Testing New Indirect CP Violation

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If new CP violating physics contributes to neutral meson mixing, but its contribution to CP violation in decay amplitudes is negligible, then there is a model independent relation between four (generally independent) observables related to the mixing: The mass splitting ($x$), the width splitting ($y$), the CP violation in mixing ($1 - |q/p|$), and the CP violation in the interference of decays with and without mixing ($\phi$). For the four neutral meson systems, this relation can be written in a simple approximate form: $y \tan \phi \approx x(1 - |q/p|)$. This relation is already tested (successfully) in the neutral K system. It provides predictions for the $B_s$ and $D$ systems. The success or failure of these relations will probe the physics that is responsible for the CP violation.

Introduction. The fact that the Standard Model depends on a single CP violating phase gives it a strong predictive power concerning CP asymmetries. The fact that CP is a good symmetry of the strong interactions makes the theoretical analysis of CP asymmetries often impressively clean. These theoretical advantages, combined with the huge experimental progress in the measurements of CP violation in $B$ decays and in the search for CP violation in $B_s$ and $D$ decays, provide a powerful probe of new physics. Observing deviations from the Standard Model predictions will not only imply the existence of new physics, but also give detailed information about features of the required new physics.

CP violation in meson decays can be classified to direct and direct CP violation. Indirect CP violation can be completely described by phases in the dispersive part of the neutral meson mixing amplitude ($M_{12}$). In contrast, direct CP violation requires that there are some phases in the decay amplitudes ($A_f$). Within the Standard Model, many CP asymmetries require to an excellent approximation – only indirect CP violation. Examples include $K \to \pi \pi$, $B \to \psi K_s$ and $B_s \to \psi \phi$. This situation persists in many – though not all – extensions of the Standard Model.

Indirect CP violation can manifest itself in two ways: CP violation in mixing, which is the source of CP asymmetries in semileptonic decays, and CP violation in the interference of decays with and without mixing, which is often the dominant effect in decays into final CP eigenstates. When there is no direct CP violation, these two manifestations are not independent of each other. They are correlated in a way that depends on the mass- and width-splittings between the two neutral meson mass eigenstates. In this work, we derive this model independent relation, and analyze its applicability and implications in each of the four neutral meson systems ($K, D, B, B_s$).

The experimental parameters. We refer here explicitly to the neutral $D$ system, but our formalism applies equally well to all four neutral meson systems. The two neutral $D$-meson mass eigenstates, $|D_1\rangle$ of mass $m_1$ and width $\Gamma_1$ and $|D_2\rangle$ of mass $m_2$ and width $\Gamma_2$, are linear combinations of the interaction eigenstates $|D^0\rangle$ (with quark content $\bar{c}u$) and $|\bar{D}^0\rangle$ (with quark content $\bar{c}u$):

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle. \quad (1)$$

The average and the difference in mass and width are given by

$$m = \frac{m_1 + m_2}{2}, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}, \quad x = \frac{m_2 - m_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}. \quad (2)$$

The decay amplitudes into a final state $f$ are defined as $A_f = \langle f|\mathcal{H}|D^0\rangle$ and $\bar{A}_f = \langle f|\mathcal{H}|\bar{D}^0\rangle$. We define a complex dimensionless parameter $\lambda_f$:

$$\lambda_f = (q/p)(\bar{A}_f/A_f). \quad (3)$$

As concrete examples, consider the doubly-Cabibbo-suppressed decay $D^0 \to K^+\pi^-$, the singly-Cabibbo-suppressed decay $D^0 \to K^0\bar{K}^-$, and the Cabibbo-favored decay $D^0 \to K^-\pi^+$. Let us assume that effects of direct CP violation are negligibly small even in the presence of new physics. On the other hand, new physics could easily generate indirect CP violation. The effects of indirect CP violation can be parameterized in the following way:

$$\lambda_{K^+\pi^-}^{-1} = r_d|p/q|e^{-i(\delta_{K^+\pi^-} + \phi)},$$

$$\lambda_{K^-\pi^+} = r_d|q/p|e^{-i(\delta_{K^-\pi^+} - \phi)},$$

$$\lambda_{K^+K^-} = -|q/p|e^{i\phi}, \quad (4)$$

where $r_d = |\bar{A}_{K^+\pi^-}/A_{K^-\pi^+}|$, $\delta_{K^+\pi^-}$ is a strong (CP conserving) phase, and $\phi$ is a weak (CP violating) universal phase. The appearance of a single weak phase that is common to all final states is related to the absence of direct CP violation, while the absence of a strong phase
in $\lambda_{K^+K^-}$ is related to the fact that the final state is a CP eigenstate.

We then have (see, for example [1]), for $\Gamma t < 1$,

$$
\frac{\Gamma[D^0(t) \to K^+\pi^-]}{\Gamma[D^0(t) \to K^+\pi^-]} = \frac{r_0^2 + r_0^i}{4} \frac{q}{p} (y' \cos \phi - x' \sin \phi) \Gamma t + \left| \frac{q}{p} \right| ^2 \frac{y'^2 + x'^2}{4} (\Gamma t)^2,
$$

$$
\frac{\Gamma[D^0(t) \to K^-\pi^+]}{\Gamma[D^0(t) \to K^-\pi^+]} = \frac{r_0^2 + r_0^i}{4} \frac{p}{q} (y \cos \phi + x \sin \phi) \Gamma t + \left| \frac{p}{q} \right| ^2 \frac{y'^2 + x'^2}{4} (\Gamma t)^2,
$$

(5)

where $r_0 = |A_{K^+\pi^-}/A_{K^+\pi^+}|$, $y' = y \cos \delta_{K\pi} - x \sin \delta_{K\pi}$, and $x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi}$, and

$$
\Gamma[D^0(t) \to K^+K^-] = e^{-\Gamma t}|A_{K^+K^-}|^2 \times \left[ 1 - \frac{|p/q|}{|y'\cos \phi - x'\sin \phi|} \Gamma t \right],
$$

$$
\Gamma[D^0(t) \to K^-K^+] = e^{-\Gamma t}|A_{K^-K^+}|^2 \times \left[ 1 - \frac{|p/q|}{|y \cos \phi + x \sin \phi|} \Gamma t \right].
$$

(6)

The relations that we derive below depend crucially on this condition. Even if, in general, there is direct CP violation in some decays, our relations apply for those modes where Eq. (10) holds.

We emphasize that the relation between the ‘theoretical’ phase $\phi_{12}$ (defined in Eq. (3)) and the ‘experimental’ phase $\phi$ (defined in Eq. (4)) is, in general, quite complicated. In particular, when $x_{12} \leq y_{12}$, as might still be the case for the neutral D system, the phase $\phi$ might be considerably smaller than $\phi_{12}$ [2]. In other words, the new physics contribution could violate CP with a phase of order one, yet $\phi$ is small.

From theory to experiment. The following expressions give the experimental parameters in terms of the theoretical ones:

$$
xy = x_{12}y_{12} \cos \phi_{12},
$$

$$
x^2 - y^2 = x_{12}^2 - y_{12}^2,
$$

$$
(x^2 + y^2) |q/p|^2 = x_{12}^2 + y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12},
$$

$$
x^2 \cos^2 \phi - y^2 \sin^2 \phi = x_{12}^2 \cos^2 \phi_{12}.
$$

(9)

To obtain the last relation, we took into account the fact that, in the absence of direct CP violation, we have for final CP eigenstates

$$
\text{Im}(\Gamma_{12}^* A_{f}/A_f) = 0, \quad \text{Re}(A_{f}/A_f) = 1.
$$

(10)

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From experiment to theory. Given experimental constraints on $x, y, |q/p|$ and $\phi$, we can use Eq. (9) to constrain $x_{12}$ and $\phi_{12}$ and subsequently the new physics model parameters. In particular, we derived the following equations for each of $x_{12}$ and $\phi_{12}$, first in terms of $x, y$ and $\phi$:

$$
x_{12}^2 = x^4 \cos^2 \phi + y^4 \sin^2 \phi
$$

$$
\sin^2 \phi_{12} = \frac{(x^2 + y^2)^2 \cos^2 \phi \sin^2 \phi}{x^4 \cos^2 \phi + y^4 \sin^2 \phi},
$$

and, second, in terms of $x, y$ and $|q/p|:

$$
x_{12}^2 = x^2 \frac{(1 + |q/p|^2)^2}{4|q/p|^2} + y^2 \frac{(1 - |q/p|^2)^2}{4|q/p|^2},
$$

$$
\sin^2 \phi_{12} = \frac{x^2 + y^2)^2(1 - |q/p|^4)}{16x^2y^2|q/p|^4 + (x^2 + y^2)^2(1 - |q/p|^4)^2}.
$$

(11)

(12)

Let us assume, as is the case for $D$ decays at present, that $x$ and $y$ are measured, while the CP violating parameters $(1 - |q/p|)$ and $\sin \phi$ are constrained to be small. For small $\sin \phi$ we obtain, to $O(\sin^2 \phi)$,

$$
x_{12}^2 = x^2 \left[ 1 + \frac{y^2(y^2 + x^2)}{x^4} \sin^2 \phi \right],
$$

$$
\sin^2 \phi_{12} = \frac{(x^2 + y^2)^2}{x^4} \sin^2 \phi.
$$

(13)
For small \((1 - |q/p|)\) we obtain, to leading order:

\[
x_{12}^2 = x^2 \left[1 + \frac{x^2 + y^2}{x^2} \left(1 - \frac{|q|}{p}\right)^2\right],
\]

\[
\sin^2 \phi_{12} = \frac{(x^2 + y^2)^2}{x^2 y^2} \left(1 - \frac{|q|}{p}\right)^2.
\]

(14)

**A model independent relation.** The fact that we are able to express the four experimental parameters in terms of three theoretical ones means that the experimental parameters fulfill a model independent relation. It depends solely on our assumption that direct CP violation can be neglected.

The relation can be extracted from Eqs. (11) and (12):

\[
\frac{(1 - |q/p|^4)^2}{\sin^2 \phi} = \frac{16(y/x)^2|q/p|^4 + [1 + (y/x)^2]^2(1 - |q/p|^4)^2}{1 + (y/x)^2 \tan^2 \phi}.
\]

(15)

The relation becomes very simple in two limits. Fortunately, each of the four neutral meson systems is subject to at least one of these two approximations. First, consider a system where

\[
y_{12} \ll x_{12}.
\]

(16)

This approximation applies to the \(B\) and \(B_s\) systems. It gives, to leading order in \(y_{12}/x_{12}\):

\[
y/x = \cos \phi_{12} y_{12}/x_{12},
\]

\[
|q/p| - 1 = (y_{12}/x_{12}) \sin \phi_{12}, \quad \tan \phi = -\tan \phi_{12}.
\]

(17)

The derivation of the sign for the CP violating observables starts from the definition of \(q/p\) (see, for example, [4]).

Second, consider a system where CP violation is small, \(|\sin \phi_{12}| \ll 1\).

(18)

This situation applies to the \(K\) system. Very recent measurements imply that it also applies (with limits of order 0.2) to the \(D\) system [3]. We obtain, to leading order in \(|\sin \phi_{12}|\),

\[
y/x = \mathrm{sign}(\cos \phi_{12}) y_{12}/x_{12},
\]

\[
|q/p| - 1 = \frac{(y/x) \tan \phi_{12}}{1 + (y/x)^2} \tan \phi = \frac{-\tan \phi_{12}}{1 + (y/x)^2}.
\]

(19)

The two sets of equations, (17) and (19), lead to the same simple relation:

\[
\frac{y}{x} = \frac{1 - |q/p|}{\tan \phi}
\]

(20)

Eq. (20) is the main theoretical result of this work. If it is found to be violated, then new physics will have to provide not only indirect CP violation, but also direct one. That would exclude many classes of candidate theories.

In what follows, we analyze the applicability and implications of this relation in each of the four neutral meson systems.

**\(K^0 - \bar{K}^0\) mixing.** The two ingredients that go into the relation (20) — small CP violation and the absence of direct CP violation — hold in the \(K \to \pi\pi\) decays. Thus, this relation should hold in the neutral \(K\) system. Neglecting direct CP violation, and defining

\[
A_0 = \langle (\pi\pi)_{I=0}|\mathcal{H}|K^0\rangle, \quad \lambda_0 = (q/p)(\bar{A}_0/A_0),
\]

(21)

the CP violating \(\epsilon\) parameter corresponds to \[6\]

\[
\epsilon = \frac{1 - \lambda_0}{1 + \lambda_0}.
\]

(22)

Then we have

\[
\Re(e) \approx \frac{1}{2}(1 - |q/p|), \quad \Im(e) \approx -\frac{1}{2} \tan \phi.
\]

(23)

The relation (20) translates into the prediction

\[
\arg(e) \approx \arctan(-x/y) = 43.5^\circ.
\]

(24)

where, for the numerical value, we used [7] \(\Delta m_K = 0.5290 \times 10^{10}\) s\(^{-1}\) and \(\Delta \Gamma_K = -1.1163 \times 10^{10}\) s\(^{-1}\). Indeed, the experimental value is [7]

\[
\arg(e) = 43.51 \pm 0.05^\circ.
\]

(25)

Thus, the relation (20) is tested in the neutral kaon system and works very well.

**\(B^0 - \bar{B}^0\) mixing.** In the neutral \(B\) system, the width difference is constrained to be small (and consistent with zero within the present accuracy), \(\Delta \Gamma/\Gamma = 0.01 \pm 0.04\), while the mass splitting is measured to be much larger, \(\Delta m/\Gamma = 0.78 \pm 0.01\). Thus \(y_{12}/x_{12} \ll 1\) and Eqs. (17) apply. One has to note, however, that the equation for \(\phi\) holds only for modes where Eq. (10) applies. Since [8, 9, 10, 11, 12]

\[
\arg(\Gamma_{12}) \approx \arg[(V_{ub} V_{td}^*)^2],
\]

(26)

the phase \(\phi\) relates to modes whose phase is dominated by \(\arg(V_{ub} V_{td}^*)\). (The weak phase of \(B \to \psi K_S\) is dominated by \(\arg(V_{ub} V_{cd}^*)\) and, therefore, \(S_{\psi K_S}\) cannot be used to test (20). The problem is that the approximation (20) gives \(1 - |q/p| = 0\) and \(\phi = 0\), so that \(y \tan \phi = x(1 - |q/p|)\) is fulfilled in a rather trivial way.

If one wants to go beyond (20), the large relative phase between \(V_{ub} V_{td}^*\) and \(V_{ub} V_{cd}^*\) has to be taken into account. It enters \(\Gamma_{12}\) and \(\bar{A}_{12}/A_{12}\) in different ways, and thus direct CP violation plays a role and (20) is violated. Nevertheless, the relation (20) could in principle provide interesting predictions if \(M_{12}\) had significant contributions from new physics carrying a new phase. Experimental data constrain, however, such contributions to be smaller than \(O(0.2)\) [13, 14], which is the same order as the direct CP violating effects in \(\Gamma_{12}\) [8, 9, 10, 11, 12].
Within the Standard Model, the discussion the $B_s$ system follows a line of reasoning that is very similar to our discussion of the $B_d$ system. However, in contrast to the $B_d$ system, a situation where the indirect CP violation is entirely dominated by new physics in $M_{12}$ is still possible for $B_s - \overline{B_s}$ mixing. Actually, recent measurements in $D_0$ and CDF provide hints at a level higher than 2$\sigma$ that this is indeed the case [5]. If so, then the relation (20) provides a very interesting probe of the new physics. Neglecting $\beta_s \approx \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$, the relation reads

$$A_{\text{SL}}^s = -\text{sign}(\cos\phi)(2y/x)S_{\psi\phi}/(1 - S_{\psi\phi}^2)^{1/2}$$

$$= -2|y/x| S_{\psi\phi}/(1 - S_{\psi\phi}^2)^{1/2}$$

(27)

where $A_{\text{SL}}^s$ is the CP asymmetry in semileptonic decays, and $S_{\psi\phi}$ is the CP violating parameter in the decays into $(\psi\phi)c\overline{p}$. The second equality assumes that neither $\Gamma_{12}$ nor $B \rightarrow c\overline{s}$ decays are significantly affected by new physics, which implies that $\text{sign}(y\cos\phi) = \text{sign}(y\cos\phi)^{\text{SM}} = +1$. The experimental data read [7] $\Delta m/\Gamma = -0.07 \pm 0.06$, $\Delta m/\Gamma = 26.1 \pm 0.5$, which give

$$y/x = -0.0014 \pm 0.0012.$$  

(28)

If the central value is approximately correct, then $S_{\psi\phi} = O(0.3)$ would imply $A_{\text{SL}}^s = O(-10^{-3})$. We can expect a significant improvement in the measurements of $y$ and of $S_{\psi\phi}$. (Hopefully, the hints for a signal in $S_{\psi\phi}$ will not disappear as the experimental accuracy improves.)

Then, we will obtain a much sharper prediction for $A_{\text{SL}}^s$. A failure of this test would imply that the new physics introduces both direct and indirect CP violation.

A relation very similar to (27) was previously presented in Refs. [13, 16]. Their relation can be written as

$$A_{\text{SL}}^s/S_{\psi\phi} = \text{Re}(\Gamma_{12}^{\text{SM}}/M_{12}^{\text{SM}})|M_{12}^{\text{SM}}/M_{12}|.$$ 

What we add here to their results are the following two points:

1. The right hand side of this relation, which is calculated from theory, can be replaced by the experimentally measurable factor $-2y/(x\cos\phi)$. Thus, this becomes a theory-independent (in both the electroweak model and QCD uncertainty aspects) relation.

2. We make it clear that a failure of this relation must imply new direct CP violation.

**$D^0 - \overline{D^0}$ mixing.** Within the Standard Model, CP violation in $D^0 - \overline{D^0}$ is negligibly small (see, for example, [17]). Thus, any signal of CP violation requires new physics. It is quite likely that such new physics will contribute negligibly to tree level decay amplitudes, though new direct CP violation is not impossible [18]. Measurements of the time dependent decay rates [16] and [15] will allow us to extract $\phi$ and $1 - |q/p|$ and put (20) to the test.

Experimentally, there has been a very significant progress in determining the mixing parameters in the neutral $D$ system [5]:

$$x = (1.00 \pm 0.25) \times 10^{-2},$$

$$y = (0.77 \pm 0.18) \times 10^{-2},$$

$$1 - |q/p| = +0.06 \pm 0.14,$$

$$\phi = -0.05 \pm 0.09.$$ 

(29)

The CP violating parameters are constrained to be small, and consistent with zero. In case, however, that CP violation is observed in the future, the fact that

$$y/x \approx 0.8 \pm 0.3$$

(30)

suggests that the CP violation in mixing is comparable in size to the CP violation in the interference of decays with and without mixing. Whether or not the relation (20) is fulfilled will teach us about the new physics and will disfavor or support models of the type discussed in Ref. [18], where direct CP violation can be generated.

**Conclusions.** CP asymmetries in neutral meson decays where direct CP violation is negligible obey a relation. The relation involves four experimentally measurable parameters and is thus independent of the electroweak model and clean of QCD uncertainties. It applies to neutral $K$ and $D$ decays in the form (20). If new physics provides a large phase to $B_s - \overline{B_s}$ mixing, then the same relation applies also to $B_s$ decays.

The phenomenological implications of this relation are the following:

- The relation is already successfully tested in $K$ decays.
- If a large CP violating effect is measured in $B_s \rightarrow \psi\phi$, then there is a clear prediction for the CP asymmetry in semileptonic decays $A_{\text{SL}}^s$ that is strongly enhanced compared to the SM.
- If, for neutral $D$ decays, CP violation in either mixing or the interference of decays with and without mixing is observed, there is a clear prediction for CP violation of the other type, of comparable size.
- If the relation fails in $D$ decays, it will be an unambiguous evidence that the new physics generates also CP violation in the decay amplitudes.

**Acknowledgments.** We are grateful to Alex Kagan for pointing out sign errors in the first version of this paper. We thank Monika Blanke for pointing out a missing factor of 2 in Eq. (27) [18]. This work is supported by the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel. The work of YG is supported by the NSF grant PHY-0757868. The work of YN is supported by the Israel Science Foundation founded by the
Israel Academy of Sciences and Humanities, the German-Israeli foundation for scientific research and development (GIF), and the Minerva Foundation. The work of GP is supported by the Peter and Patricia Gruber Award.

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