Incorporating information from source simulations into searches for gravitational-wave bursts

Patrick R Brady and Saikat Ray-Majumder
Dept. of Physics, PO Box 413
University of Wisconsin-Milwaukee, WI 53201

Abstract. The detection of gravitational waves from astrophysical sources of gravitational waves is a realistic goal for the current generation of interferometric gravitational-wave detectors. Short duration bursts of gravitational waves from core-collapse supernovae or mergers of binary black holes may bring a wealth of astronomical and astrophysical information. The weakness of the waves and the rarity of the events urges the development of optimal methods to detect the waves. The waves from these sources are not generally known well enough to use matched filtering however; this drives the need to develop new ways to exploit source simulation information in both detections and information extraction. We present an algorithmic approach to using catalogs of gravitational-wave signals developed through numerical simulation, or otherwise, to enhance our ability to detect these waves. As more detailed simulations become available, it is straightforward to incorporate the new information into the search method. This approach may also be useful when trying to extract information from a gravitational-wave observation by allowing direct comparison between the observation and simulations.

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1. Introduction

The ability to extract weak signals from noise is generally enhanced by having advanced knowledge of the expected signals [1]. For low-mass compact binary inspiral, the expected waveform can be accurately computed in the band of earth-based detectors, and matched filtering is used to search the detector output for the signals [2, 3, 4, 5]. These searches for gravitational-wave chirp signals are the archetypical example of using advanced knowledge of a waveform [2] in gravitational-wave astronomy. Once the signals are detected, the extraction of information about the astrophysical sources responsible for their generation will be of great interest. For waves from the inspiral of low-mass compact binary systems, the problem of information extraction is reasonably straightforward. A well developed formalism, based on matched filtering, exists to characterize the accuracy of parameter estimation for sufficiently strong signals [6, 7].

In general, the strongest sources of gravitational waves are composed of high density material moving at relativistic speeds in extreme gravitational fields and detailed computations of the expected waveforms are not always possible. Among the most plausible examples of such sources are supernova explosions [8] or binary black hole mergers [9]. Efforts to model these systems, and to determine the gravitational waveforms emitted by them, are under
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way [10, 11, 12, 13, 14, 15]. As the simulations become more sophisticated, the gravitational waveforms become better approximations to the waves that will arrive from real astrophysical systems. A challenge for gravitational-wave astronomy is to use the partial information from simulations to enhance signal detection and information extraction. If robust features of the waves can be identified using the simulations and those features can be associated with definite physical processes, a great deal of information might be obtained about the structure of dense matter or of spacetime itself.

Searches for gravitational-wave bursts have been made by a number of groups [16, 17, 18, 19]. These searches generally take an unbiased approach by using time-frequency methods to identify excess power in the data stream. Plausible physical waveforms are only used to determine the efficiency of the search and report an upper limit.

In this note, we outline a method to use information from numerical simulations in searches for gravitational-wave bursts. This method is based on the excess-power statistic [9, 20] and provides an algorithmic approach to incorporating the source-modeling information into searches. Moreover, it allows for the incorporation of new information as it becomes available. The theoretical formulation of the method is presented in Sec. 2 with an example of the construction in Sec. 3. Finally, other possible applications are mentioned in Sec. 4.

2. Theory

The signal $s(t)$ from a gravitational wave detector is sampled to produce a time series $s_j = s(j\Delta t)$, where $j = 0, 1, 2, \ldots, H-1$ and $\Delta t$ is the time between samples. This signal can be written as

$$s_j = n_j + h_j$$

where $n_j$ is the detector noise and $h_j$ is the (possibly absent) gravitational-wave signal. For the purposes of our presentation, we assume that the detector noise is Gaussian with joint probability distribution

$$\frac{1}{\sqrt{(2\pi)^H \det ||R||}} \exp \left\{ -\frac{1}{2} \sum_{j,k} n_j Q_{jk} n_k \right\}$$

(2)

where $R_{jk} = \langle n_j n_k \rangle$ and $\sum_k R_{jk} Q_{km} = \delta_{jm}$. This assumption is not critical to the approach, but makes for easier presentation and motivates the inner product used below. For stationary noise, the kernel $Q_{jk}$ is determined by the power-spectral density.

2.1. Search Method

Our approach is based on the excess-power search method described in Refs. [9, 20] although extensions of the present approach which might be more powerful will be described elsewhere [21]. The gravitational wave signals form a subspace $W$ of the vector space $V$ spanned by all $H$-dimensional vectors given by Eq. (1). If the subspace $W$ is also a vector space, then the optimal statistic for the detection of these gravitational waves in Gaussian noise was derived by Anderson et al. [20]. One first decomposes the signal into $s_\parallel$ which lies in $W$ and $s_\perp$ which is orthogonal to $W$, so that

$$s = s_\perp + s_\parallel$$

(3)

Then, the optimal statistic by which to identify gravitational-wave signals which are in the vector space $W$ is

$$E = (s_\parallel, s_\parallel)$$

(4)
where the inner product is defined by

\[ \langle a, b \rangle = \sum_{i,j=0}^{N-1} a_i Q_{ij} b_j . \] (5)

The key to this approach is the determination of the signal subspace \( \mathcal{W} \) and the associated operator which projects vectors from \( \mathcal{V} \) into \( \mathcal{W} \). This was already pointed out by Anderson et al. [20] where an example based on a Fourier basis was described in detail.

2.2. Constructing the signal subspace

Suppose numerical simulations yield a set of \( M \) gravitational-wave signals \( \{ h_I^j \} \) where \( I = 1, 2, \ldots, M \). Each waveform arises from a simulation with some parameters \( \mathcal{P}_I \). If the simulations are costly and only some representative sample can be completed, it is desirable to search for all signals with similar characteristics to the sample. A restricted search method might use the known waveforms as matched filters to search for waveforms which match the simulations well. This approach is extremely limited when there are only a few simulations or the simulations are known to be only qualitatively correct. The alternative approach, which we advocate here, is to use the excess power method by constructing a vector space \( \mathcal{W} \) of signals which capture the essential features of the waveforms given by the simulations. This approach also has its limitations since the space of signals generated by some class of astrophysical sources is not, in general, expected to be a vector space. Nevertheless, the projection of the detector output signal into a vector space of sufficiently small dimensions can still be a powerful method to distinguish signal from noise when combined with coincidence between detectors.

The simplest approach to determine a vector subspace would use the Gram-Schmidt method to determine an orthonormal set of vectors from the waveforms \( \{ h_I^j \} \) and hence identify \( \mathcal{W} \) as the vector space spanned by all the simulation waveforms. The orthonormal basis of this space is therefore

\[ e_J^j = \frac{\bar{h}_J^j}{\sqrt{\langle \bar{h}_J^j, \bar{h}_J^j \rangle}} \] (6)

where

\[ \bar{h}_J^j = h_J^j - \sum_{J=1}^{I-1} \langle h_J^j, e_J^j \rangle e_J^j . \] (7)

Now, any of the original simulation waveforms can be re-constructed as a sum over the basis

\[ h_J^j = \sum_J \langle h_J^j, e_J^j \rangle e_J^j . \]

In general, however, we are interested not in the exact reconstruction of the original waveform, but the identification of robust features of the waveforms. This is because the simulations may not be completely accurate representations of the physical sources. For this reason, we would like to use a smaller subspace which captures the essential features of the simulated waveforms well enough. Our approach is similar to that used when determining the grid-spacing in a matched filtering search and is most easily explained by describing the algorithmic selection of basis vectors.

First, normalize all the simulated waveforms using the inner product 5; denote the normalized waveforms by \( \hat{h}_J^j \). Suppose then, that one has used \( K < M \) of the simulated waveforms to construct an orthonormal set \( \{ e_J^j \} \) where \( J = 1, \ldots, K \); this is a basis for a vector space \( \mathcal{W}_K \subset \mathcal{W} \). Compute the norm

\[ \mu_J^L = \| \sum_{J=1}^{K} \langle \hat{h}_J^L, e_J^J \rangle e_J^J \| \] (8)
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for all waveforms which have not been used to construct the basis of $W_K$. The quantity $\mu^L$ represents the match of the $L$th remaining waveform with the nearest vector in the space $W_K$. Select the signal with $\mu = \min_L \mu^L$. The number of basis vectors is incremented by one if $\mu < \mu_{\text{min}}$ where $\mu_{\text{min}}$ is some minimal match that will be tolerated in the search. This procedure is repeated until a set of $M' \leq M$ orthonormal vectors has been identified and $\mu^L > \mu_{\text{min}}$ for all remaining waveforms which have not been used in the construction. This approach will tend to select out a subspace of signals $W$ which capture the essential features of the original set of simulated waveforms although the basis vectors may be unphysical.

For typical source modeling simulations, one expects that $M' < M$ since the waveforms produced for different parameter values often share similar qualitative behavior. As described above, the signal used to start the Gram-Schmidt construction is chosen arbitrarily and might not result in the smallest vector space of signals. Thus, the construction can be carried out repeatedly selecting a different seed signal each time. The seed signal which leads to the smallest $M'$ is deemed the best choice for the algorithm since the vector space of smallest dimension has been identified.

It should be noted that waveforms sometimes need to be shifted in time relative to each other in order to obtain the best match. In each step of the construction then, it may be necessary to shift each of the remaining waveforms in order to obtain the maximum match with the subspace $W_K$. This is done in the examples given below.

3. Supernova searches as an example of the method

The simulation of supernova explosions is extremely complicated because it relies on detailed knowledge of physics at very high densities in addition to strong field gravitational effects. To date, only simple models of gravitational-wave production in core collapse and supernova explosions have been considered [10, 11, 12]. These efforts provide guidance to the gravitational wave astronomy community in the form of waveform catalogs which represent a variety of observed behavior depending on the parameters used in a particular simulation.

3.1. Example of the Zwerger-Müller catalog

The catalog of waveforms produced by Zwerger and Müller [10] have become a reference point to calibrate searches for gravitational wave bursts [18]. The catalog provides the mass quadrupole wave amplitudes $A^I(t)$ from axi-symmetric simulations of core-collapse. The strain at the detector is given in terms of these quadrupolar amplitudes by

$$h^I(t) = \frac{1}{8} \sqrt{\frac{15}{\pi}} \sin^2 \theta A^I(t)$$

(9)

where $R$ is the distance between the detector and the source and $\theta$ is the azimuthal angle. The catalog consists of 78 different waveforms arising from representative simulations across the parameter space of the models.

Following the method outlined in the previous section, we have examined the number of basis vectors required to cover all 78 Zwerger-Müller (ZM) waveforms at various levels of minimal match $\mu_{\text{min}}$. The results are summarized in Fig. 1 for a flat noise spectrum and the LIGO-I design spectrum [22]. At fixed $\mu_{\text{min}}$, more basis vectors are generally needed for the white noise case. This is expected since high and low frequency components of the waveforms are suppressed in the inner-product by the LIGO-I noise spectrum. There is some variation in the number of basis vectors depending on the seed waveform used in the construction. It is interesting to see that only 13 waveforms are needed to cover the space of waveforms at
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Figure 1. The number of basis vectors as a function of minimal match $\mu_{\text{min}}$ for the ZM family of waveforms. The dashed line shows the results for the inner product appropriate to white noise. The triangles indicate the range of values for the inner product appropriate to the LIGO-I design using different seed waveforms.

$\mu_{\text{min}} = 0.9$. This confirms what can be seen by visually inspecting the ZM waveforms, that is, these waveforms have dominant features which are robust across many examples in the catalog.

Figure 2 shows waveform A3B3G2 from the ZM catalog plotted with the reconstructed waveform using different numbers $K$ of basis vectors. The match, as defined in Eq. 8, between the waveform and the vector space $W_K$ is also shown. As indicated in Fig. 1, the match is already greater than 0.90 for only $K = 22$ basis vectors. For completeness, the exactly reconstructed waveform is also shown when waveform A3B3G2 has been absorbed into the vector space $W_K$.

3.2. Incorporating newer waveforms in the analysis

A nice feature of this approach to using source simulation information in searches for gravitational waves is the ease with which it can be updated using new information. Moreover, the method provides an algorithmic way to identify features of the simulated waveforms which are robust from simulation to simulation.

As a concrete example of this, we consider the set of waveforms produced by Ott et al. (OBLW) [12]. As with the ZM simulations, these waveforms also represent possible wave emission from core collapse supernova explosions. The simulations explore alternate simulation methods in the numerical implementation and in accounting for the complicated physics of supernovae. When seeking guidance for the detection of gravitational waves, it is
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Figure 2. Waveform A3B3G2 from the ZM catalog (light-gray line) shown along with reconstructed waveforms for a different numbers $K$ of basis vectors. The value of the match between waveform A3B3G2 and the corresponding vector space $W_K$ is also indicated. The fourth plot on the bottom right shows the exactly reconstructed waveform once the A3B3G2 is absorbed into the vector space $W_K$.

interesting to understand how different the resulting waveform family is from the ZM family. This is important for two reasons: first, it tests the robustness of any search method that is tuned to search for waveforms in the ZM family, and second, it tests how much the waveform depends on changes in the underlying physics of the source.

We determined the number of basis vectors needed to cover the 149 waveforms from both Zwerger-Müller and Ott et al. in the same way as we determined the basis for only the ZM waveforms. At $\mu_{\text{min}} = 0.9$, one needs 33 basis vectors. In this example, we used the seed waveform that minimized the number of basis vectors for the ZM family alone. The resulting basis was constructed out of 12 waveforms from the ZM family and 21 from the OBLW family [12]; only 6 of the 12 ZM waveforms were used in the ZM-only basis set computed at this value of $\mu_{\text{min}}$. For the purpose of this note, the exact numbers are not as important as the algorithmic approach which has been taken.

4. Summary and future directions

Searches for gravitational waves using broad-band detectors and the extraction of physical insight from future detections will benefit greatly from close interaction with scientists who model sources of gravitational waves. In the case where complete and accurate information is available, it is well known that matched filtering provides the best approach to both detection
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and information extraction. For many physically complicated sources, however, there is a
dearth of information available and even the best simulations are not accurate enough to
support matched filtering as the main approach to detect the associated gravitational waves.
In this note, we have sketched an algorithmic approach to determining a vector space which
includes waveforms generated by the limited simulations and incorporates waveforms of
similar character. The benefit of this approach is that it is algorithmic and is easily updated
using newer information provided by simulations.

The supernova simulations of Zwerger-Müller [10] and Ott et al. [12] have been used
as an example to demonstrate the method. In a future publication, we will present more
detailed investigations of these and other supernova simulations along with estimates of the
performance of the approach in the presence of noise. This is particularly important since the
spiky nature of the basis vectors could increase the false alarm probability beyond a tolerable
level in real detector noise.

One of the more interesting avenues to explore with this approach is, however, the
detection of the late stages of binary black hole inspiral and merger [23]. Current efforts
in this direction have focused on identifying a detection family of waveforms for use in the
search [24]. Given the large number of templates required to span known waveform models,
it seems worthwhile to explore the method outlined here as an alternative.

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