Stringent Dilepton Bounds on Left-Right Models using LHC data

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In canonical left-right symmetric models the lower mass bounds on the charged gauge bosons are in the ballpark of 3 \(-\) 4 TeV, resulting into much stronger limits on the neutral gauge boson \(Z_R\), making its production unreachable at the LHC. However, if one evokes different patterns of left-right symmetry breaking the \(Z_R\) might be lighter than the \(W_R^\pm\) motivating an independent \(Z_R\) collider study. In this work, we use the 8 TeV ATLAS 20.3 fb\(^{-1}\) luminosity data to derive robust bounds on the \(Z_R\) mass using dilepton data. We find strong lower bounds on the \(Z_R\) mass for different right-handed gauge couplings, excluding \(Z_R\) masses up to \(\sim 3.2\) TeV. For the canonical LR model we place a lower mass bound of \(\sim 2.5\) TeV. Our findings are almost independent of the right-handed neutrino masses \((\sim 2\%\) effect) and applicable to general left-right models.

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I. INTRODUCTION

Left-Right (LR) symmetric models are popular extensions of the Standard Model (SM) and are based on the gauge group \(SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}\). They were originally motivated to explain the origin of parity violation\(^2\) of weak interactions and found to be related to the generation of light neutrino masses via the seesaw mechanisms\(^3\)\(^-\)\(^4\), linking in fact the smallness of neutrino mass with parity violation. The LR symmetry may be interpreted as the first step of an eventual unification of gauge forces as well.

While aesthetically very appealing, the theories do not predict the scale of parity restoration, leaving this question open to experiment. What is generic in LR symmetric models is the presence of right-handed currents and of gauge bosons \(W_R^\pm\) and \(Z_R\) associated with the additional \(SU(2)\) gauge group. The search for those generic features has been however unsuccessful so far. Recently, several studies have been made in TeV scale LR symmetric models exploiting meson\(^5\)\(^-\)\(^6\), collider\(^7\)\(^-\)\(^10\), flavor\(^11\)\(^-\)\(^13\) and neutrinoless double beta decay data\(^12\)\(^-\)\(^18\).

What these analyses have in common is that (i) the masses of the charged bosons \(W_R^\pm\) are smaller than that of the neutral one \(Z_R\), and that (ii) the gauge couplings of the left- and right-handed interactions \((g_L\) and \(g_R)\) are identical. This implies in particular that the effects of the \(W_R^\pm\) are the ones that matter in testing the models and in determining the scale of LR symmetry. Indeed, models typically advocate the presence of triplet and bidoublet scalars to generate the fermion and gauge boson masses. In this case both \(g_L = g_R\) and \(M_{W_R} > M_{Z_R}\) result, and the scenario has been widely explored with TeV scale breaking of \(SU(2)_R \otimes U(1)_{B-L}\) down to \(U(1)_Y\) resulting in charged and neutral gauge bosons with masses around the TeV scale. Within this symmetry breaking pattern \(W_R\) collider searches have been applied because they provide stronger constraints. For instance, CMS imposes \(M_{W_R} > M_{N_R}\) \((N_R\) being the right-handed neutrinos), which translates into \(M_{Z_R'} \sim 5.1\) TeV using the mass relation \(M_{Z_R} \approx 1.7M_{W_R}\) that holds in canonical LR models. There are also important limits stemming from electroweak data \((M_{Z_R} > 1\) TeV\), and \(K-K\) oscillations \((M_{W_R} > 4\) TeV for \(P\)-parity as the discrete LR symmetry\), which results into \(M_{Z_R} > 6.8\) TeV\(^6\).

In this respect, one can compare the limits with \(Z'\) constraints of theories in which only \(B-L\) is gauged. LEP2 imposes \(M_{Z'} \sim 6 \times g_{BL}\) TeV, where \(g_{BL}\) is the gauge coupling\(^20\). However, this limit on the \(B-L\) coupling cannot be easily applied to LR models, even though there is a gauge coupling relation \(1/g_Y^2 = 1/g_{BL}^2 + 1/g_{R}^2\). In \(U(1)_{B-L}\) theories \(g_{BL}\) solely determines the \(Z'\)-fermion couplings which are purely vectorial. In LR models, on the other hand, there are vector and axial couplings and other constant factors related to the Weinberg angle which suppress the coupling to leptons. Therefore, those extra factors should be taken into account. Indeed, the overall couplings to leptons are dwindled along with the LEP2 bound stemming from \(B-L\) theories, as one can see in Ref.\(^21\). There, the authors found a lower mass limit of 667 GeV on the \(Z'\) for \(g_R = g_L\).

We stress here the both features mentioned above, \(g_L = g_R\) and \(M_{W_R} > M_{Z_R}\), are not guaranteed in general, and the experimental searches for LR symmetry should not be limited to those assumptions. In particular one might evoke different symmetry breaking patterns yielding \(M_{Z_R} < M_{W_R}\). Hence from a general perspective, it is crucial to carry out an independent \(Z_R\)-collider study, since in this mass regime \(Z_R\) collider searches become the most effective way of constraining LR models especially when \(W_R\) is sufficiently heavy, out of reach of current experiments. Without losing generality, we use in this paper ATLAS data at 8 TeV and 20.3 fb\(^{-1}\) luminosity, to set limits on the \(Z_R\) mass of LR models using dilepton data using MadGraph5,Pythia and Delphes3, as dilepton resonance searches are the most efficient method to bound neutral...
gauge bosons that have non-negligible couplings to leptons\textsuperscript{22,23}. We emphasize that our results are quite general because they rely only on the neutral current of the $Z_R$ gauge boson.

Additionally to those limits, we present as an explicit example a model based on a two step-breaking pattern which generates $M_{Z_R} < M_{W_R}$ in a consistent way, with predicted $g_R/g_L$ ratios by forcing unification at the GUT scale.

II. LEFT-RIGHT SYMMETRIC MODEL

Left-right symmetric models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$. In addition, a discrete left-right symmetry is present implying equal values of gauge couplings for the $SU(2)_{L,R}$ gauge groups i.e. $g_L = g_R$. The quarks and leptons come as LR symmetric doublet representations $Q_{L,R} = (u,d)_L^T, (u,d)_R^T$ and $\ell_{L,R} = (\nu, e)_L^T, (\nu, e)_R^T$. In the conventional and most often studied left-right symmetric models $SU(2)_R \times U(1)_{B-L}$ is broken down to $U(1)_{Y}$ in one single step. In particular, the discrete left-right symmetry (denoted as parity or charge conjugation) and the $SU(2)_{R}$ gauge symmetry are broken at the same time and scale\textsuperscript{A}.

However, it should be noted here that the spontaneous symmetry breaking of $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_{Y}$ can be achieved either by Higgs triplets ($\Delta_L \oplus \Delta_R$ with even $B-L=2$) or Higgs doublets ($\chi_{L,R}$ with odd $B-L=-1$) or a combination of both Higgs doublets and triplets. With the simple Higgs sector comprising of a bidoublet plus $SU(2)_{L,R}$ triplets $\Delta_{L,R}$, the known formula between the right-handed charged and neutral gauge boson masses is given by

$$
\frac{M_{Z_R}}{M_{W_R}} = \frac{\sqrt{2} g_R / g_L}{\sqrt{(g_R / g_L)^2 - \tan^2 \theta_W}}.
$$

With $g_L \simeq g_R$, one can find that $M_{Z_R} = 1.7 M_{W_R}$. Thus, the existing experimental bounds on $M_{W_R}$ can be translated into more restrictive limits on $M_{Z_R}$. The aforementioned limits on the $W_R$ mass which are in the ballpark of several TeV make the $Z_R$ production unattainable at the LHC.

Albeit, in this work we consider different classes of LR models, where this mass relation does not apply. In particular, the $W_R$ mass is set to be at a scale much larger than TeV, whereas the $Z_R$ mass lies at the TeV scale. A possible way to conceive this setup is by introducing two triplet scalars $\Omega_{R,L}$ and $\Delta_{L,R}$, with $B-L = 0, -2$ respectively. With their inclusion the LR symmetry breaks down to the SM gauge group in two steps: (i) $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ breaks to $SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ at $W_R$ mass scale which is implemented through the vacuum expectation value (vev) of the heavier triplet carrying $B-L=0$, i.e. $\Omega_R$; (ii) then $U(1)_R \otimes U(1)_{B-L}$ breaks down to $U(1)_Y$ at the $Z_R$ mass scale defined by the vev of the $\delta_0$. Setting the vev of $\Omega_R$ to a very high energy scale, $W_R$ completely decouples from $Z_R$. Moreover, we need a bi-doublet ($\Phi$) to break $SU(2)_L \otimes U(1)_Y$ down to electromagnetism.

Assuming the lighter right-handed neutral gauge boson $Z_R$ acquires mass at the TeV scale, then $Z_R$ searches become the most promising ones to derive limits on the mass of this neutral boson. In the next section we discuss the $Z_R$ phenomenology and derive dilepton limits using recent ATLAS data.

Of course, ones does not have to rely on the precise symmetry breaking pattern we are proposing. The idea of having a light neutral gauge boson and a very heavy charged one is what our reasoning is mostly based on. A more detailed study of the above symmetry breaking pattern, plus analyses of the scalar potential, neutrino masses and other phenomenological consequences will be presented elsewhere\textsuperscript{20}. As far as dilepton bounds are concerned which are the focus of this work, the relevant interactions for us are the $Z_R$-fermions couplings given by

$$
\frac{g_R}{\sqrt{1 - \delta \tan^2 \theta_W}} f \gamma_\mu \left( g^L_V - g^L_A \gamma^5 \right) f Z_R^\mu,
$$

where the vector and axial couplings are defined as

$$
g^L_V = \frac{1}{2} \left\{ \left[ \frac{\delta \tan^2 \theta_W}{T_{3L} - Q^f} \right] + \left[ T_{3L}^{f'} - \delta \tan^2 \theta_W Q^f \right] \right\},
$$

$$
g^L_A = \frac{1}{2} \left\{ \left[ \frac{\delta \tan^2 \theta_W}{T_{3L}^{f'} - Q^f} \right] - \left[ T_{3L}^{f'} - \delta \tan^2 \theta_W Q^f \right] \right\},
$$

with $\delta = g^2_L / g^2_A$, the ratio of coupling strengths, $T_{3L,3R}$ being $1/2 (-1/2)$ for up (down) type fermions and $Q^f$ the respective electric charge. It is clear that the neutral current is general, and thus the coupling strengths. For this precise reason the limits that we are about to derive in the next section are applicable to a rather large class of LR models.

III. DILEPTON LIMITS

Dileptons and dijet searches have been proved to be the most effective as far as bounds on additional $Z'$ bosons are concerned unless they have negligible couplings to leptons or large branching ratios to missing energy such as dark matter\textsuperscript{27–30}. Hence, in the classes of LR symmetric models we

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{dilepton_diagram.png}
  \caption{Feynman diagram representing dilepton resonance searches at LHC.}
\end{figure}
are studying we need to consider those in order to set limits on the \(Z_R\). The neutral gauge bosons can be produced in the s-channel from \(q \bar{q}\) annihilation. To perform the search, the dilepton invariant mass line shape is studied for a localized excess of events corresponding to a new physics resonance.

To derive the dilepton limits on this specific model we simulate the process \(pp \to Z_R \to e^+e^-, \mu^+\mu^-\), plus up to two extra jets using MadGraph5 [31], and compare with the results from the ATLAS collaboration reported in Ref. [32], from where we also take the background events. For this reason we obtain the number of events in bins of the dilepton invariant mass \(M_{ll}\) as follows: \(110 - 200\) GeV, \(200 - 400\) GeV, \(400 - 800\) GeV, \(800 - 1200\) GeV, \(1200 - 3000\) GeV, \(3000 - 4500\) GeV. For the signal events we account for clustering and hadronizing jets as well as for soft and collinear QCD radiation with Pythia [33], and simulate detector efficiencies with Delphes3 [34]. In our results we used the CTEQ6L parton distribution functions computed at \(\mu_F = \mu_R = M_{Z_R}\) [35]. Following the procedure in Ref. [32], the signal events were selected by applying the cuts:

\[
\begin{align*}
    p_T(e_1) &> 40\text{ GeV}, p_T(e_2) > 30\text{ GeV}, |\eta_e| < 2.47, \\
    p_T(\mu_1) &> 25\text{ GeV}, p_T(\mu_2) > 25\text{ GeV}, |\eta_{\mu}| < 2.47, \\
    110\text{ GeV} < M_{ll} < 4.5\text{ TeV},
\end{align*}
\]

where \(l_1\) and \(l_2\) represent the hardest and next hardest lepton in the event, whereas \(M_{ll}\) is the invariant mass of the lepton pair. That being said, we simply compute the number of dilepton events for the signal and compare with the background events to derive 95\% C.L. limits on the \(Z_R\) mass.

The result is shown in Fig. 2 for \(g_R = 0.65\), 1, where the former value corresponds to the canonical LR model. For \(g_R = 0.65\) we exclude \(Z_R\) masses below \(2490\) (3250) GeV. We point out that our results are independent of the right-handed neutrino masses, as shown in the inner plot in Fig. 2 simply because the branching ratio into right-handed neutrinos is rather small. When right-handed neutrinos are kinematically available for the \(Z_R\) to decay into the limits on the \(Z_R\) change by approximately 2\%, which is basically unnoticeable in the inner graph of Fig. 2.

In order to account for several possible symmetry breaking schemes which may induce different \(g_R\) values, we show in Fig. 3 how our limits change as we vary \(g_R\). We stress that as \(g_R\) increases the \(Z_R\)-fermion couplings do not necessarily grow as one can see from Eq. (2), since there are additional \(1/g_R^2\) factors in the vector/axial couplings, explaining the shape of the figure, which is different from the one with \(W_R\) bounds discussed in Ref. [36]. From Fig. 3 we observe that dilepton data excludes \(Z_R\) masses below \(2760, 2209, 2314, 2643\) GeV for \(g_R = 0.4, 0.5, 0.6, 0.7\) respectively. So far those \(g_R\) values are simply random choices, but we stress that by embedding the symmetry breaking scheme in a \(SO(10)\) model, \(g_R\) can be predicted by enforcing gauge coupling unification as shown in the appendix for a particular example.

In summary, our limits are quite general because they rely simply on the \(Z_R\)-fermions couplings and thus are applicable to a multitude of LR models. Besides, they comprise the most stringent direct limits on the \(Z_R\) mass. We point out that the scale of the Left-Right symmetry breaking can always be pushed up to higher scales, in principle, to evade our bounds, i.e. assume heavier mediators. Concerning, collider projections, in order to determine the discovery potential at LHC 13 TeV for instance, one would have to know the fake jet rate and the dilepton efficiencies at LHC 13 TeV, which are unknown at this point. However, a rather speculative study could be done though. Since our limits lie in the ballpark of 2.5–3 TeV and heavy dilepton resonances are clean signals, it is clear that \(Z_R\) bosons with such masses will be either ruled out or observed at LHC 13 TeV.
IV. CONCLUSIONS

Canonical left-right symmetric models suffer from strong bounds on the charged gauge boson mass, which result in much stronger limits on the $Z_R$ mass due to the mass relation that holds in those models. If one explores different patterns of left-right symmetry breaking the $Z_R$ may be light enough to be produced at the LHC while the $W_R$ is way heavier, motivating an independent $Z_R$ collider study.

As proof of principle, we presented a symmetry breaking scheme which consistently generates the inverted mass hierarchy $M_Z > M_W$ with the $Z_R$ mass at the TeV scale. In the appendix, we show that through demanding gauge coupling unification and embedding the model in $SO(10)$ the value of the right-handed gauge coupling $g_R$ can be predicted, which for the example under study is in the ballpark of 0.4 for several $U(1)_{B-L}$ breaking scales. We note that models with very large $W_R$ masses have the advantage of suppressing the $W_L$-$W_R$ mixing, which generates dangerously large lepton flavor violating processes.

After showing that light $Z_R$ can be generated in LR models, we performed a collider analysis using the 8 TeV ATLAS 20.3 fb$^{-1}$ luminosity dilepton data to derive robust and stringent bounds on the $Z_R$ mass, due to the sizable $Z_R$-lepton couplings. For different $g_R/g_L$ ratios ranging from 0.4 up to 1.2 to effectively cover several different patterns of symmetry breaking, our limits in the $Z_R$ mass are given in Fig. 3. We emphasize that our results are general since they rely simply on the neutral current of the $Z_R$ gauge boson. In particular we exclude $Z_R$ masses up to $\sim 3.2$ TeV for $g_R = 1$. For $g_R = g_L$ (canonical LR model) we derive a lower mass bound of $\sim 2.5$ TeV, which is the most stringent direct limit in the literature of LR models on the $Z_R$ mass.

Our findings are almost independent of the right-handed neutrino masses due to their small branching ratio, and applicable to general left-right models. We stress that our bounds are the leading ones when $M_{Z_R} > M_{W_R}$, and complementary to the existing indirect ones stemming from $W_R$ searches in the setup $M_{W_R} < M_{Z_R}$. Either way, we provide the most stringent direct limits on the $Z_R$ mass of LR models.

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Appendix A: $SO(10)$ GUT embedding and determination of $\delta = g^2_L/g^2_R$

For the sake of completeness, let us shortly discuss a possible $SO(10)$ GUT embedding of the model from Section II which consistently predicts the $g_L/g_R$ ratio and $M_{Z_R} < M_{W_R}$. More details will be presented elsewhere [20]. We discuss here one exemplar symmetry breaking chain of $SO(10)$ GUT with the desire of having low $B - L$ breaking scale so that the extra neutral gauge boson $Z_R$ could be in the TeV range leading to interesting collider searches while decoupling the right-handed charged gauge bosons $W_R$.

The precise determination of $\delta = g^2_L/g^2_R$ (or $g_R$) depends upon the $SU(2)_R$ breaking scale and the choice of Higgs spectrum required for spontaneous symmetry breaking. Here we choose D-parity, which is broken spontaneously. Such LR models have been originally conceived in Refs. [24, 25] and recently in Refs. [16, 37–39]. We briefly point out here how the spontaneous D-Parity breaking scenario is different from usual LR model, and essentially decouples discrete and gauged left-right symmetries.

The spontaneous breaking of D-parity occurs at reasonably high energy scale along with $SU(2)_R \rightarrow U(1)_R$ breaking, simultaneously resulting in a mass of $W_R$ at high scale. Below this scale, the RG evolution of gauge couplings for $SU(2)_L$ and $SU(2)_R$ evolves differently guaranteeing the mismatch between $g_R \neq g_L$ at low energy. At a later stage, $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ breaking is achieved by $\Delta^0_R$ at the $M_{Z_R}$ scale.

As a result of spontaneous D-parity breaking mechanism, the RG evolution for both gauge couplings for $SU(2)_L$ and $SU(2)_R$ gauge group is different, resulting in different values for gauge couplings $g_L$ and $g_R$ from $M_U$ onwards up to $M_Z$ scale. In addition, the right-handed charged gauge bosons $W_R$ acquire mass around $\omega_R$ which we have chosen here to be greater than $10^{10}$ GeV making it inaccessible to high energy collider searches. We emphasize again that our bounds on the $Z_R$ mass are independent of this choice.

We fix the $B - L$ breaking scale in the range of $1 - 10$ TeV to keep the $Z_R$ mass around LHC scale. From the unification plot for gauge couplings shown in Fig. 4 the numerical values for the intermediate mass parameters and the mismatch between the two gauge couplings $g_L$ and $g_R$ are estimated to be

$$M_U = 10^{15.98} \text{ GeV}, \quad M_R = 10^{9} \text{ GeV}, \quad M_{B-L} = 5 \text{ TeV},$$

FIG. 4: Running of the coupling constants. By forcing grand unification at high scale we can predict the value of $g_R$ at the electroweak scale.
We have checked that the coupling ratio remains basically the same for $U(1)_{B-L}$ symmetry breaking scales from 1 TeV up to 100 TeV for the present analysis.

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