Super-Eddington Mass Growth of Intermediate-mass Black Holes Embedded in Dusty Circumnuclear Disks

Daisuke Toyouchi1,2,3, Kohei Inayoshi4, Takashi Hosokawa3,5, and Rolf Kuiper5

1 Kavli Institute for Astronomy and Astrophysics at Peking University, Beijing 100871, People’s Republic of China; d.toyouchi@gmail.com
2 Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU, WPI), The University of Tokyo, Chiba 277-8583, Japan
3 Theoretical Astrophysics Group, Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan
4 Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, People’s Republic of China; inayoshi@pku.edu.cn
5 Institute of Astronomy and Astrophysics, University of Tübingen, Auf der Morgenstelle 10, D-72076 Tübingen, Germany

Received 2020 September 30; revised 2020 November 19; accepted 2020 November 30; published 2021 February 1

Abstract

We perform the first three-dimensional radiation hydrodynamical simulations that investigate the growth of intermediate-mass BHs (IMBHs) embedded in massive self-gravitating, dusty nuclear accretion disks. We explore the dependence of mass accretion efficiency on the gas metallicity Z and mass injection at super-Eddington accretion rates from the outer galactic disk M_{\text{ad}}, and we find that the central BH can be fed at rates exceeding the Eddington rate only when the dusty disk becomes sufficiently optically thick to ionizing radiation. In this case, mass outflows from the disk owing to photoevaporation are suppressed, and thus a large fraction (\geq 40\%) of the mass injection rate can feed the central BH. The conditions are expressed as M_{\text{in}} > 2.2 \times 10^{-1} M_\odot \text{yr}^{-1} (1 + Z/10^{-2} Z_\odot)^{-1} (c_s/10 \text{ km s}^{-1})^{-3}, where c_s is the sound speed in the gaseous disk. With increasing numerical resolution, vigorous disk fragmentation reduces the disk surface density, and dynamical heating by formed clumps makes the disk geometrically thicker. As a result, the photoevaporative mass-loss rate rises and thus the critical injection rate increases for fixed metallicity. This process enables super-Eddington growth of BHs until the BH mass reaches M_{\text{BH}} \sim 10^{-8} M_\odot, depending on the properties of the host dark-matter halo and metal-enrichment history. In the assembly of protogalaxies, seed BHs that form in overdense regions with a mass variance of 3\sigma at z \sim 15–20 are able to undergo short periods of rapid growth and transit into the Eddington-limited growth phase afterward to be supermassive BHs observed at z > 6–7.

Unified Astronomy Thesaurus concepts: Hydrodynamical simulations (767); Supermassive black holes (1663)

1. Introduction

The formation process of supermassive black holes (SMBHs) is one of the most important puzzles in modern astrophysics. The existence of SMBHs with M_{\text{BH}} \gtrsim 10^9 M_\odot in the early universe (z \gtrsim 6–7) provides a stringent constraint for their mass-growth timescale (e.g., Fan et al. 2001; Willott et al. 2010a; Mortlock et al. 2011; Venemans et al. 2013; Wu et al. 2015; Bañados et al. 2018; Matsuoka et al. 2019; Onoue et al. 2019; Yang et al. 2020). Various models for their seed black holes (BHs) have been suggested (see, e.g., Volonteri 2012; Haiman 2013; Inayoshi et al. 2020 for a review). A natural candidate is Population III remnant BHs with a typical mass of M_{\text{BH}} \sim 10^2 M_\odot (e.g., Yoshida et al. 2008; Hosokawa et al. 2011, 2016; Susa et al. 2014; Hirano et al. 2014, 2015; Stacy et al. 2016; Sugimura et al. 2020). In this case, they must undergo substantially high accretion rates exceeding the Eddington limit to reach M_{\text{BH}} \sim 10^3 M_\odot by z \sim 7. Another possibility is producing more massive seed BHs with M_{\text{BH}} \sim 10^3–5 M_\odot via the direct collapse of massive pristine gas through formation of supermassive stars (e.g., Omukai 2001; Oh & Haiman 2002; Bromm & Loeb 2003; Hosokawa et al. 2012; Inayoshi & Omukai 2012; Inayoshi et al. 2014; Regan et al. 2014; Visbal et al. 2014; Sugimura et al. 2014, 2016; Latif et al. 2016; Umeda et al. 2016; Chon et al. 2016, 2018; Hirano et al. 2017; Wise et al. 2019) or runaway stellar mergers in dense clusters (e.g., Omukai et al. 2008; Devecchi & Volonteri 2009; Katz et al. 2015; Tagawa et al. 2015, 2020; Yajima & Khocharf 2016; Sakurai et al. 2017, 2019; Kroupa et al. 2020). With a head start in mass, the Eddington-limited accretion allows seed BHs to grow into SMBHs by z \gtrsim 6, but a high duty cycle of O(1) is still required. Therefore, in any seeding models, it is essential to know whether rapid growth of BHs could be sustained continuously in protogalaxies.

Many theoretical and numerical studies have confirmed that super-Eddington accretion flows are feasible inside the photon-trapping radius, where radiation is advected with accreting matter before escaping via diffusion (e.g., Abramowicz et al. 1988; Watarai et al. 2000; Ohsuga et al. 2005; Ohsuga & Mineshige 2011; Jiang et al. 2014; Yang et al. 2014, 2018; Sadowski & Narayan 2016). Radiation hydrodynamics (RHD) simulations that cover the BH gravitational sphere of influence showed that the mass accretion rate is generally self-regulated below the Eddington value, due to the outward thermal pressure gradient induced via photoionization and heating (e.g., Milosavljević et al. 2009a, 2009b; Park & Ricotti 2011, 2012; Jeon et al. 2012; Park et al. 2017). Accordingly, several possible scenarios of super-Eddington accretion from larger scales (∼1–10 pc) have been suggested. Inayoshi et al. (2016) showed that when a BH is embedded in sufficiently dense gas with a density n_H \gtrsim 10^5 \text{ cm}^{-3} (M_{\text{BH}}/10^4 M_\odot)^{-1}, photoionization/heating of gas is suppressed due to efficient recombination, leading to rapid mass accretion onto the BH without being impeded by radiative feedback (see also Sakurai et al. 2016; Park et al. 2016, 2020). In the intense inflow, the inward ram pressure of accreting gas substantially overcomes the sum of outward thermal pressure and radiation force. They also found that such a dense environment would be realized in the nuclei of high-z protogalaxies without prior star formation, and seed BHs that
migrate to the region within a Hubble timescale would rapidly grow into \( M_{\text{BH}} \gtrsim 10^8 M_\odot \) at hyper-Eddington rates (\( \gtrsim M_\odot \)). Moreover, anisotropic radiation emitted from the nuclear disk toward the polar regions dramatically reduces the negative feedback effect because gas accretion is allowed through the equatorial region, which is shielded against intense ionizing radiation from the accreting BH (Sugimura et al. 2017; Takeo et al. 2018). Mechanical feedback due to strong outflows launched from the disk completely evacuates the polar regions but does not affect the gas dynamics. In fact, even if a significant fraction of mass is loaded into outflows, the emergent radiation becomes less intense, and thus the super-Eddington accretion rate through the disk still holds (Takeo et al. 2020).

Although those scenarios are potentially intriguing, several simplified treatments are still imposed in their RHD simulations. One of them is the absence of angular momentum in the accreting gas that is supplied from larger galactic scales. Sugimura et al. (2018) studied the effect of angular momentum of gas and showed that mass accretion of rotating gas is suppressed from the standard Bondi rate when the centrifugal radius is larger than the BH gravitational influence radius and the angular momentum transport is inefficient. Therefore, it is crucial to quantify the efficiency of angular momentum redistribution owing to gravitational torques caused by spiral arms or turbulent motions excited within the circumnuclear disk. In the previous studies, a primordial chemical composition of gas is commonly assumed. However, recent observations have reported that the nuclear regions and host galaxies of bright quasars at \( z \gtrsim 6 \) already contain a large amount of dust (e.g., Venemans et al. 2012, 2017). The existence of heavy elements generally affects the thermal properties of gas and thus could change the mass growth of SMBHs. A series of 1D RHD simulations (Yajima et al. 2017; Toyouchi et al. 2019) have investigated accretion of dusty gas onto BHs and found that the radiative force on high-opacity, dusty accreting flows strongly regulates mass accretion onto the central BHs, so super-Eddington flows are prohibited especially for \( Z \gtrsim 10^{-2} Z_\odot \). In this paper, we extend our previous study and perform 3D RHD simulations by adopting a more realistic configuration of dusty and rotating accretion flows.

In particular, we explore the accretion dynamics at physical scales of \( \sim 0.01–1 \) pc, which roughly corresponds to the size of dusty tori or circumnuclear disks (CNDs) that are expected to play an essential role in fueling the central active galactic nuclei (AGNs; e.g., Hicks et al. 2013; Izumi et al. 2016). The dynamics of dusty nuclear disks have been extensively studied with a series of 3D hydrodynamical simulations (Wada et al. 2002, 2009, 2016, 2018) that successfully reproduce the observed spectral features of gaseous structure in low-luminosity AGNs (Izumi et al. 2018). However, they focus on the sub-Eddington AGN population in the local universe, considering already-grown massive BHs with \( M_{\text{BH}} \gtrsim 10^6 M_\odot \) and metal-enriched accretion disks with the solar abundance composition. We here focus on intermediate-mass BHs (IMBHs) with \( M_{\text{BH}} = 10^4 M_\odot \) embedded in low-metallicity environments (\( Z = 10^{-3}–10^{-1} Z_\odot \)) to study the super-Eddington mass growth of seed BHs in the early universe.

The rest of the paper is organized as follows. We first describe the numerical method and settings of our 3D RHD simulations in Section 2. The main results of our numerical simulations and a theoretical explanation for them are given in Sections 3 and 4, respectively. Based on these results, we further argue whether super-Eddington accretion can happen in the early universe in Section 5. Additionally, we provide discussions regarding physical processes that are not incorporated in our current simulations in Section 6. Finally, the summary and conclusion are given in Section 7.

## 2. Simulation Method

We utilize a hydrodynamical simulation code (\textsc{PLUTO} 4.1; Mignone et al. 2007), which has been modified to study massive star formation and the evolution of protoplanetary disks (e.g., Kuiper et al. 2010; Hosokawa et al. 2016; Nakatani et al. 2018a, 2018b; Kuiper & Hosokawa 2018; Kölligan & Kuiper 2018; Nakatani & Yoshida 2019; Fukushima et al. 2020). In particular, we make use of the specific version of the code adjusted for investigating BH accretion physics under radiative feedback (Sugimura et al. 2017, 2018; Toyouchi et al. 2019, 2020).

### 2.1. Basic Equations

We here perform three-dimensional hydrodynamical simulations to investigate the accretion dynamics of a gaseous disk surrounding a nuclear BH, which is located at the origin of the spherical coordinates \( (r, \theta, \phi) \). The basic equations of hydrodynamics that we solve are the following: the equation of continuity,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

and the equations of motion,

\[
\frac{\partial \rho v_r}{\partial t} + \nabla \cdot (\rho v_r \mathbf{v}) = -\frac{\partial P}{\partial r} + \rho \frac{v_r^2}{r} + \rho g_r,
\]

\[
\frac{\partial \rho v_\theta}{\partial t} + \nabla \cdot (\rho v_\theta \mathbf{v}) = -\frac{1}{r} \frac{\partial P}{\partial \theta} - \rho \frac{v_\theta v_r}{r} \cot \theta + \rho g_\theta,
\]

\[
\frac{\partial \rho v_\phi}{\partial t} + \nabla \cdot (\rho v_\phi \mathbf{v}) = -\frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} - \rho \frac{v_\phi v_r}{r} - \rho \frac{v_\phi v_\theta}{r} \cot \theta + \rho g_\phi,
\]

where \( \rho \) is the gas density, \( \mathbf{v} = (v_r, v_\theta, v_\phi) \) is the velocity vector, \( P \) is the gas pressure, and \( g = (g_r, g_\theta, g_\phi) \) is the external body force that includes the gravity of the central BH (\( g_{\text{BH}} = -GM_{\text{BH}}/r^2 \)), the self-gravity of gas (\( g_{\text{eg}} = -\nabla \Phi_{\text{eg}} \)), and the radiative force \( g_{\text{rad}} \) caused by absorption and scattering of photons (see Section 2.2). The gravitational potential of gaseous mass is calculated by solving the Poisson equation

\[
\Delta \Phi_{\text{eg}} = 4\pi G \rho
\]

(e.g., Mignone et al. 2007, 2012).

We solve the energy equation

\[
\frac{\partial E}{\partial t} + \nabla \cdot (H \mathbf{v}) = \rho \mathbf{v} \cdot \mathbf{g} + \rho (\Gamma - \Lambda),
\]

where \( E \) is the total (internal and kinetic) energy density, \( H \) is the enthalpy per unit volume, and \( \Gamma \) and \( \Lambda \) the specific heating and cooling rates in units of erg s\(^{-1}\) g\(^{-1}\). We set a minimum
temperature floor of 100 K and turn gas cooling off when the (local) Jeans length becomes unresolved with the longest size of each grid cell. Stellar feedback and star formation within a gravitationally unstable disk are not considered in this study, but the potential importance for BH growth is discussed in Section 6.1.

We estimate the heating and cooling rates by solving a chemical reaction network of metal-polluted gas, which is composed of the following eight species: H I, H II, He I, He II, He III, C II, O I, and e⁻. The number density of the i-th species $n_i$ is calculated with the nonequilibrium rate equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i v) = n_H R_i,$$

where $R_i$ is the sum of the reaction rate coefficients related to the i-th composition, and $n_H$ is the number density of hydrogen nuclei. The C II and O I abundances are set to $n_{\text{CII}}/n_H = 0.927 \times 10^{-3} Z/Z_\odot$ and $n_{\text{OI}}/n_H = 3.568 \times 10^{-5} Z/Z_\odot$. We here consider dust grains in metal-polluted gas, assuming that the dynamics of dust perfectly follows hydrodynamics so that a constant dust-to-gas mass ratio of 0.01 is kept. We take into account nine reactions including photoionization and collisional ionization of H I, He I, and He II and recombination of H II, He II, and He III. With the updated chemical abundances, we compute $\Lambda$ and $\Gamma$ by summing the contributions of photoelectric heating, fine-structure lines of C II and O I, free–free emission of H I, He I, and He II, and dust–gas collisional energy transfer. Other heating and cooling processes via heavy elements, such as metastable line cooling and photoionization heating, hardly affect the thermal properties of the low-metallicity gas with $Z \lesssim 0.1 Z_\odot$ considered in our study (see also Dere et al. 2009; Milosavljević et al. 2009b; Draine 2011). In addition, we neglect molecular components such as H$_2$ and CO in the low-metallicity gas because the timescale of their formation on the surface of dust grains is generally much longer than the dynamical timescale of a gaseous disk (Krumholz 2012).

Additionally, we also consider far ultraviolet (FUV) and X-ray background radiation as heating sources. Supposing an early galaxy formation phase with active star formation, we assume the background fields are 100 times stronger than in the solar neighborhood (see Toyouchi et al. 2019 for more details). With the background heating, the equilibrium gas temperature is maintained above our floor value at a gas number density of $\lesssim 10^6$ cm$^{-3}$, which is applicable to most grids except for inner disk parts in some of our simulations.

### 2.2. Radiative Feedback

Our simulation adopts a subgrid model to incorporate the radiative feedback against the accretion flow onto BHs. We suppose that the inside of $r_{\text{min}}$ is a sink region, in which a circum-BH accretion disk is contained. Mass accretion rates onto the unresolved disk $M_{\text{acc}}$ are evaluated with the inward mass flux measured at $r_{\text{min}}$ at each time step. Photons produced via mass accretion are injected from the sink, and the luminosity $L$ is described with the fitting formula given by Watarai et al. (2000):

$$L = \begin{cases} 2 \frac{L_\odot}{M} \left[ 1 + \ln \left( \frac{M}{2M_\odot} \right) \right] & (M > 2M_\odot) \\ \frac{L_\odot}{M} & (\text{otherwise}) \end{cases}$$

where $L_\odot$ and $M_\odot$ are the Eddington luminosity and mass accretion rate defined as below,

$$L_\odot = \frac{4\pi GM_{\text{BH}}c}{\kappa_{\text{T}}} = 3.3 \times 10^8 L_\odot \left( \frac{M_{\text{BH}}}{10^4 M_\odot} \right),$$

$$M_\odot = \frac{L_\odot}{\eta c^2} = 2.2 \times 10^{-4} M_\odot \text{ yr}^{-1} \left( \frac{M_{\text{BH}}}{10^4 M_\odot} \right),$$

where the opacity of Thomson scattering is $\kappa_{\text{T}} = 0.4$ cm$^2$ g$^{-1}$, and the radiative efficiency is assumed to be $\eta = 0.1$. This formula implies that the luminosity $L$ does not greatly exceed $L_\odot$ owing to the photon-trapping effect even in $M_{\text{acc}} > M_\odot$.

In this study, we assume an anisotropic radiation from the unresolved circum-BH disk,

$$F(\theta) = \frac{L}{4\pi r^2 f(\theta)} f(\theta) \propto \cos^2 \theta,$$

where the anisotropic factor is normalized according to $\int f(\theta) d\Omega = 4\pi$. This anisotropic radiation field is based on the RHD simulation of super-Eddington accretion flow by Ohshima et al. (2005), suggesting that photons are preferentially emitted perpendicular to the accretion disk plane.

We consider a power-law spectrum $L_\nu \propto \nu^{-\alpha}$ ($\alpha = 1.5$) in a UV range $6$ eV $\leq \hbar \nu \leq 1$ keV and solve the frequency-dependent radiative transfer along the radial cells. We take into account the consumption of photons by photoionization of H I, He I, and He II with the cross sections given by Osterbrock (1989) and Yan et al. (1998) and the dust attenuation with the opacity table of Weingartner & Draine (2001). In addition to the UV photons injected from the sink, we also solve the transfer of diffuse IR photons coming from the thermal dust emission with the flux-limited diffusion (FLD) approximation method. We utilize the FLD module developed by Kuiper et al. (2010, 2020), which has been applied to a lot of studies, especially for the formation of massive stars (e.g., Kuiper et al. 2011, 2012; Kuiper & Hosokawa 2018).

With the specific radiation flux $F_\nu$ calculated by the ray-tracing method, the radiative force caused by UV radiation incident from the central BH is self-consistently obtained by

$$g_{\text{rad,UV}} = \int \kappa_\nu F_\nu d\nu,$$

where $\kappa_\nu$ is the sum of the opacity via electron scattering, photoionization (bound-free transitions of H I, He I, and He II), and dust absorption.6 We also consider the radiative force via diffusive IR photons, whose flux is calculated by the FLD method,

$$g_{\text{rad,IR}} = \frac{\kappa_{\text{d,IR}} F_{\text{IR}}}{c},$$

6 Throughout this paper, we adopt the dust opacity in units of area per gas mass, assuming a dust-to-gas mass ratio of 0.01 $Z/Z_\odot$. 

Toyouchi et al.
where we adopt the Planck mean opacity for dust grains (Weingartner & Draine 2001). It is worth noting that the dust absorption of UV photons dominates the radiative force for \( Z \geq 10^{-3} Z_\odot \) and effectively modifies the Eddington limit as below,

\[
L_{E,\text{UV}} = X_{d,\text{UV}} L_E, \quad (14)
\]

\[
M_{E,\text{UV}} = \frac{L_{E,\text{UV}}}{\eta c^2}, \quad (15)
\]

\[
X_{d,\text{UV}} \equiv \frac{\kappa_T}{\kappa_T + \kappa_{d,\text{UV}}} = \left\{ 1 + 7.1 \left( \frac{Z}{10^{-2} Z_\odot} \right)^{-1} \right\}, \quad (16)
\]

where \( \kappa_{d,\text{UV}} \) is the dust absorption opacity to UV photons, for which we adopt \( \kappa_{d,\text{UV}} = 2.8 \times 10^2 (Z/Z_\odot) \text{ cm}^2 \text{ g}^{-1} \) (e.g., Yajima et al. 2017). On the other hand, the radiative force by diffuse IR photons is always subdominant compared to UV photons in the ionized region, due to the smaller dust absorption opacity. Since the IR photons, however, can propagate into the neutral medium, the resulting radiative force can affect the accretion flow in the non-ionized region.

The 1D simulations by Toyouchi et al. (2019) demonstrate the potential effect of IR radiative force by considering extremely dense environments where the UV radiative feedback no longer works due to the intense ram pressure of accreting neutral gas. This numerical experiment showed that the IR radiative force significantly regulates the gas accumulation toward the central BH, and the resulting mass accretion rate onto the BH is capped with the Eddington value evaluated for the IR radiative force,

\[
\dot{M}_{E,\text{IR}} = \frac{L_{E,\text{IR}}}{\eta_e c^2}, \quad (17)
\]

\[
L_{E,\text{IR}} = 4\pi GM_{\text{BH}} c \frac{\kappa_{d,\text{IR}}}{\kappa_{d,\text{IR}}} = 4.4 \times 10^7 \left( \frac{M_{\text{BH}}}{10^3 M_\odot} \right) \left( \frac{Z}{10^{-2} Z_\odot} \right)^{-1} L_\odot. \quad (18)
\]

In this equation, we adopt \( \kappa_{d,\text{IR}} = 30 (Z/Z_\odot) \text{ cm}^2 \text{ g}^{-1} \), supposing a dust sublimation temperature of \( T_d \sim 1500 \text{ K} \) where the opacity is expected to be highest. Thus, the IR radiative force could be dominant in neutral regions for \( Z \gtrsim 10^{-2} Z_\odot \). In this study, we extend our work to nonospheric cases. In Sections 3.1.1 and 6.2, we touch on the effect of the diffuse IR photons on the mass accretion rate onto IMBHs embedded in dense gas disks.

2.3. Numerical Setups and Boundary Conditions

In this study, we investigate the accretion flow onto BHs via dusty tori or CNDs that extend from the parsec to subparsec scale, hereafter referred to as the nuclear region. Generally, these nuclear disk structures are fed by mass transportation from the galactic disks extending over kiloparsec scales. Supposing such an external gas supply, we consider continuous mass injection at a rate of

\[
\dot{M}_\text{in} = F_{\text{in}} \dot{M}_E
\]

from the outer boundary through the equatorial region with a height of \( |z| \lesssim 0.1 \text{ pc} \), which corresponds to the thickness of a cold gas disk with \( T \sim 100 \text{ K} \). Here, we only consider the cases with \( F_{\text{in}} > 1 \) to explore the condition for the super-Eddington mass accretion. The injection velocity is set to \( V_{\text{in}} = 0.5 V_{k,\text{max}} \), where the Keplerian velocity at the outer boundary is \( V_{k,\text{max}} = \sqrt{GM_{\text{BH}}/r_{\text{max}}} \). We note that the choice of \( V_{\text{in}} \) does not affect our simulation results as long as the injection velocity is lower than the freefall value so that the inflowing gas forms a rotationally supported structure before reaching the innermost cell (see also below).

We also assume that the injected gas has a specific angular momentum perpendicular to the equatorial plane, the value of which is described as follows:

\[
\dot{J}_{\text{in}} = F_K r_{\text{max}} V_{k,\text{max}} = F_K \sqrt{GM_{\text{BH}} r_{\text{max}}}. \quad (20)
\]

We set \( F_K = 0.5 \) throughout this study. The choice of \( F_K \) is motivated by the result of high-resolution cosmological simulations of galaxy formation, where the rotational velocity of a collapsing gas is as large as half of the Keplerian velocity (Abel et al. 2002; Yoshida et al. 2008), and indeed the infall velocity becomes comparable to the rotational velocity (Inayoshi et al. 2014). The different choice of \( F_K \) has been confirmed to provide no significant impact on the time-averaged mass accretion rates at least in the range of \( F_K = 0.3-0.8 \).

At the early epoch of the simulation, the angular momentum of the injected gas produces a ring-like structure at the centrifugal radius:

\[
R_{\text{cent}} = \frac{\dot{J}_{\text{in}}^2}{GM_{\text{BH}}} = 0.63 \text{ pc} \left( \frac{F_K}{0.5} \right)^2 \left( \frac{r_{\text{max}}}{2.5 \text{ pc}} \right). \quad (21)
\]

The ring structure is fed by the gas supply from the outer boundary and eventually fragments due to the self-gravity. After that, a radially extending disk forms and drives accretion flows toward the central BH.

Note that our simulation assumes constant \( \dot{M}_{\text{in}} \) and \( \dot{J}_{\text{in}} \) throughout the computational time of \( \lesssim 10 \text{ Myr} \). In reality, the property of gas injected into the central parsec scales in galaxies can change on the timescale of 1 Myr (e.g., Hopkins & Quataert 2010). While investigating such more realistic gas inflow history from the galactic disks is essential, in this paper, we aim to acquire the fundamental knowledge for the mass growth of BHs at galactic centers by considering a simple situation.

The computational domain covers the radial range from \( r_{\text{min}} = 0.025 \text{ pc} \) to \( r_{\text{max}} = 2.5 \text{ pc} \), which resolves the Bondi radius of photoionized gas with \( T \sim 10^7 \text{ K} \), defined as

\[
R_B = \frac{GM_{\text{BH}}}{c_{s,\infty}^2} \quad (22)
\]

\[
= 0.068 \text{ pc} \left( \frac{M_{\text{BH}}}{10^4 M_\odot} \right) \left( \frac{T}{10^5 \text{ K}} \right)^{-1}, \quad (23)
\]

where we assume isothermal gas with polytropic index \( \gamma = 1 \), mean molecular weight \( \mu = 1.3 \), and sound speed \( c_{s,\infty} = \sqrt{8} \mu f_{\infty} (\mu m_{\text{H}}) = 8.1 (T_\infty/10^4 \text{ K})^{1/2} \text{ km s}^{-1} \). We basically assume an equatorial plane symmetry, in which the tangential numerical domains are \( 0 \leq \theta \leq \pi/2 \) and
The Astrophysical Journal, 907:74 (18pp), 2021 February 1

Toyouchi et al.

Table 1
Model Parameters and Results

| Model     | $Z [Z_{\odot}]$ | $F_{in}$ | UV Rad. Force | IR Rad. Force | Equatorial Sym. | $(N_r, N_{	heta}, N_z)$ | $\langle M_{acc}\rangle/M_m$ | $\langle M_{acc}\rangle/M_m$ |
|-----------|-----------------|---------|---------------|---------------|------------------|-------------------------|-----------------------------|-----------------------------|
| Z-2F2     | $10^{-2}$       | 100     | YES           | YES           | YES              | $128, 36, 72$          | 0.38                        | 0.37                        |
| Z-2F1     | $10^{-2}$       | 10      | YES           | YES           | YES              | $128, 36, 72$          | 0.001                       | 0.91                        |
| Z-2F3     | $10^{-2}$       | 1000    | YES           | YES           | YES              | $128, 36, 72$          | 0.75                        | 0.23                        |
| Z-1F2     | $10^{-1}$       | 100     | YES           | YES           | YES              | $128, 36, 72$          | 0.58                        | 0.26                        |
| Z-3F1     | $10^{-3}$       | 10      | YES           | YES           | YES              | $128, 36, 72$          | 0.11                        | 0.67                        |
| Z-3F2     | $10^{-3}$       | 100     | YES           | YES           | YES              | $128, 36, 72$          | 0.02                        | 0.96                        |
| Z-3F3     | $10^{-3}$       | 1000    | YES           | YES           | YES              | $128, 36, 72$          | 0.42                        | 0.51                        |
| Z-2F2nuv  | $10^{-2}$       | 100     | NO            | YES           | YES              | $128, 36, 72$          | 0.39                        | 0.39                        |
| Z-2F2nir  | $10^{-2}$       | 1000    | NO            | NO            | YES              | $128, 36, 72$          | 0.37                        | 0.36                        |
| Z-2F2hr   | $10^{-2}$       | 100     | YES           | YES           | YES              | $128, 36, 144$         | 0.25                        | 0.38                        |
| Z-2F2ne   | $10^{-2}$       | 100     | YES           | YES           | NO               | $128, 72, 72$          | 0.14                        | 0.72                        |
| Z-2F2hr+ne| $10^{-2}$       | 100     | YES           | YES           | NO               | $128, 72, 144$         | 0.004                       | 0.93                        |
| Z-2F3hr+ne| $10^{-2}$       | 1000    | YES           | YES           | NO               | $128, 72, 144$         | 0.02                        | 0.74                        |

0 \leq \phi \leq 2\pi. The number of grid cells in each direction are $(N_r, N_{\theta}, N_z) = (128, 36, 72)$ for the basic models introduced in Section 2.4. We adopt uniform grids in $\phi$ but logarithmic ones in $r$ and $\theta$ to realize higher resolution in the region inner and closer to the equatorial plane, which enables us to resolve the disk thickness with at least three grid cells.

2.4. Models

With RHD simulations, we investigate the gas accretion onto the IMBH with $M_{BH} = 10^4 M_{\odot}$, supposing a growing seed BH in the early universe. Table 1 summarizes the models presented in this paper. The top seven models, hereafter called the basic ones, aim to explore the dependence of the mass accretion efficiency of the IMBH on $Z$ and $F_{in}$. The first three and last two characters in the name of each model give the logarithmic value of $Z$ and $F_{in}$, respectively: our fiducial model with $Z = 10^{-2}$, $Z_{\odot}$ and $F_{in} = 10^2$ is tagged as Z-2F2.

We study the metallicity range $Z = 10^{-2} - 10^{-1} Z_{\odot}$, supposing the interstellar medium in the early galaxies where the chemical enrichment has not proceeded well. On the other hand, we assume $F_{in} = 10 - 1000$, roughly corresponding to the injection rate from the outer boundary of $M_{2f} \sim 10^{-3} - 10^{-1} M_{\odot} \text{yr}^{-1}$, realizing this gas supply rate is possible because it is much smaller than that observed in the hydrodynamical simulation of the Milky Way–like galaxy–galaxy merger by Hopkins & Quataert (2010). In Section 5, we argue when and where the situations supposed here appear in the context of galaxy evolution.

In addition to the seven basic models, we present some experimental calculations based on our fiducial model, Z-2F2. Z-2F2nuv and Z-2F2nir take into account no radiative force from direct UV and diffuse IR photons, respectively. To check the effects of numerical resolution and grid configuration, Z-2F2hr adopts twice the number of grids in $\theta$, Z-2F2ne relaxes the assumption of equatorial symmetry, and Z-2F2hr+ne is a combination of these two models. Moreover, we perform Z-2F3hr+ne corresponding to Z-2F2hr+ne but based on the Z-2F3 model. In the next section, we show the results of these seven basic and six test models.

3. Results

3.1. Overview of Numerical Results

3.1.1. Fiducial (Z-2F2) Case

Figure 1 shows the density and temperature distribution of the accretion flow obtained in our fiducial model (Z-2F2). At $t=1$ Myr, the gas injected from the outer boundary settles into a ring structure around its centrifugal radius. The gas ring has grown with time and eventually fragments due to self-gravity of the gas at $t \sim 1.4$ Myr, which forms a radially extending disk. At $t = 2$ Myr, a nonaxisymmetric spiral-arm structure forms and interacts with the surrounding gas, leading to efficient angular momentum transport and mass accretion onto the BH. Since the rapidly accreting BH releases intense radiation preferentially toward the polar regions (see Equation (11)), the gas above the disk height is ionized and evacuated. However, mass inflow is still allowed through the equatorial region, where the gas density is high enough to cool via metal line emission and maintain the marginally unstable disk structure.

The time variability of the mass accretion rate in our fiducial case is shown in the top panel of Figure 2. As expected from Figure 1, the mass accretion rate drastically rises with fragmentation of the ring at $t \sim 1.4$ Myr and begins to oscillate with peak values at $\sim 10^4 M_{\odot}$. The intermittent accretion behavior is caused by the state transition of the disk between gravitationally stable and unstable phases. In the quiescent phase, where the disk is stable and angular momentum transport is inefficient, the injected matter from the outer boundary is accumulated. Once the disk becomes massive enough to be gravitationally unstable, a large amount of the gas can fall into the central BH because of efficient angular momentum transport by the nonaxisymmetric structures. Since the burst-like accretion reduces the surface density of the disk, the gravitational instability is self-regulated, and thus the disk results in a quiescent phase. In fact, the typical interval of accretion bursts seems consistent with the orbital timescale of $\sim 0.1$ Myr at the centrifugal radius of $\sim 0.6$ pc. Even with such short variabilities, however, the time-averaged accretion rate is $\langle M_{acc}\rangle \sim 40 M_{\odot}$, as shown in Figure 2 (dashed line). This

7 We have conducted several different simulations by setting larger centrifugal radii of the injected gas, where the typical interval timescale between bursts becomes longer, but the time-averaged properties of the accreting flow are hardly affected.
implies that the BH would increase its mass by a factor of $\sim 5$ within $\sim 5$ Myr via super-Eddington accretion (although the BH mass is fixed through this simulation). Note that those rapidly growing phases would not last long because the disk tends to be gravitationally stable as the BH mass is higher than the disk mass and the shear velocity increases in the disk.

Figure 1. Time evolution of density and thermal structure in the Z-2F2 model. The left and middle columns show the face-on distributions of surface mass density and gas temperature averaged over the direction of $Z$, respectively. The right column represents the edge-on distributions of gas number density and temperature, which are not averaged but just sliced values in the $XZ$ plane. From the top to bottom panels, we show the accretion structure at $t = 1$, 1.4, and 2 Myr. The arrows denote the local flow velocities, the reference speed of which is shown at the bottom left corner in each panel. The white contours in the left and middle columns indicate the regions where the Toomre $Q$ value is $Q \equiv c_s \Omega / (\pi G \Sigma) = 1$. In the right column, the solid and dashed curves denote the layers where the neutral fraction of gas is 0.01 and 0.99. At the beginning of the simulation, injected gas from the outer boundary forms a ring structure and fragments into clumps due to its self-gravity at $t \sim 1.4$ Myr. As a result of ring fragmentation, mass accretion onto the central BH efficiently proceeds, and more intense radiation output from the accreting BH leads to a significant amount of mass loss from the disk owing to photoevaporation.

A similar episodic behavior of the mass outflow rate at the outer boundary is seen in the bottom panel of Figure 2. The outflow rate tightly correlates with the mass accretion rate over
time, implying that radiative feedback mainly drives wind mass loss from the disk surface. The time-averaged outflow rate is as high as \( \langle M_{\text{out}} \rangle \sim 40 M_\odot \), which is comparable to the BH accretion rate of \( \langle M_{\text{acc}} \rangle \). Since the outflowing matter is launched from larger radii, where the dynamical timescale is longer, the fluctuations of \( M_{\text{out}} \) are quite modest compared to those of \( M_{\text{acc}} \). We note that \( \sim 20\% \) of the gas injected from the outer boundary stays in the disk and makes it gravitationally unstable; namely, the disk mass is \( \sim 0.2 M_\odot \times 5 \text{ Myr} \sim 2 \times 10^4 M_\odot \). We note that the disk mass reaches an almost constant value by the end of the simulation, and the accretion system has been in a quasi-steady state where \( M_{\text{acc}} \simeq M_{\text{in}} - M_{\text{out}} \) is satisfied.

Here, we investigate the effects of radiative force on the accretion dynamics with the Z-2F2nuv and Z-2F2nir models, where the radiative forces due to direct UV and diffuse IR photons are ignored, respectively. As shown in Table 1, Z-2F2nuv does not significantly differ in \( M_{\text{acc}} \) and \( M_{\text{out}} \) from the fiducial case. This result implies that the strong outflow is caused by photoevaporation of accreting gas rather than the radiation force through electron scattering and dust absorption of UV photons. In fact, the radiation force does not exceed the gravitational force from the BH near the equatorial plane, due to anisotropic radiation. Therefore, the direct UV irradiation is not intense enough to repel the accretion flows. Thus, anisotropic radiation reduces the negative impacts of the radiation force on the disk dynamics and assists super-Eddington mass accretion onto the BH. On the other hand, in a dusty accretion disk, IR radiation generally could affect the disk dynamics because IR photons penetrate even near the equatorial plane in a diffusive way via absorption and reemission by dust grains. However, the Z-2F2nir model indicates that the IR radiative force also provides no significant impacts on the accretion dynamics. This is because the optical depth within the disk is less than unity, and therefore diffusive IR photons cannot be trapped within the accretion disk. Note that such an optically thin limit can break for cases with higher values of \( F_{\text{in}} \) and Z. In Section 6.2, we further discuss the effects of IR radiative force on mass transfer along accretion disks.

3.1.2. Dependence on Metallicity and Mass Injection Rate

We here investigate the effect of varying the metallicity \( Z \) and the mass injection rate \( F_{\text{in}} \) on the properties of accretion flows. Figure 3 presents the mass accretion rates for the four models with various values of \( Z \) and \( F_{\text{in}} \). For comparison, we overlay the result of the fiducial model (gray curve) in each panel (note that the simulation terminates at different times). In Figure 4, we summarize the mass accretion and outflow rate normalized by the mass injected rate, respectively (see also Table 1).

First, we compare the two models Z-2F1 (\( F_{\text{in}} = 10 \)) and Z-2F3 (\( F_{\text{in}} = 10^3 \)) with the same metallicity as in the fiducial case. With the higher injection rate, ring fragmentation occurs earlier and the BH is fed at a higher accretion rate of \( \langle M_{\text{acc}} \rangle \sim 750 M_\odot \), which is \( \sim 20 \) times higher than that in the fiducial case. This indicates that the ratio of the mass accretion rate to the injection rate from the outer boundary increases to \( \sim 75\% \). With the lower injection rate, the transition of mass accretion occurs later because it takes a longer time for the disk to become unstable. Unlike the other case, the mass accretion is much lower than the value expected from the ratio of \( F_{\text{in}} \), namely, \( \langle M_{\text{acc}} \rangle \simeq 10^{-2} M_\odot \), which is only \( \sim 0.1\% \) of the injected gas. This is because in this case, \( \sim 90\% \) of the injected mass is ejected from the disk as winds.

Next, we discuss the other two models Z-1F2 (\( Z = 0.1 Z_\odot \)) and Z-3F2 (\( Z = 10^{-3} Z_\odot \)) with the same mass injection rate as in the fiducial case. In the higher-metallicity case, the overall behavior of the mass accretion rate is similar to that in the fiducial case, except for the higher value of \( \langle M_{\text{acc}} \rangle \sim 60 M_\odot \). In contrast, with the lower metallicity, the accretion rate is suppressed and the time-averaged rate is limited at \( \langle M_{\text{acc}} \rangle \sim 2 M_\odot \). This result is opposite to the previous 1D RHD simulations of BH accretion, where the radiation force onto dust grains prevents mass accretion onto BHs as the metallicity increases (e.g., Yajima et al. 2017; Toyouchi et al. 2019). Therefore, the anisotropy of the radiation field and the geometrical effect qualitatively change the accretion dynamics at super-Eddington rates.

As summarized in Figure 4 (including two more cases Z-3F3 and Z-1F1), higher values of \( Z \) and \( F_{\text{in}} \) lead to higher mass accretion rates onto BHs, suppressing mass outflows from the disk.

3.2. Effects of Numerical Resolution and Grid Configuration

In this section, we check the numerical convergence of our simulation results, performing two higher-resolution runs with \( (N_p, N_d) = (72, 144) \) and relaxing the equatorial-symmetry assumption (Z-2F2hr+ne and Z-2F3hr+ne).
Figure 5 presents the density and temperature structure of the Z-2F2hr+ne model (i.e., $Z = 10^{-2} Z_{\odot}$ and $F_{\text{in}} = 100$). The face-on views (left panels) show an accretion disk with more clumpy structures compared to the fiducial case, because nonaxisymmetric spiral arms further fragment into those smaller clumps in the higher-resolution model. In contrast, the interclump region is too rarefied to cool down to $T \approx 10^4$ K via C II and O I fine-structure lines against dynamical heating in the disk.

The edge-on views (right panels) show that the accretion disk becomes substantially thicker than that in the fiducial case, because the equatorial symmetry is relaxed and the massive clumps dynamically heat the disk. To describe this quantitatively, we show the radial profiles of mass-weighted sound speed $c_s$ and vertical velocity dispersion $\sigma_z$ for the Z-2F2 and Z-2F2hr+ne models in Figure 6. In the high-resolution case, the gas temperature is higher everywhere, and the kinetic velocity in the z-direction exceeds the sound speed outside the centrifugal radius where dense clumps form, although the disk in the fiducial case is dynamically and thermally colder. We note that the clump properties are characterized with the mass-weighted quantities because the dense clumps orbiting at larger radii dominate the mass. Since dense clumps move subsonically in warm gas with $T \gtrsim 10^4$ K, the vertical motion of those clumps does not form shocks between the clumps and interclump media. In addition, the vertical velocity dispersion in the Z-2F2hr+ne model asymptotically approaches the Keplerian velocity in the outer regions. This suggests that a fraction of the rotational energy injected from the outer boundary is redistributed to the vertical kinetic motion and supports the clumpy disk in the vertical direction.

Gas kinetic motion in the disk affects mass accretion flows to the central BH. Figure 7 shows a negative correlation between the mass accretion efficiency and the ratio of $\sigma_z/c_s$ measured outside the centrifugal radius. Namely, the values of $\langle \dot{M}_{\text{acc}} / \dot{M}_{\text{Edd}} \rangle$ in the Z-2F2hr+ne and Z-2F3hr+ne models are reduced by a factor of $\sim 50$–100 from those in their counterpart (lower-resolution and equatorial symmetry) models, as the disk becomes dynamically hotter. We also present the results of the Z-2F2hr and Z-F2ne models, in which either the higher resolution or nonequatorial symmetry is considered. For these two models, the reduction of the mass accretion efficiency from the fiducial case is quite modest. This fact implies that the combination of clump formation and nonzero vertical motions of gas is essential to affecting the disk accretion dynamics. Thus, our simulation results still depend on the numerical resolution and grid configuration. However, even in the higher resolution cases, the qualitative dependence of mass accretion rates on the turbulent strength in the fragmenting disk as well as $F_{\text{in}}$ and $Z$. 

Figure 3. Time evolution of mass accretion rates obtained in the four basic models, Z-2F1, Z-2F3, Z-3F2, and Z-1F2. For comparison, the result of the fiducial Z-2F2 model is overlaid in each panel (gray curve).
in the marginally unstable disk. In contrast, with the lower metallicity (Z-3F2 model), such a dense, gaseous disk does not form, but the density is saturated almost at $\sim 10^4 M_\odot$ pc$^{-3}$ within the centrifugal radius. Hence, the density is expressed with the mass flux through the disk $M(R)$ as
\[
\rho(R) \approx \frac{M(R)}{4\pi c_s R^2} \approx \frac{F_{in} M_E - M_{out}(>R)}{4\pi c_s R^2} \propto R^{-\alpha},
\]
where $M_{in} = F_{in} M_E$ is the mass injection rate, and $M_{out}(>R)$ is the mass outflow rate integrated over $>R$. When the mass-loss rate is negligible, one obtains $\alpha = 2$. Therefore, the density slope becomes shallower (i.e., $\alpha < 2$) when the disk mass is removed due to radiative feedback associated with BH accretion. Thus, the higher-metallicity model loses only a small amount of gas from the inner limited region, whereas the lower-metallicity one suffers from significant outflows from the whole disk (see also Figure 4).

In the bottom panels of Figure 8, we present the radial profiles of the time-averaged disk scale height for the Z-2F2 and Z-3F2 models, where the height is defined for warm neutral gas with $T \sim 10^4$–$10^5$ K as
\[
H_w = \frac{\sum_i z_i^2 \rho_{w,i} \Delta V_i}{\sum_i \rho_{w,i} \Delta V_i},
\]
where $z_i$ is the height from the equatorial plane, $\rho_{w,i}$ is the density of warm neutral gas, and $\Delta V_i$ is the volume element at the $i$th grid cell. The summation in Equation (25) is taken only over the cells where the temperature is in the range $10^4 \, K \lesssim T \lesssim 10^5 \, K$. With the lower metallicity, the disk becomes substantially thinner in the inner region, where the disk mass undergoes photoevaporation, resulting in a smaller amount of neutral gas left in the equatorial region. To explicitly show radiation attenuation into the inner disk, we present the region where the optical depth to ionizing photons against dust absorption becomes above unity in Figure 8 (shaded regions). Here, the optical depth is calculated as
\[
\tau_{d,UV}(r, \theta) = \int_{\tau_{min}}^{r} \kappa_{d,UV} \cdot \rho(r', \theta) \, dr'.
\]
For the Z-2F2 model, since the disk thickness agrees with the boundary of $\tau_{d,UV} = 1$ at all radii, most of the disk region is shielded to ionizing photons produced from the accreting BH. In contrast, for the Z-3F2 model, the disk height is well above the optically thick region and the inner region ($R < 0.1$ pc) is heated by unattenuated radiation, driving a significant amount of mass loss. In fact, this trend of the relative position between the dust photosphere and disk height holds for all other cases: rapid accretion models (Z-1F2, Z-3F3, Z-2F3, Z-1F1) and inefficient accretion models (Z-2F1, Z-2F2hr+ne, Z-2F3hr+ne). Therefore, formation of a dense disk shielded by dust grains is required to achieve rapid mass accretion onto BHs.

4. Conditions for Rapid Mass Accretion

4.1. Implication from Simulation Results

In this section, we provide a simple analytic argument for the conditions required for rapid mass accretion. As we described, the mass accretion efficiency increases with the mass injection rate and metallicity (in Section 3.1.2) and also depends on the thickness (i.e., the vertical velocity dispersion) of a gravitationally unstable disk (in Section 3.2). For this purpose, we study the properties of the accretion disk structure and attenuated radiation field in more detail, specifically focusing on the differences in the two cases Z-2F2 and Z-3F2 as representative models that show different accretion efficiencies due to metallicity effects. For convenience, we introduce the cylindrical distance $R = r \sin \theta$ to describe the disk properties in the following discussion.

In the top panels of Figure 8, we present the radial profiles of the time-averaged gas density of neutral gas within the accretion disk for the two models. With the higher metallicity (Z-2F2 model), the gas density continuously increases inward and reaches $\sim 10^4 M_\odot$ pc$^{-3}$ near the center, although it declines near the inner boundary. Overall, the density profile follows $\rho(R) \propto R^{-\alpha}$, which indicates that the inflow velocity is characterized by the freefall velocity ($v_{ff} \sim G M_{BH} / R$) and thus $M(R) \sim R^2 v_{f} \propto (\rho c_s / \Omega) v_{f} \propto R^2$, due to efficient angular momentum transport.

Figure 4. Variations of the mass accretion and outflow efficiencies for different metallicity $Z$ and mass injection parameter $F_{in}$. The top and bottom panels indicate the time-averaged mass accretion and outflow rates normalized by the injection rates from the outer boundary as a function of gas metallicity. Cyan triangles, yellow circles, and red squares correspond to the cases with $F_{in} = 10, 100,$ and 1000. The efficiency of mass accretion (outflow) increases (decreases) for higher $Z$ and $F_{in}$.

4.2. Consideration of One-dimensional Disk Model

Next, we present a one-dimensional semianalytical model to quantify the penetration of ionizing photons into the disk and estimate the equilibrium mass accretion rate resulting from the photoevaporation effect. In the 1D model, the density profile is described with Equation (24), and for a given mass accretion
rate $\dot{M}_{\text{acc}}$ and size of an ionized region $R_{\text{HI}}$, the density slope is assumed to be

$$
\alpha(R) = \begin{cases} 
-2 - \frac{\log(\dot{M}_{\text{acc}}/\dot{M}_{\text{in}})}{\log(R_{\text{min}}/R_{\text{HI}})} & (R < R_{\text{HI}}) \\
2 & \text{(otherwise)},
\end{cases}
$$

(27)

where $R_{\text{min}}$ is the inner boundary of the disk model. We note that the density profile is assumed so that the mass flux is set to $\dot{M}_{\text{acc}}$ and $\dot{M}_{\text{in}}$ at $R = R_{\text{min}}$ and $R_{\text{HI}}$, respectively. It is worth noting that $\alpha = 2$ even at $R < R_{\text{HI}}$ if the mass outflow rate is zero (see Equation (24)).

Given that the density structure is characterized with the two values of $\dot{M}_{\text{acc}}$ and $R_{\text{HI}}$, we solve the radiation transfer equation for extreme ultraviolet (EUV) photons within the disk, considering photoionization and dust absorption. Note that we do not take into account diffusive EUV photons produced by radiative recombination of hydrogen. The number flux of ionizing photons $Q(R)$ penetrating into the disk through the midplane is calculated by solving the equation of photon-number conservation

$$
\frac{d\phi}{dR} = -f(R)\phi - g(R),
$$

(28)

where $\phi(R) = Q/\dot{Q}_0$ is the normalized photon flux, and $\dot{Q}_0$ is the unattenuated photon flux from the emission region; that is, $\phi(R_{\text{min}}) = 1$ is set. The functions $f(R)$ and $g(R)$ are given by

$$
f(R) = \kappa_{\text{d,UV}}\rho,
$$

(29)

$\kappa_{\text{d,UV}}$ and $\rho$ refer to the dust absorption coefficient and the gas density, respectively.
\[ g(R) = \frac{4\pi R^2 \alpha_B}{Q_0} \left( \frac{\rho}{m_p} \right)^2 \left( \frac{H_{\text{in}}}{R_{\text{min}}} \right). \]  

Here, \( \alpha_B \) is the case B recombination rate (the temperature is set at \( T = 7 \times 10^4 \) K), and \( H_{\text{in}} \) is the disk height at \( R = R_{\text{min}} \).

The first and second terms in the right-hand side of Equation (28) represent the effect of dust absorption and radiative recombination of hydrogen. This differential equation has an analytical solution of

\[ \phi(R) = e^{-\tau(R)} \left\{ 1 - \int_{R_{\text{min}}}^{R} g(R')e^{\tau(R')}dR' \right\}, \]

\[ \tau(R) = \int_{R_{\text{min}}}^{R} f(R')dR', \]

where \( \tau(R) \) corresponds to the optical depth for absorption of UV photons by dust.

In order to solve the radiative transfer equation, we set the photon number flux injected from the inner boundary. For anisotropic radiation set by Equation (11), the ionizing photon number flux within the disk height is estimated as

\[ Q_0 = 3 \int_{\nu_1}^{\nu_2} \frac{L}{h\nu} d\nu \int_{\pi/2-\theta_d}^{\pi/2+\theta_d} \cos^2 \theta \sin \theta d\theta \approx \frac{L}{h\nu_T} \sin^3 \theta_d \simeq \frac{L}{h\nu_T} \left( \frac{H_{\text{in}}}{R_{\text{min}}} \right)^3, \]

where \( \theta_d = \tan^{-1}(H_{\text{in}}/R_{\text{min}}) \) and \( h\nu_T = 13.6 \) eV. The choice of the inner boundary \( R_{\text{min}} \) seems somewhat arbitrary. In this work, we adopt the dust sublimation radius defined by

\[ R_{\text{sb}} = \sqrt[3]{\frac{L}{4\pi \sigma_{\text{SB}} T_d^4}} \sim 0.023 \text{ pc} \left( \frac{L}{L_{E}} \right)^{1/2} \left( \frac{M_{\text{BH}}}{10^7 M_{\odot}} \right)^{1/2} \left( \frac{T_d}{1000 \text{ K}} \right)^{-2}, \]

where \( \sigma_{\text{SB}} \) is the Stefan–Boltzmann constant. This choice is justified because at \( R < R_{\text{sb}} \), EUV radiation is not absorbed by dust and photoionization substantially dominates radiative recombination; namely, \( d\phi(R)/dR \approx 0 \) is a good approximation. By solving those nonlinear equations numerically, the size of the H II region is calculated so that \( \phi(R_{\text{HII}}) = 0 \), and the updated value of \( R_{\text{HII}} \) is used to set the density profile in Equation (27). As a result of the iterative calculations, the size of the H II region is numerically expressed as a function of the mass accretion rate: \( R_{\text{HII}} = f(M_{\text{acc}}). \)

Finally, we derive the equilibrium solution of the mass accretion rate for which mass conservation \( (M_{\text{acc}} + M_{\text{out}} - M_0 = 0) \) is satisfied. We approximately estimate the mass outflow rate from the disk surface due to photoevaporation as

\[ M_{\text{out}} = \min \left\{ M_{\text{in}}, 4\pi \epsilon_{\text{HII}} \int_{R_{\text{HII}}}^{R_{\text{in}}} \rho(R)dR \right\} \]
at $R_{\text{HI}} > R_{\text{BH,HI}}$ and otherwise $M_{\text{out}} = 0$. Here, $R_{\text{BH,HI}} = 0.1 \text{ pc}$ and $c_{s, \text{HI}} = 20 \text{ km s}^{-1}$ are the Bondi radius and the sound speed for ionized gas with $T = 7 \times 10^4 \text{ K}$, respectively. We note that the total mass outflow rate within $r < R_{\text{HI}}$ is a representative value instead of the outflow rate $M_{\text{out}}(>R)$. Using the relation $R_{\text{HI}} = F(M_{\text{acc}})$, the mass outflow rate is given by a function of $M_{\text{acc}}$, denoted $M_{\text{out}} = G(M_{\text{acc}})$. Therefore, the equilibrium value of $M_{\text{acc}}$ is obtained from the mass conservation law $M_{\text{acc}} + G(M_{\text{acc}}) = M_{\text{in}}$.

In Figure 9, we present the equilibrium values of $R_{\text{HI}}/R_{\text{BH,HI}}$ (top) and $M_{\text{acc}}/M_{\text{in}}$ (bottom) as a function of $F_{\text{in}}$ and $Z$. In the left column, the BH mass and disk sound speed are set to $M_{\text{BH}} = 10^5 \mathcal{M}_\odot$ and $c_s = 2 \text{ km s}^{-1}$, corresponding to the situation seen in our RHD simulations. With larger values of $F_{\text{in}}$ and $Z(>10^{-2} \mathcal{Z}_\odot)$, the size of the ionized region shrinks relative to the Bondi radius owing to efficient recombination and EUV absorption by dust. Since photoevaporation ceases suddenly as the ratio $R_{\text{HI}}/R_{\text{BH,HI}}$ becomes below unity (above the solid curve in each panel), most of the injected mass can feed the central BH at super-Eddington accretion rates without significant mass loss. Otherwise, the mass accretion rate is strongly suppressed and limited below the Eddington rate because of mass loss led by photoevaporation (below the solid curve in each panel). We note that the BH feeding rates obtained from the RHD simulations (filled circles) are nicely explained with this semianalytical model. In particular, we successfully demonstrate that the accretion disk with $Z = 10^{-3} \mathcal{Z}_\odot$ and $F_{\text{in}} = 100$ (Z-3F2 model) becomes optically thin to ionizing radiation and loses a larger amount of its mass owing to photoevaporation, compared to the higher-metallicity case with $Z = 10^{-2} \mathcal{Z}_\odot$ (Z-2F2), as shown in Figures 4 and 8.

In the middle panels of Figure 9, we present the case with a higher sound speed of $c_s = 6 \text{ km s}^{-1}$. This corresponds to the higher-resolution cases (Z-2F2hr+ne and Z-2F3hr+ne), in which vertical gas motions dynamically heat the accretion flow. In this case, the H\textsc{ii} region expands substantially because the gas density in the (dynamically) hotter disk decreases (see Equation (24)). As a result, even higher values of $F_{\text{in}}$ and $Z$ are required to sustain high BH accretion rates, as shown in Section 3.2 (see also Figure 7).

Moreover, to see the effect of BH mass growth, we demonstrate the case with a higher BH mass of $M_{\text{BH}} = 10^5 \mathcal{M}_\odot$ in the semianalytical model as shown in the right panels of Figure 9 (although the BH mass is fixed to $M_{\text{BH}} = 10^4 \mathcal{M}_\odot$ throughout our RHD simulation). In this case, the conditions required for rapid BH accretion are relatively moderate; namely, the solid curve moves to the lower left. This is because the larger-mass BH captures photoionized gas more effectively, and thus a denser and optically thick accretion disk forms.

In conclusion, the conditions required to avoid mass loss owing to photoevaporation (see solid curves in Figure 9) are approximately expressed as

$$F_{\text{in}} > 1000 \left( \frac{M_{\text{BH}}}{10^5 \mathcal{M}_\odot} \right)^{-1} \left( \frac{c_s}{10 \text{ km s}^{-1}} \right) \left( 1 + \frac{Z}{10^{-2} \mathcal{Z}_\odot} \right)^{-1}.$$  

(36)

---

Figure 8. Geometrical differences of the azimuthally and time-averaged ($0 \leq \phi \leq 2\pi$ and $t \geq 1.5 \text{ Myr}$) disk properties for the Z-2F2 (left) and Z-3F2 (right) models. The top and bottom panels show the radial profiles of the gas density vertically averaged within the neutral region and the disk scale height derived with Equation (25), respectively. In the bottom panel, the gray shaded area indicates the optically thick regime for dust absorption of ionizing photons. For the higher-metallicity model, a dense gaseous disk forms with minor effects of radiative feedback, whereas for the lower-metallicity one the accretion disk is too optically thin to suffer from significant mass loss via photoevaporation.
Therefore, the super-Eddington condition \( F_{\text{in}} > 1 \) combined with Equation (36) is expressed as

\[
M_{\text{BH}} < 8 \times 10^6 \left( \frac{T}{10^4 \, \text{K}} \right)^{1/2} \left( 1 + \frac{Z}{10^{-2} \, Z_{\odot}} \right)^{-1} \, M_{\odot},
\]

where \( T \) is the gas temperature in the disk. We note that this criterion for BH mass is essentially equivalent to that obtained from 1D spherically symmetric RHD simulations by Inayoshi et al. (2016), where the critical BH mass (see their Equation (36)) is derived from \( R_B < R_{\text{BH}} \) by assuming a gas density distribution of \( \rho(r) \propto r^{-2} \). The difference of the critical mass comes from a geometrical effect between disk-like accretion and spherical accretion.

### 5. Possible Sites for Super-Eddington Growth of Seed BHs

In this section, we argue where and when super-Eddington mass growth of seed BHs takes place in high-\( z \) protogalaxies. The critical conditions of Equation (36) and \( F_{\text{in}} > 1 \) are rewritten as

\[
\dot{M}_{\text{in}} > \max (\dot{M}_{\text{crit}}, \dot{M}_E),
\]

where

\[
\dot{M}_{\text{crit}} = 2.2 \times 10^{-1} \, M_{\odot} \, \text{yr}^{-1} \left( 1 + \frac{Z}{10^{-2} \, Z_{\odot}} \right)^{1/2} \left( \frac{c_s}{10 \, \text{km} \, \text{s}^{-1}} \right).
\]

Note that the condition for \( \dot{M}_{\text{crit}} > \dot{M}_E \) is equivalent to that in Equation (37). In what follows, assuming a simple galaxy evolution model, we evaluate the typical values of both \( Z \) and \( \dot{M}_E \) that depend on the properties of the host galaxies and their assembly histories.

Although we aimed to explore the growth of seed BHs at \( z > 6 \), the chemical evolution of galaxies has not been understood properly, and their observations are still limited at \( z < 4 \) (e.g., Mannucci et al. 2010; Troncoso et al. 2014; Hunt et al. 2016; Onodera et al. 2016). Instead, we here adopt a chemical-enrichment model proposed by recent cosmological simulations of galaxy formation (Sarmento et al. 2018). Their simulations successfully reproduce the statistical properties of young galaxies hosted in dark matter (DM) halos with masses of \( M_h \sim 10^{8-11} \, M_{\odot} \), such as the rest-frame UV luminosity functions observed at \( z \geq 7 \), and also predict a halo mass and metallicity relation at \( z = 7-15 \). The \( M_h-Z \) relation can be
Eddington mass growth of seed BHs would take place at formed in overdense regions with a mass variance of 3 \sigma. Similarly to the semianalytical star-forming disk model described by Thompson et al. (2005), angular momentum transport in the disk is assumed to be induced by axisymmetric spiral structures, and \( M = 0.1 \) is set based on a phenomenological prescription to describe this process (\( M \lesssim 0.2 \); see Goodman 2003). Since the mass inflow rate \( \dot{M}_{in} \) depends only on the properties of the DM halo, for the cases with \( \dot{M}_{crit} > \dot{M}_{E} \) or equivalently Equation (38), the critical condition of \( \dot{M}_{in} > \dot{M}_{crit} \) is independent of the BH mass, as shown by the blue solid curve in Figure 10. The resulting condition is given by \( \dot{M}_{E} > 10^{9} M_{\odot} \), almost independently of \( z \). This suggests that DM halos formed in overdense regions with a mass variance of 3–4\( \sigma \) become possible sites where super-Eddington mass growth of seed BHs would be led during \( z \sim 15–20 \). Such massive halos are heavier than the mass of “typical” direct-collapse BH-forming halos (see Volonteri 2012; Haiman 2013; Inayoshi et al. 2020). These facts imply that even rarer populations of seed BHs could undergo efficient mass growth immediately after their formation, as pointed out in Inayoshi et al. (2020; see also Valiante et al. 2016).

When a seed BH is embedded in the center of a massive DM halo with \( M_{h} \gtrsim 10^{9} M_{\odot} \), the seed can undergo super-Eddington growth in mass regardless of its initial mass. However, as the BH mass increases and \( M_{BH} < M_{E} \) is satisfied, the critical condition is given by \( \dot{M}_{in} > \dot{M}_{E} \), requiring an upper limit of the BH mass

\[
\dot{M}_{BH,max} \approx 1.1 \times 10^{8} M_{\odot} \left( \frac{M_{h}}{10^{9} M_{\odot}} \right)^{1/3} \left( \frac{1+z}{10} \right)^{1/2} \approx 7.9 \times 10^{7} M_{\odot} \left( \frac{T_{vir}}{10^{7} K} \right)^{1/2}.
\]

This also provides a condition for BH mass above which rapidly growing phases of seed BHs terminate due to the lack of mass reservoir in their host galaxies (see black solid curve in Figure 10). This argument for seed BH growth is consistent with a scenario proposed by Inayoshi et al. (2016), where spherically symmetric rapid mass accretion is studied. Their upper mass limit is estimated as \( M_{BH} \lesssim 1.4 \times 10^{9} M_{\odot} (T_{vir}/10^{7} K) \), which differs from Equation (43) by a factor of \( \approx 2 \) and has a stronger dependence on \( T_{vir} \) due to different accretion geometry.

We note that super-Eddington accretion does not last eternally because the BH mass growth leads to \( M_{E} > M_{in} \). The efficient BH growth terminates when the BH mass becomes as high as \( \sim 10^{8} M_{\odot} \), even in massive halos associated with a mass variance of 3–4\( \sigma \). This seems consistent with the existence of sub-Eddington, low-luminosity quasars with \( M_{BH} \gtrsim 10^{8} M_{\odot} \) at \( z \gtrsim 6 \) (e.g., Matsuoka et al. 2019; Onoue et al. 2019), but might fail to explain the existence of brighter quasars with \( L/L_{E} \sim 1 \) (e.g., Willott et al. 2010b; Mortlock et al. 2011; Wu et al. 2015; Bañados et al. 2018; Yang et al. 2020). Possibly, rapid accretion onto such SMBHs
would be induced by a large amount of gas injection into nuclear regions associated with galaxy–galaxy major mergers, as seen in cosmological hydrodynamical simulations (e.g., Hopkins & Quataert 2010). Therefore, our argument that focuses on steady mass transport through marginally stable disks would give a conservative estimate of $F_{in}$. Exploring the nature of nonsteady, violent mass accretion consistent with the outer boundary conditions set by large-scale cosmological simulations is left for future investigations.

6. Discussion and Caveats

6.1. Stellar Feedback

Intense UV radiation and energetic supernovae associated with massive star formation heat the interstellar medium and potentially induce mass loss from galactic disks (e.g., Hopkins et al. 2011, 2012; Li et al. 2015, 2017; Kim & Ostriker 2018). Cosmological hydrodynamical simulations, which explore the coevolutionary process of galaxies and SMBHs, have shown that stellar feedback substantially regulates the mass budget in the nuclear regions and suppresses mass feeding to the central BHs (e.g., Dubois et al. 2015; Habouzit et al. 2017; Anglés-Alcázar et al. 2017; Angles-Alcazar et al. 2020; Çatmabacak et al. 2020). Latif et al. (2018) also demonstrated that the combination of stellar and AGN feedback prevents seed BHs from growing to $M_{BH} \sim 10^3 M_\odot$ by $z \sim 6$. In contrast, Di Matteo et al. (2017), who adopted a quite huge simulation box of $(400 h^{-1} \text{ Mpc})^3$, found that stellar feedback is not strong enough to quench the growth of relatively massive seeds in the early epoch. They also suggested that the formation of SMBHs depends on tidal fields exerted on the host halo. In fact, in lower-spin halos, efficient gas inflows via cold streams can directly feed the nuclear region without forming large stellar disks that launch galactic outflows. The accretion efficiency under more realistic mass injection from a star-forming galactic disk will be addressed in our future work.

Active star formation (at least vigorous disk fragmentation) can still take place in the nuclear regions, as shown in our RHD simulations with a higher resolution (Z-2F2hr+ne model). The mass of clumps is as massive as $M_c \sim 100 M_\odot$, and those clumps are located at $R \sim 2\sim 3 R_{\text{cen}}$, where the density of the surrounding gas is $\Sigma \sim 5 \times 10^2 M_\odot \text{ pc}^{-2}$. Therefore, the clumps accrete from the disk at a rate of

$$M_c = \frac{3}{2} \Sigma \Omega (f_{\text{fl}} R_H)^2,$$

where the Hill radius is $R_H \equiv R(M_c/3M_{BH})^{1/3}$, and $f_{\text{fl}} \sim O(1)$ (e.g., Goodman & Tan 2004; Inayoshi & Haiman 2014). Adopting the clump properties seen in the Z-2F2hr+ne model, we estimate the rate as

$$M_c \sim 2.6 \times 10^{-4} M_\odot \text{ yr}^{-1} \times \left( \frac{f_{\text{fl}}}{1.5} \right)^2 \left( \frac{R}{1.2 \text{ pc}} \right)^2 \left( \frac{M_c}{100 M_\odot} \right)^{2/3}.$$

Assuming that all of the gas in a clump accretes into a single star, the growth rate of a newly-born protostar is given by $M_c$. The accreting protostar begins to contract by losing energy via radiative diffusion and evolves into a main-sequence star in a Kelvin–Helmholtz (KH) timescale of

$$\tau_{KH} \sim \frac{M_c}{M_c},$$

$$\simeq 0.38 \text{ Myr} \left( \frac{M_c}{100 M_\odot} \right) \left( \frac{\dot{M}_c}{2.6 \times 10^{-4} M_\odot \text{ yr}^{-1}} \right)^{-1}, \quad (46)$$

where we assume that KH contraction balances the energy input by protostellar accretion. Since this timescale is shorter than the orbital timescale $\tau_{\text{orb}} \equiv 2\pi/\Omega \sim 1 \text{ Myr}$, which is comparable to the clump migration timescale in a marginally stable disk (e.g., Inayoshi & Haiman 2014), massive main-sequence stars form within the nuclear accretion disks and contribute to stellar radiative feedback.

Contrary to the negative stellar feedback that operates in larger scales as discussed above, starburst events in the nuclear disk regions could potentially enhance mass transport through the disk, due to strong turbulence excited by supernovae in star-forming CNDs (Wada et al. 2002; Kawakatu & Wada 2008; Wutschik et al. 2013). Indeed, a positive correlation between the AGN and star formation activities in the nuclear region has been observed (e.g., Diamond-Stanic & Rieke 2012; Esquej et al. 2014), and a state transition from quiescent phases to AGNs would be triggered by nuclear starbursts (Inayoshi et al. 2020). According to a semianalytical model (Kawakatu & Wada 2009), the BH feeding rate peaks when the gas supply rate to the disk region is comparable to the gas consumption rate due to star formation, requiring a high injection rate of $M_{in} \sim 10^3 M_\odot \text{ yr}^{-1}$ over $\sim 100 \text{ Myr}$ to explain the existence of SMBHs at $z \sim 6$. Determining whether star formation in the nuclear regions has negative or positive effects on the mass growth of seed BHs is left for our future studies.

6.2. BH Radiative and Mechanical Feedback

In our RHD simulations, we treat the mass inflow rate at the inner boundary as the BH accretion rate, assuming the properties of radiative output (e.g., luminosity, spectra, and anisotropy) from the unresolved small scales. We here briefly discuss the effects of those assumptions and other types of radiative/mechanical output from the vicinity of the nuclear accreting BH.

First, we consider the IR radiative force caused by reemission from heated dust grains. Although our simulations include this effect, no significant impacts on the accretion flow are found because the disk surface density is not high enough for the gas to trap diffuse IR photons. In fact, the optical depth to IR photons toward the disk vertical direction is estimated as

$$\tau_{\text{IR}} = \kappa_{\text{d,IR}} \Sigma \simeq 0.2 \left( \frac{F_{\text{in}}}{100} \right) \left( \frac{Z}{10^{-2} Z_\odot} \right) \left( \frac{M_{BH}}{10^4 M_\odot} \right)^{1/4}, \quad (47)$$

where the surface density is estimated at $R_{\text{sh}}$, assuming the disk to be in a steady state, $\Sigma = F_{\text{in}} M_{BH} / (2 \pi R_{\text{sh}} v_{\phi})$. Therefore, unless $F_{\text{in}} > 10^4$ and $Z > 0.1 Z_\odot$ are considered, the critical conditions for rapid accretion do not change with the optical thickness of the gaseous disk against IR photons, as discussed in Section 4.

Analytical arguments by Krolik (2007) and Shi & Krolik (2008) concluded that a geometrically thick disk supported by IR radiation pressure forms around the dust sublimation radius.
Several numerical simulations have confirmed that such a disk structure produces outflows driven by IR radiation pressure onto dust and does not feed the central BH at a high rate (e.g., Dorodnitsyn & Kallman 2012; Chan & Krolik 2016). The strong IR radiation pressure within the disk height is due to the higher opacity of hotter dust that is heated by isotropic radiation emitted from the nuclear BH. In contrast, assuming that the radiation flux from the nuclear BH is highly collimated toward the poles, dust grains in the disk region are kept cold, and the IR radiation pressure is significantly reduced even for a high accretion rate of $M_{\text{acc}} \sim 0.8 M_\odot$ (Namekata & Umemura 2016). As a result, the accretion disk becomes geometrically thinner than what the previous analytical studies predicted. Even extending to higher values of $F_{\text{in}}$ and $Z$ in our case, the disk accretion dynamics is not affected by IR radiation pressure, as long as the emergent radiation flux is sufficiently anisotropic. We note, however, that if intense UV radiation from massive stars formed in a dusty disk heats dust grains near the equatorial region, a disk supported by IR radiation pressure forms in the nuclear region (e.g., Thompson et al. 2005).

In addition to radiative feedback, mechanical feedback due to outflows and disk winds launched from the vicinity of the BH would affect mass accretion in a CND region (e.g., Fabian 2012), although we do not explicitly inject mechanical momentum from the inner boundary in our simulations. One possible mechanism for launching outflows is the line-driven wind model, where UV radiation emitted from the disk around $\sim 100$–1000 $R_{\text{Sch}}$ accelerates moderately ionized metal gas, yielding a substantially high opacity via bound–bound transitions. In the wind regions, the radiative force caused by various spectral lines boosts the acceleration efficiency by several orders of magnitude above the continuum radiation force exerted through electron scattering alone (Stevens & Kallman 1990; Proga et al. 2000; Proga & Kallman 2004; Nomura et al. 2020). Another possible channel is that a highly accreting BH with super-Eddington luminosity ($L \gg L_{\text{Edd}}$) exerts a radiation force through electron scattering in an optically thick medium and produces strong outflows (e.g., Ohsuga et al. 2005; Ohsuga & Mineshige 2011; Jiang et al. 2014; Yang et al. 2014, 2018; Sadowski & Narayan 2016). In both cases, a large fraction ($\gtrsim 50\%$) of the mass injected from larger scales is loaded into outflows collimated toward the polar regions. Since the BH feeding rate is reduced by mass loading to outflows and the radiative luminosity decreases, the presence of disk winds rather promotes rapid accretion more efficiently (Takeo et al. 2020). This fact suggests that we need to make a comprehensive model of BH accretion covering the outflow launching scale and the Bondi scales in order to better understand the BH growth mechanism.

### 7. Summary and Conclusion

In this paper, we study rapid mass accretion onto IMBHs with $M_{\text{BH}} = 10^4 M_\odot$ embedded in massive, self-gravitating, dusty nuclear accretion disks, performing the first 3D RHD simulations focusing on the nuclear region of protogalaxies. Our simulations resolve the Bondi radius for hot ionized gas with a temperature of $T \sim 10^5$ K and can follow the launching process of outflows from the disk surface owing to photoevaporation, which suppresses the BH from accreting.

We here explore the dependence of mass accretion efficiency on the gas metallicity $Z$ and mass injection rate from the outer galactic disk normalized by the Eddington value $F_{\text{in}} \equiv M_{\text{in}}/M_\odot$. For this purpose, we run several numerical models (e.g., Z-2F2 and Z-2F1 models) covering a wide range of relevant parameters of $F_{\text{in}} = 10$–1000 and $Z = 10^{-3}$–$10^{-1}$ $Z_\odot$. In all cases, the nuclear disk becomes gravitationally unstable and transports mass inward owing to angular momentum transport caused by global density spiral arms. The central BH can be fed at rates exceeding the Eddington rate only when the dusty disk becomes sufficiently optically thick to ionizing radiation. In this case, a large fraction ($\gtrsim 40\%$) of the mass injection rate can feed the central BH. The critical conditions required for super-Eddington mass accretion onto BHs is given by $M_{\text{in}} > M_{\text{crit}}$, where

$$M_{\text{crit}} \equiv 2.2 \times 10^{-4} M_\odot \text{yr}^{-1} \times \left(1 + \frac{Z}{10^{-2} Z_\odot}\right)^{-1} \left(\frac{c_s}{10 \text{ km s}^{-1}}\right),$$

and $c_s$ is the sound speed in the gaseous disk. Otherwise, the disk is not obscured enough to shield intense ionizing radiation by dust absorption, mass outflows from the disk caused by photoevaporation limit the BH accretion rate to $\lesssim 1\%$–$10\%$ of the mass injection rate from the outer boundary and thus strongly prevent the BH feeding.

For the cases where a higher numerical resolution is set and the equatorial-symmetric assumption is relaxed (models Z-2F2hr+ne and Z-2F3hr+ne), vigorous disk fragmentation reduces the disk surface density, and dynamical heating by formed clumps makes the disk thickness higher. As a result, the photoevaporative mass-loss rate rises, and thus the critical injection rate increases. However, the central BH can be fed at super-Eddington rates once $M_{\text{in}} > M_{\text{crit}}$ is satisfied even if the disk becomes dynamically hot owing to clump formation.

Finally, we apply our results to the cosmological evolution of massive BHs via rapid mass accretion. With a semianalytical model, we find that super-Eddington accretion is allowed until the BH mass reaches $M_{\text{BH}} \sim 10^{7.8} M_\odot$, depending on the properties of the host DM halo and metal-enrichment history.

In the assembly of protogalaxies, seed BHs that form in overdense regions with a mass variance of $3-4\sigma$ at $z \sim 15-20$ are able to undergo short periods of rapid growth and transit into the Eddington-limited growth phase afterward to be SMBHs observed at $z > 6-7$.

The authors would like to thank Masayuki Umemura, Tohru Nagao, Shingo Hirano, and Naoki Yoshida for fruitful discussions, and Kazuyuki Sugimura and Riouhe Nakatani for their contribution to developing the numerical code. The numerical simulations were performed with the Cray XC50 at the Center for Computational Astrophysics (CICA) of the National Astronomical Observatory of Japan and with the High-performance Computing Platform of Peking University. This work is financially supported by the National Science Foundation of China (11721303, 11991052, 11950410493, 12073003: K. I.), the National Key R&D Program of China (2016YFA0400702: K.I.), and Grants-in-Aid for Basic Research by the Ministry of Education, Science and Culture of Japan (17H06360: D.T.; 16H05996, 17H01102, 19H01934: T.H.). R.K. acknowledges financial support via the Emmy Noether Research Group on Accretion Flows and Feedback in Realistic Models of Massive Star Formation funded by the German Research Foundation (DFG) under grant Nos. KU 2849/3-1 and KU 2849/3-2.
