Telling three from four neutrinos at the Neutrino Factory

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Abstract

We upgrade the study of the physical reach of a Neutrino Factory considering the possibility to distinguish a three (active) neutrino oscillation scenario from the scenario in which a light sterile neutrino is also present. The distinction is easily performed in the so-called 2+2 scheme, but also in the more problematic 3+1 scheme it can be attained in some regions of the parameter space. We also discuss the CP violating phase determination, showing that the effects of a large phase in the three–neutrino theory cannot be reproduced in a four–neutrino, CP conserving, model.

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1 Introduction

Indications in favour of neutrino oscillations \[1, 2, 3, 4\] have been obtained both in solar neutrino \[5, 6, 7, 8, 9, 10\] and atmospheric neutrino \[11, 12, 13, 14, 15\] experiments. The atmospheric neutrino data require \(\Delta m^2_{\text{atm}} \sim (1.6 - 4) \times 10^{-3} \text{ eV}^2\) whereas the solar neutrino data prefer \(\Delta m^2_{\odot} \sim 10^{-10} \text{ or } 10^{-7} - 10^{-4} \text{ eV}^2\), depending on the particular oscillation solution chosen for the solar neutrino deficit. The LSND data \[16, 17, 18\], on the other hand, would indicate a \(\nu_\mu \rightarrow \nu_e\) oscillation with a third, very distinct, neutrino mass difference: \(\Delta m^2_{\text{LSND}} \sim 0.3 - 6 \text{ eV}^2\). The LSND evidence in favour of neutrino oscillation has not been confirmed by other experiments so far \[19\]; the MiniBooNE experiment \[20\] will be able to do it in the near future. If MiniBooNE will confirm the LSND results, we would face three independent evidence for neutrino oscillations characterized by squared mass differences quite well separated. To explain the whole ensemble of data four different light neutrino species are needed; the new light neutrino is denoted as sterile, since it must be an electroweak singlet to comply with the strong bound \[21\] on the \(Z^0\) invisible decay width. We stress that all the present experimental results (LSND included) cannot be explained with three massive light neutrinos only, as it has been shown with detailed calculations in \[22\].

In order to improve our present knowledge about neutrino masses and mixings, it has been proposed to build a Neutrino Factory \[23, 24\], with two long baseline experiments running at the same time at two different distances. One of its main goal would be the discovery of leptonic CP violation and, possibly, its study \[23, 24, 25, 26\]. Previous analyses \[29, 30\] on the foreseeable outcome of experiments at a Neutrino Factory have shown that the determination of the two still unknown parameters in the three–neutrino mixing matrix, \(\theta_{13}\) and \(\delta\), will be possible, while with conventional neutrino beams it would not be so \[31\]. The scenario in which the LSND result has been confirmed (and therefore a short baseline experiment to study the four–neutrino parameter space is required) has been considered in \[32, 33\].

If the result of the MiniBooNE experiment will not be conclusive, it seems anyhow quite relevant to understand if three– and four–family models are distinguishable, and to what extent. Moreover, it is a sensible question to ask if the effect of CP violation in a three–family world could be mimicked in a four–family, CP conserving, neutrino scenario.

There are two very different classes of spectra with four massive neutrinos: three almost degenerate neutrinos and an isolated fourth one, or two pairs of almost degenerate neutrinos divided by the large LSND mass gap. The two classes of mass spectra are called for obvious reasons the 3+1 and 2+2 scheme, respectively. The experimental results were strongly in favour of the 2+2 scheme \[34\] until the latest LSND results \[18\]. The new analysis of the experimental data results in a shift of the allowed region towards smaller values of the mixing angle, \(\sin^2(2\theta_{\text{LSND}})\), reconciling the 3+1 scheme with exclusion bounds coming from other reactor and accelerator experiments \[35, 36, 37, 38, 39\]. As a consequence, the 3+1 model is (marginally) compatible with
the data [10, 11, 12, 13]. Although the 2+2 scheme gave a better fit to those data (as
was shown in [14]), the recent SNO results [10] will certainly restrict the allowed pa-
rameter region and give a considerably worse fit for this model. This is understandable
from the analysis presented in refs. [15, 16], prior to the SNO data publication, since
the SMA solution for solar neutrinos, which was more easily compatible with the limit
on sterile components in atmospheric oscillations, is at present strongly disfavoured
(see also [17, 18]).

It will certainly be impossible to tell a four–family 3+1 from a three–family scheme
in the case of very small active–sterile mixing angles: the 3+1 model reduces to a
three–family scheme for vanishing mixing with the isolated state. A discrimination
will therefore be possible only if those angles are large enough. On the other hand,
the four–family 2+2 scheme has the LSND result built–in (in the sense that it cannot
stand if LSND is not confirmed by MiniBooNE) and it does not go smoothly into a
three–family model.

We address the problem of distinction between four and three families in this paper.
In order to do that, we generated “experimental data” [28] according to a four–family
theoretical scheme without CP violation and tried to fit these data with a three–
family model having a free CP violating phase $\delta$. As a cross–check, we also proceeded
in the opposite direction, fitting with four–family formulae the data generated in a
three–family theory. The data have been generated at three distances, $L = 732, 3500,$
7332 Km, using the detector and machine parameters as defined in [28].

The main conclusions of this analysis are:

- The data generated in four–family without CP violation can be described by
  the three–family formulae in some particular zones of the four–family parameter
  space, not restricted to the obvious case of very small angles. These zones are
  reduced in size for increasing gap–crossing angles.

- Combining data taken at different distances the zones in which a description with
  the other theory is successful are generally reduced.

- An increase of the energy resolution of the detector may have a positive effect
  (reducing the zones of ambiguity), but only in the case of rather large gap–
crossing angles.

- Whenever a particular zone in the four–family parameter space gives acceptable
  $\chi^2$ when fitted in the three-family scheme, the fitted value of the three–family CP
  violating phase, $\delta$, is generally not large. In particular, this is true for $L = 3500$
  Km, whereas for $L = 7332$ Km the determination of $\delta$ is somewhat looser, due
to the overwhelming matter effects. For the shortest baseline, $L = 732$ Km, we
  have the largest spread in the values of $\delta$, although the most probable value is
  still close to zero.
• The cross-check, fitting three-family generated data in a four-family model, is consistent with the results previously obtained. Fitting data generated with a CP phase close to 90° in a CP conserving 3+1 theory is practically impossible if one combines data at two different distances.

• In the 2+2 scheme (as opposed to 3+1) the ambiguity with a three-neutrino theory is essentially absent.

In Section 2 we present the relevant formulae for the oscillation probabilities; in Section 3 we discuss the possibility to have a fit in the three-neutrino theory of “data” generated with the 3+1 model; in Section 4 we give a discussion of the previous fits, particularly regarding the value of the CP violating phase $\delta$. Section 5 is devoted to the conclusions.

2 Three and Four Neutrino Oscillations

Our ordering of the mass eigenstates corresponds to increasing absolute value of the mass, $m^2_i < m^2_j$ for $i < j$. We restrict ourselves, for simplicity, to the hierarchical 3+1 scheme ($\Delta m^2_{21} = \Delta m^2_{23} = \Delta m^2_{atm} \, \text{and} \, \Delta m^2_{43} = \Delta m^2_{LSND}$) and to the so-called “class II-B” 2+2 scheme ($\Delta m^2_{21} = \Delta m^2_{23}, \, \Delta m^2_{32} = \Delta m^2_{LSND}$ and $\Delta m^2_{43} = \Delta m^2_{atm}$). All the other spectra may be obtained changing the sign to one or more of the squared mass differences.

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [1, 2, 3, 4] for three (active) neutrinos is usually parameterized in terms of three angles $\theta_{ij}$ and one CP violating phase $\delta$ (for Majorana neutrinos, two additional CP violating phases appear, but with no effect on the oscillations):

$$U^{(3)}_{PMNS} = U_{23}(\theta_{23}) \ U_{13}(\theta_{13}, \delta) \ U_{12}(\theta_{12}) .$$

(1)

Four neutrino oscillations imply a 4 × 4 mixing matrix, with six rotation angles and three phases (for Majorana neutrinos, three additional phases are allowed). It is convenient to use two slightly different parameterizations. In the 3+1 case we write the mixing matrix as

$$U^{(3+1)}_{PMNS} = U_{14}(\theta_{14}) \ U_{24}(\theta_{24}) \ U_{34}(\theta_{34}) \ U_{23}(\theta_{23}, \delta_3) \ U_{13}(\theta_{13}, \delta_2) \ U_{12}(\theta_{12}, \delta_1) .$$

(2)

which reduces to the three-family case in the limit $\theta_{4i} \to 0$.

A more convenient parameterization for the 2+2 case, as shown in [32], is

$$U^{(2+2)}_{PMNS} = U_{14}(\theta_{14}) \ U_{13}(\theta_{13}) \ U_{24}(\theta_{24}) \ U_{23}(\theta_{23}, \delta_3) \ U_{34}(\theta_{34}, \delta_2) \ U_{12}(\theta_{12}, \delta_1) .$$

(3)
The general formula for the oscillation probability is well known:

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}[U_{\alpha,i}U_{\beta,j}U_{\alpha,j}^*U_{\beta,i}^*] \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right) \]

\[ \mp 2 \sum_{i>j} \text{Im}[U_{\alpha,i}U_{\beta,j}U_{\alpha,j}^*U_{\beta,i}^*] \sin \left( \frac{\Delta m^2_{ij} L}{2E} \right) \]  

(4)

(the lower sign applies to the antineutrino case).

In all of our numerical calculations we used the exact formulae, including matter effects, that are only important for pathlengths greater than about 2000 Km (see fig. 1 for an example in the three–family model).

![Figure 1: Transition probability in vacuum and in matter (for constant \( \rho = 2.8 \text{ g cm}^{-3} \)) in the \( \nu_e \rightarrow \nu_\mu \) (\( \bar{\nu}_e \rightarrow \bar{\nu}_\mu \)) channel for \( E = 38 \text{ GeV} \). The following parameters have been assumed: \( \theta_{12} = 22.5^\circ \), \( \theta_{23} = 45^\circ \), \( \theta_{13} = 13^\circ \), and vanishing CP violating phase.](image)

First, we recall the three–family oscillation probabilities in vacuum at distances short enough, so that the oscillations of longest length, due to \( \Delta m^2_{\odot} \), do not have time to develop \[49\]. In the three–neutrino case, neglect of solar \( \Delta m^2 \) entails no CP violation, and the oscillation probabilities are:

\[ P_3(\nu_e \rightarrow \nu_\mu) = \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right) \]  

\[ P_3(\nu_e \rightarrow \nu_\tau) = \cos^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right) \]  

\[ P_3(\nu_\mu \rightarrow \nu_\tau) = \cos^4(\theta_{13}) \sin^2(2\theta_{23}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right) \]  

\[ P_3(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta_{13}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right) \]  

\[ P_3(\nu_\mu \rightarrow \nu_\mu) = 1 - \left[ \cos^4(\theta_{13}) \sin^2(2\theta_{23}) + \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \right] \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right) \]  

(9)
The explicit expressions for the exact formulae in the four-family case are quite heavy. It may be of interest to present approximate formulae, that allow better intuition at least in their range of validity. We consider therefore the shortest distance from the neutrino source, \( L = 732 \text{ Km} \) (the CERN–LNGS and Fermilab–MINOS distance), where matter effects are negligible and we present approximate formulae for the 3+1 four-family mixing in vacuum (see also [33]). At this distance the effects of the large \( \Delta m^2_{\text{LSND}} \) oscillations are averaged and the effects of the small \( \Delta m^2_\odot \) are negligible. We also assume, in agreement with present bounds [40], \( \theta_{14} = \theta_{24} = \epsilon \) and we expand in power series in \( \epsilon \) to deduce simple expressions to be compared with eqs. (5-9). The result for vanishing CP violating phases is:

\[
P_{3+1}(\nu_e \rightarrow \nu_\mu) &= 4c_{13}^2 s_{13}^2 s_{23} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) - 8\epsilon c_{13}^2 c_{23} s_{13} s_{23} (s_{13} + c_{13} s_{23}) s_{34} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + O(\epsilon^2), \quad (10) \\
P_{3+1}(\nu_e \rightarrow \nu_\tau) &= 4c_{13}^2 c_{23}^2 s_{13}^2 c_{13} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + 4\epsilon c_{13} c_{23} s_{13} (1 - 2c_{13}^2 c_{23}) c_{34}^2 s_{34} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + O(\epsilon^2), \quad (11) \\
P_{3+1}(\nu_\mu \rightarrow \nu_\tau) &= 4c_{13}^4 c_{23}^2 c_{34}^2 s_{23} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + 4\epsilon c_{13}^2 c_{34}^2 s_{34} c_{23} s_{23} (1 - 2c_{13}^2 c_{23}) \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + O(\epsilon^2), \quad (12) \\
P_{3+1}(\nu_e \rightarrow \nu_e) &= 1 - 4c_{13}^2 s_{13}^2 \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + 8\epsilon c_{13} c_{23} s_{13} (c_{13}^2 - s_{13}^2) s_{34} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + O(\epsilon^2), \quad (13) \\
P_{3+1}(\nu_\mu \rightarrow \nu_\mu) &= 1 - 4c_{13}^2 s_{23} (1 - c_{13}^2 s_{23}) \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + 8\epsilon c_{13} c_{23} [c_{23}^2 + (-1 + 2s_{13}^2 s_{23}) s_{34} \sin^2 \left( \frac{\Delta m^2_{32} L}{4E} \right) + O(\epsilon^2). \quad (14)
\]

The three-family limit is plainly reached for \( \epsilon \) and \( \theta_{34} \rightarrow 0 \). We notice that the \( \theta_{34} \) dependence is generally suppressed with powers of \( \epsilon \), except in the \( \nu_\mu \rightarrow \nu_\tau \) channel, that is therefore the most sensitive to this angle. From the above formulae it appears that for small enough \( \epsilon \) it will be extremely difficult to distinguish a four-family 3+1 scheme from three families looking at \( \nu_\mu \rightarrow \nu_e \) transitions, regardless of the value of \( \theta_{34} \).

When the “data” generated in the 3+1 model are fitted in the three-neutrino theory, the result (if successful) fixes the three-neutrino parameters to some values, that may
be called “effective”. It will be useful for our discussion to present explicit formulae for some of them, keeping the small $\epsilon$ approximation. They can be easily derived from the equations given above, and are:

$$
\sin^2(2 \theta_{23}^{eff}) = \sin(2 \theta_{23}) \left[ \sin(2 \theta_{23}) - 4 \epsilon s_{34} \cos(2 \theta_{23}) + O(\epsilon^2) \right], \quad (15)
$$

$$
\sin^2(2 \theta_{13}^{eff}) = \sin(2 \theta_{13}) \left[ \sin(2 \theta_{13}) - 4 \epsilon s_{34} c_{23} c_{13}^2 + O(\epsilon^2) \right]. \quad (16)
$$

Contrary to the 3+1 model, the other four–neutrino option, 2+2, does not have the three–neutrino theory as a limit for small gap–crossing mixing angles. In this scheme, some of the oscillation probabilities have an expression sufficiently simple to be written down explicitly. In the following formulae, the only approximation made is to neglect the solar neutrino mass difference $\Delta m^2_{\odot}$ and to average the rapid oscillations due to $\Delta m^2_{LSND}$.

$$
P_{2+2}(\nu_e \rightarrow \nu_\mu) = 2 c_{23}^2 s_{23} c_{13} c_{24}^2 - c_{13}^2 c_{23}^2 \left[ \sin^2(2 \theta_{34}) \left( c_{24}^2 s_{23}^2 - s_{24}^2 \right) + \sin(4 \theta_{34}) c_{24} s_{24} s_{23} \right] \sin^2 \left( \frac{\Delta m^2_{23} L}{4E} \right),
$$

$$
P_{2+2}(\nu_e \rightarrow \nu_e) = 1 - 2 c_{23}^2 c_{24} \left( c_{24}^2 s_{23}^2 + s_{24}^2 \right) - \left[ \sin(2 \theta_{34}) \left( c_{24}^2 s_{23}^2 - s_{24}^2 \right) + 2 \cos(2 \theta_{34}) c_{24} s_{24} s_{23} \right] \sin^2 \left( \frac{\Delta m^2_{23} L}{4E} \right),
$$

$$
P_{2+2}(\nu_\mu \rightarrow \nu_\mu) = 1 - 2 c_{13}^2 c_{23} \left( s_{13}^2 + c_{13}^2 s_{23}^2 \right) - 4 c_{13}^4 c_{23}^2 c_{34}^2 s_{34}^2 \sin^2 \left( \frac{\Delta m^2_{13} L}{4E} \right).
$$

The oscillation probabilities involving tau neutrinos may be easily obtained: their expressions are however rather cumbersome, and we omit them here for simplicity.

It may be noted that in the limit of vanishing gap–crossing angles eq. (17) reduces to two independent two–neutrinos oscillations, $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_e \leftrightarrow \nu_e$ (that is ineffective since $\Delta m^2_{\odot} = 0$). Therefore, the possibility of confusion with a three–neutrino scheme is practically inexistent, as we will discuss later.

3 Three or four families?

In a few years from now the LSND results will be confirmed by MiniBooNE or not. If the former is the case, a short baseline experiment that could explore the LSND gap–crossing parameter space would be mandatory. In case of a non–conclusive result, the three–family mixing model will be considered the most plausible extension of the Standard Model. If this is the case, long baseline experiments will be preferred with respect to the (four–family inspired) short baseline ones. This is the scenario that we would like to explore, trying to answer the following question: will a Neutrino Factory
and corresponding detectors (designed to explore the three-family mixing model) be able to tell three neutrinos from four neutrinos?

We consider the following “reference set-up”: neutrino beams resulting from the decay of $2 \times 10^{20} \mu^+\text{s}$ and $\mu^-\text{s}$ per year in a straight section of an $E_\mu = 50$ GeV muon accumulator. An experiment with a realistic 40 Kton detector of magnetized iron and five years of data taking for each polarity is envisaged. Detailed estimates of the corresponding expected backgrounds and efficiencies have been included in the analysis \[50\]. Three reference baselines are discussed: 732 Km, 3500 Km and 7332 Km. We follow the analysis in energy bins as made in \[28, 51, 52\]. The $\nu_e \rightarrow \nu_\mu$ channel, the so-called golden channel, will be the main subject of our investigations.

Let $N^{\lambda}_{i,p}$ be the total number of wrong-sign muons detected when the factory is run in polarity $p = \mu^+, \mu^-$, grouped in energy bins specified by the index $i$, and three possible distances, $\lambda = 1, 2, 3$ (corresponding to $L = 732$ Km, $L = 3500$ Km and $L = 7332$ Km respectively). In order to simulate a typical experimental situation we generate a set of “data” $n^{\lambda}_{i,p}$ as follows: for a given value of the oscillation parameters, the expected number of events, $N^{\lambda}_{i,p}$, is computed; taking into account backgrounds and detection efficiencies per bin, $b^{\lambda}_{i,p}$ and $\epsilon^{\lambda}_{i,p}$, we then perform a gaussian (or poissonian, depending on the number of events) smearing to mimic the statistical uncertainty:

$$n^{\lambda}_{i,p} = \frac{\text{Smear}(N^{\lambda}_{i,p} \epsilon^{\lambda}_{i,p} + b^{\lambda}_{i,p}) - b^{\lambda}_{i,p}}{\epsilon^{\lambda}_{i,p}}. \quad (18)$$

Finally, the ”data” are fitted to the theoretical expectation as a function of the neutrino parameters under study, using a $\chi^2$ minimization:

$$\chi^2_\lambda = \sum_p \sum_i \left( \frac{n^{\lambda}_{i,p} - N^{\lambda}_{i,p}}{\delta n^{\lambda}_{i,p}} \right)^2, \quad (19)$$

where $\delta n^{\lambda}_{i,p}$ is the statistical error for $n^{\lambda}_{i,p}$ (errors on background and efficiencies are neglected). We verified that the fitting of theoretical numbers to the smeared (“experimental”) ones is able to reproduce the values of the input parameters.

We then proceed to fit with a three-neutrino model the “data” obtained smearing the (3+1)–neutrino theoretical input. We fixed all but two of the input parameters (in particular the CP violating phases have been put to zero) and calculated the $\chi^2$ fitting with the 3$\nu$ theory for a grid of values of the two varying parameters. In detail, we fixed the values of $\Delta m^2_{LSND} = 1$ eV$^2$, $\Delta m^2_{atm} = 2.8 \times 10^{-3}$ eV$^2$, $\Delta m^2_{\odot} = 1 \times 10^{-4}$ eV$^2$, $\theta_{12} = 22.5^\circ$ and $\theta_{23} = 45^\circ$, we also assumed $\theta_{14} = \theta_{34} = 2^\circ, 5^\circ, 10^\circ$, and allowed a variation of $\theta_{13}$ from one to ten degrees in steps of 0.5$^\circ$ and of $\theta_{34}$ up to 50$^\circ$ \[53\] in steps of 1$^\circ$. The matter effects have been evaluated assuming an average constant density

\[4\] In principle, at distances above 4000-5000 Km a more accurate earth density profile (such as the PREM \[54\]) should be used instead of the constant density approximation adopted here. This point has been thoroughly discussed in \[55\]. The qualitative features of our results, however, do not depend on this approximation.
\[ \rho = 2.8 (3.8) \text{ g cm}^{-3} \text{ for } L = 732 \text{ or } 3500 \text{ (7332) Km and a neutron fraction } Y_n = 1/2 \text{ along the neutrino path. In order to obtain results in a form that helps intuition, after checking that the conclusions of a general fit are similar, we kept two of the four parameters in the three–neutrino model to fixed values } \theta_{12} = 22.5^\circ \text{ and } \theta_{23} = 45^\circ . \text{ We allowed the remaining parameters to vary in the intervals } 1^\circ \leq \theta_{13} \leq 10^\circ \text{ and } -180^\circ \leq \delta < 180^\circ . \]

Figure 2: Oscillation probability for \( \nu_e \rightarrow \nu_\mu \) as a function of the pathlength for \( E = 38 \text{ GeV} \) and (a) \( \epsilon = 2^\circ \) or (b) \( \epsilon = 5^\circ \). See text for further details.

In order to better understand the outcome of our fits, we consider the \( \nu_\mu \rightarrow \nu_e \) conversion probability as a function of the pathlength for a fixed neutrino energy \( E = 38 \text{ GeV} \), corresponding to the effective mean energy (including the effect of the cross section) in a Neutrino Factory with 50 GeV muons, in both scenarios. We stick to the simplified approach of eqs.(10-14) and assume equal values for the angles \( \theta_{14} = \theta_{24} = \epsilon \), after checking that possible sign changes in these quantities do not modify the general behaviour of our results at the distances considered. In fig. 2a (2b) this common value is taken to be \( 2^\circ \)(5\(^\circ\)). The results for two different values of \( \theta_{34} \), \( 2^\circ \) and \( 40^\circ \), for \( \theta_{13} = 8^\circ \) and vanishing CP phases have been plotted in these figures (the wiggly lines). The smooth lines give the predictions of the three–neutrino model for \( \delta = 0 \) and \( \theta_{13}^{eff} \) as given by eq.(16).

It is convenient to write in somewhat greater detail eq.(11),

\[
P_{3+1}(\nu_e \rightarrow \nu_\mu) = 4 s_{14}^2 s_{24}^2 c_{34}^2 \sin^2 \left( \frac{\Delta m_{43}^2 L}{4E} \right) + \left[ 4 c_{13}^2 s_{13}^2 s_{23}^2 - 8 c_{13}^2 c_{23} s_{13} s_{23} (s_{13} s_{24} + c_{13} s_{23} s_{14}) s_{34} \right] \sin^2 \left( \frac{\Delta m_{23}^2 L}{4E} \right) + O(s_{14}^2, s_{24}^2, s_{14} s_{24}) ,
\]

Note that these parameter values are equal to the input (3+1 model) values. This is understandable, since at the considered distances the effects of \( \Delta m_{32}^2 \neq 0 \) are subleading, and the actual value of \( \theta_{12} \) is therefore almost irrelevant; on the other hand, from eq.(15) it appears that the maximal mixing condition \( (\theta_{23} = 45^\circ) \) holds in both theories up to corrections that are of second order in the small quantities \( \theta_{14}, \theta_{24} \) and \( \theta_{13} \).
where we explicitly show all the involved angles. We also include the term oscillating with the shortest length, due to the LSND mass difference, although it is of higher order in the expansion for small \( s_{14} \) and \( s_{24} \). For small \( L \) this term dominates the transition probability and it makes the four–neutrino prediction larger than the three–neutrino result, where this term does not exist. This effect is numerically relevant for large enough gap–crossing angles only, as it may be seen comparing fig. 2a and fig. 2b. At intermediate distances \( (L \sim 3000 \text{ Km}) \) the oscillation probabilities in the two models are quite similar: at this distance we expect therefore that it will be difficult to tell three from four neutrinos. At larger distances \( (L > 5000 \text{ Km}) \) the distinction will in general be possible for \( \theta_{34} \) large enough.

![Figure 3: Oscillation probability for \( \bar{\nu}_e \to \bar{\nu}_\mu \) as a function of the pathlength for \( E = 38 \text{ GeV} \) and (a) \( \epsilon = 2^\circ \) or (b) \( \epsilon = 5^\circ \). See text for further details.](image)

In the case of antineutrinos, a similar pattern is observed. The term oscillating with the atmospheric mass difference in matter shows a dip at \( L \simeq 8000 \text{ Km} \), evident in fig. 3a. At this distance the term oscillating with LSND frequency dominates: the average probability for \((3+1)–\)antineutrinos in matter is non-zero for \( \epsilon = 5^\circ \) and \( \theta_{34} = 40^\circ \), see fig. 3b. We also note that the oscillation probability in the antineutrino case is very small and this fact, together with the further lowering factor due to the ratio of the fluxes and cross sections, makes the total number of oscillated events essentially dominated by the neutrino contribution.

We discuss now the fits that we made to “data” generated in a \((3+1)–\)neutrino model using a three–neutrino theory. The results depend heavily on the values of the small gap-crossing angles \( \theta_{14} \) and \( \theta_{24} \), that we have assumed to be equal for simplicity. If their value is small \( (2^\circ) \), since the \((3+1)\) model has a smooth limit to the three–neutrino theory, the fit is possible for almost every value of the other parameters. This is shown in fig. 4, where in the dark regions of the “dalmatian dog hair” plot the three–neutrino model is able to fit at 68% c.l. the data generated with those parameter values. The data include five energy bins, with muons of both signs, but the first bin in each polarity has been discarded, since the expected efficiency is very low.

The reason of the blotted behaviour stands in statistical fluctuations in the smearing
of the input distributions. In the first row the data collected at three different distances ($L = 732, 3500$ and $7332$ Km) are fitted separately: the number of degrees of freedom in these cases is 6 (the $68\%$ c.l. bound is at $\chi^2 = 1.17 \times 6$). Notice that the largest distance gives a somewhat better possibility to tell three from four. In fact, for small values of $\theta_{34}$, the $\nu_e \rightarrow \nu_\mu$ transition probabilities in matter for both schemes are essentially the same whereas for larger $\theta_{34}$ they become different at large distances (see fig. \[\text{Fig. 2}\] for $E = 38$ GeV).

The second row in fig. \[\text{Fig. 4}\] shows the combined fit to two different distances ($\lambda = 1$
and $2, 1$ and $3, 2$ and $3$, respectively). Here the number of degrees of freedom is 14 (the 68% c.l. bound is at $\chi^2 = 1.14 \times 14$). Notice that the longest baseline causes a slight improvement in the discrimination when combined with the other distances. The last plot refers to using data at $\lambda = 1, 2$ and $3$ simultaneously (here we have 22 d.o.f. and the 68% c.l. bound at $\chi^2 = 1.12 \times 22$).

Increasing the number of energy bins from five to ten (i.e. assuming a detector with better resolution) for $\epsilon = 2^\circ$, we have not observed a sensible reduction of the blotted regions; we interpret this result as due to the extremely poor statistics per energy bin in this case. We do not present the corresponding figure.

On the contrary, increasing the value of $\theta_{14}$ and $\theta_{24}$, the extension of blotted regions decreases. This is shown for $\theta_{14} = \theta_{24} = 5^\circ$ in fig. 3 for five energy bins. In the first row we again present a separate fit for each baseline. Notice that for the shortest baseline we can always tell $3$ from $3+1$. Therefore, in the second row we present only the combination of $\lambda = 2$ and $3$.

![Figure 5](image)

Figure 5: Plots at 68% in the four–family plane for different baselines for $\theta_{14} = \theta_{24} = 5^\circ$. From left to right and top to bottom: $\lambda = 1$; $\lambda = 2$; $\lambda = 3$; $\lambda = 2 + 3$.

The interpretation of the results shown in the figure can be easily done following our previous considerations and fig. 2b. At the shortest distance, the term in the oscillation probability varying with the atmospheric mass difference becomes negligible, and the LSND term dominates: the $3+1$ model gives a probability larger than the three–neutrino theory and confusion is not possible. The oscillation probabilities at the
intermediate distance are very similar in the two models, and the confusion is therefore maximal in this case.

This situation might be improved if a larger number of energy bins could be attained, in that the different energy dependence in matter would help in the distinction of the two models. In fig. 6 we present the “dalmatian dog hair” plot when the “data” consist of ten energy bins. The regions of confusion in this case are considerably reduced. As a consequence, a fit with the wrong theory to the largest baseline data is only possible for low (\(\lesssim 10^\circ\)) values of \(\theta_{34}\).

Figure 6: Plots at 68 % in the four–family plane for different baselines for ten bins and \(\theta_{14} = \theta_{24} = 5^\circ\). From left to right and top to bottom: \(\lambda = 1\); \(\lambda = 2\); \(\lambda = 3\); \(\lambda = 2 + 3\).

A further increase of the gap–crossing angles to \(\theta_{14} = \theta_{24} = 10^\circ\) makes the distinction (even at 95% c.l.) of the two models possible for all values of the other, variable parameters.

A quantitative feeling of the possibility of confusion (or, with a more optimistic attitude, of discrimination) can be given by the numbers reported in Table 1. They correspond to the fraction of points, among the 969 “data” sets fitted, in which the three-neutrino model can give a good fit (at 68% c.l.). The same points have been smoothly joined to obtain the “dalmatian dog hair” plots in figs. 4, 5. The empty cells in the table correspond to situations of no possible confusion.

To complete our discussion regarding the 3+1 scheme against the three–family
\[ \lambda = 1 \quad \lambda = 2 \quad \lambda = 3 \quad \lambda = 1 + 2 \quad \lambda = 1 + 3 \quad \lambda = 2 + 3 \quad \lambda = 1 + 2 + 3 \]

|                      | \( \lambda = 1 \) | \( \lambda = 2 \) | \( \lambda = 3 \) | \( \lambda = 1 + 2 \) | \( \lambda = 1 + 3 \) | \( \lambda = 2 + 3 \) | \( \lambda = 1 + 2 + 3 \) |
|----------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 5 bins \( \epsilon = 2^\circ \) | 44%               | 54%               | 33%               | 42%               | 22%               | 27%               | 20%               |
| fig. 4               |                   |                   |                   |                   |                   |                   |                   |
| 5 bins \( \epsilon = 5^\circ \) | -                 | 54%               | 32%               | -                 | -                 | 26%               | -                 |
| fig. 5               |                   |                   |                   |                   |                   |                   |                   |
| 10 bins \( \epsilon = 5^\circ \) | -                 | 36%               | 11%               | -                 | -                 | 8%                | -                 |
| fig. 6               |                   |                   |                   |                   |                   |                   |                   |

Table 1: Fraction of “data” points generated with a four–family theory that can be fitted with a three– neutrino model.

Figure 7: Plots at 68% c.l. in the four–family plane for different baselines in the \( \nu_\mu \rightarrow \nu_\tau \) channel, for \( \theta_{14} = \theta_{24} = 2^\circ \). From left to right: \( \lambda = 1 \); \( \lambda = 2 \); \( \lambda = 3 \).

Due to unitarity, we also have

\[ P_{3+1}(\nu_\mu \rightarrow \nu_\tau) \rightarrow c_{34}^2 P_3(\nu_\mu \rightarrow \nu_\tau) \, . \]  

\[ P_{3+1}(\nu_\mu \rightarrow \nu_s) \rightarrow s_{34}^2 P_3(\nu_\mu \rightarrow \nu_\tau) \, . \]  

The region where confusion between the 3+1 model and the three–family theory is possible is that of low \( \theta_{34} \). In this case, we considered an idealized detector with constant efficiency (35%) and a very low level background \((10^{-5})\). The mass of the detector is 4 Kton and again five energy bins have been assumed. In fig. 7 we show the 68% c.l. plots for this channel, where the expected behaviour is clearly observable, for any of the considered baselines.
Up to this point, we have only discussed in this Section the 3+1 scheme. We noticed in the previous Section that it is much more difficult to reproduce “data” generated in the 2+2 four neutrino scheme with the three-family theory. We tried to do this in much the same manner as in the 3+1 case. The result can be described giving the numbers corresponding to the entries of the previous Table, that are all smaller than 5%. This maximum value is obtained for the intermediate distance, $L = 3500$ Km, and corresponds to a narrow region for quite small values of all the gap–crossing angles. Therefore we do not present the relative “dalmatian” plots, which in this case are affected by albinism.

4 CP violation vs. more neutrinos

In the previous Section, we have extensively illustrated when confusion between the three–neutrino and the (3+1)–neutrino models is possible. However, we have not presented in detail the results of a fit of the four–family “data” in the parameter space of the three–family model: in fact, the “dalmatian” plots in the previous Section represent the regions of confusion in four–family parameter space.

In three families, the free parameters that we considered are one angle, $\theta_{13}$, and the CP violating phase, $\delta$. These parameters are expected to be still unknown (or poorly known) when the Neutrino Factory will be operative. In particular, we are interested here in the following questions:

- is it possible to find a non-vanishing CP violating phase when fitting with the three–family model the 3+1 (CP conserving) “data”?
- is it possible to fit with a 3+1 (CP conserving) theory the data if their fit in the three–neutrino model gives a large CP violating phase $\delta$?

In order to answer the first question, we present in fig. 8 the 68%, 90% and 99% confidence level contours for typical three–neutrino fits (one for each distance) to 3+1 neutrino “data” generated with $\theta_{14} = \theta_{24} = 2^\circ$ and five energy bins. The parameters are best determined for the intermediate distance, 3500 Km, that is close to the optimal distance for studies of CP violation in the three–neutrino scenario [28, 57, 58, 59]. In fact, the value of the fitted phase $\delta$ stays close to zero, except for a possible ‘mirror’ region around $-160^\circ$. The existence of such mirror regions has been discussed in [52, 59]: they may be removed increasing the energy resolution [60] and/or combining data at two different distances. At the largest distance studied, 7332 Km, the uncertainty in the value of $\delta$ is higher, since the matter effects dominate over the possible effect of CP violation in the oscillations and $\delta$ is less easily determined. Finally, at the shortest

\footnote{The c.l. contours correspond to $\Delta \chi^2 = 2.30, 4.61, 9.21$, respectively, since there are two free parameters. The best fit points are represented by stars.}
distance and at 99% c.l. any value of $\delta$ is allowed. Plots at this distance with a similar behaviour have been also presented in [52]: this distance is clearly not suited for a precise determination of the CP violating phase, at least for the set-up of Neutrino Factory and detector considered here. To illustrate the representativeness of the values chosen for the plots in fig. 8 we note that, out of about 6000 successful fits, 31% (50%, 37%) for $\lambda = 1$ (2, 3) give for the CP violating phase a value $-15^\circ < \delta < 15^\circ$.

In order to answer the second question, and as a cross-check of our previous results, we adopted the converse procedure, namely we fitted “data” generated in a three-neutrino, CP violating, scenario with the formulae of the 3+1 model with CP violating phases fixed to zero. We chose for this purpose to vary the three–neutrino parameters in a region where the confusion is often not possible in the previous fit, $1^\circ \leq \theta_{13} \leq 10^\circ$ and $60^\circ \leq \delta \leq 120^\circ$. This region has only a 3% (<0.02%, 1.5%) of successful fits falling in it for $\lambda = 1$ (2, 3). Among the parameters in the 3+1 theory, $\theta_{13}$ and $\theta_{34}$ have been left free to vary, while $\theta_{14} = \theta_{24} = \epsilon$ have been kept fixed to $2^\circ$, $5^\circ$ and $10^\circ$. The corresponding plots for five energy bins and for $\epsilon = 2^\circ$ are presented in fig. 8, where the dark regions correspond to an acceptable fit (at 90% c.l.) in the 3+1 theory.

The results are precisely what was expected on the basis of the previous fits. The fit with the wrong model, with the chosen values of parameters, is possible in many a case for $L = 732$ Km, it is very difficult to obtain for the intermediate distance, $L = 3500$ Km, and at the largest $L$ an intermediate situation holds. Fitting simultaneously data at two different distances the possibility of confusion is strongly reduced and in fact it vanishes if the data at intermediate distance are used in any combination with the others. We also observed that no confusion would be possible assuming the larger values $\epsilon = 5^\circ$ or $10^\circ$.

Our result seems to imply that if the data would point to a maximal CP violating

Figure 8: Confidence level contours for typical points where the three–neutrino theory can well reproduce (3+1)–neutrino “data” at the three distances studied: from left to right, $L = 732$, 3500 and 7332 Km.
Figure 9: Examples of regions in the $(\theta_{13}, \delta)$ plane where the $(3+1)$–neutrino theory with vanishing CP violating phases can reproduce at 90% c.l. three–neutrino “data”. From left to right and top to bottom: $\lambda = 1$; $\lambda = 2$; $\lambda = 3$; $\lambda = 1 + 2$; $\lambda = 1 + 3$; $\lambda = 2 + 3$.

phase in the three–neutrino theory, it would be very difficult to describe them in a theory without CP violation, even if with more neutrinos.

5 Conclusions

The present experimental data for solar and atmospheric neutrinos give quite strong indications in favour of neutrino oscillations and of nonzero neutrino masses. In addition, the results of the LSND experiment would imply the existence of a puzzling fourth, sterile, neutrino state. The comprehensive analysis of all the other data (including the recent SNO results) seems however to disfavour both possible classes of four–neutrino spectra, the so-called 2+2 and 3+1 schemes.

An experimental set-up with the ambitious goal of precision measurement of the whole three-neutrino mixing parameter space (including the extremely important quest for leptonic CP violation) is under study. This experimental programme consists of the development of a “Neutrino Factory” (high-energy muons decaying in the straight
section of a storage ring and producing a very pure and intense neutrino beam) and
of suitably optimized detectors located very far away from the neutrino source. The
effort to prepare such very long baseline neutrino experiments will require a time period
covering this and the beginning of the following decade, in the absence of a conclu-
sive confirmation of the LSND results (that would call instead for a short baseline
experiment to investigate the four-neutrino mixing parameter space).

It is of interest to explore the capability of such an experimental set-up to discrim-
ine between a three-neutrino model and a possible scenario where a fourth neutrino
is included. Some relevant questions came to our mind. Can the experimental data be
described equally well with a three–neutrino theory or a four–neutrino model? Even
worse, is it possible for a CP-conserving three active and one sterile neutrino model to
be confused with a CP-violating three (active) neutrino theory?

In this paper we tried to answer these questions in the framework of an LSND
inspired four neutrino model, assuming a “large” squared mass difference $\Delta m^2 \approx 1 \text{eV}^2$.
However, the approach is in principle independent of the choice of the third mass gap
(in addition to the solar and atmospheric ones) and can be easily repeated for different
values of $\Delta m^2$.

The results of our analysis can be summarized as follows. Data that could be
described in four–family without CP violation can also be fitted with the three–family
formulae in some particular zones of the four–family parameter space, not restricted to
the obvious case of very small angles. These zones are reduced in size for increasing gap–
crossing angles and energy resolution of the detector. The use of $\nu_\mu \rightarrow \nu_\tau$ oscillations
through direct or indirect detection of tau lepton decays would allow a much easier
discrimination of the two theoretical hypotheses.

In the zones of the four–family parameter space that can be also described with
the three-family theory, the fitted value of the CP violating phase, $\delta$, is generally not
large. In particular, this is true for $L = 3500$ Km, whereas for $L = 7332$ Km the
determination of $\delta$ is somewhat looser. For the shortest baseline, $L = 732$ Km, we
have the largest spread in the values of $\delta$, although the most probable value is still
close to zero.

Data that can be fitted with a CP phase close to $90^\circ$ in the three–neutrino theory
cannot be described in a CP conserving 3+1 theory, provided that data at two different
distances are used.

Finally, in the 2+2 scheme (as opposed to 3+1) the ambiguity with a three–neutrino
theory is essentially absent.

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