Quark Dipole Operators in Extended Technicolor Models

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We study diagonal and transition quark dipole operators in a class of extended technicolor (ETC) models, taking account of the multiscale nature of the ETC gauge symmetry breaking and of the mixing among ETC interaction eigenstates. Because of this mixing, terms involving the lowest ETC scale can play an important role in dipole operators, and we focus on these terms. We derive from experiment new correlated constraints on the quark mixing angles and phases. Our bounds yield information on mixing angles individually in the up- and down-sectors, for both left- and right-handed quark fields and thus constrain even quark mixing parameters that do not enter in the CKM matrix. With phases of order unity, we conclude that these mixing angles are small, constraining future ETC model building, but plausibly in the range suggested by the size of the CKM elements. These values still allow substantial deviations from the standard model predictions, in particular for several CP violating quantities, including the asymmetries in $b \to s\gamma$ and $B_d \to \phi K_S$, $Re(\epsilon'/\epsilon)$, and the electric dipole moments of the neutron and the $^{199}$Hg atom.

14.60.PQ, 12.60.Nz, 14.60.St

I. INTRODUCTION

Extended technicolor (ETC) provides a framework\cite{1} for the generation of fermion masses in theories of dynamical electroweak symmetry breaking. In this paper we study diagonal and transition quark magnetic and electric, and chromomagnetic and chromoelectric, dipole moments in a class of ETC models\cite{2}-\cite{6}, taking account of the multiscale nature of the ETC gauge symmetry breaking, and mixing between ETC interaction eigenstates to form mass eigenstates for the fermions and gauge bosons. The transition electric and magnetic dipole moments contribute to processes such as $b \to s\gamma$. A CP-violating transition chromoelectric moment contributes to $Re(\epsilon'/\epsilon)$ in the kaon system, and the diagonal electric and chromoelectric dipole moments contribute to a nonzero electric dipole moment (EDM) for the neutron and atoms such as $^{199}$Hg. We work out the predictions for ETC contributions to these quantities and compare these with measurements or bounds to obtain constraints on the models. We ask whether these constraints can be satisfied without fine tuning in these models and find that they can. This is an extension to quarks of our study of lepton dipole moments in Ref.\cite{7}.

The class of ETC models in Refs.\cite{2}-\cite{6} is based on the gauge group SU(5)\textsubscript{ETC} which commutes with the standard model (SM) gauge group. It breaks sequentially to a residual exact SU(2)\textsubscript{TC} technicolor gauge symmetry, naturally producing a hierarchy of charged lepton and quark masses. Thus, SU(5)\textsubscript{ETC} $\to$ SU(4)\textsubscript{ETC} at a scale $\Lambda_1$, with the first-generation SM fermions separating from the others; then SU(4)\textsubscript{ETC} $\to$ SU(3)\textsubscript{ETC} at a lower scale $\Lambda_2$ and SU(3)\textsubscript{ETC} $\to$ SU(2)\textsubscript{TC} at a still lower scale $\Lambda_3$, with the second- and third-generation fermions separating in the same way, leaving the technifermions.

The models of Ref.\cite{6} exhibit charged-current flavor mixing, intra-family mass splittings without excessive contributions to the difference $\rho - 1$ where $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$, a dynamical origin of CP-violating phases in the quark and lepton sectors, and a potential see-saw mechanism for light neutrinos without the presence of a grand unified scale\cite{3}. A key ingredient is the use of relatively conjugate ETC representations for both down-quark and charged lepton fields\cite{6}. The choice of SU(2) for the technicolor group (i) minimizes the TC contributions to the electroweak S parameter, (ii) with a SM family of technifermions in the fundamental representation of SU(2)\textsubscript{TC}, can yield an approximate infrared fixed point\cite{8} and associated walking behavior, and (iii) makes possible the mechanism for light neutrinos.

The sequential breaking of the SU(5)\textsubscript{ETC} to SU(2)\textsubscript{TC} is driven by the condensation of SM-singlet fermions which are part of the models. At the scale $\Lambda_{\text{ETC}}$, technifermion condensates break the electroweak symmetry. The models do not yet yield fully realistic fermion masses and mixings, and they have a small number of unacceptable Nambu-Goldstone bosons arising from spontaneously broken U(1) global symmetries. Additional interactions at energies not far above $\Lambda_1$ must be invoked to give them sufficiently large masses.

Nevertheless, the models share interesting generic features, including a mechanism for CP violation, that are worth studying in their own right. We approach this study phenomenologically, relying on only the generic features and using current experimental data to constrain parameters and guide future model building within the general class.

In any ETC model, the bilinear fermion condensates

\begin{align}
W &\to \phi K_S, \\
B_d &\to \phi K_S, \\
\text{Re}(\epsilon'/\epsilon) &\to \phi K_S.
\end{align}
forming at each stage of ETC breaking have nonzero phases, providing a natural, dynamical source of CP violation. Below the electroweak symmetry breaking scale, the effective theory consists of the SM interactions, mass terms for the quarks, charged leptons, and neutrinos, and a tower of higher-dimension operators generated by the underlying ETC dynamics. Here we focus on the dimension-5 operators describing the electric/magnetic and chromoelectric/chromomagnetic dipole moments of the quarks. In a companion paper [9], we discuss the impact of dimension-6 operators.

II. QUARK MASS MATRICES

The dipole operators are related to the dimension-3 mass terms of the up-type and down-type quarks, given in general by

$$\mathcal{L}_m = -\tilde{f}_{L,j} M_{jk}^{(f)} f_{R,k} + h.c.$$ (1)

where $f$ label the ETC eigenstates of the $Q = 2/3$ and $Q = -1/3$ quarks, respectively and the indices $j,k$ label generation number. The mass matrices $M^{(f)}$ can in general be brought to real, positive diagonal form $M^{(q)}$ by the bi-unitary transformation

$$U_L^{(f)} M^{(f)} U_R^{(f)\dagger} = M^{(q)}.$$ (2)

Hence, the interaction eigenstates $f$ are mapped to mass eigenstates $q$ via

$$f_{\chi} = U^{(f)\dagger}_{\chi} q_{\chi} , \quad \chi = L,R$$ (3)

where $q = (u,c,t)$ and $q = (d,s,b)$ for $Q = 2/3$ and $Q = -1/3$ respectively. In this way, the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix entering the charged current interactions is generated:

$$V = U_L^{(u)} U_L^{(d)\dagger}.$$ (4)

Each of the matrices $U_{\chi}^{(f)}$, $\chi = L,R$, depends generally on three angles $\theta_{mn}^{(f)}$, $mn = 12, 13, 23$, and six (independent) phases. Using the conventions of [18], we write

$$U_{\chi}^{(f)} = P_{\alpha}^{(f)\chi} R_{23} (\theta_{23}^{(f)} R_{13} (\theta_{13}^{(f)} R_{12} (\theta_{12}^{(f)} R_{13} (\theta_{23}^{(f)} P_{\beta}^{(f)\chi}$$ (5)

where $R_{mn}(\theta_{mn}^{(f)})$ is the rotation through $\theta_{mn}^{(f)}$ in the $mn$ subspace, $P_{\alpha}^{(f)\chi}$ and $P_{\beta}^{(f)\chi}$ are given by

$$P_{\alpha}^{(f)\chi} = \text{diag}(e^{ia_1^{(f)\chi}}, e^{ia_2^{(f)\chi}}, e^{ia_3^{(f)\chi}}), \quad a = \alpha, \beta$$ (6)

and

$$P_{\beta}^{(f)\chi} = \text{diag}(e^{i\delta^{(f)\chi}}, 1, 1).$$

The mixing angles are typically small if the off-diagonal $M_{jk}^{(f)}$'s are more suppressed than the diagonal ones.

In ETC models, the off-diagonal entries of the quark mass matrices $M^{(f)}$ arise via mixing among the ETC gauge bosons. In the model of Ref. [6], employing a relatively conjugate ETC representation for the down-type quarks, this is true also of the diagonal elements [11].

We note that in this model, $M^{(u)}$ is hermitian, so that $U_L^{(u)} = U_R^{(u)}$, while $M^{(d)}$ is a more general complex matrix.

As for the phases, a complete theory should allow the computation of all the observable ones [10]. In this paper, having neither a complete theory nor arguments to suggest that the phases are small, we derive bounds on combinations of mixing angles and phases. We then bound the mixing angles with the assumption that the phases are generically of order unity. An important future study will be to see whether this is naturally the case. We comment further on this and the strong CP problem in section IV.

III. ELECTROMAGNETIC AND COLOR DIPOLE MOMENT MATRICES

The magnetic and electric dipole-moment matrices $D^{(f)}$ of the quarks appear in the dimension-5 operators

$$\mathcal{L}_{DM} = \frac{1}{2} \hat{f}_L D^{(f)} \sigma_{\mu\nu} f_R F_{\mu\nu}^{em} + h.c.$$ (7)

Similarly, the color (chromo-) magnetic and electric dipole-moment matrices $D^{(c)}$ enter the operators

$$\mathcal{L}_{CDM} = \frac{1}{2} \hat{f}_L T_a D^{(f)} \sigma_{\mu\nu} f_R G_{a}^{\mu\nu} + h.c.,$$ (8)

where $T_a$ and $G_{a}^{\mu\nu}$ denote a generator, and the field-strength tensor, for color SU(3).c. (We include the color SU(3)c coupling $g_s$ in our definition of $D^{(f)}$, just as it is usual to include the electromagnetic coupling e in the definition of the electric dipole moment; we note that some authors separate $g_s$ out from their definition of $D^{(f)}$.)

In the class of ETC models we consider, the ETC gauge bosons do not carry SM quantum numbers. Hence, in the respective diagrams that produce the dipole moment matrices and color dipole moment matrices, the photon and gluon couple only to the virtual (techni)fermions. Therefore,

$$D^{(f)} = \frac{g_s}{eQ_f} D^{(f)}$$ (9)

where $g_s = g_s(\mu)$ is evaluated at the appropriate scale $\mu$.

Transforming to the mass-eigenstate basis, we have

$$\hat{f}_L D^{(f)} \sigma_{\mu\nu} f_R F_{\mu\nu}^{em} + h.c. = \hat{q}_L D^{(q)} \sigma_{\mu\nu} q_R F_{\mu\nu}^{em} + h.c.,$$ (10)

where

$$D^{(q)} = U_L^{(f)} D^{(f)} U_R^{(f)\dagger}.$$ (11)
Analogously, $D^{(q)} = U_L^f D^{(f)} U_R^f - 1$. Both $D^{(q)}$ and $D^{(u)}$ are independent of $P_\beta$, $\chi = L, R$.

Decomposing $D^{(q)}$ into hermitian and anti-hermitian parts, $D^{(q)} = D^{(q)}_{H,H} + D^{(q)}_{AH}$, where $D^{(q)}_{H,AH} = (1/2)(D^{(q)} \pm D^{(q)})$, the dipole operator takes the form $(1/2)[\bar{q} D^{(q)}_{H} \sigma_{\mu \nu} q + \bar{q} D^{(q)}_{AH} \sigma_{\mu \nu} \gamma_5 q] F_{\mu \nu}$. Then the EDM of $q_j$ is

$$d_{q_j} = -i D^{(q)}_{AH,jj}.$$  

Defining $D^{(q)}_{c,AH}$ analogously, the chromo-EDM of $q_j$ is

$$d_{c,q_j} = -i D^{(q)}_{AH,jj}.$$  

We have described previously [6,7] how the mass matrices $M^{(f)}$ and dipole matrix $\tilde{D}^{(f)}$ are estimated from an underlying ETC theory and have noted how they are related in the presence of the mixing of ETC interaction eigenstates to form mass eigenstates of the fermions and gauge bosons. An important result that we need from that analysis is the relation

$$D^{(f)}_{jk} \simeq \frac{e M^{(f)}_{jk}}{\Lambda^2_{jk}}$$  

where each $\Lambda_{jk}$ is a dimensionful parameter of order the scale above which the $(j \leftrightarrow k)$ ETC propagator becomes soft. This structure reflects the fact that the leading dipole contribution contains two additional inverse factors of the ETC scale(s) relative to the mass, and that the corresponding integral is again sensitive to physics at the ETC scales. $\Lambda_{jk}$ is no greater than $\min(\Lambda_j, \Lambda_k)$, and can be less. The fact that its $(j,k)$ dependence is non-trivial implies that $D^{(f)}_{jk}$ is not, in general, $\propto M^{(f)}_{jk}$. It is therefore not diagonalized by the transformation that diagonalizes $M^{(f)}$; this transformation yields, instead, a non-diagonal and complex form for the dipole matrix $D^{(q)}$ of Eq. (11). Thus mixing has an important effect on quark dipole moments in ETC models.

For numerical estimates of the dipole matrix, we take the ETC breaking scales to be $\Lambda_1 \simeq 10^3$ TeV, $\Lambda_2 \simeq 10^2$ TeV, and $\Lambda_3 \simeq 4$ TeV, as in our previous work. Since the ETC interactions are strong at these scales, there is resultant uncertainty in the calculations; this is understood in our bounds. We focus on the contribution to each element of the dipole matrix in $D^{(q)}$ in the mass-diagonal basis of third-family physics arising at the lowest ETC scale $\Lambda_3$. Then we have

$$D^{(q)}_{jk} \simeq \frac{e Q_q m_{q_j} F^{(f)}_{jk,3}}{\Lambda^2_{3j}}$$  

where $m_{q_j} = m_t, m_b$ for the $u, d$ sectors respectively, and where $F^{(f)}_{jk,3}$ is a dimensionless function of the parameters in $U_L^{(f)}$ and $U_R^{(f)}$, of $O(1)$ for generic values of these parameters. Terms involving exchange of heavier ETC vector bosons with masses $\Lambda_1$ and $\Lambda_2$ are also present but are suppressed by the propagator mass ratios $\Lambda^2_j/\Lambda^2_3$, $j = 1, 2$. Part of this propagator suppression may be compensated for by the property that these other terms involve fewer small mixing angle factors, and hence they are not necessarily negligible; however, the $\Lambda_3$-scale terms on which we focus should provide a rough measure of the overall ETC contributions.

The experimental constraints will demand small mixing angles, so we record here the small-angle form of the function $F^{(f)}_{jk,3}$. Using Eq. (11) and Eq. (5), We have

$$F^{(f)}_{33,3} \simeq 1 + \ldots ,$$  

$$F^{(f)}_{23,3} \simeq e^{i (\alpha_{2}^{(f)L} - \alpha_{3}^{(f)L})} \theta^{(f)L}_{23} + \ldots ,$$  

$$F^{(f)}_{32,3} \simeq e^{i (\alpha_{2}^{(f)L} - \alpha_{3}^{(f)L})} \theta^{(f)L}_{23} + \ldots ,$$

and, for $j, k \neq 3$,

$$F^{(f)}_{jk,3} \simeq \eta_{jk} e^{i \left(\alpha_{j}^{(f)L} - \alpha_{k}^{(f)L}\right) - \left(\alpha_{j}^{(f)R} - \alpha_{k}^{(f)R}\right)} \theta^{(f)L}_{j3} \theta^{(f)}_{k3} + \ldots ,$$

where each expression is accurate up to a real coefficient of order unity, $\eta_{jk}$ can contain $\delta^{(f)x}$ phases, and ... denote higher order terms.

IV. THE STRONG CP PROBLEM

Before considering the phenomenology of the dimension-5 operators, with their CP violating phases, we discuss briefly the strong CP problem within the class of ETC models being considered. Can these models lead to the necessary condition

$$|\bar{\theta}| \lesssim 10^{-10}$$

where

$$\bar{\theta} = \theta - \left[ \arg(\det(M^{(u)})) + \arg(\det(M^{(d)})) \right] ,$$

with $\theta$ appearing via the topological term

$$\frac{\theta g^2}{32\pi^2} G_a \mu \nu \tilde{G}^{\mu \nu}_a$$

in the QCD Lagrangian?

The quark mass matrices $M^{(f)}$ ($f = u, d$) in the effective theory below $\Lambda_{TC}$ are generated by integrating out short-distance physics at scales ranging from $\Lambda_{TC}$ to the highest ETC scale $\Lambda_1$. Above $\Lambda_1$, all fermions are massless. Some of the global chiral symmetries are anomalous, and hence are broken by instantons. The
$F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms associated with each (nonabelian) gauge interaction may be rotated away by chiral transformations through the relevant global anomalies. In particular, this renders $\theta = 0$ for SU(3)$_c$ in the underlying theory. In the effective low energy theory, then, we have $\theta = -\arg(det(M^{(u)})) - \arg(det(M^{(d)}))$. If $\theta \neq 0$, the rotation (2) to the real diagonal mass basis will, of course, regenerate the topological term through the anomaly. In the models of Refs. [2]-[6], $M^{(u)}$ is hermitian, so $\theta$ resides in $M^{(d)}$.

More generally, the condition $|\arg(det(M^{(u)})) + arg(det(M^{(d)}))| \lesssim 10^{-10}$ can be rewritten by letting 

$$U^{(f)}_\chi = e^{i\phi^{(f)}_\chi}U^{(f)}_\chi \in U(3)$$

where $U^{(f)}_\chi \in SU(3)$ and $\chi = L, R$. Then $det(U^{(f)}_\chi) = e^{i\phi^{(f)}_\chi}$, and, from Eq. (2), 

$$det(M^{(f)}) = e^{i(-\phi^{(f)}_L + \phi^{(f)}_R)}det(M^{(f)}_{diag.}).$$

Hence, the necessary condition reads

$$|\sum_{f=u,d} (-\phi^{(f)}_L + \phi^{(f)}_R)| \lesssim 10^{-10}. \quad (25)$$

Other phases, entering the CKM matrix or the dipole operators, enter through the unimodular matrices $U^{(f)}_\chi$, and are, in this sense, distinct from the strong CP phase.

In the notation of Eq. (5),

$$det(U^{(f)}_\chi) = \exp[i \sum_j (\alpha_j^{(f)}\chi + \beta_j^{(f)}\chi)], \quad \chi = L, R \quad (26)$$

and, in terms of these quantities, Eq. (25) reads

$$\left|\sum_{f=u,d} \sum_j \left[-(\alpha_j^{(f)}L + \beta_j^{(f)}L) + (\alpha_j^{(f)}R + \beta_j^{(f)}R)\right]\right| \lesssim 10^{-10}. \quad (27)$$

(independent of $\delta^{(f)}\chi$). So in terms of these parameters, only one linear combination (a sum over flavors $j$) of the CP-violating phases $\alpha_j^{(f)\chi}$ and $\beta_j^{(f)\chi}$ is tightly constrained.

Whether a resolution of the strong CP problem will emerge in the class of models considered here is not yet clear [10]. These models include certain Nambu-Goldstone bosons, one of which can be associated with a Peccei-Quinn dynamical relaxation of $\theta$ to zero. But, as noted above, these Nambu-Goldstone bosons must be given large masses by new interactions, eliminating this approach to solving the problem. Whatever the resolution of the strong CP problem turns out to be, an important point is that the relevant phase combination, entering in Eq. (25) or equivalently Eq. (27), involves a sum over the generational phases (labelled by $j$), whereas other CP-violating phase combinations which contribute to the quantities considered here (cf. Eqs. (17)-(19)) involve differences of generational phases (and $\delta^{(f)\chi}$). One must, therefore, analyze the effects of these flavor-dependent phases, as we do here.

V. OFF-DIAGONAL DIPOLE MOMENTS

In this section, we begin our phenomenological discussion by focusing on the off-diagonal entries in the matrices $D^{(q)}$ and $D^{(\tilde{q})}$. We turn to the diagonal elements in Section V. The off-diagonal elements, both the CP-conserving and CP-violating pieces, contribute to transitions $q \rightarrow q'\gamma$ and $q \rightarrow q'g$, where $g$ denotes a gluon (which hadronizes). We derive constraints on ETC contributions to the resultant hadron decays. These complement our previous study of upper limits from the electromagnetic leptonic decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $\tau \rightarrow e\gamma$ in Ref. [7].

A. $b \rightarrow s\gamma$ and $b \rightarrow sg$

We first consider the processes $b \rightarrow s\gamma$ and $b \rightarrow sg$. The former underlies the decays $B \rightarrow X_s\gamma$, where $X_s$ denotes a semi-inclusive final state containing an $s$ quark. For this purpose, one constructs an effective Hamiltonian describing the physics at energies below the electroweak scale by integrating out the heavy $W$ and $Z$ gauge bosons and the top quark. QCD effects are then taken into account through the renormalization group (RG) running of the Wilson coefficients of this effective Hamiltonian down to the scale relevant for the physical process of interest, where the matrix elements of the operators are computed. We use the operator basis $O_k$ as in [12]. The relevant effective Hamiltonian for the processes $b \rightarrow s\gamma$ and $b \rightarrow sg$, keeping dominant terms, is

$$H \simeq \frac{G_F}{\sqrt{2}} V^*_{ts} V_{tb} \sum_{k=1}^{10} C_k O_k$$

$$+ C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} + h.c. \quad (28)$$

where $V^*_{tb}$ and $V_{ts}$ are CKM matrix elements. The $O_k$, $k = 1, \ldots, 10$ are dimension-6 four-fermion operators, while the last two operators are the dimension-5 operators of primary interest in this paper,

$$O_{7\gamma} = \frac{e m_b}{4\pi^2} \left[ s_L \sigma_{\mu\nu} b_R \right] F^{\mu\nu}_{em} \quad (29)$$

and

$$O_{8g} = \frac{g_s m_b}{4\pi^2} \left[ s_L \sigma_{\mu\nu} T_a b_R \right] G^{\mu\nu}_a. \quad (30)$$

QCD running between the electroweak scale, taken here as $m_Z$, and the scale of $B$-decays $m_b$ introduces mixing
among the Wilson coefficients, in such a way that observables determined by any particular operator \( O_k \) at the low scale depend on a combination of the Wilson coefficients at the electroweak scale.

The Wilson coefficients of the effective Hamiltonian receive contributions from SM physics as well as ETC interactions. Our focus is on the ETC contributions to the dipole operators and we thus take the first 10 Wilson coefficients, computed at the common scale \( \mathcal{O}(m_Z) \), to be determined by the SM interactions only: \( C_k = C_k^{SM} \) for \( k = 1, \ldots, 10 \). For the dipole-operator coefficients \( C_{7\gamma} \) and \( C_{8g} \) at the electroweak scale, we have
\[
C_{7\gamma, 8g} = C_{7\gamma, 8g}^{SM} + \Delta C_{7\gamma, 8g}
\]
where the increments are due to the ETC interactions and are given by
\[
\Delta C_{7\gamma} \simeq - \frac{2\sqrt{2}\pi^2 Q_f F_{23,3}^{(d)}}{GF A_3^2 V_{ts} V_{tb}}
\]
and
\[
\Delta C_{8g} \simeq - \frac{2\sqrt{2}\pi^2 F_{23,3}^{(d)}}{GF A_3^2 V_{ts} V_{tb}}.
\]

After input of the SM contributions at the electroweak scale and QCD evolution to \( \mu_b \), the Wilson coefficients can be written in the form [12],
\[
C_{7\gamma}(\mu_b) \simeq -0.3 + 0.7\Delta C_{7\gamma} + 0.09\Delta C_{8g},
\]
\[
C_{8g}(\mu_b) \simeq -0.15 + 0.7\Delta C_{8g}.
\]
The first term in each case is the SM contribution, computed using input CKM parameters based on global fits [12,14]. Each lies essentially along the real axis, because the rephasing-invariant quantity \( V_{ts} V_{tb} / (V_{ts} V_{tb}) \) has a negligibly small complex phase. In the case of \( C_{7\gamma}(\mu_b) \), the dominant ETC correction comes from the 0.7\( \Delta C_{7\gamma} \) term.

The branching ratio \( BR(B \to X_s \gamma) \) is proportional to \( |C_{7\gamma}(\mu_b)|^2 \). Experimentally, \( BR(B \to X_s \gamma) = (3.34 \pm 0.38) \times 10^{-4} \) [13], in agreement with the SM value at the 10\% level. This leads to an allowed annular region in the complex \( C_{7\gamma}(\mu_b) \)-plane, a band of width \( \pm 5\% \) relative to the SM value. The resultant constraint on the magnitude of the ETC contribution depends on its CP-violating phase. This phase depends in turn on the phase differences in the small-angle expressions Eqs. (17) and (18), which enter \( \Delta C_{7\gamma} \) through Eq. (32).

Two possibilities suggest themselves. One is that the ETC contribution to \( C_{7\gamma}(\mu_b) \) is less than about 5\% in magnitude relative to the SM value. There is then no constraint on the value of the phase, and we find
\[
|\theta_{23}^{(d)x}| < 0.02 \quad \text{for} \quad \chi = L, R.
\]

Another possibility is that the magnitude of the ETC contribution is larger, but that the phase is such as to yield a value for \( C_{7\gamma}(\mu_b) \) within the allowed annular region in the complex plane. The latter case involves some correlation between the magnitude and phase of the ETC contribution, but would allow a larger value for the mixing angles \( \theta_{23}^{(d)x} \), with \( \chi = L, R \).

To explore further the possibility of a significant relative phase between the standard model and ETC contributions to the \( b \to s \gamma \) amplitude, we consider CP-violating asymmetries in \( B \) decay. Let \( A_{CP}(i \to f) \) denote the CP-violating asymmetry in the rates for an initial particle \( i \) to decay to a final state \( f \): \( A_{CP}(i \to f) = (\Gamma_{i \to f} - \Gamma_{f \to i}) / (\Gamma_{i \to f} + \Gamma_{f \to i}) \). Current data yields \(-0.093 \leq A_{CP}(B \to X_s \gamma) \leq 0.096 \) (Belle, [13,15]) and \(-0.06 \leq A_{CP}(B \to X_s \gamma) \leq 0.11 \) (BABAR, [16]), both at the 90\% CL. These limits are consistent with the standard model, in which, using global fits to CKM parameters, this asymmetry is predicted to be \( \sim 0.005 \) [17]. This agreement constrains ETC contributions. We use the expression (e.g. [12,17])
\[
A_{CP}(B \to X_s \gamma) \simeq \frac{1}{|C_{7\gamma}(\mu_b)|^2} \left[ a_{27} Im(C_2(\mu_b)C_{7\gamma}^{\ast}(\mu_b)) + a_{87} Im(C_8(\mu_b)C_{7\gamma}^{\ast}(\mu_b)) \right]
\]
where the coefficients \( a_{ij} \) are known quantities, and where we have kept only the largest interference terms. The Wilson coefficient \( C_2 \) corresponds to the operator \( O_2 = 4[s_L \gamma_\mu c_L][S_L \gamma_\mu b_L] \), and is present already at tree level in the standard model. The ETC contribution may be computed using Eqs. (34), (35), and (28). The current bounds are such that \( |Im(F_{jk,3})| < 0.1 \) for \( jk = 32, 23 \). Hence, neglecting the special possibility of maximal destructive interference between the SM and ETC contributions to the \( b \to s \gamma \) amplitude, and the bounds on the CP-violating asymmetry for this decay together constrain \( |\theta_{23}^{(d)x}| < 0.02 \). That these mixing angles might not be too far below this range is suggested by the fact that combinations of \( \theta_{jk}^{(d)L} \) and \( \theta_{jk}^{(a)L} \) enter the CKM mixing matrix (4). In general, one might expect that the individual mixing angles \( \theta_{jk}^{(a)x} \) and \( \theta_{jk}^{(d)x} \) would be comparable to the corresponding measured CKM angle \( \theta_{jk} \) in \( V \). The measured value of the CKM angle \( \theta_{23} \) is \( \theta_{23} \simeq 0.04 \), in roughly the same range.

We remark briefly on other CP-violating observables affected by the possible ETC modification of \( C_{7\gamma} \) and \( C_{8g} \). In the standard model, time-dependent CP-violating asymmetries in the decays \( B_0^0, B_0^+ \to \phi K_S \), arising from one-loop (penguin) diagrams, are predicted to be the same as those in the tree-level decays \( B_0^0, B_0^+ \to J/\psi K_S \). ETC contributions to the \( \phi K_S \) asymmetries through \( C_{8g} \) could change this. At present, Belle and BABAR measurements of the CP-violating asymmetries...
where $\chi, \chi' = L, R$ or $R, L$.

A similar limit on the corresponding up-type angles emerges from the process $c \to u\gamma$. It leads to radiative decays such as $D^+ \to \pi^+\pi^0\gamma$, $D^+ \to \rho^+\gamma$, $D^0 \to \pi^+\pi^-\gamma$, and $D^0 \to \rho^0\gamma$. Using the limit $BR(D^0 \to \rho^0\gamma) < 2.4 \times 10^{-4}$ [18] that has been established on one of these possible decays, we derive the bound

$$|\theta_{13}^{(u)\chi} \theta_{23}^{(u)\chi'}| \lesssim 0.008$$  \hspace{1cm} (41)$$

B. Other Off-diagonal Terms

We next consider the process $s \to d\gamma$ which gives rise to radiative hyperon and meson decays. The relevant Hamiltonian is similar to that of Eq. (28), with the respective replacements of $b$ by $s$ and $s$ by $d$. Branching ratios for radiative hyperon decays are of order $10^{-3}$; for example, $BR(\Sigma^+ \to p\gamma) = (1.23 \pm 0.05) \times 10^{-3}$ and $BR(\Lambda \to n\gamma) = (1.75 \pm 0.15) \times 10^{-3}$ [18]. Angular asymmetries in decays of polarized hyperons have also been measured. A typical radiative $K$ decay is $K^+ \to \pi^+\pi^0\gamma$, with $BR(K^+ \to \pi^+\pi^0\gamma) = (2.75 \pm 0.15) \times 10^{-4}$. Although the branching ratios and asymmetries are only approximately calculable, owing to long-distance contributions, the standard model yields an acceptable fit. The relevant ETC-induced transition dipole moment is given by Eq. (15) with $jk = 12$ and $jk = 21$. We find that the ETC contributions are safely smaller than the SM one and hence these decay rates and asymmetries do not yield interesting constraints on the fermion mixing.

The process $s \to dq$ provides a tighter constraint. Its chromomagnetic and chromoelectric dipole elements produce a (virtual) gluon which then hadronizes. Most importantly, the ETC-induced transition chromo-EDM provides a new contribution to direct CP violation in $K_L \to 2\pi$ decays. The latter is measured by the quantity $Re(\epsilon'/\epsilon)$, which is determined via

$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 \simeq 1 + 6Re\left(\frac{\epsilon'}{\epsilon}\right)$$  \hspace{1cm} (38)$$

where $\eta_{+-}$ and $\eta_{00}$ are given in terms of measured amplitude ratios by $A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-) = \eta_{+-} \simeq \epsilon + \epsilon'$ and $A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0) = \eta_{00} \simeq \epsilon - 2\epsilon'$. Experimentally, $Re(\epsilon'/\epsilon) = (1.8 \pm 0.4) \times 10^{-3}$ [18]. There are uncertainties in theoretical estimates of $Re(\epsilon'/\epsilon)$ in the standard model owing to difficulties in calculating the relevant matrix elements and in choosing input values of some parameters such as the strange quark mass [19]. Nevertheless, we can deduce a rough bound from the requirement that the contribution from the ETC-induced transition quark chromo-EDM operator not be excessively large. We obtain

$$|Im(F_{12,3})| \lesssim 10^{-3} - 10^{-4}.$$  \hspace{1cm} (39)$$

This bound can be satisfied with the phase differences in $Im(F_{12,3})$ of order unity providing that

$$|\theta_{13}^{(d)\chi} \theta_{23}^{(d)\chi'}| \lesssim 10^{-3} - 10^{-4},$$  \hspace{1cm} (40)$$

C. Discussion

The study of the off-diagonal elements of the matrices $D^{(q)}$ and $D_{\chi}^{(q)}$, focusing on physics at the lowest ETC scale $\Lambda_3$, leads to constraints that can be satisfied with CP-violating phase differences of order unity and reasonable limits on the relevant mixing angles. These limits are consistent with values in a range suggested by the fact that the angles $\theta_{13}^{(d)L}$ and $\theta_{23}^{(u)L}$ determine the measured CKM angles through Eq (4).

It is worth observing that at least one combination of the phases $\alpha(f)_L$, $\beta(f)_L$, and $\delta(f)_L$ for $f = u, d$ must be of order unity. The CKM rephasing-invariant product

$$J = (1/8)\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{13})\cos\theta_{13}\sin\delta$$  \hspace{1cm} (42)$$

is quite small, $|J| \sim 10^{-5}$, but this suppression arises from the small CP-conserving mixing angles. The intrinsic CP-violating phase angle $\delta$ defined in the standard parametrization by $V_{ub} = e^{-i\delta}\sin\theta_{13}$, is not small. Indeed, current CKM fits give, for the ratio $\bar{\eta}/\bar{\rho}$, which is equal to $\tan\delta$ in the standard model, a value $\bar{\eta}/\bar{\rho} \simeq 2$ [14].
VI. DIAGONAL ELECTRIC AND CHROMOELECTRIC DIPOLE MOMENTS

The diagonal electric and chromo-electric dipole moments for the up and down quarks arising from ETC interactions provide tighter constraints on certain combinations of phase differences and mixing angles. These moments derive from the diagonal elements of $D_{\alpha}^{(q)}$ (Eq. 15). Recall that this expression is correct to leading order in ETC scales, depending explicitly on the inverse square of only the lowest ETC scale $\Lambda_3$. Thus,

$$d_u = \frac{e Q_u}{g_s} d_{c,u} \simeq \frac{e Q_u m_t}{\Lambda_3^2} \text{ Im}(F_{11,3}^{(u)})$$ (43)

$$d_d = \frac{e Q_d}{g_s} d_{c,d} \simeq \frac{e Q_d m_b}{\Lambda_3^2} \text{ Im}(F_{11,3}^{(d)})$$ (44)

with the small-angle expressions for the $F_{11,3}^{(f)}$ given by Eq. (19) and involving flavor-differences of phases.

We stress that these mixing-induced terms can be the dominant contributions to the up- and down-quark EDM’s only if the sum over all flavor-phases (in Eqs. (25) or (27)) is negligibly small, that is if the strong CP problem has been solved. We note that the pure electroweak contribution to quark electric dipole moments has been estimated to be $\lesssim 10^{-32}$ e-cm and hence is negligibly small [20,21].

CP-violating electric dipole moments such as those of the neutron and certain atoms like $^{199}$Hg receive contributions from the quark EDM’s and the quark color EDM’s. The latter lead to CP-violation in the hadronic wave function (the CEDM enters as a correction to $t$-channel gluon exchange between the bound quarks). There are also contributions from the CP-violating triple-gluon operator $c_{abc} G_a \lambda_\mu G_b^{\mu
u} G_c^{\lambda \nu}$ [22], and loop-induced CP-violating $W^+ W^- \gamma$ vertices [23].

We focus here on the quark-EDM contributions. To estimate them, one starts as in the case of the off-diagonal elements with operators defined at short distances, uses renormalization-group methods to evolve these to hadronic distance scales $\sim 1$ fm, and then computes the relevant hadronic matrix elements. For two reasons, we adopt a simpler approach here. Because the direct SM contribution to the EDM’s is many orders of magnitude smaller than the expected ETC contribution, we neglect the SM contribution here. And because the computation of the hadronic matrix elements, involving only first-generation quarks, is more uncertain than for the off-diagonal matrix elements, we also neglect the RG running of the ETC contribution.

A. Bounds from EDM’s

The current experimental upper bound on the neutron electric dipole moment $d_n$ is $|d_n| < 6.3 \times 10^{-26}$ e-cm [24]. In setting constraints, we assume that there are no accidental cancellations between different contributions to the experimentally observable EDM’s. For an estimate of the hadronic matrix element of the quark EDM operators, $\langle n| 3 f \sigma_{\mu\nu} \gamma_5 f |n\rangle$, $f = u, d$, various methods yield roughly similar results, which are comparable to the static quark model relation $d_n = (1/3)(4d_u - d_d)$. Using these estimates, we infer from the above limit on $|d_n|$ that $|\text{Im}(F_{11,3}^{(u)})| \lesssim 1 \times 10^{-6}$ and $|\text{Im}(F_{11,3}^{(d)})| \lesssim 3 \times 10^{-5}$.

The same quantities enter into the quark color EDM’s and can, in principle, be bounded from their contributions to $d_n$. Since QCD is nonperturbative at the low energies relevant here, there are uncertainties in the proportionality factors connecting the CEDM’s $d_{c,f}$, $f = u, d$, to $d_n$ (e.g., [25]), and there is the related question of what value to use for the color gauge coupling $g_s$.

The most stringent limits on these quantities are obtained from upper bounds on EDM’s of atoms, in particular from $^{199}$Hg. Experimentally, $|d_{199 Hg}| < 2.1 \times 10^{-28}$ e-cm [26], which is about a factor of 50 smaller than the upper limit on $|d_n|$. From this we obtain the bounds

$$|\text{Im}(F_{11,3}^{(u)})| \lesssim 0.3 \times 10^{-7}$$ (45)

$$|\text{Im}(F_{11,3}^{(d)})| \lesssim 0.6 \times 10^{-6}$$ (46)

We note that in the class of models Refs. [2]–[6], $\text{Im}(F_{11,3}^{(u)})$ (19) vanishes identically. This is because $M^{(u)}$ is hermitian in these models and therefore $U_{L}^{(u)} = U_{R}^{(u)}$.

From Eq. (19), we see that Eqs. (45) and (46) constrain a product of $\theta_{13}^{(d) R} \theta_{13}^{(d) L}$ times the imaginary part of a phase factor, for $f = u, d$. If the phase differences are of order unity, then

$$|\theta_{13}^{(d) R} \theta_{13}^{(d) L}| \lesssim 0.6 \times 10^{-6}$$ (47)

with a tighter bound on $|\theta_{13}^{(u) R} \theta_{13}^{(u) L}|$ if $M^{(u)}$ is not hermitian. The bound (47) is comparable to that on the corresponding product of charged-lepton mixing angles coming from the current limit on the electron EDM [7].

The above bound may be satisfied with $|\theta_{13}^{(d) L} | \simeq |\theta_{13}^{(d) R} | \lesssim 0.0008$. We note again that the corresponding angle $\theta_{13}$ in the CKM matrix $V$ has the measured value $\simeq 0.004$, only a factor of five larger. Furthermore, the expression for $\theta_{13}$ contains terms proportional to products such as $\theta_{12}^{(u) L} \theta_{23}^{(d) L}$ which could be the dominant contribution and naturally be of order 0.004. Hence, although
the bound (47) and its analogue for $f = u$ do imply quite small values for the indicated products of rotation angles, they can plausibly be satisfied in ETC models that successfully predict the CKM matrix.

We note, by contrast, that if all the above mixing angles are of order the CKM angle $\theta_{13} \simeq 0.004$, then the relevant combination of phase differences in the down sector must be rather small, $\lesssim 0.04$. The corresponding limit on the up-sector would be even smaller, of order 0.002, but would be automatically satisfied in models where $M^{(u)}$ is hermitian.

Finally, we comment that an $s$-quark (chromo-) EDM can also contribute to the $^{199}$Hg EDM. This contribution is more difficult to compute since there are no valence $s$ quarks in the nucleon. It can be roughly estimated, leading to a bound on $|\text{Im}(F^{(d)}_{22,3})|$. This bound is much weaker than the above bound on $|\text{Im}(F^{(d)}_{11,3})|$, but it could have important implications for the mixing angles $\theta_{23}^{(d)L}$ and $\theta_{23}^{(d)R}$.

B. Discussion

The quark EDM's and chromo-EDM's correspond to the diagonal elements of the matrices $D^{(q)}$ and $D_c^{(q)}$. The bounds on these quantities, in particular from the measured limit on the EDM of $^{199}$Hg, lead to tight constraints on mixing angles and/or CP-violating phase differences in the individual subsectors $q = u, d$. These derive from the contributions to the elements of $D^{(q)}$ and $D_c^{(q)}$ from physics at the lowest ETC scales $\Lambda_3$ (e.g. Eq. (15)). If the phase differences are of order unity, then the down-type mixing angles are bounded as in Eq. (47), with an even tighter bound in the up sector if $M^{(u)}$ is not hermitian. The bound (47) is comparable to that on the corresponding product of charged-lepton mixing angles coming from the current limit on the electron EDM [7].

VII. CONCLUSIONS

We have shown that in the class of ETC models [2]-[6], mixing effects significantly affect predictions for diagonal and transition magnetic and electric dipole and color dipole moments. We have used measurements of $b \rightarrow s\gamma$, $c \rightarrow u\gamma$, and $\text{Re}(\epsilon'/\epsilon)$, and limits on CP-violating electric and chromoelectric dipole moments of quarks to set new constraints on the mixing angles and phases in the unitary transformations from the quark flavor (ETC-interaction) eigenstates to the mass eigenstates. The analysis focuses on physics at the lowest ETC scale $\Lambda_3$, taken to be a few TeV to allow for the measured value of $m_t$. While there are ETC contributions to these processes involving all of the ETC scales, the terms arising at $\Lambda_3$ should provide a rough measure of the overall ETC effect.

Our bounds provide new information about quark mixings since they apply separately to the individual charge sectors $q = u$ and $q = d$. By contrast, the measured CP-conserving angles and CP-violating phase in the CKM matrix arise from the (mismatch of the) unitary transformations $U_L^{(u)}$ and $U_L^{(d)}$. Our bounds also constrain the unitary transformations $U_R^{(q)}$, $q = u, d$, which do not enter in the charged weak current and associated CKM matrix.

We make two observations about the bounds we have derived, assuming the CP-violating phase differences to be of order unity. First of all, each of the mixing angles in the diagonalization of the quark mass matrices is relatively small, a restrictive requirement for the further development of ETC models, perhaps along the lines of Refs. [2]-[6]. (The same is true for the charged leptons [7].) Secondly, while the mixing angles must be small, the bounds allow them to be "reasonable", that is, in the same range as the measured values of the CKM angles (which are expressible as combinations of mixing angles for left-handed quarks).

In the class of models of Refs. [2]-[6], small mixing angles can emerge for the up-type quark masses because the off-diagonal terms require mixing among the ETC gauge bosons, which is suppressed by ratios of ETC scales, while the diagonal terms do not. The mechanism employed in this class of models for the suppression of down-type quark masses and charged-lepton masses requires ETC gauge-boson mixing to generate diagonal as well as off-diagonal elements. Thus the ingredients for small mixing angles in the down-quark sector are not yet as evident.

Finally, we note that the experimental constraints on the dimension-five dipole operators still allow ETC models to produce sizable departures from the SM prediction, in particular for several CP violating observables. These include the EDM’s of the neutron and of $^{199}$Hg, the direct CP violation in the kaon system (measured as $\text{Re}(\epsilon'/\epsilon)$) and the CP asymmetry in the inclusive decay $b \rightarrow s\gamma$, or in the decay $B_d \rightarrow \phi K_S$. Improvements in the experimental sensitivity and reductions in the QCD uncertainties of the SM prediction could set more stringent bounds or even allow detection of ETC effects.

ACKNOWLEDGMENTS

We thank K. Lane for stimulating discussions and M. Frigerio for a comment. This research was partially supported by the grants DE-FG02-92ER-4074 (T.A., M.P.) and NSF-PHY-00-98527 (R.S.).
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