Rebar movement in seals under static loading

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Abstract. All the positive qualities of reinforced concrete as a building material are ensured mainly by the joint work of reinforcement and concrete. The forces ensuring the retention of reinforcement inside the concrete were called adhesion forces. We can say that adhesion is a combination of physical and mechanical phenomena along the contact surface of reinforcement and concrete, providing their connection and resisting the movement of reinforcement relative to concrete. To describe the real phenomena occurring in the embedment of reinforcement, the actual stress-strain state in concrete and reinforcement, a new calculation model is proposed that describes the actual mechanism of operation and failure of the embedment and methods for calculating stresses in concrete and reinforcement and displacements in the reinforcement. For this, as well as to develop methods for calculating the movement of reinforcing bars and the bearing capacity of the seal, a calculation model is proposed based on the splitting model of Professor A.S. Zalesov. To calculate the movement of reinforcement, a transformed three-line diagram of the deformation of concrete with a descending branch is proposed. Based on the developed calculation model and calculation methods, theoretical dependences are obtained for determining the displacements of the reinforcement of the bar in the seal and the bearing capacity of the reinforcement anchoring. Theoretical results are in good agreement with the experimental data of various authors.

Key words: buildings, construction, reinforced concrete, bond, crushing concrete.

1 Introduction

The essence of reinforced concrete as a material is determined mainly by the laws of interaction of reinforcement and concrete. Reinforcement slipping relative to concrete does not occur due to concrete-to-steel bond.

By bond is meant a combination of physical and mechanical phenomena on the contact surfaces of reinforcement and concrete, providing their connection and resisting movements of the reinforcement in concrete, or in other words, bond is the resistance of concrete to longitudinal movements of reinforcement.

Plain bars used to be one and only reinforcement in concrete elements before reinforcement of the periodic profile has been invented. It has been suggested that plain bars resist movement as a result of adhesive forces and friction forces arising from radial pressure during shrinkage of concrete.

In the transition to the mass use of profiled reinforcement, these ideas were tried to be maintained by introducing additional resistance due to the engagement of the protrusions of the reinforcement. But the adhesive resistance and friction on the smooth surface of the reinforcement between the protrusions are of secondary importance, and the main thing here is the engagement of the protrusions of the reinforcement, i.e. the concrete-to-steel bond of profiled reinforcement is mainly due to concrete crushing resistance.
After the mass application of profiled bars began, numerous studies were conducted [1]-[3]. After the mechanism of interaction of reinforcement with concrete was revealed, many suggestions appeared on its analytical description.

Currently, under static loading, the technical coupling theory (TCT), developed at the All-Russian Research Institute of Reinforced Concrete in Russia [4], based on the application of an analytical model, has found the greatest application. Despite the fact that the reinforcement transfers forces to concrete mainly due to crushing under the protrusions, in this model the reinforcement is modeled by a cylindrical, conditionally smooth rod, and the forces transmitted by it are replaced by stresses referred to the conditional contact surface (figure 1).

![Figure 1. Normal and elastoplastic bond laws at the static loading.](image)

The tangent component of these stresses is called "conditional adhesion stresses" \( \tau_g \), normal – "transverse pressure" \( p_g \). The distribution of these contact stresses is assumed to be axisymmetric; therefore, their values depend on only one coordinate expressing the position of the cross section along the reinforcing bar under consideration. In this case, the bond stresses are uniquely determined by the magnitude of the longitudinal displacements based on the bond law:

\[
\tau_g = F(g).
\]

There are many assumptions regarding the choice of the cohesion law in the simplest form \( \tau_g = k \cdot g \), and in the form of a power law \( \tau_g = k \cdot g^\alpha \), etc. But the most acceptable are the normal law of bond [4]:

\[
\tau_g = \frac{B \cdot \ln(1 + \alpha \cdot g)}{(1 + \alpha \cdot g)}
\]

and elastoplastic bond law [4]:

\[
\tau_g = \begin{cases} 
\tau_0 / g^\star \cdot g, & \text{if } g < g^\star \\
\tau_0, & \text{if } g \geq g^\star 
\end{cases}
\]

where \( \tau_0 = 0.35 \cdot B \) – bond strength characteristic; \( g^\star = 0.55 / \alpha \) – limit value of displacement in the elastic zone; \( \alpha, B \) – bond references:

\[
B = 0.79 \cdot R \cdot \lambda / s_r \cdot [1 - \exp(-0.75 \cdot c_r / s_r)];
\]

\[
s_r \cdot \alpha = 19 \cdot 10^8 \cdot (1 + \alpha \cdot \mu_s) \cdot d_s \cdot \lambda \cdot [(R \cdot c_r) / (E \cdot s_r)]^2;
\]

\( R \) – cubic strength of concrete; 
\( E \) – modulus of elasticity of steel reinforcement; 
\( \lambda = 7.25 cm \) – radius of influence; 
\( s_r \) – pitch of the reinforcing stud; 
\( c_r \) – width of the reinforcing stud; 
\( d_s \) – diameter of the reinforcing bar;
$\alpha_s = E/E_b$ – coefficient;
$\mu_s$ – section reinforcement percentage.

If we consider a prismatic reinforced concrete element of arbitrary cross-section with an area $A_b$, centrally reinforced by one bar $A_s \cdot \mu_s$, length $L$, to which the load is applied at the ends, then according to [4] depending on the invariant $J_0$ it can be reduced to symmetric loading $J>0$ or skew-symmetric loading $J<0$.

$$J_0 = \int [(1+\alpha_s \cdot \mu_s)/E_s \cdot \Delta \sigma_{sx}]^2 \frac{g_x}{0} F(g) dx,$$

where $F(g)$ is the function of the invariant $g$.

$$\Delta \sigma_{sx} = \sigma_{sx} - \sigma_{s,stat},$$

$$\sigma_{s,stat} = 1/(1+\alpha_s \cdot \mu_s) \cdot (\sigma_{sn} + \alpha_s \cdot \sigma_{b0} + \alpha_s \cdot \mu_s \cdot \sigma_{s0}),$$

where $\sigma_{sn}$ – reinforcement prestressing;
$\sigma_{sx}, \sigma_{bx}$ – stress in reinforcement and concrete in cross section $x$.

Depending on the type of boundary conditions, the technical theory of bond makes it possible to solve the following main problems with static loading:

1) at one end of the element the stress in the reinforcement is specified;
2) at both ends of the specified stress in the reinforcement;
3) mutual displacements are set at both ends;
4) at one end, the stress in the reinforcement is set, at the other, mutual displacements.

For calculations based on the normal law of adhesion (2) under static loading [4] tables of special functions were compiled. Depending on the boundary conditions and the case of loading, these tables determine mutual displacements, stresses in the reinforcement, and bond stresses according to (2). Defining them for several points along the length of the element, the corresponding diagrams are built, and with the static loading of the elements.

So, in the TCT, the reinforcement is modeled by a cylindrical, conditionally smooth rod, and the forces transmitted by it are replaced by the stresses referred to the conditional contact surface. TCT does not reflect the actual work of concrete and reinforcement in the seal and therefore is not able to take into account the fact that the transmission of reinforcement forces to concrete occurs mainly due to crushing under the studs.

Attention deserves the model Comité Euro-International du Béton (CEB) [5]. Coupling stresses are determined by the expressions:

$$\tau = \begin{cases} 
\tau_{max} (s/s_1)^a, & 0 \leq s < s_1; \\
\tau_{max} + (\tau_f - \tau_{max}) \left[ (s-s_2)/(s_3-s_2) \right], & s_1 \leq s < s_2; \\
\tau_f, & s_2 \leq s < s_3; \\
\tau_f, & s > s_3.
\end{cases}$$

where $a$ – empirical constant describing the formula for the adhesion/shear curve.

The CEB model claims that the ascending branch refers to the stage at which the edges of the reinforcing bars penetrate the mortar matrix. This stage is characterized by local destruction and the appearance of microcracks. Therefore, on the ascending branch, the bond stress increases nonlinearly to the point at which the shift $s$ is $s_1$ (figure 2). The horizontal equilibrium between $s_1$ and $s_2$ occurs only for reinforced concrete, in which case enhanced destruction and shear of concrete between the ribs is characteristic. At this stage, the bond stress is a constant maximum value. A descending branch refers to a decrease in bond due to the appearance of split cracks along the rods. The last horizontal part of the graph represents the residual bond supported by the minimum transverse reinforcement and corresponding to the untouched part of the structure.

A law similar to the normal law of bond (2) was proposed in [7]:

$$\tau_e = B \cdot \ln(1+a \cdot g)/(1+b \cdot g),$$

A simple approximation in the form presented in [8] also deserves attention:

$$\tau_e = \tau_{max} \cdot (2s_{max}/s)/(s_{max}^2 + s^2).$$
We can also highlight laws that closely describe the experimental data:
- power law of G. Rehm [9];
- power law of Mirza and Howde [10];
- Marty's Law [11];
- the empirical law of D.U. Watstein [12], [14];
- model for calculating the contact stresses Idda [13].

Many researchers tried to theoretically substantiate the results of experimental data, based on the postulates of material resistance, elasticity theory, and contact problems.

For example, Landgren and Gilloft [14], developed a model that takes into account bond stresses and radial deformation between the reinforcing bar and concrete. This approach allows us to describe the bond stresses at the interface between steel and concrete in three dimensions.

Fracture from cleavage, which is the main type of failure for bars in concrete, was investigated by Choi [15]. In his work, factors such as boundary properties, deformation features, and the thickness of the concrete protective layer are taken into account. Also interesting works are [22].

The bond of profiled reinforcement is mainly due to engagement, and therefore, the resistance of concrete to crushing over small areas. Thus, at present there is no adequate theory of bond, which would take into account the real modes of deformation of concrete and reinforcement in the seal, as well as the real mechanisms of its destruction.

2 Materials and methods

In modern conditions in reinforced concrete elements, as a rule, rod reinforcement of a periodic profile is used as working reinforcement. The joint operation of such reinforcement with concrete and its anchoring is ensured, usually due to bond forces. To describe the resistance to embedment of profiled reinforcement, we consider the mechanism of adhesion of reinforcement to concrete. Numerous experimental data of various researchers have indirectly established that adhesion resistance and friction on the smooth surface of the reinforcement between the protrusions are of secondary importance, and the main thing here is the engagement of the protrusions of the reinforcement, i.e. the bond of profiled reinforcement is mainly due to concrete crushing resistance.

In this regard, experimental studies of crushing with a die embedded in concrete are very obvious and useful, with the aim of ensuring the molding contact of this stamp with concrete. If during the experiment the die shape is chosen in such a way that it is close to the shape of the reinforcing ledges, and the height of the concrete ledge under the die is equal to the pitch of the ledges in the reinforcement, then the reinforcement operation will be modeled with very high accuracy. Such an
experience will make it possible to evaluate the dependence of shear stresses on the mutual displacements of the stamp relative to concrete, i.e., to determine the law of bond. Experiment was conducted by G. Rehm in his work. The prototype is shown schematically in the figure (figure 3).

![Figure 3. Test sample G. Rehm.](image)

A rod 2×2 cm in size with a rectangular cutout is concreted along the axis of the sample. One of the cutout surfaces represents a work site during crushing tests. To monitor the behavior of concrete in the contact zone, the sample has windows closed with plexiglass 10 mm thick. On the entire surface of the rod, except for the working platform, a layer of paraffin was applied, which was smelted before the tests. The inner surface of plexiglass was coated with petroleum jelly to avoid the formation of adhesive films. The results of this test are shown in table 1.

| № n/n | C, mm | h, mm | R, MPa | cot(φ_{crc}) | P_{crc}, kN | g_{rc}, mm | P_{max}, kN |
|-------|-------|-------|--------|-------------|-------------|------------|-------------|
| 1     | 2.1   | 20    | 12.5   | 3           | 4.5         | 1.9        | 10.3        |
| 2     | 2.1   | 20    | 4.5    | 1.35       | 3.0         | 1.3        | 4.1         |
| 3     | 2.1   | 20    | 3.5    | 1.26       | 1.8         | 1.3        | 5           |
| 4     | 2.1   | 20    | 45     | 1.78       | 6.4         | 2.7        | 16.2        |

When crushing with buried dies, a molding contact usually takes place, in which there is no coarse aggregate in the zone of concentration of crushing stresses. Taking the form of the stamp close to the shape of the protrusions of the reinforcement, and the height of the concrete protrusion under the stamp – the wound to the step of the protrusions in the reinforcement, in experiments [4] the work of the profiled reinforcement was simulated and it was found that the relationship between the shear stress and the mutual displacements of the stamp relative to the bulk of the concrete is close to coupling law for fittings. In this case, with these sizes and the shape of the stamp, shearing is carried out in a pure form, the main feature of which is the realization of concrete properties on a millimeter scale, namely, increased peel and shear resistance and the manifestation of intragrain shear in the shear zone. Based on the results obtained, a wedge hypothesis was put forward [19]: the pressure of the stamp on concrete is transferred in a way as if a wedge was attached to the protrusion of the stamp, transferring the load on the concrete without friction in the direction normal to its surface.

In [1], it was suggested that at the pressure of a small stamp on concrete, two main fracture mechanisms can be evolved. In both cases, it is believed that a seal zone forms in the form of a wedge.
under the stamp, which bursts the surrounding concrete. In the first mechanism, an intergranular shift is realized, and the destruction occurs due to the formation and development of cracks. In the second mechanism, intragranular shear occurs at high shear stresses inside the compaction wedge.

The formation and development of a plastic wedge leads to the appearance of major compressive stresses having a certain angle of inclination to the direction of collapse under the protrusions, and, consequently, to the development of a spread. The development of the spacer displaces the surrounding concrete from the reinforcement and, thereby, negates the adhesive resistance and friction on a smooth surface between the protrusions. As a result of this interaction between the protrusions of the reinforcement and the surrounding concrete, at some stage of loading, through-hole, internal cracks can form. The formation of these cracks saves the seal structure from the formation of a more dangerous crack – cracks along the reinforcement.

To describe the indicated real phenomena occurring in the embedment of reinforcement, the actual stress-strain state in concrete and reinforcement, a new calculation model is proposed below that describes the actual mechanism of operation and failure of the embedment and methods for calculating stresses in concrete and reinforcement and displacements in the reinforcement. The following methods are used to assess the actual stress-strain state of concrete under the protrusions and the reinforcement itself in the seal based on the developed calculation model:
1. The method of limit equilibrium.
2. The static method of the method of limiting equilibrium.
3. The kinematic method of the method of limiting equilibrium.

3 Results
The resistance of the reinforcement to the longitudinal displacements is determined mainly by the work of concrete for crushing under the protrusions. Since the studs are arranged with a certain step, and under each stud the same stress-strain state is realized, for its components in the seal it is enough to consider the work and destruction of concrete under one stud (figure 4).

![Figure 4](image.png)

**Figure 4.** The formation of a wedge seal under the protrusions of the reinforcement.

In concrete wedge forms under the stamp. The determination of the geometric parameters of this wedge is described in [4]. In this work, the wedge angle is determined by:

$$\cot(\varphi_{wd}) = 0.098 \cdot \left(\frac{c_v}{\lambda}\right)^{0.8},$$

(12)

where $\varphi_{wd}$ – wedge angle in radians; $c_v$ – stamp width in mm; $\lambda = 72.5 mm$ – radius of influence.
It is believed that this wedge presses on the surrounding concrete. Therefore, the wedge pushes the surrounding concrete from the smooth surface of the reinforcement between the protrusions. As a result, the adhesion and friction forces come to naught.

To describe the stress-strain state in the fitting of reinforcing bars under the protrusions, the authors propose a model based on the splitting model of Professor A.S. Zalesov (figure 5).

![Concrete Destruction Model](image)

**Figure 5.** Concrete Destruction Model.

The bearing capacity of the concrete section between the studs depends on the forces arising in the stretched zone between the top of the wedge and the lower stud $N_{t,wd}$, and on the tangential forces along the face of the compaction wedge $T_{wd}$.

To determine the dependencies, the method of limiting equilibrium is used. We compose the equations of equilibrium of forces on the $X$ and $Y$ axis:

\[
\Sigma Y=0: \quad R_{wd}\cos(\varphi_{wd})+T_{wd}\sin(\varphi_{wd})-N_{loc}=0; \quad (13)
\]

\[
\Sigma X=0: \quad R_{wd}\sin(\varphi_{wd})-T_{wd}\cos(\varphi_{wd})-N_{c,wd}=0; \quad (14)
\]

\[
\Sigma X=0: \quad N_{t,wd}-N_{c,wd}=0. \quad (15)
\]

Solving equations (13)-(15) together, we find the value of $N_{loc}$:

\[
N_{loc}=N_{t,wd}\cos(\varphi_{wd})+T_{wd}/\sin(\varphi_{wd}). \quad (16)
\]

At the time of the onset of fracture due to the accumulation of residual stresses, the distribution of tensile stresses is fairly uniform. The ultimate bearing capacity of the stretched zone between the top of the wedge and the lower ledge is determined:

\[
N_{t,wd}=\omega_{t}\cdot R_{h}\cdot b\cdot h, \quad (17)
\]

where $\omega_{t}=0.75$ — tensile stress diagram completeness coefficient.

$h_{t}$ — the length of the stretched zone of the element is determined:

\[
h_{t}=s_{r,c}\cdot t\cdot \tan(\varphi_{wd}). \quad (18)
\]

It is also known that at the moment of fracture onset, the distribution of shear stresses is uniform, and the ultimate shear forces along the faces of the compaction wedges are:

\[
T_{wd}=-\tau_{wd}\cdot b\cdot l_{sh}, \quad (19)
\]

where $\tau_{wd}$ — tangential stresses;

$l_{sh}$ — shear length determined:

\[
l_{sh}=c_{r}/\cos(\varphi_{wd}). \quad (20)
\]

In the limiting stage, the value of $N_{loc}$:

\[
N_{loc}=R_{h,loc}\cdot b\cdot l_{loc}, \quad (21)
\]
where \( b \) – ring die circumference;
\( l_{loc} \) – cargo area width:
\[
l_{loc} = c_r
\]
Substituting Eq. (17), (19) and (21) into Eq. (16), taking into account Eq. (18), (20) and (22), after simple transformations, we have:
\[
R_{b,loc} = 0.75 \cdot R_{bt} \cdot \frac{s_r}{c_r} \cdot \cot(\phi_{wd}) \cdot \tau_{wd} / [\sin(\phi_{wd}) \cdot \cos(\phi_{wd})].
\]
Equilibrium Eq. (13)-(15) one and only are not enough. To determine the tangential stresses \( \tau_{wd} \), it is necessary to use the kinematic method of the limit equilibrium method (figure 6).

![Figure 6. Calculation model of concrete deformation under local compression.](image)

When moving the wedge in the vertical direction, the strains are equal to:
\[
\Delta_b = \int_{c_r \cdot \tan \phi_{wd}}^{s_r} \varepsilon_{1c} (h) \, dh.
\]

The transverse movement of the wedge in concrete from the action of the compressive force flow is equal to:
\[
\nu_{zt} = \frac{1}{\tan \phi_{wd}} \int_{c_r \cdot \tan \phi_{wd}}^{s_r} \varepsilon_{1c} (h) \, dh.
\]

Moving the edges of the wedge relative to concrete during shear:
\[
\Delta_{bsh} = \frac{1}{\sin \phi_{wd}} \int_{c_r \cdot \tan \phi_{wd}}^{s_r} \varepsilon_{1c} (h) \, dh.
\]

Relative compression strains are defined as a function of:
\[
\varepsilon_{b, max} = f(\sigma_{ic, max}).
\]

According to the diagram of concrete deformation, taking into account the behavior of concrete on a small scale, i.e. knowing the value of stress in concrete under the wedge, it is possible to determine the relative deformations that occur when moving the wedge.

For this, we propose a transformed three-line diagram of concrete deformation with a descending branch (figure 7).
To determine the strains $\varepsilon_{b}^{\text{max}}$, it is necessary to establish the regularity of the change in compressive stresses along the element height, since it is incorrect to take $\sigma_{1c}^{\text{max}}=\text{const}$. Using the assumption that compressive stresses are uniformly distributed over the height of the element leads to large discrepancies with experimental data.

To determine the pressure under the tip of the wedge, we use the solution of the elementary problem of J. Boussinesq (figure 8).

$$\sigma_{x} = q \cdot \left\{ 2 \cdot (\theta_{1} - \theta_{2}) + \sin(2 \cdot \theta_{2}) - \sin(2 \cdot \theta_{1}) \right\} / (2 \cdot \pi).$$

Point M lies on the X axis:

$$\sigma_{1c}^{\text{max}} = -\left\{ (R_{b}^{\text{loc}}) / \pi \right\} \cdot [2 \cdot \theta + \sin(2 \cdot \theta)].$$

Substituting (29) in (27), and then the resulting expression in (26), we have:

$$\Delta_{\text{hub}} = (R_{b}^{\text{loc}} \cdot c_{s}) / [2 \cdot \sin(\varphi_{ub}) \cdot E_{c}] \cdot 1 / \pi \cdot \left\{ 2 \cdot \ln[\sin(\theta_{k}) / \cos(\theta_{k})] - \theta_{k} \cdot \cot(\theta_{k}) + \theta_{u} \cdot \cot(\theta_{u}) \right\}.$$

where

$$\theta_{u} = \pi / 2 - \varphi_{ub};$$

$$\theta_{k} = \arctan(c_{s} / s_{s}).$$
Knowing the absolute value of shear deformations, we determine the relative deformations:

\[ \gamma_{wd} = \Delta b_{sh}/l_{sh} = 0.5 \cdot (R_{b_{loc}}/E_b) \cdot cot(\varphi_{wd}) \cdot 1/\pi \cdot \{2 \cdot ln[(sin(\theta_{h})/cos(\theta_{h})) - \theta_k \cdot cot(\theta_{h}) + \theta_n \cdot cot(\theta_{n})] \}. \tag{30} \]

From Eq. (33) we determine the tangential stresses:

\[ \tau_{wd} = \gamma_{wd} \cdot G_b, \] \tag{34} \]

where \( G_b \) – shear modulus:

\[ G_b = E_b/[2 \cdot (1 + \nu)]. \tag{35} \]

\( \nu \approx 0.2 \) – Poisson's ratio.

Finally we have:

\[ \tau_{wd} = 5/24 \cdot R_{b_{loc}} \cdot cot(\varphi_{wd}) \cdot 1/\pi \cdot \{2 \cdot ln[(sin(\theta_{h})/cos(\theta_{h})) - \theta_k \cdot cot(\theta_{h}) + \theta_n \cdot cot(\theta_{n})] \}. \tag{36} \]

Now, substituting Eq. (36) in Eq. (23), we determine the bearing capacity of the element with local crushing:

\[ R_{b_{loc}} = 3 \cdot R_{bt} \cdot s_r/c_r \cdot cot(\varphi_{wd}) \cdot 1 \]

\[ - 4/5 \cdot 6 \cdot [1 - \{2 \cdot ln[(sin(\theta_{h})/cos(\theta_{h})) - \theta_k \cdot cot(\theta_{h}) + \theta_n \cdot cot(\theta_{n})] \} - \theta_k \cdot cot(\theta_{h}) + \theta_n \cdot cot(\theta_{n})] \}. \tag{37} \]

When the load increases from zero to a certain limit value of the load \( P_{max} \), at which the reinforcement bar is pulled out, elastic and plastic sections are observed (figure 9).

![Figure 9. Elastic and plastic sections when pulling reinforcement.](image)

Upon reaching stresses under the stamp equal to \( R_{b_{loc}} \), the termination is destroyed. The current stress level can be determined by knowing the length of the plastic section, which can be determined as:

\[ \beta \cdot L_{pl} = p_0 \cdot tanh[\beta \cdot (L - L_{pl})], \tag{38} \]

where \( \beta \) – coefficient:

\[ \beta = 2 \cdot \{(1 + \alpha_s \cdot \mu_s \cdot \tau_0)/(E \cdot d_s \cdot g^*)\}^{1/2} \tag{39} \]

\[ p_0 = \sigma_{yo} \cdot \sigma^* \tag{40} \]

\[ \sigma^* = (E \cdot \beta \cdot g^*)/(1 + \alpha_s \cdot \mu_s). \tag{41} \]

It is not possible to solve Eq. (38) explicitly, therefore, we expand the function of the hyperbolic tangent in a series:

\[ tanh(x) = x - x^3/3 + 2 \cdot x^5/15 - 17 \cdot x^7/315 + ... \tag{42} \]

For engineering accuracy, we use the first two terms of the expansion, discarding higher-order terms. In this way:

\[ \beta \cdot L_{pl} = p_0 - \beta \cdot (L - L_{pl}) + \{\beta \cdot (L - L_{pl})\}^{1/3} \tag{43} \]
Knowing the length of the plastic section, you can determine the number of reinforcing protrusions located in the plastic and elastic zones:

\[ n_{pl} = \frac{L_{pl}}{s_r} + 1; \quad n_t = n_{tot} - n_{pl} = (L - L_{pl})/s_r. \]  

We define the force, and then the pressure, falling on one protrusion in the zone of transition of the elastic zone into the plastic from the following relations:

\[ n_{pl} N_r + 0.5 n_t N_r = \sigma_{s0} A_s; \]

\[ N_r = (\sigma_{s0} A_s)/(n_{pl} + 0.5 n_t); \]

\[ \sigma_r = N_r/A_r. \]

The ultimate strains are obtained by substituting in Eq. (25) the expressions from Eq. (37):

\[ \Delta b = \frac{c_r/(2E_b)}{L/\pi} \left\{ 2\ln[\sin(\theta_c)/\cos(\theta_c)] - \theta_c \cot(\theta_c) + \theta_c \cot(\theta_c) \right\} \frac{3 R_{bt}}{s_r} \sin(\phi_{wd}) - \theta_c \cot(\theta_c) + \theta_c \cot(\theta_c). \]

To determine the movement of the entire reinforcement, you need to use the formula:

\[ g_0 = \sum g_{pl,i} + \sum g_{l,j}. \]

### 4 Discussion

As an example, we use a concrete cylinder with a height of \( H = 400\text{ mm} \), reinforcing bar with a diameter of \( d_s = 14\text{ mm} \) is embedded in the value \( L = 300\text{ mm} \). Rebar class A400, concrete class B25.

The calculation results and comparison of the data obtained with other authors are given in the next section [6]-[14].

As a result of calculating the problem using the above algorithm, the following values were obtained for the parameters: \( \sigma_{s0} \) – stress at the free end of the reinforcing bar; \( L_{pl} \) – the length of the plastic section; \( \sigma_{rc} \) – the compression stress in concrete under the wedge; \( \Delta b \) – the movement of the reinforcing bar.

#### Table 2. Calculation results.

| \( \sigma_{s0} \text{, MPa} \) | \( L_{pl} \text{, mm} \) | \( \sigma_{rc} \text{, MPa} \) | \( \Delta b \text{, mm} \) |
|----------------|----------------|----------------|----------------|
| 10             | 34.538799      | 0.4198228      | 0.067206       |
| 20             | 36.021998      | 0.8379987      | 0.134706       |
| 30             | 37.581535      | 1.2538176      | 0.201594       |
| 40             | 39.129727      | 1.6678887      | 0.26817        |
| 50             | 40.606517      | 2.0999468      | 0.33443        |
| 60             | 42.282473      | 2.4901005      | 0.400369       |
| 70             | 43.988192      | 2.889195       | 0.469884       |
| 80             | 45.514023      | 3.2042496      | 0.536061       |
| 90             | 47.161464      | 3.5082387      | 0.606937       |
| 100            | 48.830371      | 4.1101356      | 0.741436       |

| \( \sigma_{s0} \text{, MPa} \) | \( L_{pl} \text{, mm} \) | \( \sigma_{rc} \text{, MPa} \) | \( \Delta b \text{, mm} \) |
|----------------|----------------|----------------|----------------|
| 110            | 50.521756      | 4.0099122      | 0.813553       |
| 120            | 52.236391      | 4.9075392      | 0.882822       |
| 130            | 53.975093      | 5.802986      | 0.956617       |
| 140            | 55.738724      | 6.6962202      | 1.161582       |
| 150            | 57.528197      | 7.5872082      | 1.341313       |
| 160            | 59.344885      | 8.4794145      | 1.52059        |
| 170            | 61.158602      | 9.362301      | 1.699371       |
| 180            | 63.061652      | 10.246329      | 1.87683        |
| 190            | 64.964794      | 11.129756      | 1.953505       |
| 200            | 66.869266      | 12.013986      | 1.832652       |
Let us see how the size of the plastic section grows depending on the applied load on the rod (figure 10). It can be seen that the value of $L_{pl}$ is growing slowly. At the initial stage, the dependence is close to direct proportionality, which can be described as $kx+b$, after 300 MPa it begins to grow as an exponential function.

| $\sigma$, MPa | $L_{pl}$, mm | $\sigma_{c}$, MPa | $\Delta b$, mm |
|---------------|--------------|-------------------|---------------|
| 210           | 68.86639     | 8.383317          | 1.939789      |
| 220           | 70.8678      | 8.7579835         | 2.028577      |
| 230           | 72.90435     | 9.1295425         | 2.112326      |
| 240           | 74.978325    | 9.4984532         | 2.197681      |
| 250           | 77.041252    | 9.8664562         | 2.594189      |
| 260           | 79.243014    | 10.220888         | 2.68738       |
| 270           | 81.441641    | 10.583682         | 2.782124      |
| 280           | 83.633334    | 10.946364         | 2.876103      |
| 290           | 85.972477    | 11.301057         | 3.365158      |
| 300           | 88.311663    | 11.652678         | 4.168366      |

| $\sigma$, MPa | $L_{pl}$, mm | $\sigma_{c}$, MPa | $\Delta b$, mm |
|---------------|--------------|-------------------|---------------|
| 310           | 90.703722    | 12.001136         | 4.756356      |
| 320           | 93.51744     | 12.346332         | 5.311085      |
| 330           | 96.659125    | 12.683161         | 6.631509      |
| 340           | 98.229598    | 13.025908         | 7.246772      |
| 350           | 100.86729    | 13.361245         | 7.855472      |
| 360           | 103.57677    | 13.692235         | 8.457386      |
| 370           | 106.36314    | 14.019325         | 10.20999      |
| 380           | 109.23209    | 14.342346         | 10.87251      |
| 390           | 112.19003    | 14.661112         | 11.52631      |
| 400           | 115.4442     | 14.975413         | 13.66664      |

| $\sigma$, MPa | $L_{pl}$, mm | $\sigma_{c}$, MPa | $\Delta b$, mm |
|---------------|--------------|-------------------|---------------|
| 410           | 118.40283    | 15.250104         | 14.3902       |
| 420           | 121.67534    | 15.589651         | 15.09232      |
| 430           | 125.07258    | 15.889022         | 16.66459      |
| 440           | 128.60716    | 16.132731         | 18.42239      |
| 450           | 132.20786    | 16.390527         | 19.16467      |
| 460           | 136.15021    | 16.751794         | 22.17273      |
| 470           | 140.19716    | 17.026029         | 22.96134      |
| 480           | 144.16018    | 17.292571         | 23.72832      |
| 490           | 148.97063    | 17.560618         | 27.16877      |
| 500           | 153.7679     | 17.799178         | 27.96238      |

Figure 10. The plot of the length of the plastic section on the applied load.
Let us now see how the voltage changes under the tip of the wedge depending on the applied load (figure 11). As expected, stresses increase almost in proportion to the applied load. Curvature also begins after 300 MPa. Most likely this is due to changes in the reinforcement itself.

![Figure 11](image)

**Figure 11.** The stress dependence under the edge of the wedge on the applied load.

It can be seen that the growth of displacements obeys the exponential law, as was revealed by other authors (figure 12).

![Figure 12](image)

**Figure 12.** Displacement versus stress graph.

In this case, as a rule, at 10-15 mm, the bond of reinforcement to concrete is completely destroyed.

The results are consistent with the results of theoretical and experimental experiments of other scientists. The method allowed to track changes in movements at the entire stage of loading.

5 Conclusions

1. Based on the developed calculation model and calculation methods, theoretical dependences are obtained for determining the displacements of the reinforcement of the bar in the seal and the bearing capacity of the reinforcement anchoring.

2. The developed calculation model and calculation methods allow evaluating the adhesion parameters not only in the limiting stage, but also at each loading stage.

3. Theoretical results are in good agreement with the experimental data of various authors.
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