γ-ORTHOGONAL FOR K-DERIVATIONS AND K-REVERSE DERIVATIONS

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Abstract: In this paper, we introduce definitions of γ-orthogonality for two pairs of k-derivations, generalized k-derivations, and k-reverse derivations. And we present some results concerning with these notions on γ-semiprime gamma ring.

Key words: γ-semiprime gamma ring, k-derivation, generalized k-derivation, k-reverse derivation, γ-orthogonal.

1. Introduction
Let M and Γ be two additive abelian groups. M is said to be a Γ-ring in the sense of Barnes [1] if there exists a mapping of $M \times \Gamma \times M \to M$ satisfying these two conditions for all $a, b, c \in M, \alpha, \beta \in \Gamma$:

(1) $(a + b)\alpha c = a \alpha c + b \alpha c$

$a(\alpha + \beta)b = a\alpha b + a\beta b$

$a\alpha (b + c) = a\alpha b + a\alpha c$

(2) $(a\alpha b)c = a(\alpha bc)$

In addition, if there exists a mapping of $\Gamma \times M \times \Gamma \to \Gamma$ such that the following axioms hold for all $a, b, c \in M, \alpha, \beta \in \Gamma$:

(3) $(a\alpha b)c = a(\alpha bc)c$

(4) $a\alpha b = 0$ for all $a, b \in M$ implies $\alpha = 0$ where $\alpha \in \Gamma$.

Then M is called a Γ-ring in the sense of Nobusawa [2]. If a Γ-ring M in the sense of Barnes satisfies only the condition (3), then it is called weak Nobusawa Γ-ring [3]. We assume that all gamma rings in this paper are weak Nobusawa Γ-ring unless otherwise specified.

Let $M$ be a Γ-ring, $M$ is said to be a Γ-prime gamma ring if $a \Gamma M \Gamma b = 0$ with $a, b \in M$ implies that either $a = 0$ or $b = 0$ [4], and $M$ is called a Γ-semiprime gamma ring if $a \Gamma M \Gamma a = 0$ with $a \in M$ implies $a = 0$ [4]. A weak Nobusawa Γ-ring $M$ is said to be a γ-prime gamma ring if there exists a non-zero element $\gamma$ in $\Gamma$ such that $a \gamma M \gamma b = 0$ with $a, b \in M$ implies that either $a = 0$ or $b = 0$ [5] and is called a γ-semiprime gamma ring if there exists a non-zero element $\gamma$ in $\Gamma$ such that a $\gamma M \gamma a = 0$ with $a \in M$ implies that $a = 0$. And a Γ-ring M is said to be a 2-torsion free if $2a = 0, a \in M$ implies $a = 0$.

Recall that from [6], an additive mapping $d: M \to M$ is called a derivation on $M$ if $d(ab) = d(a)ab + aad(b)$ for all $a, b \in M, \alpha \in \Gamma$ and a reverse derivation on $M$ if $d(ab) = d(b)\alpha a + b\alpha d(a)$ for all $a, b \in M, \alpha \in \Gamma$ [7]. Also an additive mapping $D: M \to M$ is said to be a generalized derivation if there exists a derivation $d$ on $M$ such that $D(ab) = D(a)ab + aad(b)$ for all $a, b \in M, \alpha \in \Gamma$ [8]. In 2000, Kandamar[9] firstly introduced the notion of a k-derivation for a gamma ring in the sense of Barnes. Chakraborty and
Paul [10] introduced the notion of generalized k-derivations for gamma rings. Also in [11] presented the notion of a k-reverse derivation for a gamma ring.

In this work, we define γ-orthogonality for two pairs k-derivations, generalized k-derivations and k-reverse derivations for a weak Nobusawa gamma ring. And we obtain some results on 2-torsion free γ-semiprime Γ-ring.

2. γ-Orthogonal k-derivations

Now we introduce the notion of γ-orthogonal k-derivations as follows.

Definition (2.1). Let M be a Γ-ring. Two k1, k2-derivations d1 and d2 of Γ-ring M are said to be γ-orthogonal (γ be non-zero of Γ) if: d1(a)γMd2(b) = 0 = d2(b)γMd1(a) is satisfied for all a, b ∈ M.

Example (2.2). Let M be a Γ-ring of characteristic equal 2. we put M1 = M × M and Γ1 = Γ× Γ, we define addition and multiplication on M1 and Γ1 as follows:

\[(a1, b1) + (a2, b2) = (a1 + a2, b1 + b2),\]
\[(a1, b1)(a2, b2) = (a1a2, b1b2),\]
\[(a1, b1) + (a2, b2) = (a1 + a2, b1 + b2)\]

for all a1, a2, b1, b2 ∈ M, α, β, α2, β2 ∈ Γ. Then M1 is a Γ1-ring under these operations. Define mappings d1, d2 on M1 and k1, k2 on Γ1 as:

\[d1(a, b) = (a, 0),\]
\[d2(a, b) = (0, b),\]
\[k1(a, β) = (a, 0),\]
\[k2(a, β) = (0, β)\]

for all a, b ∈ M, α, β ∈ Γ.

Then d1 and d2 are k1, k2-derivations of M1 respectively. Let α be non-zero of Γ, we have γ = (a, 0) ∈ Γ1. Therefore d1 and d2 are γ-orthogonal of M1. We give some results below.

Theorem (2.3). Let M be a 2-torsion free γ-semiprime gamma ring and d1, d2 be two k1, k2-derivations of M such that k1(γ) = k2(γ) = 0 respectively. Then the following conditions are equivalent:

(i) d1 and d2 are γ-orthogonal.
(ii) For all a, b ∈ M, the following relation hold:
\[d1(a)γMd2(b) + d2(a)γMd1(b) = 0.\]
(iii) d1(a)γMd2(b) = 0, for all a, b ∈ M.
(iv) d1(a)γMd2(b) = 0, for all a, b ∈ M and d1d2 = 0.

Proof: (i)→(ii) By the given hypothesis, we have d1 and d2 are orthogonal derivations of a 2-torsion free semiprime ring (M, +, ·γ). Then by [12, Theorem1], we get d1(a)γMd2(b) + d2(a)γMd1(b) = 0, for all a, b ∈ (M, +, ·γ) requires d1(a)γMd2(b) + d2(a)γMd1(b) = 0.

(ii)→(iii) By the assumption, we have (M, +, ·γ) is a 2-torsion free semiprime ring and d1, d2 are derivations of ring M. Since d1(a)γMd2(b) + d2(a)γMd1(b) = 0 for all a, b of the ring (M, +, ·γ). Then d1(a)γMd2(b) = 0 for all a, b ∈ M by [12, Theorem1]. Therefore d1(a)γMd2(b) = 0 for all a, b ∈ M.

(iii)→(iv) We have d1 and d2 are derivations of the ring (M, +, ·γ) and (M, +, ·γ) is a 2-torsion free semiprime ring. By hypothesis d1(a)γMd2(b) = 0 for all a, b in ring (M, +, ·γ). Then by [12, Theorem1], we get d1(a)γMd2(b) = 0 for all a, b ∈ M and d2 = 0 requires d1(a)γMd2(b) = 0 for all a, b ∈ M and d2 = 0.

(iv)→(i) By the given hypothesis, we get d1 and d2 are derivations of a 2-torsion free semiprime ring (M, +, ·γ). Since d1(a)γMd2(b) = 0 for all a, b ∈ M and d1d2 = 0, then we have d1 and d2 are orthogonal derivations of ring (M, +, ·γ) by [12, Theorem1]. Hence d1 and d2 are γ-orthogonal of Γ-ring.

Theorem (2.4). Let M be a 2-torsion free γ-semiprime gamma ring and d1, d2 be two k1, k2-derivations of M such that k1(γ) = k2(γ) = 0 respectively. If d1d2 is a k1k2-derivation of M, then d1 and d2 are γ-orthogonal of Γ-ring M.

Proof: By the given hypothesis, we have d1 and d2 are derivations of ring (M, +, ·γ), and (M, +, ·γ) is a 2-torsion free semiprime ring. Since d1d2 is a derivation of a ring (M, +, ·γ), then by [12, Theorem2] we get d1 and d2 are orthogonal of (M, +, ·γ). Therefore d1 and d2 are γ-orthogonal of Γ-ring M.

3. γ-Orthogonal generalized k-derivations

Now we introduce the notion of γ-orthogonal k-derivations as follows.
Definition (3.1). Let $M$ be a $\Gamma$-ring. Two generalized $k_1$, $k_2$-derivations $(D_1, d_1)$ and $(D_2, d_2)$ of $\Gamma$-ring $M$ are said to be $\gamma$-orthogonal ($\gamma$ be non-zero of $\Gamma$) if: $D_1(a)\gamma M\gamma D_2(b) = 0 = D_2(b)\gamma M\gamma D_1(a)$ is satisfied for all $a, b \in M$.

Example (3.2). Let $d$ and $g$ be two $k_1$, $k_2$-derivations of a $\Gamma$-ring $M$. Consider $M_1 = M \times M$ and $\Gamma_1 = \Gamma \times \Gamma$, define addition and multiplication on $M_1$ and $\Gamma_1$ as: $(a_1, b_1) + i(a_2, b_2) = i(a_1 + a_2, b_1 + b_2)$, $(a_1, b_1)(a, b) = (a_1a, b_1b)$, $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$, for all $a_1, a_2, b_1, b_2 \in M$, $\alpha, \beta, \gamma$. If $\beta \in \Gamma$, then $M_1$ is a $\Gamma_1$-ring under these operations. We define mappings $d_1, d_2$ on $M_1$ and $k_1, k_2$ on $\Gamma_1$ as:

$$d_1(a, b) = (d(a), 0), d_2(a, b) = (0, g(b)), k_1(a, b) = (k_1(a), 0), k_2(a, b) = (0, k_2(b)),$$

for all $a, b \in M$, $\alpha, \beta \in \Gamma$. Then $d_1$ and $d_2$ are $k_1$, $k_2$-derivations of $M_1$ respectively. Moreover, if $(D, d)$ and $(G, g)$ are generalized $k_1$, $k_2$-derivations of $M$ and we define $D_1, D_2$ on $M_1$ as follows:

$$D_1(a, b) = (D(a), 0) \text{ and } D_2(a, b) = (0, G(b)) \text{ for all } a, b \in M,$$

then $(D_1, d_1)$ and $(D_2, d_2)$ are generalized $k_1$, $k_2$-derivations of $M_1$. If we take $\alpha$ be a non-zero element in $\Gamma$, then $\gamma = (\alpha, 0) \in \Gamma_1$. Therefore $D_1$ and $D_2$ are $\gamma$-orthogonal.

In the following theorem we give characterization of $\gamma$-orthogonal generalized $k$-derivations on $\Gamma$-ring.

Theorem (3.3). Let $M$ be a 2-torsion free $\gamma$-semiprime gamma ring and $(D_1, d_1)$, $(D_2, d_2)$ be two generalized $k_1$, $k_2$-derivations of $M$ such that $k_1(\gamma) = k_2(\gamma) = 0$ respectively. Then the following conditions are equivalent:

(i) $D_1$ and $D_2$ are $\gamma$-orthogonal.

(ii) For all $a, b \in M$, the following relations holds:

1. $D_1(a)\gamma D_2(b) + D_2(a)\gamma D_1(b) = 0$.
2. $D_1(a)\gamma D_2(b) + D_2(a)\gamma D_1(b) = 0$.
3. $D_1(a)\gamma D_2(b) = 0 = D_2(a)\gamma D_1(b)$ for all $a, b \in M$.
4. $D_1(a)\gamma D_2(b) = 0$ for all $a, b \in M$ and $d_1D_2 = d_2D_1 = 0$.

Proof: (i)$\rightarrow$(ii) By the given hypothesis, we have $D_1$ and $D_2$ are orthogonal generalized derivations of a 2-torsion free semiprime ring $(M, +, \gamma)$ associated with derivations $d_1$ and $d_2$ of $(M, +, \gamma)$. Then by [13, Theorem1], we get $D_1(a)\gamma D_2(b) + D_2(a)\gamma D_1(b) = 0$ and $d_1(a)\gamma D_1(b) + d_2(a)\gamma D_2(b) = 0$ for all $a, b \in M$.

(iii)$\rightarrow$(iv) By the given assumption, we have $(M, +, \gamma)$ is a 2-torsion free semiprime ring and $D_1, D_2$ are generalized derivations of $(M, +, \gamma)$ associated with derivations $d_1$ and $d_2$ of $M$. By hypothesis $D_1(a)\gamma D_2(b) + D_2(a)\gamma D_1(b) = 0$ and $d_1(a)\gamma D_2(b) + d_2(a)\gamma D_1(b) = 0$ for all $a, b \in M$. Then $D_1(a)\gamma D_2(b) = 0 = d_1(a)\gamma D_2(b)$ for all $a, b \in M$ by [13, Theorem1]. Therefore $D_1(a)\gamma D_2(b) = 0 = d_1(a)\gamma D_2(b)$, for all $a, b \in M$.

(iv)$\rightarrow$(i) By the given hypothesis, we have $D_1$ and $D_2$ are generalized derivations of the ring $(M, +, \gamma)$ associated with derivations $d_1$ and $d_2$ of ring $M$. And $(M, +, \gamma)$ is a 2-torsion free semiprime ring and since $D_1(a)\gamma D_2(b) = d_1(a)\gamma D_2(b) = 0$ for all $a, b \in M$. by [13, Theorem1], we get $D_1(a)\gamma D_2(b) = 0$ for all $a, b \in M$ and $d_1D_2 = d_2D_1 = 0$.

Lemma (3.5). Let $M$ be a 2-torsion free $\gamma$-semiprime gamma ring. If $(D_1, d_1)$ and $(D_2, d_2)$ are $\gamma$-orthogonal generalized $k_1$, $k_2$-derivations of $\Gamma$-ring $M$ such that $k_1(\gamma) = k_2(\gamma) = 0$ respectively. Then the following relations are holds.

(i) $D_1(a)\gamma D_2(b) = D_2(a)\gamma D_1(b) = 0$, hence $D_1(a)\gamma D_2(b) + D_2(a)\gamma D_1(b) = 0$, for all $a, b \in M$. 


Therefore, $Hence$

$M$ is a $\gamma$-orthogonal, and

$D_1$ and $D_2$ are $\gamma$-orthogonal, and

$D_1(a)D_2(b) = D_2(b)D_1(a) = 0$, for all $a, b \in M$.

(iii) $D_1$ and $D_2$ are $\gamma$-orthogonal, and

$D_1(a)D_2(b) = D_2(b)D_1(a) = 0$, for all $a, b \in M$.

(iv) $D_1$ and $D_2$ are $\gamma$-orthogonal.

(v) $D_1D_2 = D_2D_1 = 0$.

(vi) $D_1D_2 = D_2D_1 = 0$.

Proof: (i) By the given hypothesis, we have $D_1$ and $D_2$ are generalized derivations of a $2$-torsion free semiprime ring $(M, +, \cdot, \gamma)$ associated with derivations $d_1$ and $d_2$ of $(M, +, \cdot, \gamma)$. Since $D_1$ and $D_2$ are orthogonal of the ring $(M, +, \cdot, \gamma)$. Then by [13, Lemma 2], we get $D_1(a)D_2(b) = D_2(b)D_1(a) = 0$ and $D_1(a)D_1(b) + D_2(a)D_1(b) = 0$ for all $a, b \in M$. Therefore $D_1$ and $D_2$ are $\gamma$-orthogonal of $R$-ring $M$.

(ii) By the given assumption, we have $D_1$ and $D_2$ are orthogonal generalized derivations of a $2$-torsion free semiprime ring $(M, +, \cdot, \gamma)$ associated with derivations $d_1$ and $d_2$ of $M$. Then by [13, Lemma 2], we have $D_1$ and $D_2$ are orthogonal of the ring $(M, +, \cdot, \gamma)$. Hence by [13, Lemma 2], we get $d_1$ and $d_2$ are orthogonal of the ring $(M, +, \cdot, \gamma)$. Therefore $D_1$ and $D_2$ are $\gamma$-orthogonal.

(vi) By the hypothesis, we get $D_1$ and $D_2$ are orthogonal generalized derivations of a $2$-torsion free semiprime ring $(M, +, \cdot, \gamma)$ associated with derivations $d_1$ and $d_2$ of $(M, +, \cdot, \gamma)$. Then by [13, Lemma 2], we have $d_1$ and $d_2$ are orthogonal of the ring $(M, +, \cdot, \gamma)$. Therefore $D_1$ and $D_2$ are $\gamma$-orthogonal of $R$-ring $M$.

(vi) By the given assumption, we have $D_1, D_2$ are generalized derivations of a $2$-torsion free semiprime ring $(M, +, \cdot, \gamma)$ associated with derivations $d_1$ and $d_2$ of $M$. By hypothesis $D_1$ and $D_2$ are orthogonal of the ring $(M, +, \cdot, \gamma)$. Then by [13, Lemma 2], we get $d_1D_1 = D_2D_1 = 0$ and $d_1D_2 = D_2D_1 = 0$.

Now, for the production of generalized $k$-derivations we give the following results.

Theorem (3.6). Let $M$ be a $2$-torsion free $\gamma$-semiprime gamma ring and $(D_1, d_1), (D_2, d_2)$ be two generalized $\alpha_1, k_2$-derivations of $M$ such that $k_1(\gamma) = k_2(\gamma) = 0$ respectively. Then $D_1$ and $D_2$ are $\gamma$-orthogonal, and $D_1$ and $D_2$ are $\gamma$-orthogonal if one of the following conditions holds:

(i) $(D_1D_2, d_1d_2)$ is a generalized $k_1k_2$-derivation of $M$.

(ii) $(D_1, d_1), (D_2, d_2)$ is a generalized $k_1k_2$-derivation of $M$.

Proof: We prove (i) and the other by the same way: By the given hypothesis, we have $D_1$ and $D_2$ are generalized derivations of $2$-torsion free semiprime ring $(M, +, \cdot, \gamma)$ associated with derivations $d_1$ and $d_2$ of $(M, +, \cdot, \gamma)$. Since $D_1D_2$ is a generalized derivation of a ring $(M, +, \cdot, \gamma)$ associated with derivation $d_1d_2$ of $M$. Hence $D_1$ and $D_2$ are orthogonal of ring $(M, +, \cdot, \gamma)$, and $D_1$ and $D_2$ are orthogonal of $(M, +, \cdot, \gamma)$ by [13, Theorem 2]. Therefore $D_1$ and $D_2$ are $\gamma$-orthogonal of $R$-ring $M$.

Theorem (3.7). Let $M$ be a $2$-torsion free $\gamma$-prime gamma ring and $(D_1, d_1), (D_2, d_2)$ be two generalized $k_1, k_2$-derivations of $M$ such that $k_1(\gamma) = k_2(\gamma) = 0$ respectively. If $D_1$ and $D_2$ are $\gamma$-orthogonal, and $D_1$ and $D_2$ are $\gamma$-orthogonal, then $D_1 = d_1 = 0$ or $D_2 = d_2 = 0$.

Proof: By the given assumption, we have $(M, +, \cdot, \gamma)$ is a $2$-torsion free primering and $D_1, D_2$ are generalized derivations of the ring $(M, +, \cdot, \gamma)$ associated with derivations $d_1, d_2$ of $(M, +, \cdot, \gamma)$. By hypothesis
$D_1$ and $d_2$ are orthogonal of the ring $(M, +, \cdot_1)$, and $D_2$ and $d_1$ are orthogonal of $(M, +, \cdot_\gamma)$. Hence $D_1 = d_1 = 0$ or $D_2 = d_2 = 0$ by [13, Corollary 5].

4. $\gamma$-Orthogonal k-reverse derivations

**Definition (4.1).** Let $M$ be a $\Gamma$-ring. Two $k_1$, $k_2$-reverse derivations $d_1$ and $d_2$ of $\Gamma$-ring $M$ are said to be $\gamma$-orthogonal ($\gamma$ be non-zero of $\Gamma$) if: $d_1(a)\gamma M d_2(b) = 0 = d_2(b)\gamma M d_1(a)$ is satisfied for all $a, b \in M$.

We give an example of $\gamma$-orthogonal $k$-reverse derivations on $\Gamma$-ring.

**Example (4.2).** Let $d_1$ and $d_2$ be two $k_1$, $k_2$-reverse derivations of $\Gamma$-ring $M$. Set $M_1 = M \times M$ and $\Gamma_1 = \Gamma \times \Gamma$, define addition and multiplication on $M_1$ and $\Gamma_1$ as:

$$(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2), (a_1, b_1)(a, \beta) (a_2, b_2) = (a_1 a, a_2 b, b_1 \beta b_2), (a_1, \beta_1) + (a_2, \beta_2) = (a_1 + a_2, \beta_1 + \beta_2),$$

for all $a_1, a_2, b_1, b_2 \in M$, $\alpha, \beta, \alpha_1, \alpha_2, \beta_1, \beta_2 \in \Gamma_1$, then $M_1$ is a $\Gamma_1$-ring. Define mappings $d_1, d_2$ on $M_1$ and $k_1$, $k_2$-derivations $\Gamma_1$ as:

$$d_1(a, b) = (d_1(a), 0), d_2(a, b) = (0, d_2(b)), k_1(a, \beta) = (k_1(a), 0), k_2(a, \beta) = (0, k_2(\beta))$$

for all $a, b \in M$, $\alpha, \beta \in \Gamma_1$. Then $d_1$ and $d_2$ are $k_1, k_2$-derivations of $M_1$ respectively. Let $\beta$ be non-zero of $\Gamma$, we have $\gamma = (0, \beta) \in \Gamma_1$. Therefore $d_1$ and $d_2$ are $\gamma$-orthogonal of $M_1$.

In the following theorem we give characterization of $\gamma$-orthogonal $k$-reverse derivations on $\Gamma$-ring.

**Theorem (4.3).** Let $M$ be a 2-torsion free $\gamma$-semiprimer ring and $d_1, d_2$ be two $k_1$, $k_2$-reverse derivations of $M$ such that $k_1(\gamma) = k_2(\gamma) = 0$ respectively. Then $d_1$ and $d_2$ are $\gamma$-orthogonal of $\Gamma$-ring $M$ if and only if one of the following conditions holds:

(i) $d_1d_1 = 0$.

(ii) $d_2d_2 = 0$.

(iii) $d_1d_2 + d_2d_1 = 0$.

(iv) $d_1(a)\gamma d_1(a) = 0$, for all $a \in M$.

(v) $d_2(a)\gamma d_2(a) + d_2(a)\gamma d_1(a) = 0$, for all $a \in M$.

proof: (i) $\Rightarrow$ “$d_1$ and $d_2$ are $\gamma$-orthogonal of $M$”.

By the given hypothesis, we have $d_1$ and $d_2$ are reverse derivations of the ring $(M, +, \cdot_\gamma)$ and $(M, +, \cdot_\gamma)$ is a 2-torsion free semiprimer ring. Since $d_1d_2 = 0$. Hence $d_1$ and $d_2$ are orthogonal of the ring $(M, +, \cdot_\gamma)$ by [14, Theorem 1]. Therefore $d_1$ and $d_2$ are $\gamma$-orthogonal of $\Gamma$-ring $M$ since orthogonality of the ring $(M, +, \cdot_\gamma)$ requires $\gamma$-orthogonality of $\Gamma$-ring $M$.

“$d_1$ and $d_2$ are $\gamma$-orthogonal of $M$” $\Rightarrow$ (i).

By the given assumption, we have $d_1$ and $d_2$ are orthogonal reverse derivations of a 2-torsion free semiprimer ring $(M, +, \cdot_\gamma)$. Then $d_1d_2 = 0$ by [14, Theorem 1].

(ii) Similar way used in the proof of (i).

(iii) $\Rightarrow$ “$d_1$ and $d_2$ are $\gamma$-orthogonal of $M$”.

By the hypothesis, we have $(M, +, \cdot_\gamma)$ is a 2-torsion free semiprimer ring and $d_1, d_2$ are reverse derivations of $(M, +, \cdot_\gamma)$. Since $d_1d_2 = 0$. Then we get $d_1$ and $d_2$ are orthogonal of the ring $(M, +, \cdot_\gamma)$ by [14, Theorem 1]. Therefore $d_1$ and $d_2$ are $\gamma$-orthogonal of $\Gamma$-ring $M$.

(iv) $\Rightarrow$ “$d_1$ and $d_2$ are $\gamma$-orthogonal of $M$”.

By the given assumption, we have $d_1$ and $d_2$ are reverse derivations of the ring $(M, +, \cdot_\gamma)$ and $(M, +, \cdot_\gamma)$ is a 2-torsion free semiprimer ring. By hypothesis $d_1(a)\gamma d_2(a) = 0$, for all $a \in M$, then $d_1$ and $d_2$ are orthogonal of the ring $(M, +, \cdot_\gamma)$ by [14, Theorem 1]. Therefore $d_1$ and $d_2$ are $\gamma$-orthogonal of $\Gamma$-ring $M$.

“$d_1$ and $d_2$ are $\gamma$-orthogonal of $M$” $\Rightarrow$ (v).

By the hypothesis, we have $d_1$ and $d_2$ are orthogonal reverse derivations of a 2-torsion free semiprimer ring $(M, +, \cdot_\gamma)$, then we get $d_1(a)\gamma d_2(a) + d_2(a)\gamma d_1(a) = 0$, for all $a \in M$ by [14, Theorem 1]. Therefore $d_1(a)\gamma d_2(a) + d_2(a)\gamma d_1(a) = 0$, for all $a \in M$.

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