We study breaking and restoration of supersymmetry in five-dimensional theories by determining the mass spectrum of fermions from their equations of motion. Boundary conditions can be obtained from either the action principle by extremizing an appropriate boundary action (interval approach) or by assigning parities to the fields (orbifold approach). In the former, fields extend continuously from the bulk to the boundaries, while in the latter the presence of brane mass-terms cause fields to jump when one moves across the branes. We compare the two approaches and in particular we carefully compute the non-trivial jump profiles of the wavefunctions in the orbifold picture for very general brane mass terms. We also include the effect of the Scherk-Schwarz mechanism in either approach and point out that for a suitable tuning of the boundary actions supersymmetry is present for arbitrary values of the Scherk-Schwarz parameter. As an application of the interval formalism we construct bulk and boundary actions for super Yang-Mills theory. Finally we extend our results to the warped Randall-Sundrum background.
1 Introduction

A common feature of five-dimensional (5D) supersymmetric models are fermions propagating in the bulk of the extra dimension. In order to extract physical predictions at low energies, the four dimensional mass spectrum of those fermions has to be known. For instance supersymmetry breaking is determined by the mass spectrum of the gravitino and the existence of a zero mode signals an unbroken supersymmetry. Similarly, when gauge multiplets propagate in the bulk supersymmetry breaking is intimately linked to the existence of gaugino zero modes. In particular if supersymmetry breaking is implemented by non-trivial twist conditions, or Scherk-Schwarz (SS) mechanism [1], it acts in the same way both in the gravitino and the gaugino sectors.

The aim of this paper is to study fermions propagating in a five-dimensional space-time, with coordinates \((x^\mu, y)\), where the compact fifth dimension (with radius \(R\)) has two distinguished four-dimensional hypersurfaces located at \(y = 0\) and \(y = \pi R\). Often this space is constructed as the orbifold \(S^1/\mathbb{Z}_2\), identifying points on the circle related by the reflection of the fifth coordinate \(y \to -y\); in this approach \(y = 0\) and \(y = \pi R\) are fixed points and the resulting spacetime is a singular space without boundaries. Fields with odd parity with respect to the \(\mathbb{Z}_2\) reflections are zero at the fixed points, while the normal derivative of even fields is forced to vanish there. The treatment of fermions is complicated in the presence of brane actions localized at the boundaries. In the orbifold approach these brane actions are introduced with delta-function distributions peaked at the location of the orbifold fixed points. The latter induce discontinuities in the wave functions of the fermions, which take different values at the fixed points and infinitesimally close to them [2,3]. A possible way to avoid these problems is working from the very beginning in the fundamental region of the orbifold \([0, \pi R]\) and giving up the rigid orbifold boundary conditions (BC’s) \(^6\): the fields are then continuous and the BC’s are determined by the action principle applied to the bulk and boundary conditions. This is called the interval approach: contrary to the orbifold approach the spacetime is not singular but it has two boundaries at \(y = 0\) and \(y = \pi R\). The interval approach leads to physically equivalent results to those of the orbifold approach without any need of using, as the latter, singular functions \(^7\). To summarize in the orbifold approach one imposes fixed (orbifold) BC’s while the brane action imposes jumps, whereas in the interval approach one imposes continuity and the brane action induces the BC’s.

In a previous letter [4] we followed the interval approach and showed how one can obtain consistent BC’s by the use of the action principle \(^8\): variation of the bulk action gives rise both to a bulk term and a boundary term, while the variation of the boundary action is always localized at the branes. Imposing the whole variation to vanish the bulk

\(^6\)With some abuse of language we call boundary conditions the parity assignment for fields in the orbifold in order to make contact to the interval approach.

\(^7\)The interval approach is sometimes called “downstairs” approach while the orbifold approach is called “upstairs” approach.

\(^8\)For an alternative approach see [5].
terms give the usual bulk equations of motion (EOM) while the boundary terms give rise to the corresponding BC’s (see [6] for a recent application to symmetry breaking). In [4] it was also shown that for consistent (nontrivial) BC’s one needs to appropriately constrain the boundary action; in particular, a vanishing brane action leads to inconsistent BC’s. The BC’s can be seen to represent a point in the Riemann sphere and hence are given by complex numbers \( z_f, f = 0, \pi \). Explicit formulae for mass eigenvalues and eigenfunctions as functions of \( z_f \) can then be obtained. Whenever the BC’s at the two branes are the same, \( z_0 = z_\pi \), a zero mode exists and \( N = 1 \) supersymmetry remains unbroken. An interesting phenomenon occurs when in addition a Scherk-Schwarz [1] breaking is turned on. The latter can be implemented as a vacuum expectation value (VEV) for the auxiliary vector field \( \vec{V}_M \) which gauges the \( SU(2)_R \) automorphism symmetry [7] as \( \langle \vec{V}_M \rangle = \delta_{M5} \omega \vec{q} \), characterized by the SS parameter \( \omega \) and the SS direction (unit vector) \( \vec{q} \).

Whenever \( \vec{q} \) is aligned \(^9\) with either \( z_0 \) or \( z_\pi \), the mass spectrum becomes independent on the SS parameter \( \omega \). In particular this means that if in addition \( z_0 = z_\pi \), supersymmetry remains unbroken whatever the value of the SS parameter \( \omega \) is. This was dubbed persistent supersymmetry in Ref. [4]. As an application of the interval formalism, we write down the action of super Yang-Mills (SYM) theory. A boundary action is required both for consistent BC’s as well as (global) supersymmetry.

In the present work we also shed some light on the relation of the interval approach and the more common orbifold picture in which BC’s are fixed by the parity assignments of the fields. We show how in the latter the mass spectrum can be computed by calculating the highly generally nontrivial jump profiles across the branes. In order to consistently treat the singularities arising from the presence of delta-functions we formally use regularized delta-functions. The jumps determine the values of the wave functions an infinitesimal distance away from the branes and thus can be used as new BC’s in the general formulae for the solution to the bulk EOM. The impact of fermionic brane mass terms on the spectrum as well as its relation to the SS mechanism have been discussed before in a number of papers [2,3,8,9], with sometimes contradictory results due to incorrect treatment of the discontinuities. We believe that our present paper gives a clear and consistent treatment of this slightly involved issue by employing the most general brane Lagrangian considered so far.

The structure of the paper is as follows: section \( \text{2} \) contains the results derived in [4] on the interval approach. In section \( \text{3} \) the action of the Super Yang-Mills (SYM) theory is derived. In section \( \text{4} \) we give the treatment of the most general boundary mass terms in the orbifold picture, as outlined above. Emphasis is put on the careful (regularized) treatment of the singular profiles of the wave functions. In section \( \text{5} \) we extend our results to the case of warped (RS) geometry. Finally in section \( \text{6} \) we draw our conclusions.

\(^9\)See below for a precise definition of this term.
2 Fermions in the Interval

In this section we recall some of the results of [4]. We will take the fermions to be symplectic-Majorana spinors, although a very similar treatment holds for the case of fermionic matter fields associated to Dirac fermions. In particular we will consider the gaugino case, the treatment of gravitinos being completely analogous. The 5D spinors $\Psi^i$ satisfy the symplectic-Majorana reality condition and we can represent them in terms of two chiral 4D spinors according to

$$\Psi^i = \left( \eta^i_\alpha \bar{\chi}^{i\dot{\alpha}} \right), \quad \bar{\chi}^{i\dot{\alpha}} \equiv \epsilon^{ij} \left( \eta^j_{\beta} \right)^* \epsilon^{\dot{\alpha} \dot{\beta}},$$

(2.1)

where $\epsilon_{ij} = i (\sigma_2)_{ij}$ and $\epsilon^{im} \epsilon_{jm} = \delta^i_j$. Consider thus the bulk Lagrangian

$$\mathcal{L}_{\text{bulk}} = i \bar{\Psi} \gamma^M D_M \Psi = \frac{i}{2} \bar{\Psi} \gamma^M D_M \Psi - \frac{i}{2} D_M \bar{\Psi} \gamma^M \Psi,$$

(2.2)

where the last equation is not due to partial integration but holds because of the symplectic-Majorana property, Eq. (2.1). The derivative is covariant with respect to the $SU(2)_R$ automorphism symmetry and thus contains the auxiliary gauge connection $V_M$. The field $V_M$ is non-propagating and appears in the off-shell formulation of 5D supergravity [11]. A vacuum expectation value (VEV) 11

$$V_M = \delta^5_{M \alpha} \frac{\omega}{R} \bar{q} \cdot \bar{\sigma}, \quad \bar{q}^2 = 1$$

(2.3)

implements a Scherk-Schwarz supersymmetry breaking mechanism [1] in the Hosotani basis [7, 12]. The standard form of the SS mechanism, originally introduced for circle compactification, can be recovered by a gauge transformation $U$ that transforms away $V_M$ but twists the periodicity condition for fields charged under $SU(2)_R$ on the circle. As a matter of fact, in the interval a SS breaking term is equivalent to a suitable modification of the BC’s at one of the endpoints. The unitary vector $\bar{q}$ points toward the direction of SS breaking. We supplement the bulk action by the following boundary terms at $y = y_f$ ($f = 0, \pi$) with $y_0 = 0$ and $y_\pi = \pi R$

$$\mathcal{L}_f = \frac{1}{2} \bar{\Psi} \left( T^{(f)} + \gamma^5 V^{(f)} \right) \Psi = \frac{1}{2} \eta^i M^{(f)}_{ij} \eta^j + \text{h.c.},$$

(2.4)

where $T^{(f)}$ and $V^{(f)}$ are matrices acting on $SU(2)$ indices,

$$M^{(f)} = i \sigma_2 \left( T^{(f)} - i V^{(f)} \right)$$

(2.5)

and we have made use of the decomposition (2.1). Notice that the mass matrix is allowed to have complex entries. Without loss of generality we take it to be symmetric, which enforces $T^{(f)}$ and $V^{(f)}$ to be spanned by Pauli matrices.

10 We use the Wess-Bagger convention [10] for the contraction of spinor indices.
11 Consistent with the bulk equation of motion $d (\bar{q} \cdot \nabla) = 0$ [11].
The total Bulk + Boundary action is then given by

\[ S = S_{\text{bulk}} + S_{\text{boundary}} = \int d^5x \mathcal{L}_{\text{bulk}} + \int_{y=0}^{y=R} d^4x \mathcal{L}_0 - \int_{y=\pi} d^4x \mathcal{L}_\pi . \]  

(2.6)

The variation of the total action gives the standard Dirac-like bulk equation of motion provided that all the boundary pieces vanish. The latter are given by

\[ \delta \eta^i (\epsilon_{ij} + M_{ij}^{(f)}) \eta^j + \text{h.c.} \bigg|_{y=y_f} = 0 . \]  

(2.7)

Since we are considering unconstrained variations of the fields, the BC’s we obtain from Eqs. (2.7) are given by

\[ (\epsilon_{ij} + M_{ij}^{(f)}) \eta^j \bigg|_{y=y_f} = 0 . \]  

(2.8)

These equations only have trivial solutions (are over-constrained) unless

\[ \det (\epsilon_{ij} + M_{ij}^{(0)}) = \det (\epsilon_{ij} + M_{ij}^{(s)}) = 0 . \]  

(2.9)

Imposing these conditions we get the two complex BC’s which are needed for a system of two first order equations. Note that this means that an arbitrary brane mass matrix does not yield viable BC’s; in particular a vanishing brane action is inconsistent \(^{12}\) since \( \det (\epsilon_{ij}) \neq 0 \). \(^{13}\) However this does not imply that the familiar orbifold BC’s \( \eta_1 = 0 \) (\( \eta_2 = 0 \)) can not be achieved; in the interval approach they correspond to \( M = \sigma^1 \) (\( M = -\sigma^1 \)).

The BC’s resulting from Eqs. (2.8) are of the form

\[ (c_1^f \eta^1 + c_2^f \eta^2) \bigg|_{y=y_f} = 0 , \]  

(2.10)

where \( c_1^f, c_2^f \) are complex parameters or, setting \( z_f = -(c_1^f/c_2^f) \)

\[ (\eta^2 - z_f \eta^1) \bigg|_{y=y_f} = 0, \quad z_f \in \mathbb{C} . \]  

(2.11)

Physically inequivalent BC’s span a complex projective space \( \mathbb{CP}^1 \) homeomorphic to the Riemann sphere. In particular \( z_f = 0 \) leads to a Dirichlet BC for \( \eta_2 \), and the point at infinity \( z_f = \infty \) leads to a Dirichlet BC for \( \eta_1 \). Notice that these BC’s come from \( SU(2)_R \) breaking mass terms. Special values of \( z_f \) correspond to cases when these terms preserve part of the symmetry of the original bulk Lagrangian. In particular when both the SS and the preserved symmetry are aligned those cases can lead to a persistent supersymmetry as we will see. Once (2.8) is satisfied the values of \( z_f \) in terms of the brane mass terms are given by

\[ z_f = -\frac{M_{11}^{(f)}}{1 + M_{12}^{(f)}} = \frac{1 - M_{12}^{(f)}}{M_{22}^{(f)}} \]  

(2.12)

where the second equality holds due to condition (2.9).

\(^{12}\)In the sense that the action principle does not provide a consistent set of BC’s as boundary equations of motion.

\(^{13}\)Notice that this agrees with the methods recently used in Ref. [13].
The mass spectrum is found by solving the EOM with the BC’s (2.11). To simplify the bulk equations of motion it is convenient to go from the Hosotani basis $\Psi^i$ to the SS one $\Phi^i$, related by the transformation

$$
\Psi = U \Phi, \quad U = \exp \left( -i \vec{q} \cdot \vec{\sigma} \frac{y}{R} \right). \quad (2.13)
$$

In the SS gauge the bulk equations read

$$
i \gamma^M \partial_M \Phi = 0. \quad (2.14)
$$

We now decompose the chiral spinor $\eta^i(x, y)$ in the Hosotani basis as $\eta^i(x, y) = \varphi^i(y)\psi(x)$, with $\psi(x)$ a 4D chiral spinor. Setting $\varphi = U\phi$ we get the following equations of motion in the SS basis

$$
m\phi^i - \epsilon^{ij} \frac{d\bar{\phi}_j}{dy} = 0, \quad m\bar{\phi}_j \epsilon^{ij} + \frac{d\phi^i}{dy} = 0. \quad (2.15)
$$

The parameter $m$ in Eq. (2.15) is the Majorana mass eigenvalue of the 4D chiral spinor $^{14}$

$$
i\sigma^\mu \partial_\mu \bar{\psi} = m\psi, \quad i\bar{\sigma}^\mu \partial_\mu \psi = m\bar{\psi}. \quad (2.16)
$$

As a consequence of the transformation (2.13) the SS parameter $\omega$ manifests itself only in the BC at $y = \pi R$ $^{15}$:

$$
\zeta_0 \equiv \frac{\phi^2}{\phi^1} \bigg|_{y=0} = z_0, \quad \zeta_\pi \equiv \frac{\phi^2}{\phi^1} \bigg|_{y=\pi R} = \frac{\tan(\pi \omega)(iq_1 - q_2 - iq_3 z_\pi) + z_\pi}{\tan(\pi \omega)(iq_1 z_\pi + q_2 z_\pi + iq_3) + 1}, \quad (2.17)
$$

where $\zeta_f$ are the BC’s in the SS basis. In particular the BC $\zeta_\pi$ is a function of $\omega$, $\vec{q}$ and $z_\pi$. From this it follows that we can always gauge away the SS parameter $\omega$ in the bulk Lagrangian going into the SS basis through (2.13). However now in the new basis $\omega$ reappears in one of the BC’s.

The bulk equations have the following generic solution

$$
\phi(y) = \begin{pmatrix}
\bar{a} \cos(my) + z_0 a \sin(my) \\
-a \sin(my) + z_0 \bar{a} \cos(my)
\end{pmatrix}, \quad (2.18)
$$

where $a$ is a complex number given in terms of $z_0$ and $\zeta_\pi$:

$$
a = \frac{z_0 - \zeta_\pi}{|z_0 - \zeta_\pi|} + \frac{1 + z_0 \bar{\zeta}_\pi}{|1 + z_0 \bar{\zeta}_\pi|}. \quad (2.19)
$$

The solution (2.18) satisfies the BC’s Eq. (2.17) for the following mass eigenvalues

$$
m_n = \frac{n}{R} + \frac{1}{\pi R} \arctan \left| \frac{z_0 - \zeta_\pi}{1 + z_0 \bar{\zeta}_\pi} \right|, \quad (2.20)
$$

$^{14}$The bar acting on a scalar quantity, as e.g. $\phi_i$, and a chiral spinor, as e.g. $\bar{\psi}$, denotes complex conjugation.

$^{15}$Notice that $U(y = 0) = 1$. The roles of the branes and hence of $z_\pi$ and $z_0$ can be interchanged by considering the SS transformation $U'(y) \equiv U(y - \pi R)$. 

5
where \( n \in \mathbb{Z} \). When \( z_0 = \zeta \pi \) there is a zero mode and this corresponds to an unbroken supersymmetry. Indeed, the only sources of supersymmetry breaking reside on the branes (gaugino mass terms) and setting them to cancel each other, \( z_0 = z_\pi \), preserves supersymmetry [14]. Once supersymmetry is further broken in the bulk, an obvious way to restore it is by determining \( z_\pi \) as a function of \( z_0 \) and \( \omega \) using the relation (2.17) with \( \zeta = z_0 \). This will lead to an \( \omega \)-dependent brane-Lagrangian at \( y = \pi R \). In this case we could say that supersymmetry that was broken by BC’s (SS twist) is restored by the given SS twist (BC’s) [9].

There is however a more interesting case: suppose the brane Lagrangian determines \( z_\pi \) to be

\[
    z_\pi = z(\vec{q}) \equiv \frac{\lambda - q_3}{q_1 + iq_2}, \tag{2.21}
\]

with \( \lambda = \pm 1 \). This special value of \( z_\pi \) is a fixed point of the SS transformation, i.e. \( \zeta_f = z_f \). For \( z_\pi = z(\vec{q}) \) the spectrum becomes independent on \( \omega \). In other words, for this special subset of boundary Lagrangians, the VEV of the field \( \vec{q} \cdot \vec{V}_5 \) does not influence the spectrum. The reason for this can be understood by going back to the Lagrangian which we used to derive the BC’s. From the relation (2.12) one can see that condition (2.21) is satisfied by the mass matrix

\[
    M^{(\pi)}_{12} = \lambda q_3, \\
    M^{(\pi)}_{11} = -\lambda (q_1 + iq_2), \\
    M^{(\pi)}_{22} = \lambda (q_1 - iq_2) \tag{2.22}
\]

which can be translated into a mass term at the boundary \( y = y_\pi \) along the direction of the SS term, i.e. \( V^{(\pi)} = 0 \) and \( T^{(\pi)} = -\lambda \vec{q} \cdot \vec{\sigma} \) in the notation of Eq. (2.21). In particular this brane mass term preserves a residual \( U(1)_R \) aligned along the SS direction \( \vec{q} \). In other words the SS-transformation \( U \) leaves both brane Lagrangians invariant and \( \omega \) can be gauged away. When we further impose \( z_0 = z(\pm \vec{q}) \), i.e. \( V^{(0)} = 0 \) and \( T^{(0)} = \pm T^{(\pi)} \) the \( U(1)_R \) symmetry is preserved by the bulk. In particular if \( z_0 = z(\vec{q}) \) supersymmetry remains unbroken, although the VEV of \( \vec{q} \cdot \vec{V}_5 \) is nonzero. One could say that in this case the theory is persistently supersymmetric even in the presence of the SS twist, with mass spectrum \( m_n = n/R \). On the other hand if \( z_0 = z(-\vec{q}) \) the theory is persistently non-supersymmetric and independent on the SS twist: the mass spectrum is given by \( m_n = (n+1/2)/R \). In this case supersymmetry breaking amounts to an extra \( \mathbb{Z}_2' \) orbifolding [15].

Actually (2.22) is the most general solution of Eq. (2.21). We can set \( T^{(f)} = \vec{t}_f \cdot \vec{\sigma} \) and \( V^{(f)} = \vec{v}_f \cdot \vec{\sigma} \). The constraint (2.9) on the boundary mass-matrix translates into

\[
    \vec{t}_f^2 - \vec{v}_f^2 = 1, \quad \vec{t}_f \cdot \vec{v}_f = 0 \tag{2.23}
\]

Consider now \( f = \pi \), using an \( SU(2) \) transformation we can always rotate \( \vec{v}_\pi \) in the \( z \)-direction. As a result without loss of generality we can take \( \vec{v}_\pi = (0, 0, v_3) \). Imposing that \( \zeta_\pi = z_\pi \) and \( \vec{t}_f^2 - \vec{v}_f^2 = 1 \) we get

\[
    \vec{t}_f = \lambda \vec{q} + \vec{\theta}(v_3), \quad \lambda^2 = \pm 1 \tag{2.24}
\]
where $\vec{\theta}$ is a vector which depends on $v_3$. The last constraint $\vec{t}_f \cdot \vec{v}_f = 0$ is satisfied only if $v_3 = 0$, which gives $\vec{\theta}(v_3 = 0) \equiv 0$. Then $V^{(\pi)} = 0$ and $T^{(\pi)} = -\lambda \vec{q} \cdot \vec{\sigma}$.

3 Super Yang-Mills action in the interval

Up to now we have focused on the fermion sector spectrum. Adding the complete vector multiplet does not invalidate our conditions for supersymmetry restoration as long as the supersymmetry breaking brane mass terms are of the form given by Eq. (2.4). We would like to show the invariance of the gaugino Lagrangian, Eq. (2.6), under (global) supersymmetry. To this end we will consider the Super Yang-Mills multiplet containing the gauge field $B_M$ with field strength $G_{MN}$, the gaugino $\Psi$, the real scalar $\Sigma$ and the auxiliary $SU(2)_R$ triplet $\vec{X}$. Clearly since we are not imposing a priori any BC on the fields in the action, we have to worry about the total derivatives which arise in the variation of the bulk action. The latter is given by

$$S_{\text{SYM}}^{\text{bulk}} = \int_M \left( -\frac{1}{4} G_{MN} G^{MN} - \Sigma D^2 \Sigma - \frac{1}{2} D_M \Sigma D^M \Sigma + 2 \vec{X} \cdot \vec{X} + i \bar{\Psi} \gamma^M D_M \Psi + ig f_{ABC} \bar{\Psi}^A \Psi^B \Sigma^C \right).$$  (3.1)

Here the sum over the adjoint indices of the fields is suppressed and $D$ denotes the gauge covariant derivative. Under a global supersymmetric transformation the Lagrangian transforms into a total derivative giving rise to the supersymmetry boundary-variation:

$$\delta_\epsilon S_{\text{SYM}}^{\text{bulk}} = \int_{\partial M} \bar{\epsilon} i \gamma^5 \rho,$$  (3.2)

where $\rho$ is given by

$$\rho = \left( i \vec{X} \cdot \vec{\sigma} - \Sigma \gamma^M D_M - \frac{1}{4} \gamma^{MN} G_{MN} - \frac{1}{2} \gamma^M D_M \Sigma \right) \Psi.$$  (3.3)

To compensate for this we add the brane action

$$S_{\text{SYM}}^{\text{brane}} = \int_{\partial M} \left( 2 \vec{T}^{(f)} \cdot \Sigma \vec{X} + \frac{1}{2} \bar{\Psi} T^{(f)} \Psi \right),$$  (3.4)

which transforms into

$$\delta_\epsilon S_{\text{brane}}^{\text{SYM}} = \int_{\partial M} \bar{\epsilon} T^{(f)} \rho,$$  (3.5)

Now the supersymmetry variation at each boundary is proportional to $(1 + i \gamma^5 T^{(f)}) \epsilon(y_f)$. Denoting with $\xi$ [see Eq. (2.1)] the upper part of $\epsilon$, whenever $(\vec{T}^{(f)})^2 = 1$ these variations can cancel provided the transformation parameter satisfies the BC’s $\xi^2 = z (\vec{T}^{(f)}) \xi^1$. The only possibility is that $T^{(0)} = T^{(\pi)}$, since $\epsilon$ is constant for global supersymmetry. Notice that according to Eqs. (2.4) and (2.5), this gives rise to the same BC’s for the gaugino,
\[ \eta^2 = z(\bar{T}(f)) \eta^1. \]

The remaining EOM then fix the BC’s \( G_{\mu\delta} = \bar{X} = \Sigma = 0. \) The bottom line of the off-shell approach is that, in the presence of a boundary, at most one supersymmetry can be preserved. Global SUSY invariance for the action of a vector multiplet singles out a special boundary mass term for gauginos such that \( z_0 = z_\pi \) which is at origin of the zero mode in the spectrum [see Eq. (2.20) for \( \omega = 0 \)]. We expect there to be a locally supersymmetric extension of the action \((3.1) + (3.4)\) for \( T(0) \neq T(\pi) \). In this case the \( SU(2)_R \) auxiliary gauge connection \( \bar{V}_M \) from the supergravity multiplet gives an additional source of supersymmetry breaking. Notice that for a globally supersymmetric vacuum there must then be a solution to the Killing spinor equation

\[ \gamma^5 D_5 \epsilon(y) = 0, \quad \xi^2(y_f) = z(\bar{T}(f)) \xi^1(y_f), \quad (3.6) \]

where \( D_M \) is covariant with respect to \( SU(2)_R \). These equations coincide with the zero mode condition for the gaugino considered above.

4 **Fermions in the Orbifold**

In this section we will consider the same system of a symplectic Majorana spinor but in the more common orbifold approach. As we have seen in the previous section, in the interval framework once the action is given we obtain the bulk equations supplemented with a consistent set of BC’s. Let us now study what happens in the orbifold geometry for an \( SU(2)_R \) fermionic doublet \( \Psi^i \) where \( SU(2)_R \) is identified with the automorphism of \( N = 2 \) supersymmetry algebra. We will consider the direct product of a flat 4D Minkowski spacetime times the orbifold \( S^1/\mathbb{Z}_2 \), the (flat) geometry considered in section 2. In the \( S^1/\mathbb{Z}_2 \) orbifold the fifth coordinate runs now along the circle \( y \in [-\pi R, \pi R] \) and we assign to the spinors the following parities

\[
\begin{align*}
\eta^1(x, -y) &= \eta^1(x, y), \\
\eta^2(x, -y) &= -\eta^2(x, y), \\
\eta^1(x, \pi R - y) &= \sigma \eta^1(x, \pi R + y), \\
\eta^2(x, \pi R - y) &= -\sigma \eta^2(x, \pi R + y),
\end{align*}
\]

(4.1)

where \( \sigma = \pm 1 \). The second condition is often replaced by demanding periodicity (\( \sigma = +1 \)) or anti-periodicity (\( \sigma = -1 \)) and corresponds to an intrinsic parity for the inversion with respect to \( y = \pi R \). The orbifold Lagrangian is then given by

\[ \mathcal{L} = i \bar{\Psi} \gamma^M D_M \Psi + 2 \delta(y) \left( N^{(0)}_{ij} \eta^i \eta^j + \text{h.c.} \right) + 2 \delta(y - \pi R) \left( N^{(\pi)}_{ij} \eta^i \eta^j + \text{h.c.} \right). \]

(4.2)

Dirac-like mass terms mixing \( \eta_1 \) and \( \eta_2 \), \( N^{(0, \pi)}_{ij} \), must have an odd profile as it is obvious from the parity assignments in Eq. (4.1). If they are continuous at \( y = 0, \pi R \) they do not contribute to the brane mass Lagrangian in (4.2); therefore if they contribute they must

18In the global theory on the interval all supersymmetry breaking is encoded in the \( T(f) \) mass matrix. There is no auxiliary field \( V_M \) whose VEV could contribute to the breaking, nor can one choose a SS twist by hand, since the BC’s are uniquely fixed by the equations of motion.
possess a discontinuity at the fixed points \( y = 0, \pi R \). The simplest ansatz is that near the fixed point at \( y = y_f \) \((f = 0, \pi)\) they behave as \(^{17}\)

\[
N^{(f)}_{12} = N^{(f)}_{D} \epsilon_f(y)
\]

where \( \epsilon_0 = \epsilon(y), \epsilon_\pi = \epsilon(\pi R - y) \) and the sign function \( \epsilon(y) \) is defined as

\[
\epsilon(y) = 2 \int_{0}^{y} \delta(z) dz.
\]

In the simplest case where there is no mass localized at the orbifold fixed points (e.g. \( N^{(f)} = 0 \)) there is a straightforward correspondence between the orbifold and the interval approach: we can take the fields continuous across the orbifold fixed points. Then using the parity assignment \(^{(4.1)}\) and periodicity we have

\[
\eta^2(0^+) = 0, \quad \eta^2(\pi R^-) = 0.
\]

Being the EOM the same we recover the interval result simply using as BC \( z_f = 0 \). As a result, when no mass term is present at the orbifold fixed points the parity assignment in the orbifold is equivalent to the choice of BC’s in the interval. As soon as we turn on a mass term in the orbifold fixed point the correspondence is much more involved. First of all we have to immediately face a technical problem inherent to the orbifold construction: to give a meaning to the fixed point Lagrangian which contains the product of distributions with overlapping singularities. In order to do that often implicit or explicit assumptions regarding the continuity properties of the fields are made. We want to stress here that such assumptions can lead to inconsistencies since solutions to the EOM might not exist. On the other hand by regularizing the delta functions in \(^{(4.2)}\) one can consistently assume all fields to be smooth while only in the limit of a sharp delta the solutions to the EOM will develop discontinuities. For a “sharp” delta function, \( \epsilon(y) \) is simply the step function with value +1 \((-1)\) for \( y > 0 \) \((y < 0)\). For a “regularized” delta function \( \epsilon(y) \) is a regular odd function that therefore satisfies the property \( \epsilon(0) = 0 \). In all the expressions that follow delta and epsilon functions should be considered as regularized with the sharp limit implicitly taken at the end of the calculation.

Our strategy will thus be the following. In a first step we solve the EOM close to the branes using the orbifold BC’s \(^{(4.1)}\). Taking the limit of sharp distributions we determine the precise jump profile across the branes which will result in modified BC’s, at \( 0^+ \) and \( \pi^- \) respectively. These modified BC’s can again be encoded in complex numbers denoted by \( z_{0^+}, z_{\pi^-} \) which are functions of the brane masses. Since the bulk EOM are identical to those in the interval approach we can directly use the results from section \(^{2}\) that is Eqs. \(^{(2.18)}\) to \(^{(2.20)}\), with the only replacements \( z_0 \to z_{0^+} \) and \( z_{\pi} \to z_{\pi^-} \). This establishes the precise relation between the orbifold and interval approaches.

\(^{17}\)Of course different ansätze could be considered, as an odd power of \( \epsilon(y) \) or any other odd function. We will just consider in this paper the usual case of a linear behavior in \( \epsilon(y) \).
We use the notation of section 2, where \( \eta^i(x, y) = \varphi^i(y) \psi(x) \) are the components of the symplectic-Majorana spinors in the Hosotani basis. Going to the SS basis, \( \Phi^i, \Psi = \exp(-i \bar{q} \vec{\sigma} \omega y/R) \Phi \) we obtain the following EOM in the orbifold geometry

\[
\varepsilon^{ij} \frac{d \phi^i}{dy} + m \bar{\phi}^i - 2 \sum_{f=0,\pi} \delta(y - y_f) N^{(f)}_{ij} \phi^j = 0 \tag{4.6}
\]

To determine the precise form of the discontinuities we consider the EOM close to the fixed points at \( y = 0 \)

\[
\frac{d \varphi^1}{dy} + 2 \left( N^{(0)}_{22} \varphi^2 + N^{(0)}_{12} \varphi^1 \right) \delta(y) = 0 \tag{4.7}
\]

\[
-\frac{d \varphi^2}{dy} + 2 \left( N^{(0)}_{11} \varphi^1 + N^{(0)}_{12} \varphi^2 \right) \delta(y) = 0 \tag{4.8}
\]

Note that we have neglected the term \( m \bar{\phi}^i \) as well as the one \( \propto \delta(y - \pi) \) since they are negligible close to \( y = 0 \). This approximation becomes more and more accurate the sharper the distributions are taken, and it is in fact exact in the singular limit. At \( y = \pi R \) the equations are in the same form \textit{mutatis mutandi} 0 by \( \pi \). From here it is easy to see that making assumptions about continuity of the fields may fail. For instance, continuity at \( y = 0 \) is clearly consistent only with \( N^{(0)}_{ij} = 0 \), while the weaker assumption of only \( \varphi_1 \) smooth is inconsistent unless \( N^{(0)}_{22} = N^{(0)}_{12} = 0 \) or \( N^{(0)}_{11} = N^{(0)}_{12} = 0 \). To avoid these difficulties we proceed as follows. We can solve Eqs. (4.7) and (4.8) by assuming (close to the fixed point at \( y = 0 \)) the functional dependence \( \varphi^i = \varphi^i(\epsilon(y)) \) and using the chain rule as

\[
\frac{d \varphi^i}{dy} = 2 (\varphi^i)' \delta(y) \tag{4.9}
\]

where \( (\varphi^i)' \equiv \frac{d \varphi^i}{d \epsilon} \). Using now (4.9) we can cast Eqs. (4.7) and (4.8) as

\[
(\varphi^1)' + N^{(0)}_{22} \varphi^2 + N^{(0)}_{12} \varphi^1 = 0 \tag{4.10}
\]

\[
(\varphi^2)' - N^{(0)}_{11} \varphi^1 - N^{(0)}_{12} \varphi^2 = 0 \tag{4.11}
\]

Passing from Eqs. (4.7) and (4.8) to Eqs. (4.10) and (4.11) makes sense for any regularized delta function for which we can solve our EOM. As we said we can consistently take the sharp limit after solving the EOM. In that case, using \( \epsilon(0) = 0 \) we can solve Eqs. (4.10) and (4.11) with the BC’s \( \varphi^2(0) = 0, \varphi^1(0) = 1 \). One gets for any regularized delta function the \textit{exact} analytical solution

\[
\varphi^1(y) = e^{-N^{(0)}_{D}\epsilon^2(y)/2} \ {}_1F_1 \left[ -N^{(0)}_{11} N^{(0)}_{22} / 4 N^{(0)}_D; \frac{1}{2}; N^{(0)}_{D} \epsilon^2(y) \right] \tag{4.12}
\]

\[
\varphi^2(y) = N^{(0)}_{11} \epsilon(y) e^{-N^{(0)}_{D}\epsilon^2(y)/2} \ {}_1F_1 \left[ 1 - N^{(0)}_{11} N^{(0)}_{22} / 4 N^{(0)}_D; \frac{3}{2}; N^{(0)}_{D} \epsilon^2(y) \right] \tag{4.13}
\]

\footnote{To see this one typically integrates between 0 and 0\(^+\). Note that a discontinuous even function has the same value at 0\(^-\) and 0\(^+\) but a different one at 0.}
where \( _1F_1 \) is the Kummer confluent hypergeometric function\(^{19}\). From Eq. (4.12) and the definition of the Kummer function it is easy to check that the odd field is discontinuous at the brane unless \( N_{11}^{(0)} = 0 \). The even field makes a jump unless \( N_D^{(0)} = 0 \) and \( N_{11}^{(0)} N_{22}^{(0)} = 0 \). In fact the solutions (4.12)-(4.13) have an interesting expression in the limit where the Dirac mass \( N_D^{(0)} \to 0 \). They are given by

\[
\lim_{N_D^{(0)} \to 0} \varphi^1(y) = \cos \left( \sqrt{N_{11}^{(0)} N_{22}^{(0)}} \epsilon(y) \right) \\
\lim_{N_D^{(0)} \to 0} \varphi^2(y) = -\sqrt{\frac{N_{11}^{(0)}}{N_{22}^{(0)}}} \sin \left( \sqrt{N_{11}^{(0)} N_{22}^{(0)}} \epsilon(y) \right)
\]

Similarly we can solve the Eqs. (4.16) on the vicinity of \( \pi R \) by assuming that near the brane there is a functional dependence as \( \varphi^i = \varphi^i(\epsilon(y)) \) and using \( d\epsilon(y)/dy = -2\delta(y - \pi R) \). The solutions can be easily read off from Eqs. (4.14) and (4.15) by simply changing \( N_{ij}^{(0)} \to -N_{ij}^{(\pi)} \) and \( \epsilon_0 \to \epsilon_\pi \) (for \( \sigma = 1 \)) or by changing \( N_{ij}^{(0)} \to N_{ij}^{(\pi)} \), \( \epsilon_0 \to \epsilon_\pi \) and \( \varphi^1(y) \leftrightarrow \varphi^2(y) \), i.e. by simply label changing \( 1 \leftrightarrow 2 \) and \( 0 \to \pi \) (for \( \sigma = -1 \)).

We can now use the behaviour of the solutions close to the fixed points to read out the jumps for odd and even fields caused by the presence of the delta-functions in the EOM. We have at \( y = 0^+ \)

\[
\varphi^2(0^+) = N_{11}^{(0)} \frac{1 F_1 \left[ 1 - \frac{N_{11}^{(0)} N_{22}^{(0)}}{4 N_D^{(0)}}; \frac{3}{2}; N_D^{(0)} \right]}{1 F_1 \left[ -\frac{N_{11}^{(0)} N_{22}^{(0)}}{4 N_D^{(0)}}; \frac{1}{2}; N_D^{(0)} \right]} \varphi^1(0^+).
\]

(4.16)

Similar expressions at \( \pi^- \) are obtained by doing the before-mentioned changes. Now we can identify the values of the \( z_f \) parameters entering in the mass-formula. From the jumps at \( y = 0^+ \) one obtains:

\[
z_{0^+} = N_{11}^{(0)} \frac{1 F_1 \left[ 1 - \frac{N_{11}^{(0)} N_{22}^{(0)}}{4 N_D^{(0)}}; \frac{3}{2}; N_D^{(0)} \right]}{1 F_1 \left[ -\frac{N_{11}^{(0)} N_{22}^{(0)}}{4 N_D^{(0)}}; \frac{1}{2}; N_D^{(0)} \right]},
\]

(4.17)

while \( z_{\pi^-} \) can be obtained by the above mentioned trivial substitutions. In the particularly

---

\(^{19}\)The Kummer confluent hypergeometric function is defined by the series expansion

\[
1 F_1[a; b; z] = 1 + \sum_{k=1}^{\infty} \frac{a(a + 1) \cdots (a + k - 1) \ z^k}{b(b + 1) \cdots (b + k - 1) \ k!}.
\]
simple case of no Dirac mass one obtains the expressions

\[
\lim_{N_D^{(0)} \to 0} z_{0^+} = -\sqrt{\frac{N_{11}^{(0)}}{N_{22}^{(0)}}} \tan \left( \sqrt{\frac{N_{11}^{(0)}}{N_{22}^{(0)}}} \right), \quad \text{; (4.18)}
\]

\[
\lim_{N_D^{(e)} \to 0} z_{\pi^-} = i \sqrt{\frac{N_{11}^{(e)}}{N_{22}^{(e)}}} \left[ i \tan \left( \sqrt{\frac{N_{11}^{(e)}}{N_{22}^{(e)}}} \right) \right]^{\sigma}. \quad \text{. (4.19)}
\]

As for the solution of the EOM far from the branes, the form is identical to the one in the interval case, Eq. (2.18), but now in the mass formula one has to use the quantities \(z_{0^+}\) and \(z_{\pi^-}\), which are given by different functions of the brane masses.

To cross-check the validity of this procedure we have done a numerical integration of the full EOM, Eqs. (4.6). We indeed find that solutions only exist for the mass eigenvalues given by Eq. (2.20), where \(z_f\) are calculated from the mass parameters through Eqs. (4.17).

Let us summarize the orbifold approach. In this framework the BC’s are uniquely fixed by the parity assignments, while the boundary Lagrangian cause the wavefunctions to jump. We have shown how these jumps can be computed by regularizing the delta functions, thereby obtaining smooth wave-functions. The results are manifestly independent of the regularization, so this gives a well defined procedure. The jumps define new BC’s at \(0^+\) and \(\pi^-\) which can be used to solve the bulk equations in the interval \([0^+, \pi^-]\). If these BC’s are aligned with the Scherk-Schwarz direction, we again find the spectrum to be independent of the SS parameter.

Finally let us stress that we have computed the orbifold mass eigenvalues and eigenfunctions for arbitrary brane mass terms. Our results generalize (and agree with) the particular cases previously considered in the literature [2,3,9].

### 5 WARPED GEOMETRY

In this section we extend our previous results in the interval approach to warped geometry. The \(SU(2)_R\) automorphism group found in the off-shell bulk supergravity action in flat space is broken by the warping down to two surviving \(U(1)\)’s. One of them is gauged by the graviphoton \(A_M\) with gauge coupling \(g\), and the other \(U(1)\) invariance becomes redundant after the elimination of the auxiliary field \(\vec{V}\) in terms of the graviphoton. The gauged \(U(1)\) couples to the fermions via the off shell Lagrangian [11]

\[
\mathcal{L}_{\text{bulk}} = \frac{i}{2} \bar{\Psi} \gamma^M D_M \Psi - \frac{i}{2} D_M \bar{\Psi} \gamma^M \Psi - 2c \bar{\Psi} \vec{f} \cdot \sigma \Psi ; \quad (5.1)
\]

where \(\Psi\) is a symplectic-Majorana fermion, the field \(\vec{f}\) is an \(SU(2)_R\)-triplet auxiliary-field from the minimal supergravity multiplet, the covariant derivative is given by

\[
D_M = \partial_M - \frac{1}{4} \omega_{ABM} \gamma^{AB} - \frac{i}{2} \bar{\sigma} \cdot \vec{V}_M . \quad (5.2)
\]
and \( c \) is the bulk mass term fixed by supersymmetry in the warped space. In particular \( c = 1 \) for gauginos.

The equations of motion then imply that \([11]\)

\[
\tilde{t} = \frac{1}{4\sqrt{3}} g \tilde{q}; \tag{5.3}
\]

while for \( V_M \) one finds a relation with the graviphoton \( A_M \) as

\[
d(\tilde{q} \cdot \tilde{V} - 2gA) = 0, \quad \tilde{q}^2 = 1. \tag{5.4}
\]

which gives the solution

\[
\tilde{V}_M = 2 \tilde{q}(C_M + g A_M) \quad dC = 0. \tag{5.5}
\]

The one form \( C \) is closed, \( dC = 0 \), but not necessarily trivial, \( C = 0 \). For nonzero \( g \) it can however be absorbed into the graviphoton \( A_M \) by a shift \(^{20}\). All the Scherk-Schwarz breaking will then be encoded in the graviphoton VEV, \( \langle A \rangle = \omega dy \). The graviphoton will gauge the \( U(1)_R \subset SU(2)_R \) subgroup defined by the unit vector \( \tilde{q} \).

For the on-shell gaugino action we thus get

\[
L_{\text{bulk}} = \frac{i}{2} \bar{\Psi} \gamma^M D_M \Psi - \frac{i}{2} D_M \bar{\Psi} \gamma^M \Psi - c \frac{g}{2\sqrt{3}} \bar{\Psi} \tilde{q} \cdot \tilde{\sigma} \Psi; \tag{5.6}
\]

where the covariant derivative is now given by

\[
D_M = \partial_M - \frac{1}{4} \omega_{ABM} \gamma^{AB} - i \tilde{q} \cdot \tilde{\sigma} gA_M. \tag{5.7}
\]

Let us now turn to the bosonic sector. In order to obtain a viable phenomenology we will restrict our discussion to the Randall-Sundrum model \([16]\), which leads to \( 4D \) Minkowski spacetime. The warping in the bulk must then be balanced with the brane tensions ending up with the tunings of the Randall-Sundrum model. Moreover local supersymmetry requires the bulk (AdS) cosmological constant to be related with the fermion coupling to the graviphoton, \( \Lambda = -g^2 \). The metric in conformal coordinates can be written as

\[
ds^2 = a^2 [\eta_{\mu\nu} dx^\mu dx^\nu + du^2], \quad a(u) = (ku)^{-1}. \tag{5.8}
\]

The conformal coordinates \((x, u)\) are related with the coordinates in \([16]\) by \( u(y) = k^{-1} \exp(ky) \). The constant \( k \) is related to the gauge coupling \( g \) as \( g = \sqrt{3}k \). The graviphoton background in the conformal frame reads as

\[
A = \omega a du. \tag{5.9}
\]

Concerning the boundary Lagrangian \( T^{(0)}, V^{(0)} \) will be the mass terms in the UV brane located at \( u = u_{UV} = 1/k \) and \((u_{IR}k)^5 T^{(s)}, (u_{IR}k)^5 V^{(s)} \) the mass terms in the IR brane

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\(^{20}\)Note that this does not happen in the flat case.
Proceeding as in section 2, the BC at the IR brane is twisted by \( \omega \) as given in (2.17). Using the same notation and setting \( \hat{\omega} = \omega/k \), the bulk Dirac equation can be written in the SS frame as
\[
i \gamma^M \partial_M (a^2 \Phi) + \frac{1}{2} c k a^3 \vec{q} \cdot \vec{\sigma} \Phi = 0 \quad .
\] (5.10)
The massive KK modes associated with the solutions of (5.10) can be expressed as combinations of Bessel functions of order \( \nu = \vert c \pm 1 \vert /2 \), for \( \vec{q} = (0, 0, 1) \), where the two signs refer to the two gaugino components. Let us focus on the zero mode, its wavefunction is given by
\[
\varphi(u) = a^{-2}(u) \exp\{-\vec{q} \cdot \vec{\sigma} \log(ku)(i\hat{\omega} + c/2)\} \varphi(k^{-1}) \quad .
\] (5.11)
The condition for the existence of a zero mode coming from enforcing the BC’s is the following
\[
(z_0 - \zeta_\pi) \cosh \theta + \{-q_1(z_0 \zeta_\pi - 1) + i q_2(1 + z_0 \zeta_\pi) - q_3(z_0 + \zeta_\pi)\} \sinh \theta = 0 ;
\]
\[
\theta = \frac{1}{2} \log(u_{IR}/u_{UV}) = \frac{k\pi}{2}.
\] (5.12)
In the absence of any fine tuning, the coefficients of \( \sinh \theta \) and \( \cosh \theta \) should vanish separately and we get
\[
z_0 = \zeta_\pi = \frac{\lambda - q_3}{q_1 - i q_2} \equiv z(\vec{q}) \quad ;
\] (5.13)
the same condition as in flat space. There is however an important difference: the values of \( z_f \) are fixed by imposing that the SUSY transformations are fulfilled
\[
(z_f \delta_\epsilon \eta^1 - \delta_\epsilon \eta^2)|_{y_f} = 0 \quad .
\] (5.14)
This will lead \( z_0, z_\pi \) to depend on the bosonic tensions on the respective branes \( \tau_0, \tau_\pi \) and on the bulk cosmological constant \( g \). When further imposing one of the Randall-Sundrum tunings, \( \tau_0 = \tau_\pi \), one gets \( z_0 = z_\pi \), and when enforcing the brane-bulk balance, \( \tau_0 = \sqrt{24}g \), one obtains \( z_0 = z_\pi = z(\vec{q}) \) \(^{21}\). Therefore, persistent supersymmetry is guaranteed. Any other BC’s deviating from this value would explicitly violate local supersymmetry \(^{22}\) \^[17, 18\]. This is in contrast to the flat case, where any BC’s for the fermions respect local supersymmetry, and the breaking of \( N = 1 \) SUSY by non-aligned BC’s is spontaneous. On the other hand in the warped case, imposing local supersymmetry implies the BC’s (5.13) and supersymmetry is persistent whatever the VEV of \( \vec{q} \cdot \vec{V}_M \), Eq. (5.5), is. This fact was already noticed in Refs. \^[19–22\]. In our formalism this persistence is a consequence of the alignment of bulk and brane breaking. This alignment is due to the relation between bosonic stability in the metric and the fermionic sector via supersymmetry transformations, Eq. (5.14).

\(^{21}\)The quantity \( z(\vec{q}) \) was defined in Eq. (2.21).

\(^{22}\)The special case of flipped BC’s \^[17\] corresponds to \( z_0 = z(\vec{q}), z_\pi = \zeta_\pi = z(\vec{q}) \).
6 Conclusions

We have presented in this paper a detailed study of supersymmetry breaking and restoration when two sources of breaking are present, both Scherk-Schwarz and boundary mass terms. The mass spectrum of fermions is obtained through the equations of motion in the bulk and brane boundary conditions. These boundary conditions are extracted either in the interval approach, by extremizing the boundary action, or in the orbifold language, by assigning parities to the fields. We compare the two approaches and compute the nontrivial jump profiles of the wavefunctions in the orbifold picture for general brane mass terms. With our procedure of dealing with fermions we study in both approaches supersymmetry breaking by adding the Scherk-Schwarz breaking terms and computing the mass spectrum. We find out that for a suitable tuning of the boundary actions supersymmetry is always present for arbitrary values of the Scherk-Schwarz parameter. As an application of the interval formalism, we construct bulk and boundary actions for supersymmetric Yang-Mills theories. Finally we extend our results to the warped Randall-Sundrum background. In this case the SS-direction coincides with the gauged $U(1)_R$ subgroup, and local supersymmetry enforces alignment of the boundary conditions with the latter. Therefore any misalignment is an explicit form of supersymmetry breaking and spontaneous SS breaking becomes impossible. This conclusion agrees with the findings in Refs. [19–22]. Within our formalism, it follows from the alignment of bulk and brane breakings once one relates the Randall-Sundrum tunings in the bosonic sector with the fermionic boundary conditions.

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