Emission of gravitational waves from binary systems in the galactic center and diffraction by star clusters

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Binary systems of compact objects are strong emitters of gravitational waves whose amplitude depends on the binary orbital parameters as the component mass, the orbital semi-major axis and eccentricity. Here, in addition to the famous Hulse-Taylor binary system, we have studied the possibility to detect the gravitational wave signal emitted by binary systems at the center of our galaxy. In particular, recent infrared observation of the galactic center have revealed the existence of a cluster of stars each of which appears to orbit the central black hole in SgrA* for the stars labelled as S2 and S14, we have studied the emitted spectrum of gravitational wave and compare it with the sensitivity threshold of space-based interferometers like Lisa and Astrod. Furthermore, following recent observations, we have considered the possibility that SgrA* is actually a binary system of massive black holes and calculated the emission spectrum as a function of the system parameters. The diffraction pattern of gravitational waves emitted by a binary system by a cluster of stars has been also analyzed. We remark that this is only a preliminary-theoretical work than can acquire more interest in view of the next-coming gravitational wave astronomy era.

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I. INTRODUCTION

The nucleus of our Galaxy is only 8.5 kpc far from Earth. Consequently, it offers a unique possibility to study some physical processes with a level of details that will never be reached in external galaxies or active galactic nuclei. Thus, a consistent theoretical picture of the observed physical phenomena may allow improving not only our understanding of the galactic structure but also our general view of other galactic nuclei.

The nature of the dark object at the Galactic Center is still unclear although a super massive black hole (SMBH) of about a few million solar masses seems to be the most viable scenario. The corresponding Schwarzschild radius of such a black hole is $R_s = 2GM/c^2 \approx 10^{11}$ cm which, at the distance of $\sim 8.5$ kpc, corresponds to an angular size of a few $\mu$as. Since no present telescope has an angular resolution comparable with that required, only indirect proofs of the existence of a SMBH at the Galactic Center can be obtained.

A proof of the existence of a SMBH and its association with Sgr A* lies in assessment of the mass distribution in the central few parsec of the Galaxy $^{[10]}$. If the gravitational force is the dominant force acting in the vicinity of the SMBH, the velocities and the orbits of nearby stars strongly depend on the mass of central black hole. Hence, a number of efforts have been spent in order to map the galactic center region.

Recent ESO and Keck infrared observations have revealed the existence of a cluster of stars (see Figure 2) in the vicinity of the Galactic Center ($< 1''$) $^{[1, 7, 8, 9, 14]}$. In particular, Ghez et al. (2003) have reported on observations of a star orbiting close to the galactic center massive black hole. The star, which has been labelled as S2, with mass $M_{S2} \approx 15 M_\odot$, appears to be a main sequence star, orbiting the black hole in SgrA* with a Keplerian period of $\approx 15$ yrs. This has allowed Ghez et al. (2003) to estimate a mass of $M_{SgrA*} \approx 3.67 \times 10^6 M_\odot$ within $4.87 \times 10^{-3}$ pc. The orbital parameters for the binary system S2 - SgrA* are given in Table 11.

The relatively short periastron distance of both the S2 and S14 stars encourages the attempt of an observational campaign to look for genuine relativistic effects like the emission of gravitational waves (GWs). Starting from the Einstein equations and quadrupole radiation formula $^{[12]}$, it is possible to evaluate, in the weak field regime, the expected GW signal by the binary system SgrA*-S2 (and SgrA*-S14) and compare the emitted GW spectrum with
the threshold of next space-based interferometers like Lisa and Astrod. Moreover, according to recent observations, we have considered the possibility that SgrA* is actually a binary system of black holes orbiting the common center of mass.

In addition to the emission of gravitational waves from binary systems in the galactic center, we have also considered the effect of diffraction on a cluster of stars surrounding the central massive black hole. In fact, since in the linearized theory of General Relativity a gravitational wave acts as an ordinary electromagnetic wave, we expect that, under particular condition, it suffers diffractive effects while interacting with stars. Hence, by considering each star as a circular slit and applying the well known theory of wave diffraction, we can evaluate the expected diffraction patterns on the observer plane.

The paper is structured as follows: in Section II, we study the emission of gravitational waves from binary systems in the galactic center and compare the emitted spectrum with Lisa and Astrod sensitivity curves. Section II A, is devoted to the SgrA*-S2 (and SgrA*-S14) binary systems, while in Section II B we focus on the possibility that the galactic center hosts a massive black hole binary. In Section III A, we study the diffraction of gravitational waves by a stellar cluster possibly surrounding the SgrA* black hole. Here, by using Montecarlo techniques, we simulate the distribution of stars (each of which acts as a circular hole) at the galactic center and evaluate the diffraction patterns as observed from Earth.

II. EMISSION OF GWS BY BINARY SYSTEMS

The Theory of General Relativity predicts that a system of moving bodies emits gravitational waves. In particular, two stars on circular orbit release GWs characterized by a frequency \( \omega_n \) which is twice the orbital one (\( \omega_k \)). Stars on an elliptic orbit (with semi-major axis \( a \) and eccentricity \( e \)) emit GWs at frequencies \( \omega_n = n \omega_k \) (with \( n = 1, 2, 3... \)). In this case, the Fourier analysis gives the following expression for the released GW power in the \( n \)-th harmonic \[12\]

\[
\frac{dE}{dt}(n) = \frac{32}{5} \frac{G^4 m_1^2 m_2^2}{c^5} a^5 (m_1 + m_2) g(n, e)
\]

where

\[
g(n, e) = \frac{n^4}{32} \left\{ [J_{n-2}(ne) - 2eJ_{n-1}(ne)] + \frac{2}{n} J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right\} +
(1 - e^2) [J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)]^2 + \frac{4}{3n^2} J_n(ne)^2,
\]

where \( J_n \) (with \( n \) integer) is the usual Bessel function of \( n \) order.

Peters and Mathews \[12\], found that the orbit averaged power emitted in GW by a binary system with eccentricity \( e \) in the \( n \)-th harmonic of the orbital frequency is

\[
h(\omega_n) \simeq \frac{2}{\pi \omega_n D} \sqrt{\frac{G}{c^3}} \frac{dE(n,e)}{dt},
\]

where \( D \) is the distance of the gravitational wave source from Earth.

As an example, we may apply the previous relations in order to determine the expected gravitational wave signal form the well known Hulse-Taylor binary pulsar (PSR1913 + 16) \[10\]. In Table II we give the main orbital parameters of PSR1913 + 16 while the expected GW spectrum is shown in Figure 1 where it is compared with the Lisa and Astrod sensitivity curves (obtained with an integration time of 5 yrs). Inspection of that Figure shows that the gravitational wave signal from the binary pulsar PSR1913 + 16 is close to the threshold of Astrod but is rather far from the possibility of being observed by Lisa.

A. Gravitational waves from SgrA*-S2 (and SgrA*-S14) binary system

By using eqs. \[1\]-\[3\] it is straightforward to calculate the GW spectrum for binary systems hosted at the galactic center. In fact, as recent infrared observations have shown, a massive black hole (with mass \( M_{SgrA*} \sim 3.67 \times 10^6 M_\odot \)) is surrounded by a cluster of at least 40 stars closer than 1.2" (see Figure 2). In particular the star labelled as S2, whose orbital parameters are given in Table III is the most interesting. Since its orbit will be entirely observed within few years leading to test some relativistic effects, such as the star periastron precession, it will likely allow the acquisition of important information about the central black hole mass, its spin and the mass distribution around it.
TABLE I: Main orbital parameters and distance from Earth of the binary pulsar PSR 1913+16.

| Parameters                      | PSR 1913+16 |
|---------------------------------|-------------|
| Distance (Kpc)                  | 7.3         |
| Longest semiaxis $a$ (cm)       | $1.9 \times 10^{11}$ |
| Eccentricity $e$                | 0.617130    |
| Orbital period (hr)             | 7.7519      |
| $m_1 + m_2 (M_\odot)$          | 2.82837     |

FIG. 1: The gravitational wave spectrum expected from the Hulse-Taylor binary pulsar is shown and compared with Lisa and Astrod thresholds. The sensitivity curves have been obtained for an integration time of 5 yrs.

(see Bini et al. 2005, De Paolis et al. 2005 and references therein). In Figure 3 the GW spectrum of the S14 star (with $e = 0.97$ and $a = 15.1 \times 10^{-3}$ pc) is also given. As it is clear from the figure, none of these binary systems is a GW source detectable by the next generation of gravitational wave space-based interferometers.

B. Gravitational waves from a massive binary black hole in the galactic center

The nucleus of our Galaxy hosts a dark object whose nature is still unclear although a super massive black hole (SMBH) of $M_{BH} = 3.67 \times 10^6$ $M_\odot$ seems to be the most viable scenario. However, recent observations of the galactic center region in the radio band have pointed out a possible periodic flux variation with a period of about 106 days [11]. This periodicity may be the consequence of a Doppler shift modulation of the radio signal due to the orbital motion of a massive black hole binary system. For a system of this kind, eq. (1) becomes

$$ \frac{dE}{dt} (n) = \frac{32}{5} \frac{G^4}{c^5} \frac{M_{BH}^5}{a^5} \frac{q^2}{(1+q)^4} g(n, e) $$

TABLE II: Parameters of the binary system $S2 - SgrA^*$. 

| Parameters                      | $S2 - SgrA^*$ |
|---------------------------------|---------------|
| Distance (Kpc)                  | 8.5           |
| $M_{BH} (M_\odot)$              | $3.67 \times 10^6$ |
| $M_{S2} (M_\odot)$              | 15            |
| Longest semiaxis $a$ (pc)       | $4.87 \times 10^{-3}$ |
| Eccentricity $e$                | 0.87          |
| Orbital period (year)           | 15.78         |
FIG. 2: The orbits followed by few stars around SgrA* are shown. Data are taken from Ghez et al. 2003.

FIG. 3: Emission spectrum of gravitational waves from the binary systems S2 – SgrA* and S14 – SgrA* together with the threshold curves of LISA, ASTROD1 and ASTROD2 interferometers.

where $M_{BH} = 3.67 \times 10^6 \, M_\odot$ is the SgrA* total mass. The obtained GW spectrum is shown in Figure 4 and Figure 5 for different values of eccentricity and mass ratio $q = m_1/m_2$ (here $m_1$ and $m_2$ are the black hole masses). As one can see, the next generation of space-based interferometers should be able to detect GW from this kind of object at the galactic center and confirm, or not, the existence of a massive binary black hole.

III. DIFFRACTION OF GRAVITATIONAL WAVES BY A STELLAR CLUSTER

In 1999 Ruffa [13] has studied the diffraction of GWs by a massive black hole at the galactic center. In that work the black hole is described as a circular ring with thickness between $R_E$ and $R_E + dR_E$ ($R_E$ is the black hole Einstein radius). Ruffa showed that this phenomenon should amplify the GW amplitude by a factor of about 130. A more realistic calculation [3] has shown that the GW amplification factor is actually smaller than Ruffa’s one, due to the small chance of having source, lens (black hole) and observer (Earth) aligned.
FIG. 4: The gravitational wave spectrum from a binary system of two black holes with mass ratio $q = 0.01$ and eccentricity $e = 0.2$. The threshold curves of LISA, ASTROD1 and ASTROD2 interferometers are also shown (with integration time of 5 yrs).

FIG. 5: The gravitational wave spectrum from a binary system of two black holes with mass ratio $q = 0.9$ and eccentricity $e = 0.9$. The threshold curves of LISA, ASTROD1 and ASTROD2 interferometers are also shown (with integration time of 5 yrs).

Here, we consider the GW diffraction by a star cluster adopting the following simplifying assumptions:

1. the stars are “frozen” in the cluster, that is, their movement is neglected;
2. the cluster has a bi-dimensional distribution;
3. each star is assumed to behave as a circular hole (slit) with respect to GW diffraction (due to the Babinet principle).

The first hypothesis allows us to neglect the dynamical effects of the system on the diffraction pattern. The second one is necessary due to the long computational time needed for the simulations.

In the following two paragraph we introduce the main physical and geometrical cluster parameters. Hence, we derive a relation which makes some restrictions on these parameters in order to get GW diffraction.
A. Diffraction conditions on the cluster parameters

Starting from the hypotheses discussed above, we have studied the diffraction of GW emitted by a monochromatic GW source behind the cluster (in the linear approximation of the Einstein equations). First of all, we have assumed that the cluster mass density distribution follows a Plummer model (see Figure 6 for more details).

![Figure 6: The stellar distributions obtained for a number of stars of N = 10^2, 10^3 and 10^4, respectively. The x and y coordinates are measured in units of the cluster core radius \( r_c \). Each panel has been obtained by a Montecarlo simulation using two different seeds (one for the first row and another for the last row.)](image)

Then, if \( d_\ast \) is the average distance between two close stars and \( \lambda_{GW} \) is the GW wavelength, the Bragg formula gives the position of the first diffraction peak

\[
2d_\ast \sin \theta = \lambda_{GW}.
\] (5)

Assuming that the GW source is a binary system on circular orbit, we obtain the following condition which the cluster physical parameters should satisfy, in order to have substantial GW diffraction

\[
0 \leq 1 - \left( \frac{P_0}{P} \right)^3 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{\frac{5}{3}} \lesssim f_{BH} < 1
\] (6)

where, \( r \) is the distance from the cluster center, \( r_c \) is the cluster core radius, \( f_{BH} \) is the black hole mass fraction to the total cluster mass \( M \) and \( P_0 \) is a “critical” period defined by

\[
P_0 = \frac{4}{c} \left( \frac{4}{3} \pi \right)^{\frac{1}{3}} \left( \frac{M}{M_\odot} \right)^{-\frac{1}{3}} r_c.
\] (7)

The physical meaning of the parameter introduced above is the following: it simply gives the upper and lower limit for the period of the GW source, in order to get diffraction in the cluster central region (\( r \lesssim r_c \)). So, in terms of \( P \), the equation (7) becomes

\[
P_0 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{\frac{5}{3}} \leq P \lesssim P_0 (1 - f_{BH})^{-\frac{1}{3}} \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{\frac{5}{3}}.
\] (8)

For example, considering the cluster at the center of our Galaxy (for which we have \( M = 3.67 \cdot 10^6 \ M_\odot \), \( r_c = 5.8 \cdot 10^{-3} \ \text{pc}, \ R = 100 \ r_c \)), we finally get \( P_0 = 2.4 \cdot 10^4 \ \text{s} = 6.7 \ \text{h} \).
B. Diffraction patterns

Treating the problem in the Fraunhofer approximation, the intensity distribution on a plane screen is given by

\[ I(x, y) = I_0(x, y) \sum_{m=1}^{N} \sum_{n=1}^{N} e^{ik(\xi_m - \xi_n)x + (\eta_m - \eta_n)y} \]  

(9)

where \( I_0(x, y) \) is the diffraction pattern of a reference slit and the double sum takes into account the interference between each slit, \((\xi_m, \eta_m)\) are the coordinates of the \(m\)-th slit, \(N\) is the total number of slits, \(F\) is the distance of the cluster plane from the screen plane, and \(k\) is the GW wavenumber. For a randomly distributed slit sample, relation (9) becomes

\[ I(x, y) \simeq NI_0(x, y). \]  

(10)

Since we cannot consider a random distribution of stars, we must use the more general equation with a star distribution weighted by the Plummer function.

Now, by considering a star cluster at the galactic center with mass \(M = 3.67 \cdot 10^6 \, M_\odot\), \(N = 10, 100, 1000\) stars, \(\lambda_{GW} = 1.5 \cdot 10^{16} \, \text{cm}\), \(d_\star = 1.5 \cdot 10^{17} \, \text{cm}\), \(F = 8.5 \, \text{kpc}\) (Earth distance from the galactic center), we obtain the following diffraction patterns (see Figure 7).

FIG. 7: Each panel shows the expected diffraction pattern for \(N = 10, 100\) and 1000 stars, respectively. Here, the spatial resolution is \((1000 \, \text{pxl})^2 = (5.1 \, \text{kpc})^2\) for the two upper images, while it is \((100 \, \text{pxl})^2 = (5.1 \, \text{kpc})^2\) for the third image (on the bottom).
As evident from Figure 7, each pattern is very similar to that of a single circular slit of radius $r_c$. Hence, the Rayleigh criterion allows us to estimate the width of the first diffraction peak as

$$\delta \theta = 1.22 \frac{\lambda_G W}{r_c}. \quad (11)$$

Using the above values for the cluster parameters, we get $\delta \theta = 58$ degrees, that is $F \delta \theta = 8.7$ kpc. On the basis of this calculation, Earth will take about $42 \cdot 10^6$ years to cross the whole diffraction maximum, due to its movement around the galactic center. To obtain this result we have used an orbital velocity of about $2 \cdot 10^7$ cm s$^{-1}$ and assumed that the orbit is approximately straight in the area included in the angle $\delta \theta$. Because of the ellipticity of the orbit, it is clear that Earth’s crossing time must be slightly shorter than $42 \cdot 10^6$ years.