On Problem of Torsional Characteristics of Thin-walled Steel Beams with Web Openings

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Abstract. The current trend in the design of steel structures leads, due to the saving of the material, to the frequent use of thin-walled cold formed steel sections. The thin-walled cold-formed steel profiles are often manufactured with web holes. In the design of such a thin-walled steel members with web openings arises a question of correct determination of the real cross-sectional properties. This paper focuses on the problem of the real torsional characteristics determination, which are parameters needed for the design of members subjected to the bending with respect to lateral torsional buckling or for the design of the compressed members prone to torsional buckling or flexural torsional buckling. These torsional characteristics include St. Venant torsion constant and warping constant. In the forthcoming European Standard specifying the rules for the design of beams with holes, it is recommended to use the cross-sectional characteristics of the most weakened section of the member with web openings. This paper deals with the possibility of introduction of "substitute cross-section" whose cross-sectional characteristics are determined as the weighted average of the properties of full section and the most weakened section. The solution with substitute cross-section is validated by the series of tests focusing on the experimental verification of the both real torsional stiffness – St. Venant torsion stiffness and warping torsion stiffness. Based on the results of test series with twisted beams freely supported in torsion, the St. Venant torsion constant is being derived, respectively from the results of the test series with twisted beams fixed in torsion (the warping is restrained at both beam ends), the warping constant is being derived. Both torsional characteristics are verified by tests executed on three different lengths (L=2.0m, L=3.0m and L=4.0m) of beams with sigma cross-section with large circular web openings.

1. Introduction

The current trend in the design of steel structures leads, due to the saving of the material, to the frequent use of thin-walled cold formed steel sections. In practice are widely used various systems of thin-walled steel purlins, wall girders and floor girders of different types of cross-sectional shapes. The thin-walled cold-formed steel profiles are often manufactured with web holes. These web openings are primarily used for the installation of wiring, water or drain piping, ventilation or sprinkler systems, etc.

In the design of such a thin-walled steel members with web openings arises a question of correct determination of the real cross-sectional properties. This paper focuses on the problem of the real torsional characteristics determination which are parameters needed for the design of members subjected to the bending with respect to lateral torsional buckling [1] or for the design of the compressed members prone to torsional buckling or flexural torsional buckling. These torsional characteristics include St. Venant torsion constant and warping constant.
An example of the thin-walled steel beams with large web circular openings are the Sigma beams (figure 1) used in additionally installed built-in floors systems in warehouses [1].

![Figure 1. Geometrical dimensions of the Sigma beams](image)

2. Theoretical determination of torsional characteristics

The beams with web openings are members with variable cross-section. Insomuch that the solution of the St. Venant torsion constant and warping constant for the beams weakened by holes is not verified, the procedure for the castellated beams was used.

The calculation of the torsional characteristics is listed in the table 1. In the first step, the characteristics for the full cross-section (figure 2 - section A-A) and for the most weakened cross-section (figure 2 - section B-B) according to equations mentioned in the table 1 were determined. In the second step, the St. Venant torsion constant and warping constant are calculated for the established “substitute cross-section” as a weighted average of the full cross-section characteristics and characteristics of the most weakened cross-section.

| St. Venant torsion constant | $I_t = \frac{1}{3} \sum b_i \cdot t_i^3$ | $I_{tA}$ | $I_{tB}$ | $I_{tsub} = \frac{a \cdot I_{tA} + b \cdot I_{tB}}{a + b}$ | $I_{tsub}$ |
|----------------------------|----------------------------------|---------|---------|-----------------------------------|---------|
| Warping constant $I_\omega = \int \omega^2 \, dA$ | $I_{\omega A}$ | $I_{\omega B}$ | $I_{\omega sub} = \frac{a \cdot I_{\omega A} + b \cdot I_{\omega B}}{a + b}$ | $I_{\omega sub}$ |
|                            | $3.82 \cdot 10^9$ mm$^6$         | $3.56 \cdot 10^9$ mm$^6$ |                                   | $3.73 \cdot 10^9$ mm$^6$ |

![Figure 2. Left – Designation of sections A and B, Right – Division of beam over the length](image)
3. Experimental verification of torsional characteristics

3.1. Test arrangement

Experimental verification of the St. Venant torsion constant and warping constant was executed on the special test equipment [2]. Figure 3 shows the torsion test arrangement. The specimens were on the upper end being suspended and simply supported regarding the torsion and on the lower free end being twisted by transverse couple forces $F_z$ with the lever arm of 600 mm. The couple forces were realized by the calibrated set of 1kg weights hanging on steel cable attached to the rotating horizontal loading disk. The angle of rotation $\varphi_z$ was measured on a scale located along the circumference of a horizontal disk. The process of the loading is recorded as a dependence of the force $F_z$ [N] on the angle of rotation $\varphi_z$ [°] (rotation of the lower beam end related to the upper beam end).

Three different lengths of specimens were tested: $L=2.0m$, $L=3.0m$ and $L=4.0m$. Together 30 beams freely supported in torsion (St. Venant torsion tests) were tested (10 pcs per each beam length). Half of the specimens (in total 15, which is 5 pcs per each beam length) were modified in order to prevent the free warping at the beam ends (figure 4) and subsequently twisted with boundary conditions fixed in torsion (mixed torsion consisting of St. Venant torsion and warping torsion). The loading was realized for the shear stresses within elastic range. The loading levels are listed in Table 2.

Table 2. Levels of load $F_z$ applied in torsion tests

| Specimen length | St. Venant torsion tests | Mixed torsion tests |
|-----------------|--------------------------|---------------------|
| $L = 2.0 \, m$  | 0-10-20-30-40-50-60-70-80-90N | 0-20-60-110-160N    |
| $L = 3.0 \, m$  | 0-10-20-30-40-50-60-70N    | 0-10-20-30-40-50-60-70-80-90-110N |
| $L = 4.0 \, m$  | 0-10-20-30-40-50N         | 0-10-20-30-40-50-60-70-80N        |
The theoretical supports of the specimens are considered in the middle of the contact area between the specimen and the torsion supports. The theoretical length of specimen span $L_{\text{supp}}$ is the beam length $L$ reduced at both beam ends by value of 20 mm (half width of the contact area).

![Free warping at both beam ends](image)

**Figure 4.** Welded steel plates preventing the free warping at both beam ends

### 3.2. Test data

Each test specimen was loaded six times (three times counter-clockwise and three times clockwise); thus overall 180 St. Venant torsion loading cycles and 90 mixed torsion loading cycles has been realized. In the figure 5 are displayed the data of loading processes of the beam lengths $L=2.0\text{m}$, $L=3.0\text{m}$ a $L=4.0\text{m}$ recorded during the St. Venant torsion tests, respectively in the figure 6 data of loading processes of the beam lengths $L=2.0\text{m}$, $L=3.0\text{m}$ a $L=4.0\text{m}$ recorded during the mixed torsion tests.

![St Venant torsion test data](image)

**Figure 5.** The $F_z - \phi_z$ relation of St. Venant torsion tests for all beam lengths
3.3. Determination of St. Venant torsion constant derived from test results

For each loading step is valid the linear relationship:

$$\Delta F_{z,i} = \Delta \phi_{z,i} \pi$$

where

$$\Delta F_{z,i} = F_{z,i} - F_{z,i-1}$$

$$\Delta \phi_{z,i} = \phi_{z,i} - \phi_{z,i-1}$$

$$F_{z,i}, \phi_{z,i}$$ are couple forces and angle of rotation for the i-th loading step

$$F_{z,i-1}, \phi_{z,i-1}$$ are couple forces and angle of rotation for the previous (i-1)-th loading step

The expression for the rotation angle calculation is:

$$\Delta \phi_{z} = \frac{M \cdot L}{G \cdot I_{i}}$$

Substituting linear relationship in equation 1 into the equation 4 we get the expression for the St. Venant torsion constant determined based on the test results:

$$I_{i} = \frac{180 \cdot k_{i} \cdot d \cdot L_{\text{supp}}}{\pi \cdot G}$$

where

$$k_{i} = \frac{F_{z,i} - F_{z,i-1}}{\phi_{z,i} - \phi_{z,i-1}}$$

is slope of the linear relationship for the i-th loading step ,

$$d$$ is arm of couple forces [mm]

$$L_{\text{supp}}$$ is theoretical beam length between the torsional supports [mm],

$$G$$ is shear modulus [MPa].
The slope of the linear relationship for each loading step is calculated from the so-called average loading curve. The average loading curves are determined for the beams L=2.0m, L=3.0m and L=4.0m (in figure 7 for the beams L=2.0m). The curves pass through the points corresponding to the average values of the angle of rotation calculated separately for each load level.

The average loading curve shows the non-linear trend. The nonlinearity in the first loading step is due to the clearances of the test equipment. In higher load levels, the cause of the nonlinear behaviour of the twisted beams is the fact, that due to the excessive twist the beams behave more like a plate with membrane stresses rather than a bar. This effect is most significant in case the shortest beams L=2.0m. The table 3 summarizes the calculations of the St. Venant torsion constants for each beam length and load level. The representative results are taken from the second load level \( F_z = 20N \), in which the negative effects of test device clearances are negligible and the rotation angle of the beams does not achieve excessive values.

**Figure 7.** The plot of average loading curve for the beams L=2.0m

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**Table 3.** Determination of St. Venant torsion constant \( I_t \) derived from St. Venant torsion test results

| Load \( F_{xi} [N] \) | Beams L=2.0m \( L_{supp} = 1960 \) mm | Beams L=3.0m \( L_{supp} = 2960 \) mm | Beams L=4.0m \( L_{supp} = 3960 \) mm |
|----------------------|----------------------------------------|----------------------------------------|----------------------------------------|
|                      | \( \varphi_{xi} [\degree] \) k \([-\)] \( I_{ti} [\text{mm}^4] \) \( \varphi_{xi} [\degree] \) k \([-\)] \( I_{ti} [\text{mm}^4] \) \( \varphi_{xi} [\degree] \) k \([-\)] \( I_{ti} [\text{mm}^4] \) |
| 0                    | 0.0 - - 0.0 - - 0.0 - -               |
| 10                   | 5.2 1.94 1615 8.5 1.17 1473 10.5 0.96 1607 |
| 20                   | 9.2 2.44 2033 15.1 1.52 1912 18.6 1.23 2069 |
| 30                   | 13.0 2.68 2227 20.4 1.89 2380 25.3 1.48 2488 |
| 40                   | 16.2 3.13 2608 24.6 2.40 3013 30.5 1.95 3273 |
| 50                   | 18.8 3.79 3155 28.1 2.81 3533 34.9 2.26 3792 |
| 60                   | 21.4 3.88 3230 31.1 3.37 4227 - - - |
| 70                   | 23.4 4.93 4101 33.7 3.80 4769 - - - |
| 80                   | 25.1 5.98 4977 - - - - - - |
| 90                   | 26.4 7.30 6074 - - - - - - |

Full section \( I_{tA} = 2007 \text{mm}^4 \)  Weakened section \( I_{tB} = 1566 \text{mm}^4 \) Substitute section \( I_{t,sub} = 1863 \text{mm}^4 \)
3.4. Determination of warping constant from test results

The derivation of the relationship for warping constant calculation is based on the differential equation describing the mixed torsion (combination of St. Venant torsion and warping torsion) [3]:

\[ E \cdot I_\omega \cdot \varphi'''' - G \cdot I_t \cdot \varphi''' = m \]  

(6)

where \( m \) is the intensity of the torsion load.

Introducing the torsion parameter \( k \) defined as:

\[ \omega \cdot t \cdot I \cdot E \cdot l \cdot k \cdot I_{G} = \cdot \cdot \cdot \]  

(7)

the differential equation can be modified to:

\[ \varphi'''' - \frac{k^2}{l^2} \cdot \varphi''' = \frac{m}{E \cdot I_\omega}. \]  

(8)

The equation 8 is the simple differential equation of fourth order with constant coefficients [4,5,6].

The solution of this differential equation for the thin-walled member with open cross-section and both ends fixed in torsion (restrained warping), torque at the ends with two torsion moment \( M \) of the opposite direction (figure 8), leads to expression:

\[ \varphi_z = \frac{M}{G \cdot I_t} \left( z - \frac{l}{k} \sinh \frac{k}{2} \right) \]  

(9)

where \( M \) is torsion moment,
\( z \) is distance measured from the middle of the beam span to the examined section,
\( l \) is beam length / theoretical beam length between the torsional supports (\( l = L_{\text{supp}} \)),
\( k \) is torsion parameter defined in equation 7.

The angle of rotation in equation 9 is measured on the length corresponding to the half of the specimen length (case A in figure 8 - in this specific case the angle of rotation is in the middle of the span zero). In the tests, the upper beam end was restrained and the lower beam end was loaded with torque, which leads for the same level of applied torque to the double magnitude of the angle of rotation (case B in figure 9).

![Figure 8. Twisted beam by two opposite torques [3]](image)

![Figure 9. Rotation angles of twisted beam](image)

After the substitution of the expression for the torsion parameter (equation 7) into the equation 9, the unknown variable in the equation 9 is the warping constant \( I_\omega \), which can be determined from the iterative calculation (it is searched for such a value of warping constant for the specific magnitude of applied torsion moment that gives the angle of rotation calculated according to equation 9 equal to the angle of...
rotation measured in the torsion tests). In the calculation is used the St. Venant torsion constant for relevant beam length determined based on the results of St. Venant torsion tests. As in the case of St. Venant torsion tests, the average loading curves are constructed from the measured test data for each beam length (figure 10 shows average curve for beams L=3.0m).

![Average loading curve for beams L=3.0m](image)

The average loading curves show relatively linear trend except first loading step in case of beams L=2.0m and L=4.0m, respectively first and second loading step in case of beams L=3.0m. These loading steps are affected by the test device clearances. The table 4 states the calculations of the warping constants for each beam length. The representative results are taken from the load levels: $F_L=60N$ for beams L=2.0 (second loading step), $F_L=20-30N$ for beams L=3.0 (average taken from second and third loading step) and $F_L=20N$ for beams L=4.0 (second loading step).

**Table 4.** Determination of warping constant $I_\omega$ derived from mixed torsion test results

| Beams L=2.0m | Beams L=3.0m | Beams L=4.0m |
|--------------|--------------|--------------|
| $F_L$ [N]    | $L_{supp}$ = 1960 mm | $F_L$ [N]    | $L_{supp}$ = 2960 mm | $F_L$ [N]    | $L_{supp}$ = 3960 mm |
| $\varphi_z$ [°] | $k_1$ [-] | $\varphi_z$ [°] | $k_1$ [-] | $\varphi_z$ [°] | $k_1$ [-] |
| 0            | 0.0        | 0            | 0          | 0            | 0.0        |
| 20           | 0.6        | 32.26        | 10         | 1.0         | 9.88       | 10         | 2.0         | 4.93       |
| 60           | 1.6        | 41.96        | 20         | 1.8         | 12.16      | 20         | 3.6         | 6.21       |
| 110          | 2.7        | 45.05        | 40         | 2.6         | 13.48      | 40         | 5.2         | 6.35       |
| 160          | 3.8        | 46.58        | 60         | 3.3         | 13.19      | 60         | 8.4         | 6.42       |
| -            | -          | -            | 50         | 4.0         | 14.08      | 50         | 8.4         | 6.42       |
| -            | -          | -            | 60         | 4.8         | 13.19      | 60         | 8.4         | 6.42       |
| -            | -          | -            | 70         | 5.5         | 13.45      | 70         | 8.4         | 6.42       |
| -            | -          | -            | 80         | 6.2         | 13.86      | 80         | 12.8        | 7.58       |
| -            | -          | -            | 90         | 6.9         | 14.49      | -          | -          | -          |
| -            | -          | -            | 110        | 8.2         | 15.58      | -          | -          | -          |

$I_t$ [mm$^4$] 2033 1912 2069
$I_t$ [mm$^4$] 4.01·10$^9$ 3.89·10$^9$ 4.02·10$^9$

$I_\omega$ [mm$^6$] $4.01\cdot10^9$ Weakened section $I_{\omega B} = 3.56\cdot10^9$ mm$^6$ Substitute section $I_{\omega sub} = 3.73\cdot10^9$ mm$^6$
4. Results
The overviews of the values of St. Venant torsion constant and warping constant derived from the results of experiments are shown in the figures 11 a 12. The experimentally verified values of torsion characteristics are compared with the theoretical values calculated for the full cross-section, the most weakened section and the established substitute cross-section.

![Figure 11. The comparison chart for the St. Venant torsion constant $I_t$](image1)

![Figure 12. The comparison chart for the warping constant $I_\omega$](image2)

The comparisons in figures 11 and 12 show good match between the experimentally verified and mathematically calculated values of torsion characteristics. The resulting values of St. Venant torsion constant and warping constant comply with torsion characteristics of substitute cross-section.

5. Conclusions
In this paper the mathematical description and experimental examination of the thin-walled steel beams behaviour subjected to the St. Venant torsion and mixed torsion are reported. Based on the results of test series with twisted beams freely supported in torsion, the St. Venant torsion constant is being derived, respectively from the results of the test series with twisted beams fixed in torsion (the warping is restrained at both beam ends), the warping constant is being derived.

In the forthcoming European Standard specifying the rules for the design of beams with holes, it is recommended to use the cross-sectional characteristics of the most weakened section of the member.
with web openings. The comparisons of experimentally verified and theoretically calculated torsion characteristics shows, that this approach is conservative in this specific case of Sigma beams with large circular web openings. The real behaviour of these twisted beams corresponds more to the theoretical model of beams of “substitute cross-section” whose cross-sectional characteristics are determined as the weighted average of the properties of full section and the most weakened section. It is necessary to point out, that these conclusions should be verified for members of different cross-sectional shape, different geometry of the web openings and different arrangement of the web openings.

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