Z-lineshape versus 4th generation masses.

S. S. Bulanov *, V. A. Novikov †, L. B. Okun ‡,
ITEP, Moscow, Russia
A. N. Rozanov §,
CPPM, IN2P3, CNRS, Univ. Mediteranee, Marseilles, France
and ITEP, Moscow, Russia
M. I. Vysotsky ¶,
ITEP, Moscow, Russia.

Abstract

The dependence of the Z-resonance shape on the location of the threshold of the $N\bar{N}$ production ($N$ is the 4th generation neutrino) is analyzed. The bounds on the existence of 4th generation are derived from the comparison of the theoretical expression for the Z-lineshape with the experimental data. The 4th generation is excluded at 95% C. L. for $m_N < 46.7 \pm 0.2$ GeV.

1 Introduction

The straightforward generalization of the Standard Model through the inclusion of extra chiral generations of heavy leptons (N, E) and quarks (U, D) was studied in a number of papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. In [1, 2, 3] the analysis of deviations from the Standard Model due to 4th generation contribution was carried out in terms of $S$, $T$ and $U$ parameters for 4th generation particles being much heavier than $m_Z$. The case of new light physics

*bulanov@heron.itep.ru
†novikov@heron.itep.ru
‡okun@heron.itep.ru
§rozanov@cppm.in2p3.fr
¶vysotsky@heron.itep.ru
was investigated in \[4, 5\], using modified $S$, $T$ and $U$ parameters in order to take into account the effects of relatively light new particles. Particle and astroparticle implications of the 4th generation neutrinos were studied in \[7\]. A more thorough investigation of the properties of the 4th generation in the framework of GUT models was carried out in \[8\]. It was shown in \[9\] that unifying spins and charges in the framework of SO(1,13) group one gets four families of leptons and quarks. Possible manifestations of 4th generation particles at hadron colliders were studied in \[10\].

The bounds on the existence of the 4th generation from the analysis of the electroweak data fit were obtained in \[6, 11, 12, 13, 14\]. However, the dependence of Z-resonance shape on the contribution of 4th generation and, in particular, on the location of the threshold of $N\bar{N}$ production was not considered in \[11, 12, 13, 14\], because the results were obtained in the Breit-Wigner approximation. This approximation is valid for the thresholds of 4th generation particles production being far from $m_Z$. If the threshold location approaches $m_Z$ ($m_N \rightarrow m_Z/2$), than one gets a fast worsening of the fit (see Fig. 4 of \[13\]). It is due to the fact that the standard approach to the radiative corrections to the electroweak observables, used in \[6, 11, 12, 13, 14\], does not work in the presence of the heavy neutrino ($N$) with $m_N - m_Z/2 < \Gamma_Z$. After Taylor expansion over $p^2 - m_Z^2$, the expression for the polarization operator tends to infinity for $m_N \rightarrow m_Z/2$, because the point, at which the Taylor expansion is performed, becomes the branch point of the polarization operator.

According to the results of \[11, 12, 13, 14\] the best fit corresponds to $m_N \approx 50 \text{ GeV}$, that is why careful analysis of the region $m_N \approx m_Z/2$ is undertaken in this paper. We study the dependence of the Z-lineshape on the location of the threshold of $N\bar{N}$ production. We analyze the energy dependence of $e^+e^- \rightarrow Z \rightarrow \text{hadrons}$ cross section near Z-resonance and find that it exhibits a characteristic behavior near the threshold, a casp \(^1\). The casp is caused by the square root ($\sqrt{s - 4m_N^2}$), appearing in the contribution of 4th generation neutrino to the polarization operator of Z-boson. The form of the casp is determined by the location of the threshold with respect to $m_Z$.

\(^1\)Such behaviour of the cross section in quantum mechanics was discovered by Wigner, Baz and Breit and is discussed in textbook \[15\]. In particle physics analogous phenomenon was considered in \[19\]. Unlike cases analyzed previously, Z-boson physics is purely perturbative, allowing to get explicit formulae for cross section. However, being perturbative the variation of cross section because of the casp are small. Nevertheless, high precision of experimental data on Z production allow us to bound N mass from below.
Then we compare the theoretical expression for the Z-lineshape with the experimental data, presented in [20], using the ZFITTER [21] in order to take into account the electromagnetic corrections, and find that the 4th generation is excluded at 95% C. L. for $m_N < 46.7 \pm 0.2$ GeV. This bound depends on the masses of the charged 4th generation particles and on the mass of the higgs. Using the results of [14], we fix the mass of the charged lepton (E) and take into account that the splitting of quark masses and higgs mass are not independent. This leaves us with one free parameter – the splitting of quark masses, which we vary from 0 to 50 GeV. This variation is the source of the theoretical uncertainty in the bound on N mass as well as the uncertainties of the input parameters of ZFITTER. Note, that the $\chi^2/n_{d.o.f.}$ for the Z-lineshape with 4th generation is even better for certain region of mass values then the $\chi^2/n_{d.o.f.}$ for the SM prediction.

The paper is organized as follows. In section 2 we discuss the general behaviour of the cross-sections near threshold. In section 3 the exact formulae for the contributions of 4th generation particles to the cross section of $e^+e^- \rightarrow \text{hadrons}$ are presented. We study the behavior of the $e^+e^- \rightarrow \text{hadrons}$ cross section near the threshold of $N\bar{N}$ production in section 4. Using the result of section 3 we compare our prediction for the Z-lineshape in the presence of 4th generation with the experimental data in section 5. The conclusions are presented in section 6.

2 The cross-sections near threshold.

Let’s consider the interaction of two particles A and B, which form some resonance R, which in its turn decays into a system of particles f (see Fig. 1). The behavior of the cross section of this process near R-peak can be calculated in the general case, regardless of the exact form of interaction. We need only the partial width of particle R. The cross section near the resonance is described by the Breit-Wigner formula [21]:

$$\sigma = \frac{4\pi s^2}{I^2} \frac{2S_R + 1}{(2S_A + 1)(2S_B + 1)} \frac{\Gamma_{R\rightarrow A + B} \Gamma_{R\rightarrow f}}{(s - M^2)^2 + \Gamma^2 s^2 / M^2 M^2} s,$$

where $s = (p_A + p_B)^2$, $M$ and $\Gamma$ are mass and total width of R correspondingly, $I = 1/2 \sqrt{(s - (m_A + m_B)^2)(s - (m_A - m_B)^2)}$. $S_R$, $S_A$ and $S_B$ are spins of particles R, A and B correspondingly.
Figure 1: The reaction $A + B \rightarrow R \rightarrow f$.

Let’s consider the case when the reaction occurs not only near resonance, but also near the threshold of $N\bar{N}$ production. In order to study this effect we write explicitly the contribution of the $N\bar{N}$ loop to the propagator of particle $R$:

$$\frac{1}{s-M^2+i\Gamma s/M+\Sigma_R^{(N)}(s)},$$

(2)

the imaginary part of polarization operator $\Sigma_R^{(N)}(s)$ is connected with the $R \rightarrow N\bar{N}$ decay probability by the unitarity relation. The decay probability is proportional to $\sqrt{s-4m_N^2}$, the factor that arises from the integration over phase space. Then, if we rewrite the polarization operator as $\Sigma_R^{(N)}(s) = a+ib\sqrt{s-4m_N^2}$, and expand the expression for the propagator near $s = 4m_N^2$, it will take the following form

$$T_0 + iT_1\sqrt{s-4m_N^2},$$

(3)

where $T_0$ and $T_1$ are some functions of $a$, $b$, $s$, $m_Z^2$, $\Gamma_Z$. Then the cross-section is proportional to

$$\sigma \sim |T_0|^2 + 2\sqrt{s-4m_N^2}\Re[T_0T_1^*], \quad s > 4m_N^2$$

$$|T_0|^2 - 2\sqrt{4m_N^2 - s}\Re[T_0T_1^*], \quad s < 4m_N^2.$$  

(4)

The form of the cross-section energy behavior near threshold is defined by the value of the angle, $arg(T_0) - arg(T_1)$ (see Fig. 2). In all cases there are two branches lying on both sides of common vertical tangent. Thus, the existence of the reaction threshold leads to the appearance of the characteristic energy dependence of the cross section. The cross section near threshold is the linear function of $\sqrt{s-4m_N^2}$ with different slopes under and above $4m_N^2$. 


threshold. The existence of the square root branch point, \( s = 4m^2_N \), prevents the amplitude expansion near branch point in Taylor series.

![Figure 2: Different cases of cross section behavior near threshold. Vertical axis is \( \sigma \), while horizontal one is \( s \); dashed line crosses horizontal axis at \( 4m^2_N \).](image)

Below we will consider the case when \( R \equiv Z \).

### 3 Polarization operator.

The cross-section of \( e^+e^- \rightarrow Z \rightarrow \text{hadrons} \) near \( Z \)-resonance is well described by the Breit-Wigner formula \[21\]

\[
\sigma^\text{SM}_h = \frac{12\pi \Gamma_e \Gamma_h}{|p^2 - m^2_Z + i\Gamma^\text{SM}_Z p^2/m_Z|^2 m_Z^2}, \tag{5}
\]

where \( p = p_1 + p_2 \), \( p_1 \) and \( p_2 \) are momenta of initial electron and positron, \( m_Z \) is the mass of \( Z \) boson, \( \Gamma_e \) is the width of \( Z \rightarrow e^+e^- \) decay, \( \Gamma_h \) is the width of \( Z \rightarrow \text{hadrons} \), \( \Gamma^\text{SM}_Z \) is the total width of \( Z \) in the SM.

The 4th generation contributes to the \( Z \)-boson polarization operator. This contribution can be accounted for in the expression \[5\] by replacing the denominator:

\[
|p^2 - m^2_Z + i\Gamma^\text{SM}_Z p^2/m_Z|^2 \rightarrow |p^2 - m^2_Z + i\Gamma_Z p^2/m_Z + \Sigma^{(4th)}_Z(p^2) - \Re[\Sigma^{(4th)}_Z(m^2_Z)]|^2, \tag{6}
\]

where the subtraction is performed due to the fact that we follow the approach of \[11,12,13\] and use for \( m_Z, \alpha \) and \( G_F \) experimental values. Thus the renormalization scheme is on-shell one. The real part of the polarization operator at \( s = m^2_Z \) is subtracted in order to avoid the shifting of \( Z \)-boson mass. It is due to the fact that the real part of the polarization operator contributes to \( m_Z \).

\[
\Sigma^{(4th)}_Z(p^2) = \Sigma^N_Z(p^2) + \Sigma^E_Z(p^2) + \Sigma^{(U)}_Z(p^2) + \Sigma^{(D)}_Z(p^2) \tag{7}
\]
is the contribution of the 4th generation to Z polarization operator. The contribution of the \( N \bar{N} \) channel into Z width is taken into account by the imaginary part of \( \Sigma_Z^{(N)}(p^2) \).

Note, that \( \Gamma_Z \) in (6) includes decays of Z into particles of the first three generations. The 4th generation also influences \( \Gamma_Z \) in non-direct way. It is due to the fact that the polarization operators of gauge bosons enter the radiative corrections for \( g_A \) and \( g_V \), which in their turn enter the amplitude of the Z-boson decay into fermion-antifermion pair:

\[
M(Z \rightarrow f \bar{f}) = \frac{1}{2} \bar{f} Z \alpha \bar{\psi}_f (\gamma_\alpha g_V f + \gamma_\alpha \gamma_5 g_A f) \psi_f, 
\]

where \( \bar{f}^2 = 4\sqrt{2}G_\mu m_Z^2 = 0.54866(4) \), \( G_\mu \) is Fermi coupling constant. In the case of the Z decay into \( \nu \bar{\nu} \) the contribution of final state interaction equals zero and 

\[
\Gamma_\nu = 4\Gamma_0 (g_{V\nu}^2 + g_{A\nu}^2),
\]

where \( \Gamma_0 = G_\mu m_Z^3 / 24\sqrt{2}\pi \) is the so called ”standard” width. If we neglect the masses of neutrinos then \( g_{V\nu} = g_{A\nu} = g_\nu \). For the decay into any pair of charged leptons we get:

\[
\Gamma_l = 4\Gamma_0 \left[ g_{V\ell}^2 \left( 1 + \frac{3\bar{\alpha}}{4\pi} \right) + g_{A\ell}^2 \left( 1 + \frac{3\bar{\alpha}}{4\pi} - 6 \frac{m_l^2}{m_Z^2} \right) \right],
\]

where \( m_l \) is the mass of the lepton, \( \bar{\alpha} \equiv \alpha(m_Z^2) = [128.896(90)]^{-1} \). The situation slightly changes in the case of \( Z \rightarrow q \bar{q} \). There appear the radiative corrections \( (R_{Vq} \) and \( R_{Aq} \) due to gluon exchange and emission in the final state:

\[
\Gamma_q = 12\Gamma_0 \left( g_{Vq}^2 R_{Vq} + g_{Aq}^2 R_{Aq} \right).
\]

According to the results of [17] the one-loop expressions for \( g_{Al} \) and \( \Gamma_l = g_{Vl}/g_{Al} \) are

\[
g_{Al} = -\frac{1}{2} - \frac{3\bar{\alpha}}{64\pi s^2 c^2} V_A, \quad \Gamma_l = 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)} V_R,
\]

\[
g_{V\ell} = \frac{1}{2} + \frac{3\bar{\alpha}}{64\pi s^2 c^2} V_\nu, \quad \Gamma_\nu = 1 - 4|Q_\nu|s^2 + \frac{3\bar{\alpha}|Q_\nu|}{4\pi(c^2 - s^2)} V_{R\nu}, \quad \Gamma_q = 12\Gamma_0 \left( g_{Vq}^2 R_{Vq} + g_{Aq}^2 R_{Aq} \right),
\]

and in the case of quarks

\[
g_{Aq} = T_{3q} \left[ 1 + \frac{3\bar{\alpha}}{32\pi s^2 c^2} V_{Aq} \right],
\]

\[
\Gamma_q = 1 - 4|Q_q|s^2 + \frac{3\bar{\alpha}|Q_q|}{4\pi(c^2 - s^2)} V_{Rq},
\]
where $c \equiv \cos \theta_{\text{eff}}$ and $s \equiv \sin \theta_{\text{eff}}$.

The exact expressions for $V_A, V_R, V_{Aq}$ and $V_{Rq}$ in SM can be found in [17].

The 4th generation particles contribute to physical observables through polarization operators of gauge bosons, as it was mentioned above. This gives corrections $\delta V_i$ to the functions $V_i$ ($i = A, R$) [11].

$$\frac{3\bar{\alpha}}{16\pi s^2 c^2} \delta V_A = \Pi_Z^{(4th)}(m_Z^2) - \Pi_W^{(4th)}(0) - \Sigma_Z^{(4th)'}(m_Z^2),$$

$$\frac{3\bar{\alpha}}{16\pi s^2 c^2} \delta V_R = \left[ \Pi_Z^{(4th)}(m_Z^2) - \Pi_W^{(4th)}(0) - \Sigma^{(4th)'}(0) \right]$$

$$- \frac{sc(c^2 - s^2)}{s^2 c^2} \Pi^{(4th)}_{Z}(m_Z^2),$$

(15)

where $\Pi_Z(m_Z^2) = \Sigma_Z(m_Z^2)/m_Z^2$. Note that all singular terms, proportional to $1/\varepsilon$ ($\varepsilon = D - 4$), in right hand side of eq. (15), arising from polarization operators, cancel, as it was shown in [17]. Thus, the expressions (15) are finite. However, these formulae work well for the new particles being much heavier than Z-boson. If $m_N \rightarrow m_Z/2$ then $\Sigma_Z^{(4th)'}(m_Z^2)$ tends to infinity.

$\Sigma_Z^{(4th)'}(m_Z^2)$ comes from the Taylor expansion of Z-boson polarization operator near $m_Z$. In order to get rid of this unphysical infinity, which arise due to the fact that the expansion is performed at the branch point of the polarization operator, we use the exact expression for the contribution of 4th generation particles to the Z-boson polarization operator.

For the contribution of the 4th generation particles we get

$$\Pi_Z^{(\phi)}(p^2) = \frac{N_c \bar{\alpha}}{8\pi s^2 c^2} \left[ \Delta_\phi + 4g_{A\phi}^2 \frac{m_{\phi}^2}{p^2} F\left(\frac{m_{\phi}^2}{p^2}\right) \right.$$ 

$$- \frac{2(g_{A\phi}^2 + g_{V\phi}^2)}{3} \left(1 + 2 \frac{m_{\phi}^2}{p^2} F\left(\frac{m_{\phi}^2}{p^2}\right) + \frac{1}{3}\right),$$

(16)

where $\phi = N, E, U, D$; $N_c = 3$ for quarks and $N_c = 1$ for leptons; $\Sigma_Z^{(\phi)}(p^2) = p^2 \Pi_Z^{(\phi)}(p^2)$;

$$F(x) = \left[-2 + 2\sqrt{4x - 1} \arctan \frac{1}{\sqrt{4x - 1}}\right],$$

(17)

$\Delta_\phi$ are singular parts:

$$\Delta_\phi = 2 \left( \frac{1}{3} (g_{A\phi}^2 + g_{V\phi}^2) - 2g_{A\phi}^2 \frac{m_{\phi}^2}{p^2} \right) \frac{1}{\varepsilon - \gamma + \ln 4\pi - \ln \frac{m_{\phi}^2}{\mu^2}},$$

(18)
where \( \varepsilon \to 0, \gamma = -\Gamma'(1) = 0.577..., \) \( \mu \) is the parameter with dimension of mass, which is needed to preserve the dimensionality of the initial integral.

Let’s consider the contribution of \( N \) to \( g_A \) and \( g_V \). \( \Sigma'_Z(m_Z^2) \) arises in \( g_A \) due to the renormalization of Z-boson wave function. In order to avoid infinities we will expand near \( m_Z \) only singular part of the polarization operator, i.e.

\[
p^2 - m_Z^2 + \Sigma_Z(p^2) = (1 + \Sigma'_Z(m_Z^2)|_s)[p^2 - m_Z^2(1 - \Pi_Z(m^2)|_s) - \frac{p^2}{m_Z^2} \Pi_Z(p^2)|_{FP}],
\]

where index \( s \) denotes singular part and index \( FP \) — finite part. Then in equations (15) we should replace \( \Sigma'(m_Z^2)Z \) by \( \Sigma'(m_Z^2)|_s \) and \( \Pi(m_Z^2)Z \) by \( \Pi_Z(m_Z^2)|_s \). The combination of singular terms in the resulting expression is the same as in (15) and that’s why they cancel each other.

Let’s consider the expression for the \( e^+e^- \to \text{hadrons} \) cross section in the presence of the 4th generation. The singular part of the polarization operator is absorbed in partial widths \( \Gamma_e \) and \( \Gamma_h \) and also in total width of \( Z, \Gamma_Z \).

\[
\frac{\Gamma^0_e\Gamma^0_h}{\Gamma^0_e\Gamma^0_h} = \frac{(1 + \Sigma'_Z(m_Z^2)|_s)[p^2 - m_Z^2(1 - \Pi_Z(m^2)|_s) - \frac{p^2}{m_Z^2} \Pi_Z(p^2)|_{FP}]^2}{(1 + \Sigma'_Z(m_Z^2)|_s)[p^2 - m_Z^2(1 - \Pi_Z(m^2)|_s) - \frac{p^2}{m_Z^2} \Pi_Z(p^2)|_{FP}]^2} = \frac{(1 + \Sigma'_{(4th)}(m^2_Z)|_s)(p^2) - \Re[\Sigma'_{(4th)}(m^2_Z)]^2}{(1 + \Sigma'_{(4th)}(m^2_Z)|_s)(p^2) - \Re[\Sigma'_{(4th)}(m^2_Z)]^2}_F P^2, \tag{20}
\]

where \( \Gamma_e = \Gamma^0_e/(1 + \Sigma'_Z|m_Z|), \Gamma_h = \Gamma^0_h/(1 + \Sigma'_Z|m_Z|), \Gamma_Z = \Gamma^0_Z/(1 + \Sigma'_Z|m_Z|) \) and we used the decomposition

\[
\Sigma'_{(4th)}(p^2) = \Re[\Sigma'_{(4th)}(m^2_Z)] = (\Sigma'_{(4th)}(p^2) - \Re[\Sigma'_{(4th)}(m^2_Z)]|_s + (\Sigma'_{(4th)}(p^2) - \Re[\Sigma'_{(4th)}(m^2_Z)]|_{FP}
\]

The cancellation of the singularities is due to the fact that \( \Gamma_e, \Gamma_h \) and \( \Gamma_Z \) are proportional to \( f_0^2 \), which can be rewritten in terms of \( G_\mu, m_Z \) and polarization operators as in (17):

\[
f_0^2 = 4\sqrt{2}G_\mu m_Z^2[1 - \Pi_W(0) + \Pi_Z(m_Z^2) - D], \tag{21}
\]
where \( D \) comes from the radiative corrections to \( G_\mu \). If we divide \( \bar{f}^2 \) by \( (1 + \Sigma^{(s)}) \) then the resulting expression will be finite, due to the fact that all singular terms cancel. Thus, in the expression for the cross section of \( e^+e^- \rightarrow \text{hadrons} \) there are no singular terms.

We should note, that there is an ambiguity in the definition of singular and finite parts of the polarization operator. The constant term, proportional to \( \bar{\alpha} \), can be added to singular part and subtracted from finite part:

\[
\Pi_Z(p^2) = \left( \Pi_Z(p^2) \right|_s + a\bar{\alpha}) + \left( \Pi_Z(p^2) \right|_{FP} - a\bar{\alpha})
\]

However, this ambiguity does not affect the expression for the cross section. It is obvious from the following expressions:

\[
\Gamma_{eG}^0 = \left( \frac{\Gamma_{eG}^0}{(1 + \Sigma^{(s)})} + \frac{\bar{\alpha}(p^2 - m_Z^2)}{m_Z} + \hat{\Sigma} \right)
\]

\[
= \left( \frac{\Gamma_{eG}^0/(1 - a\bar{\alpha})^2}{(p^2 - m_Z^2)} + \frac{\bar{\alpha}(p^2 - m_Z^2)}{m_Z} + \hat{\Sigma} \right)
\]

\[
\Gamma_{i} = \frac{\Gamma_{eG}^0/(1 - a\bar{\alpha})^2}{(1 + \Sigma^{(s)})}
\]

where \( \hat{\Sigma} = (\Sigma^{(4th)}(p^2) - \Re(\Sigma^{(4th)}(m_Z^2)))_{FP} \),

\[
\Gamma_{i} = \frac{\Gamma_{eG}^0}{(1 + \Sigma^{(s)})},
\]

where \( i = e, h, Z \).

4 **Z-lineshape in the presence of 4th generation.**

According to the results of the previous section, the amplitude of \( e^+e^- \rightarrow \text{hadrons} \) is proportional to

\[
A_h \sim \left[ p^2 - m_Z^2 + i\Gamma_{Z}^0 p^2 / m_Z + \Sigma_Z^{(4th)}(p^2) - \Re(\Sigma_Z^{(4th)}(m_Z^2)) \right]^{-1}.
\]  

(22)
In order to study the behavior of the cross section near the threshold qualitatively, we shall neglect the contributions of U, D and E to $\Sigma^{(4th)}(p^2)$ as well as the contribution of all 4th generation particles to $g_A$ and $g_V$. Then the expression (22) takes the following form:

$$A_h \sim \left[ p^2 - m_Z^2 + i \Gamma_Z p^2 / m_Z + (\Sigma^{(N)}(p^2) - \Re[\Sigma^{(N)}(m_Z^2)])_F P \right]^{-1}. \quad (23)$$

Expanding this expression near the threshold of $N\bar{N}$ production ($p^2 = 4m_N^2$), $A_h \sim T_0 + iT_1 \sqrt{p^2 - 4m_N^2}$ we obtain for the cross section the following behaviour

$$\sigma_{p^2 < 4m_N^2} \sim \frac{1}{\gamma} \left( 1 + \frac{f^2 m_Z^2 (4m_N^2 - m_Z^2)}{64\pi} \sqrt{\frac{4m_N^2}{p^2} - 1} \right), \quad (24)$$

$$\sigma_{p^2 > 4m_N^2} \sim \frac{1}{\gamma} \left( 1 - \frac{f^2 m_Z^2 m_Z \Gamma_Z}{64\pi} \sqrt{1 - \frac{4m_N^2}{p^2}} \right), \quad (25)$$

where $\gamma = (4m_N^2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2$ and the second terms in the brackets in eqns. (24, 25) are proportional to $\Re[T_0 T_1^*]$ and $\Im[T_0 T_1^*]$ respectively. As it was mentioned in section 2 the form of the $p^2$ dependence of the cross section near the threshold is determined by the relative phase of $T_0$ and $T_1$. In our case we have two types of casps (see Figs. 3), which correspond to $\arg T_0 - \arg T_1$ lying in the third quadrant for $4m_N^2 < m_Z^2$ and in fourth quadrant for $4m_N^2 > m_Z^2$, or to $\Re[T_0 T_1^*]$ and $\Im[T_0 T_1^*]$ being negative for $4m_N^2 < m_Z^2$ and $\Re[T_0 T_1^*]$ being positive, while $\Im[T_0 T_1^*]$ being negative for $4m_N^2 > m_Z^2$. It can also be seen from Figs. 3 that the cross section of $e^+e^- \rightarrow hadrons$ decreases above the threshold in accordance with the unitarity.

Though the change of Z-lineshape due to these casps is very small compared to the pure Breit-Wigner curve, as it is shown in Fig. 3, this effect may manifest itself when comparing the theoretical predictions with the experimental data. It is due to the fact, that Z-lineshape is measured with very high precision. In the next section we will compare the theoretical cross section with the experimental data.
Figure 3: The dependence of relative departure of the $e^+e^-$ → hadrons cross section in the presence of 4th generation from the SM prediction on the c.m. energy of $e^+e^-$ for $m_N = 45$ GeV (a) and $m_N = 47$ GeV (b).

5 Comparison with the experiment.

The experimental data on the cross section of $e^+e^- \rightarrow \text{hadrons}$ reaction is usually presented in the form, that includes the electromagnetic corrections, i.e. initial and final state interactions and photon emission [20]. In order to compare our formulae for the cross section with experimental ones, we use the ZFITTER code [21], which takes into account these corrections. We use the following inputs:

\[ m_Z = 91.1882(22) \text{ GeV}, \quad m_t = 175(4.4) \text{ GeV}, \quad \alpha = 1/128.918(45), \]
\[ \alpha_s = 0.1182(27), \quad m_H = 120 \text{ GeV}. \]

With the reasonable assumption that the initial and final state radiation effects are not significantly modified by fourth generation we can calculate the cross section

\[ \sigma_{th} = \sigma_h \frac{\sigma_{ZF}^h}{\sigma_{SM}^h}, \quad \text{(26)} \]

where $\sigma_{ZF}^h$ is the result of ZFITTER code and

\[ \sigma_h = \frac{12\pi \Gamma_e \Gamma_h}{|p^2 - m_Z^2 + i\Gamma_Z p^2/m_Z + (\Sigma^{(4th)}(p^2) - R[\Sigma^{(4th)}(m_Z^2)])_F|_p|^2 m_Z^2}, \quad \text{(27)} \]

at values of c.m. energy, at which the experimental values of cross section were measured [20]. There are 35 experimental points from 1995 data, which
we use. These points are extracted from Fig. 2 of [20] and presented in Table 1. We took only the points corresponding to 1993-1995 set, due to the fact that they are measured with higher precision than the 1991-1993 set. Then we calculate the \( \chi^2 / n_{d.o.f.} \), where \( n_{d.o.f.} = 35 - N \), \( N \) is the number of fitted parameters\(^2\), and

\[
\chi^2 = \sum_{i=1}^{35} \left( \frac{\sigma_h^{th} - \sigma_h^{exp}}{\delta \sigma_h^{exp}} \right)^2,
\tag{28}
\]

\( \sigma_h^{exp} \) is the experimental value of cross section and \( \delta \sigma_h^{exp} \) is its error, in order to determine at what confidence level the 4th generation is excluded by the experimental data. However, the bound on N mass from below depends on the higgs mass and mass splittings between U and D quarks and between E and N leptons. The effects of varying \( m_H \), \( |m_U - m_D| \), and \( |m_E - m_N| \) are not independent. As it was shown in [14], the increase of \( |m_U - m_D| \) or \( |m_E - m_N| \) can be compensated by the increase of higgs mass. This leads to the appearance of \( \chi^2_{min} \) valleys. It can be seen from Figs. 4a) and 4b), where the dependence of \( \chi^2 \) on \( m_H, |m_U - m_D| \) (a) and \( m_H, |m_E - m_N| \) (b) is shown for \( m_N = 49 \text{ GeV} \).

Figure 4: The dependence of \( \chi^2 \) on \( m_H, |m_U - m_D| \) (a) and on \( m_H, |m_E - m_N| \).

If we use then the LEP II bound \( m_E > 100 \text{ GeV} \) and the results of [14], that the best fit of electroweak data corresponds to the light E near the bound and \( m_N \approx 50 \text{ GeV} \), then we have only two parameters, \( m_H \) and \( |m_U - m_D| \) that affect the bound on \( m_N \). As it can be seen from Fig. 5a) the best fit is acquired for \( 0.11m_H - 19.7 < |m_U - m_D| < 0.12m_H - 9.2 \). In this region of

\(^2\)In our case \( N = 1 \), because only heavy neutrino mass is a free parameter, all other parameters are fixed.
masses we calculate $\chi^2$ and find that the 4th generation is excluded at 95% C. L. for $m_N < 46.7 \pm 0.2$ GeV. The theoretical uncertainty is caused by the varying of $|m_U - m_D|$ from 0 to 50 GeV, as well as by the uncertainties of the input parameters of ZFITTER, which were also used when calculating $\sigma_h$ and $\sigma_{SM}^h$. The main contribution to the theoretical uncertainty comes from $m_t$ and $\alpha_S$. The variation from 0 to 50 GeV is chosen, because the quality of the fit is fast worsening for $|m_U - m_D| > 50$ GeV.

Figure 5: a) Exclusion plot on the plane $m_H, |m_U - m_D|$ for $m_N = 49$ GeV; $\chi^2_{\text{min}} = 0.85$ denoted by cross b) Exclusion plot on the plane $m_H, m_N$ for $|m_U - m_D| = 10$ GeV; $\chi^2_{\text{min}} = 0.85$ denoted by cross. Solid lines represents the borders of 1σ, 2σ, 3σ, 4σ and 5σ regions.

In order to illustrate the dependence of the fit quality on the higgs mass we study $\chi^2(m_N, m_H)$ (see Fig. 5b)). From this figure it is seen that the 95% C. L. bound lies below 50 GeV and slightly varies with the increase of the higgs mass near $m_N = 47$ GeV. In this figure we take $|m_U - m_D| = 10$ GeV. It can be seen from Figs. 5 a) and b) that for certain region of 4th generation particles and higgs masses the quality of the fit can be even better than in SM. According to the results of [20] the $\chi^2/n_{d.o.f.}(SM) = 1.09$, which corresponds to 2σ level, while in the presence of 4th generation it is $\chi^2_{\text{min}}/n_{d.o.f.} = 0.88$, which is inside 1σ region.

We should note, that the direct search of heavy neutrinos in $e^+e^-$ annihilation into a pair of heavy neutrinos with the emission of the initial state bremsstrahlung photon ($e^+e^- \rightarrow \gamma + \text{Nothing}$) could result in the bound
$m_N \geq 50 \text{ GeV}$ \cite{12,13} if all four LEP experiments will make a combined analysis \cite{15}.

Though this bound on $m_N$ would be slightly better than the one, obtained in the present paper, the data and the procedure used to extract the bounds are completely different and independent.

### 6 Conclusions.

In the present paper we analyzed the dependence of Z-lineshape on the location of the threshold of $N\bar{N}$ production. We studied the behavior of $e^+e^- \rightarrow \text{hadrons}$ cross section near the threshold of $N\bar{N}$ production and determined how this threshold changes the Z-lineshape. In order to find the bound on $m_N$, we compared the theoretical predictions for the Z-lineshape with the experimental data, using the exact formulae for Z polarization operator, instead of expanding it into a Taylor series near $m_Z$.

We found that the bound on $N$ mass depends on the higgs mass and the splittings of 4th generation quark and lepton masses ($|m_U - m_D|$ and $|m_E - m_N|$). However, the effects caused by them are not independent, because the increase of mass splittings can be compensated by the increase of the higgs mass, as it was shown in \cite{14}. Using the results of \cite{14} we fixed $m_E = 100$ GeV. Then we used the fact that $|m_U - m_D|$ and $m_H$ are not independent. Thus, we had one free parameter left: $|m_U - m_D|$. We varied $|m_U - m_D|$ from 0 to 50 GeV and found that the 4th generation is excluded by the experimental data at 95% C. L. for $m_N < 46.7 \pm 0.2$ GeV.

The theoretical uncertainty is caused by the varying of $|m_U - m_D|$, as well as by the uncertainties of the input parameters of ZFITTER, which were also used when calculating $\sigma_h$ and $\sigma^{SM}_h$.

### Acknowledgements

The authors are grateful to M. Yu. Khlopov and N. Mankoc Borstnik for correspondence. This work was partially supported by RFBR (grant N 00-15-96562).
References

[1] J. Erler, P. Langacker, Review of Particle Physics. The European Physical Journal C15 (2000) 95, chapter 10.6.

[2] A. Masiero et al., Phys. Lett. B355 (1995) 329.

[3] T. Inami et al., Mod. Phys. Lett. A10 (1995) 1471.

[4] N. Evans, Phys. Lett. B340 (1994) 81.

[5] P. Bamert and C. P. Burgess, Z. Phys. C66 (1995) 495.

[6] V. A. Novikov et al., Mod. Phys. Lett. A10 (1995) 1915.

[7] D. Fargion et al., Phys. Rev. D52 (1995) 1828;
   D. Fargion et al., Phys. Rev. D54 (1996) 4684;
   K. Belotsky et al., hep-ph/0210153
   D. Fargion et al., JETP Lett. 68 (1998) 685;
   D. Fargion et al., JETP Lett. 69 (1999) 434;
   K. Belotsky et al., Phys. Atom. Nucl. 65 (2002) 382;
   K. Belotsky et al., Phys. Lett. B529 (2002) 10.

[8] C. D. Froggatt and J. E. Dubicki, Proceedings to the workshops ”What comes beyond the Standard model 2000, 2001” vol. 1.

[9] A. Borstnic Bracic and N. Mankoc Borstnik, Proceedings to the workshops ”What comes beyond the Standard model”, 2000, 2001, 2002, vol 2;
   D. Lukman, A Kleppe, N. Mankoc Borstnik, Proceedings to the workshops ”What comes beyond the Standard model”, 2000, 2001, 2002, vol 2.

[10] E. Arik et al., Phys. Rev. D66 (2002) 116006;
    E. Arik et al., Phys. Rev. D66 (2002) 033003.

[11] M. Maltoni, V. A. Novikov, L. B. Okun, A. N. Rozanov, and M. I. Vysotsky, Phys. Lett. B476 (2000) 107.

[12] V.A. Ilyin et al., Phys. Lett., B503 (2001) 126; hep-ph/0006324
    V.A. Ilyin et al., Proceedings ICHEP2000 Osaka conference; hep-ph/0009167
[13] V. A. Novikov et al., Phys. Lett. B529 (2002) 111; hep-ph/0111028.
[14] V. A. Novikov et al., JETP Lett. 76 (2002) 119; hep-ph/0203132.
[15] The ALEPH Collaboration, ALEPH 2001-010; CONF 2001-007 (2001); P. Abreu et al., DELPHI Collaboration, Eur. Phys. J. C16 (2000) 53; M. Acciari et al., L3 Collaboration, Phys. Lett. B470 (1999) 268; G. Abbiendi et al., OPAL Collaboration, Eur. Phys. J. C14 (2000) 73.
[16] H.-J. He, N. Polonsky and S. Su, Phys. Rev. D64 (2001) 053004; hep-ph/0102144.
[17] V. A. Novikov et al., Rep. Prog. Phys. 62 (1999) 1275.
[18] L. D. Landau, E. M. Lifshitz, Quantum Mechanics (Moscow, Nauka, 1974) par. 147.
[19] A. I. Baz, L. B. Okun, JETP 8 (1959) 526.
[20] LEP collaborations, Combination procedure for the precise determination of Z boson parameters from results of the LEP experiments, CERN-EP/2000-153; hep-ex/0101027.
[21] D. Bardin et al., Comput. Phys. Commun., 133 (2001) 229.
[22] ALEPH Coll., Phys. Lett., B429 (1998) 201.
The experimental values of the $e^+e^- \to \text{hadrons}$ cross section, obtained by ALEPH, DELPHI, L3 and OPAL collaborations, extracted from Fig. 2 of [20]. $\sqrt{s}$ is presented in GeV, $\sigma_h$ in nanobarns. The 1993-1995 data set.

Table 1.1 ALEPH

| $\sqrt{s}$ | 89.4316 | 89.4400 | 91.1860 | 91.1980 | 91.2200 | 91.2840 |
|------------|---------|---------|---------|---------|---------|---------|
| $\sigma_h$ | 9.891   | 9.980   | 30.500  | 30.43   | 30.458  | 30.555  |
| $\delta\sigma_h$ | 0.043 | 0.044 | 0.078 | 0.032 | 0.067 | 0.13 |
| $\sqrt{s}$ | 91.2950 | 91.3030 | 92.9685 | 93.0140 |
| $\sigma_h$ | 30.678  | 30.660  | 14.300  | 14.04   |
| $\delta\sigma_h$ | 0.078 | 0.090 | 0.060 | 0.056 |

Table 1.2 DELPHI

| $\sqrt{s}$ | 89.4307 | 89.4378 | 91.186 | 91.203 | 91.28 |
|------------|---------|---------|--------|--------|-------|
| $\sigma_h$ | 9.87    | 9.93    | 30.392 | 30.50  | 30.46 |
| $\delta\sigma_h$ | 0.044 | 0.056 | 0.065 | 0.044 | 0.19 |
| $\sqrt{s}$ | 91.292  | 91.304  | 92.966 | 93.014 |
| $\sigma_h$ | 30.67   | 30.46   | 14.35  | 13.89  |
| $\delta\sigma_h$ | 0.098 | 0.086 | 0.044 | 0.045 |

Table 1.3 L3

| $\sqrt{s}$ | 89.4497 | 89.4515 | 91.206 | 91.222 | 91.297 | 91.309 |
|------------|---------|---------|--------|--------|--------|--------|
| $\sigma_h$ | 10.088  | 10.08   | 30.358 | 30.547 | 30.525 | 30.545 |
| $\delta\sigma_h$ | 0.034 | 0.034 | 0.067 | 0.034 | 0.087 | 0.067 |
| $\sqrt{s}$ | 92.983  | 93.035  |
| $\sigma_h$ | 14.231  | 13.91   |
| $\delta\sigma_h$ | 0.046 | 0.053 |

Table 1.4 OPAL

| $\sqrt{s}$ | 89.4415 | 89.45 | 91.207 | 91.222 | 91.285 | 92.973 | 93.035 |
|------------|---------|-------|--------|--------|--------|--------|--------|
| $\sigma_h$ | 9.980   | 10.044 | 30.445 | 30.46  | 30.64  | 14.27  | 13.85  |
| $\delta\sigma_h$ | 0.044 | 0.034 | 0.053 | 0.025 | 0.098 | 0.046 | 0.046 |