Non-Gaussian Signatures in the five-year WMAP data as identified with isotropic scaling indices

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ABSTRACT
We continue the analysis of non-Gaussianities in the CMB by means of the scaling index method (SIM, Räth, Schuecker & Banday 2007) by applying this method on the single Q-, V-, W-bands and the co-added VW-band of the 5-year data of the Wilkinson Microwave Anisotropy Probe (WMAP). We compare each of the results with 1000 Monte Carlo simulations mimicking the Gaussian properties of the best fit ΛCDM-model. Based on the scaling indices, scale-dependent empirical probability distributions, moments of these distributions and χ²-combinations of them are calculated, obtaining similar results as in the former analysis of the 3-year data: We derive evidence for non-Gaussianity with a probability of up to 97.3% for the mean when regarding the KQ75-masked full sky and summing up over all considered length scales by means of a diagonal χ²-statistics. Looking at only the northern or southern hemisphere of the galactic coordinate system, we obtain up to 98.5% or 96.6%, respectively. For the standard deviation, these results appear as 95.6% for the full sky (99.7% north, 89.4% south) and for a χ²-combination of both measurements as 97.4% (99.1% north, 95.5% south). We obtain larger deviations from Gaussianity when looking at separate scale lengths. By performing an analysis of rotated hemispheres, we detect an obvious asymmetry in the data. In addition to these investigations, we present a method of filling the mask with Gaussian noise to eliminate boundary effects caused by the mask. With the help of this technique, we identify several local features on the map, of which the most significant one turns out to be the well-known cold spot. When excluding all these spots from the analysis, the deviation from Gaussianity increases, which shows that the discovered local anomalies are not the reason of the global detection of non-Gaussianity, but actually were damping the deviations on average. Our analyses per band and per year suggest, however, that it is very unlikely that the detected anomalies are due to foreground effects.

Key words: cosmic microwave background – cosmology: observations – methods: data analysis

1 INTRODUCTION
The Wilkinson Microwave Anisotropy Probe (WMAP) satellite, launched in June 2001, measures the temperature anisotropy of the cosmic microwave background (CMB) radiation with surpassing accuracy, hence providing the best insight on the beginnings of our universe until now. From the first data release on, many investigations were made concerning the Gaussianity of the CMB, since such analyses give information about the nature of the primordial density fluctuations, which are the seeds of those temperature anisotropies. The statistical properties of the density fluctuations are again an important observable for testing cosmological models, especially models of inflation. Standard inflationary models predict the temperature fluctuations of the CMB to be a Gaussian random field which is isotropic and homogenous (Guth 1981; Linde 1982; Albrecht & Steinhardt 1982). Still, there also exist more complex models that allow non-Gaussianity in a scale-independent (Linde & Mukhanov 1997; Peebles 1997; Bernardeau & Uzan 2002; Bartolo, Matarrese & Riotto 2002) or in a scale-dependent way (Garriga & Mukhanov 1999; Armendariz-Picon, Damour & Mukhanov 1999). For a detailed overview...

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of the different models and a more specific survey on scale-dependent ones, see Bartolo et al. (2004) and Lo Verde et al. (2008), respectively, as well as enclosed references. In addition, topological defects like cosmic strings can induce local non-Gaussianities and influence the power spectrum (Kaiser 
Stebbins 1984; Bouchet, Bennett, 
Stebbins 1988; Turok 
& Spergel 1990; Turok 1996)Jeanerot, Rocher 
&Sakellariadou 2003). Considering this plethora of possible physical mechanisms, which may induce non-Gaussianity, studies of Gaussianity of the CMB are strongly required for testing predictions of fundamental physical theories. By comparing the results with theoretical predictions, we can evaluate which model e.g. of inflation can be accepted or rejected.

Non-Gaussianity implies the presence of any higher order correlations. Therefore, a concrete description of the characteristics of non-Gaussianity is not possible, and one has to state that it can occur in various forms. One can carry out a global analysis to search for deviations from Gaussianity (Komatsu et al. 2003; 2008; Chiang et al. 2003; Chiang. Naselsky & Coles 2007; Coles et al. 2004; O’Dwyer et al. 2004; Eriksen et al. 2007; Smith, Senatore 
& Zaldarriaga 2009; Rudjord et al. 2009). But one can also concentrate on more specific investigations (this is most often performed in addition to a general analysis), whereas we want to point out the following two:

Investigations concerning asymmetries in the CMB data were accomplished with linear (Eriksen et al. 2004a; Hansen, Banday 
& Gorski 2004b; Hansen et al. 2008; Bonaldi et al. 2007; Bernui 2008; Bernui 
& Hipolito-Ricaldi 2008; Hofuft et al. 2009) as well as non-linear methods (Eriksen et al. 2004a,b; Park 2004; Vielva et al. 2004; Hansen et al. 2004a; Land & Magueijo 2005; Pietrobon et al. 2008; Räth et al. 2009). With those methods, studies of the differences between the northern and southern hemisphere of the galactic coordinate system, naturally given by the absent region of the outmasked galactic plane, as well as a search for a preferred direction of maximum asymmetry were performed. In almost all investigations, significant asymmetries between the north and the south were detected. Thereby, it depended on the type of analysis, which hemisphere featured the larger deviations from Gaussianity and which hemisphere agreed better with the standard model. The preferred direction of maximal disparity was in most investigations found to lie close to the ecliptic axis.

Local features are another particular form of non-Gaussianity being of growing importance, e.g. for the search of topological defects like cosmic strings. Since the first detection of the famous cold spot by Vielva et al. (2004), many investigations tried to find new, or re-detect known spots with various methods [Mukherjee 
& Wang 2004; Cayón, Jin 
& Treaster 2005; Cruz et al. 2005; 2006; McEwen et al. 2005; 2006; 2008; Vielva et al. 2007; Pietrobon et al. 2008; Gurzadian 
& Kocharyan 2005; Gurzadian et al. 2009]. In doing so, several significant spots have been detected up to now.

In Räth et al. (2007), all mentioned investigations were accomplished by applying, for the first time, the scaling index method on the WMAP 3-year data. In this paper, we continue these analyses by applying the scaling index method on the WMAP 5-year observations. We search for global non-Gaussianities and asymmetries in the data and use a modified approach to detect local features.

This paper is structured as follows: In Section 2 we present the preprocessing of the WMAP data and the modality of creating the simulations. In Section 3, the scaling index method is introduced as well as a technique to cope with boundary effects. With these requisites, we are ready to perform our calculations, whose results are presented in Section 4. In this chapter, we first discuss the global investigations as well as asymmetries and focus on local features later on. All these findings are summarised in Section 5. Finally, we draw our conclusions in Section 6.

2 WMAP DATA AND SIMULATIONS

For our investigations we use the Q-, V- and W-band five-year-data of the WMAP-satellite as it is provided by the WMAP-Team. We work with the foreground-reduced maps, which use the Foreground Template Model proposed in Hinshaw et al. (2007) and Page et al. (2007) for foreground reduction. To obtain a co-added VW-map as well as single V-, W- and Q-maps, we accumulate the differencing assemblies Q1, Q2, V1, V2, W1, W2, W3, W4 via a noise-weighted sum [Bennett et al. 2003]:

\[
T(\theta, \phi) = \frac{\sum_{i \in A} T_i(\theta, \phi) / \sigma_{0,i}^2}{\sum_{i \in A} 1 / \sigma_{0,i}^2}\]  

In this equation, \( A \) characterises the set of required assemblies, e.g. for the co-added VW-map \( A = \{V1, V2, W1, W2, W3, W4\} \). The parameters \( \theta \) and \( \phi \) correspond to the co-latitude and the longitude on the sphere, while the five-year noise per observation of the different assembles, given by Hinshaw et al. (2009), is denoted by \( \sigma_0 \).

We decrease the resolution of the maps to 78432 pixels, which equals to \( N_{side} = 256 \) in the employed HEALPix-software [Gorski et al. 2005] and cut out the heavily foreground-affected parts of the sky using the KQ75-mask (Gold et al. 2009), which has to be downgraded as well. We choose a conservative downgrading of the mask by taking only all pixels at \( N_{side} = 256 \) that do completely consist of non-mask-pixels at \( N_{side} = 512 \). All downgraded pixels at \( N_{side} = 256 \), for which one or more pixels at \( N_{side} = 512 \) belonged to the KQ75-mask, are considered to be part of the downgraded mask as well. In doing so, 28.4% of the sky is removed (see upper left part of figure [1]). Finally, we remove the residual monopole and dipol by means of the appropriate HEALPix routine applied to the unmasked pixels only.

To accomplish a test of non-Gaussianity, we also need simulations of Gaussian random fields. We create 1000 simulations for each band and proceed hereby as follows: We take the best fit \( \Lambda CDM \) power spectrum \( C_l \), derived from the WMAP 5-year data only, and the respective window function for each differencing assembly \( \{Q1-Q2, V1-V2, W1-W4\} \), as again made available on the LAMBDA-website\(^1\). With these requisites, we can create Gaussian random fields mimicking the Gaussian properties of the best fit \( \Lambda CDM \)-model and including the WMAP-specific beam properties by convolving the \( C_l \)’s with the window function. For every assembly, we add Gaussian noise to these maps with a particular variance for every pixel of the sphere. This variance

1 http://lambda.gsfc.nasa.gov
2 http://healpix.jpl.nasa.gov
depends on the number of observations \( N_i(\theta, \phi) \) in the respective direction and the noise dispersion per observation, \( \sigma_{0,i} \). After this procedure, we summarize the Q-, V- and W-bands and the co-added VW-band using equation (1), decrease the resolution to \( N_{\text{side}} = 256 \), cut out the KQ75-mask and remove the residual monopole and dipole, just as we did with the WMAP-data.

\[ \alpha(p_i, r) = \frac{\sum_{j=1}^{N_{\text{pix}}} 2 \left( \frac{d_{ij}}{r} \right)^2 e^{-\left( \frac{d_{ij}}{2r} \right)^2}}{\sum_{j=1}^{N_{\text{pix}}} e^{-\left( \frac{d_{ij}}{2r} \right)^2}} \]  

where \( d_{ij} \) denotes the euclidian distance measure between the points \( p_i \) and \( p_j \), while \( r \) characterizes a scale parameter. This parameter does not draw a clear-cut line between the pixels that are included in the calculations and those that are excluded; it rather influences how each single pixel is taken into consideration for the calculation, in relation to its distance from the center pixel: For lower \( r \), only the closest pixels are important in the calculation of \( \alpha(p_i, r) \), whereas for larger \( r \), the farther distant pixels are considered as well, even though with a still lower weight than the close pixels. In our study, we use the ten scales \( r_i = 0.025, 0.05, ..., 0.25 \), \( i = 1, 2, ..., 10 \) and the radius \( R = 2 \) for the sphere. Table 1 shows for each radius \( r \) the corresponding angular scales at the position of the 90% and the

| Radius  | 0.025 | 0.050 | 0.075 | 0.100 | 0.125 |
|---------|-------|-------|-------|-------|-------|
| \([\ell_1, \ell_2]\) | [83, 387] | [41, 193] | [28, 129] | [21, 97] | [17, 77] |

| Radius  | 0.150 | 0.175 | 0.200 | 0.225 | 0.250 |
|---------|-------|-------|-------|-------|-------|
| \([\ell_1, \ell_2]\) | [14, 65] | [12, 55] | [10, 48] | [9, 43] | [8, 39] |

Table 1. The angular scales corresponding to the position of the 90% (\( \ell_1 \)) and the 10% (\( \ell_2 \)) weighting in the scaling index formula when using a given scale parameter \( r \).
10% weighting, thus giving an estimate on how the r-values relate to ℓ-bands in Fourier space.

The value of α characterises the structural components of a point distribution. For example, points in a cluster-like, filamentary or sheet-like structure lead to α ≈ 0, α ≈ 1 or α ≈ 2, respectively. A uniform distribution of points results in α ≈ 3, while points in underdense regions in the vicinity of point-like structures, filaments or walls have α > 3.

On the basis of these scale-dependent α-values, we compute simple measures such as moments and empirical probability distributions. We make use of the following scale-dependent statistics, namely the mean, the standard deviation and a diagonal χ²-statistics, to compare the results of the original WMAP data with the results of the simulations:

\[
\langle \alpha(r_k) \rangle = \frac{1}{N} \sum_{j=1}^{N} \alpha(\vec{p}_j, r_k) \tag{3}
\]

\[
\sigma_{\alpha(r_k)} = \left( \frac{1}{N-1} \sum_{j=1}^{N} \left[ \alpha(\vec{p}_j, r_k) - \langle \alpha(r_k) \rangle \right]^2 \right)^{1/2} \tag{4}
\]

\[
\chi^2_{\alpha(r_k)} = \sum_{i=1}^{2} \left[ \frac{M_i(r_k) - \langle M_i(r_k) \rangle}{\sigma_{M_i(r_k)}} \right]^2 \tag{5}
\]

where \( M_1(r_k) = \langle \alpha(r_k) \rangle, \ M_2(r_k) = \sigma_{\alpha(r_k)} \) and \( N \) denotes the number of pixels in consideration. For all analyses we will only consider the non-masked pixels of the full sky or of (rotated) hemispheres, as it will be outlined in Section 4.1. Note that we follow the reasoning of Eriksen et al. (2004b) and choose a diagonal and not the full χ²-statistics involving the inverted cross-correlation matrix, because also in our case the moments are highly correlated leading to high values in the off-diagonal elements of the cross-correlation matrix. Therefore, the matrix would converge very slowly and numerical stability would not be given. If however the chosen model is a proper description of the data, any combination of measures should yield statistically the same values for the observations and the simulations.

To obtain scale-independent variables as well, we also use three diagonal χ²-statistics, derived from \( \langle \alpha \rangle \) and \( \sigma_{\alpha} \), which sum over all utilised length scales \( r \):

\[
\chi^2_{\langle \alpha \rangle} = \sum_{k=1}^{N_r} \left[ \frac{M_1(r_k) - \langle M_1(r_k) \rangle}{\sigma_{M_1(r_k)}} \right]^2 \tag{6}
\]

\[
\chi^2_{\sigma_{\alpha}} = \sum_{k=1}^{N_r} \left[ \frac{M_2(r_k) - \langle M_2(r_k) \rangle}{\sigma_{M_2(r_k)}} \right]^2 \tag{7}
\]

\[
\chi^2_{\langle \alpha \rangle, \sigma_{\alpha}} = \sum_{i=1}^{2} \sum_{k=1}^{N_r} \left[ \frac{M_i(r_k) - \langle M_i(r_k) \rangle}{\sigma_{M_i(r_k)}} \right]^2 \tag{8}
\]

Hereby denotes \( M_1(r_k) = \langle \alpha(r_k) \rangle \) and \( M_2(r_k) = \sigma_{\alpha(r_k)} \). The number of different scale parameters \( r \) is named \( N_r \). Throughout all subsequent investigations, \( N_r \) equals ten.

Finally, to be able to access the degree of difference between the data and the simulations and hence a degree of the non-Gaussianity of the data, we use the \( \sigma \)-normalised deviation of the WMAP results of the above-mentioned statistics:

\[
S = \frac{M - \langle M \rangle}{\sigma_M} \tag{9}
\]

where in this case \( M \) refers to one of the variables defined in equation (3) to (8) respectively. \( M \) itself is calculated by using the WMAP data, while its moments result from the simulations. Note that we pass on the absolute value in this general definition to obtain positive as well as negative deviation. Although we will use the absolute value in the global investigations, the sectioning into positive and negative deviation is useful for the analysis of north-south asymmetry by means of rotated hemispheres in chapter 4.3. It also allows a better interpretation of the character of difference, since e.g. a higher mean of the scaling indices implies a more 'unstructured' arrangement of the 'pixel cloud' and vice versa. Similarly, a higher standard deviation of the indices indicates a larger structural variability.

In the tables, we also included the fraction \( p \) of the
simulations that have higher (lower) values than the data in terms of the respective calculated statistics. This percentage corresponds to a empirical significance level of the null hypothesis that the observation belongs to a Gaussian Monte Carlo ensemble.

3.2 Coping with boundary effects
The regions in the direction of the galactic plane as well as many small spots all over the WMAP map are masked out since they represent heavily foreground-affected areas which would not allow a reasonable analysis of the intrinsic background fluctuations. But this operation spoils the results of the scaling index method: Instead of a more or less uniform distribution, the \( \alpha \)-values in the regions around the mask now detect a sharp boundary with no points in the masked area, into which the scaling regions extends (see figure 2). This results in lower values of \( \alpha \). The effect can clearly be seen in the \( \alpha \)-response of the masked VW-band WMAP-data in the lower left corner of figure 1. A solution to this problem is to fill the masked areas with suitable values, that prevent the low outcome at the edges of the mask. We accomplish this by filling in (nearly) white Gaussian noise with adjusted parameters. This is performed by applying the following two steps:

At first, we fill the masked regions with Gaussian noise, whose standard deviation for each pixel corresponds to the pixel noise made available on the LAMBDA-website\(^3\):

\[
T_{\text{mask}}(\theta, \phi) \sim N(0, \sigma^2_{\theta, \phi})
\]

Here, \( \sigma_{\theta, \phi} \) denotes the pixel noise of the pixel which is located in the direction \( (\theta, \phi) \). Then, we scale the expectation value and the variance as a whole to the empirical mean \( \mu_{\text{rem}} \) and variance \( \sigma^2_{\text{rem}} \) of the remaining regions of the original temperature map:

\[
T_{\text{mask}}(\theta, \phi) = \frac{\sigma^2_{\text{rem}}}{\sigma^2_{\text{mask}}} T_{\text{mask}}^* (\theta, \phi) + \mu_{\text{rem}}
\]

with

\[
\mu_{\text{rem}} = \frac{1}{N_R} \sum_{(\theta, \phi) \in R} T(\theta, \phi)
\]

\[
\sigma^2_{\text{rem}} = \frac{1}{N_R - 1} \sum_{(\theta, \phi) \in R} (T(\theta, \phi) - \mu_{\text{rem}})^2
\]

\[
\sigma^2_{\text{mask}} = \frac{1}{N_M - 1} \sum_{(\theta, \phi) \in M} T_{\text{mask}}^*(\theta, \phi)^2
\]

where \( R \) and \( M \) stand for the non-masked and masked region of the map respectively, and \( N_R \) as well as \( N_M \) denote their number of pixels. Thus, we filled the mask with (nearly) white Gaussian noise whose mean and standard deviation equal the respective terms of the remaining map, whereby the spatial noise patterns are preserved.

With this filling technique, we obtain a complemented data set instead of just excluding the masked regions. Figure 2 shows a slice of the three-dimensional representation of the temperature fluctuations of both techniques, representing a 2D projection of the 3D point distribution used for the calculation of the scaling indices. The center region of the filled sphere now highly resembles the appearence of the remaining area, although a more uniform arrangement is visible. In the form of a mollweide projection, the filling method as well as the corresponding \( \alpha \)-response are displayed in the right column of figure 1.

The filling strategy shows obvious success in the adjustment of the scaling indices map (see the lower right panel of figure 1): The white noise leads to higher values in the \( \alpha \)-response for the pixels close to the mask as compared to the masking method. The regions around the edges of the mask feature now \( \alpha \)-responses that match far better the values of the remaining regions. Still these \( \alpha \)-values are calculated with the help of an artificial environment, but now the distortions are lower compared to the original approach. Since we apply this method to both the original WMAP data as well as to the simulations, the now smaller systematic errors in the \( \alpha \)-calculation for the ‘edge’-pixels are the same for both kinds of maps. Thus any significant deviation found in the WMAP data is due to intrinsic effects.

The most important advantage of the filling strategy

\[^3\text{http://lambda.gsfc.nasa.gov}\]
arises if one considers local features in the CMB map: For a search of local anomalies (e.g. cold spots), the filled map provides a better underlying than the original map: In a point distribution, spots as well as points at the border of the distribution show similarly low $\alpha$-values. Thus, by cutting out the mask, it is difficult to decide whether a detected local feature really exists or whether it originates from a masked area nearby. But by using the filling technique, there is no longer an edge between the masked and the non-masked regions, and anomalies in the Gaussian noise of the mask are highly improbable. Thus, any detected local feature must then originate from the data itself. Considering the amount of masked areas outside the galactic plane, this technique describes a considerable improvement for the whole sky. Due to these advantages we will only use the mask-filled maps in chapter 4.2.

4 RESULTS

4.1 Band-wise and co-added map analysis

In figure 3, the empirical probability densities $P(\alpha)$ of the scaling indices (calculated with $r = 0.2$) are displayed for the WMAP data and for the simulations, evaluated for the original and the filling method from chapter 3.2. For clarity reasons we only used the first 50 simulations in these plots. For both methods, a shift of the WMAP data to higher values can be detected, that becomes particularly apparent in the northern hemisphere of the galactic coordinate system. This indicates a more ‘unstructured’ arrangement of the underlying temperature fluctuations of the CMB data in comparison to the simulations. In addition, the histograms of the simulations are slightly broader and therewith containing a larger structural variability than the one of the WMAP data.

Comparing the non-filling and the filling method, the histograms of the latter feature a higher maximum as well as higher values for large $\alpha$, but lower probabilities for $\alpha \in [2.0, 2.5]$. The obvious reason for this shift is the fact that the filled mask does not reduce the $\alpha$-values of its surroundings as it was the case with the former method. Now, the outcome of these regions is influenced by the white noise and is therefore allocated at higher values.

If we focus on the mean values $\langle \alpha(r_k) \rangle$ of the scaling indices, and compare the results of the simulations with the WMAP data, the above mentioned shift to higher values becomes yet clearer. This can be seen in Figure 4, where the distribution of $\langle \alpha(r_k) \rangle$ for the simulations as well as the data is displayed for the five different scale parameters $r_2$, $r_4$, $r_6$, $r_8$ and $r_{10}$. These results were obtained using the full sky. For all applied scales, the distance between the average over all simulations and the result of the original WMAP map is notably similar. If we perform this analysis for the northern hemisphere only, the deviations of the original data as compared to the simulations become significantly larger.

Both the shift to higher values of the WMAP data in comparison to the simulations as well as its broader density are reflected in the $\sigma$-normalised deviations $S(r)$ of the scale-dependent statistics of the equations (3) - (5): The former is reflected in the mean and the latter in the standard deviation (and therefore both aspects in the diagonal $\chi^2$-statistics).
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Figure 5. The $\sigma$-normalised deviations $S(r)$ of the statistics of the equations (3) - (5) in absolute values for the VW-band, plotted as a function of the scale parameter $r$. The lines with "+" denote the mean, "∗" the standard deviation and the boxes the $\chi^2$-combination, each for the original (red) and the mask filling method (blue). As in figure 3, the upper diagram shows the results of a full-sky analysis, while the middle and the lower ones show the results when only concerning the northern or southern hemisphere respectively.

Figure 6. Same as figure 5 but applied to the Q- (red), V- (green) and W-band (blue). Only the results of the mask-filling method are shown while the original method is left out.

Figure 5 shows these deviations for the coadded VW-band as a function of the scale parameter $r$ for the original and the mask filling method, while figure 6 displays only the latter method, but for the three single bands. As above, the results are illustrated for the full sky as well as for the separate hemispheres. The shift to higher values of the WMAP data in the northern hemisphere in figure 3 appears now as an increased $S(r)$, especially for higher scales ($r \geq 0.125$), where the deviations of the two moments range between $2\sigma$ and $3.5\sigma$, and the $\chi^2$-combination nearly reaches a $6\sigma$-level. In the southern hemisphere, only the lowest scales show a namable $S(r)$. On larger scales, no signatures for deviations from Gaussianity are identified. In the northern hemisphere at $r = 0.15$ the frequency band which is considered to be the least foreground-contaminated one, namely the V-band. Comparing the co-added VW-band of the original approach and of the mask filling method, the $\sigma$-normalised deviations of the mean are almost identical. The standard deviation of the latter method in comparison to the former one shows a slightly lower $S(r)$ for higher scales, which is also reflected in the graph of the $\chi^2$-combination, yet the profile remains the same.

We also calculated the $\sigma$-normalised deviations $S$ and the percentages $\rho$ of the simulations with higher (lower) results of the scale-independent diagonal $\chi^2$-statistics from the equations (6) to (8), which are listed in table 2. Although the results are damped by a few unimportant scales, high deviations are still found, particularly in the northern hemisphere. For a better comparison to separate scale lengths, the respective results of the scale-dependent statistics (3) to (5) are listed in table 3 for which we used the single scale $r = 0.2$.

In general, all occurring characteristics of the figures match the findings of the analysis of the WMAP 3-year data in Räth et al. (2007). This indicates that the results are not based on some time-dependent effects. Since the 5-year data features lower error bars than the 3-year data, it is also improbable that both results are induced by noise effects only.

Evidence for north-south asymmetry in the WMAP data was already detected using the angular power spec-
The maximum value for \( S(r) \) of the \( \chi^2 \) analysis (right column of figure 7) and the empirical probability \( P \) of the scale-independent diagonal \( \chi^2 \)-statistics from the equations (4) to (6) for the single bands and methods as well as for the Full Sky and the single hemispheres.

**Table 2.** The \( \sigma \)-normalised deviations \( S \) and the empirical probabilities \( P \) of the scale-independent diagonal \( \chi^2 \)-statistics from the equations (4) to (6) for the different bands and methods as well as for the Full Sky and the single hemispheres.

|           | Full Sky (\%) | Northern Sky (\%) | Southern Sky (\%) |
|-----------|---------------|-------------------|-------------------|
| \( \chi^2(0) \): |               |                   |                   |
| VW (original) | 2.2 / 97.3   | 2.7 / 98.5        | 1.7 / 96.6        |
| VW (mask-filled) | 2.2 / 97.3 | 2.7 / 98.3        | 1.7 / 96.6        |
| Q (mask-filled) | 2.1 / 97.4   | 2.7 / 98.3        | 1.5 / 95.8        |
| V (mask-filled) | 2.0 / 97.4   | 2.6 / 98.1        | 1.5 / 96.2        |
| W (mask-filled) | 2.1 / 97.5   | 2.6 / 98.2        | 1.7 / 96.6        |

| \( \chi^2(0), \sigma_{\alpha} \): | \( S(\%) \) | \( P(\%) \) |
|-------------------------------|----------|----------|
| VW (original) | 2.3 / 99.7 | 0.6 / 81.9 |
| VW (mask-filled) | 2.3 / 99.7 | 0.8 / 97.1 |
| Q (mask-filled) | 2.2 / 99.6 | 0.6 / 81.6 |
| V (mask-filled) | 2.2 / 99.6 | 0.5 / 81.1 |
| W (mask-filled) | 2.2 / 99.6 | 0.5 / 81.0 |

**Table 3.** Same as table 2 but for the scale-dependent statistics from the equations (3) to (5) for the single scale \( r = 0.2 \).

|           | Full Sky (\%) | Northern Sky (\%) | Southern Sky (\%) |
|-----------|---------------|-------------------|-------------------|
| \( \sigma_{\alpha(0.2)} \): |               |                   |                   |
| VW (original) | 1.3 / 90.7   | 2.7 / 99.8        | 0.5 / 81.6        |
| VW (mask-filled) | 1.1 / 85.6 | 4.3 / 99.3        | 0.6 / 81.6        |
| Q (mask-filled) | 0.4 / 64.2   | 1.7 / 95.6        | 1.1 / 85.8        |
| V (mask-filled) | 1.1 / 84.9   | 2.4 / 99.0        | 0.6 / 71.4        |
| W (mask-filled) | 1.0 / 83.9   | 1.9 / 96.4        | 0.2 / 55.9        |

The \( \sigma \)-normalised deviations \( S \) and the empirical probability \( P \) of the scale-independent diagonal \( \chi^2 \)-statistics from the equations (4) to (6) for the single scale \( r = 0.2 \).
4.2 Local features

An interesting anomaly in the CMB data is that there are small regions which show very high or very low values in some local structure analysis. Vielva et al. (2004) detected the first of these regions, the well-known cold spot at \((\theta, \phi) = (147^\circ, 209^\circ)\) a few years ago by using a wavelet analysis. This Spot was re-detected several times using amongst others wavelet analysis (Mukherjee & Wang 2004; Cayón, Jin & Treaster 2005; Cruz et al. 2005, 2006), scaling indices (Kath et al. 2007) or the Kolmogorov stochasticity parameter (Gurzadyan & Kocharyan 2008). Furthermore, there have been some investigations which, in addition to the re-detection of the first spot, detected secondary spots via directional (McEwen et al. 2005, 2006, 2008) or steerable wavelets (Vielva et al. 2007), needlets (Pietrobon et al. 2008) and again the Kolmogorov stochasticity parameter (Gurzadyan et al. 2009). These spots could be the result of some yet not fully understood physical process. For the cold spot lots of theories already exist which try to explain its origin by second-order gravitational effects (Tomita 2005, Tomita & Inoue 2008), a finite universe model (Adler, Björken & Overduin 2006), large dust-filled
The results are illustrated in the upper left part of figure 8. In the lower left plot of the same figure only the deviations $S_{i,r} \leq -3.0$ are illustrated to gain yet another clearer view on the interesting areas.

The first approach clearly shows the cold spot and indicates some secondary spots in the southern as well as in the northern hemisphere. These get confirmed in the plot of the smoothing method, where we obtain a deviation of up to $-7\sigma$ for several clearly visible areas: In the southern hemisphere we detect a cold spot at $(\theta, \phi) = (124^\circ, 320^\circ)$ and another one at $(\theta, \phi) = (124^\circ, 78^\circ)$. Both were already detected with the above mentioned directional and steerable wavelet as well as with a needlet analysis. The former one is a hot spot in these investigations. In our analysis, the latter spot actually appears as two spots close to each other, which is in agreement with Pietrobon et al. (2008). We discover another southern cold spot at $(\theta, \phi) = (120^\circ, 155^\circ)$ which is very close to the mask. This spot represents a good example for the use of the mask filling method since it is situated at the edge of the non-masked region: The influence of the mask is diminishing the results of the calculation of the scaling indices in the area of this spot. This becomes obvious if one recalls the lower left plot of figure 8 in which the coordinates of the spot would be completely located in a "blue" region with low $\alpha$-values. Since the results of the scaling indices of local features show a similar, namely lower-valued, behaviour, an overlapping like that could prevent the detection of such spots close to the mask. By using the mask filling method, the detection of this cold spot on the edge of the mask is equivalent to a detection in an unmasked region, and there-

Figure 8. The pixel-wise deviations $S_{i,r}$ of the primal (upper left) and of the smoothed scaling indices map (upper right), both based on the VW-band and the scale parameter $r = 0.2$. The plot in the lower left only shows the values $\leq -3.0$ of the smoothed method. Except for the very small spots in the right part of this mapping, these regions are added to the KQ75-mask. The result is illustrated in the lower right plot.

The outcome of this procedure is shown in the upper right part of figure 8. In the lower left plot of the same figure only the deviations $S_{i,r} \leq -3.0$ are illustrated to gain yet another clearer view on the interesting areas.

The second approach smoothes the $\alpha$-maps of the original and simulated data by computing for every pixel the mean value of its surroundings given by some specified maximum distance, which equals $3^\circ$ in our analysis. We apply the pixel-wise deviations $S_{i,r}$ again on the resulting maps.
Non-Gaussian Signatures in the five-year WMAP data

Figure 9. The $\sigma$-normalised deviations of the mask-filling method for the original KQ75-mask (blue) and for the modified mask of the previous figure (green) in absolute values, plotted as a function of the scale parameter, whereby as above "+" denotes the mean, "*" the standard deviation and the boxes the $\chi^2$-combination. The full sky as well as again the single hemispheres are considered. The blue lines exactly correspond to the blue lines of figure 5.

Figure 10. Same as figure 9 but exclusive for the modified mask of figure 8 applied to the Q- (red), V- (green) and W-band (blue). This plot is associated with figure 6.

fore reliable. The spot at $(\theta, \phi) = (136^\circ, 173^\circ)$, described by McEwen et al. (2005) and Pietrobon et al. (2008), is not traced in our analysis. In the northern hemisphere, our investigation shows two other cold spots at $(\theta, \phi) = (49^\circ, 245^\circ)$ and $(\theta, \phi) = (68^\circ, 204^\circ)$, which do not correspond with the so-called northern cold spot of Gurzadyan et al. (2009), but with the results of McEwen et al. (2005), where again one of them is a hot spot. Also Pietrobon et al. (2008) locates one of these two spots. All these results were achieved with an analysis of the VW-band, but we find similar results in a single band analysis.

It is possible to define a new coordinate frame, including a new direction of the "north pole", such that all of these spots are contained in the "southern" hemisphere. This new north pole would then be located at $(\theta, \phi) = (51^\circ, 21^\circ)$.

If the considered spots really depend on some yet not completely understood, maybe secondary, physical effect, they should not be implemented in a testing for intrinsic non-Gaussianity. For this reason, we modify the KQ75-mask by additionally excluding all above mentioned spots. A small peculiarity at the edge of the mask next to the cold spot as well as three very small blurs in the right half of the lower left mollweide projection in figure 8 are not considered, since we regard their appearance as insufficient for being a distinctive feature. The modification of the KQ75-mask is illustrated in the lower right part of figure 8.

We now apply this new mask to the $\alpha$-response of both the WMAP data as well as the simulations and repeat the analysis of chapter 4.1. The results are illustrated in the figures 9 and 10 as well as in table 4. A clear increase of $S(r)$ in comparison to the former analysis is evident. This heightening is in particular present in the southern hemisphere, where we detected more local features than in the north. The largest increase takes place in the co-added VW-band, where we now reach deviations of up to 4.0 for the $\chi^2$-combination in a full-sky analysis (former maximum: 2.9) and to the extent of 6.0 in an analysis of the northern hemisphere (former maximum: 5.5). But also the single bands in figure 10 as well as all scale-independent diagonal $\chi^2$-statistics in table 4 show without exception a greater evidence for non-Gaussianity.

One could have expected to obtain higher values for $S(r)$ since the $\alpha$-response of the WMAP data in comparison to the one of the simulations featured a shift to higher values (see figure 3). By now cutting out the local features, that exclusively consist of cold spots in terms of pixel-wise devia-
features in line with later investigations of local features, where we detected more local anomalies in the southern than in the northern hemisphere.

Furthermore, we performed an analysis of local features by studying pixel-wise deviations from Gaussianity with and without a previous smoothing of the \( \alpha \)-responses. For these investigations, we exclusively applied the mask filling method which can reduce the distorting effects on measurements like the scaling indices that appear when cutting out the masked regions. This mask-filling method eliminates the diluting effects on the border and therefore allows for an analysis of local features, which show a similar behaviour of lower outcome in the scaling index method. We detected the well-known cold spot and three additional spots in the southern as well as two spots in the northern hemisphere. Except for one single spot in the south, all findings are in agreement with former results of different investigations. Since these spots could origin on some yet not completely understood physical effect, we excluded them from the data set and repeated the former analysis. Instead of obtaining lower deviations, the results show an increase of non-Gaussianity in all bands. Therefore, the discovered local anomalies are not the reason of the global detection of non-Gaussianity, but were actually dampening the deviations on average. In former isotropic wavelet-based analyses, an exclusion of detected spots lessened the significance level of indications of non-Gaussianity [Vielva et al. [2004]]. Our new findings indicate in contrast, that the isotropic scaling index method can detect several different yet complementary aspects of the structural composition of the underlying data. The results of our investigation are in agreement with the steerable wavelet-based analysis in [Wiaux et al. [2008]], where the non-Gaussianities were conserved after excluding the detected local anomalies.

### 5 SUMMARY

We performed a scaling index analysis of the WMAP 5-year data following up the investigations of [Rath et al. [2007]]. For more realistic results around the mask, we additionally implemented a mask-filling method. By comparing the \( Q \)-, \( V \)-, \( W \)- and the co-added \( VW \)-band of the WMAP data with 1000 simulated maps per band, we (re)dected strong deviations from Gaussianity as well as asymmetries in the data, which can be summarized and interpreted as follows:

- The scaling index values of the WMAP data are shifted to higher values and feature a higher variability than those of the simulations, especially in the northern hemisphere. This effect can be interpreted as less structure as well as more structural variations in the CMB signal compared to the corresponding Gaussian model. The results are confirmed by several statistics, that show deviations from Gaussianity of up to 5.9\( \sigma \) in the scale-dependent, and 5.5\( \sigma \) in the scale-independent case. These results are slightly lower applying the mask-filling method, and show high similarities within the different bands. In addition, we detected strong asymmetries by performing an analysis of rotated hemispheres: rotations pointing to northern directions show by far higher deviations from Gaussianity for the mean and the \( \chi^2 \)-analysis than rotations pointing to the south. Observing the standard deviation, we obtained a negative outcome in the north and a positive in the south. This implies that the north possesses a more consistent pattern than the simulations, while the south shows the converse behaviour. This

### 6 CONCLUSIONS

The redetection of indications for non-Gaussianity of the WMAP 3-year data analysis leads to the conclusion that the observed results are not time-depending. In contrary, we can detect even higher deviations from the simulations which mimic the Gaussian properties of the best fit \( \Lambda CDM \)-model. Therefore, it is highly improbable for the results to be caused by effects related to short-term measurements.

In addition, the coherence between the different analysed bands implies that the foreground influence plays only a minor role but that the results are very unlikely to be truly of thermal origin.

Finally, the agreement of the detected spots with former investigations confirms the existence of these local anomalies.

The two most important tasks for future studies are:
- First, to identify possible reasons for the indications of non-Gaussianity, which could be possible with the attainment of more and more precise data, e.g. with the upcoming PLANCK-mission. Second, to figure out possible sources of the observed local features and thereby solving the question, if these anomalies are due to systematics or foreground effects or indeed represent variations in the CMB signal itself.

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**Table 4.** Same as table 2, but after excluding the cold spots via the modified KQ75-mask.

|          | Full Sky | Northern Sky | Southern Sky |
|----------|----------|--------------|--------------|
| \( \chi^2_{(\alpha)} \): | (S%) | (S%) | (S%) |
| \( VW \) (mask-filled) | 2.4 / 97.6 | 2.8 / 98.4 | 2.0 / 97.2 |
| \( Q \) (mask-filled) | 2.3 / 97.5 | 2.8 / 98.3 | 1.8 / 96.9 |
| \( V \) (mask-filled) | 2.3 / 97.6 | 2.7 / 98.1 | 1.9 / 97.1 |
| \( W \) (mask-filled) | 2.4 / 97.7 | 2.6 / 98.2 | 2.0 / 97.4 |

|          | (S%) | (S%) | (S%) |
|----------|------|------|------|
| \( \chi^2_{(\alpha)} \), \( \sigma \): | | | |
| \( VW \) (mask-filled) | 2.6 / 96.7 | 4.8 / 99.8 | 0.2 / 97.2 |
| \( Q \) (mask-filled) | 1.3 / 90.6 | 2.5 / 97.0 | 0.6 / 82.1 |
| \( V \) (mask-filled) | 2.0 / 94.9 | 4.4 / 99.4 | 0.4 / 74.1 |
| \( W \) (mask-filled) | 2.2 / 96.2 | 3.1 / 98.0 | 0.4 / 78.9 |

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