Grand Yukawonification

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Abstract

Renormalizable SO(10) GUTs, extended by $O(N_g)_F$ family gauge symmetry, generate MSSM flavour structure dynamically via vacuum expectation values of “Yukawon” Higgs multiplets. SO(10) fermion Higgs channels (126, 10, 120) extend to $N_g(N_g \pm 1)/2$ dimensional representations of the family symmetry as do representations used only for GUT symmetry breaking. For concrete illustration and calculability we work with the fully realistic minimal supersymmetric GUTs (MSGUTs) based on the $210 \oplus 126 \oplus 126$ GUT Higgs system - which were already parameter counting minimal without “Yukawonification”. For $N_g = 3$ dynamical Yukawa generation reduces the matter fermion Yukawas from 15 to 3 (21 to 5) without (with) the 120 Higgs. Yukawon GUTs are thus ultra minimal in parameter counting terms. Consistent symmetry breaking is ensured by a hidden sector Bajc-Melfo superpotential with a pair of symmetric $O(N_g)$ multiplets $\phi_{ab}, S_{ab}$: of which the latter’s singlet part $S_s$ breaks supersymmetry and traceless part $\hat{S}_{ab}$ furnishes flat directions to cancel the $O(N_g)$ D-term contributions of the visible sector. $\hat{S}$ fields remain light and weakly coupled and furnish relics that may be viable light (< 50 GeV ) Cold Dark Matter as reported by DAMA/LIBRA. Hidden sector Yukawa and family gauge couplings and the large soft masses favoured by MSGUTs may also relieve the Polonyi and Moduli problems associated with $S_{ab}$.

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Renormalizable SO(10) GUTs already have a formidable array of virtues. These include 16-plet spinors to enclose entire Standard Model fermion families together with the right handed neutrinos required for seesaw neutrino masses. This provides natural quark lepton unification as well as completely realistic fermion mass relations based on just three allowed fermion mass generating Higgs irreps (in the renormalizable case): 10, 120, 126. Matter-parity((-3(B-L))) is a part (U(1)_{B-L} \subset SO(10)) GUT gauge symmetry and thus R-parity is conserved to low scales in the supersymmetric case[1]. Susy LSPs being stable are ideal WIMP Dark Matter candidates. Moreover D-flat directions involving doublet Higgs, lepton doublets and the conjugate neutrino superfields can support viable inflection point inflation scenarios at the cost of only hard parameter fine tuning[2]. SO(10) GUTs based on the 210 ⊕ 126 ⊕ 126 Higgs system [3–5] are parameter counting minimal relative to all fully realistic GUTs. The version (NMSGUT) with the 120-plet has been developed[7–9] to fit all fermion masses. It made distinctive MSSM spectra predictions(such as requiring large soft A-terms[8] in 2008 : well before the Higgs discovery promoted general acceptance of this possibility) and is thus falsifiable. It also provides a novel route to suppression of the troublesome d = 5 baryon violation violation operators of Susy GUTs[9] via the effects of threshold corrections (very significant even for small couplings due to the large number of superheavy fields) on the matching between MSSM and GUT fermion Yukawas. MSGUTs also point[10] to the unification of gauge and gravitational interactions since they provide strong indications of a physical cutoff associated with the MSGUT gauge Landau pole (due to the supersymmetry and high dimension of the 126, 126... Higgs irreps used) just above their perturbative unification scale and very close to the Planck scale.

In spite of this list of virtues Susy GUTs have so far shared the impotence of other gauge models as regards reduction in the number of matter fermion Yukawa couplings(15/21 couplings in the MSGUT/NMSGUT: or more than 50 % of the total), or the logic of their magnitudes and mixing. Faced with this plethora, model builders devise a variety of discrete family symmetries to explain features of the fermion hierarchy -specially in the case of neutrino masses[11].

Unfortunately, the combination of discrete symmetries with Grand Unification requires baroque proliferation of Higgs multiplets : usually putting paid to any chance of a calculable solution of what is generally euphemized as the “vacuum alignment problem”. In our development[3, 5–9] of minimal Susy GUTs we have consciously eschewed invocation of dis-
crete symmetries and insist upon following the logic of just SO(10) gauge symmetry. In this Letter we show how this insistence, combined with careful attention to the implications of the emergence of a light MSSM Higgs pair from the $2N_g(N_g + 1)$ (plus an additional $N_g(N_g - 1)$ pairs) in the $O(N_g)$ extended MSGUT(NMSGUT), leads to an effectively unique extension of the SO(10) gauge group by a $O(N_g)$ family gauge symmetry for the $N_g$ generation case.

The appealing speculation that the observed *dimensionless* fermion yukawa couplings actually arise via expectation values of composite or elementary fields has a long history[12]. In [13] these ‘spurion’ fields are appropriately called “Yukawa-on”s and carry representations of (continuous sub groups of) the automatic $U(N_g)^6$ family symmetry of matter fermion kinetic terms (in a theory with 6 SM flavours $(Q,L,u^c_L,d^c_L,e^c_L,\nu^c_L)$ and $N_g$ generations). They do *not* carry representations of the SM gauge group. In our work the fields that break family symmetry are called “Yukawons” and can also carry representations of the usual gauge (SM/GUT) dynamics. For continuous symmetries one must avoid problematic Goldstone bosons in the global, and gauge anomalies in the local, case. In most models the extra dimension 1 field introduced by the spurion/Yukawon $\mathcal{Y}$ into the Higgs interaction vertex requires the use of non-renormalizable terms ($\mathcal{L} = f^*\mathcal{Y}fH/\Lambda_\mathcal{Y} + ...$) suppressed by by a high scale $\Lambda_\mathcal{Y}$ associated with the presumed Yukawa-on dynamics. Estimates of $\Lambda_\mathcal{Y}$ can range from a few TeV to GUT scale values. However the only real hints for unification of flavour in the UV RG flow are found in the convergence, at high $\tan \beta$, of third generation Yukawas at GUT scales in supersymmetric theories. Although the spurion/Yukawa-on idea is attractive, implementation using a plethora of arbitrary seeming Yukawons associated with the observed fermion irrep fragments, and the arbitrary new non renormalizable dynamics, make it seem less so.

In bottom up scenarios, due to the large value of $\Lambda_\mathcal{Y}$, it seems inevitable that the family symmetry is not carried by fields responsible for the electroweak symmetry breaking and charged fermion mass generation since they cannot obtain vevs much greater than the Electroweak scale. Moreover the strong hints provided by gauge coupling unification and top-bottom-tau unification towards the link between Grand and Flavor unification are ignored in bottom-up models. The loophole provided by spontaneous neutrino Majorana mass generation via GUT scale vevs seems not to have been exploited to build a renormalizable Grand Unified Yukawon model to address the flavour problem. Thus family unification via Yukawa-ons and GUT unification have hitherto seemed orthogonal. In this Letter we outline
how Minimal SO(10) GUTs\cite{3,5,8} naturally indicate an \(O(N_g)\) family symmetry route to “Yukawaonification”: with the GUT symmetry breaking Higgs carrying family group representations and obviating the need for non-renormalizable interactions and postulation of an extraneous scale \(\Lambda_Y\) which is rather the GUT scale itself!

The Minimal SUSY SO(10) model\cite{3,4} consists of SM singlet containing Higgs irreps, \((210(\Phi) \oplus \overline{126}(\Sigma) \oplus 126(\Sigma))\), responsible for GUT symmetry breaking and right handed neutrino masses and \(10(H), 120(\Theta)\) Higgs. The \(10, 120\) have SM doublets but no SM singlets and thus cannot participate in GUT scale spontaneous symmetry breaking (SSB). Crucially the \(126(\Sigma)\) also contributes to charged lepton masses (and can implement the well known Georgi-Jarlskog mechanism for correcting second generation mass relations at the GUT scale). We found that gauging only an \(O(N_g)\) subgroup of the \(U(N_g)\) symmetry of the fermion kinetic terms is workable because the use of complex representations of a Unitary family group introduces anomalies and requires doubling of the Higgs structure to cancel anomalies and to permit holomorphic invariants to be formed for the superpotential. Worse, the Unitary symmetry enforces vanishing of half the the emergent matter Yukawa couplings. \(O(N_g)\) family symmetry suffers from none of these defects. Gauging it ensures that no Goldstone bosons arise when it is spontaneously broken. When completing the bibliography for this paper we found that \(O(3)\) with a symmetric tracefull 6-dimensional irrep has also been considered by the proponent of the (non-renormalizable) Yukawa-on (third reference in \cite{13}). However, as explained above, the two models are otherwise radically different in their construction and implications.

The GUT superpotential we take has exactly the same form as the MSGUT (See \cite{8,14–16} for comprehensive details of notation, symmetry breaking, and fermion mass generation in MSGUTs):

\[
\begin{align*}
W_{GUT} &= \text{Tr}(m\Phi^2 + \lambda\Phi^3 + M\Sigma\Sigma + \eta\Phi\Sigma\Sigma) \\
&\quad + \Phi.H.(\gamma\Sigma + \gamma\bar{\Sigma}) + M_H H.H \\
W_F &= \Psi_A.((hH) + (f\Sigma) + (g\Theta))_{AB}\Psi_B
\end{align*}
\]

We have omitted the terms in \(W_{GUT}\) containing the \(120\)-plet for simplicity. They do not affect our analysis of GUT SSB. The only innovation in the Higgs structure is that all the MSGUT Higgs fields now carry symmetric representation of the \(SO(N_g)\) family symmetry : \(\{\Phi, \Sigma, \Sigma, H\}_{AB}; A, B = 1, 2,..N_g\) (under which the matter \(16\)-plets \(\psi_A\) are vector \(N_g\)-plets).
In the NMSGUT generalization the additional $120$-plet transforms in the (antisymmetric) adjoint of $O(N_g)$. Couplings $h, f, g$ are single complex numbers while the Yukawons $(\overline{H}, \Sigma, \Theta)_{AB}$ carry symmetric $(H, \Sigma)$ and anti-symmetric $(\Theta)$ representations of $O(3)$: as required by the transposition property of the relevant $SO(10)$ invariants. Thus for $N_g = 3$ the number of real fermion mass parameters comes down from 15 ($Re[h_{AA}], f_{AB}$ (using the $U(3)$ rotations)) to just 3 ($Re[h], f$) in the MSGUT case and from 6 additional to just 2 for the $120$-plet.

An essential component of our proposal is that the single pair of light Higgs doublets $\overline{H}, H$ in the effective MSSM arise as combinations of the manifold pairs of MSSM doublets contained in the full set of fields. After imposing $\text{Det} \mathcal{H} = 0$ on the doublet mass matrix $\mathcal{H}$ one pair $\overline{H}[1, 2, -1] \oplus H[1, 2, 1]$ remains light. Its Yukawa couplings are determined by the “Higgs-fractions” specified by the left and right null eigenvectors of $\mathcal{H}$ [14, 15]. $\mathcal{H}$ depends on the family symmetry breaking SM singlet vevs $(p, a, \omega)_{AB} \in \Phi_{AB}$, $(\sigma, \bar{\sigma})_{AB} \in (\Sigma, \bar{\Sigma})_{AB}$. In this paper, for simplicity, and because the question of what modifications to the MSGUT are required to achieve realistic fermion masses is now reopened due to the proliferation of $126, \overline{126}$ vevs, we mostly ignore the $120$-plet, although it is known that this multiplet is required for fully realistic fermion masses when there is no family symmetry [7, 8]. Then the 4 dimensional $\mathcal{H}$ of the MSGUT becomes $2N_g(N_g + 1)$ dimensional. If the $120$-plet were included (NMSGUT) $\mathcal{H}$ would have dimension $N_g(3N_g + 1)$. Without $120$-plet $\Theta$ it has the form

$$
\mathcal{H} = \begin{pmatrix}
-M_H & \bar{\gamma}\sqrt{3}\Omega(w - a) & -\gamma\sqrt{3}\Omega(w + a) & -\bar{\gamma}\Omega(\bar{\sigma}) \\
\gamma\sqrt{3}\Omega(w - a) & -(2M + 4\eta\Omega(a - w)) & \varnothing_d & -2\eta\sqrt{3}\Omega(\bar{\sigma}) \\
-\bar{\gamma}\sqrt{3}\Omega(w + a) & \varnothing_d & -(2M + 4\eta\Omega(w + a)) & \varnothing_d \\
-\gamma\Omega(\sigma) & -2\eta\sqrt{3}\Omega(\sigma) & \varnothing_d & (-2m + 6\lambda\Omega(w - a)) \\
\end{pmatrix}
$$

(2)

The rows are labelled by the $N_g(N_g + 1)/2$-tuples (ordered and normalized, for a symmetric $\phi_{AB}, A, B = 1..N_g$, as $\{\phi_{11}, \phi_{22}, ... , \phi_{N_gN_g}, \sqrt{2}\phi_{12}, \sqrt{2}\phi_{13}, ..., \sqrt{2}\phi_{N_g-1N_g}\}$) containing MSSM type $\overline{H}[1, 2, -1]$ doublets from $10, 126, \overline{126}, 210$. For the columns it is for $H[1, 2, 1]$ doublets in the order $10, \overline{126}, 126, 210$. The matrix function $\Omega(\phi)$ has the
form determined by the symmetric invariant $Tr[<V> \cdot H.H]$. For $N_g = 2$ it is

$$\Omega[V] = \begin{pmatrix}
V_{11} & 0 & V_{12}/\sqrt{2} \\
0 & V_{22} & V_{12}/\sqrt{2} \\
V_{12}/\sqrt{2} & V_{12}/\sqrt{2} & (V_{11} + V_{22})/2
\end{pmatrix}$$

with labels $\{H_{11}, H_{22}, \sqrt{2}H_{12}\} \oplus \{H_{11}, H_{22}, \sqrt{2}H_{12}\}$. Imposing the zero determinant condition the fermion Yukawa couplings of the low energy theory are easily extracted by generalizing the MSGUT procedure [5, 8, 14, 15] (we show the $2 \times 2$ matrices for $N_g = 2$ the generalization to $N_g > 2$ is obvious). For the up and down type quarks we get

$$Y_u = \begin{pmatrix}
\hat{h} \hat{V}_1 + \hat{f} \hat{V}_4 & \hat{h} \hat{V}_3 + \hat{f} \hat{V}_6 \\
\hat{h} \hat{V}_3 + \hat{f} \hat{V}_6 & \hat{h} \hat{V}_2 + \hat{f} \hat{V}_5
\end{pmatrix}$$

$$Y_d = \begin{pmatrix}
\hat{h} \hat{W}_1 + \hat{f} \hat{W}_7 & \hat{h} \hat{W}_3 + \hat{f} \hat{W}_9 \\
\hat{h} \hat{W}_3 + \hat{f} \hat{W}_9 & \hat{h} \hat{W}_2 + \hat{f} \hat{W}_8
\end{pmatrix}$$

$$\hat{h} = 2\sqrt{2}h \quad ; \quad \hat{f} = -4i \sqrt{2 \over 3} f$$

The Yukawas $Y_\nu, Y_l$ for the neutrinos and charged leptons are obtained from $Y_u, Y_d$ by the replacement $\hat{f} \to -3\hat{f}$. $\hat{V}, \hat{W}$ are the normalized right and left null eigenvectors of the mass matrix $\mathcal{H}$ and contain what we have termed \cite{14, 15} the “Higgs fractions” specifying the amplitudes for the admixture of GUT Higgs doublets in the MSSM Higgs doublets.

To demonstrate the feasibility of our proposal it remains to show that SSB in such models leads to Yukawas required by the observed fermion masses: for values of the (now much reduced) GUT parameter set compatible with all other requirements such as gauge coupling unification, proton stability and constraints on exotic processes such as lepton flavour violation, cosmology etc. We have already achieved this to a considerable extent in the MSGUT (i.e. apart from the neutrino masses \cite{6, 7, 15}) and fully in the NMSGUT \cite{8, 9}. The full Yukawonification program is an order of magnitude more arduous than vanilla Minimal Supersymmetric Grand Unification. Thus in this Letter we must content ourselves with indicating the operative structure and starting the process of analysis.

Giving vevs \cite{3, 14, 15} to the SM singlets $p, a, \omega$ of the $(210, d_s[N_g])$-plet and the $\sigma \in (126, d_s[N_g]), \bar{\sigma} \in (\overline{126}, d_s[N_g])$, where $d_s[N_g] = N_g(N_g + 1)/2$, the superpotential relevant to
GUT SSB has just the MSGUT form, with sums over flavour group indices:

\[
W = \text{Tr}[m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2)] \\
+ \text{Tr}[M\bar{\sigma}\sigma + \eta(p + 3a - 6\omega)(\bar{\sigma}\sigma + \sigma\bar{\sigma})/2]
\] (5)

In the MSGUT case the entire SSB problem reduces [5] (i.e. for \(N_g = 1\) in present notation) to a cubic equation for a single complex variable \(x = -\omega\) (with just one free parameter \(\xi = M\lambda/m\eta\)). The Yukawa Ultra Minimal GUTs (YUMGUTs) proposed here generalize the remarkable analytic solution of the MSGUT [5] to the case where all Higgs fields are promoted to symmetric or antisymmetric representations family symmetry group. The F-term vanishing equations can be written as:

\[
2m(p - a) - 2\lambda a^2 + 2\lambda\omega^2 = 0 \\
2m(p + \omega) + \lambda(p + 2a + 3\omega)\omega \\
\quad \quad \quad + \lambda\omega(p + 2a + 3\omega) = 0 \\
M\sigma + \eta(\chi\sigma + \sigma\chi)/2 = 0 \\
M\bar{\sigma} + \eta(\chi\bar{\sigma} + \bar{\sigma}\chi)/2 = 0 \\
\bar{\sigma}\sigma + \sigma\bar{\sigma} = -4\eta(mp + 3\lambda\omega^2) \equiv F
\] (6)

where \(\chi \equiv (p + 3a - 6\omega)\). The homogenous equations (8,9) can be put in the more transparent form \(\Xi \cdot \hat{\Sigma} = \Xi \cdot \hat{\bar{\Sigma}} = 0\) where \(\hat{\Sigma}, \hat{\bar{\Sigma}}\) are \(N_g(N_g + 1)/2\) dimensional vectors containing the corresponding vevs: e.g. for \(N_g = 2\), \(\hat{\Sigma} = \{\sigma_{11}, \sigma_{22}, \sigma_{12}\}\); the matrix \(\Xi\) involves the combinations \(\chi_A = \chi_{AA} + \xi:\)

\[
\Xi = \begin{pmatrix}
\chi_1 & 0 & \chi_{12} \\
0 & \chi_2 & \chi_{12} \\
\chi_{12} & \chi_{12} & \chi_1 + \chi_2
\end{pmatrix}
\] (11)

Nontrivial solutions of the homogenous equations for \(\sigma, \bar{\sigma}\) exist only if \(\text{Rank}(\Xi) < N_g(N_g + 1)/2\) so that we must have \(\text{Det}(\Xi) = 0\); with additional conditions on minors for lower rank special cases. Thus one loses at least 2 out of the \(N_g(N_g + 1)\) F-equations for the variables \(\sigma, \bar{\sigma}\). In the MSGUT \((N_g = 1)\) these equations gave a linear condition \((\chi = -M/\eta)\) which supplemented the equations(6,7) and allowed the determination of \(p, a, \omega\) via the cubic
equation for $\omega$. For $N_g = 2$

$$Det[\Xi] = (\chi_1 + \chi_2)(\chi_{12}^2 - \chi_1\chi_2) = 0 \Rightarrow$$

$$\chi_1 = -\chi_2 \quad OR \quad \chi_{12} = \pm \sqrt{\chi_1\chi_2}$$

(12)

If $2 \times 2$ minors of $\Xi$ also vanish then additionally $(\chi_1 + \chi_2)\chi_1 = (\chi_1 + \chi_2)\chi_2 = \chi_{12}^2$ and $\chi_1\chi_2 = 0$. Then $\chi_1 = \chi_2 = \chi_{12} = 0$ so that $\Xi \equiv 0$. Thus $\text{Rank}[\Xi] < 2$ implies $\text{Rank}[\Xi] = 0$ so that none of the 6 $\sigma, \bar{\sigma}$ are determined. Instead we get 3 linear conditions on the 9 $(210, 3)$ vevs to supplement the 6 equations (6,7). The 3 equations (10) determine 3 combinations of $\sigma, \bar{\sigma}$. There remain only the D term conditions from the family gauge group $O(2)$ and $D_{B-L} = 0$ (the only non trivial condition from the vanishing of SO(10) D terms). In the MSGUT $D_{B-L} = 0$ requires $|\sigma| = |\bar{\sigma}|$ and since the difference of the $\sigma, \bar{\sigma}$ phases can be gauged away by $U(1)_{B-L}$ the D-terms supplement F-Terms to a nicety. We shall retain this convenience by only considering solutions with $\sigma_{AB} = \bar{\sigma}_{AB}$.

At this point an obstacle was found. When one tries to make the $O(N_g)$ D-terms vanish using the solutions of the F term vanishing conditions in the GUT sector one learns that it is a contradictory demand with only trivial solutions, since the contributions from the different $O(2)$ charge sectors have no reason to cancel and in fact never do. Faced with this no-go it is natural to look for additional fields with a dynamics such that the vevs of these fields are free to cancel the contribution $D_X$ of the GUT sector to the family gauge symmetry D terms. This requires that the F term vanishing conditions for the extra fields be sequestered from the GUT sector and their solution include a set of $O(N_g)$ fields whose vev is undetermined and free to take the GUT scale vevs required to cancel the contribution $D_X$. The requirement of sequestration naturally brings to mind the hidden sector responsible for symmetry breaking. It is known [18, 19] that there exists a class of two field superpotentials (with structure $W = S(\mu_B\phi + \lambda_B\phi^2)$) which yield potentials whose local minima break supersymmetry ($< F_S > \neq 0$) and leave the corresponding chiral scalar vev $< S >$ undetermined at tree level while a partner field $\phi$ gets a vev. Radiative corrections that determine $< S >$ then provide an alternative realization [19] of the Witten [20] model for high scale symmetry breaking triggered by a low scale symmetry breaking. On the other hand the determination of the flat direction vev by $N=1$ Supergravity corrections instead of radiative effects was already shown to be effective long ago [21, 22].

In [23] we show that coupling Bajc-Melfo fields to supergravity can resolve the flat di-
rection associated with supersymmetry breaking and fix the vevs $< S_\alpha >$. Even more to our purpose, if the fields $S, \phi$ are also promoted to (tracefull) symmetric multiplets $(S, \phi)_{ab}$ of the gauge family symmetry, then out of the additional fields (i.e the traceless symmetric multiplets $(\hat{S}, \hat{\phi})_{ab}$) the former, i.e $\hat{S}_{ab}$, are also undetermined at tree level and free to take up the slack in the $O(N_g)$ D-terms to cancel $\bar{D}_X^a$ and yield a gravity mediated scenario in which the hidden sector breaking involves breaking of family symmetry and supersymmetry. The additional fields include moduli like fields $(\hat{S}_{ab})$ which can be light enough to serve as (light Dark Matter) signals of this hidden connection between supersymmetry and family symmetry breaking. The constraints\cite{25} that make Polonyi breaking of Supergravity and string theory moduli problematic can possibly be evaded because of the rich Yukawa and gauge dynamics in the hidden sector. Via the moduli-like (but family gauge group and superpotential coupled) $\hat{S}$ modes YUMGUTs may even provide light dark matter in the sub 50 GeV mass range favoured\cite{24} by experiments such as DAMA/LIBRA. In any event for our present purposes we can use the results of \cite{23} directly and simply quote the values of the solutions found in the toy $N_g = 2$ case. This is enough to demonstrate that our mechanism for generation of flavour structure is in principle feasible. A common apprehension is that pseudo-Goldstone multiplets may arise due to the duplication of Higgs multiplets under family symmetry. However we have checked the complete spectra in the GUT sector and only present those solutions without pseudo-goldstone multiplets that ruin unification (in certain degenerate cases). This is sufficient to show that the appearance of pseudo-Goldstones is not inevitable and constitutes an existence proof for the structural possibilities we are relying on to generate flavour at the GUT scale from a family symmetric theory i.e a flavour bland theory. The determination of a set of GUT parameter values optimized to generate the observed set of Yukawa couplings, symmetry breaking, neutrino masses, B-decay and other exotic phenomenon rates requires a much more elaborate investigation based upon the generalization of the computer codes that have allowed us to derive completely realistic fits of all fermion data and distinctive predictions for Susy spectra in the context of the NMSGUT\cite{8}. A preliminary step will be to first try a toy model $N_g = 2$ aiming to fit the “real core”\cite{26} sector of the MSSM fermion hierarchy. This will be reported in sequels.
A toy solution with flavour generation

For $N > 1$ one must consider the cases that arise separately. In the $\text{Rank}[\Xi] = 0$ case, one ultimately finds that the GUT spectra in sectors that include $(SO(10)/G_{123})$ coset gauginos always include large colour and electroweak non-singlet pseudo-Goldstone multiplets that would ruin unification. Therefore we focus on the non degenerate cases where $\text{Det}[\Xi] = 0$ but $\text{Rank}[\Xi] = 2$. To show that our scenario is at least prima facie consistent, in Appendix A, we give solutions of the equations (10) with zero $D_{B-L}$ for $N_g = 2$ and the corresponding value of $\bar{D}_X$ for an set of arbitrary YUMGUT flavour bland parameters similar to those shown to be applicable in realistic fits in the NMSGUT [9]. Using these solutions, by diagonalizing the expressions (4) for matter yukawa couplings of the light MSSM Higgs pair we calculate the following values of the Yukawa coupling eigenvalues and mixing angles in the quark and lepton sectors:

$$Y_u^{\text{Diag}} = \{0.076, 0.02\} ; \quad Y_d^{\text{Diag}} = \{0.04, 0.008\}$$

$$Y_{\nu}^{\text{Diag}} = \{0.11, 0.024\} ; \quad Y_l^{\text{Diag}} = \{0.016, 0.005\}$$

$$\theta_{\text{CKM}} = 6.73^\circ ; \quad \theta_{\text{PMNS}} = 39.94^\circ$$

Thus without any attempt at optimizing these parameters we find quark and lepton mixing angles in tens of degrees and a hierarchy between generations by factors of ten. Clearly they are sufficiently close to the values in the 23 sector to inspire confidence that when a search for flavour bland GUT parameters is mounted one may be able to do much better. In Appendix B we exhibit the complete spectrum of the O(2) family symmetrized MSGUT for this solution. The crucial observation is that in spite of the family symmetry, apart from the MSSM neutral DM candidate moduli modes from $S_\pm$, there are no pseudo- Goldstones in the calculated spectrum whether MSSM charged or neutral. Thus at least the generic appearance of the effective theory is an MSSM arising from a flavour bland theory with flavour generated by family symmetry breaking. In view of the understanding conferred we call such flavour “tasty flavour” and the these models Yukawon Ultra Minimal GUTs(YUMGUTs). This mechanism, like the one found by us for suppressing $d = 5$ B violation in the NMSGUT, is crucially dependent on enforcement of the consistency conditions on the relation between the phenomenologically preferred single light pair of MSSM doublets and the large number of superheavy doublets with the same quantum numbers present in the full theory. It should
be clear that the realistic O(3) case, while considerably more complex algebraically follows the same generic logic. Therefore we expect that there may exist regions of the reduced parameter space \{λ, m, M, M_H, η, γ, g, h, f\} ⊕ \{μ_B, λ_B, W_0\} which generate fully realistic fermion Yukawas. Note that this is more than the MSGUT could do[7]. In sequels we will scan the flavour bland parameter space for sets of parameters that satisfy all phenomenological constraints. We emphasize that a great deal of algebraic and numerical investigation is required to settle such questions even in the MSGUT with its 472 Higgs fields. Here this set expands to 1416 (2832) fields in the \(N_g = 2(N_g = 3)\) case and even further for the NMSGUT generalization. Our quest for unification is based on the conviction that, as once promised by string theory, it is the number of couplings, not the number of fields, that should fall as unificatory ambitions mount.

To summarize, we have shown that Susy SO(10) theories that utilize a \(\mathbf{126}, \mathbf{126}\) pair to generate neutrino masses generically offer a radical conceptual simplification that melds the problems of the fermion flavour hierarchy and GUT symmetry breaking in a novel and appealing way. The same is likely to be true also for suitable non-supersymmetric SO(10) models. Our program for SO(10) ”Yukawonification” is eminently concrete, calculable (at least for (N)MSGUTs where much of the tedious but directly useable (see e.g eqn(2)) preparatory work on group theory[14] and mass spectra [8, 14–17] is already done) and falsifiable. Since \(O(N_g)\) contains most of the commonly fancied discrete groups, even discrete group model builders may benefit from the structural hints provided by SO(10) along with a welcome calculability as regards the “vacuum alignment” : specified here by parameters that determine, and are thus constrained by, the MSSM mass and mixing data and all other required phenomenological constraints.

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Appendix A

We present here a solution of the spontaneous symmetry breaking in the toy \(N_g = 2\) model case associated with values of matter fermion Yukawas given in eqn[13]. The
YUMGUT couplings used are

\[
\xi = 0.8719 + 0.5474i \quad ; \quad \eta = 0.4
\]

\[
\lambda = -0.038 + 0.005i \quad ; \quad \gamma = 0.32 \quad \bar{\gamma} = -1.6
\]

\[
h = 0.34 \quad ; \quad f = -0.13 \quad (14)
\]

The values of the vevs of the YUMGUT Higgs fields responsible for breaking \(SO(10) \rightarrow \) MSSM in units of \(m/\lambda \sim 10^{16} \) GeV are

\[
W = \begin{pmatrix}
0.14098 + i(-0.20299) & 0.3167 + i(0.18895) \\
0.3167 + i(0.18895) & -0.26669 + i(0.307487)
\end{pmatrix}
\]

\[
P = \begin{pmatrix}
-0.2360 - 0.20008i & 0.178688 - 0.028036i \\
0.178688 - 0.028036i & -0.22972 + 0.12022i
\end{pmatrix}
\]

\[
A = \begin{pmatrix}
-0.22996 - 0.352079i & 0.3381836 + 0.07766i \\
0.3381836 + 0.07766i & -0.447488 + 0.1202249i
\end{pmatrix}
\]

\[
\Sigma = \Sigma = \begin{pmatrix}
0.086301 - 0.236617i & 0.1972527 + 0.104141i \\
0.1972527 + 0.104141i & -0.08630175 + 0.236617i
\end{pmatrix}
\]

\[
\bar{D}_X = 2(|p_+|^2 - |p_-|^2) + 3(|a_+|^2 - |a_-|^2) + 6(|w_+|^2 - |w_-|^2) = -8.494 \quad (15)
\]

**Appendix B**

In this Appendix we give the complete superheavy spectra in units of the MSGUT scale parameter \(m/\lambda\). This is done merely to remove any prejudice that pseudo-Goldstone multiplets must occur in family symmetric models. The naming convention and MSGUT multiplicities can be found in [15].
| Field | Masses | Field | Masses |
|-------|--------|-------|--------|
| $A[1,1,4]$ | 4.093, 3.321, 0.137 | $B[6,2,5/3]$ | 0.106, 0.099, 0.091 |
| $C[8,2,1]$ | 1.727, 1.727, 1.224 | $C'[8,2,1]$ | 1.224, 0.614, 0.614 |
| $D[3,2,7/3]$ | 1.919, 1.433, 1.191 | $D[3,2,7/3]$ | 0.810, 0.205, 0.134 |
| $E[3,2,1/3]$ | 1.475, 1.043, 0.716 | $E[3,2,1/3]$ | 0.716, 0.677, 0.594, 0.506 |
| $E[3,2,1/3]$ | 0.404, 0.277, 0.087 | $E[3,2,1/3]$ | 0.073, 0.050, 0.004 |
| $F[1,1,2]$ | 1.794, 1.794, 1.681 | $F[1,1,2]$ | 1.317, 0.289, 0.228, 0.018 |
| $G[1,1,0]$ | 1.684, 1.676, 1.269 | $G[1,1,0]$ | 1.253, 0.475, 0.441, 0.329 |
| $G[1,1,0]$ | 0.264, 0.260, 0.254 | $G[1,1,0]$ | 0.010, 0.070, 0.054 |
| $G[1,1,0]$ | 0.016, 0.007, 0.002 | $h[1,2,1]$ | 4.161, 3.196, 2.049 |
| $h[1,2,1]$ | 1.289, 0.979, 0.710 | $h[1,2,1]$ | 0.540, 0.520, 0.152, 0.029, 0.010 |
| $I[3,1,10/3]$ | 0.210, 0.192, 0.003 | $J[3,1,4/3]$ | 1.889, 1.889, 0.946 |
| $J[3,1,4/3]$ | 0.740, 0.453, 0.278 | $J[3,1,4/3]$ | 0.119, 0.086, 0.021, 0.006 |
| $K[3,1,−8/3]$ | 1.591, 1.237, 0.116 | $L[6,1,2/3]$ | 1.066, 0.916, 0.757 |
| $M[6,1,8/3]$ | 1.340, 0.958, 0.493 | $N[6,1,−4/3]$ | 1.795, 1.178, 0.345 |
| $O[1,3,−2]$ | 1.127, 0.886, 0.084 | $P[3,3,−2/3]$ | 0.902, 0.754, 0.595 |
| $Q[8,3,0]$ | 0.163, 0.126, 0.083 | $R[8,1,0]$ | 0.170, 0.119, 0.107 |
| $R[8,1,0]$ | 0.086, 0.066, 0.047 | $S[1,3,0]$ | 0.090, 0.058, 0.011 |
| $t[3,1,−2/3]$ | 3.650, 3.156, 2.097 | $t[3,1,−2/3]$ | 1.747, 1.273, 1.116 |
| $t[3,1,−2/3]$ | 0.926, 0.824, 0.779 | $t[3,1,−2/3]$ | 0.541, 0.466, 0.223 |
| $t[3,1,−2/3]$ | 0.116, 0.023, 0.002 | $U[3,3,4/3]$ | 0.084, 0.070, 0.054 |
| $V[1,2,−3]$ | 0.227, 0.208, 0.003 | $W[6,3,2/3]$ | 1.693, 1.324, 0.902 |
| $X[3,2,−5/3]$ | 1.666, 1.666, 0.149 | $X[3,2,−5/3]$ | 0.102, 0.072, 0.070, 0.066 |
| $Y[6,2,−1/3]$ | 0.167, 0.118, 0.058 | $Z[8,1,2]$ | 0.100, 0.086, 0.070 |

**TABLE I:** Mass Spectrum of superheavy fields in units of $m/\lambda \sim 10^{16}$ GeV.
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