$D_S$ Mesons in Asymmetric Hot and Dense Hadronic Matter

Divakar Pathak$^\ast$ and Amruta Mishra$^\dagger$

Department of Physics, Indian Institute of Technology – Delhi,
Hauz Khas, New Delhi – 110 016, India

Abstract

The in-medium properties of $D_S$ mesons are investigated within the framework of a chiral effective model. These are observed to experience net attractive interactions in a dense hadronic medium, hence reducing the masses of the $D_S^+$ and $D_S^-$ mesons from the vacuum values. While this conclusion holds in both nuclear and hyperonic media, the magnitude of the mass drop is observed to intensify with the inclusion of strangeness in the medium. Additionally, in hyperonic medium, the mass degeneracy of the $D_S$ mesons is observed to be broken, due to opposite signs of the Weinberg-Tomozawa interaction term in the Lagrangian density. Along with the magnitude of the mass drops, the mass splitting between $D_S^+$ and $D_S^-$ mesons is also observed to grow with an increase in baryonic density and strangeness content of the medium. However, all medium effects analyzed are found to be weakly dependent on isospin asymmetry and temperature. We discuss the possible implications emanating from this analysis, which are all expected to make a significant difference to observables in heavy ion collision experiments, especially the upcoming PANDA and CBM experiments at FAIR, GSI, where matter at high baryonic densities is planned to be produced.

PACS numbers: 14.40.Lb; 21.65.Cd; 21.65.Jk; 11.30.Rd; 12.38.Lg

$^\ast$ dpdlin@gmail.com
$^\dagger$ amruta@physics.iitd.ac.in
I. INTRODUCTION

An effective description of hadronic matter is fairly common in low-energy QCD and has been vigorously pursued in various incarnations over the years [1–10]. Realizing that baryons and mesons constitute the effective degrees of freedom in this regime, it is sensible to treat QCD at low-energies as an effective theory of these quark bound states [1]. The form of these effective hadronic interactions is dictated by symmetry principles, and the physics governed predominantly by the dynamics of chiral symmetry – its spontaneous breakdown implying a non-vanishing scalar condensate $\langle \bar{q}q \rangle$ in vacuum. One naturally expects then, that the hadrons composed of these quarks would also be modified in accord with these condensates [1, 2]. But while all hadrons would be subject to medium modifications from this perspective, pseudoscalar mesons have a special role in this context. Since the light pseudoscalar mesons serve as the (almost) massless Goldstone modes related to the spontaneous breaking of chiral symmetry, an understanding of their in-medium properties, both theoretically and experimentally, provides useful inputs into our understanding of this important aspect of low energy QCD. Pions are thus, the most automatic candidates; it is observed however, that medium effects for them are weakened by the smallness of explicit symmetry breaking terms [2]. Considerably detailed analysis of medium effects have been performed over the years, for strange (kaons and antikaons) [11–14], and subsequently, charmed ($D$) pseudoscalar mesons [15–17]. With all these studies proving to be informative, the most natural direction of extension of this approach would be to analyze these medium effects for a strange-charmed system (the $D_S$ mesons). Apart from pure theoretical interest, an understanding of the medium modifications of $D_S$ mesons is important, since these can make a considerable difference to experimental observables in the (ongoing and future) relativistic heavy ion collision experiments, besides being significant in questions concerning their production and transport in such experimental situations. In a recent paper, He et al. [18] have shown that the modifications of the $D_S$ meson spectrum can serve as a useful probe for understanding key issues regarding hadronization in heavy ion collisions. It is suggested that by comparing observables for $D$ and $D_S$ mesons, it is possible to constrain the hadronic transport coefficient. This comparison is useful since it allows a clear distinction between hadronic and quark-gluon plasma behavior.

However, as far as the existing literature on this strange-charmed system of mesons is
concerned, we observe that only the excited states of $D_S$ mesons have received considerable attention, predominantly as dynamically generated resonances in various coupled channel frameworks \[19, 20\]. One must bear in mind that in certain situations, a molecular interpretation of these excited states (resonances), is more appropriate \[21\] for an explanation of their observed, larger than expected lifetimes. From this perspective, a whole plethora of possibilities have been entertained for the excited $D_S$ states, the standard quark-antiquark picture aside. These include their description as molecular states borne out of two mesons, four-quark states, or the still further exotic possibilities - as two-diquark states and as a mixture of quark-antiquark and tetra-quark states \[19\]. Prominent among these is a treatment of $D_{S0}^*(2317)$ as a $DK$ bound state \[22, 23\], that of $D_{s1}(2460)$ as a dynamically generated $D^*K$ resonance \[24\], $D_{S2}^*(2573)$ being treated within the hidden local gauge formalism in coupled channel unitary approach \[25, 26\], as well as the vector $D_{S}^*$ states and the $D_{S}^*(2632)$ resonance treated in a multichannel approach \[27\]. The former three have also been covered consistently under the four-quark picture \[28\]. So, while considerable attention has been paid to dynamically generating the higher excited states of the $D_S$ mesons, there is a conspicuous dearth of available information about the medium modifications of the lightest pseudoscalar $D_S$ mesons, $D_S(1968.5)$ \[29\], the one that we know surely, is well described within the quark-antiquark picture. In fact, to the best of our knowledge, the meager analysis of their zero-temperature spectral distributions, in Ref. \[29\], constitutes the entire existing literature about the ($J_P = 0^-$) $D_S$ mesons in a hadronic medium. This absence of available treatments is most of all, because of the lack of a proper framework where the relevant form of the interactions for the $D_S$ mesons, based on arguments of symmetry and invariance, could be written down. Clearly, such interactions would have to be based on $SU(4)$ symmetry and bear all these pseudoscalar mesons in a 16-plet representation, while baryons belong to a 20-plet representation. Of late, such formalisms have been proposed in Refs. \[30, 31\], as an extension of the frameworks based on chiral Lagrangians, where $SU(4)$ symmetry is used to write down the relevant interaction terms. However, since the mass of the charm quark is approximately 1.3 GeV \[32\], which is considerably larger than that of the up, down and strange quarks, this $SU(4)$ symmetry is explicitly broken. Hence, this formalism only uses the symmetry to derive the form of the interactions, whereas an explicit symmetry breaking term accounts for the large quark mass through the introduction of mass terms of the relevant ($D$ or $D_S$) mesons. Overall, therefore, $SU(4)$ symmetry is treated (ap-
appropriately) as being broken in this approach. Naturally then, these formalisms wipe out the reason why such an investigation for the $D_S$ mesons within chiral effective approach has not been undertaken till date, and permit this attempt to fill the void.

We organize this article as follows: in section II, we outline the Chiral $SU(3)_L \times SU(3)_R$ Model (and its generalization to the $SU(4)$ case) used in this investigation. In section III, the Lagrangian density for the $D_S$ mesons, within this model, is explicitly written down, and is used to derive their in-medium dispersion relations. In section IV, we describe and discuss our results for the in-medium properties of $D_S$ mesons, first in the nuclear matter case, and then in the hyperonic matter situation, following which, we briefly discuss the possible implications of these medium modifications. Finally, we summarize the entire investigation in section V.

II. THE CHIRAL MODEL

As mentioned previously, this study is based on a generalization of the chiral $SU(3)_L \times SU(3)_R$ model \cite{33}, to $SU(4)$. We summarize briefly the rudiments of the model, while referring the reader to Refs. \cite{33, 34} for the details. This is an effective hadronic model of interacting baryons and mesons, based on a non-linear realization of chiral symmetry \cite{35, 36}, where chiral invariance is used as a guiding principle, in deciding the form of the interactions \cite{37–39}. Additionally, the model incorporates a scalar dilaton field, $\chi$, to mimic the broken scale invariance of QCD \cite{34}. Once these invariance arguments determine the form of the interaction terms, one resorts to a phenomenological fitting of the free parameters of the model, to arrive at the desired effective Lagrangian density for these hadron-hadron interactions. The general expression for the chiral model Lagrangian density reads:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_W \mathcal{L}_{\text{BW}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{\text{scale break}} + \mathcal{L}_{\text{SB}}$$

(1)

In eqn. (1), $\mathcal{L}_{\text{kin}}$ is the kinetic energy term, while $\mathcal{L}_{\text{BW}}$ denotes the baryon-meson interaction term. Here, baryon-pseudoscalar meson interactions generate the baryon masses. $\mathcal{L}_{\text{vec}}$ treats the dynamical mass generation of the vector mesons through couplings with scalar mesons. The self-interaction terms of these mesons are also included in this term. $\mathcal{L}_0$ contains the meson-meson interaction terms, which induce spontaneous breaking of chiral symmetry. $\mathcal{L}_{\text{scalebreak}}$ introduces scale invariance breaking, via a logarithmic potential term
in the scalar dilaton field, \( \chi \). Finally, \( \mathcal{L}_{SB} \) refers to the explicit symmetry breaking term. This approach has been employed extensively to study the in-medium properties of hadrons, particularly pseudoscalar mesons \[11–17\]. As was observed in section I as well, this would be most naturally extended to the charmed (non-strange and strange) pseudoscalar mesons. However, that calls for this chiral \( SU(3) \) formalism to be generalized to \( SU(4) \), which has been addressed in \[30, 31\]. For studying the in-medium behavior of pseudoscalar mesons, the following contributions need to be analyzed \[31\]:

\[
\mathcal{L} = \mathcal{L}_{WT} + \mathcal{L}_{1^{st}\text{Range}} + \mathcal{L}_{d_1} + \mathcal{L}_{d_2} + \mathcal{L}_{\text{SME}}
\]  

(2)

In eqn.(2), \( \mathcal{L}_{WT} \) denotes the Weinberg-Tomozawa term, given by the expression

\[
\mathcal{L}_{WT} = -\frac{1}{2} \bar{B}_{ijk} \gamma^\mu \left( (\Gamma_\mu)_l^k B^{ijkl} + 2 (\Gamma_\mu)_l^j B^{ilk} \right),
\]

(3)

with repeated indices summed over. Baryons are represented by the tensor \( B^{ijk} \), which is antisymmetric in its first two indices \[30, 31\]. The indices \( i, j \) and \( k \) run from 1 to 4, and one can directly read the quark content of a baryon state, with the identification: \( 1 \leftrightarrow u, 2 \leftrightarrow d, 3 \leftrightarrow s, 4 \leftrightarrow c \). However, the heavier, charmed baryons are discounted from this analysis. In eqn.(3), \( \Gamma_\mu \) is defined as

\[
\Gamma_\mu = -\frac{i}{4} \left( u^\dagger(\partial_\mu u) - (\partial_\mu u^\dagger)u + u(\partial_\mu u^\dagger) - (\partial_\mu u)u^\dagger \right),
\]

(4)

where the unitary transformation operator, \( u = \exp(iM\gamma_5/\sqrt{2}\sigma_0) \), is defined in terms of the matrix of pseudoscalar mesons, \( M = (M^7\lambda_j/\sqrt{2}) \), \( \lambda_j \) representing the generalized Gell-Mann matrices. Further, \( \mathcal{L}_{\text{SME}} \) is the scalar meson exchange term, which is obtained from the explicit symmetry breaking term

\[
\mathcal{L}_{SB} = -\frac{1}{2} \text{Tr} \left( A_p \left( uXu + u^\dagger Xu^\dagger \right) \right),
\]

(5)

where \( A_p = (1/\sqrt{2}) \text{ diag} [m_\pi^2 f_\pi, m_\pi^2 f_\pi, (2m_K^2 f_K - m_\pi^2 f_\pi), (2m_D^2 f_D - m_\pi^2 f_\pi)] \), and \( X \) refers to the scalar meson multiplet \[16\]. Also, the first range term is obtained from the kinetic energy term of the pseudoscalar mesons, and is given by the expression:

\[
\mathcal{L}_{1^{st}\text{Range}} = \text{Tr}(u_\mu X u^\mu X + Xu_\mu u^\mu X)
\]

(6)

where \( u_\mu = -i \left( (u^\dagger(\partial_\mu u) - (\partial_\mu u^\dagger)u - (u(\partial_\mu u^\dagger) - (\partial_\mu u)u^\dagger) \right)/4 \). Lastly, the \( d_1 \) and \( d_2 \) range terms, are:

\[
\mathcal{L}_{d_1} = \frac{d_1}{4} \left( \bar{B}_{ijk} B^{ikl}(u_\mu)_l^m (u^\mu)_m^l \right)
\]

(7)
\[ \mathcal{L}_{d_2} = \frac{d_2}{2} [\tilde{B}_{ijk}(u_\mu)_l^m (B^k_{ijl} + 2(u_\mu)_m^k B^{ikl})] \]  

(8)

Adopting the mean field approximation \[4, 34\], the effective Lagrangian density for scalar and vector mesons simplifies; the same is used subsequently, to derive the equations of motion for the non-strange scalar-isoscalar meson \( \sigma \), scalar-isovector meson \( \delta \), strange scalar meson \( \zeta \), as well as for the vector-isovector meson \( \rho \), non-strange vector meson \( \omega \) and the strange vector meson \( \phi \), within this model.

The \( D_S \) meson interaction Lagrangian density and in-medium dispersion relations, as they follow from the above general formulation, are described next.

III. \( D_S \) MESONS IN HADRONIC MATTER

The Lagrangian density for the \( D_S \) mesons in isospin-asymmetric, strange, hadronic medium is given as -

\[ \mathcal{L}_{\text{total}} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} \]  

(9)

This \( \mathcal{L}_{\text{free}} \) is the free Lagrangian density for a complex scalar field (which corresponds to the \( D_S \) mesons in this case), and reads:

\[ \mathcal{L}_{\text{free}} = (\partial_\mu D^+_S)(\partial_\mu D^-_S) - m^2_{D_S} (D^+_S D^-_S) \]  

(10)

On the other hand, \( \mathcal{L}_{\text{int}} \) is determined to be:

\[ \mathcal{L}_{\text{int}} = -\frac{i}{4f_{D_S}^2} \left[ \left( 2 \left( \bar{\Xi}_0 \gamma_\mu \Xi_0 + \bar{\Xi}^- \gamma_\mu \Xi^- \right) + \bar{\Lambda}^0 \gamma_\mu \Lambda^0 + \bar{\Sigma}^+ \gamma_\mu \Sigma^+ \right. \right. 
\] 
\[ \left. + \bar{\Sigma}^0 \gamma_\mu \Sigma^0 + \bar{\Sigma}^- \gamma_\mu \Sigma^- \right) \left( D^+_S (\partial_\mu D^-_S) - (\partial_\mu D^+_S) D^-_S \right) \right] \]
\[ + \frac{m^2_{D_S}}{\sqrt{2} f_{D_S}} \left[ (\zeta' + \zeta'') \left( D^+_S D^-_S \right) \right] \]
\[ - \frac{\sqrt{2}}{f_{D_S}} \left[ (\zeta' + \zeta'') \left( (\partial_\mu D^+_S)(\partial_\mu D^-_S) \right) \right] \]
\[ + \frac{d_1}{2f_{D_S}} \left( \bar{p}p + \bar{n}n + \bar{\Lambda}^0 \Lambda^0 + \bar{\Sigma}^+ \Sigma^+ + \bar{\Sigma}^0 \Sigma^0 \right. \]
\[ + \bar{\Sigma}^- \Sigma^- + \bar{\Xi}_0 \Xi_0 + \bar{\Xi}^- \Xi^- \left( (\partial_\mu D^+_S)(\partial_\mu D^-_S) \right) \right] \]
\[ + \frac{d_2}{2f_{D_S}} \left[ \left( 2 \left( \bar{\Xi}_0 \Xi_0 + \bar{\Xi}^- \Xi^- \right) + \bar{\Lambda}^0 \Lambda^0 + \bar{\Sigma}^+ \Sigma^+ \right. \right. \]
\[ \left. + \bar{\Sigma}^0 \Sigma^0 + \bar{\Sigma}^- \Sigma^- \right) \left( (\partial_\mu D^+_S)(\partial_\mu D^-_S) \right) \right] \]  

(11)
In this expression, the first term (with coefficient $-i/4f_{D_s}^3$) is the Weinberg-Tomozawa term, obtained from eqn.(3), the second term (with coefficient $m_{D_S}^2/\sqrt{2}f_{D_S}$) is the scalar meson exchange term, obtained from the explicit symmetry breaking term of the Lagrangian (eqn.(5)), third term (with coefficient $-\sqrt{2}/f_{D_S}$) is the first range term (eqn.(6)) and the fourth and fifth terms (with coefficients $(d_1/f_{D_s}^2)$ and $(d_2/f_{D_s}^2)$, respectively) are the $d_1$ and $d_2$ terms, calculated from eqns.(7) and (8), respectively. Also, $\zeta' = \zeta - \zeta_0$, is the fluctuation of the strange scalar field from its vacuum value. The mean-field approximation, mentioned earlier, is a useful, simplifying measure in this context, since it permits us the following replacements:

\begin{align}
\bar{B}_i B_j &\rightarrow \langle \bar{B}_i B_j \rangle \equiv \delta_{ij} \rho_i^s \\
\bar{B}_i \gamma^\mu B_j &\rightarrow \langle \bar{B}_i \gamma^\mu B_j \rangle = \delta_{ij} \left( \delta_0^\mu (\bar{B}_i \gamma^\mu B_j) \right) \equiv \delta_{ij} \rho_i
\end{align}

Thus, the interaction Lagrangian density can be recast in terms of the baryonic scalar and number densities, given by the following expressions:

\begin{align}
\rho_i &= \int d^3k \ (n_i(k) - \bar{n}_i(k)), \\
\rho_i^s &= \int d^3k \ \frac{m_i^*}{E_i^s(k)} \ (n_i(k) + \bar{n}_i(k)),
\end{align}

where $n_i(k)$ and $\bar{n}_i(k)$ refer to the particle and antiparticle distribution functions [13], respectively. One can find the equations of motion for the $D^+_S$ and $D^-_S$ mesons, by the use of Euler-Lagrange equations on this Lagrangian density. The linearity of these equations follows from eqn.(11), which allows us to assume plane wave solutions ($\sim e^{i(\vec{k}.\vec{r} - \omega t)}$), and hence, ‘Fourier transform’ these equations, to arrive at the in-medium dispersion relations for the $D_S$ mesons. These have the general form:

\[-\omega^2 + \vec{k}^2 + m_{D_S}^2 - \Pi(\omega, |\vec{k}|) = 0 (16)\]

where, $m_{D_S}$ is the vacuum mass of the $D_S$ mesons and $\Pi(\omega, |\vec{k}|)$ is the self-energy of the $D_S$ mesons in the medium. Explicitly, the latter reads:

\[
\Pi(\omega, |\vec{k}|) = \left[ \left( \frac{d_1}{2f_{D_S}^2} (n_\rho^s + n_\rho^s + n_\Lambda^s + n_{\Sigma^+}^s + n_{\Xi^0}^s + n_{\Xi^-}^s + n_{\Xi^0}^s + n_{\Xi^-}^s) \right) \\
+ \left( \frac{d_2}{2f_{D_S}^2} (2(n_{\Xi^0}^s + n_{\Xi^-}^s) + n_\Lambda^s + n_{\Sigma^+}^s + n_{\Sigma^0}^s + n_{\Sigma^-}^s) \right) \\
- \left( \frac{\sqrt{2}}{f_{D_S}} (\zeta' + \zeta'_0) \right) \right] (\omega^2 - \vec{k}^2)
\]
\[
\pm \left[ \frac{1}{2 f_{D_S}^2} \left( 2 (\rho_{\Xi^0} + \rho_{\Xi^-}) + \rho_\Lambda + \rho_{\Sigma^+} + \rho_{\Sigma^0} + \rho_{\Sigma^-} \right) \right] \omega \\
\pm \left[ \frac{m_{D_S}^2}{\sqrt{2} f_{D_S}} (\zeta' + \zeta') \right]
\]

(17)

where the + and − signs in the coefficient of \( \omega \), refer to \( D_S^+ \) and \( D_S^- \) respectively, and we have used equations (12) and (13) to simplify the bilinears. In the rest frame of these mesons (i.e. setting \( \vec{k} = 0 \)), these dispersion relations reduce to:

\[- \omega^2 + m_{D_S}^2 = \Pi(\omega, 0) = 0, \]

(18)

which is a quadratic equation in \( \omega \), i.e. of the form \( A\omega^2 + B\omega + C = 0 \), where the coefficients \( A, B \) and \( C \) depend on various interaction terms in Eq. (16), and read:

\[ A = [1 + \left( \frac{d_1}{2 f_{D_S}^2 (N+H)} \sum \rho_i^s \right) + \left( \frac{d_2}{2 f_{D_S}^2} \left( 2 \sum_{\Xi} \rho_i^s + \sum_{(H-\Xi)} \rho_i^s \right) \right)] \\
- \left( \frac{\sqrt{2}}{f_{D_S}} (\zeta' + \zeta') \right) \]

(19)

\[ B = \pm \left[ \frac{1}{2 f_{D_S}^2} \left( 2 \sum_{\Xi} \rho_i + \sum_{(H-\Xi)} \rho_i \right) \right] \]

(20)

\[ C = \left[ - m_{D_S}^2 + \frac{m_{D_S}^2}{\sqrt{2} f_{D_S}} (\zeta' + \zeta') \right] \]

(21)

As before, the + and − signs in \( B \), correspond to \( D_S^+ \) and \( D_S^- \), respectively. In writing the above summations, we have used the following notation: \( H \) denotes the set of all hyperons, \( N \) is the set of nucleons, \( \Xi \) represents the Xi hyperons \( (\Xi^{-,0}) \) and \( (H - \Xi) \) denotes all hyperons other than Xi hyperons, i.e. the set of baryons \( (\Lambda, \Sigma^{+,-,0}) \) which all carry one strange quark. This form is particularly convenient for later analysis. Also, the optical potential of \( D_S \) mesons, is defined as:

\[ U(k) = \omega(k) - \sqrt{k^2 + m_{D_S}^2} \]

(22)

where \( k (= |\vec{k}|) \), refers to the momentum of the respective \( D_S \) meson, and \( \omega(k) \) represents its momentum-dependent in-medium energy.

In the next section, we study the sensitivity of the \( D_S \) meson effective mass on various characteristic parameters of hadronic matter, viz. baryonic density \( (\rho_B) \), temperature \( (T) \), isospin asymmetry parameter \( \eta = -\sum_i I_{3i} \rho_i / \rho_B \), and the strangeness fraction \( f_s = \sum_i |S_i| \rho_i / \rho_B \), where \( S_i \) and \( I_{3i} \) denote the strangeness quantum number, and the third component of isospin of the \( i \)th baryon, respectively.
IV. RESULTS AND DISCUSSION

Before describing the results of our analysis of $D_S$ mesons in a hadronic medium, we first discuss our parameter choice and the various simplifying approximations employed in this investigation. The parameters of the chiral model are fitted to the vacuum masses of baryons, nuclear saturation properties and other vacuum characteristics in the mean field approximation \cite{33, 34}. In this investigation, we have used the same parameter set that has earlier been used to study charmed ($D$) mesons within this chiral model framework \cite{31}. In particular, we use the same values of the parameters $d_1$ and $d_2$ of the range terms, fitted to empirical values of kaon-nucleon scattering lengths for the $I = 0$ and $I = 1$ channels, as were employed in earlier treatments \cite{11, 12, 16, 31}. For an extension to the strange-charmed system, the only extra parameter that needs to be fitted is the $D_S$ meson decay constant, $f_{D_S}$, which is treated as follows. By extrapolating the results of Ref.\cite{39}, we arrive at the following expression for $f_{D_S}$ in terms of the vacuum values of the strange and charmed scalar fields:

$$f_{D_S} = \frac{-(\zeta_0 + \zeta_{c0})}{\sqrt{2}}$$

(23)

For fitting the value of $f_{D_S}$, we retain the same values of $\sigma_0$ and $\zeta_0$ as in the earlier treatments of this chiral effective model \cite{12, 33}, and determine $\zeta_{c0}$ from the expression from the expression for $f_D$ in terms of $\sigma_0$ and $\zeta_{c0}$ \cite{16}, using the Particle Data Group (PDG) value of $f_D = 206$ MeV. Substituting these in eqn.\cite{23}, our fitted value of $f_{D_S}$ comes out to be 235 MeV, which is close to its PDG value of 260 MeV, and particularly, is of the same order as typical lattice QCD calculations for the same \cite{32}. We therefore persist with this fitted value $f_{D_S} = 235$ MeV in this investigation. Also, we treat the charmed scalar field ($\zeta_c$) as being arrested at its vacuum value ($\zeta_{c0}$) in this investigation, as has been considered in other treatments of charmed mesons in a hadronic context \cite{16, 31}. This neglect of charm dynamics appears natural from a physical viewpoint, since the mass of a charm quark ($\sim 1.3$ GeV \cite{32}), is quite above the hadronic physics regime that we are in. The same was verified in an explicit calculation in Ref.\cite{40}, where the charm condensate was observed to vary very weakly in the temperature range of interest to us in this regime \cite{41–43}. As our last approximation, we point out that in the current investigation, we work within the ‘frozen glueball limit’ \cite{34}, where the scalar dilaton field ($\chi$) is regarded as being frozen at its vacuum value ($\chi_0$). This approximation was relaxed in a preceding work \cite{17} within this.
chiral effective model, where the in-medium modifications of this dilaton field were found to be quite meager. We conclude, therefore, that this weak dependence only serves to justify the validity of this assumption.

We next describe our analysis for the in-medium behavior of $D_S$ mesons, beginning first with the nuclear matter ($f_s = 0$) situation, and including the hyperonic degrees of freedom only later. This approach has the advantage that many features of the $D_S$ meson in-medium behavior, common between these regimes, are discussed in detail in a more simplified context, and the effect of strangeness becomes a lot clearer.

In nuclear matter, the Weinberg-Tomozawa term and the $d_2$ range term vanish, since they depend on the number densities and scalar densities of the hyperons, and have no contribution from the nucleons. It follows from the self-energy expression that only the Weinberg-Tomozawa term differs between $D_S^+$ and $D_S^-$; all other interaction terms are absolutely identical for them. Thus, a direct consequence of the vanishing of Weinberg-Tomozawa contribution is that, $D_S^+$ and $D_S^-$ are degenerate in nuclear matter. As mentioned earlier, the $D_S$ meson dispersion relations, with $\vec{k} = 0$, eqns. (19-21), reduce to the quadratic equation form ($A\omega^2 + B\omega + C = 0$), with the following coefficients (in nuclear matter):

$$A(N) = [1 + \left(\frac{d_1}{2f_D^{D_S}}(\rho_p^s + \rho_n^s)\right) - \left(\frac{\sqrt{2}}{f_D^{D_S}}(\zeta' + \zeta'_c)\right)]$$  \hspace{1cm} (24)

$$B(N) = 0$$  \hspace{1cm} (25)

$$C(N) = \left[-m_{D_S}^2 + \frac{m_{D_S}^2}{\sqrt{2}f_D^{D_S}}(\zeta' + \zeta'_c)\right]$$  \hspace{1cm} (26)

where the subscript $(N)$ emphasizes on the nuclear matter context. For solving this quadratic equation, we require the values of $\zeta'$, as well as the scalar densities of protons and neutrons, which are obtained from a simultaneous solution of coupled equations of motion for the scalar fields, subject to constraints of fixed values of $\rho_B$ and $\eta$. The behavior of the scalar fields, so obtained, has been discussed in detail in Ref.[17]. Here, we build upon these scalar fields and proceed to discuss the behavior of solutions of the in-medium dispersion relations of $D_S$ mesons, given by equations (24) to (26). The variation of the $D_S$ meson in-medium mass, $\omega(\vec{k} = 0)$, with baryonic density in nuclear matter, along with the individual contributions to this net variation, is shown in Fig.1 for both zero and a finite temperature value ($T = 100$ MeV). It is observed that the in-medium mass of $D_S$ mesons, decreases with density, while being weakly dependent on temperature and isospin asymmetry parameter. We can understand the observed behavior through the following analysis. From equations
FIG. 1. (Color Online) The various contributions to the $D_S$ meson energies, $\omega(\vec{k} = 0)$, in nuclear matter, plotted as functions of baryonic density in units of the nuclear saturation density ($\rho_B / \rho_0$). In each case, the isospin asymmetric situation ($\eta = 0.5$), as described in the legend, is also compared against the symmetric situation ($\eta = 0$), represented by dotted lines.

From this exact solution, we expect that the in-medium mass would decrease with an increase in density, owing to the dependence of the $d_1$ range term on the scalar densities, which would increase the value of the denominator at large densities. On the other hand, the density dependence of $\zeta'$ is much more subdued in nuclear matter. It is observed that the total change in the value of $\zeta$ from its vacuum value, i.e. $(\zeta')_{\text{max}} \approx 15$ MeV. This value is acquired at a baryonic density of $\rho_B \approx 3\rho_0$, and thereafter it appears to saturate. This behavior of the strange scalar field, though ubiquitous in this chiral model [17, 33], is not just limited to it. Even in calculations employing the relativistic Hartree approximation to determine the variation of these scalar fields [44], a behavior similar to this mean field treatment is observed. We also point out that the saturation of the scalar meson exchange and first range term follows as a direct consequence of the saturation of $\zeta'$, as is implied by their proportionality in equations (11) and (26). Notice also that while the contribution for the $d_1$ term in the denominator has a coefficient that is quadratic in $(1/f_{D_S})$, that for the $\zeta'$ term is only linear in that factor. $f_{D_S}$ being a large number (235 MeV), the two would
be comparable only at small densities, where \((\rho_p^s + \rho_n^s)\) is small. At larger densities, while the terms having \(\zeta'\) saturate, the contribution from the \(d_1\) range term continues to rise. Naturally then, the high density behavior is dictated completely by this \(d_1\) term.

In order to read more into our solution, we expand the argument of the square root, in a binomial series and retain up to only the first order terms. This gives:

\[
\omega \approx m_{DS} \left( 1 + \frac{(\zeta' + \zeta'_c)}{2\sqrt{2}f_{DS}} - \frac{d_1}{4f_{DS}^2}(\rho_p^s + \rho_n^s) \right)
\]  

(28)

Moreover, since \(\rho_i^s \approx \rho_i\) at small densities, the \(d_1\) range term contribution in the above equation approximately equals \((d_1/4f_{DS})\rho_B\) at low densities. From this approximate solution, each of the features reasoned above can be seen more clearly. The first of these terms is an increasing function while the second one is a decreasing function of density. Their interplay generates the observed curve shape - the repulsive contribution being responsible for the small hump in an otherwise linear fall, at small densities \((0 < \rho_B < 0.5\rho_0)\). While the attractive \(d_1\) term would have produced a linear decrease right away, the role of repulsive contribution is to impede this decrease, hence producing the hump. At moderately higher densities, however, the contribution from the second term outweighs the first, which is why we see a linear drop with density. This is observed to be the case, till around \(\rho_B \approx 2\rho_0\). At still larger densities, the approximation \(\rho_i^s \approx \rho_i\) breaks down, though our binomial expansion is still valid. Since scalar density falls slower than number density, the term \([- (d_1/4f_{DS}) (\rho_p^s + \rho_n^s)]\) will fall faster than \([- (d_1/4f_{DS}^2) \rho_B]\). This is responsible for the change in slope of the curve at intermediate densities, where a linear fall with density is no longer obeyed. So, at intermediate and large densities, the manner of the variation is dictated by the scalar densities, in the \(d_1\) range term.

From equations (19) to (21), one would expect the mass modifications of \(D_S\) mesons to be insensitive to isospin asymmetry, since, e.g. in this nuclear matter case, the dispersion relation bears isospin-symmetric terms like \((\rho_p^s + \rho_n^s)\). (This is in stark contrast with earlier treatments of kaons and antikaons \([11, 12]\) as well as the \(D\) mesons \([16]\) within this chiral effective model, where the dispersion relations had terms like \((\rho_p^s - \rho_n^s)\) or \((\rho_p - \rho_n)\), which contributed to asymmetry.) However, the \(D_S\) meson effective mass is observed to depend on asymmetry in Fig.1, though the dependence is weak. For example, the values of \(\omega(\vec{k} = 0)\), for the isospin symmetric situation \((\eta = 0)\), are 1913, 1865 and 1834 MeV, respectively, at \(\rho_B = 2, 4\) and \(6\rho_0\) while the corresponding numbers for the (completely) asymmetric
\( (\eta = 0.5) \) situation are 1917, 1875 and 1849 MeV, respectively, at \( T = 0 \). Since the \( \eta \)-dependence of \( \mathcal{L}_{\text{SME}} \) and \( \mathcal{L}_{1^\text{st} \text{Range}} \) contributions must be identical, one can reason from Fig. 1 that this isospin dependence of \( D_S \) meson effective mass is almost entirely due to the \( d_1 \) range term \((\sim (\rho_p^s + \rho_n^s))\), which was expected to be isospin symmetric. This apparently counterintuitive behavior has been observed earlier in [17], in the context of \( D \) mesons. This is because, the value of \((\rho_p^s + \rho_n^s)\) turns out to be different for symmetric and asymmetric situations, contrary to naive expectations. Since the scalar-isovector \( \delta \) meson \((\delta \sim \langle \bar{u}u - \bar{d}d \rangle)\) is responsible for introducing isospin asymmetry in this chiral model [33], owing to the equations of motion of the scalar fields being coupled, the values of the other scalar fields turn out to be different in the symmetric \((\delta = 0)\) and asymmetric \((\delta \neq 0)\) cases. The same is also reflected in the values of the scalar densities calculated from these scalar fields, which leads to the observed behavior.

Additionally, we observe from a comparison of figures 1a and 1b that the magnitude of the \( D_S \) meson mass drop decreases with an increase in temperature. For example, in the symmetric \((\eta = 0)\) situation, at \( T = 0 \), the \( D_S \) meson mass values, at \( \rho_B = 2\rho_0, 4\rho_0 \) and \( 6\rho_0 \), are 1913, 1865 and 1834 MeV respectively, which grow to 1925, 1879 and 1848 MeV respectively, at \( T = 100 \) MeV. Likewise, with \( \eta = 0.5 \), the corresponding values read 1917, 1875 and 1849 MeV at \( T = 0 \), while the same numbers, at \( T = 100 \) MeV, are 1925, 1883 and 1855 MeV. Thus, though small, there is a definite reduction in the magnitude of the mass drops, as we go from \( T = 0 \) to \( T = 100 \) MeV, in each of these cases. This behavior can be understood, from the point of view of the temperature variation of scalar condensates, in the following manner. It is observed that the scalar fields decrease with an increase in temperature from \( T = 0 \) to \( 100 \) MeV [15, 17]. (In Ref. [15], the same effect was understood as an increase in the nucleon mass with temperature. The equity of the two arguments can be seen by invoking the expression relating the baryon mass to the scalar fields’ magnitude, \( m_i^* = -(g_{vi}\sigma + g_{i\zeta} + g_{vi}\delta) \)). In fact, we point out that a decrease in the scalar condensates, with an increase in temperature, though small in the hadronic regime \((< 170 \) MeV), is a consistent feature of all \( U(N_f)_L \times U(N_f)_R \) linear sigma models, since the same was also observed in the model-independent work of Röder et al. [40]. Since these scalar fields serve as an input in calculating the scalar densities [33], a decrease in the magnitude of the latter accompanies a decrease in the former, at larger temperatures. From the point of view of the dispersion relations, this results in a decrease in the coefficient \( A \), and owing to the inverse
dependence of $\omega$ on $A$, increases the value of $\omega(\vec{k} = 0)$ in the finite temperature case, as compared to the $T = 0$ situation. Or stating it differently, the difference of this $\omega(\vec{k} = 0)$, from the vacuum value, i.e the mass drop, decreases. Additionally, it is also observed from Fig.1 that isospin dependence of the $D_S$ meson mass, feeble anyways, weakens further with temperature. For example, as mentioned earlier, mass of the $D_S$ mesons at $\rho_B = 6\rho_0$, for the $\eta = 0$ and $\eta = 0.5$ cases, are 1834 and 1849 MeV respectively (a difference of 15 MeV), when $T = 0$, which changes to 1848 and 1855 MeV at $T = 100$ MeV (a 7 MeV difference).

This is, once again, due to a decrease in the magnitude of $\delta$, with temperature, at any fixed value of the parameters $\eta$ and $\rho_B$. In particular, the difference between the value of $\delta$ for the $\eta = 0$ and $\eta = 0.5$ cases is observed to decrease with temperature (as was shown explicitly in Ref.[17]). Since asymmetry is introduced through $\delta$, a decrease in the difference of the values of $\omega$, between symmetric and asymmetric situations, with temperature, follows naturally.

Next, we generalize our analysis by including hyperonic degrees of freedom as well, in the medium. However, as mentioned previously, in the ensuing discussion of $D_S$ mesons in hyperonic matter, we focus predominantly on the new physics arising via the introduction of strangeness in the medium, since, e.g. the weak dependence on isospin asymmetry, or a weak reduction in the mass drop magnitudes at higher temperatures, has, in principle, the same explanation that stood in the nuclear matter context.
FIG. 3. (Color Online) $D_S$ meson mass degeneracy breaking in hyperonic matter, as a function of baryonic density, for typical values of the other parameters ($T = 100$ MeV, $f_s = 0.5$ and $\eta = 0.5$). The medium masses of $D_S^+$ and $D_S^-$ are observed to fall symmetrically about $\omega_{\text{com}}$ (see text).

Fig. 2 shows the variation of the in-medium mass of the $D_S$ mesons, along with the various individual contributions, in hyperonic medium as a function of baryonic density, for typical values of temperature, isospin asymmetry parameter and strangeness fraction. The most drastic consequence of the inclusion of hyperons in the medium is that, the mass degeneracy of $D_S^+$ and $D_S^-$ now stands broken. For example, the values of $(m_{D_S^+}, m_{D_S^-})$ at $\rho_B = \rho_0, 2\rho_0, 4\rho_0$ and $6\rho_0$, are observed to be $(1948, 1953)$, $(1918, 1928)$, $(1859, 1879)$ and $(1808, 1838)$ MeV, respectively, as can be seen from the figure. Thus, except at vacuum ($\rho_B = 0$), mass difference of $D_S^+$ and $D_S^-$ is non-zero at finite $\rho_B$, growing in magnitude with density. This mass degeneracy breaking is a direct consequence of non-zero contributions from the Weinberg-Tomozawa term, which follows from equations (11), (17) and (20). The same may be reconciled with Fig. 2 from which it follows that except for this Weinberg-Tomozawa term acquiring equal and opposite values for these mesons, all other terms are absolutely identical for them. Once again, on the basis of the following analysis of the $D_S$ meson dispersion relations at zero momentum, we insist that this observed behavior is perfectly consistent with expectations.

The general solution of the equivalent quadratic equation, eqn. (18), is:

$$\omega = \frac{(-B + \sqrt{B^2 - 4AC})}{2A} \approx -\frac{B}{2A} + \sqrt{\frac{C_1}{A} + \frac{B^2}{8A\sqrt{AC_1}}} + \ldots$$  \hspace{1cm} (29)$$

where we have disregarded the negative root, and have performed a binomial expansion of
(1 + B^2/4AC_1)^{1/2}$, with $C_1 = -C$. Upon feeding numerical values, we observe that the expansion parameter $(B^2/4AC_1)$ is much smaller than unity for our entire density variation, which justifies the validity of this expansion for our analysis. The same exercise also allows us to safely disregard higher-order terms, and simply write:

$$\omega_{\text{hyp}} \approx \sqrt{\frac{C_1}{A}} - \frac{B}{2A}$$  \hspace{1cm} (30)

Further, with the same justification as in the nuclear matter case, both $(C_1/A)^{1/2}$ and $(1/A)$ can also be expanded binomially. For example, for the second term, this gives

$$\frac{|B|}{2A} = \frac{|B|}{2} + O\left(\rho_i \rho_i^s \right)$$  \hspace{1cm} (31)

where the contribution from these higher order terms is smaller owing to the large denominator, prompting us to retain only the first order terms. Here, we point out that since $B = \pm|B|$, (+ sign for $D^+_S$ and − sign for $D^-_S$), this term, which represents the Weinberg-Tomozawa contribution to the dispersion relations, breaks the degeneracy of the $D_S$ mesons.

On the other hand, the first term in eqn.(30) is common between $D^+_S$ and $D^-_S$ mesons. Thus, the general solution of the $D_S$ meson dispersion relations in the hyperonic matter context, can be written as:

$$\omega_{\text{hyp}} = \omega_{\text{com}} \mp \omega_{\text{brk}},$$ \hspace{1cm} (32)

where

$$\omega_{\text{com}} = \sqrt{\frac{C_1}{A}} \approx m_{D_S} - \left(\frac{d_1}{4f_{D_S}^2} \sum_{i}(\rho_i^s) \right) - \left(\frac{d_2}{4f_{D_S}^2} \left(2\sum_{i} \rho_i^s + \sum_{(H-\Xi)} \rho_i^s\right) + \left(\frac{\zeta' + \zeta''}{2\sqrt{2}f_{D_S}}\right)\right),$$  \hspace{1cm} (33)

$$\omega_{\text{brk}} = \frac{|B|}{2A} \approx \frac{|B|}{2} = \left[\frac{1}{4f_{D_S}^2} \left(2\sum_{i} \rho_i + \sum_{(H-\Xi)} \rho_i\right)\right]$$  \hspace{1cm} (34)

Thus, $\omega_{D^+_S} = \omega_{\text{com}} - \omega_{\text{brk}}$, and $\omega_{D^-_S} = \omega_{\text{com}} + \omega_{\text{brk}}$, (where $\omega_{\text{brk}}$ is necessarily positive), which readily explains why the mass of $D^+_S$ drops more than that of $D^-_S$, with density. Also, this formulation readily accounts for symmetric fall of the medium masses of $D^+_S$ and $D^-_S$ mesons, about $\omega_{\text{com}}$ in Fig.3. Trivially, it may also be deduced that $\omega_{\text{brk}}$ is extinguished in nuclear matter, so that the mass degeneracy of $D^+_S$ and $D^-_S$ is recovered from these equations. However, we observe that though the curve corresponding to $\omega_{\text{com}}$ is exactly bisecting the
masses of $D_S^+$ and $D_S^-$ at small densities, this bisection is no longer perfect at high densities. This can be understood in the following manner. In essence, we are comparing the density dependence of a function $f(\rho_i)$, with the functions $f(\rho_i) \pm g(\rho_i)$. Since scalar densities fall sub-linearly with the number density, it follows that the fall cannot be absolutely symmetric at any arbitrary density. In fact, since a decreasing function of scalar density will fall faster than that of a number density, one expects $\omega_{\text{com}}$ to lean away from the curve for $D_S^-$ meson and towards the curve corresponding to $D_S^+$, which is exactly what is observed in Fig. 3.

Since this disparity between $D_S^+$ and $D_S^-$ originates from the Weinberg-Tomozawa term, whose magnitude is directly proportional to the hyperonic number densities, one expects this disparity to grow with an increase in the strangeness content of the medium. The same also follows from the above formulation, since the mass splitting between the two, 

$$\Delta m = (m_{D_S^-} - m_{D_S^+}) \equiv (\omega_{D_S^-} - \omega_{D_S^+}) = -2\omega_{\text{brk}},$$

should grow with both, $f_s$ at fixed $\rho_B$ (i.e. a larger proportion of hyperons), as well as with $\rho_B$ at fixed (non-zero) $f_s$ (i.e. a larger hyperonic density), in accordance with eqn.(34). Naively, one expects this $f_s$ dependence to be shared by the two $D_S$ mesons; however, closer analysis reveals that this is not the case, as shown in Fig. 4 where we consider the $f_s$ dependence of the $D_S$ meson medium mass. It is observed that while the $f_s$ dependence for $D_S^+$ meson is quite pronounced, the same for the $D_S^-$ is conspicuously subdued. Counterintuitive as it may apparently be, this observed behavior follows from the above formulation. Since $\omega_{D_S^-} = \omega_{\text{com}} + \omega_{\text{brk}}$, at any fixed density,
FIG. 5. (Color Online) (a) A comparison of medium mass of $D_S$ mesons, in nuclear and hyperonic matter, maintaining the same values of parameters as before. (b) Comparison of the nuclear matter solution, $\omega_{\text{nucl}}$, with the $\omega_{\text{com}}$ in hyperonic matter, as observed in Fig. 3.

the first of these is a decreasing function of $f_s$, while the second increases with $f_s$. These opposite tendencies are responsible for the weak $f_s$ dependence of $D_S^-$ meson. On the other hand, because $\omega_{D_S^+} = \omega_{\text{com}} - \omega_{\text{brk}}$, the two effects add up to produce a heightened overall decrease with $f_s$ for the $D_S^+$ meson, as we observe in the figure.

A comparison of these hyperonic matter ($f_s \neq 0$) solutions with the nuclear matter ($f_s = 0$) solutions is shown in Fig. 5 for typical values of the parameters. It is observed from Fig. 5a, that at small densities, the nuclear matter curve, which represents both $D_S^+$ and $D_S^-$, bisects the mass degeneracy breaking curve, akin to $\omega_{\text{com}}$ in Fig. 3. However, at larger densities, both these $f_s \neq 0$ curves drop further than the $f_s = 0$ curve. From a casual comparison of expressions, it appears that the nuclear matter solution, given by eqn. (28), also enters the hyperonic matter solution, eqn. (33), albeit as a subset of $\omega_{\text{com}}$. It is tempting to rearrange the latter then, such that this nuclear matter part is separated out, generating an expression of the type $\omega_{\text{hyp}} = \omega_{\text{nucl}} + f(\rho_i, \rho_s^i)|_{i=H}$, which would also entail the segregation of the entire $f_s$ dependence. However, careful analysis reveals that it is impossible to achieve this kind of a relation since, in spite of an identical expression, in hyperonic matter, the RHS of eqn. (28) does not give the nuclear matter solution. This is because, the nuclear matter solution involves the scalar fields (and scalar densities computed using these scalar fields) obtained as a solution of coupled equations in the $f_s = 0$ situation, which are radically different from the solutions obtained in the $f_s \neq 0$ case. (This has been observed to be the
case, in almost every treatment of hyperonic matter within this chiral effective model, but perhaps, most explicitly in [31].) Thus, one can not retrieve the nuclear matter solution from the scalar fields obtained as solutions in the $f_s \neq 0$ case; moreover, due to the complexity of the concerned equations, it is impossible to achieve a closed-form relation between the values of the scalar fields (and hence, also the scalar densities) obtained in the two cases. In fact, the low-density similarity between Fig.3 and Fig.5a might lead one to presume that $\omega_{\text{com}}$ reduces to $\omega_{\text{nucl}}$ at small densities. This expectation is fueled further by their comparison in Fig.5b, which shows clearly that they coincide at small densities. However, in light of the above argument, it follows that they are absolutely unrelated. Their coincidence at small densities can be explained by invoking the relation $\rho_i^s \approx \rho_i$ in this regime, using which, eqn.(33) reduces to

$$\omega_{\text{com}} \approx m_{D_S} \left( 1 + \frac{(\zeta' + \zeta'_c)}{2\sqrt{2}f_{D_S}} - \frac{\kappa_1}{4f_{D_S}^2} \rho_B \right) - m_{D_S} \left( \frac{\kappa_2 \Delta}{4f_{D_S}^2} \right), \quad (35)$$

with $\kappa_1 = (d_1 + d_2) \approx 1.28d_1$, with our choice of parameters, $\kappa_2 (\equiv d_2) = 0.22\kappa_1$ and the difference term, $\Delta = (\rho^s_{\Xi^0} + \rho^s_{\Xi^-} - \rho^s_p - \rho^s_n)$. The contribution from this second term in eqn.(35) is significantly smaller than the first term at small densities, owing to the smaller coefficient, as well as the fact that this depends on the difference of densities rather than on their sum (like the first term). This first part of eqn.(35) can be likened to the approximate nuclear matter solution at small $\rho_B$, as we concluded from eqn.(28). The marginal increase of $\kappa$ above $d_1$ is compensated by a marginal increase in $\zeta'$ for the $f_s \neq 0$ case, as compared to the $f_s = 0$ situation, and hence, the two curves look approximately the same at small densities in Fig.5b. At larger densities however, this simple picture breaks down, and the fact that they are unrelated becomes evident. Nevertheless, we may conclude in general from this comparison that the inclusion of hyperonic degrees of freedom in the hadronic medium makes it more attractive, regarding $D_S$ mesons’ in-medium interactions, especially at large $\rho_B$. From the point of view of the $D_S$ meson dispersion relations, this is conclusively because of the contributions from the extra, hyperonic terms, which result in an overall increase of the coefficient $A_{\text{hyp}}$ significantly above $A_{\text{nucl}}$, particularly at large $f_s$.

Finally, we consider momentum-dependent effects by investigating the behavior of the in-medium optical potentials for the $D_S$ mesons. These are shown in Fig.6, where we consider them in the context of both, asymmetric nuclear ($f_s = 0$) as well as hyperonic matter ($f_s = 0.5$), as a function of momentum $k$ ($= |\vec{k}|$) at fixed, values of the parameters $\rho_B$, $\eta$ and
FIG. 6. (Color Online) The optical potentials of the, (a) $D_S^+$ and (b) $D_S^-$ mesons, as a function of momentum $k \equiv |\vec{k}|$, in cold ($T = 0$), asymmetric ($\eta = 0.5$) matter, at different values of $\rho_B$. In each case, the hyperonic matter situation ($f_s = 0.5$), as described in the legend, is also compared against the nuclear matter ($f_s = 0$) situation, represented by dotted lines.

$T$. As with the rest of the investigation, the dependence of the in-medium optical potentials on the parameters $T$ and $\eta$ is quite weak; hence, we neglect their variation in this context. In order to appreciate the observed behavior of these optical potentials, we observe that as per its definition, eqn. (22), at zero momentum, optical potential is just the negative of the mass drop of the respective meson (i.e. $U(k = 0) = -\Delta m(k = 0) \equiv \Delta m(\rho_B, T, \eta, f_s)$). It follows then, that at $k = 0$, the two $D_S$ mesons are degenerate in nuclear matter; moreover, it is observed that this degeneracy extends to the finite momentum regime. This is because, from equations (16) and (17), the finite momentum contribution is also common for $D_S^+$ and $D_S^-$ in nuclear matter. In hyperonic matter, however, non-zero contribution from these terms breaks the degeneracy in the $k = 0$ limit (as was already discussed before), and finite momentum preserves this non-degeneracy. Consequently, at any fixed density, the curves for different values of $f_s$ differ in terms of their y-intercept but otherwise, appear to run parallel. Further, as we had reasoned earlier for the $k = 0$ case, the effect of increasing $f_s$ on the mass drops of $D_S^+$ and $D_S^-$ is equal and opposite about the nuclear matter situation at small densities, which readily explains the behavior of optical potentials for these mesons at $\rho_B = \rho_0$. At larger densities, e.g. the $\rho_B = 4\rho_0$ case shown in Fig. 6, the lower values of optical potentials for both $D_S^+$ and $D_S^-$, as compared to the $f_s = 0$ case, can be immediately reconciled with the behavior observed in Fig. 5. In fact, the large difference between the $D_S^-$
FIG. 7. (Color Online) Mass of the $D_S^+D_S^-$ pair, compared against the (vacuum) masses of the excited charmonia states, at typical values of temperature ($T = 100$ MeV) and asymmetry ($\eta = 0.5$), for both nuclear matter ($f_s = 0$) and hyperonic matter ($f_s = 0.2$, 0.3 and 0.5) situations. The respective threshold density (see text) is given by the point of intersection of the two concerned curves.

optical potential for the $f_s = 0$ and $f_s = 0.5$ cases, as compared to that for $D_S^-$, is a by-product of their zero-momentum behavior, preserved at non-zero momenta as a consequence of equations (16) and (17).

We next discuss the possible implications of these medium effects. In the present investigation, we have observed a reduction in the mass of the $D_S$ mesons, with an increase in baryonic density. This decrease in $D_S$ meson mass can result in the opening up of extra reaction channels of the type $A \rightarrow D_S^+D_S^-$ in the hadronic medium. In fact, a comparison with the spectrum of known charmonium states [32, 45] sheds light on the possible decay channels, as shown in Fig.7. As a starting approximation, we have neglected the variation of energy levels of these charmonia with density, since we intuitively expect medium effects to be larger for an $s\bar{c}$ (or $c\bar{s}$) system, as compared to a $c\bar{c}$ system. However, this approximation can be conveniently relaxed, e.g. as was done in Refs.[31, 46] for charmonia and
bottomonia states, respectively. It follows from Fig. 7 that above certain threshold density, which is given by the intersection of the $D_S$ meson curve with the respective energy level, the mass of the $D_S^+D_S^-$ pair is smaller than that of the excited charmonium state. Hence, this decay channel opens up above this threshold density. The most immediate experimental consequence of this effect would be a decrease in the production yields for these excited charmonia in collision experiments. Additionally, since an extra decay mode will lower the lifetime of the state, one expects the decay widths of these excited charmonia to be modified as a result of these extra channels. Though it can be reasoned right away that reduction in lifetime would cause a broadening of the resonance curve, more exotic effects can also result from these level crossings. For example, it has been suggested in [47], that if the internal structure of the hadrons is also accounted for, there is even a possibility for these in-medium widths to become narrow. This counterintuitive result originates from the node structure of the charmonium wavefunction, which can even create a possibility for the decay widths to vanish completely at certain momenta of the daughter particles. Also, if the medium modifications of the charmonium states are accounted for, the decay widths will be modified accordingly (as was observed in the context of the $D$ mesons, in [31]).

Apart from this, one expects signatures of these medium effects to be reflected in the particle ratio ($D_S^+/D_S^-$). Since less massive particles are produced more easily, a production asymmetry between $D_S^+$ and $D_S^-$ mesons can be anticipated owing to their unequal medium masses, as observed in the current investigation. Also, due to the semi-leptonic and leptonic decay modes of the $D_S$ mesons [32], one also expects that, accompanying an increased production of $D_S$ mesons due to a lowered mass, there would be an enhancement in dilepton production as well. Further, we expect these medium-modifications of the $D_S$ mesons to be reflected in the observed dilepton spectra, due to the well-known fact that dileptons, with their small interaction cross section in hadronic matter, can serve as probes of medium effects in collision experiments [48, 49].

These possible experimental consequences, conjectured above, are especially interesting in wake of the upcoming PANDA [50] and CBM [51] experiments, at the future Facility for Antiproton and Ion Research (FAIR), GSI, where high baryonic densities are expected to be reached. Due to the strong density dependence of these medium effects, we expect that each of these mentioned experimental consequences would intensify at higher densities, and should be palpable in the aforementioned future experiments.
V. SUMMARY

To summarize, we have explored the properties of $D_S$ mesons in a hot and dense hadronic environment, within the Chiral $SU(3)_L \times SU(3)_R$ model, generalized to $SU(4)$. These mesons have been considered in both (symmetric and asymmetric) nuclear and hyperonic matter, at finite densities that extend slightly beyond what can be achieved with the existing and known future facilities, and at both zero and finite temperatures. Due to net attractive interactions in the medium, $D_S$ mesons are observed to undergo a drop in their effective mass. These mass drops are found to intensify with an increase in the baryonic density of the medium, while being largely insensitive to changes in temperature as well as the isospin asymmetry parameter. However, upon adding hyperonic degrees of freedom, the mass degeneracy of $D^+_S$ and $D^-_S$ is observed to be broken. The mass splitting between $D^+_S$ and $D^-_S$ is found to grow significantly with an increase in baryonic density as well as the strangeness content of the medium. Through a detailed analysis of the in-medium dispersion relations for the $D_S$ mesons, we have shown that the observed behavior, follows precisely from the interplay of contributions from various interaction terms in the Lagrangian density. We have briefly discussed the possible experimental consequences of these medium effects, which are expected to be considerably enhanced at large densities and hence, be accessible in the upcoming PANDA and CBM experiments, at the future facilities of FAIR, GSI.

ACKNOWLEDGMENTS

D.P. acknowledges financial support from University Grants Commission, India [Sr. No. 2121051124, Ref. No. 19-12/2010(i)EU-IV]. A.M. would like to thank Department of Science and Technology, Government of India (Project No. SR/S2/HEP-031/2010) for financial support.

[1] W. Weise - “Quarks, Hadrons and Dense Nuclear Matter”, Course 11 in Trends in Nuclear Physics, 100 years later, Les Houches, Session LXVI, 1996.

[2] C. M. Ko, V. Koch and G. Q. Li, Annu. Rev. Nucl. Part. Sci. 47, 505 (1997).
[3] R. S. Hayano and T. Hatsuda, Rev. Mod. Phys. 80, 2949 (2010).
[4] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515 (1997).
[5] J. A. Oller, E. Oset and J. R. Peláez, Phys. Rev. Lett. 80, 3452 (1998).
[6] E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998).
[7] A. Hayashigaki, Phys. Lett. B 487, 96 (2000).
[8] A. Kumar and A. Mishra, Phys. Rev. C 82, 045207 (2010).
[9] K. Tsushima, D. H. Lu, A. W. Thomas, K. Saito and R. H. Landau, Phys. Rev. C 59, 2824 (1999).
[10] A. Sibirtsev, K. Tsushima and A. W. Thomas, Eur. Phys. J. A 6, 351 (1999).
[11] A. Mishra, A. Kumar, S. Sanyal and S. Schramm, Eur. Phys. J. A 41, 205 (2009).
[12] A. Mishra, A. Kumar, S. Sanyal, V. Dexheimer and S. Schramm, Eur. Phys. J. A 45, 169 (2010).
[13] A. Mishra and S. Schramm, Phys. Rev. C 74, 064904 (2006).
[14] A. Mishra, S. Schramm and W. Greiner, Phys. Rev. C 78, 024901 (2008).
[15] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich, S. Schramm and H. Stöcker, Phys. Rev. C 69, 015202 (2004).
[16] A. Mishra and A. Mazumdar, Phys. Rev. C 79, 024908 (2009).
[17] A. Kumar and A. Mishra, Phys. Rev. C 81, 065204 (2010).
[18] M. He, R. J. Fries and R. Rapp, Phys. Rev. Lett. 110, 112301 (2013).
[19] A. Faessler, T. Gutsche and V. E. Lyubovitskij, Prog. Part. Nucl. Phys. 61, 127 (2008).
[20] X. Liu, Int. J. Mod. Phys. Conf. Ser. 02, 147 (2011).
[21] F. Close, Nature 424, 376 (2003).
[22] Y. Simonov and J. Tjon, Phys. Rev. D 70, 114013 (2004).
[23] P. Wang and X. G. Wang, Phys. Rev. D 86, 014030 (2012).
[24] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B 647, 133 (2007).
[25] R. Molina, T. Branz and E. Oset, Phys. Rev. D 82, 014010 (2010).
[26] R. Molina, E. Oset, T. Branz, J. J. Wu and B. S. Zou, Int. J. Mod. Phys. Conf. Ser. 02, 153 (2011).
[27] E. van Beveren and G. Rupp, Phys. Rev. Lett. 93, 202001 (2004).
[28] Y. Q. Chen and X. Q. Li, Phys. Rev. Lett. 93, 232001 (2004).
[29] M. F. M. Lutz and C. L. Korpa, Phys. Lett. B 633, 43 (2006).
[30] J. Hofmann and M. F. M. Lutz, Nucl. Phys. A 763, 90 (2005).
[31] A. Kumar and A. Mishra, Eur. Phys. J. A 47, 164 (2011).
[32] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
[33] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker and W. Greiner, Phys. Rev. C 59, 411 (1999).
[34] D. Zschiesche et al., ‘Chiral Symmetries in Nuclear Physics’, in Symmetries in Intermediate and High Energy Physics, Springer Tracts in Modern Physics, Vol. 163 (Springer-Verlag, 2000).
[35] S. Weinberg, Phys. Rev. Lett. 18, 188 (1967).
[36] S. Weinberg, Phys. Rev. 166, 1568 (1968).
[37] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2239 (1969).
[38] C. G. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2247 (1969).
[39] W. A. Bardeen and B. W. Lee, Phys. Rev. 177, 2389 (1969).
[40] D. Röder, J. Ruppert and D. H. Rischke, Phys. Rev. D 68, 016003 (2003).
[41] Y. Aoki, G. Endrödi, Z. Fodor, S. D. Katz and K. K. Szabó, Nature 443, 675 (2006).
[42] C. Bernard et al. (MILC Collaboration), Phys. Rev. D 71, 034504 (2005).
[43] A. Bazavov et al. (HotQCD Collaboration), Phys. Rev. D 85, 054503 (2012).
[44] D. Zschiesche, A. Mishra, S. Schramm, H. Stöcker and W. Greiner, Phys. Rev. C 70, 045202 (2004).
[45] M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008).
[46] A. Mishra and D. Pathak, Phys. Rev. C 90, 025201 (2014).
[47] B. Friman, S. H. Lee and T. Song, Phys. Lett. B 548, 153 (2002).
[48] R. Vogt, Ultrarelativistic Heavy-Ion Collisions (Elsevier, 2007).
[49] G. Q. Li and C. M. Ko, Nucl. Phys. A 582, 731 (1995).
[50] Official webpage, PANDA Experiment, http://www.panda.gsi.de
[51] B. Friman et al. (eds.): The CBM Physics Book, Lecture Notes in Physics 814, (Springer-Verlag, 2010).