The impact of renewable energy forecasts on intraday electricity prices
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Abstract

The shift to renewable power is accelerating at a growing pace. Contemporary energy markets undergo profound structural changes. Forecasts as to the wind and solar supply become critically important. This paper studies the impact of these forecasts on electricity market prices. We develop an intraday price model based on day-ahead auction data and errors in wind and solar power prognostications. The model manipulates empirical supply and demand curves recorded in the German EPEX SPOT SE to produce the prices. The obtained results show that forecast errors exert a non-linear impact on intraday prices. Moreover, additional wind and solar power capacities induce non-linear changes in intraday price volatility. Economical and policy implications of these findings are elaborated upon.

Keywords: Energy economics, Energy forecasting, Energy Policy, Forecasting and Prediction Methods, Renewable Resources

JEL: C53, Q21, Q41, Q47, Q48

1 Introduction

1.1 Basic idea and motivation

In an effort to curb climate change, a structural transformation of the energy sector seems unavoidable. Contemporary government policies actively foster an increased use of clean power. At the forefront of the renewable revolution are two technologies: wind and solar. Multiple indicators predict their booming future due to their continuously falling costs, widespread availability and low global warming potential. Yet, energy harnessed by wind turbines or photovoltaic panels is intermittent. This signifies the importance of load and price forecasting.

Variability of the sun and wind energy is critical in the German EPEX SPOT SE. A simplified temporal structure of this energy exchange is depicted in Figure 1. Two markets are of particular interest to us: day-ahead and continuous intraday. They differ in their temporal proximity to the point \( t \) of physical electricity delivery and in their microstructures. The former market is a non-continuous limit order book auction conducted at 12:00 a day prior to \( t \). The latter market is a continuous trading venue which closes 30 minutes prior to \( t \).

Note that prices in both markets are established before the physical delivery of electricity occurs. Wind and solar power supply forecasts are thus required to settle these prices. Therefore, the importance of forecast errors is not to be underestimated. Given that predictions are usually more precise in the intraday market, prices in this market are more accurate too (see e.g. [Weber, 2010] or [Pape, 2017]). Hence, the difference between the prices in the two markets can be at least partially explained by analyzing the forecast errors.

The influence of forecast errors on electricity market prices has already been given a thorough attention in academic literature. [Von Roon and Wagner, 2009] have explicitly demonstrated the importance of errors in wind forecasts and attempted to measure the impact of these errors on intraday prices. In a more recent study, [Kiesel and Paraschiv, 2017] show that intraday prices are...
indeed affected by errors in renewable energy forecasts and are even sensible to the signs of forecast errors. [Pape, 2017] shows that errors in wind and solar power prognostications affect not only intraday prices but also controllable electricity producers. The study of [Gürtler and Paulsen, 2018] uses panel data analysis and supports the conclusions drawn by [Kiesel and Paraschiv, 2017]. The core findings of the latter paper regarding asymmetries are though disputed by [Ziel, 2017] who agrees that forecast errors do influence intraday market prices, however, the asymmetric effects are insignificant.

The focus of the aforementioned literature was placed on the linear impact of the forecast errors. The evidence of the corresponding non-linear effects exists too. By investigating the Nord Pool data [Jónsson et al., 2010] show that electricity prices react on adjustments in wind power predictions more strongly during the day than during the night. Furthermore, the study claims that the price decreases dwindle when the level of wind penetration rises. [Hagemann, 2013] relies on the German electricity data and demonstrates that the impact of wind forecasts on electricity prices is significantly more pronounced from midnight to 8 a.m.

Nevertheless, the non-linear nature of the impact of errors in wind and solar power forecasts on the difference between day-ahead and intraday prices has not yet been studied extensively. The present paper attempts to tackle the problem and demonstrates that the impact of forecast errors depends on e.g. the sector of the merit-order curve in which the corresponding prices are realized. This conclusion follows the findings of e.g. [Ziel and Steinert, 2016] and [Coulon et al., 2014]. Furthermore, we show that forecast errors influence the volatility of intraday prices in a non-linear manner.

1.2 Intuition behind the main idea

Assume an imaginary electricity market alike the one depicted in Figure 2. Note that each side of Figure 2 shows two supply curves: a blue and a brown ones. The former one denotes a forecast of the electricity supply made in the day-ahead market. The latter one is a corresponding weighed average intraday prediction. For the matter of simplicity we suppose that the distance between the curves depends solely on the forecast error. Since the blue curve is located to the right of the brown curve it follows that the actual amount of electricity was overestimated in the day-ahead forecast. Naturally, the blue curve would be shifted towards the brown one and the day-ahead price would be closer to the intraday price be the forecast in the day-ahead market less erroneous.

4Our choice of weighted average intraday prices is motivated by [von Luckner et al., 2017].
Note that the two curves, their shapes and the distances between them are identical on both sides of Figure 2. The only difference between the two sides of the figure is the realized demand size. The discrepancy between the day-ahead and intraday prices is thus more pronounced on the right hand side of the figure even though the size of the forecast error is the same. This allows us to conclude that the impact of a forecast error on the difference between day-ahead and intraday prices may be non-linear. As the figure suggests, the discrepancy between the prices stems from the sector of a supply curve in which the prices are realized or from the shape of the merit-order curve itself.

![Figure 2: An example of a non-linear impact of a forecast error on intraday electricity prices](image)

To substantiate our idea with empirical evidence, we will elaborate several non-linear models and compare them with linear regression benchmarks. In the core of the non-linear model are manipulations with empirical supply and demand curves. The intuition behind this approach follows from [Ziel and Steinert, 2016] and Figure 2. More specifically, we do not consider the prices themselves, but rather supply and demand curves (also known as the sale and purchase curves). An intraday supply curve can thus be obtained by adjusting (i.e. shifting) a supply curve observed in the day-ahead market (i.e. day-ahead sale curve). The intersection of the shifted day-ahead sale curve with the purchase curve yields the intraday price. The magnitude and the direction of the shift are computed using errors in wind and solar forecasts and absolute amounts of wind and solar power generated at the moment of delivery. To optimize the shift size, a non-linear optimization technique is applied.

The paper is organized as follows. In the second section we describe the data and the corresponding rearrangements. We elaborate on the intuition behind the modeling approach based on
the empirical sale and purchase curves. We comment on the transformation of elastic purchase curves into their inelastic analogues using the method proposed by [Knaut and Paulus, 2016]. In the third section we introduce the pool of the intraday price models including benchmark models. We comment on the curve transformation tool developed by [Coulon et al., 2014]. In the fourth section we provide the obtained results regarding the accuracy of the models. We study the asymmetries in the impacts of forecast errors. We analyze the connection between forecast errors and the volatility of the intraday prices. We comment on economical and policy implications of the non-linear nature of forecast errors in wind and solar power prognoses.

2 Data preparation

2.1 Data description

We focus on the German-Austrian EPEX energy market and study a period between 01.31.2015 and 31.12.2017. The research is based on several datasets. One of them contains information regarding the day ahead forecasts (indexed by $F$) of renewable energy generation and the corresponding realized data (indexed by $A$). Hence, there are two pairs of parameters: $W^F$ and $W^A$, $S^F$ and $S^A$, where $W$ and $S$ stand for wind and solar power, respectively. The forecast errors can thus be written as $W^\Delta = W^A - W^F$ and $S^\Delta = S^A - S^F$.

Besides the data as to the energy supply, also the day-ahead prices $P_{t}^{DA}$ and weighted average intraday prices $P_{t}^{ID}$ were collected. In turn, the corresponding bid and ask schedules aggregated by an auctioneer to settle the prices were present in another dataset. Following the earlier chapters, the recorded bid and ask schedules are necessary to construct the supply and demand curves (i.e. sale and purchase curves).

In the next step the time spans of the datasets were adjusted. In doing so, we used a simple arithmetic average to unify the datasets to the hourly format and did not extrapolate missing data. We omitted the time points in which a price-volume observation was not available in at least one of the datasets. These manipulations shrunk the sample period to 26688 hourly time points. Moreover, we have neglected daylight saving times within the current research. Furthermore, it is crucial to note that the prices were rounded to two decimal places to spare computational time. The respective volumes were rearranged accordingly.

Figure 3 is constructed to show that day-ahead prices may deviate from intra-day prices if the amount of wind or solar power was wrongly predicted. The segment bounded by the red lines demonstrates explicitly that the intra-day prices are smaller than the day-ahead prices (upper plot) when the difference between actual and anticipated wind and solar energy generation (lower plot) is sufficiently greater than zero.

2.2 Empirical supply and demand curves approach

Let us now elaborate more thoroughly on the approach we follow. Recall that we can span combinations of the conventional sale and purchase curves for each hour given in our datasets. A corresponding example is illustrated in Figure 4. The sale and purchase curves are integral for our research due to the two main reasons. The former is that the intersections of

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5Following the regulation issued in 2008, the day-ahead and intraday prices are bound with $P_{\text{max}} = 3000 \text{ EUR}$ from above and with $P_{\text{min}} = -500 \text{ EUR}$ from below.
Figure 3: Dynamics of day-ahead and intra-day prices (upper plot), total generation of wind and solar energy (middle plot) and differences between actual and predicted wind and solar generation loads (lower plot) for a one-week sample from July, 28 to August, 03, 2016.
these curves coincide with the established day-ahead wholesale market clearing prices. Following [Ziel and Steinert, 2016], in 64 % of the cases these intersections are identical to the reported prices, in 89 % the error is less than 0.1 EUR, and in 99.8 % the error is less than 1 EUR. The reason for the errors to be present is e.g. block or other complex orders which are neglected by the model. As was mentioned in section 1.2, the latter reason is that we can shift the day-ahead sale curves to model intraday sale curves. This allows us to introduce non-linear price models in section 3.3.

![Graph of wholesale supply and demand curves](image)

**Figure 4**: Sale and purchase curves in the wholesale market at 2017-04-02 09:00:00.

From Figure 4 we can also observe that the purchase curve in the wholesale market is elastic. This poses a problem to our research. We want to shift a day-ahead sale curve to obtain an approximation for the corresponding intraday sale curve. However, market conditions change once the curve is shifted because market participants react on any changes in the price. Naturally, this problem can be avoided if the purchase curve is inelastic. The study by [Knaut and Paulus, 2016] suggests that an electricity market can be treated from two different perspectives and only in one of them the demand is elastic. More importantly, changing the perspective on the market does not alter the equilibrium price. Hence, we can transform the sale and purchase curves in our market to obtain conditions plausible for our research.

To elaborate their idea in a greater detail, consider an imaginary electricity market. Assume that this market is populated by 6 participants. Among them there are 2 utility companies, 2 conventional retailers and 2 suppliers. The demand and supply schedules of the market participants...
are depicted in Figure 5(a). Note that the supply and demand curves are represented as step functions. To ease further explanation, we will call each individual step of these curves as a sell or a buy order, respectively.

The orders of the market participants can be incorporated into a single market in 2 different ways. These 2 ways are depicted in Figures 5(b) and 5(c). The former figure illustrates a simple aggregation of the supply and demand schedules of the market participants. The latter one shows a competitive wholesale market. Note that the equilibrium prices are identical in both cases, yet the equilibrium volumes are not. More importantly thought, the demand curve is elastic only in Figure 5(b). In what follows we will elaborate on the functioning of the two markets.

Let us first focus on Figure 5(b) and the aggregated demand curve. To build up the curve, all buy orders of 6 market participants were first collected together in one pool, sorted according to the price in a descending order and then plotted. Naturally, the supply curve was created in a similar manner by processing sell orders instead of buy orders.

Figure 5(c) requires a lengthier commentary. To construct both the wholesale sale and purchase and demand curves, the orders of market participants were split into two groups. The first group incorporated demand orders, the second consisted of supply orders. This division was helpful because the trading strategy of the Utilities is somehow confusing in this case. Please note that the buy orders of the retailers were immediately placed in the first group, the sell orders of the suppliers in the second.

We will now use the example of Utility A to explain the strategies of the Utilities in the wholesale market. We first focus on the supply and demand curves of the Utility depicted in top left corner of Figure 5(a). The Utility’s buy orders located to the right of the intersection between the two curves were placed into the ‘demand orders’ group. Sell orders to the right of the intersection became a part of the ‘sell orders’ group. A possible intuition behind this grouping can be explained as follows. If considered in an autarky scenario, the Utility would not be able to fulfill these orders. Therefore, it tries to buy/sell them in the wholesale market hoping that they will be profitable for other market participants.

The buy and sell orders of the Utility to the left of the intersection do deserve a special notice too. The sell orders to the left of the intersection were placed in the ‘demand orders’ group. Intuitively, instead of producing energy and supplying it to the market, the Utility decides to act as a speculator and purchase the electricity from the market. The opposite holds for the buy orders to the left of the intersection. These orders fall into the ‘supply orders’ group. The Utility thus expects other market participants to purchase these buy orders. The Utility will thus be supplied with necessary money to act as a speculator. Hence, in the end, the Utility can fulfill all its orders without producing electricity at all.

Please note again that the prices in Figures 5(b) and 5(c) are identical. However, the equilibrium volume in Figure 5(b) is 30 MW more than that in Figure 5(c). The supply curve to the left of the equilibrium in Figure 5(c) is short of 20 MW coming from Utility A and 20 MW provided by Utility B. Yet, a 10 MW buy order from Utility A is placed into this segment of the supply curve.

Let us now concentrate on Figure 5(c). Once the market clears, Utility A realizes that it would cheaper produce electricity itself rather than purchase it (first two sell orders of Utility A). The same holds for the first two sell orders of Utility B. Moreover, the speculative gamble with the 5 EUR buy order undertaken by Utility A does not seem to be profitable too. This order (depicted in blue in the supply curve to the left of the equilibrium in Figure 5(c)) thus has to be neglected. Hence, in the end, both utilities decide to produce electricity as they would do in the Aggregated

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6Buy and sell orders do not necessarily fall into the first and second group, respectively.
Figure 5: An imaginary electricity market
Supply and Demand setting. The final volumes are thus equal in both Aggregate Supply and Demand and Wholesale Market settings.

3 Description of the models

3.1 Benchmark models

Remember that the main aim of the paper is to capture the non-linear impact exerted by errors in wind and solar power prognoses on the difference between intraday and day-ahead prices. The first benchmark model we introduce is a simple naive model with the following specification

\[ P_{naive}^t = P_t^{DA} + \varepsilon_t, \]

where \( P_{naive}^t \) stands for a day-ahead price and \( \varepsilon_t \) is an error term.

The other five models in our paper are based around the following component

\[ Z(\beta_{i:i+6}) = \beta_i + \beta_{i+1}W_t^{A+} + \beta_{i+2}W_t^A + \beta_{i+3}S_t^{A+} + \beta_{i+4}S_t^A + \beta_{i+5}W_t^A + \beta_{i+6}S_t^A, \]

where \( W_t^A \) and \( S_t^A \) are errors in wind and solar supply forecasts, respectively; \( W_t^{A+} = \max(W_t^A, 0) \) and \( S_t^{A+} = \max(S_t^A, 0) \) stand for positive errors in wind and solar supply forecasts, respectively; \( W_t^A \) and \( S_t^A \) denote absolute volumes of harvested wind and solar energy, respectively.

Note that we model positive forecast errors separately as was done in e.g. [Soysal et al., 2017], [Kiesel and Paraschiv, 2017] or [Ziel, 2017]. The component \( Z(\beta_{i:i+6}) \) will thus be incorporated into several models and different estimation techniques will be used to determine the vector of parameters \( \beta_{i:i+6} \).

The first benchmark model is a linear model \( lm_1 \) which can be characterized as follows

\[ P_t^{lm_1} - P_t^{DA} = Z(\beta_{0:6}) + \varepsilon_t. \]

The second linear model \( lm_2 \) is conceptually analogous to the first one, save for the fact that the term \( P_t^{DA} \) is shifted to the right hand side of the equation and assigned with its own \( \beta \) coefficient.

\[ P_t^{lm_2} = Z(\beta_{0:6}) + \beta_7 P_t^{DA} + \varepsilon_t. \]

To develop the non-linear models, the sale and purchase curves observed in the market must first be transformed according to the intuition described in section 2.2

3.2 Transformation of the empirical sale and purchase curves

A profound study of empirical sale and purchase curves observed in electricity markets is provided in [Ziel and Steinert, 2016]. The method we use to construct our first non-linear model is analogous to that described in this paper and depicted in Figure 2. Namely, we will subtract the amount \( Z(\beta_{i:i+6}) \) from the initial day-ahead supply forecasts. This will induce a day-ahead sale curve to be shifted to the left or to the right depending on the sign of \( Z(\beta_{i:i+6}) \). The magnitude of the shift will be additionally adjusted by solving a non-linear optimization problem and obtaining a vector of optimal \( \beta_{i:i+6} \) coefficients. The shifted day-ahead sale curve thus acts as an approximation of an intraday sale curve. The intersection of the latter curve with the purchase curve coincides with a prognosis for an intraday price.
However, recall the observed data looks as in Figure 2. Since the actual purchase curves are elastic shifting the day-ahead sale curves may lead to ambiguous results. To avoid this problem, we will use a tool developed by [Coulon et al., 2014]. This tool allows us to transform the elastic purchase curves into their inelastic analogues. The sale curves will be adjusted accordingly. Following the previous section, changing the shapes of the curves will not induce a change in the price. Therefore, the initial sale and purchase curves can be substituted with their artificially simulated analogues. The expression for the inelastic purchase curve reads

\[ Dem_{t}^{inelastic} = WSDem_{t}^{-1}(P_{\text{max}}) \]  

where a purchase curve in a wholesale market is denoted by \( WSDem \) and \( P_{\text{max}} = 3000 \) as prescribed by the regulation of EPEX. In turn, the equation for an inverse sale curve can be written as

\[ Sup_{t}^{-1}(z) = WSSup_{t}^{-1}(z) + WSDem_{t}^{-1}(P_{\text{min}}) - WSDem_{t}^{-1}(z) \]  

where a sale curve in a wholesale market is denoted by \( WSSup \) and \( P_{\text{min}} = -500 \). Please note that the above equation defines \( Sup_{t}(z) \) automatically since the function is monotonic. The example of the transformed sale and purchase curves is depicted in Figure 6.

![Figure 6](image_url)

Figure 6: A wholesale market equilibrium on 2017-04-02 08-00-00 CET (left plot) vs. its manipulated form with an inelastic demand curve (right plot).
3.3 Non-linear models

Having elaborated on the transformations of the sale and purchase curves, we can now derive the core models of the paper. We call our first non-linear model $nlm$ and specify the model as follows

$$P_{nlm}^t(\beta_{8:14}) = \text{Sup}_t(Dem_t^{inelastic} - Z(\beta_{8:14})) + \varepsilon_t$$  \hspace{1cm} (7)

where the price is namely an intersection of the shifted sale curve with the inelastic purchase curve. To estimate the optimal vector of coefficients $\beta_{8:14}$, we use an R function `optim` to solve the non-linear least squares problem $\beta_{nlm} = \arg \min_{\beta \in \mathbb{R}^7} (P_{ID}^t - P_{nlm}^t(\beta_{8}, ..., \beta_{14}))^2$. We execute the optimization function at least 3 times to ensure that the obtained results are trustworthy. To better understand the functioning of our approach in general and of the model in particular, take a look at Figure 7. The areas highlighted in blue and orange demonstrate the magnitudes of the shift induced by components $\beta_{10}W_t$ and $\beta_{12}S_t$, respectively. The intersection of the shifted sale curve with the inelastic purchase curve yields the price $P_{nlm}$.

![Figure 7: Price $P_{nlm}^t$ as the result of shifting the transformed sale curve.](image)

The second core model $cm$ is fundamentally a combination of a linear model $lm_2$ and a non-linear model $nlm$. The model $cm$ thus aims to incorporate both linear and non-linear effects. The price equation of the model can thus be written as follows

$$P_{cm}^t(\beta_{0:14}) = Z(\beta_{0:6}) + \beta_{7}P_{DA}^t + \beta_{15} \text{Sup}_t(Dem_t^{inelastic} - Z(\beta_{8:14})) + \varepsilon_t$$  \hspace{1cm} (8)

\[\text{linear component} + \text{non-linear component}\]
where the linear component coincides with the price produced by linear model \(\text{lm}_2\) and the non-
linear component is the price \(P^{nlm}\). Writing the corresponding non-linear least squares problem

\[
\hat{\beta}_{cm} = \arg \min_{\beta \in \mathbb{R}^{15}} (P^{ID}_t - P^{cm}_t(\beta_0, \ldots, \beta_{15}))^2.
\]

## 4 The obtained results

### 4.1 Model comparison

The obtained \(\beta\)-coefficients for the year 2017 are summarized in Table 1. Please note that the \(\beta\)-
coefficients of the linear models \(\text{lm}_1\) and \(\text{lm}_2\) are primarily negative. These findings are consistent
with those of e.g. [Ziel, 2017], [Kiesel and Paraschiv, 2017], [Clé et al., 2015], [Ketterer, 2014], or
[Gürtler and Paulsen, 2018] where the signs of the coefficients do resemble the ones we obtained.
The intuition behind these findings is also quite straightforward. The linear model shows a direct
relationship between the forecast errors and the market prices. Whenever the sizes of e.g. posi-
tive forecast errors grow market participants expect a higher electricity supply and market prices
decline, which is why the corresponding \(\beta\)-coefficients are negative.

Yet, the coefficients of the non-linear models are primarily positive. This observation seems
counterintuitive only at the first glance. Remember that the shifts of the sale curves are applied
to estimate the prices in the non-linear models (see e.g. Figure 7). The higher the magnitude of e.g. a positive forecast error is, the more will the merit-order be shifted to the right, and thus the lower the prices are (see e.g. [Neubarth et al., 2006], [Cludius et al., 2014], [Ketterer, 2014],
[Kiesel and Paraschiv, 2017], [Fürsch et al., 2012] or [Roldan-Fernandez et al., 2016]). Therefore,
the \(\beta\)-coefficients of the non-linear models are mostly positive. It follows that the intuition behind
the functioning of both linear and non-linear models is the same. Moreover, please note that the
coefficient \(\beta_7\) in the model \(\text{lm}_2\) and the sum \(\beta_7 + \beta_{15}\) in the model \(\text{cm}\) are significantly different
from one as indicated by corresponding t-tests (-5.7395 and -2.6509, respectively).

To support the study conducted above, we have also analyzed asymmetries in wind and solar
power forecasts. Figure 8 plots the corresponding \(\beta\)-coefficients for positive parts of the errors.
Please note that the coefficients of the linear model \(\text{lm}_2\) (left hand side of the graph) are negative,
the coefficients of the non-linear model \( nlm \) (right hand side of the graph) are positive. More importantly though, both sides of Figure 8 show that the influence of the positive forecast errors tends to diminish over the year 2017. Similar findings are in e.g. \cite{Guertler2018}.

Figure 8: \( \beta \)-coefficients for positive errors in wind and solar power prognoses

To evaluate the performance of the models, we first used MAE and RMSE measures with the following specifications

\[
\text{MAE} = \frac{1}{24D} \sum_{d=1}^{D} \sum_{h=1}^{24} |P_{d,h}^{ID} - \hat{P}_{d,h}^{ID}| \quad \text{and} \quad \text{RMSE} = \sqrt{\frac{1}{24D} \sum_{d=1}^{D} \sum_{h=1}^{24} (P_{d,h}^{ID} - \hat{P}_{d,h}^{ID})^2} \quad (9)
\]

where \( D = 364 \) and \( h \) is a hour index. Hence, we used a rolling windows study with 365 in-sample observations (year 2016) and 364 out-of-sample observations (year 2017). The window size was equal to 24 hours.

The general results of the MAE and RMSE studies applied to the whole dataset are summarized in Table 2 below. The table allows us to see that the model \( lm_2 \) has higher MAE, but lower RMSE.
values than the model $lm_1$ (the model $lm_2$ was thus used in model $cm$), the model $nlm$ fails to surpass the linear models, and the model $cm$ shows best performance of the models. It follows that the linear structures are important, yet the non-linear nature of forecast error in wind and solar power prognoses should not be neglected.

|       | Naive | $lm_1$ | $lm_2$ | $nlm$ | $cm$ |
|-------|-------|--------|--------|-------|------|
| MAE   | 4.828 | 4.265  | 4.267  | 4.332 | 4.236 |
| RMSE  | 8.048 | 7.352  | 7.286  | 7.123 | 7.097 |

Table 2: The results of MAE and RMSE tests

To determine best model among the ones considered, we used the DM-test and applied it to each of the 24 hours separately (see e.g. [Diebold and Mariano, 2002] and [Diebold, 2015]). To specify the parameters of the test, let the loss differential between the models $A$ and $B$ for hour $h$ be equal to $\delta_{d,h,A,B} = L_{d,h,A} - L_{d,h,B}$ where $L_{d,h}$ is the loss function of a model at hour $h$. The DM-test is thus $(\bar{\delta}_{h,A,B})/\sigma_{\delta_{h,A,B}} \sim N(0, 1)$ where $\bar{\delta}_{h,A,B} = \frac{1}{D} \sum_{d=1}^{D} \delta_{d,h,A,B}$ and $\sigma_{\delta_{h,A,B}}$ is the standard error which we estimate from the corresponding sample.

Figure 8 illustrates the DM-test comparison of the two best models according to the MAE and RMSE criteria: $cm$ and $lm_2$. Given the 5% confidence interval, we see explicitly that the model $cm$ outperforms the model $lm_2$, though not always significantly. This implies that (a) the combination of the two models can be used successfully to better model intraday prices and (b) the impact of the forecast error on intraday prices is of a non-linear nature.

The obtained results allow us to make the following statements. The model $cm$ (and notably even the model $nlm$) has shown a better performance than the linear model $lm_2$ for several of the 24 hours of a day. Given that the major components in our models were the errors in wind and solar power forecasts, we can conclude that the impact of these errors on the difference between day-ahead and intraday prices is non-linear. Moreover, our results could have been improved substantially if a more sophisticated optimization tool would have been used or if the adjustment was applied to the demand curve too. Therefore, we obtained the desired result even without the use of highly elaborated techniques.
4.2 Forecast errors and volatility of intraday prices

The earlier sections have demonstrated that the impact of forecast errors in wind and solar power prognoses on intraday prices is of a non-linear nature. In this section we will provide an evidence of a similar conclusion with respect to the volatility of the intraday prices. Economical and policy implications of these findings will appear straightforward.

Following e.g. [Clò et al., 2015], additional wind and solar power capacities not only induced a merit-order effect in Italy, but also increased the volatility of electricity prices. These findings are consistent with those of [Woo et al., 2011] who conduct a similar study for the electricity market in Texas. The corresponding analysis of the German electricity market is provided in [Ketterer, 2014]. The conclusions of the latter paper show that a rise in volatility of electricity prices may stem from a growing penetration of renewable resources.

Note that the amount of wind and solar power capacities is expected to exhibit a continuous growth in Germany (see e.g. [Henning and Palzer, 2015]). Following [Weber, 2010], the magnitude and the amount of the forecast errors is thus to be increased too. Moreover, the above chapters suggest that the forecast errors exert a non-linear influence on the electricity prices. Can the impact of the forecast errors on the volatility be non-linear too? And, if yes, how can this impact be mitigated?

We will first use a simple mathematical framework to answer this question. Let us assume a currently operating wind power plant. Naturally, we could have considered a photovoltaic power plant instead. Suppose that $W_t$ stands for the amount of energy harvested by the wind power plant, $\tilde{W}_t$ is an incremental wind supply from additional wind power capacities and $\gamma \geq 0$ denotes a scale factor. Generation variance of the extended wind power park can be written as

$$\text{Var}[W_t + \gamma \tilde{W}_t] = \text{Var}[W_t] + 2\gamma \rho_{[W_t;\tilde{W}_t]} \sqrt{\text{Var}(W_t) \text{Var}(\tilde{W}_t)} + \gamma^2 \text{Var}[\tilde{W}_t]$$

where $\rho$ denotes the correlation coefficient. Assuming that the variances $\text{Var}[W_t]$ and $\text{Var}[\tilde{W}_t]$ are equal allows the above expression to be represented as follows

$$\text{SD}[W_t + \gamma \tilde{W}_t] = \text{SD}[W_t] \sqrt{1 + 2\gamma \rho_{[W_t;\tilde{W}_t]} + \gamma^2}$$

greater than 1 if $\rho_{[W_t;\tilde{W}_t]} > 0$

Hence, the standard deviation of the power generation of the combined power plant increases when the correlation coefficient between the power generations of the old and the new capacities is positive. A policy implication follows: in order to decrease the standard deviation of generation, new power plants should be built such that their generation has the lowest possible correlation with generations of the already existing power plants. This problem has already been addressed in e.g. [Jónsson et al., 2010] and [Grams et al., 2017].

We can now use the models developed earlier to show the non-linear impact of forecast errors on the volatility of intraday prices. As [Weber, 2010] suggests, we can assume that the forecast errors are proportional to the standard deviation of generation. Given this assumption, we can modify the component $Z(\beta_{i;i+6})$ to test the sensitivity of our model to changes in the amounts of wind and solar power capacities. The following statement regarding the modified $Z(\beta_{i;i+6})$ can
thus be made

\[
Z_\gamma(\beta_{i;i+6}) = \beta_i + \left( \beta_{i+1} W_t^{A+} + \beta_{i+2} W_t^{A} \right) \cdot \sqrt{1 + 2 \gamma W \rho_{[W_t, \tilde{W}_t]} + \gamma^2 W} \\
+ \left( \beta_{i+3} S_t^{A+} + \beta_{i+4} S_t^{A} \right) \cdot \sqrt{1 + 2 \gamma S \rho_{[S_t, \tilde{S}_t]} + \gamma^2 S} \\
+ \beta_{i+5} W_t^A (1 + \gamma_W) + \beta_{i+6} S_t^A (1 + \gamma_S)
\]

To test our model under the new assumption, the component \( Z(\beta_{i;i+6}) \) was then replaced with the component \( Z_\gamma(\beta_{i;i+6}) \) in all of our models. We have chosen \( \gamma = \{0.1, 1, 5\} \) and \( \rho = \{0, 0.5, 0.8\} \) to conduct the study. Naturally, \( \gamma = 5 \) seems rather unrealistic. However, it allows the validity of our ideas to be shown explicitly.

Table 3 summarizes the standard deviations of the prices \( P_{lm} \) and \( P_{nlm} \) and compares them with the respective prices of the modified models. Table 4 shows the differences between the true and the modeled intraday prices for the 0.1\% quantile. Note that values in both tables exhibit same tendency: the values rise much speedier in the non-linear realm than in the linear one. Therefore, the values tend to be higher the closer they are to the bottom right corners of the tables. More importantly, the pace of the increase is clearly non-linear.

The above results support our earlier claim regarding the policy and economical implications. Building new wind and solar power capacities and neglecting their correlations with old ones will not only boost the volatility of electricity generation. It will also lead to a greater amount or magnitude of forecast errors. This, in turn, will amplify the volatility of intraday prices in a non-linear manner. Hence, whenever the system stability is of importance, the locations of new wind and solar power plants must be selected carefully.

| \( \gamma \) | \( \rho \) | \( \text{Model } lm \) | \( \text{Model } nlm \) |
|---|---|---|---|
| \( \beta_i \) | \( W_t \) | \( S_t \) | \( W_t^A \) | \( S_t^A \) |
| \( \beta_{i+1} \) | \( \beta_{i+2} \) | \( \beta_{i+3} \) | \( \beta_{i+4} \) | \( \beta_{i+5} \) | \( \beta_{i+6} \) |
| \( \gamma \) | \( \rho \) | \( \text{Wind} \) | \( \text{Solar} \) | \( \text{Wind} + \text{Solar} \) | \( \text{Wind} \) | \( \text{Solar} \) | \( \text{Wind} + \text{Solar} \) |
| \( 0.1 \) | \( 0 \) | 4.981 | 4.980 | 4.981 | 4.977 | 4.974 | 4.977 |
| \( 0.1 \) | \( 0.5 \) | 4.981 | 4.980 | 4.982 | 4.979 | 4.977 | 4.981 |
| \( 0.1 \) | \( 0.8 \) | 4.982 | 4.981 | 4.983 | 4.983 | 4.978 | 4.987 |
| \( 1 \) | \( 0 \) | 5.059 | 5.001 | 5.082 | 5.104 | 5.004 | 5.119 |
| \( 1 \) | \( 0.5 \) | 5.159 | 5.041 | 5.218 | 5.285 | 5.100 | 5.377 |
| \( 1 \) | \( 0.8 \) | 5.236 | 5.071 | 5.322 | 5.418 | 5.143 | 5.588 |
| \( 5 \) | \( 0 \) | 8.959 | 6.622 | 9.959 | 18.887 | 22.161 | 55.938 |
| \( 5 \) | \( 0.5 \) | 9.617 | 6.959 | 10.761 | 21.114 | 25.028 | 66.027 |
| \( 5 \) | \( 0.8 \) | 10.000 | 7.157 | 11.225 | 23.755 | 26.983 | 70.138 |
| \( 0 \) | \( 0 \) | 4.980 | 4.974 |

Table 3: Standard deviations of the prices \( P_{lm} \) and \( P_{nlm} \)

## 5 Conclusion

In this paper we have studied the impact of errors in wind and solar power forecasts on the difference between day-ahead and intraday prices. We based our study on manipulations with empirical supply and demand curves. We used findings of [Knaut and Paulus, 2016] and [Coulon et al., 2014] to show that it is possible to transform elastic demand curves observed in the electricity market into their inelastic analogues. This allowed us to shift day-ahead forecasted supply curves to
Table 4: The differences between true and modeled intraday prices at the 0.1% quantile

determine approximations for intraday supply curves. The magnitudes and the directions of the shifts were primarily driven by errors in wind and solar power forecasts. The intersections of the shifted supply curves with the inelastic demand curves coincided with the modeled intraday prices.

The obtained results have indicated explicitly that the impact of errors in wind and solar power forecasts is of a non-linear nature. Not only do forecast errors exert a non-linear influence on the prices, but they also affect price volatility in a non-linear manner. These conclusions can be critically important for practitioners and policy makers alike. The former group can improve their forecasts by taking non-linear effect of the forecast errors into account. The latter group must understand that expanding the fleet of wind and solar power parks may increase risks in the electricity markets significantly, unless the power generation of the new capacities is weakly or better negatively correlated with currently existing one.
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