Nonlinear effects in microwave photoconductivity of two-dimensional electron systems

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We present a model for microwave photoconductivity of two-dimensional electron systems in a magnetic field which describes the effects of strong microwave and steady-state electric fields. Using this model, we derive an analytical formula for the photoconductivity associated with photon- and multi-photon-assisted impurity scattering as a function of the frequency and power of microwave radiation. According to the developed model, the microwave conductivity is an oscillatory function of the frequency of microwave radiation and the cyclotron frequency which turns zero at the cyclotron resonance and its harmonics. It exhibits maxima and minima (with absolute negative conductivity) at the microwave frequencies somewhat different from the resonant frequencies. The calculated power dependence of the amplitude of the microwave photoconductivity oscillations exhibits pronounced sublinear behavior similar to a logarithmic function. The height of the microwave photoconductivity maxima and the depth of its minima are nonmonotonic functions of the electric field. It is pointed to the possibility of a strong widening of the maxima and minima due to a strong sensitivity of their parameters on the electric field and the presence of strong long-range electric-field fluctuations. The obtained dependences are consistent with the results of the experimental observations.

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I. INTRODUCTION

Transport properties of two-dimensional electron systems (2DES’s) subjected to a transverse magnetic field were extensively studied in late 60s and early 70s. To the best of our knowledge, the first paper on this topic was published by Tavger and Erukhimov [1]. In this paper, the dissipative conductivity (diagonal component of the conductivity tensor) associated with the tunneling between the Landau levels (LL’s) accompanied by the impurity scattering of electrons was calculated as a function of the electric and magnetic field. The dissipative current-voltage characteristic obtained considering the impurity scattering in the Born approximation is given by a non-analytical dependence $j \propto E^{-2} \exp(-E_c^2/2E_b^2)$, where $E$ is the electric field, $E_c = \hbar \Omega_c/eL$, $\hbar$ is the Planck constant, $\Omega_c$ and $L$ are the cyclotron frequency and quantum Larmor radius, respectively, and $e$ is the electron charge. Later, it was shown [2] that more strict consideration of the impurity scattering leads to the “restoration” of the Ohm’s law, so that $j \propto E$ at $E < E_b = \hbar \Gamma/eL$, where $\hbar \Gamma$ is the LL-broadening, and $j \propto E^{-1}$ when $E_b < E < E_c$. Pokrovsky et al. [3] demonstrated that the inclusion of the processes of the electron inter-LL tunneling via bound impurity states can give rise to an exponential dependence similar to that obtained in Ref. [1] but with a smaller characteristic field $E_c$. Theoretical studies of the transport in a 2DES irradiated with microwaves or under non-equilibrium conditions associated with intraband or intersubband optical excitation were carried out in Refs. [4, 7, 26, 32]. In particular, it was predicted by Ryzhii [4] (see also Ref. [7]) that microwave radiation with the frequency $\Omega$ somewhat exceeding the cyclotron frequency or its harmonics $\Lambda \Omega_c$, where $\Lambda$ is an integer, can result in the absolute negative conductivity (ANC) in a 2DES. Nonlinear microwave photoconductivity associated with the effect of photon-assisted impurity scattering of electrons was studied theoretically in Refs. [8, 9, 10]. In particular, it was demonstrated that the multi-photon processes (both virtual and real) can significantly influence the dissipative and Hall components of the conductivity tensor. Since current-voltage characteristics with ANC inevitably exhibit ranges with the negative differential conductivity (NDC), it was clear that time that uniform states of a 2DES with ANC can be unstable [11].

An interest in theoretical studies of non-linear transport in 2DES’s has revived after the observation of the breakdown of the quantum Hall regime [12] (see, for example, the articles by Heinonen, et al. [13], Balev [14], Chaubet, et al. [15, 16], Komiyama, et al. [17, 18], and review by Nachtwei [19]).

Recent observations of vanishing electrical resistance/conductance in 2DES’s caused by microwave radiation [20, 21, 22, 23] have stimulated extensive efforts to clarify the nature of the uncovered effects and triggered a new surge of theoretical papers (see, for example, Refs. [24, 25, 26, 27, 28, 29, 30, 31, 32, 33]). The occurrence of the so-called zero-resistance/conductance states is primarily considered [24, 25, 26, 27, 32] as a manifestation of the effect of ANC associated with the photon-assisted impurity scattering [4, 7, 26, 32], pos-
sibly, complicated by the scattering processes involving acoustic phonons. The role of multi-photon-induced scattering processes was evaluated in Refs. 28, 29 using models similar to that considered earlier. Vavilov and Aleiner have developed a general approach based on the quantum Boltzmann equation which provides a description of non-linear effects in 2DES. In this paper, we use more simple and transparent model of the microwave conductivity bearing in mind the goal to obtain explicit analytical formulas describing non-linear effects, namely, formulas for the dependences of the photoconductivity maxima and minima on the microwave power and the electric field (i.e., on the ac and dc fields). In particular, we demonstrate that the magnitude of the microwave photoconductivity maxima and minima is a non-monotonic function of the microwave power and the electric field. It is also shown that an increase in the microwave power and/or the electric field leads to a marked shift of the maxima and minima and an increase in their width. Due to a high sensitivity of the microwave photoconductivity to the local electric field, the observable characteristics of the 2DES can be essentially affected by long-range electric-field fluctuations.

The obtained results shed light on some features of the effects observed experimentally.

After this introduction, in Sec. II, we write down a general formula for the dissipative current based on the notion that this current is associated with the spatial displacement of the electron Larmor orbit centers caused by the photon-induced impurity scattering processes. The probability of these processes is a function of the net dc electric field (including both the applied and Hall components) and the ac microwave electric field. This probability is presented as a sum of the terms corresponding to the participation of different number of real photons. Such a technique allows to bypass the diagram (perturbation) summation. In Sec. III, we calculate the microwave photoconductivity in relatively low dc electric field (Ohmic regime) as a function of the microwave frequency and power. Section IV deals with the calculation of the microwave photoconductivity in a strong dc electric field when the latter substantially affects the scattering processes. In Sec. V, we consider the effect of microwave radiation on intra-LL impurity scattering. Section VI deals with the discussion of the obtained results and its relevance to the pattern of the experimental observations.

II. GENERAL EQUATIONS

At low temperatures $T \ll \hbar \Omega_c$ and low electric fields $E \ll E_c$, the dissipative dark current (without irradiation) is associated mainly with the electron transitions within the same LL. Under the microwave radiation, the inter-LL electron photon-assisted transition can markedly contribute to the dissipative current. The microwave radiation can also affect the intra-LL scattering processes. Therefore, the microwave photocurrent, i.e., the variation of the dissipative current caused by irradiation can be presented as

$$j_{ph} = j^{(inter)}_{ph} + j^{(intra)}_{ph},$$

where $j^{(inter)}_{ph}$ is the contribution associated with the electron transitions between the LL’s with different indices stimulated by microwave radiation, the second term in the right-hand side of Eq. (1) is the variation of the intra-LL component of the dissipative current.

For the probability of the transition between the $(N, k_x, k_y)$ and $(N', k_x + q_x, k_y + q_y)$ electron states in the presence of the net dc electric field $E = (E, 0, 0)$ perpendicular to the magnetic field $H = (0, 0, \hbar \Omega)$ and the ac microwave field $E_\Omega = (\varepsilon e_x, \varepsilon e_y, 0)$ polarized in the 2DES plane ($e_x$ and $e_y$ are the components of the microwave field polarization vector), we will use the following formula obtained on the base of the interaction representation of the operator of current via solutions of the classical equations of electron motion:

$$W_{N,k_x,k_y,N',k_x+q_x,k_y+q_y} = \frac{2\pi}{\hbar} \sum_{M} |N_i| \left| \langle \Omega_{N',N}(L^2 q^2/2) \rangle \right|^2 \times J_M^2(\xi_\Omega(q_x, q_y)) \delta[M\hbar \Omega + (N' - N) \hbar \Omega_c + e E L^2 q_y].$$

Here $N$ is the LL index, $k_x$ and $k_y$ are the electron quantum numbers, $(q_x$ and $q_y)$ are their variations due to photon-assisted impurity scattering, $q = \sqrt{q_x^2 + q_y^2}$, $e = |e|$ is the electron charge, $N_i$ is the impurity concentration, and $V_q \propto q^{-1} \exp(-d_i q)$ is the matrix element of the electron-impurity interaction, which accounts for the localization of electrons in the $z$-direction, where $d_i$ is the spacing between the 2DES and the $\delta$-doped layer. The functions characterizing the overlap of the electron initial and final states are $Q_{N,N'}(\eta) = P_{N'}^{(N'-N)}(\eta) \exp(-\eta/2)$, $P_{N}^{(N'-N)}(\eta) \propto L_{N}^{(N'-N)}(\eta)$, where $L_N^{(\eta)}(\eta)$ is the Laguerre polynomial. The LL form-factor is determined by the function $\delta(\varepsilon)$, which at a small broadening $\Gamma$ can be assumed to be the Dirac delta function. The effect of microwave radiation is reduced to the inclusion of the energy of really absorbed or emitted $M$ photons $M\hbar \Omega$ in the transition energy balance (in the argument of the function $\delta$ in Eq. (2)) and the appearance of the Bessel functions $J_M(\xi_\Omega(q_x, q_y))$, that reflects the contribution of virtually absorbed and emitted photons.

Here

$$\xi_\Omega(q_x, q_y) = \frac{e \varepsilon}{m} \left| q_x e_x + q_y e_y - i(\Omega_c/\Omega)(q_x e_y - q_y e_x) \right| / \left| \Omega_c^2 - \Omega^2 \right|^2,$$

where $m$ is the electron effective mass. Near the cyclotron resonance, the dependence $\xi_\Omega(q_x, q_y)$ is close to isotropic. Disregarding the polarization effects, we will assume in the following that

$$\xi_\Omega(q_x, q_y) = \xi_\Omega \cdot Lq.$$
Here \(\xi_0 = \sqrt{\langle \xi_0(q_x, q_y) \rangle^2}/Lq\), where the symbol \(\langle \rangle\) means averaging over the microwave field polarization, so that \(\xi_0 = (eE\sqrt{\Omega_0^2 + \Omega^2}/\sqrt{2}\pi ML)|\Omega_0^2 - \Omega^2|\). When \(\xi_0\) becomes of order of unity, the amplitude of the Larmor orbit center oscillation in the microwave field is about \(Lq\).

Taking into account that for the transitions from the LL’s with \(N \gg 1\), which play the main role in a 2DES with a large filling factor, one can put \(P_N(\eta) \approx J_\Lambda(2\sqrt{N\eta})\), the inter-LL contribution to the photocurrent can be presented in the following form:

\[
\frac{\text{j}_{\text{ph}}^{(\text{inter})}}{N^\alpha} = \sum_{N, M, \Lambda, \Lambda^\prime \geq 0} \frac{f_N(1 - f_{N+\Lambda})}{\pi} \int dq_x dq_y \frac{q_y}{q^2} \exp\left(-2d_x q - \frac{L^2q^2}{2}\right) \\
\times J_\Lambda^2(\sqrt{2N}Lq)J_{\Lambda^\prime}^2(\xi_0 Lq)\delta(M\Omega - \Lambda^\prime\Omega_\nu + eEL^2q_y),
\]

where \(f_N\) is the Fermi distribution function.

### III. OHMIC REGIME

At \(E < E_b\), assuming for definiteness that \(\delta(\omega) = \Gamma/\pi(\omega^2 + \gamma^2)\), where \(\gamma = \Gamma/\Omega_\nu\), expanding the right-hand side of Eq. (5) over \(eELq\), taking into account that

\[
J_\Lambda^2(\sqrt{2N}Lq) \approx \cos^2(\sqrt{2N}Lq - (2\Lambda + 1)/\pi)/\sqrt{2N}Lq
\]

for large \(N\), and integrating, we arrive at

\[
\frac{\text{j}_{\text{ph}}^{(\text{inter})}}{N^\alpha} \propto E\Gamma \sum_{\Lambda, M} \frac{\Theta\Lambda \mathcal{R}_M(\xi_0)(\Lambda\Omega_\nu - M\Omega)}{(\Lambda\Omega_\nu - M\Omega)^2 + \Gamma^2}. \tag{6}
\]

Here

\[
\Theta\Lambda = \sum_N f_N(1 - f_{N+\Lambda})/\sqrt{N}, \tag{7}
\]

\[
\mathcal{R}_M(z) = \int_0^\infty dxJ_M^2(z x)\exp(-\beta x - x^2/2), \tag{8}
\]

where \(\beta = 2d_x L\). When \(\beta < 1\), Eq. (8) yields

\[
\mathcal{R}_M(z) \approx \exp(-z^2)I_M(z^2), \tag{9}
\]

where \(I_M(\eta)\) is the modified Bessel function. In particular, at \(z < 1\), function \(\mathcal{R}_M(z)\) can be approximated as

\[
\mathcal{R}_M(z) \approx \exp(-z^2)\frac{z^{2M}}{2^M M!} \left[1 + \frac{z^2}{4(M + 1)}\right]. \tag{10}
\]

For \(z \gg 1\), from Eq. (9) one can obtain

\[
\mathcal{R}_M(z) \approx \frac{1}{\sqrt{2\pi z}}. \tag{11}
\]

Taking into account that \(\mathcal{E}^2 \propto p_\Omega\), where \(p_\Omega\) is the microwave power density, introducing the characteristic microwave power \(P_\Omega = m\Omega^2/2\pi \approx 21.8m\Omega^3\), where \(\alpha = \epsilon^2/\hbar c \approx 1/137\) and \(c\) is the speed of light, and using Eqs. (6) and (9), for a 2DES with \(\beta < 1\) we arrive at

\[
\frac{\text{j}_{\text{ph}}^{(\text{inter})}}{N^\alpha} \propto E\Gamma \exp\left[-P f\left(\frac{\Omega}{\Omega_\nu}\right)\right] \\
\times \left\{I_1\left(P f\left(\frac{\Omega}{\Omega_\nu}\right)\right) \sum_{\Lambda} \frac{\Theta\Lambda(\Lambda\Omega_\nu - \Omega)}{(\Lambda\Omega_\nu - \Omega)^2 + \Gamma^2} + \cdots \right\}, \tag{12}
\]

where \(P = P_\Omega/\Omega\) is the normalized microwave power and \(f(\omega) = \omega(1 + \omega^2)/(1 - \omega^2)^2\). In particular, at \(P f(\Omega/\Omega_\nu) < 1\), Eq. (12) can be presented as

\[
\frac{\text{j}_{\text{ph}}^{(\text{inter})}}{N^\alpha} \propto E\Gamma \exp\left[-P f(\Omega/\Omega_\nu)\right] \\
\times \left\{\frac{P}{2} f\left(\frac{\Omega}{\Omega_\nu}\right) \sum_{\Lambda} \frac{\Theta\Lambda(\Lambda\Omega_\nu - \Omega)}{(\Lambda\Omega_\nu - \Omega)^2 + \Gamma^2} + \cdots \right\}. \tag{13}
\]

One can see from the first term in the right-hand side of Eqs. (12) and (13) that \(j_{\text{ph}}^{(\text{inter})} = 0\) at \(\Lambda\Omega_\nu = \Omega\), whereas \(j_{\text{ph}}^{(\text{inter})} > 0\) if \(\Lambda\Omega_\nu\) slightly exceeds \(\Omega\) and \(j_{\text{ph}}^{(\text{inter})} < 0\) (i.e., the dissipative microwave photocurrent is in opposition to the electric field) when \(\Lambda\Omega_\nu\) is somewhat smaller than \(\Omega\). As it also follows from Eqs. (12) and (13), \(j_{\text{ph}}^{(\text{inter})}\) exhibits maxima and minima at \(\Omega/\Omega_\nu = \Lambda - \delta(+)\) and \(\Omega/\Omega_\nu = \Lambda + \delta(-)\), respectively, where \(0 < \delta(+) < 1\) and \(\delta(-) < 1\). These maxima and minima correspond to the electron transitions with the absorption of one real photon. Apart from the single-photon maxima and minima, there are the maxima and minima associated with the two-photon (described by the second terms in the right-hand sides of Eqs. (12) and (13)) and multiple-photon absorption processes.

Using Eqs. (12) or (13), one can find that \(\delta(\pm) \approx \sqrt{3\Gamma}/\hbar\Omega_\nu\). At moderate microwave powers, estimating the LL broadening as (see, for example, Refs. 29, 31, 32)

\[
\Gamma = \sqrt{\Delta\Omega a/\pi\tau}, \tag{11}
\]

\(\Gamma = \sqrt{\Delta\Omega a/\pi\tau}\), where \(\tau\) is the electron momentum relaxation time estimated from the electron mobility at
by all other mechanisms, one can find that scattering time which describes the difference between the
oscillatory activity as a function of the microwave power at low and increase is rather slow; the span of the photoconduc-
monics) increase with increasing microwave power. This cyclotron resonance (as well as near the cyclotron har-
of the maximum and the depth of the minimum near the
Fig. 2. One can see from Figs. 1 and 2 that the height
the cyclotron resonance and its harmonics is shown in
with the paired maxima and minima corresponding to
frequencies calculated for a 2DES with
\( \Omega / \Omega_c = 100 \text{ GHz} \), \( a = 1 \), (b) \( \Omega / \Omega_c = 50 \text{ GHz} \), \( a = 1 \) (c) \( \Omega / \Omega_c = 100 \text{ GHz} \), \( a = 10 \), and (d) \( \Omega / \Omega_c = 50 \text{ GHz} \), \( a = 10 \).

\[ H = 0 \text{ and } p_\Omega = 0 \text{ and } a \geq 1 \text{ is a semi-empirical broadening parameter which describes the difference between the scattering time } \tau \text{ and the net scattering time determined by all other mechanisms, one can find that } \delta^{(+)} \simeq \delta^{(-)} \simeq \sqrt{6a/\pi \Omega_c \tau}. \text{ Assuming } \tau = 5.8 \times 10^{-10} \text{ s}, a = 1 - 10, \text{ and } \Omega / 2\pi = 50 \text{ GHz (as in Ref. [23]) and using the above analytical estimate, we obtain } \delta^{(+)} \simeq \delta^{(-)} \simeq 0.1 - 0.3. \]

Figures 1 and 2 show the dependences of the inter-LL component of the microwave photocurrent \( j_{ph}^{(\text{inter})} \) on the inverse cyclotron frequency \( \Omega / \Omega_c \) for different broadening parameters and for different microwave powers and frequencies calculated for a 2DES with \( \beta < 1 \) using Eq. (12). It is assumed that \( \tau = 5.8 \times 10^{-10} \text{ s}. \) The oscillatory \( j_{ph}^{(\text{inter})} \) vs \( \Omega / \Omega_c \) dependence at different \( P \) with the paired maxima and minima corresponding to the cyclotron resonance and its harmonics is shown in Fig. 2. One can see from Figs. 1 and 2 that the height of the maximum and the depth of the minimum near the cyclotron resonance (as well as near the cyclotron harmonics) increase with increasing microwave power. This increase is rather slow; the span of the photoconductivity as a function of the microwave power at low and moderate powers behaves similar to a logarithmic function. However, at large powers, height of the maximum (depth of the minimum) saturates and begins to fall (see Fig. 3). Figure 3 demonstrates the variations of the photoconductivity maxima and minima with increasing microwave power. As seen from Fig. 3, the height of the first one-photon maximum (depth of the relevant minimum), corresponding to \( \Lambda = 1 \) and \( M = 1 \), falls when the microwave power increases beyond some threshold value. In this range of microwave power, the maxima (minima) corresponding to higher resonances can be comparable with that near the cyclotron resonance. At high microwave powers, the two-photon resonant maxima (minima) can increase faster than those associated with the one-photon absorption processes (compare the curve for \( \Omega / \Omega_c \pm \delta^{(+)} = 1.5 \) and the curves for \( \Omega / \Omega_c \pm \delta^{(-)} = 1 \) and 2). In particular, when approximation (9) is valid, the ratio of the two-photon maxima \( j_{ph}^{(2,3)} \) ( \( M = 2 \) and \( \Lambda = 3 \)) to the single-photon maxima \( j_{ph}^{(1,2)} \) ( \( M = 1 \) and \( \Lambda = 2 \)) calculated using Eq. (12) can be estimated as \( \max j_{ph}^{(2,3)} / \max j_{ph}^{(1,2)} \simeq 3.37 P e^{2P} \). As it follows from the latter estimate, the ratio in question, being rather small
at small $P$, markedly rises with increasing $P$. Figure 4 shows the variations of the position of the photoc conductivity maxima near the cyclotron resonance $\delta^{(+)}$ and its width at half-maximum $\Delta^{(+)}$ (normalized by $\Omega_c$) with increasing normalized microwave power $P$. One can also see from Figs. 1, 2, and 4 that the width of the resonant maximum and minimum increases with increasing microwave power. The broadening of the cyclotron absorption line at heightened microwave power associated with similar mechanism was discussed recently. Apart from this, at high microwave powers, the transition rate and, therefore, the LL broadening become larger. This can lead to an increase in the broadening parameter $a$ and, hence, in an extra increase in the maximum and minimum width when the microwave power increases. At rather high powers, the two-photon resonances can be observable. Figure 2 exhibits comparably weak maximum and minimum in the vicinity of $\Omega/\Omega_c = 1.5$ corresponding to the two-photon transition ($M = 2$ and $\Lambda = 3$). These results are consistent with experimental data by Mani, et al. [20, 23] and Zudov, et al. [21] (see also Ref. [34]). Assuming, as in Ref. [21], that the power of the microwave source and the sample cross-section are $10 - 20$ mW and $0.25$ cm$^{-2}$, respectively, and using the above formula for the characteristic microwave power $\mathcal{P}_{\Omega}$, for $\Omega/2\pi = 50$ GHz one can find that the experimental conditions correspond to max $P \lesssim (1 - 2) \times 10^{-2}$. The later values are on the order of or larger than those used in Figs. 1 and 2.

As follows from Eqs. (6) and (8), the height of the maxima and the depth of the minima determined by the function $\mathcal{R}_M(z)$ depend on the parameter $\beta$, i.e., on the thickness of the spacer separating the 2DES and the donor layer. This function is plotted in Fig. 5 for $M = 1$ (one-photon absorption) and $M = 2$ (two-photon absorption). One can see that $\mathcal{R}_1(z)$ and $\mathcal{R}_2(z)$ pronouncedly decrease with increasing $\beta$. If $\beta \gg 1$, using Eqs. (6) and (8), one can obtain instead of Eq. (12)

$$j^{(\text{inter})}_{ph} \propto E \frac{\Gamma}{\beta} \left\{ \mathcal{F}_1 \left( \frac{\sqrt{P}}{\beta} \sqrt{f \left( \frac{\Omega}{\Omega_c} \right)} \right) \sum_{\Lambda} \frac{\Theta_{\Lambda}(\Lambda \Omega_c - \Omega)}{(\Lambda \Omega_c - \Omega)^2 + \Gamma^2} \right\} \right. \right.
$$

$$+ \mathcal{F}_2 \left( \frac{\sqrt{P}}{\beta} \sqrt{f \left( \frac{\Omega}{\Omega_c} \right)} \right) \sum_{\Lambda} \frac{\Theta_{\Lambda}(\Lambda \Omega_c - 2\Omega)}{(\Lambda \Omega_c - 2\Omega)^2 + \Gamma^2} + \cdots \right\}$$

(14)

Here for $\beta \gg 1$

$$\mathcal{F}_M(z) = \int_0^\infty dx J_M^2(z, x) \exp(-x) \simeq \beta \mathcal{R}_M(z/\beta).$$

(15)

![FIG. 2: Oscillations of $j^{(\text{inter})}_{ph}$ as a function of inverse cyclotron frequency $\Omega/\Omega_c$ at different microwave powers ($\Omega/2\pi = 50$ GHz, $a = 10$). Inset shows $j^{(\text{inter})}_{ph}$ (solid line) and $j^{(\text{intra})}_{ph}$ (squares) vs inverse cyclotron frequency.](image1)

![FIG. 3: Maxima and minima of $j^{(\text{inter})}_{ph}$ corresponding to different resonances as functions of normalized microwave power $P$ ($\Omega/2\pi = 50$ GHz and $a = 10$).](image2)

![FIG. 4: Position $\delta^{(+)}$ and width at half-maximum $\Delta^{(+)}$ of near cyclotron maximum vs normalized microwave power $P$. Curves (a) - (d) correspond to photocurrent spectral dependences in Figs. 1(a) - 1(d).](image3)
At not too large \( z \), setting \( J_M(z) \approx a_M \sin(\pi z x/b_M) \), where \( a_M \) is the first maximum of the pertinent Bessel function and \( b_M \) corresponds to its first zero, one can obtain

\[
F_M(z) \approx \left( \frac{\pi a_M}{2 b_M} \right)^2 \frac{z^2}{1 + (\pi/b_M)^2 z^2}.
\]

In particular, for \( M = 1 \) Eq. (16) yields \( F_M(z) \propto z^2/(1 + 0.67 z^2) \). At very large \( z \), \( F_M(z) \) becomes a decreasing function of \( z \) similar to that given by Eq. (11). Considering Eq. (16), near the first one-photon resonance (\( \Lambda = 1 \) and \( M = 1 \)) at \( \beta \gg 1 \), from Eq. (14) we obtain

\[
j_{\text{ph}}^{(\text{inter})} \propto \frac{E P}{\beta^3} f \left( \frac{\Omega}{\Omega_c} \right) \left[ 1 + 0.67 \frac{P}{\beta^2} f \left( \frac{\Omega}{\Omega_c} \right) \right]^{-1} \times \frac{(\Omega_c - \Omega)}{(\Omega_c - \Omega)^2 + \Gamma^2}. \tag{17}
\]

Equation (14) demonstrates a decrease in the microwave photoconductivity and the height of its maxima (depth of the minima) with increasing parameter \( \beta \) and slowing down with increasing power. The former is due to a decrease in the impurity scattering rate because of a smoothening of the fluctuating electric field created by charged impurities when the spacer becomes thicker (\( d_i \) becomes larger). One can see a similarity between the roles of the inverse parameter \( \beta^2 \) and the normalized microwave power \( P \).

**IV. PHOTOCONDUCTIVITY IN STRONG ELECTRIC FIELD**

The average electric field \( \mathcal{E} \) in the experimental situations is rather moderate, so one can assume that \( E < E_b \). However, under the condition of ANC, electric-field domain structures can be formed. The value of the electric field \( E_0 \), at which the net dissipative conductivity changes its sign from negative to positive, essentially affects the magnitude of the electric field variations in the domain structures in question. Although it is unclear yet what mechanism determines the threshold electric field \( E_0 \), one can assume that it can be much larger than \( E_b \). In this case, the electric field in some regions may significantly exceed \( E_b \). As shown by Shchamkhalova and the authors, the LL broadening \( h \Gamma^2 \) and, therefore, the characteristic field \( E_b \) can substantially decrease in the electric field. Moreover, recent experiments showed that strong long range (\( \Lambda \gg L \)) fluctuations are present in 2DES’s with high electron mobility. Due to these fluctuations, the local electric field can be of a rather large magnitude (about 30 - 150 kV/cm, as estimated by Kawano, et al. \( \text{[15]} \)), tangibly affecting the electron inter-LL transitions. Hence, the inequalities \( E > E_b \) or \( E \gg E_b \) can take place in the 2DES under consideration.

At relatively large net dc electric fields \( E > E_b \), \( \delta(\omega) \) can be considered as the Dirac \( \delta \)-function. Taking into account that the transitions between high LL’s provide main contribution to the mechanism of microwave photoconductivity under consideration, and integrating over \( q_y \) in Eq. (5), we obtain

\[
j_{\text{ph}}^{(\text{inter})} \propto \sum_{\Lambda, M} \Theta_{\Lambda} \mathcal{R}_M \left( \xi \Omega, \frac{h|\Lambda \Omega_c - M \Omega|}{e \mathcal{E} L} \right) \times \left[ \frac{h^2 (\Lambda \Omega_c - M \Omega)^2}{e E^2 L} \right]^2 \exp \left[ \frac{h^2 (\Lambda \Omega_c - M \Omega)^2}{2(e \mathcal{E} L)^2} \right], \tag{18}
\]

where

\[
\mathcal{R}_M(z, y) = \int_0^\infty dx J_M^2 \left( z \sqrt{x^2 + y^2} \right) \times \frac{\exp \left[ -\beta \sqrt{x^2 + y^2} - (x^2 + y^2)/2 \right]}{(x^2 + y^2)^{3/2}}. \tag{19}
\]

At small \( \xi \Omega \), Eq. (18) coincides with that obtained a long time ago. As follows from Eq. (18), the microwave conductivity in strong electric field also exhibits an oscillatory behavior with maxima and minima at \( \Omega/\Omega_c = \Lambda - \delta^{(+)} \) and \( \Omega/\Omega_c = \Lambda + \delta^{(-)} \), respectively, for the one-photon transitions , and \( \Omega/\Omega_c = \left| \Lambda - \delta^{(\pm)} \right|/M \) and at

![Graph showing functions \( R_1(z) \) and \( R_2(z) \) for different values of parameter \( \beta \).](image-url)
considering strong electric-field fluctuations\cite{38}. \(\Omega/\Omega_c = [\Delta(+) - \delta^{(-)}]/M\) for the transitions with absorption of \(M\) photons. Here \(\delta^{(+)} \approx \delta^{(-)} \approx eEL/\hbar \Omega_c = E/E_c\). The width of the maxima and minima in question linearly increases with \(E\) as \(\Delta^{(+)} \approx \Delta^{(-)} \approx 2.24E/E_c\), while their height is proportional to \(E^{-1}\). Indeed, Eq. (18) yields the following dependence of the min/max \(j_{\text{inter}}^{(\text{intra})}\) for the one-photon absorption near the cyclotron resonance on the microwave power \(P\) :

\[
\text{min/max } j_{\text{inter}}^{(\text{intra})} \propto \frac{\exp(-Pf^{(\pm)}/E)}{E} I_1(Pf^{(\pm)}) \quad (20)
\]

where \(f^{(\pm)} = f(1 \pm E/E_c)\). Here, assuming \(\beta < 1\), we have used the following estimate: \(\mathcal{R}_1(z, 1) \approx 0.4 \exp(-z^2) I_1(z^2)\). Figure 6 shows the dependences of min/max \(j_{\text{inter}}^{(\text{intra})}\) on the electric field at different microwave power calculated using an interpolation formula leading to min/max \(j_{\text{inter}}^{(\text{intra})}\) following from Eq. (12) in the low-field region (for \(\Omega/2\pi = 100\) GHz and \(a = 1\) and \(a = 10\)) and to Eq. (20) in the high-field region.

A marked sensitivity of the resonant maxima and minima to the electric field can be crucial (together with the LL-broadening associated with the interactions other than the impurity scattering) for the explanation of their fairly wide width observed experimentally, particularly, considering strong electric-field fluctuations.\cite{38}

V. SUPPRESSION OF INTRA-LL PHOTOCONDUCTIVITY

Taking into account that, as follows from Eq. (2), the probability of the elastic impurity scattering in the presence of microwave radiation differs from that in its absence by the factor \(J_{n,0}((\Omega/\Omega_c, \xi_0)\) and generalizing the results of Ref. 2\cite{2} (see also Ref. 35\cite{35}), the dissipative current associated with the intra-LL impurity scattering can be given by

\[
j_{\text{intra}}^{(\text{intra})} \propto EN_iL^2 \sum_N f_N(1 - f_N)
\]

\[
\times \int dq_x dq_y \frac{1}{q^2} \exp\left(-2d_y - \frac{L^2q^2}{2}\right)
\]

\[
\times J_0^2(\sqrt{2N}Lq)J_0^2(\xi_0Lq) \quad (21)
\]

at \(E < E_b\), and

\[
j_{\text{intra}}^{(\text{intra})} \propto \frac{N_i}{E} \sum_N f_N(1 - f_N)
\]

\[
\times \int dq_x dq_y \frac{1}{q^2} \exp\left(-2d_y - \frac{L^2q^2}{2}\right)
\]

\[
\times J_0^2(\sqrt{2N}Lq)J_0^2(\xi_0Lq) \quad (22)
\]

at \(E \gg E_b\). Using the same procedure as in Sec. III, Eq. (21) for the case \(\beta < 1\) and \(E < E_b\), can be reduced to

\[
j_{\text{intra}}^{(\text{intra})} \approx j_{\text{dark}}^{(\text{intra})} R_0(\xi_0)
\]

\[
\approx j_{\text{dark}}^{(\text{intra})} \exp\left[-Pf\left(\frac{\Omega}{\Omega_c}\right)\right] R_0\left(\frac{\Omega}{\Omega_c}\right), \quad (23)
\]

where \(j_{\text{dark}}^{(\text{intra})} = j_{\text{dark}}^{(\text{intra})}|_{P=0}\) is the dissipative current associated with the intra-LL impurity scattering without irradiation (dark current). Using Eq. (23), we arrive at the following expression for the photoconductivity associated with the microwave-stimulated variation of the dissipative intra-LL conductivity:

\[
J_{\text{intra}}^{(\text{intra})} \approx J_{\text{dark}}^{(\text{intra})} \left\{ \exp\left[-Pf\left(\frac{\Omega}{\Omega_c}\right)\right] \cdot I_0\left(\frac{\Omega}{\Omega_c}\right) - 1 \right\}
\]

\[
\approx - \frac{3}{4} j_{\text{dark}}^{(\text{intra})} P f\left(\frac{\Omega}{\Omega_c}\right). \quad (24)
\]

The last term in the right-hand side of Eq. (24) is valid when \(P f(\Omega/\Omega_c) < 1\). As seen from Eq. (24), the contribution of the effect of microwave radiation on the elastic
intra-LL impurity scattering to the photoconductivity is negative. This contribution as a function of $\Omega/\omega_c$ exhibits a smeared minimum at the cyclotron resonance. This minimum is attributed to the suppression of the impurity scattering by microwave radiation. Such a suppression (associated with the absorption and emission of virtual phonons) is most effective when the amplitude of the Larmor orbit center oscillation is maximum, i.e., at the cyclotron resonance. The $j_{\text{ph}}^{(\text{intra})}$ at the cyclotron resonance. The $j_{\text{ph}}^{(\text{intra})}$ vs $\Omega/\omega_c$ dependence calculated using Eq. (24) is shown by squares in the inset in Fig. 2. Since this dependence does not comprise any resonant factor, it can not affect significantly the oscillatory dependences associated with the photon-assisted inter-LL transitions, although it leads to some deepening of the photoconductivity minimum near the cyclotron resonance. Formulas (23) and (24) correspond to the total suppression of the dissipative intra-LL conductivity, i.e. to $j_{\text{ph}}^{(\text{intra})} = 0$ and therefore, $j_{\text{ph}}^{(\text{intra})} = -j_{\text{dark}}^{(\text{intra})}$, at the cyclotron resonance. However, the dephasing of the electron oscillatory movement in the microwave field due to different scattering events can limit the suppression of the dissipative intra-LL conductivity. This effect can be included by the substitution of the function $f^{*}(\omega) = f(\omega)[1-\omega^2]/[(1-\omega)^2 + \gamma^2]$ for the function $f(\omega)$ introduced in Sec. III. Taking into account that $f^{*}(1) = 1/2\gamma^2$, at $P < 2(\Gamma/\omega_c)^2$ we obtain $\min j_{\text{ph}}^{(\text{intra})} \simeq j_{\text{dark}}^{(\text{intra})}[1 - P(\omega_c/\sqrt{2}\gamma)^2]$ and, consequently, $\min j_{\text{ph}}^{(\text{intra})} \simeq -j_{\text{dark}}^{(\text{intra})}P(\omega_c/\sqrt{2}\gamma)^2$. At $P \gg 2(\Gamma/\omega_c)^2$, one obtains $\min j_{\text{ph}}^{(\text{intra})} \simeq j_{\text{dark}}^{(\text{intra})}(\Gamma/\sqrt{\pi P\omega_c})$ and $\min j_{\text{ph}}^{(\text{intra})} \simeq -j_{\text{dark}}^{(\text{intra})}[1 - (\Gamma/\sqrt{\pi P\omega_c})] \simeq -j_{\text{dark}}^{(\text{intra})}$.

VI. COMMENTS

The proposed model describes the following features of the microwave conductivity in a 2DES subjected to a magnetic field:

(i) The zeroth microwave conductivity at the cyclotron resonance and its harmonics, as well as the location of the photoconductivity maxima and minima in the vicinity of the resonances with ANC in the minima.

(ii) Nonlinear dependencies of the photoconductivity on the microwave power characterized by slowing down, saturation, and even decrease in the minima/maxima magnitude with increasing microwave power.

(iii) Shift of the photoconductivity maxima and minima and their broadening with increasing microwave power and electric field.

(iv) Possibility of the suppression of the dissipative conductivity associated with the intra-LL transitions by the microwave radiation.

The positions of the photoconductivity zeros, maxima, and minima are determined by the specific features of the photon-assisted impurity scattering of electron in the magnetic field. Some of these features of the microwave photoconductivity, predicted theoretically [4, 7] and observed experimentally [21, 22] (see also Refs. [21, 22, 30]), have been discussed in the framework of different theoretical models [24, 27]. It is instructive that the effect of suppression of the impurity intra-LL scattering by microwave radiation can result in a shift of the microwave photoconductivity zero to $\Omega/\omega_c$ slightly smaller than unity.

The photon-assisted scattering on acoustic phonons also can lead to the oscillatory dependence of the dissipative conductivity with ANC at $\Omega/\omega_c$ between the cyclotron resonances and its harmonics [30, 31]. It is remarkable that this mechanism provides the photoconductivity minima and maxima approximately at the same values of $\Omega/\omega_c$, where the photon-assisted impurity scattering yields, in contrast, maxima and minima. Thus, the photon-assisted acoustic scattering can to some extent suppress the oscillations associated with the photon-assisted impurity scattering. Such a competition of the mechanisms in question can be essential because the piezoelectric acoustic scattering is one of the main scattering mechanism limiting the electron mobility in perfect 2DES’s at low temperatures [31]. The contribution of the photon-assisted acoustic scattering mechanism to the microwave conductivity markedly increases with the temperature, particularly, in the range where $T$ is comparable with the characteristic energy of acoustic phonons $hs/\xi$ ($\xi$ stands for the speed of sound). Assuming $s = 3 \times 10^5$ and $H = 1 - 2$ kG, one can obtain $T_{\text{ac}} \simeq 0.4$ K. The latter value is only slightly smaller than $T$ in the experiments. Hence, the suppression of the microwave photoconductivity oscillations and the effect of ANC (a pronounced decrease in the minima/maxima magnitude) with increasing temperature observed experimentally [20, 21] can possibly be attributed to the inclusion of the abovementioned photon-assisted acoustic scattering mechanism.

The photoconductivity considered above (and in Refs. [4, 7, 21, 22, 28, 32]) is associated with a direct effect of the microwave ac field on the the impurity scattering of electrons. However, the photon-assisted impurity scattering is complicated by the absorption of photons and, hence, some heating of the 2DES. Such a heating can lead to a variation of the dissipative conductivity associated with the scattering on impurities and acoustic phonons. The dissipative current associated with the intra-LL impurity scattering is sensitive to the temperature (mainly via the factor $\Theta_0$). This effect in part gives rise to the smearing of the Shubnikov-de Haas oscillation with increasing temperature. However, the temperature dependence of $\Theta_0$ in the range where the oscillations of microwave photoconductivity under consideration were observed experimentally is rather weak. The electron heating caused by the photon-assisted impurity scattering processes is compensated by the relaxation processes with the electron transitions from higher to lower LL’s with the emission of acoustic phonons. As shown recently [10], these relaxation processes contribute to the dissipative conductivity (their contribution is negative) complicating the spectral dependence of the microwave
photoconductivity.

A nontrivial dependences of the maxima/minima span on the microwave power are consistent with those observed experimentally [24, 26, 29] and are attributed to the multi-photon effects including the absorption and emission of real and virtual phonons (see Refs. [3, 11, 20, 24]). A relatively large width of the microwave photoconductivity maxima and minima is often identified with an extra broadening of LL’s (in comparison with the LL broadening corresponding to the electron collision time determining the electron mobility) associated with more complex interaction discussed previously [24, 26, 29]. As pointed out above, this effect can be attributed to the nonlinear effects of the ac microwave field and strong electric-field dependences of the local photoconductivity characteristics combined with strong long-range fluctuations of the electric field. In the presence of strong long range ($\lambda \gg L$) electric-field fluctuations the macroscopic properties of the 2DES, in particular, the domain structures with the scale exceeding $\lambda$ can be determined by the components of the dissipative photocurrent averaged over these fluctuations $j_{ph}(E)E_x/E$ and $j_{ph}(E)E_y/E$. Due to a strong electric field dependence of $j_{ph}$ (see Eq. (18)), the dependences of the averaged quantities on the components of the averaged electric field $E_x \propto V$ and $E_y \propto V_H$ (where $V$ and $V_H$ are the potential drop along the current and the Hall current, respectively) as well as their spectral dependences can be significantly different from those of the local dissipative current.

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