Unfolded Description of $AdS_4$ Kerr Black Hole

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Abstract

It is shown that $AdS_4$ Kerr black hole is a solution of simple unfolded differential equations that form a deformation of the zero-curvature description of empty $AdS_4$ space-time. Our construction uses the Killing symmetries of the Kerr solution. All known and some new algebraic properties of the Kerr-Schild solution result from the obtained black hole unfolded system in the coordinate-independent way. Kerr-Schild type solutions of free equations in $AdS_4$ for massless fields of any spin associated with the proposed black hole unfolded system are found.

1 Introduction

Since its discovery in 1963, the celebrated Kerr metric [1] is still a hot research topic. Being of great physical significance, it exhibits deep mathematical beauty. In particular, the Kerr-Schild form [3] of the Kerr metric illustrates the remarkable fact that Kerr black hole (BH), being an exact solution of Einstein equations, also verifies its first order fluctuational part, i.e., the second order terms vanish independently. In other words, Kerr BH solves free spin-two equations. This property suggests the relevance of perturbative study of corrections to Kerr-like solutions in such field-theoretical extensions of gravity as string theory or higher-spin (HS) gauge theory (see [4] [5] [6] [7] [8] for reviews). The Kerr-Schild form allowed Myers and Perry to generalize the four-dimensional Kerr metric to any dimension [9].

BHs with non-zero cosmological constant have also been extensively studied in the context of $AdS/CFT$ correspondence [10] [11] [12] [13] where the (anti)-de Sitter

\footnote{For the history of question we refer to the recent publication by Roy Patric Kerr [2].}
geometry plays a distinguished role. Although the generalization of the Kerr solution that describes a rotating BH in AdS$_4$ was discovered by Carter [14] soon after Kerr’s paper [1], its higher dimensional generalization has been found only recently. A generalization of the Myers-Perry solution with non-zero cosmological term in five dimensions has been given by Hawking, Hunter and Taylor-Robinson in [15] and then extended to any dimension by Gibbons, Lu, Page and Pope in [16]. These higher dimensional BHs are shown [17] to possess the hidden symmetry associated with the Yano-Killing tensor [18] which is the characteristic feature of Petrov type D metrics.

Despite considerable progress in the construction of different BH metrics, including the charged rotating Kerr-Newman solution [19], the generalization of AdS$_4$ Taub-NUT to any dimension [20] and discovery of various supergravity BHs in the series of works by Chong, Cvetic, Gibbons, Lu, Pope [21, 22], a number of problems remain open. In particular, a generalization of the well-known four-dimensional Kerr-Newman solution to charged rotating BH solution in any dimension is available neither in flat nor in (A)dS case. This suggests that some more general approaches can be useful.

Usually an analysis in the BH background uses a particular coordinate system. The aim of this paper is to reformulate the AdS$_4$ Kerr BH in a coordinate-independent way using the formalism of the unfolded dynamics [23, 24] that operates with first-order covariant field equations. We show how the AdS$_4$ Kerr BH arises as a solution of the BH unfolded system (BHUS) associated with a certain BH Killing vector.

In general, unfolded equations generalize the Cartan-Maurer equations for Lie algebras by incorporating p-form gauge potentials. They provide a powerful tool for the study of partial differential equations. More precisely, consider, following [23], a set of differential p-forms $W^A(x)$ with $p \geq 0$ and generalized curvatures $R^A$ defined as

$$R^A = dW^A + F^A(W),$$

where $d = dx^n \frac{\partial}{\partial x^n}$ is the space-time exterior differential and some functions $F^A$ are built of wedge products of the differential forms $W^B$. The functions $F^A(W)$ subjected the generalized Jacobi identity

$$F^B \frac{\delta F^A}{\delta W^B} = 0$$

guaranty the generalized Bianchi identity $dR^A = R^B \frac{\delta F^A}{\delta W^B}$ which shows, in particular, that the differential equations for $W^A(x)$

$$R^A = 0$$

are consistent. In this case the equations (1.3) are called unfolded. Note that every unfolded system is associated with some solution of the generalized Jacobi identity (1.2) on the functions $F^A(W)$ built of exterior products of the differential forms
\( W^A \) which, in turn, defines a free differential algebra \([25, 26]\) (more precisely, its generalization to the case with zero-forms included).

The reformulation of partial differential equations in the unfolded form has a number of advantages. In particular, the first-order equations formulated in terms of exterior algebra are manifestly coordinate-independent and reconstruct a local solution in terms of its values at any given point of space-time modulo gauge invariance for \( p \)-forms with \( p \geq 1 \). (For the recent summary of the properties of unfolded dynamics we refer the reader to \([27]\).)

As an example of the unfolded system consider the Killing equation in the \( AdS_4 \) space written in the local frame

\[
DK_a = \kappa_{ba}h^b, \quad D\kappa_{ab} = 2\lambda^2 K_{[a}h_{b]}, \quad D^2 = R_{ab} = \lambda^2 h_a \wedge h_b, \quad (1.4)
\]

where \( K_a \) is some Killing vector, \( \kappa_{ab} = -\kappa_{ba} \), \( D \) is the Lorentz covariant differential, \( h_a \) is the \( AdS_4 \) vierbein one-form, \( -\lambda^2 \) is the cosmological constant, and \( a, b = 0, \ldots, 3 \). The \( AdS_4 \) geometry is described by the zero curvature equations

\[
d\Omega_{ab} + \Omega_a^c \wedge \Omega_{cb} - \lambda^2 h_a \wedge h_b = 0, \quad (1.5)
\]

\[
dh_a + \Omega_a^c \wedge h_c = 0, \quad (1.6)
\]

where \( \Omega_{ab} \) is the \( AdS_4 \) Lorentz connection one-form. The first of the equations (1.4) is obviously equivalent to the \( AdS_4 \) Killing equation as it implies \( D_a K_b + D_b K_a = 0 \), while the second one follows as a consequence of Bianchi identities for the space of constant curvature. The equations (1.4)–(1.6) represent the unfolded system for the set of forms \( W_A = (h_a, \Omega_{ab}, K_a, \kappa_{ab}) \).

In this paper we show that there exists a simple deformation of the equations (1.4)–(1.6) with the same set of forms that describes the \( AdS_4 \) Kerr BH in four dimensions with the BH mass being a free deformation parameter. Since the proposed BH Unfolded System (BHUS) expresses all derivatives of fields in terms of the fields themselves, all invariant relationships on the derivatives of BH geometric quantities become direct consequences of BHUS. In particular, it manifestly expresses the Petrov type \( D \) BH Weyl tensor in terms of the Killing two-form.

The \( AdS_4 \) Killing two-form that results from the exterior differential of the one-form dual to the Killing vector plays the central role in our consideration. The integrability properties of the \( AdS_4 \) BHUS imply, in particular, the existence of the sourceless Maxwell tensor and Yano-Killing tensor, both being proportional to the Killing two-form. Let us stress that, even though the Killing two-form in \( AdS_4 \) has non-vanishing divergence, it remains related to the BH Weyl tensor and to the sourceless Maxwell field.

Although the obtained results may have different applications, the original motivation for this study was due to the search of the Kerr BH solution in the nonlinear HS gauge theory which itself is formulated in the unfolded form and \( AdS \) space (see,
e.g., reviews [5, 8] and references therein). The proposed formulation looks particularly promising in that respect. We show how the proposed BHUS makes it possible to generate Kerr-Schild like solutions of the free bosonic HS field equations for all spins $s = 0, 1 \ldots$ in $AdS_4$. Remarkably, for $s = 1$ and $s = 2$ the obtained solutions satisfy in addition the minimally coupled to gravity equations. For other spins the covariantization of Fronsdal equations [28] in the Kerr-Schild metric leads to certain nonlinear terms which are explicitly calculated.

The rest of the paper is organized as follows. First, in Section 2 we recall the basic facts about the $AdS_4$ Kerr BH solution, its Kerr-Schild metric decomposition and its Killing symmetry. In Section 3 we reconsider the Cartan formalism most appropriate for our purpose. In Section 4 we present the BH unfolded system, build the null geodesic congruence using Killing projectors, find relations between the Killing two-form, Maxwell tensor and Yano-Killing tensor. In Section 5 we reproduce the Kerr solution from $AdS_4$ space via a Kerr-Schild type algebraic shift. Finally, in Section 6 we show how Kerr-Schild type solutions for free HS equations can be obtained. Section 7 contains summary and conclusions.

2 $AdS_4$ Kerr black hole

A rotating BH of mass $M$ in $AdS_4$ admits the Kerr-Schild form [14]

$$g_{mn}(X) = \eta_{mn}(X) + \frac{2M}{U(X)}k_m(X)k_n(X), \quad g^{mn}(X) = \eta^{mn}(X) - \frac{2M}{U(X)}k^m(X)k^n(X),$$

(2.1)

where $\eta_{mn}(X)$ ($m, n = 0, \ldots, 3$) is the background $AdS_4$ metric with negative cosmological constant $-\lambda^2$ and $k^m(X)$ defines the null geodesic congruence with respect to both full metric $g_{mn}(X)$ and the background one $\eta_{mn}(X)$

$$k^m k_m = 0, \quad k^m D_m k_n = k^m D_m k_n = 0.$$  

(2.2)

Here $D$ and $D$ are full and background covariant differentials, respectively.

A useful coordinate system has the background metric of the form [16]

$$\eta^{mn} = \frac{1-a^2\lambda^2}{(1+r^2\lambda^2)(1-\lambda^2x^2-a^2)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1-\lambda^2(x^2-a^2) & -\lambda^2 x y & -\lambda^2 x z \\ 0 & -\lambda^2 x y & 1-\lambda^2(y^2-a^2) & -\lambda^2 y z \\ 0 & -\lambda^2 x z & -\lambda^2 y z & 1-\lambda^2 z^2 \end{pmatrix},$$

(2.3)

2Throughout this paper we use units with $8\pi G = 1$. 

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where the radial coordinate $r(X)$ is defined through the ellipsoid of revolution equation
\[
\frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1
\]
and $a$ is a rotational parameter. The BH has angular momentum $J = Ma$.

Components of the Kerr-Schild vector $k^m(X)$ are
\[
k^0 = \frac{1}{1 + r^2 \lambda^2}, \quad k^1 = \frac{xr - ay}{r^2 + a^2}, \quad k^2 = \frac{yr + ax}{r^2 + a^2}, \quad k^3 = -\frac{z}{r},
\]
and
\[
U(X) = r + \frac{a^2 z^2}{r^3}.
\]

Direct calculation gives
\[
\frac{2}{U} = \frac{1}{Q} + \frac{1}{\bar{Q}} = -D_m k^m = -\mathcal{D}_m k^m,
\]
where
\[
Q = r - \frac{ia z}{r}, \quad \bar{Q} = r + \frac{ia z}{r}.
\]

Note that the metric (2.1) still provides a Kerr solution after the transformation
\[
\tau(a, x, y, z, t) = (-a, -x, -y, -z, t).
\]

In other words, the Kerr-Schild Ansatz also works for the vector $n^i = \tau(k^i)$.

One can check that the Maxwell tensor
\[
F = dA^{(1)} = dA^{(2)}, \quad A_{\mu}^{(1)} = \frac{k_{\mu}}{U}, \quad A_{\mu}^{(2)} = \frac{n_{\mu}}{U}
\]
verifies the sourceless Maxwell equations both in the $AdS_4$ and in the BH geometry
\[
D_m F_{mn} = \mathcal{D}_m F_{mn} = 0.
\]

The Kerr BH has two Killing vectors, time translation $\nu^m_t$ and rotation around $z$-axis $\nu^m_{\phi}$
\[
\nu^m_t = \frac{\partial}{\partial t} = (1, 0, 0, 0), \quad \nu^m_{\phi} = \frac{\partial}{\partial \phi} = (0, y, -x, 0).
\]

Let us introduce a Killing two-form $\kappa = d\nu^m$ as the exterior derivative of the one-form $dx^m K_m$ dual to some Killing vector $K^m$. This two-form will also be called Papapetrou field. It was originally introduced in [29], where it was shown to give rise to the sourceless Maxwell tensor in Ricci flat manifolds with isometries.

A particular linear combination $\nu^m$ of the Killing vectors
\[
\nu^m = (1, a\lambda^2 y, -a\lambda^2 x, 0)
\]
that satisfies the condition $k_m \nu^m = 1$, will be used later on for the definition of the Killing two-form associated with a Kerr BH in $AdS_4$. 5
3 Cartan formalism

Let $dx^m \Omega_m^{ab}$ be an antisymmetric one-form Lorentz connection and $dx^m h_m^a$ be a vierbein one-form. These can be identified with the gauge fields of the $AdS_4$ symmetry algebra $o(3,2)$. The corresponding $AdS_4$ curvatures have the form

$$R^{ab} = d \Omega^{ab} + \Omega^{ac} \wedge \Omega_c^b - \lambda^2 h^a \wedge h^b,$$

$$R^a = d h^a + \Omega^{ac} \wedge h_c,$$  \hspace{1cm} (3.1)

where $a,b,c = 0, \ldots, 3$ are Lorentz indices.

The zero-torsion condition $R^a = 0$ expresses algebraically the Lorentz connection $\Omega$ via derivatives of $h$. Then the $\lambda$-independent part of the curvature two-form (3.1) identifies with the Riemann tensor. Einstein equations imply that the Ricci tensor vanishes up to a constant trace part proportional to the cosmological constant. In other words, only those components of the tensor (3.1) may remain non-vanishing on-shell that belong to the Weyl tensor

$$R_{ab} = \frac{1}{2} h^c \wedge h^d C_{cdab},$$

(3.3)

where $C_{abcd}$ is the Weyl tensor in the local frame, $C_{abcd} = -C_{bacd} = -C_{abdc} = C_{cdab}$.

The analysis in four dimensions considerably simplifies in spinor notation. Vector notation are translated to the spinor one and vice versa with the help of Pauli $\sigma$-matrices. For example, for a Lorentz vector $U_a$ we have

$$U_{\alpha\dot{\alpha}} = (\sigma^a)_{\alpha\dot{\alpha}} U_a, \quad U_a = \frac{1}{2} (\sigma_a)^{\alpha\dot{\alpha}} U_{\alpha\dot{\alpha}}.$$  \hspace{1cm} (3.4)

Spinor indices are raised and lowered by the $sp(2)$ antisymmetric tensors $\varepsilon_{\alpha\beta}$ and $\varepsilon_{\dot{\alpha}\dot{\beta}}$:

$$\xi_\alpha = \xi^\beta \varepsilon_{\beta\alpha}, \quad \xi^\alpha = \varepsilon^{\alpha\beta} \xi_\beta, \quad \bar{\xi}_{\dot{\alpha}} = \bar{\xi}^\dot{\beta} \varepsilon_{\dot{\beta}\dot{\alpha}}, \quad \bar{\xi}^\dot{\alpha} = \varepsilon^\dot{\alpha}\dot{\beta} \bar{\xi}_{\dot{\beta}},$$

(3.5)

where $\varepsilon_{12} = \varepsilon^{12} = 1$, $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$, $\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$.

Lorentz irreducible spinor decompositions of the Maxwell and Weyl tensors $F_{ab}$ and $C_{abcd}$ read, respectively, as

$$F_{\alpha\dot{\alpha}\beta\dot{\beta}} = \varepsilon_{\alpha\beta} \bar{F}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}} F_{\alpha\beta}, \quad C_{\alpha\dot{\alpha}\beta\dot{\beta};\gamma\dot{\gamma}} = \varepsilon_{\alpha\beta}\varepsilon_{\gamma\dot{\gamma}} \bar{C}_{\dot{\alpha}\dot{\beta};\gamma\dot{\gamma}} + \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\gamma\dot{\gamma}} C_{\alpha\beta;\gamma\dot{\gamma}},$$

(3.6)

where $\bar{F}_{\alpha\dot{\alpha}}$ and $C_{\alpha\dot{\alpha}\alpha\dot{\alpha}}$ are totally symmetric multispinors.

To describe Killing symmetries let us rewrite the Cartan equations (3.1), (3.2) in spinor notation. Lorentz connection one-forms $\Omega_{\alpha\dot{\alpha}}$, $\bar{\Omega}_{\dot{\alpha}\dot{\alpha}}$ and vierbein one-form $h_{\alpha\dot{\alpha}}$
can be identified with the gauge fields of $sp(4) \sim o(3,2)$. Vacuum Einstein equations with cosmological constant acquire the form

\[
\mathcal{R}_{\alpha \alpha} = d\Omega_{\alpha \alpha} + \Omega_\alpha^\gamma \wedge \Omega_{\gamma \alpha} = \frac{\lambda^2}{2} H_{\alpha \alpha} + \frac{1}{8} H^{\gamma \gamma} C_{\gamma \gamma \alpha \alpha}, \tag{3.7}
\]

\[
\bar{\mathcal{R}}_{\dot{\alpha} \dot{\alpha}} = d\bar{\Omega}_{\dot{\alpha} \dot{\alpha}} + \bar{\Omega}_\dot{\alpha}^\dot{\gamma} \wedge \bar{\Omega}_{\dot{\gamma} \dot{\alpha}} = \frac{\lambda^2}{2} \bar{H}_{\dot{\alpha} \dot{alpha}} + \frac{1}{8} \bar{H}^{\dot{\gamma} \dot{\gamma}} \bar{C}_{\dot{\gamma} \dot{\gamma} \dot{\alpha} \dot{\alpha}}, \tag{3.8}
\]

\[
\mathcal{R}_{\alpha \dot{\alpha}} = dh_{\alpha \dot{\alpha}} + \frac{1}{2} \Omega_\alpha^{\gamma} \wedge h_{\dot{\gamma} \dot{\alpha}} + \frac{1}{2} \bar{\Omega}_{\dot{\alpha} \dot{\gamma}} \wedge h_{\dot{\alpha} \dot{\gamma}} = 0, \tag{3.9}
\]

where $\mathcal{R}_{\alpha \beta}$, $\bar{\mathcal{R}}_{\dot{\alpha} \dot{\beta}}$ are the components of the Lorentz curvature two-form

\[
D^2 \xi_{\alpha \dot{\alpha}} = \frac{1}{2} \mathcal{R}_\alpha^{\beta} \xi_{\beta \dot{\alpha}} + \frac{1}{2} \bar{\mathcal{R}}_{\dot{\alpha}}^{\dot{\beta}} \xi_{\alpha \dot{\beta}}, \tag{3.10}
\]

and

\[
H^{\alpha \alpha} = h^{\alpha \dot{\alpha}} \wedge h^{\dot{\alpha} \dot{\alpha}}, \quad \bar{H}^{\dot{\alpha} \dot{\alpha}} = h_{\dot{\alpha} \dot{\alpha}} \wedge h^{\dot{\alpha} \dot{\alpha}}. \tag{3.11}
\]

The Killing equation is

\[
D \mathcal{V}_{\alpha \dot{\alpha}} = \frac{1}{2} h^{\gamma \dot{\alpha}} \kappa_{\gamma \alpha} + \frac{1}{2} h_{\dot{\alpha}}^\beta \bar{\kappa}_{\dot{\gamma} \dot{\alpha}}, \tag{3.12}
\]

where $\kappa_{\alpha \alpha}$ and $\bar{\kappa}_{\dot{\alpha} \dot{\alpha}}$ just represent non-zero components of first derivatives of the Killing vector $\mathcal{V}_{\alpha \dot{\alpha}}$. In vector notation this gives $D_m \mathcal{V}_n = \kappa_{mn} = -\kappa_{nm}$, hence leading to $D_m \mathcal{V}_n = 0$.

Differentiation of (3.12) with the help of (3.7), (3.8) and (3.10) yields

\[
D \kappa_{\alpha \alpha} = \lambda^2 h_{\dot{\alpha}}^\dot{\gamma} \mathcal{V}_{\gamma \alpha} + \frac{1}{4} h^{\beta \dot{\alpha}} \mathcal{V}_\beta \mathcal{C}_{\beta \alpha \alpha}, \tag{3.13}
\]

\[
D \bar{\kappa}_{\dot{\alpha} \dot{\alpha}} = \lambda^2 h_{\dot{\alpha}}^\dot{\gamma} \mathcal{V}_{\gamma \dot{\alpha}} + \frac{1}{4} h^{\beta \dot{\alpha}} \mathcal{V}_\beta \mathcal{C}_{\beta \dot{\alpha} \dot{\alpha}}. \tag{3.14}
\]

with compatibility conditions $D^{\beta \dot{\alpha}} \mathcal{C}_{\beta \alpha \alpha} = 0$, $D^{\alpha \dot{\beta}} \bar{C}_{\alpha \dot{\beta} \dot{\alpha} \dot{\alpha}} = 0$. In case $C_{\alpha \alpha \alpha \alpha} = 0$, $\bar{C}_{\dot{\alpha} \dot{\alpha} \dot{\alpha} \dot{\alpha}} = 0$ the equations (3.12)–(3.14) describe an isometry of $AdS_4$.

### 4 Black hole unfolded equations

#### 4.1 Consistency condition for Killing unfolded system

Let us investigate solutions of Einstein equations with negative cosmological constant and the Weyl tensor of the form

\[
C_{\alpha \alpha \alpha \alpha} = f(X) \kappa_{\alpha \alpha} \kappa_{\alpha \alpha}, \tag{4.1}
\]
where \( f(X) \) is some function of space-time coordinates \( X \), and \( \kappa_{a\dot{a}} \) is a Killing two-form corresponding to some Killing vector \( \mathcal{V}_{a\dot{a}} \). So, we assume at least one isometry. The Weyl tensor (4.1) is of the type \( D \) by Petrov classification [30].

From the Killing equations (3.12)–(3.14) we derive the following unfolded equations

\[
\mathcal{D}\mathcal{V}_{a\dot{a}} = \frac{1}{2} h^{\gamma\dot{a}} \kappa_{\gamma a} + \frac{1}{2} h^{\alpha\dot{a}} \bar{\kappa}_{\gamma a}, \tag{4.2}
\]

\[
\mathcal{D}\kappa_{aa} = \lambda^2 h^{\beta\gamma} \mathcal{V}_{\beta\gamma} + \frac{f}{4} h^{\beta\gamma} \mathcal{V}_{\beta\gamma} \kappa_{\beta\beta a}, \tag{4.3}
\]

\[
\mathcal{D}\bar{\kappa}_{\dot{a}\dot{a}} = \lambda^2 h^{\gamma\dot{a}} \mathcal{V}_{\gamma\dot{a}} + \frac{f}{4} h^{\beta\gamma} \mathcal{V}_{\beta\gamma} \bar{\kappa}_{\beta\dot{a}\dot{a}}, \tag{4.4}
\]

\[
\mathcal{R}_{aa} = \frac{\lambda^2}{2} H_{aa} + \frac{f}{8} H^{\beta\gamma} \kappa_{\beta\gamma a}, \tag{4.5}
\]

\[
\mathcal{R}_{\dot{a}\dot{a}} = \frac{\lambda^2}{2} \bar{H}_{\dot{a}\dot{a}} + \frac{f}{8} \bar{H}^{\beta\gamma} \bar{\kappa}_{\beta\dot{a}\dot{a}}, \tag{4.6}
\]

\[
\mathcal{D}h_{a\dot{a}} = 0, \tag{4.7}
\]

\[
d\kappa^2 = \left( \frac{\lambda^2}{2} + \frac{2}{3} f^2 \kappa^2 \right) h^{\alpha\dot{a}} \mathcal{V}_{\alpha\dot{a}} \kappa_{a}, \tag{4.11}
\]

where \( H^{aa} \) and \( \bar{H}^{\dot{a}\dot{a}} \) are defined in (3.11), \( \mathcal{R}_{aa} \) and \( \mathcal{R}_{\dot{a}\dot{a}} \) are the curvatures (3.7) and (3.8), and we use notations \( \kappa_{a\beta} \kappa_{\beta\gamma} = \kappa^2 \varepsilon_{a\gamma} \), \( \kappa_{a\alpha a\alpha} = \kappa_{aa} \kappa_{aa} \).

The system (4.2)–(4.9) has the unfolded form (1.3) and is formally consistent. We call it BH unfolded system (BHUS). It is invariant under the following transformation

\[
\tau_{\mu} : (\mathcal{V}_{a\dot{a}}, \kappa_{aa}, \bar{\kappa}_{\dot{a}\dot{a}}, f, \bar{f}) \rightarrow (\mu \mathcal{V}_{a\dot{a}}, \mu \kappa_{aa}, \mu \bar{\kappa}_{\dot{a}\dot{a}}, \mu^{-2} f, \mu^{-2} \bar{f}), \tag{4.10}
\]

where \( \mu \) is a real parameter. As we show below, the transformation (4.10) with \( \mu = -1 \), that does not affect \( f \), describes the \( \tau \)-symmetry transformation (2.9).

The equations (4.8) and (4.9) result from the Bianchi identities for the curvatures (4.5), (4.6). It can be shown that \( f(X) = f(\kappa^2) \) as a consequence of (4.3). Indeed, we have

\[
df^2 = \left( \frac{1}{12} f^2 + \frac{5f^2}{2\kappa^2} \right) h^{\alpha\dot{a}} \mathcal{V}_{\alpha\dot{a}} \kappa_{a}. \tag{4.12}
\]

Comparing (4.8) and (4.11), we obtain

\[
\frac{d\kappa^2}{2\lambda^2 + \frac{5}{3} f \kappa^2} + \frac{df}{\frac{5}{6} f^2 + 5\lambda^2 \frac{f}{\kappa^2}} = 0. \tag{4.12}
\]

Its general solution is

\[
f = 6M \frac{\mathcal{G}^3}{\kappa^2}, \tag{4.13}
\]
where the complex parameter $\mathcal{M}$ appears as an integration constant and $\mathcal{G}$ is defined implicitly by

$$\mathcal{MG}^3 - \mathcal{G}\sqrt{-\kappa^2} = \lambda^2$$  \hspace{1cm} (4.14)

and satisfies

$$d\mathcal{G} = -\frac{\mathcal{G}^2}{2\sqrt{-\kappa^2}} h^{\alpha\dot{\alpha}} \gamma^\alpha_{\dot{\alpha} \alpha}. \hspace{1cm} (4.15)$$

Analogously,

$$\tilde{f} = 6\mathcal{M} \frac{\tilde{G}^3}{\kappa^2}, \quad \mathcal{MG}^3 - \mathcal{G}\sqrt{-\kappa^2} = \lambda^2. \hspace{1cm} (4.16)$$

### 4.2 Killing projectors

Let two pairs of mutually conjugated projectors $\Pi_{\alpha\beta}^\pm$ and $\bar{\Pi}_{\dot{\alpha}\dot{\beta}}^\pm$ have the form

$$\Pi_{\alpha\beta}^\pm = \frac{1}{2}(\epsilon_{\alpha\beta} \pm \frac{1}{\sqrt{-\kappa^2}} \kappa_{\alpha\beta}), \quad \bar{\Pi}_{\dot{\alpha}\dot{\beta}}^\pm = \frac{1}{2}(\epsilon_{\dot{\alpha}\dot{\beta}} \pm \frac{1}{\sqrt{-\kappa^2}} \bar{\kappa}_{\dot{\alpha}\dot{\beta}}). \hspace{1cm} (4.17)$$

They satisfy

$$\Pi_{\alpha\beta}^\pm \bar{\Pi}_{\dot{\alpha}\dot{\beta}}^\pm = \Pi_{\alpha\gamma}^\pm, \quad \Pi_{\alpha\beta}^\pm \bar{\Pi}_{\dot{\alpha}\dot{\gamma}}^\pm = 0, \quad \Pi_{\alpha\beta}^\pm \bar{\Pi}_{\dot{\beta}\dot{\gamma}}^\pm = \bar{\Pi}_{\dot{\alpha}\dot{\gamma}}^\pm, \quad \Pi_{\alpha\beta}^\pm \bar{\Pi}_{\dot{\beta}\dot{\gamma}}^\pm = 0. \hspace{1cm} (4.18)$$

From the definition (4.17) it follows that

$$\Pi_{\alpha\beta}^\pm = -\Pi_{\beta\alpha}^\mp, \quad \Pi_{\dot{\alpha}\dot{\beta}}^\pm = -\Pi_{\dot{\beta}\dot{\alpha}}^\mp. \hspace{1cm} (4.19)$$

Hereinafter we will focus on the holomorphic (i.e., undotted) sector of the BHUS. All relations in the antiholomorphic sector result by conjugation.

From (4.17) and (4.2) and (4.3) it follows that

$$\mathcal{D}\Pi_{\alpha\dot{\alpha}}^\pm = \pm \mathcal{G}(\Pi_{\alpha\beta}^+ \Pi_{\alpha\beta}^+ + \Pi_{\alpha\beta}^- \Pi_{\alpha\beta}^-) \gamma^{\beta\dot{\gamma}} \cdot \mathcal{h}^{\beta\dot{\gamma}}. \hspace{1cm} (4.20)$$

The projectors (4.17) split the two-dimensional (anti)holomorphic spinor space into the direct sum of two one-dimensional subspaces. For any $\xi_{\alpha}$ we set

$$\xi_{\alpha}^\pm = \Pi_{\alpha}^{\mp} \xi_{\beta}, \quad \xi_{\alpha}^+ + \xi_{\alpha}^- = \xi_{\alpha}, \hspace{1cm} (4.21)$$

so that $\Pi_{\alpha}^{\mp} \xi_{\beta}^\pm = 0$. This allows us to build light-like vectors with the aid of projectors.

Indeed, consider an arbitrary vector $U_{\alpha\dot{\alpha}}^-$. Using (4.17) define $U_{\alpha\dot{\alpha}}^\pm$ and $U_{\alpha\dot{\alpha}}^{\pm\mp}$ as

$$U_{\alpha\dot{\alpha}}^\pm = \Pi_{\alpha}^{\pm} \Pi_{\dot{\alpha}}^{\pm} U_{\beta\dot{\beta}}, \quad U_{\alpha\dot{\alpha}}^{\pm\mp} = \Pi_{\alpha}^{\mp} \Pi_{\dot{\alpha}}^{\pm} U_{\beta\dot{\beta}}, \quad U_{\alpha\dot{\alpha}}^{\mp\mp} = \Pi_{\alpha}^{\pm} \Pi_{\dot{\alpha}}^{\mp} U_{\beta\dot{\beta}}. \hspace{1cm} (4.22)$$

Obviously, $U_{\alpha\dot{\alpha}}^{\pm\alpha\dot{\gamma}} = 0$ and $U_{\alpha\dot{\alpha}}^{\pm\beta\dot{\alpha}} = 0$. Then $U_{\alpha\dot{\alpha}}^-$ can be cast into the form

$$U_{\alpha\dot{\alpha}}^- = \psi_{\alpha} \bar{\zeta}_{\dot{\alpha}}, \hspace{1cm} (4.23)$$

From this it follows that

$$U_{\alpha\dot{\alpha}}^\pm U_{\beta\dot{\beta}}^\pm = U_{\alpha\dot{\alpha}}^\pm U_{\beta\dot{\beta}}^\pm, \quad U_{\alpha\dot{\alpha}}^\pm U_{\beta\dot{\beta}}^- U_{\alpha\dot{\alpha}}^\pm = -\frac{(U^- U^+)}{(U^- U^+)} U_{\alpha\dot{\alpha}}^- U_{\beta\dot{\beta}}^+, \hspace{1cm} (4.24)$$

where $(AB) = A_{\alpha\dot{\alpha}} B_{\alpha\dot{\alpha}}$.
4.3 Kerr-Schild vector in BH unfolded system

Let us identify \( \kappa_{\alpha\mu} \) in (4.17) with \( \kappa_{\alpha\mu} \) in the BHUS and introduce two null vectors

\[
k_{\alpha\mu} = \frac{2}{(V^-V^+)} V^-_{\alpha\mu}, \quad n_{\alpha\mu} = \frac{2}{(V^-V^+)} V^+_{\alpha\mu}
\]

(4.25)

with the evident property \( \frac{1}{2} k_{\alpha\mu} V^+_{\mu\alpha} = \frac{1}{2} n_{\alpha\mu} V^+_{\mu\alpha} = 1 \). It is a matter of definition which of two vectors \( k_{\alpha\mu} \) or \( n_{\alpha\mu} \) to identify with a Kerr-Schild vector. Indeed, the equations (4.2)–(4.9) are invariant under the symmetry \( \tau^{-1} \) (4.10) which acts on \( k_{\alpha\mu} \) as \( \tau^{-1}(k_{\alpha\mu}) = -n_{\alpha\mu} \). Let us choose \( k_{\alpha\mu} \) as a Kerr-Schild vector.

Obviously,

\[
k_{\alpha\mu} k^{\beta\mu} = 0.
\]

(4.26)

From BHUS and projector properties the geodesity condition follows by straightforward calculation

\[
k_{\alpha\mu} D_{\alpha\mu} k^{\beta\mu} = 0.
\]

(4.27)

In addition, \( k_{\alpha\mu} \) is the eigenvector of the Papapetrou field \( \kappa_{\alpha\mu} \)

\[
\kappa_{\alpha\beta} k^{\beta\mu} = \sqrt{-\kappa^2} k_{\alpha\mu}
\]

(4.28)

and has the following properties as a consequence of (4.2), (4.3) and (4.15)

\[
D_{\alpha\mu} k^{\alpha\mu} = -2(\mathcal{G} + \bar{\mathcal{G}}), \quad k^{\alpha\mu} D_{\alpha\mu} \mathcal{G} = 2G^2,
\]

(4.29)

\[
D_{\alpha\mu}( (\mathcal{G} + \bar{\mathcal{G}}) k^{\alpha\mu} ) = -4G\bar{G}, \quad D_{\alpha\mu}( \mathcal{G} \bar{\mathcal{G}} k^{\alpha\mu} ) = 0,
\]

(4.30)

\[
k_{\alpha} D_{\alpha\mu} k^{\gamma\mu} = \mathcal{G} k^{\gamma} k_{\alpha\gamma}.
\]

(4.31)

The BHUS (4.2)–(4.9) admits a sourceless Maxwell tensor. Indeed, consider \( F_{\alpha\mu} \) of the form

\[
F_{\alpha\mu} = \frac{G^2}{\sqrt{-\kappa^2}} \kappa_{\alpha\mu}.
\]

(4.32)

That (4.32) solves Maxwell equations can be easily verified as follows. Using (4.14), (4.25) and (4.2), (4.3) one can make sure that

\[
F_{\alpha\mu} = \frac{1}{2} D_{\alpha\mu}( (\mathcal{G} + \bar{\mathcal{G}}) k_{\alpha}^{\mu})
\]

(4.33)

and \( F_{\alpha\mu} \) is \( \tau^{-1} \)-invariant. In other words, the vector-potential \( A_m = \frac{1}{2}(\mathcal{G} + \bar{\mathcal{G}}) k_m \) gives the Maxwell tensor field \( F = dA \).

Due to (4.32), the Weyl tensor (4.1) can be rewritten in the form

\[
C_{\alpha\mu\nu\lambda} = -\frac{6M}{\mathcal{G}} F_{\alpha\mu} F_{\alpha\nu}.
\]

(4.34)
The differentiation of (4.32) with the aid of (4.3) yields
\[ \mathcal{M} D F_{\alpha\alpha} = \frac{1}{4} h^{\beta\gamma} \gamma^{\beta} \phi_{\beta\alpha\alpha} . \] (4.35)

The Maxwell equations
\[ D_{\gamma\alpha} F_{\alpha}^{\gamma} = 0 , \quad D_{\alpha\gamma} F_{\alpha}^{\gamma} = 0 \] (4.36)
are now simple consequences of (4.35).

The Maxwell tensor (4.32) is related to Killing-Yano tensor via
\[ Y_{\alpha\alpha} = i G^{3} F_{\alpha\alpha} . \] (4.37)

Indeed, (4.37) satisfies
\[ D_{\alpha\gamma} Y_{\alpha\alpha} = 0 , \quad D_{\beta\alpha} Y_{\beta\alpha} + D_{\alpha\beta} \tilde{Y}_{\beta\alpha} = 0 , \] (4.38)
which implies the Yano-Killing equation \[ \mathcal{M} \] for \( Y_{mn} = -Y_{nm} \).

As follows from (4.13), (4.14), (4.32) and (4.37), \( Y_{\alpha\alpha} \) and \( F_{\alpha\alpha} \) constitute the Papapetrou two-form
\[ \kappa_{\alpha\alpha} = \mathcal{M} F_{\alpha\alpha} + i \lambda^{2} Y_{\alpha\alpha} = \left( \mathcal{M} - \frac{\lambda^{2}}{G^{3}} \right) F_{\alpha\alpha} . \] (4.39)

5 Kerr black hole unfolded system from \( AdS_{4} \)

As a preparation to the description of the Kerr-Schild Ansatz in BHUS, let us reformulate the \( AdS_{4} \) geometry in spinor notation. The equations (1.4)–(1.6) are equivalent to the following special case of the BHUS (4.2)–(4.9) with \( f = 0 \) that describes the vacuum solution with vanishing Weyl tensor, \( i.e. \), empty \( AdS_{4} \) space

\[ DV_{\alpha\gamma} = \frac{1}{2} h_{\gamma\delta} \kappa_{\delta\alpha} + \frac{1}{2} h_{\alpha} \gamma^{\delta} \tilde{\kappa}_{\delta\gamma} , \] (5.1)
\[ D\kappa_{\alpha\alpha} = \lambda^{2} h_{\alpha} \gamma^{\gamma} V_{\alpha}^{\gamma} , \] (5.2)
\[ D\tilde{\kappa}_{\alpha\alpha} = \lambda^{2} h_{\alpha} \gamma^{\gamma} V_{\gamma\alpha} , \] (5.3)
\[ Dh_{\alpha\alpha} = 0 , \] (5.4)
\[ R_{\alpha\alpha} = d\Omega_{\alpha\alpha} + \Omega_{\alpha}^{\beta} \wedge \Omega_{\beta\alpha} = \frac{\lambda^{2}}{2} h_{\alpha\alpha} \wedge h_{\alpha}^{\alpha} , \] (5.5)
\[ R_{\alpha\alpha} = d\Omega_{\alpha\alpha} + \Omega_{\alpha}^{\beta} \wedge \Omega_{\beta\alpha} = \frac{\lambda^{2}}{2} h_{\alpha\alpha} \wedge h_{\alpha}^{\alpha} , \] (5.6)
where $h_{\alpha\dot{\alpha}}$ is the $AdS_4$ vierbein, $\Omega_{\alpha\alpha}$ and $\bar{\Omega}_{\dot{\alpha}\dot{\alpha}}$ are components of Lorentz connection, $D$ is the background Lorentz differential, $V_{\alpha\dot{\alpha}}$ is an $AdS_4$ Killing vector and $R_{\alpha\alpha}$, $\bar{R}_{\dot{\alpha}\dot{\alpha}}$ are the components of $AdS_4$ curvature two-form: $D^2 \xi_{\alpha\dot{\alpha}} = \frac{1}{2} R_{\alpha\dot{\beta}}^{\quad \beta} \bar{\xi}_{\dot{\beta}\dot{\alpha}} + \frac{1}{2} \bar{R}_{\dot{\alpha}\dot{\beta}}^{\quad \beta} \bar{\xi}_{\dot{\alpha}\dot{\beta}}$.

The $AdS_4$ Killing projectors $\Pi_{\alpha\dot{\alpha}}^{\pm}$ single out the null vector $k_{0\alpha\dot{\alpha}}$ that defines $AdS_4$ null geodesic congruence

$$k^\alpha_{0} D_{\alpha\dot{\alpha}} k_{0\beta\dot{\alpha}} = 0 ,$$

where

$$k_{0\alpha\dot{\alpha}} = \frac{2}{(V-V^\prime)} V_{\alpha\dot{\alpha}} .$$

Analogously, the Maxwell field generated by the null vector $k_{0\alpha\dot{\alpha}}$ is defined by

$$F_{0\alpha\dot{\alpha}} = - \lambda^{-2} G^3_0 k_{0\alpha\dot{\alpha}} ,$$

with

$$G_0 = - \frac{\lambda^2}{\sqrt{-\kappa^2_0}} .$$

Now we are in a position to show that the $AdS_4$ Kerr BH is a solution of BHUS (4.2)–(4.9) resulting at real $\mathcal{M}$ from some algebraic field redefinition

$$(h_{\alpha\dot{\alpha}}, V_{\alpha\dot{\alpha}}, \kappa_{0\alpha\dot{\alpha}}) \rightarrow (h_{\alpha\dot{\alpha}}, V_{\alpha\dot{\alpha}}, \kappa_{\alpha\alpha}) ,$$

that, in fact, extends the Kerr-Schild Ansatz to the full BHUS, expressing the $AdS_4$ Kerr BH entirely in terms of the $AdS_4$ background unfolded system.

Let the sets of fields $(\Omega_{\alpha\alpha}, h_{\alpha\dot{\alpha}}, V_{\alpha\dot{\alpha}}, \kappa_{\alpha\alpha})$ and $(\Omega_{\alpha\alpha}, h_{\alpha\dot{\alpha}}, V_{\alpha\dot{\alpha}}, \kappa_{0\alpha\dot{\alpha}})$ be the Lorentz connections, vierbeins, Killing vectors and Papapetrou fields of the BHUS (4.2)–(4.9) and of the vacuum equations (5.1)–(5.6), respectively. The explicit map between these two unfolded system is

$$\Omega_{\alpha\dot{\alpha}|\gamma} = \Omega_{\alpha\dot{\alpha}|\gamma} - \frac{2M G^2_0}{\sqrt{-\kappa^2_0}} k_{0\gamma\alpha\dot{\alpha}} - \frac{M G_0}{2} \left( G_0 + \bar{G}_0 \right) k_\gamma^{\quad \alpha} k_\gamma^{\quad \dot{\alpha}} ,$$

$$h_{\alpha\dot{\alpha}} = h_{\alpha\dot{\alpha}} + \frac{\mathcal{M}}{4} \left( G_0 + \bar{G}_0 \right) k_\gamma^{\quad \beta} k_\gamma^{\quad \dot{\beta}} k_{0\alpha\dot{\alpha}} ,$$

$$V_{\alpha\dot{\alpha}} = V_{\alpha\dot{\alpha}} + \frac{\mathcal{M}}{2} \left( G_0 + \bar{G}_0 \right) k_{0\alpha\dot{\alpha}} ,$$

$$\kappa_{\alpha\beta} = (1 - M \lambda^{-2} G^3_0) \kappa_{0\alpha\dot{\alpha}} .$$

As a consequence of (5.13), the metric has the Kerr-Schild form

$$g_{mn} = \eta_{mn} + \mathcal{M} (G_0 + \bar{G}_0) k_{0m} k_{0n} .$$
Another consequence is the invariance of the null congruence (5.8) and function $G_0$ under the deformation

$$G_0 = G, \quad k_{0\alpha\dot{\alpha}} = k_{\alpha\dot{\alpha}}. \quad (5.17)$$

Note that (5.17) along with (4.33) entail the equality $F_{0\alpha\alpha} = F_{\alpha\alpha}$ and thus determine the Weyl tensor (4.34) in terms of $AdS_4$ geometry.

Let us sketch the main steps of the derivation of the relations (5.12)–(5.15) in some more detail. To see that (5.12)–(5.15) indeed relate two unfolded systems and to simplify the subsequent analysis, we start with the vacuum system (5.1)–(5.6) and suppose that the deformation is geodesic, i.e., (5.11) leaves the Kerr-Schild vector and the function $G_0$ invariant (5.17), checking this afterwards.

Let us look for the Kerr-Schild deformation of the background vierbein field

$$h_{\alpha\dot{\alpha}} = h_{\alpha\dot{\alpha}} + \frac{M}{4}(G + \bar{G})h^{\beta\dot{\beta}}k_{\beta\dot{\beta}}k_{\alpha\dot{\alpha}}. \quad (5.18)$$

Imposing the zero-torsion condition on the deformed vierbein $Dh_{\alpha\dot{\alpha}} = 0$, one obtains the deformed Lorentz connection $\Omega_{\alpha\alpha} = \Omega_{\alpha\alpha} + \omega_{\alpha\alpha}$ with

$$\omega_{\alpha\dot{\alpha}|\gamma\gamma} = -h^{\dot{i}\dot{\gamma}}D_{\gamma\gamma}h_{i\alpha\dot{\alpha}}. \quad (5.19)$$

It follows then that

$$\omega_{\alpha\dot{\alpha}|\gamma\gamma} = -\frac{M}{2}D_{\gamma\gamma}((G + \bar{G})k^{\gamma\dot{\gamma}}k_{\alpha\dot{\alpha}}). \quad (5.20)$$

Using (4.26) and (4.27) one observes that the background Lorentz derivative $D$ in (5.20) can be replaced by the deformed one $D$. Using (4.2), (4.3) and (4.15) we get (5.12).

Now it is straightforward to check that the Weyl tensor admits the following representation

$$C_{\alpha\alpha\alpha\alpha} = D_{\alpha\dot{\alpha}}\omega_{\alpha\dot{\alpha}|\alpha\alpha}. \quad (5.21)$$

Note that here $D$ can again be replaced by $D$, i.e., the terms quadratic in $\omega$ cancel.

The substitution of $h_{\alpha\dot{\alpha}}$ (5.18) and $\Omega_{\alpha\alpha} = \Omega_{\alpha\alpha} - \omega_{\alpha\alpha}$ into the $AdS_4$ unfolded system (5.1)–(5.6) yields BHUS (4.2)–(4.9) with the Killing vector and the Killing two-form transformed according to (5.14) and (5.15). Finally, it is not hard to verify that (5.17) is consistent with (5.14), (5.15).

Thus, the BHUS that describes the $AdS_4$ BH geometry with non-trivial Weyl tensor results from the algebraic field redefinition of the $AdS_4$ vacuum vierbein and connection, providing Kerr-Schild type vacuum solution of Einstein equations.

The identification with the standard BH description of Section 2 requires

$$G = \frac{1}{Q}, \quad M = M, \quad (5.22)$$
with $Q$ (2.8) and the Killing vector $V^i$ (2.13). Then the Kerr-Schild vector $k^i$ determined from (5.8) coincides with (2.5), whereas $n^i$, which arises with the aid of the discrete symmetry $\tau_{-1}$, coincides with one from Section 2. Moreover, the metric (5.16) is identical to standard Kerr-Schild representation (2.1).

The Schwarzschild case with $a = 0$ is singled out by the following additional condition

$$V^\alpha_+ = V^\alpha_- = 0.$$  

(5.23)

### 6 Black hole massless fields

Let us show how the BHUS reproduces solutions to free massless field equations in $AdS_4$ for all integer spins via a Kerr-Schild type algebraic field redefinition.

Consider the traceless symmetric tensor

$$\varphi_{mm} = \frac{1}{2} (G + \bar{G}) k_m k_m.$$  

(6.1)

Taking into account (4.2), (4.3) and (4.5), the straightforward calculation yields

$$D^i D_i \varphi_{mn} - 2D^i D_i \varphi_{mn} = -6\lambda^2 \varphi_{mm}.$$  

(6.2)

In this way we obtain Einstein equations for the Kerr-Schild decomposition (5.16), or spin-2 free field equations. Note that one can use $D$ instead of $\bar{D}$ in (6.2).

The Maxwell case is analogous. It is a simple consequence of (4.33) that the vector field $\varphi_m = \frac{1}{2} (G + \bar{G}) k_m$ satisfies (4.36).

Things change for the scalar $\varphi = \frac{1}{2} (G + \bar{G})$. Using (4.2), (4.3), (4.15) and (4.36) one finds

$$D^a D_b \varphi_{\alpha a} = -4G^2 \sqrt{-k^2}.$$  

(6.3)

The substitution of (4.14) yields

$$D^a D_b \varphi_{\alpha a} = 4\lambda^2 \varphi - 4M \mathcal{D}^4.$$  

(6.4)

Thus, it reduces to $D^a D_{\alpha \dot{a}} \varphi = 4\lambda^2 \varphi$ only in the $AdS_4$ limit with $M = 0$, $\mathcal{D} = D$. This equation describes propagation of a massless scalar in $AdS_4$. Using differential properties of $k_{\alpha \dot{a}}$, one can obtain

$$D_m(k^m k^n D_n (G + \bar{G})^2) = 4(G^4 + \bar{G}^4).$$  

(6.5)

Hence, from the equation (6.4) it follows

$$D^m D_m \varphi = 2\lambda^2 \varphi - 2M D_m (\varphi^{mn} D_n \varphi).$$  

(6.6)

The second term on the r.h.s. represents the nonlinear correction to a massless scalar propagating in Kerr-Schild background. Thus, the fields $\varphi, \varphi_m, \varphi_{mm}$ verify the free
massless equations in $AdS_4$ space for spins 0, 1 and 2, respectively, \cite{28, 31}. Using BHUS one can show that\footnote{A number in parenthesis next to an index denotes a number of symmetrized indices, e.g., $\phi_{m(s)} = \phi_{m_1 \ldots m_s}$.}

$$\varphi_{m(s)} = \frac{1}{2}(G + \bar{G})k_m \ldots k_m$$

(6.7)

gives a Kerr-Schild solution for a massless integer spin–$s$ equation in $AdS_4$ background

$$D^n D_n \varphi_{m(s)} - sD_n D_m \varphi^n_{m(s-1)} = -2(s - 1)(s + 1)\lambda^2 \varphi_{m(s)} .$$

(6.8)

In the Kerr-Schild background (6.8) changes to

$$D^n D_n \varphi_{m(s)} - sD_n D_m \varphi^n_{m(s-1)} = -2(s - 1)(s+1)\lambda^2 \varphi_{m(s)} - \mathcal{M}(s-1)(s-2)D_n(\varphi^{nr}D_r \varphi_{m(s)}) .$$

(6.9)

Note that the interaction term in (6.9) vanishes only for $s = 1$ and $s = 2$.

This result suggests that the Kerr-Schild Ansatz should admit an extension to the nonlinear equations of 4d massless fields of all spins (see \cite{5} and references therein).

7 Summary and discussion

We have shown that $AdS_4$ Kerr BH admits a simple description in terms of unfolded field equations generated by a Killing vector of the background $AdS_4$ space. We considered the case of four dimensions using the spinor formalism.

Our aim was to find the description of the $AdS_4$ Kerr BH that does not refer to a particular coordinate system. Such a description is given in terms of a coordinate-independent unfolded system of differential equations that encode all properties of the $AdS_4$ Kerr BH.

The proposed approach allowed us to shown how seemingly different structures resided in the 4d BH such as, e.g., the existence of Yano-Killing tensor, the decomposition of the curvature tensor into Maxwell tensor and the Papapetrou field, naturally arise from the $AdS_4$ geometry with a distinguished Killing vector. An interesting direction for the further study is to explore the BH unfolded system on its own right, including, in particular, the case of complex deformation parameter $\mathcal{M}$ which, as we expect, should correspond to the Taub-NUT case. More generally, such an analysis can provide a powerful technique for identification of BH solutions via the algebraic properties of the corresponding solutions.

We believe that the proposed construction allows an extension to the nonlinear HS field equations also formulated in the unfolded form (see \cite{5, 7, 8} for reviews). It is worth to note that the proposed BH unfolded system allows us to build solutions not only to linearized Einstein equations which reduce in this case to Pauli-Fierz equations
but also to Fronsdal equations of all massless integer spins propagating in $AdS_4$. This fact looks particularly encouraging from the HS gauge theory perspective. Indeed, this implies that $AdS_4$ Kerr BH naturally fits linearized HS gauge theory through an algebraic field redefinition of the vacuum solution. The analysis of the BH solution in the nonlinear HS gauge theory is, however, beyond the scope of this paper and will be given elsewhere.

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