Low Cost Flexure Spring Testing

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Abstract. There are many references in the literature to fatigue testing of flexures for cryocoolers, but there is little information about the hardware required for this testing. This paper gives an overview of the requirements for flexure fatigue testing and discusses basic FE analysis of the flexures. Two different experimental methods are then described, one using commercial equipment, the other based on a simple custom-built linear oscillator. Both methods can be used for relatively high frequency testing which enables the 10 million cycles necessary to demonstrate the fatigue limit to be reached within a few days.

1. Introduction.
The “Oxford” cryocooler is defined by the use of clearance seals and flexure bearings (figure 1) to support the moving parts. The flexures have a high radial stiffness and low axial stiffness and they need to be designed for a very large number of cycles – a cryocooler operating at 50 Hz for 10 years will undergo $1.6 \times 10^{10}$ cycles.

![Figure 1. Typical Flexure Spring.](image1)

![Figure 2. Typical S-N Curve (Sandvik 7C27Mo2) [3].](image2)

There is considerable literature available on the design, material choice, analysis and fatigue life of these flexures [1][2], but very little is in the public domain with regard to the details of test methods. Testing is made easier by the fact that steels exhibit a fatigue limit whereby the stress/number of cycles (S-N) curve is horizontal from about $10^6$ cycles onwards (figure 2) [3]. Hence when fatigue...
testing flexures, if they survive $10^7$ or $10^8$ cycles then they are unlikely to fail in service. Further margin can be obtained by testing at a stroke 25% higher than the maximum operating stroke.

2. FEA and other analysis.
Historically, springs were designed empirically, with new springs being based on the successful performance of existing ones, with the geometry worked out on the basis of ratios of stroke, diameter thickness etc. Since those early days, Finite Element Analysis (FEA) has improved and become more accessible and reliable, so it is now used universally for spring design.

Care should be taken in the application of FEA analysis to flexures; the following points should be noted.

- The standard stress ‘output’ of FEA is the von Mises stress, which is strictly a criterion for yielding of material. It is based on a ‘sum of the squares’ formula, is always positive, so it does not give a realistic representation of how the stress varies throughout the stroke from a negative spring displacement to a positive displacement.
- The stress-displacement curve at some points on the spring will be non-linear, as the spring arms twist; the springs do not obey the “$\sin \theta \approx \theta$” rule typically used in structural analysis.
- At a single location, the direction of the principal stresses may vary as the spring is displaced through the full stroke due to the non-linearity of the spring.
- At some locations the stress will be fully reversing from $-S_{\text{max}}$ at one extreme to $+S_{\text{max}}$ at the other. In other locations the stress may not fully reverse. In this case the Goodman relation can be used:

$$
\sigma_e = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_{ts}}}
$$

Where $\sigma_e$ is the effective stress amplitude, $\sigma_a$ is the actual stress amplitude, $\sigma_m$ is the mean stress and $\sigma_{ts}$ is the material tensile stress.

- The failure criterion can be evaluated by using Miner’s law:

$$
\sum \frac{n_i}{N_i} = 1
$$

Where $n_i$ is the number of cycles undergone at a specific alternating stress, which would lead to failure at $N_i$ cycles.

3. Fatigue Test using vibration generator.
Testing of flexure springs requires a large displacement (5 to 20 mm stroke) with relative low forces (a few Newtons), whereas conventional fatigue testing is small displacement (< 1 mm) and higher forces (often a few kN), so typical commercial fatigue test equipment is not usually suitable. A way of using commercial equipment is to test the springs at resonance, and to use dynamic amplification such that a small stroke of the test equipment results in a high spring stroke.

Figure 3 shows a typical spring set up. Two springs are mounted a small distance apart on a cup fixed to the top of the armature of a vibration generator such as the LDS V201 or LDS V455. An AC amplifier driven by a sine wave oscillator powers the vibration generator.

The springs form a resonant system, whose resonance depends on the spring stiffness and the mass of the springs and the inner clamp rings and inner spacer. If the armature is driven at the resonant frequency of this system, then a small displacement of the armature will result in a large amplitude in the centre of the spring.
The displacements of the spring can be measured using Vernier callipers, laser displacement sensors or other means. It should be noted that most laser displacement sensors have a limited frequency range, and may give inaccurate readings at high frequency. Measurement of the amplitude of the spring is complicated by the fact that the cup is also moving, so an accurate measurement would require knowledge of the phase relationship between the displacements of the inner and outer parts of the spring. In practice, motion of the armature can be ignored, and the consequent small error in stroke measurement is accepted.

This method of testing has limitations at high frequency. Operating at high frequencies (> 100 Hz) can require high currents due to the characteristics of the vibration generator, and there is also a lot of acoustic noise generated. In one instance, even by adding mass to reduce the resonant frequency, it was not possible to achieve the desired stroke due to the amplifier current limit.

The test method is usually to set the spring running at the maximum design stroke (end stop stroke) until $10^7$ cycles has been reached. When testing at 50 Hz, this will take just over 2 days. If no failure occurs, the stroke can then be increased incrementally in steps of 5% (or more) and run for another $10^7$ cycles at each stroke. It should be noted that due to the non-linear nature of the flexure force-displacement characteristic, when the stroke is increased, the resonant frequency will change, so the drive frequency has to be adjusted. If the spring survives $10^7$ cycles at a stroke of 125% or higher, the spring can be considered to have passed the fatigue test. Sometimes it is useful to carry on beyond this point until the spring fails, which will then help to determine the safety margin of the design and to identify possible design weaknesses.

When a spring fails, the system is inherently safe, as the change in spring stiffness caused by the failure of one or more spring arms will change the resonant frequency of the system, so that it is now being driven off-resonance and the stroke of the spring will fall to a low value.

4. Fatigue testing using custom motor.

An alternative method of fatigue testing is to directly drive the spring using a simple moving magnet linear motor. The motor consists of an axially magnetised Neodymium-Boron-Iron magnet in the
centre of a pair of coils (figures 4a and 4b). Each coil has 600 turns, and the coils are connected so that the magnetic flux from each coil opposes the other. The magnet is bonded to a small cup, which is hung from a shaft attached to the centre of a pair of springs. There is no iron circuit, so the AC flux density produced by each coil is very low and the motor efficiency is also very low. However, this does not matter if the motor is driven at the resonant frequency of the mass/spring system, as the force required is very small. Typical input power to the motor is a few watts. The motor is powered by an AC amplifier, driven by an oscillator, and the displacement of the spring can be measured by either Vernier callipers or by a laser displacement sensor.

Again, failure of the spring arm leads to a change in resonant frequency, and the stroke drops. With this type of motor driven at a constant AC voltage, a failure also causes a significant change in the motor current, so simple data logging of the current will give an indication of the time of failure.

![Figure 4a](image1.png) **Figure 4a.** Sectional view through Fatigue Test Rig.  
![Figure 4b](image2.png) **Figure 4b.** Photo of Fatigue Test Rig. Vernier callipers used for stroke measurement.

The measured variables were time, frequency, voltage, current and amplitude. The latter was recorded with the use of a Vernier calliper, as shown in figure 4, the tip of which was lowered down to the spring until contact could be heard. The reading was compared with a static reading taken with the spring at rest.

5. **Test Results.**  
A typical set of test results taken from a displacer spring from ESA’s European Radioisotope Stirling Generator is shown in figure 5 below. The springs failed at an overstroke of about 40%.

When Spring Set A failed, a fracture was found in the thinnest section of one of the spring arms – this was the location in which FEA had determined the stress to be maximum. Spring Set B failed where the spring was clamped in the middle. This was not an area of high stress, and failure at this location was not expected. The cause of this failure could be a material defect, or it could be due to fretting at the clamp location. The resonant frequency for these springs was between 90 and 100 Hz,
and each test took about 4 weeks to complete, with the equipment running unattended 24 hours per day, 7 days per week. Testing to the minimum value of 125% of full stroke would take about 2 weeks.

![Fatigue Test Result Chart]

**Figure 5.** Fatigue Test Results, showing the maximum service amplitude [4].

This test method has been used with a variety of springs, and results are generally in good agreement with the results expected from FEA.

6. **Conclusion.**
Two methods of spring fatigue testing are described, one using commercial equipment and one using a custom linear motor. When testing at lower frequency either method can be used, but for higher frequency the direct drive system with a custom linear motor is recommended. When testing at high frequency, each test can be completed in about 2 weeks. When a failure occurs, the system ‘fails safe’ and with the direct drive system, the time to failure can be monitored by simple data logging of the motor drive current.

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**References**
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