Characterizing Power Support from Distribution Networks via Flexibility Area Segmentation

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Abstract—This paper proposes a framework for characterizing and managing active distribution networks (ADNs) via flexibility area segmentation. Specifically, flexible active and reactive power support at the interface with transmission networks is analyzed. The framework is fundamentally different from existing studies that either characterize ADN flexibility considering the full availability of flexible units or determine a risk-averse share of the flexibility area using robust optimization models. Such methods cannot provide sufficient information to distribution system operators (DSOs) on the structure of the flexibility area and reliability of its components. The proposed segmentation approach relies on combinatorial optimization models and classifies the ADN flexibility area’s segments by the number of flexible units activated and probabilities associated with units activation. The resulting classification enables DSOs to analyze and manage the provision of ADNs flexibility services. The applicability and scalability of the proposed framework is illustrated based on the 33-bus and 124-bus radial distribution network case studies.

Index Terms—Active distribution network, Combinatorial optimization, Distributed energy resources, Flexibility services, Probabilistic analysis, TSO-DSO coordination.

I. INTRODUCTION

The emergence of intelligent controls and distributed energy resources (DERs) brings new opportunities to enhance the efficiency and flexibility of distribution networks operation [1]. In this context, the increased controllability of the now active distribution networks (ADNs) allows managing power injections at specific points (e.g., at a substation or a feeder), making ADNs natural providers of flexibility services such as active and reactive power support and voltage control [2]. This has motivated research on the flexibility potential across entire distribution networks, which would allow distribution system operators (DSOs) to trade active and reactive power with transmission system operators (TSOs). Such arrangements enable DSOs to use part of their flexibility to support the distribution network, whereas trading the rest at the TSO/DSO interface to support the transmission network (e.g., in voltage control and congestion management) and the overall power system operation (e.g., via ancillary services).

Multiple coordination schemes have recently been proposed to enable the provision of flexibility services between DSOs and TSOs [3]–[8]. Flexibility services provided by ADNs can be evaluated by estimating a flexibility area, defined as feasible combinations of active and reactive power exchanges at the TSO/DSO interface. In early studies on distribution network flexibility area estimation, such as [9]–[11], Monte Carlo simulations were used to map the availability and cost of flexible power at the TSO/DSO interface (also called the point of common coupling). This approach requires a large number of randomly generated operating points, some of which can be infeasible and, therefore, get discarded. The resulting collection of the operating points is a P-Q capability chart comprising ADN flexibility in terms of feasible power exchanges at the TSO/DSO interface. Despite their simplicity, these methods suffer from inherent limitations related to significant computational burden and approximation errors. To overcome these limitations, more advanced flexibility area assessment techniques were developed in [12]–[15]. These techniques aim to approximate the boundary of the feasibility area by considering DERs limits and network constraints. Thus, the ADN flexibility can be estimated through a lower number of targeted simulations. Additionally, these algorithms allow explicit control over the accuracy of the approximation, which can be set as the desired level of the area’s granularity.

The flexibility areas produced with the approaches discussed above rely on aggregation of the independent P-Q ranges of each flexible unit, which are generally assumed to be fully available for flexibility services provision. This might not be reasonable in practical applications, as the firmness of each DER to deliver the full requested P-Q output is uncertain. For example, some units may not be available for flexibility services provision or may fail to provide estimated flexible power support. This problem is recognized in the existing literature. However, few attempts have been made to characterize the structure of ADN flexibility and incorporate the contributions and uncertainties associated with flexible units activation. To capture the uncertainties related to DERs, Stanković and Soder [16] proposed probabilistic reactive power capability charts that represent reactive power support limits of distribution systems as families of random variables with their associ-

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ated probabilistic density functions. Thus, this approach is able to identify different levels of flexibility with associated uncertainties. However, such levels conceal the structure of ADN flexibility and the contributions of each flexible unit. Other studies, such as [17], [18], propose characterizing the reserve provision capability area of ADN by solving a set of scenario-based robust optimization problems. However, such approaches require large data sets of scenarios to incorporate uncertainties.

The lack of the explicit components of ADN flexibility areas limits the capability to request services with desired levels of firmness (e.g., TSO might request the maximum flexible power output that can be provided with a 95% confidence level). Understanding the contribution of each unit to the flexibility areas is also relevant when delivering services that require specific metrics, such as dynamic (ramp) flexibility, duration, cost, etc. [19]. Furthermore, it is crucial to perform disaggregation of ADN flexibility to obtain dispatching commands for flexible units during real-time operation [20].

To address the mentioned research gaps, this paper proposes a new approach to characterize the flexibility services provided by ADNs based on flexibility area segmentation. For this purpose, a combinatorial optimization model is formulated to classify the segments of the flexibility area by the number of flexible units activated and probabilities associated with units activation. This approach explicitly models the contributions of each flexible unit and requires less information to build portfolios of ADN flexibility, which makes it suitable for practical applications. The resulting classification provides detailed information for DSOs to manage uncertainties associated with the use of flexibility and trade ancillary services with the desired level of firmness. Moreover, the flexibility area segmentation captures the effects of network reconfiguration, dynamic changes in DERs parameters, and probability estimation of flexible units activation.

The proposed framework is tested on the IEEE 33-bus radial system and a real 124-bus radial distribution network from the UK. The results demonstrate that flexible units’ contributions and probabilities across segments of the ADN flexibility area vary drastically, especially subject to different DERs locations and their electrical distance to the TSO/DSO interface. The numerical performance analysis shows that the flexibility area segmentation approach is scalable and can be applied to real distribution networks, considering the limitations on the number of the estimated segments and the approximation accuracy of each segment.

II. MODELLING FRAMEWORK: NETWORK FLEXIBILITY AS A COMBINATORIAL OPTIMIZATION PROBLEM

Optimal power flow (OPF) models lie at the core of the ADNs flexibility analysis. Various OPF formulations have been explored in the literature to estimate flexibility areas of distribution networks. This section introduces a novel modelling framework that describes the available flexibility as a combinatorial optimization problem. As further discussed in Section III, this framework can be used to classify flexibility areas by the number of flexible units activated and probabilities associated with units activation.

To identify feasible operating points of ADN and estimate its flexibility area, in this work, the modified DistFlow OPF model (1a)–(1j) is used. The original DistFlow OPF formulation was first proposed by Baran and Wu [21], [22] for optimizing radial distribution networks. The model defines active and reactive power flows through each branch, \( p_{ij}, q_{ij} \), as a function of generator outputs, \( p_i^G, q_i^G \), nodal voltages, \( v_i \), and branch currents, \( i_{ij} \). Note that the formulation is simplified by substituting the products of the nodal voltage variables by \( v_i^2 \) and the squared branch currents by \( i_{ij}^2 \).

In order to estimate a flexibility area, the objective function \( \min \pi_p \sum_{(i,j)\in\mathcal{L}} p_{ij} + \pi_q \sum_{(i,j)\in\mathcal{L}} q_{ij} \) should be actively managed to minimize or maximize the distribution network’s power consumption at a selected reference node, \( i^{ref} \). For this purpose, the coefficients \( \pi_p, \pi_q \) are introduced as a means to control the optimization direction. As further discussed in Section III, the optimization direction iteratively changes depending on the stage of a flexibility estimation algorithm. The TSO/DSO interface is selected in this work as the reference node to estimate the aggregated

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\begin{align*}
\text{MODEL 1 Modified DistFlow OPF [MIQCP]} \\
\text{Variables:} \\
p^G_i, q^G_i & \quad \forall i \in \mathcal{N} \\
p_{ij}, q_{ij} & \quad \forall (i,j) \in \mathcal{L} \\
v_i \quad (v_i^2 = w_i) & \quad \forall i \in \mathcal{N} \\
i_{ij} \quad (i_{ij}^2 = l_{ij}) & \quad \forall (i,j) \in \mathcal{L} \\
p^F_i, q^F_i & \quad \forall i \in \mathcal{N} \\
x^F_i \in \{0, 1\} & \quad \forall i \in \mathcal{N} \\
\text{Objective:} \\
& \min \pi_p \sum_{(i,j)\in\mathcal{L}} p_{ij} + \pi_q \sum_{(i,j)\in\mathcal{L}} q_{ij} \\
& \quad \pi_p, \pi_q \in \{-1, 0, 1\} \\
\text{Constraints:} \\
& p_i^G, q_i^G \leq p_i^{G,max}, q_i^{G,max} \quad \forall i \in \mathcal{N} \quad (1b) \\
& p^F_i, q^F_i \in \mathcal{S}_i^F \quad \forall i \in \mathcal{N} \quad (1c) \\
& p_{ij} = p_{ij}^D - p_{ij}^G - x^F_{ij} p^F_i + r_{ij} l_{ij} + \sum_{(j,k)\in\mathcal{L}} p_{jk} \quad \forall (i,j) \in \mathcal{L} \quad (1d) \\
& q_{ij} = q_{ij}^D - q_{ij}^G - x^F_{ij} q^F_i + x_{ij} l_{ij} + \sum_{(j,k)\in\mathcal{L}} q_{jk} \quad \forall (i,j) \in \mathcal{L} \quad (1e) \\
& v_i = w_i + |Z_{ij}|^2 l_{ij} - 2(r_{ij} p_{ij} + x_{ij} q_{ij}) \quad \forall (i,j) \in \mathcal{L} \quad (1f) \\
& p^2_{ij} + q^2_{ij} = l_{ij} w_i \quad \forall (i,j) \in \mathcal{L} \quad (1g) \\
& p_{ij}^2 + q_{ij}^2 \leq (S_{ij}^{max})^2 \quad \forall (i,j) \in \mathcal{L} \quad (1h) \\
& (l_{ij}^{min})^2 \leq \bar{w}_i \leq (l_{ij}^{max})^2 \quad \forall i \in \mathcal{N} \quad (1i)
\end{align*}
\]
flexibility of the entire distribution network. Nevertheless, the methodology is general and can be used to map flexibility areas in other network locations. Constraints \((1b)\) and \((1c)\) limit active and reactive power outputs of generators. Constraint \((1d)\) defines the flexible capacity at each node to a given P-Q capability set \(S_i^F\), such that each element of \(S_i^F\), \(s_i^F = [p_i^F, q_i^F]\), is a feasible flexible power output available at node \(i\). Note that \(S_i^F\) can be a compact set of any shape. However, in this work, the active and reactive power output of flexible units is bounded in a similar fashion to \((1b)\), \((1c)\). Thus, the P-Q capability of each flexible unit is specified by a box boundary. Constraints \((1e)\) and \((1f)\) define active and reactive power flows between nodes \(i\) and \(j\). For each node \(j\), net nodal active and reactive power withdrawals are given by the differences between power demands, \(p_j^D\), \(q_j^D\), generator outputs, \(p_j^G\), \(q_j^G\), and flexible unit outputs, \(p_j^F\), \(q_j^F\). Note that flexible units outputs are coupled with the corresponding binary decision variables, \(x_j^F\). Such formulation requires a unit to be activated in order to make its flexible power available. The use of these binary variables of activation decisions will be discussed in greater detail in the next sections to explore the structure of ADN flexibility areas. Active and reactive power losses in each line are represented by \(r_{ij}l_{ij}\) and \(x_{ij}l_{ij}\). Finally, the branch flow equations \((1e)\), \((1f)\) account for the power flows via lines connected to node \(j\), \(p_{jk}\) and \(q_{jk}\). The voltage relation between nodes \(i\) and \(j\) is given by \((1g)\), where \(Z_{ij} = r_{ij} + jx_{ij}\) is branch impedance. Constraint \((1h)\) defines the relation between power flows and currents. The apparent power of each line is limited in \((1i)\), and the nodal voltages are constrained in \((1j)\).

Note that the DistFlow OPF model is equivalent to the exact AC power flow equations: it accounts for active and reactive power flows and power losses. However, this equivalence holds only for distribution networks with radial topology. Other OPF models can be used to analyze ADN flexibility. For example, in [14], Capitanescu formulated an AC OPF model for ADN flexibility estimation using complex voltages in rectangular coordinates. In [12], Silva et al. exploited the AC OPF model with bus voltage variables in polar form to analyze the ADN flexibility area at the TSO/DSO interface. In this work, the focus is on distribution networks that are predominantly radial (especially at low voltage levels), and, therefore, the DistFlow formulation is opted for the flexibility modelling. As demonstrated by recent studies [15], [23], the DistFlow OPF model is accurate and computationally efficient for flexibility estimation in radial distribution networks. The power flow and voltage constraints \((1b)\)–\((1j)\) make the DistFlow model a quadratically constrained programming (QCP) problem. The modified formulation \((1a)\)–\((1i)\) contains the binary variables of flexible units activation decisions, \(x_j^F\), which makes it a mixed-integer quadratically constrained programming (MIQCP) problem. As demonstrated in the next sections, such problems can be solved accurately by modern commercial MIQCP solvers.

1An example of the flexible units classification based on the P-Q capability set boundaries can be found in [13].

### III. BUILDING FLEXIBILITY AREAS AND SEGMENTATIONS

In this section, existing methods for estimating aggregated flexibility areas of ADNs are discussed. Then, the segmentation approaches are introduced to classify flexibility areas by the number of flexible units activated and probabilities associated with units activation. These approaches will be implemented in Sections IV and V to characterize power support from distribution networks.

#### A. Aggregated Flexibility Area of ADN

An aggregated ADN flexibility area describing all feasible active and reactive power exchanges between TSO and DSO can be found by solving a sequence of OPF problems \((1a)\)–\((1j)\). Each solution provides a feasible operating point at the boundary of the flexibility area. Besides the computationally expensive random sampling algorithms (Monte Carlo simulations, such as [9]–[11]), there exist several methods for estimating flexibility areas with the desired level of granularity:

- **Radial reconstruction.** This method requires solving a sequence of OPF problems for different fixed power factors (relative to the initial operating point) at the reference node. I.e., each OPF solution provides the flexibility boundary points lying on a straight line with a given slope in the P-Q space. By rotating this line \(k\) times, it is possible to approximate the flexibility area by \(2k\) boundary points. An example of the radial reconstruction implementation can be found in [13].

- **\(\varepsilon\)-constraint method.** The first step of this method requires estimating the extreme points of the flexible area, for example, maximum and minimum reactive power exchanges at the TSO/DSO interface. Then, the identified range is divided into \(k\) intervals. For each interval, the reactive power exchange is limited by a fixed value corresponding to the interval with a given accuracy \(\varepsilon\). Thus, by dividing the flexibility area into \(k\) intervals, it is possible to approximate it by \(2k\) boundary points. Increasing the number of intervals and the accuracy of each interval estimation leads to more accurate approximations of the flexibility area. An example of the \(\varepsilon\)-constraint method implementation can be found in [14].

- **Hybrid methods.** Combinations of the radial reconstruction and the \(\varepsilon\)-constraint methods can provide more accurate approximations of the flexibility area. For example, in [15], Lopez et al. proposed the QuickFlex method, which iteratively improves the approximation by searching new boundary points that maximize the coverage of the flexibility area.

In this work, the \(\varepsilon\)-constraint method is selected to approximate ADN flexibility areas. This method is programmed by introducing inequalities at each iteration that limit power exchanges at the TSO/DSO interface. It enables straightforward control over the number of iterations and the accuracy of the approximation. Moreover, it guarantees that the extreme points of the flexibility area are identified.
B. Flexibility Area Segmentation by the Number of Flexible Units Activated

The above flexibility estimation methods have been used to analyze aggregated flexibility of ADNs. That is, all flexible units are considered fully available to participate in the flexibility services. However, in this work, the structure of the flexibility area and individual contributions by different flexible units are explicitly modeled. For this reason, the number of flexible units activated across different segments of the flexibility areas is limited by introducing the following constraint.

\[ \sum_{i \in N} x_i^f \leq X^f \quad \forall f \subseteq \mathcal{F} \]  

Thus, the sum of the binary variables representing flexible units activation decisions is restricted for each flexibility segment \( f \subseteq \mathcal{F} \), where \( \mathcal{F} \) is the total aggregated flexibility of the network. In this manner, it becomes possible to produce a mapping of the flexibility area based on the number of flexible units activated. This mapping has the following justification. While planning or providing flexibility, the DSO needs to identify flexible units available for the service. If all units are available and equally reliable, the DSO can select the lowest number of units that ensure a certain service request (e.g., from the TSO). The P-Q capability of such units forms a single segment of the flexibility area. The union of all segments comprises the resulting segmentation by the number of flexible units activated. This segmentation provides additional information to the DSO, such as the critical units and parameters of the network, control effort for flexibility provision, and technical constraints of units coordination.

C. Probabilistic Segmentation of Flexibility Area

Note that the above flexibility area segmentation approach is based on the assumption that all units are equally reliable and available for flexibility services provision. However, in practice, the firmness of different flexible units varies, e.g., due to the uncertainties of DERs. This information should be considered by DSOs when selecting the flexible units to activate when delivering flexibility services with the desired (or contracted) firmness levels.

The probabilities associated with the availability of each flexible unit can be described by a value \( R_i^f \). Then, the probability of a flexibility segment \( f \) becomes the product of the flexible units’ probabilities, \( R^f = \prod_{i \in f} R_i^f \). It follows that it is far less reliable to provide services at the boundary of the aggregated flexibility area since all flexible units have to be activated, \( R^\text{bd}(f) = \prod_{i \in f} R_i^f \). Having classified and ranked segments by their probabilities, DSO can optimize the management of its flexibility services. For example, to ensure a certain power exchange at the TSO/DSO interface, DSO can activate the minimal amount of the most reliable units. The P-Q capability of the selected units forms a single segment of the flexibility area. The union of all segments comprises the probabilistic segmentation, which could be used by DSO for planning and providing flexibility.

It is important to note that analyzing all segments of the flexibility area is a complex combinatorial problem. The number of possible flexible units combinations grows exponentially with the number of units available in the network. For example, a network with 10 flexible units has 1023 possible combinations of their P-Q capabilities. The resulting segmentation can have even a greater number of segments since flexibility areas (given by different combinations of units) can be partially overlapping. Therefore, to fully characterize the structure of the aggregated flexibility area, it is required to rank its segments by (i) resulting probabilities and (ii) operating limits in the P-Q space. This work will not attempt to solve such a problem, and, instead, presents a conceptual study to illustrate the need for analyzing the structure of ADN flexibility and the advantages of network management based on the flexibility area segmentation.

IV. Case Study: 33-bus Radial Distribution Network with 5 Flexible Units

This section illustrates the proposed methodology on an established case study, the IEEE 33-bus system, which represents a 12.66 kV radial distribution network. This case has been used in studies such as [11], [14], [17], [19], [23] to analyze the aggregated flexibility area of the distribution network. In this work, the 33-bus network is used to clearly illustrate the idea and principles of the flexibility area segmentation. A more complex distribution network will be considered in the next section.

The force-directed graph layout algorithm ForceAtlas2 [24] is exploited to visualize the topology and parameters of the network. The single line diagram in Fig. [1] interprets the case study in a compact, informative, and intuitive manner:

- First, the size of each node is set proportional to its power demand. This allows fast identification of the most loaded lines and parts of the network.

- Second, the “edge weight” parameter of each branch used in the ForceAtlas2 algorithm is defined as the line’s admittance. This enables visualizing structural proximities of the network, i.e., nodes connected via a low impedance line are located close to each other in the graph layout.

- Finally, the nodes are colored according to their voltage levels. This provides information on the voltage profile of the network and potential flexibility limitations related to voltage control.

Five identical flexible units (denoted as FU) are placed in the network. The P-Q capability of each unit \( i \) is given by \( p_i^F \in [-200, 200] \) kW and \( q_i^F \in [-200, 200] \) kVAr, where...
Thus, the units can produce or consume power within the specified symmetric box boundary. However, due to the locational effects such as power losses and voltage drops, the flexible units make different contributions to the aggregated flexibility of the network. The MIQCP model (1a)-(1j) is used to characterize these contributions and build the flexibility area segmentation by the number of flexible units activated.

The resulting segmentation is presented in Fig. 2. The light grey central segments indicate more reliable operating points where few flexible units have to be activated. Units 18 and 33 are activated first since they cause the most significant power losses changes in the network. As all of the units have identical P-Q capabilities in this example, the segments expand similarly in all directions according to the Minkowski addition [19]. However, simultaneous activation of several units can be restricted by network technical limits such as current limits and nodal voltage constraints. For example, activation of 4 and 5 units could lead to infeasible operating points (due to the voltage violations caused by increased power consumption). Such points get discarded in the plot, reducing the displayed flexibility areas.

The presented characterization of the flexibility area can be used by DSOs to manage their provision of ancillary services. The identified segments reveal the control effort needed to achieve a certain power exchange between TSO and DSO, i.e., the number of flexible units that have to be activated. Moreover, the proposed framework can be used to find the most critical units and parameters influencing the network’s flexibility. In the next section, the flexibility area segmentation is further elaborated to incorporate probabilities associated with flexible units activation.

V. CASE STUDY: 124-BUS RADIAL DISTRIBUTION NETWORK WITH 10 FLEXIBLE UNITS

This section presents an analysis for a real 124-bus 6.6 kV distribution network from the UK [23], [24]. As in the previous case study, the network is visualized using the ForceAtlas2 algorithm [24]. Fig. 3 presents the topology of the network, power demands, and nodal voltages for the initial operating point.

With the aim of increasing the complexity of the study, this time, ten flexible units are placed in the network, each with different P-Q capabilities and firmness levels. The flexible unit with the highest capacity is located at node 20. Its output power is given by $p^{F}_{20} \in [-100, 100]$ kW and $q^{F}_{20} \in [-100, 100]$ kVar. Thus, this unit cannot consume power. Its P-Q capability set is not symmetric with respect to the reference operating point.

The flexible unit with the lowest capacity is placed at node 60, with $p^{F}_{60} \in [-100, 100]$ kW and $q^{F}_{60} \in [-100, 100]$ kVar. Other flexible units in the network have identical P-Q capabilities given by $p^{F}_i \in [-200, 200]$ kW and $q^{F}_i \in [-200, 200]$ kVar, where $i \in \{14, 79, 83, 86, 99, 100, 106, 107\}$. As in the previous case study, the MIQCP model (1a)-(1j) is used to characterize the flexible units’ contributions and build the flexibility area segmentation by the number of units activated.
The segmentation presented in Fig. 4 identifies reliable operating points that can be reached by activating just a few of the units. Other operating points, e.g., at the perimeter of the aggregated flexibility area, are less reliable and require all units to be fully available for flexibility provision. Since the flexible units considered have different P-Q capabilities, the shapes of the segments are determined by the combinations of the capability sets. Thus, the segments do not expand equally in different directions. For example, the central segment is formed by the contributions of unit 20 (the most powerful unit that can decrease the network’s consumption significantly) and units 106 and 107 (the most distant units that can increase the network’s consumption and power losses). As more units get activated, the voltage regulation limits set by $v_{\min}^i = 0.94$ p.u. and $v_{\max}^i = 1.06$ p.u. restrict some of the flexible units. This effect can be observed in the right upper side of the plot, where diminishing contributions of additionally activated units occur.

Note that the network has a normally open point connecting adjusting feeders 7-1 and 7-6. The network configuration presented in Fig. 3 corresponds to the power supply via feeder 7-1. However, the normally open point can be utilized in the event of a network fault or planned outage, as well as for power losses minimization and voltage control. As illustrated in Fig. 4 (b), network reconfiguration also has a significant impact on the flexibility of the distribution network. For instance, the network’s ability to increase its power consumption changes under different network configurations. With the power supply via feeder 7-6, it requires more flexible units to be activated for the consumption increase compared to the supply via feeder 7-1. Therefore, DSOs can use the proposed segmentation framework to analyze and adjust the available flexibility.

In the previous simulations, all flexible units were considered equally reliable and available for flexibility services provision. Under this assumption, the flexibility area segmentation can be performed based on the number of units activated. However, there can be different probabilities associated with the availability of flexible units. These probabilities stem from uncertain forecasts for DERs generation, lack of data from electric vehicle aggregators, availability of consumers participating in demand response programs, etc. Considering such factors, DSOs can measure the level of firmness of each flexible unit and manage the network’s flexibility by activating...
units with high probabilities of providing flexible power. To illustrate this, different probabilities \( R_i \) were assigned to the flexible units located in the network. The unit at node 20 is considered the most reliable one with probability \( R_{20} = 0.99 \). Other units have probabilities between 0.945 and 0.985. It follows that each solitary unit is reliable enough to provide flexible power. However, the aggregated flexibility of the network is formed by different combinations of flexible units. The resulting probability of a segment can be significantly lower than units’ individual probabilities. In fact, the probability of simultaneous activation of all ten flexible units is 0.718. All possible combinations of flexible units’ P-Q capabilities can be computed and arranged by their probabilities in decreasing order \( \mathcal{R}^{f_1} \geq \mathcal{R}^{f_2} \geq \ldots \geq \mathcal{R}^{f_{10}} \). Then, the probabilistic segmentation of the flexibility area can be built iteratively, starting with the central segments with the highest probability. Additional segments are added to the segmentation only if they cover new operating points in the P-Q space. That is, if two segments overlap such that \( f_i \subseteq f_j \) and \( \mathcal{R}^{f_i} \leq \mathcal{R}^{f_j} \), the segment with the lower probability gets discarded.

The probabilistic segmentation for the networks’ flexibility area is visualized in Fig. 5(a). The central segments indicate reliable operating points that can be reached by activating a few flexible units with the highest probabilities. The probability of activating more units decreases as operating points become closer to the boundary of the aggregated flexibility area. DSO can use probabilistic segmentation to analyze and manage the flexibility available in the network. For example, DSO might plan to provide flexibility services whose estimated probability exceeds 0.9. Thus, only a fraction of the segments can be considered for network operation. These segments are depicted in Fig. 5 by a dashed outline. The area with the probability exceeding 0.9 contains much fewer operating points than the entire flexibility area. It follows that the aggregated flexibility can be significantly overestimated without considering probabilistic constraints.

The estimated probabilities of flexible units availability can change throughout the network operation. Such changes can have a major impact on the aggregated flexibility area and its segmentation. To illustrate this, it is assumed that the probability associated with unit 20, \( R_{20} \), drops from 0.99 to 0.92. I.e., the most reliable unit is turned into the least reliable one. Then, the probabilistic analysis of flexible units combinations is repeated. The modified probabilistic segmentation is displayed in Fig. 5(b). Even though the aggregated flexibility area stays the same, its components change drastically. Specifically, the probabilities of the flexibility area segments decrease. The union of the reliable segments with probabilities exceeding 0.9 (the dashed outline) shrinks compared to the simulation presented in Fig. 5(a). Thus, the proposed flexibility are segmentation framework captures the effects of dynamic changes in probabilities estimation and network parameters. DSO can update the network’s flexibility segmentation regularly to manage its flexibility services provided.

VI. NUMERICAL PERFORMANCE AND SCALABILITY

The MIQCP model \((1a)-(1j)\) with the constraints on the number of activated flexible units \((2)\) was formulated in JuMP 0.21.8 for Julia 1.6.1 programming language and solved with Gurobi 9.1.2 solver. A laptop with Intel Core i7-10510U CPU 1.80GHz and 16 GB of RAM was used for the calculations.
Fig. 5. Probabilistic segmentation of the 124-bus network flexibility area: (a) flexible unit at bus 20 is considered the most reliable one with $R_{20}^F = 0.99$; (b) probability estimation for the unit decreased down to $R_{20}^F = 0.92$. The displayed flexibility areas correspond to the network configuration with the power supply via feeder 7-1. The grey color scheme indicates probability associated with each of the segments. The dashed outlines contain the unions of the flexibility segments with probabilities greater than 0.9.

To estimate the aggregated flexibility areas and perform their segmentation, the model was solved iteratively for 50 $\varepsilon$-constrained intervals, i.e., the boundary of each segment was approximated by 100 points. The average computational time for the 33-bus case study amounts to 120 seconds, while it takes about 970 seconds to perform the flexibility area estimation for the 124-bus network. Note that the 124-bus case study contains 10 flexible units. Therefore, the flexibility area segmentation for this case requires building more segments compared to the 33-bus network with 5 units. To analyze the scalability of the segmentation framework, it is worth comparing the computational time per segment. Thus, the simulations are repeated 100 times for both case studies. The average computational times are presented in Fig. 6.

The average time for estimating each of the segments is displayed by a black marker. The computational times for all repeated simulations are visualized using violin plots. These plots, similarly to a box plot or a scatter plot, enable depicting information about the range and the density of a data set. For example, it can be observed that some peaks in the computational times rise much higher than the average values. This can be explained by the performance of the heuristics within the branch-and-bound algorithm and perturbations to the solver, which lead to different solution paths. It also follows from the plots that it is generally faster to estimate the flexibility segments when more flexible units get activated. Finding an optimal way to activate only a few of the available units could be more time-consuming. The reasoning behind this numerical performance is that there are many identical flexible units placed in the considered networks. These units make similar contributions to the flexibility areas. Thus, it takes more time to explore the P-Q capabilities of the units and activate the most effective ones.

Concerning the scalability, the average computational time per segment is 24 and 97 seconds for the 33-bus and 124-bus network case studies. Note that the 124-bus case study has twice as many flexible units installed and 3.5 times more elements in the network, which results in a 4 times slower estimation of the flexibility segments. Considering the numer-
ical performance for these case studies, it can be concluded that the flexibility area segmentation approach is scalable and can be applied to real distribution networks. The main limitations are related to the number of the estimated segments and the approximation accuracy of each segment. Therefore, to characterize large distribution networks via flexibility area segmentation, DSOs can define an acceptable level of detail.

VII. CONCLUSION

This paper, through the proposed methodology and case studies, highlights the need for a deeper analysis of the flexibility inherent in ADNs. Specifically, it is required to characterize flexibility by explicitly considering the contributions and firmness of each flexible unit. As illustrated in the case studies analysis, the flexibility of a network can change significantly depending on the network configuration, technical limits, and reliability of flexible units. By capturing these effects, the proposed flexibility area segmentation framework can enable DSOs to manage the provision of ADN flexibility services, e.g., by increasing the firmness of the services whereas reducing the number of underlying units and control actions. Moreover, the obtained segmentation can also be used as an input for risk assessment of distribution networks and optimization problems at the transmission level (e.g., to consider solutions with predefined levels of firmness), which facilitates improved information exchange between DSOs and TSOs without compromising data privacy.

Future work will aim to extend the flexibility segmentation framework to enable tracing and ranking the flexibility provided by flexible units and DERs aggregators. For this purpose, the contributions of different units to the aggregated flexibility of distribution networks will be explored through the use of game-theoretic models. Such an approach highlights the most critical units and parameters for the provision of flexibility services, which is critical information for DSOs to understand the formation and diversification of ADNs flexibility.

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