1 INTRODUCTION

Simulation of a virtual qubits on a classical computer should not encounter any insuperable difficulties, as the operating program for quantum states modelling on a classical computer has been already developed [1]. In the present study we shall dwell on a classical computer model of quantum entangled states implemented in Pascal (with Delphy) language. This is an operating program of a virtual model for two q-bits. It simulates a controlled correlation or anti-correlation of EPR-Bohm type, including the total one. As the model considered is a classical one, the Bell’s inequalities are not violated within it. The correlation factor of this classical model differs from the quantum analog of EPR state by the factor of $\sqrt{2}$. The simulated operating model of quantum entangled states and related discussions simplify understanding of quantum states and EPR-Bohm correlations. This enables starting the study of quantum algorithms and programs for quantum computer modelling on a classical computer.

Until now there has not been found any satisfactory way of modelling of quantum state patterns and their interference on a classical computer. The idea of quantum calculation put forward by R. Feynman in [2, 3] relies upon the impossibility of quantum patterns calculation on a classical computer in virtue of various reasons. At the same time, creation of quantum pattern models with the help of quantum elements would be rather straightforward. In its turn, this means that quantum elements must be created. This naturally means that these elements must interfere with each other, their correlation factor having to be nonzero. Then, alteration of a single element would alter the whole quantum pattern. This property of quantum elements is called quantum parallelism. While the value of an information unit (a bit) in a classical computer is
defined as either 0 or 1, in a quantum computer each quantum element is described by a wave function $\psi = \alpha |0\rangle + \beta |1\rangle$, meaning this element being in the state of superposition of zero and unity, $\alpha, \beta$ being the state complex amplitude (providing $|\alpha|^2 + |\beta|^2 = 1$) with probabilities $P(0) = |\alpha|^2, P(1) = |\beta|^2$.

Construction of virtual quantum states became possible due to the hypothesis on the nature of quantum states [4]. This study considers a stochastic geometrical background generating correlation (or, coherency) of various quantum non-interacting objects. The area of this background localization is called the area of coherence zone. In this area the correlation factor of various quantum microobjects is nonzero. To explain how this could occur, let us consider a physical model with the background of stochastic gravitational fields and waves representing the effect of the stochastic geometrical background, that is, the physical model with the gravitational background (i.e. gravitational fields and waves background). This means that we assume the existence of fluctuations of gravitational waves and fields in each point of the space, which are mathematically represented by metrics fluctuations. If we to discuss a possible quantization mechanism, based on solution concept by Einstein-de Brogle and use the representation of extended particles as localized self-gravitating structures, we can have the self-gravitating solution with the non-resonance quantization mechanism as result [5].

Theoretical investigation of the vacuum accounting for gravitational fields has been performed in the works by Academician Andrey D. Sakharov [6,7]. In his first paper of 1967, ”Vacuum Quantum Fluctuations in the Curved Space and Gravitation Theory” it has been stated that “it is assumed in the modern quantum field theory that the energy-momentum tensor of vacuum quantum fluctuations equal zero, and the respective action $S(0)$ is actually zero”. He has further shown that accounting for gravitational field in the vacuum, taking into consideration definition of space-time action dependence form curvature in the gravitational theory of A. Einstein (with invariants of Ricci tensor $R$ and metric tensor $g$), the action function taking on the form

$$S(R) = -\frac{1}{16\pi G} \int (dx)\sqrt{-g}R.$$  

Resultant action of all these gravitational fields with number $j$ form the functional

$$S_0(\psi) = \sum_{j=1}^{\infty} S_j,$$  

$\psi(x)$ being the external field given by the metric tensor $g_{ik}$ of the gravitational field.

Let us consider two classical particles in a field of random gravitational fields or waves. The General Theory of Relativity gives the length element in 4-dimensional Riemann space as

$$d\ell^2 = g_{ik}dx^idx^k,$$

the metric in the linear approach is

$$g_{ik} = \eta_{ik} + h_{ik},$$
\[ \eta_{ik} \] being Minkowsky metric, constituting the unity diagonal matrix. Hereinafter, the indices \( i, k, \mu, \nu, \gamma, m, n \) acquire values 0, 1, 2, 3. Indices encountered twice imply summation thereupon. Let us select harmonic coordinates (the condition of harmonicity of coordinates mean selection of concomitant frame \( \frac{\partial h_m}{\partial x_m} = \frac{1}{2} \frac{\partial h_m}{\partial x_m} \)) and let us take into consideration that \( h_{\mu\nu} \) satisfies the gravitational field equations

\[ \Box h_{mn} = -16\pi G S_{mn}, \]

which follow from the General Theory of Relativity; here \( S_{mn} \) is energy-momentum tensor of gravitational field sources with d’Alembertian \( \Box \) and gravity constant \( G \). Then, the solution shall acquire the form

\[ h_{\mu\nu} = e_{\mu\nu} \exp(ik\gamma x^\gamma) + e_{\mu\nu}^* \exp(ik\gamma x^\gamma), \]

where the value \( h_{\mu\nu} \) is called metric perturbation, \( e_{\mu\nu} \) polarization, and \( k_{\gamma} \) is 4-dimensional wave vector. We shall assume that metric perturbation \( h_{\mu\nu} \) are distributed in space with an unknown distribution function \( \rho = \rho(h_{\mu\nu}) \).

Relative displacements \( \ell \) of two particles in classic gravitational fields are described in the General Theory of Relativity by deviation equations

\[ \frac{D^2}{D\tau^2} \ell^i (j) = R_{kmn}^i (j) \ell^m \frac{dx^k}{d\tau} \frac{dx^n}{d\tau}, \]

\( R_{kmn}^i (j) \) being the gravitational field Riemann’s tensor with gravitational field number \( j \) of the stochastic gravitational fields.

Specifically, the deviation equations give the equations for two particles oscillations

\[ \ddot{\ell}^i + c^2 R_{010}^i \ell^i = 0, \quad \omega = c \sqrt{R_{010}^i}. \]

The solution of this equation has the form

\[ \ell^i (j) = \ell_0 \exp(k_a x^a + i\omega(j)t), \]

with \( a = 1, 2, 3 \). Each gravitational field or wave with index \( j \) and Riemann’s tensor \( R_{kmn}^i (j) \) shall be corresponding to the value \( \ell^i (j) \) with random modulated phase \( \Phi(j) = \omega(j)t \). If we sum all fields, we can write \( \Phi(t) = \omega(t)t \), where \( t \) is the time coordinate.

This random phase is the same for various quantum microobject in the area of this coherent background localization[8], this area being defined as the one within which the correlation factor for these particles is nonzero. Harmonic oscillations with this type phases could model entangled states.

2. MODELLING OF A VIRTUAL QUBIT ON A CLASSICAL COMPUTER

In the quantum state virtual model, a simple method of generating of two (or more) dichotomic random signals with controlled mutual correlation factor out of a single continuous stochastic process is implemented [1].
Basing on the system random number generator of the computer, a model of the stationary random process \( \Phi(t) \) with \( \langle \Phi(t) \rangle = 0 \) has been built determining the random phase evenly distributed over the interval \( 0 \div 2\pi \). Further, a random signal was generated on its basis with the help of the algorithm

\[
a(\alpha, t) = \text{sign} \{ \cos [\Phi(t) + \alpha] \},
\]

\( a \) being \( \alpha \) an arbitrary parameter. It follows from this definition that \( \langle a(\alpha) \rangle = 0 \), \( a(\alpha \pm \pi) = -a(\alpha) \), that is, the signals \( a(\alpha) \) and \( a(\alpha \pm \pi) \) are anticorrelated. At the same time, \( a(\alpha) \) and \( a(\alpha \pm \frac{\pi}{2}) \) are non-correlated signals. In the general case, correlation of signals \( a(\alpha) \) and \( a(\alpha + \Delta \alpha) \) is \( M(\Delta \alpha) = \langle x(\alpha)x(\alpha + \Delta \alpha) \rangle = 1 - 2|\Delta \alpha|/\pi \).

Therefore, out of a single stochastic process \( \Phi(t) \) it is possible to generate two (or more) stochastic dichotomic signals \( a(\alpha) \) and \( a(\alpha + \Delta \alpha) \) with an arbitrary correlation \( M(\Delta \alpha) \) between them confined within \(-1 \) and \(+1\). Hence, should the same effect \( \Phi(t) \) [1] act on several distant observers, then each of the latter ones, with the help of the "individual" local parameter \( \alpha \) will be capable of "distant influencing" the paired mutual correlation of observables. This effect is a classical analog of entangled state's correlation.

2 INITIALIZATION OF VIRTUAL QUBITS

Let us consider \( N \) virtual qubits constructed according to the above algorithm. Let us call q-bit No.1 the control, or signal, one.

If the signal qubit is green, then we assign it the value \( |0\rangle \), if it is red, we output nothing. Hence, at each output we shall get \( N \) initialized integral qubits. In calculations, red color will correspond to the value \( |1\rangle \).

3 HADAMARD TRANSFORM FOR VIRTUAL QUBITS

Let us assume we have a qubit

\[
|q\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle);
\]

here, for the state \( |0\rangle \) the probability \( P_{|0\rangle} = \frac{1}{2} \), for the state \( |1\rangle \) the probability \( P_{|1\rangle} = \frac{1}{2} \).

However, in the basis rotated by \( \frac{\pi}{4} \) the state of the virtual qubit is determined.

Let the Hadamard transform \( H \) be the state of the qubit in the basis rotated by \( \frac{\pi}{4} \). Then,

\[
H|0\rangle \rightarrow \frac{1}{2}(|0\rangle + |1\rangle)
\]

\[
H|1\rangle \rightarrow \frac{1}{2}(|0\rangle + |1\rangle).
\]
Hence, having applied the Hadamard transform to the virtual qubit $q$, we get the precisely determined value of the qubit

$$H |q\rangle = |0\rangle.$$ 

4 LOGICAL COMPONENT $CNOT$ IN A VIRTUAL QUANTUM COMPUTER

To apply the logical component $CNOT$, using of the above control qubit is required. The qubit to which the logical operation $CNOT$ is applied will be called the target qubit. The logical value of the control q-bit is not altered.

Let us consider two cases:

1. If the control qubit has the value $|1\rangle$, then the target q-bit is switched into the opposite value.
2. If the control qubit has the value $|0\rangle$, then the value of the target q-bit is not altered.

5 CONCLUSION

Operation of quantum algorithms on a virtual quantum computer in case of emulation on a classical computer, a certain time saving should be achieved in solution of certain problems. However, comparing the computation speed of real and virtual quantum computers, one could infer, basing on conclusions of Bell [10] that computation speed of a real quantum computer shall be higher than that of a virtual quantum computer. This is related to the fact that the correlation factor for real quantum states, according to Bell’s conclusions, should exceed by the factor of the respective value for classical models of these quantum states, that is $\sqrt{2}$, for virtual quantum states. On the other hand, presently there is a lot of problems related to implementations, measurements, decoherentization, etc. This gives rise to the question whether a virtual quantum computer would in the nearest future possess a gain in time in implementing quantum algorithm for quantum computation with respect to a real quantum computer. The answer could be positive due to the host of technical and technological problems with a real quantum computer impossible to be solved in the near future. However, even in the contrary case the virtual model will all the same find an application in classical computers, as it does not require alteration of latter ones, but rather, extends the capabilities of a classical computer through special program utilities described above.

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