Coherence interpretation of nonlocal quantum correlation in a delayed-choice quantum eraser

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Abstract
Bell inequality violation is a quantitative measurement tool for quantum entanglement. Quantum entanglement is the heart of quantum information science, in which the resulting nonlocal correlation between remotely separated local detectors shows a unique property of quantum mechanics. Over the last few decades, intensive researches have been conducted for the basic physics of quantum correlation as well as its potential applications to quantum technologies. Here, the role of coincidence detection is investigated for the nonlocal correlation in a simple interferometer of the delayed-choice quantum eraser. To understand the nonlocal quantum feature based on a joint-parameter relation, coincidence detection between two output photons from the interferometer is analyzed for Bell inequality violations in an inseparable intensity-product manner. Based on this understanding, a counterintuitive coherence version of the quantum entanglement generation is proposed for the use of an attenuated laser.

Introduction
Quantum entanglement is the heart of quantum information science which cannot be implemented by any classical means [1-4]. The definition of ‘classical’ represents for incoherent and individual particles [2,5]. Bell inequality violation is a test tool for a quantitative measurement of quantum entanglement [5]. Nonlocal correlation between entangled photons shows a mysterious phenomenon known as EPR paradox [6]. The EPR paradox is for the violation of local realism between space-like separated parties [7]. A common method of entangled photon generation is by a spontaneous parametric down conversion (SPDC) process [8,9]. Here, the nonlocal correlation is investigated in a Mach-Zehnder interferometer (MZI) of the delayed-choice quantum eraser [10,11]. For this, coincidence detection is coherently analyzed for polarization projection of the output photons [12,13]. Based on this interpretation, a coherence version of the nonlocal correlation is proposed and analyzed for the same quantum feature using a pair of synchronized acousto-optic modulators (AOMs) and heterodyne detection.

In 1978, Wheeler proposed a thought experiment of a delayed choice, in which post-measurements can reverse a predetermined photon characteristic [14]. Since then, many experiments have been followed to demonstrate violation of the cause-effect relation [15-19]. One example of the predetermined photon property is to use an orthogonally polarized Mach-Zehnder interferometer (MZI) to prepare distinguishable photon characteristics. The quantum eraser has been conducted for output photons through a set of polarizers to show the reversed photon property into the wave nature via coincidence detection [12,13]. In the present study, nonlocal correlation in a delayed-choice quantum eraser scheme [13] is analyzed to seek the origin of quantum features using a coherence approach. For this, polarization bases of distinguishable output photons are projected onto a rotated polarizer, resulting in interference fringes for coincidence detections. Moreover, this nonlocal interference fringe shows a joint relation between independent local parameters of the polarizers. For an equivalent coherence version, random bases of orthogonal polarizations are provided in an MZI system, where the same joint relation between output photon product is accomplished by heterodyne detection-resulting measurement-event selections.

Results
Figure 1(a) shows schematic of Sagnac interferometer-based nonlocal correlation using a type-II phase-matched nonlinear crystal, whose symmetric frequency relation in each photon pair is shown in Fig. 1(b) [13]. Figure 1(c)
shows the proposed coherence model equivalent to Fig. 1(a), whose light source is an attenuated laser. In Fig. 1(a),
the entangled photon pair has an orthogonal polarization-basis relation satisfying phase matching among pump,
signal, and idler photons [13,20]. Although photon pairs are indistinguishable on a polarizing beam splitter (PBS),
the output photons before the polarizers are turned out to be distinguishable through the PBS, resulting in no
fringes. Unlike the backward pump scheme in ref. [20], resulting in strong phase dependency between superposed
photons, the photon pairs ‘1’ and ‘2’ in Fig. 1(a) has no such relation due to $\delta f_j \neq \delta f_k$. Moreover, the path lengths
of ‘1’ and ‘2’ cannot be equal, where their path-length difference must be kept less than the ensemble coherence
c$^{-1}$. Thus, the output photon states can be represented as, e.g., $|E_A\rangle = \frac{E_0}{\sqrt{2}} (|H\rangle_2 + e^{i\varphi'}|V\rangle_1)$ and $|E_B\rangle = \frac{iE_0}{\sqrt{2}} (|V\rangle_2 + e^{i\varphi'}|H\rangle_1)$, where, $\varphi' = \varphi + \delta f_{jk}$, $\delta f_{jk} = \delta f_j - \delta f_k$, and the subscripts represent MZI paths.

Due to the locked phase between entangled photons by the phase matching of SPDC nonlinear optics [8,9],
the pair of output photons denoted by the same colors in Fig. 1(a) is phase coherent [13]. However, no such phase
coherence relation exists between different colored pairs [13,21]. Thus, this phase coherence relation does not
work for local measurements for orthogonal polarizations in each detector after the polarizers due to
$\delta f_{jk} \neq 0$ [12,13]. Because of the definite phase relation between same colored photons in Fig. 1(a), however, coincidence
measurements between independent detectors results in nonlocal fringes, representing the Bell inequality violation
[12,13]. Thus, the quantum eraser observed in ref. [12] is rooted in the MZI coherence for the superposed photon
pair according to the Hermitian operators for the same polarization basis in the backward pump scheme [20,21].
By this reason, there is no such phase dependency between different colored photons in Fig. 1(a), satisfying $\delta f_j \neq \delta f_k$.

Fig. 1. Schematics of quantum correlation. (a) Quantum model based on $\chi^{(2)}$ SPDC. (b) SPDC-generated
photon spectrum. (d) Coherence model based on coherent photons. A1/A2: synchronized acousto-optic
modulators, $D_s/D_i$: single photon detectors, HWP: 22.5°-rotated half-wave plate, L: laser, BS: nonpolarizing
50/50 beam splitter, PBS: polarizing beam splitter, $\xi$ and $\theta$ are polarizer’s rotation angles.

Analysis
In Fig. 1(a), the photon pair indicated by the same subscript (or same color) must be coherent each other whose
frequencies are oppositely detuned as shown in Fig. 1(b). This symmetric detuning $\pm \delta f_j$ of the $j^{th}$ photon pair is
represented by ‘±’ in the superscript of the polarization notation in each pair. Regardless of the SPDC bandwidth
in Fig. 1(b) and the pump photon’s absolute phase, the ensemble coherence between superposed photon pairs of
different colors cannot be accomplished due to different path lengths and $\delta f_{jk} \neq 0$ in a wide bandwidth $\Delta$. For
the same colored photons, the path-length difference has no effect especially for the coincidence detection due to
separate measurements by definition. Here, a simultaneous generation of both pairs in both paths is far less than
1% and thus are neglected for the present analysis [8,9,13,20].

In Fig. 1(b), the $j^{th}$ paired photons are oppositely detuned by $\pm \delta f_j$ from the center frequency $f_0$. This center
frequency is the half of the pump frequency according to the energy conservation law in SPDC, where its
frequency stability depends on the pump linewidth [8,9]. Due to the phase matching condition of SPDC, the
coherence relation is strictly provided among the pump, signal, and idler photons within the geometry [20]. The symmetric detuning of the entangled photon pairs plays an important role for the bandwidth-independent joint parameter relation in nonlocal correlation (discussed below).

Figure 1(c) shows a corresponding coherence model based on coherent photons from an attenuated laser. For coincidence detection, doubly bunched coherent photons are required as an input pair by definition. Three or more bunched photons given by Poisson statistics may be included in the coincidence measurements. For a small mean photon number, 〈n〉 ≪ 1, however, the multi-photon generation ratio to the single photon can be kept below 1 % [22], as in the SPDC case [8,9]. To provide random polarization bases in each MZI path, a 22.5°-rotated half-wave plate (HWP) is inserted. Using a pair of AOMs, frequency correlation (f_–; f_+) is provided for the heterodyne detection. The AOMs are coherently manipulated by a synchronized pair of rf modulators for ±δf across the laser frequency f_0. For coincidence detection between two independent detectors, a heterodyne detection technique is applied to prohibit the different-colored (same detuning) photon pairs from measurements. As analyzed for Franson-type nonlocal correlation, this measurement-event modification is the quintessence of the nonlocal correlation violating local realism (discussed below) [23].

In Fig. 1(a), output photons E_A and E_B are coherently represented as:

\[ E_A = \frac{E_0}{\sqrt{2}} (iV_1^x e^{i\varphi} e^{i\delta f_1 t} + H_2^x e^{i\delta f_2 t}) \]

\[ E_B = \frac{E_0}{\sqrt{2}} (H_1^x e^{i\varphi} e^{-i\delta f_1 t} + iV_2^x e^{-i\delta f_2 t}) \]

where the ± sign of δf can be swapped between E_A and E_B. E_0 is the amplitude of a single photon. Here, a \( \pi/2 \) phase difference between H and V is from the entangled photon pair relation [24,25]. Thus, the corresponding mean intensities measured in both MZI output ports before the polarizers are \( \langle I_A \rangle = \langle I_B \rangle = \langle I_0 \rangle \) due simply to orthogonal polarization bases, representing the Fresnel-Arago law [26]. This represents distinguishable photon characteristics in the output ports.

With the inserted polarizers (ξ, θ) in both output paths, polarization projections of the output photons onto the rotated polarization angles ξ and θ are applied to Eqs. (1) and (2):

\[ E_s = \frac{E_0}{\sqrt{2}} (iV_1^x \sin\xi e^{i\varphi} e^{i\delta f_1 t} + H_2^x \cos\xi e^{i\delta f_2 t}) \]

\[ E_i = \frac{E_0}{\sqrt{2}} (H_1^x \cos\theta e^{i\varphi} e^{-i\delta f_1 t} + iV_2^x \sin\theta e^{-i\delta f_2 t}) \]

where the inclusion of the polarization bases (H_{1,2}; V_{1,2}) is to indicate the photon’s origin for further analysis of coincidence detection below. Thus, the corresponding intensities for Eqs. (3) and (4) are as follows:

\[ I_s = \frac{I_0}{2} (iV_1^x \sin\xi e^{i\varphi} e^{i\delta f_1 t} + H_2^x \cos\xi e^{i\delta f_2 t} - iV_2^x \sin\xi e^{-i\delta f_1 t} + H_1^x \cos\xi e^{-i\delta f_2 t}) \]

\[ = \frac{I_0}{2} (1 - \sin2\xi\sin(\varphi + \delta_{jk})) \]

\[ I_i = \frac{I_0}{2} (H_1^x \cos\theta e^{i\varphi} e^{-i\delta f_1 t} + iV_2^x \sin\theta e^{-i\delta f_2 t} - iV_2^x \sin\theta e^{i\delta f_1 t} - H_1^x \cos\theta e^{-i\delta f_2 t}) \]

\[ = \frac{I_0}{2} (1 + \sin2\theta\sin(\varphi - \delta_{jk})) \]

where \( \delta_{jk} = \delta f_1 t - \delta f_2 t, \) and \( H_1V_2 \) product is allowed by the projection-driven common polarization basis. Here, the arrival time ‘t’ on the PBS from both paths is not the same due simply to the geometrical design, resulting in \( \langle \sin(\varphi \pm \delta_{jk}) \rangle = 0 \) for a wide bandwidth Δ. This relation is required for the coincidence detection to select only the same colored-photon pair. Even though the geometrical phase lock between the forward and backward photon pairs can be established by the \( \chi^{(2)} \) phase matching, the frequency detuning-caused phase difference, i.e., \( \delta_{jk} \) is random for an ensemble, resulting in an uniform intensity in each output port regardless of \( \varphi: \) \( \langle I_s \rangle = \langle I_i \rangle = \frac{I_0}{2} \) [13]. Here, the half cut in intensity is due to the 50% photon loss by the polarizer. By the way, for the backward pump scheme in ref. [12], however, the photon phase of the forward pair is influenced by the backward pair, resulting in pair-to-pair phase matching [20,21]. In this case, the local randomness is partially violated [12].

From Eqs. (3) and (4), the coincidence detection in amplitude between detectors \( D_s \) and \( D_i \) is calculated for the same colored-photon pairs in Fig. 1(a), where different colored-photon pairs are discarded by the definition of coincidence.

3
\[
R_{\text{si}}(\tau = 0) = E_x E_i = \frac{i \hbar}{2} \left( (V_1^* \sin \xi e^{i \phi_{\text{det}}} + H_1^* \cos \xi e^{i \phi_{\text{det}}}) (H_2 \cos \theta e^{i \phi_{\text{det}}} + i V_2^* \sin \theta e^{-i \phi_{\text{det}}}) + H_2 \cos \theta e^{i \phi_{\text{det}}} + i V_2^* \sin \theta e^{-i \phi_{\text{det}}}) \right),
\]

Interestingly, Eq. (7) has no relation with the SPDC bandwidth \( \Delta \) as well as \( \delta_{jk} \) due to the symmetric detuning in each entangled (same color) pair. Thus, the mean coincidence detection between two local detectors is as follows:

\[
\langle R_{\text{si}}(\tau = 0) \rangle = \langle i \hbar \rangle \left( V_1^* H_1^* \cos \theta \sin \xi e^{i 2 \phi_{\text{det}}} + H_1^* H_2 \cos \theta \sin \xi e^{-i 2 \phi_{\text{det}}} + H_2^* V_2 \cos \theta \sin \xi e^{i 2 \phi_{\text{det}}} + H_2^* V_2 \cos \theta \sin \xi e^{-i 2 \phi_{\text{det}}}) \right),
\]

From Eq. (8), it concludes that \( \langle R_{\text{si}}(\tau = 0) \rangle \) varies between \( \langle i \hbar \rangle \sin^2(\theta - \xi) \) and \( \langle i \hbar \rangle \sin^2(\theta + \xi) \) depending on \( \cos(2 \phi) \). For a fixed \( \phi = n \pi \), Eq. (8) becomes:

\[
\langle R_{\text{si}}(\tau = 0) \rangle = \frac{i \hbar}{4} \sin^2(\theta + \xi). \tag{9}
\]

Obviously Eq. (9) shows the quantum feature with an inseparable joint-parameter relation of local measurements, which cannot be obtained by local intensity products between Eqs. (5) and (6). This coherence derivation of Eq. (9) is equivalent to the observations in ref. [13]. For \( \phi = (n - 1/2) \pi \), Eq. (8) results in \( \langle R_{\text{si}}(\tau = 0) \rangle = \frac{i \hbar}{4} \sin^2(\theta - \xi) \). This also represents nonlocal correlation as in Eq. (9). Mathematically, the Bell parameter \( S \) for the \( \langle R_{\text{si}}(0) \rangle \) in Eq. (9) is \( 2 \sqrt{2} \) due to the perfect sinusoidal modulations [5,13,27]. Thus, the nonlocal quantum feature satisfying the inseparable basis product is successfully analyzed for Fig. 1(a) using the coherence approach, whose \( S \) parameter violates the Bell inequality. From this analysis, it can be said that coherence between paired photons is the origin of the spooky action at a distance as Einstein argued for the hidden variable [2].

Fig. 2. Numerical calculations for Eq. (7). (a) and (b) \( \phi = 0 \). (c) and (d) \( \phi = \frac{\pi}{4} \), \( \theta = 0 \) (blue); \( \theta = \frac{\pi}{4} \) (green); \( \theta = \frac{\pi}{2} \) (red); \( \theta = \pi \) (dotted). (e) and (f) Classical intensity product: \( I_{x,i} = \frac{1}{2} (1 - \sin 2\xi)(1 + \sin 2\theta) \).
Figures 2(a) and (b) show the numerical calculations for Eq. (9) resulting in the same nonlocal quantum feature observed in ref. [13]. Thus, the nonlocal quantum correlation, i.e., Bell inequality violation is successfully interpreted with the wave nature of quantum mechanics using pure coherence optics. The so-called mysterious quantum feature is now clearly understood as a direct result of coincidence measurements between coherently prepared paired photons. The origin of the inseparable basis product in Eq. (9) is in the coincidence detection-caused measurement-event modification. For $\varphi = \frac{\pi}{4}$, however, this joint relation between local parameters $\xi$ and $\theta$ disappears, as shown in Figs. 2(c) and (d). Unlike the typical classical feature of separable intensity products as shown in Figs. 2(e) and (f), however, Figs. 2(c) and (d) are not completely classical, because the bases are quantized in the phase domain [28], as in N00N states in a frequency domain [29]. Such a feature has already been observed in a hug scheme of the Franson-type nonlocal correlation [30].

Figure 1(c) shows a corresponding coherence model of the nonlocal correlation analyzed in Fig. 1(a). In Fig. 1(c), the coincidence detection is for oppositely detuned frequencies generated by synchronized AOMs. By the 22.5°-rotated HWP in each MZI path, each distinguishable photon becomes random in polarizations, where this photon has either $f_+$ or $f_−$. Due to the synchronized AOM manipulations for opposite frequency modulations, the same frequency-polarization correlation is established as in Fig. 1(a). The random polarizations generated by HWP in each MZI path results in only 25% contribution to coincidence measurements by the heterodyne detection. All other 75% are for the same frequency pairs, which are discarded in the heterodyne detection (see the Supplementary Materials). The discarded photon pairs do not affect the fidelity of this nonlocal correlation. In other words, measurable quantity for the heterodyne detection in Fig. 1(c) is set for different colored pairs [31]. For the heterodyne detection, the detector’s time resolution must be shorter than the inverse of the $\delta f$ ($= f_+ − f_−$). Without heterodyne detection, the fringe visibility in Fig. 2(b) drops down to 25%, belonging to the classical limit.

In Fig. 1(c), local mean values can be calculated to be uniform in both detectors as:

$$\langle I_s \rangle = \langle \frac{\delta_i}{2} (1 + \sin 2\xi \cos (\varphi - 2\delta f_i \tau)) \rangle = \langle \frac{\delta_i}{2} \rangle,$$

$$\langle I_l \rangle = \langle \frac{\delta_i}{2} (1 + \sin 2\theta \cos (\varphi - 2\delta f_l \tau)) \rangle = \langle \frac{\delta_i}{2} \rangle,$$

For this, a wide bandwidth AOM modulation is necessary to induce $(\cos(\varphi - 2\delta f_i \tau)) = 0$, otherwise local fringes are resulted. The heterodyne detection between two local detectors is obtained from Eqs. (3) and (4):

$$\langle R_{sl}(\tau = 0) \rangle = \langle \frac{\delta_{ij}}{4} \rangle \langle (V_1^x \sin \xi e^{i\varphi} e^{-i\delta f_j \tau} + H_2^x \cos \xi e^{i\delta f_j \tau}) (H_1^z \cos \theta e^{i\varphi} e^{-i\delta f_j \tau} + V_2^z \sin \theta e^{i\delta f_j \tau}) (c.c.) \rangle,$$

$$= \langle \frac{\delta_{ij}}{4} \rangle \sin^2 (\xi + \theta),$$

where $\delta f_j = \delta f_k$ in Fig. 1(c). Equation (12) shows the same feature as in Eq. (9). Thus, the coherently excited nonlocal correlation has been successfully achieved via synchronized AOM modulations and heterodyne detection. If the AOMs are not scanned but fixed at $f_\pm$, Eqs. (10) and (11) result in local fringes. Such a local fringe has been experimentally demonstrated for a quantum eraser in ref. [12] using entangled photon pairs and in ref. [22] using coherent photon pairs.

Discussion

In Fig. 1(a), the path-length difference between ‘1’ and ‘2’ is necessary to work with coincidence detection for nonlocal correlation as derived in Eq. (9) and demonstrated in Figs. 2(a) and (b). If both pairs are generated simultaneously in both paths, different colored photons are also allowed for $R_{sl}$ of the coincidence detection. This rare case leads to a separable intensity product, i.e., between Eqs. (5) and (6), resulting in Fig. 2(e) and (f), where $\delta_{jk} \neq 0$ is also allowed for coincidence detection due to different path lengths to the PBS from the $X^{(2)}$ medium. This classical feature negatively contributes to the nonlocal correlation but far less than a 1 % ratio. Thus, it is clear that coincidence detection is the essential method for nonlocal correlation, where the space-like separation does not deteriorate the mutual phase relation between paired photons. The nonlocal correlation has nothing to do with the spooky action at a distance. Instead, coincidence detection does for coherently prepared photon pairs via selective measurement events.
Coherence manipulations for random polarization bases in Fig. 1(c) were prepared by linear optics of HWP, and the prepared polarization bases were selectively measured by the heterodyne detection, resulting in the same nonlocal quantum features as in SPDC cases. Due to the polarization projection of the output photons for the delayed-choice quantum eraser, the nonlocal correlation was coherently obtained as a direct result of the wave nature of photons, whose resulting fringes were expressed by joint correlation between independent local parameters. This inherent inseparable joint-parameter relation implied in coherence between the paired photons was exposed by the action of measurement-event selections. Coherence manipulation using AOMs satisfying the symmetric frequency detuning is the origin of bandwidth independent nonlocal quantum fringes, violating classical physics. Thus, the nonlocal quantum feature is due to retrieval of a predetermined coherence relation between paired photons via coincidence detection-caused selective measurements.

Conclusion
The origin of nonlocal quantum feature was investigated using SPDC-generated entangled photon pairs in a quantum eraser scheme. For this, firstly, the phase-matched photon pairs whose polarizations are orthogonal but random were prepared in an MZI using a coherence approach. Secondly, coincidence detection between local measurements was analyzed for polarization projections via a pair of independent polarizers. Thirdly, the nonlocal quantum feature of Bell inequality violation was derived from the coincidence detection, resulting in an inseparable joint relation of local parameters. As a result, the coherence solutions for both local and nonlocal correlations were confirmed for the same quantum features observed. Finally, a coherence model of nonlocal correlation was proposed for coherent photons from an attenuated laser. The analytical solution for the coherence model was perfectly matched that of the SPDC case. In the coherence model, the same frequency-polarization correlation was provided by coherence manipulations of AOMs. The inseparable joint parameter relation between two local measurements was achieved by a coincident heterodyne-detection technique. Thus, the proposed coherence model for quantum features may open the door to deterministic quantum information science.

Method
The numerical calculations in Fig. 2 was performed by Matlab using equations appeared in the text.

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Availability of Data and Materials
The datasets used and/or analyzed during the current study available from the corresponding author on reasonable request.