Bounds on the triplet fermions in type-III seesaw and implications for collider searches

Arindam Das\textsuperscript{1,*} and Sanjoy Mandal\textsuperscript{2,†}

\textsuperscript{1}Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
\textsuperscript{2}AHEP Group, Institut de Física Corpuscular, CSIC/Universitat de València, Parc Científic de Paterna, C/ Catedrático José Beltrán, 2 E-46980 Paterna (Valencia), Spain

Abstract

Type-III seesaw is a simple extension of the Standard Model (SM) with the SU(2)\textsubscript{L} triplet fermion with zero hypercharge. It can explain the origin of the tiny neutrino mass and flavor mixing. After the electroweak symmetry breaking the light neutrino mass is generated by the seesaw mechanism which further ensures the mixings between the light neutrino and heavy neutral lepton mass eigenstates. If the triplet fermions are around the electroweak scale having sizable mixings with the SM sector allowed by the correct gauge symmetry, they can be produced at the high energy colliders leaving a variety of characteristic signatures. Based on a simple and concrete realizations of the model we employ a general parametrization for the neutrino Dirac mass matrix and perform a parameter scan to identify the allowed regions satisfying the experimental constraints from the neutrino oscillation data, the electroweak precision measurements and the lepton-flavor violating processes, respectively considering the normal and inverted neutrino mass hierarchies. These parameter regions can be probed at the different collider experiments.

\textsuperscript{*} arindam.das@het.phys.sci.osaka-u.ac.jp
\textsuperscript{†} smandal@ific.uv.es
1. INTRODUCTION

The neutrino masses and the flavor mixings are some of the missing pieces in the SM which have been observed in different experiments [1–16] consistently. Such experimental results are allowing us to think about the Beyond the Standard Model (BSM) scenarios which can explain the neutrino oscillation phenomena. A simple realization of the of the neutrino mass generation scenario was inspired by the introduction of the dimension-5 Weinberg operator [17] within the SM which led to extend the SM with an SM-singlet Majorana right handed neutrinos [18–23] which can explain the neutrino oscillation data, however, there is no experimental observation of the seesaw mechanism or no definite answer of the question of the origin of the neutrino masses. As a result, variety of models have been proposed to address this open question on the origin of the neutrino masses and the nature of the neutrinos.

Type-III seesaw is amongst such proposals where the SM is extended by an SU(2)_L right handed triplet fermion with zero hypercharge to generate small neutrino mass [24] through the seesaw mechanism. The triplet fermion consists of a charge neutral multiplet and a singly charged multiplet where the neutral multiplet participates in the seesaw mechanism to generate the tiny neutrino mass and flavor mixing after the electroweak symmetry breaking. As a result the neutral multiplets can mix with the SM neutrinos and through the mixing they can interact with the SM gauge bosons. Like the neutral multiplet, the charged multiplets can also interact with the SM gauge bosons through the mixing at the time of associated with the SM leptons. Therefore high energy colliders can study the productions of such particles when interacting with the SM gauge bosons. The charged multiplets can be also produced directly (i. e., not suppressed by the light-heavy mixing angle) in pair at various colliders from SM gauge bosons mediated process. A variety of phenomenological aspects for studying the triplet fermions at the colliders have been discussed in [25–32] followed by the experimental searches at the Large Hadron Collider (LHC) [33–41].

The rich phenomenology of the type-III seesaw model has been studied in the past addressing the effective neutrino mass including the threshold effect in [42]. The stability of the scalar potential under the perturbativity bounds for a set of degenerate triplet fermions had been studied in [43] using the evolutions of the renormalization group equations. The electroweak vacuum stability for the nonzero neutrino mass, naturalness and lepton flavor
violation have been studied in [44] for the two generations of the triples which can successfully reproduce the neutrino oscillation data for the normal and inverted orderings of the light neutrino mass spectra. Type-III seesaw has been motivated under an U(1) extension of the SM where a heavy resonantly produced pair of the triplet fermions can be successfully studied and followed by that a BSM neutral gauge boson can be probed. Type-III seesaw scenario has been realized in the grand unified theories where a triplet and a singlet fermions were proposed to be added in [45–47] where the triplet can reproduce the neutrino oscillation data being in the intermediate scale. Additionally a development of the type-III seesaw scenario was proposed in the SU(5) theory through the inclusion of the adjoint fermionic multiplet in [48] and further phenomenological analyses were performed in [48–50]. The supersymmetric version of this theory had been proposed in [51] followed by the nonsupersymmetric counterpart in [52] to find a renormalizable framework to investigate the origin of the small neutrino mass under the grand unification inspired SU(5) theory. Alternatively an inverse seesaw mechanism has been proposed in the type-III framework [53] adding a U(1)Y hyperchargeless singlet fermion and an SU(2)_{L} triplet fermion in [54] using an additional U(1) gauge group with the anomaly free scenario [55–57] to the SM. There are a verity of indirect search strategies prescribed for the type-III seesaw scenario including Lepton Flavor Violation (LFV) [58] and nonunitarity effects to [59, 60]. In this context we also mention that such studies have been made in the context of the type-I seesaw in [61–72] where only a Majorana type, heavy, and SM singlet right handed neutrino was introduced in the SM. Limits on the light heavy neutrino mixing from the Eletroweak Precision Data (EWPD) were studied in [73, 74].

In this paper we study the type-III model generalizing the Dirac Yukawa coupling following the Casas-Ibarra conjecture [75] under the constraints obtained from the nonunitary effects, LFV and EWPD applying the neutrino oscillation data. In our study we consider three degenerate generations of the SU(2)_{L} triplet fermions which are involved in the neutrino mass generations mechanism form the seesaw mechanism considering the normal and inverted hierarchies of the light neutrino masses. In the type-III seesaw the mixings between the light and heavy mass eigenstates play important roles to study the triplets at different high energy colliders, for example, proton-proton (pp), electron-positron (e^{-}e^{+}) and electron-proton (e^{-}p). There are some production processes where the production cross section of the triplet might not be affected by mixings, however, their branching ratios will
depend upon the mixings. As an example we may consider the pair production triplets (charged multiplets in pair and charged and neutral multiplets productions) where the productions processes do not depend upon the mixing directly, however, the dependence of the the mixing comes at the time of the decay of the triplets. The generation of the neutrino mass mechanism in the type-III seesaw is a type of seesaw mechanism where the Dirac Yukawa coupling is always non-diagonal which gives rise to the Flavor Non-diagonal (FND) scenario to correctly produce the neutrino oscillation data which will be considered in this article. Depending upon the constraints we will show the allowed parameter space which can be probed by the collider based experiments in the near future.

The paper is organized in the following way. In Sec. 2, we discuss the model and the interactions of the triplet fermions with the SM particles. In the Sec. 3 we discuss general parametrization of the Yukawa coupling and its effect on the different production modes and decay of the triplets. In the Sec. 4 we discuss about the branching ratios of the triplet fermions under the general parameters. We study the possibility of the displaced vertices from the type-III seesaw in Sec. 5. We discuss the results in Sec. 6 and finally conclude the article in Sec. 7.

2. MODEL

In the type-III seesaw model SM is extended by three generations of an $SU(2)_L$ triplet fermion ($\Psi$) with zero hypercharge. Inclusion of such triplets helps the generation of nonzero but tiny neutrino mass through the seesaw mechanism. The Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{SM} + \text{Tr}(\bar{\Psi} i \gamma^\mu D^\mu \Psi) - \frac{1}{2} M \text{Tr}(\bar{\Psi}\Psi^c + \bar{\Psi}^c \Psi) - \sqrt{2}(\ell_L Y_D \Psi H + H^\dagger \Psi^c D^\mu \ell_L)$$  

where $D^\mu$ represents the covariant derivative, $M$ is the Majorana mass term. $\mathcal{L}_{SM}$ is the relevant part of the SM Lagrangian. We consider three degenerate generation of the triplets. Therefore $M$ is proportional to $1_{3 \times 3}$. $Y_D$ is the Dirac Yukawa coupling between the SM lepton doublet ($\ell_L$), SM Higgs doublet ($H$) and the triplet fermion ($\Psi$). For brevity, we have suppressed the generation indices. In this analysis we represent the relevant SM candidates, the triplet fermion and its charged conjugate ($\Psi^c = C\Psi^T$) as in the following way

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad H = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}, \quad \Psi = \begin{pmatrix} \Sigma^0 / \sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0 / \sqrt{2} \end{pmatrix} \quad \text{and} \quad \Psi^c = \begin{pmatrix} \Sigma^0 c / \sqrt{2} & \Sigma^- c \\ \Sigma^+ c & -\Sigma^0 c / \sqrt{2} \end{pmatrix}$$  


After the breaking of the electroweak symmetry $\phi^0$ acquires a vacuum expectation value and we can express it as $\phi^0 = \frac{v + i h}{\sqrt{2}}$ with $v = 246$ GeV. To study the mixing between the SM charged leptons and $\Sigma^\pm$ we write the four degrees of freedom of each $\Sigma^\pm$ in terms of a Dirac spinor such as $\Sigma = \Sigma^- R + \Sigma^+ c R$ where as $\Sigma^0$ are two component fermions with two degrees of freedom. The corresponding Lagrangian after the electroweak symmetry breaking can be written as

$$-L_{\text{mass}} = \left( \bar{e}_L \Sigma_L \right) \begin{pmatrix} m_\ell & Y_D^T \nu \cr 0 & M \end{pmatrix} \begin{pmatrix} e_R \\ \Sigma_R \end{pmatrix} + \frac{1}{2} \left( \bar{\nu}_L \Sigma^0_R \right) \begin{pmatrix} 0 & Y_D^T v \sqrt{2} \\ Y_D v \sqrt{2} & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma^0_R \end{pmatrix} + \text{h.c.} \quad (3)$$

where $m_\ell$ is the Dirac type SM charged lepton mass. The $3 \times 3$ Dirac mass of the triplets can be written as

$$M_D = \frac{Y_D^T v}{\sqrt{2}}. \quad (4)$$

Diagonalizing the neutrino mass matrix in Eq. 3 we can write the light neutrino mass eigenvalue as

$$m_\nu \simeq -\frac{v^2}{2} Y_D^T M^{-1} Y_D = M_D M^{-1} M_D^T \quad (5)$$

hence the mixing between light and heavy mass eigenstates can be obtained as $\mathcal{O}(M_D M^{-1})$. Hence the light neutrino flavor eigenstate can be expressed in terms of the light and heavy mass eigenstates in the following way

$$\nu = \mathcal{A} \nu_m + V \Sigma_m \quad (6)$$

where $\nu_m$ and $\Sigma_m$ represent the light and heavy mass eigenstates respectively where $V = M_D M^{-1}$ and $\mathcal{A} = \left(1 - \frac{1}{2} \bar{\epsilon}^T \right) V_{\text{PMNS}}$ with $\bar{\epsilon} = V^* V^T$ and $V_{\text{PMNS}}$ is the $3 \times 3$ neutrino mixing matrix which diagonalizes the light neutrino mass matrix as

$$V_{\text{PMNS}}^T m_\nu V_{\text{PMNS}} = \text{diag}(m_1, m_2, m_3). \quad (7)$$

Due to the presence of $\bar{\epsilon}$ the mixing matrix ($\mathcal{A}$) becomes non-unitary, $\mathcal{A}^T \mathcal{A} \neq 1$. The charged current (CC) interactions can be expressed in terms of the mass eigenstates including the light heavy mixings as

$$-L_{\text{CC}} = \frac{g}{\sqrt{2}} \left( \bar{e} \Sigma \right) \gamma^\mu W^- \mu P_L \begin{pmatrix} (1 + \frac{\epsilon}{2}) V_{\text{PMNS}} & -\frac{Y_D^T M^{-1} v}{\sqrt{2}} \\ 0 & \sqrt{2} (1 - \frac{\epsilon}{2}) \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix}$$

$$+ \frac{g}{\sqrt{2}} \left( \bar{e} \Sigma \right) \gamma^\mu W^- \mu P_R \begin{pmatrix} 0 & -\sqrt{2} m_\ell Y_D^T M^{-2} v \\ -\sqrt{2} M^{-1} Y_D (1 - \frac{\epsilon}{2}) V_{\text{PMNS}}^* & \sqrt{2} (1 - \frac{\epsilon}{2}) \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix} \quad (8)$$
and the modified neutral current (NC) interaction for the charged sector can be written as

\[- \mathcal{L}_{\text{NC}} = \frac{g}{\cos \theta_W} \left( \bar{e} \Sigma \right) \gamma^\mu Z_\mu P_L \left( \frac{1}{2} - \cos^2 \theta_W - \epsilon \frac{Y_D^1 M^{-1} v}{\sqrt{2}} \right) \left( \frac{e}{\Sigma} \right) + \frac{g}{\cos \theta_W} \left( \bar{e} \Sigma \right) \gamma^\mu Z_\mu P_R \left( 1 - \cos^2 \theta_W - \epsilon' \frac{Y_D^1 M^{-2} v}{\sqrt{2}} \right) \left( \frac{e}{\Sigma} \right) + \left( \bar{\nu} \Sigma^0 \right) \gamma^\mu Z_\mu P_L \left( 1 - V_{\text{PMNS}}^T \epsilon V_{\text{PMNS}} \frac{V_{\text{PMNS}}^T Y_D^1 M^{-2} v}{\sqrt{2}} \right) \left( \frac{\nu}{\Sigma^0} \right) \right] (9)\]

where \( \theta_W \) is the Weinberg angle or weak mixing angle. Finally we write the interaction Lagrangian of the SM leptons, triplet fermions with the SM Higgs \( (h) \) boson. The interaction Lagrangian can be written as

\[- \mathcal{L}_H = \frac{g}{2M_W} \left( \bar{e} \Sigma \right) h P_L \left( \frac{-m_v}{v} (1 - 3\epsilon) \frac{m_\ell Y_D^1 M^{-1}}{Y_D^1 (1 - \epsilon) + M^{-2} Y_D m_\ell^2} \right) \left( \frac{e}{\Sigma} \right) + \frac{g}{2M_W} \left( \bar{e} \Sigma \right) P_R \left( \frac{-m_v}{v} (1 - 3\epsilon^*) \frac{m_\ell Y_D^1 M^{-1}}{Y_D + M^{-2} \ell M^{-2} Y_D^1 M^{-2} \ell^* Y_D^1 M^{-1}} \right) \left( \frac{e}{\Sigma} \right) + \left( \bar{\nu} \Sigma^0 \right) h P_L \left( \frac{\sqrt{2} m_v}{v} \frac{V_{\text{PMNS}}^T Y_D^1 M^{-1}}{Y_D - \frac{Y_D^1}{2} - \frac{\epsilon v}{2} Y_D} \right) \left( \frac{\nu}{\Sigma^0} \right) + \left( \bar{\nu} \Sigma^0 \right) P_R \left( \frac{\sqrt{2} m_v}{v} \frac{V_{\text{PMNS}} Y_D^1 M^{-1}}{Y_D - \frac{Y_D^1}{2} - \frac{Y_D^1}{2} \epsilon Y_D} \right) \left( \frac{\nu}{\Sigma^0} \right) \right] (10)\]

The charged multiplets of the triplet fermions can interact with photons \( (A_\mu) \). The corresponding Lagrangian derived from Eq. 1 can be written as

\[- \mathcal{L}_{\gamma \Sigma} = g \sin \theta_W \left( \bar{e} \Sigma \right) \gamma^\mu A_\mu P_L \left( \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right) \left( \frac{e}{\Sigma} \right) + g \sin \theta_W \left( \bar{e} \Sigma \right) \gamma^\mu A_\mu P_R \left( \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right) \left( \frac{e}{\Sigma} \right) \right] (11)\]

In the Eqs. 8-10 the parameters \( \epsilon = \frac{\sqrt{2}}{2} Y_D^1 M^{-2} Y_D, \ \epsilon' = \frac{\sqrt{2}}{2} M^{-1} Y_D Y_D^1 M^{-1} \) are the small quantities according to [27, 58, 59]. We neglect the effects of the higher powers (above 1) of \( \epsilon \) and \( \epsilon' \) in the calculations. Using the Eq. 8 to Eq. 10 and the expression for the mixing
we calculate the partial decay widths of \((\Sigma^0)\) as
\[
\Gamma(\Sigma^0 \rightarrow \ell^+W) = \Gamma(\Sigma^0 \rightarrow \ell^-W) = \frac{g^2|V_{\ell\Sigma}|^2}{64\pi} \left( \frac{M^3}{M_W^2} \right) \left( 1 - \frac{M_W^2}{M^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{M^2} \right)
\]
\[
\Gamma(\Sigma^0 \rightarrow \nu Z) = \Gamma(\Sigma^0 \rightarrow \bar{\nu}Z) = \frac{g^2|V_{\ell\Sigma}|^2}{128\pi \cos^2 \theta_W} \left( \frac{M^3}{M_Z^2} \right) \left( 1 - \frac{M_Z^2}{M^2} \right)^2 \left( 1 + 2 \frac{M_Z^2}{M^2} \right)
\]
\[
\Gamma(\Sigma^0 \rightarrow \nu h) = \Gamma(\Sigma^0 \rightarrow \bar{\nu}h) = \frac{g^2|V_{\ell\Sigma}|^2}{64\pi} \left( \frac{M^3}{M_h^2} \right) \left( 1 - \frac{M_h^2}{M^2} \right)^2
\]
(12)
respectively for the Majorana neutrinos. The corresponding Feynman Diagrams have been shown in Fig. 1. Similarly the partial decay widths of \((\Sigma^\pm)\) are calculated as
\[
\Gamma(\Sigma^\pm \rightarrow \nu W) = \frac{g^2|V_{\ell\Sigma}|^2}{32\pi} \left( \frac{M^3}{M_W^2} \right) \left( 1 - \frac{M_W^2}{M^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{M^2} \right)
\]
\[
\Gamma(\Sigma^\pm \rightarrow \ell Z) = \frac{g^2|V_{\ell\Sigma}|^2}{64\pi \cos^2 \theta_W} \left( \frac{M^3}{M_Z^2} \right) \left( 1 - \frac{M_Z^2}{M^2} \right)^2 \left( 1 + 2 \frac{M_Z^2}{M^2} \right)
\]
\[
\Gamma(\Sigma^\pm \rightarrow \ell h) = \frac{g^2|V_{\ell\Sigma}|^2}{64\pi} \left( \frac{M^3}{M_h^2} \right) \left( 1 - \frac{M_h^2}{M^2} \right)^2
\]
(13)
respectively. \(M_W, M_Z\) and \(M_h\) in the above expressions are the SM \(W, Z\) and Higgs boson masses respectively. The corresponding Feynman Diagrams have been shown in Fig. 2. The charged multiplet \(\Sigma^\pm\) and neutral multiplet \(\Sigma^0\) are degenerate in mass at the tree-level. This degeneracy is lifted up due to the radiative corrections induced by the SM gauge boson in the loop. The estimation of this mass difference \(\Delta M\) is found in Ref. [76] and is given by:
\[
\Delta M = \frac{\alpha_2 M}{4\pi} \left( f \left( \frac{M_W}{M} \right) - \cos^2 \theta_W f \left( \frac{M_Z}{M} \right) \right)
\]
(14)
where the function \(f\) is defined as \(f(r) = \frac{r}{2} \left( 2r^3 \ln r - 2r + \sqrt{r^2 - 4} (r^2 + 2) \ln A \right)\) and \(A = \left( r^2 - 2 - r \sqrt{r^2 - 4} \right) / 2\). This mass splitting saturates at the value \(\Delta M \approx 170\ \text{MeV}\) for mass
$M > 500$ GeV. If this mass splitting $\Delta M$ is larger than pion mass, then $\Sigma^\pm$ will have the following additional decay modes [76]

\begin{align*}
\Gamma(\Sigma^\pm \rightarrow \Sigma^0 \pi^\pm) &= \frac{2G_F^2 V_{ud}^2 \Delta M^3 f_\pi^2}{\pi} \sqrt{1 - \frac{m_\pi^2}{\Delta M^2}} \\
\Gamma(\Sigma^\pm \rightarrow \Sigma^0 e\nu_e) &= \frac{2G_F^2 \Delta M^5}{15\pi} \\
\Gamma(\Sigma^\pm \rightarrow \Sigma^0 \mu\nu_\mu) &= 0.12 \Gamma(\Sigma^\pm \rightarrow \Sigma^0 e\nu_e)
\end{align*}

which are independent of the free parameters. The corresponding Feynman Diagrams have been shown in Fig. 3. The value of the Fermi Constant, $G_F$, is $1.1663787 \times 10^{-5}$ GeV$^{-2}$, the value of the CKM parameter ($V_{ud}$) is $0.97420 \pm 0.00021$ and the decay constant of the $\pi$ meson, $f_\pi$, is 0.13 GeV from [77]. Notice that for vanishing mixing angles $V_{\Sigma\ell}$, the $\Sigma^\pm$ dominantly decay into $\Sigma^0$, hence the decay width or the decay length is determined by $\Delta M$ and is constant. On the contrary, for small mixing angles, $\Sigma^0$ decay width (decay length) is very small (very large).
The elements of the matrices $\mathcal{A}$ and $V$ in Eq. 6 can be constrained by the experimental data. In this analysis we take the global fit results at 3σ level [78] for the neutrino oscillation parameters:

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = \left[6.79 \times 10^{-5} \text{eV}^2, 8.01 \times 10^{-5} \text{eV}^2\right]$$

$$\Delta m_{23}^2 = |m_3^2 - m_2^2| = \left[2.432 \times 10^{-3} \text{eV}^2, 2.618 \times 10^{-3} \text{eV}^2\right]$$

$$\sin^2 \theta_{12} = \begin{bmatrix} 0.275, 0.350 \end{bmatrix}$$

$$\sin^2 \theta_{23} = \begin{bmatrix} 0.427, 0.609 \end{bmatrix}$$

$$\sin^2 \theta_{13} = \begin{bmatrix} 0.02046, 0.02440 \end{bmatrix}.$$  \hspace{1cm} (16)

The $3 \times 3$ neutrino mixing matrix $V_{PMNS}$ is given by

$$V_{PMNS} = \begin{pmatrix}
 c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta_{CP}} \\
 -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\
 s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13}
\end{pmatrix}\begin{pmatrix}
 1 & 0 & 0 \\
 0 & e^{i\rho_1} & 0 \\
 0 & 0 & e^{i\rho_2}
\end{pmatrix}.$$ \hspace{1cm} (17)

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. In our analysis the Dirac CP-phase ($\delta_{CP}$) is a free parameter running between the limit $[-\pi, \pi]$. However, in the recent experiments by NO$\nu$A [79] and T2K [80] indicate that $\delta_{CP}$ can be $-\frac{\pi}{2} \pm \frac{\pi}{2}$. Due to non-unitarity [59] the elements of $\mathcal{A}$ are severely constrained at 90% C. L.:

$$|\mathcal{A}^\dagger\mathcal{A}| = \begin{pmatrix}
 1.001 \pm 0.002 & < 1.1 \times 10^{-6} & < 1.2 \times 10^{-3} \\
 < 1.1 \times 10^{-6} & 1.002 \pm 0.002 & < 1.2 \times 10^{-3} \\
 < 1.2 \times 10^{-3} & < 1.2 \times 10^{-3} & 1.002 \pm 0.002
\end{pmatrix}.$$ \hspace{1cm} (18)

The diagonal elements of Eq. 18 are obtained from the precision studies of the SM weak boson where as the SM prediction is 1. The off-diagonal entries of Eq. 18 are the upper bounds obtained from the cLFV studies, for example, the constraints on the 12 and 21 elements of Eq. 18 are coming from the the $\mu \to 3e$ process [81], the constraints on the 23 and 32 elements are coming from the $\tau \to 3\mu$ process and finally the constraints on the 13 and 31 elements are originated from the $\tau \to 3e$ process respectively. These bounds are taken from [59]. The diagonal elements are obtained from LEP [77, 82]. As a result we have $\mathcal{A}^\dagger\mathcal{A} \simeq 1 - \bar{\epsilon}$ and we can calculate the constraints on $\bar{\epsilon}$ from Eq. 18 as

$$|\bar{\epsilon}| = \begin{pmatrix}
 0.001 \pm 0.002 & < 1.1 \times 10^{-6} & < 1.2 \times 10^{-3} \\
 < 1.1 \times 10^{-6} & 0.002 \pm 0.002 & < 1.2 \times 10^{-3} \\
 < 1.2 \times 10^{-3} & < 1.2 \times 10^{-3} & 0.002 \pm 0.002
\end{pmatrix}.$$ \hspace{1cm} (19)
where we have used the central values for the diagonal elements. Note that the stringent bound is given by the 12-element which is originated from the $\mu \to 3e$ eLFV process.

3. Bounds on the Mixing Angles Under the General Parametrization and Its Effect on the Decay of the Triplet Fermions

In this analysis we generalize of the Dirac Yukawa mass matrix of Eq. 4 using the Casas-Ibarra [75] conjecture as follows

$$M_D^{NH/IH} = V_{PMNS}^* \sqrt{D}^{NH/IH} O \sqrt{M},$$

(20)

where $O$ is a general orthogonal matrix and it can be written as

$$O = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos x & \sin x \\
0 & -\sin x & \cos x
\end{pmatrix}
\begin{pmatrix}
\cos [y] & 0 & \sin [y] \\
0 & 1 & 0 \\
-\sin [y] & 0 & \cos [y]
\end{pmatrix}
\begin{pmatrix}
\cos [z] & \sin [z] & 0 \\
-\sin [z] & \cos [z] & 0 \\
0 & 0 & 1
\end{pmatrix},$$

(21)

where the angles $x, y, z$ are the complex numbers. Now using $\tilde{\epsilon} = (V^*V)^{NH/IH}, (V_{\alpha i})^{NH/IH} = M_{D^{NH/IH}}M^{-1} \text{ and Eqs. 5 and 7 for the two different hierarchies we can write}

$$\tilde{\epsilon}^{NH/IH} = V_{PMNS}^* \sqrt{D}^{NH/IH} O^* M^{-1} O^T \sqrt{D}^{NH/IH} V_{PMNS}^T,$$

(22)

where NH is the normal hierarchy ($m_3 > m_2 > m_1$) and IH is the inverted hierarchy ($m_2 > m_1 > m_3$). The light neutrino mass eigenvalue matrices ($\sqrt{D}$) for the NH and IH cases are written as

$$\sqrt{D}^{NH} = \begin{pmatrix}
\sqrt{m_1} & 0 & 0 \\
0 & \sqrt{m_2^{NH}} & 0 \\
0 & 0 & \sqrt{m_3^{NH}}
\end{pmatrix}, \sqrt{D}^{IH} = \begin{pmatrix}
\sqrt{m_1^{IH}} & 0 & 0 \\
0 & \sqrt{m_2^{IH}} & 0 \\
0 & 0 & \sqrt{m_3^{IH}}
\end{pmatrix},$$

(23)

where $m_2^{NH} = \sqrt{\Delta m_{12}^2 + m_1^2}, m_3^{NH} = \sqrt{\Delta m_{23}^2 + (m_2^{NH})^2}, m_2^{IH} = \sqrt{\Delta m_{23}^2 + m_3^2}$ and $m_1^{IH} = \sqrt{(m_2^{IH})^2 - \Delta m_{12}^2}$ for the NH and IH respectively. In both cases, the triplet mass matrix is defined as $M = M(1_{3 \times 3})$ which is proportional to a $3 \times 3$ unit matrix for the three degenerate triplets. In Eq. 23 the lightest mass eigenvalue is a free parameter and bounded from the PLANCK data [83] and $m_1(m_3)$ is the lightest light neutrino mass eigenvalue for the NH (IH) case. In this analysis $\delta_{CP}$ and $\rho_{1,2}$ vary between $[-\pi, \pi]$. In this context we mention that
FIG. 4. Bounds on $\Sigma_i |V_{\Sigma i}|^2$ as a function of the $m_1(m_3)$ NH (IH) case in the left (right) panel for fixed SM lepton flavors. The red band represents electron ($e$), the blue band represents the muon ($\mu$) and the green band represents the tau ($\tau$). In this case we consider $O = 1_{3 \times 3}$ as a identity matrix. The same nature will be obtained from case when $O$ is a real orthogonal matrix. We fix the triplet mass $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data. We consider $M = 1$ TeV.

Seesaw mechanism has been extensively studied utilizing the general parametrization under the Casas-Ibarra conjecture in [84–93] and following that to study the vacuum stability in type-III seesaw with two generations of the triplet fermions using the Casas-Ibarra conjecture has been studied in [44], however, in our analysis we study three degenerate triplets under the constraints obtained from the indirect searches. We have three different choices for the orthogonal matrix in Eq. 21 as follows:

(i) $O$ is a identity matrix, $O = 1_{3 \times 3}$. In this case Eq. 20 will be

$$M_D^{\text{NH/IH}} = V_{PMNS}^* \sqrt{D_{\text{NH/IH}}} \sqrt{M}. \quad (24)$$

This will further affect the light-heavy mixing. In this case there is no dependence on $x, y, z$.

(ii) $O$ is a real orthogonal matrix with diagonal and off-diagonal entries, $(x, y, z)$ are real and vary between $[-\pi, \pi]$
(iii) \( O \) is a complex orthogonal matrix where \( x, y, z \) are the complex numbers, i.e., \( x_i + iy_i \) and \( -\pi \leq x_i, y_i \leq \pi \).

For the cases (i) and (ii) using the two hierarchies of the neutrino masses (NH and IH) we calculate the modulus square of the mixing between a triplet and the corresponding lepton flavors. Then fixing the lepton flavor, we sum over the triplets as

\[
\Sigma_i |V_{\ell \Sigma_i}|^2 = |V_{\ell \Sigma_1}|^2 + |V_{\ell \Sigma_2}|^2 + |V_{\ell \Sigma_3}|^2
\]

where \( \ell = e, \mu \) and \( \tau \). Note that \( \Sigma_i |V_{\ell \Sigma_i}|^2 \) is same if \( O \) is identity or real orthogonal matrix.

For both of these cases, \( \Sigma_i |V_{\ell \Sigma_i}|^2 \) have been plotted as a function of the lightest light neutrino mass eigenvalue in Fig. 4. The NH (IH) case is shown in the left (right) panel as a function of \( m_1 \) (\( m_3 \)) where the electron flavor is presented by the red band and the muon and tau flavors are represented by the blue and green bands. In the NH (IH) case the bounds on the electron flavor (muon and tau flavors) are stronger for the decreasing \( m_1 \) (\( m_3 \)). In this analysis we fix the triplet mass \( M = 1 \) TeV.

We also plot the individual mixing as a function of the \( m_1(m_3) \) for the NH (IH) case in the top (bottom) panel of the Fig. 5 for case (i). We find that \( |V_{\ell \Sigma_1}|^2 \) for electron (red), muon (blue) and tau (green) in the NH case are related to \( m_1 \), lower the value of \( m_1 \) lowers the individual mixing in the NH case whereas in the IH case the mixings are parallel to the horizontal axis below the PLANCK limit. In both of the cases the \( |V_{\ell \Sigma_1}|^2 \) is less stronger than the other mixings. The mixings for other two flavors overlap with each other. The nature of the \( |V_{\ell \Sigma_2}|^2 \) is same for the three flavors of the leptons in both of the NH and IH cases, where all flavors overlap with each other. On the other hand for \( |V_{\ell \Sigma_3}|^2 \) the mixing with the electron flavor is stronger than those with the other two flavors whereas \( |V_{\mu \Sigma_3}|^2 \) and \( |V_{\tau \Sigma_3}|^2 \) overlap with each other in both of the NH and IH cases, however, in the NH case all three mixings are parallel to the horizontal axis below the PLANCK limit. On the other hand in the IH case mixing decreases with the decreasing \( m_3 \).

In the following we write down the individual mixings between the \( \Sigma_1 \) and the three generations of the leptons for the case of \( O = 1_{3 \times 3} \):

\[
|V_{e \Sigma_1}|^2 = m_1 \frac{c_{12}^2 c_{13}^2}{M} \\
|V_{\mu \Sigma_1}|^2 = m_1 \frac{c_{12} s_{12} + c_{12} e^{i \delta_{CP}} s_{13} s_{23}}{M} \\
|V_{\tau \Sigma_1}|^2 = m_1 \frac{c_{12} c_{23} e^{i \delta_{CP}} s_{13} - s_{12} s_{23}}{M}.
\]
FIG. 5. Bounds on the individual mixing $|V_{i\Sigma_i}|^2$ for $O = 1_{3\times3}$ as a function of $m_1 (m_3)$ in the NH (IH) case. In this case we fix the triplet flavor ($\Sigma_i$) and find the bounds on its mixing with electron (red), muon (blue) and tau (green). We have considered $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data. We consider $M = 1$ TeV.

We write down the individual mixings between the $\Sigma_2$ and the three generations of the leptons for the case of $O = 1_{3\times3}$:

$$|V_{e\Sigma_2}|^2 = m_2 \frac{c_2^2 s_{12}^2}{M}$$
$$|V_{\mu\Sigma_2}|^2 = m_2 \frac{|c_2 c_{13} e^{i\delta_{CP}} s_{12} s_{13} + c_{12} s_{23}|^2}{M}$$
$$|V_{\tau\Sigma_2}|^2 = m_2 \frac{|c_{12} c_{23} - e^{i\delta_{CP}} s_{12} s_{13} s_{23}|^2}{M}$$

(27)

and we write down the individual mixings between the $\Sigma_3$ and the three generations of the leptons for the case of $O = 1_{3\times3}$:

$$|V_{e\Sigma_3}|^2 = m_3 \frac{s_{13}^2}{M}$$
$$|V_{\mu\Sigma_3}|^2 = m_3 \frac{c_{13}^2 s_{23}^2}{M}$$
$$|V_{\tau\Sigma_3}|^2 = m_3 \frac{c_{13}^2 c_{23}^2}{M}$$

(28)

Hence we can calculate $\Sigma_i |V_{i\Sigma_i}|^2$ from the Eqs. 26-28 for $i = 1, 2, 3$ and $\ell = e, \mu, \tau$. 

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FIG. 6. Bounds on the individual mixing $|V_{\Sigma i}|^2$ for a real orthogonal matrix $O$ as a function of $m_1(m_3)$ in the NH (IH) case. In this case we fix the triplet flavor ($\Sigma_i$) and find the bounds on its mixing with electron (red), muon (blue) and tau (green). We have considered $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data. We consider $M = 1$ TeV.

We notice that $|V_{\Sigma 1}|^2$ is proportional to $m_1$ and $|V_{\Sigma 3}|^2$ is proportional to $m_3$. Hence in the NH and IH cases the corresponding individual mixings in Eqs. 26 and 28 will tend to zero as $m_1 \to 0$ for the NH and $m_3 \to 0$ for the IH cases, which is clearly visible in Fig. 5. The behavior for the other mixings in the NH ($|V_{\Sigma 2}|^2, |V_{\Sigma 3}|^2$) and IH ($|V_{\Sigma 1}|^2, |V_{\Sigma 2}|^2$) cases do not have this behavior because they depend on $(m_2, m_3)$ for the NH and on $(m_1, m_3)$ for the IH cases respectively. They almost independent of lightest light neutrino mass eigenvalue for the respective NH ($m_1$) and IH ($m_3$) cases slightly below the PLANCK limit.

In the similar fashion we study the case (ii) where $O$ is a real orthogonal matrix of the form Eq. 21 where the elements are the real parameters. The corresponding parameter regions for individual mixing angles are shown in Fig. 6. We notice that the mixing $|V_{e\Sigma 1}|^2$ ($|V_{e\Sigma 3}|^2$) does not go to zero even with the limit $m_1 \to 0(m_3 \to 0)$ for the NH (IH) case. In the NH case the upper limit of the $|V_{e\Sigma 1}|^2$ parameter space stays below the other two mixings for the three generations of the triplets. This is opposite in the IH case.

In the following we write down the individual mixings between the $\Sigma_1$ and the three
generations of the leptons with \( O \) as real orthogonal matrix:

\[
|V_{\Sigma_1}|^2 = \frac{1}{M_{\Sigma}} c_{12} c_{13} \cos[y] \cos[z] \sqrt{|m_1|} - e^{i(\delta_{CP} - \rho_2)} s_{13} \sin[y] \sqrt{|m_3|} - c_{13} e^{-i\rho_1} s_{12} \cos[y] \sin[z] \sqrt{|m_2|}
\]

\[
|V_{\mu\Sigma_1}|^2 = \frac{1}{M_{\Sigma}} | - e^{i(\delta_{CP} - \rho_2)} s_{13} \cos[y] \sin[x] \sqrt{|m_3|} + c_{12} c_{13} \sqrt{|m_2|} (- \cos[z] \sin[x] \sin[y] + \cos[x] \sin[z]) + e^{-i\rho_1} c_{13} \sqrt{|m_2|} s_{12} \\
\cos[x] \cos[z] + \sin[x] \sin[y] \sin[z])^2
\]

\[
|V_{\nu\Sigma_1}|^2 = \frac{1}{M_{\Sigma}} e^{i(\delta_{CP} - \rho_2)} \sqrt{|m_3|} s_{13} \cos[y] \cos[y] + c_{12} c_{13} \sqrt{|m_1|} \cos[x] \cos[z] + \sin[x] \sin[y] \sin[z]) + c_{12} e^{-i\rho_1} \\
\sqrt{|m_2|} s_{12} \left( \cos[z] \sin[x] - \cos[x] \sin[y] \sin[z] \right)^2.
\]

(29)

We write down the individual mixings between the \( \Sigma_2 \) and the three generations of the leptons with \( O \) as real orthogonal matrix:

\[
|V_{e\Sigma_2}|^2 = \frac{1}{M_{\Sigma}} \sqrt{|m_1|} (-c_{23} s_{12} - c_{12} e^{i\delta_{CP}} s_{13} s_{23}) \cos[y] \cos[z] - c_{13} e^{-i\rho_2} \sqrt{|m_3|} s_{23} \sin[y] \\
- e^{-i\rho_1} \sqrt{|m_2|} (c_{12} c_{23} - e^{i\delta_{CP}} s_{12} s_{13} s_{23}) \cos[y] \sin[z])^2
\]

\[
|V_{\mu\Sigma_2}|^2 = \frac{1}{M_{\Sigma}} | - c_{13} e^{-i\rho_2} \sqrt{|m_3|} s_{23} \cos[y] \sin[x] + \sqrt{|m_1|} (-c_{23} s_{12} - c_{12} e^{i\delta_{CP}} s_{13} s_{23}) \\
(- \cos[z] \sin[x] \sin[y] + \cos[x] \sin[z]) + e^{-i\rho_1} \sqrt{|m_2|} (c_{12} c_{23} - e^{i\delta_{CP}} s_{12} s_{13} s_{23}) \\
\cos[x] \cos[z] + \sin[x] \sin[y] \sin[z])^2
\]

\[
|V_{\tau\Sigma_2}|^2 = \frac{1}{M_{\Sigma}} | c_{13} e^{-i\rho_2} \sqrt{|m_3|} s_{23} \cos[y] \cos[y] + \sqrt{|m_1|} (-c_{23} s_{12} - c_{12} e^{i\delta_{CP}} s_{13} s_{23}) \\
\cos[x] \cos[z] \sin[y] + \sin[x] \sin[z]) + e^{-i\rho_2} \sqrt{|m_2|} (c_{12} c_{23} - e^{i\delta_{CP}} s_{12} s_{13} s_{23}) \\
\cos[z] \sin[x] - \cos[x] \sin[y] \sin[z])^2
\]

(30)

and we write down the individual mixings between the \( \Sigma_3 \) and the three generations of the leptons with \( O \) as real orthogonal matrix:

\[
|V_{e\Sigma_3}|^2 = \frac{1}{M_{\Sigma}} \sqrt{|m_1|} \left( -c_{12} c_{23} e^{i\delta_{CP}} s_{13} + s_{12} s_{23} \right) \cos[y] \cos[z] - c_{13} c_{23} e^{-i\rho_2} \sqrt{|m_3|} \sin[y] \\
- e^{i\rho_1} \sqrt{|m_2|} \left( -c_{12} e^{i\delta_{CP}} s_{12} s_{13} - c_{12} s_{23} \right) \cos[y] \sin[z])^2
\]

\[
|V_{\mu\Sigma_3}|^2 = \frac{1}{M_{\Sigma}} | - c_{13} c_{23} e^{-i\rho_2} \cos[y] \sin[x] + \sqrt{|m_1|} (-c_{12} c_{23} e^{i\delta_{CP}} s_{13} + s_{12} s_{23}) (- \cos[z] \sin[x] \sin[y] \\
+ \cos[x] \sin[z]) + e^{-i\rho_1} \sqrt{|m_2|} (-c_{23} e^{i\delta_{CP}} s_{12} s_{13} - c_{12} s_{23}) (\cos[x] \cos[z] + \sin[x] \sin[y] \sin[z])^2
\]

\[
|V_{\tau\Sigma_3}|^2 = \frac{1}{M_{\Sigma}} c_{13} c_{23} e^{-i\rho_2} \sqrt{|m_3|} \cos[y] \cos[y] + \sqrt{|m_1|} \left( -c_{12} c_{23} e^{i\delta_{CP}} s_{13} + s_{12} s_{23} \right) \\
\cos[x] \cos[z] \sin[y] + \sin[x] \sin[z]) + e^{-i\rho_2} \sqrt{|m_2|} \left( -c_{23} e^{i\delta_{CP}} s_{12} s_{13} - c_{12} s_{23} \right) \\
\cos[z] \sin[x] - \cos[x] \sin[y] \sin[z])^2
\]

(31)
We calculate again $\Sigma_i |V_{\ell \Sigma_i}|^2$ from the Eqs. 29-31 and find that this is same with the case of $O = 1_{3 \times 3}$.

We notice that unlike the case of $O = 1_{3 \times 3}$, now $|V_{\ell \Sigma_1}|^2(|V_{\ell \Sigma_3}|^2)$ is a function of all three light neutrino mass eigenvalues ($m_1, m_2$ and $m_3$). Hence in the NH and IH cases the corresponding individual mixings in Eqs. 29 and 31 will not tend to zero even for $m_1 \to 0$ for the NH and $m_3 \to 0$ for the IH cases respectively when $O$ is a real general orthogonal matrix, which is clearly visible in Fig. 6. Same argument will be applicable for the mixings $|V_{\ell \Sigma_2}|^2$ and $|V_{\ell \Sigma_3}|^2$ in the NH case and the mixings $|V_{\ell \Sigma_1}|^2$ and $|V_{\ell \Sigma_2}|^2$ in the IH case. This behavior can be observed in Fig. 6. We consider $M = 1$ TeV in this analysis.

We also study the effect of the general parametrization where $O$ is an orthogonal matrix of the form given in Eq. 21 and using the case (iii). We take the most general form of the entries of the matrix as complex parameters. In this case running over the full set of the parameters we find that there is no special correlation between the mixings and $m_1(m_3)$ for the NH (IH) case. The interesting fact is due to Casas-Ibarra conjecture and for the complex orthogonal matrix, the maximum possible mixing is enhanced dramatically. Fixing the generation of the SM charged lepton and summing over the triplet generations, we plot the bounds on the mixings satisfying the neutrino oscillation data and the PLANCK limit for two hierarchic masses in Fig. 7 as a function of the lightest light neutrino mass in each hierarchy. We notice that the application of the Casas-Ibarra conjecture improves the mixing by several orders of magnitude under the applied constraints.

We show the individual mixing in Fig. 8 for the NH (IH) case as a function of the lightest light neutrino mass $m_1$ ($m_3$). At this point we mention that using Eq. 29-31 we can similarly calculate $\Sigma_i |V_{\ell \Sigma_i}|^2$ for the case (iii) using complex values of $x, y$ and $z$ and it will not be same as the case of (i) or (ii). $|V_{\ell \Sigma_i}|^2$ is now a complicated function of the light neutrino mass eigenvalues $m_i$, complex parameters $x, y, z$ and the CP violating phases $\delta_{cp}, \rho_i$. Therefore the extreme smallness of lightest mass eigenvalues $m_1 \to 0(m_3 \to 0)$ will not push the mixing to zero because the rest of the two light neutrino mass eigenvalues will not allow to do that. For the individual mixing, in each panel of Fig. 8 we show the mixings between the triplet fermion and the charged lepton. The important fact of this scenario is the upper bounds of the light heavy mixing squared which can go up to an $\mathcal{O}(10^{-5})$, however, the lower bounds stay around $\mathcal{O}(10^{-18})$. We have showed the individual mixing for the NH (IH) case in the upper (lower) of Fig. 8.
FIG. 7. Bounds on $|V_{\Sigma i}|^2$ as a function of the $m_1 (m_3)$ NH (IH) case in the left (right) panel for fixed SM lepton flavors. The red band represents electron (e), the blue band represents the muon ($\mu$) and the green band represents the tau ($\tau$). In this case we consider $O$ as a complex orthogonal matrix. We fix the triplet mass $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data.

The individual mixings for the orthogonal matrix $O$ with the complex elements can be written as Eqs. 29, 30 and 31 respectively taking $x$, $y$ and $z$ as the complex quantities having real and imaginary parts.

4. BRANCHING RATIOS OF THE TRIPLET FERMIION FOR DIFFERENT CHOICES OF THE ORTHOGONAL MATRICES

Using the three typical forms of the orthogonal matrix $O$, we calculate the bounds on the branching ratios of the $\Sigma^0$ and $\Sigma^\pm$ respectively. We consider a degenerate scenario for the three generations of the triplet fermions having mass at $M = 1$ TeV. Summing over the three generations of $\Sigma^0_i$ and $\Sigma^\pm_i$ separately, we obtain the total branching ratios of $\Sigma^0_{\text{Tot}}$ and $\Sigma^\pm_{\text{Tot}}$ respectively for the NH and IH cases.

We consider the leading mode of $\Sigma^0_i$ to $\ell^\pm W$ as this is the visible one. For $\Sigma^\pm_i$ we consider all the decay modes because where $\nu W$ is the leading mode, $\ell^\pm Z$ and $\ell^\pm h$ are the subdominant modes but visible with the charged leptons. $\text{BR}(\Sigma^0_{\text{Tot}} \to \ell^\pm W)$ for the NH (IH) case has been plotted in the top-left (top-right) panel of the Fig. 9. The muon (blue)
FIG. 8. Bounds on $|V_{\Sigma i}|^2$ as a function of the $m_1(m_3)$ NH (IH) case in the upper (lower) panel for fixed SM lepton flavors. The red band represents electron ($e$), the blue band represents the muon ($\mu$) and the green band represents the tau ($\tau$). In this case we consider $O$ as a complex orthogonal matrix. The same nature will be obtained from case when $O$ is a real orthogonal matrix. We fix the triplet mass $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data.

... and tau (green) modes are dominant over the electron (red) mode of the lepton flavors in the NH case. The IH case is opposite to the NH case. We find that the results are same for the orthogonal matrix as a identity and as a real matrix.

Corresponding total branching ratios $\text{BR}(\Sigma_{\text{tot}}^\pm)$ for the $\nu W$, $\ell^\pm Z$ and $\ell^\pm h$ modes are shown for the NH (IH) case in the top (bottom) panel of Fig. 10 for the $O$ as a $3 \times 3$ identity matrix. This is exactly same for the case when $O$ is a real orthogonal matrix. The $\nu W$ mode is shown in the left column. In this case we do not distinguish between the light neutrinos as they will be obtained as the missing energy. The $\ell^\pm Z$ ($\ell^\pm h$) mode has been shown in the second (third) column of the Fig. 10. For the $\ell^\pm Z$ and $\ell^\pm H$ modes we show the electron (red), muon (blue) and tau (green) leptons separately for the NH (top row) and IH (bottom row) cases. In the NH case muon and tau regions coincide and dominate over the electron mode. In the IH case the electron mode dominates over the muon and tau modes.

We show the individual leading branching ratio of $\Sigma_i^0$ in the Fig. 11 for the NH (IH) case in the top (bottom) panel for the orthogonal matrix as a identity matrix. For the first
FIG. 9. Total branching ratio of $\Sigma_{\text{Tot}}^0 (\sum_i \Sigma_i^0)$ into the leading $\ell W$ mode as a function of the lightest neutrino mass for the NH ($m_1$) and IH ($m_3$) cases in the left and right panels respectively. We add three generations of $\Sigma_i^0$ to obtain $\Sigma_{\text{Tot}}$. The mode containing electron is represented by the red dots, that containing muon is shown by blue dots and the tau mode is represented by green dots. This result is same for the orthogonal matrix considered to be a identity matrix and a real matrix. The corresponding result is shown upper panel. We have also considered the case where $O$ is a general orthogonal matrix. The result is shown in the lower panel. We consider $M = 1$ TeV.

generation of the neutral multiplet of the triplet ($\Sigma_1^0$) we show that the branching ratio into the electron (red) mode dominates over the muon (blue) and tau (green) flavor for the NH and IH cases. For the second generation ($\Sigma_2^0$) all the modes coincide with each other for both of the neutrino mass hierarchy. For the third generation ($\Sigma_3^0$) the muon and tau modes coincide and they dominate over the electron mode for the NH and IH cases.
FIG. 10. Total branching ratio of $\Sigma^\pm (BR(\Sigma^\pm_{\text{Tot}}))$ into the leading $\nu W$ (first column), subleading $\ell Z$ (second column) and $\ell h$ (third column) modes with respect to the lightest neutrino mass for the NH ($m_1$) and IH ($m_3$) cases in the top and bottom panels respectively. We add three generations of $\Sigma^\pm_i$ to obtain $\Sigma^\pm_{\text{Tot}}$. The mode containing electron is represented by the red dots, that containing muon is shown by blue dots and the tau mode is represented by green dots. The $\nu W$ mode is indistinguishable from the point of view of the neutrinos. This result is same for the orthogonal matrix considered to be a identity matrix and a real matrix. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

We study the individual branching ratios of the three different generations of the charged multiplet $\Sigma^\pm_1$ into $\nu W$, $\ell^\pm Z$ and $\ell^\pm h$ modes respectively where the orthogonal matrix has been considered as a identity matrix. The $\nu W$ mode is dominant over the $\ell^\pm Z$ and $\ell^\pm h$ modes. In Fig. 12 we show the different decay modes of $\Sigma^\pm_1$ for the NH (IH) case in the top (bottom) panel. For the $\nu W$ mode we do not distinguish between the neutrinos as the neutrinos will be considered as the missing momenta and hence we summed over all flavor of neutrinos. Therefore we have the single line in the first column for both of the NH and IH cases. In the $\ell^\pm Z$ and $\ell^\pm h$ modes we have almost the same nature in both of the neutrino mass hierarchies where the electron mode (red) dominates over the muon (blue) and tau (green) modes.

The behavior for the $\nu W$ mode can be obtained for the second generation of the charged
FIG. 11. Individual branching ratio of $\Sigma^0_i$ into the leading $\ell W$ mode as a function of the lightest neutrino mass for the NH ($m_1$) and IH ($m_3$) cases in the top and bottom panels respectively for the orthogonal matrix considered to be a identity matrix. The decay modes contain electron (red), muon (blue) and tau (green) for the $\Sigma_1$ (left column), $\Sigma_2$ (middle column) and $\Sigma_3$ (right column). The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

multiplet $\Sigma^\pm_2$ in the first column of the Fig. 13 for the NH and IH case. We also study the $\ell^\pm Z$ and $\ell^\pm h$ modes. We notice that the parameter regions for the three flavors coincide with each other for both the NH and IH case, see top and bottom panel of the middle column in Fig. 13.

The third generation charged triplet $\Sigma^\pm_3$ decaying into $\nu W$ show the same behavior as the other two generations, see top and bottom panel of first column in Fig. 14. For the $\ell^\pm Z$ and $\ell^\pm h$ modes we see a different behavior unlike the other two generations. In case of $\Sigma^\pm_3$ the muon (blue) and tau (green) modes dominate over the electron (red) mode. The corresponding parameter spaces for the NH (IH) case is shown in the top (bottom) panel of the second column in Fig. 14.

We have studied the case where $O$ is a general real orthogonal matrix. In this case the branching ratios of the three generations of $\Sigma^0_i$ and $\Sigma^\pm_i$ are shown in Figs. 15 and 16, respectively. For $\Sigma^0_i$ we show the leading visible mode in Fig. 15. We found that the NH and IH cases show same parameter spaces for the real orthogonal matrix. For the $\Sigma^\pm_i$ we
FIG. 12. Individual branching ratio of $\Sigma^\pm_1$ into the leading $\nu W$ (left column) and subleading $\ell^\pm Z$ (middle column), $\ell^\pm h$ (right column) modes with respect to the lightest neutrino mass for the NH ($m_1$) and IH ($m_3$) cases in the top and bottom panels respectively for the orthogonal matrix considered to be a identity matrix. The decay modes contain electron (red), muon (blue) and tau (green). The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

FIG. 13. Same as Fig. 12 but now for $\Sigma^\pm_2$. 
FIG. 14. Same as Fig. 12 but now for $\Sigma^\pm_3$.

FIG. 15. Individual branching ratio of $\Sigma^\pm_i$ into the leading $\ell W$ mode as a function of the lightest neutrino mass ($m_1, m_3$) for the two hierarchic cases (NH, IH) for the real orthogonal matrix. Red, blue and green color stand for electron, muon and tau modes, respectively. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

demonstrate the subdominant $\ell^\pm Z$ and $\ell^\pm h$ cases because they are the visible final states with the charged leptons.

We have also studied the case where $O$ is a general complex orthogonal matrix. We show the total branching ratio of the neutral multiplet into the leading mode ($BR(\Sigma^0_{\text{Tot}} \to \ell W)$) in the bottom-left (bottom-right) panel of Fig. 9 for the NH (IH) cases. The muon (blue) and tau (green) modes are dominant over the electron (red) mode of the lepton flavors in the NH case. The IH case is opposite to the NH case. We show the individual branching
FIG. 16. Individual branching ratio of $\Sigma_i^\pm$ into the subleading and visible $\ell^\pm Z$ (top panel) and $\ell^\pm h$ (bottom panel) modes with respect to the lightest neutrino mass ($m_1, m_3$) for the two hierarchic cases (NH, IH) for the real orthogonal matrix. Red, blue and green color stand for electron, muon and tau modes, respectively. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

ratio of the three generations of $\Sigma_i^0$ for the NH (IH) case in the upper(lower) panel of the Fig. 17. The decay of the three generations of the triplets into the electron dominate over the decay mode into the other two leptons in the IH case whereas the result is opposite in the NH case. Here we would like to comment that we do not show the individual or total branching ratio into the different modes for the $\Sigma_i^\pm$ because they will have exactly the same repertoire like the $\Sigma_i^0$ when $O$ is a complex orthogonal matrix.

5. DISPLACED DECAY OF THE TRIPLET FERMION

We can write the proper decay lengths of the $\Sigma_i^0$ and $\Sigma_i^\pm$ in milimeter for the NH and IH cases as follows:

$$L_{\Sigma_i^0/\Sigma_i^\pm}^{\text{NH/IH}} = \frac{1.97 \times 10^{-13}}{\Gamma_{\Sigma_i^0/\Sigma_i^\pm}^{\text{NH/IH}} [\text{GeV}]} [\text{mm}]$$

(32)
where $i$ stands for the three generations of the triplets. In this analysis we consider three types of the general orthogonal matrix ($O$) as described in Sec. 3. When $O$ is an identity matrix the proper decay lengths are shown in Fig. 18 for the NH (IH) case with respect to the lightest neutrino mass $m_1$ ($m_3$). The decay lengths of the $\Sigma^0_i$ ($\Sigma^\pm_i$) are shown in the upper (lower) panel of Fig. 18. In the NH (upper, left panel) case we see that the proper decay length of $\Sigma^0_1$ becomes inversely proportional to $m_1$ which has been represented in red. The proper decay lengths for the other two generations $\Sigma^0_2$ represented be blue and $\Sigma^0_3$ represented by green become constant when $m_1 < 10^{-2}$ eV. We estimate that for $m_1 = 10^{-4}$ eV, $L_{\Sigma^0_1}^{\text{max}} \sim 1.5$ mm whereas that for $\Sigma^0_2$ ($\Sigma^0_3$) is two (three) orders of magnitude less. This nature of $L_{\Sigma^0_i}$ can be realized from the Eq. 26. The mixings between $\Sigma_i$ and the SM leptons, $|V_{\ell\Sigma_i}|^2$, are proportional to $m_1$. Therefore when $m_1 \to 0$ the corresponding decay length of $\Sigma^0_1$ becomes very large. We have also tested this nature considering the lightest light neutrino mass $m_1$ ($m_3$) for the NH (IH) case at $10^{-6}$ eV and $10^{-10}$ eV respectively. The results are shown in the first row of Tab. I. The corresponding lengths are two and six orders
FIG. 18. Proper decay length of $\Sigma^0_i (\Sigma^{\pm}_i)$ for $O = 1_{3 \times 3}$ with respect to the lightest neutrino mass in the upper (lower) panel. We show the NH (IH) case in the left (right) panel using the neutrino oscillation data in Eq. 16. The first generation triplet is represented by the red band, the second generation is represented by blue band and the third generation is represented by green band respectively. We consider $M = 1 \text{ TeV}$. The shaded region is excluded by the PLANCK data.

magnitude larger than that for $m_1 = 10^{-4} \text{ eV}$. In the IH (upper, right panel) case we have the same scenario for the $\Sigma^0_3$ where as decay lengths of $\Sigma^0_1$ and $\Sigma^0_2$ coincide. For $\Sigma^0_3$ we notice the form of $|V_{\ell \Sigma}|^2$, proportional to $m_3$, in Eq. 26. Therefore the decay length of $\Sigma^0_3$ becomes very large when $m_3$ is very small and the corresponding benchmarks are given in the second row of Tab. I. Lower panel of Fig. 18 shows that at least for lightest neutrino mass $m_1 (m_3) > 10^{-4} \text{ eV}$, the decay length of $\Sigma^{\pm}_1 (\Sigma^{\pm}_3)$ in NH(IH) case has the same nature as the decay length of $\Sigma^0_1 (\Sigma^0_3)$ in NH(IH) case. For the lightest neutrino mass range $m_1 (m_3) \leq 10^{-4}$
eV, the behavior of the decay length of \( \Sigma_{1}^{\pm} (\Sigma_{3}^{\pm}) \) in NH(IH) case is completely different from the decay length of \( \Sigma_{0}^{0} (\Sigma_{0}^{3}) \) in NH(IH) case, see the third and fourth rows of Tab. I. This implies that for \( m_{1}(m_{3}) \leq 10^{-4} \) eV, \( L_{\Sigma_{1}^{\pm}} (L_{\Sigma_{3}^{\pm}}) \) is more or less constant. The reason for this is, in NH(IH) case as \( m_{1}(m_{3}) \to 0 \), mixing angle \( |V_{\ell \Sigma_{1}}|^{2} (|V_{\ell \Sigma_{3}}|^{2}) \to 0 \) and as a result the decay width for \( \Sigma_{1}^{\pm} (\Sigma_{3}^{\pm}) \) will be dominated by the decay modes given in Eq. 15 which is controlled by the \( \Delta M \) parameter. Hence this decay width or decay length is constant which can be noted from the benchmarks in the third or fourth row of Tab. I. We notice that in this case one can obtain large decay lengths which indicate possibilities of the displaced vertex scenarios when the decay lengths are \( \mathcal{O}(100 \text{mm}) \). Possible scenarios of the further long-livedness can also be observed when the decay lengths are \( \mathcal{O}(10^{6} \text{mm}) \).

| Decay Length [mm] | \( m_{\text{lightest}} = 10^{-6} \) eV | \( m_{\text{lightest}} = 10^{-10} \) eV |
|-------------------|----------------|----------------|
| \( L_{\Sigma_{0}^{0}} \) (NH) | [134.13, 171.03] | [1.35 \times 10^{6}, 1.74 \times 10^{6}] |
| \( L_{\Sigma_{0}^{3}} \) (IH) | [129.04, 183.71] | [1.28 \times 10^{6}, 1.83 \times 10^{6}] |
| \( L_{\Sigma_{1}^{\pm}} \) (NH) | [20.29, 20.99] | [23.9321, 23.9322] |
| \( L_{\Sigma_{3}^{\pm}} \) (IH) | [20.18, 21.17] | [23.9321, 23.9322] |

TABLE I. Benchmark for the proper decay lengths of \( \Sigma_{0}^{0} (\Sigma_{0}^{3}) \), \( \Sigma_{1}^{\pm} (\Sigma_{3}^{\pm}) \) for the NH and IH cases fitting the neutrino oscillation data in Eq. 16 when \( O = 1_{3 \times 3} \). The variation of the proper decay length represents a band due to the variation of \( \pm 3\sigma \) the oscillation data, \( \delta_{CP} \) and \( \rho_{i} \). We consider \( M = 1 \) TeV.

Similarly we consider the case when \( O \) is a real orthogonal matrix. In this case the analytical form for mixings are given in Eq. 29-31. We notice that now \( |V_{\ell \Sigma_{1}}|^{2} \) depends on all the light neutrino mass eigenvalues like \( m_{1}, m_{2} \) and \( m_{3} \). Therefore in the NH case for \( m_{1} \to 0 \), \( |V_{\ell \Sigma_{1}}|^{2} \) attains a limiting value but does not vanish. Which will be reflected in the nature of the proper decay lengths of \( \Sigma_{1}^{0} \) and \( \Sigma_{1}^{\pm} \) respectively. Similar behavior can be observed for the IH case when \( m_{3} \to 0 \), \( |V_{\ell \Sigma_{3}}|^{2} \) does not vanish due to its dependence on \( m_{1} \) and \( m_{2} \). The decay lengths of \( \Sigma_{0}^{0} (\Sigma_{1}^{\pm}) \) are shown in the upper (lower) panel of Fig. 19. We find that for lightest light neutrino mass range \( 10^{-4} \) eV \( \leq m_{1(3)} \leq 0.1 \) eV, maximum decay length can be around 1 mm. We have also considered some benchmark scenarios for very small lightest light neutrino mass, \( m_{1} (m_{3}) \) for the NH (IH) case. We fix \( m_{1} (m_{3}) \) at \( 10^{-6} \) eV and \( 10^{-10} \) eV respectively and find out the corresponding decay lengths in Tab. II fitting the
FIG. 19. Proper decay length of $\Sigma^0_i (\Sigma_i^\pm)$ when $O$ is a real and general orthogonal matrix with respect to the lightest neutrino mass in the upper (lower) panel. We show the NH (IH) case in the left (right) panel using the neutrino oscillation data in Eq. 16. The first generation triplet is represented by the red band, the second generation is represented by blue band and the third generation is represented by green band respectively. We consider $M = 1 \text{ TeV}$. The shaded region is excluded by the PLANCK data.

neutrino oscillation data from Eq. 16. In this case we have found that the minimum decay length can be as low as $O(10^{-3}\text{ mm})$ and the maximum decay length are of the same order as the case of identity orthogonal matrix $O$. The decay length can reach at a maximum value of $O(10^6\text{ mm})$ showing the possibility of a long-lived scenario. When the decay length is $O(10^{-3}\text{ mm})$, the decay of the triplet can be prompt. In that case, a comparatively large mixing can be expected.
| Decay Length $[\text{mm}]$ | $m_{\text{lightest}} = 10^{-6} \text{ eV}$ | $m_{\text{lightest}} = 10^{-10} \text{ eV}$ |
|-------------------------|---------------------------------|---------------------------------|
| $L_{\Sigma^0_1} (\text{NH})$ | [0.0027, 171.1] | [0.0028, $1.74 \times 10^6$] |
| $L_{\Sigma^0_3} (\text{IH})$ | [0.0026, 183.79] | [0.0026, $1.84 \times 10^6$] |
| $L_{\Sigma^{\pm}_1} (\text{NH})$ | [0.0029, 20.84] | [0.0025, 23.93] |
| $L_{\Sigma^{\pm}_3} (\text{IH})$ | [0.0027, 21.18] | [0.0027, 23.93] |

TABLE II. Benchmark for the proper decay lengths of $\Sigma^0_{1,3} (\Sigma^{0,\pm}_3)$ for the NH and IH cases fitting the neutrino oscillation data in Eq. 16 when $O$ is a real and general orthogonal matrix. The variation of the proper decay length represents a band due to the variation of $\pm 3\sigma$ the oscillation data, $\delta_{CP}, \rho_i$ and the parameters of the orthogonal matrix. We consider $M = 1 \text{ TeV}$.

We have also studied the effect when $O$ a complex orthogonal matrix. The real and imaginary parts of the elements of $O$ vary between $[-\pi, \pi]$. Scanning over the $\delta_{CP}$ and $\rho_{1,2}$ within the interval $[-\pi, \pi]$ simultaneously we show the range of the proper decay length for some benchmark scenarios in Tab. III fitting the neutrino oscillation data from Eq. 16. We adopt such a method for this case because there is no special pattern observed in this case after the scan. Therefore we fix the lightest light neutrino mass $m_1 (m_3)$ in the NH (IH) case at $10^{-6} \text{ eV}$ and $10^{-10} \text{ eV}$ respectively. Due to the presence of the complex orthogonal matrix there will be an improvement in the light-heavy mixings. As a result we can expect a prompt production of the triplets as expressed by the small decay lengths $\mathcal{O}(10^{-11} \text{mm})$ which represent a large mixing. On the other hand there will be some possibilities where small mixings can be observed and due to that large decay lengths $\mathcal{O}(100 \text{mm})$ can be obtained which ensure a possible displaced vertex scenario and if the decay lengths are $\mathcal{O}(10^6 \text{mm})$ then a further long-lived case might be studied.

6. DISCUSSIONS

Considering the type-III model for the three generations of the triplets we investigate the role of the mixing to study the phenomenology. We parametrize the Dirac Yukawa coupling among the triplet fermion, SM lepton doublet and the SM Higgs doublet with a general orthogonal matrix $O$. In this case we use three different choices for $O$. In this analysis we have considered $M = 1 \text{ TeV}$ and the constraints stated in Sec. 2.
First, we consider $O$ as a $3 \times 3$ identity matrix to calculate the light-heavy mixing in terms of the lightest light neutrino mass. We calculate the bounds on the mixing for two hierarchic conditions of the neutrino mass, namely, NH and IH fitting the neutrino oscillation data. For identity or real orthogonal matrix $O$, observing the nature of the mixing summed over the triplet generation we notice that the mixing can reach up to a certain lower limit when varied with respect to the lightest light neutrino mass for the NH and IH cases under the PLANCK limit. The lower limit for the $\Sigma_i |V_{\ell\Sigma_i}|^2$ can go down to $3 \times 10^{-15}$ whereas the upper limit can be one order of magnitude better under the PLANCK exclusion in the NH case. For the other two flavors in the NH case, the upper limit on the mixings can reach up to $5 \times 10^{-14}$ whereas the lower limit is slightly better for them, namely $2 \times 10^{-14}$ under the PLANCK exclusion. Alternatively if we look at the IH case, we notice that the limits on $\Sigma_i |V_{(\mu,\tau)\Sigma_i}|^2$ roughly remain the same, however, those on $\Sigma_i |V_{e\Sigma_i}|^2$ get improved. The lower and upper limits on the mixing associated with the electron flavor also improves in the IH case up to $5 \times 10^{-14}$ and $6 \times 10^{-14}$ respectively. These limits can be observed from Fig. 4.

If we notice the individual mixing $|V_{\ell\Sigma_i}|^2$ in Fig. 5 for identity orthogonal matrix $O$, we see that for the NH and IH cases the mixings $|V_{e\Sigma_1}|^2$ and $|V_{\tau\Sigma_3}|^2$ become zero as $m_1$ and $m_3$ go to zero. $|V_{\mu\Sigma_2}|^2$ has the same nature in both of the hierarchies. Below the PLANCK exclusion limit, $|V_{\ell\Sigma_2}|^2$ varies between $10^{-15}$ to $10^{-14}$ for the NH case and stays around $10^{-14}$ for the IH case. Following the same note, we notice that $|V_{e\Sigma_3}|^2$ stays around $10^{-15}$ below the PLANCK limit where as $|V_{\mu\Sigma_3}|^2$, $|V_{\tau\Sigma_3}|^2$ are one order of magnitude higher in the NH case. The scenario becomes opposite in the IH case for $|V_{\Sigma_1}|^2$ where $|V_{e\Sigma_1}|^2$ stays around

| Decay Length [mm] | $m_{\text{lightest}} = 10^{-6}$ eV | $m_{\text{lightest}} = 10^{-10}$ eV |
|------------------|-------------------------------|-------------------------------|
| $L_{\Sigma^0_1}$ (NH) | $[1.11 \times 10^{-11}, 171.2]$ | $[1.08 \times 10^{-10}, 1.74 \times 10^6]$ |
| $L_{\Sigma^0_3}$ (IH) | $[1.74 \times 10^{-11}, 183.79]$ | $[1.54 \times 10^{-11}, 1.84 \times 10^6]$ |
| $L_{\Sigma^\pm_1}$ (NH) | $[1.32 \times 10^{-10}, 20.84]$ | $[1.47 \times 10^{-10}, 23.93]$ |
| $L_{\Sigma^\pm_3}$ (IH) | $[1.41 \times 10^{-11}, 21.18]$ | $[8.23 \times 10^{-12}, 23.93]$ |

TABLE III. Benchmark for the proper decay lengths of $\Sigma_i^{0,\pm} (\Sigma_3^{0,\pm})$ for the NH and IH cases fitting the neutrino oscillation data in Eq. 16 when $O$ is a complex and general orthogonal matrix. The variation of the proper decay length represents a band due to the variation of $\pm 3\sigma$ the oscillation data, $\delta_{CP}, \rho_i$ and the parameters of the orthogonal matrix. We consider $M = 1$ TeV.
$2 \times 10^{-14}$ and the other two mixings are one order lower than that below the PLANCK exclusion. The values could be observed from Fig. 5.

Second, we consider $O$ as a $3 \times 3$ real orthogonal matrix. We find the same nature of $S_i |V_{\ell \Sigma}|^2$, as we found in the previous case and shown in Fig. 4. The individual mixing for this case is shown in Fig. 6. It is important to note that the depending upon the hierarchies the mixings are dependent upon the three light neutrino mass eigenvalues. In the NH case the mixing for the $\mu$ and $\tau$ can reach up to $2 \times 10^{-14}$ and that for $e$ flavor can reach up to $3.5 \times 10^{-15}$. This nature becomes opposite in the IH case where the maximum value of the mixing involving $e$ flavor can go up to $4 \times 10^{-14}$ where the rest of the two remain some factor below around $10^{-14}$.

Third, we consider $O$ as a $3 \times 3$ general orthogonal matrix where the entries of the matrix can be complex quantities. Using the neutrino oscillation data considering the PLANCK exclusion limit we have found that in this case there is no special correlation in the parameter space of the mixing and the lightest light neutrino mass. This happens due to the dependence of the mixing angles on the light neutrino mass eigenvalues and the complex entries of the general orthogonal matrix $O$. We notice that the highest mixing can reach $O(10^{-5})$ depending up on the generations of the triplet and SM lepton which is very high compared to the other two choices of the orthogonal matrices. The lower limit in the mixing in the different cases reach around $10^{-18}$. We mention that the limits on the mixing from the EWPD have been given in [74, 94]. We quote limits as $|V_{e \Sigma}|^2 = 3.61 \times 10^{-4}$, $|V_{\mu \Sigma}|^2 = 2.89 \times 10^{-4}$ and $|V_{\mu \Sigma}|^2 = 7.29 \times 10^{-4}$ respectively at 90% CL.

We have calculated the branching ratios for the different choices of the orthogonal matrix. Our main motivation is to present the correlation of the parameter space as a function of the lightest light neutrino mass for two different neutrino mass hierarchies. As there is no special correlation for the complex orthogonal matrix, therefore we omit its pictorial representations. For the neutral multiplet of the triplet we show the leading decay mode because it has a visible charged lepton. On the other hand for the charged multiplet we show all the possible modes because the leading mode has an associated neutrino, however, the subleading modes have associated charged leptons. At the time of generating the branching ratios we used the neutrino oscillation data, the PLANCK exclusion bound and other constraints stated in Sec. 2.

The triplet fermions can be produced at the high energy colliders from a variety of
production process. Being produced in pair or in association with SM leptons, they manifest several multilepton channels which are very clean at the colliders followed by the leptonic decay modes of the triplet fermion including $\Sigma^\pm \rightarrow W\nu, W \rightarrow \ell\nu$ or $\Sigma^\pm \rightarrow \ell Z, Z \rightarrow 2\ell$ and $\Sigma^0 \rightarrow \ell W, W \rightarrow \ell\nu$. These decay modes finally introduce events with 6, 4 and 3 leptons. A same sign dilepton channel can also be possible to study from this model. A distinguishing same sign trilepton channel will also be an interesting outcome from this model [25, 94]. Depending upon the center of mass energy and the nature of the collider a variety of sophisticated studies on the triplet fermions including lepton-jets (for collimated leptons) and fat-jet using jet substructure (for the events including the hadronic decay modes of the SM gauge bosons) can be performed [32] for a sufficiently heavy triplet for a flavor non-democratic scenario. Apart from the direct productions of the triplets, they can be produced in association with the SM leptons which are suppressed by the light-heavy mixings. Those modes are also interesting to study the bounds on the mixings.

In this article we have shown that the nature of the decay lengths justify the behavior of the mixings. In the NH case the maximum proper decay length of the $\Sigma_1$ can be very high and inversely proportional to $m_1$. This is an interesting feature that $\Sigma_1$ can show a displaced decay inside the detector or outside the detector of the high energy colliders. Similar behavior can be observed from $\Sigma_3$ in the IH case. However, we must point out that this behavior is highly dependent upon the choice of the orthogonal matrix. If the mixing becomes sufficiently large, then $\Sigma$ can have prompt decay. The detection of such a particle under the displaced (track search) or prompt decay will depend upon the nature of the detector which is currently beyond the scope of this article, however, LHC (high luminosity and/or upgraded energy), MATHUSLA detector, electron-positron ($e^-e^+$) collider and electron proton ($e^-p$) colliders could be useful to perform such studies in the near future.

7. CONCLUSION

We study the type-III seesaw model in this article where we mainly observe the allowed parameter regions for the light-heavy mixings as a function of the lightest neutrino mass. Depending upon the choice of the general Dirac Yukawa coupling of the triplet fermion with the SM lepton doublet and the Higgs doublet the allowed parameter space of the mixing changes under a variety of constraints. We also calculate the branching ratios of the
neutral and charged multiplets of the triplet fermion into leading and subleading modes to investigate the correlation with the lightest light neutrino mass eigenvalue for two different light neutrino mass hierarchies. In a continuation we have also shown the parameter space of the proper decay length of the triplet generalizing the Dirac Yukawa coupling using three different choices. This leads to an interesting property of the displaced vertex search for the triplet fermions due to their sizable proper decay length, however, we predict that for some parameter choices prompt decay of the triplet fermion can also be possible. We evaluate the mixings, branching ratios of the triplets in the different modes and hence we predict that such parameter spaces can be probed studying the decay modes (prompt or displaced) of the triplets at the different high energy colliders in the near future.

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