On the microwave response of thin superconducting films with trapped magnetic flux

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Abstract. Basing on the phenomenological model of the microwave response of thin superconducting films with trapped magnetic flux elaborated in Luzhbin et al 2005 Supercond. Sci. Technol. 18 1112-7, detailed analysis of the ac linear response of a superconducting strip-line resonator cooled down in FC and ZFC regimes is presented. Special emphasis on a nonuniform distribution of the trapped vortices and on the magnetic field and temperature dependence of the phenomenological model parameters such as critical current density, vortex pinning and viscous motion coefficients is made. For a ZFC case realistic trapped magnetic field distribution based on the critical-state model for a vortex penetration is used. Within the framework of the used model the unusual multi-peak dependence of the surface resistance of HTS films observed experimentally is explained as well as clarified conditions for its appearance.

1. Introduction
The phenomenological theory of the linear response of type-II superconductors (SC) in the mixed state developed in the last decades [1, 2, 3, 4, 5] describes the experimental data sufficiently well. Being based mainly on the idea of the induced current and its effect on the quasi-particles due to vortex motion, it can be formulated in terms of a change of the local effective resistivity of the superconducting material. Introducing into the model description various phenomenological parameters (e.g. pinning constant $\alpha_L$, viscosity $\eta$, creep rates $\tau$, $\tau_0$ [2, 3], critical current density $J_c$, elastic moduli of the flux line lattice $C_{ii}$, $i = 1, 4, 6$, etc) allows us to simultaneously incorporate into the theory various effects of the external magnetic field and the temperature starting from the ordinary flux creep to the phase transitions in the vortex ensemble.

However, a number of experiments shows unusual quasi-periodic dependence of the surface impedance of HTS films on the temperature or the external dc magnetic field [6, 7, 8], which cannot be unambiguously explained within the frameworks of the existing theoretical models.

In the current report the theoretical model of the linear response of HTS thin films with trapped vortices developed in [9] is used to model the dc magnetic field and the temperature dependence of energy losses $P_{\text{dis}}(B,T)$ connected with the oscillation of Abrikosov vortices in thin SC films cooled down in (FC) and without (ZFC) external dc magnetic field. It is shown that, under the general assumptions on the magnetic field and temperature behavior of $J_c$, $\alpha_L$ and $\eta$, the quasi-periodic form of $P_{\text{dis}}(T)$ dependence neatly corresponds to the ones obtained in [6, 7, 8]. By varying the shape of $J_c(B,T)$, $\alpha_L(B,T)$ and $\eta(B,T)$ dependences, the conditions for the appearance of nonmonotonic parts on $P_{\text{dis}}(B,T)$ curve is also clarified.
2. Theoretical consideration

Let us consider a thin SC film (with thickness \( d \leq \lambda \) and width \( 2w \gg 2\lambda^2/d \)) with a trapped magnetic flux. It is supposed that all physical quantities (current, magnetic field, etc.) are distributed along the \( y \)-axis only and averaged over the film thickness, i.e. \( J(y) = \int_{-d/2}^{d/2} J(y, z) dz/d \).

The trapped magnetic flux is produced by Abrikosov vortices distributed along the film width with some magnetic field profile \( B(y) \), which is proportional to the local density of vortices. The profile \( B(y) \) in turn depends on the cooling prehistory of the film and can in principle be found by means of magneto-optical, scanning Hall probe microscopy, or Lorentz microscopy methods.

Upon the application of an external ac current \( J_{rf}(y) \exp(-i\omega t) \), vortices are forced to oscillate around the equilibrium positions with the same frequency \( \omega \). The vortex displacement field \( u(y) \) can be found from the force balance condition \([2, 3]\), which in our case has the form \([9]\):

\[
J_{rf}(y) - \frac{2}{\pi \mu_0 d} \int_{-w}^{w} \frac{[B(\zeta)u(\zeta)]'}{\zeta - y} \sqrt{\frac{w^2 - \zeta^2}{w^2 - y^2}} d\zeta = \tilde{\alpha} \Phi_0 u(y), \tag{1}
\]

where \( \tilde{\alpha} = \alpha_L(1 - i \omega/\omega_0) \), \( \omega_0 = \alpha_L/\eta \) is the depinning frequency that lies in the range \( 10^{10} - 10^{12} \) s\(^{-1}\) \([15]\), and \( \Phi_0 \) is the flux quantum. Equation (1) models the well known ac current concentration in superconducting strip-line resonators at the film edges by a function with the square root singularities at the edges of the film, which is a rather good approximation \([10, 11]\). Moreover, since equation (1) has the finite solution over the whole interval \(-w \leq y \leq w\), it proposes a very realistic model of the linear response of thin SC films with trapped vortices.

Having found the vortex displacement field \( u(y) \) as the solution to (1), one can calculate the response of the film to the ac current; in particular, the ohmic losses \( P_{\text{dis}} \) per unit length of the film during the period of the ac field oscillations can be found according to the definition \([12]\)

\[
P_{\text{dis}} = \frac{gd}{2} \text{Re} \left( \int_{-y_{\text{edge}}}^{y_{\text{edge}}} J(y, t) E^*(y, t) dy \right) + P_{\text{edge}}, \tag{2}
\]

where all time dependent variables are supposed to have the form \( \exp(-i\omega t) \), \( g \) is defined by the geometry and operation regime of the resonator, \( y_{\text{edge}} = w - 2\lambda^2/d \), and \( P_{\text{edge}} \) corresponds to the ohmic losses at the edges of the film, i.e. for \( y_{\text{edge}} \leq |y| \leq w \), \( J(y) \) is the total current in the resonator, and \( E(y) \) is the ac electrical field generated by the current \( J(y) \). In (2) the fact that the real current distribution in the resonator is obtained by the current cut off at a distance of order \( 2\lambda^2/d \) from the edges \([11]\) is taken into account.

The general idea of the numerical solution of equation (1) that gives the approximate solution with an arbitrary specified precision can be summarized as follows \([13]\).

(i) For any smooth function \( B(y) \) that has no zeros in the interval \([-w, w]\), the function \( f(y) = B(y)u(y) \) can be well approximated by a polynomial of degree \( N \), given by \( f(y) \approx \sum_{j=0}^{N} a_j \tilde{y}^j \), where \( a_j \) are the unknown complex coefficients, \( \tilde{y} = y/w \), and \( f'(\tilde{y}) \approx \sum_{j=0}^{N} j a_j \tilde{y}^{j-1} \).

(ii) Let us write \( B(y) = B_{\text{dc}} f_B(\tilde{y}) \) and \( J_{rf}(y) = J_{\text{rf},0} f_J(\tilde{y}) \), where \( B_{\text{dc}} \) and \( J_{\text{rf},0} \) are some constants that have dimensions of magnetic field and current density, respectively, and the functions \( f_B \) and \( f_J \) define the dc magnetic field profile and the ac current distribution over the film width, respectively. In general, \( f_B(\tilde{y}) \) also depends on the external dc field \([14]\), but in the simplest case of the uniform vortex distribution \( f_B \) is a function of \( \tilde{y} \) only, and \( B_{\text{dc}} \) has the meaning of the external dc field (e.g. for FC case). For the linear response it is always possible to write \( a_j = dw \mu_0 J_{\text{rf},0} c_j \), where \( c_j \) are dimensionless unknown coefficients. Multiplying both sides of equation (1) by \( B(y) \) yields the equivalent equation

\[
f_J(\tilde{y}) f_B(\tilde{y}) - \frac{2f_B(\tilde{y})}{\pi \sqrt{1 - \tilde{y}^2}} \sum_{j=1}^{N} j c_j \varphi_j(\tilde{y}) = S(\omega) \sum_{j=0}^{N} c_j \tilde{y}^j, \tag{3}
\]
where $S(\omega) = dv/\lambda^2_c(\omega)$, $\lambda^2_c(\omega) = B_{dc}\Phi_0/\mu_0\alpha$ is the Campbell penetration depth [2, 3], and $\varphi(y) = \int_{-1}^{1} \zeta^2 \sqrt{1 - \zeta^2} d\zeta/((\zeta - \tilde{y})^3)$. (iii) Taking the values of all functions in (3) at the basic points $\tilde{y}_0, \tilde{y}_1, ... \tilde{y}_N$ which are the zeros of the Chebyshev polynomial $T_{N+1}(y) = \cos((N + 1) \arccos(y))$, one obtains a system of linear equations for determining the unknown coefficients $c_j$.

Since the parameters defining the response of resonators in this approximation (i.e. $f_J(\tilde{y})$ and $f_B(\tilde{y})$) are supposed to be known elsewhere, this model is directly applicable to any type of thin film resonator. The quasi-periodic form of $P_{\text{dis}}(S(\omega))$ dependence that follows from equations (1), (3) (see figure 1(A)) is a manifestation of the dimensional effect associated with quantization of the displacement field $u(y)$ on the width of the film [9].

To model the temperature or the dc magnetic field dependence of the microwave response in the framework of this model, one should take into account the following factors: the quantization of the displacement field $u(y)$ and $J_c$ for a film cooled down in FC and ZFC regimes in an uniform external magnetic field $B_{dc}$. For the FC case the uniform distribution of the trapped vortices is a rather good fit, i.e. $B(y) = B_{dc}$, whereas for the ZFC case the critical state model [14] is the best approximation. The original critical state model is modified in the following way: $B(y) = B_{\text{remn}}$ for $|y| < b$, and $|J(y)| = J_c$ for $b < |y| < w$ with $b = w/\cosh(\pi B_{dc}/\mu_0 J_c)$ and $B_{\text{remn}} \ll B_{dc}$ being some remnant field. This modification allows us to use the foregoing approach to model the ac response of the film in ZFC state and, as long as $B_{\text{remn}} \ll B_{dc}$, it does not affect the results; on the other hand, such modification is justified by the fact that in a lot of experiments some unscreened magnetic field of order of several oersted is always present [8]. In both cases, the following temperature [15] and the dc magnetic field [16] dependence of $\alpha_L$ and $\eta$ is supposed:

$$\alpha_L(T) = \alpha_L(0)(1 - t)^{1/3}(1 + t^2) \exp(-T/T_0), \quad \alpha_L(B) \propto B_{dc}^{n_a},$$
$$\eta(T) = \eta(0)(1 - t^2)/(1 + t^2), \quad \eta(B) \propto B_{dc}^{n_a},$$

where $t = T/T_c, T_c = 88K$, typical values of $\alpha_L(0) \simeq 3 \times 10^5$ N m$^{-2}$ and $\eta(0) \simeq (0.2 - 1.2) \times 10^{-6}$ N s m$^{-2}$, $T_0$ is a characteristic fluctuation temperature [5, 15] (with $T_0 \simeq 28K$ for YBCO), $n_a \simeq 0.55, n_\eta \simeq 1.85$ [16]. Note that the given values of $\alpha_L(0), \eta(0)$ as well as their temperature and field dependences (4) are the best fit for the numerous experimental data obtained in a number of experiments on YBCO [15, 16], whereas theoretical predictions vary from the field-independent $\eta$ for the isotropic s-wave pairing to $n_\eta = 1/2$ for the d-wave pairing, and from the field-independent $\alpha_L$ for the individual pinning of vortices to $n_a = 1$ for the collective pinning.

The temperature and the magnetic field dependence of $J_c$ is well described by the following empirical formula [17]: $J_c(B, T) = J_{c0}(B)(1 - t)^{\alpha}, \quad J_{c0}(B) = J_{c0}(0)J_B$, in the range of interest, $B^* \simeq B_{\text{eff}}(1 - t)$. $J_{c0}(0), \varepsilon, \beta$ and $B_{\text{eff}}$ are some empirical parameters, which can be independently obtained from transport, ac susceptibility and magnetization measurements. For YBCO films, the following values are typical: $\beta, \varepsilon$ of order of unity, $B_{\text{eff}}$ of order of several tesla, and $J_{c0}(0)$ of order of several MA cm$^{-2}$ [17].

3. Discussion

The energy loss dependence on the temperature $P_{\text{dis}}(T)$ for a film in FC and ZFC states is shown in figure 1(B). All the parameters $\alpha_L, \eta, \omega_0$, and $J_c$ are supposed to be temperature and magnetic field dependent according to (4) with $\alpha_L(0) = 10^5$ N m$^{-2}$ and $\eta(0) = 0.2 \times 10^{-6}$ N s m$^{-2}$.

It is found that the exact form of $P_{\text{dis}}(B, T)$ curve and the peak positions are mainly defined by $\alpha_L(0)$ and $J_{c0}(0)$ as well as the exact form of $J_c(B, T)$ dependence, while the quantities $\eta$
Figure 1. Vortex contribution to the ohmic losses in the film (A) as a function of the parameter $S_0 = dw/\lambda_0^2(\omega = 0)$, $\Omega = \omega/\omega_0$, and (B) as a function of the temperature for FC (-----) and ZFC (······) regimes for the frequency $\omega/2\pi = 1$ GHz. To avoid crowding, ZFC curves are multiplied by factor 4.

and $\omega_0$ define the peak amplitudes. Decrease of $\alpha_L(0)$ and $J_{c0}(0)$ shifts the position of the peaks to the lower temperatures and decreases their amplitude, so that the peaks cannot be detected.

Other crucial factors are the geometrical parameters of a resonator, i.e. its thickness $d$ and width $2w$. As it is seen from figure 1(A), the most pronounced peaks in the curve $P_{\text{dis}}(S_0)$ are located in the region of tens of $S_0$; thus, only if the dimensions of a resonator lie in the range corresponding to $5 \leq S_0 \leq (20 - 30)$ for the rather low frequency, the multi-peak picture similar to the ones obtained in [6, 7, 8] can be experimentally observed.

In that way, the possibility to experimentally observe the quasi-periodic (multi-peak) dependence $P_{\text{dis}}(T)$ crucially depends on the transport properties of HTS films, which can be characterized by $J_c$ and $\alpha_L$ and in turn are determined by the conditions and method of preparation of the films, as well as on the geometrical dimensions of SC resonators.

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