Reworking the Tucson-Melbourne Three-Nucleon Potential *

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Abstract. We introduce new values of the strength constants (i.e., $a$, $b$, $c$, and $d$ coefficients) of the Tucson-Melbourne (TM) $2\pi$ exchange three nucleon potential. The new values come from contemporary dispersion relation analyses of meson factory $\pi N$ scattering data. We make variational Monte Carlo calculations of the triton with the original and updated three-body forces to study the effects of this update. We remove a short-range $\pi$-range part of the potential due to the $c$ coefficient and discuss the effect on the triton binding energy.

1 Introduction

The Tucson-Melbourne (TM) three-nucleon force due to two-pion exchange has a structure which, after an expansion of the invariant $\pi N$ amplitudes in the inverse nucleon mass, was determined by the original implementation of chiral symmetry in the underlying $\pi N$ scattering amplitude. Given that structure, the strength constants (the $a$, $b$, $c$, and $d$ coefficients) are then not free parameters but depend upon the $\pi N$ scattering data base, which has improved greatly since the original determination of these coefficients. In this note, we review two recent developments in three-body force studies: i) a critical analysis of the generic structure of a $2\pi$ exchange three-body force (TBF) [1], and ii) the new TM strength constants derived from invariant $\pi N$ amplitudes [2] corresponding to the contemporary data base which includes measurements taken at the meson factories since 1980. We make updated TBF’s of the Tucson-Melbourne type which reflect one or both developments, add them to a NN force, and calculate properties of the triton in order to see the effect of these developments in a simple nuclear system.

To begin, we display the Tucson-Melbourne force (leaving out an overall momentum conserving delta function):

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*Dedicated to the 60th birthday of Walter Glöckle.*
\[ \langle p'_1 p'_2 p'_3 | W_{\pi\pi}(3) | p_1 p_2 p_3 \rangle = \]
\[ (2\pi)^3 \frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q')}{(q^2 + \mu^2)(q'^2 + \mu^2)} \frac{q^2}{4m^2} F_{\pi N N}^2(q^2) F_{\pi N N}^2(q'^2) \]
\[ \left\{ (\tau_1 \cdot \tau_2) \left[ a + bq \cdot q' + c(q^2 + q'^2) \right] + (i\tau_3 \cdot \tau_1 \times \tau_2) d(i\sigma_3 \cdot q \times q') \right\}, \]
\[ (1) \]

where \( q = p_2 - p'_2 \) and \( q' = p'_3 - p_3 \) and the pion rescatters from nucleon 3. (We refer the reader to Refs. [3, 4] for diagrams, more extensive definitions, explanations of the other two cyclic terms, etc. needed for calculation but not directly relevant to the present discussion). Now we review briefly the origin of this equation.

The approach used in the Tucson-Melbourne (TM) family of forces is based upon applying the Ward identities of current algebra to axial-vector nucleon scattering. The Ward identities are saturated with nucleon and \( \Delta (1230) \) poles. Then employing PCAC (partial conservation of the axial-vector current), one can derive expressions for the on-mass-shell pion-nucleon scattering amplitudes \[ \] which map out satisfactorily the empirical coefficients of the Hohler subthreshold crossing symmetric expansion based on dispersion relations \[ \] and, after projection onto partial waves, describe the phase shifts reasonably well \[ \]. The off-mass-shell extrapolation (needed for the exchange of virtual, spacelike pions in a nuclear force diagram) is trivial for the \( d \) coefficient. It can be taken directly from the on-mass-shell theoretical or empirical amplitude \( \bar{B}^- \) since they coincide so closely (see Appendix A of Ref. [8]). One can treat this coefficient more elaborately \[ \], but the result is the same. On the other hand, one really needs an off-shell \( \pi N \) amplitude for the important \( a, b, c \) structure of Eq. (1). This structure relies on the fact that the off-pion-mass-shell amplitude \( \bar{F}^+ \) can be written in a form which depends on measured on-shell amplitudes only. This rewriting of the PCAC/current algebra amplitude exploits a convenient correspondence between the structure of the terms corresponding to spontaneously broken chiral symmetry and the structure of the model \( \Delta \) term. To see this, we note that the nonspin flip \( t \)-channel isospin even amplitude (covariant nucleon pole term removed) is

\[ \bar{F}^+(\nu, t, q^2, q'^2) = f(\nu, t, q^2, q'^2) \frac{\sigma}{f^2_\pi} + C^+(\nu, t, q^2, q'^2) \]
\[ (2) \]

where \( \sigma \) is the pion-nucleon \( \sigma \) term, \( f_\pi \approx 93 \text{ MeV} \), and the invariant amplitude \( \bar{F}^+(\nu, t) \) is given in units of the charged pion mass (139.6 MeV). The double divergence \( q' \cdot M^+ \cdot q / f^2_\pi \) of the background axial vector amplitude denoted by \( C^+ \) contains the higher order \( \Delta \) isobar contribution. In general, \( C^+ \) must have the simple form \[ \]

\[ C^+(\nu, t, q^2, q'^2) = c_1 \nu^2 + c_2 q \cdot q' + O(q^4). \]
\[ (3) \]
On the other hand, the assumed form of the function $f$,

$$f(\nu, t, q^2, q'^2) = (1 - \beta)\left(\frac{q^2 + q'^2}{m_{\pi^+}^2} - 1\right) + \beta\left(\frac{t}{m_{\pi^+}^2} - 1\right)$$

(adapted [8, 9] for $\pi N$ scattering from the $SU(3)$ generalization of the Weinberg low energy expansion for $\pi \pi$ scattering) is such that $\bar{F}^+$ satisfies soft pion theorems (for a review see Ref. [9]), and (with the aid of Eq. (3)) the constraint at the (on-shell and measurable) Cheng-Dashen point:

$$\bar{F}^+(0, 2m_{\pi^+}^2, m_{\pi^+}^2) = \frac{\sigma}{f_{\pi^+}} + \mathcal{O}(q^4).$$

The value of $\beta$ can be determined by taking the amplitude on-shell and comparing with on-shell data extrapolated into the subthreshold region [5, 9], but it is not needed, as we will now demonstrate.

Neglecting the $\nu^2$ and $g_0$ terms in (3) because they are of the order of $(m_{\pi^+}/m)^2$ or higher, the $\bar{F}^+$ amplitude can be expanded in the three-vector pion momenta $q$ and $q'$ as follows:

$$\bar{F}^+(0, t, q^2, q'^2) = -\frac{\sigma}{f_{\pi^+}} + \frac{\sigma}{f_{\pi^+}} \frac{2\beta}{m_{\pi^+}^2} - c_2)q \cdot q' - \frac{\sigma}{m_{\pi^+}^2 f_{\pi^+}} (q^2 + q'^2)$$

The last equation explicitly exhibits the separation between the (higher order in $q^2$) $\Delta$ contribution — contained in the $c_2$ term alone — and the remaining chiral symmetry breaking terms. In ref. [8] and subsequent discussions of the TM $\pi - \pi$ force, the $c_2$ and $\beta$ constants in the coefficient of the $q \cdot q'$ term were eliminated in favor of the on-shell (measurable) quantity

$$\bar{F}^+(0, m_{\pi^+}^2, m_{\pi^+}^2, m_{\pi^+}^2)$$

$$= \left(1 - \beta\right)\frac{\sigma}{f_{\pi^+}} + \frac{c_2 m_{\pi^+}^2}{2}$$

From the expanded $\pi N$ amplitude $\bar{F}^+$ in conjunction with the $\pi NN$ vertices $F_{\pi NN}(q^2)$ and pion propagators, one constructs the three body force of Eq. (1). Comparing Eqs. (1) and (4), $(W \sim T$ and $(S - 1) = -iT$ so that $T = -F)$ one sees that

$$a = +\frac{\sigma}{f_{\pi^+}}.$$ 

The $\Delta$ constant $c_2$ contributes then to the overall coefficient “b” that has been used in nuclear calculations ($b = b_\sigma + b_\Delta$; $b_\Delta = c_2$)

$$b = -\frac{\sigma}{f_{\pi^+}^2} \frac{2\beta}{m_{\pi^+}^2} + c_2 = -\frac{2}{m_{\pi^+}^2} \left[\frac{\sigma}{f_{\pi^+}^2} - \bar{F}^+(0, m_{\pi^+}^2, m_{\pi^+}^2, m_{\pi^+}^2)\right]$$

Finally the $c$-term of Eq. (1) is given by

$$c = \frac{\sigma}{m_{\pi^+}^2 f_{\pi^+}^2} - \frac{g^2}{4m^3} + F_{\pi NN}(0) \frac{\sigma}{f_{\pi^+}^2}$$
The dominant part of $c$ comes from our ansatz Eq. (4) but a small part is due to the backward-propagating nucleon term $F_Z^+$ (“Z-graph”) $F_Z^+ = \frac{q^2}{4m^3}(q^2 + q'^2)$. This term (which also appears in the $d$ coefficient) is representation dependent and is the only local term of a consistent set of 15 terms derived some time ago [10]. We note that the term proportional to $F'_\pi NN(0)$ did not appear before in Eq. (6). This term nevertheless is inserted in $c$ because both the backward-propagating part of the nucleon pole $F_Z^+$ and the $\Delta$ couple with the pion with a (assumed the same) form factor $F_{\pi NN}(q^2)$ which is defined as $g(q^2) = gF_{\pi NN}(q^2)$. The chiral breaking $\sigma$ term has no intrinsic $q^2$ dependence (although it is multiplied by $f(\nu, t, q^2, q'^2)$). It is convenient, if not necessary, however, since part of the amplitude is due to $F_Z^+$ and $C^+$, to multiply the final amplitude by form factors, dependent upon $q^2$ and $q'^2$. Consequently, the constant term ($\sigma/f_\pi^2$, labeled “a” in the literature) attains a spurious momentum dependence from the form factors. The term proportional to $F'_{\pi NN}(0)$ in Eq. (10) is inserted to correct for this spurious momentum dependence to the orders in $q^2$ and $q'^2$ kept in the amplitude.

The new development in the structure of a $2\pi$ exchange TBF [1] lies in another look at the decomposition of the $c$-term made originally [8] to Fourier transform Eq. (1), but true in general. Begin with the schematic structure

$$W(3)|_c \propto \frac{1}{q^2 + \mu^2} \frac{1}{q'^2 + \mu^2} (q^2 + q'^2) \tau_1 \cdot \tau_2$$

and rewrite it (neglecting the isospin dependence in Eq. (11)) as

$$W(3)|_c \propto \frac{q^2}{q^2 + \mu^2} \frac{1}{q'^2 + \mu^2} + (q \leftrightarrow q')$$

$$= \frac{q^2 + \mu^2 - \mu^2}{q^2 + \mu^2} \frac{1}{q'^2 + \mu^2} + (q \leftrightarrow q')$$

$$= \left(1 - \frac{\mu^2}{SR} \right) \frac{1}{q'^2 + \mu^2} + (q \leftrightarrow q')$$

Thus the $c$-term can be decomposed into a $2\pi$-exchange term with the same operator structure as the $a$-term plus a short-range – $\pi$-range term. Without a form factor $F_{\pi NN}(q^2)$ the short-range part would be a Dirac delta function–a zero-range or contact term. This operator structure is reflected in the coordinate space representations where one always finds the coefficient $a - 2\mu^2 c$ multiplying derivatives of two “coordinate space Yukawas” see, for example, Eqs. 3.9-3.11 of Ref. [8] or Appendix A of Ref. [10]. Without a form factor $F_{\pi NN}(q^2)$ the short-range part would be a Dirac delta function–a zero-range or contact term. The Tucson-Melbourne force has an (unadorned by $\mu^2$) $c$ coefficient multiplying a derivative of a product of a delta function and a “coordinate space Yukawa” as is easily seen in the same equations.
It was the latter, rather singular, aspect of the Tucson-Melbourne force which made numerical work difficult in both coordinate space and momentum space (the operator structure is the same). In addition, the recent trend toward a low mass cutoff $\Lambda$ in
\[
F_{\pi NN}(q^2) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + q^2}
\] (13)
for pion exchange highlights the point already emphasized by the Hokkaido group [12] and, in the modern context, by the São Paulo group [13]. The contact terms (those proportional to a coordinate-space $\delta$-function and its derivatives) are spread out with increasing importance as $\Lambda$ becomes smaller and the (strong interaction) size of the nucleon grows. These groups contended that these contact terms, bringing the nucleon structure signature, should not be included in potential models.

The subject of contact terms has been revived recently with the advent of effective field theories in which contact terms are used to emulate the short distance physics, and the long distance physics, including the physics of chiral symmetry, is retained explicitly. In these effective field theories (chiral perturbation theory extended to two or more nucleons [14], contact terms abound, both in the chosen chiral Lagrangian and in the nucleon potentials. Adapting a field redefinition technique first used in pion condensation [15], Friar et al. [1] were able to demonstrate, via a field theoretic calculation with an effective chiral Lagrangian, why the contact term of Eq. (12) does not appear in the $2\pi$-three body force of chiral perturbation theory, even though that field theory can be transformed to emulate the soft pion theorems. In sum, although chiral symmetry in the form of PCAC/current algebra motivated the ansatz Eq. (4) which led to the operator structure of Eq. (12), chiral symmetry in the form of effective field theory dictates that only the $2\pi$-exchange part ($\propto a - 2\mu^2c$ for TM) should be retained in a TBF from pion exchange. One moral which can be drawn from this new insight is that chiral constraints on the off-shell scattering amplitude are not enough to determine a three-nucleon force; one must also satisfy chiral constraints on the on-shell three-nucleon $S$-matrix elements which are presumed to make up the force. This observation applies to other off-shell amplitudes embedded in nuclear force models [14].

The removal of the spurious contact term from the Tucson-Melbourne force leaves a TBF with coefficients $a' = a - 2\mu^2c$, $b$, and $d$ which has been termed TM$'$ in Ref. [1] and subsequent works. In the following section we will examine the effects in the triton of the original TM TBF and the TM$'$ TBF. We consider TM and TM$'$ with the original strength constants and with strength constants from the current $\pi N$ scattering data.

2 Numerical Results

We employ a variational Monte Carlo method developed for accurate numerical calculations of light nuclei [11]. The “Urbana-type potentials”, suited to this variational approach, take the form of a sum of operators multiplied by
functions of the interparticle distance. Following our previous study of charge symmetry breaking in light hypernuclei \[17\], and in order to compare with other TBF studies \[13, 20\], we use the Reid soft core nucleon-nucleon potential in the form of the Urbana-type Reid \(v_8\) potential \[21\]. The Reid \(v_8\) is a simplified (the sum of operators is truncated from a possible 18 \[22\] to 8 operators) NN force model which is equivalent to the original Reid soft core nucleon-nucleon potential in the lower partial waves and can produce the dominant correlations in s-shell nuclei. To be specific, the Reid \(v_8\) is obtained from the Reid soft core (RSC) potential in the singlet states \(^1S_0\) and \(^1P_1\) and the triplet states \(^3S_1 -^3D_1\) and \(^3P_2 -^3F_2\). The binding energy of the triton, calculated with exact Faddeev codes which include all partial waves \(j \leq 4\) (34 channels), is -7.59 MeV for the Reid \(v_8\) (as quoted in Table IV of \[23\]), to be compared with -7.35 MeV obtained with the original RSC \[19\]. This small discrepancy, presumably due to differences in the \(P\)-waves of the two potentials, should not affect our conclusions.

The variational method we use, with Monte Carlo evaluations of the integrals, is described in Ref. \[24\] (see also, Ref. \[25\]). Here we specify only the differences from the equations in these references. In particular, the trial nuclear wave functions have the following structure:

\[
\Psi = S \left( \prod_{i<j<k}^A f_{ijk} \right) S \left( \prod_{i<j}^A f_{ij} \right) \Phi, \tag{14}
\]

where \(\Phi\) is an antisymmetric spin-isospin state, having appropriate values of total spin and isospin, with no spatial dependence, and \(S\) is a symmetrization operator which makes 3! terms for the two-body correlation operator \(f_{ij}\) and one term for the three-body correlation operator \(f_{ijk}\). The NN correlation operator is

\[
f_{ij} = f_{ij}^c + f_{ij}^\tau \cdot \tau_i \cdot \tau_j + f_{ij}^{\sigma_i \cdot \sigma_j} + f_{ij}^{\sigma_i \cdot \tau_i \cdot \tau_j} + f_{ij}^l S_{ij} + f_{ij}^t \cdot S_{ij} \cdot \tau_i \cdot \tau_j, \tag{15}
\]

and the triplet correlation induced by the three-body force has the usual linear form suggested by the first order perturbation theory \[20\]

\[
f_{ijk} = 1 + \beta V_{ijk}, \tag{16}
\]

where \(\beta\) is a variational parameter. These pair correlations do not include the spin-orbit correlations described in Ref. \[24\], nor do our triplet correlations include the more sophisticated three-body correlations introduced by Arriaga et al. \[22\] which reduce the difference between the variational upper bound and the Faddeev binding energy of the triton to less than 2%. Both improvements would be clearly desirable, but are beyond the scope of this preliminary investigation. We do, however, include the usual central three-body correlation \(f_3\) multiplied by the correlation functions \((f_{ij}^t, f_{ij}^{\sigma_i \cdot \sigma_j}, f_{ij}^{\sigma_i \cdot \tau_i \cdot \tau_j}, f_{ij}^l, f_{ij}^t)\):

\[
f_3 = \prod_{k \neq i,j}^3 \left[ 1 - t_1 \left( \frac{R_{ij}}{R} \right) t_2 \exp(-t_3 R) \right], \tag{17}
\]
with $R = r_{ij} + r_{ik} + r_{jk}$. With these correlations we get a binding energy of $-7.28(3)$ MeV with the Reid $v_8$ alone, a number which compares favorably with variational results in Table V of Ref. [25], obtained with a slightly different trial wave function. We now demonstrate that our variational calculations track the Faddeev results of Ref. [19] and suggest that the main outline of our results (to be presented later) will reflect the properties of the Hamiltonians chosen, provided that the potentials are not too singular.

The Faddeev calculations we now examine used the RSC potential and the early parameters (labeled TM(81) here) of the Tucson-Melbourne TBF ($m_{\pi^+}a = +1.13$, $m_{\pi^+}b = -2.58$, $m_{\pi^+}c = 1.00$, and $m_{\pi^+}d = -0.753$ in units of the charged pion mass: 139.6 MeV) obtained from an interior dispersion relation (IDR) analysis of phase shifts circa 1973 [27]. Ten years ago there was little reason to look suspiciously at the $c$-term, and the goal of the exercise was to test the perturbative nature of the $\pi N$ amplitude $s$-wave terms. To this end, a restricted model was chosen with $a = c = 0$, the Faddeev eigenvalues calculated for a 34-channel solution for RSC/TM, and the solution tested by employing the resulting wave functions in a Raleigh-Ritz variational calculation. The variational result for this restricted Hamiltonian coincided with the Faddeev eigenvalue, indicating the high quality of the Faddeev wave function. Then the $a$ and $c$ terms were selectively set to their assigned values and the variational calculation was repeated. Comparison of the results shows the non-perturbative role of the $a$ and the $c$ term on the triton wave function. To test our codes and to suggest that our methods can give insight into triton binding energy effects from the proposed redefinitions of the TM force, we made a parallel set of calculations with the Reid $v_8$ and the old TM force, TM(81), with the parameters given above. The results are shown in table 1.

| Hamiltonian expectation values for RSC plus TM variations indicated and Reid $v_8$ plus TM variations indicated. |
|---------------------------------------------------------------|
| RSC/TM(81) | Reid $v_8$/TM(81) |
| $-\langle H_{n=0,c=0} \rangle$ | 9.18 | 9.12 |
| $-\langle H_{c=0} \rangle$ | 9.07 | 8.99 |
| $-\langle H_{n=0} \rangle$ | 8.46 | 8.17 |
| $-\langle H \rangle$ | 8.35 | 8.04 |

From Table 1, we first note that our variational upper bounds are always above the corresponding Faddeev eigenvalues. The qualitative agreement of the first two lines hides a slight variation in the two-body potentials chosen. The Reid $v_8$ alone has a Faddeev eigenvalue of $E_T = -7.59$ MeV compared to the Faddeev eigenvalue of the RSC which is $E_T = -7.35$. Our variational calculation of the Reid $v_8$ alone yields $E_T = -7.28(3)$ MeV; (accidently) very close to the starting point of the Faddeev calculation. Thus, the starting Hamiltonians of slightly different NN potentials and the non-singular $b$ and $d$ terms of...
TM(81) give the rather similar results of the top row. Going down the columns, we see that the effect of the $a$-term alone is to decrease the total energy by 0.11 MeV (0.13 MeV) for the RSC/TM(81) calculation and the Reidv8/TM(81) calculation respectively. The $c$-term has the effect of decreasing the triton total energy by 0.72 (0.95) MeV in the two calculations. Both the Faddeev calculation and our present variational calculation are in qualitative agreement and nearly quantitative agreement for the model three-body forces which do not include the short-range $-\pi$-range TBF from the $c$-term. On the other hand, the third and fourth rows of Table 1 do include the short-range $-\pi$-range TBF from the $c$-term. They have a discrepancy of about 0.3 MeV (three times larger than that of the non-singular Hamiltonians) and the discrepancy would be even more if the NN potentials were the same. This comparison suggests that our variational wave function is adequate for qualitative conclusions about TBF’s of $\pi$-range $-\pi$-range, but that it cannot accurately evaluate the short-range $-\pi$-range TBF and a more sophisticated [23, 25] variational trial function, beyond the scope of this work, is needed for this nearly singular term.

Now we introduce the strength constants $a'$, $b$, and $d$ which follow from the employed on-mass-shell invariant amplitudes of $\pi N$ scattering. The invariant amplitudes $\bar{F}^+(\nu, t)$ and $\bar{B}^-(\nu, t)$ are given in units of the charged pion mass (139.6 MeV). The potentials, however, use an isospin formalism instead of charge states so it would seem natural to employ the SU(2) average pion mass $(2m_{\pi^+} + m_{\pi^0})/3 = 138$ MeV in the propagators and form factors and therefore to convert the quoted values used in Eqs. (8)–(10) to these units. The results are given in Table 2. There is a rather dramatic change in the isospin even coefficients between the top two rows labeled (93) and the bottom two rows labeled (99), reflecting the difference between the invariant amplitudes from pre-meson factory data [4] and the meson factory data [5]. The recent invariant amplitudes allow further tests of the PCAC/current algebra models which underlie the Tucson-Melbourne TBF. This aspect of the new data analyses is discussed in Ref. [9]; here we confine ourselves to reworking the Tucson-Melbourne TBF with the new on-shell numbers.

The rows of the Table labeled TM(93) and TM(99) are obtained by inserting

$$\bar{F}^+(0, m_{\pi^+}^2) = -0.28m_{\pi^+}^{-1} \quad \bar{F}^+(0, 2m_{\pi^+}^2) \approx \sigma/f_{\pi}^2 = 1.03m_{\pi^+}^{-1}$$

into Eqs. (8)–(10), and using the old value of $g^2 = 179.7$; all values obtained from the phase shift analysis known as KH80 [3]. One obtains $d = -4g^2/(3m_{\pi^+}^3) - B^-(0, 0)/2m$ taking $B^-(0, 0)$ either from the model [3] amplitude or the empirical amplitude. They are the recommended values available in 1993 [3]. Two new invariant amplitude analyses of the meson factory data base paramaterized as the SP98 $\pi N$ phase shifts are in good agreement [3] and we choose Ref. [2] for the TBF force models labeled TM(99) and TM′(99). The relevant on-shell numbers from that recent analysis are

$$\bar{F}^+(0, m_{\pi^+}^2) = -0.05 \pm 0.05m_{\pi^+}^{-1} \quad \bar{F}^+(0, 2m_{\pi^+}^2) \approx \sigma/f_{\pi}^2 = 1.40 \pm 0.25m_{\pi^+}^{-1}$$

(19)
with \( B^-(0,0) \approx 8.6m^2_{\pi} \), not much changed from Appendix A of Ref. [3]. The current value of \( g^2 = 172.1 \) was input into the interior dispersion relation analysis of Ref. [2] and is therefore used in TM(99) and TM'(99).

|        | \( \mu a' \) | \( \mu^3 b \) | \( \mu^3 c \) | \( \mu^3 d \) |
|--------|-------------|-------------|-------------|-------------|
| TM(81) | -0.84       | -2.49       | 0.98        | -0.72       |
| TM'(81)| -0.84       | -2.49       | 0           | -0.72       |
| TM(93) | -0.74       | -2.53       | 0.88        | -0.72       |
| TM'(93)| -0.74       | -2.53       | 0           | -0.72       |
| TM(99) | -1.12       | -2.80       | 1.25        | -0.75       |
| TM'(99)| -1.12       | -2.80       | 0           | -0.75       |

Table 2. Expansion coefficients of the Tucson-Melbourne \( \pi - \pi \) force for \( \Lambda = 5.8\mu \). Units of SU(2) average pion mass \( \mu = 138.0 \) MeV. The \( c \) coefficient multiplies a short-range \( \pi \)-range three-body force now known to be spurious.

We follow tradition and calculate the triton properties with the cutoff in the form factor \( \Lambda/\mu = (4.1, 5.8, 7.1) \). In the publications of the Tucson-Melbourne group \( \Lambda = 5.8\mu \) has been recommended to match the Goldberger-Treiman discrepancy [25], another measure of chiral symmetry breaking [9]. The value \( \Lambda = 7.1\mu \) matches the Goldberger-Treiman discrepancy \( \Delta = 0.02 \) of the recent determinations of the \( \pi \)NN coupling constant \( g^2 = 172.1 \) [9]. We don’t know the reason others have chosen \( \Lambda = 4.1\mu \) as a test case but adopt it anyway. Please notice from Eq. (10) that \( c \), and therefore \( a' \), changes with different values of \( \Lambda \). From Eq. (10), we see that \( \mu^2c = \sigma/f_{\pi^2}(1 + F_{\pi NN}(0)) - \mu^2g^2/4m^3 = \sigma/f_{\pi^2}(1 + \Delta(1 - \Delta)) - \mu^2g^2/4m^3 \approx \sigma/f_{\pi^2}(1 + \Delta) - \mu^2g^2/4m^3 \), because the value of \( \Delta = \mu^2/\Lambda^2 \) varies only between \( \Delta = 0.06 \) and \( \Delta = 0.02 \) as \( \Lambda/\mu = (4.1, 5.8, 7.1) \) in Eq. (13). Thus, the dependence of the value of \( a' \) with \( \Lambda/\mu \) is slight, compared with the overall effect of the cutoff on the \( \pi - \pi \) force.

The results of our calculations are presented in Figure 1 as the open circles and open squares. The plotted points include Monte Carlo error bars and the lines through the symbols are drawn to guide the eye. The open circles show the calculated triton binding energy with Reid \( v_8/TM'(93) \) which has no short-range \( \pi \)-range term and the strength constants taken from twenty year old \( \pi N \) scattering data. The open squares indicate the results with the same NN potential and the updated strength constants of TM'(99). Each calculation was made variationally with the full Hamiltonian with strength constants shown in Table 2. We indicate our calculated value of the binding energy of the triton with the Reid \( v_8 \) alone \( (E_T = -7.28(3) \text{ MeV}) \) by a horizontal (sparse) dotted line and the Faddeev eigenvalue \( (E_T = -7.59 \text{ MeV}) \) by the horizontal (dense) dotted line. Our variational upper bounds are always above the corresponding Faddeev eigenvalues.

We compare our results with calculations in the literature with the old
TBF TM(81), where the lack of a prime means that the short-range $-\pi$-range term is *included*. We do not present our own variational estimates with this short-range $-\pi$-range term included as they do not reflect the true situation (see discussion of Table 1). The results of the combination RSC/TM(81) for the three cutoffs $[18]$ (already quoted in Table 1 for the cutoff $\Lambda/\mu = 5.8$) are given by the points with an *. Another Faddeev evaluation $[20]$ of the same Hamiltonian (RSC/TM(81)) is shown as stars at the three values of $\Lambda/\mu$ and the short dashed line interpolates between the calculated values.

![Figure 1](image.png)

**Figure 1.** Dependence of calculated triton binding energies on $\Lambda$ for the three-body force models TM'(93)(open circles) and TM'(99)(open squares). The NN potential was the Reid $v_{8}$ potential. Horizontal lines are the calculated value without a three-body force. Two Faddeev calculations with the NN/TBF combination RSC/TM(81) are shown for comparison. See text for details.

The models TM'(93) and TM'(99) with the spurious short-range $-\pi$-range TBF removed (open circles and open squares) give very similar binding energies.
in our calculation. The updating of the strength constants seems to have very little effect on the three nucleon bound state, once the spurious term is removed. It is difficult to estimate the effect of removing the short-range – π-range force on the binding energy with the results available in Figure 1, because both the NN potential (Reid v8 versus RSC) and the TBF (TM(81) and TM(93)) are slightly different. However, once this spurious force is removed the two models TM′(93) and TM′(99) have a similar dependence upon Λ; those two curves are shifted vertically only slightly. It is noteworthy that the dependence upon Λ is greater if the spurious short-range – π-range term is included in the TBF [18]; and significantly greater for the momentum space calculations of Ref. [20]. One would expect this as Λ increases and the singular term (in one NN separation) becomes more like a delta function. It is a nice feature that removal of the spurious term makes the Tucson-Melbourne two-pion exchange force less sensitive to the cutoff.

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