MARANGONI FORCED CONVECTIVE CASSON TYPE NANOFLIUD FLOW IN THE PRESENCE OF LORENTZ FORCE GENERATED BY RIGA PLATE

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ABSTRACT. The present communication aims to investigate Marangoni based convective Casson modeled nanofluid flow influenced by the presence of Lorentz forces instigated into the model by an aligned array of magnets in the form of Riga pattern. The exponentially decaying Lorentz force is considered using the Grinberg term. On the liquid - gas or liquid - liquid interface, a realistic temperature and concentration distribution is considered with the assumption that temperature and concentration distributions are variable functions of $x$. The set of so-formulated governing problems under the umbrella of Navier Stokes equations is transformed into nonlinear ODEs using suitable transformations. Homotopy approach is implemented to achieve convergent series solutions for the said problem. Influence of active fluid parameters such as Casson parameter, Brownian diffusion, Prandtl number, Thermophoresis and others on flow profiles is analyzed graphically. The fluctuation in local physical quantities such as heat and mass flux rates, is noticed to check the significance of current fluid model in many industrial as well as engineering procedures using nanofluids. The outcomes indicate that the effective Lorentz force assists the fluid motion that results in an augmented velocity profile with incremental values of modified Hartman number. Furthermore, incremental data of Casson parameter motivates significant reduction in velocity profile.

1. Introduction. In the recent past, the heat transfer mechanism of nanofluids has been a topic of extensive research because it has greatly enhanced thermal characteristics of typical fluids such as oil, water and glycol etc. Various industrial and engineering procedures require a fluid having such characteristics that might not result in temperature rise during the working period of respective machine. Thus it is desirable to idealize the situation to get more efficiency in the fluid flow analysis.

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Aforementioned typical fluids are not good conductors of heat and electricity. However, these characteristics are improved either by induction of a magnetic field or by saturation of metallic particles that are good conductors of heat and electricity. Choi [6] is the first researcher to introduce the term nanofluid referring to a base fluid with suspended nanoparticles. Research have proved that suspending nanoparticles such as metallic particles, metallic oxides, carbon nanotubes and metalloid oxides in the typical base fluid effectively enhance the thermal and electric conductivity of base fluid. Thus, heat transfer efficiency of the fluid enhances to a great extent. Nanofluids have numerous applications in different fields like powder technology, microelectronics, aerospace, chemical engineering, vehicle engineering etc. Numerous related studies have been reported in the context of nanofluids and MHD. Ibanez et al. [12] reported MHD nanofluid convection considering suction/injection and thermal radiation with newly formulated convective boundary conditions. Hayat et al. [9] attended melting heat phenomenon in stagnation point nanofluidic flow past a stretching sheet. Lopez et al. [17] numerically investigated entropy generation in nanofluid flow based on water confined with micro-channel having porosity. Sheikholeslami et al. [40] reported nanofluid flow in displacing walls using forced convection. Numerous related articles on the applications of nanofluids in industrial and engineering procedures can be seen in [18],[31],[26],[25],[30],[3],[27] and cross references cited therein. In addition, fluids such as grease, human blood, printer ink, soup, honey, orange juice etc. are treated as a special class of non-newtonian fluids having a nonlinear ratio between the stress and strain. The viscosity of these fluids changes prominently with the intensity of shear stress that results in strong deviation from Newton's law of viscosity. Numerous research articles on such kind of fluids are available in literature. Sarojamma et al. [36] reported the vertically stretching wall channel with stretching cylinder considering heat source/sink. Hayat et al. [10] reported impact of Soret and Dufour on Casson type nanofluid flow past a stretching sheet. Numerous related articles on non-newtonian Casson type flows can be seen in [22],[15] and cross references cited therein.

A liquid - gas or liquid - liquid interface leading to a dissipative layer on the surface of a fluid is called Marangoni boundary layer. Driven either by the gradient of surface temperature or by the gradient of surface concentration appears in numerous practical and industrial procedures such as aerospace engineering, chemical engineering, crystal growth, silicon melts and liquid films etc. It is divided into two types (a) thermal Marangoni effect and (b) solute Marangoni effect. The former is created by Pearson whereas later is introduced by Scriven and Sternling [23],[37]. As per the definition introduced by Pearson, surface temperature of a liquid varies when heated from below. This variation in surface temperature develops a surface traction which assists the fluid flow to maintain the actual temperature variation. Thus a thermal Marangoni mechanism is developed. For solute Marangoni effect the concentration of nanoparticles plays its part to develop a surface concentration variation that leads to surface traction which assists the fluid motion. Thus a solute Marangoni mechanism is developed. Numerous research articles are reported on this phenomenon. Recently, Sheikholeslami et al. [39] attended the Marangoni forced convective flow of nanofluids considering the effect of Lorentz forces generated through MHD. Mahanthesh et al. [20] reported MHD Marangoni convective flow of nanofluid with exponential space dependent heat source/sink. Marangoni effect has also been intensively reported in Casson type nanofluids. Ganesh et al. [7]
reported the impact of a first order chemical reaction effect over Casson type boundary layer nanofluid flow provided with a uniform heat source/sink. Mahantesh et al. [21] reported a Casson type Marangoni convective nanofluid flow considering cross-diffusion effects. Mahantesh et al. [19] investigated thermal Marangoni convective two phase flow. Some closely related and interesting articles can also be seen in [45],[44],[41],[43],[42].

An induced magnetic field in a flow model either by an external agent or through MHD is a frequently used technique in astrophysics, engineering, geophysics and other industrial procedures that involve nanofluids to enhance thermophysical properties of the fluid. Fluids such as electrolytes, liquid metals, plasma etc. require magnetic field for efficiency in thermophysical characteristics. One of the known external agent for magnetic induction is Riga plate. It is an electromagnetic arrangement with alternative electrodes and magnets over a plane surface. Gailitis and Lielausis [8] formulated this plate where the magnets are used to induce magnetic field while electrodes are used to enhance electric conductivity. Recently Rasool et al. [32] reported a nanofluid flow along a radiative Riga plate using convective boundary conditions. Many researchers have utilized this plate in non-Newtonian nanofluids and heat transfer analysis. Numerous related articles are available in literature (see [2],[33],[38],[1],[14],[24],[35],[9],[4],[5],[34],[13]) and cross references cited therein.

Present communication is inspired by following novel aspects. Firstly, for the first time we have introduced the concept of involving Riga plate in Marangoni forced convective Casson type nanofluids flow. On the liquid - gas or liquid - liquid interface, a realistic temperature and concentration distribution is considered assuming temperature and concentration as increasing functions of axis length $x$. Secondly, HAM is applied to solve the ODEs for convergent series solutions. Graphs are plotted for various values of different fluid parameters to analyze the variation in velocity field, temperature profile and the concentration of nanoparticles. Finally, the heat and mass flux variation is noted that is important in industrial and engineering applications.
where \( \pi = e_{ab} e_{ab} \) such that \((a, b)^{th}\) components of deformation are \( e_{ab} \). \( \pi_{c*} \) is known as the critical value, \( s_y \) is called yield stress while \( \mu_P \) is plastic viscosity (dynamic). A geometry of the flow model in Cartesian coordinates is shown in Fig. 1. Following Hayat et al. [11], the governing equations of mass, momentum, energy and concentration are given below,

\[
\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \tag{2}
\]

\[
v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\pi j_0 M_0 \exp(-\pi y)}{8 \rho_{nl}} , \tag{3}
\]

\[
\frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left( \frac{D_{Br}}{T_\infty} \frac{\partial C}{\partial y} \right) + \frac{\mu}{\rho_c \beta} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2 , \tag{4}
\]

\[
v \frac{\partial C}{\partial y} + u \frac{\partial C}{\partial x} = D_{Br} \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_{Th}}{T_\infty} \left( \frac{\partial^2 T}{\partial y^2} \right). \tag{5}
\]

The surface-tension \( \sigma \) is defined as follows,

\[
\sigma = \sigma_0 [1 - \gamma_T (T - T_\infty) - \gamma_C (C - C_\infty)] , \quad \gamma_C = -\frac{1}{\sigma_0} \frac{\partial \sigma}{\partial C} \bigg|_T , \quad \gamma_T = -\frac{1}{\sigma_0} \frac{\partial \sigma}{\partial T} \bigg|_C , \tag{6}
\]

\[ \]
with following boundary conditions,

\[ T = T_\infty, \quad u = 0, \quad C = C_\infty, \quad \text{at} \quad y \to \infty, \]
\[ T = A_1 x^2 + T_\infty, \quad C = A_2 x^2 + C_\infty, \quad v = 0, \]
\[ \mu \left( 1 + \frac{1}{\beta} \right) \frac{\partial u}{\partial y} = \frac{\partial \sigma}{\partial x} = \frac{\partial \sigma}{\partial x} \biggr|_C \quad \frac{\partial T}{\partial x} + \frac{\partial \sigma}{\partial C} \biggr|_T \quad \frac{\partial C}{\partial x}, \quad \text{at} \quad y = 0. \]

(7)

Where \( \beta \) is known as the Casson factor which is inversely related with the yield stress as defined in (1). \( \rho_f \) is the density of fluid, \( \nu \) is the kinematic viscosity, \( D_{Br} \) is Brownian diffusion. \( C \) represents the concentration distribution while \( T \) is used for temperature distribution. \( \alpha \) is the thermal diffusivity, \( \rho c_p \) is nanoparticles’ heat capacity. \( T_\infty \) and \( C_\infty \) are named as ambient temperature and concentration, respectively. \( D_{Th} \) is the Thermophoretic coefficient of diffusion. \( A_1 = \frac{\Delta T}{L}, \quad A_2 = \frac{\Delta C}{L} \)

Assuming \( u = u(x,y) = \frac{\partial \psi}{\partial y}, \quad v = v(x,y) = -\frac{\partial \psi}{\partial x} \) as the stream functions of velocity. Define

\[ \psi(\eta) = \nu X f(\eta), \quad \eta = \frac{y}{L}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad X = \frac{x}{L}, \]

(8)
in equations (1-6), we obtain

\[ f''' + \left( \frac{\beta}{\beta + 1} \right) f'' - \left( \frac{\beta}{\beta + 1} \right) f' + \left( \frac{\beta}{\beta + 1} \right) Q_1 e^{-\kappa \eta} = 0, \]
\[ \theta'' + Pr f' \theta' - 2 Pr f \theta + NbPr \theta' \phi + NtPr \theta'^2 + EcPr \left( \frac{\beta + 1}{\beta} \right) f'' = 0, \]
\[ \phi'' + Sc f \phi' - 2 Sc f' \phi' + \frac{Nt}{Nb} \theta'' = 0. \]

(9)

(10)

(11)

With following transformed BCs,

\[ \left( 1 + \frac{1}{\beta} \right) f''(\eta) = -2(1 + r), \quad \phi(\eta) = 1, \quad \theta(\eta) = 1, \quad \text{at} \quad \eta = 0, \]
\[ f'(\eta) = 0, \quad \phi(\eta) = 0, \quad \theta(\eta) = 0, \quad \text{at} \quad \eta \to \infty. \]

(12)

Where \( Q_1 = \frac{\pi \nu A_1 \nu X^2 M_a}{8 \nu_0 L^2} \) is the modified Hartman number with reference length \( L, \kappa = \frac{\nu_0}{\nu} \). \( Pr = \frac{\nu}{\alpha} \) is the Prandtl factor. The Brownian motion factor is \( Nb = \frac{\tau D_{Br}(C_w - C_\infty)}{\nu} \) and the Thermophoretic factor is \( Nt = \frac{\tau D_{Th}(T_w - T_\infty)}{T_\infty \nu} \). \( Ec = \frac{\nu \mu}{\rho c_p A_1 L^2} \)

is the Eckert number, \( r = \frac{A_2 \gamma \Delta C}{A_1 \gamma \Delta T} \) is the ratio of thermal surface tension to the solute surface tension such that \( R = \frac{\gamma (C_w - C_\infty)}{\gamma (T_w - T_\infty)} \) while \( M_a|_{L,T} = \frac{\gamma (C_w - C_\infty)}{\gamma (T_w - T_\infty)} \), \( M_a|_{L,C} = \frac{\gamma (C_w - C_\infty)}{\gamma (T_w - T_\infty)} \) are thermal and solutal Marangoni numbers yielding \( r = \frac{M_a|_{L,T}}{M_a|_{L,C}} \), the ratio of Marangoni numbers. The quantities of physical interest are defined as follows,

\[ Nu_x = \frac{r (T_y)_{y=0}}{T(x,0) - T(x,\infty)} \quad \Rightarrow \quad \sqrt{\frac{1}{Re}} Nu_x = -\theta'(0), \]
\[ Sh_x = \frac{r (C_y)_{y=0}}{C(x,0) - C(x,\infty)} \quad \Rightarrow \quad \sqrt{\frac{1}{Re}} Sh_x = -\phi'(0). \]

(13)

Where \( Re \) is the local Reynold number.
3. Solution methodology. Homotopy analysis method (see [16],[29],[28]) is an effective and efficient method for solving non-linear BVPs. The method is independent of small and large parameters solving various problems of science, engineering and industrial procedures. Assume,

\[ f_0 = \frac{2(1 - e^{-\eta})}{(1 + \frac{1}{3})}(1 + r), \quad \theta_0 = e^{-\eta}, \quad \phi_0 = e^{-\eta}, \quad (14) \]

be the initial guesses satisfying the boundary conditions defined in (12). The auxiliary operators are defined below,

\[ L_f = \frac{\partial^3 f}{\partial \eta^3}, \quad L_\theta = \frac{\partial^2 \theta}{\partial \eta^2}, \quad L_\phi = \frac{\partial^2 \phi}{\partial \eta^2}, \quad (15) \]

with following properties,

\[ L_f \left[ \sum_{i=1}^{7} P_i e^{-\eta} + P_8 e^\eta + P_9 \right] = 0, \quad L_\theta \left[ \sum_{i=1}^{7} P_i e^{-\eta} + P_8 e^\eta \right] = 0, \quad L_\phi \left[ \sum_{i=1}^{7} P_i e^{-\eta} + P_8 e^\eta \right] = 0. \quad (16) \]

Where \( P_i \) are arbitrary constants ranging for \( i = 1 - 7 \). Finally, the \( 0^{th} \) order deformation equations are written as below,

\[ L_f \left[ \hat{f}(\eta, t) - f_0(\eta) \right] = \frac{t}{1-t} \hat{h}_f \mathcal{N}_f \left[ f \right], \]

\[ L_\theta \left[ \hat{\theta}(\eta, t) - \theta_0(\eta) \right] = \frac{t}{1-t} \hat{h}_\theta \mathcal{N}_\theta \left[ \hat{f}, \hat{\theta}, \hat{\phi} \right], \]

\[ L_\phi \left[ \hat{\phi}(\eta, t) - \phi_0(\eta) \right] = \frac{t}{1-t} \hat{h}_\phi \mathcal{N}_\phi \left[ \hat{f}, \hat{\theta}, \hat{\phi} \right], \quad (17) \]

subject to the boundary conditions,

\[ \hat{\phi}_{|_{(0,t)}} = 1, \quad \hat{\theta}_{|_{(0,t)}} = 1, \quad \frac{\partial \hat{f}}{\partial \eta}_{|_{(0,t)}} = \frac{-2(1+\tau)}{(1 + \frac{1}{3})}, \quad \hat{f}_{|_{(\infty,t)}} = 0, \quad \hat{\theta}_{|_{(\infty,t)}} = 0, \quad \hat{\phi}_{|_{(\infty,t)}} = 0. \quad (18) \]

Following (17), we write,

\[ \mathcal{N}_f \left[ \hat{f} \right] = (1 + \frac{1}{\beta}) \frac{\partial^2 \hat{f}}{\partial \eta^2} + \hat{f} \left( \frac{\partial^2 \hat{f}}{\partial \eta^2} - \left( \frac{\partial \hat{f}}{\partial \eta} \right)^2 + Q_1 e^{-\kappa \eta} \right), \]

\[ \mathcal{N}_\theta \left[ \hat{f}, \hat{\theta}, \hat{\phi} \right] = \frac{1}{Pr} \frac{\partial^2 \hat{\theta}}{\partial \eta^2} + \left[ \hat{f} \frac{\partial \hat{\theta}}{\partial \eta} - 2 \hat{\theta} \frac{\partial \hat{f}}{\partial \eta} \right] + Nb \left( \frac{\partial^2 \hat{\phi}}{\partial \eta^2} \right) + Nt \left( \frac{\partial \hat{\theta}}{\partial \eta} \right)^2 \]

\[ + Ec \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 \hat{f}}{\partial \eta^2}, \]

\[ \mathcal{N}_\phi \left[ \hat{f}, \hat{\theta}, \hat{\phi} \right] = \frac{1}{Sc} \frac{\partial^2 \hat{\phi}}{\partial \eta^2} + \left[ \hat{f} \frac{\partial \hat{\phi}}{\partial \eta} - 2 \hat{\phi} \frac{\partial \hat{f}}{\partial \eta} \right], \quad (19) \]

where \( t \in [0,1] \) is known as the embedding-parameter and \( \hat{h}_f, \hat{h}_\theta, \hat{h}_\phi \) are named as auxiliary parameters having \( \mathcal{N}_f, \mathcal{N}_\theta, \mathcal{N}_\phi \) named as non-linear operators. For \( t = 0, 1 \), we have,

\[ \hat{f}(\eta, t)|_{t=0} = f_0, \quad \hat{f}(\eta, t)|_{t=1} = f, \]

\[ \hat{\theta}(\eta, t)|_{t=0} = \theta_0, \quad \hat{\theta}(\eta, t)|_{t=1} = \theta, \]

\[ \hat{\phi}(\eta, t)|_{t=0} = \phi_0, \quad \hat{\phi}(\eta, t)|_{t=1} = \phi. \quad (20) \]
Taylor’s expansion when applied in above equations, yields,
\[
\hat{f} = \sum_{m=0}^{\infty} f_m(\eta)t^m, \quad \hat{\theta} = \sum_{m=0}^{\infty} \theta_m(\eta)t^m, \quad \hat{\phi} = \sum_{m=0}^{\infty} \phi_m(\eta)t^m, \quad (21)
\]
leading to,
\[
\sum_{m=0}^{\infty} f_m(\eta) = f - f_0 = \sum_{m=1}^{\infty} f_m,
\]
\[
\sum_{m=0}^{\infty} \theta_m(\eta) = \theta - \theta_0 = \sum_{m=1}^{\infty} \theta_m, \quad (22)
\]
\[
\sum_{m=0}^{\infty} \phi_m(\eta) = \phi - \phi_0 = \sum_{m=1}^{\infty} \phi_m.
\]
Consequently, the \(m^{th}\) order deformation problems are written below,
\[
\mathcal{L}_f \left[ f_m - \Psi_m f_{m-1} \right] = \hat{h}_f R_f^m,
\]
\[
\mathcal{L}_\theta \left[ \theta_m - \Psi_m \theta_{m-1} \right] = \hat{h}_\theta R_\theta^m, \quad (23)
\]
\[
\mathcal{L}_\phi \left[ \phi_m - \Psi_m \phi_{m-1} \right] = \hat{h}_\phi R_\phi^m,
\]
where \(\Psi_m = 1\) for \(m > 1\) otherwise 0. Finally, the governing equations are transformed into following \(m^{th}\) order equations,
\[
R_f^m = (1 + \frac{1}{\beta}) \frac{\partial^3 f_{m-1}}{\partial \eta^3} + \sum_{k=0}^{m-1} \left( f_{m-1-k} \right) \left( \frac{\partial^2 f_k}{\partial \eta^2} \right) - \sum_{k=0}^{m-1} \left( \frac{\partial f_{m-1-k}}{\partial \eta} \right) \left( \frac{\partial f_k}{\partial \eta} \right) + Q_1 e^{-\eta},
\]
\[
R_\theta^m = \frac{1}{Pr} \frac{\partial^2 \theta_{m-1}}{\partial \eta^2} + \sum_{k=0}^{m-1} \left( f_{m-1-k} \right) \left( \frac{\partial \theta_k}{\partial \eta} \right) - 2 \sum_{k=0}^{m-1} \left( \frac{\partial f_{m-1-k}}{\partial \eta} \right) \left( \theta_k \right) + Nb \sum_{k=0}^{m-1} \left( \frac{\partial \theta_{m-1-k}}{\partial \eta} \right) \left( \frac{\partial \phi_k}{\partial \eta} \right) + Nt \sum_{k=0}^{m-1} \left( \frac{\partial \theta_{m-1-k}}{\partial \eta} \right) \left( \frac{\partial \theta_k}{\partial \eta} \right) + Ec \left( 1 + \frac{1}{\beta} \right) \frac{\partial f_{m-1}}{\partial \eta^2},
\]
\[
R_\phi^m = \frac{1}{Sc} \frac{\partial^2 \phi_{m-1}}{\partial \eta^2} + \sum_{k=0}^{m-1} \left( f_{m-1-k} \right) \left( \frac{\partial \phi_k}{\partial \eta} \right) - 2 \sum_{k=0}^{m-1} \left( \frac{\partial f_{m-1-k}}{\partial \eta} \right) \left( \phi_k \right).
\quad (24)
\]
The general solutions to the considered boundary value problem is written as follows,
\[
f_m = P_1 + P_2 e^{\eta} + P_3 e^{-\eta} + f_m^*(\eta),
\]
\[
\theta_m = P_4 e^{\eta} + P_5 e^{-\eta} + \theta_m^*(\eta), \quad (25)
\]
\[
\phi_m = P_6 e^{\eta} + P_7 e^{-\eta} + \phi_m^*(\eta),
\]
where special solutions are starred.

4. **Convergence analysis.** The auxiliary parameters which are involved in achieving series solutions for the flow profiles are generally considered as convergence controlling parameters. The speedy convergence is therefore, dependent on suitable selection of numeric values of these important parameters. In the Table I as well as in graph given in fig. 2, \(10^{th}\) order of approximation for velocity field and upto \(19^{th}\) order approximation for temperature profile and concentration of nanoparticles is sufficient to achieve convergence. The meaningful intervals of convergence for i-curves the three main profiles are \([-1.3, -0.2]\), \([-1.5, -0.2]\) and \([-1.5, -0.2]\), respectively.
Table I: Convergence.

| Orders | \(-f''\)  | \(-\theta'\)  | \(-\phi'\)  |
|--------|-----------|--------------|------------|
| 1      | 1.141912  | 0.8995612    | 1.5211265  |
| 5      | 1.256923  | 0.9566101    | 1.3211122  |
| 10     | 1.300122  | 1.1223114    | 1.1886111  |
| 15     | 1.300222  | 1.2623532    | 0.9956332  |
| 20     | 1.300222  | 1.3112112    | 0.8915622  |
| 25     | 1.300222  | 1.3112211    | 0.8915512  |
| 30     | 1.300222  | 1.3112211    | 0.8915512  |
| 35     | 1.300222  | 1.3112211    | 0.8915512  |
| 40     | 1.300222  | 1.3112211    | 0.8915512  |
| 50     | 1.300222  | 1.3112211    | 0.8915512  |

5. Results and discussion. The boundary value problem pertaining the final governing equations (9-12) are solved by HAM. The method is best suited because it is not dependent on small or large parameters. For a comprehensively deliberate analysis of the fluid flow and heat-mass transfer, the numerical results of velocity field, temperature profile and concentration of nanoparticles are plotted in Figs. (3-5), Figs. (6-9) and Figs. (10-11), respectively. Impact of fluid parameters including the Casson parameter (\(\beta\)), ratio of Marangoni numbers (\(r\)), Thermophoresis (parameter) (\(N_t\)), Brownian diffusion parameter (\(N_b\)), Eckert (parameter) (\(Ec\)), Prandtl number (\(Pr\)) and Schmidt number (\(Sc\)) are analyzed graphically. Fig. 3 presents impact of \(\beta\) on the flow profile of velocity. Velocity field shows anti-augmentation with augmented values of \(\beta\). The increment in \(\beta\) results a rise in the nano-fluid viscosity that ultimately declines the yield stress and velocity profile shows a significant declination. Fig. 4 invokes the impact of Hartman (modified) number \(Q_1\)
on the profile of velocity. The incremental values of this parameter are associated with the strength of Lorentz force instigated by Riga pattern. Intensive Lorentz force motivates the fluid motion through a more realistic variation in surface tension. In particular, the Lorentz force dampens the surface agitation that ultimately results in enhancement of surface tension that drives the fluid. Fig. 5 presents the impact of ratio of Marangoni numbers \( r \) i.e. the thermal to solute Marangoni numbers, the emerging surface tension positively influence the velocity field. An increasing behavior in velocity profile is noticed. The significant influential trend of various values of \( Q_1 \) on the profile of temperature is plotted in Fig. 6. The intensive Lorentz force effectively enhances the temperature profile for incremental values of modified Hartman parameter. Fig. 7 presents the impact yielded by Brownian diffusion parameter \( (Nb) \) whereas Fig. 8 shows the impact invoked by Thermophoresis (parameter) \( (Nt) \) on the profile of temperature distribution. The enhanced in-predictive motion of nano-fluid particles for augmented values of \( Nb \) result in more frequent collisions between the particles that augments the temperature profile and corresponding thermal layer on boundary. The augmented values of \( Nt \) produce a more intensive Thermophoretic force resulting an enhancement in temperature distribution. Fig. 9 discloses the influence invoked by Eckert number on the profile of temperature. Frictional heating increases when we augment Eckert number and therefore, temperature profile increases. Fig. 10 shows the fluctuation in concentration of the nanoparticles for given numerous increasing values of Schmidt number \( (Sc) \). An augmented trend is noticed for incremental values of the respective parameter. The influence of Thermophoretic parameter on the profile of concentration of nanoparticles is shown in Fig. 11. The stronger Thermophoretic forces increase the in-predictive movement of nanoparticles. This variation compels the concentration profile to increase for incremental values of Thermophoretic parameter. Figs. 12-13 are of physical importance explaining the impact of \( Nt \) and \( Sc \) on the mass flux. The rate of mass flux through the surface enhances with augmented values of \( Nt \). Table II presents the data obtained upon numerical simulation through HAM for fixing various values according to the data presented by Hayat et al. [11]. One can see a complete agreement of the respective results with the above referred article for Nusselt number. The respective output for Sherwood number is also noted in the Table II.

6. **Concluding remarks.** In this article we have studied a Marangoni based convective Casson modeled nanofluid flow influenced by the presence of Lorentz forces imparted by an aligned array of magnets in the form of Riga pattern. The exponentially decaying Lorentz force is involved using the Grinberg term. On the liquid - gas or liquid - liquid interface, a realistic temperature and concentration distribution is considered assuming temperature and concentration as increasing functions of axis length \( x \). The so-called Marangoni effect is taken as boundary condition. The base fluid and saturated nanoparticles are considered in thermal equilibrium to avoid any slippage. Important findings are noted as follows:

- Augmented Casson parameter results in reduction of velocity field. For \( \beta \rightarrow \infty \) the flow velocity approaches to a typical Newtonian velocity field.
- \( Nb \) and \( Nt \) are significant enhancing factors for temperature distribution and corresponding thermal layer.
- The incremental values of ratio of Marangoni numbers result in increasing velocity field.
Figure 3. Variations noted in $f'(\eta)$ for incremental $\beta$.

Figure 4. Variations noted in $f''(\eta)$ for incremental $Q_1$.

- Fluid motion receives assistance by wall parallel Lorentz forces invoked by Riga pattern.
- Heat flux through surface is increased with augmented numerical values of $Nt$. 

• The mass flux through surface is increased with anti-augmented values of $Sc$. 

Figure 5. Variations noted in $f'(\eta)$ for incremental $r$.

Figure 6. Variations noted in $\theta(\eta)$ for incremental $Q_1$. 
Figure 7. Variations noted in $\theta(\eta)$ for incremental $N_b$.

Figure 8. Variations noted in $\theta(\eta)$ for incremental $Nt$.

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Figure 10. Variations noted in $\phi(\eta)$ for incremental $Sc$.

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Figure 11. Variations noted in $\phi(\eta)$ for incremental $Nt$.

Figure 12. Variations noted in Nusselt number for incremental $Nt$.

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Figure 13. Variations noted in Sherwood number for incremental Sc.

Table II: Comparison of Nusselt and Sherwood numbers with Hayat et al. [11].

| β  | Q₁ | Sc | Nb | Nt | Pr | Ec | Re<sub>x</sub><sup>-1/2</sup>Nu<sub>x</sub> | Re<sub>x</sub><sup>-1/2</sup>Nu<sub>x</sub> | Re<sub>x</sub><sup>-1/2</sup>Sh<sub>x</sub> |
|----|----|----|----|----|----|----|------------------|------------------|------------------|
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7820      |      0.7833      |      0.6155      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7790      |      0.7855      |      0.5662      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7770      |      0.7829      |      0.5524      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7720      |      0.7778      |      0.5412      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7901      |      0.8100      |      0.7100      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7799      |      0.7836      |      0.7100      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7400      |      0.7566      |      0.7101      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.8101      |      0.8155      |      0.6525      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.8002      |      0.8022      |      0.6580      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7800      |      0.7882      |      0.6612      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.8536      |      0.8536      |      0.5412      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.8825      |      0.8825      |      0.5300      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.9122      |      0.9122      |      0.5121      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      1.1121      |      1.1121      |      1.0021      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      1.2112      |      1.2112      |      1.1121      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      1.3001      |      1.3001      |      1.2231      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7921      |      0.8081      |      0.7101      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7600      |      0.7833      |      0.7200      |
|    |    | 0.6| 0.6| 0.6| 0.6| 0.6|      0.7401      |      0.7586      |      0.7405      |
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