Probing the cosmic acceleration history and the properties of dark energy from the ESSENCE supernova data with a model independent method

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Abstract. With a model independent method the expansion history $H(z)$, the deceleration parameter $q(z)$ of the universe and the equation of state $w(z)$ for the dark energy are reconstructed directly from the 192 Sne Ia (type Ia supernovae) data points which are contained in the new ESSENCE (Equation of State: Supernovae Trace Cosmic Expansion) Sne Ia data and the high redshift Sne Ia data. We find that the evolving properties of $q(z)$ and $w(z)$ reconstructed from the 192 Sne Ia data seem to be weaker than those obtained from the Gold set, but stronger than those from the SNLS (Supernova Legacy Survey) set. With a combination of the 192 Sne Ia and BAO (Baryonic Acoustic Oscillation) data, a tight constraint on $\Omega_{m0}$ is obtained. At the $1\sigma$ confidence level $\Omega_{m0} = 0.278^{+0.024}_{-0.023}$, which is highly consistent with the values from the Gold + BAO and SNLS + BAO data.

Keywords: dark energy theory, supernova type Ia
1. Introduction

In order to explain the present cosmic accelerating expansion discovered firstly from the type Ia supernovae (SNe Ia) [1]–[6], dark energy (see [7]–[10] for recent reviews) is usually assumed to exist in the universe. Dark energy is an exotic energy component with negative pressure, and presumably began to dominate the evolution of the universe only recently. Although it has been studied for nearly one decade, its nature is still puzzling.

In general there are two kinds of methods for reconstructing the properties of dark energy from the observation data directly. One is where one assumes an arbitrary parametrization for the equation of state of dark energy, \( w(z) \), the potential of dark energy, \( V(z) \), the Hubble parameter, \( H(z) \), or the luminosity distance \( d_L(z) \) with some arbitrary constants. By determining these constants from the observational data, we can obtain the evolving properties of dark energy. Different ways of parametrizing have been discussed in [11]–[13]. The other is the no-parameter methods, which usually involve directly smoothing either \( d_L \), or some other quantity with some characteristic smoothing scale. Currently there are many different models for implementing this approach [14]–[16].

Recently, on the basis of smoothing the noise of supernova data over redshift, the authors in [15, 16] suggested a no-parameter method in a model independent manner to reconstruct the expansion history of our universe and the evolving properties of dark energy. In [16] two kinds of supernova data: the 182 Gold data set and 115 SNLS data set, are used firstly to reconstruct the Hubble parameter \( h(z) \) \( (h(z) = H(z)/H_0) \) and the deceleration parameter \( q(z) \). It was found that both data sets give \( q(0) < 0 \), which means that the universe is undergoing an accelerating expansion, while the Gold set seems to favour a later entering of this accelerating era than the SNLS one. However Gold and SNLS give a good consistent constraint on the present matter density parameter \( \Omega_m0 \) \( (\Omega_m0 \approx 0.276 \pm 0.023) \) when combined with the baryonic acoustic oscillation peak obtained from the large scale correlation function of a luminous red galaxy in the Sloan Digital Sky Survey (SDSS) [17]. The \( w(z) \) was also discussed with both kinds of SNe Ia data sets, and it was found, in agreement with what was obtained in [18]–[20] using some parametrized models, that the Gold slightly favours a dynamically evolving dark energy with a crossing of a phantom divide line while the SNLS does not. However, in this method, the present value of the Hubble parameter, \( H_0 \), is needed beforehand or should
be marginalized over. Since the values of $H_0$ from different observation data seem to be inconsistent and doing the marginalization wastes computer resources, in this paper, we firstly generalize this model independent approach to eliminate the impact of $H_0$ and then reconstruct the cosmic expansion history, $H(z)$, the deceleration parameter, $q(z)$, and the equation of state for dark energy $w(z)$ from the new ESSENCE Sne Ia data. Besides the 162 data points given in table 9 in [6], which contains 60 ESSENCE Sne Ia, 57 SNLS Sne Ia and 45 nearby Sne Ia, we add 30 Sne Ia detected at $0.216 < z < 1.755$ by the Hubble Space Telescope [4].

2. The method

Following a well known procedure in the analysis of large scale structure, Shafieloo et al [15, 16] use a Gaussian smoothing function rather than the top hat smoothing function to smooth the noise of the Sne Ia data directly. In order to obtain important information on interesting cosmological parameters expediently, $\ln d_L(z)$ rather than the luminosity distance $d_L(z)$ or distance modulus $\mu(z)$ is studied by the following iterative method:

$$\ln d_L(z)_n^s = \ln d_L(z)_{n-1}^s + N(z) \sum_i (\ln d_L^{\text{obs}}(z_i) - \ln d_L(z_i)_{n-1}^s) \exp \left[ -\frac{\ln^2((1+z)/(1+z_i))}{2\Delta^2} \right],$$

with a normalization parameter

$$N(z)^{-1} = \sum_i \exp \left[ -\frac{\ln^2((1+z)/(1+z_i))}{2\Delta^2} \right].$$

In equations (1) and (2) $\Delta$ is a quantity needed to be given beforehand. Since a large value of $\Delta$ leads to a smooth result but depresses the accuracy of reconstruction, and inversely for a small value of $\Delta$, it is important to choose a reasonable value of $\Delta$. Here, as in [16], we choose $\Delta = 0.6$. In equation (1), $d_L(z)_n^s$ represents the smoothed luminosity distance at any redshift $z$ after $n$ iterations. When $n = 1$, $d_L(z)_0^s$ denotes a guessed background
model and it has been shown that the results are not sensitive to the chosen value of $\Delta$ and the assumed initial guessed model [16]. In this paper we use a wCDM model with $w = -0.9$ and $\Omega_m0 = 0.28$ as this guessed background model. $\ln d_L^{\text{obs}}(z_i)$ is observed from the Sne Ia and can be expressed as

$$
\ln d_L^{\text{obs}}(z_i) = \ln 10^{[\mu^{\text{obs}}(z_i) - 42.38]} + \ln h \equiv \ln f^{\text{obs}}(z_i) + \ln h. \quad (3)
$$

Here $h = H_0/100$ and $\mu^{\text{obs}}$ is the observed distance modulus of Sne Ia. Apparently using the above method the nuisance parameter $h$ needs to be given beforehand or marginalized over. Now we generalize this method to eliminate the impact of $h$. Substituting equation (3) into equation (1), we obtain that

$$
\ln d_L(z)_n^s = \ln d_L(z)_{n-1}^s + N(z) \sum_i (\ln f^{\text{obs}}(z_i)
- \ln d_L(z_i)_{n-1}^s) \exp \left[ -\frac{\ln^2((1 + z)/(1 + z_i))}{2\Delta^2} \right] + \ln h. \quad (4)
$$

If defining $\ln d_L(z)_n^s = \ln f(z)_n^s + \ln h$, it is easy to see that

$$
\ln f(z)_n^s = \ln f(z)_{n-1}^s + N(z) \sum_i (f^{\text{obs}}(z_i) - f(z)_{n-1}^s) \exp \left[ -\frac{\ln^2((1 + z)/(1 + z_i))}{2\Delta^2} \right]. \quad (5)
$$
Figure 3. The reconstructed evolutionary curves of the deceleration parameter \(q(z)\) with the likelihood within 1\(\sigma\). The red line is the best recovered result.

When \(n = 1\),

\[
\ln f(z)_i^* = \ln f(z)_0^* + N(z) \sum_i (\ln f^{\text{obs}}(z_i) - \ln f(z)_0) \exp \left[ -\frac{\ln^2((1+z)/(1+z_i))}{2\Delta^2} \right] = \ln d_L(z)_0^* + N(z) \sum_i (\ln f^{\text{obs}}(z_i) - \ln d_L(z)_0^*) \exp \left[ -\frac{\ln^2((1+z)/(1+z_i))}{2\Delta^2} \right].
\]

Here \(d_L(z)_0^*\) is the luminosity distance of the suggested background model. Unlike [16], with iteration of equation (1), we use equation (5) to obtain the smoothed results. The advantage of doing so is that the result is independent of \(h\) (or \(H_0\)). In order to determine whether we obtain a best fit model after some iteration, we calculate, after each iteration, \(\chi^2\):

\[
\chi^2_n = \sum_i \frac{(\mu(z_i)_n - \mu^{\text{obs}}(z_i))^2}{\sigma_{\mu^{\text{obs}},i}^2}.
\]

Once this \(\chi^2\) reaches its minimum value we stop the iterative process and get the best fit result.

By differentiating the smoothed luminosity distance we can find the Hubble parameter, \(H(z)\) (not \(h(z)\)):

\[
H(z) = \left[ \frac{\mathrm{d}}{\mathrm{d}z} \left( \frac{100 f(z)}{1+z} \right) \right]^{-1},
\]
Figure 4. The constraint on $\Omega_{m0}$ from the combination of 192 Sne Ia and BAO data. The red and black lines show the derived value of $A/\sqrt{\Omega_{m0}}$, from the 192 Sne Ia data set within $1\sigma$, and the blue lines are the results from the BAO data.

which contains the information on $H_0$. Then the deceleration parameter $q(z)$ of the universe and the equation of state $w(z)$ of dark energy can be obtained:

$$q(z) = (1 + z) \frac{H'(z)}{H(z)} - 1,$$

$$w(z) = \frac{2(1 + z)/3H'/H - 1}{1 - (H_0/H)^2\Omega_{m0}(1 + z)^2}.$$

3. The results

The ESSENCE programme (Equation of State: Supernovae Trace Cosmic Expansion: an NOAO survey programme) is designed to measure the history of cosmic expansion over the past five billion years. The four-year data were released in [6], which contains 60 Sne Ia points. Here we use the 162 data points given in table 9 in [6], which contains 60 ESSENCE Sne Ia, 57 SNLS Sne Ia and 45 nearby Sne Ia. In addition, as in [21], we add 30 Sne Ia detected at $0.216 < z < 1.755$ by the Hubble Space Telescope [4].

Using these 192 Sne Ia data points, we find that when $n = 42$ a minimum value of $\chi^2$ is obtained which can be seen from figure 1. In figure 2 we show the reconstructed result for the Hubble parameter $H(z)$ with the likelihood within $1\sigma$. The red line is the best fit result and when $z = 0$ the best fit value of $H_0$ is $H_0 = 65.5$. Figure 3 shows the evolving curves of reconstructed $q(z)$ with the $1\sigma$ error bar. It is easy to see that the universe is undergoing an accelerating expansion since the present value of $q(z)$ is less than zero, and
Figure 5. The reconstructed evolutionary curves of the equation of state of dark energy, $w(z)$, within 1σ with a marginalization over $\Omega_{m0} = 0.278^{+0.024}_{-0.023}$.

The phase transition from deceleration to acceleration occurs at redshift $z \sim 0.55–0.73$ within 1σ, which is slightly later than that obtained from the Gold set ($z \sim 0.38–0.48$) but earlier than that from the SNLS set ($z > 0.7$) [16].

Figure 4 shows the constraint on the present matter density parameter $\Omega_{m0}$ with $H_0 = 65.5$ by combining the ESSENCE SNe Ia and baryonic acoustic oscillation (BAO) peak obtained from the large scale correlation function of a luminous red galaxy from the Sloan Digital Sky Survey (SDSS). For BAO data we use a model independent dimensionless parameter $A$ defined as

$$A = \frac{\sqrt{\Omega_{m0}}}{h(z_1)} \left[ \frac{1}{z_1} \int_0^{z_1} \frac{dz}{h(z)} \right]^{2/3},$$

(11)

for a flat universe, where $z_1 = 0.35$ and $A$ is measured to be $A = 0.469(n/0.96)^{-0.35} \pm 0.017$ [17]. Here $n$ is the spectral index of the primordial power spectrum and WMAP3 gives $n = 0.951$ [22]. Clearly the ESSENCE SNe Ia and BAO give a strong constraint on $\Omega_{m0}$. At the 1σ confidence level we obtain $\Omega_{m0} = 0.278^{+0.024}_{-0.023}$, which is highly consistent with the values obtained from Gold + BAO and SNLS + BAO ($\Omega_{m0} = 0.276 \pm 0.023$) [16].

In figure 5 we plot the evolving behaviour of $w(z)$ with a marginalization of $\Omega_{m0}$ over $\Omega_{m0} = 0.278^{+0.024}_{-0.023}$. The best fit (red) line shows that the ESSENCE data slightly favour an evolving dark energy with a crossing of a phantom divide line in the near past; however this evolving property is weaker than that obtained from Gold data but stronger than that from the SNSL set obtained in [16]. In addition, from figures 3 and 5 we find that the stringent constraint on $w(z)$ and $q(z)$ happens around redshift $z \sim 0.5$, which is consistent with the values obtained in [20, 23] with some parametrized models.
4. Conclusion

In this paper, with a model independent method we have reconstructed the cosmic expansion history and the properties of dark energy from recent ESSENCE Sne Ia data. We firstly obtain the evolution of $H(z)$. Then the cosmic deceleration parameter $q(z)$ and the equation of state $w(z)$ of dark energy are reconstructed. Our results show that their evolutionary property reconstructed from ESSENCE data is weaker than that from the Gold set, but is stronger than that from the SNSL one. Combining the ESSENCE Sne Ia and the BAO data, a tight constraint on $\Omega_{m0}$ is obtained. At a 1$\sigma$ confidence level $\Omega_{m0} = 0.278^{+0.024}_{-0.023}$, which is highly consistent with the values obtained from Gold + BAO and SNLS + BAO. Remarkably, like those obtained with some parametrized models [20, 23], the tight constraints on $w(z)$ and $q(z)$ seem to happen at about $z \sim 0.5$.

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References

[1] Perlmutter S et al, 1999 Astrophys. J. 517 565 [SPIRES]
[2] Riess A G et al, 1998 Astron. J. 116 1009 [SPIRES]
[3] Riess A G et al, 2004 Astrophys. J. 607 665 [SPIRES]
[4] Riess A G et al, 2007 Astrophys. J. 659 98
[5] Astier P et al, 2006 Astron. Astrophys. 447 31 [SPIRES]
[6] Wood-Vasey W M et al, 2007 Preprint astro-ph/0701041
[7] Padmanabhan T, 2006 AIP Conf. Proc. 861 179 [SPIRES]
[8] Copeland E J, Sami M and Tsujikawa S, 2006 Int. J. Mod. Phys. D 15 1753 [SPIRES]
[9] Sahni V and Starobinsky A, 2006 Int. J. Mod. Phys. D 15 2105 [SPIRES]
[10] Perivolaropoulos L, 2006 Preprint astro-ph/0601014
[11] Starobinsky A A, 1998 JETP Lett. 68 757
Huterer D and Turner M S, 1999 Phys. Rev. D 60 081301 [SPIRES]
Astier P, 2000 Preprint astro-ph/0008306
Chiba T and Nakamura T, 2000 Phys. Rev. D 62 121301 [SPIRES]
Weller J and Albrecht A, 2002 Phys. Rev. D 65 103512 [SPIRES]
Maor I, Brustein R, McMahon J and Steinhardt P J, 2002 Phys. Rev. D 65 123003 [SPIRES]
Chevallier M and Polarski D, 2001 Int. J. Mod. Phys. D 10 213 [SPIRES]
Linder E V, 2003 Phys. Rev. Lett. 90 091301 [SPIRES]
Jassal H K, Bagla J S and Padmanabhan T, 2005 Mon. Not. R. Astron. Soc. 356 L11
[12] Sahni V, Saini T D, Starobinsky A A and Alam U, 2003 JETP Lett. 77 201
Alam U, Sahni V, Saini T D and Starobinsky A A, 2003 Mon. Not. R. Astron. Soc. 344 1057
Alam U, Sahni V, Saini T D and Starobinsky A A, 2004 Preprint astro-ph/0406672
Saini T D, Raychaudhury S, Sahni V and Starobinsky A A, 2000 Phys. Rev. Lett. 85 1162 [SPIRES]
Saini T D, Weller J and Bridle S L, 2004 Mon. Not. R. Astron. Soc. 348 603
Alam U, Sahni V and Starobinsky A A, 2004 J. Cosmol. Astropart. Phys. JCAP06(2004)008 [SPIRES]
Alam U, Sahni V, Saini T D and Starobinsky A A, 2004 Mon. Not. R. Astron. Soc. 354 275
[13] Gerke B and Efstathiou G, 2002 Mon. Not. R. Astron. Soc. 335 33
Maor I, Brustein R, McMahon J and Steinhardt P J, 2002 Phys. Rev. D 65 123003 [SPIRES]
Corasaniti P S and Copeland E J, 2003 Phys. Rev. D 67 063521 [SPIRES]
Wang Y and Mukherjee P, 2004 Astrophys. J. 606 654 [SPIRES]
Nesseris S and Perivolaropoulos L, 2004 Phys. Rev. D 70 043531 [SPIRES]
Roy Choudhury T and Padmanabhan T, 2005 Astron. Astrophys. 429 807 [SPIRES]
Gong Y, 2005 Int. J. Mod. Phys. D 14 599 [SPIRES]
Wetterich C, 2004 Phys. Lett. B 594 17 [SPIRES]
Wu P and Yu H, 2006 Phys. Lett. B 643 315 [SPIRES]
Guo Z K, Ohta N and Zhang Y Z, 2005 Phys. Rev. D 72 023504 [SPIRES]
Guo Z K, Ohta N and Zhang Y Z, 2007 Mod. Phys. Lett. A 22 883 [SPIRES]
Simon J, Verde L and Jimenez R, 2005 Phys. Rev. D 71 123001 [SPIRES]

Wang Y and Lovelace G, 2001 Astrophys. J. 562 L115 [SPIRES]
Saini T D, 2003 Mon. Not. R. Astron. Soc. 344 129
Daly A and Djorgovsky S G, 2003 Astrophys. J. 597 9 [SPIRES]
Daly R A and Djorgovsky S G, 2004 Astrophys. J. 612 652 [SPIRES]
Daly R A and Djorgovsky S G, 2006 Preprint astro-ph/0609791
Wang Y and Tegmark M, 2004 Phys. Rev. Lett. 92 241302 [SPIRES]
Wang Y and Tegmark M, 2005 Phys. Rev. D 71 103513 [SPIRES]
Daly R A and Djorgovsky S G, 2005 Preprint astro-ph/0512576
Huterer D and Cooray A, 2005 Phys. Rev. D 71 023506 [SPIRES]
Huterer D and Starkman G, 2003 Phys. Rev. Lett. 90 031301 [SPIRES]
Fay S and Tavakol R, 2006 Phys. Rev. D 74 083513 [SPIRES]

Shafieloo A, Alam U, Sahni V and Starobinsky A, 2006 Mon. Not. R. Astron. Soc. 366 1081
Shafieloo A, 2007 Mon. Not. R. Astron. Soc. 380 1573
Eisenstein D J et al, 2005 Astrophys. J. 633 560 [SPIRES]
Alam U, Sahni V and Starobinsky A, 2007 J. Cosmol. Astropart. Phys. JCAP01(2007)018 [SPIRES]
Wu P and Yu H, 2007 J. Cosmol. Astropart. Phys. JCAP10(2007)014 [SPIRES]
Barger V, Gao Y and Marfatia D, 2007 Phys. Lett. B 648 127 [SPIRES]
Spergel D N et al, 2007 Astrophys. J. Suppl. 170 377
Gong Y G, 2005 Class. Quantum Grav. 22 2121 [SPIRES]
Gong Y G and Zhang Y Z, 2005 Phys. Rev. D 72 043518 [SPIRES]
Chevallier M and Polarski D, 2001 Int. J. Mod. Phys. D 10 213 [SPIRES]
Gong Y and Wang A, 2007 Phys. Rev. D 75 043520 [SPIRES]