Probing the Weak Boson Sector in $\gamma e \rightarrow Ze$

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Abstract

We study possible deviations from the standard model in the reaction $\gamma e \rightarrow Ze$ at a 500 GeV $e^+e^-$ collider. As a photon source we use a laser backscattered photon beam. We investigate the most general $\gamma Z\gamma$ and $\gamma ZZ$ vertices including operators up to energy-dimension-six which are Lorentz invariant. These vertices require four extra parameters; two are CP-conserving, $h_1^\gamma$ and $h_1^Z$, and two are CP-violating, $h_2^\gamma$ and $h_2^Z$. We present analytical expressions of the helicity amplitudes for the process $\gamma e \rightarrow Ze$ for arbitrary values of anomalous couplings. Assuming Standard Model values are actually measured we present the allowed region in the $(h_1^\gamma, h_1^Z)$ plane at the 90% confidence level. We then show how the angular correlation of the $Z$ decay products can be used to extract detailed information on the anomalous (especially CP-violating) $\gamma Z\gamma$ and $\gamma ZZ$ couplings.
1 Introduction

The Standard Model (SM) has very successfully explained all available experimental data. However, even though the properties of both the fermions and the vector bosons are predicted from a general gauge principle, only the fermionic sector and its interaction structure with gauge bosons have been rigorously tested and verified. On the contrary, the non-Abelian gauge-group structure involving the gauge-boson self couplings has yet to be probed precisely.

To investigate the non-Abelian gauge-group structure of the weak vector bosons, extensive studies[1, 2] have been devoted to the anomalous \(WWZ\) and \(WW\gamma\) couplings. But, little attention[3, 4, 5, 6] has been paid to studies of the anomalous couplings involving only the neutral \(\gamma\) and \(Z\) bosons. One reason could be that while the vertices \(WWV\) \((V = \gamma, Z)\) already appear at the tree-level in the SM, \(\gamma\)\(Z\)\(V\) vertices are forbidden due to gauge symmetry. However, adopting a general scheme to include all seven allowed anomalous trilinear vector boson couplings[1] in a phenomenological analysis, we find that two anomalous \(WWV\) couplings violate C invariance, and these C-violating terms should be related with \(ZZV\) and \(\gamma ZV\) couplings by the global SU(2) custodial symmetry[7]. From this point of view the possible trilinear couplings in the neutral boson sector are an interesting topic at high-energy hadron and \(e^+e^-\) colliders. Experimentally the processes involving neutral bosons (such as \(e^+e^- \to ZV\) and \(\gamma e \to Ze\)) allow for high precision with a clean and easily analyzable final state. Moreover, while the corresponding processes, \(e^+e^- \to W^+W^-\) and \(\gamma e \to W\nu_e\), involve many anomalous couplings, each of the \(ZZ\) and \(\gamma Z\) modes can have only two anomalous couplings (only one if CP invariance is imposed). Therefore, we can identify unambiguously the parameter from which a slight deviation originates.

Processes to which anomalous trilinear couplings of the neutral bosons could contribute include radiative \(Z\) decays, \(e^+e^- \to \gamma\gamma, Z\gamma, ZZ\), and \(\gamma e \to Ze\). Until now the structure of the trilinear couplings of the neutral bosons has been studied mainly in the radiative decays of \(Z\) bosons[5] and the processes, \(e^+e^- \to Z\gamma, ZZ\). Rather recently the process \(\gamma e \to Ze\) has been considered in Ref. [8]. However, most studies of these anomalous neutral-boson couplings were restricted to only a few couplings which satisfy CP invariance and they have not fully employed the angular correlations of the \(Z\) decay products to extract detailed information on the anomalous couplings.

In the present work we provide a more complete and systematic study of the most general dimension-six \(\gamma\)\(Z\)\(\gamma\) and \(\gamma ZZ\) vertices through the process \(\gamma e \to Ze\) (see Fig. 1) with energetic laser-backscattered photons[9] at a 500 GeV \(e^+e^-\) linear collider (NLC). We should note that the backscattered laser photon beam is especially interesting for the investigation of the anomalous \(\gamma\)\(Z\)\(\gamma\) and \(\gamma ZZ\) couplings in \(\gamma e \to Ze\); the spectrum of the scattered laser photons is hard and the luminosity of the backscattered photon beam can be as high as the luminosity of the original \(e^\pm\) beam. Furthermore, we can obtain from the initial laser beams, whose polarization is easily controlled, highly polarized backscattered
γ beams.

The paper is organized as follows. In Section 2 we give the most general form of dimension-six \( \gamma Z \gamma \) and \( \gamma ZZ \) couplings and show which constraints on these couplings come from electroweak gauge symmetry, C and P. In Section 3 we present, in a compact form, the complete helicity amplitudes, derived from the most general couplings of Section 2, for the process \( \gamma e \to Ze \). Section 4 is devoted to the presentation of all nine coefficients of the differential angular distributions of the Z decay into a massless fermion-antifermion pair, expressed in terms of the helicity amplitudes for the production process \( \gamma e \to Ze \). In Section 5 we introduce as a photon source Compton backscattered laser light for which we discuss in detail the effective luminosity. Assuming the SM values of the differential cross section are actually measured, we present, in Section 6, the allowed region of the CP-conserving couplings, \( h_{\gamma 1} \) and \( h_{Z 1} \), at the 90% confidence level. Then we investigate the angular correlations of the Z boson decay products to extract detailed information on the CP-violating anomalous couplings, \( h_{\gamma 2} \) and \( h_{Z 2} \). In Section 7 we summarize our findings.

2 Anomalous \( \gamma Z \gamma \) and \( \gamma ZZ \) couplings

The most general, Lorentz invariant effective Lagrangian[1] for \( \gamma Z \gamma \) and \( \gamma ZZ \) interactions, restricted to operators of dimension six, can be written as

\[
\mathcal{L}_{\gamma Z} = \frac{g_{\gamma Z}}{m_Z^2} \left[ \zeta_Z F_{\mu\nu} \left( \partial^\mu Z^\lambda \partial_\lambda Z^\nu - \partial^\nu Z^\lambda \partial_\lambda Z^\mu \right) + \eta_{1Z} (\partial_\mu Z^\alpha \partial_\alpha Z^\nu) \bar{F}^{\mu\nu} + \eta_{2Z} (Z_\mu \partial^2 Z_\nu) \bar{F}^{\mu\nu} \right] + \frac{g_{\gamma Z}}{m_Z^2} \left[ \zeta_{\gamma} F_{\mu\nu} F^{\nu\lambda} (\partial^\mu Z^\lambda + \partial_\lambda Z^\mu) + \eta_{1\gamma} Z_\mu (\partial^\rho \bar{F}^{\mu\nu} F_{\nu\rho}) + \eta_{2\gamma} Z_\mu (\bar{F}^{\mu\nu} \partial^\rho F_{\nu\rho}) \right],
\]

(2.1)

where \( Z^\mu \) is the \( Z \)-boson field, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and \( \bar{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \). Here we have neglected the scalar component of the \( \gamma \) and \( Z \) bosons:

\[
\partial^\mu A_\mu = 0, \quad \partial^\mu Z_\mu = 0.
\]

(2.2)

The condition (2.2) is automatic for on-shell \( Z \) and \( \gamma \). It is also valid not only for the virtual photon but also for the \( Z \) boson in the process \( \gamma e \to Ze \) in which all terms with \( \partial^\mu Z_\mu \) are in fact proportional to the electron mass and thus negligible.

We find that in momentum space the corresponding \( \gamma Z V \) vertex (for on-shell \( Z \) and \( \gamma \)) shown in Fig. 2 can be expressed in the following simple form:

\[
\Gamma_{\gamma Z V}^{\alpha\beta\mu}(q_1, q_2, P) = i \left( \frac{P^2 - m_V^2}{m_Z^2} \right) \left[ h_{1V} \varepsilon^{\mu\alpha\beta\rho} q_{1\rho} + h_{2V} (q_1^\mu g^{\alpha\beta} - q_1^\beta g^{\mu\alpha}) \right],
\]

(2.3)

where \( q_1 \) and \( q_2 \) are four-momenta of the photon and the \( Z \) boson, respectively, and \( P = q_1 - q_2 \). Terms proportional to \( P^\mu \) or \( q_2^\beta \) are omitted because they do not contribute
for $m_e \to 0$. Note that the above expression is manifestly electromagnetic gauge invariant for the on-shell photon. The coupling $h_1^V$ is CP-even and $h_2^V$ is P-even: both are C-odd. (See Table 1.) The overall factor $P^2 - m_V^2$ comes from gauge invariance for $V = \gamma$ and from Bose symmetry for $V = Z$. This factor cancels the corresponding $t$-channel pole in the process $\gamma e \to Ze$.

It is straightforward to calculate the contribution of the dim-6 operators (2.1) to the form factors $h_i^V$. We obtain the relations

$$h_1^\gamma = \zeta_\gamma, \quad h_2^\gamma = \eta_1^\gamma + \eta_2^\gamma,$$

$$h_1^Z = \zeta_Z, \quad h_2^Z = \frac{\eta_1^Z}{2} + \eta_2^Z. \quad (2.4)$$

The form factors, $h_i^V$, are real due to hermiticity and the fact that the $t$-channel momentum transfer, $P^2$, is negative in the process $\gamma e \to Ze$.

Without any loss of generality we can fix the overall coupling constants. We choose, for convenience,

$$g_{\gamma Z \gamma} = g_{\gamma ZZ} = e, \quad (2.5)$$

where $e$ denotes the positron charge, $s_W = \sin \theta_W$, $c_W = \sqrt{1 - s_W^2}$, and $\theta_W$ is the weak mixing angle of the SM.

### 3 Helicity amplitudes for $\gamma e \to Ze$

In this section we present all the helicity amplitudes for the process

$$\gamma (q_1, \lambda_1) + e^- (p_1, \sigma_1) \to Z (q_2, \lambda_2) + e^- (p_2, \sigma_2), \quad (3.1)$$

as shown in Fig. 3, with the general $\gamma Z \gamma$ and $\gamma ZZ$ couplings. The four-momentum and the helicity of each particle are shown in parentheses.

Clearly helicity amplitudes contain more information than the simple cross section for a polarized $Z$ boson. The relative phases of the amplitudes are important for the distribution of the final fermions because the interference of different $Z$ helicity states gives a nontrivial azimuthal-angle dependence. It is straightforward to include polarization of the initial $e$ and photon beams in the helicity amplitudes.

The helicity of a massive particle is not a relativistic invariant but is frame-dependent. In this paper we define the helicity of the $Z$ in the $\gamma e$ c.m. frame. Let us present the results in a compact form using the helicity basis for the $Z$. For convenience we rewrite the amplitude by extracting some kinematic factors as

$$\mathcal{M}_{\sigma_1 \lambda_1; \sigma_2 \lambda_2} (\Theta) = e g_Z \delta_{\sigma_1 \sigma_2} (1 - 1/r)^{1/2} d_{\Delta \lambda_1, \Delta \lambda_2}^J (\Theta) \tilde{\mathcal{M}}_{\sigma_1 \lambda_1; \lambda_2} (\Theta), \quad (3.2)$$

where $r = s/m_Z^2$, $\Delta \lambda_1 = \lambda_1 - \frac{1}{2} \sigma_1$, $\Delta \lambda_2 = \lambda_2 - \frac{1}{2} \sigma_2$, $J_0 = \max(|\Delta \lambda_1|, |\Delta \lambda_2|)$, and $\Theta$ denotes the scattering angle of $Z$ boson with respect to the photon direction in the $\gamma e$
c.m. frame. The coupling constant \( g_Z = e/s_W c_W \). The \( d \) functions, \( d_{\Delta \lambda_1, \Delta \lambda_2}(\Theta) \), are given in the convention of Ref. [14]. The explicit form of the \( d \) functions which are needed in the present work is found in Table 2. Of course, \( \tilde{M} \) in eq. (3.2) is not a partial wave amplitude because it may still be \( \Theta \)-dependent.

One of the three lowest-order diagrams, namely the one with the \( s \)-channel exchange of an electron has only \( J = \frac{1}{2} \) and \( J = \frac{3}{2} \) partial waves due to conservation of angular momentum conservation. On the other hand, the diagrams with \( t \)-channel \( \gamma \) or \( Z \) exchange and \( u \)-channel electron exchange may have all the partial waves with \( J \geq J_0 \).

We use the two-component spinor technique of Ref. [12] to calculate the helicity amplitudes. The results may be expressed as a sum of \( s \)-, \( u \)-, and \( t \)-channel contributions

\[
\tilde{M} = \tilde{M}_s + \tilde{M}_u + \tilde{M}_t, \tag{3.3}
\]

where the individual contributions are given by

\[
\tilde{M}_s = (v_e + \sigma a_e) A_{\sigma}^{\lambda_1 \lambda_2}, \tag{3.4}
\]

\[
\tilde{M}_u = -\frac{2(v_e + \sigma a_e)}{(1 - 1/r)(1 + \cos \Theta)} B_{\sigma}^{\lambda_1 \lambda_2}, \tag{3.5}
\]

\[
\tilde{M}_t = r[\lambda_1 h_{1\sigma} - i h_{2\sigma}] C_{\sigma}^{\lambda_1 \lambda_2}, \tag{3.6}
\]

with

\[
h_{i\sigma} = (v_e + \sigma a_e) h_i^Z - c_W s_W h_i^\gamma \quad (i = 1, 2). \tag{3.7}
\]

Here \( v_e = s_W^2 - 1/4 \) and \( a_e = 1/4 \) in the SM. The coefficients \( A_{\sigma}^{\lambda_1 \lambda_2} \) and \( B_{\sigma}^{\lambda_1 \lambda_2} \) (which arise purely from SM physics) are shown in Table 3. The \( C_{\sigma}^{\lambda_1 \lambda_2} \) coefficients (which come from for the anomalous \( \gamma Z \gamma \) and \( \gamma ZZ \) are shown in Table 4.

We first investigate the SM contributions. \( A_{\sigma}^{\lambda_1 \lambda_2} \) and \( B_{\sigma}^{\lambda_1 \lambda_2} \) are invariant under the simultaneous reversal of all particle helicities as can be seen from Table 3. The couplings \( (v_e + \sigma a_e) \) appear only as a global factor in the SM matrix elements such that the cross section for the right-handed electron is proportional to that for the left-handed electron with an overall factor \((v_e - a_e)^2/(v_e + a_e)^2 \approx 0.7\). We find that the amplitudes vanish for the electron and boson polarizations: \((\sigma; \lambda_1, \lambda_2) = (\pm; \pm, \mp), (\pm; \pm, 0)\).

For the electron and boson polarizations the total cross sections are given by

\[
\sigma_{\text{tot}}(|\cos \Theta| < 1 - \epsilon : \sigma, \lambda_1, \lambda_2) = 3R \left(\frac{v_e + \sigma a_e}{c_W s_W}\right)^2 T_{\sigma}^{\lambda_1 \lambda_2}(\epsilon, r), \tag{3.8}
\]

where

\[
R = \frac{4\pi\alpha^2}{3s} = 0.347 \left[\frac{0.5\text{TeV}}{\sqrt{s}}\right]^2 \text{[pb]}, \tag{3.9}
\]
\[ T_{++}(\epsilon, r) = T_{--}(\epsilon, r) = \left(1 - \frac{1}{r}\right)^2 \log \left(\frac{2}{\epsilon} - 1\right), \]
\[ T_{+-}(\epsilon, r) = T_{-+}(\epsilon, r) = 0, \]
\[ T_{++}(\epsilon, r) = T_{--}(\epsilon, r) = \frac{1}{r^2} \left[ \log \left(\frac{2}{\epsilon} - 1\right) - \frac{3}{2}(1 - \epsilon) \right], \]
\[ T_{+-}(\epsilon, r) = T_{-+}(\epsilon, r) = \frac{1}{2} (1 - \epsilon), \]
\[ T_{++}^0(\epsilon, r) = T_{--}^0(\epsilon, r) = 0, \]
\[ T_{+-}^0(\epsilon, r) = T_{-+}^0(\epsilon, r) = \frac{1}{r} (1 - \epsilon). \quad (3.10) \]

The results (3.8) have been compared to and are consistent with those of Denner and Dittmaier[13]. We have introduced the small angular cut \( \epsilon (|\cos \Theta| < 1 - \epsilon) \) to regularize the backward singularity caused by neglecting the small electron mass; this also allows us to neglect particles lost down the beam pipe.

We first note that, at threshold (\( s = m_Z^2 \) or \( r = 1 \)), the (\( \pm; \pm \pm \)) and (\( \pm; \pm \mp \)) cross sections vanish while the others remain finite except of course the two types which are identically zero. Second, we find that, at high energies, the dominant contributions are those where the photon and the \( Z \) boson have the same helicity. Third, we find that the cross section for longitudinal \( Z \) bosons is suppressed by at least \( 1/r \). Fig. 4 illustrates these features of the SM cross sections for left-handed electrons (integrated over \( -0.9 < \cos \Theta < 0.9 \)). The angular dependence of the differential cross sections is illustrated in Fig. 5. Clearly it is characterized by the \( u \)-channel pole in the backward region and kinematical zeros in the forward region. The (\( \pm; \mp, 0 \)) and (\( \pm; \mp, \mp \)) cross sections do not exhibit such a \( u \)-pole structure due to its cancellation by a factor in the numerator of \( u \) and \( u^2 \), respectively. On the other hand, the cross sections for (\( \pm; \pm, \pm \)) and (\( \pm; \mp, \mp \)) are finite in the forward region while those for (\( \pm; \mp, 0 \)) and (\( \pm; \pm, \mp \)) vanish as \( t \) and \( t^2 \), respectively.

Let us conclude this section with brief remarks on the consequences of CP and \( \text{CP}^\dagger \text{T} \) invariances[1] on the full amplitudes. Since the form factors \( v_e, a_e, \) and \( h^V_i \) for all \( i \) are real, \( \text{CP}^\dagger \text{T} \) invariance leads to the relation:

\[ M_{\sigma_1, \lambda_1; \sigma_2, \lambda_2} = \bar{M}^*_{-\sigma_1, -\lambda_1; -\sigma_2, -\lambda_2}, \quad (3.11) \]

up to an overall phase. (The overall phase is \( -1 \) in the convention of Ref. [12].) Here the amplitude \( \bar{M} \) is for the process \( \gamma e^+ \to Z e^+ \). On the other hand, CP invariance leads to the relation

\[ M_{\sigma_1, \lambda_1; \sigma_2, \lambda_2} = \bar{M}_{-\sigma_1, -\lambda_1; -\sigma_2, -\lambda_2}. \quad (3.12) \]

The relation (3.12) can be directly used as a test of CP conservation.
4 Angular correlations of $Z$ boson decay products

In this section we present the general angular distributions of the decay products in the sequential process (See Fig. 6.)

$$\gamma(q_1, \lambda_1) + e(p_1, \sigma_1) \rightarrow Z(q_2, \lambda_2) + e(p_2, \sigma_2),$$

$$Z(q_2, \lambda_2) \rightarrow f(k_1, \rho_1) + \bar{f}(k_2, \rho_2).$$

(4.1)

The masses of the fermions are neglected.

Since the decay processes are well understood the dependence of the cross section on the angles of the final state fermions can be extracted explicitly. The fact that the $Z$ boson has spin one allows maximally nine angle-dependent terms\[14]. The coefficient of each term can be written in terms of the production density matrix for the $Z$ boson, which may be obtained from the polarization amplitudes of Section 3.

Let us express the full amplitude as follows

$$M(p_1, \sigma_1; q_1, \lambda_1; p_2, \sigma_2; k_i, \rho_i) = D_Z(q_2^2) \sum_{\lambda_2} M(p_1, \sigma_1; q_1, \lambda_1; p_2, \sigma_2; q_2, \lambda_2) \times M_D(q_2, \lambda_2; k_1, \rho_1; k_2, \rho_2),$$

(4.2)

with the $Z$-boson propagator in the Breit-Wigner form:

$$D_Z(q^2) = (q^2 - m_Z^2 + i m_Z \Gamma_Z)^{-1}.$$  (4.3)

The production amplitude, $M$, is the sum of the contributions from three different channels as explicitly presented in Section 3.

In the limit of massless fermions the $Z$-boson decay amplitude, $M_D$, for a given final-state fermion of helicity $\tau$ simplifies to

$$M_D(\lambda_2; \tau) = g_Z C (v_f + \tau a_f) 2 \sqrt{k_1^0 k_2^0} X_D(\lambda_2; \tau).$$

(4.4)

Here $v_f$ and $a_f$ are the standard vector and axial-vector couplings. The effective color factor $C$ is 1 for leptons and $\sqrt{3}$ for quarks. The $X_D$ can be explicitly derived in a given frame.

In the $\gamma e$ c.m. frame we choose the direction of the $Z$ momentum as the positive $z$-axis and the direction of $\vec{q}_1 \times \vec{q}_2$ as the $y$-axis; the scattering $\gamma e \rightarrow Ze$ takes place in the $x$-$z$ plane. (See Fig. 7.) The production amplitude, $M$, is then a function of the scattering angle, $\Theta$, as measured between the momentum of the $\gamma$ and the momentum of the $Z$ boson in this frame. We express the decay amplitude, $M_D$, in the rest frame of the $Z$ boson, which is defined by a boost from the above $\gamma e$ c.m. frame along the $z$-axis.

In the $Z$ rest frame we parametrize the four-moment of $f$ and $\bar{f}$ as

$$k_1^\mu = \frac{1}{2} \sqrt{q_2^2}(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),$$

$$k_2^\mu = \frac{1}{2} \sqrt{q_2^2}(1, - \sin \theta \cos \phi, - \sin \theta \sin \phi, - \cos \theta).$$

(4.5)
It is rather straightforward to evaluate $X_D$ of eq. (4.4) in the $Z$ rest frame. We find

\[ X_D(\pm; +) = -\frac{1}{\sqrt{2}}(1 \pm \cos \theta)e^{\pm i\phi}, \]
\[ X_D(0; +) = X_D(0; -) = -\sin \theta, \]
\[ X_D(\pm; -) = \frac{1}{\sqrt{2}}(1 \mp \cos \theta)e^{\pm i\phi}. \]  

The polarization-summed squared matrix elements are now given by

\[
\sum |\mathcal{M}|^2 = \sum_{\sigma_1} \sum_{\lambda_1} \sum_{\sigma_2} \sum_{\rho_1} \sum_{\rho_2} |\mathcal{M}(p_1, \sigma_1; q_1, \lambda_1; p_2, \sigma_2; k_i, \rho_i)|^2 \\
= g_2^2 C^2 q_2^2 |D_Z(q_2^2)|^2 \sum_{\sigma_1} \sum_{\lambda_1} \sum_{\tau} (v_f + \tau a_f)^2 P_{\lambda_1 \lambda_2}^\sigma D_{\lambda_2 \lambda_2}^\tau. \tag{4.7}
\]

Summation over repeated indices ($\lambda_2$ and $\lambda_2'$) is implied. The production tensor reads

\[
\mathcal{P}_{\lambda_2 \lambda_2'}^\sigma = \sum_{\sigma_1} \sum_{\sigma_2} \mathcal{M}_{\sigma_1 \lambda_1, \sigma_2 \lambda_2}(\Theta) \mathcal{M}_{\sigma_1, \sigma_2 \lambda_2, \lambda_2'}^*(\Theta). \tag{4.8}
\]

and the decay tensor reads

\[
\mathcal{D}_{\lambda_2 \lambda_2'}^\sigma = X_D(\lambda_2; \tau) X_D^*(\lambda_2'; \tau). \tag{4.9}
\]

After integration over the square of the virtual $Z$ mass, $q_2^2$, and summation over the final fermion polarization, $\tau$, the differential cross section for a given photon helicity, $\lambda_1$, can be expressed in the narrow $Z$ width approximation as

\[
\frac{d\sigma(\lambda_1)}{d \cos \Theta d \cos \theta d \phi} = \frac{(1 - 1/r) 3\text{B}(Z \to f \bar{f})}{16\pi s} \frac{16}{16\pi} \sum_{i=1}^{9} F_{\lambda_1}^{\lambda_1} \mathcal{D}_i(\theta, \phi). \tag{4.10}
\]

Here the $\mathcal{D}_i$'s are the following nine orthogonal functions which are normalized to $4\pi$:

\[
\mathcal{D}_1 = 1, \\
\mathcal{D}_2 = \frac{\sqrt{5}}{2}(1 - 3 \cos^2 \theta), \\
\mathcal{D}_3 = \sqrt{3} \cos \theta, \\
\mathcal{D}_4 = \sqrt{3} \sin \theta \cos \phi, \\
\mathcal{D}_5 = \frac{\sqrt{15}}{2} \sin(2\theta) \cos \phi, \\
\mathcal{D}_6 = \frac{\sqrt{15}}{2} \sin^2 \theta \cos(2\phi), \\
\mathcal{D}_7 = \sqrt{3} \sin \theta \sin \phi, \\
\mathcal{D}_8 = \frac{\sqrt{15}}{2} \sin(2\theta) \sin \phi, \\
\mathcal{D}_9 = \frac{\sqrt{15}}{2} \sin^2 \theta \sin(2\phi). \tag{4.11}
\]
Note that after integration over the decay angles $\theta$ and $\phi$ only the coefficient $F_1^{\lambda_1}$ remains. The coefficients $F_i^{\lambda_1}$ are expressed in terms of $\mathcal{P}_{\lambda_2\lambda'_2}^{\lambda_1}$ as

$$
F_1^{\lambda_1} = \frac{2}{3} \left[ \mathcal{P}_{++}^{\lambda_1} + \mathcal{P}_{00}^{\lambda_1} + \mathcal{P}_{--}^{\lambda_1} \right],
$$

$$
F_2^{\lambda_1} = \frac{\sqrt{5}}{15} \left[ 2\mathcal{P}_{00}^{\lambda_1} - \mathcal{P}_{++}^{\lambda_1} - \mathcal{P}_{--}^{\lambda_1} \right],
$$

$$
F_3^{\lambda_1} = \frac{1}{\sqrt{3}} \left( \frac{2v_f a_f}{v_f^2 + a_f^2} \right) \left[ \mathcal{P}_{++}^{\lambda_1} - \mathcal{P}_{--}^{\lambda_1} \right],
$$

$$
F_4^{\lambda_1} = \frac{\sqrt{2}}{\sqrt{3}} \text{Re} \left[ \mathcal{P}_{+0}^{\lambda_1} - \mathcal{P}_{+0}^{\lambda_1} \right],
$$

$$
F_5^{\lambda_1} = \frac{\sqrt{2}}{\sqrt{15}} \left( \frac{2v_f a_f}{v_f^2 + a_f^2} \right) \text{Re} \left[ \mathcal{P}_{+0}^{\lambda_1} + \mathcal{P}_{-0}^{\lambda_1} \right],
$$

$$
F_6^{\lambda_1} = -\frac{2}{\sqrt{15}} \text{Re} \left[ \mathcal{P}_{+-}^{\lambda_1} \right],
$$

$$
F_7^{\lambda_1} = \frac{\sqrt{2}}{\sqrt{3}} \text{Im} \left[ \mathcal{P}_{+0}^{\lambda_1} - \mathcal{P}_{+0}^{\lambda_1} \right],
$$

$$
F_8^{\lambda_1} = \frac{\sqrt{2}}{\sqrt{15}} \left( \frac{2v_f a_f}{v_f^2 + a_f^2} \right) \text{Im} \left[ \mathcal{P}_{+0}^{\lambda_1} + \mathcal{P}_{-0}^{\lambda_1} \right],
$$

$$
F_9^{\lambda_1} = \frac{2}{\sqrt{15}} \text{Im} \left[ \mathcal{P}_{+-}^{\lambda_1} \right].
$$

(4.12)

Let us first examine the consequences of CP invariance (3.12) on the nine coefficients (4.12). Since the relevant initial $\gamma e^-$ state is not CP-invariant, the CP transformation should relate the process $\gamma e^- \rightarrow Ze^-$ to the CP-conjugate process $\gamma e^+ \rightarrow \bar{Z}e^+$. Then CP invariance leads to the following relation in the two full production and decay angular distributions:

$$
d\sigma(\lambda_1; \Theta; \theta, \phi) \xrightarrow{\text{CP}} d\bar{\sigma}(-\lambda_1; \Theta; \pi - \theta, \pi + \phi),
$$

(4.13)

where $d\sigma$ ($d\bar{\sigma}$) is the differential cross section for the process $\gamma e^- \rightarrow Ze^-(\gamma e^+ \rightarrow \bar{Z}e^+)$ including the decay of the $Z$.

We next examine the implications of having no absorptive part in the amplitude. We find that CPT invariance (3.11) leads to the relation

$$
\mathcal{P}_{\lambda_2\lambda'_2}^{\lambda_1} = \mathcal{P}_{-\lambda_2,-\lambda'_2}^{-\lambda_1},
$$

(4.14)

and for the full decay angular distributions it leads to the relation

$$
d\sigma(\lambda_1; \Theta; \theta, \phi) \xrightarrow{\text{CPT}} d\bar{\sigma}(-\lambda_1; \Theta; \pi - \theta, \pi + \phi).
$$

(4.15)
The CP and CPT\(\bar{T}\) properties of the eighteen coefficients, \(F_{\lambda}^i\) and \(\bar{F}_{\lambda}^i\) for the CP-conjugate process (\(i = 1\) to \(9\)), are listed in Table 5. It is straightforward to check that the relation (4.14) forces every CPT-odd coefficient to vanish. We emphasize once more that this is due to the negative \(t\)-channel momentum transfer and the hermiticity of the Lagrangian. As a result only nine coefficients among the eighteen original coefficients survive. Among these are six CP-conserving coefficients and three which are CP-violating.

5 Photon spectra

The calculations of the previous sections were carried out for monochromatic photons. In this section we consider realistic photon colliders with the inevitable photon energy spread. The subprocess \(\gamma e \rightarrow Ze\) is then related to \(e^+e^-\) collisions by folding the cross section with an appropriate differential \(e\gamma\) luminosity function, \(L_{e\gamma}(\hat{s})\);

\[
\sigma = \int_0^\hat{s} (d\hat{s}/\hat{s})L_{e\gamma}(\hat{s})\sigma(\hat{s}).
\]

(5.1)

Here \(\hat{s}\) is the squared c.m. energy of the \(\gamma e\) system. We may consider three different photon sources; classical bremsstrahlung, beamstrahlung\([15]\) and the Compton backscattered laser beam\([9]\). Of these three the laser backscattered beam is most interesting in our context since (1) the energy spectrum of the resulting photon beam is very hard compared to the standard bremsstrahlung photons and a typical beamstrahlung photons, (2) the effective luminosity remains as high as the original \(e^\pm\) beam, and (3) highly polarized backscattered \(\gamma\) beams are naturally produced from polarized laser beams. In light of these distinct features, we will confine ourselves to the laser backscattered photon beam for the actual numerical analysis.

To describe the machine parameters in the laser backscattering process we introduce the dimensionless variables

\[
x_0 = \frac{4E\omega_0}{m_e^2}, \quad x = \frac{\omega}{E}.
\]

(5.2)

where \(E\) is the electron or positron beam energy \(\omega_0\) is the energy of the laser photon and \(\omega\) is the energy of the scattered photon. In this work \(E = 250\) GeV. The maximum energy fraction of the scattered photon is given by

\[
x_m = \frac{x_0}{x_0 + 1}.
\]

(5.3)

The value of \(x_0\) value should be less than \(2 + 2\sqrt{2}\) to prevent a significant drop of conversion efficiency due to the onset\([9]\) of \(e^+e^-\) pair production between backscattered photons and laser photons. We take \(x_0 = 2 + 2\sqrt{2}\), which is the maximally allowed value. This means that for the 250 GeV electrons or positrons we use an initial laser beam with \(\omega_0 \approx 1.26\) eV.
The effective luminosity is very sensitive to the product of the electron helicity, $\sigma$, and the laser photon helicity, $\lambda$. A more negative average value of $\sigma \cdot \lambda$ gives a harder and more monochromatic photon spectrum. However, from the experimental point of view, the introduction of polarized electron beams may lead to new systematic errors. Additionally, the laser photons should collide with positrons, which are at present difficult to polarize, in order to use the polarized electron beam in a realistic $e^+e^-$ collider. Therefore we assume that the electron and positron beams are unpolarized. On the other hand, the laser can be easily and completely polarized, and this polarization can serve as an important experimental tool. We assume that the laser is completely polarized $|\lambda\gamma| = 1$. The final photon polarization depends on the electron or positron beam energy, and it is proportional to the initial laser beam helicity. The average helicity of the scattered laser photons is then given by

$$\xi_2 = -\lambda\gamma \frac{(2r' - 1)(2 - 2x + x^2)}{2 - 2x + x^2 - 4r(1 - r)(1 - x)}, \quad (5.4)$$

with the definition $r' = x/x_0(1 - x) < 1$. The $x$-dependence of the average photon helicity is illustrated in Fig. 8. The effective photon luminosity is then given by

$$L_{\gamma\gamma}(\hat{s}) = L_{\gamma\gamma}(xs) = \frac{x_0^2[2 - 2x + x^2 - 4r'(1 - r')(1 - x)]}{(x_0^2 - 4x_0 - 8) \log(1 + x_0) + x_0^2/2 + 8x_0 - x_0^2/2(1 + x_0)^2}. \quad (5.5)$$

Fig. 9 shows the $x$ dependence of the effective luminosity and its components according to the final photon helicity.

### 6 Discovery limits

From the discussion of Section 3 it is apparent that different anomalous $\gamma Z\gamma$ and $\gamma ZZ$ lead to deviations of different helicity amplitudes from their SM values. In order to discover and then distinguish the anomalous couplings from each other, we thus have to separate the various helicity amplitudes. As has been discussed in Section 4 the unique way to do this is to study angular distributions of the $Z$ decay products.

The complete expression for the angular distribution of the fermion-antifermion pair arising from the decay of the $Z$ boson was given in section 4. These angular distributions are particularly simple when measured in the rest frame of the parent $Z$. Experimentally this will require the direction of the $Z$-boson momentum to be reconstructed to determine $\Theta$, the angle of the $Z$ with respect to the photon beam. The momenta of the decay products (two jets or a charged lepton pair, say) can then be boosted to the rest frame of their parent, which is moving with the velocity $\beta_Z = (s - m_Z^2)/(s + m_Z^2)$ along the $Z$-boson axis.
We now cast the differential cross sections for the processes $\gamma e^{\pm} \rightarrow Ze^{\pm}$ into the following form:

\[
d\sigma(\lambda_\gamma) \sim \sum_{i=1}^{9} F_i(\lambda_\gamma; \Theta; s) D_i(\theta, \phi),
\]

\[
d\bar{\sigma}(\lambda_\gamma) \sim \sum_{i=1}^{9} \bar{F}_i(\lambda_\gamma; \Theta; s) D_i(\theta, \phi),
\]

(6.1)

where the coefficients $F_i$'s and $\bar{F}_i$'s are given by

\[
F_i(\lambda_\gamma; \Theta; s) = \int_0^1 dx L_{e\gamma}(xs) \left[ (1 + \xi_2)F_i^+(\Theta; xs) + (1 - \xi_2)F_i^-(\Theta; xs) \right],
\]

\[
\bar{F}_i(\lambda_\gamma; \Theta; s) = \int_0^1 dx L_{e\gamma}(xs) \left[ (1 + \xi_2)\bar{F}_i^+(\Theta; xs) + (1 - \xi_2)\bar{F}_i^-(\Theta; xs) \right].
\]

(6.2)

While the functions $D_i$ reflect the known dynamics of the $Z$ decay, the factors $F_i$ and $\bar{F}_i$ contain information on the dynamics of the production processes, $\gamma e^{\pm} \rightarrow Ze^{\pm}$. One can use the nine orthogonal functions, $D_i$, to extract much information on the production mechanism.

First, we discuss CP-even distributions which are in general, in the SM, non-vanishing at the tree level. In the process $\gamma e \rightarrow Ze$ we have two parameters, $h_{\gamma 1}$ and $h_{Z 1}$, which determine the size of new CP-conserving contributions.

The simplest CP-even distribution is the differential cross section $d\sigma/d\cos\Theta$, which is shown in Fig. 10 for the SM ($h_{\gamma 1} = h_{Z 1} = 0$), for $h_{\gamma 1} = 0.1$ and for $h_{Z 1} = 0.1$ at $\sqrt{s} = 500$ GeV. It is clear that the differential cross section is more sensitive to the coupling $h_{\gamma 1}$ than to the coupling $h_{Z 1}$. To make a quantitative estimate of the sensitivity of the differential cross section to the CP-conserving couplings we perform a $\chi^2$ analysis by comparing the SM predictions with those corresponding to non-vanishing anomalous couplings $h_{\gamma 1}$ and $h_{Z 1}$.

We compute the statistical errors from the following set of NLC parameters: (1) $\sqrt{s} = 0.5$ TeV, $\int L_{e+e-} = 10 fb^{-1}$, $|\cos\Theta| < 0.9$, (2) $Z$ reconstruction efficiency (including branching ratios) = 0.5, and (3) the SM cross section of the process $\gamma e \rightarrow Ze$ as measured. The systematic uncertainty is taken to be $\pm 5\%$. Then we can derive bounds on the anomalous couplings ($h_{\gamma 1}^Z, h_{Z 1}^Z$) at the 90% confidence level ($\chi^2 = 4.61$) and display the contours in the ($h_{\gamma 1}^Z, h_{Z 1}^Z$) plane. Fig. 11 shows the allowed regions for the anomalous couplings $h_{\gamma 1}^Z$ and $h_{Z 1}^Z$ as determined from the study of $\gamma e \rightarrow Ze$ using backscattered laser beams for the (a) right-handed and (b) left-handed initial laser beams, respectively, at $\sqrt{s} = 0.5$ TeV. We find that the constraint on the coupling $h_{Z 1}^Z$ is dependent a little on the laser beam helicity, but the constraint on the coupling $h_{\gamma 1}^Z$ is almost independent of the helicity. From Fig. 11 we conclude that with the backscattered laser photon beam we obtain the following constraints:

\[-0.06 < h_{\gamma 1}^Z < 0.04, \quad -0.08 < h_{Z 1}^Z < 0.08.\]

(6.3)
Next, we consider the CP-violating terms proportional to $h_2^\gamma$ and $h_2^Z$. Because of no absorptive parts the CP-violating terms contribute solely to imaginary parts of the helicity amplitudes and they would be not so large. Furthermore, the SM amplitudes are real so that the CP-violating terms have very little effect on real distributions such as $d\sigma/d\cos\Theta$. A large sensitivity can be obtained only by measuring coefficients of sines of azimuthal angles, where the relative phases between different helicity amplitudes interfere with the imaginary part due to the CP-violating terms.

We have shown in Section 4 that three CP-violating distributions can be considered in the processes $\gamma e^\pm \to Ze^\pm$. Two distributions, $F_7^{\lambda_1} - F_7^{-\lambda_1}$ and $F_8^{\lambda_1} - F_8^{-\lambda_1}$, require the interference of the longitudinally polarized $Z$ boson and the transversely polarized $Z$ boson, while the other distribution $F_9^{\lambda_1} + \bar{F}_9^{-\lambda_1}$ involves only transversely polarized $Z$ bosons. From Table 4 it is clear that the first two distributions become more sensitive than the latter distribution as the c.m. energy increases. On the other hand, the distribution $F_1^{\lambda_1} - \bar{F}_1^{-\lambda_1}$ requires the measurement of the final fermion polarization which is possible only in the $\tau$-lepton decay mode of the $Z$ boson. Without the actual polarization measurement, the effectiveness of this distribution is reduced by a polarization factor of $2v_f a_f/(v_f^2 + a_f^2)$ as can be seen clearly in eq. (4.12). Besides, charge identification is required to determine the azimuthal angle of the final fermion. Even if this requirement can be fulfilled in the charged-lepton mode of the $Z$ decay, the leptonic $Z$ decay rates are rather small and their polarization factor, $2v_f a_f/(v_f^2 + a_f^2)$, is very small (about $-0.08$). Consequently after folding the effective photon luminosity function with the $\gamma e \to Ze$ cross section we find that the most sensitive measure of CP-violation is given by the following asymmetry:

$$A_{CP}(\lambda) = \frac{F_7(\lambda) - \bar{F}_7(-\lambda)}{F_7(\lambda) + \bar{F}_7(-\lambda)}.$$  \hspace{1cm} (6.4)

The distribution $F_7$, which is the coefficient of $\sin \theta \sin \phi$, essentially denotes the up-down asymmetry of the final fermion with respect to the scattering plane.

Fig. 12 shows the $\Theta$-dependence of the CP-odd asymmetry (6.4) for $\lambda = 1$ and $h_2^\gamma = 0.01$ (solid line), for $\lambda = -1$ and $h_2^\gamma = 0.01$ (dotted line), for $\lambda = 1$ and $h_2^Z = 0.02$ (long-dashed line), and for $\lambda = -1$ and $h_2^Z = 0.02$, respectively, at $\sqrt{s} = 0.5$ TeV. A few interesting features for the CP-odd asymmetry $A_{CP}(\lambda)$ are noted. First, the asymmetry due to the coupling $h_2^\gamma$ is very sensitive to the scattering angle, $\Theta$, while the asymmetry due to the $h_2^Z$ is rather insensitive to this angle. Second, the asymmetry due to $h_2^\gamma$ is almost independent of the laser photon helicity, but the asymmetry due to $h_2^Z$ is strongly dependent on the laser photon helicity. Third, in the backward region the asymmetry is more sensitive to $h_2^\gamma$, but in the forward region the asymmetry becomes very small in both cases. Since the differential cross section is peaked by the $u$-channel pole, the CP-odd asymmetry is much more sensitive to the coupling $h_2^\gamma$ than the coupling $h_2^Z$. Consequently a deviation from the identically zero prediction of the SM is clearly visible for the $h_2^\gamma$ and the $h_2^Z$ of order of $10^{-2}$ with a left-handed initial laser beam at the backward scattering
7 Summary

In this paper we have systematically studied observable experimental distributions in the process $\gamma e^\pm \rightarrow Z e^\pm$ with photons generated by backward Compton-scattered laser light. The process could serve as a probe of possible anomalous $\gamma Z\gamma$ and $\gamma ZZ$ couplings along with the process $e^+e^- \rightarrow Z\gamma$. Since the $Z$’s decay into fermion-antifermion pairs, one may use the angular distributions of the $Z$ decay products as polarimeters to efficiently analyze the helicities of produced $Z$ bosons. Because the $Z$ decay properties are well known, a careful study of the reaction $\gamma e \rightarrow Z e \rightarrow f\bar{f}e$ therefore reveals detailed information on anomalous $\gamma Z\gamma$ and $\gamma ZZ$ couplings through the angular correlations of the final-state fermions.

More specifically we have shown that at an NLC with the c.m energy 0.5 TeV a search for anomalous moments connected with the $\gamma Z\gamma$ and $\gamma ZZ$ vertices is feasible. We have presented the allowed region for the CP-conserving anomalous couplings, $h_1^\gamma$ and $h_1^Z$, at the 90% confidence level from the measurement of the differential angular distributions for the production process $\gamma e \rightarrow Z e$.

The effects of the CP-violating anomalous couplings would not be particularly visible in the $Z$ angular distributions in the production process. However a careful study of the polar and azimuthal distributions of final-state leptons and anti-leptons and a good use of laser beam polarization enable us to isolate these CP-violating effects and to separate the contribution of $h_2^\gamma$ from the contribution of $h_2^Z$. We found that the most sensitive CP-violation asymmetry is the up-down asymmetry of the final-state fermion with the respect to the $\gamma e$ scattering plane.

We considered the sequential processes $\gamma e \rightarrow Z e$, $Z \rightarrow f\bar{f}$ only to the lowest order in electroweak interactions. It is certain that a detailed study of possible anomalous contributions should be extended to electroweak radiative corrections. In our considerations we have treated the production process separate and the decay processes separately, and primarily studied kinematical effects; it should be relatively straightforward to include radiative corrections separately for the production and decay processes. As a matter of fact, the electroweak radiative corrections\cite{13} for the production process, $\gamma e \rightarrow Z e$, have been done in Ref. \cite{13}. One also can find an extensive literature on the electroweak radiative corrections for the $Z$ decay processes. Even though these corrections should modify our amplitudes in detail, we expect that the corrections do not change the overall structure of the amplitudes.

To conclude, we found that at the NLC with a c.m. energy 0.5 TeV, an integrated luminosity $10 \text{fb}^{-1}$ and a polarized beam of backscattered photons one may obtain the bounds $-0.06 < h_1^\gamma < 0.04$ and $-0.08 < h_1^Z < 0.08$ at the 90% confidence level. The CP-violating couplings $h_2^\gamma$ and $h_2^Z$ of the order of $10^{-2}$ may easily be identified through

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the measurement of the CP-violation up-down asymmetry.

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### Tables

**Table 1** Properties of the couplings $h_i^V (V = \gamma, Z)$ under discrete transformations

| $i$ | 1 | 2 |
|-----|---|---|
| P   | − | + |
| CP  | + | − |
| C   | − | − |

**Table 2** Explicit form of the $d$ functions needed

\[
\begin{align*}
  d_{3/2,3/2}^{3/2}(\Theta) &= d_{-3/2,-3/2}^{3/2}(\Theta) = \frac{1}{2}(1 + \cos \Theta) \cos \frac{\Theta}{2} \\
  d_{3/2,1/2}^{3/2}(\Theta) &= -d_{-3/2,-1/2}^{3/2}(\Theta) = \frac{\sqrt{3}}{2}(1 + \cos \Theta) \sin \frac{\Theta}{2} \\
  d_{3/2,-1/2}^{3/2}(\Theta) &= d_{-3/2,1/2}^{3/2}(\Theta) = \frac{\sqrt{3}}{2}(1 - \cos \Theta) \cos \frac{\Theta}{2} \\
  d_{1/2,-3/2}^{3/2}(\Theta) &= d_{-1/2,3/2}^{3/2}(\Theta) = \frac{\sqrt{3}}{2}(1 - \cos \Theta) \cos \frac{\Theta}{2} \\
  d_{1/2,1/2}^{1/2}(\Theta) &= d_{-1/2,-1/2}^{1/2}(\Theta) = \cos \frac{\Theta}{2} \\
  d_{1/2,-1/2}^{1/2}(\Theta) &= -d_{-1/2,1/2}^{1/2}(\Theta) = -\sin \frac{\Theta}{2}
\end{align*}
\]
Table 3  Coefficients $A^{\lambda_1\lambda_2}_\sigma$ and $B^{\lambda_1\lambda_2}_\sigma$ for the standard model

| $(\sigma; \lambda_1, \lambda_2)$ | $A^{\lambda_1\lambda_2}_\sigma$ | $B^{\lambda_1\lambda_2}_\sigma$ |
|-------------------------------|---------------------|---------------------|
| (+; ++), (−; −−)               | −2                  | $(1 - 1/r)(1 - \cos \Theta)$ |
| (+; −−), (−; ++)               | 0                   | 0                   |
| (+; −+), (−; +−)               | 0                   | $2\sqrt{3}/(3r)$   |
| (+; --), (−; ++)               | 0                   | 2                   |
| (+; +0), (−; −0)               | $-\sqrt{2}r$        | $-\sqrt{\frac{2}{3}}(1 - 1/r)(1 + \cos \Theta)$ |
| (+; −0), (−; +0)               | 0                   | $2\sqrt{2}/\sqrt{3}r$ |


Table 4 Coefficients $C^{\lambda_1\lambda_2}_2$ for the general coupling

| $(\sigma; \lambda_1, \lambda_2)$ | $C^{\lambda_1\lambda_2}_2$ |
|-------------------------------|--------------------------|
| $(+;++)$, $(-;--)$            | 1                        |
| $(+;+-)$, $(-;+-)$            | 0                        |
| $(+;--)$, $(--;+)$            | $1/\sqrt{3}$             |
| $(+;--)$, $(--;++)$           | 1                        |
| $(++;0)$, $(--;0)$            | $\sqrt{r}/2$             |
| $(++;+0)$, $(--;+0)$          | $(r+1)/\sqrt{6r}$        |

Table 5 CP and CPT properties of the 18 coefficients $F^{\lambda_1}$'s and $\bar{F}^{\lambda_1}$'s.

| CP     | CPT  | Angular coefficients                                                                 | Number |
|--------|------|--------------------------------------------------------------------------------------|--------|
| even   | even | $F_1^{\lambda_1} + \bar{F}_1^{-\lambda_1}$, $F_2^{\lambda_1} + \bar{F}_2^{-\lambda_1}$, $F_3^{\lambda_1} + \bar{F}_3^{-\lambda_1}$ | 6      |
|        |      | $F_4^{\lambda_1} - \bar{F}_4^{-\lambda_1}$, $F_5^{\lambda_1} + \bar{F}_5^{-\lambda_1}$, $F_6^{\lambda_1} + \bar{F}_6^{-\lambda_1}$ |        |
| even   | odd  | $F_7^{\lambda_1} + \bar{F}_7^{-\lambda_1}$, $F_8^{\lambda_1} + \bar{F}_8^{-\lambda_1}$, $F_9^{\lambda_1} - \bar{F}_9^{-\lambda_1}$ | 3      |
| odd    | even | $F_7^{\lambda_1} - \bar{F}_7^{-\lambda_1}$, $F_8^{\lambda_1} - \bar{F}_8^{-\lambda_1}$, $F_9^{\lambda_1} + \bar{F}_9^{-\lambda_1}$ | 3      |
| odd    | odd  | $F_1^{\lambda_1} - \bar{F}_1^{-\lambda_1}$, $F_2^{\lambda_1} - \bar{F}_2^{-\lambda_1}$, $F_3^{\lambda_1} - \bar{F}_3^{-\lambda_1}$ | 6      |
|        |      | $F_4^{\lambda_1} + \bar{F}_4^{-\lambda_1}$, $F_5^{\lambda_1} - \bar{F}_5^{-\lambda_1}$, $F_6^{\lambda_1} - \bar{F}_6^{-\lambda_1}$ |        |
Figures

Fig. 1 Feynman diagrams which contribute to the process $\gamma e^- \rightarrow Ze^-$. The first two diagrams are the SM contribution. The blob in the last diagram denotes an anomalous vertex.

Fig. 2 Assignments of momenta and helicities for the general $\gamma ZV$ ($V = \gamma$ or $Z$) vertices.

Fig. 3 A schematic view of the process $\gamma e \rightarrow Ze$. The indices $\sigma_1$, $\sigma_2$, $\lambda_1$, and $\lambda_2$ denote particle helicities.

Fig. 4 Integrated cross sections ($-0.9 < \cos \Theta < 0.9$) versus the $\gamma e$ c.m. energy, $\sqrt{s}$, for left-handed electrons and various combinations of boson polarizations ($\lambda_1, \lambda_2$).

Fig. 5 Differential cross sections for left-handed electrons and various combinations of boson polarizations ($\lambda_1, \lambda_2$) at $\sqrt{s} = 0.5$ TeV.

Fig. 6 Schematic view of the sequential process $\gamma e \rightarrow Ze, Z \rightarrow f \bar{f}$. Shown in parentheses are the four-momenta and helicities of the particles.

Fig. 7 The coordinate system in the colliding $\gamma e$ c.m. frame. The $y$-axis is chosen parallel to $\vec{q}_1(\gamma) \times \vec{q}_2(Z)$, and thus it points out of the page. The coordinate system in the rest frame is reached from this frame by a boost along the $z$-axis.

Fig. 8 The average photon helicity of the Compton-backscattered laser light for the initial unpolarized electron beam and the initial circularly polarized laser beam ($\lambda_\gamma = 1$) at $\sqrt{s} = 0.5$ TeV.

Fig. 9 Effective photon spectra of the Compton-backscattered laser light. The dashed line is for left-handed photons and the long-dashed line for right-handed photons.

Fig. 10 Angular distribution $d\sigma/d\cos \Theta$ at $\sqrt{s} = 0.5$ TeV. Curves are shown for the SM (solid line), anomalous couplings $h_1^\gamma = 0.1$ (long-dashed line) and $h_1^Z = 0.1$ (dotted line). All the other couplings are as in the SM.

Fig. 11 Allowed regions for the anomalous couplings $h_1^\gamma$ and $h_1^Z$ from $\gamma e \rightarrow Ze$ using backscattered laser beams for (a) the right-handed initial laser beam and (b) the left-handed initial laser beam at $\sqrt{s} = 0.5$ TeV. All other couplings assume their SM values.

Fig. 12 Angular dependence of the CP-violation asymmetry, $A_{CP}(\lambda_\gamma)$, for $\lambda_\gamma = 1$ and $h_2^\gamma = 0.1$ (solid line), for $\lambda_\gamma = -1$ and $h_2^\gamma = 0.1$ (dotted line), for $\lambda_\gamma = +1$ and $h_2^Z = 0.1$ (long-dashed line), and for $\lambda_\gamma = -1$ and $h_2^Z = 0.1$ (dot-dashed line) at $\sqrt{s} = 0.5$ TeV. All other couplings are as in the SM.