Uncertainty analysis of aircraft bolt group connection clearance design using k-out-of-n model

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Abstract. Uncertainty exists in the design parameters of machinery, electronic products and other fields. The bearing performance of bolt group connection is related to the effective clearance of single bolt in the load direction, and the bolt group connection bearing reliability can be described by the k-out-of-n model. Under the condition that the load direction is determined, the bolt diameter and the positional tolerance and diameter of the hole are regarded as random variables, the uncertainty propagation analysis model of the bolt effective clearance is established, and the transmitted load probability of the single bolt is obtained. Regarding the bolt group connection of an aircraft windshield as the research object, the relationship between bearing risk and probability distribution of effective clearance is discussed. The analysis results show that the three-layer uncertainty propagation model proposed can be used for further study about bolt group connection design and fatigue life analysis.

1. Introduction

There are a lot of uncertainties in the design parameters of the mechanical and electronics products, which results in that their performance parameters are highly uncertain [1-3]. The system function reliability or failure mode probability is based on the reliability of each component according to the system work principle or the failure mechanism, the formation could be the so-called two-layer uncertainty propagation model. The model also has a certain universal significance, can be applied to all fields of the product generally [4-6]. In this case, the system reliability can be expressed as $R_s = R(R_1, R_2, ..., R_n)$, $R_s$ is the integrated function of system component reliability through the system architecture, can generally be solved by analytical method [3] or simulation method [7, 8].

In engineering, all the design parameters above should be considered to be random variables subjected to certain distribution [4, 9]. The system components reliability is determined by the design parameters and their uncertainties through the failure physical process, furthermore the uncertainties propagation to system reliability according to the different reliability model [10, 11]. Therefore, for the parameter uncertainty propagation analysis considerations, the uncertainty of these parameters should first propagate to component reliability level on the basis of the component itself, working mechanism or physics of failure process, then through system architecture propagate to the system reliability level, which formed a so-called three-layer uncertainty propagation model. The uncertainty propagation of the system parameters can be more accurately reflected by the three-layer uncertainty propagation model.

There are a large number of connection assembling with bolt and hole in the mechanical products. The size parameters of bolt and hole in the connection design are uncertain, which leads to the clearance between bolt and hole is also uncertain, causes the bearing reliability problem of the
connection. In this paper, based on the $k$-out-of-$n$ model commonly used in system reliability research, focusing on distribution probability algorithm of the bolt group clearance in the load direction [12-14]. The design parameters, such as the bolt diameter, the hole diameter and positional tolerance, are regarded as random variables, and the three-layer uncertainty propagation model of bolt group connection is established by using the concept of parameter uncertainty propagation. Considering the single bolt clearance probability distribution in the load direction, combined with the clearance transmitted load requirements, the single bolt bearing probability is determined and the bearing reliability of bolt group connection is obtained according to the $k$-out-of-$n$ model furthermore. Meanwhile, the relationship between bearing risk and the probability distribution of effective clearance is discussed, the analysis results can be used for further study about structure connection design and fatigue life analysis of bolt group connection.

2. The three-layer uncertainty propagation model of the bolt group connection

2.1. Uncertainty propagation analysis of $k$-out-of-$n$ system

The $k$-out-of-$n$ system consists of $n$ binary or multistate components, and works if and only if at least $k$ components function; otherwise while more than $n-k$ components are failure, the system doesn’t work. The $k$-out-of-$n$ system reliability can be defined as:

$$P[Z \geq k] = \sum_{i=k}^{n} C_{n}^{i} p^{i} (1 - p)^{n-i} \quad (k = 0, 1, 2, L, n)$$

(1)

where $p$ is an independent event occurrence probability or system component reliability level, affects by multiple variables, generally can be determined by the system component design parameters or event described by $Z=\left(z_{1}, z_{2}, \ldots, z_{n}\right)$ with their function $Y=g(z)$. In the case of the distribution and its characteristic parameters of $Y$ and $\left(z_{1}, z_{2}, \ldots, z_{n}\right)$ are determined, $p$ can be determined accurately. The parameters uncertainties (distribution and characteristic parameters) first propagate to $p$, then propagate to the system reliability level by $p$ through the $k$-out-of-$n$ system reliability model, that is, constitutes the three-layer $k$-out-of-$n$ uncertainty propagation model.

2.2. Bolt group connection clearance model

The bolt group connection refers to the two components $C_{1}$ and $C_{2}$ which are connected together by multiple bolts, which bear the shearing load in the same direction and is shown in figure 1.

![Figure 1. Schematic diagram of the assembly model.](image)

Where the bolt diameter and its tolerance are $d$ and $\delta_{d}$ respectively, the hole diameter and its tolerance on the connecting piece $C_{1}$, $C_{2}$ are $D$ and $\delta_{D}$. The positional tolerance of the hole on the connecting piece is $\Phi$. In the multiple bolts assembly process, due to the parameters uncertainty of the bolt diameter and the hole diameter, and the influence of the positional tolerance of the hole, the position of the bolt in the hole is stochastic. The clearance probability distribution research ideas of bolt group connection are as follows: The determined clearance between the bolt and hole in the load direction will be calculated firstly. Due to the uncertainty of bolt and hole radius, and the bolt and hole positions are also uncertain, the probability distribution of concerned clearance is obtained by Monte Carlo simulation method.
2.3. The probability distribution of bolt quantity which the concerned clearance subjects to certain requirement on the bolt group connection

The bolt group connection is composed of \( n \) identical assembly of bolt and hole, meanwhile assuming each bolt installation will not affect each other. And if the concerned clearance of bolt and hole subjects to certain requirement, it is defined as "success" or "qualified"; if the concerned clearance of bolt and hole does not subject to certain requirement, it is defined as "failure" or "unacceptable" on the contrary. For the bearing capacity concerned of bolt group connection with \( n \) bolts, and the connection is safe in the condition that if and only if at least \( k \) bolts works, means the concerned clearance of \( k \) bolts are in the specified range, the bearing reliability of bolt group connection can described by means of \( k \)-out-of-\( n \) model.

If \( X \) is the number of the bolt whose concerned clearance subjects to certain requirement on the bolt group connection with \( n \) bolts, then \( X \) is a random variable, its value may be \( 0,1,2,\ldots,k,\ldots,n \) (total \( n+1 \)). So the probability distribution of \( X \) is binomial distribution. For example, the probability that the concerned clearance of \( k \) in \( n \) bolts subjects to certain requirement, that is, the probability of \( X=k \) can be expressed as follows (the same as \( k \)-out-of-\( n \) reliability model):

\[
P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (k = 0,1,2,L ,n)
\]

Where \( p \) is the probability that the concerned clearance between bolt and hole subjects to certain requirement, and could be derived by the uncertainty propagation analysis of the concerned clearance. In addition, it also can be obtained that the probability that the concerned clearance of \( k \) and more than \( k \) in \( n \) bolts subjects to certain requirement, and the probability can be expressed as follows:

\[
P(X \geq k) = \sum_{i=k}^{n} \binom{n}{i} p^i (1 - p)^{n-i} \quad (k = 0,1,2,L ,n)
\]

3. The probability distribution analysis of effective clearance between single bolt and hole

3.1. Clearance analysis model of single bolt and hole in the load direction

Under the arbitrary external load, the installation of bolt and hole is as shown in figure 2(a), the bolt randomly fall into the hole in the actual installation process. Assuming the coordinates \( x \) axis is the same as the load direction, the large and the small circles are as the actual position of the hole and bolt respectively, the radius of hole is \( R \pm \Delta R \), the positional tolerance of the hole is \( \Phi h \), the radius of bolt is \( r \pm \Delta r \). Considering the bolt installs into hole randomly, the direction angle and distance of their centers are \( \alpha \) and \( L \) respectively. And the distance of their centers \( L \) has to do with the positional tolerance and the tolerance of bolt and hole, that is, the range of \( L \) is \( (0, \frac{h}{2} + \Delta R + \Delta r) \). Obviously, the minimum and maximum clearance between the bolt and hole are located on the line that generated by bolt cycle center and hole center, due to the geometrical relationship, the minimum clearance is:

\[
C_l=R-r-L
\]

And the maximum clearance is:

\[
C_t=R-r+L
\]

As shown in figure 2(b), the direction of hole deformation under the load and maximum clearance is not consistent, if the clearance in the load direction is too large and the hole deformation is too small, which is not enough to offset the clearance. It will result in that the hole structure load cannot be transferred to the bolt, and the connection will bearing uneven load. Therefore, the clearance between bolt and hole in the load direction is more significant in practice, which will be offset by the connection deformation firstly, as line \( AB \) shown in figure 2(b). In this paper, the clearance in the load direction is defined as the effective clearance, referred to as effective clearance, the distribution characteristics of which can provide the basis for the connection design.

The calculation method of the effective clearance between bolt and hole in the load direction shown in figure 2(b) is as follows:
1) With hole center $O_1$ and the load direction as the coordinates origin and $x$ axis respectively, we can define the $xO_1y$ coordinates system. The bolt is fallen in the hole randomly, $O_2(x_r,y_r)$ is denoted as bolt cycle center coordinates in coordinates system $xO_1y$; the line formed by the bolt and hole cycle centers is $O_1O_2$, the angle between and $x$ axis is $\alpha$, thus, $x = L\cos \alpha$, $y = L\sin \alpha$ could be derived.

2) Taking an arbitrary point $A(x,y)$ in the bolt circle, $AO_2$ is formed by point $A$ and bolt cycle center $O_2$, and the angle between $AO_2$ and $x$ axis is $\beta$, the coordinates of point $A$ can be derived as $x = x_r + r\cos \beta$, $y = y_r + r\sin \beta$;

3) The point $B$ with the same horizontal position of the point $A$ is obtained along the load direction, the coordinates of point $B$ is $(x',y')$, and its horizontal coordinates is:

$$x' = \begin{cases} \sqrt{R^2 - y'^2}, & -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2} \\ \sqrt{R^2 - y'^2}, & \frac{\pi}{2} \leq \beta \leq \frac{3\pi}{2} \end{cases}$$

(6)

Where $R$ is the hole radius;

4) The distance $|x-x'|$ between the two points $A$ and $B$ is the clearance between bolt and hole in the load direction, whose minimum value $\min|x-x'|$ is the effective clearance in the load direction, it is denoted as $C$, that is, $C = \min|x-x'|$.

(a) The clearance between bolt and hole (b) The effective clearance in the load direction

Figure 2. The clearance between bolt and hole.

3.2. The probability description of actual bolt center involved the hole positional tolerance
The bolt position is determined by the bolt center coordinates $(x_r, y_r)$, and $(x_r, y_r)$ can be determined by the direction angle $\alpha$ and the distance $L$ between hole center $O_0$ and bolt circle center $O_2$. According to the actual installation requirements, the bolt circle center always tends to the position of the hole center in the installation process, that is, the bolt cycle center should be distributed more intensive near the hole center, but should be uniform along the circumferential direction, means that the direction angle $\alpha$ is uniform distribution $U[0,2\pi]$. $x_r$, $y_r$ should subject to normal distribution, and their means should also be 0.

$$x_r = L\cos \alpha, y_r = L\sin \alpha$$

(7)
The value range of $L$ is $(0, \frac{h}{2} + \Delta R + \Delta r)$. The range of $X, Y$, are $(-L\cos \alpha, L\cos \alpha)$ and $(-L\sin \alpha, L\sin \alpha)$ with certain direction angle respectively, and whose standard deviations are $\sigma_x$ and $\sigma_y$, which can be derived according to the $3\sigma$ principle of normal distribution:

$$\sigma_x = \frac{L}{3}\cos \alpha, \quad \sigma_y = \frac{L}{3}\sin \alpha$$

(8)

We can finally get $X \sim N(0, \frac{L}{3}\cos \alpha), \, Y \sim N(0, \frac{L}{3}\sin \alpha)$. It can be seen that, the direction angle $\alpha$ is both in the expression of $X$ and $Y$. So, $X$ and $Y$ are not independent of each other. Therefore, the effective clearance probability distribution can be simulated through the function involves the random variables above by means of Monte Carlo numerical simulation method. The specific sampling process is shown as figure 3.

![Figure 3. The effective clearance sampling process](image)

4. Numerical example

4.1. Parameters input

For civil aircraft cockpit structure design, in order to make sure the dynamic load on the windshield glass is transferred to window frame structure, the windshield glass should be connected with the window frame structure through the bolt group. In order to guarantee a good bearing state of the windshield frame connecting structure after the installation, that is, there should be enough bolts to transfer load as much as possible in certain load direction. Assuming that there are 25 bolts in one side of each windshield glass, the specific parameters of bolt group are as follows: the diameter of bolt is $\Phi6.312^{0.026}_{-0.063}$ mm, means that the diameter of bolt is in the range (6.312mm, 6.338mm); the diameter of hole is $\Phi7.016^{0.07}_{-0.07}$ mm, means that the diameter of hole is in the range(7.009mm, 7.016mm);and the positional tolerance of hole is $\Phi0.608$mm. The specific probability distribution and characteristics parameters of the above parameters are shown as table 1.
Table 1. Distribution of random variables and their characteristic parameters

| Random variable                          | Distribution | Mean/mm | Standard deviation/mm |
|------------------------------------------|--------------|---------|----------------------|
| Bolt radius $r$                          | Normal       | 3.1625  | 0.00217              |
| Hole radius $R$                          | Normal       | 3.4905  | 0.00583              |
| Distance between hole center and bolt center $L$ | Normal       | 0       | 0.1013               |
| Direction angle between hole center and bolt center $\theta$ | Uniform $U[0, 2\pi]$ | $\pi$  | $\frac{\pi^2}{3}$   |

For the bolt group connection of the windshield glass and the window frame, if the effective clearance between the bolt and hole is in the range $[0.158\text{mm}, 0.546\text{mm}]$, it can be seen that the bolt will transmit the load.

4.2. Monte-carlo sampling calculation
The Monte-Carlo sampling procedure is as follows:
- Step 1: an arbitrary hole is generated according to the positional tolerance of the hole, and the center coordinates of the hole is obtained;
- Step 2: the bolt position could be sampled through the hole center;
- Step 3: calculate the effective clearance in the load direction;
- Step 4: repeat the sample $N$ times, and calculate the probability distribution of the effective clearance.

The number of samples that the effective clearance value $C$ in the load direction of single bolt are in a certain interval is denoted as $j(i)$, then, the probability that the effective clearance is in a certain interval is $p(i)=j(i)/N$. Therefore, the probability distribution density of the single bolt effective clearance in the load direction can be obtained. Furthermore, the number of samples that the effective clearances value $C$ of single bolt are in the interval $[0.158\text{mm}, 0.546\text{mm}]$ is denoted as $j$, then, the probability that the effective clearances are in the interval is $p=j/N$. At the same time, the probability that the bolt do not transmit the load is $q=1-p$.

4.3. Results and discussion
1) Probability distribution of single bolt effective clearance in the load direction
   - The probability distribution of effective clearance
     Sampling quantity selection criteria: if the relative error between the two adjacent effective clearance samples is less than 0.01%, the sample work will be completed. The probability distribution characteristics of effective clearance sampling results through $5\times10^5, 6\times10^5, 7\times10^5, 8\times10^5, 9\times10^5, 10^6$ six sampling quantity is fitted. The histograms of effective clearance under different sample quantity are as shown in figure 4.
     It can be seen from the shape of the histogram, the effective clearance all are symmetrical distribution which looks like in the form of a high middle and low tail. Through the distribution fitting curve, it is found that the effective clearance distribution is close to the normal distribution or $t$ distribution, but is not fully consistent. Furthermore 5% significant level is given, normal distribution and $t$ distribution hypothesis tests are carried out for the effective clearance through the chi-square goodness-of-fit test. Unfortunately, the hypothesis test results that the effective clearance is subjected to normal distribution or $t$ distribution should be rejected at the 5% significance level. Therefore, the above analysis shows that it is hard to use an existing distribution to describe the effective clearance distribution of single bolt. So, the empirical cumulative distribution function (Non-parametric distribution shown in figure 4) can be used to calculate the probability level that the effective clearance is in a certain interval, which can be produced by Monte-Carlo sampling.
     - The probability that single bolt cannot transmit load
       Because it is not easily obtained a reasonable and effective probability distribution of the effective clearance according to the sampling results by the curve fitting method. So, the effective clearance
probability distribution of single bolt could only be calculated the statistical method, the effective clearance probability distribution under the sample quantity $10^6$ is listed in table 2.

The probability that the effective clearance of single bolt is in the interval [0.3mm,0.35mm] is 38.5104% and also is the maximum value, as listed in table 2, and the probability that the single bolt effective clearance is less than 0.05mm is 0.1382%, the probability that the single bolt effective clearance is greater than 0.6mm is 0.0512%. Thus, most bolts are assembled in the hole center on the connection, and only a few bolts deviation center. Furthermore, the probability that the single bolt effective clearance is not in the interval [0.158mm, 0.546mm] is 2.8770%, that is, the probability that the single bolt cannot transmit the load is $p=2.8770\%$ on the bolts group connection.

![Figure 4](image)

**Figure 4.** Effective clearance distribution fitting curves of single bolt under different sampling quantity.

2) The probability that different quantity bolts cannot transmit the load

According to the three-layer $k$-out-of-$n$ uncertainty propagation model of the effective clearance of the bolt group connection, the probability that there are 10 and more than 10 bolts whose effective clearance are not in the transmitted load interval [0.158mm, 0.546mm] is $8.55 \times 10^{-10}$, less than $1 \times 10^{-9}$, and could be regarded as an extremely impossible event; The probability that there are 9 and more than 9 bolts whose effective clearance are not in the transmitted load interval [0.158mm, 0.546mm] is
1.82\times 10^8$, less than $1\times 10^7$, and could be regarded as an impossible event. The two above results mean that if the transmitted load interval of the effective clearance of the bolt group connection is given, the minimum quantity of the bearing bolts on the group connection could be determined under certain risk probability. Therefore, it could provide some instruction for the windshield bolts installation clearance design and bearing risk assessment through the effective clearance uncertainty propagation analysis results above.

| Effective clearance interval/mm | Probability | Effective clearance interval/mm | Cumulative probability |
|-------------------------------|-------------|---------------------------------|------------------------|
| 0-0.05                        | 0.001382    | 0-0.05                          | 0.001382               |
| 0.05-0.1                      | 0.005046    | 0-0.1                           | 0.006428               |
| 0.1-0.15                      | 0.015009    | 0-0.15                          | 0.021437               |
| 0.15-0.2                      | 0.038474    | 0-0.2                           | 0.059911               |
| 0.2-0.25                      | 0.087104    | 0-0.25                          | 0.147016               |
| 0.25-0.3                      | 0.181157    | 0-0.3                           | 0.328173               |
| 0.3-0.35                      | **0.385104**| 0-0.35                          | **0.713277**           |
| 0.35-0.4                      | 0.174952    | 0-0.4                           | 0.888229               |
| 0.4-0.45                      | 0.072274    | 0-0.45                          | 0.960502               |
| 0.45-0.5                      | 0.027496    | 0-0.5                           | 0.987998               |
| 0.5-0.55                      | 0.009023    | 0-0.55                          | 0.997021               |
| 0.55-0.6                      | 0.002467    | 0-0.6                           | 0.999488               |
| 0.6-0.7                      | 0.000512    | 0-0.7                           | 1                      |

5. Conclusions
Taking into account that the bearing capacity of the bolt group connection is related to the effective clearance in the load direction of single bolt, and its bearing reliability can be described by the $k$-out-of-$n$ reliability model. The bolt group connection of civil aircraft windshield frame is regarded as the analysis object. Based on the $k$-out-of-$n$ three-layer uncertainty analysis model, the relationship between the bearing risk and the effective clearance probability distribution is discussed. The analysis results can be used for further study of clearance design and fatigue life analysis of the bolt connection piece.

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