Shaping the relation between the mass of supermassive black holes and the velocity dispersion of galactic bulges

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ABSTRACT

I use the fact that the radiation emitted by the accretion disk of supermassive black hole can heat up the surrounding gas in the protogalaxy to achieve hydrostatic equilibrium during the galaxy formation. The correlation between the black hole mass $M_{BH}$ and velocity dispersion $\sigma$ thus naturally arises. The result generally agrees with empirical fittings from observational data, even with $M_{BH} \leq 10^6 M_\odot$. This model provides a clear picture on how the properties of the galactic supermassive black holes are connected with the kinetic properties of the galactic bulges.

Subject headings: Galaxies, galactic center, supermassive blackholes, velocity dispersion

1. Introduction

In the past decade, some observations have led to some tight relations between the central supermassive blackhole (SMBH) masses $M_{BH}$ and velocity dispersions $\sigma$ in the bulges of galaxies. These relations can be summarized as $\log(M_{BH}/M_\odot) = \beta \log(\sigma/200 \text{ km s}^{-1}) + \alpha$. For $10^6 M_\odot \leq M_{BH} \leq 10^9 M_\odot$, the values of $\alpha$ and $\beta$ have been estimated several times in the past 12 years: originally $(\alpha, \beta) = (8.08 \pm 0.08, 3.75 \pm 0.3)$ (Gebhardt et al. 2000) and $(8.14 \pm 1.3, 4.80 \pm 0.54)$ (Ferrarese and Merritt 2000), then $(8.13 \pm 0.06, 4.02 \pm 0.32)$ (Tremaine et al. 2002), and more recently $(8.28 \pm 0.05, 4.06 \pm 0.28)$ (Hu 2008), $(8.12 \pm 0.08, 4.24 \pm 0.41)$ (Gültekin et al. 2009), $(8.29 \pm 0.06, 5.12 \pm 0.36)$ (McConnell et al. 2011), and $(8.13 \pm 0.05, 5.13 \pm 0.34)$ (Graham et al. 2011). Generally speaking, empirical fittings show that $\alpha \approx 8$ and $\beta \approx 4 - 5$. These relations correspond to all morphological type galaxies. One can
separate the fittings into different groups such as the early-type and late-type or elliptical and spiral. For example, McConnell et al. (2011) obtain \((\alpha, \beta) = (8.38, 4.53)\) and \((7.97, 4.58)\) for the early-type and late-type galaxies respectively if they are fitted separately. The slopes are shallower than the combined one \((\beta = 5.12)\). Moreover, the slope \(\beta\) and the scatter of the \(M_{BH} - \sigma\) relation are still subject to debate, particularly at the low mass ends. Recently, Xiao et al. (2011) obtain a new \(M_{BH} - \sigma\) relation with low BH masses (below \(2 \times 10^6 M_\odot\)). They find a zero point \(\alpha = 7.68 \pm 0.08\) and slope \(\beta = 3.32 \pm 0.22\), which indicate \(\beta\) may be smaller for lower BH masses. Also, Wyithe (2006) obtained a better fit by using a log-quadratic form \(\log(M_{BH}/M_\odot) = \alpha + \beta \log(\sigma/200\ \text{km\ s}^{-1}) + \gamma'\log(\sigma/200\ \text{km\ s}^{-1})^2\) with \(\alpha = 8.05 \pm 0.06\), \(\beta = 4.2 \pm 0.37\) and \(\gamma' = 1.6 \pm 1.3\). Therefore, it is reasonable to doubt that the relation is not simply given by \(M_{BH} \propto \sigma^\beta\) with \(\beta\) just a constant for all galaxies.

The \(M_{BH} - \sigma\) relation has been derived by recent theoretical models (Silk and Rees, 1998; Adams et al., 2001; MacMillan and Henriksen, 2002; Robertson et al., 2005; Murray et al., 2005; King, 2005; McLaughlin et al., 2006; Power et al., 2011; Nayakshin et al., 2012). However, these models contain various assumptions and fail to explain the relations in the small SMBH mass regime \((\beta \approx 3.3)\). In this article, I present a model to get an exact \(M_{BH} - \sigma\) relation, which can explain the parameters \(\alpha\) and \(\beta\) in the empirical fitting in both small SMBH regime and apply to different types of galaxies. I use the fact that the strong radiation of the accretion disk of a SMBH can heat up the surrounding gas so that hydrostatic equilibrium of the latter is maintained. The cooling of the surrounding gas is mainly given by recombination, bremsstrahlung radiation and the adiabatic expansion of the gas. Without any other assumptions, the exact \(M_{BH} - \sigma\) relation is naturally obtained. In the following, I will first present the details of the model. Then I will fit our model with the data of \(\sigma\) and \(M_{BH}\) from 198 galaxies and show that it generally agrees with the empirical fitting.

2. The Accretion model of supermassive black hole and the \(M_{BH} - \sigma\) relation

It is commonly believed that all SMBHs accompany with accretion disks to emit high energy radiation during their formation. The luminosity of the disk is mainly come from the rest mass energy of the mass accretion. The accretion luminosity can be expressed as

\[
L_{\text{disk}} = \eta \dot{M} c^2, \tag{1}
\]

where \(0.05 \leq \eta \leq 0.4\) is the efficiency of the process. Let \(f_{Ed} = L_{\text{disk}}/L_{Ed}\), where \(L_{Ed} = 1.5 \times 10^{38} (M_{BH}/M_\odot) \text{ erg s}^{-1}\) is the Eddington limit of accretion, we have

\[
L_{\text{disk}} = 1.5 \times 10^{38} f_{Ed} \left(\frac{M_{BH}}{M_\odot}\right) \text{ erg s}^{-1}. \tag{2}
\]
The accretion disk of SMBH provides a large number of photons to heat up the surrounding gas in the protogalaxy during the galaxy formation. Assume that the power is mainly transmitted to the protogalaxy within a scale radius $R$ through radiation by compton scattering and photoionization. The optical depth of the gas is $\tau = n\sigma_{ph}R \ll 1$, where $n$ is the number density of the hot gas and $\sigma_{ph}$ is the effective cross section of the interaction of photons and hot gas particles, which is closed to the Thomson cross section $\sigma_{Thom}$ for zero metallicity. Therefore, the total power that can be transmitted to the protogalaxy is just $L_{disk}\tau$. In equilibrium, the heating rate is equal to the cooling rate by bremsstrahlung radiation $\Lambda_B$, recombination $\Lambda_R$, and adiabatic expansion $\Lambda_a$ (Katz et al. 1996):

$$L_{disk}\tau = \Lambda_B \sigma_{ph} R$$

\[ \Delta_B = 1.4 \times 10^{-27} \text{ erg cm}^3 \text{ s}^{-1}, \quad \Delta_B = 3.5 \times 10^{-26} \text{ erg cm}^3 \text{ s}^{-1}, \quad T, \quad p, \quad V, \quad \text{are the temperature, pressure and volume of the gas within} \ R, \ \text{respectively.} \]

The term $pdV/dt$ can be written as $pdV/dt \approx pV^{2/3}(\gamma kT/m_g)^{1/2}$ and $p = nkT$ (Muno et al. 2004; Chan and Chu 2008), where $\gamma \approx 5/3$ is the adiabatic index and $m_g$ is the mean mass of a gas particle. The Virial relation of the effective total mass of hot gas $M_g$ and $T$ within $R$ is given by (Sarazin 1988)

$$kT = f_1 \frac{G M_g m_g}{3R},$$

where $f_1$ is the virial constant. After the galactic bulge is formed, assuming spherical symmetry and by Virial theorem again, one can get

$$\sigma^2 = f_2 \frac{G M_g}{R},$$

where $f_2$ is another virial constant. From Eqs. (3), (4) and (5), and assuming $nV = M_g/m_g$, we get

$$L_{disk} = L_1 \sigma^3 + L_{21} \sigma^{2.6} (1 + L_{22} \sigma^2)^{-1} + L_3 \tau^{-1} \sigma^5,$$

where

$$L_1 = \frac{\Delta_B f_1^{1/2}}{3^{1/2} m_g^{1/2} j_2^{3/2} G k^{1/2} \sigma_{ph}},$$

$$L_{21} = \frac{\Delta_B f_1^{0.3}}{3^{0.3} m_g^{0.7} j_2^{3/3} G k^{0.3} \sigma_{ph}},$$

$$L_{22} = \frac{f_1 m_g}{3 \times 10^6 f_2 k},$$

$$L_3 = \frac{2.1 f_1^{3/2}}{j_2^{5/2} G}.$$
Take $f_1 \approx 1$ (isothermal distribution) and $f_2 \approx 1/5$ (Cappellari et al. 2006), and combine with Eq. (2), we have

$$\frac{M_{BH}}{10^8 M_\odot} = f_{Ed}^{-1} \left[ 6.2 \sigma_{200}^3 + 6.3 \sigma_{200}^{2.6} (1 + 10.4 \sigma_{200}^2)^{-1} + 0.37 \tau^{-1} \sigma_{200}^5 \right], \quad (11)$$

where $\sigma_{200} = \sigma/200$ km s$^{-1}$. Therefore, assuming $\tau \sim 0.005$, the last term dominates for $\sigma \geq 60$ km s$^{-1}$, which agrees with the observed range of $\beta$. The fitting parameters $f_{Ed}$ and $\tau$ are mainly controlled by the empirical fitting parameters $\alpha$ and $\beta$ of the observational data respectively. In Fig. 1, we get an empirical fit by using the data obtained from Greene and Ho (2006); Xiao et al. (2011); McConnell et al. (2011). The effective cross section due to metallicity may contribute to a factor of 2-3 in Eq. (11). By using the cross sections of some major metals (carbon, nitrogen, oxygen, silicon) calculated from Daltabuit and Cox (1972) and assuming metallicity of a protogalaxy is about $10^{-3}$ solar metallicity (Jappsen et al. 2009), the effective cross section is $2 \times 10^{-24}$ cm$^{-2}$, which is about $3 \sigma_{Thom}$. Also, the estimation of the $M_{BH}$ is not too reliable for $M_{BH} \leq 10^6 M_\odot$. Therefore, the fitting parameters are just an order of magnitude estimation. In Fig. 1, the functional form of Eq. (11) generally matches the observational data. The best fitted parameters are $f_{Ed} = 50$ and $\tau = 0.005$, with 4.9% rms error. By fitting with the form $\log(M_{BH}/M_\odot) = \beta \log \sigma_{200} + \alpha$, we can get $(\alpha, \beta) = (8.2, 4.5)$ with 5.1% rms error. Therefore, two functional forms can fit the data equally well. However, if we neglect the first two terms in the right hand side of Eq. (11), the best fitted line is $M_{BH}/10^8 M_\odot = 4.5 \sigma_{200}^5$ with 12% rms error. As mentioned above, the first two terms are significant when $M_{BH}$ or $\sigma$ is small. Therefore, the effective slope of the $\log M_{BH} - \log \sigma$ relation is shallower ($\beta < 5$). That means the effect of cooling by recombination and bremsstrahlung radiation should be considered especially in galaxies with low velocity dispersion.

Our result is consistent with the recent observations which indicate that many supermassive black holes may involve a long period of moderate super-Eddington accretion ($f_{Ed} \sim 10$) during their formation (Kawaguchi et al. 2004; Brian and Zhao 2004; Wang et al. 2008). On the other hand, the central number density in the Milky Way is about $0.1 - 0.5$ cm$^{-1}$ (Muno et al. 2004), which corresponds to $\tau \sim 0.001 - 0.005$ in the bulge. The best fitted $\tau$ is also consistent with the observational data.

3. Discussion

In this article, I present a new model to explain the $M_{BH} - \sigma$ relation in galaxies. The $M_{BH} - \sigma$ relation is not simply a power-law form $M_{BH} \propto \sigma^\beta$, but partially depends on $\sigma^3$ and $\sigma^5$. This exact form of the $M_{BH} - \sigma$ relation agrees with the recent observational
data, especially in the small $M_{BH}$ regime. In this model, we can obtain $3 \leq \beta \leq 5$ and $\alpha \approx 8$, which is consistent with the observational data of the grouped galaxies (small SMBH: $\alpha \approx 7.7$, $\beta \approx 3.3$; early-type: $\alpha \approx 8.4$, $\beta \approx 4.5$; late-type: $\alpha \approx 8.0$, $\beta \approx 4.6$) (Xiao et al. 2011; McConnell et al. 2011). In general, this model suggests that larger $\tau$ and $f_{Ed}$ result in smaller slope $\beta$ and normalization constant $\alpha$ in the relation. Thus, if similar galaxies have similar bulge structure and accretion disks, then the $M_{BH} - \sigma$ relation of this particular type may be tighter. It generally agrees with the observation that the $M_{BH} - \sigma$ relation in elliptical galaxies only is less scattered (Graham et al. 2011).

In this model, the evolution pattern of the supermassive black hole does not affect the function form of the relation. The only physics here is the energy balance of the gas between the heating by the radiation from accretion disk and the cooling by the free-free emission, recombination and the adiabatic expansion of the gas particles. If the black hole is still significantly accreting, the energy given out would be balanced by the cooling of gas, which gives the Eq. (11). When the black hole’s activity is switched off, the relation between the kinematic properties of the bulge and the $M_{BH}$ has already been established, which remains unchanged in Eq. (11). Therefore, the exact relation between $M_{BH}$ and $\sigma$ can definitely apply in both active and non-active galaxies.

All the parameters obtained ($f_{Ed} \sim 10$ and $\tau \sim 0.001$) are consistent with the theoretical estimation and observation. Generally, our result supports the moderate super-Eddington accretion during the SMBH formation. The variations of $f_{Ed}$ and $\tau$ within the groups of galaxies may result in the observed scatter in the $M_{BH} - \sigma$ fittings. All the above results arose from existing natural physical laws without any extra assumptions. This model provides a clear picture on how the properties of the galactic supermassive black holes are connected with the kinetic properties of the galactic bulges.

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Fig. 1.— $M_{BH,8}$ versus $\sigma$ for 198 galaxies obtained from Greene and Ho (2006); Xiao et al. (2011); McConnell et al. (2011), where $M_{BH,8} = M_{BH}/10^8 M_\odot$. The solid line is generated from Eq. (11) with $f_{Ed} = 50$ and $\tau = 0.005$. The dashed line is in the form $\log(M_{BH}/M_\odot) = \alpha + \beta \log(\sigma_{200})$ with $\alpha = 8.2$ and $\beta = 4.5$. The dotted line is $M_{BH,8} = 4.5\sigma_{200}^5$. 