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Tailoring the correlation and anticorrelation behavior of path-entangled photons in Glauber-Fock oscillator lattices

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We demonstrate that single-photon as well as biphon revivals are possible in a new class of dynamic optical systems—the so-called Glauber-Fock oscillator lattices. In these arrays, both Bloch-like oscillations and dynamic delocalization can occur which can be described in closed form. The bunching and antibunching response of path-entangled photons can be pre-engineered in such coupled optical arrangements, and the emulation of fermionic behavior in this family of lattices is also considered. We elucidate these effects via pertinent examples.

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I. INTRODUCTION

The prospect of manipulating and engineering quantum states has become an issue of great importance within the framework of quantum information and computation [1, 2]. Along these lines, several physical settings have been envisioned as viable avenues to achieve this goal. Among them, one may mention trapped-ion arrangements and optical lattices as well as spin systems and quantum dots [3, 4]. While the list of such possibilities keeps increasing with time, quantum optics has so far provided a versatile platform where such ideas can be experimentally realized and tested. As previously indicated, in optics, quantum information processing can be achieved entirely linearly, using simple passive components like beam splitters and phase shifters along with standard photodetectors and single-photon sources [5]. In this same optical realm, quantum entanglement can arise as a natural byproduct of photon interactions—a clear manifestation of their particle-wave duality. Perhaps, nowhere is this process more apparent than in the so-called Hong-Ou-Mandel two-photon interference effect [6]. In this latter configuration, photon entanglement is made possible via quantum interference—afforded after scattering from a beam splitter. Lately, optical arrays of evanescently coupled waveguides have been suggested as a possible route toward the implementation of multiport systems with moldable quantum dynamics [7]. The flexibility offered by such compact and often miniaturized optical $N \times N$ configurations is made possible by the exceptional control achievable these days in microfabrication techniques [8, 9]. In this regard, Bloch oscillations of NOON and W entangled states as well as quantum random walks have been theoretically considered and observed in such arrays [10–12]. In addition, the evolution of quantum correlations in both periodic and random (Anderson) lattices has also been investigated [13–15]. The possibility of classically emulating Jaynes-Cummings systems on such lattices has also recently been proposed [16]. The question naturally arises as to whether such multiport array systems can be utilized as a means to manipulate quantum states of light.

In this paper we investigate the propagation dynamics of nonclassical light in a new class of dynamic photonic systems—the so-called Glauber-Fock oscillator lattices. We demonstrate that Bloch-like revivals and dynamic delocalization effects can naturally occur in spite of the fact that the structure itself is semi-infinite and not periodic. Interestingly, these interactions can be described in closed form, from where one can analytically deduce the turning points of these quantum dynamics. More importantly, the bunching and antibunching response of path-entangled photons can be pre-engineered in such coupled optical arrangements. Emulating fermionic dynamics in such arrangements are also considered and compared to those expected from bosonic systems in these same arrays. Finally, the possibility of experimentally realizing such Glauber-Fock oscillator lattices is discussed.

II. SINGLE-PHOTON DYNAMICS IN GLAUBER-FOCK OSCILLATOR LATTICES

We begin our analysis by considering a semi-infinite Glauber-Fock oscillator array consisting of evanescently coupled waveguides. In this arrangement the coupling coefficients among neighboring channels vary with the square root of the site index (i.e., $C_{k, k+1} \propto \sqrt{k+1}$) [9]. For generality, we also allow this coupling to depend on the propagation distance in this lattice, in which case $C_{k, k+1} \propto f(Z)\sqrt{k+1}$ where $f(Z)$ is an arbitrary real function. In addition, we also assume that the propagation constant (local eigenvalue) of each waveguide element varies linearly with the site position. In essence, in this arrangement the refractive index is linearly increasing—in a way analogous to that of an externally biased crystal in solid state physics. Starting from these premises, one can show that, in this class of arrays, the Heisenberg equation of motion for the creation operator of a single photon in waveguide mode $k$ is given by

$$i \frac{d a_k^\dagger}{d Z} - f(Z)(\sqrt{k} + 1a_{k+1}^\dagger + \sqrt{k}a_{k-1}^\dagger) - \lambda a_k^\dagger = 0. \tag{1}$$

In the above equation, $Z$ represents the normalized propagation distance given by $Z = k_1z$, where $k_1$ stands for the coupling coefficient between sites 0 and 1, and $\lambda$ is a real constant associated with the strength of the aforementioned linear
In spite of this complexity, one can show that the evolution oscillator array can be described through the evolution matrix. This matrix cannot be simply obtained from \( \exp(-iZH) \) since the Hamiltonian of the problem is \( Z \) (or time) dependent [20]. Yet, in spite of this complexity, one can show that the evolution elements \( T_{k,n}^a \) of the system can be obtained in closed form (see appendix for details). Starting from Eq. (1), one can show that these elements are given by

\[
T_{k,n}^a = \begin{cases}
\sqrt{\frac{n!}{\pi}} A e^{\frac{\lambda}{\pi} Z} L_{k-n}^k(\Phi), & n \leq k \\
\sqrt{\frac{n!}{\pi}} B e^{\frac{\lambda}{\pi} Z} L_{n-k}^k(\Phi), & n \geq k,
\end{cases}
\]

where \( \Phi = \exp(-i\lambda Z) \exp(-D)|B|^2 \), and

\[
D(Z) = e^{-i\lambda Z} - 1,
\]

\[
B(Z) = -i \int_0^Z e^{-i\lambda Z} f'(Z')dZ',
\]

\[
C(Z) = -ie^{-i\lambda Z} \int_0^Z e^{i\lambda Z} f'(Z')dZ',
\]

\[
A(Z) = -\int_0^Z \left[ \int_0^{Z'} e^{i\lambda Z} f'(Z')dZ' \right] f(Z')e^{-i\lambda Z}dZ'.
\]

In Eqs. (3), \( L_n^k(\lambda) \) represents the associated Laguerre polynomials. In addition, Eqs. (3) imply that \( \sum_{n=0}^{\infty} |T_{k,n}|^2 = 1 \), in agreement with the fact that \( T \) is itself a unitary transformation. In order to gain insight into the quantum dynamics in this class of arrays, let us first consider the case where only a single photon is launched into the \( k \)th waveguide element. We begin by analyzing here the simplest possible scenario where the Hamiltonian of the system is \( Z \) independent, thus \( f(Z) = \kappa_0 \), where \( \kappa_0 \) is a real constant. Thus, the evolution matrix elements are given by

\[
T_{k,n} = \begin{cases}
\sqrt{\frac{n!}{\pi}} e^{(\delta+Dn)} [\Theta]^{k-n} L_{n-k}^k(\Phi), & n \leq k \\
\sqrt{\frac{k!}{\pi}} e^{(\delta+Dk)} [\Theta]^{n-k} L_{k-n}^k(\Phi), & n \geq k,
\end{cases}
\]

where

\[
\delta = (\kappa_0^2/|\lambda|^2)[i\lambda Z + \exp(-i\lambda Z) - 1],
\]

\[
\Theta = [\kappa_0 D(Z)/\lambda],
\]

\[
D(Z) = [\exp(-i\lambda Z) - 1],
\]

\[
\Phi = \{2\kappa_0[1 - \cos(\lambda Z)]/|\lambda|^2\} \exp(i\lambda Z) \exp(D).
\]
in Fig. 3, where it is shown that the probability of finding resonances. This delocalization process at resonance is depicted above of the function \(Z\) of the function \(|B(Z)|^2\). For the particular example examined here these revivals occur when the ratio \(\sigma/\lambda = P/Q\) is a rational number, and \(P\) and \(Q\) are relatively prime integers. This condition is necessary for the two oscillatory processes independently occurring in this array to lock together synchronously. From here one can deduce that \(Z = 2P\pi/\sigma\). This behavior is explicitly illustrated in Fig. 2 for \(\kappa_0 = 1, \lambda = 1, \epsilon = 0.2\) for the cases \(\sigma = 3/4, 2/3\), in (a) and (b), respectively. The dashed lines, on the other hand, represent the evolution of the \(|B(Z)|^2\) function which dictates the period of oscillations. Note that exact revivals do not occur if the ratio \(\sigma/\lambda\) is irrational. On the other hand, at the resonance \(\sigma = \lambda\), dynamic delocalization occurs. In this regime \(|B(Z)|^2 \propto Z\), and hence a drift motion is induced toward higher site indices [21]. Therefore, no pure oscillatory behavior is possible at resonance. This delocalization process at resonance is depicted in Fig. 3, where it is shown that the probability of finding the photon among the waveguides gradually shifts toward the upper side of the array oscillator.

III. QUANTUM OSCILLATIONS OF CORRELATED AND ANTICORRELATED PHOTON PAIRS

We next consider the quantum dynamics of an entangled pair of photons, launched either spatially correlated or anti-entangled into this class of Glauber-Fock oscillator arrays. As we will see, this new class of photonic lattices can tailor the correlation between the array modes, we analyze at the output the coincidence rate at waveguides \(p\) and \(q\), which is given by \(\Gamma_{p,q} = (1/W)\sum_{k \neq f} T_{p,k} T_{q,k} |Z|^2\), whereas for anticorrelated \(|\psi_A\rangle\) it becomes \(\Gamma_{p,q} = (1/W)\sum_{k \neq f} T_{p,f} T_{q,l} |Z|^2\). In order to demonstrate these effects and for comparison purposes, we will always assume here that \(W = 10\) with the excitation contained between \((f,d) = (0,0.9)\).

The evolution of the correlation map \(\Gamma_{p,q}\) as a function of distance when \(f(Z) = 1\) and \(\lambda = 0.5\) is then examined. In this case, revivals are expected at multiples of \(Z = 4\pi\). When the input state is initially correlated, the map flips and antibunching occurs at \(Z = \pi\) and \(Z = 3\pi\), while it returns to a broadened bunched state in the middle of a cycle. This situation is altered when an anticorrelated biphoton input is used. In this case the map tends to flip over to that of a bunched state at \(Z = \pi\) and \(Z = 3\pi\), while in the middle of this oscillation it attains a correlation mixture—with bunching being predominant. This evolution is altogether different from that occurring in uniform lattices [13]. The present dynamics is a direct outcome of the revivals and of the phase acquired upon reflection from the boundary of this semi-infinite Glauber-Fock oscillator array—which is absent in periodic arrays.

We next consider the evolution of correlations when two periods are simultaneously involved in the lattice [e.g., when the function \(f(Z)\) is periodic]. For this example we again take \(f(Z) = \kappa_0 + \epsilon \cos(\sigma Z)\), with \(\kappa_0 = 1, \lambda = 0.5, \epsilon = 0.2\), and \(\sigma = 3/4\), in which case the revival period is \(8\pi\). For a correlated input \(|\psi_C\rangle\) the correlation map exhibits periodic transitions from bunching to antibunching [Figs. 4(a)–4(e)]. However, at the half-cycle point the bunching is now not entirely complete [Fig. 4(e)], due to the incomplete revival of the single-photon trajectories. This scenario becomes very different when the initial biphoton state \(|\psi_A\rangle\) is anticorrelated [Figs. 4(f)–4(j)]. The correlation dynamics corresponding to both cases are shown up to half a cycle \((4\pi)\). Evidently, right after the origin, bunching is seen to occur [Fig. 4(g)] while midway in the cycle signatures of antibunching behavior appear. This latter pattern is different from that obtained before \(|(fZ)| = 1\) when only one oscillation frequency was involved in the Glauber-Fock oscillator.

We have also explored the response of this system under dynamic delocalization conditions. Figure 5 depicts again the correlations for the same parameters used in the previous case, except that here \(\lambda = \sigma = 1\). In this delocalization regime, a correlated input \(|\psi_C\rangle\) tends to initially oscillate between bunching and antibunching, as was shown in Figs. 4(a)–4(c), and eventually settles into an antibunched state [Fig. 5(a)]. On the other hand, for an anticorrelated biphoton input \(|\psi_A\rangle\) the entangled photons very quickly and irreversibly become bunched and they remain in this state [Fig. 5(b)]. The reason why delocalization itself affects the correlation evolution has to do with the fact that, in this case, the photons tend to eventually escape into the bulk of the lattice—away from the boundary. Simulations indicate that, by adjusting the two oscillation frequencies, one can at will lock the output into a particular bunching or antibunching state. In essence the presence of revivals of quantum states (or absence of revivals) respectively, of the quantity \((l + f)/2\). In order to obtain the correlation between the array modes, we analyze at the output the coincidence rate at waveguides \(p\) and \(q\), which is given by \(\Gamma_{p,q} = (1/W)\sum_{k \neq f} T_{p,k} T_{q,k} |Z|^2\), whereas for anticorrelated \(|\psi_A\rangle\) it becomes \(\Gamma_{p,q} = (1/W)\sum_{k \neq f} T_{p,f} T_{q,l} |Z|^2\). In order to demonstrate these effects and for comparison purposes, we will always assume here that \(W = 10\) with the excitation contained between \((f,d) = (0,0.9)\).
IV. CONCLUSION

In conclusion, we have shown that a new family of dynamic arrays—the so-called Glauber-Fock oscillator lattices—can be used as a way to mold the quantum evolution of path-entangled photons. In these systems, revivals and dynamic delocalization are possible—each leaving a specific mark on the correlation map. If the two oscillation periods associated with these Bloch-like oscillators are irrational with respect to each other, the dynamics become aperiodic. At this point several intriguing questions remain. For example, of interest will be to examine how such structures respond to other maximally entangled states (like NOON states) or whether they can be used to synthesize other quantum states of interest. The response of these lattices may be also useful in considering other classes of problems in other physical configurations having similar quantum analogs like those of the Bose-Hubbard or Jaynes-Cummings type with time-varying couplings [16,25].

APPENDIX

In order to obtain the solution to Eq. (1), we use operator techniques. We start by introducing the Schrödinger equation associated with Eq. (1) [26]

\[ i \frac{dT(a^\dagger,a,Z)}{dZ} = \left[ f(Z)(a^\dagger + a) + \lambda a^\dagger a \right] T(a^\dagger,a,Z), \]  

where \( a = \frac{1}{2}(x + \frac{d}{dx}) \) and \( a^\dagger = \frac{1}{2}(x - \frac{d}{dx}) \) are the annihilation and creation operators, respectively, that satisfy the relations [27]

\[ a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger |n\rangle = \sqrt{n+1}|n+1\rangle. \]  

We now introduce a transformation \( V \) which converts \( T(a^\dagger,a,t) \) into its normal form in which all the creation operators appear to the left of the annihilation operators. \( V \) transforms \( T(a^\dagger,a,Z) \) to the left of the annihilation operators. \( V \) transforms the operator differential equation into an equivalent algebraic variable differential equation. By applying \( V \) to Eq. (A1) we obtain

\[ i \frac{d\tilde{T}}{dZ} = \left[ f(Z)\left(\alpha^* + \alpha + \frac{\partial}{\partial \alpha^*}\right) + \lambda \alpha^* \left(\alpha + \frac{\partial}{\partial \alpha^*}\right) \right] \tilde{T}. \]  

This equation is easily solved by assuming a solution of the form

\[ \tilde{T}(\alpha^*,a,Z) = e^{S(a^*,a,Z)}. \]  

With this substitution, Eq. (A3) becomes

\[ i \frac{dS}{dZ} = f(Z)\alpha^* + f(Z)\alpha + \lambda \alpha^* \left(\alpha + \frac{\partial}{\partial \alpha^*}\right) + f(Z) \frac{\partial S}{\partial \alpha^*}. \]  

We now determine \( S \) by demanding a functional form that gives algebraically equivalent terms on both sides of Eq. (A5). Therefore, in this case, \( S \) must be

\[ S(\alpha^*,a,Z) = A(Z) + B(Z)\alpha + C(Z)\alpha^* + D(Z)\alpha^* \alpha, \]  

where the coefficients \( A, B, C, D \) are determined by the initial conditions.
in this case Eq. (A5) becomes

$$i \left( \frac{dA}{dZ} + \frac{dB}{dZ} \alpha + \frac{dC}{dZ} \alpha^* + \frac{dD}{dZ} a^* \alpha \right)$$

$$= \lambda \alpha^* \alpha + \lambda \alpha^*(C + D \alpha)$$

$$+ f(Z)(\alpha + \alpha^*) + f(Z)(C + D \alpha).$$  (A7)

By equating coefficients of similar terms on both sides of Eq. (A7) we readily obtain the solutions for $A(Z), B(Z), C(Z),$ and $D(Z)$; namely,

$$D(Z) = e^{-i\lambda Z} - 1,$n

$$B(Z) = -i \int_0^Z e^{-i\lambda Z'} f(Z') dZ',$$  (A8)

$$C(Z) = -e^{-i\lambda Z} B^*(Z),$$

$$A(Z) = - \int_0^Z \left[ e^{i\lambda Z} f(Z') dZ' \right] f(Z') e^{-i\lambda Z'} dZ'.$$  (A9)

Finally, after performing the Taylor series expansion of the exponentials and using the relations given in Eq. (A2), we obtain the matrix elements given in Eq. (3):

$$T_{k,n}(Z) = \langle n | e^{A} e^{C Z} | e^{D^* a^*} e^{B a} | k \rangle.$$  (A11)

where $B^*(Z)$ represents the complex conjugate of $B(Z)$. Using the normal-ordering symbol $N$, which converts a function of the commuting variables $(\alpha^* \alpha)$ into the same function of the operators $(\alpha^* a)$ (the resulting operator function being in the normal form), we can now introduce the inverse transformation $V^{-1}$ such that [20]

$$T(a^*, a, Z) = V^{-1} [T(\alpha^* \alpha, Z)] = N(e^{A+B a^*+C a^*+D^* a^*}).$$  (A10)

Then, by applying $V^{-1}$ to Eq. (A4), one obtains

$$T(a^*, a, Z) = e^{A} e^{C Z} e^{Da^*} e^{Ba}.$$  (A11)

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