Stress-strain state of the soil mass under the uniformly distributed load action adjacent to a vertical excavation

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Abstract. This article provides a solution to the problem of the stress-strain state of a soil mass adjacent to a vertical excavation when applying a uniformly distributed load on its surface. The solution of the problem was obtained by the method of trigonometric Ribere-Fileon series. The final expressions for the SSS components are provided. Contours of stresses and displacements are also shown.

1. Introduction

When excavating soil from excavations with vertical slopes under the protection of enclosing structures, a complex non-uniform stress-strain state is created in the adjacent soil mass. In the case of application of a distributed load on the soil surface in the soil mass adjacent to the working, additional normal and shear stresses, which have additional effects on the enclosing working structures, are created. This article provides an example of an analytical solution to the stress-strain state problem of a base of limited width and thickness adjacent to a vertical working under the influence of a uniformly distributed load.

It should be noted that the issue of stress distribution in the soil layer resting upon a rigid foundation was studied by many scientists - Fileon (1903), Melan (1919) [12], Marger (1931), Burmister (1956), Sovints, Davis, Taylor, Shekhter O. Ya. (1937), Gorbunov - Posadov M.I. (1946-1953) [2], Egorov K.E. (1961) [3], Tsytovich N.A. (1943) [11].

In these works, the authors note that the SSS formation in a subsoil under the local load action has a certain specificity. Under the foundation of finite width, a certain closed area is formed, within which the stresses exceed the structural strength, and in this area significant deformation of the soil occurs (Fig. 1a). Outside this area, these deformations can be neglected. The shape and size of such an area depend on the mechanical properties and on the structural strength of the soil, as well as on the load area (b = 2a). In this work, a rectangular area with the dimensions hx2l is considered as a computational one (Fig. 1b). To simulate a situation when a uniformly distributed load of width (a-b) is applied to the soil surface at a distance b from the vertical excavation, a rectangular area is considered in the range from 0 to h in height and from –l to 0 in width.
2. Problem setting

It is known that the plane problem solution of the theory of elasticity is reduced to the integration of two equilibrium equations and the equation of compatibility (continuity of deformations) with the obligatory satisfaction of the boundary conditions. In addition, this solution assumes a linear relationship between stresses and strains, i.e., the application validity of the generalized Hooke’s law [9].

It is also known that when the only volumetric force is the weight of the body, then the plane problem solution can be reduced to finding some function $\phi(x,y)$ (Airy function), which is related to stress components by the following dependencies [6-10]:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} - \rho g y; \sigma_y = \frac{\partial^2 \phi}{\partial x^2} - \rho g x; \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

(1)

and satisfies the biharmonic equation:

$$\frac{\partial^4 \phi}{\partial x^2} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

(2)

In the present work, the solution to the problem formulated is obtained by the Ribiére-Fileon trigonometric series method. The general solution of the biharmonic equation (2) can be represented as an infinite series [9-10]:

$$\phi(x,y) = \sum_{n=1}^{\infty} \left[ \cos \alpha x \left( A_n \cosh y + B_n \sinh y + C_n \sinh y + D_n \cosh y \right) + \sin \alpha x \left( A'_n \cosh y + B'_n \sinh y + C'_n \sinh y + D'_n \cosh y \right) \right]$$

(3)

The constants $A_n, B_n, \ldots, C'_n, D'_n$ are determined from the conditions on the computational domain contour. Using the stress function (2), adding, if necessary, power polynomials, it is possible to obtain the solutions to the plane problem of the theory of elasticity.
3. The formulated problem solution

To create a uniformly distributed load at a distance \( b \) from the vertical excavation, we present the design scheme as a result of the uniformly distributed loads of intensity \( q \) addition over a strip with width \( 2a \) and intensity \( \nu \) over a strip with width \( 2b \).

For the problem under consideration, we will accept the following boundary conditions at the upper and lower boundaries of the array area.

\[
\text{at } y = 0 \text{ and } y = 2h: \quad \sigma_y(x, 0) = \sigma_y(x, 2h) = q \quad \text{and} \quad (a \leq x \leq -a) \quad \text{and} \quad (-l \leq x \leq l); \quad (4)
\]

\[
\text{at } y = 0 \text{ and } y = 2h: \quad \tau_{xy}(x, 0) = 0; \quad \tau_{xy}(x, 2h) = 0
\]

If horizontal displacements at \( x = \pm l \) are absent, then we have one more boundary condition in the form:

\[
u(\pm l) = 0 \quad \nu(\pm l) \neq 0
\]

Taking into account the equations (4) - (5), we finally obtain the expressions for the stress components, \( \sigma_y(x, y) \), \( \sigma_x(x, y) \) and \( \tau_{xy}(x, y) \) in the soil:

\[
\sigma_y(x, y) = \frac{qa}{l} + \frac{4q}{\pi} \sum_{m=0}^{\infty} \sin \frac{m \pi a}{l} \left[ \frac{m \pi (y - h)}{l} \left( \frac{m \pi (y - h)}{l} + \frac{m \pi h}{l} \right) - \frac{m \pi (y - h)}{l} \right] \sin \frac{m \pi x}{l}
\]

\[
\sigma_x(x, y) = \frac{qa}{l} \nu - \frac{4q}{\pi} \sum_{m=0}^{\infty} \sin \frac{m \pi a}{l} \left[ \frac{m \pi (y - h)}{l} \left( \frac{m \pi (y - h)}{l} - \frac{m \pi h}{l} \right) - \frac{m \pi (y - h)}{l} \right] \cos \frac{m \pi x}{l}
\]

\[
\tau_{xy}(x, y) = \frac{4q}{\pi} \sum_{m=1}^{\infty} \sin \frac{m \pi a}{l} \left[ \frac{m \pi h}{l} \left( \frac{m \pi h}{l} + \frac{m \pi h}{l} \right) \cos \frac{m \pi x}{l}ight]
\]

For the problem under consideration, the expressions (6) - (8) take the form:

\[
\sigma_y(x, y) = \frac{qa}{l} \frac{qh}{l} + \frac{4q}{\pi} \sum_{m=1}^{\infty} \left[ \sin \frac{m \pi a}{l} - \sin \frac{m \pi h}{l} \right] \left[ \frac{m \pi h}{l} \left( \frac{m \pi h}{l} + \frac{m \pi h}{l} \right) \cos \frac{m \pi x}{l}ight]
\]

\[
\sigma_x(x, y) = \frac{qa}{l} \frac{qh}{l} \nu - \frac{4q}{\pi} \sum_{m=0}^{\infty} \left[ \sin \frac{m \pi a}{l} - \sin \frac{m \pi h}{l} \right] \left[ \frac{m \pi h}{l} \left( \frac{m \pi h}{l} - \frac{m \pi h}{l} \right) \cos \frac{m \pi x}{l}ight]
\]

\[
\tau_{xy}(x, y) = -\frac{4q}{\pi} \sum_{m=0}^{\infty} \left[ \sin \frac{m \pi a}{l} - \sin \frac{m \pi h}{l} \right] \left[ \frac{m \pi h}{l} \left( \frac{m \pi h}{l} - \frac{m \pi h}{l} \right) \sin \frac{m \pi x}{l}ight]
\]
These equations can be solved in the Mathcad software package and the isolines of normal $\sigma_x$, $\sigma_y$ and tangent $\tau_{xy}$ stresses in the soil mass. Figures 2-5 show the results of solving the equations (9) - (11) in the Mathcad software package.

**Figure 2.** Isolines of stresses when applying a distributed load at a distance $b$ from the vertical excavation, obtained by the equation (9): a) vertical stresses $\sigma_x$; b) horizontal stresses $\sigma_y$; c) shear stress contours $\tau_{xy}$.
Figure 2 shows that when a distributed load is applied near a vertical excavation, the normal and shear stresses are somewhat different and tend to increase when approaching the excavation boundaries. Thus, an analytical solution to the problem of the stress-strain state of a soil mass near a vertical excavation is obtained when a uniformly distributed load is applied.

4. Summary
As a result of the deformation problem analytical solution of a layer with limited width and thickness adjacent to a vertical excavation, the formulas for determining the components of normal and shear stresses when a uniformly distributed load is applied nearby a vertical excavation, are obtained. The analysis of the solution results shows that the load application nearby the excavation significantly affects the stress-strain state of the entire massif, and the stresses tend to increase when approaching the boundary of the vertical excavation.

The results of the solution can be used to determine the active pressure on the enclosing structures of the excavation to take into account additional stresses when applying a uniformly distributed load on the excavation edge from the equipment and stored building materials.

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