Local Position Invariance and Vacuum Energy Shifts

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Abstract

We discuss tests of the Einstein Equivalence Principle due to energies which are purely quantum mechanical in origin. In particular, we consider using Lamb Shift energies to test for possible quantum violations of Local Position Invariance.

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The Einstein Equivalence Principle (EEP) is foundational to our understanding of gravity. It states that (i) all test bodies fall with the same acceleration regardless of their composition (the weak equivalence principle, or WEP) and (ii) all nongravitational laws of physics take on their special relativistic form. Theories which obey the EEP, such as general relativity and Brans-Dicke Theory are called metric theories because they endow spacetime with a metric \( g_{\mu\nu} \) that couples universally to all non-gravitational fields. Non-metric theories do not have this feature: they break universality by coupling auxiliary gravitational fields directly to matter. In this context a violation of the EEP means the breakdown of either Local Position Invariance (LPI) or Local Lorentz Invariance (LLI), so that observers performing local experiments could detect effects due to their position (if LPI is violated) or their velocity (if LLI is violated) in an external gravitational environment by using clocks and rods of differing composition. Limits on LPI and LLI are set by gravitational red-shift and atomic physics experiments respectively \[1, 2\], each of which compares relative frequencies of transitions between particular energy levels that are sensitive to any potential LPI/LLI-violating effects. The dominant form of energy governing the transitions in these experiments is nuclear electrostatic energy, although violations of WEP/EEP due to other forms of energy have also been estimated \[3\].

Potential violations of the EEP due to effects which are purely quantum-mechanical in origin are not as well understood. We report here on the results of a calculation which investigates the effects Lamb-shift energies would have on violations of LPI in the context of a wide class of non-metric theories of gravity as described by the \( TH\epsilon\mu \) formalism \[4\]. The Lamb shift is the energy shift between the \( \text{2}\text{s}_{1/2} \) and \( \text{2}\text{p}_{1/2} \) states in a Hydrogen-like atom, and is entirely quantum-mechanical in origin, arising due to interactions of the electron with the quantum fluctuations of the electromagnetic field \[5\]. By considering the difference between inertial and passive gravitational masses \( m_P = m_I + \sum_A \eta_A E_A \) where \( \eta_A \) parametrizes the contribution due to the \( A \)-th kind of energy, we obtain a crude estimate of the magnitude of such potential EEP-violating effects to be

\[
\eta \equiv \frac{a_1 - a_2}{\frac{1}{2}(a_1 + a_2)} \approx 10^{-15}\eta_{\text{Lamb}} \tag{1}
\]

where \( \eta \) is the Eötvös ratio \[8\] and \( E_A \sim \frac{4\alpha \langle Z\alpha \rangle^4}{3\pi^2 n^3} m_e \ln\left(\frac{1}{Z\alpha}\right) \) for the Lamb shift. As a comparison, the expected final precision in the difference between
gravitational and inertial mass to be achieved in the STEP experiment is one part in $10^{17}$.

The $T \mu \epsilon$ formalism was constructed to study electromagnetically interacting particles in an external, static, spherically symmetric (SSS) gravitational field, whose metric tensor is given by $g_{00} = -T(U)$, $g_{ij} = H(U)\delta_{ij}$, (where $T$ and $H$ are arbitrary functions of the Newtonian gravitational potential $U = GM/r$) encompassing a wide class of non-metric (and all metric) gravitational theories. For the action of quantum electrodynamics (QED) with a spinor field $\Psi$ of rest mass $m$ and charge $e$ we have

$$I_{QED} = \int d^4x \sqrt{-g} \left[ \frac{i}{2} (\bar{\Psi} \gamma^a e^a_\mu \nabla_\mu \Psi - e^a_\mu (\nabla_\mu \bar{\Psi}) \gamma^a \Psi) - m \bar{\Psi} \Psi \right]$$

where $\nabla_\mu = \partial_\mu + \Gamma_\mu - ieA_\mu$ is the covariant derivative, $A_\mu$ the electromagnetic vector potential with field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $e^a_\mu$ is the tetrad associated with the metric $g^{\mu\nu} = \eta^{ab} e^a_\mu e^b_\nu$; $\Gamma_\mu$ is the spin connection. The $\gamma^a$ are the Dirac matrices, with $\gamma^a e^a_\mu B_\mu$.

Since we are interested in hydrogen-like atoms, we can ignore the spatial variation of $T$, $H$, $\epsilon$, and $\mu$ across the atom and evaluate each of them at the center of mass position $\mathbf{X} = 0$. In this context the $T \mu \epsilon$ formalism modifies the action (2) to

$$I'_{QED} = I_D + I_{EM}$$

where

$$I_D = \int d^4x' \bar{\Psi} (i \gamma' + e A' - m') \Psi$$

with $x'_\mu = (c_0 x_0, x)$, $A'_\mu = (A_0/c_0, \mathbf{A})$, and $m' = m_e H_0^{1/2}$ and where

$$I_{EM} = \frac{1}{2} \int (\epsilon_0 \mathbf{E}^2 - \frac{B^2}{\mu_0}) d^4x + I_{GF} = -\int d^4x' \left[ \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} (\tilde{\nabla} \cdot \tilde{A})^2 \right].$$

with $\tilde{F}_{\mu\nu} = \tilde{\partial}_\mu \tilde{A}_\nu - \tilde{\partial}_\nu \tilde{A}_\mu$, $\tilde{x}_\mu = (c_s x_0, \mathbf{x})$, $\tilde{A}_\mu = \sqrt{\epsilon_0 c_s} (A_0/c_s, \mathbf{A})$. Here $I_{GF}$ is a gauge fixing term necessary for quantization and

$$c_0 \equiv (T_0/H_0)^{1/2} \quad c_s \equiv (\mu_0 \epsilon_0)^{-1/2}$$

respectively representing the local limiting speed of massive test bodies and the local speed of photons. In metric theories (which satisfy the EEP) $\epsilon = \mu = (H/T)^{1/2}$, implying $c_0 = c_s$. 

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The energy shift of a state \( |n> \) can be expressed as

\[
\Delta E_n = \langle n | \Sigma + \Pi - \delta m | n >
\]  

(6)

where \( \Pi \) and \( \Sigma \) are the vacuum polarization and self-energy contributions, and \( \delta m \) is the counterterm for the corresponding process for a free electron. The computation of \( \Delta E_L = \text{Re}(\Delta E_n) \) is quite analogous to the usual metric case and, although quite laborious, is straightforward. Working to lowest order in \( \varepsilon = 1 - (\frac{Z}{c_0})^2 \) we find

\[
\Delta E_L = E_L \left( \frac{H_0^{1/2}}{c_0^2} \{ p_{n,l} + \bar{p}_{n,l} \} \right)
\]  

(7)

where

\[
p_{n,l} = \begin{cases} 
19/30 + \ln(c_0^2 \alpha / Z < E_{n,0} >) & l = 0 \\
3C_{j,l}/(2l + 1) + \ln(Z^2 \text{Ryd} / < E_{n,l} >) & l \neq 0
\end{cases}
\]  

(8)

\[
\bar{p}_{n,l} = -e_{n,l}/8 + 3p_{n,l}/2 + \frac{183}{80} \delta_{l,0}
\]  

(9)

and the reference energy \( < E_n > \) is defined as in the usual case \([8]\) by

\[
\ln < E_{n,0} > = \frac{\sum_r \langle r | \mathbf{p} | n > |^2 (p_0(r) - p_0(n)) \ln(|p_0(r) - p_0(n)| c_0 / \text{Ryd}^* )}{\sum_r \langle r | \mathbf{p} | n > |^2 (p_0(r) - p_0(n))}
\]  

(10)

\[
2 \left( \frac{Z \alpha}{c_0 \epsilon_0} \right)^4 m^3 \ln \left( \frac{Z^2 \text{Ryd}}{E_{n,l}} \right) = - \sum_r \langle r | \mathbf{p} | n > |^2 (p_0(r) - p_0(n)) \ln(c_0 |p_0(n) - p_0(r)|)
\]  

(11)

with \( E_L = \frac{4 \alpha (Z \alpha)^4 m_e}{3}\pi \), \( e_{n,l} = a^3_0 < n | \frac{\mathbf{p} \cdot \nabla}{|\mathbf{p}|^3} | n > \), \( a^3_0 = \frac{n^3}{H_0^{3/2} (Z \alpha m_e)^{-1}} \) and \( \text{Ryd}^* = \frac{H_0^{1/2}}{c_0 \alpha} \text{Ryd} \) being the Rydberg energy.

The physical implications of (7) are most easily parametrized in terms of anomalous mass tensors \([3]\). A breakdown of the EEP is signalled by the position and velocity dependance of the binding energy \( E_B \) of a body; this has the general form

\[
E_B(\mathbf{X}, \mathbf{V}) = E_B^0 + \delta m_{ij} U_i U_j - \frac{1}{2} \delta m_{ij} V^i V^j + \ldots
\]  

(12)
where $\mathbf{X}$ and $\mathbf{V}$ are quasi-Newtonian coordinates and velocity of the center of mass of the body. Here $\delta m_P^{ij}$ and $\delta m_I^{ij}$ are the anomalous passive gravitational and inertial mass tensors respectively, whose form depends upon the detailed internal structure of the composite body. Here $U^{ij}$ is the external gravitational potential tensor, which satisfies $U^{ii} = U$.

Violations of LPI occur whenever $\delta m_P^{ij} \neq 0$. Taking the composite body to be a hydrogen-like atom, from (7) we have calculated this to be

$$\delta m_P^{ij(L)} = \delta^{ij} \frac{E_B}{c_0^2} (A_{n,l} \Gamma_0 + B_{n,l} \Lambda_0) \equiv \delta^{ij} \alpha_{\text{Lamb}}$$

(13)

and lead to a redshift $\Delta z = (1 + \alpha_{\text{Lamb}}) \Delta U$. Here

$$A_{n,l} = 5 - B_{n,l} - 2\delta_{l,0}(1 + 3\varepsilon / 2p_{n,l}) \quad B_{n,l} = r_{n,l}(1 - \varepsilon(1 + r_{n,l}))$$

(14)

and $r_{n,l} = \tilde{p}_{n,l} / p_{n,l}$ and $E_B^L = - \frac{\varepsilon_{l,0}}{(\cos \alpha)^5} (p_{n,l} + \varepsilon \tilde{p}_{n,l})$ and the quantities $\Gamma_0$ and $\Lambda_0$ are

$$\Gamma_0 = \frac{2T_0}{T_0} (\frac{e_0'}{e_0} + \frac{T_0'}{2T_0} - \frac{H_0'}{H_0}) \quad \Lambda_0 = \frac{2T_0}{T_0} (\frac{\mu_0'}{\mu_0} + \frac{T_0'}{2T_0} - \frac{H_0'}{H_0})$$

(15)

It is an experimental challenge to design a gravitational redshift experiment using a clock sensitive to Lamb shift transition frequencies. Such an experiment would provide new observational constraints on the allowed regions of $\Gamma_0 - \Lambda_0$ parameter space, enhancing our understanding of the foundations of gravitational theory.

A computation of $\delta m_I^{ij}$ is in progress and will be reported upon in the near future.

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