Measurements of high voltage pulses with subnanosecond rise time

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Abstract. The paper deals with the application features of capacitive probes for subnanosecond voltage pulse measurements. The relation for the amplitude of distortions determined by the finite electrical length of a capacitive divider was obtained. This relation could serve for quantitative estimation of capacitive divider suitability. Probes of various designs were used for recording the high voltage pulses with subnanosecond voltage changes. The pulse shapes with voltage change durations of 0.2 - 20 ns from different voltage probes were compared. It was shown that the use of capacitive divider with high- and low-voltage arms filled with the same material is more appropriate. This divider provides the same voltage ratio for voltage changes of durations from tens to fractions of nanoseconds.

1. Introduction

There are some applications requiring the high-voltage pulses with duration of several tens of nanoseconds and with a voltage rise time in subnanosecond range, when a voltage pulse amplitude of about 1 MV. The problem for recording such pulses appears in different areas: in investigations of non-steady states and transient processes in gas discharge gaps for power switching [1-4], when researching and developing of high power microwave sources with phase lock and phase control [5,6], RF-pulses sources based on gyromagnetic nonlinear transmission lines [7], etc. High noise level unavoidably accompanies a measurement procedure at high voltage setups. It determines most requirements for a probe: high electric strength of construction, a high signal output level and low stray reactance.

The approach presented in paper [8] is quite acceptable for subnanosecond pulse measurements. The divider arms were considered as two coupled transmission lines; therefore the voltage divider ratio was defined by the wave impedances ratio of them. The divider makes it possible to record pulses almost without distortion within the time interval limited by the double transit time through the low-voltage arm. Therefore, its implementation for long pulse monitoring is problematic.

A capacitive divider satisfies the most requirements we mentioned above. However, conventional approach forces one to reduce the measuring elements of probe with the shortening of pulse rise time. Its dimensions should be of a few millimeters for measurements of subnanosecond rise time. Such miniature probe has shortcomings in our applications. The main disadvantage is that the output signal amplitude is small within relatively long monitoring intervals that causes a significant increase of the
random noise fraction in the final result. Another disadvantage is relative difficulty to calculate the
division ratio with acceptable accuracy.

We developed a new interpretation of capacitive divider, making it possible to design the probe
which is free of limitations mentioned above. The main idea is based on the known fact. A
superconductive infinitely thin layer of arbitrary dimensions placed along an equipotential surface
does not disturb the electromagnetic wave propagation. It allows one to choose measuring surface
dimensions of the divider arbitrarily when it is placed in a homogeneous medium. Simple analysis of
capacitive divider model presented in Section 2 proves a zero level of distortions for capacitive divider
with arms filled with the same material. The relation for the amplitude of distortions determined by
probe finite electrical length was obtained. It demonstrates that the level of the distortions depends
not only on the ratio of the divider electric length to the pulse rise time, but also on the difference between
electric lengths of the divider arms. The experimental setup is described in Section 3. Experimental
results presented in Section 4 attest the validity of the proposed approach. Some details of applications
of different probes are discussed in Section 5.

2. Estimation of distortion amplitude for capacitive divider
Let us begin from the main equations for conventional capacitive divider. We consider an axially
symmetric divider in a coaxial line filled with material of permittivity $\varepsilon_h$ (Fig. 1). The voltage divider is
a thin-walled hollow metal cylinder 2 separated from an outer conductor 1 by a layer of permittivity $\varepsilon_l$
$\geq\varepsilon_h$. The cylinder length is equal to $l$, and the cylinder wall thickness and the layer thickness are so
small that the divider hardly affects the pulse propagation in the line. The low-voltage arm is loaded
on a measuring circuit with impedance $\rho_s$. The connection point of the measuring circuit is equally
spaced from the divider ends.

\begin{equation}
U_0(t) = k_c \left( U_i(t) + \frac{1}{\tau_0} \int_0^t U_i(t')dt' \right), \quad k_c = \frac{C_l + C_h}{C_h},
\end{equation}

where $k_c$ is the capacitive division ratio, $\tau_0 = (C_l+C_h)\rho_s$ is the divider time constant, and $\rho_s$ is the
measuring circuit impedance. Formula (1) takes into account the influence of the measuring circuit
impedance and allows reconstruction of an actual pulse. In practice, the capacitance $C_l$ is two or three
orders of magnitude higher than $C_h$, and the integration constant $\tau_0$ can be chosen large enough to
ignore the voltage drop at the capacitance $C_l$. In this case, we derive the relations for an idealized
capacitive divider as


\[ U_0(t) \approx k_c \cdot U_i(t), \quad k_c \approx C_i/C_h, \quad C_h << C_i, \quad \tau_{\text{pulse}} << \tau_0 \]

According to the accepted approach, the divider electrical length \( \tau \) (the signal transition time through the low-voltage arm) should be much smaller than the rise time \( \tau_{\text{rise}} \). The waveform of voltage \( U_0(t) \) obtained from (1) or (2), describes the pulse shape with some methodical error, which is determined by the finite electrical length of the divider.

Let us estimate this error. Suppose the conditions (2) are satisfied and there are no losses. Let’s test the divider by pulses having infinitely short rise time, considering the divider to consist of two (low- and high-voltage) lines with wave impedances \( \rho_l \) and \( \rho_h \) and electrical lengths \( \tau_l \) and \( \tau_h \), respectively.

For analysis, it is convenient to consider the divider geometry (Fig. 2), where the difference between the electrical lengths of the divider arms is represented by an increase in the geometrical length of the low-voltage arm. The waves excited in the low-voltage arm at divider ends have opposite polarities, as it follows from the voltage matching conditions at the interfaces: \( U_i = U_h + U_l^+ \), \( U_h = U_{\text{out}} + U_l^+ \). At the left end of the divider, the incoming wave \( U_0 \) is divided into two waves \( U_l^+ \) and \( U_h \) of the same polarity. At the right end of the divider the wave of the high-voltage arm \( U_h \) is divided into the wave \( U_l \) and the output wave \( U_{\text{out}} \). The output wave is lower than the wave \( U_h \); therefore the wave exiting in the low-voltage arm \( U_l \) has to be of opposite polarity to \( U_h \).

In the accepted simplifications, the waves \( U_l \) and \( U_h^+ \) are equal in amplitude and will be fully reflected at the divider ends. The total signal \( U_i \) at the load of the measuring circuit for a long pulse with a step-like rise front is shown in Fig. 3.

The difference between the electrical lengths of the divider arms (\( \tau_l - \tau_h \)) is designated as \( \Delta \tau \). The result of testing the divider by a pulse of duration much shorter than the electrical length \( \tau_l \) is a sequence of pulses of opposite polarity (Fig. 4). The first pulse is a signal corresponding to the actual pulse. The rest of the pulses results from multiple reflections of the waves \( U_l \) and \( U_h^+ \) from the divider ends. The pulse with a steep rise time excites parasitic oscillations in the divider with a period equal to \( \tau_l \).

![Figure 3. Tests of the divider by a long pulse with steep rise time.](image)

![Figure 4. Tests of the divider by a short pulse with steep rise time](image)

Note that the results presented in Fig. 3 and 4 can be obtained if we connect two generators with high output impedance and opposite polarities to the divider ends shifted by the \( \Delta \tau \). In this case, we can estimate the parasitic oscillation amplitude \( A_m \) for an arbitrary pulse in terms of the difference \( U_i(t) - U_l(t - \Delta \tau) \). For the linear steepness the following relations are obtained

\[ A_m = (k_{\rho})^{-1}U_0 \frac{\Delta \tau}{\tau_{\text{rise}}} \approx (k_c)^{-1}U_0 \frac{\Delta \tau}{\tau_h} \frac{\tau_l}{\tau_{\text{rise}}} \Delta \tau = \frac{1}{c} \left( \sqrt{\varepsilon_l} - \sqrt{\varepsilon_h} \right), \]

where \((k_{\rho})^{-1} \approx (\rho_h/\rho_l)^{-1}\) is the impedance division ratio, and \(c\) is the vacuum speed of light. When the rise time \( \tau_{\text{rise}} \) is smaller than \( \Delta \tau \), the amplitude \( A_m \) reaches the maximum value.

If the difference \( \Delta \tau \) is equal to zero, parasitic oscillations are not exited by pulses of arbitrary duration.

3. Experimental setup
The purpose of experimental verification was to confirm the foregoing considerations. The experimental setup comprised a SINUS high-voltage pulse generator [9], a transmission line, a sharpening spark gap, and a resistive load. The pulse generator was switched onto the coaxial transmission line which was connected through the sharpening spark gap to the matched resistive load.
The transmission line was filled with transformer oil and had the wave impedance of 30 Ohm at the outer conductor radius of 4.6 cm. The pulse generator formed a “smooth” quasi-rectangular pulse with $\tau_{\text{pulse}}$ of 18 ns, $\tau_{\text{rise}}$ of 3 ns, and $U_0$ of 150 kV at the matched load. The sharpening spark gap closing time was changed within 0.15 - 0.6 ns. Voltage steep drop of subnanosecond duration $\tau_{\text{st}}$ appeared in the transmission line on the smooth pulse when the sharpening spark gap closed. Thus, the formed pulse had voltage changes of duration from 0.15 to 20 ns.

We used two main capacitive dividers: the reference divider $C_f$ and the test divider $C_{g-f}$. The capacitive dividers were made of copper foil 0.1 mm thick and had the equal areas. The divider length $l$ along the coaxial line axis was 5.5 cm. The high-voltage arms of the dividers $C_f$ and $C_{g-f}$ were filled with fluoroplastic in the form of cylinder of the same 5.5-cm length. The low-voltage arm of the test divider $C_{g-f}$ was made of glass-fiber laminate 0.2 mm thick and the reference divider $C_f$ was made of fluoroplastic film 0.08 mm thick. The capacitances of the dividers measured at a frequency of 1 kHz were $C_f = 1.47$ nF and $C_{g-f} = 1.63$ nF and corresponded to the permittivity of ~ 2.1 for fluoroplastic and ~ 5.8 for glass-fiber laminate. Thus, the arms of the reference divider $C_f$ were filled with materials similar in properties, and the arms of the test divider $C_{g-f}$ were filled with materials differing in dielectric properties. The capacitive division ratios for both dividers were close to each other and were about 300.

In the transmission line, there were two additional probes: a differential voltage probe $C_E$ and a differential current probe $C_{I\text{H}}$ which were used for verification. The probes were located in drilled holes of diameter $l = 7$ mm near the surface of the outer conductor of the coaxial line. The designs of the differential probes are shown schematically in Fig. 5. The measuring element of the voltage probe was a metal disk $I$ of diameter 6 mm and thickness 0.4 mm which was coaxially connected to the end of measuring cable.

![Figure 5. Schematics of differential probes: a) - voltage $C_E$, b) - current $C_{I\text{H}}$, $l$ - measurement element.](image)

Note that the use of the measuring element with a diameter close to the hole diameter was chosen to minimize the risk of transmission line breakdown to the measuring circuit and to increase the output signal. The measuring element of the current probe was the conductor $I$ with diameter and length of 2 mm, which was connected to the end of measuring cable and to the outer conductor of the coaxial line. All main and control probes were located in one measuring unit.

A TDS7404 digital oscilloscope with a band of 4 GHz and sampling interval of 50 ps was used. All probes were calibrated by a relatively smooth quasi-rectangular pulse ($\tau_{\text{pulse}}$ of 18 ns, $\tau_{\text{rise}}$ of 3 ns, $U_0$ of 150 kV) after averaging over 300 pulses. The waveforms taken from different probes coincided with a high accuracy.

4. Experimental results

Figure 7 shows waveforms of the voltage $U(t)$ measured using the reference divider $U_f$ and the test divider $U_{g-f}$ with the shift of 0.35 ns relative to each other. The waveforms correspond to a single pulse. There is the drop with duration of $\tau_{\text{st}} \approx 0.15$ ns on the both waveforms. It can be seen that in the test divider $C_{g-f}$ the drop excites parasitic oscillations. The oscillation amplitude is close to the voltage drop amplitude. Increasing $\tau_{\text{st}}$ decreased the oscillation amplitude. When $\tau_{\text{st}}$ was longer than 0.7 ns, no regular oscillations were detected. The oscillation period equal to $\approx 0.35$ ns corresponds to the electrical length of the low-voltage arm filled with material of permittivity $\varepsilon_l \approx 4$. In the reference divider $C_f$, irregular oscillations of small amplitude were observed. Similar oscillations were observed on the waveforms from differential probes. It leads us to the conclusion that the oscillatory process does actually occur.
Figure 7. Waveforms from \( U_f \) - reference divider and \( U_{fj} \) - test divider. Single voltage pulses with drop \( \tau_o \) of 0.15 ns.

Figure 8. Waveforms from \( U_e \) - differential probe and \( U_f \) - reference divider averaged out of 300 events.

Figure 9. Waveform of \( U_{le} \) from the differential voltage probe averaged over 300 events.

Figure 10. Waveform of \( U_{lh} \) from the differential current probe averaged over 300 events.

Figure 8 shows the result of reconstructed waveforms from the reference divider \( U_f \) and from the differential voltage probe \( U_e \) for a monitoring interval of 14 ns. Reconstruction of the waveform from the differential voltage probe was performed according to the formula (1), taking into account only the integral term. The whole formula was applied for the reference divider waveforms. Both waveforms are averaged over 300 events. The waveforms of \( U_f \) and \( U_e \) are shifted by 0.35 ns relative to each other to provide better presentation.

The average waveform \( U_{le} \) from the differential voltage probe without reconstruction is shown in Fig. 9. A similar waveform of \( U_{lh} \) from the current probe is shown in Fig. 10. Both waveforms were averaged over 300 pulses. It allowed us to reject a random noise in the data from probes. Reconstruction of current pulse shape from the current probe data has to be performed according to formula similar to (1), taking into account both the integral and linear terms. Since the reactance of the measuring element was estimated as 10 Ohm at a frequency of 1 GHz, and it is comparable to measuring circuit impedance of 50 Ohm.

It was found that the waveforms of \( U_f \) and \( U_e \) describe the pulse shape \( U(t) \) almost identically in a monitoring interval of 20 ns. Therefore, the voltage division ratios of the differential probe \( C_E \) and capacitive divider \( C_f \) are almost the same in the cases of voltage changes with duration of 0.25-20 ns. The waveforms from the probe \( C_E \) contained considerable random noise that caused the strong distortion of the reconstructed waveform from pulse to pulse. At the same time, the noise weight in data from the divider \( C_f \) was so small that the reproducibility of a pulse shape was high.

5. Discussion of results and conclusion

As follows from (1), the noise level increases proportionally to \( \tau_{\text{pulse}} / \tau_o \) for data from the differential probe and could reach an unacceptable value by the end of the monitoring interval. At the same time, for idealized capacitive divider (2) the noise level remains insignificant. Another shortcoming of
differential probe is the relative difficulty to calculate the division ratio with acceptable accuracy. This fact becomes especially important when the experimental calibration of the probe is difficult.

Despite the shortcomings of the differential probes, their use for monitoring the pulses with subnanosecond voltage changes is justified where arrangement of other type of probes is problematic. Excluding the problem of random noise, the applicability ranges of the differential probe $C_E$ and reference capacitive divider $C_f$ in our design should be considered as identical. This applicability range is determined by the diameters of the measuring cable.

The formula (1) provides the powerful tool for quantitative estimation of capacitive divider suitability. Once the amplitude of distortions is predefined, the shortest rise time for the given divider is known. Another valuable fact is that the impedance division ratio is equal to the capacitive division ratio for a divider which arms are filled with the same material. If the divider is calibrated by any available testing pulse source, then the obtained division ratio is valid for pulses with an arbitrary rise time.

The developed approach makes it possible to expand the divider length to prevent a substantial voltage drop, without any losses to suitability for recording the subnanosecond voltage changes. In the study, we obtained the relation for quantitative estimation of distortion amplitude in capacitive voltage divider. It is shown that the distortion amplitude is proportional to the electrical length difference between the low- and high-voltage arms of a divider. If the difference is close to zero, the divider electrical length could be chosen arbitrarily.

Measurements of relatively long pulses with subnanosecond voltage changes by a capacitive divider with arms filled with the same material are more appropriate. This provides the same voltage ratio for voltage changes of durations from tens to fractions of nanoseconds.

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