Minimal Angular Size of Distant Sources in Open, ΛCDM, and Scalar Field Cosmologies

J. A. S. Lima\textsuperscript{a} and J. S. Alcaniz\textsuperscript{b}

Departamento de Física Teórica e Experimental
Universidade Federal do Rio Grande do Norte
59072-970 Natal - RN - Brazil

Abstract

We propose a simple method for determining the redshift $z_m$ at which the angular size of an extragalactic source with fixed proper diameter takes its minimal value. A closed analytical expression, which is quite convenient for numerical evaluation is derived. The method is exemplified with the following FRW type expanding universes: the open matter dominated models ($\Omega_\Lambda = 0$), a critical density model with cosmological constant ($\Omega_\Lambda \neq 0$), and the class of scalar field cosmologies proposed by Ratra and Peebles. The influence of systematic evolutionary effects is briefly discussed.

\textsuperscript{a}e-mail:limajas@dfte.ufrn.br
\textsuperscript{b}e-mail: alcaniz@dfte.ufrn.br
1 Introduction

The angular size - redshift relation, $\Theta(z)$, is a kinematic test which potentially may discriminate the several cosmological models proposed in the literature. As widely known, because of the spacetime curvature, the expanding universe acts gravitationally as a lens of large focal length. Though nearby objects are not affected, a fixed angular size of an extragalactic source is initially seen decreasing up to a minimal value, say, at a critical redshift ($z_m$), after which increasing for higher redshifts. The precise determination of $z_m$, or equivalently, the corresponding minimal angular size value $\Theta(z_m)$, may constitute a powerful tool in the search for deciding which are the more realistic world models. This lensing effect was first predicted by Hoyle, originally aiming to distinguish between the steady-state and Einstein-de Sitter cosmologies [1]. Later on, the accumulated evidences against the steady state (mainly from CMBR) have put it aside, and more recently, the same is occurring with the theoretically favoured critical density FRW model [2, 3, 4, 5, 6].

The data concerning the angular size - redshift relation are until nowadays somewhat controversial, specially because they involve at least two kinds of observational difficulties. First, any large redshift object may have a wide range of proper sizes, and, second, evolutionary and selection effects probably are not negligible. The $\Theta(z)$ relation for some extended sources samples seems to be quite incompatible with the predictions of the standard FRW model when the latter effects are not taken into account [7, 8, 9]. There have also been some claims that the best fit model for the observed distribution of high redshifts extended objects is provided by the standard Einstein-de Sitter universe ($q_0 = \frac{1}{2}, \Omega_\Lambda = 0$) with no significant evolution [10]. Parenthetically, these results are in contradiction with recent observations from type Ia supernovae, which seems to ruled out world models filled only by baryonic matter, and more generally, any model with positive deceleration parameter [3, 4]. The same happens with the corresponding bounds using the ages of old high redshift galaxies [5, 11, 12].

The case for compact radio sources is also of great interest. These objects are apparently less sensitive to evolutionary effects since they are short-lived ($\sim 10^3 yr$) and much smaller than their host galaxy. Initially, the data from a sample of 82 objects gave remarkable support for the Einstein-de Sitter Universe [13]. However, some analysis suggest that Kellerman has not really detected a significant increasing beyond the minimum [14, 15, 16]. Some
authors have also argued that models where $\Theta(z)$ diminishes and after a given $z$ remains constant may also provide a good fit to Kellerman’s data. In particular, by analysing a subset of 59 compact sources within the same sample, Dabrowski et al. (1995) found that no useful bounds on the value of the deceleration parameter $q_o$ can be derived. Further, even considering that Euclidean angular sizes ($\Theta \sim z^{-1}$) are excluded at 99% confidence level, and that the data are consistent with $q_o = 1/2$, they apparently do not rule out extreme values of the deceleration parameter as $q_o \sim 5$ [15]. More recently, based in a more complete sample of data, which include the ones originally obtained by Kellermann, it was argued that the $\Theta(z)$ relation may be consistent with any model of the FRW class with deceleration parameter $\leq 0.5$ [17].

In this context, we discuss here how the critical redshift giving the turn-up in angular sizes is determined for any expanding cosmology based on the FRW geometry. An analytical expression quite convenient for numerical evaluation is derived. The approach is exemplified for three different models of current cosmological interest: (i) open matter dominated FRW universe (OCDM), (ii) flat FRW type models with cosmological constant (ΛCDM), (iii) the class of scalar field cosmologies (SF) proposed by Ratra and Peebles [18]. Hopefully, the results derived here may be useful near future, when more accurate data become available.

2 The method

Let us now consider the FRW line element ($c = 1$)

$$ds^2 = dt^2 - R^2(t)[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)] , \quad (1)$$

where $\chi, \theta,$ and $\phi$ are dimensionless comoving coordinates, $R(t)$ is the scale factor, and $S_k(\chi)$ depends on the curvature parameter ($k = 0, \pm 1$). The later function is defined by one of the following forms: $S_k(\chi) = \sinh(\chi), \chi, \sin\chi$, respectively, for open, flat and closed Universes.

In this background, the angular size-redshift relation for a rod of intrinsic length $D$ is easily obtained by integrating the spatial part of the above expression for $\chi$ and $\phi$ fixed. One finds

$$\theta(z) = \frac{D(1 + z)}{R_0 S_k(\chi)} . \quad (2)$$
The dimensionless coordinate $\chi$ is given by

$$\chi(z) = \frac{1}{H_o R_o} \int_{(1+z)^{-1}}^{1} \frac{dx}{x E(x)} ,$$  \hspace{1cm} (3)$$

where $x = \frac{R(t)}{R_o} = (1 + z)^{-1}$ is a convenient integration variable. For the three kinds of cosmological models considered here (OCDM, $\Lambda$CDM and SF) the dimensionless function $E(x)$ assume one of the following forms:

$$E_{FRW}(x) = \left[1 - \Omega_M + \Omega_M x^{-1}\right]^{1/2} ,$$  \hspace{1cm} (4)$$

$$E_{\Lambda}(x) = \left[(1 - \Omega_{\Lambda}) x^{-1} + \Omega_{\Lambda} x^2\right]^{1/2} ,$$  \hspace{1cm} (5)$$

$$E_{SF}(x) = \left[(1 - \Omega_{\phi}) x^{-1} + \Omega_{\phi} x^{4+2\alpha}\right]^{1/2} ,$$  \hspace{1cm} (6)$$

where $\Omega_M = \frac{8\pi G \rho_M}{3H_o^2}$, $\Omega_{\Lambda} = \frac{\Lambda}{3H_o^2}$ and $\Omega_{\phi} = \frac{8\pi G \rho_{\phi}}{3H_o^2}$, are the present day density parameters associated with the matter component, cosmological constant and the scalar field $\phi$, respectively. Notice that equations (5) and (6) become identical if one takes $\alpha = 0$ in the later, thereby showing that the scalar field model proposed by Ratra and Peebles may kinematically be equivalent to a flat $\Lambda$CDM cosmology.

The redshift $z_m$ at which the angular size takes the minimum value is the one cancelling out the derivative of $\Theta$ with respect to $z$. Hence, from (2) we have the condition

$$S_k(\chi_m) = (1 + z_m) S'_k(\chi_m) ,$$  \hspace{1cm} (7)$$

where $S'_k(\chi) = \frac{\partial S_k}{\partial \chi} \frac{\partial \chi}{\partial z}$, a prime denotes differentiation with respect to $z$ and by definition $\chi_m = \chi(z_m)$. To proceed further, observe that (3) can readily be differentiated yielding, respectively, for the standard FRW (matter dominated), $\Lambda$CDM and scalar field cosmologies

$$(1 + z_m) \chi'_m = \frac{(R_o H_o)^{-1}}{[1 - \Omega_M + \Omega_M (1 + z_m)]^{2}} = (R_o H_o)^{-1} F(\Omega_M, z_m) \hspace{1cm} (8)$$

$$(1 + z_m) \chi'_m = \frac{(R_o H_o)^{-1}}{[(1 - \Omega_{\Lambda})(1 + z_m) + \Omega_{\Lambda} (1 + z_m)^{-2}]^{2}} = (R_o H_o)^{-1} L(\Omega_{\Lambda}, z_m) \hspace{1cm} (9)$$
\[(1 + z_m)\chi'_m = \frac{(R_o H_o)^{-1}}{\left[ (1 - \Omega_\phi)(1 + z_m) + \Omega_\phi(1 + z_m)^{\alpha^{-1}} \right]^{\frac{1}{2}}} = (R_o H_o)^{-1} S(\Omega_\phi, \alpha, z_m). \tag{10} \]

Now, inserting the above equations into (7) we find for the cases above considered

\[ \frac{1}{(1 - \Omega_M)^{\frac{1}{2}}} \tanh \left[ (1 - \Omega_M)^{\frac{1}{2}} \int_{(1+z_m)^{-1}}^{1} \frac{dx}{x E_{FRW}(x)} \right] = F(\Omega_M, z_m), \tag{11} \]

\[ \int_{(1+z_m)^{-1}}^{1} \frac{dx}{x E_{\Lambda}(x)} = L(\Omega_\Lambda, z_m), \tag{12} \]

\[ \int_{(1+z_m)^{-1}}^{1} \frac{dx}{x E_{SF}(x)} = S(\Omega_\phi, \alpha, z_m). \tag{13} \]

The meaning of equations (11)-(13) is self evident. Each one represents an integro-algebraic equation for the critical redshift \( z_m \) as a function of the physically meaningful parameters of the models. In general, these equations cannot be solved in closed analytical form for \( z_m \). However, as one may check, if we take the limit \( \Omega_M \to 1 \) in (11), the value \( z_m = \frac{5}{4} \) is readily achieved, which corresponds to the well known standard result for the dust filled FRW flat universe. The interesting point is that expressions (11)-(13) are quite convenient for numerical evaluations. As a matter of fact, their solutions can straightforwardly be obtained, for instance, by programming the integrations using simple numerical recipes in FORTRAN.

In Fig. 1 we show the diagrams of \( z_m \) as a function of the density parameter for each kind of model. As expected, in the standard FRW model, the critical redshift starts at \( z_m = 1.25 \) when \( \Omega_M \) goes to unity. This value is pushed to the right direction, that is, it is displaced to higher redshifts as the \( \Omega_M \) parameter is decreased. For instance, for \( \Omega_M = 0.5 \) and \( \Omega_M = 0.2 \), we find \( z_m = 1.58 \) and \( z_m = 2.20 \), respectively. In the limiting case, \( \Omega_M \to 0 \), there is no minimum at all since \( z_m \to \infty \). This means that the angular size decreases monotonically as a function of the redshift. For the scalar field case, one needs to fix the value of \( \alpha \) in order to have a bidimensional plot. Given a value of \( \Omega_\phi \), the minimum is also displaced for higher redshifts when the \( \alpha \) parameter diminishes. Conversely, for a fixed value of \( \alpha \), the minimum moves for lower redshifts when \( \Omega_\phi \) is decreased. The limiting case \( \alpha = 0 \) is fully equivalent to a \( \Lambda \)CDM model. As happens in the limiting case \( \Omega_M \to 0 \),
\begin{table}[h]
\centering
\begin{tabular}{cccccc}
\hline
$\Omega_m (z_m)$ & $\Omega_\Lambda (z_m)$ & $\Omega_\phi(\alpha = 2) (z_m)$ & $\Omega_\phi(\alpha = 4) (z_m)$ & $\Omega_\phi(\alpha = 6) (z_m)$ \\
\hline
1.0 (1.25) & 1.0 ($\infty$) & 1.0 (2.16) & 1.0 (1.72) & 1.0 (1.57) \\
0.8 (1.35) & 0.8 (1.76) & 0.8 (1.65) & 0.8 (1.53) & 0.8 (1.46) \\
0.7 (1.41) & 0.7 (1.60) & 0.7 (1.55) & 0.7 (1.47) & 0.7 (1.42) \\
0.5 (1.58) & 0.5 (1.44) & 0.5 (1.42) & 0.5 (1.38) & 0.5 (1.36) \\
0.2 (2.20) & 0.2 (1.31) & 0.2 (1.30) & 0.2 (1.30) & 0.2 (1.29) \\
\hline
\end{tabular}
\caption{Critical redshift $Z_m$ in OCDM, $\Lambda$CDM, and scalar field cosmologies for some selected values of the density parameters.}
\end{table}

(\Omega_\Lambda = 0), the minimal value for $\Theta(z)$ disappears when the cosmological constant contributes all the energy density of the Universe, that is, $z_m \to \infty$ if $\Omega_M \to 0$ and $\Omega_\Lambda \to 1$ (in this connection see also [20]). For the class of models considered in this paper, the redshifts having the minimal angular size are displayed for several values of $\Omega_M$ and $\alpha$ in Table 1. As can be seen there, the critical redshift at which the angular size is a minimal cannot alone discriminate between world models since different scenarios may provide the same $z_m$ value. However, when combined with other tests, some interesting constraints on the cosmological models can be obtained. For example, when $\Omega_\phi$ is bigger than 0.55, the model proposed by Ratra and Peebles yields a $z_m$ between the standard FRW flat model and the $\Lambda$CDM cosmology. Then, supposing that the universe is really accelerating today ($q_o < 0$), as indicated recently by measurements using type Ia supernovae [3, 4], and by considering the results by Gurvits et al. [7], i.e., that the data are compatible with $q_o \leq 0.5$, the Ratra and Peebles models with $0 < \alpha \leq 4$ seems to be more in accordance with the angular size data for compact radio sources than the $\Lambda$CDM model.

5
Figure 1: Critical redshift $Z_m$ as a function of the density parameter in open, ΛCDM and scalar field cosmologies. Solid curve is the prediction for a model with nonnull cosmological constant. The same curve is also obtained as a limiting case ($\alpha \to 0$) of the scalar field cosmology proposed by Ratra and Peebles.

It is worth notice that the same procedure may be applied when evolutionary and/or selection effects due to a linear size-redshift or to a linear size-luminosity dependence are taken into account. As widely believed, a plausible way of standing for such effects is to consider that the intrinsic linear size has a similar dependence on the redshift as the coordinate dependence, i.e., $D = D_o(1 + z)^c$, being $c < 0$ (see, for instance, [10] and Refs. therein). In this case, equations (11)-(13) are still valid but the functions $F(\Omega_M, z_m)$, $L(\Omega_\Lambda, z_m)$, and $S(\Omega_\phi, \alpha, z_m)$ must be divided by a factor $(1 + c)$. The displacement of $z_m$ relative to the case with no evolution ($c = 0$) due to the effects cited above may be unexpectedly large. For example, if one takes $c = -0.8$ as found by Buchalter et al. [10], the redshift of the minimum angular size for the Einstein-de Sitter case ($\Omega_M = 1$) moves from $z_m = 1.25$ to $z_m = 11.25$. In particular, this explains why the data of Gurvits et al.
Although apparently in agreement with the Einstein-de Sitter universe, do not show clear evidence for a minimal angular size close to $z = 1.25$, as should be expected for this model.

Acknowledgments: This work was partially supported by the project Pronex/FINEP (No. 41.96.0908.00) and Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq (Brazilian Research Agency).

References

[1] F. Hoyle, in Radio Astronomy, IAU Symp. No.9, ed. R. N. Bracewell, p. 529, Stanford Univ. Press. (1959).
[2] L.M. Krauss and M.S. Turner, Gen. Rel. Grav., 27, 1137 (1995).
[3] Riess et al., AJ, 117, 707 (1999)
[4] S. Perlmutter et al., Nature, 391, 51 (1998)
[5] J.S. Alcaniz and J.A.S. Lima, ApJ Lett. in Press [astro-ph/9902298]
[6] M. Roos and S. M. Harun-or-Raschid, [astro-ph/9901234]
[7] A.R. Sandage, ARA&A, 26, 56 (1988).
[8] V.K. Kapahi, In Observational Cosmology, IAU Symposium eds. A. Hewitt, G. Burbidge, and L. Z. Fang, p.251, Dordrecht: Reidel (1987).
[9] V.K. Kapahi, AJ, 97, 1 (1989)
[10] A. Buchalter, D.J. Helfand, R.H. Becker and R.L. White, ApJ, 494, 503 (1998).
[11] J. Dunlop et al., Nature, 381, 581 (1996)
[12] L.M. Krauss, ApJ, 480, 486 (1997)
[13] K.I. Kellermann, Nature, 361, 134 (1993).
[14] Y. Dabrowski, A. Lasenby, R. Saunders, MNRAS, 277, 753 (1995).
[15] P.G. Stephanas and P. Saha, MNRAS, 272, L13 (1995).
[16] P. Cooles and G.F.R. Ellis, “Is the Universe Open or Closed?”, Cambridge Lecture Notes in Physics, Cambridge UP (1997).

[17] L.I. Gurvits, K.I. Kellermann, and S. Frey, astro-ph/9812018.

[18] B. Ratra and P.J.E. Peebles, Phys. Rev. D37, 3406 (1988).

[19] I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, London (1980).

[20] L.M. Krauss and D.N. Schramm, ApJ, 405, L43 (1993).