Understanding thermal nature of de Sitter spacetime via inter-detector interaction

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Abstract

The seminar discovery by Gibbons and Hawking that a freely falling detector observes an isotropic background of thermal radiation reveals that de Sitter space is equivalent to a thermal bath at the Gibbons-Hawking temperature in Minkowski space, as far as the response rate of the detector is concerned. Meanwhile, for a static detector which is endowed with a proper acceleration with respect to the local freely-falling detectors, the temperature becomes the square root of the sum of the squared Gibbons-Hawking temperature and the squared Unruh temperature associated with the proper acceleration of the detector. Here, we demonstrate, by examining the interaction of two static detectors in the de Sitter invariant vacuum, that de Sitter space in regard to its thermal nature is unique on its own right in the sense that it is even neither equivalent to the thermal bath in Minkowski space when the static detectors become freely-falling nor to the Unruh thermal bath at the cosmological horizon where the Unruh effect dominates, insofar as the behavior of the inter-detector interaction in de Sitter space dramatically differs both from that in the Minkowski thermal bath and the Unruh thermal bath.
I. INTRODUCTION.

De Sitter spacetime, which enjoys the same degree of symmetry as Minkowski spacetime, is one of the most typical curved spacetimes. During the past decades, it has attracted a great deal of special attention because it is, on one hand, believed to play a very important role in cosmology. Our universe, according to current observations and inflation theory, may approach de Sitter spacetime in the far past and the far future. On the other hand, it is discovered that there may exist a holographic duality between quantum gravity on de Sitter spacetime and a conformal field theory living on the boundary identified with the timelike infinity of de Sitter spacetime [1].

In a seminar work, Hawking and Gibbons discovered, by analyzing the periodicity of the imaginary direction of proper time in the propagator of scalars to which a detector moving along a timelike geodesic is nonminimally coupled, that the ratio between the probability of the detector absorbing and emitting a particle with energy \( E \) is 
\[
e^{-2\pi E \sqrt{3/\Lambda}}
\]
where \( \Lambda \) is the cosmological constant, meaning that the detector measures an isotropic background of thermal radiation with the Gibbons-Hawking temperature \( T_{GH} = \frac{1}{\pi} \sqrt{\frac{\Lambda}{12}} \) [2]. Since then, this thermal nature of de Sitter spacetime has been confirmed with other approaches such as embedding the four-dimensional de Sitter spacetime into a five-dimensional flat space [3], thermalization of a detector in de Sitter spacetime in the framework of open quantum systems [4], as well as in other physical contexts [5–11]. Particularly, studies on the spontaneous excitation rate [8], the Lamb shift [9] and the geometric phase [10] of a static or a freely falling atom [detector], as well as the Brownian motion of a particle coupled to vacuum fluctuations in de Sitter spacetime [11] show that the thermal nature of de Sitter spacetime leaves an imprint of a thermal bath on various quantum phenomena. Taking the spontaneous excitation of a ground-state atom coupled to a conformally invariant scalar field in the de Sitter invariant vacuum for an instance, if the atom is freely falling, it would spontaneously excite as if it were immersed in a thermal bath at \( T_{GH} \); while if it is static, the temperature of the thermal bath becomes \( T_S = \sqrt{T_{GH}^2 + T_U^2} \), where \( T_U \) is the Unruh temperature associated with the inherent acceleration of the static atom [8].

The aforementioned studies [2, 3, 8–11] show that as far as one detector is concerned, de Sitter spacetime is indistinguishable from a thermal bath in Minkowski space in terms of the radiative properties of the detector such as the transition rates and the Lamb shift. Questions
then arise as to what will happen if two spatially separated detectors are considered and whether de Sitter spacetime is still equivalent to a thermal bath in Minkowski spacetime in terms of physical traits of the two-detector system. The inter-detector interaction induced by vacuum fluctuations of fields the detectors are coupled to is such a trait, which can then be exploited to reveal the nature of de Sitter spacetime in addition to the transition rate of a single detector. We will examine this inter-detector interaction to see if the equivalence between the thermal bath as seen by a single detector in de Sitter spacetime and the thermal bath in Minkowski spacetime at temperature $T_S$ still holds, and if not, how they differ.

II. THE SETUP AND THE APPROACH.

We consider that two static detectors labeled by $A$ and $B$ are located at the same radial but different azimuthal coordinates in four-dimensional de Sitter spacetime which can be represented as the hyperboloid

$$z_0^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 = -\alpha^2$$  \hspace{1cm} (1)$$

embedded in five dimensional Minkowski space with the metric

$$ds^2 = dz_0^2 - dz_1^2 - dz_2^2 - dz_3^2 - dz_4^2.$$  \hspace{1cm} (2)$$

Here and after, $\alpha = \sqrt{\frac{3}{\Lambda}}$. Applying the parametrization that

$$z_0 = \sqrt{\alpha^2 - r^2} \sinh \left( \frac{t}{\alpha} \right),$$

$$z_1 = \sqrt{\alpha^2 - r^2} \cosh \left( \frac{t}{\alpha} \right),$$

$$z_2 = r \cos \theta ,$$

$$z_3 = r \sin \theta \cos \varphi ,$$

$$z_4 = r \sin \theta \sin \varphi ,$$  \hspace{1cm} (3)$$

we obtain the following static de Sitter metric,

$$ds^2 = \left( 1 - \frac{r^2}{\alpha^2} \right) dt^2 - \left( 1 - \frac{r^2}{\alpha^2} \right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2.$$  \hspace{1cm} (4)$$

Obviously, for this metric, the sphere with $r = \alpha$ is singular, and it is the so called cosmological horizon. The detectors are then assumed to be conformally coupled to a massless scalar
field \( \phi(x) \) with \( x = x(t, r, \theta, \varphi) \) in the de Sitter invariant vacuum \([12, 13]\), which satisfies
\[
\left( \nabla_\nu \nabla^\nu + \frac{1}{6} R \right) \phi(x) = 0 \tag{5}
\]
with \( R = 12\alpha^{-2} \) being the scalar curvature of de Sitter spacetime. We can expand \( \phi(x) \) in terms of a complete set of field modes \( f_k \) which are solutions of the above equation as
\[
\phi(x) = \int d^3k \left[ a_k(t) f_k(x) + a_k^\dagger(t) f_k^*(x) \right] \tag{6}
\]
with \( a_k \) and \( a_k^\dagger \) being the annihilation and creation operators with momentum \( k \).

We now model the two detectors as two two-level atoms, and denote the ground state and the excited state of atom \( \xi(= A, B) \) by \( |g_\xi \rangle \) and \( |e_\xi \rangle \) and the energy level gap by \( \omega_\xi \).

Since the radial coordinates of the two atoms are identical, the two atoms share the same proper time \( \tau \). Then the interaction Hamiltonian between the atoms and the field can be described by
\[
H_I(\tau) = \mu R^A_2(\tau) \phi(x_A(\tau)) + \mu R^B_2(\tau) \phi(x_B(\tau)) \tag{7}
\]
with \( \mu \) being a very small coupling constant, \( R^\xi_2 = \frac{i}{\tau}(R^\xi_\perp - R^\xi_\parallel) \), \( R^\xi_\perp = |g_\xi \rangle \langle e_\xi | \) and \( R^\xi_\parallel = |e_\xi \rangle \langle g_\xi | \).

Because each atom is perturbed by the fluctuating field \( \phi(x) \) in the vacuum, it emits a radiative field as a backreaction which then acts on the other atom and thus an interatomic interaction potential is resulted. We exploit the fourth-order DDC formalism \([14, 15]\) to calculate this interaction potential, which allows a distinct separation of the contributions of the vacuum fluctuations of the field and the radiation reaction of the atoms. This formalism was proposed by Dalibard, Dupont-Roc and Cohen-Tannoudji \[the DDC formalism\] \([14, 15]\) in an attempt to understand the dynamics of an atomic system coupled to the radiation field and gain insight into the radiative processes in terms of fluctuations of two interacting systems, i.e., a large reservoir and a small quantum system. The DDC formalism was then widely utilized to investigate various second-order vacuum-fluctuation-induced effects including resonant interactions \([8, 16–26]\), just to name a few. Very recently, to deal with the interaction between two ground-state atoms in interaction with fluctuating scalar fields in vacuum, which is a fourth-order perturbation effect, this formalism has been generalized from the second order in its original form to the fourth order \([27]\).

\textsuperscript{1} Throughout the paper, we exploit the units that \( \hbar = c = k_B = 1 \).
According to Ref. [27], the contribution of the vacuum fluctuations to the interatomic interaction potential between the two ground-state atoms [vf-contribution] is given by

$$
(\delta E)_{vf} = 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_A(\tau), x_B(\tau_3))\chi^A(\tau, \tau_1) \times \chi^B(\tau_2, \tau_3) + A \Rightarrow B \text{ term ,}
$$

while that of the radiation reaction of the atoms [rr-contribution] by

$$
(\delta E)_{rr} = 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_A(\tau), x_B(\tau_3))\chi^A(\tau, \tau_1) \chi^B(\tau_2, \tau_3) + 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_A(\tau_1), x_B(\tau_3))\chi^A(\tau, \tau_1) \chi^B(\tau_2, \tau_3) + 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_A(\tau_3), x_B(\tau_2))\chi^A(\tau, \tau_1) \chi^B(\tau_1, \tau_2) + 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_A(\tau_2), x_B(\tau_3))\chi^A(\tau, \tau_1) \chi^B(\tau_1, \tau_2) + 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_B(\tau_1), x_A(\tau_3))\chi^A(\tau, \tau_1) \chi^B(\tau_1, \tau_2) + 2i\mu^4 \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 C^F(x_B(\tau_2), x_A(\tau_3))\chi^A(\tau, \tau_1) \chi^B(\tau_1, \tau_2) + A \Rightarrow B \text{ terms ,}
$$

In the above two equations, $C^\xi$ and $\chi^\xi$ are the symmetric and antisymmetric statistical functions of the atoms defined by

$$
C^\xi(\tau, \tau') = \frac{1}{2} \langle g_\xi | \{ R^\xi_2 f(\tau), R^\xi_2 f(\tau') \} | g_\xi \rangle ,
$$

$$
\chi^\xi(\tau, \tau') = \frac{1}{2} \langle g_\xi | [ R^\xi_2 f(\tau), R^\xi_2 f(\tau') ] | g_\xi \rangle ,
$$

with

$$
R^\xi_2 f(\tau) = \frac{i}{2} \left[ R^\xi(\tau_0) e^{-i\omega_2 (\tau - \tau_0)} - R^\xi(\tau_0) e^{i\omega_2 (\tau - \tau_0)} \right]
$$

the free part of the atomic operator $R^\xi_2(\tau)$, and $C^F$ and $\chi^F$ are the symmetric correlation function and the linear susceptibility of the scalar field defined by

$$
C^F(x_A(\tau), x_B(\tau')) = \frac{1}{2} \langle 0 | \{ \phi^f(x_A(\tau)), \phi^f(x_B(\tau')) \} | 0 \rangle ,
$$

$$
\chi^F(x_A(\tau), x_B(\tau')) = \frac{1}{2} \langle 0 | [ \phi^f(x_A(\tau)), \phi^f(x_B(\tau')) ] | 0 \rangle
$$

with $\phi^f(x)$ being the free part of the scalar field operator not including the radiative field of the atoms and $| 0 \rangle$ the de Sitter-invariant vacuum state [12]. Adding up Eqs. (8) and (9), we then obtain the total interaction potential, $(\delta E)_{tot} = (\delta E)_{vf} + (\delta E)_{rr}$. 

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III. INTERATOMIC INTERACTION POTENTIAL IN DE SITTER SPACETIME.

To calculate the vf- and rr-contribution with Eqs. (8) and (9), we must first compute the symmetric correlation function $C^F$ and the linear susceptibility $\chi^F$ of the field along the trajectories of the atoms, which, with an appropriate choice of coordinates, can be described by $x_A(\tau) = (t_A(\tau), r_0, \varphi_A)$ and $x_B(\tau) = (t_B(\tau), r_0, \varphi_B)$ with $\varphi_\xi \in [0, \pi]$ and $\varphi_B > \varphi_A$. Combining these trajectories, the metric Eq. (4) and the following two-point correlation function of the scalar field at two arbitrary points $x$ and $x'$ [13]

$$\langle 0 | \phi^f(x) \phi^f(x') | 0 \rangle = -\frac{1}{4\pi^2} \left[ (z_0 - z'_0 - i\epsilon)^2 - \sum_{i=1}^{4} (z_i - z'_i)^2 \right]^{-1}$$

(15)

with Eqs. (13) and (14), we obtain

$$C^F(x_A(\tau), x_B(\tau')) = \frac{1}{8\pi^2} \int_0^\infty d\omega \frac{\sin [\omega F(r_0, \varphi_0)]}{H(r_0, \varphi_0)} \coth (\pi \omega \alpha \sqrt{g_{00}(r_0)}) (e^{-i\omega \Delta \tau} + e^{i\omega \Delta \tau}),$$

(16)

$$\chi^F(x_A(\tau), x_B(\tau')) = \frac{1}{8\pi^2} \int_0^\infty d\omega \frac{\sin [\omega F(r_0, \varphi_0)]}{H(r_0, \varphi_0)} (e^{-i\omega \Delta \tau} - e^{i\omega \Delta \tau}),$$

(17)

where $\Delta \tau = \tau - \tau'$, $\varphi_0 = \varphi_B - \varphi_A$, $g_{00}(r_0) = 1 - r_0^2/\alpha^2$, and

$$F(r_0, \varphi_0) \equiv 2\alpha \sqrt{g_{00}(r_0)} \sinh^{-1} \left( \frac{r_0 \sin(\varphi_0/2)}{\alpha \sqrt{g_{00}(r_0)}} \right),$$

(18)

$$H(r_0, \varphi_0) \equiv 2r_0 \sin(\varphi_0/2) \sqrt{1 + \frac{r_0^2 \sin^2(\varphi_0/2)}{\alpha^2 g_{00}(r_0)}}.$$  

(19)

Here, in expressing $C^F(x_A(\tau), x_B(\tau'))$ and $\chi^F(x_A(\tau), x_B(\tau'))$ as an integration over $\omega$, we have used the method of the Fourier transform. For brevity, we next abbreviate $F(r_0, \varphi_0)$ and $H(r_0, \varphi_0)$ which both have the dimension of length as $F$ and $H$, and $g_{00}(r_0)$ which is dimensionless as $g_{00}$.

Putting Eq. (12) in Eqs. (10) and (11), we can further simplify the two statistical functions of the atoms. Then, inserting them and Eqs. (16) and (17) into Eqs. (8) and (9), and performing the triple integrations with respect to $\tau_1$, $\tau_2$ and $\tau_3$ for an infinitely long time interval, i.e., $(\tau - \tau_0) \to \infty$, we can express the respective vf- and rr-contribution to the interaction potential as

$$\langle \delta E \rangle_{vf} = -\frac{\mu^4 \omega_A \omega_B}{64\pi^4 H^2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{\omega_2 \sin(\omega_1 F) \sin(\omega_2 F) \coth (\pi \omega_1 \alpha \sqrt{g_{00}})}{(\omega_1^2 - \omega_2^2)(\omega_1^2 - \omega_A^2)(\omega_B^2 - \omega_1^2)}$$

(20)
and

\[
(\delta E)_{rr} = -\frac{\mu^4}{32\pi^4H^2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{\omega_1 \sin(\omega_1 F) \sin(\omega_2 F)}{\omega_2^2 - \omega_1^2} \left[ \frac{\omega_1 + \omega_A + \omega_B}{(\omega_1 + \omega_A)(\omega_1 + \omega_B)(\omega_A + \omega_B)} \right. \\
\left. - \frac{\omega_A \omega_B}{(\omega_1^2 - \omega_A^2)(\omega_1^2 - \omega_B^2)} \right].
\] (21)

Finally, adding up the above two equations, we arrive at the following total interaction potential after some simplifications,

\[
(\delta E)_{tot} = -\frac{\mu^4\omega_A \omega_B}{128\pi^3H^2} \int_0^\infty du \left[ e^{-2uF} \frac{1}{(u^2 + \omega_A^2)(u^2 + \omega_B^2)} + \frac{2 \sin(2uF)}{(u^2 - \omega_A^2)(u^2 - \omega_B^2)(e^{2\pi u\alpha\sqrt{g_{00}}} - 1)} \right].
\] (22)

We next study how the interaction potential behaves as the physical separation varies in some special regimes. Let us note that the physical interatomic separation, i.e., the length of the geodesic connecting two spatial points \((r_0, \pi/2, \varphi_A)\) and \((r_0, \pi/2, \varphi_B)\), is given by

\[
\rho \equiv \rho(r_0, \varphi_0) = 2 \sqrt{\alpha^2 - r_0^2 \cos^2 \left( \frac{\varphi_0}{2} \right)} \tan^{-1}\left( \frac{r_0 \sin \left( \frac{\varphi_0}{2} \right)}{\alpha \sqrt{g_{00}}} \right)
\] (23)

with \(\varphi_0 = \varphi_B - \varphi_A\) [see the Appendix].

For an \(\alpha\) approaching infinity, both \(F\) and \(H\) are equal to \(\rho = 2r_0 \sin(\varphi_0/2)\), and the second term on the right of Eq. (22) vanishes. As a result, the total interaction potential Eq. (22) reduces to

\[
(\delta E)_{tot}^M = -\frac{\mu^4\omega_A \omega_B}{128\pi^3\rho^2} \int_0^\infty du \frac{e^{-2u\rho}}{(u^2 + \omega_A^2)(u^2 + \omega_B^2)}
\] (24)

which is exactly the interaction potential of two static atoms at a separation \(\rho\) in the flat Minkowski spacetime [28]. This is physically expected, as the de Sitter metric Eq. (4) reduces to the Minkowski metric when \(\alpha \to \infty\).

However, for a finite \(\alpha\), there does not exist a simple relation between \(F\) (\(H\)) and the physical inter-detector separation \(\rho\). In order to find how the interaction potential varies with \(\rho\), let us now analyze the behavior of the interaction potential in two special regions where an explicit relation between \(F\) (\(H\)) and \(\rho\) exists, i.e., the region far away from or very close to the cosmological horizon, where \(r_0 \ll \alpha\) or \(r_0 \lesssim \alpha\) respectively. For simplicity, we next assume that the two atoms are identical with the same transition frequency \(\omega_0\).
A. Interaction potential of two atoms far away from the cosmological horizon.

When the two atoms are located very far away from the cosmological horizon, i.e., when $r_0 \ll \alpha$, both $F$ and $H$ are approximated by $\rho \approx 2r_0 \sin(\varphi_0/2)$ and the temperature as observed by a single detector $T_S \sim T_{GH} \equiv \frac{1}{2\pi \alpha}$. A thermal wavelength $\beta_{GH} = \frac{T_{GH}}{2\pi}$ can then be defined. So $r_0 \ll \alpha$ means $r_0 \ll \beta_{GH}$. Note that there is another characteristic length for the problem under consideration, i.e., the transition wavelength of the atoms, $\lambda = \frac{2\pi \omega_0}{r_0}$.

We then find that when $\rho \ll \lambda \ll \beta_{GH}$, the vf-contribution to the interatomic interaction potential Eq. (20) can be approximated by

\[
(\delta E)_{vf} \approx \frac{\mu^4}{256\pi^3 \rho} - \frac{\mu^4 T_{GH}^2}{384\pi \omega_0^2 \rho} \left(1 - \frac{1}{4} \rho^2 \omega_0^2\right),
\]

(25)

which results in a repulsive interaction force, and the rr-contribution Eq. (21) by

\[
(\delta E)_{rr} \approx -\frac{\mu^4}{512\pi^2 \omega_0 \rho^2} + \frac{\mu^4}{256\pi^3 \rho} - \frac{\mu^4 T_{GH}^2}{384\pi \omega_0^2 \rho} \left(1 - \frac{1}{4} \rho^2 \omega_0^2\right),
\]

(26)

which leads to an attractive interaction force much greater than the vf-contribution.

Adding Eqs. (25) and (26) up, we obtain the following total interaction potential,

\[
(\delta E)_{tot} \approx -\frac{\mu^4}{512\pi^2 \omega_0 \rho^2} + \frac{\mu^4}{128\pi^3 \rho} - \frac{\mu^4 T_{GH}^2}{192\pi \omega_0^2 \rho} \left(1 - \frac{1}{4} \rho^2 \omega_0^2\right).
\]

(27)

This interaction potential is dominated by the first term coming from the rr-contribution, which is negative and proportional to $\rho^{-2}$, and it implies an attractive force between the two atoms behaving as $\rho^{-3}$. As compared with the interaction potential in a thermal bath in Minkowski spacetime [28], the first two terms on the right of Eq. (27) which are leading over the third term are identical to those in the latter case. However, the third term which is proportional to $T_{GH}^2$ and thus manifests the thermal effects of de Sitter spacetime on the interatomic interaction, displays a new separation-dependence due to the existence of the extra factor $\left(1 - \frac{1}{4} \rho^2 \omega_0^2\right)$ which is slightly smaller than unity, meaning an interaction potential slightly smaller than that in the Minkowski thermal bath. This distinction however small suggests that the thermal nature of de Sitter space is basically different from that of a thermal bath at temperature $T_S$ in Minkowski spacetime as is revealed by a single detector.

B. Interaction potential of two atoms near the cosmological horizon.

When the two atoms are very close to the cosmological horizon, i.e., $r_0 \lesssim \alpha$, $\rho \sim \pi \alpha \sin(\varphi_0/2)$, and $T_S \sim T_U \equiv \frac{a(r_0)}{2\pi}$ with $a(r_0) \equiv \frac{r_0}{\alpha \sqrt{\alpha^2 - r_0^2}}$, which is very large. Accordingly,
the characteristic wavelength $\beta_U = T_U^{-1}$ is extremely small. If we further assume $\rho \gg \lambda$, then for two atoms located very close to the cosmological horizon, we have $\beta_U \ll \lambda \ll \rho$. In this case, the vacuum fluctuations and the radiation reaction of the atoms yield almost equally important contributions to the interatomic interaction potential, which are both approximated by

$$(\delta E)_{\text{vf}} \approx (\delta E)_{\text{rr}} \approx \frac{\mu^4}{4096 \omega_0^2 T_U \rho^4 \ln(4T_U \rho)},$$

and thus

$$(\delta E)_{\text{tot}} \approx \frac{\mu^4}{2048 \omega_0^2 T_U \rho^4 \ln(4T_U \rho)},$$

which leads to a repulsive force between the two atoms. Eqs. (28) and (29) show a completely new behavior of $\sim [T_U \rho^4 \ln(4T_U \rho)]^{-1}$ for the interaction potential in sharp contrast to that of two atoms located very far away from the cosmological horizon [refer to Eqs. (25)-(27)]. More importantly, this behavior also deviates dramatically from its counterpart in the Minkowski thermal bath, as the $\text{vf}$- and $\text{rr}$-contribution and thus the total interaction potential in the latter case oscillate obviously with the interatomic separation and the interaction force between the two atoms can be either attractive or repulsive and even be null [28], while in the present case the interaction potential decays monotonically with the interatomic separation and it generates a repulsive force.

As this novel behavior emerges in the regime when the effect of proper acceleration of the static atoms with respect to the locally inertial ones dominates, one may wonder whether the thermal nature of de Sitter space as revealed by the interaction potential may approximate to that of the Unruh thermal bath associated with the proper acceleration of the static atoms. To answer this question, let us make a comparison of the result in the present case and that in the case of two atoms in synchronous uniform acceleration perpendicular to the interatomic separation in Minkowski spacetime. We then see that the behavior of the interatomic interaction we found in the present paper clearly differs from $\sim (T_U \rho^4)^{-1}$ for two uniformly accelerated atoms in Minkowski spacetime [29]. So, de Sitter spacetime as seen by two detectors in terms of the inter-detector interaction is also distinctive from an Unruh thermal bath.

Therefore, the thermal nature of de Sitter space is intrinsically different both from the thermal bath in Minkowski space and the Unruh thermal bath felt by non-inertial observers,
and in principle one can tell from the behavior of the interatomic interaction as the physical inter-detector separation varies whether the detectors are in de Sitter spacetime or in a thermal bath in Minkowski spacetime or even the detectors are uniformly accelerating themselves with respect to an inertial frame. In this sense, de Sitter spacetime is unique on its own right in regard to its thermal nature, and it is neither equivalent to a thermal bath in Minkowski spacetime nor to an Unruh thermal bath as seen by non-inertial observers. Finally, it is worth pointing out that this remarkable thermal character of de Sitter spacetime may also be revealed by examining the entanglement dynamics of a pair of detectors [30, 31]. However, it cannot be disclosed via the resonance interaction between two detectors [atoms] in one of the maximally entangled states, i.e., the symmetric or antisymmetric entangled state, although more than one detector is also involved [32], as the interaction energy is now insusceptible to the state of a field [thermal or nonthermal] [26].

IV. SUMMARY.

In this paper, we have studied the interaction potential of two static detectors [modeled as ground-state two-level atoms] at the same radial but different azimuthal coordinates which are conformally coupled to a scalar field in the de Sitter invariant vacuum. We discover that de Sitter spacetime in regard to its thermal nature as disclosed by the inter-detector interaction induced by coupling with the fluctuating vacuum fields is remarkably different both from a thermal bath in Minkowski spacetime and an Unruh thermal bath as seen by non-inertial observers. In this sense, the thermal nature of de Sitter space is unique in its own right. In principle, one can tell from the behavior of the inter-detector interaction as the physical inter-detector separation varies whether the detectors are in de Sitter spacetime or in a thermal bath in Minkowski spacetime or even the detectors are uniformly accelerating themselves with respect to an inertial frame.

Acknowledgments

This work was supported in part by the NSFC under Grants No. 11690034, No. 11875172, No. 12075084, No. 12047551, and No. 12105061; and the K.C. Wong Magna Fund in Ningbo University.
Appendix A: Derivation of interatomic proper separation.

In this Appendix, we give a derivation of the length of the geodesic connecting two atoms at \((r_0, \frac{\pi}{2}, \varphi_A)\) and \((r_0, \frac{\pi}{2}, \varphi_B)\) with \(\varphi_0 = \varphi_B - \varphi_A > 0\) in de Sitter spacetime.

With the metric Eq. (4), it is easy to deduce the Lagrangian of a particle freely falling in de Sitter spacetime,

\[
L = \frac{1}{2} \left[ \left(1 - \frac{r^2}{\alpha^2}\right) \dot{t}^2 - \left(1 - \frac{r^2}{\alpha^2}\right)^{-1} \dot{r}^2 - r^2 \dot{\theta}^2 - r^2 \sin^2 \theta \dot{\varphi}^2 \right], \tag{A1}
\]

where a dot over the coordinate variables represents the derivative with respect to an arbitrary affine parameter \(\lambda\) (\(\neq \tau\) for photons). Substituting this Lagrangian into the equation of motion of free particles

\[
\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\nu} - \frac{\partial L}{\partial x^\nu} = 0 \tag{A2}
\]

with \(x^\nu = (t, r, \theta, \varphi)\), we obtain

\[
\frac{d}{d\lambda} (r^2 \dot{\theta}) - r^2 \sin \theta \cos \theta \dot{\varphi}^2 = 0, \quad \text{for} \ x^\nu = \theta \tag{A3}
\]

\[
\left(1 - \frac{r^2}{\alpha^2}\right) \dot{t} = E_0, \quad \text{for} \ x^\nu = t \tag{A4}
\]

\[
r^2 \sin^2 \theta \dot{\varphi} = L_\varphi, \quad \text{for} \ x^\nu = \varphi \tag{A5}
\]

where \(E_0\) and \(L_\varphi\) denote the constant energy and angular momentum of the particle, respectively.

For photons travelling along a null geodesic on the plane \(\theta_0 = \frac{\pi}{2}\), we have \(ds^2 = 0\) and \(\dot{\theta}_0 = 0\), and thus

\[
\left(1 - \frac{r^2}{\alpha^2}\right) \left(\frac{dt}{d\lambda}\right)^2 - \left(1 - \frac{r^2}{\alpha^2}\right)^{-1} \left(\frac{dr}{d\lambda}\right)^2 - r^2 \left(\frac{d\varphi}{d\lambda}\right)^2 = 0. \tag{A6}
\]

Combining Eqs. (A4) and (A5) with the above equation, we arrive at

\[
dr = d\varphi \sqrt{\frac{E_0^2 r^4}{L_\varphi^2} - r^2 \left(1 - \frac{r^2}{\alpha^2}\right)}, \tag{A7}
\]

which gives rise to the trajectory after integration

\[
r(\varphi) = r_0 \cos \left(\frac{\varphi_0}{2}\right) \cos \left(\frac{1}{\cos |\varphi - \frac{\varphi_0}{2}|}\right) \tag{A8}
\]

for the geodesic connecting two points \((r_0, \frac{\pi}{2}, \varphi_A)\) and \((r_0, \frac{\pi}{2}, \varphi_B)\) with \(\varphi_B - \varphi_A = \varphi_0\).
Finally, applying Eq. (A8) in the following expression of the proper separation of the two atoms,

$$\rho(r_0, \varphi_0) = \int_{\varphi_{A}}^{\varphi_{B}} d\varphi A \sqrt{1 - \frac{r^2(\varphi)}{\alpha^2}} - \left( \frac{d r(\varphi)}{d \varphi} \right)^2 + r^2(\varphi),$$

we obtain

$$\rho(r_0, \varphi_0) = 2 \sqrt{\alpha^2 - r_0^2 \cos^2 \left( \frac{\varphi_0}{2} \right) \tan^{-1} \left( \frac{r_0 \sin \left( \frac{\varphi_0}{2} \right)}{\alpha \sqrt{g_{00}}} \right)},$$

which is accurately Eq. (23).

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