On impact by a hard cone on elasto-viscoplastic material, leading to the generation of a conical crack

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Abstract. The destruction of solid physical objects is a complex process in which mechanical, chemical, thermobaric and other matter transformations take place. Under mechanical destruction is understood the violation of the integrity of the object due to the occurrence of cracks. High-speed impact of a solid body on deformable materials is accompanied by the spread of cracks and is of a wave nature. This article presents an analysis of the dynamic stress-strain state in an elastoviscoplastic (EVP) material near the leading edge of a moving crack, approximated by a zone of continuous deformation. An analysis of the distribution of the intensity of tangential stresses and plastic deformations that occur behind the front of the longitudinal and shear head waves of a spherical shape generated by the impact of the vertex of the solid cone is carried out on the model EVP of the medium by the ray method. It is shown that the presence of a maximum of the jump of the tangential velocity component on the shear wave leads to a development with time of a jump in the displacements of the tangents to the front of the shear wave. This can be interpreted as the moment of initiation of the head part of a crack running along with the front of the elastic wave with the velocity of shear waves.

1. Introduction
A significant number of publications are devoted to the problems of formation and development of cracks. In work [1], interaction of edge cracks and factors affecting the further formation of cracks are considered. A number of illustrative examples is presented.

There are numerous experimental results [2], which indicate that a thermal shock on the ceramic surface causes cracks. It is shown that the failure pattern depends strongly on the microstructure of the material and the type of load.

Questions of the ultimate state of elastic materials with cracks are considered in detail in [3, 4, 5] from the standpoint of the static stress-strain state of an elastic medium and the use of criteria for the ultimate state of fracture or shear cracks.

The problems of the dynamics of the initiation of cracks and their propagation can not in principle be formulated in the framework of this approach. Modeling the dynamics of crack propagation requires the use in the mathematical model of the laws of elastic, viscous and plastic behavior of the material.

One of the approaches to studying the propagation of the front edges of cracks is the approximation of cracks by thin layers of continuous plastic deformation [6, 7]. In this paper, we
analyze the dynamic stress-strain state in the EVP material near the front edge of the moving crack, approximated by the zone of continuous deformation.

The initial moment of a dynamic impact by a cone on a deformable material is of a wave nature [8]. For a short time $\Delta t$, the region of the material perturbed by impact will be concentrated in a neighborhood of the radius $c\Delta t$ ($c$ is the propagation velocity of the wave) of the vertex of the cone. With a small velocity $v_0$ of the cone, the produced perturbation is reversible. However, at a sufficiently high impact speed $v_0$, irreversible plastic deformation of the material takes place.

2. Materials and methods

Let’s consider the deformed state of the material in the neighborhood of the cone vertex, bounded by the weak waves $\Sigma_1$ and $\Sigma_2$ with a singularity on their front.

For convenience, we approximate the velocity of the diffracted wave $\Sigma$ at its front by its linear approximation

$$\tilde{v}_n|\Sigma \approx v_n(\delta) + \delta \frac{\partial v_n}{\partial n}(\delta),$$

(1)

using the expansion of the function $v_n(n)$ in a Taylor series in the neighborhood of the point $n = \delta$

$$v_n(n) = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\partial v_n(\delta)}{\partial n} (n-\delta)^k.$$  

(2)

This approximation of (1) allows us to consider the diffracted wave $\Sigma$ as a strong wave [8, 9] with the intensity $v_n(\delta)$ generated by the impact of the cone. The vertex of the cone is represented by a sphere of radius $\delta$, which excludes the singularity at $n = 0$ (figure 1).

![Figure 1. The image of the surface of a cone $S$, the surface of a sphere $S_\delta$ ($\delta$ is its radius) and, generated by impact, diffracted (weak) wave $\Sigma_\delta$ and shock (strong) wave $\Sigma$](image-url)

The velocity $v_n$ of the material behind a weak sigular diffracted wave $\Sigma_1$ satisfies the conditions: $v_n|\Sigma_1 = v_n(0) = 0; \frac{\partial v_n}{\partial n}|_{n \to 0} \to \infty; \frac{\partial^2 v_n}{\partial n^2} \neq 0.$

Further on let’s confine by the piecewise-constant approximation of stresses and velocities in the zones behind the fronts of the diffracted waves $\Sigma_1$ and $\Sigma_2$. Let’s consider the EVP material as a model of the dynamic deformation of the medium. On its shock fronts, the following conditions are satisfied for jumps of stresses, strains and velocities [9, 10]:

$$[\sigma_{ij}] = \lambda [\varepsilon_{kk}] \delta_{ij} + 2\mu [\varepsilon_{ij}]; \quad (i, j = 1, 2, 3);$$
\[\varepsilon_{ij} = \frac{1}{2} ([u_{i,j}] + [u_{j,i}]); \quad [u_{i,j}] = - [v_{i}] \frac{n_{j}}{c}. \quad (3)\]

Here: \(\sigma_{ij}\) are the components of the stress tensor; \(\varepsilon_{ij}\) are the components of the strain tensor; \(\delta_{ij}\) is the Kronecker symbol; \(u_{i}\) are the displacements; \(v_{i}\) are the speeds; \(\lambda, \mu\) are the Lame elastic parameters; \(c\) is the velocity of the front of the shock wave; a comma between the lower indices \(i,j\) means the differentiation of the component \(u_{i}\) along the coordinate \(x_{j}\) \(\left(\frac{\partial u_{i}}{\partial x_{j}} = u_{i,j}\right)\); the square brackets at the functions mean the difference between their values on the right and on the left on \(\Sigma\left(\left[f\right] = f|_{\Sigma_{+}} - f|_{\Sigma_{-}}\right)\).

The initial values of the velocity of the medium behind the front \(\Sigma\) of the shock waves \(\Sigma_{1}\) or \(\Sigma_{2}\) \(\left(\left[v_{i}\right]|_{\Sigma_{-}}\right)\) are determined at the initial moment of impact from the conditions on the surface of a sphere of radius \(\delta\) approximating the vertex of the cone (figure 2).

![Figure 2. The image of the initial moment of the shock wave \(\Sigma\) coinciding with the surface of the cone (its singular point approximated by a sphere of radius \(\delta\))](image)

Let’s consider the stress state of the EVP material in the regions behind the diffracted waves \(\Sigma_{1}\) and \(\Sigma_{2}\), which are generated by the impact of the cone with the angle \(2\alpha\) at the vertex with velocity \(v_{0}\) (figure 1). This figure shows the vertex of the cone, approximated by a sphere \(S_{\delta}\) of small radius \(\delta\), and the diffracted waves generated by the impact of the vertex of the cone, at a position that appeared after a period of time \(\Delta t\) after the start of the impact. At the time of impact, the EVP material at the cone boundary \(S\) and \(S_{\delta}\) will acquire a velocity \(\tau_{s}\) that depends on the impact velocity \(v_{0}\) and the surface \(S\) properties of the cone, which causes either complete adherence of the material, or slip with friction, or complete slippage. On the surface of the sphere \(S_{\delta}\) the following relations takes place:

\[\tau_{s} = v_{n} \pi + v_{\tau} \tau, \quad \text{i.e.} \quad v_{si} = v_{n} n_{i} + v_{\tau} \tau_{i}, \quad (4)\]

where \(v_{\tau} = f v_{0} \cos \varphi; \quad v_{n} = v_{0} \sin \varphi;\)

\[\pi = (\cos \varphi \cos \psi; \cos \varphi \sin \psi; \sin \varphi);\]

\[\tau = (\sin \varphi \cos \psi; \sin \varphi \sin \psi; - \cos \varphi). \quad (5)\]

On the surface \(S\) of the cone \(\varphi = \frac{\pi}{2} - \alpha.\)

Note that the sticking coefficient \(f \in [0; 1]\). The value \(f = 1\) corresponds to the complete adhesion of the material to the surface \(S\) of the cone, and \(f = 0\) corresponds to slippage. At \(f \in (0; 1)\) different degrees of adhesion are realized.

In the case of complete slippage, the impact of a cone generates one front \(\Sigma_{1}\) of the longitudinal wave, in the case of different degrees of adhesion, a second \(\Sigma_{2}\) (shear) wave appears, which lags in its motion from the longitudinal wave \(\Sigma_{1}\).
To determine the stresses $\sigma^I_{ij}$ and $\sigma^{II}_{ij}$ in the zones behind $\Sigma_1$ and $\Sigma_2$ we use expressions (3) for stress jumps and strain rates in the regions of a piecewise constant approximation of stresses and velocities.

On $\Sigma_1$ the conditions (3) are fulfilled

$$[\sigma^I_{ij}] = \sigma^0_{ij} - \sigma^I_{ij} = - [v_n] \left( \frac{\lambda \delta_{ij} + 2\mu n_i n_j}{c_1} \right),$$

(6)

where $[v_n] = - v_0 \sin \varphi$.

On $\Sigma_2$ conditions (3) take the form

$$[\sigma^{II}_{ij}] = \sigma^I_{ij} - \sigma^{II}_{ij} = - [v_r] \frac{\mu}{c_2} \tau n_j,$$

(7)

where $[v_r] = - f v_0 \cos \varphi$.

From (6)–(7), one can obtain expressions for the components of the stress tensors $\sigma^I_{ij}$ and $\sigma^{II}_{ij}$ as functions of the velocity $v_0$ of the cone and directions of the vectors $\vec{n}, \vec{\tau}$:

$$\sigma^I_{ij} = \sigma^0_{ij} + \frac{v_0}{c_1} (\lambda \delta_{ij} + 2\mu u_i u_j) \sin \varphi;$$

(8)

$$\sigma^{II}_{ij} = \sigma^0_{ij} + \frac{v_0}{c_1} (\lambda \delta_{ij} + 2\mu u_i u_j) \sin \varphi + f \left( \frac{v_0}{c_2} \right) \mu (\tau_i n_j + \tau_j n_i) \cos \varphi.$$  

(9)

The first invariants $J_1 = \sigma_{kk}$ of the stress tensor in the zones behind $\Sigma_1$ and $\Sigma_2$ coincide

$$J^I_1 = J^{II}_1 = \sigma_{kk} = \sigma^0_{kk} + \frac{v_0}{c_1} (3\lambda + 2\mu) \sin \varphi.$$  

(10)

The deviator components of the stress tensors in the zones behind $\Sigma_1$ and $\Sigma_2$ are determined from the relations (8)–(9) as follows:

$$\sigma^I_{ij} = \sigma^0_{ij} - \frac{1}{3} \sigma^I_{kl} \delta_{ij} = \sigma^0_{ij} + \frac{v_0}{c_1} \sin \varphi \left( - \frac{2}{3} \mu \delta_{ij} + 2\mu n_i n_j \right);$$

(11)

$$\sigma^{II}_{ij} = \sigma^I_{ij} + f \frac{v_0}{c_2} \cos \varphi \mu (\tau_i n_j + \tau_j n_i).$$

(12)

It follows from (12) that the stresses in zone II differ from the stresses in zone I only in the case of friction ($f \neq 0$) on the surface of the cone.

The second invariants of the deviator of the stress tensor in the zones behind $\Sigma_1$ and $\Sigma_2$, which characterize the intensity of the tangential stresses, are calculated as follows:

$$J^I_2 = \sigma^{II}_{ij} \tau_{ij} = J^0_2 + 4 \frac{v_0}{c_1} \mu \sigma^0_{ij} n_i n_j \sin \varphi + \frac{16}{3} \left( \frac{v_0}{c_1} \mu \sin \varphi \right)^2,$$

(13)

where $\sigma^0_{ij} n_i n_j = \sigma^0_{nn}$;

$$J^{II}_2 = J^0_2 + 2f \frac{v_0}{c_1} \mu \sigma^0_{ij} n_i \tau_j \cos \varphi + 2 \left( f \frac{v_0}{c_2} \mu \cos \varphi \right)^2,$$

(14)

here $\sigma^0_{ij} n_i \tau_j = \sigma^0_{n\tau}$.

Let’s estimate the influence of the static stress state $\sigma^0_{ij}$ for the axisymmetric case on the intensity of tangential stresses $J_2$.

$$\sigma^0_{11} = \sigma_1; \quad \sigma^0_{22} = \sigma_1; \quad \sigma^0_{33} = \sigma_3; \quad \sigma^0_{ij} = 0 \text{ for } i \neq j;$$

$$\sigma^0_{11} = \sigma_1; \quad \sigma^0_{22} = \sigma_1; \quad \sigma^0_{33} = \sigma_3; \quad \sigma^0_{ij} = 0 \text{ for } i \neq j;$$

4
\[ \sigma'_{ij} = \frac{1}{3} \begin{pmatrix} \sigma_1 - \sigma_3 & 0 & 0 \\ 0 & \sigma_1 - \sigma_3 & 0 \\ 0 & 0 & 2(\sigma_3 - \sigma_1) \end{pmatrix}. \] (15)

Knowing the components of the vectors \( \bar{n} \) and \( \bar{\tau} \) (5), we define the contribution of \( \sigma'_{ij} \) to \( J_2^I \) and \( J_2^{II} \).

\[ \sigma'_{ij} n_i n_j = \bar{\sigma} (\cos^2 \varphi \cos^2 \psi + \cos^2 \varphi \sin^2 \psi - 2 \sin^2 \varphi) = \bar{\sigma} (\cos^2 \varphi - 2 \sin^2 \varphi); \] (16)

\[ \sigma'_{ij} n_i \tau_j = \bar{\sigma} (\sin \varphi \cos \varphi \cos^2 \psi + \sin \varphi \cos \varphi \sin^2 \psi + 2 \sin \varphi \cos \varphi) = \bar{\sigma} \sin \varphi \cos \varphi = \frac{\sigma}{2} \sin 2\varphi, \] (17)

Here \( \sigma = \sigma_1 - \sigma_3; \ \bar{\sigma} = \sigma \).

Let’s express the second invariant \( J_2^I \) of the deviator of the stress tensor \( \sigma'_{ij} \) in the first zone behind the diffracted longitudinal wave \( \Sigma_\delta \), taking into account the stresses \( \sigma'_{ij} \) in the unperturbed zone before \( \Sigma_\delta \) (16)–(17). We get

\[ J_2^I = J_2^0 + \frac{16}{3} \left( \frac{v_0}{c_1} \mu \sin \varphi \right)^2 + \frac{4}{3} \frac{v_0}{c_1} \mu (\sigma_1 - \sigma_3) \sin \varphi (1 - 3 \sin^2 \varphi), \] (18)

where

\[ J_2^0 = \sigma'_{ij} \sigma'_{ij} = \left( \frac{2}{3} (\sigma_3 - \sigma_1) \right)^2. \]

In the second zone, the account of the static stress state leads the expression (14) for \( J_2^{II} \) to the form

\[ J_2^{II} = J_2^I + 2 \left( f \mu \frac{v_0}{c_2} \cos \varphi \right)^2 + f \mu \frac{v_0}{c_2} (\sigma_1 - \sigma_3) \cos \varphi \sin 2\varphi. \] (19)

Here

\[ J_2^0 = \sigma'_{ij} \sigma'_{ij} = \frac{4}{9} (\sigma_1 - \sigma_3)^3. \] (20)

### 3. Results and analysis

Let’s consider the following cases.

**3.1. Analysis of the variation of the tangential stress in a non-stressed material**

We analyze the variation in the intensity of the tangential stresses \( J_2 \) behind the waves \( \Sigma_1 \) and \( \Sigma_2 \), which propagate in the unstressed material. In this case \( \sigma'_{ij} = \sigma_0^I = \sigma_0^0 = 0 \).

**3.1.1.** In the case of impact by an ideal smooth cone at \( f = 0 \), the shear wave \( \Sigma_2 \) does not arise, and behind the wave \( \Sigma_1 \), the shear stress intensity is given by

\[ \Lambda_1 = \frac{\sqrt{J_2^I}}{\mu \sqrt{16 \frac{v_0}{c_1}}} = \sin \varphi, \] (21)

so that the maximum \( J_2^I \) is attained at \( \varphi = \frac{\pi}{2} \), i.e. in the direction of impact (see figure 3).
3.1.2. In the case of a rough cone impact at $f \neq 1$, two waves arise: a longitudinal strain wave $\Sigma_1$ and a shear deformation wave $\Sigma_2$. The expression for the intensity of the tangential stresses $J^{II}_2$ behind $\Sigma_2$ takes the form

$$
\Lambda_2 = \sqrt{\frac{J^{II}_2}{\mu}} = \sqrt{\frac{v_0}{16 c_1}} \sqrt{\sin^2 \varphi + \left(\frac{c_1}{c_2} f \cos \varphi\right)^2},
$$

(22)

$\Lambda_2$ takes the maximum value in the directions $\varphi = 0$ and $\varphi = \frac{\pi}{2}$.

3.2. The influence of the static stress state of a material on the intensity of tangential stresses

The static stress state of a material influences its behavior behind $\Sigma_1$ and $\Sigma_2$ and, as follows from formulas (18)-(20), the intensity of tangential stresses $J_2$ behind $\Sigma_1$ and $\Sigma_2$ increases by a static value

$$
J_2^0 = J_{2i}^0 = \left(\frac{2}{3} (\sigma_1 - \sigma_3)\right)^2
$$
in any direction $\varphi$.

Depending on the direction $\varphi$ the additives to $J_2$ have the following form:

$$
\lambda I_2 \equiv \Delta J^I_2(\varphi) = \frac{\Delta J^I_2(\varphi)}{3 v_0 c_1} = \sin \varphi \left(1 - 3 \sin^2 \varphi\right);
$$

$$
\lambda II_2 \equiv \Delta J^{II}_2(\varphi) = \frac{\Delta J^{II}_2(\varphi)}{f v_0 c_2} = \cos \varphi \sin 2\varphi,
$$

where $\Delta J_i^j(\varphi) = J_i^j(\varphi) - J_i^j$, $i = I, II$.

The quantities $\lambda I_2$ and $\lambda II_2$ (figure 4) take the maximum values in the directions $\varphi^*_1 = \sqrt{\frac{1}{6}} \approx 24^\circ$ and $\varphi^*_2 = \sqrt{\frac{1}{3}} \approx 36^\circ$.

3.3. Distribution of residual plastic deformations after impact by a cone

Let’s investigate the distribution of residual plastic deformations remaining in the material after passage of the waves of plastic loading $\Sigma_1$ and $\Sigma_2$ for the case of a short (small $\Delta t$) impact by

Figure 3. An image of the stress $J^I_2$ intensity on the surface of the cone at the moment of impact as a function of the direction $\varphi$.
Figure 4. Graphs of the relative change in the intensity $\lambda_2$ of tangential stresses due to the static stress axisymmetric state $(\sigma_1^0, \sigma_2^0, \sigma_3^0)$ as a function of direction $\varphi$.

a cone. Directly behind $\Sigma$ plastic deformations are absent, but the rates of plastic deformations $\dot{\varepsilon}_{nn}^p|_{\Sigma_1}$ and $\dot{\varepsilon}_{nr}^p|_{\Sigma_2}$ are not equal to zero. In the linear approximation, plastic deformations $\dot{\varepsilon}_{nn}^p = \dot{\varepsilon}_{nn}^p|_{\Sigma_1} - \Delta t$ and $\dot{\varepsilon}_{nr}^p = \dot{\varepsilon}_{nr}^p|_{\Sigma_2} - \Delta t$ develop in the material (behind the wave $\Sigma$) over the time $\Delta t$ at a distance $\Delta n = c\Delta t$ behind the wave $\Sigma$ before the arrival of the waves of plastic unloading.

The rate of plastic deformation $\dot{\varepsilon}_{ij}^p$ is determined through the stress $\sigma_{ij}$ [10]

$$
\dot{\varepsilon}_{ij}^p = \dot{\varepsilon}_{ij} = \frac{\psi}{1 + \eta \psi} \sigma_{ij}',
$$

where $\psi = \frac{1}{\eta} \left( \sqrt{\frac{J_2}{k \sqrt{2}}} - 1 \right)$.

The second invariant of the deviator of the tensor of the rates of plastic deformation $\dot{I}_2 = \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p$ is expressed in terms of the second invariant of the deviator of the stress tensor as follows:

$$
\dot{I}_2 = \frac{2k^2}{\eta^2} \left( \sqrt{\frac{J_2}{k \sqrt{2}}} - 1 \right).
$$

As a measure of plastic deformation, we take a value $\Lambda_2 = \sqrt{\dot{I}_2}\Delta t \frac{\eta}{k \sqrt{2}}$, that can be represented in terms of the intensity $J_2$ of the deviator of the stress tensor as follows

$$
\Lambda_2 = \sqrt{J_2} \frac{k \sqrt{2}}{k \sqrt{2}} - 1.
$$

Plastic deformation accumulated over time $\Delta t$ on the surface of the cone, by the time of the termination of the impact process, is transported by waves $\Sigma_1^p$ or $\Sigma_2^p$ of plastic unloading, damped by geometric expansion of spherical waves and viscous friction [11]. On diffracted waves $\Sigma_1$ and $\Sigma_2$ we have

$$
\Lambda_2 = \Lambda_2^0 (\varphi) \frac{\delta}{\delta + ct} \frac{\alpha}{\mu} \exp (-\alpha t); \quad \varphi \in \left[\alpha; \frac{\pi}{2}\right],
$$

where $\alpha = \frac{4\mu^2}{3\eta \rho c^2}; \quad \rho c|^\Sigma_1 = \lambda + 2\mu; \quad \rho c|^\Sigma_2 = \mu$.

On the waves generated by the impact of the lateral surface of the cone we have

$$
\Lambda_2 = \Lambda_2^0 (\alpha) \sqrt{\frac{z \sin \alpha - \alpha}{z \sin \alpha + ct} \frac{\alpha}{\mu}} \exp (-\alpha t),
$$

where $z$ is counted from the vertex of the cone along its axis.
3.4. A note on the possibility of formation of thin zones of plastic deformation leading to the formation of cracks as discontinuities of displacements

In the direction $\varphi = \varphi^*_2$ behind the shear wave $\Sigma_2$ front, the maximum value of the residual deformation measure $\Lambda_2$ takes place and, consequently, the maximum value of the intensity $J_2$ of the second invariant of the deviator of the stress tensor and the maximum value of the jump of the tangential velocity $[v_\tau]_{\Sigma}$ takes place also, since it follows from (3)

$$J_2 = \left[ \sigma'_{ij} \right] \left[ \sigma'_{ij} \right] = \left( \frac{\mu}{c_2} [v_\tau] \right)^2. \quad (30)$$

During the time $\Delta t$ in the neighborhood of the direction $\varphi = \varphi^*_2$ on the wave $\Sigma^p_2$ of plastic unloading, the displacements of the opposite direction will occur (figure 5)

$$\Delta u_\tau = \Delta s \left. \frac{\partial v_\tau}{\partial s} \right|_{\varphi^*_2} \Delta t - \Delta s \left. \frac{\partial v_\tau}{\partial s} \right|_{\varphi^*_2} \Delta t, \quad (31)$$

which lead to disruption of the continuity of the material and development of the crack, the head part of which moves along with the shearing discharge wave $\Sigma^p_2$.

In the neighborhood of the direction $\varphi^*$, the velocity $v_\tau$ tangent to $\Sigma_2$ has a maximum, its gradients on the left and on the right from $\varphi^*$ have different signs $\left. \frac{\partial v_\tau}{\partial s} \right|_{\varphi^*} > 0; \left. \frac{\partial v_\tau}{\partial s} \right|_{\varphi^*} < 0$ (figure 6), which in time $\Delta t$ leads to tangential displacements $u_\tau$ of different signs to the left and to the right from $\varphi^*$ on $\Sigma^p$

$$\overline{u}_\tau \approx \left. \frac{\partial u_\tau}{\partial s} \right|_{\varphi^*} \Delta s \Delta t > 0; \quad \overline{u}_\tau \approx \left. \frac{\partial u_\tau}{\partial s} \right|_{\varphi^*} \Delta s \Delta t < 0,$$

so that a disruption of displacements $[u_\tau]_{\Sigma^p}$ is expected after the unloading wave.
A crack that appeared at the time $t = \Delta t$ on the surface $\Sigma^p_2$ will in time be a conical surface with an angle $2\left(\frac{\pi}{2} - \varphi^*_2\right)$. Its propagation along the generatrix of the cone will cease upon reaching the plasticity limit at the length $L = c_2 t_{\text{limit}}$

$$J_2 = 2k^2 \frac{\delta}{\delta + L} \exp\left(\frac{-\alpha L}{c_2}\right). \quad (32)$$

**Figure 6.** An image of a crack in the form of a part of a conical surface of length $L$ along the generatrix. The head part of the crack is born on $\Sigma^p_2$ and damps at the time moment $t_{\text{limit}}$ after which its propagation stops.

The above presented result on the propagation velocity of the head part of the crack agrees with the known experimental data [5, 12].

4. Conclusion
The occurrence of longitudinal and shear waves in an unstressed material is investigated depending on the properties of the surface of the cone: in the case of complete slippage, a shock generates one front $\Sigma_1$ of a longitudinal wave, in the case of different degrees of adhesion, a second (shear) wave $\Sigma_2$ appears that collapses in its motion from the longitudinal wave $\Sigma_1$.

It is shown that a static stress state $\sigma^0_{ij}$ increases the intensity of tangential stresses $J_2$ by $\Sigma_1$ and $\Sigma_2$ and for a static value $J^0_2 = J^0_2 = \left(\frac{2}{3} (\sigma_1 - \sigma_3)\right)^2$ in any direction $\varphi$.

The distribution of residual plastic deformations remaining in the material after passing through the waves of plastic loading $\Sigma_1$ and $\Sigma_2$ for the case of a short-time impact by a cone is investigated.

The possibility of formation of thin zones of plastic deformation leading to the formation of cracks as discontinuities of displacements is estimated. It should be noted that the possibility of the development of a jump in the velocities in the discontinuity jump was expressed in [7].

The patterns of propagation of the front edge of cracks are local in nature and are determined by the initial perturbation and the stressed state in front of the edge.

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