Sub-GeV Electron and Positron Channeling in Straight, Bent and Periodically Bent Silicon Crystals

G B Sushko¶, A V Korol¶§, Walter Greiner¶, A V Solov’yov§ 1
¶ Frankfurt Institute for Advanced Studies, Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany
§ St. Petersburg State Maritime University, Leninsky ave. 101, 198262 St. Petersburg, Russia
E-mail: korol@fias.uni-frankfurt.de

Abstract. Preliminary results of numerical simulations of electron and positron channeling and emission spectra are reported for straight, uniformly bent and periodically bent silicon crystal. The projectile trajectories are computed using the newly developed module [1] of the MBN Explorer package [2, 3]. The electron and positron channeling along Si(110) and Si(111) crystallographic planes are studied for the projectile energies 195–855 MeV.

1. Introduction
The basic effect of the channeling process in a crystal is in an anomalously large distance which particle can penetrate moving along a crystallographic plane (the planar channeling) or an axis (the axial channeling) and experiencing the collective action of the electrostatic field of the lattice ions [4]. The field is repulsive at small distances for positively charged particles and, therefore, they are steered into the inter-atomic region, while negatively charged projectiles move in the close vicinity of ion strings or planes.

Channeling of charged particles is accompanied by the channeling radiation [6] which arises due to the transverse motion (the channeling oscillations) of the particle inside the channel under the action of the interplanar or axial field. The intensity of the radiation depends on the type of the projectile, on its energy, and on the type of crystal and crystallographic plane (axis). In a bent crystal, the synchrotron radiation appears as a result of circular motion of the channeling particle along the bent crystallographic planes. The condition of stable channeling in bent crystals [7,8] implies the bending radius $R$ to be much smaller than the (typical) curvature radius of the channeling oscillations. Therefore, the synchrotron radiation modifies the soft-photon part of the emission spectrum. In a periodically bent crystal, additional mechanism of radiation appears to the undulating motion of channeling particles which follow the periodic bending of crystallographic planes. The concept of a crystalline undulator (CU) was formulated quite recently [9,10]. By means of CU it is feasible to produce monochromatic undulator-like radiation in the hundreds of keV up to the MeV photon energy range. The intensity and

1 On leave from A.F. Ioffe Physical-Technical Institute, St. Petersburg, Russia
2 Channeling effect can be discussed not only for crystals but, in principle, for any structured material which provides "passages", moving along which a projectile has much lower value of the mean square of the multiple scattering angle than when moving along any random direction. The examples of such materials are nanotubes and fullerites, for which the channeling effects has been also investigated, see, e.g., Ref. [5].
characteristic frequencies of the radiation can be varied by changing the type of channeling particles, the beam energy, the crystal type and the parameters of periodic bending. (see recent review [11] for more details).

In recent years, several experiments were carried out to measure the channeling parameters and the characteristics of emitted radiation of sub-GeV light projectiles. These include the attempts made [12] or planned to be made [13] to detect the radiation from a positron-based CU. More recently, a series of the experiments with straight, bent and periodically bent crystals have been carried out with 195–855 MeV electron beams at the Mainz Microtron (Germany) facility [15, 16]. The CUs, used in the experiment, were manufactured in Aarhus University (Denmark) using the molecular beam epitaxy technology to produce strained-layer $Si_{1-x} Ge_x$ superlattices with varying germanium content as described in [17,18]. Another set of experiments with diamond CUs is scheduled for the year 2013 at the SLAC facility (USA) with 10...20 GeV electron beam [19].

Theoretical support of the ongoing and future experiments as well as accumulation of numerical data on channeling and radiative processes of ultra-relativistic projectiles in crystals of various content and structure must be based on an accurate procedure which allows one to simulate the trajectories corresponding to the channeling and non-channeling regimes. Recently, a universal code to simulate trajectories of various projectiles (positively and negatively charged, light and heavy) in an arbitrary scattering medium, either structured (straight, bent and periodically crystals, superlattices, nanotubes etc) or amorphous (solids, liquids) has been developed as a new module [1] of the MBN Explorer package [2, 3]. To simulate propagation of particles through media the algorithms used in modern molecular dynamics (MD) codes were applied. Verification of the code against available experimental data as well as against predictions of other theoretical models were carried out for 6.7 GeV and 855 MeV electrons and positrons in Si(110) as well as in amorphous Si [1]. In the cited paper, critical analysis was carried out of the underlying physical model and the algorithm implemented in the recent code for electron channeling described in Refs. [20]. It was shown, that the specific model for electron–atom scattering leads in a noticeable overestimation of the mean scattering angle. As a results, the data on the dechanneling lengths presented in [20] are incorrect.

In the present paper we present new results on electron and positron channeling and emission spectra in straight, uniformly bent and periodically bent silicon crystal. The electron and positron channeling along Si(110) and Si(111) crystallographic planes are studied for the projectile energies 195–855 MeV.

2. Channeling Radiation of Electrons in Si (110)
A relevant benchmark for our simulations are the channeling spectra of 195...855 MeV electrons in Si(110) that have been addressed in previous experimental [14–16, 21] studies. To this end, we have performed extensive calculations of the particles trajectories and the emitted radiation spectra formed in $L = 50 \mu m$ crystalline silicon. Two examples of our calculations are presented in Fig. 1.

In both panels, the curves represent the spectral enhancement factor, i.e., the spectral distribution $dE/d(\hbar \omega)$ of the radiation emitted in the crystalline medium normalized by that in the amorphous silicon. The latter was calculated within the Bethe-Heitler approximation [22].

For 270 MeV incident energy (left panel) the presented spectrum, which corresponds to the aperture 0.24 mrad, was averaged over $N_0 = 3500$ simulated electron trajectories. At the crystal entrance, the initial velocity for each trajectory was parallel to the Si(110) plane and aligned along the [10, −10, 1] crystallographic direction. The error bars reflect the statistical uncertainties which correspond to the probability 0.999.

For 855 MeV electrons (right panel) the spectra were computed for the aperture 0.21 mrad and for three different directions of the initial velocity as indicated. The numbers of simulated
Figure 1. Enhancement factor for 270 MeV (left panel) and for 855 MeV (right panel) electron channeling along Si(110) plane in $L = 50 \, \mu m$ thick straight crystal. The 270 MeV data refer to the maximum emission angle $\theta_{\text{max}} = 0.24 \, \text{mrad}$, the 855 MeV data - to $\theta_{\text{max}} = 0.21 \, \text{mrad}$. Three curves on the right panel correspond to three different directions of the incident beam, as indicated.

The trajectories are: $N_0 = 3120$ for the $[10, -10, 1]$ direction, $N_0 = 1919$ for $[8, -8, 1]$, and $N_0 = 1679$ for $[13, -13, 1]$. The goal of these calculations was to analyze the sensitivity of the channeling radiation to the initial (random) direction along the (110) plane. We can state that all spectra are indistinguishable within statistical errors.

The theoretical results for the enhancement factors were compared to the experimental data [21]. The analysis of the theory-versus-experiment will be published elsewhere. Here we just note that perfect correspondence, both in the peak position and height, was found for 270 MeV incident electron beam. For 855 MeV electrons, theoretical curves reproduce the peak position but underestimate the intensity of the channeling radiation by approximately 25-30 percent. The analysis of the source of this discrepancy is currently under way.

3. 855 MeV Electrons and Positrons in Straight and Bent Si (111)

The structure of a silicon single crystal implies that a Si(111) planar channel contains two (111) planes separated by the distance $d_n = 0.784 \, \text{Å}$. Total width of the channel is $d = 3.136 \, \text{Å}$.

The presence of two planes of crystal atoms in a single channel leads to specific features of the channeling oscillations for both negatively and positively charged projectiles. These features are absent in the case of channeling along (100) or/and (110) planes.

To discuss qualitatively the channeling oscillations of an electron and a positron we refer to Figure 2, which presents the Si(111) interplanar potential $U$ calculated in the continuous approximation [4] with the use of the Molière atomic potential. Let us stress, that the results of numerical simulations presented further in this section, were not based on the continuous approximation. The latter is used for illustrative considerations only.

In the case of electron channeling (left panel in Figure 2) the interplanar potential has two wells symmetrically separated with respect to the midplane, where the potential has a local maximum $U_0$. At the boarders of the channel, i.e. at the distances $\pm d/2$, the potential has maxima $U_{\text{max}}$ which exceed $U_0$. As a result, if the transverse energy $\varepsilon_\perp$ of an electron satisfies the condition $\varepsilon_\perp < U_0$ then the channeling oscillations are restricted to one of the wells. In the case $\varepsilon_\perp > U_0$ the particle oscillates with larger amplitude within $[-d/2, d/2]$. The electron interplanar potential is strongly anharmonic, therefore, the period of oscillations depends on the amplitude. On average, the large-amplitude oscillations are slower than the small-amplitude ones in the vicinity of the local minima.
The positron interplanar potential is presented on the right panel of Fig. 2. In this case, the potential also has two wells although strongly asymmetric. Both of the wells can be approximated by parabolic dependencies. The frequency of channeling oscillations in the narrow (and shallow) well is approximately 2 times larger than that in the wide well.

Two types of channeling oscillations for electrons and positrons manifest themselves in the emission spectra calculated for small emission angle and discussed below in the paper.

The motion of 855 MeV electrons and positrons collimated at the entrance along Si(111) planes is illustrated in Fig. 3 by sets of randomly chosen simulated trajectories. The data refer to the straight crystal of the length $L = 100 \mu m$.

For positrons, noticeable are nearly harmonic oscillations. Two types of oscillations, occurring in the wide part of the channel and in the narrow as well, are clearly seen in the presented trajectories. Another feature of positron channeling through a $L = 100 \mu m$ thick crystal is a comparatively small number of the dechanneling events. This is also not surprising if one compares the crystal size with the dechanneling length $L_d \approx 700 \mu m$ for a 855 MeV positron in Si(111). The latter value can be obtained using Eq. (1.50) from [8] with the correction for
a light projectile introduced in [23]. Therefore, it is not surprising that most of the incident particles traverse the crystal in the channeling mode.

Much less regular are the channeling oscillations of electrons, see the left panel of Fig. 3. The electron trajectories exhibit a broader variety of features: channeling motion, over-barrier motion, rechanneling process, rare events of hard collisions etc. First, let us note that the dechanneling length of a 855 MeV electron in Si(111), estimated with the help of Eq. (10.1) from [24], is \( L_d \approx 23 \mu m \). Therefore, it is not surprising that the events of channeling through the whole crystal are quite rare. On the other hand, the events of rechanneling, i.e., capture to the channeling mode of an over-barrier particle, are quite common for electrons. Even the multiple rechanneling events are not rare. This phenomenon has been already noted in the simulations of the electron channeling [1,20] with a qualitative explanation provided [20] of the difference in the rechanneling rate for positively and negatively charged projectiles. Also it is worth noting a visible anharmonicity in the channeling oscillations of electrons which is a direct consequence of a strong deviation of the electron interplanar potential from a harmonic shape. As a result, the period of the oscillations varies with the amplitude. Similar to the positron channeling, two types of oscillations, related to the two wells structure of the interplanar potential (see Fig. 2 left), are clearly seen in the presented trajectories.

The simulations of the trajectories were performed for straight and bent Si(111) channels. In the former case, two crystal lengths along the incident beam direction were considered: \( L_1 = 50 \mu m \) and \( L_2 = 100 \mu m \). For the bent crystal (the uniform bending with the bending radius \( R = 33 \) mm) the simulations were carried out for \( L_1 = 50 \mu m \) in accordance with the experimental conditions at the Mainz Microtron facility [21].

The simulated trajectories were used to estimate the dechanneling length (in the case of the electron channeling) and to calculate spectral distribution of the emitted radiation.

To quantify the electron dechanneling process we calculated two penetration lengths introduced in Ref. [1]. The first one, notated below as \( L_{p1} \) was found as a mean value of the primary channeling segments, which started at the entrance and lasted till the dechanneling point somewhere inside the crystal. Generally speaking, this quantity is dependent on the angular distribution of the particles at the entrance. The \( L_{p1} \) values quoted below were obtained for a zero-emittance beam collimated initially along the (111) planar direction. The second penetration depth, \( L_{p2} \), is defined as a mean value of all channeling segments, including those which appear due to the rechanneling. In the rechanneling process an electron is captured into the channeling mode having, statistically, an arbitrary value of the incident angle \( \psi \) not greater than Lindhard’s critical angle. Therefore, \( L_{p2} \) mimics the penetration depth of the beam with a non-zero emittance \( \approx \psi_L \).

In addition to \( L_{p1} \) and \( L_{p2} \) we calculated the total channeling length \( L_{ch} \), defined as an average length of all channeling segments per trajectory.

The results for \( L_{p1}, L_{p2} \) and \( L_{ch} \), together with the calculated values of the channel acceptance \( A = N_{acc}/N_0 \) (where \( N_0 \) and \( N_{acc} \) are numbers of the incident and the accepted particles, respectively), are summarized in Table 1.

For a straight crystal, it is instructive to compare the obtained values \( L_{p1} = 18.70 \pm 0.69 \) and \( L_{p2} = 15.92 \pm 0.40 \mu m \) with the dechanneling lengths for the initial beam, \( L_{d0} = 13.57 \pm 0.12 \mu m \), and for the rechanneled particles, \( L_d = 13.69 \pm 0.07 \mu m \) obtained in Ref. [20]. The calculations performed in the cite paper were based on the peculiar model of the elastic scattering of an ultra-relativistic projectile from the crystal constituents. The model substitutes the atom with its “snapshot” image: the atomic electrons are treated as point-like charges placed at fixed positions around the nucleus. The interaction of an ultra-relativistic projectile (e.g., an electron) with each atomic constituent is treated in terms of the classical Rutherford scattering. In Ref. [1] it was demonstrated, that such a “snapshot” model noticeably overestimates the mean scattering angle in the process of elastic scattering in a single electron-atom collision. The mean square
angle for a single scattering is a very important quantity in the multiple-scattering region, where there is a large succession of small-angle deflections symmetrically distributed about the incident direction. It was noted in Ref. [1] that the “snapshot” approximation underestimates the dechanneling length of 855 MeV electrons in straight Si (110) by approximately 30 per cent. Similar to this, the Si(111) data from [20], undervalues the dechanneling length presented in Table 1: \( L_{d0} \) is less than \( L_{p1} \) by 37 ± 5 % whereas \( L_{d} \) is 17 ± 3% smaller than \( L_{p2} \).

Let us note that the obtained length \( L_{p1} = 18.70 ± 0.69 \mu \text{m} \) is in agreement with the value 18.8 \( \mu \text{m} \) evaluated recently in Ref. [25] from the solution of the Fokker-Plank equation. The simulated trajectories were used to compute spectral distribution of the emitted radiation following the formalism and algorithm described in detail in Ref. [1]. The results are presented in figures 5 – 6. The calculated spectral intensities are normalized to the Bethe-Heitler bremsstrahlung spectrum in amorphous silicon. Statistical uncertainties due to the finite number (\( \approx 3000 \ldots 4000 \)) of the analyzed trajectories are indicated by the error bars. The calculations were performed for two detector apertures: \( \theta_{\text{max}} = 0.21 \) and 2 mrad. The first value, which is close to the aperture used in the experiments with the 855 MeV electron beam [14, 15, 21], is much smaller than the natural emission angle \( \gamma^{-1} \approx 0.6 \text{ mrad} \). Therefore, the corresponding spectra refer to nearly forward emission. On the contrary, the second angle greatly exceeds \( \gamma^{-1} \), so that the cone \( \theta_{\text{max}} \) collects nearly all emitted radiation.\(^3\)

Figures 4 and 5 present the enhancement of radiation in straight silicon crystals.

First, we note that for both electrons and positrons the intensity of radiation in the oriented crystal greatly exceeds (by more than an order of magnitude) the bremsstrahlung background. The enhancement comes from the particles moving along quasi-periodic channeling trajectories, which bear close resemblance with the undulating motion. As a result, constructive interference of the waves emitted from different but similar parts of the trajectory are added coherently. For each value of the emission angle \( \theta \) the coherence is most pronounced for the radiation into harmonics, which frequencies can be estimated as (see, e.g., [24]):

\[
\omega_n = \frac{2\gamma^2 \Omega_{\text{ch}}}{1 + \gamma^2 \theta^2 + K_{\text{ch}}^2/2} n, \quad n = 1, 2, 3, \ldots ,
\]

Table 1. The penetration lengths \( L_{p1} \), \( L_{p2} \) and \( L_{\text{chan}} \) for \( \varepsilon = 855 \text{ MeV} \) electrons in straight and bent Si (111) crystals (the bending radius is 33 mm). The crystal lengths are \( L_1 = 50 \mu \text{m} \) and \( L_2 = 100 \mu \text{m} \). Also indicated are: the number of simulated trajectories \( N_0 \), the number of accepted particles \( N_{\text{acc}} \), and the acceptance \( A = N_{\text{acc}}/N_0 \).

| Crystal     | \( N_0 \) | \( N_{\text{acc}} \) | \( \alpha \) | \( L_{p1} \) (\( \mu \text{m} \)) | \( L_{p2} \) (\( \mu \text{m} \)) | \( L_{\text{chan}} \) (\( \mu \text{m} \)) |
|-------------|-----------|-----------------|----------|----------------|----------------|----------------|
| straight \( L_1 \) | 3467 | 2560 | 0.74 | 18.37 ± 0.82 | 15.48 ± 0.52 | 24.72 ± 0.83 |
| straight \( L_2 \) | 2160 | 1632 | 0.76 | 19.22 ± 1.24 | 16.38 ± 0.61 | 38.71 ± 1.76 |
| straight \( L_1 \) & \( L_2 \) | 5627 | 4192 | 0.75 | 18.70 ± 0.69 | 15.92 ± 0.40 | — |
| bent \( L_1 \) | 3603 | 2471 | 0.69 | 16.62 ± 0.77 | 15.96 ± 0.67 | 19.15 ± 1.11 |

\(^3\) The intensity of channeling radiation of 855 MeV electrons in Si(111) crystals of various thickness was calculated in [25] for large apertures. In the cited paper the channeling process was modeled on the basis of the Fokker-Plank equation.
Figure 4. Enhancement factor for 855 MeV positrons channeled in $L = 50 \mu m$ crystalline target along Si (111) planes calculated for two values of the maximum emission angle as indicated.

approximation, these quantities are dependent on the magnitude of the transverse energy which, in turn, determines the amplitude of oscillations.

Different character of channeling by positrons and electrons results in differences in the spectra of the channeling radiation.

The nearly perfect sine-like channeling trajectories of positrons lead to the emission spectrum close to that of the undulator radiation with $K^2 \ll 1$. Two peaks in the positron spectrum, see Fig. 4, is due to two types of channeling oscillations mentioned above. The peak at $\approx 2$ MeV is due to the emission in the fundamental harmonic ($n = 1$) from the trajectories corresponding to the channeling motion in the wide well of the positron Si(111) channel (see Fig. 2 right). It is more pronounced for the smaller aperture, since in this case a strong inequality $(\gamma \theta)^2 \ll 1$, valid for all angles $\theta \leq \theta_{\text{max}}$, ensures the independence of $\omega_1$ on the emission angle. The second, less accentuated peak, corresponds to the emission in the first harmonic due to the channeling motion in the narrow part of the channel. In this case, the amplitudes are smaller (this result in the decrease of the intensity) but the channeling frequencies are higher leading to the higher value of $\omega_1$. For the larger aperture, a big part of the energy is radiated into the cone $\gamma^{-1} < \theta < \theta_{\text{max}}$. For these relatively large emission angles the first harmonic energy decreases with $\theta$. As a result, the peaks become broader and less intensive.

Due to strong anharmonicity of the electron channeling oscillations, the peaks in the spectrum of channeling radiation are less pronounced even for the small aperture, see Fig. 5 left. For the large aperture $\theta_{\text{max}} = 2$ mrad the second peak is completely smeared out. The right panel in the figure illustrates the differences in the emission spectra (for the small aperture) for electron channeling in Si (111) and Si (110) channel (the calculations of the latter were performed in [1]). The Si(110) channel can be modeled as a single-well interplanar potential which leads to a single peak in the emission spectrum.

Enhancement of radiation emitted by 855 MeV positrons and electrons in uniformly bent Si (111) channels are presented in Figs. 6 and 7. The data correspond to the crystal length $L = 50 \mu m$ and the bending radius $R = 33$ cm. In a bent crystal, the channeling condition implies the centrifugal force $F_{\text{cf}} \approx \varepsilon/R$ to be smaller than the maximum interplanar force $U_{\text{max}}'$. In our case, the value of the bending parameter $C = F_{\text{cf}}/U_{\text{max}}'$ [11] is much less than one, $C \approx 0.045$, so that the channeling condition is well-fulfilled. This is reflected, in particular, by small deviation of the values of the penetration lengths $L_{p1}$, $L_{p2}$ and $L_{\text{ch}}$, calculated for an electron channeled in the bent Si(111) from those for the straight channel, see Table 1.
Figure 5. Left panel. Enhancement factors, calculated for two indicated apertures $\theta_{\text{max}}$, for 855 MeV electron channeling in $L = 50 \mu m$ straight Si crystal along (111) planes. Right panel. Enhancement factor for 855 MeV electrons channeled in $L = 50 \mu m$ straight silicon crystal along (110) and (111) planes. The data refer to $\theta_{\text{max}} = 0.21$ mrad.

Figure 6. Enhancement factor calculated for two apertures $\theta_{\text{max}}$ for 855 MeV positrons channeled along straight (dashed line) and bent (solid line) Si (111) plane. The bending radius $R = 33$ mm, the crystal length $L = 50 \mu m$. Note the double log scale in the left graph.

Let us note two features of the emission spectra formed in bent silicon crystal. First, the bending gives rise to the synchrotron radiation, since the channeled particle experiences the circular motion in addition to the channeling oscillations. This leads to the increase of the intensity in the photon energy range $\lesssim 10^2$ keV. For these energies the excess of the radiation yield from the bent channel over that from the straight channel is clearly seen in the figures (compare the solid and the dashed curves). Second, it is seen that the decrease in the intensity of the channeling radiation in the bent channel is more pronounced for the small aperture, $\theta_{\text{max}} = 0.21$ mrad than for the large one equal to 2 mrad. This feature can be understood by comparing $\theta_{\text{max}}$ to the bending angle $\Theta = L/R \approx 1.5$ mrad $\gg \gamma^{-1}$. Since the emission angle is defined with respect to the direction of the incident beam, the emission within the cone $\theta_{\text{max}} = 0.21$ mrad $\ll \Theta$ occurs, effectively, only from the initial parts of the channeling trajectories. In the opposite limit, $\theta_{\text{max}} > \Theta$, the radiation from nearly all the trajectory is detected.
4. Channeling and Radiation in Crystalline Undulators

In this section we present some of the results obtained recently with the help of the newly developed code [1] for the channeling phenomenon and radiation emitted in a crystalline undulator (CU). In CU, in addition to the channeling radiation, the undulator-type radiation appears due to the undulating motion of channeling particles which follow the periodic bending of crystallographic planes [9–11]. Two types of harmonic periodic bending of the channel centerline, which correspond to the sine and to the cosine profiles, can be considered:

\[ y(z) = a \sin \left( \frac{2 \pi z}{\lambda_u} \right), \quad y(z) = a \cos \left( \frac{2 \pi z}{\lambda_u} \right). \]  

(2)

Here, the coordinate \( z \) is measured along the straight channel, the \( y \) axis is perpendicular to the straight plane. The quantities \( a \) and \( \lambda_u \) are the bending amplitude and period and they satisfy the relation \( d < a < \lambda_u \) where \( d \) is the interplanar distance (see the cited papers for more details on the description of the CU concept).

We have performed simulation of the trajectories, the quantitative analysis of the channeling motion and computation of the spectral intensities of the radiation formed by ultra-relativistic electrons and positrons within the energy range 195 – 855 MeV in the CU with the parameters used in the experiments at at the Mainz Microtron (Germany) facility [15, 16]. The 4-periods CUs were manufactured in Aarhus University (Denmark) using the molecular beam epitaxy technology to produce strained-layer Si\(_{1-x}\)Ge\(_x\) superlattices with varying germanium content as described in [17, 18].

The following values of the CU parameters were used in the calculations:

- Channeling plane: Si(110) (the interplanar distance \( d = 1.92 \) Å)
- Crystal length: \( L = 39.6 \) μm
- Bending period: \( \lambda_u = 9.9 \) μm
- Bending amplitude: \( a = 3 \ldots 5 \) Å

In full the results of our calculations will be published elsewhere. Here we announce only few results for 355 and 855 MeV electrons which are presented in Fig. 8. The left panel compares the spectral enhancement of radiation emitted by 355 MeV electrons channeled in straight Si (110) and in the sine-shaped CU with two bending amplitudes as indicated. All three dependencies exhibit powerful maximum at about 1.7 MeV which corresponds to the channeling radiation. In the case of the undulating crystals the maxima are lower is due to the decrease in the allowed amplitude of the channeling oscillations in periodically bent channel in comparison with the
straight one [23]. However, the CU undulator radiation manifests itself as a hump in the photon energy range 40 . . . 100 keV (the vertical dashed line marks the first harmonic of the radiation in the forward direction). Hence, our simulations indicate that it is possible to observe the CU radiation even for comparatively low energies of the electron beam.

With the increase of the electron beam energy the CU radiation peak becomes more accented, as it is illustrated by the right panel. Here, the maximum at about 600 keV is seen for both sine- and cosine-shaped CUs.

5. Conclusion
Using the newly developed code [1], which was implemented as a module in the MBN Explorer package [2], we have performed the Monte Carlo simulations of trajectories of ultra-relativistic electrons and positrons in oriented straight, bent and periodically bent single Si crystals. The simulated trajectories were used as the input data for numerical analysis of the intensity of the emitted radiation. In the case of straight crystals the reduces to the channeling radiation emitted atop the incoherent bremsstrahlung background. In a bent channel the spectrum is enriched by the synchrotron radiation due to the circular motion of the projectile along the bent centerline. In a periodically bent crystal, in addition to the channeling radiation, the undulator-type radiation appears due to the periodicity of the bending. In this paper, for the first time the characteristics of the crystalline undulator radiation were computed on the basis of the Monte Carlo simulation of electron and positron trajectories.

The calculation of the spectra as well as the numerical analysis of channeling conditions and properties (acceptance, dechanneling length) have been carried out in a broad range of the beam energies, 195 . . . 855 MeV. The obtained and presented results are of interest in connection with the ongoing experiments with electron beams at Mainz Microtron [15] and with possible experiments with the positron beam [13].

Acknowledgments
We are grateful to Hartmut Backe and Werner Lauth for fruitful and stimulating discussions. The work was supported by the European Commission CUTE (the CUTE-IRSES project, grant
GA-2010-269131). The possibility to perform complex computer simulations at the Frankfurt Center for Scientific Computing is gratefully acknowledged.

References

[1] Sushko G B, Bezchastnov V G, Solov’yov I A, Korol A V, Greiner W and Solov’yov A V 2013 J. Comp. Phys. (in print)
[2] Solov’yov I A, Yakubovich A V, Nikolaev P V, Volkovets I and Solov’yov A V 2012 J. Comp. Chem. 33 2412–2439
[3] http://www.mbnexplorer.com/
[4] Lindhard J 1965 K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 34 1-64
[5] Artru X, Fomin S P, Shulga N F, Ispirian K A, Zhevago N K 2005 Phys. Rep. 412 89-189
[6] Kumakhov M A 1976 Phys. Lett. 57A 17–18
[7] Tsyganov E N 1976 Fermilab Preprint TM-682, TM-884 (Fermilab, Batavia)
[8] Biryukov V M, Chesnokov Yu A and Kotov V I 1996 Crystal Channeling and its Application at High-Energy Accelerators (Springer-Verlag, Berlin, Heidelberg)
[9] Korol A V, Solov’yov A V and Greiner W 1998 J. Phys. G: Nucl. Part. Phys. 24, L45–L53
[10] Korol A V, Solov’yov A V and Greiner W 1999 Int. J. Mod. Phys. E 8 49-100
[11] Korol A V, Solov’yov A V and Greiner W 2013 Channeling and Radiation in Periodically Bent Crystals, (Springer-Verlag Berlin Heidelberg)
[12] Baranov V T, Bellucci S, Biryukov V M, Britvich G I, Chepegin V N et al 2006 Nucl. Instrum. Methods 252 32–35
[13] Backe H, Krambrich D, Lauth W, Buonomo B, Dabagov S B, Mazzitelli G, Quintieri L, Hansen J. Lundsgaard Uggerhøj U K I, Azadegan B, Dizdar A and Wagner W 2011 Nuovo Cimento C 34 175–180
[14] Backe H, Kunz P, Lauth W and Rueda A 2008 Nucl. Instrum. Methods B 266 3835–3851
[15] Backe H, Krambrich D, Lauth W, Hansen J Lundsgaard and Uggerhøj U K I 2011 Nuovo Cimento C 34 157-165
[16] Backe H, Lauth W, Kunz P, Rueda A, Esberg A et al 2010 in Charged and Neutral Particles Channeling Phenomena (Eds: S B Dabagov, L Palumbo, A Zichichi) (Singapore, World Scientific) pp 281–290
[17] Mikkelsen U, Uggerhøj E 2000 Nucl. Instrum. Methods B 160 435–439
[18] Krause W, Korol A V, Solov’yov A V, Greiner Walter 2002 Nucl. Instrum. Methods A 483 455–460
[19] Uggerhøj U 2012 Private communication
[20] Kostyuk A, Korol A V, Solov’yov A V, Greiner Walter 2011 J. Phys. B: At. Mol. Opt. Phys. 44 075208
[21] Backe H and Lauth W 2013 Private communication
[22] Tsai Yung-Su 1974 Rev. Mod. Phys. 46 815–851
[23] Korol A V, Solov’yov A V, Greiner Walter 2001 J. Phys. G: Nucl. Part. Phys. 27 95-125
[24] Baier V N, Katkov V M, Strakhovenko V M 1998 Electromagnetic Processes at High Energies in Oriented Single Crystals (World Scientific, Singapore)
[25] Bogdanov O V and Dabagov S N 2012 J. Phys.: Conf. Ser. 357 012029