Non-Decoupling Effects of Higgs Bosons on 

\[ e^+e^- \rightarrow W^+_L W^-_L \] 
in the Two-Doublet Model

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Abstract

The non-decoupling effects of heavy Higgs bosons on the process \( e^+e^- \rightarrow W^+_L W^-_L \) are discussed in the two Higgs-doublet model. The one-loop corrections to the cross section are calculated by using the equivalence theorem and the explicit expressions of its deviation from the standard model are derived. The leading mass contributions to the deviation in expansion by \( s \) are related to the \( T \) parameter by the low-energy theorem. The next-to leading ones are free from the present data, which should be determined by future experiments. The deviation can amount to \( \sim 3 \% \) at \( \sqrt{s} = 1 \) TeV under the constraint from the present data, so that it may give useful information on the Higgs sector in cooperation with data from future linear colliders.

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1 Introduction

Radiative corrections can be one of the powerful tools for exploration of new physics in cooperation with the experimental precision data. The study of the oblique corrections, which are controlled by three parameters $S$, $T$ and $U$, has played a crucial role to determine properties of new physics with the data from LEP (and SLC) precision measurement. Standard Model (SM) has not been found to have any substantial deviation between its prediction and the data, while some types of new physics based on the dynamical breaking of electro-weak gauge symmetry have been strongly constrained by the data by virtue of their non-decoupling properties. Generally speaking, radiative corrections can be useful to constrain the new physics in which heavy mass effects do not decouple from a low energy observable.

There are models in which non-decoupling effects are expected but also are not completely constrained yet by the present data. To obtain substantial information about such models indirectly, we need to study radiative corrections to other types of observable than oblique ones, which are expected to be measured at LEP 2 or future experiments. One of these models may be an extension of SM with two Higgs doublets, in which the non-decoupling effects of heavy Higgs bosons may be expected to appear as well as those of the fermion contributions. The two Higgs-doublet model (THDM) has been studied so far with a lot of motivations such as additional CP phases, which are often required for the electro-weak baryogenesis, the strong CP problem and Minimal Supersymmetric Standard Model (MSSM). We note that in MSSM, the heavier Higgs bosons ($m_{H^0}^2$, $m_{H^+}^2$ and $m_{A^0}^2$) can become heavy only by making the soft-breaking parameter to be large because all the Higgs self-couplings are fixed to be $O(m_W^4/v^2)$. As a result, very heavy Higgs bosons, as well as all heavy super-particles, are decoupled from the low energy theory and the non-decoupling effects of these particles can be no longer expected in MSSM. We still stress that in the non-supersymmetric THDM, there remains a parameter region in which the model becomes a non-decoupling theory for heavy Higgs bosons.

One of the observables for which we can expect to have precision data next may be the scattering process $e^+e^- \rightarrow W^+W^-$. This process is not only one of the main target processes at LEP 2 but also expected to be well measured at future $e^+e^-$ linear colliders. Ahn et al. once showed that the non-decoupling effects of heavy fermions on this process can substantially
be enhanced by the factor $s/m^2_W$ in a region $m^2_W \ll \sqrt{s} < M$, where $M$ represents the heavy fermion mass. This enhancement disappears in the high energy limit, $M \ll \sqrt{s}$, because of the unitarity cancellation between $s$- and $t$-channel diagrams. The heavy mass effects on this process appear through the corrections to the triple gauge vertices (TGV’s) $W^+W^-Z^0$ and $W^+W^-\gamma$, as well as to the propagators of intermediate $Z^0$ and $\gamma$. Appelquist and Wu have studied the connection between TGV form factors and the chiral Lagrangian operators. The one-loop estimation of TGV’s has been made by several authors in SM, in the model with extra fermions such as technicolor-type theories, and in MSSM. The non-decoupling effects of heavy fermions have been one of their main interests. The possibility of the extra fermion generation and technicolr-like models, however, has already been constrained to considerable extent by careful comparison between the oblique parameters and the present precision data. On the other hand, the non-decoupling effects of additional heavy Higgs bosons have not been so studied so far. In spite of the constraint from the present data, THDM remains to have large allowed region for the Higgs boson masses and the mixing angles. Thus it is quite interesting to investigate the possibility to constrain the parameter region of THDM through the correction to the observables which are to be measured at future measurements. This is just our motivation to this work.

In this paper, we discuss the non-decoupling effects of heavy Higgs boson masses on the scattering of the polarized electron-positron into a longitudinal $W$ boson pair in THDM with a softly broken discrete symmetry. We investigate the cross section in the one-loop level ($O(1/(4\pi v)^2)$) and also estimate its deviation from SM, $\delta(s) = \sigma_{THDM}/\sigma_{SM} - 1$. Note that this quantity is related to the deviation of the TGV form factors $\kappa_V(s)$ and $g^V_1(s)$, which are defined in Ref. We obtain the explicit expressions for the leading (quadratic) and the next-to leading (logarithmic) contributions of the heavy Higgs masses to $\delta(s)$ in expansion by $s/M^2_{Higgs}$. We find that the leading contributions to $\delta(s)$ are written in terms of $\Delta \rho(= \alpha f T)$ parameter. This is regarded as a kind of the low energy theorem (Here low energy means $\sqrt{s} \ll M_{Higgs}$). It is understood from the fact
that the physical quadratic contributions of the heavy masses comes from an
unique term $\sim \beta_1 f^2 (\text{tr} T V_{\mu})^2$ in the chiral Lagrangian [13]. This term dis-
appears in the custodial $SU(2)_V$ symmetric limit [23]. The similar phenomenon
has been known to appear in the scattering such as $W^+_L W^_L \rightarrow W^+_L W^_L$ [24].
On the other hand, the next-to leading contributions include new additional
parameters other than $S, T$ and $U$. Thus these contributions can give sub-
stantial information about the new physics. In the MSSM like cases, the both
contributions turn out to vanish in the heavy Higgs limit, so that the heavy
mass effects are decoupled from the low-energy observable consistently. The
numerical study shows that the deviation $\delta(s)$ can amount to $\sim 2\text{-}3\%$ at
$\sqrt{s} = 1\text{ TeV}$ by the non-decoupling effects of the Higgs boson masses even
under the constraint from the present data as well as from the perturbative
unitarity [23]. We note that the accuracy may be expected to be smaller
than $\sim 2\%$ at $\sqrt{s} \sim 1\text{ TeV}$ by taking account of the ambiguity due to our
approximation as well as the ambiguity of the measurement by the expected
statistic and systematic errors at future $e^+ e^-$ linear colliders [26]. There-
fore we conclude that such deviation between THDM and SM on the process
may be detectable at future experiments and bring useful information on the
Higgs sector.

In Sec.2, we will summarize the results by the effective Lagrangian briefly.
THDM will be defined in Sec.3. Details of calculation will be shown in Sec.4.
The concrete expressions for the deviation from SM of the cross section are
derived and the non-decoupling properties are discussed in Sec.5. Results
will be summarized in the last section.

2 \( W^+ W^- Z^0, W^+ W^- \gamma \) vertices

The contribution of non-decoupling effects to $e^+ e^- \rightarrow W^+ W^-$ comes from
the corrections to the triple gauge vertices (TGV’s) [14] as well as the oblique-
type ones. All the corrections in other types of diagram are suppressed by the
electron masses or gauge boson masses. The effective Lagrangian for
TGV’s in the $C, P$ and $CP$ conserving case is expressed as

\[
\frac{\mathcal{L}_{WWV}^{WWV}}{g_{WWV}} = i g_1 V (W^+_{\mu} W^-_{\mu} V^{\nu} - W^-_{\mu} W^+_{\mu} V^{\nu}) + i \kappa_V W^+_{\mu} W^-_{\mu} V^{\mu\nu}
+i \frac{\lambda_V}{(4\pi v)^2} W^+_{\mu\nu} W^-_{\rho\nu} V^{\rho\mu},
\]

(1)
where $g_{WW\gamma} = -e$ and $g_{WWZ} = -e \cot \theta_W$. The tree level form factors are given in SM and also in THDM as

$$g^V_1 = \kappa_V = 1, \quad \lambda_V = 0.$$  \hfill (2)

The deviation from these tree-level values is generated at the loop level and they are denoted here as $\Delta g^V_1 = g^V_1 - 1$ and the same for others. It is known that $\kappa_\gamma$ and $\lambda_\gamma$ are related to the magnetic moment and quadrupole moment of $W^{\pm}$-bosons \[14\].

Appelquist and Wu have derived the relation between the form factors and the coefficients in the chiral Lagrangian \[15\]. From the context of the power counting method, it is expected that at one loop level there are quadratic mass contributions of inner heavy particles with the mass $M$ in $g^V_1$ and $\kappa_V$; namely $\sim O(M^2/(4\pi v)^2)$, where $v$ is the vacuum expectation value (VEV). These quadratic contributions actually appear if the new physics does not have the custodial $SU(2)_V$ invariance \[23\]. These occur through the dimension 2 operator \[15\]

$$\mathcal{L}'_1 \equiv \frac{1}{4} \beta_1 (4\pi v)^2 \left[ \text{tr}(TV_\mu) \right]^2,$$  \hfill (3)

where $V_\mu$ and $T$ are expressed in terms of the dimensionless unitary unimodular matrix field $U(x)$ as $V_\mu = (D_\mu U)U^\dagger$ and $T = U\tau_3 U^\dagger$. The dimensionless parameter $\beta_1$, which measures the breaking of $SU(2)_V$, is known to be related to $\alpha_f T$ ($= \Delta \rho$) as $\alpha_f T = 2\beta_1$, where $\alpha_f$ is the fine structure constant.

Note that the form factors, $g^V_1$, $\kappa_V$ and $\lambda_V$, should be considered as functions of the energy $\sqrt{s}$ in general. The next-to leading mass contribution in expansion by $s$ to the form factors may become important at high energy region \[16\]. Since these contributions have not been known yet, this can be expected to bring additional information for new physics in cooperation with the measurement at future linear colliders \[4\].

The helicity amplitudes for the polarized electron-positron scattering into a $W$-boson pair are expressed in terms of these form factors \[14\]. In the case of longitudinally polarized $W$-boson final states, the helicity amplitudes depend only on the combination of $g^V_1(s) + (s/2m_W^2)\kappa_V(s)$ even if there is no $C$, $P$, or $CP$ invariance in the model, where $V = \gamma$ and $Z^0$. We evaluate the correspondence to these form factors later.
3 Two Higgs-Doublet Model

Here we define THDM with a softly broken discrete symmetry; $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$. This model is the most general one in which the natural flavor conservation is realized [27]. Since the effects of CP violation disappear in the process $e^+e^- \rightarrow W_L^+W_L^-$ [14] as already mentioned in Sec. 2, we consider the CP invariant Higgs sector from the beginning. The Lagrangian of the Higgs sector is then given as

$$\mathcal{L}(\Phi_1, \Phi_2) = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + 2\mu_3^2 \text{Re}\Phi_1^\dagger \Phi_2$$

$$-\eta_1 |\Phi_1|^4 - \eta_2 |\Phi_2|^4 - \eta_3 |\Phi_1|^2 |\Phi_2|^2$$

$$-\eta_4 (\text{Re}\Phi_1^\dagger \Phi_2)^2 - \eta_5 (\text{Im}\Phi_1^\dagger \Phi_2)^2.$$ (4)

The Higgs sector then includes the eight parameters in general. Here we consider all the parameters to be free. (Note that MSSM is considered as a special case in this Lagrangian, in which the supersymmetry imposes strong relations between these parameters. In MSSM, all the quartic couplings are constrained into $O(g^2)$, where $g$ denotes weak gauge couplings. Thus in the heavy Higgs limit, which is realized by $\mu_3^2 \rightarrow \infty$, the Higgs mass-effects are decoupled from low-energy observables [10, 11].)

The Higgs doublets, both of which are assigned hypercharge as $Y = 1/2$, are parametrized as

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \sqrt{2}(v_i + h_i + i z_i) \end{pmatrix}, \ (i = 1, 2)$$

(5)

where vacuum expectation values $v_1$ and $v_2$ satisfy $\sqrt{v_1^2 + v_2^2} = v \sim 246$ GeV. The mass eigenstates are obtained by rotating the fields in the following way:

$$\begin{pmatrix} h_1^0 \\ h_2^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix},$$

(6)

$$\begin{pmatrix} w_1^+ \\ w_2^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} w^+ \\ H^+ \end{pmatrix},$$

(7)

$$\begin{pmatrix} z_1^0 \\ z_2^0 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} z^0 \\ A^0 \end{pmatrix}.$$ (8)

By setting $\tan \beta = v_2/v_1$, $w^\pm$ and $z^0$ become the Nambu-Goldstone bosons which are to be absorbed into the longitudinally polarized gauge bosons $W_L^\pm$
and $Z_L^0$, respectively. $H^\pm$ and $A^0$ are then massive charged and CP-odd neutral states. On the other hand, $h^0$ and $H^0$ are massive CP-even neutral states. The mixing angle $\alpha$ is taken in order $h^0$ to be lighter than $H^0$.

The quartic couplings are then represented by using the mass parameters, the mixing angles and VEV as

\[
\begin{align*}
\eta_1 &= \frac{1}{2v^2 \cos^2 \beta} (m_{H^0}^2 \cos^2 \alpha + m_{h^0}^2 \sin^2 \alpha) - \frac{\mu^2}{2v^2 \cos^3 \beta} \sin \beta \\
\eta_2 &= \frac{1}{2v^2 \sin^2 \beta} (m_{H^0}^2 \sin^2 \alpha + m_{h^0}^2 \cos^2 \alpha) - \frac{\mu^2}{2v^2 \sin^3 \beta} \cos \beta \\
\eta_3 &= \frac{\sin 2\alpha}{v^2 \sin 2\beta} (m_{H^0}^2 - m_{h^0}^2) + \frac{2m_{H^\pm}^2}{v^2} - \frac{2\mu_3^2}{v^2 \sin 2\beta} \\
\eta_4 &= -\frac{2m_{H^\pm}^2}{v^2} + \frac{4\mu_3^2}{v^2 \sin 2\beta} \\
\eta_5 &= \frac{2}{v^2} (m_{A^0}^2 - m_{H^\pm}^2)
\end{align*}
\]

In general, THDM does not have the custodial $SU(2)_V$ symmetry. But if $\eta_5$ is zero, the Higgs sector turns out to be $SU(2)_V$ symmetric even after the spontaneous breakdown of $SU(2)_L \otimes U(1)_Y$ gauge invariance occurs \[28\]. Therefore the mass splitting between $H^\pm$ and $A^0$ measures $SU(2)_V$-breaking in the Higgs sector \[12, 28\]. Note that there are some cases where we can take the mass splitting to be enough large within the constraint from the present data. For example, if we set $\alpha - \beta = \pi/2$ and $m_{H^\pm}^2 \sim m_{H^0}^2$, $m_{A^0}^2$ and $m_{h^0}^2$ can be chosen freely with keeping $\alpha_T T \sim 0$.

As for the Yukawa couplings, there can be two types of model in THDM (what we call, Model I and II in Ref. \[7\]) in which natural flavor conservation is realized by imposing discrete symmetries (see eq. (4)). Note that the difference between Model I and II vanishes in such the situation as we will consider later, where the mass of bottom quarks is negligible.

4 \hspace{1cm} e^+e^- \rightarrow W^+_LW^-_L in THDM

In this section, we show the one-loop calculation of the process $e^+_X e^-_Y \rightarrow W^+_LW^-_L$ in THDM, where $X$ ($Y$) is the helicity of the electron (positron). The calculation is quite simplified by making full use of the equivalence theorem
(ET) [20], which says that in the case of $\sqrt{s} \gg m_W$ the cross section for $e^+e^- \rightarrow W^+_L W^-_L$ is equivalent to that for $e^+e^- \rightarrow w^+w^-$ up to $O(m_W^2/s)$ [1].

The extension of ET to the loop level has been studied by several authors [21]. They found that some modification factors, which depend on gauge parameters, should be multiplied. He et al. showed that in SM the Landau gauge is a good choice for such the purpose here because the modification factors no longer depend on the Higgs boson mass and they can be set into unity within the approximation. We note that this situation is not changed even in the case of THDM [12, 30]. Thus the radiative correction here is calculated in the Landau gauge.

The non-decoupling effects of Higgs boson masses on $e^+e^- \rightarrow w^+w^-$ come from the corrections to the $V\mu w^+w^-$ vertices, $(V = \gamma, Z^0)$ in the $s$-channel gauge boson intermediate diagrams. Other types of diagrams (the neutrino exchanged $t$-channel and box-type diagrams) are always suppressed by powers of the electron mass or gauge boson masses. The oblique corrections in the $s$-channel diagrams are also neglected because of the suppression factor $m_W^2/M_{\text{Higgs}}^2$. (Note that this approximation is valid in the situation like $m_W^2 \ll s < M_{\text{Higgs}}^2$ or $m_W^2 \ll M_{\text{Higgs}}^2 < s$.) Thus we have only to calculate corrections to the $V\mu w^+w^-$ vertices for our purpose here. Moreover, it turns out that only the Higgs-Goldstone boson loops contribute to the vertices because the diagrams including a gauge boson loop are relatively suppressed by a factor $m_W^2/M_{\text{Higgs}}^2$ [12, 22].

The $V\mu w^+w^-$ vertices can be decomposed as

$$i\mathcal{M}_{Vww}(s) = i\mathcal{M}_{Vww}(s)(p_+ - p_-)\mu + i\mathcal{M}_{\tilde{V}ww}(s)(p_+ + p_-)\mu; \quad (14)$$

where $p_+$ ($p_-$) is the momentum of $w^+$ ($w^-$). Since the second term of RHS, which is proportional to $(p_+ + p_-)\mu$, produces the contribution suppressed by the negligible electron mass squared, we have only to calculate $\mathcal{M}_{\tilde{V}ww}(s)$ for our purpose. It is expressed by

$$i\mathcal{M}_{\tilde{V}ww}(s) = -ig_V \left\{ 1 + \mathcal{G}_V(s) + Z_w + \frac{\delta g_V}{g_V} \right\} \equiv -ig_V \Gamma_V(s), \quad (15)$$

where $\mathcal{G}_V(s)$ are the contributions of one-loop diagrams other than counterterms, $Z_w$ denotes the wave function renormalization constant for the external

\footnote{In general, the error is of $O(m_W/\sqrt{s})$. It turns out to be $O(m_W^2/s)$ in some cases including present process [31].}
$w^\pm$ lines and $\delta g_V$ are the shift of coupling constants defined by $g_V \rightarrow g_V + \delta g_V$. The tree level coupling constants $g_V$ are given by

$$g_Z = e \cot 2\theta_W, \quad g_\gamma = e,$$

where $e$ and $\theta_W$ denote the electric charge and the Weinberg angle respectively.

The scattering amplitude for the polarized $e^+e^-$ scattering into $w^+w^-$ is given in terms of $\Gamma_V(s)$ as

$$iA(e^-e^+_Y \rightarrow w^+w^-) = i e^2 \bar{\theta}_X \gamma_\mu u_X \left( \frac{\Gamma_\gamma(s)}{s} + f_{XY} \frac{\Gamma_Z(s)}{s - m_Z^2} \right) (p_+ - p_-)^\mu,$$

where $f_{XY}$ is defined according to the $e^-e^+$ helicities by

$$f_{LR} = \cot 2\theta_W, \quad f_{RL} = -\frac{\cos 2\theta_W}{2 \cos^2 \theta_W}.$$

The total cross section is then expressed in each case as

$$\sigma(e^-e^+_Y \rightarrow w^+w^-) = \frac{e^4 s}{24\pi} \left| \frac{\Gamma_\gamma(s)}{s} + f_{XY} \frac{\Gamma_Z(s)}{s - m_Z^2} \right|^2.$$

The estimation of $\Gamma_V(s)$ is performed in order. Here we concentrate into the contributions from the Higgs sector. At first, the one-loop contributions to $G_V(s)$ are calculated as

$$G_Z(s) = \frac{1}{(4\pi v)^2} \left[ \frac{2}{\cos 2\theta_W} \left\{ \tilde{C}[A^0 H^\pm H^0] \sin^2(\alpha - \beta) \right. \right.$$

$$\left. \left. \quad + \tilde{C}[A^0 H^\pm h^0] \cos^2(\alpha - \beta) \right\} \left. + \tilde{C}[w^+ H^0 w^\pm] \cos^2(\alpha - \beta) + \tilde{C}[w^0 h^0 w^\pm] \sin^2(\alpha - \beta) \right. \right.$$

$$\left. \left. \quad + \tilde{C}[H^+ H^0 H^\pm] \sin^2(\alpha - \beta) + \tilde{C}[H^\pm h^0 H^\pm] \cos^2(\alpha - \beta) \right] \right],$$

and

$$G_\gamma(s) = \frac{1}{(4\pi v)^2} \left\{ \tilde{C}[w^+ H^0 w^\pm] \cos^2(\alpha - \beta) + \tilde{C}[w^\pm h^0 w^\pm] \sin^2(\alpha - \beta) \right.$$

$$\left. \quad + \tilde{C}[H^\pm H^0 H^\pm] \sin^2(\alpha - \beta) + \tilde{C}[H^\pm h^0 H^\pm] \cos^2(\alpha - \beta) \right.$$
where \( p^2_+ = p^2_- = 0 \) is taken in the Landau gauge. The function \( \tilde{C}[123] \) is defined by

\[
\tilde{C}[123] = (m_1^2 - m_2^2)(m_3^2 - m_2^2)(C_{11} - C_{12})[123],
\]

(21)

where \( C_{11}[123] \) and \( C_{12}[123] \) can be written in terms of the \( B_0 \) and \( C_0 \) functions introduced by Passarino and Veltman [29]. We employ these notations here according to the definition in Ref. [3]. Secondly, the wavefunction renormalization \( Z_w \) is calculated up to \( \mathcal{O}(m_W^2/M_{Higgs}^2) \) as [12, 30]

\[
Z_w = -\frac{1}{(4\pi v)^2} \left\{ \frac{1}{2} (2m_{H^\pm}^2 + m_{h^0}^2 + m_{H^0}^2 + m_{A^0}^2) + \frac{m_{H^\pm}^2 m_{A^0}^2}{m_{A^0}^2 - m_{H^\pm}^2} \ln \frac{m_{H^\pm}^2}{m_{h^0}^2} + \cos^2(\alpha - \beta) \frac{m_{H^\pm}^2 m_{h^0}^2}{m_{h^0}^2 - m_{H^\pm}^2} \ln \frac{m_{H^\pm}^2}{m_{h^0}^2} + \frac{\sin^2(\alpha - \beta) m_{h^0}^2}{m_{h^0}^2 - m_{H^\pm}^2} \ln \frac{m_{H^\pm}^2}{m_{h^0}^2} \right\}
\]

(22)

Thirdly, the renormalization of the coupling constants \( \delta g_V \) is expressed as

\[
\frac{\delta g_Z}{g_Z} = \frac{1}{2 \cos^2 2\theta_W} \left( \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} - \sin^2 2\theta_W \frac{\delta \alpha_f}{\alpha_f} \right),
\]

(23)

\[
\frac{\delta g_{\gamma}}{g_{\gamma}} = \frac{1}{2} \frac{\delta \alpha_f}{\alpha_f}.
\]

(24)

where \( \delta \alpha_f \) is the shift of the fine structure constant and \( \delta m_V^2 \) are mass renormalizations defined as \( \alpha_f \rightarrow \alpha_f + \delta \alpha_f \) and \( m_V^2 \rightarrow m_V^2 + \delta m_V^2 \) (here \( V = W \) or \( Z \), respectively). Note that \( \delta \alpha_f \) is relatively suppressed by the factor \( m_W^2/M_{Higgs}^2 \). On the other hand, \( \delta m_V^2 \) are expressed up to \( \mathcal{O}(m_W^2/M_{Higgs}^2) \) as [32]

\[
\frac{\delta m_W^2}{m_W^2} = -\frac{1}{(4\pi v)^2} \left\{ \frac{1}{2} (2m_{H^\pm}^2 + m_{h^0}^2 + m_{H^0}^2 + m_{A^0}^2) + \frac{m_{H^\pm}^2 m_{A^0}^2}{m_{A^0}^2 - m_{H^\pm}^2} \ln \frac{m_{H^\pm}^2}{m_{A^0}^2} + \cos^2(\alpha - \beta) \frac{m_{H^\pm}^2 m_{h^0}^2}{m_{h^0}^2 - m_{H^\pm}^2} \ln \frac{m_{H^\pm}^2}{m_{h^0}^2} \right\},
\]

\[
\frac{\delta m_Z^2}{m_Z^2} = -\frac{1}{(4\pi v)^2} \left\{ \frac{1}{2} (m_{h^0}^2 + m_{H^0}^2 + m_{A^0}^2) + \cos^2(\alpha - \beta) \frac{m_{A^0}^2 m_{h^0}^2}{m_{h^0}^2 - m_{A^0}^2} \ln \frac{m_{A^0}^2}{m_{h^0}^2} \right\}.
\]

(25)
By inserting all these results into eq. (18), we finish to calculate the total cross sections at one loop level.

5 Non-Decoupling Effects of Higgs Bosons

Now we consider the non-decoupling effects of heavy Higgs masses on the ratio $R(s) = \sigma_{THDM}/\sigma_{SM}$, where the cross section in SM, $\sigma_{SM}(s)$, is to be calculated in the one-loop level in the same approximation manner as $\sigma_{THDM}(s)$. The one-loop corrections to $\Gamma_V$ (see eq. (15)) in SM case can be then immediately calculated as

$$\Gamma_{\gamma,Z}^{SM} = 1 + \frac{1}{(4\pi v)^2} \left\{ \tilde{C} \left[ w^+ \phi_{SM}^0 w^- \right] - \frac{1}{2} m_{\phi_{SM}}^2 \right\},$$

(26)

where $\phi_{SM}^0$ is the Higgs boson in SM. For the case of $e^-e^+$ helicity $LR$ ($RL$), the magnitude of $\sigma_{SM}(s)$ amounts at one loop level to $355 \sim 358$ (67.6 $\sim$ 68.3) fb at $\sqrt{s} = 500$ GeV and $98.9 \sim 98.5$ (19.7 $\sim$ 19.6) fb at $\sqrt{s} = 1000$ GeV for $m_{\phi_{SM}} = 140 \sim 1000$ GeV, respectively. These values are consistent with the previous results [13, 24] up to the ambiguity due to the use of the equivalence theorem. We did not include the fermion-loop contribution in these values. This is because that it takes the same form between THDM and SM, so that it consequently does not contribute to the deviation at all.

It is convenient to parametrize the ratio $R(s)$ as

$$R(s) = \frac{\sigma_{THDM}}{\sigma_{SM}} = 1 + \delta(s).$$

(27)

The deviation-function $\delta(s)$ is expressed in terms of the difference of the one loop corrections to $\Gamma_V$ between THDM and SM;

$$\delta(s) = 2\text{Re} \left[ \frac{1}{s} \left\{ \delta\Gamma_{\gamma}^{THDM} - \delta\Gamma_{\gamma}^{SM} \right\} + \frac{f_{XY}}{s - m_Z^2} \left\{ \delta\Gamma_{Z}^{THDM} - \delta\Gamma_{Z}^{SM} \right\} \right],$$

(28)

where one-loop corrections $\delta\Gamma_{V}^{model}$ are defined by $\Gamma_{V}^{model} = 1 + \delta\Gamma_{V}^{model}$. The quark loops do not contribute to $\delta(s)$ at all because of the exact cancellation between both models. As we mentioned before, $\delta(s)$ is expressed in terms of
\( \Delta g_1^V(s) \) and \( \Delta \kappa_V(s) \). We have the relation for \( m_W^2 \ll s \) as

\[
\delta(s) = \frac{4 \gamma^2}{1 + f_{XY}} \left\{ \left( 1 - \frac{\xi_{XY}}{2 \sin^2 \theta_W} \right) \left( \Delta \kappa_{THDM}^T(s) - \Delta \kappa_{SM}^T(s) \right) - \left( \Delta \kappa_{THDM}^- (s) - \Delta \kappa_{SM}^- (s) \right) \right\},
\]

where \( \xi_{LR} = 1, \xi_{RL} = 0 \) and \( \gamma = \sqrt{s}/2m_W \).

For extracting non-decoupling effects of heavy Higgs bosons, we expand \( \delta(s) \) by powers of \( s \) as

\[
\delta(s) = \delta^{(0)} + \delta^{(1)} s + O(s^2).
\]

Note that this expansion is valid only in the case with \( m_W \ll \sqrt{s} \ll M_{Higgs} \).

At one loop level \( O(1/(4\pi v)^2) \), all the non-decoupling effects are included in \( \delta^{(0)} \) and \( \delta^{(1)} \). By dimensional counting, we know that \( \delta^{(0)} \) represents the quadratic Higgs mass effects and \( \delta^{(1)} \) includes at most logarithmic ones.

We show the explicit expressions for \( \delta^{(0)} \) and \( \delta^{(1)} \) at one loop level. In calculation, we identify the lighter neutral Higgs boson \( h_0^\pm \) in THDM as SM like Higgs boson \( \phi^0_{SM} \). At first, \( \delta^{(0)} \) is calculated as

\[
\delta^{(0)} = \frac{1}{(4\pi v)^2} \frac{f_{XY}}{1 + f_{XY}} \left( \frac{2}{\cos 2\theta_W} + \frac{1}{\cos^2 2\theta_W} \right) \times \left[ F(m_{H_0^\pm}, m_{A_0}) + \sin^2(\alpha - \beta) \left\{ F(m_{H_0^\pm}, m_{H_0^\pm}) - F(m_{A_0}, m_{A_0}) \right\} \right.
\]
\[
+ \cos^2(\alpha - \beta) \left\{ F(m_{H_0^\pm}, m_{h_0}) - F(m_{A_0}, m_{h_0}) \right\} \right]
\]

where

\[
F(x, y) = \frac{x + y}{2} - \frac{xy}{x - y} \ln \frac{x}{y}.
\]

Comparing eq. (31) to the expression of the \( \Delta \rho \) \((= \alpha_f T)\) parameter [7], we have a relation

\[
\delta^{(0)} = \frac{f_{XY}}{1 + f_{XY}} \left( \frac{2}{\cos 2\theta_W} + \frac{1}{\cos^2 2\theta_W} \right) \alpha_f T.
\]
The second term in the bracket of RHS comes from $\delta g_Z$ in eq. (23). We can see that the leading effects, $\delta^{(0)}$, can be written in terms of $T$. This phenomenon is due to the low energy theorem and is understood as the concrete realizations of the fact which we discussed in Sec 2. In fact, this leading contribution $\delta^{(0)}$ vanishes if mass degeneracy between $A^0$ and $H^\mp$ exists. In this case, the Higgs sector becomes custodial $SU(2)_V$ symmetric, so that the term (3) is then forbidden. We note all the contributions to eq. (31) come from only $\delta \Gamma_Z$. As a result, the leading contribution is found not to be substantial because $T$ has already been fairly constrained by the present data.

The next-to leading contributions, $\delta^{(1)}$, may be possible candidates for the probe of the new physics. They include non-decoupling effects like $\sim \log M_{\text{Higgs}}$. These are extracted from eq. (28) as

$$\delta^{(1)} = \frac{2}{(4\pi v)^2} \left[ \left\{ G(0, m_{H^0}^2, 0) - G(0, m_{h^0}^2, 0) \right\} \cos^2(\alpha - \beta) + G(m_{H^\pm}^2, m_{h^0}^2, m_{H^\pm}^2) \sin^2(\alpha - \beta) + G(m_{H^\pm}^2, m_{h^0}^2, m_{H^\pm}^2) \cos^2(\alpha - \beta) \right] / (1 + f_{XY})$$

$$+ \frac{2f_{XY}}{(4\pi v)^2} \left[ \left\{ G(0, m_{H^0}^2, 0) - G(0, m_{h^0}^2, 0) \right\} \cos^2(\alpha - \beta) + \frac{2}{\cos 2\theta_W} \left\{ G(m_{A^0}^2, m_{H^\pm}^2, m_{h^0}^2) \sin^2(\alpha - \beta) + G(m_{A^0}^2, m_{H^\pm}^2, m_{h^0}^2) \cos^2(\alpha - \beta) \right\} \right.$$  
$$+ \left. G(m_{H^\pm}^2, m_{A^0}^2, m_{H^\pm}^2) \right] / (1 + f_{XY}) \right]$$

where $G(x, y, z)$ is the coefficient of the second term of $s$-expansion for $\tilde{C}$ function. This is expressed as

$$G(x, y, z) = \frac{1}{6(x - z)} \left\{ 2g_0(x, y, z) + g_1(x, y, z) + g_1(y, z, x) + g_1(z, x, y) \right\}$$

where the functions $g_0(x, y, z)$ and $g_1(x, y, z)$ are defined by

$$g_0(x, y, z) = \frac{-1}{(x - y)(y - z)(z - x)} \left\{ x^2 z^2 - x y z(x + z) + y^2(x^2 - x z + z^2) \right\}$$

$$+ \left. G(m_{H^\pm}^2, m_{A^0}^2, m_{H^\pm}^2) \right] / (1 + f_{XY}) \right]$$
Figure 1: The $\sqrt{s}$ dependence of $\delta(s)$ for $m_{A^0} = 800$ and 1200 GeV. The solid (dashed) lines represent $\delta(s)$ for the initial helicity states $e_L^- e_R^+$ ($e_R^- e_L^+$). The other parameters are fixed being taken account of the constraint from $T$ parameter by $m_{h^0} = 140$, $m_{H^0} = 350$, $m_{H^\pm} = 347$ GeV, and $\alpha - \beta = \pi/2$.

\[ g_1(x, y, z) = \frac{-1}{(x - z)^2(x - y)^2} x^2(y - z) \left\{ x^2 + x(y + z) - 3yz \right\} \ln x. \]  

Note that the function $G(x, y, z)$ vanishes if and only if we set $x = y$ or $y = z$. We can see from eq. (34) that $\delta^{(1)}$ does not vanish except for a few cases. One of the cases where $\delta^{(1)}$ vanishes is that with the complete degeneracy between all of the Higgs boson masses. Another one is the case with $\alpha - \beta \sim \pi/2$ and $m_{h^0}^2 \sim m_{H^\pm}^2 \sim m_{A^0}^2$. We note that the latter case occurs in MSSM with the large $m_{A^0}$ limit \[1\]. Since these cases also imply the custodial $SU(2)_V$ invariance in the Higgs sector, we find that the non-decoupling effects both $\delta^{(0)}$ and $\delta^{(1)}$ vanish simultaneously in these cases and the model then becomes a decoupling theory for Higgs boson masses \[10, 11\]. On the other hand, there are some cases in which $\delta^{(1)}$ becomes large to some extent with parameters satisfying the constraint from the present experimental data. For example, if we set $m_{H^0} \sim m_{H^\pm}$ and $\alpha - \beta = \pi/2$, the other masses $m_{h^0}$ and $m_{A^0}$ can be chosen freely with keeping $\alpha f T \sim 0$. In such cases, we can expect to obtain
Figure 2: The $m_A$ dependence of $\delta(s)$ at $\sqrt{s} = 500$ and 1000 GeV. The solid (dashed) lines represent $\delta(s)$ for the initial helicity states $e_L^{-}e_R^{+}$ ($e_R^{-}e_L^{+}$). The other parameters are fixed being taken account of the constraint from $T$ parameter by $m_{h^0} = 140$, $m_{H^0} = 350$, $m_{H^\pm} = 347$ GeV, and $\alpha - \beta = \pi/2$.

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In the situation such as the momentum $\sqrt{s}$ comparable to the largest mass of the Higgs bosons, the question whether the non-decoupling effects become relatively large or not may occur. In this case, the expansion above is no longer allowed, so that we have to investigate the non-decoupling effects only through numerical estimation. The $\sqrt{s}$ dependence of $\delta(s)$ is described in Figure 1. The parameters are chosen in order to satisfy the constraint from the present data. We here chose $\alpha - \beta = \pi/2$, $m_{h^0} = 140$, $m_{H^0} = 350$, and $m_{H^\pm} = 347$ GeV for satisfying $-3.8 \times 10^{-3} < \alpha_f T_{THDM} < 2.6 \times 10^{-4}$ \cite{3}, where $T_{THDM}$ is the additional contribution to $T$ parameter in THDM. We can see that the behavior changes according to the relative energy scale to $M_{Higgs} \sim m_{A^0}$. The deviation by non-decoupling effects is enhanced by $s$ for $\sqrt{s} < m_{A^0}$ but is reduced for very high energy region as $m_{A^0} \ll \sqrt{s}$. This behavior is consistent with the result for the fermion effects by Ahn et al. \cite{13} that the enhancement disappears in the high energy limit because of
the unitarity cancellation between $s$- and $t$-channel diagrams. In Figure 2, we show the $m_A$-dependence of $\delta(s)$ at $\sqrt{s} = 500$ GeV and 1000 GeV in the same choice for other parameters as Fig.1. We can see in Figs. 1 and 2 that the large mass difference (around $\sqrt{s}$) between $m_{A^0}$ and $m_{H^0} \sim m_{H^\pm}$ tends to produce the large deviation. At $\sqrt{s} = 1000$ GeV, it amounts to $\sim 3\%$ for $m_{A^0} = 1200$ GeV. The deviation for helicity LR is larger than that for RL in general. Note that all the parameter choice here is also taken account of the constraint from the perturbative unitarity [25].

6 Conclusion

We have discussed the non-decoupling effects of the heavy Higgs bosons on the scattering process $e^- L e^+_R$ (or $e^- R e^+_L$) $\rightarrow W^+_L W^-_L$ in THDM. The cross section has been calculated at one-loop level $O(1/(4\pi v)^2)$ by making full use of the equivalence theorem. The effects of heavy Higgs bosons have extracted in the ratio of the cross section between THDM and SM, $R(s)(= 1 + \delta(s))$. The leading (quadratic) contributions of the masses to $\delta(s)$ become to be written in terms of $T$-parameter. This phenomenon is regarded as the result by the low energy theorem and can be understood by the chiral Lagrangian approach. On the other hand, the next-to leading (logarithmic) contributions include the additional parameters other than oblique ones. The next-to leading contributions do not vanish in general except for a few cases, so that they may be useful for the indirect exploration of New physics by combining with data from future $e^+ e^-$ colliders. One of the exceptional cases is that with $\cos^2(\alpha - \beta) \sim 0$ and $m_{H^\pm}^2 \sim m_{H^0}^2 \sim m_{A^0}^2$, which corresponds to MSSM in large $m_A$ limit. In these cases, both the leading and the next-to leading contributions simultaneously vanish and the model then becomes a decoupling theory as expected. Otherwise, the non-decoupling effects on $\delta^{(1)}$ exist and can become large with keeping the constraint $\delta^{(0)} \propto \alpha_f T \sim 0$. One example for such cases may be $m_{H^\pm} \sim m_{H^0}$ and $\alpha - \beta \sim \pi/2$. Then the values of $m_{A^0}$ and $m_{H^0}$ can be taken freely with keeping $\alpha_f T \sim 0$. Actually we have numerically found that in these cases there can be the relatively large deviation from SM, which amounts to 2-3\% (see Fig.2) at $\sqrt{s} = 1$ TeV for large $m_{A^0}$ but within the constraint from the perturbative unitarity. Therefore the non-decoupling effects by heavy Higgs bosons on this process can become large to some extent, so that they may be constrained by the
data from future $e^+e^-$ linear colliders.

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