The Triple-Alpha Process and the Anthropically Allowed Values of the Weak Scale

Tesla E. Jeltema and Marc Sher

Nuclear and Particle Theory Group
Physics Department
College of William and Mary, Williamsburg, VA 23187, USA

ABSTRACT

In multiple-universe models, the constants of nature may have different values in different universes. Agrawal, Barr, Donoghue and Seckel have pointed out that the Higgs mass parameter, as the only dimensionful parameter of the standard model, is of particular interest. By considering a range of values of this parameter, they showed that the Higgs vacuum expectation value must have a magnitude less than $5.0 \times$ its observed value, in order for complex elements, and thus life, to form. In this report, we look at the effects of the Higgs mass parameter on the triple-alpha process in stars. This process, which is greatly enhanced by a resonance in Carbon-12, is responsible for virtually all of the carbon production in the universe. We find that the Higgs vacuum expectation value must have a magnitude greater than $0.90 \times$ its observed value in order for an appreciable amount of carbon to form, thus significantly narrowing the allowed region of Agrawal et al.
The anthropic principle\[1\] states that the parameters of our Universe must have values which allow intelligent life to exist. It is a principle which has existed in some form or another since the beginning of human history. It has countless formulations, many of which have religious overtones. In recent years, however, the anthropic principle has been revived as a method of explaining some fine-tuning problems. For example, Weinberg has considered\[2\] whether the principle can address the relative smallness of the cosmological constant.

In its weak form, the anthropic principle states that, because we are here to observe them, the observed properties of the universe must have values which allow life to exist. This may seem somewhat obvious or circular, but it becomes significant in some physical theories which support the existence of domains in the universe in which different parameters are applicable. In chaotic inflation models\[3\], for example, different domains may have different Higgs vacuum expectation values. These domains can be regarded as different universes. Alternatively, in regions of high gravitational curvature, new universes may, in some models, “pop” out of the vacuum; these new universes may have different values of the parameters. Thus, considering how our universe (and the life therein) would evolve if the parameters of the standard model were changed may be physically relevant.

The standard model has (including neutrino masses and mixing) some 24 parameters. Thus, any complete study of the anthropic principle would involve study of a complex 24-dimensional parameter space. In two recent papers\[4, 5\], Agrawal, Barr, Donoghue and Seckel(ABDS) noted that the Higgs mass-squared parameter is of special interest. It is the only dimensionful parameter in the model, and multiple-universe models may be more likely to have varying dimensionful couplings than varying dimensionless ones. The Higgs mass-squared parameter is also unnaturally small compared with the parameters of more general theories, such as grand unified theories.

ABDS considered the range of anthropically allowed values of the Higgs mass-squared
parameter, $\mu^2$. They considered values of this parameter ranging from $-M_{Pl}^2$ to $M_{Pl}^2$, where $M_{Pl}$ is the Planck scale, and we define the sign of $\mu^2$ to be negative in the standard model. ABDS considered both the cases $\mu^2 < 0$ and $\mu^2 > 0$. In the latter case, the electroweak gauge symmetry is still broken by quark condensation ($\langle \bar{q}q \rangle \neq 0$). For the $\mu^2 < 0$ case, they found that as one increases the Higgs vacuum expectation value $v \equiv \langle \phi \rangle = \sqrt{-\mu^2 \lambda / \chi}$ from its standard model value, $v_0$, the first major effect occurs when the deuteron becomes unbound. This occurs when $v/v_0$ reaches a value of $1.4 - 2.7$, depending on the nuclear physics model, and is due to the increasing neutron-proton mass difference. When $v/v_0$ is greater than about 5.0, all nuclei become unstable. They argue, therefore, that one must have $v/v_0 < 5.0$ (and possibly less than 2.7) in order for complex elements to form, and thus life. They also note that for $v/v_0 > 10^3$, the $\Delta^{++}$ becomes stable relative to the proton, leading to a very unusual universe indeed. For $\mu^2 > 0$, the weak scale becomes of the order of magnitude of the QCD scale, and chemical and stellar evolution become much more complicated.

One process not considered by ABDS is the triple-alpha process in stars. This process occurs when two alpha particles first fuse into beryllium ($^4He + ^4He \rightarrow ^8Be$). The beryllium has a very short lifetime (of order of $10^{-16}$ seconds), but lives long enough for further interaction with a third alpha particle ($^4He + ^8Be \rightarrow ^{12}C^*$) to produce carbon. Virtually all of the carbon in the universe is produced through this process. This process is anthropically significant because it depends very precisely on the existence of a $0^+$ resonance 7.6 MeV above the ground state in $^{12}C$. The existence of this resonance was one of the first, major successful predictions of astrophysics; being predicted by Hoyle long before the discovery of the resonance. Without this resonance, little carbon will be produced. Without carbon, it is difficult to see how life could spontaneously develop. Life, as we generally define it, requires the existence of a molecule capable of storing large amounts of information, and it is impossible for hydrogen and helium to form
such molecules. Since the existence of the resonance is a very sensitive function of the parameters of the model\[1\], one might expect it to give much more stringent bounds on $v/v_0$ than those obtained by ABDS. In this Brief Report, we examine the dependence of this process on $\mu^2$, and significantly narrow the range found by ABDS.

There have been several calculations concerning the anthropic significance of the triple-alpha process. Livio, et al.\[7\] calculated the sensitivity of the amount of carbon production to changes in the location of the $0^+$ resonance, but did not address the underlying physics behind the location of the resonance. Oberhummer, et al.\[8\] then did a detailed nuclear physics calculation of the sensitivity of the location of the resonance to the strength of the nucleon-nucleon potential. This required considering several different models for the nuclear reaction rates. They found that a change of only a part in a thousand in the strength of the nucleon-nucleon interaction will change the reaction rate of the triple-alpha process by roughly a factor of 20, and a change of two parts in a thousand changes it by roughly a factor of 400.

The strength of the nucleon-nucleon interaction, however, is a very complicated function of the many parameters of the standard model. Our objective is to relate this strength to changes in the vacuum expectation value of the Higgs boson, $v$. Changing $v$ will change the quark masses, and will also change the value of the QCD scale. Both of these are addressed by ABDS. The quark masses change in a very predictable way: $m_q \sim (v/v_0)$. The QCD scale, $\Lambda$, which is sensitive to the quark masses through threshold effects (it is assumed that the high energy value is unchanged), is found by ABDS to scale as $(v/v_0)\zeta$, where $\zeta$ varies between 0.25 and 0.3—we will take it to be 0.25 in this work. From these variations, one can calculate the variation of the relevant baryon and meson masses, and convert that into an effect on the strength of the nucleon-nucleon interaction.

The phrase “strength of the nucleon-nucleon interaction” is, of course, somewhat
ambiguous. Oberhummer, et al.\cite{8}, simply multiplied the interaction by a constant. When the meson and baryon masses change, however, the entire shape of the potential changes. A precise analysis would necessitate using this full potential in the calculation of the triple-alpha process. However, these calculations use “phenomenological” parameters, which are experimentally determined, and the variation of these parameters with \( v \) is unknown. We therefore estimate the size of the effect by finding an “average” value of the potential, defined as

\[
\langle V \rangle = \frac{\int_0^\infty V(r) |\psi(r)|^2 \, d^3 r}{\int_0^\infty |\psi(r)|^2 \, d^3 r}
\]

where \( \psi \) is the two-nucleon wavefunction, obtained by solving the Schrödinger equation, and compare this with Oberhummer, et al. We now have to determine the dependence of the potential on \( v/v_0 \).

The nucleon-nucleon potential has three main features, shown in Figure 1. There is a repulsive core, an attractive minimum and a long-range tail from one-pion exchange. We will look at two different models for the nucleon-nucleon potential. The first considers

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{potential.png}
\caption{The nucleon-nucleon potential, from Ref. \cite{9}. The \( \sigma \) is believed to be a two-pion resonance, although it may be a real, but very broad, physical state.}
\end{figure}
the repulsive core to be due to the exchange of the \( \omega \) vector meson, and the attractive minimum to be due to the exchange of the hypothetical sigma meson. Controversy exists as to whether the sigma meson is an actual particle with a large width, or simply a correlated two-pion exchange. We will assume the latter for the moment, but will show that the results will not change significantly in either case. The potential can then be written as

\[
V(r) = \frac{g_\omega}{r} \exp\left(-m_\omega r\right) - \frac{g_\sigma}{r} \exp\left(-m_\sigma r\right) - \frac{g_\pi}{r} \exp\left(-m_\pi r\right)
\]

where the \( g_i \), arising from the strong interaction van der Waals forces, are assumed to be independent of the weak scale. To find the dependence of \( V(r) \) on \( v/v_0 \), we now need to ascertain the dependence of \( m_\omega, m_\sigma \) and \( m_\pi \) on \( v/v_0 \) (as well as the dependence of the nucleon mass, due to the input into the Schrödinger equation).

The dependence of the pion mass on the weak scale is easily determined from the formula from chiral symmetry breaking, which gives \( m_\pi^2 \propto f_\pi (m_u + m_d) \). Since \( f_\pi \) varies as \( \Lambda_{QCD} \), which varies as \( (v/v_0)^\xi \), and \( m_u + m_d \) varies as \( v/v_0 \), one can see that \( m_\pi \sim (v/v_0)^{1+\xi} \). The nucleon and the \( \omega \) primarily get their masses from QCD, which scale as \( \Lambda_{QCD} \), but have small contributions from the current quark masses. In MeV, the masses are given by \( m_{\text{nucleon}} = 921(v/v_0)^\xi + 18(v/v_0) \) and \( m_\omega = 768(v/v_0)^\xi + 14(v/v_0) \), where we have taken the up and down current quark masses to be 4 and 7 MeV, respectively.

The mass of the sigma is a different matter, since it is a two-pion correlated state. We follow the work of Lin and Serot[10], who derive the mass of the \( \sigma \) in terms of the pion mass, the nucleon mass and the pion-nucleon coupling constant. By varying the masses of the pion and nucleon in their expressions, we find that \( m_\sigma \sim (v/v_0)^{0.26} \). This is not a surprising result. The \( \sigma \) mass turns out to be very insensitive to the pion mass, and thus it can only scale as the nucleon mass, which scales as \( (v/v_0)^{0.25} \). It also indicates that the result is not significantly changed if one regards the \( \sigma \) to be a real particle, since one
would expect such a particle to scale as the QCD scale, and $\Lambda_{QCD} \sim (v/v_0)^{0.25}$.

With the mass dependences, we now determine the strength of the nucleon-nucleon potential as $v$ is varied. It is found that a 1% change in $v$ affects the strength of the potential by 0.4% (in the same direction); a 10% change in $v$ affects it by 4%. To see how robust this result is, we also considered a completely different nucleon-nucleon potential, due to Maltman and Isgur[11], using six-quark states. There are two parts to the potential, a modified one-pion exchange part and a part due to residual quark-quark interactions. The latter, which is most relevant for this analysis, is entirely due to QCD, and thus its variation with $v$ only depends on the variation through $\Lambda_{QCD}$, which is determined dimensionally. The result is similar; a 1% decrease in $v$ decreases the strength of the potential by 0.6%.

Now that we have related the strength of the nucleon-nucleon potential to the dependence on $v$, we can go to the work of Oberhummer et al. who relate that to the rate of carbon production. Oberhummer et al. found that a decrease of $2 - 4\%$ in the strength of the nucleon-nucleon potential leads to the virtual elimination of carbon production (Livio, et al.[7] analyzed both 5 and 20 solar mass stars, although the result is insensitive to the precise stellar mass). Comparing with our result from the previous paragraph, we find that (conservatively taking a 4% decrease as our limit as well as the first potential model) one must have $v/v_0$ greater than 0.90. This substantially narrows the region found by ABDS, which had no effective lower bound on $v/v_0$, but only an upper bound of between 1.4 and 5.

How accurate is this result? As noted earlier, a precise determination of the effects of changing $v$ on the rate of carbon production in stars would require solving the twelve-body problem with a varying nucleon-nucleon potential (not to mention three-body forces). Oberhummer et al. just varied the overall strength of the two-body potential. A full analysis does not seem possible at this time. We have related the change in the potential
caused by the variation of \( v \) to an “average” potential strength. This “mean-field” approach is not particularly precise, but is probably the best that can be done at this time, given our lack of understanding of nuclear dynamics. The fact that two very different models of the potential give a similar bound is encouraging. Thus, our bound should be taken as a reasonable approximation to the bound that could be obtained with a full understanding of the nuclear physics involved.

We thank Dirk Walecka and Nathan Isgur for many useful conversations about the nucleon-nucleon potential, and Eric Dawnkaski for help with the computational aspects of this work. This work was supported by the National Science Foundation PHY-9900657.

References

[1] J. Barrow and F. Tipler, *The Anthropic Cosmological Principle* (Clarendon Press, Oxford 1986).

[2] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987); *Proceedings of Critical Dialogues in Cosmology*, Princeton University, June 1996; H. Martel, P. Shapiro and S. Weinberg, astro-ph/9701099.

[3] A.D. Linde, Phys. Lett. B**129**, 177 (1983); Phys. Lett. B**175**, 395 (1986); Phys. Lett. B**202**, 194 (1988).

[4] V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, Phys. Rev. Lett. **80**, 1822 (1998).

[5] V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, Phys. Rev. D**57**, 5480 (1998).

[6] F. Hoyle, D.N.F. Dunbar, W.A. Wenzel and W. Whaling, Phys. Rev. **92**, 1095 (1953).
[7] M. Livio, D. Hollowell, A. Weiss and J.W. Truran, Nature \textbf{340}, 281 (1989).

[8] H. Oberhummer, R. Pichler and A. Csoto, \textit{Proceedings of Nuclei in the Cosmos 5}, July 1998, Volos, Greece; H. Oberhummer et al., Z. Physik \textbf{A349}, 241 (1994).

[9] U. Meissner, Phys. Reports \textbf{161}, 214 (1988).

[10] W. Lin and B. Serot, Phys. Lett. \textbf{B233}, 23 (1989) and Nucl. Phys. \textbf{A512}, 637 (1990)

[11] K. Maltman and N. Isgur, Phys. Rev. D\textbf{29}, 952 (1984).