Measurement-based ground state cooling of a trapped ion oscillator

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Measurement-based cooling is a method by which a quantum system, initially in a thermal state, can be prepared probabilistically in its ground state through some sort of measurement. This is done by making a measurement that heralds the system being in the desired state. Here we demonstrate the application of a measurement-based cooling technique to a trapped atomic ion. The ion is pre-cooled by Doppler laser cooling to a thermal state with a mean excitation of \( \bar{n} \approx 18 \) and the measurement-based cooling technique selects those occasions when the ion happens to be in the motional ground state. The fidelity of the heralding process is greater than 95%. This technique could be applied to other systems that are not as amenable to laser cooling as trapped ions.

I. INTRODUCTION

Preparing a quantum system in a pure state allows the system’s quantum mechanical behaviour to be observed and exploited. Doppler laser cooling, as first demonstrated in trapped ions, makes use of the repeated absorption and emission of photons, and the associated transfer of momentum, to cool quantum systems far below their initial thermal state [11]. In order to reach the ground state, however, it is often necessary to use a sideband cooling technique after initial Doppler cooling [2, 3]. Laser cooling techniques have now been applied to a wide variety of systems, including atoms and ions [4-6], complex molecules [7-10] and mechanical oscillators [11, 12]. While laser cooling can be very successful for some systems, there exist many systems of interest for which direct application of laser cooling is either impractical or impossible, particularly if the intention is to cool the motion to the ground state. Doppler laser cooling requires a strong transition to absorb photons from the laser beam and return the system to the ground state by re-emission. Sideband cooling additionally requires a weak transition to a metastable state [2]. One or more of these may not exist or there may be some other impediment to effective cooling: for instance in a trapped molecule the additional complexity of the level structure can increase the number of photon scattering events required to reprepare the molecule by optical pumping during sideband cooling, an effect exacerbated if the trapping potential is state dependent, as each scattering event then has a higher probability of heating the molecule [13].

In a micromechanical oscillator, there may be no suitable energy level structure available for laser cooling, though direct measurement of the position and momentum quadratures using laser pulses can result in an effective temperature much less than that of the environment [14]. In a very different system also not amenable to laser cooling, the BASE collaboration aims to detect proton spin-flips in a Penning trap. In order to increase the fidelity of this detection, they first check that the proton’s cyclotron energy is sufficiently low, and if it is not, they allow it to rethermalise and then repeat the measurement [15]. In this way, spin-flip detection is only carried out when the proton has a low cyclotron energy.

In other cases an alternative method of cooling is necessary, especially if the requirement is to cool the system to its quantum mechanical ground state. Measurement-based cooling is one such method. In its simplest form, this relies on performing a measurement on a system whose result depends on its motional state. If the system starts in a thermal distribution of motional states there is a non-zero probability that it is in the ground state and so by performing a state-dependent measurement, we can select on the result of the measurement to obtain the subset of the thermal distribution which meets the requirement. After each measurement the system is allowed to rethermalise. One specific measurement outcome heralds the system being in the ground state and all other data are discarded [16-18]. Thus, measurement-based cooling enables us to probe quantum behaviour tailored to the motional ground state without the need for cooling all the population to that state. It is simpler to implement than optical sideband cooling, for example, but it has the disadvantage that because it is a probabilistic technique, not all attempts are successful. It can therefore become inefficient when the mean excitation of the motional state is large. Such probabilistic state preparation methods have already been used to prepare atomic and molecular systems into a pure internal state, for instance to initialise molecular ion systems into specific rovibrational states [19, 20]. The work presented here is an extension of these techniques to the external motional states.

One proposed cooling method considers an oscillator coupled to a two level system via a Jaynes-Cummings type Hamiltonian [16]. By using a numerically optimised time-varying coupling, Puebla et al. [16] show it is possible to perform a conditional logic operation on the two-level qubit-like system, such that the state of the qubit is unchanged if the ion is in the motional ground state, and it is flipped if in any other motional state. Here we propose a simplification of this scheme which makes use of rapid adiabatic passage to perform the logic operation, and demonstrate the use of this scheme to probabilistically prepare a single trapped ion in the ground state of one of its harmonic oscillator modes with high fidelity.
II. METHOD

Measurement-based Cooling

We consider a composite quantum system consisting of a qubit with states $|g\rangle$ and $|e\rangle$ and a harmonic oscillator mode with states $|n\rangle$, $n \geq 0$. The qubit state must be measurable with high fidelity, and for at least one of the measurement outcomes no change of the harmonic oscillator state should result as a byproduct of the measurement process. We assume that the two systems can be coupled via a controllable Jaynes-Cummings Hamiltonian:

$$H(t) = \frac{\hbar \Omega(t)}{2} \left( a \sigma_+ e^{i \delta(t) t} + a^\dagger \sigma_- e^{-i \delta(t) t} \right), \quad (1)$$

where $\Omega(t)$ and $\delta(t)$ are a suitably defined time-varying Rabi frequency and detuning.

To perform the filtering operation, a conditional logic operation must take place, with the qubit selectively excited from its initial state $|g\rangle$ to $|e\rangle$ if and only if the motion is not in the state $|0\rangle$:

$$|g\rangle |0\rangle \rightarrow |g\rangle |0\rangle , \quad (2)$$

$$|g\rangle |n \neq 0\rangle \rightarrow |e\rangle |n - 1\rangle . \quad (3)$$

The qubit state is then measured, and a measurement outcome of $|g\rangle$ heralds the oscillator being in the desired ground state. If the oscillator is initially in a thermal state characterised by a mean excitation value $\bar{n}$, the thermal distribution of the motional states is given as

$$\rho_{th} = \sum_{n=0}^{\infty} p_n |n\rangle \langle n| , \quad p_n = \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}} . \quad (4)$$

This indicates that the probability the ion is in the motional ground state is $p_0 = 1/(\bar{n} + 1)$. In the case of a qubit measurement of $|e\rangle$, the oscillator can be re-thermalised and the process repeated until the desired heralding measurement result is obtained.

This conditional operation, however, cannot be realised by a simple resonant pulse as the coupling strength of the transitions driven by the Hamiltonian of Eq. 1 varies with the motional state $n$. In Puebla et al. [16], they considered the case where only the detuning $\delta(t)$ varies, while the coupling strength has a fixed value. They showed that by using a numerical optimisation technique, a suitable form of $\delta(t)$ can be constructed to perform the required conditional operation. A simpler method to perform the operation is to use a rapid adiabatic passage (RAP), since it is able to invert population between two internal states in a way that is insensitive to the coupling strength used [21]. Rapid adiabatic passage can be performed by varying the control parameters in relatively simple ways, and is robust against errors in the form of the control parameters. This robustness may also allow RAP based techniques to be used even in the case where the oscillator is anharmonic.

The system is first prepared in the qubit ground state $|g\rangle$ with the motional states following a thermal distribution, as illustrated in Fig 1(a). Application of a RAP on the Jaynes-Cummings Hamiltonian Eq. 1 brings all the population into the excited qubit state $|e\rangle$, except when the oscillator has motional quantum number $n = 0$. This step is illustrated in Fig. 1(b) and is described by Eq. 3 above.

This process leaves the motional ground state untouched while other motional states are mapped onto a different internal state, so that qubit measurement then provides the required heralding signal to indicate a ground state cooled oscillator (Fig 1(c)).

An imperfect RAP process degrades the cooling fidelity, that is to say the conditional probability of the motion being in the ground state given a measurement of the qubit as being in $|g\rangle$, since it can leave a motionaly excited ion in the state $|g\rangle$. Using a simplified model where we assume that the probability of a successful RAP is independent of motional state $n$, and characterised by a transfer failure probability $\epsilon$, then the conditional success probability is given by

$$p(n = 0 | g) = \frac{1}{1 + \epsilon \bar{n}} . \quad (5)$$

If for a given $\epsilon$ and $\bar{n}$ this probability is lower than required then the RAP-measurement cycle can be repeated multiple times, with the operation considered successful if and only if each measurement yields a result of $|g\rangle$. For $m$ cycles,
\[
p(n = 0 | m \times g) = \frac{1}{1 + e^{m\bar{n}}}. \tag{6}
\]

Each cycle reduces the probability that a measurement result of \(|g\rangle\) is a false positive result.

### III. IMPLEMENTATION

#### Ion trap

We demonstrate the method described above using a single trapped ion. Trapped ions are an ideal system in which to first demonstrate techniques such as this due to the high degree of control possible in the system, and the degree to which they can be isolated from the environment. We will describe the ion trap system before discussing the modifications to the generic measurement-based cooling method described above that are required for the demonstration.

The experiment described in this study has been carried out in a macroscopic linear RF trap [22] (see Fig. 2 for details of the electrode structure). The trap employs a combination of static and time-varying electric fields to achieve three-dimensional confinement of \(^{40}\text{Ca}^+\) ions. The trap parameters are set such that the radial motion of the ion in a linear trap is stabilized using the Pound–Drever–Hall (PDH) technique to achieve a laser linewidth narrower than 0.40 Hz, which is a critical requirement for laser-induced coherent operations.

As shown in Fig. 2, the main spectroscopic laser at 729 nm is directed along the trap axis. This configuration maximises the Lamb–Dicke (LD) parameter \(\eta\) associated with the axial motion and nulls \(\eta\) for the radial motions. The other lasers are directed along the quantisation axis, which is defined by a weak external magnetic field of \(\approx 0.32\) mT produced by a coil placed outside the vacuum chamber. A linearly polarised 397 nm laser addresses the dipole transition \(S_{1/2} \leftrightarrow P_{1/2}\) and is used for Doppler cooling. The decay on this transition is used to discriminate between the two qubit states – if the ion is in the internal ground state \(|g\rangle\), the Doppler beam repeatedly excites the ion to the short-lived \(P_{1/2}\) state from where it decays back to the \(S_{1/2}\) state, emitting a photon; otherwise the ion is decoupled from the laser, and no photons are emitted. Emitted photons are detected by a photomultiplier tube (PMT), and the resulting photon distribution is Poissonian with different mean photon numbers depending on the projected qubit state.

State-dependent fluorescence therefore allows for discrimination between \(S_{1/2}\) \((|g\rangle\) and \(D_{5/2}\) \((|e\rangle\)). This measurement takes about 1.5 ms and has a fidelity of 99.5%. This measurement process only leaves the motional state unaffected if the system is in the excited qubit state \(|e\rangle\). (Note that this requires a small change to the scheme, which is illustrated in Fig. 2 and discussed below.)

The undesired metastable state \(D_{3/2}\) can be occupied via spontaneous decay from \(P_{1/2}\) and so this population is pumped back to \(S_{1/2}\) via the dipole transition at 866 nm during laser cooling. Any population left in \(D_{5/2}\) from coherent operations at 729 nm is pumped back to \(S_{1/2}\) via the dipole transition at 854 nm during the state preparation.

#### Applying the method to a trapped ion

The interaction between the trapped ion and the radiation is
Rapid Adiabatic Passage

Rapid adiabatic passage has been applied to many different two level quantum systems, including trapped ions [26,28]. While an ion coupled to a motional mode is not a two-level system, we operate in a parameter regime where only a single sideband is addressed and multi-level effects are negligible.

The RAP implementation used involves applying a frequency chirp to the driving laser field while the power of the laser is smoothly ramped up and down to avoid diabatic processes which can occur if the laser is abruptly switched on and off. There are many possible ways to perform these modulations of the control parameters; in our experiment we use a simple linear frequency sweep, while the amplitude modulation follows a squared-sine envelope:

\[
\delta(t) = \frac{\delta_0}{T} \left( t - \frac{T}{2} \right),
\]

\[
\Omega(t) = \Omega_{\text{peak}} \sin^2 \left( \frac{\pi t}{T} \right), \quad 0 \leq t \leq T
\]

where \( T \) is the total duration of the RAP pulse, \( \delta_0 \) is the range of the detuning chirp, and \( \Omega_{\text{peak}} \) the peak Rabi frequency.

RAP can give almost unity population transfer efficiency even when the ion is only Doppler cooled, where many different motional states are statistically occupied; this means that population transfer between two internal states can be carried out for different \( n \) using the same laser parameters, which is crucial for the measurement-based cooling method presented here.

IV. RESULTS

The performance of measurement-based cooling is heavily dependent on the fidelity of population transfer by RAP operations. In Fig. 5, the transfer efficiencies of RAP on carrier and blue-sideband transitions are presented as a function of the total duration of the driving pulses for a fixed frequency chirp and peak Rabi frequency. The simulation shows that the transfer efficiency converges to unity as the pulse length \( T \) becomes longer because the dynamics of the process become more adiabatic. Although experimental data also indicates that the transfer efficiency approaches a saturation value close to unity, the transfer efficiency still fluctuates and saturates at a value of around 95% even when the pulse time is much longer than the pulse time for which it should be very close to unity according to the simulation curve. Thus, we adopt the shortest pulse time that lies on the flat region of the simulation and experimental curves. In modelling the measurement-based cooling scheme we assume a transfer efficiency of 95%. For the carrier transition \( T = 35 \, \mu s, \Omega_{\text{peak}}/2\pi = 83 \, kHz \) and \( \delta_0/2\pi = 200 \, kHz \) while for the sideband transition \( T = 250 \, \mu s, \Omega_{\text{peak}}/2\pi = 5.8 \, kHz \) and \( \delta_0/2\pi = 40 \, kHz \). In order to set the frequency sweep range properly, we perform numerical simulations beforehand and ensure the sweep range is just big enough to avoid breakdown of the adiabaticity condition. Note that the transfer efficiency
12% (averaged over the complete dataset). A heralded success is found with a probability of 0.5. This process is repeated 900 times for each probe pulse. Rabi oscillation on either the carrier or red sideband is measured. Following this, the system is in the motional ground state. A heralding state measurement is performed, showing the Rabi oscillations on these two transitions for the ion. The blue curves show the Rabi oscillations obtained after performing one cycle of measurement-based cooling.

The success probability for this measurement-based cooling scheme can be increased by performing additional cycles of sideband RAP-measurement immediately after the first cycle, and conditioning on all measurements finding the ion in $|e\rangle$.
The probability of being in the motional state after $m$ cycles is now

$$p(n = 0|m \times e) = \frac{1}{1 + 2e^{m\bar{n}}}.$$  \hspace{1cm} (14)

The orange curves in Fig. 6 show the oscillations for double measurement ($m=2$) heralding, where 5% of the complete data set is accepted in the post-processing. Here, the data is consistent with the ion being in the ground state with high probability after this double measurement heralding. The oscillations on both the carrier and the red sideband have high visibility and slow decay, as expected. The fit to the sideband Rabi oscillation indicates that $p_0$ is increased to $\approx 0.96$.

V. DISCUSSION

The measurement-based cooling scheme presented here consists of two steps – one is to map the motional ground state and any excited motional states onto two different internal states using a sequence of RAP operations, and the other is to make a projective measurement and discard instances where the result of the measurement is the ground state $|g\rangle$. The method’s cooling efficiency therefore relies significantly on the transfer efficiency of RAP and state detection fidelity.

As presented in Sec. IV the ground state population $p_0$ after a single cycle of RAP and the state detection is still low ($\approx 0.49$). This low $p_0$ is attributed to imperfect RAP operations. Using our assumed transfer failure probability of $e = 0.05$, Eq. (11) estimates $p_0 \approx 0.56$, which is somewhat higher than the observed value. The additional cooling sequence, which consists of another RAP on the blue sideband and subsequent projective measurement, raises $p_0$, as expected from Eq. (6). However, this new value of $p_0 \approx 0.96$ is also lower than the value expected from Eq. (14), which is $\approx 0.98$. Furthermore, we do not observe any further improvements on $p_0$ when we apply any more cooling sequences.

These observations lead us to consider other sources of error. The projective measurement takes $1.5$ ms to ensure a detection fidelity greater than 99.5%. The detection fidelity is high enough to avoid false positive detection of the motional ground state, but the time taken for the detection is long enough that it is necessary to consider heating during the detection time. The heating rate of our trap was measured to be approximately 37 phonons/s. This rate implies a 5% chance that an ion in the motional ground state after the last sideband RAP pulse will no longer be in the motional ground state by the conclusion of the measurement, which leads to the increase in $p_0$. Moreover, the effect of the heating cannot be alleviated by applying more RAP operations since it occurs during the measurement, which is performed after the last RAP operation. So after our measurement-based cooling sequence, $p_0$ saturates to a value below unity no matter how many RAP operations are involved in the cooling process, and the measured heating rate is sufficient to account for the population outside the motional ground estimated from the fitting curve shown in Fig. 6.

VI. CONCLUSION

We have experimentally applied a measurement-based cooling scheme to a single trapped ion. The ion is initially prepared in a thermal state with $\bar{n} \approx 18$ using Doppler cooling, which leaves most of the population outside the motional ground state. We employ a rapid adiabatic passage to separately map the populations of the motional ground state and the rest of the motional states onto the ion’s internal excited state $|e\rangle$ and ground state $|g\rangle$ respectively. A measurement on the internal degree of freedom is then used to herald the
motional ground state. Imperfections in the rapid adiabatic passage limit the population in the motional ground state for these heralded instances, however by repeating the mapping-measurement procedure, the probability to find the system in the motional ground state is increased. In this study, the cooling is performed on a single motional mode. In principle the scheme could be used to prepare a system in the ground state of multiple modes simultaneously by applying it sequentially to each mode. In practice however the joint probability of finding all the modes initially in the ground state, for most realistic scenarios, would rapidly become extremely small as more modes were added.

The technique has been demonstrated here in a trapped ion system in which other ground-state cooling methods could be applied, however, the scheme should have wider applicability. Ground-state cooling is very difficult for trapped molecules, especially where optical trapping leads to different potentials for different molecular states, but it has been proposed to use a technique similar to that discussed here for single molecules [13]. For micro-mechanical resonators, existing cooling methods are either hard or impossible to implement. While this technique requires a Jaynes-Cummings type Hamiltonian to exist in the system, suitable couplings have already been demonstrated in micro-mechanical oscillator systems, for instance by embedding a nitrogen-vacancy center into a resonator and placing the combined system in a strong magnetic-field gradient to provide spin-motion coupling [31]. Being able to prepare a ground-state sample in these systems will allow quantum effects, which would otherwise be washed out by being in a thermal state, to be observed.

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