On the problem of solving the optimization for continuous space based on information distribution function of ant colony algorithm

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Abstract. These years, ant colony algorithm has been widely used in solving the domain of discrete space optimization, while the research on solving the continuous space optimization was relatively little. Based on the original optimization for continuous space, the article proposes the improved ant colony algorithm which is used to solve the optimization for continuous space, so as to overcome the ant colony algorithm’s disadvantages of searching for a long time in continuous space. The article improves the solving way for the total amount of information of each interval and the due number of ants. The article also introduces a function of changes with the increase of the number of iterations in order to enhance the convergence rate of the improved ant colony algorithm. The simulation results show that compared with the result in literature[5], the suggested improved ant colony algorithm that based on the information distribution function has a better convergence performance. Thus, the article provides a new feasible and effective method for ant colony algorithm to solve this kind of problem.

1 Introduction
In the early 1990s, the Italian scholars Dorigo M and Gambardella L. M. put forward a simulated evolutionary algorithm that simulated the nature ant colony behavior[1], and the algorithm obtained the optimal solution which possessed NP-hard of traveling salesman problem[2][3]. It was an intelligent algorithm that simulated the ants foraging behavior. The advantages of this algorithm were strong robustness, excellent ability of global optimization and easy to be combined with other methods, etc.[4] At present, the algorithm has got good application in many fields, meanwhile, the research results published in recent years shown that the ant colony algorithm had strong superiority in solving the problem of the discrete space optimization. However, the researches on the ant colony in solving the problem of the continuous space optimization were relatively little. There were mainly Wang L etc. who developed the process of “the amount of information retained” in discrete domain ant colony algorithm for “information distribution function” in continuous domain. They also defined the improved ant colony algorithm that used for the problem of continuous function optimization[5]; BiLchev G A etc. put forward a kind of ant colony algorithm which is combined with genetic algorithm; Gao Shang etc. raised a kind of continuous ant colony algorithm based on the strategy of network partitioning[6]; Dreo J etc. come up with a kind of continuous interaction ant colony algorithm based on intensive non hierarchical; Pourtakdou S H etc. raised the ant colony algorithm for continuous optimization; Chen Ye etc. also put forward the ant colony algorithm for continuous optimization, etc[7]. Wang Jun etc. put forward the improved ant colony algorithm that used for solving continuous function constrained optimization[8]; Zhao Haiying etc. raised the ant colony algorithm that
used for solving the normal distribution function optimization and etc\textsuperscript{[9]}. The article made some exploratory research on applying the ant colony algorithm for solving the problem of the continuous space of optimization, and put forward the improved ant colony algorithm basing on the research results of Wang L.

2 The ant colony algorithm used for the continuous space optimization

In solving the problem of the continuous space optimization, solution space is one of regional representations. Wang L defined the ant colony algorithm that used for solving the problem of continuous function optimization\textsuperscript{[5]}, the main idea of which was that the process of each solving step in the ant information retained shouldn’t aim at the discrete point set of the component, it should not only influence the current ant colony point set, but also have corresponding impact on the point of the surrounding area at the same time. Thus, the description of the retained way for the ant information should take the form of the distribution function, the peak value of which should be related to the corresponding optimization objective function values of the current ant colony’s location. Meanwhile, the optimal way of the ant colony in the solution space is a fine-tuning process. The ant colony judge the process according to the integral comparison value between the specific interval of the overall amount of information in the current ant colony’s corresponding position, not the size of information in each point or point set. After getting the corresponding information distribution function of each ant, we make the integral sum for the problem solving subspace for the sum of all the distribution function corresponding to the divided by number of ants, then compare with the integral value of the total amount of information distribution function in the entire problem space, obtaining the percentage values of the current ant colony in each subspace and the actual ants number corresponding to the current ant colony size. Thus, we can decide the moving direction of the current observing ants, making the whole ant colony move in a reasonable way according to the information inspired by the present distribution of information, and finally tend to the optimal solution of the problem. However, in this algorithm, when the operation of the whole ant colony convergence to the same intervals, if we judge the operation only basing on this intervals, there will be a situation of searching slowly or stagnant for the whole ant colony, which is bad for the convergence of optimization. Therefore, the article improves the problem solving in allusion to the amount of information in each interval and the due number of ant. Namely, we increase a function that changes with the increase of the number of iterations, so as to change the due ant numbers in the intervals, and increase the searching speed. In order to avoid the local optimal solution under the condition of large search space, avoiding the stagnant, ensuring and improving the rate of convergence, we seek out the global optimal solution to the continuous space optimization problems in the implementation process of global optimization.

3 The improved ant colony algorithm for the continuous space optimization

The optimization process of ant colony algorithm in the continuous space includes the given of the amount of information distribution function, the analysis of the information distribution status, the decision making of the ant colony’s moving direction, and etc. the improved optimal method of the article are as follow:

Step1 the ant colony makes the original distribution in the continuous space.

We divide the definition domain of the problem for N equal, among which, N is the scale of ant colony, then put one single ant \( i (i = 1 \sim N) \) in the middle of the N subinterval, while each ant has a moving subinterval, and themselves are in the moving the center of the subinterval, the length of each mobile subinterval equal 1/N. When each single ant is local in the centre of the subinterval, we define 1 as the number of ant in each subinterval. When each single ant moves, we define the actual ant number within these two adjacent subintervals according to the overlapping degree change between its moving subinterval and these two adjacent subintervals.

If the problem definition domain is \([a,b]\), then when the ant number is N, the length of each subinterval and each single ant’s moving subinterval is

\[
L = \frac{b-a}{N}
\]  
(1)
The distribution of the ant colony’s initial coordinate is

\[ x_i = a + \left( \frac{i}{N} - \frac{1}{2} \right)L \]  

(2)

The left border of its subinterval \( i \) is

\[ x_{iL} = a + (i - 1)L \]  

(3)

The right border is

\[ x_{iR} = a + iL \]  

(4)

When each single moves \( \Delta x \), the actual ant numbers corresponding to the single ant moving within the two adjacent interval \( \Delta n \) change to

\[ \Delta n = \frac{\Delta x}{L} \]  

(5)

Namely, when moving to the left, the actual ant number in the left subinterval increase \( \Delta n \), while the actual ant number in the right subinterval decrease \( \Delta n \); and vice when moving to the right.

Step 2 according to the pros and cons of the solution space position, we decide the current ant colony’s amount of information distribution.

According to the size of the current ant colony’s function value \( f(x) \) and according to the difference of the category in optimization problem, we decide the peak value of the amount of information distribution function \( M_i \), size that left by the problem. If the function is the minimum optimization, then the peak value of the amount of information distribution function is

\[ M_i = c_i - f(x_i) \]  

(6)

Among which, \( c_i \) depend on \( f(x_i) \), constant set by a probably range, moreover \( c_i > f(x_i) \); if the function is the maximum optimization, when \( f(x_i) > 0 \), then

\[ M_i = c_2 f(x_i) \]  

(7)

Among them, \( c_2 \) is set as positive constant; when \( f(x_i) < 0 \),

\[ M_i = \frac{c_3}{c_4 - f(x_i)} \]  

(8)

And \( c_2, c_4 \) are set as constant. As for the problem of function optimization in one dimensional space, the single ant’s corresponding amount of information distribution function

\[ T_i(x) = \frac{M_i e^{-k_i(x-x_i)}}{[1 + e^{-k_i(x-x_i)}]^2} \]  

(9)

Among which, \( M_i \) is the peak value, \( x_i \) is the centre deviation value, \( k_i \) is the waveform coefficient of compressibility.

Step 3 according to the current situation, we decide the due ant number in each subinterval. The actual total amount of information of the current ant number in each subinterval is

\[ I_i = IN_i + \eta I_{last} - E_v + c\phi(t) \]  

(10)

Among them, the amount of information distributed by the current ant number in the subinterval

\[ IN_i = \sum_{j=1}^{N} \frac{M_j}{1 + e^{-k_j(x_{xR} - x_j)}} - \frac{M_i}{1 + e^{-k_i(x_{xL} - x_i)}} \]  

(11)

Among which, \( \eta \) is the amount of information remaining coefficient, \( \eta I_{last} \) is the legacy of the total amount of information last time, \( E_v \) is the amount of volatile constants, \( \phi(t) \) is a function that changes with the increase of the number of iterations. In the initial stage of the search process, in order to avoid being caught in locally optimal solution, we add a small amount of negative feedback information in the initial searching stage, so as to reduce the difference between locally optimal solution and the amount of information corresponding to the worst solution, enlarging the searching scope of the algorithm; During the process, \( \phi(t) \) is increase appropriately to ensure the searching speed; In the end of the process, the optimal solution is determined basically. At this time, \( \phi(t) \) is
being increased continuously to make the algorithm convergence rapidly.

\[
\varphi(t) = \begin{cases} 
  y_1, & t \in [1, \frac{N}{10}] \\
  y_2, & t \in \left(\frac{N}{10}, \frac{3N}{10}\right) \\
  y_3, & t \in \left(\frac{3N}{10}, \frac{7N}{10}\right) \\
  y_4, & t \in \left(\frac{7N}{10}, N\right)
\end{cases} \quad (12)
\]

\( t \) is the iteration frequency, \( N \) is the total iteration frequency, \( y_1, y_2, y_3, y_4 \) are the constants set by according to the problem, and \( y_i < 0 \).

Therefore, we can obtain the ant number of the current subintervals

\[
N_{im} = \frac{I_i}{\sum_{j=1}^N I_j} N \quad (13)
\]

Step 4 According to the due ant colony distribution in each subinterval and the difference between the current ant colony distribution, we decide the moving direction of the ant number, and shift them.

According to the area where the investigated any in and the difference between the actual and due ant number, we decide the moving direction of the ant number, and then mark the coordinates change of \( \Delta x \). The movement regulation is shown in Table 1. Under other condition, the investigated ant are not moving.

Among which, \( N_{im} \) is the due ant number in each subinterval, \( N_{iML} \) is the sum of the due ant number which is on the left side of the investigated ant’s subinterval \( i \), \( N_{iMR} \) is the due ant number which is on the right side of the investigated ant’s subinterval \( i \), \( N_{irL} \) is the actual ant number in each subinterval, \( N_{irR} \) is the sum of the actual ant number which is on the left side of the investigated ant’s subinterval \( i \), \( N_{iR} \) is the sum of the actual ant number which is on the right side of the investigated ant’s subinterval \( i \).

\[
N_{iML} = \sum_{j=1}^{i-1} N_{jM}, \quad N_{iMR} = \sum_{j=i+1}^{N} N_{jM} \quad (14)
\]

\[
N_{irL} = \sum_{j=1}^{i-1} N_{jR}, \quad N_{irR} = \sum_{j=i+1}^{N} N_{jR} \quad (15)
\]

| Rules | \( N_{irL} < N_{irR} \) | \( N_{irL} > N_{irR} \) | \( N_{irL} = N_{irR} \) | Coordinate values of the investigated ant |
|-------|-----------------|-----------------|-----------------|-------------------------------------|
| 1     | < \( N_{irL} \) | \( N_{irR} \) | > \( N_{irL} \) | \( -\Delta x \) |
| 2     | \( > N_{irL} \) | < \( N_{irR} \) | \( > N_{irL} \) | \( +\Delta x \) |
| 3     | = \( N_{irL} \) | \( > N_{irR} \) | \( = N_{irL} \) | \( +\Delta x \) |
| 4     | \( > N_{irL} \) | \( > N_{irR} \) | \( = N_{irL} \) | \( -\Delta x \) |
| 5     | \( < N_{irL} \) | \( > N_{irR} \) | \( < N_{irL} \) | \( -\Delta x \) |
| 6     | \( < N_{irL} \) | \( > N_{irR} \) | \( > N_{irL} \) | \( In turn+/-\Delta x \) |
| 7     | < \( N_{irL} \) | \( > N_{irR} \) | < \( N_{irL} \) | |

After the ant colony finish a overall movement, we come back to step 2, to operate the corresponding amount of information distribution, investigate and ant number moving, and the
recirculation go on like this until the optimal solution come into being.

4 Simulation experiment
In order to verify the feasibility of the above improved ant colony algorithm in solving the problem of continuous space optimization, the article make a simulation research on nonlinear function optimization problem of continuous space optimization\textsuperscript{[5]} for references\textsuperscript{[5]}.

\[
\max f(x) = 3x^2e^{-x} \quad x \in [0,3]
\]  \hspace{1cm} (16)

We make the peak value form of each information distribution function as \( M_i = cf(x_i) \), set
\[
N = 20, \quad \eta = 0.01, \quad E_v = 70, \quad c = 250, \\
y_1 = -0.001, \quad y_2 = 0.0002, \quad y_3 = 0.002, \\
y_4 = 0.01, \quad k = 40j.
\]

( \( j \) is the optimal frequency), the circulation frequency is 1000. After the simulation calculation, we make comparison between the experimental results: improved optimization and algorithm in references\textsuperscript{[5]}. According to the random feature of ant colony algorithm, the comparison is made under the statistical significance, the contrast results are shown in Table 2.

| Algorithm               | the optimal target value | Ant colony scale | max f mean value | operation times |
|-------------------------|--------------------------|------------------|-----------------|-----------------|
| Algorithm in References \textsuperscript{[5]} | 1.624 | 20 | 1.539 | 1000 |
| Improved Optimization   | 1.624 | 20 | 1.591 | 1000 |

As we can see in the table and figure, under the same parameter settings, the article put forward an improved ant colony algorithm applying in the continuous space optimization, which is better than references\textsuperscript{[5]} in the area of convergence process. The improved optimization in the article can convergence in a faster speed, and find the optimal solution quicker, which has a good convergence. The improved optimization make the searching space in the individual’s intervals that probably have higher adaptive value, namely, narrowing the algorithm’s searching space in order to make the algorithm convergence to a total optimal solution as soon as possible, thus the convergence speed can be improved obviously.

5 Conclusion
Based on the optimization in references\textsuperscript{[5]}, the article improves the actual total amount of information and the corresponding ant number, the results of the simulation prove that the improvement for the problem of continuous space optimization have better convergence speed and performance. It provides a new method for the application of the ant colony algorithm in the continuous space optimization.

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