A new kind of classically stable static solitons called metastable quasi-topological defects (MQTD) and a systematic method to search for them is presented, with examples from realistic particle physics models. They are characterized by a topological winding number, which is not absolutely conserved so that the MQTD may be converted to radiation by quantum mechanical tunneling. The two-Higgs standard model (2HSM) supports the existence of classically stable membranes for Higgs masses consistent with present day phenomenology and with perturbative unitarity, as well as with loop corrected MSSM. We also comment upon the possibility of metastable strings in the generic 2HSM.
1 Introduction

As is well known, apart from a few isolated cases in one spatial dimension, the only known solitons are topological solitons and in order to arise either a non-trivial vacuum manifold or a non-trivial target space are necessary. The corresponding solitons are classified accordingly in two classes.

(I) The simplest paradigm of the first is the kink solution of the model $\mathcal{L} = 1/2(\partial_\mu \phi)^2 - \lambda(\phi^2 - v^2)^2/4$ for the real scalar field $\phi$ in 1+1 dimensions. The model has a non-trivial vacuum manifold (set of degenerate minima) $\mathcal{M} = \{-v, +v\}$. Field fluctuations about any of these vacua describe particles of mass $m = \sqrt{2\lambda}v$. The model admits in addition the well-known kink solution which has finite energy and interpolates smoothly between the two different vacua as one moves in space from $-\infty$ to $+\infty$. Its energy is localised over a distance of order $m$. Furthermore, one may easily be convinced that an infinite energy barrier separates the kink from the vacuum and guarantees its absolute stability\(^3\). To summarize, necessary condition for the existence of the first class of topological solitons is a non-trivial vacuum manifold and the solitons correspond to non-trivial maps of spatial infinity into the vacuum manifold. Examples of such solitons are the topological domain walls in particle physics and cosmology, the superfluid vortices, the Abrikosov vortices in superconductors and their analogue cosmic strings in relativistic models. The magnetic monopoles in 3+1 dimensions also belong to this class of topological solitons.

(II) The existence of a non-trivial field space is the necessary condition for solitons of the second kind to arise, and they correspond to non-trivial maps of space into the target space. The simplest example of a soliton of this class arises in the sine-Gordon model $\mathcal{L} = 1/2|\partial_\mu [\exp(i\Theta)]|^2 + \mu^2 \Re[\exp(i\Theta)] = 1/2(\partial_\mu \Theta)^2 + \mu^2 \cos \Theta$. The above model has a unique vacuum ($\Theta(x) = 0$), a non-trivial target space given by the set $S^1$ of possible values of the field $\exp(i\Theta)$ and supports the well-known sine-Gordon solitons, in which the angle $\Theta$ rotates by $2\pi$ as one scans space from $-\infty$ to $+\infty$. The example generalizes to higher dimensions and other models with more complicated target spaces. The Neel and Bloch domain walls in ferromagnets, the magnetic bubbles of the ferromagnetic continuum and the Belavin-Polyakov soliton in the $O(3)$ non-linear $\sigma$-model in 2+1 dimensions, as well as the skyrmions and the Hopfions in three spatial dimensions are examples of solitons which belong to this second class of topological defects.

In contrast to the case of Condensed Matter systems in which a large variety of topological solitons have been observed and studied extensively, both theoretically and experimentally, no such object has ever been observed in particle physics. Despite of this rather discouraging situation, thinking about such non-perturbative aspects of particle physics has often led to interesting physical phenomena and new ideas about our Universe. The phenomenon of catalysis of proton decay in the presence of a GUT monopole and the initial conception of the idea

\(^3\)Formally, the stability of the kink is a consequence of the conservation of the current $J_\mu = \epsilon_{\mu\nu}\partial^\nu \phi$. 
of inflation which changed our view of the very early universe, are two examples of recent
development triggered by the study of the physics of topological solitons.

Below, I will introduce a new class of defects in theories with **trivial target space and vacuum manifold** which according to the previous discussion do not support the existence of any sort of absolutely stable topological soliton. The solutions I will present are static, have finite energy and correspond to local minima of the energy functional. They are characterized by a topological quantum number which contrary to the case of topological solitons is not absolutely conserved. The defects may be converted to radiation through quantum mechanical tunnelling.

## 2 The method

The simplest system to introduce the Metastable Quasi-Topological Defects and to present the general method used in their search is a complex scalar $\Phi$ with dynamics described by

$$
\mathcal{L} = \frac{1}{2} |\partial_\mu \Phi|^2 - \frac{1}{4}\lambda(|\Phi|^2 - v^2)^2 + \mu^2 v^2 \text{Re}\Phi
$$

in 1+1 spacetime dimensions \([1]\). The potential is a tilted Mexican hat with a unique minimum, while the field space being the whole complex plane is also trivial. As a result the model at hand does not possess any kind of topological solitons. In the limit $\lambda \rightarrow \infty$ though, the generic finite energy configuration has the form $\Phi = v \exp[i\Theta(x)]$, and the target space is reduced dynamically into an effective $S^1$. The dynamics of $\Theta$ is described by the sine-Gordon model with the absolutely stable topological solitons discussed above. It is intuitively natural to expect that once we relax the parameter $\lambda$ to finite values, the soliton will not disappear altogether, but instead, there will be a critical value above which a stable solution will still exist and below which the solution will cease to exist. Indeed, it is straightforward to verify numerically that for $m/\mu \geq 6.1$ a classically stable soliton solution arises, which disappears for $m/\mu < 6.1$ \([1]\). This result has a rather obvious interpretation: The soliton under discussion may be pictured by a closed elastic rubber with equilibrium length equal to zero, tied around the central peak of the tilted potential, without friction and with one point fixed at its minimum as required by the finiteness of the energy. If the central peak is high enough (large value of the parameter $m/\mu$) the rubber cannot slip over the tip of the potential and the soliton is classically stable. If instead the central peak is low the rubber slips over and shrinks to the vacuum configuration. Of course, even the classically stable solution can decay to the vacuum quantum mechanically. It takes finite action for the rubber to follow the classically forbidden path and slip over the barrier. The decay rate per unit length is exponentially suppressed by the Euclidean action of the corresponding bounce.

Recipe: To search for metastable solitons of the kind described above in topologically trivial theories one first determines a limit of the parameters in which the target space becomes non-trivial and the reduced model leads to absolutely stable topological solitons. These solitons generically continue to exist even after relaxing the parameters away from the limit, as long as

\[4\] The only classically relevant parameter of the model is $m/\mu$ as can be seen by rescaling $\Phi \rightarrow v\Phi$ and $x \rightarrow (1/\sqrt{\lambda}v)x$. 

3 Defects in the two-Higgs Standard Model

The potential of the generic two-Higgs doublet \((H_1, H_2)\) Standard Model is

\[
V(H_1, H_2) = \lambda_1(|H_1|^2 - v_1^2/2)^2 + \lambda_2(|H_2|^2 - v_2^2/2)^2 + \lambda_3(|H_1|^2 + |H_2|^2 - (v_1^2 + v_2^2)/2)^2 + \\
+\lambda_4(|H_1|^2|H_2|^2 - (H_1^\dagger H_2)(H_1^\dagger H_1)) + \lambda_5(\text{Re}(H_1^\dagger H_2) - v_1 v_2 \cos \xi/2)^2 + \\
+\lambda_6(\text{Im}(H_1^\dagger H_2) - v_1 v_2 \sin \xi/2)^2
\]

with all parameters \(\lambda_i\) positive or zero. It is the most general form with CP and the discrete symmetry \(H_1 \rightarrow -H_1\) broken only softly, to avoid unacceptably large FCNC effects.

The gauge group of the model is \(G=SU(2) \times U(1)\) broken down to \(H=U(1)_{em}\) and it is well-known that the first and second homotopy groups of \(G/H\) are both trivial. In the absence of any discrete symmetry from the potential of the model the zeroeth homotopy group is also trivial. No topological domain wall, string or magnetic monopole exist in this theory. Since, in addition the target space of the model is trivial \((\mathbb{R}^8)\) no topological solitons of the second class arise.

**Metastable Membranes**: In the limit \(\lambda_1, \lambda_2, \lambda_4 \rightarrow \infty\) (in which all Higgses except \(A^0\) become infinitely massive) the model reduces to a 3+1 dimensional sine-Gordon model, which admits the topological domain walls mentioned several times above. According to the previous discussion, one expects the model to support the existence of such domain walls as local minima of the energy functional even after one relaxes the Higgs masses to finite values, as long as they stay "large enough". The determination of the range of values of the Higgs masses for classically stable membranes to exist is a dynamical issue which was studied numerically in Reference [3]. There one may find the plot of the membrane stability region of the parameter space. It is slightly more complicated but one may roughly state that as long as the mass ratios \(m_h/m_A^0\), \(m_{H^0}/m_{A^0}\) and \(m_{H^+}/m_{A^0}\) in the standard notation [4], are all larger than approximately 2.2, metastable quasi-topological membranes arise in the 2HSM.

Although one may envisage various other possibilities, a naive expectation is that such a membrane will be characterised by a mass of order \(\mathcal{O}(10^{10}\text{ gr/cm}^2)\) and a cosmological life-time per unit area. In such a case they are going to have desasterous consequences in our Universe, exactly like the absolutely stable topological domain walls. The curves of the membrane stability region which appear in Reference [3] should then be interpreted as providing upper bounds on the corresponding Higgs masses. Thus, for example, the set of values \(m_A = 50\text{ GeV}, m_h = 125\text{ GeV}, m_{H^0} = 160\text{ GeV and } m_{H^+} = 200\text{ GeV}\), perfectly consistent with present day experimental bounds analyzed either in the context of the generic 2HSM as well as with the one-loop corrected MSSM [4], [5], should be excluded on cosmological grounds.
In a similar fashion one may consider the limit $\lambda_1, \lambda_2, \lambda_5 \to \infty$ (in which all Higgses except $H^+$ become infinitely massive). Correspondingly, of all the components of the Higgs doublets only a unit vector field remains unfrozen, and the corresponding effective theory is the well-known $O(3)$ non-linear $\sigma$-model with its well-known Belavin-Polyakov topological solitons in two spatial dimensions. Like in the membrane case one may argue analytically and verify numerically that these solitons, slightly deformed, continue to exist even as one takes the parameters away but close to their limiting values.

It is clear from the above discussion that MQTD are guaranteed to arise in appropriate ranges of the Higgs and gauge particle masses in any model with an extended Higgs sector. Consequently, in the case of unwanted walls it is not enough to check that the model of interest does not possess a spontaneously broken discrete symmetry. One has to verify in addition that no metastable membranes exist with cosmological lifetimes. Similarly it may not be necessary to introduce extra spontaneously broken symmetries or carefully chosen target spaces in order to produce domain walls, cosmic strings or localized solitons. They may arise dynamically as metastable defects of the sort discussed here.

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