Classical and quantum depinning of a domain wall with a fast varying spin-polarized current

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(Dated: October 11, 2018)

We study in detail the classical and quantum depinning of a domain wall (DW) induced by a fast-varying spin-polarized current. By confirming the adiabatic condition for calculating the spin-torque in fast-varying current case, we show that the time-dependent spin current has two critical values that determine the classical depinning of DW. This discovery successfully explains the recent experiments. Furthermore, a feasible way is proposed to lower the threshold of spin currents and control the direction of DWs motion. Finally, the quantum properties for the depinning of DW are also investigated in this paper.

PACS numbers: 75.75.+a, 72.25.Ba, 75.30.Ds, 75.60.Ch

I. INTRODUCTION

Recently, considerable attention has been paid to displace a domain wall (DW) by a spin-polarized current in ferromagnetic nanowire and thin film. Upon the view of basic physics, this phenomenon is derived from the interaction between the conducting electrons and the local moment of the DW, which exerts a spin-torque on the DW in the adiabatic condition. By now the adiabatic spin torque is calculated in the constant current case so that the previous theoretical works are mostly focused on the relation between the constant current and the motion of the DW \[1, 2, 3, 4, 5\]. Very recently, however, several experiments discovered many intriguing phenomena by using a fast-varying spin-polarized current \[6, 7\], which motivates us to theoretically study the dynamics of DW in such a fast-varying current case.

In this paper, we first confirm the validity of adiabatic spin-torque in the case of using a fast-varying spin-polarized current, and then study the dynamics of classical and quantum depinning of DW in the biaxial ferromagnet. The validity of adiabatic condition can be verified in the fast-varying current case by the method proposed in Ref. \[1\], thus the adiabatic spin-torque can be readily obtained. Based on this result we shall show the fast-varying characteristic of the current is one of the key points to determine the classical depinning of DW, even if the time-varying of the current only persists in a short time, \(10^{-10} \sim 10^{-9}\) s. Our discovery also provides a self-contained theory for the insights proposed by L. Berger \[8\] a few years ago. The quantum depinning of the DW is also studied in this paper.

The paper is organized as follow. In Sec. II, we confirm the adiabatic condition in the case of fast varying (increasing or falling) current and ac current, and prove that the spin-torque obtained in Ref. \[1\] is also approved in our case. We then calculate in detail the system’s Lagrangian that determines the motion of DW. In doing this, we use the solution obtained in Ref. \[9\] instead of the traditional trial function \[10\] of DW to completely describe the interaction Hamiltonian, which is essential to study the quantum depinning of DW. In Sec. III, with the Lagrangian obtained in Sec. II, we study the classical depinning of the DW. The results obtained here are good in agreement with the recent experiments. The quantum depinning of the DW is investigated in Sec. IV. Finally, we briefly conclude our results in Sec. V.
II MODEL

First, let us begin with the adiabatic condition in the case of the fast-varying spin-polarized current which propagates in a one-dimensional biaxial ferromagnetic material. We assume that the average velocity of all ballistic electrons is \( \bar{v} \) so that \( ne\bar{v} = J_e \), where \( n, e, J_e \) are the density of the conducting electrons, charge of an electron and charge current density, respectively. The effective dynamical equation of the electron spin can be written as follow \[1\]

\[
i \hbar \frac{\partial \psi}{\partial t} = -JM(x(t)) \cdot \sigma \psi,
\]

where the spin-wave function \( \psi = (\phi_1, \phi_2)^T \), \( J \) is the Hunds rule coupling or in general of the local exchange and \( \mathbf{M} \) is the local magnetization. The \( 2 \times 2 \) matrix \( \mathbf{M} \cdot \sigma \) can be diagonalized with a local spin rotation \( \phi_\alpha = U_{\alpha\beta} \psi_\beta \). Thus the above equation yields

\[
i \hbar \frac{\partial}{\partial t} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} -JM_s & 0 \\ 0 & JM_s \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} + H'_{\alpha\beta} \phi_\beta,
\]

where \( H' = -i\hbar \partial_t U + \partial_t U \) is given by

\[
H' = \frac{-i\hbar}{2} \begin{pmatrix} \cos \theta \partial_x \phi & -\partial_x \theta + i \sin \theta \partial_x \phi \\ \partial_x \theta + i \sin \theta \partial_x \phi & i \cos \theta \partial_x \phi \end{pmatrix},
\]

and can be studied with perturbation theory. The transition rate \( \Gamma \) between the two spin states can be calculated in the interaction picture \[11\]

\[
\Gamma = \left| \int_0^t H_{12} dt \right|^2 = \left( \sum_{k=0}^{\infty} \frac{-i \exp(-i2JMt)}{2JM} \partial_t |H'_{12}|^2 \right)^2,
\]

where \( H'_{12} \) is the perturbation Hamiltonian \( H' \) in interaction picture. To verify the adiabatic condition, we can compare the first and second terms in above equation. Considering a fast linearly rising current \( J_e = ne\bar{v} = neat \) in our case, where \( a \) is the average acceleration of all conducting electrons, we find

\[
\frac{\Gamma_{k=1}}{\Gamma_{k=0}} = \frac{\alpha \hbar}{2 \theta JM}.
\]

Obviously the transition rate \( \Gamma \) mostly depends on the term \( k = 0 \) unless the term \( k = 1 \) is not much smaller than the one \( k = 0 \), which case, however, requires \( \frac{\Delta}{\Delta t} \sim J_e/\hbar \) according to Eq. \[5\]. In the recent experiments \[1, 12\], because the maximal current density is about or smaller than \( 10^{12} (A/m^2) \) and generally \( \frac{\Delta}{\Delta t} \leq 10^{24} (A/sm^2) \ll J_e/\hbar \), the adiabatic condition is satisfied. For the ac current case, we note that the adiabatic condition can also be satisfied unless the ac current’s frequency \( \omega_c \) is large enough that \( \hbar \omega_c \approx 2JM \). As in experiment \[6\], \( \omega_c \) is equal to the eigenfrequency \( \omega \) of the pinning potential, which is independent of the energy difference between spin-up and spin-down. Generally when \( \omega_c \approx \omega \), the value \( \hbar \omega_c \) is far different from \( 2JM \), thus the adiabatic condition is also approved in the ac current case. As a result, we can safely extend the adiabatic spin-torque to the fast-varying current case. At the same time, according to above discussion, the magnetoresistance will have two peaks with the frequency’s variety. One corresponds to the resonance of pinning potential and the other corresponds to the resonance of transition.

Now we proceed to study the dynamics of DW interacted with a fast-varying spin-polarized current. We consider an elongated ferromagnetic sample (or a microfabricated magnetic wire) which has an effective-one-dimensional (1D) DW structure. When a spin-polarized current propagates in this sample, together with an external field \( H_{ext} \) along the \( x \) axis, the energy of this system reads \[1, 4\]

\[
E = A \int d^3x (E_m + E_{int} + E_{ext}),
\]

\[
E_m = 2A((\nabla \theta)^2 + \sin \theta (\nabla \phi)^2) - HM_s (\sin \theta \cos \phi)^2 + H_M M_s \cos^2 \theta,
\]

\[
E_{int} = \frac{\hbar}{2e} J_e \nabla \phi (1 - \cos \theta), \quad E_{ext} = \hat{H}_{ext} \cdot \hat{M},
\]

\[6\]
where $A$ is the cross section of the sample, $M_s$ is the saturation magnetization, $M_z = M_s \cos \theta$, $M_x = M_s \sin \theta \cos \phi$, $H$ and $H_\perp$ are anisotropy fields which respectively correspond to easy $x$ axis and hard $z$ axis, $e$ is the electronic charge, $E_{\text{int}}$ is the interaction Hamiltonian between the magnetization and the current on the adiabatic assumption [1], which is the key point to understand the dynamics of the magnetic interacting with the spin-polarized current. Although the interaction Hamiltonian $E_{\text{int}}$ is obtained in the constant current system, it is applicable for the fast-varying current case, since the adiabatic condition can still satisfied in present case.

Before calculating the Lagrangian of the system, we shall go over the traditional soliton solution called trial function introduced by Walker [10], which will be helpful to understand our later discussion on the DW’s depinning. The trial function is

$$\phi = \phi(t), \ln \tan \frac{\theta}{2} = c(t)(x - \int_0^t v(\tau)d\tau).$$

(7)

Based on above formula the $E_{\text{int}}$ is always zero, because $\phi$ is independent on space coordinate.

![FIG. 1: A configuration of magnetization described by trial solution. The magnetization is almost in $x-z$ plane.](image)

However, since it is not the Lagrangian but the variation of Lagrangian to decide the dynamical equation of the DW, the above solution can safely describe the classical motion of the DW with the spin-torque considered [2]. But for quantum tunneling phenomenon, the tunneling rate is decided by the system’s Lagrangian, so the space-dependent character of the solution $\phi$ will be important. In trial function solution, one should use the sphere coordinate, which has an artificial singularity at south pole or north pole, say, in such case we cannot find a complete coordinate system to include both south and north pole. Noting that the spin-torque is a topological term derived from the gauge field in spin representation [1], in order to cancel the singularity, one can take the coordinate neighbour-hoods, i.e. two sets of coordinates, to describe the topological character of the system [13]. In the trial function, the magnetization points to the north pole when $x \to -\infty$ and to the south pole when $x \to \infty$, see Fig. 1. This in fact connects the north and south poles in the same position in the sphere coordinate. Under the description of the incomplete coordinate system, the solution of trial function always leads to a zero topological term $E_{\text{int}}$. Therefore, to calculate the Lagrangian of our system, we consider the solution obtained in [4]

$$\sin \theta \left( \frac{\partial \phi}{\partial t} + b_j \frac{\partial \phi}{\partial x} \right) = -\gamma \cos \theta H_\perp,$$

$$\phi = 2 \arctan[\exp\left(\frac{x - q(t)}{\Delta(t)}\right)],$$

(8)

where $\gamma$ is the gyromagnetic ratio, the coefficient of the spin-torque $b_j = \frac{\mu B P J}{c M_s}$ with $P$ and $\mu_B$ are spin polarization of the current and Bohr magneton, respectively. $q(t)$ is the center of the DW as a kink soliton and $\Delta(t)$ is the width of the DW. In this solution, the magnetization lies closely in the $x-y$ plane, so the singularity in south or north pole is readily cancelled (Fig. 2). Putting this solution [8] into the DW’s Lagrangian [9] and integrating over it, we get the form of the Lagrangian in present
of spin-torque

\[ L = S^0 - \int E = \frac{1}{2}m(\dot{q} + b J)^2 + Q\dot{\varphi} - (2A\Delta^{-1} + 2K\Delta) \]
\[ = \frac{1}{2m}(P + QA)^2 + Q\dot{\varphi} - (2A\Delta^{-1} + 2K\Delta) \]
\[ , \quad S^0 = A \int dx \frac{\hbar S_a^3}{a^2} \phi(\cos \theta - 1), \quad (9) \]

where \( m = (2\pi\gamma^2\Delta)^{-1} \) is the effective mass of a DW, \( Q \) is the effective magnetic charge at the center of the DW, and \( \nabla \varphi = H_{ext}, A = mb J/Q \). From the Eq. (9), it is clear that the effect of the spin-polarized current on a DW can be understood as a charged particle interacts with external magnetic field described by the vector potential \( A \). Since the width \( \Delta \) of the DW only changes slightly during the motion of the DW, we shall assume that \( \Delta \) keeps constant and can be neglected from DW’s lagrangian in the following discussion.

III CLASSICAL DEPINNING

With the above discussion and considering the damping term \( \frac{\alpha}{M_s} M \times \frac{\partial M}{\partial t} \) in present system, the Euler equation of the Lagrangian (9) is obtained

\[ m\ddot{q} + \alpha_0 \dot{q} + Qk + m\omega q\theta(\xi - |q|) = 0, \quad (12) \]

Noting that vector potential \( A \) is proportional to the current \( J_c \), Eq. (10) clearly shows that it is the variation of the adiabatic spin-torque that exerts a force to the DW. Therefore, with a damping term considered, if the applied current is constant, it cannot maintain the motion of the DW without external magnetic field. This result coincides with that in Ref. [2] where the terminal velocity is found to be independent of the spin torque for the constant-current case.

So far our discussion is based on an ideal sample. Practically, the defects in samples can pin the domain wall by a pinning potential. In the following, we study the classical depinning of the DW induced by a rising dc current, with the defects considered. For convenience, the pinning potential is set by

\[ V(x) = \frac{1}{2}m\omega^2 x^2 \theta(\xi - |x|), \quad (11) \]

where \( \theta(\xi - |x|) \) is the step function and \( \xi \) is the half width of the pinning potential. First, we consider applying a rising dc current on the sample without external magnetic field. For example, the current is assumed to linearly rise with time, \( A = Kt(K > 0) \). Then the dynamical equation of the DW reads

\[ m\ddot{q} + \alpha_0 \dot{q} + Qk + m\omega q\theta(\xi - |q|) = 0, \quad (12) \]

with \( \alpha_0 = 2M_s \alpha/\gamma \Delta \). For overdamping \( (\frac{\alpha_0}{2m} = \frac{\alpha M_s \gamma}{\pi} \gg \omega) \), the solution of the Eq. (12) is

\[ q(t) = C(1 - \exp(-\frac{\omega^2}{\alpha_0} t)), \quad (13) \]
where $C = \frac{QK}{mc^2}$ is the maximum displacement of the DW and proportional to $\frac{dj}{dt}$. To depin the DW, $C$ must be larger than $\xi$ and the threshold of depinning time $t_0$ satisfies $\xi = C(1 - \exp(-\frac{\omega^2}{\alpha^2}t_0))$. Thus, the minimum depinning current $J_{\text{min}}$ for different $C$ is

$$J_{\text{min}}(C) = \int_0^{t_{\text{cr}}} d(J_e) = \frac{C e M_s \alpha_0}{\mu_B P} \ln(1 - \frac{\xi}{C}),$$

$$J^c_{\text{cr}} = J_{\text{min}}(\infty) = \frac{2\alpha e M_s^2 \xi}{\gamma \Delta \mu_B P}.$$  (14)

As $C \ln(1 - \xi) = -\xi$ when $C = \infty$, it is easily to see that $J_{\text{min}}(C)$ will be fast close to $J^c_{\text{cr}}$ with the rising value of $C$ (Fig. 1). Accordingly, a current above $J^c_{\text{cr}}$ is necessary for depinning. From above discussion we conclude that there are two critical values to determine the depinning of DW by spin-polarized current. The first is $(\frac{dj}{dt})_{\text{cr}} = \frac{e M_s \xi \omega^2}{\mu_B P}(\sim C_{\text{cr}})$ which assures that the force exerted on DW is big enough for depinning. The other is $J^c_{\text{min}} = \frac{2\alpha e M_s^2 \xi}{\gamma \Delta \mu_B P}$ which indicates the interaction time should be long enough to be depin the DW. The two critical values are solely dependent on the properties of the ferromagnet. Our results are good in agreement with the recent experiment [7].

In this experiment, they observed an important phenomenon that the critical current $J_{\text{cr}}$ is independent of the current pulse durations. This can be explained by our result that $J_{\text{cr}}$ is determined by the properties of the ferromagnet. To change the current pulse duration in the experiment is equivalent to change $\frac{dj}{dt}$, e.g. if one use a small $\frac{dj}{dt}$ (but still larger than $(\frac{dj}{dt})_{\text{cr}}$), a large pulse duration (or $J_e$) will be required to depin the DW. We should also emphasize that, except for the width of the pinning filed $\xi$, the parameters in the equation $J^c_{\text{min}} = \frac{2\alpha e M_s^2 \xi}{\gamma \Delta \mu_B P}$ are observable. As a result, this equation provides us an feasible way to measure the width of the pinning potential in the ferromagnet.

Subsequently, we study the scheme to lower the threshold $J_e$ of spin currents by applying both current and external field simultaneously. Eq. (14) shows that $H_{\text{ext}} = -\nabla \phi$ has the same effect with

![FIG. 3: Abscissa $\partial J_e/(\partial J)_{\text{cr}} = \frac{dj}{dt}/(\frac{dj}{dt})_{\text{cr}}$. This figure shows that $J_{\text{min}}(C)$ is fast close to $J^c_{\text{cr}}$ with the rising value of $C$.](image)
The dynamical equation of the DW in this case is obtained as

$$m\ddot{q} + c_0\dot{q} + Q(k + H_{ext}) + m\omega q\theta(|q| - |q|) = 0. \quad (15)$$

According to Eq. (15), by applying an external field lower than critical reverse field $H_{cr}$ as well as a fast-rising current opposite to the external field, we can obtain an effective way to depinning the DW with smaller current without lack of the precision of the control. For example, when the external magnetic field is set as $H_{ext} = \frac{2}{5}H_{cr}$, the threshold current will be reduced by factor a 10. This may provide us an effective way to implement a more dense magnetic memory, since we know that the current can control the DW in a smaller spatial region than magnetic field can. For a more dense ferromagnetic memory disk than present-day’s, when the magnetic field applied on the disk is smaller than $H_{cr}$, it cannot change the information in the disk by itself but reduce the current threshold. The information is still controlled by the current, but in a easier way. Nevertheless, if the fast-rising current is in the same direction with the applied magnetic field, the threshold of the current will be larger, and the motion of DW is harder to induce. Our conclusion can also explain another important phenomenon observed in the experiment [7]: when the applied external field is smaller than $H_{cr}$, the observed direction of DW displacement will be dependent on the external field. We shall give a more detailed discussion in the following. We consider the case that the current is in the same direction with the external magnetic field. Furthermore, we note $\dot{A} = K_+\gamma$, $K_+ > 0$ to describe the rising process of the spin current, and note $\dot{A} = -K_-\gamma$, $K_- > 0$ to describe the falling process. According to the Eq. (15), the total effective field $H_{eff}$ reads

$$H_{eff} = H_{ext} \mp K_\pm, \quad (16)$$

which means that although the current is in the same direction with $H_{ext}$ during the process, the rising/falling currents try to induce the displacement of DW in the opposite/same direction with that induced by the external magnetic field. Thus, rising and falling currents separately counteract and enhance the DW’s depinning. Particularly, we may assume that $H_{ext} = \frac{2}{5}H_{cr}$ and obtain

$$J_{cr+} = \frac{3}{2}J_{cr}, \quad J_{cr-} = \frac{1}{2}J_{cr}, \quad (17)$$

where $J_{cr}$ is the threshold current without external field, $J_{cr+}$ and $J_{cr-}$ correspond to the threshold currents for the rising-current case and falling-current case, respectively. In experiment [7], because the charge current is near $J_{cr}$, when the external magnetic field is applied, rising current cannot depin the DW but failing current can. This is why in experiment the direction of DW’s displacement is solely dependent on the external magnetic field.

The effective magnetic field generated by the fast rising current can be readily estimated by using the materials parameters of permalloy: $H_\perp = 4\pi M_s = 1.0 \times 10^4\text{Oe}$ (demagnetization field has the same effect with the hard anisotropy field as we have discussed), $H = 100\text{Oe}$, $A = 1.75 \times 10^{-11}\text{J/m}$, $\gamma = 1.75 \times 10^7(\text{Oe})^{-1}\text{s}^{-1}$, $P = 0.5\text{A/cm}^2$, rising time is $0.32\text{ns}$, $J_{cr} = 6.7 \times 10^6\text{A/cm}^2$. The effective magnetic field of this rising current is then obtained $H_{eff} = 2.36\text{Oe}$.

Recently, some authors concentrated on the ac current applying on the DW [6, 14]. They concluded that the threshold current in ac case is smaller than dc case. However it is difficult to control the direction of the DW’s motion because the pinning potential is symmetry without external magnetic field in ac case. According to our result this difficulty can be resolved by applying an external magnetic field $H_{ext}$ lower than $H_{cr}$. This result can also be explained that the external magnetic field breaks the symmetry of the pinning potential and then controls the direction of DW’s displacement. Let’s assume the field $H_{ext} = \frac{5}{2}H_{cr}$ along $x$ axis. The modified potential then reads

$$V = \frac{1}{2}m\omega^2(x - \frac{1}{2}\xi)^2\theta(\xi - |x|). \quad (18)$$

Depinning occurs at the amplitude of displacement $X \geq \xi/2$ and the threshold current is $J_{cr}\text{ac}/2$ ($J_{cr}\text{ac}$ is the threshold without external field). The displacement of the DW must be in the same direction with the external field if only the current $J_e$ satisfies $\frac{J_e}{J_{cr}\text{ac}} < \frac{1}{2}$ or $\frac{J_e}{J_{cr}\text{ac}} \geq \frac{3}{2}$. So, the direction of DW motion induced by ac current can also be efficiently controlled by applying an external magnetic field.
IV QUANTUM DEPINNING

So far, both theoretical and experimental investigations are mainly focused on the DW’s classical behavior induced by a spin-polarized current, such as the classical depinning, nucleation \[4\], and so on. These works show us that the spin-polarized current can always make the same effects in DW’s behavior with the magnetic field do. As we know that the magnetic field can induce macroscopic quantum phenomena such as quantum nucleation and quantum depinning of a DW via macroscopic quantum tunnelling \[12\, 16\, 17\]. This motives us to seek how the spin-polarized current induce the quantum behavior of the DW. We here concentrate on the quantum depinning of a DW.

Considering the above discussion, the classical dynamic equation of the DW Euler equation is obtained as

\[
m\dddot{q} + Q\dot{A} - \frac{\partial u(x)}{\partial x} = QH_{\text{ext}}.
\]  

(19)

In this section, we neglect the damping term and assume that vector potential \(A\) satisfies \(A = Kt\), where \(K = \frac{m_0 P}{QeM_\text{r}}\), which means the spin-polarized current rises with time linearly. After Wick rotation, the Lagrangian and Euler equation in the imaginary time are given by

\[
L = -\frac{1}{2}m\left(\frac{dq}{d\tau}\right)^2 + u(x),
\]

(20)

\[
m\frac{d^2q}{d\tau^2} - QK - \frac{\partial u}{\partial x} = 0.
\]

(21)

Then, according to the standard instanton method \((\tau = it)\), the tunnelling rate is given by

\[
\Gamma = C_0\exp(-B_0) \quad B_0 = \frac{S_0}{\hbar}.
\]

(22)

Here \(C_0\) is the prefactor and \(S_0\) is the action of the classical solution in imaginary time corresponding to Eq. (21), which is similar with the case of ferromagnetic applied with an external magnetic field. The classical action \(S_0\) is written as

\[
S_0 = \int_{-\infty}^{\infty} d\tau \left[\frac{1}{2}m\left(\frac{dq}{d\tau}\right)^2 + u(q) - QKq\right] + (QKq + QK\tau \frac{dq}{d\tau}) + \frac{1}{2}\left\{\frac{dq}{d\tau}\right\}^2,
\]

(23)

where \(\tau_0\) is the rising time of the current. The first term in classical action \(S_0\) is the same with of quantum depinning induced by external magnetic field. The second term is a total differential which equals to \(QKq(\tau)\tau^2\) and the third term equals to \(1/2kT_0^2\). Both the later two terms are imaginary, so they just add to a phase in the action and has no effect on the tunnelling rate. Then, we reach our conclusion that a polarized current also can induce a DW tunnelling from a pinning potential.

We still consider harmonic potential in Eq. (11), with this potential and according to Eq. (23), we obtain the classical action of the DW as

\[
S_0 = \frac{1}{2}\sqrt{m_0\xi}(\xi - \frac{2\xi QK}{m\omega^2}),
\]

(24)

and the prefactor can be calculated straightforwardly \[18\]

\[
C_0 = \left(\frac{\omega}{\pi\hbar}\right)^{\frac{1}{2}}(2\sinh(\omega\tau_0))^{\frac{1}{2}}.
\]

(25)

Thus the quantum tunnelling rate can easily be obtained by the Eq. (22). The above results show that quantum tunnelling and nucleation of DW can also be induced by spin-polarized. As the current is more precise than magnetic field in controlling DW, the above conclusion is worth of further study in future works.
V. Conclusions

In conclusion we have studied the classical and quantum depinning of the one-dimensional domain wall (DW) induced by a fast-varying spin-polarized current. Firstly, we confirm the adiabatic condition of the spin-torque in fast-varying current case and predict that the magnetoresistance will appear two peaks with the frequency variety. Then, the Lagrangian of the DW based on the solution in [9]. It is interesting that an effective vector potential $A(t)$ appears in the Lagrangian due to the interaction between the DW and the spin-polarized current. This means the DW motion can be understood as a charged particle interacts with external magnetic field described by the vector potential. Together with these results, we can study the depinning of DW and find that there are two critical values for the current, $(\frac{dJ}{dt})_{cr} = \frac{eM_s\xi^2}{\mu B}\frac{a}{P}$ and $J_{cr} = \frac{2aeM_s^2}{\xi(\Delta_B)}$, to determine the classical depinning. These results can explain the experiment [7]. Furthermore, by using an external magnetic field, we propose a scheme to lower the threshold current for the DW’s depinning and control its motion direction. Finally, we discuss the quantum behaviors, and reveal that the quantum tunnelling effect of the DW can also be induced by the spin-polarized current.

ACKNOWLEDGMENTS

We thank Prof. Wu-Ming Liu, Prof. Ke Xia and Prof. Jingling Chen for their valuable discussion. This work is supported by NSF of China under grants No. 10275036, and by NUS academic research Grant No. WBS: R-144-000-071-305.

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