Applying the approximation method PAINT and the interactive method NIMBUS to the multiobjective optimization of operating a wastewater treatment plant

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Using an interactive multiobjective optimization method called NIMBUS and an approximation method called PAINT, preferable solutions to a five-objective problem of operating a wastewater treatment plant are found. The decision maker giving preference information is an expert in wastewater treatment plant design at the engineering company Pöyry Finland Ltd. The wastewater treatment problem is computationally expensive and requires running a simulator to evaluate the values of the objective functions. This often leads to problems with interactive methods as the decision maker may get frustrated while waiting for new solutions to be computed. Thus, a newly developed PAINT method is used to speed up the iterations of the NIMBUS method. The PAINT method interpolates between a given set of Pareto optimal outcomes and constructs a computationally inexpensive mixed integer linear surrogate problem for the original wastewater treatment problem. With the mixed integer surrogate problem, the time required from the decision maker is comparatively short. In addition, a new IND-NIMBUS\textsuperscript{®} PAINT module is developed to allow the smooth interoperability of the NIMBUS method and the PAINT method.

Keywords: multiple objective programming; OR in natural resources; productivity and competitiveness; simulation

1. Introduction

Many real-life problems concern multiple conflicting objectives. This means that it is not possible to find a single solution that is the best with respect to all of these objectives. Multiobjective optimization is a scientific method for dealing with these kind of problem. A general formulation for a multiobjective optimization problem with \( k \) objectives \((k \in \mathbb{N})\) is

\[
\min(f_1(x), \ldots, f_k(x)) \\
s.t. \ x \in S,
\]

where \( f_i \) are the objective functions and \( S \) is the feasible set. A vector \( x \in S \) is called a (feasible) solution. Instead of a single optimal solution that exists typically in single-objective optimization, there usually exist multiple Pareto optimal solutions to a multiobjective optimization problem. A solution \( x \in S \) is said to (Pareto) dominate another solution \( y \in S \), if \( f_i(x) \leq f_i(y) \) for all \( i = 1, \ldots, k \).

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and $f_j(x) < f_j(y)$ for at least one $j \in \{1, \ldots, k\}$. A solution $x^* \in S$ is Pareto optimal if there does not exist a solution $x \in S$ that dominates it. A vector $z = (f_1(x), \ldots, f_k(x))$ with $x \in S$ is called an outcome, and an outcome is called Pareto optimal if it is given by a Pareto optimal solution. The set of Pareto optimal outcomes is called the Pareto front.

Typically, only one preferable solution from the many Pareto optimal ones has to be chosen for implementation or only a small set of different preferable Pareto optimal solutions is sought. Finding this one or the small set of Pareto optimal solutions requires preference information about the objectives of the problem. In multiobjective optimization, it is often assumed that there exists a decision maker who is an expert in the application area and who is prepared to answer questions concerning those preferences. In this article, this whole process of finding the preferable solution(s) is called solving the multiobjective optimization problem and, when the decision maker’s involvement is emphasized, it is also referred to as the decision making process.

The type of information that is asked from the decision maker depends on the multiobjective optimization method that is used to solve the problem. Different types of multiobjective optimization methods (as categorized in Hwang and Masud [1979] and Miettinen [1999]) are no-preference methods, *a priori* methods, *a posteriori* methods and interactive methods. In no-preference methods, the decision maker is not asked any questions, but just one compromise solution is found according to some preset criteria. In *a priori* methods, the decision maker is first asked for preference information and then the best solution according to those preferences is found. The difficulty with *a priori* methods is that the decision maker may find it hard to define preferences without ever seeing feasible or Pareto optimal solutions. In *a posteriori* methods, a representative set of the Pareto optimal solutions is found from which the decision maker is allowed to choose. The difficulty with *a posteriori* methods is that generating a large enough solution set may be time-consuming and choosing from a large set of solutions may be hard, see e.g. Larichev (1992).

In this article, the ideology of interactive methods in solving multiobjective optimization problems is followed. In interactive methods, it is understood that any preference information given by the decision maker is only partial and perhaps flawed. Thus, the decision maker is allowed to explore the Pareto optimal solutions by guiding the interactive method. This allows the decision maker to learn about the problem (as argued e.g. in Miettinen, Ruiz, and Wierzbicki [2008]) and find a preferred solution without examining too many solutions. For more information about interactive methods, see e.g. Miettinen (1999) and Miettinen, Ruiz, and Wierzbicki (2008). More specifically, the so-called synchronous version of the interactive NIMBUS method (Miettinen and Mäkelä 1995, 2000, 2006) is used.

Some multiobjective optimization problems are computationally expensive, see e.g. Xu et al. (2004), Hasenjäger and Sendhoff (2005), Aittokoski and Miettinen (2008) and Hakanen, Miettinen, and Sahlstedt (2011, 2013). This may be caused, for example, by the need to use computationally expensive simulations for evaluating the objective functions. Interactive methods have an advantage over *a posteriori* methods in solving computationally expensive problems, because the decision maker may guide the search in interactive methods and, thus, potentially fewer solutions need to be computed. There is, however, a drawback. When using interactive methods, the decision maker has to wait while new solutions are computed with respect to his/her updated preferences. For computationally expensive problems, this may take a long time, which may be frustrating for the decision maker (as argued, e.g., in Korhonen and Wallenius [1996]).

In order to compute new solutions faster within the interactive method, one has to develop new approaches and one approach is using approximations. Two different approximation schemes can be identified: approximating the objective functions and approximating the Pareto front. The objective functions may be approximated with, for example, meta-models like the response surface methodologies, support vector machines or radial basis functions, see e.g. Nakayama, Yun, and Yoon (2009). These have been used in multiobjective optimization in, for example,
Wilson et al. (2001) and Nakayama, Yun, and Yoon (2009). However, this is not a straightforward task, because the approximation itself becomes a very computationally expensive task as the numbers of decision variables and objectives increase. Another approach is approximating the Pareto front. Pareto front approximations can be found in, for example, Lotov, Bushenkov, and Kamenev (2004), Ruzika and Wiecek (2005), Bezerkin, Kamenev, and Lotov (2006), Ackermann et al. (2007), Eskelinen et al. (2010), Monz et al. (2008) and Yapicioglu et al. (2011), where Lotov, Bushenkov, and Kamenev (2004), Ackermann et al. (2007), Eskelinen et al. (2010) and Monz et al. (2008) include decision making aspects connected to these. Note that, in this article, there is a difference between a Pareto front representation (a discrete set of Pareto optimal outcomes) and a Pareto front approximation (something more approximate that may contain vectors that are not outcomes of the problem, but merely approximate them).

In this article, the Pareto front approximation method PAINT (Hartikainen, Miettinen, and Wiecek 2011a, 2011b, 2012) is used to solve a computationally expensive five-objective optimization problem that models operating a wastewater treatment plant. It is shown that the PAINT method can indeed be used to speed up the iterations of an interactive method when solving computationally expensive multiobjective optimization problems. Here, the value added by the Pareto front approximation PAINT can clearly be seen: the same wastewater treatment problem has already been treated in Hakanen, Miettinen, and Sahlstedt (2013) using an interactive method, but without the PAINT method. The PAINT approximation method is application independent and can, thus, be applied to other problems also.

The PAINT method approximates the Pareto front by interpolating between a given set of Pareto optimal outcomes. The special feature of the PAINT method is that it uses a novel way to integrate knowledge about Pareto dominance into the approximation. The approach also differs from most of the other approaches for decision making with Pareto front approximations because it is also able to approximate non-convex Pareto fronts. Furthermore, the Pareto front approximation constructed with PAINT implies a multiobjective mixed integer linear surrogate problem (for the original problem) that can be solved with any applicable interactive method. A majority of the other approaches are either applicable only to convex multiobjective optimization problems or use only a custom-made procedure for choosing a preferred point on the approximation, see e.g. Hartikainen, Miettinen, and Wiecek (2011a).

As for the interactive method, the NIMBUS method and its implementation in the IND-NIMBUS® software framework (Miettinen and Mäkelä 1995, 2000, 2006) are used. The same method was used in Hakanen, Miettinen, and Sahlstedt (2013) to study the same wastewater treatment problem. Compared to the previous study in Hakanen, Miettinen, and Sahlstedt (2013), the iterations of the interactive method NIMBUS were much faster because of a surrogate problem constructed with the PAINT method. In this study, the decision maker was the same person from the engineering company Pöyry Finland Ltd. as in Hakanen, Miettinen, and Sahlstedt (2013), and thus the decision maker could compare his experiences of using the NIMBUS method to solve the problem with and without the PAINT method. This comparison of the two approaches gives a unique perspective for the study. In this article, a new IND-NIMBUS® PAINT module is developed. The module allows smooth interoperability of the PAINT approximation method and the NIMBUS method.

The aim of this article is two-fold: (1) to illustrate with an example how interactive multiobjective optimization can be applied to computationally expensive multiobjective optimization problems by using the PAINT approximation method, and (2) to find new preferable ways to operate a wastewater treatment plan, and further to learn about the problem and the interdependencies of the objectives. The approach taken in this article (i.e. using the PAINT approximation method and an interactive method together) can be directly applied in the future to solving other computationally expensive multiobjective optimization problems. This approach is also called the PAINT approach to solving computationally expensive problems in this article.
The structure of this article is as follows. After introducing the study in this section, the multiobjective optimization problem of operating a wastewater treatment plant is presented in Section 2. In Section 3, relevant details concerning the PAINT method and the PAINT approach to solving computationally expensive problems are given. The new IND-NIMBUS® PAINT module (also called the PAINT module for short) is presented in Section 4. In Section 5, it is shown how the PAINT method was used to construct a Pareto front approximation and a mixed integer linear surrogate problem for the wastewater treatment problem. In Section 5, the decision maker’s involvement in solving the problem with the PAINT module is described in detail. Finally, in Section 6, conclusions are given.

2. Operating a wastewater treatment plant

Operating a wastewater treatment plant is a complex problem with many conflicting objectives that have to be considered at the same time. In this article, a plant using the so-called activated sludge process, which is globally the most common method of wastewater treatment, is considered. The problem is modelled as a five-objective optimization problem, which was also studied previously in Hakanen, Miettinen, and Sahlstedt (2013). The approach of this article differs from the approach of Hakanen, Miettinen, and Sahlstedt (2013) because the PAINT method is used to approximate the Pareto front and to construct a surrogate problem for the original problem. In this way, the time that the decision maker has to wait while using an interactive method becomes shorter.

Figure 1 shows the schematic layout of the wastewater treatment plant considered. The wastewater treatment begins with grit removal. After grit removal, solids are separated by gravitational settling. Raw and mixed sludge removed from the primary settlers are fermented in a separate reactor and partly recycled back to the water line to provide a readily biodegradable carbon source for denitrification. The bioreactor consists of four anoxic zones, three aerobic zones and one deoxygenation zone. Nitrate-rich activated sludge is recycled from zone 8 of the bioreactor to zone 1. Return sludge and primary effluent are directed to zone 1. Methanol is injected into zone 2 to support denitrification. Excess sludge is pumped from zone 8 of the bioreactor to the

![Figure 1. A schematic layout of the wastewater treatment plant.](image-url)
beginning of the water process, from which it is removed in the primary settlers together with raw sludge. Raw and mixed sludge is thickened gravitationally into approximately 4.5% total solids prior to anaerobic digestion. Anaerobic digestion produces biogas and the biogas produced can be converted into electrical or thermal energy. The digested sludge is dewatered by centrifuges into approximately 28% total solids. The reject water from sludge treatment is pumped to the beginning of the plant. The wastewater treatment process is simulated with the commercial GPS-X simulator (see GPS-X Simulator Home Page 2014) and the model is based on the findings of Pöyry Finland Ltd. The simulations are conducted as a steady-state calculation, followed by a two-day simulation assuming constant flow and loading, corresponding to average values recorded at the real plant. For more information about the wastewater treatment plants using the activated sludge process, see e.g. Phillips et al. (2009) and Hakanen, Miettinen, and Sahlstedt (2013).

The objectives of the optimization problem are the amount of nitrogen in the effluent (in g/m³, grams per cubic meter of effluent), the aeration power consumption in the activated sludge process (in kW), the chemical consumption (in g/m³, grams per cubic meter of effluent), the excess sludge production (in kg/d, kilograms per day) and the amount of biogas production (in m³/d, cubic meters per day). Minimizing the first objective is the main goal of the activated sludge process, and the four others are connected to the operational costs. This multiobjective optimization problem allows the simultaneous consideration of the performance of the plant (through the nitrogen removal rate) and different aspects of the operational costs. Naturally, the last objective is to be maximized and the others are to be minimized. The decision variables of the problem are the percentage of inflow pumped to fermentation, the amount of excess sludge removed, the dissolved oxygen setpoint in the last aerobic zone and the methanol dose. Thus, the methanol dose is both a decision variable and an objective.

Each simulation of the wastewater treatment plant of Hakanen, Miettinen, and Sahlstedt (2013) took approximately 11 seconds on the GPS-X simulator. When the problem was previously considered in Hakanen, Miettinen, and Sahlstedt (2013), 200 simulations were run to optimize the scalarizations (i.e. single objective optimization problems, whose optimal solutions are Pareto optimal solutions to the multiobjective optimization problem) given by the interactive NIMBUS method that was used to solve the problem. This meant that each iteration of the interactive method took more than half an hour. Even though interesting solutions to the problem were found in Hakanen, Miettinen, and Sahlstedt (2013), the computational time of iterations was an inconvenience to the decision maker (according to personal communications with the authors of Hakanen, Miettinen, and Sahlstedt [2013]). Therefore, there was room for improvement using an approximation method. In addition, one can deduce from the Pareto optimal solutions computed that the problem is non-convex. Thus, the PAINT method is a suitable candidate for speeding up the iterations of the interactive method.

The preferences that are used to operate a wastewater treatment plant may change over time and, in addition, they may be dependent on the location and the economic and political environment of the wastewater treatment plant. In addition, every wastewater treatment plant is unique, and results obtained at one plant (real or simulated) cannot be directly applied at any other plant. Thus, in the treatment of this article, the aim is to find a small set of different interesting and preferable ways of operating the wastewater treatment plant, instead of merely one. The person operating a wastewater treatment plant can then decide whether to use each of the operational settings that were found or merely use it as a guideline in finding his/her own operational settings.

3. The PAINT approach to solving computationally expensive problems

In this article, the term Pareto front representation refers to a finite set of Pareto optimal outcomes and (following the terminology established in Ruzika and Wiecek [2005]) the term Pareto front
The **PAINT** approach is based on the Pareto front approximation constructed by the **PAINT** method (Hartikainen, Miettinen, andWiecek 2012). The **PAINT** method is based on the concept of an inherently non-dominated Pareto front approximation (Hartikainen, Miettinen, and Wiecek 2011b) and the mathematical concepts of Hartikainen, Miettinen, and Wiecek (2011a). The **PAINT** method interpolates between a given set of Pareto optimal outcomes in such a way that the interpolants neither dominate nor are dominated by the set of given Pareto optimal outcomes and, in addition, they are not dominated by each other (i.e. the interpolation is an inherently non-dominated Pareto front approximation, as defined in Hartikainen, Miettinen, and Wiecek [2011b]). In this article, a vector on the Pareto front approximation is called an approximate (Pareto optimal) outcome.

The general functionality of the **PAINT** method is as follows. The **PAINT** method first constructs the Delaunay triangulation (see e.g. Fortune [1997]) of the given set of Pareto optimal outcomes and then chooses the appropriate polytopes from it to the Pareto front approximation. In this article, this is realized with the Octave-based implementation (see the Octave Home Page [2014] and Eaton [2002]), which was developed during the research of Hartikainen, Miettinen, and Wiecek (2012).

The Pareto front approximation constructed with the **PAINT** method implies a computationally inexpensive mixed integer linear surrogate problem for the original problem. The Pareto front of the surrogate problem is exactly the Pareto front approximation and, thus, a preferred solution to the surrogate problem implies a preferred vector on the Pareto front approximation, which is also called a preferred approximate outcome in this article. The algorithm of the **PAINT** method and more exact details can be found in Hartikainen, Miettinen, and Wiecek (2012).

Decision making in the **PAINT** approach is described in Figure 2. In the **PAINT** approach to solving computationally expensive problems, it is assumed that there exists a set of Pareto optimal solutions to the computationally expensive problem. This set may have been generated with any a posteriori method. The **PAINT** method then interpolates between the set of given Pareto optimal outcomes and outputs the interpolation that implies a mixed integer linear surrogate problem for the original problem.

After the mixed integer linear surrogate problem has been formulated, the decision maker gets involved and uses an interactive method of his/her choice to find a preferred solution to the surrogate problem. The Pareto optimal outcomes of the surrogate problem are vectors on the Pareto front approximation, which is in the same space as the original Pareto front. Thus, the decision maker is able to give his/her preferences on them. The preferred approximate outcome is projected on the actual Pareto front of the original problem by solving an achievement scalarizing problem (see Wierzbicki [1980, 1986]) with the approximate outcome as a reference point. More details on the projection can be found in Hartikainen, Miettinen, and Wiecek (2012). Projecting the solution may take time, depending on the computational cost of the problem. However, this can be done without the decision maker’s involvement and, thus, it does not inconvenience him/her too much. In addition, if the decision maker so wishes, the decision making process can concentrate on finding multiple potentially preferred approximate outcomes and the approximate outcomes are then projected afterwards.

The projection(s) of the preferred approximate outcome(s) (i.e. Pareto optimal solution(s) to the original problem) is(are) shown to the decision maker and, if he/she is satisfied, the decision making process stops, because a preferred solution has been found. If the decision maker is not
satisfied, it is possible to update the Pareto front approximation by adding the new Pareto optimal outcome(s) to the given set of Pareto optimal outcomes and by recomputing the approximation using the PAINT method. This yields a more accurate approximation and an interactive method can again be used to find a preferred solution to the new (more accurate) surrogate problem. This process can be repeated as many times as necessary.

The PAINT method is a powerful tool as it can interpolate between any given set of Pareto optimal outcomes, \textit{i.e.} the way that the outcomes have been generated does not affect the functionality of the method. In addition, since it is based on the concept of inherent non-dominance (see Hartikainen, Miettinen, and Wiecek [2011b]), it will not provide interpolants that would mislead the decision maker. Finally, the mixed integer linear surrogate problem implied by the approximation allows one to use any interactive method for finding a preferred approximate outcome on the Pareto front approximation.

However, the PAINT method has a couple of shortcomings, already noted in Hartikainen, Miettinen, and Wiecek (2012). First, the PAINT method does not provide any information about the preimage of the Pareto front approximation in the decision space. This means that the decision maker has to project the approximate outcome (\textit{i.e.} the solution to the surrogate problem) on the Pareto front of the original problem in order to find the values of the decision variables. Second, the PAINT method cannot detect any disconnectedness in the Pareto front, but always interpolates between the outcomes whenever the interpolation is inherently non-dominated. Thus, the approximation might be inaccurate if, for example, the decision space is disconnected or the objective functions are highly non-convex. Fortunately, this was not a problem in this study.

4. A new IND-NIMBUS\textsuperscript{®} PAINT module

IND-NIMBUS\textsuperscript{®} (see Miettinen [2006bib27]) is a multi-platform optimization framework, currently available for Windows and Linux operating systems, intended to provide a flexible tool-set
for the implementation of multiobjective optimization methods. So far, the IND-NIMBUS® framework has been used to implement the synchronous NIMBUS (Miettinen and Mäkelä 1995, 2000, 2006) and the Pareto Navigator (Eskelinen et al. 2010) methods. The IND-NIMBUS® software can be connected to external sources that model the problem, such as the GPS-X simulator used for modelling the wastewater treatment plant. For this article, the IND-NIMBUS® software framework has been used to develop a so-called IND-NIMBUS® PAINT module that combines the PAINT and NIMBUS methods for computationally expensive multiobjective optimization. The PAINT module implements most of the functionalities described in Figure 2.

The synchronous NIMBUS method (Miettinen and Mäkelä 1995, 2000, 2006) is an interactive multiobjective optimization method. The NIMBUS method uses the classification of objectives as the preference information. Given a Pareto optimal solution to a multiobjective optimization problem, the decision maker can classify the objectives into classes \( I_\prec, I_\leq, I_\geq \) and \( I_{\Diamond} \) defined, respectively, as classes of objective functions that the decision maker wants to improve as much as possible, wants to improve to a given aspiration level \( \text{aspi}_j \), allows to remain unchanged, allows to deteriorate until a given reservation level \( \text{res}_j \) and allows to change freely for a while. As proposed in Miettinen and Mäkelä (2006), this preference information is converted into several different single objective subproblems with the help of different scalarizations (i.e. single-objective optimization problems) that produce Pareto optimal solutions to the multiobjective optimization problem according to the preference information. The four scalarizations in IND-NIMBUS® are the so-called standard NIMBUS scalarization

\[
\min_{i \in I_\prec \cup I_\leq \cup I_=} \max_{j \in I_\geq \cup I_{\Diamond}} \left[ \frac{f_i(x) - z_{i,Nad}^\text{ideal}}{z_{i,Nad}^\text{ideal} - z_{i,\text{ideal}}} - \frac{f_j(x) - \text{aspi}_j}{z_{j,Nad}^\text{ideal} - z_{j,\text{ideal}}} \right] + \rho \sum_{i=1}^k \frac{f_i(x)}{z_{i,Nad}^\text{ideal} - z_{i,\text{ideal}}}
\]

s.t. \( f_i(x) \leq f_i(x_c) \) for all \( i \in I_\prec \cup I_\leq \cup I_= \)

\( f_i(x) \leq \text{res}_j \) for all \( i \in I_\geq \)

\( x \in S \),

the achievement scalarizing problem of Wierzbicki (1986)

\[
\min_{i=1,\ldots,k} \max_{j=1,\ldots,k} \left[ \frac{f_i(x) - \bar{z}_i}{z_{i,Nad}^\text{ideal} - \bar{z}_i} - \frac{f_j(x) - \text{aspi}_j}{z_{j,Nad}^\text{ideal} - \text{ideal}_j} \right] + \rho \sum_{i=1}^k \frac{f_i(x)}{z_{i,Nad}^\text{ideal} - \text{ideal}_i}
\]

s.t. \( x \in S \),

the STOM scalarization (Nakayama and Sawaragi 1984)

\[
\min_{i=1,\ldots,k} \max_{j=1,\ldots,k} \left[ \frac{f_i(x) - \text{ideal}_i}{\bar{z}_i - \text{ideal}_i} \right] + \rho \sum_{i=1}^k \frac{f_i(x)}{\bar{z}_i - \text{ideal}_i}
\]

s.t. \( x \in S \),

and the GUESS (Buchanan 1997) inspired scalarization

\[
\min_{i \notin I_{\Diamond}} \max_{j=1,\ldots,k} \left[ \frac{f_i(x) - z_{i,Nad}^\text{ideal}}{z_{i,Nad}^\text{ideal} - \bar{z}_i} \right] + \rho \sum_{i=1}^k \frac{f_i(x)}{z_{i,Nad}^\text{ideal} - \text{ideal}_i}
\]

s.t. \( x \in S \).

Above, the current solution is denoted by \( x_c \), the vector \( z_{\text{ideal}}^j \) is a so-called ideal vector with \( z_{i,\text{ideal}}^i = \min_{x \in S} f_i(x) \), for \( i = 1, \ldots, k \), the vector \( z_{Nad}^i \) is the so-called Nadir vector with
The above problems can be formulated in such a way that if the feasible set $S$ is defined by linear constraints, then they are linear problems, using a standard technique of linearizing min–max type optimization problems.

In the rest of this article, the components of $\bar{z}$ are called the aspiration levels (implied by the classification). In addition, the term $\rho \sum_{i=1}^{k} [f_i(x)/ (z_i^{\text{Nad}} - z_i^{\text{ideal}})]$, where $\rho > 0$ is a small constant, is called the augmentation term, which makes sure that the optimal solutions to the above problems are Pareto optimal instead of merely weakly Pareto optimal (Miettinen 1999).

The decision maker can use one or many of the above scalarizations to generate Pareto optimal solutions, which are shown to him/her who can then see how well the desired preferences could be attained. The decision maker can choose any of the solutions as the starting point of the next iteration, i.e. classification. This iterative procedure can either start with a solution given by the decision maker or from a so-called neutral compromise solution, and it is repeated until the decision maker is satisfied with the solution at hand. Further information about the synchronous NIMBUS method using other means to direct the search process is given in Miettinen and Mäkelä (2006).

The NIMBUS method has been successfully applied to the shape design of ultrasonic transducers (Heikkola, Miettinen, and Nieminen 2006), designing a paper machine head box (Hämäläinen et al. 2003), optimal control in the continuous casting of steel (Miettinen 2007), the separation of glucose and fructose (Hakanen et al. 2007), intensity modulated radiotherapy treatment planning (Ruotsalainen et al. 2009), brachytherapy (Ruotsalainen et al. 2010) and optimizing heat exchanger network synthesis (Laukkanen et al. 2010), among others. This makes the NIMBUS method the ideal choice as the interactive method for solving the PAINT surrogate problem of the wastewater treatment plant model.

Figure 3 shows an illustration of the graphical user interface of the IND-NIMBUS® PAINT module. In the PAINT module, the decision maker can give his/her preferences concerning the surrogate problem on the left side of the PAINT module. The five horizontal bars represent the five objectives of the problem. The values of the objectives in the current solution are shown with the bars. When the decision maker clicks on any of the bars, the module interprets the decision maker’s preferences in the following way for a minimized objective:

1. if the decision maker clicks on the arrow on the left(right) side of the bar, then the objective is classified to the class $I^< (I^>)$;
2. if the decision maker clicks on the bar on the left(right) side of the current level, then the objective is classified to the class $I^\leq (I^\geq)$, where the aspiration/reservation level is given by the position clicked; and
3. if the decision maker clicks on the current level, then the objective is classified to the class $I^\equiv$.

For a maximized objective, the positions of the words ‘left’ and ‘right’ should be interchanged in the above list.

The given classification information is used to formulate a single or multiple classifications of the problem according to the decision maker’s selection with the buttons labelled ‘STD’, ‘ACH’, ‘STOM’ and ‘QUESS’ with scalarizations (2), (3), (4) and (5), respectively, depending on which buttons are active. The scalarizations are modelled using the Optimizing Programming
Language (OPL, see van Hentenryck [1999]), and these (mixed integer linear models) are solved using CPLEX (see the IBM ILOG CPLEX Optimization Studio Home Page [2014]). Optimal solutions of the scalarizations give new approximate Pareto optimal outcomes, corresponding to the preferences given by the decision maker. These approximate outcomes are shown to the decision maker. If the decision maker so wishes, he/she may reclassify the objectives of the new approximate outcome, which yields another approximate outcome.

As described in Section 3, approximate Pareto optimal outcomes can be projected on the Pareto front of the original problem using the PAINT module (using the ‘PROJECT SOLUTION’ button near the bottom of the screen). The projection of the approximate outcome, i.e. a Pareto optimal solution to the original problem, is shown to the decision maker. As mentioned, for a computationally expensive problem this may take time, but fortunately projecting an approximate outcome can be done without the involvement of the decision maker.

On the right side of the PAINT module, the given Pareto optimal outcomes and the approximate Pareto optimal outcomes are visualized. The given outcomes are shown in the upper frame and the approximate Pareto optimal outcomes that have been found during decision making are given in the lower frame. The decision maker can choose any of the given or approximate Pareto optimal outcomes as the starting point of the next NIMBUS iteration (i.e. as the basis for a new classification of objectives).

The current version of the PAINT module does not implement the construction or updating of the surrogate problem. If one wishes to update the surrogate problem using Pareto optimal outcomes obtained by, for example, projection, the decision making process must be stopped, and the surrogate problem must be manually updated using Octave. In future versions, updating the Pareto front approximation should be automatically updated in the PAINT module after projecting an approximate outcome.

5. Solving the wastewater treatment problem

In this section, it is described how the PAINT method and the IND-NIMBUS® PAINT module were used to solve the wastewater treatment problem described in Section 2. First, in Section 5.1, the construction of the Pareto front approximation with PAINT before the involvement of the decision maker is described. Then, in Section 5.2, it is described how the decision maker used the
PAINT module to find preferable approximate outcomes to the wastewater treatment problem. Finally, the projections of the approximate outcomes selected by the decision maker in the previous subsection are analysed and the preferred ways to operate the wastewater treatment plant are found in Section 5.3.

5.1. Pre-decision making phase

First, a set of 195 mutually non-dominated solutions to the wastewater problem (see the list in the electronic supplementary material) was found with the evolutionary UPS-EMO algorithm (Aittokoski and Miettinen 2010) and the GPS-X simulator. To study the optimality of these solutions, each one was locally improved using an achievement scalarizing problem (Wierzbicki 1980, 1986), which was optimized with the MATLAB™ fmincon-function with finite differences approximated gradients. This resulted in 195 mutually non-dominated solutions. The maximal improvement in the values of the achievement scalarizing problem was at most 3% so the local improvement did not cause much change. This built confidence that the final solutions were close to Pareto optimal. The outcomes given by these solutions were taken as the set of given Pareto optimal outcomes for the PAINT method. The whole process of producing this set took about three days on an Intel® Core™ 2 Duo CPU P8600, both processors running at 2.40 GHz.

After this, a Pareto front approximation based on the given set of Pareto optimal outcomes was computed with the PAINT method (see Section 3). The PAINT method chose 4272 polytopes for interpolation in the Pareto front. In order to reduce the computational complexity of the implied mixed integer linear surrogate problem, the polytopes that were subsets of larger polytopes from the approximation were removed. This resulted in a collection of 608 polytopes whose union covered the same space in \( \mathbb{R}^5 \) as that of the larger collection. In addition, all sets of vertices of the polytopes in the collection were affinely independent and, thus, the number of vertices of all the polytopes was five or less, as shown in Hartikainen, Miettinen, and Wiecek (2011a). Using the PAINT method to construct the Pareto front approximation took approximately 19 hours on an Intel® Xeon™ E5410 CPU.

The mixed integer linear surrogate problem implied by the smaller collection was equivalent to that implied by the larger collection, but it was computationally less expensive. As described in Hartikainen, Miettinen, and Wiecek (2012), the surrogate problem could be written as

\[
\begin{align*}
\text{min} & \quad (z_1, \ldots, z_5) \\
\text{s.t.} & \quad \sum_{j=1}^{608} \sum_{l=1}^{5} \lambda_{j,l} = 1 \\
& \quad \sum_{j=1}^{5} \lambda_{j,l} \leq y_j, \text{ for all } j = 1, \ldots, 608 \\
& \quad \sum_{j=1}^{608} y_j = 1, \\
\text{where} & \quad \lambda \in [0, 1]^{608 \times 5} \\
& \quad y \in \{0, 1\}^{608} \\
& \quad z_i = \sum_{j=1}^{608} \sum_{l=1}^{5} \lambda_{j,l} A_{ij} \text{ for all } i = 1, \ldots, 5,
\end{align*}
\]
where each row of the matrix $A \in \mathbb{R}^{608 \times 5}$ contained the indices of the vertices of one polytope in the smaller collection of polytopes. The component $\lambda_{j,l}$ of the matrix variable $\lambda \in \mathbb{R}^{608 \times 5}$ was, for all $j = 1, \ldots, 608$ and $l = 1, \ldots, 5$, the coefficient of the vertex $l$ of the polytope given by row $j$ in the matrix $A$. The variable $y$ determined which of the rows of the matrix $\lambda$ was non-zero. By the third constraint, only one row in the matrix $\lambda$ had non-zero elements.

Problem (6) had $608 \times 5 = 3040$ continuous variables and 608 binary variables. CPLEX was able to solve, for example, an achievement scalarizing problem for the surrogate problem in less than a second using the same laptop as for generating the alternatives. This was a tremendous improvement over solving a scalarization of the original problem, which took about half an hour with the controlled random search algorithm of Price (1983), which is implemented in the IND-NIMBUS® software framework.

### 5.2. Decision making phase

Using the PAINT module, the decision maker was able to examine the approximate outcomes and to project any of them on the Pareto front of the original problem. This entire decision making process was done within a couple of hours, which could not have been possible using the original computationally expensive problem. The decision maker aims to find a couple of interesting and preferable approximate outcomes that will then be projected on the actual Pareto front (of the original computationally expensive problem) in Section 5.3.

Before the decision maker started using the PAINT module, he was given a brief overview of the methods from the user’s perspective. He was told that a set of Pareto optimal outcomes had been computed and that a new PAINT method had been used to interpolate between those outcomes. In addition, he was informed that the outcomes given by PAINT were only approximate Pareto optimal outcomes and, thus, more computation would have to be done to find the closest real Pareto optimal outcome. Finally, he was told that the PAINT method does not unfortunately provide any information about the decision variables and those values can only be known after the real Pareto optimal solution is found. Since the decision maker had previous experience with the NIMBUS method, all of this was very clear to him. In addition, he did not find any of this too inconvenient.

The approximate Pareto optimal outcomes are given in Table 1 in the order that they were inspected by the decision maker. In each of the horizontal cells, on the first line the values of the objectives in the current solution are shown. The values in bold type indicate solutions that are actual Pareto optimal outcomes, instead of approximate outcomes. Then, on the second line, the preferences of the decision maker are given. Only two scalarizations, i.e. the NIMBUS standard scalarization and the achievement scalarizing function, were used in the decision making. If the NIMBUS scalarization was used, then the preferences are given as ‘Classification’, and if the achievement scalarizing function was used, then the ‘Aspiration’ levels are shown. On the third line for each iteration, the optimal solution for the scalarization used (i.e. the resulting approximate Pareto optimal outcome) is given.

During the decision making phase, the decision maker had different approaches to the problem, i.e. he wanted to find different kinds of solution. It could be said that the decision maker’s first agenda was to find a solution with good nitrogen removal, no matter what the cost. After the decision maker had examined different ways of obtaining good nitrogen removal, he changed his agenda and wanted to find a solution with adequate nitrogen removal and good values for other objectives. At the end of the decision making phase, the decision maker’s agenda changed again to minimizing the amount of nitrogen with a different approach from that employed at the beginning.
Table 1. The approximate Pareto optimal outcomes inspected by the decision maker with the PAINT module. ‘[P:]’ refers to a Pareto optimal outcome in the mutually non-dominated solutions given to PAINT, and ‘[A:]’ refers to an approximate outcome.

| Iteration | Issue | Iteration | Issue | Total nitrogen [gN/m³] | Blower/ wire aerator power [kW] | Methanol dosage rate [g/m³] | Mass flow total suspended solids [kgTS/d] | Total gas flow [m³/d] |
|-----------|-------|-----------|-------|------------------------|-------------------------------|-----------------------------|-------------------------------------|---------------------|
|           |       | 1         |       | 17.0                   | 419.0                         | 0.0                         | 14,600.0                           | 9,130.0             |
| 1         |       |           |       |                        |                               | 15.0                        | 14,400.0                           | 9.930.0             |
|           |       |           |       |                        |                               | 18.0                        | 15,900.0                           | 10,300.0            |
|           |       |           |       |                        |                               | 16.6                        | 14,600.0                           | 9,190.0             |
|           |       | 2         |       | 17.0                   | 419.0                         | 0.0                         | 15,100.0                           | 9,760.0             |
|           |       | 3         |       | 17.0                   | 419.0                         | 0.0                         | 15,100.0                           | 9,760.0             |
|           |       | 4         |       | 16.6                   | 418.0                         | 12.9                        | 15,100.0                           | 10,200.0            |
|           |       | 5         |       | 16.6                   | 415.0                         | 25.9                        | 15,100.0                           | 10,200.0            |
|           |       | 6         |       | 17.0                   | 419.0                         | 0.0                         | 15,100.0                           | 9,760.0             |
|           |       |           |       |                        |                               | 16.6                        | 15,100.0                           | 10,200.0            |
|           |       |           |       |                        |                               | 16.5                        | 15,100.0                           | 10,300.0            |
|           |       |           |       |                        |                               | 17.8                        | 15,100.0                           | 9,760.0             |
|           |       |           |       |                        |                               | 18.0                        | 15,100.0                           | 9,760.0             |
|           |       |           |       |                        |                               | 18.0                        | 15,100.0                           | 9,760.0             |
|           |       |           |       |                        |                               | 18.0                        | 15,100.0                           | 9,760.0             |
|           |       |           |       |                        |                               | 18.0                        | 15,100.0                           | 9,760.0             |

For the first agenda, the decision maker chose the given Pareto optimal outcome [P:74] as the starting point. This solution already had rather poor nitrogen removal. However, according to the decision maker’s evaluation, this solution was interesting, as nitrogen removal could possibly be improved by increasing methanol dosage, while maintaining good values for the other objectives. Thus, the decision maker sought to find a solution with a bigger reduction in the amount of
nitrogen, but still maintaining the energy efficiency, by giving an aspiration level of 15.0 gN/m³ for the amount of nitrogen and reservation levels of 431 kW and 24.3 g/m³ for the aeration wire power and methanol dosage, respectively. In addition, the decision maker allowed the suspended solids to change freely for a while and the amount of gas to increase to a given aspiration level 10,200 m³/d, as the amount of gas should increase as a result of the increased sludge production induced by added methanol. The result of solving the NIMBUS standard scalarization of the mixed integer linear surrogate problem with this preference information is approximate outcome [A:1]. According to the decision maker, the approximate outcome is still energy efficient, but does not have much better removal of nitrogen either. Thus, the approximate outcome is not yet preferable.

As approximate outcome [A:1] did not have good enough nitrogen removal, the decision maker gave a reservation level of 15,900 kg/d total solids for the excess sludge production and increased the aspiration level for the effluent nitrogen concentration from 15 to 15.8 gN/m³, because he felt that it was more realistic. The preferences for the other objectives remained the same. The optimal solution to the NIMBUS standard scalarization of the mixed integer linear surrogate problem with this preference information was approximate outcome [A:2]. In the new approximate outcome, the amount of nitrogen unfortunately remained the same, but there was a significant increase in the amount of gas while the amount of methanol rose to the given reservation level and the suspended solids concentration in aeration basins rose slightly, but remained well under the given reservation level. The decision maker thought that the change from approximate outcome [A:1] to [A:2] was rather good, though no better nitrogen removal was achieved. The decision maker also wanted to see whether giving an aspiration level of 12.9 g/m³ for the methanol dosage would change anything. However, solving the NIMBUS standard scalarization resulted in approximate outcome [A:3], which was exactly the same as approximate outcome [A:2].

The decision maker felt that his preferences were not exactly met in approximate outcome [A:3]. Thus, it was decided to change the scalarization into an achievement scalarizing problem, and the problem was solved with the same preferences except that the aspiration level for the excess sludge production was changed to 15,100 kg/d total solids. This resulted in approximate outcome [A:4], but there was no significant change.

For approximate outcome [A:5], the scalarization was again the standard NIMBUS scalarization and the same preferences were used as in approximate outcome [A:4]. Approximate outcome [A:5] has slightly higher gas production, but on the other hand the methanol dosage is slightly higher. However, these changes were still rather minor.

After this, the achievement scalarizing function was chosen for all the rest of the scalarizations. Next, the decision maker wanted to see a bigger change in the approximate outcome generated. Thus, the aspiration level for the methanol was raised from 12.9 to 39.2 g/m³ to gain improvement in the nitrogen removal rate, aeration power (whose aspiration level was lowered to 404 kW) and biogas production. Solving the scalarization for the mixed integer surrogate problem resulted in approximate outcome [A:6], which does have better nitrogen removal. However, according to the decision maker, the amount of methanol 39.2 g/m³ was too high to be compensated by the better nitrogen removal, which was not yet very good.

The decision maker felt that he had learned all that he could by moving from the given Pareto optimal outcome [P:74] and decided to choose another starting point. He also changed his agenda as he was now interested in finding a solution where all the other objectives but the nitrogen removal were as good as possible. That is to say, he sought to find what could be accomplished with few resources. The given Pareto optimal solution [P:63] was already a step in this direction, but the decision maker wanted to go even further and, thus, gave a bad aspiration level for the nitrogen removal and very good aspiration levels (exceeding those for the given Pareto optimal outcome [P:63]) for the others. Solving the scalarization resulted in approximate outcome [A:7]. However, the objectives that it had been sought to improve did not improve very much, although
the amount of nitrogen was almost at the upper limit. This implies that even when the amount of nitrogen can move freely, the other objectives are still conflicting.

Next, the aspiration levels for the aeration energy and the excess sludge production were raised close to the levels of those in approximate outcome [A:7]. The other preferences were kept the same. However, this did not result in a significant change from approximate outcome [A:7] to [A:8]. This means that the amount of methanol and biogas production seem to be conflicting, i.e. one cannot reduce the amount of methanol below certain limits without a reduction in the biogas production. According to the decision maker, this makes sense given his understanding of the process. This hypothesis was tested by allowing even more solids and reducing the aspiration level for biogas production, while seeking a reduction in the amount of methanol. And indeed, a reduction in the amount of methanol (from 18.9 to 11.4 g/m³) was accomplished in approximate outcome [A:9], while the amount of biogas production dropped from 9730.0 to 9480.0 m³/d.

For approximate outcome [A:10], more aeration energy was allowed (by raising the aspiration level to 452.0 kW) to seek a further reduction in the amount of methanol. More precisely, pumping to fermentation was expected to increase in order to compensate for the reduced carbon source. Increased sludge fermentation implies higher organic load to aeration basins and increased air requirement. However, this did not work as it seems that the aspirations for the other objectives were disallowing further reductions in the amount methanol. Finally, two more tests of the interdependencies between the amount of methanol and the other objectives were done. First, even better aspiration levels than in approximate outcome [A:8] for the amount of methanol and biogas production and, in addition, even worse aspiration levels than those in approximate outcome [A:8] for the other objectives were given. As both the amount of methanol and the amount of biogas production were both rather unsatisfactory in the resulting approximate outcome [A:11], the aspiration level for the amount of methanol was slightly raised to seek a satisfactory level at least in the biogas production. However, this only resulted in a small improvement in biogas production in approximate outcome [A:12].

Next, the given Pareto optimal outcome [P:170] was chosen as a starting point. According to the decision maker, this is a surprisingly good solution to something that one could call the ‘old plant’s problem’, because old plants cannot typically handle a lot of solids. In approximate outcome [A:13], an even lower level of solids was sought while it was sought to keep the other quantities at satisfactory levels by giving aspiration levels at the upper limit for the amount of nitrogen, aeration power and methanol dosage, a very low aspiration level for excess sludge production, and a rather low aspiration level for gas production. Approximate outcome [A:13] was even more satisfactory than the given Pareto optimal outcome [P:170] and a very good solution to the ‘old plant’s problem’.

As a final starting point of decision making, given Pareto optimal outcome [P:192] was chosen. From this solution, it was sought to minimize aeration power, methanol dosage and excess sludge production by giving them very good aspiration levels, while allowing the amount of nitrogen to grow and the amount of biogas produced to reduce by giving them poor aspiration levels. Solving the scalarization of the mixed integer linear surrogate problem resulted in approximate outcome [A:14]. However, the aeration power still remained too high in this solution. Thus, for approximate outcome [A:15], the aspiration level for biogas production was dropped to the lower limit. This resulted in approximate outcome [P:15] having lower biogas production and lower amounts of methanol, but aeration power that was no lower. This again emphasizes the conflict between biogas production, the amount of fermentation, and the methanol dosage as already noted earlier. In approximate outcome [A:16], the decision maker again raised the aspiration level for the methanol dosage rate (allowing larger amounts of methanol), but this did not yield much change.

Then, in approximate outcomes [A:17], [A:18] and [A:19], the decision maker was trying to find an approximate outcome, where the excess sludge production would be high and the amount of methanol low. This was interesting to him for two reasons: he wanted to find out if the process
could be operated in such a way and, in addition, if the simulator could produce such a solution. However, it seemed that such a solution does not exist and that the amount of sludge seemed to increase and decrease in unison with the amount of methanol.

Finally, the decision maker was given a possibility to ask to find actual Pareto optimal solutions that were close to the approximate outcomes. The decision maker chose approximate outcomes [A:1], [A:5], [A:6], [A:10], [A:13], [A:14] and [A:18] for projection. These approximate outcomes represented different kinds of interesting solutions and were all good in their own way. In addition, the decision maker asked to try to find a solution in which the excess sludge production would be high and the amount of methanol low.

5.3. Post-decision making phase

After decision making, the approximate outcomes chosen by the decision maker were projected onto the actual Pareto optimal front. This was done by solving the achievement scalarizing problems for the original multiobjective optimization problem with the chosen approximate outcomes as reference points. The single objective optimization problems were solved with the controlled random search algorithm of Price (1983) using 200 function evaluations and adaptive penalties (see e.g. Miettinen [2003] for the constraints).

The resulting actual Pareto optimal solutions are given in Table 2. Each of these solutions was then presented to the decision maker. These solutions will add to the knowledge of the operation of the wastewater treatment plant and that of the GPS-X simulator. In addition, having these solutions will in future help the operators of wastewater treatment plants find preferable solutions easier. These are the final best solutions for operating the wastewater treatment plant that were found in this article.

According to the decision maker, the values of the approximate outcomes and actual Pareto optimal outcomes differ from each other numerically, but not by a wide margin. Most importantly, the direction of the differences in parameter values of consecutive approximate outcomes is almost always correct, and also the specific value of differences is of the correct order of magnitude when compared to the actual Pareto optimal solutions. However, the differences between the values of effluent nitrogen and methanol dosage in consecutive approximate outcomes seem to be bigger than those in the corresponding actual Pareto optimal solutions. On the other hand, the corresponding differences in sludge production and biogas production are almost equal when comparing approximate outcomes and actual Pareto optimal solutions. Blower energy consumption seems to be the hardest to approximate; it is the only parameter in which the direction of the difference between different approximate outcomes is approximated erroneously (approximate outcomes [A:10] and [A:13]; [A:14] and [A:18]). That is, the blower energy consumption is greater in approximate outcomes [A:10] and [A:14] than in approximate outcomes [A:13] and [A:18], respectively, but between the actual corresponding Pareto optimal solutions the relation is the opposite. Despite these shortcomings, the decision maker thought that approximation of the Pareto optimal solutions does not seem to create a bias that would prevent reliable comparison of the solutions based merely on the approximation. Thus, the approach fulfilled its purpose.

The decision maker also noted that the decision making includes error from a variety of different sources: the model, the simulator, the approximation and the optimization. Each one of these may give erroneous results and, thus, induce error into decision making. According to the decision maker, differences between the decision variable values of some projected Pareto optimal solutions are small and may fit within the error limits of the simulator. Despite this, the decision maker thought that the differences in the objectives can be logically explained as consequences of changes made to the decision variables. Therefore, the actual Pareto optimal solutions in Table 2, can be considered reliable material for decision making concerning preferred operational strategy.
Table 2. The actual Pareto optimal solutions received by solving the achievement scalarizing problem for the approximate outcomes chosen by the decision maker. The values of decision variables are on the left and the values of the objectives on the right. [A:n] refer to the approximate outcomes given in Table 1. The methanol dosage rate is both a decision variable and an objective and, thus, it is in the middle.

| Projected from | Flow proportion [%] | Pumped flow of excess [m³/d] | DO set-point sludge [mgO²/l] | Methanol dosage rate [g/m³] | Total nitrogen [gN/m³] | Blower/aerator wire power [kW] | Total suspended solids mass flow [kg/d total solids] | Total gas flow [m³/d] |
|---------------|----------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------|-------------------------------|---------------------------------|------------------|
| [A:1]         | 1.7                  | 2659.9                      | 1.0                         | 11.2                        | 16.02                 | 423.2                         | 14,700.0                        | 9128.2           |
| [A:5]         | 0.7                  | 2761.5                      | 1.7                         | 18.0                        | 16.83                 | 420.3                         | 14,895.0                        | 9588.1           |
| [A:6]         | 0.5                  | 2900.8                      | 1.6                         | 30.1                        | 15.60                 | 424.2                         | 15,253.0                        | 9768.5           |
| [A:10]        | 1.3                  | 2939.2                      | 1.9                         | 8.6                         | 16.49                 | 423.8                         | 14,662.0                        | 9235.3           |
| [A:13]        | 2.3                  | 2986.9                      | 1.4                         | 3.2                         | 16.23                 | 421.8                         | 14,514.0                        | 8952.1           |
| [A:14]        | 1.0                  | 3064.6                      | 1.7                         | 11.3                        | 16.9                  | 416.1                         | 14,845.0                        | 9406.9           |
| [A:18]        | 1.5                  | 2591.9                      | 1.2                         | 3.8                         | 17.1                  | 414.7                         | 14,655.0                        | 9089.4           |

6. Conclusions

In this article, a small number of preferable compromise solutions for operating a wastewater treatment plant were found. In addition, it was shown how the interactive multiobjective method NIMBUS can be used to optimize a computationally expensive real-life problem of operating a wastewater treatment plant using the approximation method PAINT. With the help of the PAINT method, the decision making with the interactive method was not time-consuming, although the problem is computationally expensive and requires one to use a simulator in computing the values of the objectives. It is the opinion of the authors that this methodology will motivate further interactive optimization of computationally expensive multiobjective optimization problems and increase awareness among researchers of approximation methods other than surrogate models of the objective functions or the constraints. The benefit of the PAINT method is that it is tailor-made for multiobjective optimization and can be smoothly joined together with both any a posteriori method for generating the given Pareto optimal solutions and any interactive method for making decisions.

The article has multiple assets. First, the approach of using the PAINT approximation method together with an interactive method can also be used in a similar way for other computationally expensive multiobjective optimization problems. In addition, the decision maker’s behaviour in solving the problem of operating a wastewater treatment plant using an interactive method was described and this information can be used to develop the interactive method further through analysing the strengths and weaknesses of interactive methods. Finally, the Pareto optimal solutions produced offer different ways to operate a wastewater treatment plant under different conditions and with different preferences.

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Supplemental data

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