Emission rates, the Correspondence Principle and the Information Paradox

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When we vary the moduli of a compactification it may become entropically favourable at some point for a state of branes and strings to rearrange itself into a new configuration. We observe that for the elementary string with two large charges such a rearrangement happens at the ‘correspondence point’ where the string becomes a black hole. For smaller couplings it is entropically favourable for the excitations to be vibrations of the string, while for larger couplings the favoured excitations are pairs of solitonic 5-branes attached to the string; this helps resolve some recently noted difficulties with matching emission properties of the string to emission properties of the black hole. We also examine the change of state when a black hole is placed in a spacetime with an additional compact direction, and the size of this direction is varied. These studies suggest a mechanism that might help resolve the information paradox.
1. Introduction.

We can understand many properties of black holes if we use string theory as the underlying theory of quantum gravity. A key idea is to compare the properties of string theory states at small coupling with their properties at larger coupling. At small coupling we expect to understand the properties of the state from our knowledge of string theory, while at large coupling the state should behave like a black hole, and should exhibit the thermodynamic properties associated to black holes [1][2][3][4][5][6][7][8][9][10].

1.1. Extremal and near-extremal states

The most precise results have been obtained for black holes that are extremal or close to extremal. Consider the model of a black hole in type IIB string theory, in 4+1 noncompact spacetime dimensions. Out of the 10 dimensions of Minkowski space, 5 are compactified on a 5-torus $T^5 = T^4 \times S^1$. Let the coordinates $X^5, X^6, X^7, X^8$ span the $T^4$ and the coordinate $X^9$ be along the $S^1$. One way to make a black hole is to wrap $n_5$ D-5-branes on the torus $T^5$, wrap $n_w$ D-strings on circle $S^1$, and consider a momentum $p = \frac{2\pi n_p}{L}$ along the direction $S^1$. Here $L$ is the length of the circle $S^1$ and we let the volume of the $T^4$ be $V_4$.

If all the vibrations contributing to the total momentum $p$ travel in the same direction along the D-strings, then we have a BPS state, which would at larger coupling go over to an extremal black hole carrying the three charges corresponding to $n_5, n_w, n_p$. The count of string theory microstates with these charges gives an entropy that agrees with the Bekenstein-Hawking entropy of the extremal black hole [3][4]. If we have vibrations travelling on the D-strings in both directions, then we have more energy than the minimum required to carry the charges of the state, and we have a non-BPS string state which corresponds at larger coupling to a non-extremal black hole. If we are close to extremality, the extra entropy as a function of the extra energy for the string state agrees with the corresponding quantity for the non-extremal black hole [4].

The string theory state above can also absorb and emit quanta when the vibrations moving in opposite directions on the D-strings collide with each other. Since the strings are bound to the 5-branes, we assume that they can vibrate only inside the 5-branes, namely in the directions $X^5, X^6, X^7, X^8$. This leads to the fact that at leading order in the energy, out of the 10-dimensional gravitons only the $h_{ij}$ with $i, j = 5, 6, 7, 8$ can be absorbed; in other words scalars of the 4+1 dimensional spacetime theory can be absorbed but vector
particles and gravitons will be suppressed [8]. This agrees with the expectation from the black hole, which at low energy absorbs scalars but repels higher spin quanta due to a ‘centrifugal barrier’. The absorption cross section equals the area of the horizon in the classical calculation [3][4][11], and it is found that the string theory state has exactly the same absorption cross section [3].

If the momentum charge and amount of nonextremality is small compared to the other two charges, the absorption cross section arising from creation of vibration modes can be seen to agree with the classical black hole expectation not only at leading order in energy, but for all energies of the order of the temperature of the black hole [9]. This gives the agreement of greybody factors between the string theory model and the corresponding black hole. The string model emission rate is proportional to the product of the number of left moving vibrations and the number of right moving vibrations, and it is interesting that that classical greybody factor exhibits the same structure. The different numbers of left and right excitations define different left and right temperatures; the temperature of the black hole is the harmonic mean of these two temperatures.

1.2. The correspondence principle, and potential difficulties

The above results were obtained for black holes that were close to extremality. What can we say about more general black holes? A general conjecture was made by Horowitz and Polchinski about the transition between a string theory state and the black hole carrying the same charges [12]. We will be interested in the case where the string theory state is just an elementary string at small coupling \( g \), so we discuss the ‘correspondence principle’ proposed in [12] only in this context.

We consider type IIB string theory with the spacetime compactified to 4+1 dimensions as discussed above for the near extremal black hole. Let the elementary string have winding number \( n_w \) and momentum \( p = \frac{2\pi n_p}{L} \) along \( X^9 \). If we have a BPS state (all the momentum is carried by the left movers, there are no right moving vibrations on the string) then, at large \( g \), the metric of this string state would be that of an extremal black hole with zero horizon area. If we add some extra energy to make the state nonextremal, then the corresponding metric will describe a black hole with a nonsingular horizon. Let us fix the Einstein metric of the black hole in the 4+1 dimensional spacetime, and imagine reducing the coupling \( g \) while allowing the string length to increase. Around some value \( g = g_1 \) we will find that the curvature of the string metric becomes of the order of the string scale, and the string theory description of the spacetime is no longer expected to be a good one.
We will call this point \( g = g_1 \) as the ‘correspondence point’ or the ‘matching point’. What happens for \( g < g_1 \)? The conjecture of [12] is that for \( g < g_1 \) we can describe the state by using essentially the free elementary string, with charges \( n_w, n_p \) and additional vibrations to make the string energy equal to the energy of the black hole. The agreement of the black hole and free string descriptions at \( g = g_1 \) is not expected to be exact, but all physical quantities of interest should have the same order in either description.

It was shown in [12] that when one uses the correspondence principle then the entropy of the free string state at the matching point \( g = g_1 \) agrees with the entropy of the black hole, up to a factor of order unity. This provides evidence in support of the correspondence principle. But there are some indications that the matching between black holes and free string states might not be that simple:

(i) The emission rate of low energy quanta from an elementary string state is twice that expected from the corresponding black hole [13]. This is not a contradiction in itself, since factors of order unity are not fixed in the transition between the black hole and the string descriptions. But the factor of 2 can be traced to the fact that the entropy of the string state comes from vibrations of the string in all 8 directions transverse to the string, rather than the 4 directions allowed in the near-extremal models discussed above. But the restriction of the vibrations in the latter case to these 4 compact directions also implied that only 4+1 dimensional scalars will be emitted at low energy, while vectors and gravitons will be suppressed. Classically we find that not just the near extremal hole but also other holes like the Schwarzschild black hole share the property that the low energy emission is dominated by scalars. The elementary string state which has vibrations in all 8 transverse directions seems to emit vector particles and gravitons with the same rate as it emits scalars. We need to understand why this should change as we enter the black hole phase.

(ii) It was recently argued by Emparan [14] that the greybody factors suggested by emission by the string state are not the same as those required by the classical black hole. Again there is no direct conflict with the correspondence principle, since at the energies where the greybody calculation is valid the two different greybody predictions differ by a factor of order unity, and such factors are not fixed in the correspondence principle. But the string state has in general unequal number of left and right excitations, and thus unequal left and right temperatures, and the greybody factors reflect this fact. But with just two charges, the winding and the momentum, the classical hole has greybody factors with equal left and right temperatures. Again we need to understand why this kind of change should occur as we move from the free string phase to the black hole phase.
1.3. Summary of results

In this paper we do the following. We consider the toroidal compactification of 10-dimensional spacetime down to 4+1 dimensions, as described in subsection 1.1. We take an elementary string with winding charge $n_w$ and momentum charge $n_p$ in the direction $X^9$. We take $n_w \gg 1, n_p \gg 1$. At large coupling $g$ this should give a black hole with two large charges. At small coupling we have essentially the free string state, and the entropy is carried in left and right moving vibrations of the string. At the coupling increases past the value at the correspondence point we find that it becomes entropically more advantageous to put the available energy into exciting pairs of solitonic 5-branes, attached to the elementary string. Thus as we leave the free string phase (which we call phase I) and enter the black hole phase (which we call phase II) we have a change in the way that excitations are carried by the system.

It was noted in [15] that in the ‘fat black hole’ limit the excitations will be in what we have called Phase II, since this kind of excitation has the largest entropy when the excitation energy is very high. What we are noting here is that the transition from Phase I to Phase II happens at the correspondence point of [12], which is defined by the horizon size becoming string scale.

The excitation of 5-branes has the right properties to reproduce the radiation from the black hole. Let us consider the model, which we call model A, described by the charges

$$A: \text{elementary string winding} : n_w, \text{momentum} : n_p, \text{solitonic 5}−\text{brane} : n_5$$ (1.1)

By S-duality we can map this to model B which has the charges

$$B: D−\text{string winding} : n_w, \text{momentum} : n_p, D−5−\text{brane} : n_5$$ (1.2)

By a sequence of S and T dualities we can permute the three charges in model B in any way we wish [16]; in particular we can map to model C with charges:

$$C: D−5−\text{brane} : n_w, D−1−\text{brane} : n_p, \text{momentum} : n_5$$ (1.3)

The elementary string state we have gives model A with $n_5 = 0$. This thus has the same emission properties as model C (with with zero momentum charge). Such a model is known to give satisfactory Hawking radiation rates at low energy [6][9].

In the above example we imagined holding the excitation levels of the string fixed while increasing the coupling to reach the change of excitation type. We can also consider
the case where a black hole is confined to a compact circle, and then the size of this circle is reduced. Following the discussion in [12] we expect that there should be a change of excitation type when the circle size becomes small enough; we examine this transition.

In outline we perform the following steps:

(a) We review the correspondence principle of [12] for the case of an elementary string state. We note that for states with significant amount of charges the total entropy of the string at the matching point is of the same order as the entropy of the BPS state with the same charges, which in turn is of order $\sim \sqrt{n_w n_p}$.

(b) We consider the energy available for non-BPS excitations at the correspondence point, and find the number $n_{55}$ of solitonic 5-brane pairs that can be created with this energy. Note that when there are winding and momentum charges then the solitonic 5-brane excitations will be fractional [17], and it is these fractional pairs that are being counted by $n_{55}$. We find that at the correspondence point the energy which is available to the non-BPS excitations is always of the order of the mass of one full solitonic 5-brane, so that the number of fractional excitations is $\sim n_w n_p$, and the entropy of such excitations is therefore $\sim \sqrt{n_w n_p}$. Thus at the correspondence point it is equally efficient from the point of view of entropy to store the energy in the vibrations of the string or in the creation of 5-brane pairs. For smaller coupling we show that the string vibrations are more efficient while for larger coupling the 5-brane pairs are more efficient.

(c) We note that in Phase II (where the excitations are the 5-brane pairs) the emission properties agree with the properties expected of the black hole carrying two large charges.

(d) We examine some effects of the gravitational field of the string at the correspondence point. In particular we observe that at the horizon the length of the circle where the string is wrapped is such that the energy of a free string carrying the given charges would be minimised. We discuss the relation of this result with properties of the absorption of higher partial waves by the black hole.

(e) We examine the change in the state of the 4+1 dimensional black hole when an additional circle is compactified and the size of this circle is reduced. We find that when the size of the compact circle becomes $\sim r_0$ ($r_0$ is the nonextremality length scale of the hole) then it is entropically favourable to use the non-BPS energy to excite Kaluza-Klein monopoles wrapping around the new compact direction.

(f) We examine how the occurrence of fractional charges may lead to a change in the picture of how a black hole absorbs an incoming quantum. The occurrence of a large length
scale due to fractional charges may help resolve the information retrieval issue for black holes.

(g) We examine some consequences of a recent postulate for the quantised spectrum of excitations of the winding-momentum-5-brane system. We observe that this quantisation appears to describe Phase II but not Phase I.

The plan of the paper is the following. In section 2 we review the arguments of [14] concerning the disagreements of greybody factors between the elementary string and the black hole. In section 3 we review the correspondence principle in our case of interest and demonstrate point (a) above. In section 4 we demonstrate point (b). Section 5 discusses point (c). Section 6 concerns point (d). Section 7 discusses point (e). Section 8 examines point (f). Section 9 is a general discussion. Point (g) is discussed in the Appendix.

2. Emission properties of the elementary string.

In this section we review the behavior of emission rates in the context of the correspondence principle. One issue is that the spins of the quanta emitted from the elementary string at weak coupling are not those that we expect the black hole to emit at low energies; this issue was mentioned in the introduction. We discuss now the details of the greybody spectrum that do not seem to agree between the black hole and the elementary string; here we will try to paraphrase some of the arguments of [14].

Let us consider the case of the 4+1 dimensional black hole where we have two large charges:

\[ r_1 > r_p >> r_5, r_0 \]  

where

\[ r_1^2 \sim G_N^{(5)} M_1 \sim (g^2 V L^{-1} L(S)^5) n_w L T(S) \sim g^2 V^{-1} n_w L(S)^6 \]  

In our notation the tension of the elementary string is \( T(S) = \frac{1}{2\pi \alpha'} = \frac{2\pi}{L(S)^2} \), so that \( L(S) = 2\pi \sqrt{\alpha'} \) and under T-duality a compact direction of length \( A L(S) \) goes to a length \( A^{-1} L(S) \). We have taken \( r_1 > r_p \) without loss of generality, since the two charges can be interchanged by a T-duality. For low energy quanta the greybody factors were computed in [3]. Here low energy means that

\[ \lambda >> r_1 \]  

Thus

\[ \omega \sim \lambda^{-1} \ll V_4^{1/2} g^{-1} n_w^{-1/2} L(S)^{-3} \]  

Consider the process where the elementary string absorbs the incoming quantum. If we take the coupling to be very weak and thus ignore any redshift effects, then the change in level of the string is given through

$$\delta(M^2) \sim M\delta M = M\omega \sim n_wLT(S)\omega \sim T(S)\delta N_R$$  \hspace{1cm} (2.5)$$

So

$$\delta N_R \sim n_wL\omega \ll n_wLV_4^{1/2}g^{-1}n_w^{-1/2}L(S)^{-3} \sim n_w^{1/2}LV_4^{1/2}g^{-1}L(S)^{-3}$$  \hspace{1cm} (2.6)$$

But the temperature of the right movers is

$$T^*_R \sim N_R^{1/2}$$  \hspace{1cm} (2.7)$$

(Here we are referring to the temperature for the distribution of oscillator levels on the world sheet; this is a dimensionless temperature.) To find \(N_R\), note that extra mass over extremality for the classical black hole is

$$\delta M = \frac{(2\pi)^3LV_4r_0^2}{g^2L(S)^8} \sim \frac{LV_4}{g^2L(S)^6}$$  \hspace{1cm} (2.8)$$

where in the last step we have set \(r_0 \sim L(S)\) for the correspondence point. If this mass were to be carried in vibrations, the level would be

$$N_R \sim M_s\delta MT(S)^{-1} \sim \frac{n_wL^2V_4}{g^2L(S)^6}$$  \hspace{1cm} (2.9)$$

Note that if we take \(g \ll 1\), \(\frac{L}{L(S)} \sim 1\), \(\frac{V_4}{L(S)^4} \sim 1\), then we have \(N_R \gg n_w \gg 1\), so that we can use thermodynamic arguments. But from (2.6),(2.7)

$$\frac{\delta N_R}{T^*_R} \ll 1$$  \hspace{1cm} (2.10)$$

so that we will see no interesting greybody factors. (The left movers have \(\delta N_L = \delta N_R\), and \(T^*_L > T^*_R\), so they also give \(\delta N_L/T^*_L \ll 1\).)

On the other hand we know that in the present domain of parameters the black hole does have nontrivial greybody factors which come from a set of effective left movers and a set of effective right movers. Thus we appear to have a disagreement, but we note that one effect that we have ignored is the redshift, which for large charges is significant even at the coupling where the string turns into a black hole.
As long as we have $N_R << N_L$, we will have $T_R^* << T_L^*$, and the greybody factors will reflect these unequal temperatures. On the other hand if we have only two charges nonzero, we know from [9] that the emission from the classical hole is described by a product of left and right thermal factors with equal temperatures.

The string can have equal left and right temperatures if $N_R \sim N_L$. But note that the difference $N_L - N_R = n_p n_w$ is fixed by the charges. So to have $N_R \sim N_L$ we would need to have much larger $N_R, N_L$ than those implied by the correspondence principle analysis where the mass of the black hole was equated to the mass of the free string.

While it may well be that when we take into account the redshift effects the string must have much larger $N_R, N_L$ than that expected from the free string analysis, taking these large values will not allow the string entropy $S \sim \sqrt{N_R} + \sqrt{N_L}$ to equal the black hole entropy. Thus the greybody factors do not agree very well with the correspondence principle, where we equate the properties of the black hole to the properties of a string at the point where the black hole description ceases to be adequate [14].

3. The correspondence principle and non-BPS entropy

Let us examine the calculation of the correspondence principle for the case that will be of interest to us. The spacetime is $M^5 \times T^5 = M^5 \times T^4 \times S^1$. The string theory state is that of one elementary string, which can carry winding and momentum charges along the $S^1$ direction.

The black hole solution corresponding to these charges and some amount of nonextremality is given by the following Einstein metric $G_E$ and 5-dimensional dilaton $\Phi$:

$$ds_5^2 = -f^{-2/3}(1 - \frac{r_0^2}{r^2})dt^2 + f^{1/3}[\{(1 - \frac{r_0^2}{r^2})^{-1}dr^2 + r^2d\Omega_3^2\}]$$ (3.1)

$$f = [1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}][1 + \frac{r_0^2 \sinh^2 \sigma}{r^2}]$$ (3.2)

$$e^{-2\Phi} = (1 + \frac{r_0^2 \sinh^2 \alpha}{r^2})^{1/2}(1 + \frac{r_0^2 \sinh^2 \sigma}{r^2})^{1/2}$$ (3.3)

The 5-d string metric $G_S = G_E e^{4\Phi/3}$ is

$$ds_5^2 = -\{1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}(1 + \frac{r_0^2 \sinh^2 \sigma}{r^2})\}^{-1}(1 - \frac{r_0^2}{r^2})dt^2 + \{[(1 - \frac{r_0^2}{r^2})^{-1}dr^2 + r^2d\Omega_3^2\]}$$ (3.4)
The curvature at the horizon of the string metric becomes of order the string scale when \( r_0 \sim L(S) \). Thus the black hole description is reasonable if \( r_0 >> L(S) \), but we expect that there is an alternative description in terms of a string theory state at \( r_0 << L(S) \).

The mass is

\[
M = \frac{(2\pi)^3 LV_4 r_0^2}{2g^2 L(S)^8} [\cosh(2\alpha) + \cosh(2\sigma) + 1]
\]  

(3.5)

The charges are

\[
n_w = \frac{(2\pi)^2 V_4 r_0^2}{2g^2 L(S)^6} \sinh(2\alpha)
\]  

(3.6)

\[
n_p = \frac{(2\pi)^2 L^2 V_4 r_0^2}{2g^2 L(S)^8} \sinh(2\sigma)
\]  

(3.7)

The Bekenstein-Hawking entropy of the hole is

\[
S = \frac{(2\pi)^4 LV_4 r_0^3}{g^2 L(S)^8} \cosh \alpha \cosh \sigma
\]  

(3.8)

The extremal state with the same charges has the mass

\[
M_{ex} = \frac{2\pi n_w L}{L(S)^2} + \frac{2\pi n_p}{L}
\]  

(3.9)

We can obtain this result by taking the limit \( r_0 \rightarrow 0 \) and \( \alpha, \sigma \rightarrow \infty \) in such a way as to keep the charges (3.6), (3.7) fixed. If we compute the entropy for the extremal configuration the same way we get \( S_{ex} = 0 \) since in this limit

\[
S_{ex} = (2\pi)^2 \frac{r_0}{L(S)} \sqrt{n_w n_p}
\]  

(3.10)

and \( r_0 \rightarrow 0 \) while the other quantities are held fixed. But we can trust the horizon geometry to give the entropy only for \( r_0 > L(S) \). If we put [2]

\[
r_0 = \frac{1}{\sqrt{2\pi}} L(S)
\]  

(3.11)

rather than \( r_0 = 0 \) in (3.10) then we get

\[
S_{ex} = 2\sqrt{2\pi} \sqrt{n_w n_p}
\]  

(3.12)

which agrees with the entropy of the BPS state of the free string carrying charges \( n_p, n_w \) (note that the effective central charge for the free string is 12).
If we equate the mass (3.3) to the mass of a free string state with winding number \( n_w \) and momentum of \( n_p \) units, then we get for the left and right oscillator excitation numbers

\[
N_R = \left[ \frac{(2\pi)^2 L V_4 r_0^2}{4 g^2 L(S)} \right]^2 [3 + 2 \{ \cosh(2(\alpha - \sigma)) + \cosh(2\alpha) + \cosh(2\sigma) \}] \tag{3.13}
\]

\[
N_L = \left[ \frac{(2\pi)^2 L V_4 r_0^2}{4 g^2 L(S)} \right]^2 [3 + 2 \{ \cosh(2(\alpha + \sigma)) + \cosh(2\alpha) + \cosh(2\sigma) \}] \tag{3.14}
\]

We can take without loss of generality \( \alpha \geq \sigma \geq 0 \). We take \( g << 1 \), and the compactification scales to be order string scale; the exact scales will drop of our final estimates. We take the case where we have two large charges at the correspondence point

\[
\alpha \gg 1, \sigma \gg 1, \text{ for } r_0 \sim L(S) \tag{3.15}
\]

For convenience of presentation in the calculation below we also take \( \alpha - \sigma >> 1 \), though this is not essential to the argument (dropping this restriction just introduces factors of order unity in the relations below).

Then the fact that \( \alpha \gg 1, \sigma \gg 1 \) when \( r_0 \sim L(S) \) gives using (3.6)(3.7) that \( n_w >> 1, n_p >> 1 \).

From (3.13)(3.14) we find that

\[
\frac{N_L}{N_R} \approx e^{2\sigma} >> 1 \tag{3.16}
\]

The entropy of the free string state is

\[
S_{st} = 2\pi \sqrt{2} [\sqrt{N_L} + \sqrt{N_R}] \tag{3.17}
\]

The entropy of the extremal string state carrying the same charges was given in (3.12). The fraction of the entropy that can be attributed to the non-BPS excitations is measured by

\[
\frac{S_{st} - S_{ex}}{S_{ex}} \approx e^{-\sigma} \approx \sqrt{\frac{N_R}{N_L}} << 1 \tag{3.18}
\]

Thus we see that in the case at that we have taken (two large charges at the correspondence point) most of the string entropy at the correspondence point is actually the BPS entropy, which in turn is \( \sim \sqrt{n_p n_w} \).
4. The transition from the string to the black hole

In this section we compare the entropy that can be carried by vibrations of the string with that which can be carried by excitation of solitonic 5-brane pairs.

4.1. Entropies of excitations

The mass available above extremality is, from (3.5) and (3.9)

$$M - M_{ex} \equiv \delta M \approx \frac{(2\pi)^3 LV_4 r_0^2}{2g^2L(S)^8}$$

(4.1)

The mass of a pair of fractional 5-branes is

$$m_{5\bar{5}} = \frac{2V_4 L 2\pi}{L(S)^6 g^2 n_p n_w}$$

(4.2)

We now need to set $r_0 \sim L(S)$ to be at the correspondence point. For convenience let us set $r_0 = L(S)(\sqrt{2\pi})^{-1}$, which is the value obtained in (3.11). Then the number of fractional 5-brane pairs is

$$n_{5\bar{5}} = \frac{\delta M}{m_{5\bar{5}}} \approx \frac{n_p n_w}{2}$$

(4.3)

The entropy of these pairs is

$$S_{5\bar{5}} = 2\pi[(\sqrt{n_{5\bar{5}}} + \sqrt{n_{5\bar{5}}})] = 4\pi \sqrt{\frac{n_p n_w}{2}} = 2\pi \sqrt{2\sqrt{n_p n_w}}$$

(4.4)

where we have used that the effective central charge for these excitations is 6. If we had excited no 5-brane pairs but had put all the energy into vibrations of the string, the entropy would have been, using (3.12), (3.18)

$$S \approx S_{ex} = 2\pi \sqrt{2\sqrt{n_p n_w}}$$

(4.5)

so that we get the same entropy at the matching point (3.11) for the two different ways of carrying the excitations.

Now let us consider the change of the entropy in the two cases when we add a small extra bit of energy. We hold fixed the coupling $g$, the moduli and the charges, but have a small increase in $r_0$. The condition that the charges are fixed gives

$$\delta \alpha = -\frac{\delta r_0}{r_0} \tanh(2\alpha)$$

(4.6)
\[ \delta \sigma = - \frac{\delta r_0}{r_0} \tanh(2\sigma) \]  

(4.7)

For the case when the excitations are vibrations of the string, we have

\[ \delta S_{st} = \delta [2\sqrt{2}\pi (\sqrt{N_R} + \sqrt{N_L})] \approx \pi \sqrt{2} \frac{\delta N_R}{\sqrt{N_R}} \approx \frac{2\sqrt{2} \pi^3 L V_4 r_0 \delta r_0}{g^2 L^{(S)} \alpha} \]  

(4.8)

where we have used that \( N_R << N_L \), and the inequalities (3.13). For the case where the excitations are 5-brane pairs,

\[ \delta S_{5\bar{5}} = \delta [2\pi (\sqrt{n_{5\bar{5}}} + \sqrt{n_{5\bar{5}}})] = 2\pi \frac{\delta n_{5\bar{5}}}{\sqrt{n_{5\bar{5}}}} = \frac{(2\pi)^4 L V_4 r_0^2 \delta r_0}{4g^2 L^{(S)} \alpha} e^{\alpha + \sigma} \]  

(4.9)

The ratio is

\[ \frac{\delta S_{5\bar{5}}}{\delta S_{st}} = \frac{\sqrt{2} \pi r_0}{L^{(S)} \alpha} e^\sigma \]  

(4.10)

If we set \( r_0 \) to the value (3.11) which we have used for the correspondence point then we get

\[ \frac{\delta S_{5\bar{5}}}{\delta S_{st}} = e^\sigma \]  

(4.11)

Since \( e^\sigma > 1 \) we see that for \( r_0 > \frac{L^{(S)}}{\sqrt{2} \pi} \) we have \( S_{5\bar{5}} > S_{st} \) while for \( r_0 < \frac{L^{(S)}}{\sqrt{2} \pi} \) we have \( S_{5\bar{5}} < S_{st} \).

4.2. Interpretation

We have studied above in detail the case of [12] that pertains to large winding and momentum charges. Instead of focusing on the curvature of the metric we have focused on the microscopically most efficient way to carry the entropy. The solitonic 5-branes are heavy when \( g \) is small, but it is interesting that the values of \( g \) and the moduli where they start becoming relevant is also the set of parameters where the curvilinear metric is starting to be a good description of the black hole. More generally when we put a string theory state in a compact space and change the coupling then at some point the state begins to feel the effects of compactification and the excitation spectrum changes [18].

By duality we can map the case studied above to model C [13]. In the extremal configuration we have \( n_w \) D-5-branes and \( n_p \) D-strings bound to these D-5-branes. Clearly the entropy is very small if these D-strings are joined up to one long string; there will instead be a microcanonical ensemble of bound states of various winding numbers, and this ensemble has the entropy \( \sim \sqrt{n_p n_w} \).
Now if we add a small amount of nonextremal energy, we do not expect things to change much. But beyond a certain amount of nonextremality it would be more advantageous for the D-strings to join up to one long string, so that the momentum excitations can occur in a fraction \(1/(nPn_w)\) of one unit of momentum in the \(S^1\) direction. As argued in [13] for large excitation energies this will be the favoured mode of excitation; what we note here is that the changeover occurs exactly at the correspondence point.

[The fact that we have an exact rather than an approximate agreement of entropies at the correspondence point (4.4), (4.5) is not a significant fact; this was arranged for convenience by the choice (3.11). We have matched the entropy of the string to the black hole entropy in the choice (3.11) (this choice actually concerned the extremal case (3.12), but the extremal and near extremal entropies are very close by (3.18)). On the other hand we know that the entropy of the near extremal three charge system agrees with the entropy of the corresponding black hole [4]. Thus we have arranged for the two entropies to agree exactly by the choice of the correspondence point.]

5. Emission properties at the correspondence point

5.1. Spins of emitted quanta

Consider our case where we have two large charges at the correspondence point. If we have emission from the free string, then all the 8 directions transverse to the string are on equal footing, and so we emit 5-dimensional scalars, vectors and gravitons at low energy. But from our discussion of the above sections at the correspondence point where the physics of the string becomes the physics of a black hole we have instead the low energy excitations as the 5-brane pairs. By duality we can map this case (which is model A) to the model C. The excitations map to momentum and antiparticle modes. But in this latter model we know that we emit at low energy 5-dimensional scalars \(h_{ij}, i, j = 5, 6, 7, 8\), while the 5-dimensional vectors \(h_{i\mu}, B_{i\mu}\) and the 5-dimensional gravitons \(h_{\mu\nu}\) are suppressed. Reversing the sequence of dualities, we find that the scalars \(h_{ij}\) of model C map to the same scalars in model A, the \(h_{i\mu}\) and \(B_{i\mu}\) are interchanged, and the \(h_{\mu\nu}\) also maps to itself. So we see that only 5-dimensional scalars will be emitted at by the elementary string once it reaches the correspondence point and passes into the black hole phase.
5.2. Leading order emission rate

In [13] it was found that the low energy emission rate from the free string was twice what would be expected from the black hole with the same charges. But if the excitations that collide and emit quanta are the 5-brane pairs then as in the above subsection, the calculation of emission rates becomes under duality the collision of momentum modes in model C, and here we know that the emission rate does agree with the semiclassical calculation of Hawking radiation. Thus the factor of 2 found in [13] will disappear at the correspondence point, at least for the elementary string that has large $n_w, n_p$.

5.3. Greybody factors

It was argued in [14] that when there are two large charges on the elementary string then we have difficulties matching the greybody factors at the correspondence point. The left and right temperatures of a free string would be unequal in this situation, while the classical cross section demands equal temperatures. But if we note that at the correspondence point the non-BPS excitations are not the vibrations of the string but the solitonic 5-brane pairs then we find that the left and right temperatures for these excitations are equal. This can be seen again from the same duality as used above.

6. Gravitational effects

One of the interesting effects noted in [12] was that if we consider the gravitational field of the string state at the correspondence point, then the redshift effects will not be small if there are two large charges, and this redshift in fact implies that the asymptotic temperature maps to the Hagedorn temperature at the horizon. Here we note some other effects of the gravitational field of the black hole, in relation to the string theory state which gives rise to the hole.

6.1. The size of $S^1$ at the horizon

The 10 dimensional string metric that describes the hole with elementary string winding and momentum charges is

\[
ds^2_5 =\left[1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right]^{-1}[-dt^2 + (dX^9)^2 + \frac{r_0^2}{r^2}(\cosh \sigma dt + \sinh \sigma dX^9)^2 + (1 - \frac{r_0^2}{r^2}) dX_i dX^i] + [(1 - \frac{r_0^2}{r^2})^{-1} dr^2 + r^2 d\Omega^2] \tag{6.1}
\]
Consider the extremal limit $r_0 \to 0$, $\alpha, \sigma \to \infty$. In this limit the length of the $X^9$ direction at the horizon is

$$L_H = L \frac{\cosh \sigma}{\cosh \alpha} \to \sqrt{\frac{n_p}{n_w}} L(S)$$

(6.2)

Thus this length becomes independent of the length $L$ at infinity. But we can obtain the same length $L_H$ by the following investigation. Consider the free string wrapped on the circle, so that there is no effect of gravity and the metric is flat. Let us ask what is the value of $L'$, the length of the circle for which the energy of the string is minimised. (We hold fixed the tension of the string and the charges $n_w, n_p$.) Then we find that we must minimise

$$M(L') = n_w L'T(S) + \frac{2\pi n_p}{L'}$$

(6.3)

with respect to $L'$, which gives

$$L'_{\text{min}} = L(S) \sqrt{\frac{n_p}{n_w}}, \quad n_w L'_{\text{min}} T(S) = \frac{2\pi}{L(S)} \sqrt{n_p n_w},$$

$$\frac{2\pi n_p}{L'_{\text{min}}} = \frac{2\pi}{L(S)} \sqrt{n_p n_w}, \quad M_{\text{min}} = \frac{4\pi}{L(S)} \sqrt{n_p n_w}$$

(6.4)

Thus we get $L'_{\text{min}} = L_H$, so that the circle size at the horizon is such that in a free theory it would minimise the mass for the given charges. We also note that for this special length the free string state has equal mass contributions from the winding and momentum charges.

### 6.2. A comment on the absorption of angular momentum

In [19][20][21] the absorption of angular momentum by black holes was studied. In [20][21] it was noted that if an effective string model was to be used for the absorption, then the tension of this string would have to be $\sim (r_1 r_5)^{-1}$ since the classical cross section is a function of the product $r_1 r_5$. If we take the absorbing element to be a D-string with its naive tension then the tension would be $\sim r_5^{-2}$.

One possibility is that the details of the bound state of D-1-branes and D-5-branes at weak coupling is such that the requisite tension is effectively produced at low energies. Here we consider another possibility for the source of a tension that is symmetric in $r_1$ and $r_5$.

From the analysis of the above subsection we note that if we have two large charges, then the near the horizon geometry is such that if we placed the charges here then they
would have equal contributions to the local mass. When we have a low energy wave incident on a black hole, the wave is oscillatory at infinity, essentially non-oscillatory over the scales $r_1, r_5$ and then oscillatory in the near horizon region due to the increasing blueshift. In the calculation of the leading order absorption cross section the tension of the effective string drops out [6], but it may be that for subleading effects we need to use an effective tension that includes effects of gravity and uses the near horizon geometry for the effective string analysis of absorption. In that case the above discussion suggests a reason why $r_1$ and $r_5$ enter in a simple symmetric combination in the absorption cross section, since now these would correspond the two large charges.

6.3. The non-BPS mass blueshifted to the horizon

Consider the system with large charges $n_w, n_p$, with a small amount of non-BPS excitation, which brings the system to the vicinity of the correspondence point. The mass above extremality is then

$$\delta M \approx \frac{(2\pi)^3 LV_4^2 r_0^2}{2g^2 L^{(S)^3}}$$

If we consider this mass blueshifted to the horizon, then we would need to multiply (6.3) by the factor

$$\nu \cosh \alpha \cosh \sigma \approx \nu \frac{g^2 L^{(S)^7}}{LV_4 r_0^2 (2\pi)^2 \sqrt{n_w n_p}}$$

where following [12] we have replaced $(1 - \frac{r_0^2}{r^2})^{-1/2}$ by a quantity $\nu \sim 1$. The mass (6.3) blueshifted to the horizon is

$$\delta M_H = \sqrt{n_w n_p} \frac{\pi}{L^{(S)}} \nu$$

We observe that this quantity is of the same order as the mass $M_{min}$ (6.4) of the BPS state of the elementary string wrapped at the horizon and carrying the charges of the hole. The interpretation of this coincidence is not clear.

7. The ‘crushing’ transition

Suppose we have a black hole in $D$ space-time dimensions, and we compactify one additional direction on a circle. It was noted in [12] that from the viewpoint of classical geometry if the size of this circle is much larger than the horizon then we essentially get a $D$-dimensional hole, while if the circle is smaller than horizon size then we expect the stable solution to be $D - 1$ dimensional hole. This happens because for small compactification
radius the latter solution gives larger horizon area for the same mass, and is thus expected to be the stable solution. In the compactification of branes on a circle, it was argued in [12] that these two geometries had microscopic explanations, in terms of ‘unwrapped’ and ‘wrapped’ branes respectively.

We wish to analyse from a microscopic viewpoint this kind of transition for the case where we have a black hole in 4 spacetime dimensions, and a fifth direction is compactified and taken to be large or small.

Let us start with the 4-dimensional hole. We let the black hole have three charges, corresponding to the charges of our model A. The horizon is nonsingular because there is a small amount of nonextremal energy. When an additional direction is compactified in model A, on a circle of length $L'$, the extra kind of excitation that is available is pairs of Kaluza-Klein monopoles [22].

The mass above extremality for the 4-dimensional hole is

$$M - M_{ex} \equiv \delta M = \frac{(2\pi)^2 LL'V_4 r_0}{2g^2 L(S)^8}$$

The mass of a pair of monopoles is

$$m_{m\bar{m}} = \frac{2(2\pi)LL'^2 V_4}{g^2 L(S)^8}$$

Thus the number of pairs of monopoles that can be created by the mass (7.1) is

$$f = \frac{\delta M}{m_{m\bar{m}}} = \frac{\pi r_0}{2L'}$$

First let us take the case where the energy above extremality is used to create excitations of the three charge system (i.e. there is no excitations of the monopoles). The analogue of (3.18) says that the entropy is essentially the extremal one,

$$S_3 \approx 2\pi \sqrt{n_1 n_2 n_3}$$

Now take the case that the non-BPS energy goes to creating the monopole-antimonopole pairs. The entropy of this four charge system is

$$S_4 = 4\pi \sqrt{n_1 n_2 n_3 f}$$

We have used in both cases the fact that $c = 6$ [23]
The entropies $S_3, S_4$ agree when

$$2\sqrt{f} = 1, \quad f = \frac{1}{4}, \quad r_0 = \frac{L'}{2\pi} \quad (7.6)$$

Following arguments similar to those in section 4, we can show that the entropy of the kind $S_4$ dominates when $r_0 >> \frac{L'}{2\pi}$, while the entropy $S_3$ dominates when $r_0 << \frac{L'}{2\pi}$. Thus the excitations that ‘see’ the extra compactified direction come into play just when the size of this direction equals the scale $r_0$.

We get a similar result if we start with the metric of the 5-dimensional hole and take one extra compactified direction whose length $L'$ we vary. The mass above extremality is

$$M - M_{\text{ex}} \equiv \delta M \approx \frac{(2\pi)^3 LV_4 r_0^2}{2g^2 L^{(S)} 8} \quad (7.7)$$

Thus the number of created monopole pairs is

$$f = \frac{\delta M}{m_{\text{min}}} = \frac{(2\pi)^2 r_0^2}{4L'^2} \quad (7.8)$$

Again we obtain equality of $S_3, S_4$ when

$$f = \frac{1}{4}, \quad r_0^2 = \frac{L'^2}{(2\pi)^2}, \quad r_0 = \frac{L'}{2\pi} \quad (7.9)$$

To interpret the scale $r_0$ in say (7.9) we note that the three kinds of charges are symmetric under U-duality. The Einstein metric is

$$ds_5^2 = -f^{-2/3}(1 - \frac{r_0^2}{r^2})dt^2 + f^{1/3}[1 - \frac{r_0^2}{r^2}]^{-1}dr^2 + r^2d\Omega^2] \quad (7.10)$$

$$f = [1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}][1 + \frac{r_0^2 \sinh^2 \sigma}{r^2}][1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}] \quad (7.11)$$

In analogy to (3.3) we define

$$e^{-2\tilde{\Phi}} = (1 + \frac{r_0^2 \sinh^2 \alpha}{r^2})^{1/2}(1 + \frac{r_0^2 \sinh^2 \sigma}{r^2})^{1/2}(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2})^{1/2} \quad (7.12)$$

Then we define the metric $\tilde{G} = G_e e^{4\tilde{\Phi}/3}$

$$ds^2 = -f^{-1}(1 - \frac{r_0^2}{r^2})dt^2 + [(1 - \frac{r_0^2}{r^2})^{-1}dr^2 + r^2d\Omega^2] \quad (7.13)$$

In this metric $r_0$ is the size of the horizon.
8. A conjecture on the absorption process

In a semiclassical picture a quantum infalling into a black hole falls smoothly through the horizon into the interior of the hole, thus trapping itself causally from the outside world to which it can send no information. The Hawking radiation that takes away its energy arises from redshift of the vacuum modes near the horizon, so that an impure quantum state is forced to result at the end of the evaporation process.

If this fate is to be avoided by the string theory black hole then it does not appear to be enough that there be a suitable theory of quantum gravity at the planck scale; the above argument does not get invalidated by the presence of small scale local fluctuations of the spacetime [24]. Nor does it help that the string scale may be somewhat longer than the planck length, since the black hole horizon scale can be taken as big as we wish. What we would like is a length scale that grows with the size of the hole. Such a length scale can arise from the property of fractionation of branes [17][15], and we give a schematic model that invokes this physics below. (It was pointed out in [15] that fractionation gives rise to long strings, but the physics and scales involved there do not appear to be the same as those that will arise in our analysis.)

Let us consider absorption into the extremal 4+1 dimensional hole for convenience. Our basic postulate will be the following. When an incoming quantum comes at a distance $L'$ from the hole, then in some sense the situation is like the one where we compactify an additional direction with length $L'$. If we had compactified another direction then we could have excited pairs of (fractional) monopoles-antimonopoles wrapping around this new circle; this is what we used in the last section. We assume that the incoming quantum can also act as a ‘peg’ around which the new kind of excitation can wrap. We do not know how to justify this assumption in any rigorous way.

Because of fractionation, the monopole excitations can be quite light; in fact we will see that an adequate excitation can arise just from the kinetic energy of confining the quantum to within a horizon radius. If this happens, then we do not have the picture of a quantum freely falling through the horizon; instead the structure of the hole rearranges itself somewhat and monopole pairs emerge to wrap around the quantum. With such an ‘active’ mode of absorption it is plausible that the information of the quantum can be transmitted to the emerging radiation.
8.1. Outline of calculation

(a) The extremal hole itself has no energy available to create the non-BPS monopole pairs. Thus this energy must be supplied by the incoming quantum; let the energy used be $\delta M$. This energy can create some number $f$ of monopole pairs; we expect to get a fractional number of pairs, $f \ll 1$.

(b) In the calculations of the earlier sections we have taken the excitation type to be of entirely of one kind or entirely of another kind, and then made a comparison of entropies. While this if fine for locating the rough transition point between configurations, in the present case we expect that there will be only a small change of excitation type when a small quantum arrives. Thus assume for convenience that we are in model A and that the excitations of the extremal hole are given by counting the fractional momentum modes that run of a string of length $n_5 n_w L$. Upon arrival of the quantum let a fraction $\mu$ of these modes still contribute to the entropy in this form, while a fraction $(1 - \mu) \ll 1$ of the momentum modes bind to one state (thereby losing entropy) but giving rise to monopole pairs that occur in units of $[n_5 n_w n_p (1 - \mu)]^{-1}$ of a full pair (thereby increasing entropy). We do not know $\mu$ a priori, but we extremise the entropy over $\mu$ and find what the best arrangement of excitations would be.

(c) We require that the increase in entropy which occurs upon rearrangement of excitations be such that the entropy increase by at least order unity. This would indicate that the postulated process is dynamically probable, and not just energetically possible. Note that the entropy increase of order unity means that we double the available states; it is not enough to ask that that states go from a large number $N$ to $N + 1$ since in that case there is a very small likelihood of reaching the new excitation within a dynamical time scale of the system.

(d) The entropy increase depends on the available energy, so we ask what value of $\delta M$ would produce the order unity increase in entropy. We find that the required value of $\delta M$ is $\sim R_H^{-1}$, where $R_H$ is the radius of the horizon. If we try to confine a particle trajectory to make it enter the hole, then we expect this to be the minimum energy that would accompany the particle. Thus the absorption appears allowed by such a mechanism for all infalling quanta.
8.2. Calculations

Let the supplied energy be $\delta M$. Let this be enough to create a fraction $f$ of a complete monopole pair. Thus

$$f = \frac{\delta M}{m_{m\bar{m}}} \quad (8.1)$$

Let a fraction $\mu < 1$ of the quanta of the $n_3$ charge be distributed in the manner required to maximise the entropy of the three charge system, and let the remainder $1 - \mu$ be bound up into one state, thus allowing a fractionation of monopoles by the factor $n_1 n_2 n_3 (1 - \mu)$. The total entropy of this set of states is

$$2\pi \sqrt{n_1 n_2 n_3 \mu} + 4\pi \sqrt{n_1 n_2 n_3 (1 - \mu) f} \quad (8.2)$$

Let us extremise this with respect to $\mu$. Then we get

$$\frac{1}{2\sqrt{\mu}} - \frac{2\sqrt{f}}{2\sqrt{1 - \mu}} = 0 \quad (8.3)$$

$$f = \frac{1 - \mu}{4\mu}, \quad \mu = \frac{1}{1 + 4f}, \quad 1 - \mu = \frac{4f}{1 + 4f} \approx 4f \quad \text{for } f \ll 1 \quad (8.4)$$

[Note that if $f >> 1$, then we have $\mu << 1$ and we are in ‘Phase II’ where the excitations are monopole pairs. If $f << 1$ then $1 - \mu << 1$ and we are in ‘Phase I’ where most of the entropy comes from the distribution that gives the BPS entropy of the three charge system.]

We will be interested in the case $f << 1$. In that case, the entropy gain by taking $\mu$ to be its optimal value, rather than unity, is

$$2\pi \sqrt{n_1 n_2 n_3 [(1 + 4f)^{-1/2} + 2\sqrt{\left(\frac{4f}{1 + 4f}\right)f - 1}] \approx 2\pi \sqrt{n_1 n_2 n_3 [-2f + 4f]} = 4\pi \sqrt{n_1 n_2 n_3 f} \quad (8.5)$$

We would like this extra entropy to be order unity. So we have

$$4\pi \sqrt{n_1 n_2 n_3} f = 1, \quad f = \frac{1}{4\pi \sqrt{n_1 n_2 n_3}} \quad (8.6)$$

Thus we need

$$\frac{\delta M}{m_{m\bar{m}}} = \frac{1}{4\pi \sqrt{n_1 n_2 n_3}}, \quad \delta M = \frac{m_{m\bar{m}}}{4\pi \sqrt{n_1 n_2 n_3}} \quad (8.7)$$
Taking $m_{m\bar{m}}$ from (7.2),

$$
\delta M = \frac{2(2\pi)LL'2V_4}{g^2L^{(s)^8}4\pi\sqrt{n_1n_2n_3}} \tag{8.8}
$$

Note that

$$
2\pi\sqrt{n_1n_2n_3} = A/G_{(5)} = \frac{2\pi^2R^3_H32\pi^2LV_4}{g^2L^{(s)^8}} \tag{8.9}
$$

Thus

$$
\delta M = \frac{L'^2}{32\pi^3R^3_H} \tag{8.10}
$$

If we put $L' = R_H$ (the quantum is in the range of the horizon) then we get

$$
\delta M = \frac{1}{32\pi^3R_H} \tag{8.11}
$$

But $\sim R^{-1}_H$ is the minimum energy that will accompany the quantum localised within a distance of order the horizon size. Thus when the incoming quantum is of the order of the horizon distance away then we can create fractional monopole pairs using its energy, such that the entropy gain by creating these pairs is order unity, and the process is thus seen to be probable and not just possible.

8.3. Notes on the above calculation

(a) We have allowed the available non-BPS energy to form monopoles pairs that wrap around the incoming quantum, but we have not allowed this energy to be used to excite the non-BPS excitations of the three charge system itself. The latter excitations, which are just the excitations of the right moving momentum modes in the above example, would in fact have a higher entropy than the monopole pairs. But we can imagine that the incoming quantum cannot transfer its energy to these momentum modes directly, while it can transfer it to the monopole pairs since these pairs are the ones that see the location of the quantum. After the quantum has been absorbed, the energy can be transferred to the right moving momentum modes, which would be entropically more favourable, and would also be in accord with the effective absorption process at weak coupling [1].

(b) We have used the formulae for the entropy of fractional excitations in a domain where a very small fraction for more than one charge is present (eq. (8.2)). This issue may need a more careful analysis.
(c) The smallest energy quantum that can be absorbed by the extremal hole has an energy much lower than $\sim R_H^{-1}$ [25]. But we have considered the infall of a well defined trajectory rather than the absorption of a monochromatic wave, and here it seems more reasonable to use the scale $R_H^{-1}$ as the minimum energy that the quantum must have to fall in. The geometric picture that we have tried to make of the absorption process pertains to such localised trajectories.

(d) The most unclear step of course is the argument that the infalling quantum sets a scale which can be taken as a compactification scale for the generation of pairs of the fourth charge. The location of the quantum will change with time, and when it enters deep within the hole then we expect that the energy has been converted to the right moving vibrations.

9. Discussion

In this paper we have considered the case of the string state with large momentum and winding charges. The transition from the black hole to the weakly coupled string, which happens when the horizon is string scale [12], has also a simple microscopic description. This transition point is characterised by parameters and a degree of nonextremality such that for smaller energies (or weaker coupling) it is entropically more advantageous to store the non-BPS energy in the form of left and right moving vibrations of the string, while for larger excitation energies (or larger coupling) it is entropically better to unify all the winding and momentum modes to one bound state, and to excite fractional pairs of solitonic 5-branes to carry the non-BPS energy.

As pointed out in [12] the rate of growth of entropy as a function of the energy is different for the free string and for the black hole; thus agreement can be obtained only at the ‘correspondence’ point. The black hole entropy in the near extremal case is known to agree with the entropy of the three-charge system, so it is not a surprise that the point where the non-BPS excitations will change from being string vibrations to being solitonic-5-brane pairs will also be the correspondence point.

But with this microscopic picture, we see that there is no reason for the properties of emission (spins, greybody factors etc.) to agree between the string phase and the black hole phase. In fact for large charges, we may term the change of excitation type at the correspondence point as a phase transition, since the degrees of freedom that which are manifested undergo a change. While we do not understand the physics of emission from the
strongly coupled black hole phase it is gratifying that the three charge model which does reproduce the right low energy emission at weak coupling is entropically favoured from the correspondence point onwards into the black hole phase.

It has been recently noted that emission rates fail to agree in significant ways for black branes at the correspondence point [26]. It would be worth investigating if in this case too there is a change of excitation type that occurs at the correspondence point (for example the excitation of a pair of higher dimensional branes).

We have also investigated the change in state of a black hole when an additional direction is compactified and made smaller so that the hole is ‘crushed’. In accordance with the expectation in [12] there is a change of the entropically favoured state at a certain radius of compactification; we find this radius.

With regard to the information paradox, we note that two places where strings differ from usual quantum gravity plus matter theories are (i) the fractionation of quanta by other quanta [17],[15] which gives rise to new scales depending of the number of particles present and (ii) the occurrence of a $U(N)$ gauge theory when $N$ quanta of the same type come close together [27] which encourages the quanta to spread out from each other. We have speculated on a mechanism using fractionation in this paper, suggesting a more dynamical black hole absorption process than a simple infall into a smooth geometry. Some arguments for the existence of a long length scale using the enhanced gauge symmetry were given in [28]; these two effects may be closely related in the black hole problem.

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Appendix A. Quantisation of the three-charge system

To be able to fully quantify the transition between Phase I and Phase II we need a quantisation of the winding-momentum-5-brane system analogous to the quantisation of the elementary string. A proposal in this direction was given in [29]. In this Appendix we (i) perform algebraic manipulations to obtain a mass formula in terms of excitation level, the charges and the moduli (ii) verify that for a unit increase in excitation number the mass increase in a certain limit is what would be expected by analogy with the elementary string (iii) observe that the left and right temperatures in this quantisation agree with the expectation of what we have called Phase II but not with Phase I.

We will assume that we are in model C, though by duality the same results hold for any model.

For the elementary string the mass formula is

\[ m^2 = (n_wLT(S) + \frac{2\pi n_p}{L})^2 + 8\pi T(S)N_R \]  \hfill (A.1)

For later reference we write

\[ Q_w = n_wLT(S), \quad Q_p = \frac{2\pi n_p}{L} \]  \hfill (A.2)

where the quantities \( Q_w, Q_p \) give the mass of the BPS state with the given winding and momentum charge respectively.

The neutral excitations are measured by one integer \( \delta N_R = \delta N_L \), and not by different numbers that correspond to winding and momentum excitations. But if the winding is the dominant contribution to the mass in (A.1) then the excitations have a spectrum that looks like the spectrum of momentum modes on a single long string [17]:

\[ \delta m \approx \frac{8\pi T(S)}{2m} \approx \frac{4\pi T(S)}{n_wLT(S)} = \frac{4\pi}{n_wL} \]  \hfill (A.3)

Now we consider the quantisation of [29] of the three charge system but for simplicity restrict ourselves to the case of no angular momentum. Let the mass be written as

\[ M = \frac{\mu}{2} \sum_i \cosh(2\delta_i) \]  \hfill (A.4)

Define the effective charges

\[ Q_i = \frac{\mu}{2} \sinh(2\delta_i) \]  \hfill (A.5)
The entropy is then
\[ S = 2\pi\mu^{3/2} \prod_i \cosh(\delta_i) \] (A.6)

Then the system is described by left and right oscillator numbers, with
\[ N_R = \frac{\mu^3}{4} \left[ \prod_i \cosh(\delta_i) - \prod_i \sinh(\delta_i) \right]^2 \] (A.7)
\[ N_L = \frac{\mu^3}{4} \left[ \prod_i \cosh(\delta_i) + \prod_i \sinh(\delta_i) \right]^2 \] (A.8)

In the notation used in [30][29] and adopted in this appendix
\[ \prod_i Q_i = \prod_i n_i \] (A.9)

which is equal to setting to unity the following quantity of units (length)^3:
\[ \frac{g^2 L^{(S)^8}}{(2\pi)^3 L V_4} = 1 \] (A.10)

A.1. The mass spectrum

With some manipulations we can write
\[ M = \frac{1}{2} \sum_i [\mu^2 + 4Q_i^2]^{1/2} \] (A.11)
\[ N_R = \frac{1}{32} \prod_i [(\mu^2 + 4Q_i^2)^{1/2} + \mu] + \frac{1}{32} \prod_i [(\mu^2 + 4Q_i^2)^{1/2} - \mu] - 2 \prod_i Q_i \]
\[ = \frac{1}{16} \prod_i [\mu^2 + 4Q_i^2]^{1/2} + \frac{\mu^2}{8} M - 2 \prod_i Q_i \] (A.12)

We then get
\[ [N_R - \frac{\mu^2}{8} M + 2 \prod_i Q_i]^2 = \frac{1}{256} \prod_i (\mu^2 + 4Q_i^2) \] (A.13)

This is a cubic in \( \mu^2 \), so we can solve it explicitly for \( \mu^2 \) as a function of \( M, N_R \), and the charges \( Q_i \). Substituting in (A.11) we get a relation \( f(M, Q_i, N_R) = 0 \) which is analogous to (A.3).
A.2. A check on the mass spectrum

Consider the case

\[ \mu \ll Q_3 \ll Q_1, Q_2 \]  

(A.14)

For concreteness let \( Q_3 \) correspond to D-5-branes, \( Q_1 \) to D-strings and \( Q_2 \) to momentum. Following what we saw in (A.3) we wish to see that if we make a unit change in \( N_R \) then the change in mass of the soliton complex should approach the mass of a fractional 5-brane pair, if the winding and momentum charges are large. Note that this pair should consist of fractional branes, due to the presence of the other two charges.

In (A.12), let \( \delta N_R = 1, \delta Q_i = 0 \). Then with (A.14),

\[ \mu \delta \mu \approx \frac{8Q_3}{Q_1Q_2} \]  

(A.15)

\[ \delta M \approx \frac{1}{4Q_3} \mu \delta \mu = \frac{2}{Q_1Q_2} \]  

(A.16)

Using (A.10) we convert this mass change to the notation used elsewhere in this paper:

\[ \delta M = 2 \frac{2\pi LV}{gL(s)^6 n_w n_p} \]  

(A.17)

which is seen to be exactly the mass of a fractional D-5-brane pair. Thus we have recovered the analogue of (A.3), which provides one consistency check of the quantisation.

A.3. The domain of applicability of the quantisation

In this quantisation the left and right temperatures are [29]

\[ T_{R,L}^{-1} = \pi \mu^{1/2} \left[ \prod \cosh \delta_i \pm \prod \sinh \delta_i \right] \]  

(A.18)

Thus if one charge (say \( Q_3 \)) is zero, then

\[ T_R^{-1} = T_L^{-1} = \pi \mu^{1/2} \cosh \delta_1 \cosh \delta_2 \]  

(A.19)

This equality of temperatures is expected of excitations in Phase II in our language, but in Phase I the left and right temperatures are not equal. Thus we see that the quantisation proposal of [29] covers Phase II but not Phase I. So it appears that the proposal cannot be used as a rigorous quantisation of the complete three-charge system.

The reason that the proposal naturally covers Phase II is straightforward: it was derived from a study of black hole properties which as we have seen pertain to Phase II. The classical hole is described by the limit of large charges. In this situation the dominant contribution comes from the effect of fractionation of one charge by the other charges, so we naturally pick up the physics of Phase II.
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