The Production of Gluino Pairs in High Energy $e^+e^-$ Collisions

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Abstract

In the Minimal Supersymmetric Standard Model (MSSM), the process $e^+e^- \to \tilde{g}\tilde{g}$ is mediated by quark/squark loops, dominantly of the third generation, where the mixing of left- and right-handed states can become large. Taking into account realistic beam polarization effects, photon and $Z^0$-boson exchange, and current mass exclusion limits, we scan the MSSM parameter space for various $e^+e^-$ center-of-mass energies to determine the regions, where gluino production should be visible.
1 Introduction

In the Minimal Supersymmetric Standard Model (MSSM) [1, 2], the exclusive production of gluino pairs in $e^+e^-$ annihilation is mediated by $s$-channel photons and $Z^0$-bosons, which couple to the gluinos via triangular quark and squark loops. In this Talk, we report about a recent study [3] of the potential of high-energy linear $e^+e^-$ colliders for the production of gluino pairs within the MSSM. Similar studies have been performed earlier for the production of sfermion pairs [4, 5]. Taking into account realistic beam polarization effects, photon and $Z^0$-boson exchange, and current mass exclusion limits, we scan the MSSM parameter space for various $e^+e^-$ center-of-mass energies to determine the regions, where gluino production should be visible. Furthermore we clarify the theoretical questions of the relative sign between the two contributing triangular Feynman diagrams, of the possible presence of an axial vector anomaly, and the conditions for vanishing cross sections – three related issues, which have so far been under debate in the literature.

2 Results

The scattering process

\[ e^- (p_1, \lambda_1) \ e^+ (p_2, \lambda_2) \rightarrow \tilde{g}(k_1) \ \tilde{g}(k_2) \]  

with incoming electron/positron momenta $p_{1,2}$ and helicities $\lambda_{1,2}$ and outgoing gluino momenta $k_{1,2}$ proceeds through the two Feynman diagrams A and B in Fig. 1 with $s$-channel photon and $Z^0$-boson exchange and triangular quark and squark loops. Higgs boson exchange is not considered due to the negligibly small electron Yukawa coupling, but it could well be relevant at muon colliders. The process occurs only at the one-loop level, since the gluino as the superpartner of the gauge boson of the strong interaction couples neither directly to leptons nor to electroweak gauge bosons. Our analytical results are presented in Ref. [3] in full detail. Taking into account chiral squark mixing, we find that pairs of identical (Majorana) gluinos are produced by a parity violating axial vector coupling induced by mass differences between the chiral squarks and the axial vector coupling of the $Z^0$-boson. The (mass-independent) ultraviolet singularities contained in the loop integrals cancel among the two contributing Feynman diagrams. As we have checked explicitly (even for complex squark mixing matrices), adding the two amplitudes induces not only a cancellation of the ultraviolet singularities and of the logarithmic dependence on the scale parameter, but also a destructive interference of the finite remainders. This happens separately for each weak isospin partner. Our analytical results have been obtained in two independent analytical calculations and have been implemented in a compact Fortran computer code. As a third independent cross-check, we have recalculated the production of gluino pairs in $e^+e^-$ annihilation with the computer algebra program FeynArts/FormCalc [6] and found numerical agreement up to 15 digits.

Our calculations involve various masses and couplings of SM particles, for which we use the most up-to-date values from the 2002 Review of the Particle Data Group [7]. In particular, we evaluate the electromagnetic fine structure constant $\alpha(m_Z) = 1/127.934$ at the mass of the $Z^0$-boson, $m_Z = 91.1876$ GeV, and calculate the weak mixing angle $\theta_W$ from the tree-level expression $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$ with $m_W = 80.423$ GeV. Among the
Fermion masses, only the one of the top quark, $m_t = 174.3$ GeV, plays a significant role due to its large splitting from the bottom quark mass, $m_b = 4.7$ GeV, while the latter and the charm quark mass, $m_c = 1.5$ GeV, could have been neglected like those of the three light quarks and of the electron/positron. The strong coupling constant is evaluated at the gluino mass scale from the one-loop expression with five active flavors and $\Lambda_{LO}^{n_f=5} = 83.76$ MeV, corresponding to $\alpha_s(m_Z) = 0.1172$. A variation of the renormalization scale by a factor of four about the gluino mass results in a cross section uncertainty of about $\pm 25\%$. Like the heavy top quark, all SUSY particles have been decoupled from the running of the strong coupling constant.

We work in the framework of the MSSM with conserved $R$-(matter-) parity, which represents the simplest phenomenologically viable model, but which is still sufficiently general to not depend on a specific SUSY breaking mechanism. Models with broken $R$-parity are severely restricted by the non-observation of proton decay, which would violate both baryon and lepton number conservation. We do not consider light gluino mass windows, on which the literature has focused so far and which may or may not be excluded from searches at fixed target and collider experiments [7]. Instead, we adopt the current mass limit $m_{\tilde{g}} \geq 200$ GeV from the CDF [8] and D0 [9] searches in the jets with missing energy channel, relevant for non-mixing squark masses of $m_{\tilde{q}} \geq 325$ GeV and $\tan \beta = 3$. Values for the ratio of the Higgs vacuum expectation values, $\tan \beta$, below 2.4 are already excluded by the CERN LEP experiments, although this value is obtained using one-loop corrections only and depends in addition on the top quark mass [10]. In order to delimit the regions of large gluino cross sections, we have scanned the MSSM parameter space over $\tan \beta \in [1.6; 50]$, the Higgs mass parameter $\mu \in [-2; 2]$ TeV, the trilinear coupling $A_q \in [-6; 6]$ TeV, and the squark mass parameter $m_{SUSY} \in [200; 2000]$ GeV. We found that the cross section is visible only for parameter choices resulting in large squark mass splittings, specified below, and that its sensitivity to individual parameter choices is small.

If not stated otherwise, we will present unpolarized cross sections for a $\sqrt{s} = 500$ GeV linear $e^+e^-$ collider like DESY TESLA, gluino masses of $m_{\tilde{g}} = 200$ GeV, and squark masses $m_{\tilde{q}} \approx m_{\tilde{Q}} = m_{\tilde{B}} = m_{\tilde{U}} = m_{\tilde{C}} = m_{\tilde{D}} = m_{\tilde{T}} = m_{SUSY} = 325$ GeV. We will
Figure 2: Dependence of the photon and $Z^0$-boson contributions to the process $e^+e^- \rightarrow \tilde{g}\tilde{g}$ on the right-handed top squark mass parameter $m_{\tilde{T}}$. The photon contribution (dashed curve) is dominated by top (s)quarks and cancels for $m_{\tilde{t}_L} = m_{\tilde{t}_R}$.

consider two cases of large squark mass splittings: I.) On the one hand, the masses of the superpartners of left- and right-handed quarks need not be equal to each other. In this scenario we will vary the right-handed up-type squark mass parameters $m_{\tilde{u}, \tilde{c}, \tilde{t}}$ between 200 and 1500 GeV. II.) On the other hand, the superpartners of the heavy quarks can mix into light and heavy mass eigenstates. This alternative is restricted by the CERN LEP limits on the light top and bottom squark masses, $m_{\tilde{t}_1} \geq 100$ GeV and $m_{\tilde{b}_1} \geq 99$ GeV \[11\], and on SUSY one-loop contributions \[12, 13, 14\] to the $\rho$-parameter, $\rho_{\text{SUSY}} < 0.0012$ \[7\]. In this case we assume the maximally allowed top squark mixing with $\theta_{\tilde{t}} = 45.2^\circ$, $m_{\tilde{t}_1} = 110$ GeV, and $m_{\tilde{t}_2} = 506$ GeV, which can be generated by choosing appropriate values for the Higgs mass parameter, $\mu = -500$ GeV, and the trilinear top squark coupling, $A_t = 534$ GeV. For small values of $\tan \beta$, mixing in the bottom squark sector remains small, and we take $\theta_{\tilde{b}} = 0^\circ$. Although the absolute magnitude of the cross section depends strongly on the gluino mass and collider energy, the relative importance of the different contributions is very similar also for higher gluino masses and collider energies.

First we examine the conditions found in Ref. \[3\] for vanishing of the photon and $Z^0$-boson contributions, restricting ourselves to the third generation. Since we expect the photon contribution to cancel for equal left- and right-handed squark masses, we vary the right-handed top squark mass parameter, $m_{\tilde{T}} \simeq m_{\tilde{t}_R}$, between 200 and 1500 GeV, but keep $m_{\tilde{t}_L} \simeq m_{\tilde{Q}} = m_{\tilde{B}} = m_{\text{SUSY}} = 325$ GeV fixed (case I), since top and bottom squarks generally interfere destructively due to their opposite charge and weak isospin quantum numbers. As can be seen from Fig. \[2\], the photon contribution cancels
Mixing Angle Dependence of $t\bar{t}$ and $b\bar{b}$ Loop Contributions

Figure 3: Mixing angle dependence of the $\tilde{t}$ (dashed) and $\tilde{b}$ (dotted) loop contributions to the process $e^+e^- \rightarrow \tilde{g}\tilde{g}$, which interfere destructively (full curve), except for $\theta_{\tilde{t}} \approx 45.2^\circ$, where the imaginary parts of the amplitudes interfere constructively. Mixing in the $\tilde{b}$ sector (dot-dashed curve) enhances the cross section only slightly.

indeed for $m_{\tilde{t}} \approx m_{\tilde{t}_R} = m_{\tilde{t}_L} \approx m_{\text{SUSY}}$, i.e. for all flavors $q$ with equal squark masses. Due to their charge, top (s)quarks contribute four times as much as bottom (s)quarks, whose contribution is even more suppressed by the condition $m_{\tilde{b}_L} \approx m_{\tilde{b}_R}$. The $Z^0$-boson contribution can never cancel, since $m_t \gg m_b$, and therefore it depends only weakly on $m_{\tilde{t}}$, but it can become minimal for $m_{\tilde{t}} \approx m_{\tilde{t}_R} = m_{\tilde{t}_L} = m_{\tilde{b}_R} = m_{\tilde{b}_L} \approx m_{\text{SUSY}}$. As $m_{\tilde{t}}$ gets significantly larger (or smaller) than $m_{\text{SUSY}}$, the photon contribution starts to dominate over the $Z^0$-boson contribution. If only $m_{\tilde{t}}$ differs from $m_{\text{SUSY}}$, the third generation contributes almost 100% to the total cross section. However, if $m_{\tilde{U}} = m_{\tilde{C}} = m_{\tilde{t}}$ are varied simultaneously, all three generations contribute to the total cross section, which can therefore become significantly larger.

When $m_{\tilde{U}} = m_{\tilde{C}} = m_{\tilde{t}} = m_{\text{SUSY}}$ and large mass splittings are generated only by mixing in the top squark sector (case II), photon contributions are suppressed by more than two orders of magnitude. Fig. 3 shows that the $Z^0$-boson contributions from top and bottom squarks interfere destructively due to opposite values of their weak isospin quantum numbers, except for $\theta_{\tilde{t}} \approx 45.2^\circ$, where the imaginary parts of the amplitudes interfere constructively. It is therefore advantageous to keep the bottom squark mass splitting small. As is also evident from Fig. 3, mixing in the bottom squark sector is of little importance. Note that the central region with maximal top/bottom squark mixing is excluded by the CERN LEP limits on $m_{\tilde{t}_1}, m_{\tilde{b}_1}$, and the $\rho$-parameter. When $m_{\text{SUSY}}$ and the diagonal elements of the squark mixing matrix become much larger than the quark
masses and the off-diagonal elements of the matrix, the role of squark mixing is reduced, as expected. Squarks from the first two generations contribute at most 10% at low $m_{\text{SUSY}}$ and are otherwise strongly suppressed.

At future linear $e^+e^-$ colliders it will be possible to obtain relatively high degrees of polarization, i.e. about 80% for electrons and 60% for positrons. For these degrees of polarization, we show in Fig. 4 a scan in the center-of-mass energy of a future $e^+e^-$ collider for various gluino masses and maximal top squark mixing (case II). The cross section rises rather slowly due to the factor $\beta^3$ for $P$-wave production of the gluino pairs. For $m_{\tilde{g}} = 200$ GeV we observe an interesting second maximum, which arises from the intermediate squark pair resonance at $\sqrt{s} = 2m_{\text{SUSY}} = 650$ GeV. At threshold, the cross section depends strongly on the gluino mass and is largest for $m_{\tilde{g}} = 200$ GeV, which we consider to be the lowest experimentally allowed value. It drops fast with $m_{\tilde{g}}$, so that for $m_{\tilde{g}} > 500$ GeV no events at colliders with luminosities of 1000 fb$^{-1}$ per year can be expected, irrespective of their energy. Smaller squark mixing (cf. Fig. 3) or larger values of $m_{\text{SUSY}}$ will reduce the cross section even further. Far above threshold, it drops off like $1/\sqrt{s}$ and becomes independent of the gluino mass.

The slow rise of the cross section can be observed even better in Fig. 5, where the sensitivity of a $\sqrt{s} = 500$ GeV collider like DESY TESLA to gluino masses around 200 GeV has been plotted. For the CERN LHC experiments, a precision of $\pm 30 \ldots 60$ (12 \ldots 25) GeV is expected for gluino masses of 540 (1004) GeV. If the masses and mixing angle(s) of the top (and bottom) squarks are known, a precision of $\pm 5 \ldots 10$ GeV can be achieved at DESY TESLA for $m_{\tilde{g}} = 200$ GeV and maximal top squark mixing with an integrated
As has already been mentioned above, the Majorana nature of the gluino leads to a vanishing forward-backward asymmetry. In order to establish this feature experimentally, the asymmetry has to be measured with an accuracy of at least 7–10%, which are the values relevant for Dirac fermions such as up- (charm) and down-type (strange/bottom) quarks [7]. As a consequence, the production cross section must be known with about the same precision. In view of the results obtained in Fig. 5, such a measurement appears to be extremely difficult if not impossible.

3 Conclusions

In this Talk, we have reported on a recent study of gluino pair production at future $e^+e^-$ colliders [3]. In this study, we have resolved a long-standing discrepancy in the literature about the relative sign of the quark and squark loop contributions to the production of gluino pairs in $e^+e^-$ annihilation. We have found that the ultraviolet divergence cancels for each squark flavor separately and not between weak isospin partners. Our results rely on two completely independent analytical calculations and one computer algebra calculation. Furthermore, we have investigated the prospects for precision measurements of gluino properties, such as its mass or its Majorana fermion nature, at future linear
$e^+e^-$ colliders. We have taken into account realistic beam polarization effects, photon and $Z^0$-boson exchange, and current mass exclusion limits. Previously, only light gluinos at center-of-mass energies up to the $Z^0$-boson mass had been investigated. Within the general framework of the MSSM, we have concentrated on two scenarios of large left-/right-handed up-type squark mass splitting and large top squark mixing, which produce promisingly large cross sections for gluino masses up to 500 GeV or even 1 TeV. Gluino masses of 200 GeV can then be measured with a precision of about 5 GeV in center-of-mass energy scans with luminosities of 100 fb$^{-1}$/point. However, when both the left-/right-handed squark mass splitting and the squark mixing remain small, gluino pair production in $e^+e^-$ annihilation will be hard to observe, even with luminosities of 1000 fb$^{-1}$/year.

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