The heavy quark’s self energy from moving NRQCD on the lattice
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We present a calculation of the heavy quark’s self energy in moving NRQCD to one-loop in perturbation theory. Results for the energy shift and external momentum renormalisation are discussed and compared with non-perturbative results. We show that the momentum renormalisation is small, which is the result of a remnant of re-parameterisation invariance on the lattice.

1. INTRODUCTION

The study of semi-leptonic $B$ decays is currently of great interest as it provides us with a test of the Standard Model. A key process is that of $B \to \pi$ semi-leptonic decay, however, because the $B$ meson is much heavier than the final states, the recoil momenta of the $\pi$ meson can be large. This leads to both problems of small signal to noise ratios and significant discretisation errors if $a p_\pi$ is large. Computations involving the simulation of heavy quarks at low momentum can be dealt with using the NRQCD formalism \cite{1}, where the dynamics of interest are isolated by re-expressing the Lagrangian as an expansion in $v/c$. Although this framework for dealing with heavy quarks reduces the impact of discretisation errors, it is not designed to deal with large momenta. In this case, a suitable formalism to use is moving NRQCD (mNRQCD) \cite{2,3,4} which allows the momentum to be shared more evenly between the initial and final states. mNRQCD is developed in two stages, the first of which involves a choice of lattice frame such that the $B$ meson is moving, thus reducing the magnitude of the pion’s recoil momentum. Secondly, an effective field theory must be constructed for the heavy quark with non-zero velocity. In mNRQCD, the heavy quark’s momentum is written as

$$P_Q^\mu = M_Q u^\mu + k^\mu$$

where $u^\mu = \gamma(1, v)$ is the 4-velocity of the $b$ quark which is treated exactly, and $k^\mu$ is the small internal momentum (of the heavy quark) inside the $B$ meson which is discretised on the lattice.

As with the non-moving case, mNRQCD is constructed from a set of non-renormalisable interactions, each of which is accompanied by a coupling coefficient which can be calculated perturbatively. The heavy quark’s self energy in NRQCD has been calculated to $O(\alpha_s)$ \cite{5}. Here we present a calculation of the one-loop renormalisation parameters that appear in the mNRQCD Lagrangian. In addition to the overall energy shift, mass and wavefunction renormalisation, we must also now calculate the renormalisation of the external momentum. This renormalisation is close to one because of the remnants of re-parameterisation invariance on the lattice.

2. COMPUTING THE SELF ENERGY

In this section, we present the Lagrangian used in this calculation and define the renormalisation parameters.

2.1. Perturbation theory

The Lagrangian is defined as

$$a L_{\text{NRQCD}} = \psi^\dagger(x) \psi(x) - \psi^\dagger(x + a \xi) H \psi(x)$$

where

$$H = \frac{1}{u_0} \left( 1 - a H_0 \right)^n U^\dagger(x) \left( 1 - a H_0 \right)^n$$

and

$$H_0 = \frac{1}{2 \gamma M} \left( \Delta^{(2)} - |\mathbf{v} \cdot \nabla|^2 \right) - i \mathbf{v} \cdot \nabla$$
In Eqn. 11 n is a positive parameter that stabilises the high momentum modes in the heavy quark propagator. The mean-field parameter $u_0$ is used to improve the match between lattice and continuum theories. The gluon fields are discretised using the Wilson plaquette gauge action. This Lagrangian includes all spin-independent corrections up to $O(1/M, v^2)$.

The Feynman rules are calculated by expanding the $U$ fields in terms of the coupling $g$. Following a Fourier transform to momentum space, the quark (gluon) propagator is defined as the inverse of the term that is bilinear in the quark (gluon) fields. Vertices of different order are selected according to their order in $g$. The rules are defined in the Feynman gauge and the infra-red divergence is regulated by introducing a small gluon mass. Note that in the limit $v \to 0$, we recover the Feynman rules for the corresponding NRQCD action.

2.2. Outline of the calculation

The heavy quark’s self energy $\Sigma$, represented diagrammatically in Fig. 1 along with tadpole counterterms from the $u_0$ factors, is defined by writing the inverse quark propagator $G^{-1}(k)$ in the form

$$G^{-1}(k) = Q^{-1}(k) - a\Sigma(k)$$

where $Q^{-1}$ is the free propagator. The self energy can also be expanded for small $k$ to give

$$a\Sigma(k) = \alpha_s \left( \Omega_0 - ik_0\Omega_1 + \frac{k^2}{2\gamma M}\Omega_2 + v.k\Omega_v + \cdots \right)$$

(3)

Combining the expansion of $\Sigma$ with the quark propagator, to $O(\alpha_s)$, gives

$$G^{-1}(k) \approx -ik_0 - \alpha_s\Omega_0 + \alpha_s ik_0\Omega_1 + \frac{k^2}{2\gamma M}$$

$$- \alpha_s \frac{k^2}{2\gamma M}\Omega_2 + v.k - \alpha_s v.k\Omega_v + \cdots$$

(4)

Re-arranging this expression, we obtain

$$G^{-1} = Z_p \left( -i(k_1 - i\alpha_s\Omega_0) + \frac{k^2}{2\gamma M_P} + \frac{P_r \cdot k}{2\gamma M_P} + \cdots \right)$$

(5)

Figure 1. Diagrams contributing to the self energy. The broken lines represent temporal gluons and the curly lines represent spatial gluons.

The renormalisation parameters are then defined as

$$Z_P = 1 - \alpha_s(\Omega_0 + \Omega_1)$$

(6)

$$Z_M = 1 + \alpha_s(\Omega_2 - \Omega_1) + \alpha_s(\Omega_v - \Omega_1)$$

(7)

$$Z_P = 1 - \alpha_s(\Omega_v - \Omega_2)$$

(8)

where $M_r = Z_M M$ and $P_r = Z_P P$ ($P = \gamma Mv$). The energy shift is defined as $E_0 = -\alpha_s\Omega_0$. We also note that since the tadpole counterterms are the same for $\Omega_2$ and $\Omega_v$, these factors will cancel in $Z_P$. In order to compute the renormalisation parameters, we need to isolate the coefficients, $\Omega_n$, of individual terms appearing in Eqn. 3. This is equivalent to taking derivatives of the diagrams with respect to an appropriate choice of momentum.

The quark propagator is zero for $t \leq 0$, so by choosing the momentum flow appropriately, poles in the quark propagator can be avoided. The diagrams have internal loop momentum $q$ and contain a pole in the gluon propagator in the $q_0$ plane. The integrals are therefore evaluated analytically in the complex plane ($z = e^{iq_0}$) which leads to a 3-D integral that can be evaluated numerically using the VEGAS package. The calculations were performed at $aM = 2.0$ with several different values of the stabilisation parameter and velocity. These values match those used in the
Figure 2. The simulated energy \((E_{\text{sim}})\) and the physical binding energy \((E_{\text{sim}} - E_0)\gamma_r\) for a heavy-light meson with \(aM = 2\).

Numerical calculations \(^6\) which were carried out on Wilson glue configurations at \(\beta = 5.7\). We take \(\alpha_s(1/a) = 0.35\). The results are checked for invariance under \(v_j \rightarrow -v_j\).

3. PRELIMINARY RESULTS

Preliminary results for the heavy-light binding energy \(M_{\text{kin}} - Z_M M = (E_{\text{sim}} - E_0)\gamma_r\) as a function of the bare velocity are presented in Fig. 2. The simulated energy \(^6\) clearly exhibits velocity dependence as \(v/c\) approaches 1. After subtraction of the energy shift, the physical binding energy is independent of \(v\), as it should be.

Preliminary values for the renormalisation of the external momentum \(\gamma Mv\) as a function of the bare velocity are presented in Fig. 3. Up to lattice artefacts, the Lagrangian has an invariance under arbitrary shifts of momentum between \(\gamma Mv\) and \(k\), and this protects \(Z_P\) from becoming very different from 1 \(^7\). \(Z_P\) from perturbation theory and simulations disagree at the level of systematic errors in this calculation.

4. CONCLUSIONS

We have presented preliminary results for the energy shift and external momentum renormalisation for a heavy quark in moving NRQCD. Comparison with numerical simulation is encouraging.

The full set of renormalisation parameters will be presented in a forthcoming publication \(^7\). Future work will incorporate higher order terms in the Lagrangian and include the renormalisation of current operators for \(B \rightarrow \pi\) decay.

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