Tunnel shape optimization method considering the interaction between lining and soil

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Abstract. Recent tunnel shape optimization considers only the stress distribution in the soil around the tunnel or the lining’s mechanical performance and frequently ignores the influence of the interaction between the lining and soil. Thus, this study presents a tunnel shape optimization method considering this interaction. First, the concept of a lining rational arch axis is employed to derive an analytical expression for the tunnel shape, which is subjected to earth pressure recommended by the International Tunneling Association (ITA). Further, the rationality of the obtained tunnel shape is verified using a finite element method (FEM) simulation, which considers the interaction between the lining and soil. Thus, an iterative FEM is further proposed to minimize the lining’s eccentricity. The tunnel shape is gradually modified via the iterative FEM by offsetting the lining axis along the normal vector using the corresponding eccentricity to obtain the optimal tunnel shape. The results reveal that, with the optimal tunnel shape based on the rational arch axis, the tunnel lining is mainly subjected to axial compressive force and exhibits good performance in terms of deformation and bending moment.

1. Introduction

The exploration of underground space has rapidly developed following socio-economic growth and
increasing demand of humans for living space, as well as the design theory and construction methods of tunnels have also been fully developed. However, certain tunnel challenges are yet to be fully resolved. Some tunnel segments are subjected to excessive bending moments under earth pressure and other loadings, resulting in several issues such as lining cracking and segment damage, thus the tunnel’s stability is severely affected [1-2]. However, these issues are usually associated with the tunnel shape to some extent. In tunnel engineering, various tunnel shapes are adopted [3], although the circular tunnel is usually preferred due to convenient construction and function layout requirements.

Currently, segment assembly technology is widely applied in the construction of circular tunnels, which has resulted in higher requirements for the bending resistance of the segments and their joints. Therefore, some scholars have conducted several experimental investigations on the tunnel lining structure’s mechanical performance [4-7], which focus on the bending characteristics of the segments and their joints. The lining mechanical performance is closely related to the adopted tunnel shape. If the tunnel shape is unreasonable, an excessive bending moment may arise on the lining under earth pressure. In the tunnel design, the shape of the tunnel must be evaluated and optimized to minimize the bending moment on the lining, so that the optimum mechanical performance is achieved.

The research on the tunnel’s rational arch axis is a classical problem [8-11]. Ma Yuchuan [8] studied the tunnel shape based on the rational arch axis concept, considering the elastic resistance coefficient of the surrounding rock, and obtained the curve equation of the lining matching the rational arch axis. Liu Xiaobing [9] optimized the objective function by minimizing the tunnel’s section area and obtained the tunnel shape that satisfied the building boundary and allowable lining eccentricity requirements. Dai Huijuan et al. [10] derived the equations of the large-span shallow-buried arch structure and verified the rationality of its shape using a finite element method (FEM) program. Huang Dawei et al. [11] derived the analytical expression for the tunnel lining axis and obtained the formula for the key parameters of this tunnel shape. However, the influences of the lining–soil interaction on the optimal tunnel shape were not discussed in these investigations.

Another approach to optimize the tunnel profile is based on the stress distribution in the soil around the tunnel [12-18]. The stress distribution at any position around the hole in the plane is usually determined using the complex variable function with the angle-preserving mapping method for this class of problems. The mapping function of the optimal tunnel shape is obtained based on the specific stress concentration criteria, and then the tunnel optimal shape can be achieved. The achieved optimal tunnel shape varies with different stress concentration criteria. Bjorkman et al. [12, 13] calculated the harmonic hole for infinite elastic planes based on the complex variable function method to obtain the tunnel optimal shape that satisfies the boundary conditions. Based on the harmonic hole concept, Wang et al. [14, 15] developed the harmonic shapes in a nonlinear elastic plane using the method of complex variables. However, the lining mechanical performance was not considered in the aforementioned studies.

In addition to the two aforementioned optimization approaches, Ren et al. [19, 20] proposed an evolutionary structural optimization method for the optimal shape of underground excavation. In this method, an initial hole is created in a 2- or 3-dimensional numerical model based on the FEM program, and the elements with lower mean principal stress around the hole are gradually deleted to obtain the tunnel optimal excavation shape. These types of studies are mainly concerned with the stress
concentration in the soil around the tunnel, whereas the mechanical performance of the lining itself is not analyzed.

Therefore, a tunnel shape optimization method is presented in the study, which considers the interaction between lining and soil. First, the concept of a rational arch axis is employed to derive the analytical expression for the tunnel shape, which is subjected to the earth pressure recommended by the International Tunneling Association (ITA). The formulas for the axial force and radial deformation at any lining position are also provided in this study. Further, the rationality of the obtained tunnel shape is verified using FEM simulation. Further, considering the interaction between lining and soil, an iterative FEM is proposed to minimize lining eccentricity. Using the iterative FEM, the tunnel lining is gradually approximated to a rational arch axis by offsetting the lining along the normal vector by the corresponding eccentricity, then the optimal tunnel shape is obtained.

2. Rational arch axis of the lining under ITA load mode

2.1. Equation of rational arch axis

Assuming that the tunnel is buried at a certain depth below the ground, the lining is subjected to the earth pressure recommended by ITA [21, 22], as shown in Figure 1. To investigate the optimal tunnel shape, the following assumptions are made in this study: (1) the soil is an isotropic continuous medium; (2) the thickness of the lining is ignored; and (3) the lining is weightless.

The tunnel shape can be arranged symmetrically along the central axis since the loading on the lining is in symmetry. This investigation adopts a half-lining structure with unit width, and its height and lateral width are $h$ and $f_0$, respectively. Figure 2 shows the sketch of the lining calculation model. Where $p_1$ and $p_2$ are the vertical earth pressures at the top and bottom of the tunnel, respectively, $q_1$ and $q_2$ are the lateral pressures of the ground, and $F_1$ and $F_2$ represent the axial force at two ends of the lining.

The lining deformation is minimal when the tunnel shape conforms to the rational arch axis, and the lateral pressure of the ground can be equated to static earth pressure, as presented in Equations 1 and 2. The vertical earth pressure at the bottom of the lining can be expressed as Equation 3, regardless of the lining weight.

\[
q_1 = k_0 p_1 
\]  
(1)

\[
q_2 = k_0 (p_1 + \gamma h)
\]  
(2)

\[
p_1 = p_2
\]  
(3)

where $\gamma$ is the soil weight and $k_0$ denotes the coefficient of static earth pressure.

For the lining in Figure 2, the following geometric boundary conditions should be satisfied:

\[
(y)_{x=0} = 0 \quad \text{and} \quad (y)_{x=h} = 0
\]  
(4)

\[
\left( \frac{dx}{dy} \right)_{x=0} = 0 \quad \text{and} \quad \left( \frac{dx}{dy} \right)_{x=h} = 0
\]  
(5)
In principle, the tunnel lining is a statically indeterminate structure. The lining is only subjected to axial forces when the tunnel shape conforms to a rational arch axis. Because of the geometry and load symmetry, at a position $x = 0$ and $x = h$, the lining structure is only subjected to horizontal axial forces, and its shear force and bending moment are zero, thus the axial force of this section can be calculated using the static equilibrium condition. The bending moments at Points $A$ and $B$ can be expressed as Equations 6 and 7, respectively, which are equal to zero.

$$\sum M_x = -F_2 h - \frac{1}{2} p_2 f_0^2 + \int_0^h \left( q_1 + \frac{q_2}{h} - \frac{q_0}{h} \right) x \, dx + \frac{1}{2} p_1 f_0^2 = 0$$

$$\sum M_y = F_1 h + \frac{1}{2} p_1 f_0^2 - \int_0^h \left( q_1 + \frac{q_2}{h} - \frac{q_0}{h} \right) (h-x) \, dx - \frac{1}{2} p_2 f_0^2 = 0$$

Equations 1, 2, 6, and 7 are solved to obtain the axial force of the lining at Points $A$ and $B$, as shown in Equations 8 and 9.

$$F_1 = \frac{1}{6} k_0 (3p_1 + \gamma h) h$$

$$F_2 = \frac{1}{6} k_0 (3p_1 + 2\gamma h) h$$

Supposedly, the lining lateral width, $f_0$, corresponds to the horizontal coordinate $x = x_k$. When $x \leq x_k$, the bending moment at any position of the lining can be expressed as Equation 10, whereas the bending moment for $x > x_k$ is given by Equation 11. These two equations can satisfy the boundary conditions.

$$\sum M_D = F_1 x - \frac{1}{2} p_1 y^2 - \frac{1}{2} q_1 x^2 - \frac{1}{6h} (q_2 - q_1) x^3$$

$$\sum M_E = -F_2 (h-x) + \frac{1}{2} p_2 y^2 + \frac{1}{6h} (q_1 h + 2q_2 h - q_1 x + q_2 x) (h-x)^2$$
By allowing the bending moment at any position on the lining to be zero, the rational arch axis equation of the lining structure is further obtained, as presented in Equations 12 and 13.

\[ Q_1(x,y) = \frac{1}{6} k_0 \gamma x^3 - \frac{1}{2} k_0 p_x x^2 + \frac{1}{6} k_0 (\gamma h^2 + 3 p_i h) x - \frac{1}{2} p_i y^2 = 0 \]  

\[ Q_2(x,y) = \frac{1}{6} k_0 \gamma x^3 + \frac{1}{2} k_0 p_x x^2 - \frac{1}{6} k_0 (\gamma h^2 + 3 p_i h) x + \frac{1}{2} p_i y^2 = 0 \]  

where \( Q_1(x,y) \) and \( Q_2(x,y) \) denote the rational arch axis equations of the lining at the position of \( x \leq x_k \) and \( x > x_k \), respectively.

These two expressions are differentiated by only one negative sign (presented in Equation 14), therefore, the lining rational arch axis equation can be expressed uniformly as \( Q(x,y) \), shown in Equation 15, and thus Equation 16 is derived.

\[ Q_2(x,y) = Q_1(x,y) = \frac{1}{6} k_0 \gamma x^3 + \frac{1}{2} k_0 p_x x^2 - \frac{1}{6} k_0 (\gamma h^2 + 3 p_i h) x + \frac{1}{2} p_i y^2 = 0 \]  

\[ y^2 = \frac{- k_0 \gamma x^3 - 3 k_0 p_x x^2 + k_0 (\gamma h^2 + 3 p_i h) x}{3 p_i} \]  

where \( Q(x,y) \) denotes the rational arch axis equation of the entire lining.

According to the calculation sketch, a maximum value for the lining rational arch axis exists at \( x = x_k \). Thus, Equation 17 is derived by differentiation, and the horizontal coordinate of the maximum point is solved as shown in Equation 18.

\[ \frac{d\left(y^2\right)}{dx} = - \frac{k_0 \left(3 \gamma x^2 + 6 p_x x - \gamma h^2 - 3 p_i h \right)}{3 p_i} = 0 \]  

\[ x_k = \frac{\sqrt{3 \gamma ^2 h^2 + 9 p_i \gamma h + 9 p_i^2}}{3 \gamma} \]  

2.2. Internal force and lining deformation

Based on the lining rational arch axis obtained above, the axial force at any position of the lining can be calculated by static equilibrium and can be expressed as Equation 19.

\[ F(x,y) = \sqrt{p_i^2 y^2 + \left(\frac{h k_0 (3 p_i + \gamma h)}{6} - \frac{k_0 x (2 p_i + \gamma x)}{2}\right)^2} \]  

where \( F(x,y) \) denotes the axial force in the lining.

In addition, the lining deformation can be calculated from the axial force. Notably, lining deformation is a plane strain problem. Taking point \( (x_k, 0) \) as the tunnel center, the radial deformation at any point on the lining with respect to the tunnel center-point can be stated as Equation 20, with the direction toward the lining center.
\[ u_r(x,y) = (1 - \nu^2) \left( \int_0^y \frac{F(x,y)}{EA} \, dy \right)^2 + \left( \int_0^y \frac{F(x,y)}{EA} \, dy \right)^2 \]  

(20)

where \( u_r(x,y) \) denotes the radial deformation of the lining, whereas \( EA \) and \( \nu \) denote the lining section axial stiffness and lining Poisson's ratio, respectively.

2.3. Case analysis

We assume that the tunnel is buried with a depth of 20 m and a height of 6 m, and the lining thickness is 0.3 m. Table 1 shows the materials of soil and lining for this tunnel.

**Table 1. Material properties used in finite element model.**

| Category | Property          | Value          |
|----------|-------------------|----------------|
| Soil     | Volume weight     | 19 kN/m³       |
|          | Young’s modulus   | 2 \times 10^7 kN/m² |
|          | Poisson’s ratio   | 0.33           |
| Lining   | Young’s modulus   | 3.55 \times 10^7 kN/m² |
|          | Poisson’s ratios  | 0.20           |

Calculated from Equations 1, 2, 16, and 18, the following relevant parameters can be obtained: \( p_1 = 380 \text{kPa}, p_2 = 380 \text{kPa}, x_k = 3.07 \text{ m}, \) and \( f_0 = 2.275 \text{ m}. \) Thus, the equation of the rational arch axis for the lining can be calculated, as presented in Equation 21.

\[ Q(x,y) = x^3 + 60x^2 - 396x + 120y^2 = 0 \]  

(21)

The complete lining rational arch axis is obtained from Equation 21, and an egg-like tunnel shape is achieved (Figure 3). The lining bending moments and shear forces are zero everywhere for this type of tunnel defined by the rational arch axis. The axial force at any position of the lining can be calculated from Equation 19. The maximum value of the axial force is 864.7 kN, occurring at lining width direction, whereas the minimum value is 627.0 kN, occurring at the top of the lining. The radial deformation at any point of the lining centerline is calculated from Equation 20. The maximum displacement of the lining centerline is 0.22 mm, occurring at the top of the lining, whereas the radial displacement at the lining width direction is only 0.15 mm.

3. FEM analysis considering the interaction between the lining and the soil

The numerical simulation of the tunnel with the shape presented in Equation 21 is conducted based on the FEM program. The FEM model is a plane strain model with a model size of 100 m \( \times \) 100 m (Figure 4), and the material parameters of each part are mentioned in Table 1. The plane strain element is assigned to the soil mass in the model, and the beam element to the lining, of which Young’s modulus and Poisson’s ratio are converted from plane stress to plane strain by Equations 22 and 23. Notably, there is no relative sliding between the lining and soil.

\[ E_1 = \frac{E}{1 - \nu^2} \]  

(22)
where the parameters $E$ and $\nu$ are Young’s modulus and Poisson’s ratio, respectively, whereas $E_i$ and $\nu_i$ are respectively Young’s modulus and Poisson’s ratio for plane strain model.

$$v_i = \frac{\nu}{1 - \nu} \quad (23)$$

In this FEM model, the initial soil ground stress equilibrium is first executed, and then the tunnel analysis is performed to obtain the lining radial deformation and internal force distribution. Figure 5 presents the lining bending moment and eccentricity diagrams. Figure 6 shows the axial forces, which are presented by the magenta solid and blue-dashed lines. Figure 7 shows the lining radial deformation distribution, which is represented by the dashed line, and the magenta centerline and blue centerline line indicate the analytical and FEM solutions, respectively. Figure 5 shows that the maximum bending moment is $67.8$ kNm, occurring at the bottom of the tunnel. Figure 6 shows that the lining maximum axial force is $980.3$ kN and its minimum axial force is $614.1$ kN, which is consistent with the calculation result of Equation 19 (as magenta solid line shown in Figure 6), whereas Figure 7 shows the radial deformation of the lining axis caused by bending moment and axial force, which differs from the calculation result of Equation 20 (presented by the magenta centerline line).

In comparison, a FEM numerical simulation of the circular tunnel with an equal sectional area is conducted to obtain the lining deformation and bending moment. Figure 8 shows a magenta solid line and blue centerline, which indicates the lining bending moment and deformation, respectively. The lining bending moment for the egg-like tunnel is $67.8$ kNm, which is significantly smaller than that of $251.5$ kNm for a circular tunnel; whereas the tunnel’s maximum radial deformation is $1.61$ mm, which is also smaller than the radial deformation of $6.7$ mm for the circular tunnel. Thus, it is verified that the egg-like shape for a tunnel under the earth pressure recommended by ITA is rational because the axial force is predominant for the lining.
Figure 5. Bending moment and eccentricity of the lining.

Figure 6. Axial force of lining.

Figure 7. Lining deformation.

Figure 8. Deformation and bending moment of the circular lining.

Figure 5 indicates that a small lining bending moment still exists for the tunnel with a cross-section shape determined by Equation 21. Furthermore, the tunnel shape optimization is conducted
subsequently to modify the aforementioned tunnel shape, such that the lining can be approximated to the real rational arch axis, considering the interaction between the lining and soil.

4. Approximation optimization of rational arch axis

To obtain the optimal tunnel shape, an iterative FEM is employed to minimize lining eccentricity, which is an approach to gradually approximate the lining axis to the real rational arch axis.

The lining axis obtained using Equation 21 is discretized into a series of segments, with the coordinate \((X_i^0, Y_i^0)\) for the node, \(i\), and the bending moment and axial force at this node can be calculated using FEM, where the bending moment is positive, subjecting the outer lining fiber in tension, and axial force is positive, subjecting the lining section in compression. Thus, the lining eccentricity at this node is calculated from Equation 24. Given that the unit vectors, \(\overrightarrow{p}_{i-1}\) and \(\overrightarrow{p}_i\), are respectively perpendicular to segments adjacent to the node, \(i\), and the lengths of the segments are \(s_{i-1}\) and \(s_{i+1}\), the normal vector, \(n_i\), at this node (Figure 9) can be expressed in Equation 25 with the directional cosine, \(n_x\) and \(n_y\). The coordinate of node \(i\) is acquired based on Equation 26, by offsetting nodes along the vector, \(n_i\), using the corresponding eccentricity. In addition, a new tunnel shape is obtained after offsetting, which is further scaled for its height to equal that of the initial design. Further, the finite element analysis of the new model is conducted.

\[
e_i^0 = M_i^0/F_i^0
\]

\[
n_i = \frac{s_{i-1} \overrightarrow{p}_{i-1} + s_i \overrightarrow{p}_i}{s_{i-1} + s_i}
\]

\[
(X_i, Y_i) = \left(X_i^0 - e_i^0 n_x Y_i^0 - e_i^0 n_y \right)
\]

\[
\max |e_i| \leq 0.05t
\]

Where \(e_i^0\) denotes the lining eccentricity at node \(i\), \(M_i^0\) and \(F_i^0\) denote the bending moment and the axial force, respectively, in this position; \(n_x\) and \(n_y\) are the directional cosines of the normal vector, \(n_i\); \(e_i\) denotes the lining eccentricity at node \(i\) for the modified tunnel; \(t\) is the lining thickness, and the other symbols are mentioned in the previous paragraph.

**Figure 9.** Normal of node at lining axis

Based on the iterative FEM, the lining axis is continuously modified until the criterion is satisfied (Equation 27), thus obtaining the optimal tunnel shape (Figure 10). The bending moment, deformation, and eccentricity of the modified lining are presented in Figures 11–13.
Figures 10–13 show that the optimized lining has significantly improved in terms of deformation and bending moment. The maximum lining eccentricity decreases from 21.4 to 5.7 mm, which is less
than 0.05 times of lining thickness after two optimizations. Further, the maximum lining bending moment is 3.9 kNm, and the maximum deformation is only 0.31 mm for the optimal tunnel.

5. Conclusion
Based on a rational arch axis concept, an analytical equation is derived for the tunnel shape, which is subjected to earth pressure recommended by ITA. On this basis, the rationality of the egg-like tunnel is verified using a FEM simulation, and an optimal tunnel shape is obtained using an iterative FEM. The following conclusions are obtained from the study:

1) In comparison with a circular tunnel, the egg-like shaped tunnel is rational in mechanical performance, in which the axial force is predominant for the lining.

2) The optimal tunnel modified from the egg-like shaped tunnel based on the iterative FEM, exhibits a significant improvement in mechanical performance, with a maximum bending moment of only 3.9 kNm and a maximum eccentricity of 5.7 mm.

3) The iterative FEM proposed in this study can provide theoretical guidance and technical reference for shape optimization in tunnel design.

Acknowledgments This work was supported by National Natural Science Foundation of China (Grant No. 51909259, 52079135), the International Partnership Program of Chinese Academy of Sciences (Grant No. 131551KYSB20180042), the Science and Technology R & D Project of China State Construction International Holdings Limited (No. CSCI-2020-Z-21), and Ningbo Public Welfare Science and Technology Planning Project (No. 2019C50012).

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