Time-scales of stellar rotational variability and starspot diagnostics

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Accepted 2017 October 12. Received 2017 October 11; in original form 2017 July 21

ABSTRACT
The difference in stability of starspot distribution on the global and hemispherical scales is studied in the rotational spot variability of 1998 main-sequence stars observed by Kepler mission. It is found that the largest patterns are much more stable than smaller ones for cool, slow rotators, whereas the difference is less pronounced for hotter stars and/or faster rotators. This distinction is interpreted in terms of two mechanisms: (1) the diffusive decay of long-living spots in activity complexes of stars with saturated magnetic dynamos, and (2) the spot emergence, which is modulated by gigantic turbulent flows in convection zones of stars with a weaker magnetism. This opens a way for investigation of stellar deep convection, which is yet inaccessible for asteroseismology. Moreover, a subdiffusion in stellar photospheres was revealed from observations for the first time. A diagnostic diagram was proposed that allows differentiation and selection of stars for more detailed studies of these phenomena.

Key words: diffusion – turbulence – stars: activity – stars: interiors – starspots.

1 INTRODUCTION
Spots in stellar photospheres are extensively studied as an indicator of physical processes in stars and as a proxy of sunspot phenomenology. Nevertheless, particular features in evolution of starspot patterns and their physical drivers in stars of various types are insufficiently explored. For example, the measurements of spot lifetime lead to unexpectedly high estimates of magnetic diffusivity in some stars, supposing an anomalous magnetic diffusion related to the famous giant convection cells (Bradshaw & Hartigan 2014). Besides magnetic cycles, the shorter time-scales of starspot variability traditionally were interpreted in terms of diffusive decay and spot disruption by the shearing of stellar differential rotation (e.g. Bradshaw & Hartigan 2014 and therein). However, the distinction between these effects rarely was a subject of practical study (Hall & Henry 1994). Mainly both the mechanisms were considered in stellar modelling without distinction and clear diagnostics (e.g. Isik, Schüssler & Solanki 2007 and therein). Additionally, the spot emergence frequency was recognized as the third factor affecting the lifetime of starspot pattern.

The importance of the emergence modulation for the evolution of spot pattern was confirmed in our solar and stellar studies (Arkhypov, Antonov & Khodachenko 2013; Arkhypov et al. 2015a,b, 2016). In particular, a diagnostic method has been proposed there to identify the manifestations of the aforementioned mechanisms of starspot variability. In this Letter, we describe several important express-results of application of this method to an extended set of main-sequence stars, which opens for research community the new ways to probe the stellar physics.

2 METHOD AND DATA SET
Detailed descriptions of the used spectral-autocorrelation method can be found in our previous papers (Arkhypov et al. 2015a,b, 2016). Therefore, we briefly describe here only its key ideas.

We analyse the rotational modulation of the stellar radiation flux F (PDCSAP_FLUX from the Kepler mission archive1), which reflects the longitudinal distribution of spots.

Like in our previous studies (Arkhypov et al. 2015a,b, 2016), we consider here the squared amplitudes A12 and A22 of the light-curve rotational harmonics with periods P and P/2, respectively, where P is the period of stellar rotation. Following this approach, we found the time-scales of their variability τ1 = −P/ln [r1(P)] and τ2 = −P/ln [r2(P)]. Here, r1(P) and r2(P) are the autocorrelation functions of the chronological series of the A12 or A22, respectively, at the time lag of one rotational period P. This method is based on the simplest approximation of the logarithm of autocorrelation function at the shortest lag: ln (r_m) ≈ −Δt/τ_m (for details, see in

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1 https://exoplanetarchive.ipac.caltech.edu/
Arkhyypov et al. (2016), where \( m = 1 \) or 2 is the harmonic number, and \( \Delta t = P \) is the lag.

The squared amplitudes of harmonics are used in the analysis because of their statistical proportionality to the solar spot number (Arkhyypov et al. 2016). Analogously, Messina et al. (2003) used as an activity index the maximum amplitude (\( A_{\text{max}} \)) of rotational variations of stellar light curve, which is a proxy of our amplitude \( A_1 \) of the fundamental harmonic. They found that their index of various main-sequence stars \((0.1 < P < 20 \, \text{d}; \text{M4V to F5V})\) is related to normalized X-ray luminosity: \( L_x / L \propto A_{\text{max}}^m \), where \( b \approx 2 \), and \( L \) is the stellar bolometric luminosity. The ratio \( L_x / L \) is widely used as a standard activity index (e.g. Wright et al. 2011), related to spots (Wagner 1988; Ramesh & Rohini 2008). Since \( A_1^2 \) and \( A_2^2 \) are naturally the major contributors to \( A_{\text{max}}^m \), the approximate relation \( L_x / L \propto A_{\text{max}}^2 \) supports applicability of \( A_1^2 \) and \( A_2^2 \) as stellar activity indexes for the non-solar-type stars too. Moreover, we found that our activity index \( A_1^2 \) is related to the Rossby number similarly to the X-ray ratio \( R_x = L_x / L \) (see table 3 in Arkhyypov et al. 2016), supporting the general statistical proportionality \( L_x / L \propto A_1^2 \). Note that the measurements of time-scales \( \tau_1 \) and \( \tau_2 \) are insensitive to this proportionality. For example, the variation cycle of generalized index \( A_\nu^m \) with harmonic number \( m = 1, 2 \) and the constant \( \gamma \) has the same duration (i.e. the same time-scale) at any \( \gamma \).

According to Arkhyypov et al. (2015a, 2016), the effects of starspot variability can be distinguished using the ‘gradient’ ratio

\[
\beta_{12} = \frac{\log(\tau_2) - \log(\tau_1)}{\log(2) - \log(1)}
\]

In fact, this ratio measures the extent to which the spot distribution evolves faster on smaller scale compared to the largest scales.

For example, the Kolmogorov’s theory of turbulence (e.g. Lang 1974) predicts a universal relationship between the characteristic size of turbulent eddies (\( L \)) and the time-scale of their variability (\( \tau_L \)): \( \tau_L \approx L^2/3 \) or \( \tau_w \propto m^{-2/3} \) (taking into account that \( L \propto m^{-1} \)). Substituting this proportionality into equation (1), one can obtain

\[
\beta_{12} = -2/3 \approx -0.67.
\]

The horizontal diffusion of magnetic elements in photosphere effectively decreases our activity indexes \( A_m^2 \) \((m = 1, 2)\), when the average displacement of magnetic elements is about the longitudinal period of harmonic \( 2n/m \). For the normal diffusion, this happens during the characteristic time \( \tau_m \approx (2n/m)^2/\eta \) (where \( \eta \) is the diffusion coefficient), i.e. \( \tau_m \propto m^{-2} \). In this case, equation (1) gives \( \beta_{12} = -2 \).

In highly magnetized sub-photospheric plasma, like porous media, or in a network flow, the dependence of a squared displacement of magnetic element \( x \) on time \( t \) can differ from the linear one. In general case, it may be represented as \( x^2 \approx t^\alpha \), where \( \alpha \) is a constant. Therefore, the squared displacement \( x^2 \approx (2n/m)^2 = \eta_\tau_m^2 \) corresponds to the noticeable decrease of the activity index \( A_m^2 \) during the harmonic time-scale \( \tau_m \). In this case, \( \tau_m \approx m^{-2/\alpha} \) and, according equation (1), \( \beta_{12} = -2/\alpha \). For example, the subdiffusion of magnetic elements with \( \alpha = 0.6 \pm 0.2 \) was found in the solar photosphere at the spatial scale of supergranulation \((\sim 10^4 \, \text{km})\), which is related to the material network flow that traps these elements in the conjunction points (Iida 2016).

The differential rotation of a star stretches an activity region, or a complex of active regions, over the longitudinal harmonic scale \( \Lambda_m = 2\pi/m \) during the time \( \tau_m = \Lambda_m / \Delta \Omega \), where \( \Delta \Omega \) is a typical variation of angular velocity \( \Omega \) over the latitudinal extension of the activity region or complex. Hence, the time-scale of the considered feature blurring (i.e. \( A_m^2 \) damping) is \( \tau_m \propto m^{1/3} \) that corresponds to \( \beta_{12} = -1 \) in equation (1).

A stellar activity cycle modulates the total spot number with the identical period or time-scale for all rotational harmonics of a light curve. This means \( \beta_{12} = 0 \).

The aforementioned predictions are tested below by measuring the parameter \( \beta_{12} \) for the main-sequence stars. For this purpose, we combined our previously analysed data set (Arkhyypov et al. 2016) that contains the light curves of 1361 main-sequence stars observed by the Kepler space observatory, with the light curves of additionally selected 637 slow rotators. In summary, the extended sample includes 1998 stars with \( 0.5 < P < 30 \, \text{d} \) (according to measurements in Nielsen et al. 2013; McQuillan, Mazeh & Agrain 2014) and the effective temperatures \( 3227 \leq T_{\text{eff}} \leq 7171 \, \text{K} \) (according to Huber et al. (2014) in the Mikulski Archive for Space Telescopes).

All selected stars belong to the main sequence (surface gravity \( g(\text{cm} \, \text{s}^{-2}) > 4 \) in the used catalogues). The availability of a high-quality light curve without any interferences (i.e. no detectable short period pulsations or double periodicity from companions) was a special criterion for the compiling of analysed sample of stars. Further details on the star selection and light curve preparing (i.e. removing of gaps, flares, artefacts, trend) and processing are described in Arkhyypov et al. (2015b, 2016).

### 3 DIAGNOSTIC DIAGRAM

After processing of the light curves, we have measured the parameter \( \beta_{12} \) for the selected stars. Fig. 1(a) shows the \( P - T_{\text{eff}} \) distribution of smoothed values (\( \beta_{12} \)) as a result of \( \beta_{12} \) averaging over individual stars with \( \log(P) < -0.2 < \log(P) < \log(P) + 0.2 \) and \( \log(T_{\text{eff}}) - 0.05 < \log(T_{\text{eff}}) < \log(T_{\text{eff}}) + 0.05 \) in a sliding window with the central values of stellar rotation period \( P \) and effective temperature \( T_{\text{eff}} \). In average, 117 (up to 400) individual estimates of \( \beta_{12} \) appeared within this sliding window. The average standard error of \( \beta_{12} \) is \( \sigma = 13 \) per cent. The errors \( \sigma = [(\sigma_1 - \beta_{12})^2/3]^{1/2} \) in the individual windows of averaging does not exceed 15 per cent in the majority (83 per cent) of cases for \( \beta_{12} \). The total distribution of \( \sigma / \beta_{12} \) is shown in Fig. 1(c). One can see that the found pattern of \( \beta_{12} \) is mainly reliable, excluding the corners of the diagram with a depleted stellar population there (Fig. 1d) and increased relative errors (Fig. 1c).

Fig. 1(a) reveals an interesting pattern, consisting of the two regions with an increased (dark colour) and decreased (brighter tint) value of \( \beta_{12} \). Fig. 1(b) depicts these regions in terms of predictions for different mechanisms of starspot variability. The used intervals for schematics of \( \beta_{12} \) correspond to \( \pm \sigma \). Since the difference between \( \beta_{12} \approx -2/3 \) in the ‘turbulent’ region (black colour in Fig. 1b) and the ‘sub-diffusion’ region with \( \beta_{12} < -2.3 \) (grey) is more than 12.5\( \sigma \), these diagram details are significant. Fig. 2(a) demonstrates that the discussed regions in the diagnostic diagram are separated approximately along the lines, corresponding to the Rossby number \( \text{Ro} = P / \tau_{\text{MLT}} = 0.13 \), which was found as a border between the saturated and unsaturated stellar magnetism (Wright et al. 2011). To construct these border lines, we used two versions of the turnover time \( \tau_{\text{MLT}} \) in the mixing length theory (MLT):

- (a) the blue line is based on the classical definition \( \tau_{\text{MLT}}(B - V) \) by Noyes et al. (1984), where the standard colour index \( B - V \) is related to \( T_{\text{eff}} \) (Flower 1996);

\[ \beta_{12} \approx -1 \]
of selected stars in typical light-curve amplitude and measured rotational periods lead to the dispersion of the autocorrelation accuracy, hence, to the scattering of the estimates in (a); (d) distribution of studied stars in the same frame as in (a)–(c).
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Figure 2. Backgrounds of the diagnostic diagram: (a) the border lines at \( \text{Ro} = P / \tau_{\text{MLT}} \leq 0.13 \) between saturated and unsaturated stellar magnetism (‘red’: \( \tau_{\text{MLT}} \) by Wright et al. 2011, ‘blue’: \( \tau_{\text{MLT}} \) by Noyes et al. 1984) on the \( \langle \beta_{12} \rangle \) pattern; (b) the value \( \langle \log (\tau_1) \rangle \) averaged in the same sliding windows as in (a); (c) the relative error \( \sigma_{\tau_1} / \langle \log (\tau_1) \rangle \) of estimates \( \langle \log (\tau_1) \rangle \) in (a).

observed difference in \( \langle \beta_{12} \rangle \) for hot-slow and cold-fast rotators is not a result of the selection effect. The quasi-symmetric core of the distribution of \( \beta_{12} \) estimates in Fig. 3(b) and the correspondence of the histogram peaks to the predicted value \( \beta_{12} = -2/3 \) demonstrate insignificance of the selection effect for our conclusions. Since the decrease of \( \beta_{12} \) due to the light-curve noise is statistically insignificant in the considered stars with unsaturated magnetism, this effect must be weaker and negligible in the high-amplitude light curves of saturated stars. Hence, the decreased \( \beta_{12} \) for cold rotators can be interpreted as real manifestation of the sub-diffusion decay of starspot patterns in stars with saturated magnetism.

4 DISCUSSION AND CONCLUSIONS

In general, we found that the global spot distribution evolves slower than the hemispherical one. Particularly, the largest scale is much more stable than the smaller one for cold-slow rotators, whereas the difference is less pronounced for hotter stars and/or faster rotators. The corresponding gradient ratio \( \langle \beta_{12} \rangle < 0 \) appears an applicable indicator for a dominating mechanism of starspot pattern evolution. Thus, the constructed diagnostic diagram in Fig. 1(b) opens the way to differentiate stars with respect to the processes which control the stochastic variability of starspot pattern. For example, it is first found that the sub-diffusion \( (-3.0 < \beta_{12} < -2.3 \) in Fig. 1(a), hence, \( 0.7 < \alpha < 0.9 \)) is a dominating process in the decay of activity complexes in stars with saturated magnetism and certain values \( T_{\text{eff}} \) and \( P \). Note that the solar sub-diffusion is a result of deceleration of the normal diffusion process in the converging sub-photospheric plasma flows between supergranules at the scale \( \sim 10^5 \) km (Iida 2016). Analogously, the sub-diffusion at much larger scales up to \( \sim 10^6 \) km \( (m = 1 \text{ and } 2) \) requires regular mega-flows.
in the sub-photospheric plasma. Such flows, converging in active regions, are found in the Sun (Hindman, Haber & Toomre 2009).

The turbulent convection generates non-regular sub-photospheric flows at the local height scale \( m \gg 1 \) in the standard MLT approach. For example, in the solar photosphere the turbulent cascade has been described at scales smaller than that of supergranules, i.e. \( m > 200 \) (Abramenko et al. 2001; Stenflo 2012). However, Figs 1(b), 3(b) and (c) argue for the turbulence manifestation \( (\beta_{12} \approx -0.67) \) at \( m = 1 \) and 2 in solar-type stars \( (4500 \lesssim T_{\text{eff}} \lesssim 6900 \text{ K} \) and \( 6 \lesssim P \lesssim 30 \text{ d} \)). This incommensurability of the photospheric and found turbulent convection means that the last is the manifestation of plasma mixing in deep layers of the stellar convection zones. The connection between starspot pattern and the deep convection was predicted in numerical models of magnetic tube emergence which take into account the modulation effect from the convective flows (e.g. Weber, Fan & Miesch 2013). However, the deep-mixing effect is masked when its time-scale becomes shorter than the typical spot lifetime in the stars with saturated magnetism.

Figs 1(a) and (b) do not reveal any signs of dominating manifestation of magnetic cycles. Note, the period \( (>10^3 \text{ d}) \) of a typical activity cycle, which standardly described in terms of MFT (Brandenburg & Subramanian 2005), must be much longer than the time-scales of the aforementioned processes. Our autocorrelation method for the estimation of \( \tau_m \) is focused on the shortest time-scale, i.e. on the activity complex decay or turbulence manifestation.

Therefore, the described diagnostic approach appears as a useful instrument for future studies of starspot phenomenology, magnetic diffusion and flows at stellar photospheres as well as in deep layers of convection zones.

ACKNOWLEDGEMENTS

This work was performed as a part of the project P25587-N27 of the Fonds zur Förderung der wissenschaftlichen Forschung, FWF. We also acknowledge the FWF projects S11606-N16, S11604-N16, S11607-N16 and I2939-N27. MLK acknowledges grant 14.616.21.0084 of the Ministry of Education and Science of the Russian Federation. This research has made use of the NASA Exoplanet Archive, which is operated by the California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program. Some of the data presented in this paper were obtained from the Mikulski Archive for Space Telescopes (MAST). STScI is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. Support for MAST for non-HST data is provided by the NASA Office of Space Science via grant NNX09AF08G and by other grants and contracts.

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