Lepton flavor violating $Z$-decays in supersymmetric seesaw model

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In supersymmetric seesaw model, the large flavor mixings of sleptons induce the lepton flavor violating (LFV) interactions $\ell_i \ell_j V (V = \gamma, Z)$, which give rise to various LFV processes. In this work we examine the LFV decays $Z \to \ell_i \ell_j$. Subject to the constraints from the existing neutrino oscillation data and the experimental bounds on the decays $\ell_j \to \ell_i \gamma$, these LFV $Z$-decays are found to be sizable, among which the largest-rate channel $Z \to \tau \mu$ can occur with a branching ratio of $10^{-8}$ and may be accessible at the LHC or GigaZ experiment.

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I. INTRODUCTION

It is well known that the Standard Model (SM) predicts an unobservably small branching ratio for any lepton flavor violating (LFV) process, such as $\ell_j \to \ell_i \gamma$ or $Z \to \ell_i \ell_j$. In some extensions of the SM the LFV processes may be significantly enhanced [1–3]. One example of these extensions is the popular weak-scale supersymmetry (SUSY). In SUSY models the LFV interactions $\ell_i \ell_j V (V = \gamma, Z)$ [4–7] receive two kinds of additional loop contributions: One is from the charged-current lepton-sneutrino-chargino couplings; the other is from the flavor mixings of charged sleptons. While the former is a common feature of all SUSY models accommodating right-handed neutrinos, the latter is sizable only in some specific realizations of SUSY, such as the minimal supergravity model (mSUGRA) [8] with seesaw mechanism to generate the tiny masses for light neutrinos. The mechanism is realized by introducing right-handed neutrino superfields [1,2] with very heavy Majorana masses. In such a framework the flavor diagonality of charged sleptons is usually assumed at the Planck scale, but the flavor mixings at weak scale are inevitably generated through renormalization equations since there is no symmetry to protect the flavor diagonality. Such flavor mixings of charged sleptons generated at weak scale are proportional to neutrino Yukawa coupling, which may be as large as top quark Yukawa coupling due to seesaw mechanism, and are enhanced by a large factor $\log(M_P^2/M^2)$ ($M_P$ is Planck scale and $M$ is the neutrino Majorana mass). Therefore, the popular mSUGRA with seesaw mechanism predicts large flavor mixings of sleptons at weak scale, which will reveal their effects through some LFV processes in collider experiments.

The aim of this article is to examine the LFV $Z$-decays $Z \to \ell_i \ell_j$ induced by slepton flavor mixings in the mSUGRA seesaw model. Given the possibility of the extremely accurate measurement of $Z$-decays in future experiments, the decays $Z \to \ell_i \ell_j$ may serve as a sensitive probe for such a new physics model.

We will use the existing neutrino oscillation data and the experimental bounds on the decay $\ell_j \to \ell_i \gamma$ to constrain the model parameters, and then evaluate the branching ratios of $Z \to \ell_i \ell_j$. We find that, subject to the current constraints, the channel $Z \to \tau \mu$ can occur with a branching ratio as large as $10^{-8}$ and thus may be accessible at LHC [9] or the GigaZ option of TESLA at DESY [10].

This article is organized as follows. In section II, we briefly describe the SUSY seesaw model with minimal CP-violation in the right-hand neutrino sector and discuss the induced flavor mixings between sleptons. In section III, we present the analytic results for the SUSY contributions to the branching ratio of $Z \to \ell_i \ell_j$. In section IV, we present the correlation between the process $Z \to \ell_i \ell_j$ and $\ell_j \to \ell_i \gamma$. In section V, we evaluate the numerical size of the branching ratio of $Z \to \ell_i \ell_j$. Finally in section VI, we give our conclusion.

II. SUPERSYMMETRIC SEESAW MODEL AND CHARGED SLEPTON MIXINGS

A. Supersymmetric seesaw model

The seesaw mechanism [11] provides an elegant explanation for the tiny masses of light neutrinos, which implies that new physics scale is about $10^{14}$ GeV. However, a non-symmetric seesaw model suffers from a serious hierarchy problem [1], which can be automatically solved in the SUSY framework.

In supersymmetric seesaw model with $N$ right-handed neutrino singlet fields $\nu^c_R$, additional terms in the superpotential arise [1]:

$$ W_\nu = -\frac{1}{2} \nu^c_R \nu^c_R \hat{M}_R^c + \nu^c_R \nu^c_R \hat{H}^c, $$

(1)

where $\hat{M}$ is $N \times N$ mass matrix for the right-handed neutrino, and $L$ and $H_2$ denote the left-handed lepton
and the Higgs doublet with hypercharge $-1$ and $+1$, respectively. At energies much below the mass scale of the right-handed neutrinos, the superpotential leads to the following mass matrix for the left-handed neutrinos:

$$\mathbf{M}_\nu = \mathbf{m}_D^T \mathbf{M}^{-1} \mathbf{m}_D = \mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu (v \sin \beta)^2 .$$  \hspace{1cm} (2)

Obviously, the neutrino masses tend to be light if the mass scale $\mathcal{M}$ of the matrix $\mathbf{M}$ is much larger than the scale of the Dirac mass matrix $m_D = \mathbf{Y}_\nu (H^0_2) = \mathbf{Y}_\nu v \sin \beta$ with $v = 174$ GeV and $\tan \beta = (H^0_2)/H^0_1$. The matrix $\mathbf{M}_\nu$ can be diagonalized by the MNS matrix $\mathbf{U}_\nu$:

$$\mathbf{U}_\nu^T \mathbf{M}_\nu \mathbf{U}_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) ,$$  \hspace{1cm} (3)

where $m_{\nu i}$ are the light neutrino masses.

### B. Slepton flavor mixings

The mass matrix of the charged sleptons is given by

$$\mathbf{m}^2_\ell = \begin{pmatrix} m^2_{\ell LL} & m^2_{\ell LR} & m^2_{\ell RR} \end{pmatrix},$$  \hspace{1cm} (4)

with

$$m^2_{\ell LL} = \mathbf{m}_L^2 + \left[ \mathbf{m}_R^2 + (\mathbf{m}_R^2 - \mathbf{m}_L^2) (1 + s_W^2) \right] \cos 2\beta \mathbf{1},$$  \hspace{1cm} (5)

$$m^2_{\ell LR} = m^2_{\ell LR} + \left[ (\mathbf{m}_R^2 - \mathbf{m}_L^2) - \mathbf{m}_L^2 \right] \cos 2\beta \mathbf{1},$$  \hspace{1cm} (6)

$$m^2_{\ell RR} = \mathbf{A}_{\ell \mu} \cos \beta - m_{\ell \mu} \tan \beta \mathbf{1},$$  \hspace{1cm} (7)

where $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, $\theta_W$ is the Weinberg angle and $\mathbf{1}$ is unit $3 \times 3$ matrix in generation space. In mSUGRA model it is assumed that at the Planck scale the soft-breaking parameters satisfy

$$m^2_L = m^2_R = m_0, \quad m_{H_1} = m_{H_2} = m_0, \quad \mathbf{A}_\ell = \mathbf{A}_0 \mathbf{Y}_\ell, \quad \mathbf{A}_\nu = \mathbf{A}_0 \mathbf{Y}_\nu .$$  \hspace{1cm} (8)

In general, the lepton Yukawa couplings $\mathbf{Y}_\ell$ and $\mathbf{Y}_\nu$ cannot be diagonalized simultaneously. It is usually assumed that $\mathbf{Y}_\ell$ is flavor diagonal but $\mathbf{Y}_\nu$ is not. In this basis the mass matrix of the charged sleptons is flavor diagonal at Planck scale. However, when evolving down through renormalization group (RG) equations (see Appendix A) to weak scale, such flavor diagonality is broken. In the leading-log approximation, we have [2]

$$\delta(m^2_{\ell i})_{jj} \simeq -1 \left[ \frac{1}{8 \pi^2} (3m_0^2 + A_0^2) \mathbf{Y}_\nu^0 \mathbf{Y}_\nu^0 \right] \ln \left( \frac{M_P}{\mathcal{M}} \right) ,$$  \hspace{1cm} (9)

$$\delta(m^2_{\ell R})_{ij} \simeq 0 ,$$  \hspace{1cm} (10)

$$\delta(A_{\ell i})_{jj} \simeq -\frac{3}{16 \pi^2} A_{0 \ell} \mathbf{Y}_\nu^0 \mathbf{Y}_\nu^0 \ln \left( \frac{M_P}{\mathcal{M}} \right) ,$$  \hspace{1cm} (11)

where $\mathbf{Y}_\nu^0 = \mathbf{Y}(M_P)$.

The flavor non-diagonal mass matrix $\mathbf{m}_\ell^2$ in Eq.(4) at weak scale can be diagonalized by a unitary matrix $\mathbf{S}_\ell$

$$\mathbf{S}_\ell \mathbf{m}_\ell^2 \mathbf{S}_\ell^T = \text{diag}(m^2_{\ell i}) .$$  \hspace{1cm} (12)

Such a unitary rotation of slepton fields is to induce the flavor-changing neutral-current vertices: $\bar{\chi}_\alpha^0 \ell_i \ell_X$ and $Z\ell_X \ell_Y$.

In supersymmetric seesaw model, there exist right-handed neutrinos with the same order masses as the heavy Majorana neutrinos. However, due to their large masses, they do not give significant contributions to the considered LFV processes. Therefore, only the left-handed sneutrinos need to be take into account, whose mass matrix is given by

$$\mathbf{m}_\nu^2 = \mathbf{m}_L^2 + \frac{1}{2} m_Z^2 \cos 2\beta \mathbf{1} ,$$  \hspace{1cm} (13)

Due to the non-diagonal contribution $\delta(m^2_{\nu i})_{jj}$ in Eq.(9), $\mathbf{m}_\nu^2$ is flavor non-diagonal at weak scale and needs to be diagonalized by a unitary matrix $\mathbf{S}_\nu$

$$\mathbf{S}_\nu \mathbf{m}_\nu^2 \mathbf{S}_\nu^T = \text{diag}(m^2_{\nu i}) .$$  \hspace{1cm} (14)

Such a unitary rotation of sneutrino fields results in the charged-current flavor-changing vertex: $\bar{\chi}_\alpha^0 \ell_i \ell_X$.

### C. The form of neutrino Yukawa coupling

As shown in Eqs.(9) and (11), the flavor mixings of charged sleptons are proportional to neutrino Yukawa couplings. Lack of knowledge of the neutrino Yukawa couplings results in numerous speculations on their possible forms. Different forms may lead to different flavor mixings. In this work we consider a scenario called as minimal CP violating seesaw model which has two heavy Majorana neutrinos with the Dirac mass matrix $\mathbf{m}_D$ parameterized as [12]

$$\mathbf{m}_{\ell}^T = \mathbf{Y}_\ell^T (H^0_2) = \mathbf{U}_L \mathbf{m} \mathbf{V}_R, \quad \mathbf{m} = \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix} ,$$  \hspace{1cm} (15)

where

$$\mathbf{V}_R = \begin{pmatrix} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{pmatrix},$$  \hspace{1cm} (16)

with mixing angle $\theta_R$ and CP violating phase $\gamma_R$ for the heavy Majorana neutrinos concerning directly with leptogenesis [12]. The matrix $\mathbf{U}_L$ appearing in Eq. (15) reads

$$\mathbf{U}_L = \mathbf{O}_{23} (\theta_{123}) \mathbf{O}_{12} (\theta_{123}, \delta_2) \mathbf{O}_{12} (\theta_{123}) \mathbf{P} (-\gamma_{1/2}) ,$$  \hspace{1cm} (17)

where $\mathbf{P}(-\gamma_{1/2}) = \text{diag}[1, \exp(-i\gamma_{1/2}), \exp(i\gamma_{1/2})]$, and $\mathbf{O}_{ij}$ and $\mathbf{U}_{ij}$ denote the rotations in $(i,j)$ plane.
Without loss of generality, \(m_{2,3}\) in Eq. (15) are chosen to be real, positive and \(m_2 < m_3\). As Eqs. (2) and (15) are used, the mass matrix for the light neutrinos in this model can be further expressed as

\[
M_\nu = U_L m V_R M^{-1} V_R^T m^T U_L^T.
\]

And the MNS matrix in (3) is found to be a product of matrices,

\[
U_\nu = U_L K_R,
\]

where \(K_R = K_R(\theta, \phi, \alpha)\) is a unitary matrix. Therefore, now Eq. (3) can be rewritten as

\[
K_R^\dagger m V_R M^{-1} V_R^T m^T K_R^* = \text{diag}[m_{\nu_1}, m_{\nu_2}, m_{\nu_3}].
\]

From this equation, one can learn both \(K_R\) and \(m_{\nu_i}\) are independent of the choice of \(U_L\).

It is noticeable that the special form (15) for the neutrino Yukawa couplings matrix \(Y_\nu\) implies [12]:

1. One of the neutrinos is massless, i.e., \(m_1 = 0\).
2. The quantity \(Y_\nu^\dagger Y_\nu\) is only dependent on 3 mixing angles \(\theta_{L12}, \theta_{L13}, \theta_{L23}\) and a CP violating phase \(\delta_L\) in \(U_L\),

\[
(Y_\nu^\dagger Y_\nu)_{ij} = \frac{m_{\nu_i}^2 (U_L^\dagger U_L)_{ij} + m_{\nu_j}^2 (U_L^\dagger U_L)_{ij}}{(v \sin \beta)^2}.
\]

3. For small mixing angles \(\theta_{L13}\) and \(\theta\), the light neutrinos mixing matrix \(U_\nu\) takes a simplified form similar to the mixing matrix introduced in [13]:

\[
U_\nu \simeq \begin{pmatrix}
 c_{L12} & s_{L12} e^{-i\phi_L} & s_{L13} e^{-i\theta_L} \\
-s_{L12} c_{L23} & c_{L12} c_{L23} & s_{L12} s_{L23} e^{-i\phi_L} \\
 s_{L13} c_{L23} & -s_{L12} c_{L23} e^{-i\theta_L} & c_{L23}
\end{pmatrix} P(\alpha')
\]

where \(\phi' = \phi + \gamma_L\), \(\alpha' = \alpha - \gamma_L/2\) and \(s_x \equiv \sin x\), \(c_x \equiv \cos x\). In this case, the angles in \(Y_\nu^\dagger Y_\nu\) can be related directly to the corresponding neutrino mixing angles and determined by neutrino experiments.

### III. THE LFV DECAYS \(Z \to \ell_1 \bar{\ell}_j\)

The flavor changing interactions in slepton sector discussed in the preceding section, namely the couplings \(\tilde{\chi}_R^0 \ell_1 \bar{\ell}_j\) and \(Z \to \ell_1 \bar{\ell}_j\) from charged slepton mixings as well as \(\tilde{\chi}_R^0 \ell_i \nu_j\) from sneutrino mixings, can induce the LFV processes \(Z \to \ell_1 \bar{\ell}_j\), as shown in Fig. 1. The relevant Feynman rules can be derived straightforwardly from the analysis in the preceding section. Our analytic results will be expressed in terms of the constants \(\theta_{\alpha\beta}\), \(G_{XY}\) and \(C_{i\alpha X}^{L,R}\) defined in Fig. 2, whose explicit expressions can be found in [7,14].

The calculation of the diagrams in Fig. 1 results in an effective \(Z \ell_1 \ell_j\) vertex:

\[
\mathcal{M} = ig e_{\mu} \bar{u}_{\ell_1} (p_1) \Gamma^\mu u_{\ell_j}(p_2)
\]

with \(e_{\mu}\) being the polarization vector of Z-boson, \(p_1(p_2)\) the momentum of \(\ell_1(\ell_j)\), and \(\Gamma^\mu\) given by

\[
\Gamma^\mu = \frac{\alpha_{em}}{\sin^2 \theta_W} \left[ g^\mu \left( f_{1L} P_L + f_{1R} P_R \right) + \bar{\nu}_{\ell_1} \Gamma_{\ell_1} \nu_{\ell_j} \left( f_{2L} P_L + f_{2R} P_R \right) \right],
\]

where \(P_{R,L} = \frac{1}{2} (1 \pm \gamma_5)\), \(g = e / \sin \theta_W\) and \(k = p_1 - p_2\) is the momentum transfer. The form factors \(f_{1L}, f_{1R}, f_{2L}\) and \(f_{2R}\) were extracted previously.
and $f_{2R}$ arising from the calculation of the loop diagrams in Fig. 1 are listed as follows.

**Contribution of Fig. 1(a):**

\[ f_{1L}^a = G_{XY} C_{1aX}^L \left[ -2 C_{24}^a C_{J_{aY}}^L \right. \\
\left. + m_{t_{a}} m_{\alpha} (C_{0}^a + C_{11}^a + C_{12}^a) C_{J_{aY}}^L \right], \]

\[ f_{2L}^a = G_{XY} C_{24R}^a \left[ m_{o} (C_{0}^a + C_{11}^a + C_{12}^a) C_{J_{aY}}^L \right. \\
\left. - m_{t_{a}} (C_{12}^a + C_{22}^a + C_{23}^a) C_{J_{aY}}^L \right]. \]

**Contribution of Fig. 1(b):**

\[ f_{1L}^b = C_{1aX}^L C_{J_{bX}}^L \left[ g_{0}^L m_{\alpha} m_{\beta} C_{0}^b \right. \\
\left. + g_{0}^R \left( m_{2}^2 C_{23}^b - 2 C_{24}^b + \frac{1}{2} \right) \right] \\
+ C_{1aX}^L C_{J_{bX}}^L g_{0}^L m_{t_{a}} \left( C_{0}^b + C_{11}^b + C_{12}^b \right), \]

\[ f_{2L}^b = C_{1aX}^L C_{J_{bX}}^L \left[ g_{0}^L m_{\alpha} m_{\beta} C_{0}^b \right. \\
\left. + g_{0}^R \left( m_{2}^2 C_{23}^b - 2 C_{24}^b + \frac{1}{2} \right) \right] \\
+ C_{1aX}^L C_{J_{bX}}^L g_{0}^R m_{t_{a}} \left( C_{0}^b + C_{11}^b + C_{12}^b \right). \]

**Contribution of Fig. 1(c) plus Fig. 1(d):**

\[ f_{1L}^c = C_{1aX}^L \left[ m_{o}^a (B_{1}^1 - B_{0}^2) C_{J_{aY}}^L - B_{1}^1 C_{J_{aY}}^L \right] g_{L}, \]

\[ f_{1R}^c = 0. \]

In the above, $g_{L} = (1 - 2 \sin^2 \theta_W)/(2 \cos \theta_W)$, and $B_{0,1}^i = B(-p_i; m_{2}^2, m_{X}^2, m_{X})$, $C_{0,ij} = C_{ij}(-p_1, -p_2; m_{2}^2, m_{X}^2, m_{X}^2)$ and $C_{0,ij}^b = C_{ij}(-p_1, -p_2; m_{2}^2, m_{X}^2, m_{X}^2)$ are the Feynman loop integral functions [15]. Terms proportional to the lepton masses $m_{t_i}$ are neglected. The right-handed form factors from the vertex loops are obtained from the corresponding left-handed ones in (25)-(28) by the substitution $L \leftrightarrow R$.

The branching ratio of $Z \to \ell_1 \bar{\ell}_1$ (including its charge-conjugate channel) is then given by

\[ \text{Br}(Z \to \ell_1 \bar{\ell}_1) = \left( \frac{\alpha_{em}}{4\pi^2} \frac{e}{\sin^2 \theta_W} \right)^3 \frac{m_{Z}}{\Gamma_{Z}} \left[ |f_{1L}|^2 + |f_{1R}|^2 \right. \\
\left. + m_{Z}^2 \left( |f_{2L}|^2 + |f_{2R}|^2 \right) \right]. \]

where $f_{1L,R} = \sum_{a,b,c} f_{1L,R}^a$ and $\Gamma_Z$ denotes the total decay width of $Z$ boson.

Although the above results are sufficient to allow for numerical calculations, we would like to derive an analytical expression for the branching ratio by considering the limit $m_S \gg m_Z$ where $m_S$ represents the mass of any internal sparticle in the loops in Fig.1. In this case the loop functions can be much simplified and we use the mass-insertion approximation in our derivation. In such a limit, the chargino mass matrix

\[ M_{\tilde{\chi}^\pm} = \left( \begin{array}{cc} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{array} \right) \]

is nearly diagonal. Here $\mu$ is the mass parameter appearing in the term $\mu H_1 H_2$ in superpotential and $M_2$ is the $SU(2)$ gaugino mass parameter. The matrices $U$ and $V$ which diagonalize $M_{\tilde{\chi}^\pm}$ will be unit ones for $\mu > 0$, and the chargino masses are given by

\[ m_{\tilde{\chi}^\pm} = M_2, \quad m_{\tilde{\chi}^\pm} = |\mu|. \]

The symmetric neutralino mass matrix

\[ M_{\tilde{\chi}^0} = \left( \begin{array}{cccc} M_1 & 0 & -m_{Z} s_{W} c_{\beta} & m_{Z} c_{W} c_{\beta} \\ 0 & M_2 & m_{Z} c_{W} s_{\beta} & -m_{Z} s_{W} s_{\beta} \end{array} \right) \]

can be diagonalized by a unitary matrix $N$

\[ N = \left( \begin{array}{cc} 1 & 1 \\ \sqrt{2} e^{i \frac{\pi}{4}} & -\sqrt{2} e^{i \frac{\pi}{4}} \end{array} \right). \]

The corresponding mass eigenvalues are given by

\[ m_{\tilde{\chi}^0_{1,2}} = M_{1,2}, \quad m_{\tilde{\chi}^0_3} = m_{\tilde{\chi}^0_4} = |\mu|. \]

When using mass-insertion method, one should note the fact that, for any matrix $M = M^0 + M^1$, where $M^0 = \text{diag}(m_1, m_2, \cdots, m_n)$ and $M^1$ has no diagonal elements, if matrix $T$ can diagonalize the matrix $M$, $M^T M = \text{diag}(m_1, m_2, \cdots, m_n)$, then at leading order for an arbitrary function $f$

\[ T_{ik} f(m_k) T_{kj} = \delta_{ij} f(m_0) + M_{ij}^1 f(m_1, m_0) \]

with

\[ f(x, y, z_1 \cdots z_n) = f(x, z_1 \cdots z_n) - f(y, z_1 \cdots z_n) \]

After a straightforward calculation we obtain an analytical expression for the branching ratio

\[ \text{Br}(Z \to \ell_1 \bar{\ell}_1) = \left( \frac{\alpha_{em}}{4\pi^2} \frac{e}{s_{W}^2} \frac{m_{Z}}{\Gamma_{Z}} \right)^3 \frac{m_{Z}^2}{M_2^2} \left| f_1(x_1, x_1) \right. \\
\left. - 2 f_2(x_1, x_1) - \frac{1}{2} + s_{W}^2 f_1(x_1, x_1) \right. \\
\left. + f_3(x_1, x_1) \right. \\
\left. + \frac{1}{2} s_{W}^2 - s_{W}^2 \right. \\
\left. \frac{M_2}{M_1} \left( \frac{f_3(x_1, x_1)}{c_{W}^4} \right) \right. \\
\left. - \frac{1}{2} f_3(x_1, x_1) \right. \\
\left. - 3 \frac{s_{W}}{c_{W}^4} f_3(x_1, x_1) \right. \]
Here $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, $x_I = (m_L^2)_{11}/M^2$, $x_I' = (m_L^2)_{11}/M^2$, and

\[
f_1(x) = \frac{1}{x-1} \left( 1 - \frac{x}{x-1} \ln x \right),
\]

\[
f_2(x) = \frac{1}{4(x-1)} \left( 1 - \frac{x^2}{x-1} \ln x \right),
\]

\[
f_3(x) = \frac{1}{(x-1)} \left( 1 + \frac{x^2 - 2x}{x-1} \ln x \right),
\]

(40)
and $f_i(x, y)$ can be obtained through Eq. (38).

**IV. COMPARISON OF LFV Z-DECAYS WITH LEPTON DECAYS**

Now we compare the LFV Z-decays with lepton decays. Using a similar procedure in the preceding section, we can easily calculate the decay width for $\ell_j \to \ell_1 \gamma$ by setting $g = e$, $g^{\nu, R}_{\alpha \beta} = 1$ with $\alpha = \beta$, $G_{XY} = 1$ with $X = Y$ in Fig. 2, and $f_{1L, 2R} = 0$ in (24). Meanwhile one should also note the fact that neutrinos in Fig. 1(a) and neutralinos in 1(b) do not couple to photon and that the self-energy diagrams do not contribute to dipole operators. The branching ratios of $\ell_j \to \ell_1 \gamma$ are obtained as

\[
\frac{\text{Br}(\ell_j \to \ell_1 \gamma)}{\text{Br}(\ell_j \to \ell_1 \nu_j \nu_1)} = \frac{6 \alpha \sin m_{\nu}^2}{m_{\nu_j}^2} \left( |f_{2L}^2| + |f_{2R}^3|^2 \right). \tag{41}
\]

Here the form factors are given by [7]

\[
f_{2L} = \sum_{k=a,b} \frac{1}{m_{k}^2} C_{\nu, X}^{R(k)} \left[ m_{a} C_{\nu, X}^{L(k)} F_{a} + m_{\ell_j} C_{\nu, X}^{R(k)} F_{b} \right], \tag{42}
\]

\[
f_{2R} = f_{2L} \left| e_{R} \right|^2, \tag{43}
\]

where

\[
F_{a}^2(x_a) = m_{a}^2 \left( C_{a}^{\alpha} + C_{a}^{\alpha'} + C_{a}^{\alpha''} \right) \left[ \left( a_x + 1 \right) + \frac{a_x}{a_x - 1} \ln a_x \right], \tag{44}
\]

\[
F_{b}^2(x_b) = -m_{b}^2 \left( C_{b}^{\alpha} + C_{b}^{\alpha'} + C_{b}^{\alpha''} \right) \left[ \left( b_x + 1 \right) + \frac{b_x}{b_x - 1} \ln b_x \right], \tag{45}
\]

\[
F_{1}^{L}(x_1) = m_{c}^2 \left( C_{c}^{\alpha} + C_{c}^{\alpha'} + C_{c}^{\alpha''} \right) \left[ \left( c_x + 1 \right) + \frac{c_x}{c_x - 1} \ln c_x \right] \tag{46}
\]

\[
F_{1}^{R}(x_1) = m_{d}^2 \left( C_{d}^{\alpha} + C_{d}^{\alpha'} + C_{d}^{\alpha''} \right) \left[ \left( d_x + 1 \right) + \frac{d_x}{d_x - 1} \ln d_x \right], \tag{47}
\]

with $a_x = m_{\ell_j}^2 / m_{a}^2$, $a_x = m_{\ell_j}^2 / m_{b}^2$, $b_x = m_{\ell_j}^2 / m_{c}^2$, and $d_x = m_{\ell_j}^2 / m_{d}^2$. Unlike the form factors for the Z-decays which contain terms not proportional to the small lepton mass [see Eqs. (25) and (28)], the form factors for $\ell_j \to \ell_1 \gamma$ are always proportional to the small lepton mass $m_{\ell_j}$. In this case, the off-diagonal elements in the mass matrices of chargino and neutralino are no longer negligible, especially when $\tan \beta$ is large. In fact, the terms $m_{a} C_{\nu, X}^{L(b)} C_{\nu, X}^{R(b)}$ in $f_{2R}^b$ receive the contribution from the wino-Higgsino mixing, which can be enhanced by $\tan \beta$. So for a large $\tan \beta$, the contribution of $f_{2R}^b$ is dominant and the branching ratios are given by [2]

\[
\text{Br}(\ell_j \to \ell_1 \gamma) \approx \text{Br}(\ell_j \to \ell_1 \nu_j \nu_1) \frac{6 \alpha \sin m_{\nu}^2}{m_{\nu_j}^2} \left( |f_{2L}^2| + |f_{2R}^3|^2 \right) \tag{48}
\]

\[
= \text{Br}(\ell_j \to \ell_1 \nu_j \nu_1) \frac{6 \alpha \sin m_{\nu}^2}{m_{\nu_j}^2} \left( \frac{\mu}{M_2^2} \right)^2 \times \left| \frac{1}{2} F_1^a(x_1, x_j) - F_1^b(x_1, x_j) \right|^2 \times \left| \frac{\delta(m_{\ell_j}^2)}{M_2^2} \right|^2 \times \frac{\tan^2 \beta}{(1 - \frac{\mu^2}{M_2^2})^2},
\]

where $x_1 = (m_{\ell_j}^2)_{11}/\mu^2$.

Comparing $\text{Br}(\ell_j \to \ell_1 \gamma)$ with $\text{Br}(Z \to \ell_1 \bar{\ell}_j)$, we find:

1. The dipole transitions in (24), the only operators contributing to $\ell_j \to \ell_1 \gamma$, do not give dominate contributions to decays $Z \to \ell_1 \bar{\ell}_j$ due to heavy sparticle mass suppression;

2. $\text{Br}(Z \to \ell_1 \bar{\ell}_j)$ is not sensitive to $\tan \beta$, whereas $\text{Br}(\ell_j \to \ell_1 \gamma)$ can be enhanced by large $\tan \beta$;

3. The ratio $\text{Br}(Z \to \ell_1 \bar{\ell}_j)/\text{Br}(\ell_j \to \ell_1 \gamma)$ is independent of the heavy Majorana sector introduced by seesaw mechanism.

**V. NUMERICAL RESULTS**

In our numerical calculation we consider the constraints from current neutrino oscillation experiments and the experimental bounds on LFV lepton decays.

1. **Neutrino oscillation experiments:**

The SK Collaboration [16] showed that the $\nu_\mu$ created in the atmosphere oscillates into $\nu_\tau$ with almost maximal mixing, $\sin(2 \theta_{atm}) \approx 1$ and the neutrino mass-square difference is $\Delta m_{\nu_{atm}}^2 \approx (2 - 4) \times 10^{-3} \text{ eV}^2$. The second mass-square difference and mixing angle are found to be $\Delta m_{\nu_{sol}}^2 = (3 - 15) \times 10^{-5} \text{ eV}^2$, $\sin(2 \theta_{sol}) = 0.7 \sim 0.9$ from solar neutrino experiments [17,18]. For the third mixing angle, only the upper bound is obtained from the reactor neutrino experiments [19,20]: $\sin^2 2 \theta_{rea} < 0.1$ for $\Delta m_{\nu_{atm}}^2 \approx 3 \times 10^{-3} \text{ eV}^2$.

Although there exists a possibility that neutrino masses are quasi-degenerate, in this work we take the
normal mass order $m_{\nu 1} < m_{\nu 2} < m_{\nu 3}$ with values: \(^2\)

$$m_{\nu 1} = 0, \quad m_{\nu 2} = \sqrt{\Delta m^2_{sol}}, \quad m_{\nu 3} = \sqrt{\Delta m^2_{atm}}.$$  \hspace{1cm} (49)

The mixing angles are fixed to be

$$\theta_{L12} = \theta_{sol} = 30^\circ, \quad \theta_{L23} = \theta_{atm} = 45^\circ.$$  \hspace{1cm} (50)

Further, we restrict $\theta_{L13} < 10^0$. Then $(Y^\dagger_{\nu} Y_{\nu})_{IJ}$ in Eq. (21) are given by

$$(Y^\dagger_{\nu} Y_{\nu})_{12} \simeq \frac{\sqrt{2}}{4v^2 \sin^2 \beta} \left( \frac{\sqrt{3}}{2} m^2_2 + \sin 2\theta_{L13} m^2_3 \right),$$  \hspace{1cm} (51)

$$(Y^\dagger_{\nu} Y_{\nu})_{13} \simeq \frac{\sqrt{2}}{4v^2 \sin^2 \beta} \left( -\frac{\sqrt{3}}{2} m^2_2 + \sin 2\theta_{L13} m^2_3 \right),$$  \hspace{1cm} (52)

$$(Y^\dagger_{\nu} Y_{\nu})_{23} \simeq \frac{1}{4v^2 \sin^2 \beta} (2m^2_3 - m^2_2).$$  \hspace{1cm} (53)

The dependence of the parameter $(Y^\dagger_{\nu} Y_{\nu})$ on CP phase $\delta_L$ is very weak and thus has been neglected.

The experimental upper bound on $m_{3}\mu \gamma$ is very weak and thus has been neglected.

$$\text{Br}(\tau \rightarrow (\epsilon, \mu) \gamma) < (2.7, 1.1) \times 10^{-6},$$

$$\text{Br}(Z \rightarrow \tau \bar{\mu}) < 1.2 \times 10^{-5},$$

$$\text{Br}(Z \rightarrow (\mu, \tau) \bar{e}) < (1.7, 9.8) \times 10^{-6}.$$  \hspace{1cm} (55-57)

In addition, the explanation of the observed lepton number asymmetry by seesaw mechanism gives a lower bound for heavy Majorana neutrinos $M_1 > 10^{11}$ GeV [12]. Taking into the constraints mentioned above and fixing the right-handed neutrino masses as $M_1 = 10^{13}$ GeV, $M_2 \approx 10^{15}$ GeV, we solve the full RG equations listed in Appendix A numerically based on the work of [26], where the experimental bounds from $b \rightarrow s \gamma$ and $g_\mu - 2$ have been already taken into account. Although the processes $Z \rightarrow \ell_1 \ell_2$ are closely correlate to $\ell_1 \rightarrow \ell_1 \gamma$ and there is a quite stringent bound on $m_{2} \rightarrow e\gamma$, our numerical results show that there exists a scenario with $m_2 \ll m_3$ and a very small $\theta_{L13}$, in which a large branching ratio for $Z \rightarrow \tau \bar{\mu}$ is obtained.

In Fig. 3 we show the branching ratios of $Z \rightarrow \ell_1 \ell_2$ and $\ell_1 \rightarrow \ell_1 \gamma$ versus the common scalar mass $m_0$. From Fig. 3 we have the following observations:

1. With fixed $m_{1/2}$ and tan $\beta$, both $\text{Br}(Z \rightarrow \tau \bar{\mu})$ and $\text{Br}(\tau \rightarrow (\epsilon, \mu) \gamma)$ reach their maximum values as $m_0 \simeq 1000$ GeV, then drop slowly as $m_0$ gets larger.

2. The branching ratio of $Z \rightarrow \tau \bar{\mu}$ can be as large as $10^{-8}$.

Since $5.5 \times 10^9$ Z-bosons will be produced at the LHC [9] and the possible sensitivity of GigaZ to $Z \rightarrow \tau \bar{\mu}$ will be up to $10^{-8}$ [10], the mode $Z \rightarrow \tau \bar{\mu}$ will be accessible at both the LHC and TESLA GigaZ.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Branching ratios of $Z \rightarrow \ell_1 \ell_2$ and $\ell_1 \rightarrow \ell_1 \gamma$ versus the common scalar mass $m_0$. Other parameters are fixed to be $m_{1/2} = 800$ GeV, $A_0 = 0$, tan $\beta = 10$, $m_2 = 10$ GeV, $m_3/m_2 = 30$ and $\theta_{L13} = 0$. The dash line in (b) is the experimental upper bound on $m_{3} \rightarrow e\gamma$.}
\end{figure}

(2) Experimental bounds on LFV lepton decays:

LFV lepton decays have been searched in several experiments and the current bounds are given by [22–25]

$$\text{Br}(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11},$$

$$\text{Br}(\tau \rightarrow (\epsilon, \mu) \gamma) < (2.7, 1.1) \times 10^{-6},$$

$$\text{Br}(Z \rightarrow \tau \bar{\mu}) < 1.2 \times 10^{-5},$$

$$\text{Br}(Z \rightarrow (\mu, \tau) \bar{e}) < (1.7, 9.8) \times 10^{-6}.$$  \hspace{1cm} (54-57)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Same as Fig. 3, but versus the mixing angle $\theta_{L13}$ with $m_0 = 500$ GeV.}
\end{figure}

In general, the impact of RG evolution on neutrino masses and mixing angles can be large, however, it is small for a hierarchy of neutrinos we chosen [21].

\text{Br}(\tau \rightarrow (\epsilon, \mu) \gamma) < (2.7, 1.1) \times 10^{-6},$$

$$\text{Br}(Z \rightarrow \tau \bar{\mu}) < 1.2 \times 10^{-5},$$

$$\text{Br}(Z \rightarrow (\mu, \tau) \bar{e}) < (1.7, 9.8) \times 10^{-6}.$$  \hspace{1cm} (55-57)

It is noticeable that the branching ratios are sensitive to the mixing angle $\theta_{L13}$ except for the processes $Z \rightarrow \tau \bar{\mu}$ and $\tau \rightarrow \mu \gamma$. As an illustration, we plot the dependence
on $\theta_{13}$ in Fig. 4. We see that to satisfy the experimental constraint on $\mu \rightarrow e\gamma$, the mixing angle $\theta_{13}$ must be quite small. Therefore, a joint measurements for LFV $Z$-decays and lepton decays will set strong constraints on the model parameter space.

VI. CONCLUSIONS

We evaluated the lepton flavor violation $Z$ decays in the framework of supersymmetric seesaw model at first time. Although different forms of neutrino couplings may lead to different size of LFV $Z$ decays, we emphasize that it is important to study how large the rate for the LFV can be for some typical cases and analyze the possibility to observe $Z \rightarrow \ell\ell\gamma$ in future experiments. From our calculation results we conclude that, subject to the constraints from the existing neutrino oscillation data and the experimental bounds on the decays $\ell_\ell \rightarrow \ell_\ell\gamma$, the $Z$-decays $Z \rightarrow \ell\ell_\ell$ may be still sizeable in supersymmetric seesaw model, among which the largest-rate channel $Z \rightarrow \tau\mu$ can occur with a branching ratio of $10^{-8}$ and thus may be accessible at the LHC and GigaZ experiment.

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APPENDIX A: RENORMALIZATION GROUP EQUATIONS IN SUSY SEESAW MODEL

In this appendix we present additional contributions to the RG equations of some parameters in supersymmetric seesaw model due to non-zero neutrino interactions. The detailed description of these equations can be found in [1,2]. At one-loop level, the RG equations are given as follows.

(1) For Yukawa couplings:

$$\frac{dY_\nu}{dt} = \frac{Y_\nu}{16\pi^2} \left( T_2 - g_1^2 - 3g_2^2 + 3Y_\nu^\dagger Y_\nu + Y_{\ell}^\dagger Y_{\ell} \right),$$

$$\frac{dY_{\ell}}{dt} = \frac{Y_{\ell}}{16\pi^2} Y_\nu^\dagger Y_\nu,$$

$$\frac{dY_U}{dt} = \frac{Y_U}{16\pi^2} Tr(Y_\nu^\dagger Y_\nu),$$

where $t = \ln \mu$, with $\mu_\nu$ being the renormalization scale, and $T_2 = Tr(3Y_\nu^\dagger Y_U + Y_{\ell}^\dagger Y_{\ell})$. $Y_U$ is the Yukawa coupling matrix for up-type quarks, and $g_1$, $g_2$ and $g_3$ are the $U(1)_Y$, $SU(2)$ and $SU(3)$ gauge coupling constants, respectively.

(2) For soft parameters:

$$\frac{dm^2_\ell}{dt} = \frac{1}{16\pi^2} \left[ m_L^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_L^2 + 2Y_\nu^\dagger m^2_\nu Y_\nu + 2m^2_\nu Y_\nu + Y_\nu^\dagger m^2_\nu Y_\nu + 2Y_\nu^\dagger Y_\nu m^2_\nu Y_\nu \right] + 2\Delta_{A_{\nu}} M_1,$$

$$\frac{dm^2_{2\tilde{H}_d}}{dt} = \frac{1}{16\pi^2} \left[ m^2_\nu Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m^2_\nu + 2Y_\nu^\dagger m^2_\nu Y_\nu \right] + 2\Delta_{A_{\nu}} M_1,$$

$$\frac{dA_{\nu}}{dt} = \frac{1}{16\pi^2} \left[ (2Y_\nu^\dagger Y_\nu A_{\nu} + A_{\nu} Y_\nu^\dagger Y_\nu) \right],$$

$$\frac{dA_\ell}{dt} = \frac{1}{16\pi^2} \left[ (2Y_\nu^\dagger Y_\nu A_{\nu} + A_{\nu} Y_\nu^\dagger Y_\nu) \right].$$

(3) For neutrino masses [27]:

$$\frac{dM_{\nu}}{dt} = \frac{1}{16\pi^2} \left[ M(Y_\nu^\dagger Y_\nu) + 2Y_\nu^\dagger Y_\nu M_\nu \right],$$

$$\frac{dM_{\nu}}{dt} = \frac{1}{16\pi^2} \left[ M(Y_\nu^\dagger Y_\nu) + 2Y_\nu^\dagger Y_\nu M_\nu \right].$$

Note that the above RG equations are valid for the running from $M_P$ to $M$. Below the scale $M$, the RG equations are the same except that the couplings of the right-handed neutrinos do not appear.

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