Stabilization of Robots With a Regulator Containing the Sigmoid Mapping

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ABSTRACT Actuators nonlinearities are unknown external perturbations in robots, which are unwanted because they can severely limit their performance. This research is focused on the stabilization of robots subject to actuators nonlinearities with a regulator containing the sigmoid mapping. Our regulator has the following three main characteristics: a) a sigmoid mapping is used to ensure boundedness of the regulator law terms, b) the chattering is reduced by the usage of the saturation mapping instead of the signum mapping, and c) the stabilization is ensured by the Lyapunov analysis. Finally, we evaluate our regulator for the stabilization of two robots.

INDEX TERMS Stabilization, regulator, sigmoid, mapping, robot.

I. INTRODUCTION

The nonlinear uncertainties and external perturbations are unwanted characteristics in nonlinear models because they can severely limit their performance or damage their components; this fact has been drawing much interest in the community for a long time [1]–[4]. The linear quadratic regulator is one approach used to reach constant paths in linear models, it is called optimization [5]–[8]. Different to the mentioned research, a regulator is one approach used to reach constant paths in nonlinear models subject to nonlinear uncertainties or external perturbations, it is called stabilization [2], [3].

There is some research of regulators focused on the stabilization of nonlinear models subject to nonlinear uncertainties or external perturbations. Authors addressed regulators for stabilization of microgrids in [9], [10], and [11]. In [12]–[14], and [15], authors focused fuzzy sliding model regulators of robotic exoskeletons and robotic manipulators. Authors employed neural network sliding mode regulators of wheeled aerial vehicles and robotic manipulators in [16]–[18], and [19]. In [20]–[22], and [23], authors detailed sliding mode regulators based on observers of robotic exoskeletons and quadrotors. Authors analyze proportional derivative sliding model regulators of overhead cranes and inverted pendulums in [24]–[26], and [27]. In [28]–[30], and [31], authors addressed robust sliding model regulators of parallel manipulators and quadrotors. Authors discussed regulators for stabilization of multiple converters in [32], [33], and [34].

The aforementioned research is divided in two big groups where [16]–[19], [21], [22], [24]–[26], [29], [32]–[34] are focused on the stabilization of nonlinear models subject to nonlinear uncertainties, and [9]–[15], [20], [23], [27], [28], [30], [31] are focused on the stabilization of nonlinear models subject to external perturbations. It is important to note that in most of the cases the nonlinear uncertainties or external perturbations are unknown. Hence, the stabilization of nonlinear models where the nonlinear uncertainties or external perturbations are unknown is of great interest.

Actuators nonlinearities are a kind of external perturbations in the robots nonlinear models yielded by the interaction of actuators with the environment [1]–[4]. This research is focused on the stabilization of robots subject to actuators nonlinearities with a regulator containing the sigmoid mapping. Our regulator has the following three main characteristics: a) we utilize the sliding mode in our regulator to compensate the actuators nonlinearities and gravity terms, b) we also use...
the proportional derivative approach in our regulator to reach the stabilization of the robots positions; and c) we ensure the stabilization of the regulator error by the Lyapunov analysis.

There are other two issues in a regulator that could limit its performance: a) when boundedness of the regulator law terms is not ensured, and b) when chattering of the signum mapping is increased. The mentioned issues are solved in our regulator as following: a) taking into account that the sigmoid mapping is used in a neural network to ensure its boundedness [16]–[19], we use the sigmoid mapping to ensure boundedness of the regulator law terms, b) taking into account that the saturation mapping is used to reduce its chattering without ensuring its stabilization [28]–[31], we reduce the chattering by the usage of the saturation mapping while we ensure the stabilization.

This research is structured as following. In section II, we present the nonlinear model of robots, and the proportional derivative and sliding mode regulators. In section III, we present a regulator containing the sigmoid mapping for the stabilization of robots. In section IV, we evaluate our regulator for the stabilization of two robots. In section V, we express the conclusions and future research.

II. SOME PROPERTIES OF ROBOTS

In this section we describe some properties of robots such as their nonlinear model, and the proportional derivative and sliding mode regulators.

We define the nonlinear model for the robots with $n$ degrees of freedom in the joint space as [1], [2], [4]:

$$ Q(p)\ddot{p} + C(p, \dot{p})p + O(p) = \tau, \quad (1) $$

$p \in \mathbb{R}^{n \times 1}$ as the position, $\dot{p} \in \mathbb{R}^{n \times 1}$ as the speed in the robot, $Q(p) \in \mathbb{R}^{n \times n}$ as the robot inertia matrix which is symmetric and positive definite, $C(p, \dot{p}) \in \mathbb{R}^{n \times n}$ as the centripetal and Coriolis terms, and $O(p)$ as the gravity terms, $\tau$ as the actuators nonlinearities.

We express the states and actuators nonlinearities as:

$$ w_1 = p \in \mathbb{R}^{n \times 1}, $$
$$ w_2 = \dot{p} \in \mathbb{R}^{n \times 1}, $$
$$ e = \tau \in \mathbb{R}^{n \times 1}, $$
$$ w_1 = \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix}^T = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^T, $$
$$ w_2 = \begin{bmatrix} w_{21} \\ w_{22} \end{bmatrix}^T = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^T. $$

Then, we rewrite the nonlinear model of the equation (1) as:

$$ w_1 = w_2, $$

$$ Q(w_1)w_2 + C(w_1, w_2)w_2 + O(w_1) = e, \quad (3) $$

$Q(w_1), C(w_1, w_2), O(w_1)$ as in (1).

We represent the actuators nonlinearities $e$ as [1]–[4]:

$$ e = \begin{cases} n_r (v - w_r) & v \geq w_r \\ 0 & w_l < v < w_r \\ n_l (v - w_l) & v \leq w_l, \end{cases} \quad (4) $$

$n_r, n_l, w_r$ and $w_l$ as constant terms for the actuators nonlinearities. Note that $v$ is the input of the actuators nonlinearities. We show the actuators nonlinearities in Figure 1.

The actuators nonlinearities of robots are symmetric as $n_r = n_l$ in (4); consequently, we can express the actuators nonlinearities $e$ of (4) as:

$$ e = \begin{cases} n_l (v - w_r) & v \geq w_r \\ 0 & w_l < v < w_r \\ n_l (v - w_l) & v \leq w_l, \end{cases} \rightarrow e = \begin{cases} n_l v & v \geq w_r \\ n_l v & w_l < v < w_r + \begin{cases} -n_l w_r & v \geq w_r \\ -n_l v & w_l < v < w_r \end{cases} \quad (5) \end{cases} $$

after some arranges the actuators nonlinearities $e$ of (5) are expressed as:

$$ e = n_l v - h(v), \quad h(v) = \begin{cases} n_l w_r & v \geq w_r \\ n_l v & w_l < v < w_r \end{cases} \quad (6) $$

with $h(v)$ as the nonlinearity. We note that the nonlinearity $h(v)$ are bounded as:

$$ |h(v)| \leq T, \quad (7) $$

We will use the next property in a posterior section to reach the stabilization of our regulator.

**Property 1:** We express the centripetal and Coriolis matrix as skew-symmetric and this matrix complies the relationship:

$$ w^T \left( Q(w_1) - 2C(w_1, w_2) \right) w = 0, \quad (8) $$

$$ w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}^T, Q(w_1), \text{ and } C(w_1, w_2) \text{ as in (3)}. $$

Now, we express the proportional derivative and sliding mode regulators because they will be used for the results in a posterior section.

We detail a proportional derivative regulator as [24], [25]:

$$ v = \frac{1}{n_l} (-K_p \tilde{w}_1 - K_d \tilde{w}_2), \quad (9) $$

FIGURE 1. The actuator nonlinearities.
\( \hat{\omega}_1 = w_1 - w^d_1 \in \mathbb{R}^{n \times 1}, \hat{\omega}_1 \in \mathbb{R}^{n \times 1} \) as the position regulator error, \( w_1 \in \mathbb{R}^{n \times 1} \) as the position, \( w^d_1 \in \mathbb{R}^{n \times 1} \) as the constant desired position, \( \hat{\omega}_2 = w_2 - w^d_2 \in \mathbb{R}^{n \times 1} \) as the speed regulator error, \( K_p, K_d \in \mathbb{R}^{n \times n} \) as positive definite, symmetric and constant matrices, \( w^d_2 \in \mathbb{R}^{n \times 1} \) as the desired reference, \( n_l \) as a actuators nonlinearities term.

We detail a sliding mode regulator as [2, 3]:

\[
\nu = \frac{1}{n_l} \left( -K_p \hat{\omega}_1 - K_d \hat{\omega}_2 - K_{\text{sign}}(\hat{\omega}_2) \right),
\]

\[
\text{sign}(\hat{\omega}_2) = \begin{cases} 
1 & \hat{\omega}_2 > 0 \\
0 & \hat{\omega}_2 = 0 \\
-1 & \hat{\omega}_2 < 0,
\end{cases}
\]

III. A REGULATOR CONTAINING THE SIGMOID MAPPING

We represent the gravity terms \( O(w_1) \) of (3) bounded as:

\[
|O(w_1)| \leq \bar{O},
\]

We take into account the stabilization case in this research due to we use the desired speed states as \( w^d_2 = 0 \), and the desired references as constant. We detail a regulator containing the sigmoid mapping \( v \) as:

\[
v = \frac{1}{n_l} \left( -K_p \hat{\omega}_1 \right) K_{\text{sign}}(\hat{\omega}_2),
\]

\[
sat(\hat{\omega}_2) = \begin{cases} 
1 & \hat{\omega}_2 > 1 \\
\hat{\omega}_2 & \hat{\omega}_2 \leq 1 \\
-1 & \hat{\omega}_2 < -1,
\end{cases}
\]

\[
b(\hat{\omega}_2) = \frac{1 - \exp(-2\hat{\omega}_2)}{1 + \exp(-2\hat{\omega}_2)},
\]

\[
\hat{\omega}_1 = w_1 - w^d_1 \in \mathbb{R}^{n \times 1}, \hat{\omega}_1 \in \mathbb{R}^{n \times 1} \text{ as the position regulator error,} w_1 \in \mathbb{R}^{n \times 1} \text{ as the position,} w^d_1 \in \mathbb{R}^{n \times 1} \text{ as the constant desired position,} \hat{\omega}_2 = w_2 \in \mathbb{R}^{n \times 1} \text{ as the speed regulator error,} K_p, K_d \in \mathbb{R}^{n \times n} \text{ as positive definite, symmetric and constant matrices,} sat(\cdot) \text{ as the saturation mapping,} b(\cdot) \text{ as the sigmoid mapping,} K \text{ as a constant such as} \bar{O} + \bar{h} \leq K, \bar{O} \text{ as in} (11), \bar{h} \text{ as in} (7), n_l \text{ as a actuators nonlinearities term. It is important to note that we do not know the behavior of} O(w_1), h(v) \text{ and we utilize their upper bounds} \bar{O}, \bar{h}.
\]

Remark 2: Since \( w_2 \) will reach to \( w^d_2 \) and \( w^d_2 = 0 \), it yields \( \hat{\omega}_2 = w_2 - w^d_2 = 0 \); consequently, \( \hat{\omega}_2 \) is bounded, and since \( b(\hat{\omega}_2) \) and \( sat(\hat{\omega}_2) \) are also bounded, it yields that the regulator law terms \( v \) for a regulator containing the sigmoid mapping (12) are bounded.

In Figure 2 we show a regulator containing the sigmoid mapping called RSM for the stabilization of robots called Robot.

Now, we will detail the stabilization of the regulator error.

**Theorem 1:** The stabilization of the regulator error in the close-loop model of a regulator containing the sigmoid mapping (12) and robots (3) is ensured, and the speed regulator error \( \hat{\omega}_2 \) will comply with:

\[
limit_{T \to \infty} \|\hat{\omega}_2\|^2 = 0,
\]

\( T \) as the final time, \( \hat{\omega}_2 = w_2, |O(w_1)| \leq \bar{O}, |h(v)| \leq \bar{h}, \) and \( \bar{O} + \bar{h} \leq K.\)

**Proof:** We represent the Lyapunov candidate as:

\[
L_1 = \frac{1}{2} \hat{\omega}_2^T Q(w_1) \hat{\omega}_2 + \frac{1}{2} b(\hat{\omega}_2)^T K_{p} b(\hat{\omega}_2),
\]

\( Q(w_1) \) as the positive definite matrix of (3) and \( K_{p} \) as the positive definite matrix of (12). We take into account \( \hat{\omega}_2 = w_2 \) and we substitute (12) into (3), we obtain the closed-loop model as:

\[
Q(w_1) \hat{\omega}_2 + C(w_1, w_2) \hat{\omega}_2 + O(w_1) = e = n_l v - h(v)
\]

\[= n_l \frac{1}{n_l} \left( \left( 1 - b(\hat{\omega}_1)^T \right)^T K_p b(\hat{\omega}_1) - K_d \hat{\omega}_2 ( -K_{\text{sat}}(\hat{\omega}_2)) - h(v),\right)
\]

\[\Rightarrow Q(w_1) \hat{\omega}_2 = - \left( 1 - b(\hat{\omega}_1)^T \right)^T K_p b(\hat{\omega}_1) - K_d \hat{\omega}_2 - C(w_1, w_2) \hat{\omega}_2.
\]

We use the fact \( \hat{\omega}_2 = w_2 \), we express the derivative of (14) as:

\[
L_1 = \hat{\omega}_2^T Q(w_1) \hat{\omega}_2 + \frac{1}{2} \hat{\omega}_2^T Q(w_1) \hat{\omega}_2 + \frac{1}{2} \hat{\omega}_2^T (1 - b(\hat{\omega}_1)^T)^T K_p b(\hat{\omega}_1),
\]
\[ \dot{\hat{w}_1} = \hat{w}_1 - \hat{w}_1^d = w_2 - \hat{w}_2 \] and \[ \dot{\hat{w}}_2 = w_2. \]

We substitute the last equation of (15) into (16) as:
\[ \begin{align*}
\dot{L}_1 & = \dot{\hat{w}}_1^2 \left[ - \left( 1 - b(\hat{w}_1)^2 \right)^T K_p b(\hat{w}_1) - K_d \hat{w}_2 \\
& \quad - O(w_1) - h(v) - K_{sat}(\hat{w}_2) - C(w_1, w_2) \right] \\
& \quad + \frac{1}{2} \hat{w}_1^T \hat{Q}(w_1) \dot{\hat{w}}_2 + \hat{w}_1^T \left( 1 - b(\hat{w}_1)^2 \right)^T K_p b(\hat{w}_1), \\
\Rightarrow \dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2 - \hat{w}_1^T O(w_1) - \hat{w}_1^T h(v) - \hat{w}_1^T K_{sat}(\hat{w}_2) \\
& \quad + \frac{1}{2} \hat{w}_1^T \hat{Q}(w_1) \dot{\hat{w}}_2 + \hat{w}_1^T \left( 1 - b(\hat{w}_1)^2 \right)^T K_p b(\hat{w}_1),
\end{align*} \]

after some arranges, \( L_1 \) of (17) is expressed as:
\[ \begin{align*}
L_1 & = -\hat{w}_1^T K_d \hat{w}_2 - \hat{w}_1^T O(w_1) - \hat{w}_1^T h(v) - \hat{w}_1^T K_{sat}(\hat{w}_2) \\
& \quad + \frac{1}{2} \hat{w}_1^T \left( 1 - b(\hat{w}_1)^2 \right)^T K_p b(\hat{w}_1),
\end{align*} \]

We use (8) of Property 1 in the equation of (18) as:
\[ \begin{align*}
\dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2 - \hat{w}_1^T O(w_1) - \hat{w}_1^T h(v) - \hat{w}_1^T K_{sat}(\hat{w}_2),
\end{align*} \]

From (11) \( O(w_1) + h(v) \leq |O(w_1)| + |h(v)| \leq \mathcal{O} + \mathcal{H} \leq K, \) and from (12) \( sat(\hat{w}_2) = \begin{cases} 1 & \hat{w}_2 > 1 \\ \hat{w}_2 & |\hat{w}_2| \leq 1 \end{cases}, \) we represent the three cases of the saturation mapping.

1) If \( \hat{w}_2 > 1, \) then \( sat(\hat{w}_2) = 1 \) and \( \hat{w}_2 = \hat{w}_2, \) we substitute in (19) as:
\[ \begin{align*}
\dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2 + \hat{w}_1^T |\hat{w}_2|^T \mathcal{O} + |\hat{w}_2|^T \mathcal{H} - |\hat{w}_2|^T K, \\
\Rightarrow \dot{L}_1 & \leq -\hat{w}_1^T K_d \hat{w}_2,
\end{align*} \]

2) If \( |\hat{w}_2| \leq 1, \) then \( sat(\hat{w}_2) = \hat{w}_2 \) and \( \hat{w}_1^T \hat{w}_2 = |\hat{w}_2|^T |\hat{w}_2|, \) we substitute in (19) as:
\[ \begin{align*}
\dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2 + |\hat{w}_2|^T \mathcal{O} + |\hat{w}_2|^T \mathcal{H} - \hat{w}_1^T \hat{w}_2 K, \\
\Rightarrow \dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2 + |\hat{w}_2|^T \mathcal{O} + |\hat{w}_2|^T \mathcal{H} - |\hat{w}_2|^T |\hat{w}_2| K, \\
\Rightarrow \dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2,
\end{align*} \]

due to in this case \( |\hat{w}_2| \leq 1. \)

3) If \( \hat{w}_2 < -1, \) then \( sat(\hat{w}_2) = -1 \) and \( \hat{w}_2 = -\hat{w}_2, \) we substitute in (19) as:
\[ \begin{align*}
\dot{L}_1 & = -\hat{w}_1^T K_d \hat{w}_2 - (|\hat{w}_2|^T) O(w_1) \\
& \quad - (|\hat{w}_2|^T) h(v) - (|\hat{w}_2|^T) K (-1), \\
\Rightarrow \dot{L}_1 & \leq -\hat{w}_1^T K_d \hat{w}_2 + |\hat{w}_2|^T \mathcal{O} + |\hat{w}_2|^T \mathcal{H} - |\hat{w}_2|^T K,
\end{align*} \]

The three cases yield the same inequality, from (20), (21), (22) we express:
\[ \dot{L}_1 \leq -\hat{w}_1^T K_d \hat{w}_2, \]

From [2], [3], the stabilization of the regulator error is ensured. We integrate (23) from \( 0 \) to \( T \) as:
\[ \begin{align*}
\int_0^T \hat{w}_1^T K_d \hat{w}_2 dt \leq L_{1,0} - L_{1,T} \leq L_{1,0},
\end{align*} \]

and applying the lim sup to both sides of the last inequality of (24) is:
\[ \limsup_{T \to \infty} \left( \frac{1}{T} \int_0^T \hat{w}_1^2 dt \right) \leq \frac{L_{1,0}}{\lambda_{\min}(K_d)} \left( \limsup_{T \to \infty} \frac{1}{T} \right) = 0, \]

If \( T \to \infty, \) then \( \| \hat{w}_1 \|^2 = 0, \) and it is (13).

Remark 3: Our regulator (12) requires to comply with conditions (11), (7) to be applied for the stabilization of robots (3), (4).

IV. RESULTS
In this section, we evaluate a regulator containing the sigmoid mapping of (12) denoted as RSM, a proportional derivative regulator of (9), (24), (25), and a sliding mode regulator of (10), (2), [3] denoted as SM, for the stabilization of the scara and two link robots. Our goal in the regulators is that the paths of the states in robots must follow the paths of desired constant references as fast as possible. The scara and two link robots are chosen due to they are written as (3) and could be employed in pick and place, screwed, printed circuits boards, packaging and labeling, etc. We mainly use the MATLAB software for the results. We utilize the mean square error (MSE), the root mean square error (RMSE), the mean absolute error (MAE), and the mean absolute percent error (MAPE) for the evaluations as:
\[ \begin{align*}
MSE & = \left( \frac{1}{T} \int_0^T \hat{w}_1^2 dt \right), \\
RMSE & = \left( \frac{1}{T} \int_0^T \hat{w}_1^2 dt \right)^{\frac{1}{2}}, \\
MAE & = \left( \frac{1}{T} \int_0^T |\hat{w}_1| dt \right),
\end{align*} \]
The scara robot has three degrees of freedom, it has two rotary positions and speeds or nonlinearities.

**A. SCARA ROBOT**

The scara robot has three degrees of freedom, it has two rotary joints and two links configured in horizontal position, it has one linear joint and one link configured in vertical position. We express the scara robot of the Figure 3.

We write the scara robot as (3) and we detail it as:

\[
\begin{align*}
\dot{w}_1 &= w_2, \\
Q(w_1)w_2 + C(w_1, w_2)w_2 + O(w_1) &= e, \\
Q(w_1) &= \begin{bmatrix}
q_{11}(w_1) & q_{12}(w_1) & q_{13}(w_1) \\
q_{21}(w_1) & q_{22}(w_1) & q_{23}(w_1) \\
q_{31}(w_1) & q_{32}(w_1) & q_{33}(w_1)
\end{bmatrix}, \\
C(w_1, w_2) &= \begin{bmatrix}
c_{11}(w_1, w_2) & c_{12}(w_1, w_2) & c_{13}(w_1, w_2) \\
c_{21}(w_1, w_2) & c_{22}(w_1, w_2) & c_{23}(w_1, w_2) \\
c_{31}(w_1, w_2) & c_{32}(w_1, w_2) & c_{33}(w_1, w_2)
\end{bmatrix}, \\
O(w_1) &= \begin{bmatrix}
o_1(w_1) & o_2(w_1) & o_3(w_1)
\end{bmatrix}^T,
\end{align*}
\]

and:

\[
\begin{align*}
q_{11}(w_1) &= J_{13} + m_2 l_2^2 + m_3 (l_1^2 + l_2^2) + l_1 C_2 (m_3 l_2 + m_4 l_2), \\
q_{12}(w_1) &= q_{21}(w_1) = \left( m_3 l_2^2 + m_4 l_2^2 \right) + l_1 C_2 (m_3 l_2 + m_4 l_2), \\
q_{22}(w_1) &= J_3 + \left( m_3 l_2^2 + m_4 l_2^2 \right), \\
q_{33}(w_1) &= m_4, \\
\end{align*}
\]

the other terms of \(Q(w_1)\) are zero,

\[
\begin{align*}
c_{11}(w_1, w_2) &= -2l_1 S_2 (m_3 l_2 + m_4 l_2) w_{22}, \\
c_{12}(w_1, w_2) &= -l_1 S_2 (m_3 l_2 + m_4 l_2) w_{22}, \\
c_{12}(w_1, w_2) &= 2l_1 S_2 (m_3 l_2 + m_4 l_2) w_{21},
\end{align*}
\]

the other terms of \(C(w_1, w_2)\) are zero,

\[
o_3(w_1) = -m g
\]

the other terms of \(O(w_1)\) are zero. \(e\) as actuators nonlinearities, \(w_1\) as positions, \(w_2\) as speeds, \(m_2, m_3,\) and \(m_4\) as the masses of the links one, two, and three, \(w_{11} = \theta_1, \ w_{12} = \theta_2,\) as the angles of the joints one and two in rad, \(w_{13} = l_3\) as the length of the link three, in \(m, g\) is the acceleration gravity constant. \(l_1 = l_2 = 0.3\ m, l_3 = l_{1/2}, \ l_{22} = l_{22}/2, m_2 = m_3 = m_4 = 0.3\ kg, J_{13} = J_1 + J_2 + J_3, J_1 = 0.0208\ kgm^2, J_2 = J_3 = 0.0127\ kgm^2,\) and \(g = 9.81\ m/s^2, n_r = n_l = 0.5, w_r = 0.5,\) and \(w_l = -0.5\) as the actuators nonlinearities terms.

PD of [24], [25] is expressed by equation (9) with parameters \(K_p = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 500 \end{bmatrix}, K_d = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}.\)

SM of [2], [3] is expressed by equation (10) with parameters \(K_p = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 500 \end{bmatrix}, K_d = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, K = \begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \end{bmatrix}.\)

RSM of this research is expressed by equation (12) with parameters \(K_p = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 500 \end{bmatrix}, K_d = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, K = \begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \end{bmatrix}.\)

We evaluate the actuators nonlinearities in the Figure 4, we evaluate the positions in the Figure 5, we evaluate the speeds in the Figure 6, we show the MSE of (26), the RMSE of (27) in the Table 1, the MAE of (28), and the MAPE of (29) in the Table 2 for the scara robot.

In the Figure 5, since the position and speed of RSM reach better the paths of the constant desired references than the position and speed of PD and SM, we can see that RSM is more efficient than PD and SM. In the Figures 4 and 6, in the RSM the chattering of the actuators nonlinearities and speeds is reduced, while in the SM the chattering of the actuators nonlinearities and speeds is not reduced, and in the PD the actuators nonlinearities and speeds are not stabilized. In the
Table 1 and Table 2, since the MSE, RMSE, MAE, and MAPE for the RSM are smaller than for PD and SM, we can show that RSM is more efficient than PD and SM.

B. TWO LINK ROBOT

The two link robot has two degrees of freedom, it has two rotary joints and two links configured in vertical position. We express the two link robot of the Figure 7.

We write the two link robot as (3) and we detail it as:

\[
\begin{align*}
  w_1 &= w_2, \\
  Q(w_1)w_2 + C(w_1, w_2)w_2 + O(w_1) &= e,
\end{align*}
\]

where

\[
\begin{align*}
  Q(w_1) &= \begin{bmatrix} q_{11}(w_1) & q_{12}(w_1) \\ q_{21}(w_1) & q_{22}(w_1) \end{bmatrix}, \\
  C(w_1, w_2) &= \begin{bmatrix} c_{11}(w_1, w_2) & c_{12}(w_1, w_2) \\ c_{21}(w_1, w_2) & c_{22}(w_1, w_2) \end{bmatrix}, \\
  O(w_1) &= \begin{bmatrix} o_1(w_1) \\ o_2(w_1) \end{bmatrix}^T, \\
  e &= e, \\
  w_1 &= \theta_1 \quad \text{and} \quad w_2 = \theta_2 \quad \text{as the angles of the joints one and two in rad}, \\
  m_2 &= 0.34 \text{ kg}, \quad l_2 = 0.293 \text{ m}, \quad l_2 = l/2, \\
  J_1 &= J_1 + J_2, \quad J_2 = 0.0208 \text{ kgm}^2, \quad J_1 = 0.0127 \text{ kgm}^2, \quad g = 9.81 \text{ m/s}^2, \quad n_r = n_l = 0.5, \quad w_r = 0.5, \quad w_l = -0.5 \quad \text{as the actuators nonlinearities terms}.
\end{align*}
\]

the other terms of \( Q(w_1) \) are zero,

\[
\begin{align*}
  q_{11}(w_1) &= J_1 + m_2 l_c^2 C_2, \\
  q_{22}(w_1) &= J_2 + m_2 l_c^2 C_2, \\
  c_{12}(w_1, w_2) &= -m_2 l_c^2 S_2 w_2, \\
  c_{21}(w_1, w_2) &= m_2 l_c^2 C_2 w_2, \\
  o_2(w_1) &= m_2 g l_c C_2, \quad (34)
\end{align*}
\]

the other terms of \( O(w_1) \) are zero. \( \tilde{e} \) as actuators nonlinearities, \( w_1 \) as positions, \( w_2 \) as speeds, \( m_2 \) as the mass of the link two in kg, \( w_{11} = \theta_1 \) and \( w_{12} = \theta_2 \) as the angles of the joints one and two in rad, \( g \) is the acceleration gravity constant, \( J_1 \) and \( J_2 \) as the inertia in kgm\(^2\), \( C_2 = \cos(w_{12}), \quad S_2 = \sin(w_{12}), \quad m_2 = 0.34 \text{ kg}, \quad l_2 = 0.293 \text{ m}, \quad l_2 = l/2, \quad J_1 = J_1 + J_2, \quad J_2 = 0.0208 \text{ kgm}^2, \quad J_1 = 0.0127 \text{ kgm}^2, \quad g = 9.81 \text{ m/s}^2, \quad n_r = n_l = 0.5, \quad w_r = 0.5, \quad w_l = -0.5 \quad \text{as the actuators nonlinearities terms}.

PD of [24], [25] is expressed by equation (9) with parameters

\[
K_p = \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix}, \quad K_d = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}.
\]
SM of [2], [3] is expressed by equation (10) with parameters
\[ K_p = \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix}, ~ K_d = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, ~ K = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}. \]

RSM of this research is expressed by equation (12) with parameters
\[ K_p = \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix}, ~ K_d = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, ~ K = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}. \]

We evaluate the actuators nonlinearities in the Figure 8, we evaluate the positions in the Figure 9, we evaluate the speeds in the Figure 10, we show the MSE of (26), the RMSE of (27) in the Table 3, the MAE of (28), and the MAPE of (29) in the Table 4 for the two link robot.

In the Figure 9, since the position and speed of RSM reach better the paths of the constant desired references than the position and speed of PD and SM, we can see that RSM is
more efficient than PD and SM. In the Figures 8 and 10, we can see that in the RSM the chattering of the actuators nonlinearities and speeds is reduced, while in the SM the chattering of the actuators nonlinearities and speeds is not reduced, and in the PD the actuators nonlinearities and speeds are not stabilized. In the Table 3 and Table 4, since the MSE, RMSE, MAE, and MAPE for the RSM are smaller than for PD and SM, we can show that RSM is more efficient than PD and SM.

V. CONCLUSION

In this research, we were focused on the stabilization of robots subject to actuators nonlinearities with a regulator containing the sigmoid mapping. In the results with respect to a proportional derivative regulator and a sliding mode regulator, since the position and speed of our regulator reach better the paths of the constant desired references, and the chattering in our regulator is reduced, we showed that our regulator is more efficient for the stabilization of two robots. Our regulator illustrates the viability, efficiency, and potential especially important in robots subject to actuators nonlinearities. Our discussed method could also be applied to solve other issues in robots like Coulomb friction, or backlash. As a future research, we will modify our discussed regulator using that some parameters are approximated by the intelligent systems.

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