Black hole demography: from scaling relations to models

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Abstract
In this contributed paper, I review our current knowledge of the local black hole (BH) scaling relations and their impact on the determination of the local BH mass function. I particularly emphasize the remaining systematic uncertainties impinging upon a secure determination of the BH mass function and how progress can be made in this direction. I then review and discuss the evidence for a different time evolution for separate BH–galaxy scaling relations, and how these independent empirical evidences can be reconciled with the overall evolution of the structural properties of the host galaxies. I conclude discussing BH demography in the context of semi-empirical continuity accretion models, as well as more complex evolutionary models, emphasizing the general constraints we can set on them.

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(Some figures may appear in colour only in the online journal)

1. Introduction
Supermassive black holes (BHs) reside at the centre of most, if not all, massive galaxies. The masses of BHs are tightly correlated with properties of their galactic hosts, especially the velocity dispersions and masses of their stellar bulges [1–4].

However, while it is now clear that relations between BHs and their hosts exist, it is still a matter of hot debate in the recent literature what their slopes, intrinsic scatters and evolution with redshift, truly are. This is partly due to the still limited working samples of independent BH mass measurements available, as detailed kinematical studies are limited to mostly nearby galaxies. Nevertheless, the number of secure BH measurements has increased over the years and advances have been achieved.

Given the undoubted progress made in the recent literature [5], paralleling the strong interest of the astronomical community in the broad topic of BH demography, it is valuable to summarize some of the key recent results and try to insert them in a coherent context. The aim
of this contributed paper is thus to provide the reader with a short guide through a variety of different empirical and theoretical recent results, revisited with the intent of connecting them within a plausible, broad physical framework.

2. Recent developments on the local black hole scaling relations

In the local Universe, BH masses $M_{\text{BH}}$ correlate with several global properties of the host galaxies. One of the most studied ones is between BH mass and host stellar (bulge or total) luminosity/mass [6]. The usual trend usually quoted in the literature is a linear correlation between BH mass and host bulge luminosity, with a slope close to unity and a normalization of about $1-2 \times 10^{-3}$ [3, 7]. Recent studies performed in the NIR/IR bands, less affected by dust extinction and more sensitive to total stellar mass, tend to confirm this trend with a slope of about $\sim 0.9$ [8, 9], or slightly shallower $\sim 0.8$ [105]. The scatter of this relation, initially claimed to be around 0.5 dex in bluer bands [10], has been somewhat reduced to $\sim 0.3-0.4$ dex in the latest calibrations [8, 9, 11], making it closer to the scatter in the $M_{\text{BH}}-\sigma$ relation.

Possibly one of the newest proposals in this respect has been put forward by the author of [12], who claimed for a net break in the BH–bulge mass relation, dependent on the host galaxy profile. Core Sérsic galaxies, mostly dominating above $M_{\text{BH}} \gtrsim 2 \times 10^9 M_\odot$, will continue following a linear relation between BH mass and bulge mass, while Sérsic galaxies at lower masses tend to follow a quadratic relation with host bulge mass. This claim was further quantified by the authors of [13].

On more general grounds, it has been several times emphasized in the very recent years that BH scaling relations tend to exhibit larger scatters especially at the low BH-mass end, where the hosts preferentially become later-type systems. Reference [14] reviewed this topic claiming that BHs correlate differently with different galaxy components. In particular, they stressed that any correlation with disc-grown pseudobulges or halo dark matter are very weak, implying no close co-evolution. In fact, as pointed out by [15], at face value, the possible large scatter in local BH–bulge mass relation induced when including all measurements with no restriction on galaxy type, is hard to reproduce by models in which the fuelling of BHs closely follows the growth of their host bulges, such as in late bar-instability modes (see section 5).

The correlation with velocity dispersion $\sigma$ continues to hold the record as the tightest correlation. The authors of [16] have recently re-analysed all the correlations between BH mass and host galaxy property, including the Sérsic index, circular velocity, and galaxy dynamical and effective masses. They confirm that there is no evidence for a tighter correlation than the one between $M_{\text{BH}}$ and $\sigma$, at least for the so-called classical bulges and that the correlation with large-scale quantities such as the circular velocity is weaker. They then tested the need for a third parameter in the BH scaling relations, confirming that the fundamental plane of BHs [17] is mainly driven by $\sigma$, with a small tilt due to the effective radius.

The authors of [16] also claimed a poor correlation between the BH mass and Sérsic index, at variance with previous findings. The authors of [18] revisited the issue of the relation between BH mass and the central light concentration of the surrounding bulge, quantified by the Sérsic index $n$. They claimed that a clear correlation exists, although with significant scatter. They then discuss how this relation is consistent with what would be derived by combining the $M_{\text{BH}}-L_{\text{sph}}$ and $L_{\text{sph}}-n$ relations, and conclude on how, for the same central light concentration, the correlation with BH mass could change with the galaxy profile.
3. Probing the local black hole mass function

Improved measurements of the local BH scaling relations are clearly fundamental to further advance in our true knowledge of the co-evolution between BHs and galaxies. Moreover, more secure BH scaling relations can potentially set interesting constraints on the number density of BHs as a function of mass and time, providing in turn useful terms of comparisons for models. For instance, some groups place a stronger emphasis on BH growth through gas accretion [19, 20], while others claim that growth by mergers plays an important role especially for the most massive BHs [21], thus possibly impacting the shape of the high mass end BH mass function [22]. Similarly, growth histories characterized by distinctive time and/or mass-dependent BH accretion rates can easily yield at $z = 0$, quite different BH number densities at low to intermediate mass scales [23].

The advent of large, well-studied, galaxy surveys, such as the Sloan Digital Sky Survey (SDSS), has allowed through time an improved understanding of the local demography of galaxies in terms of luminosity, stellar mass, size and velocity dispersion [24, 25]. Coupling this information with the above-mentioned BH–galaxy scaling relations, then allows us to calibrate the total mass distribution of BHs [26, 27]. The most basic procedure for calculating the BH mass function has been to assume that all galaxies host one BH. The local BH mass function is then derived by converting the galaxy distribution $\Phi(y)$, expressed as a function of a given measured variable $y$ (e.g., the stellar velocity dispersion or bulge luminosity/stellar mass), into a BH mass function by assuming a corresponding empirical $M_{\text{BH}}–y$ relation. Specifically, the BH mass function is computed via the equation [28]

$$\Phi(M_{\text{BH}}) = \int \Phi(y) \frac{1}{\sqrt{2\pi\eta^2}} \exp \left[-\frac{(M_{\text{BH}} - [a + by])^2}{2\eta^2}\right] dy$$

which also accounts for the scatter $\eta$ in the $M_{\text{BH}}–y$ relation.

Alternatively, one could directly use a complete galaxy catalogue, assign a BH to each galaxy via the chosen empirical scaling relation and then determine BH mass functions directly using the $V_{\text{max}}$ weight appropriate for each galaxy [29, 30]. In a statistical sense, the two methods should provide equivalent results.

3.1. The normalization issue

Obtaining good estimates of the BH mass function is not a trivial task, and in fact different approaches and assumptions may lead to different answers. For example, several groups point to significant discrepancies both in the shape and normalization of the BH mass function when adopting different scaling relations [30–32].

To highlight this point, in this section I show two estimates of the local BH mass function derived from up-to-date scaling relations and galaxy functions. More in detail, I use the early-type luminosity and velocity dispersion functions by [24], converted into BH mass functions using the BH mass–$r$ band luminosity, $M_{\text{BH}}–M_r$, and BH mass–velocity dispersion, $M_{\text{BH}}–\sigma$, relations of early-type galaxies by [32] and [11], respectively. The BH mass function $\pm 1 \sigma$ error bars are computed via Monte Carlo simulations [33]. In practice, I collect the results of 1000 realizations, in which the parameters of the $M_{\text{BH}}–y$ relation, its scatter $\eta$ and the Poisson errors associated with $\Phi(y)$ are allowed to vary within their measured uncertainties. For each bin of BH mass, I then compute the median and $1 \sigma$ errors associated with the distribution of $\log \Phi(M_{\text{BH}})$. The result, shown in figure 1, proves the importance of well calibrating the local scaling relations in order to estimate a more reliable BH mass function. The example in figure 1 is particularly meaningful in two respects. First, the statistics of galaxies in stellar mass and velocity space is homogeneous, being derived from the same exact SDSS subsample.
Second, the analysis is restricted to a subsample dominated by ellipticals; thus no bulge correction has been included in the calculation. We only convert from the SDSS $r$-band to the $R$ band in [32] adopting an average colour correction of $r - R = 0.2$ [34]. This implies that the differences in the final BH mass functions, highly significant in some bins ($\gtrsim 2\sigma$), are mainly driven by our choice of scaling relations. Similar, if not larger, discrepancies would have been obtained by using, for instance, stellar masses in place of galaxy luminosities. An analogous BH mass function results in fact by using, for example, the $M_{\text{BH}}-M_{\text{star}}$ by [8], and assuming an average correction of 0.25 dex, typical of massive galaxies [24], to convert from dynamical masses to a Chabrier initial mass function [35].

### 3.2. The low mass end

When dealing with lower mass BHs, a number of uncertainties clearly arise depending on the increasing difficulty in measuring both low-mass BHs and their hosts’ properties. For example, velocity dispersions become progressively less well defined in the later type, lower mass galaxies, with their bulge components becoming gradually less prominent and thus difficult to measure.

When moving to lower mass systems, one of the major concerns, as anticipated in section 2, is that possibly not all galaxies may follow the same tight scaling relations. In particular, a number of independent groups seem to find that especially for galaxies hosting the so-called pseudobulges [36], there is a near absence of correlation between BH mass and velocity dispersion/bulge stellar mass of the host, with BHs being often significantly less massive than what predicted by extrapolations of the scaling relations characterizing classical bulges [8, 16, 36–38]. The authors of [37] claimed that pseudobulges follow scaling relations of a factor of $\sim 3$ lower in normalization, while the authors of [36] do not find signs for any evident correlation for BHs residing in pseudobulges. The authors of [11] find that the late-type galaxies follow an $M_{\text{BH}}-\sigma$ relation with a similar slope and scatter as for the early-type galaxies, but with a normalization lower by a factor of $\sim 3$. Also, the break in the $M_{\text{BH}}-M_{\text{star}}$ relation advocated by [13] could at some level be reconciled with the findings by [11, 37]. In fact, imposing a
steepening of the relation at the low-mass end would tend to better line up with the data in the later-type galaxies.

Moreover, it has been suggested that the vast majority of low-mass galaxies may host pseudobulges [39]. Under this working assumption, the authors of [15] showed that assuming that all late-type galaxies host pseudobulges, the BH mass function could lead to a significantly reduced number density at low masses. Properly studying the impact of pseudobulges on the local BH mass function is far from trivial. One possibility could be to start from the Hubble-dependent stellar mass function by, e.g., [24], and then to correct it adopting the stellar mass-dependent fractions by [39]. However, the latter approach suffers from the significant uncertainties associated with the method of using stellar masses to infer BH masses [8, 23].

Following the strategy pursued by [15], to highlight the possible impact of pseudobulges, we present a modified BH mass function in figure 2. The red, solid lines bracket the 1σ uncertainty region for the local BH mass function derived on the assumption that all local galaxies follow the early-type $M_{BH}-\sigma$ relation by [11]. The blue, solid lines mark instead the 1σ uncertainty region inferred by assuming that BH masses in Sa galaxies are negligible. As discussed in [40, 41] and preliminarily quantified in [15], allowing for a significant fraction of BHs to be hosted in pseudobulges can indeed have a major impact on the local BH mass function, as shown in figure 2, decreasing the number density by a factor of $\sim 2$ around $M_{BH} \sim 10^8 M_\odot$, up to nearly an order of magnitude for $M_{BH} \lesssim 10^6 M_\odot$. We thus conclude that determining a secure estimate of the local BH mass function requires detailed knowledge of the role of pseudobulges, as well as bulge fractions, velocity dispersions, nuclear star clusters [42], breaks or Hubble-dependent variations in the BH scaling relations, etc. What has been calibrated in this work (and most of the previous ones) may safely be considered as an actual upper limit to the true function describing the demography of local BHs.

It is also valuable to point out that in very recent years an additional interesting issue has been raised, namely the possible existence of numerous ultra-massive BHs, which lie above the classical scaling relations, as discussed in, e.g., [43]. This may further complicate the exact determination of the BH mass function at the highest masses [44]. More, in general,
a degeneracy exists between the true intrinsic scatter and/or the exact slope in the scaling relations, as both can similarly shape the high-mass end of the BH mass function.

Finally, it is relevant to stress that variations at the very low or very high mass end of the local BH mass function have a relatively moderate impact on the global BH mass density peaking by BHs around $M_{\text{BH}} \sim 1–3 \times 10^8 M_\odot$ [23]. Thus, the global uncertainties quoted by, e.g., $\rho_{\text{BH}} \sim 3–5.5 \times 10^5 M_\odot \text{Mpc}^3$ remain quite a valid broad range of uncertainty, as further shown by, e.g., [22, 30].

4. The evolution of black hole–host scaling relations

Beyond the local Universe, a variety of statistical studies on the BH accretion history (discussed in section 5.1) support the view that the redshift evolution of median BH accretion and star formation rate track each other [23, 33, 45], consistent with the general idea that massive BHs and their host galaxies may indeed co-evolve at some level. However, the latter are hints derived from integrated quantities and are affected by systematic uncertainties (although recent studies carried out by, e.g., [22, 46] continue to support a close link even in the ratio between average BH accretion rate and host galaxy star formation rate).

One way to test BH–galaxy co-evolution, and at the same time to begin to explore the evolution of the BH mass function, is by direct and indirect measurements of the cosmic evolution of the scaling relations. A variety of studies have tried in the last decade or so to infer the degree of evolution in the scaling relations of BHs, in particular the BH–stellar mass and BH–velocity dispersion relations. A positive/negative evolution in these scaling relations could physically imply that, on average, BHs grow faster/slower than their host galaxies, thus suggesting some non-parallel evolution between the two systems.

Among the first, the authors of [47] measured the BH-to-host galaxy mass ratio in a sample of radio-loud active galactic nuclei (AGNs) in the redshift range $0 < z < 2$ supporting a strong evolution close to $M_{\text{BH}}/M_{\text{star}} \propto (1+z)^2$. More recently, their work was confirmed by different groups working with larger and different types of samples [48, 49].

As pointed by the authors of [50, 51], however, an increasing scatter at higher redshifts in the scaling relations could clearly mimic/increase the effect of evolution in the normalization in flux-limited samples. At intermediate redshifts $0.4 \lesssim z \lesssim 2.5$, the constraints from quasar clustering under the assumption of a monotonic mean relation between quasar luminosity and host halo mass support independent evidence of a relatively large lognormal scatter [52]. Based on a large and multiwavelength sample extracted from zCOSMOS, the authors of [53] claimed instead a much milder, but still significant, evolution of the type $M_{\text{BH}}/M_{\text{star}} \propto (1+z)^{0.68\pm0.12}$, along with a sign for a possible increase in the scatter.

However, any strong evolution in the scatter should break down or somewhat stabilize at redshifts higher than $z \sim 3$, at least for the most massive BHs. Luminous quasars at these redshifts have a very high large scale correlation length [54–56], consistent with that characterizing the spatial distribution of dark matter haloes as massive as $M_H \sim 10^{13} M_\odot h^{-1}$ [57], in the assumption that quasar hosts are an unbiased tracer of the underlying population of haloes of similar mass [58]. As first pointed out by the authors of [59], this extremely high clustering amplitude, combined with the corresponding space density, constrains the dispersion in the quasar luminosity–host halo mass relation to be less than 50% at 99% confidence, for the most conservative case of a 100% duty cycle. In other words, all the haloes of mass equal and above $M_H \sim 10^{13} M_\odot h^{-1}$ at $z \gtrsim 4$ should be active, i.e., hosting very luminous quasars, and the scatter in their luminosity–halo mass relation should be at least as small as the one in the local Universe measured for the $M_{\text{BH}}–\sigma$ relation.
Figure 3. Solid line with filled squares is the BH accreted mass density at each redshift obtained from the convolution of the age-dependent, early-type velocity dispersion function, convolved with a redshift-dependent $M_{\text{BH}}-\sigma$ relation, with a normalization scaling as $(1+z)^{\alpha}$. Long-dashed and dot-dashed lines are instead the BH mass densities inferred from the integration of the AGN luminosity functions and a fixed radiative efficiency. The match in time between the two independent mass densities sets the constraint $\alpha \lesssim 0.3$, i.e., nearly absent apparent evolution in the relation. Taken from [61].

The authors of [60] further elaborated on the proposal by those of [59], showing that reproducing the observed luminosity function and the high clustering of $z > 3$ quasars also requires a high ratio between the radiative efficiency $\epsilon$ and the luminosity in Eddington units $\lambda \equiv L/L_{\text{Edd}}$. In other words, a radiative efficiency of $\epsilon \gtrsim 0.2-0.3$ was favoured, for quasars radiating at a significant fraction of their Eddington limit. Their method was based on predicting the evolution of the BH mass function directly from the evolution of the halo mass function, once a given $M_{\text{BH}}-M_{H}$ relation was specified. The implied growth of BHs was then used to predict the luminosity function through a continuity equation and an input mean radiative efficiency and Eddington ratio.

While claims for a relatively strong evolution in the BH–host galaxy mass relation are numerous, several studies performed by a variety of groups with direct and indirect techniques consistently failed in detecting any parallel significant evolution in the $M_{\text{BH}}-\sigma$ relation. The authors of [61] adopted the local velocity dispersion function of spheroids, together with their inferred age distributions, to predict the velocity function at higher redshifts. Taking the normalization of the $M_{\text{BH}}-\sigma$ relation to evolve as $(1+z)^{\alpha}$, they computed the BH mass function associated with the velocity dispersion function at each redshift, and compared to the cumulative BH mass density inferred from the integrated quasar luminosity function (see section 5.1). This comparison, insensitive to the assumed duty cycle or Eddington ratio of quasar activity, favoured a relatively mild redshift evolution, with $\alpha \sim 0.33$, with a positive evolution as strong as $\alpha \gtrsim 1.3$ excluded at more than the 99% confidence level.

The results in [61] are shown in figure 3, where the filled squares indicate the BH-accreted mass density at each redshift obtained from the convolution of the age-dependent, early-type velocity dispersion function, convolved with a redshift-dependent $M_{\text{BH}}-\sigma$ relation, while the long-dashed and dot-dashed lines are instead the BH mass densities inferred from the integration of the AGN luminosity functions and a fixed radiative efficiency. We here note that
the basic assumptions made by [61] were that most of the stars in each nearby spheroid formed in a single episode and the velocity dispersion $\sigma$ remained nearly constant at later epochs. However, if the velocity dispersions of bulged galaxies increase at higher redshifts paralleling their apparently strong decrease in sizes [62, 63], the constraints on very low values of $\alpha$ would clearly become even tighter. More recently, the authors of [64] performed a similar exercise, comparing accreted mass functions with estimates of the local mass functions extrapolated to higher redshifts assuming some evolution in the scaling relations. Their results yield a positive evolution for the correlation with stellar mass consistent with [53], and null, or even negative results (anti-correlated with redshift) for velocity dispersion, in full agreement with those of [61].

Another piece of independent evidence in support of a null evolution in the $M_{\text{BH}}-\sigma$ relation comes from direct spectral fitting in the SDSS. The authors of [65] measured BH masses and velocity dispersions from broad and narrow emission lines in the quasar spectra of the SDSS Data Release 7, respectively, finding minimal change in the relation for BHs in the range $10^{7.5} < M_{\text{BH}}/M_\odot < 10^9$ up to $z = 1.2$.

The main conclusion of this section is that there seems to be growing evidence for a significant positive evolution in the normalization of the $M_{\text{BH}}-M_{\text{star}}$ relation, at least up to redshift $z \sim 2$, and some, possibly nonlinear and/or mass-dependent evolution in the scatter around it. However, there is no apparent sign for any significant evolution in the normalization, and possibly also scatter (otherwise we would have seen some evolution), of the $M_{\text{BH}}-\sigma$ relation. How can this be possible?

In order to properly consider the problem, one should insert the issue of redshift evolution in BH scaling relations within the broader context of structural evolution of massive galaxies. It is now well established that early-type galaxies show a strong half-light radius evolution when scaling up with redshift, becoming progressively more compact by a factor of a few at redshift $z \gtrsim 1$ with respect to their local counterparts of similar stellar mass [66, 67]. A reduction in size at fixed stellar mass should be paralleled by some increase in the velocity dispersion. Thus, at fixed BH mass, one would expect, if anything, a decrease in the normalization of the $M_{\text{BH}}-\sigma$ relation, more or less strong depending on how much mass is actually accreted onto the BH during the gas-rich, high-redshift phase of massive galaxy formation [19, 21]. However, if BHs are effectively mapped to lower stellar masses naturally characterized by lower velocity dispersions, this could conspire to erase or even reverse the predicted evolution in the observed $M_{\text{BH}}-\sigma$ relation, thus reconciling the separate observations on the disparate degree of evolutions on BH mass, and velocity dispersion/size of the host galaxy. This line of thought has been recently more quantitatively confirmed by [63], who self-consistently computed sizes and velocity dispersions within the [68] semi-analytic model, finding a positive evolution in the $M_{\text{BH}}-M_{\text{star}}$ relation, and milder, but still positive, in the $M_{\text{BH}}-\sigma$ relation (see figure 4).

5. How to use black hole demography to constrain models

5.1. Semi-empirical, continuity equation models

Semi-empirical models, although clearly more limited in scope than more advanced galaxy formation models, can still provide useful physical insights into galaxy–BH evolution, which can in turn be further interpreted within more advanced galaxy evolution models. To this purpose, the original proposal by the authors of [28, 69] of matching the local and accreted BH mass densities to limit the average radiative efficiency of BHs has been recently developed by the authors of [22] into a more comprehensive semi-empirical model.
The demography of the BH population through time is numerically computed by self-consistently solving the following continuity equation [70]:

$$\frac{\partial n_{BH}}{\partial t}(M_{BH}, t) = -\frac{\partial \langle \dot{M}_{BH} \rangle n_{BH}(M_{BH}, t)}{\partial M_{BH}}.$$  \hspace{1cm} (2)

Here $\langle \dot{M}_{BH} \rangle$ is the mean accretion rate (averaged over the active and inactive populations) of the BHs of mass $M_{BH}$ at time $t$ [71].

The authors of [22] generalized these continuity equation models to allow for any input mass and/or redshift-dependent radiative efficiency, and observationally motivated Eddington ratio distributions $P(\lambda | M_{BH}, z)$. Through this advanced semi-empirical approach, these authors found that reproducing the high observed fractions of active galaxies at low redshift requires a characteristic Eddington ratio that steadily declines at late times, in a possible mass-dependent manner (more massive BHs having lower $\lambda$ than less massive counterparts at similar epochs). In other words, at fixed mass, BHs become progressively less efficient in time at shining at high luminosities, either because the triggering mechanisms become rarer [72], and/or they change with time [73] and/or simply because the fuelling rate continuously drops [74, 75].

Figure 5 presents the cumulative BH mass density as a function of redshift for two continuity equation models taken from [22]. The models share the same input radiative efficiency and Gaussian Eddington ratio distributions, but differ in having the characteristic $\lambda$ (i.e., the median of the Gaussian), in one case constant at 1/3 (left panel) and steadily decreasing with cosmic time (from Eddington to strongly sub-Eddington) in the other case (right panel). The solid lines show the cumulative total BH mass density as a function of redshift, while the other lines indicate the mass density accreted in selected bins of current BH mass, as labelled. The main relevant feature arising from the comparison between the two panels is that in the former case (constant $\lambda$), BHs more massive than $M_{BH} \gtrsim 10^8 M_\odot$ stop accreting below $z \lesssim 1$, while in the latter (decreasing $\lambda$) they continue to significantly grow in mass, at the expense of the less massive ones which grow much less. The solid grey square indicates the systematic uncertainties in the total local BH mass density estimated by the authors of [23].

Figure 2 reports the predictions of the $z = 0$ BH mass function predicted by the two above-mentioned accretion models, compared with the empirical estimates discussed in section 2.
Figure 5. Cumulative BH mass density as predicted from the continuity equation models of [63].
Left: results for a model with input Gaussian Eddington ratio distribution and constant with time.
Right: the same Gaussian model as in the left panel, with peak $\lambda$ steadily decreasing with cosmic time. The solid lines represent the total cumulative BH mass density as a function of redshift, while the other lines mark the contributions from BHs in different mass bins, as labelled. The grey bar indicates the values and systematic uncertainties in the total local mass density in BHs estimated by [23]. When the characteristic $\lambda$ decreases, the low-mass BHs accrete less mass, while the more massive accrete more, especially at $z \lesssim 1$.

The no evolution model (long-dashed line) well matches the local BH mass function inferred on the assumptions that all galaxies follow similar scaling relations (solid, red lines). The model with decreasing characteristic $\lambda$ produces instead significantly less numerous BHs at the low-mass end. Decreasing $\lambda \propto L/M_{\text{BH}}$, in fact, maps a given luminosity to more massive and rarer BHs, thus progressively limiting the growth of less massive BHs boosting the accretion onto the most massive ones. As extensively discussed and reviewed by the authors of [22], a decreasing Eddington ratio is currently favoured by several direct observational signatures, such as the Eddington ratio distributions of local SDSS active galaxies [76] and the high fraction of massive active galaxies [77].

5.2. More complex evolutionary models

Besides accretion models, a variety of more or less complex models for the evolution of the BH population have been put forward in the literature along the years. Most of them are based on triggering mechanisms associated with mergers and/or flybys [21, 74, 78–82], as well as in situ processes, such as more or less strong disc instabilities [21, 83], or other processes [20, 84]. Models to probe BH cosmological evolution via gas accretion and mergers can be highly sophisticated, especially if BHs are modelled within the already complex, and still not fully understood, net of host galaxies and dark matter haloes [85, 86]. Due to the diverse physical assumptions and the non-trivial degeneracies induced by the large set of underlying parameters characterizing and shaping galaxy formation models, it is still very hard to constrain the successful physical models of galaxy and BH formation.

To mention some of the most recent results in the topic, the authors of [87] within the context of a full semi-analytic model for galaxy formation suggest a scenario in which disc instabilities are the main driver for moderately luminous Seyfert galaxies at low redshift, while major mergers are the main trigger for luminous active galaxies. Similar conclusions were reached by the authors of [73] in the context of a semi-empirical model, based on combining an observationally motivated AGN triggering rate and a theoretical AGN light curve. They found
major mergers to be insufficient to account for the entire AGN population, and claimed non-merger processes, such as secular mechanisms, to be the dominant AGN triggering mechanism at $z \lesssim 1$–$1.5$.

While the basic notion that major mergers may not explain the full AGN demography seems to be confirmed by many independent studies [80, 81, 88], minor mergers have been found to still represent a rather successful mechanism to reproduce the full AGN luminosity function and clustering properties [57, 78, 89], as well as their connection to star formation rates [82].

The authors of [15] have analysed the predictions of two state-of-the-art hierarchical galaxy formation models. In the first one, BHs grow only via major and minor mergers [68], while in the second one [90] BHs are allowed to grow also via disc instabilities. Their study highlighted the fact that the model in which BHs always closely follow the growth of their host bulges, also during late disc instabilities (i.e., bars), produces too narrow a distribution of BHs at fixed stellar mass to account for the numerous low-mass BHs now detected in later-type galaxies (see section 2). Models with a looser connection between BH growth and bar instability instead predict the existence of a larger number of under-massive BHs, in better agreement with the observations. Simulations and direct observations support the presence and growth of stellar bars in gas-poorer systems [91], thus possibly disfavouring a strong link between BH gas fuelling and stellar bar growth. On the other hand, clumpy accretion of gas clumps towards the centre of gas-rich and turbulent, high-redshift discs, could still represent a viable mechanism to grow classical bulges and their central BHs [83, 92].

It is thus clear that the secure knowledge of the slopes and intrinsic scatters of BH scaling relations can impose valuable constraints to alternative models of BH evolution. For example, the exact slope in the $M_{BH}$–$\sigma$ relation could allow us to discern between a radiative against a momentum-driven AGN feedback [93]. If instead repeated BH dry mergers are the primary cause behind the growth of the most massive BHs, then BH scaling relations should progressively tighten with increasing mass [94, 95] (but see also [96]). However, the present sparse sample of local BHs offers only tentative evidence for this [11].

Additional, independent constraints on BH models can be derived from AGN clustering. On empirical grounds, as reviewed by the authors of [26], sharp AGN clustering measurements offer a unique constraint to the duty cycles of active galaxies and scatters around the median BH scaling relations [97, 98]. As anticipated in section 4, reproducing the strong observed clustering of $z = 4$ quasars, for example, requires duty cycles close to unity and minimal scatter between luminosity and halo mass. On the other hand, reconciling the lower values of the correlation length of luminous quasars at $z \approx 1.5$ requires a significant scatter between luminosity and halo mass to lower the predicted clustering amplitude [52, 99].

Merger models can broadly reproduce the clustering of quasars at nearly all epochs and scales [80, 89]. This is because mergers are most efficient in haloes of masses around $M_H \sim 3 \times 10^{12} M_\odot$, which is the typical mass scale inferred from direct clustering measurements of quasars of nearly all luminosities and redshifts [72]. However, in recent years there has been mounting evidence for the clustering of x-ray AGN [100], to be significantly higher than the corresponding values of optically selected AGN at the same redshifts [101–103], consistent with dark matter halo masses up to one order of magnitude higher than those typical of optical quasars. While clearly larger samples are needed to better control eventual systematic observational biases, such as cosmic variance [104], it is still worth exploring some possible physical causes behind these apparent discrepancies. The authors of [86] claim that an additional channel for BH accretion is required, namely hot-halo mode, which is disassociated with the cold accretion during disc instabilities and galaxy mergers discussed above. In their model the hot-halo mode becomes prominent in dark matter haloes
with masses greater than $M_{\text{BH}} \gtrsim 10^{12.5} M_\odot$, giving rise to a distinct class of moderate luminosity AGN that inhabit rich clusters and superclusters.

6. Conclusions

In this contributed paper, I have reviewed several key topics on the demography of BHs. I started by reviewing the latest results on the local scaling relations between BHs and their host galaxies, concluding that there is still significant mismatch among the results of different groups, mainly because of the still limited sample available. This in turn poses challenges to determine a complete and secure calibration of the local BH mass function which still presents a systematic uncertainty in the normalization and shape. The latter is especially true at the low-mass end, where the role of BHs in later-type galaxies and/or pseudobulges is still not properly understood. I then continued reviewing the continuous cumulative evidence for an evolving $M_{\text{BH}}$–$M_{\text{star}}$ relation at high redshifts, and a non-evolving $M_{\text{BH}}$–$\sigma$ relation, which would be consistent with the coupled strong evolution of the size function of massive spheroids. Continuity accretion models can account for the full local BH mass function, and favour steadily decreasing Eddington ratios with cosmic time. More complex models based on mergers and/or strong high-redshift disc instabilities are consistent with the local demography, while models with bar instabilities tend to be disfavoured.

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