The nuclear symmetry energy around or below saturation density has been extensively studied and roughly pinned down, while its behavior at suprasaturation densities is rather uncertain. Related experimental studies are being carried out or planned at facilities that offer radioactive beams worldwide. Especially, $\pi$ measurements in medium nuclei $^{132}\text{Sn}+^{124}\text{Sn}$ collisions at 300 or 200 MeV/nucleon incident beam energies are ongoing at Radioactive Isotope Beam Facility (RIBF) in RIKEN/Japan with the physical goal of probing the nuclear symmetry energy at high densities. However, our studies first show that the probe of $\pi^{-}/\pi^{+}$ ratio in $^{132}\text{Sn}+^{124}\text{Sn}$ reactions at 300 or 200 MeV/nucleon incident beam energies just probes the slope of the symmetry energy around or below saturation density. Only the $\pi^{-}/\pi^{+}$ ratio in heavy reaction system at relatively high incident beam energies, such as heavy nuclei $^{208}\text{Pb}+^{208}\text{Pb}$ collision at 600 MeV/nucleon, can substantially probe the symmetry energy at suprasaturation densities.

I. INTRODUCTION

The nuclear symmetry energy, i.e., the single nucleonic energy change as one replaces protons in nuclear matter with neutrons [1], has been extensively studied in nuclear physics community [2-4], simply because in a density range of 0.1 ~ 10 times nuclear saturation density, the symmetry energy determines the birth of neutron stars and supernova neutrinos [5], a range of neutron star properties such as cooling rates, the thickness of the crust, the mass-radius relationship, and the moment of inertia [6, 7]. The nuclear symmetry energy also plays crucial role in the evolution of core-collapse supernova [8] and astrophysical r-process nucleosynthesis [9].

To constrain the symmetry energy in broad density regions, besides the studies in astrophysics [12-14], many terrestrial experiments are being carried out or planned using a wide variety of advanced new facilities, such as the National Superconducting Cyclotron Laboratory (NSCL) and the Facility for Rare Isotope Beams (FRIB) in the US [15], the ASY-EOS experiment at GSI in Germany [16], the Radioactive Isotope Beam Facility (RIBF) at RIKEN in Japan [17, 18], the Cooling Storage Ring on the Heavy Ion Research Facility at IMP (HIRFL-CSR) in China [19], the Korean Rare Isotope Accelerator (KoRIA) in Korea [20].

With great efforts, the nuclear symmetry energy and its slope around saturation density of nuclear matter have been roughly pinned down [21, 22]. Owing to complexity of the nuclear force, the density-dependent symmetry energy by the power-law fit at lower density and higher density may require different exponents [23], the symmetry energy at high densities is thus still very controversial [24, 25]. Nowadays, many sensitive observables have been identified as promising probes of the symmetry energy, such as the $\pi^{-}/\pi^{+}$ ratio [26-33], energetic photon as well as $n[34, 35]$, the neutron to proton ratio $n/p[36, 37]$, $t/He$ [38, 39], the isospin fractionation $\eta[40, 41]$, the isodynamic energy changes as one replaces protons in nuclear matter [1], has been extensively studied in nuclear physics community [2-4], simply because in a density range of 0.1 ~ 10 times nuclear saturation density, the symmetry energy determines the birth of neutron stars and supernova neutrinos [5], a range of neutron star properties such as cooling rates, the thickness of the crust, the mass-radius relationship, and the moment of inertia [6, 7]. The nuclear symmetry energy also plays crucial role in the evolution of core-collapse supernova [8] and astrophysical r-process nucleosynthesis [9].

II. THE METHODOLOGY AND RESULTS

In order to know in which density region the $\pi^{-}/\pi^{+}$ ratio show maximum sensitivity to the symmetry energy, one way is similar to the studies in Refs. [54, 55], i.e., in the whole density region ($0 < \rho < \rho_{\text{max}}$) one chooses a specific form of the density-dependent symmetry energy as the standard calculation. To get the relative sensitivity of one observable in different density regions, one may change the density-dependent symmetry energy in different density regions, respectively. Figure IV(a) shows the relative sensitivity of symmetry energy sensitive observable $\pi^{-}/\pi^{+}$ ratio in different density regions. We use the symmetry energy parameter $\gamma = 2$ in the density-dependent symmetry energy form $E_{\text{sym}} = 32(\rho/\rho_0)^\gamma$ as
the standard calculation and change the parameter $\gamma$ from 2 to 0.5 in different density regions to obtain a series of $\pi^-/\pi^+$ ratios. The choices of the symmetry energy parameters $\gamma = 2$ and 0.5 are just for the convenience of research. It is seen that below 0.5$\rho_0$ or above 1.5$\rho_0$, effects of the symmetry energy on the $\pi^-/\pi^+$ ratio are in fact negligible. The effects of the symmetry energy in density regions of 0.5$\rho_0$ to $\rho_0$ and $\rho_0$ to 1.5$\rho_0$ are both obvious and roughly equal. And the corresponding values of the $\pi^-/\pi^+$ ratio are both higher than the standard calculation with $\gamma = 2$. This is understandable since the respective changes of $\gamma$ from 2 to 0.5 in the two density regions in fact soften the total density-dependent symmetry energy. The soften symmetry energy causes a high value of the $\pi^-/\pi^+$ ratio $^{20}$. The roughly equal effects of the symmetry energy in the two density regions of 0.5$\rho_0$ to $\rho_0$ and $\rho_0$ to 1.5$\rho_0$ reveal that, the symmetry energy sensitive observable $\pi^-/\pi^+$ ratio in $^{132}$Sn$^{+124}$Sn reaction at 300 MeV/nucleon in fact just probes the density-dependent symmetry energy around saturation density $\rho_0$.

To confirm the above results, the other method to demonstrate the same question is adding the symmetry energy from zero to maximum density step by step.

For each case, because the symmetry energy changes smoothly as the density increases, this method seems more acceptable. Figure 1(b) shows the relative sensitivity of the $\pi^-/\pi^+$ ratio in different density regions by adding the symmetry energy $E_{\text{sym}} = 32(\rho/\rho_0)^\gamma$ from zero to maximum density step by step. As expected, the value of the $\pi^-/\pi^+$ ratio almost does not change when adding the symmetry energy in density regions 0-0.5$\rho_0$ or 1.5$\rho_0$-2$\rho_0$. While in density regions of 0.5$\rho_0$ to $\rho_0$ and $\rho_0$-1.5$\rho_0$, the effects of the symmetry energy are both obvious. And the effects of the symmetry energy below saturation density seem to be somewhat larger than that above saturation density. The nuclear symmetry potential repels neutrons and attracts protons, causes neutron-deficient matter in heavy-ion collisions which corresponding a low value of the $\pi^-/\pi^+$ ratio. We thus see the value of the $\pi^-/\pi^+$ ratio generally decreases step by step as the symmetry energy added step by step.

For the $^{132}$Sn$^{+124}$Sn reaction, we in fact have changed incident beam energies from 200 to 600 MeV/nucleon and find that the $\pi^-/\pi^+$ ratio mainly probes the high-density symmetry energy only when the incident beam energies are larger than 400 MeV/nucleon, and the larger the better. But, conversely, the effects of the symmetry energy gradually decrease as the incident beam energies increase. Changing the impact parameter and kinematic cuts of pion emission in $^{132}$Sn$^{+124}$Sn reaction do not evidently affect the physical result of probed density region of the symmetry energy by the $\pi^-/\pi^+$ ratio.

While in fact the $\pi^-/\pi^+$ ratio in $^{132}$Sn$^{+124}$Sn reaction at incident beam energy 300 MeV/nucleon probes the slope of the symmetry energy around saturation density. This point can be seen from Figure 2.
try energy parameter \( \gamma = 0 \), which corresponding a zero slope of the symmetry energy, one can see that the symmetry energy does not affect the \( \pi^-/\pi^+ \) ratio, although the value of the symmetry energy is not zero in the density region \( 0-2\rho_0 \). Whereas with the symmetry energy parameter \( \gamma = 2 \), which corresponding a large slope of the symmetry energy, the effects of the symmetry energy on the \( \pi^-/\pi^+ \) ratio are clearly shown. In fact, it is the isospin-dependent gradient slope force caused by the symmetry potential in nuclear matter, which pushes neutrons and protons to move discriminatorily in matter. Therefore, it is not surprising to see the symmetry energy with \( \gamma = 0 \) has no effects on the \( \pi^-/\pi^+ \) ratio while the symmetry energy with \( \gamma = 2 \) greatly affects the value of the \( \pi^-/\pi^+ \) ratio, although around saturation density both cases have symmetry energy values of about 32 MeV.

To see why the \( \pi^-/\pi^+ \) ratio in \( {}^{132}\text{Sn}+{}^{124}\text{Sn} \) reaction at incident beam energy 300 MeV/nucleon does not probe the symmetry energy at high densities, we plot Figure 3 pion freeze-out local baryon density as a function of time in \( {}^{132}\text{Sn}+{}^{124}\text{Sn} \) reaction at 300 MeV/nucleon and \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at 600 MeV/nucleon.

To probe the high-density symmetry energy by the \( \pi^-/\pi^+ \) ratio, it seems that one has to use heavier reaction system and with relatively high incident beam energies. Figure 3(b) shows the pion freeze-out local baryon density as a function of time in \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at 600 MeV/nucleon. Compared with the light reaction system \( {}^{132}\text{Sn}+{}^{124}\text{Sn} \) at 300 MeV/nucleon, the maximum pion freeze-out local baryon density in \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at 600 MeV/nucleon is above \( 2\rho_0 \), which is much larger than that of the light system. The \( \pi^-/\pi^+ \) ratio in heavy reaction system at high beam energies is thus expected to probe the nuclear symmetry energy at high densities.

![Figure 3: Pion freeze-out local baryon density as well as central maximum baryon density as a function of time in \( {}^{132}\text{Sn}+{}^{124}\text{Sn} \) reaction at 300 MeV/nucleon and \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at 600 MeV/nucleon.](image)

**FIG. 3:** Pion freeze-out local baryon density as well as central maximum baryon density as a function of time in \( {}^{132}\text{Sn}+{}^{124}\text{Sn} \) reaction at 300 MeV/nucleon and \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at 600 MeV/nucleon.

To see why the \( \pi^-/\pi^+ \) ratio in \( {}^{132}\text{Sn}+{}^{124}\text{Sn} \) reaction at incident beam energy 300 MeV/nucleon does not probe the symmetry energy at high densities, we plot Figure 4 relative sensitivity of the symmetry energy sensitive observable \( \pi^-/\pi^+ \) ratio as a function of kinetic energy (in the frame of center mass) in \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at incident beam energy 600 MeV/nucleon.

**FIG. 4:** Relative sensitivity of the symmetry energy sensitive observable \( \pi^-/\pi^+ \) ratio as a function of kinetic energy (in the frame of center mass) in \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at incident beam energy 600 MeV/nucleon.

Figure 4 shows the relative sensitivity of \( \pi^-/\pi^+ \) ratio to the density-dependent symmetry energy in \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at incident beam energy 600 MeV/nucleon. It is seen that the symmetry energy in the density region \( 0-\rho_0 \) does not affect the value of \( \pi^-/\pi^+ \) ratio much compared with that in the density region \( \rho_0-3\rho_0 \). The effects of the symmetry energy at suprasaturation densities are much larger than that below saturation density. And above \( 1.5\rho_0 \), the symmetry energy still has clear effects on the \( \pi^-/\pi^+ \) ratio. We also simulated the \( {}^{208}\text{Pb}+{}^{208}\text{Pb} \) reaction at 300 MeV/nucleon, and find that the \( \pi^-/\pi^+ \) ratio mainly probes the symmetry energy around saturation density. Therefore, to probe the symmetry energy at suprasaturation densities, it seems that one should carry out heavy reaction system experiments and with relatively high incident beam energies.

It is instructive to show time evolution of the total \( (\pi^-/\pi^+)_{\text{like}} \) ratio in light and heavy reaction systems, respectively. Taking into account the dynamics of
resonance production and decays,

\[ \frac{\pi^-}{\pi^+}(t_{\text{late}}) = \frac{\pi^- + \Delta^- + \Delta^0}{\pi^+ + \Delta^+ + \Delta^0} \]

This ratio naturally becomes the final $\frac{\pi^-}{\pi^+}$ ratio after all resonances have decayed. From Figure 5(a), it is seen that, in $^{132}\text{Sn}+^{124}\text{Sn}$ reaction at 300 MeV/nucleon, the effects of the symmetry energy in the density region $0-0.5\rho_0$ are larger than that in the density region $1.5\rho_0-2\rho_0$. The effects of the symmetry energy below saturation density are also obviously smaller than that above saturation density. Figure 5 clearly shows that, to use the $\frac{\pi^-}{\pi^+}$ ratio as the probe of the high-density symmetry energy, using the heavy system and at relatively higher incident beam energies is a preferable way.

It is worth mentioning that, although the complicated transport model simulation integrally shifts the value of the $\frac{\pi^-}{\pi^+}$ ratio in heavy-ion collision [56], the density region that the symmetry energy sensitive observable $\frac{\pi^-}{\pi^+}$ ratio probes should be almost the same. This is because the maximum pion freeze-out local baryon density in $^{132}\text{Sn}+^{124}\text{Sn}$ reaction at 300 MeV/nucleon given by the complicated transport model [56] is about $1.05\rho_0$ (reaches its maximum at about 12.5 fm/c) and about $1.95\rho_0$ (reaches its maximum at about 9.5 fm/c) for the $^{208}\text{Pb}+^{208}\text{Pb}$ at 600 MeV/nucleon, which are both similar to that shown in Figure 5.

III. CONCLUSIONS

In summary, to probe the high-density behavior of the symmetry energy, frequently used observable $\frac{\pi^-}{\pi^+}$ ratio is not always suitable. Below 400 MeV/nucleon incident beam energies, for light or medium nucleus-nucleus reaction systems, the $\frac{\pi^-}{\pi^+}$ ratio mainly probes the symmetry energy around or below saturation density. For the $^{132}\text{Sn}+^{124}\text{Sn}$ reactions at 300 or 200 MeV/nucleon, which is being carried out at RIKEN/Japan, the produced $\frac{\pi^-}{\pi^+}$ ratio just probes the slope of the density-dependent symmetry energy around or below saturation density. To probe the high-density symmetry energy by the $\frac{\pi^-}{\pi^+}$ ratio, the heavy reaction system at relatively high beam energies such as $^{208}\text{Pb}+^{208}\text{Pb}$ at 600 MeV/nucleon is preferable.

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