Investigation on Terminal Correction in Theoretical Model of the Microperforated Panel Absorber

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Abstract. Microperforated panel (MPP) absorber is promising sound absorbing structure for the noise reduction. It is extremely especially to construct accurate theoretical sound absorbing model of the MPP structure for design and development of an excellent sound absorber. Thus, modification and validation of the terminal correction in theoretical sound absorption model of the MPP absorber is investigated in this research, which aimed to improve its accuracy and reasonability. Firstly, based on the Maa’s theory, classic sound absorption model of the MPP absorber was presented and treated as basic reference of the whole research. Secondly, through a set of verification experiments, necessity and importance of the research on reasonability of terminal correction was revealed. For the superposition of several monolayer MPP absorbers, when its sound absorption coefficient was derived as the multilayer MPP absorber through the transfer matrix method, the terminal corrections for the former n-1 monolayer MPPs should be removed, because length of these MPPs was 0. Thus, it was essential to take length of the rear cavity into account for the terminal corrections in the theoretical sound absorption model of the MPP absorber.

1. Introduction
Microperforated panel (MPP) absorber is promising sound absorbing structure for the noise reduction [1]. Besides the excellent low-frequency sound absorption performance, the MPP has the advantages of elegant appearance, outstanding machinability, fine washability, and low fabrication cost as well [2]. Meanwhile, relative to the other common porous sound absorbing material, such as the porous metal and metalfiber, the MPP absorber is easy to design, fabricate, manufacture, assemble, transport, maintain, and replace. Moreover, there are few limits for the materials of the panels to fabricate the MPP and their sound absorption performance is hardly affected by physical and chemical properties of the panel, which indicate that many special MPP absorbers can be developed through selecting the proper panel, such as the transparent MPP through using glass as the panel, colorful MPP by utilizing polymethyl methacrylate as the panel, stealthy MPP through painting absorbing coating on the panel, and so on. Therefore, the MPP absorber is one of the research focuses in the field of acoustics. In order to increase the sound absorption bandwidth and decrease the occupied space, multilayer MPP has been developed and its structural parameters are optimized for a better sound absorption property. Structural parameters of the multilayer MPP absorbers were optimized by Bravo et al. [3] through the impedance translation approach, by Ruiz [4] et al. through the simulated annealing, by Zhao et al. [5] through the genetic algorithm, by Yang et al. [6] through the cuckoo search algorithm, and so on. Meanwhile, in the interest of overcoming the shortage of single sound absorbing material, many composite structures had been promoted, which consisted of MPP. Low-frequency sound absorption of the MPP absorber...
was enhanced by Zhao et al. [7] through combining with parallel mechanical impedance. Shen et al. [8] had optimized the combination structure of MPP and porous metal to develop the sound absorber with constraint conditions for the low-frequency noise reduction. The physical mechanisms involved in the sound attenuation and absorption of the MPP backed with the anisotropic fibrous material were studied by Bravo and Maury [9]. Li et al. [10] had attempted to enhance the low-to-mid frequency sound absorption using parallel-arranged perforated plates with the extended tubes and porous material. Bai et al. [11] proposed the microperforated compressed porous metal panel absorber by combining the porous metal and MPP into one, and Duan et al. [12] had further optimized and improved its sound absorption property with variable target frequency range.

The classic sound absorbing model for the MPP absorber is proposed by Maa [1], based on which theoretical sound absorption coefficients of the MPP absorber can be derived through calculating its acoustic impedance. Both the acoustic resistance and acoustic mass, which correspond to real part and imaginary part of the acoustic impedance respectively, have the terminal corrections, which aim to improve accuracy and veracity of the constructed theoretical sound absorbing model. For the acoustic resistance, the terminal correction is generated by the friction loss of the air flowing along the panel when the sound wave goes in and out of the micro-hole. Meanwhile, with regard to the acoustic mass, the terminal correction represents acoustic radiation at the terminal of the micro-hole. However, the terminal corrections are conditional and their reasonability must be taken into consideration when the MPP forms the multilayer MPP absorber or the composite sound absorbing structure. Sheng [13] had investigated variation of the terminal correction when the MPP was closely pasted with absorptive thin layer, result of which indicated that the additional acoustic impedance could be adjusted through selecting proper parameters of the thin layer. Moreover, it had been proved by Yang et al. [14] that the relative error between the theoretical data and the experimental data would increase when there were some tiny rear cavities in the multilayer MPP absorber. Therefore, modification and validation of the terminal correction in theoretical sound absorption model of the MPP absorber is investigated in this research, which aimed to improve its accuracy and reasonability.

2. Classic sound absorption model of the MPP absorber

According to the Maa’s theory [1], the theoretical sound absorption model of the monolayer MPP absorber could be constructed through the transfer matrix method based on equivalent circuit, which was obtained by the Eqs. (1) and (2). Here $\alpha$ is the theoretical sound absorption coefficient of the monolayer MPP absorber; $T_{11}$ and $T_{21}$ in the Eq. (1) were two components in the total transfer matrix $T_{total}$, as shown in the Eq. (2); $\rho$ and $c$ in the Eq. (1) were density and sound speed of the air with condition of room temperature and atmospheric pressure, 1.21 kg/m$^3$ and 340 m/s respectively; $Tr_{mpp}$ and $Tr_{cav}$ were transfer matrix of the MPP and that of the rear cavity, respectively, which could be calculated by the Eqs. (3) and (4) respectively [1].

$$\alpha = \frac{4 \text{Re} \left( \frac{T_{11}}{T_{21}} \frac{1}{\rho c} \right)}{1 + \text{Re} \left( \frac{T_{11}}{T_{21}} \frac{1}{\rho c} \right) + \text{Im} \left( \frac{T_{11}}{T_{21}} \frac{1}{\rho c} \right)}$$

(1)

$$T_{total} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = Tr_{mpp} \cdot Tr_{cav}$$

(2)

$$Tr_{mpp} = \begin{bmatrix} 1 & Zs \\ 0 & 1 \end{bmatrix}$$

(3)
\[
T_{res} = \begin{bmatrix}
sin \left( \frac{\omega D}{c} \right) & j \rho c \sin \left( \frac{\omega D}{c} \right) \\
-j \sin \left( \frac{\omega D}{c} \right) & \cos \left( \frac{\omega D}{c} \right)
\end{bmatrix}
\] (4)

In the Eq. (3), \( Z_s \) was acoustic impedance rate of the MPP, which included the real part \( R \) and the imaginary part \( X \), as shown in the Eq. (5). In the Eq. (4), \( j \) was symbol of the imaginary number, \( j = \sqrt{-1} \); \( \omega \) was the angular frequency, which could be obtained by the Eq. (6), and \( f \) and \( \pi \) were the sound frequency and circular constant respectively; \( D \) was length of the rear cavity.

\[
Z_s = R + jX
\] (5)

\[
\omega = 2\pi f
\] (6)

The real part \( R \) and the imaginary part \( X \) could be calculated by the Eqs. (7) and (8) respectively. Here \( \mu \) was viscosity coefficient of the air with the condition of room temperature and atmospheric pressure, \( 1.506 \times 10^{-5} \text{ m}^2/\text{s} \); \( \nu \) was the temperature conduction coefficient of the metal panel with the condition of room temperature and atmospheric pressure, \( 2.0 \times 10^{-5} \text{ m}^2/\text{s} \); \( e \) was the perforating rate, which could be derived by the Eq. (9); The \( d \), \( t \), and \( b \) in the Eqs. (7) to (9) were diameter of the hole, thickness of the panel, and distance between the neighboring holes, respectively; \( k_r \) in the Eq. (7) was the acoustic resistance constant, which could be obtained by Eq. (10); \( k_m \) in the Eq. (8) was the acoustic mass constant, which could be derived by Eq. (11). In the Eqs. (10) and (11), \( k \) is the perforated panel constant, which can be calculated by Eq. (12) [1].

\[
R = \frac{32(\mu + \nu)}{\varepsilon} \frac{\rho}{d^2} \cdot k_r
\] (7)

\[
X = \frac{\varepsilon}{4} \cdot k_m
\] (8)

\[
\varepsilon = \frac{\pi}{4} \left( \frac{d}{b} \right)^2
\] (9)

\[
k_r = \sqrt{1 + \frac{k_m^2}{32} + \frac{\sqrt{2}}{8} k \cdot \frac{d}{t}}
\] (10)

\[
k_m = 1 + \left( 9 + \frac{k_m^2}{2} \right)^{0.85} + 0.85 \cdot \frac{d}{t}
\] (11)

\[
k = \sqrt{\frac{\omega}{\mu + \nu}} \cdot \frac{d}{2}
\] (12)

3. Existing problems for the terminal correction

It could be observed from the Eq. (10) that there was terminal correction \( \frac{\sqrt{2}}{8} k \frac{d}{t} \) for the acoustic resistance constant, which were generated from friction loss of the air, because there would be flow along the panel when the air accessed the microtubule. Meanwhile, judging from the acoustic mass constant in the Eq. (11), there also existed the corresponding terminal correction \( 0.85 \frac{d}{t} \), which was generated from the acoustic radiation of the terminal at the two ends. Furthermore, it could be found that the two terminal corrections were determined by the diameter of the hole \( d \) and thickness of the panel \( t \), and it had no relationship with length of the cavity \( D \). However, no matter the friction loss or the acoustic radiation, the precondition of their appearances was that there had a cavity. When length of the cavity was close to 0, the terminal correction would be affected. A simple analysis to explain the
existing problems for terminal correction was conducted. One single layer MPP absorber could be considered as the superposition of some monolayer MPP absorbers, as shown in the Figure 1.

![Diagram](image)

Figure 1. Schematic diagram of single layer MPP absorber and that of the corresponding equivalent.

For the single layer MPP absorber in Figure 1(a), the diameter of the hole, thickness of the panel, distance between the neighboring holes, and length of the rear cavity were $d_0$, $t_0$, $b_0$, and $D_0$, respectively. Meanwhile, for the equivalent $n$ monolayer MPPs in the Figure 1(b), the diameter of the hole, thickness of the panel, and distance between the neighboring holes were $d_0$, $t_0/n$, and $b_0$, respectively. Moreover, in the Figure 1(b), length of the rear cavity for the $n^{th}$ monolayer MPP was $D_0$, and those for the other monolayer MPPs were 0. In fact, the exhibited absorber in the Figure 1(b) was completely same with that in the Figure 1(a).

From the classic sound absorption model of the MPP absorber, the theoretical sound absorption coefficient of the MPP absorber in the Figure 1(a) could be calculated, and that in the Figure 1(b) could be derived as the multilayer MPP absorber. Taking $d_0 = 0.2\, mm$, $t_0 = 0.4\, mm$, $b_0 = 1\, mm$, and $D_0 = 20\, mm$ for example, the studied layer number $n$ was 1, 2, 4, 8, 10, and 20 respectively, and the calculated sound absorption coefficients in frequency range of 100-2000 Hz for various superposition of the monolayer MPP absorbers were shown in the Figure 2. It could be found that for the various superposition of monolayer MPP absorbers, distribution of corresponding sound absorption coefficient was different. Along with increase of the layer number, the peak sound absorption frequency moved to low frequency direction, and the peak sound absorption coefficient decreased gradually. Meanwhile, the theoretical average sound absorption coefficient decreased from 0.6580 to 0.4951 when the layer number increased from 1 to 20. However, the seamless division of the single layer MPP from Figure 1(a) to Figure 1(b) was an assumption, and sound absorption coefficients of various superposition of the monolayer MPP absorbers should have no difference, because them two absorbers were same.
4. Additional modification factors

The present question was how to quantitatively describe the influence of the length of the rear cavity to the terminal corrections. Referring to the introduction of correction factors to improve the accuracy of the Johnson-Champoux-Allard model for sound absorption coefficient of the porous material by Kino et al. [15] and by Yang et al. [16] respectively, an additional modification factor \( \eta(D,t) \) was introduced to the terminal corrections, and it was decided by length of the rear cavity \( D \), thickness of the panel \( t \), and frequency of the incident wave \( f \) together. From the analysis in the section 3, it could be found that the additional modification factor \( \eta(D,t,f) \) was 0 when \( D = 0 \), as shown in the Eq. (13). Meanwhile, it was easy to find that when \( D \neq t \), the additional modification factor \( \eta(D,t,f) \) should be 1, as shown in the Eq. (14). These were two boundary constraint conditions.

\[
\eta(D,t,f) = 0 \quad D = 0 \tag{13}
\]
\[
\eta(D,t,f) = 1 \quad D \neq t \tag{14}
\]

Meanwhile, the additional modification factor \( \eta(D,t) \) should obey the following three principles: (1) It should be monotonically increasing along with the increase of the length of the cavity \( D \); (2) It should be monotonically increasing along with the increase of the frequency of the incident wave \( f \); (3) It should be monotonically decreasing along with the increase of the thickness of the panel \( t \). Referring to the introduced correction factors to improve accuracy of the Johnson-Champoux-Allard model for the sound absorption coefficient, the selected additional modification factor \( \eta(D,t,f) \) was established, as shown in Eq. (15). Here \( \lambda \) was the modification constant, which was a negative number.

\[
\eta(D,t,f) = 1 - e^{-\lambda \frac{D}{t}} \quad \lambda < 0, D \geq 0, t > 0, f > 0 \tag{15}
\]

5. Conclusions

The undesired difference was generated from the unreasonable terminal correction in the theoretical modeling process. Although it seemed that the calculation of the terminal correction \( \frac{\sqrt{\delta}}{8k} \frac{d}{f} \) for the acoustic resistance constant in the Eq. (10) and that of the terminal correction \( 0.85 \frac{d}{f} \) for the acoustic mass constant in the Eq. (11) had no relationships with length of the rear cavity \( D \), the rear cavity with a certain length was essential condition for the terminal corrections. For the superposition of several monolayer MPP absorbers in the Figure 1(b) with the parameters, when its sound absorption coefficient was derived as the multilayer MPP absorber by the transfer matrix method, the terminal corrections for the former \( n - 1 \) monolayer MPPs should be removed, because length of these MPPs
was 0. Thus, it was essential to take length of the rear cavity into account for the terminal corrections in the theoretical sound absorption model of the MPP absorber.

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