Research on Random Modelling of Computer Multi-sample Redundancy in Nonlinear Mathematics Stochastic Finite Element

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Abstract. The stochastic problems of engineering structures are generally simulated by stochastic finite element method. Based on the non-linear mathematical random method, the paper proposes the concept of multi-sample redundancy based on Bayesian network and the concept of redundancy compression algorithm. A Monte Carlo stochastic finite element method based on Bayesian network redundancy compression algorithm is established. The modelling process of Bayesian network redundancy compression algorithm is explained in detail through two examples. The Bayesian network redundancy compression algorithm model provides a new strategy for multi-sample system reliability analysis, which can conveniently implement system reliability calculation, fault diagnosis, importance analysis and other applications. If the constructed Bayesian network redundancy compression algorithm satisfies that all non-root nodes have two parent nodes, the reliability solving process of the multi-sample system only needs O(Nm^3) computational complexity.

Keywords. Nonlinear mathematics, Bayesian networks, finite element analysis, multi-sample redundancy compression algorithm, modelling and simulation.

1. Introduction

The multi-sample system consists of multiple continuous and non-overlapping time zones. The system configuration, success criteria, and component behaviour are different in different stages. In a multi-sample system, not only do multiple components have correlations in the same stage, but the same component also has correlations between different stages. The existence of this complex correlation makes the reliability analysis of multi-sample systems difficult.

For dynamic multi-sample systems, the Markov chain model is currently used for modelling. The Markov chain model is an effective modelling tool in reliability engineering. Its advantage is that it can accurately describe the dependencies between components in a phase and the dependencies of components across phases [1]. The Markov chain model independently analyses the Markov chain at each stage, and the initial state probability of each stage comes from the analysis results of the previous stage. In addition, the Markov chain at each stage can also be integrated into a single Markov chain consortium composed of state space. The reliability of the multi-sample system is the sum of the probabilities of all working states in the Markov chain. The above two methods are essentially...
processing their Markov chains in stages, and obtain the reliability parameters of the multi-sample system from the Markov chain in the final stage.

In order to make the reliability analysis process truly fall into the description of the system model instead of solving complex mathematical problems, this paper proposes a new multi-sample system reliability analysis model based on nonlinear mathematical Bayesian networks. Bayesian network redundancy the redundancy compression algorithm aims to simplify the modelling process of multi-sample system reliability analysis, reduce the computational complexity of the model, and support multi-sample system analysis under general conditions [2]. The Bayesian network redundancy compression algorithm model describes the multi-sample system as a nonlinear mathematical Bayesian network, which can solve the reliability of the multi-sample system with an efficient calculation method. Using the unique reasoning mechanism of nonlinear mathematical Bayesian networks, the Bayesian network redundancy compression algorithm model is also suitable for more complicated applications such as system fault diagnosis and importance analysis.

2. Bayesian network and its application in reliability analysis

A nonlinear mathematical Bayesian network is a directed acyclic graph, in which nodes represent random variables (in nonlinear mathematical Bayesian networks, usually "nodes" are equivalent to "random variables"), and directed edges represent conditional independence relationship. The root node refers to the node without a parent node, the leaf node refers to the node without child nodes, and the other nodes are called intermediate nodes. For a nonlinear mathematical Bayesian network composed of discrete variable nodes, the root node has a prior probability table (PPT), and the values in the table indicate the probability of the root node in different states; non-root nodes have a conditional probability table (CPT), The values in the table indicate the probability that the node is in a different state given the combination of values of the parent node.

If nodes and directed edges are used to represent the components of the system and the relationship between them, then the nonlinear mathematical Bayesian network describes the conditional independence relationship between variables in the system, that is, the node is given its parent node. Independent of its non-descendant node conditions [3]. The nonlinear mathematical Bayesian network reveals the complete joint probability distribution (JPD) of all variables, so that all probability-related problems can be inferred through the marginal solution mechanism. The existence of the conditional independence relationship between variables reduces the parameters required to determine JPD, thereby simplifying the probability model of the variables in the system. The JPD of all variables \(X_1, X_2, \ldots, X_n\) in the system can be expressed as

\[
P[X_1, X_2, \ldots, X_n] = \prod_{i=1}^{n} P[X_i | \text{pa}(X_i)]
\]

Among them, \(\text{pa}(X_i)\) represents the parent node of node \(X_i\).

In recent years, the application of nonlinear mathematical Bayesian network models in the field of reliability analysis has gradually attracted attention. The research results show that, no matter in the modelling ability or the analysis ability, the nonlinear mathematical Bayesian network model has significant advantages over the fault tree, reliability block diagram and other models, and has less complexity. In the DT nonlinear mathematical Bayesian network, the root node represents the system components, the intermediate nodes represent the relationship between a series of components, and the leaf nodes represent the entire system. DT nonlinear mathematical Bayesian network discretizes the system task time into \(m\) time periods [4]. If the system task time is \(T\), the width of each time period is \(\Delta=T/m\). Correspondingly, each node has \(m+1\) state, and each state represents the behaviour of the node in the corresponding time period.

There are many nodes in DT nonlinear mathematics Bayesian networks of complex systems, and the relationship between nodes will become extremely close due to the complex behavior of the system. However, compared with the modeling method based on the state space, the DT nonlinear mathematical Bayesian network model maps the transition of the system state to the conditional probability table.
attached to the node. This use of multiple local behaviors to describe the transition of the global state will greatly reduce the complexity of the model. In addition, the reasoning method of DT nonlinear mathematical Bayesian network is intuitive and concise. It is easy to realize rapid analysis and processing by computer, and avoids the problem of solving complex differential equations.

3. Reliability analysis model of Bayesian network redundancy compression algorithm

3.1. The generation method of Bayesian network redundancy compression algorithm
Each stage of a multi-sample system can be expressed by a DT nonlinear mathematical Bayesian network. This article refers to this DT nonlinear mathematical Bayesian network used to express the correlation of components within the stage as a phase-nonlinear mathematical Bayesian network. Similar to FT, the phase-nonlinear mathematical Bayesian network is a representation method of the system behavior in the phases of a multi-sample system, and can be obtained by FT conversion in each phase. For example, the root node in a nonlinear mathematical Bayesian network can represent the basic events in FT, the intermediate nodes represent various static/dynamic gates in FT and the basic events that have a dependency relationship with the root node, and the leaf nodes represent FT The top event.

The nonlinear mathematical Bayesian network generated according to the above process is the Bayesian network redundancy compression algorithm. Figure 1 shows the process of constructing a Bayesian network redundancy compression algorithm for a 2-stage multi-sample system, where the second stage is represented by a dynamic fault tree (DFT). In the first stage, components A and B work in parallel, and the system will fail only when A and B fail at the same time. In the second stage, component B serves as A's cold reserve; after A fails, B starts to work; only when A and B both fail, the system will fail. Phase-nonlinear mathematical Bayesian network corresponding to the two stages.

A Bayesian network redundancy compression algorithm generated by two phase-nonlinear mathematical Bayesian networks, where T1 (T2) represents the top event of phase 1 (2), and S represents a multi-sample system.

![Figure 1. The generation process of Bayesian network redundancy compression algorithm](image)

3.2. Reliability analysis method of Bayesian network redundancy compression algorithm
The time of each phase is divided into m time periods, so that the entire task time is divided into mn time periods, where n is the number of phases. In the Bayesian network redundancy compression algorithm, the component nodes in the first stage have m+1 state. The first m states indicate that the component fails in the m-th time period, and the last state (marked as m+1) indicates that the component does not fail in the first stage [5]. Corresponding to component nodes, the other intermediate nodes and leaf nodes in the first stage also have m+1 state.

In the remaining stages, each component has m+2 states. The first state indicates that the component has failed in the previous stage and is marked with 0. The next m states indicate that the component fails in
the mth time period of this stage, and is identified by \((j-1)m+i\), where \(i\) is the time period number and \(j\) is the stage number \((0 < i \leq m, 1 < j \leq n)\). The last state indicates that the component has not failed in this stage, and is identified by \(mj+1\). Corresponding to component nodes, other intermediate nodes and leaf nodes in these stages also have \(m+2\) states. Assuming that the life of all components obeys an exponential distribution, let the failure rate of A and B \(\lambda A = \lambda B = 0.02 h^{-1}\), the number of time periods in the stage \(m=2\), and the time period length \(\Delta = 1 h\), then the PPT of \(A_1\) or \(B_1\) can be According to the following formula

\[
\Pr\{A_i = k\} = F(k \cdot \Delta) = 1 - e^{-\lambda A \Delta} \\
\Pr\{A_i = 3\} = 1 - \Pr\{A_i = 1\} - \Pr\{A_i = 2\}
\]

Among them, \(k < 3\). The complete PPT is shown in Table 1.

| Table 1. PPT of \(A_1(B_1)\) |
|-----------------------------|
| \(A_1(B_1)\) | 1 | 2 | 3 |
|-----------------------------|
| 0.01980                    | 0.01941 | 0.96079 |

If the component fails in the first stage, it will not continue to work in the subsequent stages. Therefore, if the state of \(A_1\) is 1 or 2, the probability that \(A_2\) is in state 0 is 1. On the premise that component A does not fail in stage \(j-1\), the conditional probability that A is in state \(i\) in stage \(j\) is calculated by the following formula:

\[
P_{(A_{j,i})} = \frac{F(((j-1)m+i) \cdot \Delta) - F(((j-1)m+1+i) \cdot \Delta)}{1 - F((j-1)m \cdot \Delta)}
\]

If the life of the components obeys the exponential distribution, then the formula (3) can be sorted into:

\[
P_{(A_{j,i})} = \frac{e^{-\lambda_A ((j-1)m+i) \Delta} - e^{-\lambda_A ((j-1)m+i+1) \Delta}}{1 - e^{-\lambda_A (j-1)m \Delta}} = F(i \cdot \Delta) - F((i-1) \cdot \Delta)
\]

The CPT of \(A_2\) is shown in Table 2:

| Table 2. CPT of \(A_2\) |
|--------------------------|
| \(A_1\) | 1 | 2 | 3 |
|--------------------------|
| 0                        | 1 | 1 | 0 |
| 3                        | 0 | 0 | 0.0198 |
| 4                        | 0 | 0 | 0.01941 |
| 5                        | 0 | 0 | 0.96079 |

Consider the computational complexity expression \(\max(O(m^p) , O(Nm^{p+r}))\). In \(O(m^p)\), \(n\) actually represents the number of parent nodes of the leaf node of the Bayesian network redundancy compression algorithm, and the size of \(O(Nm^{p+r})\) also mainly depends on the value of \(p\), so we can get the conclusion is as follows: The number of parent nodes will greatly affect the computational efficiency of the Bayesian network redundancy compression algorithm model. As shown in Figure 2, when the 4-input AND gate is converted into a nonlinear mathematical Bayesian network, the final conversion result should be a nonlinear mathematical Bayesian network composed of cascaded nodes as shown.
Figure 2. A simplified example of a nonlinear mathematical Bayesian network

In most cases, the system's Bayesian network redundancy compression algorithm can be constructed into a nonlinear mathematical Bayesian network topology that meets the above requirements. At this point, in order to obtain the reliability of the multi-sample system, the required calculation amount will become $\max \left(O\left(m^3\right), O\left(Nm^{3/2}\right)\right) = O\left(Nm^3\right)$. Therefore, the computational complexity of the Bayesian network redundancy compression algorithm model does not increase exponentially with the system scale. Compared with the Markov chain model, the latter state space is $2^q$, and $q$ is the number of all variables in the system. The comparison of the two reliability analysis models is shown in Table 3.

Table 3. Comparison of Bayesian network redundancy compression algorithm model and Markov chain model

|                           | Bayesian network redundancy compression algorithm model | Markov chain model |
|---------------------------|-------------------------------------------------------|-------------------|
| Modelling process         | Simple and intuitive                                  | Complex and error-prone |
| the complexity             | In most cases $O\left(Nm^3\right)$                    | $O\left(2^q\right)$ |
| Repairable system          | not support                                           | stand by          |
| Solving algorithm          | Simple DT nonlinear mathematical Bayesian network model | Complex differential equation |
| Accuracy                   | Approximate solution                                  | Analytical solution or numerical solution    |

It can be seen from Table 3 that, in addition to not supporting the reliability analysis of repairable multi-sample systems, the Bayesian network redundancy compression algorithm model is better than the Markov chain model in terms of modelling process, complexity, and solution algorithm. Great advantage. In addition, the accuracy of the reliability analysis results obtained by the Bayesian network redundancy compression algorithm model is determined by the parameter $m$. The value of $m$ is a compromise between the calculation accuracy and the time and space consumed [6]. This compromise
provides us with a Ashgate flexible solution. Solving the Markov chain model usually requires solving complex differential equations. Although various simplification methods can be used to speed up the solution, the analysis of the Markov chain model of a complex system requires a lot of time and space costs.

4. Finite element simulation analysis

Figure 3 shows that there are 3 sub-modules in the multi-sample system, namely trigger gate module T, CPU module and CPUT module. Among them, CPUT represents the functional correlation between the CPU unit and the trigger gate. From this point of view, CPUT is essentially an OR gate between T and CPU. In fact, in BN, if you don't need T to trigger the failed main P and backup B at the same time, you can equivalently trigger the CPUWSP gate as a whole.

![Figure 3. CPU module and DFT equivalent BN](image)

Using the Runge-Kutta algorithm of the Matlab tool to solve the equation of state of the system, the numerical solution of the reliability of the system at the end of the third stage can be obtained as \( R_{\text{Markov}} = 0.950861 \), and the calculation process takes 0.047s. For different values of \( m \), the Bayesian network redundancy compression algorithm calculates the reliability \( R \) of AES at the end of the third stage, and the relative error with \( R_{\text{Markov}} \) is shown in Table 4. It can be seen from Table 4 that as \( m \) increases, the value of \( R \) gradually approaches the numerical solution. Even when \( m=5 \), the relative error with \( R \) remains within 1.5%. Therefore, in actual application, in order to reduce the computational complexity and obtain higher computational accuracy, \( m=7 \) can be set.

| \( m \) | 3          | 5          | 7          | 8          | 10         |
|--------|------------|------------|------------|------------|------------|
| Reliability R | 0.962690   | 0.961721   | 0.955821   | 0.952634   | 0.951987   |
| Elapsed time (s) | 0.017s     | 0.052s     | 0.732s     | 1.781      | 9.394      |
| Relative error \( \left( \frac{R - R_{\text{Markov}}}{R_{\text{Markov}}} \right) \times 100\% \) | 1.24%     | 1.14%     | 0.52%     | 0.19%     | 0.12%     |

Table 4. Reliability calculated using Bayesian network redundancy compression algorithm model under different \( m \)
5. Conclusion
This paper proposes a multi-sample system reliability analysis model based on nonlinear Bayesian network. Bayesian network redundancy compression algorithm aims to provide a complete and efficient method for multi-sample system reliability analysis. The Bayesian network redundancy compression algorithm model uses the Bayesian network to describe the multi-sample system, so that the Bayesian network can be used as a powerful modelling and analysis tool to complete the reliability analysis of the multi-sample system.

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