Kinematic modelling of a 3-axis NC machine tool in linear and circular interpolation

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Abstract Machining time is a major performance criterion when it comes to high-speed machining. CAM software can help in estimating that time for a given strategy. But in practice, CAM-programmed feed rates are rarely achieved, especially where complex surface finishing is concerned. This means that machining time forecasts are often more than one step removed from reality. The reason behind this is that CAM routines do not take either the dynamic performances of the machines or their specific machining tolerances into account. The present article seeks to improve simulation of high-speed NC machine dynamic behaviour and machining time prediction, offering two models. The first contributes through enhanced simulation of three-axis paths in linear and circular interpolation, taking high-speed machine accelerations and jerks into account. The second model allows transition passages between blocks to be integrated in the simulation by adding in a polynomial transition path that caters for the true machining environment tolerances. Models are based on respect for path monitoring. Experimental validation shows the contribution of polynomial modelling of the transition passage due to the absence of a leap in acceleration. Simulation error on the machining time prediction remains below 1%.

Keywords High-speed machining · Linear interpolation · Circular interpolation · Polynomial transition

1 Introduction

High-speed machining centres allow for extremely high feed rates to be programmed. However, when machining molds or dies, dynamic performances of the machines do not always allow such feed rates to be reached. Indeed, according to the quality sought, the segments making up the machining path are often extremely short and in such conditions the feed rate reached by the machine will then be limited by the NC interpolation time or even the capabilities in jerk or acceleration of the axes [1, 2]. The feed rate will then not be constant, leading to considerably lower productivity, a variation in tangential cutting forces and impaired quality [3]. Many publications relating to the search to reduce the number of feed rate changes base their research on the use of NURBS interpolations [4–6] or B-spline [7]. However, in the industrial world, linear and circular interpolation remain the most frequently used methods on many workpieces. Precise simulation of this type of movement is therefore essential. The aim of this work is therefore to propose a comprehensive model intended to simulate the position, feed rate, acceleration and jerk in three-axis linear and circular interpolation taking the machine/NC combination parameters into account.

At present, NC machine manufacturers [8] propose a displacement law on the axes in trapezoid acceleration. This type of command has been studied in the literature by a number of authors writing on uniaxial paths with
null initial and final feed rates [9, 10]. This movement involves seven phases (Fig. 1; Table 1):

- On phase 1,

\[
\begin{align*}
J_i(t) &= J_{\text{max},i}, \\
A_i(t) &= A_{0,i} + J_{\text{max},i}(t - T_0) \\
V_i(t) &= V_{0,i} + A_{0,i}(t - T_0) \\
&\quad + \frac{1}{2} J_{\text{max},i} (t - T_0)^2 \\
X_i(t) &= X_{0,i} + V_{0,i}(t - T_0) \\
&\quad + \frac{1}{2} A_{0,i} (t - T_0)^2 \\
&\quad + \frac{1}{6} J_{\text{max},i} (t - T_0)^3
\end{align*}
\tag{1}
\]

- On phase 2,

\[
\begin{align*}
J_i(t) &= 0 \\
A_i(t) &= A_{\text{max},i} \\
V_i(t) &= V_i(T_1) + A_{\text{max},i}(t - T_1) \\
X_i(t) &= X_i(T_1) + V_i(T_1)(t - T_1) \\
&\quad + \frac{1}{2} A_{\text{max},i} (t - T_1)^2
\end{align*}
\tag{2}
\]

- On phase 3,

\[
\begin{align*}
J_i(t) &= -J_{\text{max},i} \\
A_i(t) &= A_i(T_2) - J_{\text{max},i}(t - T_2) \\
V_i(t) &= V_i(T_2) + A_i(T_2)(t - T_2) \\
&\quad - \frac{1}{2} J_{\text{max},i} (t - T_2)^2 \\
X_i(t) &= X_i(T_2) + V_i(T_2)(t - T_2) \\
&\quad + \frac{1}{2} A_i(T_2) (t - T_2)^2 \\
&\quad - \frac{1}{6} J_{\text{max},i} (t - T_2)^3
\end{align*}
\tag{3}
\]

**Table 1** Nomenclature

| **Kinematic and dynamic parameters** |
|-----------------|------------------|
| \(\mathcal{J}\) | Jerk vector      |
| \(\mathcal{A}\) | Acceleration vector |
| \(\mathcal{V}\) | Feed rate vector |
| \(\mathcal{X}\) | Position vector |
| \(J_{\text{max},i}\) | Maximum jerk limited by machine dynamics on the axis \(i\) |
| \(A_{\text{max},i}\) | Maximum acceleration limited by the machine dynamics on the axis \(i\) |
| \(A_{0,i}, V_{0,i}\) | Acceleration, feed rate and initial position on the axis \(i\) |
| \(V_F\) | Programmed feed rate |
| \(V_F'\) | Feed rate reached if \(V_F\) is not achieved |
| \(V_{c,i}\) | Feed rate set on the axis \(i\) |
| \(V_{\text{In}}\) | Feed rate of entry into a block |
| \(V_{\text{Out}}\) | Feed rate exiting a block |
| \(V_{\text{Out}}'\) | Feed rate exiting a block if \(V_{\text{Out}}\) is not reached |
| \(V_{\text{disc}}\) | Maximum feed rate for entering a discontinuity |
| \(V_f\) | Feed rate entering a discontinuity limited by jerk |
| \(V_a\) | Feed rate entering a discontinuity limited by acceleration |
| \(\tau_i\) | Duration of phase \(i (\tau_i = T_i - T_{i-1})\) |

**Circular transitions in linear interpolation**

| \(A, O, B\) | Theoretical programmed path |
| \(A_i, O_i, B_i\) | Path described by the machine |
| \(TIT, tol_x\) | Method to define point \(Q\) in accordance with \(tol_y\) the programming method |
| \(R\) | Radius of the arc inserted on transition |
| \(l_1, l_2\) | Length covered before and after the transition |
| \(\beta\) | Angle formed by segments \([AO]\) and \([OB]\) |
| \(a_i, k_i\) | Normal acceleration and tangential jerk in steady state |

**Circular transitions in circular interpolation**

| \(R_1, R_2\) | Radii of circles before and after a circle-circle transition |

**Parameters used in circular interpolation**

| \(O\) | Centre of circle |
| \(R\) | Radius of circle |
| \(P(t)\) | Current point |
| \(\alpha\) | Angle covered by the circle arc |
| \(\theta(t)\) | Current angle |
| \(J_i\) | Curvilinear jerk limited by the axis \(i\) |
| \(J_C\) | Curvilinear jerk |
| \(A_i\) | Curvilinear acceleration limited by the axis \(i\) |
| \(A_C\) | Curvilinear acceleration |

| **Polynomial transitions in linear interpolation** |
|-----------------|------------------|
| \(\mathcal{R}\) | Frame \((\mathcal{X}, \mathcal{Y}, \mathcal{Z})\) |
| \(A, O, B\) | Programmed theoretical path |
| \((x_A, y_A, z_A)\) | Coordinates of \(\overline{OA}\) in the frame \(\mathcal{R}\) |
| \((x_B, y_B, z_B)\) | Coordinates of \(\overline{OB}\) in the frame \(\mathcal{R}\) |
| \(M\) | Point of entry into the discontinuity |
| \(N\) | Point of exit from the discontinuity |
| \(Q\) | Effective point of passage in the discontinuity |
| \(T\) | Time of passage in the discontinuity |
| \(L\) | Distance \(OM\) |
| \(P(t)\) | Current point |
| \(\phi_i, \theta_i\) | Spherical coordinates of the point \(i\) |
| \(tol_x, tol_y\) | Tolerance of position on the axes |
| \(V_{M}, V_{N}\) | Feed rates for entry on \(M\) and exit on \(N\) |