Infinite-dimensional manifolds related to $C$-spaces

Oryslava Polivoda, Mykhailo Zarichnyi

Abstract. Haver’s property $C$ turns out to be related to Borst’s transfinite extension of the covering dimension. We prove that, for a uncountably many countable ordinals $\beta$ there exists a strongly universal $k_\omega$-space for the class of spaces of transfinite covering dimension $< \beta$. In some sense, our result is a $k_\omega$-counterpart of Radul’s theorem on existence of absorbing sets of given transfinite covering dimension.

1. Introduction

The notion of metric $C$-space was introduced by W. Haver [9]. He applied these spaces to the theory of retracts. In [1] the property $C$ was defined for all topological spaces. The $C$-spaces play an important role in the dimension theory.

P. Borst [7] introduced a transfinite extension of the covering dimension $\dim$ which characterizes property $C$.

T. Radul [13] proved that there exists an uncountable set of countable ordinals $\beta$ such that there exist noncountable-dimensional pre-Hilbert spaces.
$D_\beta$ which are absorbing spaces (in the sense of Bestvina and Mogilski [6]) for the class of compacta with $\dim C$ less than $\beta$.

In some sense, our main result is a counterpart of Radul’s theorem in the category of $k_\omega$-spaces. We prove that, for an uncountable set of countable ordinals $\beta$, there exists a $k_\omega$-space which is strongly universal for the class of compacta with $\dim C$ less than $\beta$.

2. Preliminaries

A family $\mathcal{V}$ of subsets of a space $X$ is said to refine a family $\mathcal{U}$ of subsets of $X$ if for each element $V \in \mathcal{V}$ there exists $U \in \mathcal{U}$ for which $V \subset U$. A family $\mathcal{V}$ is said to star-refine a family $\mathcal{U}$ if for every $U \in \mathcal{U}$ there exists $V \in \mathcal{V}$ such that $U' \subset V$ for any $U' \in \mathcal{U}$ such that $U \cap U' \neq \emptyset$.

A family $\mathcal{V}$ of subsets of $X$ is called disjoint if every two elements of $\mathcal{V}$ are disjoint and is open if each element of $\mathcal{V}$ is open. The family of all open coverings of a space $X$ is denoted by $\text{cov}(X)$.

If $\mathcal{U}$ is a family of subsets in a metric space, then we define

$$\text{mesh}(\mathcal{U}) = \sup\{\text{diam}(U) \mid U \in \mathcal{U}\}$$

(as usual, $\text{diam}(U)$ is the diameter of $U$).

**Definition 2.1.** A space $X$ has property $C$ (briefly is a $C$-space) if for each sequence $\{\alpha_n \mid n \in \mathbb{N}\}$ of open coverings of $X$ there exists a sequence $\{\beta_n \mid n \in \mathbb{N}\}$ of open disjoint families such that each family $\beta_n$ refines $\alpha_n$ and $\bigcup_{n=1}^{\infty} \beta_n \in \text{cov}(X)$.

We will need the following properties of compact metrizable $C$-spaces (see [1]), where they are proved for more general class of spaces).

**Proposition 2.2.** Every closed subspace of a $C$-space is a $C$-space.

**Proposition 2.3.** If $X = \bigcup_{n=1}^{\infty} X_n$, where $X_n$ is a $C$ space for any $n \in \mathbb{N}$, then $X$ is a $C$-space.

**Corollary 2.4.** Let $A$ be a closed subset of a compact metrizable $C$-space $X$. Then the quotient space $X/A$ is a $C$-space.

**Proof.** Let $\{U_n \mid n \in \mathbb{N}\}$ be a countable family of neighborhoods of $A$ such that $A \cap \bigcap_{n=1}^{\infty} U_n$. Then $X/A$ is the sum of the sets homeomorphic to $X\setminus U_n$ and a singleton.

The following statement is proved in [8] for the paracompact spaces.
Proposition 2.5. Let \( f \) be a closed map from a compact metrizable space \( X \) onto a \( C \)-space \( Y \). If \( f^{-1}(y) \) has property \( C \) for each \( y \in Y \), then \( X \) is a \( C \)-space.

We will need the following result by T. Radul [13].

Theorem 2.6. For each \( \alpha < \omega_1 \) there exists a compact metrizable \( C \)-space \( L_\alpha \) which contains topologically each compact metrizable space \( K \) with \( \dim C K \leq \alpha \).

2.7. Dimension \( \dim C \). P. Borst [7] introduced the transfinite extension \( \dim C \) of the covering dimension. We recall some necessary definition. Let us start with the ordinal number \( \text{Ord} \).

Given a set \( L \), we denote by \( \text{Fin} L \) the collection of all finite, nonempty subsets of \( L \). Let \( M \) be a subset of \( \text{Fin} L \). For \( \sigma \in \{ \emptyset \} \cup \text{Fin} L \), put

\[
M^\sigma = \{ \tau \in \text{Fin} L \mid \sigma \cup \tau \in M \text{ and } \sigma \cap \tau = \emptyset \}.
\]

We write \( M^a \) instead of \( M^{(a)} \).

Define the ordinal number \( \text{Ord} M \) inductively as follows:

1. \( \text{Ord} M = 0 \) if and only if \( M = \emptyset \),
2. \( \text{Ord} M \leq \alpha \) if and only if for every \( a \in L \), \( \text{Ord} M^a < \alpha \),
3. \( \text{Ord} M = \alpha \) if and only if \( \text{Ord} M \leq \alpha \) and \( \text{Ord} M < \alpha \) is not true,
4. \( \text{Ord} M = \infty \) if and only if \( \text{Ord} M > \alpha \) for every ordinal number \( \alpha \).

Let \( X \) be a topological space and \( K(X) \) denote the set of the all locally finite coverings of \( X \). Put

\[
M_{K(X)} = \left\{ \{ \alpha_i \}_{i=1}^n \in \text{Fin} K(X) \mid \text{there are no open disjoint families } \beta_i, i = 1, \ldots, n, \text{ such that } \beta_i \text{ refines } \alpha_i \text{ and } \bigcup_{i=1}^n \beta_i \text{ covers } X, n \in \mathbb{N} \right\}.
\]

Definition 2.8. For any topological space \( X \) we set

\[
\dim C X = \text{Ord} M_{K(X)}.
\]

Remark that the dimension \( \dim C \) coincides with classical covering dimension \( \dim \) for finite-dimensional spaces [7] and for any compact metric space \( K \) \( \dim C K \) exists if and only if \( K \) has property \( C \).

Also, it is an easy consequence of the definition that if \( A \) is a closed subset of \( X \), then \( \dim C A \leq \dim C X \).

Let us denote by \( \mathcal{D}(\beta) \) the class of compact metric spaces with \( \dim C \) less than \( \beta \). We say that a topological space \( Y \) is \( \mathcal{D}(\beta) \)-universal if \( Y \) contains topologically all compacta from \( \mathcal{D}(\beta) \).
Theorem 3 proved that for each ordinal $\alpha < \omega_1$ there exists an ordinal $\beta$, $\alpha \leq \beta < \omega_1$, and a $C$-compact metric space $X$ such that $\dim_C X = \beta$ and $X$ is $D(\beta)$-universal.

Lemma 2.9. There exists a function $h: \omega_1 \to \omega_1$ such that, for any compact metric $C$-space $X$ and any closed subset $A$ of $X$, $\dim_C(X/A) \leq h(\dim_C X)$.

Proof. Let $K$ be a universal space for compact metric spaces $X$ with $\dim_C X \leq \alpha$. Let $\exp K$ denote the hyperspace of $K$, i.e., the space of all nonempty closed subsets of $X$ endowed with the Vietoris topology (see, e.g., [11]). There exists a continuous map of the Cantor discontinuum $C$ onto $\exp X$.

Let $Z = C \times K$ and $B = \{(c, x) | x \in f(c)\} \subset C \times K$. Clearly, $B$ is a closed subset of $Z$. Since $C$ is zero-dimensional, $Z$ is a $C$-space. We let $h(\alpha) = \dim_C(Z/B)$.

Now, suppose that $Y$ is a compact metrizable space and $\dim_C Y \leq \alpha$. We may assume that $Y \subset K$. If $F$ is a nonempty closed subset of $Y$, then $F \in \exp Y \subset \exp K$ and there exists $c \in C$ such that $f(c) = F$. Then $Y/F$ is, clearly, homeomorphic to $(\{c\} \times Y)/(\{c\} \times F)$, and therefore is homeomorphic to a subset of $Z/B$. We conclude that $\dim_C(Y/F) \leq \dim_C(Z/B) \leq h(\alpha)$. \qed

Remark 2.10. Actually, no example is known witnessing that $h$ is not the identity map.

3. Results

Recall that an absolute retract ($AR$-space) is a space $X$ which is a retract of every metric space containing $X$ as a closed subset.

Proposition 3.1. Let $X$ be a compact metrizable $C$-space. Then there exists a compact metrizable $C$-space $\hat{X}$ that contains a topological copy of $X$ and is an $AR$-space.

Proof. We assume that $X$ is a metric space. Define inductively a sequence $(U_i)$ of open covers of $X$ as follows. Let $U_1 = \{X\}$. If $U_j$ is already defined for $j < i$, let $U_i$ be a cover of $X$ which star-refines $U_{i-1}$ and such that $\text{mesh}(U_{i-1}) \leq 2^{-i}$.

Assume that $X$ is isometrically embedded into a Banach space $L$ which, in turn, is identified with the subset $L \times \{0\}$ of $L \times [0, 1]$. Let $N(U_i)$ be the nerve of the cover $U_i$. Assume also that $N(U_i)$ is a subpolyhedron of $L \times \{2^{1-i}\}$ with the following property: for any $U \in U_i$, the vertex of $N(U_i)$
Let there be an uncountable set \( \mathbb{R} \) and is a compact metrizable AR-space. The projection of \( \hat{L} \) map such that every its preimage is a countable union of the singleton \( P \). Then, clearly, the map is the diagonal \( \tilde{t} \). We assume as well that the mapping cylinder consists of all linear segments in \( L \times [2^i, 2^{i-1}] \).

Finally, let \( \hat{L} = L \cup \bigcup_{i=1}^{\infty} K_i \). Using standard arguments we show that \( \hat{L} \) is a compact metrizable AR-space. The projection of \( \hat{L} \) onto \([0, 1]\) is a closed map such that every its preimage is a C-space (either \( L \) or a polyhedron). Therefore, \( \hat{L} \) is a C-space. \( \square \)

**Proposition 3.2.** For any \( \alpha < \omega_1 \) there exists a pointed compact metrizable C-space \( (\hat{L}, \ast) \) that contains a topological copy of each pointed compact metrizable C-space \( (K, \ast) \) with \( \dim C K \leq \alpha \).

**Proof.** Let \( L_\alpha \) be a universal space for compact metrizable C-spaces \( K \) with \( \dim C K \leq \alpha \). Denote by \( \hat{L} \) the quotient space \( (L_\alpha \times L_\alpha)/\Delta \), where \( \Delta \) is the diagonal \( \{(x, x) \mid x \in L_\alpha \} \subset L_\alpha \times L_\alpha \). The set \( \Delta \) is regarded as the base point of \( \hat{L} \). Denote by \( q : L_\alpha \times L_\alpha \rightarrow \hat{L} \) the quotient map.

Suppose that \( (K, x_0) \) is a compact metrizable C-space with \( \dim C K \leq \alpha \). Then, clearly, the map \( f : K \rightarrow \hat{L} \) defined by the formula \( f(x) = q(x, x_0), \) \( x \in K \), is a pointed embedding.

Finally, remark that \( \hat{L} \) is a C-space. Indeed, one can represent \( \hat{L} \) as the countable union of the singleton \( \{q(\Delta)\} \) and the spaces \( q((L \times L) \setminus U_i) \), where \( \{U_i \mid i \in \mathbb{N}\} \) is a countable base neighborhoods of \( \Delta \) in the product \( L \times L \), and then apply Propositions 2.2 and 2.3. \( \square \)

Recall that a topological space \( X \) is said to be a \( k_\omega \)-space if \( X = \lim_{\to} X_i \), where \( (X_i) \) is an increasing sequence of its compact subspaces.

We say that a space \( X \) is strongly \( D(\beta) \)-universal (resp. locally strongly \( D(\beta) \)-universal) if for every compact metric space \( A \) with \( \dim C (A) < \beta \) and every embedding \( f : B \rightarrow X \) of its closed subset \( B \) into \( X \) there exists an embedding \( \tilde{f} : A \rightarrow X \) (resp. an embedding \( \tilde{f} : U \rightarrow X \), where \( U \) is a neighborhood of \( B \) in \( A \)) that extends \( f \).

**Theorem 3.3.** There is an uncountable set \( \Phi \subset \omega_1 \) such that, for every \( \beta \in \Phi \) there exists a strongly \( D(\beta) \)-universal \( k_\omega \)-space \( K_\beta \) which is the countable direct limit of an increasing sequence of compact spaces from the class \( D(\beta) \).

**Proof.** Let \( \alpha < \omega_1 \). Let \( X_1 = \{\ast\} \) be any compact metrizable C-space with \( \dim C X_1 = \alpha \). Suppose that compact metrizable C-spaces \( X_i \) are already constructed for all \( i < n \).
By Proposition 3.1, there exists a compact metric $C$-space $\hat{X}_{n-1}$ which contains $X_{n-1}$ and is an absolute retract.

By Proposition 3.2, there exists a pointed compact $C$-space $(Y_{n-1}, \ast)$ which is universal for pointed compact metric spaces of dim $C$ not exceeding $h(\alpha)$. Take $X_n = \hat{X}_{n-1} \times Y_{n-1}$.

Let $X = \lim X_n$ and let $\beta = \sup\{\dim (X_n) + 1 \mid n \in \mathbb{N}\}$. Then clearly $\beta \geq \alpha$.

We are going to show that $X$ is strongly $\mathcal{D}(\beta)$-universal. Suppose that $(A, B)$ is a pair of compact metric spaces and dim $C A < \beta$. Suppose also that $f : B \rightarrow X$ is an embedding. Since $X$ is a $k_\omega$-space, there exists $n \in \mathbb{N}$ such that $f(B) \subset X_n$. Since $\hat{X}_n$ is an AR-space, there is a continuous extension $f' : A \rightarrow \hat{X}_n$.

Let $q : A \rightarrow A/B$ be the quotient map. By Proposition 3.2, there exists an embedding $g : A/B \rightarrow Y_n$ such that $q(B) = \ast$.

Finally, define $\tilde{f} : A \rightarrow X_{n+1}$ by the formula $\tilde{f}(x) = (f'(x), gq(x))$. It is easy to see that $\tilde{f}$ is an embedding that extends $f$. 

The following is a characterization theorem for the spaces $K_\beta$.

**Theorem 3.4.** Let $X$ be a $k_\omega$-space and $X$ is countable direct limit of compact metric spaces from the class $\mathcal{D}(\beta)$. Then the following properties are equivalent:

1. $X$ is strongly $\mathcal{D}(\beta)$-universal;
2. $X$ is homeomorphic to $K_\beta$.

**Proof.** We apply the back and forth argument used in [14] as well as in another publications. For the sake of reader’s convenience, we provide some details.

Let $Y = \lim Y_n$, where $Y_1 \subset Y_2 \subset \ldots$ is a sequence of compact spaces such that $Y_n \in \mathcal{D}(\beta)$ for every $n \in \mathbb{N}$.

Let $m_1 = 1$. There exists an embedding $f_1 : Y_{m_1} \rightarrow K_\beta$. Since $Y_{m_1}$ is compact, there is $n_1 \in \mathbb{N}$ such that $f_1(Y_{m_1}) \subset X_{n_1}$. Moreover, since $Y$ is strongly $\mathcal{D}(\beta)$-universal, there exists an embedding $g_1 : X_{n_1} \rightarrow Y$ such that $g_1|f_1(Y_{m_1}) = f_1^{-1}$. Then by compactness of $X_{n_1}$, there exists $m_2 > m_1$ such that $g_1(X_{n_1}) \subset Y_{m_2}$.

Continuing in this way we obtain a commutative diagram

$$
\begin{array}{cccccccc}
Y_{m_1} & \rightarrow & Y_{m_2} & \rightarrow & Y_{m_3} & \rightarrow & \ldots \\
\downarrow f_1 & & \downarrow f_2 & & \downarrow f_3 & & \downarrow g_3 \\
X_{n_1} & \leftarrow & X_{n_2} & \leftarrow & X_{n_3} & \leftarrow & \ldots,
\end{array}
$$

in which $m_1 < m_2 < \ldots$, $n_1 < n_2 < \ldots$, $f_j, g_j$ are embeddings, $j \in \mathbb{N}$. 

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Then
\[ Y \simeq \varinjlim_{m} Y_{m} = \varinjlim_{j} Y_{m_{j}} = \varinjlim_{n} \{ Y_{m_{1}} \xrightarrow{f_{1}} X_{n_{1}} \xrightarrow{g_{1}} Y_{m_{2}} \xrightarrow{f_{2}} X_{n_{2}} \xrightarrow{g_{2}} \ldots \} \]

\[ = \varinjlim_{j} X_{n_{j}} = \varinjlim_{n} X_{n} = K(\beta). \]

A $K_{\beta}$-manifold is a Hausdorff space which is locally homeomorphic to open subsets in $K_{\beta}$. We will assume that the $K_{\beta}$-manifolds are $k_{\omega}$-spaces.

The following is a characterization theorem for the $K_{\beta}$-manifolds.

**Theorem 3.5.** Let $X$ be a $k_{\omega}$-space and $X$ is countable direct limit of compact metric spaces from the class $D(\beta)$. Then the following properties are equivalent:

1. $X$ is locally strongly $D(\beta)$-universal;
2. $X$ is a $K_{\beta}$-manifold.

Actually, the proof of this result can be performed along the line of the proof of [14, Theorem 1.3]. And, similarly as in [14], we obtain the following Open Embedding Theorem (see also [10]).

**Theorem 3.6.** Any $K_{\beta}$-manifold can be embedded into the space $K_{\beta}$ as an open set.

4. UNIVERSAL MAPS

The following notion is introduced in [15]. Let $\mathcal{K}_{fd}$ denote the class of metrizable finite-dimensional compacta. A map $f : X \to Y$ is called strongly $\mathcal{K}_{fd}$-universal if, for any if for every embedding $\alpha : B \to X$ of a closed subset $B$ of a space $A \in \mathcal{K}_{fd}$ and any map $\gamma : A \to Y$ with $f\alpha = \gamma|B$ there is an embedding $\bar{\alpha} : A \to X$ such that $f\bar{\alpha} = \gamma$ and $\bar{\alpha}|B = \alpha$.

Let $\mathbb{R}^{\infty}$ be the direct limit of the sequence
\[ \mathbb{R} \to \mathbb{R} \times \{0\} \subset \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R} \times \{0\} \subset \cdots \]

Let $Q$ denote the Hilbert cube $[-1, 1]^{\omega}$. By $Q^{\infty}$ we denote the direct limit of the sequence
\[ Q \to Q \times \{0\} \subset Q \times Q \to Q \times Q \times \{0\} \subset \cdots \]

A strongly $\mathcal{K}_{fd}$-universal map $f : \mathbb{R}^{\infty} \to Q^{\infty}$ is constructed in [15].

**Theorem 4.1.** There is a strongly $\mathcal{K}_{fd}$-universal map from $\mathbb{R}^{\infty}$ to $K_{\beta}$.

**Proof.** Without loss of generality we may assume that $K_{\beta}$ is embedded in $Q^{\infty}$ as a closed subset. Let $f : \mathbb{R}^{\infty} \to Q^{\infty}$ be an $\mathcal{K}_{fd}$-universal map. Define $X = f^{-1}(K_{\beta})$ and let $f'$ denote the restriction $f' = f|X : X \to K_{\beta}$. Since
\(K_\beta\) is closed in \(Q^\infty\), we see that \(X\) is a \(k_\omega\)-space. Clearly, \(X = \lim X_i\), where \(X_i \in K_{fd}\) for all \(i\). Since \(K_\beta\) is an absolute extensor, the space \(X\) satisfies the conditions of the characterization theorem for \(\mathbb{R}^\infty\) (see [14]).

Also, this implies the strong \(K_{fd}\)-universality of \(f'\). □

Remark also that a strongly \(K_{fd}\)-universal map from Theorem 4.1 is unique up to homeomorphism.

5. Remarks

In connection with Radul’s results on existence of \(D(\beta)\)-absorbing sets in the sense of Bestvina and Mogilski [6] the following question arises.

**Question 5.1.** Are there ordinals \(\beta < \omega_1\) for which both \(D_\beta\) and \(K_\beta\) exist? Is there a bitopological characterization of this pair in the spirit of Banakh and Sakai [4]?

Recall that in [4] a characterization of the bitopological space \((\mathbb{R}^\infty, \ell^2_f)\) is given.

We expect the negative answer to the following question related to Theorem 4.1.

**Question 5.2.** Is there a strongly \(D(\beta)\)-universal map \(K(\beta) \to Q^\infty\)?

Also, we conjecture that the universal map from Theorem 4.1 is not locally self-similar. (Here, a map \(\pi: X \to Y\) is said to be *locally self-similar* if for every point \(x \in X\) and every neighborhood \(U \subset X\) of \(x\) there is a neighborhood \(V \subset U\) of \(x\) such that the map \(\pi|V: V \to \pi(V)\) is homeomorphic to \(\pi\). It is proved in [3] that the universal map \(\mathbb{R}^\infty \to Q^\infty\) is not locally self-similar.)

Banakh and Repovš [3] proved that there exists a linear realization of the universal map \(\mathbb{R}^\infty \to Q^\infty\). It looks plausible that such a realization can be found for the universal map from Theorem 4.1. This would provide another construction of the spaces \(K_\beta\), namely as a linear topological space.

**Question 5.3.** Are there free topological groups, semigroups, semilattices etc. homeomorphic to \(K_\beta\)?

See, e.g., [2, 5, 15] for various results concerning infinite-dimensional manifolds in topological algebra.

Finally, remark that some of the results of this note are announced in [12]; here they are given with new proofs.
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Oryslava Polivoda
Ukrainian Academy of Printing, 19 Pip Holoskomy Str., 79000 Lviv, Ukraine
Email: shabor@ukr.net

Mykhailo Zarichnyi
Department of Mechanics and Mathematics, Lviv National University, Universytetska Str., 1, Lviv, 79000, Ukraine
Email: zarichnyi@yahoo.com
ORCID: orcid.org/0000-0002-6494-2289