Research Article

Hans Joakim Skadsem*, Amare Leulseged, and Eric Cayeux

Measurement of Drilling Fluid Rheology and Modeling of Thixotropic Behavior

https://doi.org/10.1515/arh-2019-0001
Received Sep 25, 2018; accepted Jan 08, 2019

Abstract: Drilling fluids perform a number of important functions during a drilling operation, including that of lifting drilled cuttings to the surface and balancing formation pressures. Drilling fluids are usually designed to be structured fluids exhibiting shear thinning and yield stress behavior, and most drilling fluids also exhibit thixotropy. Accurate modeling of drilling fluid rheology is necessary for predicting friction pressure losses in the wellbore while circulating, the pump pressure needed to resume circulation after a static period, and how the fluid rheology evolves with time while in static or near-static conditions. Although modeling the flow of thixotropic fluids in realistic geometries is still a formidable future challenge to be solved, considerable insights can still be gained by studying the viscometric flows of such fluids.

We report a detailed rheological characterization of a water-based drilling fluid and an invert emulsion oil-based drilling fluid. The micro structure responsible for thixotropy is different in these fluids which results in different thixotropic responses. Measurements are primarily focused at transient responses to step changes in shear rate, but cover also steady state flow curves and stress overshoots during start-up of flow. We analyze the shear rate step change measurements using a structural kinetics thixotropy model.

Keywords: Thixotropy; drilling fluids; stress overshoot

PACS: 47.57.-s; 83.10.Gr; 83.60.Pq; 83.60.Rs

1 Introduction

During a drilling operation, drilling fluid is pumped from pits at the surface installation down the drill string and through nozzles at the bit, which is at the very bottom of the drill string. From there, drilling fluid flows back up toward the surface in the annulus between the drill string and the wall of the newly drilled hole. In the annulus, the drilling fluid balances formation stresses by providing hydraulic pressure to the wellbore wall and hole cleaning by transporting rock fragments and drilled cuttings toward the surface. In addition, the drilling fluid lubricates contact points between the drill string and the formation, thereby limiting the torque necessary to apply at the surface in order to rotate the string.

The two most common types of drilling fluids are water-based (WBM) and invert emulsion oil-based (OBM) drilling fluids. In WBM, water or brine is the continuous phase, while in OBM a base oil is the continuous phase emulsified with water or brine. Additional components such as polymers, lubricants, filtration reduction additives and weighting material are added to meet the diverse functional requirements to modern drilling fluids. Drilling fluids are usually designed to be shear thinning yield stress fluids: The shear thinning characteristic limits friction pressure losses while circulating at high flow rates, while the yield stress and gel behavior provides solid carrying capacity also at static and near-static conditions.

For hydraulic pressure predictions, drilling fluids are often treated as time-independent, generalized Newtonian fluids in the form of power law, Bingham plastic or Herschel-Bulkley fluids [1, 2]. However, most drilling fluids are also thixotropic, meaning that the fluid rheology exhibits a transient response to changing shear conditions, and that the rheological properties of the fluids are sensitive to recent shear history [3]. Drilling fluid thixotropy can manifest itself in various ways during a drilling operation. As the fluid flows in the annulus between drill string and formation to the surface, variations in the cross sectional area caused by e.g. drill string tool joints and formation irregularities affect the local shear rate in the fluid, and thereby perturb the state of the micro structure
in the fluid. The drill string often display combinations of axial, torsional and lateral drill string vibrations while drilling which result in unsteady flow conditions and both inertial and thixotropic effects. Frigaard et al. [4] point out that drilling fluid thixotropy is generally of benefit for drilling conventional short and medium distance wells, as increased viscosity and gelling during static conditions help keep drilled cuttings in suspension, while shear thinning leads to lower viscosity and efficient drilling conditions while circulating. In more challenging extended reach wells, thixotropy can contribute to pressure variations that may result in formation fracturing or collapse of the hole [4].

Drilling fluid thixotropy and yielding has been the topic of several recent papers. Thixotropic behavior of a model oil-based drilling fluid was investigated by Herzhaft et al. [5], who showed that a thixotropic model could explain both yielding and thixotropy of the drilling fluid, and proposed a method of fitting model parameters to oilfield viscometer measurements. Ragouilliaux et al. [6] used the same model to interpret creep tests and MRI velocimetry of a model oil-based drilling fluid, and showed how a relatively simple single-structure thixotropic model describes the observed viscosity bifurcation when subjecting the fluid to constant shear stresses below the yield stress.

Tehrani and Popplestone [7] showed that a similar single-structure thixotropic model could predict the effects of time and recent shear history on drilling fluid rheology. Maxey [8] demonstrated drilling fluid thixotropy for four model fluids by subjecting the fluids to sudden step changes in the shear rate and by measuring the shear stress hysteresis while ramping the shear rate from low to high, followed by a downward ramp from high to low shear rates. Similar thixotropic hysteresis curves were measured by Schulz et al. [9], who also found that the effective viscosity of weighted drilling fluids decrease continuously over a period of more than 1 hour when sheared at a constant, high shear rate of 1022 s\(^{-1}\). A similar, prolonged reduction of effective viscosity at high shear rates was also observed recently by Cayeux and Leulseged [10] for weighted drilling fluids. Cayeux and Leulseged introduced a new model for thixotropy based on modifying the steady state Herschel-Bulkley model by introducing a structure parameter and allowing for gel strength development while the fluid is static. The effect of thixotropy and gel breaking on the restart of drilling fluid flow in pipes was considered by Negrão et al. [11], who fitted stress overshoot measurements to a structural kinetics thixotropy model that was later used as rheology input for numerical solution of one-dimensional mass and momentum conservation equations. The effect of thixotropy and elasticity on the start-up flow was found to be most prominent when start-up occurs at low shear rates [11].

As pointed out by Saasen [12], oil-based and water-based drilling fluids build their viscosities differentially. WBM are normally based on polymers as viscosifiers, while the rheology of OBMs are based on emulsified water or brine droplets in combination with organoclays [13]. This results in generally different dynamic rheological behavior even if their steady state viscosities are similar. Motivated by this observation, we study one water-based (WBM) and one invert emulsion oil-based (OBM) drilling fluid, and perform a series of steady state and transient rheological tests to compare the rheological response of the two systems. We investigate the thixotropic response of the two fluids to rapid step-wise changes in the shear rate, and we measure the flow start-up following static periods of varying durations. In an attempt to model the rheometric flow of the two drilling fluids, we fit measurements of the steady state flow curve and the thixotropy measurements to the structural kinetics model proposed by Dulait and Mewis [14].

## 2 Experiment methods

We have formulated the two drilling fluids to have comparable flow curves, and using standard components to build the two systems. The OBM is based on EDC 99 base oil as the continuous phase and with a CaCl\(_2\) brine emulsified in the base oil. Organoclay acts in combination with the water droplets as the main viscosifying agent in the OBM. A liquid emulsifier and a common filtration reduction agent are the final two ingredients in the OBM. The WBM is based on a potassium chloride brine, with a xanthan gum as viscosifier. Common lubrication and filtration control agents are the final two ingredients in the WBM.

We note that for the drilling operation, drilling fluids are usually weighted with barite or similar weighting material in order to balance formation stresses and pore pressure by maintaining a certain hydrostatic pressure in the wellbore. We have deliberately excluded weighting material from the fluid compositions listed above, primarily in order to avoid the risk of altered effective rheological properties over time due to sedimentation or particle migration to low shear regions in the Couette geometry.

The rheological measurements are performed with an Anton Paar MCR301 rheometer using a smooth coaxial cylinder Couette geometry with rotating inner bob of outer diameter 26.653 mm, and an open cup with smooth in-
ner wall of diameter 28.910 mm. All measurements have been performed at constant fluid temperature of 293.15 K and with the rheometer operating in controlled shear rate mode (CSR).

3 Steady state flow curve

We begin the rheological characterization of the two fluids by measuring the steady state shear stress as function of shear rate. The drilling fluids are first subjected to a constant high shear rate of 1021 s\(^{-1}\) for 900 seconds, followed by measurement of shear stress for shear rates varying from 1021 s\(^{-1}\) and down to 0.1 s\(^{-1}\). For each of the fluids, the measurement sequence was first performed with 20 seconds measurement duration at each shear rate, then repeated with 10 seconds duration at each shear rate to confirm repeatability.

The measurements are fitted to the constitutive Herschel-Bulkley model [15]:

\[ \tau = \tau_y + K \dot{\gamma}^n, \]

where \(\tau\) and \(\tau_y\) are the shear stress and the yield stress, respectively, \(\dot{\gamma}\) is the shear rate, \(K\) is the consistency index and \(n \leq 1\) is the shear thinning index. Results for the water-based and the oil-based drilling fluids are shown in Figure 1, where the points are rheometer measurements and lines are Herschel-Bulkley model fittings for each fluid. The solid lines are associated with shear stress measurements while dashed lines correspond to effective viscosities. The Herschel-Bulkley parameters are listed in Table 1 and estimated using the method of least squares. The parametrizations agree well with the measurements over the range of shear rates. Especially the WBM is highly shear thinning, resulting in low effective viscosity at high shear rates. The low-end rheology is comparable and both fluids exhibit a small yield stress of approx. 2 Pa. The model parametrizations overestimate the shear stresses at shear rates lower than approx. 1 s\(^{-1}\). The measurement sequences using 10 seconds alternatively 20 seconds per shear rate are practically identical, but even longer durations may have improved the agreement between model and measurements at low shear rates. We repeat that the measurements have been obtained with a smooth bob and container walls, so apparent slip may also affect the measured shear stress in the low shear rate range.

Finally, to link the flow curve shear rate measurements to typical wall shear rates in the annulus between the drill string and the formation, we consider axial fluid flow in two different concentric annulus geometries, each consisting of a 5 1/2” outer diameter drill string in a hole with inner diameters of 8 1/2” and 12 1/4”, respectively. These are typical combinations of drill string and wellbore diameters in drilling operations for hydrocarbon or geothermal energy production. We assume the drill string is not rotating, and use the semi-analytical result of Hanks [16] to calculate the axial velocity profile and the shear rate at the drill string wall for the Herschel-Bulkley parametrizations in Table 1 at different constant flow rates. The calculated inner wall shear rates as functions of flow rate are shown in Figure 2 for the wide (hole diameter 12 1/4” and drill string diameter 5 1/2”) and the narrow annulus (hole diameter 8 1/2” and drill string diameter 5 1/2”). The colored regions indicate typical flow rates during drilling of the 8 1/2” hole (2000 - 2800 l/min) and the 12 1/4” hole (3000 - 3500 l/min). The solid lines correspond to the estimated wall shear rates for the two drilling fluids in the narrow concentric annulus between the drill string and the 8 1/2” hole wall. The dashed lines represent the wall shear rates in the wider annulus between the drill string and the 12 1/4” hole; at equal flow rates, the wall shear rates are significantly lower in the 12 1/4” hole, as expected.

![Figure 1: Steady state shear stresses and viscosities for the two drilling fluids. The solid lines are associated with shear stress measurements while dashed lines correspond to effective viscosities.](image)

| Parameter | OBM | WBM |
|-----------|-----|-----|
| \(\tau_y\) | 1.86 Pa | 2.32 Pa |
| \(K\) | 0.19 Pa·s\(^n\) | 0.62 Pa·s\(^n\) |
| \(n\) | 0.74 | 0.48 |
of approximately 100-200 s\(^{-1}\) at relevant flow rates while drilling. In this wide annulus geometry, it is primarily the low end of the flow curve in Figure 1 that is relevant for pressure loss predictions. In this case, utilizing mainly high shear rate measurements for Herschel-Bulkley or Bingham model parametrization can result in very poor pressure loss predictions, as the significant shear thinning apparent at lower shear rates would not be captured in the parametrizations. In the narrow annulus, the wall shear rates can go up to several hundred reciprocal seconds, and a much wider interval of the flow curve is relevant for pressure loss predictions while drilling.

### 4 Stress overshoots

To limit particle settling during periods of static conditions, most drilling fluids gel and develop a micro structure over time when they are left to rest. In the field, the so-called gel strength of drilling fluids is measured in a viscometer by first destructuring the fluid at high shear rate, then letting the fluid rest for a specific time, typically 10 seconds and 10 minutes. After this period of rest, the fluid is subjected to a rapid shear rate step from 0 s\(^{-1}\) to 5.1 s\(^{-1}\). The gel strength is recorded as the maximum shear stress during start-up of flow.

We have implemented a similar sequence to measure the evolution of shear stress during start-up of flow in the rheometer. In all cases, the fluids were subjected to a constant shear rate of 1021 s\(^{-1}\) for 5 minutes. The rotation is next set to zero for the specified resting time. The start-up of flow is then performed by increasing the shear rate linearly from 0 s\(^{-1}\) to 2 s\(^{-1}\) over the course of 0.25 seconds and 0.5 seconds for the OBM and the WBM, respectively. Following the initial ramping, the shear rate is subsequently maintained at 2 s\(^{-1}\) for an additional 30 seconds. A shorter shear rate ramp interval was selected for the OBM in order to reach the target shear rate of 2 s\(^{-1}\) before the maximum shear stress occurs during start-up.

As discussed by e.g. Mujumdar et al. [17], the stress evolution during start-up flow of a gelled fluid can be explained in terms of the combined elastic and viscous responses. The elastic stress increases with increasing strain up to the “yield” point of the fluid, after which it decreases due toward a lower “post-yield” value. The viscous stress response increases monotonically during start-up flow. The combined elastic and viscous stress responses during start-up flow of thixotropic fluids will typically show a maximum value (stress overshoot) due to the yielding transition from a primarily elastic to a primarily viscous response. The measurement results are shown in Figure 3a and Figure 3b for the WBM and the OBM, respectively, where the measured shear stress is plotted as function of accumulated strain in the fluid. As anticipated, the measured shear stress during flow start-up increases with longer resting times. We observe that the stress build-up in the OBM occurs at a quicker rate compared to the more elastic yielding of the WBM. The black vertical line in Figure 3a and Figure 3b indicates the approximate strain at which the maximum shear stress is measured, and this is greatest for the WBM, where it is found to be close to 1.7. The strain is approximately 0.65 for the OBM. We note a certain, small dependence on the resting time for the strain at which maximum stress occurs. The maximum stress overshoot values measured during flow start-up is plotted as function of resting time in Figure 4. The maximum values for both fluids increase logarithmically with resting time at a nearly equal rate. A similar logarithmic increase in gel strength as function of resting time was reported by Bjørkevoll et al. [18], who measured gel strength of a 10 g/l Laponite solution using an oilfield Fann 35 viscometer. We note that the stress overshoot values are larger for the WBM, even though both fluids have comparable steady state yield stress values, as reported in the previous section.
5 Thixotropy

Next, to investigate the thixotropic response of the two drilling fluids, a sequence of shear rate steps has been performed, by stepping primarily from a high shear rate to a lower shear rate. In the following we discuss the implemented stepping sequence, and assess thixotropic modeling of the transient response to a change in shear rate.

5.1 Shear rate stepping sequence

The measurement sequence begins with a 10 minute pre-shear at constant shear rate of 10^2 s^{-1}. The fluid is next sheared at a constant high shear rate for 60 seconds to allow the fluid to approach the steady state viscosity at the current shear rate. Next, the shear rate is stepped down to a low shear rate and the shear stress is recorded over a period of 60 seconds to monitor the thixotropic response of the fluid at the lower shear rate. Afterward, the shear rate is stepped up to a different high shear rate which is again maintained for 60 seconds before stepping the shear rate down to the same low shear rate as before. Example measurements from a shear rate step is provided in Figure 5:

The viscosity of both fluids equal the steady state viscosity at the high shear rate before stepping down. Following the decrease in shear rate, the viscosity of the fluids gradually increases as the fluids become relatively more structured at this lower shear rate.
By stepping down from different high shear rates, the thixotropic response to different step magnitudes can be compared for the same final, low shear rate. We focus primarily on the thixotropic response to a reduction in shear rate, as this produces a more visible thixotropic response compared to increasing shear rates. As high shear rates we have investigated 51.1 s⁻¹, 102.1 s⁻¹, 170.2 s⁻¹, 341 s⁻¹ and 510.7 s⁻¹, and as low shear rates we have fixed 5.1 s⁻¹ and 10.2 s⁻¹. To achieve a controlled change in shear rate, the shear rate is stepped down linearly at a rate of approximately 500 s⁻¹ per second.

A typical example of the thixotropic response following a rapid step down in shear rate is shown in Figure 6, where the shear rate has been stepped down from an initial value of 170.2 s⁻¹ to a final value of 5.1 s⁻¹. Similar responses were observed when performing the other steps as well. The solid lines are least squares fit to a function of the form [19]

\[
\tau(t) = \tau_f \left( 1 - \exp \left[ \frac{t}{\tau} \right]^m \right),
\]

where \( T \) is a characteristic thixotropic relaxation time and \( \tau_f \) is the steady state shear stress at the final shear rate. With the exception of the first few seconds after stepping down the shear rate, an empirical equation of the form (2) with \( m < 1 \) captures the long-term thixotropic response quite well. As the structural build-up after decreasing the shear rate is in the form of a stretched exponential (2), we focus our thixotropic modeling of the measurements using the structural kinetics model proposed by Dullaert and Mewis [14], which allows for a stretched exponential evolution towards steady state. This is a so-called Type I thixotropy model, where elasticity and thixotropy are added to a viscoplastic stress equation [20], and the model has previously been used to model the stress overshoots of a drilling fluid by Negrão et al. [11] with reasonable agreement between rheological measurement and model predictions. Below we briefly introduce the Dullaert and Mewis (DM) model and proceed by presenting thixotropic measurements and compare with model predictions.

## 5.2 Structural kinetics model

In the DM model, the equation of state for the shear stress is defined as follows:

\[
\tau(\lambda, \dot{\gamma}) = \lambda \left[ G_0 \gamma_e(\lambda, \dot{\gamma}) + \eta_{st,0} \dot{\gamma} \right] + \eta_{\infty} \dot{\gamma},
\]

where \( \lambda \in [0, 1] \) is a parameter describing the state of the fluid micro structure, \( \lambda = 1 \) corresponds to a fully structured fluid while \( \lambda = 0 \) corresponds to a fully destructured fluid. The elastic contribution to the shear stress is modeled as a Hookean spring where \( G_0 \) is the shear modulus of the fully structured fluid. The factor \( \gamma_e \) is the elastic strain in the deformable aggregate structure. \( \eta_{st,0} \) is the viscosity increment of the fully structured fluid and \( \eta_{\infty} \) is the limiting viscosity of the completely destructured fluid [14]. In steady state, Eq. (3) is similar to the Bingham plastic constitutive model for viscoplastic fluids; time-dependence is introduced via the elastic strain \( \gamma_e \) and the micro structure parameter \( \lambda \).

To close the model, kinetic equations are proposed for the structure parameter and for the elastic strain. The following kinetic equation is assumed for the structure parameter:

\[
\frac{d\lambda}{dt} = \frac{1}{\beta} \left[ -k_1 \dot{\gamma} \lambda + \left( k_2 \dot{\gamma}^{0.5} + k_3 \right) (1 - \lambda) \right],
\]

where \( k_1, k_2 \) and \( k_3 \) are fitting constants that represent respectively shear-induced breakdown, shear-induced build-up and Brownian build-up of the fluid structure. The prefactor \( \beta \) generates the stretched exponential approach toward steady state conditions as shown in Figure 6, which is indicative of a distribution of relaxation times [14]. Finally, the kinetic equation for the elastic stress is defined as follows:

\[
G_0 \frac{d\gamma_e}{dt} = \left( \frac{k_4}{\beta} \right)^{\beta} \left[ \tau(\lambda, \dot{\gamma}) \tau_{y,ss} - \tau_{ss}(\dot{\gamma}) \right] G_0 \gamma_e,
\]

where \( \tau_{ss}(\dot{\gamma}) \) is the steady state shear stress at shear rate \( \dot{\gamma} \), and \( \tau_{y,ss} = \lim_{\dot{\gamma} \to 0} \tau_{ss}(\dot{\gamma}) \). The prefactor \( (k_4/\beta)^{\beta} \) produces a stretched exponential response also for the evolution of the elastic strain. We observe that the steady state fixed point of Eq. (5) equals \( G_0 \gamma_e^* = \tau_{y,ss} \), i.e. the elastic
strain approaches the critical strain given by the steady state yield stress \( \gamma_c = \gamma_{ss}/G_0 \), [21].

We note that the time-dependent prefactor \( t^\beta \) in Eqs. (4) and (5) means that we need to treat the individual shear rate steps individually, and define the initial time \( t = 0 \) as the point when the shear rate is stepped down. Wei et al. recently proposed a time-invariant formulation of a similar structural kinetics model, that applies to arbitrary flows [22].

### 5.3 Parameter estimation

Model fitting is performed by a nonlinear least squares approach, where both the steady state flow curve and the thixotropic shear rate step sequences are combined to estimate the eight model parameters \( k_1, k_2, k_3, k_4, \beta, \eta_{ss,0}, \eta_\infty \) and \( \tau_{ss} \) of the Dullaert-Mewis model. The term \( G_0 \) is taken as the storage modulus from the oscillatory amplitude sweep. For completeness, the least squares model parameters for the OBM and the WBM are listed in Table 2.

**Table 2:** Dullaert-Mewis model parameter estimates.

| Parameter           | OBM     | WBM     |
|---------------------|---------|---------|
| \( k_1 (s^\beta) \) | 0.084   | 0.46    |
| \( k_2 (s^{\beta-1/2}) \) | 0.063 | 0.082 |
| \( k_3 (s^{\beta-1}) \) | 1.40 | 0.73 |
| \( k_4 (Pa^{-1/\beta} s^{1-1/\beta}) \) | \( 7.4 \cdot 10^{-4} \) | \( 1.3 \cdot 10^{-5} \) |
| \( \beta \)         | 0.17    | 0.17    |
| \( \tau_{ss} (Pa) \) | 1.81    | 2.37    |
| \( \eta_{ss,0} (Pa\cdot s) \) | 0.26  | 2.32 |
| \( \eta_\infty (Pa\cdot s) \) | 0.024  | 0.0034 |

### 5.4 Oil-based drilling fluid

We next consider the model fitting to shear rate steps for the OBM. In Figure 7a and Figure 7b we present measurements and model prediction for the five steps down from a high shear rate to 5.1 s\(^{-1}\) and to 10.2 s\(^{-1}\), respectively.

In all cases, the shear stress increases with time since the fluid evolves from a relative destructured state to a more structured state at this lower shear rate. The timescale associated with the thixotropic response is longer when stepping down to 5.1 s\(^{-1}\) compared to the steps down to 10.2 s\(^{-1}\). The structural kinetics model fits in Figure 7a and Figure 7b are found to represent the thixotropic response satisfactory, both in terms of the thixotropic time scale associated with the shear rate step as well as the quantitative evolution of the shear stress toward the steady state value at the two low shear rates. The coefficient of determination, \( R^2 \), is greater than 0.7 for all the shear rate step sequences in Figure 7a and Figure 7b, with an average value of 0.82. Reversing the shear rate step direction by starting from a low shear rate and ramping up to a higher shear rate are also well described by the structural kinetics model, as seen in Figure 8. The thixotropic effect is smaller when increasing the shear rate, and this is well captured by the model parametrization given in Table 2.
Shear stress, $\tau$ (Pa)

| Time since ramping up shear rate (s) | Shear stress, $\tau$ (Pa) |
|--------------------------------------|--------------------------|
|                                      | 5.1 s$^{-1} \rightarrow 102.1$ s$^{-1}$ |
|                                      | 5.1 s$^{-1} \rightarrow 341$ s$^{-1}$ |
|                                      | 5.1 s$^{-1} \rightarrow 510.7$ s$^{-1}$ |
| Time since ramping down shear rate (s)| Shear stress, $\tau$ (Pa) |
|                                      | 5.11 s$^{-1} \rightarrow 5.1$ s$^{-1}$ |
|                                      | 341 s$^{-1} \rightarrow 5.1$ s$^{-1}$ |
|                                      | 170.2 s$^{-1} \rightarrow 5.1$ s$^{-1}$ |
|                                      | 510.7 s$^{-1} \rightarrow 5.1$ s$^{-1}$ |

**Figure 8**: Measurements and model predictions for shear stresses in the OBM following shear rate steps from a shear rate of 5.1 s$^{-1}$ to different final high shear rates.

### 5.5 Water-based drilling fluid

Turning next to the WBM and the shear rate steps down to respectively 5.1 s$^{-1}$ and 10.2 s$^{-1}$, we obtain the results presented in Figure 9a and Figure 9b.

The match is now less satisfactory than for the OBM in Figure 7a and Figure 7b. The steady state shear stresses toward the end of the measurement series are well-represented by the model, but the thixotropic time scale and the evolution of the shear stress to the final value are not matched equally well as for the other drilling fluid. The coefficient of determination, $R^2$, is not greater than 0.6 for any of the shear rate step sequences in Figure 9a and Figure 9b. Measurements of the WBM is seen to approach the steady state shear stress more slowly than the oil-based drilling fluid, with the measured shear stress exhibiting an increasing trend even at the end of the 60 second measurement at the low shear rates. To further evaluate the model parametrization in Table 2, we have attempted to simulate the flow start-up experiments in Figure 3a and Figure 3b, which were not used as basis for the parameter estimation. The model parametrizations do not produce a stress overshoot for either fluid, but rather a smooth transition from an unstressed initial state up to the steady state shear stress. Inclusion of stress overshoot measurements and possibly also other transient rheology measurements in the data set used for model fitting could have improved the determination of the eight model parameters. Although several of the published thixotropy models can predict stress overshoot behavior, we are not aware of such models that also capture the logarithmic increase in gel strength with resting time.

**Figure 9**: Measurements and model predictions for shear stresses in the WBM following shear rate steps down to shear rates of 5.1 s$^{-1}$ and 10.2 s$^{-1}$.

### 5.6 Thixotropic time scales

Finally, we evaluate the time from stepping down to the target shear rates and until the shear stress is within 95% of the steady state value, using the rheometer measurements. This is plotted in Figure 10 for the two fluids, where the shear rate step change scale is logarithmic.

The lines are fitting functions that are taken to be logarithmic in the shear rate step length. Solid lines are associated with measurements at target shear rate 5.1 s$^{-1}$, while dashed lines are associated with measurements at 10.2 s$^{-1}$. We observe that the time duration is nearly twice as long for the steps down to 5.1 s$^{-1}$ compared to the steps to 10.2 s$^{-1}$, suggesting that the thixotropic time scale may be inversely proportional to the target shear rate.

Depending on the flow rate and the radial clearance between the drill string and the formation, the axial bulk velocity is typically of the order of one meter per second.
while drilling. For the narrow annulus between a 8 1/2" hole and a 5 1/2" drill string considered in Figure 2, the wall shear rate is about 230 s\(^{-1}\) over the drill pipe body when circulating the WBM at 1000 l/min. Individual drill pipes are connected by tool joints approximately every 9 meter, and the outer diameter of tool joints is often some 25-30% larger than the drill pipe body. For the example above, the wall shear rate would be of the order of 700 s\(^{-1}\) when circulating past a tool joint. Although the shear rate change is large, and of the order of 500 s\(^{-1}\) over the tool joint, the relatively high shear rates involved imply that the drilling fluid is in a highly destructured state exhibiting limited thixotropic behavior. Suddenly decreasing the flow rate to near quiescent conditions would produce a longer and arguably more pronounced thixotropic response.

### 5.7 Effect of weighting material

The rheological measurements presented so far in the paper have all been obtained for drilling fluids without solid weighting material. As mentioned in section 2, weighting material was deliberately excluded from the fluid formulations to avoid the risk of particle sedimentation in the measurement geometry over the course of long measurement series. In the field, virtually all drilling fluids are weighted to a specific mass density in order to achieve a hydrostatic pressure in the well required for balancing formation stresses and the pore pressure. Thus, the effect of weighting material on the thixotropic behavior of drilling fluid would be highly relevant to investigate. As we will show in this section, addition of solids indeed results in non-trivial effects on the transient rheological behavior of the two drilling fluids investigated in this paper.

For both the water-based and the oil-based drilling fluids, barium sulphate or barite is added to achieve a fluid mass density of 1250 kg/m\(^3\). As the initial mass density of the OBM is less than the WBM prior to adding barite, more barite needs to be added to the OBM to reach the target mass density of 1250 kg/m\(^3\). Next, the weighted fluids are subjected to the same shear rate stepping sequence as previously and presented in section 5.1. First, the effect of weighting material on the shear stress of the WBM when stepping from the highest shear rate to the lowest shear rate, and vice versa can be seen in Figure 11. In the plot to the left, we show the measured shear stress for stepping down the shear rate, while stepping up the shear rate is shown in the plot to the right. At low shear rates, there is no discernible effect of the weighting material at this concentration. At the highest shear rate, weighting material results in a significantly longer transition toward a steady state shear stress compared to the unweighted base fluid. It is not clear whether this is due to gradual particle sedimentation within the measurement geometry, or whether particles have a more complex influence on the polymeric microstructure resulting in the longer thixotropic response. The average effective viscosities at these two shear rates are similar for both the unweighted and the weighted composition.

![Figure 11: Measured shear stresses at a high shear rate of 1021 s\(^{-1}\) followed by a step change in shear rate to 5.1 s\(^{-1}\) (left), and the reverse step sequence from 5.1 s\(^{-1}\) to 1021 s\(^{-1}\) (right). The fluid is at rest at the very start of the measurement sequence (t = 0 seconds). Measurements are shown for the unweighted water-based drilling fluid, and the same fluid with addition of weighting material (barite, BaSO\(_4\)) giving fluid mass density of 1250 kg/m\(^3\).](image-url)
WBM above, in that the weighted composition clearly has increased effective viscosities at these shear rates. Further, both the weighted and the unweighted compositions exhibit a similar transition toward steady state shear stress.

Figure 12: Measured shear stresses at a high shear rate of 1021 s\(^{-1}\) followed by a step change in shear rate to 5.1 s\(^{-1}\) (left), and the reverse step sequence from 5.1 s\(^{-1}\) to 1021 s\(^{-1}\) (right). The fluid is at rest at the very start of the measurement sequence (t = 0 seconds). Measurements are shown for the unweighted water-based drilling fluid, and the same fluid with addition of weighting material (barite, BaSO\(_4\)) giving fluid mass density of 1250 kg/m\(^3\).

Interestingly, addition of weighting material at this concentration increases the effective viscosity of the OBM but not the WBM. From section 3, steady state measurements showed the WBM to be highly shear thinning (n = 0.48), whereas the OBM exhibits less shear thinning (n = 0.74). In addition, the concentration of barite is higher in the OBM compared to the WBM, as mentioned above. We speculate that the different effective viscosity behavior may be traced to these differences between the fluids. The presence of particles that slip relative to the bulk fluid give rise to high local shear rates. In the highly shear thinning WBM, high local shear rates and shear thinning may compensate for the viscosifying effect of particles in the fluid, resulting in no net effect on the effective viscosity. Within the OBM, which is less shear thinning, the viscosifying effect of particles results in a net increase in the effective viscosity. Particles influence not only the local velocity fields in the fluid however, but their presence may also have a more complex interaction with the polymeric or clay-droplet based micro structure in the fluids. The effect of particles on steady state and transient rheological behavior of drilling fluids is a topic of on-going research.

6 Conclusions

We have studied transient and steady state rheology of a water-based and an invert emulsion oil-based drilling fluid. The steady state flow curves are overall well-represented by the constitutive Herschel-Bulkley model. The measurements deviate from the model at low shear rates, which may be due to apparent slip effects or insufficient measurement time. Although the two drilling fluids have comparable steady state viscosities, stress overshoot and thixotropy measurements indicate they build viscosity differently [12]. The thixotropic responses to rapid changes in shear rate are well characterized by the structural kinetics model of Dullaert and Mewis for the OBM, but less satisfactory for the polymeric-based WBM. As the model consists of eight parameters, several different parameter combinations may produce model predictions that fit the data set used for parameter estimation, but may perform poorly when attempting to simulate other transient rheometric flows.

Acknowledgement: The authors acknowledge the Research Council of Norway, ConocoPhillips, AkerBP, Equinor and Wintershall for financing the work through the research centre DrillWell – Drilling and Well Centre for Improved Recovery, a research cooperation between IRIS, NTNU, SINTEF and UiS. We are very grateful to M-I SWACO Norge, a Schlumberger company, for providing the ingredients necessary to mix the drilling fluids used in this study.

References

[1] Bourgoyne Jr. A. T., Millheim K. K., Chenevert M. E., Young Jr. F. S., Applied Drilling Engineering, Society of Petroleum Engineers, 1986.
[2] Livescu S., Mathematical modeling of thixotropic drilling mud and crude oil flow in wells and pipelines - A review, J. Pet. Sci. Eng., 2012, 98-99, 174–184.
[3] Caenn R., Darley H. C. H., Gray G. R., Composition and properties of drilling and completion fluids, Gulf Professional Publishing, 2011.
[4] Frigaard I. A., Paso K. G., de Souza Mendes P. R., Bingham’s model in the oil and gas industry, Rheol Acta, 2017, 56, 259–282.
[5] Herzhaft B., Ragouilliaux A., Coussot P., How To Unify Low-Shear-Rate Rheology and Gel Properties of Drilling Muds: A Transient Rheological and Structural Model for Complex Wells Applications, in: IADC/SPE Drilling Conference, 21-23 February, Miami, Florida, USA, 2006, pp. 1–9, IADC/SPE 99080.
[6] Ragouilliaux A., Herzhaft B., Bertrand F., Coussot P., Flow in-stability and shear localization in a drilling mud, Rheol Acta, 2006, 46, 261–271.

[7] Tehrani A., Popplestone A., Modelling the gelling properties of water-based drilling fluids, in: AADE National Technical Conference and Exhibition, New Orleans, Louisiana, American Association of Drilling Engineers, 2009, pp. 1–8, AADE 2009-NTCE-12-02.

[8] Maxey J., Thixotropy and Yield Stress Behavior in Drilling Fluids, in: AADE National Technical Conference and Exhibition, Houston, Texas, April 10-12, 2007, American Association of Drilling Engineers, 2007, pp. 1-10, AADE-07-NTCE-37.

[9] Schulz A., Strauß H., Reich M., Modern rheological analysis of drilling fluids, in: Proceedings of the ASME 2013 32nd International Conference on Ocean, Offshore and Arctic Engineering, June 9-14, Nantes, France, 2013, pp. 1–10, OMAE2013-11580.

[10] Cayeux E., Leulseged A., Modelling of Drilling Fluid Thixotropy, in: Proceedings of the ASME 2018 37th International Conference on Ocean, Offshore and Arctic Engineering, 2018, pp. 1–14, OMAE 2018-77203.

[11] Negra o C. O. R., Franco A. T., Rocha L. L. V., A weakly compressible flow model for the restart of thixotropic drilling fluids, J. Non-Newtonian Fluid Mech., 2011, 166, 1369–1381.

[12] Saasen A., Annular Frictional Pressure Losses During Drilling - Predicting the Effect of Drillstring Rotation, Journal of Energy Resources Technology, 2014, 136, 034501.

[13] Herzhaft B., Rousseau L. N., Moan M., Bossard F., Influence of Temperature and Clays/Emulsion Microstructure on Oil-Based Mud Low Shear Rate Rheology, SPE Journal, 2003, 8, 211 – 217.

[14] Dullaert K., Mewis J., A structural kinetics model for thixotropy, J. Non-Newtonian Fluid Mech., 2006, 139, 21–30.

[15] Herschel W. H., Bulkley R., Konsistenzmessungen von Gummibenzolösungen, Kolloid Zeitschrift, 1926, 39, 291 – 300.

[16] Hanks R. W., The Axial Laminar Flow of Yield-Pseudoplastic Fluids in a Concentric Annulus, Ind. Eng. Chem. Process Des. Dev., 1979, 18, 488 – 493.

[17] Mujumdar A., Beris A. N., Metzner A. B., Transient phenomena in thixotropic systems, J. Non-Newtonian Fluid Mech., 2002, 102, 157–178.

[18] Bjørkevoll K. S., Rommetveit R., Aas B., Gjeraldstveit H., Merlo A., Transient gel breaking model for critical wells applications with field data, in: SPE/IADC Drilling Conference, 19-21 February, Amsterdam, Netherlands, Society of Petroleum Engineers, 2003, pp. 1–8, SPE/IADC 79843.

[19] Dullaert K., Mewis J., Thixotropy: Build-up and breakdown curves during flow, J. Rheol., 2005, 49, 1213–1230.

[20] de Souza Mendes P. R., Thompson R. L., A critical overview of elasto-viscoplastic thixotropic modelling, J. Non-Newtonian Fluid Mech., 2012, 187-188, 8–15.

[21] Mewis J., Wagner N. J., Thixotropy, Adv. Colloid Interface Sci., 2009, 147-148, 214–227.

[22] Wei Y., Solomon M. J., Larson R. G., Quantitative nonlinear thixotropic model with stretched exponential response in transient shear flows, J. Rheol., 2016, 60, 1301–1315.