Quantum fluctuations of some gravitational waves*

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Abstract

We review our previous work on the calculation of the vacuum expectation value of the stress-energy tensor for a scalar particle in the background metric of different types of spherical impulsive, spherical shock and plane impulsive gravitational waves. We get non-zero vacuum fluctuations only if we use the spherical impulsive wave as the background. We take a detour in de Sitter space for regularizing our result both in the ultraviolet and the infrared regions.

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INTRODUCTION

We know that the methods for the quantization of Einstein’s theory of gravitation are not at their final stage at the moment. There are many models which incorporate this theory into the models for elementary particles. One can cite all the string, membrane, M models, etc. which are extensively studied. The final word is not still said in these models.

In the absence of a generally accepted method for the quantization of gravitation, semi classical methods were used to extract information about the theory, which were very useful in the absence of a full quantization. We can calculate the fluctuations in the stress-energy tensor of a particle that propagates through universes described by different metrics. Extensive work in this direction was done in the seventies stressing the phenomena of particle production in these metrics. This work is described in the books written by Birrell and Davies, Fulling and Wald. Essentially we are confronted by a problem of a particle in an external field, as the metric contribution can be written as an external potential. The n-point functions where n exceeds two can be found by a simple application of the Wick’s theorem in quantum field theory. Kuo and Ford show the calculation of the four-point function, thereby variance in the stress-energy tensor using this idea.

In the eighties and nineties people applied the same method to problems in the presence of gravitational waves, or to the effects of different topological structures in the space-time manifold. Vacuum fluctuations in the presence of cosmic strings is an example of such work.

We were interested in the same problem in the presence of spherical waves created by the snapping of cosmic strings. We calculated the vacuum-expectation-value, VEV, of the stress-energy tensor, $< T_{\mu\nu} >$, for several types of spherical impulsive and shock waves, described by different warp functions. There is a general argument backed up by a explicit calculation which prohibits the polarization of the vacuum in the presence of plane waves. It was not known whether the same obstruction prevailed in the spherical wave case, though; so, the expression for $< T_{\mu\nu} >$ was calculated for special cases of the Nutku-Penrose metric. We found that although a finite expression can not be extracted in the first order calculation, the second order calculation indeed gives a finite
Here we had to take a detour in the de Sitter space and take the appropriate limit to go back to the Minkowski space at the end of the calculation.

The same method was applied to the spherical shock wave given by Nutku. In this case we were not able to extract a finite expression even in second order.

We, then, applied our method to the impulsive plane waves. Relying on the Deser-Gibbons work, we were not anticipating a finite result at this point. It seems that the method we proposed for this calculation passed this test. We could not get a finite contribution to the vacuum fluctuations if we had a plane impulsive wave propagating in the Minkowski space. If we have an impulsive plane wave in the de Sitter universe, though, we get a finite result which is proportional to the cosmological constant. We also find that the presence of impulsive pp waves does not change this result.

In Section II we first calculate $<T_{\mu\nu}>$ for an impulsive wave in first order perturbation theory for a special form of the warp function. In Section III we do the similar calculation for a specially simple form of the same function in second order perturbation theory. Section IV is devoted to the similar problem in the de Sitter background. Here we show how one can extract a finite expression for $<T_{\mu\nu}>$ first by performing the calculation in de Sitter space and then taking the appropriate limit to land in Minkowski space. In Section V we do the similar calculation for the shock wave and point to the absence of a finite term in the expression for $<T_{\mu\nu}>$ even in second order in perturbation theory. In Section VI the similar calculation is repeated for an impulsive plane wave and it is shown how the proposed method fails to extract a finite expression for the vacuum expectation value of the stress-energy tensor in the background metric of a plane wave. This result is in agreement with the result given by Deser and Gibbons. We also show how this calculation can be extended to pp waves and give the result for this case. We end up with few concluding remarks.

II. Spherical impulsive wave: first-order calculation

The Nutku-Penrose metric for spherical gravitational waves formed as a result of snapped cosmic strings is given by

$$ds^2 = 2dudv - u^2d\zeta^2 + v\Theta(v)f(\zeta)d\zeta^2.$$
Here $v$ is the retarded time, $u$ is a Bondi-like luminosity distance, $\zeta$ is the angle of stereographic projection on the sphere. $f$ is the Schwarzian derivative of an arbitrary holomorphic function $h(\zeta)$ which describes the way the cosmic string is snapped and is called the warp function. Only one component of the Weyl tensor is not zero. It is given by

$$\Psi_4 = \frac{\delta(v)}{u} f,$$

exhibiting the characteristic behaviour of an impulsive wave. We expect similar behaviour for the vacuum expectation value of the stress-energy tensor $< T_{\mu\nu} >$, for a massless scalar particle propagating in the background metric of such a wave if we get any finite value for this expression.

We choose a particular form for the warp function $h$ to perform an explicit calculation. Our experience with the different forms we chose shows that the form of the expression for $< T_{\mu\nu} >$ is not sensitive to the actual choice for $h$. The presence of the Dirac delta function and the presence of a finite expression depends only on the general properties of the metric. The functional form of the warp function only effects the dependence on $\zeta, \bar{\zeta}$. This fact allows us to choose a special form for this function that will make our calculations as simple as possible.

Our first choice corresponds to a “rotating” string and is given by

$$h = (\zeta)^{1+i\epsilon} \tag{3}$$

which gives

$$f(\zeta) = \frac{-i\epsilon}{\zeta^2} \tag{4}$$
in first order in $\epsilon$. With this choice we rotate the shear of the metric by $\pi/2$ compared with the real exponent case. We take the rotating string because the calculations are somewhat simpler in this case. Here $\epsilon$ is a small number which can be positive or negative, and will be used as the perturbative expansion parameter.

First we go to real coordinates

$$\zeta = \frac{1}{\sqrt{2}} (x + iy) \tag{5}$$
and write the metric, up to first order in $\epsilon$, for $v > 0$,

$$ds^2 = 2dudv - \left( u^2 - \frac{8\epsilon vuxy}{(x^2 + y^2)^2} \right) dx^2 - \left( u^2 + \frac{8\epsilon vuxy}{(x^2 + y^2)^2} \right) dy^2 - \frac{8\epsilon vu}{(x^2 + y^2)^2}(x^2 - y^2)dxdy.$$ 

The d’Alembertian operator $\Box$ times $\sqrt{-g}$, written in the background of this metric reads

$$\sqrt{-g}\Box = 2u^2 \frac{\partial^2}{\partial u \partial v} + 2u \frac{\partial}{\partial v} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{8\epsilon}{u(x^2 + y^2)^2} \left[ xy \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) - (x^2 - y^2) \frac{\partial^2}{\partial x \partial y} \right].$$

For technical reasons we first couple a massive scalar field to this metric and take the limit mass going to zero at the end of the calculation. We use conformal coupling. The Ricci scalar is zero in the Minkowski case. The form of this coupling only matters in the de Sitter case.

We first compute the Green’s function by summing the eigenvalues of the operator

$$G_F = \sum_{\lambda} \frac{\phi_{\lambda}(x)\phi_{\lambda}^*(x')}{\lambda}$$

where

$$L\phi_{\lambda} = \lambda\phi_{\lambda},$$

$$L = \sqrt{-g}[\Box + m^2].$$

We calculate $< T_{\mu\nu} >$ by taking the appropriate derivatives of $G_F$. Since our computation is in first order in $\epsilon$, we expand $L, \phi_{\lambda}$ and $\lambda$ and write

$$(L_0 + \epsilon L_1)(\phi_0 + \epsilon \phi_1 + ...) = (\lambda_0 + \epsilon \lambda_1 + ...)(\phi_0 + \epsilon \phi_1 + ...).$$

This gives us

$$L_0\phi_0 = \lambda_0\phi_0,$$

$$L_1\phi_0 + L_0\phi_1 = \lambda_1\phi_0 + \lambda_0\phi_1,$$

where

$$L_0 = \left( 2u^2 \frac{\partial^2}{\partial u \partial v} + 2u \frac{\partial}{\partial v} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + 2m^2 u^2 \right),$$

$$L_1 = \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right)^2.$$
\[
\phi_0 = \frac{e^{iRv} e^{ik_1 x} e^{ik_2 y} e^{-iK}}{u \sqrt{2R(2\pi)^2}}
\]

\[\lambda_0 = k_1^2 + k_2^2 - K\]

\[L_1 = \frac{8v}{u} \left[ \frac{xy}{(x^2 + y^2)^2} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) - \frac{x^2 - y^2}{(x^2 + y^2)^2} \frac{\partial^2}{\partial x \partial y} \right],\]

\[\lambda_1 = (\phi_0, L_1 \phi_0) = 0.\]

We write \(\phi_1 = \phi_0 f.\)

The form of the operator \(L_1\) given in eq. 17 suggests the ansatz

\[f = v f_1(x, y, u) + f_2(x, y, u)\]

for \(f.\) This ansatz gives us two coupled equations

\[L' f_1 = \frac{8v}{u} \left[ \frac{xy}{(x^2 + y^2)^2} (-k_1^2 + k_2^2) + \frac{(x^2 - y^2)k_1k_2}{(x^2 + y^2)^2} \right],\]

\[L' f_2 = -2u^2 \frac{\partial f_1}{\partial u} - \left( \frac{iK}{R} + \frac{2im^2u^2}{R} \right) f_1,\]

where

\[L' = 2u^2 iR \frac{\partial}{\partial u} - 2i \left( k_1 \frac{\partial}{\partial x} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}.\]

Here we take mass only as an infrared regulator and from this point on equate it to zero everywhere except in the zeroth order solution \(\phi_0.\) We will take the limit where it goes to zero at the end of our calculation.

At this point we see that it is convenient to make the change of variables and use \(z = x + iy, \overline{z} = x - iy,\) and \(s = \frac{1}{u}.\) In terms of these variables we get

\[L' = -2iR \frac{\partial}{\partial s} - 2i \left[ (k_1 + ik_2) \frac{\partial}{\partial z} + (k_1 - ik_2) \frac{\partial}{\partial \overline{z}} \right] - 4 \frac{\partial^2}{\partial z \partial \overline{z}} \]

\[L' f_1 = -2is \left( \frac{(k_1 + ik_2)^2}{z^2} - \frac{(k_1 - ik_2)^2}{\overline{z}^2} \right)\]
with the immediate solution

\[ f_1 = R(ln z - \ln z') - \left( \frac{(k_1 + ik_2)s}{z} - \frac{(k_1 - ik_2)s}{z'} \right) \]

\[ f_2 = -\frac{Ks}{2R} \left( \frac{1}{z} - \frac{1}{z'} \right) - \frac{i}{2} (ln z - ln z') + K \left( \frac{zln z - z'}{k_1 + ik_2} - \frac{zln z'}{k_1 - ik_2} \right). \]

If we go back to the real variables the solutions read

\[ f_1 = 2R \tan^{-1} \frac{y}{x} - 2iK \frac{k_2 x - k_1 y}{x^2 + y^2} \]

\[ f_2 = -\frac{(iKs}{R} \tan^{-1} \frac{y}{x} + \frac{iK}{k_1^2 + k_2^2} \left( 2(k_1 x + k_2 y)(\tan^{-1} \frac{y}{x} - y) \right) \]

\[ + (k_1 y - k_2 x) \left( (\ln x^2 + y^2) - 2x \right) \]

To form the Green’s function \( G_F \), we have to calculate

\[ G_F^{(1)} = \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} dK \int_{-\infty}^{\infty} dR \frac{\phi_0^\lambda(x) \phi_1^\lambda(x') + \phi_0^\lambda(x') \phi_1^\lambda(x)}{K - k_1^2 - k_2^2}, \]

where \( \phi_0^\lambda \) and \( \phi_1^\lambda \) are the zeroth and the first order solutions respectively. We have to multiply \( f_1 \) and \( f_2 \) by the operator

\[ L = i \int_{0}^{\infty} d\alpha \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} dK \frac{1}{(8\pi^2)^2 u u'} \int_{-\infty}^{\infty} dR e^{-\frac{iK(x-x')}{2R}} \]

\[ \times e^{iR(v-v')} e^{ik_1(x-x')} e^{ik_2(y-y')} e^{i\alpha(K-k_1^2-k_2^2)} e^{\frac{im^2}{4R}} (\frac{1}{4} - \frac{1}{R}) \]

At this point we are looking for finite contributions to the \( < T_{\mu\nu} > \). To calculate them we have to first calculate the Green’s function, and then differentiate it to obtain the vacuum expectation value of the stress-energy tensor. Since we expect a non zero contribution, if any, only from \( < T_{vv} > \), taking the coincidence limit in \( x \) and \( y \) before we calculate \( G_F \) will not change the end result because we have to differentiate with respect to \( v \) and \( v' \) only. If these terms are zero in the coincidence limit in \( x \) and \( y \) for the Green function, they will be also zero for \( < T_{\mu\nu} > \).

A straightforward calculation shows that if we take the coincidence limit for \( x \) and \( y \), i.e. take \( x = x' \), \( y = y' \), we find

\[ G_F^{(1)} = [A (\Theta(v) + \Theta(v')) \left( \frac{v \Theta(v) - v' \Theta(v')}{(u - u')(v - v')} \right) tan^{-1} \frac{y}{x}, \]

\[ B \left( \frac{v \Theta(v) - v' \Theta(v')}{(u - u')(v - v')} \right) tan^{-1} \frac{y}{x}. \]
Here $A, B$ are numerical constants. We could not extract a finite portion out of these expressions in the coincidence limit.

One can show that if we go one order higher, we may get finite contributions if we take a detour in de Sitter space. Since this calculation is quite involved, we first use a simpler form of the warp function and demonstrate how this method works.

III. Spherical impulsive wave: second-order calculation

In this section we take the warp function $h = e^{\alpha \zeta}$, which results in $f = -\frac{\alpha^2}{2}$, resulting in a metric
\[ ds^2 = 2dudv - \frac{1}{4} \left( dx^2 (2u - \nu \alpha^2 \Theta(v))^2 + dy^2 (2 + \nu \alpha^2 \Theta(v))^2 \right). \]

The d’Alembertian operator in this metric reads
\[ \Box = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial u} (g^{\mu\nu} \sqrt{-g}) \partial_v = 2 \partial_{uv} + \left( \frac{1}{u - \frac{\alpha^2 v}{2}} + \frac{1}{u + \frac{\alpha^2 v}{2}} \right) \partial_v + \frac{\alpha^2}{2} \left( \frac{1}{u + \frac{\alpha^2 v}{2}} - \frac{1}{u - \frac{\alpha^2 v}{2}} \right) \partial_u - \frac{1}{(u - \frac{\alpha^2 v}{2})^2} \partial_x^2 - \frac{1}{(u + \frac{\alpha^2 v}{2})^2} \partial_y^2. \]

for the exact operator. We multiply this expression, eq. 34, by $\sqrt{-g}$ which is equal to $u^2$ and expand the operator up to second order in $\alpha^2$:
\[ L^{II} = 2u^2 \partial_u \partial_v + 2u \partial_v - \partial_x^2 - \partial_y^2 - \frac{\alpha^2 v}{u} (\partial_x^2 - \partial_y^2) - \alpha^4 \left( \frac{v}{2} \partial_u - \frac{v^2}{u} \partial_v + \frac{3}{2} \frac{v^2}{u^2} (\partial_x^2 + \partial_y^2) \right). \]

Here, again, we will first compute the Green’s function for this operator, and then differentiate it to get the vacuum expectation value of the stress-energy tensor. We obtain the same zeroth order solution as given in eq. 15 and make the same ansatz $\phi^{(1)} = \phi^{(0)} f$ since the first order operator is of the same form as previous case. We see that $f$ just modulates the zeroth-order solution, and does not essentially change it in a radical manner.

$f$ obeys the differential equation
\[ L_2 f = \frac{v}{u} (k_2^2 - k_1^2) \]

where $L_2$ is defined as
\[ L_2 = \left( -2iR \frac{\partial}{\partial s} - 2i \left( k_1 \frac{\partial}{\partial x} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - 2 \frac{\partial^2}{\partial s \partial v} + iK \frac{\partial}{\partial v} \right). \]

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Here the modes $k_1, K_2, R, K$ are as given in eq. 16, $s = \frac{1}{v}$. We make the ansatz for $f$ similar to the one given in eq. 20. $f = v f_1(s, x, y) + f_2(s, x, y)$. This yields equations similar to the ones given in eq.s 21 and 22. We get

$$L' f_1 = s(k_2^2 - k_1^2),$$  \hspace{1cm} 38

$$L' f_2 = \left(2 \frac{\partial}{\partial s} + \frac{iK}{R}\right) f_1$$  \hspace{1cm} 39

where $L'$ is as defined in eq. 23. These equations are simply integrated over with the result

$$f = -ivs^2(k_2^2 - k_1^2) + i(k_1^2 - k_2^2) \left(\frac{is^2}{4R^2} + \frac{Ks^3}{24R^3}\right).$$  \hspace{1cm} 39

To get the Green’s function at this order we operate on this function by $L$ given in eq. 31. The result is seen to yield

$$G_{FK1}^{(i)} = [(x - x')^2 - (y - y')^2] \left(A_1 \frac{s^2v\Theta(v) - s'^2v'\Theta(v')}{(s - s')^2} + A_2 \frac{\Theta(v)s^2 + \Theta(v')s'^2}{(s - s')^2}\right) + A_3 \frac{s^3\Theta(v) - s'^3\Theta(v')}{(s - s')^3},$$  \hspace{1cm} 40

where

$$[\ ] = (u - u')(v - v') - \frac{uu'}{2} ((x - x')^2 + (y - y')^2)$$  \hspace{1cm} 41

and $A_1, A_2, A_3$ are constants. We see that this result is of the Hadamard form. We find that all these terms have the same type of ultraviolet singularity as the flat part. We could not find a finite part of this expression.

If we go one order higher, we end up with the differential equation

$$L_2 g = v^2 \left(\frac{iRs}{2} + 3s^2(k_1^2 + k_2^2) + (k_1^2 - k_2^2)^2 \left(\frac{-is^3}{2R}\right)\right) + v \left(\frac{-s}{2} + \frac{iKs^2}{4R} - \frac{(k_1^2 - k_2^2)^2s^3}{4R^2} + \frac{iK(K_1^2 - k_2^2)^2s^4}{24R^3}\right)$$  \hspace{1cm} 42

when we make the ansatz $\phi_2 = \phi_0 g$. We take

$$g = v^2 g_1(x, y, s) + vg_2(x, y, s) + g_3(x, y, s).$$  \hspace{1cm} 43
Going through similar steps we find

\[ g_1 = -\frac{s^2}{8} + \frac{is^3(k_1^2 + k_2^2)}{2R} + \frac{(k_1^2 - k_2^2)^2s^4}{32R^2}, \]

\[ g_2 = -\frac{i3s^2}{8R} - \frac{13Ks^3}{12R^2} + \frac{is^4}{32R^3} ((k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2)) \]
\[ + \frac{s^5K}{160R^4}(k_1^2 - k_2^2)^2, \]

\[ g_3 = \frac{3s^2}{8R^2} - \frac{i55Ks^3}{16R^3} - \frac{s^4}{32R^4} \left( (k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2) + \frac{13K^2}{3} \right) \]
\[ + \frac{is^5K}{320R^5} \left( 3(k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2) \right) - \frac{K^2s^6(k_1^2 - k_2^2)^2}{1920R^6}. \]

We see that when the Green’s function is calculated using these functions we get no infrared divergences for \( g_1 \). When we use \( g_2 \), we have to perform integrals of the type

\[ \int_0^\infty \frac{d\alpha}{\alpha} \exp \left[ -\frac{i(u - u')(v - v')}{uu'\alpha} \right] \]

which results in a soft infrared divergence. If we use an infrared regulator mass \( m^2 \), we get terms that go as \( \log \left( \frac{m^2(u - u')(v - v')}{} \right) \). \( m^2 \) cancels out when derivatives are taken to obtain \( < T_{\mu\nu} > \).

The infrared divergences are more severe for \( g_3 \). There we have to perform integrals of the type

\[ \int_0^\infty d\alpha \exp \left[ -\frac{i(u - u')(v - v')}{uu'\alpha} \right] \]

which are linearly divergent at the upper limit for the massless case. If we use a massive field as an infrared cut-off, we get terms that go as \( \frac{1}{m^2} \) as \( m \) tends to zero. This term multiplies the whole expression and does not drop out on differentiation.

**IV. Going to de Sitter space**

Here we treat the same problem for an impulsive wave in de Sitter space. It is known that an exact solution can be found for an impulsive wave propagating in de Sitter space if we multiply the Minkowski space solution by the conformal factor \( \frac{1 + \frac{\Lambda uv}{6}}{} \). This is reflected to the \( G_F \) by factors multiplying the expression from both sides.

\[ G_F^S = \left( 1 + \frac{\Lambda uv}{6} \right) G_F^M(x, x') \left( 1 + \frac{\Lambda u'v'}{6} \right), \]
where $G^S_F$ and $G^M_F$ are the de Sitter and Minkowski space Green functions respectively.

Note that
\[
\left(1 + \frac{\Lambda uv}{6}\right)\left(1 + \frac{\Lambda u'v'}{6}\right) = \left(1 + \frac{\Lambda UV}{6}\right)^2 + \frac{\Lambda}{12}(u-u')(v-v')
\]
where $U = \frac{u+u'}{2}, V = \frac{v+v'}{2}$. Using this expression we see that for $G^S_F^{(1)}$ we get terms in the coincidence limit that give a well-defined finite expression for $<T^{(1)}_{\mu\nu}>$. Here the regularization is performed using the point-splitting method, exactly in the same way one would do for the zeroth part of the de Sitter universe. One sees that for the warp function we studied in Section II, we get a finite contribution. If we use $h = \left(\zeta^1+i\delta+\epsilon\right)$ we get
\[
<T_{\nu\nu}^{(1)}> = -\frac{\Lambda^2 u\delta(v)}{2(96)^2\pi} \left(\delta\log(x^2 + y^2) - 4\epsilon\tan^{-1}\frac{y}{x}\right).
\]
This result is finite, contrary to the case for propagating in Minkowski space. This is no contradiction to our previous results, since this expression vanishes as $\Lambda$ goes to zero. One can also show that /14 for $h = \left(\frac{\zeta-1}{\zeta+1}\right)^{1+i\sqrt{2}\epsilon}$ we get
\[
<T_{\nu\nu}^{(1)}> = -\frac{u\epsilon\Lambda^2}{(96)^2\pi} \left(2x\tan^{-1}\frac{y}{x+1} - 2x\tan^{-1}\frac{y}{x-1} - 2y\log\left(\frac{(x-1)^2 + y^2}{(x+1)^2 + y^2}\right)\right)\delta(v).
\]
All of these calculations are in first order perturbation theory. We saw in Section III that there are problems in the second order calculation due to the appearance of infrared divergences for the special warp function used. One can show that these infrared divergences are generic for the Nutku-Penrose impulsive metric /15 in second order and all can be tamed to give a finite expression for $<T_{\mu\nu}>$ in the following way.

The solution for the same type of wave in de Sitter space has the multiplicative factor $\frac{\Lambda}{m^2}$ multiplying a finite expression. This fact suggests taking the curvature of the de Sitter space $\Lambda$ proportional to the infrared parameter $m^2$. These two terms both have the same dimensions, as were introduced only as a technical aid to get rid of the ultraviolet and infrared divergences. Our end result should be independent of them. This is achieved if we take them to be proportional to one another, and let them go to zero at the same rate. The
finite proportionality constant that may appear is absorbed in the perturbation expansion parameter, \( \alpha^2 \) for the case studied in Section III.

Studying this particular case, one sees that applying the operator \( L \) as given in eq. 31 on the expression \( g_3 \), eq.(46), results in divergent terms of the form

\[
\frac{\Theta(v) s^2}{(s - s')^2 m^2} + \frac{\Theta(v') s'^2}{(s - s')^2 m^2}
\]

in Minkowski space. If we go to de Sitter space, we have to multiply this solution by the factor \( \Lambda \frac{(u - u')(v - v')}{u^3} \). This operation gives rise to a finite expression when both \( \Lambda \) and \( m^2 \) are taken to zero at the same rate.

The result of this exercise is that by performing the calculation in de Sitter space and taking the limit where the curvature of this space goes to zero at the same rate as the infrared parameter, we indeed get

\[
<T_{vv} > \propto \frac{\delta(v)}{u^3}
\]

which is proportional to the sole non-zero component of the Riemann tensor, as anticipated in the first place.

In the next section we see that this property is very special to the impulsive spherical waves and is not shared with the spherical shock or impulsive plane and impulsive pp waves.

V. Spherical shock wave

In first order perturbation theory both the impulsive and the shock wave cases showed similar behaviour. The two solutions are essentially different, though. The shock wave solution has a dimensional constant which is lacking in the impulsive wave solution. Since in quantum field theory, models with dimensional and dimensionless constants belong to different classes, we thought similar distinction between these two models may exist. Relying on these motivations we planned to check whether there is a qualitative difference between the two solutions exhibited by their behaviour at higher orders.

We will show that the infrared divergences which may be cancelled via a detour in de Sitter space are absent in the shock wave calculation. Another point of difference is the
importance attributed to the homogenous solutions in these two cases. The homogenous solutions give just the free Greens function for the impulsive case, whereas they result in a totally different contribution to the Greens function in the shock wave. This is an artifact of the presence of a dimensional coupling constant in the latter case. What may be more interesting is the fact that just the contribution of the first order calculation contributes to $< T_{\mu\nu} >$ in de Sitter space. The higher order terms cancel out when the VEV of the stress-energy tensor is computed.

We start with the metric /12,

$$ds^2 = 2Pdudv + 2uP\zeta d\zeta dv + 2uP\zeta d\bar{\zeta} dv - 2u^2 d\zeta d\bar{\zeta}. \quad 55$$

Here we use the same variables as those given for the impulsive wave; i.e., $u$ is a Bondi-type luminosity distance, $v$ is a timelike coordinate that can be regarded as retarded time and $\zeta$ and $\bar{\zeta}$ replace the angles on the stereographic projection of the sphere. $P$ is defined by $P = \frac{1}{|h\zeta|}$, where $h$ is an arbitrary function of the argument $\zeta + gv\Theta(v)$. $g$ is the dimensional coupling constant, with dimensions of mass and $\Theta$ is the Heavyside unit step function. Nutku /12 shows that the identifying factor in this metric is one component of the Weyl tensor,

$$\Psi_4 = -\frac{1}{uP}[PP\zeta\zeta - P\zeta\zeta P]\Theta(v). \quad 56$$

When we write the d’Alembertian operator in the this background metric, we get

$$\sqrt{-g}\Box = L = 2u^2 \frac{\partial^2}{\partial u \partial v} + 2u \frac{\partial}{\partial v} + 2uP\zeta \frac{\partial}{\partial u} - 2u^2 \frac{P\zeta}{P} \frac{\partial^2}{\partial u^2} - 4u \frac{P\zeta}{P} \frac{\partial}{\partial u}$$

$$+ 2u \left(P\frac{\partial}{\partial \zeta} + P\frac{\partial}{\partial \bar{\zeta}}\right) \frac{\partial}{\partial u} - 2P \frac{\partial^2}{\partial \zeta \partial \bar{\zeta}}. \quad 57$$

In our particular case we take $h = (\zeta + gv\Theta(v))^{1+i\delta}$, where $\delta << 1$ and is the expansion parameter.

We expand the operator $L$ in powers of $\delta$.

$$L = L_0 + \delta L_1 + \delta^2 L_2 + ... \quad 58$$

If we use real variables, $\zeta = \frac{x+iy}{\sqrt{2}}$, we get

$$L_0 = 2u^2 \frac{\partial^2}{\partial u \partial v} + 2u \frac{\partial}{\partial v} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}. \quad 59$$
\[
L_1 = -\frac{2u}{x^2 + y^2} \left( y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y} \right) \frac{\partial}{\partial u} - \tan^{-1} \frac{y}{x} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right),
\]
\[
L_2 = -\frac{1}{x^2 + y^2} \left( u \frac{\partial}{\partial u} + u^2 \frac{\partial^2}{\partial u^2} + 2\tan^{-1} \frac{y}{x} \left( y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y} \right) u \frac{\partial}{\partial u} \right)
+ \frac{1}{2} \left( 1 - (\tan^{-1} \frac{y}{x})^2 \right) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right).
\]

We expand both the solutions and the eigenvalue in terms of \( \delta \),
\[
\phi = \phi_0 + \delta \phi_1 + \delta^2 \phi_2 + \ldots, \quad \lambda = \lambda_0 + \delta \lambda_1 + \delta^2 \lambda_2 + \ldots.
\]

Up to second order we get the coupled set of equations
\[
L_0 \phi_0 = \lambda_0 \phi_0,
\]
\[
L_1 \phi_0 + L_0 \phi_1 = \lambda_1 \phi_0 + \lambda_0 \phi_1
\]
\[
L_2 \phi_0 + L_1 \phi_1 + L_0 \phi_2 = \lambda_2 \phi_0 + \lambda_1 \phi_1 + \lambda_0 \phi_2
\]

We make the ansatz \( \phi_1 = f \phi_0, \phi_2 = h \phi_0 \). The ansatz for \( \phi_1 \) results in the equation
\[
\left[ -2 \frac{\partial^2}{\partial s \partial v} - 2iR \frac{\partial}{\partial s} + \left( \frac{iK}{R} \frac{\partial}{\partial v} \right) - 2i \left( k_1 \frac{\partial}{\partial z} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial y^2} \right] f = \frac{2i}{z^2 + y^2} (k_1 y - k_2 z) \left( \frac{iKs}{2R} - 1 \right) - (k_1^2 + k_2^2) \tan^{-1} \left( \frac{y}{z} \right)
\]

We recognize that when we plug these ansatze into our equations, we get inhomogenous equations where on the right hand side the variable \( v \) always appears in the combination \( z = x + gv \) for \( v > 0 \). \( v \) is not an independent variable in \( f \) and \( h \). Thus we substitute to new set of variables \( u, z, y \). This necessitates replacing all \( v \) derivatives by \( g \frac{\partial}{\partial z} \). This makes an expansion in powers of \( g \) possible. We take
\[
f = f_0(z, y, u) + gf_1(z, y, u),
\]
\[
h = h_0(z, y, u) + gh_1(z, y, u).
\]

\( \phi_0 \) is given by, as in the previous cases
\[
\phi_0 = \frac{\exp[i(k_1 x + k_2 y + Rv - \frac{K}{2Rv})]}{u \sqrt{|R|(2\pi)^2}}.
\]

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where $K, k_1, k_2, R$ are the separation constants which act as eigenfrequencies to be integrated over to find the Greens Function. For $z = x + g\Theta(v)\nu, s = \frac{1}{\nu}$ we find

$$L_0 f_0 = I$$

$$L_1 f_0 + L_1 f_0 = 0$$

where

$$L_0 = -2iR \frac{\partial}{\partial s} - 2i \left( k_1 \frac{\partial}{\partial z} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

$$I = \frac{2i}{(z^2 + y^2)}(k_1 y - k_2 z) \left[ \frac{iKs}{2R} - 1 \right] - (k_1^2 + k_2^2)\tan^{-1}\frac{y}{z}$$

We see that if we use the variables $z + iy$ and $z - iy$ the right hand side can be written as a sum of functions of this two variables. We also write the differential operator on the left hand side in terms of these new set of variables and after some algebra we find

$$f_0 = \left[ \frac{Ks}{4R} + \frac{i}{2} \right] \ln \frac{z + iy}{z - iy} - \left[ \frac{K + k_1^2 + k_2^2}{4} \right] \left( \frac{1}{(k_1 + ik_2)} \right) \left[ (z + iy)\ln(z + iy) - (z + iy) \right]$$

$$- \left( \frac{1}{k_1 - ik_2} \right) \left[ (z - iy)\ln(z - iy) - (z - iy) \right]$$

$$f_1 = \left( \frac{iK}{2R} + \frac{K^2 s}{8R^2} \right) \left( \ln(z + iy) \right) \left( \frac{\ln(z + iy)}{k_1 + ik_2} - \frac{\ln(z - iy)}{k_1 - ik_2} \right)$$

$$- \frac{K}{4R} \left( \frac{1}{2(k_1^2 + k_2^2)} + \frac{K}{(k_1^2 + k_2^2)^2} \right) \left( (k_1 - ik_2)^2 ((z + iy)\ln(z + iy) - (z + iy)) \right.$$

$$- (k_1 + ik_2)^2 ((z - iy)\ln(z - iy) - (z - iy)) \right).$$

The Green function in first order for the warp function chosen above is given in reference 16. The expression given in this paper was calculated by a different method and also includes the contribution coming from the homogenous solution. Here we calculate them separately. We can show that we do not get a finite contribution to the vacuum expectation value of the stress-energy tensor. When the contribution only from the inhomogenous equations (eq.s 69,70) is taken, the Green function is given by a product of two factors one
is which always equals zero when the limit in \( x = x', y = y' \) is taken without equating \( u, v \) to \( u', v' \) respectively.

In second order in \( \delta \), we can reduce the differential equation to the system

\[
\mathcal{L}_0 h_0 = M_0, \tag{76}
\]
\[
\mathcal{L}_0 h_1 + \mathcal{L}_1 h_0 = M_1, \tag{77}
\]

where

\[
\mathcal{L}_0 = -2iR \frac{\partial}{\partial s} - 2i \left( k_1 \frac{\partial}{\partial z} + k_2 \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial y^2}, \tag{78}
\]
\[
\mathcal{L}_1 = -2 \frac{\partial^2}{\partial s \partial z} + \frac{iK}{R} \frac{\partial}{\partial z}. \tag{79}
\]

Here \( s = \frac{1}{u} \). \( M_0 \) and \( M_1 \) are given as

\[
M_0 = \frac{1}{2} K - \frac{1}{z^2 + y^2} \left( 1 - \frac{3iKs}{2R} - \frac{K^2 s^2}{4R^2} \right) \left( 1 - 2i(k_1 y - k_2 z)\tan^{-1}\frac{y}{z} \right) + K \left( \frac{5}{2} - \frac{iKs}{2R} \right) \left( \tan^{-1}\frac{y}{z} \right)^2
\]
\[
-2K \left( \frac{\log(z^2 + y^2)}{2} - 1 \right) (k_1 y - k_2 z) + \tan^{-1}\frac{y}{z}(k_1 z + k_2 y)
\]
\[
\times \left( \left( 1 - \frac{iKs}{R} \right) \frac{k_1 y - k_2 z}{(z^2 + y^2)K} - \frac{i}{2} \tan^{-1}\frac{y}{z} \right)
\]
\[
- \frac{i}{z^2 + y^2} \left( 1 - \frac{iKs}{2R} \right) (k_1 z + k_2 y)\log(z^2 + y^2) + 2(k_2 z - k_1 y)\tan^{-1}\frac{y}{z} \right)
\]
\[
M_1 = \frac{2iK}{(z^2 + y^2)R} \left( 1 - \frac{iKs}{4R} \right) \tan^{-1}\frac{y}{z} - \frac{2k_1}{(z^2 + y^2)R} \left( 1 - \frac{iKs}{R} - \frac{K^2 s^2}{8R^2} \right)
\]
\[
- \left( 2k_1 \tan^{-1}\frac{y}{z} - k_2 \log(z^2 + y^2) \right)
\]
\[
\left( \frac{-iK^2 s}{8R^2} + \frac{5K}{4R} \right) \tan^{-1}\frac{y}{z} - i(k_1 y - k_2 z) \left( 1 - \frac{iKs}{R} - \frac{K^2 s^2}{8R^2} \right)
\]
\[
+ \left( k_1^2 - k_2^2 \right) \left( 2\tan^{-1}\frac{y}{z} + \log(z^2 + y^2) - 2y \right)
\]
\[
- 2k_1 k_2 \left( z\log(z^2 + y^2) - 2y\tan^{-1}\frac{y}{z} - 2z \right)
\]
\[
\times \left( \frac{3Ki}{8R} \tan^{-1}\frac{y}{z} - \frac{3}{4(z^2 + y^2)R} (k_1 y - k_2 z)(1 - \frac{iKs}{2R}) - \frac{3i}{4(z^2 + y^2)R} \left( 1 - \frac{iKs}{2R} \right) \right)
\]
\[
\times \left( (k_1^2 - k_2^2) \left( z \log(z^2 + y^2) - 2ytan^{-1} \frac{y}{z} \right) + 2k_1k_2 \left( y \log(z^2 + y^2) + 2z tan^{-1} \frac{y}{z} \right) \right)
\]

In these expressions we took the ‘mass-shell’ condition, which is imposed in the calculation of the Greens function; i.e. we set \( k_1^2 + k_2^2 \) equal to \( K \). One can check that after we perform the \( K \) and \( k_1, k_2 \) integrations the effect of these two expressions are exactly the same.

We see that, contrary to the impulsive wave calculation, in both \( M_0 \) and \( M_1 \), there are no terms that are independent of \( z \) and \( y \) except a single term which is proportional to \( K \).

To be able to obtain terms in \( < T_{\mu\nu} > \) that diverge as the infrared parameter goes to zero, we need inverse powers of \( R \) which are not multiplied by \( K \) or \( k_1^2, k_2^2 \). Each inverse power of \( R \) means a higher infrared divergence, resulting in terms that go \( m_1^2, 1, \log m_1^2, \frac{1}{m_1^2}, \frac{1}{m_1^4}, \) etc...as the power of \( \frac{1}{R} \) is increased, whereas each power of \( K, k_1^2, k_2^2 \) means one lower order in the same divergence. In free space the power of \( m \) is zero. There is no infrared divergence.

In reference 15, we generated these divergences at second order and then cancelled them with the cosmological constant of the de Sitter solution. Our mechanism for obtaining these infrared divergences was as follows. We isolated the \( -2iR \frac{\partial}{\partial s} \) in the operator \( L_0 \) from the others and equated it to the term which did not contain \( z \) or \( y \).

\[
-2iR \frac{\partial}{\partial s} h'_0 = cs
\]

where \( c \) can be a function of \( v \) but not that of \( z \) and \( y \). Then \( h'_0 = \frac{ics^2}{4R} \) has an extra power of \( 1/R \) compared to the other terms. The second iteration gives us \( h'_1 \propto s^3/R^2 \). Such a term will induce \( 1/m^2 \) factor in the expression for the Greens Function , \( G_F \), and this infrared mass will be retained in \( < T_{\mu\nu} > \).

For the shock wave solution all the terms in \( M_0 \) and \( M_1 \) are either functions of \( z \) and \( y \) or are multiplied by \( K \). We can not isolate a part of the operator \( L_0 \) and equate it to a single term on the RHS. We may still study the shape of solutions. Let us make the ansatz

\[
h_0 = s^2 h_{00} + sh_{01} + h_{02}.
\]

We find that

\[
h_{00} = -\frac{4K^2}{32R^2} \left( tan^{-1} \frac{y}{z} \right)^2 ;
\]
\[ h_{01} = -\frac{3iK}{4R} \left( \tan^{-1} \frac{y}{z} \right)^2 \]

etc. These solutions show that we are not getting powers of \( R \) in the denominator with no powers of \( K \) in the numerator. This fact makes it clear that there is no way of acquiring \( \frac{1}{m^s} \) in the Green function, since we need an extra \( R^{-2} \) factor, which necessitates at least one power in the first integration. On RHS only the combination \( \frac{K^2}{R^2} \) and \( \frac{K}{R} \) exist. \( \frac{K^2}{R^2} \) gives exactly the singularity structure as the free case, and \( \frac{K}{R} \) gives a logarithmic divergence which is cancelled in the \( <T_{\mu\nu}> \) calculation.

At this point we note that we can find solutions of equations 76 and 77 even if \( M_0 \) and \( M_1 \) are set to zero. These are the homogenous solutions of the problem which give a non trivial contribution for the shock wave calculation. These solutions of the homogenous equations generated the only non-zero part of the Green function when we took \( x = x' \), \( y = y' \) in the first-order calculation. Since \( M_0 \) and \( M_1 \) are independent of \( v \) we can assume a powers series expansion in \( v \) for a chosen order in \( g \). For the sake of illustration we take a solution in third order in \( g \) and write the expansion as

\[ f^{(1,3)}_H = g^3 (v^3 f^{(1,3)}_1 + v^2 f^{(1,3)}_2 + v f^{(1,3)}_3 + f^{(1,3)}_4). \]

Here \( f^{(1,3)}_1(s, z, y) \) has dimension zero, \( f^{(1,3)}_2(s, z, y) \) has dimension minus one, etc. Inverse powers of \( v \) are excluded by the regularity at \( v = 0 \). One can show that taking powers of \( v \) higher than that of \( g \) do not give results that differ from the free case. A similar expansion in the impulsive case would go as

\[ f^{(1,3)}_H = (v^3 R^3 f^{(1,3)}_1 + v^2 R^2 f^{(1,3)}_2 + ...) \]

where \( f^{(1,3)}_1 \) etc., have the same dimensions as above, since the only free dimensional parameters are \( v \) and \( R \). This gives the result of the flat, Minkowski, space Green function.

Keeping track of powers of \( v \) we get a system of four equations. We note that the first of these equations

\[ \mathcal{L}_o f^{(1,3)}_1 = 0 \]

has a solution for any function

\[ F = F \left( \frac{s}{R} (k_1 \pm ik_2) - (z \pm iy) \right). \]
We can also show that the singularity behaviour of the Greens Function is independent of the form of $F$. At the end we get, for the worse infrared poles the expressions

$$G_{H}^{(1,H)} = 2\pi c_1 \left( \frac{v^3 \Theta(v) + v'^3 \Theta(v')}{(u - u')(v - v')} \right),$$

$$G_{H}^{(2,H)} = 2\pi c_2 \left( \frac{uv^2 \Theta(v) + u'v'^2 \Theta(v')}{(u - u')^2} \log(2m^2(u - u')(v - v')) \right),$$

$$G_{H}^{(3,H)} = 2\pi c_3 \left( \frac{u^2v \Theta(v) + u'^2v' \Theta(v')}{(u - u')^4 m^2} \right),$$

$$G_{H}^{(4,H)} = 2\pi c_4 \left( \frac{u^3 \Theta(v) + u'^3 \Theta(v')}{(u - u')^6 m^4} \right).$$

Here $c_i$ are functions of $x$ and $y$, depending on the form for $F$ used. $m$ is the infrared mass. If we use a linear function for $F$, $c_i$ is proportional to $y$ or $x$.

Upon symmetric differentiation with respect to $v$ and $v'$, the terms with $\frac{1}{m^2}$ and $\frac{1}{m^4}$ vanish. We find a finite contribution only if we go to the de Sitter space, i.e. multiply by the factor $(1 + \frac{\Lambda v}{6})(1 + \frac{\Lambda v'}{6})$. In this case we get

$$< T_{vv} > = g^3 \left( f_1 \Lambda v \Theta(v) + f_2 \Lambda^2 uv^2 \Theta(v) \right)$$

which goes to zero with $\Lambda$ when we go back to Minkowski background. In this expression $f_1$ are regular functions of $x, y$. One can also show that any terms with less diverging powers of $m^{-2}$ in $G_{H}^{(4,H)}$ do not give a finite contribution even in de Sitter background.

Here we tried to show that two qualitative differences exist between the shock and the impulsive wave solutions proposed by the same group $^{10,12}$. In the shock wave solution the infrared divergences which may be used to tame the ultraviolet divergences to result in finite contributions to $< T_{vv} >$ are absent in second order perturbation theory. We can not find finite contributions to $< T_{vv} >$ in Minkowski space. If we go to de Sitter space, though, we get a finite contribution which is proportional to $\Theta$ function, which is the signature of a shock wave solution.

The homogenous solutions, in the shock wave, give contributions to the Greens function expression which are different from the free case. These solutions also give a finite contribution to $< T_{vv} >$ in de Sitter space. The presence of these nontrivial solutions is
only due to the dimensional coupling constant. The presence of $g$ in the expansion makes it necessary to have an extra power of $\frac{1}{R}$ in the solution which results in a nontrivial term in $G_F$.

VI. Plane impulsive wave

As the last example here we take the metric describing an impulsive plane wave /17,

$$ds^2 = 2dudv - |d\zeta + q\zeta v\Theta(v)d\zeta|^2.$$ 95

If we take $q = g\frac{\zeta^2}{2}$ we get a plane wave. If the power is higher than quadratic we get pp waves /18. The d’Alembertian operator in this metric is written as

$$L = 2\partial_u\partial_v - \frac{2vg^2}{1 - v^2g^2}\partial_u - \frac{1}{(1 + vg)^2}\partial_x^2 - \frac{1}{(1 - vg)^2}\partial_y^2$$ 96

If we expand up to second order in the coupling constant $g$, we get

$$L \approx 2\partial_u\partial_v - 2vg^2\partial_u - (1 + 3(vg)^2)(\partial_x^2 + \partial_y^2 + 2vg(\partial_x^2 - \partial_y^2)).$$ 97

The zeroth-order solution is similar to the previous cases, eq. (15) resulting in a Green function that goes as

$$A = \frac{1}{(u - u')(v - v') - \frac{1}{2}[(x - x')^2 + (y - y')^2]}$$ 98

for constant $A$. We expand the solution in powers of $g$ and take the first order solution as $\phi^{(1)} = f\phi^0$. It is straightforward to solve for $f$ and we get

$$f = \frac{(k_1^2 - k_2^2)u}{2iR} \left[ v + i\frac{Ku}{4R^2} \right]$$ 99

For the second order solution we again take $\phi^{(2)} = \phi^{(0)}h$. Here $h = v^2h_1(x, y, u) + vh_2(x, y, u) + h_3(x, y, u)$. A straightforward calculation gives us

$$h_1 = \frac{3i}{2R}(k_1^2 + k_2^2)u - \frac{u^2}{4R^2}(k_2^2 - k_1^2)^2,$$ 100

$$h_2 = \frac{u}{R^2}(\frac{K}{2} - 3(k_1^2 + k_2^2)) - \frac{3iu^2}{4R^3}((k_1^2 - k_2^2)^2 + K(k_1^2 + k_2^2)) + \frac{K(k_1^2 - k_2^2)^2u^3}{8R^4},$$ 101
\[ h_3 = \frac{iu}{R^3} \left( \frac{K}{2} - 3(k_1^2 + k_2^2) \right) + \frac{u^2}{R^3} \left( \frac{1}{8} \left( 3(k_1^2 - k_2^2)^2 + 3K(k_1^2 + k_2^2) - K^2 \right) \right. \]
\[ \left. + \frac{iu^3}{8R^5} \left( 2K(k_1^2 - k_2^2)^2 + K^2(k_1^2 + k_2^2) \right) - \frac{K^2(k_1^2 - k_2^2)^2 u^4}{64R^6} \right]. \]

Here we see that a peculiar thing happens. We almost get a finite expression, but we have one power of \((u - u')\) too many in the denominator. If we go to de Sitter space to cancel both the ultra-violet and infrared divergences, we see that the finite part of \(< T_{vv} >\) goes as

\[ < T_{vv} > \propto -2 \frac{\Lambda^2}{m^2} \Theta(v) \]

which is finite only in de Sitter space. One power of the curvature cancels with the infrared parameter since we take \(\Lambda \propto m^2\), but the remaining power takes the contribution to zero when we go back to Minkowski space.

This result which is in accord with general arguments of Deser \cite{6,7} is a check that our method does not contradict any known results.

One can show that this result does not change in the presence of a pp-wave background. Whether a wave is plane or pp type depends only on the form of the function \(q(\zeta)\) in the metric. The general behaviour of the expression for the vacuum expectation value of the stress-energy tensor does not depend on the form of the function \(q\). This form only changes an overall factor which can not decide whether the whole expression is finite or null. The same behaviour was already seen in the different warp functions we have used for the spherical wave.

**CONCLUSION**

Here we reviewed the work on the vacuum fluctuations for the stress-energy tensor for a scalar particle in the background of different impulsive and shock wave metrics. We found a finite fluctuation only for the spherical impulsive wave metric, whereas our result was null for the spherical shock and plane impulsive waves. We attribute the difference in the two cases to the presence of the dimensionless and dimensional constants.

If we calculate the fluctuations for a conformal metric, fluctuations should be absent \cite{2}. We first perform perturbation theory about the Minkowski space, and our perturbations are not strong enough to overcome the restrictions imposed by conformal symmetry. If we
go to de Sitter space, and perform perturbation around that metric, we do not have this
obstruction. We always find finite fluctuations in that metric. Note that de Sitter space
also tames our ultra-violet divergences. It turns out that if we have dimensional coupling
constants, we have more severe ultra-violet divergences which are tamed only with having
higher powers of the curvature scalar of de Sitter space, multiplying our expressions for
the fluctuations. This happens in the two latter metrics, plane and pp waves, we have
studied. The resulting infrared divergences are not strong enough to cancel these scalar
curvature factors to give us a finite result.

In the spherical shock wave solutions, we could not generate such divergences in the
inhomogenous case. For the homogenous solutions, the ultra-violet divergences could not
be tamed by going to de Sitter space. Cancellation of the infrared divergence, say in
eq.92, by cancelling it by the $\Lambda$ term was not sufficient to obtain a finite expression.

The presence of non-trivial homogenous solutions was another novel feature of the
shock waves. It was amusing that only one of them survived in the expression for $<T_{\mu\nu}>$.
If many of them survived, we would have an ambiguity in defining this quantity. It was
also interesting that no matter how many orders in the coupling constant one expands,
one gets one and only one surviving term, the one obtained already in first order.

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