Concluding Remarks: Neutrino Mixing, Majorana CP-violation, \((\beta\beta)_{0\nu}\)-Decay and Beyond

S. T. Petcov

Scuola Internazionale Superiore di Studi Avanzati and INFN, I-34014 Trieste, Italy

E-mail: petcov@sissa.it

Abstract. The problem of determination of the nature - Dirac or Majorana, of massive neutrinos is discussed. The physics potential of experiments, searching for \((\beta\beta)_{0\nu}\)-decay, for providing information on the type of \(\nu\)-mass spectrum, absolute scale of \(\nu\)-masses and on the Majorana CP-violating phases in the PMNS neutrino mixing matrix \(U\), is reviewed. The possibility that the CP-violation necessary for the generation of the baryon asymmetry of the Universe is due exclusively to the Majorana CP-violating phase(s) in \(U\), is also briefly discussed.

1. Introduction

It is both an honor and a pleasure for me to speak at this Symposium organized in honor of Frank Avignone, Ettore Fiorini and the late Peter Rosen. I met Frank first in 1986, Ettore - in 1979 and Peter Rosen - in 1983. However, their names were familiar to me from some of the publications I was reading in the middle of the 1970’s when I was a Ph. D. student. I was interested in neutrino physics and Frank, Ettore and Peter were actively working in this field. Their “passion” for neutrinos did not diminish with time. For Peter Rosen it continued until his last days. All three of them marked the field with their achievements: Frank and Ettore - in experimental neutrino physics, and Peter - in the theory of neutrinos.

Frank played an important role in the interpretation of the results of the \(\bar{\nu}_e - e^-\) elastic scattering experiment of Gurr, Reines and Sobel at Savannah River Reactor. In 1978 he became interested in improving the sensitivity of Ge detectors for fundamental physics. In the early 1980s Frank started to work with R. Brodzinski and the Pacific Northwest Laboratory Group in lowering the background in Ge detectors by several orders of magnitude. In 1987 the group had published the first terrestrial sensitive search for cold dark matter (CDM), which eliminated massive Dirac neutrinos as a possible dominant component of the CDM in the halo of our galaxy. Later Frank with collaborators used these unique detectors to search for axions from the Sun. Starting in the late 80’s, Frank has been actively involved in experiments searching for neutrinoless double beta \((\beta\beta)_{0\nu}\) - decay: IGEX, CUORICINO, CUORE, MAJORANA. IGEX, in which Frank played a leading role, and CUORICINO, distinguished themselves with the limits obtained, while CUORE and MAJORANA belong to the next generation of detectors under preparation at present. Our hopes to determine the nature of neutrinos and thus resolve one of the most challenging among the fundamental problems in neutrino physics, are associated

1 Also at: INRNE, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria.
with these new detectors. The MAJORANA project is to a large extent (if not entirely) due to Frank.

Ettore Fiorini’s involvement in neutrino physics began in 1963, when he started the first $(\beta\beta)_{0w}$-decay experiment with $^{76}Ge$. In 1970’s Ettore participated in one of the most remarkable experiments in neutrino physics - “Gargamelle” at CERN, in which the neutral current weak interaction was discovered. Later in the second half of 1980’s Ettore played a very active role in the GALLEX solar neutrino experiment, in which for the first time the lower energy component of the solar neutrino flux was observed. However, he never stopped thinking about how to improve the sensitivity of the experimental searches for $(\beta\beta)_{0w}$-decay. Intrigued by a suggestion in a preprint by G. Mitselmakher from JINR, Dubna, Russia, that the electrons from the $(\beta\beta)_{0w}$-decay might be detected thermally, Ettore together T. Niinikoski devised an approach that was applied in the first successful physics experiment using thermal spectrometers. In this experiment a new lower limit on the $(\beta\beta)_{0w}$-decay lifetime of $^{130}Te$ was obtained. The novel technique of thermal detection of beta-decay electrons was further developed and implemented in the successful CUORICINO experiment. It will be used in the CUORE detector which is under construction. Ettore is the leader of both these experiments.

Peter Rosen made pioneering contributions in the theory of $(\beta\beta)_{0w}$-decay. I was lucky to have met him in the early 1980’s: the discussions with him were important for my scientific development. I also had the privilege to work with Peter on few scientific projects. As a curiosity let me mention that the results of one of these projects, done in collaboration with Boris Kayser, were widely known and quoted, but were never published. It is among the most quoted unpublished scientific works. Peter had also a strong indirect influence on my subsequent work with collaborators on the neutrino physics aspects of the $(\beta\beta)_{0w}$-decay.

The above sketchy description of the scientific activity of Frank, Ettore and Peter is far from being complete, of course. Frank and Ettore trained and continue to train young scientists, held important administrative positions, were (and still are) members of various scientific policy Committees, etc. Peter, in particular, held a high position in DOE, with responsibilities directly related to the funding of High Energy Physics.

Since many years Frank and Ettore are the driving force in the field of experimental searches for $(\beta\beta)_{0w}$-decay. Most of the scientific achievements of Peter are related to the development of the theory of $(\beta\beta)_{0w}$-decay: for many years Peter was one of the very few leading experts in this field. In view of this most of my talk will be devoted to the $(\beta\beta)_{0w}$-decay. I will review the current understanding of the neutrino physics aspects of $(\beta\beta)_{0w}$-decay and the physics potential of the $(\beta\beta)_{0w}$-decay experiments.

Establishing whether the neutrinos with definite mass $\nu_j$ are Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e., spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the origin of $\nu$-masses and mixing and the underlying symmetries of particle interactions (see, e.g. [1]). Let us recall that the neutrinos $\nu_j$ with definite mass $m_j$ will be Dirac fermions if particle interactions conserve some additive lepton number, e.g., the total lepton charge $L = L_e + L_\mu + L_\tau$. If no lepton charge is conserved, the neutrinos $\nu_j$ will be Majorana fermions (see, e.g. [2]). The massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [3], which also provides an attractive explanation of the smallness of neutrino masses and, through the leptogenesis theory [4], of the observed baryon asymmetry of the Universe. The observed patterns of neutrino mixing and of neutrino mass squared differences driving the solar and the dominant atmospheric neutrino oscillations can be related to Majorana massive neutrinos and the existence of an approximate symmetry in the lepton sector corresponding to the conservation of the non-standard lepton charge $L' = L_e - L_\mu - L_\tau$ [5]. Determining the nature (Dirac or Majorana) of massive neutrinos $\nu_j$ is one of the fundamental and most challenging problems in the future studies of neutrino mixing [1].
Extensive studies have shown that the only feasible experiments having the potential of establishing the Majorana nature of massive neutrinos at present are the $(\beta\beta)^{0\nu}$-decay experiments searching for the process $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$ (for reviews see, e.g. [2,6–8]). The observation of $(\beta\beta)^{0\nu}$-decay and the measurement of the corresponding half-life with sufficient accuracy, would not only be a proof that the total lepton charge is not conserved, but might provide also unique information on the i) type of $\nu_j$ mass spectrum, [9, 10] (see also [11]), ii) absolute scale of $\nu_j$ masses (see, e.g. [12]), and iii) Majorana CP-violating (CPV) phases [9, 13–15] (see also, e.g. [16–18]).

In what follows we will consider the case of 3-$\nu$ oscillations, for which we have compelling evidence from experiments with solar, atmospheric, reactor and accelerator neutrinos (see, e.g. [1, 19] for extensive lists of references):

$$\nu_{iL} = \sum_{j=1}^{3} U_{ij}\nu_{jL}, \quad l = e, \mu, \tau,$$

where $\nu_{jL}$ is the field of neutrino $\nu_j$ having a mass $m_j$ and $U$ is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix [20], $U \equiv U_{\text{PMNS}}$. Assuming that the massive neutrinos $\nu_j$ are Majorana particles, we will use the standard parametrisation of the PMNS matrix:

$$U_{\text{PMNS}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \text{diag}(1, e^{i\frac{\pi}{2}}, e^{i\frac{\pi}{2}})$$

(2)

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violation (CPV) phase and $\alpha, \beta$ are two Majorana CPV phases [21]. The existing neutrino oscillation data allow us to determine the parameters which drive the solar neutrino and the dominant atmospheric neutrino oscillations, $\Delta m_{23}^2 = \Delta m_{21}^2$, $\sin^2 \theta_{12}$, and $|\Delta m_{31}^2| = |\Delta m_{32}^2| \equiv |\Delta m_{A}^2|$, $\sin^2 2\theta_{23}$, with relatively good precision, and to obtain rather stringent limits on the CHOOZ angle [22] $\theta_{13}$ (see, e.g. [23, 24]). The best fits values and the 95% C.L. allowed ranges of $|\Delta m_{A}^2|$, $\sin^2 2\theta_{23}$, $\Delta m_{23}^2$ and $\sin^2 \theta_{13}$ read:

$$\begin{align*}
|\Delta m_{A}^2|_{\text{BF}} &= 2.5 \times 10^{-5} \text{ eV}^2, \\
\sin^2 2\theta_{23} &\geq 1.0, \\
\sin^2 \theta_{13} &\geq 0.90, \\
\Delta m_{23}^2 &\geq 8.0 \times 10^{-5} \text{ eV}^2, \\
7.3 \times 10^{-5} &\leq \sin^2 2\theta_{23} \leq 8.5 \times 10^{-5} \text{ eV}^2, \\
0.26 &\leq \sin^2 \theta_{13} \leq 0.36.
\end{align*}$$

(3-6)

A combined 3-$\nu$ oscillation analysis of the global data gives [23, 24]

$$\sin^2 \theta_{13} < 0.040 \ (0.056) \quad \text{at} \quad 95\% \ (99.73\%) \ \text{C.L.}$$

(7)

The existing data allow a determination of, e.g. $\Delta m_{23}^2$, $\sin^2 \theta_{23}$ and $|\Delta m_{A}^2|$ at 3$\sigma$ with an error of approximately 12%, 24%, and 28%, respectively. Future oscillation experiments will improve considerably the precision on these basic parameters: the indicated 3$\sigma$ errors can be reduced to [25] 4%, 10% and 12%. There are several proposals for reactor experiments which could improve the current limit on $\sin^2 \theta_{13}$ by a factor of (5-10) (see, e.g. [26]). Two of these experiments - Double CHOOZ and Daya Bay [27, 28], are under preparation at present. Oscillation experiments will provide data on the Dirac CPV phase $\delta$ as well - the latter is a source of CP-violation in $\nu$-oscillations (see, e.g., [29, 30]). However, these experiments cannot give information on the absolute scale of neutrino masses. They are insensitive to the nature - Dirac or Majorana, of massive neutrinos $\nu_j$ and, correspondingly, to the Majorana CPV phases present in $U$ [21, 31].
If $\nu_j$ are Majorana fermions, getting experimental information about the Majorana CPV phases in $U_{PMNS}$ would be a remarkably difficult problem [9,12,14,15,32]. The phases $\alpha$ and $\beta$ can affect significantly the predictions for the rates of (LFV) decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. in a large class of supersymmetric theories with the see-saw mechanism of $\nu$-mass generation [33]. The Majorana CPV phase(s) in the PMNS matrix can play the role of the leptogenesis CPV parameter(s) at the origin of the baryon asymmetry of the Universe [34,35] (see also [36]).

2. Majorana Neutrinos and $(\beta \beta)_{0\nu}$-Decay

Under the assumptions of 3-$\nu$ mixing, of massive neutrinos $\nu_j$ being Majorana particles, and of $(\beta \beta)_{0\nu}$-decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos $\nu_j$ having masses $m_j \lesssim$ few MeV, the $(\beta \beta)_{0\nu}$-decay amplitude has the form (see, e.g. [9]): $A(\beta \beta)_{0\nu} \sim |m|$, where $M$ is the corresponding nuclear matrix element (NME) which does not depend on the neutrino mixing parameters, and

$$|<m>| = |m_1[U_{e1}]^2 + m_2[U_{e2}]^2 e^{i\alpha} + m_3[U_{e3}]^2 e^{i\beta}|,$$

is the effective Majorana mass in $(\beta \beta)_{0\nu}$-decay; $|U_{e1}|=c_{12} c_{13}$, $|U_{e2}|=s_{12} c_{13}$, $|U_{e3}|=s_{13}$. In the case of CP-invariance one has [37], $\eta_{21} \equiv e^{i\alpha}=\pm 1$, $\eta_{31} \equiv e^{i\beta}=\pm 1$, $\eta_{21(31)}$ being the relative CP-parity of Majorana neutrinos $\nu_{2(3)}$ and $\nu_1$.

The predicted value of $|<m>|$ depends strongly on the type of neutrino mass spectrum [9,10]. Depending on the sign of the neutrino mass squared difference which drives the atmospheric neutrino oscillations, $\Delta m^2_\alpha \equiv \Delta m^2_{31} \equiv \Delta m_{31}^2 > 0$, the spectrum (in the standard convention) can be with normal neutrino mass ordering, $m_1 < m_2 < m_3$ ($\Delta m^2_\alpha \equiv \Delta m^2_{31} > 0$), or with inverted neutrino mass ordering, $m_3 < m_1 < m_2$ ($\Delta m^2_\alpha \equiv \Delta m^2_{33} < 0$). Depending on the sgn($\Delta m^2_\alpha$) and the value of the lightest neutrino mass, i.e., the absolute $\nu$-mass scale, $m_{\nu_{\min}} \equiv m_{\min}$, the $\nu$-mass spectrum can be

- **Normal Hierarchical (NH):** $m_1 \ll m_2 \ll m_3$, $m_2 \approx (\Delta m^2_{31})^{1/2} \sim 0.009$ eV, $m_3 \approx |\Delta m^2_{31}|^{1/2} \sim 0.05$ eV;

- **Inverted Hierarchical (IH):** $m_3 \ll m_2 \ll m_1$, with $m_{1,2} \approx |\Delta m^2_{31}|^{1/2} \sim 0.05$ eV;

- **Quasi-Degenerate (QD):** $m_1 \approx m_2 \approx m_3 \approx m$, $m_j \gg |\Delta m^2_{31}|$, $m \approx 0.10$ eV.

Information on the absolute scale of neutrino masses can be derived in $^8$H $\beta$-decay experiments [38,39] and from cosmological and astrophysical data. The most stringent upper bounds on the $\bar{\nu}_e$ mass were obtained in the Troitzk [38] and Mainz [39] experiments:

$$m_{\bar{\nu}_e} < 2.3 \text{eV} \text{ at 95\% C.L.}$$

We have $m_{\bar{\nu}_e} \approx m_{1.2.3}$ in the case of the QD $\nu$-mass spectrum. The KATRIN experiment [39] is planned to reach a sensitivity of $m_{\bar{\nu}_e} \sim 0.20$ eV, i.e. it will probe the region of the QD spectrum. The CMB data of the WMAP experiment, combined with data from large scale structure surveys (2dFGRS, SDSS), lead to an upper limit on the sum of $\nu_j$ masses (see, e.g. [40]):

$$\sum_j m_j \equiv \Sigma < (0.4-1.7) \text{ eV} \text{ at 95\% C.L.}$$

Data on weak lensing of galaxies, combined with data from the WMAP and PLANCK experiments, may allow $\Sigma$ to be determined with an uncertainty of $\delta \sim 0.04$ eV [40].

It proves convenient to express [41] the three neutrino masses in terms of $\Delta m^2_{31}$ and $\Delta m^2_{33}$, measured in neutrino oscillation experiments, and the absolute neutrino mass scale determined by $m_{\nu_{\min}}$ \footnote{For a detailed discussion of the relevant formalism see, e.g. [8,9].}. In both cases of $\nu$-mass spectrum with normal and inverted ordering one has (in the convention we use): $\Delta m^2_{31}=\Delta m^2_{21} > 0$, $m_2=(m_1^2 + \Delta m^2_{31})^{1/2}$. For normal ordering,
$\Delta m_{A}^{2} = \Delta m_{31}^{2} > 0$ and $m_{3} = (m_{1}^{2} + \Delta m_{31}^{2})^{1/2}$, while if the spectrum is with inverted ordering, $m_{\text{MIN}} = m_{3}$, $\Delta m_{2}^{2} = \Delta m_{23}^{2} > 0$ and $m_{1} = (m_{3}^{2} + \Delta m_{23}^{2} - \Delta m_{31}^{2})^{1/2}$. Thus, given $\Delta m_{A}^{2}$, $\Delta m_{2}^{2}$, $\theta_{13}$ and $\theta_{13}$, $|<m>|$ depends on $\text{min}(m_{j})$, Majorana phases $\alpha$, $\beta$ and the type of $\nu$-mass spectrum.

The problem of obtaining the allowed values of $|<m>|$ given the constraints on the parameters following from $\nu$-oscillation data, and more generally of the physics potential of $(\beta\beta)_{0w}$-decay experiments, was first studied in [41] and subsequently in a large number of papers\(^3\). Detailed analyses were performed more recently in [15, 18, 42]. The results are illustrated in Fig. 1.

Figure 1. The value of $|<m>|$ as a function of $m_{\text{MIN}} \equiv \text{min}(m_{j})$, obtained using i) the 95\% C.L. allowed ranges of $\Delta m_{A}^{2}$, $|\Delta m_{2}^{2}|$, $\sin^{2}\theta_{13}$ and $\sin^{2}\theta_{13}$ (left panel), and ii) prospective 2\sigma uncertainty in $|<m>|$, corresponding to input 1-$\sigma$ experimental errors in $\Delta m_{3}^{2}$, $\Delta m_{A}^{2}$ and $\sin^{2}\theta_{13}$ of 2\%, 2\% and 4\% and $\sin^{2}\theta_{13} = 0.01 \pm 0.006$ (right panel). The regions shown in red/grey correspond to violation of CP-symmetry. (From [42].)

1. The main features of the predictions for $|<m>|$ are [9, 10, 12] (Fig. 1, left panel):
   i) for NH spectrum, $|<m>| \cong (|\Delta m_{A}^{2}|/2) s_{12}^{2} + (|\Delta m_{A}^{2}|/2) s_{12}^{2} e^{-i(\alpha - \beta)} \lesssim 0.006$ eV;
   ii) for IH spectrum, $|<m>| \cong (|\Delta m_{A}^{2}|/2) s_{12}^{2} + (|\Delta m_{A}^{2}|/2) s_{12}^{2} e^{-i(\alpha - \beta)}$, thus $(|\Delta m_{A}^{2}|/2) s_{12}^{2} \lesssim |<m>| \lesssim (|\Delta m_{A}^{2}|/2)$, or 0.013 eV $\lesssim |<m>| \lesssim 0.055$ eV, the bounds corresponding to the CP-conserving values of $\alpha = 0$; \(\pi\);
   iii) for QD spectrum, $|<m>| \cong m_{0} (1 - \sin^{2}2\theta_{13} \sin^{2}\alpha/2)^{1/2}$, $m_{0} \gtrsim |<m>| \gtrsim m_{0} \cos 2\theta_{13} \gtrsim 0.03$ eV, with $m_{0} \gtrsim 0.1$ eV, $m_{0} < 2.3$ eV [39] or $m_{0} < 0.5$ eV [40].

For IH (QD) spectrum we have: $\sin^{2}(\alpha/2) \cong (1 - |<m>|^{2}/m_{0}^{2}) / \sin^{2}2\theta_{13}$, $m_{0}^{2} = |\Delta m_{A}^{2}| / (m_{0}^{2})$. Thus, a measurement of $|<m>|$ (and $m_{0}$ for QD spectrum) allows us to determine $\alpha$.

The experimental searches for $(\beta\beta)_{0w}$-decay have a long history [6]. The best sensitivity was achieved in the Heidelberg-Moscow $^{76}\text{Ge}$ experiment [43]: $|<m>| < (0.35 - 1.05)$ eV (90\% C.L.), where a factor of 3 uncertainty in the relevant NME (see, e.g. [44]) is taken into account. The IGEX collaboration has obtained [45]: $|<m>| < (0.33 - 1.35)$ eV (90\% C.L.). A positive signal at $> 3\sigma$, corresponding to $|<m>| = (0.1 - 0.9)$ eV, is claimed to be observed in [46], while a more recent analysis reports evidence at 6\sigma for $(\beta\beta)_{0w}$-decay with $|<m>| = 0.32 \pm 0.03$ eV [47]. Two experiments, NEMO3 (with $^{100}\text{Mo}$ and $^{82}\text{Se}$) [48] and CUORICINO (with $^{130}\text{Te}$) [49], designed to reach sensitivity to $|<m>| \sim (0.2 - 0.3)$ eV, obtained the following limits: $|<m>| < (0.7 - 1.2)$ eV [48] and $|<m>| < (0.16 - 0.68)$ eV [49] (90\% C.L.), where estimated uncertainties in the NME are accounted for. Most importantly, a number of projects aim at sensitivity to $|<m>| \sim (0.01 - 0.05)$ eV [50]: CUORE ($^{130}\text{Te}$), GERDA ($^{76}\text{Ge}$), SuperNEMO ($^{100}\text{Mo}$), EXO ($^{136}\text{Xe}$), MAJORANA ($^{76}\text{Ge}$), XMASS ($^{136}\text{Xe}$), etc. These experiments will probe the region

\(^{3}\) An extensive list of references on the subject is given in [8].
corresponding to IH and QD spectra and test the positive result claimed in [46].

The existence of significant lower bounds on \( |<m>| \) in the cases of IH and QD spectra \([10]\), which lie either partially (IH spectrum) or completely (QD spectrum) within the range of sensitivity of the next generation of \((\beta \beta)_{0\nu}\)-decay experiments, is one of the most important features of the predictions of \( |<m>| \). These minimal values are given, up to small corrections, by \( \Delta m^4_1 \cos 2\theta_1 \) and \( m_0 \cos 2\theta_1 \). According to the combined analysis of the solar and reactor neutrino data \([23,24]\), i) the possibility of \( \cos 2\theta_1 = 0 \) is excluded at \( \sim 6\sigma \), ii) the best fit value of \( \cos 2\theta_1 \) is \( \cos 2\theta_1 = 0.40 \), and iii) at 95\% C.L. one has \( \sin^2 \theta_{13} = 0 \) (0.02), \( \cos 2\theta_\odot \gtrsim 0.28 \) (0.28). The quoted results on \( \cos 2\theta_\odot \) together with the range of possible values of \( |\Delta m^2_1| \) and \( m_0 \) lead to the conclusion about the existence of significant and robust lower bounds on \( |<m>| \) in the cases of IH and QD spectrum \([10,11]\). At the same time one can always have \( |<m>| \ll 10^{-3} \) eV in the case of spectrum with normal ordering \([12]\). As Fig. 1 indicates, \( |<m>| \) cannot exceed \( \sim 6 \) meV for NH \( \nu\)-mass spectrum. This implies that \( \text{max}(|<m>|) \) in the case of NH spectrum is considerably smaller than \( \text{min}(|<m>|) \) for the IH and QD spectrum. This opens the possibility of obtaining information about the type of \( \nu\)-mass spectrum from a measurement of \( |<m>| \neq 0 \) \([10]\). In particular, a positive result in the future \((\beta \beta)_{0\nu}\)-decay experiments with \( |<m>| > 0.01 \) eV would imply that the NH spectrum is strongly disfavored (if not excluded).

Prospective experimental errors in the values of the oscillation parameters (Fig. 1, right panel), in \( |<m>| \) and the sum of neutrino masses, and the uncertainty in the relevant NME, can weaken but do not invalidate these results \([11,14,15]\).

As Fig. 1 indicates, a measurement of \( |<m>| \gtrsim 0.01 \) eV would either \([12]\) i) determine a relatively narrow interval of possible values of the lightest \( \nu\)-mass \( m_{\text{MIN}} \), or ii) would establish an upper limit on \( m_{\text{MIN}} \). If an upper limit on \( |<m>| \) is experimentally obtained below 0.01 eV, this would lead to a significant upper limit on \( m_{\text{MIN}} \).

The possibility of establishing CP-violation in the lepton sector due to Majorana CPV phases has been studied in \([12,32]\) and in much greater detail in \([14,15]\). It was found that it is very challenging: it requires quite accurate measurements of \( |<m>| \) (and of \( m_0 \) for QD spectrum), and holds only for a limited range of values of the relevant parameters. More specifically \([14,15]\), establishing at 2\( \sigma \) CP-violation associated with Majorana neutrinos in the case of QD spectrum requires for \( \sin^2 \theta_\odot = 0.31 \), in particular, a relative experimental error on the measured value of \( |<m>| \) and \( m_0 \) smaller than 15\%, a “theoretical uncertainty” \( F \lesssim 1.5 \) in the value of \( |<m>| \) due to an imprecise knowledge of the corresponding NME, and value of the relevant Majorana CPV phase \( \alpha \) typically within the ranges of \( \sim (\pi/4 - 3\pi/4) \) and \( \sim (5\pi/4 - 7\pi/4) \) (Fig. 2).

The knowledge of NME with sufficiently small uncertainty\(^4\) is crucial for obtaining quantitative information on the \( \nu\)-mixing parameters from a measurement of \((\beta \beta)_{0\nu}\)-decay half-life.

3. Baryon Asymmetry from CP-Violating Majorana Phases in \( U_{\text{PMNS}} \)

We will discuss next briefly the interesting possibility that the CP-violation necessary for the generation of the baryon asymmetry of the Universe, \( Y_B \), in the leptogenesis scenario can be due exclusively to the Majorana (and/or Dirac) CPV phases in the PMNS matrix, and thus can be directly related to the low energy CP-violation in the lepton sector. Let us recall that leptogenesis \([52]\) is a simple mechanism which allows us to explain the observed baryon asymmetry of the Universe. The simplest scheme in which this mechanism can be implemented is the “seesaw” (type I) model of neutrino mass generation \([3]\). In its minimal version it includes the Standard Model (SM) plus two or three right-handed (RH) heavy Majorana neutrinos, \( N_j \). Thermal leptogenesis (see, e.g. \([54]\)) can take place, e.g. in the case of a hierarchical spectrum

\(^4\) Encouraging results, in what regards the problem of calculation of the NME, were reported recently in \([44]\).

\(^5\) A possible test of the NME calculations is discussed in \([51]\).
of heavy Majorana neutrino masses, \( M_1 < M_2 < M_3 \). The lepton asymmetry is produced in out-of-equilibrium lepton number and CP nonconserving decays of the lightest RH Majorana neutrino, \( N_1 \), mediated by the neutrino Yukawa couplings, \( \lambda \). The lepton asymmetry thus generated is converted into a baryon asymmetry by \((B-L)\)-conserving but \((B+L)\)-violating sphaleron interactions [53] which exist within the SM. In grand unified theories the masses of the heavy Majorana neutrinos are typically (by a few to several orders of magnitude) smaller than the scale of unification of the electroweak and strong interactions, \( M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \). This range coincides with the range of values of \( M_j \), required for a successful thermal leptogenesis [54].

At energies below the heavy Majorana neutrino mass \( M_1 \) and of the electroweak symmetry breaking scale, the neutrino Yukawa couplings generate a Majorana mass for the LH flavour neutrinos:

\[
m^\nu = v^2 A^T M^{-1} \lambda = U^* m U^T,
\]

where \( M \equiv \text{Diag}(M_1, M_2, M_3) \), \( m \equiv \text{Diag}(m_1, m_2, m_3) \), \( M_j > 0 \), \( m_k \geq 0 \), \( U \equiv U_{\text{PMNS}} \), and \( v = 174 \text{ GeV} \) is the Higgs field vacuum expectation value. In our further discussion it is convenient to use the “orthogonal parametrisation” of the matrix of neutrino Yukawa couplings:

\[
\lambda = v^{-1} \sqrt{M R \sqrt{m U^T}}, \quad R R^T = R^T R = 1,
\]

where \( R [55] \) is, in general, a complex matrix.

The possibility of existence of a connection between leptogenesis and the low energy CP-violation in the lepton (neutrino) sector has been widely discussed in recent years, the conclusion being generically negative (see, e.g. [54,56] and the references quoted therein). Recent progress in the understanding of the importance of lepton flavour effects in leptogenesis [57,58] (for earlier discussion see [59]) lead to the realization [34] that the CP-violation necessary for the generation of the baryon asymmetry of the Universe, \( Y_B \), can be due exclusively to the Majorana or/and Dirac CPV phases in the PMNS matrix, and thus can be directly related to the low energy CP-violation in the lepton sector (e.g. in \( \nu \)-oscillations, etc.). In the cases when the only source of CP-violation is respectively the Majorana or the Dirac phases in \( U_{\text{PMNS}} \), there exists a correlation between the baryon asymmetry \( |Y_B| \) and i) the effective Majorana mass in \((\beta\bar{\beta})_{10}\)-decay, \( |<m>| \), or ii) the rephasing invariant \( J_{\text{CP}} \) controlling the magnitude of CP-violation in neutrino oscillations [29].

As was shown in [57,58], in the case of a hierarchical spectrum of the heavy Majorana neutrinos \( N_j \), \( M_1 \ll M_2 \ll M_3 \), the flavour effects in leptogenesis can be significant for \( 10^8 \text{ GeV} \lesssim M_1 \lesssim (0.5 - 1.0) \times 10^{12} \text{ GeV} \). If the requisite lepton asymmetry is produced in this regime, the CP-violation necessary for successful leptogenesis can be provided entirely by the low energy Majorana CPV phases in \( U_{\text{PMNS}} \) [34]. Indeed, suppose that the mass of \( N_1 \) lies in the interval of interest, \( 10^9 \text{ GeV} \lesssim M_1 \lesssim 10^{12} \text{ GeV} \), and that the matrix \( R \) has real and/or purely imaginary elements: we are interested in the case when the CP-violation necessary for leptogenesis is due exclusively to the CPV phases in \( U_{\text{PMNS}} \). Under these assumptions \( Y_B \) generated via leptogenesis can be written as [57,58] \( |Y_B| \equiv 3 \times 10^{-3} |\epsilon_\tau \eta| \), where \( \epsilon_\tau \) is the CPV asymmetry in the \( \tau \) flavour (lepton charge) produced in \( N_1 \)-decays \(^6\):

\[
\epsilon_\tau = -\frac{3 M_1}{16 \pi v^2} \text{Im} \left( \sum_{j<k} m_j^{1/2} m_k^{3/2} U_{e,j}^* U_{e,k} R_{1j} R_{1k} \right)/\sum_{i} m_i |R_{1i}|^2.
\]

\(^6\) We have given the expression for \( Y_B \) normalised to the entropy density, see, e.g. [34].
\[ \eta \text{ is the efficiency factor [58], } |\eta| \equiv |\eta(0.71\tilde{m}_2) - \eta(0.66\tilde{m}_\tau)|, \text{ \tilde{m}_2, \tilde{m}_\tau \text{ being the wash-out mass parameters, } \tilde{m}_2 = \tilde{m}_\nu + \tilde{m}_\mu, \tilde{m}_l = |\sum_j m_j \, R_{1j} \, U_{e j}|^2. \text{ An approximate analytic expression for } |\eta| \text{ is given in [57,58].} \]

Consider the specific example of IH light neutrino mass spectrum, \( m_3 \ll m_{1,2} \cong (\Delta m^2_{\text{atm}})^{1/2} \).

Under the simplifying assumptions of \( m_3 \cong 0 \) and \( R_{13} \cong 0 \) (\( N_3 \) decoupling), leptogenesis can be successful for \( M_1 \cong 10^{12} \text{ GeV} \) only if \( R_{11} R_{12} \) is not real [34,60], so we consider the case of purely imaginary \( R_{11} R_{12} = i \kappa |R_{11} R_{12}| \), \( \kappa = \pm 1 \). The requisite CP-violation can be due to the Majorana phase \( \alpha \) in \( U_{\text{PMNS}} \).

If we set \( s_{13} = 0 \), the maximum of \( |Y_B| \) for, e.g. \( \kappa = -1 \), is reached for [34] \( |R_{11}|^2 \cong 1.4 \) \((|R_{12}|^2 = |R_{11}|^2 - 1 = 0.4) \), and \( \alpha \cong 2\pi/3,4\pi/3 \), and at the maximum \( |Y_B| \cong 1.5 \times 10^{-12}(\Delta m^2_{\text{atm}})^{1/2}/0.05 \text{ eV} (M_1/10^{10} \text{ GeV}) \) (Fig. 2, left panel). The observed baryon asymmetry, \( |Y_B| \cong (8.0 - 9.2) \times 10^{-11} \), can be reproduced if \( M_1 \cong 5.3 \times 10^{10} \text{ GeV} \). Since both \( |Y_B| \) and the effective Majorana mass \( |<m>| \) depend on the Majorana phase \( \alpha \), there exists a correlation between the values of \( |Y_B| \) and \( |<m>| \) (Fig. 2).

As was shown in [34], we can have successful leptogenesis in the case of IH spectrum also if the source of CP-violation is the Dirac phase \( \delta \) in \( U_{\text{PMNS}} \). In this case values of \( |\sin \theta_{13} \sin \delta| \gtrsim 0.02 \), or \( |J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3} \), are required. Values of \( |J_{\text{CP}}| \) as small as \( 4.6 \times 10^{-3} \), can be probed in \( \nu \)-oscillation experiments at neutrino factories [61]. Since both \( Y_B \) and \( J_{\text{CP}} \) depend on \( s_{13} \) and \( \delta \), there exists a correlation between the values of \( |Y_B| \) and \( J_{\text{CP}} \) [34] if the other relevant parameters are fixed.

Similar results can be obtained [34] for the NH light neutrino mass spectrum, as well as in the case of quasi-degenerate in mass heavy Majorana neutrinos.

4. Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is one of the most formidable and pressing problems in today’s neutrino physics. The (\( \beta\beta \))\textsubscript{0ν}–decay experiments have the potential of establishing the Majorana nature of neutrinos with definite mass. If the latter are Majorana particles, the (\( \beta\beta \))\textsubscript{0ν}–decay experiments can provide information on the type of \( \nu \)-mass spectrum, on the absolute scale of neutrino masses and on the Majorana CPV phases present in the neutrino mixing matrix \( U \). The CP-violation in leptogenesis, necessary for the generation of the baryon asymmetry of the Universe, can be due exclusively to the Majorana and/or Dirac CPV phase(s) in \( U \). These results underline the importance of understanding
the status of the CP-symmetry in the lepton sector and, correspondingly, of the experimental searches for leptonic CP-violation at low energies.

The developments described briefly in this talk would have pleased Peter and I am sure he would have contributed to at least some of them.

To Frank and Ettore: Happy Anniversary! Thank you for being the driving force in the field of experimental ($\beta\beta$) decay research and inspiration for the younger generation of researchers working in this field. My best wishes for personal happiness and many professional successes.

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References
[1] S.T. Petcov, Nucl. Phys. B (Proc. Suppl.) 143 (2005) 159.
[2] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 67.
[3] P. Minkowski, Phys. Lett. B 67 (1977) 421.
[4] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.
[5] S.T. Petcov, Phys. Lett. B 110 (1982) 245;
P.H. Frampton et al., Nucl. Phys. B 687 (2004) 31.
[6] A. Morales and J. Morales, Nucl. Phys. Proc. Suppl. 114 (2003) 141.
[7] C. Aalseth et al., hep-ph/0412300.
[8] S.T. Petcov, Physica Scripta T121 (2005) 94;
Pascali S and S.T. Petcov, hep-ph/0308034.
[9] S.M. Bilenky, S. Pascoli and S.T. Petcov, Phys. Rev. D 64 (2001) 053010 and 113003.
[10] Pascoli S, Petcov S T and Wolfenstein L 2002 Phys. Lett. B 524 319.
[11] Choubey S and Petcov S T 1980 Phys. Lett. B 94 495.
[12] Apollonio M et al. 1999 Phys. Lett. B 466 415.
[13] Schwetz T, Tortola M and Valle J W F 2008 Preprint arXiv:0804.4857.
[14] Espaci J et al. 2004 Preprint hep-ex/0402041.
[15] Bandyopadhyay A et al. 2005 Phys. Lett. B 608 115;
Bandyopadhyay A et al. 2008 Preprint arXiv:0804.4857.
[16] Choubey S and Petcov S T 2004 Phys. Lett. B 594 333;
Bandyopadhyay A et al. 2005 Phys. Rev. D 72 072002;
Huber P et al. 2004 Phys. Rev. D 70 073014.
[17] Anderson K et al. 2004 Preprint hep-ex/0402041.
[18] Ardellier F et al. [Double Chooz Collaboration] 2006 Preprint hep-ex/0606025.
[19] See, e.g., Heeger K M 2006, talk given at Neutrino’06 International Conference, June 13 - 19, 2006, Sant Fe, U.S.A.
[20] Krastev P I and S.T. Petcov S T 1988 Phys. Lett. B 205 84;
[30] Bandyopadhyay A et al. 2007 Preprint arXiv:0710.4947;
  Albright C et al. 2004 Preprint physics/0411123;
  Itow Y et al. 2001 Preprint hep-ex/0106019.
[31] Langacker P et al. 1987 Nucl. Phys. B 282 589.
[32] Barger V et al. 2002 Phys. Lett. B 540 247;
  De Gouvea A, Kayser B and Mohapatra R 2003 Phys. Rev. D 67 053004.
[33] Pascoli S, Petcov S T and Yaguna C E 2003 Nucl. Phys. B 738 219;
  Petcov S T and Shindou T 2006 Phys. Rev. D 74 073006.
[34] Pascoli S, Petcov S T and Riotto A 2007 Phys. Rev. D 75 083511 and Nucl. Phys. B 774 1.
[35] Molinaro E et al. 2008 Nucl. Phys. B 797 94.
[36] Petcov S T and Shindou T 2006 Preprint hep-ph/0605204.
[37] Wolfenstein L 1981 Phys. Lett. B 107 77;
  Bilenky S M et al. 1984 Nucl. Phys. B 247 61;
  Kayser B 1984 Phys. Rev. D 30 1023.
[38] Lobashev V et al. 2003 Nucl. Phys. A 719 153c.
[39] Eitel K et al. 2005 Nucl. Phys. Proc. Suppl. 143 197.
[40] Tegmark M 2005 Preprint hep-ph/0503257;
  Hannestad S et al. 2006 Preprint astro-ph/0603019.
[41] Petcov S T and Smirnov A Yu 1994 Phys. Lett. B 322 109.
[42] Pascoli S and Petcov S T 2008 Phys. Rev. D 77 113003.
[43] Klapdor-Kleingrothaus H V 2001 et al., Nucl. Phys. Proc. Suppl. 100 309.
[44] RodinV A et al. 2003 Phys. Rev. C 68 044302 and Preprint nucl-th/0503063;
  Caullier E et al. 2007 Preprints arXiv:0709.2137 and arXiv:0709.0277 (nucl-th).
[45] Aalseth C E et al. 2000 Phys. Atomic Nuclei 63 1225.
[46] Klapdor-Kleingrothaus H V et al. 2004 Phys. Lett. B 586 198.
[47] Klapdor-Kleingrothaus H V and Krivosheina I V 2006 Mod. Phys. Lett. A 21 1547.
[48] Barabash A et al. 2004 JETP Lett. B 80 377.
[49] Amaboldi C et al. 2008 Preprint arXiv:0802.3439.
[50] Avignone A 2005 Nucl. Phys. Proc. Suppl. 143 233.
[51] Bilenky S M and Petcov S T 2004, Preprint hep-ph/0405237.
[52] Fukugita M and Yanagida Y 1986 Phys. Lett. B 174 45.
[53] Kuzmin V A, Rubakov V A and Shaposhnikov M E 1985 Phys. Lett. B 155 36.
[54] Buchmuller W et al. 2002 Nucl. Phys. B 643 367; Buchmuller W et al. 2005 Annals Phys. 315 305.
[55] JCasas J A and Ibarra A 2001 Nucl. Phys. B 618 171.
[56] Pascoli S, Petcov S T and Rodejohann W 2003 Phys. Rev. D 68 093007.
[57] Abada A. et al. 2006 JCAP 0604 (2006) 004;
  Nardi E et al. 2006 JHEP 0601 (2006) 164.
[58] Abada A. et al. 2006 JHEP 0609 (2006) 010.
[59] Barbieri R et al. 2000 Nucl. Phys. B 575 61.
[60] Petcov S T et al. 2006 Nucl. Phys. B 739 208.
[61] See, for instance, De Lellis G et al. 2005 “Neutrino factories and superbeams”, Proceedings, 7th International Workshop, NuFact05, Frascati, Italy, June 21-26, 2005.