Experiments with Kundt’s tube

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Abstract. We present some lab science experiments that are intended to be performed by undergraduate university students of scientific and technological disciplines. We come out with an experimental study of waves in Kundt’s tube by varying the absorption characteristics of its closed end. The measurement of the resonant frequencies permits to determine the speed of sound in the tube. The impedance tube-standing wave method (ISO-10534-1) is applied to study the normal absorption coefficient of acoustics insulators. The setup includes basic lab equipment available in the undergraduate laboratory: a tube, a speaker, a microphone, a digital function generator and an oscilloscope.

1. Introduction
The Kundt’s tube (see Figure 1) permits to investigate the propagation of sound waves in a tube. A loudspeaker produces an acoustic wave, which travels down the pipe and reflects from its end. The phase interference between the incident and reflected waves results in the formation of a standing wave pattern in the pipe.

In the introductory undergraduate laboratory, the experiments that are commonly performed with the Kundt’s tube are the study of the standing wave patterns in closed or open tubes (when 100% of the incident wave is reflected) and the measurement of the speed of sound [1-9].

If a sample of absorbing material is placed at the end of the tube, some of the incident sound energy is absorbed by the sample, then the incident and reflected waves have different amplitudes. The total sound field in the pipe is the superposition of a standing wave and a wave travelling in the tube-axis direction. The ISO-10534-1 provides an experimental method for calculating the absorption coefficient of the sample [10]. This experimental procedure can be easily reproduced by using ordinary equipment in undergraduate laboratories.

In this work, we propose to perform an experimental study of standing waves in Kundt’s tube varying the absorption characteristics of its closed end. The proposal is suitable for undergraduate students of engineering, architecture and other scientific fields that need a good understanding of vibrations and waves and materials’ properties.

Figure 1. Kundt’s tube.
2. Theory

Sound waves striking a surface are either reflected or absorbed. The amount of energy reflected or absorbed depends on acoustic properties of the surface.

In a Kundt’s tube, one of the tube ends is bounded by a sound source (x=0) and the other by a sample (x=L). A plane sound wave propagates along the tube axis. If a progressive wave is impeded by a barrier, it can be reflected at that location. If the tube has boundaries on both sides, standing waves and resonance phenomena will occur [11].

The resulting sound field in the tube, when excited using pure tones, is:

\[ p = p_+ + p_- = p_0 [e^{-jkx} + re^{+jkx}] \]  

(1)

The first term describes a wave moving in the +x direction and the second summand, one moving in the -x direction; \( p_0 \) describes the amplitude of the wave travelling toward the reflector. The equation takes into consideration that the returning wave (-x direction) may be attenuated relative to the approaching wave by a reflection coefficient \( r \). The total sound field can be written:

\[ p = 2p_0 r \cos(kx) + p_0 (1 - r) e^{-jkx} = p_s + p_f \]  

(2)

where \( p_s \) is a standing wave and \( p_f \) is a wave travelling in the x-direction. The combination of standing and travelling waves is seen in the ripple of the locally measured rms-value:

\[ \langle p^2 \rangle_{\text{rms}} = \frac{1}{2} |p|^2 = \frac{1}{2} |p|^2 [1 + R^2 + 2R \cos(2kx + \phi)] \]  

(3)

where \( R = |r| \).

For total absorption, \( R=0 \), the sound field is only given by the progressive wave \( p_f \) and the rms-value is constant. For total reflection, \( R=1 \), the sound field is dominated by the standing wave \( p_s \) and the rms-value represents space dependence with large fluctuations.

For partial reflection, \( 0<R<1 \), only a part of the incident wave is reflected. As a result, there is more energy moving to the right than what is being reflected to the left, so there is a net energy flow to the right. The resulting sound field is given by equation (2): a standing wave that appears to advance toward the right. The locations where the wave pattern reaches a maximum (or minimum) value are fixed in space. The rms-value represents space dependence with smaller fluctuations.

For a visualisation of these different combinations, ‘Acoustics and Vibration Animations’ of D. A. Russell can be consulted [12].

2.1. Resonant frequencies

In general, not all the multiple waves reflected between the ends of the tube will be in phase, and the amplitude of the wave pattern will be small. However, at certain frequencies, all the reflected waves are in phase, resulting in a very high amplitude standing wave. These frequencies are called resonant frequencies.

Resonant frequencies depend on the length of the tube \( L \). The resonance states also depend on whether the ends of the tube are open or closed. For a closed tube (a tube open at one end and closed at the other), resonance occurs for the following frequencies:

\[ f_n = n \frac{v}{4L} = nf_1 \quad n = 1,3,5, \ldots \]  

(4)

where \( v \) is the speed of sound in the tube. Note that only odd resonance modes appear.

2.2. Normal absorption coefficient

In architectural acoustics, the energy absorption coefficient or absorption coefficient is [13]:

\[ \alpha = 1 - \frac{I_r}{I_i} \]  

(5)

where \( I_i \) and \( I_r \) are the incident intensity and the reflected intensity, respectively. The absorption coefficient of materials varies from 0 (total reflection) to 1 (total absorption). The absorption
The coefficient does not depend on the incident intensity. In general, $\alpha$ is a function of the frequency of the incident wave and of the incident angle. The normal incidence value is denoted $\alpha_n$.

In the Kundt’s tube, the maximum and minimum values of the locally measured rms-value (equation 3) are:

$$p_{\text{max}}^2 = \frac{1}{2}p_0^2[1 + R^2 + 2R] = \frac{1}{2}p_0^2[1 + R]^2$$  \hspace{1cm} (6)

$$p_{\text{min}}^2 = \frac{1}{2}p_0^2[1 + R^2 - 2R] = \frac{1}{2}p_0^2[1 - R]^2$$  \hspace{1cm} (7)

From equations (6) and (7):

$$R = \frac{p_{\text{max}} - p_{\text{min}}}{p_{\text{max}} + p_{\text{min}}} = \frac{(p_{\text{max}}/p_{\text{min}}) - 1}{(p_{\text{max}}/p_{\text{min}}) + 1}$$  \hspace{1cm} (8)

and the normal absorption coefficient:

$$\alpha_n = 1 - R^2 = 4 \left(\frac{p_{\text{max}}}{p_{\text{min}}}\right)^2 \left[\left(\frac{p_{\text{max}}}{p_{\text{min}}}\right) + 1\right]^2$$  \hspace{1cm} (9)

The quotient $(p_{\text{max}}/p_{\text{min}})$ is called “Standing Wave Ratio” and gives the name to the method used for the determination of the normal absorption coefficient [10].

3. Experimental

The PASCO WA-9612 Resonance System has been used [14]. This equipment includes a plastic tube (inside diameter 31.4 mm, overall length 0.90 m), a speaker, a miniature microphone that can be mounted on the end of the probe arm, a digital function generator capable of driving the speaker and an oscilloscope (see Figure 2).

![Figure 2. PASCO WA-9612 Resonance System](image)

![Figure 3. Samples used in this work.](image)
Samples of four acoustic sound insulators have been used (see Figure 3). All the materials were porous absorbers. Sample 1 was a foam. Samples 2 and 3 combined different polyurethane foams compacted and compressed (reconstituted foam). The density of sample 2 was 150 kg·m\(^{-3}\) and that of sample 3 was 80 kg·m\(^{-3}\). Sample 4 was a recycled product of industrial waste. Samples were circular cylinders with both faces flat and parallel and sides smooth.

3.1. Resonant frequencies
The procedure for measuring the resonant frequencies was [14]:
- Mount the speaker at one end of the tube and the sample under test at the opposite end. Connect the microphone output to the vertical oscilloscope input and the function generator output to the horizontal oscilloscope input. Set the function generator to sinusoidal at approximately 80 Hz. Adjust the amplitude of the function generator until the sound from the speaker is clearly audible, but not loud.
- Increase the frequency slowly and listen carefully for a relative maximum in the sound level. This relative maximum indicates a resonance mode in the tube. Adjust the frequency carefully to find the lowest frequency at which a relative maximum occurs. Record the value of this lowest resonant frequency \(f_{\text{low}}\).
- Raise the frequency slowly until you find a new resonant frequency. Record the value of this resonant frequency \(f_{\text{res}}\).
- Continue finding still higher resonant frequencies.
- Divide each of the resonant frequencies \(f_{\text{res}}\) by the lowest resonant frequency \(f_{\text{low}}\). If the results give a series of whole numbers, \(f_{\text{low}}\) is the fundamental frequency of the closed tube: \(f_{\text{low}} = f_1\).
- If the results do not give a series of whole numbers, you may not have found the lowest resonant frequency for the tube. If this is the case, use the results to determine the lowest resonant frequency of the tube: \(f_1 < f_{\text{low}}\).

3.2. Normal absorption coefficient
The procedure for measuring \(\alpha_n\) at a frequency \(f\) was [15-17]:
- Mount the speaker at one end of the tube and the sample under test at the opposite end. Connect the microphone output to the vertical oscilloscope input and the function generator output to the horizontal oscilloscope input. Set the function generator to sinusoidal at the selected frequency.
- Insert the microphone into the tube and place it near the sample. Move the microphone along the tube until a pressure maximum is detected. Measure the value of pressure at this point in the oscilloscope, \(p_{\text{max}}\).
- Move the microphone along the tube until the minimum nearest to the sample is detected. This minimum is chosen in order to minimize a possible error caused by sound attenuation along the tube. Measure the value of pressure at this point in the oscilloscope, \(p_{\text{min}}\).
- Determine the value of the normal absorption coefficient using equation (9).
- Repeat the measurement at least 20 times and calculate the average value of the normal absorption coefficient and the corresponding measurement error.
- Compare the results with those obtained with the commercial system ACUPRO that conforms to standards ISO 10534-2 and ASTM E-1050.

4. Results

4.1. Resonant frequencies
Table 1 shows the resonant frequencies obtained for the sample 1. The results of dividing each resonant frequency \(f_{\text{res}}\) by the lowest measured resonant frequency (\(f_{\text{low}}=304.5\) Hz) do not give a series of whole numbers, so \(f_{\text{low}}\) is not the fundamental frequency \(f_1\). Taking into account that the closed tube produces only odd resonant modes, it is easy to conclude that the first measured resonant frequency, \(f_{\text{low}}\), corresponds to \(n=3\).
Table 1. Resonant frequencies obtained for the sample 1.

| f_{res} (Hz) | f_{res} / f_{low} | n  |
|--------------|-------------------|----|
| f_{low}=304.5| 1.0               | 3  |
| 475.5        | 1.6               | 5  |
| 661.5        | 2.2               | 7  |
| 845.5        | 2.8               | 9  |
| 1030.5       | 3.4               | 11 |
| 1221.0       | 4.0               | 13 |
| 1410.5       | 4.6               | 15 |
| 1599.5       | 5.3               | 17 |
| 1777.5       | 5.8               | 19 |
| 1972.0       | 6.5               | 21 |
| 2149.5       | 7.1               | 23 |
| 2342.5       | 7.7               | 25 |
| 2521.5       | 8.3               | 27 |
| 2720.5       | 8.9               | 29 |
| 2894.0       | 9.5               | 31 |
| 3081.0       | 10.1              | 33 |
| 3268.5       | 10.7              | 35 |
| 3426.0       | 11.3              | 37 |
| 3650.0       | 12.0              | 39 |
| 3832.0       | 12.6              | 41 |
| 4023.0       | 13.2              | 43 |
| 4242.5       | 13.9              | 45 |
| 4405.0       | 14.5              | 47 |
| 4600.0       | 15.1              | 49 |

Figure 4 shows the resonant frequencies versus n for all the samples. Comparing the obtained linear fits with equation (4), one can obtain the value of the fundamental frequency f_1:

- Sample 1: f_{res} = 93.54n + 4.39 \cong f_1n \rightarrow f_1 = 93.54 \text{ Hz}
- Sample 2: f_{res} = 94.00n + 6.21 \cong f_1n \rightarrow f_1 = 94.00 \text{ Hz}
- Sample 3: f_{res} = 94.36n - 3.03 \cong f_1n \rightarrow f_1 = 94.36 \text{ Hz}
- Sample 4: f_{res} = 94.25n + 0.28 \cong f_1n \rightarrow f_1 = 94.25 \text{ Hz}

The obtained fundamental frequency is similar for the four materials, as expected. One can obtain the speed of sound in the tube using equation (4) and the mean value of f_1:

\[ f_1 = \frac{v}{4L} \rightarrow v = 4Lf_1 = 4 \times 0.9 \times 94.04 = 338.5 \text{ m/s} \]
4.2. Normal absorption coefficient

Figure 5 shows the results obtained for the normal absorption coefficient versus frequency for samples 2 and 3. As the experimental method requires single frequency measurements, only the ‘third octave band-centre frequencies’ between 100 Hz and 5000 Hz were selected. However, taking into account the limits for measurements specified by Bruel & Kjær [18] for the Standing Wave Apparatus Type 4002 (diameter 29.0 mm frequency range 800 Hz–6500 Hz; diameter 99 mm frequency range 90 Hz–1800 Hz) the frequency range between 100 and 800 Hz has been discarded for experiments with the PASCO tube (diameter 31.4 mm).

Results show that $\alpha_n$ grows with the frequency until it reaches a value close to 0.90. Then it remains practically constant until 5000 Hz. Sample 2 is a very good sound absorber from 2500 Hz to 5000 Hz. Sample 3 is a very good sound absorber from 1600 Hz to 5000 Hz.
Unfortunately, no ‘reference’ sound absorbing material exists with which to contrast the experimental data obtained with our system in a wide frequency range. Considering this comparison imperative, the sound absorption coefficient of the same samples has been measured using a commercial system [16] conforms to standards ISO 10534-2 and ASTM E-1050 [17]. These measurements correspond to the continuous lines of Figure 5. The results obtained for the normal absorption coefficient of two acoustic reconstituted foams (samples 2 and 3) using the PASCO system shows quite good agreement with the results obtained with the commercial system.

As equation (9) shows, normal absorption coefficient $\alpha_n$ depends on two variables ($p_{\text{max}}$ and $p_{\text{min}}$) which have independent errors. The uncertainty of $\alpha_n$ depends on $(1/p_{\text{min}})$. When $p_{\text{min}}$ decreases (low signal-to-noise ratio) the uncertainty of the obtained result increases. This fact appears more often in experiments involving measurements on thin samples at lower frequencies.

5. Conclusion

The design and study of undergraduate laboratory courses is an area of increasing interest in the physics education community. In this work, an experimental study of standing waves in Kundt’s tube varying the absorption characteristics of its closed end is described and checked for four acoustics insulators. The experimental set up uses ordinary equipment in undergraduate laboratories. These experiments permit students to deepen their knowledge of the science of sound and materials’ properties.

The study of the resonant frequencies shows similar results in all the experiments. This is an expected result because the resonant frequencies depend on the length of the tube and the speed of sound (see equation 4) but not on the characteristics of the closed end.

The procedure for the measurement of the normal absorption coefficient is based on the “Standing Wave Ratio” method. The effect of the frequency on the sound absorption coefficient has been analysed. This method requires single frequency measurements, so it is fairly time consuming. Therefore, we recommend the “third octave band-centre frequencies” between 800 Hz and 5000 Hz to perform the measurements. For comparison, the sound absorption coefficient of the same samples has been measured using a commercial system. This comparison is imperative because no “reference” sound absorbing material exists with which to contrast the experimental data. Reasonably good agreement between the results obtained by the two methods has been found in the frequency range 800 Hz-5000 Hz. A deeper study of the sound absorption coefficient using this technique can be found in reference [15].

6. References

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