UNITARITY, GHOSTS AND NONMINIMAL TERMS IN LORENTZ VIOLATING QED

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The unitarity of a Lorentz-invariance violating QED model with higher-order Myers and Pospelov photons coupled to standard fermions is studied. As expected, we find ghost states associated to the higher-order terms that may lead to the loss of unitarity. An explicit calculation to check perturbative unitarity in the process of electron-positron scattering is performed and it is found to be possible to be preserved.

1. Introduction

In recent years, higher-order operators have become the object of intense study in the search for possible effects of Lorentz invariance violation. These Planck-mass suppressed higher-order operators allows to describe new physics beyond those obtainable from renormalizable operators, that is, operators with mass dimension four or less. For example, the higher-order effective theory may involve additional degrees of freedom associated to ultra-high energies which do not converge perturbatively to the normal ones when taking the limit of the dimensionless parameters in the effective terms to zero. Lee and Wick studied these exotic modes in the context of negative metric theories and in spite of the ghost states that appear, they showed that unitarity can be preserved by demanding all stable particles to be positive norm states.

Here we check perturbative unitarity in a QED consisting of higher-order Myers and Pospelov photons and standard fermions.
2. The Myers and Pospelov model

The Myers-Pospelov Lagrangian density for photons is given by

\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{\xi}{2M_P} n_\mu \epsilon^{\mu\lambda\sigma} A_\nu (n \cdot \partial)^2 F_{\lambda\sigma}, \] (1)

where \( n \) is a four-vector defining a preferred reference frame, \( M_P \) is the Planck mass and \( \xi \) is a dimensionless parameter.

We can always select a real basis of four-vectors \( e_\mu^{(a)} \) to be orthonormal and to satisfy the properties described in Ref. 8. In analogy with the left and right handed polarizations of usual electrodynamics we can switch to a basis of complex four-vectors \( \varepsilon_\mu^\lambda \) and define the orthogonal projectors \( P^\lambda_{\mu\nu} \) as

\[ \varepsilon_\mu^\lambda = \frac{1}{\sqrt{2}} (e_\mu^{(1)} + i \lambda e_\mu^{(2)}), \quad P^\lambda_{\mu\nu} = -\varepsilon_\mu^\lambda \varepsilon_\nu^{*\lambda}, \] (2)

where \( \lambda = \pm \). To derive the dispersion relation we can expand the gauge field in terms of this complex basis and replace in the equations of motion to arrive at

\[ (k^2)^2 - 4g^2 (n \cdot k)^2 \left( (n \cdot k)^2 - n^2 k^2 \right) = 0, \] (3)

in agreement with the work in Ref. 9.

3. Unitarity

Here we check perturbative unitarity in the process of electron-positron scattering \( e^+ e^- \rightarrow e^+ e^- \). For this we use the optical theorem which relates the imaginary part of the forward scattering amplitude \( \mathcal{M}_{ii} \) with the total cross section as

\[ 2 \text{Im} \mathcal{M}_{ii} = \sum_m \int d\Pi_m |\mathcal{M}_{im}|^2, \] (4)

where the sum runs over all intermediate physical states.

Considering the QED extension model the amplitudes that contribute to the \( S \)-matrix are the direct amplitude

\[ \mathcal{M}^{\text{dir}} = (-ie)^2 \int d^4k \delta^4(p_1 - p'_1 - k) \hat{U}_\mu G_{\mu\nu}(k), \] (5)

and the exchange amplitude

\[ \mathcal{M}^{\text{ex}} = (-ie)^2 \int d^4k \delta^4(p_1 + p_2 - k) \hat{V}_\mu V^\nu G_{\mu\nu}(k), \] (6)
where $\tilde{U} = N_{p_x} N_{p'_x} \bar{u}(p_2) \gamma^\mu v(p'_2)$, $U = N_{p'_x} N_{p_1} \bar{u}(p'_1) \gamma^\nu u(p_1)$ and $\tilde{V} = N_{p'_x} N_{p_x} \bar{u}(p'_2) \gamma^\nu v(p_2)$, $V = N_{p_x} N_{p_1} \bar{v}(p_2) \gamma^\nu u(p_1)$ and where $N_p = \sqrt{E_p}$ is the usual fermionic normalization constant.

Let us start with the left hand side of the unitarity condition (4). A similar calculation has been given in the minimal sector of the Standard-Model Extension, see Ref. 10. To simplify we will consider the lightlike case where we have a ghost state with frequencies $\omega^0_\lambda$ and two photons with frequencies $\omega^1_{1,2}$, see Ref. 8, and the propagator

$$G_{\mu\nu}(k) = -\sum_\lambda \frac{P^\lambda_{\mu\nu}(k)}{k^2 + 2g\lambda(n \cdot k)^3 + i\epsilon},$$

where and we have included the $i\epsilon$ prescription.

We are interested in the imaginary part of the forward scattering amplitude, therefore let us set $p'_1 \rightarrow p_1$ and $p'_2 \rightarrow p_2$. Moreover, we can see that the direct process does not contribute since the virtual photon can never be on shell for non-zero external momenta, hence $\text{Im}[\mathcal{M}^{\text{dir}}] = 0$. Let us find the contribution of the exchange process and substitute the propagator (7) in (6)

$$\mathcal{M}^{\text{ex}} = e^2 \int \frac{d^4k}{(2\pi)^4} \delta^4(p_1 + p_2 - k) V^{\mu\nu} V^{\ast\nu\mu} \sum_\lambda \frac{P^\lambda_{\mu\nu}(k)}{k^2 + 2g\lambda(n \cdot k)^3 + i\epsilon}. \tag{8}$$

Because only the poles can contribute to the imaginary part and due to energy conservation encoded in $\delta^4(p_1 + p_2 - k)$, we have that only the positive poles of the virtual photon have a chance to contribute. We can discard the ghost contribution since its energy $|\omega^0_\lambda| \sim 1/2g$ lies beyond the region of validity of the effective theory. That is, the external fermions will always satisfy the condition $p_{01} + p_{02} < |\omega^0_\lambda|$. Hence, we have

$$2\text{Im}[\mathcal{M}^{\text{ex}}] = -e^2 \int dk^0 \int \frac{d^3k}{(2\pi)^3} \delta^4(p_1 + p_2 - k) V^{\mu\nu} V^{\ast\nu\mu} \sum_\lambda \frac{P^\lambda_{\mu\nu}(k)}{2g\lambda(k_0 - \omega^\lambda_0)(k_0 - \omega^\lambda_2)},$$

$$= e^2 \int \frac{d^3k}{(2\pi)^3} \delta^4(p_1 + p_2 - k) V^{\mu\nu} V^{\ast\nu\mu} \sum_\lambda \frac{\varepsilon^\lambda_{\mu\nu} \varepsilon^{\ast\lambda}_{\nu\mu}}{2g\lambda(\omega^\lambda_1 - \omega^\lambda_0)(\omega^\lambda_1 - \omega^\lambda_2)},$$

$$= \int \frac{d^3k}{(2\pi)^3} \delta^2(p_1 + p_2 - k) \sum_\lambda |\mathcal{M}_\lambda|^2, \tag{9}$$

where we have used the notation $\mathcal{M}_\lambda = (-i\epsilon)N_{k,\lambda} V^{\mu\nu} \varepsilon^\lambda_{\mu\nu}$ for the physical process $\mathcal{M}_{\text{phys}}(e^+e^- \rightarrow \gamma)$ and we have introduced the normalization con-
stant \( N_{k,\lambda} = \frac{1}{\sqrt{2g_\lambda(\omega_1 - \omega_0)(\omega_1 - \omega_2)} } \). Finally we have

\[
2\text{Im}[\mathcal{M}] = \int \frac{d^3k}{(2\pi)^3} \delta^2(p_1 + p_2 - k) |\mathcal{M}_{\text{phys}}|^2 ,
\]

and therefore the unitarity condition is satisfied in this scattering process.

4. Conclusions

With an explicit calculation we have verified that the unitarity condition in the process of electron-positron scattering at tree level order is satisfied. A next step is to verify unitarity to order \( e^2 \) that will require to analyze more diagrams. Some of them contain loops where the ghosts can appear off-shell, thus, introducing an extra difficulty. Checking the unitarity condition to these order will give us a robust support in order to make physical predictions in the theory.

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