Emergence of two-phase behavior in markets through interaction and learning in agents with bounded rationality

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Phenomena which involves collective choice of many agents who are interacting with each other and choosing one of several alternatives, based on the limited information available to them, frequently show switching between two distinct phases characterized by a bimodal and an unimodal distribution respectively. Examples include financial markets, movie popularity and electoral behavior. Here we present a model for this biphasic behavior and argue that it arises from interactions in a local neighborhood and adaptation & learning based on information about the effectiveness of past choices.

1 Introduction

The behavior of markets and other social agglomerations are made up of the individual decisions of agents, choosing among a number of possibilities open to them at a given time. Let us consider the example of binary choice, where the agent can make one of two possible decisions, e.g., to buy or to sell. If each agent makes a choice completely at random, the outcome will be an unimodal distribution, a Gaussian to be precise, of the collective choice (i.e., the sum total of all the individual decisions), at whose mean value the distribution will have its peak. In our example this implies that, on the average, there are equal numbers of buyers and sellers.

However, empirical data in financial markets [1, 2], movie popularity [3] and electoral behavior [4] indicate that there is another phase, corresponding to the agents predominantly choosing one option over the other. This is reflected in a bimodal distribution of the collective choice (Fig. 1).

To account for this we argue that, in a society, agents make choices based on their personal beliefs as well as opinions of their neighbors about the possible outcomes of a choice. These beliefs are not fixed but evolve over time according to changing circumstances, based on previous choices (adaptation)
Fig. 1. Examples of empirical bimodal distributions. (Left) The distribution of opening week gross earning, $G_O$ (scaled by the average value for a particular year, $<G_O>$) for movies released in the USA during the period 1999-2003. The different symbols correspond to individual years, while the curve represents the average over the entire period. (Right) Frequency histogram of vote share for the Democratic Party candidate in each House district at the 2000 US Federal House of Representatives election.

and how they accorded with those of the majority (learning). We propose a model of collective choice dynamics where each agent has two variables associated with it, one corresponding to its choice and the other corresponding to its belief regarding the possible outcome of the choice.

The bounded rationality of the agents in our model is due to the limited information available to the agent at a given point of time. However, subject to this constraint, the agent behaves deterministically. One of the striking observations obtained from the model is that although each agent may behave rationally and change their beliefs (and hence their choices) periodically, the collective choice may get polarized and remain so for extremely long times (e.g., the entire duration of the simulation).

2 The Model

Our model is defined as follows. Consider a population of $N$ agents, each of whom can be in one of two choice states $S = \pm 1$ (e.g., to buy or to sell, to vote Party A or Party B, etc.). In addition, each agent has a personal preference or belief, $\theta$, that is chosen from a uniform random distribution initially. At each time step, every agent considers the average choice of its neighbors at the previous instant, and if this exceeds its belief, makes the same choice; otherwise, it makes the opposite choice. Then, for the $i$-th agent, the choice dynamics is described by:

$$S_i^{t+1} = \text{sign}(\sum_{j \in N} J_{ij} S_j^t - \theta_i^t),$$

(1)

where $N$ is the set of neighbors of agent $i$ ($i = 1, \ldots, N$), and $\text{sign}(z) = +1$, if $z > 0$, and $= -1$, otherwise. The coupling coefficient among agents, $J_{ij}$, is
assumed to be a constant (= 1) for simplicity and normalized by $z (= |\mathcal{N}|)$, the number of neighbors. In a lattice, $\mathcal{N}$ is the set of spatial nearest neighbors and $z$ is the coordination number.

The individual belief $\theta$ in turn evolves, being incremented or decreased at each time step, according to the agent’s choice:

$$
\theta_{i}^{t+1} = \theta_{i}^{t} + \mu S_{i}^{t+1} + \lambda S_{i}^{t}, \text{ if } S_{i}^{t}M^{t} < 0,
= \theta_{i}^{t} + \mu S_{i}^{t+1}, \text{ otherwise},
$$

where $M^{t} = (1/N)\sum_{j} S_{j}^{t}$ is the collective choice of the entire community at time $t$. The adaptation parameter $\mu$ is a measure of how frequently an agent switches from one decision to another. Belief also changes according to whether the previous choice agreed with the majority decision. In case of disagreement, the belief is increased/decreased by a quantity $\lambda$ that measures the relative importance of global feedback (e.g., through information obtained from the media). The desirability of a particular choice is assumed to be related to the fraction of agents in the community choosing it; hence, at any given time, every agent is trying to coordinate its choice with that of the majority.

3 Results

Although some analytical results can be obtained under mean field theory, here we present only numerical simulation results for the case where the agents are placed on a two-dimensional regular lattice (see Ref. [5] for details). Note that, in absence of either adaptation or global feedback ($\mu = \lambda = 0$) the model reduces to the well-studied random field Ising model.

![Fig. 2. (Left) Probability distribution of the collective choice $M$ at different values of the global feedback parameter $\lambda$. A phase transition from a bimodal to an unimodal distribution occurs as $\lambda \to 0$. The simulation results shown are for $100 \times 100$ agents interacting in a 2-D lattice for 50,000 iterations. The adaptation rate is $\mu = 0.1$. Compare with Fig. 1a in Ref. [1]. (Right) Spatial pattern in the choice behavior for $1000 \times 1000$ agents interacting in a square lattice after $10^5$ iterations with $\mu = 0.1$ and $\lambda = 0.05$.](image)
In the presence of adaptation but absence of learning ($\mu > 0, \lambda = 0$), starting from an initial random distribution of choices and personal preferences, we observe only very small clusters of similar choice behavior and the average choice $M$ fluctuates around 0. In other words, at any given time equal number of agents have opposite choice preferences (on an average). Introduction of learning in the model ($\lambda > 0$) gives rise to significant clustering as well as a non-zero value for the collective choice $M$. We observe a phase transition of the probability distribution of $M$ from an unimodal to a bimodal form as a result of the competition between the adaptation and global feedback effects (Fig. 2 (left)).

The collective choice switches periodically between a positive value and a negative value, having an average residence time which diverges with $\lambda$ and with $N$. For instance, when $\lambda$ is very high relative to $\mu$, $M$ gets locked into one of two states (depending on the initial condition), corresponding to the majority preferring either one or the other choice. This is reminiscent of lock-in in certain economic systems subject to positive feedback [6]. The existence of long-range correlations in the choice of agents in the bimodal phase often results in striking spatial patterns such as vortices and spiral waves [Fig. 2 (right)] after long times. These patterns often show the existence of multiple domains, with the behavior of agents belonging to a particular domain being highly correlated and slaved to the choice behavior of an “opinion leader”.

We have also introduced partial irrationality in the model by making the choice dynamics stochastic. Each agent may choose the same as or opposite to that of its neighbors if their overall decision exceeds its personal belief, according to a certain probability function with a parameter $\beta$ that is a measure of the degree of reliability that an agent assigns to the information it receives. For $\beta \to 0$, the agent ignores all information and essentially chooses randomly; in this case, expectedly, the distribution becomes unimodal. Under mean-field theory, one sees that the bimodal distribution occurs even for $\lambda = 0$ as $\beta \to \infty$; however, as $\beta$ is gradually decreased a phase transition to the unimodal distribution is observed.

4 Discussion and Summary

Our model seems to provide an explanation for the observed bimodality in a large number of social or economic phenomena, e.g., in the initial reception of movies, as shown in the distribution of the opening gross of movies released in theaters across the USA during the period 1997-2003 [3]. Bimodality in this context implies that movies either achieve significant success or are dismal box-office failures initially. We have considered the opening, rather than the total, gross for our analysis because the former characterizes the uncertainty faced by the moviegoer (agent) whether to see a newly released movie, when there is very little information available about its quality. Based on the model presented here, we conclude that, in such a situation the moviegoers’ choice depends not only on their neighbors’ choice about this movie, but also on how well previous action based on such neighborhood information agreed with
media reports and reviews of movies indicating the overall or community choice. Hence, the case of $\lambda > 0$, indicating the reliance of an individual agent on the aggregate information, imposes correlation among agent choice across the community which leads to bimodality in the opening gross distribution.

Our model also provides justification for the two-phase behavior observed in the financial markets wherein volume imbalance clearly shows a bimodal distribution beyond a critical threshold of local noise intensity [1]. In contrast to many current models, we have not assumed a priori existence of contrarian and trend-follower strategies among the agents [7]. Rather such behavior emerges naturally from the micro-dynamics of agents’ choice behavior.

Similar behavior possibly underlies biphasic behavior in election results. The distribution of votes in a two-party election will show an unimodal pattern for elections where local issues are more important than the role of the mass media (hence $\lambda = 0$) and a bimodal distribution for elections where voters are more reliant on media coverage for individual-level voting cues ($\lambda > 0$).

One can also tailor marketing strategies to different segments of the population depending on the role that global feedback plays in their decisions. Products, whose consumers have $\lambda = 0$, can be better disseminated through distributing free samples in neighborhoods; while for $\lambda > 0$, a mass media campaign blitz will be more effective.

In summary, we have presented here a model of the emergence of collective choice through interactions between agents who are influenced by their personal preferences which change over time through processes akin to adaptation and learning. We find that introducing these effects produce a two-phase behavior, marked by an unimodal distribution and a bimodal distribution of the collective choice, respectively. Our model explains very well the observed two-phase behavior in markets, not only in the restricted context of financial markets, but also, in a wider context, movie income and election results.

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