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Enhanced Electron EDM with Minimal Flavor Violation

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Abstract. The latest data from the ACME experiment have led to the most stringent limit to date on the electric dipole moment \( d_e \) of the electron. Nevertheless, the standard model (SM) prediction for \( d_e \) is many orders of magnitude below the new result, making this observable a powerful probe for physics beyond the SM. We perform a model-independent study of \( d_e \) in the SM plus right handed neutrinos and its extension with the seesaw mechanism under the framework of minimal flavor violation (MFV). We find that \( d_e \) crucially depends on whether neutrinos are Dirac or Majorana fermions. In the Majorana case, \( d_e \) can reach its measured bound, which therefore constrains the scale of MFV to be above a few hundred GeV. We also consider extra \( CP \)-violating sources in the Yukawa couplings of the right-handed neutrinos. Such new sources can have important effects on \( d_e \).

1. Introduction

Electric dipole moments (EDMs) constitute highly sensitive indicators for the presence of new sources of the violation of charge parity (\( CP \)) and time reversal (\( T \)) symmetries beyond the standard model (SM) of particle physics [1, 2, 3, 4]. Recently the ACME experiment [5], which searched for the electron EDM, \( d_e \), utilizing the polar molecule thorium monoxide, has reported a fresh result of

\[
|d_e| = (−2.1 ± 3.7_{\text{stat}} ± 2.5_{\text{syst}}) \times 10^{-29} \text{ cm},
\]

which corresponds to an upper bound of

\[
|d_e| < 8.7 \times 10^{-29} \text{ cm} \quad \text{at 90\% confidence level.}
\]

This is more stringent than the previous best limit by about an order of magnitude, but still way above the SM expectation for \( d_e \), which is at the level of \( 10^{-44} \text{ cm} \) [6]. Hence there is plenty of room between the current bound on \( d_e \) and its SM value where potential new physics may be detected in future measurements.

Extra ingredients beyond the SM can boost \( d_e \) considerably with respect to its SM prediction, even up to its experimental limit. Such tremendous enhancement of \( d_e \) may hail from various origins depending on the specifics of the new physics models. Therefore it is desirable to carry out an analysis of \( d_e \) beyond the SM which deals with some general features of the physics in a model-independent fashion. This turns out to be feasible under the framework of the so-called minimal flavor violation (MFV) which presupposes that the sources of all flavor-changing neutral currents (FCNC) and \( CP \) violation reside in renormalizable Yukawa couplings [7, 8]. This offers a systematic method to organize and study possible SM-related flavor- and \( CP \)-violating new interactions.

Here we discuss an MFV treatment of \( d_e \) recently performed in Refs. [9, 10] within the SM with three right-handed neutrinos and its extension with the neutrino seesaw mechanism. The relevant contributions arise from effective dipole operators. As will be shown below, the predicted...
size of \( d_e \) depends significantly on whether light neutrinos are Dirac or Majorana in nature. In the Majorana case, \( d_e \) can be as large as its experimental bound, which therefore limits the scale of MFV to be above a few hundred GeV or higher [9, 10].

2. Leptonic MFV

In the SM slightly expanded with the addition of three right-handed neutrinos, the renormalizable Lagrangian for lepton masses is given by

\[
\mathcal{L}_m = -(Y_\nu)_{kl} \bar{L}_{k,L} \nu_{l,R} H - (Y_\nu)_{kl} \bar{L}_{k,L} E_{l,R} H - \frac{1}{2} (M_\nu)_{kl} \bar{\nu}_{k,R} \nu_{l,R} + \text{H.c.} ,
\]

(1)

where \( k, l = 1, 2, 3 \) are summed over, \( L_{k,L} \) represents left-handed lepton doublets, \( \nu_{l,R} (E_{l,R}) \) denotes right-handed neutrinos (charged leptons), \( Y_{\nu,e} \) are matrices for the Yukawa couplings, \( H \) is the Higgs doublet, \( \bar{H} = i\tau_2 H^* \), and \( M_\nu \) is the Majorana mass matrix for \( \nu_{l,R} \). The \( M_\nu \) part is essential for the seesaw mechanism to generate light neutrino masses [11].

Since it is still unknown whether light neutrinos are Dirac or Majorana particles, we deal with the two possibilities separately. If neutrinos are of Dirac nature, the \( M_\nu \) terms in Eq. (1) are absent, and according to the MFV hypothesis for leptons [8] the Lagrangian is formally invariant under the global group \( SU(3)_L \times SU(3)_\nu \times SU(3)_E = G_\ell \times U(1)_L \times U(1)_\nu \times U(1)_E \) with \( G_\ell = SU(3)_L \times SU(3)_\nu \times SU(3)_E \). This entails that the three generations of \( L_{k,L}, \nu_{k,R}, \) and \( E_{k,R} \) transform as fundamental representations of \( SU(3)_{L,\nu,E} \), respectively,

\[
L_L \rightarrow V_L L_L , \quad \nu_R \rightarrow V_{\nu} \nu_R , \quad E_R \rightarrow V_E E_R , \quad V_{L,\nu,E} \in SU(3) ,
\]

(2)

whereas the Yukawa couplings are spurions transforming according to

\[
Y_\nu \rightarrow V_L Y_\nu V_L^\dagger , \quad Y_e \rightarrow V_L Y_e V_L^\dagger .
\]

(3)

Taking advantage of the invariance under \( G_\ell \), we work in the basis where

\[
Y_e = \frac{\sqrt{2}}{v} \text{diag}(m_e, m_\mu, m_\tau) ,
\]

(4)

with \( v \approx 246 \text{ GeV} \) being the vacuum expectation value of \( H \), and the fields \( \nu_{k,L}, \nu_{k,R}, E_{k,L}, \) and \( E_{k,R} \) refer to the mass eigenstates. We can then express \( L_{k,L} \) and \( Y_\nu \) in terms of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \( U_{\text{PMNS}} \) as

\[
L_{k,L} = \left( \begin{array}{c}
(U_{\text{PMNS}})_{kl} \nu_{k,L} \\
E_{k,L}
\end{array} \right) , \quad Y_\nu = \frac{\sqrt{2}}{v} U_{\text{PMNS}} \bar{m}_\nu , \quad \bar{m}_\nu = \text{diag}(m_1, m_2, m_3) ,
\]

(5)

where \( m_{1,2,3} \) are the light neutrino eigenmasses and in the standard parametrization [12]

\[
U_{\text{PMNS}} = \left( \begin{array}{ccc}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23} & c_{12}c_{23} - s_{12}s_{23} & s_{13}e^{i\delta} \\
s_{12}s_{23} - c_{12}c_{23} & s_{12}s_{23} & c_{23}c_{13}
\end{array} \right) ,
\]

(6)

with \( \delta \) being the CP violation phase, \( c_{kl} = \cos \theta_{kl}, \) and \( s_{kl} = \sin \theta_{kl} \).

If neutrinos are of Majorana nature, the \( M_\nu \) part in Eq. (1) is allowed. As a consequence, for \( M_\nu \gg M_D = \nu Y_\nu/\sqrt{2} \) the seesaw mechanism [11] becomes operational involving the 6×6 neutrino mass matrix

\[
M = \left( \begin{array}{cc}
0 & M_D \\
M_D^T & M_\nu
\end{array} \right)
\]

(7)
in the \((U_{\text{PMNS}}^*\nu_L, \nu_R)^T\) basis. The resulting mass-matrix for the light neutrinos is

\[ m_\nu = -\frac{\nu^2}{2} Y_\nu M_\nu^{-1} Y_\nu^T = U_{\text{PMNS}} \hat{m}_\nu U_{\text{PMNS}}^T, \]

where now \(U_{\text{PMNS}}\) contains the diagonal matrix \(P = \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)\) multiplied from the right, \(\alpha_{1,2}\) being the Majorana phases. It follows that \(Y_\nu\) in Eq. (5) is no longer valid, and one can instead take \(Y_\nu\) to be [13]

\[ Y_\nu = \frac{i\nu}{v} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O M_\nu^{1/2}, \]

where \(O\) is a matrix satisfying \(OOT = 1\) and \(M_\nu = \text{diag}(M_1, M_2, M_3)\). As we see later, \(O\) can give rise to new \(CP\)-violation effects besides those from \(U_{\text{PMNS}}\).

To arrange nontrivial two-lepton FCNC and \(CP\)-violating interactions satisfying the MFV principle, one assembles an arbitrary number of the Yukawa coupling matrices \(Y_\nu \sim (3, 3, 1)\) and \(Y_e \sim (3, 1, 3)\) as well as their Hermitian conjugates to devise the \(G_\ell\) representations \(\Delta_{\ell\ell} \sim (8, 1, 1)\), \(\Delta_{e\ell} \sim (1, 8, 1)\), \(\Delta_{e\ell} \sim (1, 1, 8)\), \(\Delta_{e\ell} \sim (3, 3, 1)\), and \(\Delta_{e\ell} \sim (3, 1, 3)\), combines them with two lepton fields to arrive at the \(G_\ell\)-invariant objects \(L_L\Delta_{\ell\ell} L_L\), \(\nu_R\Delta_{e\ell}\nu_R\), \(\bar{E}_R\Delta_{e\ell} E_R\), \(\bar{\nu}_R\Delta_{e\ell}\bar{L}_L\), and \(\bar{E}_R\Delta_{e\ell} L_L\), attaches appropriate numbers of the Higgs field \(H\) and SM gauge fields to form singlets under the SM gauge group, and also contracts all Lorentz indices. Since fermion EDMs flip chirality, only the last combination, \(E_R\Delta_{e\ell} L_L\), pertains to our examination of \(d_\ell\). We can choose to write \(\Delta = Y_\nu Y_\nu^T\) with \(\Delta\) consisting of products of \(A = Y_\nu Y_\nu^T + B = Y_e Y_e^T\).

Formally \(\Delta\) comprises an infinite number of terms, \(\Delta = \sum \xi_{ijk} A^i B^j A^k \cdots\). The MFV hypothesis implies that the coefficients \(\xi_{ijk}\) have to be real because otherwise they would introduce new \(CP\)-violating sources beyond what is already contained in \(Y_\nu,\nu\). Using the Cayley-Hamilton identity \(X^3 = X^2 TrX + X[TrX^2 - (TrX)^2]/2 + 1 DetX\) for a 3×3 invertible matrix \(X\), this infinite series can be resummed into a finite number of terms [14]:

\[ \Delta = \xi_1 I + \xi_2 A + \xi_3 B + \xi_4 \nu^2 + \xi_5 \nu^2 B + \xi_6 AB + \xi_7 BA + \xi_8 ABA + \xi_9 B^2 A + \xi_{10} BAB + \xi_{11} AB^2 + \xi_{12} ABA^2 + \xi_{13} A^2 B + \xi_{14} AB^2 A + \xi_{15} B^2 AB + \xi_{16} AB^2 A^2 + \xi_{17} B^2 A^2 B, \]

where \(I\) is a 3×3 unit matrix. Although \(\xi_{ijk}\) are real, the reduction of the infinite series into the 17 terms can make the coefficients \(\xi_{e}\) in Eq. (10) complex due to imaginary parts among the traces of the matrix products \(A^i B^j A^k \cdots\). Such imaginary contributions turn out to be proportional to the Jarlskog invariant \(\text{Im} Tr(A^2 B A B^2) = (i/2) \text{Det}[A, B] \ll 1\) [15]. The implication is that, with all \(\xi_{ijk}\) being expected to be at most of \(O(1)\), the impact of \(\xi_{e}\) on \(d_\ell\) is suppressed by a factor of \(m_\nu^2/m^2\nu^4\) compared to the contribution from \(ABA^2\) which has the smallest number of suppressive factor \(Y_e\) among the products in Eq. (10) that contribute to \(d_\ell\). Therefore, hereafter we ignore \(\text{Im} \xi_{e}\).

3. Lepton EDMs with MFV

The EDM \(d_l\) of a lepton \(l\) is described by \(\mathcal{L}_d = -(i\xi_{e}/2) |\sigma^{\nu}\gamma_5| F_{\nu\nu}\), where \(F_{\nu\nu}\) is the photon field strength tensor. In the MFV framework, this arises at lowest order from the operators in the effective Lagrangian [8]

\[ \mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \left( g E_R Y_e^\dagger \Delta_1 \sigma_{\nu\nu} H^\dagger L_L B^{\nu\nu} + g E_R Y_e^\dagger \Delta_2 \sigma_{\nu\nu} H^\dagger \tau_j L_L W_j^{\nu\nu} \right) + \text{H.c.}, \]

where mass scale \(\Lambda\) characterizes the heavy new physics underlying these flavor-violating interactions, \(W\) and \(B\) denote the SM gauge fields with coupling constants \(g\) and \(g'\), respectively, \(\tau_j\) are Pauli matrices, and \(\Delta_{1,2}\) are of the form in Eq. (10) with generally different \(\xi_{e}\)’s.
Expanding Eq. (11), we find that the charged-lepton EDMs are proportional to $\text{Im}(Y_{e}^\dagger A_{1,2})_{kk}$, but that in $\Delta_{1,2}$ only the $ABA^2$ and $AB^2A^2$ terms are relevant. Thus for the electron

$$d_{e} = \frac{\sqrt{2} e v}{\Lambda^2} \left[ \xi_{12}^\ell \text{Im}(Y_{e}^\dagger A_{2})_{11} + \xi_{16}^\ell \text{Im}(Y_{e}^\dagger A_{2}^2)_{11} \right],$$

(12)

where $\xi_{r}^\ell = \xi_{r}^{(1)} - \xi_{r}^{(2)}$. If neutrinos are Dirac particles, we obtain from Eqs. (5) and (12)

$$d_{e}^{D} = \frac{32 e m_{e}}{\Lambda^2} \left[ \xi_{12}^\ell \frac{2(m_{1}^{2} - m_{2}^{2})}{v^{2}} \xi_{16}^\ell \frac{(m_{1}^{2} - m_{2}^{2})(m_{1}^{2} - m_{3}^{2})(m_{2}^{2} - m_{3}^{2})}{v^{8}} \right] J_{\ell},$$

(13)

invariant for $U_{PMNS}$. In the Majorana neutrino case, if the right-handed neutrinos $\nu_{k,R}$ are degenerate, $M_{\nu} = M\mathbb{I}$, and $O$ is a real orthogonal matrix, from Eq. (9) we have

$$A = \frac{2}{v^{2}} M U_{PMNS} \tilde{m}_{\nu} U_{PMNS}^{\dagger},$$

(14)

and consequently

$$d_{e}^{M} = \frac{32 e M^{2} m_{e}}{\Lambda^2 v^{8}} (m_{1}^{2} - m_{2}^{2})(m_{2}^{2} - m_{3}^{2})(m_{3}^{2} - m_{1}^{2}) \xi_{12}^\ell J_{\ell},$$

(15)

neglecting the $\xi_{16}^\ell$ term. Since $m_{k} \ll M$, obviously $d_{e}^{D}$ is highly suppressed relative to $d_{e}^{M}$.

In the preceding paragraph, $d_{e}$ depends on the $CP$-violating Dirac phase $\delta$ in $U_{PMNS}$, and the Majorana phases $\alpha_{1,2}$ therein do not enter. However, if $\nu_{k,R}$ are not degenerate, nonzero $\alpha_{1,2}$ can lead to an extra effect on $d_{e}$ even with a real $O \neq \mathbb{I}$. If $O$ is complex, its phases may induce an additional contribution to $d_{e}$, whether or not $\nu_{k,R}$ are degenerate. We explore these scenarios numerically later.

To evaluate $d_{e}$, we need the values of the various pertinent quantities, such as the elements of $U_{PMNS}$ as well as the masses of neutrinos and charged leptons. In Table 1, we have listed $\sin^{2}\theta_{kl}$ and $\delta$ from a recent fit to global neutrino data [16]. Most of these numbers depend on whether neutrino masses fall into a normal hierarchy (NH) or an inverted one (IH). Since the Majorana phases $\alpha_{1,2}$ remain undetermined, we will select specific values for them in our illustrations.

For Dirac neutrinos, we scan the empirical ranges of the parameters from Table 1 to maximize $d_{e}^{D}$ in Eq. (13). The result in the NH or IH case is $d_{e}^{D} = 1.3 \times 10^{-99} \xi_{12}^\ell (\text{GeV}^{2}/\Lambda^{2}) e\text{cm}$. This is negligible compared to the latest experimental upper bound [5].

Table 1. Results of a recent fit to the global data on neutrino oscillations [16]. The neutrino mass hierarchy may be normal ($m_{1} < m_{2} < m_{3}$) or inverted ($m_{3} < m_{1} < m_{2}$).

| Observable     | NH         | IH         |
|----------------|------------|------------|
| $\sin^{2}\theta_{12}$ | 0.308 ± 0.017 | 0.308 ± 0.017 |
| $\sin^{2}\theta_{23}$ | 0.425^{+0.029}_{-0.027} | 0.437^{+0.059}_{-0.029} |
| $\sin^{2}\theta_{13}$ | 0.0234^{+0.0022}_{-0.0018} | 0.0239 ± 0.0021 |
| $\delta/\pi$ | 1.39^{+0.33}_{-0.27} | 1.35^{+0.24}_{-0.39} |
| $\delta m^{2} = m_{2}^{2} - m_{1}^{2}$ | (7.54^{+0.26}_{-0.22}) \times 10^{-5} \text{eV}^{2} | (7.54^{+0.26}_{-0.22}) \times 10^{-5} \text{eV}^{2} |
| $\Delta m^{2} = |m_{3}^{2} - (m_{1}^{2} + m_{2}^{2})/2|$ | (2.44^{+0.08}_{-0.06}) \times 10^{-3} \text{eV}^{2} | (2.40 ± 0.07) \times 10^{-3} \text{eV}^{2} |
If neutrinos are Majorana fermions, in contrast, $d_e$ can be sizable. We start with the simplest possibility that $\nu_{k,R}$ are degenerate, $M_\nu = MA$, and the $O$ matrix in Eq. (9) is real. Thus $d_e$ is already given in Eq. (15). Scanning again the empirical parameter ranges in Table 1 to maximize $d_e^M$, we then obtain for $m_1 = 0$ ($m_3 = 0$) in the NH (IH) case

$$\frac{d_e^M}{e \text{ cm}} = 47 (5.2) \times 10^{-24} \left( \frac{\mathcal{M}}{10^{15} \text{ GeV}} \right)^3 \left( \frac{\text{GeV}}{\Lambda} \right)^2, \quad \hat{\Lambda} = \frac{\Lambda}{|c_{12}|^{1/2}}, \quad (16)$$

where $\mathcal{M}$ is specified below. Hence $|d_e|_{\exp} < 8.7 \times 10^{-29} \text{ e cm}$ [5] translates into

$$\hat{\Lambda} > 0.74 (0.24) \text{ TeV} \left( \frac{\mathcal{M}}{10^{15} \text{ GeV}} \right)^{3/2}. \quad (17)$$

Since $d_e^M$ in Eq. (16) is proportional to $\mathcal{M}^3$, one might naively think that $d_e^M$ can easily reach its measured bound, which would therefore constrain $\hat{\Lambda}$ to an arbitrarily high level with a very large $\mathcal{M}$. However, the size of $\mathcal{M}$ is capped based on the condition that the series in Eq. (10) which supposedly includes arbitrarily high powers of $A$ and $B$ must converge [9, 15]. Therefore, in this work, we require that the biggest eigenvalue of $A$ not exceed 1. Accordingly, we arrive at $\mathcal{M} = 6.16 (6.22) \times 10^{14} \text{ GeV}$ in Eq. (16) for the NH (IH) case, roughly similar to the expected seesaw scales in some grand unified theories. Hence $\hat{\Lambda} > 360 (120) \text{ GeV}$ from Eq. (17). For comparison with later examples, these $\mathcal{M}$ numbers imply $d_e^M \hat{\Lambda}^2 = 1.1 (0.13) \times 10^{-23} \text{ e cm}$.

With $\nu_{k,R}$ again degenerate, $M_\nu = MA$, but $O$ complex, we have in Eq. (12)

$$\Lambda = \frac{2}{v^2} MU_{\text{PMNS}} \bar{m}_\nu^{1/2} O O^{\dagger} \bar{m}_\nu^{1/2} U_{\text{PMNS}}^{\dagger} \quad (18)$$

In general $OO^{\dagger} = e^{2iR}$, where $R$ is a real antisymmetric matrix with nonzero elements denoted by $r_1 = R_{12} = -R_{21}$, $r_2 = R_{13} = -R_{31}$, and $r_3 = R_{23} = -R_{32}$. Since $OO^{\dagger}$ is not diagonal, the Majorana phases in $U_{\text{PMNS}}$ can also enter $\Lambda$ if $\alpha_{1,2} \neq 0$. We concentrate first on the $CP$-violating effect of $O$ by setting $\alpha_{1,2} = 0$. For illustrations, we pick two possible sets of $r_{1,2,3}$, namely, (i) $r_1 = -r_2 = r_3 = -\rho$ and (ii) $r_1 = 2r_2 = 3r_3 = \rho$, and employ the central values of the data in Table 1. In Figure 1 we present the resulting $d_e^M \hat{\Lambda}^2$ versus $\rho$ for the NH (IH) of light neutrino masses with $m_{1(3)} = 0$. Since $\delta$ is not yet well-determined, we also depict the variations of $d_e^M$ over the one-sigma ranges of $\delta$ quoted in Table 1 with the lighter blue and red bands. Within these bands, the blue and red solid curves belong, respectively, to the NH and IH central values in the table. We also graph the (dashed) curves for $\delta = 0$ to reveal the $CP$-violating role of $O$ alone.

With $\alpha_{1,2} = 0$, the $CP$-violating effect of $O$ can still occur even if it is real, provided that $\nu_{k,R}$ are nondegenerate, in which case

$$\Lambda = \frac{2}{v^2} U_{\text{PMNS}} \bar{m}_\nu^{1/2} O M_\nu O^{\dagger} \bar{m}_\nu^{1/2} U_{\text{PMNS}}^{\dagger} \quad (19)$$

from Eq. (9). For instance, assuming that $O$ is real, $O = e^R$ with (a) $r_1 = -r_2 = r_3 = -\rho$ and (b) $r_1 = 2r_2 = 3r_3 = \rho$, and that $M_\nu = M \text{ diag}(1, 0.8, 1.2)$, we show the resulting $d_e^M \hat{\Lambda}^2$ versus $\rho$ in Figure 2, where only the $\delta \neq 0$ curves are nonvanishing and the sinusoidal behavior of $d_e$ is visible. As in the previous figure, we also display the variations of $d_e^M$ over the one-sigma ranges of $\delta$ from Table 1.

All of these results in Figures 1 and 2 clearly demonstrate that $O$ can generate potentially significant extra effects of $CP$ violation which can exceed those of $\delta$. The latter point is most
Figure 1. Dependence of $d_{\nu}^M$ times $A^2 = \Lambda^2 / |\xi|^2$ on the $O$-matrix parameter $\rho$ for zero Majorana phases, $\alpha_{1,2} = 0$, degenerate $\nu_{k,R}$, and complex $O$ with (a,b) $r_1 = -r_2 = r_3 = -\rho$ and (c,d) $r_1 = 2r_2 = 3r_3 = \rho$, as discussed in the text. The lighter blue and red bands reflect the one-sigma ranges of $\delta$, while the solid and dashed curves correspond, respectively, to its central values and to $\delta = 0$.

Figure 2. Dependence of $d_{\nu}^M A^2$ on $\rho$ for zero Majorana phases, $\alpha_{1,2} = 0$, nondegenerate $\nu_{k,R}$ with $M_{\nu} = M \text{diag}(1,0.8,1.2)$, and real $O = e^R$ with (a) $r_1 = -r_2 = r_3 = -\rho$ and (b) $r_1 = 2r_2 = 3r_3 = \rho$, as discussed in the text. The lighter blue and red bands reflect the one-sigma ranges of $\delta$, while the solid curves correspond to its central values.

noticeable in Figure 1(b,d) from comparing the IH $\delta \neq 0$ regions at $\rho \sim 0$ with the extreme values of the corresponding IH $\delta = 0$ curves.

To see the effect of the Majorana phases, we entertain a couple of possibilities: (i) $M_{\nu} = M \mathbf{1}$ and $O = e^{R}$ and (ii) $M_{\nu} = M \text{diag}(1,0.8,1.2)$ and $O = e^{R}$, both with the two sets of $r_{1,2,3}$ chosen earlier. Fixing $\alpha_1 = 0$ and $\rho = 1/2$, we depict the resulting dependence of $d_{\nu}^M$ on $\alpha_2$ in Figures 3 and 4 for nonzero $\delta$ within its one-sigma ranges from Table 1 and also for $\delta = 0$. It is
Figure 3. Dependence of $d^2_{\nu_e} \hat{\Lambda}^2$ on $\alpha_2$ for $\alpha_1 = 0$, degenerate $\nu_{k,R}$, and $O = e^{iR}$ with (a) $r_1^2 = -r_2 = -1/2$ and (b) $r_1 = 2r_2 = 3r_3 = 1/2$, as explained in the text. The bands and curves have the same meanings as in preceding figures.

Figure 4. The same as Figure 3, except $\nu_{k,R}$ are nondegenerate with $M_\nu = M \text{diag}(1, 0.8, 1.2)$ and $O = e^{iR}$.

evident from these graphs that the Majorana phases produce additional important $CP$-violating impact on $d_e$ beyond $\delta$.

For the EDMs of the muon and tau lepton, one could do a similar analysis. However, their experimental limits are still much weaker than $|d_e|_{\text{exp}}$. Consequently, they lead to bounds on $\hat{\Lambda}$ which are not competitive to that in the electron case [9, 10].

4. Conclusions
We have investigated the electron EDM, $d_e$, under the MFV framework and found that $d_e$ can reach its experimental limit if neutrinos are Majorana in nature. Moreover, from the latest data on $d_e$, we infer that the MFV scale has to be a few hundred GeV or higher. We demonstrate that $d_e$ has the potential to probe not only the Dirac phase in the lepton mixing matrix, but also the Majorana phases therein, as well as extra $CP$-violation sources in the Yukawa couplings of the right-handed neutrinos.

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