Nonlinear interactions of stationary planetary waves during February 2016 sudden stratospheric warming

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Abstract. An analysis of nonlinear wave-wave and wave-mean flow interactions has been carried out during a sudden stratospheric warming (SSW) event in February 2016. This approach is based on a study of the perturbed potential enstrophy (Ertel’s potential vorticity squared) balance equation. The results obtained by using UK Met Office reanalysis data are presented. It has been shown that an increase in the nonlinear interactions occurs at higher-middle latitudes of the boreal stratosphere during the SSW. It is noted that it is necessary to include the stationary planetary wave with zonal wave number m=3 in the analysis of major warmings.

1. Introduction
In the stratosphere, there is an anti-correlation between changes in the amplitudes of stationary planetary waves with zonal wave numbers 1 and 2 (SPW1 and SPW2) due to nonlinear wave-wave interactions. To interpret the observed behavior of SPW amplitudes, it is useful to consider the nonlinear interactions of SPWs with the zonal mean flow and between the SPWs with different zonal wave numbers. Using this approach, the conservation of the perturbed potential enstrophy is investigated. In this case the terms responsible for nonlinear interactions in the balance equation of the potential enstrophy (Ertel’s potential vorticity squared) are calculated [1].

To obtain the balance equation of perturbed potential enstrophy, we use the conservation equations of Ertel’s potential vorticity (EPV) perturbation for SPW1 and SPW2 and multiply these equations by the perturbed EPV for each SPW. The result is a general form of the eddy enstrophy balance in a log-pressure coordinate system:

\[
\frac{\partial}{\partial t} (P'^2/2) + P' \left( \vec{V}' \cdot \vec{\nabla} P' \right) + P' \left( \vec{V}' \cdot \vec{\nabla} \vec{P}' \right) + P' \left( \vec{\nabla}' \cdot \vec{\nabla} P' \right) = \vec{P}' \cdot \vec{Q}',
\]

where \( P' \) and \( \vec{P} \) are the perturbed and zonally averaged components of Ertel’s potential vorticity; \( \vec{V}' \) and \( \vec{V} \) are the perturbed and zonally averaged components of the wind vector; \( \vec{Q}' \) represents the
perturbation of diabatic sources and the sinks and terms describing the subscale contributions to the momentum equation [2]. The expression for Ertel’s potential vorticity is as follows:

\[ P = \omega_a \cdot \bar{\nabla} \theta / \rho_0, \]

where \( \omega_a \) is the absolute vorticity, \( \theta \) is the potential temperature, and \( \rho_0 \) is the background density.

The first term, \( A \), denotes the wave transience and can be defined as a measure of wave activity variability [3]. The next terms, \( B \) and \( C \), describe the wave-wave and wave-mean flow interactions, respectively. The term \( D \) is responsible for the advective transport of potential enstrophy. The right-hand side term \( E \) describes the changes in perturbed enstrophy due to diabatic heating. This term involves subscale contributions to the momentum equation including momentum deposition by gravity and inertial-gravity waves.

Equations for SPW1 and SPW2 similar to equation (1) may be obtained using the secondary wave generation method. If a signal consisting of two cosine waves with zonal wavenumbers and frequencies \((m_1, \omega_1)\) and \((m_2, \omega_2)\) passes through some quadratic system, the output of this system will contain the secondary waves \((2m_1, 2\omega_1), (2m_2, 2\omega_2), (m_1-m_2, \omega_1-\omega_2), \) and \((m_1+m_2, \omega_1+\omega_2)\) [4, 5]. Thus, secondary SPWs with zonal wave number 2 (SPW2) are generated as a result of the nonlinear SPW1-SPW3 interaction and SPW1 self-interaction. Secondary SPW1 arise due to SPW1-SPW2 and SPW2-SPW3 interactions.

2. Data
To show the behavior of nonlinear terms characterizing the wave-wave and wave-mean flow interactions in the balance equations of perturbed potential enstrophy for SPW1 and SPW2, considering secondary wave generation the UK Met Office assimilated fields were used. Since the activity of SPWs increases before and decreases after the SSW, the major SSW in February 2016 was chosen for analysis. The UK Met Office data are currently unique in terms of the upper boundary height. In November 2009 the upper boundary of the UK Met Office model was extended to 0.01 hPa, i. e., practically in to the mesosphere. To investigate the dynamic processes observed during 2015-2016 winter months in the Northern Hemisphere, the assimilated meteorological fields were decomposed at each latitude and altitude into Fourier-series taking into account zonal harmonics with wave number \( m = 0-3. \) Figure 1 demonstrates the time evolution of the amplitude of a planetary wave with \( m=1 \) in geopotential height (the upper panel), the mean zonal wind at latitude 62.5°N (the middle panel), and the change in the zonal mean temperature at latitude 87.5°N (the lower panel). One can see two strong increases in the amplitude of the SPW1 on the 28th of January and 8th of February, which was accompanied by the reversal of the zonal mean flow in the stratosphere. As a consequence, a sudden stratospheric warming was observed at the beginning of February. This SSW event was attended by significant increases in the amplitudes of SPW2 and SPW3 (see Figure 2).
Figure 1. Time-altitude cross-sections of the amplitude of zonal harmonic with $m = 1$ in geopotential height and the mean zonal wind at latitude 62.5°N (upper and middle panels) for January-March 2016; change of the zonal mean temperature at 87.5°N (lower panel). UK Met Office data.
Figure 2. Time-altitude cross-sections of the amplitude of zonal harmonics with $m = 2$ (upper panel) and $m = 3$ (lower panel) in geopotential height at latitude 62.5°N for January-March 2016. UK Met Office data.

The winter of 2015-2016 is also characterized by a very strong final warming at the beginning of March. This event was preceded by increases in the SPW2 and SPW3 amplitudes. At the same time, there was no a substantial rise in the amplitude of SPW1, which is usually observed during such warming events. The analysis of this SSW is out of scope of the present paper.

3. **Nonlinear interactions during the SSW event in February 2016**

Using UK Met Office data, the terms of equation (1) were calculated and averaged over 2-6 of February (Figures 3 and 4).
Figure 3. Latitude-height cross sections of terms in the potential enstrophy balance for SPW1: transience (a), SPW1-mean flow interaction (b), SPW1-SPW2 (c) and SPW2-SPW3 (d) interactions. Units: $10^{12} \text{ (kg} \cdot \text{m}^{-3})^2 \cdot \text{PVU}^2$/day, where $1 \text{PVU}=10^6 \cdot \text{K} \cdot \text{m}^2 \cdot \text{kg}^{-1} \cdot \text{s}^{-1}$. 
Figure 4. Latitude-height cross-sections of terms in the potential enstrophy balance for SPW2: transience (a), SPW2-mean flow interaction (b), self-interaction of SPW1 (c) and SPW1-SPW3 interaction (d). Units: $10^{12} (\text{kg} \cdot \text{m}^{-3})^2 \cdot \text{PVU}^2 / \text{day}$, where $1\text{PVU}=10^6 \cdot \text{K} \cdot \text{m}^2 \cdot \text{kg}^{-1} \cdot \text{s}^{-1}$.

The calculation results show that the warming event in February 2016 was accompanied by an extremely strong increase in the wave-wave and wave-mean flow interactions in the stratosphere in higher-middle latitudes. During the considered time interval a decrease in the SPW1 wave activity was attended by a rise in the SPW2 wave activity (see Figures 3 (a) and 4 (a)).

The wave-mean flow interaction (Figures 3 (b) and 4 (b)) requires special attention. The greatest influence on the mean flow is exerted by SPW2 with a strong maximum in the region where SPW1 has a minimum in the higher-middle latitudes. Considering the results for the wave-wave interaction (Figures 3 (c, d) and 4 (c, d)), it can be noted that the order of magnitude indicates a significant SPW3 contribution to the generation of secondary waves during the SSW in February 2016. Strengthening of the wave-wave interaction in the case of SPW1 generation takes place in higher-middle latitudes and in the case of SPW2 generation, throughout all middle latitudes.

4. Summary and conclusions
In the present paper, nonlinear interactions during a SSW event at the beginning of February 2016 were analyzed by using the perturbed potential enstrophy balance equation. The above-obtained results showed the contributions of the transience, wave-wave, and wave-mean flow interaction terms
to this equation. It is noted that the SSW event is described by an increase in all types of interactions in the stratosphere. The possibilities for further study, both analytical and numerical, are immense. For example, SPW3 should be involved in the analysis in the case of strong sudden stratospheric warmings. This implies the derivation of an equation (1) taking into account the secondary planetary wave generation method for SPW3. Additionally, a similar approach can be used to investigate the final warmings, especially during their anomalous development (e.g., as observed in March 2016). An identical approach can be used to analyze the nonlinear interactions between higher-frequency planetary waves (for instance, atmospheric tides) themselves or/and with stationary planetary waves during a SSW.

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