Many-body corrections to cyclotron resonance in graphene

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Abstract. We report our recent study of cyclotron resonance in graphene, with focus on the many-body effect on the resonance energies. The genuine many-body corrections turn out to derive from vacuum polarization, specific to graphene, which diverges at short wavelengths and which requires renormalization of velocity and, for bilayer graphene, interlayer coupling as well. As a result, the renormalized velocity and interlayer coupling strength run with the magnetic field, and many-body corrections are uniquely determined from one resonance to another. Theory is compared with the experimental data for both monolayer and bilayer graphene.

1. Introduction
Graphene and its multilayer give rise to rich spectra of cyclotron resonance, with resonance energies varying from one transition to another within the electron band or the hole band, and, notably, even between the two bands [1]. This is in sharp contrast with conventional quantum Hall (QH) systems with a parabolic dispersion, where cyclotron resonance takes place between adjacent Landau levels, hence at a single frequency $\omega_c = eB/m^*$ which, according to Kohn’s theorem [2], is unaffected by Coulomb interactions [3]. The nonparabolic electronic spectra in graphene evade Kohn’s theorem and offer the possibility to detect the many-body corrections to cyclotron resonance [4, 5, 6].

Experiment has so far verified, via infrared spectroscopy, some characteristic features of cyclotron resonance in monolayer [7, 8] and bilayer [9] graphene. Data generally show no clear sign of the many-body effect, except for one [7] on monolayer graphene.

In this paper we would like to report our recent study of many-body corrections to cyclotron resonance in graphene and its bilayers [10]. We construct an effective theory of cyclotron resonance within the single-mode approximation and point out the following:

(1) The genuine nonzero many-body corrections arise from vacuum polarization, specific to graphene, which diverges logarithmically at short wavelengths.

(2) Both intralayer and interlayer coupling strengths require renormalization, which then allows one to determine many-body corrections uniquely from one resonance to another. The renormalized velocity $v_0^{\text{ren}}$ and inter-layer coupling $\gamma_1^{\text{ren}}$ thereby run with the magnetic field $B$, decreasing gradually with increasing $B$.

(3) Theory is favorably compared with the experimental data on graphene.
2. Monolayer graphene

Monolayer graphene supports as charge carriers massless Dirac fermions at two inequivalent valleys (K and K’), and they, in a magnetic field, lead to a tower of unequally-spaced Landau levels |n, y0⟩ of energy

\[ \epsilon_n = s_n \omega_c \sqrt{|n|} \quad \text{with} \quad \omega_c = \sqrt{2} v_0/\ell, \]  

labeled by integers \( n = \pm 0, \pm 1, \pm 2, \ldots \), and the center coordinate \( y_0 \equiv \ell^2 p_x \) with the magnetic length \( \ell \equiv 1/\sqrt{eB} \); \( s_n \equiv \text{sgn}[n] \rightarrow \pm 1 \); the “light velocity” \( v_0 \sim 10^6 \text{ m/s} \) leads to the basic cyclotron energy \( \omega_c \equiv \sqrt{2} v_0/\ell \approx 36.3 \times 10^3 \text{ m/s} \). Each Landau level is fourfold degenerate (in spin and valley) when Zeeman coupling is ignored. Supplying a tiny asymmetry in sublattices reveals that the \( n = 0 \) level is doubly degenerate and electron-like with \( \epsilon_0 = 0_+ \) at one valley while it is hole-like with \( \epsilon_0 = -0_+ \) at another valley. See Fig. 1(a).

Let us expand the electron field \( \psi(x, t) \) in terms of the Landau-level eigenmodes \( \psi_n(y_0, t) \) and write the charge density \( \rho_{-p}(t) = \int d^2x \, e^{ip \cdot x} \psi \dagger \psi \) as [11]

\[ \rho_{-p} = \sum_{k, n = -\infty}^{\infty} g_{kn}^p R_{kn}^p, \quad R_{kn}^p = e^{-\frac{i}{2} \ell^2 p^2} \int dy_0 \psi_n^\dagger(y_0, t) e^{ip \cdot r} \psi_n(y_0, t), \]  

where \( r = (i\ell^2 \partial/\partial y_0, y_0) \) stands for the center coordinate. The coefficient matrix \( g_{kn}^p \) is given by

\[ g_{kn}^p = \frac{1}{2} b_k b_n (f_{kn}^{[k]} - s_k s_n f_{kn}^{[n]}) \quad \text{with} \quad b_0 = \sqrt{2} \quad \text{and} \quad b_n = 1 \quad \text{for} \quad n \neq 0, \]  

where \( s_n = \text{sgn}[n] \rightarrow \pm 1; \ f_{kn}^{[k]} = \sqrt{n/k!} (-\ell p/\sqrt{2})^{k-n} L_n^{(k-n)}(\ell^2 p^2) \) for \( k \geq n \geq 0 \), and \( f_{kn}^p = (f_{kn})^\dagger; \ p = p_x - ip_y \). The Coulomb interaction is written as

\[ H_{\text{Coul}} = \frac{1}{2} \sum_p v_p : \rho_{-p} \rho_p :, \]  

where \( v_p = 2\pi \alpha/(\epsilon_0 |p|) \) with \( \alpha = e^2/(4\pi \epsilon_0) \approx 1/137 \) and the substrate dielectric constant \( \epsilon_b \).

3. Cyclotron resonance and renormalization

Let us note that the charge operator \( \rho_{kn}^p = g_{kn}^p R_{kn}^p \) in Eq. (2) annihilates an electron at the nth level \( L_n \) and creates one at the kth level \( L_k \). One may thus associate it with the inter-Landau-level transition \( L_n \rightarrow L_k \), and calculate the Coulomb corrections \( \propto H_{\text{Coul}} \) to the excitation energy using the single-mode approximation in the standard way. Actually, the calculation is done in a purely algebraic manner if one notes that the charge operators \( R_{kn}^p \) obey the \( W_\infty \) algebra.

The result is the following [10]: The cyclotron-resonance energy for a general \( L_n \rightarrow L_k \) channel with the Landau levels filled up to \( n = n_f \) is written as \( \epsilon_{\text{exc}}^{k = 0} = \epsilon_b - \epsilon_a + \triangle_{k = 0}^{b - a} \) with

\[ \triangle_{k = 0}^{b - a} = \sum_p v_p e^{-\frac{i}{2} \ell^2 p^2} \left( \sum_{n \leq n_f} (|g_{kn}^p|^2 - |g_{kn}^{0n}|^2) - g_{kn}^0 g_{kn}^{0n} \right). \]  

Note that \( n_f = 1, 0_+, -1, -2, -3 \) correspond to the filling factors \( \nu = 4 n_f + 2 = 6, 2, -2, -6, -10 \), respectively. The \( \sum_{n \leq n_f} (|g_{kn}^p|^2 - |g_{kn}^{0n}|^2) \) term refers to the change in quantum fluctuations, via the \( a \rightarrow 0 \) transition, of the filled states.

For standard QH systems this correction \( \triangle_{k = 0}^{b - a} \) vanishes for each transition to the adjacent level, \( L_n \rightarrow L_{n+1} \), according to Kohn’s theorem. Let us consider it for the \( 0 \rightarrow 1 \) transition in graphene. We use \( g_{00}^0 = 1, g_{10}^0 = -\ell p/2 \) and \( g_{11}^0 = 1 - \ell^2 p^2/4 \), and note the following:
Any nonzero shift $\Delta \epsilon_{\mathbf{k}=0}^{1-0}$ comes from the quantum fluctuations of the Dirac sea, and actually diverges logarithmically with the number $N_L$ of filled Landau levels in the sea,

$$\Delta \epsilon_{\mathbf{k}=0}^{1-0} = \sum_{\mathbf{p}} v_0 \epsilon \sqrt{2} \epsilon^2 \mathbf{p} \sum_{-N_L \leq n \leq -1} (|g_{\mathbf{p}}^{0n}|^2 - |g_{\mathbf{p}}^{1n}|^2) = v_c C_N,$$

(6)

$$C_N \approx (\sqrt{2}/8) (\log N_L - 1.017),$$

$$v_c \equiv \alpha/(\epsilon_b \ell) \approx (56.1/\epsilon_b) \sqrt{B[T]} \text{meV}.$$

This divergence in $C_N$ derives from short-wavelength vacuum polarization and is present even for $B = 0$, i.e., in free space [12]. It thus does not make sense to discuss the magnitude of $C_N$. The legitimate procedure is to renormalize $v_0$ by rescaling

$$v_0 = Z_v v_0^{\text{ren}}$$

(8)

and to put reference to the cutoff into $Z_v$.

The renormalized velocity $v_0^{\text{ren}}$ is defined by referring to a specific resonance. Let us take the $0 \rightarrow 1$ resonance and choose to absorb the entire $O(V_c)$ correction at some reference scale (e.g., at magnetic field $B_0$) into $Z_v$, i.e., we write

$$\epsilon_{\mathbf{k}=0}^{1-0}|_{\nu=2} = \epsilon_1 + \Delta \epsilon_{\mathbf{k}=0}^{1-0} = \sqrt{2} v_0^{\text{ren}}|_{B=B_0} \equiv \omega_c^{\text{ren}}|_{B}.$$

(9)

The renormalized velocity then depends on $B$, or runs with $B$,

$$v_0^{\text{ren}}|_{B} = v_0^{\text{ren}}|_{B_0} - \frac{\alpha}{8\epsilon_b} \log(B/B_0),$$

(10)

decreasing gradually for $B > B_0$. We denote $v_0^{\text{ren}}|_{B}$ as $v_0^{\text{ren}}$ and $\omega_c^{\text{ren}}|_{B}$ as $\omega_c^{\text{ren}}$ for short.

The resonance energies $\epsilon_{\mathbf{k}=0}^{\nu-0}$ for all other channels, as illustrated in Fig. 1 (a), are made finite by this renormalization. The resulting corrections $\propto V_c$ then make sense as genuine observable corrections. In particular, for several intraband channels one finds

$$\epsilon_{\mathbf{k}=0}^{2-1}|_{\nu=6} = \epsilon_{\mathbf{k}=0}^{2-1}|_{\nu=-6} = (\sqrt{2} - 1) \{\omega_c^{\text{ren}} - 0.264 V_c\},$$

$$\epsilon_{\mathbf{k}=0}^{3-2}|_{\nu=10} = \epsilon_{\mathbf{k}=0}^{3-2}|_{\nu=-10} = (\sqrt{3} - \sqrt{2}) \{\omega_c^{\text{ren}} - 0.358 V_c\}.$$
Cyclotron resonance in graphene is governed by the selection rule $\Delta|n| = \pm 1$, and, as a result, graphene supports interband cyclotron resonances. Some typical channels yield

$$
\epsilon_{k=0}^{2-1}|_{\nu=-2} = (\sqrt{2} + 1) \left\{ \epsilon_{c0}^{\text{ren}} + 0.122 V_c \right\},
$$

$$
\epsilon_{k=0}^{1-2}|_{\nu=-2} = (\sqrt{2} + 1) \left\{ \epsilon_{c0}^{\text{ren}} + 0.155 V_c \right\},
$$

$$
\epsilon_{k=0}^{1-2}|_{\nu=-6} = (\sqrt{2} + 1) \left\{ \epsilon_{c0}^{\text{ren}} + 0.084 V_c \right\},
$$

$$
\epsilon_{k=0}^{2-3}|_{\nu=-10} = (\sqrt{2} + \sqrt{3}) \left\{ \epsilon_{c0}^{\text{ren}} + 0.044 V_c \right\}.
$$

(12)

It is now clear that cyclotron resonance is best analyzed by plotting the rescaled energies $\epsilon_{k=0}^{b-a} = s_b \sqrt{|B|} - s_a \sqrt{|a|}$ as a function of $\sqrt{B}$ or $B$. The Coulombic many-body effect will be seen as a variation of the characteristic velocity $v_0^{\text{ren}}[1 + O(V_c)]$ from one resonance to another, and a deviation of $\epsilon_{c0}^{\text{ren}}$ from the $\sqrt{B}$ behavior would indicate the running of $v_0^{\text{ren}}$ with $B$.

Actually, experiment by Jiang et al. [7] did observe a small deviation of the $1 : (1+\sqrt{2})$ ratio of $\epsilon_{k=0}^{b-a}$ to $\epsilon_{k=0}^{b-1}$ well outside of the experimental errors under magnetic fields $B = (6 \sim 18)$ T. Such data are included in Fig. 1 (b), which plots $\epsilon_{k=0}^{1-0}$ and $\epsilon_{k=0}^{b-1} = (\sqrt{2} + 1)$ as a function of $\sqrt{B}$. A small increase of $v_0^{\text{ren}}$ in $\epsilon_{k=0}^{b-1}/(\sqrt{2} + 1)$, relative to $\epsilon_{k=0}^{b-0}$, is roughly consistent with Eq. (12) which suggests a $0.122 V_c/\epsilon_{c0}^{\text{ren}} \sim 4\%$ increase in $v_0^{\text{ren}}$ (since $V_c/\epsilon_{c0}^{\text{ren}} \sim 0.3$).

This feature is more pronounced in Fig. 1 (c), which plots the same data in units of $\omega_c = \sqrt{2} v_0^{\text{ren}}(B=10 T) / \ell \propto \sqrt{B}$. There a dotted curve represents a possible profile of the running of $v_0^{\text{ren}}$ with $B$, and, especially, the $(-1 \rightarrow -2)$ data (with smaller error bars) suggests such running.

The data are thus consistent (in sign and magnitude) with the present estimate of the many-body effect. In this connection, let us note that an earlier experiment on thin epitaxial graphite [13] also observed the same pair of resonances, with apparently no deviation from the $1 : (1+\sqrt{2})$ ratio. This measurement was done under relatively low magnetic fields $B = (0.4 \sim 4)$ T, and it could be that a small deviation, under larger error bars, simply escaped detection, apart from the potential difference between thin graphite and graphene.

More precise measurements of cyclotron resonance, especially in the high $B$ domain, would be required to pin down the many-body effect in graphene. In this respect, the comparison between interband and intraband resonances from the same initial state, such as the $(-2 \rightarrow \pm 1)$ resonances and $(-3 \rightarrow \pm 2)$ resonances, would provide a clearer signal for the many-body effect. One would expect sizable variations in $v_0^{\text{ren}}$ for such pairs, $\Delta R(-2 \rightarrow \pm 1) \approx 6 \%$.

4. Cyclotron resonances in bilayer graphene

In bilayer graphene interlayer coupling $\gamma_1 \equiv \gamma_{AB} \sim (0.3 - 0.4)$ eV modifies the intralayer linear spectra to yield, in the low-energy branches $|\epsilon| < \gamma_1$, quasiparticles with a parabolic dispersion. They, in a magnetic field, support a variety of cyclotron resonances, as listed in Fig. 2 (a).

One can again use the SMA formula (5) to calculate the interlevel excitation energies. It turns out that not only $v_0$ but also $\gamma_1$ undergo infinite renormalization and, rather unexpectedly, the divergent terms are the same for both of them to $O(V_c)$ at least. We thus make the common rescaling $v_0 = Z v_0^{\text{ren}}$ and $\gamma_1 = Z \gamma_1^{\text{ren}}$, and eventually learn that the excitation spectrum is cast in the form

$$
\epsilon_{k=0}^{b-a} = (s_b \eta_b - s_a \eta_a) (\omega_c^{\text{ren}} + \Delta c^{ba} V_c),
$$

(13)

where $\eta_k$ are some given functions of $v_0^{\text{ren}}/\gamma_1^{\text{ren}}$ and $\Delta c^{ba}$ are uniquely fixed as genuine quantum corrections.

The many-body effect is expected to be sizable in bilayer graphene. The existing data by Henriksen et al. [9] generally appear to defy good fit by theory but certainly suggest nontrivial features of many-body corrections, such as running with $B$; see Fig. 2.
Figure 2. (a) Cyclotron resonance in bilayer graphene. (b) Resonance energies; real curves, with $v_0 = 1.15\times10^6\text{m/s}$, $\gamma_1 = 0.35\text{ eV}$ and $V_c = 0$; dotted curves, with $\gamma_1 = 0.38\text{ eV}$ and $V_c \approx 5.6\sqrt{B[T]}\text{ meV}$ (or $\epsilon_b \approx 10$). The experimental data are reproduced from Ref. [9]. (c) Electron data, reorganized in the form $\epsilon_k^{\text{ib}} = a_k - b_s\gamma_b - s_a\gamma_a$ and plotted in units of $\omega_c = \sqrt{2v_0/\ell^2}$; for clarity the data points are slightly shifted in $B$. (d) Theoretical expectation according to Eq. (13).

5. Concluding remarks
In this paper we have studied many-body corrections to cyclotron resonance in graphene, with emphasis on the need for renormalization. More precise measurements of cyclotron resonance are highly desired. Of particular interest are experiments that compare interband and intraband resonances from the same initial states, which would clarify the many-body effect with minimal uncertainties.

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References
[1] Abergel D S L and Fal’ko V I 2007 Phys. Rev. B 75 155430
[2] Kohn W 1961 Phys. Rev. 123 1242
[3] For many-body corrections to cyclotron resonance in quantum Hall systems see, Asano K and Ando T 1998 Phys. Rev. B 58 1485
[4] Iyenga A, Wang J, Fertig H A and Brey L 2007 Phys. Rev. B 75 125430
[5] Bychkov Yu A and Martinez G 2008 Phys. Rev. B 77 125417
[6] Viola Kusminskiy S, Campbell D K and Castro Neto A H 2009 Euro. Phys. Lett. 85 58005
[7] Jiang Z, Henriksen E A, Tung L C, Wang Y-J, Schwartz M E, Han M Y, Kim P and Stormer H L 2007 Phys. Rev. Lett. 98 197403
[8] Deacon R S, Chuang K-C, Nicholas R J, Novoselov K S and Geim A K 2007 Phys. Rev. B 76 081406(R)
[9] Henriksen E A, Jiang Z, Tung L-C, Schwartz M E, Takita M, Wang Y-J, Kim P and H. L. Stormer 2008 Phys. Rev. Lett. 100 087403
[10] Shizuya K 2010 Phys. Rev. B 81 075407
[11] Shizuya K 2007 Phys. Rev. B 75 245417
[12] González J, Guinea F and Vozmediano M A H 1994 Nucl. Phys. B 424 [FS] 595
[13] Sadowski M L, Martinez G, Potemski M, Berger C and de Heer H A 2006 Phys. Rev. Lett. 97 266405