Negative Quasiprobabilities Enhance Phase Estimation in Quantum-Optics Experiment

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Abstract: Uncertainty principles limit measurement precision, via operator noncommutation. Wielded correctly, noncommutation can boost precision. We relate metrological enhancement with negative quasiprobabilities, quantum extensions of probabilities. In a phase measurement, we amplify the precision per detected photon by two orders of magnitude.

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1. Introduction

Quantum metrology aims to develop better techniques for measurement and estimation problems [1, 2]. The maximum sensitivity to a small change in a parameter $\theta$ of a quantum state $\rho(\theta)$ is quantified by quantum Fisher Information (QFI) $I(\theta)$, and QFI determines the lower bound for the variance of every unbiased estimator $\theta_0$ of $\theta$ by the Cramer-Rao bound, $\text{var}(\theta_0) \geq 1/I(\theta)$. However, the information can be distilled into fewer trials with proper filtering, which can increase per-input information arbitrarily [3]. The filter has to be noncommutative, i.e. applying the filter at the different parts of the setup must have a different outcome. This non-commutation can be quantified by the negativity of the Kirkwood-Dirac distribution [4] of the system.

We demonstrated this distillation phenomenon with our filtering method partially postselected amplification (PPA) [5]. The information gained from the filtered trials can exceed conventional per–input-trial limits, but the lowered success probability counterbalances the total information [6].

![Fig. 1: Partially Postselected Amplification for polarimetry](image)

Single photons generated by a heralded–single-photon source (HSPS) enter a polarizing–beam-displacer (PBD0) and exit vertically polarized ($|1\rangle$). The half-waveplate (HWP0) with an optic axis angled $45^\circ$ above the horizontal axis rotates the photons’ polarization by an angle $\theta(\alpha) - \pi$. HWP0 is tilted away from normal incidence through an angle $\alpha$ about its optic axis. The partial polarizer consists of a polarizing–beam-displacer interferometer, followed by a beam block in the undisplaced port. The horizontal-polarization transmission amplitude, $t$ with $|t| \in [0, 1]$, is controlled by a half-waveplate (HWP2) inside the interferometer. All horizontally polarized photons are discarded when $|t| = 0$ and kept when $|t| = 1$.

2. Partially Postselected Amplification

Let a unitary $U(\theta) = \exp(i\theta A)$ imprint $\theta$ on an input state with the observable $A$. With the optimal input state $|0\rangle$, the imprinted state $U(\theta)|0\rangle = |\Psi(\theta)\rangle$ carries the most QFI possible without postselection, $I(\theta) = \Delta^2$, $\Delta$ being the difference between maximum and minimum eigenvalues of $A$. We apply a filter represented by a Kraus operator $K(t) = t|0\rangle\langle 0| + |1\rangle\langle 1|$, wherein $|t| \in [0, 1]$. For any $|t| < 1$, the filter does not commute with the generator $A$ and enables noncommutative filtering. The postselected state carries the QFI $I(\theta) = [\Delta |t| p^\text{ps}(\theta,t)]^2$, $p^\text{ps}(\theta,t)$ being the success probability of postselection.
3. The Relation Between QFI and Nonclassicality Gap

Let \(\{|a\rangle\}_a\) and \(\{|a'\rangle\}_{a'}\) denote copies of an \(A\) eigenbasis and let \(\{K_f\}_f\) denote the Kraus operators for the possible outcomes of the filter. The information-bearing state \(\rho(\theta)\) is represented by the Kirkwood-Dirac quasiprobabilities \(\tilde{\rho}_{\rho(\theta)}(a, f, a') := \text{Tr}(\{a'\rangle\langle a'|K_fK_f^\dagger\langle a|\rho(\theta))\). Conditioning on a filter outcome \(f\) induces the conditional Kirkwood-Dirac distribution

\[
\tilde{\rho}_{\rho(\theta)}(a, a'| f) := \tilde{\rho}_{\rho(\theta)}(a, f, a')/\sum_{a, a'} \tilde{\rho}_{\rho(\theta)}(a, f, a').
\]

These quasiprobabilities are positive if \(A\) and \(K_f^\dagger K_f\) commute on the support of \(\rho(\theta)\) [7]. If \(\Delta \theta < \pi\) and \(|t|^2 < 1\), \(\tilde{\rho}_{\rho(\theta)}(a, a'| f)\) has negative elements, and the postselected QFI exceeds \(\Delta^2\).

Let \(x\) denote the vector of arguments for a Kirkwood-Dirac distribution \(\{\tilde{\rho}(x)\}_x\). Define the nonclassicality gap as the greatest difference between quasiprobabilities’ absolute squares:

\[
\max_{x} \left\{ |\tilde{\rho}(x)|^2 \right\} - \min_{x} \left\{ |\tilde{\rho}(x)|^2 \right\}.
\]

The gap > 1 only if a quasiprobability \(\notin [0, 1]\). For any postselection operator \(K_+\), the nonclassicality gap is proportional to the optimal input state’s postselected QFI:

\[
I(\theta) = 4\Delta^2 \times \left[ \max_{a, a'} \left\{ |\tilde{\rho}_{\rho(\theta)}(a, a'| +)\right| ^2 \right\} - \min_{a, a'} \left\{ |\tilde{\rho}_{\rho(\theta)}(a, a'| +)\right| ^2 \right\}
\]

We obtain the conditional quasiprobabilities and QFIs from the tomography of the states. Figure 2 compares the nonclassicality gap with the QFI. The estimated QFI and nonclassicality gap are consistent with the theoretical QFI [Fig. 2(a)]. Thus, our experiment corroborates the relationship (2) between enhanced precision and quasiprobability negativity.

![Fig. 2: Information per detected state (a) and per input state (b) vs. magnitude of the postselection parameter, \(|t|\). The 4 times the nonclassicality gap and experimental QFI are within the error bars. Error bars denote the geometric standard error of 4 independent runs. Without postselection, our estimates are shot-noise–limited to the per–input-state precision 1 rad\(^{-2}\). As we increasingly filter (as \(|t|\) decreases), the per–detected-state QFI steadily increases when \(\theta \approx 0\). Filtering too much decreases the QFI when \(\tan(\theta/2) < |t|\). Despite sacrificing little per–input-state information, the smallest \(|t|\) and \(\theta\) provide a per–detected-state information > 200 rad\(^{-2}\).](image-url)

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