Heat model of a spindle support of a precision metal cutting machine

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Abstract. Based on the use of the electrothermal analogy in describing the heat transfer process, an engineering technique has been developed to determine the thermal state of the supports of spindle assemblies, which does not require "heavy" software packages based on the application of the finite element method. The design features of modern control rooms made it possible to reasonably present the thermal model in the form of a flat model. The principles of dividing the blocks of a flat model into enlarged elements are proposed, taking into account the features of the geometry of the calculated structure and heat sources. A technique has been developed for determining the thermal resistance of an element of a calculated thermal model taking into account boundary conditions. The use of enlarged elements of the thermal model makes it easy to assess the influence of technological features of processing and assembly of individual structural parts on its thermal state. The presented methodology for developing a thermal model can be used to estimate stationary thermal processes of any axisymmetric structures with insignificant temperature variations.

1. Introduction

The operational characteristics of the metal-cutting machine depend significantly on the temperature conditions in which its main subsystem, the spindle unit (SU), operates. Based on this, there are reasonable recommendations that limit the values of the temperature criterion. So, for example, the permissible excess temperature of the outer rings of the spindle bearings of precision machines, depending on the accuracy, is (30...35) °C [1].

The creation of mathematical models for describing heat transfer in heat-loaded elements encounters enormous difficulties associated with finding a compromise between the completeness of the model being developed and the possibility of its numerical implementation [2, 3].

Currently, the finite element method (FEM) has been widely used in research and in the design of spindle assemblies [4, 5, 6, 7]. Using it, you can describe any area, since, for example, triangles and tetrahedra easily cover even complex objects. To increase the accuracy of calculations in the necessary subdomains, one can easily increase the density of the computational grid. However, its use is possible only when using computers with large resources, the availability of special software packages (ANSYS, Solid Works), it requires considerable time to develop a model and set boundary conditions, especially when modeling nodes consisting of several parts. These shortcomings are leveled when it is justifiably necessary to obtain a detailed picture of the thermal field, making it possible to make further design decisions.
In the case when the design criterion requires evaluating only the temperature of the most important and critical structural element (the outer ring of the spindle bearing) and modeling the design and technological solutions regarding the shape, size of individual parts and the conditions for their connection, it is convenient to use a simpler model based on its presentation as a set of blocks [8].

When developing thermal models of spindle assemblies for precision metal cutting machines, a number of features should be taken into account.

Firstly, in precision machines with finishing machining using cutting fluids, heat fluxes in the cutting zone have an insignificant effect on the temperature of the SU and the formation of its temperature field is provided mainly by heat release in the spindle bearings.

Secondly, the parts included in the design of the SU, have contact on one or more surfaces with adjacent parts.

The third feature are small temperature fluctuations. In this regard, in the analysis of thermal processes in machines, it is possible to take the thermal conductivity coefficients practically independent of temperature.

Thus, the thermal model of the SU can be represented by a set of elements with constant heat conductivity coefficients (depending only on the material), contact areas of parts between each other (contact thermal resistance zones) and surfaces free from such contact on which convection processes are carried out with air or liquid medium (oil) and radiation.

2. The use of electrothermal analogy in the description of the heat exchange process
In accordance with the Fourier law in bodies, when the temperature of its individual sections is not the same, a heat flux occurs [2]:

\[ P = \frac{(t_1 - t_2)}{F}, \]

where \( F = \frac{(x_1 - x_2)}{(\lambda S)} \) is the heat coefficient; \( \lambda \) - coefficient of thermal conductivity; \( S \) - isothermal surface area; \( x_1; x_2 \) - coordinates of the location of the isothermal surface; \( t_1; t_2 \) - temperature at points corresponding to coordinates \( x_1; x_2 \).

Comparing the dependence of the electric current on a certain section of the circuit on the voltage and resistance of this section, described by Ohm's law,

\[ I = \frac{(U_1 - U_2)}{R}, \]

with expression (1), it is easy to establish analogies between electric and thermal parameters: electric current - heat flux (heat flux power); electrical voltage - temperature; electrical resistance is the heat coefficient.

Provided that between the considered isothermal surfaces there are no drains and additional energy sources, the thermal coefficient is called the thermal resistance.

By analogy with electric circuits, the value \( \sigma_T = \lambda/(x_1 - x_2) \) is called thermal (thermal) conductivity with a dimension \((\text{W/(m}^2 \text{K)})\), and its inverse value \( R_T = (x_1 - x_2)/\lambda \) is called thermal (thermal) resistance with a dimension \((\text{m}^2 \text{K}/\text{W})\).

Along with the indicated values of \( \sigma_T \) and \( R_T \), the values of total (absolute) thermal conductivity \( \sigma_{TA} \) with dimension \((\text{W/K})\) and total (absolute) thermal resistance \( R_{TA} \) with dimension \((\text{K/W})\) are used [9]:

\[ \sigma_{TA} = \sigma_T S = \frac{\lambda S}{x_1 - x_2}; \quad R_{TA} = R_T / S = \frac{(x_1 - x_2)}{(\lambda S)}. \]

Then equation (1) can be written in the integral form [9]:

\[ P = \sigma_{TA} \left( t_1 - t_2 \right) = \left( t_1 - t_2 \right) / R_{TA}. \]

Thus, based on the electrothermal analogy, the heat exchange process can be represented by a thermal model, the elements of which are sources and receivers of thermal energy, and thermal resistance (conductivity). The thermal model has branches and nodes. The branches of the model are its sections, consisting of one or more resistances, in which the magnitude of the heat flux is the same.
The nodes of the model are the junction of two or more branches. Each node of the thermal model is assigned a specific temperature.

Variables in the calculated thermal model (heat fluxes and temperature) obey Ohm and Kirchhoff’s laws, which makes it possible to calculate the temperature at any point in the system. The calculation is reduced to the preparation and solution of the heat balance equations for the nodal points, similar to the Kirchhoff equation for a branched electric circuit.

The thermal model can be used to solve a direct heat engineering problem, that is, calculate power from a known surface temperature, or to solve an inverse heat engineering problem, that is, determine a surface temperature from a known input power, which in practice has to be done more often.

In the design analysis, taking into account the axisymmetric design of the SU, we can consider the calculation model in the form of thin platinum of constant thickness $H$, located along the spindle’s diametrical section on one side of its axis. Given the same conditions for the distribution of heat in the plates, we can assume that heat transfer between the plates does not occur.

The flat model is divided into blocks corresponding to the parts included in the design of SU.

For the design of the front support of the spindle assembly with one bearing (Figure 1), a flat thermal model is shown for determining the temperature of the outer ring of the bearing (Figure 2) and the division of the blocks into rectangular elements (Figure 3).

![Figure 1. The design of the front support of the spindle assembly.](image)

![Figure 2. Blocks of a flat thermal model of the outer ring of the bearing of the front support of the spindle assembly.](image)
Figure 3. The division of the blocks of the flat thermal model into rectangular elements and the designation of the nodes of the thermal model.

It is convenient to further divide the blocks into rectangular elements (shown in dashed lines in Fig. 3) so that four elements contact in the corners of the rectangles not located on the free contour of the block. Similarly, the creation of the model occurs when several bearings are used in the support (Fig. 4).

Figure 4. Development of a thermal model of the front spindle support with three bearings: a - construction; b - blocks; c - breaking blocks into rectangular elements.
For ball bearings, we assume that the source of heat in the bearing is a cylindrical surface, determined by the size of the groove along the corresponding ring (0.008 m in Figure 2).

It is convenient to place the nodes of the developed thermal model in the center of a rectangular element (Figure 3). Then, for any \( i \)-rd element (Figure 5), having thermal resistance \( R^T_i \), the thermal resistance of its sides is \( R^C_{i-1}, R^C_{i-2}, R^C_{i-3}, R^C_{i-4} \) (connections with associated parts and the environment) and the thermal resistance of coupled elements \( R^N_i; R^P_i; R^l_i; R^g_i \), the cell of the thermal model will be presented in the form shown in Figure 6, \( a \), where \( R^T_{i-n-g}, R^T_{i-l-p} \) are the thermal resistance of the element body along the corresponding trajectory.

Using the rules of the electrical thermal analogy, the resistances arranged in series can be added up and then the cell of the thermal model will have the form shown in Fig. 6, \( b \), where

\[
R^T_{i-n} = 0.5R^T_{i-n-g} + R^C_{i-n} + 0.5R^T_{i-n},
\]  

(5)
\[ R_{i-p} = 0.5R_{i-l-p}^T + R_{i-p}^C + 0.5R_p^T; \quad (6) \]
\[ R_{i-g} = 0.5R_{i-n-g}^T + R_{i-g}^C + 0.5R_g^T; \quad (7) \]
\[ R_{i-l} = 0.5R_{i-l-p}^T + R_{i-l}^C + 0.5R_{i-l}^T. \quad (8) \]

The thermal resistance of element \( R_i^T \), by analogy with electrical resistance, is determined by the dependence \[ R_i^T = \int \frac{dl}{\lambda S(l)}, \quad (9) \]
where \( dl \) is an element of the coordinate line of the heat flux; \( \lambda \) - coefficient of thermal conductivity of the material; \( S(l) \) is the cross-sectional area perpendicular to the heat flux at a point \( l \) of the coordinate line; 1 and 2 - isothermal zones between which there is a body part characterized by thermal resistance.

For a rectangular element, thermal resistance depends on the trajectory of the heat flux. For a rectangular element, provided that the heat flux enters the element through one side, and exits through the opposite
\[ R_b^T = \frac{b}{\lambda S} \int \frac{dl}{\lambda a H}, \quad (10) \]
where \( H \) is the thickness of the plate; \( b \) - the dimensions of the element along the propagation of the heat flux.

Similarly, we get
\[ R_a^T = \frac{a}{\lambda S} \int \frac{dl}{\lambda b H}. \quad (11) \]

Then for the central element in Fig. 5 we have
\[ R_{i-n-g}^T = \frac{b}{\lambda a H}; \quad R_{i-l-p}^T = \frac{a}{\lambda b H}. \quad (12) \]

Heat from the surface of one part to the mating surface can be transferred in the following ways: thermal conductivity through the point of direct contact; thermal conductivity through the medium filling the space between the protrusions and roughnesses of the contacting surfaces; convective heat transfer by the medium filling this space; radiant heat transfer between surfaces.

The last two methods due to their insignificant share in the overall balance of heat transfer can be ignored.

As was established in [11], solids touch each other only with the vertices of the roughness profiles, the contact area of which is 0.01...0.1% of the nominal. In this regard, we can assume that almost the entire heat flux is transmitted through the air gap. In preliminary design estimates, we can assume that the thickness of the air gap in the contact is on average half the maximum distance between the roughness troughs. To more fully take into account the factors affecting the formation of CTR, you can use the studies presented in [12, 13].

In accordance with Newton’s law [10], the amount of heat transferred to the environment by a heated surface per unit time (that is, the heat transfer power) is proportional to the difference in surface and medium temperatures and the value of the heat transfer surface. Thus, if the model element is not in contact with a solid, but with a gaseous medium (air), then the heat transfer capacity of \( P_T \) to the environment
\[ P_T = \alpha(T_S - T_A)S, \quad (13) \]
where \( \alpha \) is the coefficient of convective heat transfer from the surface to the medium, W/(m²·K); (for air with natural convection \( \alpha = 5...25 \) W/(m²·K)); \( T_S \) - surface temperature of a solid body; \( T_A \) - temperature of a gaseous medium (air); \( S \) - area of a heat-transfer surface.
Thermal resistance to the process of convective heat transfer from the surface $S$ of the body to the environment \[10\], K/W:

$$R_{T\alpha} = \frac{1}{\alpha S}.$$  \hspace{1cm} (14)

Using the electrothermal analogy and Kirchhoff’s law and drawing up the equations of equilibrium of heat fluxes in its nodes, we determine the temperatures in the elements of the thermal model:

$$\begin{align*}
\sigma_{1-2}(t_1 - t_2) &= P; \\
\sigma_{2-1}(t_2 - t_1) + \sigma_{2-4}(t_2 - t_4) + \sigma_{2-3}(t_2 - t_3) + \sigma_{2-15}(t_2 - t_{15}) &= 0; \\
\sigma_{3-2}(t_3 - t_2) + \sigma_{3-B}(t_3 - t_B) + \sigma_{3-17}(t_3 - t_{17}) + \sigma_{3-16}(t_3 - t_{16}) &= 0; \\
\sigma_{4-2}(t_4 - t_2) + \sigma_{4-6}(t_4 - t_6) + \sigma_{4-5}(t_4 - t_5) + \sigma_{4-14}(t_4 - t_{14}) &= 0;
\end{align*}$$

\hspace{1cm} (15)

3. **Approbation**

The temperature calculation results (°C) for the nodes of the thermal model with the spindle dimensions shown in Fig. 2, for an average bearing diameter of 100 mm operating at a speed of 2000 rpm, are shown in Fig. 7.

**Figure 7.** Temperatures in the nodes of the calculation model at a spindle speed of 2000 rpm.

The resulting thermal model makes it easy to evaluate the effect of the processing quality of the contact surfaces of SU parts on the formation of the temperature field. So, for example, at a spindle speed of 6000 rpm, reducing the thermal resistance by half in the contact of the bearing ring with the parts of the control unit (thermal conductivity $\sigma_{3-17}$; $\sigma_{3-16}$; $\sigma_{2-15}$; $\sigma_{4-14}$; $\sigma_{4-5}$) reduces the temperature $t_2$ (outer bearing ring) by $3.0^\circ$.

4. **Conclusion**

The developed engineering technique for determining the thermal state of the supports of spindle assemblies does not require the use of "heavy" software packages based on the use of the finite element method. The enlarged elements of the thermal model make it easy to assess the influence of technological features of processing and assembly of individual structural parts on its thermal state and outline measures that reduce the temperature of bearings. The proposed methodology for developing a
thermal model can be used to study stationary thermal processes of any axisymmetric structures with insignificant temperature variations.

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