Interference of holon strings in 2D Hubbard model

Chang-Yan Wang$^{1,*}$ and Tin-Lun Ho$^{*}$

Department of Physics, The Ohio State University, Columbus, OH 43210, United States of America

E-mail: wang.10183@buckeyemail.osu.edu and jasontlho@gmail.com

Received 23 August 2023, revised 1 December 2023
Accepted for publication 17 January 2024
Published 1 February 2024

Abstract

The 2D Hubbard model with large repulsion is an important problem in condensed matter physics. At half filling, its ground state is an antiferromagnet (AMF). The dope AMF below half filling is believed to capture the physics of high $T_c$ superconductors. The fermion excitation of this dope AMF is theorized as splitting up into holons and spinons that carry charge and spin separately. It is believed that these exotic holons and spinons are the origins of the unusual properties of high $T_c$ superconductors. Despite the interests in holons and spinons, the direct observations of these excitations remain difficult in solid state experiments. Here, we show that with the rapid advances in the experimental techniques in cold atoms, the direct observation of holons is possible in quantum quench dynamic processes in cold atom settings. We show that the well-known holon-strings generated by the motion of a holon as well as their interferences can be detected by the measurements spin–spin correlations and demonstrate the presence of the Marshall phase associated with a holon string reflecting an underlying AMF background. Moreover, we show that the interferences of the holon strings make a holon propagate anisotropically, with a diffusion pattern clearly distinct from that of spinless fermions. At the same time, we show that these interferences lead to a large suppression in magnetic order in the region swept through by the strings (even to about 95% for some bond). We further demonstrate the Marshall phase of the holon-strings by comparing the dynamics of holon in the $tJ$ model with that of the so-called $\sigma tJ$-model, which is the $tJ$ model with the Marshall phase removed. The holons in these models propagate entirely differently.

Keywords: Hubbard model, holon, holon string, quantum dynamics

1. Introduction

The Fermi Hubbard model is one of the most important models in condensed matter physics. It has been intensely studied since its appearance [1]. The model has a very simple form, consisting a term describing electron hopping on a lattice and a term describing short range interaction between electrons. There is considerable evidence showing that the model can exhibit a wide range of phenomena in various parameter regime, such as metal–insulator transition, antiferromagnetism, superconductivity, etc [2]. The 2D Hubbard model with strong repulsion is of particular interest, as it is believed to capture the key physics of high-$T_c$ superconductivity [3]. Yet despite decades of studies, the model remains unsolved.

For simplicity, we shall consider the 2D Hubbard model on a square lattice. For strong repulsion, the system at half filling has one electron per site, with an antiferromagnetic (AFM) ground state. The central question has been how this ground state is changed as the system is doped below half filling. The problem is challenging because the fermion excitations of a doped AFM are believed to split up into ‘holons’ and ‘spinons’, which carry charge and spin separately. They are very different from the excitations of a Fermi liquid, which carry both spin and charge. The unusual properties of the spinons and holons are believed to be the cause of many
unusual properties of high $T_c$ superconductors including the ‘strange metal’ behavior [3]. Yet despite their central role in theoretical studies, there are no known ways to observe them directly in current solid state experiments. However, an exciting possibility has emerged in the last few years. With the rapid advances in cold atom experiments, one can now simulate with great precision the Fermi Hubbard model using ultra-cold fermions in optical lattices, and observe the AFM order at half filling [4–8]. Moreover, with the development of atom microscope that can image atoms with single site resolution, one can now image the many-body wavefunction of a quantum state with unprecedented detail.

In the case of a holon, it is supposed to leave behind it a string of ‘wrong’ spins as it moves through the AFM background. Recently, Markus Greiner’s group tried to identify the holon strings by comparing the images of a doped AFM with a classical AFM background, and had concluded their presence by comparing the observed images with a theoretical model [9]. Propagation of holon has also been studied in [10–12]. More recently, one of us (TLH) has introduced a method to identify the holon string directly from the spin density of the system without making reference to the results of specific theories [13]. The method relies only on an exact property of the Heisenberg AFM—that its ground state obeys the Marshall sign rule. In [13], it is showed that if the holon moves along the x or y axis of a square lattice, then its string has a very clear signature—that the spin correlation of neighboring sites is ferromagnetic in the immediate vicinity of the string, while being AFM everywhere else. This behavior follows from a key property of the holon string—that it carries a phase reflecting the Marshall sign of the fluctuating AFM background. We shall refer to this phase as the ‘Marshall’ phase [13, 14].

In this paper, we focus on the interference effect of holon strings and how they are reflected in the spin correlation of nearest neighbors. The interference effects occur when a holon travels away from the x or y axis. In this case, there are many strings of the same length connecting the initial and the final position of the holon. As we shall see, the different Marshall phases of different strings will lead to a strong suppression of nearest neighbor spin correlations, as if driving the system towards a spin-liquid. We should mention that the ground state wavefunction of a holon has also been studied for finite systems with periodic boundary condition [15] and for ladders systems [16]. Here, rather than focusing on the ground state, we study the propagation of holons in physical environments created in current experiments. After this study on the interference of strings, we further demonstrate the effect of the Marshall phase by comparing our results with those of the so-called $\sigma tJ$-model [17], which is the Hubbard in strong repulsion limit but with the Marshall phase stripped off. In this case, a single holon in an AFM background behaves more like a fermion in an empty lattice, as if recovering the Fermi liquid behavior. In other words, the Marshall phase is responsible for the non-Fermi behavior of the holon.

Before proceeding, we mention that the study of a single hole in a half-filled Hubbard model has a long history [15, 18–27]. Most of these studies focus on the ground state. In particular, the ground state in the infinite $U$ limit is proven to be ferromagnetic, (i.e. the Nagaoka state) [28]. Our work, however, is to study the dynamical evolution of a single hole when it is introduced into the anti-ferromagnetic ground state of a half-filled Hubbard model. This set up can be easily achieved using ultra-cold fermions in optical lattice. Our goal is not to demonstrate the Nagaoka ground state through quantum evolution, but to observe the formation of holon strings during quantum evolutions through the measurement of spin–spin correlations.

Moreover, we would like to point out that the propagation of holons and spinons has been studied experimentally in 1D Hubbard model using time resolved observation [29]. Such imaging technique is precisely what is needed for our proposed measurements. We also note that the spin correlations of the equilibrium states containing a mobile doublons [30] as well as magnetic polarons [31] have been studied. Our study focuses on holons and their quantum dynamics.

2. $IJ$ model, Marshall sign, and amplitudes for holon propagation

We start with the Hubbard model,

$$H = -t \sum_{\langle i,j \rangle, \sigma} c^\dagger_i(\sigma) c_j(\sigma) + U \sum_i n^\dagger_i(\uparrow) n^\dagger_i(\downarrow).$$

(1)

The first term describes the hopping of a fermion $c_j(\sigma)$ with spin $\sigma$ hopping from site $i$ to a neighboring site $j$, $t > 0$, $U$ is the on-site interaction between opposite spins, and $n^\dagger_i(\sigma) = c^\dagger_i(\sigma) c_i(\sigma)$. For strong repulsion ($U \gg t$), each site can at most be occupied by one fermion. The Hubbard model then reduces to the $IJ$-model $H_{IJ} = T + H_J$:

$$H_J = J \sum_{\langle i,j \rangle} S_i \cdot S_j, \quad J = \frac{\tilde{t}}{U} > 0,$$

(2)

$$T = -t \sum_{\langle i,j \rangle, \sigma} \sigma^\dagger_i(\sigma) \tau_{\langle i,j \rangle}(\sigma) \cdot \tau_{\langle i,j \rangle}(\sigma) = c^\dagger_i(\sigma) \{ 1 - n_{-\sigma}(i) \},$$

(3)

where $S_i = c^\dagger_i(\sigma) \sigma_{\mu\nu} c_i(\sigma)/2$. $H_J$ is the nearest neighbor AFM Heisenberg Hamiltonian with spin interaction $I$, and $T$ is the hopping of fermions subject to the constraint of no double occupancy. This constraint is the origin of the intricate transport of the system. At half filling, $T$ vanishes. The $IJ$-model reduces to the AFM Heisenberg model $H_J$. Experimentally, an immobile hole at a selected site can be created by piercing through it with a focused blue-detuned laser. Note that $T$ remains zero as long as the hole is immobile.

We shall denote the spin states of the Heisenberg system with an immobile hole at $\mathbf{R}$ as $|\nu;\mathbf{R}\rangle$, where $\nu \equiv (1,2,2,\ldots)$ represents the spin configuration where the fermion at site $i$ has spin $\nu_i$, $\nu_i = \uparrow, \downarrow$. Let $|\Psi;0\rangle$ be a state with exactly one fermion per site and an immobile hole at the origin, it has the expansion
where $N^j_\nu$ is the number of down-spins in configuration $\nu$ in one of the two sub-lattices of the square lattice (denoted as $a$). We shall denote $|\nu, 0\rangle$ as the Marshall spin basis, and the wavefunction $\hat{\psi}(\nu, 0)$ as the ‘modified wavefunction’. The advantage of the modified representation equation (5) is that for the ground state $|G, 0\rangle$ with the immobile hole at the origin, the modified ground state wavefunction is positive $\hat{\psi}([G, 0]) \geq 0$ [13], which is the generalization of the Marshall theorem [32]. Any states $|\psi, 0\rangle$ that satisfies Marshall rule means $\hat{\psi}(\nu, 0) > 0$.

To allow the hole to move, we remove the focused laser at time $\tau = 0$. The quantum state at later time is

$$|\psi(\tau)\rangle = e^{-i\tilde{H}_0\tau}\hat{\psi}([G, 0])$$

(7)

$$\check{\psi}(\mu, R; \tau) = \sum_{\nu, \mu} \hat{\psi}(\mu, R; \nu) e^{-iH_0\tau} |\nu, 0\rangle$$

(8)

Although $\hat{\psi}(\nu, 0)$ is positive, $\check{\psi}(\mu, R; \tau)$ needs not to be because of the holon propagator.

As discussed in [13], the Marshall sign of a state $\Phi$ shows up in the ‘exchange overlap’ of opposite spins at neighboring sites $(i,j)$, $\rho_{ij} = \langle c_i^\dagger c_j \rangle \langle c_i^\dagger c_j \rangle$. For systems without SU(2) symmetry, such as the $\sigma I$ model we discuss later, the exchange correlation can be obtained using the interference method discussed in [13]. To measure the spin–spin correlation of the state when the hole has moved to $R$ after time $\tau$, one can first take many images of the spin density after the hole is released for a time $\tau$, and then post select from these images the subset where the hole has arrived at $R$.

3. Holon strings in the AFM background

3.1. Marshall phase effects on holon propagation

Exact calculation of the propagation amplitude $\check{\psi}(\mu, R; \tau)$ is formidable. However, the situation is simplified considerably when $J/t \ll 1$, where spins flips are much slower than the motion of holes. In this limit, which is satisfied in current experiments [6], one can expand equation (8) in powers of $J/t$. To the lowest order, one can replace the $H_0$ by $\mathcal{T}$, and equation (8) becomes

$$\check{\psi}(\mu, R; \tau) \approx \sum_{\nu} \Gamma(\mu, R; \nu, 0; \tau) \check{\psi}(\nu, 0)$$

(10)

$$\Gamma(\mu, R; \nu, 0; \tau) = \sum_{\nu = 0,1,2,...} \left( \frac{i\tau}{t} \right)^n \left( \frac{\nu}{n!} \right) \check{\psi}(\mu, R; \nu, 0; \tau)$$

(11)

where $\check{\psi}(\nu, 0)$ is the transition amplitude of the hole reaching $R$ from $0$ through $n$ nearest neighbor hops,

$$\check{\psi}(\mu, R; \nu, 0; \tau) = \left( \frac{-T}{t} \right)^n |\nu, 0\rangle.$$ (12)

The string connecting $0$ to $R$ through $n$-hops will be referred to as an $n$-string, and is denoted by the sequence of $n + 1$ sites $(R_0, R_1, \ldots, R_n)$, where $R_0 = 0, R_n = R$. See figures 1(a) and (b). Successive $R$'s are nearest neighbors. It is clear from equation (12) that $\check{\psi}(\nu, 0)$ is non-vanishing only when the initial spin configurations $\mu$ and $\nu$ are identical ($\mu = \nu$) on all the sites $i$ not on the string. For the spins on the string, the final spin configuration is given by the initial one sliding along the string by one lattice site, i.e. $\mu_i = \nu_{i+1}$ for $i = 0, 1, 2, \ldots, n - 1$. With this relation between the initial and the final spin configurations, the value of $\check{\psi}(\nu, 0)$ is [13, 14],

$$\check{\psi}(\nu, 0) = \sum_{n-string} \left( -1 \right)^{N^\downarrow_{n-string}} \check{\psi}(\nu, 0).$$ (13)

where $N^\downarrow_{n-string}$ is the number of down spins on the $n$-string, and the factor $\left( -1 \right)^{N^\downarrow_{n-string}}$ (which is $\pm 1$) is the Marshall phase of the string.

From equation (12), it is clear that $\check{\psi}(\nu, 0)$ is non-zero only when the length of the string $n$ exceeds a minimum value $n^* \geq R_x + R_y$, which is the minimum number of hops from 0 to $R$ through $\mathcal{T}$. See figures 1(c) and (d) The number of such strings is $L_R = (R_x + R_y) / (R_x, R_y)$. The area swept through by these strings is a rectangle $(\Lambda)$ of size $R_x \times R_y$. If the amplitude $\check{\psi}$ is approximated by the leading term $\check{\psi}(\nu^*)$, we then have

$$\check{\psi}(\mu, R; \tau) \approx \sum_{\nu} \Gamma(\mu, R; \nu, 0; \tau) \check{\psi}(\nu, 0).$$ (14)

From the constraints on spin configurations imposed by $\check{\psi}(\nu, 0)$ we have just discussed, it is clear from equation (14) that the final state $\check{\psi}$ and the initial state $\check{\psi}$ have identical spin configurations for all sites $i$ outside $\Lambda$, i.e. $\mu_i = \nu_i$. Consequently, the exchange overlap $\rho_{ij}$ (or the spin correlation $\langle S_i^x S_j^x \rangle$) of both states are the same for all neighboring pairs $(i,j)$ outside $\Lambda$. (See figure 1(c)). On the other hand, if $i$ is inside $\Lambda$ and $j$ is inside, then the spin at site $j$ of the two states $\check{\psi}$ and $\check{\psi}$ need not be the same. This is because some strings in $\Lambda$ will pass through site $j$, each of which will change the spin $\nu_j$ of the original AFM configuration $(\nu)$ by the sliding motion along its respective path as discussed before, while carrying its Marshall phase. For strings with opposite Marshall phases, they interfere destructively and hence weaken the original AFM order.

Similar weakening occurs with both both $i$ and $j$ are inside $\Lambda$. However, the case when $R$ is on the $x$ or $y$ axis is an exception. In this case, there is only one string of minimum length, which
The behaviors of spin correlations of neighboring sites with different overlap with \( \Lambda \) remain unchanged. Moreover, since the number of strings grows rapidly as \( n \) increases, and their contributions to \( \Gamma^{(n)} \) tend to cancel each other. As a result, for time interval \( \tau < 1/t \), the magnitudes of \( \Gamma^{(n)} \) decreases rapidly with \( n \). As we shall see, the leading term \( \Gamma^{(n^*)} \) provides a good approximation to the full amplitude \( \Gamma \).

In the next section, we shall present numerical results to demonstrate all the effects mentioned above. We shall show that when the holon propagates away from the \( x \)-axis, the interference of holon strings strongly reduces the magnitude of spin–spin correlation within the region swept over by the holon strings, even reducing the AFM order by 95% for some bonds.

### 3.2. Switching off the Marshall phase, the \( \sigma tJ \)-model

Another way to demonstrate the Marshall phase is to study the motion of the holon with the Marshall phase is ‘switched off’. This can be achieved by changing the kinetic energy in the \( tJ \)-model to \( \mathcal{T} = -t \sum \langle \mu_i, \sigma \rangle \sigma^\dagger \sigma^\dagger \langle \mu_i, \sigma \rangle \sigma^\dagger \sigma^\dagger \langle \mu_{i-1}, \sigma \rangle \sigma_{i-1} \rangle \), resulting in the so-called \( \sigma tJ \)-model [17]. Although we do not yet have a simple way to generate this Hamiltonian in cold atom experiments, the model is worth studying because it strips off the Marshall phases of the original model. At half filling, \( H_{\rho \mu} \) again reduces to the Heisenberg AFM since \( \mathcal{T} \) also vanishes. The amplitude for the propagation of a hole is still given by \( \Gamma \) and \( \Gamma^{(n)} \) in equations (11) and (12) with \( \mathcal{T} \) replaced by \( \mathcal{T} \). It is easy to see that when a hole travels from \( 0 \) to \( R \) through a particular path (or \( n \)-string), the hopping \( \mathcal{T} \) accumulates a phase \( (-1)^n \) cancelling exactly the same factor in equation (11) arising from AFM background. Consequently, all strings amplitudes add coherently, and the amplitude \( \langle \mu|\psi^\dagger\mathcal{T}^n|\sigma \rangle \) is simply the total number of strings connecting \( 0 \) and \( R \). This feature is identical to that of a spinless fermion in an empty square lattice, although in that case there is no spin sums. Due to the difference in the phase coherence of the strings, the behavior of holons in the \( \sigma tJ \)-model is very different from that in the \( tJ \)-model, which we shall show in the next section.

### 4. Numerical results

#### 4.1. Holon density and spin–spin correlation

We have performed numerical calculations for the density distribution and spin correlations of the holon after it has traveled over a time interval \( \tau \). We consider a \( 5 \times 5 \) square lattice. The lattice sites are labeled as 0–24 as shown in figure 2(a). The center is at site-12. Initially, the hole is held fixed at the center. All other sites are occupied by a fermion. We have computed the AFM ground state \( G(\mu, 0) \) using exact diagonalization [33, 34]. This result allows us to calculate the wavefunction \( \Psi(\mu, R; \tau) \) as an expansion of \( \tau \) as shown in equations (10) and (11); and hence the holon distribution at time \( \tau, n(R, \tau) = N^{-1} \sum_{\mu} |\Psi(\mu, R; \tau)|^2 \), where \( N \) is the normalization constant.

In figures 2(a) and (b), we show the holon distribution \( n(R, \tau) \) for the \( tJ \)-model and the \( \sigma tJ \)-model respectively, with the wavefunction \( \Psi(\mu, R; \tau) \) calculated up to 8th order of \( \tau \).
model is closer to the spinless fermion than to the ally measurable. Note also that the holon density of the figures is the nearest neighbor and the next nearest neighbor. See the τ contrast to the fermion model has a Marshall phase in holon propagation, in figure distribution of a spinless fermion on an empty lattice, shown in is much less anisotropic. In fact, it resembles more the density figure (or the holon travel at an angle less than 90 degrees from the center. This is due to the destructive interference of the strings when is very anisotropic, being strongest in the σ direction.

Figure 2. The density distribution of the holon after released from the center (site-12) for time τ = 0.8/t on a 5 × 5 lattice with open boundary condition: Lattice sites are labeled as 1, 2, 3, etc. They are represented by a square. The number on site R is the holon density n(R, τ). (a) and (b) are the results for the τJ and στJ model respectively. (c) is the result for a spinless fermion on empty lattice. The quantum state is calculated by keeping up to eighth order in τR.

Figure 3. The densities n(R, τ) in figure 2 are plotted as a function of τ for different position R. (a) The holon returns to the origin (site-12). (b) The holon arrives at the nearest neighbor (site-13). (c) The holon arrives at the next nearest neighbor (site-18). The results for holons in the τJ-model and the στJ-model are shown in green and red. The result for a spinless fermion in an empty lattice is shown in blue. At time τ = 0.8/t, the distinction between τJ-model and the στJ-model is apparent. For the τJ-model, the holon propagates more efficiently along x (case (b)) than along the diagonal (case (c)), with probability ratio ∼ 0.14/0.05. In contrast, the corresponding ratio for the στJ-model is ∼ 0.08/0.08. As explained in the text, the strong anisotropy of holon propagation in the τJ-model is due to the destructive interference of the holon strings. It is also pointed out in the text that the propagation of a holon in the στJ-model (red curve) is similar to that of a single fermion in an empty lattice (blue curve) because the propagators in both cases have similar phase coherence. In (a), the τR ∈ [0.4, 0.8] part of the main plot is plotted in the inset using log scale.

Figure 2(a) shows that the holon propagation in the τJ-model is very anisotropic, being strongest in the x or y direction. This is due to the destructive interference of the strings when the holon travel at an angle less than 90 degrees from the x (or y) axis as discussed in the previous section. In contrast, figure 2(b) shows that the holon propagation in the στJ-model is much less anisotropic. In fact, it resembles more the density distribution of a spinless fermion on an empty lattice, shown in figure 2(c). This is because neither στJ-model nor the spinless fermion model has a Marshall phase in holon propagation, in contrast to the τJ-model.

Figures 3(a)–(c) show the density n(R, τ) at different time τ when the holon is found is at the center 0, at the nearest neighbor and at the next nearest neighbor from the center. When τ reaches 0.8, the difference between the τJ- and the στJ-model is very apparent when the final position R is the nearest neighbor and the next nearest neighbor. See figures 3(b) and (c). These differences should be experimentally measurable. Note also that the holon density of the στJ-model is closer to the spinless fermion than to the τJ-model. The return probabilities to the origin of all three cases are quite similar, as shown in figure 3(a).

4.2. Marshall phase and spin correlations

With the holon wavefunction Ψ(μ, R; τ) calculated in (section 4.1), we can evaluate the spin correlation ⟨S_i S_j⟩ for neighboring sites (i, j). In figure 4(a), we show this correlation of the initial AFM ground state |G; 0⟩ with the immobile hole at site-6 (which is the origin 0). The small squares represent the lattice sites. The number on the rectangle linking two sites is the value spin correlation of these two sites. Blue and red color correspond to negative and positive sign. For this AFM state, the correlations of all neighboring sites are negative, consistent with the Marshall sign rule. The typical magnitude of the correlation is around −0.12.

Figure 4(b) shows the spin correlation when the final position of the holon is along x (at site-9). In this case, the shortest path connecting the initial and final position is a straight line. The numerical result confirms the features discussed in the
previous section, and in [13], i.e. $\langle S_i^z S_j^z \rangle$ is negative if $i$ and $j$ are both outside or on the string; and is positive if on one of them ($i$ or $j$) is on the string. The magnitude of the ferromagnetic correlations (the red bonds) is weaker than that of the background AFM by a factor of 2. The exchange overlap of the $\sigma tJ$-model (figure 4(c)) is very different. It remains negative for all neighboring sites. This is due to the phase coherence of all strings of the same length as discussed before. The magnitude of the spin correlation in the vicinity of the string, however, is weaker than that of the AFM background by roughly a factor of 4.

Figure 5(a) shows the spin correlation when the final position of the holon (site-13) is not on the x-axis. In this case, the strings connecting site-6 to site-13 have lengths $n = 3, 5, 7, \ldots$. The pattern of spin correlation is very different from that in figure 4(b) when the final position is along x. We note that the spin correlations of some neighboring sites (such as the ‘bonds’ between (17–12), (12–7), (7–2), (11–6)) are reduced from the original AFM value by almost a factor of 20. This large reduction shows the strong destructive interference effects of the holon strings.

In figure 5(b), we show the different contributions that make up the spin–spin correlation for the final state when the hole is at site-13 at time $\tau = 0.8/\tau$. The horizontal axis shows the nearest neighbor pair of interest. The values of the spin correlations of the initial state and the final state (when the hole is at site-6 and site-13) are given in blue and red color respectively. These values are calculated from equation (11) by including up to seventh order in $\tau \tau$. On the same figure, we also show (in green color) the corresponding values for the final state including only the shortest strings (i.e. $n = n^* = 3$) contributions. The fact that the blue and green dots almost overlap with each other shows that the $n = 5$ and $n = 7$-strings are less important.

The $n = 3$ strings connecting site-6 to site-13 sweep through a rectangle $\Lambda$ of size $2 \times 1$ (including sites - 6, 7, 8, 11, 12, 13). The pairs of neighboring sites inside (and outside) of $\Lambda$ are collected on the left (and the right) hand side of the vertical dashed line. For the pairs outside $\Lambda$, there are little differences in the spin correlations between the initial and the final states, (i.e. red and blue dots). In contrast, the differences are significant for the neighboring sites inside $\Lambda$, a feature discussed in the previous section.

To show quantitatively the different contributions to the string interference, we write the quantum state in equation (10) as

$$|\Psi^{(n)}\rangle = \sum_\alpha |\Psi^{(n)}_\alpha\rangle,$$

where ‘$\alpha$’ labels the strings. The spin correlation can be written as

$$\langle \Psi^{(n)} \big| S_i^z S_j^z | \Psi^{(n)} \rangle = D + I,$$

where $D$ and $I$ are the ‘diagonal’ and the ‘off-diagonal’ terms,

$$D = \sum_\alpha \langle \Psi^{(n)}_\alpha \big| S_i^z S_j^z | \Psi^{(n)}_\alpha \rangle,$$

$$I = \sum_\alpha \sum_{\beta \neq \alpha} \langle \Psi^{(n)}_\alpha \big| S_i^z S_j^z | \Psi^{(n)}_\beta \rangle.$$

The off-diagonal terms represent the interference of the holon strings.

In figure 5(b), the spin correlation $D + I$ and the diagonal terms are plotted as green and purple dots respectively. We see that these values are close to each other for the neighboring sites overlapping with $\Lambda$, i.e. those pairs on left-hand side of vertical dashed line. This means the interference contribution $I$ is small, precisely the destructive interference physics we have discussed before when one or both of neighboring sites $(i, j)$ are inside $\Lambda$.

On the other hand, we have shown previously that the spin configurations of the final and the initial state are identical outside $\Lambda$, independent of the strings inside it. This means both diagonal and off-diagonal contributions are present, and their sum is equal the original AFM correlation, as seen on agreement between the green and red data in figure 5(b).
two holes are introduced in the antiferromagnetic background, the phase in the Hubbard model in following way: if initially the holons evolution could be related to the stripe phase of Hubbard model \[\text{(1)}\], one may expect that an initial state with one then the role played by the holon string in this thermalization process still needs further study. Beside the single holon evolution, which suffices for our aim here. However, it would be also interesting to investigate the fate the holon after long time evolution. If the holon do thermalize finally, it would be also interesting to investigate the fate the holon after long time evolution. According to the eigenstate thermalization hypothesis \[\text{(3)}\], one may expect that an initial state with one holon in the antiferromagnetic background will thermalize after long time of evolution. If the holon do thermalize finally, then the role played by the holon string in this thermalization process still needs further study. Beside the single holon evolution, it is also worth to study multi-holon evolution which could be related to the stripe phase of Hubbard model \[\text{(36)}\]. For example, the holons evolution could be related to the stripe phase in the Hubbard model in following way: if initially two holes are introduced in the antiferromagnetic background, when they evolve, the interference between the holon strings originating from these two holes could result in stripe pattern of charge density, just like the stripe pattern in the double slits experiment due to the interference between photons from the two slits. However, these questions are beyond the scope of this paper, we shall leave them for future study.

5. Conclusion

We have studied the motion of a single hole in the \(t\)-model in the limit of slow spin motion, \(J/t \ll 1\). In this limit, the propagation of a holon generates strings equipped with a Marshall phase, which depends on the spin configurations in the underlying antiferromagnetic state. The interference of these strings leads to a holon propagation much more anisotropic than the that of free fermions. It also reduces substantially the antiferromagnetic correlation in the region swept through by the strings. We further demonstrate the effect of the Marshall phase by considering the so-called \(\sigma tJ\)-model. The spin dependent hopping of this model removes completely the Marshall phase of the antiferromagnet, making the holon propagation similar to that of a spinless fermion. Our method can be generalized to study multi-holon transport, and include the effect of spinons, which we shall discuss elsewhere.

In this paper, we mainly focus on the early stage of holon evolution, which suffices for our aim here. However, it would be also interesting to investigate the fate the holon after long time evolution. According to the eigenstate thermalization hypothesis \[\text{(35)}\], one may expect that an initial state with one holon in the antiferromagnetic background will thermalize after long time of evolution. If the holon do thermalize finally, then the role played by the holon string in this thermalization process still needs further study. Beside the single holon evolution, it is also worth to study multi-holon evolution which could be related to the stripe phase of Hubbard model \[\text{(36)}\].

Figure 5. (a) The distribution of spin correlation of the state at time \(\tau = 0.8/t\) with the holon at site \(i = 13\), starting from the state at figure \(4\). The shortest string connecting the initial and final position (site-6 and site-13) has length \(n = 3\). These \(n = 3\) strings sweep through a rectangle \(\Lambda\) covering sites 6, 7, 8, 11, 12, 13. For nearest neighbors \((i-j)\) that are inside \(\Lambda\), such as \((6-11)\), \((7-12)\), as well as those connected to it, such as \((7-2)\), \((12-17)\), their spin correlations are strongly suppressed, due to the destructive interference of the holon strings. (b) Comparison of spin correlations between the initial state at figure \(4\), and final state with final holon at site \(i = 13\) at time \(\tau t = 0.8\). The horizontal axis labels of neighboring pairs \([i-j]\) of interest. The final state is calculated by expanding equation (10) up to 7th order in \(\tau t\). The result for the final state calculated with the only shortest strings \((n = 3)\) in equation (10) is shown in green. Its closeness to the 7th order result (red dots) shows the dominance of the \(n = 3\) terms over the \(n = 5\) and 7 terms. To the right and the left of the vertical dashed line are the neighboring pairs outside \(\Lambda\) and those overlapping with it. For those to the right (outside \(\Lambda\)), the spin correlations of the final state is essentially unchanged from the initial ones. For those to the left (overlapping with \(\Lambda\)), the differences in spin correlation between the initial and final state are significant. This is the result of destructive interference of the strings that pass though one of the site (or both) in the neighboring pair \((i-j)\) as discussed in section 2. As discussed in section 3, the spin correlation of the final state (the green dots) is made up of ‘diagonal’ and ‘off-diagonal’ terms, \(\langle SS_i^z \rangle = D_0 + \tau Z_0\). Here, we have also plotted the direct term in purple. One sees that \(D_0\) is close to the full value \(\mathcal{D}_0 + I\) for the nearest neighbors that overlap with \(\Lambda\) (i.e. those to the right of the vertical dashed line). It means the interference of holon string is close to complete destruction, with \(I \sim 0\).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

We thank Professor Zheng-Yu Weng for helpful discussions, and the kind support of the MacMaster Funds.

Note added. This work has been presented as part of the doctoral dissertation of one of the authors (CYW) \[\text{(37)}\].

ORCID ID

Chang-Yan Wang https://orcid.org/0000-0002-7783-9205

References

[1] Hubbard J and Flowers B H 1963 Proc. R. Soc. A 276 238
[2] Tasaki H 1998 J. Phys.: Condens. Matter 10 4353
[3] Anderson P W 1997 The Theory of Superconductivity in the High-Tc Cuprates (Princeton University Press)
