Generalization of the model of conflict between two armed groups

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Abstract

The conflicts between armed groups often go on for years. The classical model of such conflicts accounts for the number of participants and for the technology level of the equipment of the groups. Below we extend this model in order to account for events that are present for limited time. As examples we discuss three kinds of such events: inclusion of reserves, presence of epidemics and use of non-conventional weapons. We show that if such events are not handled properly by the leaders of the groups the corresponding group can lose the conflict.

Key words: conflict, attrition, ambush, combat, mathematical models

1 Introduction

Population dynamics deals with coexistent animal or human populations [1-4]. The most antagonistic relations among animal populations are the predator-prey ones. Human populations are different as they can compete economically or politically [5-8] or they can support fighting among armed groups [9-10]. Several decades ago Richardson and Lanchester applied the idea for mathematical modeling of arms races and military combats [11-12]. In our unstable world today such kind of modeling becomes highly actual [13-14].

Usually the number of members and the quality of their equipment are the most important characteristics of the armed groups. A general model of conflict between two such groups (called the ”Red group” and the ”Blue group”) is

\[
\frac{dB}{dt} = F(B, R; b, r), \quad \frac{dR}{dt} = G(B, R; b, r)
\]

where \(R(t)\) and \(B(t)\) are the numbers of armed members of the two groups; \(b\) and \(r\) account for the technology level of the equipment of the two groups;
and $F$ and $G$ are linear or nonlinear functions, depending on the character of the conflict. Epstein [15] discussed the following particular case of the model (1)

$$\frac{dR}{dt} = -bBc_1c_2R, \quad \frac{dB}{dt} = -rRc_3c_4$$

where $c_{1,2,3,4}$ are real nonnegative coefficients.

Below we generalize the model (1) by addition of terms which describe the influence of different events acting for limited time. In section 3 we discuss three simple cases of such events: inclusion of reserves, epidemics and use of non-conventional weapons. Several concluding remarks are summarized in section 4.

2 Generalization of the model (1)

Let us introduce the function

$$V(t, t_1, t_2, \mu, \nu) = \Theta(t_1)\left\{1 - \exp[\nu(t_1 - t)]\right\} \exp[\Theta(t_2)\nu(t_2 - t)]$$

where $t_2 > t_1$ and $\Theta$ is the Heaviside theta function. $V$ describes the following process: at $t_1$ $V$ grows from 0 to almost 1 and this growth is controlled by the parameter $\mu$. At $t = t_2$ $V$ begins an exponential decrease from 1 to 0. This decrease is controlled by the parameter $\nu$ (see Fig. 1).

By means of the function $V$ we can include different effects that act for limited time. Three examples are

1. Inclusion of reserves: $N_1$ times from $t_1^{(i)}$ until $t_2^{(i)}$ for the Blue group (amplitude $B_i$) and $N_4$ times from $t_1^{(p)}$ until $t_2^{(p)}$ for the Red group (amplitude $R_p$)

$$\sum_{i=1}^{N_1} B_i V(t, t_1^{(i)}, t_2^{(i)}, \mu_i, \nu_i), \quad \sum_{p=1}^{N_4} R_p V(t, t_1^{(p)}, t_2^{(p)}, \mu_p, \nu_p)$$

2. Epidemics: $N_2$ times from $t_1^{(j)}$ until $t_2^{(j)}$ for the Blue group (amplitude $C_j$) and from $t_1^{(q)}$ until $t_2^{(q)}$ for the Red group (amplitude $E_q$)

$$\sum_{j=1}^{N_2} C_j V(t, t_1^{(j)}, t_2^{(j)}, \mu_j, \nu_j), \quad \sum_{q=1}^{N_5} E_q V(t, t_1^{(q)}, t_2^{(q)}, \mu_q, \nu_q)$$

3. Using of non-conventional weapons: $N_3$ times from $t_1^{(k)}$ until $t_2^{(k)}$ against the Blue group (amplitude $D_k$) and $N_6$ times from $t_1^{(v)}$ until $t_2^{(v)}$ against
Figure 1: The function $V(t, T_1, t_2, \mu, \nu)$. Fig. 1a: long switch-on fast switch-off ($\mu = 0.1$, $\nu = 5.0$). Fig. 1b: fast switch-on, fast switch-off ($\mu = \nu = 5.0$). $t_1 = 2.5, t_2 = 50$. Fig. 1c: fast switch-on, fast switch-off, $t_1 = 2.5, t_2 = 80$.

the Red group (amplitude $H_v$).

$$\sum_{k=1}^{N_3} D_j V(t, t_1^{(k)}, t_2^{(k)}, \mu_k, \nu_k), \quad \sum_{v=1}^{N_6} H_v V(t, t_1^{(v)}, t_2^{(v)}, \mu_v, \nu_v)$$

$D_k$ and $H_v$ can depend on parameters characterizing the kind of the non-conventional weapon.

The general system of model equations becomes

$$\frac{dB}{dt} = F(B, R; b, r) + \sum_{i=1}^{N_1} B_i V(t, t_1^{(i)}, t_2^{(i)}, \mu_i, \nu_i) -$$

$$- \sum_{j=1}^{N_2} C_j V(t, t_1^{(j)}, t_2^{(j)}, \mu_j, \nu_j) - \sum_{k=1}^{N_3} D_j V(t, t_1^{(k)}, t_2^{(k)}, \mu_k, \nu_k)$$

(4) $$\frac{dR}{dt} = G(B, R; b, r) + \sum_{p=1}^{N_4} R_p V(t, t_1^{(p)}, t_2^{(p)}, \mu_p, \nu_p) -$$
\begin{equation}
- \sum_{q=1}^{N_5} E_j V(t, t_1^{(q)}, t_2^{(q)}, \mu_q, \nu_q) - \sum_{v=1}^{N_6} H_j V(t, t_1^{(v)}, t_2^{(v)}, \mu_v, \nu_v)
\end{equation}

3 Epidemics, reserves, non-conventional weapons

3.1 Epidemics in a position conflict

Let us assume that an epidemics situation exists for the Blue group from time \( t_1 \) to the time \( t_2 \). The system of model equations become

\begin{equation}
\frac{dB}{dt} = -rR - CV(t, t_1, t_2, \mu, \nu), \quad \frac{dR}{dt} = -bR
\end{equation}

where we assume that the amplitude \( C \) connected to the epidemics is a constant. Figure 2 illustrates the danger of epidemics. We have a Blue group that at \( t = 0 \) is two times larger than the Red group and we have an epidemics that starts at \( t = 20 \) and begins to decrease its death toll at \( t = 80 \). At moderate amplitude of epidemics - Fig. 2a - the Blue group still wins but if the amplitude of epidemics is large and there are no effective countermeasures the two times larger Blue group loses the conflict as it can be seen in Fig. 2b.

Figure 2: Influence on an epidemics on the result of a position conflict. At \( t = 0 \) the Blue group is two times numerous as the Red group: \( B_0 = 200000, \quad R_0 = 100000 \). The epidemics starts at \( t = 20 \) and it begins do decay rapidly at \( t = 80 \). \( \mu = \nu = 3 \) and the firepower effectiveness of the two groups is the same \( b = r = 0.0025 \). Figure (a) : amplitude of disease \( C = 1000 \) - Blue group still wins. Figure (b): amplitude of disease \( C = 2000 \) - the Blue group losses the conflict.
### 3.2 Using reserves to counter an attack

Now let a five time larger Red group attacks the Blue group and the Blue group has some reserve members that can be used in the conflict. We shall consider here the case of including all reserves at once. The system of equations becomes

\[ \frac{dR}{dt} = -bRB, \quad \frac{dB}{dt} = -rRB + B_1 V(t, t_1, t_2, \mu, \nu) \]

The result from the attack depends on the quantity and the speed of introduction of the reserves of the Blue group. Several outcomes are presented in Fig. 3. If the reserves are enough but are not introduced fast enough the attack of the Red group is successful. If the reserves are introduced fast enough the Blue group stops the attack and wins. These observations confirm the existence of thresholds in the nonlinear systems. If the change of the system is below the threshold the system eliminates the perturbation (as in Fig. 3a). If the perturbation is large enough however (over the threshold) the system state can be changed (as in Fig. 3c).

### 3.3 Nuclear strike in course of a position conflict

Now let a very large Red group fights much smaller Blue group. Let the Blue group has no alternative of use of non-conventional weapons and let it performs a nuclear strike on the Red group. The system of model equations is

\[ \frac{dB}{dt} = -rR, \quad \frac{dR}{dt} = -bB - H_1 V(t, t_1, t_2, \mu, \nu) \]

Results from this scenario are shown in Fig. 4. Here again we demonstrate existence of a threshold. If the nuclear strike is not massive enough (if the intensity of the strike is below the threshold) there is no result - the larger group wins. But if the strike is massive enough the situation changes and the smaller Blue group wins.

### 4 Concluding remarks

More than 500 years ago Nicolo Machiavelli wrote that the people start a war at will but end it when they can. In other words it is important to know the consequences of the possible scenarios of a conflict. This can be achieved even on the basis of simple mathematical models. May be the most important conclusion from our results is that the armed group which defend a territory must not be reduced too much. If this happens then it is extremely difficult
Figure 3: Influence of reserves on an attack. Red group of 500 000 man attacks Blue group of 100 000 man. $b = 10^{-6}$, $r = 5 \cdot 10^{-7}$. Blue group introduces reserves from $t = 10$ till $t = 20$. $\mu = \nu = 1$. Figure (a): amplitude $B_1 = 10^4$. The Red army group. Figure (b): amplitude $B_1 = 1.7 \cdot 10^7$. The Blue group stops the attack. Figure (c): amplitude $B_1 = 2.5 \cdot 10^4$. Disastrous defeat for the Red group.

and costly to fight more numerous enemy. Another conclusion is that if the dynamics of the armed conflict become nonlinear then the accounting for the thresholds is crucial for the success. For example the inclusion of reserves must be massive or the nuclear strike must be large enough. Otherwise the conflict can be lost.

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Figure 4: Influence on nuclear strike on position conflict. Red group of 900,000 man attacks Blue group of 100,000 man. $\mu = \nu = 5$. The strike is between $t = 5$ and $t = 6$. $b = 10^{-2}$, $r = 2 \cdot 10^{-3}$. Figure (a): $H_1 = 5 \cdot 10^5$. Insufficient strength of the strike. The Red group wins. Figure (b): $H_1 = 8 \cdot 10^5$. The strength of the nuclear strike is sufficient. The Blue group wins.

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