Lexicase Selection at Scale

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ABSTRACT
Lexicase selection is a semantic-aware parent selection method, which assesses individual test cases in a randomly-shuffled data stream. It has demonstrated success in multiple research areas including genetic programming, genetic algorithms, and more recently symbolic regression and deep learning. One potential drawback of lexicase selection and its variants is that the selection procedure requires evaluating training cases in a single data stream, making it difficult to handle tasks where the evaluation is computationally heavy or the dataset is large-scale, e.g., deep learning. In this work, we investigate how the weighted shuffle methods can be employed to improve the efficiency of lexicase selection. We propose a novel method, fast lexicase selection, which incorporates lexicase selection and weighted shuffle with partial evaluation. Experiments on both classic genetic programming and deep learning tasks indicate that the proposed method can significantly reduce the number of evaluation steps needed for lexicase selection to select an individual, improving its efficiency while maintaining the performance.

CCS CONCEPTS
- Mathematics of computing → Genetic programming; - Computing methodologies → Machine learning.

KEYWORDS
genetic programming, lexicase selection, parent selection, deep learning, image classification.

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1 INTRODUCTION
Genetic programming (GP) is a general methodology that has been widely embraced to tackle various kinds of problems including program synthesis, symbolic regression, statistical modeling, and machine learning. GP evolves programs by applying operations such as mutation and parent selection, mimicking the natural evolution processes, to the population of programs. Among many techniques that have been developed to determine the performance of individuals in parent selection, lexicase selection [13, 26] and its variants [9, 12, 13, 21] have been demonstrated to have more promising and robust performance than others [6, 7, 15] in a number of applications. The key idea of the lexicase selection is to gradually eliminate candidates by evaluating the population on each case in a randomly shuffled sequence of data. This procedure has been shown to bolster the diversity and generality in populations, which enhance the performance of evolved programs.

Some recent work also shows that lexicase selection can be applied in rule-based learning systems [1], symbolic regression [20], machine learning [18, 19], evolutionary robotics [14, 17], and deep learning [5], helping enhance model performance and generalization. Ding and Spector [5] propose gradient lexicase selection, a variant of lexicase selection that shows promising results on improving the generalization of deep neural networks (DNNs). Despite these recent successes, one potential drawback of the lexicase selection algorithm comes to our attention, especially for problems that involve computationally-heavy programs and large-scale datasets. Modern computing hardware, such as GPU clusters, have moved towards single-instruction multiple-data (SIMD) architectures. Since lexicase selection operates in a sequential single-data-stream fashion, it hardly can be optimized with parallel computing to reduce its running time. For example, in Ding and Spector [5], each lexicase selection process involves running inference of a deep neural network on at most 50,000 training cases per generation, with a batch size of just 1.

In this work, we aim to explore ways to speed up lexicase selection while preserving its core behavior of performing sequential evaluation of training cases for selection. An essential observation of the lexicase selection algorithm is that the training cases are considered in randomly-shuffled ordering, with cases earlier in the ordering receiving more attention than those later in the ordering. It is straightforward to presume that if the early training cases have more selection pressure, the selection process may be terminated earlier with a reduced number of evaluations on training cases. With this intuition, we revisit the weighted shuffle methods [29] that shuffle the training cases in a non-uniform manner for lexicase selection, with the goal of finding a weighted shuffle that does not negatively affect problem-solving performance but can improve the average running time of lexicase selection.
While Troise and Helmuth [29] did extensive experiments and comparisons of different shuffling methods, a prominent missing piece is that they only investigate if weighted shuffling has an effect on the final performance, but not the algorithm running time. There were two potential reasons: 1) the computation needs for classic GP problems are usually small, so the common implementation of lexicase selection assumes that the population has already been evaluated by all the training cases, in which case the selection is performed on the error vectors; 2) the weight assignment for weighted shuffling is based on those error vectors produced by a complete evaluation. However, when using lexicase selection for problems with computationally-heavy evaluation process and/or large-scale datasets, e.g., deep learning problems, it is obvious to see that such evaluate-then-select paradigm is no longer appropriate as it will introduce a lot of redundancy in computation.

Our goal is to reduce the number of evaluations needed for selection, in which case only the evaluations that are necessary for the selection process will be performed. This also means that only partial evaluation of the training set will be available for assigning weights to each training case for the weighted shuffling. To solve the above issues, we propose Fast Lexicase Selection: a new variant of lexicase selection method using weighted shuffle with partial evaluation. Our method performs evaluation and selection simultaneously. It can also assign and update weights for shuffling training cases based on evaluating only a portion of the training cases. We also extend the idea of our method to other variants of lexicase selection, e.g., gradient lexicase selection [5], to target on problems such as deep learning.

For experiments, we test the proposed algorithm on both the classic GP problems and the benchmark deep learning problem (image classification), where the latter involves large-scale datasets and heavy computation. Empirical results show that the proposed algorithm has a significant improvement in efficiency with a reduced number of evaluations, while maintaining similar performance to the regular lexicase selection.

2 BACKGROUND AND RELATED WORK

2.1 Preliminaries of Lexicase Selection

Lexicase selection considers performance on individual training cases as opposed to aggregated fitness or accuracy metrics [13, 26]. This has the benefit of including some semantic information about the performance of the population to potentially help guide search. Due to the lack of aggregation, lexicase selection tends to select specialist individuals that trade being somewhat good at all cases for being elite on a subset of them [10, 11]. This tends to have beneficial effects on behavioral diversity [9, 22], and success rates when compared to tournament and fitness-proportionate selection [13].

The most recent work [5] using this selection method proposed an evolutionary framework, gradient lexicase selection, that combines stochastic gradient descent and lexicase selection to improve the generalization of deep neural networks.

For every selection event, the training cases are shuffled into a random order that will be used to select one parent. Using this random case ordering, lexicase selection filters down the population by iteratively removing all individuals that do not exhibit exactly the best performance on each case, in the order they are presented. When a single individual remains, it is returned as the selected parent. If all the cases have been traversed and there are still individuals in the population, a random individual from the remaining group is returned.

One potential issue with lexicase selection and its variants in general, however, is that the selection process operates in a sequential single-data-stream fashion. In other words, the algorithm’s time complexity can not be efficiently reduced by using modern parallel computing architectures. When the training cases for selection are large scale and the evaluation of each training case is computationally expensive, lexicase selection may have significant disadvantages when it comes to computation. In this work, our goal is to improve the running time of lexicase selection while maintaining its core behavior of looking at each individual case instead of using aggregated fitness metrics.

It is important to note that in lexicase selection, the cases are shuffled between selection events, so different cases are given different priority over the course of the selection process. In general, training cases that show up earlier have more impact on the selection process, since they are responsible for filtering out the most individuals from the population. It is likely that cases that come at the end have little to no effect as the selected individual could have been returned before these cases are ever seen.

2.2 Improving Selection Efficiency

There is a large historical literature surrounding tweaking selection methods to improve efficiency in GP. Gathercole and Ross [8] introduce Dynamic Subset Selection, where subsets of the training data that are harder are sampled and used for selection in tournament selection. Poli and Langdon [23] ran GP backwards, only evaluating individuals that would eventually be selected to be part of a tournament selection pool. More recently, Chitty [2] harnesses the power of parallel computing in order to speed up tournament selection. These methods have shown great strengths in increasing the efficiency of selection methods in GP.

More specifically for lexicase selection, de Melo et al. [3] introduced Batch Tournament Selection (BTS), a hybrid of tournament and lexicase selection, which attempt to incorporate the idea of lexicase selection into tournament selection to improve both efficiency and quality of solutions. Aenugu and Spector [1] proposed Batch Lexicase Selection, which is a variant of lexicase selection that considers batched data during selection events. However, Ding and Spector [5] find that batch lexicase selection does not perform as well as lexicase selection on some large-scale problems, such as image classification. In this work, our method aims to directly increase the efficiency of lexicase selection, which can be further extended to the above hybrid methods.

2.3 Weighted Shuffle

Lexicase selection with weighted shuffle [29] is an attempt to change the way cases are shuffled every selection event. Instead of uniform random shuffling, weighted shuffling techniques bias the resultant ordering of cases based on a given metric. In this work, we will be considering only the weighted and ranked shuffling methods, with the Number-of-Zeros (easy first) and Number-of-Nonzeros (hard first) bias metrics.
Weighted shuffle. Weighted shuffle assigns a weight to each training case based on the chosen bias metric. When selection requires a shuffled list of cases to be built, cases are chosen iteratively where cases with higher weights have a higher chance of being selected. This case shuffling technique could be thought of as a roulette wheel, where cases with a higher weight have a larger slice of the roulette wheel. Every time we need to pick a case to add to the list, we update the case probabilities and re-roll the wheel. This entire process is repeated for every selection event in a generation, using the same case weights at the beginning.

Ranked shuffle. Ranked shuffle first ranks the cases based on the chosen bias metric, and sorts the cases using this ranking. Then, an upper bound is randomly selected between 1 and the number of training cases, inclusive. The selected case index is then chosen randomly from 1 to the upper bound, inclusive. The case at this selected index is added as the first case in the shuffled case list. This process is repeated for the remaining cases, building up the shuffled case ordering. Ranked shuffle is likely to place cases with low rank first in the final case ordering as these low ranks are likely to be between 1 and the selected upper bound. This method differs from Weighted shuffle due to it ignoring the magnitude of differences in weight between cases.

Bias Metrics. The bias metrics used in the weighted shuffle selection process inform the shuffling method of how to assign weights to each training case. Past implementations of weighted shuffle have had access to how every individual in the population performed on each training case before selection begins. Therefore, this information can be utilized in the selection process as follows. The Number-of-Zeros bias metric counts the number of individuals that achieve zero error on a given training case. Shuffling using this metric would result in a bias towards easier cases earlier in the case ordering as easy cases likely have more individuals achieving perfect scores on them. The Number-of-Nonzeros bias metric counts the number of individuals that do not have zero error on a given training case. This metric would bias the case ordering towards placing harder cases earlier in the shuffle. For the applications studied in this work, we propose a general system to update these case weights without evaluating every individual on every case (as this process might be computationally intensive when performing large-scale evolutionary optimization.

While Troise and Helmuth [29] compared these weighted shuffling methods by referring to their solution rates and behavioral diversity, an empirical study comparing these methods with respect to efficiency was not performed.

3 METHOD

In this section, we first introduce a new method, weighted shuffle with partial evaluation, that can assign and update weights of training cases with a partially-observed performance measure. We then apply this method to lexicase selection and propose two new variants of the algorithm, fast lexicase selection and fast gradient lexicase selection, for which the computation cost may be effectively reduced by having less training cases required to be evaluated.

Algorithm 1: Fast Lexicase Selection for Parent Selection

Data:
- cases - a sequence of all the data samples to be used in selection with default ordering
- candidates - the entire population of programs
- \( W \) - a weight vector for all the cases

Result:
- an individual program to be used as a parent
- an updated weight vector \( W \)

\[
\text{shuffled\_cases} \leftarrow \text{Weighted\_Shuffle}(\text{cases}, W)
\]

for case in shuffled\_cases do
  \( i \leftarrow \text{the index of case in cases} \)
  results \( \leftarrow \text{evaluate candidates on case} \)
  \( W[i] \leftarrow \text{Bias\_Metric(results)} \)
  candidates \( \leftarrow \text{the subset of the current candidates that have exactly best performance on case} \)
  if candidates contains only one single candidate then
    return candidate, \( W \)
  end

candidate \( \leftarrow \) a randomly selected individual in candidates
return candidate, \( W \)

3.1 Weighted Shuffle with Partial Evaluation

Our goal is to harness the benefit of weighted shuffle to reduce the number of evaluations needed to be performed on training cases. In our case study, and the work performed by [29], the weighted shuffle method requires all the training cases to be evaluated before the lexicase selection process, which is not actually reducing the number of individual program evaluations in practice.

The weighted shuffle method scores each training case based on some bias metric that summarizes the performance of the population on that specific case. In this work, we consider the scenario that not all the training cases are evaluated during selection at each generation. We propose a novel method, weighted shuffle with partial evaluation, to handle such a situation.

We begin by initializing a weight vector \( W = [w_1, w_2, \cdots, w_n] \) as the weights assigned to each of the \( n \) training cases, with some pre-defined bounds \( w_i \in [w_{\text{min}}, w_{\text{max}}], i = 1, 2, \cdots, n \). With a larger value of \( w \), the training case is more likely to appear earlier when shuffled.\(^1\) The value of \( w_{\text{min}} \) and \( w_{\text{max}} \) depends on the bias metrics one choose to use. In general, we assume that \( w_{\text{min}} > 0 \) to ensure that easy cases still have possibility to come early in the ordering.

Since none of the training cases have been evaluated prior to the selection event, we propose two options for assigning the initial weights: 1) Default-Min: let \( w_1 = w_2 = \cdots = w_{\text{min}} \), such that the unevaluated cases are default to be least likely to appear early among all the cases; 2) Default-Max: \( w_1 = w_2 = \cdots = w_{\text{max}} \), meaning the unevaluated cases are most likely to appear early in the shuffled queue of training cases.

\(^1\)The shuffling method used in this work is the standard weighted sampling from the multinomial probability distribution, as defined by the weights. We use the PyTorch implementation of the function torch.utils.data.WeightedRandomSampler.
Selection lexicase selection, Fast Lexicase Selection
tion method into the algorithm pipeline, we propose two variants of
By introducing the proposed weighted shuffle with partial evalu-
3.2 Fast Lexicase Selection
sequence of training cases based on the current weight vector
$\mathcal{W}$, which is a variant of lexicase selection that focuses on optimizing
depth neural networks. An outline of fast gradient lexicase selection is shown in Alg. 2.

4 EXPERIMENTS ON GP PROBLEMS
In order to compare the efficiency of the above selection methods in a well established field, we performed GP runs evolving solutions to two different program synthesis problems. We introduce a new measure to evaluate the efficiency of said selection strategies and also verify that the relative success rate results match those found by Troise and Helmuth [29]. Outlined below is the experimental setup and the results from these experiments.

4.1 Experimental Setup
Problems. The two problems chosen for this investigation come from the first General Program Synthesis Benchmark Suite [12]. This benchmark suite contains programming problems that deal with a range of data types and programming constructs. The two problems chosen from this benchmark suite are Mirror Image and Last Index of Zero. Solution programs to Mirror Image receive two vectors of integers, and return a Boolean indicating whether the vectors are the reverse of each other. For Last Index of Zero, solutions must take as input a vector of integers (one of which is a zero), and return the index of the last zero in the vector. The datasets we used consist of input and output cases for these problems, as outlined in the benchmark suite [12].

PushGP. We use the PushGP system to evolve solutions to the problems described above. PushGP is a GP system that evolves programs written in the Push programming language [25, 28]. The Push programming language is a stack-based language that facilitates the use of multiple data types and complex programming paradigms such as iteration and recursion [27]. We used an implementation of PushGP, Clojush2 that is written in Clojure. Table 1 outlines the system parameters used for the Mirror Image and Last Index of Zero problems.

Algorithm 2: Fast Gradient Lexicase Selection for Optimizing Deep Neural Networks

Data:
- data - the whole training dataset in a default order
- candidates - set of $p$ instances of the DNN model
- $\mathcal{W}$ - a weight vector for all the samples in data

Result:
- an optimized DNN model

// K training epochs
set $w_{\text{min}} = 1$ and $w_{\text{max}} = p + 1$
initialize $\mathcal{W}$ with $w_{\text{min}}$ (Default-Min) or $w_{\text{max}}$ (Default-Max)
for epoch = 1 : $K$
subsets ← $p$ equal-size subsets obtained through random sampling from the entire data without replacement
apply gradient descent and backpropagation to optimize each of the $p$ candidates on each of the $p$ subsets respectively
shuffled_cases ← Weighted_Shuffle(data, $\mathcal{W}$)
parent ← None
for case in shuffled_cases do
	$i$ ← the index of case in data
	results ← evaluate candidates on case
	$W[i]$ ← Bias_Metric(results)
	candidates ← the subset of candidates that have exactly best performance on case
	if candidates contains only one single candidate then
		parent ← candidate break
	end
if parent is None then
	parent ← a randomly selected individual in candidates
end
candidates ← set of $p$ instances of the DNN model copied with the same parameters as parent
return parent

For each generation, we first perform weighted shuffle on the sequence of training cases based on the current weight vector $\mathcal{W}$. Let $p^*$ denote the population size and $p \in [0, p^*]$ to be the size of current population (after potential removal of some offspring by the ongoing selection event). During the selection process, after each evaluation step on training case $i$, we update its assigned weight $w_i$ according to the performance of the current population, by using some bias metrics.

3.2 Fast Lexicase Selection
By introducing the proposed weighted shuffle with partial evaluation method into the algorithm pipeline, we propose two variants of lexicase selection, Fast Lexicase Selection and Fast Gradient Lexicase Selection.
Measures. We propose a new measure, the average number of evaluations, in attempt to measure the efficiency of the various selection methods discussed above. This metric is a rough measure of the computational effort required to perform the lexicase selection procedure to select individuals. Instead of using wall-clock times like explored in previous selection strategy comparisons [3, 8, 23], we use a measure that provides a hardware-agnostic efficiency comparison. We define an evaluation as the act of either computing the error for an individual (by evaluating the Push program they produce) on a specific case, or looking up its error on this case from a different selection. To calculate the number of evaluations for a single selection event, we add together the number of individuals remaining after every case is visited. For example, if there are 10 individuals in the population, and 3 remain after visiting the first case, and 1 remains (and is selected) after visiting the second case, then the number of evaluations for this selection event is $10 + 3 = 13$. This metric represents the number of times (in an isolated selection event) a new individual would have to be evaluated on a new case from scratch.

It is important to note that the selections we are performing in our GP runs are not isolated selection events as we can (and do) reuse evaluations of an individual on a specific training case from other selection events in the same generation to help increase performance. This is because it is sometimes the case in lexicase selection that one individual is evaluated on the same training case twice or more in a generation during the selection of different parents. Furthermore, the PushGP implementation we are using evaluates all individuals on all training cases at the beginning of every generation, for use in all selection events. Nevertheless, we believe that measuring the average number of evaluations is still a meaningful metric that can be broadly used to compare the efficiency of selection procedures. For different implementations of PushGP, or uses of lexicase selection outside of GP like those described in this paper and beyond, this metric can be used as a much closer correlate of run time. The lower the average number of evaluations, the more efficient the selection procedure is likely to be. This measure can also be thought of as how effective the cases are at filtering down the population size to more computationally manageable numbers.

To calculate the number of evaluations for a given generation, we sum together the number of evaluations performed in every selection event in that generation. For our GP runs, we average these values across all of the runs that are still active at this generation. This means that runs finishing early will not deceptively decrease the number of comparisons occurring during that generation. Along with this, we will also measure whether the various weighted shuffling methods in fast lexicase selection are comparable to lexicase selection in terms of success rate. The number of successes in the following experiments refers to the number of runs (out of the 50 total) that successfully evolve a solution with perfect scores on all the held out training cases.

4.2 Results

Table 2 presents the differences in success rate across these selection methods. None of the weighted or ranked shuffling methods performed significantly better than lexicase selection. Also, none of the hard-first metrics performed significantly worse than lexicase selection. These findings align with those found by Troise and Helmuth [29].

Figures 1 and 2 show how the average number of comparisons per generation are affected by these different selection methods for the Mirror Image and Last Index of Zero problems, respectively. It is clear from both of these figures that ranked shuffle with Number-of-Nonzeros is consistently performing less evaluations per generation than any of the other methods.

These results show that there are significant improvements in efficiency when using a method based on weighted shuffling with a hard-first metric. We also find that using hard-first shuffling methods have comparable performance (in terms of success rate) to randomly shuffled lexicase selection in GP. This result leads us to believe that using weighted shuffle in other applications of lexicase selection, especially ones with computationally intensive evaluations, would potentially reduce the overall computational cost of lexicase selection, while maintaining the numerous benefits it provides. In order to verify this claim, we repeat the above experiments in a different, more computationally intensive, domain: image classification with deep neural networks.

| Parameter                          | Value |
|------------------------------------|-------|
| runs per problem                   | 50    |
| population size                    | 1000  |
| maximum generations                | 300   |

| Genetic Operator                  | Probability |
|-----------------------------------|-------------|
| alteration                        | 0.2         |
| uniform mutation                  | 0.2         |
| uniform close mutation            | 0.3         |
| uniform mutation followed by alteration | 0.5 |

Table 1: PushGP system parameters for the Mirror Image and Last Index of Zero problems. These parameters were picked as a rule of thumb to better compare to prior work.

| Selection Method               | Success Rate (/50 Runs) |
|--------------------------------|-------------------------|
|                                | Mirror Image | Last Index of Zero |
| Lexicase                       | 43           | 20               |
| Weighted - Num-Nonzeros        | 45           | 11               |
| Ranked - Num-Nonzeros          | 46           | 21               |
| Weighted - Num-Zeros           | 48           | 2                |
| Ranked - Num-Zeros             | 35           | 5                |

Table 2: Success Rates of GP runs on the Mirror Image and Last Index of Zero problems, utilizing a variety of selection techniques. Underlined are the results that were significantly worse than lexicase selection. No results were significantly better than lexicase selection across either problem.
Figure 1: Average number of evaluations performed in a given active generation over evolutionary time while solving the Mirror Image problem. The hard first metrics (those using the Number-of-Nonzeros bias metric) are shown in red and the easy first metrics (Number-of-Zeros) are shown in blue. The weighted shuffling technique is presented as dotted lines, and the ranked shuffling technique is shown in solid lines. Lexicase selection is shown in black. Ranked shuffling using the Number-of-Nonzeros bias metric consistently performs less comparisons per generation than all other shuffling methods, including lexicase selection (random shuffling).

Figure 2: Average number of evaluations performed in a given active generation over evolutionary time while solving the Last Index of Zeros problem.

5 EXPERIMENTS ON IMAGE CLASSIFICATION

In this section, we present the experimental results of testing the proposed fast gradient lexicase selection method on the image classification problem, which is a popular problem that has been extensively studied in computer vision and deep learning literature. We use a common network architecture, VGG [24], and test our method on a benchmark dataset, CIFAR-10 [16]. The dataset comprises of 32 × 32 pixel real-world RGB images of common objects (50,000 for training, 10,000 for testing).

5.1 Bias Metrics and Weight Assignment

In this experiment, we consider the same two bias metrics as we did for the GP problems: Number-of-Zeros (Num-Zeros) and Number-of-Nonzeros (Num-Nonzeros). For the image classification problem, the model returns a probability distribution of all the classes for each input case. So, we take the one with largest probability as the discrete prediction output. The Num-Zeros metric \( n_{\text{zeros}} \) counts the number of individuals in the current population that achieve zero error, i.e., make the correct prediction, on the given training case \( i \). We therefore assign the according weight as \( W[i] = n_{\text{zeros}} + 1 \), with +1 to avoid zero weights. This metric can be interpreted as giving easier cases more weight, so they tend to appear earlier in the sequence of training cases. Alternatively, the Num-Nonzeros metric \( n_{\text{nonzeros}} \) counts the number of individuals that achieve non-zero error, which, in this experiment is referred to making a wrong prediction. This metric, as opposed to \( n_{\text{zeros}} \), gives more priority to hard cases.

5.2 Implementation Details

We follow the general experiment setups as described in [4, 5]. The network architecture used in this work is VGG [24]. We use a population size \( p = 4 \) for both gradient lexicase selection and fast gradient lexicase selection. We set the total number of epochs as \( 200(p + 1) \).

5.3 Results

We evaluate the proposed fast gradient lexicase selection method against the original lexicase selection on the image classification problem. Analyses on both speed and accuracy are presented as follows.

Speed. First, we compare the number of evaluations performed among all the design choices of ways to initialize default weight and bias metrics. The count of evaluations performed in each generation can be viewed as the theoretical runtime of lexicase selection, under the assumption that a single evaluation step, e.g., inference of a DNN on a single input case, takes significantly more time than other operations such as comparing values.

The raw evaluation counts per generation is shown in Fig. 3. We can see that in this problem, most of the evaluations happen in the last few generations. Fast gradient lexicase selection in general ends up with fewer generations for evolution, and requires fewer evaluations to be performed in each generation.

To have a better view of the results, we create another view of Fig. 3 with the last 50 generations of each method, aligned by the end. We focus on those later generations because early generations require many fewer evaluations, and thus does not contribute much to the total runtime. We also perform a moving average of 20 generations to smooth the values of y-axis. As shown in Fig. 4, we can see a clear comparison between the methods. The fast gradient lexicase
selection with Default-Max and Num-Nonzeros requires significantly fewer evaluations throughout generations. These results align well with our findings for GP, as described in Sec. 4.2.

Accuracy: While the results show that fast gradient lexicase selection is significantly more efficient than the regular method, we also need to test whether the reduced number of evaluations has an effect on the performance on the image classification task. Table 3 shows quantitative results of model performance. We can see that the performance of all the methods are similar. To statistically illustrate this, for each of the fast gradient lexicase selection methods, we perform the one proportion z-test against the regular gradient lexicase selection.

Let $p$ be the accuracy of tested method (fast gradient lexicase selection) and $p_0$ be the hypothesized accuracy of the baseline method (gradient lexicase selection). For the z-test, we use the following null hypotheses:

$$H_0 : p = p_0$$

and the left-tailed alternative hypothesis:

$$H_1 : p < p_0$$

If the p-value that corresponds to the test statistic is less than the significance level, we can reject the null hypothesis. From Table 3, we can see that if we choose the common significance level 0.05, none of the methods have a p-value less than 0.05. So, we conclude that the performance of fast gradient lexicase selection (with any design choice considered in this work) is not significantly different from the performance of regular gradient lexicase selection.

6 DISCUSSION

When using lexicase selection, some cases have a large impact on how many individuals are filtered out of the selection pool, while other cases have little to no impact. Additionally, some cases are synonymous (or nearly synonymous), in that they test similar inputs and require very similar behavior of individuals to perform well on. While one synonymous case may filter the selection pool, consequent cases may have little effect.

Our primary motivation of using weighted shuffle for lexicase selection is to bias the shuffle such that cases that have more impact on the filtering of the selection pool more often occur near the start of the shuffled list of cases. By doing so, the selection pool size will shrink more rapidly, requiring fewer evaluations during selection. Before this work, it was an open question whether certain shuffling techniques or bias metrics would result in more rapid filtering of the selection pool, and if so, whether they would also have any effect on problem-solving performance.

Our results, both using PushGP for program synthesis and deep neural networks for image classification, indicate that weighted shuffle does have a positive impact on running time while not negatively affecting problem-solving performance compared to uniform shuffle. In particular, methods that weight cases that are more difficult to answer correctly more heavily (Num-Nonzeros) used the fewest comparisons or evaluations, and therefore better running time.

More difficult cases are likely passed by fewer individuals in the population than easier cases. When putting more emphasis on these cases by having them biased to occur earlier in the weighted
Table 3: Image classification results. We report the percentage accuracy (acc.) for all the methods. For each of the fast gradient lexicase selection methods, we also report the p-value of one proportion z-test against the regular gradient lexicase selection. A p-value greater than the significance level (0.05) means there is no significance different between the two methods in performance.

| Method                        | acc. | p-val |
|-------------------------------|------|-------|
| Gradient Lexicase Selection   | 93.34| -     |
| Fast Lexicase - Nonzeros, Default-Max | 93.19| 0.276 |
| Fast Lexicase - Nonzeros, Default-Min | 92.99| 0.085 |
| Fast Lexicase - Zeros, Default-Max | 93.29| 0.421 |
| Fast Lexicase - Zeros, Default-Min | 93.17| 0.25  |

shuffle, they likely remove more individuals from the selection pool, keeping around the smaller group of individuals who perform better on them. However, this result is not obvious a priori; it might have been the case that all individuals perform equally poorly on some difficult cases, meaning that they apply less filtering power on the selection pool, and that putting easier cases first (or a uniform shuffle) would produce faster filtering.

Even though biasing easier cases earlier (Num-Zeros) did not reduce the number of evaluations of gradient lexicase selection as much biasing harder cases first, it did nevertheless produce significantly fewer evaluations than uniform shuffle in our deep learning experiments. While not intuitive, we suppose that both easier and harder cases may result in increased filtering pressure compared to average cases. A possible explanation is that the benefit of weighted shuffling may actually come from the behavior of focusing on some set of cases (no matter easy or hard ones), which leads to faster evolution compared to looking at random cases.

We also find that the initialization of case weights is important to consider when performing fast gradient lexicase selection. When using a metric that places higher weight on hard cases, it seems to be beneficial to initialize all the cases to be maximally difficult with Default-Max. This might be because there is an initial bias towards looking at cases that have not been seen yet (as these are set as being hard) and therefore leads to a better initial exploration of the case space. As we iteratively assign weights (that might be less than or equal to the max), the probability we select these seen cases becomes less than that for an unseen (maximally hard) case. We find that this explanation is also supported by the easy first metric. When using a shuffling method that places emphasis on easy cases, such as Number-of-Zeros, initializing cases to be minimally difficult would place an initial bias towards looking at cases that have not been considered yet. This might explain why Default-Max outperforms Default-Min for the Number-of-Nonzeros bias metric, and the opposite for the Number-of-Zeros metric in our fast gradient lexicase selection experiments.

7 CONCLUSION AND FUTURE WORK

In this work, we demonstrate the merit of using weighted shuffle to increase the efficiency of the selection procedure. In order to overcome the efficiency disadvantage of lexicase selection for large-scale evolutionary computation, we propose fast lexicase selection, which is an attempt to combine weighted shuffle and online weight updating with lexicase selection. The experimental results on classic GP problems show that there are significant improvements in efficiency when using the hard-first weighted shuffling methods, and the performance is still comparable to regular lexicase selection.

We further extend the idea to the gradient lexicase selection method, which can better demonstrate the enhanced efficiency of fast lexicase selection in computationally intensive domains. We test our method on a large-scale image classification problem. Experimental results show that fast lexicase selection with different weighting methods outperforms regular lexicase selection in terms of efficiency, without hindering accuracy. More specifically, the hard-first metric with the initialization of cases to be maximally hard gives the best efficiency. We also find that when using the fast lexicase selection method, initializing cases as being maximally hard when using a hard-first metric, or minimally hard when using an easy-first metric seems to be beneficial in decreasing the number of evaluations needed for every generation.

Through this work, we highlight the importance of using the information about how the population performs on training cases in the selection procedure to identify cases with higher selection pressure, and thus improve the efficiency of the selection procedure. This is especially important in large scale evolutionary optimization problems, where each evaluation step might be costly, or the total number of evaluations might be large. For future work, we look forward to the exploration of better ways to extract the incremental information about the relationship between population and training data, in order to improve the algorithm in terms of both efficiency and performance.

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