I. INTRODUCTION

The Maxwell’s equations in vacuum can be written as

\[ \nabla \cdot \mathbf{E} = \frac{\mu_0 \mathbf{B}}{\epsilon_0}, \tag{1} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2} \]
\[ \nabla \cdot \mathbf{B} = 0, \tag{3} \]
\[ \frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{4} \]

where \( \mathbf{E} \) is the electric field vector, \( \mathbf{B} \) is the magnetic induction vector, \( \rho_0 \) is the density field of electric charges, \( \epsilon_0 \) is the dielectric constant of vacuum, \( \mu_0 \) is magnetic permeability of vacuum, \( t \) is time, \( \nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \) is the Hamilton operator.

The main purpose of this paper is to derive the above mentioned Maxwell’s equations in vacuum based on a continuum mechanics model of vacuum and a singularity model of electric charges.

The motive of this paper is to seek a mechanism of electromagnetic phenomena. The reasons why new mechanical models of electromagnetic field are interesting may be summarized as follows.

Firstly, there exists some electromagnetic phenomena that could not be interpreted by the present theories of electromagnetic field, e.g., the spin of electrons \([1,2]\), the Aharonov-Bohm effect \([3,4]\). New theories of electromagnet phenomena may view these problems from new angles.

Secondly, there exists some inconsistencies and inner difficulties in the classical electrodynamics, e.g., the inadequacy of Liénard-Wiechert potentials \([3,6,7]\). New theories of electromagnetic phenomena may overcome such difficulties.

Thirdly, there exists some divergence problems in quantum electrodynamics \([8]\). In Dirac’s words, ‘I must say that I am very dissatisfied with the situation, because this so-called good theory does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics.’ New theories of electromagnetic phenomena may open new ways to solve such problems.

Fourthly, since quantum theory shows that vacuum is not empty and has physical effects, e.g., the Casimir effect \([9,10,11,12]\), it is valuable to reexamine the old concept of electromagnetic aether.

Fifthly, from the point view of reductionism, Maxwell’s theory of electromagnetic field can only be regarded as a phenomenological theory. Although Maxwell’s theory is a field theory, the concept of field is different from that of continuum mechanics \([13,14,15,16]\) because of the absence of a continuum. Thus, from the point view of reductionism, the mechanism of electromagnetic phenomena remains an unsolved problem in physics \([17]\).

Sixthly, one of the puzzles in physics is the problem of dark matter and dark energy (refer to, for instance, \([18,19,20,21,22,23,24,25,26]\)). New theories of electromagnetic phenomena may provide new ideals to attack this problem.

Finally, one of the tasks of physics is the unification of the four fundamental interactions in the universe. New theories of electromagnetic phenomena may shed some light on this puzzle.

To conclude, it seems that new considerations on electromagnetic phenomena is needed. It is worthy keeping an open mind with respect to all the theories of electromagnetic phenomena before the above problems been solved.

Now let us briefly review the long history of the mechanical interpretations of electromagnetic phenomena.

According to E. T. Whittaker\([17]\), Descartes was the first to bring the concept of the aether into science by suggesting that it has mechanical properties according to E. T. Whittaker\([17]\). Descartes believed that every physical phenomenon could be interpreted in the construction of a mechanical model of the universe. William Watson and Benjamin Franklin introduced the one-fluid theory of electricity independently in 1746 \([17]\). Henry Cavendish attempted to explain some of the principal phenomena of electricity by means of an elastic fluid in 1771 \([17]\). Not contented with the above mentioned one-fluid theory of electricity, du Fay, Robert Symmer and C. A. Coulomb
developed a two-fluid theory of electricity from 1733 to 1789 [17].

Before the unification of electromagnetic phenomena and light phenomena by Maxwell in 1860s, light phenomena were also studied independently based on Descartes’ scientific research program of the mechanical theory of nature. John Bernoulli introduced a fluidic aether theory of light in 1752 [17]. Euler believed that all electrical phenomena is caused by the same aether that propagates light. Furthermore, Euler attempted to explain gravity in terms of his single fluidic aether [17].

In 1821, in order to explain polarisation of light, A. J. Frenshel proposed an aether model which is able to transmit transverse waves. After the advent of this successful transverse wave theory of light of Frenshel, those imponderable fluid theories were abandoned. Frenshel’s dynamical theory of a luminiferous aether had an important influence on the mechanical theories of nature in the nineteenth century [17]. Inspired by Frenshel’s luminiferous aether theory, numerous dynamical theories of elastic solid aether were established by Stokes, Cauchy, Green, MacCullagh, Boussinesq, Riemann and William Thomson, see, for instance, [17].

Thomson’s analogies between electrical phenomena and elasticity helped to inspire James Clark Maxwell to establish a mechanical model of electrical phenomena [17]. Strongly impressed by Faraday’s theory of lines of forces, Maxwell compared the Faraday’s lines of forces with the lines of flow of a fluid. In 1861, in order to obtain a mechanical interpretation of electromagnetic phenomena, Maxwell established a mechanical model of a magneto-electric medium. Maxwell’s magneto-electric medium is a cellular aether, looks like a honeycomb. Each cell of the aether consists of a molecular vortex surrounded by a layer of idle-wheel particles. In a remarkable paper published in 1864, Maxwell established a group of equations, which were named after his name later, to describe the electromagnetic phenomena.

In 1878, G. F. FitzGerald compared the magnetic force and the velocity in a quasi-elastic solid of the type first suggested by MacCullagh [17]. FitzGerald’s mechanical model of the electromagnetic aether were studied by A. Sommerfeld, by R. Reiff and by Sir J. Larmor later [17].

Because of some dissatisfactions with the mechanical models of the electromagnetic aether and the success of the theory of electromagnetic field, the mechanical world view was replaced by the electromagnetic world view gradually. Therefore, the concepts of a luminiferous aether and an elastic solid aether were replaced by the concepts of an electromagnetic aether or the electromagnetic field. This paradigm shift in scientific research was attributed to many scientists, including Faraday, Maxwell, Sir J. Larmor, H. A. Lorentz, J. J. Thomson, H. R. Hertz, Ludwig Lorenz, Emil Wiechert, Paul Drude, Wilhelm Wien, etc., see, for instance, [17].

In a remarkable paper published in 1905, Einstein abandoned the concept of aether [27]. However, Einstein’s assertion did not cease the exploration of aether (refer to, for instance, [17, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]). Einstein changed his attitude later and introduced his new concept of ether [38, 39]. In 1979, A. A. Golebiewska-Lasta observed the similarity between the electromagnetic field and the linear dislocation field [28]. V. P. Dmitriyev have studied the similarity between the electromagnetism and the linear elasticity since 1992 [32, 33, 34, 35, 36, 37]. H. Marmanis established a new theory of turbulence based on the analogy between electromagnetism and turbulent hydrodynamics in 1998 [33]. D. J. Larson derived the Maxwell’s equations from a simple two-component solid-mechanical aether in 1998 [33]. D. Zareski gave an elastic interpretation of electrodynamics in 2001 [30]. I regret to admit that it is impossible for me to mention all the works related to this field in history.

Inspired by the above mentioned works, we show that the Maxwell’s equations of electromagnetic field can be derived based on a continuum mechanics model of vacuum and a singularity model of electric charges.

II. CLUES OBTAINED FROM DIMENSIONAL ANALYSIS

According to Descartes’ scientific research program which was based on his mechanical view of nature, the electromagnetic phenomenon must be and can be interpreted based on the mechanical motions of aether particles.

Therefore, all the physical quantities appearing in the theory of electromagnetic field must be mechanical quantities.

Thus, in order to establish a successful mechanical model of electromagnetic field, we should undertake a careful dimensional analysis (refer to, for instance, [41]), of electric field vector $\mathbf{E}$, magnetic induction vector $\mathbf{B}$, the density field of electric charges $\rho$, the dielectric constant of vacuum $\varepsilon_0$, magnetic permeability of vacuum $\mu_0$, etc.

It is known that the Maxwell’s equations (11) in vacuum can also be expressed as [1]

$$\nabla^2 \phi + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{\rho}{\varepsilon_0}, \quad (5)$$

$$\nabla^2 \mathbf{A} - \nabla(\nabla \cdot \mathbf{A}) - \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left( \nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \right) = -\mathbf{j}, \quad (6)$$

where $\phi$ is the scalar electromagnetic potential, $\mathbf{A}$ is the vector electromagnetic potential, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

Noticing the similarity between the Eq. (6) and the equation (83) of momentum conservation of elastic solids, it is natural to speculate that the dimension of the vector electromagnetic potential $\mathbf{A}$ of aether has the same of
the dimension of the displacement vector \( \mathbf{u} \) of an elastic solid.

In 1846, W. Thomson compared electric phenomena with elasticity. He pointed out that the elastic displacement \( \mathbf{u} \) of an incompressible elastic solid is an possible analogy with the vector electromagnetic potential \( \mathbf{A} \) [17].

Inspired by this clue, let us set out to investigate in this direction further in the following sections.

III. A VISCO-ELASTIC CONTINUUM MODEL OF VACUUM

The purpose of this section is to establish a visco-elastic continuum mechanical model of vacuum.

In 1845-1862, Stokes suggested the aether might behave like a glue-water jelly [42, 43, 44]. He believed that the aether would act like a fluid with respect to translational motion of large bodies through it, but would still possess elasticity to produce small transverse vibrations.

Following Stokes, we propose a visco-elastic continuum model of vacuum.

Hypothesis 1 Suppose the universe is filled with a kind of continuously distributed material.

This material may be called aether for convenience.

In order to establish a continuum mechanical theory of aether, we must introduce some hypotheses based on experimental data of the macroscopic behavior of vacuum.

Hypothesis 2 We suppose that the all the mechanical quantities of the aether under consideration, such as density, displacements, strains, stresses, etc., are piecewise continuous functions of space and time. Furthermore, we suppose that the material points of the aether remain one-to-one correspondence with the material points before deformation happens.

Hypothesis 3 We suppose that the material of the aether under consideration is homogeneous, that is \( \partial \rho/\partial y = \partial \rho/\partial z = \partial \rho/\partial x = 0 \), where \( \rho \) is the density of aether.

Hypothesis 4 Suppose that the deformation processes of aether are isothermal processes. We neglect the thermal effects.

Hypothesis 5 Suppose the deformation processes is not influenced by the gradient of stress tensor.

Hypothesis 6 We suppose that the material of the aether under consideration is isotropic.

Hypothesis 7 We suppose that the deformation of the aether under consideration is small.

Hypothesis 8 We suppose that there are no initial stress and strain in the body under consideration.

When aether is subjected to a set of external forces, the relative positions of the aether particles forming the body changes.

In order to described the deformation of the aether, let us introduce a Cartesian coordinate system \( \{o, x, y, z\} \) or \( \{o, x_1, x_2, x_3\} \) which is static relative to the aether. Now we may introduce the definition of displacement vector \( \mathbf{u} \) of every point in the aether as

\[
\mathbf{u} = \mathbf{r} - \mathbf{r}_0,
\]

where \( \mathbf{r}_0 \) is the position of the point before the deformation, \( \mathbf{r} \) is the position of the point after the deformation.

The displacement vector may be written as \( \mathbf{u} = u_i \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k} \) or \( \mathbf{u} = u_i \mathbf{i} + u_j \mathbf{j} + u_k \mathbf{k} \), where \( i, j, k \) are the three unit vectors along the three coordinates.

The gradient of the displacement vector \( \mathbf{u} \) is the relative displacement tensor \( u_{ij} = \partial u_i/\partial x_j \).

We can decompose the tensor \( u_{ij} \) into two parts, symmetric \( \varepsilon_{ij} \) and skew-symmetric \( \omega_{ij} \) (refer to, for instance, [14, 15, 46]).

\[
\varepsilon_{ij} = \frac{1}{2} (u_{ij} + u_{ji}) + \frac{1}{2} (u_{ij} - u_{ji}) = \varepsilon_{ij} + \omega_{ij}
\]

\[
\varepsilon_{ij} = \frac{1}{2} (u_{ij} + u_{ji}), \quad \omega_{ij} = \frac{1}{2} (u_{ij} - u_{ji}),(9)
\]

The symmetric tensor \( \varepsilon_{ij} \) represents pure deformation of the body at a point and is called strain tensor (refer to, for instance, [14, 15, 46]). The matrix form and the indicial notation of strain tensor \( \varepsilon_{ij} \) are

\[
\varepsilon_{ij} = \begin{pmatrix}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} & \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\
\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} & \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} \right) \\
\frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) & \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} \right) & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial z}
\end{pmatrix}
\]

The strain-displacements equations can be obtained from Eq. (10)

\[
\varepsilon_{11} = \frac{\partial u}{\partial x}, \quad \varepsilon_{12} = \varepsilon_{21} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right),
\]

\[
\varepsilon_{22} = \frac{\partial v}{\partial y}, \quad \varepsilon_{23} = \varepsilon_{32} = \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right),
\]

\[
\varepsilon_{33} = \frac{\partial w}{\partial z}, \quad \varepsilon_{31} = \varepsilon_{13} = \frac{1}{2} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right).
\]

For convenience we can introduce the definitions of mean strain \( \varepsilon_m \) and strain deviator \( \varepsilon_{ij} \)

\[
\varepsilon_m = \frac{1}{3} (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}).
\]

\[
\varepsilon_{ij} = \varepsilon_{ij} - \varepsilon_m = \begin{pmatrix}
\varepsilon_{11} - \varepsilon_m & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} - \varepsilon_m & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} - \varepsilon_m
\end{pmatrix}
\]
When the aether is deformed, internal forces arise due to the deformation. The the indicial notation of the stress tensor $\sigma_{ij}$ is

$$
\sigma_{ij} = \begin{pmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{pmatrix}
$$

(14)

For convenience we can introduce the definitions of mean stress $\sigma_m$ and stress deviator $s_{ij}$ as

$$
\sigma_m = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}),
$$

(15)

$$
s_{ij} = \sigma_{ij} - \sigma_m = \begin{pmatrix}
\sigma_{11} - \sigma_m & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} - \sigma_m & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_m
\end{pmatrix}.
$$

(16)

Now let us turn to study the constitutive relation.

An elastic Hooke solid responds instantaneously with respect to a external stress. A Newtonian viscous fluid responds to a shear stress by a steady flow process.

In 19th century, people began to notice that some materials showed time dependence in their elastic response with respect to external stress. When materials like pitch, gum rubber, polymeric materials, hardened cement and even glass were loaded, an instantaneous elastic deformation was followed by a continuous slow flow or creep.

Now this time-dependent response is called viscoelasticity (refer to, for instance, [47, 48, 49]). Materials exhibits both instantaneous elastic elasticity and creep characteristics is called viscoelastic materials [48, 49]. Viscoelastic materials were studied long ago by Maxwell-liquid and are damped in the course of time. Maxwellian relaxation time.

Another definition of derivative operation fulfil the the principle of objectivity [48, 53]. Unfortunately, there is no unique definition of derivative operation fulfil the the principle of objectivity presently [48].

As an enlightening example, let us recall the description in [47] about a simple shear experiment. We suppose

$$
\frac{d\sigma_l}{dt} = \frac{\partial \sigma_l}{\partial t} t\frac{de_l}{dt} = \frac{\partial e_l}{\partial t},
$$

(22)

where $\sigma_l$ is the shear stress, $e_l$ is the shear strain.

Therefore, Eq. (21) becomes

$$
\frac{\partial e_l}{\partial t} = \frac{1}{2\eta}\sigma_l + \frac{1}{2G}\frac{\partial \sigma_l}{\partial t}.
$$

(23)

Integration of Eq. (23) gives

$$
\sigma_l = \epsilon \eta e_l \left(\sigma_0 + 2G \int_0^t \frac{de_l}{dt} e \eta \, dt\right).
$$

(24)

If the shear deformation is kept constant, i.e. $\partial e_l/\partial t = 0$, we have

$$
\sigma_l = \sigma_0 e^{-\frac{\sigma_l}{\eta G}}.
$$

(25)

Eq. (25) shows that the shear stresses remain in the Maxwell-liquid and are damped in the course of time.

We see that $\eta/G$ must have the dimension of time. Now let us introduce the following definition of Maxwellian relaxation time $\tau$

$$
\tau = \frac{\eta}{G}.
$$

(26)
Therefore, using Eq. (26), Eq. (21) becomes

$$ \frac{s_{ij}}{\tau} + \frac{ds_{ij}}{dt} = 2G \frac{de_{ij}}{dt}. \tag{27} $$

Now let us introduce the following hypothesis

**Hypothesis 9** Suppose the constitutive relation of the aether satisfies Eq. (27).

Now we can derive the equation of momentum conservation based on the above hypotheses [9].

Let $T$ be the characteristic time scale of an observer. When the observer’s time scale $T$ is the same order of the period of wave motion of light, the Maxwellian relaxation time $\tau$ is a relatively a large number. Thus, the first term of Eq. (27) may be neglected. Therefore, the observer concludes that the strain and the stress of the aether satisfies the generalized Hooke’s law.

The generalized Hooke’s law [14, 52]

$$ \sigma_{11} = \lambda \theta + 2G \varepsilon_{11} $$
$$ \sigma_{22} = \lambda \theta + 2G \varepsilon_{22} $$
$$ \sigma_{33} = \lambda \theta + 2G \varepsilon_{33} $$
$$ \sigma_{12} = \sigma_{21} = 2G \varepsilon_{12} = 2G \varepsilon_{21} $$
$$ \sigma_{13} = \sigma_{31} = 2G \varepsilon_{13} = 2G \varepsilon_{31} \tag{28} $$

where $\lambda = Y \nu / [(1 + \nu)(1 - 2 \nu)]$ is Lamé constant, $\theta$ is the volume change coefficient. The definition of $\theta$ is $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The following relationship are useful

$$ G = \frac{Y}{2(1 + \nu)}, \quad K = \frac{Y}{3(1 - 2\nu)}, \tag{29} $$

where $K$ is the volume modulus.

It is known that the equations of momentum conservation are (refer to, for instance, [14, 46, 52, 54, 55]),

$$ \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} + f_x = \rho \frac{\partial^2 u}{\partial t^2}, \tag{30} $$
$$ \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} + f_y = \rho \frac{\partial^2 v}{\partial t^2}, \tag{31} $$
$$ \frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} + f_z = \rho \frac{\partial^2 w}{\partial t^2}. \tag{32} $$

where $f_x, f_y$ and $f_z$ are the volume force density exerted on the aether.

The tensor form of the equations (30, 32) of momentum conservation can be written as

$$ \sigma_{ij,j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{33} $$

Noticing Eq. (28), Eqs. (30, 32) can also be written as

$$ 2G \left( \frac{\partial \varepsilon_{11}}{\partial x} + \frac{\partial \varepsilon_{12}}{\partial y} + \frac{\partial \varepsilon_{13}}{\partial z} + \frac{\partial \theta}{\partial x} + f_x = \rho \frac{\partial^2 u}{\partial t^2}, \tag{34} \right) $$
$$ 2G \left( \frac{\partial \varepsilon_{21}}{\partial x} + \frac{\partial \varepsilon_{22}}{\partial y} + \frac{\partial \varepsilon_{23}}{\partial z} + \frac{\partial \theta}{\partial y} + f_y = \rho \frac{\partial^2 v}{\partial t^2}, \tag{35} \right) $$
$$ 2G \left( \frac{\partial \varepsilon_{31}}{\partial x} + \frac{\partial \varepsilon_{32}}{\partial y} + \frac{\partial \varepsilon_{33}}{\partial z} + \frac{\partial \theta}{\partial z} + f_z = \rho \frac{\partial^2 w}{\partial t^2}. \tag{36} \right) $$

Using Eq. (11), Eqs. (34, 35, 36) can also be expressed by means of displacement $\mathbf{u}$

$$ G\nabla^2 \mathbf{u} + (G + \lambda) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + f_x = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \tag{37} $$

The vector form of the above equations (37) can be written as (refer to, for instance, [14, 46, 52, 54, 55]),

$$ G\nabla^2 \mathbf{u} + (G + \lambda) \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}. \tag{38} $$

When there are no body force in the aether, Eqs. (38) reduces to

$$ G\nabla^2 \mathbf{u} + (G + \lambda) \nabla (\nabla \cdot \mathbf{u}) = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}. \tag{39} $$

From a theorem of Long [45, 56], there exist a scalar function $\phi$ and a vector function $\mathbf{R}$ such that $\mathbf{u}$ is represented by

$$ \mathbf{u} = \nabla \psi + \nabla \times \mathbf{R} \tag{40} $$

and $\phi$ and $\mathbf{R}$ satisfy the following wave equations

$$ \nabla^2 \phi - \frac{1}{c_l^2} \frac{\partial^2 \phi}{\partial t^2} = 0, \tag{41} $$
$$ \nabla^2 \mathbf{R} - \frac{1}{c_l^2} \frac{\partial^2 \mathbf{R}}{\partial t^2} = 0, \tag{42} $$

where $c_l$ is the velocity of longitudinal waves, $c_l$ is the velocity of transverse waves. The definitions of these two elastic wave velocities are (refer to, for instance, [45, 46, 54, 55]),

$$ c_l = \sqrt{\frac{\lambda + 2G}{\rho}}, \quad c_t = \sqrt{\frac{G}{\rho}}. \tag{43} $$

$\psi$ and $\mathbf{R}$ is usually called the scalar displacement potential and the vector displacement potential respectively.
IV. DEFINITION OF POINT SOURCE AND SINK

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then these isolated points are called velocity singularities usually. Point sources and sinks are examples of velocity singularities.

Definition 10 Suppose there exists a singularity at point \( P_0 = (x_0, y_0, z_0) \) in a continuum. If the velocity field of the singularity at point \( P = (x, y, z) \) is

\[
\mathbf{v}(x, y, z, t) = \frac{Q}{4\pi r^2} \hat{r},
\]

where \( r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \), \( \hat{r} \) denotes the unit vector directed outward along the line from the singularity to the point \( P = (x, y, z) \), then we call this singularity a point source if \( Q > 0 \) or a point sink if \( Q < 0 \). \( Q \) is called the strength of the source or sink.

Suppose a static point source with strength \( Q \) locates at the origin \((0,0,0)\). In order to calculate the volume leaving the source per unit time, we may enclose the source with an arbitrary spherical surface \( S \) with radius \( a \). A calculation shows that

\[
\oiint_S \mathbf{u} \cdot d\mathbf{S} = \iint_S \frac{Q}{4\pi a^2} \hat{r} \cdot n dS = Q, \tag{45}
\]

where \( n \) denotes the unit vector directed outward along the line from the origin of the coordinates to the field point \((x, y, z)\). Equation \( \text{[45]} \) shows that the strength \( Q \) of a source or sink evaluates the volume of the fluid leaving or entering a control surface per unit time.

For the case of continuously distributed point sources or sinks, it is useful to introduce the following definition of volume density \( \rho_s \) of point sources or sinks,

\[
\rho_s = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V}, \tag{46}
\]

where \( \Delta V \) is a small volume, \( \Delta Q \) is the sum of strengths of all the point sources or sinks in the volume \( \Delta V \).

V. A POINT SOURCE AND SINK MODEL OF ELECTRIC CHARGES

The purpose of this section is to propose a point source and sink model of electric charges.

Let \( T \) be the characteristic time of a observer of a electric charge in the aether. We may suppose that the observer’s time scale \( T \) is very large compares to the Maxwellian relaxation time \( \tau \). So the Maxwellian relaxation time \( \tau \) is a relatively a small number and the stress deviator \( s_{ij} \) changes very slowly. Thus, the second term in the left of Eq.\( \text{[27]} \) may be neglected. According to this observer, the constitutive relation of the aether may be written as

\[
s_{ij} = 2\eta \frac{de_{ij}}{dt}, \tag{47}
\]

Therefore, the observer concludes that the aether behaves like the Newtonian-fluid under his time scale.

In order to compare fluid motion with electric fields, Maxwell introduced an analogy between sources or sinks and electric charges \( \text{[17]} \). Inspired by Maxwell, we may introduce the following

Hypothesis 11 Suppose all the electric charges in the universe are sources or sinks in the aether. We define a source as a negative electric charge. We define a sink as a positive electric charge. The electric charge quantity \( q_e \) of an electric charge is defined as

\[
q_e = -kQ \rho Q, \tag{48}
\]

where \( \rho \) is the density of the aether, \( Q \) is called the strength of the source or sink, \( k_Q \) is a positive dimensionless constant.

A calculation shows that the mass \( m \) of an electric charge is changing with time as

\[
\frac{dm}{dt} = -\rho Q = \frac{q_e}{k_Q}, \tag{49}
\]

where \( q_e \) is the electric charge quantity of the electric charge.

We may introduce a hypothesis that the masses of electric charges are changing so slowly relative to the time scaler of human beings that they can be treated as constants approximately.

For the case of continuously distributed electric charges, it is useful to introduce the following definition of volume density \( \rho_e \) of electric charges

\[
\rho_e = \lim_{\Delta V \to 0} \frac{\Delta q_e}{\Delta V}, \tag{50}
\]

where \( \Delta V \) is a small volume, \( \Delta q_e \) is the sum of strengths of all the electric charges in the volume \( \Delta V \).

From Eq.\( \text{[40]} \), Eq.\( \text{[48]} \) and Eq.\( \text{[50]} \), we have

\[
\rho_e = -k_Q \rho_s. \tag{51}
\]

VI. DERIVATION OF THE MAXWELL’S EQUATIONS IN VACUUM

The purpose of this section is to derive the Maxwell’s equations based on the above visco-elastic continuum model of vacuum and the singularity model of electric charges.
Now let us to derive the continuity equation of the aether from mass conservation. Consider an arbitrary volume $V$ bounded by a closed surface $S$ fixed in space. Suppose there are electric charges continuously distributed in the volume $V$. The total mass in volume $V$ is

$$M = \iiint_V \rho dV,$$  

(52)

where $\rho$ is the density of the aether.

The rate of increase of the total mass in volume $V$ is

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t} \iiint_V \rho dV.$$

(53)

Using the Ostrogradsky–Gauss theorem (refer to, for instance, [16, 57, 58, 59, 60, 61, 62]), the rate of mass outflow through the surface $S$ is

$$\iiint_S (\mathbf{u} \cdot \mathbf{n}) dS = \iiint_V \nabla \cdot (\rho \mathbf{u}) dV,$$

(54)

where $\mathbf{v}$ is the velocity field of the aether.

The definition of the velocity field $\mathbf{v}$ is

$$v_i = \frac{\partial u_i}{\partial t}, \text{ or } \mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}.$$

(55)

Using Eq.(51), the rate of mass created inside the volume $V$ is

$$\iiint_V \rho \mathbf{u} dV = \iiint_V -\frac{\rho_e}{k_Q} dV.$$

(56)

Now according to the principle of mass conservation, and making use of Eq.(52), Eq.(54) and Eq.(55), we have

$$\frac{\partial}{\partial t} \iiint_V \rho dV = \iiint_V -\frac{\rho_e}{k_Q} dV - \iiint_V \nabla \cdot (\rho \mathbf{v}) dV$$

(57)

Since the volume $V$ is arbitrary, from Eq.(57) we have

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = -\frac{\rho_e}{k_Q}.$$

(58)

According to Hypothesis 11 the aether is homogeneous, that is $\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = 0$. Thus, Eq.(58) becomes

$$\nabla \cdot \mathbf{v} = -\frac{\rho_e}{k_Q \rho}.$$

(59)

According to Hypothesis 11 and Eq.(49), the masses of positive electric charges are changing since the strength of a sink evaluates the volume of the aether entering the sink per unit time. Thus, the momentum of a volume element $\Delta V$ of the aether containing continuously distributed electric charges moving with an average speed $\mathbf{v}_e$ is changing. The increased momentum $\Delta \mathbf{P}$ of the volume element $\Delta V$ during a time interval $\Delta t$, that is,

$$\Delta \mathbf{P} = \rho(\mathbf{v}_s \Delta V \Delta t) \mathbf{v}_e = -\frac{\rho_e}{k_Q} \Delta V \Delta t \mathbf{v}_e$$

(60)

Therefore, the equation of momentum conservation Eq.(48) of the aether should be changed as

$$G \nabla^2 \mathbf{u} + (G + \lambda) \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}.$$

(61)

In order to simplify the Eq.(61), we may introduce an additional hypothesis as

**Hypothesis 12** We suppose that the aether is incompressible, that is $\theta = 0$.

From Hypothesis 12 we have

$$\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial v_x}{\partial y} + \frac{\partial w_x}{\partial z} = \theta = 0$$

(62)

Therefore, the vector form of the equation of momentum conservation Eq.(61) reduces to the following form

$$G \nabla^2 \mathbf{u} + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}.$$

(63)

According to the Stokes-Helmholtz resolution theorem (refer to, for instance, [45, 54]), which states that every sufficiently smooth vector field may be decomposed into irrotational and solenoidal parts, there exist a scalar function $\phi$ and a vector function $\mathbf{R}$ such that $\mathbf{u}$ is represented by

$$\mathbf{u} = \nabla \psi + \nabla \times \mathbf{R}.$$

(64)

Now let us introduce the definitions

$$\nabla \phi = k_E \frac{\partial}{\partial t}(\nabla \psi), \quad \mathbf{A} = k_E \nabla \times \mathbf{R},$$

(65)

$$\mathbf{E} = -k_E \frac{\partial \mathbf{u}}{\partial t}, \quad \mathbf{B} = k_E \nabla \times \mathbf{u},$$

(66)

where $\phi$ is the scalar electromagnetic potential, $\mathbf{A}$ is the vector electromagnetic potential, $\mathbf{E}$ is the electric field intensity, $\mathbf{B}$ is the magnetic induction, $k_E$ is a positive dimensionless constant.

From Eq.(61), Eq.(65) and Eq.(66), we have

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

(67)

and

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0.$$
Based on Eq. (65) and noticing
\[ \nabla^2 \mathbf{u} = \nabla (\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}), \]
\[ \nabla^2 \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}), \]
and \( \nabla \cdot \mathbf{u} = 0, \nabla \cdot \mathbf{A} = 0 \), we have
\[ k_E \nabla^2 \mathbf{u} = \nabla^2 \mathbf{A}. \] (72)

Therefore, using Eq. (72), Eq. (63) becomes
\[ \frac{G}{k_E} \nabla^2 \mathbf{A} + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \] (73)

Using Eq. (71), Eq. (73) becomes
\[ - \frac{G}{k_E} \nabla \times (\nabla \times \mathbf{A}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \] (74)

Now using Eq. (71), Eq. (74) becomes
\[ - \frac{G}{k_E} \nabla \times \mathbf{B} + \mathbf{f} = \frac{\rho_e \mathbf{v}_e}{k_Q}. \] (75)

It is natural to speculate that there are no other body forces or surface forces exerted on the aether. Thus, we have \( \mathbf{f} = 0 \). Therefore, Eq. (75) becomes
\[ \frac{k_Q G}{k_E} \nabla \times \mathbf{B} = \frac{k_Q \rho}{k_E} \frac{\partial \mathbf{E}}{\partial t} + \rho_e \mathbf{v}_e. \] (76)

Now let us introduce the following definitions
\[ \mathbf{j} = \rho_e \mathbf{v}_e, \quad \epsilon_0 = \frac{k_Q \rho}{k_E}, \quad \frac{1}{\mu_0} = \frac{k_Q G}{k_E}. \] (77)

Therefore, Eq. (76) becomes
\[ \frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \] (78)

Noticing Eq. (60) and Eq. (77), Eq. (59) becomes
\[ \nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}. \] (79)

Now we see that Eq. (88), Eq. (59), Eq. (78) and Eq. (79) coincide with the Maxwell’s equations (14).

VII. MECHANICAL INTERPRETATION OF THE ELECTROMAGNETIC WAVE

It is known that from the Maxwell’s equations (14), we can obtain the following equations (refer to, for instance, [1])
\[ \nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0} \nabla \rho_e + \mu_0 \frac{\partial \mathbf{j}}{\partial t}, \] (80)
\[ \nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{H}}{\partial t^2} = -\frac{1}{\mu_0} \nabla \times \mathbf{j}. \] (81)

Eq. (80) and Eq. (81) are the electromagnetic wave equations with sources in the aether. In the source free region where \( \rho_e = 0 \) and \( \mathbf{j} = 0 \), this equations reduce to the following equations
\[ \nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \] (82)
\[ \nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0. \] (83)

Eq. (82) and Eq. (83) are the electromagnetic wave equations without sources in the aether.

From Eq. (82), Eq. (83) and Eq. (77), we see that the velocity \( c_0 \) of electromagnetic waves in vacuum is
\[ c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{G}{\rho}}. \] (84)

From Eq. (83) and Eq. (84), we see that the velocity \( c_0 \) of electromagnetic waves in the vacuum is the same as the velocity \( c_t \) of the transverse elastic waves in the aether.

Now we may regard electromagnetic waves in the vacuum as the transverse waves in the aether. This idea was first introduced by Fresnel in 1821 [17].

VIII. CONCLUSION

It is an old idea that the universe may be filled with a kind of continuously distributed material which may be called aether. Following Stokes, we propose a visco-elastic constitutive relation of the aether. Following Maxwell, we propose a fluidic source and sink model of electric charges. Thus, the Maxwell’s equations in vacuum are derived by methods of continuum mechanics based on this continuum mechanical model of vacuum and the singularity model of electric charges.

IX. DISCUSSION

There exists some interesting theoretical, experimental and applied problems in the fields of continuum mechanics, the classical electrodynamics, quantum electrodynamics and other related fields involving this theory of electromagnetic phenomena. It is an interesting task to generalize this theory of electromagnetic phenomena in the static aether to the case of electromagnetic phenomena of moving bodies.

* Electronic address: wangxs1999@yahoo.com

[1] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1963).
