On the modeling of cross-section and longitudinal section of pipes

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Abstract. This paper deals with the construction of the various inflate-deflate pipe patches with its center curve of a line segment. The construction is formulated by the cross-section, the longitudinal section and the line segment center curve of the pipe patches. The objectives of the research are to model the cross-section curves of the patches by using the polar coordinate, and the longitudinal section of the trigonometry, Bézier, and Hermit curves to enable designing the various models of pipe parts. The study found that some polar formulas and regular polygon sides can evaluate the various cross-section forms of the pipes, and the function cosine, sinus, Bézier and cubic Hermit can draw, respectively, the longitudinal section forms of the pipes. We have tested as well these formulas, and the results show that they are very useful to design the pipes in which its center curves are lines. In the future works, we need to study the pipe modeling with its center curves of space curves.

1. Introduction

Some methods related to pipes modeling have been introduced. We can simulate the physical model of transitional pipeline parts whose cross-sections that have polygonal form and made of materials that cannot be wrinkled or stretched [1]. This geometrical concept is taken from the classical line geometry, and it can be directly applied to the creation of a computer algorithm that generates the transitional surfaces between two polygons. Using rational functions in polar form can be presented several theoretical results, and an algorithm for properly plotting curves parametrized [2]. They are effective to identify phenomena which are typical of these curves, like the existence of infinitely many self-intersections, spiral branches, limit points or limit circles, but it may be deficient of the cross-section pipes design. Pipe connection to multiple pipes that are constructed by freeform cross-sections and arbitrary poses can be undertaken by connecting every two pipe ends of totally three using side patches. Then two holes inside them can be covered by using hole-filling patches [3]. To provide good tamper localization accuracy has been proposed topology authentication for piping isometric drawings that are in robust against local similarity transformations and invariant to the stretching operation on pipes [4]. These introduced schemes can be employed to authenticate topology integrity for each of the drawings derived from the model individually in industry practices. Then, comparative study of bend pipe for a circular section and ovality induced bend pipes have been presented for analyzing a standalone pipe of specified dimensions [5]. To build a complete pipeline, we can connect the pipe parts. We continuously join two adjacent center curves of the pipe parts, and by using the same formula, we define its coincidence cross-section boundary curves of the pipes. Then, the joining of the longitudinal section boundary curves of the pipe parts can be done [6].
Constructing the whole pipe needs to consider some aspects such as outer surface forms and length of pipes, cross-section form, and longitudinal shapes of pipes. Meanwhile, these methods introduced generally focused on the combining and connecting small parts of the pipes to obtain the length and to get the branches topology of the pipes. Different from these introduced methods, the discussion in this paper will be addressed on formulating and designing the cross-section and longitudinal shapes of the pipe parts with its center curve of the line segment. This method will offer some polar formulas and regular polygon sides that can design various cross-section forms of the pipes, and the function cosine, sinus, Bézier and cubic Hermit that can draw, respectively, the longitudinal section forms of the pipes. Also, they can use to model the different thickness of that pipes in the cross-section direction.

This paper is organized in the following sections. In the first section, we will discuss the formulation of pipe cross-section shapes. In the second we define the longitudinal pipe shapes. Finally, the results will be summarized in the conclusion section.

2. Formulation of Pipe Cross-section Shapes

Let two constant unity vectors \( \mathbf{v}_1 = <a_1, a_2, a_3> \) and \( \mathbf{v}_2 = <b_1, b_2, b_3> \) perpendicular in space that define a vertical plane \( \alpha = [\mathbf{v}_1, \mathbf{v}_2] \). Every vector \( \mathbf{v} \) in this plane can be expressed as \( \mathbf{v} = x \mathbf{v}_1 + y \mathbf{v}_2 \) with \( x, y \) the real scalars. The proposed problem in this section is how to construct the various pipe cross-section shapes in the plane \([\mathbf{v}_1, \mathbf{v}_2] \) that are defined by the parametric curves \( \mathbf{P}(v) \) in the formula

\[
\mathbf{P}(v) = r(v)\, [\cos \varphi \, \mathbf{v}_1 + \sin \varphi \, \mathbf{v}_2]
\]

with \( \varphi = 2\pi v \) and \( 0 \leq v \leq 1 \). First of all, we choose a simple case when the curves \( \mathbf{P}(v) \) are in the natural form of a circle, a limacon or a rose. If the scalar functions \( r(v) \) represent the polar coordinate of the circle, the limacon and the rose [7], then we can formulate it under the parametric form in the plane \([\mathbf{v}_1, \mathbf{v}_2] \) respectively

\[
\begin{align*}
P_1(v) &= r_1(v)\, [\cos \varphi \, \mathbf{v}_1 + \sin \varphi \, \mathbf{v}_2] \quad (2a) \\
P_2(v) &= r_2(v)\, [\cos \varphi \, \mathbf{v}_1 + \sin \varphi \, \mathbf{v}_2] \quad (2b) \\
P_3(v) &= r_3(v)\, [\cos \varphi \, \mathbf{v}_1 + \sin \varphi \, \mathbf{v}_2] \quad (2c)
\end{align*}
\]

with \( r_1(v) = a; \ r_2(v) = a \pm b \cos \varphi \) or \( r_2(v) = a \pm b \sin \varphi; \ r_3(v) = a \cos (n \varphi) \) or \( r_3(v) = a \cos (n \varphi). \) The value \( a, b \) are positive real constants, \( \varphi = 2\pi v \) with \( 0 \leq v \leq 1 \) and \( n \) is the number of defined rose leaves.

To obtain other forms of pipe cross-section, we can choose the function \( r(u) \) using adding, subtracting or multiplying among \( r_1(v), r_2(v), \) and \( r_3(v). \) For example, when we choose the orthogonal unit vectors \( \mathbf{v}_1 = <0,1,0>, \mathbf{v}_2 = <0,0,1> \) and the combination of \( r_1(v) \) and \( r_3(v) \) such that

\[
\begin{align*}
r_6(v) &= 5 \cos (4\varphi) + \sin (7\varphi); \quad & r_7(v) &= 5 + \frac{1}{2b} \cos (4\varphi) + \sin (7\varphi); \\
r_8(v) &= 3 \cos (3\varphi); \quad & r_9(v) &= [2 + \cos (9\varphi)] + [3 + \cos (2\varphi)];
\end{align*}
\]

the equation (1) will give the results as they are shown in Figure 1a, 1b, 1c, 1d respectively. Some other results of the combined formula are shown in Figure 1e, 1f with the formula

\[
\begin{align*}
&\begin{align*}
r_{10}(v) &= -2 + \sin (\varphi) - \cos (\varphi); \quad & r_{11}(v) &= 3 + \sin (\varphi) + \cos (2\varphi); \\
r_{12}(v) &= -2 - \cos (2\varphi) - \cos (4\varphi); \quad & r_{13}(v) &= 2 - \sin (\varphi) + \cos (4\varphi).
\end{align*}
\]
Furthermore, our discussion focused on the determination of this scalar function \( r(v) \) in the polar coordinate form that can model and create the various pipe cross-sections shapes.

![Figure 1. Some examples to represent the cross-sections of pipe](image)

Let a regular polygon \( P_1P_2...P_n \) with its center of gravity \( O \) (Figure 2a). Consider in the triangle \( O P_1 P_2 \), the ray \( O P_1 \) as a polar axe at the origin \( O \) and two points \( P_1, P_2 \) of the polar form \( P_1(r_o,0) \) and \( P_2(r_o, \frac{2\pi}{n}) \). The segment \( OC_1 \) intersect at the middle point \( D \) of segment \( P_1P_2 \) such that \( OC_1 \) perpendicular to \( P_1P_2 \) and two segments \( C_1P_1 \) and \( C_1P_2 \) are respectively perpendicular to the segment \( OP_1 \) and \( OP_2 \). Then we can calculate the line segment equation \( P_1P_2, P_1C_1 \) and \( P_2C_1 \) in the polar coordinate as follows. Let any point \( X \) that is lied on the segment \( P_1P_2 \). The ray \( OX \) form the angle \( \phi \) to the polar axe and the measure of the angle \( \angle XOD = (\phi - \frac{1}{2n}\pi) \). So the polar equation of segment \( OX \) can be determined by the trigonometric relation between the angles and the sides of the right triangle \( XOD \) that is \( OX = r_X = \frac{OP_2}{\cos(\phi - \frac{1}{2n}\pi)} \). By using the same method of calculation, if the points \( Y \) and \( Z \) respectively lie on the segments \( P_1C_1 \) and \( P_2C_1 \), we can determine \( OY = r_Y = \frac{OP_2}{\cos(\phi - \frac{1}{2n}\pi)} \) and \( OZ = r_Z = \frac{OP_2}{\cos(\phi - \frac{1}{2n}\pi)} \). Thus the polar equation of the segment \( P_1P_2, P_1C_1 \) and \( P_2C_1 \) are respectively

\[
\begin{align*}
    r_4(v) &= r_X = \frac{r_o \cos \left( \frac{1}{2} \pi + \frac{2\pi}{n} v \right)}{\cos \left( \phi - \frac{1}{2} \pi + \frac{v}{n} \right)} \\
    r_5(v) &= r_Y = r_o \sec \phi \\
    r_6(v) &= r_Z = r_o \sec \left( \frac{1}{2} \pi - \phi \right)
\end{align*}
\]

From the equation (3a), we can express the regular polygon \( P_1P_2...P_n \) in the parametric form

\[
\begin{align*}
P_4(v) &= r_7(v) \left[ \cos \left( \frac{\pi}{n} + i \frac{2\pi}{n} v \right) v_1 + \sin \left( \frac{\phi}{n} + i \frac{2\pi}{n} v \right) v_2 \right] \\
\text{with } r_7(v) &= \frac{r_o \cos \left( \frac{1}{2} \pi + \frac{2\pi}{n} v \right)}{\cos \left( \frac{1}{2} \pi - \phi \right)}
\end{align*}
\]

for \( i = 0, 1, \ldots, n-1 \) and \( \phi = 2 \pi v \) with \( 0 \leq v \leq 1 \). On the other hand, from equations (3b) and (3c), we can construct the triangles \( \Delta C_1P_1P_2, \Delta C_2P_2P_3, \ldots, \Delta C_nP_nP_1 \) on all sides of the polygon \( P_1P_2...P_n \) by drawing the segments

\[
\begin{align*}
P_5(v) &= r_8(v) \left[ \cos \left( \frac{1}{2} \pi + i \frac{2\pi}{n} v \right) v_1 + \sin \left( \frac{1}{2} \pi + i \frac{2\pi}{n} v \right) v_2 \right] \\
P_6(v) &= r_9(v) \left[ \cos \left( (2i - 1) \frac{\pi}{n} + \frac{1}{2} \phi \right) v_1 + \sin \left( (2i - 1) \frac{\pi}{n} + \frac{1}{2} \phi \right) v_2 \right]
\end{align*}
\]
with \( r_0(\nu) = r_0 \cdot \sec \left( \frac{\pi}{n} \right) \) for \( i = 0, 1 \ldots n - 1 \) and \( \phi = 2 \pi \nu \) with \( 0 \leq \nu \leq 1 \). Using the equation (4) and (5) for \( n = 9 \) and \( r_0 = 12 \) will get the polygon and the triangles in Figure 2b.

In the right triangle \( ODP \), we have \( DP_0 = r_0 \sin \left( \frac{\pi}{n} \right) \) and the side \( P_1 P_2 = 2P_0 = 2r_0 \sin \left( \frac{\pi}{n} \right) \).

Meanwhile in the right triangle \( OP_1 C_1 \) we find the relation \( OC_1 = r_C = r_0 \sec \left( \frac{\pi}{n} \right) \) and \( P_1 C_1 = r_0 \tan \left( \frac{\pi}{n} \right) \). When \( W(r, \phi) \) states any point on the circle of the centre \( C \left( \frac{r_C}{r_0}, \frac{\phi}{2\pi\nu} \right) \) with the radius \( P_1 C_1 \), then the polar coordinate equation of the circle can be determined by Cartesian coordinate frame, that is \((x - x_C)^2 + (y - y_C)^2 = P_1 C_1^2 \) or \((r \cdot \cos \phi - r_C \cdot \cos \left( \frac{\pi}{n} \right))^2 + (r \cdot \sin \phi - r_C \cdot \sin \left( \frac{\pi}{n} \right))^2 = r_C^2 \cdot t^2 \cdot g^2 \left( \frac{\pi}{2}, \frac{\pi}{n} \right) \).

Thus we obtain

\[
\begin{align*}
\sin \left( \frac{\pi}{n} \right) = r_0 & \cdot \sqrt{\cos \phi \cdot t^2 \cdot g^2 \left( \frac{\pi}{2}, \frac{\pi}{n} \right) - 1}. \\
\end{align*}
\]

Base on this equation, we can construct the circles with the centre point at \( C_1, C_2, \ldots, C_n \) that pass exactly two consecutive angle points of the polygon \( P_1 P_2 \ldots P_n \) in the parametric formula

\[
P_i(\nu) = r_{i0}(\nu) \cdot \left[ \cos \left( \phi + 2 \cdot \frac{\pi}{n} \right) \right] v_1 + \sin \left( \phi + 2 \cdot \frac{\pi}{n} \right) v_2
\]

for \( i = 0, 1, \ldots, n - 1 \) and \( \phi = 2 \pi \nu \) with \( 0 \leq \nu \leq 1 \). In case of \( n = 14 \), we get the circles in Figure 2c. Furthermore, from the equation (4), (5), and (7), we can draw Figure 2d.

Consider a fixed radius of circle \( t \) in the interval \( 0 \leq t \leq P_1 P_2 \). When each angle points of the polygon \( P_1 P_2 \ldots P_n \) is the centre of the circle with the radius \( t \), then the polar form of the circles can be determined as follows. Let \( P_{i+1}(r_0, i \cdot \frac{\pi}{n}) \) the polar coordinate of angle point \( P_{i+1} \) for \( i = 0, 1, \ldots, n - 1 \). Meanwhile \( T(r, \phi) \) is any point of the circle of centre \( P_{i+1} \). So, in the Cartesian coordinate frame, we will have the circle equation \((r \cdot \cos \phi - r_0 \cdot \cos \left( \frac{\pi}{n} \right))^2 + (r \cdot \sin \phi - r_0 \cdot \sin \left( \frac{\pi}{n} \right))^2 = t^2 \).

Thus we obtain the parametric formula

\[
P_i(\nu) = r_{i1}(\nu) \cdot \left[ \cos \left( (2i + 1) \cdot \frac{\pi}{n} \right) \right] v_1 + \sin \left( (2i + 1) \cdot \frac{\pi}{n} \right) v_2
\]

with \( r_{i1}(\nu) = r_0 \cdot \cos \left( (2i + 1) \cdot \frac{\pi}{n} \right) \cdot v_1 + \sin \left( (2i + 1) \cdot \frac{\pi}{n} \right) v_2 \) for \( i = 0, 1, \ldots, n - 1 \) and \( \phi = 2 \pi \nu \) with \( 0 \leq \nu \leq 1 \). In case of \( n = 8 \) combinations, we will find the example Figures 2e,f.

\[
\begin{align*}
\text{Figure 2. Representation of some different boundary curves of pipe cross-sections}
\end{align*}
\]
The real functions $r_i(u)$ for $i = 1, 2, 3, ..., 12$ of equation (2) up to (8) offer many design parameter to model the pipe cross-section form. We can determine the different value of $r_o$ and $r_i(u)$ to represent the various exterior and interior cross-section boundaries of pipe (Figure 3a,b,c,d). Moreover, we can determine the different value of angle $\phi$, $r_o$ and $r_i(u)$ in order to obtain the various curve segment profiles of the pipe cross-section (Figure 3e,f,g,h).

![Figure 3. Some models of the interior-exterior curves of pipe cross-sections](image)

3. Formulation of Pipe Longitudinal Shapes

We will discuss about the formulation of the pipe longitudinal shape using the real function $\rho(u)$ of the interval $0 \leq u \leq 1$ (Figure 4a). This function will determine the radius measure of the pipe in $u$ value direction that will format the longitudinal shape of the pipe. Because of the application reason, we choose $\rho(u)$ of the simple real function, that are the function cosine, sinus, Bézier and cubic Hermit respectively in the form [8]

\[
\rho_3(u) = a + b \cos(2\pi u) + c \sin(2\pi u) \quad (9a)
\]

\[
\rho_2(u) = P_o(1 - u)^3 + 3P_1(1 - u)^2u + 3P_2(1 - u)u^2 + Pu^3 \quad (9b)
\]

\[
\rho_3(u) = \rho_3(0)H_1(u) + \rho_3(1)H_2(u) + \rho_3(0)^3H_3(u) + \rho_3(1)^3H_4(u) \quad (9c)
\]

with $a, b, c, r$ real constant, $(b + c) < a$ and $H_1(u) = 2u^3 - 3u^2 + 1$; $H_2(u) = 2u^3 - 3u^2$; $H_3(u) = u^3 - 2u^2 + u$; $H_4(u) = u^3$ and $0 \leq u \leq 1$. The real value $\rho_3(0)$, $\rho_3(1)$, $\rho_3(0)^3$, $\rho_3(1)^3$, and $P_i$ for $i = 0, 1, ..., 3$ are determined. The values $\rho_3(0)$ and $\rho_3(1)$ represent respectively the gradient value of tangent line at the point $(0, \rho_3(0))$ and $(1, \rho_3(1))$.

Figure 4b illustrates the equation (9a) for the value $a = 2$, $b = 1$ and $c = 1$. Figure 4c draws equation (9b) for $P_o = 2$, $P_1 = 4$, $P_2 = 1$, and $P_3 = 2$. Figure 4d shows the illustration of equation (9c) for $\rho_3(0) = 2$, $\rho_3(1) = 2$, $\rho_3(0)^3 = 8$ and $\rho_3(1)^3 = 10$. Furthermore, inspired by the cubic Hermit curve form, we formulate the quartic real function $\rho(u)$ with the same representation of the equation (9b) but it can offer one more fixed control point at $(u_x, \rho_x)$ with $0 < u_x < 1$ and $\rho_x = \rho(u_x)$ that lie between the end points $(0, \rho(0))$ and $(1, \rho(1))$ under the constrains as follows.

Consider the quartic real function $\rho(u)$ in the algebraic form \(q(u) = a_0u^4 + a_1u^3 + a_2u^2 + a_3u + a_4\) with $0 \leq u \leq 1$ and the first derivation of the variable $u$ is $q'(u) = 4a_0u^3 + 3a_1u^2 + 2a_2u + a_3$. We fix the boundary conditions $q(0) = q_0 = a_0$; $q(1) = q_1 = a_4 + a_3 + a_2 + a_1 + a_0$ with its tangent line gradient values at $(0, q(0))$ and $(1, q(1))$ are $q''(0) = q_0'' = a_1$ and $q''(1) = q_1'' = 4a_4 + 3a_3 + 2a_2 + a_1$ respectively. Then along the function $q(u)$ between the end points $(0, q(0))$ and $(1, q(1))$, we give an additional condition [5], namely a free control point $(u_x, q(u_x))$ for modeling the shape of the graph $q(u)$ in the following form \(q(u_x) = q_x = a_4u_x^4 + a_3u_x^3 + a_2u_x^2 + a_1u_x + a_0\) with the fixed value $u_x$ in the
According to the boundary conditions, we obtain the equations of the variables $a_0$, $a_1$, $a_2$, $a_3$, and $a_4$ in the form

$$ q_o = a_o; \quad q_o' = a_1; $$
$$ q_1 = a_4 + a_3 + a_2 + q_o'^2 + q_o^2; \quad q_4 = a_2u_x^4 + a_3u_x^3 + a_2u_x^2 + q_o^2u_x + q_o^3; $$
$$ q_4' = 4a_4 + 3a_3 + 2a_2 + q_o^4. $$

The solution of this equation system is

$$ a_0 = q_o, \quad a_1 = q_o', $$
$$ a_2 = (a - 3)q_o + bq_x + (c + 3)q_1 + (d - 2)q_o^2 + (e - 1)q_4^2, $$
$$ a_3 = (2 - 2a)q_o - 2bq_x - (2c + 2c)q_1 + (1 - 2d)q_o^2 + (1 - 2e)q_4^2, $$
$$ a_4 = aq_o + bq_x + cq_1 + dq_o^2 + eq_4^2 $$

where

$$ a = \frac{(-2a^2 - 2u_x^2 + 1)}{u_x^2 - 2u_x^2 + u_x^2}, \quad b = \frac{1}{(u_x^2 - 2u_x^2 + u_x^2)}, \quad c = \frac{(2a^2 - 2u_x^2)}{(u_x^2 - 2u_x^2 + u_x^2)}, \quad d = \frac{(-u_x^2 + u_x^2)}{(u_x^2 - 2u_x^2 + u_x^2)}, \quad e = \frac{(-u_x^2 + u_x^2)}{(u_x^2 - 2u_x^2 + u_x^2)}. $$

If the values of $a_0$, $a_1$, $a_2$, $a_3$, and $a_4$ are inserted to the former real function $q(u)$, then the quartic real function in algebraic form $q(u)$ can be written in the geometric form

$$ q_4(u) = q(u) = q_o[au^4 + (2 - 2a)u^3 + (a - 3)u^2 + 1] + q_4[bu^4 - 2bu^3 + bu^2] $$
$$ + q_4[cu^4 - (2 + 2c)u^3 + (c + 3)u^2] + q_4[dau^4 + (1 - 2d)u^3 + (d - 2)u^2 + u] $$
$$ + q_4[eau^4 + (1 - 2e)u^3 + (e - 1)u^2] $$

or in more general, we can write it

$$ q_2(u) = q(u) = q_o[au^4 + (2 - 2a)u^3 + (a - 3)u^2 + 1] + q_2[bu^4 - 2bu^3 + bu^2] $$
$$ + q_2[cu^4 - (2 + 2c)u^3 + (c + 3)u^2] + q_2[dau^4 + (1 - 2d)u^3 + (d - 2)u^2 + u] $$

with $\sigma$ and $\tau$ as fixed real scalars.

In the quartic equation (10), if we give the value $q_o = 2$, $q_1 = 1$, $q_o' = 5$, $q_4' = 2$ but at $u_x = 0.4$ we fix $q_x$ of the different value $q_x = 3$, $q_x = 4$ and $q_x = 6$, then the curve will pass at the different points (0.4,3), (0.4,4) and (0.4,6) that are shown in Figure 4e. On the other hand, when those values $q_x$ are fixed at $u_x = 0.7$, then the curve will change and be deformed in the right direction (Figure 4f). This mean that we can modify the curve shape with changing the value $u_x$ or $q_x$.

![Figure 4. Some examples to represent the longitudinal pipe shapes](image-url)
4. Implementation of the Methods

Let two unit constant vectors \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) perpendicular in space and determine a unit vector \( \mathbf{l} = \mathbf{v}_1 \wedge \mathbf{v}_2 \). Based on this data vectors, we can define a unit line segment in space \( \mathbf{L}(u) = \langle x_0, y_0, z_0 \rangle + u \mathbf{l} \) and in more general, the line segment can be defined by

\[
\mathbf{L}(u) = \langle x_0, y_0, z_0 \rangle + \lambda \ u \mathbf{l} \tag{12}
\]

where \( \lambda \) is a positive real constant and \( 0 \leq u \leq 1 \). The constant vector \( \langle x_0, y_0, z_0 \rangle \) and the vector \( \mathbf{l} \) are respectively the origin and the unit direction vector of the line segment. Now consider each point of the line segment curve \( \mathbf{L}(u) \) as the centre point of the defined circle of radius in the plane \([\mathbf{v}_1, \mathbf{v}_2]\) along the direction of parameter \( u \). Then using the equation (1), (12) and the determination of the real functions \( \rho(u) \) and \( r(v) \) in the equation (2) until (11), we can model the various shapes of the parametric tubular surface in the form

\[
\mathbf{T}_s(u, v) = \mathbf{L}(u) + \mathbf{r}_s(u,v), [\cos(\phi) \mathbf{v}_1 + \sin(\phi) \mathbf{v}_2] \tag{13}
\]

with \( \mathbf{L}(u) = \langle x_0, y_0, z_0 \rangle + \lambda \ u \mathbf{l} \) and \( \mathbf{r}_s(u,v) = \rho(u)r(v) \). These real functions \( \rho(u) \) and \( r(v) \) are elected among the \( \rho_1(u), \rho_2(u), ..., \rho_4(u) \) and \( r_1(v), r_2(v), ..., r_4(v) \) in the equation (2) until (11), the value \( \phi = 2\pi v \) and \( 0 \leq u \leq 1 \).

As a validation of equation (13), let the vectors \( \mathbf{v}_1 = \langle 0, 1, 0 \rangle, \mathbf{v}_2 = \langle 0, 0, 1 \rangle \) and \( \mathbf{L}(u) = \langle 10, 0, 0 \rangle + 15u. \langle 1, 0, 0 \rangle \). If we choose \( \rho(u) = 1.5 \) and \( r(v) = 2 + \sin(2\pi v) + \cos(4\pi v) \) such that \( \mathbf{r}_s(u,v) = 1.5 \ [2 + \sin(2\pi v) + \cos(4\pi v)] \), equation (13) will yield Figure 5a. When we change \( \rho(u) \) in the form the Bézier and Hermit \( \rho(u) = 2(1-u)^3 + 12(1-u)^2u + 3(1-u)u^2 + 2u^3 \) and \( \rho(u) = (2u^3-3u^2+1) + 2(-2u^3+3u^2) + 8(u^3-2u^2+u) + 10(u^3-u^2) \) that are shown at Figure 4c and 4d, we will find Figure 5b and 5c, respectively. Figure 5d, 5e and 5f show the graph of of the quartic real function and its application to model the tubular shapes. Meanwhile, Figure 5d, 5e and 5f are the tubular surface shapes that are defined respectively by \( r(v) \) of the formula \( r_d(v), r_f(v) \) and \([r_1(v), r_f(v)]\).

Based on the numerical calculation results, we have some interpretations as follows. Equation (1) up to equation (8) can design effectively various cross-section forms of the pipes. They are very applicable to construct the inner or outer cross-section curves of the pipes in single (Figure 5a, b, c, e), double (Figure 5d, e, f, i) or triple layers such that their thickness can be adjusted to the desired shapes. Equation (9) up to equation (11) can be applied to model the inner or outer curves of pipes in a longitudinal direction. They will design shape fluctuation of the inner or outer pipes along the direction of the center curve of pipes (Figure 5d, e, f, l). Finally, equation (13) as a combination of both formulas are handy to design all shapes and thickness of the pipes in a linear direction.

![Figure 5. Pipe patches design with the centre curves of line segment](image-url)
5. Conclusions

This paper formulated the various functions that can be used to design many shapes of the cross-sections and longitudinal sections of pipe patches. Using the polar coordinate formulas of equation (1) up to equation (8), we can design effectively various inner or outer cross-section curves of the pipes in single, double or triple layers such that their thickness can be adjusted to desired shapes. To design shape fluctuation of the inner or outer pipes along the direction of their center curve of pipes can be used, respectively, the trigonometry, Bézier and Hermit curves of equation (9) up to equation (11). Finally, equation (13) as a combination of these formulas are handy to design all shapes and thickness of the pipes in a linear direction.

The pipeline design methods in the cross and longitudinal directions have been introduced, the exciting thing to discuss ahead is how to model the pipes when its center curves are not a line segment. Furthermore, the center curves can be defined by Hermit or Bézier curve.

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