Note on Reheating in G-inflation

Hossein Bazrafshan Moghaddam,† Robert Brandenberger,‡ and Jun’ichi Yokoyama§

1Department of Physics, McGill University, Montreal, QC, H3A 2T8, Canada
2Physics Department, McGill University, Montreal, QC, H3A 2T8, Canada, and Institute for Theoretical Studies, ETH Zürich, CH-8092 Zürich, Switzerland
3Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan,
Department of Physics, Graduate School of Science, The University of Tokyo, Tokyo, 113-0033, Japan
Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU), UTIAS, WPI, The University of Tokyo, Kashiwa, Chiba, 277-8568, Japan

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We study particle production at the end of inflation in kinetically driven G-inflation model and show that, in spite of the fact that there are no inflaton oscillations and hence no parametric resonance instabilities, the production of matter particles due to a coupling to the evolving inflaton field can be more efficient than pure gravitational Parker particle production.

I. INTRODUCTION

Reheating, namely the transition from the period of inflation [1], during which the energy-momentum tensor is dominated by the coherent inflaton field, to the radiation phase of Standard Big Bang cosmology, is an important aspect of inflationary cosmology. Without such an energy transfer, inflation would produce a cold empty universe and would not be a viable early universe scenario. On the other hand, there will inevitably be gravitational particle production of any non-conformal field which lives in the space-time of an inflationary universe [2] (see [3] for a classic review of quantum field theory in curved space-time). The energy density produced by this mechanism by the end of the inflationary phase will be of the order $H^4$, where $H$ is the Hubble expansion rate during inflation. Note that this energy scale is parametrically suppressed compared to the energy density $\rho_I$ during inflation:

$$\frac{H^4}{\rho_I} \sim \frac{H^2}{m_{pl}^2},$$

where $m_{pl}$ is the Planck mass. This ratio is bounded to be smaller than $10^{-8}$ based on the upper bound on the strength of gravitational radiation produced during inflation [4].

In simple scalar field models of inflation, based on a slowly rolling scalar field with canonical kinetic term in the action, there is a much more efficient energy transfer mechanism which can reheat the universe. In the presence of any coupling between the matter field (here modelled as a scalar field $\chi$) and the inflaton field $\phi$ there is a parametric resonance instability which causes $\chi$ field fluctuations to grow exponentially during the phase after inflation when $\phi$ oscillates coherently about its ground state value [5]. Although this process does not directly produce a thermal state of matter particles, it efficiently transfers the energy density from the inflaton to matter, typically in a period which is short compared to a Hubble expansion time. This process is now called preheating [6–9] (see [10] for recent reviews). It produces a state after inflation in which the matter energy density

$$\rho_m \sim \rho_I$$

after inflation is not suppressed compared to the energy density $\rho_I$ during inflation, in contrast to what is obtained [1] if only gravitational particle production is operative.

Simple slow-roll inflation based on an action with a canonical kinetic term is at the moment consistent with the data we have. In fact, the scenario made a number of successful predictions (spatial flatness, slight red tilt [11] to the spectrum of cosmological perturbations, etc.) [12–14]. The scenario also predicts a red tilt in the spectrum of gravitational waves [16].

There are alternatives to the inflationary paradigm of early universe cosmology (see e.g. [17] for reviews). One of these alternatives, String Gas Cosmology [18] (see also [19]), while consistent with all current observations of scalar cosmological perturbations [20], predicts a slight blue tilt in the spectrum of gravitational waves [21]. In the context of inflationary cosmology with vacuum initial conditions and with matter obeying the Null Energy Condition (NEC), one always obtains a red tilt [2].

However, it was pointed out in [22] (see also [23]), that by introducing Galileon type terms (in particular kinetic

References:

1 But see [15] for a different view.

2 This is different than the scalar spectrum for which either a red or a blue tilt can be obtained, although the simplest slow-roll models of inflation also predict a red tilt of the scalar spectrum.
terms) in the action of a scalar field $\phi$, it is possible to obtain an inflationary model in which matter violates the NEC and hence a blue tensor tilt is possible $^3$. This model is called $G$-inflation. In this model, inflation is driven by the kinetic term in the action which at early times has the “wrong” sign and hence can lead to the violation of the NEC. Nevertheless thanks to the Galileon-type terms, the stability of fluctuations is maintained even in the presence of NEC violation contrary to the case of $k$-inflation $^{[25]}$. Inflation terminates at a scalar field value above which the sign of the kinetic term reverts back to its canonical form. This leads to a transition from an inflationary phase to a standard kinetic-driven phase with equation of state $w = 1$, where $w$ is the ratio of pressure to energy density. The energy density in $\phi$ then decreases as $a(t)^{-6}$, where $a(t)$ is the cosmological scale factor.

Since there is no phase during which $\phi(t)$ oscillates there is no possibility of preheating. As discussed in $^{[22]}$, the production of regular matter particles after Galileon inflation is still possible by the gravitational Parker particle production mechanism. However, the resulting matter energy density will be suppressed as in $^{[1]}$. The question we ask in this note is whether in the presence of a coupling between matter and the inflaton there is nevertheless some non-gravitational channel which transfers energy to matter faster than what can be achieved by gravitational effects.

In the following we point out that there is indeed a channel for direct particle production, and we derive conditions on the coupling constant for which this direct channel is more efficient than Parker particle production $^4$. Our analysis is based on the general framework set out in $^{[5]}$.

We begin with a brief review of G-inflation, move on to a discussion of the particle production mechanism we use, before presenting the calculations applied to our model. We work in natural units in which the speed of light, Planck’s constant and Boltzmann’s constant are set to 1.

II. G-INFLATION

The original G-inflation $^{[22]}$ is based on a scalar field $\phi$ minimally coupled to gravity with an action

$$\mathcal{L} = K(\phi, X) - G(\phi, X) \Box \phi, \quad (3)$$

where $X$ is the standard kinetic operator

$$X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi, \quad (4)$$

and $K$ and $G$ are general functions of $\phi$ and $X$. See $^{[29]}$ for its generalized version. The special property of this class of Lagrangians is that the resulting equations of motion contain no higher derivative terms than second order $^{[30]}$. In the case $K = K(X)$ and $G(\phi, X) \propto X$ the action has an extra shift symmetry (“Galilean symmetry”) and these Lagrangians were introduced and studied in $^{[31]}$.

The model of kinetically driven G-inflation $^{[22]}$ is based on choosing

$$K(\phi, X) = -A(\phi)X + \delta K, \quad (5)$$

with

$$A(\phi) = \tanh[\lambda(\phi_c - \phi)], \quad (6)$$

and

$$G(\phi, X) = \tilde{g}(\phi)X = \tilde{g}X. \quad (7)$$

Here $\lambda$ and $\tilde{g}$ are coupling constants and $\delta K$ includes higher order terms in $X$ which are important during inflation. After the sign of the linear kinetic term in the action is flipped at $\phi = \phi_c$, they soon become negligible and do not affect our analysis of reheating.

We consider homogeneous and isotropic cosmological solutions resulting from this action. As shown in $^{[22]}$, for $\phi < \phi_c$ there are inflationary trajectories for which the quasi-exponential expansion of space is driven by the wrong-sign kinetic term. Inflation ends at $\phi = \phi_c$, and for $\phi > \phi_c$ the background becomes that of a kinetic-driven phase with $w = 1$, $a(t) \sim t^{1/3}$ and

$$\dot{\phi}(t) \sim \frac{1}{t}. \quad (8)$$

We call this stage the kination regime of the model. Since the energy density in $\phi$ decays so rapidly, eventually the kination regime will end and regular radiation and matter will begin to dominate. The energy density at which this transition happens determines the reheating temperature of the Universe.

Knowledge of the reheating temperature is important for various post-inflationary processes such as baryogenesis or the possibility of production of topological defects. It may also be possible to directly probe the physics of the phase between the initial thermal stage and the hot Big Bang phase with precision observations (see e.g. $^{[32]}$).

Regular matter and radiation are produced by gravitational particle production. However, if this is the only mechanism, then the reheating temperature will be low as it is suppressed by $^{[1]}$. In the following we will assume that there is a direct coupling between matter (described by a free massless scalar field $\chi$) and the inflaton field $\phi$. We consider two possible couplings. The first is of the form

$$\mathcal{L}_I = \frac{1}{2} g^2 \phi \chi^2, \quad (9)$$

where $g$ is a dimensionless coupling constant. Note that we have chosen a derivative coupling of $\phi$ with $\chi$ to preserve the invariance of the interaction Lagrangian under

$^3$ It is still possible to distinguish String Gas Cosmology from G-Inflation by considering non-Gaussianities or consistency relations $^{[24]}$.

$^4$ A similar channel is operative in the “emergent Galileon” scenario of $^{[26]}$ - see $^{[27]}$. Particle-induced particle production has also recently been studied in a bouncing cosmology in $^{[25]}$. 

shifting of the value of $\phi$ (which is part of the Galilean symmetry. The disadvantage of this coupling is that it violates the symmetry $\phi \rightarrow -\phi$. The second coupling obeys this symmetry but involves non-renormalizable interactions. It is

$$\mathcal{L}_I = -\frac{1}{2} M^{-2} \dot{\phi}^2 \chi^2,$$  

(10)

where $M$ is a new mass scale which is expected to be smaller than the Planck mass. These couplings open up non-gravitational channels for the production of $\chi$ particles. In the following we will study the conditions under which these direct production channels are more efficient than the gravitational particle production channel.

### III. INFATON-DRIVEN PARTICLE PRODUCTION

Assuming that the Lagrangian for the matter field $\chi$ has canonical kinetic term, then the Lagrangian for $\chi$ is that of a free scalar field with a time dependent mass, the time dependence being given by the interaction Lagrangians [9] or [10]. Each Fourier mode $\chi_k$ of $\chi$ evolves independently, the equation of motion is

$$\ddot{\chi}_k + 3H \dot{\chi}_k + \left(\frac{k^2}{a^2} - g^2 \dot{\phi}^2\right) \chi_k = 0 .$$  

(11)

or

$$\ddot{\chi}_k + 3H \dot{\chi}_k + \left(\frac{k^2}{a^2} + M^{-2} \dot{\phi}^2\right) \chi_k = 0 ,$$  

(12)

depending on the form of the interaction Lagrangian. The effects of the expansion of space can be pulled out by rescaling the field

$$X_k \equiv a^{-1} \chi_k .$$  

(13)

Then, in terms of conformal time $\tau$ (which is related to physical time $t$ by $dt = a(t)d\tau$), the equation of motion becomes

$$X''_k + \left( k^2 - g^2 \dot{\phi} a^2 - \frac{a''}{a} \right) X_k = 0 ,$$  

(14)

or

$$X''_k + \left( k^2 + M^{-2} \dot{\phi}^2 a^2 - \frac{a''}{a} \right) X_k = 0 ,$$  

(15)

where a prime denotes a derivative with respect to $\tau$.

The qualitative features of the equations of motion (14) or (15) are well known from the theory of cosmological perturbations (see e.g. [33] for an in-depth review and [34] for a brief overview): In the absence of the interaction term, $X_k$ will oscillate on sub-Hubble scales, i.e. scales for which

$$k^2 > \frac{a''}{a} \sim \mathcal{H}^2 ,$$  

(16)

whereas the mode function $X_k$ is squeezed on super-Hubble scales, i.e.

$$X_k \sim a .$$  

(17)

Following [5], we will treat the effects of the interaction term in leading order Born approximation, i.e. we write

$$X \equiv X_0 + X_1 ,$$  

(18)

(here and in the following we will drop the subscript $k$) where $X_0$ is the solution of the equation in the absence of interactions, i.e. a solution of

$$X''_0 + \left( k^2 - \frac{a''}{a} \right) X_0 = 0 ,$$  

(19)

solving the initial conditions of the problem, and $X_1$ is the solution of the inhomogeneous equation

$$X''_1 + \left( k^2 - \frac{a''}{a} \right) X_1 = g^2 \dot{\phi} a^2 X_0$$  

(20)

or

$$X''_1 + \left( k^2 - \frac{a''}{a} \right) X_1 = -M^{-2} \dot{\phi}^2 a^2 X_0$$  

(21)

(with vanishing initial conditions) obtained by taking the interaction term in (14) or (15) to the right hand side of the equation and replacing $\chi_k$ by the “unperturbed” solution $X_0$.

The inhomogeneous equation (20) (or (21)) can be solved by the Green’s function method

$$X_1(\tau) = \int_{\tau_i}^{\tau} d\tau' G(\tau, \tau') g^2 a^2(\tau') \dot{\phi}(\tau') X_0(\tau') ,$$  

(22)

or

$$X_1(\tau) = -\int_{\tau_i}^{\tau} d\tau' G(\tau, \tau') M^{-2} a^2(\tau') \ddot{\phi}(\tau') X_0(\tau') ,$$  

(23)

where the Green’s function $G(\tau, \tau')$ is determined in terms of the two fundamental solutions $u_1$ and $u_2$ of the homogeneous equation via

$$G(\tau, \tau') = W^{-1} (u_1(\tau) u_2(\tau') - u_2(\tau) u_1(\tau')) ,$$  

(24)

where $W$ is the Wronskian

$$W = u_1(\tau) u_2'(\tau) - u_2(\tau) u_1'(\tau) .$$  

(25)

In the above, the time $\tau_i$ is the time when the initial conditions are imposed. In our case it is the end of the period of inflation.

The condition that direct particle production is more efficient than gravitational particle production is

$$X_1(\tau) > X_0(\tau)$$  

(26)

at some time $\tau > \tau_i$ before the time when the kinetic phase would be terminated by gravitational particle production alone.
IV. ANALYSIS

We now apply the formalism of the previous section to our specific Galileon inflation model. We are interested in super-Hubble modes for which the $k^2$ term in the equation of motion (11) can be neglected. The fundamental solutions are then

$$u_1(\tau) = \left(\frac{\tau}{\tau_i}\right)^{1/2}, \quad u_2(\tau) = \left(\frac{\tau}{\tau_i}\right)^{1/2} \ln \left(\frac{\tau}{\tau_i}\right),$$

and hence the Wronskian becomes

$$W = \frac{1}{\tau_i},$$

and the Green’s function is

$$G(\tau, \tau') = (\tau \tau')^{1/2} \ln \left(\frac{\tau'}{\tau}\right).$$

The contribution $X_1(\tau)$ induced by the direct coupling between $\phi$ and $\chi$ thus becomes

$$X_1(\tau) = g^2 \int_{\tau_i}^{\tau} d\tau' (\tau \tau')^{1/2} \ln \left(\frac{\tau'}{\tau}\right) \phi(\tau') a^2(\tau') X_0(\tau'),$$

or

$$X_1(\tau) = -\frac{1}{M^2} \int_{\tau_i}^{\tau} d\tau' (\tau \tau')^{1/2} \ln \left(\frac{\tau'}{\tau}\right) \phi(\tau') a^2(\tau') X_0(\tau'),$$

For $X_0(\tau)$ we can take the dominant solution of the homogeneous equation

$$X_0(\tau') = X_0(\tau_i) \left(\frac{\tau'}{\tau_i}\right)^{1/2} \ln \left(\frac{\tau'}{\tau_i}\right).$$

Making use of the scaling $\dot{g}$ of $\dot{\phi}$ and after a couple of lines of algebra we obtain the approximate result (keeping only the contribution from the upper integration limit)

$$X_1(\tau) \approx g^2 \dot{\phi}(\tau_i) \tau^2 X_0(\tau_i),$$

or

$$X_1(\tau) \approx -M^{-2} \dot{\phi}(\tau_i) \tau_i^2 \left(\frac{\tau}{\tau_i}\right)^{1/2} X_0(\tau) \ln \left(\frac{\tau}{\tau_i}\right).$$

If we take the initial time $\tau_i$ to correspond to the end of inflation, we have

$$\dot{\phi}(\tau_i) \approx H(\tau_i) m_{pl},$$

where $H(\tau_i)$ is the value of $H$ at the end of inflation. In this case

$$X_1(\tau) \approx g^2 m_{pl} \tau_i \left(\frac{\tau}{\tau_i}\right)^{3/2},$$

or

$$X_1(\tau) \approx -\left(\frac{m_{pl}}{M}\right)^2 X_0(\tau).$$

The criterion (26) for direct particle production to dominate over gravitational particle production then becomes (up to logarithmic factors)

$$g^2 > \frac{H(\tau_i)}{m_{pl}} \left(\frac{t_i}{\tau_i}\right);$$

or

$$\left(\frac{m_{pl}}{M}\right)^2 > 1.$$

Note that for the second interaction term, particle production via direct interactions dominates within one Hubble expansion time (the time interval after which the contribution from the lower integration end can be neglected), provided that $M < m_{pl}$, a condition which has to be satisfied if we are to trust the effective field justification of the interaction term.

Once $X_1(\tau)$ starts to dominate over $X_0(\tau)$, the Born approximation ceases to be valid. At that point, the coupling term in the equation of motion for $X$ will become the dominant one, and an approximation to (14) (we will first focus on the case of the first interaction term) which is self-consistent for long wavelength modes (for which the $k^2$ term in the equation is negligible) is

$$X'' = g^2 \dot{\phi}^2 X = 0.$$

An approximate solution of this equation is

$$X(\tau) = A(\tau) e^{\dot{f}(\tau_i)^{3/4} \tau_i},$$

with

$$\dot{f} = \frac{4}{3} \left(g^2 \dot{\phi}(\tau_i)\right)^{1/2}.$$

Inserting this ansatz (41) and (42) into (40) we find an equation for the amplitude $A(\tau)$

$$A'' + \frac{3}{2} \dot{f} \tau^{-1/4} \tau_i^{-3/4} A' - \frac{3}{16} \dot{f} \tau^{-5/4} \tau_i^{-3/4} A = 0,$$

which both for $\dot{f} \tau_i < 1$ and $\dot{f} \tau_i > 1$ has a dominant solution which is constant in time.

From (41) and (42) we see that there is quasi-exponential growth of $X$ which becomes important once

$$\dot{f} \tau_i \left(\frac{\tau}{\tau_i}\right)^{3/4} > 1,$$

which in terms of physical time is

$$\frac{t}{\tau_i} > \dot{f}^{-2} \tau_i^{-2}.$$
In the above we are implicitly assuming that $\hat{\tau}_i < 1$. If $\hat{\tau}_i > 1$ then reheating via direct particle production is instantaneous on a Hubble time scale and the reheating temperature is given by the energy density at the end of inflation.

Returning to the case $\hat{\tau}_i < 1$, we see that once the time $t$ is larger than the one given by (45), the energy transfer from the inflaton to matter is exponentially fast and will immediately drain all of the energy from the inflaton. Hence, the “reheating time” $t_{RH}$ is

$$t_{RH} \sim t_i (\hat{\tau}_i)^{-2},$$

and since the energy density between $t_i$ and $t_{RH}$ decreases as $a(t)^{-6} \sim t^{-2}$ we have

$$\rho(t_{RH}) \sim \rho(t_i) (\hat{\tau}_i)^4.$$  \hfill (47)

Making use of $\rho(t_i) = H^2(t_i) m_{pl}^2$ (up to a numerical factor) and $\rho(t_{RH}) \sim T_{RH}^4$ we finally obtain the reheating temperature $T_{RH}$ to be

$$T_{RH} \sim \hat{\tau}_i (H(t_i)m_{pl})^{1/2}$$ \hfill (48)

which is larger than the reheating temperature $H(t_i)$ which would be obtained if only gravitational particle production were effective, provided that

$$\hat{\tau}_i > \left( \frac{H(t_i)}{m_{pl}} \right)^{1/2}. \hfill (49)$$

In the case of the second coupling, the conclusions are similar. Once the coupling term in the equation of motion dominates over the expansion term, the equation can be approximated as (changing the sign of the coupling term)

$$X'' - M^{-2} \dot{\phi}^2 a^2 X = 0.$$ \hfill (50)

Since

$$\dot{\phi}^2(a^2) = \dot{\phi}^2(\tau_i) \left( \frac{T_i}{T} \right)^2,$$ \hfill (51)

the equation has power law solutions with an exponent $\Delta$ given by

$$\Delta = \frac{1}{2} \left[ 1 \pm \sqrt{1 + 4R^2} \right].$$ \hfill (52)

where

$$R \equiv \frac{m_{pl}}{M}.$$ \hfill (53)

We see that if $M \ll m_{pl}$, then the power of the dominant solution is $\Delta \gg 1$ and this means that there is complete energy transfer from the inflaton to $\chi$ within one Hubble expansion time. Hence, the reheating temperature is given by the energy density at the end of inflation, i.e.

$$T_{RH} \sim (H(t_i)m_{pl})^{1/2}.$$ \hfill (54)

V. CONCLUSIONS AND DISCUSSION

We have derived the condition under which direct particle production in G-inflation dominates over gravitational particle production. The discussion also applies to k-inflation [23]. We consider two possible interaction Lagrangians, namely (9) and (10). We first study the onset of matter particle production from the direct coupling using the Born approximation. We find that for both interaction terms we consider the direct particle production channel eventually dominates. This happens within one Hubble expansion time for the coupling (10), whereas in the case of (9), the time when direct particle production starts to dominate depends on the coupling constant $g$.

Once direct particle production begins to dominate over gravitational particle production we must use a different approximation scheme to solve the equation of motion. We can now neglect the squeezing term in the equation of motion. We provide solutions of the resulting approximate equations of motion and show that once direct particle production begins to dominate, the energy transfer from the inflaton to the matter fields will be almost instantaneous. This allows us to estimate the value of the reheating temperature, the temperature of matter once the inflaton field has lost most of its energy density to particle production. In the case of the second interaction term (10), the reheating temperature is given by the energy density at the end of inflation, in the case of the first interaction term (9), it is reduced by a factor which involves the interaction coupling constant $g$.

In the present work we have studied the production of particles which correspond to an entropy fluctuation direction. There is the danger (see e.g. [35] for an initial study and [36] for recent work) that the induced entropy fluctuations might induce too large curvature perturbations, as it does in the model studied in [37]. A study of this question will be the focus of future work.

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Appendix

In this Appendix we compare our calculation of particle production in G-inflation with what is obtained using more standard methods of quantum field theory in curved space-time (see e.g. [2] [3] [5]). Recall the equation of motion for our canonical field \( X \). In the case of the second coupling which we consider in the main text this equation is

\[
X'' + \left( k^2 + M^{-2} \beta^2 a^2 - \frac{\alpha''}{a} \right) X_k = 0. \tag{55}
\]

We will compare the energy density in produced particles in the initial stages of particle production (before back-reaction becomes important).

Starting from vacuum initial conditions, we can obtain the energy density from the Bogoliubov coefficients \( \beta_k \) which describe how the solution of (55) that initially corresponds to the vacuum can at late times \( \tau \) be decomposed into positive and negative frequency modes (see e.g. [2] for a textbook treatment):

\[
\rho(\tau) = \frac{1}{2\pi^2 a^4(\tau)} \int_0^\infty |\beta_k|^2 k^3 dk. \tag{56}
\]

The Bogoliubov coefficient \( \beta_k \) is given by

\[
\beta_k = \frac{i}{2k} \int_{-\infty}^{\infty} e^{-2ik\tau} V(\tau) d\tau, \tag{57}
\]

where in the case of the equation (55)

\[
V(\tau) = \left( 12 \frac{m_{pl}^2}{M^2} - 2 \right) \frac{1}{2\pi^2} \tau. \tag{58}
\]

(for times in the kination phase). The first term on the right hand side of this equation represents particle production via the particle interactions, whereas the second term corresponds to gravitational particle production. From this equation it is already clear that for \( M \ll m_{pl} \) particle interactions will dominate the energy transfer from the inflaton field to matter.

Following [37], the expression (56) for the energy density can be rewritten as

\[
\rho(\tau) = \frac{-1}{32\pi^2 a^4(\tau)} \int_{-\infty}^{\tau} d\tau_1 d\tau_2 \ln(\mu (\tau_1 - \tau_2)) V'(\tau_1) V'(\tau_2) \tag{59}
\]

where \( \mu \) is a regularization scale which has been introduced to remove the ultraviolet divergence. Note that the derivative of \( V \) in the above equation is with respect to conformal time.

If we are interested in particle production due to the squeezing of the mode wave functions on super-Hubble scales, we have the natural regularization scale \( \mu = \bar{H}(\tau) \), the Hubble scale at the end of inflation. Making use of this cutoff, it can be shown that the order of magnitude of \( \rho(\tau) \) obtained from (59) agrees with what is obtained using the method we have used in the main text, namely

\[
\beta_k(\tau) \sim \frac{X_k(\tau)}{X_k^{vac}} \tag{60}
\]

where \( X_k^{vac} \) is the vacuum value of the canonical variable.

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