Efficient Model-Driven Network for Shadow Removal

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Abstract

Deep Convolutional Neural Networks (CNNs) based methods have achieved significant breakthroughs in the task of single image shadow removal. However, the performance of these methods remains limited for several reasons. First, the existing shadow illumination model ignores the spatially variant property of the shadow images, hindering their further performance. Second, most deep CNNs based methods directly estimate the shadow free results from the input shadow images like a black box, thus losing the desired interpretability. To address these issues, we first propose a new shadow illumination model for the shadow removal task. This new shadow illumination model ensures the identity mapping among un-shadowed regions, and adaptively performs fine grained spatial mapping between shadow regions and their references. Then, based on the shadow illumination model, we reformulate the shadow removal task as a variational optimization problem. To effectively solve the variational problem, we design an iterative algorithm and unfold it into a deep network, naturally increasing the interpretability of the deep model. Experiments show that our method could achieve SOTA performance with less than half parameters, one-fifth of floating-point of operations (FLOPs), and over seventeen times faster than SOTA method (DHAN).

Introduction

Shadows, which are caused by light being blocked by objects, widely exist in various natural scenes. Shadows present a substantial challenge for computer vision applications such as tracking (Sanin, Sanderson, and Lovell 2010; Guo et al. 2020) and object detection (Cucchiara et al. 2003). Consequently, removing shadows is a crucial preprocessing step in computer vision, and has attracted extensive research attention. Currently, deep CNNs-based methods (Qu et al. 2017; Wang, Li, and Yang 2018; Cun, Pun, and Shi 2020; Le and Samaras 2020; Liu et al. 2021c) dominate this field and achieve state-of-the-art performance. However, these approaches have some inherent problems:

Simplified assumption in the existing shadow illumination model. Recently, many researchers try to further explore the physical properties of the shadow phenomenon. In (Le and Samaras 2019, 2020), the illumination model of the

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Figure 1: Comparison among different models PSNR performance, number of models parameters and FLOPs. Shadow removal results are achieved on ISTD dataset.

transformation relationship between the shadow-free image $$I_{ns}$$ and shadow image $$I_s$$ is explicitly expressed as

\[
I_{lit} = \omega \ast I_b + b, \tag{1}
\]

\[
I_{ns} = \alpha \ast I_s + (1 - \alpha) \ast I_{lit}, \tag{2}
\]

where $$I_{lit}$$ is the relit result of transforming $$I_l$$ with six constants ($$\omega = [\omega_R, \omega_G, \omega_B], b = [b_R, b_G, b_B]$$) for different image channels. The matte $$\alpha$$ balances the shadow effects in penumbra areas. Given the above formula, they design networks to predict different parameters in two stages. In the first stage, this illumination model strongly assumes that all areas of the umbra are equally affected by shadows, which largely ignores the spatial-variant property of shadow images. We argue that directly predicting a uniform mapping function for each pixel in umbra regions.

Lacking sufficient interpretability of CNNs. Deep CNNs significantly promote the single image shadow removal performance in conjunction with the large-scale datasets of shadow/non-shadow pairs. However, many deep CNNs-based methods (Qu et al. 2017; Wang, Li, and Yang 2018; Cun, Pun, and Shi 2020) mainly focus on designing the network architectures, and then adopt an end-to-end training strategy to directly learn the mapping function between $$I_s$$ and $$I_{ns}$$ to remove shadows. This learning paradigm makes the deep network act as a black box and ignores the intrin-
sic prior knowledge of the shadow image itself, resulting in weak interpretability and limited performance.

**Insufficient use of shadow mask information.** Current high-quality datasets for the shadow removal task, e.g., Image Shadow Triplets Dataset (ISTD) (Wang, Li, and Yang 2018), have provided corresponding shadow masks. In order to utilize the shadow mask location information, many deep CNNs-based shadow removal methods (Qu et al. 2017; Wang, Li, and Yang 2018; Le and Samaras 2019, 2020; Liu et al. 2021c) directly concatenate the shadow image and mask as inputs, and then send them into the deep network. However, they may still mistake the dark albedo material areas like shadows, thus resulting in undesired artifacts in the results (as shown in Figure 5). Therefore, we argue that the mask information used to assist the network in locating shadow regions has not been fully exploited.

To alleviate the aforementioned issues, we propose an interpretable and effective deep network by combining the advantages of both model-driven methods and data-driven CNNs-based approaches. Specifically, we first propose a new shadow illumination model for this specific shadow removal task. Our proposed model integrates the shadow mask to design a constraint item, which could achieve the identity mapping among non-shadow regions pixels and adaptively fine-grained mapping between shadow region pixels and shadow-free counterparts within one stage. Secondly, based on the new illumination model, we reformulate the de-shadow task as a variational optimization model, composed of the favorable data fidelity term and the shadow-related prior term. The favorable data fidelity ensures that estimated results are consistent with the proposed shadow illumination model, while the other terms learn shadow-relevant priors for removing shadow. To effectively solve the variational model, we design an optimization algorithm with the gradient descent strategy (Ruder 2016) and unfold it into deep CNNs. In this way, the operation process of our network is highly consistent with the optimization algorithm, thus the interpretability of the network has been well improved. Additionally, we construct a basic Dynamic Mapping Residual Block (DMRB), which conforms to the new shadow illumination model, to further improve the performance. The contributions of our paper are as follows:

- We propose a new illumination model at a fine-grained level for shadow removal. We verify the previous shadow illumination model neglects the spatially-variant property of the shadow images through statistical analysis. Our model is more comprehensive than the previous model, making the subsequent solution process more accurate.
- We propose an efficient model-driven network for shadow removal. Our network is built on the designed iterative optimization algorithm of shadow variational model, which largely increases its interpretability.
- We further design a Dynamic Mapping Residual Block (DMRB) as the basic module in our network, which is our proposed shadow illumination model-inspired. Compared with standard form Residual Block, DMRB can enhance our model performance without increasing additional parameters.
- Extensive experiments indicate that our method achieves leading shadow removal performance in terms of quantitative metrics, inference efficiency, and visual quality.

**Related Work**

**Shadow Removal**

Shadow removal is one of the fundamental tasks in the computer vision field. Traditional shadow removal methods (Shor and Lischinski 2008; Finlayson, Hordley, and Drew 2002; Wu, Zhang, and Kumar 2012) rely on the image intrinsic priors (e.g., image gradients, illumination) and user interaction. For example, (Gong and Cosker 2014) designs the on-the-fly learning method by providing two rough user inputs for the pixels of the shadow and the lit area. Guo, Dai, and Hoiem (2012) combine pairwise relationships between shadow regions and non-shadow regions pixels to obtain illumination conditions.

Recently, there are many works to employ deep CNNs to remove shadows and achieve great breakthroughs. (Wang, Li, and Yang 2018; Qu et al. 2017) have proposed the public large-scale dataset (e.g., ISTD) to train the network in an end-to-end manner. Ding et al. introduces the recurrent adversarial network to predict shadow mask and shadow-free images. (Hu et al. 2019a) novelty leverages direction-aware spatial context to promote shadow detection and removal performance. (Le and Samaras 2019) constructs a shadow illumination model and designs networks to remove shadow effects. (Cun, Pun, and Shi 2020) synthesizes realistic shadow images through adversarial learning to boost network performance. (Fu et al. 2021a) transfers shadow removal task as multi-exposure images fusion problem. Moreover, (Hu et al. 2019b; Le and Samaras 2020; Liu et al. 2021c) attempt to train their network with unpaired data.

**Deep Unfolding Methods**

Deep unfolding methods usually integrate the advantages of model-driven methods (e.g., clearly interpretable) and the advantages of CNNs-based methods. (e.g., powerful learning capability) They typically contain these steps: (1) unfold the specific iteration optimization algorithms, e.g., iterative shrinkage-threshold algorithm (Beck and Teboulle 2009; Fu et al. 2019) and half-quadratic splitting algorithm (Afonso, Bioucas-Dias, and Figueiredo 2010; Zhang, Gool, and Timofte 2020); (2) parameterize the unrolled models; (3) update the learnable parameters via CNNs. Current many methods (Zhang et al. 2017; Dong et al. 2018; Mu et al. 2018; Li et al. 2019; Liu et al. 2019; Wang et al. 2020; Fu et al. 2021b; Liu et al. 2021a) in other computer vision tasks build different iterative optimization algorithms based on their physical models. Such as, dehazing method (Liu et al. 2019) unfolds the iterative algorithm, which solves the components of the well-known haze image physical formulation (Narasimhan and Nayar 2000; Fattal 2008), into CNNs.

However, such deep unfolding methods have not been explored on the shadow removal task. To our best knowledge, this is the first attempt to combine the model-driven optimization strategy with the CNNs-based method to achieve shadow removal.
Methodology

In this section, we illustrate our proposed shadow illumination model, the iterative solution to the minimization variational model, and the corresponding unfolding networks.

Shadow Illumination Model

Here we first revisit the previous illumination formula of modeling shadow removal process. Early traditional methods (Liu and Gleicher 2008; Wu et al. 2007; Arbel and Hel-Or 2010) have been exploring shadow removal based on the simplified physical image formulation, in which shadow images $I_s$ are modeled as the element-wise multiplication of a shadow matte $S_m$ and a shadow-free image $I_{ns}$:

$$I_s = S_m \ast I_{ns}.$$  
(3)

This Eqn.(3) is evolved from a universal image formation equation, proposed by (Barrow et al. 1978) as

$$I = R \ast L.$$  
(4)

where image $I$ is obtained by element-wisely multiplying the reflectance $R$ and luminance $L$. In (Qu et al. 2017), they further transform Eqn.(3) into log space as

$$\log(I_s) = \log(S_m) + \log(I_{ns}).$$  
(5)

They directly infer the mapping function between the shadow image $I_s$ and its matte $S_m$ via the deep CNNs. However, such simplified Eqn.(3) and (5) do not specifically consider the shadow image characteristics, such as the difference between shadow and non-shadow regions.

(Le and Samaras 2019) firstly proposes a shadow illumination model to separately process shadow/non-shadow regions and fuse them into the second stage as

$$I_{lit} = \omega \ast I_s + b,$$  
(6)

$$I_{ns} = \alpha \ast I_s + (1 - \alpha) \ast I_{lit}.$$  
(7)

Their modeling function assumes that the shadow effects are completely consistent in the umbra areas, thus they obtain the relit results through the linear transformation with only six constants $(\omega = [\omega_R, \omega_G, \omega_B], b = [b_R, b_G, b_B])$. We argue that applying the consistent transformation for all pixels of the umbra areas is unreasonable. We also prove our view through a simple but effective statistics analysis on the ISTD dataset, as shown in Figure 2. Following (Le and Samaras 2019, 2020), the shadow mask $M$ can be decomposed into $M_{umbra}$ and $M_{penumbra}$ (shadow boundary areas). We utilize pixel erosion operation to obtain $M_{umbra}$ in Figure 2. Secondly, we could obtain the pixel ratio map of umbra regions $R$ through

$$R = \frac{I_{ns}}{I_s} \ast M_{umbra}.$$  
(8)

Inspired by the aforementioned models, we propose a new shadow illumination model, which considers the spatially-variant property in umbra areas and exploits existing shadow mask information. The new model can be written as

$$I_{ns} = (1 + A) \ast I_s,$$  
(9)

where $M$ means the shadow mask that shadow regions are 1 and the rest are 0, and $A$ is the corresponding learned illumination transformation map. Note that we impose a constraint on $A$, which enforces value of the non-shadow pixel equals its value in the input image and guarantees identical mapping among non-shadow areas. This physically plausible transformation also conforms to the goal of the shadow removal task: recovering shadow regions’ information and maintaining the non-shadow regions’ information.

Variational Model and Optimization Algorithm

Generally, we formulate this problem as a maximum a-posteriori estimation problem, which is written as

$$I_{ns} = \operatorname*{arg max}_{I_{ns}} \log P(I_{ns} | I_s)$$
$$= \operatorname*{arg max}_{I_{ns}} \log P(I_{ns}) + \log P(I_{ns}),$$  
(10)

where $\log P(I_{ns} | I_s)$ indicates the data likelihood and $\log P(I_{ns})$ characterizes the intrinsic prior knowledge of $I_{ns}$. Based on our proposed shadow illumination Eqn. (9) and the Eqn. (10), shadow removal problem is transferred to solve the following minimization variational model:

$$\arg \min_{I_{ns}} D(I_s, I_{ns}, A) + \lambda \phi(I_{ns}, A),$$
$$\text{s.t.} \quad g(A) = 0,$$  
(11)

where $\lambda$ is a weight parameter and $D(\cdot)$ denotes the favorable data fidelity term, defined as

$$D(I_s, I_{ns}, A) = \frac{1}{2} \left\| I_s - \frac{I_{ns}}{1 + A} \right\|_2^2.$$  
(12)

For the convenience of writing, we employ $g(A)$ to present the constraint term: $\left\| (1 - M) \ast A \right\|_2$. We further convert it
into a non-constrained optimization problem as
\[
\arg\min_{\text{A}, \text{I}_{\text{ns}}} D(\text{I}_s, \text{I}_{\text{ns}}, \text{A}) + \beta g(\text{A}) + \lambda \varphi(\text{A}),
\]  
(13)
where \(\beta\) is the weight parameter. Since the data fidelity items \(D(\cdot)\) and \(g(\cdot)\) are both quadratic constraints, they are differentiable. Assume that \(\varphi(\cdot)\) is differentiable, the optimization objection (13) can be addressed through the gradient descent scheme (Ruder 2016). At every iteration stage, we need to alternately update \(\text{A}\) and \(\text{I}_{\text{ns}}\) as follows:
\[
\begin{aligned}
\text{A}^{n+1} &= \text{A}^n - \eta_\text{A} (\nabla_{\text{A}} D^n + \beta \nabla_{\text{A}} g(A^n) + \lambda \nabla_{\text{A}} \varphi(A^n)), \\
\text{I}_{\text{ns}}^{n+1} &= (1 + \text{A}^{n+1}) \ast I_s,
\end{aligned}
\]  
(14)
where \(\eta_\text{A}\) indicates the step size of \(\text{A}\); \(\nabla\) indicates the gradient operator; \(n\) indicates the current iteration number and the maximum iteration number is \(K\).

There are two unresolved problems for the above iteration process in Eqn. (14). One is computing the objective function gradients. The corresponding gradients of the quadratic items \(D(\cdot)\) and \(g(\cdot)\) are obviously easy to calculate. Followed (Liu et al. 2019), the counterpart of \(\varphi(\cdot)\) can be performed with a deep CNNs. \(\nabla_{\text{A}} \varphi(A^n)\) could be directly learned from our training dataset. The other problem is to obtain the initial values for the iteration algorithm. As the CNNs can generate reasonable results, we leverage CNNs technique to estimate the initial results \(\text{A}^0\) and \(\text{I}_{\text{ns}}^0\).

**Unfolding Network Design**

As shown in Figure 3, we build a deep CNNs model following the proposed iteration algorithm 1. Specifically, the network contains one initialization stage and \(K\) iterative stages, which is corresponding to the total number of iterations of our proposed optimization algorithm. Except for the initial stage, each subsequent stage consists of two sub-steps, respectively corresponding to Eqn. (14) to update the learned transformation map \(\text{A}\) and shadow-free \(\text{I}_{\text{ns}}\).

In Figure 3, one place to deploy CNNs is to obtain the

Figure 3: (a) Overview illustration of our proposed efficient model-driven network, which conforms to our designed iterative algorithm 1. (b) The basic structure of one of the iterative stages. Except for the first initialization stage, each subsequent stage consists of two sub-steps, respectively corresponding to Eqn. (14) to update the transformation map \(\text{A}\) and shadow-free \(\text{I}_{\text{ns}}\).
Algorithm 1: Iterative algorithm for shadow removal.

1: Initialization:
   hyperparameters $\eta_A = 0.01, \beta = 0.01, \lambda = 0.01$; $n \leftarrow 0$; maximum iterations $K = 4$; $A^0, I_{ns}^0 = \mathcal{N}_{\text{init}}(I_s, M)$;
2: repeat
3:   $n \leftarrow n + 1$
4:   $A^{n+1} = A^n - \eta_A(\nabla_A D^n + \beta \nabla_A g(A^n) + \lambda N_{A}^{n+1})$
5:   $I_{ns}^{n+1} = (1 + A^{n+1}) \ast I_s$
6:   Update $\mathcal{N}_{\text{init}}$ with Eqn.(16)
7: until $n + 1 = K$
Output: $I_{ns}^K$ and $A^K$

Figure 4: (a) Images illustration of our proposed shadow illumination model (Eqn. (9)); (b) Basic block in $\mathcal{N}_A$, derived from ResNet and Mobilenet-v2; (c) Basic block in $\mathcal{N}_{\text{init}}$, a model-informed module from (a).

Experiments

Implementation Details

We implement our network in the PyTorch framework on the PC with a single NVIDIA GeForce GTX 1080Ti GPU. In the training phase, we adopt the Adam optimizer (Kingma and Ba 2014) with a batch size of 2 and the patch size of 256 × 256. The initial learning rate is $5 \times 10^{-5}$ and changes with Cosine Annealing scheme (Loshchilov and Hutter 2016). The CNNs parameters are randomly initialized and the model converges well after 150 epochs. For the hyperparameters, the weights $(\eta_A, \beta, \lambda)$ in Equations (14) are initialized as 0.01 and these parameters can be automatically updated during the training phase in an end-to-end manner. The maximum iteration number $K$ is empirically set to 4 as a trade-off between speed and accuracy.

Dataset and Evaluation Metrics

**ISTD dataset** ISTD is proposed in (Wang, Li, and Yang 2018), which is the first public benchmark that could be used to train shadow detection and removal. ISTD totally consists of 1870 image triplets (shadow image, shadow-free image, shadow mask), including 135 various scenes with different shadow shapes. This dataset has been divided into 1330 triplets for training and 540 triplets for testing.

**SRD dataset** SRD is proposed in (Qu et al. 2017), containing 2680 and 408 pairs of images for training and testing respectively. SRD does not provide masks, we employ the public SRD shadow masks from (Cun, Pun, and Shi 2020).

For evaluation on ISTD and SRD datasets, we compute the root mean square error (RMSE) between the estimated result and ground truth image in the LAB color space. For fair comparisons, we directly adopt the MATLAB evaluation codes from (Fu et al. 2021a). Following previous methods (Liu et al. 2021c; Fu et al. 2021a; Le and Samaras 2020, 2019), we employ results with a resolution of 256 × 256 for evaluation in this paper. In the Table 1 and 2, we report the evaluation results about the shadow (S) regions, non-shadow (NS) regions, and all images (ALL). For the RMSE metric, the lower value means the better result. In addition, we also compute the PSNR and SSIM (Wang et al. 2004) for quantitative assessment in RGB color space.

**Shadow Removal Evaluation on ISTD Dataset**

We first compare our ISTD results with current state-of-the-art shadow removal methods, including 2 traditional methods (Guo, Dai, and Hoiem 2012; Gong and Cosker 2014) and 9 CNNs-based methods (Wang, Li, and Yang 2018; Hu et al. 2019a,b; Le and Samaras 2019, 2020; Cun, Pun, and Shi 2020; Liu et al. 2021b,c; Fu et al. 2021a). For fair comparison, all the comparison results or metrics values are provided by the original authors. Since the pre-trained model and shadow removal results of MaskShadow-GAN (Hu et al. 2019b) are not publicly available, we directly adopt the metrics values from (Fu et al. 2021a). Quantitative comparisons are shown in Table 1. We surpass the SOTA methods on the PSNR metric by a large margin, whether in the shadow region, non-shadow region, or the entire region. Additionally, we compare visual results in the Figure 5, in which our method achieves better visual performance than other methods. (Le and Samaras 2020; Fu et al. 2021a) may misprocess relatively dark non-shadow regions, bringing undesired artifacts in their estimated results. It turns out that their models fail to make full use of the shadow mask information, even though their network inputs contain shadow masks.

We further compare the different models efficiency and complexity on ISTD results, as shown in Figure 1 and Table 6. Our method achieves leading shadow removal performance in terms of quantitative metrics, inference efficiency, and model interpretability. We also evaluate our model on the AISTD dataset (Le and Samaras 2019). Our PSNR ↑ and RMSE ↓ are 33.36 dB and 3.42, while metric values of SOTA method (Fu et al. 2021a) are 29.44 dB and 4.23.

**Shadow Removal Evaluation on SRD Dataset**

We also compare the methods (Qu et al. 2017; Hu et al. 2019a; Cun, Pun, and Shi 2020; Fu et al. 2021a), which provide the pre-trained model or processed results on the SRD dataset. During the evaluation, we adopt the public SRD shadow masks provided by (Cun, Pun, and Shi 2020). The comparison results can be found in Table 2. It is obvious that our method achieves the best performance on all metrics.

**Ablation Study**

**Ablation Study of Iterative Stage Numbers** The effectiveness of the iteration numbers is tested by comparing the performance on the PSNR and RMSE metrics. Table 5 demonstrates the effect of the stage number $K$ on the shadow removal performance of our network. $K = 0$ represents...
Table 1: Quantitative comparisons of the SOTA methods on the ISTD datasets. The best and the second results are boldfaced and underlined, respectively. ‡ means results copied from Auto-Exposure (Fu et al. 2021a).

| Method                  | Shadow Region (S) | Non-Shadow Region (NS) | All image (ALL) |
|-------------------------|-------------------|------------------------|-----------------|
|                         | PSNR↑  SSIM↑  RMSE↓| PSNR↑  SSIM↑  RMSE↓   | PSNR↑  SSIM↑  RMSE↓|
| Original input          | 22.40  0.9361  32.10 | 27.32  0.9755  7.09   | 20.56  0.8934  10.88 |
| Guo et al. PAMI’12      | 27.76  0.9643  18.65 | 26.44  0.9664  7.76   | 23.08  0.9198  9.26   |
| Gong et al. BMVC’14     | 30.14  0.9727  13.54 | 26.98  0.9730  7.20   | 24.71  0.9260  8.03   |
| ST-CGAN CVPR’18         | 33.74  0.9808  9.99  | 29.51  0.9576  6.05   | 27.44  0.9291  6.65   |
| MaskShadow-GAN‡ ICCV’19 | -     -      12.67   | -     -      6.68    | -     -      7.41    |
| SP-M-Net ICCV’19        | 32.16  0.9812  10.30 | 31.26  0.9690  5.04   | 25.08  0.9429  7.79   |
| DSC T-PAMI’19           | 34.64  0.9835  8.72  | 26.40  0.9702  7.47   | 29.00  0.9438  5.59   |
| Gong et al. BMVC’14     | 30.88  0.9794  11.32 | 25.42  0.9639  8.01   | 23.89  0.9340  8.37   |
| DSC T-PAMI’19           | 34.64  0.9835  8.72  | 26.40  0.9702  7.47   | 29.00  0.9438  5.59   |
| Auto-Exposure CVPR’21   | 34.71  0.9752  7.91  | 28.61  0.8799  5.51   | 27.19  0.8456  5.88   |
| Ours                    | 36.95  0.9867  8.29  | 31.54  0.9779  4.55   | 29.85  0.9598  5.09   |

Table 2: Quantitative comparisons of the SOTA methods on the SRD datasets. The best and the second results are boldfaced and underlined, respectively. ‡ means results copied from Auto-Exposure (Fu et al. 2021a).

| Method                  | Shadow Region (S) | Non-Shadow Region (NS) | All image (ALL) |
|-------------------------|-------------------|------------------------|-----------------|
|                         | PSNR↑  SSIM↑  RMSE↓| PSNR↑  SSIM↑  RMSE↓   | PSNR↑  SSIM↑  RMSE↓|
| Original input          | 18.96  0.8710  36.69 | 31.47  0.9750  4.83   | 18.19  0.8295  14.05 |
| Guo et al.‡ PAMI’12     | -     -      29.89   | -      -      6.47    | -     -      12.60   |
| DeshadowNet‡ CVPR’17    | -     -      11.78   | -      -      4.84    | -     -      6.64    |
| DSC T-PAMI’19           | 30.65  0.9602  8.62  | 31.94  0.9650  4.41   | 27.76  0.9033  5.72   |
| DHAN AAAI’20            | 33.67  0.9777  7.16  | 34.79  0.9789  3.91   | 30.51  0.9492  4.87   |
| Auto-Exposure CVPR’21   | 34.26  0.9663  8.55  | 30.59  0.9445  5.74   | 27.74  0.8933  6.50   |
| Ours                    | 34.94  0.9797  7.44  | 35.85  0.9819  3.74   | 31.72  0.9523  4.79   |

Figure 5: Visual comparisons with SOTA methods on ISTD datset. (a) to (d) are the estimated results from SOTA methods: DSC (Hu et al. 2019a), Param+M+D-Net (Le and Samaras 2020), DHAN (Cun, Pun, and Shi 2020), and G2R (Liu et al. 2021c).
Therefore, we empirically set $K$ as the default setting throughout all experiments. We find that our network is not sensitive to the value of $\gamma_{n_{\text{reg}}}$, as long as the range of $\gamma_{n_{\text{reg}}}$ is between $1e^-5$ and $1e^-7$. Therefore, we empirically set $\gamma_{n_{\text{reg}}} = 1e^-6$ ($n = 0, 1, \cdots , K$) as the default setting throughout all experiments.

**Ablation Study of the Imperfect Mask** In Figure 7, we provide a visualization comparison with the imperfect shadow mask as input. Our method still could remove the shadows and recover the underlying contents.

**Ablation Study of Inference Time** In Table 6, we compare part SOTA methods’ average inference time on IST-D testing dataset. Our method’s inference speed is over 17 times faster than DHAN (Cun, Pun, and Shi 2020).

### Conclusion

In our paper, we verify the previous shadow illumination models exists drawbacks, *e.g.*, ignoring the spatially-variant property of shadow images. We further propose a new shadow illumination model, which considers spatially-variant property and shadow mask information. Utilizing the new shadow illumination model, we reformulate the shadow removal task as a variational optimization problem. To this end, we build an iterative optimization algorithm to address it. This iteration optimization algorithm is unrolled into deep CNNs, so that our method enjoys the advantages of both model-driven methods (*e.g.*, well interpretability) and data-driven CNNs-based methods (*e.g.*, powerful learning capability). Extensive experiments demonstrate that our method achieves leading shadow removal performance with fewer network parameters and faster inference speed.
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