On Holographic description of the Kerr-Newman-AdS-dS black holes

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ABSTRACT: In this paper, we study the holographic description of the generic four-dimensional non-extremal Kerr-Newman-AdS-dS black holes. We find that if focusing on the near-horizon region, for the massless scalar scattering in the low-frequency limit, there exists hidden conformal symmetry on the solution space. Similar to the Kerr case, this suggests that the Kerr-Newman-AdS-dS black hole is dual to a two-dimensional CFT with central charges $c_L = c_R = \frac{6a(r_+ + r_*)}{k}$ and temperatures $T_L = \frac{k(r_+^2 + r_*^2 + 2a^2)}{4\pi\alpha\Xi(r_+ + r_*)}$, $T_R = \frac{k(r_+-r_*)}{4\pi\alpha\Xi}$. The macroscopic Bekenstein-Hawking entropy could be recovered from the microscopic counting in dual CFT via the Cardy formula. Using the Minkowski prescription, we compute the real-time correlators of the scalar, photon and graviton in near horizon geometry of near extremal Kerr-AdS-dS black hole. In all these cases, the retarded Green’s functions and the corresponding absorption cross sections are in perfect match with CFT prediction. We further discuss the low-frequency scattering of a charged scalar by a Kerr-Newman-AdS-dS black hole and find the dual CFT description.
1. Introduction

The Kerr/CFT correspondence conjectures that a Kerr black hole with mass $M$ and angular momentum $J$ is dual to a 2D CFT with central charges $c_L = c_R = 12J$ and temperatures $T_L = M^2/2\pi J, T_R = \sqrt{M^4 - J^2}/2\pi J$. This correspondence was first proposed in [2] by studying the near-horizon geometry of extreme Kerr black hole (NHEK) [1], and was then improved by the subsequent study [4, 3], especially on the near-extremal Kerr black holes. Support of this conjecture has been found in the perfect match of the macroscopic Berenstein-Hawking entropy of the black hole with the conformal field theory entropy computed by the Cardy formula. See [5] for some further studies of the Kerr/CFT correspondence as well as generalizations to other spacetime which contain a warped AdS structure.

Further support of the correspondence was found in the study of the superradiant scattering off the extreme Kerr black holes [6]. In this case, the Kerr black hole actually becomes near-extremal, and correspondingly the right-moving sector of dual CFT is excited [4]. In the near-horizon limit, the modes of interest are the ones near the super-radiant bound. It was shown in [6] that the bulk scattering amplitudes were in precise agreement with the CFT descriptions whose form are completely fixed by the conformal invariance. Similar discussions have been generalized to charged Kerr-Newman [7], multi-charged Kerr
and higher dimensional near-extremal Kerr black holes. In all these cases, perfect agreements with the dual CFT descriptions have been found.

In [11], it was shown that the real-time correlators of various perturbations in near-extremal Kerr(-Newman) black hole could be computed directly from the bulk, following the Minkowski prescription proposed in AdS/CFT [9] and successfully used in the warped AdS/CFT correspondence [10]. It allowed us to perform a test directly on the CFT correlators and the real-time correlators as obtained by holography. The results are in perfect agreement with the CFT predictions. The similar prescription [13] has been applied to calculate the three-point functions in the Kerr/CFT correspondence [12].

The support of the Kerr/CFT correspondence for generic non-extremal Kerr black holes only appeared very recently. In a remarkable paper [14] the authors argued that the existence of conformal invariance in a near horizon geometry is not a necessary condition, instead the existence of a local conformal invariance in the solution space of the wave equation for the propagating field is sufficient to ensure a dual CFT description. The observation indicates that even though the near-horizon geometry of a generic Kerr black hole could be far from the AdS or warped AdS spacetime, the local conformal symmetry on the solution space may still allow us to associate a CFT description to a Kerr black hole. Both the microscopic entropy counting and the low frequency scalar scattering amplitude in the near region support the picture. The similar treatment has been successfully applied to the higher-dimensional Kerr black hole [15], the RN black hole [16], the Kerr-Newman black hole [17, 18]. The hidden conformal symmetry in the solution space also allows us to use the Minkowski prescription to compute the real-time correlators [18, 19]. For other related study on hidden conformal symmetry in the Kerr/CFT correspondence, see [20, 21, 22].

In this paper, we would like to study the holographic description of four-dimensional Kerr-Newman-AdS-dS black holes. The extreme case has been studied in [23], where a dual CFT description of the extreme Kerr-Newman-AdS-dS black holes has been suggested. It would be interesting to see if the picture could be pushed away from the extreme limit. For the generic non-extremal Kerr-Newman-AdS-dS black holes, the function determining the black hole horizons is quartic, which makes the problem intractable. Nevertheless, we find that in the near-horizon region there still exists a conformal symmetry acting on the solution space of radial wave function for the massless scalar scattering off the black hole in the low frequency limit. This is quite different from the treatment on other black holes in the literature. For the low frequency scattering of a Kerr black hole, one may just consider the “Near” region with $r\omega << 1$. It was argued in [14] that the hidden conformal symmetry originate from the arbitrariness in choosing the matching surface between the “Near” and “Far” region. In our case, we have to focus on the near-horizon region, which is much more restricted than the “Near” region. Actually, the scattering problem in the near-horizon region of the Kerr-Newman-AdS-dS black hole is not well-defined. Nevertheless, the study of the hidden conformal symmetry of the radial equation in the near horizon region is still fruitful. The reason originates from the universal property of the black hole, which suggests that much of the black hole property is captured by the near horizon geometry of the black hole. The studies of the entropy, the Hawking radiation and the attractor mechanism all support this picture. In a sense, our study gives another support to this universal picture.
Firstly we investigate the low-frequency limit of a scalar scattering off a Kerr-AdS-dS black hole. We find that for a massless scalar, there is a hidden $SL(2, R) \times SL(2, R)$ conformal symmetry acting on the solution space of the radial wave function in the near-horizon region. The conformal coordinate transformation allows us to read the corresponding left and right temperatures in the dual CFT:

$$T_L = \frac{k(r_+^2 + r_2^2 + 2a^2)}{4\pi a \Xi(r_+ + r_*)}, \quad T_R = \frac{k(r_+ - r_*)}{4\pi a \Xi}.$$ \hspace{1cm} (1.1)

We calculate the central charge of the dual CFT by studying the near-NHEK geometry and find that

$$c_L = c_R = \frac{6a(r_+ + r_*)}{k}. \hspace{1cm} (1.2)$$

Here $r_+$ is the horizon and $r_*, k$ are parameters depending on the properties of the black hole. Honestly speaking, these central charges are derived only in the extremal and near-extremal black holes cases. As in the Kerr case, we expect that they still make sense for the generic non-extremal black holes. At the first looking, the above relations are different from the ones found in the literature. But actually they can reduce to the known ones without trouble. In a sense, the above relations is universal, holding in all cases which have been studied. As the first nontrivial check, with (1.1,1.2), we recover the macroscopic Bekenstein-Hawking entropy of generic Kerr-Newmann-AdS-dS black holes from the Cardy formula.

Next we use the Minkowski prescription of the AdS/CFT correspondence to calculate the real-time correlators for the scalar in the near-horizon geometry of near-extremal Kerr-AdS-dS (near-NHEKS) and find perfect match with the CFT prediction. The reason we focus on the near-NHEKS is that as we move away from the near horizon region, the radial equation changes to a much involved form and is hard to solve. Only for near-extremal case, the radial equation is workable. As a consequence, we have to focus on the frequencies near the super-radiant bound. Then we turn to the superradiant scattering of vector and gravitational perturbations off a near-extremal Kerr-AdS-dS black hole. Using the prescription proposed in [11], we compute the real-time correlators for these perturbations and find perfect agreement with the CFT prediction.

Finally we study the holographic description of generic Kerr-Newman-AdS-dS black hole. We discuss the scattering of a charged scalar off the black hole. Once again, in the near-horizon region, we find the hidden conformal symmetry, which allow us to associate a CFT description of the black hole. We discuss the charged scalar superradiant scattering off a near-extremal Kerr-Newman-AdS-dS black hole, and find that the real-time correlator and greybody factor are in good match with the CFT prediction.

In the next section, we study the low frequency scalar scattering off the Kerr-AdS-dS black hole. In the near-horizon region, the wave function takes a form of hypergeometric function, suggesting a underlying conformal invariance. In section 3, we show the hidden $SL(2, R) \times SL(2, R)$ symmetry acting on the solution space of the massless scalar wave function. In section 4, we discuss the microscopic description of generic non-extremal Kerr-AdS-dS black hole. We obtain the central charges of the near-extremal black holes, and
assume that they take the same form for generic black holes. We compute the CFT entropy via the Cardy formula and find it in perfect agreement with the black hole entropy. We also give a brief review of the real-time correlators in 2D CFT. In section 5, we discuss the scattering of scalar, vector and gravitational perturbations off the near-extremal Kerr-AdS-dS black hole. We compute the real-time correlators from the Minkowski prescription, and find the agreements with the CFT predictions. In section 6, we study the holographic description of a Kerr-Newman-AdS-dS black hole. We end with some discussions in section 7.

2. Scalar scattering off a Kerr-AdS-dS black hole

For a four-dimensional Kerr-AdS-dS black hole, its metric takes the following form in Boyer-Lindquist-type coordinates

\[ ds^2 = -\frac{\Delta_r}{\rho^2} \left( dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right)^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta}{\rho^2} \sin^2 \theta \left( a dt - \frac{(r^2 + a^2)}{\Xi} d\phi \right)^2, \]

where

\[ \Delta_r = (r^2 + a^2)(1 + \frac{r^2}{l^2}) - 2Mr, \]
\[ \Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2 \theta, \]
\[ \rho^2 = r^2 + a^2 \cos^2 \theta, \]
\[ \Xi = 1 - \frac{a^2}{l^2}. \]

(2.2)

Here \( l^{-2} \) is the renormalized cosmological constant, which is positive for dS and negative for AdS. When \( l^{-2} = 0 \), the above metric reduces to the one of a Kerr black hole. The physical mass and angular momentum of the black hole are related to the parameter \( M \) by

\[ M_{\text{phy}} = \frac{M}{\Xi^2}, \quad J = \frac{aM}{\Xi^2}. \]

(2.3)

The black hole horizons are decided by the positive roots of \( \Delta_r = 0 \), among which the outer one is denoted by \( r_+ \). And the Hawking temperature, entropy and angular velocity of the horizon are respectively

\[ T_H = \frac{r_+ (1 + \frac{r_+^2}{l^2} + \frac{3r_+^2}{l^2} - \frac{a^2}{l^2})}{4\pi(r_+^2 + a^2)}, \]
\[ S = \frac{\pi(r_+^2 + a^2)}{\Xi}, \]
\[ \Omega_H = \frac{a\Xi}{r_+^2 + a^2}. \]

(2.4)

For simplicity, let us consider a complex massless scalar field scattering off the Kerr-AdS-dS black hole. With the ansatz

\[ \bar{\Phi} = e^{-i\omega t + i\Phi} \Phi, \]

(2.5)
where \( \omega \) and \( m \) are the quantum numbers, the wave equation is of the form

\[
\partial_r \Delta_r \partial_r \Phi + \frac{(\omega(r^2 + a^2) - ma \Xi)}{\Delta_r} \Phi + \frac{1}{\sin \theta} \partial_\theta (\Delta_\theta \sin \theta \partial_\theta \Phi) - \frac{(m \Xi)^2}{\Delta_\theta \sin^2 \theta} \Phi + \left( \frac{2m \omega \Xi}{\Delta_\theta} - \frac{a^2 \omega^2 \sin^2 \theta}{\Delta_\theta} \right) \Phi = 0.
\]

(2.6)

Let \( \Phi = R(r)S(\theta) \), the above equation could be decomposed into the angular part and the radial part. The angular part is of the form

\[
\frac{1}{\sin \theta} \frac{d}{d \theta} \left( \Delta_\theta \sin \theta \frac{d}{d \theta} S \right) - \frac{(m \Xi)^2}{\Delta_\theta \sin^2 \theta} S + \left( \frac{2m \omega \Xi}{\Delta_\theta} - \frac{a^2 \omega^2 \sin^2 \theta}{\Delta_\theta} \right) S + KS = 0.
\]

(2.7)

Here \( K \) is the separation constant. The radial part of the wave function is of the form

\[
\partial_r (\Delta_r \partial_r R) + V_R R = 0
\]

(2.8)

with

\[
V_R = -K + \frac{(\omega(r^2 + a^2) - ma \Xi)^2}{\Delta_r}.
\]

(2.9)

As we are interested in the low frequency limit, \( \omega a \ll 1 \), the \( \omega^2 \) term in the angular equation could be neglected. One important consequence is that the separation constant is independent of \( \omega \). Another key point in finding the hidden conformal symmetry in the Kerr case is to focus on the “Near” region \( r \omega \ll 1 \), which allows us to simplify the radial equation such that it could be rewritten in terms of the \( SL(2, R) \) quadratic Casimir. The same treatment does not work in the case of Kerr-AdS-dS black hole as the function \( \Delta_r \) is quartic. Nevertheless, we find that if we focus on the near-horizon region then we find the hidden conformal symmetry again. Obviously this region is much restricted than the “Near” region discussed before. In the near-horizon region, we can expand the function \( \Delta_r \) to the quadratic order of \( r - r_+ \),

\[
\Delta_r = (r^2 + a^2)(1 + \frac{r^2}{l^2}) - 2Mr,
\]

\[
\simeq k(r - r_+)(r - r_*)
\]

(2.10)

where \( r_+ \) is the outer horizon, and

\[
k = 1 + \frac{a^2}{l^2} + \frac{6r_+^2}{l^2},
\]

(2.11)

\[
r_* = r_+ - \frac{1}{kr_+} \left( r_+^2 - a^2 + \frac{3r_+^4}{l^2} + \frac{a^2 r_+^2}{l^2} \right).
\]

(2.12)

Note that in general \( r_* \) is not the inner horizon. Only in the cases that \( \Delta_r \) are quadratic, which happens in the Kerr, Kerr-Newman and RN case, \( r_* \) coincides with the other horizon. Moreover, we have also

\[
r_+^2 \geq \frac{l^2}{6} \left( \sqrt{(1 + a^2/l^2)^2 + 12a^2/l^2} - (1 + a^2/l^2) \right),
\]

(2.13)
in which the equality holds in the extreme case.

As we are in the near-horizon region, for simplicity, we just focus on the case with also \( r_+ \omega \ll 1 \). Now the radial equation could be simplified even more

\[
\partial_r (r - r_+) (r - r_*) \partial_r \mathcal{R}(r) + \frac{r_+ - r_*}{(r - r_+)} A \mathcal{R}(r) + \frac{r_+ - r_*}{(r - r_*)} B \mathcal{R}(r) + C \mathcal{R}(r) = 0, \tag{2.14}
\]

with

\[
A = \frac{(ma \Xi - \omega (r_+^2 + a^2))^2}{k^2 (r_+ - r_*)^2}, \\
B = -\frac{(ma \Xi - \omega (r_*^2 + a^2))^2}{k^2 (r_+ - r_*)^2}, \\
C = -\frac{K}{k}. \tag{2.15}
\]

The equation (2.14) has the solution

\[
\mathcal{R}(z) = z^\alpha (1 - z)^\beta F(a, b, c; z) \tag{2.16}
\]

with \( z = \frac{r - r_+}{r - r_*} \) and

\[
\alpha = -i \sqrt{A}, \quad \beta = \frac{1}{2} (1 - \sqrt{1 - 4C}), \tag{2.17}
\]

and

\[
c = 1 + 2\alpha, \quad a = \alpha + \beta + i \sqrt{B}, \quad b = \alpha + \beta - i \sqrt{B}. \tag{2.18}
\]

Note that the radial equation (2.14) holds only at the very near horizon region. However, to discuss the asymptotic behavior of the radial wave function, one has to move away from the near-horizon region, which could make the expansion to the quadratic order problematic. Nevertheless, if one consider the near-horizon region of the near-extremal black holes, one can still apply the same treatment. From the definition \( z = \frac{r - r_+}{r - r_*} \), only when \( r_+ \simeq r_* \), one do not need to move very far from the horizon to get \( z \to 1 \). In this case, one can discuss the scattering amplitudes in the Near-NHEKS geometry, as we will show in section 5.

3. Hidden conformal symmetry

In this section, we show that the radial equation (2.14) could be written in terms of the \( \text{SL}(2, \mathbb{R}) \) quadratic Casimir.

From the conformal coordinates

\[
\begin{align*}
\omega^+ &= \sqrt{\frac{r - r_+}{r - r_*}} e^{2\pi T_R \phi + 2n_R t}, \\
\omega^- &= \sqrt{\frac{r - r_+}{r - r_*}} e^{2\pi T_L \phi + 2n_L t}, \\
y &= \sqrt{\frac{r_+ - r_*}{r - r_*}} e^{\pi (T_L + T_R) \phi + (n_L + n_R) t},
\end{align*}
\]

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we can locally define the vector fields

\[ H_1 = i \partial_+ \]
\[ H_0 = i \left( \omega^+ \partial_+ + \frac{1}{2} y \partial_y \right) \]
\[ H_{-1} = i (\omega^+ \partial_+ + \omega^- y \partial_y - y^2 \partial_-) \] (3.1)

and

\[ \tilde{H}_1 = i \partial_- \]
\[ \tilde{H}_0 = i \left( \omega^- \partial_- + \frac{1}{2} y \partial_y \right) \]
\[ \tilde{H}_{-1} = i (\omega^- \partial_- + \omega^+ y \partial_y - y^2 \partial_+) \] (3.2)

These vector fields obey the \( SL(2, R) \) Lie algebra

\[ [H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \quad [H_{-1}, H_1] = -2i H_0, \] (3.3)

and similarly for \( (\tilde{H}_0, \tilde{H}_{\pm 1}) \). The quadratic Casimir is

\[ \mathcal{H}^2 = \tilde{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) \]
\[ = \frac{1}{4} (y^2 \partial_y^2 - y \partial_y) + y^2 \partial_\pm \partial_- \] (3.4)

In terms of \((t, r, \phi)\) coordinates, the Casimir becomes

\[ \mathcal{H}^2 = (r - r_+) (r - r_*) \frac{\partial^2}{\partial r^2} + (2r - r_+ - r_*) \frac{\partial}{\partial r} \]
\[ + \frac{r_+ - r_*}{r - r_+} \left( \frac{n_L - n_R}{4\pi G} \partial_\phi - \frac{T_L - T_R}{4G} \partial_t \right)^2 - \frac{r_+ - r_*}{r - r_+} \left( \frac{n_L + n_R}{4\pi G} \partial_\phi - \frac{T_L + T_R}{4G} \partial_t \right)^2 \] (3.5)

where \( G = n_L T_R - n_R T_L \). We find that with the following identification

\[ n_R = 0, \quad n_L = -\frac{1}{2(r_+ + r_*)} \]
\[ T_R = \frac{k(r_+ - r_*)}{4\pi a \Xi}, \quad T_L = \frac{k(r_+^2 + r_*^2 + 2a^2)}{4\pi a \Xi (r_+ + r_*)} \] (3.6)

the radial equation (2.14) of the scalar, is the same as

\[ \tilde{\mathcal{H}}^2 \mathcal{R}(r) = \mathcal{H}^2 \mathcal{R}(r) = K \mathcal{R}(r). \] (3.7)

In other words, the scalar Laplacian is just the \( SL(2, R) \) Casimir.

As pointed out in [14], the vector fields are not globally defined. In fact, due to the periodic identification \( \phi \sim \phi + 2\pi \), the \( SL(2, R) \times SL(2, R) \) symmetry is spontaneously broken down to \( U(1)_L \times U(1)_R \) subgroup. As a result, we can identify the left and right temperatures in the dual CFT.
4. Microscopic description

From the identification \( (1.1) \), we know the corresponding left and right temperatures in the dual CFT. In order to have a microscopic description of the black hole, we need to determine the central charges of the dual CFT. For the extremal and near-extremal Kerr black holes, the central charges were derived from the asymptotic symmetry group of the NHEK and near-NHEK geometry\([2, 4]\), similar to the study of BTZ black hole in three-dimensional gravity\([24]\). For a generic non-extremal Kerr black hole, there is no derivation on the central charges. It was conjectured that the central charges in the near-extremal case could be generalized to the generic cases\([14]\). This treatment has been proved to be valid in the Kerr and also Kerr-Newman black holes\([18]\). Here we will follow the same logic.

We start from the near-extreme black hole and consider its near-horizon geometry. Similar to the Kerr case, let us try the following coordinate transformation

\[
\begin{align*}
  r & = \frac{r_+ + r_*}{2} + \epsilon r_0 \tilde{r}, & r_+ - r_* = \lambda \epsilon r_0, & t = \frac{r_0}{\epsilon}, & \phi = \tilde{\phi} + \Omega_H t / \epsilon \\
  r_+ - r_* & = \lambda \epsilon r_0, & t = \frac{r_0}{\epsilon}, & \phi = \tilde{\phi} + \Omega_H t / \epsilon \\
  \end{align*}
\]

(4.1)

then we have the near-horizon geometry of near-extremal Kerr-AdS-dS black hole

\[
\begin{align*}
  ds^2 = \Gamma(\theta) \left( -\left( \frac{\hat{r} - \lambda}{2} \right) (\hat{r} + \frac{\lambda}{2}) d\hat{t}^2 + \frac{d\hat{r}^2}{(\hat{r} - \frac{\lambda}{2})(\hat{r} + \frac{\lambda}{2})} + \alpha(\theta) d\theta^2 \right) + \gamma(\theta)(d\hat{\phi} + \hat{p} dt)^2
\end{align*}
\]

(4.2)

where

\[
\begin{align*}
  \Gamma(\theta) &= \frac{\rho^2_+ r_0^2}{r_+^2 + a^2}, & \alpha(\theta) &= \frac{r_+^2 + a^2}{\Delta r_0^2}, & \gamma(\theta) &= \frac{\Delta r^2 \sin^2 \theta (r_+^2 + a^2)^2}{\rho^2_+ r_0^2}, \\
  \hat{p} &= \frac{ar^2_0 \Xi (r_+ + r_*)}{(r_+^2 + a^2)^2}, & \rho^2_+ &= r_+^2 + a^2 \cos^2 \theta, & r_0^2 &= \frac{r_+^2 + a^2}{k}.
\end{align*}
\]

(4.3)

(4.4)

The same geometry has been discussed in \([26]\).

From the general argument in \([23]\), the central charge should be

\[
\begin{align*}
  c_L = c_R &= 6p \int_0^\pi d\theta \sqrt{\Gamma(\theta) \alpha(\theta) \gamma(\theta)} \\
  &= 6a \frac{r_+ + r_*}{k}.
\end{align*}
\]

(4.5)

This result looks different from the one obtained in the literature. However, note that in the extreme limit, we have

\[
\begin{align*}
  c_L = c_R &= 6a \frac{r_+^2 + a^2 + \frac{a^2 r_+^2}{r_+^2} + 9r_+^4}{r_+ k^2},
\end{align*}
\]

(4.6)

which is consistent with the result found in \([23, 29]\). Honestly speaking, the central charge \((4.3)\) is derived in the near-extremal limit, its robustness for the generic non-extremal black holes needs to be checked. One support to this identification is that it gives correct black hole entropy, as we will show very soon.

Another subtle issue is on the right central charge. Here we actually assume that the left and right central charges should be same, similar to the Kerr case. One support
evidence is that when we take \( l^{-2} = 0 \) limit, the above central charges reduce to the ones in the Kerr case. We find no reason to break the symmetry between the left- and right-movers on this issue.

The Kerr/CFT correspondence suggests that the Kerr-AdS-dS black hole is dual to a CFT with central charges \([4.3]\) at finite temperature \((T_L, T_R)\) given in \([3.6]\). This should be true for every value of angular momentum.

### 4.1 Thermodynamics

As a first check of this conjecture in the Kerr-AdS-dS case, we show that the entropy of the black hole could be recovered from dual CFT. The Cardy formula gives the microscopic entropy

\[
S = \frac{\pi^2}{3} (c_LT_L + c_RT_R). \tag{4.7}
\]

From the central charges \([4.3]\) and the temperatures \([3.6]\), we have

\[
S = \frac{\pi (r_+^2 + a^2)}{\Xi} \tag{4.8}
\]

which is in perfect agreement with the macroscopic Bekenstein-Hawking area law for the entropy of the Kerr-AdS-dS black hole.

To determine the conjugate charges, we begin with the first law of thermodynamics

\[
\delta S = \frac{\delta M - \Omega \delta J}{T_H} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \tag{4.9}
\]

The solution is

\[
\delta E_L = \frac{r_+^2 + r_+^2 + 2a^2}{2a\Xi} \delta M,
\]

\[
\delta E_R = \frac{r_+^2 + r_+^2 + 2a^2}{2a\Xi} \delta M - \delta J. \tag{4.10}
\]

If we identify

\[
\delta M = \omega, \quad \delta J = m,
\]

\[
\omega_L = \frac{r_+^2 + r_+^2 + 2a^2}{2a\Xi} \omega, \quad \omega_R = \frac{r_+^2 + r_+^2 + 2a^2}{2a\Xi} \omega - m, \tag{4.11}
\]

we have

\[
\delta E_L = \omega_L, \quad \delta E_R = \omega_R. \tag{4.12}
\]

### 4.2 Correlators in 2D CFT

Another support to the Kerr/CFT conjecture is on the study of the scattering amplitudes. We will focus on the scattering in the near-NHEKS region. Before we analyze the scattering amplitudes, let us give a brief review on the correlators in the dual 2D CFT.

In a 2D conformal field theory(CFT), one can define the two-point function

\[
G(t^+, t^-) = \langle \mathcal{O}_\phi^\dagger(t^+, t^-) \mathcal{O}_\phi(0) \rangle, \tag{4.13}
\]
where \( t^+, t^- \) are the left and right moving coordinates of 2D worldsheet and \( \mathcal{O}_\phi \) is the operator corresponding to the field perturbing the black hole. For an operator of dimensions \((\hbar_L, \hbar_R)\), charges \((q_L, q_R)\) at temperatures \((T_L, T_R)\) and chemical potentials \((\mu_L, \mu_R)\), the two-point function is dictated by conformal invariance and takes the form \([27]\):

\[
G(t^+, \tau^-) \sim (-1)^{h_L + h_R} \left( \frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left( \frac{\pi T_R}{\sinh(\pi T_R \tau^-)} \right)^{2h_R} e^{i q_L \mu_L t^+ + i q_R \mu_R \tau^-}. \tag{4.14}
\]

The CFT absorption cross section could be defined with the two-point functions, following Fermi’s golden rule:

\[
\sigma_{\text{abs}} \sim \int dt^+ d\tau^- e^{-i \omega_R \tau^- - i \omega_L t^+} [G(t^+ - i\epsilon, \tau^- - i\epsilon) - G(t^+ + i\epsilon, \tau^- + i\epsilon)] \tag{4.15}
\]

Then after being changed into momentum space, the absorption cross section is

\[
\sigma \sim T_L^{2h_L - 1} T_R^{2h_R - 1} \sinh \left( \frac{\omega_L - q_L \mu_L}{2T_L} + \frac{\omega_R - q_R \mu_R}{2T_R} \right) \left| \Gamma \left( h_L + \frac{i(\omega_L - q_L \mu_L)}{2\pi T_L} \right) \right|^2 \left| \Gamma \left( h_R + \frac{i(\omega_R - q_R \mu_R)}{2\pi T_R} \right) \right|^2. \tag{4.16}
\]

The retarded correlator \( G_R(\omega_L, \omega_R) \) is analytic on the upper half complex \( \omega_{L,R}\)-plane and its value along the positive imaginary \( \omega_{L,R}\)-axis gives the Euclidean correlator:

\[
G_E(\omega_{L,E}, \omega_{R,E}) = G_R(i\omega_{L,E}, i\omega_{R,E}), \quad \omega_{L,E}, \omega_{R,E} > 0. \tag{4.17}
\]

At finite temperature, \( \omega_{L,E} \) and \( \omega_{R,E} \) take discrete values of the Matsubara frequencies

\[
\omega_{L,E} = 2\pi m_L T_L, \quad \omega_{R,E} = 2\pi m_R T_R, \tag{4.18}
\]

where \( m_L, m_R \) are integers for bosonic modes and are half integers for fermionic modes.

In a 2D CFT, the Euclidean correlator \( G_E \) is obtained by a Wick rotation \( t^+ \rightarrow i\tau_L, \ t^- \rightarrow i\tau_R \), and is determined by the conformal symmetry. At finite temperature the Euclidean time is taken to have period \( 2\pi/T_L, 2\pi/T_R \) and via analytic continuation the momentum space Euclidean correlator is given by \([28]\)

\[
G_E(\omega_{L,E}, \omega_{R,E}) \sim T_L^{2h_L - 1} T_R^{2h_R - 1} e^{i \tilde{\omega}_{L,E} \tau_L} e^{i \tilde{\omega}_{R,E} \tau_R} \cdot \Gamma(h_L + \frac{\tilde{\omega}_{L,E}}{2\pi T_L}) \Gamma(h_R + \frac{\tilde{\omega}_{L,E}}{2\pi T_L}) \Gamma(h_R + \frac{\tilde{\omega}_{R,E}}{2\pi T_R}). \tag{4.19}
\]

where

\[
\tilde{\omega}_{L,E} = \omega_{L,E} - iq_L \mu_L, \quad \tilde{\omega}_{R,E} = \omega_{R,E} - iq_R \mu_R. \tag{4.20}
\]

It is remarkable that since the absorption cross section is closely related to the retarded Green’s function. Actually \( P_{\text{abs}} = \text{Im}(G_R) \). It seems that the retarded Green’s function encodes more information. However via the spectral theorem, it could be true that the absorption cross section can determine the retarded Green’s function uniquely. We should not take the greybody factor and the real-time correlators as the independent check of the Kerr/CFT correspondence. Nevertheless, it would still be valuable to compute the real-time correlators directly in the framework of AdS/CFT correspondence.
5. Superradiant scattering off Kerr-AdS-dS black hole

In \cite{11}, it was shown that in the Kerr/CFT correspondence, one can compute the real-time correlators from usual prescription in AdS/CFT correspondence. In \cite{18}, we showed that this is still true for the low-frequency scattering off generic non-extremal Kerr(-Newman) black holes. Here we apply the same prescription to compute the retarded Green’s functions in the near-NHEKS geometry.

As we discussed before, to study the scattering problem we have to move away from the near-horizon region. For a generic non-extremal black hole, the radial wave function becomes intractable. However, for the near-extremal Kerr-AdS-dS black hole, we can still investigate the scattering in the near-horizon region, similar to the study in the Kerr(-Newman) case\cite{6, 7, 11}.

From the coordinate transformation (4.1), we have
\[
\tilde{\Phi} = e^{-i\omega t + im\phi} \Phi = e^{-i(\omega - m\Omega_H)\hat{t}} + im\hat{\phi} \Phi,
\]
(5.1)

This indicates that we need to focus on the frequencies near the superradiant bound
\[
\omega - m\Omega_H = \hat{\omega} \frac{\epsilon}{r_0},
\]
with \(\hat{\omega}\) being finite.

5.1 Scalar scattering

For the massless scalar scattering in the near-NHEKS region, the angular equation of the wave function is of the form
\[
\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \Delta_{\theta} \sin \theta \frac{d}{d\theta} \right) S - \frac{(m\Xi)^2}{\Delta_{\theta} \sin^2 \theta} S + \frac{m^2 a \Xi \Omega_H}{\Delta_{\theta}} \left( 2 - \frac{\sin^2 \theta}{r_+^2 + a^2} \right) S + \hat{K} S = 0.
\]
(5.3)

The separation constant \(\hat{K}\) is different from the one in the low-frequency limit. Therefore, the conformal weight of the fields change correspondingly.

For the radial part of the wave function, the solution is just
\[
\mathcal{R}(z) = z^{\hat{\alpha}}(1 - z)^{\hat{\beta}} F(\hat{a}, \hat{b}, \hat{c}; z)
\]
(5.4)

with
\[
z = \frac{\hat{\omega} - \lambda/2}{\hat{\omega} + \lambda/2}
\]
and
\[
\hat{\alpha} = -i \sqrt{\hat{A}}, \quad \hat{\beta} = \frac{1}{2} \left( 1 - \sqrt{1 - 4\hat{C}} \right),
\]
(5.5)

and
\[
\hat{c} = 1 + 2\hat{\alpha}, \quad \hat{a} = \hat{\alpha} + \hat{\beta} + i \sqrt{-\hat{B}}, \quad \hat{b} = \hat{\alpha} + \hat{\beta} - i \sqrt{-\hat{B}},
\]
\[
\hat{A} = \frac{\hat{\omega}^2}{\hat{\lambda}^2}, \quad \hat{B} = -\left( \frac{\hat{\omega}}{\hat{\lambda}} - \frac{2r_+ m\Omega_H}{k} \right)^2, \quad \hat{C} = \frac{\hat{K}}{k}.
\]
(5.6)

For a scalar field in a black hole background, the prescription for two-point real-time correlators was first proposed in \cite{3}. It could be simplified as follows. For the scalar wave
function satisfying the ingoing boundary condition at the black hole horizon, its asymptotic behavior is
\[ \phi \sim A r^{h-1} + B r^{-h}. \] (5.7)
Then taken \( A \) as the source term and \( B \) as the response term, the two-point retarded correlator is just
\[ G_R \sim \frac{B}{A}. \] (5.8)
For the scalar in the near-NHEKS geometry, its retarded Green’s function is just
\[ G_R \sim \frac{\Gamma(1-2\hat{h})}{\Gamma(2\hat{h}-1)} \frac{\Gamma\left(\hat{h} + i \frac{\omega_L}{2\pi T_L}\right) \Gamma\left(\hat{h} + i \frac{\omega_R}{2\pi T_R}\right)}{\Gamma\left(1 - \hat{h} + i \frac{\omega_L}{2\pi T_L}\right) \Gamma\left(1 - \hat{h} + i \frac{\omega_R}{2\pi T_R}\right)} \] (5.9)
with the identifications
\[ \hat{T}_L = \frac{k}{4\pi r_+ \Omega_H}, \quad \hat{T}_R = \frac{k r_0}{4\pi a \Xi} \chi, \] (5.10)
\[ \hat{\omega}_L = m, \quad \hat{\omega}_R = \frac{r_0 k}{a \Xi} \left( \hat{\omega} - \frac{\lambda r_+ m \Omega_H}{k} \right) \epsilon, \] (5.11)
\[ \hat{h} = \frac{1}{2} (1 + \sqrt{1 - 4C}) \] (5.12)
This is in good match with the CFT prediction (4.17).
It is remarkable that the identifications (5.10) are exactly the same as the ones (3.6)(4.11), which were obtained respectively from the conformal coordinate transformation in the low-frequency limit and the first law of thermodynamics for generic non-extremal black hole. This provides another nontrivial evidence to support the Kerr/CFT correspondence for the Kerr-AdS-dS black holes.
Note also that the identifications (5.10,5.11) are slightly different from the ones used in \([6, 7, 11]\), in which the right temperature was set to be finite in the scaling limit. To accommodate the Kerr/CFT correspondence for generic non-extremal black hole, the right temperatures and the right frequencies are set to be very small.

5.2 Photons and gravitons scattering
To study the perturbations with nonvanishing spin, one has to apply the Newman-Penrose formalism \([29]\). For the Kerr-AdS-dS black hole, this problem has been discussed in \([31]\) (see also Khanal:1983vb). It turned out that the equations of motion of the perturbations can be decomposed into the angular part and the radial part. The wave function is of the form
\[ \Psi^s = e^{-i\omega t + im\phi} R^s(r) S^s(\theta). \] (5.13)
\( \Psi^s \) are related to the electromagnetic field strength and Weyl tensor for spin-1 and spin-2 perturbations. The angular and radial functions satisfy the Teukolsky master equations. The angular part takes the form:
\[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \Delta_\theta \frac{d}{d\theta} S^s(\theta) \right) + \left( K^s - sN(\theta) - \frac{M(\theta)^2}{\Delta_\theta \sin^2 \theta} \right) S^s(\theta) = 0, \] (5.14)
with
\[
M(\theta) = m \Xi - \omega a \sin^2 \theta + s \cos \theta \left(1 + \frac{a^2}{l^2} - \frac{2a^2 \cos^2 \theta}{l^2}\right),
\]
\[
N(\theta) = 1 + \frac{a^2}{l^2} + 4\omega \cos \theta - \frac{6a^2 \cos^2 \theta}{l^2}.
\] (5.15)

The radial wave function is
\[
\left(D_{-s/2} D_{s/2} \right) R^s = 0,
\] (5.16)
where
\[
D_n = \partial_r + \frac{iH_r}{\Delta_r} + n \frac{\Delta'}{\Delta_r},
\]
\[
D_{-n} = \partial_r - \frac{iH_r}{\Delta_r} + n \frac{\Delta'}{\Delta_r},
\]
\[
H_r = am \Xi - (r^2 + a^2)\omega.
\] (5.17)

Here $K^s$ is the separation constant, satisfying
\[
K^s(s) = K^s(-s).
\] (5.18)

For the scattering in the near-NHEKS geometry, we need to consider the frequencies near the superradiant bound. In this limit, the $\omega$ dependent terms in the angular equation could be evaluated at the superradiant bound. As a result the separation constant is independent of $\omega$.

In fact, for the spin 2 case, there are extra terms proportional to the cosmological constant in the angular and radial equation. In the angle equation, this term may change the value of separation constant. In the radial equation, such a term is quadratic to $r^2$. However, in the near-horizon region, such a term contribute a constant and may induce a change of the conformal weight. In the following, we will not keep the track of such terms, since they are not relevant for our analysis.

Similar to the scalar case, we will focus on the near horizon region $r \rightarrow r_+$. For the superradiant scattering in the near-NHEKS geometry, the radial equation reduces to
\[
\frac{d}{d\hat{r}} \left(\hat{r} - \frac{\lambda}{2}\right) \left(\hat{r} + \frac{\lambda}{2}\right) \frac{dR^s(\hat{r})}{d\hat{r}} + \frac{\lambda}{\hat{r} - \lambda/2} A^s R^s(\hat{r}) + \frac{\lambda}{\hat{r} + \lambda/2} B^s R^s(\hat{r}) + C^s R^s(\hat{r}) = 0
\] (5.19)
with
\[
A^s = \left(\frac{\hat{\omega}}{\lambda} - \frac{is}{2}\right)^2,
\]
\[
B^s = -\left(\frac{\hat{\omega}}{\lambda} + \frac{is}{2} - \frac{2m\Omega_H r_+}{k}\right)^2,
\]
\[
C^s = s - \frac{K^s}{k} + \frac{i4sm\Omega_H r_+}{k} + \left(\frac{2m\Omega_H r_+}{k} - is\right)^2.
\]
The solution satisfying the ingoing boundary condition at the horizon could be once again written in terms of hypergeometric function

$$ R_s = z^{\alpha_s} (1 - z)^{\beta_s} F(a_s, b_s, c_s; z), \quad (5.20) $$

where \( z = \frac{\hat{r} - \lambda/2}{\hat{r} + \lambda/2} \) and

$$
\begin{align*}
\alpha_s &= -i \frac{\hat{\omega}}{\lambda} - \frac{s}{2} \\
\beta_s &= \frac{1}{2} (1 - \sqrt{1 - 4\hat{C}}) \\
a_s &= \beta_s - s - i \frac{\hat{\omega}_L}{2\pi \hat{T}_L} \\
b_s &= \beta_s - i \frac{\hat{\omega}_R}{2\pi \hat{T}_R} \\
c_s &= 1 - s - i \frac{2\hat{\omega}}{\lambda} \quad (5.21)
\end{align*}
$$

Here \( \hat{C} = C^s(\omega_s) \), where \( \omega_s = m\Omega_H \) is the frequency saturating the superradiant bound.

The asymptotic behavior of the radial wave function is

$$ R^s(r) \sim A_1^{s} r^{h_s - 1} + A_2^{s} r^{-h_s}, \quad (5.22) $$

where

$$
\begin{align*}
h_s &= \frac{1}{2} (1 + \sqrt{1 - 4\hat{C}^s}) \\
A_1^{s} &= \frac{\Gamma(2h_s - 1)}{\Gamma(-s + h_s - i \frac{\hat{\omega}_L}{2\pi \hat{T}_L}) \Gamma(h_s - i \frac{\hat{\omega}_L}{2\pi \hat{T}_L})} \\
A_2^{s} &= \frac{\Gamma(1 - 2h_s)}{\Gamma(-s + 1 - h_s - i \frac{\hat{\omega}_R}{2\pi \hat{T}_R}) \Gamma(1 - h_s - i \frac{\hat{\omega}_R}{2\pi \hat{T}_R})}
\end{align*}
$$

One may calculate the absorption cross sections following the way in [7] and compare the results with the CFT prediction. It turns out to be in perfect match. We will not present the details here. Instead, we give an alternative derivation from the retarded Green’s functions.

For the vector and gravitational perturbations, the prescription has been proposed in [11]. If the radial wave function of the perturbation with the spin \( s \) satisfying the ingoing boundary condition at the black hole horizon has asymptotic behavior as

$$ R^s(r) \sim A_1^{s} r^{h_s - 1} + A_2^{s} r^{-h_s}, \quad (5.23) $$

then the retarded Green’s function could be

$$ G^s_{\hat{r}} \sim \frac{A_2^{s}}{A_1^{s}}. \quad (5.24) $$
In our case, this leads to
\[
G_R^s \sim \frac{\Gamma(1 - 2h^s)}{\Gamma(2h^s - 1)} \frac{\Gamma \left( -s + h^s - i \frac{\hat{\omega}_L}{2\pi T_L} \right)}{\Gamma \left( s + 1 - h^s - i \frac{\hat{\omega}_R}{2\pi T_R} \right)} \frac{\Gamma \left( h^s - i \frac{\hat{\omega}_R}{2\pi T_R} \right)}{\Gamma \left( 1 - h^s - i \frac{\hat{\omega}_R}{2\pi T_R} \right)},
\]
(5.25)
where \(\hat{T}_{L,R}\hat{\omega}_{L,R}\) are the ones suggested in [5.10,5.11]. Note that in Kerr-AdS-dS case, the chemical potentials \(\mu_{L,R}\) are absent. With the conformal weights of the fields being identified as
\[
h^s_R = h^s, \quad h^s_L = h^s_R - s,
\]
(5.26)
the above retarded Green’s function agrees precisely, up to a normalization factor, with the CFT result (4.19) at the Matsubara frequencies. The cross section can be read directly from the above Green’s function
\[
\sigma^s \sim \text{Im}(G_R^s) \sim \frac{1}{(\Gamma(h^s_R - 1))^2} \sinh \left( \frac{\hat{\omega}_L}{2T_L} + \frac{\hat{\omega}_R}{2T_R} \right) \times \left| \Gamma \left( h^s_L + i \frac{\hat{\omega}_L}{2\pi T_L} \right) \right|^2 \left| \Gamma \left( h^s_R + i \frac{\hat{\omega}_R}{2\pi T_R} \right) \right|^2.
\]
(5.27)
They agree with the CFT result.

6. Holographic description of Kerr-Newman-AdS-dS black hole

For a four-dimensional Kerr-Newman-AdS-dS black hole, its metric takes the following form in Boyer-Lindquist-type coordinates[25]
\[
ds^2 = - \frac{\Delta_r}{\rho^2} \left( dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right)^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta}{\rho^2} \sin^2 \theta \left( a dt - \frac{(r^2 + a^2)}{\Xi} d\phi \right)^2,
\]
(6.1)
where
\[
\Delta_r = (r^2 + a^2)(1 + \frac{r^2}{l^2}) - 2Mr + q^2, \quad q^2 = q_e^2 + q_m^2
\]
\[
\Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2 \theta,
\]
\[
\rho^2 = r^2 + a^2 \cos^2 \theta,
\]
\[
\Xi = 1 - \frac{a^2}{l^2}.
\]
(6.2)
Here \(l^{-2}\) is the renormalized cosmological constant, which is positive for dS and negative for AdS. When \(l^{-2} = 0\), the above metric reduces to the one of a Kerr-Newman black hole. The physical mass, angular momentum and charges of the black hole are related to the parameter \(M, q_{e,m}\) by
\[
M_{\text{ADM}} = \frac{M}{\Xi^2}, \quad J = \frac{aM}{\Xi^2}, \quad Q_{e,m} = \frac{q_{e,m}}{\Xi}.
\]
(6.3)
The gauge potential and its field strength are respectively
\[
A = - \frac{qe r}{\rho^2} (dt - \frac{a \sin^2 \theta}{\Xi} d\phi) - \frac{qm \cos \theta}{\rho^2} (adt - \frac{r^2 + a^2}{\Xi} d\phi),
\]
\[
F = - \frac{1}{\rho^4} (qe (r^2 - a^2 \cos^2 \theta) + 2qm r a \cos \theta) (dt - \frac{a \sin^2 \theta}{\Xi} d\phi) \wedge dr
\]
\[
+ \frac{1}{\rho^4} (qm (r^2 - a^2 \cos^2 \theta) - 2qe r a \cos \theta) \sin \theta d\theta \wedge (adt - \frac{r^2 + a^2}{\Xi} d\phi).
\]
(6.4)

In the following, for simplicity, we just focus on the case with only electric charge \( q_e = q, q_m = 0 \). The Hawking temperature, entropy and angular velocity of the horizon are respectively
\[
T_H = \frac{r_+ (1 + \frac{a^2}{r_+^2} + \frac{3a^2}{r_+^2} - \frac{a^2 + q_e^2}{r_+^2})}{4\pi (r_+^2 + a^2)},
\]
\[
S = \frac{\pi (r_+^2 + a^2)}{\Xi},
\]
\[
\Omega_H = \frac{a \Xi}{r_+^2 + a^2}.
\]
(6.6)

The electric potential \( \Phi \), measured at infinity with respect to the horizon, is
\[
\Phi_e = A_\mu \xi^\mu|_{r \to \infty} = \frac{qe r_+}{r_+^2 + a^2},
\]
(6.7)

where \( \xi = \partial_t + \Omega_H \partial_\phi \) is the null generator of the horizon.

For a scalar with charge \( e \) and mass \( \mu \), the Klein-Gordon equation is
\[
(\nabla_\mu + ieA_\mu)(\nabla^\mu + ieA^\mu)\tilde{\Phi} - \mu^2 \tilde{\Phi} = 0.
\]
(6.8)

In the following, we just focus on the massless case. With the ansatz
\[
\Phi = e^{-i\omega t + im\phi} S(\theta) R(r),
\]
(6.9)

where \( \omega \) and \( m \) are the quantum numbers, the wave equation could be decomposed into an angular part and a radial part. The angular part has the form:
\[
\frac{1}{\sin \theta} \partial_\theta \sin \theta \Delta_\theta \partial_\theta S(\theta) - \frac{(m^2 + l^2)}{\Delta_\theta} S(\theta) + \frac{2ma \Xi \omega - a^2 \omega^2 \sin^2 \theta}{\Delta_\theta} S(\theta) = K_q S(\theta),
\]
(6.10)

where \( K_q \) is the separation constant. And the radial wave-function is of the form
\[
\partial_r \Delta_r \partial_r + \frac{(H - eqr)^2}{\Delta_r} R - K_q R = 0.
\]
(6.11)

In the near horizon region, the radial equation could be simplified even more
\[
\partial_r (r - r_+)(r - r_+ \Delta_\theta \partial_r \partial_r R(r) + \frac{r_+ - r_+}{(r - r_+)} A^q R(r) + \frac{r_+ - r_+}{(r - r_+)} B^q R(r) + C^q R(r) = 0,
\]
(6.12)
with

\[ A^q = \frac{(\omega (r_+^2 + a^2) - ma\Xi - eqr_+)^2}{k^2(r_+ - r_*)^2}, \]

\[ B^q = -\frac{(\omega (r_+^2 + a^2) - ma\Xi - eqr_*)^2}{k^2(r_+ - r_*)^2}, \]

\[ C^q = \frac{e^2q^2}{k^2} - \frac{K_q}{k}, \] (6.13)

In fact, there is also a hidden conformal symmetry acting on the solution space. For the neutral scalar, the above radial equation could be rewritten in terms of the \( SL(2, R) \) quadratic Casimir. The discussion is the same as the one we did in section 3. In the end, we have the same temperature identification (3.6).

Similarly, from the near horizon geometry of the near-extremal Kerr-Newman-AdS-dS black holes, we can read out the central charges of the dual CFT. They turn out to be of the same form as (4.5). With them and the temperatures (3.6), we recover the Bekenstein-Hawking entropy of the black hole from the microscopic counting via the Cardy formula.

From the first law of thermodynamics, \( \delta S = \frac{\delta M - \Omega \delta J - \Phi \delta Q}{T_H} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R} \), we find that

\[ \delta E_L = \omega_L - q_L \mu_L, \quad \delta E_R = \omega_R - q_R \mu_R, \] (6.14)

where we have applied the identification

\[ \delta M = \omega, \quad \delta J = m, \quad \delta Q = e, \]

\[ \omega_L = \frac{r_s^2 + r_+^2 + 2a^2}{2a\Xi}\omega, \quad \omega_R = \frac{r_s^2 + r_+^2 + 2a^2}{2a\Xi}\omega - m, \] (6.15)

\[ q_L = q_R = \delta Q = e, \]

\[ q_L = q_R = \frac{q(r_+ + r_s)}{2a\Xi(r_+ + r_s)}, \quad \mu_R = \frac{q(r_+ + r_s)}{2a\Xi}. \] (6.16)

(6.17)

As in usual Kerr/CFT correspondence, we can associate a CFT description to a generic non-extremal Kerr-Newman-AdS-dS black hole. The dual CFT has the temperatures \((T_L, T_R)\) as (3.6), the central charges (4.11) and the chemical potential (6.17).

To lend more support to the above conjecture, let us study the scattering off the black hole more carefully. We focus on the scattering in the near-horizon geometry of near-extremal Kerr-Newman-Kerr-AdS-dS (near-NHEKNS). As before, we need to focus on the frequencies near the superradiant bound,

\[ \omega \sim \omega_s = m\Omega_H + e\Phi_e. \] (6.18)

In order to study the superradiant scattering in the near-NHEKNS region, it is convenient to introduce the coordinate transformation (4.1). The wave function of the radial equation is then

\[ R(z) = z^{\alpha_q}(1 - z)^{\beta_q} F(a_q, b_q, c_q; z) \] (6.19)

with \( z = \frac{r - \lambda/2}{r + \lambda/2} \),

\[ \alpha_q = -i\sqrt{A^q}, \quad \beta_q = \frac{1}{2}(1 - \sqrt{1 - 4\tilde{C}^q}), \] (6.20)
and

\[ c_q = 1 + 2\alpha_q, \quad a_q = \alpha_q + \beta_q + i\sqrt{-B_q}, \quad b_q = \alpha_q + \beta_q - i\sqrt{-B_q} \]

\[ \hat{A}_q = \frac{\hat{\omega}^2}{\lambda^2}, \quad \hat{B}_q = -(\frac{\hat{\omega}}{\lambda} - \frac{2m\Omega_{H}r_+}{k} + \frac{eqa^2 - r_+^2}{k a^2 + r_+^2}) \]

The solution behaves asymptotically as

\[ R(r) \sim A_1^q r^{h_q - 1} + A_2^q r^{q} \]

where \( h_q \) is the conformal weight of the scalar field

\[ h_q = 1 - \beta_q = \frac{1}{2}(1 + \sqrt{1 - 4C_q}). \]

Taking the \( A_1^q \) as the source and \( A_2^q \) as the response, the retarded Green’s function is just

\[ G_R \sim \frac{A_2^q}{A_1^q} \]

\[ = \frac{\Gamma(1 - 2h_q)}{\Gamma(2h_q - 1)} \frac{\Gamma \left( h_q + i \left( \frac{2m\Omega_{H}r_+}{k} - \frac{eqa^2 - r_+^2}{k a^2 + r_+^2} \right) \right) \Gamma \left( h_q + i \left( 2\frac{\hat{\omega}}{\lambda} - \frac{2m\Omega_{H}r_+}{k} + \frac{eqa^2 - r_+^2}{k a^2 + r_+^2} \right) \right)}{\Gamma \left( 1 - h_q + i \left( \frac{2m\Omega_{H}r_+}{k} - \frac{eqa^2 - r_+^2}{k a^2 + r_+^2} \right) \right) \Gamma \left( 1 - h_q + i \left( 2\frac{\hat{\omega}}{\lambda} - \frac{2m\Omega_{H}r_+}{k} + \frac{eqa^2 - r_+^2}{k a^2 + r_+^2} \right) \right)} \]

\[ = \frac{\Gamma\left( 1 - 2h_q \right)}{\Gamma\left( 2h_q - 1 \right)} \frac{\Gamma \left( h_q + i\omega k - q_{\mu \nu} \xi_{\mu \nu} \right)}{\Gamma \left( 1 - h_q + i\omega k - q_{\mu \nu} \xi_{\mu \nu} \right)} \frac{\Gamma \left( h_q + i\omega_{R} - q_{\mu \nu} \xi_{\mu \nu} \right)}{\Gamma \left( 1 - h_q + i\omega_{R} - q_{\mu \nu} \xi_{\mu \nu} \right)} \]

(6.24)

with the identification (6.15-6.17) and (3.6). It is in consistent with the CFT prediction. So is the absorption cross section.

### 7. Discussions

In this paper, we showed that there existed a hidden conformal symmetry in the low-frequency scattering off the Kerr(-Newman)-AdS-dS black holes as well. Different from the Kerr or Kerr-Newman case, we had to focus on the near-horizon region, as the function deciding the horizon is quartic. In the near-horizon region, the radial equation of the wave function could be rewritten as the \( SL(2, R) \) quadratic Casimir, indicating a hidden conformal symmetry acting on the solution space. This local conformal symmetry is broken by periodic identification in the configuration space, which allows us to read out the temperatures of the dual CFT. Consequently, we would like to suggest that a generic 4D Kerr-AdS-dS black hole is dual to a 2D CFT with the temperatures

\[ T_L = \frac{k(r_+^2 + r_+^2 + 2a^2)}{4\pi a^2 (r_+ + r_+^2)}, \quad T_R = \frac{k(r_+ - r_+^2)}{4\pi a^2} \]

(7.1)

and the central charges

\[ c_L = c_R = \frac{6a(r_+ + r_+^2)}{k} \]

(7.2)
which were derived by studying the near-NHEK geometry. Here $r_+$ is the outer horizon of the black hole, while $k$ and $r_*$ are determined by (2.1) (2.2). For the Kerr-Newman-AdS-dS black holes, we have similar holographic picture with addition of chemical potentials to the dual CFT.

We presented the evidence to support this holographic picture. The first evidence is that for a generic black hole, the macroscopic entropy could be recovered from the microscopic counting on the degeneracy in CFT via the Cardy formula. The second evidence is from the study on various kinds of superradiant scattering off the near-extremal black holes. We found that the real-time correlators and so the absorption cross sections were in perfect match with the CFT prediction, under the identification of the quantum numbers derived from the first law of thermodynamics.

In our study, the hidden conformal symmetry and its corresponding conformal coordinates were investigated in the low-frequency limit. The first law of thermodynamics allows us to identify the frequencies and chemical potential in the CFT dual to a generic black hole. However, when we discussed the scattering off the near-extremal black hole, we focused on the frequencies very near the superradiant bound. Thus the perfect match of various real-time correlators with the CFT predictions provides strong support to the picture that a generic Kerr(-Newman)-AdS-dS black hole has a holographic 2D CFT description.

There is a significant difference between our investigation and the other existing one. For the black holes studied in this paper, we had to focus on the near-horizon region to look for the hidden conformal symmetry. It would be interesting to ask why we have such conformal symmetry, since for generic non-extremal black holes we have no freedom in choosing the matching region. Actually we can only solve the radial equation in the near horizon region and cannot discuss the scattering issue as the asymptotic behavior is not well-defined in this case. Only in the near-extremal case, we can zoom in the near-horizon region and study its scattering amplitude. Nevertheless, the radial equation in the near horizon region has the hidden conformal symmetry which allows us to read the dual left and right temperatures. This is in spirit in accordance with the universal property of the black hole, which suggests that the black hole properties such as entropy and Hawking radiation are determined merely by its near-horizon geometry [32, 33].

There are other kinds of black holes which may have dual CFT description. For example, the higher-dimensional extremal Kerr-AdS-dS black holes have been shown to have CFT descriptions. It would be interesting to apply the treatment developed in this paper to study the holographic description of the generic non-extremal black holes in these cases.

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