Plastic deformation of 2D crumpled wires

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Abstract
When a single long piece of elastic wire is injected through channels into a confining two-dimensional cavity, a complex structure of hierarchical loops is formed. In the limit of maximum packing density, these structures are described by several scaling laws. In this paper this packing process is investigated but using plastic wires which give rise to completely irreversible structures of different morphology. In particular, the plastic deformation from circular to oblate configurations of crumpled wires is experimentally studied, obtained by the application of an axial strain. Among other things, it is shown that in spite of plasticity, irreversibility and very large deformations, scaling is still observed.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
In the last two decades, crumpled surfaces have been the subject of increasing interest in theoretical and applied physics in connection with fractals, mechanical properties, nontrivial scaling laws, packing processes, anomalous relaxation, condensation of stress, among other aspects [1–14]. On the other hand, heterogeneous structures of crumpled wires, with a quasi-one-dimensional topology, were much less studied. However, it is known that ill-defined compression procedures of wires in three-dimensional space give rise to robust scaling laws for this type of disordered system [15]. Only recently rigid, non-compact, heterogeneous crumpled structures of wires of circular shape were obtained in two dimensions (briefly, 2D-CW) from packing processes of a single layer of wire injected through channels into a quasi-two-dimensional cavity [16–22]. In particular, in the first studies of 2D crumpled structures wires of copper or steel and nylon fishing lines were used [16, 17, 19], which give rise to quasi-reversible or almost perfectly reversible packing structures of loops of the type illustrated in figure 1(a). The degree of reversibility mentioned here can be evaluated, for instance, from the level of recovery or uncoiling of the wire when the confining wall of the cavity is withdrawn. Thus, considerable elastic energy remains stored in the cavity in different degrees after a long period if wires of these materials are used to generate the crumpled structures. The low-dimensional packing of an elastic wire in a quasi-two-dimensional cavity of circular or square shape generates complex configurations of a cascade of loops with a fractal dimension $D = 1.9 \pm 0.1$ as obtained from box-counting and mass–radius measurements [16]. Although the mass of wire in these heterogeneous packing structures distributes in an essentially two-dimensional support, from the practical point of view the maximum packing fraction observed within the cavity is significantly less than unity. In fact, it is close to 0.15 (i.e. much smaller than typical packing fractions of 0.82–0.84 obtained with the random close packing of discs), irrespective of the material of the wire, the angle formed between the injection channels and other details [16, 17, 19].

Differently, in this work use is made of plastic wires of the alloy Pb\textsubscript{0.40}Sn\textsubscript{0.60}, which in turn give rise to strongly irreversible structures of 2D-CW of the type exemplified in figure 1(b). When compared with the elastic case, crumpled structures of a plastic wire present both a higher packing fraction (in our experiments, it is close of 0.30) and a different morphology for the individual loops, as well as for the heterogeneous distribution of the loops, as will be detailed in the next section. Besides its intrinsic interest, low-dimensional structures of crumpled wires can, in principle, be related to important topics in applied physics such as, for instance, self-avoiding walks and polymer configurations on the plane [23], or to random network motifs of atoms

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or molecules adsorbed on solid surfaces [24]. Additionally, 
crumpled wires could be of interest in the study of the packing 
of genetic material in chromosomes and viral capsids [21] and 
to describe the statistical aspects of meandering nanotubes in 
a planar confining geometry [25].

In this paper, using experiments and scaling arguments 
the plastic deformation of the nearly circular configurations 
of 2D-CW (figure 1(b)) into oblate configurations (figure 1(c)) 
is studied, obtained as a consequence of the application of 
an axial strain. The morphologies shown in figures 1(b) and 
(c) rigorously present a spontaneous break of the rotational 
and translational symmetries of the free space, albeit present 
scaling symmetry in several aspects. Figure 1(c), in particular, 
represents a structure with less porosity halfway towards a 
more condensate state akin to a disordered solid.

The structure of the paper is as follows: in section 2 
the experiments of deformation of 2D-CW are described in 
detail, in section 3 the experimental results are presented and 
discussed and in section 4 there is a summary of the main 
results.

2. Experimental details

In figure 2(a) the experimental setup used to deform axially and 
irreversibly the crushed wires of the alloy Pb_{0.40}Sn_{0.60} is shown. 
This wire, which is commonly used in welding in electronic 
devices, had a diameter of \( \zeta = 1.5 \) mm, and the internal 
circular cavity of the cell where the initial configurations 
of crushed wires were generated (not shown) had a radius of 
\( R = 75 \) mm. Figure 2(b) illustrates the main physical variables measured in the experiment: the height \( y \) and the equatorial diameter \( x \) of the system.

![Figure 1](image1.png)

**Figure 1.** Photographs of rigid heterogeneous hierarchical structures of loops obtained from forced injection of wire of diameter \( \zeta \) in a 
 quasi-two-dimensional cavity of height \( h \) and radius \( R \): (a) copper wire, \( \zeta = 1 \) mm, \( h = 1.1 \) mm and \( R = 100 \) mm; (b) Pb_{0.40}Sn_{0.60} wire, 
 \( \zeta = 1.5 \) mm, \( h = 1.6 \) mm and \( R = 75 \) mm. (c) Photograph of the sample shown in (b) after axial deformation of 69%.

![Figure 2](image2.png)

**Figure 2.** (a) Experimental setup used to deform axially and irreversibly the crushed wires of the alloy Pb_{0.40}Sn_{0.60} of the type shown in 
figure 1(b). (b) The main physical variables measured in the experiment: the height \( y \) and the equatorial diameter \( x \) of the system.
the nearly circular 2D-CW. After the jammed stage is reached, the 2D-CW configurations are transferred to the rectangular cell which is disposed vertically in the press frame as indicated in figure 2(a).

The press shown in figure 2(a) has a transparent vertical cell formed from two rectangular pieces of glass 1.2 cm thick parallelly disposed and separated by a distance of 1.5 mm allowing only a single layer of crumpled wire. The cavity of the cell was polished in order to reduce the friction. The cavity and wire operated in the dry regime, free of any lubricant. The press has a rigid metallic piece that fits with precision the cell was polished in order to reduce the friction. The cavity of the cell formed from two rectangular pieces of glass 1.2 cm thick parallelly disposed and separated by a distance of 1

Figure 3. (a) Main plot: mass (M)–size (R) dependence for all samples of undeformed 2D-CW studied in this paper at the maximum packing density (figure 1(b)). The power-law fit for the ensemble, $M \sim R^D$, has $D = 1.93 \pm 0.12$. The scaling region covers typically the interval $0.15 < R (\text{cm}) < 5.0$. The inset shows the corresponding box-counting plot $N(\epsilon)$ leading to the same exponent $D$ which is obtained from the mass–size scaling. (b) Mass ($m$)–size ($r$) data after averaging in all blobs associated with the structures of the type shown in figure 1(c) (mean vertical deformation $\langle \delta y \rangle = 0.65 \pm 0.04$). The best fit obtained indicates that $m \sim r^D$, with $D = 1.90 \pm 0.10$. The upper left inset shows a typical distribution of blobs for 2D-CW at high strain. The lower right inset exhibits the corresponding box-counting plot for 2D-CW at high strain leading to the same exponent $D$ previously reported.

3. Results and discussion

Figure 3(a) shows in the main plot the average mass ($M$)–size ($R$) dependence for the ensemble of 2D-CW studied in this paper when the jammed state is reached (figure 1(b)). The inset shows the corresponding box-counting plot [16,19]. Both figures are compatible with a fractal dimension $D = 1.93 \pm 0.12$, as previously observed by using much less irreversible crushed structures made with wires of copper and steel, as well as with nylon fishing lines [16,19]. Interestingly, the fractal mass–size dimension seems insensitive to the elastic properties of the wire and to the degree of irreversibility involved. This supports the idea that in crumpled systems, $D$ is heavily dependent on the self-avoidance forces and on the fixed quasi-one-dimensional topology of the wire [14]. In the context of the present discussion, it is interesting to mention that it was recently noticed that, in nature, DNA in many bacteriophage viral capsids presents a similar mass–size scaling relation in the form of DNA length $\sim$ DNA mass $\sim$ (capsid size)$^{1.9}$ [21]. Of course, this is a surprising result because a viral capsid is a three-dimensional cavity and, in principle, we should expect a mass–size exponent close to 3. Furthermore, the striking independence of the fractal mass–size exponent with the degree of repacking of the wires is observed, i.e. on the magnitude of the strain $\delta y$, as suggested by figure 3(b). This figure exhibits the average mass of wire $m(r)$ within the type of circular blobs shown in the upper left inset of the figure, as a function of the radial variable $r$ defined at the interior of each blob. The power-law fit in this figure gives $m \sim r^{1.94 \pm 0.1}$, which is indicative of the nearly two-dimensional character of the distribution of mass within the blobs after averaging in 24 equivalent blobs.

If the 2D-CW is compressed along an axis, its initial approximate circular symmetry shown in figure 1(b) is broken. The process of transference of mass of wire perpendicular to the direction $y$ of the strain is described in figure 4. This plot shows the average equatorial diameter $x$ indicated in figure 2(b) plus the statistical fluctuations as a function of $1 - |\delta y|$, in a
The Poisson ratio \([ 27]\) is defined as the ratio of the transversal and the axial strains introduced in section 2, \(\nu = -\delta_x/\delta_y = (x - 2R)/(2R - y)\). The negative sign is included in the expression so that it will always be positive, since \(\delta_x\) and \(\delta_y\) will always be of opposite sign. For very small strain, the Poisson ratio is a constant belonging to the intervals \((-1, 0.5)\), for 3D systems, and \((-1, 1)\), for 2D systems [28]. As a matter of illustration, in the linear elastic limit, the interval \(0.25 < \nu < 0.35\), includes the Poisson ratio of most alloys, irons and composites. Commercially pure gold has \(\nu = 0.42\); lead and tin, both being part of the alloy used in the wire (figures 1(b) and (c)), have in the same elastic limit Poisson ratios of 0.44 and 0.36, respectively. Differently, for high strain, the Poisson ratio is a function of the strain, as exemplified below for 2D-CW. As there is coupling between both strains, \(\delta_x\) and \(\delta_y\), one can plot \(\nu\) as a function for instance of \(|\delta_x|\). This is done in figure 5, which shows the average experimental Poisson ratio \(\nu\) plus statistical fluctuations. The fluctuation bars in figure 5 represent the full interval of variability of \(\nu\) for all samples used in the experiment. Although these fluctuations are not too small, because they reflect the high level of heterogeneity of the structures, the average values of \(\nu\) are quite well described by a linear increasing function of \(|\delta_x|\), from \(|\delta_x| \approx 0.1\) to \(|\delta_x| \approx 0.66\).

Finally, the dependence of the packing fraction of the 2D-CW with the vertical strain was investigated. This quantity changes from \(p_0 = 0.31 \pm 0.03\), for \(\delta_y = 0\), to \(p = 0.52 \pm 0.03\), for \(|\delta_x| = 0.65 \pm 0.05\); that is, it increases by 67% in this interval of strain. It is interesting to notice that this interval of variability of packing fraction for 2D-CW when the axial strain varies in a wide interval is close to the variability observed in the critical probability \(p_c\) in 2D bond percolation for the most studied lattices \((p_c = 0.34, 0.49\) and 0.64 for triangular, square and honeycomb lattices) [29]. The value \(p = 0.52\) obtained for the largest strains in our experiment represents only 62% of the maximum packing fraction for the random close packing of discs in 2D or 57% of the packing fraction \(\pi/2\sqrt{3} = 0.9068\), . . . of the triangular close packing of discs, the densest possible arrangement of equal discs. Using the data in figure 5 and the experimental dependence \(p(\delta_x)\) we obtained the dependence of the packing fraction with the Poisson ratio. Thus, the packing fraction \(p\) is an increasing function of \(\nu\) as shown in figure 6. The Poisson ratio and the packing fraction \(p\) or the porosity \(1 - p\) are basic properties of materials in general. For disordered materials such as the crumpled wires discussed here, the relationship between these quantities is of particular recent interest [28, 30]. In fact, the plot in figure 6 obtained for our 2D structures of loops and lamellae illustrated in figures 1(b) and (c) is reminiscent of a similar plot shown in [30] for a planar ensemble of cellular structures in soft media acting as force dipoles.
4. Summary and conclusions

Besides its intrinsic interest, low-dimensional structures of crumpled wires could be related to random network motifs of atoms or molecules adsorbed on solid surfaces [24], to the study of DNA packing in chromosomes and viral capsids [21] and to describe the statistical aspects of meandering nanotubes in planar confining geometries [25], among others. Here, the geometric changes observed in packings of crumpled wires of circular shape have been experimentally studied when such structures are confined in a 2D cell and an axial compressive strain is applied. In particular, one can observe scaling laws connecting variables of physical interest such as (i) mass and size, (ii) relative lateral expansion and axial strain, (iii) Poisson function and strain and (iv) volume fraction as a function of the Poisson ratio, among others. Surprisingly, critical exponents as fractal dimensions were found to be independent of the strain in a very large interval of variability.

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