On the onset of runaway stellar collisions in dense star clusters - II. Hydrodynamics of three-body interactions

Evghenii Gaburov, James C. Lombardi, Jr., and Simon Portegies Zwart

1 Leiden Observatory, Leiden Observatory, the Netherlands
2 Astronomical Institute “Anton Pannekoek”, University of Amsterdam, the Netherlands
3 Section Computational Science, University of Amsterdam, the Netherlands
4 Department of Physics, Allegheny College, USA

ABSTRACT
The onset of runaway stellar collisions in young star clusters is more likely to initiate with an encounter between a binary and a third star than between two single stars. Using the initial conditions of such three-star encounters from direct $N$-body simulations, we model the resulting interaction by means of Smoothed Particle Hydrodynamics (SPH). Our code implements new equations of motion that allow for efficient use of non-equal mass particles and is capable of evolving contact binaries for thousands of orbits. We find that, in the majority of the cases considered, all three stars merge together. In addition, we compare our SPH calculations against those of the sticky-sphere approximation. If one is not concerned with mass loss, then the sticky sphere approach gives the correct qualitative outcome in approximately 75% of the cases considered. Among those cases in which the sticky-sphere algorithm identifies only two particular stars to collide, the hydrodynamic calculations find the same qualitative outcome in about half of the instances. If the sticky-sphere approach determines that all three stars merge, then the hydrodynamic simulations invariably agree. However, in such three star mergers, the hydrodynamic simulations reveal that: (1) mass lost as ejecta can be a considerable fraction of the total mass in the system (up to $\sim 25\%$); (2) due to asymmetric mass loss, the collision product can sometimes receive a kick velocity that exceeds $10\;\text{km/s}$, large enough to allow the collision product to escape the core of the cluster; and (3) the energy of the ejected matter can be large enough (up to $\sim 3 \times 10^{50}\;\text{erg}$) to remove or disturb the inter cluster gas appreciably.

1 INTRODUCTION
Stars are born in clusters, which upon formation are generally dense and massive. In recent years it has become clear that clusters remain bound even after losing a considerable fraction of their mass due to primordial out-gassing (Baumgardt & Kroupa 2007). The subsequent dynamical evolution of these clusters leads to a state of core collapse (Portegies Zwart et al. 2007), almost irrespective of the number of primordial binaries (Portegies Zwart et al. 2004); primordial binaries do, however, appear to delay the collapse of the core (Fregeau et al. 2003; Heggie et al. 2006). In addition, clusters with appropriate initial conditions may form a very massive star by means of runaway stellar collisions (Portegies Zwart et al. 2004). Such an object has been hypothesised to be a progenitor of an intermediate mass black hole (however, see Glebbeek et al. (2009)).

Even if binaries are not present at the birth of a star cluster, they can form via three-body encounters during the process of core collapse. Indeed, the expansion of the cluster core after deep gravothermal collapse (Sugimoto & Bettwieser 1983) is mediated by binaries, regardless of the presence or absence of a primordial population. During post-core collapse evolution, a cluster may enter a phase of gravothermal oscillations (Cohn et al. 1989), allowing periods of high interaction rate and providing further opportunity for binaries and single stars to interact closely.

Analytic expressions describing encounters between a binary and a third star, all treated as point masses, have been derived for various portions of parameter space (Heggie 1975; Hut 1983; Heggie & Hut 1993). In addition, complementary numerical surveys have been performed in the point mass approximation by a number of authors (Harrington 1970; Hut & Bahcall 1983; Hills 1992). During triple encounters, however, individual stars may approach close enough to each other that the approximation of point-particle dynamics breaks down: the size and internal structure of the stars then play a major role in determining the outcome of the encounter. Consequently, some numerical studies have augmented the point-mass treatment with simplified models that incorporate several hydrodynamic effects (McMillan 1986; Fregeau et al. 2004). Large-scale $N$-body simulations of clusters have demonstrated the ubiquity of resonance interactions in dynamically unstable triples (Portegies Zwart et al. 1999) – the scenarios that ultimately may lead to the coalescence of all three stars (Fregeau et al. 2004). The accu-
rate modelling of the details under which triples merge, and whether or not two or all three stars in an encounter participate in the merger, has a profound consequence for the occurrence of collision runaways (Portegies Zwart & McMillan 2002; Freitag et al. 2006) and whether or not such runaways can lead to the formation of binaries among intermediate mass black holes (Girani et al. 2006).

The first three-dimensional hydrodynamical calculations of encounters between a binary and a single star were performed by Cleary & Monaghan (1990) with the smoothed particle hydrodynamics (SPH) method. However, computational constraints at that time limited their work to a very small number of SPH particles (usually 136 per star) and to $n = 1.5$ polytropes, appropriate only for white dwarfs or extremely low mass main sequence stars. Subsequent hydrodynamic treatments of three-body interactions typically confined themselves to scenarios in which at least one of the stars was a compact object and therefore could be treated as a point mass (e.g., Davies et al. 1993, 1994, Davies et al. 1998, and Adams et al. 2004) consider three-body encounters between a binary and a red giant star as a mechanism for destroying red giants near the centres of dense stellar systems. Their hydrodynamic simulations follow the fluid of the red giant envelope during the encounter, with the red giant core and both components of the binary being treated as point masses. Because only the red giant envelope is treated hydrodynamically, the only mergers that can result are those which form a binary of the two point masses surrounded by a common envelope donated from the red giant envelope.

Numerous hydrodynamic simulations of colliding stars have studied the structure of the merger product (Benz & Hills 1987, Davies et al. 1993, 1994, Lombardi et al. 1995, Davies et al. 1998, Lombardi et al. 2002, Freitag & Benz 2005, Gaburov et al. 2008). In some cases the evolution of these collision products is studied further (Suzuki et al. 2007, Glebbeek et al. 2009), especially within the context of the formation and evolution of blue stragglers (Sills et al. 2001, 2005, Glebbeek et al. 2008, Glebbeek & Pols 2008). Such collision studies, however, have been focused on encounters between two single stars, ignoring for the time being that collisional cross sections and rates can be large for systems consisting of three or more stars.

The scenario of triple-star mergers among low-mass main-sequence stars has been previously considered by Lombardi et al. (2003) using SPH. Their calculations indicate that the collision product always has a significantly enhanced cross-section and that the distribution of most chemical elements within the final product is not sensitive to many details of the initial conditions. They, however, concentrated solely on low mass stars and treated the triple star merger as two separate, consecutive parabolic collisions.

Recently, Gaburov et al. (2008) performed an extensive and detailed study to investigate the circumstances under which a first collision between stars occurs. Using direct N-body integration with sticky spheres of realistic stellar sizes, they argued that binaries tend to catalyse collisions. In their simulations the binaries that are formed during core collapse tend to interact with an incoming star, which subsequently merges with one of the binary components. The results of Lombardi et al. (2003) suggest that the hydrodynamics of such interactions are unlikely to keep the binary itself undamaged. Instead, it is quite likely that the stellar material that is expelled during a collision engulfs the system in a common envelope, leading to the merger of all three stars.

In this paper, we introduce a new implementation of SPH and apply it to follow accurately the hydrodynamics of encounters between hard binaries and intruders. We concentrate on cases involving massive main-sequence stars, such as those found in young star clusters, treating all three stars simultaneously and with realistic orbital parameters determined from a dynamical cluster calculation. In particular, the initial conditions are selected from the set of $N$-body simulations carried out by Gaburov et al. (2008), but with the internal structure of the stars now being determined by a stellar evolution code. A comprehensive survey of triple-star collisions would need to explore an enormous amount of parameter space, but here we focus on a number of representative cases. In total, we selected 40 encounters from the simulations of Gaburov et al. (2008). Among these are random selections, as well as some that are specifically chosen because of their relevance for the subsequent $N$-body evolution or because of their uncertain outcome given the relatively simple treatment of mergers in the $N$-body simulations.

This paper is structured as follows. In §2 we introduce our new formulation of SPH, which allows efficient use of non-equal mass particles, as well as our approach for relaxing single stars. In §3 we describe how we model close and contact binary star systems, and we demonstrate the stability of these systems for at least the time interval of interest. The set of initial conditions for the three-body collisions are presented in §4. Finally, §5 presents, while §6 discusses, the results of our calculations.

2 METHODS AND CONVENTIONS

2.1 SPH code

Smoothed Particle Hydrodynamics is the most widely used hydrodynamics scheme in the astrophysics community. It is a Lagrangian particle method, meaning that the fluid is represented by a finite number of fluid elements or “particles.” Associated with each particle $i$ are, for example, its position $\mathbf{r}_i$, velocity $\mathbf{v}_i$, and mass $m_i$. Each particle also carries a purely numerical smoothing length $h_i$ that determines the local spatial resolution and is used in the calculation of fluid properties such as acceleration and density. See Monaghan (1992) and Rasio & Lombardi (1999) for reviews on SPH. The code which we used in this work was presented in Lombardi et al. (2006). However, we modify the dynamical equations to allow the efficient use of a range in particle masses. This modification is presented in Appendix A.

2.2 Choice of units

Throughout this paper, numerical results are given in units where $G = M_\odot = R_\odot = 1$, where $G$ is the Newtonian gravitational constant and $M_\odot$ and $R_\odot$ are the mass and radius of the Sun. The units of time, velocity, and energy
Figure 1. Stellar radius versus mass at 2 Myr, as given by the TWIN stellar evolution code, for the stars considered in this paper.

are then

\[ t_u = \left( \frac{R^3}{GM} \right)^{1/2} = 1594 \text{ s}, \]

\[ v_u = \left( \frac{GM}{R^2} \right)^{1/2} = 437 \text{ km s}^{-1}, \]

\[ E_u = \frac{GM^2}{R} = 3.79 \times 10^{48} \text{ erg}. \]

2.3 Relaxing a single star

Before initiating a triple star collision, we must first prepare an SPH model for each star in isolation. To compute the structure and composition profiles of our parent stars, we use the TWIN stellar evolution code \cite{Eggleton71,Glebbeek08,Pols08} from the MUSE software environment \cite{PortegiesZwart09}. We evolve main-sequence stars with initial helium abundance \( Y = 0.28 \) and metallicity \( Z = 0.02 \) for a time \( t = 2 \text{ Myr} \), a small enough age that even the most massive stars in a star cluster are still on the main sequence. The mass-radius relation which results from these calculations is shown in Figure 1.

Initially, we place the SPH particles on a hexagonal close packed lattice, with particles extending out to a distance only a few smoothing lengths less than the full stellar radius. After the initial particle parameters have been assigned according to the desired profiles from TWIN, we allow the SPH fluid to evolve into hydrostatic equilibrium. During this calculation, we include the artificial viscosity contribution to the SPH acceleration equation so that energy is conserved, and we do not include a drag force on the particles. For the relaxation calculations of massive stars, we do, however, implement a method to keep low mass particles from being pushed to large radii: namely, during the initial stages of the relaxation, we implement a variation on the XSPH method \cite{Monaghan92,Monaghan02}, in which the velocity used to update positions is the average of the actual particle velocity and the desired particle velocity (zero). All our relaxed models remain static and stable when left to dynamically evolve in isolation.

This approach allows us to model the desired profiles very accurately, and we present an example in Figure 2 where we plot desired profiles and SPH particle data for a 19.1\( M_\odot \) star. The structure and composition profiles of the SPH model closely follow those from TWIN profiles, and the model remains stable when evolved dynamically.

3 PREPARING A BINARY

In this section, we present our algorithm to model the close binary systems that are used in most of the triple star collisions (see §5). The first step in creating a binary is to relax each of the two stellar components in isolation, as described in the previous section. In the case of detached binaries, we place these relaxed stellar models along the \( z \) axis with their centres of mass separated by the desired separation \( r \). For contact binaries, however, we begin with the stars well sep-
arated and gradually decrease the semi-major axis until the desired separation is achieved, in order to minimise oscillations initiated by tidal forces. In all cases, the centre of mass of the system remains fixed in space, which we choose to be the origin.

During the binary relaxation process, the positions of the particles within each star are adjusted at each timestep by simple uniform translations along the binary axis, such that the separation between the centres of mass equals the desired separation $r$. Simultaneously, the angular velocity $\Omega_{\text{orb}}$ defining the co-rotating frame is continuously updated, such that the net centrifugal and gravitational accelerations of the two stars cancel exactly:

$$\Omega_{\text{orb}}^2 = -\frac{1}{2} \left( \sum_{i=1}^{N} m_i \dot{v}_{x,i} + \sum_{j=2}^{N} m_j \dot{v}_{x,j} \right),$$

where $\sum_{i}^{j}$ symbolizes a sum over all particles in star $j$. Here, the Cartesian coordinate $x$ is measured along to the binary semi-major axis; $\dot{v}_{x,i}$ is the acceleration of particle $i$ parallel to the axis of the binary in an inertial frame. A centrifugal acceleration is given to all particles such that the system approaches a steady state corresponding to a synchronised binary. As in the relaxation process of a single star, we also include the artificial viscosity contribution to the SPH acceleration equation.

This approach allows us to create close binaries that remain in dynamically stable orbits for many hundreds of orbits, if not indefinitely. An example is presented in Fig. 3 and Fig. 4. In Fig. 3 we plot column densities of a contact binary both before and after dynamical evolution through over 600 orbits. In Fig. 4 we show time evolution of various energies for the same binary. The epicyclic oscillations, with a period of 650 time units, are clearly visible. The fact that the epicyclic period is more than an order of magnitude larger than the orbital period of 35 time units underscores how close this binary is to the dynamical stability limit. As a binary approaches this limit, the epicyclic period would formally approach infinity (Rasio & Shapiro 1994). The innermost dynamically stable orbit then marks the transition when the squared frequency of the epicyclic oscillations passes from a positive to a negative value, so that the qualitative behaviour of perturbations changes from oscillatory to exponential. Throughout the calculation, the perturbations remain small and actually damp with time: the internal energy $U$ remains constant to within about 0.03%, the gravitational energy $W$ to within about 0.008%, and the kinetic energy to within about 0.2%. Meanwhile, the total energy is conserved within about 0.0004%.

In another example, we relaxed a contact binary with a 92.9M$_\odot$ and a 53.3M$_\odot$ star with semi-major axis of 43.8R$_\odot$. In Fig. 5 we show snapshots for every 50 time units (0.92 days) of the binary during and after relaxation process. We began the relaxation process of a binary with an initial semi-major axis of 55.8R$_\odot$, and we decrease it to 43.8R$_\odot$ in 500 time units (9.2 days). The top-most left panel shows a binary during relaxation at a time of 350 units (6.5 days), and the semi-major axis at this time is equal to 48.0R$_\odot$. It is possible to notice commencement of the mass transfer form the primary onto the secondary. At the time of 500 units (9.22 days), when the semi-major axis becomes 43.8R$_\odot$, we stop the relaxation and dynamically evolve the system in the inertial frame. At this time half of the secondary star is already submerged in the fluid of the primary star. By the time of 650 units (12 days), the secondary star is completely engulfed in the fluid of the primary star. The bottom-most right panel shows a binary at the time of 17300 units (319 days), and the semi-major axis maintains its value of 43.8R$_\odot$. In Fig. 6 we show energy and semi-major axis of the binary as a function of time.

Figure 3. A contact binary consisting of a 12.2M$_\odot$ primary and a 6.99M$_\odot$ secondary both at the end of the relaxation (upper frame) and after dynamical evolution through more than 600 orbits (lower frame). Colours represent column density, measured in g cm$^{-3}$ on a log scale, along lines of sight perpendicular to the orbital plane.
4 INITIAL CONDITIONS

The parameter space of three-body encounters is immense, leaving no hope to be completely covered with SPH simulations. The approach we take here is to study part of it by using the initial conditions obtained from direct N-body simulation. In particular, we take initial condition for three-body collisions from Gaburov et al. (2008) who carried out an extensive set of N-body simulations of young star clusters. In these simulations the stars were modelled as hard spheres with a given mass and corresponding radius. A collision occurs when two spheres experience physical contact, or in other words, when the separation between the centre of these spheres is equal to the sum of their radii. This treatment of collisions, known as the sticky sphere approximation, conserves total mass and momentum.

In this paper, however, we resolve the stellar structure and focus on isolated close three-body interactions. This can be justified since usually such interactions last less than a year, and therefore local conditions hardly change on such a short timescale. All three-body interactions we split in two groups: the interaction between a binary and a single star, and the interaction between three single stars which are in the middle of a resonant interaction. The latter case is straightforward to model, as we need to prepare only relaxed single star models, as described in §2, and then assign the appropriate initial positions and velocities to each of the stars. The actual dynamical interaction process is then modelled using the SPH code.

In the case of an interaction between a binary and a single star, we initially relax the binary as described in §3. The binary separation is taken from the N-body simulations. Because most of the binaries have separations of a few stellar radii, tidal circularisation plays an important role, and therefore eccentricity of these binaries is nearly equal to zero. In some of the cases, the synthetic stellar evolution part of N-body calculations predict a binary separation too small

Figure 5. The relaxation and dynamical evolution of a close binary between 92.9M⊙ and 53.3M⊙ with the semi-major axis equal to 43.8R⊙. The orbital period of the binary is 2.78 days (150.6 time units). The calculation switches from a corotating frame to an inertial frame at a time of 9.22 days.
Figure 4. Internal energy $U$, gravitational potential energy $W$, kinetic energy $T$ and total energy $E$ versus time $t$ for the dynamical evolution of the contact binary shown in Fig. 3. The orbital period is 35 time units, while the epicyclic period is 650 time units.

Figure 6. Internal energy $U$, gravitational potential energy $W$, kinetic energy $T$, total energy $E$ and semi-major axis $a$ versus time $t$ for the dynamical evolution of an isolated close binary consisting of a $92.9M_\odot$ primary and a $53.3M_\odot$ secondary star. All quantities remain within 0.2% of their initial value throughout the simulation of more than 100 orbits, highlighting the ability of our code to evolve stably even those binaries in deep contact. The small increase in the total energy occurs due to a few low mass particles that are escaping to infinity.

Figure 7. The orientation of a binary and an intruder star to be dynamically stable, and in such cases we relax an SPH model of the binary near the smallest possible semi-major axis such that the binary remains stable or quasi-stable, such that the merger time-scale is at least a few thousand time units.

Table 1 lists the initial positions and trajectories in a way that is meant to aid the mental visualisation of each case: for example, comparing the periastron separation $r_{p,b}$ to the binary semimajor axis $a_{12}$ provides an indication of where within the binary the intruder strikes. The ratio $E_{ib}/|E_{12}|$ gives a measure of how much energy is being brought to the system by the intruder, relative to the binding energy of the binary. A negative value of $E_{ib}/|E_{12}|$ implies that the intruder star is bound to the binary, otherwise it is initially unbound. We note, however, that the magnitude of this ratio is much less than unity, which corresponds to a nearly parabolic encounter between the intruder and the binary star. Indeed, in almost all of these cases the trajectory of the intruder about the binary is nearly parabolic ($0.9 < e < 1.1$). The last column indicates the angle of approach of the intruder toward the binary, with $0 \leq \theta \leq 180^\circ$. More precisely, the angle $\theta$ is the angle between the angular momentum vector $L_{12}$ of the binary and the angular momentum vector $(L_3)$ of the intruder calculated about the center of mass of the binary (Fig. 7). For example, $\theta = 0$ corresponds to coplanar trajectories with the intruder orbiting the binary in the same direction (clockwise or counterclockwise) as the binary is orbiting; $\theta = 90^\circ$ corresponds to the third star incident on the binary from a direction perpendicular to the orbital plane of the binary; and $\theta = 180^\circ$ again corresponds to coplanar trajectories, although now the intruder approaches with an angular momentum that is antiparallel to that of the binary. All of our initial binaries are on nearly circular orbits ($e_{12} < 0.02$), with the exception of case 249 ($e_{12} = 0.41$).

We initiate two types of hydrodynamic calculations. The first type, which comprises the majority of our calculations, consists of a co-rotating binary intruded upon by a third star. In these situations, a circular binary is relaxed as we described in §3. If it is a contact binary, then the circular orbit is maintained, with the orbital plane and phase being shifted to match those of the desired initial conditions. If the binary is detached, then the velocity of each star is adjusted to give not only the desired orbital orientation and phase, but the eccentricity as well. In this way, we account for tidal bulging in the binary components. The third star, relaxed as described in §3, is initially not rotating and sepa-
Table 1. In the first column, we present the case identification number. The second and third columns show the masses of the binary components, while the fourth column gives the mass of the intruder. The fifth column gives the semimajor axis $a_{12}$ of the binary. Columns 6 and 7 gives the periastron separation $r_{p,ib}$ and eccentricity $e_{ib}$, respectively, of the equivalent two-body Kepler orbit between the intruder and the center of mass of the binary. Column 8 gives the ratio of the energy $E_{ib}$ in this orbit of the intruder and binary to the binding energy $|E_{12}|$ of the binary itself. Column 9 gives the angle $\theta$, in degrees, between the angular momentum of the binary and the angular momentum of the intruder about the binary (see Fig. 7).

Table 2. The masses, in solar masses, of single stars which participate in the resonance interaction. The first column show the case number, while the following columns give the masses of participating stars.

5 RESULTS

In this section we report on the results of 40 simulations of different encounters between three stars. In terms of computational time, most of the runs are performed with $N \sim 10^4$ and lasted somewhere between one and two weeks on a modern PC equipped with an MD-GRAPE3 (Fukushige et al. 1996) card or an NVIDIA GPU (Hamada & Iitaka 2007; Portegies Zwart et al. 2007; Belleman et al. 2008; Gaburov et al. 2009) for both self-gravity calculation. Our higher resolution calculations ($N \sim 10^5$) typically require a few months to complete; the total number of integration steps are usually between $10^5$ and $10^6$. 

Hydrodynamics of three-body interactions
| id  | method | outcome                  | speed [km/s] | $f_L$ | $E_{ej}$ [10^{48} \text{erg}] |
|-----|--------|--------------------------|--------------|-------|-------------------------------|
| 201 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 0.8, 352     |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,3),2$ | 0            |       |                               |
|     |        | 14118 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 28296 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 113046 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
| 202 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 0.7, 520     |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,3),2$ | 0            |       |                               |
|     |        | 6138 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 11466 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 22380 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 91956 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
| 203 | pm     | $(1,2,3) \rightarrow (1,2,3) \rightarrow (1,2),3$ | 6.24, 480    |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,3),2$ | 0            |       |                               |
|     |        | 10398 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | 0.13                         |
| 204 | pm     | $(1,2,3) \rightarrow (1,3),2$ | 40.6, 143    |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,3),2$ | 0            |       |                               |
|     |        | 11946 $(1,2,3) \rightarrow (1,3),2$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 60024 $(1,2,3) \rightarrow (1,2,3)$ | $< 0.1$      | $< 0.001$ | 12.                          |
| 206 | pm     | $(1,2,3) \rightarrow (1,2,3) \rightarrow (1,2),3$ | 60.7, 136    |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2,3)$ | 0            |       |                               |
|     |        | 14475 $(1,2,3) \rightarrow (1,2,3) \rightarrow (1,2,3)$ | 1.2          | 0.048 | 13.                          |
| 207 | pm     | $(1,2,3) \rightarrow (1,2,3)$ | 3.31, 308    |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2,3)$ | 0            |       |                               |
|     |        | 9492 $(1,2,3)$ | 3.7, 345 | 0 | 0                           |
| 208 | pm     | $(1,3),2$ | 1.02, 555 |       |                               |
|     | ss     | $(1,3),2$ | 0            |       |                               |
|     |        | 15018 $(1,3),2$ | $< 0.1$      | $< 0.001$ | 0.22                         |
| 211 | pm     | $(1,3),2$ | 91.4, 824 |       |                               |
|     | ss     | $(1,3),2$ | 0            |       |                               |
|     |        | 11028 $(1,3),2$ | 53.1         | 203   |                               |
| 212 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 34.3, 139    |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2),3$ | 0            |       |                               |
|     |        | 22080 $(1,2,3) \rightarrow (1,2),3$ | 3.6          | 0.17  | 66.                          |
| 213 | pm     | $(1,2),3$ | 1.82, 724 |       |                               |
|     | ss     | $(1,2),3$ | 0            |       |                               |
|     |        | 13314 $(1,2),3$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 125130 $(1,2),3$ | $< 0.1$      | $< 0.001$ | 1.5                          |
| 214 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 95, 348      |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2),3$ | 0            |       |                               |
|     |        | 11016 $(1,2,3) \rightarrow (1,2),3$ | 1.4          | 0.14  | 3.9                          |
| 217 | pm     | $(1,2,3) \rightarrow (1,2,3) \rightarrow (1,2),3$ | 0.7, 713     |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2,3)$ | 0            |       |                               |
|     |        | 17442 $(1,2,3) \rightarrow (1,2,3)$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
|     |        | 139656 $(1,2,3) \rightarrow (1,2,3)$ | $< 0.1$      | $< 0.001$ | $< 0.1$                       |
| 219 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 49, 255      |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2),3$ | 0            |       |                               |
|     |        | 14160 $(1,2,3) \rightarrow (1,2),3$ | 8.1          | 0.038 | 33.                          |
| 220 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 166, 778     |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2),3$ | 0            |       |                               |
|     |        | 20178 $(1,2,3) \rightarrow (1,2),3$ | 14.          | 0.062 | 130                          |
|     |        | 46296 $(1,2,3) \rightarrow (1,2),3$ | 11.          | 0.062 | 130                          |
| 222 | pm     | $(1,2,3) \rightarrow (1,2),3$ | 38.3, 246    |       |                               |
|     | ss     | $(1,2,3) \rightarrow (1,2),3$ | 0            |       |                               |
|     |        | 17076 $(1,2,3)$ | 48, 310 | 0 | 0                           |
| 223 | pm     | $(1,3),2 \rightarrow (1,2,3) \rightarrow (1,2),3$ | 43, 454     |       |                               |
|     | ss     | $(1,3),2 \rightarrow (1,2,3) \rightarrow (1,2),3$ | 0            |       |                               |
|     |        | 12456 $(1,3),2 \rightarrow (1,2,3) \rightarrow (1,2),3$ | 12.          | 0.25  | 14.                          |
| 224 | pm     | $(1,2),3$ | 0.858, 323 |       |                               |
### Hydrodynamics of three-body interactions

| Source | Reaction | Time, # | Energy, eV | Width, keV |
|--------|----------|---------|------------|------------|
| ss (1,2,3) → (1,3,2) | 0.63, 20.8 | 0.096, 7.9 | 0.010, 3.7 |
| 14472 (1,2,3) → (1,2,3) → (1,3,2) → (1,3,2) | 0.94, 0.023 | 3.4 |
| 227 pm (1,3,2) → (1,3,2) | 19, 55 | |
| ss (1,3,2) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 1008 (1,3,2) → (1,2,3) → (1,2,3) | 3.6, 0.27 | 2.3 |
| 231 pm (2,3,1) → (1,2,3) → (1,2,3) | 71, 72 | |
| ss (2,3,1) → (1,2,3) | 1.3, 163 | |
| 13554 (2,3,1) → (1,2,3) → (1,2,3) → (1,2,3) | 0.25, 0.036 | 1.9 |
| 232 pm (2,3,1) → (1,2,3) → (1,2,3) | 106, 107 | |
| ss (2,3,1) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 13110 (2,3,1) → (1,2,3) → (1,2,3) | 4.0, 0.067 | 20.0 |
| 233 pm (2,3,1) → (1,3,2) | 18.7, 487 | |
| ss (2,3,1) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 13020 (2,3,1) → (1,2,3) → (1,2,3) → (1,2,3) | 3.7, 0.17 | 20 |
| 236 pm (1,2,3) → (1,2,3) → (2,3,1) | 39, 58 | |
| ss (1,2,3) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 12672 (1,2,3) → (1,2,3) → (1,2,3) → (1,2,3) | 5.8, 0.14 | 25.0 |
| 241 pm (2,3,1) → (1,2,3) → (1,2,3) | 68, 128 | |
| ss (2,3,1) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 19956 (2,3,1) → (1,2,3) → (1,2,3) | 8.0, 0.086 | 32.0 |
| 242 pm (1,2,3) | 6.5, 856 | |
| ss (1,2,3) → (1,3,2) → (1,3,2) | 0.067, 20.0 |
| 10224 (1,2,3) → (1,3,2) → (1,3,2) → (1,3,2) | 0.26, 0.016 | 2.0 |
| 245 pm (2,3,1) → (1,2,3) → (1,2,3) | 31.2, 23.9 | |
| ss (2,3,1) → (1,3,2) → (1,3,2) | 0.067, 20.0 |
| 16884 (2,3,1) → (1,2,3) → (1,2,3) → (1,2,3) | 5.3, 0.027 | 26.0 |
| 246 pm (1,2,3) | 3.1, 194 | |
| ss (1,2,3) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 5232 (1,2,3) → (1,2,3) → (1,2,3) | 0.017, 7.6 |
| 10554 (1,2,3) → (1,2,3) → (1,2,3) | 11.8, 691 | 3.7 |
| 21040 (1,2,3) → (1,2,3) → (1,2,3) | 1.1, 0.023 | 6.3 |
| 42294 (1,2,3) → (1,2,3) → (1,2,3) | 0.71, 0.020 | 5.3 |
| 84642 (1,2,3) → (1,2,3) → (1,2,3) → (1,2,3) | 3.4, 0.096 | 7.9 |
| 249 pm (1,3,2) → (1,3,2) → (1,3,2) | 0.06, 30 | |
| ss (1,3,2) → (1,2,3) | 0.06, 30 |
| 10374 (1,3,2) → (1,2,3) | < 0.1, < 0.001 | < 0.1 |
| 82812 (1,3,2) → (1,2,3) | < 0.1, < 0.001 | < 0.1 |
| 250 pm (1,2,3) → (1,2,3) → (1,2,3) | 2.3, 324 | |
| ss (1,2,3) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10254 (1,2,3) → (1,2,3) → (1,2,3) | 0.4, < 0.001 | 0.54 |
| 253 pm (1,2,3) | 2.0, 212 | |
| ss (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10374 (1,2,3) → (1,2,3) | 1.0, 0.032 | 5.3 |
| 256 pm (1,3,2) → (1,3,2) → (1,3,2) | 39, 234 | |
| ss (1,3,2) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10200 (1,3,2) → (1,2,3) → (1,2,3) → (1,2,3) | 7, 0.15 | 11.0 |
| 257 pm (1,2,3) → (1,2,3) | 11, 257 | |
| ss (1,2,3) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10296 (1,2,3) → (1,2,3) → (1,2,3) | 2.9, 0.087 | 36.0 |
| 258 pm (1,3,2) | 0.8, 79 | |
| ss (1,3,2) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10194 (1,3,2) → (1,2,3) → (1,2,3) → (1,2,3) | 0.6, < 0.001 | 2.0 |
| 259 pm (1,2,3) | 52, 353 | |
| ss (1,2,3) → (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10272 (1,2,3) → (1,2,3) → (1,2,3) → (1,2,3) | 5, 0.085 | 30.0 |
| 260 pm (1,2,3) → (1,2,3) → (1,2,3) | 31.0, 340 | |
| ss (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 22518 (1,2,3) → (1,2,3) | 44, 456 | 0.024 | 6.6 |
| 261 pm (1,2,3) → (1,2,3) | 39, 198 | |
| ss (1,2,3) → (1,2,3) | 0.067, 20.0 |
| 10092 (1,2,3) → (1,2,3) | 7.1, 0.020 | 22.0 |
| Time  | Treatment | Case | Initial State | Final State | N_{SPH} | $v_\infty$ | $\frac{M_{\text{loss}}}{M_0}$ (t=200) | Energy Ejected ($10^{48}$ erg) |
|-------|------------|------|----------------|-------------|----------|-----------|-------------------------------------|-------------------------------|
| 262   | pm         | (1,2,3) → (1,2),3 | (1,2,3) → (1,2),3 | 69, 226   |          |           | 0                                    |                               |
| 10314 | ss         | (1,2,3) → (1,2),3 | (1,2,3) → (1,2),3 | 68, 223   | 0.011    | 2.1       |                                     |                               |
| 267   | pm         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 403, 905  |          |           | 0                                    |                               |
| 14934 | ss         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 9.6       | 0.063    | 67.        |                                     |                               |
| 298   | pm         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 60, 290   |          |           | 0                                    |                               |
| 13818 | ss         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 7         | 0.21     | 52.        |                                     |                               |
| 299   | pm         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 128, 797  |          |           | 0                                    |                               |
| 10194 | ss         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 6.9       | 0.15     | 310        |                                     |                               |
| 102540| ss         | (1,3),2 → (1,2),3 | (1,3),2 → (1,2),3 | 2.7       | 0.16     | 330        |                                     |                               |

Table 3: Summary of the 40 simulations for three-star interactions. The first column gives the case identification number. The second column either gives the number $N$ of SPH particles used to simulate this case, or names the treatment as “pm” (point mass) or “ss” (sticky spheres). The third column summarizes the interaction that resulted by listing all changes in the state of the system. The fourth column lists the projected speed(s) at infinity of the resulting object(s), in units of km s$^{-1}$. The fifth column gives the fractional mass loss 200 time units after the final change of state, while the sixth column lists the total energy ejected, in units of $10^{48}$ erg, at that same time.
5.1 Selected cases

In Table 3 we summarise the outcomes of all collisions from Tables 1 and 2. Binaries are represented by (1, 2) while resonances are represented by (1, 2, 3), with the masses satisfying $M_1 > M_2 > M_3$. The merger product between stars 1 and 2, due either to a binary coalescence or a direct collision, is represented using braces, $\{1, 2\}$, where the more massive component at the time of the merger is listed first. In addition, the notation can be embedded. Consider, for example, case 242 with the following interaction sequence:

(1, 2), 3 → (1, 2, 3) → $\{1, 3\}, 2$ → $\{2, \{1, 3\}\}$. The initial state (1, 2), 3 represents a primary 1 and a secondary 2 in a binary being intruded upon by the least massive star 3. The (1, 2, 3) indicates that there is neither an immediate retreat of the intruder nor an immediate merger, but instead the three stars move in a resonance interaction. The state ($\{1, 3\}, 2$) means that the intruder has merged with the primary, leaving the merger product in a binary with the secondary star. Finally, $\{2, \{1, 3\}\}$ indicates that these two remaining objects coalesce. Note that in this final state, the secondary star indicated to the left of $\{1, 3\}$ within the outer braces, because the former was more massive at the time of the merger due to dynamical mass transfer during the final stages of binary inspiral.

Here, we present several cases in greater details. First, in Figures 8 and 9 we show trajectories and column density plots respectively from calculations of case 232, in which a $12.2 + 6.99 M_\odot$ binary collides with a $19.1 M_\odot$ intruder. The set up of the initial conditions for these three particular stars is described in §2 (Figs. 2, 3 and 4). Figure 9a shows the three bodies shortly after the start of the calculation. Figure 9b shows the three bodies just prior to the impact and merger of the intruder and the secondary from the binary. The first apocentre passage in the resulting binary star is shown in Figure 9c, while Figure 9d shows the binary in the process of merger. In Figure 9e we show the snapshot shortly after the fluid from the three stars has merged into a single object, and finally, Figure 9f shows a snapshot from near the end of our calculation: the merger product has drifted away from the origin due to asymmetric mass loss. In this case, the merger product has little angular momentum, and the calculated mass loss quickly asymptotes to a constant value of approximately 2.6 $M_\odot$ (see Fig. 10).

As another example, we consider case 260 in which a massive binary (92.9 and 53.3 $M_\odot$) is perturbed by a less massive intruder (13.3 $M_\odot$). Here, the intruder is a catalyst which triggers binary merger. In Figure 11 we show time evolution of energies (left panel) and global quantities (right panel), such as the masses of individual stars and ejected fluid. In Figure 12 we present time snapshots for this run. Figure 12a shows a snapshot at the beginning of the simulations, and Figure 12b at the moment of closest approach between the intruder star and the binary. The binary merger process is shown in Figures 12c, 12d and 12e. It can be seen that fluid is gradually lost from the L2 Lagrangian point. Finally, the merged binary is shown in Figure 12f. In contrast to case 232, binary orbital angular momentum is converted into spin of the product, explaining the elongated shape of the collisions product in Figure 12. One may also notice that the collision product is quickly drifting away from the centre with a velocity of 14 km/s. Most of this kick velocity...
Figure 9. Column density along lines of sight perpendicular to the $xy$ plane at various times for the same hydrodynamic calculation of case 232 presented in Figure 8.

comes from the escaping intruder star rather than from the asymmetric mass ejection.

The second episode of mass ejection, which occurs after the binary merger as can be seen in the right panel of Figure 11, is an artifact of the artificial viscosity used in SPH. Initially, the collision product is in the state of both differential rotation and hydrostatic equilibrium. Artificial viscosity tends to transfer angular momentum from the rapidly rotating shells to slower ones (Lombardi et al. 1999), and this forces the product to be a solid rotator. Since the inner regions of the collision product are spinning much faster than the outer ones, the angular momentum is transferred outwards. The net effect is that the inner regions of the collision product contract, because of loss of the rotational support, but the outer regions expand because of the continuously increasing supply of angular momentum, and this case can be seen in Fig. 13. Eventually, these outer regions become unbound and escape, and this results in the second episode of the mass loss. The mass loss we report in Table 1 is before this second episode but after the first, corresponding to the plateau $M_{\text{ejecta}} \approx 4M_\odot$ near $t = 7500$ in Figure 11.

From the data of Table 3 it is evident that when only two stars merge the mass loss remains below a few percent, and often considerably smaller. It is known that mass loss in a parabolic collision between two main-sequence stars is small (Freitag & Benz 2005; Gaburov et al. 2008). The mass loss percentage is typically larger in cases where all three stars ultimately merge, exceeding 10% in the hydrodynamic simulations of cases 212, 214, 223, 233, 236, 256, and 298. The hydrodynamic evolution in these more extreme cases is qualitatively similar: the first merger event is between the most massive star and one of the other two, and typically occurs after a short resonant interaction. The resulting merger product is enhanced in size by shock heating and rotation, leaving its outermost layers loosely bound. The third star, often after flung out to a large distance, can experience several periastron passages through the envelope of the first merger product before ultimately donating its fluid to the mix. In the process, substantial amounts of gas are ejected from the diffuse envelope at every periastron passage.

An example of this type of interaction is summarised in Figures 14 and 15 for case 298, which involves a 56.7$M_\odot$ + 25.3$M_\odot$ binary is intruded upon by a 28.1$M_\odot$ star. The features of these curves can be associated with events during the encounter. In this situation the intruding star initiates a short lived resonance that ends with the induced merger of the binary components near $t = 400$. As can be seen in the middle frame of Figure 15, approximately 2$M_\odot$ of fluid is
Hydrodynamics of three-body interactions

Figure 11. In the left panel, we show the time evolution of internal energy $U$, gravitational potential energy $W$, kinetic energy $T$ and total energy $E$ for case 260, while on the right panel we display the evolution of stellar masses and separations. The top frame shows the masses of components 1, 2, and 3, represented by blue, green, and red curves, respectively. The middle frame plots the amount of mass ejected (solid curve) and bound in a circumbinary envelope (dotted curve). The bottom curves show the separations between components 1 and 2 (green), between 1 and 3 (red), and between 2 and 3 (black).

Figure 14. Internal energy $U$, gravitational energy $W$, kinetic energy $T$, and total energy $E$ versus time for case 298. Peaks in $T$ and associated dips in $W$ correspond to close passes or mergers between the stars. Note that the total energy is conserved to about 0.2% over the interval shown.
units (50 hours) and a separation of $26R_\odot$. The calculation for this case lasted more than $3.3 \times 10^6$ iterations and covered a timespan of over 80000 time units (over 4 years simulation time). During this calculation, total energy and angular momentum were conserved to better than 0.1%.

5.2 The effect of numerical resolution

Because of the longevity of three-body interactions, most of our simulations are limited to $N \approx (1–2) \times 10^4$ particles. Even with this relatively low number of particles, a single simulation may take a few weeks to complete, as it typically needs to span at least several thousand time units. To test whether our results are affected by numerical artifacts, we recalculated a few of the simulations in high resolution. In most cases, the results are only weakly dependent on the resolution. In particular, a case of interest is case 204, which begins with three single stars in the middle of the resonance interaction. In Figure 13, we present the time evolution of the energy for two resolutions. One may see from the kinetic energy plot that the first close interaction occurs at $t \approx 75$. The further behaviour of the three stars bear characteristics of typical resonant interactions, with kinetic and gravitational potential energy exhibiting aperiodic oscillations of different magnitudes until $t \approx 200$. At this time two of the three stars merge (Table 3) and binary continues to decay.

In the high resolution case (right panel in Figure 13), the merger occurs somewhat earlier than in the low resolution case. Because this kind of interaction is chaotic, it is well known that the details at the level of trajectories are resolution sensitive (Davies et al. 1993; Freitag & Benz 2005). However, the final outcome is consistent between the two
resolutions: all three stars eventually merge. Moreover the mass and energy of the ejecta, as well as the kick velocity of the merger product, change by at most a factor of two. In Figure 19 we show the time evolution of masses of three stars, the mass of ejected fluid and the separation between stars.

Another interesting case is 299, in which a massive binary ($52.3 +16.9 \, M_\odot$) is intruded upon by a massive star ($52.3\, M_\odot$). In Figures 20 and 21 we show the time evolution of energies and global quantities, such as the masses of stars, the ejecta mass, and the stellar separations. Even though there are some differences, the general agreement between these two simulations is excellent. The merger between two of the three stars (the intruder and the primary of the binary) occurs at $t \approx 110$, and further binary decay lasts for more than 1200 units. Mass loss and energy of ejected fluid are consistent between these two runs of different resolution.

In Figure 22, we examine the effects of resolution for four separate simulations of case 202 with the number of particles varying by a factor of 15 from the lowest resolution treatment to the highest resolution. The agreement is excellent, with even the lowest resolution simulation capturing all important aspects of the orbital dynamics. The small bump in the kinetic energy $T$ shortly after the time $t = 100$ corresponds to the absorption of the $0.120M_\odot$ star into the $57.9M_\odot$ star, which excites oscillations in the merger product that are visible in the internal energy $U$ and gravitational potential energy $W$ curves. The merger product is left orbiting the $29.9M_\odot$ star in a stable binary with eccentricity $e = 0.583$ and semimajor axis $a = 127R_\odot$: the peaks in $T$
Figure 18. Internal energy $U$, gravitational potential energy $W$, kinetic energy $T$, and total energy $E$ versus time $t$ for two simulations of case 204 that differ in resolution: $N = 11946$ (left panel) and 60024 (right panel).

Figure 19. Masses and separations versus time for two simulations of case 204: $N = 11946$ (left panel) and 60024 (right panel). The top frame shows the masses of components 1, 2, and 3, represented by blue, green, and red curves, respectively. The middle frame plots the amount of mass ejected (solid curve) and bound in a circumbinary envelope (dotted curve). The bottom curves show the separations between components 1 and 2 (green), between 1 and 3 (red), and between 2 and 3 (black).

and simultaneous dips in $W$ correspond to the periastron passages.

In Figure 23 we show the projected trajectories of the three stars in case 246 of masses 42.2, 38.3 and 1.37$M_\odot$, as calculated with a point-mass integrator (top left frame), by using sticky spheres (top right frame) and with the hydrodynamics code (bottom four frames) with different resolution. In all cases, the 1.37$M_\odot$ intruder approaches the circular binary on a hyperbolic trajectory with eccentricity $e = 1.09$. In the point mass approximation, the intruder reaches a minimum separation of 4.90$R_\odot$ from the secondary and then slingshots back outward on a trajectory with eccentricity $e = 1.05$. The interaction increases the semimajor axis of the binary slightly to 28.5$R_\odot$, while also perturbing its eccentricity to $e = 0.0424$. In the sticky sphere approximation, a merger between the intruder and the secondary of the binary occurs during the initial pericenter passage,
followed shortly thereafter by a second merger with the primary.

The case plays out qualitatively differently when the hydrodynamics is followed. The intruder again passes to a minimum separation of about 5$R_\odot$ from the core of the secondary, well within its 11$R_\odot$ stellar radius, and then begins to retreat. The impact, however, transfers energy into oscillations of the secondary and the intruder is not moving fast enough to escape further than about 40$R_\odot$ from the secondary. The hydrodynamic calculations indicate that the intruder makes a second pericenter passage through the secondary, but these calculations deviate depending on the resolution: the resulting trajectories do not converge as the number of particles is increased up to $N = 84642$ due to the chaotic nature of the orbits.

In the case of our relatively low-resolution $N = 10554$ calculation of case 246, the intruder is shot out to a distance of over 100$R_\odot$. Finally, the intruder makes one final pass


Figure 17. Like Fig. 15 and Fig. 16, but for case 261, and with time plotted on a logarithmic scale so that the long term evolution and circularization of the resulting binary can be more easily observed.

Figure 22. Internal energy $U$, gravitational potential energy $W$ and kinetic energy $T$ versus time $t$ for four simulations of case 202 that differ in resolution: $N = 6138$ (bottom curve), 11466 (second from bottom), 22380 (third from bottom), 91956 (top). The energy scale on the left axis corresponds to the low resolution $N = 6138$ case; the other energy curves have been offset by 10, 20, and 30 energy units to facilitate the comparison.

Figure 24. Evolution versus time of, from the top of the figure to the bottom, the stellar masses, mass in common envelope (dotted curve) as well as in ejecta (solid curve), semimajor axis $a_{12}$ of the binary, and eccentricity $e_{12}$ of the binary ($t < 4500$) as well as eccentricity $e_i$ of the third star as it departs from the merger product ($t > 4500$) for the $N = 10554$ SPH calculation of case 246.

through the secondary, and is ejected out of the system on a trajectory with $e = 1.3$. The removal of orbital energy from the binary initiates a mass transfer instability. The primary cannibalizes the secondary and, as the binary merges, 0.06$M_\odot$ of material is ejected. At this time, the blue and green curves in Figure 23 merge into a single blue curve (see the lower right hand corner of the middle left frame). Masses and orbital parameters for this calculation are shown in Figure 24.

The $N = 21204$ and 42294 calculations of case 246 yield qualitatively similar results. After the third pericenter passage of the intruder through the secondary, the two stars merge. The resulting binary, surrounded by an envelope of gas removed from the secondary by the impacts, ultimately merges. In our highest resolution calculation of this case ($N = 84642$), the intruder does not immediately merge with either star in the binary, but rather the three stars move around one another in a long-lived resonance interaction before ultimately all three stars merge.

6 DISCUSSION AND CONCLUSIONS

We present a set of hydrodynamical simulations of 40 close encounters between three stars. The initial conditions are taken from the high-precision direct $N$-body simulations of Gaburov et al. (2008), who studied the onset of collision runaway in young star clusters. Most of the collisions (31) involve a massive binary star intruded upon by, generally, a lower mass star. The rest of the collisions (9) occur between three single stars which are in the middle of the resonant
Figure 23. Trajectories projected onto the xy plane for case 246 as calculated in a pure point mass approximation (top left), in a sticky sphere approximation (top right), by our hydrodynamics code with \( N = 10554 \) (middle left), with \( N = 21204 \) (middle right), with \( N = 42294 \) (bottom left), and with \( N = 84642 \) (bottom right). We adopt the convention that the trajectory of the most massive star is represented by the blue curve, the intermediate mass star by the green curve, and the lowest mass star by the red curve. The initial conditions are marked by squares, while the final position of an object before it merges with another one is marked by a 5-point asterisk.

interaction. All the simulations were carried out with both the SPH method and in the sticky sphere approximation.

If only initial and final states are of interest, the sticky sphere method provides the appropriate outcome of the encounter in about 3 out of every 4 cases. In the cases where sticky spheres result in a merger between three stars, our hydrodynamic simulations tend to give a similar result. However, if one is interested in mass loss, close inspection reveals that in a considerable amount of mass can be ejected in double mergers. In addition, the collision product acquires a kick velocity, which is usually a result of the asymmetric mass ejection. The kick velocity can be sufficiently high to eject the merger product to the cluster halo and even to escape. In cases where only two stars merge and the third escapes, the kick velocity is large enough that the collision product could be ejected out of the star cluster completely. Therefore, it is not completely unreasonable to expect collision products to be observed in the outer regions of young star clusters. The Pistol star in the Quintuplet cluster (Figer et al. 1998) may well be a merger product resulting from an encounter between a single and a binary star.

The sticky sphere approximation, however, fails in several cases. On occasion, this approximation predicts the formation of a binary with a merger product as one of the components (cases 214, 253, 260 and 262), an interesting outcome from either an observational or theoretical point of view. Detailed hydrodynamic modelling of the same cases, however, show that a complete merger is a more likely outcome, if the interaction is mild; otherwise, the outcome is two unbound stars. In another case, the sticky sphere method predicts either one (case 207) or two collisions (case 222) in a system, but the hydrodynamic simulations predict a fly-by. These are the cases for grazing encounters which result in the ejection of the intruder star. If the semi-major axis of the binary is sufficiently large, binaries tend to avoid mergers and become eccentric instead.

For those situations in which the sticky sphere algorithm predicts a single merger event, the result is incorrect in almost half of the situations. It is important to keep in mind that the condition for a merger in the sticky sphere approximation is energy independent, and therefore if two stars with large enough velocities have a grazing collision, this method will incorrectly predict a complete merger.

Thus in an environment with high velocity dispersion,
such as galactic nuclei in which the velocity dispersion is typically at least an order of magnitude larger than in the cores of young massive star clusters, the sticky sphere approximation may fail more often. In such environments, the merger cross-section is reduced, as grazing interactions between stars may not necessarily lead to mergers (Freitag & Benz 2005). While this could be improved by a more sophisticated effective radius of the merger product (we use simply $R_{\text{eff}}$) as some of our rotating collision products are gradually losing mass even at the termination of our hydrodynamical calculations. The reason for this mass loss is due to spurious transport of angular momentum outward caused by artificial viscosity (Lombardi et al. 1999), as described in §5. The precise timescale of this effect depends on numerical parameters and the treatment of artificial viscosity. For example, in case 220, the progression of the stellar collisions is essentially the same in the $N=20178$ and $N=46296$ calculations. In the higher resolution simulation, however, the angular momentum transport and resulting mass loss in the final collision product progresses more slowly. It is worth noting, however, that physical angular momentum transport mechanisms, such as stellar winds and magnetic braking, would have a similar qualitative effect but on a longer timescale (Sills et al. 2003).

All of our collision products possess some amount of angular momentum. In some cases, the angular momentum is large enough that the shape of the collision product substantially deviates from spherical symmetry. Evolving such an object is a challenge for stellar evolution codes, given that even the evolution of non-rotating massive collisions product is a formidable task (Glebbeek 2008). In addition, there still exist problems on even hydrodynamical grounds, as some of our rotating collision products are gradually losing mass even at the termination of our hydrodynamical calculations. The reason for this mass loss is due to spurious transport of angular momentum outward caused by artificial viscosity (Lombardi et al. 1999), as described in §5. The precise timescale of this effect depends on numerical parameters and the treatment of artificial viscosity. For example, in case 220, the progression of the stellar collisions is essentially the same in the $N=20178$ and $N=46296$ calculations. In the higher resolution simulation, however, the angular momentum transport and resulting mass loss in the final collision product progresses more slowly. It is worth noting, however, that physical angular momentum transport mechanisms, such as stellar winds and magnetic braking, would have a similar qualitative effect but on a longer timescale (Sills et al. 2003).

Stellar collisions in a young dense star cluster are expected to occur in the first few million years of the cluster lifetime (Portegies Zwart et al. 1999). At this age, the star cluster may still be embedded in a natal gas (Lada & Lada 2003), and therefore if the ejecta is energetic enough the state of the gas may be considerably disturbed, and such mechanism has recently been proposed within the context of globular clusters (Umbreit et al. 2008). In the case of young star clusters, our results suggest that ejecta emanating from stellar collisions is energetic enough to significantly disturb and even eject the remaining gas. Indeed, a young massive star cluster with a star formation efficiency of about 50% has about $10^{59} - 10^{60}$ ergs in binding energy of the remaining gas. Our results show that the energy of the ejected fluid in stellar collisions exceeds $10^{65}$ ergs, and in two cases (cases 220 and 299) even $10^{60}$ ergs. Since collisions are expected to occur in the core of a star cluster, it would be just a matter of a few collisions to significantly perturb or largely expel the natal gas from the central region. In the case of a runaway merger (Portegies Zwart et al. 2004; Freitag et al. 2006), we therefore expect that the gas will be expelled form the central regions before the end of runaway.

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APPENDIX A: DERIVATION OF SPH EQUATIONS OF MOTION

The use of non-equal mass particles in the simulations allows us to resolve both the core and the envelope of parent stars. However, during the merger process, particles of significantly different mass from two or more parent stars mix, and the standard constraint between density and the smoothing length, $h_i = f(\rho_i, C_i)$, becomes inappropriate. Such a constraint naturally involves a constant with dimensionality of mass, $C_i$. This constant is usually determined during the set up of the initial conditions and therefore reflects the initial mass resolution of particle $i$, that is the initial total mass of the neighbours of that particle. However, as the particle migrates from one region to another, the mass resolution of the particle should adapt to its new environment. If this does not happen, the particle may have too few or too many neighbours, depending on whether it migrates into a region with, respectively, an average particle mass significantly larger or smaller than its initial environment. To mend this, we present a new approach that keeps the number of neighbors roughly constant. Here, we can draw an analogy with finite-difference hydrodynamics, either on fixed or moving meshes: the number of neighbouring cells that a given grid cell interacts with is also roughly constant (exactly constant on a fixed mesh) and is, to some degree, independent of the local fluid conditions.

We propose a continuous constraint between an estimate of the number of neighbours and the smoothing length. Relaxing the condition that the neighbour number estimate be an integer, we weight each neighbour with a function that depends on its distance from the particle, $G(r_{ij}/h_i)$, where $r_{ij}$ is separation between the particle $i$ and the neighbour $j$. Using such a weight function, we estimate the number of neighbours of a given particle $i$ as

$$N_i = \sum_j G(|r_i - r_j|, h_i) \equiv \sum_j G_{ij}(h_i). \quad (A1)$$

We find empirically that the following function provides satisfactory results:

$$G(x, h) \equiv \frac{1}{(4h - 4|x - h|, h)}, \quad (A2)$$

where $0 \leq x < 2h$, otherwise it is equal to zero, and

$$V(x, h) \equiv 4\pi \int_0^x x^2 W(x, h) \, dx. \quad (A3)$$

Here, $W(x, h)$ is an SPH smoothing kernel with a compact support of $2h$. We use the kernel of Monaghan & Lattanzio.
Using the SPH definition of density,
\[ \rho_j = \sum_k m_k W(|r_j - r_k|, h_j) \equiv \sum_k m_k W_{jk}(h_j), \]  
(A7)
we derive its gradient
\[ \frac{d\rho_j}{dr_i} = \sum_k m_k \nabla_i W_{ik}(h_j) \delta_{ij} + m_i \nabla_i W_{ij}(h_j) \]
\[ + \sum_k m_k \frac{\partial W_{jk}(h_j)}{\partial h_j} \frac{dh_j}{dr_i}. \]  
(A8)
Differentiating Eq. (A1) with respect to \( r_i \) we find
\[ \chi_j \frac{dh_j}{dr_i} = - \sum_k \nabla_i G_{jk}(h_j) \delta_{ij} - \nabla_i G_{ij}(h_j), \]  
(A9)
where
\[ \chi_j \equiv \sum_k \frac{\partial G_{jk}(h_j)}{\partial h_j}. \]  
(A10)
With these equations, it is straightforward to derive accelerations due to pressure
\[ a_{h,i} = - \sum_j m_j \frac{\partial}{\partial h_i} \left[ \nabla_i W_{ij}(h_i) - \frac{\omega_i}{\chi_j m_j} \nabla_i G_{ij}(h_j) \right], \]  
(A11)
and due to gravity
\[ a_{g,i} = - \frac{1}{2} \sum_j m_j \left[ \nabla_i g_{ij}(h_i) + \nabla_i g_{ij}(h_j) \right] \]
\[ + \frac{1}{2} \sum_j m_j \frac{\Psi_i}{\chi_i m_i} \nabla_i G_{ij}(h_i), \]  
(A13)
Here, we define two more quantities:
\[ \omega_j \equiv \sum_k m_k \frac{\partial W_{jk}(h_j)}{\partial h_j}, \]  
(A16)
and
\[ \Psi_i \equiv \sum_k m_k \frac{\partial g_{ik}(h_i)}{\partial h_i}. \]  
(A17)
Following the approach of Monaghan (2002) (see their §2.3), we find the rate of change of the specific internal energy to be
\[ \frac{du_i}{dt} = \frac{P_i}{\rho_i^2} \sum_j m_j (v_i - v_j) \cdot \left[ \nabla_i W_{ij}(h_i) - \frac{\omega_i}{\chi_j m_j} \nabla_i G_{ij}(h_i) \right], \]  
(A18)
which guarantees conservation of total energy and entropy in the absence of shocks. In order to handle shock waves while maintaining energy conservation, we augment these equations with artificial viscosity (Monaghan 1997). For the calculations of this paper, we implement a variation on the artificial viscosity term proposed by Balsara (1995):
\[ \Pi_{ij} = \frac{\left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right)}{\alpha \mu_{ij} + \beta \mu_{ij}^2}, \]  
(A19)
with \( \alpha = 1 \) and \( \beta = 2 \). In our treatment,
\[ \mu_{ij} = \frac{(v_i - v_j) \cdot (r_i - r_j)}{|r_i - r_j|} \frac{f_i + f_j}{c_i + c_j}, \]  
(A20)
if \( (v_i - v_j) \cdot (r_i - r_j) < 0 \); otherwise \( \mu_{ij} = 0 \). Here \( c_i \) is the sound speed at particle \( i \). See Lombardi et al. (2006) for the definition of the form factor \( f_j \), and for additional details on how the artificial viscosity is incorporated.

The evolution equations are integrated using a symplectic integrator with shared symmetrised timesteps, as in Springel (2005). Our shared timestep is determined as

\[
\Delta t = \min_i \left[ \left( \Delta t_{i,1}^{-1} + \Delta t_{i,2}^{-1} \right)^{-1} \right], \tag{A21}
\]

where for each SPH particle \( i \), we use

\[
\Delta t_{i,1} = C_{N,1} \frac{h_i}{\max_j \left[ \kappa_{ij} \right]}, \tag{A22}
\]

with

\[
\kappa_{ij} \equiv \left[ \frac{p_i}{\rho_i^2} + \frac{1}{2} \Pi_{ij} \right]^{1/2}, \tag{A23}
\]

and

\[
\Delta t_{i,2} = C_{N,2} \frac{u_i}{|du_i/dt|}. \tag{A24}
\]

For the simulations presented in this paper, \( C_{N,1} = 0.2 \) to 0.3 and \( C_{N,2} = 0.05 \). The \( \max_j \) function in equation (A22) refers to the maximum of the value of its expression for all SPH particles \( j \) that are neighbors with \( i \). The denominator of equation (A22) is an approximate upper limit to the signal propagation speed near particle \( i \). The incorporation of \( \Delta t_2 \) enables us to treat shocks without drastically decreasing the timestep during intervals in which the flow is subsonic.

APPENDIX B: INITIAL CONDITIONS

In Table B we summarize the raw initial conditions of our calculations in order to facilitate comparisons with any future works.

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Table B1. The first column gives the case identification number. The second, third, and fourth columns give the masses $M_1$, $M_2$, and $M_3$ of the colliding stars. Columns 5 through 7 and columns 8 through 10 give the position and velocity, respectively, of star 1 in Cartesian coordinates. Likewise, Columns 11 through 13 and columns 14 through 16 give the position and velocity of star 2. The position and velocity of star 3 can be determined from the constraints that the center of mass be at the origin and that the net momentum is zero. All quantities are in solar units.

| id | $M_1$ | $M_2$ | $M_3$ | $x_1$ | $y_1$ | $z_1$ | $v_{x,1}$ | $v_{y,1}$ | $v_{z,1}$ | $x_2$ | $y_2$ | $z_2$ | $v_{x,2}$ | $v_{y,2}$ | $v_{z,2}$ |
|----|------|------|------|------|------|------|----------|----------|----------|------|------|------|----------|----------|----------|
| 201 | 84.1 | 27.1 | 0.250 | 81.6 | 33.9 | -7.56 | -0.00102 | 0.0690 | 0.0777 | -254. | -108. | 22.0 | 0.00207 | -0.206 | -0.237   |
| 202 | 57.9 | 29.9 | 0.120 | 52.5 | 13.8 | -37.1 | 0.0150 | 0.148 | -0.0508 | 102. | -26.4 | 71.9 | -0.0302 | 0.124 | 0.150   |
| 203 | 47.1 | 36.3 | 1.09  | -7.32 | 8.64 | -9.06 | 0.474 | -0.398E-01 | -0.469 | 10.5 | -10.5 | 12.3 | -0.0302 | 0.150 | 0.124   |
| 204 | 24.6 | 21.9 | 0.206 | 17.0 | 46.8 | 20.8 | -0.850 | 0.404E-03 | -0.386 | 23.4 | 52.8 | 11.0 | 0.791 | -0.631 | 0.180   |
| 205 | 42.2 | 18.2 | 0.651 | 9.29  | 1.86 | -1.35 | -0.434E-02 | -0.596E-02 | -0.412 | 19.6 | -8.11 | 0.774 | 0.293E-02 | 0.524 | 0.452   |
| 206 | 86.7 | 0.513 | 0.161 | -1.86 | 0.139 | 0.382E-02 | 0.489E-02 | 0.505E-02 | 298. | 220. | -21.2 | -0.524 | 0.452 | 0.355   |
| 207 | 61.7 | 18.4 | 8.89  | -28.1 | 1.04  | 13.5 | 0.384E-01 | -0.676E-01 | 0.288E-01 | 117. | -5.11 | 46.2 | -0.553 | 0.355 | 0.452   |
| 208 | 87.6 | 27.1 | 22.7  | 29.8 | 18.3 | -7.61 | 0.204 | 0.155E-01 | 0.212 | 15.4 | 6.21 | 71.9 | -0.0302 | 0.124 | 0.150   |
| 209 | 76.8 | 13.6 | 0.277 | -2.97 | -3.52 | -2.06 | 0.199 | -0.810E-01 | 17.9 | 16.8 | 11.8 | 0.769 | -0.131 | 0.046   |
| 210 | 17.8 | 16.8 | 9.49  | 2.63  | 13.0 | 8.76 | 0.322 | -0.150 | -6.44 | -3.78 | 2.14 | 0.264 | -0.002 | 0.011   |
| 211 | 22.6 | 28.9 | 0.110 | -11.9 | 4.92 | -0.182 | -0.0434 | -0.969E-01 | -0.412 | 117. | -5.11 | 46.2 | -0.553 | 0.355 | 0.452   |
| 212 | 36.6 | 22.0 | 0.200 | -5.22 | -4.58 | 2.42 | 0.328 | -0.136 | 0.427 | 55.2 | 16.7 | -66.3 | -0.208 | 0.097 | 0.504   |
| 213 | 48.1 | 28.9 | 0.110 | -11.9 | 4.92 | -0.182 | -0.0434 | -0.969E-01 | -0.412 | 117. | -5.11 | 46.2 | -0.553 | 0.355 | 0.452   |
| 214 | 22.6 | 28.9 | 0.110 | -11.9 | 4.92 | -0.182 | -0.0434 | -0.969E-01 | -0.412 | 117. | -5.11 | 46.2 | -0.553 | 0.355 | 0.452   |

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