Numerical modeling of the dynamics of multi-storey buildings with elastoplastic seismic insulators

E V Zenkov, V I Sobolev  
Department of Theoretical Mechanics and Strength of Materials, Irkutsk National Research Technical University, 83, Lermontov, Irkutsk, Russian Federation

Abstract. The work is devoted to the problems of reducing the intensity of seismic impacts on multi-storey buildings. The main difficulties in solving the problems of seismic isolation of buildings and structures lie in the field of creating mathematical models of the dynamic interaction of the systems "foundation - seismic isolation device - construction". One of the most effective ways to reduce the level of seismic effects on buildings is to equip them with special devices - seismic isolators, a successful way of implementing which is the use of elastoplastic systems. The method of calculation for seismic actions of multi-storey buildings equipped with elastoplastic seismic insulators is proposed. Numerical modelling of the alternating loading of which uses the rheological model of N N Davidenkov. Modelling of dynamic processes is carried out by means of a mathematical description of the dynamics of two linear multidimensional subsystems that approximate the structures located under seismic insulators and above seismic insulators. The dynamics equations are formed on the basis of the D'Alembert principle. Horizontal seismic effects are considered. The presented technique makes it possible to determine the movements of the nodes of the system at any time of the seismic action specified in the form of a digitized seismogram. The above algorithm is implemented as a software module Proxima. The results of calculations are presented on the example of a model of a 24-storey building designed for the conditions of building in the city of Irkutsk.

1. Introduction

In conditions of dense building of modern cities and increase in the price of land for construction, many designers and customers make bets on increasing the number of storeys of buildings. In areas where there is a danger of earthquakes, an increase in the number of floors of a building can cause difficulties in ensuring the seismic stability of structures [1-3]. One of the possibilities to prevent the destruction of buildings and structures under seismic influences is to equip them with special devices - seismic isolators [7-9], using various physical principles with the formation of hysteresis loops during the alternating loading process [11-13].

Among the most promising are the elastoplastic seismic isolators intended for pumping out the potential energy of the building and reducing the intensity of the stress-strain state of the structures in the process of seismic action. The effect of suppressing seismic influences when using elastoplastic devices is based on the manifestation of residual deformations that decrease the restoring elastic force when deviating from the equilibrium position and, thus, forming a hysteresis loop when the direction of movement is changed [4, 9, 10].

The current practice of calculating buildings and structures for seismic actions uses a predominantly spectral technique, which assumes that the linear elastic-dissipative elements in the
calculated structures have nonzero rigidity [3, 6]. This condition is necessary to ensure the static stability of the calculated system. At the same time, the introduction of the geometrically variable systems into the design scheme leads to the degeneracy of the stiffness matrices, as a result of which calculation by the spectral technique becomes impossible. The introduction of nonlinear kinematic or elastoplastic elements leads to the inevitable coupling of vibrational modes capable of performing energy exchange. In this case, it is impossible to reduce the initial multiply connected system of equations to separate differential equations using a solution by the spectral method. A numerical method for calculating buildings with elastoplastic seismic insulators is proposed, based on the simulation of nonstationary processes of dynamic interaction of nonlinearly coupled linear subsystems.

2. Method of numerical calculation of building dynamics with seismic insulators

The rheological model of N N Davidenkov [10, 11], which led to the construction of a numerical model of the building on the basis of discrete approximations, is based on the development of the method for calculating the dynamics of a building equipped with elastoplastic seismic insulators [14, 15]. In the general case, the numerical calculation method for building dynamics with seismic insulators can be represented as a sequence of the following stages:

1) Using the known methods [16, 17] and existing computational complexes, finite element sampling of the building's structures is carried out;
2) The discrete dynamic system is divided into two linear parts: the Lower Linear System (LLS) is located below the seismic insulators and the Upper Linear System (ULS) located above the seismic insulators (Figure 1).

3) On the basis of existing software complexes we obtain the stiffness matrix of two linear systems (LLS and ULS);
4) Determine the inertial parameters of the associated ground and inertial parameters, concentrated in the non-support nodes of the system;
5) We simulate the work of an elastoplastic seismic insulator by introducing an elastoplastic element into the design scheme according to N N Davidenkov's model [10, 11]. A uniaxial stress state is considered. The model of the element (see Figure 1, I) consists of an infinite number of arms. In each arm, the spring of rigidity $E_d$ is connected in series with the ideal dry-friction damper of the value $E_h$. The stiffness characteristics of all the springs $E$ are the same, the yield strengths of the springs $H$ are different and distributed continuously with a probability density $p(h)$.

The law of deformation of an arbitrary arm has the form:

$$d\sigma = E(\varepsilon - \varepsilon_h)dh$$

$$E(\varepsilon - \varepsilon_h)dh = Ehd\text{sgn}\varepsilon_h$$

where $d\sigma$ – the stress in an arbitrary arm $h$, $\varepsilon$ – the total deformation of the arm, which is the same for all arms, $\varepsilon_h$ – the plastic deformation in the arm $h$. In the equality (1), the function $\text{sgn}\varepsilon_h$ is used in the following sense: the function $\text{sgn}\varepsilon_h$ is equal to $(+1)$ at a positive plastic deformation rate and $\text{sgn}\varepsilon_h$ is equal to $(-1)$ at a negative plastic deformation rate, at zero plastic deformation rate $\text{sgn}\varepsilon_h$ in the interval $(-1; +1)$. With this definition of $\text{sgn}$, the system of equations (1) is suitable for describing the processes of loading and unloading the arm, both in the absence of plastic deformations and in the presence of them. Summing the efforts in all the arms of the model in accordance with the probability density $p(h)$, we obtain

$$\sigma = E\int_0^\varepsilon (\varepsilon - \varepsilon_h) p(h)dh$$

$${\varepsilon - \varepsilon_h = h\text{sgn}\varepsilon_h}$$

or

$$\sigma = E\varepsilon - E\int_0^\varepsilon \varepsilon_h p(h)dh$$

$${\varepsilon = h\text{sgn}\varepsilon_h + \varepsilon_h}$$

In expression (2), the first term represents the elastic force following Hooke's law, the second term describes the dissipation effect. Equation (3) serves to determine the plastic deformation in an arbitrary arm $h$. The system of equations (3) is suitable for describing the processes of loading and unloading a material according to any law in time.

To construct the unloading curve, the Masing principle [10, 18] will be used, which states that the unloading curve can be obtained from the deformation curve by means of coordinate transformation. The shape of the discharge curve does not depend on the strain attained during loading. The form of the bilinear elastoplastic model of the loading-unloading curve is shown in Figure 2.

![Bilinear model of elastoplastic loading-unloading curve](image)

6) A system of equations of dynamic equilibrium of a discrete model of a building is formed on the basis of the D'Alembert principle [3, 19]. In this case, the state of the elastoplastic elements is uniquely determined by the mutual movement of the upper and lower supports of the seismic isolators using the analytical expressions described. In this discrete version of the model, the inertial parameters (masses and moments of mass inertia relative to the vertical) are considered to be concentrated at the center of mass of each overlap taking into account the masses of the adjacent vertical structures. To form the systems of equations for the dynamic state of a building subject to horizontal influences, we introduce two coordinate systems:
Mechanical Science and Technology Update (MSTU-2018) IOP Publishing
IOP Conf. Series: Journal of Physics: Conf. Series 1050 (2018) 012102  doi:10.1088/1742-6596/1050/1/012102

- an absolute coordinate system, rigidly connected with the resting position of the building;
- a relative coordinate system, rigidly connected with the movement of the ground environment.

We denote the coordinates of the position of the model nodes in the absolute coordinate system through \( x_{\alpha}, y_{\alpha} \), and the relative coordinates are \( x, y \). The dynamic equilibrium of a building when moving a ground environment can be described in vector form by a system of differential equations:

\[
M \frac{d^2 V}{dt^2} + R(V) = 0, \tag{4}
\]

where \( V_a = (x_{1\alpha}, y_{1\alpha}, \varphi_{1\alpha}, x_{2\alpha}, y_{2\alpha}, \varphi_{2\alpha}, \ldots, x_{n\alpha}, y_{n\alpha}, \varphi_{n\alpha})^T \) – the vector of displacements of the nodes of the system in the absolute coordinate system, \( V = (x, y, \varphi, x_1, y_1, \varphi_1, x_2, y_2, \varphi_2, \ldots, x_n, y_n, \varphi_n)^T \) – the displacement vector of the system nodes in the relative coordinate system, \( M = \text{diag} (m_1, m_1, I_1, m_2, I_2, \ldots, m_n, I_n) \) – the diagonal matrix of inertial parameters, in which \( m_i, I_i \) – the mass and moment of inertia of the solid mass (overlap) with the number \( i \) [5, 20].

The vector-valued function \( R(V) \) being generally nonlinear, contains linear expressions formed with the help of the previously defined rigidity matrices \( R_a \) and \( R_e \) for LLS and ULS. For inclusion in the system of equations of dynamics, the matrix \( R_e \) expands taking into account the compliance of the support node, which is now a node characterizing the movements of the upper support part of the seismic insulators. Such expansion is carried out on the basis of the equilibrium equations of ULS in projections on the X, Y axis and reactive moments in the horizontal plane. Thus, the matrix \( R_e \) increases its order by 3 and becomes triply degenerate. Let \( V_a \) be the extended matrix \( R_a \).

If the matrix \( R_a \) has the dimension \( k \), then the current number of the reference node of the ULS located on the seismic insulators for \( V_a \) and \( V \) is \( k + 1 \). In this way

\[
V_a = V + V_g, \tag{5}
\]

where \( V_g \) – the displacement vector of the model reference point in the absolute coordinate system.

Taking into account equality (4), the system of equations (3) will have the form:

\[
M \frac{d^2 V}{dt^2} + R(V) = -M \frac{d^2 V_g}{dt^2} \tag{6}
\]

The components of the vector \( V_g \) can be specified in the form of earthquake records along the directions of the X, Y axes and rotations of \( \varphi \) with respect to the vertical. In the system of equations (5), the vector function \( R(V) \) has the form:

\[
R(V) = \begin{bmatrix}
R_a \cdot V_a + R_k(\Delta V_{k+1}) & 0 \\
0 & R_{vr} \cdot V_{vr} + R_{k+1}(\Delta V_{k+1})
\end{bmatrix}, \tag{7}
\]

where \( R_k(\Delta V_{k+1}) = (0, 0, 0, \ldots, r_{\alpha k}(\Delta V_{k+1}), r_{\beta k}(\Delta V_{k+1}), r_{\gamma k}(\Delta V_{k+1}))^T \),

\[ R_{k+1}(\Delta V_{k+1}) = (r_{\alpha k+1}(\Delta V_{k+1}), r_{\beta k+1}(\Delta V_{k+1}), r_{\gamma k+1}(\Delta V_{k+1}), \ldots, 0, 0, 0)^T, \]

\( \Delta V_{k+1} \) – a vector formed from the differences in the linear displacements of the upper and lower reference points of the seismic insulators,

\( r_{\alpha k}(\Delta V_{k+1}), r_{\beta k}(\Delta V_{k+1}), r_{\gamma k}(\Delta V_{k+1}) \) – projections of elastoplastic reactions in the horizontal linear and angular connections of the node with the number \( k \) of the LLS,

\( r_{\alpha k+1}(\Delta V_{k+1}), r_{\beta k+1}(\Delta V_{k+1}), r_{\gamma k+1}(\Delta V_{k+1}) \) – projections of elastoplastic reactions in the horizontal linear and angular connections of the node with the number \( k + 1 \) ULS.

The values of \( R_k(\Delta V_{k+1}), R_{k+1}(\Delta V_{k+1}) \) are determined by the algorithms outlined in step 5. The solution of the system of equations (5) is carried out by the Runge-Kutta method [15, 21]. At each step of integration with respect to the independent time variable \( t_i \) the components of the vectors \( V, \frac{dV}{dt}, \frac{d^2V}{dt^2} \) are determined.

The presented method of calculating the dynamics of the building makes it possible to determine the movements of the nodes of the system at any time during the action of a real seismogram. However, this is not sufficient to estimate the loss of seismic impact points. It is necessary to determine how much the seismic effect reduces the introduction of an elastoplastic seismic insulator into the building structure. This estimate of the loss of seismic impact points can be performed by the values of the maximum acceleration of the reference node of the ULS, which are considered external seismic influences on the ULS.. The algorithm for calculating the dynamics of the building is implemented in the form of a software module Proxima.
3. Practical implementation of numerical methods for calculating a building with seismic insulators

The Proxima software module is designed to simulate the dynamics of load-bearing structures of a building with elastoplastic seismic insulating supports, with seismic actions.

The structure of the initial data of the Proxima module includes the stiffness and inertial parameters of the bearing structures of the building, the location and parameters of the seismic insulators, the accelerogram of the seismic action, the angle of the direction of action, the correction factor, and the calculation settings. The source data can be imported from a pre-prepared text file, or specified in a table form directly in the program itself.

The results of building dynamics simulation using the Proxima module are presented in the form of trajectories in a tabular form for each degree of freedom of each node in the calculation scheme (Figure 3). The results of the calculation are the acceleration trajectories ($a$), velocities ($V$) and displacements ($u$) of the dynamically calculated building circuit nodes, as well as the trajectory (graphics) of the dynamic forces acting on the nodes of the calculation circuit.

For each trajectory, the maximum and minimum value is reached, which it reaches at a given time interval. The results of the calculation can be exported to a text file, and then used in other systems of engineering analysis.

![Figure 3: Page "Results" after the calculation](image)

Directly below the field with the counter is a table in which the trajectories and graphs corresponding to the chosen degree of freedom are displayed. The table contains eight columns (see Figure 3). The number of rows in the table depends on the size of the time interval on which the trajectories (the integration segment of the system of differential equations) and the values of the interval for preserving the trajectories are calculated.

The first column contains time marks with a step equal to the interval of preservation of dynamic trajectories. In the second column are the values of the relative displacement of the degree of freedom, corresponding to the time marks. In the third column, the relative velocity is placed, in the fourth column - relative acceleration, in the fifth - the inertial force, in the sixth - the reaction in the connection, in the seventh - the external force acting on the link, in the eighth - the absolute acceleration.

The first line of the table shows the names of physical quantities located in the corresponding columns. In the second and third lines, respectively, the minimum and maximum values of the corresponding quantity are located on the entire time interval.

In Figure 4 shows an accelerogram of seismic effects for modelling the dynamics of a building model consisting of 24 floors, designed for building conditions in the city of Irkutsk.
Figure 4. Graphical representation of the earthquake accelerogram (9 points)

The results of calculating the dynamics of the building with the Proxima module are shown in Figure 5 in the form of the trajectory of displacement ($u$) of the nodes of the dynamic design of the building at a height of 74.4 meters from the base (reference part).

Figure 5. The trajectory of moving the nodes of the building's settlement diagram at the level of 74.4 meters

The results of calculations can be used to evaluate the performance of elastoplastic seismic insulators. For this type of insulator, the main criterion is the pumping out of the earthquake energy due to the so-called plastic effect, that is, the more the seismic insulator works in the plastic stage, the more effectively the earthquake energy is pumped out. On the other hand, there are restrictions on movements beyond which the insulator ceases to satisfy the construction of the bearing capacity.

4. Conclusion
The proposed methodology allows performing calculations of multi-storey buildings equipped with elastoplastic seismic insulators for seismic actions, represented by seismogram records, without using the decomposition procedures of multimass dynamical systems in their own vibrational forms. This circumstance relieves the calculator of doubts connected with the estimation of the errors of such transformations.

Analysis of the movements of the top of the seismic isolator relative to its support part makes it possible to evaluate the effectiveness of using the seismic isolation system. It is obvious that the efficiency of such a system is determined by the measure of the manifestation of plastic deformations, which is caused by hysteresis phenomena and associated energy losses due to vibrations. The latter circumstance makes it possible to perform a quantitative estimation of the level of seismic isolation and to carry out the selection of seismic isolation variants for the given operating conditions.
5. References

[1] Bershtein S A 1938 Fundamentals of the Dynamics of Structures (Moscow: Gosstroyizdat) p 160
[2] Borges D F, Ravara A 1978 Design of reinforced concrete structures for seismic regions Structures (Moscow: Stroiizdat) p 135
[3] Claf R, Penzien J 1979 Dynamics of constructions (Moscow: Stroiizdat) p 319
[4] Sobolev V I, Gaskin V V 2003 Transformation of seismic influences on multi-storey buildings with vibration isolating foundations Proceedings of the VIth Russian National Conference on Earthquake-Resistant Construction and Seismic Zoning (Sochi) p 67
[5] Sobolev V I, Gaskin V V 1999 Numerical studies of buildings under seismic influences given by oscillograms 3rd regional conference on seismic resistance. construction and seismic zoning (Sochi) p 3
[6] Koreneva B G, Rabinovich I M 1984 Dynamic calculation of buildings and structures Handbook of the designer (Moscow: Stroiizdat) p 303
[7] Ishlinsky A Yu 1985 Plasticity Overview. Mechanics. Ideas. Tasks. Applications (Moscow: Nauka) pp 258-271
[8] Nashif A, Jones D, Henderson J 1988 Dampening of oscillations (Moscow: Mir) p 448
[9] Sobolev V I, Gotovsky S I 2003 Dynamics of seismic manifestations in multi-storey buildings equipped with kinematic foundations Problems of Mechanics of Modern Machines: Proceedings of the Second International Conference (Ulan-Ude: GISU) chapter 2 p 6
[10] Palmov V A 1976 Oscillations of elastoplastic bodies (Moscow: Science) p 328
[11] Davidenkov N N 1938 On the scattering of energy in vibrations (ZhTF) vol 8 pp 43-45
[12] Boevkin V I, Pavlov Yu N 1971 Dynamic absorber with dry friction (Moscow: MWU) No 153 pp 110-117
[13] Dubkov S V 1988 The method of elastic solutions for transversely isotropic elastoplastic shells Numerical analysis, mathematical modeling and their application in mechanics (Moscow: Mosk. University) pp 41 - 42
[14] Sobolev V I, Gaskin V V 1998 Fluctuations in multi-storey buildings with seismic influences given by oscillograms Bulletin of IrSTU. Series "Construction" (Irkutsk: IrSTU) pp 112-116
[15] Bakhvalov N S 1973 Numerical methods (Moscow: Nauka) p 631
[16] Bate K, Wilson E 1982 Numerical Analysis Methods and Finite Element Method (Moscow: Stroiizdat) p 447
[17] Argyris J H, Boni B, Hinderlang V 1982 Finite element analysis of two- and three dimensional elastoplastic frames - the natural approach Comp. Meth. Appl. Mech., vol 35 No 2 pp 221-248
[18] Annin B D 1988 Development of methods for solving elastoplastic problems Mechanics and scientific and technical progress. Mechanics of a deformable solid (Moscow: Science) chapter 3 pp 123-136
[19] Mandelstam A I 1955 Lectures on fluctuations (Moscow: Academy of Sciences of the USSR) p 503
[20] Sobolev V I, Gaskin V V, Snitko A N 1996 Numerical study of multi-storey buildings under seismic influences given by oscillograms Methods of potential and finite elements in the automation of research of engineering structures (St. Petersburg) p 1
[21] Golberg S M, Zakharov A Yu, Filippov S S 1976 On some numerical methods for solving nonlinear systems of ordinary differential equations (Moscow: IPM AN USSR) No 12 p 48