Seismic Response Analysis of Cable-stayed Bridges with Cable Forces Optimized Parameters based on a New Type of Strongly Sub-feasible SQP Method

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Abstract. A new type of strongly sub-feasible SQP (Strong Quadratic Programming) method is used to establish a nonlinear model to realize the automatic search of the rational cable force of the cable-stayed bridge. The objective function is established based on the stress and displacement of the main girder and the bridge tower. The cable force of the stay cable is the design variable, the stress and displacement of the main girder and the bridge tower are the constraints, and the numerical model of the earthquake force is analyzed under different cable loading conditions. The results show that the new type of strongly sub-feasible SQP method is suitable for the complicated cable force optimized parameters of large-scale cable-stayed bridges, the calculated results are in good agreement with the designed cable force, and the bending moment of the main beam in the bridge state is uniform. The main tower is close to the axial compression force, and structural deformation is much smaller than the specification limits. Under the excitation of earthquake, the optimized cable force can effectively improve the uneven distribution of bending moment of the main beam, limit the longitudinal mid-span deformation and the lateral deformation of the main tower, so that the whole structure is safer in the earthquake.

1. Introduction
The reasonable state of the bridge [1-2] is a significant problem in the design process, and it is also an important criterion to judge the quality of the design of cable-stayed bridges. Modern cable-stayed bridges have developed from sparse cable system to the present dense cable system. The structure type is mostly high-order statically indeterminate structure, and the changes of cable force have a great influence on the stress state of the bridge. Cable forces can be adjusted to obtain optimal performance of cable-stayed bridge under the condition that layout is determined. Thus, the cable force optimized parameters become the key problem in the process of determining the reasonable status of the bridge.

Earthquake disaster which has an enormous destructive power occurred frequently in recent years. The damage to the bridge which is a lifeline project not only endangers people's lives and property but also delays the rescue work. In the past decades, the cable-stayed bridge has developed rapidly in the world because of its excellent ability to span and graceful line shape. So the seismic capacity of cable-stayed bridge has become a common concern problem in academia [3-5]. And the influence of cable force optimized parameters on seismic response of the cable-stayed bridge [6] has become a hot research direction because of its important role in the formation of internal structural forces.

Domestic and foreign scholars have done a lot of research on the cable force optimization method of cable-stayed bridge and developed some methods. For example, the mechanics concept of the rigid
support continuous beam method \cite{7} is clear, and the calculation process is simple, but the optimized cable force of the complex structure is usually not uniform \cite{8}. The zero displacement method \cite{9} is reasonable for main girder, but cable forces of local positions are not reasonable. Especially for the cantilever structure and the cast structure, the zero displacement method cannot be used; the minimum bending energy method \cite{10} is used to obtain smaller bending moments and uniform cable forces \cite{11}, but it can only consider the effects of the dead loads, and cannot consider the case of live load. The establishment of objective functions of the feasible domain method \cite{12} is convenient and easy to operate, but the error rate of the constraint function is high; various factors can be included in the influence matrix method \cite{13}, and it is a complete algorithm. But it is only applicable to linear structure \cite{14}. First-order optimization method \cite{15-16} genetic algorithm \cite{17-18} and sequential quadratic programming method \cite{19} are suitable for solving the nonlinear programming problem which contains multiple elements, working conditions and design variables. These three methods all can consider dead load, live load and geometric nonlinear factors, but there are differences in convergence and computational complexity. At present, the application of these three methods is still seldom used due to the high requirements of the algorithm design and the programming ability.

The cable force of the cable stayed bridge plays an important decisive role in the internal forces of the whole structure. Therefore, it is an important research direction to analyze the influence of cable forces on the seismic response of cable-stayed bridges. In this paper, the new type of strongly sub-feasible SQP method is used to obtain the set of optimal cable forces of a cable-stayed bridge based on Matlab and the analysis of an example shows that this algorithm is reasonable. The improvement of this method is that the optimization process is stable, efficient, and can obtain reasonable cable forces without manual intervention, and the cable force error is controlled within 5%. Taking the bending strain energy as the objective function and considering a variety of different constraints, the impact of the overall structure on the cable force is considered more comprehensively. The optimized structure is more reasonable, and the obtained bridge state is more suitable for engineering practice. The conclusion can supply references for similar projects.

\section*{2. The principle of the new type of strongly sub-feasible SQP algorithm}

The sequence quadratic programming (SQP) algorithm originated from Wilson's Newton-lagrange method. Sequential quadratic programming method is the most efficient algorithm for solving constrained optimization problems. Since Wilson proposed this method, the domestic and foreign scholars have conducted extensive research on the SQP method. In 2001, Jian Jinbao \cite{20} proposed a strongly sub-feasible SQP method by applying generalized projection technique in strongly sub-feasible directions method. Tao Hai \cite{21} firstly proposed that the SQP method can be applied to the cable force optimization problem of cable-stayed bridge, and the method is proved to be feasible through numerical examples. However, the general strongly sub-feasible SQP method theory cannot guarantee iterative points must fall into the feasible region. And the numerical computation is very huge in each iteration. A new type of strongly sub-feasible SQP method is proposed by Tang Chunming \cite{22} based on the above general strongly sub-feasible SQP method. The initial point of the new method is arbitrary, and there is no need to establish a complex penalty function. The method has global and super linear convergence. In each iteration, the proposed method only needs to solve one QP sub-problem, and it can ensure that the iteration point is in the feasible region after finite iterative calculation. In addition, the algorithm constructs a new Armijo curve \cite{23} search and two new display correction directions constructed by the same inverse matrix, which reduces the iterative computation and overcomes the Marotos effect that would appear in the traditional strong sub-feasible SQP method.

The algorithm is as follows:

General inequality constrained optimization problem can be expressed as:

\begin{equation}
\min f(x)
\end{equation}

\text{(1a)}
s.t. $g_j(x) \leq 0, j \in I = 1, 2, ..., n$ \hspace{1cm} (1b)

Where $f(x)$ is the objective function; $x$ is the design variable; $I$ is the subscript set; $g_j(x)$ is the constraint function, where $j$ is the number of the constraint function. For the $k$th iteration point $x_k \in \mathbb{R}^n$, mark and define as follows:

\[
\begin{align*}
I^-(x_k) &= \{ j \in I : g_j(x) \leq 0 \} \\
I^+(x_k) &= \{ j \in I : g_j(x) > 0 \}
\end{align*}
\] \hspace{1cm} (2)

\[
\varphi(x) = \max \{ 0, g_j(x), j \in I \} \hspace{1cm} (3)
\]

You can solve the inequality constrained optimization problem (1) by solving the quadratic programming sub-problem of the following formula (4), and finally get the main search direction $d_k^\phi$.

\[
\min \nabla f(x_k)^T d + \frac{1}{2} d^T B^\phi d \hspace{1cm} (4a)
\]

\[
\begin{align*}
\begin{cases}
g_j(x_k) + \nabla g_j(x_k)^T d &\leq 0, j \in I^+_k \\
g_j(x_k) + \nabla g_j(x_k)^T d &\leq \varphi_k, j \in I^-_k
\end{cases}
\end{align*}
\] \hspace{1cm} (4b)

In the formula, $B^\phi \in \mathbb{R}^{n \times n}$ is the approximation of the Hessen matrix at $x_k$ for the Lagrange function of problem (1).

However, since there is always a feasible solution to the problem (4), $d=0$, $d_k^\phi$ is the optimal solution, but it is not necessarily a feasible direction. In this algorithm, we use the generalized projection technique to obtain a new modified display correction direction and give a new High-order display correction direction $d_k$, references for specific calculation of high order display correction directions [22].

The iterative formula of design variables is

\[
x^{k+1} = x^k + \lambda^k d^k + \lambda_2^k d_k^\phi
\] \hspace{1cm} (5)

In the formula, the step size $\lambda^k$ is obtained by constructing a new Armijo curve. When the formula is $(d_k^\phi, \varphi_k) = (0, 0)$, it is the optimal solution to problem (1).

3. Cable force optimized parameters under the reasonable state
The reasonable state which includes stress state and linear state of cable-stayed bridge is the goal of cable force optimization of cable-stayed bridge. Reasonable stress state is that the mechanical properties of the cable-stayed bridge under stress state can meet requirements of the bearing capacity limit and the normal use limit. For example, the bending moment distribution of the main beam is uniform, the axial force meets the standard limit, the axial force of the main tower is mainly dominated, and the cable force range meets the standard requirements; Reasonable linear state is required to ensure that the geometric line shape of the main beam is reasonable. Also consider the requirements of economic reasons, in the case of ensuring that the above two are reasonable, the total cost should be close to the minimum. To achieve the above-mentioned conditions, the selection of the objective function and the constraint condition becomes the key problem of the algorithm.
3.1. Selection of objective function
The stress state plays a critical role in the reasonable bridge state. The minimum strain energy of the main tower and girder is taken as the objective function, not only the influence of the bending moments of the main girder and tower can be counted, but also the axial force of the main girder and tower is considered. The calculated cable forces can ensure the internal force of the entire structure is uniform and the minimum, and ensure the rationality of the force state.

3.2. Selection of constraint conditions
Firstly, considering the state of the bridge and the strength and fatigue of the cable during operation, the upper and lower values of the cable forces are limited. At the same time, considering the engineering practice, the initial cable force and cable forces at operation stage are all positive. Secondly, the calculated deformation of the various parts of the cable-stayed bridge can directly reflect the design rationality of the whole bridge. Therefore, the lateral displacements of the tower and the vertical displacements of intersection points of the main beam and inclined cable are limited.

Cable-stayed cable design optimization of cable is the key issue, and the necessary pre-work for full-bridge internal force analysis. A mathematical programming model is established by using a new type of strongly sub-feasible SQP method to solve the cable force optimization problem in this paper. Combined with MIDAS/Civil software to build the objective function and the constraint condition function with the advantages of good operation performance and the interaction between data and EXCEL. In such ways, the cycle time of modeling is reduced greatly. The technical route flow chart of the algorithm is shown in Figure 1.

![Flowchart of the automatic cable force adjustment algorithm](image-url)
4. Cable force optimization example

4.1. Engineering situation Cable force optimization example

A cable-stayed bridge with couple tower and three-span beam, whose span distribution is 70m+ 125m+ 70m and the length of 265m, is selected as the research object. The count of stay cable is 24, and its distribution is fan-shaped. The type of girder section is a plate edge girder. The width of the girder is 35m, and the height of the end section is 1m. Concrete mark is C40. The section of the main tower is variable section hollow box. Cross-section of the tower shown in Figure 2. The numbers of cables are T6~T1, T7~T12, T12~T7, T1~T6 from left to right, corresponding to the cables from side span to mid-span. The cable number distribution is shown in Figure 3. The material properties of the component are shown in Table 1.

| Component        | E (GPa) | Moment of Inertia (m^4) | Area (m^2) |
|------------------|---------|-------------------------|------------|
| Beam             | 32.5    | 5.4                     | 14.7       |
| Cable            | 195     | 0                       | 0.02       |
| Upper main tower | 31.5    | 25.4                    | 9.5        |
| Lower main tower | 31.5    | 51.6                    | 17.6       |

The finite model is simulated by the fish bone model in MIDAS/Civil2012, which has 69 nodes, 66 beam elements, and 24 truss elements. The boundary conditions are fixed boundaries, as shown in Figure 4. The modeling process divides the length of the main beam unit into one, which can reduce the calculation of subsequent work.

The objective function of optimization is to select the sum of bending strain energy of bridge girder and bridge tower. The constraint condition is the upper and lower limit of the material that the cable force range takes the standard requirement. The vertical allowable displacement at the intersection of girder and stay cable is 10 mm. The displacement of the top of the tower is reasonable, set to allow the displacement along the bridge to 10 mm. Because of the small span, fewer cables and less geometric nonlinearity [24-25], we consider the optimal solution in the linear case.

4.2. Programming model

(1) Objective function: The sum of bending strain energy of girder and main tower is selected as
optimization object function. The objective function is constructed by finite element discrete model in Midas/Civil.

$$f(x) = \sum_{i=1}^{n} \frac{L_i}{4EI_i} (ML_i^2 + MR_i^2) = \{ML\}^T \{B\}\{ML\} + \{MR\}^T \{B\}\{MR\}$$

$$= \{ML_0 + [C_L] \{X\}\}^T \{B\}\{ML_0 + [C_L] \{X\}\} + \{MR_0 + [C_R] \{X\}\}^T \{B\}\{MR_0 + [C_R] \{X\}\}$$

$$[B] = \begin{bmatrix}
\frac{L_1}{4EI_1}, & 0, & \cdots, & 0 \\
0, & \frac{L_2}{4EI_2}, & \cdots, & 0 \\
\cdots, & \cdots, & \cdots, & \cdots \\
0, & 0, & \cdots, & \frac{L_n}{4EI_n}
\end{bmatrix}$$

(6)

Where $n$ is the count of structural elements; $L$, $E$, and $I$ are respectively length, EX and moment of Inertia. $ML_i$, $MR_i$, $ML_0$, $MR_0$, $CL$, $CR$ are the left and right bending moment of element, the left and right bending moment before cable adjustment and influence matrix of left and right bending moment generated by cable tension. $X$ is adjustment vector. The cable force is selected as $X$.

(2) Constraint function: According to specification, value ranges of the cable force are upper and lower limits of material, and values are all positive. The vertical allowable displacement of girder and stay cable is 10mm. The horizontal allowable displacement of main tower is set to 10mm to make sure displacement of main tower is reasonable.

$$X_{\text{min}} < \{X\} < X_{\text{max}}$$

$$0 < \{X\}$$

$$-10 \leq g_j(x) \leq 10$$

(10)

(3) To abolish a nonlinear inequality constraint optimization problem is as follows:

$$\min f(x)$$

s.t. $g_j(x) \leq 0, j \in I = 1, 2, ..., n$

(11a)

Where $f(x)$ is the objective function; $x$ is the design variable; $I$ is the subscript set; $g_j(x)$ is the constraint function, where $j$ is the number of the constraint function.

4.3. Convergence Analysis of the Optimization Process

Consider the above example 20 repeated callback calculations to verify the stability of the algorithm and summarize the rules of the search results. Each callback calculation takes different initial values and the same conditions, each iteration step is set to 400 steps. The convergence curve of the objective function is shown in Figure 4, setting the convergence criterion:
\[
D = \left\| \frac{d^0}{d^0} \right\|_2 \\
D \leq 10^{-4}
\]

(12)

Figure 5. Convergence curve of objective function

As can be seen from Figure 5, the convergence \[26-27\]. Speed of the algorithm is very fast in the early period. Almost all the calculations reduce the iteration target to about 100 in 50 steps. Then, the convergence speed starts to decline and stabilize. After 100 steps, it enters the fast convergence stage again and approaches the convergence state of the target. Each trial reaches the convergence state before 300 steps, and the objective function is reduced to a similar range, which proves that the algorithm is reliable and reasonable.

4.4 The result of cable force optimized parameters

According to symmetry of shaft, the half of structure is analyzed. After cable forces optimization, the cable forces of bridge are obtained in the condition that the bending strain energy of girder and main tower is minimal. The contrast between designed cable forces and optimized cable forces is shown in Figure 6.

Figure 6 shows that cable forces are well distributed in general, and gradually enlarged from short to long cable. Max cable force is 7926kN and the cable is No.5 of the side span. Min cable force is 5225kN and the cable is No.7 of mid-span. Calculated cable forces are consistent with design value, and error is within 5%. Dead load moments of the main girder and tower under optimum state are shown in Figure 7 and 8. These figures show that the moment of the girder is well distributed in general. Beam bending moment distribution is shown in Figure 9. The max moment is 7797kN·m, and the bottom moment of main tower is 480kN·m. The stress status of the main tower is close to the status of axial compression, which meets the design requirements.

Figure 6. The contrast between design cable forces and optimized cable tensions
The main tower of the bridge as a compression member, cable force is optimized to bring the force close to the axial compression state, reducing its damage caused by excessive local bending moment. The axial force of the bridge tower is shown in Figure 10.

Vertical displacements of the main girder under optimum state are shown in Figure 11. The main girder has a downward displacement, and the max displacement is 5.5mm, which is less than 10mm. The horizontal displacement of the node in the top of the main tower is 1.1mm, which is much less than the limiting value in the standard.

Figure 7. Dead load moments of the main girder under optimum state
Figure 8. Dead load moments of the main tower under optimum state
Figure 9. Beam moment diagram distribution
Figure 10. Tower Bridge Axial Force
Figure 11. Vertical displacements of the main girder
5. Seismic response analysis

The model for seismic analysis is shown in Figure 4, and there are three kinds of working conditions. Condition one: the cable does not increase cable forces; Condition two: each cable bears the same initial cable force of 1000kN; Condition three: give the calculated reasonable initial cable force of the bridge to each cable. Dynamic time history analysis method is used to calculate the seismic response, and the direct integration method is used in the dynamic time history analysis. In the calculation, the Rayleigh model is adopted, and the damping constant is calculated by two-step self-seismic frequency of the structural vibration. The control frequency damping ratio of cable-stayed bridge is 0.05, and the damping constants $\alpha = 0.620$, $\beta = 0.003$. The input seismic wave is the EL-Centro wave which is representative in the seismic design of the structure, and the acceleration time history curve is shown in Figure 12.

![Figure 12. Time-history of input ground motion](image1)

![Figure 13. Moments of the main girder](image2)

After the dynamic analysis, the maximum bending moments of the main beam in three conditions are shown in Figure 13. The comparison of condition one and condition two shows that the bending moments of the main beam have a decreasing tendency by considering the cable force, and the maximum negative moment is reduced from 62687kN•m to 52358kN•m, and the difference between the two is 15%. The maximum positive moment is reduced from 56578kN•m to 51089kN, and the difference between the two conditions is 10%. The comparison of condition one and condition three shows that the positive and negative moments are significantly reduced, respectively, down to 24693kN•m and 3254kN•m, and the maximum difference is more than 50%. From the above data, we can draw a conclusion that the cable forces can weaken the moment seismic response of cable-stayed bridge, and the effect of optimized cable forces is significant.

Displacements of different positions under the earthquake action in three different conditions are shown in Table 2. The deformation in condition one is similar to the deformation in condition two, but the latter is smaller. The main girder has a larger downward displacement in middle span than on both sides, and the main tower has a larger horizontal displacement. Compared with the previous two conditions, the maximum displacement under the third condition decreased significantly. The maximum displacement is reduced from 312mm to 45mm. The deformations of the main girder and tower in X direction are consistent. We can draw the following conclusions that the optimized cable forces can reduce the displacements of the main girder and tower, and it can also solve the problem that structure displacements are uneven.

| Direction | Position     | Condition 1 | Condition 2 | Condition 3 |
|-----------|--------------|-------------|-------------|-------------|
| X         | Mid-span     | 61          | 61          | 61          |
|           | tower top    | 163         | 151         | 73          |
|           | Maximum deformation | 163 | 151 | 73 |
6. Conclusion
With the increase in span, the structural patterns and design requirements of the cable-stayed bridge become increasingly complex, an optimized method of finished bridge state which has better precision, higher computing speed and wider application range, becomes a research focus. For the purpose of accuracy, effectiveness and engineering practicality, the new type of strongly sub-feasible SQP method is raised in solving cable forces optimization problems. Combined with Matlab and MIDAS/Civil, a nonlinear programming model is built and solved optimally to obtain reasonable cable forces of cable-stayed bridge. Then, based on optimization results of cable force, a dynamic seismic analysis of the structure is performed under conditions of different cable forces. The conclusion is as follow:

(1) A new type of strongly sub-feasible SQP method could be used in cable force optimization problems of the cable-stayed bridge. The method, which is more convenient and has stronger operability combined with MIDAS/Civil, could be used in solving reasonably finished bridge state problem of the cable-stayed bridge with more design variable and structure elements and complex conditions.

(2) Under the action of earthquakes, considering the effect of cable forces, moment and displacement responses of the mid-span of the main girder are decreased. Horizontal displacement of the main tower is more reasonable. Displacement distribution of structure intends to be well distributed. Optimal cable forces have positive effects on the seismic response analysis of cable-stayed bridge.

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References
[1] Kohei Furukawa HSTE. Studies on optimization of cable-prestressing for cable-stayed bridges[M]. Bangkok: Asian Institute of Technology, 1987:67-75.
[2] A. Kasuga, H.Arai, J. E. Breen, K. Furukawa. Optimum Cable-Force Adjustment in Concrete Cable-stayed Bridges. Journal of Structure Engineering, April, 1995,12(4):685-694.
[3] Zong Zhouhong, Huang Xueyang,Li Yale, et al.2016. Study of collapse failure control of long span cable-stayed bridges under strong earthquake excitation. Bridge Construction46(1):24-29. (in Chinese)
[4] Song Li, Zhang Min, Qin Sifeng.2014. Pushover analysis for large span cable-stayed bridge. Journal of Liao ning Technical University(Natural Science)33(4):481-484 (in Chinese)
[5] Hoon Yoo, Ho_Sung Na, Dong-Ho Choi, Approximate Method for Estimation of Collapse Loads of Steel Cable-Stayed Bridges[J]. Journal of Constructional Steel Research,2012,(72):143-154
[6] Nakayama H, Kaneshige K, Takemoto S, et al. An application of a multi-objective programming technique to construction accuracy control of cable-stayed bridges(European Journal of Operational Reasearch),2005,87(3):731-738
[7] D. Janjic, M. Pircher, H. Pircher.2003. Optimization of Cable Tensioning in Cable-Stayed Bridges,(Journal of Bridge Engineering),(8):131-137

\[
\begin{array}{cccc}
\text{Mid-span} & 312 & 269 & 1 \\
\text{tower top} & 7 & 7 & 8 \\
\text{Maximum deformation} & 312 & 269 & 45 \\
\end{array}
\]
[8] Lee T Y, Kim Y H, Kang S B. 2008. Optimization of Tensioning Strategy for Asymmetric Cable-stayed Bridge and its Effect on Construction Process. Structural and Multidisciplinary Optimization35(6):623-629.

[9] P.H.Wang,T.C.Tseng,C.G.Yang.1993.Initial Shape Of Cable-Stayed Bridges. Computers & Structures46(6):1095-1106.

[10] Yang Xiyao,Yang Shunping,Cai Min.2014. Determining of Rational Completion Cable Forces of Cable-Stayed Bridges by Minimum Bending Energy Method.Anhui Architecture(2):112-113. (in Chinese)

[11] Sung Y C, Chang D W, Teo E H. 2006. Optimum post-tensioning cable forces of Mau-Lo His cable-stayed bridge. Engineering Structure28(10):1407-1417.

[12] Guo Zhong qun, Xie Zhihua, Zhao Kui, Xie Liang, Lin Xu. 2012. The cable tension optimization based on the feasible domain method. Journal of Jiangxi University of Science and Technology33(3): 10-13. (in Chinese)

[13] X.Liu.2011. Extension of Kleiser and Schumann’s influence-matrix method for generalized velocity boundary conditions. Journal of Computational Physics230(2011):7911-7916

[14] Zhang Junfeng, Ding Zhiwei, Luo Xuechong. 2011. Optimization of Cable Forces for Cable-stayed Bridges Based on the Influence Matrix Method. Transporation Science&Technology246(3): 4-6. (in Chinese)

[15] Venkat Lute,Akhil Upadhyay,K.K. Singh. 2009 Computationally efficient analysis of cable-stayed bridge for GA-based optimization [J]. Engineering Applications of Artificial Intelligence, 22(4). 243-249.

[16] Shahria Alam M.,Junjun Guo,Wancheng Yuan,Xin zhi Dang,.2019 Cable force optimization of a curved cable-stayed bridge with combined simulated annealing method and cubic B-Spline interpolation curves[J]. Engineering Structures, 201(2):162-171.

[17] Wu xiao, Xiao Rucheng. 2014. Optimization of cable force for cable-stayed bridges with mixed stiffening girders based on genetic algorithm. Journal of Jiangshu University (Natural Science Edition)35(6): 722-726. (in Chinese)

[18] Vanderplaats Garret N 2006. Structural optimization for statics, dynamics and beyond[J]. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 28(3):683-692.

[19] Tao Hai, Shen Xiangfu. 2006. Strongly subfeasible sequential quadratic programming method of cable tension optimization for cable-stayed bridges. Chinese Journal of Theoretical and Applied Mechanics38(3): 381-384. (in Chinese)

[20] Jian Jinbao. 2001. A super linearly and quadratically convergent SQP algorithm for inequality constrained optimization. Acta Mathematica Scientia21(2): 268-277. (in Chinese)

[21] Tong mingchun. 2008. Researches on strongly sub-feasible methods and sequential quadratically constrained quadratic programming algorithms. Shanghai : Shanghai University. (in Chinese)

[22] Wu shuhua.2016. An Improved Conjugate Gradient Method with Global Convergence Property Under the Armijio Line Search.Chongqing. Journal of Hubei University for Nationalities( Natural Science Edition)34(4):394-397. (in Chinese)

[23] PH Wang, HT Lin, TY Tang 2002. Study on nonlinear analysis of a highly redundant cable-stayed bridge. Computers and Structures, 80:165-182.

[24] Frode Martinsen, Lorentz T. Biegler, Bjarne A. Foss 2005. A new optimization algorithm with application to nonlinear MPC[J].Modeling, Identification and Control, 26(1):375-386.
[26] Y. Diouane, S. Gratton, L. N. Vicente 2015. Globally convergent evolution strategies for constrained optimization [J]. Computational Optimization and Applications, 62(2):232-243.

[27] Yeniay Ozgur 2005. A comparative study on optimization methods for the constrained nonlinear programming problems [J]. Mathematical Problems in Engineering, 2005(2):153-168.