exits the platform or intersects with one of the mesh triangles. For the full intersection case, the reflected beam will be generated, and the information of the mesh triangle will be stored for further field calculation. The generation of reflected beam is shown in Figure 5a.

![Figure 5](image)

**Figure 5.** (a) The generation of reflected beams. (b) The generation of hit beams, e.g., IDE, IEF, IFGH, and miss beams, e.g., CIH, DAE, and GFB.

When it comes to the partial-intersection case, the situation is quite complicated. First, a beam may have partial intersections with several adjacent mesh triangles, if all the normal of the intersected triangles are the same, the beam can be treated as the full intersection without partition. Otherwise, as shown in Figure 5b, the original beam should be dynamically split into several new hit beams and miss beams according to the projection of the mesh triangle on the beam base plane using the partition method introduced in [7]. The number of corner ray directions of the beam are limited to three or four because of the
consistency, and both are supported by the partition method and the beam tracing used in the MADBT method.

And for the hit beams, reflected beams will be generated. After that, all current beams, including reflected beams and miss beams will continue to be traced and adaptively divided until they exit the platform, or the order of reflection is larger than the threshold. At the same time, the beam tree that records the tracing procedure for each initial beam is generated.

Generally, the number of beams will increase for considering the multi-bounce effects within the BT region. In actual applications, this problem will not bring an unacceptable negative impact to the efficiency of the proposed method for several reasons: First, not all beams with the partial intersection need to be split since the first condition of the partial intersection. Second, the large platforms discussed in this paper are not the enclosing objects with antennas inside, and most beams will be reflected into the free space within a finite order of reflection. Third, considering diminishing returns in multiple bounces, a proper threshold of the maximum order of the reflection can control the number of the beams while keeping a reasonable accuracy.

Throughout the entire BT process, each beam is adaptively and recursively subdivided according to the intersections with the mesh triangles of the BT region. In addition, the tracing process of the MADBT method is only related to the geometry of the BT region. Therefore, the MADBT method is insensitive to the mesh size. Thus, the platform can be modeled with extremely coarse mesh if no loss of geometry information of the platform. It can be further concluded that the iterative MLFMA-MADBT method is mesh-independent while keeping high accuracy.

3.4. Iterative MLFMA-MADBT Method

Although the total far field can be obtained by summing up the contributions from the MLFMA region and the BT region, the MLFMA region may be disturbed by the induced field from the BT region and the whole electromagnetic (EM) system cannot reach stable status. Figure 6 shows the couple intersection between the MLFMA region and BT region.

![Figure 6. The couple intersection between the MLFMA region and the BT region.](image)

The near field $E_{\text{near}}$ from the MLFMA region acts as the incident field of the initial beams launched towards the BT region; the near field $E_{\text{PO near}}$ from the BT region can disturb the field of the MLFMA region. Thus, an iterative process should be introduced to handle the coupling between the two regions.

3.4.1. Field Calculation from the MLFMA Region to the BT Region

After applying the MADBT method to the BT region, the field of the BT region can be gained as follows:

1. The incident field of each initial beam is approximately equal to the field of the MLFMA region at the beam’s center and can be calculated by (1).
2. For each hit beam, the incident field of its reflected beam can be calculated by geometrical optics (GO). Then the current reflected beam can be treated as a new incident beam, which can be recursively used to generate new reflected beam for the next reflection until it exits the platform or the number of reflections reaches the threshold. Thus, the incident field of all beams can be calculated recursively by traversing the beam tree of each initial beam.

3. After the calculation of the incident field of each hit beam, the near and far field of the BT region can be calculated by PO integral [22].

Please note that the mesh size for PO integral should be less than 0.3λ (λ indicates the wavelength) for accuracy. Thus, if the size of a hit beam is too large for the PO integral, it is necessary to split the hit beam into several new small beams of 0.3λ during the integral process.

3.4.2. Disturbance from the BT Region to the MLFMA Region

For the interaction coupled back from the BT region to the MLFMA region, the induced PO near field $E_{PO \ near}$ affecting the MLFMA region [23] can be taken into account by adding the additional exciting voltages to the voltage vector of the MLFMA. Figure 7 shows two triangles associated with the $i$th common edge in SWG basis functions.

![Triangular pair](image)

Figure 7. The division of the MLFMA and BT regions for the whole structure.

The additional exciting voltages $\Delta V$ can be calculated by the center point approximation method given as follows:

$$\Delta V_i = \frac{l_i}{2}(E_{i \ \text{inc, } c^+} \cdot \rho_i^{c^+} + E_{i \ \text{inc, } c^-} \cdot \rho_i^{c^-})$$ (7)

where $\Delta V_i$ is the additional excitation voltage on the $i$th common edge, $l_i$ is the length of the $i$th common edge, $E_{i \ \text{inc, } c^+}$ and $E_{i \ \text{inc, } c^-}$ are the induced near field $E_{PO \ near}$ from the BT region at each triangle center which is the superposition of PO near field from all the small hit beams in the BT region, $\rho_i^{c^+}$ and $\rho_i^{c^-}$ are the vectors in the triangular pairs between the center point and the vertex opposite to the common edge on each triangle.

Solving the matrix Equation (9) excited by the modified voltages in (8), we can get the new current $I_{MLFMA}$.

$$V_i = V + \Delta V_i$$ (8)

$$Z_{MLFMA} I_{MLFMA} = V$$ (9)
3.4.3. Iterative Process

An iterative process will be implemented from (7) to (9) to consider the coupling interaction between the MLFMA region and BT region. When the equivalent surface currents of the MLFMA region reach stable status, the field generated by the whole structure become stable, too. Thus, the current continuity at the boundary between the MLFMA region and BT regions can be guaranteed. In addition, the iteration stop criteria is defined as

\[ \frac{\| I_{i}^{\text{MLFMA}} - I_{i-1}^{\text{MLFMA}} \|}{\| I_{i-1}^{\text{MLFMA}} \|} < \varepsilon, \quad i = 1, 2, \ldots, I_{\text{iter}} \]   \hspace{1cm} (10)

where the norm is defined in the 2-norm form, \( i \) stands for the \( i \)th iterative process and \( \varepsilon \) is the threshold value.

3.5. Far Field Calculation

After the iterative process, the entire EM system reaches stable status. Therefore, the far field of the MLFMA region can be obtained by (1), and the far field of the BT region can be obtained by summing up the PO fields scattered from each small hit beam. Thus, the total far field can be obtained as the superposition of contributions from both the MLFMA region and the BT region.

4. Numerical Results

In this section, several numerical examples are presented to demonstrate the accuracy and efficiency of the proposed iterative MLFMA-MADBT method in this paper. In addition, the numerical results of the proposed method are compared with those of the MLFMA and the MLFMA-PO. In additional, the set of the transition region is the same for the MLFMA-PO method and the proposed method.

For the MLFMA region in all examples, the mesh sizes are approximately 0.1\( \lambda \). For the BT region used in the iterative MLFMA-MADBT, because of the mesh-independent character of the MADBT method, the mesh sizes are extremely coarse for the BT region and the specific sizes are given in each example, respectively. As for the PO region used in the MLFMA-PO, the mesh sizes are 0.3\( \lambda \).

All computations were carried out on a workstation with a 2.6GHz Core E5-4620 v2 CPU equipped with 256GB RAM and all digits were stored in double precision.

4.1. Patch Antenna Array Installed on an Airplane

An airplane model which has a complex patch antenna array with 16 \( \times \) 18 (288) elements installed above the top is studied. The layout and feeding structure of the patch antenna array is shown in Figure 8a–c. The length of the airplane is 17.7 m, the wingspan is 15.6 m and the height is 4.9 m, as illustrated in Figure 9a. The dimension of the patch antenna array is 0.74 m \( \times \) 1.17 m \( \times \) 0.004 m and the magnitude of each element is 1 \( Idl(\text{Am}) \). The work frequency of the antenna is 3 GHz.

As the antenna is placed about 0.2 m (2\( \lambda \)) above the airplane, there is no need to partition a transition region between the airplane (BT region) and antenna (MLFMA region). The mesh size is about 1\( \lambda \) for the BT region in the iterative MLFMA-MADBT method.

The far-field radiation patterns of the model in XOZ plane are shown in Figure 9b. According to the radiation patterns and the comparison mean, standard deviation, the result of the proposed iterative MLFMA-MADBT method agrees well with the result based on the MLFMA method, which suggests that the proposed method is correct and valid even for platforms with curved surfaces and modeled by coarse mesh.
Figure 8. (a) The layout of the patch antenna array. (b) The plan view of a single patch antenna. (c) The 3D view of a single patch antenna.

Figure 9. The model and far-field radiation patterns in XOZ plane of the patch antenna array installed above the top of an airplane. (a) Model (b) Comparison result.
4.2. Wire Antenna Mounted on a Trihedral Corner Reflector

As displayed in Figure 10a, a wire antenna is mounted on a trihedral corner reflector whose side length is 1 m. Its working frequency of the antenna is 3 GHz and the magnitude is 1 \( I_{dl} (A_m) \). A part of the trihedral corner reflector near the antenna is divided into the transition region. For the proposed iterative MLFMA-MADBT method, the mesh size of the remaining trihedral corner (BT region) is about 3\( \lambda \).

![Figure 10a](image_url)

### Figure 10. The model and far-field radiation patterns in XOZ plane of the wire antenna mounted on a trihedral corner reflectors. (a) Model (b) Comparison result.

The far-field radiation patterns in the XOZ cut-plane obtained from the MLFMA, the MLFMA-PO and the proposed iterative MLFMA-MADBT methods are plotted in Figure 10b. The proposed method gets smaller value of mean and standard deviation than the MLFMA-PO, which indicates that the proposed method has a higher accuracy and is more advantageous than the MLFMA-PO especially for the platform with multiple reflectors.

Furthermore, for the wire antenna mounted on a trihedral corner reflector, the number of mesh elements, memory usage and total calculation time of the three methods are listed in Table 1. It is obvious that the iterative MLFMA-MADBT method has the best performance on memory usage and time consumption among the three methods.

### Table 1. Calculation time and memory requirements of different methods for the wire antenna mounted on a trihedral corner reflector.

| Methods           | Mesh Elements | Memory (MB) | Time (s) |
|-------------------|---------------|-------------|----------|
|                   | MLFMA | PO/BT |         |         |
| MLFMA             | 119874 | /     | 4270.14 | 66.02   |
| MLFMA-PO          | 171   | 13907 | 85.90   | 56.47   |
| MLFMA-MADBT       | 171   | 115   | 29.29   | 8.01    |
4.3. Wire Antenna Mounted on a Realistic Ship

Another example is a wire antenna mounted on a realistic ship model. The magnitude of the antenna is 1 \( \text{Idl}(\text{Am}) \). The length of the realistic ship is 16.48 m and the height is 4.78 m, as shown in Figure 11a. The antenna works at a frequency of 2 GHz. The mast of the ship connected with the antenna is totally allocated as the transition region. The remaining part of ship (BT region) is modeled by mesh size of 1\( \lambda \) for the iterative MLFMA-MADBT method.

![Figure 11](image)

Figure 11. The model and far-field radiation patterns in XOZ plane of the wire antenna mounted on a realistic ship. (a) Model (b) Comparison result.

The far-field radiation patterns in the XOZ cut-plane calculated by the MLFMA, the MLFMA-PO and the proposed iterative MLFMA-MADBT methods are displayed in Figure 11b. Compared with the MLFMA method, the proposed method can reduce the mean and standard deviation of the MLFMA-PO method from 6.93 to 4.95, from 9.54 to 7.51, respectively. Furthermore, the proposed method can keep good accuracy even with extremely coarse mesh for the flat platform.

Furthermore, for the wire antenna mounted on the realistic ship model, the number of mesh elements, memory requirement and the computation time of the three methods are given in Table 2. Compare to the MLFMA method, the proposed iterative MLFMA-MADBT method can reduce the requirement for the memory from 81.46 GB to 0.64 GB and computation time from 4882.03 s to 229.01 s. AS for the MLFMA-PO method, the memory requirement of the MLFMA-PO method is about 5 times of the proposed method and about 10 times on the time consumption. It is obvious that the proposed method not only has high predominance in memory usage but also uses the least time among these three methods.

**Table 2.** Calculation time and memory requirements of different methods for the wire antenna mounted on realistic ship model.

| Methods       | Mesh Elements | Memory (MB) | Time (s) |
|---------------|---------------|-------------|----------|
|               | MLFMA PO/BT   |             |          |
| MLFMA         | 3,250,756     | 81,458.10   | 4882.03  |
| MLFMA-PO      | 21,384        | 359,071     | 2178.81  |
| MLFMA-MADBT   | 21,384        | 650.33      | 229.01   |
5. Conclusions

A new iterative MLFMA-MADBT hybrid method is proposed to solve the complex radiation problem of the antennas mounted on large platforms. A transition region surrounding the junction of the antenna and the platform is strictly considered, which can guarantee high accuracy. The MLFMA region is regarded as a point source and the pyramid beams correlated with the source are modeled by the FAFFA-MLFMA to compute the interaction from the MLFMA region to the BT region. Then the MADBT method for the radiation analysis is applied to consider the multi-bounce effects within the platforms without any matrix operation. The numerical results show that the computations of the proposed method consume much less memory and time than the MLFMA and the MLFMA-PO. Furthermore, under the premise of good efficiency, the proposed method obtains obviously better accuracy than the MLFMA-PO especially for the corner reflector structures, which indicates that the proposed method is especially suitable for the complex platforms with multiple reflections. In addition, it is necessary to point out that the iterative MLFMA-MADBT method is mesh-independent not only for the flat platform but also the platform with curved surfaces.

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