On radiative muon capture in hydrogen

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Abstract

We analyze the radiative capture of the negative muon in hydrogen using amplitudes derived within the chiral Lagrangian approach. Besides the leading and next to leading order terms, given by the well-known Rood-Tolhoek Hamiltonian, we extract from these amplitudes the corrections of the next order in $1/M$ ($M$ is the nucleon mass). In addition, we estimate within the same formalism also the $\Delta(1232)$ isobar excitation effects and processes described by an anomalous Lagrangian. Using of the parameters of the model obtained from the analysis of the pion photoproduction, which restricts the arbitrariness of the $\pi N \Delta$ and $\gamma N \Delta$ vertices off-shell, allows us to explain two times more of the discrepancy between the value $g_P^{PCAC}$ of the induced pseudoscalar form factor $g_P$, predicted by partial conservation of the axial current, and of $g_P$ extracted from the recent TRIUMF experiment, than the standard approach. Varying these parameters independently, one can remove the discrepancy completely.

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I. INTRODUCTION

It is well known [1] that the charged weak interaction of the nucleon with a lepton is described by the weak hadron current

\[ J^a_{W,\mu}(q_1) = J^a_{V,\mu}(q_1) + J^a_{A,\mu}(q_1), \]

(1.1)

where the vector part is given by the matrix element of the isovector Lorentz 4–vector current operator between the nucleon states,

\[ \hat{J}^a_{V,\mu}(q_1) = i \left( g_V(q_1^2)\gamma_\mu - \frac{g_M(q_1^2)}{2M}\sigma_\mu\nu q_1^\nu \right) \frac{\tau^a}{2}, \]

(1.2)

and the axial–vector part is analogously,

\[ \hat{J}^a_{A,\mu}(q_1) = i \left( -g_A(q_1^2)\gamma_\mu\gamma_5 + i\frac{g_P(q_1^2)}{m_l}q_1^\mu\gamma_5 \right) \frac{\tau^a}{2}. \]

(1.3)

Here a is the isospin index, \( m_l \) is the lepton mass and the 4–momentum transfer is given by \( q_{1\mu} = p'_{\mu} - p_\mu \), where \( p'_{\mu} \) (\( p_\mu \)) is the 4–momentum of the final (initial) nucleon.

The least known of the four form factors entering the currents Eqs. (1.2) and (1.3) is the induced pseudoscalar form factor, \( g_P(q_1^2) \), in the axial–vector current \( \hat{J}^a_{A,\mu} \). Actually, its presence in the axial–vector current (1.3) tests our understanding of the basic strong and weak interaction processes, such as the strong \( \pi NN \) vertex and the weak pion decay. Elementary calculations lead to

\[ g_P(q_1^2) = -2g_{\pi NN}f_\pi m_l \Delta_\pi(q_1^2), \]

(1.4)

where \( \Delta_\pi(q_1^2) \) is the pion propagator, \( g_{\pi NN} = 13.05 \) is the pseudoscalar \( \pi NN \) coupling constant and \( f_\pi = 92.4 \text{ MeV} \) is the pion decay constant.

The matrix element of the axial current \( \hat{J}^a_{A,\mu} \) should satisfy partial conservation of the axial current (PCAC). It is easy to obtain that

\[ \bar{u}(p')q_{1\mu}\hat{J}^a_{A,\mu}u(p) = \bar{u}(p') \left[ 2Mg_AF_A(q_1^2) - \frac{g_P(q_1^2)}{m_l}q_1^2 \right] \gamma_5 \frac{\tau^a}{2} u(p). \]

(1.5)

It is seen from this equation, that if

\[ \tilde{g}_P(q_1^2) = -2g_{\pi NN}f_\pi m_l \frac{1}{q_1^2} \left[ 1 + \frac{Mg_A}{g_{\pi NN}f_\pi} F_A(q_1^2) \right], \]

(1.6)
is subtracted from $g_P(q_1^2)$, then indeed, PCAC is valid. Here we put

$$g_A(q_1^2) = g_A F_A(q_1^2),$$  
(1.7)

with $g_A \equiv g_A(0) = -1.267$. In the chiral model [2,3], the axial form factor is of the monopole form

$$F_A(q_1^2) = \frac{m_{a_1}^2}{m_{a_1}^2 + q_1^2},$$  
(1.8)

where $m_{a_1}$ is the mass of the axial vector meson $a_1(1260)$. Then for the $\tilde{g}_P(q_1^2)$, Eq. (1.7), we have

$$\tilde{g}_P(q_1^2) = -2g_{\pi NN}f_{\pi} m_t \left[ q_1^2 + \left( 1 + \frac{M g_A}{g_{\pi NN}f_{\pi}} \right) m_{a_1}^2 \right] \Delta_{F_a}^{a_1}(q_1^2).$$  
(1.9)

However, this equation for $\tilde{g}_P(q_1^2)$ cannot be used, because of the singularity for $q_1^2 = 0$, which shows its presence in the hadron radiative part of the radiative muon capture amplitude (RMC) for large photon momentums $k$ and it is close to the physical region for the ordinary muon capture (OMC), because of the large value of the axial meson mass. So our model cannot be used beyond the exact Goldberger–Treiman relation and we take

$$\tilde{g}_P(q_1^2) = -2g_{\pi NN}f_{\pi} m_t \Delta_{F_a}^{a_1}(q_1^2),$$  
(1.10)

which is in agreement with [4].

The best way to search for the effect of the form factor $g_P(q_1^2)$ is the muon capture. In the elementary process of OMC in the hydrogen,

$$\mu^- + p \rightarrow \nu_\mu + n,$$  
(1.11)

according to Eq. (1.4), the value of the induced pseudoscalar form factor $g_P$ is

$$g_P^{OMC}(p) \equiv g_P(q_1^2 = 0.877m_\mu^2) = -\frac{2g_{\pi NN}f_{\pi} m_\mu}{0.877m_\mu^2 + m_\pi^2} = 6.87 g_A = -8.71,$$  
(1.12)

and for $\tilde{g}_P(q_1^2)$, Eq. (1.10), we have

$$\tilde{g}_P^{OMC}(p) \equiv \tilde{g}_P(q_1^2 = 0.877m_\mu^2) = -\frac{2g_{\pi NN}f_{\pi}}{0.877m_\mu^2 + m_{a_1}^2} = 0.13 g_A = -0.16,$$  
(1.13)

which is a correction of $\approx 2\%$ to the $g_P^{OMC}(p)$, Eq. (1.12). The resulting value is
The axial form factor of the nucleon has recently been measured by the \( p(e, e'\pi^+)n \) reaction in Ref. [5]. The dipole form of the form factor was used and the extracted axial mass \( m_A = 1.077 \pm 0.039 \) GeV. For this form factor,

\[
\tilde{g}_P(q_1^2) = -2g_{\pi NN}f_\pi m_l \frac{2m_A^2 + q_1^2}{(m_A^2 + q_1^2)^2},
\]

and analogously with Eqs. (1.13) and (1.14) we have

\[
\tilde{g}_P^{OMC}(p) = 0.34 g_A = -0.43,
\]

and

\[
g_P^{PCAC}(p) = -8.28,
\]

respectively. Both values of \( g_P^{PCAC}(p) \) are in reasonable agreement with the calculations of \( g_P^{PCAC}(p) \) within the framework of the heavy baryon chiral perturbation theory (HBChPT), [6–8].

Very flat dependence of the capture rate on \( g_P \) in the OMC by the proton, Eq. (1.11), provides its world average value [9] with an error of \( \approx 20 \% \) and particular experiments have an error larger by a factor of \( \approx 2 \).

Recently, a very precise experimental study of the muon capture by \(^3\)He [10,11],

\[
\mu^- + ^3He \rightarrow \nu_\mu + ^3H,
\]

yields the transition rate

\[
\Gamma_{\text{exp}} = 1494 \pm 4 \text{ s}^{-1},
\]

which allowed [12] an extraction of the value of \( g_P \) with an accuracy of \( \approx 20 \% \) from this experiment alone,

\[
\frac{g_P}{g_P^{OMC}(^3He)} = 1.05 \pm 0.19,
\]

where for the reaction Eq. (1.18)

\[
g_P^{OMC}(^3He) \equiv g_P^{OMC}(q_l^2 = 0.954 m_\mu^2) = 6.68 g_A.
\]
Analogously with Eq. (1.14), this value of $g_P$ differs slightly from the one demanded by PCAC by about the same amount as for the reaction Eq. (1.11). A contribution of 20% due to the meson exchange current effect turned out to be essential to get the calculated transition rate

$$\Gamma_{th} = 1502 \pm 32 \text{s}^{-1}, \quad (1.22)$$

into agreement with the data Eq. (1.19). Let us note that a further improvement of the extracted value of $g_P$ is hindered by an uncertainty of $\approx 2\%$ in calculations [12] which will be difficult to improve. The main uncertainty arises from the less known parameters of the $\Delta$ excitation processes.

Another attractive tool to extract the value of $g_P^{PCAC}$ is the RMC by the proton,

$$\mu^- + p \longrightarrow \nu_\mu + \gamma + n. \quad (1.23)$$

As is well known [13], the RMC amplitude contains the pseudoscalar form factor $g_P$ in the form

$$g_P^{L(N)} = -x \frac{2g_{\pi NN}f_\pi m_\mu}{(q^{L(N)})^2 + m_\pi^2} \rightarrow x g_P^{OMC}(p) \frac{0.877m_\mu^2 + m_\pi^2}{(q^{L(N)})^2 + m_\pi^2} \left[1 - \frac{(q^{L(N)})^2 + m_\pi^2}{(q^{L(N)})^2 + m_\pi^2} F \left(\frac{q^{L(N)}}{m_\mu}\right)\right], \quad (1.24)$$

where we implemented the correction of Eq. (1.6) and

$$F \left(\frac{q^{L(N)}}{m_\mu}\right) = 1/ \left(\frac{(q^{L(N)})^2 + m_{\Delta^0}^2}{(q^{L(N)})^2 + m_{\Delta^+}^2}\right), \quad (1.25)$$

for the chiral model [2,3] and

$$F \left(\frac{q^{L(N)}}{m_\mu}\right) = \frac{(q^{L(N)})^2 + 2m_A^2}{(q^{L(N)})^2 + m_A^2}, \quad (1.26)$$

for the dipole form of the form factor. As it is seen from Eq. (1.24), the form factor $g_P$ depends either on the square of the four-momentum transfer $q^L = p - p' = \nu + k - \mu = -q_1$ (characterizing the muon radiation process) or on the $q^N = \nu - \mu = q^L - k$ (for the hadron radiation). For large photon momentums $k$, $(q^L)^2 \approx +m_\mu^2$, whereas $(q^N)^2 \approx -m_\mu^2$, which enhances the hadron radiation amplitude by a factor of $\approx 3$. This enhancement makes the reaction (1.23) particularly interesting. On the other hand, the dependence on $g_P$ of the effective form factors $g_i$, entering the effective RMC Hamiltonian, appears only from the order $O(1/M)$, which makes the isolation of the dependence of the photon spectrum on $g_P$ difficult\footnote{In Ref. [14], it is proposed to isolate the effect due to the hadron radiative amplitude in a very difficult polarization experiment.}. The factor $x$ in Eq. (1.24) is used to study the change of the photon spectrum and capture rates by scaling $g_P$. 
The theory of the RMC was elaborated by many authors during the decades (see Refs. [1], [13,15–26] and references therein).

The nuclear Hamiltonian, suitable for use in nuclear physics calculations of the RMC processes, was provided first by Rood and Tolhoek [19]. It contains the leading and the next to leading order terms in $1/M$ derived from the conserved RMC amplitude given by a set of the Feynman diagrams. Christillin and Servadio [21] rederived in an elegant way the RMC amplitude obtained earlier by Adler and Dothan [4] using the low energy theorems. This amplitude is written in terms of elastic weak form factors and pion photoproduction amplitude, up to terms linear in $k$ and $q$. It was also found [21] that higher order terms cannot be obtained using this method. Recently, this amplitude was produced [3] from a chiral Lagrangian of the $N\pi\rho\omega\rho_1$ system. It satisfies the corresponding continuity equations and the consistency condition exactly. Higher order terms follow without any restriction. It was shown that the leading order terms coincide with those given by the low energy theorems. However, higher order terms differ, which is given by a different prescription to pass towards higher energies.

The above mentioned set of the relativistic Feynman diagrams was used by Fearing [22] to calculate the photon energy spectrum for the reaction Eq. (1.23). This work was later extended by Beder and Fearing [26] by considering also the contribution from the $\Delta$ excitation processes. A recent comparison of the TRIUMF experiment [27,28] with the Beder–Fearing calculations provided a value of $g^{OMC}_P(p)$ which is enhanced by $\approx 50\%$ in comparison with the value of Eq. (1.12), which corresponds to using $g_P$ from Eq. (1.24) with $x = 1.5$. This is the so–called ‘$g_P$ puzzle’.

In connection with the presence of the factor $x$ in Eq. (1.24), it should be noted that

(i) Referring only to the change of $g^{OMC}_P(p)$ is confusing. As it is seen from Eq. (1.24), the whole form factor $g_P$ is scaled.

(ii) In the experiment [27,28], the high energy part of the photon spectrum is measured. Then increasing $x$ simulates processes enhancing this part of the spectrum.

In our opinion, the variation of $x$ can be considered as a tool to study an uncertainty in our knowledge of $g_P$ due to a restricted experimental accuracy. Any real difference from $x = 1$ would mean violation of PCAC.

Searching for the processes enhancing the high energy part of the photon spectrum has recently been performed within the concept of HBChPT by several authors [29–33]. Ando and Min [31] considered one–loop order correlations to the tree approximation and confirmed the existing discrepancy. Bernard, Hemmert and Meißner (BHM) calculated [29] both ordinary

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2 See also the discussion in Ref. [29].
and radiative muon capture on the proton in an effective field theory of pions, nucleons and Δ isobars by using the small scale expansion [34]. According to [29], the most probable explanation of the problem is a combination of many small effects. Besides the photon spectra, BHM present the numerical results also for the singlet (Λ_s) and triplet (Λ_t) capture rates. This will enable us to compare our calculations with those by BHM in a more detail. Here we only note the difference of ≈ 10% in Λ_t. As we shall see later, about half of this difference arises from the use of an approximate equation for the neutrino energy in Ref. [29].

In Ref. [32], a possible explanation of the discrepancy was suggested that a fraction of spin 3/2 orthomolecular pµp state in liquid hydrogen can exist. The analysis of the experimental photon spectrum [27,28] yielded 10 to 20% of this state. However, this is in sharp contrast with the existing calculations [35,36], which give zero fraction of this state. As noted very recently in Ref. [33], a new analysis restricts the fraction of spin 3/2 orthomolecular pµp state to at most 5%.

Finally, let us comment on Ref. [37], where Cheon and Cheoun reported on the derivation of an additional term from a chiral model, which does not appear in the standard approach to the RMC on the proton and which generates a large contribution to the photon spectrum. As it was shown in Ref. [38], the report [37] suffers from two flaws. First, the derivation of this terms contains an algebraic error due to an incorrect application of the covariant derivative in Eq. (14). After removing it, the effect is reduced by a factor of ~ 5. The second flaw in Ref. [37] is related to the introduction of the pseudovector πNN coupling by the vertex L_1 of Eq. (18), which yields the desired term. However, the equivalent passage from one type of the πNN coupling to another one is guaranteed only by the Foldy–Dyson unitary transformation. As shown in Ref. [38], when this transformation is applied to a chiral model with the pseudoscalar πNN coupling, the pseudovector πNN coupling appears in the resulting Lagrangian, which does not contain the incriminating term, however. Besides, the presence of this term in the RMC amplitude violates the Ward–Takahashi identity derived in Ref. [3]. Later attempt to improve the situation [39] suffers from the same shortcomings. The report [37] was also criticised in Ref. [40].

This situation makes the expectation of the result from the next TRIUMF experiment in helium,

\[ \mu^- + ^3He \rightarrow \nu_\mu + \gamma + ^3H, \]  \hspace{1cm} (1.27)

with a particular tension. However, one should keep in mind also complications analogous to those in reaction Eq. (1.18) and non-negligible meson exchange current effects are to be expected, which makes the analysis much more difficult. It is clear that the Beder–Fearing relativistic formalism is not applicable in calculations with the realistic 3N wave

\[^3\text{See also a discussion in Ref. [14].}\]
functions and a consistent non–relativistic approach should be developed. Here we make an independent step in this direction by making the non–relativistic reduction of the amplitudes derived in Ref. [3] from a chiral Lagrangian of the $N\pi\rho\omega a_1$ system. As a result, we get an effective Hamiltonian which is close to the one obtained by Rood and Tolhoek [19] but not identical with it. We also apply the constructed effective Hamiltonian to compute both the capture rates and the photon energy spectra for the reaction Eq. (1.23) and for various spin states. Our reduction provides more terms of the order $(1/M)$ and $(1/M^2)$ than Rood and Tolhoek present. Added to the leading order terms, they should reproduce with a good accuracy results given in Ref. [22]. Another set of terms of the order $O(1/m_\rho^2) \approx O(1/M^2)$ is produced by reduction of additional relativistic amplitudes following from our chiral Lagrangian. We shall call it hard pion (hp) correction. Numerically, it enhances the photon spectra by 2–4%.

Next we include the $\Delta$ isobar using again the formalism of chiral Lagrangians developed in Refs. [2,42], which we extend by adopting results of Refs. [43–45]. Then the resulting $N\Delta\pi\rho a_1$ Lagrangian consists of three terms and is characterised by three couplings and four arbitrary parameters $A, X, Y, Z$. In its turn, each term contains a tensor of the form

$$\Theta_{\mu\nu}(B) = \delta_{\mu\nu} + \left[ \frac{1}{2}(1 + 4B)A + B \right] \gamma_\mu \gamma_\nu, \quad B = X, Y, Z,$$  

(1.28)

which ensures the independence of the $\Delta$ contribution to the S–matrix on the parameter $A$. The choice $A = -1$ simplifies the $\Delta$ propagator considerably. The parameters $X, Y, Z$, which reflect the off–shell ambiguity of the massive spin 3/2 field were found [43–47] by analyzing the data on pion photoproduction. The value of these parameters depends on how the pion photoproduction amplitude is unitarized. This model does not require the use of the Breit–Wigner form of the $\Delta$ propagator.

In the calculations of the $\Delta$ excitation effect in the reaction Eq. (1.23), Beder and Fearing [26] took a model for needed vertices with $\Theta_{\mu\nu} = \delta_{\mu\nu}$, the Breit-Wigner form of the $\Delta$ propagator and the needed $\gamma N\Delta$ coupling from Ref. [44], thus introducing an inconsistency into calculations. This model provides about 7% effect from the $\Delta$ excitation to the photon spectrum.

The $\pi N\Delta$ and $\gamma N\Delta$ vertices including the off–shell parameters $X, Y$ and $Z$ were discussed in Ref. [13] and the $\pi N\Delta$ vertex of the form Eq. (1.28) was considered also in the small scale expansion [34]. However, the dependence of the results on these parameters was not exploited in any of the calculations performed within the framework of HBChPT.

Let us note that besides the adopted model [13–45], other models [50,51] were developed.

\footnote{These corrections up to the order $(1/M^2)$ were discussed in Ref. [41].}

\footnote{This model describes well also the latest data on the $\pi^0$ electroproduction on the proton [18].}
to describe the production of pions on protons by the electromagnetic interaction. All these models consider the same non–resonant Lagrangian of the \(N\pi\rho\omega\) system, but differ principally in the treatment of the \(\Delta\) isobar and in the method of unitarization of the \(\pi N\) amplitude.

We have also analyzed the contribution due to amplitudes constructed from an anomalous Lagrangian of the \(\pi\rho\omega a_1\) system [52,53]. We have found that the influence of this contribution on the photon energy spectrum is not significant. Earlier estimate of a contribution which arises from the Wess–Zumino–Witten part of the anomalous Lagrangian was reported in Ref. [54].

One can find in the literature an attempt to study the form factor \(g_P\) in the reaction of electroproduction of charged soft pions off the proton [29,55],

\[
e + p \longrightarrow e' + \pi^+ + n.
\] (1.29)

The starting point of this attempt is the soft pion production amplitude given as [56]

\[
f_\pi M^{\alpha \beta}_{\lambda \lambda}(q,k) \rightarrow \int dq e^{-iqy} T \left( j^\mu_{\lambda \alpha}(y) j^{\mu \nu}_{\lambda \lambda}(0) \right) |p\rangle + \varepsilon \epsilon_{\nu j m} F_{\mu \lambda}(0) |p\rangle .
\] (1.30)

The matrix element of the time–ordered product of the two currents is related to the RMC amplitude by the time reversal. The form factor \(g_P\) is contained on the right–hand side of Eq. (1.29) in the matrix element of the axial current. If one admits that in the soft pion limit only the nucleon Born terms contribute to the divergence of the current–current amplitudes, then one has the pion production amplitude which can provide information on \(g_P\). However, when one of the currents is axial, a contribution to the divergence of the current–current amplitudes from the pion pole term in the t–channel survives even in the soft pion limit [57–60]. A part of this contribution cancels the induced pseudoscalar term in the axial current and the remaining part is just the pion pole production amplitude, as one can expect intuitively. Then in the soft pion regime, the reaction Eq. (1.29) is suitable to study the weak axial nucleon form factor \(F_A(k^2)\) and the electromagnetic form factor (the electromagnetic radius) of the charged pion, but not to extract any information on \(g_P\).

In our opinion, the RMC reactions and particularly reactions (1.23) and (1.27) are at present the only available tool to study the form factor \(g_P\) as a function of the momentum transfer.

In order to compare our effective form factors with the results of Ref. [19], we define in Sect. [4] the effective Hamiltonian analogously and we consider the velocity independent part only. Then in Sect. [11], we present the results for the form factors following from our amplitudes [3] up to the order \(O(1/M^2)\). Further we deal with the contribution to \(g_i\)'s from the \(\Delta\) excitation amplitudes of our model and we compare our effective weak \(N\Delta\) vertex with the one used in Ref. [29]. Finally, we discuss the RMC amplitudes stemming from the anomalous Lagrangian.
In Sect. IV, we give the numerical results for the capture rates and we present various photon spectra. Our conclusions are presented in Sect. V. Our main result is an enhancement up to \( \approx 7\text{–}15\% \) relative to the calculations without including the \( \Delta \) isobar, which can explain twice as much of the discrepancy in comparison with the standard approach between the \( g_P^{PCAC} \) and \( g_P \) extracted from the experiment \cite{27,28}, if we take the values of the parameters of the model found from the data on pion photoproduction \cite{14,47}. If we are allowed to change these parameters independently, we can generate for the value of \( g_P \approx g_P^{PCAC} \) the photon spectrum, which is in the region of the photon energies \( k \geq 60 \text{ MeV} \) close to the spectrum, that should correspond to the experimental one.

Our triplet capture rate agrees with the one calculated very recently by BHM up to 10\%. A half of this discrepancy can be attributed to an incorrect integration over the phase volume in Ref. \cite{29}.

II. EFFECTIVE HAMILTONIAN FOR RMC

In presenting the effective Hamiltonian, we follow Rood and Tolhoek \cite{19}. Then the velocity independent part is

\[
H^{(0)}_{\text{eff}} = \frac{1}{\sqrt{2m_\mu}} \left( 1 - \sigma_i \cdot \hat{\nu} \right) [g_1 (\sigma_i \cdot \bar{\varepsilon}) + g_2 (\sigma \cdot \bar{\varepsilon}) + g_3 i (\sigma \cdot \bar{\varepsilon} \times \sigma_i) + g_4' (\sigma_i \cdot \bar{\varepsilon}) \left( \sigma \cdot \hat{k} \right) + g_5' (\sigma \cdot \hat{\nu} + \bar{\varepsilon} \cdot \bar{\nu}) + g_6' (\bar{\varepsilon} \cdot \bar{\nu}) + g_7' (\sigma \cdot \hat{\nu}) + g_8' (\sigma_i \cdot \bar{\varepsilon} \times \sigma_i) + g_9' (\sigma_i \cdot \bar{\varepsilon} \times \sigma_i) + g_{10}' (\sigma \cdot \hat{k} + \bar{\varepsilon} \cdot \bar{\nu}) + g_{11}' (\sigma \cdot \hat{\nu} + \bar{\varepsilon} \cdot \bar{\nu}) + g_{12}' (\sigma \cdot \hat{\nu} + \bar{\varepsilon} \cdot \bar{\nu})].
\]

(2.1)

Here \( \sigma_i \) (\( \sigma \)) are the lepton (nucleon) spin Pauli matrices and \( \hat{\nu} \) (\( \hat{k} \)) is the unit vector in the direction of the neutrino (photon) momentum vector \( \bar{\nu} \) (\( \bar{k} \)). Not all the form factors are independent. Using equation

\[
\bar{\varepsilon}_\lambda = -i \lambda \left( \hat{k} \times \bar{\varepsilon}_\lambda \right), \quad \bar{\varepsilon}_\lambda = \frac{1}{\sqrt{2}} \left( \hat{i} - \lambda \hat{j} \right),
\]

(2.2)

one gets redefinitions:

\[
g_2 \to g_2 - \lambda (g_7' + yg_7''), \quad g_{10}' \to g_{10}' + \lambda g_7''', \quad g_8' \to g_8' + \lambda g_3, \quad g_4' \to g_4' - \lambda g_3,
\]

(2.3)

where \( y = (\hat{\nu} \cdot \hat{k}) \). The last two terms in (2.1) are new in comparison with \cite{19}.
III. CONTRIBUTION TO $H_{\text{EFF}}^{(0)}$ FROM THE AMPLITUDES OF THE CHIRAL LAGRANGIAN OF THE $N\Delta\pi\rho\omega A_1$ SYSTEM

Here we discuss our amplitudes and contributions to $g_i$’s. We start by presenting briefly the part of the RMC amplitude derived earlier [3] without $\Delta$’s, referring for details to Sect. 3 of that paper. Then we deal with the amplitudes describing the $\Delta$ excitation processes and we compare our effective vertices with those of Ref. [26]. Finally, we discuss the amplitudes stemming from the anomalous Lagrangian.

A. The RMC amplitude without $\Delta$’s

Besides the muon radiative part $M^a(k, q)$, the amplitude $T^a(k, q)$ [3] consists of three terms representing the hadron radiative amplitude

$$T^a(k, q) = \frac{eG}{\sqrt{2}} \left\{ M^a(k, q) + l_\mu(0)\epsilon_\nu(k) \left[ M_{\mu\nu}^B(a_1; k, q) + M_{\mu\nu}^a(\pi; k, q) + M_{\mu\nu}^a(a_1; k, q) \right] \right\},$$

(3.1)

The amplitude $M_{\mu\nu}^B(a_1; k, q)$ consists of the nucleon Born terms and of some related contact amplitudes. The amplitude $M_{\mu\nu}^a(\pi; k, q)$ contains the mesonic amplitude $M_{\mu\nu}^{m.c.}(\pi; k, q)$ and all contact terms where the electroweak vertex is connected with the nucleon by the pion line. The amplitude $M_{\mu\nu}^a(a_1; k, q)$ has graphically a similar structure as the amplitude $M_{\mu\nu}^a(\pi; k, q)$ with the pion line changed for the $a_1$ meson one. These amplitudes satisfy separately continuity equations when contracted with the four momentum transfer $q_\mu$ of the weak vertex.

Since our model respect vector dominance and PCAC, the sum of the hadron radiative amplitudes satisfies exactly the following Ward–Takahashi identities

$$q_\mu \left[ M_{\mu\nu}^B + M_{\mu\nu}^a(\pi) + M_{\mu\nu}^{c.t.}(a_1) \right] = if_\pi m_\pi^2 \Delta_F^\pi(q) \mathcal{M}_{\pi,\nu}^a + i\varepsilon^{ab} \bar{u}(p') \gamma^\mu \mathcal{J}_{\nu}^b(q_1) u(p),$$

(3.2)

$$k_\nu \left[ M_{\mu\nu}^B + M_{\mu\nu}^a(\pi) + M_{\mu\nu}^{c.t.}(a_1) \right] = i\varepsilon^{ab} \bar{u}(p') \gamma^\mu \mathcal{J}_{\nu}^b(q_1) u(p).$$

(3.3)

Besides, the monopole electroweak form factors with $m_\nu = m_\rho$ and $m_A = m_{a_1}$ appear naturally in our amplitudes. Let us note that Eq. (3.3) guarantees the gauge invariance of the model. The consistency condition [21] for our amplitudes is

$$\Delta_F^\pi(q) k_\nu \mathcal{M}_{\pi,\nu}^a = \Delta_F^\pi(q_1) i\varepsilon^{ab} M^b_\pi.$$
Here $\mathcal{M}_{\pi, \nu}^a$ is the radiative pion absorption amplitude, $M_N^b$ is the pseudoscalar $\pi NN$ vertex and $q = k + q_1$.

The leading amplitudes are the nucleon Born terms $M_{\mu \nu}^B, a(k, q)\text{ 6}$ (corresponding to $M(b), M(c), M(d)$ in Ref. [12]), the amplitudes $M_{\mu \nu}^a(\pi, 1)$, $M_{\mu \nu}^a(\pi, 2)$ and $M_{\mu \nu}^a(5)$ (the sum of them corresponds to $M(e)$ in Ref. [12]), and the mesonic amplitude $M_{\mu \nu, c, a}^m$ (in correspondence with $M(f)$ in Ref. [12]). As discussed above, besides these amplitudes, other contact terms appear.

The low energy theorems allow one [1, 21], by applying current conservation and PCAC to a general amplitude, to determine consistently the amplitude for the RMC in terms of elastic weak form factors and pion photoabsorption amplitude, up to terms linear in $k$ and $q$. As shown in Ref. [21], higher order terms cannot be predicted. Since our amplitudes satisfy exactly current conservation and PCAC, we can obtain terms of any desired order. We now present the expansion of our non–resonant amplitudes up to the order $\mathcal{O}(1/M^2)$.

1. Corrections up to the order $\mathcal{O}(1/M)$

The non-relativistic reduction of these amplitudes yields the following contributions up to the order $\mathcal{O}(1/M)$ to the form factors $g_i$

\[ g_1 = -\lambda_+ g_V^L \left[ 1 + \frac{s}{2M} \cdot \hat{k} \right] - g_V^N \eta + g_A^N \lambda \eta \mu \nu, \]

\[ g_2 = -\lambda_+ g_A^L - g_P^N \eta + g_V^N \lambda \eta \mu \nu - g_A^N \eta, \]

\[ g_3 = -\lambda_+ g_A^L + g_M^N \eta + g_V^N \eta - g_A^N \lambda \eta \mu \nu, \]

\[ g_4 = \lambda_+ g_V^N - g_V^N \lambda \eta \mu \nu + g_P^N \left[ \lambda_- + \frac{\nu}{m_\mu} (1 - y) \lambda_+ \right], \quad g_4' = g_4 \frac{k}{2M}, \quad g_4'' = g_4 \frac{\nu}{2M}. \]

\[ g_5 = \lambda_+ g_V^L + g_M^N \lambda \eta \mu \nu + g_V^N \eta \mu \nu + g_M^N \lambda \eta, \quad g_5' = g_5 \frac{\nu}{2M}, \]

\[ g_6 = \lambda_+ g_V^L + (g_P^N + g_A^N) \lambda \eta \mu \nu + g_V^N \eta - g_V^N (1 + 2 \mu_n), \quad g_6' = g_6 \frac{\nu}{2M}, \]

\[ g_7 = \lambda_+ (g_V^L + g_M^L) + (g_P^N + g_A^N) \lambda \eta \mu \nu - g_V^N \eta, \quad g_7' = g_7 \frac{k}{2M}, \quad g_7'' = g_7 \frac{\nu}{2M}, \]

\[ g_8 = -\lambda_+ (g_V^L + g_M^L) - g_V^N \lambda \mu \nu, \quad g_8' = g_8 \frac{k}{2M}, \quad g_8'' = g_8 \frac{\nu}{2M}, \]

\[ g_9 = \lambda_+ (g_V^L + g_M^L) + (g_V^N + g_M^N) \lambda \eta \mu \nu - 2 g_A^N \eta \mu_n, \quad g_9' = g_9 \frac{\nu}{2M}, \]

\[ g_{10} = g_V^N \eta \frac{4M \nu}{m_\nu^2 + (q^*)^2} + \lambda_+ g_P^L \frac{\nu}{m_\mu}, \quad g_{10}' = g_{10} \frac{k}{2M}, \quad g_{10}'' = g_{10} \frac{\nu}{2M}, \]

\[ ^6\text{For notations see Sect. 3 of Ref. [3].} \]
\begin{equation}
    g_{11} = \lambda + g_P \frac{\nu}{m_\mu}, \quad g_{11}' = g_{11} \frac{k}{2M}, \quad g_{11}'' = g_{11} \frac{\nu}{2M}.
\end{equation}

Here our notations mostly follow Ref. [19]:

\[ \vec{s} = \vec{k} + \vec{\nu}, \quad \eta = \frac{m_\mu}{2M}, \quad \lambda_\pm = \frac{1}{2} (1 \pm \lambda). \]

In addition we have

\[ \mu_V = 1 + \mu_p - \mu_n \equiv 1 + \kappa_V, \quad \mu_S = 1 + \mu_p + \mu_n \equiv 1 + \kappa_S. \]

Besides the obvious momentum dependence of the form factors \( g_P^{L(P)} \) given in Eq. (1.24), all other nucleon weak vector and axial–vector form factors are assumed to have either the monopole momentum dependence, which naturally appears in our model, with \( m_V = m_\rho \) and \( m_A = m_a \), or, for the sake of comparison, the dipole one with \( m_V = 0.843 \text{ GeV} \) and \( m_A = 1.077 \text{ GeV} \).

2. Corrections of the order \( \mathcal{O}(1/M^2) \)

Here we have two groups of contributions. The first one arises from the expansion of the amplitudes considered above by one order more in \( 1/M \) which leads to

\begin{align*}
    \left( \frac{2M}{\eta} \right)^2 \Delta g_1 &= g_M^N \left( k - \mu_V \vec{\nu} \cdot \hat{k} \right) - \left( g_V^N \mu_S - g_A^N \lambda \eta \mu_n \right) \left( \vec{s} \cdot \hat{k} \right), \\
    \left( \frac{2M}{\eta} \right)^2 \Delta g_2 &= -g_M^N \lambda \left( \vec{\nu} \cdot \hat{k} \right) + 2 \mu_n \left( g_A^N + \lambda g_V^N - g_P^N \right) \left( \vec{s} \cdot \hat{k} \right), \\
    \left( \frac{2M}{\eta} \right)^2 \Delta g_3 &= -g_M^N \lambda \mu_n \left( \vec{s} \cdot \hat{k} \right) - 2 g_V^N \mu_n \left( \vec{\nu} \cdot \hat{k} \right), \\
    \left( \frac{M}{\nu \eta} \right)^2 \Delta g_4 &= -g_M^N \lambda \mu_S - 2 g_V^N \lambda \mu_n, \\
    \left( \frac{M}{\nu \eta} \right)^2 \Delta g_8 &= g_V^N \lambda \mu_n, \\
    \left( \frac{M}{\nu \eta} \right)^2 \Delta g_{10}' &= \left( g_P^N - g_A^N \right) \mu_n.
\end{align*}

The main part of the contribution to the photon spectrum arises from the terms proportional to \( g_A^N \) and \( g_V^N \) in \( \Delta g_2 \). These terms appear due to the neutron recoil induced by the time
component of the weak current. Actually, the terms $\Delta g_4''$, $\Delta g_4'$ and $g_{10}'$ contribute in the order $O(1/M^3)$. We have verified that they change the singlet capture rate by $\approx 10\%$ and the triplet capture rate by $\approx 0.8\%$, which is the reason to keep them. They also arise presumably from the neutron recoil.

The second group of corrections to order $O(1/m^2_\rho) \approx O(1/M^2)$ (the hp correction) stems from some contact terms present in the hadron radiative part of the amplitude Eq. (3.1). It is discussed in Sect. 4 of Ref. [3]. Here we quote the results of the non–relativistic reduction

$$
\Delta g_1 = -2 g_L \left( \frac{2M}{m_\rho} \right)^2 \eta \frac{k}{2M}, \quad \Delta g_2 = -\frac{g_A^N}{2} \left( \frac{2M}{m_\rho} \right)^2 \eta \frac{2k + y_\nu}{2M}. \tag{3.8}
$$

**B. The RMC amplitude with $\Delta$’s**

We derive the RMC amplitudes arising due to the $\Delta$ excitations from chiral Lagrangians [42–45]. They correspond to the standard nucleon Born terms with the $\Delta$ isobar instead of nucleon in the intermediate state. The needed Lagrangian reads

$$
L_{N\Delta\pi\rho a_1}^M = \frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi}_{\mu} T \mathcal{O}_{\mu\nu}(Z) \Psi \cdot (\partial_\nu \bar{\pi} + 2 f_{\pi} g_{\rho} \bar{a}_\nu) - g_{\rho} \frac{G_1}{M} \bar{\Psi}_{\mu} \tilde{T} \mathcal{O}_{\mu\eta}(Y) \gamma_5 \gamma_\nu \Psi \cdot \bar{\rho}_{\eta\nu} + h. c. \tag{3.9}
$$

Here $\tilde{T}$ is the operator of the transition spin. Another possible term in the $\rho N\Delta$ vertex is suppressed by one order in $1/M$ and it does not contribute in any sizeable manner [26].

We take the operator $\mathcal{O}_{\mu\nu}(B)$ in a form [43–45]

$$
\mathcal{O}_{\mu\nu}(B) = \delta_{\mu\nu} + C(B) \gamma_\mu \gamma_\nu, \tag{3.10}
$$

$$
C(B) = \frac{1}{2} (1 + 4B) A + B. \tag{3.11}
$$

A choice $A = -1$ simplifies considerably [43] the propagator of the $\Delta$.

The coupling constant $f_{\pi N\Delta}$ is not well known and the values for $f_{\pi N\Delta}^2/4\pi$ from the interval between 0.23 and 0.36 can be found in the literature [12]. From the dispersion theory [71], $f_{\pi N\Delta}^2/4\pi \approx 0.30$ and $f_{\pi N\Delta}^2/4\pi \approx 0.35$ from the decay width [12]. Also a good fit to the 33 phase shift was obtained in Refs. [44,45] by using $f_{\pi N\Delta}^2/4\pi \approx 0.314$. The new data on pion photoproduction prefer $f_{\pi N\Delta}^2/4\pi \approx 0.371$ [47]. The ranges of the other relevant parameters of the model are [45,47]

$$
-0.8 \leq Z \leq 0.7, \quad -1.25 \leq Y \leq 1.75, \quad 1.97 \leq G_1 \leq 2.65. \tag{3.12}
$$
Our radiative amplitude with the $\Delta$ excitation can be written analogously with the nucleon Born term $M_{\mu \nu}^{B,a}(1)$ \cite{3} as

$$M_{\mu \nu}^{\Delta,a} = -\bar{u}(p') \left[ \left( \hat{J}_{W,\mu \alpha}(-q) \right)^{+} S_{F}^{\alpha \gamma}(Q) \hat{J}_{em, \nu \gamma}(k) \left( T^{+} \right)^{a} T^{a} \right. + \left. \left( \hat{J}_{em, \nu \gamma}(-k) \right)^{+} S_{F}^{\alpha \gamma}(P) \hat{J}_{W, \mu \alpha}(q) \left( T^{+} \right)^{3} T^{a} \right] u(p), \tag{3.13}$$

Here the weak $N\Delta$ vertex reads

$$\hat{J}_{W, \mu \alpha}(q) = \hat{J}_{V, \mu \alpha}(q) - \hat{J}_{A, \mu \alpha}(q), \tag{3.14}$$

with the vector part defined as

$$\hat{J}_{V, \mu \alpha}(q) = \frac{i}{G_{1}} \left( \frac{m_{\pi}}{M} \right) m_{\pi}^{2} \Delta_{\mu \alpha}(q) \left( q_{\beta} \delta_{\mu \lambda} - q_{\lambda} \delta_{\mu \beta} \right) \mathcal{O}_{\alpha \beta}(Y) \gamma_{5} \gamma_{\lambda}, \tag{3.15}$$

and with the axial–vector part of the form

$$\hat{J}_{A, \mu \alpha}(q) = \frac{f_{\pi} f_{N\Delta}}{m_{\pi}} \left[ m_{\pi}^{2} \Delta_{\mu \alpha}^{a}(q) - q_{\mu} q_{\lambda} \Delta_{\alpha \lambda}(q) \right] \mathcal{O}_{\alpha \lambda}(Z). \tag{3.16}$$

Further the electromagnetic $\gamma N\Delta$ vertex is

$$\hat{J}_{em, \nu \gamma}(k) = -\hat{J}_{V, \nu \gamma}(k, k^{2} = 0) = -i \left( \frac{G_{1}}{M} \right) \left( k_{\beta} \delta_{\nu \lambda} - k_{\lambda} \delta_{\nu \beta} \right) \mathcal{O}_{\gamma \beta}(Y) \gamma_{5} \gamma_{\lambda}. \tag{3.17}$$

Finally, $S_{F}^{\alpha \gamma}(p)$ is the $\Delta$ isobar propagator. With the choice $\mathcal{O}_{\alpha \beta} = \delta_{\alpha \beta}$, our amplitudes Eqs. (3.14)–(3.16) coincide in the form with the ones obtained in Ref. \cite{26} from the study of the weak $N-\Delta$ vertex in the reaction $\nu d \rightarrow \mu^{-} \Delta^{++} n$.

For the divergence of the resonant amplitude $M_{\mu \nu}^{\Delta,a}$ from Eq. (3.13) we have

$$q_{\mu} M_{\mu \nu}^{\Delta,a} = i f_{\pi} m_{\pi}^{2} \Delta_{\mu \lambda}^{\pi}(q^{2}) M_{\mu, \nu}^{\Delta,a}. \tag{3.18}$$

Here the associated resonant radiative pion absorption amplitude $M_{\pi, \nu}^{\Delta,a}$ is,

$$M_{\pi, \nu}^{\Delta,a} = -\bar{u}(p') \left[ \left( \hat{M}_{\pi, \alpha}^{\Delta}(-q) \right)^{+} S_{F}^{\alpha \gamma}(Q) \hat{J}_{em, \nu \gamma}(k) \left( T^{+} \right)^{a} T^{a} \right. + \left. \left( \hat{J}_{em, \nu \gamma}(-k) \right)^{+} S_{F}^{\alpha \gamma}(P) \hat{M}_{\pi, \alpha}^{\Delta}(q) \left( T^{+} \right)^{3} T^{a} \right] u(p), \tag{3.19}$$

\footnote{For a recent study of this reaction see \cite{38}.}
and the $\pi N\Delta$ vertex reads

$$\hat{M}_{\pi,\alpha}(q) = i \frac{f_{\pi N\Delta}}{m_\pi} q_\lambda \mathcal{O}_{\alpha\lambda}(Z).$$

(3.20)

We now present the contributions from the amplitude Eq. (3.13) to the form factors $g_i$. They are

$$\Delta g_1 = \frac{2}{3} \lambda (C_+ - C_-) C \left\{ 1 + (1 - R) [C(Y) + C(Z) + 2(2 + R)C(Y)C(Z)] \right\},$$

$$\Delta g_2 = \frac{1}{3} (C_+ + C_-) C(1 - R) \left\{ - (1 + 2R) + 2(1 - 2R)C(Y) + 2(1 - R)C(Z) + 4(2 - R)C(Y)C(Z) \right\},$$

$$\Delta g_3 = \frac{2}{3} \lambda (C_+ + C_-) C \left\{ 1 + (1 - R) [C(Y) + C(Z) + 2(2 + R)C(Y)C(Z)] \right\},$$

$$\Delta g_4' = -\Delta g_8' = (C_+ + C_-) C,$$

$$\Delta g_6 = -\lambda \nu \frac{g_P^N}{3M g_A^N} (C_+ - C_-) C \left\{ 1 + (1 - R) [C(Y) + C(Z) + 2(2 + R)C(Y)C(Z)] \right\},$$

$$\Delta g_{10}' = (C_+ + C_-) C \nu \frac{g_P^N}{6M g_A^N} \left\{ 1 - 2(1 - R) [C(Y) + C(Z) + 2(2 + R)C(Y)C(Z)] \right\},$$

(3.21)

where

$$C = \frac{-4 f_{\pi N\Delta}}{3 m_\pi} G_1 y k, \quad R = M/M_\Delta,$$

$$C_+^{-1} = \left\{ (M_\Delta - M) + \frac{2M}{M_\Delta + M} \left[ m_\mu - \nu + \frac{m_\mu}{2M} \left( 2\nu - m_\mu - \frac{2\nu}{m_\mu}(\nu + y k) \right) \right] \right\},$$

$$C_-^{-1} = \left\{ (M_\Delta - M) + \frac{2M}{M_\Delta + M} \left[ \nu - m_\mu + \frac{m_\mu}{2M} (2\nu - m_\mu) \right] \right\}.$$

(3.22)

According to the concept developed in Refs. [43–45], we take the mass of the $\Delta$ isobar real.

C. The RMC amplitude from an anomalous Lagrangian of the $\pi\rho\omega a_1$ system

We have considered so far the amplitudes where a natural parity does not change in any vertex. The natural parity of a particle is defined as $P (-1)^J$, where $P$ is the intrinsic parity and $J$ is the spin of the particle. Some amplitudes of this kind relevant for the process under study are presented in Fig. [11].
FIG. 1. (a),(b) – the radiative hadron amplitudes obtained from the anomalous Lagrangian of the $\pi\rho\omega a_1$ system, Eq. (3.23); in (a), $B=\pi$ or $a_1$; (c) – the associated radiative pion absorption amplitude. These amplitudes satisfy PCAC.

The starting point is an anomalous Lagrangian of the $\pi\rho\omega a_1$ system \[53\], \[52\] constructed within the approach of hidden local symmetries \[64,65\]. The electromagnetic interaction in such a system was first considered in Ref. \[66\] and the relevant constants $\tilde{c}_i$ were extracted from the data as well. The weak interaction was incorporated explicitly in Ref. \[52\] and the refit of the constants to the modern data \[67\] was made in Ref. \[53\].

The Lagrangian reads

$$L_{an} = 2ig_\rho\varepsilon_{\kappa\lambda\mu\nu} \left[ (\partial_\kappa \omega_\lambda ) \left( g_\rho \bar{\rho}_\mu - e\bar{\nu}_\mu \right) + \left( g_\rho \omega_\kappa - \frac{1}{3} e B_\kappa \right) (\partial_\lambda \bar{\rho}_\mu) \right]$$

$$\cdot \left[ c_7 \left( \frac{1}{f_\pi} \partial_\nu \bar{\pi} + e\bar{A}_\nu \right) + c_8 \left( \frac{1}{2} e\bar{A}_\nu - g_\rho \bar{a}_\nu \right) \right]$$

$$+ 2ie\varepsilon_{\kappa\lambda\mu\nu} \left[ \left( \frac{1}{3} \partial_\kappa B_\lambda \right) \left( g_\rho \bar{\rho}_\mu - e\bar{\nu}_\mu \right) + \left( g_\rho \omega_\kappa - \frac{1}{3} e B_\kappa \right) (\partial_\lambda \bar{\nu}_\mu) \right]$$

$$\cdot \left[ c_9 \left( \frac{1}{f_\pi} \partial_\nu \bar{\pi} + e\bar{A}_\nu \right) + c_{10} \left( \frac{1}{2} e\bar{A}_\nu - g_\rho \bar{a}_\nu \right) \right],$$

(3.23)

where besides the meson fields, the external vector isoscalar $B_\mu$ and isovector $\bar{V}_\mu$ and axial vector isovector $\bar{A}_\mu$ fields are also included. The constants $\tilde{c}_i$ are \[53\]

$$\tilde{c}_7 = 8.64 \times 10^{-3}, \quad \tilde{c}_8 = -1.02 \times 10^{-1}, \quad \tilde{c}_9 = 9.23 \times 10^{-3}, \quad \tilde{c}_{10} = 1.29 \times 10^{-1}.$$

(3.24)

The axial RMC amplitudes arising from the anomalous Lagrangian $L_{an}$, Eq. (3.23), are
\[ M_{\mu\nu}^a(1) = i \frac{g^2}{3} \varepsilon_{\mu\nu\beta\sigma} k_\eta q_\sigma q_\mu \Delta_F^\pi(q) \Delta_{\beta\lambda}^\rho(q_1) \bar{u}(p') \left( \gamma_\lambda - \frac{\kappa_V}{2M} \sigma_\lambda q_1 \right) \tau^a u(p), \]
\[ M_{\mu\nu}^a(2) = -i \frac{g^2}{3} \varepsilon_{\mu\nu\beta\sigma} k_\eta \Delta_{\beta\lambda}^\rho(q_1) \bar{u}(p') \left( \gamma_\lambda - \frac{\kappa_V}{2M} \sigma_\lambda q_1 \right) \tau^a u(p), \quad (3.25) \]

and they correspond to the processes presented in Figs. 1(a) and 1(b). Together with the radiative pion absorption amplitude of Fig. 1(c)

\[ M_{\pi,\nu}^a = -i \frac{g^2}{f_\pi} \varepsilon_{\nu\beta\gamma} k_\eta q_\sigma \Delta_{\beta\lambda}^\rho(q_1) \bar{u}(p') \left( \gamma_\lambda - \frac{\kappa_V}{2M} \sigma_\lambda q_1 \right) \tau^a u(p), \quad (3.26) \]

the amplitudes Eq. (3.25) satisfy PCAC:

\[ q_\mu \left[ M_{\mu\nu}^a(1) + M_{\mu\nu}^a(2) \right] = if_\pi m_\pi^2 \Delta_F^\pi(q) M_{\pi,\nu}^a. \quad (3.27) \]

The contribution from the anomalous amplitudes Eq. (3.25) to the total hadron radiative amplitude Eq. (3.1) for the reaction Eq. (1.23) is given as

\[ T_{an}^a = \frac{eG}{\sqrt{2}} h_\mu(0) \varepsilon_\nu \left( \bar{c}_\gamma + \bar{c}_g \right) \left[ M_{\mu\nu}^a(1) + M_{\mu\nu}^a(2) \right]. \quad (3.28) \]

In Ref. [8], a contribution arising from the Wess–Zumino–Witten anomalous Lagrangian was estimated. Graphically, it corresponds to our Fig. 1(b) with the pion instead of the rho meson and with the vector interaction \( V^a_\mu \) instead of the axial one. The associated amplitude depends on the momentum transfer \( q_L \) and therefore, it does not possess the enhancement factor \( \approx 3 \) for large photon momentums. For illustration, we present the contribution to one of the form factors

\[ \Delta g'_4 = -\eta \frac{k^2}{8\pi^2 f_\pi^2} \frac{\lambda k + y\nu}{2m_\mu} g_P^L, \quad (3.29) \]

all other contributions have a similar structure. It is also seen that these contributions are of the order \( \mathcal{O}(1/M^3) \), because \( 8\pi^2 f_\pi^2 \sim M^2 \).

In our case, it is the amplitude related to the graph Fig. 1(a), which is \( q^N \)-dependent. We present from the calculated contributions to \( g_i \) arising from the amplitudes Eq. (3.25) only the one for the form factor \( g_2 \), the other ones are suppressed by one order in \( 1/M \)

\[ \Delta g_2 = -\frac{g_\rho^2}{3g_A} \left( 2M/m_\rho \right)^2 \left( 1 + \kappa_V \right) \eta \frac{k}{2M} \frac{s^2}{4M^2} g_P^N + \frac{g_\rho^2}{3} \left( \frac{2M}{m_\rho} \right)^2 \left( 1 + \kappa_V \right) \eta \frac{k \cdot s}{4M^2}. \quad (3.30) \]
For comparison, we keep also the \( g_N^2 \)-dependent contribution. Using the KSFR relation
\[
2f_\pi^2 g_P^2 = m_\rho^2,
\]
we can rewrite this contribution to the form
\[
-\frac{1 + \kappa_V}{g_A} \eta \frac{k}{M} \frac{s^2}{12f_\pi^2} g_P^2.
\]

Taking into account that \( 12f_\pi^2 \approx M^2/10 \) we can see that the \( g_N^2 \)-dependent contribution is larger than the \( g_P^2 \)-dependent one for large values of \( k \) by a factor \( \approx 20 \). However, it is not enough to influence the photon spectrum, because of an additional factor \((\tilde{c}_7 + \tilde{c}_9)\) in the amplitude Eq. (3.28) (see below). It is seen from Eq. (3.30) that the first term on the right-hand side is suppressed in comparison with the second one arising from the contact amplitude \( M_{\mu\nu}^a(2) \). As we shall see later, the second term contributes to the triplet capture rate by an amount of \( \approx -0.2\% \).

Let us note that the sum of the vector RMC amplitudes arising from the anomalous Lagrangian Eq. (3.23) is zero with a good accuracy.

One can obtain more general result for \( \Delta g_2 \), Eq. (3.30), by a change
\[
1/3 \to g_{\rho 1}/g_{\omega 1}, \quad \kappa_V \to g_{\rho 2}/g_{\rho 1},
\]
which corresponds to the model used in Refs. [44,45] for describing the pion photoproduction amplitude in the t channel. In this model, the \( \pi \rho \gamma \) and \( \pi \omega \gamma \) amplitudes are effectively the same as those obtained from our anomalous Lagrangian Eq. (3.23) and the \( \rho NN \) and \( \omega NN \) vertices contain four constants \( g_{\rho 1}, g_{\rho 2}, g_{\omega 1} \) and \( g_{\omega 2} \), which are the free parameters, obtained together with other free parameters of the model from a fit to the data.

Compared with the sets of the form factors given in Eqs. (3.7) and (3.8), the \( g_i \)'s, Eq. (3.30), are even larger. However, due to the values of \( \tilde{c}_i \), Eq. (3.24), the factor \((\tilde{c}_7 + \tilde{c}_9) \approx 1.8 \times 10^{-2} \) makes the contribution from the amplitude \( T_{\mu\nu}^a \), Eq. (3.28), small.

### IV. RESULTS

Using the Hamiltonian \( H_{eff}^{(0)} \), Eq. (2.1), and the sets of the form factors \( g_i \), Eqs. (3.3), (3.7), (3.8), and (3.21), we have calculated the capture rates and the photon energy spectra for the RMC in a muon-hydrogen system described by a spin density matrix \( \rho_\xi \) \((\xi = s, t)\)
\[
\frac{d\Lambda_\xi}{dk} = \frac{1}{4\pi^2} \left( \alpha^2 G_F \cos \theta_c m/m_\mu \right)^2 M_n k \int_{-1}^{+1} dy \frac{\nu_0}{W + k(y - 1)} Tr \left\{ (1 - \vec{\sigma} \cdot \hat{\nu}) \times H_{eff}^{(0)} \rho_\xi \left[ H_{eff}^{(0)} \right]^+ \right\}.
\]
Here $\alpha$ is the fine structure constant, $G_F$ is the Fermi constant, $\cos \theta_c$ is the Cabibbo angle, $m$ is the reduced mass of the $\mu - p$ system, the neutrino energy is determined by the energy conservation

$$\nu_0 = \frac{W}{W + k(y - 1)}(k_{\text{max}} - k) \approx \left[ 1 + \frac{k}{M_p}(1 - y) + \frac{k}{M_p^2}(y - 1) \right. \times (m_\mu + k(y - 1))] (k_{\text{max}} - k) ,$$

(4.2)

where the maximum photon energy is given as

$$k_{\text{max}} = \frac{W^2 - M_p^2}{2W} , \quad W = m_\mu + m_\mu ,$$

(4.3)

and $M_p(n)$ is the proton (neutron) mass. The singlet and triplet spin density matrices are

$$\rho_s = \frac{1}{4}(1 - \vec{\sigma} \cdot \vec{\sigma}_l) , \quad \rho_t = \frac{1}{4}(1 + \frac{1}{3} \vec{\sigma} \cdot \vec{\sigma}_l) .$$

(4.4)

We have also calculated the capture rates and spectra for the ortho- and paramolecular $\mu p$ states and for the mixture of muonic states relevant to the TRIUMF experiment [27] [28]. The ortho ($\Lambda_o$)- and paramolecular ($\Lambda_p$) capture rates are given in terms of $\Lambda_s$ and $\Lambda_t$ as

$$\Lambda_o = 0.756 \Lambda_s + 0.253 \Lambda_t , \quad \Lambda_p = 0.286 \Lambda_s + 0.857 \Lambda_t ,$$

(4.5)

and the capture rate $\Lambda_T$, relevant to the TRIUMF experiment [27] [28], is

$$\Lambda_T = 0.061 \Lambda_s + 0.854 \Lambda_o + 0.085 \Lambda_p .$$

(4.6)

Now we present numerical results for the capture rates.

### A. Capture rates

Here we present the results for the capture rates calculated in various models. If not stated otherwise, we use the monopole form factors and we put $x = 1$ in Eq. (1.24).

A) We first discuss the results obtained in the model with the $\Delta$ isobar kept on–shell. We give the singlet and triplet capture rates in a more detail, in order to see explicitly various contributions,
\[ \Lambda_s \times 10^3 = 0.40(0) + 1.65(-1) + 1.29(-2) + 0.11(hp) - 0.07(al) + 0.05(\Delta) \\
= 3.43 \times (3.51) \text{ s}^{-1}, \quad (4.7) \]
\[ \Lambda_t \times 10^3 = 43.7(0) + 53.1(-1) + 3.7(-2) + 0.9(hp) - 0.2(al) + 2.2(\Delta) \\
= 103.0 (103.4) \text{ s}^{-1}. \quad (4.8) \]

Here on the right–hand side of Eqs. (4.7) and (4.8), the number \( n (n=0,-1,-2) \) in the brackets means the order of the contribution in \( O(1/M^n) \) and hp (al) and \( \Delta \) mean the contributions from the hard pion form factors Eq. (3.8) (from the form factors Eq. (3.30)) and from the form factors Eq. (3.21) due to the \( \Delta \) isobar excitation processes, respectively. These were calculated using the parameters

\[ \frac{f_{pN\Delta}}{4\pi} = 0.371, \quad G_1 = 2.525, \quad Y = Z = -0.5. \quad (4.9) \]

The choice of the parameters \( Y \) and \( Z \) is such that only the terms proportional to \( \delta_{\mu\nu} \) in Eq. (1.28) contribute (the \( \Delta \) is on–shell). The contribution of the \( \Delta \) excitation to \( \Lambda_t \) is \( \approx 2\% \), which is in agreement with BHM. The numbers in the brackets on the right–hand side of Eqs. (4.7) and (4.8) are obtained using Eq. (1.24) for \( g_L(N)P \) without the correction \( \tilde{g}_L(N) \) included.

For the other capture rates we have

\[ \Lambda_o = 28.7 \times 10^{-3} \text{ s}^{-1}, \quad \Lambda_p = 89.3 \times 10^{-3} \text{ s}^{-1}, \quad \Lambda_T = 32.3 \times 10^{-3} \text{ s}^{-1}. \quad (4.10) \]

For the dipole form factors, analogously with Eqs. (4.7) and (4.8) we obtain

\[ \Lambda_s = 3.28 (3.50) \times 10^{-3} \text{ s}^{-1}, \quad \Lambda_t = 101.5 (102.5) \times 10^{-3} \text{ s}^{-1.} \quad (4.11) \]

Let us compare our results with available calculations. The singlet and triplet capture rates were calculated earlier by Opat [18]. He obtained \( \Lambda_s = 4.96 \times 10^{-3} \text{ s}^{-1} \) and \( \Lambda_t = 90.0 \times 10^{-3} \text{ s}^{-1} \).

Very recent calculations [29] yield \( \Lambda_s = (2.90–3.10) \times 10^{-3} \text{ s}^{-1} \) and \( \Lambda_t = (112–114) \times 10^{-3} \text{ s}^{-1} \). Having in mind that \( \Lambda_s \) results as the difference of two large and almost equal numbers, the agreement between our value Eq. (4.7) of \( \Lambda_s \) and the BHM value can be considered as satisfactory. However, the difference of \( \approx 10\% \) between the triplet capture rates is too large. A half of this discrepancy can be understood by checking the integration over the phase volume. We use for the neutrino momentum Eq. (4.2) with \( k_{\text{max}} \) from Eq. (1.3), while BHM employ for \( k_{\text{max}} \) equation (4.37), which is in our notations

\[ k_{\text{max}} = m_\mu (1 + \frac{m_\mu}{2M_N})(1 + \frac{m_\mu}{M_N})^{-1} \approx m_\mu (1 - \frac{m_\mu}{2M_N} + \frac{m_\mu^2}{2M_N^2}). \quad (4.12) \]
where \( M_N = (M_p + M_n)/2 \) is the nucleon mass. From Eq. (1.3), one obtains \( k_{\text{max}} = 99.15 \text{ MeV} \), while from Eq. (4.12) one has \( k_{\text{max}} = 100.3 \text{ MeV} \) and instead of \( \Lambda_t = 103.0 \times 10^{-3} \text{s}^{-1} \) one obtains \( \Lambda_t = 108.0 \times 10^{-3} \text{s}^{-1} \), which is larger by \( \approx 5\% \). Using in Eq. (4.12) for \( \nu_0 \) the expansion Eq. (4.12) for \( k_{\text{max}} \), one obtains

\[
\nu_0 = m_\mu - k - \frac{m_\mu^2}{2M_N} + \frac{k}{M_N} (1-y)(m_\mu - k) + \frac{1}{2M_N^2} [m_\mu + k(y-1)]
\times \left[ m_\mu^2 + 2k(m_\mu - k)(y-1) \right].
\] (4.13)

In Ref. [29], Eq. (4.39) is used for \( \nu_0 \). It retains terms up to the order \( O(1/M_N) \), which yields in our case \( \Lambda_t = 107.2 \times 10^{-3} \text{s}^{-1} \). But the source of the remaining difference of \( \approx 5\% \) between the results for the \( \Lambda_t \) is not clear.

B) The results of calculations without the \( \Delta \) excitation effect are

\[
\Lambda_s = 3.38 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_t = 100.8 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_T = 31.6 \times 10^{-3} \text{s}^{-1}.
\] (4.14)

C) We now present the capture rates for the same case as in B), but for the value of the parameter \( x = 1.5 \). The rates are

\[
\Lambda_s = 8.35 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_t = 116.6 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_T = 39.8 \times 10^{-3} \text{s}^{-1}.
\] (4.15)

The strong dependence of the capture rates on \( g_P \) was already known to Opat [18].

D) The capture rates, calculated for the same parameters as in A), but \( Y = 1.75 \) and \( Z = -0.8 \), are

\[
\Lambda_s = 3.28 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_t = 106.0 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_T = 33.0 \times 10^{-3} \text{s}^{-1}.
\] (4.16)

In this case, the effect of the \( \Delta \) excitation is for the values of \( Y \) and \( Z \), allowed by the inequalities of Eq. (3.12), maximal and it is \( \approx 5\% \) for \( \Lambda_t \) and \( \approx 4\% \) for \( \Lambda_T \).

E) The capture rates, calculated as in D, but \( Z = -1.95 \),

\[
\Lambda_s = 5.93 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_t = 116.7 \times 10^{-3} \text{s}^{-1}, \quad \Lambda_T = 38.1 \times 10^{-3} \text{s}^{-1}.
\] (4.17)

These capture rates are close to those calculated in the case C) for \( x = 1.5 \). In order to achieve the same enhancement in the rates, one should put \( \Delta \) isobar more off–shell than it

\[\text{8There is factor 2 missing in the denominator of the third term on the right–hand side of this equation.}\]
is needed in pion photoproduction processes. On the other hand, the values of the off-shell parameters $X$, $Y$, $Z$ depend strongly on whether the pion production amplitude is unitarized or not and on the method of unitarization. Let us note that our choice of the parameters of the model is not optimal. In order to extract an optimal set of these parameters from the data, one should use a minimization procedure.

Similar enhancement in the capture rates as in the case E) can be achieved by taking $f_{\pi N\Delta}^2/4\pi \approx 20$ or $G_1 \approx 20$, which is an amplification of $\approx 7$ in the $\pi N\Delta$ coupling and of $\approx 8$ in the constant $G_1$, which is much more than the enhancement factor of $\approx 2.5$ for the parameter $Z$.

B. Photon spectra

Photon spectra, corresponding to the capture rates calculated in the model A of the previous section, are presented in Fig. 2. The $\Delta$ isobar is included, but the choice of the parameters $Y = Z = -0.5$ is such that the isobar is on-shell. As it is seen from Fig. 2, our spectra are in a close correspondence with those of Fig. 3 of Ref. [26]. However, our spectrum for the triplet state of the $\mu - p$ system (long-dashed curve) differs from the analogous spectrum of Fig. 5 [29], as it should be, because the triplet capture rates differ significantly.

FIG. 2. Photon spectra calculated in the model A of the previous section, the $\Delta$ isobar excitation effect is included; the dotted, long-dashed, dot-dashed and dashed curves correspond, respectively, to the singlet, triplet, ortho- and para $\mu p$ molecule spin combinations, the solid curve correspond to the mixture of muonic states relevant to the TRIUMF experiment [27,28].
FIG. 3. Relative difference in % for the spectra calculated with (model A) and without (model B) the $\Delta$ excitation included; the dotted, long–dashed, dot–dashed and dashed curves correspond, respectively, to the singlet, triplet, ortho- and para $\mu\mu\mu$ molecule spin combinations, the solid curve correspond to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

FIG. 4. Relative difference in % for the spectra calculated with (model D) and without (model B) the $\Delta$ excitation included; the dotted, long–dashed, dot–dashed and dashed curves correspond, respectively, to the singlet, triplet, ortho- and para $\mu\mu\mu$ molecule spin combinations, the solid curve correspond to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

The percentage change in the spectra when the $\Delta$ excitation effect is taken into account is presented in Fig. 3. The $\Delta$ excitation effect was calculated according to the model A of the previous section. The case without this effect corresponds to the model B. This change in
the spectra due to the $\Delta$ was first calculated by Beder and Fearing [26]. Our Fig. 3 is in a good agreement with Fig. 4 of Ref. [26], but it differs from the analogous Fig. 6 of Ref. [29].

Similar calculations are presented in Fig. 4 and Fig. 5 using instead of the model A the spectra of the model D and E, respectively. As it is seen, by putting the $\Delta$ isobar off–shell, both the singlet and triplet spectra are changed sensibly.

![Graph showing relative difference in % for the spectra calculated with (model E) and without (model B) the $\Delta$ excitation included.

**FIG. 5.** Relative difference in % for the spectra calculated with (model E) and without (model B) the $\Delta$ excitation included; the dotted, long–dashed, dot–dashed and dashed curves correspond, respectively, to the singlet, triplet, ortho– and para $\mu\mu p$ molecule spin combinations, the solid curve correspond to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

In Fig. 6, we show how the photon spectra, relevant to the mixture of the muonic states for the TRIUMF experiment, depend on the parameters of our model. The dotted curve corresponds to the model B (no $\Delta$, $x=1$), the dashed curve is the photon spectrum for the case A ($\Delta$ on–shell, $Y = Z = -0.5$), the dot–dashed curve is calculated using the model D: $\Delta$ is off–shell, the parameters $Y' = 1.75$, $Z = -0.8$ are at the boundary of the region Eq. (3.12) allowed by the pion photoproduction data [44–47]. In this case, about two times more discrepancy is explained in comparison with the dashed curve. The dependence of the photon spectrum on the change in $g_P$ is illustrated by the long–dashed curve, calculated within the model C (no $\Delta$, $x=1.5$). We have found that this curve, normalized to 14.5 counts at $k = 60$ MeV, follows closely the solid curve of Fig. 4 [28]. All other curves in this and in the next figure are multiplied by this normalization factor. The long–dashed curve closely follows the solid curve from $k \approx 60$ MeV, which corresponds to the model E: $\Delta$ is off–shell, the parameter $Y = 1.75$ is the same as in the case D, the parameter $Z = -1.95$.

Evidently, this picture is in agreement with the one obtained by comparing the Beder–Fearing model [26] with the experiment [27,28]: the experimental photon spectrum can be satisfactorily described with $g_P$ of the form Eq. (1.24) only for $x \approx 1.5$ and only a small part
of the discrepancy could be explained by calculating the spectra by including the on-shell ∆ isobar. As it is seen from the solid curve, putting the ∆ isobar off-shell, one can describe the experimental data quite well.

FIG. 6. Influence of the ∆ isobar parameters on the photon spectra corresponding to the mixture of muonic states in TRIUMF experiment [27,28]. For the explanation of the curves see text.

As we have just found, our model can explain about two times more of the discrepancy between the $g_P^{PCAC}$, Eq. (1.14), and of $g_P$ extracted from the experiment [27,28], if one restricts oneself to the set of the parameters from Eq. (3.12). They were extracted in Refs. [44–47] from the data on pion photoproduction by using unitarized multipoles arising from the pion photoproduction amplitude. The problem with the unitarity appears in the pion photoproduction, because of the pion–nucleon interaction in the final state. It is true that the time reversed of the pion production amplitude is connected with the hadron radiative part of our RMC amplitude by the continuity equations Eqs. (3.2), (3.18), and (3.27). However, we do not need to unitarize our amplitude. Therefore it is not clear, how the inequalities Eq. (3.12) are restrictive for the problem considered here. It is seen that the model can reproduce reasonably the experimental photon spectrum [27,28] in the region $k \geq 60$ MeV for the PCAC value of $g_P$, Eq. (1.14), if the values of the parameters $Y$ and $Z$ are taken to be $Y = 1.75$ and $Z = -1.95$.

In Fig. 4, we show the dependence of the calculations on uncertainties in our knowledge of $g_P$ and of the admixture $\xi$ of the $S = 3/2$ orthomolecular $p\mu p$ state. As discussed recently [33], admixture of the $S = 3/2$ orthomolecular $p\mu p$ state changes the molecular capture rate to [33]

$$\Lambda'_o = \xi \Lambda_o(1/2) + (1 - \xi)\Lambda_o(3/2) ,$$

(4.18)
where $\Lambda_o(1/2)$ is $\Lambda_o$ of Eq. (1.5) and $\Lambda_o(3/2) = 1.009 \Lambda_t$. It was found in [33] that the data on OMC in hydrogen requires $g_P \leq 1.2 g_P^{P,AC}$ or $\xi \geq 0.95$. This restriction on $g_P$ is in agreement with our Eq. (1.20) for the OMC in $^3$He. The dependence on the uncertainty in $g_P$ and on $\xi$ is illustrated by the long–dashed, dot–dashed and dashed curves. The numbers in the brackets are for the total capture rate, for the partial capture rate in the interval $(60 - k_{\text{max}}) \text{MeV}$ and for the capture rate in counts for this interval, respectively. Otherwise, the unit for the capture rates is $10^{-3} \text{s}^{-1}$. For the sake of illustration, we assigned a 10% error [28] to a set of ‘data’ represented by 14 points of the spectrum C. The number of counts 276, related to this curve is to be compared with the number of counts 286, which can be read off the histogram presented in Fig.4 [28]. As it is seen, all curves lay already inside the $2\sigma$ bound.

FIG. 7. Dependence of the photon spectra, corresponding to the mixture of muonic states relevant to the TRIUMF experiment [27,28], on various parameters of the calculations; the dotted curve is for the reference and corresponds to the model A (32.3/7.1/196); the long–dashed curve is calculated for the model D, but $\xi = 0.95$ (36.3/8.1/225); the dot–dashed curve is obtained within the model D, but $x = 1.2$ (35.9/8.6/239); the dashed curve– the model D, but $x = 1.2$ and $\xi = 0.95$ (39.4/9.3/259); the solid curve corresponds to the model C (39.7/9.9/276). For the details see the text.

V. SUMMARY

In this paper, we have presented the capture rates and the photon energy spectra for the RMC in hydrogen, calculated using the effective Hamiltonian $H_{\text{eff}}^{(0)}$, Eq. (2.1), where the form factors $g_i$ are obtained from the amplitudes derived from the chiral Lagrangian of the $N\Delta\pi\rho\omega a_1$ system. The non–resonant part of the Lagrangian contains the normal and anomalous Lagrangians of the $N\pi\rho\omega a_1$ system interacting with the external electromagnetic
and weak fields by the associated one–body currents \[2,26,29,42,52,53\]. In the expansion of the amplitudes in \(1/M\), we keep all terms up to the order \(O(1/M^2)\).

For the resonant part of our Lagrangian, we have extended the standardly used model \[2,26,29,12\] by adopting results of the model developed by Olsson and Osypowski \[43\] and by Davidson, Mukhopadhyay and Wittman \[44,45\], allowing the \(\Delta\) isobar to be off–shell.

The calculated triplet capture rate differs by \(\approx 10\%\) from the one derived quite recently within the HBChPT approach in Ref. \[29\]. About a half of this discrepancy can be understood by the use of an approximate equation for the neutrino momentum in Ref. \[29\]. The origin of the rest of the discrepancy is not clear.

In the model, restricting the \(\Delta\) isobar on–shell, our spectra are close to those obtained earlier by Beder and Fearing \[26\]. Our full model, that includes off–shell \(\Delta\) isobar, can explain two times more of the discrepancy between the PCAC value of \(g_P\), Eq. (1.14), and of \(g_P\) extracted from the experiment \[27,28\], if one restricts oneself to the values of the parameters of the model extracted from the data on the pion photoproduction off the nucleon. Let us note that taking into account existing uncertainty in \(g_P^{PCAC}\) extracted from the OMC in the hydrogen and \(^3\text{He}\), and in the parameter \(\xi\), regulating the admixture of the \(S = 3/2\) orthomolecular \(\text{p}\mu\text{p}\) state, one finds that this model is, actually, in reasonable agreement with the data \[27,28\].

Moreover, if one is allowed to vary the parameters of the model independently, the experimental photon spectrum can be described for the induced pseudoscalar form factor \(g_P\) of Eq. (1.24) without any scaling \((x = 1)\). It would be difficult to find any physics behind the scaling of \(g_P\), which would mean violation of PCAC. On the other hand, the part of our full model, containing the electroweak interaction of the off-shell \(\Delta\) isobar, is widely used to describe successfully the pion photoproduction data, which provides a strong basis for confidence in our results. For the reaction of the RMC in the hydrogen, Eq. (1.11), it is the only model known so far, providing enough enhancement in the high energy region of the photon spectrum.

In conclusion we note that the reactions of the RMC in the hydrogen and \(^3\text{He}\) are at present the only available effective tool for the study of the form factor \(g_P\) as a function of the momentum transfer. Therefore, more efforts, both theoretical and experimental, are highly desirable.

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