Tuning the periodic V-peeling behavior of elastic tapes applied to thin compliant substrates

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Abstract

In this paper, we investigate the periodic peeling behavior of opposing symmetric peeling fronts involving an elastic tape peeled off from a deformable substrate of finite thickness, backed onto a rigid foundation.

We treat the problem by means of an energetic formulation, and we found that, depending on the values of the initial detached length $l$, substrate thickness $h$, and peeling periodicity $\lambda$, the translational invariance of the peeling process is lost and restored, as the elastic interaction between the peeling fronts is limited by the substrate thickness. Indeed, given $h$ and $\lambda$, a critical value of the detached length can be found, which is able to prevent unstable peeling of the tape under a fixed applied load, thus resulting in enhanced adhesion strength, with respect to the classical Kendall’s solution for peeling from a rigid substrate. On the other hand, given the geometrical system configuration (i.e. the detached length $l$) the load necessary to trigger the peeling can be minimized by conveniently tuning the ratio $h/\lambda$. This feature might be of interest for the development of innovative designs for future biomedical devices, such as Transdermal Drug Delivery Systems or wound dressing, requiring low peel adhesion for safe successive removals.

Keywords: V-peeling, adhesion, finite thickness, pressure sensitive adhesives, elastic coupling.
I. INTRODUCTION

Nowadays, it is well established that peeling process of thin tapes and membrane is one of the most important detachment mechanism, both in natural and human-related applications. Moving from the pioneeristic studies by Kaelble [1, 2] and Kendall [3], who set the thermodynamic framework to describe peeling processes, it is now clear that the interplay between adhesive interfacial interactions and elastic deformations of the contacting bodies plays a key role on the overall peeling behavior [4–10]. Aiming at understanding and, possibly, mimicking their extraordinary adhesive performance, several experimental observations [11–13] on arachnids and reptiles toes have, indeed, suggested that hierarchically arranged structures of hairs or spatulae may play a key role in the attachment and detachment process. Theoretical studies [14–16], and experimental bio-inspired investigations [17–20, 22] have, then, confirmed that very compliant hierarchical structures entail large intimate contact area between the contacting bodies, thus resulting in high van der Waals short-range adhesive interactions even in the case of rough substrates [23–26].

However, although ensuring enhanced adhesive performances, these structures also allow for easy detachment, thus allowing the animals to rapidly walk and climb. To this regard, successive investigations [27, 28, 30, 31], specifically devoted to the attachment and detachment mechanics of gecko toes, have shown that the detachment process is governed by peeling, and that a non-negligible interplay exists between the toe adhesive performance and the peeling angle of the multiple spatulae and fibrils, thus leading to both high adhesive strength and low peel adhesion. Similarly, the effect of friction on adhesion has been experimentally assessed, showing that interfacial shear stress nonlinearly affect the pull-off load and may even slightly increase the adhesive performance of thin fibrillar pads [29], as indeed predicted in a recent theory [32].

Following these studies, several attempt have been made to mimic this combined high adhesive-low peeling behavior in industrial applications. For instance, several climbing robots [21, 22] equipped with bio-inspired attachment pads [20, 34–37] have been developed, all exploiting the superior adhesion provided by very compliant mushroom- and hairy-shaped end structures. Peeling was then employed to easily control the detachment process of such systems.

However, most of the existing studies, only focus on the case of elastic thin structures, hierarchically arranged, in adhesive contact with significantly stiffer substrates, thus usually considered as rigid. Although this assumption holds true for a large class of practical applications, peculiarly demanding problems exist in which the effect of the substrate elasticity on the peeling behavior cannot be neglected. It is the case, for instance, of thin protective peelable packaging for food and general soft components, in which the peeled tape is stiffer than the substrate.

Of course, dealing with a single elastic tape peeled away from an elastic substrate would not affect Kendall’s results [3], due to the translational invariance of the elastic field in the substrate, which therefore does not contribute to the energy balance of the advancing peeling. However, for instance, if we assume a viscoelastic behavior for, at least, one among the tape and the substrate the peeling behavior may turn out significantly altered, as due to viscous dissipation in the materials the process is now velocity dependent. It is the case, for instance, of the peeling of a viscoelastic tape from a rigid substrate [38], in which increasing peeling force with the peeling rate is reported. Similarly, the more complex case of an elastic tape peeled away from a viscoelastic adhesive has been firstly treated in Ref. [39].
where enhanced adhesive toughness has been reported, and then in Ref. [10], where, in the framework of linear viscoelasticity, the authors define specific scaling laws for the process.

Also the number of peeling fronts involved in the peeling process, as well as the specific spatial arrangement of them, may strongly affect the peeling performances. Moving from natural observation of biological systems [41, 42] (e.g. web anchors), a specific adhesion mechanism has been identified which relies on multiple pattern of opposing double-sided peeling fronts (V-peeling). Both theoretical [10] and experimental [43] studies have indeed shown that dealing with V-peeling from rigid substrates, both geometrical and hierarchical optimizations can be performed in order to increase the maximum sustainable load. Further investigations on the same V-peeling configuration have addressed the case of uniform cohesive interactions at the interface [44], also taking into account for the tape large deformations and frictional sliding effects [45] which, indeed, significantly increase the pull-off load.

Definitely less has been done in the case of V-peeling from deformable substrates. Of course, in this case, also the elastic energy stored in the substrate has to be taken into account in the overall energy balance governing the peeling evolution, since, as the peeling advances and the fronts moves apart from each other, the elastic field in the substrate changes. A first attempt to investigate such peculiar scenario has been made in Ref. [46] by relying on computationally demanding Finite Element (FE) simulations. However, a more comprehensive investigation on this topic has been recently provided in Ref. [47] where, by relying on reliable Boundary Element (BE) calculations, the case of periodic V-peelings from an elastic half-space is treated, eventually finding that a non-negligible interactions between the multiple peeling fronts occurs through the elastic fields in the substrate. This entails that the results strongly depend on the distance between the peeling fronts, thus making the process no longer translationally invariant.

To this regard, it is important to observe that there exist a large class of applications in which the elasticity of the substrate is crucial in determining the overall adhesive performance of the system. Among the others, this is the case of biomedical applications such as pressure sores dressing and Transdermal Drag Delivery Systems (TDDS). Several attempts have been made to analyze in details the peel adhesion performance of these Pressure Sensitive Adhesives (PSA) [51–54], however the usual half-space assumption falls short in describing the role played by the substrate deformability in the adhesive behavior for such applications. Indeed, for instance, TDDS usually stuck on sufficiently thin biological tissues confined by the underlaying stiffer bones. Since high holding power and low peel adhesion are essential requirements for the medical functionality of such systems [55–57], the possibility to explore innovative designs relying on periodically arranged V-peeling fronts in order to reduce the peeling resistance can be important for future developments. To this regard, a significant effort has been made, in both elastic and viscoelastic contact mechanics, to clarify that, in case of bodies of finite thickness, an additional lengthscale (i.e. the body size) competes in defining the system behavior [48–50], leading to non-negligible effects in terms of contact stiffness and area.

Here, we present a study on the physical behavior of the multiple periodically distributed double V-shaped opposing peeling of an elastic thin tape from an elastic substrate of finite thickness, backed onto a rigid foundation, a configuration that mimics the above scenarios. We perform our study in the framework of linear elasticity, considering quasi-static reversible thermodynamic transformations, in the same path already drawn by Kendall [3]. The displacement field in the elastic substrate is solved by means of a Green’s function approach, by exploiting the formalism recently given in Ref. [58]. The results here presented provide an
improved understanding of the effect of the geometrical parameters on the peeling behavior of periodic arrangements, which can potentially be used to minimize/maximize the peeling load for different applications.

II. FORMULATION

Figure 1 shows a sketch of the system under investigation: a thin elastic tape of thickness $d$ is periodically peeled away from an elastic layer of thickness $h$, which is backed onto a rigid foundation. The detachment occurs via multiple V-shaped opposite peeling fronts under periodic normal loads $2P$, whose spatial periodicity is $\lambda$. Since in each periodic cell, both the loads and system geometry are symmetric, we will conveniently focus our study on half of the periodic cell. We can easily define the overall load on each peeling front as $F = P/(\sin \alpha)$, with $\alpha$ being the peeling angle between the tape and the substrate. Further, by defining $\varepsilon = F/(Ebd)$ the tape elongation and $l$ the detached length, simple geometrical arguments \cite{47} show that $l(1 + \varepsilon) \cos \alpha = l$, which finally gives the following implicit expression of the peeling angle $\alpha$

$$P - Ebd(\tan \alpha - \sin \alpha) = 0 \quad (1)$$

where $E$ is the Young modulus of elasticity of the tape, and $b$ is the transverse width.

FIG. 1: The system configuration under periodic V-peeling of an elastic tape of thickness $d$ from an elastic substrate of thickness $h$.

In this work, the description of the peeling behavior relies on the energetic formulation firstly proposed in Refs [1–3]; thus we focus on linear elasticity, neglecting any large deformation effect. Of course, dealing with very soft materials, large deformations effects might slightly change the quantitative evolution of the process; however, as shown in previous works on similar topics \cite{59, 60}, the main physical picture would not be affected, and qualitative inferences can still be drawn from linear theory. Similarly, detailed studies have shown that dynamic effects may significantly alter the peeling behavior, also giving rise to instabilities during the peeling evolution \cite{61–66}. However, since our study aims at investigating the fundamental mechanisms linked to geometric effects (e.g. thickness), here we focus on quasi-static conditions, thus neglecting any dynamic effect (i.e. we assume that the peeling fronts propagation occurs far slower than the speed of sound).
As already observed in Ref. [47], since here the tape is peeled away from an elastic substrate via a periodic V-peeling process, the geometrical configuration changes as the detachment fronts move apart. This implies that the elastic field within the substrate is no longer stationary, thus the translational invariance typical of Kendall’s peeling is lost. Under these conditions, the elastic energy term associated with the deformation of the substrate have to be taken into account in defining the total potential energy of the system $U_{tot}$.

Therefore we have

$$U_{tot} = U_{el,t} + U_{el,s} + U_P + U_{ad}$$

(2)

where $U_{el,t}$ and $U_{el,s}$ are the elastic energy stored in the tape and in the substrate, respectively. $U_P$ is the potential energy associated with the normal load $P$, and $U_{ad}$ is the adhesion energy. We assume the elastic tape sufficiently thin to neglect any bending contribution to its deformation (see Ref [62]). The resulting deformation is of pure stretch, thus $U_{el,t} = F_2 l / (2 E b d)$. Similarly, the surface adhesion energy is $U_{ad} = l b \Delta \gamma$, where $\Delta \gamma$ is the work of adhesion, also known as the Dupré energy of adhesion.

For what concerns the energetic term associated to the deformation of the elastic substrate, building on the physical arguments already given in Refs. [39, 47, 68], we assume that the load $F$ acting on each branch of the V-shaped tape is balanced by a uniform distribution of normal $p = F \sin \alpha / (2 ab)$ and tangential $q = \pm F \cos \alpha / (2 ab)$ tractions localized at the tip of the detachment fronts over a region of size $2a \approx d$ (see fig. 1). Indeed, very thin tapes show vanishing bending stiffness ($\propto d^3$), thus leading to highly concentrated normal interfacial stresses (see Refs. [68, 69]); furthermore, dimensional arguments show that tangential tractions must interest a region of size $(E/E_s)^{1/2} \sqrt{\lambda d}$, with $E_s$ corresponding to the substrate’s elastic modulus. Although the assumption of $2a \approx d$ may be restrictive in some case, we expect this not to affect our assessment of the physical scenario under consideration, provided that $2a + l \ll \lambda / 2$.

Since the substrate is thin, we expect the total interfacial displacement $u^t(x) = (u_x^t, u_y^t)$ to be function of both the layer thickness $h$ and loads periodicity $\lambda$ [58, 73, 74]. In order to calculate $u^t(x)$, we conveniently rely on the superposition of the periodic displacement fields $u^{r,l}(x)$ independently related to the $p$ and $q$ stresses of each of the two adjacent peeling fronts ($r, l$ indicate for the right and left detachment fronts). From Fig. 1 it follows that

$$u^t(x) = u^r(x-l) + u^l(-x-l),$$

where $u^{r,l}$ can be found by relying on a Green’s function approach based on the fundamental periodic solutions given in Ref. [58] for the case of an elastic layer of finite thickness backed onto a rigid foundation (see Appendix A). Finally, the elastic energy stored in the substrate can be found as

$$U_{el,s} = \frac{1}{2} b \int_{l-a}^{l+a} \sigma \cdot u^t dx$$

(3)

being $\sigma = (q, p)$.

In the same framework, the potential energy associated to the applied vertical load $P$ is given by

$$U_P = -P \left[ u_y^t(l) + \delta \right]$$

(4)

where $u_y^t(l)$ is the substrate normal displacement of the point of application of $P$, and $\delta = l \tan \alpha$. 

5
Following Ref. [70], for isothermal reversible transformations, the peeling process equilibrium requires that

$$\frac{\partial U_{tot}}{\partial A} = \frac{1}{b} \frac{\partial U_{tot}}{\partial l} = 0$$

(5)

where $A$ is the detached tape area.

Since, from Eq. (2), $U_{tot}$ is sensitive to both the vertical load $P$ and the detached length $l$, Eq. (5) allows to determine whether, for a given value of the controlled parameter $P$, a specific value of $l$ exists able to ensure equilibrium.

III. RESULTS

Several works have shown that, dealing with elastic substrates of finite thickness in periodic contacts, a peculiar role is played by the ratio $h/\lambda$, as the thickness $h$ represents a limit to the elastic interaction length [73, 74]. In the case of rough contacts, for instance, this implies that asperities whose distance is larger than $h$ do not interact with each other. For this reason, we do expect the ratio $h/\lambda$ to significantly affect the V-peeling behavior, thus we focus out study on different values of $h$ and $\lambda$.

Furthermore, several experimental studies on human-related soft tissues for biomedical applications have reported an almost incompressible behavior of the tissues [71, 72], thus here we focus on a substrate with Poisson’s ratio $\nu_s = 0.5$. Regarding the tape, we assume $d = 10^{-4}$ m, $E = 730$ MPa, and $b = 0.015$ m, as in several of the available commercial adhesive tapes. We define the elasticity ratio $\chi = E/E_s$ between the tape and substrate elastic moduli, and the dimensionless peeling load $\tilde{P} = P/P_0$, with $P_0$ being the Kendall’s peeling load in the case of rigid substrates [3].

In Fig. 2a, the total energy $U_{tot}$ is plotted against the dimensionless detached length $l/\lambda$ for different values of the substrate thickness, at fixed peeling load $P/P_0 = 2$. Calculations refer to $\lambda = 0.1$ m, and $\Delta \gamma = 74$ J/m$^2$. (b) A physical scheme of the interaction between adjacent peeling fronts.

In Fig. 2a the total energy $U_{tot}$ is plotted against the dimensionless detached length $l/\lambda$, at given dimensionless peeling load $\tilde{P} = 2$. A critical detached length $l_{cr}$ exists where the equilibrium condition of Eq. (5) is satisfied. However, since $\partial^2 U_{tot}/\partial l^2 < 0$, $l_{cr}$ represents an
unstable equilibrium condition \( l > l_{cr} \). This means that, for \( l > l_{cr} \) the system is no longer able to sustain the load, and unstable peeling propagation occurs up to complete detachment; on the contrary, for \( l < l_{cr} \), peeling is prevented and, in the ideal case of perfectly reversible process, a self-healing reattachment would be observed. Of course, in real applications, several sources of dissipation can be identified in the proximity of the peeling front (i.e. non-conservative adhesive bonds braking, high elastic strain rate, dynamic effects) able to prevent the self-healing behavior, which indeed reduces to simple non-advancing peeling for \( l < l_{cr} \). Furthermore, in Fig. 2a we can also appreciate the effect of the elastic layer thickness \( h \). Specifically, by focusing on the thinner layer, we observe that significant interaction between the peeling fronts through the deformable substrate occurs only for sufficiently small, and large, values of \( l \). Indeed, dimensional arguments indicate that, due to the finiteness of the elastic substrate thickness, the elastic fields associated to each of the peeling fronts can only affect a circular region of the elastic substrate of radius \( r \approx h \). The mechanism of such an interaction is clearly shown in top sketch in Fig. 2a for \( l \lesssim h \), peeling fronts A and B interact, whereas for \( l \gtrsim \lambda/2 - h \), cracks A and C interact instead. In the intermediate range of value of \( l \), all the peeling fronts are independent of each other, thus resulting in local translational invariance of the substrate deformation, and in turn in a linear trend of the total energy (i.e constant energy release rate), as predicted by the Kendall theory [3]. Notably, under this condition, peeling propagation would occur for \( \tilde{P} > 1 \). Increasing the layer thickness leads to a different scenario (see the bottom sketch in Fig. 2b), as for \( h \gtrsim \lambda \) all the peeling fronts are able to mutually interact regardless of the value of \( l \).

The effect of the system configuration in terms of both the spatial periodicity \( \lambda \) of the peeling loads and the substrate thickness \( h \) on the critical detached length \( l_{cr} \), at fixed load \( P/P_0 = 1.1 \). Two different elasticity ratios have been considered: (a) \( \chi = 100 \), and (b) \( \chi = 1000 \).

FIG. 3: Surface representations of the effect of the peeling spatial periodicity \( \lambda \) and substrate thickness \( h \) on the critical detached length \( l_{cr} \), at fixed load \( P/P_0 = 1.1 \). Two different elasticity ratios have been considered: (a) \( \chi = 100 \), and (b) \( \chi = 1000 \).

The effect of the system configuration in terms of both the spatial periodicity \( \lambda \) of the peeling loads and the substrate thickness \( h \) on the critical detached length \( l_{cr} \) is shown in Figs. 3 for two different values of the elasticity ratio \( \chi \). Regardless of the values of \( \lambda \) and \( h \), we observe that the softer the substrate, the tougher the peeling, as very compliant substrates enhance the peeling fronts interaction, thus leading to larger critical detached length before peeling occurs (i.e. larger \( l_{cr} \) at given load). Further, increasing \( \lambda \) and \( h \) leads to the asymptotic non-periodic half-space peeling behavior (see Ref. [47]). Interestingly, a non-monotonic trend of \( l_{cr} \) with respect to \( h \) is also reported.
FIG. 4: (a) The non monotonic trend of the critical detached length $l_{cr}$ as a function of the substrate thickness $h$, for different values of the peeling load $P$ and elasticity ratio $\chi = E/E_s$. (b) the general trend of normal displacements of the elastic substrate induced by generic $p = q > 0$ (see Fig. 1) tractions for different substrate thickness $h$.

Such a peculiar behavior is even more evident in Fig. 4a where the critical detached length $l_{cr}$ is shown as a function of the substrate thickness $h$ for different peeling loads and elasticity ratios $\chi$. In order to qualitatively explain this interesting feature of interacting multiple peeling process involving deformable substrate of finite thickness, in Fig. 4b we plot the general trend of normal displacement field of the substrate for different values of $h/\lambda$ due to a distribution of normal and tangential tractions such as those related to the single crack A of Fig. 2b. We observe that reducing $h$ the size $\xi$ of the significantly deformed region of the layer surface reduces too, as $\xi \approx h^{0.38}$. Indeed, given the detached length of the V-peeling process (i.e. the distance between the two peeling fronts A and B in Fig. 2b), due to elastic interactions, the peeling fronts will affect each other differently depending on the substrate thickness $h$, thus modifying both the amount of elastic energy stored in the substrate, and the overall potential energy associated to the normal peeling load ($P$ see Eqs. (3-4)). Consequently, in Fig. 4a we note that, regardless of $\tilde{P}$ and $\chi$, this non-monotonic effect occurs approximately at $l_{cr} \approx h$, where the displacement fields induced by the opposing peeling fronts present the highest degree of interaction with each other. Finally, reducing $\chi$ leads to less pronounced effects of $h$, as, given the normal load, stiffer substrates entail smaller displacements and in turn smaller amount of elastic energy stored (i.e. it selectively affects only the terms $U_P$ and $U_{el,s}$ in Eq. (2)).

In Fig. 5a we show the effect of the spatial periodicity $\lambda$ of the periodic V-peelings on the peeling toughness, as indeed $l_{cr}$ is plotted against $h$ for different value of $\lambda$. We observe that, given the peeling load, increasing $\lambda$ entails larger detached length tolerable before unstable peeling propagation. In other words, the interaction between the fronts A and C in Fig. 2b reduces the overall peeling toughness, whereas the interaction between the V-peeling fronts (A and B) is beneficial in terms of higher tolerable detached length. Of course, per each value of the layer thickness $h$, a further periodicity increase beyond $\lambda \geq h$ does not change the physical scenario, as the asymptotic aperiodic behavior is approached.

In Fig. 5b, instead, we show the effect of the $\lambda$ and $h$ on the system in terms of the critical peeling load $\tilde{P}_{cr}$ above which unstable peeling propagation occurs, given a value of $l$. In this case, we observe that the peeling load can be minimized by tuning the substrate
thickness $h$ and system periodicity $\lambda$. Specifically, the reduction of both $\lambda$ and $h$ leads to a reduction of $\tilde{P}_{cr}$. However, depending on the value of $\lambda$, non-trivial results are found, as a global minimum of $\tilde{P}_{cr}$ as a function of $h$ is reported. These results show how the V-peeling configuration can be exploited for innovative optimization strategies to tailor the peel adhesion of several systems. It is the case, for instance, of both wound dressing and TDDS in biomedical medical applications, whose critical peeling load might be conveniently reduced by relying on an innovative V-peeling removal mechanism, which adjusts the value of $\lambda$ given a certain skin thickness $h$, thus reducing the possibility of injuries in successive removals.

IV. CONCLUSIONS

In this paper, we investigate the peeling behavior of a thin elastic tape peeled off from an elastic substrate of finite thickness. The peeling process occurs by means of symmetrically V-peeling fronts, periodically distributed over the substrate. Due to the specific system configuration, the translational invariance of the peeling process is lost and restored, depending on the specific value of the detached length, substrate thickness, and spatial periodicity.

As a consequence, the overall results show a certain dependency on these parameters, in terms of both peel-off load and toughness. In general, we found that the sustainable peeling load depends on the detached length; however, the ratio between the substrate thickness and spatial peeling periodicity affects this trend, giving rise to local minima. Such a peculiar feature offers the opportunity to conveniently tune both the substrate thickness and spatial peeling periodicity in order to minimize the peeling load. We hope this research provides the fundamental understanding needed to develop new adhesive solutions, possibly opening new design opportunities to enhance the peeling behavior of, for instance, wound dressing and TDDS in biomedical applications, as well as on general purpose packaging films.
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Appendix A: Elastic surface displacements of thin layers under periodic uniform normal and tangential tractions

We focus on the problem shown in Fig 6 of an elastic substrate of thickness $h$ subjected to a distribution of uniform normal $\sigma_2$ and tangential $\sigma_1$ tractions acting over a strip of size $2a$.

![Elastic substrate diagram](image)

FIG. 6: The elastic problem at hand: an elastic layer of thickness $h$ is loaded with a periodic distribution of uniform normal and tangential tractions over a strip of size $2a$.

The spatial periodicity of the loads is $\lambda$.

Further, we assume the tractions $\sigma_1$ and $\sigma_2$ to be periodically distributed, with periodicity $\lambda$. Following Ref. [58], by means of the the convolution product between the Green’s tensor $G(x)$ and the interfacial stress vector $\sigma = (\sigma_1, \sigma_2)$ (see Fig. 6), one can find the total surface displacement vector $v = (v_1, v_2)$ as

$$v(x) - v_m = \int_{\Omega} ds G(x-s) \sigma(s); \quad x \in [0, \lambda] \quad (A1)$$

where $\Omega$ is the domain of integration (i.e. $\Omega = [-a, a]$), $v_m$ is the mean plane displacement vector and the components of the the Green’s tensor are given by [58]

$$G_{11}(x) = \frac{2(1-\nu^2_s)}{\pi E_s} \left[ \log \left| 2 \sin \left( \frac{kx}{2} \right) \right| + \sum_{m=1}^{\infty} A_m(kh) \frac{\cos (m kx)}{m} \right], \quad (A2)$$

$$G_{12}(x) = -G_{21}(x) = \frac{1+\nu_s}{\pi E_s} \left[ \frac{1-2\nu_s}{2} [\operatorname{sgn}(x) \pi - kx] - \sum_{m=1}^{\infty} B_m(kh) \frac{\sin (m kx)}{m} \right], \quad (A3)$$

$$G_{22}(x) = \frac{2(1-\nu^2_s)}{\pi E_s} \left[ \log \left| 2 \sin \left( \frac{kx}{2} \right) \right| + \sum_{m=1}^{\infty} C_m(kh) \frac{\cos (m kx)}{m} \right]. \quad (A4)$$
where \( k = 2\pi/\lambda \), and the quantities \( A_m \), \( B_m \), and \( C_m \) are given by

\[
A_m (kh) = 1 - \frac{2khm + (3 - 4\nu_s) \sinh (2khm)}{5 + 2(khm)^2 - 4\nu (3 - 2\nu_s) + (3 - 4\nu_s) \cosh (2khm)},
\]

\[
B_m (kh) = \frac{4 (1 - \nu_s) \left[ 2 + (khm)^2 - 6\nu_s (3 - 2\nu_s) + (3 - 4\nu_s) \cosh (2khm) \right]}{5 + 2(khm)^2 - 4\nu_s (3 - 2\nu_s) + (3 - 4\nu_s) \cosh (2khm)},
\]

\[
C_m (kh) = 1 + \frac{2khm - (3 - 4\nu_s) \sinh (2khm)}{5 + 2(khm)^2 - 4\nu (3 - 2\nu_s) + (3 - 4\nu_s) \cosh (2khm)}
\]

By recalling that we refer to a uniform distribution of tractions, after substituting Eqs. (A2-A4), Eq. (A1) can be conveniently rewritten in terms of the interfacial displacements \( u = v(x) - v_m = (u_1, u_2) \)

\[
u_1 (x) = q [Z_{11} (x + a) - Z_{11} (x - a)] + p [Z_{12} (x + a) - Z_{12} (x - a)]
\]

\[
u_2 (x) = -q [Z_{12} (x + a) - Z_{12} (x - a)] + p [Z_{22} (x + a) - Z_{22} (x - a)]
\]

where the functions \( F_1 (x) \) and \( F_2 (x) \) are so defined

\[
Z_{11} (x) = \frac{2 (1 - \nu_s^2)}{\pi E_h k} \left( -Cl_2 (kx) + \sum_{m=1}^{\infty} A_m (kh) \frac{\sin (mkx)}{m^2} \right)
\]

\[
Z_{12} (x) = \frac{1 + \nu_s}{\pi E_s k} \left( \frac{1 - 2\nu_s}{2} \Phi (kx) + \sum_{m=1}^{\infty} B_m (kh) \frac{\cos (mkx)}{m^2} \right)
\]

\[
Z_{22} (x) = \frac{2 (1 - \nu_s^2)}{\pi E_h k} \left( -Cl_2 (kx) + \sum_{m=1}^{\infty} C_m (kh) \frac{\sin (mkx)}{m^2} \right)
\]

being

\[
Cl_2 (x) = - \int_0^x \log |2 \sin (t/2)| dt
\]

the Clausen function of order 2 (Ref. [75]), and

\[
\Phi (x) = \int_0^x [\text{sgn} (t) \pi - t] dt
\]

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