Analytical and regression models of glass rod drawing process

L B Alekseeva

St. Petersburg Mining University, 21 line, 2, St. Petersburg, 199106, Russia

E-mail: lbalek@rambler.ru

Abstract. The process of drawing glass rods (light guides) is being studied. The parameters of the process affecting the quality of the light guide have been determined. To solve the problem, mathematical models based on general equations of continuum mechanics are used. The conditions for the stable flow of the drawing process have been found, which are determined by the stability of the motion of the glass mass in the formation zone to small uncontrolled perturbations. The sensitivity of the formation zone to perturbations of the drawing speed and viscosity is estimated. Experimental models of the drawing process, based on the regression analysis methods, have been obtained. These models make it possible to customize a specific production process to obtain light guides of the required quality. They allow one to find the optimum combination of process parameters in the chosen area and to determine the required accuracy of maintaining them at a specified level.

1. Introduction

The structure of any fiber-optic element (FOE) includes single-core and multi-core rods (light guides) with certain geometric and optical characteristics. From the very beginning of the development of fiber optics, the dominant problems of optics were the stability of the diameter of original rods and the small light attenuation in them. While the second problem is being successfully solved, the problem of super-constancy of the light guide diameter still exists. To solve it, the constant control and monitoring of the production process of fabrication of light guides are required. The complexity of control-related tasks is due to the fact that the quality of the products obtained depends on multiple factors, which was taken into account at a high level of uncertainty. Online control of the process is implemented on the basis of monitoring the production situation by means of processing the data, including the results of instrument use. The volume and timeliness of the information obtained may be insufficient. Formation of optimum control of the production process is possible by increasing the level of the information and analytical support. In this case, the information is formed from matching the analysis of current measurements with the calculated values formed on the basis of the initial data and mathematical models. Therefore, the need to develop the automated control of the manufacture of light guides leads to the need to study the production process as an object of control, which makes it possible to identify the latent dynamics of the ongoing processes.

During development of automated production facilities, problematic issues have been revealed, which are hard to be solved empirically. These issues include: forecasting and optimizing the process parameters; stabilization of its characteristics; finding correlations between the input and output parameters of the process; optimum arrangement of the control systems; estimation of the level of interference which does not affect the quality of control.
Solution of arising problems and accordingly decision making when designing technical complexes, i.e. choosing control schemes, measuring tools, actuating devices, calculating settings of regulators, estimating quality of control systems in conditions of uncertainty, are possible on the basis of creation of mathematical models, adequately describing the production process and allowing one to identify the latent dynamics of ongoing processes.

2. A typical drawing plant

A typical drawing plant (Figure 1), built according to the traditional design, consists of frame 1, glass block guides 2, heating furnace 3, OZhS cooling rate control device 4, outer diameter sensor 5, shell thickness sensor 6, drawing mechanism 7, OZhS cutting device 8. Fiber parts with high resolution can only be obtained by using the light guides with identical geometry. Deviations of individual elements of geometry (cross sectional shapes, dimensions, etc.) lead to deterioration of the frequency-contrast characteristics of a component, the appearance of microstructural noise and various defects in the working area of the components. The degree of deviation of the light guide geometry in terms of certain parameters characterizes the quality of the light guides’ geometry.

The parameters of the component defining the possibility of its use in a certain device impose the requirements for the geometry of single light guides, since any fiber component is formed by multiple light guides. Proper laying of light guides in the finished component, defined by their geometry, creates favorable conditions for obtaining a specified micro-geometry in the finished component and, therefore, its performance characteristics.

In industry, in the manufacture of the vast majority of fiber-optic parts, a total inspection of the light guides is applied, followed by their selective assembly. It extremely complicates the production process and increases the cost of the finished component. The need for accurate manufacture of the original light guides has been especially sharply manifested in the fabrication of micro-channel systems based on the principle of channel electronic multipliers [1]. The number of channels of capillaries with the diameter of 10 µm in such a system reaches about $10^7$ pcs with the thickness of the partition between the channels of 1 ... 2 µm.

![Figure 1. Typical plant for mass](image1)

![Figure 2. A diagram of a one-dimensional flow of the glass light guide drawing](image2)

The conducted studies have shown that micro-channel systems shall consist of the channels manufactured with an accuracy of ±2%, which provides uniformity of amplification over the working
field of the plate not worse than ±5%. Such accuracy is a normal indicator for the contrast sensitivity of a human eye.

In view of this, a light guide with the diameter of 1 mm should have a diameter tolerance not exceeding ±20 µm. Multistaging of the redrawing process and the possibility of additional geometric distortions (elongation, flattening) during subsequent operations make it necessary to set the tolerance 2 ... 3 times stricter.

The physical state of the glass mass in the formation zone defines the conditions for the stable flow of the drawing process and magnitudes of “responses” to various disturbing and control impacts. Therefore, a mathematical description of the process of deformation of the glass mass is of importance. Multiple works have been devoted to the mechanics of deformation of viscoplastic media.

3. Analytical model of the control object

The process of formation of the light guide geometry takes place in the formation zone, which is a transition from the heated glass mass to the light guide.

To study the processes occurring in this zone, the mathematical models based on the continuum mechanics are proposed, including three groups of equations: the equations of equilibrium of forces, the continuity equations and the equations determining the physical state of the medium [2].

The formation zone is a body of rotation. Therefore, cylindrical coordinate system \( rz \) is used, where \( z \) is the axial coordinate; \( r \) is the radius of the circle in the section with coordinate \( z \). Two coordinates are sufficient since one-dimensional motion is considered (Fig. 2).

If one assumes that the velocity, viscosity, and stresses in the formation zone are functions of time and the axial coordinate, then the system of resolving equations is reduced to three equations (one-dimensional flow model).

Thus, for the one-dimensional model, there is a system of three equations:

\[
\frac{\partial S}{\partial t} + \frac{\partial (vS)}{\partial z} = 0; \\
\rho \left[ \frac{\partial (vS)}{\partial t} + \frac{\partial (v^2S)}{\partial z} \right] = \rho S g + \frac{\partial (pS)}{\partial z}; \\
p + \lambda \left( \frac{\partial p}{\partial t} + v \frac{\partial p}{\partial z} \right) = 3 \mu \frac{\partial v}{\partial z}. 
\]

where \( t \) is the time; \( \upsilon, p \) - the speed and the stress in the section of the formation zone being considered, respectively; \( S \) is the area of the section under consideration; \( \rho \) is the density of the glass mass, which is assumed to be constant; \( \mu \) is the dynamic viscosity of the glass mass; \( \lambda = \mu / G \) is the relaxation time; \( G \) is the modulus of elasticity of the glass mass.

4. Study of the stability of the motion of the glass mass in the formation zone

The motion is considered stable if, at small uncontrolled perturbations, the formation zone retains its original shape. The study is conducted using the direct Lyapunov method [3].

Stability is estimated using the equation of the first approximation, which in most cases gives the correct answer to the question of the stability of motion.

At the first step, let us study the stability of a viscous medium (the Newton model). When studying the stability, let us consider a non-stationary process.

Boundary conditions of the task are:

\[ S(0,t) = S_0; \upsilon(0,t) = 0; \upsilon(L,t) = \dot{L} = \upsilon_d, \]

where \( S_0 \) is the area of the initial section of the formation zone; \( \dot{L} \) is a certain time function, depending on drawing speed \( \upsilon_d \).

The results obtained make it possible to draw the qualitative conclusions. For example, in order to obtain a more precise geometry of the light guides, it is necessary that the glass mass in the formation
zone has the properties of a viscoelastic body. Such state can be achieved by lowering the temperature in the formation zone. It predetermines the choice of the method of manufacturing the light guides from a workpiece. When using the spinneret method, special conditions for the passage of the viscous glass mass through the spinnerets shall be created.

To solve the control tasks in the selected area of parameters, it is necessary to know the static and dynamic properties of the control object [4].

The static properties are determined by the sensitivity of the process to various kinds of perturbations, including the process parameters, in the steady state. Knowledge of these properties is the basis for choosing the measuring instruments and regulating and actuating devices. To develop a methodology for estimating this kind of sensitivity, it is necessary to study the steady motion of the glass mass. However, at the first step it is necessary to study the process of reaching a steady state, in which the configuration of the formation zone is stable. It will allow determining length \( L \) of the formation zone as one of the monitored parameters (Fig. 2). To obtain a solution, let us use a system of equations for a one-dimensional flow.

5. Regression model

The regression equation at the first step takes the form [5]:

\[
y = b_0 x_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2 ,
\]

where \( x_1, x_2 \) - normalized values of control factors; \( b_0 \) is the free term equal to the output when \( x = 0 \); \( b_1, b_2 \) - coefficients indicating the influence of factors \( x_1, x_2 \) on the process; \( b_{12} \) - a coefficient indicating the interaction of factors \( x_1, x_2 \); \( x_0 = 1 \) is a dummy variable.

The experiments have been conducted on a typical drawing plant consisting of a mechanism for feeding a workpiece into a furnace, an oven, and a drawing mechanism (Fig. 1). In the feed mechanism, a screw-and-nut gear is used, and in the drawing mechanism - a chain drive with two rows of chains to which the carriages with grips are attached. The glass of X 230 grade has been drawn. The diameter of the workpiece was 22 mm; the diameter of rod being drawn was 0.6 mm.

Control factors are: \( \tilde{x}_1 \) - drawing speed; \( \tilde{x}_2 \) - temperature in the workpiece heating zone. In the first series of experience, the following has been admitted: \( \tilde{x}_{1m} = 1.5 \) m/min; \( \Delta \tilde{x}_1 = 0.5 \) m/min; \( \tilde{x}_{2w} = 615^\circ \text{C}; \Delta \tilde{x}_2 = 10^\circ \text{C}. \)

The light guide being drawn has been cut into 800 mm lengths. In each mode, 12 lengths have been sampled, for which the maximum dispersion of the \( d \) diameter of the cross-section of light guide \( \left( y = \Delta d \right) \) has been measured.

As a result of the experiment, the following values of the coefficients have been obtained:
\[
\begin{align*}
b_0 &= 5.40; & b_1 &= -1.25; & b_2 &= 1.25; & b_{12} &= -1.41. 
\end{align*}
\]

Estimates of the regression equation coefficients have been found to be as follows: line-by-line dispersions \( (S_j^i) \) \( S_1^1 = 1.056, S_2^1 = 0.854, S_1^2 = 0.932, S_2^2 = 0.576 \); reproducibility dispersion \( S^i \left( \bar{y} \right) = 0.855; \) mean value dispersion \( S^i \left( \bar{\bar{y}} \right) = 0.071 \); standard error of coefficients in the regression equation \( S \left( b \right) = 0.133. \)

The value of the Student's coefficient with confidential probability \( P = 0.95 \) and the number of degrees of freedom equal to \( f = N(n-1) = 4(12-1) = 44 \) is \( t_p = 2.02 \).

Since \( |b_{on}| = |b_1| = 1.25 > 0.133 \cdot 2.02 = 0.269 \), all the coefficients in the regression equation are significant.

The average response value at zero values of the control factors is \( \bar{y}_0 = 4.75 \). Zero level dispersion is \( S^i \left( \bar{y}_0 \right) = 0.138. \)

Weighted average of two dispersion values is:
\[ \tilde{S}_2 = \frac{(N-1)S^2(b) + (n-1)S^2(\bar{y}_n)}{N + n - 2} = \frac{(4-1) \cdot 0.133^2 + (12-1) \cdot 1.38^2}{4 + 12 - 2} = 0.112. \]

Let us check the inequality:
\[ |\bar{y}_n - b_i| = |4.75 - 5.40| = 0.65 < \sqrt{\frac{\tilde{S}_2 (N + n)/(N/n)}{P}} = 3.3. \]

Therefore, the model obtained is adequate.

As a result, the following has been obtained: the dispersion of inadequacy is \( S^2_d = 6.25 \); the design value of the Fisher criterion for degrees of freedom \( f_1 = 1 \) and \( f_2 = 11 \) with the \( P = 0.99 \) level has been found as \( F(1.11) = 4.0. \) Since \( F_{calc} > F \), the linear model without taking into account the interaction of factors is inadequate. It has been proven that in the chosen range of change of control factors, the process model is described by the regression equation.

The obtained model has shown a significant influence of the process parameters on the interrelation response. Thus, with increase of the drawing speed and the decrease of the temperature in the heating zone (hence, with increase of viscosity), the deviation of the diameter of light guide \( \Delta d \) decreases. Such conclusion coincides with the results of the experiments of [6].

6. Normalization of control factors

On the basis of the model found, described by the selected and tested polynomial, it is possible to determine the required accuracy of maintaining the control factors at a specified level [7], [8]. This accuracy is defined by the allowable response values.

**Figure 3. Areas for determining the control factors**

Equation (2) corresponds to the three-dimensional factor space: the values of the factors are determined along two axes, and the response value is determined along the third axis. But in order to normalize the parameters, it is possible not to expand to three-dimensional space, but to be satisfied with a plane. To do it, it is necessary to dissect the response surface by planes, parallel to the \( x_1 0 x_2 \) plane and to project the resulting lines onto this plane. To obtain such projections, it is necessary to set
the \( y = \Delta d \) values in the regression equation and plot on plane \( x_0 \) graphs \( x_1 = x_1(x_2) \), which enable each of the factors to be normalized taking into account their interaction. Let us illustrate this possibility using the results of the second series of experiments. A light guide with a diameter of 2 mm was drawn out from the glass of F-4 grade; the diameter of the workpiece was 22 mm. It has been assumed that: \( \bar{x}_{\text{av}} = 2.5 \text{ m/min}; \Delta \bar{x}_1 = 1 \text{ m/min}; \bar{x}_2 = 930 \text{ C}; \Delta \bar{x}_2 = 15^\circ \text{ C} \). After processing the results of the experiments, it has been obtained (in \( \mu \text{m} \)): \( b_0 = 38; b_1 = -22; b_2 = 10; b_{12} = 30 \).

In Fig. 3, the projections of the response surface onto the \( x_0 \) plane are plotted. Curves 1, 2, 3, 4 correspond to \( \Delta d = 40, 30, 20, 10 \mu \text{m} \).

**7. Consideration of an example**

Let the maximum allowable deviation of the diameter of the light guide be \( \Delta d = 10 \mu \text{m} \). This deviation corresponds to curve 4 in Fig. 3. Let us choose the initial value of the control factor \( x_1^* = -0.5 \) in which the initial value of the control factor corresponds to \( x_1^* = 0.61 \). The response does not exceed 10 \( \mu \text{m} \), if the variation ranges of the factors are \( \Delta x_1^w \) and \( \Delta x_2^w \). That is, the variation limits of the factors are:

\[
0.61 \leq x_1 \leq 1; \quad -1 \leq x_2 \leq -0.5. 
\]

Or, taking the natural values, there is:

\[
3.11 \frac{\text{m}}{\text{min}} \leq \bar{x}_1 \leq 3.50 \frac{\text{m}}{\text{min}}; \quad 915 \text{ C} \leq \bar{x}_2 \leq 922.5 \text{ C}. 
\]

In Fig. 3, the area of the allowable variations of the factors is shaded. The experiment has shown that not only the values of the control factors, but also the tolerances of these factors are correlated. Indeed, with increasing \( \Delta x_1^w \), \( \Delta x_2^w \) decreases.

**Conclusion**

It is shown that the formation zone is extremely sensitive to perturbation of the drawing force. It is explained by the fact that the drawing force is a complex parameter associated with the light guide drawing speed and with the viscosity of the glass mass in the formation zone.

There are areas of the main process parameters, in which the drawing process can be carried out and the conditions for a stable process flow under uncontrolled external perturbations have been determined.

A mathematical model of the light guide formation zone has been proposed, which is built on the basis of general equations of continuum mechanics. It has been shown that the most favorable conditions for obtaining the light guides with stable geometry are formed when the glass mass in the formation zone has the properties of a visco-elastic body. It allows justifying the choice of manufacturing methods creating the required physical state of the glass mass.

Experimental models of the drawing process, based on the regression analysis methods, have been obtained. These models make it possible to customize a specific production process to obtain light guides of the required quality. They allow finding the optimal combination of the process parameters in the selected area of parameters and determining the required accuracy of maintaining them at a given level.

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