δ-GLMB filter based on DI in a clutter

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Abstract: For the problem that the performance of existing multi-target tracking algorithm’s serious degrades in a dense clutter environment, a novel Doppler information assistant δ-generalised labelled multi-Bernoulli (DI-δ-GLMB) filter is proposed. By introducing DI, a new measurement likelihood function is established, and the improved update equation based on the δ-GLMB filter framework is derived. In addition, a sequential Monte Carlo implementation method is given under the non-linear model. Simulation results show that compared with the DI probability hypothesis density filter and the standard δ-GLMB filter, the estimation of the proposed algorithm is more accurate and stable.

1 Introduction

In multiple target tracking (MTT), a set of observed data is given to the filter to estimate the trajectories and the time-varying number of targets. It is widely applied in many fields, such as battlefield surveillance, air defence, and anti-missile. The classical MTT method is based on data association, which divides the tracking process into two steps: data association and state estimation. As the number of targets and clutter increases, the complexity of data associations increases exponentially. The MTT technology based on the random finite set (RFS) can effectively avoid complex data association processes and become the research hotspot in the field of MTT. Although the number of clutter increases, the performance of the famous probability hypothesis density (PHD) filter, cardinality probability hypothesis density (CPHD) filter, and multi-target multi-Bernoulli (MeMBer) filter severely attenuates. In MTT application, position measurements are typically utilised. In order to enhance the performance in a clutter environment, amplitude information (AI) and Doppler information (DI) can be further considered [1, 2]. AI of the signal was introduced into the PHD filter to improve the accuracy of the target state estimation [3, 4], but the variance increased as the target increases. Also AI was introduced into the CPHD and MeMBer filter to improve the tracking performance under a low signal-to-noise ratio scene in [5, 6].

However, in practical application, the amplitude of the echo signal is affected by many factors such as the distance between the target and the sensor and the radar cross-section. Compared with AI, DI is only related to the motion state and has strong stability. In [7, 8], DI was introduced into the GM-PHD filter. The proposed DI-PHD filter effectively suppressed the effect of clutter false alarm measurements and improved the estimated accuracy. In [9], the algorithm introduced DI into the CPHD filter, which improved the tracking performance effectively under the linear motion model.

Most of the existing algorithms have improved the traditional PHD/CPHD and MeMBer filters by introducing AI and DI. However, when the clutter is more densely distributed, the performance of the improved filters is still poor, and the trajectories cannot be directly estimated. In response, Vo et al. [10–12] proposed δ-generalised labelled multi-Bernoulli (δ-GLMB) filter by combining multiple hypothesis tracking and RFS theory, which improved the estimated accuracy and directly formed track. For these reasons, the δ-GLMB filter was widely used in applications such as manoeuvring targets tracking [13] and multi-sensor information fusion [14, 15].

In this paper, we propose a solution to the Bayes multi-target filter for target tracking in a dense clutter environment. Specifically, we introduce DI into the δ-GLMB filter, deduce new measurement update equation, and give a sequential Monte Carlo (SMC) implementation method under the non-linear motion model. Finally, the proposed DI assistant δ-GLMB (DI-δ-GLMB) filter is verified via numerical examples.

2 System models

2.1 Target state model

Suppose that at time k there are $N_k$ targets, and each target state $x^0_i$ contains the positions $p^0_i$ and $p^0_i$ and the velocities $\dot{p}^0_i$ and $\dot{p}^0_i$, so

$$x^0_i = \begin{bmatrix} p^0_i, \dot{p}^0_i, p^0_i, \dot{p}^0_i, p^0_i, \dot{p}^0_i \end{bmatrix}^T, \quad i = 1, 2, \ldots, N_k$$

Then a multi-target state is represented by RFS

$$X_k = \{x^0_i\}, \quad i = 1, 2, \ldots, N_k$$

The time evolution of the RFS $X_k$ can be modelled as

$$X_k = \bigcup_{x^0_i \in X_{k-1}} S_{x^0_i \rightarrow x^0_i} \cup B_k$$

where $S_{x^0_i \rightarrow x^0_i}$ is the survival target RFS at time $k$ with probability $p_s(x^0_i)$, and $B_k$ represents the birth target RFS. Tracks of targets are assumed to follow a discrete-time linear dynamic model:

$$x_{k|k-1} = F_k \cdot x_{k-1} + v_k$$

where $F_k$ is the transition matrix, $v_k \sim N(0; Q_k)$ is the white Gaussian noise with covariance $Q_k$. Then the probability density function of a transition is given by

$$f_{x_{k|k-1}}(x_k | x_{k-1}) = N(x_k; F_k x_{k-1}, Q_k)$$
2.2 Measurement model

Suppose that at time $k$ there are $M_k$ measurements. Then each measurement which contains the position $z_{j,k}^0$ and the Doppler $(\nu_j,k)$ is expressed as

$$z_j^k = [z_{j,k}^0; \nu_j,k], \quad j = 1, \ldots, M_k$$

A multi-target measurement is represented by RFS

$$Z_k = \Theta(X_k) \cup K_k$$

where $K_k$ is the clutter RFS, $\Theta(X_k)$ is the RFS of the detected target-originated measurements with detection probability $p_{D,k} < 1$.

Suppose that the position measurement from target $x_k$ at time $k$ is linearly related to the target state:

$$z_{c,k} = H_{c,k} x_k + w_k$$

where $w_k \sim N(w_k; 0, R_{c,k})$ is the position observation noise with covariance $R_{c,k}$, $H_{c,k}$ is the position observation matrix which is given by

$$H_{c,k} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Then the position measurement likelihood density function is

$$g(z_{c,k} | x_k) = N(z_{c,k}; H_{c,k} x_k, R_{c,k})$$

The Doppler measurement from the target can be expressed as

$$z_{d,k} = h_{d}(x_k) + n_k$$

where $n_k \sim N(n_k; 0, R_{d,k})$ is the Doppler observation noise with covariance $R_{d,k}$ and $h_d(\cdot)$ is given by

$$h_d(x_k) = \frac{p_{x,k} x_k}{\sqrt{p_{x,k}^2 + p_{x,k} + p_{y,k}}} + \frac{p_{y,k}}{\sqrt{p_{x,k}^2 + p_{y,k}}}$$

The Doppler measurement likelihood density function is given by

$$g(z_{d,k} | x_k) = N(z_{d,k}; h_d(x_k), R_{d,k})$$

From the measurement models (8) and (11), the joint measurement likelihood density function can be expressed as

$$g(z_k | x_k) = g(z_{c,k} | x_k) g(z_{d,k} | x_k)$$

$$= N(z_{c,k}; H_{c,k} x_k, R_{c,k}) N(z_{d,k}; h_d(x_k), R_{d,k})$$

Similarly, the joint clutter density can be expressed as

$$c(z_k) = c(z_{c,k}) \cdot c(z_{d,k})$$

where $c(z_{c,k})$ is the spatial clutter measurement density and $c(z_{d,k})$ is the Doppler clutter density.

3 DI-δ-GLMB filter

Multi-target states can be represented by labelled RFS:

$$X = \{(x, l) \} \in X \times L, \quad i = 1, 2, \ldots, N$$

where $x$ is the kinematic feature, $l$ is the label, $X$ is the state space, and $L$ is the label space.

Suppose that the prior density at time $k - 1$ is a δ-GLMB form, which is given by

$$p_{x_{k-1}}(X) = \Delta(X) \times \sum_{(l, j) \in \mathcal{S}(X \setminus L_{k-1})} \left[ o_{h_{k-1} \setminus (I; \nu)}(\cdot; \nu) \cdot \delta_{\pi}(\cdot; \nu) \right]$$

where

$$\Delta(X) = \delta_{\pi}(\cdot; \nu), \quad \delta_{\pi}(Y) = \begin{cases} 1 & X = Y \\ 0 & \text{else} \end{cases}$$

Each $I \in \mathcal{S}(L_{k-1})$ represents a set of tracks, $\mathcal{S}(\cdot)$ is the class of finite subsets, $\Theta$ is a discrete space which represents the history of track to measurement associations, $\nu \in \Theta$ is the realisation of $\Theta$, $O(I; \nu)$ is weight, and $p(\cdot; \nu)$ is a probability density function.

Suppose the birth density is a δ-GLMB form:

$$p_{x_0}(X) = \sum_{l \in \mathcal{S}(L_{k-1})} o_{h_{0,l} \setminus (I; \nu)}(\cdot; \nu) (\pi(l), \nu)$$

where

$$o_{h_{0,l} \setminus (I; \nu)}(L) = \prod_{i \in l} (1 - r_{i,k}) \prod_{c \in L_i} \frac{1 \leftarrow \leftarrow \frac{1}{1 - r_{i,k}}.$$
4. Implementation of the DI-δ-GLMB filter

Bayesian multi-target filters are mainly implemented with the Gaussian mixture (GM) and SMC methods. The GM method is only suitable for linear Gaussian cases, so the SMC method is used in this paper.

4.1 Initialisation

For an SMC approximation, suppose that each single target density at time \( k - 1 \) is represented as a set of weighted samples \( \{(x_{k-1}^{(i)}), w_{k-1}^{(i)}\}_{i \in N_{k-1}} \). Then the prior density is given by

\[
\{r_{k-1}^{(i)}(x_{k-1}^{(i)}, x_{k-1}^{(i)}), \omega_{k-1}^{(i)}\}_{i \in N_{k-1}}
\]

(21)

where \( r_{k-1}^{(i)} \) is the existence probability, \( N_{k-1} \) is the number of particles.

Similarly, the birth density is represented by

\[
\{(r_{k-1}^{(i)}, x_{k-1}^{(i)}, x_{k-1}^{(i)}), \omega_{k-1}^{(i)}\}_{i \in N_{k-1}}
\]

(22)

4.2 Prediction

At time \( k \), the prediction density becomes

\[
\{r_{k-1}^{(i)}, (x_{k-1}^{(i)}, x_{k-1}^{(i)}), \omega_{k-1}^{(i)}\}_{i \in N_{k-1}}
\]

(23)

where

\[
r_{k-1}^{(i)} = r_{k-1}^{(i)} \eta_{k-1}(l), \quad \eta_{k-1}(l) = \sum_{i \in N_{k-1}} p_{k-1}(x_{k-1}^{(i)}, x_{k-1}^{(i)})w_{k-1}^{(i)}
\]

\[
(x_{k-1}^{(i)}) \sim q(\cdot|x_{k-1}^{(i)}, l, Z_{k})
\]

\[
w_{k-1}^{(i)} = \frac{f_{k-1}(x_{k-1}^{(i)}, x_{k-1}^{(i)})p_{k-1}(x_{k-1}^{(i)}, x_{k-1}^{(i)})w_{k-1}^{(i)}}{q(x_{k-1}^{(i)}, x_{k-1}^{(i)}; Z_{k})}
\]

\[
w_{k-1}^{(i)} = \sum_{i \in N_{k-1}} \omega_{k-1}^{(i)}
\]

\[
x_{k-1}^{(i)} \sim b(\cdot|Z_{k})
\]

4.3 Update

At time \( k \), the update density is given by

\[
\{r_{k-1}^{(i)}(x_{k-1}^{(i)}, x_{k-1}^{(i)}), \omega_{k-1}^{(i)}\}_{i \in N_{k-1}}
\]

(24)

where

\[
r_{k-1}^{(i)} = \sum_{(i, \zeta) \in F_{k-1} \times \Theta} \omega_{k-1}(x_{k-1}^{(i)}, \zeta)
\]

\[
\omega_{k-1}(x_{k-1}^{(i)}, \zeta) \propto \prod_{l \in N_{k-1}} \pi_{l}(x_{k-1}^{(i)})(1 - \delta_{l}) \prod_{l \in N_{k-1}} \lambda_{l}(x_{k-1}^{(i)})
\]

\[
\psi_{k-1}^{(i)}(x_{k-1}^{(i)}) = \frac{f_{k-1}(x_{k-1}^{(i)})q(x_{k-1}^{(i)}; l, Z_{k})}{\sum_{i \in N_{k-1}} \omega_{k-1}^{(i)}}
\]

\[
w_{k-1}^{(i)} = \sum_{i \in N_{k-1}} \omega_{k-1}^{(i)}
\]

In order to simplify the calculation, during the prediction and update process, the particles are trimmed and combined; also the k-shortest path is used to limit the number of components. Multi-target states can be estimated from the multi-target posterior marginal multi-target estimator. The joint multi-target estimator and marginal multi-target estimator are Bayes optimal, but hard to calculate. In this paper, a suboptimal solution is used based on the marginal multi-target estimator.

5 Numerical example

Consider a set of multi-target trajectories on the two-dimensional region \([-1000, 1000] \times [-1000, 1000] \text{m}^2\). The maximum Doppler speed is 35 m/s. The transition matrix and the modelling error covariance are given by

\[
F = \begin{bmatrix}
1 & \frac{\sin(\theta T_s)}{\theta} & 0 & \frac{1 - \cos(\theta T_s)}{\theta} \\
0 & \cos(\theta T_s) & 0 & -\sin(\theta T_s) \\
0 & 1 - \cos(\theta T_s) & 1 & \frac{\sin(\theta T_s)}{\theta} \\
0 & \sin(\theta T_s) & 0 & \cos(\theta T_s)
\end{bmatrix}
\]

and

\[
Q = \alpha^2 \begin{bmatrix}
T_s^3/3 & T_s^2/2 & 0 & 0 \\
T_s^2/2 & T_s & 0 & 0 \\
0 & T_s^2/3 & T_s/2 & 0 \\
0 & T_s^2/2 & T_s & 0
\end{bmatrix}
\]

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where \( \theta = 1.5 \pi / 180 \text{ rad/s} \), \( \sigma_v = 0.1 \text{ m/s} \), and \( T_s = 1 \text{ s} \). The probabilities of target survival and detection are \( p_s = 0.95 \) and \( p_d = 0.9 \), respectively. The initial states of the targets are described in Table 1.

The measurement error standard deviation is given by \( R = \text{diag}(10; 10; 0.5) \). Clutters follow the Poisson distribution with \( \lambda = 60 \) in the surveillance region. The spontaneous birth is a multi-Bernoulli process with density

\[
\sum_{i=1}^{mB_i} \mathcal{N}(x_{B_i}; m_{B_i}, P_{B_i}),
\]

where \( P_{B_i} = \text{diag}(20, 1, 20, 1) \), \( r_{B_i} = 0.03 \). True trajectories and measurements of a single trial are shown in Fig. 1.

Fig. 2 shows the results of a single trial using the standard \( \delta \)-GLMB algorithm and the proposed DI-\( \delta \)-GLMB algorithm. It can be seen that although the standard \( \delta \)-GLMB algorithm can estimate trajectories under a dense clutter environment, the number of targets is overestimated. The estimation of the DI-\( \delta \)-GLMB filter is more accurate.

To verify the performance of the algorithm, 100 Monte Carlo experiments were performed on the DI-PHD, \( \delta \)-GLMB, and DI-\( \delta \)-GLMB algorithms. Fig. 3 illustrates the variance of the cardinality estimation. The results show that the estimate variance of the DI-PHD algorithm is larger from 20 to 70 s. This is due to the fact that the DI-PHD algorithm is based on the Poisson distribution hypothesis, so the variance is proportional to the cardinality of targets. The estimated variances of the \( \delta \)-GLMB and DI-\( \delta \)-GLMB algorithms are independent of the number of targets. Compared with the DI-PHD and \( \delta \)-GLMB algorithms, the variance of the proposed DI-\( \delta \)-GLMB algorithm is the smallest.

Fig. 4 shows the average optimal sub-pattern assignment (OSPA) [16] distances of the DI-PHD, \( \delta \)-GLMB, and DI-\( \delta \)-GLMB filters. The DI-\( \delta \)-GLMB algorithm provides the minimum OSPA distances as expected. However, similar to the \( \delta \)-GLMB algorithm, when the target disappears, a large spike appears. This is because the \( \delta \)-GLMB filter uses the multi-frame measurement information to calculate the hypothesis probability.

### Table 1 States of the tracks

| Target index | Appear time | Disappear time | Initial states       |
|--------------|-------------|----------------|----------------------|
| 1            | 3           | 65             | [500; − 11; 400; − 5] |
| 2            | 5           | 68             | [450; − 15; 8; 5]    |
| 3            | 7           | 71             | [0; − 15; 150; − 20] |
| 4            | 10          | 100            | [600; − 5; 200; 10]  |
| 5            | 15          | 100            | [− 500; 10; − 300; 15]|
6 Conclusion

In general, the Doppler frequency of a manoeuvring target is stable over a certain period of time, while clutter does not have this feature. Therefore, DI can be used to distinguish measurements originated from targets and clutter. This paper proposed the DI-δ-GLMB filter by introducing DI into the δ-GLMB filter. Simulation experiments show that the algorithm is suitable for the non-linear motion model. Compared with DI-PHD and the standard δ-GLMB filter, the proposed algorithm is more stable and accurate.


7 References

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