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Scott Dietrich, William Mayer, Sean Byrnes, Sergey Vitkalov, A. Sergeev, Anthony T. Bollinger, and Ivan Božović
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Frequency dispersion of nonlinear response of thin superconducting films in 
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Scott Dietrich, William Mayer, Sean Byrnes, and Sergey Vitkalov
Physics Department, City College of the City University of New York, New York 10031, USA

A. Sergeev
SUNY Research Foundation, SUNY at Buffalo, Buffalo, NY14226, USA

Anthony T. Bollinger and Ivan Božović
Brookhaven National Laboratory, Upton NY 11973, USA

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The effects of microwave radiation on transport properties of atomically thin La$_{2-x}$Sr$_x$CuO$_4$ films were studied in the 0.1-13 GHz frequency range. Resistance changes induced by microwaves were investigated at different temperatures near the superconducting transition. The nonlinear response decreases by several orders of magnitude within a few GHz of a cutoff frequency $\nu_{cut} \approx 2$GHz. Numerical simulations that assume ac response to follow the dc V-I characteristics of the films reproduce well the low frequency behavior, but fail above $\nu_{cut}$. The results indicate that 2D superconductivity is resilient to high-frequency microwave radiation, because vortex-antivortex dissociation is dramatically suppressed in 2D superconducting condensate oscillating at high frequencies.

Transport properties of thin superconducting films have attracted much interest due to a fascinating physical phenomenon, the Berezinskii-Kosterlitz-Thouless (BKT) transition, predicted to occur at the critical temperature $T_c$ lower than the temperature of the superconducting transition in bulk samples [1–4]. The BKT phase transition originates from long-range (logarithmic) interactions between vortex excitations in a two-dimensional (2D) superconducting condensate. Below $T_c$, the dominant thermal excitations are vortex-antivortex (V-AV) pairs. A superconducting current does not move pairs, thus producing no energy dissipation. However, the current can break some of V-AV pairs, generate free vortices, set them in motion via the Lorentz force, and thus make the transport dissipative [5–7]. The induced dissociation of V-AV pairs depends on the current strength and thus results in an extraordinary violation of the Ohm’s law. With this motivation, the strong nonlinear response to the electric current in thin superconducting films was investigated extensively [8–11]. Despite significant progress, important many-body and edge effects in the V-AV pair dissociation are still under debate [12, 13].

Note that the majority of studies of the nonlinear transport properties of atomically thin La$_{2-x}$Sr$_x$CuO$_4$ films, over a broad frequency range from dc to 13 GHz. The experiments indicate a dramatic decrease of the nonlinear response at high drive frequencies, suggesting significant reduction of the V-AV pair dissociation in the oscillating superconducting condensate.

The experiments were performed on thin La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) films synthesized by Atomic-Layer-by-Layer Molecular Beam Epitaxy, providing precise atomically thin layers [16–18]. On the extreme level of control, delta-doping in a single CuO$_2$ layer has been demonstrated [17]. Recently a linear ac response of such films in the BKT state has been studied [19, 20]. Present samples have three distinct layers. The top and the bottom layers, each 5 unit cells (UC) thick, are made of strongly overdoped ($x = 0.41$) normal metals. The sample A, with 5 UC thick inner layer of La$_{1.72}$Sr$_{0.28}$CuO$_4$, shows the BKT transition at $T_c \approx 7$K [21]. The sample B with 1.5 UC thick inner layer of La$_{1.80}$Sr$_{0.20}$CuO$_4$ has $T_c \approx 5$K. Below we study nonlinear response of the BKT state at $T > T_c$.

The films were patterned into the shape of Hall-bar devices with the width $W = 200 \mu$m and the distance between the voltage contacts $L = 800 \mu$m. A direct current $I$ was applied through a pair of current contacts, and the longitudinal dc voltage $V$ was measured between the potential contacts. The sample and a calibrated thermometer were mounted on a cold copper finger in vacuum. The electromagnetic (EM) radiation was guided by a rigid coaxial line and applied to samples as shown in the upper insert in Fig. 1. A 50 $\Omega$ resistor terminates the end of the coax and provides the broaden bandwidth of the EM circuit. The EM power $P$ and amplitude of the microwave voltage $V_m$ at the end of the coax were measured in-situ using the non-selective bolometric response of the 50 $\Omega$ resistor [21]. In what follows, the
measured nonlinear response is normalized with respect to the calibrated EM power $P$, which takes into account all the effects of EM transmission (reflection) to (from) the sample stage.

Fig. 1 shows the dependence of the voltage $V$ on the current $I$ taken at different powers $P$ of a low frequency radiation. The thick solid line presents the $V-I$ dependence with no radiation applied. The EM radiation increases the resistance as shown by thin solid lines. To evaluate numerically the radiation effect the electrical connection (coupling) between the coax and the sample was approximated by a high-frequency contact resistance $R_c$, which determines the total voltage $V_S(t)$ and current $I_S(t)$ applied to the sample, assuming that the electromagnetic response follows the $dc$ nonlinear $V-I$ dependence [21]. The time averaged $< V_S(t) >$ and $< I_S(t) >$ are denoted by the open circles in Fig. 1. The resistance $R_c$ was used as the single fitting parameter for each computed curve, providing a good agreement between the low frequency experiments and the simulations.

At small currents, both the experiment and the simulations display a linear relation between the current $I$ and voltage $V$ - Ohm’s Law. Fig. 2b presents the dependence of the ohmic resistance on microwave (MW) power ($\nu = 1.5$ GHz) taken at different temperatures. The filled dots are the slopes of the $V-I$ dependencies at small currents. The power dependence varies considerably with the temperature. Close to the superconducting state the microwave-induced resistance variations are strong, whereas near the normal state the variations are weak. The shape of the dependence changes with the temperature. In particular, at $T = 6$ K the dependence looks more linear than at $T = 6.5$ K.

Significant changes of the power dependence are found in the response to microwaves with different frequencies. Fig. 2a shows the power dependencies of the resistance obtained at frequencies $\nu = 0.37$, 1.5, and 3 GHz. At low frequency $\nu = 0.37$ GHz, the dependence is in a good agreement with the numerical simulation. At high frequencies, the nonlinear response is much weaker and has a different functional form. To highlight the difference, the values of MW power at frequency 3 GHz were scaled down by a factor of 2. A comparison of power dependencies indicates that at a low power the nonlinear response is considerably weaker at a high frequency (3 GHz) than at a low frequency (0.37 GHz). At a high power the strength of the high frequency nonlinearity is restored.

Fig. 3 shows the dependence of radiation-induced variations of the $dc$ voltage $\Delta V$ on $dc$ bias $I$ taken at different frequencies and power levels, as labeled. At small currents, the response is linear, indicating no observable $ac$ rectification in the device. The left panel demonstrates the effect of the low-frequency radiation ($\nu = 0.575$ GHz) on the resistance. All three curves are in very good agreement with the ones obtained by numerical simulations for the same radiation powers [21]. The simulations replicate all the details of the experiments including the shift of the observed maximum with increased power using single fitting parameter $R_c = 63 \pm 2$ $\Omega$. The right panel shows the effect of the high frequency ($\nu = 3.4$ GHz) radiation. One can see that the high-frequency response does not
follow the dc V-I curve and cannot be explained by a reduction (re-scaling) of the MW current.

At small currents, the linear part of the voltage variations \( \Delta V(I) \sim I \) is related to the change of the ohmic resistance: \( \Delta R = R(P) - R(0) = \Delta V(I)/I \). Figure 4 displays the behavior of the resistance variation \( \Delta R \) with the power at different frequencies. At a small power, the induced resistance variations are proportional to the power. At higher powers, the dependence is weaker. At higher frequencies, the transition to weak power dependence occurs at a higher power. At the highest power levels all dependencies converge (see Fig. 2a).

An analysis indicates several distinct features of the dc nonlinear response presented in Fig. 1. At small currents the V - I dependence is well approximated by a combination of linear and cubic terms [15]. The dependence is presented below:

\[
V = R_0 \cdot I + \gamma I^3, \tag{1}
\]

where \( R_0 \) is the Ohmic resistance, the coefficient \( \gamma \) is a constant. The high-current behavior is in agreement with the one expected within the BKT scenario: \( V \sim I^\alpha \) [5, 9, 10]. The exponent \( \alpha(T) \) decreases from 8 to 1 as the temperature increases, indicating a BKT transition at \( \alpha(T_c) = 3 \) [21]. In accordance with Eq. (1) at small ac currents \( I_0(t) \), the voltage variations \( \Delta V(I) = -V(I + I_0(t)) - V(I) \approx 3 \cdot I \cdot < I_0(t)^2 > \) are proportional to the dc bias \( I \), which agrees with Fig. 3, and proportional to the RF power \( P \sim I^2 \), which agrees with Fig. 4. The decrease of the nonlinear response shown in Fig. 3 at high dc biases and the transition to a weaker power dependence presented in Fig. 4 are the results of the high current response \( V \sim I^\alpha \) at \( \alpha < 2 \).

Fig. 5 presents the frequency dependence of the nonlinear response, taken at different temperatures and small powers \( P \). At low temperatures, 9 K \( \leq T \leq 11 \) K, the response decreases by 3-4 orders in magnitude, as the frequency increases above few GHz. This is in a good agreement with the reduction of the dc nonlinearity by 3-4 order of magnitude observed between the regimes of V-AV depairing at 10 K and electron heating in the normal state (see Fig. 3b in [15]).

To be sure that the observed effect is not related to a strong decrease in the microwave coupling, we have evaluated the MW current through samples by investigating the MW reflection in the same setup [21]. These studies indicate some frequency dispersion in the MW current. However, the dispersion is nearly uniform and significantly smaller (a variation by a factor of 3-4) than the observed reduction of the nonlinear response by 3-4 orders of magnitude. Independent measurements of MW voltage, current, and the nonlinear response in several samples allow us to conclude that the observed reduction of the nonlinearity is of fundamental nature.

Since the reduction is strong, we associate it with the frequency suppression of the dominant mechanism in BKT regime - the current induced V-AV pair dissociation. If in the course of high-frequency oscillation the distance between the vortex and antivortex within a pair does not exceed a critical distance \( l_c \), then the pair can survive. At higher power, the amplitude of V-AV oscillations may exceed the critical distance \( l_c \), making the response similar to the one obtained at low frequency. It

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FIG. 3: (Color online). The change of voltage \( \Delta V = V(P) - V(0) \) due to applied EM radiation as a function of the dc bias \( I \) at different microwave powers as labeled. The solid lines are experimental data; the open circles present numerical simulations at \( R_c = 63 \pm 2 \Omega \). Comparison of the left and right panels shows the agreement between experiment and simulation at low frequencies, but serious disparity at high frequencies. Sample A.

FIG. 4: (Color online) The power dependence of radiation induced resistance variations \( \Delta R = R(P) - R(0) = \Delta V/I \) obtained at small dc biases \( \Delta V \sim I \) (see Fig. 3). The solid line is the dependence \( \Delta R \sim P \) expected for small power \( P \). Open circles are numerical simulations at \( R_c = 62 \Omega \). Sample A.
corresponds to Fig. 2a and Fig. 4.

An analysis of the oscillating vortex motion inside a V-AV pair in the presence of dc depairing current shows rich physics, analogous to the Kapitza pendulum with a vibrating pivot point [22]. Position $x_v$ of a vortex with the mass $M$ moving in a medium with the viscosity $\eta$ under effect of a time-dependent external potential $U(r, t)$ is given by

$$M \frac{d^2x_v}{dt^2} + \eta \frac{dx_v}{dt} = -\frac{\partial U(r, t)}{\partial r},$$

(2)

The potential $U$ is a sum of V-AV interaction and a potential of Lorentz force [7]: $U(r, t) = 2q_0^2 \ln(r/\xi) - q_0 J_s(t)r$, where $\xi$ is a size of the vortex core and $q_0^2 = \pi n_s^2 h^2 / 4m$, where $n_s$ and $2m$ are density and the mass of superconducting carriers. We express the supercurrent density $J_s = J_{dc} + J_\omega$ as a sum of the dc bias and current oscillations $J_\omega$ at angular frequency $\omega = 2\pi v$. An analysis of ac vortex motion for critical pairs near the saddle point of the potential ($\partial U/\partial r \approx 0$) shows a reduction of the vortex displacement at high frequencies: $x_v^{ac} \approx q_0 J_\omega / (-M \omega^2 + i\omega \eta)$. Moreover, at $\omega > \omega_c = \eta / M$, the ac depairing is strongly suppressed due to the phase shift between the vortex displacement and the barrier height. At the moments when the potential $U(t)$ reaches a minimum, the displacement is the smallest, thus preventing the V-AV dissociation. Finally at high frequencies a nonlinear analysis [22] of Eq.(2) indicates an effective attractive force inside the pair, which is proportional to the square of the vortex displacements: $\delta F_{eff} \sim \frac{\eta^2}{\omega^2} < (x_v^{ac}(t))^2 > \sim \frac{\omega^2}{\eta^2}$. These effects reduce the pair dissociation at high driving frequencies. Although the origin of the cutoff frequency $\nu_{cut}$ requires further research, we note that an evaluation of the frequency $\nu_c = \eta / (2\pi M)$ is in a good agreement with our data. Using the Stephen-Bardeen viscosity [23] and considering the vortex mass $M$ as a total mass of electrons in the core [24] we get the characteristic frequency of $\nu_c = 2$ GHz at the core radius of 8 nm, which is approximately two times bigger than the superconducting coherence length $\xi$. The above analysis points toward the crucial importance of the displacement-force phase relations in the nonlinear $ac$ response of the bulk BKT state supporting substantially the fundamental origin of the observed phenomenon.

In summary, strong nonlinear response to low frequency radiation is observed in atomically thin superconducting films, in BKT state. The response decreases by several decades within a few GHz above the cutoff frequency $\nu_{cut} \approx 2$ GHz. This indicates that 2D superconductivity is quite resilient to the high frequency radiation because of a strong reduction of the vortex-antivortex dissociation in oscillating 2D superconducting systems. This general conclusion is in agreement with the results of the linear response studies of the BKT state in thin disordered films of traditional superconductors. In particular, detailed investigations of the conductivity in the range 9-120 GHz show only frequency-dependent Drude absorption without any measurable dissipation due to vortex-antivortex dissociation [25].

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\* Corresponding author: vitkalov@sci.ccny.cuny.edu

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