

\textbf{Z}_c(3900) \text{ as a Four-Quark State}

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\textbf{Abstract}

Using the method of QCD Sum Rules, we derive the correlator $\Pi^Z$ for a state consisting of two charm quarks and two light quarks, $c\bar{d}u\bar{c}$, and carry out a Borel transform to find $\Pi^Z(M_B)$. From this we find the solution that $M_B \simeq 3.9 \pm 0.2$ GeV, showing that the $Z_c(3900)$ is a tetra-quark state.

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\section{Introduction}

Recently $e^+e^-$ collision experiments by BESIII[1] and Belle[2] Collaborations have found a state at about 3,900 MeV, called the $Z_c(3900)[1]$, that might be a four quark state, $|c\bar{d}u\bar{c}>[3]

For many years states in the energy region of 3,900-4,500 MeV, near the $D\bar{D}$ threshold, there have been found possible tetra-quark states. See, e.g., Ref[4], for a theoretical study of the $X(3872)$ as a charm tetra-quark about one decade ago. Recently, there have been studies of the $Z_c(3900)$ as a $D\bar{D}^*$ molecular state[5, 6], vector/axial vector Charmonium state[7] using the method of QCD Sum Rules as in the present study. See these references for references to earlier publications, as well as a review of New Charmonium States via QCD Sum Rules[8]

In our study of the $Z_c(3900)$ as possibly a $|c\bar{d}u\bar{c}>$ state we use the method of QCD sum rules[9]. Our approach differs from earlier studies[5, 6] in that our correlator corresponds to a four-quark (tetra-quark) rather than a $D\bar{D}^*$ molecular state. First we find the correlator in momentum space, and then as a function of the Borel mass, $M_B$, and see if it has a minimum near the value of 3.9 GeV, similar to our study of heavy quark hybrid meson states[10].

In Section II we briefly review the method of QCD sum rules, and derive the correlator for our four-quark model. In Section III we find the correlator as a function of the Borel mass in the region near 4,000 MeV; and then from a plot find the minimum value of $M_B$. In Section IV we discuss the results and conclusions.
2 The $|c\bar{d}\bar{c}u>\,$ state and QCD Sum Rules

The method of QCD sum rules\cite{9} for finding the mass of a state $A$ starts with the correlator,
\[ \Pi^A(x) = \langle 0 | T[J_A(x)J_A(0)] | 0 \rangle , \] (1)
with $|0\rangle$ the vacuum state and the current $J_A(x)$ creating the states with quantum numbers $A$:
\[ J_A(x)|0\rangle = c_A|A\rangle + \sum_n c_n|n;A\rangle , \] (2)
where $|A\rangle$ is the lowest energy state with quantum numbers $A$, and the states $|n;A\rangle$ are higher energy states with the $A$ quantum numbers, which we refer to as the continuum. One then carries out a Borel transform to reduce the importance of the continuum and higher order diagrams.

For our theory of the $Z_c(3900)$ as a tetra-quark state $|c\bar{d}\bar{c}u>\,$, we use the current $J_{Z_c}$, which creates a $J^{PC}=1^{-+}$ tetra-quark state:
\[ J_{Z_c} = \bar{d}\gamma_5c\bar{c}\gamma_5u , \] (3)
with the correlator
\[ \Pi^Z(x) = \langle 0 | T[J_{Z_c}(x)J_{Z_c}(0)] | 0 \rangle . \] (4)

Note that as was emphasized by J-R. Zhang in the Summary of Ref\cite{5} the current and correlator used for a QCD sum rule study of the $Z_c(3900)$ as a $\bar{D}D^*$ molecular state, with a current and correlator given by
\[ J_{\bar{D}D^*}^\mu = (\bar{Q}_a i\gamma_5q_a)(\bar{q}_b\gamma^\mu Q_b) \]
\[ \Pi_{\bar{D}D^*}^{\mu\nu} = i < 0 | T[J_{\bar{D}D^*}^\mu(x)J_{\bar{D}D^*}^{\nu+}(0)] | 0 > , \] (5)
with $Q,q$ charm,light ($u,d$) quarks and $a,b$ color indices, is just one possible theoretical interpretation of the $Z_c(3900)$ state, and is not the same at a tetra-quark state. We emphasize that with our current, Eq(3) and correlator, Eq(4), we are using a QCD sum rule to explore the possibillity that the $Z_c(3900)$ is a tetra-quark state, which is not completely orthogonal to but is quite different from a $\bar{D}D^*$ molecular state, by finding the mass using the tetra-quark correlator.

Using Wick’s Theorem to express $\Pi^Z(x)$ in terms of the quark propagators, and taking the Fourier transform, one obtains the correlator in momentum space:
\[ \Pi^Z(p) = \int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^3} Tr[S_d(k_1)\gamma_5S_c(k_2)\gamma_5]Tr[S_c(k_3)\gamma_5S_u(p+k_1-k_3+k_3)\gamma_5] , \] (6)
with $S_q(k)$ a quark propagator,

$$S_q(k) = \frac{k + m_q}{k^2 - m_q^2},$$  \hspace{1cm} (7)$$

where $k = \sum \gamma^\alpha k_\alpha$, with $\gamma^\alpha$ a Dirac matrix.

The correlator is illustrated in Fig. 1

Note that higher order terms are very small, as pointed out in Ref[6].

![Figure 1: $c, \bar{c}$ are charm, anticharm quarks. $u, \bar{d}$ are up, antidownquarks](image)

Finally, we carry out a Borel Transform[9]

$$B\Pi^Z(p) = \Pi^Z(M_B)$$  \hspace{1cm} (8)

### 3 The Correlator $\Pi^Z(p) \rightarrow \Pi^Z(M_B)$

From Eqs(6,7), carrying out the traces, one finds

$$\frac{\Pi^Z(p)}{16} = \int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^3} \frac{mM(mM - k_1 \cdot k_2) + k_3 \cdot (p + k_1 - k_2 + k_3)(k_1 \cdot k_2 - mM)}{(k_1^2 - m^2)(k_2^2 - M^2)(k_3^2 - M^2)((p + k_1 - k_2 + k_3)^2 - m^2)}$$  \hspace{1cm} (9)

Since the $k_1 \cdot k_2$ term vanishes via the momentum integrals and the $k_3 \cdot (p + k_1 - k_2 + k_3)$ term vanishes via the Borel transform, one needs to evaluate two terms;

$$I_1(p) = 16 \int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^3} \frac{m^2M^2}{(k_1^2 - m^2)(k_2^2 - M^2)(k_3^2 - M^2)((p + k_1 - k_2 + k_3)^2 - m^2)}$$  \hspace{1cm} (10)

$$I_4(p) = 16 \int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^3} \frac{k_1 \cdot k_2 k_3 \cdot (p + k_1 - k_2 + k_3)}{(k_1^2 - m^2)(k_2^2 - M^2)(k_3^2 - M^2)((p + k_1 - k_2 + k_3)^2 - m^2)}$$  \hspace{1cm} (11)

with $M = M(\text{charm quark}) \simeq 1.5 \text{ GeV}$ and $m = m(\text{u, d quark}) \simeq 4 \text{ MeV}$. 

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For the evaluation of Eqs(10,11) one uses
\[ I_H(p) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)((p - k)^2 - M^2)} = \frac{1}{(4\pi)^2} \frac{1}{(2p^2 - 4M^2)} \int_0^1 d\alpha \frac{1}{\alpha(1-\alpha)p^2 - M^2}, \] (12)
and
\[ I_{Hh}(p) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)((p - k)^2 - m^2)} = \frac{1}{(4\pi)^2} \left[ \frac{1}{\alpha(1-\alpha)(1-\beta)M^2} \right] \int_0^1 d\alpha \frac{1}{\alpha(1-\alpha)p^2 - (1-\alpha)m^2 - \alpha M^2} + \frac{5}{2} - \frac{3M^2 - m^2}{2p^2} \ln(M^2/m^2). \] (13)

Carrying out the integrals in Eqs(10,11), one finds
\[ I(1) = -\frac{16m^2M^2}{(4\pi)^6} \int_0^1 d\alpha \frac{5(1 - 0.8\alpha(1-\alpha))}{\alpha(1-\alpha)^2} \ln(M^2/m^2)I_{Hh}(p)_{m=0} + \frac{m^2 + 2\alpha(1-\alpha)}{(2(1-\beta))}I_{Hh}(p)_{m^2} - \frac{m^2 + 2\alpha(1-\alpha)M^2}{2m^2(1-\alpha)}I_{Hh}(p)_{m^2} + \frac{m^2 + 2\alpha(1-\alpha)M^2}{2m^2(1-\alpha)}I_{Hh}(p)_{m^2} \] (14)
with \( m^2 = (1-\beta)m_1^2 + \beta M^2 \) and \( m^2 = m^2/(\beta(1-\beta))^2 \); and
\[ I(4) = -\frac{32}{(4\pi)^2} \frac{m^2}{m^2} \int_0^1 d\alpha \frac{5\alpha(1-\alpha)}{\alpha(1-\alpha)^2} \left[ \ln(M^2/m^2)I_{Hh}(p)_{m^2} + (m^2 - 2\alpha(1-\alpha)M^2)(m^2 - 2\alpha(1-\alpha)M^2) \right] \gamma M^2 - 7\bar{m}^2/4 \] (15)
with \( m^2 = (M^2 + (1 - 2\gamma)m_2^2)/\gamma(1 - \gamma) \), \( m^2 = (M^2 + (\gamma - 1)m_2^2)/\gamma(1 - \gamma) \), and \( m^2 = m^2/(\alpha(1-\alpha)) \).

Taking the Borel transform one obtains (after removing common factors)
\[ B(1) = -5.0 \frac{1}{\alpha^2} \frac{1}{1-\alpha} \int_0^1 d\alpha \int_0^1 d\beta(1 - 0.8\alpha(1-\alpha)) \ln(M^2/m^2) \] (16)
\[ + (m^2 + 2\alpha(1-\alpha)M^2)/(2(1-\beta)m^2 - \beta \alpha(1-\alpha)M^2) \] (16)
\[ (M^2 - 2\alpha(1-\alpha)M^2)e^{-\alpha M^2/(2\beta M^2)} \] (16)
\[ + \int_0^1 d\gamma(1/2)[(1+\beta)m^2 + (1-\alpha)(\alpha + \beta)M^2 - (m^2 + 2\alpha(1-\alpha)M^2)\alpha(1-\alpha) - (1-\beta)m^2 - \beta \alpha(1-\alpha)M^2] \] (16)
\[ (3m^2 - M^2 + (1 - \beta)m^2 - \alpha(1-\alpha)M^2)/2 + (m^2 - M^2)\gamma(1 - \gamma) \] (16)
\[ e^{-((1-\beta)m^2 - \alpha(1-\alpha)M^2)/[\alpha(1-\alpha)M^2]} \] (16)
$$\mathcal{B}I_4(p) = -\int_0^1 d\alpha \int_0^1 d\gamma \int_0^1 (5\alpha(1-\alpha) - 7/2)((5\alpha(1-\alpha)\gamma(1-\gamma) - 7/4)m^2$$

$$+ (m^2 - M^2\alpha(1-\alpha)\gamma(1-\gamma) + \frac{((m^2 - M^2\alpha(1-\alpha)\gamma(1-\gamma))^2\lambda(1-\lambda)}{2((1-\lambda)m^2 + \lambda\alpha(1-\alpha)\gamma(1-\gamma)M^2)}$$

$$\frac{1}{2}((1-\lambda)m^2 - \lambda\alpha(1-\alpha)\gamma(1-\gamma)M^2))$$

$$e^{-(1-\lambda)m^2 + \lambda\alpha(1-\alpha)\gamma(1-\gamma)M^2/\alpha(1-\alpha)\gamma(1-\gamma)\lambda(1-\lambda)M^2_\Pi}$$

$$-((3(\gamma - 1)m^2 + \alpha(1-\alpha)(1-\gamma(1-\gamma)) - \frac{1}{2}\tilde{M}^2\tilde{M}^2 - \frac{1}{2}((\gamma - 1)m^2 +$$

$$\alpha(1-\alpha)(1-\gamma(1-\gamma))M^2)^2 e^{-M^2/[\alpha(1-\alpha)\gamma(1-\gamma)\lambda(1-\lambda)M^2_\Pi]}$$

$$\mathcal{B}I_2(p)(p) \quad = \quad \Pi^Z(M_B) = \mathcal{B}I_1(p) + \mathcal{B}I_4(p).$$

The correlator as a function of the Borel mass, $\Pi^Z(M_B)$ is shown in Fig. 2 below. Note that the result for the mass is given by $M_B$ at the minimum in the plot of $\Pi^Z(M_B)$, and the error by the shape of the plot near the minimum[9], as is discussed in detail in Ref[10].

![Figure 2](image-url)

Figure 2: $\Pi(M_B)$ is the 4-quark correlator, a function of the Borel Mass $M_B$, in units of MeV
4 Results and Conclusions

From Figure 2, the mass of the $|c\bar{d}c\bar{u}\rangle$ state is $3900 \pm 200$ MeV, in agreement with the state found in the recent BESIII[1] and Belle[2] experiments. From this we conclude that the conjecture of these two collaborations is correct, that the $Z_c(3900)$ is a tetra-quark state. For decades experimentalists and theorists have attempted to find tetra-quark states, so this is a very important discovery.

Our plans for future research include estimates of the decay probabilities of our tetra-quark theory of the $Z_c(3900)$ to compare with experimental results, which requires a three-point correlator.

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