Investigation on Multiple Algorithms for Multi-Objective Optimization of Gear Box

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Abstract. The field of gear design is an extremely important area in engineering. In this work a spur gear reduction unit is considered. A review of relevant literatures in the area of gear design indicates that compact design of gearbox involves a complicated engineering analysis. This work deals with the simultaneous optimization of the power and dimensions of a gearbox, which are of conflicting nature. The focus is on developing a design space which is based on module, pinion teeth and face-width by using MATLAB. The feasible points are obtained through different multi-objective algorithms using various constraints obtained from different novel literatures. Attention has been devoted in various novel constraints like critical scoring criterion number, flash temperature, minimum film thickness, involute interference and contact ratio. The output from various algorithms like genetic algorithm, fmincon (constrained nonlinear minimization), NSGA-II etc. are compared to generate the best result. Hence, this is a much more precise approach for obtaining practical values of the module, pinion teeth and face-width for a minimum centre distance and a maximum power transmission for any given material.

1. Introduction

Mechanical design includes an optimization process in which designers consider certain objectives and constraints. Design of gear box is an art as well as a science of engineering. Designers, based on the principles and their knowledge about the gears, lay out a gear for a particular application. The community of engineers know that the principles of engineering alone cannot suggest a good design. It is the designer’s expertise suggests good design. The problem with the conventional design procedures is that it gives out only a single solution. Optimization is the act of obtaining the best result for the given circumstances. However, design optimization of a complete mechanical assembly leads to a complicated objective function with a large number of design variables. So, it is a better practice to apply optimization techniques on individual components or intermediate assemblies rather than a complete assembly.

Traditional gear design involves computations based on tooth bending strength, tooth surface durability, tooth surface fatigue, interference, efficiency, and so on. Gear design involves a number of empirical formulas, various graphs and tables which lead to a complicated design procedure. Manual design is very difficult considering the above facts and thus, there is a need for the computer aided design for gears. With the help of computers, iterative designing can be performed. Also, the design
variables which satisfy the given conditions can be determined. As the optimization problem involves objective functions and constraints that are not stated as explicit functions of the design variables, it is hard to solve it through classical optimization methods. Moreover, the increasing demand for compact, efficient, and reliable gears forces the designers to use optimal design methodology.

The design of gears is a highly complicated task. The need to develop light weight, quiet and more reliable gear designs has resulted in a variety of changes in the design process. Here, mathematical programming techniques, generally known as numerical optimization methods, are investigated to provide a reliable design methodology for gears. These techniques offer a logical approach to design automation. It also handles a wide variety of design variables and constraints which are difficult to visualize using graphical methods. Design optimization of spur gears is to reduce the size and weight, and increase the power transmission capacity of the gear. As a part of this study, gear optimization programs have been developed for spur gears. The programs evaluate a wide variety of functions required by the optimizer. Typical design variables include the number of pinion teeth, module and face width. Design objectives are maximum power and minimum centre distance. Constraints may be imposed on flash temperature, film thickness, involute interference, scoring and contact ratio.

The problem is solved by following two ways:
(i) Using optimization toolbox of MATLAB,
(ii) Using code developed for multi-objective optimization (NSGA-II) technique.

2. Algorithms for Optimization

2.1. Genetic Algorithm

Genetic engineering is a field of tremendous growth. It is applicable to a variety of areas. People are more interested in this field because of its possibility to build up the next generation of easier and faster algorithms. They are also being used in multi-disciplinary design methods for optimal design. Genetic algorithm (GA) is a set of evolutionary algorithms, and non-deterministic and random search methods. It is an adaptive search optimization algorithm. It is based on the principles of natural selection and natural genetics. It utilizes the theories of evolution and natural selection in order to solve a problem within a complex solution space. In the process of evolution, reproduction concentrates on high fitness individuals by exploiting the available fitness information. Hence, each individual in the population receives a measure of its fitness in the environment.

Genetic Algorithm maintains a population of structures that are evolved upon the rules of selection and other operations generally known as ‘search operators’ like recombination and mutation. Recombination and mutation disturb those individuals providing general trial and error method for exploration. GA begins with a population of randomly generated strings that represents the problem and there possible solutions. Thereafter, each of these strings is evaluated to find its fitness. If a satisfactory solution, based on the acceptability or search stoppage criterion exists, the search is terminated. If not, the initial population is subjected to genetic evolution to procreate the next generation of candidate solutions. The genetic method of procreation uses the population as the input. The members of the population are processed by the four main GA operators – reproduction, crossover, mutation and inversion – to create the progenies for the next generation of candidate solutions. The resultant progenies are then evaluated and tested for termination.

2.2. NSGA-II

In this case, the problem is considered as a multi-objective problem. So, both objectives are treated together. In general, in case of multi-objective optimization, the objectives are conflicting. So, a single solution cannot be accepted as the best solution. Instead, a set of solutions is obtained which are better than the other solutions in terms of both objectives which are called Pareto optimal solutions. Since evolutionary algorithms are population based, they are the natural choice for solving this kind of
problem. In NSGA-II, the iterative procedure starts from an arbitrary population of solutions and gradually the algorithm converges to a population of solutions lying on the Pareto optimal front with higher diversity. The operators applied are the same as those of GA, namely, selection, crossover, and mutation. The tournament selection operator is applied which also takes care of constraints. However, in case of multi-objective optimization, additional task is to obtain solutions, which are as diverse as possible. For that, the sharing function approach is used. Crossover and mutation operators are applied as usual. The major problem with the inbuilt ‘fmincon’ function of MATLAB is that it considers all the variables real. As a result, one has to round the optimum value of integer variable to the nearest integer. However, GA can deal with both types of variables, integer and real, very easily by choosing appropriate string length.

2.3. Fmincon
Fmincon finds the minimum of a constrained non-linear multivariable function. It is based on a sequential quadratic programming. It finds a constrained minimum of a scalar function of several variables starting from an initial estimate. This is commonly known as constrained non-linear optimization. Fmincon may only give local solutions. When the problem is infeasible, fmincon attempts to minimize the maximum constraint value. The objective functions and the constraint functions must both be continuous and real-valued, that is why they cannot return complex values.

3. Design optimization
In this optimization problem for spur gear design, objective functions can be stated as:
Maximize: Power transmission
Minimize: Centre distance

Subjects to limit on
1) Flash temperature
2) Specific film thickness
3) Scoring criterion
4) Contact ratio
5) Involute interference

4. Design variables
The selection of the independent design variables, that are necessary to characterize the design of the system, is the key element in formulating an optimization problem. Generally, it is good to choose those variables that have a significant impact on the objective function. Based on this criterion design variables are chosen to be the number of pinion teeth (z1), module (m) and face-width (b).

5. Objective function
5.1. Centre distance
In general, the most desirable gear set is the smallest one that will perform the required job. Because, smaller gears are less expensive, easier to manufacture, requires less material and needs only less space to operate.

\[ C = m \times z_1 \times (1 + G) / 2 \]

m = module
\( z_1 = \) no. of teeth on pinion
G = gear ratio

5.2. Power
It is desirable to increase the power output.
\[
P = \frac{F_1 \cdot m \cdot z_1 + 2 \cdot \pi \cdot N_1}{2 \cdot 60}
\]

\( F_1 \) = Tangential load (N)
\( N_1 \) = RPM of pinion

6. Constrains

6.1. Flash temperature

Flash temperature should be within the limit, since the oil properties change when it exceeds the temperature limit.

\[
T_f = T_b + \frac{C_t \cdot f \cdot F_t \cdot (V_1 - V_2)}{b \cdot (\sqrt{V_1} + \sqrt{V_2}) \cos \theta}
\]

Where,

- \( T_b \) = Temperature of blank surface in contact zone (°F)
- \( C_t \) = Material constant for conductivity, density and specific heat (0.0528 for straight petroleum oil)
- \( f \) = Coefficient of friction
- \( F_t \) = Tangential driving load (lb)
- \( b \) = Face width (inches)
- \( V_1 \) = Rolling velocity of pinion at point of contact tips (fps)
- \( V_2 \) = Rolling velocity of gear at point of contact tips (fps)
- \( V_s = (V_1 - V_2) \) = Sliding velocity (fps)
- \( F_e \) = Face width in contact (in)
- \( B \) = Width of band of contact (in)
- \( \Theta \) = Pressure angle
- \( C \) = Centre distance in inches

\[
B = 0.00054 \sqrt{\frac{W_t \cdot \rho_1 \cdot \rho_2}{F_e \cdot (\rho_p + \rho_g) \cos \theta}}
\]

\[
V_1 = \frac{n_p \cdot \pi \cdot \rho_1}{360}
\]

\[
V_2 = \frac{n_p \cdot \pi \cdot \rho_2}{360}
\]

\( \rho_1 = \rho_p \) and \( \rho_2 = \rho_g - z \);

\( \rho_p = \sqrt{r_0^2 - (r \cos \theta)^2} \)

\( \rho_g = \sqrt{R_0^2 - (R \cos \theta)^2} \)

Length of the line of action,

\( z = \rho_p + \rho_g - C \sin \Theta \)

6.2. Specific film thickness:

To avoid scoring failure, specific film thickness > 1

Specific film thickness,

\[
\lambda = \frac{h_{\text{min}}}{\sigma}
\]

Where,

\[
\sigma = \sqrt{\sigma_p^2 + \sigma_g^2}
\]
σ_p = Average roughness of pinion (rpm)
σ_g = Average roughness of gear (rpm)
h_{min} = Minimum oil film thickness (in)

Minimum elastohydrodynamic (EHD) film thickness,

\[ h_{min} = \frac{2.65 \alpha^{0.54} (V_0^* u)^{0.7} e^{-0.03 R^0.43}}{W^{0.13}} \]

Where,
\( \alpha = \) Pressure viscosity coefficient
\( V_0 = \) Absolute velocity
\( W = W/b = \) Specific loading,
\( \frac{1}{E} = \frac{1}{2} \left[ \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right] \)
\( u = \frac{1}{2} [u_1 + u_2] \)

\( u_1 \) and \( u_2 \) are the rolling velocity of pinion and gear at the point of contact.

\[ u_1 = \frac{\pi n_p}{30} \rho_1 \]
\[ u_1 = \frac{\pi n_g}{30} \rho_2 \]
\[ R = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \]

6.3. Scoring criterion number
To avoid scoring failure,

Scoring criterion number < Critical scoring number.

Scoring criterion number = \[ ((F_t/b)^{3/2})/[(n_p)^{1/2}/(P_d)^{1/4}] \]

Where,
\( F_t = \) Tangential driving load (lb)
\( b = \) Face width (in)
\( n_p = \) Speed of pinion (rpm)
\( P_d = \) Diametric pitch

6.4. Contact ratio
For standard spur gear, contact ratio should be greater than 1.4 and normally less than 2.
i.e., \( 1.4 < x < 2 \)

\[ m_p = \frac{\left( r_1 a_1 - r_2 d_1 + r_2 a_2 - c \sin \theta \right) p}{\pi \cos \theta} \]

Where,
\( r_{a1} = r + (1/P_d) \)
\( r_{a2} = R + (1/P_d) \)
\( r_{b1} = r \cos \Theta \)
\( r_{b2} = R \cos \Theta \)
6.5. **Involute interference**

Involute interference occurs when the pinion tooth makes contact with the gear tooth before the involute portion of the pinion comes within the range i.e., when the contact occurs below the base circle of the pinion on the non-involute portion of the flank.

For no involute interference,

\[
\text{Csin} \theta > \sqrt{\frac{r_{a2}^2}{r_{b2}^2} - r_{a2}^2}
\]

7. **Formulation of problem**

**Inputs:**
- Permissible bending stress = 191.6Mpa
- Permissible contact stress = 455Mpa
- Pinion RPM (N1) = 960
- Gear RPM (N2) = 320

Design vector: \( x = (m, z_1, b) \)

\( 3 \leq m \leq 10 \)
\( 18 \leq z_1 \leq 24 \)
\( 30 \leq b \leq 70 \) (mm)

**Objective function to be minimized:**

\[
F(1) = m\times z_1\times (1+G)/2
\]

**Objective function to be maximized:**

\[
F(2) = \frac{F_t\times m\times z_1\times 2\times \pi \times N_1}{2\times 60}
\]

**Constrains to be satisfied:**

- C (1): Flash temperature \( \leq 176 \)
- C (2): 1 \( \leq \) specific film thickness \( \leq 10 \)
- C (3): Scoring criterion number \( \leq 8000 \)
- C (4): 1.4 \( \leq \) contact ratio \( \leq 2 \)
- C (5): For no involute interference,

\[
\text{Csin} \theta > \sqrt{\frac{r_{a2}^2}{r_{b2}^2} - r_{a2}^2}
\]

8. **Results and Discussion**

| Table 1. Comparison of outputs from Algorithms. |
|-----------------------------------------------|
| **Variables** | **GA1** | **GA2** | **Fmincon** | **NSGA-II** |
|----------------|---------|---------|-------------|--------------|
| Module         | 6.061   | 6.853   | 6.441       | 7.642        |
| Number of Pinion Teeth | 18.91   | 23.35   | 18.00       | 19.87        |
| Face width(mm) | 70.00   | 66.89   | 66.84       | 69.99        |
| Power(kW)      | 24.32   | 46.49   | 23.69       | 43.65        |
| Centre distance (mm) | 229.5   | 320.2   | 231.7       | 304.0        |

GA1 – single objective genetic algorithm.
GA2 – multi objective genetic algorithm.
Fig 1 and fig 2 shows the pareto-front obtained for NSGA II and multi-objective genetic algorithm. X-axis represents reciprocal of power and Y-axis represents centre distance. Since both the algorithms are designed for minimization problems and our objective is to maximize power, we took reciprocal of power instead of power. By taking reciprocal of power in X-axis, we got perfect minimization curve as desired. From the pareto-front obtained from both algorithms, we found the utopia point, which gives the best-optimized result for our problem. Utopia point is the point, which is nearer to origin in the minimization curve obtained for the given problem. From the utopia point in the NSGA II graph, we got the best value of power as 43.469KW and centre distance as 304 mm. From the utopia point in GA multi-objective, we got value of power as 46.49 KW and centre distance of 320 mm.

9. Conclusion

The single-speed spur gear design optimization problem considered in this paper provides an adequate challenge to almost every aspect of optimization. The design variables for the multi-objective optimization are module, no of teeth and face width of pinion. Objective functions include reducing the center distance and maximizing the power transmitting capacity of the gear box. Both the objective functions are conflicting in nature, which creates challenge to designer to choose the values of design variables. Various novel constraints like flash temperature, minimum film thickness, scoring criterion number, involute interference and contact ration are used to find the optimum solution. In the present study we use algorithms like genetic (single and multi-objective), NSGA II and fmincon for obtaining optimized results.

By comparing four algorithms for the optimization process, we found that multi-objective Genetic algorithm provides the optimized results. This provides the maximum power output with an optimized centre distance for the gears. GA multi-objective provides 49% increase in power when compared to fmincon and 6% increase in power when compared to NSGA II. fmincon algorithm depends on starting point, and thus the results depend on different starting points that are given by the user. Even though the centre distance provided by single objective genetic algorithm is minimal, it shows a remarkable loss in power output.
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