Towards Understanding Asynchronous Advantage Actor-critic: Convergence and Linear Speedup

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Abstract

Asynchronous and parallel implementation of standard reinforcement learning (RL) algorithms is a key enabler of the tremendous success of modern RL. Among many asynchronous RL algorithms, arguably the most popular and effective one is the asynchronous advantage actor-critic (A3C) algorithm. Although A3C is becoming the workhorse of RL, its theoretical properties are still not well-understood, including its non-asymptotic analysis and the performance gain of parallelism (a.k.a. linear speedup). This paper revisits the A3C algorithm and establishes its non-asymptotic convergence guarantees. Under both i.i.d. and Markovian sampling, we establish the local convergence guarantee for A3C in the general policy approximation case and the global convergence guarantee in softmax policy parameterization. Under i.i.d. sampling, A3C obtains sample complexity of $O(\epsilon^{-2.5}/N)$ per worker to achieve $\epsilon$ accuracy, where $N$ is the number of workers. Compared to the best-known sample complexity of $O(\epsilon^{-2.5})$ for two-timescale AC, A3C achieves linear speedup, which justifies the advantage of parallelism and asynchrony in AC algorithms theoretically for the first time. Numerical tests on synthetic environment, OpenAI Gym environments and Atari games have been provided to verify our theoretical analysis.

1. Introduction

Reinforcement learning (RL) has achieved impressive performance in many domains such as robotics (Lillicrap et al., 2016; Mnih et al., 2015) and video games (Mnih et al., 2016). However, these empirical successes are often at the expense of significant computation. To unlock high computation capabilities, the state-of-the-art RL approaches rely on sampling data from massive parallel simulators on multiple machines (Nair et al., 2015; Mnih et al., 2016; Assran et al., 2019; Chen et al., 2018). Empirically, these approaches can significantly reduce training time when implemented in an asynchronous manner. One popular method that achieves the state-of-art performance is the asynchronous variant of the actor-critic (AC) algorithm, referred to as A3C (Mnih et al., 2016).

A3C builds on the original AC algorithm (Konda, 2002). At a high level, AC simultaneously performs policy optimization (a.k.a. the actor step) using the policy gradient (PG) method (Sutton et al., 2000) and policy evaluation (a.k.a. the critic step) using the temporal difference learning (TD) algorithm (Sutton, 1988). To ensure scalability to large state-action spaces, both actor and critic steps can combine with various function approximation techniques. To ensure stability, AC is often implemented in a two time-scale fashion, where the actor step runs in the slow timescale and the...
critic step runs in the fast timescale. Similar to other on-policy RL algorithms, AC uses samples generated from the target policy. Thus, data sampling is entangled with the learning procedure, which generates significant overhead. To speed up the sampling process of AC, A3C introduces multiple workers with a shared policy, and each worker has its own simulator to perform data sampling. The shared policy can be then updated using samples collected from multiple workers.

Despite the empirical success achieved by A3C, to the best of our knowledge, its theoretical property is not well-understood. The following theoretical questions remain unclear: Q1) Under what assumption does A3C converge? If so, does it converge to the global optimal solution? Q2) What is its convergence rate? Q3) Can A3C obtain benefit (or linear speedup) using parallelism and asynchrony?

For Q3, we are interested in the training time linear speedup with \( N \) workers, which is the ratio between the training time using a single worker and that using \( N \) workers. Since asynchronous parallelism mitigates the effect of stragglers and keeps workers busy, the training time speedup can be measured roughly by the sample complexity (i.e., computational) linear speedup (Lian et al., 2016):

\[
\text{Speedup}(N) = \frac{\text{sample complexity with one worker}}{\text{average sample complexity per worker with } N \text{ workers}}. \tag{1}
\]

If \( \text{Speedup}(N) = \Theta(N) \), the speedup is linear, and the training time roughly reduces linearly as the number of workers increases. This paper aims to answer this question, towards the goal of providing theoretical justification for the empirical successes of parallel and asynchronous RL.

1.1. Related works

The PG method and its global convergence. The global optimality of the stationary points of policy optimization problems has been shown in (Bhandari and Russo, 2019). Then the finite-time convergence rate for exact PG method with softmax policy was established in (Agarwal et al., 2020) by utilizing a gradient-dominance type result under relative entropy regularized objective function. Later, (Mei et al., 2020) extended this result to the entropy regularized setting and established linear convergence rate for exact PG method under softmax parameterization. Later, (Bhandari and Russo, 2021) has proved linear convergence rate for general class PG methods. In the stochastic setting, (Zhang et al., 2019) has established local optimal convergence for stochastic PG with unbiased rollout and increasing step sizes, and (Wang et al., 2019) has established the global convergence of stochastic neural PG with increasing batch of i.i.d. samples. Later, (Zhang et al., 2021) proved that the minibatch version of PG achieves global convergence with the help of relative entropy regularization. But none of them consider the global convergence of the AC method. On the application side, the PG method has been broadly applied in various settings; see e.g. (Chai and Lau, 2019, 2020; El-Laham and Bugallo, 2021; Cervino et al., 2021). In (Chai and Lau, 2019), the actor critic method was used to jointly optimize the trajectory, transmission and caching content delivery of the unmanned aerial vehicles. In (Chai and Lau, 2020), the policy gradient method was used to jointly optimize the streaming rate and transmission power. In (El-Laham and Bugallo, 2021), the PG method was used to help the learning of a distribution adaptation strategy. In (Cervino et al., 2021), the PG method is used in a multitask learning algorithm which seeks to improve generalization to new tasks.

Analysis of AC algorithm. AC method was first proposed by (Borkar and Konda, 1997; Konda, 2002), with asymptotic convergence guarantees provided in (Borkar and Konda, 1997; Konda, 2002; Bhatnagar et al., 2009). It was not until recently that the non-asymptotic analyses of AC have
be established. The finite-sample guarantee for the batch AC algorithm has been established in (Yang et al., 2018; Kumar et al., 2019; Fu et al., 2020) with i.i.d. sampling. Later, in (Qiu et al., 2019), the finite-sample analysis was established for the double-loop nested AC algorithm under the Markovian setting. An improved analysis for the Markovian setting with minibatch updates has been presented in (Xu et al., 2020a) for the nested AC method. More recently, (Xu et al., 2020b; Wu et al., 2020) have provided the first finite-time analyses for the two-timescale AC algorithms under Markov sampling, with both \( \tilde{O}(e^{-2.5}) \) sample complexity, which is the best-known sample complexity for two-timescale AC. Through the lens of bi-level optimization, (Hong et al., 2020) has provided finite-sample guarantees for two-timescale AC, when a natural policy gradient step is used in the actor. Recently, (Fu et al., 2020) also analyzed the single-timescale AC algorithm under an exact critic oracle. On a less relevant line of research, AC-based multi-agent RL has been studied in (Zhang et al., 2018; Christianos et al., 2020; Qu et al., 2020). However, none of the existing works has analyzed the effect of the asynchronous and parallel updates in AC.

**Parallel and distributed RL methods.** In (Mnih et al., 2016), the original A3C method was proposed and became the workhorse in empirical RL. Later, (Babaeizadeh et al., 2017) has provided a GPU-version of A3C which significantly decreases training time. Recently, the A3C algorithm is further optimized in modern computers by (Stooke and Abbeel, 2019), where a large batch variant of A3C with improved efficiency is also proposed. In (Espeholt et al., 2018), an importance weighted distributed AC algorithm IMPALA has been developed to solve a collection of problems with one single set of parameters. A gossip-based distributed AC algorithm has been proposed in (Assran et al., 2019) which achieves performance competitive to A3C. Additionally, distributed RL is closely related to the multi-agent RL, both of which have a broad range of applications (Kar et al., 2013; Sadeghi et al., 2018; Wu et al., 2021). In (Kar et al., 2013), a distributed algorithm based on Q-learning was proposed and was shown to achieve convergence under a sparse communication network. In (Sadeghi et al., 2018), an asynchronous caching approach which utilized PG to find an optimal caching policy was developed. A robust decentralized TD learning method was proposed in (Wu et al., 2021) to defend against malicious agents in a multi-agent network.

**Asynchronous stochastic optimization.** For solving general optimization problems, asynchronous stochastic methods have received much attention recently. Due to the possible speedup that can be achieved by asynchronous optimization, it has also been extensively applied to various machine learning areas including RL (Mnih et al., 2016; Sadeghi et al., 2018) and distributed learning (Wu et al., 2018). The study of asynchronous stochastic methods can be traced back to 1980s (Bertsekas and Tsitsiklis, 1989). With the batch size \( M \), (Agarwal and Duchi, 2011) analyzed asynchronous SGD (async-SGD) for convex functions, and derived a convergence rate of \( O(K^{-\frac{1}{2}}M^{-\frac{1}{2}}) \) if delay \( K_0 \) is bounded by \( O(K^{\frac{1}{2}}M^{-\frac{1}{2}}) \). This result implies linear speedup. (Feyzmahdavian et al., 2015) extended the analysis of (Agarwal and Duchi, 2011) to smooth convex with nonsmooth regularization and derived a similar rate. Recent studies by (Lian et al., 2016) improved upper bound of \( K_0 \) to \( O(K^{\frac{1}{2}}M^{-\frac{1}{2}}) \). However, all these works have focused on the single-timescale SGD with a single variable, which cannot capture the stochastic recursion of the AC and A3C algorithms. To best of our knowledge, non-asymptotic analysis of asynchronous two-timescale SGD has remained unaddressed, and its speedup analysis is an uncharted territory.

**1.2. This work**

In this context, we revisit A3C with TD(0) for the critic update. The goal is to provide non-asymptotic guarantee and linear speedup justification for this popular algorithm.
Our contributions. Compared to the existing literature on both the AC algorithms and the async-SGD, our contributions can be summarized as follows.

c1) We revisit two-timescale A3C and establish its convergence rates with both i.i.d. and Markovian sampling. We first proves the local convergence rate for A3C in the general function approximation case, and then proves that A3C achieves global convergence for the softmax policy parameterization. To the best of our knowledge, this is the first non-asymptotic convergence result for asynchronous parallel AC algorithms.

c2) We characterize the sample complexity of A3C. In the i.i.d. setting, A3C achieves a sample complexity of \( \mathcal{O}(\epsilon^{-2.5}/N) \) per worker, where \( N \) is the number of workers. Compared to the best-known complexity of \( \mathcal{O}(\epsilon^{-2.5}) \) for i.i.d. two-timescale AC (Hong et al., 2020), A3C achieves linear speedup, thanks to the parallelism and asynchrony. In the Markovian setting, if delay is bounded, the sample complexity of A3C matches the order of the non-parallel AC algorithm (Wu et al., 2020).

c3) We test A3C on a synthetic environment to verify our theoretical guarantees with both i.i.d. and Markovian sampling. We also test A3C on the classic control tasks and Atari Games.

Technical challenges. Compared to the recent analysis of nonparallel two-timescale AC in (Wu et al., 2020; Xu et al., 2020b; Hong et al., 2020), several new challenges arise due to the parallelism and asynchrony.

Markovian noise coupled with asynchrony and delay. The analysis of two-timescale AC algorithm is non-trivial because of the Markovian noise coupled with both the actor and critic steps. Different from the nonparallel AC that only involves a single Markov chain, A3C introduces multiple Markov chains (one per worker) that mix at different speeds. This is because at a given iteration, workers collect different number of samples and thus their chains mix to different degrees. As we will show later, the worker with the slowest mixing chain will determine the convergence.

Linear speedup for SGD with two coupled sequences. Parallel async-SGD has been shown to achieve linear speedup recently (Lian et al., 2016; Sun et al., 2017). Different from async-SGD, asynchronous AC is a two-timescale stochastic semi-gradient algorithm for solving the more challenging bilevel optimization problem (see (Hong et al., 2020)). The errors induced by asynchrony and delay are intertwined with both the actor and critic updates via a nested structure, which makes the sharp analysis more challenging. Our linear speedup analysis should be also distinguished from that of mini-batch async-SGD (Lian et al., 2017), where the speedup is a result of variance reduction thanks to the larger batch size generated by parallel workers.

2. Preliminaries

2.1. Markov decision process and policy gradient

A Markov decision process (MDP) can be described by \( \mathcal{M} = \{S, A, \mathcal{P}, R, \gamma\} \), where \( S \) is the state space, \( A \) is the action space, \( \mathcal{P}(s'|s,a) \) is the probability of transitioning to \( s' \in S \) from state \( s \in S \) and action \( a \in A \), \( r(s,a,s') \) is the reward associated with the transition \( (s,a,s') \), and \( \gamma \in [0,1) \) is a discount factor. Throughout the paper, we assume the reward \( r \) is upper-bounded by a constant \( r_{\text{max}} \). A policy \( \pi : S \rightarrow \Delta(A) \) is defined as a mapping from the state space \( S \) to the probability distribution over the action space \( A \).

Considering discrete time \( t \) in an infinite horizon, a policy \( \pi \) can generate a trajectory \( (s_0,a_0,\ldots) \) with \( a_t \sim \pi(\cdot|s_t) \) and \( s_{t+1} \sim \mathcal{P}(\cdot|s_t,a_t) \). Given a policy \( \pi \), we define the state and state action value
functions as

\[
V_\pi(s) := \mathbb{E}\left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t, s_{t+1}) \mid s_0 = s \right],
\]

\[
Q_\pi(s, a) := \mathbb{E}\left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t, s_{t+1}) \mid s_0 = s, a_0 = a \right]
\]

where \( \mathbb{E} \) is taken over the trajectory \((s_0, a_0, s_1, a_1, \ldots)\) generated under policy \( \pi \). With the above definitions, the advantage function is \( A_\pi(s, a) := Q_\pi(s, a) - V_\pi(s) \). With \( \eta \) denoting the initial state distribution, the discounted state visitation measure induced by policy \( \pi \) is defined as \( d_\pi(s) := (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \mathbb{P}(s_t = s \mid s_0 \sim \eta, \pi) \). We also overload the notation and define the state-action visitation distribution \( d_\pi(s, a) := (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \mathbb{P}(s_t = s \mid s_0 \sim \eta, \pi) \pi(a|s) \). In the case where \( \pi \) is parameterized by \( \theta \), we use \( d_\theta \) as shorthand notations for \( d_\pi \).

The goal of RL is to find an optimal policy \( \pi^* \) defined as \( \pi^* \in \arg \max_\pi J(\pi) := (1 - \gamma) \mathbb{E}_{s \sim \eta} [V_\pi(s)] \), with the optimal return defined as \( J^* := \max_\pi J(\pi) \). When the state and action spaces are large, finding the optimal policy \( \pi \) becomes computationally intractable. To overcome the inherent difficulty of learning a function, the policy gradient methods search the best performing policy over a class of parameterized policies. We parameterize the policy with parameter \( \theta \in \mathbb{R}^d \), and solve the optimization problem as

\[
\max_{\theta \in \mathbb{R}^d} J(\theta) \quad \text{with} \quad J(\theta) := (1 - \gamma) \mathbb{E}_{s \sim \eta}[V_{\pi_\theta}(s)].
\]

To maximize \( J(\theta) \) with respect to \( \theta \), one can update \( \theta \) using the policy gradient (Sutton et al., 2000)

\[
\nabla J(\theta) = \mathbb{E}_{s, a \sim d_\theta}[A_{\pi_\theta}(s, a) \psi_\theta(s, a)],
\]

where \( \psi_\theta(s, a) := \nabla \log \pi_\theta(a|s) \). Since computing \( \mathbb{E} \) in (4) is expensive if not impossible, popular policy gradient-based algorithms iteratively update \( \theta \) using stochastic estimate of (4) such as REINFORCE (Williams, 1992) and G(PO)MDP (Baxter and Bartlett, 2001).

It is also a common practice to adopt regularization and augment the objective function to

\[
J_\lambda(\theta) := J(\theta) - \lambda \mathbb{E}_{s \sim \eta_p} \left[ D_{KL}(\pi_\theta(\cdot|s) \mid \pi_p(\cdot|s)) \right]
\]

with a regularization constant \( \lambda \geq 0 \). Here \( \eta_p \) is a prior distribution of states, \( \pi_p \) is a prior policy. The regularization term encourages \( \pi_\theta \) to imitate \( \pi_p \), incorporating prior knowledge into training process. When \( \pi_p \) and \( \eta_p \) are set as uniform distributions, the regularization term is reduced to the relative-entropy regularization widely analyzed in the literature (Agarwal et al., 2020; Bhandari and Russo, 2019; Zhang et al., 2021). Moreover, the regularization prevents degenerate solutions that can lead to the pitfall of certain policy parametrization (Bhandari and Russo, 2019). Given \( \pi_p \) and \( \eta_p \), we use \( R(\theta) \) as a shorthand notation of \( -\mathbb{E}_{s \sim \eta_p} [D_{KL}(\pi_p(\cdot|s) \mid \pi_\theta(\cdot|s))] \).

2.2. Actor-critic with value function approximation

Both REINFORCE and G(PO)MDP-based policy gradient algorithms rely on a Monte-Carlo estimate of the value function \( V_{\pi_\theta}(s) \) and thus \( \nabla J(\theta) \) by generating a trajectory per iteration. However, policy gradient methods based on Monte-Carlo estimate typically suffer from high variance and large
sampling cost. An alternative way is to recursively refine the estimate of $V_{\pi \theta}(s)$. For a policy $\pi \theta$, it is known that $V_{\pi \theta}(s)$ satisfies the Bellman equation (Sutton and Barto, 2018), that is

$$V_{\pi \theta}(s) = \mathbb{E}_{a \sim \pi \theta(\cdot|s), s' \sim P(\cdot|s,a)} \left[ r(s, a, s') + \gamma V_{\pi \theta}(s') \right].$$

(6)

In practice, when the state space $\mathcal{S}$ is prohibitively large, one cannot afford the computational and memory complexity of computing $V_{\pi \theta}(s)$ and $A_{\pi \theta}(s, a)$. To overcome this curse-of-dimensionality, a popular method is to approximate the value function using function approximation techniques. Given the state feature mapping $\phi(\cdot): \mathcal{S} \rightarrow \mathbb{R}^{d'}$ for some $d' > 0$, we approximate the value function linearly as $V_{\pi \theta}(s) \approx \hat{V}_{\omega}(s) := \phi(s)^\top \omega$, where $\omega \in \mathbb{R}^{d'}$ is the critic parameter.

Given $\pi \theta$, the task of finding the best $\omega$ such that $V_{\pi \theta}(s) \approx \hat{V}_{\omega}(s)$ is usually addressed by TD learning (Sutton, 1988). Given $\pi \theta$, the task of finding the best $\omega$ such that $V_{\pi \theta}(s) \approx \hat{V}_{\omega}(s)$ is usually addressed by TD learning (Sutton, 1988). Formally, we first define

$$A_{\theta, \phi} \triangleq \mathbb{E}_{s \sim \mu_{\pi \theta}, s' \sim P_{\pi \theta}} [\phi(s)(\gamma \phi(s') - \phi(s))]^\top,$$

(7a)

and the critic gradient $g(x_k, \omega_k) := \hat{\delta}(x_k, \omega_k) \nabla \hat{V}_{\omega_k}(s_k)$. We update the parameter $\omega$ via

$$\omega_{k+1} = \Omega_{R_\omega} \left( \omega_k + \beta g(x_k, \omega_k) \right).$$

(10)

where $\beta$ is the critic step size, and $\Omega_{R_\omega}$ is a projection operator that projects a vector to an $l_2$ norm ball with radius $R_\omega$. The projection step is often used to control the norm of gradient. In AC, it prevents the actor and critic updates from going too far in the ‘wrong’ direction; see e.g., (Konda, 2002; Wu et al., 2020; Xu et al., 2020b; Zou et al., 2019).
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Using the definition that $A_{π_θ}(s, a) = E_{s' \sim P}[r(s, a, s') + γV_{π_θ}(s')] - V_{π_θ}(s)$, we can also rewrite (4) as

$$\nabla J(θ) = E_{s, a, s' \sim P}[(r(s, a, s') + γV_{π_θ}(s') - V_{π_θ}(s)) \psi_θ(s,a)].$$

Leveraging the value function approximation, we can then approximate the regularized policy gradient as

$$\hat{∇} J_λ(θ) = \hat{∇} J(θ) + λ\hat{∇} R(θ) = \hat{δ}(x, θ, ω) \psi_θ(s,a) + λψ_θ(x^p).$$

(11)

where $v(x, θ, ω)$ is an estimator of $∇ J(θ)$, and $x^p := (s^p, a^p, s^p_{(|s^p)})$. Then it is easy to check that $ψ_θ(x^p)$ is an unbiased estimator of $∇ R(θ)$. This gives rise to the policy update

$$θ_{k+1} = θ_k + α(v(x_k, θ_k, ω_k) + λψ_θ(x^p)),$$

(12)

where $α$ is the stepsize for the actor update. To ensure convergence when simultaneously performing critic and actor updates, the stepsizes $α$ and $β$ often decay at two different rates, which is referred to the two-timescale AC (Konda, 2002; Wu et al., 2020).

3. A3C Implementation

To speed up the training process, AC can be implemented over $N$ workers in a shared memory setting without coordinating among workers (Mnih et al., 2016). Each worker has its own simulator to perform sampling, and then collaboratively updates the shared policy $π_θ$ using AC updates. As there is no synchronization after each update, the policy used by workers to generate samples may be outdated, which introduces staleness.

Notations on samples. Subscription $t$ in $x_t$ and $x^p_t$ indicates the sample is generated in $t$th local iteration of a worker. When Markovian sampling is used, subscription $t$ in $x_t = (s_t, a_t, s_{t+1})$ also indicates that it is the $t$th transition of the local Markov chain. We use $k$ to denote the global counter (or iteration), which increases by one whenever a worker finishes the actor and critic updates in the shared memory. We use subscription $(k)$ in $(s_{(k)}, a_{(k)}, s'_{(k)})$ and $(s^p_{(k)}, a^p_{(k)})$ to indicate the samples used in the $k$th update.

Algorithm flow. Specifically, we initialize $θ_0, ω_0$ in the shared memory. Each worker will initialize the simulator with initial state $s_0$. Without coordination, workers will load $θ$, $ω$ in the shared memory. The worker then generates samples with either i.i.d. or Markovian sampling method. In Markovian sampling case, we maintain separate Markov chains for actor and critic. For critic, we

Figure 1: Implementation of A3C with two workers.
where \( \tau \)

As indicated by (7), the desired sampling distribution of critic is \( \pi \).

Algorithm 1 A3C: each worker’s view.

1: **Global initialize:** Global counter \( k = 0 \), initial \( \theta_0, \omega_0 \) in the shared memory.

2: **Worker initialize:** Counter \( t = 0 \). Sample \( s_0 \sim \eta, \hat{s}_0 \sim \eta \).

3: **for** \( t = 0, 1, 2, \ldots \) **do**

4: Read \( \theta, \omega \) in the shared memory.

5: **option 1** (i.i.d. sampling):

6: \( x_t = (s_t \sim \mu_{\pi_\theta}, a_t \sim \pi_{\theta_k} (\cdot | s_t), s_t^\prime \sim \mathcal{P}(\cdot | s_t, a_t)) \).

7: \( \hat{x}_t = (\hat{s}_t \sim d_{\pi_\theta}, \hat{a}_t \sim \pi_{\theta_k} (\cdot | \hat{s}_t), \hat{s}_t^\prime \sim \mathcal{P}(\cdot | \hat{s}_t, \hat{a}_t)) \).

8: **option 2** (Markovian sampling):

9: \( x_t = (s_t, a_t \sim \pi_{\theta} (\cdot | s_t), s_{t+1} \sim \mathcal{P}(\cdot | s_t, a_t)) \).

10: \( \hat{x}_t = (\hat{s}_t, \hat{a}_t \sim \pi_{\theta} (\cdot | \hat{s}_t), s_{t+1}^\prime \sim \mathcal{P}(\cdot | \hat{s}_t, \hat{a}_t)) \).

11: With probability \( \gamma \): \( \hat{s}_{t+1} = s_{t+1}^\prime \); Otherwise: \( \hat{s}_{t+1} \sim \eta \).

12: Compute \( g(x_t, \omega) = \delta(x_t, \omega) \nabla \omega \hat{V}_\omega (s_t) \).

13: Compute \( v(\hat{x}_t, \theta, \omega) = \delta(\hat{x}_t, \omega) \psi_\theta (\hat{s}_t, \hat{a}_t) \).

14: Compute \( \psi_\theta (x_t^p) \) with \( x_t^p = (s_t^p \sim \eta_p, a_t^p \sim \pi_p (\cdot | s_t^p)) \).

15: In the shared memory, perform update (13).

16: **end for**

generate samples following the original transition kernel \( \mathcal{P} \). While the actor’s chain can be viewed as evolving under a transition kernel \( \hat{\mathcal{P}} = \gamma \mathcal{P} + (1 - \gamma) \eta \). At each iteration, we have a probability of \( 1 - \gamma \) to reset the chain, thus taking the initial state distribution into account. If the actor’s chain evolves under \( \mathcal{P} \) like critic, asymptotically the initial distribution \( \eta \) is forgotten, which will introduce an asymptotic error. Once samples are obtained, each worker locally computes the gradients, and then updates the parameters in shared memory asynchronously by

\[
\omega_{k+1} = \Pi_{R_\omega} \left( \omega_k + \beta g(x_{(k)}, \omega_{k-\tau_k}) \right) \tag{13a}
\]

\[
\theta_{k+1} = \theta_k + \alpha \left( v(\hat{x}_{(k)}, \theta_{k-\tau_k}, \omega_{k-\tau_k}) + \lambda \psi_{\theta_{k-\tau_k}} (x_{(k)}^p) \right) \tag{13b}
\]

where \( \tau_k \) is the delay in the \( k \)-th actor and critic updates. See A3C in Algorithm 1 and Figure 1.

**Parallel sampling.** The AC update (10) and (12) uses samples generated “on-the-fly” from the target policy \( \pi_{\theta} \), which brings overhead. Compared with (10) and (12), the A3C update (13) allows parallel sampling from \( N \) workers, which is the key to linear speedup. We consider the case where only one worker can update parameters in the shared memory at the same time and the update cannot be interrupted. In practice, (13) can also be performed in a mini-batch fashion.

**Separate sampling protocols.** In Algorithm 1, we maintain separate sampling protocols for actor and critic. This is due to the mismatch between the actor and critic sampling distribution. As indicated by (7), the desired sampling distribution of critic is \( \mu_{\pi_\theta} \). The policy gradient (4) requires sampling from \( \mu_{\pi_\theta} \). However, \( \mu_{\pi_\theta} \) and \( \mu_{\pi_\theta} \) are in general different, and the difference is non-diminishing. Therefore, if one uses the same samples for actor and critic, either the actor or the critic update will have a non-diminishing bias.

To mitigate the asymptotic bias, it is just natural to choose different sampling protocols for actor and critic. Our theoretical analysis justifies this choice by proving that such sampling method gives unbiased stochastic gradients asymptotically. We also provide experiments to demonstrate the superiority of the separated sampling methods.
4. Convergence Analysis of A3C

In this section, we analyze the convergence of A3C in both i.i.d. and Markovian settings. Throughout this section, $\mathcal{O}(\cdot)$ contains constants that are independent of $N$ and $K_0$.

To analyze the performance of A3C, we make the following assumptions.

**Assumption 2** There exists $K_0$ such that the delay at each iteration is bounded by $\tau_k \leq K_0$, $\forall k$.

Assumption 2 ensures the viability of analyzing the asynchronous update; see the same assumption in e.g., (Lian et al., 2016; Assran et al., 2019; Wu et al., 2018). In practice, the delay usually scales as the number of workers, that is $K_0 = \Theta(N)$.

**Assumption 3** For any $\theta, \theta' \in \mathbb{R}^d$, $s \in S$ anda $\in A$, there exist constants $C_\psi, L_\psi, L_\pi$ such that: i) $\|\psi_\theta(s,a)\|_2 \leq C_\psi$; ii) $\|\psi_\theta(s,a) - \psi_{\theta'}(s,a)\|_2 \leq L_\psi\|\theta - \theta'\|_2$; iii) $|\pi_\theta(a|s) - \pi_{\theta'}(a|s)| \leq L_\pi\|\theta - \theta'\|_2$.

Assumption 3 is common in analyzing policy gradient-type algorithms which has also been made by e.g., (Agarwal et al., 2020; Zhang et al., 2019). This assumption holds for many policy parameterization methods such as tabular softmax policy (Agarwal et al., 2020), Gaussian policy (Doya, 2000) and Boltzmann policy (Konda and Borkar, 1999).

**Assumption 4** For any $\theta$, assume the Markov chains with transition kernels $\mathcal{P}$ and $\hat{\mathcal{P}}$ are irreducible and aperiodic under policy $\pi_\theta$. Then there exist constants $\kappa > 0$ and $\rho \in (0,1)$ such that

$$
\sup_{s \in S} d_{TV}(\mathbb{P}(s_t \in \cdot | s_0 = s, \pi_\theta, \mu_{\pi_\theta}) \leq \kappa \rho^t,
$$

(14a)

and

$$
\sup_{s \in S} d_{TV}(\mathbb{P}(s_t \in \cdot | \hat{s}_0 = s, \pi_\theta, d_{\pi_\theta}) \leq \kappa \rho^t.
$$

(14b)

where $s_t$ is the $t$th state of the Markov chain with transition kernel $\mathcal{P}$, and $\hat{s}_t$ is the $t$th state of the Markov chain with transition kernel $\hat{\mathcal{P}}$.

Assumption 4 assumes the Markov chain mixes at a geometric rate. This assumption has also been made by other analysis on Markovian sampling; see e.g. (Bhandari et al., 2018; Wu et al., 2020). It is worth noting that the second part of our assumption, that is (14b), holds as long as $\gamma < 1$.

We define the critic approximation error as

$$
\epsilon_{app} := \max_{\theta \in \mathbb{R}^d} \sqrt{\mathbb{E}_{s \sim \mu_\theta} |V_{\pi_\theta}(s) - \hat{V}_{\omega_\theta}(s)|^2}
$$

(15)

where $\mu_\theta$ is the stationary distribution under $\pi_\theta$ and $\mathcal{P}$. This error captures the quality of the critic function approximation; see also (Qiu et al., 2019; Wu et al., 2020, 2021). When the MDP is tabular and the feature matrix is full-rank, the value function $V_{\pi_\theta}$ is in the span of the features. In this case, we have $\epsilon_{app} = 0$.

We first give the convergence result of critic update.

**Theorem 1 (Critic convergence)** Suppose Assumptions 1–4 hold. Consider Algorithm 1 with i.i.d. sampling and $\hat{V}_{\omega}(s) = \phi(s)^\top \omega$. Select step size $\alpha = K^{-\frac{3}{2}}$ and $\beta = K^{-\frac{2}{2}}$. Then it holds that

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \|\omega_k - \omega_{\theta_0}\|_2^2 = \mathcal{O}\left(\frac{K_0^2}{K^2}\right) + \mathcal{O}\left(\frac{K_0}{K^{\frac{3}{2}}}\right) + \mathcal{O}\left(\frac{1}{K^{\frac{5}{4}}}\right).
$$

(16)
Given the critic convergence, we can present the convergence result of actor update.

**Theorem 2 (Actor convergence)** Under the same assumptions of Theorem 1, select step size $\alpha = K^{-\frac{2}{5}}$ and $\beta = K^{-\frac{3}{5}}$. Then it holds that

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\| \nabla J_{\lambda}(\theta_k) \right\|_{2}^{2} = \mathcal{O} \left( \frac{1}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K_{0}^{2}}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K_{0}}{K^{\frac{3}{5}}} \right) + \mathcal{O} \left( \epsilon_{\text{app}} \right).
$$

(17)

If $K_{0} = \Theta(N) = \mathcal{O}(K^{\frac{1}{5}})$, then it holds that

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\| \nabla J_{\lambda}(\theta_k) \right\|_{2}^{2} = \mathcal{O} \left( K^{-\frac{2}{5}} \right) + \mathcal{O} \left( \epsilon_{\text{app}} \right)
$$

(18)

where $\mathcal{O}(\cdot)$ contains constants independent of $N$ and $K_{0}$.

**Corollary 1 (Linear speedup)** To reach $\epsilon$-accuracy in (18), the required number of iterations is $\mathcal{O}(\epsilon^{-2.5})$. Since each iteration of A3C only uses one sample (one transition), the sample complexity is $\mathcal{O}(\epsilon^{-2.5})$, which matches the state-of-the-art sample complexity of two-timescale AC running on one worker. Then under A3C, the average sample complexity per worker is $\mathcal{O}(\epsilon^{-2.5}/N)$ which indicates linear speedup in (1). The negative effect of parameter staleness introduced by parallel asynchrony vanishes asymptotically with the step size. Vanished staleness allows for parallel computing from workers to speedup the training process.

**Remark 1 (Comparison to async-SGD analysis)** Different from async-SGD (e.g., (Lian et al., 2016)), the optimal critic parameter $\omega_{0}^{*}$ is constantly drifting as $\theta$ changes at each iteration. This necessitates setting the actor update to be at a faster time scale than the critic. In this sense, the policy is static relative to the critic asymptotically. In actor update, the gradient $v(x, \theta, \omega)$ is biased because of inexact value function. The bias introduced by the critic optimality gap and the function approximation error correspond to the last two terms in (17).

### 4.1. Convergence result with Markovian sampling

**Theorem 3 (Critic convergence)** Suppose Assumptions 1–4 hold. Consider Algorithm 1 with Markovian sampling and $\hat{V}_{\omega}(s) = \phi(s)^{\top}\omega$. Select step size $\alpha = K^{-\frac{2}{5}}$ and $\beta = K^{-\frac{3}{5}}$. Then it holds that

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\| \omega_k - \omega_{k}^{*} \right\|_{2}^{2} = \mathcal{O} \left( \frac{1}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K_{0}^{2} \log^{2} K^{*}}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K_{0} \log K}{K^{\frac{3}{5}}} \right).
$$

(19)

The following theorem gives the convergence rate of actor update in Algorithm 1.

**Theorem 4 (Actor convergence)** Under the same assumptions of Theorem 3, select step size $\alpha = K^{-\frac{2}{5}}$ and $\beta = K^{-\frac{3}{5}}$. Then it holds that

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\| \nabla J_{\lambda}(\theta_k) \right\|_{2}^{2} = \mathcal{O} \left( \frac{K_{0}^{2} \log^{2} K^{*}}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K_{0} \log K}{K^{\frac{3}{5}}} \right) + \mathcal{O} \left( \epsilon_{\text{app}} \right).
$$

(20)
If we further assume $K_0 = \Theta(N) = \mathcal{O}(K^{\frac{1}{2}})$. It holds that

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} ||\nabla J(\theta_k)||^2_2 = \tilde{O} \left( K_0 K^{-\frac{3}{2}} \right) + \mathcal{O}(\epsilon_{\text{app}}) \tag{21}
$$

where $\tilde{O}(\cdot)$ hides constants and the logarithmic order of $K$.

Different from i.i.d. sampling, the stochastic gradients $g(x, \omega_k)$ and $v(x, \theta, \omega_k)$ are biased for Markovian sampling, and the bias decreases as the chain mixes. The mixing time corresponds to the logarithmic terms $\log K$ in (19) and (20). Because of asynchrony, at a given iteration, workers collect different number of samples and their chains mix to different degrees. The worker with the slowest mixing chain will determine the rate of convergence. The product of $K_0$ and $\log K$ in (19) and (20) appears due to the slowest mixing chain. As the last term in (19) dominates other terms asymptotically, the convergence rate degrades as the number of workers increases. While the theoretical linear speedup is difficult to establish in the Markovian setting, we will empirically test it in Section 5.

**Remark 2 (Challenges compared to AC analysis)** Unlike synchronous AC, A3C introduces asynchrony and delay in both the actor and critic updates. At each iteration $k$, the delayed parameters will introduce extra error in $g(x, \omega_k - \tau_k) - g(x, \omega_k)$ and $v(x, \theta_k - \tau_k, \omega_k - \tau_k) - v(x, \theta_k, \omega_k)$. Furthermore, it also causes delays in sampling since samples are drawn from the delayed policy $\pi_{\theta_k - \tau_k}$ instead of $\pi_{\theta_k}$. This delay will get amplified as every state on the Markov chain is generated by policies with different delays. At local counter $t$ (tth transition on local Markov chain), we compare the chain transition in synchronous and asynchronous settings:

**sync** : $s_t \xrightarrow{\theta_t} a_t \xrightarrow{\mathcal{P}} s_{t+1} \xrightarrow{\theta_{t+1}} a_{t+1} \cdots$

**async** : $s_t \xrightarrow{\theta_{k-\tau_k}} a_t \xrightarrow{\mathcal{P}} s_{t+1} \xrightarrow{\theta_{k+d_t-\tau_k+d_t}} a_{t+1} \cdots$

where $k$ is the global counter at which the local Markov chain takes tth transition, $\tau_k$ is the delay of policy used to generate tth local transition, $d_t$ is the number of global updates between two local transitions. Clearly, the parameter delay makes the Markov chain more difficult to analyze.

### 4.2. Global convergence under structured problem

A3C is a gradient ascent type algorithm, thus can only achieve local convergence under a generally non-concave objective function $J_\lambda(\theta)$ w.r.t. $\theta$. However, under some special structured problem, A3C can be shown to achieve global convergence. In this section, we consider the class of MDP which has finite state space and action space. Suppose the policy is parameterized by the softmax function:

$$
\pi_{\theta}(a|s) = \frac{\exp(\theta_{s,a})}{\sum_{s,a} \exp(\theta_{s,a})} \tag{22}
$$

where $\theta \in \mathbb{R}^{||S|| \times ||A||}$ and $\theta_{s,a}$ is the policy parameter corresponds to pair $(s, a)$. The softmax policy class cannot represent deterministic policies with finite $\theta$. To avoid driving $\theta$ to infinity, it is crucial to penalize the deterministic policies with the regularization term introduced in (5). To do so, we set
the priors $\eta_p$ and $\pi_p$ as uniform distribution on state and action space, then the objective function can be rewritten as

$$ J_\lambda(\theta) = J(\theta) + \frac{\lambda}{|S||A|} \sum_{s,a} \log \pi_\theta(a|s) + \lambda \log |A|. $$

Define the state feature matrix $\Phi' := [\phi(s^1), \phi(s^2), \ldots, \phi(s^{|S|})]^T \in \mathbb{R}^{|S| \times d'}$ of which rows are features. We make the following assumption on $\Phi'$.

**Assumption 5** For any eligible $\theta$, there exists $\omega_\theta \in \mathbb{R}^{d'}$ such that $\Phi' \omega_\theta = V_{\pi_\theta}$.

This assumption assumes that the value function $V_{\pi_\theta}$ can be accurately approximated by linear functions. For the assumption to hold, it suffices to select a squared full-rank feature matrix $\Phi'$. It is worth noting that when this assumption does not hold, our result in Theorem 5 holds with an extra error term, which is the function approximation error $\epsilon_{\text{app}}$.

To establish global convergence, a gradient-dominance type condition was proven in (Agarwal et al., 2020):

**Lemma 1** With softmax policy parameterization and uniform priors, if $\| \nabla J_\lambda(\theta) \|_2 \leq \frac{\lambda}{2|S||A|}$, then $J^* - J(\theta) \leq \epsilon_\lambda := \frac{2\lambda}{1 - \gamma} \frac{\epsilon_{\text{app}}}{\eta}$.

Figure 2: Algorithm 1 with separate chain sampling (option 2) vs shared chain sampling (setting $\tilde{x}_t = x_t$ in the algorithm). The asymptotic error roughly scales proportionally to $1 - \gamma$. With a smaller $\gamma$, the objective function $J$ becomes more shortsighted, and thus initial state distribution (restarting the chain) plays a more important role. If the actor shares the sample with critic, then a lack of chain restarting will introduce an unavoidable asymptotic error that grows larger as $\gamma$ becomes smaller. Separate chain sampling works thanks to the random restarting with a probability scaling with $\gamma$.

Figure 3: Convergence results of A3C with i.i.d. sampling in synthetic environment.
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Figure 4: Convergence results of A3C with Markovian sampling in synthetic environment.

Figure 5: Speedup of A3C in OpenAI gym classic control task (Carpoles).

For an arbitrary accuracy $\epsilon$, if we set $\lambda = \frac{(1-\gamma)\epsilon}{2\|\frac{d\pi}{\eta}\|_{\infty}}$, then we have $\epsilon_\lambda = \epsilon$. Note that in order for $\|\frac{d\pi}{\eta}\|_{\infty}$ to be finite, we need $\eta(s) > 0$ for any $s \in \mathcal{S}$, which can be assumed without loss of generality.

In the case where $\eta > 0$ does not hold, one can start with an exploratory initial state distribution $\eta' > 0$ like in (Agarwal et al., 2020), and our result still holds. This lemma allows us to establish connection between the gradient norm and optimality gap, giving rise to the following theorem.

**Theorem 5** Suppose Assumptions 1,2 and 4–5 hold. Consider Algorithm 1 with softmax policy and linear critic function $\hat{V}_\omega(s) = \phi(s)^\top \omega$. Select step size $\alpha = K^{-\frac{2}{3}}$, $\beta = K^{-\frac{2}{3}}$ and let $K_0 = \Theta(N) = O(K^{\frac{2}{3}})$, then it holds for i.i.d. sampling

$$J^* - \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[J(\theta_k)] = \mathcal{O} \left( \lambda^{-2}K^{-\frac{2}{3}} \right) + \epsilon_\lambda, \quad (24a)$$

and for Markovian sampling

$$J^* - \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[J(\theta_k)] = \tilde{\mathcal{O}} \left( \lambda^{-2}K_0K^{-\frac{2}{3}} \right) + \epsilon_\lambda. \quad (24b)$$

5. Numerical Experiments

We test the impact of separate sampling and the speedup property of A3C in both synthetically generated and Gym environments. The tests on synthetic environment were performed in a 16-core CPU computer, and those on Atari games were run in a 4 GPU computer.
5.1. Separate sampling protocol

We compare the separate chain sampling method in Algorithm 1 with the shared chain sampling method. The shared chain method is simply using the same sample for both actor and critic, i.e., setting \( \hat{x}_t = x_t \) in Algorithm 1.

To clearly demonstrate the impact of sampling, we mitigate the impact from other sources such as delay and MDP non-ergodicity by considering a synthetic environment with 1 worker. In this test, we use the tabular softmax policy parameterization. The synthetic MDP has a state space \( |S| = 10 \), an discrete action space of \( |A| = 4 \). State features each has a dimension of 10. Elements of the transition matrix, the reward and the state features are randomly sampled from a uniform distribution over \( (0, 1) \).

It can be clearly observed from Figure 2 that using the same sample for actor and critic leads to an asymptotic error scaled with choice of \( \gamma \). The intuitive explanation is in the caption of Figure 2. Although when \( \gamma \to 1 \), the error is small (but still exists), we want our algorithm design to not restrict the choice of \( \gamma \), and thus adopt the separate chain sampling method.

5.2. Linear speedup

**Experiment settings.** For the synthetic environment, we used linear value function approximation and tabular softmax policy (Agarwal et al., 2020). For CartPole, we used a 3-layer MLP with 128 neurons and sigmoid activation function in each layer. The first two layers are shared for both actor and critic network. For the Atari games, we used a convolution-LSTM network. For network details, see (Dgriff, 2018).

For the separate sampling protocol test, we have \( \alpha = 0.6 \) and critic step size \( \beta = 0.7 \), along with \( \lambda = 0.3 \). For the speedup tests in synthetic environment, we set actor step size \( \alpha_k = \frac{0.05}{(1+k)^{0.5}} \) and critic step size \( \beta_k = \frac{0.05}{(1+k)^{0.5}} \). In tests of CartPole, we run Algorithm 1 with a minibatch of 20 samples. We update the actor network with a step size of \( \alpha_k = \frac{0.01}{(1+k)^{0.6}} \) and critic network with a step size of \( \beta_k = \frac{0.01}{(1+k)^{0.6}} \). See Table 1 for hyper-parameters in Atari game tests.
Figure 8: Speedup of A3C in OpenAI Gym Atari game (Pong).

| Hyper-parameters               | Value       |
|-------------------------------|-------------|
| Number of workers             | 1,2,4,8,16  |
| Optimizer                     | Adam        |
| Step size                     | 0.00015     |
| Batch size                    | 20          |
| Discount factor               | 0.99        |
| Entropy coefficient           | 0.01        |
| Frame size                    | $80 \times 80$ |
| Frame skip rate               | 4           |
| Grayscaling                   | Yes         |
| Training reward clipping      | [-1,1]      |

Table 1: Hyper-parameters of A3C in the Atari games.

**Synthetic environment.** We first test the speedup property of A3C in a synthetic environment with $|S| = 100$, $|A| = 5$ and state feature with dimension 10. The reward and transition matrix of the MDP are randomly generated in the same way as that in section 5.1. We evaluate the convergence of actor in terms of the average reward and the critic in terms of the gap $\|\omega_k - \omega_{\theta_k}\|_2$.

Figures 3 and 4 show the training time and sample complexity of running A3C with i.i.d. sampling and Markovian sampling respectively. The speedup plot is measured by the number of samples needed to achieve a target running average reward under different number of workers. All the results are average over 10 Monte-Carlo runs. Figure 3 shows that the sample complexity of A3C stays the same with different number of workers under i.i.d. sampling. Also, it can be observed from the speedup plot of Figure 3 that the A3C achieves roughly linear speedup, which is consistent with Corollary 1. The speedup of A3C with Markovian sampling shown in Figure 4 is roughly linear when number of workers is small.

**OpenAI Gym environments.** We also test the speedup property of A3C with neural network parametrization in the classic control (Carpoles) and the Atari (Breakout and Pong) environments. In Figures 5-8, each curve was averaged over 5 Monte-Carlo runs with 95% confidence interval. Figures 5-8 show the speedup of A3C under different number of workers, where the average reward is computed by taking the running average of test rewards. The speedup is respectively measured by the number of samples and training time needed per worker to achieve a target average reward. Although not justified theoretically, Figures 5-8 suggest that the sample complexity speedup is roughly linear, and the runtime speedup slightly degrades when the number of workers increases.
This is partially due to hardware limit. Similar observation was obtained in async-SGD (Lian et al., 2016, Fig. 4).

6. Conclusions

This paper revisits the A3C algorithm. With linear value function approximation, the convergence of the A3C algorithm has been established under both i.i.d. and Markovian sampling settings. Under i.i.d. sampling, A3C achieves linear speedup compared to the best-known sample complexity of AC, theoretically justifying the benefit of parallelism and asynchrony for the first time. Under Markov sampling, such a linear speedup can be observed in most benchmark tasks. One limitation of this paper is that theoretical linear speedup cannot be established in the Markovian setting. This motivates two interesting directions: i) developing new tools of analyzing two-timescale SGD with Markov sampling; and, ii) designing better algorithms than A3C to achieve better speedup.

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Supplementary Material

Appendix A. Preliminary Lemmas

A.1. Geometric mixing

The operation $p \otimes q$ denotes the product between two distributions $p(x)$ and $q(y)$, i.e. $(p \otimes q)(x, y) = p(x) \cdot q(y)$.

**Lemma 2** Suppose Assumption 4 holds. For any $\theta \in \mathbb{R}^d$, we have

$$\sup_{s_0 \in S} d_{TV}(\mathbb{P}((s_t, a_t, s_{t+1}) \in \cdot | s_0, \pi_{\theta}), \mu_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P}) \leq \kappa \rho^t. \quad (25a)$$

and

$$\sup_{s_0 \in S} d_{TV}(\mathbb{P}((\hat{s}_t, \hat{a}_t, s'_{t+1}) \in \cdot | s_0, \pi_{\theta}), d_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P}) \leq \kappa \rho^t. \quad (25b)$$

where $(s_t, a_t, s_{t+1})$ is the $t$th transition on the Markov chain with transition kernel $\mathcal{P}$. $(\hat{s}_t, \hat{a}_t)$ is the $t$th state-action pair on Markov chain with transition kernel $\hat{\mathcal{P}}$, and $s'_{t+1} \sim \mathcal{P}(\cdot | \hat{s}_t, \hat{a}_t)$.

**Proof** We start with

$$\sup_{s_0 \in S} d_{TV}(\mathbb{P}((s_t, a_t, s_{t+1}) = \cdot | s_0, \pi_{\theta}), \mu_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P})$$

$$= \sup_{s_0 \in S} d_{TV}(\mathbb{P}(s_t = \cdot | s_0, \pi_{\theta}) \otimes \pi_{\theta} \otimes \mathcal{P}, \mu_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P})$$

$$= \sup_{s_0 \in S} \frac{1}{2} \int_{s \in S} \sum_{a \in A} \int_{s' \in S} |\mathbb{P}(s_t = ds|s_0, \pi_{\theta})\pi_{\theta}(a|s)\mathcal{P}(ds'|s, a) - \mu_{\theta}(ds)\pi_{\theta}(a|s)\mathcal{P}(ds'|s, a)|$$

$$= \sup_{s_0 \in S} \frac{1}{2} \int_{s \in S} \left|\mathbb{P}(s_t = ds|s_0, \pi_{\theta}) - \mu_{\theta}(ds)\right| \sum_{a \in A} \pi_{\theta}(a|s) \int_{s' \in S} \mathcal{P}(ds'|s, a)$$

$$= \sup_{s_0 \in S} d_{TV}(\mathbb{P}(s_t = \cdot | s_0, \pi_{\theta}), \mu_{\theta})$$

$$\leq \kappa \rho^t. \quad (26)$$

Inequality (25a) along with the fact that the stationary distribution of the Markov chain with transition probability $\hat{\mathcal{P}}$ and policy $\pi_{\theta}$ is simply $d_{\theta}$ immediately implies (25b). This completes the proof. ■

For the use in the later proof, given $K > 0$, we first define $m_K$ as:

$$m_K := \min \{ m \in \mathbb{N}^+ \mid \kappa \rho^{m-1} \leq \min\{\alpha, \beta\} \}, \quad (27)$$

where $\kappa$ and $\rho$ are constants defined in (4). $m_K$ is the minimum number of samples needed for the Markov chain to approach the stationary distribution so that the bias incurred by the Markovian sampling is small enough.
A.2. Auxiliary Markov chain

The auxiliary Markov chain is a virtual Markov chain with no policy drifting — a technique developed in (Zou et al., 2019) to analyze stochastic approximation algorithms in non-stationary settings. We provide an analysis here for completeness.

**Lemma 3** Under Assumption 2 and Assumption 3, consider the update (13) in Algorithm 1 with Markovian sampling. For a given number of samples \( m \), consider the Markov chain of the worker that contributes to the \( k \)th update:

\[
\begin{align*}
    s_{t-m} &\xrightarrow{\theta_{k-d_m}} a_{t-m} \xrightarrow{\mathcal{P}} s_{t-m+1} \\
    &\xrightarrow{\theta_{k-d_{m-1}}} a_{t-m+1} \cdots s_{t-1} \xrightarrow{\theta_{k-d_{1}}} a_{t-1} \xrightarrow{\mathcal{P}} s_t \xrightarrow{\theta_{k-d_0}} a_t \xrightarrow{\mathcal{P}} s_{t+1},
\end{align*}
\]

where \((s_t, a_t, s_{t+1}) = (s(k), a(k), s'(k))\), and \(\{d_j\}_{j=0}^m\) is some increasing sequence with \(d_0 := \tau_k\).

Given \((s_{t-m}, a_{t-m}, s_{t-m+1})\) and \(\theta_{k-d_m}\), we construct its auxiliary Markov chain by repeatedly applying \(\pi_{\theta_{k-d_m}}\):

\[
\begin{align*}
    s_{t-m} &\xrightarrow{\theta_{k-d_m}} a_{t-m} \xrightarrow{\mathcal{P}} s_{t-m+1} \\
    &\xrightarrow{\theta_{k-d_{m-1}}} a_{t-m+1} \cdots s_{t-1} \xrightarrow{\theta_{k-d_{1}}} a_{t-1} \xrightarrow{\mathcal{P}} s_t \xrightarrow{\theta_{k-d_0}} a_t \xrightarrow{\mathcal{P}} s_{t+1}.
\end{align*}
\]

Then we have:

\[
\begin{align*}
    d_{TV} \left(\mathbb{P}\left((s_t, a_t) \in \cdot | \theta_{k-d_m}, s_{t-m+1}\right), \mathbb{P}\left((\tilde{s}_t, \tilde{a}_t) \in \cdot | \theta_{k-d_m}, s_{t-m+1}\right)\right) \\
    \leq \frac{1}{2} |A| L_{\pi} \sum_{t=\tau_k}^{d_m} \mathbb{E} \left[ ||\theta_{k-1} - \theta_{k-d_m}||_2 | \theta_{k-d_m}, s_{t-m+1}\right].
\end{align*}
\]

**Proof** Throughout the lemma, all expectations and probabilities are conditioned on \(\theta_{k-d_m}\) and \(s_{t-m+1}\). We omit this condition for convenience.

With \(\tilde{x}_t := (\tilde{s}_t, \tilde{a}_t, \tilde{s}_{t+1})\), first we have

\[
\begin{align*}
    d_{TV} \left(\mathbb{P}(s_{t+1} \in \cdot), \mathbb{P}(\tilde{s}_{t+1} \in \cdot)\right) &\leq \frac{1}{2} \int_{s' \in S} \left| \mathbb{P}(s_{t+1} = ds') - \mathbb{P}(\tilde{s}_{t+1} = ds') \right| \\
    &\leq \frac{1}{2} \int_{s' \in S} \left| \sum_{a \in A} \mathbb{P}(s_t = ds, a_t = a, s_{t+1} = ds') - \mathbb{P}(\tilde{s}_t = ds, \tilde{a}_t = a, \tilde{s}_{t+1} = ds') \right| \\
    &\leq \frac{1}{2} \int_{s' \in S} \left| \sum_{a \in A} \mathbb{P}(s_t = ds, a_t = a, s_{t+1} = ds') - \mathbb{P}(\tilde{s}_t = ds, \tilde{a}_t = a, \tilde{s}_{t+1} = ds') \right| \\
    &\leq \frac{1}{2} \int_{s' \in S} \sum_{a \in A} \int_{s'' \in S} \left| \mathbb{P}(s_t = ds, a_t = a, s_{t+1} = ds') - \mathbb{P}(\tilde{s}_t = ds, \tilde{a}_t = a, \tilde{s}_{t+1} = ds') \right| \\
    &\leq d_{TV} \left(\mathbb{P}(x_t \in \cdot), \mathbb{P}(\tilde{x}_t \in \cdot)\right),
\end{align*}
\]

(29)
where the second last equality is due to Tonelli’s theorem. Next we have

\[ d_{TV}(\mathbb{P}(x_t \in \cdot), \mathbb{P}(\overline{x}_t \in \cdot)) \]

\[ = \frac{1}{2} \int_{s \in S} \sum_{a \in A} \int_{s' \in S} \left| \mathbb{P}(s_t = ds, a_t = a, s_{t+1} = ds') - \mathbb{P}(\overline{s}_t = ds, \overline{a}_t = a, \overline{s}_{t+1} = ds') \right| \]

\[ = \frac{1}{2} \int_{s \in S} \sum_{a \in A} \left| \mathbb{P}(s_t = ds, a_t = a) - \mathbb{P}(\overline{s}_t = ds, \overline{a}_t = a) \right| \int_{s' \in S} \mathbb{P}(s_{t+1} = ds'|s_t = ds, a_t = a) \]

\[ = \frac{1}{2} \int_{s \in S} \sum_{a \in A} \left| \mathbb{P}(s_t = ds, a_t = a) - \mathbb{P}(\overline{s}_t = ds, \overline{a}_t = a) \right| \]

\[ = d_{TV}(\mathbb{P}((s_t, a_t) \in \cdot), \mathbb{P}((\overline{s}_t, \overline{a}_t) \in \cdot)). \] (30)

Due to the fact that \( \theta_{k-r_k} \) is dependent on \( s_t \), we need to write \( \mathbb{P}(s_t, a_t) \) as

\[ \mathbb{P}(s_t, a_t) = \int_{\theta_{k-r_k} \in \mathbb{R}^d} \mathbb{P}(s_t, \theta_{k-r_k}, a_t) \]

\[ = \int_{\theta \in \mathbb{R}^d} \mathbb{P}(s_t) \mathbb{P}(\theta_{k-r_k} = d\theta | s_t) \pi_{\theta_{k-r_k}}(a_t | s_t) \]

\[ = \mathbb{P}(s_t) \mathbb{E}[\pi_{\theta_{k-r_k}}(a_t | s_t)]. \] (31)

Then we have

\[ d_{TV}(\mathbb{P}((s_t, a_t) \in \cdot), \mathbb{P}((\overline{s}_t, \overline{a}_t) \in \cdot)) \]

\[ = \frac{1}{2} \int_{s \in S} \sum_{a \in A} \left| \mathbb{P}(s_t = ds) \mathbb{E}[\pi_{\theta_{k-r_k}}(a_t = a | s_t = ds)] \mathbb{E}[s_t = ds] - \mathbb{P}(\overline{s}_t = ds) \pi_{\theta_{k-r_m}}(\overline{a}_t = a | \overline{s}_t = ds) \right| \]

\[ \leq \frac{1}{2} \int_{s \in S} \sum_{a \in A} \left| \mathbb{P}(s_t = ds) \mathbb{E}[\pi_{\theta_{k-r_k}}(a_t = a | s_t = ds)] \mathbb{E}[s_t = ds] - \mathbb{P}(s_t = ds) \pi_{\theta_{k-r_m}}(a_t = a | s_t = ds) \right| \]

\[ + \frac{1}{2} \int_{s \in S} \sum_{a \in A} \left| \mathbb{P}(s_t = ds) \pi_{\theta_{k-r_m}}(\overline{a}_t = a | \overline{s}_t = ds) - \mathbb{P}(\overline{s}_t = ds) \pi_{\theta_{k-r_m}}(\overline{a}_t = a | \overline{s}_t = ds) \right| \]

\[ = \frac{1}{2} \int_{s \in S} \mathbb{P}(s_t = ds) \sum_{a \in A} \left| \mathbb{E}[\pi_{\theta_{k-r_k}}(a_t = a | s_t = ds)] \mathbb{E}[s_t = ds] - \pi_{\theta_{k-r_m}}(a_t = a | s_t = ds) \right| \]

\[ + \frac{1}{2} \int_{s \in S} \left| \mathbb{P}(s_t = ds) - \mathbb{P}(\overline{s}_t = ds) \right|. \] (32)

Using Jensen’s inequality, we have

\[ d_{TV}(\mathbb{P}((s_t, a_t) \in \cdot), \mathbb{P}((\overline{s}_t, \overline{a}_t) \in \cdot)) \]

\[ \leq \frac{1}{2} \int_{s \in S} \mathbb{P}(s_t = ds) \sum_{a \in A} \mathbb{E} \left[ \left| \pi_{\theta_{k-r_k}}(a_t = a | s_t = ds) - \pi_{\theta_{k-r_m}}(a_t = a | s_t = ds) \right| \right] \mathbb{E}[s_t = ds] \]

\[ + \frac{1}{2} \int_{s \in S} \left| \mathbb{P}(s_t = ds) - \mathbb{P}(\overline{s}_t = ds) \right| \]

\[ \leq \frac{1}{2} \int_{s \in S} \mathbb{P}(s_t = ds) \sum_{a \in A} \mathbb{E} \left[ \left| \theta_{k-r_k} - \theta_{k-r_m} \right| s_t = ds \right] + \frac{1}{2} \int_{s \in S} \left| \mathbb{P}(s_t = ds) - \mathbb{P}(\overline{s}_t = ds) \right| \]

\[ = \frac{1}{2} |A| L_x \mathbb{E} \left[ \left| \theta_{k-r_k} - \theta_{k-r_m} \right| s_t = ds \right] + d_{TV}(\mathbb{P}(s_t \in \cdot), \mathbb{P}(\overline{s}_t \in \cdot)) \] (33)
We first give a proposition regarding the which completes the proof.

Suppose Assumption 1, 3 and 4 hold. For any Proposition 2

Since \(d TV(\mathbb{P}(s, a) \in \cdot), \mathbb{P}(s, \tilde{a} \in \cdot)) = 0\), recursively applying (34) for \(\{t - 1, \ldots, t - m\}\) gives

\[
\begin{align*}
    d TV(\mathbb{P}(s, a) \in \cdot), \mathbb{P}(s, \tilde{a} \in \cdot)) &\leq \frac{1}{2} |A| L_\pi \sum_{j=0}^{m} \mathbb{E}\|\theta_{k-d_j} - \theta_{k-d_m}\|_2 \\
    &\leq \frac{1}{2} |A| L_\pi \sum_{i=\tau_k}^{d_m} \mathbb{E}\|\theta_{k-i} - \theta_{k-d_m}\|_2,
\end{align*}
\]

which completes the proof.

A.3. Lipschitz continuity of actor and critic

We first give a proposition regarding the \(L_\lambda\)-Lipschitz continuity of the regularized policy gradient under proper assumptions, which has been shown by (Zhang et al., 2019; Agarwal et al., 2020).

**Proposition 1** Suppose Assumption 3 hold. For any \(\theta, \theta' \in \mathbb{R}^d\), we have \(\|\nabla J_\lambda(\theta) - \nabla J_\lambda(\theta')\|_2 \leq L_\lambda\|\theta - \theta'\|_2\), where \(L_\lambda\) is a positive constant.

We provide a justification for Lipschitz continuity of \(\omega_\theta^*\) in the next proposition.

**Proposition 2** Suppose Assumption 1, 3 and 4 hold. For any \(\theta_1, \theta_2 \in \mathbb{R}^d\), we have

\[
\|\omega_{\theta_1} - \omega_{\theta_2}^*\|_2 \leq L_\omega\|\theta_1 - \theta_2\|_2,
\]

where \(L_\omega := 2r_{\max} |A| L_\pi (\lambda^{-1} + \lambda^{-2}(1 + \gamma))(1 + \log \rho \kappa^{-1} + (1 - \rho)^{-1}).\)

**Proof** We use \(A_1, A_2, b_1\) and \(b_2\) as shorthand notations of \(A_{\pi_{\theta_1}}, A_{\pi_{\theta_2}}, b_{\pi_{\theta_1}}\) and \(b_{\pi_{\theta_2}}\) respectively. By Assumption 1, \(A_{\theta, \phi}\) is invertible for any \(\theta \in \mathbb{R}^d\), so we can write \(\omega_{\theta}^* = -A_{\theta, \phi}^{-1}b_{\theta, \phi}\). Then we have

\[
\|\omega_1^* - \omega_2^*\|_2 = \| - A_1^{-1}b_1 + A_2^{-1}b_2\|_2 \\
= \| - A_1^{-1}b_1 - A_1^{-1}b_2 + A_1^{-1}b_2 - A_2^{-1}b_2\|_2 \\
= \| - A_1^{-1}(b_1 - b_2) - (A_1^{-1} - A_2^{-1})b_2\|_2 \\
\leq \|A_1^{-1}(b_1 - b_2)\|_2 + \|(A_1^{-1} - A_2^{-1})b_2\|_2 \\
\leq \|A_1^{-1}\|_2 \|b_1 - b_2\|_2 + \|A_1^{-1} - A_2^{-1}\|_2 \|b_2\|_2 \\
= \|A_1^{-1}\|_2 \|b_1 - b_2\|_2 + \|A_1^{-1}(A_2 - A_1)A_2^{-1}\|_2 \|b_2\|_2 \\
\leq \|A_1^{-1}\|_2 \|b_1 - b_2\|_2 + \|A_1^{-1}\|_2 \|b_2\|_2 \|A_2 - A_1\|_2 \\
\leq \lambda^{-1} \|b_1 - b_2\|_2 + \lambda^{-2} r_{\max} \|A_1 - A_2\|_2,
\]

(36)
We first define the exact TD update as:
\[ C \]
We also define constant \( \tau \) where the last inequality follows Assumption 1, and the fact that
\[
\|b_2\|_2 = \left\| \mathbb{E}[r(s, a, s')\phi(s)] \right\|_2 \leq \mathbb{E} \left\| r(s, a, s')\phi(s) \right\|_2 \leq \mathbb{E} \left[ |r(s, a, s')||\phi(s)||_2 \right] \leq r_{\text{max}}.
\]

Denote \((s^1, a^1, s'^1)\) and \((s^2, a^2, s'^2)\) as samples drawn with \( \theta_1 \) and \( \theta_2 \) respectively, i.e. \( s^1 \sim \mu_{\theta_1}, \ a^1 \sim \pi_{\theta_1}, \ s'^1 \sim \mathcal{P} \) and \( s^2 \sim \mu_{\theta_2}, \ a^2 \sim \pi_{\theta_2}, \ s'^2 \sim \mathcal{P}. \) Then we have
\[
\|b_1 - b_2\|_2 = \left\| \mathbb{E}[r(s^1, a^1, s'^1)\phi(s^1)] - \mathbb{E}[r(s^2, a^2, s'^2)\phi(s^2)] \right\|_2 \\
\leq \sup_{s,a,s'} \|r(s, a, s')\phi(s)\|_2 \|\mathbb{P}((s^1, a^1, s'^1) \in \cdot) - \mathbb{P}((s^2, a^2, s'^2) \in \cdot)\|_{TV} \\
\leq r_{\text{max}} \|\mathbb{P}((s^1, a^1, s'^1) \in \cdot) - \mathbb{P}((s^2, a^2, s'^2) \in \cdot)\|_{TV} \\
= 2r_{\text{max}}d_{TV}(\mu_{\theta_1} \otimes \pi_{\theta_1} \otimes \mathcal{P}, \mu_{\theta_2} \otimes \pi_{\theta_2} \otimes \mathcal{P}) \\
\leq 2r_{\text{max}}|A|L_\pi(1 + \log_\rho \kappa^{-1} + (1 - \rho)^{-1})\|\theta_1 - \theta_2\|_2, \quad (37)
\]
where the first inequality follows the definition of the total variation (TV) norm, and the last inequality follows Lemma A.1. in (Wu et al., 2020). Similarly we have:
\[
\|A_1 - A_2\|_2 \leq 2(1 + \gamma)d_{TV}(\mu_{\theta_1} \otimes \pi_{\theta_1}, \mu_{\theta_2} \otimes \pi_{\theta_2}) \\
= (1 + \gamma)|A|L_\pi(1 + \log_\rho \kappa^{-1} + (1 - \rho)^{-1})\|\theta_1 - \theta_2\|_2. \quad (38)
\]
Substituting (37) and (38) into (36) completes the proof.

\[ \textbf{Appendix B. Proof of Main Theorems} \]

\[ \textbf{B.1. Proof of Theorem 1} \]

We first define the exact TD update as:
\[
\bar{g}(x, \omega) := \mathbb{E}_{s \sim \mu_{\theta}, a \sim \pi_{\theta}, s' \sim \mathcal{P}}[g(x, \omega)]. \quad (39)
\]

We also define constant \( C_\delta := r_{\text{max}} + (1 + \gamma)\max\{r_{\text{max}}/(1 - \gamma), R_{\omega}\}, \) and we immediately have
\[
\|g(x, \omega)\|_2 \leq |r(x) + \gamma\phi(s')^\top \omega - \phi(s)^\top \omega| \leq r_{\text{max}} + (1 + \gamma)R_{\omega} \leq C_\delta \quad (40)
\]
and likewise, we have \( \|\bar{g}(x, \omega)\|_2 \leq C_\delta. \)

The critic update in Algorithm 1 can be written as:
\[
\omega_{k+1} = \Pi_{R_{\omega}}(\omega_k + \beta \bar{g}(x_{(k)}, \omega_{k-\tau_k})), \quad (41)
\]
where \( \tau_k \) is the delay of the parameters used in evaluating the \( k \)th stochastic gradient, and \( x_{(k)} := (s_{(k)}, a_{(k)}, s'_{(k)}) \) is the sample used to evaluate the stochastic gradient at \( k \)th update.
Proof Using $\omega_k^*$ as shorthand notation of $\omega_{\theta_k}^*$, we start with the optimality gap
\[
\|\omega_{k+1} - \omega_{k+1}^*\|_2^2
= \|\Pi_{R_{\omega_k}}(\omega_k + \beta g(x_k, \omega_{k-\tau_k})) - \omega_{k+1}^*\|_2^2
\leq \|\omega_k + \beta g(x_k, \omega_{k-\tau_k}) - \omega_{k+1}^*\|_2^2
\]
\[
= \|\omega_k - \omega_k^*\|_2^2 + 2\beta \langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) \rangle + 2 \langle \omega_k - \omega_k^*, \omega_k^* - \omega_{k+1}^* \rangle
+ \|\omega_k^* - \omega_{k+1}^* + \beta g(x_k, \omega_{k-\tau_k})\|_2^2
\]
\[
= \|\omega_k - \omega_k^*\|_2^2 + 2\beta \langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) \rangle
+ 2 \langle \omega_k - \omega_k^*, \omega_k^* - \omega_{k+1}^* \rangle + 2 \|\omega_k^* - \omega_{k+1}^*\|_2^2 + 2C_0^2\beta^2. \tag{42}
\]
The second term in (42) can be decomposed as
\[
\langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) \rangle = \langle \omega_k - \omega_k^*, \overline{g}(\theta_k, \omega_k) \rangle + \langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) - g(x_k, \omega_k) \rangle
+ \langle \omega_k - \omega_k^*, g(x_k, \omega_k) - \overline{g}(\theta_k, \omega_k) \rangle. \tag{43}
\]
We first bound $\langle \omega_k - \omega_k^*, \overline{g}(\theta_k, \omega_k) \rangle$ in (43) as
\[
\langle \omega_k - \omega_k^*, \overline{g}(\theta_k, \omega_k) \rangle = \langle \omega_k - \omega_k^*, \overline{g}(\theta_k, \omega_k) - g(\theta_k, \omega_k) \rangle
\leq -\lambda \|\omega_k - \omega_k^*\|_2^2; \tag{44}
\]
where the first equality is due to $\overline{g}(\theta, \omega_0^*) = A_{\theta, \phi} \omega_0^* + b = 0$, and the last inequality follows Assumption 1.

We then bound the term $\langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) - g(x_k, \omega_k) \rangle$ in (43) as
\[
\langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) - g(x_k, \omega_k) \rangle = \langle \omega_k - \omega_k^*, (\gamma \phi(s') - \phi(s))\phi(s) \rangle \end{align}
\[
\leq (1 + \gamma)\|\omega_k - \omega_k^*\|_2 \|\omega_{k-\tau_k} - \omega_k\|_2
\leq (1 + \gamma)\|\omega_k - \omega_k^*\|_2 \left\| \sum_{i=k-\tau_k}^{k-1} (\omega_{i+1} - \omega_i) \right\|_2
\leq (1 + \gamma)\|\omega_k - \omega_k^*\|_2 \sum_{i=k-\tau_k}^{k-1} \beta \|g(x_i, \omega_{i-\tau_k})\|_2
\leq 2C_0K_0\beta \|\omega_k - \omega_k^*\|_2, \tag{45}
\]
where the last inequality follows the definition of $C_\delta$ in (40). Substituting (45) and (44) into (43) gives
\[
\langle \omega_k - \omega_k^*, g(x_k, \omega_{k-\tau_k}) \rangle \leq -\lambda \|\omega_k - \omega_k^*\|_2^2 + 2C_0K_0\beta \|\omega_k - \omega_k^*\|_2
+ \langle \omega_k - \omega_k^*, g(x_k, \omega_k) - \overline{g}(\theta_k, \omega_k) \rangle. \tag{46}
\]
Next we jointly bound the third and fourth term in (42) as
\[
\langle \omega_k - \omega_k^*, \omega_k^* - \omega_{k+1}^* \rangle + \| \omega_k^* - \omega_{k+1}^* \|_2^2
\leq \| \omega_k - \omega_k^* \|_2 \| \omega_k^* - \omega_{k+1}^* \|_2 + \| \omega_k^* - \omega_{k+1}^* \|_2
\leq 2L_\omega \| \omega_k - \omega_k^* \|_2 \| \theta_k - \theta_{k+1} \|_2 + 2L_\omega^2 \| \theta_k - \theta_{k+1} \|_2^2
\leq 2L_\omega C_p \alpha \| \omega_k - \omega_k^* \|_2 + 2L_\omega^2 C_p^2 \alpha^2,
\]
where constant $C_p := C_\delta C_\psi + \lambda C_\psi$. The second inequality is due to the $L_\omega$-Lipschitz continuity of $\omega_k^*$ shown in Proposition 2, and the last inequality follows the fact that
\[
\| \theta_k - \theta_{k+1} \|_2 = \alpha \| \hat{\delta}(x(k), \omega_{k-r_k}) \psi_{\theta_{k-r_k}}(s(k), a(k)) + \lambda \psi_{\theta_{k-r_k}}(x_{k}) \|_2 \leq \alpha C_p.
\]
(47)

Substituting (46) and (47) into (42), and taking expectation on both sides yield
\[
\mathbb{E} \| \omega_{k+1} - \omega_{k+1}^* \|_2^2 \leq (1 - 2\lambda \beta) \mathbb{E} \| \omega_k - \omega_k^* \|_2^2 + 2\beta (C_1 \alpha + C_2 K_0 \beta) \mathbb{E} \| \omega_k - \omega_k^* \|_2
\leq 2\beta \mathbb{E} \langle \omega_k - \omega_k^*, g(x(k), \omega_k) - \mathcal{G}(\theta_k, \omega_k) \rangle + C_q \beta^2,
\]
where $C_1 := L_\omega C_p, C_2 := 2C_\delta$ and $C_q := 2C_\delta^2 + 2L_\omega^2 C_p^2 \alpha^2$.

For brevity, we use $x \sim \theta$ to denote $s \sim \mu_\theta, a \sim \pi_\theta$ and $s' \sim \mathcal{P}$ in this proof. Consider the third term in (49) conditioned on $\theta_k, \omega_k, \theta_{k-r_k}$. We bound it as
\[
\mathbb{E} \left[ \langle \omega_k - \omega_k^*, g(x(k), \omega_k) - \mathcal{G}(\theta_k, \omega_k) \rangle \right| \theta_k, \omega_k, \theta_{k-r_k}
\\leq \| \omega_k - \omega_k^* \|_2 \| \mathcal{G}(\theta_k, \omega_k) - \mathcal{G}(\theta_k, \omega_k) \|_2
\leq 2R_\omega \mathbb{E}_{x \sim \theta_{k-r_k}} \| g(x, \omega_k) \|_2 \| \mu_{\theta_{k-r_k}} \otimes \pi_{\theta_{k-r_k}} \otimes \mathcal{P} - \mu_{\theta_k} \otimes \pi_{\theta_k} \otimes \mathcal{P} \|_{TV}
\leq 4R_\omega \mathcal{C}_d \mathcal{C}_\psi \| \mu_{\theta_{k-r_k}} \otimes \pi_{\theta_{k-r_k}} \otimes \mathcal{P} - \mu_{\theta_k} \otimes \pi_{\theta_k} \otimes \mathcal{P} \|_{TV}
\leq 4R_\omega \mathcal{C}_d \mathcal{C}_\psi \| \mu_{\theta_{k-r_k}} \otimes \pi_{\theta_{k-r_k}} \otimes \mathcal{P} - \mu_{\theta_k} \otimes \pi_{\theta_k} \otimes \mathcal{P} \|_{TV},
\]
(50)

where second last inequality follows the definition of TV norm and the last inequality uses the definition of $C_\delta$ in (40).

Define constant $C_3 := 2R_\omega C_\delta |\mathcal{A}| L_\pi (1 + \log_p \kappa^{-1} + (1 - \rho)^{-1})$. Then by following the third term in (Wu et al., 2020, Lemma A.1), we can write (50) as
\[
\mathbb{E} \left[ \langle \omega_k - \omega_k^*, g(x(k), \omega_k) - \mathcal{G}(\theta_k, \omega_k) \rangle \right| \theta_k, \omega_k, \theta_{k-r_k}
\leq 4R_\omega \mathcal{C}_d \mathcal{C}_\psi \| \mu_{\theta_{k-r_k}} \otimes \pi_{\theta_{k-r_k}} \otimes \mathcal{P} - \mu_{\theta_k} \otimes \pi_{\theta_k} \otimes \mathcal{P} \|_{TV}
\leq C_3 \| \theta_{k-r_k} - \theta_k \|_2
\leq C_3 \sum_{i=k-r_k}^{k-1} \alpha \| g(x_i, \omega_{i-r_k}) \|_2
\leq C_3 C_\delta K_0 \alpha,
\]
(51)
We first define the ‘optimal’ TD target as:

\[ \omega^* \]

This completes the proof.

The update in Algorithm 1 can be written as:

\[ \text{convergence proof.} \]

For brevity, we use \[ \lambda \]

Taking total expectation on both sides of (51) and substituting it into (49) yield

\[
E\|\omega_{k+1} - \omega_{k+1}^*\|_2^2 \leq (1 - 2\lambda \beta)E\|\omega_k - \omega_k^*\|_2^2 + 2\beta \left( \alpha \frac{C_1}{\beta} + C_2K_0\beta \right)E\|\omega_k - \omega_k^*\|_2^2 + 2C_3C_4K_0\beta \alpha + C_4\beta^2. \]

(52)

which along with the fact \[ \alpha = \frac{1}{(K+1)^\delta} \] and \[ \beta = \frac{1}{(K+1)^\delta} \] implies

\[
\frac{1}{K} \sum_{k=1}^{K} E\|\omega_k - \omega_k^*\|_2^2 \leq O \left( \frac{K^2}{K^\delta} \right) + O \left( \frac{K_0}{K^\delta} \right) + O \left( \frac{1}{K^\delta} \right). \]

(53)

This completes the proof.

B.2. Proof of Theorem 2

We first define the ‘optimal’ TD target as:

\[ \delta(x, \theta) := r(s, a, s') + \gamma V_{\pi_\theta}(s') - V_{\pi_\theta}(s). \]

The update in Algorithm 1 can be written as:

\[ \theta_{k+1} = \theta_k + \alpha \left( \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) + \lambda \psi_{\theta_{k-\tau_k}}(x^p(k)) \right). \]

(54)

For brevity, we use \[ \omega_{k}^* \] as shorthand notation of \[ \omega_{\theta_k}^* \] in this proof. Then we are ready to give the convergence proof.

**Proof** From \[ L_\lambda \]-Lipschitz continuity of regularized policy gradient shown in Proposition 1, we have:

\[
J_\lambda(\theta_{k+1}) \geq J_\lambda(\theta_k) + \langle \nabla J_\lambda(\theta_k), \theta_{k+1} - \theta_k \rangle - \frac{L_\lambda}{2} \| \theta_{k+1} - \theta_k \|_2^2
\]

\[
= J_\lambda(\theta_k) + \alpha \left( \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) - \hat{\delta}(\hat{x}(k), \omega_k^*) \right) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) \right)
\]

\[
+ \alpha \left( \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) + \lambda \psi_{\theta_{k-\tau_k}}(x^p(k)) \right) - \frac{L_\lambda}{2} \| \theta_{k+1} - \theta_k \|_2^2
\]

\[
\geq J_\lambda(\theta_k) + \alpha \left( \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) - \hat{\delta}(\hat{x}(k), \omega_k^*) \right) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) \right)
\]

\[
+ \alpha \left( \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) + \lambda \psi_{\theta_{k-\tau_k}}(x^p(k)) \right) - \frac{L_\lambda}{2} C_p^2 \alpha^2,
\]

where the last inequality follows the definition of \[ C_p \] in (48).

Taking expectation on both sides of the last inequality yields

\[
E[J_\lambda(\theta_{k+1})] \geq E[J_\lambda(\theta_k)] + \alpha E \left[ \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) - \hat{\delta}(\hat{x}(k), \omega_k^*) \right) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) \right]
\]

\[
+ \alpha E \left[ \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \omega_{k-\tau_k}) \psi_{\theta_{k-\tau_k}}(\hat{s}(k), \hat{a}(k)) + \lambda \nabla R(\theta_{k-\tau_k}) \right] - \frac{L_\lambda}{2} C_p^2 \alpha^2.
\]

(55)
where we used the fact that $\mathbb{E}[\psi_{\theta_k-\tau_k}(x^p_{(k)})|\theta_k-\tau_k] = \nabla R(\theta_k-\tau_k)$.

We first decompose $I_1$ as

$$
I_1 = \mathbb{E}\left< \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k-\tau_k) - \hat{\delta}(\hat{x}(k), \omega_k^*) \right) \psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k)) \right>
$$

\begin{align*}
&= \mathbb{E}\left< \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k-\tau_k) - \hat{\delta}(\hat{x}(k), \omega_k^*) \right) \psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k)) \right>
\end{align*}

$$
+ \mathbb{E}\left< \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k) - \hat{\delta}(\hat{x}(k), \omega_k^*) \right) \psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k)) \right>.
$$

(56)

We bound $I_1^{(1)}$ as

\begin{align*}
I_1^{(1)} &= \mathbb{E}\left< \nabla J_\lambda(\theta_k), \left( \gamma \phi(\hat{s}(k)) - \phi(\hat{s}(k)) \right)^\top (\omega_k-\tau_k - \omega_k) \psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k)) \right>
\geq -\mathbb{E}\left[ \|\nabla J_\lambda(\theta_k)\|_2 \|\gamma \phi(\hat{s}(k)) - \phi(\hat{s}(k))\|_2 \|\omega_k-\tau_k\| \|\psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k))\|_2 \right]
\geq -(1 + \gamma)C_\psi \mathbb{E}\left[ \|\nabla J_\lambda(\theta_k)\|_2 \|\omega_k-\tau_k\| \|\psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k))\|_2 \right]
\geq -(1 + \gamma)C_\psi C_\delta K_0 \beta \mathbb{E}\|\nabla J_\lambda(\theta_k)\|_2.
\end{align*}

(57)

where the last inequality follows

\begin{align*}
\|\omega_k-\tau_k\|_2 &= \left\| \sum_{i=k-\tau_k}^{k-1} (\omega_{i+1} - \omega_i) \right\|_2 \\
&\leq \sum_{i=k-\tau_k}^{k-1} \|\beta g(x_i, \omega_i-\tau_k)\|_2 \\
&\leq \beta K_0 C_\delta,
\end{align*}

(58)

where the second inequality is due to (40).

Then we bound $I_1^{(2)}$ as

\begin{align*}
I_1^{(2)} &= \mathbb{E}\left< \nabla J_\lambda(\theta_k), \left( \gamma \phi(\hat{s}(k)) - \phi(\hat{s}(k)) \right)^\top (\omega_k^* - \omega_k) \psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k)) \right>
\geq -\mathbb{E}\left[ \|\nabla J_\lambda(\theta_k)\|_2 \|\gamma \phi(\hat{s}(k)) - \phi(\hat{s}(k))\|_2 \|\omega_k^* - \omega_k\| \|\psi_{\theta_k-\tau_k}(\hat{s}(k), \hat{a}(k))\|_2 \right]
\geq -(1 + \gamma)C_\psi \mathbb{E}\|\nabla J_\lambda(\theta_k)\|_2 \|\omega_k^* - \omega_k\|_2.
\end{align*}

(59)

Collecting lower bounds of $I_1^{(1)}$ and $I_1^{(2)}$ gives

\begin{align*}
I_1 &\geq -2C_\psi \mathbb{E}\left[ \|\nabla J_\lambda(\theta_k)\|_2 \left( C_\delta K_0 \beta + \|\omega_k^* - \omega_k\|_2 \right) \right].
\end{align*}

\begin{align*}
&\geq -\frac{1}{2} \mathbb{E}\|\nabla J_\lambda(\theta_k)\|_2^2 - 2C_\psi^2 \mathbb{E}\left[ (C_\delta K_0 \beta + \|\omega_k^* - \omega_k\|_2)^2 \right].
\end{align*}

\begin{align*}
&\geq -\frac{1}{2} \mathbb{E}\|\nabla J_\lambda(\theta_k)\|_2^2 - 4C_\psi^2 C_\delta^2 K_0^2 \beta^2 - 4C_\psi^2 \mathbb{E}\|\omega_k - \omega_k^*\|_2^2,
\end{align*}

(60)
where the second and third inequality follow Young’s inequality.

Now we consider $I_2$. We first decompose $I_2$ as

$$I_2 = \mathbb{E} \left< \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \omega_k^*) \psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k)) + \lambda \nabla R(\theta_{k - \tau_k}) \right>$$

$$= \mathbb{E} \left< \nabla J_\lambda(\theta_k), \left( \frac{ \hat{\delta}(\hat{x}(k), \omega_k^*) - \hat{\delta}(\hat{x}(k), \omega_{k - \tau_k}^*) }{ \omega_k^* - \omega_{k - \tau_k}^* } \right) \psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k)) \right>$$

$$+ \mathbb{E} \left< \nabla J_\lambda(\theta_k), \frac{ \hat{\delta}(\hat{x}(k), \theta_{k - \tau_k}) }{ \omega_k^* - \omega_{k - \tau_k}^* } \psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k)) \right>$$

$$+ \mathbb{E} \left< \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \theta_{k - \tau_k}) \psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k)) + \lambda \nabla R(\theta_{k - \tau_k}) - \nabla J_\lambda(\theta_k) \right> \right> + \| \nabla J_\lambda(\theta_k) \|^2.$$

(61)

We bound $I_2^{(1)}$ as

$$I_2^{(1)} = \mathbb{E} \left< \nabla J_\lambda(\theta_k), \left( \frac{ \hat{\delta}(\hat{x}(k), \omega_k^*) - \hat{\delta}(\hat{x}(k), \omega_{k - \tau_k}^*) }{ \omega_k^* - \omega_{k - \tau_k}^* } \right) \psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k)) \right>$$

$$= \mathbb{E} \left< \nabla J_\lambda(\theta_k), \left( \frac{ \gamma \phi(\hat{s}(k)) - \phi(\hat{s}(k)) }{ \omega_k^* - \omega_{k - \tau_k}^* } \right) \psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k)) \right>$$

$$\geq -\mathbb{E} \left[ \left\| \nabla J_\lambda(\theta_k) \right\|^2 \left( \gamma \phi(\hat{s}(k)) - \phi(\hat{s}(k)) \right) \right] \frac{ \left\| \omega_k^* - \omega_{k - \tau_k}^* \right\|_2 {\psi_{\theta_k - \tau_k} (\hat{s}(k), \hat{a}(k))} \right\|^2$$

$$\geq -L_V C_{\psi}(1 + \gamma) \mathbb{E} \left\| \omega_k^* - \omega_{k - \tau_k}^* \right\|^2$$

$$\geq -L_V L_{\omega} C_{\psi}(1 + \gamma) \mathbb{E} \left\| \theta_k - \theta_{k - \tau_k} \right\|^2$$

$$\geq -L_V L_{\omega} C_{\psi}(1 + \gamma) K_0 \alpha,$$

(62)

where $L_V := \max_{t \geq 0} C_{\psi} + \lambda C_{\psi}$ is the trivial upper bound of $\left\| \nabla J_\lambda(\theta) \right\|_2$ and $\left\| \nabla J(\theta) \right\|_2$. The second last inequality follows from Proposition 2 and the last inequality uses (48) as

$$\left\| \theta_k - \theta_{k - \tau_k} \right\|^2 \leq \sum_{i=k-\tau_k}^{k-1} \left\| \theta_{i+1} - \theta_i \right\|^2$$

$$= \sum_{i=k-\tau_k}^{k-1} \alpha \left\| \hat{\delta}(\hat{x}_i, \omega_i) \psi_{\theta_i - \tau_i} (\hat{s}_i, \hat{a}_i) \right\|^2$$

$$\leq \sum_{i=k-\tau_k}^{k-1} \alpha C_p \leq C_p K_0 \alpha.$$

(63)
We bound $I_2^{(2)}$ as

$$I_2^{(2)} = \mathbb{E} \left< \nabla J_\lambda(\theta_k), \left( \delta(\hat{x}(k), \omega_{k-r}\theta_k) - \delta(\hat{x}(k), \theta_{k-r}) \right) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \right>$$

$$\geq -\mathbb{E} \left[ \left\| \nabla J_\lambda(\theta_k) \right\|_2 \left\| \delta(\hat{x}(k), \omega_{k-r}\theta_k) - \delta(\hat{x}(k), \theta_{k-r}) \right\|_2 \left\| \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \right\|_2 \right]$$

$$\geq -L_V C_\psi \mathbb{E} \left[ \delta(\hat{x}(k), \omega_{k-r}\theta_k) - \delta(\hat{x}(k), \theta_{k-r}) \right]$$

$$= -L_V C_\psi \mathbb{E} \left[ \gamma \left( \phi(s'_k)^T \omega_{k-r} - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) + V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) - \phi(s'_k)^T \omega_{k-r} \right) \right]$$

$$\geq -L_V C_\psi \left[ \gamma \mathbb{E} \left[ \phi(s'_k)^T \omega_{k-r} - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) + \mathbb{E} \left[ V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) - \phi(s'_k)^T \omega_{k-r} \right] \right] \right]$$

$$\geq -L_V C_\psi \left[ \gamma \mathbb{E} \left[ \phi(s'_k)^T \omega_{k-r} - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) \right] \right]$$

$$\geq -L_V C_\psi (1 + \gamma) \epsilon_{\text{app}}. \quad (64)$$

We bound $I_2^{(3)}$ as

$$I_2^{(3)} = \mathbb{E} \left< \nabla J_\lambda(\theta_k), \delta(\hat{x}(k), \theta_{k-r}) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) + \lambda \nabla R(\theta_{k-r}) - \nabla J_\lambda(\theta_k) \right>$$

$$= \mathbb{E} \left< \nabla J_\lambda(\theta_k), \mathbb{E} \left[ \delta(\hat{x}(k), \theta_{k-r}) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \Big| \theta_{k-r}, \theta_k \right] \right> + \lambda \nabla R(\theta_{k-r}) - \nabla J_\lambda(\theta_k)$$

$$= \mathbb{E} \left< \nabla J_\lambda(\theta_k), \mathbb{E}_{(\hat{s}(k), \hat{a}(k)) \sim \pi_{\theta_{k-r}}} \left[ A_{\pi_{\theta_{k-r}}} \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \right] + \lambda \nabla R(\theta_{k-r}) - \nabla J_\lambda(\theta_k) \right>,$$}

(65)

where we used the fact that

$$\mathbb{E} \left[ \delta(\hat{x}(k), \theta_{k-r}) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \Big| \theta_{k-r}, \theta_k \right]$$

$$= \mathbb{E}_{(\hat{s}(k), \hat{a}(k)) \sim \pi_{\theta_{k-r}}} \left[ (r(\hat{s}(k), \hat{a}(k), \hat{s}'(k)) + \gamma V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) \right) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \Big| \theta_{k-r}, \theta_k \right]$$

$$= \mathbb{E}_{\hat{s}'(k) \sim \mathcal{P}} \left[ \mathbb{E}_{(\hat{s}(k), \hat{a}(k)) \sim \pi_{\theta_{k-r}}} \left[ (r(\hat{s}(k), \hat{a}(k), \hat{s}'(k)) + \gamma V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) \right) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \Big| \theta_{k-r}, \theta_k \right]$$

$$= \mathbb{E}_{\hat{s}'(k) \sim \mathcal{P}} \left[ Q_{\pi_{\theta_{k-r}}}(\hat{s}(k), \hat{a}(k)) - V_{\pi_{\theta_{k-r}}}(\hat{s}(k)) \right) \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \Big| \theta_{k-r}, \theta_k \right]$$

$$= \mathbb{E}_{\hat{s}'(k) \sim \mathcal{P}} \left[ A_{\pi_{\theta_{k-r}}} \psi_{\theta_{k-r}}(\hat{s}(k), \hat{a}(k)) \Big| \theta_{k-r}, \theta_k \right]$$

Continuing from (65),

$$I_2^{(3)} = \left< \nabla J_\lambda(\theta_k), \nabla J_\lambda(\theta_{k-r}) - \nabla J_\lambda(\theta_k) \right>$$

$$\geq -\left\| \nabla J_\lambda(\theta_k) \right\|_2 \left\| \nabla J_\lambda(\theta_{k-r}) - \nabla J_\lambda(\theta_k) \right\|_2$$

$$\geq -L_V L_\lambda \left\| \theta_{k-r} - \theta_k \right\|_2 \geq -L_V L_\lambda C_p K_0 \alpha, \quad (66)$$

where the second last inequality is due to $L_\lambda$-Lipschitz continuity of policy gradient shown in Proposition 1, and the last inequality follows (63).
Collecting lower bounds of $I_2^{(1)}$, $I_2^{(2)}$ and $I_2^{(3)}$ gives

$$I_2 \geq -D_1 K_0 \alpha - L_V C_\psi (1 + \gamma) \varepsilon_{app},$$

where constant $D_1 := L_V L_\omega C_\psi C_p (1 + \gamma) + L_V L_\lambda C_p$.

Substituting (60) and (67) into (55) yields

$$\mathbb{E}[J_\lambda(\theta_{k+1})] \geq \mathbb{E}[J_\lambda(\theta_k)] + \frac{\alpha}{2} \mathbb{E}||\nabla J_\lambda(\theta_k)||_2^2 - 4C_\psi^2 \alpha \mathbb{E}||\omega_k - \omega_k^*||_2^2 - 4C_\psi^2 C_\theta^2 K_0^2 \alpha \beta^2$$

$$- 2L_V C_\psi \varepsilon_{app} \alpha - (D_1 K_0 + \frac{L_\lambda}{2} C_p^2) \alpha^2. \tag{68}$$

After telescoping, we have

$$\sum_{k=1}^{K} \frac{1}{2} \mathbb{E}||\nabla J_\lambda(\theta_k)||_2^2 \leq \frac{1}{\alpha} \left( J^* - J_\lambda(\theta_{K_0}) \right) + 4C_\psi^2 \sum_{k=1}^{K} \mathbb{E}||\omega_k - \omega_k^*||_2^2 + 4K C_\psi^2 C_\theta^2 K_0^2 \beta^2$$

$$+ 2KL_V C_\psi \varepsilon_{app} + K(D_1 K_0 + \frac{L_\lambda}{2} C_p^2) \alpha. \tag{69}$$

Select $\alpha = K^{-\frac{3}{4}}$ and $\beta = K^{-\frac{2}{5}}$, we have

$$\frac{1}{K} \sum_{k=1}^{K} \mathbb{E}||\nabla J_\lambda(\theta_k)||_2^2 = O\left( \frac{1}{K^{\frac{3}{5}}} \right) + O\left( \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}||\omega_k - \omega_k^*||_2 \right) + O\left( \frac{K^2}{K^{\frac{5}{4}}} \right) + O\left( \frac{K_0}{K^{\frac{9}{5}}} \right) + O(\varepsilon_{app}). \tag{70}$$

This completes the proof. \[\blacksquare\]

### B.3. Proof of Theorem 3

Given the definition in Section B.1, we now give the convergence proof of critic update in Algorithm 1 with linear function approximation and Markovian sampling. **Proof** By following the derivation of (49), we have

$$\mathbb{E}||\omega_{k+1} - \omega_{k+1}^*||_2^2 \leq \left( 1 - 2\lambda \beta \right) \mathbb{E}||\omega_k - \omega_k^*||_2^2 + 2\beta \left( C_1 \frac{\alpha}{\beta} + C_2 K_0 \beta \right) \mathbb{E}||\omega_k - \omega_k^*||_2$$

$$+ 2\beta \mathbb{E}\left( \omega_k - \omega_k^*, g(x_{(k)}, \omega_k) - \overline{g}(\theta_k, \omega_k) \right) + C_\theta^2 \beta^2, \tag{71}$$

where $C_1 := C_p L_\omega$, $C_2 := C_0 (1 + \gamma)$ and $C_\theta := 2C_\psi^2 + 2L_\psi^2 C_\theta^2 \max(\alpha)^2$. $\beta^2$

Now we consider the third item in the last inequality. For some $m \in \mathbb{N}^+$, we define $M := (K_0 + 1)m + K_0$. Following Lemma 4 (to be presented in Section C.1), for some $d_m \leq M$ and positive constants $C_4, C_5, C_6, C_7$, we have

$$\mathbb{E}\left( \omega_k - \omega_k^*, g(x_{(k)}, \omega_k) - \overline{g}(\theta_k, \omega_k) \right)$$

$$\leq C_4 \mathbb{E}||\theta_k - \theta_{k-d_m}||_2 + C_5 \sum_{i=\tau_k}^{m-d_m} \mathbb{E}||\theta_{k-i} - \theta_{k-d_m}||_2 + C_6 \mathbb{E}||\omega_k - \omega_{k-d_m}||_2 + C_7 \kappa \rho^{m-1}$$

$$\leq C_4 \sum_{i=\tau_k}^{k-1} \mathbb{E}||\theta_{i+1} - \theta_i||_2 + C_5 \sum_{i=\tau_k}^{m-d_m} \sum_{j=k-d_m}^{k-1} \mathbb{E}||\theta_{j+1} - \theta_j||_2 + C_6 \sum_{i=\tau_k}^{k-1} \mathbb{E}||\omega_{i+1} - \omega_i||_2 + C_7 \kappa \rho^{m-1}$$

$$\leq C_4 d_m C_p \alpha + C_5 (d_m - \tau_k)^2 C_p \alpha + C_6 d_m C_\theta \beta + C_7 \kappa \rho^{m-1}$$

$$\leq (C_4 M + C_5 M^2) C_p \alpha + C_6 M C_\theta \beta + C_7 \kappa \rho^{m-1}, \tag{72}$$
where the last inequality is due to $\tau_k \geq 0$ and $d_m \leq M$.

Further letting $m = m_K$ which is defined in (27) yields

$$
\mathbb{E} \left\{ \omega_k - \omega_k^*, g(x(k), \omega_k) - \mathcal{G}(\theta_k, \omega_k) \right\} \\
= (C_4 M + C_5 M^2 K) C_p \alpha + C_0 C_\delta M K \beta + C_\gamma \rho m_K^{-1} \\
\leq (C_4 M + C_5 M^2 K) C_p \alpha + C_0 C_\delta M K \beta + C_\gamma \alpha,
$$

(73)

where $M_K = (K_0 + 1) m_K + K_0$, and the last inequality follows from $m_K = \mathcal{O}(\log K)$.

Substituting (73) into (71) gives

$$
\mathbb{E} \left\| \omega_{k+1} - \omega_{k+1}^* \right\|_2^2 \leq (1 - 2 \lambda \beta) \mathbb{E} \left\| \omega_k - \omega_k^* \right\|_2^2 + 2 \beta \left( C_1 \frac{\alpha}{\beta} + C_2 K_0 \beta \right) \mathbb{E} \left\| \omega_k - \omega_k^* \right\|_2 \\
+ 2 \beta \left( (C_4 M + C_5 M^2 K) C_p \alpha + C_0 C_\delta M K \beta + C_\gamma \alpha \right) + C_q \beta^2.
$$

(74)

Select $\alpha = K^{-\frac{3}{5}}$ and $\beta = K^{-\frac{2}{5}}$. After telescoping, we have

$$
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\| \omega_k - \omega_k^* \right\|_2^2 = \mathcal{O} \left( \frac{1}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K^2 \log^2 K}{K^{\frac{2}{5}}} \right) + \mathcal{O} \left( \frac{K_0 \log K}{K^{\frac{2}{5}}} \right).
$$

This completes the proof.

B.4. Proof of Theorem 4

Given the definition in section B.2, we now give the convergence proof of actor update in Algorithm 1 with linear value function approximation and Markovian sampling method.

Proof By following the derivation of (55), we have

$$
\mathbb{E}[J_\lambda(\theta_{k+1})] \geq \mathbb{E}[J_\lambda(\theta_k)] + \alpha \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \omega_k - \tau_k) \psi_{\theta_k - \tau_k}(\hat{s}(k), \hat{a}(k)) \right\rangle \\
+ \alpha \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \hat{\delta}(\hat{x}(k), \omega_k^*) \psi_{\theta_k - \tau_k}(\hat{s}(k), \hat{a}(k)) + \lambda \nabla R(\theta_{k-\tau_k}) \right\rangle - \frac{L_1}{2} C_p^2 \alpha^2.
$$

(75)

The item $I_1$ can be bounded by following (60) as

$$
I_1 \geq -\frac{1}{2} \mathbb{E} \left\| \nabla J_\lambda(\theta_k) \right\|_2^2 - 4C_\psi^2 C_\delta^2 K_0 \beta^2 - 4C_\psi^2 \mathbb{E} \left\| \omega_k - \omega_k^* \right\|_2^2.
$$

(76)
Next we consider $I_2$. We first decompose it as

$$I_2 = \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \delta(x(k), \omega_k^*) \psi_{\theta_{k-r_k}}(s(k), a(k)) + \lambda \nabla R(\theta_{k-r_k}) \right\rangle$$

$$= \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \left( \delta(x(k), \omega_k^*) - \delta(x(k), \theta_k) \right) \psi_{\theta_{k-r_k}}(s(k), a(k)) \right\rangle$$

$$+ \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \delta(x(k), \theta_k) \psi_{\theta_{k-r_k}}(s(k), a(k)) - \nabla J(\theta_k) \right\rangle$$

$$+ \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \lambda \nabla R(\theta_{k-r_k}) - \lambda \nabla R(\theta_k) \right\rangle + \mathbb{E} \| \nabla J_\lambda(\theta_k) \|^2_2.$$  \hspace{1cm} (77)

For some $m \in \mathbb{N}^+$, define $M := (K_0 + 1)m + K_0$. Following Lemma 5, for some $d_m \leq M$ and positive constants $D_2, D_3, D_4, D_5, I_2^{(1)}$, can be bounded as

$$I_2^{(1)} = \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \left( \delta(x(k), \omega_k^*) - \delta(x(k), \theta_k) \right) \psi_{\theta_{k-r_k}}(s(k), a(k)) \right\rangle$$

$$\geq -D_2 E\|\theta_{k-r_k} - \theta_{k-d_m}\|_2 - D_3 E\|\theta_{k} - \theta_{k-d_m}\|_2 - D_4 \sum_{i=k-d_m}^{k-r_k} E\|\theta_i - \theta_{k-d_m}\|_2$$

$$- D_5 \kappa \rho^{m-1} - L_V C_\psi(1 + \gamma) \epsilon_{\text{app}}$$

$$\geq -D_2(d_m - \tau_k) C_p \alpha - D_3 d_m C_p \alpha - D_4(d_m - \tau_k)^2 C_p \alpha$$

$$- D_5 \kappa \rho^{m-1} - (1 + \gamma) L_V C_\psi \epsilon_{\text{app}},$$  \hspace{1cm} (78)

where the derivation of the last inequality is similar to that of (72). By setting $m = m_K$ in (78), and following the fact that $d_{m_K} \leq M_K$ and $r_k \geq 0$, we have

$$I_2^{(1)} \geq -D_2 M_K C_p \alpha - D_3 M_K C_p \alpha - D_4 M_K^2 C_p \alpha - D_5 \kappa \rho^{m_K-1} - (1 + \gamma) L_V C_\psi \epsilon_{\text{app}}$$

$$= -\left( (D_2 + D_3) C_p M_K + D_4 C_p M_K^2 \right) \alpha - D_5 \kappa \rho^{m_K-1} - (1 + \gamma) L_V C_\psi \epsilon_{\text{app}}$$

$$\geq -\left( (D_2 + D_3) C_p M_K + D_4 C_p M_K^2 \right) \alpha - D_5 \alpha - (1 + \gamma) L_V C_\psi \epsilon_{\text{app}},$$  \hspace{1cm} (79)

where the last inequality is due to the definition of $m_K$.

Following Lemma 6, for some positive constants $D_6, D_7$ and $D_8$, we bound $I_2^{(2)}$ as

$$I_2^{(2)} = \mathbb{E} \left\langle \nabla J_\lambda(\theta_k), \delta(x(k), \theta_k) \psi_{\theta_{k-r_k}}(s(k), a(k)) - \nabla J(\theta_k) \right\rangle$$

$$\geq -D_6 E\|\theta_{k-r_k} - \theta_{k-d_m}\|_2 - D_7 E\|\theta_{k} - \theta_{k-d_m}\|_2 - D_8 \sum_{i=\tau_k}^{d_m} E\|\theta_{k-i} - \theta_{k-d_m}\|_2 - D_9 \kappa \rho^{m-1}.$$  \hspace{1cm} (80)

Similar to the derivation of (79), we have

$$I_2^{(2)} \geq -\left( D_6 C_p M_K + D_7 C_p M_K + D_8 C_p M_K^2 \right) \alpha - D_9 \alpha.$$  \hspace{1cm} (80)
Term $I_2^{(3)}$ can be bounded as

$$I_2^{(3)} \geq -\lambda L V \| \nabla R(\theta_k) - \nabla R(\theta_{k-\tau_k}) \|_2 \geq -\lambda L V L_\psi \| \theta_k - \theta_{k-\tau_k} \|_2 \geq -\lambda L V L_\psi K_0 C_p \alpha.$$  

Collecting the lower bounds of $I_2^{(1)}$, $I_2^{(2)}$ and $I_2^{(3)}$ yields

$$I_2 \geq -D_K \alpha - (1 + \gamma) L V C_\psi \epsilon_{\text{app}} + \mathbb{E} \| \nabla J_\lambda(\theta_k) \|^2_2,$$  

where we define $D_K := C_p (D_4 + D_8) M_\psi^2 + C_p (D_2 + D_3 + D_6 + D_7) M_K + \lambda L V L_\psi K_0 C_p + D_5 + D_9$ for brevity.

Substituting (76) and (82) into (75) yields

$$\mathbb{E}[J_\lambda(\theta_{k+1})] \geq \mathbb{E}[J_\lambda(\theta_k)] + \frac{\alpha}{2} \mathbb{E} \| \nabla J_\lambda(\theta_k) \|^2_2 - 4C_\psi^2 \alpha \mathbb{E} \| \omega_k - \omega_k^c \|^2_2 - 4C_\psi^2 C_\delta^2 K_0^2 \alpha \beta^2 - 2L_\psi C_\psi \epsilon_{\text{app}} \alpha - D_K \alpha^2.$$  

Choose step size $\alpha = K^{-\frac{3}{8}}$, $\beta = K^{-\frac{3}{8}}$. With $D_K = \mathcal{O}(M_\psi^2) = \mathcal{O}(K_0^2 \log^2 K)$, the last inequality implies

$$\frac{1}{K} \sum_{k=1}^K \mathbb{E} \| \nabla J_\lambda(\theta_k) \|^2_2 = \mathcal{O}\left( \frac{1}{K} \sum_{k=1}^K \mathbb{E} \| \omega_k - \omega_k^c \|^2_2 \right) + \mathcal{O}\left( \frac{K_0^2 \log^2 K}{K} \right) + \mathcal{O}(\epsilon_{\text{app}}).$$  

This completes the proof.

\[\Box\]

B.5. Proof of Theorem 5

\textbf{Proof} We define an event $E_k$ as $\| \nabla J_\lambda(\theta_k) \| \leq \frac{\lambda}{2|A||A^\top|}$ and its complement $E^c_k$ as $\| \nabla J_\lambda(\theta_k) \| > \frac{\lambda}{2|A||A^\top|}$. We use $1_{E_k}$ to indicate whether the event happens or not, i.e. $1_{E_k} = 1$ if $E_k$ happens and $1_{E_k} = 0$ if $E^c_k$ happens. Then we have for the optimality gap:

$$\sum_{k=1}^K \mathbb{E} [ J^* - J(\theta_k) ] = \sum_{k=1}^K \mathbb{E} [ (J^* - J(\theta_k)) 1_{E_k} ] + \sum_{k=1}^K \mathbb{E} [ (J^* - J(\theta_k)) 1_{E^c_k} ]$$

$$\leq \frac{2\lambda}{1 - \gamma} \left\| \frac{d_{\pi^*}}{\eta} \right\|_\infty \sum_{k=1}^K \mathbb{E} [ 1_{E_k} ] + \sum_{k=1}^K \mathbb{E} [ (J^* - J(\theta_k)) 1_{E^c_k} ]$$

$$\leq \frac{2\lambda}{1 - \gamma} \left\| \frac{d_{\pi^*}}{\eta} \right\|_\infty \sum_{k=1}^K \mathbb{E} [ 1_{E_k} ] + J^* \sum_{k=1}^K \mathbb{E} [ 1_{E^c_k} ]$$

$$\leq \frac{2\lambda}{1 - \gamma} \left\| \frac{d_{\pi^*}}{\eta} \right\|_\infty K + J^* \sum_{k=1}^K \mathbb{E} [ 1_{E^c_k} ],$$

where the first inequality follows from Lemma 1.
Now it suffices to bound \( \sum_{k=1}^{K} \mathbb{E}[1_{E_k}^c] \).

\[
\sum_{k=1}^{K} \mathbb{E}\|\nabla J_\lambda(\theta_k)\|^2 \geq \sum_{k=1}^{K} \mathbb{E}\left[\|\nabla J_\lambda(\theta_k)\|^2 1_{E_k^c}\right] \\
\geq \frac{\lambda^2}{4|\mathcal{S}|^2|\mathcal{A}|^2} \sum_{k=1}^{K} \mathbb{E}\|\nabla J_\lambda(\theta_k)\|^2 \tag{86}
\]

Substituting the above inequality into (85) and dividing both sides by \( K \) give

\[
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[J^* - J(\theta_k)] \leq \frac{2\lambda}{1-\gamma} \frac{d_{m*}}{\eta} \|\zeta\| + \frac{4|\mathcal{S}|^2|\mathcal{A}|}{\lambda^2} \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}\|\nabla J_\lambda(\theta_k)\|^2 \tag{87}
\]

It is known that the softmax policy satisfies Assumption 3, thus we immediately know that Theorem 2 and Theorem 4 hold. Furthermore, by assumption 5, we have \( \phi(s) \top \omega^*_\theta = V_{\omega^*_\theta}(s) \), leading to \( \epsilon_{\text{app}} = 0 \). Applying Theorem 2 and 4 to (87) completes the proof. \( \blacksquare \)

Appendix C. Supporting Lemmas

C.1. Supporting Lemmas for Theorem 3

Lemma 4 For any \( m \geq 1 \) and \( k \geq (K_0 + 1)m + K_0 + 1 \), we have

\[
\mathbb{E}\left\langle \omega_k - \omega^*_\theta, g(x(k), \omega_k) - \overline{g}(\theta_k, \omega_k) \right\rangle \leq C_4 \mathbb{E}\|\theta_k - \theta_{k-d_m}\|_2 + C_5 \sum_{i=0}^{d_m} \mathbb{E}\|\theta_{k-i} - \theta_{k-d_m}\|_2 \\
+ C_6 \mathbb{E}\|\omega_k - \omega_{k-d_m}\|_2 + C_7 \kappa \rho^{m-1},
\]

where constant \( d_m \leq (K_0 + 1)m + K_0 \), and \( C_4 := 2C_5 L_\omega + 4R_\omega C_\delta |\mathcal{A}| L_\pi (1 + \log_\rho \kappa^{-1} + (1 - \rho)^{-1}) \)

\( C_5 := 4R_\omega C_\delta |\mathcal{A}| L_\pi \) and \( C_6 := 4(1 + \gamma)R_\omega + 2C_\delta \). \( C_7 := 8R_\omega C_\delta. \)

Proof Consider the collection of random samples \( \{x(k-K_0), x(k-K_0+1), \ldots, x(k)\} \). Suppose \( x(k) \) is sampled by worker \( n \), then due to Assumption 2, \( \{x(k-K_0), x(k-K_0+1), \ldots, x(k-1)\} \) will contain at least another sample drawn by worker \( n \). Therefore, \( \{x(k-(K_0+1)m), x(k-(K_0+1)m+1), \ldots, x(k-1)\} \) will contain at least \( m \) samples from worker \( n \).

Consider the Markov chain formed by \( m+1 \) samples in \( \{x(k-(K_0+1)m), x(k-(K_0+1)m+1), \ldots, x(k)\} \):

\[
s_{t-m} \xrightarrow{\theta_{k-d_m}} a_{t-m} \xrightarrow{\theta_{k-d_{m-1}}} a_{t-m+1} \cdots s_{t-1} \xrightarrow{\theta_{k-d_1}} a_{t-1} \xrightarrow{\theta_{k-d_0}} a_t \xrightarrow{P} \xrightarrow{P} s_{t+1},
\]

where \( (s_t, a_t, s_{t+1}) = (s(k), a(k), s'(k)) \), and \( \{d_j\}_{j=0}^m \) is some increasing sequence with \( d_0 := \tau_k \).

Suppose \( \theta_{k-d_m} \) was used to generate the \( k_m \)th update, then we have \( x_{t-m} = x_{(k_m)} \). Following Assumption 2, we have \( \tau_{km} = k_m - (k - d_m) \leq K_0 \). Since \( x_{(km)} \) is in \( \{x(k-(K_0+1)m), \ldots, x(k)\} \), we have \( k_m \geq k - (K_0+1)m \). Combining these two inequalities, we have

\[
d_m \leq (K_0 + 1)m + K_0. \tag{88}
\]
Given \((s_{t-m}, a_{t-m}, s_{t-m+1})\) and \(\theta_{k-d_m}\), we construct an auxiliary Markov chain as Lemma 3:

\[
s_{t-m} \xrightarrow{\theta_{k-d_m}} a_{t-m} \xrightarrow{\mathcal{P}} s_{t-m+1} \xrightarrow{\theta_{k-d_m}} a_{t-m+1} \cdots \xrightarrow{\theta_{k-d_m}} a_{t} \xrightarrow{\mathcal{P}} s_{t} \xrightarrow{\theta_{k-d_m}} a_{t} \xrightarrow{\mathcal{P}} s_{t+1}.
\]

For brevity, we define

\[
\Delta_1(x, \theta, \omega) := \langle \omega - \omega^*_\theta, g(x, \omega) - \overline{g}(\theta, \omega) \rangle.
\]

Throughout this proof, we use \(\theta, \theta', \omega, \omega', x\) and \(\bar{x}\) as shorthand notations of \(\theta_k, \theta_{k-d_m}, \omega_k, \omega_{k-d_m}, x_t\) and \(\bar{s}_t := (s_t, a_t, s_{t+1})\), respectively.

First we decompose \(\Delta_1(x, \theta, \omega)\) as

\[
\Delta_1(x, \theta, \omega) = \frac{\Delta_1(x, \theta, \omega) - \Delta_1(x, \theta', \omega)}{I_1} - \frac{\Delta_1(x, \theta', \omega) - \Delta_1(x, \theta', \omega')}{I_2} + \frac{\Delta_1(x, \theta', \omega') - \Delta_1(\bar{x}, \theta', \omega')}{I_3} + \frac{\Delta_1(\bar{x}, \theta', \omega')}{I_4}.
\]

We bound \(I_1\) in (89) as

\[
\Delta_1(x, \theta, \omega) - \Delta_1(x, \theta', \omega) = \langle \omega - \omega^*_\theta, g(x, \omega) - \overline{g}(\theta, \omega) \rangle - \langle \omega - \omega^*_{\theta'}, g(x, \omega) - \overline{g}(\theta', \omega) \rangle
\]

\[
\leq |\langle \omega - \omega^*_\theta, g(x, \omega) - \overline{g}(\theta, \omega) \rangle - \langle \omega - \omega^*_{\theta'}, g(x, \omega) - \overline{g}(\theta, \omega) \rangle| + |\langle \omega - \omega^*_{\theta'}, g(x, \omega) - \overline{g}(\theta, \omega) \rangle - \langle \omega - \omega^*_{\theta'}, g(x, \omega) - \overline{g}(\theta', \omega) \rangle|.
\]

For the first term in (90), we have

\[
|\langle \omega - \omega^*_\theta, g(x, \omega) - \overline{g}(\theta, \omega) \rangle - \langle \omega - \omega^*_{\theta'}, g(x, \omega) - \overline{g}(\theta, \omega) \rangle| = |\langle \omega^*_\theta - \omega^*_{\theta'}, g(x, \omega) - \overline{g}(\theta, \omega) \rangle|
\]

\[
\leq \|\omega^*_\theta - \omega^*_{\theta'}\|_2 \|\overline{g}(\theta, \omega)\|_2 \leq 2C_{\delta} \|\omega^*_\theta - \omega^*_{\theta'}\|_2,
\]

\[
\leq 2C_{\delta} L_\omega \|\theta - \theta'\|_2,
\]

where the last inequality is due to Proposition 2.

We use \(x \sim \theta'\) as shorthand notations to represent that \(s \sim \mu_{\theta'}, a \sim \pi_{\theta'}, s' \sim \mathcal{P}\). For the second term in (90), we have

\[
\leq 2R_\omega \sup_x \|g(x, \omega)\|_2 \|\mu_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P} - \mu_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P}\|_{TV}
\]

\[
\leq 2R_\omega C_{\delta} \|\mu_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P} - \mu_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P}\|_{TV}
\]

\[
= 4R_\omega C_{\delta} d_{TV} (\mu_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P}, \mu_{\theta} \otimes \pi_{\theta} \otimes \mathcal{P})
\]

\[
\leq 4R_\omega C_{\delta} |A| L_{\pi}(1 + \log_{\rho} \kappa^{-1} + (1 - \rho)^{-1}) \|\theta - \theta'\|_2,
\]

(92)
where the third inequality follows the definition of TV norm, the second last inequality follows (40), and the last inequality follows (Wu et al., 2020, Lemma A.1).

Collecting the upper bounds of the two terms in (90) yields

$$I_1 \leq [2C_δ L_ω + 4R_ω C_δ |A| L_π (1 + \log_ρ κ^{-1} + (1 - ρ)^{-1})] \|θ - θ'\|_2. \quad (93)$$

Next we bound $\mathbb{E}[I_2]$ in (89) as

$$\mathbb{E}[I_2] = \mathbb{E}[\Delta_1(x, θ', ω) - \Delta_1(x, θ', ω')]$$

$$= \mathbb{E} [⟨ω - ω^*_g, g(x, ω) - g(x, r, ω')⟩ - ⟨ω - ω^*_g, g(x, ω') - g(θ', ω')⟩]$$

$$\leq 2R_ω \mathbb{E} |∥g(x, ω) - g(x, ω')∥_2 + ∥g(θ', ω') - g(θ', ω)∥_2|$$

$$\leq 2R_ω \mathbb{E} |∥g(x, ω) - g(x, ω')∥_2 + ∥E_{x-θ'}[g(x, ω)] - E_{x-θ'}[g(x, ω)]∥_2|$$

$$= 2R_ω (1 + γ)E|∥ω - ω'∥_2 + (1 + γ)E|∥ω - ω'∥_2|$$

$$= 4R_ω (1 + γ)E|∥ω - ω'∥_2. \quad (94)$$

We bound the first term in (94) as

$$\mathbb{E} [⟨ω - ω^*_g, g(x, ω) - g(x, ω')⟩ - ⟨ω - ω^*_g, g(x, ω') - g(θ', ω')⟩]$$

$$= \mathbb{E} [⟨ω - ω^*_g, g(x, ω) - g(x, ω') + g(θ', ω') - g(θ', ω')⟩]$$

$$\leq 2R_ω (1 + γ)E|∥ω - ω'∥_2 + (1 + γ)E|∥ω - ω'∥_2|$$

$$= 4R_ω (1 + γ)E|∥ω - ω'∥_2. \quad (95)$$

Collecting the upper bounds of the two terms in (94) yields

$$\mathbb{E}[I_2] \leq 4(1 + γ)R_ω + 2C_δ E|∥ω_k - ω_{k-d_m}∥_2. \quad (97)$$

We first bound $I_3$ as

$$\mathbb{E}[I_3] = \mathbb{E} [\Delta_1(x, θ', ω') - \Delta_1(x, θ', ω')|\theta', ω', s_{t-m+1}]$$

$$\leq \mathbb{E} [\Delta_1(x, θ', ω')|\theta', ω', s_{t-m+1}] - E \sum_{x} |\Delta_1(x, θ', ω')|P(x ∈ \cdot |θ', ω', s_{t-m+1}) - P(x ∈ \cdot |θ', ω', s_{t-m+1})|TV$$

$$\leq 8R_ω C_δ d_{TV} (P(x ∈ \cdot |θ', s_{t-m+1}), P(x ∈ \cdot |θ', s_{t-m+1})), \quad (98)$$

where the second last inequality follows the definition of TV norm, and the last inequality follows

$$|\Delta_1(x, θ', ω')| \leq ||ω - ω^*_g||_2 ||g(x, ω') - g(θ', ω')||_2 \leq 4R_ω C_δ. \quad (99)$$
By following (28) in Lemma 3, we have

\[ d_{TV} \left( \mathbb{P}(x \in \cdot | \theta', s_{t-m+1}), \mathbb{P}(\tilde{x} \in \cdot | \theta', s_{t-m+1}) \right) \leq \frac{1}{2} |A| L_{\pi} \sum_{i=r_k}^{d_m} \mathbb{E} \| \theta_{k-i} - \theta_{k-d_m} \|_2 \theta', s_{t-m+1} \).

Substituting the last inequality into (98), then taking total expectation on both sides yield

\[ \mathbb{E}[I_3] \leq 4 R_\omega C_\delta |A| L_{\pi} \sum_{i=r_k}^{d_m} \mathbb{E} \| \theta_{k-i} - \theta_{k-d_m} \|_2. \]  

(100)

Next we bound \( I_4 \). Define \( \overline{x} := (\overline{s}, \overline{a}, \overline{s'}) \) where \( \overline{s} \sim \mu_{\theta'}, \overline{a} \sim \pi_{\theta'} \) and \( \overline{s'} \sim \mathcal{P} \). It is immediate that

\[ \mathbb{E}[\Delta_1(\overline{x}, \theta', \omega')|\theta', \omega', s_{t-m+1}] = \langle \omega' - \omega^*_\theta' \mathbb{E} g(\overline{x}, \omega')|\theta', \omega', s_{t-m+1} - \overline{g}(\theta', \omega') \rangle = \langle \omega' - \omega^*_\theta' \mathbb{E} g(\theta', \omega') - \overline{g}(\theta', \omega') \rangle = 0. \]  

(101)

Then we have

\[ \mathbb{E}[I_4|\theta', \omega', s_{t-m+1}] = \mathbb{E} \left[ \Delta_1(\overline{x}, \theta', \omega') - \Delta_1(\overline{x}, \theta', \omega')|\theta', \omega', s_{t-m+1} \right] 
\leq \mathbb{E} \left[ \Delta_1(\overline{x}, \theta', \omega')|\theta', \omega', s_{t-m+1} \right] - \mathbb{E} \left[ \Delta_1(\overline{x}, \theta', \omega')|\theta', \omega', s_{t-m+1} \right] 
\leq \sup_x \| \Delta_1(x, \theta', \omega') \| \mathbb{E} \| \mathbb{P}(\overline{x} \in \cdot | \theta', s_{t-m+1}) - \mathbb{P}(\overline{x} \in \cdot | \theta', s_{t-m+1}) \|_{TV} 
\leq 8 R_\omega C_\delta d_{TV} \left( \mathbb{P}(\overline{x} \in \cdot | \theta', s_{t-m+1}), \mathbb{P}(\overline{x} \in \cdot | \theta', s_{t-m+1}) \right) 
= 8 R_\omega C_\delta d_{TV} \left( \mathbb{P}(\overline{x} \in \cdot | \theta', s_{t-m+1}), \mu_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P} \right), \]  

(102)

where the second inequality follows the definition of TV norm, and the third inequality follows (101).

The auxiliary Markov chain with policy \( \pi_{\theta'} \) starts from initial state \( s_{t-m+1} \), and \( \overline{s}_i \) is the \((m-1)\)th state on the chain. Following Lemma 2, we have:

\[ d_{TV} \left( \mathbb{P}(\overline{x} \in \cdot | \theta', s_{t-m+1}), \mu_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P} \right) 
= d_{TV} \left( \mathbb{P} \left( (\overline{s}_i, \overline{a}_t, \overline{s}_{t+1}) \in \cdot | \theta', s_{t-m+1} \right), \mu_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P} \right) \leq \kappa \rho^{m-1}. \]  

(103)

Substituting the last inequality into (102) and taking total expectation on both sides yield

\[ \mathbb{E}[I_4] \leq 8 R_\omega C_\delta \kappa \rho^{m-1}. \]  

(104)

Taking total expectation on (89) and collecting bounds of \( I_1, I_2, I_3, I_4 \) yield

\[ \mathbb{E} \left[ \Delta_1(x, \theta, \omega) \right] \leq C_4 \mathbb{E} \| \theta_k - \theta_{k-d_m} \|_2 + C_5 \sum_{i=r_k}^{d_m} \mathbb{E} \| \theta_{k-i} - \theta_{k-d_m} \|_2 
+ C_6 \mathbb{E} \| \omega_k - \omega_{k-d_m} \|_2 + C_7 \kappa \rho^{m-1}, \]  

(105)

where \( C_4 := 2 C_\delta L_\omega + 4 R_\omega C_\delta |A| L_{\pi} (1 + \log_\rho \kappa^{-1} + (1 - \rho)^{-1}) \), \( C_5 := 4 R_\omega C_\delta |A| L_{\pi} \), \( C_6 := 4 (1 + \gamma) R_\omega + 2 C_\delta \) and \( C_7 := 8 R_\omega C_\delta \).
C.2. Supporting Lemmas for Theorem 4

Lemma 5  For any \( m \geq 1 \) and \( k \geq (K_0 + 1)m + K_0 + 1 \), we have

\[
\mathbb{E} \left( \nabla J_{\lambda}(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k^t) - \delta(\hat{x}(k), \theta_k) \right) \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \right) \geq -D_2 \mathbb{E} \| \theta_{k-d_k} - \theta_{k-d_m} \|_2
\]

\[
- D_3 \mathbb{E} \| \theta_k - \theta_{k-d_m} \|_2 - D_4 \sum_{i=\tau_k}^{d_m} \mathbb{E} \| \theta_k - \theta_{k-d_m} \|_2 - D_5 \kappa_1 \rho^{m-1} - L_V C_\psi(1 + \gamma) \epsilon_{app},
\]

where \( D_2 := 2L_V L_\psi C_\delta \), \( D_3 := (2C_\delta C_\psi L_\lambda + L_V C_\psi (L_\omega + L_V)(1 + \gamma)) \), \( D_4 := 2L_V C_\psi C_\delta |A| L_\pi \) and \( D_5 := 4L_V C_\psi C_\delta \).

Proof  For the worker that contributes to the \( k \)th update, we construct its Markov chain:

\[
\hat{s}_{t-m} \xrightarrow{\theta_{k-d_m}} \hat{a}_{t-m} \xrightarrow{\hat{P}} \hat{s}_{t-m+1} \xrightarrow{\theta_{k-d_m}} \hat{a}_{t-m+1} \cdots \xrightarrow{\hat{P}} \hat{s}_{t-1} \xrightarrow{\theta_{k-d_m}} \hat{a}_{t-1} \xrightarrow{\hat{P}} \hat{s}_t \xrightarrow{\theta_{k-d_m}} \hat{a}_t \xrightarrow{\hat{P}} \hat{s}_{t+1},
\]

where \( (s_t, a_t, \hat{s}_{t-1}) = (\hat{s}(k), \hat{a}(k), s_{t-1}') \), and \( \{d_j\}_{j=0}^m \) is some increasing sequence with \( d_0 := \tau_k \). By (88) in Lemma 4, we have \( d_m \leq (K_0 + 1)m + K_0 \).

Given \( \hat{s}_{t-m}, \hat{a}_{t-m}, \hat{s}_{t-m+1} \) and \( \theta_{k-d_m} \), we construct an auxiliary Markov chain:

\[
\tilde{s}_{t-m} \xrightarrow{\theta_{k-d_m}} \tilde{a}_{t-m} \xrightarrow{\tilde{P}} \tilde{s}_{t-m+1} \xrightarrow{\theta_{k-d_m}} \tilde{a}_{t-m+1} \cdots \xrightarrow{\tilde{P}} \tilde{s}_{t-1} \xrightarrow{\theta_{k-d_m}} \tilde{a}_{t-1} \xrightarrow{\tilde{P}} \tilde{s}_t \xrightarrow{\theta_{k-d_m}} \tilde{a}_t \xrightarrow{\tilde{P}} \tilde{s}_{t+1}.
\]

First we have

\[
\begin{aligned}
&\left\langle \nabla J_{\lambda}(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k^t) - \delta(\hat{x}(k), \theta_k) \right) \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \right\rangle \\
&= \left\langle \nabla J_{\lambda}(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k^t) - \delta(\hat{x}(k), \theta_k) \right) \left( \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \right) \right\rangle \\
&+ \left\langle \nabla J_{\lambda}(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k^t) - \delta(\hat{x}(k), \theta_k) \right) \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \right\rangle.
\end{aligned}
\]

We first bound the fist term in (106) as

\[
\begin{aligned}
&\left\langle \nabla J_{\lambda}(\theta_k), \left( \hat{\delta}(\hat{x}(k), \omega_k^t) - \delta(\hat{x}(k), \theta_k) \right) \left( \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \right) \right\rangle \\
&\geq -\|J_{\lambda}(\theta_k)\|_2 \| \hat{\delta}(\hat{x}(k), \omega_k^t) - \delta(\hat{x}(k), \theta_k) \| \| \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \|_2 \\
&\geq -\|J_{\lambda}(\theta_k)\|_2 \left[ \| \hat{\delta}(\hat{x}(k), \omega_k^t) \| + \| \delta(\hat{x}(k), \theta_k) \| \right] \| \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \|_2 \\
&\geq -L_V \left[ \| \hat{\delta}(\hat{x}(k), \omega_k^t) \| + \| \delta(\hat{x}(k), \theta_k) \| \right] \| \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \|_2 \\
&\geq -2L_V C_\delta \| \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) \|_2 \\
&\geq -2L_V L_\psi C_\delta \| \theta_{k-d_m} - \theta_{k-d_m} \|_2,
\end{aligned}
\]

where the last inequality follows Assumption 3 and second last inequality follows

\[
|\hat{\delta}(x, \omega_k^t)| \leq |r(x)| + \gamma \| \phi(s') \|_2 \| \omega_k^t \|_2 + \| \phi(s) \|_2 \| \omega_k^t \|_2 \leq r_{max} + (1 + \gamma) R_u \leq C_\delta,
\]

\[
|\delta(x, \theta) - \delta(x, \theta)| \leq |r(x)| + \gamma |V_{\tau_{\theta}}(s')| + |V_{\tau_{\theta}}(s)| \leq r_{max} + (1 + \gamma) R_u \leq C_\delta.
\]
Substituting (107) into (106) gives
\[
\left\langle \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\tilde{x}(k), \omega^*_k) - \delta(\hat{x}(k), \theta_k) \right) \psi_{\theta_k-\tau_k}(\hat{s}(k), \bar{a}(k)) \right\rangle 
\geq -2L_V L_\psi C_\delta \|\theta_{k-\tau_k} - \theta_{k-d_m}\|_2 + \left\langle \nabla J_\lambda(\theta_k), \left( \hat{\delta}(\tilde{x}(k), \omega^*_k) - \delta(\hat{x}(k), \theta_k) \right) \psi_{\theta_{k-d_m}}(\hat{s}(k), \bar{a}(k)) \right\rangle.
\]
(108)

Then we start to bound the second term in (108). For brevity, we define
\[
\Delta_2(x, \theta) := \left\langle \nabla J_\lambda(\theta), \left( \hat{\delta}(x, \omega^*_\theta) - \delta(x, \theta) \right) \psi_{\theta_{k-d_m}}(s, a) \right\rangle.
\]

In the following proof, we use \( \theta, \theta', \omega^*_\theta, \omega^*_\theta, x \) as shorthand notations for \( \theta_k, \theta_{k-d_m}, \omega^*_k, \omega^*_k, x_t \) respectively. We also define \( \overline{x} := (\overline{x}, \overline{x}, \overline{x}) \), where \( \overline{x} \sim d_{\theta'}, \overline{a} \sim \pi_{\theta'} \) and \( \overline{x} \sim \mathcal{P} \). Define \( \overline{x} := (\tilde{s}_{t-d_m}, \tilde{a}_{t-d_m}, \tilde{s}'_{t-d_m}) \) where \( \tilde{s}_{t-d_m}, \tilde{a}_{t-d_m} \) are state and action on the auxiliary Markov chain, and \( \tilde{s}'_{t-d_m} \sim \mathcal{P}(\tilde{s}_{t-d_m}, \tilde{a}_{t-d_m}) \) is a virtual sample. We decompose the second term in (108) as
\[
\Delta_2(x, \theta) = \underbrace{\Delta_2(x, \theta) - \Delta_2(x, \theta')}_{I_1} + \underbrace{\Delta_2(x, \theta') - \Delta_2(\overline{x}, \theta')}_{I_2} + \underbrace{\Delta_2(\overline{x}, \theta') - \Delta_2(\overline{x}, \theta')}_{I_3} + \underbrace{\Delta_2(\overline{x}, \theta')}_{I_4}.
\]

We bound the term \( I_1 \) as
\[
I_1 = \left\langle \nabla J_\lambda(\theta), \left( \hat{\delta}(x, \omega^*_\theta) - \delta(x, \theta) \right) \psi_{\theta'}(s, a) \right\rangle - \left\langle \nabla J_\lambda(\theta'), \left( \hat{\delta}(x, \omega^*_\theta) - \delta(x, \theta') \right) \psi_{\theta'}(s, a) \right\rangle 
\geq -\|\nabla J_\lambda(\theta) - \nabla J_\lambda(\theta')\|_2 \|\delta(x, \omega^*_\theta) - \delta(x, \theta)\|_2 \|\psi_{\theta'}(s, a)\|_2 
\geq -2C_\delta C_\psi \|\nabla J_\lambda(\theta) - \nabla J_\lambda(\theta')\|_2
\]
(110)

For the first term in \( I_1 \), we have
\[
\left\langle \nabla J_\lambda(\theta), \left( \hat{\delta}(x, \omega^*_\theta) - \delta(x, \theta) \right) \psi_{\theta'}(s, a) \right\rangle - \left\langle \nabla J_\lambda(\theta'), \left( \hat{\delta}(x, \omega^*_\theta) - \delta(x, \theta) \right) \psi_{\theta'}(s, a) \right\rangle 
\geq -2C_\delta C_\psi \|\nabla J_\lambda(\theta) - \nabla J_\lambda(\theta')\|_2
\]
where the last inequality is due to the \( L_\lambda \)-Lipschitz continuity of policy gradient shown in Proposition 1.
For the second term in $I_1$, we have

$$
\langle \nabla J_\lambda(\theta'), \left( \delta(x, \omega^\theta) - \delta(x, \theta) \right) \psi_P(s, a) \rangle - \langle \nabla J_\lambda(\theta'), \left( \delta(x, \omega^\theta) - \delta(x, \theta') \right) \psi_P(s, a) \rangle
$$

$$
= \left \langle \nabla J_\lambda(\theta'), \left( \delta(x, \omega^\theta) - \delta(x, \omega^\theta) + \delta(x, \theta') - \delta(x, \theta) \right) \psi_P(s, a) \right \rangle
$$

$$
\geq -L_V C_P \left \| \delta(x, \omega^\theta) - \delta(x, \omega^\theta) + \delta(x, \theta') - \delta(x, \theta) \right \|_2
$$

$$
\geq -L_V C_P \left \| \gamma \phi(s^T) (\omega^\theta - \omega^\theta) + \phi(s^T) (\omega^\theta - \omega^\theta) + \gamma V_{\pi^\theta}(s') - \gamma V_{\pi^\theta}(s') + V_{\pi^\theta}(s) - V_{\pi^\theta}(s) \right \|
$$

$$
\geq -L_V C_P \left \| \gamma \| \omega^\theta - \omega^\theta \|_2 + \| \omega^\theta - \omega^\theta \|_2 + \gamma |V_{\pi^\theta}(s') - V_{\pi^\theta}(s')| + |V_{\pi^\theta}(s) - V_{\pi^\theta}(s)| \right \|
$$

$$
\geq -L_V C_P \left \| \gamma L_\omega \| \theta - \theta' \|_2 + L_\omega \| \theta - \theta' \|_2 + \gamma L_V \| \theta - \theta' \|_2 + L_V \| \theta - \theta' \|_2 \right \|
$$

$$
= -L_V C_P (L_\omega + L_V)(1 + \gamma) \| \theta - \theta' \|_2,
$$

(111)

where the last inequality is due to the $L_\omega$-Lipschitz continuity of $\omega^\theta$ shown in Proposition 2 and $L_V$-Lipschitz continuity of $V_{\pi^\theta}(s)$. Collecting the upper bounds of $I_1$ yields

$$
I_1 \geq -(2C_\delta C_\psi L_\lambda + L_V C_\psi (L_\omega + L_V)(1 + \gamma)) \| \theta - \theta' \|_2.
$$

(112)

First we bound $I_2$ as

$$
\mathbb{E}[I_2] = \mathbb{E} \left[ \Delta_2(x, \theta') - \Delta_2(\bar{x}, \theta') \mid \theta', \hat{s}_{t-m} \right]
$$

$$
\geq - \mathbb{E} \left[ \Delta_2(x, \theta') \mid \theta', \hat{s}_{t-m} \right] - \mathbb{E} \left[ \Delta_2(\bar{x}, \theta') \mid \theta', \hat{s}_{t-m} \right]
$$

$$
\geq -\sup_x \left \| \Delta_2(x, \theta') \right \|_TV \mathbb{E} \left[ \mathbb{P}(x \in \cdot | \theta', \hat{s}_{t-m}) - \mathbb{P}(\bar{x} \in \cdot | \theta', \hat{s}_{t-m}) \right]
$$

$$
\geq -4L_V C_\psi C_\delta d_{TV} \left( \mathbb{P}(x \in \cdot | \theta', \hat{s}_{t-m}), \mathbb{P}(\bar{x} \in \cdot | \theta', \hat{s}_{t-m}) \right)
$$

$$
= -4L_V C_\psi C_\delta d_{TV} \left( \mathbb{P}(s_t, \hat{s}_{t-m} \mid \theta, \hat{s}_{t-m}), \mathbb{P}(\tilde{s}_{t-m}, \hat{s}_{t-m} \mid \theta', \hat{s}_{t-m}) \right)
$$

$$
\geq -2L_V C_\psi C_\delta |A| L_{\pi} \sum_{k=1}^{d_m} \mathbb{E} \left[ \| \theta_{k-i} - \theta_{k-d_m} \|_2 \mid \theta', \hat{s}_{t-m} \right],
$$

(113)

where the second inequality is due to the definition of TV norm, the last inequality follows (28) in Lemma 3, and the second last inequality follows the fact that

$$
\| \Delta_2(x, \theta') \|_2 \leq \| \nabla J_\lambda(\theta') \|_2 \| \delta(x, \omega^\theta) - \delta(x, \theta') \| \psi_P(s, a) \|_2 \leq 2L_V C_\delta C_\psi.
$$

(114)

Taking total expectation on both sides of (113) yields

$$
\mathbb{E}[I_2] \geq -2L_V C_\psi C_\delta |A| L_{\pi} \sum_{k=1}^{d_m} \mathbb{E} \left[ \| \theta_{k-i} - \theta_{k-d_m} \|_2 \right].
$$

(115)

Next we bound $I_3$ as

$$
\mathbb{E}[I_3] = \mathbb{E} \left[ \Delta_2(\bar{x}, \theta') - \Delta_2(\bar{x}, \theta') \mid \theta', \hat{s}_{t-m} \right]
$$

$$
\geq - \mathbb{E} \left[ \Delta_2(\bar{x}, \theta') \mid \theta', \hat{s}_{t-m} \right] - \mathbb{E} \left[ \Delta_2(\bar{x}, \theta') \mid \theta', \hat{s}_{t-m} \right]
$$

$$
\geq -\sup_x \left \| \Delta_2(x, \theta') \right \|_TV \mathbb{E} \left[ \mathbb{P}(x \in \cdot | \theta', \hat{s}_{t-m}) - \mathbb{P}(\bar{x} \in \cdot | \theta', \hat{s}_{t-m}) \right]
$$

$$
\geq -4L_V C_\psi C_\delta d_{TV} \left( \mathbb{P}(x \in \cdot | \theta', \hat{s}_{t-m}), \mathbb{P}(\bar{x} \in \cdot | \theta', \hat{s}_{t-m}) \right)
$$

$$
= -4L_V C_\psi C_\delta d_{TV} \left( \mathbb{P}(s_t, \hat{s}_{t-m} \mid \theta, \hat{s}_{t-m}), \mathbb{P}(\tilde{s}_{t-m}, \hat{s}_{t-m} \mid \theta', \hat{s}_{t-m}) \right)
$$

$$
\geq -2L_V C_\psi C_\delta |A| L_{\pi} \sum_{k=1}^{d_m} \mathbb{E} \left[ \| \theta_{k-i} - \theta_{k-d_m} \|_2 \right].
$$

(116)
where the second inequality is due to the definition of TV norm, and the last inequality follows (114).

The auxiliary Markov chain with policy \( \pi_{\theta'} \) starts from initial state \( s_{t-m+1} \), and \( \bar{s}_t \) is the \((m-1)\)th state on the chain. Following Lemma 2, we have:

\[
d_{TV} \left( P(x \in \cdot | \theta', \bar{s}_{t-m+1}), d_{\theta'} \otimes \pi_{\theta'} \otimes \hat{P} \right) = d_{TV} \left( P \left( (\bar{s}_t, \bar{a}_t, \bar{s}_{t+1}) \in \cdot | \theta', \bar{s}_{t-m+1} \right), d_{\theta'} \otimes \pi_{\theta'} \otimes P \right) \leq \kappa \rho^{m-1}. \tag{117}
\]

Substituting the last inequality into (116) and taking total expectation on both sides yield

\[
E[I_3] \geq -4L_V C_{\psi} C_{\delta} \kappa \rho^{m-1}. \tag{118}
\]

We bound \( I_4 \) as

\[
E[I_4 | \theta'] = E \left[ \left\langle \nabla J_\lambda(\theta'), \left( \hat{\delta}(\bar{x}, \omega_{\theta'}) - \delta(\bar{x}, \theta') \right) \psi_{\theta'} (s, a) \right\rangle | \theta' \right]
\]

\[
\geq -L_V C_{\psi} E \left[ \left| \hat{\delta}(\bar{x}, \omega_{\theta'}) - \delta(\bar{x}, \theta') \right| | \theta' \right]
\]

\[
= -L_V C_{\psi} E \left[ \gamma \left( \phi(\bar{s})^T \omega_{\theta'} - V_{\pi_{\theta'}}(\bar{s}) \right) + V_{\pi_{\theta'}}(\bar{s}) - \phi(\bar{s})^T \omega_{\theta'} \right] \left| \theta' \right]
\]

\[
\geq -L_V C_{\psi} \left( \gamma E \left[ |\phi(\bar{s})^T \omega_{\theta'} - V_{\pi_{\theta'}}(\bar{s})| \left| \theta' \right| \right] + E \left[ |V_{\pi_{\theta'}}(\bar{s}) - \phi(\bar{s})^T \omega_{\theta'}| \left| \theta' \right| \right] \right)
\]

\[
\geq -L_V C_{\psi} \left( \gamma \sqrt{E \left[ |\phi(\bar{s})^T \omega_{\theta'} - V_{\pi_{\theta'}}(\bar{s})|^2 \left| \theta' \right| \right]} + \sqrt{E \left[ |V_{\pi_{\theta'}}(\bar{s}) - \phi(\bar{s})^T \omega_{\theta'}|^2 \left| \theta' \right| \right]} \right)
\]

\[
= -L_V C_{\psi} \left( \gamma \sqrt{E_V [\phi(\bar{s})^T \omega_{\theta'} - V_{\pi_{\theta'}}(\bar{s})]^2} + \sqrt{E_V [V_{\pi_{\theta'}}(\bar{s}) - \phi(\bar{s})^T \omega_{\theta'}]^2} \right)
\]

\[
\geq -L_V C_{\psi} (1 + \gamma) \epsilon_{app}, \tag{119}
\]

where the second last inequality follows Jensen’s inequality.

Taking total expectation on both sides of (108), and collecting lower bounds of \( I_1, I_2, I_3 \) and \( I_4 \) yield

\[
E \left[ \nabla J_\lambda(\theta), \left( \hat{\delta}(\bar{x}(k), \omega_k^*) - \delta(\bar{x}(k), \theta_k) \right) \psi_{\theta_{\kappa-\tau_k}} (\hat{s}(k), \hat{a}(k)) \right]
\]

\[
\geq -D_2 E \| \theta_{\kappa-\tau_k} - \theta_{\kappa-d_m} \|_2 - D_3 E \| \theta_k - \theta_{\kappa-d_m} \|_2 - D_4 \sum_{i=\tau_k}^{d_m} E \| \theta_{k-i} - \theta_{k-d_m} \|_2
\]

\[
- D_5 \kappa \rho^{m-1} - L_V C_{\psi} (1 + \gamma) \epsilon_{app}, \tag{120}
\]

where \( D_2 := 2L_V L_{\psi} C_{\delta}, D_3 := (2C_{\delta} C_{\psi} L_{\lambda} + L_V C_{\psi} (L_{\omega} + L_V) (1 + \gamma)), D_4 := 2L_V C_{\psi} C_{\delta} |A| L_{\pi} \) and \( D_5 := 4L_V C_{\psi} C_{\delta}. \)

\textbf{Lemma 6} For any \( m \geq 1 \) and \( k \geq (K_0 + 1)m + K_0 + 1 \), we have

\[
E \left[ \nabla J_\lambda(\theta_k), \delta(\bar{x}(k), \theta_k) \psi_{\theta_{\kappa-\tau_k}} (\hat{s}(k), \hat{a}(k)) - \nabla J(\theta_k) \right]
\]

\[
\geq -D_6 E \| \theta_{\kappa-\tau_k} - \theta_{\kappa-d_m} \|_2 - D_7 E \| \theta_k - \theta_{\kappa-d_m} \|_2 - D_8 \sum_{i=\tau_k}^{d_m} E \| \theta_{k-i} - \theta_{k-d_m} \|_2 - D_9 \kappa \rho^{m-1},
\]

where \( d_m \leq (K_0 + 1)m + K_0, D_6 := L_V C_{\delta} L_{\psi}, D_7 := C_{\delta} L_{\lambda} + (1 + \gamma)L_{\pi}^2 C_{\psi} + 2L_V L_{\lambda}, D_8 := L_V C_{\delta} |A| L_{\pi} \) and \( D_9 := 2L_V C_{\delta}. \)
Proof For the worker that contributes to the $k$th update, we construct its Markov chain:

$$
\hat{s}_{t-m} \xrightarrow{\theta_{k-d_m}} \hat{a}_{t-m} \xrightarrow{\hat{P}} \hat{s}_{t-m+1} \xrightarrow{\theta_{k-d_{m-1}}} \hat{a}_{t-m+1} \cdots \hat{s}_{t-1} \xrightarrow{\theta_{k-d_1}} \hat{a}_{t-1} \xrightarrow{\hat{P}} \hat{s}_t \xrightarrow{\theta_{k-d_0}} \hat{a}_t \xrightarrow{\hat{P}} \hat{s}_{t+1},
$$

where $(s_t, a_t, \hat{s}_{t+1}) = (\hat{s}(k), \hat{a}(k), \hat{s}'(k))$, and $\{d_j\}_{j=0}^m$ is some increasing sequence with $d_0 := \tau_k$. By (88) in Lemma 4, we have $d_m \leq (K_0 + 1)m + K_0$.

Given $(\hat{s}_{t-m}, \hat{a}_{t-m}, \hat{s}_{t-m+1})$ and $\theta_{k-d_m}$, we construct an auxiliary Markov chain:

$$
\tilde{s}_{t-m} \xrightarrow{\theta_{k-d_m}} \tilde{a}_{t-m} \xrightarrow{\tilde{P}} \tilde{s}_{t-m+1} \xrightarrow{\theta_{k-d_{m-1}}} \tilde{a}_{t-m+1} \cdots \tilde{s}_{t-1} \xrightarrow{\theta_{k-d_1}} \tilde{a}_{t-1} \xrightarrow{\tilde{P}} \tilde{s}_t \xrightarrow{\theta_{k-d_0}} \tilde{a}_t \xrightarrow{\tilde{P}} \tilde{s}_{t+1}.
$$

First we have

$$
\begin{align*}
\left\langle \nabla J_\lambda(\theta_k), \delta(\hat{x}(k), \theta_k) & \psi_{\theta_{k-d_k}}(\hat{s}(k), \hat{a}(k)) - \nabla J(\theta_k) \right\rangle \\
= \left\langle \nabla J_\lambda(\theta_k), \delta(\tilde{x}(k), \theta_k) & \left( \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\tilde{s}(k), \tilde{a}(k)) \right) \right\rangle \\
& + \left\langle \nabla J_\lambda(\theta_k), \delta(\tilde{x}(k), \theta_k) \psi_{\theta_{k-d_m}}(\tilde{s}(k), \tilde{a}(k)) - \nabla J(\theta_k) \right\rangle.
\end{align*}
\tag{121}
$$

We bound the first term in (121) as

$$
\begin{align*}
\left\langle \nabla J_\lambda(\theta_k), \delta(\tilde{x}(k), \theta_k) \left( \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\tilde{s}(k), \tilde{a}(k)) \right) \right\rangle \\
& \geq -\| \nabla J_\lambda(\theta_k) \|_2 \| \delta(\tilde{x}(k), \theta_k) \|_2 \| \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \psi_{\theta_{k-d_m}}(\tilde{s}(k), \tilde{a}(k)) \|_2 \\
& \geq -L_V C_\delta \| \theta_{k-d_k} - \theta_{k-d_m} \|_2 \\
& \geq -L_V C_\delta \| \theta_{k-d_k} - \theta_{k-d_m} \|_2 \\
& \geq -L_V C_\delta \| \theta_{k-d_k} - \theta_{k-d_m} \|_2 \\
& \geq -L_V C_\delta \| \theta_{k-d_k} - \theta_{k-d_m} \|_2, \tag{122}
\end{align*}
$$

where the last inequality follows Assumption 3. Substituting (122) into (121) gives

$$
\begin{align*}
\left\langle \nabla J_\lambda(\theta_k), \delta(\tilde{x}(k), \theta_k) \left( \psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \nabla J_\lambda(\theta_k) \right) \right\rangle \\
& \geq -L_V C_\delta \| \theta_{k-d_k} - \theta_{k-d_m} \|_2 + \left\langle \nabla J_\lambda(\theta_k), \delta(\tilde{x}(k), \theta_k) \psi_{\theta_{k-d_m}}(\tilde{s}(k), \tilde{a}(k)) - \nabla J_\lambda(\theta_k) \right\rangle. \tag{123}
\end{align*}
$$

Then we start to bound the second term in (123). For brevity, we define

$$
\Delta_3(x, \theta) := \left\langle \nabla J_\lambda(\theta), \delta(x, \theta) \psi_{\theta_{k-d_m}}(s, a) - \nabla J(\theta) \right\rangle.
$$

Throughout the following proof, we use $\theta, \theta', x$ as shorthand notations of $\theta_k, \theta_{k-d_m}, x_t$ respectively. Define $\bar{x} := (\bar{s}_{t-d_m}, \bar{a}_{t-d_m}, \bar{s}'_{t-d_m})$ where $\bar{s}_{t-d_m}, \bar{a}_{t-d_m}$ are state and action on the auxiliary Markov chain, and $\bar{s}'_{t-d_m} \sim \mathcal{P}((\bar{s}_{t-d_m}, \bar{a}_{t-d_m})$ is a virtual sample.

We decompose $\Delta_3(x, \theta)$ as

$$
\Delta_3(x, \theta) = \Delta_3(x, \theta) - \Delta_3(x, \theta') + \Delta_3(x, \theta') - \Delta_3(\bar{x}, \theta') + \Delta_3(\bar{x}, \theta').
$$
We first bound $I_1$ as
\[
|I_1| = |\Delta_3(x, \theta) - \Delta_3(x, \theta')| \\
= |\langle \nabla J_\lambda(\theta), \delta(x, \theta)\psi_p(s, a) \rangle - \langle \nabla J_\lambda(\theta'), \delta(x, \theta')\psi_p(s, a) \rangle + \langle \nabla J_\lambda(\theta'), \nabla J(\theta') \rangle| \\
\leq |\langle \nabla J_\lambda(\theta), \delta(x, \theta)\psi_p(s, a) \rangle - \langle \nabla J_\lambda(\theta'), \delta(x, \theta')\psi_p(s, a) \rangle| + |\langle \nabla J_\lambda(\theta), \nabla J(\theta) \rangle - \langle \nabla J_\lambda(\theta'), \nabla J(\theta') \rangle| \\
\leq |\langle \nabla J_\lambda(\theta), \delta(x, \theta)\psi_p(s, a) \rangle - \langle \nabla J_\lambda(\theta'), \delta(x, \theta')\psi_p(s, a) \rangle| + 2L_V L_\lambda \| \theta - \theta' \|_2, \tag{124}
\]
where the last inequality uses the fact that $\langle \nabla J_\lambda(\theta), \nabla J(\theta) \rangle$ is $2L_V L_\lambda$-lipschitz continuous. We bound the first term in (124) as
\[
|\langle \nabla J_\lambda(\theta), \delta(x, \theta)\psi_p(s, a) \rangle - \langle \nabla J_\lambda(\theta'), \delta(x, \theta')\psi_p(s, a) \rangle| \\
\leq |\langle \nabla J_\lambda(\theta), \delta(x, \theta)\psi_p(s, a) \rangle - \langle \nabla J_\lambda(\theta), \delta(x, \theta')\psi_p(s, a) \rangle| + \langle \nabla J_\lambda(\theta), \delta(x, \theta')\psi_p(s, a) \rangle| \\
+ |\langle \nabla J_\lambda(\theta), \delta(x, \theta')\psi_p(s, a) \rangle - \langle \nabla J_\lambda(\theta'), \delta(x, \theta')\psi_p(s, a) \rangle| \\
= |\langle \nabla J_\lambda(\theta), (\delta(x, \theta) - \delta(x, \theta'))\psi_p(s, a) \rangle| + |\langle \nabla J_\lambda(\theta) - \nabla J_\lambda(\theta'), \delta(x, \theta')\psi_p(s, a) \rangle| \\
\leq L_V C_\psi \| \delta(x, \theta) - \delta(x, \theta') \| + C_p \| \nabla J_\lambda(\theta) - \nabla J_\lambda(\theta') \|_2 \\
= L_V C_\psi |(V_{\pi_p}(s') - V_{\pi_p}(s')) + V_{\pi_p}(s) - V_{\pi_p}(s)| + C_p \| \nabla J_\lambda(\theta) - \nabla J_\lambda(\theta') \|_2 \\
\leq L_V C_\psi |(V_{\pi_p}(s') - V_{\pi_p}(s')) + V_{\pi_p}(s) - V_{\pi_p}(s)| + C_p \| \nabla J_\lambda(\theta) - \nabla J_\lambda(\theta') \|_2 \\
\leq (C_p L_\lambda + (1 + \gamma) L_V^2 C_\psi) \| \theta - \theta' \|_2. \tag{125}
\]
Substituting the above inequality into (124) gives the lower bound of $I_1$:
\[
I_1 \geq - \left( C_p L_\lambda + (1 + \gamma) L_V^2 C_\psi + 2L_V L_\lambda \right) \| \theta - \theta' \|_2.
\]
First we bound $I_2$ as
\[
\mathbb{E}[I_2|\theta', \hat{s}_{t-m+1}] = \mathbb{E} \left[ \Delta_3(x, \theta') - \Delta_3(x, \theta')|0', \hat{s}_{t-m+1} \right] \\
\geq - \mathbb{E} \left[ \Delta_3(x, \theta')|0', \hat{s}_{t-m+1} \right] - \mathbb{E} \left[ \Delta_3(x, \theta')|0', \hat{s}_{t-m+1} \right] \\
\geq - \sup_x |\Delta_3(x, \theta')| \begin{vmatrix} \mathbb{P}(x \in |\theta', \hat{s}_{t-m+1}) - \mathbb{P}(x \in |\theta', \hat{s}_{t-m+1}) \end{vmatrix}_{TV} \\
\geq -2L_V (C_p + L_V) \| \mathbb{P}(x \in |\theta', \hat{s}_{t-m+1}) - \mathbb{P}(x \in |\theta', \hat{s}_{t-m+1}) \|_{TV} \\
\geq -2L_V (C_p + L_V) \| A |L_\pi \sum_{i=1}^{d_m} \mathbb{E}[\| \theta_{k-i} - \theta_{k-d_m} \|_2 |0', \hat{s}_{t-m+1}] \tag{126}
\]
where the second inequality is due to the definition of TV norm, the last inequality is due to (28) in Lemma 3, and the second last inequality follows the fact that
\[
|\Delta_3(x, \theta')| \leq \| \nabla J_\lambda(\theta) \|_2 (\| \delta(x, \theta)\psi_{k-d_m}(s, a) \|_2 + \| \nabla J(\theta) \|_2) \leq L_V (C_p + L_V). \tag{127}
\]
Taking total expectation on both sides of (126) yields
\[
\mathbb{E}[I_2] \geq -L_V (C_p + L_V) |A|L_\pi \sum_{i=1}^{d_m} \mathbb{E}[\| \theta_{k-i} - \theta_{k-d_m} \|_2]. \tag{128}
\]
Define \( \mathcal{F} := (\mathcal{S}, \mathcal{A}, \mathcal{P}) \), where \( \mathcal{S} \sim d_{\theta'}, \mathcal{A} \sim \pi_{\theta'} \) and \( \mathcal{P} \sim \mathcal{P} \). Then we can further decompose \( I_3 \) as

\[
\mathbb{E}[I_3|\theta', \hat{s}_{t-m+1}] = \mathbb{E}[\Delta_3(\mathcal{X}, \theta') - \Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}] + \mathbb{E}[\Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}].
\]  

The first term in (129) can be bounded as

\[
\mathbb{E}[\Delta_3(\mathcal{X}, \theta') - \Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}] 
\geq - \mathbb{E}[\Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}] - \mathbb{E}[\Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}]
\]

\[
\geq - \sup_x |\Delta_3(x, \theta')| \|\mathbb{P}(\hat{x} \in \cdot|\theta', \hat{s}_{t-m+1}) - \mathbb{P}(\mathcal{X} \in \cdot|\theta', \hat{s}_{t-m+1})\|_{TV}
\]

\[
\geq -2L_V(C_p + L_V)d_{TV} (\mathbb{P}(\hat{x} \in \cdot|\theta', \hat{s}_{t-m+1}), \mathbb{P}(\mathcal{X} \in \cdot|\theta', \hat{s}_{t-m+1}))
\]

\[
= -2L_V(C_p + L_V)d_{TV} (\mathbb{P}(\hat{x} \in \cdot|\theta', \hat{s}_{t-m+1}), d_{\theta'} \otimes \pi_{\theta'} \otimes \mathcal{P}) \leq \kappa \rho^{m-1}.
\]  

Substituting the last inequality into (130) yields

\[
\mathbb{E}[\Delta_3(\mathcal{X}, \theta') - \Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}] \geq -2L_V(C_p + L_V)\kappa \rho^{m-1}.
\]  

The second term in (129) is simply

\[
\mathbb{E}[\Delta_3(\mathcal{X}, \theta')|\theta', \hat{s}_{t-m+1}] = \mathbb{E}[\langle \nabla J_\lambda(\theta'), \delta(\mathcal{X}, \theta')\psi_{\theta'}(\bar{s}, \bar{a}) - \nabla J(\theta') \rangle |\theta', \hat{s}_{t-m+1}] = 0.
\]  

Substituting (131) and (132) into (129) yields

\[
\mathbb{E}[I_3|\theta', \hat{s}_{t-m+1}] \geq -2L_V(C_p + L_V)\kappa \rho^{m-1}.
\]  

Taking total expectation on \( \Delta_3(x, \theta) \) and collecting lower bounds of \( I_1, I_2, I_3 \) yield

\[
\mathbb{E}[\Delta_3(x, \theta)] \geq - (C_p L_\lambda + (1 + \gamma)L^2_\psi C_\psi + 2L_V L_\lambda) \mathbb{E}\|\theta_k - \theta_{k-d_m}\|_2
\]

\[
- L_V(C_p + L_V)|A|L_\pi \sum_{i=\tau_k}^{d_m} \mathbb{E}\|\theta_{k-i} - \theta_{k-d_m}\|_2 - 2L_V(C_p + L_V)\kappa \rho^{m-1}.
\]  

Taking total expectation on (123) and substituting the above inequality into it yield

\[
\mathbb{E}\left(\nabla J_\lambda(\theta_k), \delta(\hat{x}(k), \theta_k)\psi_{\theta_{k-d_m}}(\hat{s}(k), \hat{a}(k)) - \nabla J(\theta_k)\right)
\]

\[
\geq -D_6\mathbb{E}\|\theta_{k-d_m} - \theta_{k-d_m}\|_2 - D_7\mathbb{E}\|\theta_k - \theta_{k-d_m}\|_2 - D_8 \sum_{i=\tau_k}^{d_m} \mathbb{E}\|\theta_{k-i} - \theta_{k-d_m}\|_2 - D_9 \kappa \rho^{m-1},
\]  

where \( D_6 := L_V C_\lambda L_\psi, D_7 := C_p L_\lambda + (1 + \gamma)L^2_\psi C_\psi + 2L_V L_\lambda, D_8 := L_V(C_p + L_V)|A|L_\pi, \) \( D_9 := 2L_V(C_p + L_V) \).