The time evolution of coherent atomic system and probe light in an EIT medium

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The adiabatic solutions of Maxwell-Bloch equation governing the three-level EIT medium is presented. The time evolution of the density matrix elements of the EIT system and the probe light is thus investigated by using the adiabatic approximation formulation and the slowly varying envelope condition.

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I. INTRODUCTION

Recently, many theoretical and experimental investigations show that the control of phase coherence in multilevel atomic ensembles will give rise to many novel and striking quantum optical phenomena, such as the coherent population trapping [1], laser without inversion and electromagnetically induced transparency (EIT) [2–4], in the wave propagation of near-resonant light. According to the theoretical analysis of multilevel atomic phase coherence, the requirement of the occurrence of EIT is such that the strength of coupling light is much stronger than that of probe light [4,5]. Under this condition, the EIT atomic vapor allows the probe light to propagate without dissipation through the medium. Due to its unusual quantum coherent character, the discovery of EIT has so far led to many new peculiar effects and phenomena [4], some of which are believed to be useful for the development of new techniques in quantum optics. More recently, the physical effects associated with EIT observed experimentally include the ultraslow light pulse propagation [6,7] and light storage [8,9] in atomic vapor, and atomic ground state cooling [10].

With the development of the quantum information (quantum computation, quantum communication and quantum measurement), the search for new ways to manipulate photon states becomes increasingly important [11,12]. In the area of EIT, historically, such a possibility might first be considered by Ling et al., who investigated the “electromagnetically induced grating” in homogeneously broadened media [13]. In this work there is a strong coupling standing wave, interacting with three-level Lambda-type (or cascade-type) atoms. This can diffract a weak probe field, which propagates along a direction normal to the standing wave, into high-order diffractions. As stated by Arve et al., many researches of EIT including the freezing light pulses [14,15], time-dependent group velocity [16,17], delayed probe light [8], stopping a pulse and restarting it in the opposite direction (creating a time reversed version of the original probe pulse) [18] have been limited to electromagnetic propagation in one dimension only [19], namely, the investigators of these studies did not consider how to change the direction of the probe light. For this reason, Arve et al. lifted the restriction to the one dimension and considered the Maxwell-Bloch dynamics in two or more dimensions of three-level EIT system, by using the adiabatic approximation and the slowly varying envelope approximation. In their treatment, however, they did not take into account the decay terms in the dynamical equations. Since in a typical EIT experiment [20,21], the spontaneous decay rate of the excited state of the three-level systems may be of the same order of magnitude as that of the Rabi frequencies of the probe and coupling optical fields, the contribution of the decay terms in the Maxwell-Bloch dynamics should also be taken into consideration. In the present paper, we obtain the adiabatic solutions of the Maxwell-Bloch equations, i.e., the expressions for the time evolution of both the probe light and the off-diagonal density matrix elements of the atomic system. The formulation presented here can apply to the study of the variation of the propagation direction (diffraction) of the optical fields in the EIT medium.

II. MAXWELL-BLOCH EQUATIONS GOVERNING THE THREE-LEVEL EIT SYSTEM

Consider a three-level Λ-type atomic ensemble interacting with two resonant laser beams, the Rabi frequencies of which are denoted by \(\Omega_e\) and \(\Omega_p\), respectively. In such a Λ-type atomic system, levels \(|c\rangle\) and \(|b\rangle\) are the ground states, and \(|a\rangle\) the excited state. The laser beam which couples levels \(|a\rangle\) and \(|b\rangle\) is called the probe light (\(\Omega_p\)). Another laser beam is termed the coupling light (\(\Omega_c\)), which couples levels \(|a\rangle\) and \(|c\rangle\). The schematic diagram for the above Λ-type system is depicted in Fig. 1.
If we apply the adiabatic approximation (i.e., $|d\rho/dt| \ll |\Omega_{\rho}|$) to the three-level system, then the Bloch equation can be rewritten in the form
\begin{align}
0 &= \text{Im} \left( \Omega_{\rho b}^* \rho_{ab} + \Omega_{\rho c}^* \rho_{ac} \right) - \gamma_{aa} \rho_{aa},
0 &= \frac{i}{2} \left( |\Omega_c| \rho_{bc} + |\Omega_c^*| \rho_{cb} \right) - \gamma_{bb} \rho_{bb},
0 &= \frac{i}{2} \left( |\Omega_p| \rho_{bc} + |\Omega_p^*| \rho_{cb} \right) - \gamma_{ac} \rho_{ac},
\dot{\rho}_{bb} &= \text{Im} \left( \Omega_p^* \rho_{ba} \right) - \gamma_{bb} \rho_{bb},
\dot{\rho}_{bc} &= \frac{i}{2} \left( \Omega_p^* \rho_{ac} - |\Omega_c^*| \rho_{cb} \right) - \gamma_{bc} \rho_{bc},
0 &= \text{Im} \left( \Omega_c^* \rho_{ca} \right) - \gamma_{cc} \rho_{cc},
\end{align}
where dot denotes the time derivative. In order to let level $|b\rangle$ into a dark state (trapped state) and thus realize the transparency effect for the probe light, the magnitude of Rabi frequency $|\Omega_c|$ should be much greater than $|\Omega_p|$. Generally speaking, $\Omega_{\rho b}^*$ in the first equation in (1) can be ignored compared with the term $\Omega_{\rho c}^* \rho_{ac}$. It follows from the first and last equations in (1) that the relation $\gamma_{aa} + \gamma_{cc} \rho_{cc} = 0$ can be satisfied, and consequently the ratio $\rho_{aa}/\rho_{cc} = -\gamma_{cc}/\gamma_{aa}$ is obtained.

Assume that the initial state of the excited level $|a\rangle$ of the EIT medium under consideration is nearly empty, and the two ground states $|c\rangle$ and $|b\rangle$ are occupied according to the dark state conditions, i.e., $\rho_{bb} \approx 1$ and $\rho_{aa} \approx 0$. This, therefore, means that the term $\rho_{bb} \rho_{ab}$ in the second equation in (1) approximately equal unity. So, we can obtain the following relation between the density matrix elements $\rho_{bb}$ and $\rho_{ab}$
\begin{equation}
\rho_{bb} = \frac{-\Omega_p + 2i \gamma_{ab} \rho_{ab}}{\Omega_c},
\end{equation}
In the meanwhile, it follows from the fifth equation in (1) that the relation between $\rho_{ba}$ and $\rho_{bc}$ is
\begin{equation}
\rho_{ba} = \frac{2i \left( \Omega_p^* \rho_{bc} + \gamma_{bc} \rho_{bc} \right)}{\Omega_c},
\end{equation}
where $\rho_{ba}$ and $\rho_{bc}$ are the complex conjugates to $\rho_{ab}$ and $\rho_{cb}$, respectively. Note that Eqs. (2) and (3) constitute a set of coupling equations. In order to see how $\rho_{bc}$ and $\rho_{ba}$ evolve in the presence of the probe and coupling lasers, we should first obtain their respective equations. With the help of Eqs. (2) and (3), one can arrive at
\begin{equation}
\frac{\partial}{\partial t} \rho_{bc} + \left( \gamma_{bc} + \frac{\Omega_{\rho c}^* \Omega_c}{4 \gamma_{ab}} \right) \rho_{bc} + \frac{\Omega_{\rho c}^* \Omega_c}{4 \gamma_{ab}} = 0
\end{equation}
and
\begin{equation}
\frac{\partial}{\partial t} \rho_{ba} + \left( \gamma_{bc} + \frac{\Omega_{\rho c}^* \Omega_c}{4 \gamma_{ab}} \right) \rho_{ba} + \frac{i}{2 \gamma_{ab}} \left( \frac{\partial}{\partial t} \Omega_{\rho c}^* + \gamma_{bc} \Omega_{\rho c} \right) = 0.
\end{equation}
As in the adiabatic approximation, the Rabi frequency $\Omega_c$ of the coupling laser can be considered a constant in time, one can derive $\rho_{bc}(t)$ from Eq. (4), i.e.,
\begin{equation}
\rho_{bc}(t) = e^{-\lambda t} \left[ \int_0^t \frac{\Omega_{\rho c}^* \Omega_c}{4 \gamma_{ab}} e^{\lambda t'} dt' + \rho_{bc}(0) \right]
\end{equation}
with $\lambda = \gamma_{bc} + \Omega_{\rho c}^* \Omega_c / (4 \gamma_{ab})$. In the meanwhile, one can obtain the expression for $\rho_{ba}(t)$ from Eq. (5)
\begin{equation}
\rho_{ba}(t) = e^{-\lambda t} \left[ \int_0^t \frac{\partial}{\partial t} \Omega_{\rho c}^* (t') + \gamma_{bc} \Omega_{\rho c} (t') \right] e^{\lambda t'} dt' + \rho_{ba}(0)e^{-\lambda t}.
\end{equation}
It should be noted that although the Rabi frequency $\Omega_c$ is a time-dependent quantity, in the adiabatic case, it can be viewed as a constant number. Such a result will be theoretically validated by using the Maxwellian equations in the following. Thus, the parameter $\lambda$ in Eqs. (6) and (7) is considered a constant number.

In the above, we consider the Bloch equation governing the three-level EIT atomic medium. In what follows, we will discuss the Maxwellian equations
\begin{equation}
\begin{cases}
\left( \frac{c^2 \nabla^2 - \frac{\partial^2}{\partial t^2}}{\epsilon_0} \right) \frac{1}{2} \chi_{E P} e^{i \omega t} (k - \mathbf{r}, -\mathbf{r}, t) = \frac{N_{\rho c}}{\epsilon_0} \frac{\partial}{\partial t} \rho_{ac},
\left( \frac{c^2 \nabla^2 - \frac{\partial^2}{\partial t^2}}{\epsilon_0} \right) \frac{1}{2} \chi_{E P} e^{i \omega t} (k, -\mathbf{r}, t) = \frac{N_{\rho c}}{\epsilon_0} \frac{\partial}{\partial t} \rho_{ab},
\end{cases}
\end{equation}
of the coupling and probe lasers in an EIT medium, where $N$ and $\epsilon_0$ denote the atomic number density (total number of atoms per volume) and the electric permittivity in vacuum, respectively. Note that here $\rho_{ac}(t)$ and $\rho_{ab}(t)$ can be rewritten as follows $\rho_{ac}(t) = \rho_{ac}(0) \exp \left[ \frac{i}{\epsilon_0} (\omega_{ac} t - k_{ac} \cdot \mathbf{r}) \right]$, $\rho_{ab}(t) = \rho_{ab}(0) \exp \left[ \frac{i}{\epsilon_0} (\omega_{ab} t - k_{ab} \cdot \mathbf{r}) \right]$. By using the slowly varying envelope approximation (i.e., the second order derivatives of the envelopes $\chi_{E P}$ are negligible compared with the other terms on the left-handed sides of the above Maxwellian equations), Eq. (8) can be reduced to the first order differential equation
\begin{equation}
\begin{cases}
\epsilon \chi_{E} \cdot \nabla \chi_{E} + \frac{\partial}{\partial t} \chi_{E} = i \omega N_{\rho c} \rho_{ac},
\epsilon \chi_{P} \cdot \nabla \chi_{P} + \frac{\partial}{\partial t} \chi_{P} = i \omega \rho_{ab},
\end{cases}
\end{equation}
where
\begin{equation}
\rho_{ab}(t) = e^{-\lambda t} \int_0^t \frac{1}{-2i \gamma_{ab}} \left( \frac{\partial}{\partial t} \Omega_{\rho c} (t') + \gamma_{bc} \Omega_{\rho c} (t') \right) e^{\lambda t'} dt' + \rho_{ab}(0) e^{-\lambda t}.
\end{equation}
Here $\rho_{ac}$ and $\rho_{ba}$ stand for the transition dipole matrix moments. Note that because of the initial condition (e.g., the excited state $|a\rangle$ is nearly empty and, moreover, the population probability of level $|c\rangle$ is negligibly small), at least for the case of investigating the transient optical properties of EIT media, $\rho_{ac}$ can be taken to be zero [19]. Thus the wave equation of the coupling light $\chi_{E}$ takes the form of a source-free field equation. Since such an equation is simple, we will not further consider it. Instead, in the next section we will concentrate our attention on the wave equation of the probe light.
where $\Omega_p = \mathbf{p}_{ab} / \hbar$ with $\mathbf{p}_{ab}$ being the transition dipole moment matrix moment of $ab$ transition.

It is apparently seen that if we have solved the solution $\Omega_p$ of Eq. (11), then according to (6) and (7), the information on the time evolution of the medium polarization ($\rho_{bc}$ and $\rho_{ba}$) can therefore be easily obtained. Such a study can apply to the investigation of the adiabatic storing of a light pulse by the EIT mechanism [19].

### III. Time Evolution of the Probe Light

The solution $\Omega_p$ of Eq. (11) takes the form

$$\Omega_p(r, t) = f \left( r - \mathbf{k}_p \int_0^t v_g(t') dt' \right),$$

(12)

where $\mathbf{k}_p$ is a unit vector defined as $\mathbf{k}_p = \mathbf{k}_p / |\mathbf{k}_p|$, and $v_g(t)$ the group velocity of the probe light in the EIT medium. Here $f$ is a certain function. It is readily verified that the spatial derivative of $\Omega_p$ can be expressed in terms of the time derivative of $\Omega_p$, i.e.,

$$\mathbf{k}_p \cdot \nabla \Omega_p = -\frac{1}{v_g} \frac{\partial}{\partial t} \Omega_p.$$

(13)

Thus, Eq. (11) can be rewritten as

$$\left( 1 - \frac{c}{v_g} \right) \frac{\partial}{\partial t} \Omega_p = i \frac{\omega_p N_{pba} p_{ab}}{\epsilon_0 \hbar} \rho_{ab}.$$

(14)

Further calculation (i.e., calculating the time derivative of the above equation) yields

$$\frac{v_g}{v_g} \frac{\partial}{\partial t} \Omega_p + \left( 1 - \frac{c}{v_g} \right) \frac{\partial^2}{\partial t^2} \Omega_p = i \frac{\omega_p N_{pba} p_{ab}}{\epsilon_0 \hbar} \frac{\partial}{\partial t} \rho_{ab}.$$

(15)

In accordance with Eq. (5), we have

$$\frac{\partial}{\partial t} \rho_{ab} = -\lambda \rho_{ab} + i \frac{\sigma}{2 \gamma_{ab}} \left( \frac{\partial}{\partial t} \Omega_p + \gamma_{bc} \Omega_p \right).$$

(16)

Substitution of Eq. (14) into (16) yields

$$\frac{\partial}{\partial t} \rho_{ab} = -\lambda \left( i \frac{\omega_p N_{pba} p_{ab}}{\epsilon_0 \hbar} \right)^{-1} \left( 1 - \frac{c}{v_g} \right) \frac{\partial}{\partial t} \Omega_p$$

$$+ i \frac{\sigma}{2 \gamma_{ab}} \left( \frac{\partial}{\partial t} \Omega_p + \gamma_{bc} \Omega_p \right).$$

(17)

It follows from Eqs. (15) and (17) that the Rabi frequency of the probe field, $\Omega_p$, agrees with the following equation

$$\frac{\partial^2}{\partial t^2} \Omega_p + \zeta \frac{\partial}{\partial t} \Omega_p + \zeta \Omega_p = 0,$$

(18)

where the coefficients involved are defined as

$$\zeta = \frac{1}{\epsilon_0 h} \left( \frac{\omega_p N_{pba} p_{ab}}{\epsilon_0 \hbar} \right),$$

(19)

In this paper, we consider the homogeneous EIT media only, where the group velocity of the probe light is independent of time, i.e., $v_g(t) = 0$. Thus the coefficients $\zeta$ and $\zeta$ in Eq. (18) are constant. Such a choice will simplify the problem under consideration, since the expression (12) can be reduced to the simple form $\Omega_p(r, t) = f \left( r - \mathbf{k}_p v_g t \right)$.

Apparently, the general solution of Eq. (18) may be of the form

$$\Omega_p(r, t) = \hat{\Omega}_{p+}(r) \exp(\eta_+ t) + \hat{\Omega}_{p-}(r) \exp(\eta_- t),$$

(20)

where $\hat{\Omega}_{p\pm}(r)$ is defined

$$\hat{\Omega}_{p\pm}(r) = \Omega_{p\pm}(0, 0) \exp \left( \sigma \mathbf{k}_p \cdot \mathbf{r} \right).$$

(21)

with $\Omega_{p\pm}(0, 0) = \Omega_{p\pm}(\mathbf{r} = 0, t = 0)$. Here $\sigma$ is a parameter that characterizes the field amplitude shape. Keeping $\mathbf{k}_p \cdot \mathbf{k}_p = 1$ in mind, we rewrite the time-dependent factor $\exp(\eta \pm t)$ in Eq. (20) as follows

$$\exp(\eta \pm t) = \exp \left[ -\sigma \mathbf{k}_p \cdot \mathbf{k}_p \left( \frac{\eta \pm \eta}{\sigma} \right) t \right].$$

(22)

Hence Eq. (20) is rewritten

$$\Omega_p(r, t) = \hat{\Omega}_{p+}(0, 0) \exp \left\{ \sigma \mathbf{k}_p \cdot \left[ r - \mathbf{k}_p \left( \frac{\eta \pm \eta}{\sigma} \right) t \right] \right\}$$

$$+ \hat{\Omega}_{p-}(0, 0) \exp \left\{ \sigma \mathbf{k}_p \cdot \left[ r - \mathbf{k}_p \left( \frac{\eta \pm \eta}{\sigma} \right) t \right] \right\}. \tag{23}$$

Comparing the expression (23) with (12), one can arrive at the group velocity of the probe light

$$v_g = -\frac{\eta \pm}{\sigma},$$

(24)

where $\eta \pm = (-\zeta \pm \sqrt{\zeta^2 - 4 \gamma_{bc}}) / 2$, which is expressed in terms of the parameters $\zeta$ and $\zeta$ of Eq. (18). Note that here $v_g$ is constant, according to (19), we have

$$\zeta = \lambda + \frac{\omega_p N_{pba} p_{ab}}{2 \gamma_{bc} \epsilon_0 \hbar} \left( 1 - \frac{c}{v_g} \right).$$

(25)

Further calculation shows that the group velocity $v_g$ satisfies the following quadratic equation

$$\Delta v_g^2 - \sigma v_g + \zeta = 0.$$

(26)

For convenience, we set $\zeta = \lambda + \beta(1 - c/v_g)^{-1}$, $\zeta = \beta \gamma_{bc}(1 - c/v_g)^{-1}$, where

$$\beta = \frac{\omega_p N_{pba} p_{ab}}{2 \gamma_{bc} \epsilon_0 \hbar}.$$
and rewrite Eq. (26) as
\[ v_k^2 - \frac{\beta + \lambda + \sigma c}{\sigma} v_k + \frac{\lambda c + \beta \gamma_{bc}}{\sigma^2} = 0, \] (28)
the two roots of which take the form
\[ v_{k\pm} = \frac{\beta + \lambda + \sigma c \pm \sqrt{(\beta + \lambda + \sigma c)^2 - 4(\lambda c + \beta \gamma_{bc})}}{2\sigma}. \] (29)

Note that in a typical EIT experiment, the relationships between the parameters \( \gamma_{ab}, \gamma_{bc} \) and \( \Omega_c \) are as follows: \( \gamma_{ab} \approx \Omega_c \approx 10^8 \text{ s}^{-1}, \gamma_{bc} \approx 0.01 \gamma_{ab} \) and \( \omega_p \approx 10^{15} \text{ s}^{-1} [20,21] \). Under these conditions, one can verify that \((\beta + \lambda + \sigma c)^2 > 4(\lambda c + \beta \gamma_{bc})\). Thus, the group velocity, \( v_{k-} \), of the probe laser in the EIT medium is reduced to
\[ v_{k-} = \frac{\beta + \lambda + \sigma c}{\sigma (\beta + \lambda + \sigma c)} \frac{\Omega_c^* \Omega_c}{\eta_g - \frac{4(\lambda c + \beta \gamma_{bc})}{(\beta + \lambda + \sigma c)^2}} \] (30)
It is worth pointing out that the above result (30) is self-consistent, as it can be validated by using the formula of group velocity, \( v_g = c/(n + \omegadn/d\omega) \), where the optical refractive index \( n(\omega) = \sqrt{1 + \chi(\omega)} \). Here, the electric susceptibility for the probe laser with a mode frequency \( \omega \) in a steady EIT system is of the form [22]
\[ \chi(\omega) = -\frac{N\rho_{ab} \rho_{ab}}{\epsilon_0 \hbar} \left( \frac{\omega - \omega_{ab} + i\gamma_{bc}}{\omega - \omega_{ab} + i\gamma_{bc}} \right) \frac{\Omega_p^* \Omega_p}{\Omega_p^* + \Omega_p} \] (31)

In addition, it should be noted that the group velocity \( v_{k+} (v_{k+} = -\eta_-/\sigma) \) should be ruled out, since it does not satisfy the relation \( \eta_- = (\zeta - \sqrt{\zeta^2 - 4\epsilon})/2 \). So, the only retained group velocity of the probe laser in the EIT medium is \( v_{k-} \).

Eqs. (12) and (13) shows that the adiabatic solutions obtained in this paper can deal with the problems of two- or three-dimensional wave propagation, including diffraction. As is shown in Eq. (30), the coherent control of the probe light can be realized: specifically, the change of \( \Omega_c \) will have an influence on the probe group velocity (and hence the parameters \( \zeta \) and \( \zeta \)). Thus, according to Eqs. (12), (13) and (18), the variation of the direction of propagation of the probe light may arise in response to the change of the field strength of the coupling laser.

IV. OFF-DIAGONAL DENSITY MATRIX ELEMENTS

In the previous section, we considered the wave propagation of the probe laser. In this section, we will discuss the time evolution of the off-diagonal density matrix elements of the EIT atomic system. By inserting the expression (20) into Eqs. (6) and (7), one can obtain the explicit expressions for \( \rho_{bc} \) and \( \rho_{ba} \) in the time-evolution process, i.e.,
\[ \rho_{bc}(t) = \frac{\Omega_{bc}^*}{4\gamma_{ab}} ([ \eta_+ + \lambda ] - \frac{1}{\Omega_{bc}^*} (e^{\eta_+ t} - e^{-\lambda t}) + (\eta_- + \gamma_{bc}) (\eta_+ + \lambda) - \frac{1}{\Omega_{bc}^*} (e^{\eta_- t} - e^{-\lambda t})] + \rho_{bc}(0) e^{-\lambda t}, \] (32)

In a typical EIT experiment \((N = 10^{18} \text{ m}^{-3} [21])\), one can have \( \zeta \gg \zeta + \sqrt{\zeta^2 - 4\epsilon} \), so that \( \zeta \) is less than \( \gamma_{bc} \). In the meanwhile, \( \lambda = \gamma_{bc} + \Omega_c^2 \Omega_c / (4\gamma_{ab}) \), which is two or three orders of magnitude larger than \( \gamma_{bc} \) in a typical EIT experiment [20,21]. This, therefore, means that when the time \( t \) is taken the several Rabi oscillation periods \((\approx 10^{-8} \text{ s})\), \( e^{\eta_+ t} \approx 1 \) and \( e^{-\lambda t} \approx 0 \). Thus, \( \rho_{ba} \) in (32) approximately equals
\[ \rho_{ba} \approx \frac{1}{2\gamma_{ab}} \frac{\gamma_{bc} \Omega_{p+}^*}{\lambda \Omega_{p+}}, \] (33)
or
\[ \rho_{ba} \approx -\frac{i}{\gamma_{bc}^2 + \Omega_{p+}^2} \frac{\gamma_{bc} \Omega_{p+}^*}{\lambda \Omega_{p+}}. \] (34)

According to Eq. (20), we have \( \Omega_p(r, t) \approx \tilde{\Omega}_{p+}(r), \Omega_p(r, t) \approx \tilde{\Omega}_{p-}(r) \). Thus, by using the definition of the dipole-transition electric susceptibility
\[ \chi = \frac{2N\rho_{ab}^2}{\epsilon_0 \hbar} \rho_{ab}, \] (35)
one can arrive at the electric susceptibility at probe frequency, i.e.,
\[ \chi(\omega_p) = \frac{N\rho_{ab}^2}{\epsilon_0 \hbar} \frac{i\gamma_{bc}}{\gamma_{ab} \gamma_{bc} + \Omega_{p+}^2}. \] (36)
It should be noted that this expression is consistent with (31), so long as the relation \( \omega = \omega_{ab} \) (the resonance condition) is inserted into (31). It is thus believed that the obtained adiabatic solutions (32) of Maxwell-Bloch equation governing the three-level EIT medium may be self-consistent. This set of solutions can be utilized to consider the information storage in a atomic vapor.

V. MAGNETIC-DIPOLE TRANSITION AND PERMEABILITY FOR THE PROBE LIGHT

In the previous sections, we have treated the propagation of the optical fields in the three-level system, we did not
consider the magnetic-dipole transition, but the electric-dipole transition. It is believed that the contribution of the magnetic-dipole transition to the wave propagation of the probe light should be taken into account in the case of a stronger coupling field in the EIT medium. According to the appendix to this paper, the density matrix elements $\rho_{ab}$ and $\rho_{cb}$ satisfy the following matrix equation

$$\begin{align*}
\frac{\partial}{\partial t} \begin{pmatrix} \rho_{ab} \\ \rho_{cb} \end{pmatrix} &\approx \begin{pmatrix} -\frac{i}{2}\Omega_{ab} & \frac{i}{2}\Omega_{bc} \\ \frac{i}{2}\Omega_{bc} & -\frac{i}{2}\Omega_{cb} \end{pmatrix} \begin{pmatrix} \rho_{ab} \\ \rho_{cb} \end{pmatrix} + \begin{pmatrix} \frac{i}{2}\Omega_{p} \\ 0 \end{pmatrix} . \end{align*}$$

(37)

Here, $\Gamma_{ab} = \gamma_{ab} + i\Delta_{ab}$ and $\Gamma_{bc} = \gamma_{bc} + i(\Delta_{ab} - \Delta_{ac})$, where $\gamma_{ab}$ and $\gamma_{bc}$ represent the spontaneous decay rate of level $|a\rangle$ and the dephasing rate (nonradiative decay rate) of $|c\rangle$, respectively. It can be readily verified that the steady solution of Eq. (37) takes the following form

$$\begin{align*}
\rho_{ab} &= \frac{i\Omega_{ab}[\gamma_{bc} + i(\Delta_{ab} - \Delta_{ac})]}{2\left(\gamma_{ab} + i(\Delta_{ab} - \Delta_{ac})\right) + \Omega_{bc}^2 + i\Omega_{p}}, \\
\rho_{cb} &= -\frac{\Omega_{bc}^*}{4\left(\gamma_{ab} + i(\Delta_{ab} - \Delta_{ac})\right) + \Omega_{bc}^2 + i\Omega_{p}}. \\
\end{align*}$$

(38)

Apparently, there exists a relation between $\rho_{cb}$ and $\rho_{ab}$, i.e.,

$$\rho_{cb} = \frac{i}{2}\frac{\Omega_{bc}^*}{\gamma_{ab} + i(\Delta_{ab} - \Delta_{ac})}\rho_{ab}.$$  

(39)

Note that since the level pairs $|a\rangle-|c\rangle$ and $|a\rangle-|b\rangle$ can be coupled to two laser fields, the parity of level $|a\rangle$ is different from both $|b\rangle$ and $|c\rangle$. If, for example, $|a\rangle$ possess an odd parity, $|b\rangle$ and $|c\rangle$ will have an even parity. Thus, the electric-dipole matrix elements $p_{ab} = \langle a|c|b\rangle \neq 0$ and $p_{cb} = \langle c|e|b\rangle = 0$, and the magnetic-dipole matrix elements $m_{ab} = \langle c|e/2m_{c}|b\rangle \neq 0$ and $m_{cb} = \langle a|e/2m_{e}|b\rangle = 0$, where $L$ and $S$ denote the operators of the orbital angular momentum and spin of electrons, respectively. So, it is possible for the nearly resonant probe laser to cause the electric-dipole transition between levels $|a\rangle$ and $|b\rangle$ as well as the magnetic-dipole transition between levels $|c\rangle$ and $|b\rangle$ in the three-level atomic medium. The electric-dipole transition ($|a\rangle-|b\rangle$) and the magnetic-dipole transition ($|c\rangle-|b\rangle$) will yield the electric polarizability and the magnetic susceptibility at the probe frequency, respectively. In general, the dimensionless ratio $m_{cb}/p_{abc}$ is of the order of magnitude as the electric dipole moment $2m_{cb}^*\rho_{cb}$. Thus, the order of magnitude of the density matrix element $\rho_{cb}$ may be larger than that of $\rho_{ab}$. This, therefore, means that the magnetic dipole moment $(2m_{cb}^*\rho_{cb})$ may possibly have the same order of magnitude as the electric dipole moment $(2\rho_{ab}^*\rho_{ab})$.

V. CONCLUDING REMARKS

We treat the wave propagation of the probe light in a three-level EIT medium and then obtain the solutions of Maxwell-Bloch equations (containing the decay terms) by
using the adiabatic approximation under the slowly varying envelope condition. The expressions for the time evolution of both the probe light and the off-diagonal density matrix elements are presented, which can be used to consider the variation of the propagation vector of the probe light. This property, i.e., the change of the propagation direction and polarization of light in an EIT medium by using coherent control may be important for the controllable manipulation of optical fields (e.g., light storage), as well as for the quantum information processing. Under the conditions of parameters which are taken as in the ordinary EIT experiments, the results (e.g., the group velocity and the electric susceptibility for the probe light) obtained here can be reduced to the ones derived from the steady solutions of Bloch equations [22].

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Appendix

The three-level density matrix satisfies the following equation

$$
\frac{\partial}{\partial t} \hat{\rho} = -i \left[ \hat{H}, \hat{\rho} \right] - \frac{1}{2} \{ \Gamma, \hat{\rho} \}
$$

(A1)

with the Hamiltonian

$$
\hat{H} = \begin{pmatrix}
\omega_a & -\frac{1}{2} \Omega_{ab} & -\frac{1}{2} \Omega_{ac} \\
-\frac{1}{2} \Omega_{ba} & \omega_b & 0 \\
-\frac{1}{2} \Omega_{ca} & 0 & \omega_c
\end{pmatrix},
$$

(A2)

where $\Omega_{ab}, \Omega_{ac}$ denote the Rabi frequencies of two optical fields (the probe and coupling lasers) coupled to the $ab$ and $ac$ transitions, respectively. $\Gamma$ is a diagonal decay matrix of the levels, which agrees with $\langle i|\Gamma|j \rangle = \gamma_{ij} \delta_{ij}$. $\omega_a$, $\omega_b$ and $\omega_c$ are the level frequencies of $|a\rangle$, $|b\rangle$ and $|c\rangle$, respectively. If the density matrix can be rewritten as $\hat{\rho}(t) = \rho(t) \exp \left[ \frac{i}{\hbar} (\omega_i t - \mathbf{k}_j \cdot \mathbf{r}) \right]$, then it follows from Eq. (A2) that

$$
\begin{align*}
\dot{\rho}_{aa} &= \text{Im} (\Omega_{pa} \rho_{pb} + \Omega_{pa} \rho_{pc}) - \gamma_{aa} \rho_{aa}, \\
\dot{\rho}_{ab} &= -i (\Delta_{ab} \rho_{ab} + \frac{1}{2} (\omega_c \rho_{bc} + \Omega_{pc} (\rho_{bc} - \rho_{ac}))) - \gamma_{ab} \rho_{ab}, \\
\dot{\rho}_{ac} &= -i (\Delta_{ac} \rho_{ac} + \frac{1}{2} (\Omega_{pc} \rho_{bc} + \omega_c (\rho_{cc} - \rho_{ca}))) - \gamma_{ac} \rho_{ac}, \\
\dot{\rho}_{bb} &= \text{Im} (\Omega_{pb} \rho_{pa}) - \gamma_{bb} \rho_{bb}, \\
\dot{\rho}_{bc} &= -i (\Delta_{bc} - \Delta_{ab}) \rho_{bc} + \frac{1}{2} (\Omega_{pc} \rho_{ac} - \omega_c (\rho_{cc} - \rho_{ca}))) - \gamma_{bc} \rho_{bc}, \\
\dot{\rho}_{cc} &= \text{Im} (\Omega_{pc} \rho_{ca}) - \gamma_{cc} \rho_{cc}.
\end{align*}
$$

(A3)

Here $d$ denotes the derivative with respect to time. $\Omega_{pa} (= \Omega_{ab})$ and $\Omega_{pc} (= \Omega_{ac})$ are the Rabi frequencies of the probe and coupling light, respectively. The frequency detunings are defined as follows: $\Delta_{ab} = \omega_a - \omega_b - \omega_{ab}$, $\Delta_{ac} = \omega_a - \omega_c - \omega_{ac}$ and $\Delta_{bc} = \Delta_{ac} - \Delta_{ab}$. It should be noted that if the probe and coupling light fields are in resonance with the $ab$ and $ac$ transitions, respectively, namely, the following conditions are satisfied: $\Delta_{ab} = \Delta_{ac} = \Delta_{bc} = 0$, Eqs. (A3) can be simplified into Eq. (1) that has been considered in this paper.

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