Configuration of smart phase-swapping switches in low-voltage distribution systems based on sequenced participation indices

Lixia Cao¹, Guoliang Feng², Xingong Cheng² and Luhao Wang²

Abstract

The smart phase-swapping switches are used to rapidly change the phases of single-phase loads online in low-voltage distribution systems. They can reduce the three-phase imbalance indices. However, the effectiveness of phase-swapping operations is determined by not only the control strategy but also by the quantity and locations of smart phase-swapping switches. In this paper, a configuration method is proposed to determine the preferable quantity and locations of smart phase-swapping switches with considerations of economic benefits and operational requirements. Based on historical load information, the active and reactive powers of the loads are used to formulate the current imbalance index. The configuration problem is modeled as a multiobjective optimization that minimizes the current imbalance indices of all nodes and phase-swapping operations. The problem is solved by the particle swarm optimization algorithm to obtain the phase-swapping participation index of each single-phase load. The loads with high phase-swapping participation indices are preferably equipped with smart phase-swapping switches. The simulation results verify that the proposed method is effective and easy to be implemented in practical applications.

Keywords

Three-phase imbalance index, single-phase load, smart phase-swapping switch, optimal configuration, online control

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Introduction

Three-phase four-wire wiring is widely applied in Chinese low-voltage distribution systems in which a large number of single-phase loads exist. Three-phase imbalance is increasingly serious because the loads usually located unevenly in the three phases. Moreover, the single-phase generators such as some photovoltaics deteriorate the imbalance.

In distribution systems, three-phase imbalance produces negative- and zero-sequence currents, which causes extra power loss and may trip protective devices. Furthermore, it increases the risk of overloading and limits the loading capability of utility facilities. Numerous efforts have been made to mitigate three-phase imbalance, which can be roughly classified into three categories: feeder reconfiguration, power regulation, and phase-swapping.

Feeder reconfiguration has long been used to alter the topological structure of distribution feeders by changing the open/closed status of sectionalizing and tie switches in medium voltage networks. The authors utilized the feeder reconfiguration technologies to reduce power loss and three-phase imbalance. However, feeder reconfiguration generally improves phase balancing at the system level, while the other categories such as power compensation and phase-swapping balance the loads at the feeder level.

Parallel power-electronic devices such as static var compensator (SVC) and distribution static compensator (DSTATCOM) are used to compensate the unbalanced loads. They are usually installed at the low-voltage sides of some distribution transformers and mitigate the local three-phase imbalance. However, SVC is unable to transfer active power among phases.

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which makes it less effective for balancing the loads than DSTATCOM. Meanwhile, the power consumption of DSTATCOM is high because of the high switching frequency, which brings more power loss at some circumstances.

Phase-swapping is a direct and effective way to balance the loads at the feeder level. The authors\textsuperscript{16–22} regulated the phases of loads to evenly allocate loads across three phases of distribution feeders. However, the traditional manual phase-swapping methods were unable to regulate the loads in real time. And manual operations inevitably caused power outages and increased labor costs. In recent years, the smart phase-swapping switches (SPSSs) have been applied to balance the loads of the feeders in some distribution areas.\textsuperscript{20,21} The SPSS is easily installed at a single-phase load and can automatically change the phase of the load within 20 ms. The online phase-swapping operations of SPSSs hardly interrupt power supply and have no influence on residential loads.\textsuperscript{16} Besides, it is characterized by its small size, low power loss, and low price relative to aforementioned devices for load balancing.

However, the SPSSs are impossible to be installed at every load considering economic benefits in practical applications. The authors\textsuperscript{19,20} installed the switches at some nodes based on experience. It is obvious that the effectiveness of SPSSs is influenced by their configuration, that is, quantity and locations. But how to determine the quantity and locations of SPSSs is not solved theoretically so far. Therefore the study of their configuration problem is of both theoretical and applied interest. It is a long-term planning problem in consideration of the economic benefits and operational requirements involved. As mentioned in Ding and Loparo,\textsuperscript{6} 1 year is the most commonly used metric for distribution system planning, as this amount of time can be used to capture the effects of time-varying loads. Then, the optimization model for the configuration of SPSSs is established on the basis of the historical load information throughout a year.

The main contributions of this paper are as follows:

1. The optimization model for the configuration of SPSSs based on the historical load information is established, which provides a model for reducing the current imbalance indices of all nodes and phase-swapping operations.
2. A configuration method based on sequenced phase-swapping participation indices (SPPIs) is proposed to solve the appropriate quantity and locations of the SPSSs.
3. The proposed method is fast and easy to be implemented in practical applications.

This paper is organized as follows. The mathematical formulation of the configuration problem for SPSSs in the low-voltage distribution systems is described in section “Formulation of the multiobjective optimization for SPSS configuration.” In section “Proposed method based on sequenced PSPIs,” a configuration method based on sequenced PSPIs is proposed. Simulation results are given to show the effectiveness of the proposed method in section “Case studies.” Section “Conclusion” gives some concluding remarks.

Formulation of the multiobjective optimization for SPSS configuration

Control strategy for phase balancing

An automatic control system of SPSSs in a low-voltage distribution area is composed of an online terminal at the secondary side of a distribution transformer and SPSSs at some single-phase loads as shown in Figure 1.

The SPSS is controllable as shown in Figure 2. The control strategy for SPSSs is briefly described as follows:

1. The online control terminal calculates the three-phase imbalance index of each node according to the load information at regular intervals. If some nodal imbalance index exceeds a limit, the terminal executes optimization and then sends the phase-swapping instructions to the corresponding SPSSs.
2. The SPSS receives remote phase-swapping instructions from the online control terminal. The switch inside the SPSS disconnects the
single-phase load with original phase line at the zero-crossing point of contact current, and then connects the single-phase load with new phase line at the zero-crossing point of the contact voltage. The single-phase load is freely switched among A, B and C phases. Thus, the three-phase imbalance of the whole low-voltage distribution area is mitigated after all SPSSs operate phase-swapping.

It is obvious that the quantity and locations of the SPSSs inevitably influence the effectiveness of the phase-swapping operations. The target of this paper is to propose a fast and effective method to solve configuration problem of SPSSs.

Configuration principles for SPSSs

Minimization of total current imbalance indices of all nodes in a distribution system. According to IEEE 1159-2009,\(^\text{22}\) the three-phase current imbalance index $\varepsilon$ in AC power systems is defined as

$$\varepsilon = \frac{I_2}{I_1}$$

where $I_1$ and $I_2$ denote the root mean squares (RMS) of positive- and negative-sequence currents. $\varepsilon$ is difficult to be obtained in practice for the following reasons. First, the real-time measurement data, particularly phasor measurements are not available in the low-voltage distribution systems. On this occasion, $I_1$ and $I_2$ cannot be directly calculated with current phasor data according to the Symmetrical Component Method. Second, if $\varepsilon$ is obtained through power flow calculation, it will be computationally costly and time-consuming for the long-term planning problem of SPSS configuration to be solved. Therefore, it is necessary to think out a simple and practical method to approximately calculate $\varepsilon$. As the load information, such as active and reactive powers and phase-states, is available in the low-voltage distribution systems via electric power marketing system, we can formulate the current imbalance index on the basis of load information.

It is proved in Appendix 1 that the nodal voltage imbalance index is much less than the current imbalance index in the three-phase low-voltage distribution systems. The nodal phase voltages are reasonably assumed to be symmetrical, that is, $U_b = a^2 U_A$, $U_C = aU_A$, where $U_A$, $U_B$ and $U_C$ are A, B and C phase voltage phasors, respectively; $\alpha = e^{i20^\circ} = - (1/2) + j(\sqrt{3}/2)$, $\alpha^2 = e^{i40^\circ} = -(1/2) - j(\sqrt{3}/2)$. Let $S_A$, $S_B$ and $S_C$ represent the complex powers of phase loads. Then

$$\tilde{S}_A = U_A I_A^* = P_A + jQ_A$$
$$\tilde{S}_B = U_B I_B^* = P_B + jQ_B$$
$$\tilde{S}_C = U_C I_C^* = P_C + jQ_C$$

where $I_A^*$, $I_B^*$ and $I_C^*$ are respectively the conjugate values of phase currents $I_A$, $I_B$ and $I_C$; $P_A$ and $Q_A$, $P_B$ and $Q_B$, $P_C$ and $Q_C$ are the active and reactive powers of phase loads. Every phase current can be generally expressed as

$$I_\phi = \frac{S_\phi}{U_\phi} = \frac{1}{U_\phi^2} (P_\phi - jQ_\phi) U_\phi$$

where $\phi \in \{A, B, C\}$, $U_\phi$ is the rated phase voltage.

The positive- and negative-sequence currents are expressed as

$$i_1 = \frac{1}{3} (i_A + aI_B + a^2 I_C)$$
$$i_2 = \frac{1}{3} (i_A + a^2 I_B + a I_C)$$

Substituting equation (3) into equation (4), we can obtain

$$I_1 = \frac{\tilde{S}_A}{3U_\phi^2} [(P_A + P_B + P_C) - j(Q_A + Q_B + Q_C)]$$
$$= \frac{\tilde{S}_A}{3U_\phi^2} (P_1 - jQ_1)$$
$$= \frac{S_1}{3U_\phi^2} \tilde{U}_A$$

where $P_1$, $Q_1$ denote total active and reactive powers of the loads respectively; $S_1$ is the conjugate value of total complex power $\tilde{S}_1$. Let $P_1 = P_A + P_B + P_C$, $Q_1 = Q_A + Q_B + Q_C$ and $\tilde{S}_1 = P_1 + jQ_1$. The RMS of positive sequence current can be written as

$$I_1 = \frac{S_1}{3U_\phi}$$
$$S_1 = \sqrt{P_1^2 + Q_1^2}$$

We can similarly obtain the negative-sequence current $I_2$ as follows by substituting equation (3) into equation (4).
\[ I_2 = \frac{\dot{U}_A}{3U_\phi} \left\{ \left[ P_A - \frac{1}{2}P_B - \frac{1}{2}P_C + \frac{\sqrt{3}}{2}(Q_B - Q_C) \right] - j \left[ Q_A - \frac{1}{2}Q_B - \frac{1}{2}Q_C - \frac{\sqrt{3}}{2}(P_B - P_C) \right] \right\} \]  

(7)

For simplification, three symbols \( P_2, Q_2 \) and \( S_2 \) are introduced. They are expressed as

\[ P_2 = P_A - \frac{1}{2}P_B - \frac{1}{2}P_C + \frac{\sqrt{3}}{2}(Q_B - Q_C) \]

\[ Q_2 = Q_A - \frac{1}{2}Q_B - \frac{1}{2}Q_C - \frac{\sqrt{3}}{2}(P_B - P_C) \]

\[ S_2 = P_2 + jQ_2 \]

It can be derived from equations (7) and (8) that

\[ I_2 = \frac{\dot{U}_A}{3U_\phi} (P_2 - jQ_2) = \frac{S_2}{3U_\phi} \dot{U}_A \]  

(9)

Therefore

\[ I_2 = \frac{S_2}{3U_\phi} \]

\[ S_2 = \sqrt{P_2^2 + Q_2^2} \]  

(10)

Substituting equations (6) and (10) into equation (1), then the nodal three-phase current imbalance index can be rewritten as

\[ e = \frac{I_2}{I_1} = \frac{S_2}{S_1} = \frac{\sqrt{[P_A - P_B/2 - P_C/2 + \sqrt{3}(Q_B - Q_C)/2]^2 + [Q_A - Q_B/2 - Q_C/2 - \sqrt{3}(P_B - P_C)/2]^2}}{\sqrt{(P_A + P_B + P_C)^2 + (Q_A + Q_B + Q_C)^2}} \]  

(11)

where nodal phase active powers \( P_A, P_B, P_C \) and reactive powers \( Q_A, Q_B, Q_C \) are calculated as follows

\[ P_{\phi k} = \sum_{k=1}^{K} P_{\phi k} \]

\[ Q_{\phi k} = \sum_{k=1}^{K} Q_{\phi k} \]  

(12)

where \( K \) is the quantity of loads at this node, \( P_{\phi k} \) and \( Q_{\phi k} \) are the active and reactive powers per phase of the \( k \)th load. For computational convenience, the square of \( e \) is used in this paper

\[ e^2 = \frac{[P_A - P_B/2 - P_C/2 + \sqrt{3}(Q_B - Q_C)/2]^2 + [Q_A - Q_B/2 - Q_C/2 - \sqrt{3}(P_B - P_C)/2]^2}{(P_A + P_B + P_C)^2 + (Q_A + Q_B + Q_C)^2} \]  

(13)

Let \( \bar{S}_k, P_k \) and \( Q_k \) denote the complex, active and reactive powers of the \( k \)th load, respectively, and \( \bar{S}_{\phi k}, P_{\phi k} \) and \( Q_{\phi k} \) denote the phase complex powers of the \( k \)th load. Then \( P_{\phi k} \) and \( Q_{\phi k} \) in equation (12) are calculated considering the following three cases:

**Case 1:** If \( \bar{S}_k \) is a single-phase load and equipped with a SPSS, a phase-state vector \( X_k \) is introduced to denote the phase that the load is connected with \( X_k = (1 \ 0 \ 0)^T \) as the load is connected with phase \( A, \bar{S}_k = (0 \ 1 \ 0)^T \) for phase \( B \) and \( \bar{S}_k = (0 \ 0 \ 1)^T \) for phase \( C \). Then, the phase complex powers \( \bar{S}_{\phi k}, \bar{S}_{\phi k} \) and \( \bar{S}_{\phi k} \) are functions of \( X_k \) and can be calculated as follows

\[
\begin{pmatrix}
\bar{S}_{\phi k}(X_k) \\
\bar{S}_{\phi k}(X_k) \\
\bar{S}_{\phi k}(X_k)
\end{pmatrix} = \begin{pmatrix}
P_{\phi k}(X_k) + jQ_{\phi k}(X_k) \\
P_{\phi k}(X_k) + jQ_{\phi k}(X_k) \\
P_{\phi k}(X_k) + jQ_{\phi k}(X_k)
\end{pmatrix} = (P_k + jQ_k)X_k
\]  

(14)

**Case 2:** If \( \bar{S}_k \) is a three-phase load, then its equivalent phase complex powers are

\[
\bar{S}_{\phi k} = \bar{S}_{\phi k} = \bar{S}_{\phi k} = \frac{P_k}{3} + j\frac{Q_k}{3}
\]  

(15)

**Case 3:** If \( \bar{S}_k \) is an interphase load, its equivalent phase loads can be computed according to Appendix 2. For example, if \( \bar{S}_k \) is between phases \( A \) and \( B \), its equivalent phase loads are

\[
\bar{S}_{\phi k} = \bar{S}_{\phi k} = \bar{S}_{\phi k} = \frac{P_k}{3} + j\frac{Q_k}{3}
\]  

(16)

where \( \theta_k \) is the power factor angle of the \( k \)th load, and \( \tan \theta_k = Q_k/P_k \). For a load between phases \( B \) and \( C \), or between phases \( C \) and \( A \), the method in Appendix 2 is also applicable.
It can be seen from equations (12) to (16) that the nodal $\epsilon^2$ can be simply and fast calculated based on the load information such as active and reactive powers, phase-states. As the phase-states of the single-phase loads can be changed by the SPSSs, $\epsilon^2$ is the function of the phase-states. For a distribution system with $N$ nodes and $M$ single-phase loads, among them $M'$ single-phase loads are equipped with SPSSs, one objective for the configuration of SPSSs is minimizing the current imbalance indices of all nodes, which can be modeled as

$$\min \sum_{n=1}^{N} \epsilon^2_n(X_1, \ldots, X_M)$$

where $X_m$ is the phase-state vector of the $m$th ($m = 1, 2, \ldots, M'$) load, $\epsilon_n$ is the three-phase current imbalance index of the $n$th node, and $\epsilon^2_n$ is calculated according to equation (13).

**Minimization of phase-swapping operations.** The less a SPSS operates, the longer it serves. It is necessary to decrease phase-swapping operations to reduce the costs and load fluctuation. For the $m$th ($m = 1, 2, \ldots, M'$) load, its phase-states before and after the phase-swapping operation are, respectively, denoted by $X_m$ and $X_m'$. A change index $d_m$ is introduced to denote whether or not the load phase changes. If $X_m = X_m'$, the switch does not operate, then $d_m = 0$. Otherwise, $d_m = 1$. If $X_m$ is known and $X_m'$ is variable, $d_m$ is the function of $X_m$. The switch configuration should minimize the phase-swapping operations, which is formulated as

$$\min \sum_{m=1}^{M'} d_m(X_1, \ldots, X_M)$$

**Mathematical model**

From the above control strategy and two principles, it can be concluded that the configuration of SPSSs should minimize the total square of three-phase imbalance index and the phase-swapping operations. The problem is modeled as the following multiobjective optimization

$$\begin{cases} \min \sum_{n=1}^{N} \epsilon^2_n(X_1, \ldots, X_M) \\ \min \sum_{m=1}^{M'} d_m(X_1, \ldots, X_M) \end{cases}$$

(19)

**Proposed method based on sequenced PSPIs**

The particle swarm optimization (PSO) algorithm, which has strong parallel search ability and universality, is employed to solve the nonlinear discrete multiobjective optimization of equation (19). The two objective functions in equation (19) are first normalized in the manner divided by their maximal values as follows

$$\begin{align*}
\min & \frac{1}{N} \sum_{n=1}^{N} \epsilon^2_n(X_1, \ldots, X_M) \\
& \min \frac{1}{M'} \sum_{m=1}^{M'} d_m(X_1, \ldots, X_M) \\
& \quad (20)
\end{align*}$$

where $\epsilon_{n,0}$ is the initial three-phase current imbalance index of the $n$th node before phase-swapping operation, $\sum_{n=1}^{N} \epsilon^2_{n,0}(X_1, \ldots, X_M)$ is the maximal value of $\sum_{n=1}^{N} \epsilon^2_n(X_1, \ldots, X_M)$. $M'$ is the maximal value of $\sum_{m=1}^{M'} d_m(X_1, \ldots, X_M)$. Then equation (20) can be expressed as the following compact form

$$F(X_1, \ldots, X_M) = \min \left( \frac{1}{N} \sum_{n=1}^{N} \epsilon^2_{n,0}(X_1, \ldots, X_M) + \frac{1}{M'} \sum_{m=1}^{M'} d_m(X_1, \ldots, X_M) \right)$$

(21)

where $\alpha_1$ and $\alpha_2$ are the weights of objective functions. Equation (21) is the fitness function of the particles.

Because the loads are time-varying, the configuration of SPSSs is a long-term load-driven planning problem. This problem is solved based on 1-year historical load information including active and reactive powers, phase-states. The load information is sampled at $\Delta t$ intervals and $T$ groups are available. It is first assumed that all $M$ single-phase loads are equipped with SPSSs. Then the phase-state variables compose a particle $X$. The configuration method based on sequenced PSPIs is step-by-step described as follows:

**Step 1:** Set the maximum of iteration $\xi$, the swarm size, learning factors $c_1$ and $c_2$, maximal and minimal inertia weight coefficients $\omega_{\text{max}}$ and $\omega_{\text{min}}$. Input $T$ groups of sampled load information. For the first group, initialize the phase-state variable $X_m (m = 1, 2, \ldots, M)$. Set the sample number $t = 1$.

**Step 2:** Compute the fitness value as the initial local-best fitness $p_{\text{best}}$ and global-best fitness $g_{\text{best}}$ based on the $t$th group of load information. Let the iteration number $s = 1$. Randomly initialize the particle swarm and search velocities of the particles.

**Step 3:** Compute the fitness value of each particle. Update the local-best fitness $p_{\text{best}}$ that is the best fitness achieved so far (up to the $s$th iteration) by the particular particle $X$, and global-best fitness $g_{\text{best}}$ that is the best fitness achieved so far (up to the $t$th iteration) by the swarm. Update the positions of particles and generate new particle swarm as follows
\[
\begin{align*}
\vec{V}^t + 1 &= \omega \vec{V} + c_1 \vec{r}_1 \cdot [\vec{P} - \vec{X}] + c_2 \vec{r}_2 \cdot [\vec{G} - \vec{X}] \\
\vec{X}^t + 1 &= \vec{X}^t + \vec{V}^t + 1
\end{align*}
\]

where \( \vec{V} \) is the search velocity of \( \vec{X} \); \( \omega \) is inertia weight coefficient and \( \omega = \omega_{\text{max}} - \frac{s \times ((\omega_{\text{max}} - \omega_{\text{min}})/\xi)}{\xi} \); \( \vec{P} \) is the local-best solution, a solution having the local-best fitness \( p_{\text{best}} \); \( \vec{G} \) is the global solution, a solution having the global-best fitness \( g_{\text{best}} \). \( r_1 \) and \( r_2 \) are the random variables ranged in \([0, 1] \).

**Step 4:** Repeat Step 3 until \( \xi \) is reached. The optimal solution is composed of the optimal phase-states \( X_1^t, X_2^t, \ldots, X_M^t \) of single-phase loads. Then the change indices \( d_1, d_2, \ldots, d_M \) can be obtained according to the change of phase-states. \( X_1^t, X_2^t, \ldots, X_M^t \) are taken as the initial phase-states of single-phase loads before the \((t + 1)\)th group of phase-swapping operations.

**Step 5:** Repeat Steps 2–4 until the optimal solution for the \( T \)th group of phase-swapping operations is obtained. The PSPI of the \( m \)th \((m = 1, 2, \ldots, M)\) single-phase load is defined as follows

\[
PSPI_m = \sum_{t=1}^{T} d_{m,t}
\]

where \( d_{m,t} \) is the change index of the \( m \)th single-phase load after the \( t \)th group of phase-swapping operations that is determined according to the descriptions above equation (18). The PSPIs of \( M \) single-phase loads are sequenced in descending order. The higher the \( PSPI_m \) is, the higher priority does the corresponding load has to be equipped with a SPSS.

**Step 6:** Install the SPSSs in sequence, and evaluate the effectiveness of various configurations of SPSSs as follows:

1. For \( M' (1 \leq M' \leq M) \) SPSSs installed, the objective function equation (21) are recalculated on the basis of \( T \) groups of load information.
2. Calculate the average objective function value (AOFV) as follows

\[
AOFV = \frac{\sum_{t=1}^{T} F_t(X_1, \ldots, X_{M'})}{T}
\]

where \( F_t(X_1, \ldots, X_{M'}) \) is the objective function value for the \( t \)th group of phase-swapping operations that is calculated according to equation (21). The configuration with smaller AOFV indicates the quantity and locations of SPSSs with better effectiveness.

**Case studies**

The evaluation of the proposed method for SPSS configuration has been performed on an actual distribution area shown in Figure 3. The primary feeder is three-phase four-wire wiring. There are 12 nodes. The secondary side of the distribution transformer is number 1 and other nodes are numbered in order of feeder-lateral. There are 40 single-phase loads which are numbered in accordance with nodes as shown in the small boxes in Figure 3. In other words, the first number in the small box in Figure 3 indicates which node the load is located at, and the second number is sequenced from 1. The load information is sampled per 15 min and 35,040 groups of load information can be obtained throughout a year. The parameters of distribution lines

![Figure 3. Topology of a low-voltage distribution area with 12 nodes.](image-url)
and partial load data of the distribution system are listed in Appendix 3. The proposed method is coded in MATLAB and simulations are executed on a 2.3-GHz AMD Ryzen-7 3700U CPU with 8-GB main memory running under Windows environment. Set \( \alpha_1 = 0.65 \), \( \alpha_2 = 0.35 \) as minimization of the three-phase imbalance is the primary objective for SPSS configuration. Set \( j = 300 \), \( c_1 = c_2 = 2 \), \( v_{\text{max}} = 0.9 \), \( v_{\text{min}} = 0.4 \) and the swarm size is 150 particles.\(^{24,25} \)

The quantity of SPSSs is not generally more than half of the single-phase loads in the distribution area in consideration of economic benefit. For the system in Figure 3, the optimal locations of SPSSs determined by the proposed configuration method are listed in Table 1 as the quantity of SPSSs increases from 0 to 20. The AOFV is calculated according to equation (24) where \( T = 35,040 \). The variations of AOFV relative to the previous configuration are also given in Table 1.

The average current imbalance index of all nodes, phase-swapping operations of SPSSs and active power loss of distribution lines are shown in Table 2. They are respectively calculated according to equations (25)–(27) where \( M' \) is the quantity of SPSSs, \( e_{n,t} \) is the three-phase current imbalance index of the \( n \)th node calculated according to equation (11), and \( P_{\text{loss},t} \) is the active power loss of distribution lines after the \( t \)th group of phase-swapping operations.

The AOFV and average current imbalance index of node 1 calculated according to equation (28) are plotted in Figure 4 where \( e_{1,t} \) is the three-phase current imbalance index of the node 1 after the \( t \)th group of phase-swapping operations.

As shown in Tables 1 and 2, the AOFV, average current imbalance index of all nodes, and active power loss of distribution lines are significantly reduced as the quantity of SPSSs increases. It can be seen from Figure 4 that the average current imbalance index of node 1 slightly fluctuates while the AOFV goes down all along. Since the variation of AOFV in Table 1 becomes evidently smaller as the quantity of SPSSs is more than 12, it is preferable to equip 12 single-phase loads with SPSSs as shown in Table 1 in consideration of economic benefit. On this occasion, the average current imbalance index of all nodes is 5.66%, the average phase-swapping operations is 3.32, the average active power loss of distribution lines is 16.85 kW which is reduced by 26.8% relative to the original system without SPSSs. Therefore, the configuration of SPSSs considerably improves the power quality and economic benefit of the distribution area.

### Table 1. Optimal locations of SPSSs in the proposed method.

| Quantity of SPSSs | Optimal locations | AOFV | Variation of AOFV |
|-------------------|-------------------|------|------------------|
| 0                 | –                 | 0.650| –                |
| 1                 | 72                | 0.650| 0                |
| 2                 | 72, 92            | 0.650| 0                |
| 3                 | 123, 72, 92       | 0.650| 0                |
| 4                 | 123, 72, 92       | 0.623| –0.027           |
| 5                 | 123, 111, 72, 92  | 0.595| –0.028           |
| 6                 | 123, 111, 72, 91, 92 | 0.578| –0.017           |
| 7                 | 123, 103, 111, 72, 91, 92 | 0.564| –0.014           |
| 8                 | 123, 103, 111, 72, 91, 92 | 0.549| –0.015           |
| 9                 | 123, 103, 111, 72, 91, 92 | 0.524| –0.025           |
| 10                | 123, 103, 111, 72, 91, 92 | 0.512| –0.012           |
| 11                | 123, 103, 111, 72, 91, 92 | 0.498| –0.014           |
| 12                | 123, 103, 111, 72, 91, 92 | 0.470| –0.028           |
| 13                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.459| –0.011           |
| 14                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.448| –0.011           |
| 15                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.442| –0.006           |
| 16                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.433| –0.009           |
| 17                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.428| –0.005           |
| 18                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.422| –0.006           |
| 19                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.416| –0.006           |
| 20                | 54, 123, 103, 104, 111, 113, 115, 72, 63, 91, 92, 94 | 0.409| –0.007           |

SPSS: smart phase-swapping switches; AOFV: average objective function value.
If the three-phase current imbalance index $e$ is obtained through power flow calculation by use of the back/forward sweep algorithm instead of equation (11), the consumed time is compared with that of the proposed method in Table 3. The consumed time of the method with power flow calculation is obviously much more than the proposed one because the power flow

![Figure 4. AOFV and average current imbalance index of node 1.](image)

Table 2. Average current imbalance index of all nodes, phase-swapping operations, and active power loss of distribution lines for different SPSS configurations.

| Quantity of SPSSs | Average current imbalance index of all nodes (%) | Average phase-swapping operations | Average active power loss of distribution lines (kW) |
|-------------------|-----------------------------------------------|-----------------------------------|-----------------------------------------------|
| 0                 | 12.44                                        | 0.00                             | 23.03                                        |
| 1                 | 12.43                                        | 0.02                             | 23.01                                        |
| 2                 | 12.43                                        | 0.02                             | 23.01                                        |
| 3                 | 12.42                                        | 0.03                             | 23.00                                        |
| 4                 | 12.72                                        | 0.77                             | 22.18                                        |
| 5                 | 11.00                                        | 1.24                             | 21.87                                        |
| 6                 | 10.60                                        | 1.55                             | 21.74                                        |
| 7                 | 10.27                                        | 1.83                             | 21.54                                        |
| 8                 | 9.99                                         | 2.10                             | 21.40                                        |
| 9                 | 7.88                                         | 2.64                             | 20.08                                        |
| 10                | 7.68                                         | 2.82                             | 19.83                                        |
| 11                | 6.89                                         | 3.11                             | 18.78                                        |
| 12                | 5.66                                         | 3.32                             | 16.85                                        |
| 13                | 5.37                                         | 3.51                             | 16.63                                        |
| 14                | 5.18                                         | 3.60                             | 16.38                                        |
| 15                | 5.05                                         | 3.76                             | 16.29                                        |
| 16                | 4.77                                         | 3.91                             | 14.24                                        |
| 17                | 3.76                                         | 4.27                             | 14.21                                        |
| 18                | 3.61                                         | 4.65                             | 14.31                                        |
| 19                | 3.57                                         | 4.66                             | 14.31                                        |
| 20                | 3.55                                         | 4.79                             | 14.41                                        |

SPSSs: smart phase-swapping switches.

Table 3. Average consumed time for solving equation (21) with a group of load information.

| Solution method for equation (21) | Method with power flow calculation | Proposed method |
|-----------------------------------|-----------------------------------|----------------|
| Consumed time (s)                 | 23.4                              | 3.9            |
calculation is an iterative procedure while equation (11) is not.

For 1-year planning problem of SPSS configuration, the procedure of solving equation (21) needs to be repeated for 35,040 times. Then the consumed time of the method with power flow calculation is excessive. Therefore, the proposed method is fast and easy to be implemented in practical applications.

Conclusion

The effectiveness of phase-swapping operations is determined by both the control strategy and the quantity and locations of SPSSs. Two principles for SPSS configuration are presented to obtain the quantity and locations of SPSSs. One is minimizing total current imbalance indices of all nodes in a distribution system, the other is minimizing the phase-swapping operations. The historical load information is used to formulate the nodal three-phase current imbalance index. Then the multiobjective optimization model is set up and solved by the PSO algorithm to obtain the PSPIs of single-phase loads. The sequenced PSPIs are proposed to determine the appropriate quantity and locations of SPSSs. The numerical studies indicate that the proposed method effectively decreases active power loss as well as current imbalance indices. Moreover, it is easy to be implemented in practical applications, which makes the method significant for improving the power quality and economic benefit of distribution systems. Since the three-phase imbalance is affected by the network topology and allocation of loads, if a new load is added or the network topology is changed, the optimal configuration may change. However, the existing SPSSs can still be controlled online to balance the loads at the feeder level.

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28. GB/T 15543-2008. Power quality: three-phase voltage unbalance.

**Appendix 1**

According to GB/T 15543-2008, the nodal voltage imbalance index in three-phase AC power systems is approximately computed as follows

\[ e_U = \frac{\sqrt{3} I_2 U_L}{S_k} \times 100\% \tag{29} \]

where \( e_U \) is the nodal voltage imbalance index, \( I_2 \) is the RMS of negative-sequence current, \( U_L \) is rated phase to phase voltage, \( S_k \) is nodal three-phase short-circuit capacity. Equation (29) can be further converted to

\[ e_U = \frac{\sqrt{3} I_2 U_L \cdot I_1}{S_k} \times 100\% \]

\[ = \frac{\sqrt{3} U_L I_1 \cdot I_2}{S_k} \times 100\% \]

\[ = \frac{I_1}{S_k} \cdot \varepsilon \times 100\% \]

\[ = \frac{I_1}{I_k} \cdot \varepsilon \]

where \( \varepsilon \) is the three-phase current imbalance index, \( I_1 \) is the RMS of positive-sequence current, \( I_k \) is the RMS of periodic component of the nodal three-phase short-circuit current. Because \( I_1 \ll I_k \) for the low-voltage distribution systems, then

\[ e_U \ll \varepsilon \tag{31} \]

**Appendix 2**

For an interphase load \( S_{k1} \) located between phases \( \phi_1 \) and \( \phi_2 \) as shown in Figure 5(a), where \( \phi_1 \) and \( \phi_2 \) denote

![Figure 5. An interphase load: (a) schematic diagram of an interphase load and (b) phasor diagram.](image-url)
$A$ and $B$, or $B$ and $C$, or $C$ and $A$, its complex power is computed as

$$S_k = U_{\phi_1}I_{\phi_2} = P_k + jQ_k$$  (32)

It can be divided into two equivalent single-phase powers $S_{\phi_1}$, $S_{\phi_2}$ which can be expressed as

$$S_{\phi_1} = U_{\phi_1}I_{\phi_1} = P_{\phi_1} + jQ_{\phi_1}$$
$$S_{\phi_2} = U_{\phi_2}I_{\phi_2} = P_{\phi_2} + jQ_{\phi_2}$$  (33)

The phase voltages are assumed symmetrical. It can be seen from Figure 5 that $I_{\phi_1} = I_{\phi_2} = I_{\phi_3}$, $U_{\phi_1} = \sqrt{3}U_{\phi_1} \cdot e^{30^\circ}$. Then $S_k$ can be written as

$$S_k = P_k + jQ_k$$
$$= \sqrt{3}e^{30^\circ}U_{\phi_1}I_{\phi_1}$$
$$= \sqrt{3}e^{30^\circ}(P_{\phi_1} + jQ_{\phi_1})$$  (34)

So we obtain

$$P_{\phi_1} + jQ_{\phi_1} = (P_k + jQ_k) \cdot e^{(-30^\circ)}$$  (35)

At last we can derive

$$P_{\phi_1} = P_k \left( \frac{\sqrt{3}}{6} \tan \theta_k + \frac{1}{2} \right)$$  (36)
$$Q_{\phi_1} = P_k \left( \frac{1}{2} \tan \theta_k - \frac{\sqrt{3}}{6} \right)$$

where $\theta_k$ is the power factor angle of the load and $\tan \theta_k = Q_k/P_k$. Because $P_{\phi_2} + jQ_{\phi_2} = P_k + jQ_k - (P_{\phi_1} + jQ_{\phi_1})$, then

$$P_{\phi_2} = P_k \left( -\frac{\sqrt{3}}{6} \tan \theta_k + \frac{1}{2} \right)$$  (37)
$$Q_{\phi_2} = P_k \left( \frac{1}{2} \tan \theta_k + \frac{\sqrt{3}}{6} \right)$$

### Appendix 3

The distribution lines in the test system in Figure 3 are copper cables with phase resistance and reactance values shown in Table 4. The resistance and reactance values of neutral lines are 0.0350 and 0.0175 $\Omega$, respectively.

Partial sampled load information is listed in Table 5.

| Table 4. Line section parameters of the distribution system with 12 nodes. |
|---------------------------------------------------------------|
| Line section number | Start node of line section | End node of line section | Phase resistance ($\Omega$) | Phase reactance ($\Omega$) |
|---------------------|---------------------------|--------------------------|-----------------------------|---------------------------|
| 1                   | 1                         | 2                        | 0.0886                      | 0.0443                    |
| 2                   | 2                         | 3                        | 0.0443                      | 0.02215                   |
| 3                   | 3                         | 4                        | 0.0641                      | 0.03205                   |
| 4                   | 4                         | 5                        | 0.0312                      | 0.0156                    |
| 5                   | 2                         | 6                        | 0.0961                      | 0.04805                   |
| 6                   | 6                         | 7                        | 0.0273                      | 0.01365                   |
| 7                   | 2                         | 8                        | 0.087                       | 0.0435                    |
| 8                   | 8                         | 9                        | 0.0203                      | 0.01015                   |
| 9                   | 3                         | 10                       | 0.0132                      | 0.0066                    |
| 10                  | 3                         | 11                       | 0.0138                      | 0.0069                    |
| 11                  | 4                         | 12                       | 0.0352                      | 0.0176                    |
Table 5. Partial sampled load information.

| Load number | Active power (W) | Power factor | Initial phase | Load number | Active power (W) | Power factor | Initial phase |
|-------------|------------------|--------------|---------------|-------------|------------------|--------------|---------------|
| 51          | 2961.58          | 0.90         | B             | 71          | 1617.77          | 0.90         | A             |
| 52          | 2357.70          | 0.90         | C             | 72          | 1634.83          | 0.91         | B             |
| 53          | 2313.90          | 0.88         | C             | 73          | 2951.83          | 0.91         | C             |
| 54          | 1735.92          | 0.89         | A             | 74          | 1924.43          | 0.91         | A             |
| 55          | 2900.37          | 0.89         | A             | 75          | 3012.79          | 0.91         | B             |
| 121         | 1732.61          | 0.88         | B             | 61          | 1788.99          | 0.92         | B             |
| 122         | 1821.88          | 0.92         | A             | 62          | 1505.33          | 0.91         | A             |
| 123         | 2315.95          | 0.88         | C             | 63          | 1473.97          | 0.90         | A             |
| 124         | 2436.53          | 0.92         | C             | 64          | 1712.04          | 0.90         | B             |
| 125         | 1763.16          | 0.89         | A             | 65          | 1732.52          | 0.90         | C             |
| 101         | 3002.77          | 0.91         | B             | 91          | 3043.71          | 0.93         | B             |
| 102         | 1345.92          | 0.92         | A             | 92          | 1767.80          | 0.90         | A             |
| 103         | 2375.50          | 0.91         | C             | 93          | 2922.70          | 0.89         | A             |
| 104         | 2275.79          | 0.88         | A             | 94          | 3011.07          | 0.91         | B             |
| 105         | 2987.10          | 0.91         | C             | 95          | 2417.28          | 0.92         | C             |
| 111         | 2374.24          | 0.90         | B             | 81          | 1446.58          | 0.93         | C             |
| 112         | 1119.26          | 0.91         | B             | 82          | 2345.04          | 0.90         | A             |
| 113         | 2360.82          | 0.90         | A             | 83          | 2365.85          | 0.90         | C             |
| 114         | 1764.66          | 0.89         | C             | 84          | 1803.26          | 0.92         | C             |
| 115         | 2302.63          | 0.91         | A             | 85          | 2990.53          | 0.91         | B             |