Photon polarization with anomalous right-handed top couplings in $B \to K_{\text{res}}\gamma$

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The effect of anomalous right-handed top couplings on the photon polarization in $B \to K_{\text{res}}\gamma$ is investigated. It is recently reported that the photon polarization can be measured through the up-down asymmetry of the photon direction relative to the subsequent $K_{\text{res}}$ decay plane. We find that the anomalous couplings can severely affect the photon polarization without spoiling the well measured branching ratio of $B \to X_s\gamma$. Different features from other scenarios are also discussed.

Radiative $B$ decay of $B \to X_s\gamma$ plays important roles in testing the standard model (SM) and constraining the new physics. Not only the inclusive and exclusive branching ratios but also polarization of the emitted photon have been extensively studied [1-4]. In the SM, photons from $b \to s\gamma$ are predominantly left handed up to $\mathcal{O}(m_s/m_b)$. A smart way of measuring the photon polarization in $B \to K_{\text{res}}\gamma$ is recently proposed [5]. In this approach, hadronic three body decay of $K_{\text{res}} \to K^\pm\pi^\mp$ (or $\rho K) \to K\pi\pi$ is essential to construct a triple vector product $\mathbf{p}_\gamma \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$. Here $\mathbf{p}_\gamma$ is the photon momentum and $\mathbf{p}_1, \mathbf{p}_2$ are two of the daughter hadron momenta, measured in the $K_{\text{res}}$ rest frame. A remarkable result is that the so called photon polarization parameter $\lambda_\gamma$, which encodes the left-right asymmetry of the emitted photon polarization, is universally determined by the Wilson coefficients of the effective Hamiltonian. In the SM, the predominant left-handedness implies that $\lambda_\gamma = -1 + \mathcal{O}(m_s^2/m_b^2)$. The authors of [5] predicted that the integrated up-down asymmetry in $K_{1}(1400) \to K\pi\pi$ is $(0.33\pm0.05)\lambda_\gamma$, which is quite large compared to other $K_{\text{res}}$ decays. Estimated number of $BB$ pairs required to measure the asymmetry is about $10^8$, which is already within the reach of current $B$ factories. A stringent test of SM as well as constraints on new physics by the photon polarization is therefore near at hand.

In this Letter, we investigate the effects of anomalous right-handed couplings on the photon polarization parameter. The left-right (LR) symmetric model and the minimal supersymmetric standard model (MSSM) can provide such new couplings. The LR model is one of the natural extensions of the SM, based on the $SU(2)_L \times SU(2)_R \times U(1)$ gauge group. Besides the usual left-handed quark mixing, right-handed quark mixing is also possible in the LR model. Without a manifest symmetry between the left- and right-handed sectors, the right-handed quark mixing is not necessarily the same as the left-handed quark mixing governed by the CKM paradigm. Consequently, there are additional right-handed charged current interactions with couplings different from the left ones, which are suppressed by the heavy extra $W$ boson [6].

In the unconstrained MSSM (uMSSM), the gluino-involved loop can contribute to the "wrong" chirality operator through the left-right squark mixing [7]. There can be a special case where $W$, Higgs, chargino, and gluino contributions to the ordinary Wilson coefficient $C_{12}$ tend to cancel each other while the "wrong" chirality coefficient $C'_{12}$ gives the dominant contribution.

In the present analysis, however, we just concentrate on the anomalous right-handed top quark couplings $tbW$ and $tsW$ interacting with coupling $C_{12}$. While the branching ratio is estimated to be $\text{Br}(t \to sW) \sim 1.6 \times 10^{-5}$, the large number of top quarks at LHC will enable us also to measure the $sW$ process and provide a chance to probe the $tsW$ coupling directly.

We introduce dimensionless parameters $\xi_t$ and $\xi_b$ where the anomalous right-handed $tsW$ and $tbW$ couplings are encapsulated, respectively. Up to the leading order of $\xi$, ordinary Wilson coefficients are modified through the loop-function corrections proportional to $\xi$. On the other hand, there appear new chiral-flipped operators in the effective Hamiltonian. The corresponding new Wilson coefficients are proportional to $\xi_t$, with new loop functions. It was recently shown that there is a parameter space of $(\xi_t, \xi_s)$ where the discrepancy of $\sin 2\beta$ between $B \to J/\psi K$ and $B \to \phi K$ is well explained while satisfying the $B \to X_s\gamma$ constraints [8]. Though the allowed values of $\xi$ are rather small, modified Wilson coefficients of the color-magnetic and electromagnetic operators $O_{11}, O_{12}$ involve large enhancement factor of $m_t/m_b$. Fortunately, the photon polarization parameter $\lambda_\gamma$ depends only on the Wilson coefficient $C_{12}$ and its chiral-flipped partner $C'_{12}$, irrespective of the species of $K_{\text{res}}$. Thus the photon polarization parameter $\lambda_\gamma$, or the up-down asymmetry of the emitted photon is quite sensitive to the new right-handed couplings.

Let us first consider the effective Lagrangian containing possible right-handed couplings

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=s,b} V_{tb} V_{tq}^{\dagger} \bar{\gamma}^\mu (P_L + \xi_q P_R) q W^+_\mu + \text{h.c.} , \quad (1)$$

where $P_{L,R}$ are the usual chiral projection operators.
With new dimensionless parameters $\xi_{b,s}$, the effective Hamiltonian for the radiative $B$ decays has the form of

$$H_{\text{rad}} = \frac{-4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[ C_{12}(\mu) O_{12}(\mu) + C_{12}'(\mu) O_{12}'(\mu) \right],$$

$$O_{12} = \frac{e}{16\pi^2} m_b s \sigma_{\mu \nu} b F^{\mu \nu},$$

and $O_{12}'$ is the chiral conjugate of $O_{12}$. After matching at $\mu = m_W$, the Wilson coefficients are given by, in the SM, \cite{9}

$$C_{12}(m_W) = F(x_t) = \frac{x_t(7 - 5x_t - 8x_t^2)}{24(x_t - 1)^3} - \frac{x_t^2(2 - 3x_t)}{4(x_t - 1)^4} \ln x_t,$$

$$C_{12}'(m_W) = 0,$$

where $x_t = m_t^2/m_W^2$. Turning on the right-handed $\bar{b}bW$ and $\bar{t}sW$ couplings, the Wilson coefficients are modified as

$$C_{12}(m_W) \rightarrow F(x_t) + \frac{\xi_{b} m_t}{m_b} F_R(x_t),$$

$$C_{12}'(m_W) \rightarrow \xi_{s} \frac{m_t}{m_b} F_R(x_t),$$

with the new loop function \cite{8,10}

$$F_R(x) = \frac{-20 + 31x - 5x^2}{12(x - 1)^2} + \frac{x(2 - 3x)}{2(x - 1)^3} \ln x.$$

Scaling down to $\mu = m_b$ is accomplished by the usual renormalization group (RG) evolution. We use the RG improved Wilson coefficients of \cite{8}.

Next consider the radiative $B$ decay of $B \rightarrow K_{\text{res}}^{(*)}$. The photon polarization in this process is naturally defined as follows:

$$\lambda^{(*)}_\gamma = \frac{|A_R^{(*)}|^2 - |A_L^{(*)}|^2}{|A_R^{(*)}|^2 + |A_L^{(*)}|^2},$$

where $A_{L(R)}^{(*)} \equiv A(B \rightarrow K_{\text{res}}^{(*)} \gamma_{L(R)})$ is the weak amplitude for left(right)-polarized photon. The authors of \cite{5} simply argued that

$$\langle K_{\text{res}}^{(*)} \gamma_{L,R} | O_{12}' | B \rangle = (-1)^{f_t - 1} P_t \langle K_{\text{res}}^{(*)} \gamma_{L,R} | O_{12} | B \rangle,$$

where $J_t(P_t)$ is the resonance spin (parity). It means that $|A_R^{(*)}|/|A_L^{(*)}| = |C_{12}'|/|C_{12}|$, and further

$$\lambda^{(*)}_\gamma = \frac{|C_{12}'|^2 - |C_{12}|^2}{|C_{12}'|^2 + |C_{12}|^2} \equiv \lambda_\gamma.$$

Thus the photon polarization parameter $\lambda_\gamma$ is independent of the $K_{\text{res}}$ states and is universally determined by the Wilson coefficients. It is also free from the hadronic uncertainty which usually originates from the weak form factors. The relation (7) ensures that the common form factors are involved in $A_{L}$ and $A_{R}$, being canceled in the ratio. In the SM, $\lambda_\gamma \approx -1$ (+1 for $b \rightarrow s \gamma$) since $|C_{12}'|/|C_{12}| \approx m_s/m_b$.

To relate $\lambda_\gamma$ with the physical observable, a full analysis including the strong decays of $K_{\text{res}}$ must be implemented. As mentioned before, there needs at least three hadrons in the final state to see the up-down asymmetry. One can readily calculate the angular distribution of $B \rightarrow K \pi \pi$ \cite{5}, and find that terms containing the photon polarization parameter $\lambda_\gamma$ are proportional to the up-down asymmetry of the emitted photon momentum with respect to the $K \pi \pi$ decay plane.

![Contour plot of $\lambda_\gamma$](image)

FIG. 1. Contour plot of $\lambda_\gamma$, for (a) $|\xi_s| = 0.012$ and (b) $|\xi_s| = 0.001$. Concentric circles correspond to, from outside to inside, $\lambda_\gamma = -0.9, -0.8, \ldots, 0$ in (a) and $\lambda_\gamma = -0.99, -0.98, \ldots, -0.9$ in (b), respectively. Shaded ring is the allowed region from $B \rightarrow X_s \gamma$.

Now let us examine the effects of newly introduced $\xi_{b,s}$ on $\lambda_\gamma$. New couplings $\xi_{b,s}$ are strongly constrained by the branching ratio $\text{Br}(B \rightarrow X_s \gamma)$ and the $CP$ asymmetry.
$A_{CP}(B \to X_s \gamma)$. We use the weighted average of the branching ratio [7,8]

$$\text{Br}(B \to X_s \gamma) = (3.23 \pm 0.41) \times 10^{-4}, \quad (9)$$

from the measurements of Belle [11], CLEO [12], and ALEPH [13] groups. The $CP$ violating asymmetry in $B \to X_s \gamma$, defined by

$$A_{CP}(B \to X_s \gamma) = \frac{\Gamma(B \to X_s \gamma) - \Gamma(B \to X_s' \gamma)}{\Gamma(B \to X_s \gamma) + \Gamma(B \to X_s' \gamma)}, \quad (10)$$

is measured by CLEO [14]:

$$A_{CP}(B \to X_s \gamma) = (-0.079 \pm 0.108 \pm 0.022)(1.0 \pm 0.030). \quad (11)$$

The explicit expressions of the branching ratio and the $CP$ asymmetry are given in [15] in terms of the evolved Wilson coefficients at the $\mu = m_b$ scale. We adopt the constraints on $\xi$ established in [8] from these experimental and theoretical results at 2$\sigma$ C.L.:

$$-0.002 < \text{Re}\xi_b + 22|\xi_b|^2 < 0.0033,$$

$$-0.299 < \frac{0.27\text{Im}\xi_b}{0.095 + 12.54\text{Re}\xi_b + 414.23|\xi_b|^2} < 0.141,$$

$$|\xi_s| < 0.012. \quad (12)$$

Figure 1 shows the contour plot of $\lambda_s$ for various values of $\xi_b,s$. Shaded ring denotes the allowed region by (12). Since the measured $CP$ asymmetry has rather large errors, constraints on $\xi$ are mainly from the $\text{Br}(B \to X_s \gamma)$. In Fig. 1, two contours for small and large value of $\xi_s$ are shown as an illustration. As one can expect from (8), deviation of $\lambda_s$ from $-1$ can be sizable if $\xi_s$ is large (Fig. 1 (a)). On the other hand, if $\xi_s$ is very small, $\lambda_s \approx -1$, irrespective of $\xi_b$ (Fig. 1 (b)).

As can be seen in Fig. 1 (a), large value of $\lambda_s$ can be obtained in the region of $\text{Im}\xi_b = 0$. In Fig. 2, we give plots of $\lambda_s$ vs $\xi_s$ for different values of $\xi_b$. Here we assumed that $\xi_b,s$ are all real for simplicity. Shaded bands are the allowed region by (12).

In this case, a large deviation of $\lambda_s$ from $-1$ is possible in the right-hand-side band. This is quite natural since one can expect large $\lambda_s$ in the region of large $\xi_s$ and small $\xi_b$, by the inspection of (8). We have

$$-1 \leq \lambda_s \lesssim -0.12. \quad (13)$$

It should be noticed that current experimental bounds on $B \to X_s \gamma$ do not allow the different sign of $\lambda_s$ compared to the SM prediction at 2$\sigma$ level. Note that the upper bound of $\lambda_s$ is chosen at the edge point of the allowed parameter space, say, $(\xi_b, \xi_s) = (-0.0021, 0.012)$. If the new couplings are flavor-blind, i.e. $\lambda_s = \lambda_s$, then $\lambda_s \approx -0.96$ for $\xi_b,s = -0.002$. Thus a large amount of reduction in $|\lambda_s|$ implies that the anomalous right-handed couplings are flavor dependent.

It is quite interesting to compare our result with that of the uMSSM [7]. The authors of Ref. [7] proposed the "$C'_{12}$-dominated" scenario, where the total contribution to $C_{12}$ is negligible while the main contribution to the $Br(b \to s\gamma)$ is given by $C'_{12}$. This is possible when the chargino, neutralino, and gluino contributions to $C_{12}$ are canceled out by the $W$ and Higgs contributions. Now that the size of $C_{12}$ is very small, they expect $\lambda_s \approx +1$ as an extreme case, quite contrary to the SM predictions. This is also very distinguishable from our result, since (13) does not allow the sign flip of $\lambda_s$. Thus the sign of $\lambda_s$ is a very important landmark indicating which kind of new physics is involved, if exists. In case of $\text{sgn}(\lambda_s) > 0$, models which produce only the anomalous right-hand top vertices would be disfavored. At least we might need new particles, or new mechanism to make the $C'_{12}$ dominant.

As introduced earlier, the integrated up-down asymmetry of $K_1(1400) \to K^*\pi, \rho\pi$ is reported to be $(0.33 \pm 0.05)\lambda_s$ [5], where the uncertainty is a combined one from the uncertainties of $D$- and $S$-wave amplitudes in the $K^*\pi$ channel, and $\rho\pi$ amplitude. Within the SM where $\lambda_s \approx -1$, it means that about 80 charged and neutral $B$ and $\bar{B}$ decays into $K^\pm\pi^0$ are needed to measure an asymmetry of $-0.33$ at 3$\sigma$ level. The authors of [5] estimated that at least $2 \times 10^7$ $B\bar{B}$ pairs of both neutral and charged are required, with the use of $\text{Br}(B \to K_1(1400)\gamma) = 0.7 \times 10^{-5}$ and $\text{Br}(K_1(1400) \to K^*\pi) = 0.94 \pm 0.06$.

If the anomalous right-handed couplings were present, we would need more $B\bar{B}$ pairs because new couplings will reduce the value of $|\lambda_s|$. For example, in case of $\lambda_s = -0.5$, we need 4 times larger number of $B\bar{B}$ pairs ($8 \times 10^7$) to see the 3$\sigma$ deviation of the up-down asymmetry from both zero and the SM prediction. Fortunately, this is already within the reach of current $B$ factories [16]. If the deficiency of the up-down asymmetry were found, then the direct searches of the anomalous top couplings at the LHC in coming years would be very exciting to check the consistency.

In conclusion, we have analyzed the effects of anomalous right-handed top couplings on the photon polarization in radiative $B \to K_{res}\gamma$ decays. The photon polarization parameter $\lambda_s$ defined by the ratio of the relevant
Wilson coefficients is a useful observable for measuring the photon helicity. It is found that the new couplings can reduce $|\lambda_\gamma|$ significantly, compared to the SM prediction of $\lambda_\gamma \approx -1$, while satisfying the strong constraints from the measured branching ratio and $CP$ asymmetry of $B \to X_s \gamma$. We also find that the anomalous right-handed top couplings would not produce different sign of $\lambda_\gamma$ from the SM prediction. This is a crucial point to distinguish our case from other scenarios such as uMSSM where an extreme value of $\lambda_\gamma = +1$ can be possible through the "$C_{12}'$-dominated" mechanism. The importance of present work also lies in the fact that current $B$ factories can produce enough $BB$ pairs to analyze the photon polarization in $B \to K_{res} \gamma$.

The author is grateful for Kang-Young Lee’s valuable comments on the Wilson coefficients. He also gives thanks to Heyoung Yang for helpful discussions. This work was supported by the BK21 program of the Korean Ministry of Education.

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