The ρ Meson and the Thermal Behavior of an Effective Hadronic Coupling Constant

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Vector Meson Dominance ideas together with a Finite Energy QCD sum rule allows for the determination of the \( \rho \) dependence of the effective hadronic coupling constant \( g_{\rho \pi \pi}(q^2, T) \) in the space-like region. It turns out that \( g_{\rho \pi \pi}(q^2, T) \) vanishes at the critical temperature \( T_c \), independently of \( q^2 \). A comparison with a previous independent QCD determination of the electromagnetic pion form factor at finite temperature supports the validity of Vector Meson Dominance at finite temperature. We find also that the pion radius increases with \( T \), having a divergent behavior at \( T_c \).

In this talk \cite{1}, we will consider the thermal behavior of the effective hadronic coupling \( g_{\rho \pi \pi}(q^2, T) \). From the phenomenological point of view, this is an important issue since this coupling is related to the electromagnetic pion form factor which plays an important role in the production of dileptons, in relativistic heavy ion collisions, due to pion annihilation in the normal hadronic phase. \cite{2,3}.

To answer this question we will use Finite Energy QCD sum rules (FESR) at finite T for the three point function that involves the rho-meson interpolating current plus two axial divergences. We will start first with the determination of \( g_{\rho \pi \pi}(q^2) \) at zero temperature (see \cite{3} for an earlier determination using Laplace Sum Rules). In this way we will establish the normalizations, checking also the validity of Vector Meson Dominance (VMD) here. This will be done through a comparison with a finite temperature \( g_{\rho \pi \pi}(q^2, T) \). From the latter does fit the data at \( T = 0 \) very well, we can adopt it as the benchmark \( F_\pi(Q^2, T) \) at finite T.

Later we will compare our result at finite temperature with the same direct determination \cite{4}.

\cite{1} talk given at the QCD Euroconference 97, Montpellier 3-9 July 1997

Finding an agreement between the two expressions which can be taken as evidence in support of VMD at finite T.

Let us consider the \( T = 0 \) correlator

\[ \Pi_\mu(q) = i^2 \int d^4x d^4ye^{-iqy} e^{iq'x} \langle 0 | T(j_\mu^\rho(x) J^\rho_\mu(y) j_\pi(0)) | 0 \rangle = \Pi_1(q^2) P_\mu + \Pi_2(q^2) q_\mu \] (1)

where \( J^\rho_\mu(y) \) and \( j_\pi(x) \) are the usual \( \rho \)- and \( \pi \)-meson currents. \( q_\mu = (p' - p)_\mu \) and \( P_\mu = (p' + p)_\mu \).

To leading order in \( \alpha_s \) and in the quark masses, the imaginary part of this three point function comes only from the perturbative triangle diagram and is given by

\[ Im\Pi_\mu|_{\text{QCD}} = \frac{3}{4} \frac{(m_\mu + m_\rho)^2}{[s + s' + Q^2]^2 - 4ss']^2} \times [-Q^2 ss' P_\mu + ss'(s - s')q_\mu] \] (2)

where \( s = p^2, s' = p'^2 \), and \( Q^2 = -q^2 \geq 0 \). For the hadronic part of the analysis we saturate with a pion intermediate state and use the current-field identity \( j_\mu^\pi = \frac{M_\rho^2}{f_\rho} \rho_\mu^\rho \), being a an isospin index and \( \rho_\mu^\rho \) the rho-meson field. Experimentally \( f_\rho = 5.0 \pm 0.1 \).
For the hadronic spectral function, e.g. \( \text{Im} \Pi_1 \), we get

\[
\text{Im} \Pi_1(s, s', Q^2)|_{HAD} = -2f_\pi^2 \mu_\pi^4 \frac{M_\rho^2}{M_\rho^2 + Q^2} \\
\times \frac{g_{\rho\pi\pi}(Q^2)}{f_\rho} \pi^2 \delta(s - \mu_\pi^2) \delta(s' - \mu_\pi^2) \\
+ \theta(s - s_0) \theta(s' - s'_0) \text{Im} \Pi_1(s, s', Q^2)|_{QCD} \tag{3}
\]

In the above expression \( f_\pi = 93.2 \text{MeV} \) and \( g_{\rho\pi\pi}(M_\rho^2) = 6.06 \pm 0.03 \). Note that, according to the philosophy of QCD sum rules, we assume that for \( s \) and \( s' \) values above the continuum thresholds the hadronic spectral function can be expressed in terms of QCD degrees of freedom. Due to the radial excitations of the rho-meson, \( g_{\rho\pi\pi} \) actually is a form factor, i.e. a function of \( Q^2 \). In the dual model \( \text{EVMD} \) this effect is taken into account. Note that in addition to the pion pole there are other terms, as for example the \( a_1 \) meson. Here, however, we shall include these in the hadronic continuum provided that the thresholds \( s_0 \simeq s'_0 > 1 - 3 \text{GeV}^2 \). Now we can proceed in the standard fashion, i.e., invoking Cauchy’s theorem we construct the lowest dimensional FESR

\[
\int_0^{s_0} \int_0^{s'_0} \text{Im} \Pi_1(s, s')|_{HAD} \text{dsds'} = \\
\int_0^{s_0} \int_0^{s'_0} \text{Im} \Pi_1(s, s')|_{QCD} \text{dsds'} \tag{4}
\]

where \( s_0 \) and \( s'_0 \) are the continuum thresholds, i.e. the onset of perturbative QCD. In this way we get

\[
g_{\rho\pi\pi}(Q^2) = \frac{3}{8\pi^2} \frac{f_\pi^2}{M_\rho^2} Q^2 (Q^2 + M_\rho^2) I(q^2) \tag{5}
\]

where

\[
I(q^2) = \frac{s_0}{16} (3 + \frac{s_0}{Q^2}) + \frac{1}{8} (s_0 + \frac{3}{4} Q^2) \ln \frac{Q^2}{Q^2 + 2s_0} \tag{6}
\]

We have used the Gell-Mann, Oakes and Renner (GMOR) relation. The result for \( I(q^2) \) was obtained after a double integration in a triangle in the \( s, s' \) plane. It is important to remark that other shapes for the integration region do not introduce appreciable differences in the numerical results. According to Extended Vector Meson Dominance (EVMD), where a \( Q^2 \)-dependence of the effective coupling \( g_{\rho\pi\pi} \) is allowed, the electromagnetic pion form is given by

\[
F_\pi(q^2)|_{EVMD} = \frac{M_\rho^2}{M_\rho^2 + Q^2} \frac{g_{\rho\pi\pi}(Q^2)}{f_\rho} \tag{7}
\]

which, after substituting the result from the FESR leads us to

\[
F_\pi(q^2)|_{EVMD} = \frac{3}{8\pi^2} \frac{f_\pi^2}{M_\rho^2} Q^2 I(q^2) \tag{8}
\]

where \( I(q^2) \) was defined in Eq. (6). This result should be compared with a previous independent determination of \( F_\pi \) based on a three-point function involving the electromagnetic current and two axial currents, \( \gamma \), which projects the form factor directly, making no use of VMD, i.e.

\[
F_\pi(q^2) = \frac{1}{16\pi^2} \frac{1}{f_\pi^2} \frac{1}{(1 + Q^2/2s'_0)^2} \tag{9}
\]

It is remarkable that Eqs. (8) and (9), in spite of looking structurally very different, are numerically very similar for \( Q^2 \geq 0.5 \text{GeV}^2 \); if we take \( s_0 \simeq 2.18 \text{GeV}^2 \), and \( s'_0 \simeq 1 \text{GeV}^2 \). The latter values lead to a very good fit of the data above 1 GeV². It should be stressed, since the correlators are different, that the onset of the continuum need not be the same in both cases. All one knows is that \( s_0(s'_0) \) should roughly be in a region where the resonances lose prominence, and the hadronic continuum takes over. Somewhere between 1 – 3 GeV². The good numerical agreement between both results should be considered as a reflection of the validity of EVMD.

Our purpose here, however, is not another fit to the data at \( T = 0 \) but rather the thermal evolution of the \( \rho\pi\pi \) form factor. Therefore, let us reconsider the FESR at finite temperature. The thermal corrections to \( \Pi_1(Q^2)|_{QCD} \) can be calculated using the finite temperature fermion propagators. Since this is a one loop calculation we do not need the full machinery of Thermo Field Dynamics. For the imaginary part we find

\[
\text{Im} \Pi_1(s, s', Q^2, T) = \text{Im} \Pi_1(s, s', Q^2, 0) \\
\times F(s, s', Q^2, T) \tag{10}
\]
where

\[
F(s, s', Q^2) = 1 - n_1 - n_2 - n_3 + n_1 n_2 + n_1 n_3 + n_2 n_3
\]  
(11)

\[
n_1 \equiv n_2 \equiv n_F(\frac{1}{2T} \sqrt{\frac{x+y}{2}})
\]  
(12)

\[
n_3 \equiv n_F(\frac{Q^2 + (x-y)/2}{2T \sqrt{x+y/2}})
\]  
(13)

\(n_F(x) = (1 + e^x)^{-1}\) is the Fermi-Dirac factor and \(x = s + s', y = s - s'\). Concerning the hadronic side, both \(f_\pi\) and \(<qq>\) will develop a temperature dependence, and so will \(S_0\). We can safely ignore the small variations of \(M_\rho(T)\) from QCD sum rules [8] and other methods [9]. The temperature behavior of the asymptotic freedom threshold can be obtained from an independent FESR associated to a two-point function correlator involving axial-vector currents [10] [11]. Here \(f_\pi(T)\) is an input. It turns out that for temperatures not too close to the critical temperature \(T_c\), the following scaling relation holds to a very good approximation [12].

\[
\frac{f_\pi^2(T)}{f_\pi^2(0)} \sim \frac{<\overline{q}q>_T}{<\overline{q}q>} \sim \frac{s_0(T)}{s_0(0)}
\]  
(14)

We will make use of the results of [13] for \(f_\pi(T)\) and \(<\overline{q}q>_T\) in the chiral limit and also for \(m_q \neq 0\). In addition we invoke the GMOR relation at finite \(T\). It only gets modified at next to leading order in the quark masses [12]. For a different recent analysis of the same subject, with essentially the same numerical behavior see also [14].

The \(T \neq 0\) FESR now reads

\[
g_{\rho \pi \pi}(Q^2, T) = \frac{3}{8\pi^2} \frac{f_\pi^2(T) Q^2}{<\overline{q}q>_T^2 M_\rho^2} (Q^2 + M_\rho^2) \times I(q^2, T)
\]  
(15)

where now

\[
I(Q^2, T) = \frac{1}{8} \int_0^{s_0(T)} dx \int_{-x}^{x} dy \frac{(x^2 - y^2)}{Q^2 + 2xQ^2 + y^2}^{3/2} \times F(x, y, Q^2, T)
\]  
(16)

and the integration in Eq.(16) must be done numerically. In Fig.(1) we show the behavior of the ratio \(R_1 = g_{\rho \pi \pi}(Q^2, T)/g_{\rho \pi \pi}(Q^2, 0)\) for \(f_\pi(T)\) and \(<\overline{q}q>_T\) in the chiral limit (curve (a)) as well as for \(m_q \neq 0\) (curve(b)) for \(Q^2 = 1 GeV^2\). Higher values of \(Q^2\) give similar results.

Figure 1.

It is important to remark that the vanishing of \(R_1\) at or near the critical temperature is basically \(Q^2\) - independent, providing analytical evidence for deconfinement. The good agreement between the pion form factor using \(g_{\rho \pi \pi}(Q^2)\) plus VMD and that obtained without invoking VMD, persists at finite temperature.

Finally, an extrapolation to \(Q^2 = 0\) allows for a determination of the \(\rho \pi \pi\) root mean squared radius. The ratio \(<r^2>_T(T)/<r^2>_0(0)\) is well defined, in spite of the fact that \(<r^2>_T\) is divergent at any temperature due to mass singularities. It turns out that this ratio increases with increasing \(T\) until the critical temperature where it diverges,
as we could have expected from an intuitive point of view, thus signalling deconfinement. See \cite{13} for more details.

**Acknowledgements:** The work of (CAD) and (MSF) has been supported in part by the FRD (South Africa), and that of (ML) by Fondecyt (Chile) under grant No. 1950797, and Fundación Andes under grant No. C-12999/2.

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**Discussions**

**H. G. Dosch,** Inst. Theor. Phys., Heidelberg.

*Can you comment on the objections of B. Ioffe and coworkers that one should not use thermal quark propagators, but rather take into account the pionic heat bath?*

**M. Loewe**

*We have investigated the validity of the thermal OPE in the framework of two exactly solvable models: the two-dimensional $\sigma$-model $O(N)$, in the large $N$ limit, and the Schwinger model, showing that the thermal dependence of the perturbative part cannot be absorbed into the condensates. No confusion arises in these cases. This point strongly supports the thermal QCD sum rules program. It would be extremely bizarre if duality and the OPE suddenly would be no longer valid, as soon as we increase the temperature in a few millikelvins. Both descriptions are complementary instead of being contradictory. The pion basis is well suited to determine the temperature dependence of the vacuum condensates at low $T$. It does not make use of QCD-hadron duality. The quark-gluon basis, on the other hand, allows for an extension of the QCD sum rule program.*

**M. Neubert,** CERN

*The previous speaker (S. Mallik) has shown that for finite $T$ new additional condensates enter in the QCD sum rules. Would these effects have an impact on your analysis?*

**M. Loewe**

*In fact we have not used these new condensates. Remember that for FESR, because of the dimensions involved, not all possible new condensates will contribute to a particular FESR. However we have checked that the new condensates are negligible from the numerical point of view.*
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9709224v1