The String Worldsheet as the Holographic Dual of SYK State

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Recent studies of the SYK (Sachdev-Ye-Kitaev) model [1] and its gravitational dual have excited the research enthusiasm in both sides of the condensed matter [2] and the holographic dualities [3]. The SYK model is a strongly interacting quantum system that is solvable at large N, which leads to new insights in quantum gravity and conformal field theories. One conjecture on the duality relies on the similarity between SYK model and the AP (Almheiri-Polchinski) model with the near AdS\(^2\) geometry in dilation gravity [4], as they share the same symmetry and saturate the universal chaos bound.

The studies of chaos bound and butterfly effects were initiated in [6] and have been generalized to various gravitational setups [7]. In the holographic theories with Einstein gravity duals one finds that the Lyapunov exponent \(\lambda_L\) has a universal bound [8],

\[
\lambda_L \leq \lambda_\beta \equiv \frac{2\pi}{\beta}. \tag{1}
\]

The other known model which saturates the same bound is the SYK model [1]. Thus, it is expected that the two dimensional dilation Einstein gravity which contains a near AdS\(^2\) sector could be a dual theory of the SYK model. However, the exact gravitational dual formula of SYK model is still unclear [8, 9].

Two recent papers shade some new lights on the dual models of the SYK states [10, 11], with an string worldsheet embedded in an higher dimensional AdS spacetime.

I. INTRODUCTION

Recently these models are used to study the interaction between the heavy quark (or Brownian particle) and the strongly coupled thermal plasma. In [10, 11], the authors firstly evaluate the Lyapunov exponent from correlation function, and find that \(\lambda_L = 2\pi/\beta\), which saturates the universal chaos bound. The other models saturating this bound are the black holes and SYK states. From these observations, we then conjecture that the action of the string worldsheet in an AdS background could be a candidate dual description of the SYK model.

In this paper, we will show that the 0 + 1 dimensional SYK state can be naturally thought as the “SYK quasi-particle”. The dual description could be an open string connecting the black brane horizon and the AdS boundary. We will prove that the fluctuation of the string embedded in charged BTZ black hole is dual to a one dimensional system which has an asymptotic scaling symmetry. This leads that its IR fixed point is governed by the same quadratic Schwarzian action, which is the IR effective theory of SYK model. Considering the open string worldsheet also has natural reparametrization symmetry, we conjecture that the action of the string worldsheet is a dual description of SYK state.

II. THE SYMMETRIES

To argue two different theories could be dual to each other, one starting point is to confirm that they share the same symmetry. In the IR limit, SYK model shows emergent conformal symmetry that is explicitly and spontaneously broken into SL(2,R). The residual exact symmetry is obvious when we express its effective action as Schwarzian action in Euclidean signature [3, 5],

\[
S_{Sch} := -\frac{1}{g_s^2} \int_0^\beta d\tau \{ f(\tau), \tau \}, \quad \frac{1}{g_s^2} = \frac{\alpha_S N}{\mathcal{F}}, \tag{2}
\]
Now making the small fluctuation means reparametrization of the worldsheet coordinates which $\epsilon$ expanding in the reparametrization $f$ case when $\tilde{\sigma}$ two-copy counterpart of conformal symmetry in SYK reparametrization can be apparently considered as $g$ embedding coordinates into the target spacetime with $\sigma, \tau$ under $(\sigma, \tau)$ where $ab, \beta$ are the embedding coordinates into the target spacetime with a,b, $\sigma, \tau$ under $(\sigma, \tau)$ where $ab, \beta$ are the embedding coordinates into the target spacetime with a,b.

The dynamics of an open string follows from the Nambu-Goto action is invariant under SL(2,R) transformation or Schwarzian derivative is invariant under SL(2,R) transformation of SYK model also exhibits the same pattern in explicit and spontaneous symmetry breaking [4].

Now let us consider an alternative dual description of SYK model, the open string with a worldsheet horizon. The dynamics of an open string follows from the Nambu-Goto action of the worldsheet

$$ S_{NG} = -\frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det h_{ab}}, $$

where $h_{ab} = g_{\mu\nu} \partial_\sigma X^\mu \partial_\tau X^\nu$ is the induced metric on the worldsheet with $a, b = \sigma, \tau$, and $X^\nu(\tau, \sigma)$ are the embedding coordinates into the target spacetime with metric $g_{\mu\nu}$. The Nambu-Goto action is invariant under reparametrization of the worldsheet coordinates which means

$$ S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tilde{\sigma} d\tilde{\tau} \sqrt{-\det h_{ab}}, $$

under $(\sigma, \tau) \rightarrow (\tilde{\sigma}(\sigma, \tau), \tilde{\tau}(\sigma, \tau))$. This kind of reparametrization can be apparently considered as two-copy counterpart of conformal symmetry in SYK model and includes the SL(2,R) symmetry as the special case when $\tilde{\sigma} = a\sigma + b\tau, \tilde{\tau} = c\sigma + d\tau$.

**Small Reparametrization.** — Let us first consider the small reparametrization of SYK model. If we make the reparametrization $f(\tau) = \tan \frac{\pi \tau}{\beta} \rightarrow \tan \frac{\pi \tilde{\tau}(\tau)}{\beta}$, then the Schwarzian action (2) becomes

$$ S_{Sch} = \frac{1}{2g_s} \int_0^\beta d\tau \left[ \left( \frac{\tilde{f}}{f} \right)^2 - \left( \frac{2\pi}{\beta} \right)^2 \left( \frac{\dot{f}}{f} \right)^2 \right]. $$

Now making the small fluctuation $g(\tau) = \tau + \epsilon(\tau)$ and expanding in $\epsilon(\tau)$, we get a quadratic action,

$$ S_{Sch}^{(2)} = \frac{1}{2g_s} \int_0^\beta d\tilde{\tau} \left[ \left( \frac{\ddot{\tau}}{\tau} \right)^2 - \left( \frac{2\pi}{\beta} \right)^2 \left( \frac{\dot{\tau}}{\tau} \right)^2 \right]. $$

As we have fixed the fluctuation on the fixed parametrization $g(\tau) = \tau$, the quadratic action in terms of fluctuation $\epsilon(\tau)$ loses the SL(2,R) symmetry. However, this quadratic action has an new scaling symmetry. To see this, let us first make a rescaling on time $\tau = \tilde{\tau} \mu$, then Eq. (7) reads,

$$ \frac{1}{2g_s^2 \mu^3} \int_0^{\beta/\mu} d\tilde{\tau} \left[ \left( \frac{\ddot{\tau}}{\tau} \right)^2 - \left( \frac{2\pi}{\beta} \right)^2 \mu^2 \left( \frac{\dot{\tau}}{\tau} \right)^2 \right], $$

which is different from its original action in Eq. (7) and can lead a different equation of motion. If we apply the following combinations under the rescaling,

$$ \tilde{\tau} = \frac{\tau}{\mu}, \quad \tilde{\beta}(\mu) = \beta/\mu, \quad \tilde{\epsilon}(\tilde{\tau}) = \epsilon(\tau) \mu^{-3/2}, $$

with the new time $\tilde{\tau}$ and the new variable $\tilde{\epsilon}(\tilde{\tau})$, the action action (7) becomes

$$ \frac{1}{2g_s^2} \int_0^{\beta} d\tau \left[ \left( \frac{\ddot{\tau}}{\tau} \right)^2 - \left( \frac{2\pi}{\beta} \right)^2 \left( \frac{\dot{\tau}}{\tau} \right)^2 \right]. $$

We see that it is just as the same as the action (7). The transformation (9) is only the symmetry of quadratic Schwarzian. This is a new symmetry and is not contained in the SL(2,R) symmetry. The scaling transformation (9) shows that the conformal dimension of $\epsilon(\tau)$ is $3/2$. Because of this scaling symmetry, the quadratic Schwarzian actions of different temperatures are equivalent to each other.

If it is true as what we proposed, that the open string worldsheet action is a candidate dual description of SYK model, then its fluctuation can also give the symmetry of Eq (9) and the dual boundary theory should be equivalent to Eq. (7). In the following, we will show that the fluctuation of an open string in AdS black brane is dual to a one dimensional system which has an asymptotic scaling symmetry just like the transformation (9). This symmetry leads to an IR theory, which is just the quadratic Schwarzian action of SYK shown in Eq. (7).

**III. ACTION OF THE WORLDSHEET**

We begin with the black brane solution in 2+1 dimensional Maxwell-Einstein gravity with a negative cosmological constant. The generalization to higher dimensions is straightforward. The metric of the charged BTZ black brane is given by

$$ ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 dx^2, $$

where

$$ f(r) = 1 - \frac{r_h^2}{r^2} \left[ 1 + q^2 \ln \left( \frac{r}{r_h} \right) \right]. $$

The horizon is located at $r = r_h$, and the Hawking temperature of the black brane is $T = \frac{1}{\beta} = \frac{(2-q^2)r_h}{4\pi}$, which is identified as the temperature of the dual states on the AdS boundary $r \rightarrow \infty$. Charge parameter $q^2 < 2$ so that the dual temperature is large than zero. In the context of
AdS/CFT correspondence [12], the black brane is dual to a thermal bath on the boundary, and an open string connecting the horizon and boundary could be interpreted as a dual particle state, such as the heavy quark [13] or Brownian particle [14]. In the following we will show that the properties of the dual “particle” behave as the well studied SYK state.

**Worldsheet Metric.** — For an open string that hangs from the AdS boundary to the horizon of the black brane, we choose the static gauge \((r, \sigma) = (t, r)\) and parametrize the embedding of the string as \(X^\mu = \{t, r, x(t, r)\}\). Then the position of the dual particle is given by \(\epsilon(t) = x(t, r_c)\), where \(r_c \rightarrow \infty\) is an UV cut-off. For the static particle in average \((\epsilon(t)) = 0\), and the solution of the corresponding static string is \(x(t, r) = 0\). The induced metric on the string worldsheet embedded in the black brane (11) is an AdS\(_2\) black hole

\[
d s_{\text{ws}}^2 = g_{ab} d\sigma^a d\sigma^b = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)},
\]

with the same \(f(r)\) in Eq. (12). We will consider the perturbations of the static string in the bulk, which also induce the perturbations on top of this worldsheet metric [11].

**String Fluctuations** — Now let us consider the string fluctuations in the Nambu-Goto action (4). For simplicity, we consider the perturbation along one transverse direction, and fix one parameterization through setting \(\sigma = r\). The fluctuation with such fixed background breaks the original reparametrization as well as the SL(2,R) symmetry. Up to the leading quadratic order of the perturbations \(x(t, r)\), it is given by

\[
S_{\text{NG}} \simeq -\frac{1}{2\pi \alpha'} \int dr dt \left[ 1 - \frac{1}{2f(r)} (\dot{x})^2 + \frac{r^4 f(r)}{2} (\dot{x}^2)^2 \right],
\]

(14)

where \(\dot{x} \equiv \partial x(t, r) / \partial t\), and \(x' \equiv \partial x(t, r) / \partial r\). This action is divergent because of the constant term in the action (14) and the UV asymptotic behavior of \(x(t, r)\). Both these two divergences can be canceled by following counterterm,

\[
S_{\text{ct}} := \frac{1}{2\pi \alpha'} \int_{r = r_c} \sqrt{-\gamma} dt .
\]

(15)

\(\gamma\) is the induced one dimensional metric at the cut-off boundary of the worldsheet \(r = r_c\). We will define the renormalized on-shell action of the worldsheet as \(S_{\text{ren}} = S_{\text{NG}} + S_{\text{ct}}\). Let us make an periodic boundary condition in time \(x(t, r) \sim x(t + \Delta_0, r)\), we extract the following quadratic order of the Nambu-Goto action of the worldsheet

\[
S_{\text{NG}}^{(2)} = -\frac{1}{4\pi \alpha'} \int_{r_h}^{r_c} dr \int_{-\Delta_0/2}^{\Delta_0/2} dt \left[ r^4 f(r)(\dot{x})^2 - \frac{1}{f(r)} (\dot{x})^2 \right] .
\]

(16)

As we work in the Lorentz signature, this period has nothing to do with the inverse temperature \(1/T = \beta\). The value of \(\Delta_0\) will be determined later to match the quadratic order of the Schwarzian action (7) in Euclidean signature. Let us make a Fourier’s transformation \(x(t, r) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} b_n(r) e^{i\lambda_n t}\) with \(\lambda_n := 2\pi n / \Delta_0\), then the renormalized quadric order action of the string worldsheet is

\[
S_{\text{ren}}^{(2)} := S_{\text{NG}}^{(2)} + S^{(2)}_{\text{ct}} = \sum_{n=-\infty}^{\infty} s_n ,
\]

(17)

with the \(n\)-th induced action,

\[
s_n = \frac{1}{4\pi \alpha'} \left\{ \int_{r_h}^{r_c} dr \left[ \frac{1}{4} b_n b_n' - r^4 f b_n b_n' \right] \right. \nonumber \]

- \(\lambda_n^2 b_n b_n' \right\} .
\]

(18)

We see that \(s_{-n} = s_n\). Since \(x(t, r)\) is real valued, we see that \(b_{-n} = b_n^*\). Thus the induced action (18) has a global \(U(1)\) symmetry with the following conserved current,

\[
J_n (r) = i r^4 f (b_n b_n' - b_n b_n').
\]

(19)

In order to see what fluctuation of bulk open string \(x(t, r)\) corresponds in the boundary, let us write down the equation of motion for \(x(t, r)\) in terms of \(b_n(r)\), which reads,

\[
b_n'' + (r^4 f)' b_n' = \frac{\lambda_n^2 b_n}{r^4 f^2} = 0 .
\]

(20)

At the horizon, we impose the in-falling boundary condition,

\[
b_n(r_h) = \chi_n e^{i\lambda_n r}, \quad r_* := \int_{r_h}^{r_\infty} \frac{dr}{r^2 f} .
\]

(21)

\(\chi_n\) is a finite constant determined by \(x(t, r)\) at the horizon. Putting the equation of motion (20) into the action (18) yields the formula,

\[
(4\pi \alpha') s_n = - \left. b_{-n} r^4 f b_n' \right|_{r_h} - r_c \lambda_n^2 b_n b_{-n} |_{r_c} ,
\]

(22)

with the UV cut-off \(r_c \gg r_h\).

On the other hand, from equation of motion (20), the function \(b_n(r)\) has the following asymptotic solution at the boundary,

\[
b_n(r) = \epsilon_n [ (1 + \frac{\lambda_n^2}{2r^2}) \cdots - \frac{\psi(\lambda_n)}{3r^3} (1 + \cdots) ] .
\]

(23)

And at the horizon, the ingoing condition implies

\[
\left. b_{-n} r^4 f b_n' \right|_{r_h} = i r_h^2 \lambda_n |\chi_n|^2 .
\]

(24)

In (23), the constant \(\epsilon_n\) is determined by boundary value...
of \( x(t, r) \) in the following way,
\[
\lim_{r_c \to \infty} x(t, r_c) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \varepsilon_n e^{i\lambda_n t}.
\]  
(25)
Since the dual boundary describes a heavy quark oscillating in the thermal system, so the high frequency modes will be suppressed by \( e^{-\lambda_n/T} \) for large \( n \) and lower temperature \( T \). This means \( \varepsilon_n \) will be suppressed exponentially for large \( n \).

Notice that the renormalized on-shell action (22) reads,
\[
(4\pi\alpha')s_n = ir^2\lambda_n|\chi_n|^2 - b_{-n}^4 f'b_n'|_{r_c} - r_c\lambda_n^2 b_{n-1}^4 r_c^4.
\]  
(26)
The conserved current \( J_n \) defined in Eq. (19) implies that \( \text{Im}[b_{-n}^4 f'b_n'] \) is a constant so we see that \( \text{Im}[b_{-n}^4 f'b_n']_{r=r_c} = r_c^2\lambda_n|\chi_n|^2. \) Thus, the action (26) becomes
\[
(4\pi\alpha')s_n = -\varepsilon_n B(\lambda_n)\varepsilon_n.
\]  
(27)
Here \( B(\lambda_n) = \text{Re}[\psi(\lambda_n)] \), and \( B(\lambda_n) \) has the following expansion in terms of \( \lambda_n \),
\[
B(\lambda_n) = c_0 + c_2\lambda_n^2 + c_4\lambda_n^4 + \cdots.
\]  
(28)
The coefficients \( c_0, c_2, c_4, \cdots \) are independent of \( \lambda_n \). One can check that for any given charge \( q \), we always have \( c_0 = 0 \). Thus, putting (27) and (28) into (17), we can find that the total renormalized on-shell action of the worldsheet finally reads,
\[
S^{(2)}_{\text{ren}} = \frac{1}{4\pi\alpha'} \sum_{n=-\infty}^{\infty} \varepsilon_n \sum_{k=1}^{\infty} c_{2k}\lambda_n^{2k}\varepsilon_n.
\]  
(29)
The exponential decline of \( \varepsilon_n \) makes the summation to be well-defined. Using the inverse transformation of Fourier’s series, which change from \( \varepsilon_n \) in phase space to \( \varepsilon(t) \) in the real space, Eq. (29) then becomes
\[
S^{(2)}_{\text{ren}} = \frac{1}{2g_s^2} \int_0^{\Lambda_0} dt \left[ -M_0(\dot{\varepsilon})^2 + (\ddot{\varepsilon})^2 + \sum_{k=3}^{\infty} \ddot{c}_{2k} \left( \frac{d^k\varepsilon}{dt^k} \right)^2 \right].
\]  
(30)
Here we have defined \( 1/(2\pi\alpha'c_4) = 1/g_s^2 \) in order to compare with the Schwarzian action in Eq. (2), and \( M_0 = -c_2/c_4, \ddot{c}_{2k} = c_{2k}/c_4 \). We have shifted the time by \( t \to t+\Delta_0/2 \). From the “bulk-boundary” correspondence in holography [14], \( \varepsilon(t) \) is the boundary operator dual to the bulk field \( x(t, r) \). The renormalized action (30) governs the dynamics of dual operator. Coefficients \( c_4 \) and \( -c_2 \) are both assumed to be positive here. Later on, we will give numerical evidence that they are indeed positive when \( q \neq 0 \).

So far, the period \( \Delta_0 \) for time \( t \) has been assumed to be arbitrary. Now let us make a rescaling such that \( \ddot{t} = t/\mu, \dddot{\varepsilon}(\ddot{t}) = \varepsilon(t)\mu^{-3}. \) Then the action (30) reads,
\[
S^{(2)}_{\text{ren}} = \frac{1}{2g_s^2} \int_0^{\Delta_0/\mu} dt \left[ -M_0\mu^2(\dot{\varepsilon})^2 + (\ddot{\varepsilon})^2 + \sum_{k=3}^{\infty} \ddot{c}_{2k}\mu^{3-2k} \left( \frac{d^k\varepsilon}{dt^k} \right)^2 \right].
\]  
(31)
We see that if
\[
M(\mu) = M_0\mu^2, \quad \Delta(\mu) = \Delta_0/\mu, \quad \delta = 3/2,
\]  
(32)
then the action (31) will have an asymptotic scaling invariance when \( \mu \to \infty \), which implies the following renormalization equation,
\[
\frac{d}{d\mu} (M\Delta^2) = 0.
\]  
(33)
Now take the initial value of \( \Delta_0 \) to satisfy \( M_0\Delta_0^2 = 4\pi^2 \), then at the IR limit (\( \mu \to \infty \)), we can drop the higher order terms, and the action (31) reads,
\[
S^{(2)}_{\text{ren}} = \frac{1}{2g_s^2} \int_0^\Delta dt \left[ (\dot{\varepsilon})^2 - \left( \frac{2\pi}{\beta} \right)^2 (\ddot{\varepsilon})^2 \right].
\]  
(34)
In order to compared with the quadratic Schwarzian action (7) in Euclidean signature, let’s change (34) into the Euclidean signature by the replacement \( \ddot{t} \to -i\dot{t}, \Delta \to -i\beta \), then we find that the Euclidean IR action reads,
\[
S^{(2)}_{\text{ren}} = \frac{1}{2g_s^2} \int_0^\beta d\tau \left[ (\dot{\varepsilon})^2 - \left( \frac{2\pi}{\beta} \right)^2 (\ddot{\varepsilon})^2 \right].
\]  
(35)
This is nothing but the quadratic Schwarzian action in (7). The asymptotic symmetry (32) is just the rescaling symmetry (9) if we make an identification about the boundary operator \( \dddot{\varepsilon}(\ddot{t}) \) and reparametrization variable \( \varepsilon(\tau) \). The period \( \beta \) is not the temperature of bulk hole and may be different from the period in (7). However, because of the scaling symmetry, the quadratic Schwarzian actions are equivalent to each other for all the values of \( \beta \).

Finally, we will show the numerical evidence that the coefficients \( -c_2 \) and \( c_4 \) in (29) are positive. We first rewrite Eq. (20) by the replacement \( b_n(r) \propto e^{i\lambda_n r}R_n(r) \). Then the equation of motion for \( R_n(r) \) is,
\[
R''_n(r) + \left( \frac{r^4f'}{r^4f} + \frac{2i\lambda_n}{r^4f} \right) R'_n(r) + \frac{2i\lambda_n}{r^4f} R_n(r) = 0.
\]  
(36)
Because of the scaling symmetry of black brane metric (11), we can set \( r_h = 1 \) when solving Eq. (36) numerically. The ingoing boundary condition for \( R_n(r) \) at the horizon is just he regular condition for \( R_n(r) \). Near the AdS boundary, we can just set \( R_n(\infty) = 1 \), and the asympt-
totic solution for $R_n(r)$ reads,
\[ R_n(r) = 1 + \frac{i\lambda_n}{r} - \frac{\chi(\lambda_n) - i\lambda_n q^2 \ln r}{3r^3} + O\left(\frac{\ln r}{r^4}\right). \] (37)

Then the value of $B(\lambda_n)$ can be expressed as,
\[ B(\lambda_n) = \text{Re}[\chi(\lambda_n)]. \] (38)

For the case $q = 0$, Eq. (36) can be solved exactly and $b_n(r) = e^{i\lambda_n r}(1 + i\lambda_n/r)$, which gives $B(\lambda_n) = 0$. This means that we cannot obtain the non-trivial action in Eq. (35) for open string worldsheet action in the neutral BTZ black hole. Thus, we need to consider the case when $q \neq 0$, which can be solved numerically. The numerical results of $B(\lambda_n)$ in terms of $\lambda_n$ for different charge $q$ are shown in Fig. 1. From Eq. (37), we see that $B(\lambda_n)$ is the even function of $\lambda_n$. The value of $c_2$ and $c_4$ can be obtained by fitting the value of $B(\lambda_n)$ for small $\lambda_n$. For the case that $q = 1$, the result shows that $c_2 \approx -1.8, c_4 \approx 1.8$. We have scad $q = \{0.1, 0.4, 0.7, 1.0, 1.4\}$ and found that $c_2 < 0$ and $c_4 > 0$ in all cases.

**FIG. 1.** Left panel: Numerical values of $B(\lambda_n)$ for small $\lambda_n$ when $q = \{0.1, 0.4, 0.7, 1.0, 1.4\}$. Right panel: The fitting result for $B(\lambda_n)$ and $\lambda_n$ when $q = 1$, which shows that $c_2 \approx -1.8$ and $c_4 \approx 1.8$.

**IV. DISCUSSIONS**

Let us make a brief conclusion. We showed nontrivial evidence that the worldsheet of an open string could be considered as the dual description of the SYK state. More precisely, we proved that the fluctuation of an open string embedded in charged BTZ black brane is dual to a $0 + 1$ dimensional state which has an asymptotic scaling symmetry. This leads to the fact that its IR fixed point of the Nambu-Goto action can be reduced to the quadratic Schwarzian action, which is also the low energy effective action of the SYK model. Considering the open string worldsheet also has the $\text{SL}(2, \mathbb{R})$ symmetry, we conjecture that the renormalized on-shell action of the string worldsheet could be a candidate dual description of the SYK model. In fact there exist some other pieces of evidence, which favor our conjecture. For example, it has been shown in [10, 11] that the worldsheet with a horizon saturates the universal chaos bound, as the same as the SYK model. Thus, the gravity is not necessary to be included in the dual descriptions of SYK models.

If the dual theory of SYK model can be described by the string worldsheet, it is interesting to compare it with the previous physical interpretations of the dual boundary state dual to the string in AdS, such as “heavy quark” [13], or “Brown particle”. Brownian motion of the particle in the AdS/CFT is studied in [14], and it is quite promising to relate the random force on the Brownian particles with the random coupling in the SYK states. Since the holographic dual of an open string in AdS can be assumed as the SYK quasiparticle attached at the string end point, the entanglement between two SYK particles is expected to be described by the holographic EPR pair [16]. Then the well studied holographic dual model of the EPR pair can also be interpreted as the interaction between two “SYK quasiparticles”. Further more, multi “SYK quasiparticles” are expected to be dual to multi strings in the AdS. In the continuous limit, the $p + 1$ dimensional “SYK material” is dual to the $(p + 1) + 1$ dimension brane in higher dimensional AdS, such as the “SYK chain” or “SYK layer” [17, 18]. Although the string worldsheet has the AdS$_2$ black hole solution, it seems also interesting to introduce the Maxwell field in higher dimensional gravity, which is used to realize the extreme black hole with the near horizon geometry as $\text{AdS}_2 \times \text{R}_{d-1}$ [18]. Thus, there are rich physics to be explored on the dynamics of open string in such a background, and it is interesting to compare them carefully with the SYK models.

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