Asymptotic Expansion
for the Magnetoconductance Autocorrelation Function

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Abstract

We complement a recent calculation (P.B. Gossiaux and the present authors, Ann. Phys. (N.Y.) in press) of the autocorrelation function of the conductance versus magnetic field strength for ballistic electron transport through microstructures with the shape of a classically chaotic billiard coupled to ideal leads. The function depends on the total number $M$ of channels and the parameter $t$ which measures the difference in magnetic field strengths. We determine the leading terms in an asymptotic expansion for large $t$ at fixed $M$, and for large $M$ at fixed $t/M$. We compare our results and the ones obtained in the
previous paper with the squared Lorentzian suggested by semiclassical theory.

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1 Introduction

Lately, the transport of ballistic electrons through microstructures with the shape of a classically chaotic billiard has received much attention [1, 2]. One observable investigated both experimentally and theoretically is the conductance autocorrelation function \( C(\Delta B) \). With \( g(B) \) the dimensionless conductance, \( B \) the external magnetic field, and \( \delta g(B) = g(B) - \bar{g}(B) \) the difference between \( g(B) \) and its mean value, \( C \) is defined as \( C(\Delta B) = \frac{\delta g(B) \delta g(B + \Delta B)}{} \). The bars indicate an average which experimentally is taken over the Fermi energy or the applied gate voltage and, in theories which simulate the chaotic billiard in terms of an ensemble of random matrices, over that ensemble. For ideal coupling between leads and microstructure, \( C \) depends only on the number \( M \) of channels in the leads, and on \( t \), a measure of \( \Delta B \) defined below.

The exact dependence of \( C \) on \( M \) and \( t \) is not known. In the semiclassical approximation, applicable for \( M \gg 1 \), \( C \) was found to have the form of a squared Lorentzian [3]. This result was confirmed by calculations using the supersymmetry technique [4, 5]. However, the supersymmetric nonlinear sigma model leads to expressions for \( C(B) \) which so far have resisted all attempts at an exact evaluation. In the actual experiments [6, 7], \( M \) is quite small. These facts and the desire to attain a deeper understanding of the supersymmetry technique prompted us [8] to calculate analytically the leading terms in the asymptotic expansion of \( C \) for small \( t \) at fixed \( M \). In the present paper, we complement this study by calculating analytically the leading terms in the asymptotic expansion of \( C \) for large \( t \) and fixed \( M \), and for large \( M \) and fixed \( t/M \).

Starting point is the random matrix model for the Hamiltonians of the chaotic microstructure for two values of the external field \( B \). Our approach is identical to that taken in Ref. [8]. However, we introduce a new parametrization of the coset manifold suitable for large values of \( t \). We pay particular attention to the Efetov–Wegner terms (Refs. [9] to [16]) generated by this procedure.

In order to save space, we keep the introductory part as brief as possible and refer the reader to our previous paper [8]. In Section 2, we start with the form of the autocorrelation function obtained after averaging over the ensemble, after using the saddle–point approximation, and after integration over the massive modes. Unless otherwise stated, all quantities appearing in that section are defined as in Ref. [8]. The parametrization of the coset
manifold, and the integral theorem used in calculating the asymptotic formulae, are presented in Section 3. Simplications of the integral theorem are discussed in Section 4. The asymptotic formula for large $t$ at fixed $M$ is derived in Section 5. Section 6 contains the conclusions. Additional mathematical details including the derivation of the asymptotic formula for large $M$ at fixed $t/M$ are given in several Appendices.

2 Supersymmetric Form of the Autocorrelation Function

The two magnetic field strengths are denoted by $B^{(1)}$ and $B^{(2)}$. We write $g(B^{(i)}) = g^{(i)}$ with $i = 1, 2$. The autocorrelation function

$$C(t, M) = \delta g^{(1)} \delta g^{(2)} = g^{(1)} g^{(2)} - g^{(1)} g^{(2)}$$

(1)

depends on $M$, the total number of channels in both leads, and on the parameter $t$ which is related to the area $A$ of the billiard by $\sqrt{t} = k |B^{(1)} - B^{(2)}| A/(2\phi_0)$. Here $k$ is a numerical constant of order unity and $\phi_0 = h c/e$ is the elementary flux quantum [17]. For sufficiently large values of $B^{(1)}$ and $B^{(2)}$, we have $g^{(1)} = g^{(2)} = M/2$, while $g^{(1)} g^{(2)}$ is given by [8]

$$g^{(1)} g^{(2)} = \int \mathcal{D}\mu(T) \exp \left( -\frac{t}{2} < (Q \tau_3)^2 > \right) R(Q) \det g^{-M} (1 + Q L).$$

(2)

Here, $Q = T^{-1} L T$. In the sequel, we use brackets $< ... >$ as a shorthand notation for the graded trace. The matrices $T$ belong to the coset space $U(2, 2/4)/ U(2/2) \times U(2/2)$ where $U(2/2) \times U(2/2)$ is the subgroup of matrices in $U(2, 2/4)$ which commute with $L$. The symbol $\mathcal{D}\mu(T)$ denotes the invariant integration measure for the coset space. The 8N dimensional graded matrices $L$ and $\tau_3$ are given by

$$L_{\alpha \alpha', \alpha' \alpha} = (-)^{(1+p)} \delta_{\alpha \alpha'} \delta_{r r'} \delta_{p p'}, \quad (\tau_3)_{\alpha \alpha', \alpha' \alpha} = (-)^{(1+r)} \delta_{\alpha \alpha'} \delta_{r r'} \delta_{p p'}.$$  

(3)

The supersymmetry index $\alpha$ distinguishes ordinary complex (commuting) integration variables ($\alpha = 0$) and Grassmann (anticommuting) variables ($\alpha = 1$). Later, the two values 0 and 1 of the supersymmetry index $\alpha$ will also be denoted by $b$ and $f$, respectively. The index $r = 1, 2$ refers to the two
The index $p = 1, 2$ refers to the retarded and the advanced propagator, respectively. Finally, $R(Q)$ denotes the source term given by the sum

$$R(Q) = \sum_{j=1}^{4} N_j R_j(Q),$$

where $N_1 = M^2/2, N_2 = M^4/4, N_3 = N_4 = M^3/4$, and where

$$R_1(Q) = \langle GI^{(11)} GI^{(22)} > GI^{(12)} GI^{(21)} > ,$$
$$R_2(Q) = \langle GI^{(11)} GI^{(12)} > GI^{(21)} GI^{(22)} > ,$$
$$R_3(Q) = \langle GI^{(11)} GI^{(22)} GI^{(21)} GI^{(12)} > ,$$
$$R_4(Q) = \langle GI^{(11)} GI^{(12)} GI^{(21)} GI^{(22)} > ,$$

with

$$G(Q) = (1 + QL)^{-1}.$$  

The matrices $I^{(rp)}$ are given by

$$I^{(rp)}_{\alpha''\gamma''\gamma',\alpha'\gamma'\gamma} = (-1)^{(1+\alpha')} \delta_{r'r'} \delta_{p'p'} \delta_{\alpha'\alpha''} \delta_{\gamma''\gamma}.$$  

The graded matrices $Q$ can be parametrized in terms of 32 variables, half of them commuting, the others, anticommuting. In calculations of the average two-point function, one typically deals with a total of 16 integration variables. Except for this increase in the number of variables and for the form of the source terms (which are, of course, specific to our problem), the form of our result in Eqs. (1) and (2) is quite standard. In spite of this similarity, the increase in the number of variables renders a full analytical evaluation of the graded integral (4) very difficult. As pointed out in the Introduction, our analytical work is restricted to two asymptotic expansions of this integral. We evaluate the leading term of the asymptotic expansion of $C(t, M)$ in inverse powers of $t$ and fixed $M$, and the leading term of the asymptotic expansion of $C(t, M)$ in inverse powers of $M$ and fixed $t/M$. Progress in this calculation depends crucially on the proper choice of the 32 variables used in parametrizing the matrices $Q$.

3 Parametrization of the Saddle–Point Manifold

In order to calculate the autocorrelation function for large $t$, we need a new parametrization of the saddle–point manifold different from the one used in
our previous paper \[8\]. Motivated by the paper by Altland, Iida and Efetov \[18\], we write our coset matrices \( T \) as products of matrices \( T_I \) obtained by exponentiating the coset generators anticommuting with \( \tau_3 \), and matrices \( T_0 \) obtained by exponentiating the coset generators commuting with \( \tau_3 \),

\[
T = T_I T_0 ,
\]

(8)

where \( T_0 \) and \( T_I \) are the matrices

\[
T_0 = \begin{pmatrix}
t_{11} & t_{12} & 0 & 0 \\
t_{21} & t_{22} & 0 & 0 \\
0 & 0 & t_{11} & t_{12} \\
0 & 0 & t_{21} & t_{22}
\end{pmatrix}, \quad T_I = \begin{pmatrix}
t_{11} & 0 & 0 & t_{12} \\
0 & t_{22} & t_{21} & 0 \\
0 & t_{12} & t_{11} & 0 \\
t_{21} & 0 & 0 & t_{22}
\end{pmatrix},
\]

(9)

with

\[
t_q^{21} = k(t_q^{12})^\dagger , \quad t_{11} = (1 + t_q^{12}t_q^{21})^{1/2} , \quad t_{22} = (1 + t_q^{21}t_q^{12})^{1/2}
\]

(10)

for \( q = 1, 2, 3, 4 \) and

\[
k = \text{diag}(1, -1) .
\]

(11)

In writing the matrices we label the rows of matrices of dimensions 2, 4, and 8 by the indices \( \alpha, (p\alpha) \) and \( (r\rho\alpha) \), respectively. The indices follow in lexicographical order. The matrices of dimensions 4 and 8 are presented in block form. By construction, the matrix \( T_0 \) (\( T_I \)) does (does not) commute with \( \tau_3 \). Eq. (9) shows that the matrices \( T_0 \) and \( T_I \) are constructed from the GUE coset matrices

\[
T_q = \begin{pmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{pmatrix} ,
\]

(12)

the matrix \( T_0 \) from the matrices \( T_1 \) and \( T_2 \), the matrix \( T_I \) from \( T_3 \) and \( T_4 \). The matrix elements \( (t_q^{12})_{\alpha\alpha'} \) and their conjugates \( (t_q^{12})^{*}_{\alpha\alpha'} \) represent the Cartesian coordinates of \( T_q \). We denote this set of coordinates by \( x_q \). The invariant measure for integration over the coset matrices \( T = T_I T_0 \) has the form

\[
\mathcal{D}\mu(T) = \prod_q \mathcal{D}\mu_G(T_q) \mathcal{P}(T_3, T_4) ,
\]

(13)

where \( \mathcal{D}\mu_G(T_q) \) is the invariant measure for integration over the GUE coset matrices \( T_q \), and \( \mathcal{P}(T_3, T_4) \) is a function of eigenvalues of \( t_3^{12}t_3^{21} \) and \( t_4^{12}t_4^{21} \) given later. In Cartesian coordinates,

\[
\mathcal{D}\mu_G(T_q(x_q)) = \prod_{\alpha\alpha'} d(t_q^{12})_{\alpha\alpha'}d(t_q^{12})^{*}_{\alpha\alpha'} .
\]

(14)
In spite of the simplicity of this expression, the Cartesian coordinates are not well suited for our calculation. Therefore, we follow Efetov [9] and change to polar coordinates.

### 3.1 Polar Coordinates

We introduce polar coordinates $z_r = (\theta^r, \phi^r, \theta^f, \phi^f, \gamma^1_r, \gamma^1_r^*, \gamma^2_r, \gamma^2_r^*)$ for the matrix $T_0$ by the transformation

$$
t^{12}_r = u^1_r \lambda^{12}_r (u^2_r)^{-1}, \quad t^{21}_r = u^2_r \lambda^{21}_r (u^1_r)^{-1}
$$

(15)

where $u^p_r, \lambda^{12}_r, \lambda^{21}_r$ are defined by

$$
u^1_r = \exp \left( \begin{array}{ccc} 0 & \gamma^1_r \\ \gamma^{1*}_r & 0 \end{array} \right), \quad u^2_r = \exp \left( \begin{array}{ccc} 0 & \gamma^{2*}_r \\ \gamma^2_r & 0 \end{array} \right),
$$

(16)

$$
\lambda^{12}_r = i \sin(\theta_r/2)e^{i\phi_r}, \quad \lambda^{21}_r = i \sin(\theta_r/2)e^{-i\phi_r},
$$

(17)

$$
\theta_r = \text{diag}(\theta^r, \theta^f), \quad \phi_r = \text{diag}(\phi^r, \phi^f).
$$

(18)

The coordinates $\theta^r, \phi^r$ are commuting variables, the coordinates $\gamma^p_r, \gamma^{p*}_r$ anticommuting variables. The relation $t^{21}_r = k(t^{12}_r)^*$ implies $\lambda^{21}_r = k(\lambda^{12}_r)^*$ so that $\theta_r = -k\theta^*_r$ whereas $\phi_r = \phi^*_r$. Substituting Eqs. (15) into Eqs. (13), (14) yields

$$
T_0 = U_0 \Lambda_0 U_0^{-1}
$$

(19)

where

$$
U_0 = \text{diag}(u^1_1, u^2_1, u^1_2, u^2_2), \quad \Lambda_0 = \begin{pmatrix} 
\lambda^{11}_1 & \lambda^{12}_1 & 0 & 0 \\
\lambda^{21}_1 & \lambda^{22}_1 & 0 & 0 \\
0 & 0 & \lambda^{11}_2 & \lambda^{12}_2 \\
0 & 0 & \lambda^{21}_2 & \lambda^{22}_2 
\end{pmatrix},
$$

(20)

with $\lambda^{11}_r = \lambda^{22}_r = \cos(\theta_r/2)$. We insert this expression for $T_0$ in $T = T_I T_0$ and get

$$
T = U_0 \tilde{T}_I \Lambda_0 U_0^{-1},
$$

(21)

where

$$
\tilde{T}_I = U_0^{-1} T_I U_0
$$

(22)
has the same form as the matrix $T_I$ but with $t^{pp',q}_3, t^{pp',q}_4$ replaced by

$$
\bar{t}^{pp',q}_3 = (u^p_p)^{-1} t^{pp',q}_3 u^{p'}_{p'}, \quad \bar{t}^{pp',q}_4 = (u^p_p)^{-1} t^{pp',q}_4 u^{p'}_{p'}.
$$

(23)

The indices $\hat{i}$ are defined by $\hat{i} = 2, 1$ for $i = 1, 2$ respectively. We replace the Cartesian coordinates $x_s$ of $T_s$ by the Cartesian coordinates $\tilde{x}_s$ of $\tilde{T}_s$. The Berezinian of this coordinate transformation is equal to one. Since for $s = 3, 4$ the matrices $\tilde{t}_s^{12} \tilde{t}_s^{21}$ and $t_s^{12} t_s^{21}$ have the same eigenvalues, the measure has the same form in the old and in the new coordinates. We suppress the tildes. Finally we pass from the Cartesian coordinates $x_s$ of $T_I$ to the polar coordinates $z_s = (\theta^b_s, \theta^f_s, \phi^b_s, \phi^f_s, \gamma^1_s, \gamma^2_s, \gamma^3_s, \gamma^4_s)$ . We do this in the same way as in the transformation from $x_r$ to $z_r$. Thus we get

$$
T_I = U_I \Lambda_I U_I^{-1},
$$

(24)

where $U_I, \Lambda_I$ denote the counterparts of $U_0, \Lambda_0$ defined by

$$
U_I = \text{diag}(u^1_3, u^2_4, u^1_4, u^2_3), \quad \Lambda_I = \begin{pmatrix}
\lambda^1_3 & 0 & 0 & \lambda^2_3 \\
0 & \lambda^2_4 & \lambda^2_3 & 0 \\
0 & \lambda^1_4 & \lambda^1_3 & 0 \\
\lambda^2_3 & 0 & 0 & \lambda^2_3
\end{pmatrix},
$$

(25)

with $\lambda^1_3 = \lambda^2_3 = \cos(\theta_s/2)$. Collecting the results yields the matrix $T$ as function $T = T(z)$ of the polar coordinates $z = (z_1, z_2, z_3, z_4)$,

$$
T = U_0 U_I \Lambda_I U_I^{-1} \Lambda_0 U_0^{-1}.
$$

(26)

For all $q$, the matrices $u^p_q, \lambda^1_q$ and $\lambda^2_q$ have the form shown in Eqs. (16) and (17). By construction, the matrices $U_0, \Lambda_0$ depend solely on the coordinates $z_{12} = (z_1, z_2)$, the matrices $U_I, \Lambda_I$ solely on the coordinates $z_{34} = (z_3, z_4)$. Moreover, $U_0, U_I$ depend only on the anticommuting variables, $\Lambda_0, \Lambda_I$ only on the commuting variables. Substituting Eq. (24) into $Q = T^{-1}LT$ and making use of the commutativity of $U_0, U_I$ with $L$ yields

$$
Q = U_0 \Lambda_0^{-1} U_I \Lambda_I^{-1} L \Lambda_I U_I^{-1} \Lambda_0 U_0^{-1}.
$$

(27)

This is the expression of the matrix $Q$ in polar coordinates.

The integration measure $d\mu (z) = D\mu (T(z))$ has the form

$$
d\mu (z) = \prod_q d\mu_G (z_q) \rho (\theta_{34})
$$

(28)
where the $d\mu_G(z_q) = D\mu_G(T_q(z_q))$ denote the GUE measures given by

\[ d\mu_G(z_q) = d[z_q]\rho_G(\theta_q) \]  \hspace{1cm} (29)

with

\[ d[z_q] = d[\theta_q]d[\chi_q] , \quad \rho_G(\theta_q) = \prod_{q} \sin \theta_q^\alpha < \cos \theta_q >^{-2} , \]  \hspace{1cm} (30)

\[ d[\theta_q] = \prod_{q\alpha} d\theta_q^\alpha , \quad d[\chi_q] = \prod_{q\alpha} d\phi_q^\alpha \prod_p d\gamma_q^p d\gamma_q^{p*} , \]  \hspace{1cm} (31)

and where $\rho(\theta_{34}) = \mathcal{P}(T_3(z_3), T_4(z_4))$ denotes the density

\[ \rho(\theta_{34}) = \prod_q (\cos \theta_3^\alpha + \cos \theta_4^\alpha)^2 \prod_{q,\alpha\alpha'} (\cos \theta_3^\alpha + \cos \theta_4^\alpha')^{-2} . \]  \hspace{1cm} (32)

From these equations,

\[ d\mu(z) = d[z] \prod_q \rho_G(\theta_q) \rho(\theta_{34}) , \quad d[z] = \prod_q d[z_q] . \]  \hspace{1cm} (33)

The ordinary parts of the angles $\theta^h_q$ are integrated over the positive imaginary axis, the ordinary parts of the phases $\phi^a_q$ over an interval of length $2\pi$. The ordinary parts of $\theta^f_q$ are integrated over the interval $(0, \pi)$, those of $\theta^f_q$ over the interval $(0, \pi/2)$.

An essential simplification of the calculation results from the observation that the matrices $T$ possess a high degree of symmetry. We consider similarity transformations induced by the idempotent permutation matrices

\[ J_{12} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} , \quad J_{34} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} . \]  \hspace{1cm} (34)

It is easy to see that $U_0, \Lambda_0, U_f$ and $\Lambda_f$ satisfy the symmetry relations

\[ J_{12}U_0(z_{12})J_{12} = U_0(w_{21}) , \quad J_{12}\Lambda_0(z_{12})J_{12} = \Lambda_0(w_{21}) , \]  \hspace{1cm} (35)

\[ J_{12}U_f(z_{34})J_{12} = U_f(w_{34}) , \quad J_{12}\Lambda_f(z_{34})J_{12} = \Lambda_f(w_{34}) , \]  \hspace{1cm} (36)

\[ J_{34}U_0(z_{12})J_{34} = U_0(w_{12}) , \quad J_{34}\Lambda_0(z_{12})J_{34} = \Lambda_0(w_{12}) , \]  \hspace{1cm} (37)

\[ J_{34}U_f(z_{34})J_{34} = U_f(w_{43}) , \quad J_{34}\Lambda_f(z_{34})J_{34} = \Lambda_f(w_{43}) , \]  \hspace{1cm} (38)
where \( w_q = (\theta_q^b, \psi_q^b, \theta_q^f, \psi_q^f, \beta_q^1, \bar{\beta}_q^1, \beta_q^2, \bar{\beta}_q^2) \) represents a second set of polar coordinates defined by

\[
\psi_q^\alpha = -\phi_q^\alpha, \quad \beta_q^p = \gamma_q^p, \quad \bar{\beta}_q^p = \gamma_q^p.
\] (37)

For later developments it is important to consider the coordinates \( w_q \) on the same footing as the coordinates \( z_q \). The Berezinian of the coordinate transformation from \( z \) to \( w \) is equal to unity, and the coordinates \( w \) are integrated with the same measure over the same domain as the coordinates \( z \). Inserting the symmetry relations (35) and (36) into Eq. (26) yields the symmetry relations

\[
T(z_{12}, z_{34}) = J_{12} T(w_{21}, w_{34}) J_{12} = J_{34} T(w_{12}, w_{43}) J_{34}.
\] (38)

Since the permutation matrices \( J_{12}, J_{34} \) anticommute with the matrix \( L \), the corresponding symmetry relations for the matrices \( Q = T^{-1} LT \) read

\[
Q(z_{12}, z_{34}) = -J_{12} Q(w_{21}, w_{34}) J_{12} = -J_{34} Q(w_{12}, w_{43}) J_{34}.
\] (39)

### 3.2 Integral Theorem

Eqs. (29) and (30) show that the measures \( d\mu_G(z_q) \) contain nonintegrable singularities located at \(< \cos \theta_q >= 2(|\lambda_q^b|^2 + |\lambda_q^f|^2) = 2 < t_q^{12} t_q^{21} >= 0 \). As we evaluate the integral

\[
\mathcal{I}[F] = \int \mathcal{D}\mu(T) F(Q)
\] (40)

for some function \( F(Q) \) over the coset space in polar coordinates, the singularities cause the occurrence of additional terms (the Efetov–Wegner terms). These terms can be found by applying the method used in Ref. \[8\]. We write the integral in Cartesian coordinates and exclude an infinitesimal neighbourhood \(|(t_q^{12})_{11}|^2 + |(t_q^{12})_{22}|^2 < \varepsilon \) of singularities of \( d\mu_G(z_q) \) from the domain of integration. Making use of Berezin’s theory \[13\] of coordinate transformations in superintegrals over functions which do not vanish on the boundary of the integration region, we change to polar coordinates, let \( \varepsilon \) go to \( 0^+ \), and obtain

\[
\mathcal{I}[F] = \int d[z] \rho(\theta_{34}) \prod_q \left( \delta(z_q) + \rho_G(\theta_q) P(z_q) \right) F(Q).
\] (41)

Here \( \delta(z_q) \) denotes the \( \delta \) function of the polar coordinate \( z_q \), and \( P(z_q) \) is the projector on the subspace of functions of 4th order in the anticommuting
coordinates $\gamma_q^1, \gamma_q^{1*}, \gamma_q^2$ and $\gamma_q^{2*}$. For a function $f(z_q)$ of $z_q$ which is regular at $z_q = 0$ and whose Taylor expansion in the anticommuting variables terminates with the 4th order term $f_4(z_q)$, we have

$$\int d[z_q] \delta(z_q) f(z_q) = f(0), \quad P(z_q) f(z_q) = f_4(z_q).$$

(42)

The product over the four factors $\left(\delta(z_q) + \rho_G(\theta_q)P(z_q)\right)$ in the integral (41) yields a sum of sixteen terms. We specify these terms by the indices $a = (a_1a_2a_3a_4)$. We put $a_q = 0$ ($a_q = 1$) if the term contains $\delta(z_q)$ ($\rho_G(\theta_q)P(z_q)$, respectively). This yields

$$\mathcal{I}[F] = \sum_{a_1,a_2,a_3,a_4=0,1} \mathcal{I}_{a_1a_2a_3a_4}[F]$$

(43)

where

$$\mathcal{I}_{a_1a_2a_3a_4}[F] = \int d[z]\rho_{a_1a_2a_3a_4}(\theta_{12}, \theta_{34}) \prod_q^{(0)} \delta(z_q) \prod_q^{(1)} P(z_q) F(Q)$$

(44)

with

$$\rho_{a_1a_2a_3a_4}(\theta_{12}, \theta_{34}) = \prod_q^{(1)} \rho_G(\theta_q)\rho(\theta_{34}).$$

(45)

The product denoted by $\prod_q^{(0)} (\prod_q^{(1)})$ extends over those values of $q$ for which $a_q = 0$ ($a_q = 1$, respectively) and equals unity if none of the $a_q$'s meets the definition. Eq. (43) is the desired decomposition of the integral (41) into surface and volume terms. The integral $\mathcal{I}_{1111}[F]$ extends over the entire coset space and represents the volume term. The remaining fifteen integrals represent the boundary or Efetov–Wegner terms. The term $\mathcal{I}_{0000}[F]$ is equal to the value of $F(Q(z))$ at $z = 0$, $\mathcal{I}_{0000}[F] = F(L)$. For brevity, we use the symbol $\mathcal{I}_a$ for the integrals $\mathcal{I}_{a_1a_2a_3a_4}$.

4 The Efetov–Wegner Terms

In our case, the integrand $F$ in the integral $\mathcal{I}[F]$ has the form given in Eqs. (2) to (6). We write $F$ as the product

$$F = K R D,$$

(46)
where $K$ denotes the coupling term,

$$K = e^{-(t/2)<(Q\tau_3)^2>},$$  \hspace{1cm} (47)

$R$ denotes the source term \textbullet, and $D$ denotes the function

$$D = \det g^{-1} (1 + Q L) = e^{-M<\ln(1 + Q L)>.}$$  \hspace{1cm} (48)

We show that for this form of $F$ only four of the integrals $\mathcal{I}_a[F]$ give a nonvanishing contribution.

### 4.1 The Integrals $\mathcal{I}_a[F]$

We rewrite Eq. (27) for $Q$ in the form

$$Q = U_0 \Lambda_0^{-1} Q_I \Lambda_0 U_0^{-1}, \quad Q_I = U_I \Lambda_I^{-1} L \Lambda_I U_I^{-1}$$ \hspace{1cm} (49)

and use this form to rewrite the terms $K$, $R$, and $D$. For the coupling term $K$, we use that $U_0, \Lambda_0$ and $U_I$ commute with $\tau_3$. As a consequence, we have $< (Q \tau_3)^2 >= < (Q_I \tau_3)^2 >= < (\Lambda_I^{-1} L \Lambda_I \tau_3)^2 >$. Using Eqs. (25) and (17), we find

$$K = e^{-\frac{t}{2} <(Q_I \tau_3)^2>} = e^{2t \sum_s <\sin^2 \theta_s>}. \hspace{1cm} (50)$$

The exponent of the coupling term $K$ thus depends only on the angles $\theta_s$. The functions $R$ and $D$ both depend on $Q$ via the matrix $(1 + Q L)^{-1}$. We write $Q_I = L + \delta Q_I$ and use Eq. (19). We note that $U_0$ commutes with $L$, introduce the matrix $G_0 = (1 + \Lambda_0^{-1} L \Lambda_0 L)^{-1}$, note that $G_0$ commutes with $\Lambda_0^{-1}$ and that $G_0 \Lambda_0^{-1}$ is diagonal and therefore commutes with $L$. We find

$$(1 + Q L)^{-1} = U_0 \Lambda_0^{-1} G_0 (1 + \delta W)^{-1} \Lambda_0 U_0^{-1}$$ \hspace{1cm} (51)

where

$$\delta W = \delta Q_I G_0 L \hspace{1cm} (52)$$

with

$$\delta Q_I = Q_I - L , \quad G_0 = (1 + \Lambda_0^{-1} L \Lambda_0 L)^{-1}. \hspace{1cm} (53)$$

Inserting the expression for $(1 + Q L)^{-1}$ in the defining Equation (3) for $R$ we find that the anticommuting variables $\gamma_\mu^p, \gamma^\mu_p$ appear only via the matrices

$$I_0^{(rp)} = U_0^{-1} I^{(rp)} U_0 \hspace{1cm} (54)$$
The explicit form of these matrices as functions of $\gamma_p^p, \gamma_s^{ps}$ can be found with the help of Eqs. (16) and (7). We obtain
\[ \prod_{q}^{(0)} \delta(z_q) \prod_{q}^{(1)} P(z_q) RD = \prod_{q}^{(0)} \delta(z_q) \prod_{q}^{(1)} P(z_q) SD , \] (55)
where
\[ S = \sum_j N_j S_j \] (56)
and
\[ S_1 = <V I_0^{(11)} V I_0^{(22)}>, \]
\[ S_2 = <V I_0^{(11)} V I_0^{(12)}>, \]
\[ S_3 = <V I_0^{(11)} V I_0^{(22)} V I_0^{(21)} V I_0^{(12)}>, \]
\[ S_4 = <V I_0^{(11)} V I_0^{(12)} V I_0^{(21)} V I_0^{(22)}>, \] (57)
with
\[ V = (1 + \delta W)^{-1} G_0 . \] (58)
The anticommuting variables $\gamma_p^p, \gamma_s^{ps}$ appear only in the matrix $\delta W$. For $D$, we use Eqs. (18), (51), (58), (20) and (17), and obtain
\[ D = D_0 e^{-M<\ln(1+\delta W)>} , \] (59)
where
\[ D_0 = e^{-M<\ln(1+\Lambda_0^{-1} L \Lambda_0 L)>} = e^{-M \sum \phi <\ln(1+\cos \theta_r)>} \] (60)
depends only on $\theta_r$. Again, the anticommuting variables appear only in $\delta W$. For the integral $I_a[F]$, we have
\[ I_a[F] = \int \frac{d[z]}{\rho_a} \prod_{q}^{(0)} \delta(z_q) \prod_{q}^{(1)} P(z_q) KSD , \] (61)
with $K$, $S$ and $D$ given by Eqs. (50), (54) and (59).

For later use we also give explicit expressions for $\delta Q_I$ and $G_0$ in terms of $\theta_q, \phi_q$ and $u_p^p$. Using the formulae for $U_0, \Lambda_0, U_I, \Lambda_I$ in Eqs. (20), (25) and (17) yields
\[ \delta Q_I = \begin{pmatrix} \delta Q_3^{11} & 0 & 0 & \delta Q_3^{12} \\ 0 & \delta Q_4^{22} & \delta Q_4^{21} & 0 \\ 0 & \delta Q_4^{12} & \delta Q_4^{11} & 0 \\ \delta Q_3^{21} & 0 & 0 & \delta Q_3^{22} \end{pmatrix}, \quad G_0 = \frac{1}{2} \begin{pmatrix} G_1^{11} & G_1^{12} & 0 & 0 \\ G_1^{21} & G_1^{22} & 0 & 0 \\ 0 & 0 & G_2^{11} & G_2^{12} \\ 0 & 0 & G_2^{21} & G_2^{22} \end{pmatrix} , \] (62)
where $\delta Q_{s \rho}^{\rho' j}$, $G_{r \rho}^{\rho' j}$ denote the matrices

$$
\delta Q_{s}^{11} = u_{s}^{1} (\cos \theta_s - 1) (u_{s}^{1})^{-1}, \quad \delta Q_{s}^{12} = i u_{s}^{1} \sin \theta_s e^{i \phi_{s}} (u_{s}^{2})^{-1},
$$

$$
\delta Q_{s}^{21} = -i u_{s}^{2} \sin \theta_s e^{-i \phi_{s}} (u_{s}^{1})^{-1}, \quad \delta Q_{s}^{22} = u_{s}^{2} (1 - \cos \theta_s) (u_{s}^{2})^{-1}, \quad (63)
$$

$$
G_{r}^{1} = i \tan(\theta_r/2) e^{i \phi_r}, \quad G_{r}^{2} = i \tan(\theta_r/2) e^{-i \phi_r}, \quad G_{pp}^{\rho} = 1. \quad (64)
$$

### 4.2 Symmetries of $I_{a}[F]$

The symmetry properties of our coset matrices imply symmetries of the integrals $I_{a}[F]$. We use Eq. (61) for $I_{a}[F]$, the symmetry properties in Eqs. (63) of $U_{0}, \Lambda_{0}, U_{1}, \Lambda_{1}$, and we pass from the coordinates $z$ to the coordinates $w$. We conclude that

$$
S_{j}(z_{12}, z_{34}) = S_{j'}(w_{21}, w_{34}) \quad (65)
$$

where $j' = 1, 2, 4, 3$ for $j = 1, 2, 3, 4$ respectively. With $N_{j'} = N_{j}$, this yields $S(z_{12}, z_{34}) = S(w_{21}, w_{34})$. Similarly, we conclude that

$$
D(z_{12}, z_{34}) = D(w_{21}, w_{34}). \quad (66)
$$

\[\text{From Eq. (65) we have } \rho_{a_{1}a_{2}a_{3}a_{4}}(\theta_{12}, \theta_{34}) = \rho_{a_{2}a_{1}a_{3}a_{4}}(\theta_{21}, \theta_{34}), \text{ so that}
\]

$$
I_{a_{1}a_{2}a_{3}a_{4}}[F] = \int d[w] \rho_{a_{2}a_{1}a_{3}a_{4}}(\theta_{21}, \theta_{34}) \times \prod_{q}^{(0)} \delta(w_{q}) \prod_{q}^{(1)} P(w_{q}) K(\theta_{34}) S(w_{21}, w_{34}) D(w_{21}, w_{34}). \quad (67)
$$

The coordinates $w_{2}$ are integrated over the same domain as the coordinates $w_{1}$, and the coordinates $w_{q}$ over the same domain as their counterparts $z_{q}$. Therefore, the integral on the r.h.s. of Eq. (67) equals the integral $I_{a_{1}a_{2}a_{3}a_{4}}[F]$. Using the symmetry properties in Eqs. (66), we can repeat the argument to show that $S(z_{12}, z_{34}) = S(w_{12}, w_{34})$ and $D(z_{12}, z_{34}) = D(w_{12}, w_{34})$. We recall that $\rho_{a_{1}a_{2}a_{3}a_{4}}(\theta_{12}, \theta_{34}) = \rho_{a_{1}a_{2}a_{4}a_{3}}(\theta_{12}, \theta_{43})$ and find that in this case the coordinate transformation yields the integral $I_{a_{1}a_{2}a_{4}a_{3}}[F]$. Combining these results shows that the integrals $I_{a}[F]$ satisfy the symmetry relations

$$
I_{a_{1}a_{2}a_{3}a_{4}}[F] = I_{a_{1}a_{2}a_{3}a_{4}}[F] = I_{a_{1}a_{2}a_{4}a_{3}}[F] = I_{a_{2}a_{1}a_{4}a_{3}}[F]. \quad (68)
$$

These symmetry relations simplify the sum over the Efetov–Wegner terms in Eq. (63) to

$$
I[F] = I_{1111}[F] + 2I_{1110}[F] + 2I_{1011}[F] + I_{1100}[F]
+ 4I_{1010}[F] + I_{0011}[F] + 2I_{1000}[F] + 2I_{0010}[F] + I_{0000}[F]. \quad (69)
$$
4.3 Vanishing $\mathcal{I}_a[F]$

The sum in Eq. (39) simplifies even further: Due to the vanishing of some of the $\mathcal{I}_a[F]$’s, it reduces to four terms. We first consider the case $a = (1110)$. The integration over $\delta(z_4)$ yields $z_4 = 0$. Using Eqs. (62) to (64) for $\delta Q_I$ and $G_0$, we find that

$$V = \frac{1}{2} \begin{pmatrix}
1 & x_1 & 0 & u_3^1 x_3 (u_3^2)^{-1} y_2 \\
x_1 & 1 & 0 & 0 \\
0 & 0 & 1 & x_2 \\
u_3^2 x_3 (u_3^1)^{-1} y_1 & 0 & \bar{x}_2 & 1
\end{pmatrix}, \quad (70)$$

and that

$$\text{det} \left(1 + \delta W\right) = (1 - x_3^f \bar{x}_3^f) (1 - x_3^b \bar{x}_3^b)^{-1}. \quad (71)$$

Here, $x_q, \bar{x}_q$ and $y_q$ denote the matrices

$$x_q = i \tan(\theta_q/2) e^{i\phi_q}, \bar{x}_q = i \tan(\theta_q/2) e^{-i\phi_q}, y_q = 1 - x_q \bar{x}_q. \quad (72)$$

The only part of $V$ which contributes to $I_0^{(rp)} V I_0^{(r'p')}$ is the block $V_{r,p,r'p'}$. According to Eq. (71), $V_{12,21} = V_{21,12} = 0$, and the only nonzero part of $S$ is the term $N_2 S_2$. However, $S_2$ contains neither $\gamma_3^p$ nor $\gamma_3^{p*}$, and Eq. (71) shows that the same is true for the function $D$. Thus $P(z_3) S D = 0$, and the integral $\mathcal{I}_{1110}[F]$ is equal to zero. For $a = (1010)$, $z_2 = z_4 = 0$ and, therefore, $V_{12,21} = V_{21,12} = V_{21,22} = V_{22,21} = 0$. Thus $S = 0$, and $\mathcal{I}_{1010}[F] = 0$. For the same reason, $\mathcal{I}_{1000}[F] = \mathcal{I}_{0010}[F] = \mathcal{I}_{0000}[F] = 0$. The sum in Eq. (39) simplifies to

$$\mathcal{I}[F] = \mathcal{I}_{1111}[F] + 2 \mathcal{I}_{1011}[F] + \mathcal{I}_{1100}[F] + \mathcal{I}_{0011}[F]. \quad (73)$$

The term $\mathcal{I}_{1100}[F]$ is independent of $t$ and yields the limit $t \to \infty$ of $\mathcal{I}[F]$. For this term, $K = 1$, $D = D_0$, and $S = N_2 S_2 = N_2 \Pi_{r,p} \gamma_r^{p*} \gamma_r^p \Pi_r < x_r \bar{x}_r >$. Integrating over $\phi_r, \gamma_r$ and $\gamma_r^*$ we are left with

$$I_{1100}[F] = (M^4/4) \int \prod_r d[\theta_r] \prod_r \rho_G(\theta_r) < \tan^2(\theta_r/2) > D_0 = M^2/4. \quad (74)$$

This is the disconnected part of $g^{(1)} g^{(2)}$. Thus, the autocorrelation function $C(t, M)$ is given by

$$C(t, M) = \mathcal{I}_{1111}[F] + 2 \mathcal{I}_{1011}[F] + \mathcal{I}_{0011}[F]. \quad (75)$$

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5 The Limit of Large $t$ at Fixed $M$

In this section, we expand the autocorrelation function $C(t, M)$ for fixed $M$ asymptotically in inverse powers of $t$, and evaluate the leading term. In the limit $t \gg 1$, the integrals $\mathcal{I}_a[F]$ appearing on the r.h.s. of Eq. (73) are dominated by the contribution of the neighbourhood of the surface $\theta_s^\alpha = 0$ where $Q_I = L$, and where the graded trace in the exponent of the coupling term

$$K = e^{-(t/2)\langle Q_I \tau_3 \rangle^2}$$

is equal to zero.

5.1 Asymptotic Expansion

We start from Eq. (61), introduce the rescaled angles $\tilde{\theta}_s^\alpha = t^{1/2} \theta_s^\alpha$, and pass from the coordinates $z_s$ to the coordinates $\tilde{z}_s = (\tilde{\theta}_s^b, \phi_s^b, \phi_s^f, \gamma_s^1, \gamma_s^2, \gamma_s^3)$ with $d[z_s] = d[\tilde{z}_s] t^{-1}$ and $P(z_s) = P(\tilde{z}_s)$. We expand $t^{-2} \rho_a, K, V$ and $D$ in powers of $t^{-1/2}$,

$$t^{-2} \rho_a = \sum_{n_\rho} t^{-n_\rho/2} \rho_a^{(n_\rho)} , \quad K = \sum_{n_k} t^{-n_k/2} K^{(n_k)} ,$$

$$V I_0^{(rp)} = \sum_{n_{rp}} t^{-n_{rp}/2} V^{(n_{rp})} I_0^{(rp)} , \quad D = \sum_{n_d} t^{-n_d/2} D^{(n_d)} .$$

The coefficients $\rho_a^{(n_\rho)}, K^{(n_k)}, V^{(n_{rp})}, D^{(n_d)}$ are functions of $z_r, \tilde{z}_s$. Collecting terms of the same order in $t^{-1/2}$, we get

$$\mathcal{I}_a[F] = \sum_n t^{-n/2} \mathcal{I}_a^{(n)}[F] .$$

Here $\mathcal{I}_a^{(n)}[F]$ denotes the sum of integrals

$$\mathcal{I}_a^{(n)}[F] = \sum_j \sum_{n_\rho n_k n_{12} n_{21} n_{22}} \mathcal{I}_a^{(n_\rho; n_k; n_{12}; n_{21}; n_{22})}[F_j]$$

with

$$\mathcal{I}_a^{(n_\rho; n_k; n_{12}; n_{21}; n_{22})}[F_j] = \int \prod_r d[z_r] \prod_s d[\tilde{z}_s] \rho_a^{(n_\rho)}$$

$$\times \prod_r (0) \delta(z_r) \prod_r (1) P(z_r) \prod_s P(\tilde{z}_s) K^{(n_k)} N_j \delta_j^{(n_\rho)} D^{(n_d)} .$$

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Here, \( n_\sigma = (n_{11} n_{12} n_{21} n_{22}) \), and \( S_j^{(n_{\sigma})} \) are given by Eqs. (57) with \( V I_0^{(\rho \rho)} \) replaced by \( V (n_{\rho}) I_0^{(\rho \rho)} \),

\[
S_1^{(n_{\sigma})} = < V^{(n_{11})} I_0^{(11)} V^{(n_{22})} I_0^{(22)} > < V^{(n_{12})} I_0^{(12)} V^{(n_{21})} I_0^{(21)} > ,
\]
\[
S_2^{(n_{\sigma})} = < V^{(n_{11})} I_0^{(11)} V^{(n_{12})} I_0^{(12)} > < V^{(n_{21})} I_0^{(21)} V^{(n_{22})} I_0^{(22)} > ,
\]
\[
S_3^{(n_{\sigma})} = < V^{(n_{11})} I_0^{(11)} V^{(n_{22})} I_0^{(22)} V^{(n_{21})} I_0^{(21)} V^{(n_{12})} I_0^{(12)} > ,
\]
\[
S_4^{(n_{\sigma})} = < V^{(n_{11})} I_0^{(11)} V^{(n_{12})} I_0^{(12)} V^{(n_{21})} I_0^{(21)} V^{(n_{22})} I_0^{(22)} > .
\]

(81)

The order of \( S_j^{(n_{\sigma})} \) is \( n_\sigma = \sum_{\rho \rho} n_{\rho \rho} \). The sum over \( n_\rho, n_k, n_{\rho \rho} \) and \( n_d \) is restricted by the condition \( n_\rho + n_k + n_\sigma + n_d = n \). Since \( K \) is given by the exponential (78), all integrals contain the exponential factor

\[
K^{(0)} = e^{2 \sum_{\kappa} \langle (\theta_\kappa^s)^2 \rangle} .
\]

(82)

On extending the domain of integration region over \( \tilde{\theta}_s^f \) from zero to infinity, the series (78) yields an asymptotic expansion for \( I_a[F] \). Since \( \rho_a, K, S \) and \( D \) are even functions of \( \theta_\kappa^s \), only terms with even \( n \) appear, and the expansion proceeds in inverse powers of \( t \). Thus,

\[
C(t, M) = \sum_{n=1}^{\infty} t^{-n} \left( I_{1111}^{(2n)}[F] + 2 I_{1011}^{(2n)}[F] + I_{0011}^{(2n)}[F] \right) .
\]

(83)

To calculate the expansion coefficients \( I_a^{(2n)}[F(Q)] \), we use the symmetry properties of \( U_0, \Lambda_0, U_I, \Lambda_I \). It follows (see Appendix 14) that the integrals \( I_a^{(n_{\rho}; n_k; n_{\rho \rho}; n_d)}[F_j] \) satisfy the symmetry relations

\[
I_{a_{12}11}^{(n_{\rho}; n_k; n_{12} n_{21} n_{22} n_\sigma n_d)}[F] = I_{a_{21}11}^{(n_{\rho}; n_k; n_{22} n_{21} n_{12} n_\sigma n_d)}[F_j] = I_{a_{12}11}^{(n_{\rho}; n_k; n_{12} n_{21} n_{22} n_\sigma n_d)}[F_j] = I_{a_{12}11}^{(n_{\rho}; n_k; n_{22} n_{21} n_{12} n_\sigma n_d)}[F] ,
\]

(84)

where \( j' = 1, 2, 4, 3 \) for \( j = 1, 2, 3, 4 \), respectively.

To find the leading term of the expansion we recall two properties of \( I_a^{(n_{\rho}; n_k; n_{\rho \rho}; n_d)}[F_j] \): (i) This expression is nonzero only when \( \sum \rho_s P(z_s) S_j^{(n_{\rho})} D^{(n_d)} \) is nonzero and (ii) the expression does not vanish when integrated over the phases \( \phi_\kappa^s \). Inspecting the explicit expressions of the relevant low–order terms \( S_j^{(n_{\rho})} D^{(n_d)} \) as given by the matrices \( (\delta Q_{s}^{pp'})^{(n)} \) and \( G_{r}^{pp'} \), we find that these conditions are first met for \( n_\sigma + n_d = 4 \). Thus, the leading term \( C_{t \gg 1}(t, M) \) of the expansion is of order \( t^{-2} \) and given by

\[
C_{t \gg 1}(t, M) = t^{-2} \left( I_{1111}^{(4)}[F] + 2 I_{1011}^{(4)}[F] + I_{0011}^{(4)}[F] \right) .
\]

(85)
For each of the $\mathcal{I}_a^{(2)}[F]$ in Eq. (85), the sum on the r.h.s. of Eq. (79) reduces to the terms with $n_\rho = n_k = 0$.

### 5.2 The Leading Term $C_{t\gg 1}(t, M)$

We first calculate the volume integral

$$\mathcal{I}_{1111}^{(4)}[F] = \sum_j \sum_{n_1n_2n_3n_4} \mathcal{I}_{1111}^{(0; n_1, n_2, n_3, n_4)}[F_j].$$

(86)

We suppress the tildes above $z_s$ and $\theta_s$. Inspection of the explicit expressions of the relevant terms $S_j^{(n_4)} D^{(n_4)}$ shows that the nonvanishing contributions to $\sum_s P(z_s) S_j^{(n_4)} D^{(n_4)}$ stem solely from the terms which are linear in all four matrices $\delta Q_s^{(12)}(1)$, $\delta Q_s^{(21)}(1)$ and which for each $r$ contain the same number of $G_r^{(12)}$ and of $G_r^{(21)}$. We restrict the sum over $\mathcal{I}_{1111}^{(0; n_1, n_2, n_3, n_4)}[F_j]$ accordingly, employ the symmetry relations Eq. (80), and find

$$\mathcal{I}_{1111}^{(4)}[F] = \mathcal{I}_{1111}^{(0; 0, 0, 0, 1, 1, 1, 1, 0)}[F_1]$$

$$+ \mathcal{I}_{1111}^{(0; 0, 0, 0, 0, 0, 0, 2, 2)}[F_2] + 2 \mathcal{I}_{1111}^{(0; 0, 0, 0, 0, 2, 2, 2, 0)}[F_2]$$

$$+ 2 \mathcal{I}_{1111}^{(0; 0, 0, 2, 2, 0, 0, 0)}[F_2] + 2 \mathcal{I}_{1111}^{(0; 0, 0, 0, 0, 0, 1, 0, 1, 2)}[F_3] + 4 \mathcal{I}_{1111}^{(0; 0, 0, 0, 0, 1, 2, 1, 0)}[F_3].$$

(87)

The integrals $\mathcal{I}_{1111}^{(0; n_1, n_2, n_3, n_4)}[F_j]$ have the form shown in Eq. (80). Explicit expressions in terms of polar coordinates for the $V^{(n_4)}$, $D^{(n_4)}$ contributing to Eq. (87) are given in Appendix 7.2. Substituting these formulae and applying the projectors $P(z_r)$ yields

$$\mathcal{I}_{1111}^{(4)}[F] = (M^2/64) \int d[z] \rho^{(0)}_{1111} K^{(0)} D_0 \prod_{r \neq r'} \gamma^{p_r p_{r'}} P$$

$$\times \left( \begin{array}{c}
2 < X_3^{(1)} y_2 X_3^{(1)} y_1 > < X_4^{(1)} y_1 X_4^{(1)} y_2 > \\
+ M^4 < y_1 > < y_2 > < X_3^{(1)} \bar{x}_2 X_4^{(1)} \bar{x}_1 > < \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} x_2 > \\
+ 4 M^3 < y_1 > < X_3^{(1)} \bar{x}_2 y_2 X_4^{(1)} \bar{x}_1 > < X_3^{(1)} x_1 X_4^{(1)} x_2 > \\
- 2 M^2 < y_1 > < X_3^{(1)} y_2 \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} y_2 X_4^{(1)} \bar{x}_1 > \\
+ 2 M^2 < X_3^{(1)} \bar{x}_2 X_4^{(1)} \bar{x}_1 y_1 > < \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} x_2 y_2 > \\
+ 2 M^2 < X_3^{(1)} \bar{x}_2 y_2 X_4^{(1)} \bar{x}_1 y_1 > < \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} x_2 > \\
- 4 M < X_3^{(1)} y_2 X_3^{(1)} x_1 X_4^{(1)} y_2 X_4^{(1)} \bar{x}_1 y_1 > \end{array} \right),$$

(88)
where $X_s^{(1)}, \bar{X}_s^{(1)}$ denote the matrices

$$X_s^{(1)} = (\delta Q_s^{12})^{(1)} = i u_s^1 \theta_s e^{i \phi_s (u_s^2)^{-1}},$$

$$\bar{X}_s^{(1)} = (\delta Q_s^{21})^{(1)} = -i u_s^2 \theta_s e^{-i \phi_s (u_s^1)^{-1}},$$

and where

$$x_r = G_r^{12}, \quad \bar{x}_r = G_r^{21}, \quad y_r = 1 - x_r \bar{x}_r$$

are the matrices introduced in Eq. (72). The density $\rho^{(0)}$ is given by

$$\rho^{(0)}_{1111} = \prod_r \rho_G(\theta_r) \prod_s \rho_G^{(0)}(\theta_s), \quad \rho_G^{(0)}(\theta_s) = 4 \prod_\alpha \theta_s^\alpha < (\theta_s)^2 >^{-2},$$

and $P$ denotes the projector $P = \prod_s P(z_s)$ which we suppress in the sequel. First we integrate over $\phi_s, \gamma_s, \gamma_s^*$.

This can be done making use of

$$\int \prod_s d[\chi_s] \langle X_3^{(1)} A_I X_3^{(1)} B_I \rangle < X_4^{(1)} A_{II} \bar{X}_4^{(1)} B_{II} >$$

$$= \prod_s < (\theta_s)^2 > < A_I > < B_I > < A_{II} > < B_{II} > ,$$

$$\int \prod_s d[\chi_s] \langle X_3^{(1)} A_I X_4^{(1)} B_I \rangle < \bar{X}_3^{(1)} A_{II} \bar{X}_4^{(1)} B_{II} >$$

$$= \prod_s < (\theta_s)^2 > < A_I B_{II} > < B_I A_{II} > ,$$

$$\int \prod_s d[\chi_s] \langle X_3^{(1)} A_I \bar{X}_3^{(1)} B_I \bar{X}_4^{(1)} A_{II} X_4^{(1)} B_{II} >$$

$$= \prod_s < (\theta_s)^2 > < A_I > < A_{II} > < B_I B_{II} > .$$

These formulae are valid for all two-dimensional graded matrices $A_I, B_I, A_{II}, B_{II}$ which are independent of $\phi_s, \gamma_s, \gamma_s^*$, and can be verified with the help of

$$\int d_s^p d \gamma^p_s (u_s^p)_{\alpha_1 \alpha_2} (u_s^p)_{\alpha_1' \alpha_2'}^{-1} = (-)^{p+\alpha_2} \delta_{\alpha_1 \alpha_2'} \delta_{\alpha_2 \alpha_1'} (2\pi)^{-1}.$$}

Integrating over $\phi_r, \gamma_r, \gamma_r^*$ then simplifies the volume term to an “eigenvalue” integral over the angles $\theta_q^\alpha$.

$$T_{1111}[F] = (M^2/64) \int \prod_q d[\theta_q] \rho^{(0)}_{1111} K^{(0)} D_0 \prod_s < (\theta_s)^2 >$$

$$\times \left( < y_1 >^2 \left( (M_x^4 + 2M_x^2 + 2) < y_2 >^2 - 4M^3 < y_2 (1 - y_2) > \right) - 4M < y_1 (1 - y_1) > \right) < y_2 >^2 - M < y_2 (1 - y_2) > .$$

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Since the exponentials $K^{(0)}, D_0$ factorize into products of exponentials each depending on one of the $\theta_q$, the integration over angles simplifies to the evaluation of two-dimensional integrals over $\theta_q$. The integrals over $\theta_r$ can be done as in Ref. [8]. With

$$\int d[\theta_r] \rho_G(\theta_r) e^{-M <\ln(1+\cos \theta_r)>} < y_r >^2 = (1-M^2)^{-1},$$

$$\int d[\theta_r] \rho_G(\theta_r) e^{-M <\ln(1+\cos \theta_r)>} < y_r (1-y_r) > = M(1-M^2)^{-1} \quad (95)$$

and

$$\int d[\theta_s] \rho_G^{(0)}(\theta_s) e^{2<(\theta_s)^2>} < (\theta_s)^2 >$$

$$= 4 \int_0^\infty d\theta_s^b \int_0^\infty d\theta_s^f e^{2<(\theta_s)^2>\theta_s^b\theta_s^f} < (\theta_s)^2 >^{-1} = 1/2, \quad (96)$$

the result is

$$I^{(4)}_{1111}[F] = \frac{M^2}{256} \frac{M^4 - 2M^2 + 2}{(M^2 - 1)^2}. \quad (97)$$

The boundary terms can be evaluated in the same way, see Appendix 7.2.

For $a = (1011)$ we have $z_2 = 0$, which reduces the number of contributing integrals. We find

$$I^{(4)}_{1011}[F] = I^{(0;0;1111;0)}_{1011}[F_1] + I^{(0;0;0022;0)}_{1011}[F_2] + 2I^{(0;0;0121;0)}_{1011}[F_3]$$

$$= -\frac{M^2 M^2 - 2}{256 M^2 - 1}. \quad (98)$$

For $a = (0011)$ we have $z_1 = z_2 = 0$, and only one integral contributes,

$$I^{(4)}_{0011}[F] = I^{(0;0;1111;0)}_{0011}[F_1] = \frac{M^2}{128}. \quad (99)$$

We thus find that the leading term is

$$C_{t>1}(t, M) = t^{-2} \left( I^{(4)}_{1111}[F] + 2I^{(4)}_{1011}[F] + I^{(4)}_{0011}[F] \right)$$

$$= \frac{1}{4} \frac{M^4}{(M^2 - 1)^2} \left( \frac{M}{8t} \right)^2. \quad (100)$$

This expression is discussed in the next Section 6.
6 Summary and Conclusions

We have investigated the magnetoconductance autocorrelation function for ballistic electron transport through microstructures having the form of a classically chaotic billiard. The structures were assumed to be connected to ideal leads carrying few channels. Assuming ideal coupling between leads and billiard, we have described the system in terms of a random matrix model.

The autocorrelation function depends only on the field parameter $t$, specified by the field (flux) difference, and on the channel number $M$. Using the supersymmetry technique, we have calculated analytically the leading terms in the asymptotic expansion for large $t$ at fixed $M$, and, in Appendix 7.3.2, for large $M$ at fixed $t/M$. To this end, we have employed a new parametrization of the coset space which is particularly convenient for the present problem of partly broken symmetry. Applying Berezin's theory of coordinate transformations in superintegrals, we succeeded in formulating the relevant integral theorem, and in identifying volume and boundary (Efetov–Wegner) terms. Symmetry properties of the coset matrices helped us in considerably simplifying the evaluation of these terms. We believe that the method developed in this paper is of general interest for the supersymmetry technique, and we hope it will be helpful in other cases. We have shown that the Efetov–Wegner terms are essential for details of the large $t$ behaviour at small $M$.

To discuss our results, we construct a function $C_{SL}^L(t, M)$ of the form of a squared Lorentzian which has the same leading term at large $t$ as our result \((100)\). We use the value of the autocorrelation function at the origin $t = 0$ given by the variance $M^2/[4(M^2 - 1)]$ of the dimensionless conductance $g(B)$, see Ref. [19]. This yields

$$C_{SL}^L(t, M) = \frac{M^2}{4(M^2 - 1)} \left( 1 + \frac{8t}{M^2} \sqrt{M^2 - 1} \right)^{-2}. \quad (101)$$

In our earlier paper [8], we found that the leading terms in the asymptotic expansion of the autocorrelation function for small $t$ are

$$C(t, M) = \frac{M^2}{4(M^2 - 1)} - \frac{4M^3}{(M^2 - 1)^2} t$$
$$+ \frac{16M^2 (3M^4 - 11M^2 + 36)}{(M^2 - 1)^2 (M^2 - 4) (M^2 - 9)} t^2 + O(t^3). \quad (102)$$
The squared Lorentzian consistent with the first two terms is
\[
C_{II}^{SL}(t, M) = \frac{M^2}{4(M^2 - 1)} \left( 1 + \frac{8t}{M} \frac{M^2}{M^2 - 1} \right)^{-2}.
\]
(103)

The two squared Lorentzians are different. This shows that we cannot find a function with the shape of a squared Lorentzian which has the same leading terms as the exact correlation function for both small and large values of \( t \). In other words, the exact shape of the autocorrelation function is not a squared Lorentzian. However, with increasing \( M \), the differences between \( C_{SL}^{I}(t, M) \) and \( C_{SL}^{II}(t, M) \) decrease, and for \( M \gg 1 \), the semiclassical result
\[
C_{SL}(t, M) = \frac{1}{4} \left( 1 + \frac{8t}{M} \right)^{-2}
\]
(104)
does have the leading terms required by Eqs. (100) and (102). Moreover, the numerical differences between the three functions \( C_{SL}, C_{SL}^{I}, \) and \( C_{SL}^{II} \) are small even at small \( M \). From the numerical simulations discussed in Ref. [8], the difference between the exact RMT autocorrelation function and the best squared Lorentzian fit does not exceed 5% in magnitude even for the smallest value of \( M, M = 2 \).

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7 Appendices

7.1 Symmetry Properties of the Integrals

\( \mathcal{I}_{a_1a_211}^{(n_1;n_2;n_3;n_4)} [F_j] \)

Similar to the symmetry properties of \( I_a[F] \) discussed in Subsection 4.2, these properties follow from the symmetries of the matrices \( U_0, \Lambda_0, U_I, \Lambda_I \) under the similarity transformations by the permutation matrices \( J_{12} \) and \( J_{34} \), and can be found in the same way. The integrals are given by Eq. (84). We use Eqs. (83) and change from the coordinates \( z_r, \tilde{z}_s \) to the coordinates \( w_r, \tilde{w}_s \),
where \( \tilde{w}_s = (\theta^a_r, \psi^a_r, \tilde{\theta}^a_r, \psi^a_r, \beta^a_s, \tilde{\beta}^a_s, \beta_s^2, \tilde{\beta}_s^2) \). Since the expansion terms satisfy the same relations as the expanded functions, we have

\[
S^{(n_{11}n_{12}n_{21}n_{22})}_j(z_{12}, \tilde{z}_{34}) = S^{(n_{22}n_{21}n_{12}n_{11})}_{j'}(w_{21}, \tilde{w}_{34})
\]  \hspace{1cm} (105)

with \( j' = 1, 2, 3, 4 \) for \( j = 1, 2, 3, 4 \), respectively. Similarly,

\[
D^{(n_4)}(z_{12}, \tilde{z}_{34}) = D^{(n_4)}(w_{21}, \tilde{w}_{34})
\]  \hspace{1cm} (106)

With \( \rho^{(n_p)}_{a_1a_211}(\theta_{12}, \tilde{\theta}_{34}) = \rho^{(n_p)}_{a_2a_11}(\theta_{21}, \tilde{\theta}_{34}) \) we finally find that

\[
\mathcal{I}^{(n_p;n_k;n_{12}n_{21}n_{22};n_d)}_{a_1a_211}(F_j)
= \int \prod_r d[w_r] \prod_s d[\tilde{w}_s] \rho^{(n_p)}_{a_1a_211}(\theta_{12}, \tilde{\theta}_{34}) \prod_r^{(0)} \delta(w_r) \prod_r^{(1)} P(w_r)
\times \prod_s P(\tilde{w}_s) K^{(n_k)}(\tilde{\theta}_{34}) N_j S^{(n_{22}n_{21}n_{12}n_{11})}_{j'}(w_{21}, \tilde{w}_{34}) D^{(n_4)}(w_{21}, \tilde{w}_{34}).
\]  \hspace{1cm} (107)

Since the coordinates \( w_2 \) are integrated over the same domain as \( w_1 \), and the coordinates \( w_r, \tilde{w}_s \) over the same domain as \( z_r, \tilde{z}_s \), the integral on the r.h.s. of Eq. (107) is the integral \( \mathcal{I}^{(n_p;n_k;n_{12}n_{21}n_{22};n_d)}_{a_1a_211}[F_j] \). The same procedure can be repeated using the symmetry properties with respect to \( J_{34} \) instead of those with respect to \( J_{12} \). In this case, the coordinate transformation yields the integral \( \mathcal{I}^{(n_p;n_k;n_{12}n_{21}n_{22};n_d)}_{a_1a_211}[F_j] \). Summarizing and combining the results we find that the integrals \( \mathcal{I}^{(n_p;n_k;n_{12}n_{21};n_d)}_{a_1a_211}[F_j] \) satisfy the symmetry relations in Eqs. (83).

### 7.2 The Integrals \( \mathcal{I}^{(4)}_a[F] \)

For the volume term \( \mathcal{I}^{(4)}_{11111}[F] \), the functions \( V^{(1)}, V^{(2)}, D^{(2)}, D^{(4)} \) take the form

\[
V^{(1)} = \frac{1}{4}
\left(\begin{array}{cccc}
0 & 0 & 0 & X_3^{(1)} y_2 \\
0 & 0 & -X_4^{(1)} y_2 & 0 \\
0 & X_4^{(1)} y_1 & 0 & 0 \\
-X_3^{(1)} y_1 & 0 & 0 & 0
\end{array}\right),
\]

\[
V^{(2)} = -\frac{1}{8}
\left(\begin{array}{cccc}
0 & X_3^{(1)} x_2 X_4^{(1)} y_1 & 0 & 0 \\
X_4^{(1)} x_2 X_3^{(1)} y_1 & 0 & 0 & 0 \\
0 & 0 & 0 & X_4^{(1)} x_1 X_3^{(1)} y_2 \\
0 & 0 & X_3^{(1)} x_1 X_4^{(1)} y_2 & 0
\end{array}\right),
\]

23
\[ D^{(2)} = (MD^{(0)}/4) < \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} x_2 >, \]
\[ D^{(4)} = (M^2 D^{(0)}/16) < X_3^{(1)} \bar{x}_2 X_4^{(1)} \bar{x}_1 > < X_3^{(1)} x_1 X_4^{(1)} x_2 >, \]

where \( X_s^{(1)}, \bar{X}_s^{(1)} \) are the matrices given in Eq. (89) and \( x_r, \bar{x}_r, y_r \) the matrices introduced in Eq. (72). For brevity, we present here only those parts of \( D^{(2)}, D^{(4)} \) which give a nonvanishing contribution to the integral.

For the boundary terms, inserting the matrices \( V^{(n)} \) and applying the projectors \( \prod_r \rho^{(1)} P(z_r) \) yields

\[
I^{(4)}_{1011} [F] = \left( \frac{M^2}{64} \right) \int d[\theta_1] \prod_s d[\theta_s] \rho^{(0)}_{1011} K^{(0)} D_0 \prod_p \gamma_1 \gamma_1^* \times \left( -2 < X_3^{(1)} k \bar{X}_3^{(1)} y_1 > < X_4^{(1)} y_1 \bar{X}_4^{(1)} k > + M^2 < y_1 > < X_3^{(1)} k \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} k x_1 > + 2M < X_3^{(1)} k \bar{X}_3^{(1)} x_1 \bar{X}_4^{(1)} k x_1 \bar{x}_1 > \right),
\]

\[
I^{(4)}_{0011} [F] = \left( \frac{M^2}{32} \right) \int \prod_s d[\theta_s] \rho^{(0)}_{0011} K^{(0)} D_0 \times < X_3^{(1)} k \bar{X}_3^{(1)} k > < X_4^{(1)} k \bar{X}_4^{(1)} k >.
\]

These integrals can be done using Eqs. (93) and (96).

### 7.3 The Limit of Large \( M \) at Fixed \( t/M \)

This limit has been considered, and the leading term worked out, previously in Refs. [4, 5]. Thus, the present Appendix provides a test of our procedure. At the same time, comparison between the two approaches shows the relative simplicity of the present approach. We proceed in the same way as in Section [3]. Many steps are similar and will not be repeated. We expand the autocorrelation function \( C(t, M) \) in inverse powers of \( M \) and evaluate...
the leading term. In the limit where \( M, t \gg 1 \) at \( t/M \) fixed and arbitrary, the integrals \( \mathcal{I}_a[F] \) appearing in the sum (112), i.e. the integrals with \( a = (1111), (1011), (0011) \), are dominated by the contribution of the neighbourhood of the surface \( \theta_\alpha^r = \theta_s^\alpha = 0 \) where \( Q = L \) and where the graded traces in the exponents of

\[
K = e^{-(t/2)(Q^{\alpha q})^2}, \quad D = e^{-M <\ln(1 +QL)>}
\]

are equal to zero.

### 7.3.1 Asymptotic Expansion

We start from Eq. (61), rescale the angles \( \tilde{\theta}_q^\alpha = M^{1/2}\theta_q^\alpha \), and pass from the coordinates \( \tilde{z}_q \) to the coordinates \( \tilde{z}_q = (\tilde{\theta}_q^b, \tilde{\theta}_q^f, \tilde{\phi}_q^f, \tilde{\gamma}_q^1, \tilde{\gamma}_q^{1*}, \tilde{\gamma}_q^2, \tilde{\gamma}_q^{2*}) \), with

\[
d(z)\rho_a \prod_r^{(0)} \delta(z_r) = d(\tilde{z}) M^{-1} \rho_a \prod_r^{(0)} \delta(\tilde{z}_r)
\]

for \( l = \sum_q a_q \). At fixed \( t/M \), we expand \( M^{-1} \rho_a, K, V \) and \( D \) in powers of \( M^{-1/2} \). The expansion coefficients \( \rho_a^{(n_\rho)}, K^{(n_k)}, V^{(n_{\nu p}), D^{(n_d)}} \) are functions of \( \tilde{z}_q \). We take into account that the \( N_j \) also depend on \( M \), and write \( N_j = M^{\nu_j/2}N_j^{(\nu_j)} \), with \( \nu_j = 4, 8, 6, 6 \) for \( j = 1, 2, 3, 4 \), respectively. We collect all terms of the same order in \( M^{-1/2} \) and get

\[
\mathcal{I}_a[F] = \sum_n M^{-n/2} \mathcal{I}_a^{(n)}[F].
\]

Here \( \mathcal{I}_a^{(n)}[F] \) denotes the sum of integrals

\[
\mathcal{I}_a^{(n)}[F] = \sum_j \sum_{n_\rho,n_k,n_{\nu p},n_{\nu q}} \mathcal{I}_a^{(n_\rho,n_k,n_{\nu p},n_{\nu q})}[F_j]
\]

with

\[
\mathcal{I}_a^{(n_\rho,n_k,n_{\nu p},n_{\nu q})}[F_j] = \int d(\tilde{z}) \rho_a^{(n_\rho)} \times \prod_r^{(0)} \delta(\tilde{z}_r) \prod_r^{(1)} P(\tilde{z}_r) \prod_s P(\tilde{z}_s) K^{(n_k)} N_j^{(\nu_j)} D^{(n_d)}. \quad (113)
\]

For fixed \( j \), the sum over \( n_\rho, n_k, n_{\nu p}, n_d \) is restricted by the condition \( n_\rho + n_k + n_{\nu p} + n_d = n + \nu_j \). All integrals contain the exponential factor \( K^{(0)} D^{(0)} \) with

\[
K^{(0)} = e^{(2/M) \sum_s <(\tilde{\theta}_s)^2>}, \quad D^{(0)} = e^{(1/4) \sum_q <(\tilde{\theta}_q)^2>}. \quad (114)
\]

On extending the domain of integration region over \( \tilde{\theta}_q^f \) from zero to infinity, the series (112) yields an asymptotic expansion of \( \mathcal{I}_a[F] \). Since \( \rho_a, K, S \)
and $D$ are even functions of $\theta^a_q$, only the terms with even $n$ appear, and the expansion proceeds in inverse powers of $M$,

$$C(t, M) = \sum_{n=1}^{\infty} M^{-n} \left( T_{1111}^{(2n)}[F] + 2T_{1011}^{(2n)}[F] + T_{0011}^{(2n)}[F] \right).$$  \hspace{1cm} (115)

The integrals $I_{a(1)}^{(n_{\rho}; n_k; n_v; n_d; \nu_j)}[F]$ satisfy the same symmetry relations as the integrals $I_{a(2)}^{(n_{\rho}; n_k; n_v; n_d)}[F_j]$, cf. Eq. (84). The leading term of the expansion is of the order $M^0$, and is given by the sum

$$C_{M^0}(t, M) = T_{1111}^{(0)}[F] + 2T_{1011}^{(0)}[F] + T_{0011}^{(0)}[F].$$  \hspace{1cm} (116)

### 7.3.2 The Leading Term $C_{M^0}(t, M)$

The nonvanishing projections $\sum_s P(z_s) S_{j}^{(n_{\rho})} D^{(n_d)}$ stem solely from the terms which are linear in all four matrices $(\delta Q^1_{s})^{(1)}$, $(\delta Q^2_{s})^{(1)}$ and which for each $r$ contain the same number of $(G^1_{s})^{(1)}$ and of $(G^2_{s})^{(1)}$. We employ the symmetry properties of $I_{a(1)}^{(n_{\rho}; n_k; n_v; n_d; \nu_j)}[F]$ and find that

$$T_{1111}^{(0)}[F] = T_{1111}^{(0;0;1111;4:8)}[F] + 4T_{1111}^{(0;0;1113;2;8)}[F] + 2T_{1111}^{(0;0;1331;0;8)}[F] + 2T_{1111}^{(0;0;1111;2;6)}[F],$$  \hspace{1cm} (117)

$$T_{1011}^{(0)}[F] = T_{1011}^{(0;0;1133;0;8)}[F] + 2T_{1011}^{(0;0;1131;0;6)}[F],$$

$$T_{0011}^{(0)}[F] = T_{0011}^{(0;0;1111;0;4)}[F].$$  \hspace{1cm} (118)

For the volume term $T_{1111}^{(0)}[F]$,

$$V^{(1)} = \frac{1}{4} \begin{pmatrix} 0 & 2x_1^{(1)} & 0 & X_3^{(1)} \\ 2\bar{x}_1^{(1)} & 0 & -\bar{X}_4^{(1)} & 0 \\ 0 & X_4^{(1)} & 0 & 2x_2^{(1)} \\ -\bar{X}_3^{(1)} & 0 & 2\bar{x}_2^{(1)} & 0 \end{pmatrix},$$

$$V^{(3)} = -\frac{1}{8} \begin{pmatrix} 0 & X_3^{(1)} & x_2^{(1)} & X_4^{(1)} & 0 & 0 & 0 \\ X_4^{(1)} & x_2^{(1)} & X_3^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{x}_1^{(1)} & X_3^{(1)} \\ 0 & 0 & \bar{x}_3^{(1)} & X_4^{(1)} & 0 & 0 & 0 \end{pmatrix}. $$
\[ D^{(2)} = (D^{(0)}/4) < \vec{X}_{3}^{(1)} x_{1}^{(1)} \vec{X}_{4}^{(1)} x_{2}^{(1)} > , \]
\[ D^{(4)} = (D^{(0)}/16) < X_{3}^{(1)} \vec{x}_{2}^{(1)} X_{4}^{(1)} x_{1}^{(1)} > < X_{3}^{(1)} x_{1}^{(1)} \vec{X}_{4}^{(1)} x_{2}^{(1)} > , \]

where \( X_{s}^{(1)}, \vec{X}_{s}^{(1)} \) are the matrices introduced in Eq. (89), and \( x_{r}^{(1)}, \vec{x}_{r}^{(1)} \) denote the matrices
\[ x_{r}^{(1)} = (G_{r}^{12})^{(1)} = i(\theta_{r}/2)e^{i\phi_{r}}, \quad \vec{x}_{r}^{(1)} = (G_{r}^{21})^{(1)} = i(\theta_{r}/2)e^{-i\phi_{r}} . \quad (119) \]

We present only those parts of \( V^{(3)}, D^{(2)}, D^{(4)} \) which give nonzero contribution to the integrals. The densities \( \rho_{a}^{(0)} \) have for all \( a \) the form
\[ \rho_{a}^{(0)} = \prod_{r}^{\varphi} \rho_{G}^{(0)}(\theta_{r}), \quad \rho_{G}^{(0)} \text{ given in Eq. (91)}. \]
The integrals are evaluated in the same way as in Subsection 5.2. The integration over the eigenvalues is done making use of
\[ \int d[\theta_{r}] \rho_{G}^{(0)}(\theta_{r}) e^{\frac{1}{4} < (\theta_{r})^2 >} < y_{r}^{(2)} > = 1 , \]
\[ \int d[\theta_{r}] \rho_{G}^{(0)}(\theta_{r}) e^{\frac{1}{4} < (\theta_{r})^2 >} < (\theta_{r})^2 > = -1 \quad (120) \]
and
\[ \int d[\theta_{s}] \rho_{G}^{(0)}(\theta_{s}) e^{\frac{1}{4} < (\theta_{s})^2 >} < (\theta_{s})^2 > = 4 [1 + (8t/M)]^{-1} . \quad (121) \]
The contributions stemming from the integrals appearing in the second row of Eq. (117) cancel mutually, as well as the contributions of the two boundary terms. Thus the evaluation of the leading term reduces to the evaluation of the integral
\[ \mathcal{I}_{111}^{(0;0;111;4;8)}[F_{2}] = \int d[z] \rho_{1111}^{(0)} K^{(0)} \]
\[ \times N_{2}^{(8)} < V^{(1)} I_{0}^{(11)} V^{(1)} I_{0}^{(12)} > < V^{(1)} I_{0}^{(21)} V^{(1)} I_{0}^{(22)} > D^{(4)} \]
\[ = (1/64) \int d[z] \rho_{1111}^{(0)} K^{(0)} D^{(0)} \prod_{r} < y_{r}^{(2)} >^{2} \]
\[ \times \prod_{r} \gamma_{r}^{p} = \gamma_{r}^{p} < X_{3}^{(1)} \vec{x}_{2}^{(1)} X_{4}^{(1)} x_{1}^{(1)} > < \vec{X}_{3}^{(1)} x_{1}^{(1)} \vec{X}_{4}^{(1)} x_{2}^{(1)} > \]
\[ = (1/64) \int d[z] \rho_{1111}^{(0)} K^{(0)} D^{(0)} \prod_{r} < y_{r}^{(2)} >^{2} \prod_{s} < (\theta_{s})^2 > . \quad (122) \]
The integrals over \( \theta_{r} \) yield the unit diffusion factor independent of \( t/M \), the integrals over \( \theta_{s} \) yield the cooperon factor \( 16 [1 + (8t/M)]^{-2} \). The result is
\[ C_{M>>1}(t, M) = \frac{1}{4} \left( 1 + \frac{8t}{M} \right)^{-2} . \quad (123) \]
This result agrees with the one obtained by Efetov \cite{4} and by Frahm \cite{5} in the framework of supersymmetry, and with the squared Lorentzian found in the semiclassical approach \cite{3}.

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