Experimental/clinical evaluation of EIT image reconstruction with $\ell_1$ data and image norms

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Abstract. Electrical impedance tomography (EIT) image reconstruction is ill-posed, and the spatial resolution of reconstructed images is low due to the diffuse propagation of current and limited number of independent measurements. Generally, image reconstruction is formulated using a regularized scheme in which $\ell_2$ norms are preferred for both the data misfit and image prior terms due to computational convenience which result in smooth solutions. However, recent work on a Primal Dual-Interior Point Method (PDIPM) framework showed its effectiveness in dealing with the minimization problem. $\ell_1$ norms on data and regularization terms in EIT image reconstruction address both problems of reconstruction with sharp edges and dealing with measurement errors. We aim for a clinical and experimental evaluation of the PDIPM method by selecting scenarios (human lung and dog breathing) with known electrode errors, which require a rigorous regularization and cause the failure of reconstructions with $\ell_2$ norm. Results demonstrate the applicability of PDIPM algorithms, especially $\ell_1$ data and regularization norms for clinical applications of EIT showing that $\ell_1$ solution is not only more robust to measurement errors in clinical setting, but also provides high contrast resolution on organ boundaries.

1. Introduction

Electrical impedance tomography (EIT) shows promise to help patient ventilation in an intensive care unit (ICU) by monitoring the distribution of inspired air in mechanically ventilated patients. However, EIT image reconstruction is an ill-posed inverse problem. Due to limited independent measurements, the spatial resolution of reconstructed images is low. The Jacobian (sensitivity) matrix is ill-conditioned, which implies that measurement noise or errors due to electrode detachment and patient movement cause solutions to be unstable with large image artifacts. In order to stabilize the problem, regularization techniques are commonly used by introducing forms of filtering to reduce these artifacts. Regularization introduces a priori information regarding smooth conductivity changes which limits reconstructing sharp images. However, sharp edges are physiologically realistic and are desired to image better inter-organ boundaries and step changes in the tissue conductivity.

Total Variation (TV) regularization (the use of $1$-norm on the regularization term) was demonstrated on simulation and clinical data [1], which allowed the reconstruction of sharp image transitions. The Primal Dual-Interior Point Method (PDIPM) was proposed by [2], where $\ell_1$ and $\ell_2$ norms were applied to data and regularization terms using simulated data only. The simulated results of the paper suggested that $\ell_1$ norm (Least-absolute values) on both data
and regularization terms improved image resolution and provided robustness to noise and data outliers (errors).

In this paper, we evaluated the PDIPM algorithms [2] on the experimental and clinical data by applying \( \ell_1 \) and \( \ell_2 \) norms for both the data-fidelity and regularization terms. Subsequently, we analyze and compare the algorithms’ management of measurement errors in the data.

2. Methodology

2.1. Formulation of PDIPM

In EIT, the measurements (data) are the surface potentials on the boundary of the object. Let \( f(x) = y \) represent the nonlinear relationship between the data (voltage data) and model parameters (electrical properties), \( x \in \mathbb{R}^N \) and \( y \in \mathbb{R}^M \), respectively. In difference EIT, \( y \) is the normalized difference data \( y = (v - v_r)/v_r \) between the current measured voltages \( v \) and the reference measurements \( v_r \). \( x \) is the image of relative conductivity changes between the background (or reference) conductivity \( \sigma_r \) and the corresponding conductivity \( \sigma \) for the measurement \( v \).

Generally, the number of the unknown model parameters are much more than the available number of data, i.e., \( M < N \), which results in an underdetermined set of equations. The corresponding inverse problem of recovering \( x \) from \( y \) based on the mapping \( f \) can be stated in the form of a minimization problem as follows:

\[
\arg \min_x \{ J(x) := ||f(x) - y||_p^p + \lambda ||L(x - x_0)||_n^n \}, \tag{1}
\]

where the \( ||f(x) - y||_p^p \) is the data fidelity term and \( ||L(x - x_0)||_n^n \) is the regularization term, and \( x_0 \) is a prior conductivity distribution (in our case the initial estimate was set to zero); \( L : \mathbb{L} \in \mathbb{R}^{M \times N} \) is usually referred to as the regularization matrix and constructed based on the a priori information about the model parameters. The trade-off between these two terms is determined by the regularization parameter \( \lambda \). \( p \) and \( n \) respectively specify the types of norms for the data fidelity and regularization terms. The characteristics of the solutions are dependent on the choice of the norms:

- Using \( \ell_2 \)-norm for both data and regularization terms is widely used and the most convenient way regarding computational time and convergence stability. The resultant solutions are commonly referred to as the least-squares solution with Tikhonov regularization.
- Using a \( \ell_1 \)-norm instead of \( \ell_2 \)-norm in the regularization term allows high frequency components in the solutions to be preserved. For instance, when the regularization matrix is constructed as a gradient operator, the variations in the solution are penalized. Such a scheme is referred to as the total variation (TV) regularization with an appropriate choice of \( L \). The regularization method preserves the edge information in the restored images, while filtering out noise.
- The \( \ell_1 \)-norm of the data fidelity is much more resilient to outliers than the \( \ell_2 \)-norm since it does not square the measurement misfit. Thus, it is usually preferred to suppress errors originating from outliers.
- For \( \ell_1 \ell_1 \) case, the solutions are resilient to data outliers because of the \( \ell_1 \)-norm of the data fidelity term and the \( \ell_1 \)-norm of the regularization term permits reconstruction of sharp edges. However, it is computationally challenging due to the non-differentiability of the absolute value function at a number of points.

2.2. Evaluation Data Set

We considered difference imaging to reflect the physiological changes during the breathing for a certain time duration. We used EIT data acquired during mechanically ventilation of humans.
and dogs. These data are known to contain a certain level of electrode errors in the measurement data.

**Human data:** The human breathing data were based on [3], where pediatric patients with ALI/ARDS were recruited for an open lung treatment protocol. The experimental protocol was consisted of a baseline ventilation stage, a lung recruitment stage with sequentially increased airway pressure, and a PEEP (Positive end-expiratory pressure) titration stage with sequentially decreased airway pressure. A tidal volume of 6 mL/kg of body weight was used as baseline ventilation using volume-controlled mode. During the PEEP titration stage, the PEEP level was decreased sequentially to the lowest possible setting. The EIT system used for the measurement was the Goe-MT II EIT device (CareFusion, Hoechberg, Germany) operated in single frequency, where 16 electrodes were used, and adjacent stimulation and measurement patterns were applied.

**Dog data:** 16-electrode EIT system was used to take measurements of conductivity changes due to lung ventilation and lung fluid instillation in mongrel dogs [4]. ECG was used to record cardiac activity and synchronize EIT measurements 100 ms after the QRS peak of the ECG. Measurements were taken before and after 100 ml of fluids (5 % bovine albumin and Evans blue dye) were injected to a lobe of the right lung at the presence of 700 ml tidal volume. The same reference data was used before fluid instillation.

3. Results

Fig. 1 shows the time difference EIT image of a human thorax. A series of $\lambda$ values were tested for the first measurement data (R1) taken from a ventilated patient for the protocol steps (R-R4 and T1-T4). The $\lambda$ value that produces good reconstructed image was heuristically selected and fixed for the rest of the data sets. Different $\ell$-norms have different fixed $\lambda$ values.

![Figure 1](image_url)  
**Figure 1:** Time difference images of a human thorax for the PDIPM algorithms with four different $\ell$-norms on the data and regularization terms. R1–R4 are the protocol steps of lung recruitment stage and T1–T4 are the protocol steps of the PEEP titration stage.

Fig. 1 showed that the $\ell_2$-norm on the regularization term produced smooth reconstruction while the $\ell_1$-norm on the regularization term produced high contrast conductivity distributions (sharper images). Interestingly, at the protocol steps R3 and T1, $\ell_2\ell_2$ and $\ell_2\ell_1$ produced a clinically unrealistic (failed) reconstruction, while $\ell_1\ell_2$ and $\ell_1\ell_1$ showed expected images. Further analysis showed 3 times higher noise levels at the two steps compared to the rest of the protocol steps.

Fig. 2 illustrates the use of PDIPM algorithms for imaging the conductivity changes in dog’s lung immediately after fluid injection (row 1) and 60 minutes after the fluid injection (row 2) with the presence of certain level of electrode errors.
Figure 2: Reconstructed cross-section of conductivity changes in dog’s lung right after injection (row 1) and 60 minutes after the injection (row 2). Difference images were reconstructed using PDIPM algorithms, where the presence of electrode errors in the measured data affected all reconstructions, except those using the l1 norm on the data term (robust reconstruction).

As shown in Fig. 2, $\ell_2\ell_2$ and $\ell_2\ell_1$ showed large image artifacts as they suffered from noise from measurement system. $\ell_1\ell_2$ and $\ell_1\ell_1$ reconstructed cleaner and meaningful images, where particularly $\ell_1\ell_1$ produced sharp images with little image artifacts.

4. Discussion
In this study, we evaluated PDIPM algorithms for the EIT real-measurement on clinical patient and experimental dog data (with electrode errors) using $\ell_1/\ell_2$ data and image norms. The results (Fig. 1 and 2) showed that $\ell_1\ell_2$ and $\ell_1\ell_1$ are less affected by data outliers because of 1-norm for the data term, while $\ell_2\ell_1$ and $\ell_1\ell_1$ produces sharper reconstructed images because of 1-norm on the regularization term. $\ell_1\ell_1$ provided both sharp conductivity image and robustness to data outliers. However, $\ell_1$ solution is not efficient compared to Gauss-Newton based solution, since $\ell_1$ solution involves with the minimization of a non-differentiable function, thus requires more computation time and more iterations to reach the convergence. Most traditional image reconstruction is based on $\ell_2$ norm, which produces smooth solutions without clear edges and is sensitive to measurement noise (data outliers). Nevertheless, $\ell_2\ell_2$ takes least computation time as it is the one-step linearized image reconstruction, which is also widely used in EIT imaging.

PDIPM algorithms with 1-norm ($\ell_1\ell_1$) present significant advantages in medical applications of EIT, since high contrast sharper images are more desired in clinical scenario. In reality, noise from the acquisition system, and patient movement or electrode errors are unavoidable, while measurement noise has been modeled as Gaussian. This greatly affects reconstructed image quality (especially $\ell_2$ data norm solution). This paper demonstrated the advantages of $\ell_1$ norm solutions in clinical applications of EIT, where $\ell_1$ solution is not only more robust to unavoidable measurement errors in clinical setting, but it also could provide high contrast resolution.

5. References
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