A GREAT model comparison against the cosmological constant

Rubén Arjona, Llorence Espinosa-Portales, Juan García-Bellido, and Savvas Nesseris
Instituto de Física Teórica UAM-CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
(Dated: May 3, 2022)

Recently, a covariant formulation of non-equilibrium phenomena in the context of General Relativity was proposed in order to explain from first principles the observed accelerated expansion of the Universe, without the need for a cosmological constant, leading to the GREA theory. Here, we confront the GREA theory against the latest cosmological data, including type Ia supernovae, baryon acoustic oscillations, the cosmic microwave background (CMB) radiation, Hubble rate data from the cosmic chronometers and the recent $H_0$ measurements. We perform Markov Chain Monte Carlo analyses and a Bayesian model comparison, by estimating the evidence via thermodynamic integration, and find that when all the aforementioned data are included, but no prior on $H_0$, the difference in the log-evidence is $\sim -9$ in favor of GREA, thus resulting in overwhelming support for the latter over the cosmological constant and cold dark matter model (ΛCDM). When we also include priors on $H_0$, either from Cepheids or the Tip of the Red Giant Branch measurements, then due to the tensions with CMB data the GREA theory is found to be statistically equivalent with ΛCDM.

I. INTRODUCTION

Our understanding of the expanding universe is anchored in the geometric description provided by Einstein’s theory of General Relativity (GR). On the one hand, its approximate symmetries, i.e., homogeneity and isotropy at large scales, determine its background space-time to be described by a Friedmann-Lemaître-Robertson-Walker (FLRW) metric. On the other hand, its matter content is responsible for the dynamics of the scale factor, which tracks the growth of length-scales in the geometric expansion, as described by the Friedmann equations.

The currently accepted realization of FLRW cosmology is given by the Λ – Cold Dark Matter (ΛCDM) model. According to it, baryonic matter and radiation make up only a small portion of the present content of the universe. Instead, its expansion is dominated by two components which lack a fully satisfactory microscopic description. First, a cosmological constant, usually denoted by Λ, which is added to Einstein’s field equations to account for the observed late-time accelerated expansion of the universe. Second, cold (low temperature) dark (without electromagnetic interactions) matter, which was required originally to explain anomalies in the galactic rotation curves but is nowadays consistent with many other early- and late-time cosmological observables.

Even though ΛCDM seems to be the best fit to observations, the existence of a cosmological constant has been challenged on theoretical grounds. Consequently, a plethora of alternatives have been explored, which fall systematically into two groups. First, modified gravity (MG) theories attempt to deliver new dynamics at large, cosmological, scales, while leaving invariant smaller scales at which GR has been thoroughly probed. Second, dark energy (DE) models propose the addition of exotic components which lack a fully satisfactory microscopic description.

I. INTRODUCTION

Our understanding of the expanding universe is anchored in the geometric description provided by Einstein’s theory of General Relativity (GR). On the one hand, its approximate symmetries, i.e., homogeneity and isotropy at large scales, determine its background space-time to be described by a Friedmann-Lemaître-Robertson-Walker (FLRW) metric. On the other hand, its matter content is responsible for the dynamics of the scale factor, which tracks the growth of length-scales in the geometric expansion, as described by the Friedmann equations.

The currently accepted realization of FLRW cosmology is given by the Λ – Cold Dark Matter (ΛCDM) model. According to it, baryonic matter and radiation make up only a small portion of the present content of the universe. Instead, its expansion is dominated by two components which lack a fully satisfactory microscopic description. First, a cosmological constant, usually denoted by Λ, which is added to Einstein’s field equations to account for the observed late-time accelerated expansion of the universe. Second, cold (low temperature) dark (without electromagnetic interactions) matter, which was required originally to explain anomalies in the galactic rotation curves but is nowadays consistent with many other early- and late-time cosmological observables.

Even though ΛCDM seems to be the best fit to observations, the existence of a cosmological constant has been challenged on theoretical grounds. Consequently, a plethora of alternatives have been explored, which fall systematically into two groups. First, modified gravity (MG) theories attempt to deliver new dynamics at large, cosmological, scales, while leaving invariant smaller scales at which GR has been thoroughly probed. Second, dark energy (DE) models propose the addition of exotic components which lack a fully satisfactory microscopic description.

Recently, a first-principles explanation of cosmic acceleration has been proposed by two of us. This is the General Relativistic Entropic Acceleration (GREA) theory. It is not based on MG or DE. Rather, it is based on the covariant formulation of non-equilibrium formulation of thermodynamics. Entropy production during irreversible processes necessarily has an impact on Einstein field equations. This suggests the idea that entropy production or, equivalently, information coarse graining, gravitates. As such, it affects the space-time geometry.

In FLRW cosmology, irreversible processes inevitably contribute with an acceleration term to the Friedmann equations. In GREA, it is the sustained growth of the entropy associated with the cosmic horizon in open inflation scenarios that explains current cosmic acceleration.

The goal of this paper is to test the full viability of the GREA theory at the background level and compare it with the ΛCDM, against available cosmological data. To that end we consider several datasets: type Ia supernovae, baryon acoustic oscillations (BAO), cosmic microwave background radiation (CMB) and recent determinations of $H_0$. We find that, when all of them are included and no prior on $H_0$ is assumed, Bayesian evi-
ence strongly favors the GREA theory, with difference in log-evidence $\sim 9$. It is to our knowledge the first time an alternative to $\Lambda$CDM performs so remarkably. When priors on $H_0$ are included, however, GREA is statistically equivalent to a cosmological constant and future precision tests are required.

This paper is organized as follows. In Sec. II we review the the covariant formulation of non-equilibrium thermodynamics and the GREA theory built thereupon. In Sec. III we describe the cosmological data used in our analysis. In Secs. IV and V we present our results. We finish with our conclusions in Sec. VI.

II. THE GREA THEORY

A. Entropic forces in General Relativity

The GREA theory [6] is build upon the covariant formulation of non-equilibrium thermodynamics in GR [7]. This formalism provides a rigorous synthesis of the variational formulation of GR and the second law of thermodynamics. As a result, it predicts the emergence of entropic forces associated to any out-of-equilibrium phenomenon, i.e., any increase in entropy. The Einstein field equations are modified by the introduction of term that encodes such a force

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G (T_{\mu\nu} - f_{\mu\nu}) ,$$

where $f_{\mu\nu}$ is the entropic force tensor. Its precise form is obtained in the Arnowitt-Deser-Misner (ADM) formalism from the relation between the time evolution of the spatial metric and the local production of entropy. When applied to homogeneous and isotropic cosmology, it leads to the modified Friedmann equations

$$H^2 = \frac{8\pi G}{3} \left( \rho - \frac{k}{a^2} \right) ,$$

$$\frac{\dot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3p - \frac{H^2}{a^2} \right) .$$

In this setup, the cosmic fluid satisfies the out-of-equilibrium continuity equation

$$\dot{\rho} + 3H(\rho + p) = \frac{T \dot{S}}{a^3} ,$$

One concludes from the form of the entropic force in the second Friedmann equation that entropy production leads in general to a positive contribution to the acceleration of the universe.

There are two sources of entropy that fit naturally in the variational formalism. On the one hand, the matter Lagrangian may depend on the entropy or entropy density. We call this bulk entropy. On the other hand, one may be assign entropy to horizons, as inspired by black hole thermodynamics. This is achieved by adding a Gibbons-Hawking-York (GHY) term that is then interpreted as thermodynamic contribution to the action. We call this boundary entropy.

Bulk entropy is produced during cosmic expansion during certain out-of-equilibrium processes, such as (p)reheating, phase transitions or gravitational collapse. However, most of the expansion history of the universe is adiabatic and deviations from it are expected to be short-lived. This means that, although it may provide interesting phenomenology, it seems unable to explain the current accelerated expansion of the universe. On the contrary, boundary entropy can undergo a sustained increase that becomes relevant only at recent times.

B. Cosmic acceleration from boundary entropy

Let us consider an open universe nucleated in de Sitter space, i.e. in eternal inflation [8]. Inside the true vacuum bubble, local space-time as seen by a comoving observer is essentially flat if inflation lasts long enough, e.g. of order $N \sim 70$ e-folds. Nevertheless, the bubble walls are still located at a finite coordinate distance and, thus, we can define a true casual horizon with $\sqrt{-k} = a_0 H_0$. Inspired by this scenario we propose a GHY thermodynamic term that induces an entropic contribution satisfying [6]

$$\rho_H a^2 = \frac{T_H S_H}{a} = \frac{1}{2G} \frac{\sinh(2a_0 H_0 \eta)}{a_0 H_0} ,$$

$$\frac{\Omega_K}{1 - \Omega_K} = e^{-2N} \left( \frac{T_{th}}{T_{eq}} \right)^2 (1 + z_{eq}) ,$$

where $\eta$ is the conformal time, $\Omega_K$ is the curvature parameter inside the inflated patch, $T_{th}$ is the reheating temperature, $T_{eq}$ and $z_{eq}$ are, respectively, the temperature and redshift at matter-radiation equality. We now introduce, for convenience, the time coordinate $\tau = a_0 H_0 \eta$ and denote with primes the derivatives w.r.t. to $\tau$. Then the second Friedmann equation becomes

$$\left( \frac{a'}{a_0} \right)^2 = \Omega_M \frac{a}{a_0} + \Omega_K \frac{a^2}{a_0^2} + \frac{4\pi}{3} \Omega_K^{3/2} \frac{a^2}{a_0^2} \sinh(2\tau) ,$$

where $\Omega_M$ is the matter density parameter.

Thus, the expansion of the universe is affected by the increase in entropy of the causal horizon. Since the causal horizon keeps growing, the entropic term eventually dominates and leads to a late-time cosmic acceleration. Contrary to a cosmological constant, however, the entropic term is diluted with the expansion, albeit at a slower rate than radiation and dust, and the universe ends in Minkowski space-time in the far future.

From the mathematical point of view, this modified second Friedmann equation is a differential equation in re-scaled conformal time $\tau$. It is, however, an integro-differential equation in cosmic time $t$, unlike the usual second Friedmann equation. Physically, this is related to the nature of the entropic term associated to the causal horizon: it builds up as the expansion proceeds.
III. THE DATA

Here we present in detail the compilations of data we use in our analysis.

A. The $H(z)$ data

First, we consider the Hubble rate data, which are obtained via two complementary ways. The first one is from the redshift drift of distant objects over long periods of time, usually on the order of a decade. This is possible as in the FLRW metric the Hubble parameter $H(z)$ can be related to the rate of change of redshift with respect to time, i.e. $H(z) = -rac{1}{1+z} \frac{dz}{dt}$ [1]. In particular, the $H(z)$ data are determined via the differential age method using the evolution of $D_A$, 4000, which is a spectral feature of very massive and passive galaxies. The systematics in this case mainly come from the metallicity, via the M11 and BC03 models discussed in Ref. [10]. However, it has been shown that the systematics can be kept under control by implementing strict selection criteria [10].

On the other hand, some measurements also come from the clustering of galaxies or quasars, which is a probe of the Hubble expansion via the determination of the BAO in the radial direction [11]. Furthermore, we assume that the $H(z)$ data are uncorrelated with each other. Finally, here we will make use of the compilation from Ref. [12] that contains 36 points in the redshift range $0.07 \leq z \leq 2.34$ and which are in the form ($z_i, H_i, \sigma_{H_i}$), as is shown in Table I.

B. The SnIa data

We also use the Pantheon supernovae type Ia data (SnIa) compilation of Ref. [22] of 1048 Supernovae Ia points in the redshift range $0.01 < z < 2.26$, along with their covariance matrix. The apparent magnitude $m_B$ of the SnIa points is given by

$$m_B = 5 \log_{10} \left[ \frac{D_L(z)}{1\text{Mpc}} \right] + 25 + M_B,$$

where $D_L(z)$ is the luminosity distance and $M_B$ the absolute magnitude. Finally, the parameter $M_B$ is marginalized over, according to the recipe in Appendix C of Ref. [23].

C. The BAO

The compilation of BAO data used in our analysis includes points from 6dFGS [24], WiggleZ [18], the MGS, ELG, LRG, quasars and DR12 galaxy samples BAO points from the completed SDSS-IV eBOSS survey [25], the year 3 DES [26, along with the Lyman-$\alpha$ (Ly$\alpha$) absorption and quasars, auto and cross correlation points from Ref. [27].

| $z$ | $H(z)$ | $\sigma_H$ | Ref. |
|-----|--------|------------|------|
| 0.07 | 69.0 | 19.6 | 14 |
| 0.09 | 69.0 | 12.0 | 15 |
| 0.12 | 68.6 | 26.2 | 14 |
| 0.17 | 83.0 | 8.0 | 15 |
| 0.179 | 75.0 | 5.0 | 16 |
| 0.2 | 72.9 | 29.6 | 16 |
| 0.27 | 77.0 | 14.0 | 16 |
| 0.28 | 88.8 | 36.6 | 16 |
| 0.35 | 82.7 | 8.4 | 17 |
| 0.352 | 83.0 | 14.0 | 16 |
| 0.3802 | 83.0 | 13.5 | 16 |
| 0.4 | 95.0 | 17.0 | 16 |
| 0.4004 | 77.0 | 10.2 | 16 |
| 0.4247 | 87.1 | 11.2 | 16 |
| 0.44 | 82.6 | 7.8 | 16 |
| 0.44497 | 92.8 | 12.9 | 16 |
| 0.4783 | 80.9 | 9.0 | 16 |

TABLE I. The $H(z)$ data used in our analysis (in units of km s$^{-1}$Mpc$^{-1}$). This compilation, which was presented in Ref. [12], is partly based on those of Refs. [10] and [13].

In what follows, we will briefly discuss the functions which are used to describe the BAO data. A key quantity is the ratio of the sound horizon at the drag redshift $r_s(z_d)$ to the so-called dilation scale $D_V(z)$:

$$d_z = \frac{r_s(z_d)}{D_V(z)},$$

where the comoving sound horizon is

$$r_s(z_d) = \int_{z_d}^\infty c_s(z) \frac{dz}{H(z)},$$

where the redshift at the dragging epoch $z_d$ is given for example by Eq. (4) of [29], however to actually evaluate the integral of Eq. (9) we will use the fitting formula from Ref. [29], which is obtained via machine learning improved fits of the full recombination history, resulting in

$$z_d = \frac{1 + 428.169 \omega_b^{0.256459} \omega_m^{0.616388} + 925.56 \omega_m^{0.751615}}{\omega_m^{0.714129}},$$

and which is accurate up to $\sim 0.001\%$ [29]. In Eq. (8) we also defined the dilation scale $D_V(z)$, which is given by

$$D_V(z) = \left[ (1 + z)^2 D_A(z)^2 \frac{cz}{H(z)} \right]^{1/3},$$

where $D_A(z)$ is the angular diameter distance. Finally, we can also define the Hubble and comoving angular di-
ameter distances, via
\[ D_H(z) = c/H(z), \]
\[ D_M(z) = (1 + z)D_A(z). \] (12) (13)

Next we describe the actual BAO data. In particular, the 6dFGs and WiggleZ points are given by
\[ \frac{z}{d_z}, \sigma_{d_z} \]
0.106 0.336 0.015
0.44 0.073 0.031
0.60 0.0726 0.0164
0.73 0.0592 0.0185

where their inverse covariance matrix is
\[ C_{ij}^{-1} = \begin{pmatrix}
0.015 & 0 & 0 \\
0 & 1040.3 & -807.5 & 336.8 \\
0 & -807.5 & 3720.3 & -1551.9 \\
0 & 336.8 & -1551.9 & 2914.9 \\
\end{pmatrix} \] (15)

with the \( \chi^2 \) is then given by
\[ \chi^2_{6dFGs, Wig} = V^i C_{ij}^{-1} V^j, \] (16)
where the difference vector is given by \( V^i = d_{z,i} - d_z(z_i) \).

The BAO measurements for MGS and eBOSS ELGs are given by \( D_V/r_s = 1/d_z \) via
\[ \frac{z}{1/d_z}, \sigma_{1/d_z}, \]
0.15 4.65676 0.168135
0.85 18.33 0.595

and the \( \chi^2 \) is
\[ \chi^2_{MGS, ELG} = \sum_{i=1}^{2} \left( \frac{1/d_{z,i} - 1/d_z(z_i)}{\sigma_{1/d_z,i}} \right)^2. \] (17)

The BAO data from DES year 3 are of the form \( D_M(z)/r_s \) with \( [z, D_M(z)/r_s, \sigma_{D_M(z)/r_s}] = (0.835, 18.92, 0.51) \) and the \( \chi^2 \) given by
\[ \chi^2_{DES} = \left( \frac{D_{M,i}/r_s - D_M(z_i)/r_s}{\sigma_{D_{M,i}/r_s}} \right)^2. \] (18)

We also include the eBOSS LRG data, which are given by \( (z, D_M/r_s, D_H/r_s) = (0.698, 17.8581, 19.3261) \) with an inverse covariance matrix
\[ C_{ij}^{-1} = \begin{pmatrix}
10.4515 & 2.14754 \\
2.14754 & 3.96466 \\
\end{pmatrix}, \] (19)

so that the \( \chi^2 \) is
\[ \chi^2_{LRG} = V^i C_{ij}^{-1} V^j, \] (20)
where the difference vector is
\[ V^i = [D_{M,i} - D_M(z_i), D_{H,i} - D_H(z_i)]/r_s. \] (21)

Similarly, the eBOSS QSO points are given by \( (z, D_M/r_s, D_H/r_s) = (1.48, 30.6876, 13.2609) \) with an inverse covariance matrix
\[ C_{ij}^{-1} = \begin{pmatrix}
1.84606 & -1.0342 \\
-1.0342 & 3.86146 \\
\end{pmatrix}, \] (22)
so that the \( \chi^2 \) is
\[ \chi^2_{QSO} = V^i C_{ij}^{-1} V^j, \] (23)
where the difference vector is
\[ V^i = [D_{M,i} - D_M(z_i), D_{H,i} - D_H(z_i)]/r_s. \] (24)

We also include the BAO data from Lyα and the cross/auto correlations with the quasars, which are of the form \( f_{BAO} = (D_H/r_s, D_M/r_s) \) and are given by
\[ \frac{z}{f_{BAO}} \]
2.334 8.99 0.429418
0.38 10.2341 24.9806
0.51 13.366 22.3166

with a correlation coefficient \( \rho = -0.45 \), so that the \( \chi^2 \) given by
\[ \chi^2_{Lyα} = V^i C_{ij}^{-1} V^j, \] (25)
where the difference vector is
\[ V^i = [D_{M,i} - D_M(z_i), D_{H,i} - D_H(z_i)]/r_s. \] (26)

Finally, the eBOSS DR12 galaxy samples data are of the form \( f_{BAO} = (D_M/r_s, D_H/r_s) \) and are given by
\[ \frac{z}{f_{BAO}} \]
2.334 37.5 2.77308

with an inverse covariance matrix
\[ C_{ij}^{-1} = \begin{pmatrix}
52.584 & 5.15947 & -20.0391 & -3.54599 \\
5.15947 & 2.8048 & -2.10831 & -1.61178 \\
-20.0391 & -2.10831 & 36.8787 & 5.7886 \\
-3.54599 & -1.61178 & 5.7886 & 4.64349 \\
\end{pmatrix}, \] (27)

while the difference vector is
\[ V^i = [D_{M,0.38}, D_{H,0.38}, D_{M,0.51}, D_{H,0.51}]/r_s - [D_M(0.38), D_H(0.38), D_M(0.51), D_H(0.51)]/r_s, \] (28)
with the \( \chi^2 \) given by
\[ \chi^2_{DR12} = V^i C_{ij}^{-1} V^j. \] (29)

Finally, the total \( \chi^2 \) is then given by
\[ \chi^2_{BAO} = \chi^2_{6dFGs, Wig} + \chi^2_{MGS, ELG} + \chi^2_{DES} + \chi^2_{LRG} + \chi^2_{QSO} + \chi^2_{Lyα} + \chi^2_{DR12}. \] (30)
Note that in the latter equation we assume that the data are independent with each other, thus we can simply add the $\chi^2$ terms. However, since some of the points are derived by the same survey, inevitably there will be common overlapping galaxies between the datasets, which will result to strong covariances, which is clearly a limitation in our analysis.

For example for the WiggleZ data the correlations between the points is given by the covariance matrix $C_{ij}$, thus we have included this information in our analysis. However, overall the full correlations are not publicly available and it is impossible to correctly estimate a covariance matrix, even if a few attempts have been made in the literature, e.g. for a similar discussion for the growth-rate data see Ref. [30].

D. The CMB shift parameters

The main effects of the new entropy terms will be twofold: one on the background Friedmann equation given by Eq. (4) and another on possible contributions to the perturbations as seen by Eq. (5). Currently, a perturbation theory for the GREA model is not readily available, thus in this work we only focus on the background contributions and leave the full perturbation analysis for future work.

Thus, we can use the so called CMB shift parameters [31,32]. Furthermore, this simplifies the analysis as most Boltzmann codes calculate the conformal time, after having calculated the Hubble parameter, which make modifications of codes like CAMB or CLASS highly non-trivial. The CMB shift parameters encapsulate the geometric information in the CMB spectrum, via the location of the peaks and are in a sense a compressed form of the CMB likelihood. They are given by

$$R \equiv \sqrt{\Omega_m h^2} r(z_{\text{rec}}) / c, \quad (34)$$
$$l_a \equiv \pi r(z_{\text{rec}}) / r_s(z_{\text{rec}}), \quad (35)$$

where $r_s(z_{\text{rec}})$ is the sound horizon at recombination and $z_{\text{rec}}$ is the redshift at recombination, which can be calculated by the fitting formula of Ref. [29].

As here we are interested in non-flat universes we use the Planck 2018 chains base_omegak_plikHM_TTTEEE_lowl_lowE_lensing to estimate the data vectors for $(R, l_a, \Omega_k h^2, h)$. Note that the curvature is in fact included in our compressed likelihood as the Planck 2018 chains we used include a free curvature parameter, as denoted by the name of the chain. Thus, the curvature appears directly in the likelihood, since the parameters $R$ and $l_a$ given by Eqs. (34)-(35) depend explicitly on $\Omega_k$. Following then the procedure of Refs. [31,32] we find

$$v = \begin{pmatrix} 1.7448 \\ 302.21792 \\ 0.02249 \\ 0.63549 \end{pmatrix}, \quad (36)$$

while the covariance matrix is

$$C_v = 10^{-8} \times \begin{pmatrix} 2604.4383 & 16594.36494 & -58.52126 & 4633.20089 \\ 16594.36494 & 738151.92316 & -410.26313 & 20120.28532 \\ -58.52126 & -410.26313 & 2.58145 & -93.88730 \\ 4633.20089 & 20120.28532 & -93.88730 & 49803.48059 \end{pmatrix}. \quad (37)$$

Thus, the difference vector can be written as

$$V = [R, l_a, \Omega_k h^2, h] - v, \quad (38)$$

thus, the $\chi^2$ for the CMB data can be written as

$$\chi^2_{\text{CMB}} = V C_v^{-1} V. \quad (39)$$

E. The Riess $H_0$ prior

We also use the $H_0$ measurement from Ref. [33], which comes from a sample of 75 Milky Way Cepheids, which were used to recalibrate the extragalactic distance ladder. This approach gives

$$H_0^{(\text{R})} = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (40)$$

Thus, the $\chi^2$ term is just

$$\chi^2_{H_0} = \left( \frac{H_0^{(\text{R})} - H_0}{\sigma_{H_0^{(\text{R})}}} \right)^2, \quad (41)$$

where the Hubble parameter today is given by $H_0 = 100 h$ in the $\Lambda$CDM model and by evaluating Eq. (6) at $\tau = \tau_0$, i.e. at today, for the GREAT model.

F. The TRGB $H_0$ prior

Finally, we also include the $H_0$ measurement from Ref. [34], which comes from the Tip of the Red Giant Branch (TRGB) method using stars in the Large Magellanic Cloud (LMC). This approach gives

$$H_0^{(\text{TRGB})} = 69.6 \pm 0.8 \text{ (stat)} \pm 1.7 \text{ (syst)} \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (42)$$

Then, the $\chi^2$ term is just

$$\chi^2_{H_0} = \left( \frac{H_0^{(\text{TRGB})} - H_0}{\sigma_{H_0^{(\text{TRGB})}}} \right)^2, \quad (43)$$

where the Hubble parameter today is given by $H_0 = 100 h$ in the $\Lambda$CDM model and by evaluating Eq. (6) at $\tau = \tau_0$, i.e. at today, for the GREAT model.
TABLE II. The values of both the linear and the logarithmic Jeffreys’ scale.

| $B_{ij}$ | $\ln B_{ij}$ | Evidence |
|---------|---------------|-----------|
| < 3     | < 1.1         | Weak      |
| < 20    | < 3           | Definite  |
| < 150   | < 5           | Strong    |
| > 150   | > 5           | Very Strong |

IV. MCMC

In this section we present the results of our Markov Chain Monte Carlo (MCMC) analysis after fitting the data described in Sec. III. Our total likelihood function $L_{\text{tot}}$ can be given as the product of the various likelihoods as

$$L_{\text{tot}} = L_{\text{SnIa}} \times L_{\text{BAO}} \times L_{\text{H(z)}} \times L_{\text{cmb}} \times L_{\text{H_0}},$$

which can also be translated to the total $\chi^2$ via $\chi^2_{\text{tot}} = -2 \ln L_{\text{tot}}$ or

$$\chi^2_{\text{tot}} = \chi^2_{\text{SnIa}} + \chi^2_{\text{BAO}} + \chi^2_{\text{H(z)}} + \chi^2_{\text{cmb}} + \chi^2_{\text{H_0}}. \tag{44}$$

Our $\chi^2$ is given by Eq. (44) and the parameter vectors for both the $\Lambda$CDM and GREAT models are given by: $p_{\text{Model}} = (\Omega_m, \Omega_b, h, \Omega_k, \cdots)$. Then, the best-fit parameters and their uncertainties were obtained via an MCMC code written by one of the authors. Moreover, we assumed priors for the parameters of the $\Lambda$CDM model given by $\Omega_{m0} \in [0.01, 0.5]$, $\Omega_b h^2 \in [0.015, 0.035]$, $\Omega_k \in [-0.1, 0.1]$, $h \in [0.5, 1]$, while for the GREAT model we chose $\Omega_{m0} \in [0.01, 0.5]$, $\Omega_b h^2 \in [0.015, 0.035]$, $\Omega_k \in [0.00001, 0.1]$, $h \in [0.5, 1]$. Finally, we obtained approximately $O(10^7)$ points for each of the models.

In order to compare the quality of fit between the models, we use Bayesian model comparison by means of the evidence $B$. The latter is calculated as the integral of the evidence is very strong. Note however, that it was shown in Ref. 38 that the Jeffreys’ scale has to be interpreted with care, especially in the case of nested models, as it may result to biased conclusions.

Finally, for easy reference we show the particular values of both the linear and the logarithmic Jeffreys’ scale in Table II.

V. RESULTS

Here we present the results of our analysis for both the $\Lambda$CDM and the GREAT models, using the methodology described in the previous sections. In all cases in the Tables that follow we will show the mean values, $1\sigma$ errors of the parameters for the GREAT and $\Lambda$CDM models respectively, along with the minimum $\chi^2$, the log-evidence $\log Z(1)$ and the difference of the log-evidence with respect to the $\Lambda$CDM model $\Delta \log Z(1)_{\Lambda,i} \equiv \log Z(1)_{\Lambda} - \log Z(1)_{i}$.

Similarly, in the figures we will always show the 68.3%, 95.5% and 99.7% confidence contours for the GREAT (left panel) and $\Lambda$CDM (right panel) models respectively. In all cases, the black points will correspond to the mean values of the parameters from the MCMC, the blue shaded regions will be the confidence levels, while the red points will correspond to the Planck 2018 best-fit ($\Omega_{m0}, \Omega_b h^2, \Omega_k, H_0) = (0.315, 0.0224, 0.001, 67.4)$, with $H_0$ given in units of km s$^{-1}$ Mpc$^{-1}$.

First, we consider the case when we include all of the data, except the priors on $H_0$, as they may be in some tension with other data [39 40]. In particular, in Table II we provide the results for the relevant parameters of the two models and as can be seen, in this case the
The 68% and 95% confidence contours for the GREAT (left panel) and ΛCDM (right panel) models respectively, including all the data, but no prior on $H_0$. The red points/dashed lines correspond to the Planck best-fit ($\Omega_{m,0}, \Omega_{b,0} h^2, \Omega_{k,0}, H_0) = (0.315, 0.0224, 0.001, 67.4)$, where $H_0$ is given in units of km s$^{-1}$ Mpc$^{-1}$.

**TABLE III.** Here we present the results of the MCMC analysis when not including any $H_0$ prior. In particular, we show the mean values, 1σ errors of the parameters for the GREAT and ΛCDM models respectively, along with the minimum $\chi^2$ and the log-evidence $\log Z(1)$, see appendix A and the difference of the log-evidence with respect to the ΛCDM model $\Delta \log Z(1)_{\Lambda,i}$, which give a Bayes ratio of $B_{\Lambda,G} = \exp \left[ \Delta \log Z(1)^{\Lambda,G} \right] = \exp (-9.006) \sim 1/8150$, thus resulting in very strong evidence in favor of the GREAT model according to the Jeffreys’ scale [38]. Note that $H_0$ is given in units of km s$^{-1}$ Mpc$^{-1}$.

| Model | $\Omega_{m,0}$ | $\Omega_{b,0} h^2$ | $\Omega_k,0$ | $H_0$ | $\chi^2_{\text{min}}$ | $\Delta \log Z(1)_{\Lambda,i}$ |
|-------|----------------|------------------|-------------|-------|----------------|------------------|
| ΛCDM  | 0.3057 ± 0.0056 | 0.0224 ± 0.0002  | 0.0012 ± 0.0018 | 68.08 ± 0.58 | 1075.63 | -557.515 |
| GREAT | 0.3522 ± 0.0190 | 0.0225 ± 0.0001  | 0.0010 ± 0.0002 | 68.38 ± 0.48 | 1071.35 | -548.509 |

**TABLE IV.** The breakdown of the $\chi^2$ for the two models and the different datasets used in our analysis, in the case of not including any $H_0$ prior. The best-fit parameters from the MCMC are given in Table III. As can be seen, the main contribution in the difference of the $\chi^2$s comes from the CMB and to a lesser extent from the $H(z)$ and BAO data, while the values for the SnIa are practically the same.

| Model  | CMB | BAO | H(z) | $\chi^2_{\text{tot}}$ |
|--------|-----|-----|------|------------------|
| ΛCDM   | 4.28 | 13.99 | 1034.84 | 22.52 | 1075.63 |
| GREAT  | 0.07 | 14.39 | 1034.82 | 22.10 | 1071.35 |

As this case gives the strongest result in favor of GREAT, we also analyse in more detail what piece of experimental data is contributing to this improvement over the ΛCDM model. In particular, as can be seen in Table IV there is a difference of $\chi^2$ of $\sim 4.3$ between ΛCDM ($\chi^2 = 1075.63$) and GREAT ($\chi^2 = 1071.35$) and the different datasets contribute in different ways. Specifically, the main effect comes from the CMB data ($\delta\chi^2 \sim 4.21$) and to a much lesser degree from the $H(z)$ data ($\delta\chi^2 \sim 0.42$). On the other hand the BAO favor ΛCDM slightly ($\delta\chi^2 \sim -0.4$) and the $\chi^2$ for the SnIa is practically the same.

Second, we also consider the case where we include all the data, along with the Riess $H_0$ prior of Ref. [33]. In Table V we provide the results for the relevant param-
FIG. 2. The 68.3%, 95.5% and 99.7% confidence contours for the GREAT (left panel) and ΛCDM (right panel) models respectively, including all data and the Riess H₀ prior. The red points/dashed lines correspond to the Planck best-fit \((\Omega_{m,0},\Omega_{b,0}h^2,\Omega_k,0) = (0.315,0.0224,0.001,67.4)\), where \(H_0\) is given in units of km s\(^{-1}\) Mpc\(^{-1}\).

| Model  | \(\Omega_{m,0}\) | \(\Omega_{b,0}h^2\) | \(\Omega_k,0\) | \(H_0\) | \(\chi^2_{\text{min}}\) | \(\Delta\log Z(1)\) | \(\log Z(1)\) |
|--------|-----------------|-----------------|--------------|---------|----------------|--------------------|----------------|
| ΛCDM   | 0.2995 ± 0.0051 | 0.0224 ± 0.0002 | 0.0017       | 68.85 ± 0.53 | 1088.79 -557.588 | 0                  |                |
| GREAT  | 0.3350 ± 0.0155 | 0.0225 ± 0.0001 | 0.0008       | 68.98 ± 0.44 | 1083.39 -557.974 | 0.386              |                |

TABLE V. Here we present the results of the MCMC analysis when we include all the available data and the Riess H₀ prior, as discussed in the previous sections. In particular, we show the mean values, 1σ errors of the parameters for the GREAT and ΛCDM models respectively, along with the minimum \(\chi^2\) and the log-evidence \(\log Z(1)\), see appendix A and the difference of the log-evidence with respect to the ΛCDM model \(\Delta \log Z(1)\). The latter give a Bayes ratio of \(B_{\Lambda,G} = \exp [\Delta \log Z(1)_{\Lambda,G}] = \exp (0.386) \sim 1.47\), thus resulting in the two models being considered statistically equivalent according to the Jeffreys’ scale \([38]\). Note that \(H_0\) is given in units of km s\(^{-1}\) Mpc\(^{-1}\).

eters of the two models and as can be seen, the thermodynamic integration gives a Bayes ratio of \(B_{\Lambda,G} = \exp [\Delta \log Z(1)_{\Lambda,G}] = \exp (0.386) \sim 1.47\), thus resulting in the two models being considered statistically equivalent according to the Jeffreys’ scale, see Table II and Ref. \([38]\). The corresponding confidence contours are given in Fig. 2.

Furthermore, in Fig. 3 we show the confidence contours for the \(w_0, w_a\) parameters of the \(w_0w_a\)CDM model, which has an equation of state \(w(a) = w_0 + w_a(1 - a)\) \([11]\) \([32]\). As can be seen, as predicted by GREAT, the point \((w_0, w_a) = (-0.946, -0.318)\) \([6]\), denoted by an orange star in the plot, is very close to the best-fit of the model and in good agreement with observations in this case.

Finally, we also consider the case with all the data and the TRGB \(H_0\) prior of Ref. \([34]\). In Table VI we provide the results for the relevant parameters of the two models and as can be seen, the thermodynamic integration gives a Bayes ratio of \(B_{\Lambda,G} = \exp [\Delta \log Z(1)_{\Lambda,G}] = \exp (-0.373) \sim 0.689\), thus resulting in the two models being considered statistically equivalent according to the Jeffreys’ scale, see Table II and Ref. \([38]\). The corresponding confidence contours are given in Fig. 4.

VI. CONCLUSIONS

The matter and energy content of the universe can only be inferred indirectly from the light that reaches us from distant sources which are affected by the expansion of the universe. It is therefore needed to interpret those measurements in the context of a given framework. We have assumed a spatially curved, homogeneous and isotropic universe and determined the parameters of the model that best fit the currently available data, from CMB to Large Scale Structure (LSS), SNIa and local rate of ex-
The CPL model for the w star to the prediction of GREAT (best-fit value, the red dot to the ΛCDM model and the orange data and the Riess prior. The black dot corresponds to the

FIG. 3. The 68.3% and 95.5% confidence contours for the CPL model for the w0, wa parameters, when including all data and the Riess prior. The black dot corresponds to the best-fit value, the red dot to the ΛCDM model and the orange star to the prediction of GREAT (w0, wa) = (−0.946, −0.318) [6].

TABLE VI. Here we present the results of the MCMC analysis when we include all the available data and the TRGB H0 prior, as discussed in the previous sections. In particular, we show the mean values, 1σ errors of the parameters for the GREAT and ΛCDM models respectively, along with the minimum χ2 and the log-evidence log Z(1), see appendix A and the difference of the log-evidence with respect to the ΛCDM model Δ log Z(1)Λ, i  ΛCDM i, GREA = log (Z(1))i = log (Z(1))Λ − log (Z(1))i. The latter give a Bayes ratio of BΛ,G = exp [Δ log Z(1)Λ, G] = exp (−0.373) ~ 0.689, thus resulting in the two models being considered statistically equivalent according to the Jeffreys’ scale [38]. Note that H0 is given in units of km s−1 Mpc−1.

| Model   | Ωm,0 | Ωb,0 h2 | Ωk,0 | H0  | χ2 min | log Z(1) | Δ log Z(1)Λ,i |
|---------|-------|----------|-------|------|--------|----------|----------------|
| ΛCDM   | 0.3047 ± 0.0052 | 0.0224 ± 0.0001 | 0.0015 ± 0.0017 | 68.20 ± 0.54 | 1076.23 | -550.484 | 0 |
| GREAT   | 0.3502 ± 0.0157 | 0.0225 ± 0.0001 | 0.0010 ± 0.0002 | 68.46 ± 0.45 | 1071.74 | -550.111 | -0.373 |

ACKNOWLEDGEMENTS

The authors acknowledge support from the Research Project PGC2018-094773-B-C32 and the Centro de Excelencia Severo Ochoa Program SEV-2016-0597. S. N. also acknowledges support from the Ramón y Cajal program through Grant No. RYC-2014-15843. The work of L.E.P. is funded by a fellowship from “La Caixa” Foundation (ID 100010434) with fellowship code.
FIG. 4. The 68.3%, 95.5% and 99.7% confidence contours for the GREAT (left panel) and ΛCDM (right panel) models respectively, including all data and the TRGB prior on $H_0$. The red points/dashed lines correspond to the Planck best-fit $(\Omega_m, \Omega_b, h^2, \Omega_k, H_0) = (0.315, 0.0224, 0.001, 67.4)$, where $H_0$ is given in units of km s$^{-1}$ Mpc$^{-1}$.

LCF/BQ/IN18/11660041 and the European Union Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 713673.

Appendix A: Thermodynamic integration for the Bayesian evidence

In order to estimate the evidence integral, we can use thermodynamic MCMC integration [35, 36]. To do so, we define the evidence as a function of the inverse temperature $\beta = 1/T$ as follows:

$$ Z(\beta) = \int d^n x \mathcal{L}(x)^\beta p(x), \quad (A1) $$

where $x$ are the $n$ parameters of the model, the likelihood is $\mathcal{L}(x)$ and finally the prior $p(x)$ is assumed to be normalized, i.e. $\int d^n x p(x) = 1$. Then, the actual Bayes factor, i.e. the evidence, of the model is just $Z(1)$. Furthermore, it is easy to show that

$$ \frac{d \ln Z}{d \beta} = \frac{1}{Z(\beta)} \int d^n x (\ln \mathcal{L}) \mathcal{L}(x)^\beta p(x) = \langle \ln \mathcal{L} \rangle_\beta, \quad (A2) $$

where $\langle \ln \mathcal{L} \rangle_\beta$ is the average log-likelihood over the posterior at an inverse temperature $\beta$. Since $Z(0) = 1$, as the prior is normalized, then we get

$$ \ln Z(1) = \int_0^1 d\beta \langle \ln \mathcal{L} \rangle_\beta. \quad (A3) $$

The integral in the last expression can be calculated by estimating the average log-likelihood of each chain at a given inverse temperature and then performing the integral numerically.

[1] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri, and D. Scolnic, Astrophys. J. 876, 85 (2019), arXiv:1903.07603 [astro-ph.CO].
[2] N. Aghanim et al. (Planck), Astron. Astrophys. 641, A6 (2020) [Erratum: Astron. Astrophys. 652, C4 (2021)], arXiv:1807.06209 [astro-ph.CO].
[3] M. Kunz, S. Nesseris, and I. Sawicki, Phys. Rev. D 92, 063006 (2015) [arXiv:1507.01486 [astro-ph.CO]].
[4] R. Arjona and S. Nesseris, JCAP 11, 042, arXiv:2001.11420 [astro-ph.CO].
[5] R. Arjona and S. Nesseris, Phys. Rev. D 101, 123525 (2020) [arXiv:1910.01529 [astro-ph.CO]].
[6] J. Garcia-Bellido and L. Espinosa-Portales, Phys. Dark Univ. 34, 100892 (2021), arXiv:2106.16014 [gr-qc].
[7] L. Espinosa-Portales and J. Garcia-Bellido, Phys. Dark Univ. 34, 100893 (2021), arXiv:2106.16012 [gr-qc].
[8] A. D. Linde, M. Sasaki, and T. Tanaka, Phys. Rev. D59, 123522 (1999), arXiv:astro-ph/9901135 [astro-ph].
[9] R. Jimenez and A. Loeb, Astrophys. J. 573, 37 (2002), arXiv:astro-ph/0106145 [astro-ph].
[10] M. Moresco, L. Pozzetti, A. Cimatti, R. Jimenez, C. Maraston, L. Verde, D. Thomas, A. Citro, R. Tojeiro, and D. Wilkinson, JCAP 1605 (05), 014, arXiv:1601.01701 [astro-ph.CO].
[11] E. Gaztanaga, A. Cabre, and L. Hui, Mon. Not. Roy. Astron. Soc. 399, 1663 (2009), arXiv:0807.3551 [astro-ph].
[12] R. Arjona, W. Cardona, and S. Nesseris, Phys. Rev. D99, 043516 (2019), arXiv:1811.02469 [astro-ph.CO].
[13] R.-Y. Guo and X. Zhang, Eur. Phys. J. C76, 37 (2002), arXiv:astro-ph/0106145 [astro-ph].
[14] M. Moresco et al., JCAP 1208, 006 arXiv:1201.3609 [astro-ph.CO].
[15] C.-H. Chang and Y. Wang, Mon. Not. Roy. Astron. Soc. 435, 255 (2013), arXiv:1209.2010 [astro-ph.CO].
[16] C. Blake et al., Mon. Not. Roy. Astron. Soc. 425, 405 (2012), arXiv:1204.3674 [astro-ph.CO].
[17] L. Anderson et al. (BOSS), Mon. Not. Roy. Astron. Soc. 441, 24 (2014), arXiv:1312.4877 [astro-ph.CO].
[18] M. Moresco, Mon. Not. Roy. Astron. Soc. 450, L16 (2015), arXiv:1503.01116 [astro-ph.CO].
[19] T. Delubac et al. (BOSS), Astron. Astrophys. 574, A50 (2015), arXiv:1404.1801 [astro-ph.CO].
[20] D. M. Scolnic et al., Astrophys. J. 859, 101 (2018), arXiv:1710.00845 [astro-ph.CO].
[21] A. Conley et al. (SNLS), Astrophys. J. Suppl. 192, 1 (2011), arXiv:1104.1443 [astro-ph.CO].
[22] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, Mon. Not. Roy. Astron. Soc. 416, 3017 (2011), arXiv:1106.3366 [astro-ph.CO].
[23] S. Alam et al. (eBOSS), Phys. Rev. D 103, 083533 (2021), arXiv:2007.08991 [astro-ph.CO].
[24] T. M. C. Abbott et al. (DES), (2021), arXiv:2107.04646 [astro-ph.CO].
[25] H. du Mas des Bourboux et al., Astrophys. J. 901, 153 (2020), arXiv:2007.08995 [astro-ph.CO].
[26] D. J. Eisenstein and W. Hu, Astrophys. J. 496, 605 (1998), arXiv:astro-ph/9709112 [astro-ph].
[27] A. Aizpuru, R. Arjona, and S. Nesseris, Phys. Rev. D 104, 043521 (2021), arXiv:2106.06428 [astro-ph.CO].
[28] S. Alam, S. Ho, and A. Silvestri, Mon. Not. Roy. Astron. Soc. 456, 3743 (2016), arXiv:1509.05034 [astro-ph.CO].
[29] Y. Wang and P. Mukherjee, Phys. Rev. D 76, 103533 (2007), arXiv:astro-ph/0703780.
[30] Z. Zhai and Y. Wang, JCAP 07, 005 arXiv:1811.07425 [astro-ph.CO].
[31] A. G. Riess, S. Casertano, W. Yuan, J. B. Bowers, L. Macri, J. C. Zinn, and D. Scolnic, Astrophys. J. Lett. 908, L6 (2021), arXiv:2012.08534 [astro-ph.CO].
[32] W. L. Freedman, B. F. Madore, T. Hoyt, I. S. Jang, R. Beaton, M. G. Lee, A. Monson, J. Neely, and J. Rich 10.3847/1538-4357/ab7339 (2020), arXiv:2002.01550 [astro-ph.GA].
[33] M. Beltran, J. Garcia-Bellido, J. Lesgourgues, A. R. Lidke, and A. Slosar, Phys. Rev. D 71, 063532 (2005), arXiv:astro-ph/0501477.
[34] N. Lartillot and H. Philippe, Systematic Biology 55, 195 (2006), https://academic.oup.com/sysbio/article-pdf/55/2/195/106355/106355106355500437222.pdf.
[35] M. V. John and J. V. Narlikar, Phys. Rev. D 65, 043506 (2002), arXiv:astro-ph/0111122.
[36] S. Nesseris and J. Garcia-Bellido, JCAP 08, 036, arXiv:1210.7652 [astro-ph.CO].
[37] G. Efstathiou, Int. J. Mod. Phys. D 10, 213 (2001), arXiv:astro-ph/0009008 [gr-qc].
[38] E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003), arXiv:astro-ph/0208512 [astro-ph].