Evaluation of the accuracy of the ship location determined by GPS global positioning system on a given sea area

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Abstract. The current location of navigation and navigation depends heavily on global satellite navigation systems, mainly GPS (Global Position System). In actual maritime practice, the position of the ship is determined to be the most probable position and will be the center of the circle of probability containing the ship's position. However, this is not entirely accurate because the exact position on the chart depends on many factors such as deviation of the geodetic system, the accuracy of the chart, etc. On the other hand, the probability circle contains the location. The ship is scalar, the radius of error of the probability circle depends on many factors, etc. Therefore, determining the probability location with the highest accuracy is a rather complex problem. In particular, in the waters where the safety system is not complete, the level of survey is not good: Solomon Islands, Papua New Guinea, Indonesia, Cambodia, Vietnam, etc. actual errors of GPS location operating above. The chart is much larger than the error recommended by the manufacturer. In the context of the article, the author proposes a method for determining the probability area containing the actual ship position and its orientation to improve high safety in manoeuvring and navigating vessels at sea, testing on coastal Vung Tau - Vietnam.

1. Introduction
Maritime is now moving towards the integration of electronic maritime. Safe ship operation becomes simple but potentially involves many risks due to the carelessness, ignorance of the duty officer and the limitations of electronic equipment. Locate the ship and assess the accuracy of the location and basic maritime operations when navigating the ship at sea. Although the methods of locating traditional ships by navigation radar or sight are still highly accurate and reliable, they are the methods of determining the main ship's position (primary methods) as recommended by the Organization. International Customs (IMO) [1]. However, marine officers mainly use the method of determining the location of the ship by GPS satellite navigation system at sea. This method is highly accurate when navigating ships on well-secured waters. Coastal areas, when the safety system is limited, the charts are unreliable, it is necessary to have methods to assess the error of the ship's position not caused by the GPS system itself.

Location errors of navigation satellite systems today reach 10-25m (95%), and preferably 1m - 5m (95%) for GPS differential systems [2]. However, when performing the operation on the chart to guide the ship, in addition to the errors of the system itself, there are many types of errors that appear in reality such as; geodetic datum of the chart is not compatible, the chart is built. built with unreliable survey sources, the accuracy of the chart is not high. On the ship, marine officers often determine the area of probability of ship's position is a circle whose center is the definite position, the radius is the error of the GPS navigation system. The method of determining the area of probability is limited as recognizing that the defined position is the center of the probability circle and the direction with the highest probability
of location is not known. Meanwhile, the defined position will be in a probability area depending on the actual impact factors mentioned above. This can affect the safety of navigation, the most likely occurrence is the deviation caused by stranding due to misconception about the error of the ship position. Within the scope of the paper, the author focuses on the general method of determining the actual area of the ship's position received from the GPS receiver and its orientation, testing on the coastal Vung Tau – Vietnam.

2. Assess the error of ship’s position received from GSP receiver on board

Today, navigational navigation is mainly based on the position obtained from the global navigation system, mainly the GPS system. The error of the GPS position published in documents and instructions for receiver use is \( R_{95} = 10 \div 25m \) (95%), depending on the manufacturer [3]. However, it is the accuracy of the GPS system, there are still errors due to the influence of other factors that need to be considered and the probability circle is scalar.

The adjustment of errors to be able to manipulate the exact position of the ship determined by the GPS receiver on the chart depends on the specific conditions and the nature of the impact error. Some basic errors can be limited such as: selection of 2D / 3D ship position determination mode, HDOP (Horizontal Dilution of Precision) geometry errors, errors due to heterogeneous geodetic systems, errors due to initial parameter settings are not correct, etc. Among them, errors due to the geodetic system of the chart and the incompatible GPS system need special attention when manipulating the specified position, specifically as follows:

- Diagrams using the geodetic system WGS84 (WGS72) are compatible with the geodetic system of the GPS system so there are no errors due to different geodetic systems or small errors, which can be ignored [4].
- Charts using the local survey system are not WGS84 / 72 (or for the amount of error correction due to the difference of geodetic system), the GPS receiver will have a correction mode, switch between geodetic or scholar systems. The customs official directly corrects the coordinates received from the GPS receiver before manipulating the chart or calculating the position errors to have a safe navigation plan [5].
- In special cases, charts do not have a clear geodetic system, there is no correction information, only a warning of location errors due to different geodetic systems is unable to be determined [4]. At that time, the position of the ship operating on the chart did not evaluate the accuracy and unreliability. If the travel area cannot be determined by another method, the GPS position received in this case will not meet the standards for maritime safety accuracy (IMO) [6-7].

Normally, the GPS position after adjusting the difference of coordinates due to different geodetic systems will be manipulated on the chart to guide the ship. The accuracy of position M is determined by the area of the probability that the ship's position is a circle centered on F, radius R95 (95%) is the position error of the GPS system (figure 1).

![Figure 1. GPS positioning error.](image1)

![Figure 2. Actual probability area.](image2)
Determining the probability of such a ship’s position has the following limitations: it is assumed that F is the center of the circle of error and the area of probability is not oriented.

The position of determining F in addition to the error of the GPS system itself is subject to a number of influences due to various practical causes. Then, F will be within the circle (O, RL). If the assumption is that the ship’s actual position is determined on the circle (O, RL), then the probability area will be the center circle O, the radius R = R_L + R_95 will contain the ship position. In fact, the center O is assumed that when finding the position error for a sea area, it is not possible to obtain the center coordinates O at the time when the location of F is performed. Therefore, the actual probability area must be a circle centered on F, the radius calculated by the formula R_F = 2 R_L + R_95 (figure 2).

3. The mathematical basis for calculating the area of probability containing the ship's location

The area of probability that contains the ship position is the smallest circle containing the ship’s positions received from the GPS receiver at a fixed position. When the test data set is large enough, ensuring both space and time in a certain sea area, the probability circle will be highly reliable. The general method of determining the probability circle containing the ship’s position includes the following steps:

Step 1: Develop a test location data set
At a fixed position, proceed to determine the position of the ship by the GPS receiver continuously and put it on the chart. Suppose the set of ship positions received is \( A = \{F_1, F_2, F_3, \ldots, F_n\} \) with \( F_i \) is the definite position (t), coordinates \((\varphi_i, \lambda_i)\). The area of probability containing the ship position is the smallest circle with center O, radius M, containing the set of points A.

Step 2: Building theodolite network containing set of points A
Latitude limit: \( \varphi_{\text{min}} \div \varphi_{\text{max}} \), the distance between two consecutive latitudes is 0’001, which matches the accuracy of the latitude received from the GPS receiver.

\[
\varphi_i = \varphi_{\text{min}}, \varphi_{i+1} - \varphi_i = 0'001
\] (1)

Longitude limit: \( \lambda_{\text{min}} \div \lambda_{\text{max}} \), the distance between two consecutive longitudes is 0’001, which corresponds to the accuracy of the longitude received from the GPS receiver (figure 3).

\[
\lambda_i = \lambda_{\text{min}}, \lambda_{i+1} - \lambda_i = 0'001
\] (2)

Figure 3. Theodolite network contains a definite position.

Step 3: Develop a formula to calculate the distance between two points in theodolite
The set of intersections between meridians and parallels is \( B = \{O_{00}, O_{01}, O_{02}, \ldots, O_{ij}, \ldots, O_{xy}\} \)
The point where the latitude \( \varphi_i \) and longitude \( \lambda_j \) intersection is denoted \( O_{ij} (\varphi_i, \lambda_j) \).

The distance (d) from the point \( O_{ij} \in B \) to any ship position \( F_t \in A \) is calculated by Euclidean function

\[
d_i = \sqrt{k^2(\varphi_i - \varphi_j)^2 + (\lambda_i - \lambda_j)^2}
\]

Increase the proportion of chains along the meridian [8]:

\[
k = \frac{MP}{\varphi}.
\]

Progress Latitude:

\[
MP = a \ln \left( \frac{\varphi}{4} + \frac{j}{2} \left( 1 - e \sin j \right)^{\frac{1}{2}} \right)
\]

with “a” is the major semi-axis and e is the eccentricity of the earth's ellipsoid.

The relationship between the corresponding latitude and longitude is graphed in figure 4 [9]:

![Figure 4. Graph of changes in Meridional Parts.](image)

The value (k) can look up table 1 with the argument of latitude \( \varphi \) [9]:

| Latitude | Meridional Parts (MP) | Increasing Factor (IF) |
|----------|-----------------------|------------------------|
| 0°       | 0                     | -                      |
| 10°      | 600.15                | 1.00                   |
| 20°      | 1219.45               | 1.02                   |
| 28°      | **1743.48**           | **1.04**               |
| 30°      | 1880.23               | 1.05                   |
| 40°      | 2612.57               | 1.09                   |
| 50°      | 3463.03               | 1.15                   |
| 60°      | 1415.50               | 1.25                   |
| 70°      | 5954.92               | 1.42                   |
| 80°      | 8367.48               | 1.74                   |
| **85°**  | **10760.93**          | **2.11**               |
| 89°      | 16305.71              | 3.05                   |
| 89°5     | 18692.92              | 3.48                   |
| 89°8     | 21848.57              | 4.06                   |
| 89°9     | 24235.71              | 4.49                   |
| **90°0** | **128543.35**         | **23.80**              |
Step 4: Determine the area of probability containing the set of points A

Calculate the distance from \(O_{00}\) the point to all points \(F_t \in A\) \((t = 1, 2, 3, \ldots, n)\) according to formula (3), determine the maximum distance: \(M_{00} = \max \{O_{00}F_t\}\) (5)

\((O_{00}, M_{00})\) is the smallest center circle \(O_{00}\) containing the set A.

Calculate the distance from \(O_{01}\) the point to all points \(F_t \in A\) \((t = 1, 2, 3, \ldots, n)\) according to formula (3), determine the maximum distance: \(M_{01} = \max \{O_{01}F_t\}\) (6)

\((O_{01}, M_{01})\) is the smallest center circle \(O_{01}\) containing the set A.

Continue to calculate the final point is the distance from the point \(O_{xy}\) to all points \(F_t \in A\) \((t = 1, 2, 3, \ldots, n)\), determining the maximum distance: \(M_{xy} = \max \{O_{xy}F_t\}\) (7)

\((O_{xy}, M_{xy})\) is the smallest center circle \(O_{xy}\) containing the set A.

Determine the minimum value in the set of the largest radii defined above: \(\min \{M_{ij}\} = M_{pq}(R_p)\)

The area of probability that contains the ship’s main position is a center circle \(O_{pq}\) and radius \(M_{pq}\).

Step 5: Determine the distance from the ship’s location to different directions

Considering the area of the probability of the ship’s location being a circle, it is convenient to calculate, manipulate and use in actual navigation. However, the circle is scalar so the probability of the ship’s position is large, which is not favorable for the captain to lead the ship into dangerous narrow areas, the maritime safety system is less reliable and not [10-12]. There are shore targets to apply the method of determining the position of the ship by radar or aiming. Determining the orientation of the area of probability of ship position will help the captain have more bases when deciding to lead the ship with a single GPS location that is not reliable. Therefore, the author proposed the method of determining the orientation of the area of probability of ship position for a given sea area as follows:

From the probability circle \((O_{pq}, M_{pq})\), construct the set of axes of the direction of the probability area that passes through the center, \((O_{pq})\), separated from each other \((1^\circ)\): \(D = \{D_1, D_2, D_3, \ldots, D_{180}\}\). According to the result in step 4, the mind \((O_{pq})\) has the coordinates \((\phi_p, \lambda_q)\). The distance from any ship position \(F_t \in A\) to the axis \(D_i \in D\) \((D_i\) the direction \(i^\circ)\) is calculated as follows (figure 5).

![Figure 5. Calculate the orientation of probability area.](image)

The distance \(O_{pq}F_t\) is calculated by the formula (3):
\[ O_{pq} F_i = \sqrt{(\phi_t - \phi_p)^2 + (\lambda_t - \lambda_q)^2} \quad (8) \]

\[ \tan \alpha = \frac{|\lambda_t - \lambda_q|}{k|\phi_t - \phi_p|} \Rightarrow \alpha = \atan \left( \frac{|\lambda_t - \lambda_q|}{k|\phi_t - \phi_p|} \right) \quad (9) \]

\[ \tan \beta = \frac{h_{it}}{O_{pq} F_i} \Rightarrow h_{it} = \frac{\tan \beta}{O_{pq} F_i} \text{ với } \beta = i - \alpha \quad (10) \]

**Step 6: Determine the orientation of probability area**

Based on the results of step 5, in turn summing the squares of the distances of the ship positions \( F_i \in A \) to the directions \( D_i \in D \), we have:

The sum of squares of the distance from the position of the ship in episode A to \( D_1 \): \( S_1 = \sum_{i=1}^{n} (h_{1i})^2 \)

The sum of squares of the distance from the position of the ship in episode A to \( D_2 \): \( S_2 = \sum_{i=1}^{n} (h_{2i})^2 \)

Turn to the sum of squares of the distance from the position of the ship to the direction \( D_{180} \) is \( S_{180} = \sum_{i=1}^{n} (h_{180i})^2 \)

The direction with the largest ship position distribution density will have the least squared total distance (S). Assuming the minimum value is calculated \( S_k = \min \left\{ S_1, S_2, S_3, \ldots, S_{180} \right\} \), the most probable direction \( D_k (k^0) \) will be axial.

**4. Develop a program to determine the probability area containing ship’s locations received from GPS receivers**

Based on the mathematical mentioned in section 3, the author built a program to calculate the probability area containing ship location determined by the global positioning system GPS. Testing the program many times on land with data obtained on Vung Tau - Vietnam waters at different locations, at different times for relatively homogeneous results.

![Figure 6](image-url)

**Figure 6.** The program interface determines the area of the ship's probability location.

Figure 5 illustrates a test case with a database set of locations identified by the FURUNO GPS / WASS NAVIGATOR receiver on VOSCO STAR in Vung Tau - Vietnam. With 250 ship's locations identified continuously every 1 minute from 09:00 on November 15, 2019, the results obtained when running the program (figure 6):
Center of probability circle: position: \( O_{07}(10^024'737\, N, 107^007'231\, E) \)

Radius: \( M_{07}(R_{07}) = 126m \)

The most probable direction: \( D_{07} : 075^0 - 255^0 \)

Thus, according to the experimental results, the area of probability containing the ship's position determined by the GPS receiver in Vung Tau - Vietnam sea area is a probability circle with a radius of \( RL = 126m \). The area of probability that contains the actual ship position will be the center circle \( F \) (defined position) and radius: \( RF = 2\, RL + R95 \approx 2 \times 126 + 15 = 267m \). The direction with the highest probability of appearing on the ship is axis \( 0750 \, – \, 2550 \).

5. Conclusion
The author has solved the problem of assessing the accuracy of the ship’s position determined by the global GPS system in reality. Studying mathematical basis to calculate probability area containing ship's position, proposing a method for determining the area of probability containing ship's position in reality. Develop a program to calculate the probability circle containing the ship's position. Based on that research, the program test was conducted many times on the test data set collected from coastal areas of Vung Tau - Vietnam with relatively homogeneous results. The actual test results on the original ship in the coastal area of Vung Tau - Vietnam received a radius of error with the probability (95%) of approximately 126 meters and the most probable direction is the axis \( 0750 \, – \, 2550 \). Although the test data set is still limited, it can confirm the method of calculating the area of probability of ship's position determined by GPS receiver and its orientation has solved the research problem that the paper posed. The calculation program for the results demonstrates the applicability of the method proposed by the paper. Above is a general method and testing with limited data set in coastal areas of Vung Tau - Vietnam. The author will continue to make additional surveys on the sea area, the time and the number of ship locations determined to improve the reliability of the method. The proposed method is fully applicable and necessary for marine areas with limited maritime safety systems in the world.

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