Precise Relative Orbit Determination of Twin GRACE Satellites

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Abstract When formation flying spacecrafts are used as platform to gain earth oriented observation, precise baselines between these spacecrafts are always essential. Gravity recovery and climate experiment (GRACE) mission is aimed at mapping the global gravity field and its variation. Accurate baseline of GRACE satellites is necessary for the gravity field modeling. The determination of kinematic and reduced dynamic relative orbits of twin satellites has been studied in this paper, and an accuracy of 2 mm for dynamic relative orbits and 5 mm for kinematic ones can be obtained, whereby most of the double difference onboard GPS ambiguities are resolved.

Keywords GRACE; gravity field modeling; formation flying spacecraft; relative orbit determination

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Introduction

In accordance with high precision and continuous requirements in time and position reference for their geodetic application, Low Earth Orbit (LEO) satellite for precise orbit determination and navigation are equipped with more dual-frequency GPS (global positioning system) receivers. Spacecraft formation flying is currently considered as a key technology for advanced geodetic application in oriented space missions. Compared to large individual spacecraft, the distribution of sensor systems among multiple platforms offers improved flexibility and redundancy, shorter times to mission, and the prospect of being more cost effective.\[1\]

The gravity recovery and climate experiment (GRACE) was launched on 17 March 2002, with objectives including the mapping of the global gravity field and its variation for a nominal mission of 5 years. The GRACE mission consists of two identical formation flying spacecraft in a near polar, near circular orbit with an initial altitude about 500 km.\[2\] The onboard payload of twin GRACE satellite consists primarily of the GPS receiver and the K band ranging (KBR) system. The former can be used for precise orbit determination, while the second one, with a precision of better than 10 μm, can be a useful validation for the relative orbit determination of the twin GRACE satellites with a nominal separation of 220 km.

The objective of this contribution is to develop a strategy for high precision relative orbit determination of formation flying satellites, using dual fre-
quency onboard GPS observations. This method can be used not only in GRACE satellite mission but also in the near future scientific satellite mission like the planned synthetic aperture radar (SAR) interferometry formation consisting of the TerraSAR-X and TanDEM-X satellites for example. In order to obtain a vertical mapping accuracy of 2 m or better, which can only be achieved if the relative distance between both spacecraft is known within 2 mm (1-dimensional RMS)\(^2\). The method developed has been realized in the PANDA (position and navigation data analyst) software, which was developed at GNSS (Global Navigation Satellite System) Research Center of Wuhan University, China\(^4\) and was validated using 101-days GRACE data in this paper.

1 Processing strategy

1.1 Observation functions

When GPS measurements are used to determine precise orbit of LEO satellites or long baseline of ground stations, in order to absorb the ionosphere effect, the well-known ionosphere-free observation is generally used:

\[
\begin{align*}
J_{ck}^i &= \frac{f_1^2}{f_1^2 - f_2^2} P_{ik}^i - \frac{f_2^2}{f_1^2 - f_2^2} P_{ik}^2 \\
&= \rho_i^i + c(\Delta \delta_{ik} - \Delta \delta_i^i) + \epsilon_{\delta,i} \\
L_{ck}^i &= \frac{f_1^2}{f_1^2 - f_2^2} L_{ik}^i - \frac{f_2^2}{f_1^2 - f_2^2} L_{ik}^2 = \rho_i^i + c(\Delta \delta_{ik} - \Delta \delta_i^i) \\
&+ \lambda_i B_i^i + \epsilon_{\lambda,i} \\
\rho_i^i &= \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}
\end{align*}
\]

Where \( P_{ik}^i \) is the ionosphere-free combination of code observation \( P_{ik}^i \) with frequency \( f_i \) and \( P_{ik}^2 \) with frequency \( f_2 \); \( L_{ik}^i \) is the ionosphere-free combination of phase observation \( L_{ik}^i \) and \( L_{ik}^2 \); \( c \) is the speed of light in vacuum; \( \Delta \delta_{ik} \) is the receiver clock offset and \( \Delta \delta_i^i \) is the clock offset of GPS satellite; \( B_i^i \) is the ambiguity of ionosphere-free combination; \( \rho_i^i \) is the corrected geometric range between GPS satellite \( j \) \((x_j^i, y_j^i, z_j^i)\) and receiver \( k \) \((x_i, y_i, z_i)\), and the corrections include relativistic effect and geometric phase center, etc.

1.2 Data cleaning

High quality of the onboard GPS data is necessary for POD (precision orbit determination) of LEO satellite. Therefore, proper methods for data screening have to be applied to detect outliers and bad measurement. In this research, the data cleaning of the GPS observations consists of four steps.

Step 1 The raw RINEX data are cleaned using automatic editing strategy of Blewitt (1990) by fitting ionosphere and wide-lane observation at the preprocessing stage. With the help of ionosphere-free phase fitting, we successfully rescue data with high ionosphere effect or bad quality pseudorange. Cycle slips are repaired or flagged if they cannot be repaired, and bad observations are removed.

Step 2 Discrete positions of the LEOs were calculated through single point positioning method using only pseudorange measurement. Preliminary dynamic orbits were obtained by fitting these discrete positions. The force models used include Gravity field model EIGEN-CG03C up to 120 degree, N-body force model using DE405 ephemeris of Sun, Moon, and all the planets, solid earth tides, pole tide, and relativistic effect model according to IERS standards, ocean tides CSR 2.0, atmospheric drag and lift\(^6\) in DTM94, solar radiation force integrated over all surface areas of the satellite, and 1-cpr empirical acceleration in three directions. There are 13 parameters that are estimated per arc(24h), which include six initial orbit parameters (position and velocity), one Cd parameter of atmospheric drag, and six empirical 1-cpr acceleration parameters.

Step 3 GPS data were screened epoch by epoch with the preliminary dynamic LEO orbits fixed. Since GPS orbit and clock parameters are fixed, so the clock offset of receiver \( k \) can be estimated with the set of \( n \geq 2 \) observations:

\[
c\delta t_i(t) = \frac{1}{n} \sum_{j=1}^{n} (P_{ij}^i(t) - (\rho_i^i(t) - c\delta t^i(t)))
\]

The residuals of code observation at the epoch \( t \) are calculated with function

\[
res_{\delta}^i = P_{ij}^i(t) - (\rho_i^i(t) + c\delta t_i(t) - c\delta t^i(t))
\]

If the standard deviation of these residuals exceeds a predefined threshold, the epoch is regarded as a
problem epoch, and the code observation that contributes the dominating error is identified as an outlier. If necessary, this process is repeated to reject multiple outliers at the same epoch. The skips in phase observations are detected by analyzing residuals of the epoch-differenced observations:

\[
\text{res}_{k,i} = L^i_k(t_{i-1},t_i) - (\rho_k^i(t_{i-1},t_i) + c\delta t_k(t_{i-1},t_i))
\]  

(4)

Where the epoch-differenced clock offset of receiver \(k\) is estimated with phase observation as

\[
c\delta t_k(t_{i-1},t_i) = \frac{1}{N} \sum_{n=1}^{N} (L^i_n(t_{i-1},t_i) - (\rho_k^i(t_{i-1},t_i) - c\delta t_k(t_{i-1},t_i)))
\]  

(5)

Using the similar detecting processing with code observations, the skips of phase observation can be identified with a smaller predefined threshold. A data cleaning approach similar to Step 3 was applied by Kroes et al.\(^{[1]}\), which was conformed effective and stable.

**Step 4** Reduce-dynamic orbit determination is applied in this stage using both code and phase observation. The residuals screening procedure is used to analyze postresiduals of reduced-dynamic orbit determination for detecting small skips and outliers that cannot be found in above stages. The force models used in this step is the same ones as used in Step 2. The parameters estimated per arc include six initial orbit parameters (position and velocity), four \(C_d\) parameters of atmospheric drag (one per 6 hours), and 96 empirical 1-cpr acceleration parameters (six per 1.5 hours).

After Step 4, Step 3 can be done again by fixing the obtained reduced-dynamic orbits instead of preliminary dynamic orbits. The thresholds for code and phase differenced observation are changed to very small ones since the reduced dynamic orbits are much more precise than the preliminary dynamic orbits.

### 1.3 Ambiguity fixing between two GRACE satellites

In order to obtain better baseline of two GRACE satellites, an effective ambiguity method, which has been applied in global GPS network data processing,\(^{[7]}\) is adopted to resolve the ambiguity between twin GRACE satellites. In this approach, real-valued zero-difference ambiguities of the ionosphere-free carrier-phase linear combination are estimated prior to ambiguity fixing stage. A matrix \(D\), similar to the DD-operator, can be defined to map the zero-difference ambiguities into a maximum set of independent DD-ambiguities. For example, ambiguity between two receivers, \(k\) and \(l\), and two satellites \(i\) and \(j\) can be translated as

\[
B_{ik,j} = \left[ \begin{array}{ccc} 1 & -1 & 1 \\ B_{i,k} & B_{j,k} & B_{i,l} & B_{j,l} \end{array} \right]
\]  

(6)

With the \(D\)-matrix, the normal equation for the zero-difference ambiguities is transformed into an equivalent one for the DD-ambiguities. The ionosphere-free DD-ambiguities are expressed as combination of wide-lane and narrow-lane ambiguities:

\[
B_{w,k,j} = \frac{f_1}{f_1 + f_2} B_{ik,j} + \frac{f_2}{f_1 - f_2} B_{il,j}
\]  

(7)

Where \(B_{w,k,j}\) and \(B_{n,k,j}\) are wide-lane and narrow-lane ambiguities, respectively.

Melbourne-Wubbena method\(^{[8]}\) is used to fix the wide-lane ambiguities. The average and STD of wide-line ambiguities are estimated using the cleaned code and phase observation:

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} \, dt
\]  

(9)

where \(B\) and \(\sigma\) is the estimation and STD of wide-line ambiguity, and \(N\) is the nearest integer of \(B\).

For a given confidence level \(\alpha\), e.g., 0.1% as usual,\(^{[9]}\) the ambiguity can be fixed to the nearest integer if \(P\) is larger than \(1-\alpha\), otherwise, it should remain unfixed.

With fixed wide-line ambiguity, the narrow-lane and the related STD can be derived according to Eq. (7) as
\[ B_{nk}^{ij} = \frac{f_1 + f_2}{f_1} B_{nk}^{ij} - \frac{f_2}{f_1} N_{nk}^{ij} \]  
\[ \sigma_{B_{nk}^{ij}} = \frac{f_1 + f_2}{f_1} \sigma_{B_{nk}^{ij}} \]

Where \( N_{nk}^{ij} \) is the fixed integer value of the wide-lane ambiguity; \( B_{nk}^{ij} \) and \( \sigma_{B_{nk}^{ij}} \) are the estimated value, and the STD of ionosphere-free ambiguity was calculated based on the real-valued solution. The fixing decision for narrow-lane ambiguity can be made with the same way as for the wide-lane (Eq. 9).

If both the wide- and narrow-lane are fixed, the ionosphere-free double ambiguity can be reconstructed with the integer ambiguities with Eq.(7). Then, a constraint to the related ZD-ambiguities can be built up with Eq.(6) as

\[ \frac{f_1}{f_1 + f_2} N_{nk}^{ij} + \frac{f_2}{f_1} N_{nk}^{ij} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & \end{bmatrix} \begin{bmatrix} B_{ki}^i \\ B_{ki}^j \\ B_{ki}^k \\ B_{ki}^l \end{bmatrix} \]  

The constraint is imposed to the normal equation with original float ZD-ambiguities to get fixed solution.[7]

2 Result and discussion

Approximately 100 days of GPS data (9, July–19 October 2003) were processed. The final IGS ephemeris products provides GPS orbits and clock offsets data in SP3 formats with a reported accuracy of better than 5 cm and 0.1 ns (IGSCB 2005). IGS orbit products with 15 min interval can be interpolated to any interval with high quality, while the 5 min clock products cannot be interpolated precisely because of their random character.[10] For improving the accuracy, high-rate (30 s) clock solutions of CODE are used.

Both reduced dynamic and kinematic orbit determination are done in this contribution. In this paper, the orbits of two satellites are obtained simultaneously, so the ambiguity fixing constraint is used to improve the orbits of two satellites. This method is different from the other approach as always done, with which then relative baseline are determined with one reduce dynamic orbit fixed.[1] In order to assess the absolute and relative precision of the calculated two GRACE orbits, SLR and KBR data are used.

For the SLR validation, first of all, tropospheric mapping functions, according to Marini and Murray, are used to correct the SLR measurements. An elevation cut-off angle of 10 degrees has been applied to the SLR ground stations. Second, orbits are interpolated to obtain positions at the epochs of SLR normal points using 11-degree Lagrange interpolation method. Therefore, the SLR residuals can be obtained. The residuals larger than 20 cm are excluded from statistics. Table 1 gives an overview of the SLR statistics for the 101 days period. The total RMS is approximately 3.3 cm for kinematic and 2.8 cm for reduced dynamic GRACE orbits, which are comparative and even better than the products of TUM and JPL covering the same period.[1]

Table 1 Mean and RMS of SLR for GRACE satellite kinematic orbits over the period day 190-290 of 2003

|        | Mean (cm) | RMS (cm) |
|--------|-----------|----------|
| GRACE_A: (kin./red.) | -1.05/-0.52 | 3.27/2.79 |
| GRACE_B: (kin./red.) | -1.35/-0.72 | 3.18/2.62 |

Since the dual-frequency KBR system is able to measure the distances between the two satellites within a precision of about 10 μm, which is much more precise than GPS computed distances, the KBR measurements can be used to independently evaluate the along-track component of the GPS baselines. The baselines constructed directly from both dynamic and kinematic orbits of twin GRACE satellites are compared with KBR measurements. The dynamic and kinematic relative orbits in accordance with KBR data in 2.2 mm and 4.64 mm separately. Fig.1 gives an overview of the daily RMS of the KBR residuals (KBR measurements minus computed distances from dynamic and kinematic orbits) over the 101 days period. It can be found that the daily RMS of fixed solutions are very stable and only vary between approximately 1 mm and 4 mm for dynamic orbits and from 4 to 6 mm for the kinematic orbits. Considering the claimed noise level of BlackJack carrier-phase observation (less than or equal to 5 mm), the accuracy of the relative position (approximate 5 mm in along-track direction) indicates that we have a high quality of the observation models, and the ambiguities are fixed effectively.

3 Conclusion

This paper is devoted to the development of efficient
and stable algorithms for the determination of precise relative dynamic and kinematic relative orbit determination of LEOs for geodetic application. Both reduced dynamic and kinematic relative orbit determination are studied using 101-days GRACE data. The results are validated with SLR and KBR data to access the absolute and relative precision. The SLR data validation shows that the kinematic and reduced dynamic orbits have a precision of 3.3 cm and 2.7 cm in radial direction. The relative precision along the direction of the two satellites is about 2.2 mm and 4.6 mm with reduced dynamic and kinematic relative orbit determination approach.

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