Model-independent inference of laser intensity

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An ultrarelativistic electron beam passing through an intense laser pulse emits radiation around its direction of propagation into a characteristic angular profile. Here we show that measurement of the variances of this profile in the planes parallel and perpendicular to the laser polarization, and the mean initial and final energies of the electron beam, allows the intensity of the laser pulse to be inferred in a way that is independent of the model of the electron dynamics. The method presented applies whether radiation reaction is important or not, and whether it is classical or quantum in nature, with accuracy of a few per cent across three orders of magnitude in intensity. It is tolerant of electron beams with broad energy spread and finite divergence. In laser-electron beam collision experiments, where spatiotemporal fluctuations cause alignment of the beams to vary from shot to shot, this permits inference of the laser intensity at the collision point, thereby facilitating comparisons between theoretical calculations and experimental data.

I. INTRODUCTION

Electromagnetic fields of extraordinary strength are produced at the focus of modern high-power lasers [1], inducing nonlinear classical, even quantum, dynamics of particles and plasmas [2–4]. The amplitude of these fields, expressed covariantly through the normalized vector potential $a_0$, plays an essential role in determining which regime is explored. However, it remains difficult to diagnose in situ the intensity reached in experiments. This is particularly acute for experiments at or beyond the current intensity frontier, which will explore the transition to the nonlinear quantum regime [5, 6]. As the dynamics are not fully understood, it is important to know what $a_0$ is reached for comparison between competing theoretical descriptions and experimental data. Furthermore, the method by which $a_0$ is determined should not be sensitive to the underlying physics, particularly if the latter is the subject of the experiment.

The method presented here is based on the collision of an ultrarelativistic electron beam with the laser pulse; this geometry has been already been exploited as a high-energy photon source [7–9] and a probe of radiation reaction [10, 11]. Measurement of the angular profile of the resulting radiation has been proposed as a means of determining the peak intensity of a laser pulse [12, 13] (the former demonstrated experimentally in [9]), as has measurement of the electron scattering angles [14]. However, the results presented in [12–14] depend critically on the model assumed for the electron dynamics. The appropriate choice of model depends on the intensity of the laser to be probed: for example, at very high intensity, radiation reaction and quantum effects are expected to become important, if not dominant [4]. A method that does not require such an assumption to be made would be a useful complement to methods that are model-dependent, providing stronger evidence that a particular regime has been reached.

Here we show that the laser intensity can be inferred in a model-independent way, using the angular profile of the radiation emitted in the collision of the laser with an electron beam, in combination with the mean initial and final energies of the beam. We derive analytical predictions for the size of the radiation profile and the energy change of the electron beam, treating the laser as a pulsed plane electromagnetic wave, that can be combined so as to eliminate an explicit dependence on classical radiation reaction. We show that this model-independence applies to a high degree of accuracy under quantum models of radiation reaction as well.

Examining our method in a more realistic scenario, where the tight focussing of the laser and finite size of the electron beam are taken into account, we find that it yields a model-independent estimate of the laser intensity at the collision point, averaged over the electron-beam size. This is complementary to methods aimed at determining the peak intensity itself, by measurement of the ionization level of heavy atoms [15], Thomson scattering of low-energy electrons present in the focal volume [16], or by detailed characterization of the laser structure, gathered over hundreds of shots [17]. In conjunction with these, our method provides a means of determining the shot-to-shot overlap between laser pulse and electron beam.

II. ANALYTICAL RESULTS

Consider an electron (charge $-e$ and mass $m$) with Lorentz factor $\gamma \gg a_0$ oscillating in a linearly polarized, plane electromagnetic wave that has normalized amplitude $a_0$ and frequency $\omega_0$. Over a single cycle of the wave, the angle between the electron momentum and the laser axis is $\theta(\phi) = a_0 \sin \phi/\gamma$, and the electron’s quantum parameter $\chi(\phi) = 2\gamma a_0 \omega_0 |\cos \phi|/m$. Here $\phi$ is the phase

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and the angle $\theta$ lies in the plane defined by the laser’s electric field and wavevector; we refer to angles in this plane as being parallel ($\parallel$) to the laser polarization. We work throughout in natural units where $\hbar = c = 1$. The distribution of energy radiated per unit angle $d\mathcal{E}_{\text{rad}}/d\theta$ may be calculated by integrating the Larmor power, which is proportional to $\chi^2(\phi)$, over the cycle and assuming that the radiation is strongly beamed along the electron’s instantaneous momentum, i.e. at phase $\phi$ the emission angle is $\theta(\phi)$. We then normalize the result by the total radiated energy to obtain $d\mathcal{E}_{\text{rad}}/d\theta = 2\gamma\sqrt{1-(\gamma\theta/a_0)^2}/(\pi a_0)$ for $|\theta| < a_0/\gamma$. The variance of the distribution is $\sigma^2_{\parallel} = \int \theta^2 \, d\mathcal{E}_{\text{rad}} = a_0^2/(4\gamma^2)$.

To incorporate a pulse envelope $f(\phi)$ and the effect of radiation reaction into this result, we use the fact that the variances introduced each cycle may be added linearly. The contribution of each cycle of the pulse to the total variance is $a_0^2 \int f^2(\phi)/[4\gamma^2(\phi)]$, weighted by $\gamma^2(\phi)f^2(\phi)$. The weighting comes from the Larmor power, which is proportional to the square of the instantaneous Lorentz factor $\gamma(\phi)$ and the local electric field, which is proportional to $f(\phi)$. $\gamma(\phi)$ is obtained by solution of the Landau-Lifshitz equation [18], which accounts for the self-consistent radiative energy loss.

The total variance, in the direction parallel to the laser polarization, can be expressed compactly in terms of the final Lorentz factor $\gamma_f$ and integrals over the pulse envelope:

$$\sigma^2_{\parallel} = \frac{a_0^2}{4\gamma_f\gamma_i} \int \int f^4(\phi) \, d\phi + \sigma^2_{\perp}. \tag{1}$$

The second term in eq. (1) accounts for the contributions of the intrinsic divergence of the radiation and any initial divergence of the beam electrons, which we assume to be cylindrically symmetric. These are the only contributions in the direction perpendicular to the laser polarization and wavevector [19]:

$$\sigma^2_{\perp} = \frac{5}{8\gamma_f\gamma_i} + \delta^2. \tag{2}$$

In both eqs. (1) and (2), we have [20]

$$\gamma_f = \frac{\gamma_i}{1 + R\gamma_i}, \quad R = \frac{2\alpha a_0^2\omega_0}{3m} \int_0^{\infty} f^2(\phi) \, d\phi, \tag{3}$$

where $\alpha = e^2/(4\pi)$ is the fine-structure constant.

In the absence of radiation reaction, $\alpha = 0$ and $\gamma_i = \gamma_f$. If the intensity profile $f^2(\phi)$ is a Gaussian with full-width-at-half-maximum (FWHM) duration $\tau$, we have $\int f^2(\phi) \, d\phi = \omega_0 \tau \sqrt{\pi/(4\ln 2)}$ and $\int f^4(\phi) \, d\phi = [\int f^2(\phi) \, d\phi]^2/\sqrt{2}$. Notice that the radiation profile is elongated along the polarization direction; strictly, this result is valid for $a_0 \gtrsim 1$, as we have in this work, otherwise the profile would be dipolar in shape [21].

A comparison of eqs. (1) and (2) to the results of simulations is shown in fig. 1. In these, the plane-wave laser pulse has a Gaussian temporal envelope with FWHM duration $\tau = 40$ fs and a wavelength of 0.8 μm. The energies of the beam electrons are normally distributed, with a mean of 500 or 1000 MeV and standard deviation 50 MeV in both cases; the initial divergence is $\delta = 2$ mrad. Three models of the dynamics are considered: no radiation reaction (RR), i.e. the Lorentz force only; classical RR in the Landau-Lifshitz prescription; and a fully stochastic, quantum model using probability rates calculated in the locally constant field approximation [22].

Radiation is generated by Monte Carlo sampling of the classical synchrotron spectrum in the former two cases and the quantum synchrotron spectrum in the latter, as is appropriate. This method is applicable at high intensity and at high harmonic order in both the classical [23, 24] and quantum regimes [25, 26], where the photon formation length becomes much smaller than the wavelength of the driving laser [22]. Such photons dominate the power spectrum, which is the object of analysis in this work, even for relatively low $a_0$ [27]. We use a synchrotron spectrum that is differential in both energy and angle in our particle-tracking code, thereby resolving the intrinsic divergence of the radiation around the electron’s instantaneous velocity vector [19, 28]. (Full details are given in the Supplemental Material.)

We find excellent agreement between the analytical predictions and the simulation results for classical and no RR. It is clear that that radiation reaction leads to increased broadening of the angular profile in the plane of polarization. Furthermore, the classical and quantum models give generally similar results, even though the broadening is smaller for the latter because the radiated power is reduced. In the plane perpendicular to the laser polarization, the variance is dominated by the initial divergence of the electron beam and radiation reaction effects are weaker.
The effect of a finite energy spread on the variances is surprisingly small. Consider a beam of electrons, in which the initial Lorentz factors $\gamma_i$ are distributed as $dN_i/d\gamma_i$. The variance of the radiation angular profile, in the direction parallel to the laser polarization, for the beam as a whole, is obtained by integrating eq. (1), weighted by $(\gamma_i - \gamma_f) dN_i/d\gamma_i$, over all $\gamma_i \gg 1$. The weighting reflects the fact that electrons with higher $\gamma_i$ radiate more energy and therefore contribute more to $\sigma_{\|}^2$. We obtain

$$\sigma_{\|}^2 = \frac{a_0^2}{4\sqrt{2}} \left[ \langle \gamma_i \rangle \langle \gamma_f \rangle + \text{cov}(\gamma_i, \gamma_f) \right]^{-1} + \sigma_{\perp}^2,$$  \hspace{1cm} (4)

where $\langle \gamma_{i,f} \rangle$ are the mean initial and final Lorentz factors. The covariance term $\text{cov}(\gamma_i, \gamma_f) \leq \Delta_i \Delta_f \leq \Delta_i^2$, where the $\Delta_{i,f}$ are the standard deviations of $\gamma_{i,f}$; the equality holds when radiation reaction is neglected. Even if $\Delta_i$ is as large as $\langle \gamma_i \rangle / 3$, i.e. the beam has close to 80% energy spread (full width at half max), the change in $\sigma_{\|}$ is at most 5% and we can safely neglect any effect of the initial energy spread. As an example, we show in fig. 2 that $\sigma_{\|}$ and $\sigma_{\perp}$ are unchanged when the energy spread is increased from 0 to 0.2 of the mean initial energy.

III. INTENSITY INFERENCE

A. Plane waves

We now show that the angular profile and eq. (4) can be used to obtain the laser intensity, i.e. the collision $a_0$, in a model-independent way. The key points are that the analytical result is given in terms of the mean initial and final energies and that the covariance term is negligible. Even though eq. (4) is derived assuming classical radiation reaction, it is exactly true under no RR (in which case $\langle \gamma_i \rangle = \langle \gamma_f \rangle$) and it holds true to a high degree of accuracy under the quantum model.

This being the case, $a_0$ is fixed by the mean initial and final electron energies, which can be measured; any explicit dependence on radiation reaction effects is absorbed into the latter quantity. To demonstrate this, we determine what $a_0$ must have been for a set of simulations, given the $\sigma_{\|}, \sigma_{\perp}$, and the mean initial and final electron energies obtained from those simulations [29], using:

$$a_0^2 = 4\sqrt{2} \langle \gamma_i \rangle \langle \gamma_f \rangle (\sigma_{\|}^2 - \sigma_{\perp}^2).$$  \hspace{1cm} (5)

We vary the energy spectra of the electron beams, their initial divergence, the laser intensity and duration, and the model of RR used to calculate the dynamics. The laser pulse is modelled as a plane EM wave. In addition to the three models introduced earlier, we also consider a modified classical model, in which radiation losses are continuous, but the strength of the Landau-Lifshitz force is reduced by the Gaunt factor $g(\chi) \leq 1$ [30], and the photon energies are sampled from the quantum synchrotron spectrum (see Supplemental Material). This has been used to describe recent experimental results [11]. It ensures that the power spectrum is quantum-corrected, but neglects stochastic effects. The inferred $a_0$ is plotted against the true $a_0$ in fig. 3.

We find that using eq. (5) to infer the laser $a_0$ is accu-
rate to within a few per cent across the range of parameters explored, whether radiation reaction is classical or quantum in nature, or absent. If no radiation reaction, or classical RR, is used in the simulations, the agreement is near perfect; when a quantum-corrected model is used instead, the error grows with increasing electron beam energy and $a_0$. This suggests that, while $\gamma$-dependent corrections can be made to eqs. (1) and (2), most of their effect is encapsulated in the dependence on the final energy $\gamma_f$. For example, in a collision between an electron beam with mean energy $500$ MeV (standard deviation $5$ MeV, orange points in fig. 3) and a laser pulse with $a_0 = 100$, going from the classical to the quantum model increases the mean final energy by a factor of $1.58$, but decreases the parallel variance $\sigma_F^2$ by a factor of $1.43$. Therefore the inferred $a_0 = 105$ is close to the actual $a_0 = 100$, which is the classical result.

For $a_0 > 50$, it is advisable to use electron beams of lower energy to minimize the importance of quantum corrections to eq. (5): reducing the mean initial energy to $200$ MeV (red points in fig. 3) from $1000$ MeV (green points), reduces the error in the inferred $a_0$ by more than a factor of two. The difficulty associated with doing so is that a detector with larger acceptance angle is required to capture the whole radiation profile, which has characteristic size $\propto a_0/(\gamma_f \gamma_f)^{1/2}$. Our simulation results indicate that capturing all photons with $\theta_{\text{max}} < 2a_0/\gamma_f$ is necessary for accurate determination of $\sigma_F^2$. This angle is almost independent of $\gamma_f$ at high $a_0$ due to radiative losses, where it grows as $a_0^2$. These radiative losses also mean that we do not necessarily have $\gamma > a_0$ throughout the laser pulse, as assumed in our earlier derivation. As such, we expect the method presented here to be limited by the reflection threshold $\gamma_f \simeq a_0$, above which the re-acceleration of the decelerated electrons by the laser pulse becomes significant [31, 32]. In fact, quantum effects intercede before this is reached. Nevertheless, the region $5 \lesssim a_0 \lesssim 150$ shown in fig. 3 is relevant for a wide variety of laser-electron scattering experiments at existing, and planned, high-intensity laser facilities.

B. Focussed fields

We now consider the application of our results in a more realistic configuration, where we take into account the spatiotemporal structure of a tightly focussed laser pulse and the finite size of the electron beam. It is clear that, ideally, the electron beam should be much smaller than the laser focal spot, in the same way that any probe must be smaller than the system to be probed. If this is not the case, the inferred $a_0$ will be smaller than the true $a_0$, as the radiation profile will have been averaged over the spatial profile of the electron beam. This is still a useful quantity, as it represents an $a_0$ that is characteristic of the collision as a whole. Indeed, we show that if a transverse offset is introduced between electron beam and laser pulse, the reduction in the inferred $a_0$ allows the imperfect overlap to be identified.

Consider the collision of an electron beam with a laser pulse that has wavelength $0.8$ μm and FWHM duration $30$ fs, which is focussed to a spot size of $w_0 = 2$ μm, where $w_0$ is the radius at which the intensity falls to $1/e^2 \simeq 0.14$ of its peak value. The electron beam has a cylindrically symmetric, Gaussian charge distribution of radius $r_b = 0.5$ μm and length $\ell_b = 5.0$ μm; it has mean energy $750$ MeV (standard deviation $100$ MeV), rms divergence $3$ mrad, and is offset from the laser axis by a perpendicular distance $x_b$. The angular distributions of the emitted photons and electron energy spectra for this configuration are shown in fig. 4, for $a_0 = 30$ and assuming quantum radiation reaction. The inferred $a_0$ are $a_0^{\text{inf}} = 28.2$ and $16.7$ for $x_b = 0$ and $w_0$ respectively. The former is consistent with the true value of $a_0$; the reduction in the latter case is evidence that electron beam has not interacted with the most intense part of the laser pulse. Notice that the average energy loss of the electrons and the angular profile of the radiation are both reduced in size.

We can estimate the reduction due to finite-size effects as follows. An individual electron of the beam, with transverse displacement $x, y$, encounters a peak normalized laser amplitude $a(x, y) = a_0 \exp[\frac{-((x^2 + y^2))/w_0^2}].$ As the radiation profile is an integrated signal, the $a_0$ inferred from it is $a_0^{\text{inf}} = \sqrt{(a^4)/(\alpha^2)}$ to lowest order in $\alpha$, where the average is taken over the distribution of $x, y$. Then

$$a_0^{\text{inf}} = a_0 \sqrt{\frac{P}{Q}} \exp[\frac{-\xi^2}{(PQ)}],$$

where $P = 1 + 4\rho^2$, $Q = 1 + 8\rho^2$, $\rho = r_b/w_0$ and

![FIG. 4. (a) Energy radiated per unit solid angle by an electron beam in a collision with a laser pulse that has peak $a_0 = 30$. White arrows indicate the standard deviations of the distributions. (b) Energy spectra of the electrons before the collision (grey, dashed) and after, when the beam offset from the laser axis is $x_b = 0$ (blue) and $x_b = w_0$ (orange). Arrows indicate the mean energy.](image-url)
\[ \xi = x_b / w_0. \] First we confirm that eq. (5) provides a model-independent prediction of the laser \( a_0 \) by repeating the simulations shown in fig. 4 for different models of radiation reaction: fig. 5 shows that the inferred \( a_0 \) is consistent across all four models tested, at a peak \( a_0 < 150 \). We also find that the reduction in the inferred \( a_0 \) due to the finite size of the electron beam and the transverse offset is in agreement with eq. (6).

For the highest \( a_0 \) shown in fig. 5, the three radiation-reaction models, while consistent with each other, separate from the ‘no RR’ result. This is due to ponderomotive scattering, which is the radial expulsion of electrons from a focussed field by intensity gradients, and therefore a source of angular deflection absent in a plane wave. Such deflection is amplified by radiation losses, which reduce \( \gamma \) and so the rigidity of the electron beam. Consequently, the radiation profile is broader for a focussed field than for a plane wave with the same peak \( a_0 \) and eq. (5) overestimates the intensity. The error grows to 10\% at \( a_0 \approx 150 \), for a focal spot size \( r_0 = 2.0 \, \mu \text{m} \). Nevertheless, taking this as the upper bound on the validity of the method we have presented, we conclude that it does infer \( a_0 \) in a model-independent way across approximately three orders of magnitude in laser intensity, as shown in fig. 3.

Given a separate estimate, or measurement, of the peak laser intensity, the transverse offset \( x_b \) could be inferred from the reduction of \( a_{0\text{inf}} \) from \( a_0 \). Furthermore, as the overlap between laser pulse and electron beam varies from shot to shot due to imperfect pointing stability, accumulating the distribution of \( a_{0\text{inf}} \) over a large series of collisions could indicate systematic effects such as the finite size of the electron beam.

IV. CONCLUSIONS

We have shown that by probing an intense laser pulse with a relativistic electron beam, measuring the angular size of the emitted radiation and the initial and final beam energies, the normalized amplitude \( a_0 \) at the collision point may be inferred in a model-independent way. By ‘model-independent’, we mean that across three orders of magnitude in laser intensity, \( 5 \leq a_0 \leq 150 \), different models for the electron dynamics yield a consistent value for the inferred \( a_0 \) that is accurate to within a few per cent. As the best choice of model depends on the intensity, relaxing the requirement that one must be assumed \( \text{a priori} \) means that our method provides strong evidence that a particular intensity range has been reached. This is particularly useful for experiments intended to distinguish between radiation reaction models, as this becomes feasible in a reduced number of successful collisions if \( a_0 \) can be measured independently [33]. The quantities necessary to use eq. (5), our main result, can be measured on a shot by shot basis without additional theoretical modelling. This allows the variation in the collision \( a_0 \) due to shot-to-shot fluctuations to be identified, including the effect of a systematic error in alignment. We emphasize that our method is complementary to model-dependent analysis of the interaction, using, for example, the largest angle of the radiation angular profile [12], or coincidence measurements of the radiation and electron energy spectra [10].

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