Endogenous fertility in a growth model with public and private health expenditures

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Abstract We build an overlapping-generations model that incorporates endogenous fertility choices, in addition to public and private expenditures on health. Following the seminal analysis of Bhattacharya and Qiao (J Econ Dyn Control 31:2519–2535, 2007) we assume that the effect of public health investment is complementary to private health expenditures. We find that this effect reinforces the positive impact of the capital stock on aggregate saving. Furthermore, we show that this complementarity can provide an additional explanation behind a salient feature of demographic transition; that is, the fertility decline along the process of economic growth.

Keywords Fertility · Economic growth · Health expenditures

JEL Classification J13 · O41

1 Introduction

One of the salient features of demographic transition is the decline of the fertility rate along the process of economic development (e.g., Kirk 1996; Ehrlich and Lui 1997; Galor 2012). Among the various explanations that have emerged while trying to explain this outcome, a prominent place belongs to those that attribute it to improvements in the population’s health status at more advanced stages of an economy’s development process. A common
feature to these analyses is that such improvements are either exogenous or they are driven by an aggregate externality according to which public health expenditures improve the population’s health characteristics. In Blackburn and Cipriani (2002) the provision of public health services improves life expectancy and, thus, raises the effective return on education. As a result, households respond by reducing their expenditures during the earlier stages of their lifetime—among them, the expenditures for child rearing. A similar mechanism pervades the analysis of Zhang and Zhang (2005), the difference being that life expectancy is an exogenously given parameter. Kalemli-Ozcan (2003) assumes that child survival follows an exogenous stochastic process and shows that a reduction in child mortality induces a reduction in the number of children reared and a corresponding increase in the parental investment to each child’s human capital. In Soares (2005), parents derive utility, not only from their surviving offspring, but also from the length of each surviving child’s lifespan. He shows that improvements in these health characteristics can generate a demographic transition as the economy develops.1

None of the aforementioned analyses, however, considers private health spending as an additional factor determining an agent’s health status despite the fact that, in reality, private expenditure appears to represent a significant part of total health spending in many economies. For example, a recent publication by the World Health Organisation (2010) reports that private health expenditures amount to roughly 50% of the total in the United States, they range from 20–40% of total health expenditures in some countries of the European Union, while in many less-developed countries they contribute to more-than-half of total health expenditures.

The more recent papers by Strulik (2008) and Manuelli and Seshadri (2009) are notable exceptions since they take account of the importance of private health expenditure in their analyses of fertility choices. Strulik (2008) builds a two-period overlapping-generations model with subsistence consumption and infant survival. The latter includes an exogenous component—positively related to average income per capita among others—and an endogenous one, capturing the resources that parents devote towards the health care of their offspring. He finds that the relationship between fertility and income is an inverted U-shape due to two opposing effects. On the one hand, higher income relaxes the strain imposed by subsistence consumption and leaves more resources available for child rearing. On the other hand, an increase in income induces parents to devote more resources towards their children’s health care in order to take advantage of the increased public health spending and, thus, reduce the infant mortality rate even further. They do this at the expense of their fertility rate. Manuelli and Seshadri (2009) calibrate a general, multi-period model in which both fertility decisions and agents’ lifetimes are endogenous—the latter being determined by health capital that is supported

1For empirical support on the negative relation between improved health status and fertility rates, see Finlay (2007).
by private spending. They find that their model fits well to the data and can explain major cross country differences in fertility rates and life expectancy at birth. Their model allows them to attribute these differences to changes in either Total Factor Productivity or taxes on labour income.

In this paper, we offer an alternative and novel mechanism via which the presence of private health spending can account for the incidence of fertility decline at higher stages of economic development. In particular, we consider both private and public health expenditures in the manner suggested by the seminal analysis of Bhattacharya and Qiao (2007). In their paper, they analyse a growth model in which public health investment is supportive to the effectiveness of the expenditures that individuals incur for the improvement of their health status. They find that an increase in the public provision of health services induces individuals to reduce their saving and, correspondingly, increase the resources they devote towards health improvements. As a result, the dynamics of capital intensity become non-monotonic and may admit periodic (endogenous limit cycles) or even aperiodic (i.e. chaotic) equilibria. Nevertheless, the focus of their paper is not related to demographic change, that is why they do not consider endogenous fertility.

Our model utilises the main idea of Bhattacharya and Qiao (2007)—that is, public health investment being complementary to (optimally chosen) private health spending—but modifies their set-up in the following respects: (a) we assume that individuals are reproductive during their young adulthood and they choose the number of their offspring in an optimal fashion; (b) we allow individuals to consume during both periods of their adulthood, and not only during their old age; (c) rather than considering private health expenses as incurred during young adulthood, with the purpose of improving life expectancy, we assume that individuals incur their health expenses during the final period of their lifetime in order to improve their utility-enhancing health status.

Our results can be summarised as follows. We find that, rather than generating a trade-off between saving and private health spending, the provision of public health services induces an increase in both private health spending and saving. This is due to the fact that agents wish to have more resources when old in order to take advantage of the higher productivity of public health services and, consequently, improve their quality of life during the final stage of their lifetime. In addition to the motive for increased saving, this effect has ramifications for the fertility choices of reproductive agents. Particularly, they will try to mitigate the strain of higher saving on their first period consumption by increasing the time they devote to earning labour income in order to increase the resources available for consumption. As a result, they are induced to bear and raise fewer children during their young adulthood as a means of consumption smoothing over their lifetime.

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2This particular extension is not critical for the subsequent results. These remain similar even if individuals care only about old-age consumption.
The assumption according to which individuals incur health expenses when old is not critical for the result according to which fertility declines are attributed to the interaction between private and public health expenditure. We show this formally by solving a similar model, but assuming that health status takes the form of capital, given that private health spending is incurred by young agents. The only result that changes is the one concerning the marginal propensity to save which, similarly to Bhattacharya and Qiao (2007) in which the young devote resources to health improvements, falls as a result of an increase in public health services provision. Furthermore, we show that our main results are robust to an alternative health production technology with similar characteristics, that is a technology whereby the efficiency of private health spending is supported by the provision of public health services.

Given the above, our analysis joins the strand of literature that attributes the decline in the fertility rate to improvements in the health status of reproductive agents. Nevertheless, there is an important difference between our analysis and the aforementioned body of literature. In particular, we demonstrate the importance of the interplay between public and private health spending in generating the decline in fertility rates—an idea that has eluded the attention of existing theories on the nexus between economic growth and demographic transition. This importance has actually been already identified by Cigno (1998) in his analysis of endogenous fertility in a static model with endogenous infant mortality. Assuming that infant mortality falls as a result of public health spending, he argues that there is no a priori clear mechanism between the latter and the resources that parents devote towards their children’s health. In particular, public and private health expenditures may be either complements or substitutes depending on how parents form their decisions. In the former case, the optimal fertility rate increases whereas in the latter case it declines.

Concerning the link between private and public health expenditure, our assumption is admittedly not as general as the one utilised by Cigno (1998). Our focus is on the particular case whereby the effectiveness of private health expenditure is enhanced in an environment where the productivity of the sector that offers health services is supported by higher public spending. Nevertheless, we are able to offer a different dimension in comparison to Cigno (1998) since the complementary effect of public health expenditure on its private counterpart is responsible for declining rather that increasing fertility. In addition, another difference of our model is that it is a dynamic one with particular emphasis on the issues of capital accumulation and economic growth.

There is another important difference of our setting compared to some of the aforementioned literature (Cigno 1998; Kalemli-Ozcan 2003; Soares 2005; Strulik 2008). That is why we need to stress that our intention is not to provide another link of health and fertility based on the issue of infant mortality. While

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3Of course, the intuition differs as it will transpire in Section 5 where we provide the formal analysis.
the latter is indubitably a very important aspect of demographic transition, our purpose is to offer an alternative, intuitive, but yet unexploited mechanism that attributes variations of fertility choice on health issues that, while not necessarily fatal, they could have significant repercussions for an adult’s quality of life and well-being (e.g., chronic conditions and illnesses; various forms of physical injuries and disabilities; depression and anxiety disorders etc.). We believe that this mechanism is potentially important and, thus, worthy of consideration. Therefore, we have focused on it by following other analyses (e.g., Becker et al. 1990; Tamura 1996; Galor and Weil 1996, 2000; de la Croix and Doepke 2003) that abscond from infant mortality when considering the issue of endogenous fertility.

The rest of the paper is organised as follows. In Section 2, we outline the basic set-up of the economy. Section 3 derives the model’s equilibrium and Section 4 analyses the dynamics of capital accumulation as well as the demographic transition. In Section 5, we show that our results on the link between fertility choice and the interactions between private and public health spending can remain robust under some modified set-ups. Section 6 concludes.

2 The economy

Time is discrete and indexed by $t = 0, 1, \ldots \infty$. We consider an economy which produces a homogeneous commodity and is inhabited by reproductive individuals who live for three periods and belong to overlapping generations. The three periods of an individual’s lifetime are childhood, young adulthood and old adulthood. Agents make decisions only after they reach their adulthood. These decisions are dictated by the desire to maximise their lifetime utility function

$$u' = \ln (c_t) + \gamma \ln (n_t) + \beta \ln (h_{t+1}c_{t+1}), \gamma > 0, \beta \in (0, 1),$$

or, equivalently,

$$u' = \ln (c_t) + \gamma \ln (n_t) + \beta [\ln (c_{t+1}) + \ln (h_{t+1})],$$

subject to the constraints

$$c_t = (1 - \tau)(1 - qn_t)\omega_t - s_t, q > 0, \tau \in (0, 1),$$

$$c_{t+1} = r_{t+1}s_t - x_{t+1},$$

$$h_{t+1} = Hx_{t+1}^{\delta}, H > 0, \delta \in (0, 1).$$

In the previous expressions, $c_t$ denotes consumption during young adulthood, $c_{t+1}$ denotes consumption during old adulthood, $n_t$ is the number of children raised by a young adult, $\omega_t$ is the market wage per unit of labour, $s_t$ denotes saving, $r_{t+1}$ is the gross interest on saving, $h_{t+1}$ is the old adult’s health status and $x_{t+1}$ is the old adult’s spending towards health improvements. When young, each person is endowed with a unit of time which she allocates between
raising children and providing labour services. Raising each child requires \( q \) units of time. Therefore, the young adult will use her remaining time to earn labour income—an income that is subject to a flat tax rate \( \tau \). She divides her disposable income between consumption and saving. The latter is deposited to a financial intermediary with the purpose of providing the agent with retirement income when she becomes an old adult. When old, the agent can potentially face some health problems which she can tackle by using part of her retirement income for the improvement of her health status. The remaining part of retirement income is used so as to satisfy her consumption needs. Note that by augmenting the utility from old age consumption by health status—that is, writing the last term of lifetime utility as \( \ln (h_{t+1}c_{t+1}) \)—we assume that improved health increases an old agent’s quality of life. The higher the health status is, the greater the utility enjoyed for given levels of consumption (see Pitt 1990).

With respect to the link between public and private health expenditures, we follow Bhattacharya and Qiao (2007) in using the expression in Eq. 4 for which it is assumed that

\[
\varepsilon_{t+1} = Z(p_{t+1}),
\]

where

\[
p_{t+1} = \frac{g_{t+1}}{N_t}.
\]

The function \( Z(p_{t+1}) \) in Eq. 5 satisfies \( Z(0) = 1 \), \( Z(\infty) = \bar{\varepsilon} > 1 \), \( Z' > 0 \), \( Z'' < 0 \), \( Z'(0) = \phi > 0 \) and \( Z'(\infty) = 0 \). Given these, it is straightforward to establish that \( Z(p_{t+1}) > Z'(p_{t+1}) p_{t+1} \) holds. In Eq. 6, the variable \( g_{t+1} \) is the stock of public capital devoted to health services and \( N_t \) is the population of those agents who are young in period \( t \) and therefore will be old during \( t+1 \). The presence of the variable \( N_t \) is meant to capture a congestion-type effect. In particular, those who are young in period \( t \) will access public health services when they become old, i.e. in period \( t+1 \). We assume that, for given \( g_{t+1} \), a larger population of agents mitigates the benefit accrued to each agent individually. Thus, public health is non-excludable but rival.

Given the above, the assumptions illustrated through Eqs. 4, 5 and 6 provide a mechanism through which the public investment in health services is complementary and supportive to private health expenditures. Particularly,

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4For a similar assumption on health expenses being incurred during old adulthood, see Gutiérrez (2008).

5We assume that \( \delta \bar{\varepsilon} < 1 \) in order to ensure the concavity of \( h_{t+1} \) with respect to \( x_{t+1} \). A functional form that satisfies all these conditions is \( Z(p_{t+1}) = 1 + \psi p_{t+1} + \theta p_{t+1} \) with \( \varphi = \delta - 1 \).

6The idea that public health spending contributes to some type of capital formation is intuitive once we think of spending on hospitals, medical equipment, support for medical research and training etc.
the former promotes the effectiveness of the latter in improving the agent’s health status during old adulthood.\footnote{We can think of many examples that justify this assumption. The presence of qualified professionals—in the national health system—that offer support and advice on various difficulties that may emerge while people are trying to quit smoking (e.g., cravings etc.) may provide an incentive for smokers to seek and buy treatments that support Nicotine Replacement Therapy (patches, gums etc.). Clinical depression can be combated more effectively if sufferers combine antidepressant medication with appropriate counselling by qualified psychiatrists—counselling that is sometimes offered by professionals employed in the national health system. See Bhattacharya and Qiao (2007) for further examples in support of this conjecture.} This can be formally expressed through

$$\frac{dh_{t+1}}{dx_{t+1} h_{t+1}} x_{t+1} = \delta e_{t+1} = \delta Z \left( \frac{g_{t+1}}{N_t} \right),$$

i.e., the elasticity of health status with respect to private health spending is increasing to the stock of public capital that the government devotes towards health services.

Of course, there is a notable difference between our setting and that of Bhattacharya and Qiao (2007) when it comes to the modelling of health improvements. In their model, an agent’s health spending occurs during her youth because the main motive is to increase her life expectancy. In our model, the old individuals are those that devote resources towards health improvements as they try to enhance the quality of life during the final period of their existence. Certainly, this is not an alien assumption. Life expectancy, albeit hugely important, is by no means the only factor that determines the health status of a person. There is a variety of nonfatal medical conditions that can cause great discomfort and mitigate the quality of life unless treated effectively. Examples include chronic conditions and illnesses such as bronchitis; osteoporosis; prostatitis; periodontitis; dermatitis; diabetes; and various forms of physical injuries and disabilities. Furthermore, there are mental illnesses that have significant implications for a person’s emotional well-being—for example, depression and anxiety disorders.

Another reason why we opt for this specification is related to an interesting outcome concerning the dynamics of capital accumulation. Although in Bhattacharya and Qiao (2007) the public provision of health services reduces the marginal propensity to save, the different timing of private health spending in our model results in a positive relation between public spending and the marginal propensity to save. As we shall see later, this effect reinforces the monotonicity of the economy’s dynamics thus ruling out the emergence of periodic equilibria and endogenous fluctuations. Instead, in our case we may have multiple path-dependent equilibria. Given our focus on demographic aspects, this is actually a welcomed aspect in our analysis. In Section 5, we show that our results on the link between fertility choice and the interactions between private and public health spending can remain robust even under the assumption that individuals devote resources for health improvements when they are young. What changes in this case is the result concerning the saving...
rate which becomes an inverse function of the economy’s capital stock. As we want to abscond from limit cycles, we have opted for the formulation discussed in the preceding part of the analysis.

In any time period $t$, there is a large number (normalised to one) of competitive firms who combine labour from young adults, $L_t$, and capital from financial intermediaries, $K_t$, so as to produce $Y_t$ units of output according to

$$Y_t = AK^\alpha_t L_t^{1-\alpha}, \quad 0 < \alpha < 1.$$  

(8)

In equilibrium, labour demand will be equal to labour supply. The latter is given by the total labour units devoted by the economy’s young adults. Thus,

$$L_t = (1 - qn_t) N_t.$$  

(9)

Firms who maximise profits will equate the marginal product of each input with the respective marginal cost. Taking account of Eq. 9, profit maximisation leads to

$$\omega_t = (1 - \alpha) AK^\alpha_t L_t^{\alpha} = (1 - \alpha) Ak^\alpha_t (1 - qn_t)^{-\alpha},$$  

(10)

and

$$r_t = \alpha AK^{\alpha - 1}_t L_t^{1-\alpha} = \alpha Ak^{\alpha - 1}_t (1 - qn_t)^{1-\alpha},$$  

(11)

where $k_t = K_t / N_t$ is the stock of capital per worker.

### 3 Equilibrium

A young adult will choose quantities for $c_t, n_t, s_t, c_{t+1}$ and $x_{t+1}$ to maximise Eq. 1 subject to Eqs. 2–4, taking $\omega_t, r_{t+1}$ and $\epsilon_{t+1}$ as given. After some straightforward algebra, the first order conditions associated with an agent’s optimal problem allow us to derive the solutions for saving, fertility and private health expenditures. These are

$$s_t = \frac{\beta (1 + \delta \epsilon_{t+1})}{1 + \beta (1 + \delta \epsilon_{t+1})} (1 - \tau) \omega_t (1 - qn_t),$$  

(12)

$$n_t = \frac{\gamma / q}{1 + \beta (1 + \delta \epsilon_{t+1}) + \gamma},$$  

(13)

and

$$x_{t+1} = \frac{\beta \delta \epsilon_{t+1}}{1 + \beta (1 + \delta \epsilon_{t+1})} r_{t+1} s_t.$$  

(14)

Now, we shall assume that the government uses its collected revenues in period $t$ to finance the formation of public capital that will be available next period, i.e. during $t + 1$, according to a balanced budget rule. Given Eq. 9, we have

$$g_{t+1} = \tau L_t \omega_t = \tau (1 - qn_t) N_t \omega_t.$$  

(15)
Combining Eq. 15 with Eqs. 5, 6 and 10 leads to

$$\varepsilon_{t+1} = Z \left( \tau \left(1 - \alpha\right) Ak_t^\alpha \left(1 - qn_t\right)^{1-\alpha} \right).$$

(16)

Substituting Eqs. 16 and 10 in Eqs. 12 and 13 yields

$$s_t = \beta \left[ 1 + \delta Z \left( \tau \left(1 - \alpha\right) Ak_t^\alpha \left(1 - qn_t\right)^{1-\alpha} \right) \right]$$

$$\times (1 - \tau) \left(1 - \alpha\right) Ak_t^\alpha \left(1 - qn_t\right)^{1-\alpha},$$

(17)

$$n_t = \frac{\gamma/q}{1 + \beta \left[ 1 + \delta Z \left( \tau \left(1 - \alpha\right) Ak_t^\alpha \left(1 - qn_t\right)^{1-\alpha} \right) \right] + \gamma}.$$  

(18)

These solutions allow us to derive

**Proposition 1** The economy’s saving is increasing in the stock of capital per worker while the fertility rate is decreasing in the stock of capital per worker.

Proof See the Appendix.  

The intuition behind the result of Proposition 1 is the following. A higher capital stock increases the government’s revenues and allows a greater provision of public capital towards health services. As a result, the effectiveness of private health expenditures increases and old adults will find optimal to devote more resources towards them. Naturally, this implies that young adults will find desirable to have more resources available at the beginning of their old adulthood. Indeed, they can achieve this by saving a larger fraction of the disposable income they earn when young. Therefore, agents decide to limit the resources they keep during their reproductive period. This is an outcome to which they respond by reducing the number of children they rear and, correspondingly, increasing their effort to earn labour income. They do so because they try to counteract the negative effect on their consumption during youth and smooth out their consumption profile over their lifetime. The latter effect reinforces the positive effect of the capital stock on aggregate saving because the increase in labour leads to higher output.

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8Depending on different parameter values, $n_t$ can become less than one in the steady state. However, this possibility can be ruled out by an appropriate parameter restriction—particularly, if the parameter $q$ is sufficiently low. Alternatively, rather than normalising the total endowed time to unity, we could have endowed young individuals with a larger time endowment—so large, to guarantee that fertility is always above one. Qualitatively, none of our results would be affected.
4 Economic growth, health and endogenous fertility

With Proposition 1, we have established that
\[ s_t = s(k_t), \quad s'(k_t) > 0. \quad (19) \]
and
\[ n_t = n(k_t), \quad n'(k_t) < 0. \quad (20) \]
Now, let us use the financial market equilibrium, \( K_{t+1} = s_t N_t \), together with the growth rate of the population, \( N_{t+1} = n_t N_t \), to get
\[ k_{t+1} = s_t \frac{n_t}{n_t} = s(k_t) \frac{n(k_t)}{n(k_t)}. \quad (21) \]
Substituting Eqs. 17, 19 and 20 in Eq. 21 we get
\[ k_{t+1} = \beta \left[ 1 + \delta Z (\tau (1 - \alpha) A k_t^a (1 - qn(k_t))^{1-a}) \right] \]
\[ \times \frac{\eta k_t^a (1 - qn(k_t))^{1-a}}{n(k_t)} = \psi (k_t), \quad (22) \]
where \( \eta = A (1 - \alpha) (1 - \tau). \)
Equation 22 describes the dynamics of capital accumulation. We can use this to derive the economy’s long-run equilibrium and to trace its transitional dynamics towards it. Formally, we analyse these issues in

**Proposition 2** For \( k_0 > 0 \), the economy will asymptotically converge to at least one long-run equilibrium \( \hat{k} \) such that \( \hat{k} = \psi(\hat{k}) \) and \( 0 < \psi'(\hat{k}) < 1. \) In the transition towards this steady-state equilibrium, the economy will grow at a positive, but declining, rate over time as long as \( k_0 < \hat{k}. \)

**Proof** See the Appendix. \( \square \)

The transition graph, manifested in Eq. 22, rises monotonically because saving is increasing and fertility is decreasing in the stock of capital per worker. To understand the importance of this outcome in this particular setting, we re-emphasize the fact that, despite public health investment being complementary to private health expenditures, the economy will not admit periodic equilibria (endogenous cycles). If anything, this complementarity actually enhances the monotonicity of capital dynamics in our model because it increases the marginal propensity to save as the economy develops. For this reason, and despite the complex nature of Eq. 22, the model’s dynamics may be qualitatively similar to those of the canonical OLG model (see Figs. 1 and 2). As we explained earlier, this stark contrast to the previously established result of Bhattacharya and Qiao (2007) rests on our assumption that the old, rather than the young, are actually those who devote resources towards health improvements that enhance their overall quality of life.
However, it is also possible that the dynamics in Eq. 22 admit multiple, path-dependent steady-state equilibria—a possibility illustrated in Fig. 3. The reason for this outcome is the bi-directional effects between saving, fertility and the capital stock. On the one hand, higher saving and lower fertility increase the rate of capital formation; on the other hand, the higher capital stock increases the marginal propensity to save and reduces the fertility rate due to the complementary effect of public health investment on its private counterpart. Correspondingly, these bi-directional effects (which, by the way, do not permeate the canonical OLG model with a unitary elasticity of intertemporal substitution) could generate points of inflexion on the transition graph.

Fig. 1 The dynamics of capital accumulation (unique steady-state)

Fig. 2 A unique steady-state (with points of inflexion on the transition graph)
Fig. 3 The possibility of multiple equilibria

graph that may be responsible for the multiplicity of equilibria. Note that, in this case, at least one of these interior equilibria will be unstable. For example, in the case where three interior equilibria exist (see Fig. 3) the middle one will be unstable. Effectively, it will emerge as an endogenous threshold that determines whether (depending on initial conditions) the economy will converge either to the low- or the high-income equilibrium in the long-run.

Unfortunately, the complexity of the dynamics in Eq. 22, coupled with the implicit nature of the functions $Z(\cdot)$ and (unavoidably) $n(\cdot)$, do not allow us to derive analytical conditions under which a situation similar to the one depicted in Fig. 3 may emerge. The only outcomes that can be proven analytically is that the equilibrium at the origin is unstable and that at least one stable steady-state equilibrium exists (if multiple equilibria exist, this is the highest one in value).

Nevertheless, the main point of our analysis is not the possibility of path-dependent equilibria. Instead, our purpose is to use this framework so as to provide a novel explanation behind the fertility decline observed in economies that reach more advanced stages of their development process. This is formally shown in

**Proposition 3** Suppose that $k_0 < \hat{k}$ and that there is no $\tilde{k} \in (k_0, \hat{k})$ such that $\tilde{k} = \psi(\tilde{k})$. Then for $t = 0, 1, 2, \ldots$, it is $n_0 > n_1 > n_2 > \cdots$. This demographic transition is solely associated with the complementarity between public and private health expenditures.

**Proof** It follows as a corollary of the results established in Propositions 1 and 2. Note that if $\varepsilon_{t+1} = 1\forall t$, i.e. if public health spending is not complementary to its private counterpart, then $n_t = \frac{\gamma/q}{1+\beta(1+\delta)+\gamma} = \hat{n} \forall t$ and demographic change does not occur. 

$\Box$
Our analysis shares some similarities with existing theories that reproduce fertility declines in the process of economic development. However, it also has important differences in comparison to them. The similar aspect is that we attribute the decline in the fertility rate to improvements in the health status of reproductive agents. The difference emerges from the fact that we exemplify the importance of the interplay between public and private health spending—an idea that has eluded the attention of most existing theories. Hence, the current set-up improves our understanding on the underlying forces behind some striking facts of demographic transition (Fig. 4).

5 Alternative approaches

5.1 Private health spending during youth

Consider the model of Section 2, with the only difference being that the young adults are those who devote resources with the purpose of forming their health capital. Thus, Eqs. 2, 3 and 4 change to

\[ c_t = (1 - \tau) (1 - qn_t) \omega_t - s_t - x_t, \]  
\[ c_{t+1} = r_{t+1} s_t, \]  
\[ h_{t+1} = H x_t^\delta, \]  
respectively.
It is straightforward to establish that the solution of the model leads to the following equilibria for fertility, saving and health spending:

\[ n_t = \frac{\gamma / q}{1 + \beta [1 + \delta Z (\tau (1 - \alpha) Ak_t^\alpha (1 - q n_t)^{1 - \alpha})] + \gamma} \],

(26)

\[ s_t = \frac{\beta}{1 + \beta [1 + \delta Z (\tau (1 - \alpha) Ak_t^\alpha (1 - q n_t)^{1 - \alpha})] + \gamma} (1 - \tau) \omega_t, \]

(27)

\[ x_t = \frac{\beta \delta Z (\tau (1 - \alpha) Ak_t^\alpha (1 - q n_t)^{1 - \alpha})}{1 + \beta [1 + \delta Z (\tau (1 - \alpha) Ak_t^\alpha (1 - q n_t)^{1 - \alpha})] + \gamma} (1 - \tau) \omega_t. \]

(28)

As it is evident, the solution for fertility in Eq. 26 is identical to the corresponding expression in Section 2 (i.e. Eq. 18). Therefore, the model’s result concerning the negative effect of \( k_t \) on fertility and the fact that this effect exists only due to the interaction between public and private health spending remain intact. The intuition however is different. Once more, an increase in \( k_t \) will allow the provision of more publicly provided health services for a given tax rate. Individuals respond by increasing their health expenditure at the expense of their current consumption. Now, however, agents can retain a more uniform pattern of consumption over time, by reducing both their saving rate and the number of children they decide to give birth to. The latter effect is the outcome of their decision to provide more labour and, thus, increase the available resources during young adulthood.

A notable difference in this scenario is the fact that, as in Bhattacharya and Qiao (2007), the marginal propensity to save is decreasing in the stock of physical capital. Note that this effect will actually be reinforced by the increase in labour supply, due to lower fertility, as the capital stock increases.

5.2 An alternative form for the health generation function

Once more, let us consider the model of Section 2. Now, however, we consider a different functional form for health status. In particular, we replace Eq. 4 by

\[ h_{t+1} = (1 - \delta) p_{t+1} + f (\delta p_{t+1}) x_{t+1}, \]

(29)

where \( \delta \in [0, 1) \), \( f' (\cdot) > 0 \) and \( f (0) = f > 0 \). The variable \( p_{t+1} \) is still given by Eq. 6 as it captures the benefit from public health services. The idea behind the health generation function in Eq. 29 is that the benefit from public health spending has two components. On the one hand, some public health provision is beneficial to health status irrespective of whether individuals contribute resources towards their health improvements or not. On the other hand, part of the economy’s public services support the effectiveness of private health expenditure by increasing the productivity of the health sector—an effect that is captured through the presence of the increasing function \( f (\cdot) \). Naturally, \( \delta \in [0, 1) \) provides a flexible parameterisation of the relative strength of these two effects.
Another change in comparison to the model of Section 2, is related to the production technology which now takes the form of

\[ Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}. \] (30)

The variable \( A_t \) indicates some type of labour-augmenting technological progress. Following Frankel (1962) and Romer (1986), we assume that this is related to the average capital-labour ratio according to a learning-by-doing externality. That is

\[ A_t = \Psi \frac{K_t}{L_t}, \Psi > 0. \] (31)

The reason why we opt for this specification here is because, despite the fact that we assume logarithmic preferences, the presence of the term \((1 - \delta) p_{t+1}\) in Eq. 29 means that optimal decisions will depend on the interest rate. To maintain analytical tractability, we use a production technology, commonly employed in the endogenous growth literature, whose main property is that the marginal product of capital is not inversely related to its stock. In this case, the marginal product of capital is constant at

\[ r_t = \frac{\alpha}{(1-\alpha)} k_t. \]

Solving this model and using Eqs. 6, 9, 15 and 32 in the solutions, it is straightforward to establish the following results for saving, fertility and private health spending:

\[ s_t = \frac{2\beta \left(1 - \tau\right) - \frac{\hat{r}^{-1}(1-\delta)\tau}{f(\delta \tau (1-\alpha) \Psi^{1-\alpha} k_t)}}{1 + 2\beta} (1 - \alpha) \Psi^{1-\alpha} k_t, \] (33)

\[ n_t = \frac{1}{q} \times \frac{\gamma \left[ \hat{r} (1+2\beta+\gamma)(1-\tau) \right]^{-1}(1-\delta)\tau}{f(\delta \tau (1-\alpha) \Psi^{1-\alpha} k_t)} \times \frac{1 + z f(1+2\beta+\gamma)(1-\tau)}{f(\delta \tau (1-\alpha) \Psi^{1-\alpha} k_t)} = n(k_t), \] (34)

\[ x_{t+1} = \frac{\beta \hat{r} \left(1 - \tau\right) - \frac{(1+\beta)(1-\delta)\tau}{f(\delta \tau (1-\alpha) \Psi^{1-\alpha} k_t)}}{1 + 2\beta} (1 - \alpha) \Psi^{1-\alpha} k_t. \] (35)

In these results, we assume that \( f(0) = f \) is sufficiently high to guarantee a strictly positive solution for health spending. Given this, we can use Eq. 34 to verify that \( n'(k_t) < 0 \) if and only if \( \delta > 0 \). In the limiting scenario where \( \delta = 0 \), we have \( n'(k_t) = 0 \) as well. In other words, the decline of the fertility rate as the economy grows depends crucially on the idea that (part of) public health expenditures enhance the productivity of the health sector and, consequently, boost the efficiency of private health spending. This is exactly the result we established in our original model (see Proposition 3).
6 Conclusions

We have established a new mechanism in explaining a salient feature of demographic transition. In particular, the decline of fertility during the process of growth is attributed to the complementary effect of public health investment on private health expenditures—an effect that has been introduced in the manner of the seminal analysis of Bhattacharya and Qiao (2007; see Eqs. 4–7). As the economy grows, the public capital available for health services increases and improves the effectiveness of private health expenditures. Old adults will find that it is optimal for them to increase the resources they devote towards the improvement of their health status. In order to ensure the availability of these resources, they reduce their expenditures when young in order to save more. Given that childrearing costs are among these expenditures, reproductive young adults will reduce the number of children they give birth to. Consequently, the fertility rate declines.

Furthermore, our model shows that as long as some health expenses are incurred by the old, the complementary impact of public health spending on its private counterpart need not be a source of economic instability. Actually, it reinforces the monotonicity of the economy’s dynamics by motivating individuals to save a larger part of their labour income—thus, supporting capital accumulation.

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Appendix

Proof to Proposition 1

Using Eq. 18, we can define

\[ J(qn_t, k_t) = qn_t - \frac{\gamma}{1 + \beta \left[ 1 + \delta Z \left( \tau (1 - \alpha) Ak_t^\alpha (1 - qn_t)^{1-\alpha} \right) + \gamma \right] + \gamma}. \]  

(36)

We can also use Eqs. 6, 10 and 15 to write

\[ \tau (1 - \alpha) Ak_t^\alpha (1 - qn_t)^{1-\alpha} = p_{t+1}. \]  

(37)

From Eq. 36 we have

\[ J_{k_t}(\cdot, \cdot) = \frac{\gamma \beta \delta Z (\tau (1 - \alpha) Ak_t^\alpha (1 - qn_t)^{1-\alpha}) \tau (1 - \alpha) A\alpha k_t^{\alpha - 1} (1 - qn_t)^{1-\alpha}}{\left[ 1 + \beta \left[ 1 + \delta Z \left( \tau (1 - \alpha) Ak_t^\alpha (1 - qn_t)^{1-\alpha} \right) \right] + \gamma \right]^2} > 0. \]  

(38)
Alternatively, we can substitute Eq. 37 in Eq. 38 to write

\[
J_{k_t} (\cdot, \cdot) = \frac{\gamma \beta \delta \alpha Z'(p_{t+1}) p_{t+1}}{\left[1 + \beta \left[1 + \delta Z (p_{t+1}) \right] + \gamma \right]^2 k_t} > 0. \tag{39}
\]

From Eq. 36, we can also derive

\[
J_{q_n t} (\cdot, \cdot) = 1 - \frac{\gamma \beta \delta Z' (\tau (1 - \alpha) A k_t^a (1 - q n_t)^{1-\alpha}) (1-\alpha)}{\left[1 + \beta \left[1 + \delta Z (\tau (1 - \alpha) A k_t^a (1 - q n_t)^{1-\alpha}) \right] + \gamma \right]^2 (1 - q n_t)}, \tag{40}
\]

to which we can substitute Eq. 37 to get

\[
J_{q_n t} (\cdot, \cdot) = 1 - \frac{\gamma \beta (1 - \alpha) Z'(p_{t+1}) p_{t+1}}{\left[1 + \beta \left[1 + \delta Z (p_{t+1}) \right] + \gamma \right]^2 (1 - q n_t)}. \tag{41}
\]

Substituting Eqs. 18 and 37 in Eq. 41, we can write the latter as

\[
J_{q_n t} (\cdot, \cdot) = 1 - \frac{\gamma (1 - \alpha) \beta \delta Z' (p_{t+1}) p_{t+1}}{\left[1 + \beta \left[1 + \delta Z (p_{t+1}) \right] + \gamma \right]} > 0, \tag{42}
\]

which is positive because \( Z'(p_{t+1}) p_{t+1} < Z (p_{t+1}) \) holds. Now, we can combine the results in Eqs. 42 and 38, and apply the implicit function theorem to Eq. 36. This yields

\[
\frac{dq_{n t}}{d k_t} = - \frac{J_{k_t} (\cdot, \cdot)}{J_{q_n t} (\cdot, \cdot)} < 0, \tag{43}
\]

Therefore, given that \( q \) is a fixed parameter, we can conclude that \( n_t = n (k_t) \) such that \( n' (k_t) < 0 \). Finally, we can use the previous analysis to write Eq. 17 as

\[
s_t = \frac{\beta \left[1 + \delta Z (\tau (1 - \alpha) A k_t^a (1 - q n (k_t))^{1-\alpha}) \right]}{1 + \beta \left[1 + \delta Z (\tau (1 - \alpha) A k_t^a (1 - q n (k_t))^{1-\alpha}) \right]} (1 - \tau) (1 - \alpha) \times A k_t^a (1 - q n (k_t))^{1-\alpha},
\]

from which it is straightforward to establish that \( ds_t/d k_t > 0 \). \( \Box \)
Proof to Proposition 2

From Eqs. 18 and 20, we can establish that \( n(0) = \frac{\gamma}{1 + \beta(1 + \delta) + \gamma} \) and \( n(\infty) = \frac{\gamma q}{1 + \beta(1 + \delta) + \gamma} \). Combining these with Eq. 22, we see that \( \psi(0) = 0 \) and \( \psi(\infty) = \infty \). Furthermore, it is

\[
\psi'(k_i) = \frac{\eta}{\frac{\beta}{1 + \beta}[1 + \delta Z(\tau(1 - \alpha)Ak_i^a(1 - qn(k_i))^{1-\alpha})]}
\times \frac{\theta(k_i) + k_i^a(1 - qn(k_i))^{1-\alpha}}{n(k_i)}
\times \frac{\beta \delta Z'(\tau(1 - \alpha)Ak_i^a(1 - qn(k_i))^{1-\alpha})\tau(1 - \alpha)A\theta(k_i)}{[1 + \beta[1 + \delta Z(\tau(1 - \alpha)Ak_i^a(1 - qn(k_i))^{1-\alpha})]]^2}
\times \frac{\beta[1 + \delta Z(\tau(1 - \alpha)Ak_i^a(1 - qn(k_i))^{1-\alpha})]}{1 + \beta[1 + \delta Z(\tau(1 - \alpha)Ak_i^a(1 - qn(k_i))^{1-\alpha})]} \times \frac{k_i^a(1 - qn(k_i))^{1-\alpha}}{[n(k_i)]^2n'(k_i)},
\]

where

\[
\theta(k_i) = \alpha k_i^{a-1}(1 - qn(k_i))^{1-\alpha} - k_i^a(1 - \alpha)qn'(k_i)(1 - qn(k_i))^{-\alpha}.
\]  

Clearly, \( \psi'(k_i) > 0 \). Now, combine Eqs. 39 and 42 to write Eq. 43 as

\[
\frac{dq_{nt}}{dk_t} = -\frac{\gamma\beta\delta\alpha Z'(p_{t+1}) p_{t+1}[1 + \beta[1 + \delta Z(p_{t+1})]]}{[1 + \beta[1 + \delta Z(p_{t+1})] + \gamma][1 + \beta[1 + \delta Z(p_{t+1})]] - \gamma\beta\delta(1 - \alpha)Z'(p_{t+1}) p_{t+1}} \times \frac{1}{k_t[1 + \beta[1 + \delta Z(p_{t+1})] + \gamma]}.
\]

Given \( Z'(0) = 0 \) and \( Z'(\infty) = 0 \), we can combine the expression above together with Eq. 37, \( n(0) = \frac{\gamma}{1 + \beta(1 + \delta) + \gamma} \) and \( n(\infty) = \frac{\gamma q}{1 + \beta(1 + \delta) + \gamma} \) to establish that

\[
n'(k_i) = \begin{cases} -\infty & \text{for} \ k_t = 0 \\ 0 & \text{for} \ k_t \to \infty \end{cases}.
\]

Combining Eqs. 45 and 46 with Eq. 44, we infer that

\[
\psi'(k_i) = \begin{cases} \infty & \text{for} \ k_t = 0 \\ 0 & \text{for} \ k_t \to \infty \end{cases}.
\]

Thus, we conclude that there must be at least one \( \hat{k} \in (0, \infty) \) such that \( \hat{k} = \psi(\hat{k}) \) and \( \psi'(\hat{k}) < 0 \), i.e. \( \hat{k} \) is a stable steady-state equilibrium.
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