Determination of Power and Kinematic Wheel Parameters when Rolling against a Drum

T A Balabina¹, D S Simonov¹, V R Rogov¹, A N Mamaev², A S Vorontsov³

¹Department of Machinery Parts and Theory of Mechanisms, Moscow Automobile and Road State Technical University (MADI), 64, Leningradsky Ave., Moscow, 125319, Russia
²Moscow Polytechnic University, 38, B. Semyonovskaya Street, Moscow, 107023, Russia
³Department of Materials Science and Resource-Saving Technologies, Yanka Kupala State University of Grodno, 22, Ozheshko str, Grodno, 230023, Belarus
E-mail: dimsimonov94@mail.ru

Abstract. Car tire tests, including rolling resistance tests, are often performed on drum stands. Car tests on drum stands of various designs are also becoming more widespread: with a different number of drums, their locations and types of drives. However, in the literature there are no works describing the process of interaction of the wheel with the drum. In this regard, there is a need to consider the mechanics of the interaction of an elastic wheel with a drum and determine its kinematic and power characteristics, which is the task of this study. The mechanics of rolling the wheel against the drum is considered, taking into account the frictional interaction of the pair “elastic wheel – rigid drum” using the theory of preliminary displacement of the wheel elements in the zone of contact with the drum. This made it possible, inter alia, to obtain relationships for determining the rolling resistance caused by hysteresis losses in the wheel material, the tangential force acting in the contact, the moment on the wheel, the power of friction losses in the contact, the coordinates of the boundary of the adhesion and sliding sections in the contact of the wheel with the drum, relative loss of speed (slippage) of the wheel. The interaction of the wheel with two drums is also considered.

1. Introduction
Car tire tests, including rolling resistance tests, are often performed on drum stands. Car tests on drum stands of various designs are also becoming more widespread: with a different number of drums, their location and type of drive.

However, the process of rolling a wheel against a drum remains poorly understood – in foreign and Russian literature, there is no analysis of the mechanics of rolling a wheel against a drum, although this requires, with regard to the above, close study including the determination of both power and kinematic parameters of the wheel.

2. Main Part
Currently, issues related to the mechanics of rolling a wheel on a flat surface are considered quite widely [1–24].

The rolling mechanics of an elastic wheel against a drum are the same as when the wheel is rolling on a flat rigid supporting surface [1–5].
When a driven wheel is loaded, only with a normal load, due to imperfect elasticity of the material, there is a loss of internal friction in the wheel material (hysteresis), which causes the appearance of a moment of resistance $M_f$ and the appearance of a rolling resistance force $F_f$ – the longitudinal tangential force acting in contact of the wheel with the base in the opposite direction to the wheel rotation. A similar rolling resistance force also arises in a brake wheel, loaded in comparison with a driven wheel, with an additional braking torque $M_t$. The presence of this force leads to slipping of the elements of its treadmill relative to the base in the contact zone and to the loss of the angular velocity of the wheel.

When the drive wheel is rolling, the movement of which occurs under the influence of the torque $M_k$, a leading (traction) force arises in the contact, directed along the wheel rotation. As in the previous case, this force causes slippage of the treadmill elements of the wheel in the zone of contact with the base and to the loss of the linear velocity of the wheel axis.

The mechanism of occurrence of sliding of the surface elements of the wheel relative to the base is considered in detail in [1–5]. Using the scheme of the reversed mechanism “elastic wheel – rigid base”, on the basis of the theory of preliminary displacement, it is shown that with steady rolling, the wheel surface elements entering the contact zone are not yet “prepared” for perceiving tangential forces and at the same time pressed to the base normal force, they begin to move without sliding, while receiving tangential displacements (directed opposite to rolling for the brake and driven wheels, and in the direction of rolling – for the driving wheel). As the interlocked elements of the wheel and the base move in the inverted mechanism in the contact zone, their tangential displacements increase, therefore, the tangential friction force acting between the interlocked elements also increases. In the place of contact, where the increased friction force reaches its ultimate in adhesion, a breakdown occurs and on the entire part of the contact located beyond the breakdown point, regardless of whether it is in the zone of decreasing or increasing normal pressures, sliding occurs.

With an increase in the loss of wheel speed and a corresponding increase in the tangential force acting in the contact, the slip zone increases, as well as the power of friction losses in the contact, which characterizes the wear rate of the treadmill and partly the wheel rolling resistance.

The tangential displacements of the points of the treadmill in the contact zone, due to the implementation of the tangential force in the contact, are determined [1–5] by the relationship:

$$U = (a - x) \left( \frac{\omega_{wh} r_{wh}^f}{V} - 1 \right)$$

(1)

where $a$ is the half-length of the wheel area with a rigid supporting surface, $x$ is the distance from the beginning of the contact area to the point of the wheel in the contact zone, $r_{wh}^f$ is the radius of free rolling, $V = \omega_{wh} r_{wh}^f$ is the speed of the wheel axis in the inverted mechanism, $r_{wh}$ is the rolling radius of the wheel.

As applied to wheel rolling against a rigid drum, $V = V_d = \omega_d r_d$, where $\omega_d$ and $r_d$ are the angular velocity and radius of the drum. As a result, the tangential displacements of the surface points of the elastic wheel, due to the implementation of tangential force in contact with the drum, in the adhesion section can be represented by the expression:

$$U = (a - x) \left( \frac{\omega_{wh} r_{wh}^f}{\omega_d r_d} - 1 \right) = \tilde{\xi} (a - x)$$

(2)

where

$$\tilde{\xi} = \frac{\omega_{wh} r_{wh}^f}{\omega_d r_d} - 1$$

(3)

$\tilde{\xi}$ – relative speed difference.

With a known value of $\tilde{\xi}$, the ratio of the angular velocities of the wheel and drum will be equal to:

$$\frac{\omega_{wh}}{\omega_d} = (1 + \tilde{\xi}) \frac{r_d}{r_{wh}}$$

(4)
Based on the proportionality of the tangential stresses (specific tangential forces) to tangential displacements, we can write, that the tangential stresses due to the implementation of the tangential force in the contact are:

\[ q_t = \lambda U = \lambda \zeta (a - x) \tag{5} \]

where \( \lambda \) is the tangential stiffness coefficient of the wheel, defined \([1]\) as:

\[ \lambda = \frac{\lambda_{th} r_d}{r_d + r} = \frac{1.5qr}{a^2} \frac{1}{1+\frac{r}{r_d}}. \tag{6} \]

Under a parabolic law, the distribution of normal pressures along the length of the contact area, the coordinate of the boundary of the adhesion and slip sections, determined from the equality \( q_t = \mu q_n \), can be represented by the relationship \([1–3]\):

\[ x_B = -a \pm 2 \frac{\lambda}{\mu q_n}. \tag{7} \]

The moment on the drum, due to the action of the tangential force, is equal to:

\[ M_t = 2b \int_{-a}^{a} r_d q_t^s dx + 2b \int_{x_B}^{a} r_d q_t dx = \left[ 2b \int_{-a}^{a} q_t^s dx + 2b \int_{x_B}^{a} q_t dx \right] r_d = F_t r_d. \tag{8} \]

In the last formula, the value in square brackets equal to the algebraic sum of all the specific tangential forces in the contact is called the circumferential traction force (Figure 1):

\[ F_t = 2b \left[ \frac{\lambda \zeta}{2} (a - x_B)^2 \pm \frac{1}{3} \mu q_n (2a^3 + 3a^2 x_B - x_B^3) \right] \tag{9} \]

Figure 1. Forces in contact of the wheel with the drum

Substituting in (9) the expression \( \zeta = \pm \mu q_n (a + x_B) \), obtained from (7), after transformations, we obtain an equation whose solution gives a relationship for finding the coordinate of the boundary of the adhesion and slip sections:

\[ x_B = a \left( 1 - 2 \frac{1}{a} \sqrt{1 - \frac{F_t}{\mu \sigma_2}} \right). \tag{10} \]

As a result:

\[ \zeta = \pm 2 \mu q_n \left( 1 - \frac{3}{\sqrt{1 - \frac{F_t}{\mu \sigma_2}}} \right) \tag{11} \]

or, taking into account the expressions for \( q_n \) \([1 \ldots 5]\), (2) and (6):

\[ \zeta = \pm \frac{\mu a}{r} \left( \frac{1}{r_d} \right) \left( 1 - 3 \frac{1 - \mu q_n}{\mu \sigma_2} \right). \tag{12} \]
The last expression, if we do not take into account the saturation coefficient of the treadmill pattern \( s \) (for a wheel without a treadmill pattern \( s = 1 \)), coincides with a similar formula obtained by G. Fromm \([6, 7]\) (the difference lies only in the degree of radical: G. Fromm – square root), and then R.V. Virabov \([1]\) for the friction gear, consisting of two cylinders.

With a known relationship between \( \xi \), the ratio of the angular velocities of the elastic wheel and the rigid drum as a function of the traction force \( F_t \) and normal load realized in the contact in accordance with formulas (3), (11) and (12) can be represented as:

\[
\frac{\omega_{wh}}{\omega_d} = \frac{r_d}{r_{wh}} \left[ 1 \pm \frac{2\mu q_n a}{\lambda} \left( 1 - \sqrt{1 - \frac{F_t}{\mu F_z}} \right) \right] = \frac{r_d}{r} \left[ 1 \pm \frac{\mu a}{s} \left( \frac{r}{r_d} - 1 \right) \left( 1 - \sqrt{1 - \frac{F_t}{\mu F_z}} \right) \right].
\] (13)

For small (compared with the adhesion limit \( F_t^\text{max} = \mu F_z \)) tangential forces, the last formulas can be simplified if the expression \( \sqrt{1 - \frac{F_t}{\mu F_z}} \) is expanded in a power series, then discarding second-order negligible values:

\[
\frac{\omega_{wh}}{\omega_d} = \frac{r_d}{r} \left[ 1 - \frac{a}{3s} \left( \frac{r}{r_d} + 1 \right) \frac{F_t}{F_z} \right].
\] (14)

In the latter expressions, the force \( F_t \) is positive for the drive wheel and negative for the driven and braking.

The friction power losses in the contact of the elastic wheel with the rigid drum, due to the implementation of the traction force \( F_t \) and normal load realized in the contact in accordance with formulas (3), (11) and (12) can be represented as:

\[
\frac{\omega_{wh}}{\omega_d} = \frac{r_d}{r} \left[ 1 - \frac{a}{3s} \left( \frac{r}{r_d} + 1 \right) \frac{F_t}{F_z} \right].
\] (15)

The point of application of the resulting tangential force in the contact (Fig. 3) is found from the relationship \([8]\):

\[
x_{F_t} = a \left( 1 - \frac{\mu F_z}{F_t} \right) (1 - \sqrt{1 - \frac{F_t}{\mu F_z}})
\] (18)

which for small tangential forces compared to the ultimate in adhesion can be represented in a simplified form, if we expand \( \sqrt{1 - \frac{F_t}{\mu F_z}} \) in a power series:

\[
x_{F_t} = a \left( 1 - \frac{\mu F_z}{F_t} \right) \frac{F_t}{3\mu F_z} = \frac{a}{3} \frac{F_t}{\mu F_z} - 1.
\] (19)

The arm of the tangential force relative to the center of the wheel is found from geometric considerations (Figure 2):

\[
h_t = O_d O_{wh} \cos \alpha_t - r_d = (r_d + r_d) \cos \alpha_t - r_d
\] (20)

where the angle

\[
\alpha_t = x_{F_t} / r_d.
\] (21)

After the transformations we get:

\[
h_t = r_d \left( 1 - x_{F_t}^2 / 2r_d^2 \right).
\] (22)

As with the rolling of an elastic wheel along a flat supporting surface, in the case of the wheel interacting with a rigid drum, there is a redistribution of normal pressures along the length of the contact area and, as a result, a shift in the point of application of the normal reaction of the drum relative to the center of the wheel (Fig. 2).

The arm of the normal reaction of the drum relative to the center of the wheel is:

\[
h_n = (r - h_t) F / F_n.
\] (23)
We consider the case [9], when an elastic wheel rolls against two supporting drums (Fig. 3). The wheels of a car, in particular, are installed in this way during testing on drum stands.

$e'$ and $e''$ denote the distance between the center of the wheel and the centers of the left and right drums, respectively:

$$e' = r' + r_a', \quad e'' = r'' + r_a''$$

where $r'$ and $r''$ – the shortest distance from the axis of the wheel to the left and right drums.

**Figure 2.** The arms of the forces acting on the wheel

**Figure 3.** Forces acting on the wheel when rolling against two drums.

Under the action of the moment $M_\omega$ applied to the wheel, the normal reactions of the drums are shifted by the values of $h'$ and $h''$:

$$h' = h'_n + h'_0 \quad h'' = h''_n + h''_0$$

(24)

where $h'_n$ and $h''_n$ are the displacements of normal reactions due to the realization of tangential forces $F'_z$ and $F''_z$ in the contact; $h'_0$ and $h''_0$ are the displacements of normal reactions due to hysteresis in the tire material, which can be calculated as:

$$h'_0 = f_0 r \left(1 + \frac{r}{r_d} \right) \frac{a'(z)}{area \gamma'}$$

(25)

where $a'$ and $a''$ are the half-lengths of the contact area of the wheel with the drums under the action of normal forces $F'_z$ and $F''_z$, $area \gamma'$ – the half-length of the contact area on the plane under the action of the forces $F'_z = F'_n$ and $F''_z = F''_n$.

The points of application of tangential forces are determined from formulas (18) (for small tangential forces, from (19)).

The angles $\gamma'$, which determine the displacement of the point of application of the normal reactions of the drums, can be found from geometric considerations:

$$\gamma' = \arcsin \frac{h'_n + h'_0}{e'} \quad \gamma'' = \arcsin \frac{h''_n + h''_0}{e''}$$

(26)

It should be noted that with a rigid kinematic connection between two drums, the distances between the wheel axis and the axes of the drums, as well as normal reactions in the contacts, would be different. The arms of the circumferential and normal forces, hysteresis losses, power of friction losses in the contact, sizes of the contact areas will also be different.

When the wheel is mounted on two freely rotating unconnected drums, the normal reactions from the side of both drums and the dimensions of the contact pads are the same.
Under the action of the moment $M_{wh}$, the center of the wheel is located asymmetrically relative to the axes of the drums.

The rolling resistance of the driven wheel, as already noted, is determined by the moment of rolling resistance due to hysteresis losses in the wheel material. The value of this moment is more convenient to determine [10] through the hysteresis loss power $M_H = P_H / \omega_H$.

To determine the power of hysteresis losses in the material of an elastic wheel rolling along a rigid drum, we will take into account only the normal deformation of the wheel, which can be represented as the sum of terms (Figure 4): $W = W' + W''$.

![Figure 4](image_url) Normal stresses $q_n$ and deflection $w$ when the wheel is pressed against the drum.

Since:

$$W' = \frac{a^2-x^2}{2r} \text{ and } W'' = \frac{a^2-x^2}{2r_d}$$

then:

$$W = \frac{a^2-x^2}{2} \left( \frac{1}{r} + \frac{1}{r_d} \right).$$  \hspace{1cm} (27)

The hysteresis power loss can be found using the following relationship:

$$P_H = \beta_H \int_0^a q_n \left| \frac{dW}{dt} \right| dx \: 2b$$  \hspace{1cm} (28)

where $\beta_H$ is the hysteresis loss coefficient; $dW/dt$ – wheel deformation rate:

$$\frac{dW}{dt} = \frac{dW}{dx} \frac{dx}{dt} = \frac{dW}{dx} V_{ang.} = \frac{dW}{dx} \omega_{wh} r$$  \hspace{1cm} (29)

Here, the $x$ coordinate lies on the OX axis (Fig. 4); $\frac{dx}{dt} = V_{ang.} = \omega_{wh} r$ – since the change in the normal deformation $dW/dt$ as the treadmill element moves deeper into the contact occurs at a speed equal to the peripheral speed of the wheel.

In view of (27):

$$\frac{dW}{dt} = -x \left( \frac{1}{r} + \frac{1}{r_d} \right) \omega_{wh} r.$$  \hspace{1cm} (30)

As a result:

$$P_H = \frac{3}{16} \beta_H F_n a \omega_{wh} r \left( \frac{1}{r} + \frac{1}{r_d} \right).$$  \hspace{1cm} (31)

With the relationship found for $P_H$, the moment of hysteresis in the tire material can be represented as:

$$M_H = \frac{P_H}{\omega_{wh}} = \frac{3}{16} \beta_H a F_n \left( 1 + \frac{r}{r_d} \right).$$  \hspace{1cm} (32)
Then the arm shift of the normal reaction of the drum will be equal to:

$$h_0 = \frac{M_H}{F_n} = \frac{3}{16} \beta_H a \left( 1 + \frac{r}{r_d} \right).$$  \hspace{1cm} (33)

According to [4, 5]:

$$3\beta_H a_{\text{area}} / 16 = f_0 r_{\text{wh}}^f \equiv f_0 r$$

then

$$h_0 = f_0 r \left( 1 + \frac{r}{r_d} \right) \frac{a}{a_{\text{area}}}. \hspace{1cm} (34)$$

Here, $a_{\text{area}}$ is the half-length of the contact of the wheel with a flat rigid supporting surface at the same load $F_n$.

Knowing the arm $h_0$, we can find the relationship for the tangential force (it is the rolling resistance of the driven wheel), due to hysteresis:

$$F_{\tau_0} = F_n f_0 \left( 1 + \frac{r}{r_d} \right) \frac{a}{a_{\text{area}}}. \hspace{1cm} (35)$$

Since the ratio of rolling resistance to normal force $F_{\tau_0}/F_n = f_0^d$ is the coefficient of rolling resistance of the wheel, for the case of rolling of an elastic wheel against a rigid drum in this case:

$$f_0^d = f_0 \left( \frac{r}{r_d} + 1 \right) \frac{a}{a_{\text{area}}}. \hspace{1cm} (36)$$

For $r_d \rightarrow \infty$, relationships (32), (35), (36) lead to the expressions derived for the case of rolling of an elastic wheel along a flat rigid supporting surface. Comparison of these expressions with the above relationships leads to the conclusion that both the moment of hysteresis and the strength and coefficient of rolling resistance of the driven elastic wheel against the rigid drum, caused by hysteresis, increase in $a(1+r/r_d)/a_{\text{area}}$ times compared to rolling the same wheel on a flat rigid surface.

An increase in rolling resistance against the drum leads to a difference in the drag coefficients determined on the drum and when the wheel moves along a flat supporting surface. When using the coefficients of resistance to lateral withdrawal and rolling resistance, obtained experimentally on a drum stand, for the case of movement of the wheel on a flat supporting surface, appropriate correction factors should be introduced.

The interaction of an elastic wheel with a rigid supporting surface is described in more detail in the references cited at the end of this article [1–24].

3. Conclusion

1. Relationships that make it possible to calculate the power and kinematic parameters of a wheel while rolling against one or two rigid drums are obtained.

2. The coefficient of rolling resistance of the driven wheel against a rigid drum increases in $a(1+r/r_d)/a_{\text{area}}$ times compared to rolling on a flat rigid surface.

3. An increase in rolling resistance against a drum leads to a difference in the drag coefficients determined on the drum and when the wheel moves along a flat supporting surface.

4. When using the coefficients of resistance to lateral withdrawal and rolling resistance, obtained experimentally on a drum stand, for the case of movement of the wheel on a flat supporting surface, appropriate correction factors should be introduced.

References

[1] Virabov R V 1982 Traction Properties of Friction Gears (Moscow: Engineering)

[2] Virabov R V and Mamaev A N 1980 Analysis of Kinematic and Power Relationships When Rolling an Elastic Wheel on a Rigid Base Mechanics of machines 57 101–6
[3] Virabov R V and Mamaev A N 1978 Determination of Power Losses for Friction in the Contact of a Friction Pair – A Wheel with a Pneumatic Tire-Rigid Base *Infinitely variable transmission: interuniversity* 3 61–7

[4] Virabov R V, Mamaev A N and Balabina T A 2010 General Issues of the Interaction of an Elastic Wheel with a Rigid Supporting Surface AAI “Automobile and Tractor Engineering”: *Abstracts of the International Scientific and Practical Conference* (Moscow) pp 73–80

[5] Balabina T A and Mamaev A N 2014 The Rolling Mechanics of an Elastic Wheel on a Rigid Supporting Surface *Compilation: Engineering: trends, prospects and development technologies* pp 20–25

[6] Fromm H 1927 Berechnung des Schlupfes beim Rollendeformierbaren Scheiben *Math. Und mech. Bd.* 7

[7] Fromm N 1928 Arbeitsverlust, Formanderungen und Schlupf beim Rollen von treibenden und gebremsten Radern oder Scheiben. Beitrag zur Analyse der Reibungsgesetze *Technische Physik* 9

[8] Mamaev A N, Virabov R V, Portuguese V M and Chepurnoy S I 2010 Determination of Power and Kinematic Characteristics of an Elastic Wheel when Rolling against a Rigid Drum *Sat Proceedings of the international scientific conference dedicated to the 145th anniversary of MSTU MAMI* (Moscow: MSTU MAMI)

[9] Abuzov V I and Mamaev A N 2012 Rolling of an Elastic Wheel on Two Rigid Drums *Autom. prom.* 10 p 19

[10] Mamaev A N 2010 Resistance to Rolling of a Driven Wheel on a Rigid Drum *Sat Proceedings of the international scientific conference dedicated to the 145th anniversary of MSTU MAMI* (Moscow: MSTU MAMI)

[11] Virabov R V and Mamaev A N 1983 The Effect of Toroidal Elastic Wheels on Uneven Wear across the Width of the Treadmill *Proceedings of universities. Engineering* 9 94–7

[12] Virabov R V and Mamaev A N 1987 To the Question of the Smallest Value of the Coefficient of Rolling Resistance of an Elastic Wheel on a Rigid Horizontal Surface *Proceedings of the universities. Engineering* 10 85–8

[13] Mamaev A N 1982 Determination of the Tangential Elasticity Coefficient of a Wheel with a Toroidal Treadmill Shape *Izvestiya Vuzov. Engineering* 10 80–6

[14] Mamaev A N, Vukolova G S and Dmitrieva L N 1999 The Influence of the Form of the Adopted Law of the Distribution of Normal Pressures in Contact of the Wheel with a Rigid Base on the Calculated *Collection of scientific papers dedicated to the 60th anniversary of the reconstruction of MAMI* (Moscow: Publishing. MSTU MAMI) pp 112–8

[15] Virabov R V, Mamaev A N, Marinkin A P and Yuryev Yu M 1986 The Influence of the Rolling Mode of the Elastic Wheel on the Value of Lateral Force during Lateral Withdrawal *Bulletin of mechanical engineering* 1 33–5

[16] Virabov R V and Mamaev A N 1980 Analysis of Power Relations when Rolling a Driven Elastic Cylindrical Wheel along a Curved *Path Mechanics of machines* 57 105–12

[17] Virabov R V and Mamaev A N 1980 Investigation of Contact Phenomena during Curvilinear Rolling of a Toroidal Wheel *Proceedings of the universities. Engineering* 2 33–8

[18] Virabov R V and Mamaev A N 1980 Determination of Forces and Moments Acting on a Toroidal Wheel during Curvilinear Rolling *Proceedings of the universities. Engineering* 3 30–4

[19] Mamaev A N, Sazanov I V and Nazarov Yu P 1990 Determination of the Power Characteristics of an Elastic Wheel during Rolling by Withdrawal along a Curved Path *Problems of tires and rubber-cord materials. Strength and Durability: Materials of the II All-Union Symposium* (Moscow: NIISHP) 50–6

[20] Mamaev A N 1999 The Influence of Tire Operating Conditions on the Wear of Their Treadmill *Truck* 9 12–4

[21] Mamaev A N 1982 The Influence of the Design Parameters of Elastic Wheels on the Magnitude of their Deflection and the Dimensions of the Contact Area with a Rigid Base *Safety and..."
reliability of the car: Interuniversity. Sat scientific works 203–11

[22] Mamaev A N and Alepin E A 1980 Determination of the Dimensions of the Contact Area and Deflection of the Wheel with a Rubber Tire with Static Pressing of the Wheel to a Rigid Base Machine Science: Sat scientific works 251 82–5

[23] Mamaev A N 1982 On the Determination of the Hysteresis Loss Coefficient of Highly Elastic Rolling Elements NIINavtoprom 779

[24] Balabina T A, Mamaev A N and Chepurnoy S I 2013 Determination of the Ratio of Camber and Toe Angles of Elastic Wheels, Providing the Least Rolling Resistance News of MSTU "MAMI" 1(15)