Vibrations of a viscoelastic dam–plate of a hydro-technical structure under seismic load

A A Tukhtaboev*, F Turaev², B A Khudayarov², E Esanov³, and K Ruzmetov⁴

¹Namangan Civil Engineering Institute, Namangan, Uzbekistan
²Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 100000 Tashkent, Uzbekistan
³Tashkent State Technical University named after I.Karimov, 100095 Tashkent, Uzbekistan
⁴Tashkent State Agrarian University, 100140 Tashkent, Uzbekistan

*Email: t.fozil86@mail.ru

Abstract. One of the characteristic features of the development of the hereditary theory is the wide possibilities for describing the dynamic processes of deformation of various materials. However, due to the lack of an adequate mathematical apparatus, the implementation of these possibilities is in many cases difficult, especially when studying nonlinear dynamic processes. In recent years, the power of computing has increased interest in nonlinear problems. Under these conditions, it is important to create and develop such effective methods of solution that could be applied to the widest possible class of problems. In this work, a mathematical model of the problem of the dynamics of thin-walled structures is constructed taking into account the hereditary properties of the material. Using the Bubnov-Galerkin method under various boundary conditions, the problem under consideration is reduced to solving systems of integro-differential equations. The analysis of the influence of various properties of the construction material on the amplitude-frequency characteristics is carried out.

1. Introduction

When solving energy and water management problems in Uzbekistan, one of the main tasks is the creation of economical and reliable structures of mountain hydro-technical structures, taking into account the fact that the construction site presents a zone of high seismicity. The design of hydro-technical structures subject to potential earthquakes significantly depends on their dynamic characteristics and the vibration processes over time. Therefore, the need arises to proceed to dynamic theory of earthquake resistance.

The intensity of structures vibrations under dynamic influences substantially depends on the degree of energy dissipation in them. It can be expected that the higher the energy dissipation in the structure, the less intense the resonant vibrations at a given level of excitation.

Theoretical description of the processes of strain during vibrations of rigid bodies and structures, taking into account internal friction, is often limited to studying the general laws of the external manifestation of the dissipation mechanism. Hypotheses and linear models of frequency-independent internal friction [1, 2] are widely used in solving the problems of the dynamics of thin-walled
structures. These hypotheses, reflecting the manifestation of elastic imperfections in the materials, do not describe the creep of strains and relaxation of stresses, called “hereditary properties”.

The foundations of the modern hereditary theory of viscoelasticity, which reflects almost all the features of the quasistatic and dynamic behavior of the material, are found in classical works of Boltzmann and Volterra. The hereditary theory of viscoelasticity is more general and it describes more accurately the mechanism of dissipation in the materials [3-7].

It is well known that the rheological properties of the medium significantly affect the strain process as a whole, i.e. all three of its stages: elastic, plastic and the stage of destruction. Therefore, the problems of hereditary-deformable systems have attracted special attention of researchers in recent years [8 - 17].

This work is devoted to the numerical solution of the problem of vibrations of a dam-plate made of a homogeneous viscoelastic isotropic material.

2. The mathematical model of the problem

Let us investigate the problem of vibrations of a viscoelastic dam-plate made of a homogeneous isotropic material. The inertia forces acting on the dam-plate are generated from the motion and dam deformation, and the hydrodynamic pressure of water arises from the dam motion as a rigid body and the dam strain.

The mathematical model of the problem with respect to the transverse deflection \( w_1 = w_1(x,y,t) \), under known assumptions, taking into account the hereditary properties of the material of the dam, is reduced to solving equations of the form

\[
D\left(1 - R^*\right)K^4w_1(x,y,t) + \rho_1h \frac{\partial^2 (w_1 + w_0)}{\partial t^2} - \rho \frac{\partial \varphi_1}{\partial t} \bigg|_{x=0} = - \\
\rho \left( \frac{\partial \varphi_0}{\partial t} + \frac{1}{2} \left( \frac{\partial \varphi_0}{\partial z} \right)^2 + \frac{\partial \varphi_0}{\partial y} \right) \bigg|_{x=w_0(t)} = 0, \tag{1}
\]

where \( w_1(x,y,t) \) is the deflection of the dam-plate; \( h \) is the thickness of the dam; \( \rho_1 \) is the density of the dam material; \( \rho \) is the density of water; \( \varphi_0(x,y,z,t) \) is the function of the potential of fluid rate arising from dam-plate strain; \( \varphi_0(x,y,t) \) is the function of the potential fluid flow rate, arising from the dam motion as a rigid body; \( w_0(t) \) is the law of base motion during an earthquake:

\[
w_0(t) = a_0e^{-\varepsilon_0 t} \sin \omega_0 t; \tag{2}
\]

where \( a_0 \) is the initial maximum amplitude; \( \varepsilon_0 \) - the soil attenuation coefficient; \( \omega_0 \) - frequency of soil vibrations; \( t \) - time. All these values are determined from the analysis of the earthquake seismogram of corresponding intensity.

3. Solution methods

Solution of integro-differential equations (1), is given in the form

\[
w_1(y,z,t) = \sum_{k=1,3,...}^{\infty} C_k(t)w_k(y,z), \tag{3}
\]

where \( C_k = C_k(t) \) is the sought for time functions; coordinate functions \( w_k(y,z) \) satisfy the boundary conditions for fixing the dam-plate edges.

Let us examine constructions with the following boundary conditions:

1. Edges \( z = \pm \alpha \) are freely supported
\[ w_1 = 0, \frac{\partial w_1}{\partial y} + \mu \frac{\partial w_1}{\partial z} = 0 \]

2. Edge \( y = 0 \) is rigidly fixed,
\[ w_1 = 0, \frac{\partial w_1}{\partial y} = 0 \]  \hspace{1cm} (4)

3. Edge \( y = b \) is free.
\[ \frac{\partial^2 w_1}{\partial y^2} + \mu \frac{\partial^2 w_1}{\partial z^2} = 0, \frac{\partial w_1}{\partial y} + (2 - \mu) \frac{\partial^3 w_1}{\partial z^2 \partial y} = 0 \]

Functions \( w_k(y,z) \) satisfying these boundary conditions of fixation are taken in the form
\[ w_k(y,z) = V_k(y) H_k(z), \]
where
\[ H_k(z) = \cos \frac{k \pi z}{2b}; \]
\[ V_k(y) = V_{ik}(y) + E_k V_{2k}(y). \]

Here
\[ E_k = \frac{1}{V_{ik}(b)} \left[ V_{ik}^2(b) - \left( \frac{k \pi}{2a} \right) V_{ik}(b) \mu \right], \]
\[ V_{ik}(y) = \frac{1}{k} \sin \frac{k \pi y}{2b}, \quad \frac{1}{k+2} \sin \frac{(k+2) \pi y}{2b}, \quad V_{2k}(y) = \cos \frac{k \pi y}{2b} - \cos \frac{(k+2) \pi y}{2b}. \quad k = 1,3,5,\ldots \]

So, the expression for function \( w_k(y,z) \), is determined and has the form
\[ w_k(y,z) = \cos \frac{k \pi z}{2a} \left[ \frac{1}{R} \sin \frac{k \pi y}{2b} \frac{1}{k+2} \sin \frac{(k+2) \pi y}{2b} + E_k \left( \cos \frac{k \pi y}{b} - \cos \frac{(k+2) \pi y}{b} \right) \right]. \]  \hspace{1cm} (5)

Substituting (5) and (3) into equation (1) and applying the Galerkin-Bubnov procedure to determine the unknowns \( C_k = C_k(t), k = 1,3,5,\ldots \), the following system of integro-differential equations is obtained:
\[ \sum_{k=1,3,\ldots}^{\infty} \left[ L_{mk} \ddot{C}_k(t) + \omega^2 (1 - R^2) M_{mk} C_k(t) \right] + a_0 \omega^2 N_m(t) = 0. \]  \hspace{1cm} (6)

Here
\[ C_k(0) = C_{0k}, \quad \dot{C_k}(0) = \dot{C}_{0k}, \quad k,m = 1,3,5,\ldots \]

\[ L_{mk} = \frac{1}{ab} \left[ \int_0^b \int_{y-a}^{y+a} w_k(y,z)w_m(y,z) \, dy \, dz + \frac{2}{k \pi} \rho \left( \frac{b}{h} \right) \times \right] \]
\[ \int_0^b \int_{y-a}^{y+a} (z-a) w_k(y,z) \cos \gamma_k y \, dy \, dz \left[ \int_0^b \int_{y-a}^{y+a} (z-a)^2 \cos^2 \gamma_k y \, dy \, dz \right]^2; \]

\[ M_{mk} = \frac{b^3}{a \pi^2} \int_0^b \int_{y-a}^{y+a} \nabla^4 w_k(y,z)w_m(y,z) \, dy \, dz; \]
\[
N_m(t) = \frac{1}{ab a_0 \omega^2} \int_0^a \int_0^b \left[ \ddot{w}_0(t) - \frac{\rho}{h} \frac{1}{\alpha^2} \left( \frac{\partial \phi_0}{\partial t} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi_0}{\partial \nu} \right)^2 \right]_{x=w_0(t)} dy dz
\]

where \( \omega \) is the frequency of the basic mode of vibration:

\[
\omega = \sqrt{\frac{D}{\rho h \left( \frac{\pi}{b} \right)^4}}
\]

In calculations, the Koltunov–Rzhanitsyn kernel is used:

\[
R(t) = A t^{\alpha-1} \exp(-\beta t), \quad A, \beta > 0, \quad 0 < \alpha < 1.
\]

Integration of the system of equations (6), obtained on the basis of numerous approximations of deflections, was performed using a numerical method [4, 6, 7, 11].

4. Numerical solution and discussion of results

The graphs of curves \( w \left( \frac{1}{2}, \frac{1}{2}, t \right) \) for different values of the viscosity parameter \( A \) are presented in Figure 1. An analysis of the results obtained shows that at the initial stage of time, the solutions of elastic and viscoelastic problems differ little from each other. Over time, the vibrations at \( A = 0 \) occur closer to the harmonic law, and with increasing \( A \), the amplitude and frequency of vibrations decrease significantly.

The effect of the hydrodynamic pressure of water on the dam behavior was investigated. Figure 2 shows the graphs of curves \( w \left( \frac{1}{2}, \frac{1}{2}, t \right) \) for various values of parameter \( \rho / \rho_1 \). The results obtained here show that in the initial point in time, the curves almost coincide, and over time they differ significantly from each other. An analysis shows that with an increase in the parameter value \( \rho / \rho_1 \), the amplitude of dam-plate vibrations decreases. So, an account for hydrodynamic pressure of water leads to a decrease in the vibration amplitude, and the frequency of vibrations does not change significantly.

Figure 3 shows the graphs of curves \( w \left( \frac{1}{2}, \frac{1}{2}, t \right) \) for various values of parameter \( \lambda \). With increasing values of \( \lambda \), the amplitude of vibrations decreases and a phase shift to the right is observed.

Figure 4 shows the graphs of curves \( w \left( \frac{1}{2}, \frac{1}{2}, t \right) \) for various values of rheological parameter \( \alpha \). An analysis of results shows that an increase in the value of this parameter leads to an increase in the amplitude and frequency of vibrations.

Figure 5 shows the graphs of curves \( w \left( \frac{1}{2}, \frac{1}{2}, t \right) \) for various values of rheological parameter \( \beta \). An analysis of results shows that taking parameter \( \beta \) into account does not significantly affect the amplitude and frequency of vibrations of a dam-plate.

When calculating the deflection value by formula (1), the first five harmonics were held \( (N=5) \). The calculations showed that a further increase in term number does not significantly affect the amplitude of dam-plate vibrations (Figures 6 and 7).
Figure 1. \( \alpha = 0.25; \beta = 0.05; \lambda = 1; \mu = 0.3; \rho / \rho_1 = 1/2.4; A = 0(1); 0.05(2); 0.1(3). \)

Figure 2. \( A = 0.05; \alpha = 0.25; \beta = 0.05; \lambda = 1; \mu = 0.3; \rho / \rho_1 = 0(1); 1/5(2); 1/2.4(3). \)

Figure 3. \( A = 0.05; \alpha = 0.25; \beta = 0.05; \mu = 0.3; \rho / \rho_1 = 1/2.4; \lambda = 1(1); 1/5(2); 2(3). \)
Figure 4. \( A = 0.05; \quad \beta = 0.05; \quad \lambda = 1; \quad \mu = 0.3; \quad \rho / \rho_1 = 1/2.4; \quad \alpha = 0.25(1); \quad 0.5(2); \quad 0.75(3). \)

Figure 5. \( A = 0.05; \quad \alpha = 0.25; \quad \lambda = 1; \quad \mu = 0.3; \quad \rho / \rho_1 = 1/2.4; \quad \beta = 0.05(1); \quad 0.075(2); \quad 0.1(3). \)

Figure 6. \( A = 0.05; \quad \alpha = 0.25; \quad \lambda = 1; \quad \mu = 0.3; \quad \rho / \rho_1 = 1/2.4; \quad \beta = 0.05; \quad N = 1(1); \quad 3(2); \quad 5(3). \)

Figure 6. \( A = 0.05; \quad \alpha = 0.25; \quad \lambda = 1; \quad \mu = 0.3; \quad \rho / \rho_1 = 1/2.4; \quad \beta = 0.05; \quad N = 5(1); \quad 7(2). \)
5. Conclusions
Mathematical models of the dynamics problems of a dam-plate of constant and variable thickness are constructed taking into account:
- hereditary properties of the material of construction;
- inertia forces arising from the dam-plate motion as a rigid body and from its deformation.
Based on the approximate Galerkin-Bubnov method in combination with the numerical method:
- the methods have been developed for solving interconnected integro-differential equations of Volterra type;
- a numerical algorithm has been developed that allows one to study the problems of oscillations of a dam of constant thickness, taking into account the hereditary properties of the material of structures.
It was revealed that taking into account the hereditary properties of the material of the structure leads to a decrease in the amplitudes and frequencies of oscillations.

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