Half-skyrmion picture of single hole doped high-\(T_c\) cuprate

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The single hole doped CuO\(_2\) plane is studied by the O(3) non-linear \(\sigma\) model and its CP\(^1\) representation. It is argued that the spin configuration around a Zhang-Rice singlet is described by a half-skyrmion. The form of the excitation spectrum of the half-skyrmion is the same as that in the \(\pi\)-flux phase.

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In the phase diagram of high-temperature superconductors, the most established phase is apparently the antiferromagnetic long-range ordered phase, or the Néel ordered phase of undoped parent compounds. Due to a large charge-transfer gap\(^{[1]}\), a hole at each copper site in the CuO\(_2\) plane is localized to form a spin \(S = 1/2\) moment. There is an antiferromagnetic spin-exchange interaction \(J\) between the copper site spins and the system is described by the antiferromagnetic Heisenberg model on the square lattice, \(H_{AFH} = J \sum_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j\). Antiferromagnetic spin wave dispersions observed by neutron scattering experiments are in quite good agreement with the spin wave theory with the value of \(J\) determined by Raman scattering\(^{[2]}\). High-temperature superconductivity occurs upon moderate hole doping on the Néel ordering phase.

For understanding the mechanism of high-temperature superconductivity, it is necessary to describe the doped holes. However, for the purpose of figuring out the proper picture, considering a moderately doped system does not appear to be promising because such a system is quite complicated and indeed there is no consensus on the description of the system. Contrastingly, a system with only one doped hole is much simpler and there seems to be much hope to understand the physics because it is closest to the well-established antiferromagnetic long-range ordered phase. Experimentally angle resolved photoemission spectroscopy (ARPES)\(^{[3, 4, 5]}\) on undoped compounds provide valuable information on the single hole doped systems\(^{[6]}\).

As an effective theory of the antiferromagnetic Heisenberg model \(H_{AFH}\), the O(3) non-linear \(\sigma\) model (NL\(\sigma\)M) has been studied extensively\(^{[7]}\). The model is derived from \(H_{AFH}\) by applying Haldane’s mapping\(^{[8]}\): 

\[
S = \frac{\rho_0^2}{2} \int_0^\beta d\tau \int d^2 \mathbf{r} \left[ (\nabla \mathbf{n})^2 + \frac{1}{c_{sw}^2} \left( \frac{\partial \mathbf{n}}{\partial \tau} \right)^2 \right],
\]

where \(\rho_0\) is the bare spin stiffness constant on the length scale of the lattice constant, which is taken to be unity, and \(c_{sw}\) is the spin-wave velocity. The integral with respect to the imaginary time \(\tau\) is carried out up to the inverse temperature \(\beta\). (Hereafter we take the unit \(\hbar = 1\).)

The unit vector \(\mathbf{n}\) represents the directions of staggered spin moments. The low-energy and the long-length scale behaviors of \(H_{AFH}\) are well described by NL\(\sigma\)M as confirmed by experiments\(^{[2]}\) and quantum Monte Carlo simulations\(^{[3]}\).

There is a useful form for the NL\(\sigma\)M, which is called CP\(^1\) model\(^{[9]}\). Introducing complex fields \(\zeta_\sigma(x)\) with \(x = (c_{sw}\tau, \mathbf{r})\) through \(\mathbf{n} = \sum_{\sigma=\mp} \zeta_{\sigma} \sigma \zeta_{\sigma} \mathbf{r}\), with \(\sigma^\alpha (\alpha = x, y, z)\) representing Pauli matrices, we obtain 

\[
S = \left( \rho_0^2 / 2 c_{sw} \right) \int d^3 x \left[ (\partial_\mu \zeta_\sigma)^2 + (\bar{\zeta}_\sigma \partial_\mu \zeta_\sigma)^2 \right].
\]

Performing a Stratonovich-Hubbard transformation for the second term in the square brackets, we obtain the CP\(^1\) model:

\[
S = \frac{1}{2g} \int d^3 x \left( (\partial_\mu - i \alpha_\mu) \zeta_\sigma(x) \right)^2,
\]

where \(g = c_{sw} / 2 \rho_0^2\) and \(\langle \alpha_\mu(x) \rangle = \langle i \sum_\sigma \bar{\zeta}_\sigma(x) \partial_\mu \zeta_\sigma(x) \rangle\) at the saddle point. The field \(\alpha_\mu\) is a U(1) gauge field associated with a local gauge symmetry, \(\zeta_\sigma(x) \rightarrow \zeta_\sigma(x) \exp(i \chi(x))\). Thus, the NL\(\sigma\)M is equivalent to the boson system with the U(1) gauge field interaction.

The CP\(^1\) model is also derived from the Schwinger boson mean field theory (SBMFT)\(^{[10]}\). In the theory, the spin is represented in terms of boson operators \(z_\sigma^\dagger\) and \(z_\sigma\) as \(\mathbf{S}_j = \frac{1}{2} \sum_\sigma z_\sigma^\dagger \sigma \sigma' z_\sigma'\), with the constraint \(\sum_{\sigma=\mp} z_\sigma^\dagger z_\sigma = 1\). The SBMFT Hamiltonian is given by

\[
H_{SB} = -(J/2) \sum_{(i,j)} \left[ \Delta^2 (z_{\uparrow i} z_{\downarrow j} - z_{\downarrow i} z_{\uparrow j}) + \text{h.c.} - |\Delta|^2 \right] + \sum_{j\sigma} \lambda \left( z_{j\sigma}^\dagger z_{-\sigma} - 1 \right),
\]

with \(\Delta\) and \(\lambda\) being the mean field parameters. Here \(\Delta = \langle z_{\uparrow i} z_{\downarrow j} - z_{\downarrow i} z_{\uparrow j} \rangle\) describes the Schwinger boson pairing\(^{[11-13]}\). After Fourier transforming, the Schwinger boson quasi-particle excitation spectrum is obtained by a Bogoliubov transformation\(^{[14]}\):

\[
\omega_k = \sqrt{\lambda^2 - J^2 |\Delta|^2} (\sin k_x \pm \sin k_y)^2,
\]

where the plus sign is for \(k_x k_y > 0\) and the minus sign is for \(k_x k_y < 0\). In the Néel ordering phase, \(\lambda = 2J |\Delta|\) and the Schwinger bosons are gapless at zone face centers (±\(\pi/2\),±\(\pi/2\)).

The Néel ordering state is a consequence of Bose-Einstein condensation of the Schwinger bosons at those points\(^{[11-13]}\). One can derive the CP\(^1\) model after some algebra\(^{[11]}\). The relation between the Schwinger boson fields and \(\zeta_\sigma(x)\) and \(\zeta_\sigma(x)\) in Eq.\(^{[2]}\) is \(z_{j\sigma} = \zeta_{j\sigma}\) at one sublattice and \(\zeta_{j\sigma} = z_{j\sigma}^\dagger\) at the other sublattice. One
Schwinger boson carries spin 1/2 and the unit gauge charge. Therefore, pairs of the Schwinger bosons that describe the low-lying excitations carry integer spins and the gauge charge two.

Both of NLσM and SBMFT are, so to speak, bosonic description of the system. Meanwhile, one can construct a fermionic theory by representing the $S = 1/2$ spins in terms of fermions. Mean field analysis was carried out by Affleck and Marston\[16\] and the $\pi$-flux phase was proposed. The state is characterized by a flux $\pi$ penetrating each plaquette with alternating directions. The quasi-particle excitations are gapless at $(\pm \pi/2, \pm \pi/2)$. More advanced analysis based on an effective theory that includes fluctuations around the mean field state through a U(1) gauge field shows that a mass term is induced in the spectrum dynamically. This phenomenon is associated with the Néel ordering\[14, 15\].

It has been shown experimentally that doped holes reside primarily on oxygen sites. Zhang and Rice suggested\[14\] that there is strong correlations of forming a singlet pair between oxygen p-orbital holes and copper d-orbital holes. The t-J model was proposed based on this picture. Although the single hole problem has been studied mostly by this model, the focus is mainly on frustration effects induced by hopping of the Zhang-Rice singlet\[20, 21\]. However, not so much attention has been paid on the spin configuration around the Zhang-Rice singlet.

In this paper, it is argued that in the single hole doped system the doped hole induces a half-skyrmion which is schematically shown in Fig. 1. The half-skyrmion is a spin texture characterized by a half of a topological charge.

We start with considering the spin configuration around a static Zhang-Rice singlet. The moving case shall be considered later. A Zhang-Rice singlet is composed of a d-orbital hole state and the four oxygen hole states around the copper ion. Constructing the Wannier functions, the operator for the Zhang-Rice singlet is given by\[10\] $\frac{1}{2}\sum_{\sigma} (\phi_{ij} d_{ji} - \phi_{ji} d_{ij})$, where $\phi_{ij\sigma}$ is the symmetric combination state of the four oxygen hole states around the copper ion at the site $j$\[14\]. The d-orbital state of the Zhang-Rice singlet is given by the superposition of the up-spin state and the down-spin state. To be specific, we assume that in the Néel ordered state before hole doping the ordered spin moment at the site $j$ is in the direction of the positive z-axis. Then, the d-orbital state with up-spin does not affect the neighboring spins so much.

In contrast, the down-spin state affects the neighboring spins to change their directions. From the analysis of the NLσM below, we show that a spin configuration, which is characterized by a non-zero topological charge, is created around the down-spin state. This spin configuration is called skyrmion\[6\]. For the static case, the energy is given by $E = \frac{\theta_0}{2} \int d^2 r (\nabla n)^2$. The field equation is $\nabla^2 n - (n \cdot \nabla^2 n) n = 0$. Solutions are divided into sectors characterized by a topological charge: $Q = \frac{1}{4\pi} \int d^2 r \epsilon_{\mu\nu} n \cdot (\partial_\mu n \times \partial_\nu n)$, where $\epsilon_{xy} = -\epsilon_{yx} = 1$ and $\epsilon_{xx} = \epsilon_{yy} = 0$. The energy in each sector has the lower bound: $E \geq 4\pi |\theta_0| |Q|$. The equality is satisfied if and only if $\partial_\mu n = \pm \epsilon_{\mu\nu} (n \times \partial_\nu n)$\[22\]. The lowest energy state is obtained by solving this equation with the boundary conditions: $n \to +\hat{z}$ at infinity and $n = -\hat{z}$ at $r = ry$. In terms of a variable $w = (n_x + i n_y)/(1 - n_z)$, the equation turns out to be the Cauchy-Riemann equations\[22\]. Therefore, $w$ is an analytic function of $z = x + iy$ or $z^* = x - iy$. The lowest energy state satisfying the boundary conditions is $w_\ell = [(x - x_j) + i(y - y_j)]/\lambda$ and $w_{as} = [(x - x_j) - i(y - y_j)]/\lambda$ with $\lambda$ being a constant. In the vector $n$ representation, these solutions are

$$n = \left( \frac{2\lambda(x - x_j)}{|r - r_j|^2 + \lambda^2}, \pm \frac{2\lambda(y - y_j)}{|r - r_j|^2 + \lambda^2}, \frac{|r - r_j|^2 - \lambda^2}{|r - r_j|^2 + \lambda^2} \right),$$

where the plus sign is for $\omega_s$ and the minus sign is for $w_{as}$.

Now we consider the superposition of the uniform state $n = +\hat{z}$ for the up-spin state and the skyrmion spin configuration\[8\] created by the down-spin state. The NLσM analysis suggests that a spin configuration with non-zero topological charge $Q$, where $0 < |Q| < 1$, is formed with $n_{as} \geq 0$ for any site $\ell$. The value of the topological charge $Q$ is determined through the flux representation in terms of the CP$^1$ gauge field $\alpha_\omega$: $Q = \frac{1}{2\pi} \int d^2 r (\nabla \times \alpha)$. The spin configuration with $Q$ corresponds to the flux $2\pi Q$. Due to Bose-Einstein condensation of the bosons $\zeta_\omega(x)$, the value of the flux $2\pi Q$ is not arbitrary. Since low-lying excitations are pairs of these bosons, the flux quantum is $\pi$ analogous to conventional BCS superconductors. From the constraint on $Q$, i.e., $0 < |Q| < 1$, we
conclude that \(2\pi|Q| = \pi\), that is, \(|Q| = 1/2\). Therefore, the spin configuration is the half-skyrmion, which is equivalent to the skyrmion with the core, and its energy is \(E_0 = 2\pi \rho_0^2\). This conclusion can be reached if we assume that the staggered magnetization vanishes at the perimeter of the core. The analysis of the NLoM gives the half-skyrmion solution as was shown by Saxena and Dandoloff [22]. They considered the two-dimensional ferromagnetic Heisenberg model in the context of the quantum Hall systems. For the static case, the field equation is the same. So we can apply their analysis to the antiferromagnetic case as well.

The moving half-skyrmion is obtained from the static half-skyrmion by applying a Lorentz transformation because the action \(\Gamma\) and \(\lambda\) are Lorentz invariant with \(c_{sw}\), the speed of “light.” From the calculation of the energy-momentum tensors, we find \(e_k^0 = \sqrt{c_{sw}^2 (k_x^2 + k_y^2) + E_0^2}\). For the \(S = 1/2\) antiferromagnet, the bare spin-wave velocity is \(c_{sw} = \sqrt{2J}\) and the bare spin stiffness is \(\rho_0^2 = J/4\). Renormalization effect leads to \(c_{sw} \to \sqrt{2Z}\) and \(\rho_0^2 \to Z J/4(\equiv \rho)^[3]\). Thus, the renormalized dispersion is given by \(e_k = J \sqrt{2Z^2 (k_x^2 + k_y^2) + (\pi\rho/2)^2}\). Since the doped hole is fermion, this relativistic dynamics of the half-skyrmion is described by a Dirac fermion Lagrangian density:

\[
\mathcal{L} = \sum_\sigma \overline{\psi}_\sigma (\gamma_\mu \partial_\mu + mc_{sw}^2) \psi_\sigma ,
\]

with \(\overline{\psi} = \psi^\dagger \gamma_0 \rho_0 \) and \(mc_{sw}^2 = \pi Z J/2\). The \(\gamma\) matrices satisfy \(\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}\) in the Euclidean space time. Here \(\sigma\) is an index for the sign of the topological charge. The number of components of the Dirac fermion field is four because there are a positive energy state and negative energy state and the origin of the Dirac fermion dispersion is at either \(k_1 = (\pi/2, \pi/2)\) or \(k_2 = (-\pi/2, \pi/2)\). The latter is implied from the fact that the Schwinger bosons are gapless at those points in the symmetry broken Néel ordering phase. Note that the dynamics is associated with the properties of the parent compound, namely, the Mott insulator. In this sense, the fermion \(\psi\) is different from quasi-particles in the conventional Fermi liquid.

The lattice version of the action is derived by discretizing Eq. 4. As is well-known in the lattice gauge-field theory [24], there is a fermion-doubling problem when one introduces a lattice for a Dirac fermion. This subtlety does not make any difficulty in our case because of the antiferromagnetic correlations that naturally introduce two species of Dirac fermions with half of the Brillouin zone (the magnetic Brillouin zone). Discretizing Eq. 4 and then performing Fourier transformation, we obtain

\[
\mathcal{L} = \sum_\sigma \overline{\psi}_\sigma \begin{pmatrix} mc_{sw}^2 + \partial_x & \cos k_x + i \cos k_y \\ -\cos k_x + i \cos k_y & mc_{sw}^2 - \partial_y \end{pmatrix} \psi_\sigma .
\]

Here we have used the fact that the origin of the Dirac fermion dispersion is at \(k_{1,2}\). The dispersion relation is given by

\[
\epsilon_k^\pm = \pm \sqrt{c_{sw}^2 (\cos^2 k_x + \cos^2 k_y) + (mc_{sw})^2}.
\]

The dispersion curve \(\epsilon_k^\pm - mc_{sw}^2\) is qualitatively in good agreement with the experimentally observed dispersion by Wells et al.

In our theory, \(Z_x\) and \(Z_y\) are determined by the parameters of the undoped system if one neglects spin wave effects on the self-energy of the half-skyrmion. For the \(S = 1/2\) antiferromagnetic Heisenberg model, the values of \(Z_x\) and \(Z_y\) are estimated as \(Z_x = 1.17\) and \(Z_y = 0.72\). Quantum Monte Carlo simulations [22] and a series expansion analysis [27]. Substituting these values into Eq. 4, we find that the band width along \((0, 0)\) to \((\pi, \pi)\) is \(\sim 1.5 J\) and the Dirac fermion mass is \(m \sim 1.13 J\). Experimentally, the band width is estimated to \(2.2 J\). This 30% discrepancy would be associated with the deviation of the real system from the NLoM description. There is also self-energy corrections on the mass \(m\) due to spin waves. We have estimated it to \(m J = 1.13/(1 - 0.12c_{sw}^2)\), where \(c_A\) is a coupling parameter between the half-skyrmion and the antiferromagnetic spin waves. The coupling parameter \(c_{sw}\) turns out to be a gauge charge in the dual theory below.

The effective action for the half-skyrmion can be derived by duality mappings [27]. In the CP1 theory, the half-skyrmion solution residing at the origin has the following form \(\varphi(x) = \frac{\lambda_{\mu\nu} + i\varphi_{\mu} \exp(\pm i\theta)}{\sqrt{2}}\), where \(u^* \cdot v = 0\), and \(|u| = |v| = 1\). (For the solution 3 with setting \(r_j = 0\), these parameters are \(v_\uparrow = u_\downarrow = 1\) and \(u_\uparrow = v_\downarrow = 0\).) If we take \(|v_\uparrow| = |v_\downarrow| \neq 0\), then the staggered moments lie in the plane with the same direction at infinity. In this boundary condition, \(\varphi(x) \sim \varphi_{\sigma} \exp(\pm i\theta)\) at \(r \gg \lambda\). We write \(z_\mu(x) = \rho_0^{1/2} \exp(i\theta(x))\), where \(\phi(x) = \phi_\sigma(x) + i\varphi_{\mu}(x)\). Here \(\phi_\sigma(x)\) is a non-singular function of \(x\) and \(\varphi_{\mu}(x)\) describes the vortex (half-skyrmion) field: \(\varphi_{\mu}(x) \sim q_{\sigma} \tan^{-1}(y - y_0)/(x - x_0)\), where \(q_\sigma\) is the sign of the topological charge and \((x_0, y_0)\) denotes the position of the vortex. (Note that \(x_0\) and \(y_0\) are functions of the imaginary time.) In applying the duality mapping, a dual gauge field is introduced. The coupling between the gauge field and the half-skyrmion has the form of the minimal coupling. Thus, we obtain the following QED3 Lagrangian density in the continuum,

\[
\mathcal{L} = \sum_\sigma \overline{\psi}_\sigma \begin{pmatrix} \gamma_\mu \partial_\mu - i q_\sigma A_\mu \end{pmatrix} + m|\psi_\sigma + \frac{1}{4e_A^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 .
\]
respect to the one “photon” vertex part is evaluated numerically and the first order term with respect to the two “photon” vertex part is computed analytically. While the former has negligible effect, the latter leads to \( \Sigma_\mu = -(i\sqrt{2e}\alpha/16) \sum_\mu \gamma_\mu \sin(k_\mu a) \int \frac{d^2k}{(2\pi)^2} \frac{1}{\sqrt{1-(\cos k_x+\cos k_y)/2}} \). The mass renormalization due to this term is \( m \to m/(1-0.12\alpha^2) \).

A similar action of Eq. [8] is obtained in the fermionic representation of the \( S=1/2 \) antiferromagnetic Heisenberg model \( \text{(17, 18)} \) based on the \( \pi \)-flux phase \( \text{(10)} \) with dynamically induced mass \( m \), through the interaction with a gauge field. (The condition of this dynamical mass generation \( \text{(25)} \) is that the number of Dirac fermion species is less than \( 32/\pi^2 \approx 3.2 \). This is the case for the QED\(_3\) theory of the antiferromagnet \( \text{\( \text{(15, 20)} \)} \).) From exact diagonalization studies it was shown \( \text{(30)} \) that the bond spin currents which characterize the \( \pi \)-flux phase is reproduced by a skyrmion-like spin texture \( \text{(31)} \). From the viewpoint of the spin-charge separation \( \text{(32)} \), Baskaran suggested \( \text{(33)} \) that a half-skyrmion can be seen as deconfined spinons based on an analysis of an \( n \)-skyrmion solution.

In summary, we have argued that, within the effective theory approaches to the \( S=1/2 \) Heisenberg antiferromagnet, the doped hole induces the half-skyrmion spin texture through the formation of the Zhang-Rice singlet in the single hole doped system. This picture is consistent with rapid suppression of the Néel ordering by the doped holes because the half-skyrmion behaves as a vortex in the condensate. It would be interesting to extend the picture to the slightly doped regime.

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K. A. Gschneidner Jr., L. Eyring, and M. B. Maple (North Holland, 2000).

[1] See, for example, M. Imada, A. Fujimori, and Y. Tokura, Rev. Mod. Phys. 70, 1039 (1998).

[2] See, for example, T. E. Mason, in Handbook on the Physics and Chemistry of Rare Earths, Eds.

[3] B. O. Wells et al., Phys. Rev. Lett. 74, 964 (1995).

[4] F. Ronning et al., Science 282, 2067 (1998).

[5] A. Damascelli, Z. Hussain, and Z. X. Shen, Rev. Mod. Phys. 75, 473 (2003).

[6] S. Chakravarty, B. I. Halperin, and D. R. Nelson, Phys. Rev. B 39, 2344 (1989).

[7] F. D. M. Haldane, Phys. Lett. 93A, 464 (1983).

[8] B. B. Beard et al., Phys. Rev. Lett. 80, 1742 (1998); J.-K. Kim and M. Troyer, Phys. Rev. Lett. 80, 2705 (1998).

[9] R. Rajaraman, Solitons and Instantons (North-Holland, 1987).

[10] D. P. Arovas and A. Auerbach, Phys. Rev. B 38, 316 (1988).

[11] N. Read and S. Sachdev, Phys. Rev. Lett. 62, 1694 (1989); Phys. Rev. B 42, 4568 (1990).

[12] A. Chubukov, Phys. Rev. B 44, 12318 (1991).

[13] T.-K. Ng, Phys. Rev. B 52, 9491 (1995); T.-K. Ng, Phys. Rev. Lett. 82, 3504 (1999).

[14] D. Yoshioka, J. Phys. Soc. Jpn. 58, 3733 (1989).

[15] J. E. Hirsh and S. Tang, Phys. Rev. B 39, 2850 (1989); M. Raykin and A. Auerbach, Phys. Rev. Lett. 70, 3808 (1993).

[16] I. Affleck and J. B. Marston, Phys. Rev. B 37, 3774 (1988).

[17] J. B. Marston, Phys. Rev. Lett. 64, 1166 (1990).

[18] D. H. Kim and P. A. Lee, Ann. Phys. 272, 130 (1999).

[19] F. C. Zhang and T. M. Rice, Phys. Rev. B 37, 3759 (1988).

[20] B. I. Shraiman and E. D. Siggia, Phys. Rev. Lett. 61, 467 (1988).

[21] C. Kane, P. A. Lee, and N. Read, Phys. Rev. B 39, 6880 (1989).

[22] A. A. Belavin and A. M. Polyakov, JETP Lett. 22, 245 (1975).

[23] A. Saxena and R. Dandoloff, Phys. Rev. B 66, 104414 (2002).

[24] See, for example, J. B. Kogut, Rev. Mod. Phys. 55, 775 (1983).

[25] R. B. Laughlin, Phys. Rev. Lett. 79, 1726 (1997).

[26] R. R. P. Singh, Phys. Rev. B 39, 9760 (1989).

[27] M. P. A. Fisher and D. H. Lee, Phys. Rev. B 39, 2756 (1989).

[28] R. D. Pisarski, Phys. Rev. D 29, 2423 (1984); T. Appelquist, D. Nash and L. C. R. Wijewardhana, Phys. Rev. Lett. 60, 2575 (1988).

[29] See also Z. Tešanović, O. Vafek, and M. Franz, Phys. Rev. B 65, 180511(R), 2002.

[30] R. J. Gooding, Phys. Rev. Lett. 66, 2266 (1991).

[31] P. B. Wiegmann, Phys. Rev. Lett. 60, 821 (1988); J. P. Rodriguez, Phys. Rev. B 39, 2906 (1989); A. Auerbach, B. E. Larson and G. N. Murphy, Phys. Rev. B 43, 11515 (1991); S. Haas et al., Phys. Rev. Lett. 77, 3021 (1996); E. C. Marino, Phys. Rev. B 61, 1588 (2000).

[32] P. W. Anderson, Science 235, 1196 (1987).

[33] G. Baskaran, Phys. Rev. B 68, 212409 (2003).