Frozen magnetic response in mesoscopic superconductors

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Abstract. For a bulk type II superconducting sample at low temperature, the magnetic field can penetrate in the form of a single quantum fluxoid, for bulk samples this fluxoides are arranged in a hexagonal lattice, this so-called Shubnikov-Abricosov state or vortex state and takes place between the first and the second critical thermodynamics magnetic fields. Under the first magnetic critical field is present the Meissner-Oschenfeld state. For mesoscopic samples, the magnetic response can present very interesting properties due the proximity effect of in-homogeneous boundary conditions and the presence of dots, anti-dots and/or impurities. The superconducting state in a mesoscopic sample with dot/anti-dot/dot/pillar/trench is calculated within the nonlinear Ginzburg-Landau equations. We predict that the critical magnetic fields and magnetization depends on strongly of the nature, geometry, size of the defects and the boundary conditions used.

1. Introduction
The superconducting-magnetic properties of samples with magnetic-structured-topological pinning or anti-pinning cores have attracted over the past years [1, 2]. The present possibility of controlling the magnetic and physics properties of the superconducting condensate (vortex state, critical fields and currents) has made from superconductors of size of the coherence or penetration length the principal experimental and theoretical systems for studies [3,5]. Is known that when the magnetic-structural-topological defects are placed into the sample, the vortex matter depends on their configuration, density and the pinning array [2]. In previous works one of the authors of this paper studied the superconducting properties of a Niobium prism with its lateral surfaces in contact with deferent kinds of metallic and/or superconducting materials, the authors found that the second thermodynamic field increase when a superconducting-superconducting interface is considered and a slow entry of the magnetic field is observed in the metallic regions of the boundary [6]. In addition, they studied the influence of several thermal gradients on the vortex configuration and the thermodynamics properties of a low critical temperature nanoscopic superconducting long square prism [7]. The field for the first vortex penetration presents a slow dependence on the exponent of the gradient and the size of the sample [7]. The magnetic susceptibility of a superconductor immersed in a magnetic field is calculated numerically using the Ginzburg-Landau model; the authors found that the magnetic susceptibility shown a quasi-periodic modulation at the vortex transition fields [8]. Baelus found that increasing...
field, the number of vortices simultaneously entering the sample depends on the presence and the positions of the surface defects [9]. Now, in this work we solved the time dependent Ginzburg-Landau equations for calculating the magnetic response in several mesoscopic superconducting samples with anti-dot/dot/ pillar/trench. We found a strong/weak dependence of the critical fields and magnetization on the size, boundary conditions, geometry and natures of these defects.

2. Theoretical Method
The time dependent Ginzburg-Landau (TDGL) equations for isotropic superconductors, which govern the superconducting order parameter and the vector potential in the zero electric potential gauge, are given by [10-14]:

\[ \frac{\partial \Psi}{\partial t} - D^2 \Psi - \nabla^2 \Psi + \Psi = 0 \] (1)

\[ \vec{J}_T = -\frac{\partial \vec{A}}{\partial t} = \vec{J}_s + \vec{J}_n \] (2)

Where \( D = i \nabla + \vec{A} \), \( \vec{J}_T \) is the total current, \( \vec{J}_n = k^2 \nabla \times \nabla \times \vec{A} \) is the normal current, and \( -\text{Re}(\overline{\Psi D \Psi}) = \vec{J}_s \) is the super-current. The time dependent Ginzburg Landau equations were rescaled as follows: \( \Psi \) in units of \( \Psi_\infty(0) \), lengths in units of the coherence length \( \xi(0) \), the external applied magnetic field \( H_e \) in units of the second critical thermodynamics field in the bulk \( H_{c2}(0) \), potential vector \( \vec{A} \) in units of \( H_{c2}(0) \xi(0) \), temperatures in units of the critical temperature \( T_c \). The TDGL equations are complemented with the general boundary conditions for the order parameter: \( D \Psi = \Psi / b \), \( b \) is deGennes extrapolation length and simulate the interface with another material. The magnetization \( -4\pi M = \vec{B} - \vec{H}_e \) (here \( \vec{B} \) is the induction, the spatial average of the local magnetic field) is [15]:

\[ -4\pi M_n = \frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} h_{i,j}^n - H_e \] (3)

3. Results and discussion
In this work, we report the dependence of several kinds of defects on the magnetization and thermodynamics critical fields in a superconducting nanoscopic flat disk and parallelepiped submerged in an applied magnetic field. In thin disk limit, we considered a superconducting disk of Nb of radius \( R = 1 \mu m \), and thickness \( d \approx 45 \text{ nm} \) at \( T = 0 \). The TDGL equations are solved numerically in two different scenarios. One is that in which the system has invariance along the z-direction in which the external field is applied; this is the case of a circular cross section. In this scenario, we have a bi-dimensional system and demagnetization effects are not present, that is, the local magnetic field outside the sample is equal to the external applied magnetic field [16]. The second scenario is the thin film limit, for which the first TDGL equation (for the order parameter) is solved only at the plane perpendicular to the direction of the applied magnetic field, and the second equation (for the potential vector) is solved by substituting the super-current density with a superconducting sheet current density. The Studied cases are:

- Case 1: A triangular and circular central dot (anti-dot) in a thin disk.
3.1. Case 1
For a very thin disk of variable thickness, the TDGL equations can be rewritten as:

$$\frac{\partial \Psi}{\partial t} - \frac{1}{\mu} \left( \nabla + \vec{A}_0 \right) \cdot \mu \left( \nabla + \vec{A}_0 \right) \Psi + \left| \Psi \right|^3 - \Psi = 0$$  \hspace{1cm} (4)

$\mu(r, \theta)$ is a function which describes the thickness of the sample. We considered $\mu(r, \theta) = 1$ everywhere, except at dot/anti-dot position in the disk which is simulated by using $\mu(r, \theta) = 0.85$ for the dot and $\mu(r, \theta) = 1.25$ for the anti-dot. Initially, we analysed the first entrance of vortices into the sample, for disk with circular and triangular defects (dot and anti-dot) and thin disk without defect, we found that all samples exhibit the first jump of the magnetization at equal magnetic field, for the second vortex entry, we found differences in the location of peaks for different values of applied magnetic field, the vorticity decreases when the anti-dot is used, this fact is due to the anti-dot generate greater opposition to the vortices entrance, while the opposite occur with the dot \[17\]. The vortices always (never) are located into the dot (anti-dot) thus, the pinning force is present in the dot and the force of vortex–vortex interaction overcomes the repulsive force exerted by the anti-dot. The superconductor/normal transition field or second critical field is the same for all studied samples.

3.2. Case 2
Thin disk with regions on its surface at different critical temperature described by the function $\mu(r, \theta)$. For this case, the first TDGL can be rewritten as:

$$\frac{\partial \Psi}{\partial t} = \left( \nabla + \vec{A}_0 \right) \cdot \mu \left( \nabla + \vec{A}_0 \right) \Psi + \left| \Psi \right|^3 - \Psi$$  \hspace{1cm} (5)

We considered $\mu(r, \theta) = 1$ everywhere, except for $\mu(r, \theta) = 0.85$ for the superconductor at lower critical temperature and $\mu(r, \theta) = 1.25$ for the superconductor at higher critical temperature. In our analysis, we found that the first transition field is independent of the defect geometry and the kind of the defect used. We found differences in the magnetization curve when different kinds of defects are studied at low magnetic applied fields, but the second critical field does not depend on the geometry and nature of the defect. In another hand, in the down branch of the magnetic field at zero magnetic fields, the magnetization zero for the hexagon case, while for a pentagonal defect into the pentagon defect is different to zero (paramagnetism).

3.3. Case 3
In this case, the used TDGL equation is the same equation used for the case 1. Here, we analysed the influence of the boundary conditions on the magnetization and critical magnetics fields. The general boundary conditions for a superconductor are $D \Psi / \nu = \Psi / b$ (b is the extrapolation deGennes length). We will assume either a superconductor/vacuum external interface (b $\rightarrow \infty$), and the surface of the
sample in a completely normal state (b = 0). We found that the second critical field is independent of the geometry and height of the defects in the disk [18]. Moreover, the first vortex penetration field depends on the pinning or anti-pinning force of the defects; this field is lower for a barrier in both geometries. This means that the barrier repeals the first penetration than the trench, which was expected, but surprisingly, it does not depend on the geometry used [18]. For both pentagonal and hexagonal cases, the second critical field occurs at the same value. The first vortex penetration field and second critical fields dependent on the boundary condition. These fields are equals when we use b → ∞ for all cases, whereas if we take b = 0, we have the first field is lower for the trench case that for barrier case, also the second field is the same for both geometries. If we compare these results with previous works [19-20], we found that the first and second critical fields for a superconducting disk with a square, circular or triangular trench are independent of the shape of the defects and depends strongly on the boundary conditions.

3.4. Case 4
A parallelepiped with a square pillar on its top surface: The geometry of the problem includes the demagnetization effects, so the magnetic field in the surface is higher that the applied magnetic field. The superconducting domain covers the parallelepiped of thickness c and lateral sizes a and b. The dimension of no superconducting and superconducting region is A × B × C. The pillar dimensions are W × W × a, the first and second GL can be rewritten as:

\[ \frac{\partial \Psi}{\partial t} = \left( \nabla + \frac{A}{\mu} \right)^2 + \left| \Psi \right|^2 - \Psi + i \left( \nabla \psi - \frac{A}{\mu} \right) \cdot \nabla \mu \]  

\[ \mathbf{J}_T = - \frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_S + \mathbf{J}_n \]  

\[ J_s = 0 \] out of the superconducting area and just the normal current is taken in count [16]. Again, we considered \( \mu(r, \theta) = 1 \) everywhere, except \( \mu(r, \theta) = 1.25 \) for the pillar. We analyzed the diamagnetic response of the superconductor both from the three and bi dimensional cases. We found that the first penetration field and the field corresponding to the maximum of magnetization depend strongly on the size and dimensionality of the sample, being bigger for bi-dimensional samples. One difficulty in making a comparison between the bi and three models on equal footing comes from the fact that most of the configurations with equal vorticity belong to distinct values of the external applied magnetic field. This is so, because in the bi-dimensional model, the demagnetization effect not is considered and requires a larger applied field to enhance the shielding currents and consequently produces the same picture observed in the three dimensional model at a larger value of the external magnetic field [16]. We have found that the presence of the pillar perturbation diminishes the difference of the first penetration field from approximately the half between them [21].

4. Conclusion
In this contribution, we analysed the magnetic response of a mesoscopic superconducting sample (disk and parallelepiped) with several kinds of internal structural defects (dots/anti-dot/trench/pillar) and different boundary conditions in presence of an external homogenous magnetic field, by solving numerically the 2D and 3D non-linear Ginzburg–Landau differential equations. Our results show that magnetic properties of all studied samples depend on the dimensions, nature, the geometry of the imperfections, and boundary conditions used in each system. The small size of the sample (mesoscopic regime) determines strongly the influence of the defects on the critical fields and magnetization. Also, the pinning (anti-pinning) force of these defects determines the surface energy barrier and subsequently the magnetic field for the first vortex penetration.
Acknowledgments
The authors thank Edson Sardella by useful discussions.

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