Creating quantum correlations through local non-unitary memoryless channels

Francesco Ciccarello and Vittorio Giovannetti

NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, Piazza dei Cavalieri 7, I-56126 Pisa, Italy

(Dated: January 26, 2013)

We show that two qubits, initially in a fully classical state, can develop significant quantum correlations as measured by the quantum discord (QD) under the action of a local memoryless noise (specifically we consider the case of a Markovian amplitude-damping channel). This is analytically proven after deriving in a compact form the QD for the class of separable states involved in such a process. We provide a picture in the Bloch sphere that unambiguously highlights the physical mechanism behind the effect regardless of the specific measure of QCs adopted.

The existence of states where two or more systems are correlated in a way unattainable in classical physics ranks among the most puzzling and yet distinctive features of quantum mechanics. Such possibility is commonly pictured in terms of an extra amount correlations, usually referred to as quantum correlations (QCs), which a multipartite system can possess in addition to those of a merely classical nature. Until recently, the scrutiny of QCs has been almost ubiquitously intertwined with investigations on entanglement [1] and the pivotal role that while classicality always entails separability the reverse is in general untrue (a state is entangled iff it is non-separable). Entanglement thereby is not the only form in which QCs can occur. Such finding brought about a widened perspective, which is currently prompting a growing number of researchers to advance the field along various lines. A prominent one is the quest for faithful easy-to-handle indicators of QCs [4,5]. Among those proposed so far, quantum discord (QD) [3] is having a considerable impact despite its explicit calculation is usually demanding even for two qubits (i.e. a pair of two-dimensional systems). Yet, evidence of its ability to capture QCs not detected by entanglement has been supplied in various frameworks such as one-qubit quantum computation (even experimentally) [6] and quantum phase transitions [1].

Another major concern that soon arose is to assess how QCs, according to such a novel paradigm, are affected by non-unitary dynamics. These typically stem from the interaction with an environment, a process where entanglement either [9]. Also, preexisting QCs can exhibit an increasing behavior at some stages of their time evolution in the presence of non-Markovian local channels as recently demonstrated for both entanglement and QD [10]. In such instances, the increase of QCs stems either from the ability of a common memoryless reservoir to mediate an effective interaction or as a memory effect of local environments. The question is now raised: Can the interaction with a bath which is both local and memoryless enhance QCs? The answer is well-known to be negative for entanglement, which cannot grow under any local quantum maps [1]. As for the full amount of QCs, however, the issue is not as much trivial since as stressed above even separable states may feature some quantunness [3]. As far as local unitaries are concerned, though, QCs’ measures, such as QD, cannot increase [3,5]. In some respects, this conclusion might be expected to be strengthened with noisy local operations. However, QCs without entanglement can arise under a local non-unitary operation owing to its ability to map orthogonal into non-orthogonal states [5]. In this work, we present a simple paradigmatic process, where entanglement is absent throughout, which clearly testifies that QCs can even be entirely created (or increased) solely via the interaction with a bath which is both local and memoryless. We provide rigorous and comprehensive insight into such effect in a way that makes transparent the underlying physical mechanism. Although we focus on QD, it will become clear that the essential physical effect takes place regardless of the specific measure of QCs used.

To begin with, we briefly recall the definition of QD. Given two systems A and B in a state \( \rho \), this measures the discrepancy between the mutual information \( I \) and the classical correlations \( C \) associated with \( \rho \) [3]. A local measurement on \( B \) in a given orthonormal basis can be specified by a complete set of projectors \( \{ B_k \} \), where \( k \) indexes a possible outcome. If \( k \) is recorded with probability \( p_k = Tr[B_kB_k] \) the overall system collapses onto the (normalized) state \( \rho_k = (B_kB_k) / p_k \). Then the QD \( D^- \) can be expressed as [3]

\[
D^-(\rho) = S(\rho_B) - S(\rho) + \min_{\{B_k\}} \sum_k p_k S(\rho_k) .
\] (1)

In Eq. (2), \( \rho_B = Tr_A \rho \) is the reduced density operator that describes the state of B. \( S(\sigma) = -Tr(\rho \log_2 \sigma) \) is the Von Neumann entropy of an arbitrary state \( \sigma \) and the infimum in the last term is evaluated over all the possible sets \( \{ B_k \} \). As for the QD \( D^+ \) involving measurements on A with associated projectors \( \{ A_k \} \), this is obtained from (1) through replacement of B with A. In general, \( D^- \neq D^+ \) [3].

Specifically, in the process under study A and B are two qubits and the only involved states have the separable form

\[
\rho = \frac{1}{2} ( |0\rangle \langle 0 | \otimes \tau_{00} + |1\rangle \langle 1 | \otimes \tau_{11} ) ,
\] (2)

where \( \tau_{0(1)} \) are generic density matrices while the orthonormal set of vectors \( \{ |0\rangle, |1\rangle \} \) is the usual local computational ba-
sis. The states are commonly dubbed as classical-quantum states owing to \( D^c = 0 \) as is immediate to check \( \text{[12]} \), while in general \( D^c \neq 0 \). We will thereby set \( D \equiv D^c \) henceforth.

Consider now the initial state
\[
\rho_0 = \frac{1}{2} |0\rangle \langle 0| \otimes [|+\rangle \langle +| + |1\rangle \langle 1| \otimes |-\rangle \langle -|],
\]
i.e. in the light of (2) \( \tau_0 = 1 \langle +| + \rangle \) and \( \tau_1 = 1 \langle -| - \rangle \), where \( \langle \pm \rangle = (|0\rangle \pm |1\rangle)/\sqrt{2} \). Although manifestly correlated, state (3) is fully classical since it is diagonal in the basis resulting from the tensor product between \(|0\rangle_{A}, |1\rangle_{B}\) and \(|+\rangle_{B}, |-\rangle_{B}\), namely two local orthogonal bases \( \text{[13]} \). Thus \( D(\rho_0) = 0 \).

Assume now that while \( A \) is well protected from the external environment so is not \( B \), which is in contact with a dissipative Markovian bath. In such a case, the system dynamics as a function of time \( t \) is fully described by an amplitude-damping channel \( \text{[2]} \). This has an associated quantum map \( E_p \) that transforms state \( \rho_0 \) according to
\[
E_p(\rho_0) = E_0 \rho_0 E_0^\dagger + E_1 \rho_1 E_1^\dagger,
\]
where \( E_0 = |0\rangle \langle 0| + \sqrt{1-p^2} |1\rangle \langle 1| \) and \( E_1 = \sqrt{p^2} |0\rangle \langle 0| \) are the associated Kraus operators while \( p \) is a probability that grows with time \( t \) according to \( p = 1 - e^{-\gamma t} \) (\( \gamma \) is a relaxation rate). As anticipated, \( E_p(\rho_0) \) evidently belongs to class (2) for any \( p \) (i.e. \( \forall t \)) since \( E_0 \) and \( E_1 \) act on \( B \) only (hence entanglement never appears throughout).

In Fig. 1(a), we plot \( D \) [cf. Eq. (1)] against \( p \) and the rescaled time \( yt \) as resulting from numerical evaluation \( \text{[14]} \). \( D(p) \), which is initially null as discussed above, at a first stage grows and eventually decays to zero as \( p \rightarrow 1 \), i.e. for \( t \gg \gamma^{-1} \). This marks a profound difference between \( D \) and entanglement in that a local non-unitary and memoryless channel is able to create QCs previously absent. Remarkably, the dissipative dynamics is merely detrimental to the classical correlations \( C = I - D \), where \( I = S(\rho_A) + S(\rho_B) - S(\rho) \) is the mutual information \( \text{[2]} \). \( C \) indeed exhibits a monotonic decay vanishing for \( p \rightarrow 1 \), i.e. \( yt \rightarrow \infty \), as shown in Fig. 1(b).

In the following, we make this result rigorous by analytically deriving \( D(p) \) and \( C(p) \) so as to reproduce Fig. 1. To this aim, we first explicitly derive in a compact form the \( D \) of states (2) when \( \tau_0 \) and \( \tau_1 \) have the same purity. Next, we present a picture in terms of trajectories in the Bloch sphere clearly highlighting the physical mechanism that causes \( D \) to necessarily grow in the present process.

Among states (2) a prominent instance is the resource state for the B92 quantum cryptography protocol \( \text{[2]} \), which reads
\[
\rho_{B92} = \frac{1}{2} (|0\rangle \langle 0| \otimes |0\rangle \langle 0| + |1\rangle \langle 1| \otimes |+\rangle \langle +|) \equiv D. \]
Similarly to the popular BB84 \( \text{[2]} \) the above is among those quantum protocols where the exploited quantum resource is not entanglement, which is is fully absent. Rather, it harnesses one-way QCs stemming from the non-distinguishability of states \(|0\rangle \) and \(|+\rangle \) \( \text{[13]} \). Later on, we will indeed show that \( \rho_{B92} \) possesses the maximum allowed QD within family (2) with \( \tau_0 \) and \( \tau_1 \) having equal purities. Also, states such as (2) can allow for quantum equal purities. Also, states such as (2) can allow for quantum rotation, which cannot affect the QD like any local unitary operation \( \text{[6]} \). Using this along with \(|0\rangle \langle 0| = (1 + \sigma_z)/2 \) and \(|1\rangle \langle 1| = (1 - \sigma_z)/2 \), state (2) can be arranged as (from now on we drop subscripts \( A \) and \( B \))
\[
\rho = \frac{1}{2} \left[ I \otimes I + I \otimes [(a_1 \sigma_1 + a_3 \sigma_3) + \sigma_3 \otimes (b_1 \sigma_1 + b_3 \sigma_3)] \right]
\]
with
\[
a_1 = b_1 = \frac{s_1 \sin \varphi}{2}, \quad a_3 = \frac{s_3 \cos \varphi}{2}, \quad b_3 = \frac{s_3 \sin \varphi}{2},
\]
where we have carried out the replacements \( s_1 = s_1 \sin \varphi \), \( s_1 = s_1 \cos \varphi \), \( \varphi \) is the angle between \( s_0 \) and \( s_1 \). Next, in the light of \( \text{[1]} \) we need to calculate how \( \rho \) is transformed when a generic Von Neumann measurement is performed on system \( B \), i.e. \( \rho_k \), as well as the associated probability \( p_k \). As
in [16] we use the property that the projector corresponding to any such measurement can be expressed as $\Pi \otimes B_k$ with $B_k = V \Pi_k V'$, where $\Pi_k = |k\rangle \langle k|$ ($k = 0, 1$) and $V$ is a generic one-qubit unitary. We thus expand $p_k \rho_k = (\Pi \otimes B_k) \rho (\Pi \otimes B_k)$ as
\[ p_k \rho_k = (\Pi \otimes V)(\Pi \otimes \Pi_k)(\Pi \otimes V') \rho (\Pi \otimes V)(\Pi \otimes \Pi_k)(\Pi \otimes V') \].

The unitary $V'$ transforms each Pauli matrix $\sigma_j$ according to $V'\sigma_j V = \sum_{\gamma} v_{ij} \sigma_{j'}$, where $v_{ij}$ are real numbers satisfying the constraint $v_{ij}^2 + v_{ij}^2 + v_{ij}^2 = 1$, for every $j = 1, 2, 3$. Also, for $k = 0, 1$, $\Pi_k \sigma_j \Pi_k = \Pi_k \sigma_j \Pi_k = 0$ while $\Pi_k \sigma_j \Pi_k = f_k \Pi_k$, where $f_0 = 1$ and $f_1 = -1$. This along with (4) yield (5) in the simpler form
\[ p_k \rho_k = (\Pi \otimes V) \left[ \frac{\mu_k + v_k \sigma_3}{4} \otimes \Pi_k(\Pi \otimes V') \right] \rho (\Pi \otimes V') (\Pi \otimes V) \].

where $\mu_k = 1 + f_k (a_1 v_{13} + a_2 v_{33})$, $v_k = f_k (b_1 v_{13} + b_2 v_{33})$. (7)

As $V' \Pi_k V'$ represents a pure state and each Pauli matrix is traceless it is immediately checked that $p_k = \mu_k / 2$. Therefore, we obtain that $\rho_k = (I + v_k \sigma_3 / \mu_k) / 2(\otimes V) V'$ whose eigenvalues are $(1 \pm v_k / \mu_k) / 2$ (each two-fold degenerate). The quantity to minimize entering the last term of (11) thus reads
\[ \delta(\Pi_1, \Pi_3) = \sum_{k=0,1} p_k S(\rho_k) = \sum_{k=0,1} \mu_k^2 + v_k^2 \sigma_3^2 / 4 \left( 1 - \frac{v_k^2}{\mu_k^2} \right) \],

where $h(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$ is the binary Shannon entropy function and on the l.h.s. we have highlighted the dependence on variables $\Pi_1$ and $\Pi_3$. The identity $\sum_{k=0,1} ^2 v_k^2 = 1$ (see above) yields that $\Pi_1$ and $\Pi_3$ must fulfill $v_{13}^2 + v_{33}^2 = 1 - v_{23}^2 \leq 1$, i.e. they belong to the unit circle. To work out the infimum of $\delta$ we accomplish the linear transformation $x = a_1 \Pi_1 + a_2 \Pi_3$ and $y = b_1 \Pi_1 + b_2 \Pi_3$. This way, (8) now becomes a universal function of $x$ and $y$, which we call $\delta(x, y)$, i.e. it no longer depends on parameters $[a_i, b_j]$ specifying states $\tau_0$ and $\tau_1$ [cf. Eq. (5)].

In Fig. 2(a) we plot $\delta$ against $x$ and $y$. Note that its actual domain of definition is the square having side length $1/\sqrt{2}$ and vertices at points $(\pm 1, 0)$ and $(0, \pm 1)$ [this is because of the logarithms of $1 \pm x y / \sqrt{2}$, see Eqs. (7)–(8)]. Also, $\delta$ is an even function of $x$ and $y$ for any set value of $y(x)$ as it is also clear from its dependence on $x$ and $y$ through $\mu_k$ and $v_k$ [see Eqs. (7)–(8)]. It takes value 1 for $x = y = 0$ and decreases as the distance from the origin grows. Its concavity is minimal along the $x$-axis, where the function is fully flat, and maximum along the $y$-axis [see the sail-like shape in Fig. 2(a)]. As for the domain $\mathcal{D}$ within which (8) is to be minimized, i.e. the region in the new reference frame corresponding to the unit circle $v_{13}^2 + v_{33}^2 \leq 1$ (see above), it is immediately checked through the inverse transformation that this is an elliptic region given by
\[ \mathcal{D} = \{ x, y : A x^2 + 2B x y + C y^2 \leq 1 \}, \] where
\[ A = \left( \frac{2 B}{|y_0 y_1|/\sin \delta} \right)^2, \quad B = \left( \frac{2 C}{|y_0 y_1|/\sin \delta} \right)^2, \quad C = \left( \frac{2 y_0 y_1}{|y_0 y_1|/\sin \delta} \right)^2. \]

As is evident from Fig. 2(a), owing to the concavity of $\delta$ its infimum under constraint (8) necessarily lies on the boundary of $\mathcal{D}$, i.e. the ellipse obtained from (9) by turning the inequality into an identity. While such ellipse is centered at $x = y = 0$, its own axes in general do not coincide with those defining the reference frame. Yet, in the case that $B = 0$ [cf. Eqs. (9) and (10)] the ellipse is not rotated with respect to the $x$- and $y$-axis. This circumstance physically occurs when $\tau_0$ and $\tau_1$ have the same purity, i.e. $s_0 = s_1$ [see Eq. (10)]. Henceforth, we will focus on such a case, which is enough for the scopes of the present work and allows for a prompt analytical derivation of the infimum of (8) under constraint (9). Hence, by setting $s_0 = s_1 = s$ coefficients (10) become $A = 1/r_1^2$, $B = 0$, $C = 1/r_2^2$, where $r_1 = s / \sin (\varphi / 2)$ and $r_2 = s / \cos (\varphi / 2)$ are the ellipse semi-axis lengths along the $x$- and $y$-axis, respectively.

Now, because of the shape of $\delta(x, y)$ [see Fig. 2(a)] when the main axes and the ellipse own axes are collinear $\delta(x, y)$ takes its minimum at the points $(0, x, y)$. This is evident from Figs. 2(b) and (c), where we plot the restriction of $\delta$ to region $\mathcal{D}$ in a paradigmatic case such that $r_2 > r_1$ (b) and one featuring $r_2 < r_1$ (c). The minimum of (8) within region (9), i.e. the last term in Eq. (11) thus reads
\[ \min_{\mathcal{D}} \delta(x, y) = \delta(0, r_1) = \left( 0, s \frac{\cos (\varphi / 2)}{2} \right) = h\left( \frac{1 + \sin (\varphi / 2)}{2} \right), \]

where we used that $\mu_k = 1 + f_k x$ and $v_k = f_k y$ [cf. Eqs. (7)–(8)].

As for $S(\rho_B)$, the trace over $A$ of (9) is obtained as $\rho_B = (I + \alpha_1 (\sigma_1 + \sigma_3) / \sqrt{2})$, whose eigenvalues with the help of (8) are found as $(1 \pm s / \cos (\varphi / 2)) / 2$. Hence, $S(\rho_B) = h\left( 1 \frac{1 + \sin (\varphi / 2)}{2} \right) / 2$. Using again Eqs. (8) and (10), the eigenvalues of $\rho$ are calculated as $(1 \pm s / 2)$, each being two-fold degenerate yielding that $S(\rho) = 1 - h(1 / (1 + s)) / 2$. Using these results along with (11) in the light of (12), we find the QD of any state (2) such that $\tau_0$ and $\tau_1$ have the same Bloch-vector length $s$ in the compact form
\[ D = h\left( \frac{1 + \sin (\varphi / 2)}{2} \right) + h\left( \frac{1 - \sin (\varphi / 2)}{2} \right) - h\left( \frac{1}{2} \right) = -1. \]

As for the classical correlations $C = I - D$, using that $S(\rho_A) = S(\Pi_A) = 1$ we find that
\[ C = 1 - h\left( \frac{1 + \sin (\varphi / 2)}{2} \right). \]

In Fig. 3 we plot $D$ (a) and $C$ (b) vs. $s$ and $\varphi$ as given by Eqs. (12) and (13). For a given $s$, i.e. for fixed purity, $D$ solely depends on $|\varphi - \pi / 2|$ exhibiting its maximum value when $\varphi = \pi / 2$. As $|\varphi - \pi / 2|$ grows the QD progressively decreases until it vanishes at $\varphi = 0, \pi$. On the other hand, for a given $\varphi$, $D$ increases with the purity at a rate that grows as $\varphi$ approaches $\pi / 2$. Within the present class of states, the

FIG. 3. (Color online) $D$ (a) and $C$ (b) vs. $s$ and $\varphi$.  

(a)  
(b)
be created in the course of the process. In essence, the local dissipative channel transforms $|+\rangle\langle+|$ and $|−\rangle\langle−|$ so as to reduce their distinguishability, which unavoidably gives rise to QCs without entanglement [3,13]. Note that the process non-unitarity is crucial since distinguishability as measured by the scalar product is unaffected by any unitary [3,13]. Also, note that based on the definition of classically correlated states [13] the above reasoning shows that QCs will be surely created regardless of the specific measure chosen to quantify them.

In summary, in this Letter we asked whether QCs can develop as a result of local non-unitary dynamics, an unattainable phenomenon with entanglement. After deriving the QD for the class of involved states, we analytically proved that this indeed can occur for two qubits, initially in a fully classical state, under a local memoryless amplitude-damping channel. Also, we showed that the mechanism behind the QCs’ birth can be readily grasped in the Bloch-sphere picture.

All these phenomena are arguably not restricted to qubits. The generalization to continuous-variable systems in a way that local bosonic Gaussian maps now play the role of the amplitude-damping channel is under ongoing investigations.

We thank R. Fazio and M. Paternostro for comments, and acknowledge support from FIRB IDEAS through project RBID08B3FM.

Note added.-During completion of this work we became aware of a related manuscript [19].

[1] R. Horodecki, et al. Rev. Mod. Phys. 81, 865 (2009).
[2] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, U. K., 2000).
[3] L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001); H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001).
[4] J. Oppenheim, et al. Phys. Rev. Lett. 89, 180402 (2002); B. Groisman, S. Popescu, and A. Winter, Phys. Rev. A, 72, 032317 (2005); S. Luo, Phys. Rev. A, 77, 022301 (2008); B. Dakić, V. Vedral, and I. C. V. Brukner, Phys. Rev. Lett., 105, 190502 (2010); D. Girolami, M. Paternostro, and G. Adesso, arXiv:1008.4136.
[5] K. Modi et al., Phys. Rev. Lett., 104, 080501 (2010).
[6] A. Datta, A. Shaji, and C. M. Caves, Phys. Rev. Lett. 100, 050502 (2008); B. P. Lanyon, et al. Phys. Rev. Lett. 101, 200501 (2008).
[7] T. Werlang, et al. Phys. Rev. A 80, 024103 (2009); J. Maziero, et al. Phys. Rev. A 80, 044102 (2009); L. Mazzola, J. Pitlo, and S. Maniscalco, Phys. Rev. Lett. 104, 200401 (2010).
[8] J.-B. Yuan, L.-M. Kuang, and J.-Q. Liao, J. Phys. B At. Mol. Opt. Phys. 43, 165503 (2010); F. Altintas and R. Eryigit, arXiv:1105.2222 [quant-ph].
[9] D. Braun, Phys. Rev. Lett. 89, 277901 (2002); F. Benatti, R. Floreanini, and M. Piani, ibid. 91, 070402 (2003); T. S. Cubitt, et al. ibid. 91, 037902 (2003).
[10] B. Bellomo, R. Lo Franco, and G. Compagno, Phys. Rev. Lett. 99, 160502 (2007); F. F. Fanchini, et al. Phys. Rev. A 81, 052107 (2010).
[11] M. S. Sarandy, Phys. Rev. A 80, 022108 (2009).
[12] By definition, $D^* \geq 0$. To prove that $D^* \equiv 0$ it suf-

FIG. 4. (Color online) Trajectories of $\mathcal{E}_p(\pm\langle\pm\rangle)$ (blue solid line) and $\mathcal{E}_p(\langle\pm\rangle\langle\pm\rangle)$ (red dashed line) on the XZ plane of the Bloch sphere.

QD thus takes its maximum value $D_{\text{max}} \approx 0.202$ [about three times larger than the maximum value attained in Fig. 1(a)] for $s = 1$ and $\varphi = \pi/2$, which correspond to the B92 resource state $\rho_{\text{B92}}$ introduced previously. This feature is in accordance with Ref. [17] (where a different QCs’ measure was used). As for C. Fig. 3(b) shows that this maximum is for $s = 1$ and $\varphi = \pi$ and decreases when either of such two parameters is reduced.

We are now in a position to provide a comprehensive explanation for the effect in Fig. 1. Using the above discussed Kraus decomposition of $\mathcal{E}_p$ it is straightforwardly found that $\mathcal{E}_p(\pm\langle\pm\rangle) = (1 \pm \sqrt{1-p} \sigma_1 + p\sigma_3)/2$. Hence, $s_{0x} = -s_{1x} = \sqrt{1-p}$, while $s_{0c} = s_{1c} = p$ [note that this immediately shows that $s_0 \equiv s_1 = s$ for any $p$]. Using this, we end up with

$$s(p) = \sqrt{1+p(p-1)}, \quad \varphi(p) = \pi - 2 \arctan\left(p/\sqrt{1-p}\right). \quad (14)$$

By replacing (14) in Eqs. (12) and (13), the functions $D(p)$ and $C(p)$ so obtained reproduce the plots in Fig. 1.

To shed light on the physical mechanism behind the effect, in Fig. 4 we plot on the XZ plane of the Bloch sphere the parabolic trajectories of the Bloch vectors corresponding to $\mathcal{E}_p(\pm\langle\pm\rangle)$. Initially, the two vectors have unit length pointing towards opposite directions of the X-axis. Hence, $s = 1$ and $\varphi = \pi$, which yields that $D = 0$ and $C = 1$ [cf. Eqs. (12) and (13) and Fig. 3]. When $p = 1$, i.e. for $\varphi \gg 1$, either state is mapped onto $|0\rangle\langle0|$, i.e. the North Pole, which gives that $s = 1$, $\varphi = 0$ and thereby $D = C = 0$ [Eq. (12) indeed shows that the state becomes evidently uncorrelated]. At the intermediate stage, map $\mathcal{E}_p$ acts in a way that the two Bloch vectors move along symmetrical parabolic trajectories (see Fig. 4) yielding that, in particular, $0 < \varphi < \pi$. On the other hand, the purity shows a dip with $s$ taking its minimum $s_{\text{min}} = \sqrt{3}/2$ for $p = 1/2$ in accordance with (13). In the light of Eq. (12) and Fig. 3(a), this necessarily brings about that $D > 0$ since $0 < \varphi < \pi$ and the Bloch vector length, although shrinking to some extent, keeps finite throughout. This clarifies why discord must necessarily
fices to select $A_0 = |0\rangle_\Lambda \langle 0|$ and $A_1 = |1\rangle_\Lambda \langle 1|$ as projectors. Then $p_0 = p_1 = 1/2$ and $\sum_k p_k S(\rho_k) = 1/2 \sum_{k=0,1} S(\sigma_k)$. Also, $S(\rho_0) = S(\mathbb{I}/2) = 1$ while $S(\rho) = 1 + 1/2 \sum_k S(\sigma_k)$. Use of (1) then gives $D^* = 0$.

[13] B. Groisman, D. Kenigsberg, and T. Mor, \textit{arXiv:quant-ph/0703103}

[14] We used that any set of one-qubit orthonormal projectors can be expressed as $\{M_k = |\Psi_k\rangle \langle \Psi_k| (k = 0, 1)\}$ with $|\Psi_0\rangle = (\cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle)/\sqrt{2}$ and $|\Psi_1\rangle = (e^{-i\phi} \sin \theta |0\rangle - \cos \theta |1\rangle)/\sqrt{2}$, where $\theta \in [0, \pi/2]$ and $\phi \in [0, 2\pi]$. We then discretized $D$ through a mesh of $\theta$ and $\phi$, took its minimum value throughout and eventually checked its stability against the number of grid points.

[15] S. Boixo \textit{et al.}, \textit{arXiv:1105.2768}

[16] S. Luo, Phys. Rev. A \textbf{77}, 042303 (2008); M. Ali, A. R. P. Rau, and G. Alber, Phys. Rev. A \textbf{81}, 042105 (2010).

[17] S. Gharibian \textit{et al.}, Int. J. Quantum Info. \textbf{9}, 1701 (2011).

[18] The task to spoil the distinguishability of $\{|\pm\rangle \langle \pm|\}$ giving rise to QCs could also be accomplished via an engineered local quantum operation on $B$ (see [5] and references therein). It is quite remarkable, though, that a \textit{spontaneously occurring} dissipative dynamics, typically regarded as undesirable for the sake of quantum information processing, can be effective either.

[19] S. Campbell \textit{et al.}, Phys. Rev. A \textbf{84}, 052316 (2011).