Individual Vortex Manipulation and Stick-Slip Motion in Periodic Pinning Arrays

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We numerically examine the manipulation of vortices interacting with a moving trap representing a magnetic force tip translating across a superconducting sample containing a periodic array of pinning sites. As a function of the tip velocity and coupling strength, we find five distinct dynamic phases, including a decoupled regime where the vortices are dragged a short distance within a pinning site, an intermediate coupling regime where vortices in neighboring pinning sites exchange places, an intermediate trapping regime where individual vortices are dragged longer distances and exchange modes of vortices occur in the surrounding pins, an intermittent multiple trapping regime where the trap switches between capturing one or two vortices, and a strong coupling regime in which the trap permanently captures and drags two vortices. In some regimes we observe the counterintuitive behavior that slow moving traps couple less strongly to vortices than faster moving traps; however, the fastest moving traps are generally decoupled. The different phases can be characterized by the distances the vortices are displaced and the force fluctuations exerted on the trap. We find different types of stick-slip motion depending on whether vortices are moving into and out of pinning sites, undergoing exchange, or performing correlated motion induced by vortices outside of the trap. Our results are general to the manipulation of other types of particle-based systems interacting with periodic trap arrays, such as colloidal particles or certain types of fractional systems.

I. INTRODUCTION

Vortices in type-II superconductors interacting with ordered or disordered substrates represent an outstanding example of a condensed matter system with competing interactions, since the vortex-vortex repulsion favors a hexagonal lattice while the substrate ordering can favor different lattice symmetries, leading to commensurate-incommensurate transitions\textsuperscript{[1,2]} depinning phenomena in the presence of an external drive\textsuperscript{[3,4]} and order-disorder transitions\textsuperscript{[5,6]}\textsuperscript{[1,2,4,5,8,9,31,12]}. In addition to these basic science issues, vortex motion and pinning are relevant to a variety of applications such as critical current optimization\textsuperscript{[11]} while there are a number of proposals for using individual vortex manipulation to test aspects of statistical physics\textsuperscript{[13,14]} or to create new types of vortex logic devices\textsuperscript{[15,16]}. It has also been proposed that vortices in particular materials can support Majorana fermions\textsuperscript{[17,18]} and that individual vortex manipulation and exchange could be used to create certain types of quantum braiding phenomena for quantum computing operation\textsuperscript{[19,20]}

A growing number of experiments have demonstrated individual vortex manipulation using various techniques such as local magnetic field\textsuperscript{[21]}, magnetic force tips\textsuperscript{[22,23]}\textsuperscript{[21]},\textsuperscript{23},\textsuperscript{24} optical methods\textsuperscript{[25,26]}\textsuperscript{[25,26]} local mechanical applied stress\textsuperscript{[27] and tunneling microscope tips\textsuperscript{[28,29]}}. Numerous related works describe the dynamics of individually manipulated or dragged colloidal particles moving through glass\textsuperscript{[30] or crystalline system\textsuperscript{[31,32]}}\textsuperscript{[31,32]}\textsuperscript{[31,32] where the fluctuations of the probe particle can be used to induce local melting or to study changes in the viscosity across an order-disorder transition. Understanding the different kinds of dynamics associated with particle manipulation on periodic substrates is relevant for vortices in superconductors\textsuperscript{[33,34]} or Bose-Einstein condensates\textsuperscript{[35]}\textsuperscript{[35]}, as well other particle based systems with periodic substrates such as skyrmions\textsuperscript{[36]}\textsuperscript{[36]} ions on optical traps\textsuperscript{[37]} colloidal particles\textsuperscript{[38,39]} and nanofriction systems where individual atoms or molecules can be dragged with a tip\textsuperscript{[40]}. In many of the previous numerical works on the local manipulation of dragged particles, the trap used for manipulation is strong enough to permanently bind a single particle and drag it under a constant force. A more accurate model of current experiments on vortices in a type-II superconductor is a trap of fixed strength moving at fixed velocity that can couple to or decouple from an individual vortex. Vortices dragged by such a trap can either move at the average velocity of the trap or decouple and fall away from the trap, and the trapping of multiple vortices is also possible.

Here we consider a trap with a finite confining force or strength moving across a superconductor containing a periodic array of pinning sites. As a function of trap strength and velocity we identify five generic dynamic phases and several subphases. At low coupling or high trap velocities we find a decoupled phase (I) where the trap can only shift a vortex within a pinning site but cannot depin the vortex. For larger coupling or smaller tip velocities, there is an intermediate coupling phase (II) where a single vortex can be dragged out of the pinning site but is trapped by the next pinning site it encounters in an exchange process. In the intermediate trapping phase (III), vortices can be dragged over a distance of several lattice constants and additional vortex exchange modes arise in adjacent pinning sites. For stronger coupling, there is an intermittent multiple trapping phase (IV) in which the trap alternates between capturing one and two vortices, producing telegraph noise in the trap force fluctuations. At the strongest coupling and lowest trap velocities we find a strong coupling phase (V) where the trap permanently captures two vortices. These
phases are associated with distinct signatures in the force fluctuations exerted on the moving trap, such as stick-slip signals associated with vortices exiting and entering pinning sites or exchanging positions in the trap. We observe nonmonotonic behavior in which the trapping effectiveness increases as the trap velocity decreases, but for the highest trap velocities the system is always in a decoupled phase. We map the dynamic phases as a function of coupling strength, trap velocity, and the angle between the driving direction and the pinning lattice symmetry direction.

II. SIMULATION AND SYSTEM

We consider a two-dimensional system with periodic boundary conditions in the $x$ and $y$-directions containing $N_v$ vortices modeled as point particles interacting with a square periodic pinning array. The magnetic field applied perpendicular to the sample plane is set to the matching field $B = B_0$ at which the number of vortices equals the number of pinning sites. We introduce a trap of radius $R_{tr}$ that moves across the sample, representing a magnetic force microscope (MFM) tip as illustrated schematically in Fig. 1(a). The MFM tip creates a localized potential with a finite trapping force that can capture one or more vortices, and it travels at a constant velocity $V_{tr}$ at an angle $\theta$ with respect to the $x$-axis symmetry direction of the pinning lattice. The dynamics of vortex $i$ are determined by the overdamped equation of motion

$$\eta_i \frac{d\mathbf{r}_i}{dt} = F_{i vv} + F_{i vp} + F_{i tr}.$$  (1)

Here $\mathbf{r}_i$ is the position of vortex $i$ and we set the damping coefficient $\eta = 1$. All forces are measured in units of $f_0 = \phi_0^2/(2\pi \mu_0 \lambda^2)$ where $\phi_0$ = $\hbar/2e$ is the flux quantum and $\lambda$ is the London penetration depth. The first term on the right hand side describes the repulsive vortex-vortex interactions, $F_{i vv} = \sum_{j=1}^{N_v} K_1(r_{ij}) \mathbf{r}_{ij}$, where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$, $\mathbf{r}_{ij} = (r_{ij} - r_{ij})/r_{ij}$, and $K_1$ is the modified Bessel function of the second kind. The pinning forces arise from a square lattice of finite range harmonic wells, $F_{i vp} = -\sum_{k=1}^{N_p}(F_p/r_p)(\mathbf{r}_p - \mathbf{r}_k^{(p)})\Theta(r_p - |\mathbf{r}_i - \mathbf{r}_k^{(p)})|$, where $F_p = 0.3$ is the maximum pinning force, $r_p = 0.3$ is the pin radius, $\mathbf{r}_k^{(p)}$ is the location of the $k$-th pinning site, and $\Theta$ is the Heaviside step function. The force from the moving trap $F_{i tr}$ has the same form as the pinning interaction but with a maximum trapping force of $F_{tr}$ and a trapping radius $R_{tr} = 0.5$. The trap translates at a constant velocity of $v_{tr}$. We consider a 20 x 20 square pinning array at a field of $B/B_0 = 1.0$, where $B_0$ is the matching field at which there is one vortex per pinning site. The pinning lattice constant is $a = 1.0$ and we measure all distances in terms of $\lambda$. We initialize the system with each pinning site filled with a vortex. Figure 1(b) schematically illustrates a 4 x 4 subsection of the sample showing the motion of the trap, which is dragging a single vortex. We measure the vortex displacements in and outside of the trap as well as the time series of the force fluctuations on the moving trap. During an individual run we translate the trap a total distance of $D_x = 300a$ in the $x$ direction, corresponding to a total distance of $D_x/cos(\theta)$ in the driving direction. Throughout this work we describe distances in terms of their projections into the $x$ direction.

III. RESULTS

We define the time and location at which an individual vortex $i$ becomes captured by the trap as $(t_{in}^i, \mathbf{r}_{in}^i)$, and the corresponding time and location at which vortex $i$ escapes from the trap as $(t_{out}^i, \mathbf{r}_{out}^i)$. We can then write the capture length $C_i = |\mathbf{r}_{out}^i - \mathbf{r}_{in}^i|/cos(\theta)/a$ as a measure of the distance the vortex travels inside the trap projected into the $x$ direction and normalized by the pinning lattice constant $a$. In Fig. 2(a) we plot $C_i$ versus the trap velocity $v_{tr}$ for a trap with $F_{tr} = 1.0$ moving at an angle of $\theta = 30^\circ$ with respect to the $x$ axis of the pinning array. We observe a clear drop in $C_i$ for $v_{tr} > 0.375$ when the system enters the decoupled phase I in which the trap moves too rapidly to capture any of the pinned vortices. In Fig. 3(a) we show the vortex and pinning site locations along with the trajectories of the vortices and the trap over a fixed period of time for the system in Fig. 2(a) in the decoupled phase I at $v_{tr} = 0.5$. Vortices...
in the pinning sites wiggle a small amount as the trap passes over them but they do not depin.

For $0.19 < v_{tr} < 0.375$, we find an intermediate coupling phase II in which the trap captures a vortex and drags it a projected distance of approximately $2a$ before exchanging places with a pinned vortex. For $0.012 < v_{tr} < 0.19$, the system is in the intermediate trapping phase III where vortices can be dragged a distance of several lattice constants and additional vortex exchanges occur among the sites close to the trap. For $v_{tr} < 0.012$, vortices are able to escape more easily from the slow trap so $C_I$ drops even as the overall amount of displacement $d$ in the system remains high.

FIG. 2: (a) The capture length $C_I$, a measure of the average distance a vortex is dragged by the trap, vs trap velocity $v_{tr}$ in a system with $F_R = 1.0$ where the trap moves at an angle of $\theta = 30^\circ$ with respect to the $x$ axis of the pinning array. The points marked A, B, and C correspond to $v_{tr}$ values at which the images in Fig. 3 were obtained. All distances have been projected into the $x$ direction. (b) The total displacements $d$ of all the vortices over a time interval during which the trap translates by $D_x = 300a$ vs $v_{tr}$. Above $v_{tr} = 0.375$, we find a transition to the decoupled phase I in which the trap does not drag any vortices. In the intermediate coupling phase II for $0.19 < v_{tr} < 0.375$, an individual vortex can be dragged by the trap a distance of $2a$ before exchanging places with a pinned vortex. For $0.012 < v_{tr} < 0.19$, the system is in the intermediate trapping phase III where vortices can be dragged a distance of several lattice constants and additional vortex exchanges occur among the sites close to the trap. For $v_{tr} < 0.012$, vortices are able to escape more easily from the slow trap so $C_I$ drops even as the overall amount of displacement $d$ in the system remains high.

For $0.19 < v_{tr} < 0.375$, we find an intermediate coupling phase II in which the trap captures a vortex and drags it a projected distance of approximately $2a$ before exchanging places with a pinned vortex. In Fig. 3(b), the vortex trajectories in phase II at $v_{tr} = 0.2$ extend from pin to pin following the motion of the trap. For $v_{tr} < 0.19$ we find an intermediate trapping phase III where individual vortices remain inside the trap for distances greater than $2a$ but are not permanently trapped. Simultaneously, vortex exchange motions emerge in the surrounding pinning sites, as illustrated in Fig. 3(c) for $v_{tr} = 0.02$. Figure 2(a) indicates that there is an optimal trapping velocity $v_{tr} = 0.012$ corresponding to the peak in $C_I$ where the vortex can on average be trapped for distances as large as $18a$ before exchanging places with a pinned vortex. For $v_{tr} < 0.012$, $C_I$ drops dramatically when the trap velocity becomes so slow that vortices have enough time to escape from the trap or exchange with neighboring pinned vortices. In contrast, for $v_{tr} > 0.012$, the trapped vortex can remain trapped since it does not have enough time to exchange with another vortex. As $v_{tr}$ increases above 0.012, $C_I$ drops as the trapped vortex experiences larger displacements until the system reaches the I-III transition where the trapped vortex always exchanges with a pinned vortex.

To measure the global effect of the trap, in Fig. 2(b) we plot the scaled net total projected displacement $d$ of all the vortices $d = a^{-1} \sum_{i=0}^{N} |r_i(t_0 + \tau) - r_i(t_0)| \cdot \hat{x}$ versus $v_{tr}$, where $\tau = D/(v_{tr} \cos(\theta))$ is the time required for the trap to translate a projected distance of $D = 300a$. Above the I-II transition at $v_{tr} = 0.375$, $d$ drops to zero. In the intermediate coupling phase II, the trap is never empty, and there is a plateau with $d = 300$ throughout the phase II region of $0.19 < v_{tr} < 0.375$. No individual vortex travels this distance with the trap; instead,
as shown in Fig. 2(a), vortices translate an average distance of \( C_I = 2a \) before encountering a pinning site and exchanging with the pinned vortex. The surrounding vortices remain pinned and do not contribute to \( d \). In the intermediate trapping phase III for \( v_{tr} < 0.19 \), \( d \) increases with decreasing \( v_{tr} \) as vortices surrounding the trap begin to depin from the pinning sites and undergo rotational exchange motions of the type illustrated in Fig. 3(c) at \( v_{tr} = 0.02 \).

We find transitions among the different phases as a function of trap strength \( F_{tr} \) as well as trap velocity. Figure 2(a,b) shows \( C_I \) and \( d \) versus \( v_{tr} \) for driving at \( \theta = 30^\circ \) in the same system from Fig. 2 with a smaller \( F_{tr} = 0.5 \). Both \( C_I \) and \( d \) are less than one, and the system remains in the decoupled phase I for all values of \( v_{tr} \). When we instead consider a larger trap strength \( F_{tr} = 1.8 \), Fig. 2(c,d) shows that phase II appears for \( v_{tr} > 0.3 \), while for \( v_{tr} < 0.03 \) the system is in phase V and the trap always contains two vortices. In Fig. 3(a) we illustrate the vortex trajectories in phase V at \( v_{tr} = 0.02 \), where a multi-vortex exchange process occurs in the vortices adjacent to the trap. The two trapped vortices produce a repulsion that is strong enough to depin the vortex in the pin traversed by the trap along with those in a pair of neighboring pins on either side of the trap. The three depinned vortices form a cascading loop of reoccupancy, and one of them moves to occupy the pinning site behind the trap that was previously vacated. For \( 0.018 < v_{tr} < 0.3 \) we find the intermediate trapping phase III, while for \( 0.03 < v_{tr} < 0.018 \) an intermittent multiple trapping phase IV occurs in which trap alternates between capturing one or two vortices. Figures 3(b) and (c) illustrate typical phase IV and phase III trajectories, respectively. Phase IV contains several subregimes. When \( v_{tr} \) is close to 0.1, the trap permanently captures one vortex and exchanges a second vortex with each pinning site it passes, while at larger \( v_{tr} \), both trapped vortices exchange places with vortices in the pinning sites as the trap moves.

In Fig. 4(a) we plot a heat map of the total displacements \( d \) as a function of trap strength \( F_{tr} \) versus trap velocity \( v_{tr} \) for a driving angle of \( \theta = 30^\circ \) in which we highlight the locations of phases I through V. For \( F_{tr} < 0.75 \), the system is in the decoupled phase I. The effect of changing the angle of drive on \( d \) appears in the \( F_{tr} \) versus \( v_{tr} \) heat maps in Fig. 7(a,b,c) for \( \theta = 0^\circ, 15^\circ, \) and \( 45^\circ \), which trace the evolution of the five different phases. In each case, the transition lines generally shift to higher values of \( F_{tr} \) with increasing \( v_{tr} \). For \( \theta = 0^\circ \) in Fig. 7(a), the trap does not start dragging vortices out of the pinning sites until \( F_{tr} > 1.25 \), and we observe a variety of additional subphases that are not present at larger \( \theta \). Previous work for vortices driven over square periodic pinning
arrays at $\theta = 0^\circ$ showed a series of distinct dynamical phases associated with positive or negative jumps in the velocity-force curves, and the dynamics we observe in Fig. 7(a) is consistent with this type of behavior. In Fig. 7(a) we plot $d$ versus $v_{tr}$ for the $\theta = 0^\circ$ sample at $F_{tr} = 1.8$, where we find numerous jumps at small $v_{tr}$, while in Fig. 8(b), $d$ versus $v_{tr}$ at $F_{tr} = 1.8$ and $\theta = 45^\circ$ has a smoother behavior at small $v_{tr}$ and a step marking the II-III transition at $v_{tr} = 0.25$.

In order for a system in the decoupled phase I to reach the intermediate coupling phase II, in which the trap is able to drag vortices from pin to pin, a vortex must remain inside the trap during the entire time required for the trap to traverse the distance $d_p$ between pins. The trap can interact with the finite size pin once it is within a distance $r_p$ of the pin, giving $d_p = (a - r_p)/C$, where $C = 1$ for $\theta = 0^\circ$ and $C = \sin \theta$ for $0^\circ < \theta < 90^\circ$. The time required to travel this distance is $\delta t_1 = d_p/v_{tr}$. During the entire interval $\delta t_1$, the vortex experiences two competing forces: the trapping force $F_{tr}$ that pulls the vortex toward the center of the trap, and a harmonic restoring force $F_r$ from the background vortex lattice that pulls the vortex toward its equilibrium lattice position. There is also a velocity-dependent drag term $\eta v_{tr}$ that represents an effective viscosity produced by dynamic rearrangements of the background vortex lattice. During the time interval $\delta t_2 = 2r_p/Cv_{tr}$ spent traversing a pin the vortex is also subjected to the pinning force $F_p$. The maximum displacement the vortex can experience under these forces while traveling between pinning sites is then $\Delta r = \delta t_1(F_{tr} - F_r - \eta v_{tr})/\eta - \delta t_2 F_p/\eta$. When $\Delta r \leq r_p$, the vortex remains inside the pin and cannot be captured by the trap, placing the system in phase I, but when
\[ \Delta r > r_p, \] the trap can pull the vortex out of the pin and carry it to the next pin, and the system enters phase II. Thus, we predict that the I-II transition should occur when \( r_p = (a-r_p)(F_{tr}-F_r-\eta v_{tr})/C\eta v_{tr}-2r_p F_p/C\eta v_{tr}, \) which can be written as

\[ F_{tr} = \frac{Cr_p\eta v_{tr}+2r_p F_p}{a-r_p} + F_r + \eta v_{tr}. \quad (2) \]

To obtain an estimate of the value of \( F_{tr} \), first consider the \( \theta = 0^\circ \) case. As the trap passes over a pinned vortex, which we call vortex A, it tends to shift vortex A away from the center of the pin toward the leading or trailing edge of the trap, depending on whether the trap is just arriving at the pinning site or just leaving it. Vortex-vortex interactions cause a similar shift in the position of the neighboring pinned vortices due to the high symmetry of the \( \theta = 0^\circ \) motion. Vortex A can thus be regarded as sitting inside three potential wells: the trap potential, the pin potential, and a harmonic potential produced by the surrounding vortex lattice that provides the restoring force \( F_r \). When \( \theta = 0^\circ \), this restoring force is dominated by interactions from the four closest vortices that are a distance \( a \) from vortex A. The magnitude of the restoring force, to lowest order, can be estimated as

\[ F_r = \frac{K_1(a)}{K_1(a+\delta r)} \] for \( a = 0.005 \), we obtain \( F_r = 1.017 \) for \( \theta = 0^\circ \). The motion becomes less symmetric once \( \theta > 0^\circ \) and the four closest vortices no longer contribute to an effective harmonic confining potential due to their asymmetric distortions. Instead, the next four closest vortices that are a distance \( \sqrt{2}a \) from vortex A maintain sufficient symmetry in their motion to produce a weaker harmonic confining force. We thus obtain

\[ F_r = \frac{K_1(\sqrt{2}a)}{K_1(\sqrt{2}a+\delta r)} \] for \( \delta r = 0.005 \) and \( 0^\circ < \theta < 90^\circ \).

From Eq. 2, the predicted \( v_{tr} = 0 \) intercepts are \( F_{tr} = 1.27 \) for \( \theta = 0^\circ \) and \( F_{tr} = 0.716 \) for \( 0^\circ < \theta < 90^\circ \) due to the different values of \( F_{tr} \) in the two cases. At \( v_{tr} = 0.5 \), the I-II transition line is predicted to fall at \( F_{tr} = 1.99 \) for \( \theta = 0^\circ \), \( F_{tr} = 1.27 \) for \( \theta = 15^\circ \), \( F_{tr} = 1.32 \) for \( \theta = 30^\circ \), and \( F_{tr} = 1.36 \) for \( \theta = 45^\circ \). These predictions are in general agreement with the results in Figs. 6 and 7, but are not exact due to our neglect of higher-order contributions to \( F_r \) and the influence of the non-symmetric arrangement of the closest trapped vortices. For the \( \theta = 30^\circ \) system in Fig. 8(c,d) with \( F_{tr} = 1.8 \), Eq. 2 predicts a I-II transition at \( v_{tr} = 0.9 \), outside the range of values we consider, while there is no value of \( v_{tr} \) satisfying Eq. 2 when \( F_{tr} = 0.5 \) and \( \theta = 30^\circ \) since the \( v_{tr} \) intercept falls at \( F_{tr} = 0.716 \), indicating that the system in Fig. 8(a) should never undergo an I-II transition. For \( \theta = 45^\circ \) in Fig. 7(c), the I-II line separating the decoupled and partially coupled phases is slightly steeper than in the \( \theta = 30^\circ \) and \( \theta = 15^\circ \) samples, as expected from Eq. 2 due to the larger value of \( \sin \theta \). The \( \theta = 0^\circ \) and \( F_{tr} = 1.8 \) sample in Fig. 8(a) is predicted to have a I-II transition at \( v_{tr} = 0.37 \), while for the \( \theta = 45^\circ \) and \( F_{tr} = 1.8 \) sample the I-II transition is predicted to fall at \( v_{tr} = 0.83 \).

In Fig. 9(a) we show the vortex and trap trajectories in phase II for a sample with \( F_{tr} = 1.8 \) and \( \theta = 0^\circ \).
at \( v_{tr} = 0.35 \). Individual vortices are trapped over a distance of one lattice constant, moving along a one-dimensional path defined by the trap trajectory and inducing few to no perturbations in the surrounding vortices before exchanging positions with the next pinned vortex along the path of the trap. At \( v_{tr} = 0.1 \) in Fig. 9(b), the perturbations to the surrounding vortices are stronger, while at \( v_{tr} = 0.02 \) in Fig. 9(c), there is continuous plastic mixing of the vortices in the two rows of pins on either side of the trap trajectory. For driving along \( \theta = 45^\circ \), Fig. 9(d) shows that in phase II at \( F_{tr} = 1.8 \) and \( v_{tr} = 0.35 \), motion occurs along the diagonal with some distortions of the vortices in the adjacent pinning sites.

**IV. FORCE FLUCTUATIONS**

We next examine the time series of the \( x \) direction forces \( f_x \) experienced by the trap as it moves in the different phases. In Fig. 10(a) we plot a representative time series of \( f_x \) for the \( \theta = 30^\circ \) system from Fig. 2(a) and Fig. 3(a) in the decoupled phase I at \( F_{tr} = 1.0 \) and \( v_{tr} = 0.5 \). We find a pronounced stick-slip character in \( f_x \) with a strong asymmetry of sudden increases and gradual decreases. The slow drops in \( f_x \) occur when the moving trap is dragging a vortex inside a pinning site and the force from the pinning site is resisting the pull of the trap, while the rapid increases correspond to intervals when the vortex decouples from the trap and drops back into the pinning site. Figure 10(b) shows that the probability distribution function \( P(f_x) \) has a spike at \( f_x = 0 \) produced by the time periods during which there is no vortex inside the trap. There is a local maximum in \( P(f_x) \) near \( f_x \approx -0.6 \), the value of the \( x \) component of the average decoupling force \( F_{dc} \) at which the vortex escapes from the trap. At decoupling, the vortex is at the edge of the pinning site, where it experiences a force of magnitude \( F_p \), and it is a distance \( r_p \) from the center of the trap, where it is subjected to a force of magnitude \( F_{tr} (r_p / R_{tr}) \). There is also a drag force contribution of \( C \eta v_{tr} \) from the background of pinned vortices. This gives a decoupling force of \( F_{dc} = -F_p - F_{tr} (r_p / R_{tr}) + C \eta v_{tr} \), which for \( \theta = 30^\circ \), \( F_p = 0.3 \), and \( v_{tr} = 0.5 \) gives \( F_{dc} = -0.65 \), in agreement with the location of the local maximum in \( P(f_x) \). As \( v_{tr} \) increases, we find that the local maximum in \( P(f_x) \) shifts to lower absolute values of \( f_x \).

In Fig. 11(a,b) we plot \( f_x(t) \) and \( P(f_x) \) in phase II at \( v_{tr} = 0.2 \) for a sample with \( \theta = 30^\circ \) and \( F_{tr} = 1.0 \). There is no longer a peak in \( P(f_x) \) at \( f_x = 0 \) since the trap always contains one vortex. We find a periodic signal in \( f_x(t) \) containing both stick-slip features and additional smoother oscillations between pairs of force spikes. The force spike pairs arise when the trap captures a new vortex or drops a trapped vortex. Since a trap with \( F_{tr} = 1.0 \) is not strong enough to confine two vortices, every time the trap captures a vortex it sheds the previously captured vortex. The process of bringing a trapped vortex close to a pinned vortex, followed by capture of the pinned vortex, produces a peak in \( P(f_x) \) at \( f_x = -0.4 \). The smooth oscillations occurring on a longer time scale correspond to the transport of a vortex between pinning sites by the trap, since at \( \theta = 30^\circ \) the trap passes over a pinning site in every other column of the pinning array. When the trap passes between two pinned vortices at a distance \( a/2 \), the trapped vortex must cross an energy barrier generated by the repulsive vortex-vortex forces, giving a second peak in \( P(f_x) \) at \( f_x = -0.2 \).

In Fig. 11(c,d) we show \( f_x(t) \) and \( P(f_x) \) for \( v_{tr} = 0.05 \)
in phase III for the $\theta = 30^\circ$ and $F_{tr} = 1.0$ system from Fig. 11(a,b). At this low trap velocity, the trapped vortex produces a larger perturbation of the surrounding vortices as it moves, resulting in the appearance of additional peaks in $P(f_x)$. The highest peak in $P(f_x)$ at $f_x = 0.45$ results when the strongly trapped vortex passes through a pinning site and pushes the pinned vortex out of its way without escaping from the trap.

In phase IV, illustrated for a sample with $\theta = 30^\circ$ and $F_{tr} = 1.8$ at $v_{tr} = 0.12$ in Fig. 12(a), $f_x(t)$ shows a strong telegraph noise signal in which two states arise when the trap alternates between dragging one or two vortices. In phase V at $v_{tr} = 0.02$, the trap always captures two vortices and the telegraph noise is lost. (c) In phase IV at $v_{tr} = 0.048$, there is a transient signature when the trap initially drags one vortex but then captures a second vortex, producing a clearly visible jump in $f_x$.

### V. DISCUSSION

Within the particle description we use, the number of vortices and their shape is held fixed; however, it is possible that a sufficiently strong trapping force could induce vortex shape distortions that could change the dynamics. Additionally, if the trap is strong enough, then in the multiple vortex trapping phases IV and V, the trapped vortices may merge and form multiquantum states. Our results apply to the limit in which the trap is weak enough that such distortions do not occur. In experiments, it is likely that the tip speed will be in the limit of low $v_{tr}$; however, the phase diagrams of Figs. 6 and 7 indicate that most of the phases can be accessed even at the lowest trap velocities by varying the trap strength. It is also possible to use a stationary trap of fixed strength and apply a current so that all of the vortices flow past the trap in order to exert forces on it. When the vortices are moving fast enough that the trap cannot capture a vortex, the system will be in the decoupled state. Our results should be general to other systems of particles interacting with periodic trap arrays, such as colloidal particles in optical or gravitational lattices, where the interactions between colloids can be of magnetic form with a $1/r^3$ behavior or of screened Coulomb or Yukawa form.
VI. SUMMARY

We have numerically examined vortex manipulation in superconductors with a periodic array of pinning sites by a local moving trap. We find five distinct phases depending on the trap strength and velocity. In phase I, which appears for low trap strength or large trap velocity, the vortices are decoupled from the trap, which can move a vortex within a pinning site but cannot depin it. The distribution of forces experienced by the trap has a peak at zero force corresponding to time intervals during which the trap is moving between adjacent pinning sites and contains no vortex. In the intermediate coupling phase II, the trap drags a vortex out of a pinning site and then exchanges that vortex with another pinned vortex. Within phase III we find a counterintuitive effect in which the trap couples more strongly to a single vortex at higher velocities than at lower velocities, since at lower velocities there is enough time for a pinned vortex to complete an exchange with the trapped vortex. Phases II and III both exhibit stick-slip fluctuations of the force experienced by the trap that correlate with vortex exchange events and with the entry and exit of vortices from the trap. Phase IV is an intermittent multiple trapping regime in which the trap alternates between capturing one or two vortices, producing a telegraph noise signature in the trap force fluctuation signal. In phase V, where the trap is strongly coupled and always captures two vortices, the telegraph noise signal is lost. We map the evolutions of these phases for varied trap coupling strength, trap velocity, and the angle of trap motion with respect to the $x$ symmetry axis of the pinning array. For a given trap coupling force, transitions among the phases occur as a function of increasing trap velocity. Our results should be general to other types of particle systems with a periodic substrate subjected to a moving local trap, such as colloidal particles, skyrmions, or ions on optical traps.

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