Zero-Velocity Detection – A Bayesian Approach to Adaptive Thresholding

Johan Wahlström¹, Isaac Skog²*, Fredrik Gustafsson³**, Andrew Markham¹, and Niki Trigoni¹

¹Department of Computer Science, University of Oxford, Oxford OX1 2JD, UK
²S3 Research AB, 116 29 Stockholm, Sweden
³Department of Electrical Engineering, Linköping University, 581 83 Linköping, Sweden
*Senior Member, IEEE
**Fellow, IEEE

Abstract—A Bayesian zero-velocity detector for foot-mounted inertial navigation systems is presented. The detector extends existing zero-velocity detectors based on the likelihood-ratio test, and allows, possibly time-dependent, prior information about the two hypotheses – the sensors being stationary or in motion – to be incorporated into the test. It is also possible to incorporate information about the cost of a missed detection or a false alarm. Specifically, we consider an hypothesis prior based on the velocity estimates provided by the navigation system and an exponential model for how the cost of a missed detection increases with the time since the last zero-velocity update. Thereby, we obtain a detection threshold that adapts to the motion characteristics of the user. Thus, the proposed detection framework efficiently solves one of the key challenges in current zero-velocity-aided inertial navigation systems: the tuning of the zero-velocity detection threshold. A performance evaluation demonstrates that the proposed detection framework outperforms current state-of-the-art detectors.

Index Terms—Zero-velocity updates, foot-mounted inertial navigation, indoor localization, posterior odds ratio, adaptive thresholding.

I. INTRODUCTION

Zero-velocity-aided inertial navigation is one of the most promising technologies for indoor positioning in environments without pre-installed infrastructure [1], [2]. The basic concept is illustrated in Fig. 1. A key component of this technology is the zero-velocity detector, which identifies when the sensors are stationary and zero-velocity updates can be applied. Hence, a variety of detectors have been proposed, many of which can be derived as generalized likelihood ratio tests [3].

One of the problems with existing zero-velocity detectors is that the optimal detection threshold varies significantly with the pedestrian’s walking speed and motion mode (walking, jogging, running, etc.), the placement of the sensor, the type of shoe, and the walking surface. If the threshold on the likelihood ratio is too large, the detector will not be able to detect stationary instances when the user is running. If the threshold is too small, the detector will produce false zero-velocity instances [4]. Consequently, several methods for designing adaptive thresholds have been proposed.

The most common approach to adaptive thresholding is to first use some heuristic or ad-hoc solution for estimating or classifying the speed or motion mode of the user. Based on the result, the detector selects a threshold value that has been optimized, using ground truth data, for that specific speed or motion class [6–9]. However, other methods for robust zero-velocity detection under varying gait conditions have also been explored. The authors in [10] used accelerometer measurements to detect the beginning and end of individual steps, and then applied different detectors and thresholds to different parts of the gait cycle; [11] made the detection using a long short-term memory (LSTM) neural network; [12] designed zero-velocity detectors that used a hidden Markov model (HMM) to represent different stages of the gait cycle; [13] allowed the threshold to vary with the temporal variance of the accelerometer measurements; and [14] held the threshold fixed while dynamically adapting the window length of the samples used to compute the detection statistic.

Unfortunately, it is not clear how these methods relate to the existing theory on zero-velocity detection using likelihood ratio tests. Moreover, proposed methods tend to require large data sets collected at different gait speeds and motion modes to calibrate the threshold values or other design parameters. Additionally, methods that attempt to infer speed or motion mode are always limited by the accuracy of the speed estimation or motion mode classification that precede the threshold selection.

This paper presents a zero-velocity detector based on the posterior odds ratio. The proposed detection framework demonstrates how the threshold, used in established detectors based on the likelihood ratio, can be factorized as a product of (i) the inverse prior odds ratio, quantifying the prior probability of a zero-velocity detection; and (ii) a loss factor, quantifying the cost of incorrect detections. The primary contributions are:

- A theoretical justification of adaptive zero-velocity detection within the established framework for zero-velocity detection based on the likelihood ratio test.
- An application-specific interpretation of the threshold used in traditional methods for zero-velocity detection.
II. BAYESIAN ZERO-VELOCITY DETECTION

Consider the problem of determining whether an inertial measurement unit (IMU) is stationary or not given the measurements \( z_n = \{ y_n \}_{k=n}^{N-1} \), collected over \( N \) sampling instances. Here, \( y_n \) denotes the inertial measurements at sampling instance \( n \). The problem can be formalized as the binary classification problem of choosing between the two hypotheses

\[
\begin{align*}
\mathcal{H}_0 &: \text{IMU is moving} \\
\mathcal{H}_1 &: \text{IMU is stationary}. \tag{1}
\end{align*}
\]

In what follows, a method for performing the classification by minimizing the conditional risk is described.

A. Minimum-Error-Rate Classification

The performance of a Bayesian detector is quantified using the conditional risk

\[
R(\mathcal{H}_i | z_n) = \sum_{j=0}^{1} \lambda_{ij} p(\mathcal{H}_j | z_n).
\]

The conditional risk \( R(\mathcal{H}_i | z_n) \) is the expected incurred loss when deciding on hypothesis \( \mathcal{H}_i \) given the data \( z_n \). Here, \( \lambda_{ij} \) represents the loss incurred as a result of deciding on hypothesis \( \mathcal{H}_i \) when the true hypothesis is \( \mathcal{H}_j \). To minimize the conditional risk, we should decide hypothesis according to

\[
\frac{p(\mathcal{H}_i | z_n)}{p(\mathcal{H}_0 | z_n)} \geq \eta
\]

where \( \eta \equiv (\lambda_{10} - \lambda_{00})/(\lambda_{01} - \lambda_{11}) \). Throughout the paper, all denominators will be assumed to be nonzero.

B. Relation to Established Zero-Velocity Detectors

By using \( p(\mathcal{H}_0) = 1 - p(\mathcal{H}_1) \) and the factorization

\[
\frac{p(\mathcal{H}_1 | z_n)}{p(\mathcal{H}_0 | z_n)} = \frac{p(z_n | \mathcal{H}_1)}{p(z_n | \mathcal{H}_0)} \cdot \frac{p(\mathcal{H}_1)}{p(\mathcal{H}_0)},
\]

the detection rule in (1) can be written as

\[
L(z_n) \equiv \frac{p(z_n | \mathcal{H}_1)}{p(z_n | \mathcal{H}_0)} \eta \geq \gamma \tag{5}
\]

where the detection threshold is

\[
\gamma \equiv \frac{1 - p(\mathcal{H}_0)}{p(\mathcal{H}_1)} \cdot \eta
\]

\[
= \frac{1 - p(\mathcal{H}_1)}{p(\mathcal{H}_0)} \cdot \frac{\lambda_{10} - \lambda_{00}}{\lambda_{01} - \lambda_{11}}.
\]

Thus, performing a zero-velocity detection using the posterior odds ratio in (5) is equivalent to performing a likelihood ratio test with a threshold dependent on the hypothesis prior \( p(\mathcal{H}_i) \) and the loss factor \( \eta \).

C. Modeling the Loss Factor

As shown in [3], many commonly applied zero-velocity detectors can, given different assumptions about the prior knowledge of the sensor signals, be derived as (generalized) likelihood ratio tests of the same form as (5). These include the acceleration-moving variance detector, the acceleration-magnitude detector, the angular rate energy detector, and the stance hypothesis optimal detection (SHOE) detector. Hence, the threshold in these detectors can be interpreted using (5). What is more, the relationship in (5) provides a theoretically sound way to design adaptive thresholds for these detectors. Next, we discuss how this can be done by letting the hypothesis prior \( p(\mathcal{H}_i) \) and the loss factor \( \eta \) be time-dependent.

D. Modeling the Hypothesis Prior

The hypothesis prior will be dependent on what information is available in each specific scenario. In a zero-velocity-aided aided inertial navigation system implemented using a Bayesian filter such as the extended Kalman filter, the system calculates a navigation solution in terms of a statistical distribution. This statistical distribution can be used to calculate a prior for the detector. Hence, consider the case when an extended Kalman filter is used, which at sampling instant \( k \) provides the velocity estimate \( \hat{v}_k \) and velocity error covariance \( \mathbf{S}_k \). A measure of how close the system is to have zero velocity, weighted by the uncertainty of the velocity estimate, is then given by \( \xi_k = \mathbf{S}_k^{-1} \hat{v}_k \). Following the ideas of logistic regression, this measure can be mapped to the probability of the system being stationary via the logistic function. Thus, the prior is then set as

\[
p(\mathcal{H}_i) = \frac{1}{1 + e^{p(\xi_k) \cdot \lambda_i}},
\]

where \( p(\xi_k) \) provides the velocity estimate \( \hat{v}_k \) and velocity error covariance \( \mathbf{S}_k \). A measure of how close the system is to have zero velocity, weighted by the uncertainty of the velocity estimate, is then given by \( \xi_k = \mathbf{S}_k^{-1} \hat{v}_k \). Following the ideas of logistic regression, this measure can be mapped to the probability of the system being stationary via the logistic function. Thus, the prior is then set as

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\]

1Reproducible research: The data and the code used in the experiments are available under an open-source licence at www.openshoe.org.
where $\beta_1$ and $\beta_2$ are design parameters.

For $\xi_k$ to be a reliable measure of how close the system is to have zero velocity, the covariance $S_k$ must reflect the true uncertainty of the velocity estimate $\hat{v}_k$. If this cannot be guaranteed, the uninformative prior $p(H_1) = 1/2$ may be a better choice.

### E. Parameter Selection

Given the cost factor in (7) and the prior in (8), the logarithm of the detection threshold becomes

$$
\log \gamma_k = c_1 + c_2 \Delta t_k + c_3 \xi_k
$$

where $c_1 = \beta_2 + \log \alpha$, $c_2 = -\theta$, and $c_3 = \beta_1$ (with an uninformative prior, $c_3$ still holds but with $c_1 = \log \alpha$, $c_2 = -\theta$, and $c_3 = 0$). Hence, it remains to select the parameters $c_1$, $c_2$, and $c_3$. One possibility is to estimate them using ground truth data. However, this would require the collection of training data. Therefore, in what follows, a semi-heuristic parameter selection method will be presented.

When the IMU is perfectly stationary the detector should with high probability decide on hypothesis $H_1$, so that frequent zero-velocity updates are applied. This can be achieved by enforcing the condition $p(\log L(\varphi_n^*) < c_1) = \epsilon$, where $\epsilon$ is small and $\varphi_n^*$ is the output of the IMU when it is perfectly stationary. Here, we have used that when the foot has zero velocity, $\xi_k$ tends to be very small. Similarly, it is reasonable to expect a zero-velocity update after completing a step. In mathematical terms, this gives us $p(\log L(\varphi_n^*) < c_1 + c_2 \Delta t) = \epsilon$, where $\Delta t$ is the approximate time length of a step and $\varphi_n^*$ is the output from the IMU during the midstance of normal gait. In this way, most steps during normal walking will result in detected zero-velocity instances. When using an uninformative prior, one may select the parameters $c_1$ and $c_2$ by using these two conditions with $c_3 = 0$. However, when using the prior in (5) we need a third condition to perform the parameter selection. To this end, note that the detector should decide on hypothesis $H_2$ during the swing phase. Therefore, we impose the condition $p(\log L(\varphi_n^*) < c_1 + c_2 \Delta t/2 + c_3 \xi_k^*) = 1 - \epsilon$ where $\varphi_n^*$ and $\xi_k^*$ are typical values during the swing phase of $\varphi_n$ and $\xi_k$, respectively. Detectors obtained by choosing the parameters in this way with $\epsilon = 0.05$ are illustrated in Fig. 2.

### III. EXPERIMENTS

The performance of the proposed detector was benchmarked against a detector with a fixed threshold using data from [17]. The data set was collected with a MicroStrain 3DM-GX2 IMU and consists of 40 data recordings; half with normal gait (5 km/h) and the other half with fast gait (7 km/h). In each recording, the user walked one lap along a closed-loop trajectory with an approximate length of 84 m. The zero-velocity detection was performed using the likelihood ratio of the SHOE detector (see [3]) with a window length of 20 m (5 samples).

The performance improvement may be explained in several ways. On the one hand, the adaptive threshold prevents the navigation system from running too long without a zero-velocity update, and thereby prevents the estimation errors from growing too large before the next zero-velocity update. On the other hand, the adaptive threshold prevents the navigation system from using an excessive number of zero-velocity updates, and thereby reduces the negative impact that modeling errors in the pseudo velocity measurements have on the navigation solution.

### IV. CONCLUSIONS

This paper has developed a Bayesian zero-velocity detector using the posterior odds ratio. Experiments indicated that the proposed detector provides a significantly lower positioning error than state-of-the-art detectors where an adaptive threshold is selected based on the motion mode of the user. Further, the proposed method does not rely on learning the relationship between the user motion and the optimal threshold, and therefore avoids the time-consuming data collection that has been necessary in previous methods for adaptive thresholding.

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Fig. 3. Position error of zero-velocity-aided inertial navigation using a fixed and an adaptive threshold.

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