Mathematical modelling of beam dynamics in
diamagnetic confinement regime of open trap

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Abstract. The paper is devoted to the mathematical modelling of a beam injected into an open magnetic trap in the diamagnetic mode. The two-dimensional axially symmetric hybrid numerical model with the kinetic approximation for the ion component of the plasma and the MHD approximation for magnetized electrons is used. In the numerical experiments, the possibility of formation of the cavity in the magnetic field was shown. A detailed study of the evolution of the ion beam continuously injected into the trap is done, the dependencies on time, moment of injection and beam temperature are presented. The obtained results may be used for the data analysis of the laboratory experiments in open magnetic systems.

1. Introduction
One of the important problems in plasma physics is the problem of controlled thermonuclear fusion, and particularly, construction of the fusion reactor based on an open magnetic trap. A new mode of the plasma confinement in the asymmetrical magnetic trap was proposed in [1]. The theoretical estimates in [1] demonstrate the principal possibility of the development of the fusion reactor with moderate parameters of the size and the magnetic field and high ratio \( \beta \) of the plasma pressure to the magnetic field pressure. An open trap with permanent particle injection is considered with injection power exceeding the loss rate. When the plasma pressure equals the pressure of the magnetic field of the trap, the mirror ratio on the axis formally becomes infinity, which implies complete longitudinal confinement. The concept includes the formation of an axisymmetric region oblong in the longitudinal direction with a thin boundary. The region of bubble shape is occupied by a dense plasma and has inside it a weak magnetic field.

At present, the structure of the so-called diamagnetic “bubble” was studied in [2, 3, 4]. The research on the high-\( \beta \) plasma as a result of neutral beam injection at C-2W device (TAE Technologies, USA) is carried out [5]. The new trap CAT (BINP SB RAS, Russia) is constructed and the last preparations for the experiments are done [6]. The diamagnetic confinement regime is planned to be studied in the later stages of the experiment.

The present paper belongs to a cycle of works on the mathematical modelling of the open magnetic systems in the diamagnetic regime [7, 8, 9] and presents the study of the ion beam behaviour on time, moment of injection and beam temperature. The second section of the paper contains a description of the equations and methods. We describe the results of numerical experiments in the third section and sum up them in the section Conclusion.
2. Equations and methods

We consider axially symmetric cylindrical trap of sizes \([R_{\text{max}} \times L_{\text{max}}]\) in cylinder coordinate system. The two coils at the ends of the trap produce the magnetic field with the mirror ratio on the axis \(R_{m} = H_{\text{max}}|_{r=0}/H_{0} = 2\), where \(H_{0}\) is the value of the magnetic field in the middle of the axis \(C = (0, L_{\text{max}}/2)\). The trap is filled with cold background hydrogen plasma, its density is \(n = n_{0}\) at the initial time moment. We do not take into account the processes of the ionization and consider the ions appearing at the point \(C\) and moving to the sidewalls of the trap [8]. The plasma is quasi-neutral and the ion and electron densities are equal: \(n_{i} = n_{e} = n\).

We use dimensionless units and following scaling factors:

- time: \(t_{0} = 1/\omega\), where \(\omega = \omega_{ci} = eH_{0}/cm_{i}\) is the ion gyrofrequency,
- length: \(L_{0} = c/\omega_{pi}\), where \(\omega_{pi} = \sqrt{4\pi n_{0}e^{2}/m_{i}}\) is the ion plasma frequency,
- velocity: the Alfven velocity \(V_{A} = H_{0}/\sqrt{4\pi m_{i}n_{0}}\)

The motion equations of the ions are following:

\[
\frac{d\vec{r}_{i}}{dt} = \vec{v}_{i},
\]

\[
\frac{d\vec{v}_{i}}{dt} = \vec{F}_{i},
\]

where \(\vec{v}_{i}, \vec{r}_{i}\) are the velocities and the coordinates of the ions, and

\[
\vec{F}_{i} = \vec{E} + [\vec{v}_{i}, \vec{H}] - \kappa \vec{j}/n
\]

is the Lorentz force combined with the force of the friction between the ion and the electron components, \(\kappa = cm_{e}/eH_{0}\tau_{ei}, \tau_{ei}\) is the characteristic ion-electron collision time [10].

From the equation for the currents the electron velocities \(\vec{V}_{e}\) can be found:

\[
\vec{j} = n(\vec{V}_{i} - \vec{V}_{e}).
\]

The mean ion velocity \(\vec{V}_{i}\) and the density \(n_{i}\) are defined by the distribution function \(f_{i}(t, \vec{r}, \vec{v})\):

\[
n(\vec{r}) = \int f_{i}(t, \vec{r}, \vec{v})d\vec{v},
\]

\[
\vec{V}_{i}(\vec{r}) = \frac{1}{n(\vec{r})} \int f_{i}(t, \vec{r}, \vec{v})\vec{v}d\vec{v}.
\]

The electron component is considered to be massless:

\[
\vec{E} + [\vec{V}_{e}, \vec{H}] + \frac{\nabla p_{e}}{2n} - \kappa \frac{\vec{j}}{n} = 0
\]

and its temperature \(T_{e}\) can be calculated according to the following equation

\[
n\left(\frac{\partial T_{e}}{\partial t} + (\vec{V}_{e} \nabla)T_{e}\right) = (\gamma - 1) \left(2\kappa \frac{\vec{j}^{2}}{n} - p_{e} \text{div}\vec{V}_{e}\right),
\]

where the adiabatic index \(\gamma = 5/3\), \(p_{e} = nT_{e}\) is the pressure of the electrons.

The displacement currents are assumed to be negligible, Maxwell’s equations are used for the the electric field \(\vec{E}\) and magnetic field \(\vec{H}\):
\[ \frac{\partial \vec{H}}{\partial t} = -\text{rot} \vec{E}, \quad (9) \]
\[ \text{rot} \vec{H} = \vec{j}, \quad (10) \]

The initial magnetic field is defined from the potential of the coils, and the electric field \( \vec{E} = 0 \). The boundary conditions for the magnetic field may be omitted when using staggered grids. The tangential components of the electric field are zero on the boundaries and \( E_\phi = 0 \) on the axis \( r = 0 \). The particles may leave the domain through the ends \( z = 0, z = Z_{\text{max}} \) of the trap and be reflected from the sidewall \( r = R_{\text{max}} \).

We use finite-difference explicit schemes on the staggered grids for the solution of the hybrid model equations with the particle-in-cell method [11, 12]. The parallel algorithm includes the domain and particle decomposition. The computation algorithms of the ion velocities and the coordinates, the densities and the mean ion velocities are modified to enable vectorization [13]. All these methods help us to perform computations with the small time step (needed for the satisfaction of the scheme stability condition) for the long time periods (tens of time units) with the increasing number of the model particles in the computations due to the permanent injection.

A detailed description of the model realization can be found in [9].

3. Numerical results

In the numerical experiments the value of the magnetic field \( H_0 = 2kG \) in the point C and the background ion density \( n_0 = 10^{12} \text{cm}^{-3} \) are the expected parameters of laboratory experiments. The scaling factors for the corresponding dimensionless units are following: \( t_0 = 5.2 \cdot 10^{-8} \text{sec} \) \( (\omega = 1.9 \cdot 10^7 \text{sec}^{-1}) \), \( L_0 = 22.8 \text{cm} \) \( (\omega_{pi} = 1.3 \cdot 10^9 \text{sec}^{-1}) \), \( V_A = 4.4 \cdot 10^8 \text{cm/sec} \). In the model problem a bunch of ions entering into the trap at the point C is considered as the injected beam. The shape of the bunch is defined by the idea of its uniform density in the absence of the electromagnetic fields, thus in the cylinder geometry \( \sim 70\% \) of the particles have the radial component of the velocity greater than the longitudinal one. We consider \( 0.28 \cdot 10^{18} \) ions entering into the domain with velocity magnitude \( |\vec{V}| = 0.2V_A \) and the ion beam temperature \( T_b \) varying from 0 to \( 10^4 \text{ eV} \). The domain has sizes \( R_{\text{max}} = 4L_0, Z_{\text{max}} = 12L_0 \).

On the initial stage the particles are injected into magnetic field domain with \( |\vec{H}| \sim 1 \). For the sufficiently high beam current the injection leads to the formation of the cavity in the magnetic field. In fig. 1,2 the magnitude \( |\vec{H}| \) and the beam ion coordinates \( (z, r) \) with the black dots at moments of time \( T = 6.4 \) and \( T = 14.4 \) respectively are presented. The left figure demonstrates the particles injected into the trap before \( T = 0.4 \), the right one - the particles injected at \( T_{\text{inj}} \in [2.0, 2.4] \).

**Figure 1.** Coordinates \((z, r)\) of the particles with \( T_{\text{inj}} < 0.4 \) at \( T = 6.4 \).

**Figure 2.** Coordinates \((z, r)\) of the particles with \( T_{\text{inj}} \in [2.0, 2.4] \) at \( T = 14.4 \).
While the particles with low radial speed manage to move out from the cavity and head for the mirrors, the beam part with high radial speed is trapped with the magnetic field and rotate within the cavity (fig. 1). The particles injected at the domain at later times move in the low magnetic field domain ($|\vec{H}| \leq 10\%$) meeting the cavity boundary with almost their initial velocity and may move out from the cavity (fig. 2).

The fig. 3,4 represent the ion distribution in the central transversal section of the trap $L_c - \delta \leq z \leq L_c + \delta$ with $L_c = L_{\text{max}}/2$ and $\delta = 0.02$ at the moment of time $T = 1.6$. With the colored points the beam ion distribution in the phase space $(r, V_r)$ is shown in fig. 3 and in the phase space $(r, V_\phi)$ in fig. 4. The colour corresponds to the particle injection moment $T_{\text{inj}}$. The crosses represent the background ion coordinates in the corresponding phase space. The black lines represent the absolute value of the magnetic field at the plane $z = L_c$, its scale is located on the right side of the figures.

The figures demonstrate, that the early injected particles (yellow colour) lose their radial speed and gain the initially zero angular speed due to the magnetic field influence. The gain decreases with the moment of injection $T_{\text{inj}}$, the “later” particles (purple color) maintain the initial velocity $|\vec{V}| = 0.2$ since the cavity with its small values of the magnetic field expands (for $|\vec{H}| \leq 10\%H_0$ the radius $R \approx 0.17$). For the small injection times the beam particles of the central section remain within the cavity.

Similarly to fig. 3, the ion distribution and the magnitudes $|\vec{H}|$ are shown in fig. 5,6 for the moment of time $T = 6.4$. A part of the early injected ions is involved in the Larmor rotation within the cavity, however, the particles injected at $T_{\text{inj}} \in [2, 2.5)$ increase their $r$-coordinates ($r \approx 0.65$), having overcome the magnetic barrier earlier and moving outside the cavity.

At the initial time moment the magnetic field magnitudes in the central section do not

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**Figure 3.** The ion distribution in the phase space $(r, V_r)$ and $|\vec{H}|$ for $z = L_c$ at $T = 1.6$.

**Figure 4.** The ion distribution in the phase space $(r, V_\phi)$ and $|\vec{H}|$ for $z = L_c$ at $T = 1.6$.

**Figure 5.** The ion distribution in the phase space $(r, V_r)$ and $|\vec{H}|$ for $z = L_c$ at $T = 6.4$.

**Figure 6.** The ion distribution in the phase space $(r, V_\phi)$ and $|\vec{H}|$ for $z = L_c$ at $T = 6.4$. 
exceed \( H_0 = 1 \), the cavity formation is accompanied with the increase of the field magnitudes: \( H_{\text{max}} = 1.27 \) for \( T = 1.6 \), and \( H_{\text{max}} = 1.25 \) for \( T = 6.4 \). The ions of the initially cold background plasma become ousted from the domain of the magnetic cavity and gather near the border of the cavity.

Having introduced the beam temperature we study the beam dynamics dependence on it. Now the injected beam at the initial time moment is considered to have the Maxwell velocity distribution with the temperature \( T_b \). In fig. 7 the beam structures at the central section \( z \in [L_c - \delta, L_c + \delta] \) with the same \( \delta \) for the different beam temperatures are presented. The beam coordinates \((r, V_r)\) in the phase space at moments \( T = 4 \) (right column), \( T = 16 \) (central column) and \( T = 128 \) (left column) are marked with blue color. The black line represents the dependence of the magnitude \(|\vec{H}|\) on \( r\)-coordinate. The scale of the magnetic field is located on the right side of the figures.

**Figure 7.** The beam dynamics with \( T_b = 0, 10, 100 \) eV.

The increase of the velocity divergence leads to the higher kinetic energy and the increase of the beam size. However, the characters of the beam dynamics in these cases resemble each other: we observe the initial deceleration of the particles in the radial direction, the separation of the particles with the magnetic barrier, which remain in the cavity and move out of it (\( r \approx 0.9 \) for \( T = 16 \)) and the bunch formation in front of the cavity, where particles rotate in the higher magnetic field (\( r \geq 1.8 \) for \( T = 128 \)).
In fig. 8 the dependence of the cavity size on the initial beam temperature is presented. The size of the cavity is defined as coordinate $r$, where the magnetic field in the central section reaches $H_c = 10\% H_0$. The moment of time $T = 16$ was taken to calculate the cavity size $R_c$. The logarithmic scale is used for the temperature axis. The horizontal line represents the cavity size $R_c$ for $T_b = 0$. At the initial stage of the evolution the temperature increase leads to the non-linear growth of the cavity size.

The computations were performed with spatial steps $h_r = h_z = 0.04$ and time step $\tau = 4\cdot10^{-6}$. The number of the background ions $N_{bg} = 1.2 \cdot 10^5$, and $N_{inj} = 10^5$ particles were injected per time unit.

4. Conclusion
The injected beam dynamics in the open magnetic trap was analyzed with methods of mathematical modelling. The possibility of formation of the cavity in the magnetic field during the interaction of the injected beam - background plasma was shown in series of numerical experiments. The detailed analysis demonstrates that the early injected beam particles are trapped with the magnetic field in case of a high ratio of the radial speed to the longitudinal one, and leave the cavity in case of small values of this ratio. The collective dynamics of the injected particles includes the rotation within the expanding domain of the magnetic field cavity, however, some part of the beam overcomes the magnetic barrier and remains outside the cavity. The beam temperature increase leads to the non-linear growth of the cavity size with the similar character of the beam dynamics. The obtained results may be used for the data analysis of the laboratory experiments in open magnetic systems.

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