Determination of the spatial TDR-sensor characteristics in strong dispersive subsoil using 3D-FEM frequency domain simulations in combination with microwave dielectric spectroscopy

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Abstract
The spatial sensor characteristics of a 6 cm TDR flat band cable sensor section was simulated with finite element modelling (high frequency structure simulator—HFSS) under certain conditions: (i) in direct contact with the surrounding material (air, water of different salinities, different synthetic and natural soils (sand–silt–clay mixtures)), (ii) with consideration of a defined gap of different size filled with air or water and (iii) the cable sensor pressed at a borehole-wall. The complex dielectric permittivity \( \varepsilon^*(\omega, \tau_i) \) or complex electrical conductivity \( \sigma^*(\omega, \tau_i) = i\omega\varepsilon^*(\omega, \tau_i) \) of the investigated saturated and unsaturated soils was examined in the frequency range 50 MHz–20 GHz at room temperature and atmospheric pressure with a HP8720D-network analyser. Three soil-specific relaxation processes are assumed to act in the investigated frequency–temperature–pressure range: one primary \( \alpha \)-process (main water relaxation) and two secondary \( (\alpha', \beta) \)-processes due to clay–water–ion interactions (bound water relaxation and the Maxwell–Wagner effect). The dielectric relaxation behaviour of every process is described with the use of a simple fractional relaxation model. 3D finite element simulation is performed with a \( \lambda/3 \) based adaptive mesh refinement at a solution frequency of 1 MHz, 10 MHz, 0.1 GHz, 1 GHz and 12.5 GHz. The electromagnetic field distribution, S-parameter and step responses were examined. The simulation adequately reproduces the spatial and temporal electrical and magnetic field distribution. High-lossy soils cause, as a function of increasing gravimetric water content and bulk density, an increase in TDR signal rise time as well as a strong absorption of multiple reflections. An air or water gap works as a quasi-waveguide, i.e. the influence of the surrounding medium is strongly reduced. Appropriate TDR-travel-time distortions can be quantified.

Keywords: lossy dielectrics, finite element modelling, HFSS, dielectric spectroscopy, fractional relaxation

(Some figures in this article are in colour only in the electronic version)
1. Introduction

Soil science, geophysical prospecting, agriculture, hydrology, archeology and geotechnical engineering have benefited greatly from developments in radio and microwave technology. Electromagnetic techniques are used to estimate soil and rock physical characteristics such as water content, density and porosity [25, 27, 36, 46]. Both invasive methods, such as time domain reflectometry [15, 39, 48] and cross borehole radar [10], and non-invasive methods, such as capacity methods [25, 27, 45] and ground penetrating radar [2, 6, 13, 24, 41] are used. Common to all these techniques is the fact that electromagnetic wave interaction depends on the dielectric properties of rock or soil deposit through which it travels, which are influenced by chemical composition, mineralogy, structure, porosity, geological age and forming conditions. Besides, several additions like ubiquitous water have an effect on the dielectric properties.

In particular, knowledge of the spatial and temporal variability of water saturation in soils is important to obtain improved estimates of water flow (and its dissolved components) through the vadose zone. Due to its accuracy and potential for automated measurement, TDR has become one of the standard methods to measure the spatial and temporal variability of water content in laboratory soil cores and experimental field plots [15]. For this purpose, the object of numerous experimental and theoretical investigations is the development of general dielectric mixing models for a broad class of soil textures and structures [42, 48, 49]. Mostly, these empirical, numerical or theoretical models are based on the assumption of a constant dielectric permittivity of the soil as a function of volumetric water content in a narrow frequency range around 1 GHz [4, 9, 38, 43, 47]. However, the strong frequency dependence in the dielectric relaxation behaviour below 1 GHz due to a certain amount of swelling clay minerals in nearly each real soil is considered only insufficiently [15, 25, 30, 32].

The type of multi-scale structure renders the analysis of dynamic data in clays rather complex. The problem has been addressed both by experimental and modelling techniques. Besides broadband dielectric spectroscopy on clay–water suspensions, microscopic simulations of clays have been an active field of research since the late 1980s and began with simulations at ambient temperature and pressure, of clays with various cationic species. More recently, several studies appeared dealing with non-ambient conditions (increased temperature and pressures), which are primarily linked to the issue of storage of radioactive waste or bore-hole stability [34]. Previous experimental and modelling results suggest that clay–water systems have multiple relaxation processes, such as interfacial polarizations around the clay particles and rotational relaxation of bound and free H2O. Therefore, the dielectric behaviour is expected to be complicated. Useful and precise dielectric information may only be obtained when each relaxation process is extracted from the complicated overall behaviour based on the measurement of the complex dielectric permittivity over a broad frequency range and at high resolutions [21, 22, 32, 33].

In the present study, the dielectric relaxation behaviour of the investigated saturated and unsaturated soils was examined in the frequency range 50 MHz–20 GHz. To parametrize the dielectric spectra three soil-specific relaxation processes are assumed to act as a function of water saturation and porosity in the investigated frequency–temperature–pressure range: one primary α-process (main water relaxation) and two secondary (α′, β)-processes due to clay–water–ion interactions (bound water relaxation and the Maxwell–Wagner effect). The dielectric relaxation behaviour of every process is described with the use of a simple fractional relaxation model [16, 19, 21–23]. The chosen approach enables a characterization of the dielectric relaxation behaviour with the separation in the observed relaxation processes depending on the porosity and the water content.

A simplification frequently utilized in TDR applications is the use of an idealized equivalent circuit for the sensor without consideration of losses due to the skin-effect or radiation from the sensor as well as the assumption of a homogeneous sensitivity distribution along the sensor [15, 20, 28, 44]. In addition, a frequently arising problem in various applications is the direct contact between sensor, e.g. a flexible band cable, and the surrounding medium. An air or water gap between sensor and soil leads to dramatic under or overestimation of water content. A suitable tool for an examination of this specific problems offers three-dimensional numeric finite element simulation [28]. For these reasons, in this study the spatial sensor characteristics of a 6 cm TDR flat band cable sensor section was simulated with electromagnetic finite element modelling (Ansoft-HFSS™, high frequency structure simulator). In order to carry out the finite element calculations as realistically as possible, the measured frequency-dependent dielectric permittivity was considered. Moreover, the simulations were performed under certain conditions: (i) in direct contact with surrounding material, (ii) with consideration of a defined gap of variable size filled with air or water and (iii) cable sensor pressed at a borehole-wall.

2. Theoretical background

Time domain reflectometry measures the propagation velocity of a broadband step voltage pulse (typical values: rise time \( t_r \approx 70 \) ps, sampling increment \( \Delta t \approx 20 \) ps) with a bandwidth of around 20 kHz to 25 GHz (Nyquist frequency: \( f_{\text{max}} = 0.5/\Delta t \)). But due to the limitations of used connectors, type of the TDR device, coaxial cable type and length the effective bandwidth is reduced distinctly [31]. Under atmospheric conditions the velocity of this signal is a function of the frequency \( \omega = 2\pi f \) and temperature \( T \) dependent effective relative complex permittivity \( \varepsilon_{\text{eff}}(\omega, T) = \varepsilon_{\text{eff}}(\omega, T) - j\sigma_{\text{dc}}(\omega, T) \) of the material through which it travels. The overall losses \( \varepsilon_{\text{eff}}(\omega, T) = \varepsilon_{\text{eff}}^\prime(\omega, T) + \sigma_{\text{dc}}(T) \) of the material which have to be considered result from the dielectric losses \( \varepsilon_{\text{eff}}^\prime(\omega, T) \) and the conductive losses \( \sigma_{\text{dc}}(T) \) due to a direct current electrical conductivity \( \sigma_{\text{dc}}(T) \). Here, \( \varepsilon_0 \) is the permittivity of free space and \( j^2 = -1 \) is the imaginary unit. It is often convenient to consider the analogy of propagation phase velocity and attenuation of an electromagnetic plane wave (see [6, 15, 40, 48]):
of an electromagnetic wave in a real medium will propagate at a group velocity according to the Rayleigh equation [11, 12]:

\[
v_g = \frac{d\omega}{dk} = v_p \left[ 1 - \frac{f}{v_p} \frac{df}{d\omega} \right]^{-1}
\]

where \( k \) denotes the wave number. The flat band cable of length \( l \) consists of three strip conductors embedded in a polyethylene band (see [20, 27] for details). The effective group or phase velocity of the signal \( v_p \) in a perfect dielectric (pure real dielectric constant \( \varepsilon_r = \varepsilon'_r \) const without dispersion and conducting losses) surrounding the cable sensor is in principle only a crude approximation of the real material properties especially at frequencies \( f < 1 \) GHz and \( f > 10 \) GHz (cf figure 1)

\[
v_p = \frac{2f}{\tau} = \frac{c}{\sqrt{\varepsilon_r}}
\]

where \( \tau \) is two way travel time. Considering anomalous dispersion equation (4) is referred to as a high frequency approximation of phase velocity (figure 1). In contrast, the high frequency attenuation approximation \( \beta_h(\omega, T) \) for real soils frequently used in ground penetrating radar applications works considerably well (cf [6], figure 1):

\[
\beta_h(\omega, T) = \frac{\omega \varepsilon_r' \varepsilon''(\omega, T)}{\varepsilon'(\omega, T)} Z_0 \epsilon_0 \frac{\sqrt{\varepsilon'(\omega, T)}}{\sqrt{\varepsilon''(\omega, T)}} \frac{1}{2}
\]

with impedance \( Z_0 = c \mu_0 \) of vacuum. We now consider the soil as a four-phase medium composed of air, quartz grain, water, and clay. In the particular case of spatial TDR the surrounding medium in the direction of the band cable is described by a relative effective permittivity \( \varepsilon_{eff}(x, \omega, T, p) \).

Herein, \( h \) denotes the Planck constant, \( k_B \) the Boltzmann constant, \( \kappa_i \approx 1 \) the transmission coefficient, \( R \) the gas constant and \( E_{act}(T, p) = \Delta G_i(T, p) + T \Delta S_i(T, p) \) activation energy with free enthalpy \( \Delta G_i(T, p) \) and activation entropy \( \Delta S_i(T, p) \) of the \( i \)th process [21, 22]. Dielectric loss spectra of saturated and unsaturated soils very often show a marked deviation from simple Debye behaviour [17, 19, 22, 25]. Based on the theory of fractional time evolutions Hilfer [16] derived a Jonscher type function [23] for the complex frequency dependent dielectric permittivity of amorphous and glassy materials,

\[
\varepsilon_{eff}(\omega, \tau) - \varepsilon_{\infty} = \frac{\Delta \varepsilon_i(T)}{(\jmath \omega \tau_i)^\alpha} + (\jmath \omega \tau_i)^{\beta}
\]

\[
\varepsilon_{\infty} = (\jmath \omega \tau_i)^{\alpha} + (\jmath \omega \tau_i)^{-\beta}
\]

where \( \alpha, \beta \) are stretching exponents 0 ⩽ \( \alpha, \beta \) similar to the familiar empirical Havriliak and Negami [14], Cole–Cole [3].
well as the gravimetric water content. Here, we present our results for synthetic soil SB50 after calibration with open, short and load standards.

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After each dielectric measurement the bulk density \( \rho \) and to a constant volume. Care was taken to pack the soil in the transmission line to a homogeneous bulk density \( \rho \).

The measured complex S-parameter values \( S_{ij} \) were used to calculate complex dielectric permittivity with commercial software (HP 85070/71C Materials Measurement Software) after calibration with short, open and load standards.

Different natural and synthetic soils were investigated. Here, we present our results for synthetic soil SB50/50. It is a mixture of 50 wt% sand (grain size <2 mm) and 50 wt% bentonite (calcigel: 71 wt% Ca-dioctahedral smectite, 9 wt% illite/dioctahedral mica, 1 wt% kaolinite, 1 wt% chlorite, 9 wt% quartz, 5 wt% feldspar, 2 wt% calcite, 2 wt% dolomite). Ca-dioctahedral smectites are clay minerals, i.e., they consist of individual crystallites the majority of which are less than 2 \( \mu \)m in the largest dimension. The crystal structure of clays has been established from x-ray diffraction studies for almost all types of common clays. Smectite crystallites themselves are three-layer clay minerals. Individual clay layers consist of fused sheets of octahedra of Al\(^{3+}\) or Mg\(^{2+}\) oxides and tetrahedra of Si\(^{4+}\) oxides. Substitution of Al\(^{3+}\), Mg\(^{2+}\) or Si\(^{4+}\) with lower valence ions results in an overall negative charge on the layers, which is then compensated by cationic species (counterions, in the current case Ca\(^{2+}\)) between clay layers (interlayers) ([29], figure 3).

Under increasing relative humidity the compensating ions become hydrated and the spacing between individual layers increases. While the detailed swelling characteristics of a smectite (interlayer spacing as a function of relative humidity) depends crucially on the charge of the clay layers (magnitude and localization) and the nature of the compensating ion, in general, it occurs in three stages. Swelling begins in a step-wise manner (discrete layers of water formed in the interlayer, states referred to as monolayer/monohydrated, bilayer/bihydrated, etc), becomes continuous thereafter and in the extreme a colloidal suspension of clay particles (<10 \( \mu \)m in size) is formed. Beyond the microscopic scale, aggregates of aligned clay layers form particles of the order of 10–1000 nm in size, with porosities on the meso- (8–60 nm) and macro scale (>60 nm) ([34] and citations in it). In table 1 the physical and chemical properties of the used bentonite calcigel are summarized.

Three relaxation processes are assumed to act in the investigated frequency–temperature–pressure range: one primary \( \alpha \)-process (main water relaxation) and two secondary (\( \alpha' \), \( \beta' \))-processes due to clay–water–ion interactions (bound
The effective permittivity of a multiphase soil-mixture can be determined by the complex relative permittivity of water \( \varepsilon_{\text{water}} \), bound water \( \varepsilon_{\text{bound}} \), the contribution due to clay–water–ion interaction \( \varepsilon_{\text{clay}} \) as well as the real and constant permittivity of quartz grain \( \varepsilon_{\text{sand}} \) and air [4, 21, 22, 36, 43, 46, 47]. The dielectric relaxation behaviour of each process is described by a fractional relaxation model according to (7) considering relaxation time distributions \( H(\tau) \). This allows the complete spectrum to fit as a function of water content \( \theta_g \) and bulk density \( \varphi \) at constant temperature and pressure with the use of a generalized dielectric response (GDR, figures 4 and 5):

\[
\varepsilon_{\text{eff}}(\omega) - \varepsilon_{\infty} = \sum_{i=1}^{3} \frac{\Delta \varepsilon_i}{(j\omega\tau_i)^\alpha_i + (j\omega\tau_i)^\beta_i} - \frac{\varepsilon_{dc}}{\omega\varepsilon_0} \tag{8}
\]

A shuffled complex evolution metropolis (SCEM-UA) algorithm is used to find best GDR fitting parameters [15, table 2]. This algorithm is an adaptive evolutionary Monte Carlo Markov chain method inspired by the SCE-UA global optimization algorithm of [7] and combines the strengths of the Metropolis algorithm [35], controlled random search [37], competitive evolution [18], and complex shuffling [7] to obtain an efficient estimate of the most optimal parameter set, and its underlying posterior distribution, within a single optimization run. The resulting relative error of each parameter is less than 3%.

The relaxation parameters obtained from the GDR-fit are presented in figure 6. The relaxation strength \( \Delta \varepsilon_i \) of each process and the relative high frequency permittivity \( \varepsilon_{\infty} \) depend on moisture content. Above a gravimetric water content of \( \approx 30 \text{ wt}\% \) the \( \alpha' \)-process decreases and the \( \alpha \)-process strongly increases. This suggest an increase of the primary \( \alpha \)-relaxation at the expense of the bound water process \( \alpha' \). In contrast, at the highest saturation level of 0.66 at a porosity of 0.53 the primary \( \alpha \) and low frequency \( \beta \)-process decrease. The relaxation time \( \tau_1 \) and the distribution parameter \( \beta_i \) are nearly constant. An exception represents sample 50-0 which shows clear deviation from the general trend because of the very low gravimetric water content of 0.6 wt%. Without swelling, increasing saturation reduces the gas-filled pore space, while the effective pore space available for the fluid becomes larger. This leads to the simple calibration models mentioned in the introduction to determine the volumetric water content with dielectric measurements. Due to the swelling of the clay minerals in the process of hydration the distribution of immobile and effective pore space cannot be considered as constant. A circumstance which complicate a careful realization of accurate measurements is the fact that a multitude of variables of the measurement conditions and the sample preparation essentially determine the intensity of the swelling and the resulting change of the pore structure [1].

The results show the potential of the chosen approach but a detailed explanation of this complex behaviour is beyond the scope of this paper. In general, there is a need for further systematic investigations by broadband dielectric spectroscopy of saturated and unsaturated soils under consideration of the swelling process and with utilization of microscopic modelling.
Figure 6. Top: relaxation strength $\Delta \varepsilon_i$ of the $i$th process in comparison to the relative high frequency permittivity $\varepsilon_\infty$ as a function (left) of gravimetric water content and (right) water saturation $S_w$. Bottom: apparent direct current electrical conductivity $\sigma_{dc}$.

Table 2. Parameters of the three relaxation processes from GDR-fitting ($i = [\alpha, \alpha', \beta]$): gravimetric water content $\theta_g$, bulk density $\varrho$, porosity $\Phi$, water saturation $S_w$ and volumetric water content $\theta_V = S_w/\Phi$ of SB-50/50.

| Parameter | 50-0 | 50-1 | 50-2 | 50-3 | 50-4 | 50-5 | 50-6 | Mikrosol |
|-----------|------|------|------|------|------|------|------|---------|
| $\theta_g$ (wt%) | 0.6 | 5.0 | 11.4 | 18.1 | 28.1 | 33.5 | 41.5 | 25.47 |
| $\varrho$ (g cm$^{-3}$) | 1.35 | 1.14 | 1.37 | 1.34 | 1.75 | 1.52 | 1.66 | 1.55 |
| $\Phi$ | 0.5 | 0.6 | 0.55 | 0.59 | 0.53 | 0.62 | 0.64 | 0.56 |
| $S_w$ (vol%) | 0.02 | 0.09 | 0.25 | 0.34 | 0.66 | 0.54 | 0.63 | 0.22 |
| $\theta_V$ (vol%) | 0.81 | 5.42 | 13.84 | 19.86 | 35.36 | 33.86 | 40.3 | 39.48 |
| $\varepsilon_\infty$ | 1.03 | 1.15 | 1.23 | 2.07 | 3.38 | 3.77 | 6.69 | 1.26 |
| $\Delta \varepsilon_\alpha$ | 0.86 | 0.05 | 0.01 | 0.13 | 5.44 | 3.67 | 19.57 | 16.73 |
| $\tau_\alpha$ (ps) | 4.99 | 9.18 | 8.44 | 7.43 | 6.06 | 8.74 | 9.8 | 9.25 |
| $1 - \beta_\alpha$ | 0.004 | 0.004 | 0.07 | 0.025 | 0.075 | 0.091 | 0.095 | 0.002 |
| $\alpha_\alpha$ (fixed) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Delta \varepsilon_\alpha'$ | 0.25 | 2.16 | 3.18 | 5.06 | 10.31 | 19.7 | 5.77 | 2.09 |
| $\tau_\alpha'$ (ps) | 5.37 | 9.64 | 9.34 | 9.63 | 9.93 | 9.7 | 9.32 | 0.27 |
| $1 - \beta_\alpha'$ | 0.004 | 0.081 | 0.081 | 0.075 | 0.003 | 0.082 | 0.034 | 0.048 |
| $\alpha_\alpha'$ (fixed) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\Delta \varepsilon_\beta$ | 0.05 | 1.57 | 4.9 | 11.06 | 43.35 | 62.17 | 74.99 | 0.87 |
| $\tau_\beta$ (ns) | 98.46 | 0.5 | 0.6 | 0.51 | 0.52 | 0.65 | 0.83 | 22.6 |
| $1 - \beta_\beta$ | 0.65 | 0.99 | 0.56 | 0.69 | 0.66 | 0.68 | 0.74 | 0.92 |
| $\alpha_\beta$ (fixed) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\sigma_{dc}$ (mS cm$^{-1}$) | 6.68E-5 | 0.03 | 0.05 | 0.75 | 3.37 | 4.62 | 5.31 | 0.12 |

4. HFSS-simulations

The transfer or scattering function $S_{ij}(\omega)$ of the flat band cable section (figure 7) was simulated by finite element modelling (commercial software from Ansoft: high frequency structure simulator—HFSS$^1$) under certain conditions: (i) in direct contact with the surrounding material (air, water of various salinities, different synthetic and natural soils (sand–silt–clay mixtures)), (ii) with consideration of a defined gap of various size (total 2 mm, 3 mm, 5 mm, 7 mm or 10 mm) filled with air or distilled water and (iii) cable sensor pressed at a borehole-wall.

In HFSS tangential element basis function interpolates field values from both nodal values at vertices and on edges. Surfaces of the structure (air-box) in the $yz$ and $xy$ planes are radiation boundaries and the second-order radiation boundary condition is used

$$\nabla \times \vec{E}_t = jk_0 \vec{E}_t - \frac{j}{k_0} \nabla \times (\nabla \times \vec{E}_t) + \frac{j}{k_0} \nabla (\nabla \cdot \vec{E}_t)$$

where $\vec{E}_t$ is the component of the E-field that is tangential to the surface and $k_0$ is the free space phase constant. The second-order radiation boundary condition is an approximation of free space. The accuracy of the approximation depends on the distance between the boundary and the object from which the radiation emanates. For this reason a sensitivity analysis was carried out for a total height of the structure between

$\text{1 HFSS is a standard simulation package for electromagnetic design and optimization.}$
Determination of the spatial TDR-sensor characteristics

Figure 7. Model geometry of (top) flat band cable surrounded by saturated and unsaturated soil with a gap filled with air or water and (bottom) cable sensor pressed at a borehole-wall with air or soil as borehole filling.

20 mm and 50 mm depending on the material properties. The influence of the boundary layer for the simulation into air can be neglected for a distance from the cable sensor greater than 12 mm, so a minimum height of 25 mm is used.

Surfaces in the $xz$ plane are wave ports. HFSS assumes that each wave port is connected to a semi-infinitely long waveguide that has the same cross-section and material properties as the port. When solving for the S-parameters, HFSS assumes that the structure is excited by the natural field patterns (modes) associated with these cross-sections. The 2D field solutions generated for each wave port serve as boundary conditions at those ports for the 3D problem. The final field solution computed must match the 2D field pattern at each port.

The simulation is performed with a $\lambda/3$ based adaptive mesh refinement at a solution frequency of 1 MHz, 10 MHz, 0.1 GHz, 1 GHz and 12.5 GHz. Broadband complex S-parameters are calculated with an interpolating sweep in frequency range 1 MHz–12.5 GHz with extrapolation to dc. The electromagnetic field distribution, S-parameter and step response (200 ps rise time) of the structure were computed in reflection and transmission mode.

5. Discussion

The simulation adequately reproduces the spatial and temporal electrical and magnetic field distributions in comparison with 2D-FE investigations of [20] (figure 8). Figure 9 represents reflection and transmission factors as well as the corresponding TDR waveform for the sand–bentonite mixture at various water contents and bulk densities in reflection and transmission mode. As a reference material the simulation results for the cable sensor surrounded by air are included. As expected, the appropriate resonances in the reflection coefficient $S_{11}$ shift with rising water content to deeper frequencies and the attenuation increases. In the time domain the onset travel-time as well as the rise time of the TDR signals are analysed. As is clearly recognizable in both reflection and transmission mode, the onset time increases and the rise times decreases with rising water content.

Qualitatively the numerical calculation shows that the sensitivity characteristic of the cable sensor changes along the sensor as a function of the dielectric relaxation behaviour of the surrounding material (figure 8). The investigated high-lossy sand–bentonite mixture causes, as a function of increasing gravimetric water content $\theta_g$ and bulk density $\rho$, an increase in TDR signal rise time as well as strong absorption of multiple reflections. This leads to a frequency dependent decrease of spatial resolution and penetration depth (sensitivity region around the sensor) along the flat band cable.

Figure 8. Electric field distribution at the rate of 12.5 GHz for the investigated flat band cable surrounded by air, sand–bentonite mixture (SB 50/50-4) with $\theta_g = 28.14$ wt% and $\rho = 1.79$ g cm$^{-3}$ as well as a defined 3 mm air or water gap. Left, cross section; right, longitudinal section of the middle conductor.
Figure 9. Left: input return loss magnitude or reflection coefficient $S_{11}$ and forward transmission or transmission coefficient $S_{21}$; right: TDR-waveform in reflection and transmission mode for simulated flat band cable structure, surrounded by air and sand bentonite mixture of varying water content and bulk density (see table 2).

Figure 10. Real effective relative permittivity $\varepsilon_{\text{eff}} = 2/l_{\text{onset}}$ according to equation (4) plotted against TDR rise time $t_{\text{rise}}$ (in reflection mode) for all sensor configurations and investigated cases.

As a consequence the gap works as a quasi-waveguide, i.e. the influence of the surrounding medium is strongly reduced but changes in dielectric properties along the cable sensor are reproduced.

6. Conclusion

In this study, the spatial sensor characteristics of a 6 cm TDR flat band cable section is simulated with finite element modelling in combination with dielectric spectroscopy.

For this purpose the dielectric relaxation behaviour of saturated and unsaturated soils is examined in the frequency range 50 MHz–20 GHz. The dielectric relaxation behaviour is described with the use of a generalized fractional relaxation model. With this approach the frequency dependent dielectric permittivity is determined based on a parametrization of each relaxation process as a function of water content and porosity. This enables a development of improved calibration strategies. However, there is a need for further systematic investigations by broadband dielectric spectroscopy of saturated and unsaturated soils under consideration of the swelling process and with utilization of microscopic modelling.

The three-dimensional numeric finite element simulation in HFSS provides an informative basis of the sensor characteristics under consideration of the frequency dependence of the measured complex dielectric permittivity. It is shown that an air or water gap between sensor and soil leads to dramatic under or overestimation of water content already for a gap thickness of 0.25 mm above and below the cable sensor. Therefore, the application of the flat cable as a moisture sensor requires an accurate installation technique of cable-like elements (especially for long sensors). Moreover, the spatial sensitivity characteristics of the cable sensor change along the sensor as a function of the dielectric relaxation behaviour.
of the surrounding material. For this reason, the precise determination of soil moisture profiles requires an improved TDR-waveform analysis strategy under consideration of the change of the sensitive area along the cable sensor.

A disadvantage of the used FEM with HFSS is the lack of consideration of exchange of material between the air or water-filled gap and the surrounding medium. In addition, to achieve sufficient accuracy the numerical simulations with HFSS requires a high degree of computational cost in terms of computational time and memory. Further theoretical, numerical, and experimental investigations in conjunction with reconstruction algorithms have to point out to what extent the accuracy of water content and porosity profiles can be determined in strong dispersive, high loss materials.

References

[1] Agus S S and Schanz T 2005 Comparison of four methods for measuring total suction Vadose Zone J. 4 1087–95
[2] Annan A P, Waller W M, Strangway D W, Rossiter J R, Redman J D and Watts R D 1975 The electromagnetic response of a low-loss, 2-layer, dielectric earth for horizontal electric dipole excitation Geophysics 40 285–98
[3] Cole K S and Cole R H 1941 Dispersion and absorption in dielectrics J. Chem. Phys. 9 341–51
[4] Cosenza Ph and Tabbagh A 2004 Electromagnetic determination of clay water content: role of the microporosity Appl. Clay Sci. 26 21–36
[5] Davidson D and Cole R 1951 Dielectric relaxation in glycerol, propylene glycol, and n-propanol J. Chem. Phys. 19 1484–90
[6] Davis J L and Annan A P 1989 Ground-penetrating radar for high-resolution mapping of soil and rock stratigraphy Geophys. Prospect. 37 531–51
[7] Duan Q, Sorooshian S and Gupta V 1992 Effective and efficient global optimization for conceptual rainfall-runoff models Water Resour. Res. 28 1015–31
[8] Engelhardt I, Finsterle S and Hofstee C 2003 Experimental and numerical investigation of flow phenomena in nonisothermal, variably saturated bentonite-crushed rock mixtures Vadose Zone J. 2 239–46
[9] Evert S R and Parkin G W 2005 Advances in soil water content sensing: the continuing maturation of technology and theory Vadose Zone J. 4 986–91
[10] Fechner T, Bömer F, Richter T, Yaramakav and Weihnacht B 2004 Near Surf Geophys. 23 150–9
[11] Forkmann B and Petzold H 1989 Prinzip und Anwendung des Gesteinsraders zur Erkundung des Nahbereichs (Leipzig: VEB Deutscher Verlag für Grundstoffindustrie)
[12] Greiner W 1998 Classical Electrodynamics (Berlin: Springer)
[13] Hanafy S and Hagrey S A al 2006 Ground-penetrating radar tomography for soil-moisture heterogeneity Geophysics 71 K9–K18
[14] Havriliak S and Negami S 1967 A complex plane model of glassy systems Phys. Rev. E 65 061510
[15] Heekstra P and Delaney A 1974 Dielectric properties of soils at UHF and microwave frequencies J. Geophys. Res. 79 6999–708
[16] Holland J H 1975 Adaptation in Natural and Artificial Systems (Ann Arbor, MI: The University of Michigan Press)
[17] Hollender F and Tillard S 1998 Modeling ground-penetrating radar wave propagation and reflection with the Jonscher parameterization Geophysics 63 1933–42
[18] Ichioka T, Kawase M, Yagi K, Yamakawa J and Fukada K 2003 Effects of the counterion on dielectric spectroscopy of a montmorillonite suspension over the frequency range 105–109 Hz J. Colloid Interface Sci. 268 121–6
[19] Kellners T J, Robinson D A, Shouse P J, Ayars J E and Skaggs T H 2005 Frequency dependence of the complex permittivity and its impact on dielectric sensor calibration in soils Soil Sci. Soc. Am. J. 69 67–76
[20] Kohlauch R 1847 Ann. Phys., Lpz. 12 393
[21] Kupfer K 2005 Electromagnetic Aquametry (Berlin: Springer)
[22] Leidenberger P, Oswald B and Roth K 2005 Efficient reconstruction of dispersive dielectric profiles using time domain reflectometry (TDR) Hydrology Earth Syst. Sci. 2 1449–502
[23] Liebau F 1985 Structural Chemistry of Silicates (Berlin: Springer)
[24] Logsdon S D 2000 Cation and water content effects on dipole rotation activation energy of smectites Soil Sci. Soc. Am. J. 68 1586–91
[25] Logsdon S D 2000 Effect of cable length on time domain reflectometry calibration for high surface area soils Soil Sci. Soc. Am. J. 64 54–61
[26] Logsdon S D 2005 Soil dielectric spectra from vector network analyzer data Soil Sci. Soc. Am. J. 69 983–9
[27] Logsdon S D and Laird D A 2004 Electrical conductivity spectra of smectites as influenced by saturating cation and humidity Clays Clay Miner. 52 411–20
[28] Malikova N, Cadene A, Marry V, Dubois E, Turq P, Zanotti J-M and Longeville S 2005 Diffusion of water in clay—microscopic simulation and neutron scattering Chem. Phys. 317 226–35
[29] Metropolis N, Rosenbluth A W, Rosenbluth M N, Teller A H and Teller E 1953 Equation of state calculations by fast computing machines J. Chem. Phys. 21 1087–92
[30] Robinson D A, Jones S B, Wraith J M, Or D and Friedman S P 2003 A review of advances in dielectric and electrical conductivity measurement in soils using time domain reflectometry Vadose Zone J. 2 444–75
[31] Robinson D A, Schaap M G, Or D and Jones S B 2005 On the effective measurement frequency of time domain reflectometry in dispersive and nonconductive dielectric materials Water Resour. Res. 41 2007/1–9
[41] Rossiter J R, Strangway D W, Annan A P, Watts R D and Redman J D 1975 Detection of thin layers by radio interferometry Geophysics 40 299–308
[42] Roth C H, Malicki M A and Plagge R 1992 Empirical evaluation of the relationship between soil dielectric constant and volumetric water content as the basis for calibrating soil moisture measurements by TDR J. Soil Sci. 43 1–13
[43] Saarenketo T 1998 Electrical properties of water in clay and silty soils J. Appl. Geophys. 40 73–88
[44] Schlaeger S 2005 A fast TDR-inversion technique for the reconstruction of spatial soil moisture content Hydrol. Earth Syst. Sci. 9 481–92
[45] Seyfried M S and Murdock M D 2004 Measurement of soil water content with a 50-MHz soil dielectric sensor Soil Sci. Soc. Am. J. 68 394–403
[46] Shen L C, Savre W C, Price J M and Athavale K 1985 Dielectric properties of reservoir rocks at ultra-high frequencies Geophysics 50 629–704
[47] Sihvola A 2000 Electromagnetic Mixing Formulae and Applications (IEE Electromagnetic Waves Series no 47)
[48] Topp G C, Davis J L and Annan A P 1980 Electromagnetic determination of soil water content: measurement in coaxial transmission lines Water Resour. Res. 16 574–82
[49] Topp G C, Zegelin S and White I 2000 Impacts of the real and imaginary components of relative permittivity on time domain reflectometry measurements in soils Soil Sci. Soc. Am. J. 64 1244–52
[50] Williams G and Watts D C 1970 Non-symmetrical dielectric relaxation behaviour arising from a simple empirical decay function Trans. Faraday Soc. 66 80–5