Three-dimensional slope stability predictions using artificial neural networks

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Abstract
To enable assess slope stability problems efficiently, various machine learning algorithms have been proposed recently. However, these developments are restricted to two-dimensional slope stability analyses (plane strain assumption), although the two-dimensional results can be very conservative. In this study, artificial neural networks are adopted and trained to predict three-dimensional slope stability and a program, SlopeLab has been developed with a graphical user interface. To reduce the number of variables, groups of dimensionless parameters to express stability of slopes in classic stability charts are adopted to construct the neural network architecture. The model has been trained with a dataset from slope stability charts for fully cohesive and cohesive-frictional soils. Furthermore, the impact of concave plan curvature on slope stability that is usually found by excavation in practice is investigated by introducing a dimensionless parameter, relative curvature radius. Slope stability analyses have been conducted with numerical calculations and the artificial neural networks are trained with dimensionless data. The performance of the trained artificial neural networks has been evaluated with the correlation coefficient \( R \) and root mean square error \( \text{RMSE} \). High accuracy has been found in all the trained models in which \( R > 0.999 \) and \( \text{RMSE} < 0.15 \). Most importantly, the proposed program can help engineers to estimate 3D effects of a slope quickly from the ratio of the factors of safety, \( \text{FS}_{3D}/\text{FS}_{2D} \). When \( \text{FS}_{3D}/\text{FS}_{2D} \) is large (such as larger than 1.2), a 3D numerical modelling on slope stability analyses that can consider complex 3D geometry and boundary condition is advised.

KEYWORDS
artificial neural networks, factor of safety, stability charts, three-dimensional end effects, three-dimensional slope stability analysis

1 | INTRODUCTION

Slope stability analysis is a common task in geotechnical engineering due to its essential role in maintaining the stability of natural slopes and designing man-made structures such as railway embankments, dams and highways etc. The stability of
slopes is quantified with the factor of safety (FS). In order to simplify the calculation process for FS, a vast majority of slope stability analyses in engineering practice are performed in two-dimensions (2D) under plane strain assumption, in which the slope is assumed to be infinitely wide such that three-dimensional (3D) end effects are negligible. However, numerous natural and constructed practical slopes exhibit a complex geometric configuration and a 3D state. From reported slopes in the literature, relative differences between the two-dimensional factor of safety (FS2D) and three-dimensional factor of safety (FS3D) are found greater than 50%. Therefore, the prediction based on 2D analysis can be excessively conservative and economically unjustified. 3D slope stability analysis should be used for accurate slope stability prediction, especially for the slopes with evident 3D end effects or complex slope geometries.

Limit equilibrium method (LEM) is perhaps the oldest calculation method to predict the stability of slopes. The common limit equilibrium techniques are the ordinary method of slices, Bishop simplified, Spencer, Janbu and Morgenstern-Price. In LEMs, assumptions on the shape and location of the failure surface need to be made. In contrast, the failure mechanism at the critical equilibrium state can be obtained automatically from finite element numerical results with the strength reduction method (SRM). The factor of safety is found with the ratio of the soils’ actual shear strength to the reduced strength at failure. Owing to its flexibility and robust performance, its application has been increasingly popular. An alternative to the strength reduction method is the limit analysis method, in which the soils are treated as a perfectly plastic material obeying an associated flow rule. Its accuracy and powerful performance have been demonstrated in analysing the stability of various slopes, such as the seismic rock slope, the two-layered cohesive soil slope and the slope with cracks. The advanced numerical methods can provide an accurate FS for slopes under complex conditions, and much detailed information is available from the numerical results. However, the calculation normally takes much longer time. Therefore, machine learning aided slope stability methods may be considered as an alternative when an FS is merely required for a slope.

In recent years, machine learning has received increasingly more attention in slope stability analyses. The accurate results have been demonstrated with various techniques such as extreme gradient boosting method, Gaussian process regression, relevance vector machine, artificial neural networks, extreme learning machine and hybrid machine learning techniques. Most recently, hybrid intelligent methods for predicting FS of slopes under seismic conditions are employed in. An artificial neural network to investigate inhomogeneous soil slope stability under undrained conditions is developed in. Machine learning aided probabilistic stability analysis of slopes in spatially variable soils has been conducted in. However, as far as the authors know, the application of machine learning algorithms on slope stability analysis is restricted to two-dimensions. To better predict the stability of slopes, machine learning aided slope stability analysis algorithms need to be extended to three-dimensions.

In this study, artificial neural networks are adopted and trained for assessing the stability of 3D slopes. The straight slopes in both purely cohesive and cohesive-frictional soils are considered first before concave slopes in plan view. The effect of plan curvature of slopes is measured with a dimensionless parameter, the relative curvature radius of the slope and the stability analyses are conducted for a series of slopes using the strength reduction method. To reduce the number of variables in machine learning models, groups of dimensionless parameters based on slope stability charts are used. As a result, a very good predictive performance can be achieved even with a relatively small training dataset. In addition, a program, SlopeLab, is developed for easy use by wrapping the trained neural networks with a graphical user interface. A ratio of the safety factor, FS3D/FS2D, can also be obtained from the program, providing a quick reference on 3D effects of a slope. The program’s predictions have been validated with a series of examples.

2 | ARTIFICIAL NEURAL NETWORKS WITH DIMENSIONLESS VARIABLES

Artificial neural networks (ANN) are a computing model inspired by the biological neural networks in animal brains. It has been used in the geotechnical engineering community for various challenges. The architecture of ANN consists of the input layer, the output layer and several hidden layers, as demonstrated in Figure 1. The data is processed and transmitted between neurons via connection links. During the learning process, neurons’ weight is adjusted. Generally, more difficult problems require more neurons, and even more layers. Given very limited dimensionless parameters used in this work, one-hidden-layer feed forward neural networks are examined firstly. The number of neurons is increased until a satisfying result is reached.

For a conventional ANN aided slope stability prediction approach, FS is treated as the output. The input is the slope factors that play a role in determining FS, commonly involving soils’ shear strength (e.g., cohesion and internal friction angle), soils’ unit weight and slope geometry, commonly described by slope height, slope width, slope depth and the slope...
angle. Therefore, around seven independent variables need to be used as the input, resulting in high complexity of learning and computing.

Slope stability charts have been used in practice as a quick reference before personal computers become widely available.52–54 While computational methods have made most stability charts obsolete, dimensionless slope parameters are formulated with a clear relationship in the charts. The charts have been redeveloped with more results obtained from advanced numerical methods.20,55,56 In this study, dimensionless parameters based on stability charts are used, leading to a reduced number of independent variables and less complexity of the ANN model, as demonstrated in the next sections.

A dataset for the ANN is required to be randomly split into two independent subsets: the training dataset and the testing dataset. The ANN is trained with the training data, while the testing dataset is used to verify its predictive performance. In this study, 80% of the whole data is used for the training dataset while 20% is used for the testing dataset. The Levenberg-Marquardt algorithm57 is employed to train the neural networks and the mean square error is minimised in the training process. The performance of the trained ANN is evaluated with the correlation coefficient $R$ and root mean square error $RMSE$:

\[
R = \sum_{i=1}^{n} \frac{(C_i - \bar{C})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{n}(C_i - \bar{C})^2} \sqrt{\sum_{i=1}^{n}(P_i - \bar{P})^2}}
\]

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (C_i - P_i)^2}
\]

where $C$ is calculated solutions, $P$ is predicted results, $\bar{C}$ and $\bar{P}$ are the mean value and $n$ is the size of the dataset. $R$ is a measure of the linear relationship between two variables. A value of 1 indicates a perfect positive correlation while 0 represents that there is no linear correlation between the variables. $RMSE$ is a measure of absolute errors of predicted valuables. A lower $RMSE$ represents a better performance of trained models and a value of 0 indicates a perfect fit.

\section{Three-dimensional Slope Stability Assessment Using Artificial Neural Networks}

The slope geometry for straight slopes is shown in Figure 2, in which the $x$–$z$ dimension can be extended by a distance in the $y$-direction to simulate the 3D boundary conditions. The shear strength of soils is described on the basis of the Mohr-Coulomb failure criterion. The purely cohesive slopes are studied first before the slopes in cohesive-frictional soils. At last, the three-dimensional stability assessment of concave slopes in plan view is conducted. The ANN models are trained with dimensionless data based on stability charts.
3.1 | Purely cohesive slopes

The model geometry of purely cohesive slopes is shown in Figure 2 involving the slope height $H$, the slope width $B$, the slope angle $\beta$ and the slope depth $D$.

To present the results in a dimensionless manner, following, a stability number is given in the dimensionless form:

$$N_s = \frac{c_u}{\gamma H F S} \quad (3)$$

where $N_s$ is the stability number, $c_u$ is the undrained shear strength, $\gamma$ is the unit weight, and $H$ is the slope height.

In the literature, a series of stability charts have been developed with a wide range of slope angles $\beta$, the width ratio ($B/H$) and the depth ratio ($D/H$). $B/H$ is increased from 1, 2, 3, 5 to $\infty$ (2D plane strain assumption). For example, dark lines in Figure 3 show three- and two-dimensional chart solutions for slopes with $B/H = 1$ and $\infty$, respectively.

To examine the chart solutions in, the strength reduction method with a finite element software, Plaxis 3D is employed. A slope is used as the first example with the following parameters: $\beta = 45^\circ$, $B = 4.5$ m, $H = 4.5$ m, $D = 6$ m, $\gamma = 16$ kN/m$^3$ and $c_u = 30$ kPa. For the second slope, the slope angle $\beta$ is set to $30^\circ$ and the slope depth $D$ is changed to 9 m, and the rest of the parameters remain the same. From the charts, $c_u/\gamma HF S$ of the two slopes reads 0.11 and 0.09, respectively. Therefore, $FS3D$ is calculated as 3.79 and 4.58, respectively. The numerical results of finite element strength reduction for two slopes show that $FS3D = 3.62$ and 4.62, respectively. The comparison of the two methods is plotted in Figure 3a, indicating a good agreement is achieved. Furthermore, a conventional 2D slope stability analysis is performed using the limit equilibrium method with the program, SLOPE/W. The factor of safety of the two slopes above is $FS2D = 2.41$ and 2.42, respectively. The results in comparison with the chart solutions are shown in Figure 3b, indicating a good accuracy of the chart. In addition, the ratio of the safety factors of the two slopes, $FS3D/FS2D$, is calculated as 1.50 and 1.91, respectively, showing that the 2D analysis underestimates the stability of the two slopes greatly.

As the stability charts are verified, a total of 168 3D slope cases and 42 2D slope cases are collected and used for training the artificial neural networks. Thanks to the adopted dimensionless variables, the number of inputs of the ANN analysis is reduced to three: $D/H$, $\beta$ and $B/H$. The dimensionless stability number $c_u/\gamma HF S$ is considered as the output. Figure 4 shows results of the trained ANN indicating that the predicted results agree with the calculated data very well with correlation coefficient $R$ approximately equalling to 1. Meanwhile, all data lie in a straight line with the slope of 1.

3.2 | Cohesive-frictional soil slopes

The geometry model as shown in Figure 2 is still used in this section. Since the slip surface of cohesive-frictional slopes is relatively shallow, commonly above the slope toe, $D/H$ may be not required to be treated as an independent variable.

Stability charts for cohesive-frictional soil slopes regard a dimensionless parameter $FS/\tan \varphi$ as a function of a dimensionless group $c/\gamma H \tan \varphi$ for a variety of $\beta$ and $B/H$, where $c$ is the cohesion and $\varphi$ is the frictional angle. The stability charts presented in are developed based on numerical results of limit analysis. A wide range of dimensionless parameters are used in the charts and $\beta$ is increased from $15^\circ$ to $90^\circ$ with an interval of $15^\circ$. For example, dark lines in Figure 5 illustrate the stability chart for the slope with a slope angle $\beta = 30^\circ$. 
To examine the accuracy of the stability charts, chart solutions are compared with numerical results from the finite element strength reduction method. The chosen example adopts the following parameters $\beta = 30^\circ$, $H = 4.5\ m$, $D = 6\ m$, $\gamma = 16\ kN/m^3$, $c = 10\ kPa$ and $\phi = 32^\circ$, and $B/H$ of slopes is increased from 1, 2, 4 to $\infty$ (2D plane strain assumption). It can be estimated from Figure 5 that as $B/H$ of the slopes raises, $FS/\tan \phi$ reads 5.46, 4.66, 4.40 and 4.14, respectively. Thus, $FS$ is found to be 3.41, 2.91, 2.75 and 2.59, respectively. The numerical results give $FS = 3.33$, 2.92, 2.74 and 2.60, respectively. The comparison is plotted in Figure 5 showing a satisfying agreement between the chart solutions and the numerical results.

After the charts are validated, a total of 1254 cohesive-frictional slope cases are taken from the charts. Independent variables $c/\gamma H \tan \phi$, $\beta$ and $B/H$ are used as inputs of ANN, and a dimensionless number $FS/\tan \phi$ is considered as the output. After the ANN is trained, its performance is evaluated, as demonstrated in Figure 6. A great performance of the trained neural networks is concluded with the correlation coefficient $R$ being very close to 1.

### 3.3 Concave slopes in plan view

The design of stable concave slopes with plan curvature such as open-pit mine slopes is of significance in practical engineering. However, to date, few systematic studies have been conducted on stability prediction of concave slopes with a wide range of slope parameters. In this work, the shear strength reduction method with Plaxis 3D and the limit equilibrium method were implemented to predict the factor of safety for concave slopes in plan view.
FIGURE 4 Performance of the trained ANN: (a) 3D undrained slopes and (b) 2D undrained slopes ($B/H = \infty$)

FIGURE 5 Stability chart and numerical solutions for cohesive-frictional slopes with $\beta = 30^\circ$

method with SLOPE/W are employed for three- and two-dimensional slope stability analyses. Figure 7 shows 3D slope geometry for the defined problem, in which the radius of the plan curvature is denoted by $r$. The typical boundary condition for the slope analysis is adopted. The top surface of the slope is set free, the bottom and side surfaces are fully fixed, and the symmetric surface is fixed in the normal direction. It is necessary to note that for steep slopes that have non-circular slip surfaces, the two-dimensional stability analysis is conducted using the finite element strength reduction method.

Based on the numerical results, the stability charts are presented in Figure 8 for homogeneous concave slopes with plan curvature, in which a wide range of slope angles $\beta$, relative curvature radius of slopes ($r/H$), and the dimensionless parameter $c/\gamma H \tan \varphi$ is utilized. The results show that an increase of the radius of the plan curvature reduces slope stability. When slope angles increase from $15^\circ$ to $90^\circ$, $FS/\tan \varphi$ decreases continuously. In addition, unit weight and slope height have a negative effect on slope stability.

A total of 234 slope cases from numerical results are collected to train the ANN. The input includes $\beta$, $r/H$ and $c/\gamma H \tan \varphi$, while a dimensionless number $FS/\tan \varphi$ is used as the output. The performance of the trained ANN is shown
FIGURE 6  Performance of the trained ANN: (a) 3D cohesive-frictional slopes and (b) 2D cohesive-frictional slopes

FIGURE 7  Model geometry for concave slopes with plan curvature

in Figure 9. The results indicate that a very good performance of the trained ANN is achieved as $R$ is greater than 0.999 and calculated data and predicted results lie closely in a straight line with the slope of 1.

4  |  APPLICATION EXAMPLES

Based on the trained ANN, a program, SlopeLab is built with a user-friendly graphical user interface. As shown in Figure 10, $FS_{2D}$ and $FS_{3D}$ of the slope can be easily obtained after setting a slope’s parameters. Furthermore, a ratio of the factor of safety $FS_{3D}/FS_{2D}$ is displayed, which can be used to find if 3D end effects are significant or not. For example, if $FS_{3D}/FS_{2D} > 1.5$, sophisticated 3D slope stability analyses may be conducted to account for 3D end effects. Comparing to the classic stability charts, selection of the chart and troublesome manual estimation are prevented. A series of examples have been employed to validate the proposed program and to demonstrate its easy use.

4.1  |  An undrained slope

The first example is taken from, which is a slope in ‘undrained clay’ with $\gamma = 16 \text{ kN/m}^3$, $c_u = 14.4 \text{ kPa}$ and $\beta = 26.57^\circ$ (1:2 slope) whose geometry parameters include $B/H = 4$ and $D/H = 1.5$. A stability analysis using the finite element strength reduction method has been conducted for this slope yielding $FS_{3D} = 1.45$. 
Stability charts can provide a solution but a cumbersome manual procedure is required. Since the stability charts are presented based on slopes of $B/H = 1, 2, 3, 5$ and $\beta = 7.5^\circ, 15^\circ, 22.5^\circ, 30^\circ, 45^\circ, 60^\circ$ and $75^\circ$, the slope with $B/H = 4$ and $\beta = 26.57^\circ$ cannot be found straightforwardly. Instead, four most relevant cases are determined, and their $c_u/\gamma H F_S$ and $F_S$ are resolved in sequence as listed in Table 1. It turns out that $F_S$ of the slope is between 1.43 and 1.67 from the results. Finally, linear interpolation of the four slopes' $F_S$ gives an estimated value $F_S = 1.54$.

In contrast, it is very simple to use the proposed program. For undrained slopes, the value of cohesion is equal to the undrained shear strength and a zero frictional angle is used. The default value for the plan curvature radius $r$ is infinite, representing straight slope geometry. After setting the parameters on the left side, two- and three-dimensional factors of safety of the slope are obtained immediately, as shown in Figure 10. The result shows $F_S3D = 1.63$, which agrees well with...
FIGURE 9  Performance of the trained ANN: (a) 3D concave slopes and (b) 2D slopes

FIGURE 10  Results for the undrained slope by the proposed program

the numerical results and the chart solution. In addition, $FS_{3D}/FS_{2D} = 1.29$ indicating that the stability of this slope is underestimated by 2D solution by greater than 20%.

4.2  A cohesive-frictional soil slope

The second example is a homogeneous slope in cohesive-frictional soils. For the slope model, $\gamma = 18$ kN/m$^3$, $c = 4$ kPa, $\varphi = 31^\circ$, $\beta = 37^\circ$ and $B/H = 1.5$. A stability analysis using the finite element strength reduction method has been conducted yielding $FS_{3D} = 1.75$. Since the stability charts$^{55}$ are presented based on $B/H = 0.5, 0.6, 0.8, 1.0, 2.0, 3.0, 5.0, 10$ and $\beta = 15^\circ$, $30^\circ, 45^\circ, 60^\circ, 75^\circ$ and $90^\circ$, to estimate the stability of slope with $B/H = 1.5$ and $\beta = 37^\circ$, four closest slopes need to be determined from different curves. Table 2 shows the manual procedure of the chart solution. The results show that $FS$ is between 1.38
| TABLE 1 | Chart solutions of the undrained slope |
|----------|---------------------------------|
| $\beta = 22.5^\circ$ | $\beta = 30^\circ$ |
| $c_u/\gamma FS (B/H = 3)$ | 0.12 | 0.13 |
| $c_u/\gamma FS (B/H = 5)$ | 0.13 | 0.14 |
| $FS (B/H = 3)$ | 1.67 | 1.54 |
| $FS (B/H = 5)$ | 1.54 | 1.43 |

Estimated $FS$: 1.54 ($B/H = 4$ and $\beta = 26.57^\circ$)

| TABLE 2 | Chart solutions of the cohesive-frictional slope |
|----------|---------------------------------|
| $\beta = 30^\circ$ | $\beta = 45^\circ$ |
| $FS/\tan \phi (B/H = 1)$ | 3.65 | 2.53 |
| $FS/\tan \phi (B/H = 2)$ | 3.24 | 2.29 |
| $FS (B/H = 1)$ | 2.19 | 1.52 |
| $FS (B/H = 2)$ | 1.95 | 1.38 |

Estimated $FS$: 1.78 ($B/H = 1.5$ and $\beta = 37^\circ$)

and 2.19, and linear interpolation gives an estimated $FS$ of 1.78. Following the same procedure, the 2D solution is resolved as estimated $FS_{2D} = 1.56$.

After setting the slope’s parameters in the program, Figure 11 shows the program’s results: $FS_{3D} = 1.69$ and $FS_{2D} = 1.53$, which is in a very good agreement with the numerical results and chart solutions.

### 4.3 | A concave slope in plan view

The third example involves a homogenous concave slope with $\gamma = 18$ kN/m$^3$, $c = 22.5$ kPa, $\phi = 32^\circ$, $\beta = 50^\circ$, $H = 4$ m and $r = 10$ m. Table 3 shows the manual procedures dedicated to estimating $FS$ of the slope with $r/H = 2.5$ and $\beta = 50^\circ$. The chart solutions give $FS_{3D} = 3.31$. Figure 12 shows results from the proposed program: $FS_{3D} = 3.15$ and $FS_{2D} = 2.91$. In addition, the slope’s stability analyses are also performed using numerical methods. The calculation results yield factors of safety: $FS_{3D} = 3.28$ and $FS_{2D} = 2.90$. A satisfying agreement has been found between the program’s results, chart solutions and numerical results, indicating the high accuracy of the trained ANN.

![Graph showing slope parameters and stability factors](image-url)
TABLE 3  Chart solutions of the concave slope

|                  | $\beta = 45^\circ$ | $\beta = 60^\circ$ |
|------------------|---------------------|---------------------|
| $FS/\tan\phi$ ($r/H = 2$) | 5.70               | 4.67               |
| $FS/\tan\phi$ ($r/H = 3$) | 5.55               | 4.56               |
| $FS$ ($r/H = 2$)            | 3.56               | 2.92               |
| $FS$ ($r/H = 3$)            | 3.47               | 2.85               |

Estimated $FS$: 3.31 ($r/H = 2.5$ and $\beta = 50^\circ$)

FIGURE 12  Results for the concave slope by the proposed program

5  | CONCLUSION

Artificial neural networks (ANN) are trained for three-dimensional slope stability predictions in this study. The training dataset is constructed with dimensionless parameters based on stability charts so that the number of variables is reduced and a wide range of slope parameters are guaranteed. Furthermore, for ease of use, a program, SlopeLab, is developed by wrapping the trained ANN with a graphical user interface. After setting the slope parameters, three- and two-dimensional factors of safety, $FS_{3D}$ and $FS_{2D}$, can be obtained immediately. In addition, the three-dimensional end effects of slopes can be evaluated in the result, $FS_{3D}/FS_{2D}$. This would help engineers to decide if a 2D stability analysis solution is justifiable for a slope or not.

The approach applies to slopes in purely cohesive soils and cohesive-frictional soils and concave slopes with plan curvature. A remarkable performance of the trained ANN is demonstrated with the measured correlation coefficient being very close to 1. The program has been adopted for a series of numerical examples, and numerical results and chart solutions have been derived for the purpose of validation. The results show that the proposed program can predict the stability of slopes with high accuracy.

The pore-water pressure conditions are not considered in this work. For future studies, this factor can be taken into account with an additional factor, the pore-water pressure coefficient representing a simplified average pore-water condition in a slope stability analysis. The slopes in this study are assumed to be homogenous and constant Mohr-Coulomb shear strength parameters are used.

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DATA AVAILABILITY STATEMENT

The data used to support the findings of this study are available from the corresponding author upon request.

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