We investigate the $B$ meson light-cone distribution amplitudes in the heavy-quark limit which are relevant for the QCD factorization approach for the exclusive $B$ meson decays. We derive exact relations between two- and three-particle distribution amplitudes from the QCD equations of motion and heavy-quark symmetry constraint. As solution of these relations, we give representations for the quark-antiquark distribution amplitudes in terms of independent dynamical degrees of freedom. In particular, we find that the Wandzura-Wilczek-type contributions are determined uniquely in analytic form in terms of $\bar{\Lambda}$, a fundamental mass parameter of heavy-quark effective theory, and that both leading- and higher-twist distribution amplitudes receive the contributions of multi-particle states with additional gluons.
Recently systematic methods based on the QCD factorization have been developed for the exclusive $B$ meson decays into light mesons $[1, 2, 3, 4, 5]$ (for other approaches see, e.g. $[6]$). Essential ingredients in this approach are the light-cone distribution amplitudes for the participating mesons, which constitute nonperturbative long-distance contribution to the factorized amplitudes $[7]$. As for the light mesons ($\pi$, $K$, $\eta$, $\rho$, $\omega$, $K^*$, $\phi$) appearing in the final state, systematic model-independent study of the light-cone distributions exists for both leading and higher twists $[8, 9, 10, 11]$. On the other hand, unfortunately, the light-cone distribution amplitudes for the $B$ meson are not well-known at present and provide a major source of uncertainty in the calculations of the decay rates. In this Letter, we demonstrate that heavy-quark symmetry and constraints from the equations of motion determine a unique analytic solution for the $B$ meson light-cone distribution amplitudes within the two-particle Fock states. We also derive the exact integral representations for the effects of higher Fock states with additional gluons. A complete set of the $B$ meson distribution amplitudes are constructed in terms of independent dynamical degrees of freedom, which satisfies all relevant QCD constraints.

In the heavy-quark limit, the $B$ meson matrix elements obey the heavy-quark symmetry, and is conveniently described by the heavy-quark effective theory (HQET) $[12, 13]$. Following Refs. $[14, 2, 3]$, we introduce the quark-antiquark light-cone distribution amplitudes $\tilde{\phi}_\pm(t)$ of the $B$ meson in terms of vacuum-to-meson matrix element of nonlocal light-cone operators in the HQET:

$$
\langle 0 | \bar{q}(z) \Gamma h_v(0) | \bar{B}(p) \rangle = -\frac{i f_B M}{2} \text{Tr} \left[ \gamma_5 \Gamma \frac{1+\not{t}}{2} \left\{ \tilde{\phi}_+(t) - \frac{\tilde{\phi}_+(t) - \tilde{\phi}_-(t)}{2t} \right\} \right].
$$

(1)

where $z^2 = 0$, $v^2 = 1$, $t = v \cdot z$, and $p^\mu = M v^\mu$ is the 4-momentum of the $B$ meson with mass $M$. $h_v(x)$ denotes the effective $b$-quark field, $b(x) \approx \exp(-im_b v \cdot x)h_v(x)$, and is subject to the on-shell constraint, $\not{v} h_v = h_v$ $[12, 13]$. $\Gamma$ is a generic Dirac matrix and, here and in the following, the path-ordered gauge factors are implied in between the constituent fields. $f_B$ is the decay constant defined as usually as

$$
\langle 0 | \bar{q}(0) \gamma^\mu \gamma_5 h_v(0) | \bar{B}(p) \rangle = i f_B M v^\mu,
$$

(2)

so that $\tilde{\phi}_\pm(t = 0) = 1$. The behavior of the RHS of eq.(1) for a fast-moving meson, $t = v \cdot z \to \infty$, shows that $\tilde{\phi}_+$ is the leading-twist distribution amplitude, whereas $\tilde{\phi}_-$ has subleading twist $[14]$.

It is well-known that the equations of motion impose a set of relations between distribution amplitudes $[8, 9, 10, 11]$. To derive these relations, the most convenient
method is to start with the exact operator identities between the nonlocal operators:

\[
\frac{\partial}{\partial x^\mu} \bar{q}(x) \gamma^\mu \Gamma h_v(0) = \bar{q}(x) \tilde{\nabla} \Gamma h_v(0) + i \int_0^1 du u \bar{q}(x) g G_{\mu\nu}(ux) x^\nu \gamma^\mu \Gamma h_v(0) , \tag{3}
\]

\[
\frac{\partial}{\partial x^\mu} \bar{q}(x) \Gamma h_v(0) = -\bar{q}(x) D_\mu h_v(0) + i \int_0^1 du (u - 1) \bar{q}(x) g G_{\mu\nu}(ux) x^\nu \Gamma h_v(0)
+ \partial_\mu \{ \bar{q}(x) \Gamma h_v(0) \} , \tag{4}
\]

where \( x^\mu \) is not restricted on the light-cone. \( D_\mu = \partial_\mu - igA_\mu, \tilde{D}_\mu = \partial_\mu + igA_\mu \) are the covariant derivatives, \( G_{\mu\nu} = (i/g)[D_\mu, D_\nu] \) is the gluon field strength tensor, and

\[
\partial_\mu \{ \bar{q}(x) \Gamma h_v(0) \} \equiv \frac{\partial}{\partial y^\mu} \bar{q}(x + y) \Gamma h_v(y) \bigg|_{y \to 0} \tag{5}
\]

stands for the derivative over the total translation. These identities simply describe the response of the nonlocal operators to the change of the interquark separation and/or total translation.

By taking the vacuum-to-meson matrix element of these relations, the operators involving the equations of motion vanish. Going over to the light-cone limit \( x_\mu \to z_\mu \), the remaining terms can be expressed by the appropriate light-cone distribution amplitudes \( \tilde{\phi}_+, \tilde{\phi}_- \), etc. The terms given by an integral of quark-antiquark-gluon operator are expressed by the three-particle distribution amplitudes corresponding to the higher-Fock components of the meson wave function. Through the Lorentz decomposition of the three-particle light-cone matrix element, we define the four functions \( \tilde{\Psi}_V(t, u), \tilde{\Psi}_A(t, u), \tilde{X}_A(t, u) \) and \( \tilde{Y}_A(t, u) \) as the independent three-particle distributions:

\[
\langle 0 | \bar{q}(z) g G_{\mu\nu}(uz) z^\nu \Gamma h_v(0) | \bar{B}(p) \rangle = \frac{1}{2} f_B M \text{Tr} \left[ \gamma_5 \Gamma \frac{1 + \gamma^\mu}{2} \left\{ (v_\mu \not\! t - \gamma_\mu) \left( \tilde{\Psi}_A(t, u) - \tilde{\Psi}_V(t, u) \right) \right. \right.
- i \sigma_\mu\nu z^\nu \tilde{\Psi}_V(t, u) - z_\mu \tilde{X}_A(t, u) + \frac{z_\mu}{t} \not\! z \tilde{Y}_A(t, u) \left. \right\} \right] . \tag{6}
\]

This is the most general parameterization compatible with Lorentz invariance and the heavy-quark limit.

By using \( \bar{q} \tilde{\nabla} = 0 \) for the light quark and substituting eqs.(1) and (6), the first identity (3) yields the two differential equations connecting the two-particle distributions \( \tilde{\phi}_+ \) and \( \tilde{\phi}_- \) with the three-particle distribution amplitudes:

\[
\tilde{\phi}_-'(t) - \frac{1}{t} \left( \tilde{\phi}_+(t) - \tilde{\phi}_-(t) \right) = 2t \int_0^1 du u \left( \tilde{\Psi}_A(t, u) - \tilde{\Psi}_V(t, u) \right) , \tag{7}
\]
\[
\frac{\partial^2 \Phi_+(t)}{\partial z^2} - \frac{\partial^2 \Phi_-(t)}{\partial z^2} - \frac{1}{t} \left( \Phi_+(t) - \Phi_-(t) \right) + 4t \frac{\partial \Phi_+(t)}{\partial z^2} = 2t \int_0^1 du \left( \bar{\psi}_A(t,u) + 2 \bar{\psi}_V(t,u) + \bar{X}_A(t,u) \right),
\]
(8)
where \( \Phi_\pm(t) = d\Phi_\pm(t)/dt \), and we introduced a shorthand notation
\[
\frac{\partial \Phi_+(t)}{\partial z^2} \equiv \frac{\partial \Phi_+(t,x^2)}{\partial x^2} \bigg|_{x^2 \to 0} .
\]
(9)

Here, via \( \Phi_+(t) \to \Phi_+(t,x^2) \), we extend the definitions in eq.(1) to the case \( z \to x \) \( (x^2 \neq 0) \), since the derivative in the LHS of eq.(3) has to be taken before going to the light-cone limit.

Next, we proceed to the second identity (4). To use the HQET equation of motion \( \nu \cdot DH_v = 0 \) \[12, 13\], we contract the both sides of eq.(4) with \( \nu_\mu \). After similar manipulations as above, we finally obtain another set of two differential equations:
\[
\frac{\partial \Phi_+(t)}{\partial z^2} - \frac{\partial \Phi_-(t)}{\partial z^2} + i\bar{\Lambda} \Phi_+(t) + 2t \left( \frac{\partial \Phi_+(t)}{\partial z^2} - \frac{\partial \Phi_-(t)}{\partial z^2} \right) = 2t \int_0^1 du \left( \bar{\psi}_A(t,u) + \bar{X}_A(t,u) \right),
\]
(10)
where
\[
\bar{\Lambda} = M - m_b = \frac{iv \cdot \partial \langle 0 | q \Gamma_h | B(p) \rangle}{\langle 0 | q \Gamma_h | B(p) \rangle}
\]
(12)
is the usual “effective mass” of meson states in the HQET \[15, 14, 13\].

Eqs.(7)-(11) are exact in QCD in the heavy-quark limit, and are the new results. In the approximation that the three-particle amplitudes in the RHS are set to zero (“Wandzura-Wilczek approximation”), the system of differential equations (7), (8) is equivalent to that found in Ref. \[2\]. By combining eqs.(8) and (10), we can eliminate the term \( \partial \Phi_+(t)/\partial z^2 \) as
\[
\tilde{\Phi}_+(t) + \tilde{\Phi}_-(t) + 2i \bar{\Lambda} \tilde{\Phi}_+(t) = -2t \int_0^1 du \left( \bar{\psi}_A(t,u) + \bar{X}_A(t,u) + 2u \bar{\psi}_V(t,u) \right) .
\]
(13)
Now, from a system of equations (7) and (13), we can obtain the solution for \( \tilde{\Phi}_+ \) and \( \tilde{\Phi}_- \). For this purpose, it is convenient to work with the momentum-space distribution amplitudes \( \Phi_\pm(\omega) \) defined by
\[
\tilde{\Phi}_\pm(t) = \int d\omega \ e^{-i\omega t} \Phi_\pm(\omega) .
\]
(14)
Here $\omega v^+$ has the meaning of the light-cone projection $k^+$ of the light-antiquark momentu
in the $B$ meson. The singularities in the complex $t$-plane are such \hfill (15)

\begin{equation}
\frac{\omega d\phi_+}{d\omega} + \phi_+ = I(\omega),
\end{equation}

\begin{equation}
(\omega - 2\bar{\Lambda}) \phi_+ + \omega \phi_- = J(\omega),
\end{equation}

where $I(\omega)$ and $J(\omega)$ denote the “source” terms due to three-particle amplitudes as

\begin{equation}
I(\omega) = 2 \int_0^\omega d\rho \int_{\omega-\rho}^{\infty} \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} [\Psi_A(\rho, \xi) - \Psi_V(\rho, \xi)],
\end{equation}

\begin{equation}
J(\omega) = -2 \int_0^\omega d\rho \int_{\omega-\rho}^{\infty} \frac{d\xi}{\xi} [\Psi_A(\rho, \xi) + X_A(\rho, \xi)]
-4 \int_0^\omega d\rho \int_{\omega-\rho}^{\infty} \frac{d\xi}{\xi} \frac{\partial \Psi_V(\rho, \xi)}{\partial \xi}.
\end{equation}

We solve eqs. (16) and (17) with boundary conditions $\phi_\pm(\omega) = 0$ for $\omega < 0$ or $\omega \to \infty$. Obviously, the solution can be decomposed into two pieces as

\begin{equation}
\phi_\pm(\omega) = \phi_{\pm}^{(WW)}(\omega) + \phi_{\pm}^{(g)}(\omega),
\end{equation}

where $\phi_{\pm}^{(WW)}(\omega)$ are the solution of eqs. (13) and (17) with the source terms set to zero, $I(\omega) = J(\omega) = 0$, i.e., in the Wandzura-Wilczek approximation. $\phi_{\pm}^{(g)}(\omega)$ denote the pieces induced by the source terms.

First, let us discuss $\phi_{\pm}^{(WW)}$. Eq. (16) alone, with $I(\omega) = 0$, is equivalent to a usual Wandzura-Wilczek type relation derived in Ref. [2]:

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \int_{\omega}^{\infty} d\rho \frac{\phi_{+}^{(WW)}(\rho)}{\rho}.
\end{equation}

Now combining eqs. (16) and (17), we are able to obtain the analytic solution explicitly as

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{+}^{(WW)}(\omega) = \frac{\omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega),
\end{equation}

\begin{equation}
\phi_{-}^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\Lambda^2} \theta(2\bar{\Lambda} - \omega)
for $\omega \geq 0$. These are normalized as $\int_0^\infty d\omega \phi^{(WW)}_\pm(\omega) = 1$. They vanish for $\omega > 2\bar{\Lambda}$, and we note that $2\bar{\Lambda}$ is actually the kinematical upper bound of $\omega$ allowed for the two-particle Fock states of the $B$ meson in the heavy quark limit.

The solution for $\phi^{(g)}_\pm(\omega)$ can be obtained straightforwardly, and reads:

$$\phi^{(g)}_+(\omega) = \frac{\omega}{2\bar{\Lambda}} \Phi(\omega),$$  \hfill (24)

$$\phi^{(g)}_-(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}} \Phi(\omega) + \frac{J(\omega)}{\omega},$$  \hfill (25)

where $\omega \geq 0$ and

$$\Phi(\omega) = \theta(2\bar{\Lambda} - \omega) \left\{ \int_0^\omega d\rho \frac{K(\rho)}{2\bar{\Lambda} - \rho} - \frac{J(0)}{2\bar{\Lambda}} \right\} - \theta(\omega - 2\bar{\Lambda}) \int_\omega^\infty d\rho \frac{K(\rho)}{2\bar{\Lambda} - \rho} - \int_\omega^\infty d\rho \left( \frac{K(\rho)}{\rho} + \frac{J(\rho)}{\rho^2} \right),$$  \hfill (26)

with

$$K(\rho) = I(\rho) + \left( \frac{1}{2\bar{\Lambda}} - \frac{d}{d\rho} \right) J(\rho).$$  \hfill (27)

These functions obey $\int_0^\infty d\omega \phi^{(g)}_\pm(\omega) = 0$, so that the total amplitudes are normalized as $\int_0^\infty d\omega \phi_\pm(\omega) = \tilde{\phi}_\pm(0) = 1$. The solution (20) with eqs. (22)-(27) for the distribution amplitudes is exact and presents the principal result of our paper. Eq. (20) might exhibit discontinuity at $\omega = 2\bar{\Lambda}$, but this does not constitute a problem because physical observables are given by convolution integrals of distribution amplitudes with smooth coefficient functions, i.e., the sharp behavior is averaged over. Namely, the distribution amplitudes should generally be understood as distributions (in the mathematical sense).

In terms of the Mellin moments ($n = 0, 1, 2, \cdots$),

$$\langle \omega^n \rangle_\pm = \int_0^\infty d\omega \omega^n \phi_\pm(\omega) = \int_0^\infty d\omega \omega^n \phi^{(WW)}_\pm(\omega) + \int_0^\infty d\omega \omega^n \phi^{(g)}_\pm(\omega)$$

$$\equiv \langle \omega^n \rangle^{(WW)}_\pm + \langle \omega^n \rangle^{(g)}_\pm,$$  \hfill (28)

our solution reads:

$$\langle \omega^n \rangle^{(WW)}_+ = \frac{2}{n + 2} (2\bar{\Lambda})^n, \quad \langle \omega^n \rangle^{(WW)}_- = \frac{2}{(n + 1)(n + 2)} (2\bar{\Lambda})^n,$$  \hfill (29)

$$\langle \omega^n \rangle^{(g)}_+ = \frac{2}{n + 2} \sum_{i=0}^{n-1} (2\bar{\Lambda})^{-i} \sum_{j=1}^{n-i} \binom{n-i}{j} \left( \left\{ \binom{n+1-j}{j+1} \right\} + 1 \right) \Psi_A j^{n-i}$$

5
\[ + (n + 2 - i) [X_A]_j^{n-i} + (n + 3 - i) \frac{j}{j+1} [\Psi_V]_j^{n-i} \] 

\[ \langle \omega^n \rangle_{(g)} = \frac{1}{n+1} \langle \omega^n \rangle_{+} - \frac{2n}{n+1} \sum_{j=1}^{n-1} \left( \frac{n-1}{j} \right) \frac{j}{j+1} \left( [\Psi_A]_{j}^{n-1} - [\Psi_V]_{j}^{n-1} \right). \] 

where \( \binom{i}{j} = i!/[j!(i-j)!] \), and we introduced the double moments of the three-particle distributions as

\[ [F]_j^i = \int_0^\infty d\omega \int_0^\infty d\xi \ \omega^{i-j} \xi^{j-1} F(\omega, \xi), \quad (F = \{\Psi_V, \Psi_A, X_A\}). \]

For a few low moments, eqs. (28)-(32) give

\[ \langle \omega \rangle_{+} = \frac{4}{3} \bar{\Lambda}, \quad \langle \omega \rangle_{-} = \frac{2}{3} \bar{\Lambda}, \]

\[ \langle \omega^2 \rangle_{+} = 2\bar{\Lambda}^2 + \frac{2}{3} \lambda_E^2 + \frac{1}{3} \lambda_H^2, \quad \langle \omega^2 \rangle_{-} = \frac{2}{3} \bar{\Lambda}^2 + \frac{1}{3} \lambda_H^2. \]

Here we have used

\[ [\Psi_A]_1^1 = \frac{1}{3} \lambda_E^2, \quad [\Psi_V]_1^1 = \frac{1}{3} \lambda_H^2, \quad [X_A]_1^1 = 0, \]

where \( \lambda_E \) and \( \lambda_H \) parameterize the two independent reduced matrix elements of local quark-antiquark-gluon operators of dimension 5, and are related to the chromoelectric and chromomagnetic fields in the B meson rest frame [14],

\[ \langle 0| \bar{q} g E \cdot \alpha \gamma_5 h_v | \bar{B}(p = 0) \rangle = f_B M \lambda_E^2, \]

\[ \langle 0| \bar{q} g H \cdot \sigma \gamma_5 h_v | \bar{B}(p = 0) \rangle = i f_B M \lambda_H^2, \]

with \( E^i = G^{0i}, \ H^i = -\frac{1}{2} \epsilon^{ijk} G^{jk} \), and \( \alpha = \gamma^0 \gamma \). The results (33) and (34) exactly coincide with the relations obtained by Grozin and Neubert [14], who have derived their relations by analyzing matrix elements of some local operators. Our results (28)-(32) from nonlocal operators give generalization of theirs to \( n \geq 3 \).

An interesting feature revealed by our results is that the leading-twist distribution amplitude \( \phi_+ \) as well as the higher-twist \( \phi_- \) contains the three-particle contributions. This is in contrast with the case of the light mesons, where the leading-twist amplitudes correspond to the “valence” Fock component of the wave function, while the higher-twist amplitudes involve contributions of multi-particle states.

In the higher-twist distribution amplitudes of light mesons [8, 9, 10, 11], the contributions of multi-particle states with additional gluons have been generally important and broadened the distributions, but they have typically produced corrections less
than $\sim 20\%$ to the main term given by the Wandzura-Wilczek contributions. We note that, in the present case, there exists a rough estimate $\lambda_E^2/\Lambda^2 = 0.36 \pm 0.20$, $\lambda_H^2/\Lambda^2 = 0.60 \pm 0.23$ by QCD sum rules [14] (see eq.(24)), but any estimate of the higher moments is not known. It is obvious that further investigations are required to clarify the effects of multi-particle states. In the applications to the physical amplitudes, the evolution effects including the three-body operators also enter the game. All these further developments for going beyond the Wandzura-Wilczek approximation can be exploited systematically starting from the exact results in this paper, as it has been done for light mesons.

Inspired by the QCD sum rule estimates, Grozin and Neubert [14] have proposed model distribution amplitudes $\phi_{GN}^+(\omega) = \left(\omega_{\omega_0}\right) \exp\left(-\omega_{\omega_0}\right)$, $\phi_{GN}^-(\omega) = \left(\frac{1}{\omega_{\omega_0}}\right) \exp\left(-\frac{\omega}{\omega_{\omega_0}}\right)$, where $\omega_0 = 2\Lambda/3$. The shape of their model distributions is rather different from that of the Wandzura-Wilczek contributions (22) and (23), except the behavior $\phi_{GN}^+(\omega) \sim \omega$, $\phi_{GN}^-(\omega) \sim \text{const}$, as $\omega \to 0$. Actually, such behavior when the light-antiquark becomes “soft” is suggested by the corresponding behavior of the light-mesons [8, 9, 10, 11]. But we note that the gluon correction (24) to $\phi_+(\omega)$ would modify such behavior if $J(0) = -2 J_0^\infty(d\xi/\xi)(\Psi_A(0, \xi) + X_A(0, \xi)) \neq 0$.

To summarize, the solution in this paper provides the powerful framework for building up the $B$ meson light-cone distribution amplitudes and their phenomenological applications. Our results represent the quark-antiquark distribution amplitudes in terms of independent dynamical degrees of freedom, and satisfy the constraints from the equations of motion exactly. The other essential constraints are imposed by heavy-quark symmetry, which lead to the reduced set of the distribution amplitudes and thus allow us to determine the Wandzura-Wilczek contributions explicitly in analytic form. The Wandzura-Wilczek contributions give the effects corresponding to the valence distributions, while the corrections due to the additional gluons are also obtained as exact integral representations involving the three-particle distribution amplitudes. A detailed study of the gluon corrections requires systematic treatment of the three-particle distributions, and will be presented elsewhere.

We have treated the light-cone distribution amplitudes for the pseudoscalar $B$ meson. In the heavy-quark limit, the distribution amplitudes of the vector meson $B^*$ is related with those of the $B$ meson thanks to heavy-quark spin symmetry [14], so that the solution in this paper also determines a complete set of the $B^*$ meson distribution.

\[ \phi_{\omega_{\omega_0}}(0) \text{ of eq.(23) is finite even if } J(0) \neq 0. \]
Among the four differential equations (7)-(11) derived in this paper, we have utilized only a system of two equations to obtain the solution (20). Now, with our explicit solution, the other two equations can be used to determine $\partial \tilde{\phi}_\pm(t)/\partial z^2$ (see eq.(13)), i.e., the transverse momentum dependence of the distributions, which are necessary for computing the power corrections to the exclusive amplitudes.

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References

[1] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000).

[2] M. Beneke and T. Feldmann, Nucl. Phys. B 592, 3 (2001).

[3] D. Pirjol, hep-ph/0101045.

[4] M. Beneke, T. Feldmann and D. Seidel, hep-ph/0106067.

[5] S. W. Bosch and G. Buchalla, hep-ph/0106081.

[6] P. Ball and V. M. Braun, Phys. Rev. D 58, 094016 (1998).

[7] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. 112, 173 (1984).

[8] V. M. Braun and I. E. Filyanov, Z. Phys. C 48, 239 (1990).

[9] P. Ball, JHEP 9901, 010 (1999).

[10] P. Ball, V. M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B 529, 323 (1998).

[11] P. Ball and V. M. Braun, Nucl. Phys. B 543, 201 (1999).

[12] N. Isgur and M. B. Wise, Phys. Lett. B 232, 113 (1989); 237, 527 (1990).

[13] M. Neubert, Phys. Rept. 245, 259 (1994).
[14] A. G. Grozin and M. Neubert, Phys. Rev. D 55, 272 (1997).
[15] A. F. Falk, M. Neubert and M. Luke, Nucl. Phys. B 388, 363 (1992).
[16] M. Neubert, Phys. Rev. D 46, 1076 (1992).
Erratum to: “B meson light-cone distribution amplitudes in the heavy-quark limit”
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There was an error in Eq. (6) of our Letter. The correct equation is

\[
\langle 0 | \bar{q}(z) g G_{\mu\nu}(uz) z^\nu \Gamma h_v(0) | \bar{B}(p) \rangle = \frac{1}{2} f_B M \text{Tr} \left[ \gamma_5 \Gamma \frac{1 + \gamma^\mu}{2} \left\{ (v_\mu z - t \gamma_\mu) \left( \bar{\Psi}_A(t, u) - \bar{\Psi}_V(t, u) \right) - i \sigma_{\mu\nu} z^\nu \bar{\Psi}_V(t, u) - z_\mu \bar{X}_A(t, u) + \frac{z_\mu}{t} \bar{Y}_A(t, u) \right\} \right],
\]

which involves the four functions \( \bar{\Psi}_V(t, u) \), \( \bar{\Psi}_A(t, u) \), \( \bar{X}_A(t, u) \) and \( \bar{Y}_A(t, u) \) as the independent three-particle distribution amplitudes. Due to this change, Eq. (11) must be also replaced by

\[
\tilde{\phi}_+^\prime(t) = \tilde{\phi}_-^\prime(t) + \left( i \Lambda - \frac{1}{t} \right) \left( \tilde{\phi}_+(t) - \tilde{\phi}_-(t) \right) + 2t \left( \frac{\partial \tilde{\phi}_+(t)}{\partial z^2} - \frac{\partial \tilde{\phi}_-(t)}{\partial z^2} \right) = 2t \int_0^1 du (u - 1) \left( \bar{\Psi}_A(t, u) + \bar{Y}_A(t, u) \right).
\]

All other equations and the conclusions presented in the Letter remain unchanged.