On the Effect of Nonlinear Energy Sink Damping in Seismic Vibration Attenuation

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Abstract Nonlinear Energy Sinks (NESs) have been proposed for passively reducing the amplitude of vibrations in different types of structures. The main advantage of NES over traditional Tuned Mass Dampers (TMDs) lies in its capability to redistribute the vibrating energy inside a primary structure, what effectively reduces the amplitude of the structure oscillations over a wide range of frequencies. However, the performance of an NES can be substantially affected even by small variations on input energy as in the case of buildings under seismic ground excitation. In this work it is shown that the NES energy sensibility can be significantly reduced by properly selecting the NES damping coefficient. A three stories shear building model subject to seismic ground excitation is used to numerically study the effect that NES damping has on its vibration reduction performance.

Keywords Nonlinear Energy Sink · Passive vibration control · Targeted Energy Transfer

1 Introduction

NESs have gained popularity as vibration palliatives because they have no preferred natural frequency and can resonate with any mode of the primary structure to which they are connected. An NES reduces vibrations in two different forms. In the first form, one-directional energy transfer is generated from the structure to the NES where the vibrating energy is consumed by NES damping. This process is called Targeted Energy Transfer (TET) [11]. In the second form, the NES induces energy transfer from the structure’s low-frequency modes to the high-frequency ones where the vibrating energy is dissipated by the internal damping of the structure [2]. Since NESs are not frequency-tuned absorbers, their main advantage over the traditional TMD is their power to dissipate vibrating energy in a broader range of frequencies [8]. NESs are energy-dependent absorbers characterized by thresholds where effective NES designs regions can be very close to low efficiency ones [17]. It is then frequent that even small changes in energy levels can reduce significantly the performance of previously optimized NESs designs.

Previous studies have proposed the use of NESs for the mitigation of seismic induced vibrations. Since seismic ground excitation is characterized by random energy fluctuations, the performance of previously optimized NES absorbers could be dramatically reduced when the exciting seismic energy is slightly modified. In Luo et al. [9] a nine-floor structure coupled to an array of NESs is excited using different historic earthquake motions. For specific NES dampings coefficients, the experiments display robust vibration mitigation at different energy levels. Ahmadi et al. [1] use scaled historic earthquakes to optimize a NES attached to the top story of a building. Nucera et al in [13] use a two degree-of-freedom structure coupled to a Vibro-Impact NES to study the system’s response to four historic earthquakes. All these works demonstrate the NES potential to reduce seismic induced vibrations but do not
addressed the effect that damping has on NES performance robustness.

NES damping has an important effect on the vibration attenuation. In a numerical study, Savva et al. [16] varied the damping content of an NES absorber to study its effect on the reduction of torsional oscillations produced in a vehicle powertrain while the engine runs at idle conditions. Similarly, in Wierschem et al. [17], numerical analysis is used to obtain the optimal NES parameters that induces the maximum vibration attenuation in a two-story building. The optimal NES design is then tested using a seismic load reduced in amplitude by 50 percent while decreasing the natural frequency of the primary structure in 15 percent. Results confirm that for this specific load reduction, acceptable NES performance is maintained. No other load variations were considered.

The present work analyzes the effect that NES damping has on reducing seismic vibrations in shear buildings. This work also explores the use of multiple NESs to improve absorber robustness. A three-story building is excited using scaled and randomly generated ground accelerograms. The oscillation amplitude of the top floor is then compared for the structure with and without NES attachments. NES performance sensibility is studied in two different NES arrangements. The NES damping is varied and the accelerograms are scaled to find damping regions where the NES performance is less sensible to variations of the seismic energy.

The paper continues as follows. In section two, the dynamic equations of the structure with and without NESs are defined. In section three, the method used for the generation of stochastic accelerograms is described. In section four, optimal parameters for the undamped NES case are obtained. In section five, the sensitivity of the optimized NES designs to energy variations is studied. In section six, a robustness index is used to obtain an optimal NES damping value. Section seven shows the response of the optimized damped NES system to different seismic ground accelerograms. Finally the conclusions are presented.

2 Dynamic models

The three-story building defined in [4] and shown in Figure 1 is used in this numerical analysis. The dynamic equation of this system is:

\[ M(\ddot{x} + I_3\dot{x}_g) + C\dot{x} + Kx = 0 \]  

(1)

where \( \mathbf{x}^T = [x_1 \ x_2 \ x_3] \) is the vector of displacements for each of the three floors, \( x_g \) is the ground displacement and \( I_3 \) is the \((3 \times 3)\) identity matrix. The mass, the stiffness and the damping matrices are respectively:

\[
\begin{align*}
M &= \begin{bmatrix} 2100 & 0 & 0 \\ 0 & 2100 & 0 \\ 0 & 0 & 2100 \end{bmatrix} \text{ kg} \\
C &= \begin{bmatrix} 14175 & -10500 & 0 \\ -10500 & 21000 & -10500 \\ 0 & -10500 & 10500 \end{bmatrix} \text{ Ns/m} \\
K &= \begin{bmatrix} 3869950 & -2607500 & 0 \\ -2607500 & 5215000 & -2607500 \\ 0 & -2607500 & 2607500 \end{bmatrix} \text{ N/m}
\end{align*}
\]

(2)

(3)

(4)

The three natural natural frequencies of this system are 1.97, 6.30 and 9.87 Hz.
Two different NES configurations attached to the aforementioned primary structure are analyzed. The first configuration includes a cubic NES connected to the third floor of the primary structure (See Figure 2). It is assumed that the NES mass is substantially smaller than the mass of the floor to which it is attached. Each NES is designed to be non-parasitic; by this, it is recognized the basic feature of the NES of not adding mass to the primary structure, since the combined mass of the NES and the floor to which it is attached is equal to the corresponding mass floor of the primary structure [2]. The dynamic equation describing the primary structure coupled to the first NES configuration is:

$$\mathbf{\ddot{M}}(\ddot{x} + \mathbf{I}_1 \ddot{x}_3) + \mathbf{C}\ddot{x} + \mathbf{Kx} + \mathbf{F}(\ddot{x}) = 0 \quad (5)$$

where $\mathbf{x}^T = [x_1 \ x_2 \ x_3 \ x_3]$ with $\ddot{x}_3$ as the displacement of the NES connected at the third floor and $\mathbf{I}_1$ is the $(4 \times 4)$ identity matrix. The mass, stiffness and damping matrices of this system are respectively:

$$\mathbf{M} = \begin{bmatrix} 2100 & 0 & 0 & 0 \\ 0 & 2100 & 0 & 0 \\ 0 & 0 & 2100 & 0 \\ 0 & 0 & 0 & \bar{m}_3 \end{bmatrix} \quad (6)$$

$$\mathbf{C} = \begin{bmatrix} 14175 & -10500 & 0 & 0 \\ -10500 & 21000 & -10500 & 0 \\ 0 & -10500 & 10500 + \bar{c}_3 & -\bar{c}_3 \\ 0 & 0 & -\bar{c}_3 & \bar{c}_3 \end{bmatrix} \quad (7)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K} \\ \mathbf{z}^T \\ 0 \end{bmatrix} \quad (8)$$

where $\mathbf{z} = [0 \ 0 \ 0]$, $\bar{m}_3$ is the inertia of the NES connected to the third floor and $\bar{c}_3$ is the corresponding NES damping. The vector of restoring forces provided by the nonlinear NES spring is:

$$\mathbf{F}(\ddot{x}) = \begin{bmatrix} 0 \\ 0 \\ \hat{k}_3(x_3 - \ddot{x}_3)^3 \end{bmatrix} \quad (9)$$

where $\hat{k}_3$ is the stiffness coefficient of the nonlinear cubic spring connected to the third floor.

The second configuration studied has two NESs connected respectively to the second and to the third floors of the primary structure (See Figure 3). Similarly as in the previous configuration, in this case both NESs are designed to be non-parasitic. The dynamic equation describing the primary structure coupled to the two-NES configuration is:

$$\mathbf{M}(\ddot{x} + \mathbf{I}_2 \ddot{x}_3) + \mathbf{C}\ddot{x} + \mathbf{Kx} + \mathbf{F}(\ddot{x}) = 0 \quad (10)$$

where $\ddot{x}_3$ is the displacement of the NES connected at the second floor and $\bar{c}_2$ is the corresponding NES damping. The vector of restoring nonlinear forces provided by the NES springs is:

$$\mathbf{F}(\ddot{x}) = \begin{bmatrix} 0 \\ \hat{k}_2(x_2 - \ddot{x}_2)^3 \\ \hat{k}_2(x_2 - \ddot{x}_2)^3 \\ \hat{k}_3(x_3 - \ddot{x}_3)^3 \\ \hat{k}_3(x_3 - \ddot{x}_3)^3 \end{bmatrix} \quad (14)$$

where $\hat{k}_2$ is the stiffness coefficient of the nonlinear cubic spring connected to the second floor.
3 Generation of Seismic Ground Acceleration

This section describes the method applied to generate the input seismic ground acceleration used in the dynamic equations (1), (5) and (10). Similar methods are shown in [10],[5],[14] and [15]. This method is based on the classical kernel presented in [18] and [12]. The obtained accelerograms are scaled to produce the desired peak ground acceleration. The building code used is the Eurocode 8 defined in the European standard EN 1998-1 [6]. The seismic ground acceleration is computed using the equation:

\[ \ddot{x}_g(t) = \Psi(t) \times g(t) \]  

where envelop signal \( \Psi(t) \) [4], is given by:

\[ \Psi(t) = \begin{cases} \left( \frac{1}{t_1} \right)^2 & 0 \leq t \leq t_1 \\ 1 & t_1 < t \leq t_2 \\ e^{-a(t-t_2)} & t > t_2 \end{cases} \]  

where \( t_1 \) is the time when the signal reaches its maximum value, \( t_2 \) is the time when the signal begins to decrease towards the extinction of the earthquake and \( a \) is the decreasing exponential rate. Overall, \( a, t_1, \) and \( t_2 \) determine the envelope of the earthquake signal. The function \( g(t) \) is given by:

\[ g(t) = \sqrt{2} \sum_{k=1}^{n} \sqrt{S(k \Delta f) \Delta f \cos[2\pi k \Delta f t + \phi(k)]} \]  

where \( \Delta f \) is the frequency step width, \( n \) is the number of frequency steps in the signal bandwidth, and \( \phi(k) \) is a uniformly distributed random phase. The spectral density for the \( i^{th} \) iteration, is defined as

\[ S_i(f) = S_{i-1}(f) \left[ \frac{EAS(f)}{EAS(f)} \right] \]  

where \( EAS(f) \) is the horizontal elastic response spectrum and is given by

\[ EAS(f) = \begin{cases} 2.5 f^2 \eta S a_g T_C T_D & 1 \leq f < \frac{1}{T_D} \\ 2.5 f \eta S a_g T_C & \frac{1}{T_D} \leq f < \frac{1}{T_C} \\ 2.5 f \eta S a_g & \frac{1}{T_C} \leq f < \frac{1}{T_B} \\ S a_g \left( \frac{2 \eta - 1}{f T_B} \right) & f \geq \frac{1}{T_B} \end{cases} \]  

where \( \eta \) is the damping correction factor, \( S \) is the soil factor, \( a_g \) is the design ground acceleration and \( T_B, T_C \) and \( T_D \) are corner periods in the spectrum. \( EAS_i(f) \) is the elastic response spectra for the \( i^{th} \) iteration. Algorithm 1 summarizes the required steps.

Figure 4 shows the Response Spectra Acceleration (RSA) for a type 1 earthquake in ground type E with a duration 30 seconds and \( a_g = 0.6g \). Figure 5 shows the corresponding accelerogram signal \( \ddot{x}_g(t) \).

![Elastic acceleration response spectrum](image)

**Algorithm 1** Stochastic Accelerogram Generation

1: Variable initialization
2: for \( iteration = 1, 2, \ldots \) do
3: \[ g(t) = \sqrt{2} \sum_{k=1}^{n} \sqrt{S(k \Delta f) \Delta f \cos[2\pi k \Delta f t + \phi(k)]} \]
4: \[ \ddot{x}_g(t) = \Psi(t) \times g(t) \]
5: Determine \( EAS_i(f) \)
6: \[ S_i(f) = S_{i-1}(f) \left[ \frac{EAS_i(f)}{EAS_i(f)} \right] \]
7: end for
8: PGA scaling

In the next section the Arias Intensity \( (i_A) \) is used to quantify the earthquake acceleration power [3]. The Arias Intensity is defined as:

\[ i_A = \pi \frac{3}{2g} \int_0^T \ddot{x}_g(t) \, dt \]  

where \( g \) is the gravity constant (9.81 m/s\(^2\)). For the acceleration signal in Figure 5, \( i_A = 6.5 \text{ m/s} \) with \( \ddot{x}_g(t) \) being sampled at \( 4f_u \) samples/s, \( f_u = 20 \text{ Hz} \) (\( f_u \) is the upper cutoff frequency of the earthquake).
4 NES Tuning

Tuning procedures focus in finding the NES optimal parameters (stiffness and inertia) that mitigate the vibrations in primary structures subject to deterministic, impulsive excitation or random loading [7]. The tuning method proposed here, aims to minimize the sum of the squares of the velocities for all the floors in the structure using a specific earthquake record. Simulations shown that the smaller is this sum, the less is the amplitude of oscillations exhibited in the structure. It is initially assumed that there is not damping in the connection between the NES and the corresponding story. The rationale for optimizing undamped NESs is that this matches to the case of maximum energy redistribution within the modal space of the structure [2].

The tuning of the NES absorber in Eq. 5, and shown in Fig. 2 (primary system with one NES at the third floor), is carried out by maximizing the index:

$$P_1 = \int_0^t \sum_{i=1}^{3} (v_{i,p})^2 dt - \int_0^t \sum_{i=1}^{3} (v_{i,1})^2 dt$$

(21)

where $t$ is the time duration of the earthquake, $v_{i,p}$ is the velocity of the $i^{th}$ story of the primary structure defined in Eq. 1 (No-NES) and $v_{i,1}$ is the velocity of the $i^{th}$ story of the structure with one NES attached to the third floor (One-NES).

A large number of NES stiffness and inertia combinations are executed using MATLAB/Simulink. The goal is to find the NES parameter combination that maximizes the performance index defined in Eq. 21. The NES inertia $\tilde{m}_3$, is varied between 1 to 10 percent of the inertia of the third story ($M_3 = 2100 \text{ kg}$). The NES stiffness $\tilde{k}_3$ is varied between $1 \times 10^2 \text{ N/m}^3$ to $1 \times 10^8 \text{ N/m}^3$. In total, $1 \times 10^5$ different $\tilde{k}_3$-$\tilde{m}_3$ pair combinations are examined. The ground accelerogram signal shown in Figure 5 is used to excite both structures. This acceleration profile lasts 30 seconds and it is characterized by a maximum value of $0.6g \text{ m/s}^2$ with $i_A = 6.5 \text{ m/s}$.

The simulation results are summarized in the contour plot shown in Fig. 6. The $x$ axis indicates the variation in the NES inertia. The $y$ axis indicates the variation in the NES stiffness coefficient. The colorbar on the right-hand side of the contour plot, indicates the performance index $P_1$ achieved. This contour plot shows that there are regions of stiffness and inertia combinations providing effective vibration reduction when the earthquake shown in Fig. 5 is applied to the structure in Fig 2. The maximum value of the performance index $P_1$ obtained is $3.1 \text{ m}^2/\text{s}$. This value is obtained when the NES inertia $\tilde{m}_3 = 180 \text{ kg}$ and the corresponding NES stiffness is $\tilde{k}_3 = 1.67 \times 10^6 \text{ N/m}^3$.

Figure 7 shows the displacement of the third floor of the building structures with one NES at the third floor (Fig. 2) and without NES (Fig. 1). Both systems are excited using the ground acceleration shown in Fig. 5. As it is shown in Fig. 7, substantial vibration reduction is achieved when a properly tuned single cubic NES is attached to the third floor. It can be seen that after 10 seconds, the amplitude of oscillations are reduced from peak to peak values close to 160 millimeters to peak to peak values around 40 millimeters for the rest of the earthquake duration. Similar behavior is exhibited in the other two floors of the structure.
An additional NES is attached to the second story as it is shown in Fig 3. The NES inertia $\tilde{m}_2$ and the NES stiffness $\tilde{k}_2$, are varied using similar ranges as in the previous case ($\tilde{m}_2$ between 1 to 10 percent of the floor inertia $M_2$ and $\tilde{k}_2$ between $1 \times 10^2$ $N/m^3$ to $1 \times 10^8$ $N/m^3$. The goal is to maximize the performance index:

$$P_2 = \int_0^t \sum_{i=1}^3 (v_{i,1})^2 dt - \int_0^t \sum_{i=1}^3 (v_{i,2})^2 dt$$

where $t$ is the time duration of the earthquake, $v_{i,1}$ is the velocity of the $i^{th}$ story of the structure with one NES attached to the third floor (using the optimal parameters for $\tilde{k}_3$ and $\tilde{m}_3$) and $v_{i,2}$ is the velocity of the $i^{th}$ story of the structure with two NESs. In this second system the previously obtained NES parameters $\tilde{k}_3$ and $\tilde{m}_3$ are maintained constant while the parameters $\tilde{k}_2$ and $\tilde{m}_2$ are varied. Among all the $1 \times 10^5$ parameter combinations simulated, $\tilde{m}_2 = 210$ kg and $\tilde{k}_2 = 9.5 \times 10^6$ $N/m^3$ maximize the index $P_2$ for this particular ground acceleration excitation.

Figure 8 compares the third floor displacement for the system with two NESs (Fig. 3) and the system with no NES (Fig. 1). The addition of this second NES provides a slight vibration reduction improvement when it is compared to the one NES system (See Fig. 7). Since both NESs attachments were tuned assuming no damping, its performance is highly sensitive to energy variations. If the input ground acceleration (with an Arias intensity of 6.5 m/s) is slightly scaled down to obtain a ground acceleration with an Arias intensity of 6.46 m/s, the NES vibration attenuation completely disappears. This effect is shown in Fig. 9.

The next section analyzes the effect that the NES damping has on its sensitivity to energy variation. It is shown that this sensitivity can be reduced by properly tuning the NES damping. This damping cannot be too high in order to keep active the modal energy redistribution effect induced by the NES into the primary structure. Conversely, the NES damping cannot be too low otherwise the system will exhibit high sensitivity to energy variations. Through simulation it is also shown that if the proper NES damping is used, the system with two NESs exhibits a more robust performance than the system with one NES.

5 NES Sensitivity to Input energy variation

NES damping has a substantial effect on its capability to reduce the vibrations induced in the primary structure. The maximum energy redistribution within the modal space of the primary structure is achieved when the NES is undamped [5] (a physical impossible condition). However, numerical simulations have show that the lower the NES damping is, the higher the NES performance, is affected by variations of the input energy. In this section, the ground acceleration signal shown in Fig. 5, is progressively scaled down to generate a set of $1 \times 10^3$ acceleration profiles with different Arias intensities. Each signal is used to excite the three building structures (No-NES, One-NES and Two-NES). For six different damping factors, the performance indexes $P_1$ and $P_2$ defined respectively in equations (21) and (22) are computed. Figure 13, shows how the performance indexes for both systems change when the Arias Intensity is reduced from 6.5 m/s to 2 m/s.
Figure 10, shows that at low NES damping values (e.g. $C_N \leq 100$) performance indexes $P_1$ and $P_2$ changes abruptly even with small variations of the Arias Intensity factor. In such case, even negative (vibration amplification) or very low performance index values can be obtained. Simulations also show that this sensibility is progressively reduced when the NES damping is gradually increased (e.g $C_N \geq 400$). Since higher NES damping values reduce the modal energy redistribution effect inside the primary structure, the performance index also decreases if the NES damping coefficient is sufficiently high.

The advantage of adding a second NES is also highlighted in Fig. 10. In section four, it was shown that for the undamped NES case and when $i_A = 6.5 \text{ m/s}$, the addition of second NES provides negligible vibration reduction improvement when it is compared to the system with one NES ($P_2 \approx P_1$). Conversely, (See Fig 10) at higher NES damping values ($C_N \geq 400$) and for all $i_A \leq 6.5 \text{ m/s}$, the Two-NES performance clearly surpass the One-NES performance ($P_2 > P_1$).

6 NES Robustness Index

In this section an additional criterion is used to quantify the sensibility of the NES performance to energy variations. Fig. 10 shows that the abrupt changes on the $P_1$ or $P_2$ curves (occurring at low NES dampings) reduce the area under them. It is then possible to claim that a large area under any these curves is a good indication that the NES system is less sensible to energy variations. The area under any of these two curves defines the Robustness Index:

$$R_I = \int_0^t P_i \, dt$$

where $P_i$ is any of the two performance indexes defined in Eq. (21) for the one NES system or Eq. (22) for the two NES system and the $t$ is the duration of the ground acceleration signal.

The index $R_I$ is plotted on Fig. 11. To obtain this plot, the area under the $P_1$ and $P_2$ is calculated using 400 different NES damping factors in the range $0 \leq C_N \leq 1 \times 10^4$. Recall that $P_1$ and $P_2$ are computed by progressively scaling down $1 \times 10^5$ ground acceleration profiles. The benefit of using a second NES is also recognized in this plot. The Two-NES system always exhibits a greater $R_I$ factor than the One-NES system. For the Two-NES system, the $R_I$ factor is maximum around the range $500 \leq C_N \leq 2000$ and progressively decreases at lower dampings. (Due to high sensibility to energy variations) or at higher dampings (Because the modal energy redistribution effect stops).
The Performance Index $P_2$ at different NES dampings and Arias Intensities is shown in Fig. 12. The dark spots around the range $0 \leq C_N \leq 200$ indicates the abrupt performance variations characteristic at low NES damping. Conversely, $P_2$ is consistent at higher damping values $C_N \geq 400$ and maximum inside the range $500 \leq C_N \leq 2000$. A good NES performance is achieved if the NES damping is selected around this last range. Fig. 13, compares the third floor displacement of the primary structure and the Two-NES system when $C_N = 880$ Ns/m and $i_A = 6.46$ m/s. A substantial vibration reduction is achieved when it is compared to the undamped case shown in Fig. 9.

Fig. 14 shows the two forms of instantaneous power dissipation activated by the NESs. In the first form (Fig. 14a), the power is dissipated by the main structure as a consequence of the modal energy redistribution induced by the NESs. In the second form (Fig. 14b), power is dissipated by the two NES dampers, after the energy is irreversibly transferred and trapped into the NESs.
7 Response to different Ground Accelerations

The structures are excited using four statistically different ground acceleration profiles (I, II, III, and IV), each with different $i_A$. The NESs parameters are the optimal values selected on previous sections. Fig. 15 compares the third floor displacement for the No-NES with the Two-NES systems when $C_N = 880$. For all of the four ground acceleration profiles, the proposed NESs design provide positive vibration reduction.

Fig. 16 compares the response of the No-NES system with the undamped Two-NES system. The ground acceleration profiles (I, II, III, and IV) are used to excite the undamped structure ($C_N = 0$). As it is expected, the low NES damping significantly affects the robustness of the NES performance and for this particular four cases, the NESs fail to provide considerable vibration reduction. It can be seen that in some cases even harmful vibration amplification is obtained.
8 Conclusions

NES performance is substantially affected even by negligible variations on the input energy. This work presented a numerical analysis of the effect that NES damping has on its capability to reduce vibrations in structures subject to ground seismic excitations. It has been numerically demonstrated that the NES sensitivity to energy changes can be greatly reduced by properly adjusting the NES damping. Although low damping factors can maximize the modal energy redistribution inside the primary structure (what increasing the vibrating energy dissipation), the resultant NES design is highly sensible to energy variations. Higher dampings values, properly selected, can reduce the NES sensitivity while inducing energy dissipation through both structural damping and NES damping. In this work it is also shown that the use of multiple NES tuned properly at different energy levels, increases the NES robustness to energy variations, which is also an additional form to achieve reliable vibration dissipation at different input energy levels.

Conflict of interest

The authors declare that they have no conflict of interest.

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Availability of data and material

Research data for this paper are available on request from Eliot Motato.

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