Limiting stiffness coefficients analysis of texture foil journal bearing

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Abstract. The purpose of this study is to explore the limiting stiffness coefficients of a foil journal bearing with the texture bump profile of top foil. The limiting stiffness coefficients are evaluated based on simplified compressible Reynolds equation for large bearing numbers with high speeds of foil journal bearing. A limiting pressure gradient solution for a texture bump of top foil bearing is analyzed. The analytical model accounts for the top foil texture bump profile and bottom foil bump compliance. Results of linearized nondimensional stiffness coefficients obtained using infinitesimal perturbation method are compared for various top foil texture bump profiles. The influence of top foil texture bump extent and height on the limiting stiffness coefficients of a foil journal bearing are investigated.

1. Introduction
Foil bearings are compliant bearing surfaces lubricated with air/gas. Heshmat et al. [1] investigated the static performance of bump foil bearings considering elastic deformation of the compliant foil structure. Based on Lund’s [2] perturbation method, Peng and Carpino [3] determined the stiffness and damping coefficients of elastically supported foil bearing. The effects of compliance of elastic support, coulomb friction, membrane and bending effects in a foil bearing are considered. Radil et al. [4] investigated the effect of an optimum radial clearance on the maximum load carrying performance of foil bearing. Peng and Khonsari presented the pressure and load capacity of foil bearings under limiting conditions using analytical [5] and numerical [6] methods. The limiting conditions for high bearing numbers (journal operating at high speed) are obtained by simplifying the compressible Reynolds equation. Based on the analysis of Peng and Khonsari [5] for limiting load capacity of foil bearing, Sawicki and Rao [7] derived foil bearing dynamic coefficients. Rubio and San Andres [8] investigated the effects of geometrical and operating parameters on the foil bearing performance using theoretical and experimental methods. Kim and San Andres [9] analyzed the double-stage bump foil characteristics on improvement in bearing load capacity, stiffness and damping coefficients. Della Corte [10] presented the challenges and opportunities of turbo-machineries operating with foil bearings. Samanta et. al. [11] highlighted the developmental path for tackling different challenges associated with the foil bearing technology and their evaluation over the last five decades.

The purpose of this study is to investigate limiting stiffness coefficients of texture bump foil journal bearing. A limiting pressure and pressure gradient solution for a texture bump foil bearing is presented.
2. Analysis

The necessary conditions for very high speed operation of foil journal bearing ($\Lambda \to \infty$) obtained by simplifying the compressible Reynolds equation is [5]

$$\frac{\partial (PH)}{\partial \theta} = 0$$  \hspace{1cm} (1)

The schematic of texture bump foil is shown in figure 1. The texture bump region extends along circumferential direction from the position fixed to negative X-axis (load line depicted in Fig. 2). The texture bump region is considered on the top foil surface. The texture bump foil bearing nondimensional film thickness is:

$$H = 1 + \varepsilon \cos(\theta - \phi) - H_b + \alpha(P_o - 1)$$  \hspace{1cm} (2)

The steady state pressure and pressure gradients are obtained by infinitesimal perturbation [2] of the simplified compressible Reynolds equation. The pressure distribution and film thickness for infinitesimally small perturbations about the journal steady state is:

$$P = P_o + P_x \Delta X + P_y \Delta Y$$  \hspace{1cm} (3)
$$H = H_o + H_x \Delta X + H_y \Delta Y$$  \hspace{1cm} (4)

where

$$H_o = 1 + \varepsilon_o \cos(\theta - \phi_o) - H_b + \alpha(P_o - 1), \quad H_x = \cos \theta + \alpha P_x, \quad H_y = \sin \theta + \alpha P_y$$  \hspace{1cm} (5)

Substituting Eqs. (3-4) in Eq. (1), the steady state and dynamic simplified compressible Reynolds equations are

$$\frac{\partial (P_o H_o)}{\partial \theta} = 0$$  \hspace{1cm} (6)
$$\frac{\partial}{\partial \theta} \left( P_o H_o + P_x H_x \right) = 0$$  \hspace{1cm} (7)
$$\frac{\partial}{\partial \theta} \left( P_o H_o + P_y H_y \right) = 0$$  \hspace{1cm} (8)

The texture bump foil bearing limiting pressure distribution is obtained by solving Eq. (6) using the pressure boundary conditions at inlet ($\theta = 0$) as [5]

$$P_o = \frac{1 + \varepsilon_o \cos \phi_o - H_b}{1 + \varepsilon_o \cos(\theta - \phi_o) - H_b + \alpha(P_o - 1)}$$  \hspace{1cm} (9)
The limiting pressure is simplified from Eq. (9) as
\[ P_o = \frac{-(1+\varepsilon_o \cos(\theta-\phi_o)-H_b-\alpha) + \sqrt{(1+\varepsilon_o \cos(\theta-\phi_o)-H_b-\alpha)^2 + 4\alpha(1+\varepsilon_o \cos\phi_o-H_b)}}{2\alpha} \] (10)

Substituting Eq. (5) in Eqs. (7-8), reduce to
\[ \frac{\partial}{\partial \theta}(P_o \cos \theta + P_x(\alpha P_o + H_o)) = 0 \] (11)
\[ \frac{\partial}{\partial \theta}(P_o \sin \theta + P_y(\alpha P_o + H_o)) = 0 \] (12)

The boundary conditions are:
\[ P_o = 1; \quad P_x = P_y = 0 \text{ at } \theta = 0 \] (13)
\[ P_x = \frac{1-P_o \cos \theta}{1+2\alpha P_o+\varepsilon_o \cos(\theta-\phi_o)-H_b} \] (14)
\[ P_y = \frac{-P_o \sin \theta}{1+2\alpha P_o+\varepsilon_o \cos(\theta-\phi_o)-H_b} \] (15)

The nondimensional pressure is integrated to obtain the limiting nondimensional load as
\[ \left\{ \begin{array}{l} F_x \\\ \\
F_y \end{array} \right\} = \int_{\theta=0}^{\theta=2\pi} P_o \left\{ \begin{array}{l} \cos \theta \\
\sin \theta \end{array} \right\} d\theta \] (16)

The bearing stiffness coefficients \( K_{xx}, K_{yy}, K_{xy}, K_{yx} \) are evaluated from pressure perturbations as
\[ \left\{ \begin{array}{l} K_{ij} \\
K_{ji} \end{array} \right\} = \int_{\theta=0}^{\theta=2\pi} P \left\{ \begin{array}{l} \cos \theta \\
\sin \theta \end{array} \right\} d\theta \text{ for } j = x, y \] (17)

3. Results and discussion
The limiting values of load capacity for a texture bump foil bearing are presented. Figures 1-2 show the nondimensional load capacity of a texture bump foil journal bearing. The nondimensional load capacity \( W \) increases with increasing nondimensional texture bump height \( H_b \) for a critical value of texture bump extent \( (\theta_t) \). The nondimensional load capacity \( W \) does not vary with increasing nondimensional texture bump height \( H_b \) at higher bump foil compliance coefficient \( (\alpha=10) \).

![Figure 3. Nondimensional load capacity (\( \alpha=1 \))](image3.png)

![Figure 4. Nondimensional load capacity (\( \theta_t=180^{\circ} \))](image4.png)
The limiting values of nondimensional stiffness coefficients \((K_{ij})\) for texture bump foil bearing are presented in Figs. 5-8. The nondimensional stiffness coefficients \((K_{ij})\) showed higher variation with increasing nondimensional texture bump height \((H_b)\) at lower eccentricity ratio \((\epsilon=0.2)\). The nondimensional stiffness coefficients perpendicular to load line \((K_{ij})\) showed variation with increasing extent of texture bump region \((\theta_t)\). The nondimensional pressure gradients with perturbation along the direction perpendicular to load line \((P_y)\) are significantly influenced by the extent of texture bump region \((\theta_t)\) from position fixed to negative X-axis. Similar to the nondimensional load capacity \((W)\), as bump foil compliance coefficient increases from \(a=1\) to \(a=10\), the nondimensional stiffness coefficients \((K_{ij})\) does not show considerable variation with increasing nondimensional texture bump height \((H_b=0-1.5)\).

**Figure 5.** \(K_{ij} (\epsilon=0.2, \alpha=1)\).

**Figure 6.** \(K_{ij} (\epsilon=0.8, \alpha=1)\).

**Figure 7.** \(K_y (\epsilon=0.2, \theta_t=180^\circ)\).

**Figure 8.** \(K_y (\epsilon=0.8, \theta_t=180^\circ)\).
4. Conclusions
This paper presents the influence of texture bump height on the limiting values of nondimensional load capacity and nondimensional stiffness coefficients of a foil journal bearing. The limiting solutions are based on the compressible Reynolds equation for high bearing numbers. A texture bump region extent with critical values of extent along the circumferential direction on bearing surface provides higher load capacity. The location and extent of bump influences the nondimensional stiffness coefficients of a texture bump foil bearing at low eccentricity ratios.

The texture bump region has potential to enhance the foil bearing characteristics at low eccentricity ratio operations.

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Nomenclature

- \( C \): Radial clearance, m
- \( h, H \): Film thickness, m; \( H = h/C \)
- \( H_b \): Nondimensional height of texture bump
- \( k_{ij}, K_{ij} \): Stiffness coefficients, Ns/m; \( \bar{K}_{ij} = k_{ij}C / p_a LR \), \( K_{ij} = \bar{K}_{ij} / W \), \( i, j = x, y \)
- \( L \): Length of bearing, m
- \( p_a \): Ambient pressure, N/m²
- \( p, P \): Pressure, N/m²; \( P = p / p_a \)
- \( R \): Radius of journal, m
- \( x, y, X, Y \): Coordinates with respect to bearing center, m; \( X = x/C \), \( Y = y/C \)
- \( \Delta X, \Delta Y \): Displacements (nondimensional) in \( x \) and \( y \) directions from the equilibrium position
- \( \alpha \): Bump foil compliance coefficient
- \( \varepsilon \): Eccentricity ratio
- \( \theta \): Circumferential coordinate measured from the position fixed to the negative X-axis
- \( \phi \): Attitude angle
- \( \omega \): Angular velocity, rad/s
- \( \Lambda \): Bearing number; \( \Lambda = 6 \mu \omega R^2 / p_a C^2 \)
- \( w, W \): Static load, N; \( W = w / p_a LR \)
- \( \theta_i \): Angle coordinate measured from position fixed to negative X-axis for texture bump region

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