Model Independent Tests
for Time Reversal and CP Violations
and for CPT Theorem
in $\Lambda_b$ and $\bar{\Lambda}_b$ Two Body Decays

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Abstract
Weak decays of beauty baryons like $\Lambda_b(\bar{\Lambda}_b)$ into $\Lambda(\bar{\Lambda})$ and $V(J^P=1^-)$, where both decay products are polarized, offer interesting opportunities to perform tests of time reversal and CP violations and of CPT invariance. We propose a model independent parametrization, via spin density matrix, of the angular distribution, of the polarizations and of some polarization correlations of the decay products. The transverse component of the polarization and two polarization correlations are sensitive to time reversal violations. Moreover several CP- and CPT-odd observables are singled out.

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1 Introduction

The interest in CP violations (CPV) and time reversal violations (TRV) has been increasing in last years[1-10]. The reason is that, although some CPV and also a direct TRV[11] have been detected experimentally, the nature of such symmetry violations has not yet been clarified. More precisely, the prediction of the size of the violation in some weak decays is strongly model dependent, which stimulates people to search for signals of new physics[1, 2, 5, 4, 10, 12, 13] (NP), beyond the standard model (SM). For example, the decays involving the transition

\[ b \rightarrow s \]  

present CPV parameters, like the $B^0 - \bar{B}^0$ mixing phase[10, 13] and the transverse polarization of spinning decay products of $\Lambda_b[1]$, which are very small in SM predictions, but are considerably enhanced in other models. In particular, recent signals of NP have been claimed in B decays: the CP violating phases of $B \rightarrow \pi K[10]$ and $B_s \rightarrow \Phi J/\Psi[13]$ may be considerably greater than predicted by SM. Also $\Lambda_b$ decays[1, 3, 6, 8] are suggested as new sources of CPV and TRV parameters, especially in view of the abundant production of this resonance in the forthcoming LHC accelerator.

As regards direct TRV, only one evidence[11, 14, 15, 16, 17, 18] has been given so far, and assuming the Bell-Steinberger[19] relation, which might be violated to few percent[20]. Lastly the CPT theorem, valid for local field theories, has been tested to a great precision in the neutral kaon decay[21], but not in other situations: for example, it has never been checked in decays involving the $b$-quark, furthermore a meaningful size of uncertainty remains in $K^\pm \rightarrow \pi^\pm \pi^0[20]$.

The aim of the present paper is to suggest model independent tests of TRV, CPV and CPT invariance in hadronic $\Lambda_b$ and $\bar{\Lambda}_b$ decays of the type

\[ \Lambda_b(\bar{\Lambda}_b) \rightarrow \Lambda(\bar{\Lambda})V, \]  

$V$ denoting a $J^P = 1^-$ resonance, either the $J/\psi$ or a light vector meson, like $\rho^0, \omega$. Each resonance decays, in turn, to more stable particles, like, e. g., $\Lambda \rightarrow p\pi^-$,
$J/\psi \to \mu^+\mu^-$, so that one has to consider a typical cascade decay. A previous paper[8] had been devoted to the subject. Now we parametrize, by means of the spin density matrix (SDM), the angular distribution and the polarizations of the decay products, without introducing any dynamic assumption at all. Then we study the behavior of these observables under CP and T, singling out those which are sensitive to T, CP and CPT violations. Our approach resembles the one proposed by Lee and Yang[22] and by Gatto[23] many years ago, to use hyperon decays for the same tests. However, as we shall see, a hadronic two-body weak decay involving two spinning particles in the final state - never proposed before - presents some advantages over hyperon decays[22, 23, 12], where one of the two final particles is spinless.

In sect. 2 we derive the expressions of the spin density matrices, angular distribution and polarizations of the decay products in the above mentioned decays. In sect. 3 we present a parametrization of the angular distribution and of polarizations. In sect. 4 we suggest tests for TRV, CPV and CPT. Lastly we conclude with some remarks in sect. 5.

2 Angular Distribution and Polarization Vectors of the Decay Products

In order to deal with the angular distribution and the polarization of the intermediate resonances, Λ and $\Lambda_b$, coming from $\Lambda_b$ decay, the best suited method consists of applying the relativistic helicity formalism pioneered by Jacob and Wick[24] and reformulated later by Jackson[25] (see also[26, 27]). This formalism presents some advantages:

(i) thanks to its definition, $\lambda = \vec{j} \cdot \hat{p}$, where $\vec{j} = \vec{\ell} + \vec{s}$ and $\hat{p} = \vec{p}/|\vec{p}|$, the helicity of a particle of spin $\vec{s}$ and momentum $\vec{p}$ does not depend on its orbital angular momentum $\vec{\ell}$ and it is rotationally invariant;

(ii) $\lambda$ equals the spin projection along $\vec{p}$ in the resonance rest frame.

These physical properties can be applied just to cascade decays of the type described
above, \textit{i.e.},

\[ R_0 \rightarrow R_1 + R_2, \text{ followed by } R_1 \rightarrow a_1 + b_1 \text{ and } R_2 \rightarrow a_2 + b_2, \quad (3) \]

provided we take, in the rest frame of the resonance \( R_1 \) or \( R_2 \), the quantization axis parallel to its momentum in the \( R_0 \) rest frame. The helicity of \( R_i \) \((i = 1,2)\), computed in the \( R_0 \) rest frame, is equal to the projection of its total angular momentum along this quantization axis in the \( R_i \) rest frame. In our case we identify \( R_0 \) with \( \Lambda_b \), \( R_1 \) with \( \Lambda \) and \( R_2 \) with \( V \).

In the following, the formalisms of helicity and SDM will be intensively used by specifying different rest frames.

\section{2.1 Spin Density Matrices}

In the standard detector frame the \( z \)-axis is taken parallel to the incident proton beam. For our aims it is more convenient to define a different frame, through the three mutually orthogonal unit vectors

\[
\vec{e}_z = \frac{\vec{p}_p \times \vec{p}_b}{|\vec{p}_p \times \vec{p}_b|}, \quad \vec{e}_x = \frac{\vec{p}_p}{|\vec{p}_p|}, \quad \vec{e}_y = \vec{e}_z \times \vec{e}_x.
\]

Here \( \vec{p}_p \) and \( \vec{p}_b \) are, respectively, the proton momentum and the \( \Lambda_b \) momentum. If produced by means of strong interactions - as usually assumed for \( \Lambda, \Sigma, \Xi, \ldots \) hyperons -, the \( \Lambda_b \) is polarized along \( \vec{n} \). Therefore we find it suitable to choose the quantization axis along \( \vec{e}_z = \vec{n} \).

\( \Lambda_b \) SDM

We denote, here and in the following, the \( \Lambda_b \) spin by \( J \), with \( J = 1/2 \). Therefore the \( \Lambda_b \) SDM reads

\[
\rho^{\Lambda_b} = \frac{1}{2}(1 + 2\vec{P}^{\Lambda_b} \cdot \vec{\sigma}). \quad (4)
\]

Here \( \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \), \( \sigma_i \) are the Pauli matrices and \( \vec{P}^{\Lambda_b} \) is the polarization vector of \( \Lambda_b \). Defining a \( \Lambda_b \) rest frame, whose axes are oriented like those in the laboratory frame, the components of \( \vec{P}^{\Lambda_b} \) result in

\[
P_{z}^{\Lambda_b} = \frac{1}{2}(\rho_{++}^{\Lambda_b} - \rho_{--}^{\Lambda_b}), \quad P_{x}^{\Lambda_b} = \Re(\rho_{+-}^{\Lambda_b}), \quad P_{y}^{\Lambda_b} = -\Im(\rho_{+-}^{\Lambda_b}). \quad (5)
\]
\(\rho_{M,M'}^{\Lambda b}\) are the matrix elements of \(\rho^{\Lambda b}\), \(M,M' = \pm\) denoting the values of the third component of the \(\Lambda b\) spin along the quantization axis. \(\rho^{\Lambda b}\) verifies the normalization condition \(\text{Tr}(\rho^{\Lambda b}) = \rho_{++} + \rho_{--} = 1\). The components of the polarization vector are regarded as external parameters. Note that, if parity is conserved in the production process, we have \(P_x^{\Lambda b} = P_y^{\Lambda b} = 0\).

**SDM of the \(\Lambda-V\) System**

The intermediate state in a cascade decay of the type (3) is a composite one, consisting of the two spinning particles \(\Lambda\) and \(V\). The SDM of this state is given by

\[
\rho^f = \mathcal{M}\rho^\Lambda_b\mathcal{M}^\dagger,
\]

where \(\mathcal{M}\) is the (unitary) operator which describes the decay considered. The matrix elements of the SDM \(\rho^f\) are obtained from (6) by projecting the operators involved in that expression onto the initial and final states. The latter ones are characterized by a given three-momentum in the \(\Lambda_b\) center-of-mass system and by a pair of helicities, \(\lambda_1\) and \(\lambda_2\), corresponding to each resonance \(R_1\) and \(R_2\). Therefore the SDM of this two-particle system is endowed with two pairs of indices, i.e.,

\[
\rho^f_{\lambda_1\lambda'_1\lambda_2\lambda'_2} = \sum_{M,M'} F^{JM}_{\lambda_1\lambda_2}(\theta,\phi) \rho_{M,M'}^{\Lambda b} F^{JM*}_{\lambda'_1\lambda'_2}(\theta,\phi),
\]

\[
F^{JM}_{\lambda_1\lambda_2}(\theta,\phi) = <\theta,\phi; \lambda_1,\lambda_2|\mathcal{M}|JM>.
\]

Here \(\theta\) and \(\phi\) are, respectively, the polar and azimuthal angle of the momentum of the \(\Lambda\) resonance in the \(\Lambda_b\) rest frame. Note that the average value of a given operator \(\mathcal{O}\) over the mixing of states defined by SDM (7) reads

\[
\langle \mathcal{O} \rangle = \sum_{\lambda_1\lambda'_1\lambda_2\lambda'_2} \rho^f_{\lambda_1\lambda'_1\lambda_2\lambda'_2} \mathcal{O}_{\lambda_1\lambda'_1\lambda_2\lambda'_2}.
\]

From now on we shall denote this sum over two pairs of indices with the usual symbol of "Trace", i.e., \(\text{Tr}(\rho^f \mathcal{O})\).

Angular momentum conservation demands

\[
\Lambda' = \lambda_1 - \lambda_2 \quad \Lambda'' = \lambda'_1 - \lambda'_2,
\]
where $\Lambda', \Lambda'' = \pm 1/2$. Therefore the expression (7) of the SDM can be conveniently transformed to

$$\rho_{\lambda, \lambda'}^{JM} = \sum_{M, M'} F_{\lambda, \lambda'-\Lambda}(\theta, \phi) \rho_{M, M'}^{\Lambda, \Lambda'} F_{\lambda', \Lambda''}(\theta, \phi).$$  \hfill (11)

The helicity formalism implies

$$F_{\lambda, \lambda'}^{JM}(\theta, \phi) = N_J A_{\lambda, \lambda'}^J D_{M, M'}(\phi, 0).$$  \hfill (12)

Here $N_J = \sqrt{(2J + 1)/4\pi}$, $D_{M, M'}^J$ is a rotation matrix element and

$$A_{\lambda, \lambda'}^J = 4\pi \left( \frac{M_b}{p} \right)^{1/2} < J, M; \lambda_1, \lambda_2 | M | J, M >$$  \hfill (13)

is the rotationally invariant decay amplitude, $M_b$ being the $\Lambda_b$ rest mass and $p$ the momentum of $\Lambda$ in the $\Lambda_b$ rest frame.

Now we sum over the indices $M$ and $M'$ in the expression (11), taking into account eqs. (5), and recalling the well-known properties of the $D$-functions[27]. As a result we get

$$\rho_{\lambda, \lambda'}^{JM} = \frac{1}{4\pi} \left[ A_{\lambda, \lambda'-\Lambda}^{\Lambda, \Lambda'} A_{\lambda'-\Lambda, \lambda'-\Lambda}^{\Lambda, \Lambda'} (1 + 4\Lambda' P_1^{\Lambda_b}) \delta_{\lambda'-\Lambda''} 
+ 2 A_{\lambda, \lambda'-\Lambda}^{\Lambda, \Lambda'} A_{\lambda'-\Lambda, \lambda'-\Lambda}^{\Lambda, \Lambda'} (P_2^{\Lambda_b} + 2i\Lambda' P_3^{\Lambda_b}) \delta_{\lambda'-\Lambda''} \right].$$  \hfill (14)

Here we have dropped the index $J$ from the $A$-amplitudes and the index 1 from helicities. Moreover we have set

$$P_1^{\Lambda_b} = \vec{P}^{\Lambda_b} \cdot \hat{p}, \quad P_2^{\Lambda_b} = \vec{P}^{\Lambda_b} \cdot \vec{e}_N, \quad P_3^{\Lambda_b} = \vec{P}^{\Lambda_b} \cdot \hat{r},$$  \hfill (15)

where $\hat{p}$ is the unit vector in the direction of the $\Lambda$ momentum and

$$\vec{e}_N = \vec{e}_T \times \hat{p}, \quad \vec{e}_T = \frac{\vec{n} \times \hat{p}}{|\vec{n} \times \hat{p}|}, \quad \hat{r} = \vec{e}_T \times \vec{n}.$$  \hfill (16)

Note that the first term of the expression (14) corresponds to the cases where either $\lambda' = \lambda_1$, $\lambda'' = \lambda_2$ (if $\lambda' = \lambda$) or $\lambda' \neq \lambda_1$, $\lambda'' \neq \lambda_2$ (if $\lambda' = -\lambda$). Conversely the second term corresponds to the cases where either $\lambda' = \lambda_1$, $\lambda'' \neq \lambda_2$ (if $\lambda' = \lambda$) or $\lambda' \neq \lambda_1$, $\lambda'' = \lambda_2$ (if $\lambda' = -\lambda$). We have to take into account such combinations in calculating...
average values of operators, according to eq. (9). In the case of observables connected to \( V \) it is convenient to re-express the SDM as

\[
\rho^f_{\mu\mu',N'N''} = \frac{1}{4\pi} \left[ A_{\mu+N',\mu} A^*_{\mu'+N',\mu'} (1 + 4\Lambda' P_1^{\Lambda_b}) \delta_{N',N''} + 2 A_{\mu+N',\mu} A^*_{\mu'-N',\mu'} (P_2^{\Lambda_b} + 2i\Lambda' P_3^{\Lambda_b}) \delta_{N'-,N''} \right],
\]

(17)

with the constraint \(|\mu + \Lambda'| = |\mu' \pm \Lambda'| = 1/2\).

### 2.2 Angular Distribution

The angular distribution of the decay products, \( W(\theta, \phi) \), can be deduced from the SDM, according to the formulae

\[
W(\theta, \phi) = Tr \rho^f.
\]

(18)

Taking into account eq. (14) or (17), we get

\[
W(\theta, \phi) = \frac{1}{4\pi} (G_W + \Delta G_W P_1^{\Lambda_b}),
\]

(19)

with

\[
\begin{align*}
G_W &= |A_{1/2,0}|^2 + |A_{-1/2,-1}|^2 + |A_{1/2,1}|^2 + |A_{-1/2,0}|^2, \\
\Delta G_W &= 2 \left( |A_{1/2,0}|^2 + |A_{-1/2,-1}|^2 - |A_{1/2,1}|^2 - |A_{-1/2,0}|^2 \right).
\end{align*}
\]

(20)

(21)

We may also obtain the respective projections over the polar and azimuthal angles:

\[
\begin{align*}
W_p(\theta) &= \frac{1}{2} (G_W + \Delta G_W P_2^{\Lambda_b} \cos \theta), \\
W_a(\phi) &= \frac{1}{2\pi} [G_W + \Delta G_W (P_x^{\Lambda_b} \cos \phi + P_y^{\Lambda_b} \sin \phi)].
\end{align*}
\]

(22)

(23)

It is worth noting the crucial role played by the initial polarization of \( \Lambda_b \) in both the polar and azimuthal projections. In particular, the \( \phi \)-dependence disappears if parity is conserved in the production reaction of this resonance.
2.3 Polarization Vectors

In order to compute the polarization vector of each resonance $R_i$, a special frame has to be defined, by means of three mutually orthogonal unit vectors. For the $\Lambda$ resonance we have

\[ \hat{z}' = \vec{e}_L, \quad \hat{y}' = \vec{e}_T, \quad \hat{x}' = \vec{e}_N. \]

The $\Lambda$ polarization vector is decomposed like

\[ \vec{P}_\Lambda = P_L \vec{e}_L + P_T \vec{e}_T + P_N \vec{e}_N, \]

where $P_L$, $P_T$ and $P_N$ are defined, respectively, as the longitudinal, transverse and normal component of $\vec{P}_\Lambda$. For the $V$-resonance we have

\[ \vec{P}_V = P_L \vec{e}'_L + P_T \vec{e}'_T + P_N \vec{e}'_N, \]

with $\vec{e}'_L = -\vec{e}_L$, $\vec{e}'_T = -\vec{e}_T$, $\vec{e}'_N = \vec{e}_N$.

In these particular frames [28] we have, for each resonance,

\[ \vec{P}_{R_i} = \frac{\text{Tr}(\rho^I \vec{s})}{\text{Tr}(\rho^I)}, \quad \text{whence} \quad \vec{P}_{R_i} \, W(\theta, \phi) = \text{Tr}(\rho^I \vec{s}), \]  

(24)

where $\vec{s} \equiv (s_x, s_y, s_z)$ denotes the spin vector operator.

Polarization Vector of $\Lambda$

We calculate the components of the polarization vector of $\Lambda$ by exploiting eq. (14) of the SDM and eq. (24). The longitudinal component reads

\[ W(\theta, \phi) P^\Lambda_L(\theta, \phi) = \frac{1}{4\pi} (G^\Lambda_L + \Delta G^\Lambda_L P^\Lambda_1), \]  

(25)

where

\[ 2G^\Lambda_L = |A_{1/2,0}|^2 - |A_{-1/2,-1}|^2 + |A_{1/2,1}|^2 - |A_{-1/2,0}|^2, \]  

(26)

\[ \Delta G^\Lambda_L = |A_{1/2,0}|^2 - |A_{-1/2,-1}|^2 - |A_{1/2,1}|^2 + |A_{-1/2,0}|^2. \]  

(27)

As to the transverse component, the previous formulae yield

\[ W(\theta, \phi) P^\Lambda_T(\theta, \phi) = \frac{1}{4\pi} (G^\Lambda_T P^\Lambda_2 + \Delta G^\Lambda_T P^\Lambda_3), \]  

(28)
where

\[
G^\Lambda_T = -2 \Im \left( A_{1/2,0}A_{1/2,0}^* + A_{1/2,1}A_{1/2,-1}^* \right),
\]
\[
\Delta G^\Lambda_T = 2 \Re \left( A_{1/2,0}A_{1/2,0}^* - A_{1/2,1}A_{1/2,-1}^* \right),
\]

Lastly, the normal component yields

\[
W(\theta, \phi) P^\Lambda_N(\theta, \phi) = \frac{1}{4\pi} (G^\Lambda_N P^\Lambda_2 + \Delta G^\Lambda_N P^\Lambda_3),
\]

where

\[
G^\Lambda_N = 2 \Re \left( A_{1/2,0}A_{1/2,0}^* + A_{1/2,1}A_{1/2,-1}^* \right),
\]
\[
\Delta G^\Lambda_N = -2 \Im \left( A_{1/2,0}A_{1/2,0}^* - A_{1/2,1}A_{1/2,-1}^* \right).
\]

**Polarization Vector of \( V \)**

In order to calculate the components of the polarization vector of \( V \) we exploit eq. (17) of the SDM and take into account the expression of \( s \) for spin-1 particles[29]. We have

\[
W(\theta, \phi) P^V_L(\theta, \phi) = \frac{1}{4\pi} (\Delta G^V_L + G^V_L P^A_1),
\]
\[
W(\theta, \phi) P^V_T(\theta, \phi) = \frac{1}{4\pi} (G^V_T P^A_2 + \Delta G^V_T P^A_3),
\]
\[
W(\theta, \phi) P^V_N(\theta, \phi) = \frac{1}{4\pi} (\Delta G^V_T P^A_2 - G^V_T P^A_3).
\]

Here

\[
G^V_L = -2(|A_{-1/2,-1}|^2 + |A_{1/2,1}|^2),
\]
\[
\Delta G^V_L = |A_{1/2,1}|^2 - |A_{-1/2,-1}|^2,
\]
\[
G^V_T = -2\sqrt{2} \Im (A_{1/2,1}A_{1/2,0}^* - A_{-1/2,-1}A_{-1/2,0}^*),
\]
\[
\Delta G^V_T = 2\sqrt{2} \Re (A_{1/2,1}A_{1/2,0}^* + A_{-1/2,-1}A_{-1/2,0}^*).
\]

**Polarization Correlations**
Now we define the following four polarization correlations, similar to those considered by Chiang and Wolfenstein[30]:

\[
W(\theta, \phi)P_{TT(NN)}(\theta, \phi) = \frac{1}{2} Tr \left[ \rho^{f(x)} \sigma_{y(x)}^{A} s_{x(y)}^{V} \right],
\]

(41)

\[
W(\theta, \phi)P_{TN(NT)}(\theta, \phi) = \frac{1}{2} Tr \left[ \rho^{f(x)} \sigma_{y(x)}^{A} s_{x(y)}^{V} \right].
\]

(42)

These observables are related to the angular correlations of the decay products of the \(\Lambda\) and \(V\) resonance, similar to those considered in refs. [31, 32, 33] and measured in experiments quoted in ref. [33].

Substituting expression (14) or (17) into eqs. (42), we get

\[
W(\theta, \phi)P_{TT}(\theta, \phi) = \frac{1}{4\pi}(G_{TT} + \Delta G_{TT}P_{1}^{A_{b}}),
\]

(43)

\[
W(\theta, \phi)P_{NT}(\theta, \phi) = \frac{1}{4\pi}(G_{TN} + \Delta G_{TN}P_{1}^{A_{b}}),
\]

(44)

\[
W(\theta, \phi)P_{TN}(\theta, \phi) = \frac{1}{4\pi}(G_{TN} + \Delta G_{TN}P_{1}^{A_{b}}),
\]

(45)

\[
W(\theta, \phi)P_{NN}(\theta, \phi) = -\frac{1}{4\pi}(G_{TT} + \Delta G_{TT}P_{1}^{A_{b}}),
\]

(46)

with

\[
G_{TT} = -\frac{1}{\sqrt{2}} \text{Re}(A_{1/2, -1}A^{*}_{1/2, 0} + A_{1/2, 1}A^{*}_{1/2, 0}),
\]

(47)

\[
\Delta G_{TT} = -\sqrt{2} \text{Re}(A_{-1/2, -1}A^{*}_{1/2, 0} - A_{1/2, 1}A^{*}_{1/2, 0}),
\]

(48)

\[
G_{TN} = \sqrt{2} \text{Im}(A_{-1/2, -1}A^{*}_{1/2, 0} + A_{1/2, 1}A^{*}_{1/2, 0}),
\]

(49)

\[
\Delta G_{TN} = \frac{1}{\sqrt{2}} \text{Im}(A_{-1/2, -1}A^{*}_{1/2, 0} - A_{1/2, 1}A^{*}_{1/2, 0}).
\]

(50)

3 Parametrization of Observables

In this section we write a model independent parametrization, based on the previous formulae, of the angular distribution, of the polarization of \(\Lambda\) and of the polarization correlations \(P_{TT}\) and \(P_{TN}\). In particular, we describe such observables in terms of a minimum number of independent parameters. The polarization components of \(V\) can be expressed as functions of such parameters, as is straightforward to see from eqs (34) to (40).
The formulae of the angular distribution and of the polarization of Λ can be rewritten as

\[ W(\theta, \phi) = \frac{1}{4\pi} G_W (1 + 2 P^A_1 \alpha_W), \] (51)

\[ \mathcal{P}^A(\theta, \phi) = \frac{1}{1 + 2 P^A_1 \alpha_W} \left[ C_L \bar{e}_L + C_T \bar{e}_T + C_N \bar{e}_N \right], \] (52)

with

\[ C_L = B_L (1 + 2 P^A_1 \alpha_L), \quad C_T = B_T (P^A_2 + 2 P^A_3 \alpha_T), \] (53)

\[ C_N = B_N (P^A_2 + 2 P^A_3 \alpha_N) \] (54)

and

\[ B_L = \frac{G^A_L}{G_W}, \quad B_T = \frac{G^A_T}{G_W}, \quad B_N = \frac{G^A_N}{G_W}, \] (55)

\[ \alpha_L = \frac{\Delta G^A_L}{2G^A_L}, \quad \alpha_T = \frac{\Delta G^A_T}{2G^A_T}, \quad \alpha_N = \frac{\Delta G^A_N}{2G^A_N}. \] (56)

As for the polarization correlations, we have

\[ P_{TT} = \frac{1}{1 + 2 P^A_1 \alpha_W} B_{TT} (1 + 2 P^A_1 \alpha_{TT}), \] (57)

\[ P_{TN} = \frac{1}{1 + 2 P^A_1 \alpha_W} B_{TN} (1 + 2 P^A_1 \alpha_{TN}), \] (58)

where

\[ B_{TT} = \frac{G_{TT}}{G_W}, \quad B_{TN} = \frac{G_{TN}}{G_W}, \] (59)

\[ \alpha_{TT} = \frac{\Delta G_{TT}}{2G_{TT}}, \quad \alpha_{TN} = \frac{\Delta G_{TN}}{2G_{TN}}. \] (60)

The parameters which appear in eqs. (51) to (60) are not all independent of one another, they fulfil the following relations:

\[ B_L^2 (1 - \alpha_L)^2 + B_{TT}^2 (1 - \alpha_{TT})^2 + B_{TN}^2 (1 - \alpha_{TN})^2 = (1 - \alpha_W)^2, \] (61)

\[ B_L^2 (1 + \alpha_L)^2 + B_{TT}^2 (1 + \alpha_{TT})^2 + B_{TN}^2 (1 + \alpha_{TN})^2 = (1 + \alpha_W)^2, \] (62)

\[ (\frac{1}{2} \alpha_W - B_L)^2 + (B_T - 2 \alpha_N B_N)^2 + (B_N - 2 \alpha_T B_T)^2 = \frac{1}{4} (1 - 2 \alpha_L B_L)^2, \] (63)

\[ (\frac{1}{2} \alpha_W + B_L)^2 + (B_T + 2 \alpha_N B_N)^2 + (B_N + 2 \alpha_T B_T)^2 = \frac{1}{4} (1 + 2 \alpha_L B_L)^2. \] (64)
The first two equations allow to express some of the parameters just introduced as functions of a more restricted number of other, independent, parameters. We propose the following parametrization, similar to previous conventions in hyperon decays[22]:

\begin{align}
\alpha_L &= \frac{1 - \xi_L}{1 + \xi_L}, \quad \alpha_{TT} = \frac{1 - \xi_{TT}}{1 + \xi_{TT}}, \quad \alpha_{TN} = \frac{1 - \xi_{TN}}{1 + \xi_{TN}}, \\
\xi_L &= \xi_W \frac{\cos \psi_-}{\cos \psi_+}, \quad \xi_{TT} = \xi_W \frac{\sin \psi_- \cos \beta_-}{\sin \psi_+ \cos \beta_+}, \\
\xi_{TN} &= \xi_W \frac{\sin \psi_- \sin \beta_-}{\sin \psi_+ \sin \beta_+}, \quad \xi_W = \frac{1 - \alpha_W}{1 + \alpha_W}, \\
B_L &= \frac{1 \pm \alpha_W \cos \psi_+}{1 \pm \alpha_L \cos \psi_+}, \quad B_{TT} = \frac{1 \pm \alpha_W \sin \psi_+ \cos \beta_+}{1 \pm \alpha_{TT} \sin \psi_+ \cos \beta_+}, \\
B_{TN} &= \frac{1 \pm \alpha_W \sin \psi_+ \sin \beta_+}{1 \pm \alpha_{TN} \sin \psi_+ \sin \beta_+}, \\
B_T &= \frac{1}{4}(\Gamma_+ \cos \varphi_+ + \Gamma_- \cos \varphi_-), \quad B_T \alpha_T = \frac{1}{4}(\Gamma_+ \sin \varphi_+ - \Gamma_- \sin \varphi_-), \\
B_N &= \frac{1}{4}(\Gamma_+ \sin \varphi_+ + \Gamma_- \sin \varphi_-), \quad B_N \alpha_N = \frac{1}{4}(\Gamma_+ \cos \varphi_+ - \Gamma_- \cos \varphi_-), \\
\Gamma &= \left[ (1 - 2\alpha_L B_L)^2 - (\alpha_W - 2B_L)^2 \right]^{1/2}.
\end{align}

Then the angular distribution, the Λ polarization and the polarization correlations are expressed as functions of the 10 independent parameters \(P_{1ab}, P_{2ab}, P_{3ab}, \alpha_W, \psi_\pm, \beta_\pm, \varphi_\pm\).

4 \ TRV, CPV and CPT Tests

Here we illustrate properties of the observables illustrated in the preceding sections under discrete transformations and suggest possible tests for violation of relative symmetries.

4.1 \ T Violations

The rotationally invariant amplitudes introduced in sect. 2 transform under time reversal (TR) in such a way that[27]

\[ A_{\lambda_1, \lambda_2} A_{\lambda'_1, \lambda'_2}^* \rightarrow A_{\lambda_1, \lambda_2}^* A_{\lambda'_1, \lambda'_2}. \]  

(73)

This follows from the antiunitary character of the TR and from helicity invariance under this operation. Then eqs. (28), (29), (30), (35), (44), (45), (49) and (50)
imply, together with eqs. (15), that the transverse polarizations $P^\Lambda_T$ and $P^V_T$ and the polarization correlations $P_{TN}$ and $P_{NT}$ change sign under TR. Such equations imply, together with eq. (20) and the second eqs. (55) and (59), that also the parameters $B_T$ and $B_{TN}$ change sign under the same operation. Therefore nonzero values of such observables are signatures of TRV. These are promising for detecting possible effects of NP, according to the considerations of refs.[1, 5, 4]. Quite analogous properties are shared by two-body decays of $\bar{\Lambda}_b$.

In this connection it is worth remembering that also the transverse polarization of the muon in $K^+$ decays to $\pi^0\mu^+\nu_\mu$ and to $\gamma\mu^+\nu_\mu$ has been indicated as a possible signature of TRV[21].

It is important to stress that, in order to get TRV observables, two different polarizations are needed, either $\Lambda_b$’s and $\Lambda$’s or $V$’s, or the simultaneous measurement of $\Lambda$ and $V$ polarizations. In particular we observe that these polarizations are connected to T-odd pseudoscalar triple products. For example, we have

$$P_{TN} - P_{NT} \propto \langle \vec{s}^V \times \vec{\sigma}^\Lambda \cdot \hat{p} \rangle,$$

(74)

brackets denoting average. Similarly, by combining $\Lambda_b$’s and $\Lambda$’s polarizations, according to formulae (28) and (15), we can perform the following triple products:

$$P^3_{3T} P^\Lambda_T \propto \langle \sigma^{3T} \rangle \vec{r} \times \langle \sigma^\Lambda \vec{r} \rangle \cdot \hat{p}$$

$$P^2_{2T} P^\Lambda_T \propto \langle \sigma^\Lambda_N \rangle \vec{e}_N \times \langle \sigma^\Lambda_T \rangle \cdot \hat{p},$$

(75)

where we have set, for the sake of brevity, $\sigma_r = \vec{\sigma} \cdot \vec{r}$ and so on. Other authors already proposed T-odd triple products[31, 38, 1, 4, 5], but those considered here are rid of effects of final state interactions[31, 39]. Moreover, we ascertain $a posteriori$ that some T-odd pseudoscalar[31, 32, 30, 33] triple products are unequivocally connected to TRV.

### 4.2 CP Violations

The CP transformation causes, according to the usual phase conventions[34, 27],

$$A_{\lambda_1,\lambda_2} \to -\overline{A}_{-\lambda_1,-\lambda_2},$$

(76)
where the barred amplitude refers to the $\bar{\Lambda}_b$ decay. Then, taking into account eqs. (55), (56), (59) and (60), together with the definitions given in subsects. 2.2 and 2.3 of the quantities defined in these expressions, we find that the following parameters are useful for detecting possible CP violations:

\[
R_W = \frac{G_W - \bar{G}_W}{G_W + \bar{G}_W}, \quad R_L = \frac{B_L + \bar{B}_L}{B_L - \bar{B}_L},
\]
\[
R_N = \frac{B_N - \bar{B}_N}{B_N + \bar{B}_N}, \quad R_{TT} = \frac{B_{TT} - \bar{B}_{TT}}{B_{TT} + \bar{B}_{TT}},
\]
\[
\gamma_W = \frac{\alpha_W + \bar{\alpha}_W}{\alpha_W - \bar{\alpha}_W}, \quad \gamma_L = \frac{\alpha_L + \bar{\alpha}_L}{\alpha_L - \bar{\alpha}_L},
\]
\[
\gamma_T = \frac{\alpha_T + \bar{\alpha}_T}{\alpha_T - \bar{\alpha}_T}, \quad \gamma_N = \frac{\alpha_N + \bar{\alpha}_N}{\alpha_N - \bar{\alpha}_N},
\]
\[
\gamma_{TT} = \frac{\alpha_{TT} + \bar{\alpha}_{TT}}{\alpha_{TT} - \bar{\alpha}_{TT}}, \quad \gamma_{TN} = \frac{\alpha_{TN} + \bar{\alpha}_{TN}}{\alpha_{TN} - \bar{\alpha}_{TN}}.
\]

Any nonzero value of the above defined ratios - defined conformally to the usual conventions[12, 36, 37, 31] - would be a signature of CP violation and also, possibly, of NP[13]. The ratios $B_T + \bar{B}_T$ to $B_T - \bar{B}_T$ and $B_{TN} + \bar{B}_{TN}$ to $B_{TN} - \bar{B}_{TN}$ have not been considered, since the sums are CP-odd and the differences are CPT-odd, therefore both quantities may be, in principle, nearly zero. In any case, the sums may be used as further tests for CP violations.

### 4.3 CPT Tests

The ratios (77) to (81) are even under time reversal, therefore they can also be suitably employed in tests of the CPT theorem. Moreover it follows from the discussion above that also $B_T - \bar{B}_T$ and $B_{TN} - \bar{B}_{TN}$ are good parameters for testing the theorem.

We note that polarization of muons from semileptonic decays of $K^\pm$ had been proposed by Lee and Wu[35] as a possible test for CPT violation.

### 5 Concluding Remarks

We conclude this note with some remarks about the method suggested.

A) Our analysis is completely model independent and is also independent of spurious effects[1, 4, 5, 6, 38] caused by final state interactions[31, 39], which may flaw,
in principle, other kinds of tests proposed[38, 4, 20]. In particular, we stress that our tests for TRV do not rely on any assumptions. Our calculation can be used as an input for calculating the model predictions of the observables considered here[8, 9].

B) It is important to note that the TRV tests based on $\Lambda_b$ polarization are similar to those proposed for hyperon decays[23, 12],

$$\Lambda \rightarrow p \pi^-, \quad \Sigma \rightarrow \Lambda \pi, \quad \Xi \rightarrow \Lambda \pi.$$  \hspace{1cm} (82)

However in our case we may also consider the polarization correlations[30, 31], which provide a TRV test independent of the polarization of the parent resonance. Decays of the type (2) are very suitable for detecting possible TRV, as pointed out also by other authors in studying CP violations[4, 5].

C) The observables considered in the present letter are very sensitive to NP, since they are rid of unpleasant effects of Wilson’s coefficients[40]. These quantities have been considered even more convenient than $B^0 - \bar{B}^0$ mixing phases[4].

D) Reactions similar to those studied here have been proposed also by other authors[41, 42] in a different context, in occasion of LHC forthcoming run. Then it appears not unrealistic to suggest to measure also some of the observables considered in the present note, that is, the angular distribution and the polarization of at least one of the decay products.

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