A new phase unwrapping method

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Abstract. This work is devoted to an elimination of the phase discontinuities problem of the reconstructed object wavefront. Novel robust phase unwrapping method is proposed. It is based on the phase shift implementation of the reconstructed wavefront. The unwrapped phase field of the amplitude-phase object is presented. Comparison with another known methods is shown and discussed. Proposed method resistant to noise and object phase field inhomogeneities. Its advantages are robustness and simplicity.

1. Introduction
Holography is the universal method for the complete reconstruction of the object wavefront [1]. Nowadays digital holography successfully replaces the analog one and becomes more popular due to the rapid development of digital technologies. Hologram recording is performed with the use of CCD or CMOS matrices, and holograms subsequent processing is realized on computers [2]. The advantage of using digital technology is significant acceleration of the hologram recording and processing owing to the absence of the chemical preparation of the recording media [3, 4], and numerical reconstruction of the object wavefront greatly simplifies holograms post-processing. Due to the universality of the holography scientific interest to this method is not quenched, but rather attracting more attention of researchers [5, 6, 7, 8, 9].

The main goal of holographic research methods is an absolute phase calculation of object wavefronts. This task is especially difficult because of the specificity of the holographic technique for the phase reconstruction: it always comes down to calculation of inverse trigonometric functions. As a result, it gives the phase values, that differ from the absolute through the periodicity of these functions: \( \Theta_{\text{abs}} = \Theta_{\text{w}} + 2\pi \cdot n \), \( \Theta_{\text{abs}} \) – is the absolute phase of the object wavefront, \( \Theta_{\text{w}} \) – wrapped phase, \( n \) – integer number. Thus, after reconstruction the absolute phase is divided into separated regions with phase values in the region \((-\pi, \pi]\) (these values depend on a period of trigonometric function used in holographic reconstruction), and researchers’ task is to unwrap these phase regions in a single continuous field. Unwrapping is complex problem due to the presence of a noise in registration systems and phase inhomogeneities of the object.

Phase unwrapping problem was set long ago, and many methods were suggested to solve it. The main task of the phase unwrapping is a choice of the place where the phase of the field should be shifted. In real experiment conditions such choice is often complicated by a phase noise which can lead to erroneous phase unwrapping shifts. Since phase noise often have a higher frequency than the desired signal, initial filtering of the wrapped phase field is the easiest and most intuitive way to simplify unwrapping, for example it could be done by the Fourier filtering [11], or least
square method [12]. But the filtering could result in a loss of the useful high-frequency signal and smoothing of the whole wavefront. Also there is popular method that based on the finding of local phase errors by a specific scanning of the phase wavefront [13], but this method has difficulties when spatial size of errors is much larger than the size of recording system pixel. An alternative to the search of local errors method is an approach based on the energy minimization algorithm with initial assumption that the phase difference between adjacent pixels is less than the value of \( \pi \) radians [14], in fact this assumption is not correct in many real experiment cases. There is also a hardware method for obtaining of the absolute phase by utilization of two or more radiation light wavelengths with close values [15]. Increasing of the wavelengths number gives possibility to create a synthetic wavelength with bigger value, and therefore it is possible to enlarge continuous phase regions of the reconstructed wavefront. Often in this method the unwrapping procedure is not required or reduced to the simplest one. It should be noted, however, that the holograms registration with different light wavelengths greatly complicates this technique both in a hardware and software. Despite the significant number of proposed methods for phase unwrapping problem solving, there is no unequivocal reconstruction method of the absolute phase from holograms of objects with phase inhomogeneities.

Especially it is hard to unwrap the phase of the amplitude-phase object wavefront, that reconstructed with high resolution, because amplitude field creates phase inhomogeneities in areas of complete absorption of the light radiation. The higher resolution the bigger number of such inhomogeneities. Such high resolution wavefront of the amplitude-phase object (the wing of a fly) is shown in figure 1. It was reconstructed by phase-shifting digital holography [16].

\[
\Theta = \arctan \left[ \frac{I_{3\pi/2} - I_{\pi/2}}{I_{\theta} - I_{\pi}} \right]
\]  

The phase of the object wavefront was calculated by equation 1, where \( \Theta \) - object wavefront phase, \( I_p \) - holograms with shifted reference beam by 0, \( \pi/2 \), \( \pi \) and \( 3\pi/2 \) radians. Arctangent has a period of \( \pi \) radians, but with proper consideration of numerator and denominator signs, the phase has the period of \( 2\pi \). Actually this procedure could be considered as a first step in the unwrapping, because it decreases the number of wrapped lines twice.

Figure 1. Amplitude (left) and phase (right) of the object wavefront

2. Method description

This paper presents a phase unwrapping method by a Phase Shift of the Reconstructed Field (PSRF) that shows reliable unwrapping results for amplitude-phase objects. Since wrapped phase field is defined by a periodic function, it is possible to add a constant phase to whole wrapped phase field, thus realizing a shift of the entire field without disturbing its general order. It is obvious that addition of a half period of the function will result in phase shift of the field,
and ruptured area of the field will be replaced by the continuous area. Steps of the PSRF method application to phase field of the amplitude–phase object are shown in figure 2. Fig. 2(a) represents the initial phase reconstruction of the object wavefront: \( E = A \exp^{i \Theta} \) (where \( E \) – object wavefront, \( A \) – amplitude, \( \Theta \) – phase), and Fig. 2(b) shows the same wavefront’s phase field, but with an addition of the phase constant \( \pi \) to its phase: \( E = A \exp^{i(\Theta+\pi)} \).

**Figure 2.** Method PSRF. Wrapped phase field (a), phase field shifted by \( \pi \) rad (b), phase field after stitching of continuous areas (c), resulting unwrapped phase field (d). Black squares mark out regions of the undisturbed phase.

As it shown in Fig. 2(b) corrupted region moved to another part of the field. Thus, the addition of the phase shift to whole field leads to a realization of the phase unwrapping by stitching only undisturbed areas of the phase field (areas marked by rectangles in Fig. 2(a) and (b)). The stitching result is presented in Fig. 2(c). Final unwrapped phase field is presented in Fig. 2(d), where the phase additive constant is added only to the shifted phase area. The \( \pi \) radians shift is a half period of the wrapped function, therefore, this shift will always lead to a replacement of the ruptured region by continuous, that makes it possible to assemble a complete picture of the absolute wavefront phase by a successive stitching of shifted continuous fragments.

### 3. Results and discussion

The main problem of all unwrapping methods is phase discontinuities presence in the reconstructed wavefront. The cause of this discontinuities is in high noise levels of the hologram and/or characteristics of investigated objects. To evaluate the effectiveness of the PSRF method, results of the phase unwrapping by modern methods: Goldstein [13] and PUMA [14], are presented in Fig. 3. Detailed analysis of absolute phases, obtained by these methods, shows that the large number of non-smooth phase regions leads to erroneous unwrapping results. Processing by Goldstein in this case leads to unsatisfactory results due to false detection of discontinuities in the spatial phase distribution. Method PUMA, based on energy minimization, shows better result than the Goldstein method. However, it provides the distorted unwrapped phase due to assumption the difference between adjacent pixels smaller than \( \pi \) radians. This assumption leads to a smoothing of existing phase inhomogeneities and unjustified unwrapped phase jump on the border of the object. The proposed PSRF method do not form phantom phase shifts, and unwrapping is reliably performed even in the case of the phase inhomogeneities presence, through the implementation of a global phase shift, without a consideration of the individual field points.

For the qualitative comparison in Fig. 4 longitudinal cross-sections are presented: initial phase field distribution (1) and unwrapped phase fields by methods: (2) – Goldstein, (3) – PUMA, (4) - PSRF. Fig. 4(a) presents cross-sections from the part of the undisturbed phase, that corresponds to dashed line from fig. 3(a), and fig. 4(b) presents cross-sections from the
Figure 3. Reconstructed object phases – wrapped (a) and unwrapped by: Goldstein (b), PUMA (c), PSRF (d)

object phase field, dotted line from fig. 3(a). As it can be seen from the cross sections shown in
Fig.4(a), in undisturbed conditions all methods work reliably and accurately, unwrapping results
are performed equally. But Goldstein’s and PUMA’s results from the area with many phase
inhomogeneities, related to characteristics of the object, are erroneous (Fig.4(b)). Goldstein
shows good results only on small areas of the phase field, but the whole phase field is unwrapped
wrong. Due to small height of the object the phase shift on the "object–undisturbed wavefront”
line should not be bigger than $\pi$ (see fig.4) , but PUMA recognized it as about $8\pi$ (fig.4(b)).
This error is explained by that the border of the object is not transparent, and it creates
inhomogeneities on the phase field, but PUMA considers this border as place of the phase shift
and wrongly unwraps it. The proposed PSRF method is free of such shortcomings and gives
legitimate result. Thus, it is shown that the PSRF method can be used to eliminate phase
discontinuities as for undisturbed (Fig.4(a)), so for noised (Fig.4(b)) wavefronts. Furthermore
PSRF could be realized in real-time unwrapping owing to relatively simple calculations.

4. Conclusion
I have described the new phase unwrapping method based on the use of the phase shift of the
reconstructed wrapped field. This method demonstrates unwrapping robustness to a noise and
features of the object. Due to the fact that phase information more often lost in high amplitude
areas of the radiation absorption, phase anomalies take place in these areas and make difficulties
Figure 4. Longitudinal cross-sections of phase fields: (a) – 50-th row pixels (dashed line from fig. 3, a), (b) – 440-th row (dotted line from fig. 3, a). Here 1 – cross-section of wrapped phase, and cross-sections of the unwrapped phase by methods: 2 – Goldstein, 3 – PUMA, 4 – PSRF.

to existing unwrapping methods. Based on the demonstrated results, proposed PSRF method is the most promising in amplitude-phase object cases.

References
[1] D. Gabor, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 197, 454 (1949).
[2] J.W. Goodman, R.W. Lawrence, APL. 11, 77 (1967).
[3] V.P. Smaev, Y.A. Vavilova, A.D. Galpern, Opt.-Mekhan. Prom.3, 44 (1990).
[4] V.P. Smaev, A.D. Gal’Pern, Yu.A. Vavilova, A.A. Paramonov, “Some Features of Production of Monolayers for Multilayer Photographic Materials” Opt. and Spectr. 76, 738 (1994).
[5] A.V. Belashov, N.V. Petrov, I.V. Semenova, Opt. Express, 22, 28363 (2014).
[6] V. Katkovnik, I.A. Shevkunov, N.V. Petrov, K. Egiazarian, Opt. Lett. 40(10), 2417 (2015).
[7] S. Hakim, M. Yamaguchi, and F. Kimura, Information Optics (WIO), 2015 14th Workshop on. IEEE, 1 (2015).
[8] D. Merrill, R. An, J. Turek, D.D. Nolte, Appl. Opt. 54, A89 (2015).
[9] P.A. Cheremkhin, I.A. Shevkunov, N.V. Petrov, Physics Procedia 73, 301 (2015).
[10] W.J. Hwang, H.Y. Chen, C.J. Cheng, Appl. Opt. 54(1), A67 (2015)
[11] M. Kujawinska, J. Wojciak, Opt. Lasers Eng. 14, 325 (1991).
[12] R. Goldstein, H. Zebreker, C. Werner, Radio science, 23, 713 (1988).
[13] R. Bioucas-Dias, G. Valadão, Image Processing, IEEE Transactions on 2007, 16, 698 (2007).
[14] Y. Li, W. Xiao, E. Fan, Appl. Opt. 53, 979 (2014).
[15] I. Yamaguchi, T. Zhang, Opt. Lett. 22, 1268 (1997).