Single Parameter Logarithmic Image Processing for Edge Detection

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SUMMARY Considering the non-linear properties of the human visual system, many non-linear operators and models have been developed, particularly the logarithmic image processing (LIP) model proposed by Jourlin and Pinoli, which has been proved to be physically justified in several laws of the human visual system and has been successfully applied in image processing areas. Recently, several modifications based on this logarithmic mathematical framework have been presented, such as parameterized logarithmic image processing (PLIP), pseudo-logarithmic image processing, homomorphic logarithmic image processing. In this paper, a new single parameter logarithmic model for image processing with an adaptive parameter-based Sobel edge detection algorithm is proposed. By using an image noise estimation method to evaluate the noise level of image, the adaptive parameter in the single parameter LIP model is calculated based on the noise level and grayscale value of a corresponding image area, followed by the single-parameter LIP-based Sobel operation to overcome the noise-sensitive problem of classical LIP-based Sobel edge detection methods, especially in the dark area of an image, while retaining edge sensitivity. Compared with the classical LIP and PLIP model, the given single parameter LIP achieves satisfactory results in noise suppression and edge accuracy.

key words: image edge detection, single parameter logarithmic image processing, image noise measurement

1. Introduction

With the development of image processing techniques, it has been found that image processing frameworks should be physically consistent with human visual laws and models to achieve better performance. Since the 1980’s, much attention has been paid to non-linear image processing models, such as the multiplicative homomorphic image processing (MHIP) model, the log-ratio image processing (LRIP) model[9], the unified model for human brightness perception, which use a set of non-linear operators to simulate the properties of the human visual system to process bounded intensity images. Particularly, the logarithmic image processing (LIP) model, which entails several laws and characteristics of human brightness perception, such as brightness scale inversion, the saturation characteristic, Weber’s and Fechner’s laws, and the psychophysical contrast notion, has become an important part of non-linear image processing.

In the classical LIP model, an image is formed by a uniform light \( F_{\text{max}} \) passing through a semi-transparent object according to transmittance perception. The gray tone of an image is the absorbance of the semi-transparent object with a scalar factor \( M \), so the relationship of the gray tone value \( g(x) \) of each spatial point \( x \) and its corresponding gray scalar value \( f(x) \) in an image is represented as follows:

\[
f(x) = M \left( 1 - \frac{F(x)}{F_{\text{max}}} \right) = M - g(x)
\]

Here, \( F(x) \) is the amount of light absorbed by the semi-transparent object at a particular position \( x \), and \( M \) is the maximum grayscale in an imaging system. \( M \) equals 255 when an 8-bit imaging system is used.

Since its publication, the LIP model has found a series of successful applications in background removal [2], image enhancement [3], edge detection and image segmentation [4], [5], and image data compression [6]. Recently, a series of modified LIP models have been published. In the homomorphic LIP model [7], [8], gray tone values of an image defined as \([0, M]\) are linearly transformed onto a standard set \((-1, 1)\), and the results of all operations in this model, such as addition, subtraction, scalar multiplication, are bounded in the limited range \((-1, 1)\). The results are then linearly remapped into the image gray scale value \([0, M]\) without clipping to reduce the information loss. But this method is similar to traditional log-ratio image processing [9], wherein the definitions of fundamental isomorphism are almost the same, and the main shortcoming of this model is not based on any physical model or related to any physical phenomena [10].

In the pseudo-logarithmic image processing model [11], [12], the image gray scale value is firstly scaled to the range \([0, 1]\), and is then applied to image processing according to pre-defined operators without converting to the gray tone space, and the results of pseudo-LIP operations are also constrained in the limited range \([0, 1]\). But the definition of subtraction between two image pixels, take \( a \) and \( b \) for example, is available only when \( a > b \) in this model. Otherwise, the definition of this operation leads to an unbounded and nonmonotonic result.

The parameterized logarithmic image-processing framework proposed by K. Panetta [13]–[16] is a parameterization modification of the classical LIP model, using four different parameters to replace the fixed value \( M \) in gray tone conversion, addition, subtraction, and constant
multiplication, and one other parameter for the exponential value of an isomorphic function, which yields better image fusion and enhancement results. But the calculation of a global optimum solution for five parameters is considered to be excessively time consuming.

Another parameterized logarithmic image processing model, which is called the Generalized Logarithmic Image Processing Model, was proposed by Guang Deng [25] based on the Gigavision Sensor Model. The key parameters in this model are physically interoperate as the number of sub-pixels that make up a pixel and the threshold for the output of sub-pixel, but the method of how to choose these parameters is not given in this paper.

Considering that the property and value range of parameters in PLIP were not defined, in this paper, the distributive law, the subtractive law, and the isomorphic property of the PLIP model are firstly analyzed, and then a new single parameter logarithmic image processing model is presented, using one parameter but not five in PLIP to present the algebraic and logarithm relationship in addition operation to ensure the completeness of this model. Then, an adaptive parameter-based edge detection algorithm is proposed, by using an image noise estimation method to evaluate the noise intensity in an image, wherein the adaptive parameter of the single parameter LIP model is calculated based on the noise intensity and grayscale value of the corresponding image area, followed by a single parameter LIP-based Sobel edge detection algorithm to suppress the influence of image noise and improve edge accuracy. Compared with the classical LIP and PLIP models, the given single parameter LIP achieves satisfactory results in noise suppression and edge accuracy.

The rest of this paper is organized as follows. Section 2 briefly reviews the basic operations and properties of the PLIP model. Section 3 presents the proposed single parameter logarithmic image processing model. Section 4 introduces a single parameter LIP-based edge detection algorithm, with an adaptive parameter choosing method based on image noise intensity and pixel gray scale value. Section 5 gives the experimental results and evaluations with two different edge assessment methods of the proposed method. Finally, Sect. 6 gives a summary of this paper and briefly discusses some future works.

2. Properties of the PLIP Model

In this section, we briefly review the basic operations and properties of the PLIP model. Considering that the relationship and value range of five parameters in PLIP were not defined, and the order of addition and subtraction operations should have no impact on the results of image processing, in this paper, the law of distributive and the subtraction property are examined to determine the relationship of parameters $\gamma$, $\lambda$, and $k$. Then a basic logical property of equality and the monotonicity of PLIP addition are also evaluated to determine the relationship of parameters $\mu$ and $\gamma$. Finally, the property of fundamental isomorphism in PLIP with parameter $\beta$ is analyzed, to determine the space property which is algebraically isomorphic with regard to the gray tone space.

2.1 Definition of the PLIP Model

The addition of two images using the classical LIP model always leads to a darker result than the originals, which results in images that are too dark overall. When using linear arithmetic addition, the added images are always brighter than the originals. In order to set up a link between classical LIP and the linear arithmetic operation, the PLIP model was proposed by K. Panetta [13]–[16], which, by using five parameters, $\mu$, $\gamma$, $\lambda$, $k$, and $\beta$, allows for fine tuning of the fundamental isomorphism function $\varphi$, which is primarily used for multiplication and is defined as follows:

$$ g(i,j) = \mu - f(i,j) $$

$$ g_1 \oplus g_2 = g_1 + g_2 - \frac{g_1 g_2}{\gamma} $$

$$ g_1 \ominus g_2 = k g_1 - g_2 $$

$$ c \otimes g_1 = \lambda - \lambda \left(1 - \frac{g_1}{\lambda}\right)^c $$

Where $\mu$, $\gamma$, $k$, and $\lambda$ are parameters that can be trained for the system to get the optimum performance. The value $\mu$ is used to link the relation between gray tone and image grayscale value, which is irrelevant in gray tone operation. Additionally, the coefficient $\beta$ in (1) is proposed for the fundamental isomorphism function $\varphi$, which is primarily used for multiplication and is defined as follows:

$$ \varphi(g) = -\lambda \left[\ln \left(1 - \frac{g}{\lambda}\right)^\beta\right] $$

$$ \varphi^{-1}(g) = \lambda \left[1 - \exp \left(-\frac{g}{\lambda}^{1/\beta}\right)\right] $$

The PLIP has extended the properties of classical LIP, with the following basic properties:

1) The PLIP operations can generate more cases between the two extreme cases of the classical LIP and the linear arithmetic operations with different parameters $\mu$, $\gamma$, $k$, and $\lambda$, using $\mu = \gamma = k = \lambda = M$ and $\beta = 1$ for classical LIP, and $\gamma = k = \lambda = \infty$ and $\beta = 1$ for linear arithmetic.

2) If the input gray tone is within the range $[0, \gamma]$, the result of the PLIP addition and scalar multiplication will be within the range $[0, \gamma]$, which expands the upper border $M$ in classical LIP to $\gamma$.

3) If the input gray tone is within range $[0, k]$, the result of the PLIP subtraction will be within the range $(-\infty, k]$, which expands the upper border $M$ in classical LIP to $k$.

2.2 Distributive Law for PLIP ($\gamma$ and $\lambda$)

The distributive law is an algebraic property which defines
that the sum of two numbers times a third number is equal
to the sum of each addend times the third number. When
using PLIP operators, the addition and constant multiplica-
tive operators and the parameters $\gamma$ and $\lambda$ are involved cor-
responding to the distributive law. The constant value 2 is
exemplarily used to establish the relationship of these two
parameters, according to (3), and the left part in (2) is ex-

diagonal (3) while the right part is expanded in (4).

\[
2 \otimes g_1 \oplus 2 \otimes g_1 = (2 + 2) \otimes g_1 = 4 \otimes g_1 \tag{2}
\]

\[
2 \otimes g_1 \oplus 2 \otimes g_1 = (\lambda - \lambda \left(1 - \frac{g_1}{\lambda}\right)^2) = \left(1 - \frac{g_1}{\lambda}\right)^2 \tag{3}
\]

\[
4 \otimes g_1 = \lambda - \lambda \left(1 - \frac{g_1}{\lambda}\right)^4 \tag{4}
\]

By comparing (3) and (4), (2) is validated only when $\gamma$
equals $\lambda$.

2.3 Subtractive Property for PLIP ($\gamma$ and $k$)

Considering that the equation $(a - b) + c = a - (b - c)$ is a very
important subtractive property which presents the relation-
ship of addition and subtraction, this property is represented
in PLIP operators as (5), and the left part in it is expanded as (6) while
the right part is shown in (7).

\[
(g_1 \odot g_2) \odot g_3 = g_1 - (g_2 - g_3) \tag{5}
\]

\[
(g_1 \odot g_2) \odot g_3 = \frac{k(g_1 - g_2 + g_3) - kg_1g_3 + kg_2g_3(\gamma - k)}{k - g_2} \tag{6}
\]

\[
g_1 \odot (g_2 \odot g_3) = \frac{k(g_1 - g_2 + g_3) - kg_1g_3}{k - g_2} \tag{7}
\]

By comparing (6) and (7), the conclusion that (5) is

2.4 Monotonicity Property of PLIP Addition ($\mu$ and $\gamma$)

Considering that the image addition is a monotonical pro-
cess, which means that the output result is monotonically increased or decreased with the increase of input value, the
output greyscale $F(f_1, f_2)$ of image addition with PLIP op-
erators is represented as follows:

\[
F(f_1, f_2) = F(f_1, f_2) = f_1 + f_2 - \mu + \frac{(\mu - f_1)(\mu - f_2)}{\gamma} \tag{8}
\]

Where $f_1$ and $f_2$ represent the grayscale of the input
image within the range $[0, M]$. Considering that the function $F(f_1, f_2)$ is a monotonic process, the grayscale $F(f_1, f_2)$
equals $F(f_1, f_2)$ if and only if $f_2$ equals $f_2$.

While from analyzing (8), another condition $\mu - \gamma = f_1$
will also lead to the establishment of the equation $F(f_1, f_2) = F(f_1, f_2)$ for an arbitrary $f_2$ and $f_2$ greyscale
value, which is contrary to the monotonically property of
image addition.

From another perspective of (8), the minimum and
maximum greyscale values of $F(f_1, f_2)$ are different with
the change of $\mu$ and $\gamma$, and the extrema are obtained with
different input greyscales $f_1$, $f_2$, as shows in (9).

\[
F_{\text{min}}(f_1, f_2) = \begin{cases} F(0, M) = F(M, 0) & \text{if } \mu - \gamma > 0 \\ F(0, 0) & \text{if } \mu - \gamma \leq 0 \end{cases} \tag{9}
\]

\[
F_{\text{max}}(f_1, f_2) = \begin{cases} F(0, 0) & \text{if } \mu - \gamma \geq M/2 \\ F(M, M) & \text{if } \mu - \gamma \leq M/2 \end{cases}
\]

When $\mu - \gamma > 0$, the monotonicity problem arises, caus-
ing the addition values $F(M, M) > F(0, 0) > F(0, M)$. Tak-
ing $\mu = 355, \gamma = 255$ for example, the values $F(255, 255) =
194, F(0, 0) = 139, and F(0, 255) = 39$. Only when
$\mu - \gamma \leq 0$ is the monotonic property of PLIP addition
validated.

2.5 The Fundamental Isomorphism $\varphi$ and $\beta$

Moreover, it has been shown that the LIP fundamental iso-
morphisms $\varphi$ is physically justified. The real-valued func-
tion $\varphi(g)$ expresses the optical density [17], [18] of the im-
age $f$, and the use of the exponential value $\beta$ has no physical
meaning.

The range of fundamental isomorphism $\varphi(g)$ is
$(-\infty, +\infty)$. When $\beta$ is chosen to be an even integer, tak-
ing 2 for instance, the range of $\varphi(g)$ is limited to $(-\infty, 0]$, and
$\varphi(g)$ is no longer an isomorphic function because the
mapping from gray tone space to spatial $(-\infty, 0]$ is no longer
bijective.

For a gray tone value $g$ within the range $[0, \lambda)$, the log-
arithm part of fundamental isomorphism $\ln(1 - g/\lambda)$ in clas-
sical LIP gets a negative value. When $\beta$ is chosen as an even integer, taking 2 for instance, the logarithm part $\ln(1 - g/\lambda)$
of $\varphi(g)$ in PLIP is converted to a positive value, which makes
$\varphi^{-1}(\varphi(g))$ no longer equal to $g$, so the inverse function
$\varphi^{-1}$ is no longer valid.

Further, when $\beta$ is a decimal value, the value of $\varphi(g)$
is converted from a real range to a complex domain. When
$\beta = 1.1$ and $g = 100$, the value $\varphi(g)$ becomes $-112.636.6i$,
which has no physical meaning in image processing.

2.6 The Relationship of PLIP Parameters

From the analysis above, the following conclusion can be
drawn:

1) The value $\mu$ should be no larger than $\gamma$, otherwise the
monotonicity problem occurs.
2) From the transmittance view of LIP, the value of $\mu$
is the maximum of light intensity, which should not be
less than the maximum value of image $M$.
3) $\gamma$, $k$, and $\lambda$ should be equal to satisfy the distributive
law and the subtraction property.
As opposed to the conclusion in PLIP [14], the value of $\beta$ should be set to 1, otherwise, the predefined $\varphi(g)$ is no longer an isomorphic function, and the inverse function $\varphi^{-1}$ fails. Further, a decimal value of $\beta$ will probably lead to complex operation of image processing.

### 3. Single Parameter LIP Model

On the basis of the restriction of parameters in PLIP models, in this section, the five parameters in PLIP are replaced by a single parameter to ensure completeness of the model and physically constancy with the nature of the image. Then the characteristics of the Sobel operator based on the proposed model are analyzed.

#### 3.1 Single Parameter LIP Model

From the previous discussion in Sect. 2, the values $\gamma$, $k$, and $\lambda$ should be equal to satisfy the distributive law and the subtraction property, and the value of $\beta$ should be set to 1 to ensure the bijective mapping and the real-valued range of the mapping function. The value $\mu$ used to map the image grey level to the gray tone space should set to $M$, just like the classical LIP, to fit the transmittance model and physically constancy within the nature of the image. Therefore, the single parameter LIP model with one parameter $\gamma$ is proposed as follows:

$$
g(i, j) = \mu - f(i, j)$$

$$
g_1 \oplus g_2 = g_1 + g_2 - \frac{g_1g_2}{\gamma}$$

$$
g_1 \oplus g_2 = \gamma \frac{g_1 - g_2}{\gamma - g_2}$$

$$
c \ominus g_1 = \gamma - \gamma \left(1 - \frac{g_1}{\gamma}\right)^c$$

$$
\varphi = -\gamma \left[ \ln \left(1 - \frac{g}{\gamma}\right) \right]$$

$$
\varphi^{-1} = \gamma \left[ 1 - \exp \left(-\frac{g}{\gamma}\right) \right]
$$

For the value $\gamma$ to be no less than the value $\mu$, which is set to be equal to $M$, the value range of $\gamma$ is $[M, +\infty)$. Considering that the range of gray tone which is directly mapped from the image grayscale value is $[0, M]$, and the minimum negative in the single parameter LIP model is now set to $-M\gamma/(\gamma - M)$, unlike the negative infinity in classical LIP, this operator decreases the range of subtraction, which make the computation of the subtraction operator much more efficient.

While with the increase of the value $\gamma$, the range of subtraction is decreased, leading to lower resolution. Because only one parameter is used, it is much easier than PLIP to find the global optimum value $\gamma$ for different image processing algorithms with various images.

#### 3.2 The Interpretation of Parameter $\lambda$

Supposing that the value of $\lambda$ in (1) is no less than $M$, the isomorphic forms of a given gray tone value $g$, with $\beta = 1$ can be rewritten in the Taylor series as follows:

$$
\varphi(g) = -\lambda \left[ \ln \left(1 - \frac{g}{\lambda}\right) \right] = \lambda \left[ \frac{g}{\lambda} + \frac{1}{2} \left(\frac{g}{\lambda}\right)^2 + \frac{1}{3} \left(\frac{g}{\lambda}\right)^3 + \cdots \right]
$$

(10)

While a linear combination of arithmetic operation and classical LIP isomorphic formation can also expressed in (11), where $k$ is the weight of classical LIP, $b_1$ and $b_2$ are the weights of the arithmetic operation, respectively.

$$
\varphi'(g) = -M \left[ \ln \left(1 - \frac{g}{M}\right) \right] * k + b_1 * g + b_2 * g^2$$

$$
= M k \left[ \frac{g}{M} + \frac{1}{2} \left(\frac{g}{M}\right)^2 + \frac{1}{3} \left(\frac{g}{M}\right)^3 + \cdots \right] + b_1 * g + b_2 * g^2
$$

(11)

When fitting (10) using (11), the weight coefficients in (11) are calculated to achieve the minimum errors both in terms of local and global errors.

$$
k = \frac{M^2}{\lambda^2} \quad b_1 = 1 - \frac{M^2}{\lambda^2} \quad b_2 = \frac{1}{\lambda} \left(1 - \frac{M}{\lambda}\right)
$$

(12)

When the weight coefficients are chosen as (12), the 1st, 2nd, and 3rd orders of $\varphi(g)$ and $\varphi'(g)$ are exactly the same, and from the 4th order, these two functions are slightly different. Considering that the $b_2$ has quite a small value and can be ignored, the $\varphi'(g)$ can be expressed as follows:

$$
\varphi'(g) = -M \left[ \ln \left(1 - \frac{g}{M}\right) \right] * \left(\frac{M}{\lambda}\right)^2 + \left(1 - \left(\frac{M}{\lambda}\right)^2\right)g
$$

(13)

The relative difference between the $\varphi(g)$ and its approximation $\varphi'(g)$ are calculated by using (14), and the results are shown in Fig. 1.

$$
Diff(\lambda, g) = \left| \frac{\varphi(\lambda, g) - \varphi'(\lambda, g)}{\varphi(\lambda, g)} \right|
$$

(14)

It can be seen that the difference in these two functions is quite small, especially for most of the image gray tone value $g$. A large difference occurs when $g$ is close to $M$, while the parameter $\lambda$ is somewhat larger than $M$, as show in Fig. 1 (a). While from Fig.1 (b), it can be seen that the difference starts from 0, increases rapidly, and reaches its maximum at about $\lambda = 350$, then the difference is reduced, and eventually diminishes with the increasing of parameter $\lambda$. Most of the mean difference is below 5 % when $\lambda$ is close to $M$.

It can be known that the addition of the PLIP model can be regarded as a combination of linear arithmetic operation and classical LIP with weight $(1 - (M/\lambda)^2)$ and $(M/\lambda)^2$,
separately. With the increase of parameter $\gamma$, the weight of the logarithm addition decreases. The maximum logarithm addition is achieved at $\gamma = M$, which equals the classical LIP addition.

4. Single Parameter LIP Based Sobel Operator

4.1 Single Parameter LIP Based Sobel Operator

The classical LIP based Sobel algorithm has been introduced [19]. It functions the same as the standard Sobel, using the LIP operators instead of classical linear arithmetic to calculate the horizontal gradient $G_x$ and vertical gradient $G_y$ of the image intensity at each point by using the kernels shown in Fig. 2. The magnitude $E$ of the gradient is then calculated to determine whether the given pixel is at an edge or not according to (15).

$$E = \sqrt{G_x^2 + G_y^2}$$

In this paper, the classical linear arithmetic to calculate the horizontal gradient $G_x$ and vertical gradient $G_y$ is replaced by the single parameter LIP operators. It is easy to deduce that the single parameter LIP based Sobel operator $G_x$ as shown in (16) and the structure of $G_y$ are almost the same, which can be easily deduced.

$$G_x = \gamma \frac{(g_1 \odot 2 \odot g_4 \odot g_7) - (g_3 \odot 2 \odot g_6 \odot g_9)}{(g_1 \odot 2 \odot g_4 \odot g_7)}$$

$$= \gamma \frac{(\gamma - g_1)(\gamma - g_7)(\gamma - g_4)^2}{(\gamma - g_3)(\gamma - g_9)(\gamma - g_6)^2}$$

$$E = \sqrt{G_x^2 + G_y^2}$$

Here, the variables from $g_1$ to $g_9$ are the gray tone value within a $3 \times 3$ block counted in the order of left to right, and up to down. Considering that the structures of expression $G_x$ and $G_y$ are approximately the same, the expression $G_x$ is used to analyze the performance of the single parameter LIP based Sobel. Considering that the value range of $\gamma$ is $[M, \infty)$, let $\gamma = M + \Delta$ for instance, when $\Delta = 0$, the single LIP is converted to classical LIP. Combined with the mapping function between gray tone space and image grayscale value, the expression $G_x$ can be rewritten as follows:

$$G_x = (M + \Delta) - (M + \Delta) \frac{(\Delta + f_1)(\Delta + f_2)(\Delta + f_4)^2}{(\Delta + f_3)(\Delta + f_6)(\Delta + f_9)^2}$$

where the variables from $f_1$ to $f_9$ are the image grayscale values corresponding to the gray tone values from $g_1$ to $g_9$.

From (17), it can be seen that with the increasing of number $\Delta$, the decrease of the image gradient, when $\Delta = 0$, the maximum gradient is obtained, which equals the result of classical LIP; the limitation of this equation with $\Delta \to \infty$ is $(2f_4 + f_1 + f_4 - f_6 - f_3 - f_6)$, which equals the result of the arithmetic operation.

Along with the growth of the value $\Delta$, the gradient becomes more robust with regard to image noise, especially in the area where the image grayscale is low. Taking an extreme situation for instance, a black flat area with image grayscale $f_1 = f_7 = f_3 = f_9 = f_6 = k$ and $f_4$ is infected by noise with magnitude $k + N$, the $G_x$ can be written as:

$$G_x = (M + \Delta) - (M + \Delta) \frac{(\Delta + f_1)(\Delta + f_2)(\Delta + f_4)^2}{(\Delta + f_3)(\Delta + f_6)(\Delta + f_9)^2}$$

When $\Delta = 0$, which means the classical LIP is used, any small noise in a black area (the grayscale value is low) will cause a huge gradient value in the edge detection process. On the other hand, a large $\gamma$ will decrease the sensitivity of the gradient to the image noise, and when $\gamma$ goes to infinity, the gradient will close the minimum arithmetic result $2 \times N$, while the local grayscale also contributes to suppress the image noise, as shown in Fig. 3. The larger the local grayscale value is, the smaller the $\Delta$ value is required to achieve the equivalent result.

Figure 4 (b) and (c) show the comparison of $\gamma = 255$, which equals the classical LIP, and $\gamma = 1026$, using a synthetic figure which is corrupted by white additive Gaussian noise. It can be seen that while $\gamma = 255$, the result shows that all image edges are detected, but it is fragile
with regard to the image noise, while with a higher value, \( \gamma = 1026 \), it can effectively remove the influence of image noise, but it is then less sensitive to the image edge.

So the proper choice of value \( \gamma \) can suppress the influence of image noise while improving the accuracy and sensitivity of image edge detection.

### 4.2 Adaptive Parameter Choosing Based on Image Noise Estimation

Considering that the chosen value \( \gamma \) in edge detection is a trade-off between sensitivity and noise suppression, a larger value of \( \gamma \) tends to increase noise suppression, while having less sensitivity of the edge. In this section, an adaptive parameter for the chosen method based on image noise estimation is proposed, wherein the image content and noise are firstly analyzed, then the roughly noise standard deviation is estimated, and the parameter \( \gamma \) in single parameter LIP model is chosen according to the image noise and local grayscale. At an area with a higher grayscale or lower image noise, a lower value \( \gamma \) is used to improve the edge sensitivity. Otherwise, a higher value \( \gamma \) is used to suppress the image noise.

Considering that the accurate estimation of the noise level would require a very sophisticated prior model for images, in this paper, the noise standard deviation is estimated using the MAD estimator:

\[
\sigma_{est} = \text{mad}(I) = \frac{\text{median}(|I - \text{median}(I)|)}{0.6745}
\]

Because the \( \gamma \) not only depends on the intensity of image noise, but also the image grayscale concerned. In the area with higher grayscale or lower image noise, a lower value of parameter \( \gamma \) is used to achieve higher sensitivity on the image edge. Meanwhile, in the black area of the image, a higher \( \gamma \) value is used to suppress the noise. Then the local average grayscale \( A(i, j) \) of the image is calculated, and with the window size \((2C + 1) \times (2C + 1)\), usually \( 5 \times 5 \) is used. Combined with the estimated noise standard deviation \( \sigma_{est} \), the adaptive parameter value \( \gamma(i, j) \) for each pixel is calculated as follows:

\[
\gamma(i, j) = M \left( 1 + \alpha \frac{\sigma_{est}}{A(i, j) + \theta} \right) = M \left( 1 + \frac{\alpha}{\text{SNR}_{Local}} \right)
\]

\[
A(i, j) = \frac{1}{(2C + 1) \times (2C + 1)} \sum_{m=-C}^{C} \sum_{n=-C}^{C} I(i + m, j + n)
\]

where \( \alpha \) is a weight parameter which is used to determine the impact of image noise and local image mean grayscale on \( \gamma \), and \( \theta \) is a small value of 0.01 to avoid the zero division problem. The ratio of estimated noise standard deviation \( \sigma_{est} \) to average local grayscale \( A \) can be considered as the local signal to noise ratio (SNR_{Local}) of the input image.

### 5. Experimental Results

To illustrate the effectiveness of the proposed method, the single parameter LIP based Sobel operator with adaptive parameter choosing is used to detect edges in two kinds of images as shown in Fig. 5. Some popular images, such as “Lena”, “Peppers”, “rose” and “steel” are used to demonstrate the visual effect of the proposed model, and 4 kinds of synthetic figures with known edges are used for the quantitative measurement of accuracy and robustness.

#### 5.1 Measurement of Edge Quality

Consider that the assessment of edge detection [20], [21] performance obeys the following three important criteria: first, all real edges should be found and to have no false edges; second, the edges should be located in the correct place; and third, no duplicated edges should be found for a single edge. Hence, in this paper, the two kinds of edge quality measurement methods are used, including Pratt’s Figure of Merit to measure the accuracy of edge, and PSNR to measure the edge quality from the aspect of information theory.

Pratt’s Figure of Merit is used to compare the result of an edge detection algorithm to the known ground truth [22]. This measurement obeys the three important edge assessment criteria of detection, localization, and spurious response. It returns a normalized evaluation result between 0 and 1 based upon the quality of the detected edge, with 1 being the idea result which is completely the same as the ground truth. Pratt’s Figure of Merit is computed as follows:

\[
F = \frac{1}{\max(N_I, N_A)} \sum_{k=1}^{N_A} \frac{1}{1 + \eta d^2(k)}
\]

where \( N_I \) is the number of actual edges calculated based on the ground truth image, \( N_A \) is the number of detected edges, \( d(k) \) is the deviation from the \( k \)th actual edge to the corresponding detected edge, and \( \eta \) is a scaling constant set to 0.1 in this paper (following the Pratt’s original work).

Another quality measure for the robustness of edge detection schemes to the noise is the PSNR value, which is obtained by calculating the peak signal to noise ratio between
the edge images $E(i, j)$ which is infected by noise and the original ground truth image $G(i, j)$. In this paper, the additive white Gaussian noise (AWGN) is used to verify the robustness of the proposed edge detection algorithm against image noise, when the noise level of the image increases, a good edge detection algorithm should suppress the effect of image noise and precisely obtain the edge.

The mean square error ($mse$) which indicates the difference of the detected edge and the ground truth image is calculated, and then compared with the peak signal value, which is 255 in image, to get PSNR value.

\[
PSNR = 20 \times \log_{10} \frac{255}{\sqrt{mse}}
\]

\[
mse = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} [E(i, j) - G(i, j)]^2
\]

Here, $m$ and $n$ are the value of rows and columns in the input image. In this paper, the PSNR and Pratt’s Figure of Merit are calculated between the original ground truth image and the different edge detection operators with various image noise level, to demonstrate the accuracy and robustness of the proposed method.

5.2 Performance Evaluation

When the proposed edge detection algorithm is compared with classical LIP, and PLIP with, which $\gamma = k = \lambda = 1026$ is said to be the best value [23], images with different noise standard deviations $\sigma$ such as 0 and 6 are used to test the accuracy and noise suppression ability of each algorithm under different circumstances, and the weight parameter $\alpha$ is equal to the estimated image noise standard deviation $\sigma_{est}$.

In Fig. 6, the general image “Lena” is used. Considering that the ground truth edge of this image is hard to obtain, the capacity of each algorithm is evaluated by the human eye. The first row of Fig. 6 is a result obtained when no noise is added to the image with different LIP algorithms and different parameter values, and the second row is the result when the noise standard deviation $\sigma = 6$.

In the first row, the classical LIP and the proposed single parameter LIP with different parameter values achieved approximately the same results, while determining all possible edges in the image, and performed much better than PLIP, in which the edges of the vertical wood shelf and the contour of hair are lost. Further, in the dark area of the source image, such as the mirror frame and hairs in the “Lena” image, a small amount of noise was still there when using classical LIP, while in the edge detected by the proposed single parameter LIP, there is no noise in the corresponding area, which shows that the best result is achieved by the proposed algorithm.

With the growth of image noise, the result of classical LIP drops rapidly, and the area where the grayscale is low in the source image is filled by noise. But in the image of the proposed single parameter LIP and PLIP with a constant $\gamma$ value, the image noise causes a small of amount of influence on the image edge with the increase of the image noise standard deviation, so most image edges are detected.

In Fig. 7, the general image “Peppers” is used. The first row of Fig. 7 is a result obtained when no noise is added to the image with different LIP algorithms and different parameter values, and the second row is the result with a noise standard deviation $\sigma = 6$.

Just like the result of the general image “Lena”, the classical LIP is quite sensitive to image noise, while the PLIP lost some edges “Peppers”. The single parameter LIP with different $\alpha$ values achieved approximately the same result, but a larger value of $\alpha$ is less sensitive to the image edge, and a smaller value of $\alpha$ is more sensitive to the image noise, so the $\alpha = \sigma_{est}$ seems get the balance of edge sensitivity and noise suppression.
Fig. 6  The edges of general “Lena” image with noise variance $\sigma = 0$ and 6, for different algorithm, such as classical LIP, proposed single-parameter LIP ($\alpha = 2$, 12, and $\sigma_{est}$) and PLIP ($\gamma = k = \lambda = 1026$), to test their accuracy and noise suppression ability.

Fig. 7  The edges of general “peppers” image with noise variance $\sigma = 0$ and 6, for different algorithm, such as classical LIP, proposed single-parameter LIP ($\alpha = 2$, 12, and $\sigma_{est}$) and PLIP ($\gamma = k = \lambda = 1026$), to test their accuracy and noise suppression ability.

And the first row of Fig. 8 is the result when no noise is added to the image with different LIP algorithms. Unlike the result of Fig. 8 (e), which is achieved by PLIP, the propose SLIP with $\alpha = \sigma_{est}$ achieves the best result in Fig. 8 (d), which remains the edge of leaves and stem of rose at the bottom of the image, and unlike the classical LIP in Fig. 8 (a), there is few false edges in Fig. 8 (d).

The second row of Fig. 8 is the result of Fig. 5 (c) added with gauss noise ($\sigma = 6$) by using different LIP algorithms. Just like the result above, the classical LIP is quite sensitive to image noise, while the PLIP lost some edges, and the proposed SLIP with $\alpha = \sigma_{est}$ achieves the best result in Fig. 8 (i), which shows good noise suppression capability and remains the accuracy of edge detection.

And the Fig. 9 achieves the same result, the proposed SLIP achieves good result, not only the edge of steel sections, but also the thread of the steels in the image are detected. And SLIP achieves the best result while $\alpha = \sigma_{est}$, which get the balance of edge sensitivity and noise suppression compare to the PLIP and classical LIP.

From Fig. 10 to Fig. 13, the synthetic figure is used for the ground truth edge can be determined for quantitative measurement of accuracy and robustness using (19) and (20). The same as in Fig. 6, the first row and the second are the results of source image additive noise with $\sigma = 0$ and 6, separately.

In the first row of Fig. 10, the classical LIP and proposed single parameter LIP achieved approximately the
same results, while in PLIP, the detection of edges of ellipses and polygons failed in Fig. 10(e), because the PLIP with $\gamma = 1026$ decreased the sensitivity of the image edge.

With the increase of the image noise, taking Fig. 10(f) for example, the classical LIP seems more sensitive to image noise in the place where the image grayscale is low, while in the place where the image grayscale is relatively higher, the noise is better suppressed.

The proposed SLIP seems to take both noise suppression and edge accuracy into account, and the edge in Fig. 10(i) is approximately the same as the Fig. 10(d), which means most noise is suppressed by the proposed algorithm while the image edge is preserved. The choice of a different parameter $\alpha$ seems have a slight influence on the result. Although the PLIP with $\gamma = 1026$ suppressed the image noise the same as the single parameter LIP, the sensitivity to the image edge is relatively lower. The single parameter LIP shows much better results in edge accuracy and continuity, while having the same ability in noise suppression.

Approximately the same conclusions are also achieved while using “Synthetic figure 2”, “Synthetic figure 3”, “Synthetic figure 4” from Fig. 11 to Fig. 13, where in the proposed single parameter LIP with weight parameters $\alpha = \sigma_{est}$ get best similar results, which are much better than the classical LIP and PLIP both in image edge detection and noise suppression.

5.3 Quantitative Results

Figure 14 shows the quantitative results of PSNR and Pratt’s Figure of Merit of the 4 synthetic figures, which is given in Fig. 5(e)∼(h), compared to the ground truth image by using different algorithms and different parameter values, such as
classical LIP, PLIP (γ = k = λ = 1026) and proposed single parameter LIP. The horizontal axis is the standard deviation of image noise, and the vertical axis is quantitative results. In the first row of Fig. 14, the result of PSNR is presented, and the vertical axes is result of PSNR value according to Eq. (20), while in the second row, the value of Pratt’s Figure of Merit is given, and vertical axis is the Pratt’s Figure of Merit according to Eq. (19).

Considering that a big α will leading to an algebraic operation, in this experiment, three different value of α, 2, 12, and σ_{est} is chosen to demonstrate its impaction.

From the image pairs in Fig. 14, it can be seen that the tendency of these figures is almost the same. When there is no image noise, the result of classical LIP is quite good, which is shown in red lines in Fig. 14, while the PLIP is less sensitive to the image edge for a large value γ and brings it down. With the increase of image noise, the result of classical LIP drops rapidly.

While the value of PLIP tends to be the same as proposed SLIP while the image noise is big, because with the increasing of image noise, the adaptive γ tends to the value 1026, or may be even higher, as shown in black lines in Fig. 14, but achieves worst result when there is no noise in image.

And the proposed SLIP achieved similar result with different value of α. But small value of α, such as α = 2, is not capable of handling large image noise, which can be seen from the blue line in Fig. 11, which also dropped while
Fig. 12  The edges of “Synthetic figure 3” with noise variance $\sigma = 0$ and 6, for different algorithms, such as classical LIP, proposed single-parameter LIP ($\alpha = 2$, 12, and $\sigma_{est}$), and PLIP ($\gamma = k = \lambda = 1026$), to test their accuracy and noise suppression ability.

Fig. 13  The edges of “Synthetic figure 4” with noise variance $\sigma = 0$ and 6, for different algorithms, such as classical LIP, proposed single-parameter LIP ($\alpha = 2$, 12, and $\sigma_{est}$), and PLIP ($\gamma = k = \lambda = 1026$), to test their accuracy and noise suppression ability.

Fig. 14  The result of PSNR, Pratt’s Figure of Merit and normalized mean value for the synthetic figure comparing to the ground truth image for different algorithm, such as classical LIP, proposed single-parameter LIP ($\alpha = 2$, 12, and $\sigma_{est}$) and PLIP ($\gamma = k = \lambda = 1026$).
the image noise increased. With a large value of $\alpha$, such as $\alpha = 12$ which is presented in green lines in Fig. 14, it can achieve good results when the image noise level is high, but it cannot achieve a good result when the image noise level is low, such as $\alpha = 2$ does.

Further, $\alpha = \sigma_{\text{est}}$ performs much better than when using a constant value of $\alpha$, as it can respond fast enough to the change of image noise level. When the image noise level is low, a small value of $\alpha$ has sufficient sensitivity to the image edge, and with the increase of image noise, a large value of $\alpha$ suppresses the effect of noise and achieves the best result of all, which can be seen from the purple lines in Fig. 14.

6. Conclusion

In this paper, a self-contained and physical constant single parameter LIP model is proposed, using one parameter which presents the relationship of arithmetic and logarithmic operation in gray tones. Compared with the PLIP, this proposed single parameter LIP model is much more flexible, and can easily find the global optimum value for different image processing algorithms with various images.

In addition, we combined the proposed model with the Sobel operator to detect edges in an image, with an image noise estimation algorithm to evaluate the noise level of the image, followed by adaptive parameter choosing to get the image content relevant single parameter to suppress the influence of image noise, which improves the accuracy and sensitivity of image edge detection.

From the experiment results, single parameter LIP shows higher sensitivity on edge detection with regard to the PLIP model, and much better noise suppression capacity than the classical LIP model.

Furthermore, the performance of image enhancement of the proposed model will be studied, considering that the LIP model expands the range of mathematical operation, and remapping criteria from operation to display domain and the image quality assessment in accordance with the human visual system will also be explored.

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