Nonlocal Quantization Principle in Quantum Field Theory and Quantum Gravity

Martin Kober
Capstone Institute for Theoretical Research
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In this paper a nonlocal generalization of field quantization is suggested. This quantization principle presupposes the assumption that the commutator between a field operator and the operator of the canonical conjugated variable referring to other space-time points does not vanish as it is postulated in the usual setting of quantum field theory. Based on this presupposition the corresponding expressions for the field operators, the eigenstates and the path integral formula are determined. The nonlocal quantization principle also leads to a generalized propagator. If the dependence of the commutator between operators on different space-time points on the distance of these points is assumed to be described by a Gaussian function, one obtains that the propagator is damped by an exponential. This leads to a disappearance of UV divergences. The transfer of the nonlocal quantization principle to canonical quantum gravity is considered as well. In this case the commutator has to be assumed to depend also on the gravitational field, since the distance between two points depends on the metric field.

I. INTRODUCTION

The most important problem of contemporary fundamental theoretical physics is the unification of quantum theory and general relativity. A quantum theoretical setting of general relativity as usual quantum field theory in analogy to the description of the other fundamental interactions is not possible, since such a description leads to divergencies, which cannot be cured. This fact seems to indicate that the problem of incorporating general relativity to a quantum theoretical description of all fundamental interactions is directly connected to the question, whether the basic postulates of quantum field theory in its usual setting can be transferred to such a more fundamental description of nature. These basic postulates are related to the idea of the basic structure of space-time. General relativity is a background independent theory, which in contrast to the usual setting of quantum field theory, on which the other fundamental interactions of the standard model of particle physics are based, does not presuppose a nondynamical metric structure of space-time. Quantum theory on the other hand contains a constitutive element of nonlocality, which becomes manifest in the famous Einstein-Podolsky-Rosen thought experiment for example. The basic postulates of general quantum theory itself, this means the general Hilbert space formalism in the setting of Dirac without relation to additional field theoretic elements, does not contain special assumptions about a space-time structure. Therefore it seems to be possible that a quantum theoretical description of general relativity as well as a unification of all fundamental interactions appearing in nature could indeed require a decisive modification of the postulates usually presupposed in quantum field theory. This holds especially for the basic assumption of local commutativity or microscopic causality, which implies that the commutator or anticommutator respectively of field operators at two separated space-time points vanishes.

In accordance with the above explanations in the present paper is suggested a generalization of the quantization principle as it is presupposed in quantum field theory and as it is transferred to canonical quantum gravity. Usually, the commutator or anticommutator respectively between the field and the corresponding canonical conjugated momentum is assumed to be proportional to the delta function. In the present paper this quantization principle is extended in such a way that the delta function is replaced by a general function, which depends on the distance between the space-time points the corresponding operators refer to. This implies that the commutator between the field operator and the corresponding operators describing the canonical conjugated quantity on two separated space-time points does not vanish anymore. Such a quantization principle establishes a nonlocal structure of the quantum description of a theory, violates the postulate of local commutativity and thus can be called nonlocal quantization principle. A natural assumption for the function replacing the delta function is a Gaussian function, which as limiting case merges the delta function. It will be shown that under this assumption one obtains a propagator, which corresponds to the propagator of usual quantum field theory multiplied with an exponential damping factor depending on the squared three momentum. This implies that this generalized quantum field theory contains no ultra violet divergences. The obtained propagator corresponds to the propagator within the coherent state approach to noncommutative geometry, which has been developed in [1,2,3], extended in [4,5] and transferred to the gravitational field in [6]. In [7] also appears a propagator of this structure. It is also possible to formulate this generalized quantization principle within the path integral formalism. Such a formulation is obtained by replacing the usual inner product between field eigenstates and eigenstates of the canonical conjugated variables by the generalized one according to the nonlocal quantization principle. This results in a generalized path integral expression. If the nonlocal quantization principle is considered as a fundamental description of nature, then one has of course...
to assume that it is also valid for the quantization of the gravitational field. Accordingly it has to be postulated a nonlocal generalization of the commutator between the three metric and its canonical conjugated momentum in quantum geometrodynamics. With respect to the quantization of the gravitational field, the intricacy arises that the distance between two space-time points depends on the metric field. Therefore the commutator also depends on the three metric itself. A Generalized quantization principle in canonical quantum gravity and quantum cosmology has already been considered in \[8\]. Other considerations of a generalized quantization principle with respect to the variables of quantum cosmology and black holes can be found in \[0\],\[10\]. The considerations in \[8\],\[0\],\[10\] have transferred the generalized uncertainty principle in quantum mechanics, see \[11\],\[12\],\[13\] for example, which leads to a minimal length, to the variables describing the gravitational field. Another generalization of the quantization principle in canonical quantum gravity, which is based on quaternions and has also been transferred to \(N = 1\) supergravity, has been developed in \[14\]. However, these generalizations of the quantization principle in canonical quantum gravity in contrast to the approach of the present paper remain local. Nonlocality has been considered in the context of quantum field theory \[15\],\[16\],\[17\],\[18\],\[19\],\[20\],\[21\],\[22\],\[23\],\[24\],\[25\],\[26\],\[27\],\[28\],\[29\],\[30\],\[31\],\[32\],\[33\],\[34\],\[35\],\[36\],\[37\],\[38\],\[39\],\[40\],\[41\],\[42\],\[43\],\[44\],\[45\],\[46\],\[47\],\[48\],\[49\],\[50\],\[51\],\[52\],\[53\],\[54\],\[55\],\[56\],\[57\],\[58\],\[59\],\[60\],\[61\],\[62\],\[63\],\[64\],\[65\],\[66\],\[67\],\[68\],\[69\],\[70\] and also of black holes \[71\]. But the concept of a nonlocal quantization principle has not been studied yet.

The paper is structured as follows: The generalized formalism of quantum field theory based on this nonlocal quantization principle for a scalar field is considered at the beginning. Subsequently, the corresponding generalized expression for the propagator is calculated. Especially the mentioned special case is regarded, where the function the commutator depends on is assumed to be a Gaussian function leading to a propagator containing an exponential damping function, which depends on the squared three momentum with negative sign of the exponent as additional factor. This means that the corresponding quantum field theory contains no ultra violet divergences. After this the nonlocal quantization principle is considered in the context of canonical quantum gravity and the generalized representations of the operators as well as the generalized constraints are formulated. At the end the nonlocal quantization principle is transferred to the path integral formula.

II. NONLOCAL QUANTIZATION PRINCIPLE

In this section is presented the basic idea of the nonlocal quantization principle in a general way. According to the explanations given in the introduction a generalization of the fundamental quantization principle with respect to the quantization of fields is suggested. This quantization principle omits the postulate of microscopic causality. The generalized nonlocal quantization principle is obtained by the following transition:

\[
\left[ \hat{\Phi}(x), \hat{\Pi}(y) \right] = i\delta^3(x-y) \quad \longrightarrow \quad \left[ \hat{\Phi}(x), \hat{\Pi}(y) \right] = i\alpha^3(x-y),
\]

(1)

where \(\hat{\Phi}(x)\) denotes a field operator, \(\hat{\Pi}(x)\) denotes the corresponding canonical conjugated operator, \(x\) and \(y\) denote three-vectors, \(\delta^3(x-y)\) denotes the three-dimensional delta function referring to a spacelike hypersurface and \(\alpha^3(x-y)\) denotes a general function on the spacelike hypersurface, which is symmetric in \(x\) and \(y\) and thus just depends on the distance of the points \(x\) and \(y\). This assumption is natural, if homogeneity and isotropy of space shall be maintained. The field operator \(\hat{\Phi}(x)\) and the corresponding canonical conjugated operator \(\hat{\Pi}(x)\) are generalizations of the usual operators, which shall be denoted with \(\phi(x)\) and \(\pi(x)\) and obey the usual commutation relations:

\[
\left[ \phi(x), \pi(y) \right] = i\delta^3(x-y).
\]

(2)

The generalized operators can be represented in a specific way by using the usual operators. One possibility is the representation, in which the generalized field operator \(\hat{\Phi}(x)\) is equal to \(\phi(x)\) and the generalized canonical conjugated operator is modified,

\[
\hat{\Phi}(x) = \hat{\phi}(x), \quad \hat{\Pi}(x) = \int d^3z \, \alpha^3(x-z) \, \hat{\pi}(z),
\]

(3)

and another possibility is the representation, where the generalized canonical conjugated operator is equal to \(\pi(x)\) and the generalized field operator is modified,

\[
\hat{\Phi}(x) = \int d^3z \, \alpha^3(x-z) \, \hat{\phi}(z), \quad \hat{\Pi}(x) = \hat{\pi}(x).
\]

(4)
The momentum eigenstates in the representation, where the momentum operator is generalized and where the usual operators are represented with respect to the field, looks as following:

$$|\Pi[\Phi(x)]\rangle = \exp \left[ i \int d^3x \, d^3y \, \bar{\alpha}^3(x-y)\Phi(x)\Pi(y) \right], \quad (5)$$

where $\bar{\alpha}^3(x-y)$ is defined by the following relation:

$$\int d^3z \, \alpha^3(x-z)\bar{\alpha}^3(z-y) = \delta(x-y), \quad (6)$$

and $\Pi(x)$ denotes the momentum eigenvalue corresponding to the momentum eigenstate. This can be seen as follows:

$$\hat{\Pi}(x)\langle \Pi \rangle = -i \int d^3y \, \alpha^3(x-y) \frac{\delta}{\delta \Phi(y)} \exp \left[ i \int d^3w \, d^3z \, \bar{\alpha}^3(w-z)\Phi(w)\Pi(z) \right]$$

$$= \int d^3y \, d^3w \, d^3z \, \alpha^3(x-y)\bar{\alpha}^3(w-z)\delta(w-y)\Pi(z) \exp \left[ i \int d^3w \, d^3z \, \bar{\alpha}^3(w-z)\Phi(w)\Pi(z) \right]$$

$$= \int d^3z \, \bar{\alpha}^3(y-z)\Pi(z) \exp \left[ i \int d^3w \, d^3z \, \bar{\alpha}^3(w-z)\Phi(w)\Pi(z) \right]$$

$$= \Pi(x) \exp \left[ i \int d^3w \, d^3z \, \bar{\alpha}^3(w-z)\Phi(w)\Pi(z) \right] = \Pi(x)\langle \Pi \rangle. \quad (7)$$

The corresponding position eigenstate is given by

$$|\Phi[\Phi(x)]\rangle = \delta \left( \Phi[\Phi(x)] - \Phi'[\Phi(x)] \right), \quad (8)$$

where $\Phi(x)$ denotes the eigenvalue to the position eigenstate. The inner product for two states referring to the field can be defined as

$$\langle \Psi[\Phi(x)]|\Omega[\Phi(x)]\rangle = \int d\Phi(x) \, d^3x \, \Psi[\Phi(x)] \Omega[\Phi(x)]. \quad (9)$$

This implies that the inner product of a momentum eigenstate with a position eigenstate is given by

$$\langle \Phi|\Pi \rangle = \exp \left[ i \int d^3w \, d^3z \, \bar{\alpha}^3(w-z)\Phi(w)\Pi(z) \right]. \quad (10)$$

III. DERIVATION OF THE PROPAGATOR IN QUANTUM FIELD THEORY WITH NONLOCAL QUANTIZATION PRINCIPLE

The general concept of the nonlocal quantization principle can now be studied in the context of a scalar field theory. In this section the corresponding propagator is calculated in its general shape meaning that no special assumption for the special choice of the quantization function is made so far. To determine the propagator, it is necessary to formulate the generalized expression for the plane wave expansion of the scalar field. The following expression for the quantum field

$$\hat{\Phi}(x, t) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2\omega_p}} \left( a(p) \int d^3z \, \alpha^3(x-z)e^{ipx-i\omega_p t} + a^\dagger(p) e^{-ipx+i\omega_p t} \right) \quad (11)$$
and its canonical conjugated variable

\[
\hat{\Pi}(x, t) = \int \frac{d^3 p}{\sqrt{(2\pi)^3 2\omega_p}} \left( a(p) \int d^3 z \; \alpha^3(x - z)e^{ipx - \omega_p t} - a^\dagger(p)e^{-ipx + i\omega_p t} \right),
\]

(12)

where \(a(p)\) and \(a^\dagger(p)\) are the usual creation and annihilation operators fulfilling the commutation relations

\[
[a(p), a^\dagger(p')] = \delta(p - p'),
\]

(13)

obey the generalized commutation relations given in (11). This can be shown as follows:

\[
\begin{align*}
\left[ \hat{\Phi}(x, t), \hat{\Pi}(y, t) \right] &= \frac{i}{(2\pi)^3} \int \frac{d^3 p}{\sqrt{2\omega_p}} \int \frac{d^3 p'}{\sqrt{2\omega_p}} \int d^3 z \left[ [a(p), a^\dagger(p')] \alpha^3(x - z)e^{ip'y - ip' z} + [a(p'), a^\dagger(p)] \alpha^3(y - z)e^{ip'x - ipz} \right] \\
&= \frac{i}{2(2\pi)^3} \int \frac{d^3 p}{\sqrt{2\omega_p}} \int \frac{d^3 p'}{\sqrt{2\omega_p}} \int d^3 z \left[ \delta^3(p - p') \alpha^3(x - z)e^{ipy - ip' z} + \delta^3(p' - p) \alpha^3(y - z)e^{ip'x - ipz} \right] \\
&= \frac{i}{2} \int d^3 z \left[ \alpha^3(x - z)\delta^3(y - z) + \alpha^3(y - z)\delta^3(x - z) \right] \\
&= \frac{i}{2} \left[ \alpha^3(x - y) + \alpha^3(y - x) \right] \\
&= i\alpha^3(x - y).
\end{align*}
\]

(14)

The corresponding propagator of the generalized quantum field (11) is obtained by inserting it to the general expression for the propagator in terms of the quantum field, which is given by

\[
G(x - y) = \langle 0 | T \left[ \hat{\Phi}(x)\hat{\Phi}(y) \right] | 0 \rangle,
\]

(15)

where \(T\) denotes the time ordering operator \(|0\rangle\) denotes the vacuum state. This leads to the following expression:

\[
G(x - y) = \int \frac{d^3 p}{(2\pi)^3 2\omega_p} \int d^4 z \left[ \theta(x_0 - y_0)\alpha^4(x - z)e^{-ipz + ipy} + \theta(y_0 - x_0)\alpha^4(y - z)e^{-ipz + ipy} \right],
\]

(16)

where the function \(\alpha^4(x - y)\) has been defined as follows:

\[
\alpha^4(x - y) = \alpha^3(x - y)\delta(x_0 - y_0).
\]

(17)

By following the usual procedure of expressing the \(\theta\)-function by an integral

\[
\theta(t) = \lim_{\epsilon \to 0} \frac{1}{2\pi i} \int dE \frac{e^{-iEt}}{E + i\epsilon},
\]

(18)

the expression of the propagator can be transformed to

\[
G(x - y) = i \int \frac{d^3 p \; dE}{(2\pi)^3 2\omega_p} \frac{1}{E + i\epsilon} \int d^4 z \left[ \alpha^4(x - z)e^{ip(z - y) - i(\omega_p + E)(z_0 - y_0)} + \alpha^4(y - z)e^{ip(z - x) - i(\omega_p + E)(z_0 - x_0)} \right].
\]

(19)

Changing the variables according to,

\[
E' = E + \omega_p,
\]

(20)
leads to

\[ G(x - y) = i \int \frac{d^3p}{(2\pi)^4} \frac{dE'}{2\omega_pE'} \frac{1}{\omega_p + i\epsilon} \int d^4z \left[ \alpha^4(x - z)e^{i\mathbf{p}(x - y) - iE'(z_0 - y_0)} + \alpha^4(y - z)e^{i\mathbf{p}(z - x) - iE'(z_0 - x_0)} \right] \]

\[ = i \int \frac{d^3p}{(2\pi)^4} \frac{dE'}{2}\left[ \frac{e^{i\mathbf{p}(x - y) - iE'(z_0 - y_0)}}{\omega_pE' - \omega_p^2 + i\epsilon} \alpha^4(x - z) + x \leftrightarrow y \right] \]

\[ = i \int \frac{d^3p}{(2\pi)^4} \frac{dE'}{2}\left[ \frac{e^{i\mathbf{p}(x - y) - iE'(z_0 - y_0)}}{\sqrt{\mathbf{p}^2 + m^2E'} - \mathbf{p}^2 - m^2 + i\epsilon} \alpha^4(x - z) + x \leftrightarrow y \right]. \quad (21) \]

By setting

\[ p_0 = E' \]

one obtains the following generalized expression for the propagator in position space:

\[ G(x - y) = i \int \frac{d^3p}{(2\pi)^4} \frac{d^4z}{2} \left[ \frac{e^{i\mathbf{p}(x - y) - ip_0(z_0 - y_0)}}{\sqrt{\mathbf{p}^2 + m^2p_0 - \mathbf{p}^2 - m^2 + i\epsilon}} \alpha^4(x - z) + x \leftrightarrow y \right] \]

\[ = i \int \frac{d^4p}{(2\pi)^4} \frac{d^4z}{2} \left[ \frac{e^{i\mathbf{p}(z - y)}}{\mathbf{p}^2 - m^2 + i\epsilon} \alpha^4(x - z) + x \leftrightarrow y \right]. \quad (23) \]

**IV. SPECIAL MODEL FOR THE NONLOCAL QUANTIZATION PRINCIPLE**

The generalized nonlocal quantum field theory shall now be considered for a special choice of the quantization function \( \alpha^3(x - y) \) appearing in \([1]\). Since this function should be symmetric in \( x \) and \( y \), should become smaller, if the distance becomes larger, should vanish at infinity and should merge the delta function in the limiting case, a natural choice seems to be a Gaussian function,

\[ \alpha^3(x - y) = \frac{e^{-\frac{1}{2}(x - y)^2}}{\Delta \sqrt{2\pi}}, \quad (24) \]

where \( \Delta \) describes the width of the Gaussian function. Accordingly \( \alpha^4(x - y) \) reads

\[ \alpha^4(x - y) = \delta(x_0 - y_0) \frac{e^{-\frac{1}{2}(x - y)^2}}{\Delta \sqrt{2\pi}}. \quad (25) \]

If the expression for \( \alpha^4(x - y) \) given in \( [25]\) is inserted into \( [20]\), one obtains the expression for the corresponding special propagator

\[ G(x - y) = i \int \frac{d^4p}{(2\pi)^4} \frac{d^4z}{2} \left[ \frac{e^{i\mathbf{p}(z - y)}}{\mathbf{p}^2 - m^2 + i\epsilon} \delta(x_0 - z_0) \frac{e^{-\frac{1}{2}(x - y)^2}}{\Delta \sqrt{2\pi}} + x \leftrightarrow y \right]. \quad (26) \]

By using the substitution

\[ w = x - z, \quad (27) \]

this expression can be transformed to
\[ G(x - y) = i \int \frac{d^4 p \, d^4 w}{2(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - m^2 + i\epsilon} \delta(w_0) e^{-\frac{\Delta p^2}{2\pi}} + x \leftrightarrow y \]
\[ = i \int \frac{d^4 p \, d^3 w \, dw_0}{2(2\pi)^4} \frac{1}{\Delta \sqrt{2\pi}} \left[ e^{ip(x-y)} \delta(w_0) e^{-\frac{\Delta p^2}{2\pi} + ipw + x \leftrightarrow y} \right] \]
\[ = i \int \frac{d^4 p \, d^4 w}{2(2\pi)^4} \frac{1}{\Delta \sqrt{2\pi}} \left[ e^{ip(x-y)} \frac{e^{-\frac{\Delta p^2}{2\pi} + ipw + x \leftrightarrow y}}{p^2 - m^2 + i\epsilon} \right]. \quad (28) \]

By substituting \( w \) through \( a \) according to
\[ a = \frac{w}{\sqrt{2\Delta}} - i \sqrt{\frac{\Delta}{2}} p, \]
the corresponding integral can be solved and one obtains the final expression for the propagator represented in position space of the quantum field theory with the nonlocal quantization principle under consideration of the special choice \[ \overrightarrow{24} \] for the quantization function
\[ G(x - y) = i \int \frac{d^4 p \, d^4 a}{(2\pi)^4 \sqrt{\pi}} e^{-\frac{\Delta p^2}{4\pi}} \left[ e^{ip(x-y)} \frac{e^{-\Delta a^2}}{p^2 - m^2 + i\epsilon} + x \leftrightarrow y \right] \]
\[ = i \int \frac{d^4 p}{(2\pi)^4} \pi \Delta e^{-\frac{\Delta p^2}{4}} \left[ e^{ip(x-y)} + e^{ip(y-x)} \right] \frac{e^{-\Delta a^2}}{p^2 - m^2 + i\epsilon} \]
\[ = i\pi \Delta \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} e^{-\frac{\Delta p^2}{4}} \frac{e^{-\Delta a^2}}{p^2 - m^2 + i\epsilon} \]
\[ = i\Delta \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} e^{-\frac{\Delta p^2}{4}} \frac{e^{-\Delta a^2}}{p^2 - m^2 + i\epsilon} \quad (30) \]

The corresponding expression for the propagator represented in momentum space is accordingly given by
\[ G(p) = \frac{i2\pi \Delta e^{-\frac{\Delta p^2}{4}}}{p^2 - m^2 + i\epsilon}. \quad (31) \]

This means that this quantum field theory in analogy to the situation in noncommutative geometry contains no ultra violet divergences, since the propagator contains the damping factor \( e^{-\frac{\Delta p^2}{4}} \). The special shape of the generalized propagator corresponds to the one calculated based on the coherent state approach to noncommutative geometry \[ \overrightarrow{1, 2, 3} \]. In \[ \overrightarrow{5} \], where the additional assumption of noncommuting momenta appears, has been found a generalization of this propagator even containing no infrared divergences. Accordingly the presented generalized quantum field theory, which is based on the concept of a nonlocal quantization principle represents at least under certain conditions a promising candidate for an alternative theory to noncommutative geometry.

V. CANONICAL QUANTUM GRAVITY WITH NONLOCAL QUANTIZATION PRINCIPLE

If the nonlocal quantization principle is assumed to be a fundamental principle of nature, then of course it has also been transferred to the quantum description of the gravitational field. In this section the corresponding generalization of quantum geometrodynamics is considered. In quantum geometrodynamics space-time is foliated into a time coordinate and a spacelike hypersurface. Accordingly the metric field can be expressed in the following way:
\[ g_{\mu\nu} = \begin{pmatrix} N_a N_a - N^2 & N_b \\ N_c & h_{ab} \end{pmatrix}, \quad (32) \]
where \( h_{ab} \) denotes the three metric on the spacelike hypersurface, \( N \) denotes the lapse function and \( N_a \) is the shift vector. The corresponding canonical conjugated momentum \( \pi^{ab} \) reads
\[ a_{ab} = \frac{\partial L}{\partial h_{ab}} = \frac{\sqrt{h}}{16\pi G} (K_{ab} - Kh_{ab}), \]  
(33)

with \( K_{ab} \) describing the extrinsic curvature

\[ K_{ab} = \frac{1}{2N} \left( \dot{h}_{ab} - D_aN_b - D_bN_a \right). \]  
(34)

Since the generalization of the delta function should because of homogeneity and isotropy of space just depend on the distance of the two space points of the field operator and its canonical conjugated operator and with respect to the canonical quantum description of general relativity the distance of two space points depends on the three metric itself, the commutator depends on the three metric as well. This situation is similar to the generalized quantization principle which has been developed in [8]. The distance \( D \) between two points, \( x \) and \( y \), on the spacelike hypersurface on which the three metric \( h_{ab} \) is defined, is given by

\[ D(x, y, h) = \int_x^y d\lambda \sqrt{h_{ab}(\lambda)} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda}. \]  
(35)

In case of the variables of quantum geometrodynamics, this leads to the following generalized commutation relation between the components of the three metric and its corresponding canonical conjugated quantity

\[ [h_{ab}(x), p^{cd}(y)] = \frac{i}{2} \left( \delta_a^c \delta_b^d - \delta_a^d \delta_b^c \right) \alpha^3 [D(x, y, h)]. \]  
(36)

The corresponding nonlocal operators can be expressed as follows:

\[ \hat{h}_{ab}(x)|\Psi[h]\rangle = h_{ab}(x)|\Psi[h]\rangle, \quad \hat{n}_{ab}(x)|\Psi[h]\rangle = -i \int d^3y \alpha^3 [D(x, y, h)] \frac{\delta}{\delta h_{ab}(y)}|\Psi[h]\rangle. \]  
(37)

And the corresponding quantum constraints are obtained by inserting the operators \( 37 \) to the corresponding classical constraints

\[ \hat{H}(x)|\Psi[h]\rangle = \left[ 16\pi GG_{abcd} \hat{h}(x) \hat{h}_{ab}(x) \hat{p}^{cd}(x) - \frac{\sqrt{h(x)}}{16\pi G} \left( R[h(x)] - 2\Lambda \right) \right] |\Psi[h]\rangle = 0, \]

\[ = \left[ 16\pi GG_{abcd} h(x) \int d^3y \int d^3z \alpha^3 [D(x, y, h)] \alpha^3 [D(x, z, h)] \frac{\delta^2}{\delta h_{ab}(y) \delta h_{cd}(z)} \right] |\Psi[h]\rangle = 0, \]

\[ \hat{H}_a(x)|\Psi[h]\rangle = -2D_b \hat{p}^b_a(x)|\Psi[h]\rangle = -2D_b \hat{h}_{ac}(x) \int d^3z \alpha^3 [D(x, y, h)] \frac{\delta}{\delta h_{bc}(y)}|\Psi[h]\rangle = 0, \]  
(38)

where \( G_{abcd} \) is defined according to

\[ G_{abcd} = \frac{1}{2\sqrt{h}} \left( h_{ac}h_{bd} + h_{ad}h_{bc} - h_{ab}h_{cd} \right). \]  
(39)

If the special case of the assumption of a Gaussian function for the quantization function is considered, this function is in case of quantum geometrodynamics of the following shape:

\[ \alpha^3 [D(x, y, h)] = e^{-\frac{1}{2} \frac{|D(x, y)|^2}{\Delta}}, \]  
(40)

where the squared difference between the space-time points has to be replaced by the distance function \( D(x, y, h) \) depending on the three metric \( h_{ab} \), too.
VI. PATH INTEGRAL QUANTIZATION

Of course it has to be possible to formulate the generalized quantization principle, which is given in (II) by generalization of the usual commutation relations belonging to canonical quantization, within the path integral formalism as well. To obtain such a formulation in the framework of path integrals, first the general expression of the transition amplitude between two states separated by an infinitesimal time interval has to be modified according to the generalization of the canonical quantization condition. The transition amplitude between a state \( |\Phi(t)\rangle \) at time \( t \) and a state \( |\Phi'(t+dt)\rangle \) at time \( t+dt \) is given by

\[
\langle \Phi'(t+dt)|\Phi(t) \rangle = \int d\Pi \langle \Phi' | \exp \left[ -i dt \int d^3x \, \mathcal{H} \left( \Phi(x), \Pi(x) \right) \right] |\Pi \rangle \langle \Pi | \Phi' \rangle 
\]

\[
= \int d\Pi \langle \Phi' | \exp \left[ -i dt \int d^3x \, \mathcal{H} \left( \Phi(x), \Pi(x) \right) \right] |\Pi \rangle \langle \Pi | \Phi' \rangle. \tag{41}
\]

Usually the inner product between an arbitrary eigenstate of the field operator and an arbitrary eigenstate of the momentum operator reads \( \langle \Pi | \Phi \rangle = \exp \left[ i \int d^3x \, \Phi(x) \Pi(x) \right] \). In case of the nonlocal quantization principle the general inner product is given by (III). Inserting (II) to (II) leads to

\[
\langle \Phi'(t+dt)|\Phi(t) \rangle = \int d\Pi \exp \left[ -i dt \int d^3x \, \mathcal{H} \left( \Phi(x), \Pi(x) \right) + i \int d^3w \, d^3z \, \tilde{\alpha}^3(w-z) \left( \Phi'(w) - \Phi(w) \right) \Pi(z) \right]. \tag{42}
\]

By using the expression of the transition amplitude for an infinitesimal time interval in case of the nonlocal quantization principle, the corresponding transition amplitude for a non-infinitesimal time interval can be determined as usual in the following way:

\[
\langle \Phi(t_b)|\Phi(t_a) \rangle = \lim_{N \to \infty} \int d\Phi_1 \ldots d\Phi_N \langle \Phi(t_b)|\Phi_N \rangle \ldots \langle \Phi_1|\Phi(t_a) \rangle 
\]

\[
= \lim_{N \to \infty} \int \prod_{k=1}^N d\Phi_k \prod_{k=0}^{N} d\Pi_k \exp \left[ i \sum_{k=1}^{N+1} \int d^3x \left( \int d^3z \, \tilde{\alpha}^3(x-z) \left( \Phi_k(x) - \Phi_{k-1}(x) \right) \Pi_{k-1}(z) \right. \right. 
\]

\[
- \mathcal{H} \left( \Phi_k(x), \Pi_{k-1}(x) \right) \left) \right] 
\]

\[
= \int \mathcal{D} \left[ \Phi(x) \right] \mathcal{D} \left[ \Pi(x) \right] \exp \left[ i \int_{t_a}^{t_b} dt \int d^3x \left( \int d^3z \, \tilde{\alpha}^3(x-z) \Pi(x) \Phi(z) - \mathcal{H} \left( \Phi(x), \Pi(x) \right) \right) \right]. \tag{43}
\]

This represents the analogue generalization from the usual quantization principle to the nonlocal quantization principle in the path integral formalism.

VII. SUMMARY AND DISCUSSION

In this paper has been suggested a generalization of the concept of quantization within quantum field theory. The generalization consists in the assumption that the commutator between the operator describing the field, which is quantized, and the operator describing its corresponding canonical conjugated momentum does not vanish, if the two operators refer to different space-time points, as it is presupposed in the usual setting of quantum field theory. Accordingly the delta function appearing on the right hand side of the quantization condition usually, has to be replaced by another function, which does not vanish, if the space-time points the two operators within the commutator refer to are not equal. Because of the postulate of homogeneity and isotropy of space, this function should be assumed just to depend on the distance between the two space coordinates. According to this nonlocal quantization principle generalized field operators have to be defined fulfilling this generalized quantization conditions. Based on the generalized expression of a scalar field operator, the corresponding generalized expression for the propagator has been calculated, which of course directly depends of the generalization of the delta function appearing in the generalized quantization condition. If the natural assumption is made that the quantization function is described by a Gaussian function, then one obtains a special shape for the propagator with an exponential containing the negative square of the three momentum as damping factor. Thus the propagator corresponds to the propagator found in
the coherent state approach of noncommutative geometry and contains no ultra violet divergences. This is a very important hint that this generalization of quantum field theory omitting the postulate of locality with respect to the basic quantization condition could indeed represent a promising candidate for a generalization of quantum field theory similar to quantum field theory on noncommutative space-time. Since such a quantization principle should be interpreted as fundamental property of nature, it has analogously to be transferred to the quantum description of all field theories including general relativity. This implies the necessity to generalize the canonical quantum description of general relativity as well. In this paper has been given a consideration with respect to quantum geometrodynamics, which has not been elaborated completely yet. But the quantization function has in each case to be assumed to depend on the gravitational field, since the distance between to space points depends on the gravitational field. This leads to a similarity to a generalized quantization principle within canonical quantum gravity \[8\] in accordance with a generalized uncertainty in quantum mechanics, where the right hand side of the quantization condition depends on the operators themselves. It has also to be possible to incorporate the nonlocal generalization of the quantization principle to path integrals. This has been done by replacing the usual inner product between an arbitrary eigenstate of the field operator and an arbitrary eigenstate of its canonical conjugated variable by the corresponding generalized one. Based on this generalization of path integrals it is in principle possible to incorporate the nonlocal quantization principle to gravity by using the path integral formalism. In general it could be very interesting with respect to further research projects, to formulate a full quantum description of general relativity based on this concept. Perhaps because of the ultraviolet behaviour of the commutator it can even contribute to circumvent the problem of non-renormalizability of the quantum description of gravity in the framework of usual quantum field theory. An additional important task would be to determine an upper limit for the parameter $\Delta$ within the Gaussian function, since the generalization of the quantization principle must not get in conflict with the usual description of quantum field theory at low energies.

[1] A. Smilajagic and E. Spallucci, J. Phys. A 36 (2003) L517 \[arXiv:hep-th/0308193\].
[2] A. Smilajagic, E. Spallucci, J. Phys. A A36 (2003) L467. \[hep-th/0307217\].
[3] A. Smilajagic and E. Spallucci, J. Phys. A 37 (2004) 1 \[Erratum-ibid. A 37 (2004) 7169\] \[arXiv:hep-th/0406174\].
[4] W.-H. Huang, K.-W. Huang, Phys. Lett. B670 (2009) 416-420. \[arXiv:0808.0024 [hep-th]\].
[5] M. Kober and P. Nicolini, Class. Quant. Grav. 27 (2010) 245024 \[arXiv:1005.3293 [hep-th]\].
[6] M. Kober, Class. Quant. Grav. 28 (2011) 225021 \[arXiv:1107.1071 [hep-th]\].
[7] L. Modesto and P. Nicolini, Phys. Rev. D 81 (2010) 104040 \[arXiv:0912.0220 [hep-th]\].
[8] M. Kober, Int. J. Mod. Phys. A 27 (2012) 1250106 \[arXiv:1109.4620 [gr-qc]\].
[9] B. Majumder, Phys. Lett. B 699 (2011) 315 \[arXiv:1104.3488 [gr-qc]\].
[10] B. Majumder, Phys. Lett. B701 (2011) 384-387. \[arXiv:1105.5314 [gr-qc]\].
[11] M. Maggiore, Phys. Lett. B 319 (1993) 83 \[arXiv:hep-th/9309034\].
[12] A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D 52 (1995) 1108 \[arXiv:hep-th/9412167\].
[13] H. Hinrichsen and A. Kempf, J. Math. Phys. 37 (1996) 2121 \[arXiv:hep-th/9510144\].
[14] M. Kober, “Quaternionic Quantization Principle in General Relativity and Supergravity”
[15] H. Yukawa, Phys. Rev. 77 (1950) 219.
[16] H. Yukawa, Phys. Rev. 80 (1950) 1047.
[17] V. A. Alekseev and G. V. Efimov, Commun. Math. Phys. 31 (1973) 1.
[18] R. Marnelius, Phys. Rev. D 10 (1974) 3411.
[19] W. F. Wreszinski, Rept. Math. Phys. 8 (1975) 229.
[20] A. Z. Dubnickova and G. V. Efimov, Czech. J. Phys. B 26 (1976) 1301.
[21] P. Bandyopadhyay and S. Roy, Int. J. Theor. Phys. 15 (1976) 323.
[22] D. Buchholz and J. T. Lopuszanski, Lett. Math. Phys. 3 (1979) 175.
[23] G. V. Efimov and K. Namsrai, Theor. Math. Phys. 50 (1982) 144
[24] D. Buchholz, J. T. Lopuszanski and S. Ralsztyn, Nucl. Phys. B 263 (1986) 155.
[25] P. D. Mannheim, Nuovo Cim. A 93 (1986) 185.
[26] E. C. Marino, Phys. Rev. D 38 (1988) 3194.
[27] A. D. Mironov and A. V. Zabrodin, J. Phys. A 23 (1990) L493.
[28] E. C. Marino and J. E. Stephany, Phys. Rev. D 39 (1989) 3690.
[29] D. Bernard and A. Leclair, Commun. Math. Phys. 142 (1991) 99.
[30] N. J. Cornish, Mod. Phys. Lett. A 7 (1992) 1895.
[31] N. J. Cornish, Int. J. Mod. Phys. A 7 (1992) 6121.
[32] D. G. Barci, L. E. Oxman and M. Rocca, Int. J. Mod. Phys. A 11 (1996) 2111 \[hep-th/9503101\].
[33] M. A. Solovev, J. Math. Phys. 39 (1998) 2635.
[34] G. Esposito and C. Stornaiolo, Int. J. Mod. Phys. A 15 (2000) 449 \[hep-th/9907212\].
[35] R. Amorim and J. Barcelos-Neto, J. Math. Phys. 40 (1999) 585 \[hep-th/9902014\].
[36] M. I. Shirokov, Class. Quant. Grav. 19 (2002) 3821.
