\[ \pi N N \text{ and } \pi N \Delta \text{ formfactors determined from a microscopic model} \]

\textbf{for } \pi N \text{ scattering} \\

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\textbf{Abstract}

We determine the \( \pi N N \) and \( \pi N \Delta \) formfactors from the \( P_{11} \) resp. \( P_{33} \) partial wave of \( \pi N \) scattering by dressing corresponding bare vertices with the help of \( \pi N \) non–pole contributions. The underlying model is based on meson exchange, and involves nucleon and delta–isobar pole and crossed–pole terms together with correlated \( \pi \pi \)–exchange in the \( J^P = 0^+ (\sigma) \) and \( 1^- (\rho) \) channel. The results are very similar for \( \pi N N \) and \( \pi N \Delta \) and can be roughly parametrized by a monopole with cutoff mass \( \gtrsim 500 \text{ MeV} \), with some variation due to model dependencies. Thus the formfactors are much less soft than derived before for the \( \pi N N \) case by Saito and Afnan using the same procedure but different \( \pi N \) interaction models.
The strong $\pi NN$ (and $\pi N\Delta$) vertex plays an important role everywhere in nuclear physics. Therefore a precise knowledge of corresponding formfactors at these vertices is essential in order to reach a combined and consistent understanding of nuclear phenomena.

Recently Saito and Afnan [1] determined the $\pi NN$ formfactor from the $P_{11}$ partial wave of $\pi N$ scattering. Within their $\pi N$ interaction model a bare $\pi NN$ formfactor in the $(s$–channel) nucleon pole term gets dressed by phenomenological separable non-pole terms, through the iteration in a Lippmann–Schwinger equation. Their result for the dressed formfactor is extremely soft, corresponding to a monopole cutoff mass $\Lambda_{\pi NN}$ much less than 400 MeV, which is one reason why their resulting three–body force contribution to the triton binding energy turns out to be extremely small ($\simeq -2$ keV). Moreover, starting from completely different bare formfactors (monopole masses of 1822 and 323 MeV, respectively), the authors of Ref. [1] were able to show that within their model the corresponding dressed formfactors had quite similar, extremely soft behavior indicating that the requirement to fit the experimental data puts considerable constraints on the dressed formfactor.

On the other hand there is numerous information [2], also from QCD lattice calculations [3], that the $\pi NN$ formfactor should be characterized by a monopole cutoff mass around 800 MeV. This is not nearly as soft as found in Ref. [1] and thus provides much less suppression of the $\pi NN$ vertex. (Of course it is still soft compared to the hard $\pi NN$ formfactors used in most $NN$ boson exchange models; for example, in the full Bonn potential [4], $\Lambda_{\pi NN} = 1.3$ GeV).

Therefore the question arises (which was put already in Ref. [1]) whether such a determination of the $\pi NN$ formfactor from empirical $P_{11}$ $\pi N$ scattering phase shifts as performed in Ref. [1] provides indeed an unambiguous, extremely soft result (being in contrast to other informations [2,3]) or whether this is a special feature of the $\pi N$ model used in Ref. [1]. It is the purpose of this letter to address this issue, by starting from an alternative $\pi N$ interaction [5].

This model developed recently by our group in Jülich is based on meson exchange. It contains, apart from nucleon– (N) and delta–isobar (Δ) s–channel pole terms, non–pole
pieces consisting of crossed N and Δ exchange and correlated ππ exchange in the 0+ (σ) and 1− (ρ) channels as visualized in Fig. 1. A satisfactory description of all πN scattering phase shifts below pion production is achieved. It is important to note that the non-pole pieces which determine the πNN formfactor now act with a unified set of parameters in all partial waves and are therefore strongly tested by the simultaneous description of all S and P waves. (This is not true in Ref. [1] since there only the P11 data put a constraint on the dressed πNN formfactor).

In order to derive the renormalized πNN formfactor we start from the ‘dressed’ vertex function \( v^{\pi NN} \)

\[
v^{\pi NN} \equiv v^{\pi NN}_0 + T^{\text{non-pole}} G^{\pi N} v^{\pi NN}_0 ,
\]
with the bare vertex \( v^{\pi NN}_0 \) given by (\( p \) being the relative \( \pi N \) momentum)

\[
v^{\pi NN}_0(p) = \sqrt{3} \frac{f^{\pi NN}_0}{m_N \sqrt{4\pi}} \frac{1}{\sqrt{2\pi}} \frac{E_N(p) + \omega_{\pi}(p) + m_N}{\sqrt{E_N(p)\omega_{\pi}(p)(E_N(p) + m_N)}} p F^{\text{bare}}_{\pi NN}(p)
\equiv \mathcal{F}^{\pi NN}(p) F^{\text{bare}}_{\pi NN}(p).
\]

Here \( F^{\text{bare}}_{\pi NN}(p) \) denotes the bare \( \pi NN \) formfactor. (Since the nucleon is a P–wave resonance the momentum dependence of \( v^{\pi NN}_0 \) is essentially given by the factor of \( p \)). The \( \pi N \) model of Ref. [3] is based on time–ordered perturbation theory; therefore the \( \pi N \) propagator \( G^{\pi N} \) has to be chosen accordingly. The non–pole amplitude \( T^{\text{non-pole}}_{\pi N} \) is generated by iterating the non–pole part of the potential, \( V^{\text{non-pole}}_{\pi N} \). After partial wave decomposition, Eq. (1) reads explicitly

\[
v^{\pi NN}(p, Z) = v^{\pi NN}_0(p) + \int_0^\infty q^2 dq T^{\text{non-pole}}_{\pi N}(p, q; Z) \frac{1}{Z - E_N(q) - \omega_{\pi}(q) - i\epsilon} v^{\pi NN}_0(q) ,
\]

\( Z \) being the \( \pi N \) starting energy. The physical formfactor \( F^{\pi NN}(p, Z) \) is then obtained from \( v^{\pi NN}(p, Z) \) by dividing out the momentum dependence already inherent in the bare \( \pi NN \) vertex. It is a function of both \( Z \) and \( p \) and is normalized to unity at the physical nucleon pole, i.e. at \( Z = m_N \). Consequently

\[
F^{\pi NN}(p, Z) = \frac{v^{\pi NN}(p, Z)}{v^{\pi NN}(p_0, Z = m_N)} \frac{\mathcal{F}^{\pi NN}(p_0)}{\mathcal{F}^{\pi NN}(p_0, Z = m_N)},
\]

3
where \( p_0 \) is the on–shell momentum belonging to \( Z = m_N \).

Later we will also present results for the dressed \( \pi N \Delta \) formfactor. All formulas given so far hold correspondingly; only \( v_0^{\pi NN} \) has to be replaced by \( v_0^{\pi N\Delta} \) given by

\[
v_0^{\pi N\Delta}(p) = \frac{1}{\sqrt{6\pi m_{\pi}}} \sqrt{\frac{E_N(p) + m_N}{E_N(p)\omega_\pi(p)}} \sqrt{\frac{E_N(p) + m_N}{E_N(p)\omega_\pi(p)}} p \ F_{\pi N\Delta}^{\text{bare}}(p)
\]

\[
\equiv F^{\pi N\Delta}(p) \ F_{\pi N\Delta}^{\text{bare}}(p).
\] (5)

In the following we use two different \( \pi N \) models 1, 2 (for details, see Ref. [5]), which are based on the same dynamical input (cf. Fig. 1), but differ in the parametrization of the formfactors in both the pole and non–pole contributions. The predictions of these models for the \( \pi N \) partial waves of relevance for the calculation of the \( \pi NN \) and the \( \pi N\Delta \) formfactor are given in Fig. 2.

Results for the dressed \( \pi NN \) formfactor evaluated according to Eq. (4), at \( Z = m_N \) and as function of \( p \), are shown in Fig. 3 (a), in comparison to a simple monopole parametrization, \( F(p) = (\Lambda^2 - m_{\pi}^2)/(\Lambda^2 + p^2) \). Corresponding results for the \( \pi N\Delta \) formfactor are shown in Fig. 3 (b). In principle the \( \pi N\Delta \) formfactor defined analogously to Eq. (4) is a complex quantity since the delta–isobar lies within the physical range of \( \pi N \) scattering. It turns out however, that the imaginary part of the formfactor is very small at \( Z = m_\Delta \). Therefore we restrict our discussion to its real part at this place. For both models the \( \pi NN \) and the \( \pi N\Delta \) formfactor have very similar structure. Deviations occur at smaller momenta, mainly since the normalization points are different (\( p_0 = 227.3 \) MeV for \( \pi N\Delta \) and \( p_0 = i 137.3 \) MeV for \( \pi NN \)).

In Fig. 4, we compare our predictions for the \( \pi NN \) formfactor to corresponding results obtained in Ref. [4]. (In order to enable a comparison in the latter case our results have been normalized to unity in Fig. 4 at \( p = 0 \).) The results have the following main features:

(i) For both models 1, 2 the renormalized \( \pi NN \) formfactor is much less soft than found in Ref. [4], though still softer than the presently favored monopole with a cutoff mass of 0.8 GeV.

(ii) Results for model 1 and 2 are considerably different although the non–pole amplitude
is generated from identical dynamics and is constrained by all $\pi N$ partial waves. Obviously the remaining freedom in parametrizing the (bare) formfactors in the pole terms and those in the non-pole contributions has substantial effects. Finally, we would like to point out that inclusion of further processes in the non-pole part of the interaction model (like e.g. coupling to the reaction channel $\pi \Delta$) might also lead to a change of the dressed $\pi N N$ formfactor.

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FIGURES

FIG. 1. Diagrams included in the $\pi N$ potential.

FIG. 2. $\pi N$ scattering phase shifts in the $P_{11}$ and the $P_{33}$ partial wave, as function of the pion laboratory momentum. The solid (dashed) lines are the results obtained in model 1 (2) of Ref. [5]. The empirical information is taken from Ref. [6].

FIG. 3. $\pi NN$ (a) and $\pi N\Delta$ formfactor (b) as function of the square of the pion momentum in the $\pi N$ c.m. system. The solid (dash–dotted) lines denote the predictions resulting from model 1 (2) of Ref. [5]. The dashed (dotted) lines represent a conventional monopole formfactor with the cutoff mass $\Lambda = 500$ (700) MeV.

FIG. 4. $\pi NN$ formfactor (normalized to unity for $p = 0$) as function of the pion momentum in the $\pi N$ c.m. system. The solid (dash–dotted) line denotes our prediction resulting from model 1 (2) of Ref. [5], whereas the dotted (dashed) line is the result of model PJ (M1) of Ref. [5].
C. Schuetz and K. Holinde: Fig. 1
C. Schuetz and K. Holinde: Fig. 2
C. Schuetz and K. Holinde: Fig. 3

(a) 

(b) 

$F_{\pi NN}(p, Z=m_N)$ 

$F_{\pi N\Delta}(p, Z=m_{\Delta})$ 

$p^2 [\text{GeV}^2]$
C. Schuetz and K. Holinde: Fig. 4