IS THERE EVIDENCE FOR DARK ENERGY EVOLUTION?

XUHENG DING$^1$, MAREK BIESIADA$^{1,2}$, SHUO CAO$^1$, ZHENGXIANG LI$^1$, AND ZONG-HONG ZHU$^1$

$^1$ Department of Astronomy, Beijing Normal University, Beijing 100875, China
$^2$ Department of Astrophysics and Cosmology, Institute of Physics, University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland

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ABSTRACT

Recently, Sahni et al. combined two independent measurements of $H(z)$ from BAO data with the value of the Hubble constant $H_0 = H(z = 0)$ in order to test the cosmological constant hypothesis by means of an improved version of the Om diagnostic. Their result indicated considerable disagreement between observations and predictions of the $\Lambda$ cold dark matter ($\Lambda$CDM) model. However, such a strong conclusion was based only on three measurements of $H(z)$. This motivated us to repeat similar work on a larger sample. By using a comprehensive data set of 29 $H(z)$, we find that discrepancy indeed exists. Even though the value of $\Omega_m h^2$ inferred from the Om$^2$ diagnostic depends on the way one chooses to make summary statistics (using either the weighted mean or the median), the persisting discrepancy supports the claims of Sahni et al. that the $\Lambda$CDM model may not be the best description of our universe.

Key words: cosmology: observations – dark energy – methods: statistical

1. INTRODUCTION

The discovery of the accelerating expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999) brought us a mystery that has become one of the most important challenges for modern cosmology and theoretical physics. Since then, this phenomenon has been confirmed in manifold ways using different probes such as supernovae Ia, acoustic peaks in the cosmic microwave background radiation (CMBR) (Bernardis et al. 2000; Spergel et al. 2003), and baryon acoustic oscillations (BAOs) (Eisenstein et al. 2005, 2011) in the distribution of the large scale structure. Until now, all this observational evidence was concordant with the simplest assumption that a non-vanishing cosmological constant $\Lambda$ exists. Although this model is the simplest, it is not theoretically satisfactory. There is a huge discrepancy if one tries to motivate $\Lambda$ as a zero-point quantum vacuum energy. Therefore, alternative explanations have been proposed that invoke the scalar field settling down in an attractor (Ratra & Peebles 1988). This motivated us to push forward the phenomenological picture of the so-called dark energy, described as a fluid with a barotropic equation of state $p = w \rho$, where $w$ can either be constant—the so-called “quintessence” (Peebles & Ratra 1988a; as a value characteristic for the fixed point attractor, noted as XCDM parameterization)—or evolving in time (Chevalier & Polarski 2001; Linder 2003) since the scalar field could be expected to evolve in time (noted as CPL, parameterization). The main drawback of such an approach is that it makes an explicit assumption about the dark energy before its specific model (a quintessence or evolving equation of state) can be tested on observational data. Moreover, alternatives to dark energy such as modified gravity (Dvali et al. 2000; Bengochea & Ferraro 2009; Sotiriou & Faraoni 2010; Nojiri & Odintsov 2011) cannot be easily tested within this phenomenon. All observational tests of quintessence pinpoint its value close to $w = -1$ (within the uncertainties) which is equivalent to the cosmological constant. On the other hand, tests of a cosmic equation of state varying in time are much less restrictive and do not allow us to make a decisive statement whether the dark energy equation of state has evolved or not.

Therefore, we clearly need alternative probes capable of discriminating between the cosmological constant and evolving dark energy that do not rely on the dark energy assumption and its equation of state parameterization. One promising approach to such a probes has been initiated by Sahni et al. (2008) and developed further in Shafileo et al. (2012). By properly rearranging the equation for the Hubble function in the flat $\Lambda$ cold dark matter ($\Lambda$CDM) model: $H^2(z) = H_0^2[\Omega_m(1 + z)^3 + 1 − \Omega_\Lambda]$, they noticed that the so-called Om$^2$ diagnostic (where $\dot{h} = H(z)/H_0$):

$$\text{Om}(z) = \frac{\dot{h}^2(z) - 1}{(1 + z)^3 - 1}$$

should be constant and exactly equal to the present mass density parameter if the $\Lambda$CDM model is the true one: $\text{Om}(z)_{\Lambda\text{CDM}} = \Omega_{m,0}$. This is remarkable and differentiates between the $\Lambda$CDM and other dark energy models (including evolving dark energy). Let us note that essentially the same idea has also been formulated by Zunckel & Clarkson (2008) who called it “a litmus test” for the canonical $\Lambda$CDM model. Developing this method (Shafileo et al. 2012), they also considered a generalized two-point diagnostic,

$$\text{Om}(z_1, z_2) = \frac{\dot{h}^2(z_1) - \dot{h}^2(z_2)}{(1 + z_1)^3 - (1 + z_2)^3},$$

which should also be equal to $\Omega_{m,0}$ within the $\Lambda$CDM model but has an advantage that having $H(z)$ measurements at $n$ different redshifts, one has $n(n-1)/2$ two-point diagnostics, hence a considerably increased sample, for inference.

In their latest paper, Sahni et al. (2014) used three accurately measured values of $H(z)$ to perform this test. These were: the $H(z = 0)$ measurement by Riess et al. (2011) and Ade et al. (2014), the $H(z = 0.57)$ measurement from SDDS-DR9 (Samushia et al. 2013), and the latest $H(z) = 2.34$ measurement from the $L_{\text{Virgo}}$ forest in SDSS-DR11 (Delubac et al. 2015). They found that all three values of the two-point diagnostics
Om(z, z_i)h^2 were in strong disagreement with the \( \Omega_{m,0}h^2 \) reported by Planck (Ade et al. 2014). It has been noted (Sahni et al. 2014; Delubac et al. 2015) that such a result could be in disagreement not only with the \( \Lambda \)CDM model but with other dark energy models based on general relativity. Because this conclusion could be of paramount importance for dark energy studies, an update of this test with a larger sample of \( H(z) \) is essential.

### 2. DATA, METHODOLOGY, AND RESULTS

As a basic data set, we used a sample of 29 \( H(z) \) measurements taken from the compilation of Chen et al. (2013) modified in the following way: one data point at \( z = 0.6 \) from Blake et al. (2012) was added and two data points from Gaztañaga et al. (2009) were withdrawn. The reason for deleting the aforementioned two points is that these results have been debated in subsequent papers, e.g., by Miralda-Escudé (2009), Kazin et al. (2010), and Cabré & Gaztañaga (2011). For the sake of consistency with Sahni et al. (2014), we have also taken the latest BAO measurement by Delubac et al. (2015) \( H(z = 2.34) = 222 \pm 7 \) instead of \( H(z = 2.3) = 224 \pm 8 \) from Busca et al. (2013). After these changes, our data are essentially like the ones used by Farooq & Ratra (2013) with the Busca et al. (2013) measurement replaced by that of Delubac et al. (2015). During the analysis, we also made assessments of sub-samples of the larger data set, as we will explain further. Part of the data comes from cosmic chronometers—spectroscopy of galaxies assumed to evolve passively (Jimenez & Loeb 2002). Hereafter, this differential age approach will be noted as “DA” for short. The other part of the data comes from BAO—including the data points used by Sahni et al. (2014). Data are summarized in Table 1. Then we proceed in exactly the same way as Sahni et al. (2014), i.e., for each pair of redshifts \((z_i, z_j)\) we calculate the improved \( Om \) diagnostic:

\[
Om h^2(z_i, z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1 + z_i)^3 - (1 + z_j)^3} \tag{3}
\]

where \( h(z) = H(z)/100 \text{ km sec}^{-1} \text{ Mpc}^{-1} \) is the dimensionless Hubble parameter. In the particular case of the \( \Lambda \)CDM model, the improved diagnostic Equation (3) should be equal to \( \Omega_{m,0}h^2 \) which luckily is the quantity best constrained by the CMBR data, e.g., from Planck (Ade et al. 2014). Because the sample of 29 data points leads to 406 different pairs, we summarize our calculations in Figure 1. One can see that the distribution of inferred \( \Omega_{m,0}h^2 \) is skewed and centered around the different value that was reported by Sahni et al. (2014).

If one is to make summary statistics, one can do it in two ways. First, the straightforward way would be to calculate the weighted mean

\[
Om h^2_{(w,m)} = \frac{\sum_{i=1}^{29} \sum_{j>i}^{29} Om h^2(z_i, z_j) / \sigma^2_{Om h^2,ij}}{\sum_{i=1}^{29} \sum_{j>i}^{29} 1 / \sigma^2_{Om h^2,ij}} \tag{4}
\]

and the standard deviation

\[
\sigma_{(w,m)} = \left( \sum_{i=1}^{29} \sum_{j>i}^{29} 1 / \sigma^2_{Om h^2,ij} \right)^{-1/2} \tag{5}
\]

Table 1

| \( z \) | \( H(z) \) | \( \sigma_H \) | Method |
|---|---|---|---|
| 0.07 | 69 | 19.6 | DA |
| 0.1 | 69 | 12 | DA |
| 0.12 | 68.6 | 26.2 | DA |
| 0.17 | 83 | 8 | DA |
| 0.179 | 75 | 4 | DA |
| 0.199 | 75 | 5 | DA |
| 0.2 | 72.9 | 29.6 | DA |
| 0.27 | 77 | 14 | DA |
| 0.28 | 88.6 | 36.6 | DA |
| 0.35 | 82.7 | 8.4 | BAO |
| 0.352 | 83 | 14 | DA |
| 0.4 | 95 | 17 | DA |
| 0.44 | 82.6 | 7.8 | BAO |
| 0.48 | 97 | 62 | DA |
| 0.57 | 92.9 | 7.8 | BAO |
| 0.593 | 104 | 13 | DA |
| 0.6 | 87.9 | 6.1 | BAO |
| 0.68 | 92 | 8 | DA |
| 0.73 | 97.3 | 7 | BAO |
| 0.781 | 105 | 12 | DA |
| 0.875 | 125 | 17 | DA |
| 0.88 | 90 | 40 | DA |
| 0.9 | 117 | 23 | DA |
| 1.037 | 154 | 20 | DA |
| 1.1 | 168 | 17 | DA |
| 1.43 | 177 | 18 | DA |
| 1.53 | 140 | 14 | DA |
| 1.75 | 202 | 40 | DA |
| 2.34 | 222 | 7 | BAO |

Note. These are essentially the data of Farooq & Ratra (2013), with the BAO measurement at the largest redshift \( H(z = 2.34) \) taken from Delubac et al. (2015).

where

\[
\sigma^2_{Om h^2,ij} = \frac{4 \left( h^2(z_i) \sigma^2_{H(z_i)} + h^2(z_j) \sigma^2_{H(z_j)} \right)}{\left( (1 + z_i)^3 - (1 + z_j)^3 \right)^2} \tag{6}
\]

and \( \sigma_{H(z)} \) denotes the uncertainty of the \( i \)th measurement. We also assume that redshifts are measured accurately. However, this well known and often used approach relies on several strong assumptions: a statistical independence of the data, no systematic effects, and a Gaussian distribution of the errors. These assumptions, especially the Gaussianity of errors, are not valid here. Hence the weighted mean of

\[
Om h^2_{(w,m)} = 0.1253 \pm 0.0021 \tag{7}
\]

is not a reliable measure, as one can see from the histogram in Figure 1. We will comment more on this non-Gaussianity in a moment.

Given that the assumptions mentioned are not valid, we took another much more robust approach to calculating the median. This approach was pioneered by Gott et al. (2001) and then used by others, e.g., quite recently by Crandall & Ratra (2013). The robustness of this method stems from the fact that if systematic effects are absent, half of the data is expected to be higher and another half lower than the median. Then, as the
number of measurements $N$ increases, the calculated median approaches its true value. Thus, the median has a clear and robust meaning without the need to assume anything about the error distribution. From the definition of the median, the probability that any particular measurement, in other words, one of $N$ independent measurements is higher than the true median is 50%. Consequently, the probability that $n$ observations out of the total $N$ is higher than the median follows a binomial distribution, $P = 2^{-N} N! / [n! (N-n)!]$. This allows one to calculate the confidence regions (e.g., 68% confidence region) of the median value estimated from the sample in a simple manner. Proceeding this way, we have obtained

$$\text{Om}^2_{\text{median}} = 0.1550^{+0.0065}_{-0.0072} \quad (8)$$

In order to facilitate a comparison between the inferred values of $\Omega_m h^2$ obtained from two statistical approaches and the Planck data, we display the results in Figure 2.

The inconsistency between the $\text{Om}^2(z_i, z_j)$ diagnostic calculated on the $H(z)$ data and the Planck value of $\Omega_m h^2$ as well as the mutual inconsistency between weighted averaging and median statistic schemes motivated us to make some more detailed tests. First, we recalculated $\text{Om}^2$ for three sub-samples: one excluding the highest redshift $z = 2.34$ measurement, one using only DA data, and one using only BAO data. Results are visualized in Figure 2 and shown in more detail in Table 2. One can see that the $z = 2.34$ point had a large effect on the weighted average value—dropping this point one achieves agreement with the $\Lambda CDM$ Planck value. However, the question remains whether the weighted average scheme is appropriate. Therefore, following Chen et al. (2003) and Crandall et al. (2014), we have drawn histograms of the distribution of our measurements as a function of the number of standard deviations $N_e$ away from central estimates (the weighted mean and the median, respectively). Because of limited space, we do not show them here, but report in Table 2 the corresponding percentage of the distribution falling within $\pm 1\sigma$ i.e., $|N_e| < 1$. One clearly sees that they strongly deviate from the Gaussian 68% expectation. We also tested the $N_e$ distribution with the Kolmogorov–Smirnov test which strongly rejected the hypothesis of Gaussianity in each sub-sample (with $p$ values ranging from $10^{-4}$ to $10^{-7}$). Therefore, our conclusion is that weighted average scheme is not appropriate here and the median statistics are more reliable. Both statistical methods, the weighted mean and the median, produce similar results for the BAO data, but with the addition of DA data, these two schemes give drastically different results. This may suggest the existence of some systematic error in the DA data. It is not obvious by itself because the nonlinear relation between input variables ($H(z)$) underlying the $\text{Om}^2$ diagnostic may be the source of asymmetric uncertainties in the latter. However, the fact that BAO and DA data deviate from the $\Lambda CDM$ expected result in opposite directions strongly supports the suggestion of unaccounted for systematics. This will be the subject of a separate study.

### 3. CONCLUSIONS

In this paper, we attempted to assess the $\text{Om}^2(z_i, z_j)$ diagnostic introduced and developed by Shafileo et al. (2012). The main reason for doing so was the recent paper by Sahni et al. (2014) where they claimed that recent precise measurements of expansion rates at different redshifts suggest a severe disagreement with the $\Lambda CDM$ model. We repeated this on a much more comprehensive data set of 29 $H(z)$ obtained by two techniques: DA and BAO. One can see from Table 1 that the uncertainties of $H(z)$ obtained using these two methods are different. Even within the same methodology (DA), uncertainties are different from case to case. The $\text{Om}^2(z_i, z_j)$ diagnostic, involving the ratio of certain differences (see Equation 3) calculated on our data, produces an asymmetric distribution. This means that the weighted mean is not a reliable summary measure. Therefore, we used a more robust approach for calculating the median.

Our result is that the value of $\text{Om}^2$ inferred from the $\text{Om}^2$ diagnostic is indeed in disagreement with the one obtained by Planck (assuming the $\Lambda CDM$ model). In our case, this disagreement is not as severe as that in Sahni et al. (2014) ($\text{Om}^2 = 0.122 \pm 0.01$ versus $\text{Om}^2_{\text{Planck}} = 0.1426 \pm 0.0025$). Even though the inferred value is sensitive to the way one chooses to calculate summary statistics—the weighted mean value is lower and the median value is higher than that obtained by Planck—they are both different from each other. The non-Gaussianity in the data suggests that the approach of using median statistics is more appropriate, so this tension cannot be alleviated by excluding high-redshift data.

This supports the claims of Sahni et al. (2014) that the concordance model ($\Lambda CDM$) might not be the true or even the best one describing our universe. Therefore, we also performed a quick test of whether the XCDM or CPL models (simplest evolving equation of state parameterization) best fitted to the Planck or Wilkinson Microwave Anisotropy Probe (WMAP) data agree better with the $H(z)$ data. In such a case, the $\text{Om}^2(z_i, z_j)$ diagnostic, defined in Equation (3), will no longer be a single number $\Omega_m h^2$, but rather $\text{Om}^2(z_i, z_j) = \Omega_m h^2 + (1 - \Omega_m) h^2 \frac{(1 + z_i)(1 + z_j) - (1 + z_i)(1 + z_j)}{(1 + z_i)^2 - (1 + z_j)^2}$ for XCDM and $\text{Om}^2(z_i, z_j) = \Omega_m h^2 + (1 - \Omega_m) h^2 \frac{(1 + z_i)(1 + z_j)^2 - (1 + z_i)(1 + z_j)}{(1 + z_i)^2 - (1 + z_j)^2}$ for CPL parameterization.
by subtracting the right-hand sides from the left-hand sides. Because such residuals inherit non-Gaussianity from $\Omega_m h^2$, we summarized our findings with the weighted average $R_{(w,m)}$ and the median $R_{(median)}$. If a particular model (XCDM or CPL) agreed better with the $H(z)$ data than with $\Lambda$CDM, then its $R$ should have been closer to zero than $R(\Lambda$CDM)$_{(w,m)} = -0.0173 \pm 0.0033$ or $R(\Lambda$CDM)$_{(median)} = 0.0124^{+0.0076}_{-0.0076}$. In the XCDM model we took the $w = -1.0507^{+0.0469}_{-0.0507}$ parameter according to Cai et al. (2014) best fit to Planck+WMAP9 data. For the CPL parameterization of the equation of state, we used the values $w_0 = -1.17^{+0.13}_{-0.12}$, $w_a = 0.35^{+0.50}_{-0.49}$ best fitted to WMAP + CMB+ BAO+$H_0$+SNe according to Hinshaw et al. (2013).

The results are $R(\Lambda$CDM)$_{(w,m)} = -0.0176 \pm 0.0025$ and $R(\Lambda$CDM)$_{(median)} = 0.0151^{+0.0068}_{-0.0068}$. For the CPL varying equation of state parameterization, we get $R(CPL)$_{(w,m)} = $0.0275 \pm 0.0073$ or $R(CPL)$_{(median)} = -0.0517^{+0.0149}_{-0.0077}$. We see that they do not reconcile the discrepancy but their performance is even worse than $\Lambda$CDM. However, this is not a decisive conclusion because what remains to be done is finding

![Figure 2](image-url)

**Figure 2.** Values of $\Omega_m h^2$ calculated from the $\Omega_m h^2$ diagnostic as a weighted mean of the data and as the median value. The result from the Planck experiment is shown for comparison. Bands display the 68% confidence regions. The upper left figure corresponds to the full sample of $n = 29$ data points, the upper right shows the results when the $z = 2.34$ point was dropped ($n = 28$), the lower left corresponds to DA data only ($n = 23$), and the lower right figure shows BAO data only ($n = 6$).

| Sample             | $\Omega_m h^2$ ($w_m$) | $|N_c(w_m)| < 1$ | $\Omega_m h^2$ (median) | $|N_c$(median)| < 1 |
|--------------------|------------------------|----------------|------------------------|----------------|
| Full sample ($n = 29$) | 0.1253 ± 0.0021       | 80.54%         | 0.1550^{+0.0065}_{-0.0072} | 75.62%         |
| $z = 2.34$ excluded ($n = 28$) | 0.1404 ± 0.0047       | 77.78%         | 0.1682^{+0.0075}_{-0.0074} | 82.80%         |
| DA only ($n = 23$) | 0.1448 ± 0.0057       | 77.47%         | 0.1852^{+0.0079}_{-0.0072} | 86.56%         |
| BAO only ($n = 6$) | 0.1231 ± 0.0045       | 100%           | 0.1218^{+0.0033}_{-0.0031} | 100%           |

**Table 2**

Values of the $\Omega_m h^2$ Diagnostic Central Values (Weighted Mean and Median) and Their “Non-Gaussianity” Indicated by the Percentage of the Distribution Falling within $|N_c| < 1$ for the Main Sample and Different Sub-samples

Note. Observations of the CMB from PLANCK inform us that $\Omega_m h^2_{\text{Planck}} = 0.1426 \pm 0.0025$. 

The Astrophysical Journal Letters, 803:L22 (5pp), 2015 April 20 Ding et al.
the values of the equation of state parameters \( w \) or \((w_0, w_a)\) best fitted to the \( H(z) \) data (according to \( Omh^2 \) diagnostics). This will be the subject of a separate study.

Now that we have confirmed the discrepancy between \( \Lambda \)CDM and the \( H(z) \) data, the origin of this discrepancy should be studied in greater detail. One reason for the discrepancy could be that our phenomenological description of an accelerated expansion with \( \Lambda \)CDM is incorrect. However, we pointed out that the conclusion (more specifically the direction of this discrepancy) depends on the statistical approach taken, so one should investigate possible systematics in both methods—DA and BAO—and their effect on the conclusion. This is the subject of an ongoing study.

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