Dark energy without fine tuning

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Abstract: We present a two-field model that realises inflation and the observed density of dark energy today, whilst solving the fine-tuning problems inherent in quintessence models. One field acts as the inflaton, generically driving the other to a saddle-point of the potential, from which it acts as a quintessence field following electroweak symmetry breaking. The model exhibits essentially no sensitivity to the initial value of the quintessence field, naturally suppresses its interactions with other fields, and automatically endows it with a small effective mass in the late Universe. The magnitude of dark energy today is fixed by the height of the saddle point in the potential, which is dictated entirely by the scale of electroweak symmetry breaking.
1 Introduction

Explaining the current accelerated expansion of the Universe [1, 2] with a cosmological constant \( \Lambda \) requires an unacceptable amount of fine tuning, due to the extreme smallness of the observed density of dark energy \( \rho_\Lambda \approx 2.5 \cdot 10^{-47} \text{ GeV}^4 \) [3–6]. A possible and popular mechanism for alleviating this fine tuning is to construct a theory where the energy of the vacuum is strictly zero, forbidding a \( \Lambda \) term in the Lagrangian. For example, this is often achieved by invoking some additional (often unspecified) symmetry. The current observation of an approximately constant density of dark energy is then explained by the gradual dynamics of fields [7].

Similarly to cosmic inflation, many popular models for explaining dark energy contain slowly-rolling scalar fields [8, 9]. These are usually referred to as “quintessence” models. However, they frequently suffer from issues related to fine tuning and naturalness. Even if not always the case [10–13], predictions are often highly sensitive to the initial conditions. Successful quintessence generally requires very small (i.e. fine-tuned) parameters, such as masses of the order of \( m \sim 10^{-33} \text{ eV} \). Similarly, couplings of the quintessence field to other fields, which should otherwise be allowed by the known gauge and Lorentz symmetries of the Standard Model (SM), must also be heavily suppressed or screened [14–17] in order to avoid fifth-force constraints [18–20; see also recent experimental proposals 21, 22].

A rather unexpected connection was recently discovered [23] between the observed dark energy density \( \rho_\Lambda \), cosmic inflation and electroweak symmetry breaking. This relation is

\[
\rho_\Lambda \approx \frac{v^8}{P_c M_P^4}, \tag{1.1}
\]
where $P_\zeta \approx 2.2\cdot 10^{-9}$ is the observed amplitude of the spectrum of primordial perturbations at microwave background scales [24], $M_P$ is the reduced Planck mass and $v = 246$ GeV is the vacuum expectation value (VEV) of the Higgs field $h$. The relation (1.1) is more than just a curious coincidence between parameters; in many models this combination arises as the magnitude of the potential energy at electroweak symmetry breaking (EWSB) left over from the interplay of the Higgs boson and the inflaton [23].

Unfortunately, (1.1) is not a panacea for all fine-tuning problems of quintessence. Even if (1.1) can generate the scale of dark energy, successful quintessence still requires a small enough effective mass, the suppression of couplings to other fields, and fine-tuned initial conditions. The main result of this work is to show that all these issues can be naturally resolved in a two-field model, whilst simultaneously explaining the observed magnitude of dark energy via (1.1).

Throughout this paper, we use positive sign conventions for the metric, Riemann tensor and Einstein equation $(+, +, +)$ [25].

## 2 Dark energy scale from inflation and EWSB

The tree-level potential for the Higgs at $T = 0$ can be conveniently parameterised as

$$V(h) = \frac{\lambda}{4} (h^2 - v^2)^2,$$

where the vacuum energy is assumed to strictly vanish at $h = v$. This in accordance with the discussion in the introduction; we will take it as an assumption that the potential energy strictly vanishes in the vacuum $h = v$, with the hope that in a more complete theory it has a more fundamental motivation. The crucial feature for our purposes is that the potential energy from the Higgs changes at EWSB. Above the critical temperature $T_{EW}$ at which the mass parameter of the Higgs potential $m^2_H(T_{EW}) = 0$, the potential is minimised at $h = 0$. At $T < T_{EW}$, we see that $m^2_H(T < T_{EW}) < 0$, such that the potential recovers its familiar Mexican-hat shape and $h = 0$ is rendered a local maximum of the potential [26].

As the thermal corrections do not affect the value of $V(0)$, this can be summarised as

$$V(h_{\text{min}}) = \begin{cases} V(0) = \frac{\lambda}{4} v^4, & T > T_{EW} \\ V(v) = 0, & T < T_{EW} \end{cases}.$$  

To understand how inflation, the Higgs and EWSB can lead to a potential energy of magnitude (1.1) following EWSB, consider the following action for the inflaton $\phi$:

$$S = - \int d^4 x \sqrt{-g} \left\{ \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + U(\phi, h) \right\} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + c \frac{\phi}{M_P} V(h).$$

In addition to the familiar quadratic piece defining the bare inflaton mass $m$, the potential of this theory possesses a linear Planck-suppressed coupling of the inflaton to the Higgs,

$$U(\phi, h) = \frac{1}{2} m^2 \phi^2 + c \frac{\phi}{M_P} V(h).$$

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where $c$ is a number characterising the strength of this interaction.

Before EWSB, the Higgs has non-zero vacuum energy $V(0) = (\lambda/4)v^4$. This causes the minimum of the inflaton potential to be very slightly displaced from the origin,

$$U'(\phi_0,0) = 0 \iff \phi_0 = -\frac{cV(0)}{M_Pm^2} = -\frac{c\lambda v^4}{4M_Pm^2}; \quad T > T_{EW}.$$  \hfill (2.5)

After EWSB, once the Higgs reaches its (new) minimum $h = v$, a second rolling of the inflaton is triggered starting from $\phi = \phi_0$. The initial value of the potential energy at the beginning of this rolling phase is

$$U(\phi_0,v) = \frac{c^2\lambda^2v^8}{32M_P^2m^2}; \quad T < T_{EW}.$$  \hfill (2.6)

For the quadratic model of inflation, $m \approx 6 \cdot 10^{-6}M_P \sim 10^{-1}\sqrt{\rho_\Lambda}M_P$. Choosing $|c| \approx 1.7$ and using $M_P = 2.43 \cdot 10^{18}$ GeV, $\lambda = 0.129$ and $v = 246$ GeV, we see that

$$\rho_\Lambda \approx \frac{(10c\lambda v^4)^2}{32M_P^2\rho_\Lambda} \approx 2.5 \cdot 10^{-47}\text{GeV}^4,$$  \hfill (2.7)

which agrees with the observed value of $\rho_\Lambda$ [6]. Dark energy with the magnitude (1.1) is thus a direct prediction of inflation and electroweak physics in this setup, as long as the potential energy does not change significantly after EWSB. The relation (2.6) is a consequence of the specific linear coupling to the Higgs $\phi M_PV(h)$. Here we specifically focus on the relevant interactions for our mechanism to work, tacitly staying within the vicinity of the minimum $\phi_0$, where the linear term dominates. For now, we will assume that problems such as boundedness of the potential from below are resolved by additional, unspecified, higher-order terms in the potential. In Section 5 we present a realistic scenario that indeed has no such issues, due to the presence of additional higher-order interactions. The symmetries of the theory also permit a portal term $\phi_2h^2$, which should therefore also be present. However, such a term has only a very small impact on the end result, as the mass of the inflaton is significantly larger than the electroweak scale. On the other hand, a linear term of the form $\phi M^3$ with a scale $M^3 \gtrsim \frac{1}{M_P}V(0) \sim \frac{v^4}{M_P}$ would spoil the relation (2.6).

Qualitatively the same features as in (2.4) are present in the Starobinsky model of inflation [27], with the added bonus that a coupling between the Higgs and the inflaton is generated automatically [23]. This is a feature often present in those scalar-tensor theories where the Planck mass is promoted to a function of a field, e.g. [28].

Unfortunately, even if the dark energy density $\rho_\Lambda$ somewhat miraculously arises as a function of known and observable scales, the inflaton remains much too heavy to act as a quintessence field. For it to roll slowly today, its mass must be much smaller than the current Hubble rate. This is an extremely small number, which one can estimate by neglecting all but dark energy in the current cosmic energy budget

$$H_0^2M_P^2 \sim \rho_\Lambda \iff H_0 \sim \frac{\sqrt{\rho_\Lambda}}{M_P} \sim 10^{-33}\text{eV} \ll m.$$  \hfill (2.8)

Fortunately, there are ways around this problem.
Slow rolling from a non-canonical kinetic term

The general idea of ameliorating the problems of quintessence with non-canonical kinetic terms has existed for some time [see e.g. 29–33]. In this vein, let us now modify the theory characterised by the action (2.3) by taking the kinetic term to have a non-canonical form

\[(\partial_\mu \phi)^2 \rightarrow \left( \frac{M_P}{\phi} \right)^2 (\partial_\mu \phi)^2 \equiv (\partial_\mu \chi)^2, \tag{3.1}\]

with the same notation as introduced in Appendix A. In the minimum defined by (2.5), the potential after EWSB now has the behaviour

\[\tilde{U}(\chi_0, v) \sim \rho \Lambda; \quad \tilde{U}''(\chi_0, v) = \frac{4 \tilde{U}(\chi_0, v)}{M_P^2} \sim H_0^2, \tag{3.2}\]

which is precisely of the type required for quintessence of the thawing variety [34], due to the enormous stretching of the potential in the canonical coordinate ($\chi$) caused by the pole at the origin in $\phi$. By introducing an $O(1)$ coupling in front of the non-canonical kinetic term (3.1), one may evade current observational bounds on the parameter of state for dark energy [23]. We explore these bounds quantitatively for our two-field model in Section 4.

The modification (3.1) not only makes the mass of $\chi$ effectively small (see Appendix A), but also suppresses all interactions with other fields. Consider, for example, a Yukawa coupling of the form $g \phi \bar{\psi} \psi$. Close to the minimum (2.5) in the canonical variable $\chi$, the effective Yukawa coupling $\tilde{g}$ is

\[\tilde{g} \sim \sqrt{\frac{\rho \Lambda}{m^2 M_P^2}} g, \tag{3.3}\]

which constitutes an interaction with a completely negligible strength. This is very similar to the suppression of interactions that occurs in $\alpha$-attractor models of inflation [35, 36].

Unfortunately, by solving one problem we have created another one. The term (3.1) will also make it extremely difficult for the inflaton field to reach the minimum (2.5) in the first place. In the early Universe, a very light field will not roll freely, but will be stopped by Hubble friction long before ever reaching such a minimum. Decay via interactions (required for reheating) is also virtually impossible, due to the manifest suppression of all interactions, as seen e.g. in the example of the Yukawa interaction (3.3). This is of course not a problem unique to our scenario, but rather a common feature of quintessence models (albeit not all quintessence models, as discussed in Section 1). Avoiding this usually requires very careful fine-tuning of the initial field conditions in the early Universe, such that the initial value of the quintessence field is already very close to its minimum.

To avoid this issue more naturally, we need a mechanism that first allows the field to reach the minimum unhindered, and only then triggers the non-canonical kinetic behaviour (3.1). In Ref. [23], this was dubbed the bait-and-switch mechanism, and several examples were also provided. The most natural one comes by coupling (3.1) with the Higgs, such that the kinetic term is canonical up until EWSB, providing ample time for the relaxation into the minimum. Although not problematic at the classical/mean-field level, coupling the kinetic term to the Higgs introduces interactions that are likely already excluded by
collider bounds (on e.g. invisible Higgs decays). The other examples provided in Ref. [23] were more for illustrative purposes, and not expected to be easily realised in top-down approaches. In Ref. [37], a mechanism designed to extend that of Ref. [23] was presented, requiring a very large non-minimal coupling between the Higgs and the scalar curvature of gravity, of the order $|\xi| \sim 10^{32}$.

In the next Section, we show that this issue has a natural resolution in models where inflation and dark energy are not sourced by the same field (as we have assumed so far).

4 A two-field model: assisted relaxation

So far we have only discussed a situation where inflation and dark energy are given by the same field $\phi$, i.e. quintessential inflation [38]. The mechanism that we now present relies on the relation (1.1) and the stretching of the potential by a non-canonical kinetic term of the form (3.1), but with the crucial difference that it includes a second field in addition to the inflaton. It is this additional field that sources dark energy in the late Universe, in contrast to the scenario discussed in Ref. [23]. What we will show is that the dynamics of the inflaton can assist and effectively force the relaxation of a field with a flat potential into the pre-EWSB minimum. This provides a natural resolution of the initial-value problem, leading to a model that does not suffer from any of the usual fine-tuning issues of quintessence.

Our mechanism likely has many manifestations, but for simplicity we will first focus on the following action

$$S = -\int d^4x \sqrt{-g}\left\{\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + b \frac{M_P^2}{\phi^2 + \sigma^2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} m^2 \sigma^2 + c \frac{\sigma}{M_P} V(h)\right\},$$

$$\equiv -\int d^4x \sqrt{-g}\left\{\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \tilde{U}_\sigma(\chi, h)\right\} \equiv S_\phi + S_\sigma,$$  \hspace{1cm} (4.1)

where $\phi$ is the inflaton and $\sigma$ the quintessence field. The constants $b$ and $c$ are $O(1)$ dimensionless couplings. Crucially, we assume that $\sigma$ couples to the Higgs similarly to (2.4), allowing the field to possess a non-zero VEV before EWSB.

If $\phi > M_P$ during inflation, as expected for quadratic inflation, the non-canonical kinetic term in (4.1) effectively makes $\sigma$ heavy enough to roll, even when close to $\sigma = 0$ (see Appendix A). This means that for a large range of initial conditions, if inflation lasts long enough $\sigma$ will be driven towards its minimum by the inflaton, due to the kinetic coupling. We refer to this process as assisted relaxation.

When inflation has ended and the Universe has reheated, $\phi = 0$, and the same kinetic term will make $\sigma$ exponentially light. This allows $\sigma$ to source dark energy precisely as discussed in Section 3. Furthermore, as we showed in Section 2, a linear coupling to the Higgs potential and a tree-level mass of the order required for successful inflation ($m \sim 10^{-6} M_P$) leads to an excellent match to observations; this is indeed why we chose the masses of $\phi$ and $\sigma$ to be identical. Conversely, if the underlying inflationary model is not quadratic, this is also the mass scale that should be chosen for $\sigma$ in order to avoid spoiling the relation (2.7).
Let us explicitly show the mechanism at work for the model (4.1). Deep in inflation, \( \phi \gg M_P \), and we approximately have

\[
S \approx - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \left( \frac{b M_P}{\phi} \right)^2 \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} m^2 \sigma^2 + \frac{1}{2} \left( \frac{b M_P}{\phi} \right)^2 \partial_\mu \chi \partial^\mu \chi + \frac{1}{2} \sigma^2 \chi^2 + \frac{c \lambda v^4}{4 b M_P^2} \phi \chi \right\},
\]

where \( \sigma \sim \phi \chi / (b M_P) \), and neglecting the derivatives of \( \phi \) in the change of variables (as they are subleading in the slow-roll expansion). An important assumption in (4.2) is that the Higgs is at its minimum. This is not particularly constraining, as it can be arranged via a portal coupling to the inflaton, a non-minimal coupling to curvature or ultimately, by having the possible Higgs condensate decay before the inflaton, during reheating.

Solving the equation of motion for \( \sigma \) (or \( \chi \)) from (4.2) whilst holding \( H \) and \( \phi \) constant, we see that the average energy density of the quintessence field, \( \rho_\sigma \sim \frac{1}{2} m^2 \sigma^2 \), dilutes approximately as either

\[
\rho \propto e^{-3N} \quad \text{or} \quad \rho \propto e^{-\frac{3}{b^2}N},
\]

where \( N = H t \) is the number of e-folds since the beginning of inflation. Here we have made the approximation that the energy density is completely dominated by the inflaton. The first case corresponds to a heavy field oscillating around its minimum, i.e. acting as normal matter [see e.g. 39]. The second case corresponds to a light field slow-rolling down its potential. The two cases are approximately distinguished by whether or not the slow roll parameter

\[
\eta_\sigma = \frac{\ddot{U}_\sigma(\chi, 0)}{3H^2} = \frac{\phi^2}{2 \frac{m^2}{M_P^2} \phi^2} = \frac{2}{b^2}
\]

is larger (leading to the first case) or smaller than unity (leading to the second case).

As a representative initial condition, let us choose \( \sigma = M_P \). Setting \( b = 1 \) for simplicity, leading to a coherently oscillating \( \sigma \), we have the approximate behaviour

\[
\rho \propto \sigma^2 \propto e^{-3N} \quad \Rightarrow \quad \sigma - \sigma_0 = \sigma + c \frac{\lambda v^4}{4 M_P m^2} = \left( M_P + c \frac{\lambda v^4}{4 M_P m^2} \right) e^{-\frac{3N}{2}} \approx M_P e^{-\frac{3N}{2}}.
\]

This translates into a minimal required duration of inflation for our mechanism to work, as inflation must continue long enough for the quintessence field to relax into the minimum. In terms of e-folds, this is

\[
e^{-\frac{3N}{2}} \lesssim \left| c \right| \frac{\lambda v^4}{4 M_P^2 m^2} \sim \sqrt{\frac{\rho_\Lambda}{M_P m}} \quad \Rightarrow \quad \text{for } \left| c \right| = 1, \ N \gtrsim 83.
\]

Importantly, this number is not very sensitive to the initial condition for \( \sigma \) in units of \( M_P \). If we take the standard picture of chaotic inflation at face value, around eighty e-folds is much less than the expected duration of inflation [40]. Inflation is therefore generically expected to drive the \( \sigma \) field to its minimum, for a wide range of initial conditions.
also that although we have assumed an initial hierarchy $\phi \gg \sigma$, this can be expected to arise more or less automatically: as visible in (4.2), the effective mass for $\chi$ is generally larger than $m$ deep into inflation, making $\sigma$ roll faster than $\phi$ and hence quite generically leading to $\phi \gg \sigma$.

Suppose that one instead takes $b^2 = 10$, which corresponds to the case of a light, slowly-rolling field. The left-hand side of (4.6) then becomes $e^{-\frac{2}{5}N}$, leading to the requirement that $N \gtrsim 312$. Again, this is not an unrealistic number.

After EWSB, the inflaton has long decayed and the Higgs has rolled to its minimum. In this epoch, the action for the quintessence field $\sigma$ therefore becomes

$$S_\sigma \approx - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left( \frac{b M_P}{\sigma} \right)^2 \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} m^2 \sigma^2 \right\} \equiv - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{1}{2} m^2 M_P^2 e^{\frac{2}{5}N} \right\},$$

(4.7)

with the initial condition at EWSB

$$\sigma_0 = \pm M_P e^{\frac{\chi_0}{M_P}} = - \frac{c \lambda v^4}{4 M_P^2 m^2},$$

(4.8)

where “±” refers separately to the cases where $c > 0$ (−) or $c < 0$ (+). Choosing $|c| \approx 2.1$ and $m \approx 6 \cdot 10^{-6} M_P$, and again that $M_P = 2.43 \cdot 10^{18}$ GeV, $\lambda = 0.129$ and $v = 246$ GeV, this gives

$$\rho_\Lambda = U_\sigma(\chi_0, v) = \frac{1}{2} m^2 \sigma_0^2 = \frac{c^2 \lambda^2 v^8}{32 M_P^2 m^2} \approx 2.5 \cdot 10^{-47}$ \text{GeV}^4. \quad (4.9)$$

The deviation from a strictly constant $\rho_\Lambda$ can be parameterised in standard fashion with

$$X \equiv M_P \frac{U'_\sigma(\chi, v)}{U_\sigma(\chi, v)} = \frac{2}{b}. \quad (4.10)$$

The corresponding equation of state for dark energy is constant and slightly larger (less negative) than $-1$,

$$w \equiv -1 + \delta w = \frac{X^2 - 6}{X^2 + 6} = \frac{2 - 3b^2}{2 + 3b^2}. \quad (4.11)$$

Taking e.g. $b = 1$ leads to $w = -0.2$, and $b^2 = 10$ gives $w = -0.875$. The current observational bound of $\delta w \lesssim 0.12$ at 90% confidence [41] means that our scenario is consistent with existing constraints for $b \gtrsim 3.2$.

5 A more realistic model

When writing (4.1), we simply assumed that the $\sigma$ field is coupled to the SM only through a very particular Planck-suppressed linear coupling to the Higgs potential. From a model-building point of view, however, this interaction is also somewhat non-trivial to achieve. Specifically, similar couplings to the inflaton linear in $\sigma$ will likely spoil the mechanism. In this section we will discuss a model where the coupling structure is better justified. The model that we will present here also possesses a symmetry in the UV limit between the
inflaton $\phi$ and quintessence field $\sigma$, giving further motivation for the idea that $\phi$ and $\sigma$ may have similar masses.

Let us postulate the following action, $S = \int \sqrt{-g} \mathcal{L}$, for a combined theory of dark energy and cosmic inflation:

$$-\mathcal{L} = -\frac{1}{2} M_p^2 e^{\frac{\phi}{M_p}} R + \frac{3}{4} e^2 e^{\frac{\sigma}{M_p}} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} b^2 e^{-\phi/M_p} \left( e^{-\sigma/M_p} - 1 \right)^{-2} \partial_\mu \sigma \partial^\mu \sigma$$

$$+ \alpha^2 M_p^4 \left[ \left( e^{-\phi/M_p} - 1 \right)^2 + \left( e^{-\sigma/M_p} - 1 \right)^2 \right] \left( e^{\frac{\sigma}{M_p}} \right)^2 + V(h) + \cdots. \quad (5.1)$$

Here we choose $\alpha$ such that $\phi$ leads to successful inflation, and $b$ and $c$ are again dimensionless couplings, with a similar interpretation as in (4.1). The first term is reminiscent of a dilaton theory and will introduce a linear coupling to the Higgs potential, which is manifest in the Einstein frame. Denoting Einstein-frame quantities with an overline and making use of the standard relations
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$$g_{\mu\nu} = \Omega^{-2} \overline{g}_{\mu\nu} \Rightarrow \sqrt{-g} = \Omega^{-4} \sqrt{-\overline{g}},$$

$$R = \Omega^2 \left[ R - 2 \overline{g}^{\mu\nu} \partial_\mu (\ln \Omega^2) \partial_\nu (\ln \Omega^2) \right],$$

and choosing the Weyl scaling function as

$$\Omega^2 = e^{\frac{\sigma}{M_p}},$$

this gives (dropping the overlines)

$$-\mathcal{L} = -\frac{1}{2} M_p^2 R + \frac{3}{4} e^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} b^2 e^{-\phi/M_p} \left( e^{-\sigma/M_p} - 1 \right)^{-2} e^{\frac{-\sigma}{M_p} - 1} \partial_\mu \sigma \partial^\mu \sigma$$

$$+ \frac{3}{4} e^2 \partial_\mu \sigma \partial^\mu \sigma + \alpha^2 M_p^4 \left[ \left( e^{-\phi/M_p} - 1 \right)^2 + \left( e^{-\sigma/M_p} - 1 \right)^2 \right]$$

$$+ \left( e^{\frac{-\sigma}{M_p}} \right)^2 V(h) + \cdots, \quad (5.2)$$

where the dots on the last line signify all other SM contributions not written explicitly in (5.1). When $\sigma \lesssim M_p$ the last term introduces a linear coupling to the Higgs potential as required. The $e^{-2\sigma/M_p}$ term multiplying the potential also automatically avoids couplings between $\phi$ and $\sigma$ in the Einstein frame.

5.1 The limit $\phi \gg M_p, \sigma \gg M_p$

In this limit the theory (5.2) simplifies approximately to

$$-\mathcal{L} \approx -\frac{1}{2} M_p^2 R + \frac{3}{4} e^2 (\partial_\mu \phi \partial^\mu \phi + \partial_\mu \sigma \partial^\mu \sigma) + \alpha^2 M_p^4 \left[ \left( 1 - e^{-\phi/M_p} \right)^2 + \left( 1 - e^{-\sigma/M_p} \right)^2 \right], \quad (5.3)$$

so in this region we have inflation from both fields. This limit is obviously symmetric under the interchange $\phi \leftrightarrow \sigma$, which may indeed be taken as a motivation for choosing the mass scale $\alpha M_p$ to be the same for both fields, as we have.

\footnote{Here we drop a $\Box$ term, as in an unbounded space it can be removed by partial integration.}
5.2 The regime $\phi \gg M_P, \sigma \lesssim M_P$

\[-\mathcal{L} \approx -\frac{1}{2} M_P^2 R + \frac{3}{4} e^2 (\partial_\mu \phi \partial^\mu \phi + \partial_\mu \sigma \partial^\mu \sigma) + \alpha^2 M_P^4 \left(1 - e^{-\phi/M_P}\right)^2 + \alpha^2 M_P^2 \sigma^2 \]
\[+ \left(1 - \frac{2\sigma}{M_P}\right) \frac{\lambda v^4}{4} + \cdots . \]

(5.4)

In this regime, we have Starobinsky-type single-field inflation from $\phi$, which successfully matches observations for $\alpha \sim 10^{-5}$. Furthermore, the $\sigma$ field will be driven towards the minimum at

$$\sigma_0 = \frac{c \lambda v^4}{4 \alpha^2 M_P^2}.$$  

(5.5)

The condition for slow-roll of $\sigma$ is $\eta_\sigma \ll 1$. Taking the canonically-normalised field $\chi = \sqrt{\frac{3}{2}} c \sigma$, we see that

$$\eta_\sigma = \frac{\ddot{U}_\sigma(\chi)}{3 \dot{H}^2} = \frac{8}{3 c^2},$$

or that the field rolls slowly only for $c \gtrsim 1$.

In order for the approximation (5.4) to be valid, the pre-factor in front of the quintessence field’s kinetic term $(\partial \sigma)^2$ in the first line of (5.2) must not grow large and flatten the potential when $\sigma \to \sigma_0$. We can then use this to place a crude lower bound on the initial value of the inflaton field:

$$\frac{1}{2} b^2 e^{-\phi/M_P} \left(e^{-\sigma_0/M_P} - 1\right)^{-2} e^{-c \sigma_0 / M_P} \lesssim 1$$

$$\approx \frac{1}{2} e^{-\phi/M_P} \left(\frac{c \lambda v^4}{4 \alpha^2 M_P^4}\right)^{-2} \lesssim 1$$

$$= e^{-\phi/M_P} \frac{M_P^4}{4 \rho_\Lambda} \lesssim 1$$

$$\implies \phi \gtrsim \ln \left(\frac{M_P^4}{4 \rho_\Lambda}\right) M_P \sim 276 M_P .$$

(5.7)

We see that in this case — unlike in the scenario described in Section 4 — we do require a mild hierarchy amongst the initial field values for the model to work, with $\phi \sim \mathcal{O}(100) M_P$ for $\sigma \sim M_P$.

5.3 The regime $\phi = 0, \sigma \ll M_P$

After the inflaton has decayed, we have

$$-\mathcal{L} \approx -\frac{1}{2} M_P^2 R + \frac{1}{2} b^2 \left(\frac{M_P}{\sigma}\right)^2 \partial_\mu \sigma \partial^\mu \sigma + \alpha^2 M_P^2 \sigma^2 + \left(1 - \frac{2\sigma}{M_P}\right) V(h) + \cdots ,$$

(5.8)

which leads to qualitatively identical behaviour to (4.7) in the late Universe. That is, the quintessence field will start slow-rolling from (5.5) following EWSB, sourcing dark energy in the late Universe in a manner consistent with observations if $b$ and $c$ are chosen to have appropriate $\mathcal{O}(1)$ values.
6 Conclusions

We have set out a general mechanism for explaining the observed magnitude of dark energy without any fine tuning, based on a theory with two interacting scalar fields. We described the basic mechanism, its predictions for the equation of state $w$ and corresponding observational bounds in Section 4. We then introduced a more realistic realisation of the mechanism in Section 5.

In particular, we point out that the form of the kinetic term that we propose in (5.1) is likely more realistic than the one postulated in (4.1). Indeed, theories beyond Einstein gravity can be generically parameterised in the form \[ S_{\mathrm{GE}} = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - e^{-2F(\phi)} \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\phi, \sigma) \right]. \] \[ (6.1) \]

For the appropriate choice of $F(\phi)$ and potential, this expression clearly leads to the same qualitative features as the $\sigma$ kinetic term in (5.2).

The model that we have presented successfully explains the strength of dark energy without fine tuning, via the inflation-assisted relaxation of a quintessence field and electroweak symmetry-breaking. Our model avoids all of the fine-tuning issues that traditionally dog quintessence, including the need for a small effective mass, the initial value problem, and the need to forbid interactions of the quintessence field with other fields. It therefore poses significant interest for model building. Indeed, we have also shown that the critical aspects of the theory are expected to be achievable in well-motivated top-down constructions. This is especially true given that the non-canonical term that we have introduced is simply one example that realises the mechanism, and it is not difficult to write down other operators with the same qualitative features.

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A Non-canonical Kinetic terms

Let us first study a simple non-interacting Lagrangian, but with a non-canonically normalised kinetic piece

\[ S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} C^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 \right\} \equiv - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} C^2 \partial_\mu \phi \partial^\mu \phi + U(\phi) \right\}, \]

\[ (A.1) \]

where $C$ is a number. We can define a canonically normalised field variable $\chi$ simply by demanding

\[ C^2 \partial_\mu \phi \partial^\mu \phi \equiv \partial_\mu \chi \partial^\mu \chi \quad \Rightarrow \quad \chi = C \phi, \]

\[ (A.2) \]
leading to

\[
S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{1}{2} m^2 \chi^2 \right\} \equiv - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \tilde{U}(\chi) \right\}. \tag{A.3}
\]

There are two crucial features arising from having a non-canonical kinetic term: 1) the value of the potential at some given \( \phi_0 = \chi_0/C \) is the same in both coordinates

\[
U(\phi_0) = \frac{1}{2} m^2 \phi_0^2 = \frac{1}{2} \frac{m^2}{C^2} \phi_0^2 = \tilde{U}(\chi_0). \tag{A.4}
\]

2) the slope and specifically the effective mass changes

\[
U''(\phi_0) = m^2, \quad \tilde{U}''(\chi_0) = \frac{m^2}{C^2}. \tag{A.5}
\]

In a nutshell, factors larger than \( \mathcal{O}(1) \) in front of kinetic pieces make the field lighter and smaller than \( \mathcal{O}(1) \) make it heavier.

These features persist also when the kinetic term is multiplied by a function. As an example let us consider the theory

\[
S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left( \frac{M_P}{\phi} \right)^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 \right\}, \tag{A.6}
\]

giving the canonical variable

\[
\frac{d\phi}{d\chi} = \frac{\phi}{M_P} \Rightarrow \phi = M_P e^{\chi/M_P}, \tag{A.7}
\]

\[
\tilde{U}''(\chi) = \frac{4}{M_P^2} \tilde{U}(\chi) = 2 m^2 e^{2\chi/M_P}. \tag{A.8}
\]

So clearly, when \( \phi \ll M_P \) (\( \chi \ll 0 \)) the field is very light, whereas for \( \phi \gg M_P \) (\( \chi \gg 0 \)) the field is heavy. At the point where the kinetic prefactor is precisely unity \( \phi = M_P \) (\( \chi = 0 \)) there is virtually no change in effective mass.

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