Theory and observations of galactic dark matter

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Received August 14, 1998; accepted —

Abstract. Sir James Jeans’s (1902 and 1929) linear, acoustic, theory of gravitational instability gives vast errors for the structure formation of the early universe. Gibson’s (1996) nonlinear theory shows that nonacoustic density extrema produced by turbulence are gravitationally unstable at turbulent, viscous, or diffusive Schwarz scales \(L_{ST}, L_{SV}, L_{SD}\), independent of Jeans’s acoustic scale \(L_J\). Structure formation began with decelerations of \(10^{46}\) kg protosuperclusters in the hot plasma epoch, 13,000 years after the Big Bang, when \(L_{SV}\) decreased to the Hubble (horizon) scale \(L_H \equiv ct\), where \(c\) is light speed and \(t\) is time, giving \(10^{42}\) kg protogalaxies just before the cooled plasma formed neutral H-He gas at 300,000 years. In \(10^4\) years this primordial gas condensed to \(10^{23} - 10^{25}\) kg \(L_{SV} - L_{ST}\) scale objects, termed “primordial fog particles” (PFPs), that have either aggregated to form stars or still persist as micro-brown-dwarf (MBD) galactic “dark matter.” Using a precisely measured 1.1-year time delay, Schild (1996) suggested from continuous microlensing of quasar Q0957 + 561 A,B that the mass of the \(10^{42}\) kg lens galaxy is dominated by \(10^{23} - 10^{25}\) kg “rogue planets ... likely to be the missing mass”. A microlensing event seen at three observatories confirms Schild’s (1996) claims, and supports Gibson’s (1996) prediction that PFPs comprise most of the dark matter at galactic scales. The Tyson and Fischer (1995) mass profile of a \(10^{46}\) kg dense galaxy cluster suggests a superdiffusive, nonbaryonic, dark matter cluster-halo (CH) with \(L_{CH} \approx L_{SD} \gg L_{GH}\). Therefore nonbaryonic dark matter should be negligible at galaxy-halo, \(L_{GH}\), scales.

Send offprint requests to: C. H. Gibson
Key words: Cosmology: theory – dark matter – early Universe – gravitational lensing – large-scale structure formation of the Universe

1. Introduction

Most cosmologists agree that 99.7 to 99.9% of the mass of the universe is “dark matter,” the smallest part ordinary “baryonic” matter (electrons, protons and neutrons) and the largest part some unidentified, virtually collisionless, “nonbaryonic” material, that behaves like (and may just be) neutrinos \( \text{Carr 1994, McGaugh and de Blok 1998} \). This view of the world is required for galaxies to be bound by gravity with observed rotation rates, and for the universe to be marginally bound by gravity (flat) with observed chemical abundances matching the standard hot Big Bang scenario with inflation \( \text{Peebles 1993, Kolb and Turner 1994, Silk 1994, Guth 1997} \).

How is it possible for so much matter to be “missing” from galaxies and the universe? We suggest the “dark matter” paradox is mostly the result of inappropriate fluid mechanical theories being applied to highly scrambled and turbulent (viscous, diffusive, nonlinear) self-gravitational condensation situations of the actual universe, giving vastly incorrect predictions for the times, masses, and forms of matter structures as they evolved soon after the Big Bang under the influence of gravity and real fluid effects. Unfortunately, the field of cosmology \( \text{Weinberg 1972, Silk 1989, Peebles 1993, Padmanabhan 1993, Kolb and Turner 1994, Silk 1994 etc.} \) has relied exclusively on linear, inviscid, isentropic fluid theory (following \text{Jeans 1902, 1929}) by which self-gravitational condensation of a continuous body of gas with constant density \( \rho \) and constant pressure \( p \) may occur only on scales \( L_C \) larger than the Jeans acoustic length scale

\[
L_C \geq L_J \equiv \frac{V_S}{(\rho G)^{1/2}}
\]

where \( V_S \) is the speed of sound in the gas, and \( G \) is Newton’s gravitational constant \( 6.7 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \). Jeans’s condensation criterion in Eq. \( 1 \) follows from a linear perturbation stability analysis (LPSA) of the hydrodynamic momentum equations, with the “barotropic” assumption that pressure and density perturbations depend only on each other. Either the LPSA method or the barotropic assumption will reduce the problem to one of acoustics, without turbulence, without turbulent mixing, and without any validity whatsoever as a description of gravitational structure formation in cosmology or astrophysics.

Jeans’s criterion \( 1 \) forbids condensation of baryonic matter (by itself) to form such observed structures as galaxies and stars, but requires gravitational facilitation by a previous condensation of nonbaryonic matter in the plasma epoch that is impossible according to the present fluid mechanical theory. By hypothesis, cold dark matter (CDM) does not
respond to radiation pressure and consists of nonrelativistically moving massive particles with small enough $L_J$ to clump and collect galaxy masses of baryonic gas in gravitational potential wells (McGaugh and de Blok 1998). This idea fails because nonbaryonic material must be very weakly collisional or it would have been detected, with a diffusivity $D_{NB}$ much too large to permit condensation on galaxy scales because $L_G \ll L_{SD}$, independent of $L_J$ (§4). From the new theory and resulting nonlinear cosmology, gravitational decelerations of baryonic matter begin in the plasma epoch, first with protosupercluster and finally galaxy masses, at $L_{SV}$ length scales (Gibson 1996, 1997ab). The Jeans criterion of equation (4) is discarded. Instead, gravitational condensation may occur only on scales $L_C$ larger than the maximum Schwarz gravitational condensation scale $L_{SX}$

$$L_C \geq (L_{SX})_{\text{max}} \neq L_J; X = V, T, M, D$$

(2)
determined by viscous, turbulent, or magnetic forces, or by molecular diffusivity (§3 Eqs. 27, 28, 29, 20). The idea that radiation pressure, thermal pressure, or any other kind of “pressure support” prevents gravitational condensation is one further misconception (see §2.4) associated with the Jeans theory, among many.

In the following we review (§2) and revise (§3) the linear theory of self-gravitational condensation. A very different scenario for gravitational structure formation emerges at every stage of cosmological evolution (§4) that we term “nonlinear cosmology”. Nonlinear cosmology is as different from linear cosmology as a turbulent flow is different from a laminar flow, and for the same reasons. Use of the Jeans theory to describe gravitational condensation is equivalent to using the linearized Navier Stokes equations to describe flow of a real fluid at high Reynolds number, where the actual turbulent flow is vastly different from the laminar solutions obtained from the linearized equations. We focus on the time 300 000 years after the Big Bang when the cooling plasma neutralized to form primordial gas. By the new theory, the gas rapidly condensed at a scale $L_{SV}$ determined by viscous and gravitational forces to form a “fog” of H-He “particles” which still persist as “micro-brown-dwarfs” (MBDs) in galaxy halos as the dominant form of dark matter (Gibson 1996). The predicted mass of these “primordial fog particles” (PFPs) coincides with that of the objects noted by Schild 1996, from quasar-microlensing observations in the double image quasar Q0957+561 A,B, as “rogue planets ... likely to be the missing mass” of the quasar lens galaxy. New observations for Q0957+561 A,B are presented (§5) from three observatories, and all support the Schild 1.1-year time delay between images A,B and his microlensing interpretation from the image twinkling frequency. In (§6) the Tyson and Fischer (1995) dense galaxy cluster tomography data is interpreted using the new theory to estimate the diffusivity $D_{NB}$ of the nonbaryonic component of the dark matter. We find $D_{NB}$ is trillions of times larger than $D_B$ for baryonic matter, suggesting the galactic dark matter for the inner halo is mostly baryonic because any nonbaryonic
matter would diffuse to outer halo or cluster halo ($L_{SD}$) scales. Summary and conclusions are provided in (§7).

2. What is wrong with the Jeans gravitational instability theory?

Jeans's theory fails to accurately describe gravitational structure formation because it is linear, whereas gravitational structure formation is a very nonlinear process. Linear perturbation assumptions (e.g.: $\rho = \rho_0 + \rho'$) and the barotropic assumption $p = p(\rho)$ both reduce the problem to one of acoustics, as shown in detail by Kolb and Turner (1994, p342). Eulerian equations of Newtonian motion describing a perfect fluid are assumed, which neglect viscous forces or the gravitational or molecular diffusion of density. By dropping the convection term $v \cdot \nabla \rho$ in the density conservation equation, turbulent mixing is precluded. By dropping the inertial-vortex force $v \times \omega$ in the momentum equation, turbulence is precluded, where $v$ is velocity and $\omega$ is vorticity.

In the LPSA approach, density $\rho'$, pressure $p'$, velocity $v'$, and gravitational potential $\phi'$ perturbation equations, formed by dropping second order perturbation terms, are cross differentiated with respect to time and space and combined to give

$$\frac{\partial^2 \rho'}{\partial t^2} - V_S^2 \nabla^2 \rho' = 4\pi G \rho_0 \rho', \quad (3)$$

where $\rho'$ is the density perturbation about the mean density $\rho_0$, $t$ is time, and $V_S$ is the sound speed. Solutions are of the form

$$\rho'(r, t) = A \rho_0 \exp[-i k \cdot r + i \omega_S t], \quad (4)$$

where $r$ is position, and wavenumber $k$ and frequency $\omega_S$ satisfy the dispersion relation

$$\omega_S^2 = V_S^2 k^2 - 4\pi G \rho_0, \quad (5)$$

with $k \equiv |k|$. If $\omega_S$ is imaginary, the mode grows exponentially; if $\omega_S$ is real, the $\rho'$ mode propagates as a sound wave. For $k$ less than some critical value, $k_J$, $\omega_S$ will be imaginary, where

$$k_J = (4\pi G \rho_0 / V_S^2)^{1/2}, \quad (6)$$

giving the Jeans wavenumber criterion for gravitational instability. The usual physical explanation of this mathematical result is that very large wavelength, $\lambda$, sound waves provide density maxima with propagation time periods, $\lambda / V_S$, larger than the gravitational condensation, or “free-fall”, time, $\tau_G \equiv (\rho G)^{-1/2}$, so they can trigger gravitational condensation, but shorter waves cannot. We see below (§2.1.1) that even this interpretation of the Jeans theory is incorrect. The initial condensation scale, $L_{IC}$, is equal to $L_J$, but its physical basis has nothing to do with acoustics.
Nonacoustic density maxima and minima, particularly the cold spots and hot spots and helium spots and hydrogen spots that must exist in abundance in the primordial gas due to past or present turbulent mixing, are gravitationally unstable to condensation and void formation, but are assumed out of existence by the linear Jeans theory. Reynolds numbers of the primordial gas are well above critical \((Re_{PG} \approx 10^9)\) so the flow will certainly be turbulent at the Kolmogorov scale \(L_K\) and larger (Gibson 1996). Turbulence is defined as an eddy-like state of fluid motion where the inertial-vortex forces \(\mathbf{v} \times \mathbf{\omega}\) of the eddies are larger than any other forces that tend to damp the eddies (Gibson 1991). Turbulence first appears at \(L_K \equiv (\nu^3/\varepsilon)^{1/4}\) and cascades to larger scales by an eddy pairing mechanism, where \(\varepsilon\) is the viscous dissipation rate. The Reynolds number \(Re \equiv (\nu \times \mathbf{\omega})/(\nu \nabla^2 \mathbf{v})\) is the ratio of inertial-vortex to viscous forces, where \(\nu\) is the kinematic viscosity. Temperature fluctuations \(\delta T/T \approx 10^{-5}\) have been detected in the primordial gas by the COBE (COsmic Background Explorer) satellite. These give density gradients that will be scrambled by the turbulence to produce nonacoustic density microstructure which is unstable to gravitational condensation and void formation at length scales determined by fluid mechanical constraints (i.e.: Eq. 2), independent of the sound speed \(V_S\) or the Jeans scale \(L_J\).

Jeans showed that without gravitation, acoustic density perturbations propagate in the continuous gas with sound speed \(V_S\), but with gravitation the speed decreases to zero as the wavelength increases to \(L_J\) and becomes imaginary for larger values (\(\S\) 2.3 Eq. 11). This result of the Jeans analysis has been widely interpreted as proof that no condensation can occur on scales smaller than \(L_J\), as proposed by Jeans 1902, 1929. However, this interpretation is incorrect because gravitational condensation on nonacoustic density maxima to form “primordial fog particles” (PFPs) and gravitational expansion of nonacoustic density minima to form voids was not and could not be considered in the linear Jeans analysis, and cannot be described by LPSA with or without the barotropic assumption. Even when several terms that Jeans neglected are included and the barotropic assumption is dropped, the acoustic mode at the Jeans scale is still the only one that is unstable (Ned Wright, personal communication 1997). By definition, the LPSA method neglects nonlinear terms of the momentum and density conservation equations, but these are crucial to the formation of turbulence and the creation by turbulence of nonacoustic density extrema, saddle points, and other zero density gradient configurations that determine the gravitational creation of structure (\(\S\) 3.2).

Although LPSA has a long history in the fluid dynamics literature, the technique has generally been supplanted for problems involving turbulence and turbulent mixing by universal similarity methods, pioneered by Kolmogorov and Obukhov, involving model-
based regimes of critical length, time, and scalar field scales and self-similar, nonlinear, cascades over wide space-time ranges [Gibson 1991].

2.1. Can anything in the Jeans theory be salvaged?

According to the new, nonlinear, gravitational condensation theory, presented here, the Jeans condensation criterion is completely irrelevant, and is replaced by new criteria based on the Schwarz fluid mechanical scales. The equation for the Jeans scale can be written

\[ L_J \equiv V_S / (\rho G)^{1/2} \approx \left( \frac{p}{\rho} / \rho G \right)^{1/2} \approx \left( \frac{RT}{\rho G} \right)^{1/2} \]  \tag{7}

where \( p \) is pressure, \( R \) is the gas constant for the particular ideal gas or mixture of gases, and \( T \) is the temperature. Let us examine the two forms on the right.

2.1.1. Jeans’s initial condensation scale \( L_{IC} \)

From the last term of Eq. (7), the Jeans scale \( L_J \) can be reinterpreted as the initial gravitational condensate size \( L_{IC} \) in an infinite, motionless gas with uniform \( R, \rho, p \) and \( T \) values, so that

\[ L_{IC} \equiv \left( \frac{RT}{\rho G} \right)^{1/2} \approx L_J. \]  \tag{8}

The fact that the mass of globular clusters of stars closely matches the Jeans mass at decoupling is the strongest evidence supporting the Jeans theory. However, we see from the definition of \( L_{IC} \) that the physical basis of the initial condensation mass of the primordial gas to form proto-globular-clusters (PGCs) is an artifact of the ideal gas law \( p/\rho = RT \). In an isothermal gas this relation permits nonacoustic density maxima to increase and nonacoustic density minima to decrease with precisely compensating pressure changes which maintain constant temperature in quasi-equilibrium during this initial condensation process, so that radiative heat transfer cannot compensate for the gravitational effects. This initial condensation instability, and \( L_{IC} \), have nothing to do with acoustics or Jeans’s LPSA theory.

2.1.2. Jeans’s hydrostatic scale \( L_{HS} \)

Now consider the penultimate term in Eq. (7) that includes the pressure \( p \). Condensations on the numerous and inevitable nonacoustic density maxima within an initial \( L_{IC} \) sized Jeans “droplet” are not prevented, as often assumed, by the constant internal pressure \( p \) or constant temperature \( T \) of the \( L_{IC} \) droplet or by its small hydrostatic pressure gradients. The strongest nonacoustic density maxima are in effect absolutely unstable with respect to gravitational condensation in a motionless fluid because their sizes are
limited only by molecular diffusivity or hydrodynamic forces at the appropriate Schwarz scale \((L_{SX})_{\text{max}} \ll L_{IC}\) (Eq. 2), as discussed in the following (3), and immediately begin to collect mass from their surroundings by gravitational forces. As the densities of such nonacoustic condensates increase and their sizes decrease, their individual core internal pressures respond by increasing to values that decelerate the condensing gas and eventually provide internal hydrostatic equilibrium when their supply of mass ceases upon the arrival of a void. Their local Jeans scales \(L_J\) therefore decrease as an effect of the condensation, not as a cause, so \(L_J\) in this case should be reinterpreted as a hydrostatic length scale \(L_{HS}\); that is,

\[
L_{HS} \equiv \left(\frac{|p/\rho|}{\rho G}\right)^{1/2} \approx L_J
\]

where it is assumed that the object is near hydrostatic equilibrium surrounded by a void with zero pressure. Again, the hydrostatic equilibrium process, and \(L_{HS}\), have nothing to do with acoustics or Jeans’s LPSA theory. Neither have the concepts of “pressure support” or “thermal support” commonly used to justify Jeans’s theory.

### 2.2. Problems with previous attempts to salvage the Jeans theory

The usual assumptions are incorrect (Chandrasekhar 1951, 1961; Bonazzola et al. 1987, 1992) that a constant internal pressure of a gas prevents gravitational condensation and that other forces (turbulence, viscous, magnetic) simply add additional “pressures” which increase the minimum condensation scale. For example, Chandrasekhar (1951) proposed a minimum condensation scale

\[
L_{IC} \equiv \left(\frac{V_S^2 + V^2/2}{(\rho G)}\right)^{1/2} \geq L_J
\]

where \(V\) is a characteristic turbulent velocity with a “turbulence pressure,” \(p_T/\rho = V^2/2\). One problem with this expression is that the “sonic pressure,” \(p_S/\rho = V_S^2\), should be dropped since \(p_S\) is constant on the scale of the condensate and larger, and a constant pressure provides no force to resist condensation. Other forces prevent condensation, but at larger or smaller scales that are independent of the initial value of \(L_{IC} \approx L_J\). Correct expressions for minimum condensation scales result when the \(p_S/\rho = V_S^2\) term is not included and various expressions for the limiting stresses divided by density are used. The correct “turbulence pressure” depends on the length scale of the condensation, as shown in the derivation of \(L_{ST}\) in Section 3.1. The largest fluid mechanical stress or the diffusivity of the gas determines the initial condensation scale for a nonacoustic density nucleus, and this scale will be larger than subsequent local hydrostatic scales \(L_{HS} \approx L_J\).
2.3. What Jeans thought he was doing

Jeans (1929, Chapter XII) was concerned with disputing the “conjecture” that the luminous cores of the “great nebulae” might be star-clouds rather than gas clouds. At the time, telescopes could not resolve stars in elliptical galaxies or in the cores of spiral galaxies, both of which Jeans believed were composed of hot luminous gas without any stars. He speculated that the sources of the hot gas emerging at such spiral galaxy cores might be connections to other universes. Several arguments were put forth by Jeans to prove his erroneous hypotheses, including the linear, acoustic, analysis leading to $L_J$.

Based on his (1902) acoustic gravitational theory, Jeans (1929, p349) explained that elliptical nebulae and the cores of spiral nebulae could not condense to form stars because $V_S$ would be too large at the high gas temperatures he assumed must exist in these luminous regions; that is, temperatures giving $L_J$ values larger than the region sizes. Only in the spiral arms, which he suggested were thrown out into the cold by centrifugal forces, could stars form, since radiative cooling of the spiral arms should reduce $V_S$ and $L_J$ to values much smaller than those in the hot core, and smaller than the sizes of the arms, so that condensation to form stars could begin. Of course we now know that elliptical “nebulae” and the cores of spiral “nebulae” are not starless luminous gas but galaxies of $\approx 10^{11}$ stars, that spiral galaxy arms are not thrown out by centrifugal forces, and that the individual “stars” Jeans identified in the arms of spiral nebulae were actually star-burst knots of millions of stars.

Jeans (1929, p345) assumed that “the medium has found a way of acquiring kinetic energy in this way indefinitely,” but since the velocity of any affected acoustic waves rapidly becomes zero, little or no increase in kinetic energy could actually result. Any condensations on acoustic scales $L_J$ might generate more sound waves on the same scale, thus amplifying the Jeans scale condensation effect by resonance. However, acoustic density maxima become nonacoustic density maxima if they accumulate significant quantities of mass by gravitational condensation since they will also accumulate the momentum per unit mass of the medium, which is zero rather than $V_S$. Jeans (1929, p348) himself showed that sound waves do not propagate with velocity $V_S \approx (dp/d\rho)^{1/2}$ when the gravitational term is retained in the momentum equation, but with velocity $V_S(\lambda)$ given by

$$V_S(\lambda) = [\rho G (\lambda_J^2 - \lambda^2)/\pi]^{1/2},$$

where $\lambda_J^2 = \pi V_S^2 / \rho G = \pi L_J^2$. Thus, self-gravitational acoustic density maxima for sound waves with wavelengths $\lambda \approx \lambda_J$ are equivalent to nonacoustic density maxima because $V_S(\lambda_J) = 0$ from Eq. (11).
2.4. What about pressure support and thermal support?

The Jeans criterion is sometimes derived (Shu 1982, p395, Problem 16.14; Weinberg 1993, p176) by suggesting that the internal pressure (or internal energy) of an infinite, homogeneous gas can resist gravitational condensation. However, a constant internal pressure within a region cannot resist gravitational condensation in its interior. Pressure forces require pressure gradients. Terms like “pressure support” and “thermal support” may apply to a star where the external pressure is negligible, but not to the condensation and void instabilities existing within a large body of nearly homogeneous, constant pressure gas with embedded nonacoustic density minima and maxima, and their associated minimum and maximum saddle points. At maximum saddle points, large gravitational tearing stresses develop to form voids that enclose the associated density maximum, and at the maximum point gravitational condensation forces monotonically increase with the condensation. Such “pressure support” and “thermal support” derivations of $L_J$ are good examples of bad dimensional analysis; that is, dimensional analysis based on an incorrect, misleading, or non-existent physical model. Setting the condensation length $L_C$ equal to a function of $\rho$, $G$, and $p$ gives $L_C$ proportional to $(p/\rho)^{1/2}/(\rho G)^{1/2}$, so $L_C \approx L_J$ since $(p/\rho)^{1/2} \approx V_S$. What correct physical model is represented? This hydrostatic analysis only makes sense if the surroundings have zero pressure and density and $p$ is the internal hydrostatic pressure.

As discussed previously, $L_{HS} \approx L_J$ for any gravitationally bound gaseous object can be interpreted as a reflection of its hydrostatic equilibrium, which ultimately is determined by a complex energy balance. Hydrostatic equilibrium is established in a spherical gas blob between pressure gradients and gravitational forces in the radial direction for this model. The gravitational force

$$F_G = G \times \rho L^3 \times \rho L^3 / L^2 = \rho^2 G L^4$$

is balanced by the pressure force

$$F_P = p \times L^2$$

so

$$L_{HydroStatic} \equiv L_{HS} \equiv [(p/\rho)/\rho G]^{1/2} \approx V_S/(\rho G)^{1/2} \equiv L_J$$

where $F_G = F_P$ and $V_S$ is the speed of sound.

Neither internal pressures nor hydrostatic pressure gradients can prevent gravitational condensation on embedded nonacoustic density nuclei if $L_{ST}$, $L_{SV}$ and $L_{SD}$ are $\ll L_J$. On the other hand, no condensations on scale $L_J$ are possible if $L_{ST}$, $L_{SV}$ or $L_{SD}$ are $\gg L_J$. Thus, $L_J$ may overestimate or underestimate the minimum scale of gravitational condensation and is therefore quite irrelevant as a gravitational criterion: Eq. (1) is incorrect, obsolete, and misleading, and is replaced by Eq. (2).
3. A new, nonlinear, theory of gravitational structure formation

A very different self-gravitational condensation theory and cosmology results when condensation is permitted to occur on nonacoustic density nuclei (Gibson 1988, 1996, 1997ab); i.e., points produced by turbulent mixing of density in the primordial gas with maximum and minimum density that are either geometrically symmetric and therefore nearly stationary with respect to the fluid motion, or which move toward symmetric stationary points by molecular diffusion (Gibson 1968). According to the new theory, the criterion for self-gravitational condensation of density fluctuations on scale $L_C$ is

$$L_C \geq (L_{SX})_{\text{max}}; \ X = V, T, M, D$$

(15)

where subscript $X$ of $L_{SX}$ shows whether gravitational condensation is limited by viscous, turbulent, or magnetic forces, or by molecular diffusion, respectively (Gibson 1996).

Density extremum points in great numbers are produced by turbulent mixing of temperature, species concentration and any other fluid property perturbations that affect density, and may persist as fossil turbulence long after the turbulence that produced them has disappeared. The nonlinear advection terms $v \cdot \nabla \rho$ in the density conservation equation

$$\frac{\partial \rho}{\partial t} + v \cdot \nabla \rho = [D_\rho - L^2 \tau_G] \nabla^2 \rho$$

(16)

and $(v \cdot \nabla)v = [\nabla(v^2/2) - v \times \omega]$ in the momentum conservation equation

$$\frac{\partial v}{\partial t} = -\nabla \left( p/\rho + v^2/2 \right) + v \times \omega + \nu \nabla^2 v + F_G + F_M,$$

(17)

are crucial to the turbulence and turbulent production of nonacoustic density extremum points, where $D_\rho$ is the molecular diffusivity of density, $L^2 \tau_G$ is the gravitational diffusivity, $L$ is the distance to the nucleus, $\tau_G \equiv (\rho G)^{-1/2}$ is the gravitational free fall time, $F_G$ is the gravitation force, $F_M$ is the magnetic force, and $p$ is pressure.

Theories like Jeans’s that drop the nonlinear terms fail to correctly describe gravitational condensation in fluids that are turbulent or that have been turbulent, with viscous forces and molecular and gravitational diffusivity of density which may affect the smallest possible gravitational condensation sizes. Magnetic forces $F_M$ are presumably not important in the primordial gas condensation, but certainly arise once star formation begins.

Before nonlinear gravitational condensation and void production begins, the density diffusivity $D_\rho$ simply represents a weighted average molecular diffusivity of temperature and helium concentration, depending on what linear combination of Fick’s law for concentration and Fourier’s law for temperature dominates the initial small density fluctuations at scales smaller than $(L_{SX})_{\text{max}}$. The sources of density for a fluid particle (the right
hand side of Eq. (16) are gravity diffusion, thermal diffusion (cooling), or helium diffusion from the surroundings. Sources and sinks of temperature and helium concentration are neglected, so the equation does not apply to the final stages of condensation (e.g., in stars) where these may be important. At scales larger than \((L_{SX})_{\text{max}}\), gravitational forces become important and the effective density diffusivity \(D_{\text{eff}} \equiv [D_{\rho} - L^2 \tau_G]\) becomes negative. Gravity driven “diffusion” up density gradients causes density maxima to grow without limit to form PFPs, and density minima to decrease without limit to form voids. Any CDM clumps on scales \(L \leq L_{SD}\) would simply diffuse away.

In natural fluids, density, temperature, and species concentration extrema are produced by turbulence, and persist in numerous quantities at Batchelor scales \(L_B \equiv (D/\gamma)^{1/2}\) of the turbulence or fossil turbulence (Gibson 1968, 1986, 1988), where \(D\) is the molecular diffusivity of such conserved scalar quantities and \(\gamma\) is the local rate of strain. Fossil turbulence means fluctuations in hydrophysical fields like density and vorticity produced by turbulence that persist after the fluid is no longer turbulent at the scale of the fluctuation. Most of the density microstructure of the ocean and atmosphere is fossil density turbulence (Gibson 1986, 1991, 1996). The Batchelor scale reflects a local equilibrium between \(\gamma\), which produces more zero gradient points by stretching and compressing strong points till they split, and \(D\) which damps out weak ones and expands the size of strong ones to an equilibrium size \(L_B\) for all Prandtl number values \(Pr = \nu/D\). More information about the kinematics of zero gradient points, lines and minimal gradient surfaces is given in Gibson (1968) and in the derivation of the diffusive Schwarz scale \(L_{SD}\) that follows below (Eq. [20]).

In the nearly motionless and homogeneous primordial gas, the strongest nonacoustic density minima are absolutely unstable to gravitationally driven expansion to form voids, and the strongest nonacoustic maxima are absolutely unstable to gravitationally driven condensation to form dense gaseous objects (PFPs), both with hydrodynamically or diffusively limited length scales \((L_{SX})_{\text{max}}\) much smaller than \(L_J\). Proto-PFP masses \((M_{SX})_{\text{min}} = [(L_{SX})_{\text{max}}]^3 \rho \ll M_J = (L_J)^3 \rho\) are \(10^{11} - 10^{13}\) less than the Jeans mass \(M_J\), contrary to the Jeans theory. In the chaotic field of numerous nonacoustic density maxima and minima in competition for mass and space, the strongest density maxima will dominate and absorb weaker neighbors as they condense to the maximum density permitted by their mass (contrary to fragmentation theories of the first condensation; e.g., Low and Lynden-Bell 1976), just as the density minima with the smallest density values devour neighboring weaker minima to form voids with the maximum size and minimum density permitted by the PFP objects these voids evolve to surround and contain.

It is crucial to recognize that not all density maxima in fluids are associated with sound waves. In natural fluids, most \(\rho^{\text{max}}\) are not. Density depends not only on pressure,
but on temperature and species concentration. As previously discussed, nonacoustic density maxima $\rho_{\text{max}}$ result from turbulent scrambling (stirring, mixing, and diffusion) of temperature and concentration gradients (Gibson 1968, Gibson et al. 1988). Nonacoustic density maxima move with the fluid velocity if the density distribution about the maximum is symmetric, or will diffuse with respect to the fluid toward a position of symmetry and then move with the fluid velocity. If the velocity of the fluid is $v$, the velocity of a density maximum $v_{\rho_{\text{max}}}$ is given by

$$v_{\rho_{\text{max}}} = v - D \left[ \left( \begin{array}{c} \rho_{j1} \\ \rho_{11} \\ \rho_{j2} \\ \rho_{22} \\ \rho_{j3} \\ \rho_{33} \end{array} \right), \left( \begin{array}{c} \rho_{jj1} \\ \rho_{jj2} \\ \rho_{jj3} \end{array} \right) \right] = v + v_{\rho_{\text{max}}}^D,$$

(18)

where the coordinate system is locally aligned with the principal axes of the tensor $\rho_{ij}$ with principal values $\rho_{11}$ etc., $\rho_{jj}$ represents $\nabla^2 \rho$, and $\rho_{jj}$ etc. are the components of the vector $\nabla (\rho_{jj})$. One can show from the equation that the direction of the diffusive velocity is toward a position of symmetry where $\nabla (\rho_{jj}) = 0$ and $v_{\rho_{\text{max}}} = v$.

The velocity of an isodensity surface $v_\rho$ with diffusivity $D$ is given (Gibson 1968) by

$$v_\rho = v - D \left( \frac{\nabla^2 \rho}{|\nabla \rho|} \right) s = v + v_\rho^D,$$

(19)

where $s$ is the unit vector of $\nabla \rho$. Neglecting convection, a nonacoustic density maximum of scale $L$ will grow when the damping diffusive velocity $|v_\rho^D| \approx D/L$ is of the same order as the velocity of gravitational collapse (on a density maximum), or expansion (to form a void from a density minimum), $L (\rho G)^{1/2}$. Equality of the two velocities occurs at a diffusive-gravitational scale termed the “diffusive Schwarz radius”

$$L_{SD} \equiv \left( \frac{D^2}{\rho G} \right)^{1/4},$$

(20)

which represents the scale of the smallest nonacoustic density maximum that can grow by gravitational condensation when the molecular diffusivity of the density is $D$ and the flow is “quiescent”. The dynamical equivalent of $L_{SD}$ in turbulent mixing theory is the Obukhov-Corrsin scale $L_{OC} \equiv (D^3/\varepsilon)^{1/4}$, which is obtained (Gibson 1968) by setting the diffusive velocity $D/L$ of a scalar field with diffusivity $D$ equal to the Kolmogorov (turbulence) velocity $(\varepsilon L)^{1/3}$ at scale $L$. For strongly diffusive scalars with $Pr \geq 1$, $L_{OC}$ represents the smallest sized turbulent eddy that can scramble zero gradient configurations (ZGCs) from a region of uniform scalar gradient.

Gravitational condensation of nonbaryonic fluids, which have enormous diffusivity values $D_{NB}$, is prevented by diffusion for nonacoustic density nuclei, assuming $L_{SD}$ is larger than Schwarz scales $L_{SV}$ or $L_{ST}$ that would otherwise provide the condensation criterion. This new length scale is related to the gravitational-inertial-viscous length scale $L_{GIV} \equiv (\nu^2/\rho G)^{1/4}$ (Gibson 1996) by

$$L_{SD} = L_{GIV} Pr^{-1/2},$$

(21)
where the generalized Prandtl number $Pr \equiv \nu/D$ (if $D$ is not the diffusivity of temperature $\alpha$ then the ratio might also be called the Schmidt number). The minimum scale of self-gravitational condensation is therefore $L_{SD}$ if the flow is “quiescent”, where

$$Quiescent\ flow\ criteria\ [\langle L_{SX} \rangle_{max} = L_{SD}]: \gamma \leq (\rho G)^{1/2} Pr^{-1} \text{ and } \varepsilon \leq \rho GD,$$

so that the condensation is limited by diffusive smoothing rather than viscous or turbulent forces. For gases where diffusion of heat, mass, and momentum occur by particle collisions, the diffusivities of these quantities are all approximately equal, so $Pr \approx 1$ and $L_{SD} \approx L_{GIV}$.

### 3.1. Schwarz scale nonacoustic nuclei for non-quiescent flows

Consider an infinite, non-quiescent volume of gas with density $\rho$ containing nonacoustic density nuclei; that is, density maxima $\rho^{max}$ subject to viscous and turbulence forces without compensating pressure fields that cause them to propagate as sound waves. Suppose two density nuclei separated by a distance $L$ have amplitudes $\Delta \rho$ proportional to $\rho$. The gravitational force between the nuclei

$$F_G \approx (\rho G \rho L^3 \rho L^3)/L^2 \approx G \rho^2 L^4$$

will be resisted either by viscous forces

$$F_V \approx \rho \nu \gamma L^2$$

by turbulence inertial-vortex forces

$$F_T \approx \rho V(L)^2 L^2 \approx \rho (\varepsilon L)^2 L^2 \approx \rho \varepsilon^{2/3} L^{8/3},$$

where the Kolmogorov theory is used to estimate velocity differences, or by magnetic forces

$$F_M \approx H^2 L^2$$

where $H$ is the magnetic field strength. According to Kolmogorov’s theory the velocity difference between points separated by a distance $L$ in turbulence with dissipation rate $\varepsilon$ is $V(L) \approx (\varepsilon L)^{1/3}$ if $L$ is larger than the Kolmogorov length scale $L_K \equiv (\nu^3/\varepsilon)^{1/4}$. According to turbulent mixing theory (Gibson 1991), if the magnetic field $H$ is mixed by turbulence as a passive scalar, its variance $H^2 \approx \chi_H \varepsilon^{-1/3} L^{2/3}$, where $\chi_H$ is the diffusive dissipation rate of the variance.

Equating the gravitational and viscous forces gives the viscous Schwarz radius

$$L_{SV} \equiv (\nu \gamma / \rho G)^{1/2},$$

equating the gravitational and turbulence forces gives the turbulent Schwarz radius

$$L_{ST} \equiv \varepsilon^{1/2} / (\rho G)^{3/4},$$
and equating gravitational and magnetic forces gives the magnetic Schwarzschild radius

\[
L_{SM} \equiv \left( \frac{H^2}{\rho^2 G} \right)^{1/2} = \left( \frac{\chi H}{\varepsilon^{1/3} \rho^2 G} \right)^{3/4}.
\]  

(29)

A close dynamical similarity exists between \( L_{ST} \) and the Ozmidov length scale

\[
L_R \equiv \left( \frac{\varepsilon}{N^3} \right)^{1/2}
\]

of stratified turbulence theory (Gibson 1986, 1996). The Ozmidov scale of stably stratified turbulence corresponds to the scale where buoyancy forces \( F_B \approx \rho N^2 L^4 \) balance inertial-vortex forces of turbulence \( F_I \approx \rho \varepsilon^{2/3} L^{8/3} \). In the self-gravitational case the Väisälä frequency \( N = \left[ \left( \frac{g}{\rho} \right) \partial \rho / \partial z \right]^{1/2} \) is replaced by the \( N_\rho = (\rho G)^{1/2} \), where \( g \) is the gravitational force per unit mass in the \( z \) direction (down); so that \( L_{ST} \) could be written

\[
L_{ST} = \left( \frac{\varepsilon}{N_\rho^3} \right)^{1/2}.
\]

Just as turbulence is prevented by buoyancy forces of stable stratification in the vertical direction on scales larger than \( L_R \), turbulence is prevented by gravitational forces of a density gradient in space on scales larger than \( L_{ST} \) (Gibson 1988). Condensation on nonacoustic density nuclei takes place at \( L_{SV} \) if the Reynolds number is subcritical at the condensation scale, and at \( L_{ST} \) if it is supercritical (Gibson 1996), where

\[
L_{SV} \geq L_{SD} = L_{SV} \left( \frac{\gamma \tau G P r}{\rho} \right)^{-1/2}
\]

and

\[
L_{ST} \geq L_{SD} = L_{ST} \left( \frac{\rho G D}{\varepsilon} \right)^{1/2}
\]

for non-quiescent flows, independent of \( L_J \).

### 3.2. How nonlinear gravitational condensation works

Figure 1 schematically illustrates the new theory. The mechanism of turbulence production of nonacoustic density maxima and minima is shown on the left. A uniform density gradient is distorted by a turbulent convective episode (eddy) that ceases (for simplicity) soon after the distorted isopycnal surface becomes diffusively unstable and splits into multiply connected surfaces with maximum points (plus), minimum points (minus), and associated maximum saddle points and minimum saddle points (triangles). Zero gradient density configurations may be characterized by the signs of the eigenvalues \( E \) of the density Hessian \( \nabla \nabla \rho \) (Gibson 1968). Maximum points have \( sgn E = (- - -) \), minimum points \( (+ + +) \), maximum saddle points \( (- - +) \), minimum saddle points \( (+ + -) \), and saddle lines of doublets \( (+0-) \). Without gravity, no other zero gradient configurations are quasi-stable; for example, zero gradient volumes and minmax saddles with \( sgn E = (000) \), zero gradient surfaces with \( sgn E = (+00) \) and \( (-00) \), and lines with \( sgn E = (+0+) \) and \( (-0-) \) immediately split up into the quasi-stable configurations. This might be termed nonlinear perturbation stability analysis (NLPSA) applied to dynamically passive turbulent mixing of scalar fields like temperature (Gibson 1968), compared to LPSA, which fails to describe the turbulent mixing process without gravity just as it fails if gravity is important.
The gravitational force per unit mass $F_G = -\nabla \phi$ changes direction at such zero gradient points, where the gravitational potential $\phi$ is determined by density from Poisson’s equation

$$\nabla^2 \phi = 4\pi G \rho,$$

(30)

$\rho$ is density, and $G$ is Newton’s gravitational constant. Close to density maxima, maximum magnitude $F_G$ vectors point toward the maxima, and close to density minima, maximum $F_G$ forces point away. Along the principle axis of a maximum saddle pointing toward the maximum the $F_G$ forces reverse and act to pull the saddle apart to form a void, and succeed at scales larger than $(L_{SX})_{max}$ where other forces and diffusion cannot prevent it (Fig. 1, bottom right). Gravitational forces per unit area along this axis increase without limit as the angle between the cones of the maximum-saddle point approaches zero, overwhelming pressure gradients and other forces that might resist void formation.

For diffusive decay without gravity or convection, shown at the top right of Fig. 1, the maximum and minimum saddle points (triangles) merge to form a minmax saddle (double triangles) which immediately expands to a saddle line (filled circles) surrounding the doublet where the plane intersects the sphere to form lobes. Because $D_\rho$ is positive, the decay of the density microstructure is monotonically back toward the original uniform density gradient configuration without any zero gradient structures.

The evolutions of a density doublet, minimum point-minimum saddle, and maximum point-maximum saddle under the influence of gravity and diffusion, shown at the bottom of Fig. 1, are very different than their decays with diffusion alone. Because density moves up gradient due to gravitational forces, the result is similar to what would happen if the sign of the density diffusivity $D_\rho$ (henceforth $D$) were reversed. It is assumed that the Jeans scale $L_J$ of the gas is much larger than the size of the density doublet $L$, which is larger than the maximum Schwarz scale $(L_{SX})_{max}$ so that gravitational condensation or void production is not inhibited by molecular diffusion or hydrodynamic forces. For the primordial gas example of interest, the diffusive time scale $\tau_D = L^2 / D$ is $\approx 30 \times 10^6$ years, compared to $\tau_G = (\rho G)^{-1/2}$ of $\approx 4 \times 10^6$ years or $\tau_{PFP} = (L_{SX})_{max} / V_S$ about $10^3$ years. All of the matter contained in the doublet collapses into the density maximum due to the imbalance of gravitational forces caused by the doublet, in stages shown in Fig. 1. Gravity causes the saddle line to collapse back to the down-gradient minmax saddle configuration shown, which is diffusively unstable but which gravity stabilizes. The density maximum monotonically increases in magnitude and shrinks in size until it is completely engulfed by the void, cutting off the mass supply to the isolated gas particle that continues to collapse to form a PFP. The surrounding void propagates as a rarefaction wave with sonic velocity $V_S$ away from the density minimum point in
all directions driven by radial buoyancy forces, forming a nearly spherical cavity that expands until it reaches the nearest neighboring void.

Isolated density minima larger than \((L_{SX})_{\text{max}}\) form expanding voids (Fig. 1, bottom right). Isolated density maxima larger than \((L_{SX})_{\text{max}}\) are engulfed by expanding voids that form at their associated maximum saddle points, where gravitational forces in opposite directions along the axis toward the maximum point split the maximum saddle point to form the minimum point lobe and saddle line of a doublet. As before, the minimum point lobe of the doublet wraps around the contracting maximum point lobe as the saddle line moves down gradient, forming a minmax saddle (double triangles) and an expanding void surrounding the PFP. Split maxima larger than \((L_{SX})_{\text{max}}\) that are engulfed by expanding voids form binary PFPs orbiting about each other in the void. Numerous other permutations and combinations of these processes are possible as the continuous primordial gas shatters into PFP mass objects in a continuous void due to the nonlinear gravitational instability of the turbulence generated nonacoustic density extrema.

3.3. Evolution of cold spots, hot spots, and acoustic density extrema

We can qualitatively demonstrate the absolute gravitational instability of nonacoustic density maxima and minima to condensation and void formation, respectively, by considering the evolution of constant pressure cold spots and hot spots starting from rest. Figure 2 shows schematically the gravitationally driven evolution (left to right) of a cold spot nonacoustic density maximum in a large body of continuous, initially motionless gas with constant initial pressure \(p\) (top), compared to the evolution of a hot spot nonacoustic density minimum with constant initial pressure \(p\) (middle). The gravitational instability of both these nonacoustic extrema is contrasted with the decay of an acoustic density maximum and its compensating pressure maximum in the same uniform ambient conditions (bottom).

The gas is assumed to be a primordial mixture of hydrogen and helium at primordial gas temperatures and pressures so that the gas is transparent and obeys the perfect gas law. The initial “spot” sizes \(L\) are assumed much smaller than \(L_J\) but larger than \((L_{SX})_{\text{max}}\). Initially, gravitational forces and heat flux vectors are radially away from the hot spot (left, top) and radially toward the cold spot (left, middle), as shown in Fig. 2, driving monotonically increasing velocities \(v\) (center to right) toward the density maximum (top) and away from the density minimum (middle). Because no hydrostatic pressure \(\delta p\) exists, \(L'_{HS} = 0 \ll L\), as shown (top left) so the condensation continues.

Both the density excess of the cold spot (top) and the density deficit of the hot spot (middle) grow until their differences from ambient temperature \(T\) are erased (center).
The cold spot is warmed by a combination of heat transfer and compression, and the hot spot is cooled by a combination of heat transfer and expansion. The pressure outside the collapsing cold spot core decreases due to the increasing inward velocity, from Bernoulli’s equation \( p/\rho + v^2/2 \approx \text{constant} \). Pressure increases near the center of the former cold spot as the velocity goes to zero and as its density increases from the inward flow of mass, but \( L'_{HS} \ll L \) so condensation continues as shown (top center) in Fig. 2. Similarly, the pressure decreases in response to the velocity and decreased density of the former hot spot (middle) as gravity forces from the surrounding higher density gas pull it to a larger volume. Heat transfers cease and reverse as the cold spot and hot spot reach ambient temperature, in Fig. 2 (center middle). Rapid initial heat transfer does not erase either of the density extrema, it just spreads them out and may reduce their central density amplitudes and temperature differences temporarily.

The constant pressure initial condition is realistic. If instead, the density were initially constant the pressure would necessarily be below ambient for the cold spot and above ambient for the hot spot, and would rapidly approach the constant pressure initial conditions shown in Fig. 2 (top middle left) after emission of sound waves, with slight warming of the cold spot and cooling of the warm spot due to the loss and gain of acoustic energy, respectively.

For the now-isothermal former cold spot (top center Fig. 2), the density maximum produces a radially inward gravitational force \( F_G \) and a radially inward gas velocity \( v \) everywhere that increase and accelerate indefinitely. Compressive work will heat the gravitationally condensed gas and hydrostatic pressures will build up to decelerate the condensing gas, but the radius will monotonically decrease and the mass within any radius, and the temperature, will monotonically increase until either thermonuclear processes begin and a star is born, or a nearby growing void cuts off the supply of mass to form a PFP. The dissipation rate \( \varepsilon = \rho^{5/6} G^{3/2} M^{2/3} \) corresponding to star formation with mass \( M = 10^{29} \) kg is \( 10^{-11} \) m\(^2\) s\(^{-3}\), compared to only \( \varepsilon \approx 10^{-14} - 10^{-15} \) m\(^2\) s\(^{-3}\) in the primordial gas estimated from COBE measurements \cite{Gibson1996}. Therefore, we can expect PFPs to form from the primordial gas, rather than small stars.

Radiative heat transfer cools the now-hot former-cold spot and increases the rate of compaction to form the proto-PFP (right top), but is not a necessary condition for the condensation to continue. The now-isothermal former hot spot (middle center Fig. 2) is also absolutely unstable in the homogeneous surrounding environment shown. The decreased density produces a radially outward gravitational force \( F_G \) and a radially outward gas velocity \( v \) everywhere that increase and accelerate indefinitely, or until the nearest PFP is encountered. Radiative heat transfer from the warmer surroundings heats the now-cold former-hot spot and increases the rate of expansion to form the proto-void
shown on the right. The expansion is accelerated by heat transfer, but the heat transfer is not a necessary condition for expansion since the void formation accelerates with or without heat transfer.

The former cold and hot spots evolve toward the “proto-PFP” and “proto-void” configurations, as shown on the top and middle right of Fig. 2. Both processes are monotonic, irreversible, and inexorable. The collapse to form a PFP continues until the supply of matter vanishes with the arrival at the PFP of a nearby growing void. The growth of a void ceases when it encounters a PFP. The PFP is then gravitationally bound with length scale $L_{HS}$, and will continue to compact as it cools by radiative heat transfer toward ambient temperature.

In contrast, the Jeans acoustic density maximum shown (bottom left) of Fig. 2 first collapses because its compensating pressure maximum expands, and then overshoots to form sound waves that propagate radially outward with velocity $V_S$. These move more than a wavelength $L$ in a time small compared to $\tau_G$ because $L \ll L_J$ by hypothesis, so their amplitude decreases as their energy propagates to larger volumes, and they eventually dissipate by viscous forces and vanish, as recognized by Jeans (1902, 1929). If any mass is accumulated during the propagation of such acoustic density maxima, the acoustic nuclei will produce nonacoustic density maxima and minima because the ambient momentum of the condensed material is conserved and zero. Thus, even if only sonic perturbations were available in primordial gases, they would trigger condensation of the gas to PFPs by the same mechanisms as those illustrated in Fig. 2; that is, self-gravitational condensation and void formation triggered by nonacoustic density extrema on scales $L_J \gg L \approx (L_{SX})_{\text{max}}$ limited by hydrodynamic forces or diffusion, contrary to the Jeans theory.

### 3.4. Evolution of “cannonball” and “vacuum-beachball” density perturbations

The isothermal (former) cold and hot spots (center, top and middle) are functionally equivalent to local “cannonball” (mass $M'$) and “vacuum beachball” (mass $-M'$) initial density conditions, which are obviously unstable to gravitational condensation and void formation in an infinite homogeneous gas at scales independent of and smaller than $L_J$, contradicting the Jeans criterion for gravitational instability. Helium and hydrogen maximum points have the same effects where they dominate the density extrema rather than temperature. Near a helium maximum point that causes a density maximum larger than $(L_{SX})_{\text{max}}$, gravitational forces are imbalanced and cause a monotonic growth in the density to form a proto-PFP under primordial gas conditions, just as hydrogen concentration maxima that cause density minima larger than $(L_{SX})_{\text{max}}$ will produce proto-voids. Again, compressive heating and expansion cooling
cannot prevent the condensation and void formation. Growths are accelerated by radiative heat transport but do not require it. The momentum equation starting from rest is \( \partial v_r / \partial t + M' G / r^2 \approx 0 \) neglecting magnetic, viscous, inertial-vortex, and dynamic pressure gradient forces, so the radial velocity \( v_r \approx -M' G t / r^2 \). Density changes occur only near \( r = 0 \), where \( dM'/dt \approx -4 \pi \rho v_r \approx -4 \pi \rho M' G t \) by the conservation of mass, so \( M'(t) \approx M'(0) \exp[2 \pi \rho G t^2] \) increases or decreases exponentially with \((t/\tau_G)^2\) depending on the sign of the initial density perturbation \( M'(0) \) and the ‘free-fall’ time \( \tau_G \equiv (\rho G)^{-1/2} \).

4. Consequences of the new condensation theory: nonlinear cosmology

4.1. Plasma epoch deceleration of protosuperclusters to protogalaxies

During the plasma epoch following the Big Bang, in the beginning of structure formation, the radiation dominated speed of sound was some fraction of the speed of light \((\approx c/3)\), giving \( L_J \) values larger than the Hubble scale \( L_H = ct \). Taking \( V_S = 10^8 \) m s\(^{-1}\) and \( \rho \approx 10^{-18} \) kg m\(^{-3}\) [Weinberg 1972] gives \( L_J \approx 10^8 / (10^{-18} \cdot 6.7 \cdot 10^{-11})^{1/2} = 1.2 \times 10^{22} \) m, larger than \( L_H \equiv ct = 3 \times 10^8 \cdot 10^{13} = 3 \times 10^{21} \) m, assuming \( t \approx 10^{13} \) s \( \approx 300,000 \) y at the plasma to gas transition. Therefore, no gravitational deceleration or condensation of baryonic matter is possible during the plasma epoch by the Jeans criterion because density information has insufficient time to be transmitted over scales as large as \( L_J \).

After the plasma neutralizes to form gas the Jeans scale is still very large because the gas is very hot. Taking \( V_S \approx 5.6 \times 10^3 \) m s\(^{-1}\) gives \( L_J \approx 5.6 \times 10^5 / (10^{-18} \cdot 6.7 \cdot 10^{-11})^{1/2} = 6.8 \times 10^{17} \) m, so \( M_J \approx (6.8 \times 10^{17})^3 \cdot 10^{-18} = 3.2 \times 10^{35} \) kg, or \( 1.6 \times 10^5 M_\odot \). Thus, structure formation models that rely completely on the Jeans criterion and baryonic matter produce structures much too slowly to match observations. Padmanabhan 1993 assumes that the hypothetical, weakly interacting, nonbaryonic dark matter decouples from radiation and begins to condense during the plasma epoch in order to provide the potential wells required to explain the observed baryonic matter condensation despite the Jeans criterion.

“Weakly interacting massive particles” (WIMPs) provide adjustable \( L_J \) values, by adjusting the speed (hot versus cold dark matter) and mass of the “WIMP” particles, so that the nonbaryonic dark matter can guide the baryonic condensation to match observations [Klypin et al. 1997]. Suggested WIMP particles [Padmanabhan 1993] have masses in the range \( 10^{-38} \) to \( 10^{-24} \) kg. “Axion” particles [Weinberg 1993, p186] may be so numerous that a particle mass as little as \( 10^{-41} \) kg would supply all the nonbaryonic dark matter necessary for a flat universe.

However, one problem (among others) with this means of resolving the Jeans dilemma is that the nonbaryonic dark matter is generically weakly-interacting, similar to neutrinos (it may consist entirely of massive neutrinos), with detectable collision cross sections
of order $10^{-40}$ m$^2$ or less (otherwise WIMPs could be observed). Such small collision probabilities imply extremely long mean free paths $L_{mfp}$ and long times between collisions $\tau_{col}$. If the $L_{mfp}$ and $\tau_{col}$ values are smaller than the space-time scales over which momentum is to be transferred, then viscous momentum diffusion occurs. Kinematic viscosities $\nu_{WIMP}$ for “WIMP” gases are enormous compared to $\nu_{baryonic}$ for baryonic gases because

$$\nu \approx L_{mfp} \times V_{particle}$$

(31)

where momentum is transferred by particle collisions, or

$$\nu \approx \frac{a \times T^4 \times \tau_{col}}{\rho}$$

(32)

where momentum is transferred by radiation (Weinberg 1972, p57). The radiation pressure is $a T^4$, $a$ is the black body constant $7.56 \times 10^{-16}$ J m$^{-3}$ K$^{-4}$, and $T$ is the radiation temperature. Condensation of WIMP dark matter fluids during the plasma epoch as proposed by Padmanabhan (1993) is subject to the new viscous-diffusive-gravitational condensation criteria of Eq. (3), and the requirement that the condensation scale $L_C$ be less than the Hubble scale $L_H$

$$L_H = ct \geq L_C \geq L_{SV}, L_{SD}$$

(33)

which will not be satisfied early in the plasma epoch where $t$ is small and $[\nu \approx D]_{NB}$ are very large since $[L_{SV}, L_{SD}]_{NB}$ for nonbaryonic fluids are likely to be greater than $L_H$ for most of this epoch and much greater than proto-galaxy scales $L_G$ for all of it, depending on the diffusivity $D_{NB}$ and kinematic viscosity $\nu_{NB}$ as a function of time.

Little is known of the fluid mechanics of the plasma epoch, except that temperature, density and viscous diffusivities must have been large to smooth the temperature and velocities to the small values observed by the Cosmic Background Explorer (COBE) satellite in the cosmic microwave background radiation (Silk 1994), requiring very weak or no turbulence. Large viscous dissipation rates are also suggested by the huge entropy observed, with indicated initial viscosities $\mu$ of order $10^{59}$ kg m$^{-1}$ s$^{-1}$ or larger (Brevik and Heen 1994) that produce highly subcritical Reynolds numbers (Gibson 1996).

Without the Jeans constraint, and without strong turbulence, only large viscosities or diffusivities can prevent structure formation in the plasma epoch (an unfortunate misconception of linear cosmology is that the expansion of the universe will damp away all turbulence, whereas expansion actually triggers and drives turbulence unless constrained by viscosity). The magnitude of the viscosity $\nu \approx D$ existing then can be inferred from the mass of the largest structures existing now (superclusters), since these presumably were the first objects to form by gravitational deceleration because they were the first
that could possibly form (Gibson 1996, 1997b). Supercluster masses can be estimated because supervoid sizes are known. The scale of the largest super voids are observed to be $\approx 10^{24}$ m (Kolb and Turner 1994), substantially smaller than the present horizon scale of $\approx 10^{26}$ m. From the critical density of $\rho_{\text{crit}} = 10^{-26}$ kg m$^{-3}$ and $L_{SV}^3$ we infer a supercluster mass $\approx 10^{46}$ kg that reflects the first deceleration mass when $L_{SV}^3 \times \rho$ decreased to equal the increasing Hubble mass $M_H = (ct)^3 \times \rho$. Solutions of Einstein’s equations (Weinberg 1972, Table 15.4) give the decreasing density $\rho(t)$ of the expanding universe.

Therefore, setting $(ct)^3 \times \rho(t) = 10^{46}$ kg and solving for $t$, we find the time when viscous forces first permitted gravitational structure formation in the universe to be $t \approx 4 \times 10^{11}$ s, or 13000 y. From the density at this time of $\rho_{\text{cond}} \approx 5 \times 10^{-15}$ kg m$^{-3}$ and the rate-of-strain $\gamma \approx t^{-1}$, it follows that the kinematic viscosity $\nu$ was about $1.9 \times 10^{27}$ m$^2$ s$^{-1}$ by setting $L_{SV} \approx L_H = ct$. The Reynolds number of the flow $c^2t/\nu$ is about 20, which is below critical, so $L_{SV}$ is the appropriate condensation length scale (Eq. 3) assuming it is determined by convection rather than diffusion. Setting $\nu \approx D$ gives $L_{SD} \approx 6 \times 10^{19}$ m, which is less than $L_{SV}$, consistent with our assumption that viscous forces set the minimum condensation scale. Such a large value of $\nu \approx D$ is about $10^2$ larger than the photon viscosity $\nu \approx c/n\sigma_T = 3 \times 10^8/1.5 \times 10^{11} \times 0.67 \times 10^{-28} = 3 \times 10^{25}$ m$^2$ s$^{-1}$ expected for Thomson scattering with the estimated electron density $n$ and Thomson scattering cross section $\sigma_T$, suggesting the possibility that the baryonic matter may somehow be coupled to the more diffusive nonbaryonic matter. Since neutrinos appear to have mass from the Neutrino-98 announcement in Japan, the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism coupling massive neutrinos to electrons appears to be a physical possibility worthy of future study. The possibility of neutrinos as dark matter is discussed by Spergel 1997, p225.

Extrapolation of decreasing $L_{SV}$ with decreasing temperature from about $10^5$ to 3000 K gives a decreasing mass of the decelerations from $10^{46}$ kg (the mass of a protosupercluster) to about $10^{42}$ kg (the mass of a protogalaxy) at the plasma to gas transition (Gibson 1990), with a nested foam topology. The average protosupercluster expansion velocity to date is about $10^7$ m s$^{-1}$, compared to $10^4$ m s$^{-1}$ for protogalaxies. The first true gravitational condensation with increasing density happens soon after transition.

4.2. When the universe turned to fog: the origin of baryonic dark matter

In the present paper, we are primarily interested in a very special initial condition caused by a very special event, where both the condition and event are unique in the history of the universe; i.e., the vast nearly homogeneous regions of hot (3000 K), dense ($10^{-18}$ kg m$^{-3}$), nearly motionless ($v \leq 10^{-5}c$), primordial hydrogen (75 %) and helium (25 %)
gas formed soon after the plasma-gas transition when the photons of the plasma epoch decoupled from the free electrons of the plasma as they combined with H and He ions to form neutral gas, and when the smallest gravitational condensation mass permitted by hydrodynamic forces decreased by a factor of $10^{18}$ from that of a protogalaxy to that of a PFP. Soon after that moment, nearly all of the baryonic matter of the universe turned to primordial fog particles (PFPs). From our new theory and observations, most of these now-frozen, planetary-mass PFPs still persist invisibly in massive galaxy halos as most of the baryonic dark matter.

By our new theory, gravitational condensation previous to decoupling was limited by enormous viscous forces to $L_{SV}$ scales corresponding to protogalaxies (PGs) which immediately began fragmentation to form proto-globular-clusters (PGCs) at the initial condensation scale $L_{IC} \equiv (RT/\rho G)^{1/2} \approx L_J$ (Gibson 1996). Simultaneous fragmentation occurred on the nonacoustic density nuclei separated by Batchelor scales $L_B$, at scales no smaller than the largest Schwarz scale $(L_{SX})_{max}$. The ratio of Jeans to Batchelor scales $L_J/L_B = (RT\gamma/\rho GD)^{1/2}$ is more than $10^4$, substituting known primordial $R$, $\rho$, and $D$ values and assuming a minimum rate-of-strain $\gamma \approx 1/t$, where $t = 10^{13}$ s is the age of the universe at plasma-gas transition (300 000 years).

Minimum condensation scales (the Schwarz scales $L_{SX}$) for these nonacoustic nuclei are determined by fluid mechanical forces or molecular diffusion, and are also smaller by about $10^4$ than the Jeans scale $L_J$ for primordial conditions. Remarkably, this ancient hydrodynamic state has been measured, and is known to be extremely quiet, weakly turbulent ($\delta v/c \ll 0.2$ at horizon scales), and almost perfectly homogeneous, from COBE observations of the cosmic background radiation at the time of photon decoupling and plasma-gas transition, so the calculation is possible. Temperature, and therefore velocity fluctuations, were observed to be $\delta T/T \approx 10^{-5} \gg \delta v/c$ on large $(0.1L_H)$ scales (Silk 1994). Because He nucleosynthesis is sensitive to temperature, it seems likely that fluctuations of He and H concentration may also have existed at the plasma-gas transition as a possible source of nonacoustic density extrema, although they have not been detected.

The initial $M_{IC} = (RT/G)^{3/2} \rho^{-1/2} \approx 10^{35}$ kg condensation to form proto-globular-cluster (PGC) droplets with size $L_{IC} \equiv (RT/\rho G)^{1/2} \approx L_J$ determined by the temperature $T$, gas constant $R$ and density $\rho$ of this hot primordial gas is caused by the ideal gas law, not the Jeans scale $L_J$. According to our present theory, condensations should occur simultaneously at $(L_{SX})_{max}$ scales within PGC droplets on nonacoustic density nuclei with $(L_{SX})_{max} \approx L \ll L_{IC}$, and the condensations are not prevented in any way by the approximately constant internal pressure and temperature within the PGC
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droplets as usually assumed from the Jeans theory. For the weak primordial turbulence conditions and the homogeneous primordial gas, such fog-like condensates are estimated to have moon-planet \((10^{23} - 10^{25})\) kg masses (Gibson 1996). All of the baryonic matter of the universe rapidly condensed to form such primordial fog particles (PFPs) by this theory soon after the plasma-gas transition. Rather than requiring the “free fall” time \(\tau_G = (\rho G)^{-1/2} \approx L_J/V_S\) of the primordial gas of a few million years from the Jeans theory, PFPs form in only \(L_{PFP}/V_S \approx 10^3\) years corresponding to the time for the sonic rarefaction waves of void formation to traverse average PFP separation distances.

Therefore, since only a small fraction of the cooling, compacting, and thus increasingly collisionless PFPs in their merging voids are likely to form luminous stars, the baryonic dark matter of the galaxies consists of any remaining non-aggregated primordial fog particles. The formation of the first stars must have taken place soon after the formation of PFPs, while the PFPs were large and gaseous, densely packed, and relatively collisional. Double stars with numerous gassy planets should terminate the accretion cascade within PGCs, except possibly for a superturbulent gas ball with growing \(L_{ST}\) size at the PG core, rapidly accreting PFPs but with excellent convective and radiative heat transfer outward. The small primordial stars of PGCs must have formed early if they were to form at all, perhaps in a few 10 million years, since the free fall time \(\tau_G = (\rho G)^{-1/2}\) increases as the density of the expanding universe decreases. By this time the universe was quite cold, dark, and still quiet, with temperature \(T \approx 30\) K at \(t \approx 10^{15}\) s (30 million years) and with the total mass to luminous mass ratio \(M/L\) nearly infinity. A few sparks of weakly colliding PFPs and PFP agglomerates, all still in completely gaseous states well above the triple point temperature of hydrogen, might have begun weakly lighting the skies of the late dark ages of the universe, along with a few dimly glowing, gassy brown dwarfs, sparking as they collect mass from the \(10^5\) PFP gas-balls necessary for them to become small stars. Brilliant gamma ray bursts appear daily as the superturbulence of distant galaxy cores become relativistic and can no longer prevent superstar formation, with immediate collapse to black holes of PGC or multiple PGC mass. If no structures had formed in the universe by the present time it might be too late. The time scale \(\tau_G = (\rho G)^{-1/2}\) for the present average density of the expanding universe \((10^8\) less than primordial) exceeds thirty billion years, twice its estimated age.

Nonacoustic density maxima and minima in the constant pressure, homogeneous and continuous primordial gas are local regions that are either colder or hotter than the surroundings, and therefore more or less dense. The strongest such hot and cold spot density extrema are the most unstable to gravitationally driven condensation and expansion and will develop most rapidly. Radiant and conductive heat transfer may slow or speed up the condensation and expansion of such extrema, but not stop them once they start.
Figure 3 shows the process of primordial-fog-particle (PFP) formation at the plasma-gas transition based on the present revised theory of self-gravitational condensation. On the left is a protogalaxy (PG) droplet of plasma at transition density \( \rho_o \approx 10^{-18} \) to \( 10^{-17} \) kg m\(^{-3}\) and \( T_o \approx 3000\) K at time at \( t_o \) (Weinberg 1972). The Jeans scale \( L_J \) is larger than the Hubble scale \( L_H = ct \), so no condensation is possible in the plasma epoch by the Jeans theory for baryonic matter, as shown by the comparison of critical length scales of the process in the upper left hand corner of Fig. 3. Cold dark matter theories (Padmanabhan 1993) assume that weakly interacting massive particle (WIMP) nonbaryonic matter decouples from radiation during the plasma epoch, thus reducing the WIMP sound velocity to values less than \( c \), where \( c \) is the velocity of light, so that the Jeans scale \( L_{J-WIMP} \leq L_H \), permitting WIMP condensations to occur. These hypothetical WIMP condensations then serve to drive the formation of protogalaxies by providing gravitational potential wells into which the baryonic matter falls. However, by the present theory and estimates of WIMP diffusivities this model of galaxy formation is unnecessary and unworkable. We show evidence in the following that such WIMP condensations are prevented by the extremely large generic diffusivity and long condensation times of the WIMP material, so that \( L_{SD-WIMP} \gg L_{PG} \), where \( L_{SD-WIMP} \) is the Schwarz diffusive scale for WIMP fluid and \( L_{PG} \) is the size of a protogalaxy at \( t_o \). Galaxy size “WIMP-PFPs” would simply diffuse away. They are not necessary for protogalaxy formation because the revised condensation theory predicts \( L_{PG} \approx L_{HS} \approx L_{SV} \approx L_{ST} \) for plasma at transition to gas conditions (Gibson 1996), as shown in Fig. 3.

The pressure distribution for the gravitationally bound protogalaxy (PG) is shown on the left bottom of Fig. 3, with pressure \( p_{HS} \approx p_{PG} \) to preserve hydrostatic equilibrium of the plasma protogalaxy by radiation pressure. Proto-globular-cluster (PGC) droplets form from the PG droplet at scale \( L_{IC} \), as shown in the center of Fig. 3. Each of the PGCs then fragment to form smaller scale PFPs, as shown on the right of Fig. 3, according to the condensation and void formation mechanisms described in the last subsection and shown in Fig. 2. The pressures in the proto-PFPs will rapidly become much greater than the original PGC droplets \( p_{PGC} \). Temperatures of the PFPs are larger than \( T_o \) or \( T_{Voids} \), so heat transfer from the PFPs to cooler regions of space will accelerate their compaction, as shown on the right of Fig. 3. Because of the quiet initial conditions and the gentle condensation process, turbulence should continue to be weak, so the hydrostatic Jeans scale is approximately equal to the Schwarz viscous and turbulent scales \( L_{HS} \approx L_{SV} \approx L_{ST} \) for the proto-PFPs, as shown. This means that during the compaction process, viscous, turbulence, pressure, and gravitational forces are all in approximate balance.
Protosupercluster to protogalaxy mass gravitational decelerations of the rate of expansion of the plasma universe provide patterns for subsequent structure formation in the primordial gas. Within the protogalaxy $10^{42}$ kg “droplets,” further decelerations occur on acoustic nuclei with proto-globular-cluster mass $10^5 M_\odot$, reflecting the classical Jeans gravitational instability. Only protosupercluster, protocluster, and perhaps protogalaxy size structures are within the resolution of the COsmic Background Explorer satellite (COBE), which shows weak velocity fluctuations (Bunn et al. 1996) and therefore weak turbulence existed at that time (Gibson 1996) at scales no larger that $1/10 L_H$.

Within all these structures, condensation on nonacoustic nuclei should proceed at $L_{SV}$ scales $\ll L_J$, and the entire baryonic mass of the universe should condense to a “primordial fog” of particles of scale $L_{SV}$. The mass of such viscous primordial fog particles is given by

$$M_{PFP} = L_{SV}^3 \times \rho = \left[ \frac{\mu}{G} \right]^{\frac{1}{2}} \times \frac{1}{\rho^2}. \quad (35)$$

Assuming $T \approx 3000$ K, $\gamma = 1/t$, and density $\rho \approx 10^{-17}$ kg m$^{-3}$ (the density of a globular cluster), gives

$$M_{PFP} = \left[ \frac{4.4 \times 10^{-5} \times 10^{-13}}{6.67 \times 10^{-11}} \right]^{\frac{1}{2}} \times \frac{1}{[10^{-17}]^2} = 1.7 \times 10^{23} \text{ kg}, \quad (36)$$

where the dynamical viscosity $\mu$ was obtained from standard tables. This moon-mass estimate of $M_{PFP}$ is a probable lower bound, appropriate to non-turbulent, $L_{IC}$ scale, proto-globular-cluster (PGC) mass “droplets” of gas, which is where the first star formation is most likely to occur. Outside the PGC mass “droplets” the density will be less, so that the $M_{PFP}$ masses will be greater. The average density of the universe at $t = 10^{13}$ s was $\rho \approx 10^{-18}$ kg m$^{-3}$ (Weinberg 1972, Table 15.4) giving $M_{PFP} \approx 10^{25}$ kg, or earth mass, $M_\oplus$.

The larger the mass, the larger the separation distance, and the smaller the collision probability of PFPs, especially over time as they become colder and more compact. protogalaxy droplets with smaller $\rho$ values, or larger $\gamma$ and $\varepsilon$ values, might form only massive, widely separated PFPs that collide too rarely to form stars, leaving such “ghost galaxies” of collisionless “rogue Jupiters” forever dark. A large population of Low Surface Brightness Galaxies (LSBGs) have been detected (Bothun 1997) with properties one might expect for galaxies with intermediate “rogue Neptune” $M_{PFP}$ values barely small enough to aggregate; that is, LSBGs have small stars, low metal concentrations, low gas concentrations, are relatively un-evolved, may be enormous (100 kpc Malin 1 is 30 times the size of the Milky Way), and as a population may have more total mass than the total mass of conventional galaxies.

Formation of PFPs inhibits formation of turbulence and slows the rate of structure formation, contrary to standard fragmentation models of initial star for-
mation (Low and Lynden-Bell 1976, Rees 1976, Silk 1982). PFPs should condense to stars slowly by the weakly-collisional, clustering cascade of stellar dynamics (Binney and Tremaine 1987), since the mass must increase by a factor of $10^6$ to reach $10^{29}$ kg, or $0.1M_\odot$, required for the first small star to form. Such clusters are termed “robust associations of massive baryonic objects” (RAMBOs) by Moore and Silk 1995, who anticipate severe sampling problems due to clustering if RAMBOs dominate the halo dark matter (Gibson and Schild 1998).

Previous fragmentation models (Silk 1994) imply a strongly turbulent initial star formation process involving all existing gas to produce massive, metal-free, stars (termed Population III) and thus a monotonic growth of the metal/hydrogen ratio since metals form at the expense of hydrogen, but neither Population III-stars nor the expected metal/hydrogen growth have been observed. By our PFP-dark-matter model the metal fraction in stars of ancient and recent galaxies should be rather constant since star formation, and thus metal production, produces a proportional amount of primordial material by evaporation of PFPs. Significant numbers of large, Population III, metal-free stars of pure primordial gas never existed by the present model. Formation of “fog particles” in the primordial gas damped out its already weak turbulence. Nearly all of the original PFPs should still persist in the halos of conventional galaxies as their baryonic dark matter. Some galaxies and possibly clusters of galaxies may exist with virtually all of their original PFPs intact, and practically no stars.

5. A quasar-microlensing event recorded at three observatories

The discussion in the preceding sections leads to the conclusion that baryonic dark matter dominating galaxy rotation and cluster motions should consist of a remnant population of “primordial fog particles” (PFPs). These hydrogenous objects should have masses in the range $10^{-5}M_\odot$ to $10^{-7}M_\odot$, and should be evident in searches for galaxy halo populations by microlensing. Have the particles been detected?

The search for MAAssive Compact Halo Object (MACHO) particles in the Halo of our Galaxy by microlensing stars of the Large Magelanic Cloud (LMC) has resulted in negative but controversial results. The classic Alcock et al. (1995abcde) papers of the MACHO collaboration usually report lens masses $m \geq 0.1M_\odot$, for which event times $t_{sm} \approx 130\sqrt{m/M_\odot}$ (days) are several months. Star-microlensing events by objects in the PFP mass range last about an hour, and are therefore difficult to detect. The corresponding TwQSO quasar-microlensing time $t_{qsm} \approx 3\times10^4\sqrt{m/M_\odot}$ (days) is 9.6 days.

Alcock et al. (1996) report that for a limited subsample of their data, where several exposures of rapid succession were considered, the low detection rates indicate non-detection of sufficient mass to make PFP’s the entire mass of a standard spheroidal dark
matter Halo. Renault et al. (1998) reach the same conclusion from a more intensive search of a smaller area. The combined MACHO and EROS (Expérience de Recherche d’Objets Sombres) collaborations focus on small-planetary-mass objects such as PFPs in excluding a population with mass $M_p = (10^{-7} - 10^{-3}) M_\odot$ as more than 25% of the missing halo mass within 50 kpc of the Galaxy center (the distance to the LMC), or $M_p = (3.7 \times 10^{-7} - 4.5 \times 10^{-5}) M_\odot$ having more than 10%.

However, these exclusions are based on the assumption that the objects are distributed homogeneously. This is most unlikely for such a small-mass population of PFPs, which are hydrogenous and primordial, and consequently distributed as a complex array of nested clumps due to their nonlinear, gravitational-accretion-cascade for a wide range of mass to form stars. For small $M_p$ values, the number density $n_p$ is likely to become a lognormal random variable with intermittency factor $I_p \equiv \sigma^2_{\ln[n_p]} = 0.5 \ln[M_\odot/M_p] = 8.1$ (Gibson and Schild 1998). For a lognormal random variable, the mean to mode ratio is $\exp[3I_p/2] = 1.8 \times 10^5$ for $I_p = 8.1$. A small number of independent samples of $n_p$ gives an estimate of the mode (the most probable value) of a random variable, which is what is estimated by MACHO/EROS $n_p$ measurements since the LMC occupies only about 0.1% of the sky and only a fraction of this was sampled. Thus, an exclusion of $10^{-7} M_\odot$ objects as $\leq 0.1 M_{\text{halo}}$ from an estimate of the mode of $n_p$ is inconclusive, since the mean PFP halo mass could be $\geq 1.8 \times 10^4 M_{\text{halo}}$ from such measurements. Therefore, we believe that the interpretation of the MACHO/EROS statistics is highly model dependent, and as yet inconclusive.

Results from the quasar-microlensing programs are perhaps more relevant. Whereas the star-microlensing searches are in a (dimensionless) surface mass density realm of $10^{-6.5}$, the quasar microlensing optical depth (dimensionless) is greater than unity for the TwQSO system, so the observations are much simpler. A microlensing event or two should be underway in one of the two observed images at all times. However this results in some complications of interpretation as well, and the microlensing program has required the determination of the quasar lens’ time delay. With emerging agreement on a 1.1 year time delay, Schild 1996 examined the microlensing presumably originating in the lens galaxy of the gravitational lens system, and concluded that dominant mass of microlensing particles had masses around $10^{-5.5} M_\odot$. The Schild results suggesting the detection and identification of the dark matter required confirmation from independent observations (Sky and Telescope 1996).

A heightened interest in the TwQSO system (Q0957 + 561 A,B) resulted from a prediction by Kundic et al. 1995 that a rapid decline in the quasar’s brightness should be seen in Feb.—Mar. 1996 in the second arriving B image, based upon observation of the event in the first arriving A image in December 1994. Because the time delay was
still controversial, with values of 1.1-years (Schild 1990, Pelt et al. 1995) and 1.4-years (Lehar et al. 1991, Press et al. 1991), it appeared that observations during February and May 1996 would settle the time delay issue. Thus at least 3 observatories undertook monitoring programs to observe the predicted event.

At Mount Hopkins, the 15 year monitoring program on the 1.2 m telescope continued, with observations made by scheduled observers on 109 nights. Four observations were made each night with a Kron-Cousins R filter, and the observations averaged together for a published nightly mean brightness value. The quasar brightness was referenced to 5 nearby stars whose brightnesses were checked relative to each other to ensure stability of the magnitude zero point. The data are plotted in Figures 4 and 5 of this report, and data for the first season showing the brightness drop in the first-arriving A component have been published by Schild and Thomson 1997. Data for the second arriving B component in the second year will be published in our own report about the time delay.

The Princeton data were obtained by Kundic et al. 1995 using the 3.5m Apache Point telescope with g and r filters on the Gunn photometric system. Their data are not published, but data for the first season were posted on a World-Wide-Web site listed in the Kundic report. We have converted these data to a standard R filter using the relations given in Kent 1985. When we compare the Princeton results to Mt. Hopkins data for the same dates, we find an rms residual of 0.029 mag for component A and 0.018 mag for component B. Unexpectedly, we find the origin of this disagreement not to be so much in random errors as in a systematic drift in the apparent zero points in the course of the observing season. Data for the second observing season have not been presented in tabular form, but a plot of the data posted at the Princeton WWW site has allowed us to compare the results. We have taken the Princeton data plot, separated the two colors of data, and rescaled data for the Mt. Hopkins and Canary Island (Oscoz et al. 1996) groups to make the comparisons in Figures 4 and 5.

The Canary Island data are posted at the WWW site given in the report by Oscoz et al. 1996. They were obtained with the 0.8m telescope using standard R filters and local comparison stars. Because data were obtained in response to the Princeton challenge of Kundic et al. 1995, the Canary Island group reports data for the second season only. We have compared their data with the Mt. Hopkins data with the assumption that any Canary Island datum taken within 24 hours of a Mt. Hopkins observation had agreeing dates, and the rms deviations of the two data sets for our 15 agreeing dates is 0.013 mag for image A and 0.016 mag for B. The error estimates listed at the WWW web site are considerably larger, averaging 0.022 for A and 0.020 for B. Because the Canary Island—Mt. Hopkins comparison must have some error contribution from Mt. Hopkins, it is clear that the posted Canary Island error estimates are too large by a factor of
approximately 2. In our plots of the Canary Island data, Figure 4, we have used the original posted error estimates.

We show in Figure 4 a comparison of the available photometries for the first observing season. In the upper plot, the Mt. Hopkins data are shown with a magnitude scale and zero point for a standard R filter. The Princeton data have been shown with an arbitrary offset of 0.2 mag. These magnitudes are determined from a transformation from the Princeton Gunn g,r photometric system, using the transformation equations determined by Kent 1985. Error bars are shown strictly according to the estimates of the authors. The data are superimposed in the bottom panel of Figure 4, and the error bars are suppressed for clarity. It may be seen that the data agree about as well as predicted from the errors. One artifact that may be noticed is that there appear to be several points, mostly in the Princeton data, markedly below the mean trend. It is surprising that these discrepant points are in the sense of brightness deficiency, because the two principal error sources, cosmic rays and merging of the two quasar images due to bad seeing effects, both tend to make the images brighter. The A component data in the lower panel will be compared to the observations of image B in Figure 6.

In Figure 5 we show 3 data sets in the upper panel, with their associated error bars. However as noted previously, we do not actually have the tabulated data for the Princeton team, and we have scaled the results of Mt. Hopkins and the Canary Island groups to the Princeton data as posted in a plot released by the Princeton team. Thus the plotted magnitudes are on the Gunn photometric system, which differs from standard R by a zero point offset and a color term of 0.15 mag. In other words, \( R = r - 0.15(g-r) + \text{Const.} \) Since image B varied only from 1.071 to 1.142, a variation of 0.07 mag, we conclude that the scatter introduced into the comparison of r and R magnitudes has a full amplitude of 0.01 magnitudes, or a scatter of at most 0.005 magnitudes around a mean offset. Thus we have simply combined the Princeton r magnitudes with an arbitrary zero point offset in the comparison with the Mt. Hopkins and Canary Island R magnitudes in the bottom panel of Figure 5.

We find in Figure 5 (bottom) good evidence that the brightness drop predicted by Kundic et al. 1995 did indeed occur at around Julian Date 2450130. The brightness in the R band did indeed drop almost 0.1 magnitudes, and time delays of 423 days (Oscoz et al. 1997) and 416 days (Kundic et al. 1997) are determined. However, a remarkable thing happened at the end of this event, or immediately afterward; a strong microlensing event was observed, principally in the Mt. Hopkins data. The event may be seen as a strong downward spike centered on J.D. 2450151. Although the event was primarily seen in the Mt. Hopkins data, the brightness did not recover to the expected level for another 30
days, and for the remainder of this discussion, we refer to this event as the 3-observatory microlens.

A much better perspective on the 3-observatory microlens comes from inspection of Figure 6, where we plot data for both observing seasons combined with the 416-day time delay of Kundic et al. (1997). In this plot the open and filled symbols refer to the first and second observing seasons, exactly as in Figures 4 and 5. We consider that from J.D. 2 450 151 to 2 450 180 the data records are sufficiently discrepant to conclude that a microlensing event of 30 to 40 days duration and asymmetrical profile occurred. It is of course possible that more than one event was occurring at this time. It is likely that another event was seen at 2 450 220 ± 10 days, again seen by 3 observatories. A few other significant discrepancies may be recognized in this fascinating combined data record, and it is not surprising that the time delay has been so difficult to determine because of the influence of this complex pattern of microlensing. On the other hand, with the time delay now measured, the microlensing provides a powerful probe of the mass distribution of objects in the lens galaxy, and perhaps elsewhere (Schild 1996). The mass of the object causing the 35 day duration microlensing event at J.D. 2 450 151 is $10^{-6} M_{\odot} = 2 \times 10^{24}$ kg.

We now pose the question of the significance level of the detection of microlensing. We avoid questions of a posteriori statistics by phrasing a test as follows. A dramatic microlensing event was seen covering dates J.D. 2 450 150-70. During the previous year, a brightness record was obtained that covered the same time interval. If we average and smooth the brightness record for the previous year, at what level of statistical significance can we say each observatory noted a departure in the second year? Posed this way, we can easily determine that each of the three observatories observed a departure attributed to microlensing of at least $10 \sigma$, where the standard deviation $\sigma$ has been estimated for the individual data points of each observatory. Thus we conclude that each of three observatories has obtained as at least a $10 \sigma$ result that the second arriving image has brightness departures attributed to microlensing, because they were not seen in the first arriving image.

The observation by 3 observatories of the same microlensing event confirms the conclusion by Schild (1996) that low amplitude rapid microlensing is routinely seen in the Q0957 system. Interpreted as a manifestation of microlensing at large optical depth, the result implies the existence of a large population of masses in the range $10^{-6} M_{\odot}$, as predicted for the initial condensation mass within the primordial gas (PFPs) in (§3).
6. Nonbaryonic dark matter at galactic scales

What can be said of the nonbaryonic, weakly collisional, matter that makes up nearly all of the mass of the universe by most cosmologies (Silk 1994)? Tyson and Fischer (1995) have produced the first calibrated dark matter profile of a dense galaxy cluster, by a tomographic inversion of 6000 gravitational arcs from 4000 background galaxies. The Abell 1689 cluster mass is estimated as \(10^{45} \text{ kg}\), the redshift \(z = 0.18\), the density is \(\rho \approx 5 \times 10^{-21} \text{ kg/m}^3\), and the dark/luminous mass ratio is \(400 h^{-1} \approx 800\). For comparison, average (HST Deep Field) galaxies examined by the same method (Dell’Antonio and Tyson 1996) with mass \(\approx 2 \times 10^{42} \text{ kg}\) show dark/luminous mass ratios of only \(11^{4} h \approx 6\) within a radius of \(\approx 3 \times 10^{20} \text{ m (10 kpc)}\). The Abell 1689 dark mass distribution is smooth and strongly concentrated within a radius of \(6 \times 10^{21} \text{ m (200 kpc)}\) from the cluster center. Assuming an equilibrium between diffusivity, or turbulent or viscous forces, and gravitational forces gives a method for estimating the kinematic viscosity of the nonbaryonic dark matter (WIMP, NB) fluid, by setting the cluster radius equal to the maximum of \(L_{SD}, L_{ST},\) or \(L_{SV}\). For \(L_{SV}\),

\[
\nu_{WIMP} = \nu_{NB} \approx L_{SV}^2 \times \frac{\rho G}{\gamma} \approx \frac{M G}{L_{SV} \gamma} = \frac{10^{45} \times 6.7 \times 10^{-11}}{6 \times 10^{22} \times 3 \times 10^{-18}} \approx 10^{30} \text{ m}^2 \text{ s}^{-1},
\]

(37)

where \(\gamma\) is assumed to be that of the universe about 3 By before present.

Such a large value of \(\nu_{NB}\) is consistent with the weakly interacting nature assumed for the nonbaryonic dark matter fluid. The Reynolds number of the flow is about \(2 \times 10^{-4}\), so the non-turbulent assumption would be justified. The value \(\nu_{NB} \approx 10^{30} \text{ m}^2 \text{ s}^{-1}\) is an upper bound, and much less than \(c^2 t \approx 10^{34} \text{ m}^2 \text{ s}^{-1}\), which is the upper bound for physically meaningful \(\nu\) or \(D\) values within the horizon at that time. Smaller estimated values for \(\nu_{NB}\) by one or two orders of magnitude might be possible based on the observations, since the effective \(\gamma\) for the embedded galaxies of the cluster could be larger than \(1/t\) and \(L_{SV}\) indicated by the mass distribution could also be somewhat larger. If we assume \(\nu_{NB} \approx D_{NB}\) then we can test whether \(L_{SD} \leq L_{SV}\) as is required. We find \(L_{SD} \approx 4 \times 10^{22} \text{ m (1.3 Mpc)}\) by this method, much larger than the size of the core of the cluster \(6 \times 10^{21} \text{ m (200 kpc)}\). Therefore it appears that diffusion rather than viscous forces of the expanding universe may be limiting the size of the cluster to \(L_{SD}\). Substituting the measured size and density gives \(D \approx 2 \times 10^{28} \text{ m}^2 \text{ s}^{-1}\) for \(D_{NB} \approx \nu_{NB}\). If turbulence forces, diffusion, and gravitational forces are all in equilibrium with \(L_{SD} = L_{ST}\) then \(\varepsilon \approx 10^{-3} \text{ m}^2 \text{ s}^{-3}\). This value is close to \(\varepsilon \approx V^3/L\) estimated from dispersion velocities of \(2 \times 10^6 \text{ m}^2 \text{ s}^{-1}\) (Tyson and Fischer 1995), giving Reynolds numbers near critical with \(D_{NB} \approx \nu_{NB}\), but it is \(10^{11}\) greater than \(\varepsilon \leq 10^{-14} \text{ m}^2 \text{ s}^{-3}\) for the weak turbulence levels of the universe at the plasma-gas transition inferred from the COBE CMR measurements (Gibson 1996).
From $D_{NB} \approx m_{NB}/\rho \sigma_{NB}$ it is possible to calculate an effective collision cross section $\sigma_{NB}$ for the nonbaryonic fluid if the particle mass $m_{NB}$ can be assumed. Recent measurements suggest that neutrinos have mass, which would be $m_\nu \approx 10^{-35}$ kg for neutrinos to produce a flat universe. Substituting this, the cluster density $\rho$ and $D_{NB} \approx 10^{28}$ m$^2$ s$^{-1}$ gives $\sigma_{NB} = m_\nu/\rho D_{NB} \approx 10^{-43}$ m$^2$ assuming the nonbaryonic dark matter of the dense cluster consists of such massive neutrinos. This small cross section supports the assumption that the dense cluster fluid must be nonbaryonic.

If we accept $D_{NB} \approx \nu_{NB} \approx 10^{28}$ m$^2$ s$^{-1}$ as representative, it follows that such a large diffusivity would prevent condensation of most of the nonbaryonic dark matter on smaller scale objects like galaxies and could explain why the observed dark/luminous mass ratio for inner regions of galaxies (10 kpc) $\approx 6$ is so much smaller than the ratio for galaxy clusters (200 kpc) $\approx 800$. The averaged mass of nonbaryonic dark matter fluid compared to baryonic mass at galaxy scales may be estimated by

$$\frac{M_{NB \text{ dark matter}}}{M_{\text{galaxy}}} \approx \frac{\rho_{\text{universe}}}{\rho_{\text{galaxy}}} \approx \frac{10^{-26}}{10^{-20}} = 10^{-6},$$

which shows nearly all galactic dark matter is probably baryonic. A contrary view that galaxy halos cannot be baryonic is presented by Hegyi and Olive (1986), but their arguments and references rely crucially on the Jeans gravitational condensation theory and do not consider the possibility that condensation on nonacoustic nuclei is limited by $L_{SV}$, $L_{ST}$, $L_{SM}$, or $L_{SD}$ rather than $L_J$.

Figure 7abc summarizes the predictions of the present theory (7a,7c) of self-gravitational structure formation compared to that of the Jeans theory (7b). Viscous forces resist condensation in the initial stages of expansion of the hot plasma of the Big Bang, but some decelerations on protosupercluster to protogalaxy scales occur. A wavy line marks the plasma-gas transition (photon decoupling) in 7a and 7b. The Jeans theory in 7b is questionable because it relies on nonbaryonic-galaxy-nuclei condensing in the plasma epoch, which should be prevented by viscous forces on this super-viscous fluid, with $L_{SV-WIMP} > L_H \equiv ct$. Condensation of Jeans mass proto-globular-cluster droplets within protogalaxy droplets of gas is shown on the left of 7c, with embedded formations of primordial fog particles (PFPs). PFPs become increasingly compact and collisionless as they cool, so their aggregation should have been slow and gentle to form the first small stars from growing “robust associations of massive baryonic objects” or RAMBOs rather than rapid and turbulent to form large, short-lived, stars from turbulent gas at scales $L_{ST}$. Without PFP formation the Jeans mass proto-globular-cluster droplets of 6c would have been consumed by starbursts and supernova, leaving strongly turbulent conditions, with dense molecular gas clouds and dust clouds typical of spiral galaxy disks, that would have prevented the formation of the small, ancient, metal-free,
population II stars observed in dense, spherically-symmetric globular clusters, and left large quantities of metallic ashes (materials other than H and He) that are not observed.

7. Summary and conclusions

We conclude that gravitational structure formation is generally determined by either diffusion or viscous, magnetic, or turbulent forces on nonacoustic density nuclei at diffusive, viscous, magnetic or turbulent Schwarz scales $L_{SD}$, $L_{SV}$, $L_{SM}$ and $L_{ST}$. These scales are completely independent of the acoustic Jeans scale $L_J$. An initial condensation scale $L_{IC} \equiv (RT/\rho G)^{1/2} \approx L_J$ for a much larger body of gas with uniform pressure and density ratio $p/\rho = RT$. This initial condensation process also has nothing to do with the Jeans gravitational instability theory. Condensed bodies of gas in hydrostatic equilibrium have a hydrostatic scale $L_{HS} \equiv [(p/\rho)/\rho G]^{1/2} \approx L_J$, but again the relation to the Jeans scale has nothing to do with acoustics or Jeans's LPSA theory. $L_{HS}$ is an effect and not a cause of the condensation.

Jeans's theory of gravitational condensation fails because it is linear. Linear perturbation stability analysis cannot be applied to gravitational condensation because it drops the nonlinear inertial vortex force terms that cause turbulence, and the nonlinear density convective terms required for turbulent mixing to form nonacoustic density nuclei and their associated zero gradient configurations (ZGCs), such as density extrema and associated saddle points, which actually trigger and guide gravitational structure formation at gravitational condensation scales, $L_{SX}$, determined by fluid mechanical forces or diffusion (Gibson 1996). Because $D_{eff}$ may be negative ($\S$), ZGCs exhibit complex and remarkable gravitational instabilities that are quite independent of the Jeans $L_J$ criterion, as illustrated in Fig. 1, with rapid structure formation at the largest Schwarz scale $(L_{SX})_{max}$. We show in Fig. 2 that the strongest of such density extrema in the unique continuous gas produced at the primordial plasma-gas transition are absolutely unstable to the formation of proto-PFP condensates and proto-voids on length and mass scales determined by the Schwarz condensation limits $L_{SX}$ and $M_{SX}$, that are smaller than $L_J$ and $M_J$ by factors of about $10^4$ and $10^{12}$, respectively. Jeans’s acoustic criterion (Eq. 1) should be abandoned and replaced by appropriate fluid mechanical criteria (Eq. 2).

“Cannonball” and “vacuum beachball” nuclei on scales $(L_{SX})_{max} \leq L \ll L_J$ were introduced as perhaps the most easily understandable counterexamples to Jeans’s 1902, 1929 claim that no condensation can take place on scales smaller than $L_J$ in a homogeneous, motionless, gaseous continuum. Using Jeans’s initial conditions, these nuclei clearly trigger condensation and void formation by gravity at any scale $(L_{SX})_{max} \leq L \ll L_J$, contrary to the Jeans criterion (Eq. 1) for $L \ll L_J$. So do any other nonacoustic density nuclei. We found in Fig. 2 that cold-maxima and hot-minima density extrema...
monotonically condense to form proto-PFPs and expand to form proto-voids, respectively, with an intermediate state without heat transfer as they change to hot-maxima and cold-minima that qualitatively mimic their cannonball-beachball counterparts as absolutely unstable PFP and void nuclei.

The minimum condensation scale in quiescent fluids, with \( \gamma \leq (\rho G)^{1/2}(D/\nu) \) and \( \varepsilon \leq \rho GD \), is determined by diffusivity at a new length scale \( L_{SD} \) termed the diffusive Schwarz scale. Nonbaryonic fluids have enormous diffusivities \( D_{NB} \), and thus enormous \( L_{SD} \) values. Cosmological models based on early nonbaryonic condensations are strongly affected. The new magnetic Schwarz scale, \( L_{SM} \), is probably not important for the electrically neutral condensations of greatest interest in the present paper, but will be important in strongly ionized supernova shocked gas clouds or near galactic nuclei jets, for example.

By the new self-gravitational condensation theory, the primordial gas of hydrogen and helium condenses immediately after the plasma-gas transition to “primordial fog particles” (PFPs) within Jeans mass proto-globular clusters and the protogalaxy droplets of gas partially formed in the plasma epoch along with protoclusters and protosuperclusters. The structure formation sequences for the present theory and the Jeans theory are contrasted in Figure 7abc. Galaxy formation mechanisms by Jeans’s theory fail because diffusivity and viscous forces prevent condensation of the nonbaryonic-galaxy-nuclei required to form galaxies [Padmanabhan 1993]. Thus, computer modeling based on cold dark matter, hot dark matter, and mixed dark matter with n-body interactions of such nonbaryonic-galaxy-nuclei objects to match observations of large scale structure are unnecessary and confuted by the present theory.

According to the \( (L_{SX})_{max} \leq L \leq L_H \) condensation criterion, the most massive baryonic structures of the universe begin forming at a millionth of the present age rather than a tenth, with densities larger by billions and \( \tau_G \) values of millions of years rather than billions. If the age of the universe were a year, then superclusters and supervoids began decelerating at thirty seconds and the galaxies in an hour as plasma. At two hours the plasma turned to gas and the gas turned to fog, but a fog so rarified, and with such small particles after their freezing at three hours, that its particles become invisible. Only with the powerful backlighting of a quasar’s brightness and the powerful leverage of a quasar’s distance, which together increase the probability of a PFP microlensing a quasar by a factor of a million over that for a PFP microlensing a nearby star, has it been possible to reveal these primordial fog particles, which have provided the materials for all the stars and which likely comprise the dominant component of the missing mass of galaxies, at least within their inner halos (50 kpc).

Just as the twinkling of stars is caused by atmospheric density fluctuations, twinkling of quasars is caused by dark matter objects of galaxy halos. Any intrinsic variability of a
lensed quasar may be removed by subtraction when the time delay between two images is precisely known, so that the dominant mass of the objects in the halo can be estimated from the spectrum (Schild 1996). The Schild (1990, see also Schild and Thomson 1997) time delay of 1.1-years for the TwQSO quasar is herein reconfirmed by data from two more independent observatories. Two 3-observatory microlensing episodes were observed, also confirming the Schild (1996) interpretation that the lens galaxy mass is dominated by “rogue planet” mass objects that we suggest are PFP galactic dark matter.

The Tyson and Fischer (1995) calibrated measurement of the mass profile of the dense galaxy cluster Abell 1689 is considered in view of the proposed gravitational condensation theory. By setting the dark matter condensation scale equal to the diffusive Schwarz radius $L_{SD}$, the diffusivity of the nonbaryonic dark matter fluid $D_{NB}$ is estimated to be $\approx 10^{28}\text{ m}^2\text{s}^{-1}$. Such a large value is consistent with the diffusivity and viscosity one might expect for weakly-interacting-massive-particle (WIMP) nonbaryonic dark matter fluids $D_{NB} \approx \nu_{NB}$, but inconsistent with baryonic $D_B \approx \nu_B \approx 10^{15}\text{ m}^2\text{s}^{-1}$ values. These large $D_{NB} \approx \nu_{NB}$ values imply that only a small fraction of galaxy halo dark matter is likely to be nonbaryonic for the inner halo within, say, about 50 kpc of the core, although it may dominate the outer halo if it extends to $L_{SD}$ scales of $\approx 100$ kpc.

A long, nonlinear, clustering cascade is required for PFPs to aggregate by factors of millions in mass to form stars. Thus, PFP dark matter will aggregate to form nested clumps, or RAMBOs (Moore and Silk 1995), which become more difficult to detect the longer the cascade. Probability distributions for random variables $X$ undergoing self-similar nonlinear cascades over many decades become highly non-Gaussian, and $X$ becomes an extremely intermittent lognormal. This strongly reduces the probability of microlensing along a single line of sight (the optical depth) for PFPs clustering to form stars for our random variable $X = \text{PFPs per unit volume}$. The optical depth for intermittent PFPs (in RAMBOs) will be much smaller than for such homogeneously distributed micro-MACHOs by star-microlensing, so that the average density of these objects in the halo may be underestimated by the star-microlensing method. Because star-microlensing interpretations which exclude small-planetary-mass objects (PFPs) as the Galaxy halo mass have not accounted for undersampling errors associated with clumping of such small objects (Alcock et al. 1998), we consider such exclusions insecure.

Confirmation by three observatories of the 1.1-year time delay and PFP-period microlensing with 10 $\sigma$ certainty of TwQSO quasar-microlensing reported herein gives strong support to the Schild 1996 claim that the lens galaxy mass is “dominated by rogue planets ... likely to be the missing mass”, and the Gibson 1996 prediction of the origin of such objects as “primordial fog particles”.

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Acknowledgements. We wish to acknowledge advice and discussions with a number of colleagues, particularly Barnaby Rickett, Paul Libby, Norris Keeler, Norbert Peters and Peter Bradshaw. We are grateful for useful comments of anonymous referees and thank Ned Wright for providing an extended version of Jeans's linear perturbation analysis. This work was carried out under the auspices of the Society for Statistical Geometry.

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Figure 1. Schematic illustration of the new turbulent gravitational structure formation theory. Nonacoustic density maxima and minima are produced by a turbulent eddy, shown on the left, which distorts constant density surfaces of a uniform density gradient until they become diffusively unstable and break up to form density extrema and associated saddle points. Neighboring saddle points (triangles) form a saddle line (circles) and a doublet. Without convection or gravitational forces the doublet decays by diffusion and vanishes, as shown top right. With gravitation at scales larger than \((L_{SX})_{\text{max}}\), the maximum density lobe contracts to form a PFP and the minimum density lobe expands to form a void that surrounds the PFP as the saddle line contracts to form a minmax saddle point (double triangle) down gradient (center). At the bottom, a minimum point larger than \((L_{SX})_{\text{max}}\) forms a void, and a maximum point larger than \((L_{SX})_{\text{max}}\) forms a doublet, which forms a PFP in a void as before.

Figure 2. Condensation of a “cold spot” nonacoustic density maximum to form a proto-PFP (top) and expansion of a “hot spot” nonacoustic density minimum to form a proto-void (middle) for a continuous, otherwise homogeneous, static, perfect gas representing the primordial universe after transition from the Big Bang plasma, compared to the decay of an acoustic density perturbation smaller than the Jeans scale \(L_J\) (bottom) that propagates away and vanishes by viscous dissipation. Both these examples of nonacoustic density maxima and minima initial conditions are absolutely unstable. They cause imbalances in the initially uniform, static, mass distribution, producing forces and accelerations toward and away from the nonacoustic density maximum and minimum (left, top and middle), respectively, with intermediate states (center, top and middle) of nearly uniform temperature where the directions of heat transfer reverse. After reversal, the collapse of mass toward the former cold spot accelerates, due to both the increasing mass within any radius and the increased average density of the proto-PFP caused by the cooling heat transfer. The expansion of former hot spot (middle) is also accelerated for all radii by both the increasing size and the reversed heat transfer to form the expanding proto-void (right, middle). The absolute instability of nonacoustic density maxima and minima in a homogeneous gas continuum may also be understood by replacing the cold and hot spot density extrema by a “cannon ball” and a “vacuum beach ball”, giving the same results as those shown at the right (top, middle), respectively.

Figure 3. Application of the new self-gravitational condensation theory to the formation of primordial-fog-particles (PFPs) from protogalaxy primordial gas droplets soon after the plasma-gas transition 300,000 years after the Big Bang.

Figure 4. Data for the A (northern) gravitational lens image, recorded in the October 1994 - June 1995 observing season. In the upper panel, brightness estimates with error bars are shown as triangles for the Schild and Thomson (1997) data, and as circles for
the Kundic et al. (1995) data. The R magnitude scale is for the Schild and Thomson
data and the Kundic et al. data is arbitrarily offset 0.2 mag to permit comparison. In
the lower panel, the data are shown superimposed and without error bars, to show the
generally good agreement, especially around the date of the large quasar brightness drop
at 2449715. In Figures 4, 5, and 6 the most significant 2 digits of the Julian date have
been suppressed for clarity.

Figure 5. Data for the B (southern) component recorded in the Nov. 1995–June 1996
observing season. In the upper panel, filled squares with error bars are from Oscoz et al.
1996, Triangles are from Schild and Thomson (1997, in preparation), and circles are from
the WWW plot at the site reported by Kundic et al. (1995). In the lower panel, data
from the three observatories are plotted without error bars. Generally good agreement is
shown in the comparison, and distinct brightness trends are seen in all three data sets.

Figure 6. Data from the lower panels of Figures 4 and 5 are shown superimposed
with the same symbol definitions as previously, and for a 416 day time delay. It may
immediately be seen that there is generally good agreement, and that the second arriv-
ing B image (solid symbols) generally follows the pattern of fluctuation exhibited the
year before in the first-arriving A image (open symbols). However there are important
differences; around Julian Date 2450150-70 a strong brightness drop occurred that had
not been seen in the first arriving A image. Similarly, around J.D. 2450220 the records
differ systematically by several percent.

Figure 7. Comparison of gravitational structure formation in the universe for the
present theory (7a) and the Jeans acoustic theory (7b), and details of galaxy for-
mination (7c). The theories diverge in the hot plasma epoch (left 7a, 7b) soon after
the Big Bang. By the present theory, nonbaryonic (WIMP) matter that dominates
the total mass of the universe is superdiffusive and superviscous (see Section 4) with
either $L_{SP}$ or $L_{SV}$ values larger than the Hubble, or causal, scale of the universe
$L_H = ct$ during the plasma epoch, preventing condensation to form the WIMP-galaxy-
nuclei assumed and permitted in many cold, hot, and mixed “dark matter” theories
(Padmanabhan 1993, Klypin et al. 1997) since $L_{J-WIMP} < L_H$. Such assumptions and
theories appear to be questionable, as indicated (7b). By the present theory (7a), bary-
onic dark matter is mostly H-He objects $\approx 10^{-6} M_\odot$ that condensed as “primordial fog
particles” (PFPs) soon after the plasma to gas transition (and photon decoupling). The
Jeans theory permits no baryonic condensation in the plasma epoch, but condensation
of baryonic plasma is permitted by the present theory on nonacoustic nuclei when $L_{SV}$
decreases to $L_H$ or less at $t \approx 10,000$ years to form a nested-foam topology of pro-
tosupercluster to protogalaxy mass associations (Gibson 1996) which turn to PFP fog
at decoupling. The nonbaryonic dark matter forms $L_{SD}$ scale superhalos in diffusive-
gravitational equilibrium with the baryonic supercluster and cluster structures as they evolve to form protogalaxy-droplets of plasma, that turn to gas after the wavy line of Fig. 7a. The detailed evolution of such a protogalaxy-droplet of gas to form PGCs, PFPs, RAMBOs, and eventually stars is shown in Fig. 7c.