Theoretical results are presented which show that noiseless phase quadrature amplification is possible, and limited experimentally only by the efficiency of the phase detection system. Experimental results obtained using a Nd:YAG laser show a signal gain of 10\,dB and a signal transfer ratio of $T_s = 0.9$. This result easily exceeds the standard quantum limit for signal transfer. The results also explicitly demonstrate the phase sensitive nature of the amplification process.

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I. INTRODUCTION

The transmission of small optical signals carried by coherent light is fraught with difficulty. Attenuation of the optical power causes degradation of the signal-to-noise ratio as the signal recedes into the quantum noise of the beam. A simple method to rectify this problem would be to amplify the signal. However this is not a trivial exercise as phase insensitive amplifiers, such as laser amplifiers, introduce excess noise \cite{1}. In the limit of high gain, the noise penalty for a phase insensitive amplifier is 3\,dB, i.e. a halving of the signal-to-noise ratio. This is known as the standard quantum limit (SQL) for phase insensitive amplifiers. To overcome this problem, phase sensitive amplification is required \cite{2}. This can entail a non-linear process, as in the case of the optical parametric amplifier, where one observable is amplified while the conjugate is de-amplified \cite{3}.

A far simpler method has been demonstrated by Lam et al. \cite{4} using positive electro-optic feed-forward. In this scheme, a part of the input beam is tapped off to the homodyne detection system (H1) by a beamsplitter of transmissivity $\varepsilon$. The remaining light is passed through an electro-optic phase modulator, EOM, which is controlled by the signal detected in H1. Using this method with amplitude signals, Lam et al. achieved a signal transfer ratio of $T_s = 0.88$.

In this work we use an analogous feed-forward network to show better-than-SQL amplification of phase quadrature signals. To our knowledge, this is the first demonstration of phase signal amplification superior to the amplifier SQL.

In any experiment which generates phase quadrature signals close to the quantum noise limit (QNL) our system could be used to make this signal robust optical loss. Specific suggestions for the application of phase feed-forward include quantum non-demolition measurements \cite{5} and continuous variable teleportation \cite{7,8}.

II. THEORY

We will model the system shown in figure 1. At the input to the system is a beam containing some phase signal, some of which is tapped off to the homodyne detection system (H1) by a beamsplitter of transmissivity $\varepsilon$. The remaining light is passed through an electro-optic phase modulator, EOM, which is controlled by the signal detected in H1.

A theoretical model of the behaviour of this system will be generated using linearized operators. This is the same approach used previously in \cite{2} and \cite{5}. The time domain annihilation operator for the input beam to our system, $\hat{A}_\text{in}$, will be written as

$$\hat{A}_\text{in} = \bar{A}_\text{in} + \delta \hat{A}_\text{in}$$  \hspace{1cm} (1)

where $\bar{A}$ is the mean value of the field amplitude and $\delta \hat{A}$ is the time dependent component of the field with an expectation value of 0. Embedded in this time dependant component is the signal information and quantum noise carried by the beam. After traversing the beam path through the beamsplitter and EOM, the field operator $\hat{A}_\text{f}$ for the output can be written as

$$\hat{A}_\text{f} = \sqrt{\varepsilon} \hat{A}_\text{in} + \sqrt{\varepsilon} \delta \hat{A}_\text{in} - \sqrt{1-\varepsilon} \hat{\nu}_b + i \delta \hat{r}$$  \hspace{1cm} (2)

where $\hat{\nu}_b$ is the vacuum input due to the beamsplitter and $\delta \hat{r}$ is a modulation imposed on the beam by the EOM. The operator $\delta \hat{r}$ will be a function of fluctuations detected in the homodyne detection system, H1.
A homodyne system, such as that shown in figure 1, measures a particular quadrature amplitude of a low power beam by mixing it with a much more intense local oscillator beam on a 50/50 beamsplitter \( \hat{A}_l \). The phase difference between the local oscillator and the signal beam, \( \theta \), determines which quadrature of the signal beam is measured. The output of the system is the difference of the photocurrents from the two photodiodes. The form of this current may be calculated by finding the difference of the photon number operators (i.e. \( \hat{A}_l^\dagger \hat{A}_l \)) for the fields incident on each of the detectors. To simplify matters we will linearize the equations by neglecting all terms greater than first order in the fluctuation operators. The form of the subtracted photocurrent is found to be

\[
\hat{i}_s = 2\sqrt{\eta_d \eta_h} \hat{A}_s \cos \theta + \sqrt{\eta_h \eta_d (1 - \varepsilon)} \delta \hat{X}_{A_{in}}^\theta \\
+ \sqrt{\eta_d} \left( \sqrt{\eta_h \varepsilon} \delta \hat{X}_{\nu_h}^\theta + \sqrt{1 - \eta_h} \delta \hat{X}_{\nu_h}^\theta \right) \\
+ \frac{\sqrt{1 - \eta_d}}{\sqrt{2}} (\delta \hat{X}_{\nu_\text{d1}} + \delta \hat{X}_{\nu_\text{d2}}),
\]

where we have defined the general quadrature of an operator \( \hat{z} \) to be

\[
\delta \hat{X}_{\nu}^\theta = e^{-i\theta} \hat{z} + e^{i\theta} \hat{z}^\dagger.
\]

for convenience of notation, the amplitude quadrature (\( \theta = 0 \)) will be written with no superscript, while the phase quadrature (\( \theta = \pi/2 \)) will be written as \( \hat{X}_{\nu}^- \). The quantum noise terms \( \delta \hat{X}_{\nu_{\text{d1,d2}}} \) result from the efficiency \( \eta_d \) of the two photodetectors in the homodyne system. The mode-matching efficiency of the homodyne system \( \eta_h \) also gives rise to a source of quantum noise, namely \( \delta \hat{X}_h^\theta \). The mode-matching efficiency, \( \eta_h \), is given by the square of the fringe visibility in the homodyne system \( \eta_h \) i.e.

\[
\text{Homodyne efficiency} = \eta_h = \left( \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \right)^2
\]

where \( I_{\text{min/max}} \) is the minimum/maximum power of the output of one arm of the homodyne system as measured with equal power in the signal and local oscillator arms. To measure the phase quadrature with homodyne detection, we require that \( \theta = \pi/2 \). This means that the DC component of the current in equation 3 is zero. Experimentally, this provides a means for locking the homodyne system to the phase quadrature.

The form of the modulation \( \delta \hat{r} \) imposed by the EOM will be given by a convolution of \( \delta \hat{i}_s \), the time dependent part of the current \( i_s \), and the time response of the feed-forward electronics, \( k(t) \). We may therefore write

\[
\delta \hat{r}(t) = \int_0^t k(u) \delta \hat{i}_s(t - u) du.
\]

Combining equations 2, 3 and 5, as well as choosing the phase of the homodyne detection system to be \( \pi/2 \) so that the phase quadrature is being fed-forward, the final form of the fluctuations in the output beam of the EOM will be given by

\[
\delta \hat{A}_f = \sqrt{\varepsilon} \delta \hat{A}_{\text{in}} - \sqrt{1 - \varepsilon} \delta \hat{r}_h \\
+ \int_0^t k(u) \left[ \sqrt{\eta_h \eta_d (1 - \varepsilon)} \delta \hat{X}_{A_{\text{in}}}^\theta (t - u) \\
+ \sqrt{\eta_d} \left( \sqrt{\eta_h \varepsilon} \delta \hat{X}_{\nu_h}^\theta (t - u) + \sqrt{1 - \eta_h} \delta \hat{X}_{\nu_h}^\theta (t - u) \right) \\
+ \frac{\sqrt{1 - \eta_d}}{\sqrt{2}} \left( \delta \hat{X}_{\nu_{\text{d1}}} (t - u) + \delta \hat{X}_{\nu_{\text{d2}}} (t - u) \right) \right] du.
\]

Considering the general quadrature, \( \phi \), of \( \delta \hat{A}_f \) and taking the Fourier transform of the operators we obtain

\[
\delta \hat{X}_{\phi}^\theta = K(\omega) \sin \phi \sqrt{\eta_h \eta_d (1 - \varepsilon)} \delta \hat{X}_{A_{\text{in}}}^- + \sqrt{\varepsilon} \delta \hat{X}_{A_{\text{in}}}^\phi \\
+ K(\omega) \sin \phi \sqrt{\eta_d} \left[ \sqrt{\eta_h \varepsilon} \delta \hat{X}_{\nu_h}^\theta - \sqrt{1 - \varepsilon} \delta \hat{X}_{\nu_h}^\phi \right] \\
+ K(\omega) \sin \phi \left[ \sqrt{\eta_d} \left( \delta \hat{X}_{\nu_{\text{d1}}} + \delta \hat{X}_{\nu_{\text{d2}}} \right) \right],
\]
where $\tilde{X}_z$ is the Fourier transform of $\hat{X}_z$. The spectrum of $\hat{A}_f$, normalized to the quantum noise limit ($\text{QNL}$), can now be obtained for arbitrary quadrature phase angle by evaluating $V_{\phi}^A = \langle |\delta \hat{X}_\phi| \rangle^2$. Using the result that the spectra of the quantum noise sources is just 1, we obtain

$$V_{\phi}^A = \sqrt{\frac{V_{\phi}^A}{\sin^2 \alpha + V_{A_{in}}^- \cos^2 \alpha}} \times$$

$$\left( \varepsilon \cos^2 \phi + \left| \sqrt{\varepsilon} + K(\omega) \sqrt{\eta_h \eta_d (1 - \varepsilon)} \right|^2 \sin^2 \phi \right) + |K(\omega) \sqrt{\eta_h \eta_d \varepsilon} - \sqrt{1 - \varepsilon}|^2 \sin^2 \phi$$

$$+ (1 - \varepsilon) \cos^2 \phi + |K(\omega)|^2 (1 - \eta_d \eta_h) \sin^2 \phi$$

(9)

where the angle $\alpha$ is given by

$$\tan \alpha = \left( 1 + \frac{K(\omega) \sqrt{\eta_h \eta_d (1 - \varepsilon)}}{\sqrt{\varepsilon}} \right) \tan \phi.$$

It has also been assumed that the amplitude noise of the input beam, $\hat{A}_{in}$, is at the quantum limit. To investigate the action of the feed-forward system on the phase quadrature, we take $\phi = \pi/2$ so that

$$V_{\phi}^A = \left( \varepsilon \cos^2 \phi + \left| \sqrt{\varepsilon} + K(\omega) \sqrt{\eta_h \eta_d (1 - \varepsilon)} \right|^2 \sin^2 \phi \right)$$

$$+ |K(\omega) \sqrt{\eta_h \eta_d \varepsilon} - \sqrt{1 - \varepsilon}|^2 \sin^2 \phi$$

$$+ (1 - \varepsilon) \cos^2 \phi + |K(\omega)|^2 (1 - \eta_d \eta_h) \sin^2 \phi$$

(10)

From equation (10) it is clear that a value of the electronic gain may be chosen such that the second term in equation (10) becomes 0. Physically this may be explained by the division of the beamsplitter vacuum fluctuations $\delta \hat{\nu}_b$. The component of $\delta \hat{\nu}_b$ imposed on the signal beam (i.e. the beam which goes through the EOM) is anti-correlated with the component of $\delta \hat{\nu}_b$ imposed on the light passed to the homodyne system $H1$. The result is that that for a unique value of positive feed-forward gain, the EOM will be driven such that the signal it imposes exactly cancels the beamsplitter vacuum fluctuations originally introduced to the signal beam. If we assume ideal homodyne and detector efficiency, the value of the gain required for cancellation is

$$K(\omega) = \sqrt{\frac{1 - \varepsilon}{\varepsilon}}.$$

(11)

With this gain, the phase noise of the beam at the output of the feed-forward modulator is given by

$$V_{\phi}^A = \frac{V_{A_{in}}^-}{\varepsilon}.$$  

(12)

Assuming perfect in-loop detection, the system can now been seen to behave as a noiseless amplifier with signal amplification of $\varepsilon^{-1}$.

When detection losses are considered, matters become a little more complex. The optimum gain is no longer that which gives total cancellation of the beamsplitter vacuum noise. This is because the extra noise due to detection scales with feed-forward gain. The result is that the optimum gain level is less than that found for ideal detection. To evaluate the performance of this system with imperfect detection we define a signal transfer ratio $T_s$. This is given by the ratio of the signal-to-noise at the output of the system to the signal-to-noise at the input of the system, i.e.

$$T_s = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}}.$$  

(13)

The optimum value of $T_s$ occurs at a gain of

$$K(\omega) = \sqrt{\frac{\eta_h \eta_d (1 - \varepsilon)}{\varepsilon}},$$

(14)
for which

\[ T_s = \varepsilon(1 - \eta_h \eta_d) + \eta_h \eta_d. \]  

We note that with ideal detection we obtain \( T_s = 1 \) indicating perfect signal transfer.

When using a feed-forward system as a signal amplifier the optical power of the signal beam is attenuated by a factor of \( \varepsilon \). This power loss can be overcome by using an injection locking system \cite{10}. The final output beam can then be an optical state with identical power to the original state, but with an amplified signal.

### III. THE EXPERIMENT

An experiment was set up as shown in figure 2. The light source was a single mode Nd:YAG laser pumped by a diode laser array. The output power of the laser was 300mW at 1064nm. Most of the light was dumped at the 96% beamsplitter. Of the remaining light, 99.75%, was tapped off for the local oscillator beams \( LO1 \) and \( LO2 \). The remaining light formed the signal beam for the phase feed-forward. The electronics used for the feed forward consisted of two high gain amplifiers and a bandpass filter to ensure sufficient gain at the signal frequency. A variable electronic attenuator was used to adjust the overall gain of the loop.

A phase signal was imposed on the beam using the phase modulator \textit{EOM} 1. A frequency of \( 25MHz \) was chosen because the laser was at the \textit{QNL} at this frequency. The signal was measured using the homodyne detection systems \( H1 \) and \( H2 \). The homodyne detector \( H1 \) consisted of two identical low noise detectors with a quantum efficiency of \( \eta_d = 0.91 \pm 0.02 \) [1]. The homodyne efficiency was measured to be \( \eta_h = 0.94 \pm 0.02 \). System \( H2 \) contained similar detectors also with a quantum efficiency of \( \eta_d = 0.91 \pm 0.02 \). The homodyne efficiency of \( H2 \) was 0.88 ± 0.02.

The homodyne system \( H1 \) was locked to the phase quadrature using the DC voltage from the subtraction of the photocurrents of the homodyne system as an error signal. As discussed previously in relation to equation 13, when the DC voltage from \( H1 \) is 0, the detection system measures the phase quadrature.

The phase modulation imposed by \textit{EOM} 1 is shown in figure 2 as measured using a spectrum analyzer (Hewlett-Packard 3589 A). The upper trace shows the signal level with 100% of the signal beam directed into the locked homodyne system \( H1 \). The signal is observed to be 8.0 ± 0.4dB above the quantum noise, shown at 0dB. Taking detection efficiency into account means that the inferred signal level is 8.6 ± 0.4dB above the \textit{QNL}.

We then altered our system so that 80% of the light was directed to \( H1 \) while the remaining 20% was passed through \textit{EOM} 2 to the homodyne detector \( H2 \). This corresponds to \( \varepsilon = 0.2 \) in the above theory. The signal measured by \( H1 \) was used to drive \textit{EOM} 2 through the feed-forward loop. The result is shown in figure 2. Homodyne system \( H2 \) was swept and signals are therefore plotted as a function of the phase of \( LO2 \). Trace \( i \) shows the signal power at 25MHz on the phase quadrature to be 17.6 ± 0.2dB above the \textit{QNL}. The signal gain of the system is therefore \( \approx 10dB \).

The signal level measured after feed-forward must be compared to the noise observed at a frequency slightly removed from the signal frequency. Trace \( ii \) of figure 2 shows the phase quadrature noise power as measured at 25.0003MHz to be at 9.5 ± 0.4dB above the \textit{QNL}. The difference between traces \( i \) and \( ii \) when the phase of \( LO2 \) is \( \pi/2 \) shows the fed-forward signal to be 8.1 ± 0.4dB above the noise floor.

A theoretical fit of this data may be obtained using equation 3. This is also shown in figure 3. The only free parameter in this fit is the gain \( K(\omega) \) which we take to be a constant \( K \) at a fixed frequency of 25MHz. The value of \( K \) to give this fit is 3.2, which is 70% (or 2.4dB) larger than the calculated optimum. However with high signal gain the system is insensitive to detuning from the calculated optimum [2].

Importantly, figure 2 demonstrates that the system is truly phase sensitive. The amplitude quadrature of the signal beam, as measured at phase angles of 0 and \( \pi \), is shown to be quantum noise limited.

From an operational point of view, the system is very successful. We measure an input signal of 8±0.4dB above the noise, and retrieve a signal 8.1±0.4dB above the noise. This new signal is almost 18dB above the \textit{QNL} instead of 8dB and therefore more immune to attenuation. To rigorously determine how our system rates as a noiseless amplifier, we employ the signal transfer ratio defined in equation 4. In this equation, \( SNR_{in} \) is the inferred signal-to-noise ratio (not the detected) of the input beam and \( SNR_{out} \) is the inferred signal to noise of the output beam. Considering the homodyne and detection efficiency of \( H1 \), we can infer an input signal-to-noise ratio of \( SNR_{in} = 6.2 ± 0.7 \). From the efficiency of the homodyne system \( H2 \) we find the inferred \( SNR_{out} \) to be 5.6 ± 0.6, therefore giving \( T_s = 0.90 ± 0.14 \). The uncertainty in this result is largely due to the swept operation of \( H2 \). We expect that a locked homodyne system would allow more accurate determination of \( T_s \).

Using equation 3 we find that the detector and homodyne efficiencies of \( H1 \) limit the maximum achievable \( T_s \) to 0.88, which is in agreement with our experimental results. The maximum transfer coefficient achievable with a P1A with 10dB signal gain is 0.53, which our system easily exceeds.
IV. CONCLUSION

We have developed a theoretical model of a phase feed-forward network and shown that it can behave as a noiseless phase quadrature amplifier in the limit of ideal phase homodyne phase signal detection. Experimental results with a Nd:YAG laser source demonstrate the practicality of the system. A signal transfer ratio of $T_s = 0.9 \pm 0.14$ with a signal gain of $10\,dB$ was measured. The relative simplicity of this system and the demonstrated insensitivity to non-optimum gain make this system an ideal add-on to any experiment where close to $QNL$ phase signals need to be made robust to optical loss. Finally, these results indicate that quantum non-demolition and continuous variable teleportation experiments relying on electro-optic control of the phase quadrature are feasible.

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FIG. 1. A schematic diagram showing the components required for noiseless phase quadrature amplification using phase feed-forward.

FIG. 2. The layout of our phase feed-forward experiment.
FIG. 3. Trace i shows a plot of the input phase signal at 25MHz. This measurement was made with 100% of the signal beam directed into the homodyne detector $H_1$ and is therefore a measurement of $V_{Ai}$. Trace ii show the quantum noise. The spectrum analyser was set on a resolution bandwidth of 100Hz and video bandwidth 3Hz.
FIG. 4. Trace i shows a plot of the phase signal measured at 25MHz with 80% of the light directed into H1 and the feed-forward optimised. Trace ii show the noise level at 25.0003MHz. The theoretical fit to the data was obtained using equation 9. The spectrum analyser was set on a resolution bandwidth of 100Hz and video bandwidth 3Hz. This plot is an average of 3 sweeps.