Production of $K$ mesons in exclusive $B_c$ decays

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ABSTRACT: The paper is devoted to investigation of $K$ mesons production in exclusive $B_c$ decays $B_c \to \psi^{(')}K^+K^-\pi^+$ and $B_c \to \psi^{(')}K^+\pi^+\pi^-$. In the framework of resonance approximation we obtain theoretical predictions for branching fractions of these decays and present distributions over different kinematical variables.
1 Introduction

Recently there was a large progress in theoretical and experimental study of $B_c$ mesons. In addition to measurement of its mass and lifetime, which is in excellent agreement with theoretical predictions, the branching fractions of some decays were also determined. One can mention, for example, semileptonic decays $B_c \rightarrow J/\psi \mu \nu$ and exclusive decays into vector charmonium and a system of light mesons, i.e. $B_c \rightarrow J/\psi \pi$, $B_c \rightarrow \psi(2S)\pi$, $B_c \rightarrow J/\psi + 3\pi$, etc. Production of $K$-mesons in exclusive $B_c$ decays, however, was not yet considered. Below we give theoretical analysis of these reactions.

2 Exclusive $B_c$ decays

Exclusive $B_c$ meson decays into charmonia and semileptonic pair or a set of light mesons take special place. On the partonic level in the factorization approximation such processes proceed via weak decay of $b$ quark into $c$. The virtual $W$ boson afterwards hadronizes into final system of light particles, while $c\bar{c}$ pair transforms into charmonium meson. Typical diagram of such process is shown in Fig.1. The amplitude of the reaction can be written in the form

$$\mathcal{M}(B_c \rightarrow \psi^{(')} + R) = \frac{G_F V_{bc}}{\sqrt{2}} a_1(m_b) \mathcal{H}^\mu \epsilon^{(R)}_\mu,$$

(2.1)

where $G_F$ is the Fermi constant, $V_{bc}$ is the quark mixing matrix element, Wilson coefficient $a_1(m_b) \approx 1.14$ describes the effect of QCD corrections [1], while vectors $\mathcal{H}^\mu$ and $\epsilon^{(R)}_\mu$ describe $B_c \rightarrow \psi^{(')}W$ and $W \rightarrow R$ transitions.
Figure 1. Feynman diagram for $B_c \to \psi'(c) + \mathcal{R}$ decay

### Table 1. $B_c \to J/\psi W$ formfactors

| $B_c \to J/\psi$ | $B_c \to \psi(2S)$ |
|------------------|-------------------|
| $q^2 = 0$        | $q^2 = q^2_{\text{max}}$ |
| $A_0$            | $0.60$            | $0.15$ |
| $A_1$            | $0.63$            | $0.14$ |
| $A_2$            | $0.69$            | $0.13$ |
| $V$              | $1.0$             | $0.3$  |

Let us first discuss the first vertex. It is usually written in terms of $B_c$ formfactors and one of popular parameterizations has the form

$$
\mathcal{H}^\mu = 2M_2 A_0 \left(q^2\right) \frac{q^\mu \langle q \epsilon \rangle}{q^2} + \left(M_1 + M_2\right) A_1 \left(q^2\right) \left(\epsilon^\mu - \frac{q^\mu \langle q \epsilon \rangle}{q^2}\right) - A_2 \left(q^2\right) \frac{\langle q \epsilon \rangle}{M_1 + M_2} \left(p_1 + p_2 - \frac{M_1^2 - M_2^2}{q^2}\right) - \frac{2iV \langle q^2 \rangle}{M_1 + M_2} \epsilon_{\mu \nu \alpha \beta} \epsilon^\nu p_1^\alpha p_2^\beta ,
$$

where $p_{1,2}$ and $M_{1,2}$ are momenta and masses of initial and final heavy quarkonia, $q = p_1 - p_2$ is the transferred momentum, $\epsilon_\mu$ is the polarization vector of $\psi'(c)$ meson, and $A_{0,1,2} \left(q^2\right)$, $V \left(q^2\right)$ are the formfactors. It is clear that these functions cannot be determined from perturbative theory, so some other approach should be used for example QCD sum rules [2–6], different potential quark models [7–11], light-front models [12–14], etc. In our paper we will use formfactors stets presented in papers [5] and [15] (in the following they will be labeled as SR and PM respectively). In table 1 we show values of these formfactors at different points.

From the presented in this table data it is clear that in the case of $B_c \to J/\psi$ transition $q^2$ dependence and of formfactors are similar for different models, the main difference is in the overall normalization. In the case of $B_c \to \psi(2S)$ transition, on the contrary, the form...
of distribution depends strongly on the choice of the model. This difference is caused by the fact, that the wave function of excited charmonium meson should have a node that is not represented in QCD sum rules results. The role of this node is discussed, for example, in paper [16].

The last factor in eq. (2.1), i.e. the amplitude of $W \to R$ transition, is also nonperturbative. In our paper we shall use resonance approximation that gives good results in the case of $B_c \to J/\psi + 3\pi$ decay [17, 18]. The same method was used later in papers [16, 19, 20] to study some other exclusive decays of $B_c$ meson. In this approach the amplitude of the process is written as an amplitude of subsequent decays of virtual resonances with suitable quantum numbers. For example, in the case of $B_c \to J/\psi + 3\pi$ decay the reaction $W \to a_1 \to \rho \pi \to 3\pi$ was used. For reactions considered in the present paper we take into account also contributions of $K_1(1270)$, $K_1(1400)$, and $K^*$ mesons.

The branching fractions of $B_c$ decays are often normalized to branching fraction of $B_c \to \psi(\prime)\pi$ reactions. The latter quantity can easily calculated using presented above expressions with effective polarization vector $\epsilon^{(\pi)}_\mu$ equal to

$$\epsilon^{(\pi)}_\mu = f_\pi k_\mu \delta \left( q^2 - m^2_\pi \right).$$

(2.3)

Here $k_\mu$ and $m_\pi = 140$ MeV are momentum and mass of produced pion, while $f_\pi \approx 130$ MeV is its mesonic constant. Numerical values of branching fractions of $B_c \to \psi(\prime)\pi$ decays for different sets of formfactors are shown in table 2.

### Table 2. Branching fractions of exclusive $B_c$ decays (in %)

|   | $B_c \to J/\psi + R$ | $B_c \to \psi(2S) + R$ |
|---|---------------------|----------------------|
| $\pi$ | 0.17                | 0.0066               |
| $K\pi\pi$ | 0.03               | 0.0011               |
| $KK\pi$ | 0.081              | 0.0023               |

#### 3 $B_c \to \psi(\prime) + K\pi\pi$

Let us first consider the $B_c \to \psi(\prime) + K\pi\pi$ decay. In this case typical diagrams for $W \to K\pi\pi$ transition are shown in Fig.2. From quantum numbers of virtual resonances one can determine the amplitude of this reaction:

$$\epsilon^{(K\pi\pi)}_\mu = g_{K(1270)\rho} D_{K(1270)}(q) D_\rho(k_1 + k_2)(k_1 - k_2)_{\mu} +$$

$$g_{K(1270)K^*} D_{K(1270)}(q) D_{K^*}(k_2 + k_3)(k_2 - k_3)_{\mu} +$$

$$(K_1(1270) \to K_1(1400)),$$

(3.1)

where $k_{1,3}$ are momenta of $\pi^+$, $\pi^-$ and $K^+$ mesons respectively and we introduce the shorthand notation for meson propagator

$$D_A(p) = \frac{1}{p^2 - M^2_A + i M_A \Gamma_A},$$

(3.2)
Coupling constants $g_{K\rho}$, $g_{KK^*}$ can be determined from analysis of corresponding decays of $K$ mesons, but we prefer to use directly experimental $q^2$ distributions from $\tau \rightarrow \nu_\tau + KK\pi$ decay, as it was done for $B_c \rightarrow J/\psi + 3\pi$ reaction in papers [17, 21]. From the fit of presented in ref.[22] data we obtain the following values of these constants:

$$
g_{K(1270)\rho} = -4.14 \times 10^{-2} \text{GeV}^3, \quad g_{K(1270)K^*} = 0.17 \text{GeV}^3, \quad g_{K(1400)\rho} = 0.13 \text{GeV}^3, \quad g_{K(1400)K^*} = 0.24 \text{GeV}^3.
$$

(3.3)

(3.4)

Using presented above expressions one can calculate the branching fractions of $B_c \rightarrow \psi^{(i)} K^+\pi^+\pi^-$ decays. Our results for different choice of formfactor set are shown in the third row of table 2. After normalization to $B_c \rightarrow \psi^{(i)}\pi$ branching fractions we obtain the ratios

$$
\frac{\text{Br} (B_c \rightarrow J/\psi + K\pi\pi)}{\text{Br} (B_c \rightarrow J/\psi\pi)} = 0.18, \quad \frac{\text{Br} (B_c \rightarrow \psi(2S) + K\pi\pi)}{\text{Br} (B_c \rightarrow \psi(2S)\pi)} = 0.16
$$

(3.5)

for SR model of form factors and

$$
\frac{\text{Br} (B_c \rightarrow J/\psi + K\pi\pi)}{\text{Br} (B_c \rightarrow J/\psi\pi)} = 0.17, \quad \frac{\text{Br} (B_c \rightarrow \psi(2S) + K\pi\pi)}{\text{Br} (B_c \rightarrow \psi(2S)\pi)} = 0.047
$$

(3.6)

for PM. Distributions over invariant masses of $K\pi$ and $\pi\pi$ pairs are presented in Fig.3, 4, 5. A notable peak in the last figure is caused by $K^*$ meson contribution (see diagram 2b). One can easily see that in the case of $J/\psi$ meson in the final state branching fractions ratio and forms of distributions do not depend on the choice of formfactors model, while in the case of $B_c \rightarrow \psi(2S) + K\pi\pi$ decays modification of formfactor’s set changes theoretical predictions significantly.

4 $B_c \rightarrow \psi^{(i)} + KK\pi$

Using the same approach one can consider $B_c \rightarrow \psi^{(i)} K^+K^-\pi^+$ decays. In this case decay chains $W \rightarrow a_1 \rightarrow KK^* \rightarrow KK\pi$ and $W \rightarrow a_1 \rightarrow \phi\pi \rightarrow KK\pi$ can give contributions (see...
Figure 3. Distribution over $m_{\pi^+\pi^-}$ in the case of $B_c \rightarrow J/\psi + K\pi\pi$ (left figure) and $B_c \rightarrow \psi(2S) + K\pi\pi$ (right figure) decays. Solid and dashed lines correspond to SR and PM formfactor sets respectively.

Figure 4. Distribution over $m_{K^+\pi^+}$ in the case of $B_c \rightarrow \psi(\prime) + K\pi\pi$ decay. Notations are same as in Fig.3.

Figure 5. Distribution over $m_{K^+\pi^-}$ in the case of $B_c \rightarrow \psi(\prime) + K\pi\pi$ decay. Notations are same as in Fig.3.

diagrams shown in Fig.6), but the last reaction should be suppressed due to OZI rule and G-parity violation, so we will not take it into account. As for diagram shown in Fig.6(a),
the corresponding amplitude can be written as

$$\epsilon^{(KK\pi)}_\mu = g_{KK\pi} D_{a_1}(q) D_{K^*}(k_1 + k_3)(k_1 - k_3)_\mu,$$

where the constant $g_{KK\pi}$ was determined from $\tau \to \nu_\tau + KK\pi$ decay. The branching fractions of $B_c \to \psi^{(')} + KK\pi$ decays, obtained using presented above expressions are listed in the last row of table 2, and the ratios to $\text{Br}(B_c \to \psi^{(')}\pi)$ for different form factor sets are equal to

$$\frac{\text{Br}(B_c \to J/\psi + KK\pi)}{\text{Br}(B_c \to J/\psi\pi)} = 0.49, \quad \frac{\text{Br}(B_c \to \psi(2S) + KK\pi)}{\text{Br}(B_c \to \psi(2S)\pi)} = 0.34 \quad (4.1)$$

for SR set of form factors and

$$\frac{\text{Br}(B_c \to J/\psi + KK\pi)}{\text{Br}(B_c \to J/\psi\pi)} = 0.47, \quad \frac{\text{Br}(B_c \to \psi(2S) + KK\pi)}{\text{Br}(B_c \to \psi(2S)\pi)} = 0.053 \quad (4.2)$$

for PM model. It is seen that these values are greater than the corresponding ratios in the case of $B_c \to \psi^{(')} + KK\pi$ decays. The reason is that the latter processes are suppressed by Cabibbo factor $V_{us}$. In our model, this factor is contained in the $W \to K\pi\pi$ transition amplitude. Distributions over invariant masses $m_{K+K-}$, $m_{K+\pi+}$ and $m_{K-\pi+}$ are shown in Fig.7, 8, 9 respectively.

5 Conclusion

Let us summarize briefly the results of our note. In the framework of factorization approximation the branching fractions of $B_c \to \psi^{(')} + K\pi\pi$ and $B_c \to \psi^{(')} + KK\pi$ decays were calculated. These branching fractions depend on form factors of $B_c \to \psi^{(')}W$ vertices (in our analysis we use FF models presented in papers [5, 15]) and amplitudes of $W \to K\pi\pi$, $W \to KK\pi$ transitions. The latter amplitudes were written in the resonance approximation with contributions of $K_1(1270)$, $K_1(1400)$, $K^*(890)$, $a_1$ and $\rho$ mesons taken into account. Corresponding coupling constants were determined from analysis of $\tau$ lepton decays. We believe that experimental study of discussed in our paper decays can give additional information about both $B_c \to \psi^{(')}$ form factors and physics of light mesons.
Figure 7. Distribution over $m_{K^+K^-}$ in the case of $B_c \to \psi^{(1')} K K \pi$ decays. Notations are same as in Fig.3.

Figure 8. Distribution over $m_{K^+\pi^+}$ in the case of $B_c \to \psi^{(1')} K K \pi$ decays. Notations are same as in Fig.3.

Figure 9. Distribution over $m_{K^+\pi^-}$ in the case of $B_c \to \psi^{(1')} K K \pi$ decays. Notations are same as in Fig.3.

Acknowledgments

The author would like to thank A. Likhoded, I. Belyaev and V. Egorychev for useful and fruitful discussions. The work was financially supported by Russian Foundation for Basic
Research (grant #10-00061a), the grant of the President of Russian Federation (grant #MK-3513.2012.2), and FRRC grant.

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$B_c^+ \rightarrow J/\psi + K^+ K^- \pi^+$

$B_c^+ \rightarrow \psi(2S) + K^+ K^- \pi^+$