Magneto-electric coupling in zigzag graphene nanoribbons

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Zigzag graphene nanoribbons can have magnetic ground states with ferromagnetic, antiferromagnetic, or canted configurations, depending on carrier density. We show that an electric field directed across the ribbon alters the magnetic state, favoring antiferromagnetic configurations. This property can be used to prepare ribbons with a prescribed spin-orientation on a given edge.

Introduction—Expanding techniques that can achieve electrical control of spin is a key goal of both metal and semiconductor spintronics [1, 2]. In metal spintronics, for example, electrical spin-transfer torque [3] research seeks to amplify the potential [4] of technologies based on giant magnetoresistance [5] and tunnel magnetoresistance. [6] The aim of research on dilute magnetic semiconductors [7] is to create semiconductor materials in which magnetic properties are as sensitive to doping and external gate potentials as electrical properties. Recent interest in the spin Hall effect [8] and the topological magnetoelectric effect [9] is motivated by the possibility of interesting magnetoelectric effects in non-magnetic materials. In this context it is interesting to address the possibility of interesting magnetoelectric effects in graphitic material. The physics of zigzag graphene ribbons and edge terminations has received considerable attention recently [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. Magnetism is expected in any graphitic material containing edge terminations, for example highly defected bulk graphitic material. [41, 42, 43, 44]. Thanks to progress in structural control of graphene flakes and related materials [45], prospects for mastering graphitic magnetism have improved. The perfect zigzag nano-ribbon studied in this Letter may be viewed as a model system in which graphitic magnetism is exhibited in its simplest and most essential form.

The magnetic ground state of a zigzag nanoribbon has collective moments localized near its edges. In the absence of a transverse electric field, a doped nanoribbon has either full or partial orientational alignment [46, 47] between moments on opposite edges, as illustrated schematically in Fig. [1]. We show that an external electric field applied across the ribbon can control the relative orientation angle $\theta$, and that this property can be used to prepare ribbons with a prescribed spin-orientation on a given edge.

Hubbard Model Mean-Field Theory—Since ab initio density-functional and simpler model Hamiltonian approaches make essentially identical predictions [48, 49, 50], we base our analysis of zigzag ribbon magnetism on a Hubbard model which allows the underlying physics to be identified more clearly. The success of the Hubbard model has been shown to be due to the essentially local character of edge magnetism in graphene ribbons [51]. The Hubbard model mean-field Hamiltonian,

$$
H_\sigma = -\gamma_0 \sum_{(i,j)} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i c_{i\sigma}^\dagger c_{i\sigma} (v_{ext} + e\varepsilon y_i) + \frac{U}{2} \sum_i \left[ \langle c_{i\sigma}^\dagger c_{i\sigma} \rangle c_{i-\gamma}^\dagger c_{i+\gamma} - \langle c_{i\sigma}^\dagger \gamma \rho_{\sigma\gamma} c_{i\sigma} \rangle \cdot c_{i\sigma}^\dagger \mu \rho_{\gamma\mu} c_{i\sigma} \right]$$

has a term which represents hopping between nearest neighbor $\pi$-orbitals with amplitude $\gamma_0 = 2.6$ eV, an external potential term which accounts for the transverse electric field, and a mean-field interaction term. The operator $c_{i\sigma}^\dagger$ creates a $\pi$ orbital electron at site $i$ with spin $\sigma$, $\varepsilon$ is the transversal electric field, $y_i$ is the position of lattice site $i$ along the ribbon width and $\rho_{\sigma\gamma}$ represents the elements of the three Pauli matrices. As a convenience we include a constant term $v_{ext} = -U$ in the external potential which removes the interaction with a unit charge on each site from the mean-field quasiparticle energy. (All spin indices in Eq. [1] are summed over.) Note that the mean-field interaction energy of an electron on site $i$ is spin-dependent and proportional to the density of opposite spin electrons. Following Yazyev et al. [13] we choose $U = 3$ eV a value slightly larger than estimates based on the

![FIG. 1: Schematic representation of the influence of a lateral electric field on zigzag ribbon magnetism. Upper panel. Non-collinear ground state of a doped zigzag ribbon illustrating the angle $\theta$ between moment orientations on opposite edges. The red box indicates the unit cell of the ribbon. Lower panel. A lateral electric field drives the angle $\theta$ to $\pi$, an antiferromagnetic configuration similar to that of an undoped ribbon.](image)
local density approximation used in some previous Hubbard model analyses \[19, 21, 47\]. We have used 1200 \( k \)-points in the Brillouin zone for the self-consistent calculations. Zig-zag edge magnetism is very sensitive\[47\] to the net charge density of the ribbon \( \delta n \), which we refer to as doping whether due to chemical dopants or gate voltages and measure per repeat distance \( a = 2.46 \AA \) along the edge. The corresponding areal density is \( \delta \pi n_{2D} = \delta n/W \) where the ribbon width \( W = \sqrt{3}Na/2 \) and \( N \) is the number of atom pairs per ribbon unit cell.

FIG. 2: Edge state bands of a zigzag ribbon with \( N = 20 \), nearest-neighbor hopping \( \gamma_{0} = 2.6eV \) and on site repulsion \( U = 3eV \) under a transverse electric field of \( \mathcal{E} = 0.2eV/\text{nm} \). Left panel: In the AF case high-spin (see text) bands shift toward the Fermi energy and the low-spin bands move away. The wavefunctions of the occupied bands shift density to the low-energy edge. Right panel: In the F case \( \Delta_{\mathcal{E}} \) shifts the wavevector at which majority and minority spin bands cross to slightly larger values of \( |k| \). The dotted lines show the bands at \( \Delta_{\mathcal{E}} = 0 \).

**Edge-Only Model**—The influence of a transverse electric field on electronic structure in neutral ribbons has been studied previously for both armchair \[48\] and zigzag cases \[16, 49, 50\] using density functional theory. The essentials of zigzag-ribbon magnetism and of the transverse-field magnetoelectric effect are captured by an edge-state-only model \[19, 21\]; the qualitative discussion below refers mainly to this model and to the special case of collinear magnetic states (\( \theta = 0 \) or \( \theta = \pi \)), but the numerical calculations and the phase-diagram results are based on solutions of the full non-collinear \( \pi \)-band Hubbard model self-consistent field equations. In the edge-only model the spin-dependent mean-field Hamiltonian for collinear states takes the form \[21, 51\]

\[
H_{\sigma} (k) = (\sigma \Delta_{c}(k) + \Delta_{\mathcal{E}}(k)) \tau_{z} + (\Delta_{0}(k) + h_{z}) \sigma f + t(k) \tau_{z},
\]

(2)

Here the \( \tau_{0} \) are Pauli matrices which act on the which edge degree of freedom. The terms proportional to \( \tau_{z} \) in Eq. (2) therefore represent the difference in energy between left (\( \tau_{z} \rightarrow 1 \)) and right (\( \tau_{z} \rightarrow -1 \)) edges for electrons of spin \( \sigma \), whereas the term proportional to \( \tau_{x} \) represents the momentum-dependent inter-edge hopping amplitude. Zig-zag edge magnetism follows from the property \[21\] that \( t(k) \) vanishes rapidly with ribbon width in the part of the Brillouin-zone \( (2\pi/3a < |k| < \pi/a) \) in which edge states reside. In Eq. (2), \( \Delta_{\mathcal{E}}(k) \) captures the difference between exchange energies on opposite edges, which vanishes in the \( \theta = 0 \) (F) state in the absence of a transverse field, whereas \( \Delta_{0}(k) \) captures the spin-dependence of the edge average, which vanishes in the \( \theta = \pi \) (AF) state. (An irrelevant spin and edge independent exchange energy has been dropped from \( H_{\sigma} \)). Both exchange energies are large only for the edge-states \( |k| > 2\pi/3a \). \( h_{z} \) accounts for Zeeman coupling to the ribbon spins by an external magnetic field when present.

**Magnetoelectric Coupling in Undoped Ribbons**—The eigenenergies of this Hamiltonian are

\[
E^{\pm}_{\sigma}(k) = \sigma (\Delta_{0}(k) + h_{z}) \pm \sqrt{[(\sigma \Delta_{c}(k) + \Delta_{\mathcal{E}}(k))^{2} + t^{2}(k)]},
\]

(3)

Note that there are always four distinct eigenvalues in the ferromagnetic case, whereas the antiferromagnetic state bands occur in doubly-degenerate pairs when \( \Delta_{\mathcal{E}} \rightarrow 0 \). For undoped ribbons the lowest two edge states bands are normally fully occupied. In Fig. 2 we plot ribbon band structures for both AF and F states of a neutral ribbon calculated using a constant transverse electric field of \( \mathcal{E} = 0.2 \text{ V/nm} \). States that are shifted down (up) in energy relative to the \( \Delta_{\mathcal{E}} = 0 \) case are localized on the low (high) potential side of the ribbon. (Note that a constant field generates a \( k \)-dependent \( \Delta_{\mathcal{E}} \) because of the \( k \)-dependence of the degree of edge state localization.) In the F-state the Fermi energy is pinned to a band-crossing near \( |k| = 2\pi/3a \) between the higher energy majority spin band and the lower energy minority spin band. In the AF case we refer to the spin-orientation which dominates occupied states on the low (high) potential side of the ribbon as the low-spin (high-spin). (In Fig. 2, ↓ is the low-spin.) A transverse spin shifts
In Fig. 4 we compare the transverse field dependence of the field required to close the AF-state gap in the undoped calculations and confirm the expected magnetoelectric effect. The transverse field will close the indirect gap, creating a half-metallic band structure with only high-spin bands crossing the Fermi level. This is the magnetoelectric effect discussed in earlier\cite{16, 22, 50} work. The energy difference between F and AF states\cite{21} is relatively unchanged by a transverse field. Below we show that in doped ribbons a transverse field tilts the competition between F and AF states in favor of the latter, yielding a distinct and stronger magnetoelectric effect.

Magneto-electric coupling at finite doping—We consider for definiteness the case of n-type ribbons in which carriers are added by gate doping. In the absence of a transverse electric field, doping favors\cite{46, 47} the gapless F state over the gapful AF state. The transition between undoped AF and doped F states occurs continuously by varying the relative orientation angle $\theta$ between its AF ($\theta = \pi$) and F ($\theta = 0$) end points. The effect we discuss in this paper is based on the following simple observation concerning the edge-state bands plotted in Fig. 2. In the AF case the conduction band states which are occupied upon doping are high-spin antibonding states, which are localized on the low-energy side of the ribbon. For the F state, on the other hand, there are occupied states in two bands, one localized on the high-energy side and one localized on the low-energy side. The net effect is that a transverse field favors the AF state in doped zigzag ribbons. In Fig. 3 we plot the relative angle between spin polarizations on opposite edges vs. lateral electric field for a series of different doping values. These results were obtained by non-collinear spin self-consistent field calculations and confirm the expected magnetoelectric effect. In Fig. 4 we compare the transverse field dependence of the energy difference between F and AF states for doped and undoped systems. The electric field strength required to convert F states into AF states in doped ribbons is much smaller than the field required to close the AF-state gap in the undoped case.

Discussion—Typical results for the edge-state bands of both electron and hole doped zigzag ribbons with a transverse field strong enough to induce the $\theta = \pi$ state are illustrated in Fig. 5. Because of the partial occupation of the highest unoccupied band of the n-doped case and the lowest unoccupied band in the p-doped case, the $\theta = \pi$ state has an overall spin-polarization proportional in magnitude to the doping. Note that these $\theta = \pi$ states are always half-metallic. This ferromagnetic component of the order allows Zeeman coupling from an external magnetic field to fix the spin-orientation on each edge. It follows from Fig. 5 that the majority spins are high-energy spins in the n-doped case and low-energy spins in the p-doped case. For a fixed magnetic field direction, the spin-orientations on both edges can therefore be switched with a gate voltage, which changes the sign of the carrier density. This remarkable property of zigzag edge magnetism has no parallel of which we are aware in any other magnetic system.

In closing we remark that the one-dimensional character of zigzag edge magnetism works against robust collective spin properties. As analyzed in more detail elsewhere\cite{18, 22}, the consequences of reduced dimensionality are somewhat mitigated by the substantial stiffness of zigzag edge moments. Nevertheless, robust magnetism in graphitic nanostructures will likely require exchange-coupled

\[ \Delta E = E_{AF} - E_{NC}, \delta n = 0.06 \]

\[ \Delta E = E_{AF} - E_{F}, \delta n = 0 \]

\[ \Delta E = E_{NC} - E_{F}, \delta n = 0.06 \]
two-dimensional ribbon networks. The rather unique properties of graphitic magnetism discussed in this Letter motivate an effort to realize structures of this type.

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