Dufour and Soret effects on MHD convection of Oldroyd-B liquid over stretching surface with chemical reaction and radiation using Cattaneo-Christov heat flux

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Abstract: The main purpose of this investigation is to present the effects of Dufour and Soret on MHD convection of an Oldroyd-B liquid over a stretching surface with Cattaneo-Christov heat flux model in the appearance of radiation and chemical reaction. The governing systems of non-linear PDE’s are reconstructed into non-linear ODE’s. The resulting intricate non-linear boundary value problem is soluted through homotopy analysis method. The influence of various physical parameters on velocity, concentration, temperature profiles and dimensionless version of local skin friction, local mass and heat transfer rates are discussed through graphs.

1. Introduction
The study of heat transfer for an incompressible liquid past a stretching surface has attracted many investigators by interest of its industrial applications. Such as petroleum industry, geothermal problems, cooling of nuclear reactors. Hayat et al.[1] provides MHD 2D steady flow of an Oldroyd-B liquid with Cattaneo-Christov model. Similar studies in this region was found in Eswaramoorthy et al.[2] and Hayat et al. [3]. Bhuvaneswari et al. [4] examined the heat transfer aspects of the free convection stream of a heat generating liquid towards a semi-infinite inclined surface impacted in a porous medium. Imtiaza et al. [5] explored the 2D flow of third grade liquid over a linear stretchy sheet with chemical reaction. Dufour and Soret effects on the mixed convection flow of an Oldroyd-B liquid was analyzed by Ashraf et al.[6]. Jagan et al.[7] investigate the non-linear thermal radiation effect on three-dimensional MHD convective flow of nanoliquid over a non-linear stretchy sheet with porous medium. Heat transfer and convective flow with chemical reaction is applied in many technical and industrial progress. Few more studies in this area are seen in Karthikeyan et al.[8], Ruhaila et al.[9] and Sivasankaran et al.[10].

Inspired by the above applications and surveys explained, the purpose of this present work is to explore the effects of chemical reaction and radiation on 2D magneto-hydrodynamic flow of an Oldroyd-B liquid with Cattaneo-Christov model in appearance of Dufour and Soret effects.

2. Mathematical formulation of the problem
Consider the MHD flow of an Oldroyd-B liquid over a linear stretchy sheet. The velocity components \((u_1, v_1)\) are taken in the \((x, y)\)-directions. The velocity of sheet is assumed as \(U_W = ax\) where \(a > 0\) is the stretching rate. The two temperatures on and apart from the surface are expressed by \(T_w\) and \(T_c\) with \(T_w > T_c\). The magnetic field of strength \(B_m\) is applied upright to the stretchy surface. The electric and induced magnetic fields are neglected. We obtain the following governing equations

\[
\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0
\]  

\[
u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} + A_1 \left( u_1^2 \frac{\partial^2 u_1}{\partial x^2} + v_1^2 \frac{\partial^2 u_1}{\partial y^2} + 2u_1 v_1 \frac{\partial^2 u_1}{\partial x \partial y} \right) = \frac{\mu}{\rho} \frac{\partial^2 u_1}{\partial y^2}
\]

\[-\mu A_2 \left( u_1 \frac{\partial^2 u_1}{\partial x^2} + v_1 \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial u_1}{\partial x} \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial u_1}{\partial y} \frac{\partial^2 v_1}{\partial y^2} \right) - \frac{\sigma B_m^2}{\rho} \left( b + A_1 \frac{\partial u_1}{\partial y} \right) \]

\[
\rho c_p \left( u_1 \frac{\partial T}{\partial x} + v_1 \frac{\partial T}{\partial y} \right) = -\nabla q
\]
Then, the boundary conditions are

\[ u_1 = u_{1w}, \quad v_1 = 0, \quad c = c_{w}, \quad T = T_w \text{ at } y = 0 \]

\[ u_1 \to 0, \quad v_1 \to 0, \quad c \to c_{\infty}, \quad T \to T_{\infty} \quad \text{as} \quad y \to \infty \] (5)

where \( \mu, \rho, \alpha, A_1, A_2, \sigma, c, c_p, c_w, c_{\infty}, D_m \) and \( k_T \) are the kinematic viscosity, liquid density, relaxation time, retardation time, electrical conductivity, concentration, specific heat, wall concentration, ambient liquid concentration, diffusion coefficient and first order chemical reaction parameters respectively. Using Cattaneo-Christov heat flux theory, we obtain the following energy equation

\[
\nu \frac{\partial T}{\partial x} + \nu_t \frac{\partial T}{\partial y} + \lambda \left( u_1 \frac{\partial^2 T}{\partial x^2} + v_1 \frac{\partial^2 T}{\partial y^2} \right) + \left( u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} \right) = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{D_m k_T}{c_i c_p} \frac{\partial^2 c}{\partial x^2}
\] (6)

Consider the following similarity transformations

\[ \eta = \sqrt{\frac{a}{\nu}} y, \quad u_1 = ax \frac{\partial f}{\partial \eta}, \quad v_1 = -\sqrt{\frac{a}{\nu}} x \frac{\partial f}{\partial \eta}, \quad \theta(\eta) = \frac{T - T_{\infty}}{T_{\infty} - T_{\infty}}, \quad \phi(\eta) = \frac{c - c_{\infty}}{c_{w} - c_{\infty}} \] (7)

Apply equation (7) in equations (2), (4) and (6) we have

\[
f^* + f f^* - f^2 + \alpha_1 \left( 2ff' f^* - 2f^2 f' \right) + \beta_1 \left( f^2 - ff' \right) - M \left( f^* - \alpha_1 ff' \right) = 0 \] (8)

\[
\left( 1 + \frac{4}{3} Rd \right) \theta^* + Prf \theta' - PrT \theta^* - Prf \theta^* + ff' \theta^* + PrDf \phi^* = 0
\] (9)

\[
\frac{1}{Sc} \phi^* + f \phi' - Cr \phi + Sr \theta^* = 0
\] (10)

with boundary conditions

\[
f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \text{ at } y = 0
\]

\[
f'(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \text{ as } y \to \infty
\] (11)

Here we declare the non-dimensional variables with respect to \( \alpha_1 = A_1 a \) and \( \beta_1 = A_2 a \) are the relaxation time & retardation time parameters respectively, \( M = \sigma B_0^2 / \rho a \) the Hartmann number, \( Pr = \rho c_p / k \) the Prandtl number, \( Rd = (4\sigma^2 T_{\infty}^3) / (kk') \) the radiation parameter, \( \gamma = \lambda a \) the non-dimensional thermal relaxation time, \( D_f = \frac{D_m k_T}{\mu c_i c_p T_{\infty} - T_{\infty}} \) the Dufour number, \( Cr = \frac{k_m}{a} \) the chemical reaction parameter, \( Sc = \mu / D_m \) the Schmidt number, \( Sr = \frac{D_m k_T (T_{\infty} - T_{\infty})}{c_i c_p (c_w - c_{\infty})} \) the Soret number.

The dimensionless forms of skin friction, heat transfer rate and local mass transfer rate are represented below

\[
Re^\frac{1}{2} C_{f_x} = \frac{1 + \alpha_1}{1 + \beta_1} f'(0), \quad Re^\frac{1}{2} \frac{N u_x}{u_1} = -\left( 1 + \frac{4}{3} Rd \right) \phi'(0), \quad Re^\frac{1}{2} Sh_x = -f'(0)
\]
3. HAM Solutions
The initial guesses for homotopy solutions are defined as $f_0 = 1 + e^{-\eta}$, $\theta_0 = e^{-\eta}$ and $\phi_0 = e^{-\eta}$ the auxiliary linear operators $L_f$, $L_\theta$ and $L_\phi$ are derived as $L_f = f''(\eta) - f'(\eta)$, $L_\theta = \theta''(\eta) - \theta'(\eta)$ and $L_\phi = \phi''(\eta) - \phi'(\eta)$ with satisfying the following properties $L_f[E_1 + E_2 e^{\eta} + E_3 e^{-\eta}] = 0$, $L_\theta[E_4 e^{\eta} + E_5 e^{-\eta}] = 0$, $L_\phi[E_6 e^{\eta} + E_7 e^{-\eta}] = 0$, where $E_j (j = 1 - 7)$ denote the arbitrary conditions. 

The auxiliary parameters $h_f$, $h_\theta$ and $h_\phi$ play an vital part for convergence series solutions. The $h$-profiles are drawn at $15^{\text{th}}$ order of approximations to accomplish valid ranges of parameters (see Figure 1). The admissible values of $h_f$, $h_\theta$ and $h_\phi$ are $-1.3 \leq h_f \leq -0.3$, $-1.0 \leq h_\theta \leq -0.3$, $-1.2 \leq h_\phi \leq -0.3$. More over the solutions converge in the total range of $\eta$. When $h_f = h_\theta = h_\phi = -0.7$. Table 1 displays the order of approximations of HAM solutions.

![Figure 1. h-curves for $h_f$, $h_\theta$ and $h_\phi$](image)

4. Results and discussion
The efficient numerical calculations are execute for various values of $\alpha_1$, $\beta_1$, $Rd$, $Cr$, $D_f$ and $Sr$ on velocity, temperature and concentration profiles with fixed values of $M = 0.5$, $\alpha_1 = 0.1$, $\beta_1 = 0.2$, $\gamma = 0.5$, $Pr = 1.0$, $Sc = 0.9$, $Rd = 0.3$, $Cr = 1.0$, $D_f = 0.5$ and $Sr = 0.3$. Figure 2 show that relaxation time($\alpha_1$) and retardation time($\beta_1$) have inverse effects on the velocity distribution $f'(\eta)$. Figure 3(a) depicts that extensive values of radiation constant($Rd$) on temperature component the thermal and momentum boundary layer thickness rises. The influence of Dufour number($D_f$) on temperature distribution is plotted in Figure 3(b) it is noted that an enlargement in Dufour number the temperature enhances. Figure 4(a) illustrates the influence of chemical reaction($Cr$) on concentration profiles. For larger values of chemical reaction reduces in species the concentration. Figure 4(b) demonstrates the effects of Soret number($Sr$) on the concentration distribution. From this figure it is identify that an enhances in the Soret number the concentration of the liquid and solutal boundary layer thickness enhances. From Figure 5(a)&5(b) sketched for relaxation time($\alpha_1$), retardation time($\beta_1$) with magnetic field parameter($M$) on skin friction co-efficient. It is observed that $\alpha_1$ and $\beta_1$ has opposite effects on local skin friction. Figure 6(a) is for the influence of radiation constant($Rd$) and non thermal relaxation time constant($\gamma$). Larger values of $Rd$ lead to enhance the Nusselt number. Similar behavior is observed for $\gamma$. Figure 6(b) is plotted for the influence of Dufour number($D_f$) and non thermal relaxation time ($\gamma$). Here Nusselt number diminishes via Dufour number. Characteristics of $M$ and $Sr$ with $Cr$ on Sherwood number are displayed in Figure 7(a) & 7(b). The local mass transfer rate diminishes by uplifting the values of $M$, $Sr$ and $Cr$. 

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Figure 2. Influence of $\alpha_1$ and $\beta_1$ on velocity profile $f'(\eta)$.

Figure 3. Influence of $R_d$ and $D_f$ on temperature profile $\theta(\eta)$.

Figure 4. Influence of $C_r$ and $S_r$ on concentration profile $\phi(\eta)$.
Figure 5. Influence of $\alpha_1$ and $\beta_1$ on skin friction coefficient $Re^{\frac{1}{2}}C_{f_{x}}$.

Figure 6. Influence of $Rd$ and $D_f$ on local Nusselt number $Re^{\frac{1}{2}}Nu_x$.

Figure 7. Influence of $M$ and $Sr$ on local Sherwood number $Re^{\frac{1}{2}}Sh_x$. 
Table 1: Order of approximations when \( M = 0.5, \beta_1 = 0.2, \alpha_1 = 0.1, \gamma = 0.5, Pr = 1.0, Sc = 0.9,\nRd = 0.3, Cr = 1.0, Df = 0.5, Sr = 0.3 \) and \( h = -0.7 \)

| Order of approximation | \(-f''(0)\) | \(-\theta'(0)\) | \(-\phi'(0)\) |
|------------------------|-------------|----------------|--------------|
| 1                      | 1.6333      | 0.6266         | 0.9727       |
| 5                      | 1.1944      | 0.5253         | 1.0595       |
| 10                     | 1.1944      | 0.5217         | 1.0625       |
| 16                     | 1.1944      | 0.5216         | 1.0625       |
| 20                     | 1.1944      | 0.5216         | 1.0625       |
| 30                     | 1.1944      | 0.5216         | 1.0625       |
| 40                     | 1.1944      | 0.5216         | 1.0625       |
| 50                     | 1.1944      | 0.5216         | 1.0625       |

5. Final report
The magneto-convection of an Oldroyd-B liquid under the effects of Cattaneo-Christov heat flux and chemical reactions in attendance of Dufour and Soret effects are investigated. The thermal and momentum boundary layer thickness enhances with upgrading radiation constant. The distribution of temperature of the liquid enhances with Dufour number increases. The concentration diminishes for the larger chemical reaction constant. Finally, the solutal boundary layer thickness enhances with Soret number.

6. References
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