A novel SPMSM sensorless strategy based on parameter identification

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Abstract. In recent years, with the application of Permanent-magnet synchronous motor (PMSM) has been wildly appeared in many industrial areas, sensorless control method of PMSM is being a research hotspot in last two decades. This paper propose a novel speed sensorless control method for SPMSM, which consists of model adaptive reference speed (MARS) sensorless method and Recursive Least Square (RLS) parameter identification method. We use RLS method to identify the inductance and take the estimated inductance value to MARS control and the fundamental control method. Finally we use the estimated speed and position value as feedback to make a close loop control frame. This novel method can successfully make the whole control system operate using the estimated speed and position value as feedback meanwhile overcome the flaw that control frame is sensitive with the parameters. We use Matlab/Simulink to implement simulation and the results could prove this method’s feasibility.

1. Introduction

Permanent-magnet synchronous motor (PMSM) has been wildly used in all electrical industrial areas, such as electrical vehicle, wind power and other new energy industries because of its high power density, high effectiveness and more reliability. The PMSM can be separated into two types according to whether its rotor has salient effect. One is named surface permanent-magnet synchronous motors because its rotor has no salient effect; Another is the interior permanent-magnet synchronous motors called IPMSM which has salient effect. As we all know, the PMSM need high precision sensor to detect its rotor position to achieve the close-loop control. Normally, the mechanical sensor like encoder costs high, and it can’t adapt the environment variation well which causes the whole control system more vulnerable. Therefore, the sensorless control method of PMSM has gradually attracted lots of researchers’ and scholars’ attention.

For now, there are mainly two categories speed sensorless control methods for SPMSM operating at medium and high speed, one is by estimating the BEMF (back electromotive force) to calculate the rotor electrical speed and position like directly calculation method and SMO (Sliding Mode Observer) [1, 2], SMO sensorless method can make system insensitive with parameter in some extent and estimate rotor speed and position successfully, but the nature of SMO is changed structure method which means its estimated speed value has discontinuity and chattering phenomena, even though lots of efforts have been made based on conventional SMO to reduce speed chattering in some extent, inasmuch as the SPMSM control system needs the high precise rotor speed and position value, the result of using SMO sensorless method in close-loop control system is unsatisfactory. A new method...
named Super-Twisting Sliding Mode Observer has been proposed by recent researches [3, 4], this method uses a second-order sliding surface on the basis of traditional SMO and has a good performance in chartering phenomenon, but it’s difficult to implement in hardware because of its complicated algorithm.

Another category makes use of the SPMSM model to estimate rotor speed, like MARS(Model Adaptive Reference Speed) [5, 6] and extend Kalman observer, this kind of method has high estimated precise therefore we can use its estimated speed and position value to replace the mechanical sensor, inasmuch as they rely on the system models’ accuracy, which means once the system parameters changed this kind of method will lose efficacy.

To solve the problem above, this paper presents a novel speed sensorless control strategy, which combines the RLS (Recursive Least Square) to indentify the inducctance online with MARS. Finally, we complete the parameter close-loop control and speed sensorless close-loop control, the simulations’ results have proved this method’s effectiveness.

2. Speed sensorless method

The mathematic model of SPMSM in (d-q) frame is given by Equation (1)

\[
\begin{align*}
U_d &= R_i + L_s \frac{di_d}{dt} - W_L i_q \\
U_q &= R_i + L_s \frac{di_q}{dt} + W_L i_d + W_e \Psi_f 
\end{align*}
\]

(1)

\(U_d, U_q\) represent the d-q axis voltage; 
\(R\) represents stator resistance; 
\(i_d, i_q\) represent the d-q axis current; 
\(W_e\) represents the rotor electrical speed; 
\(L_s\) represents the d-q axis inductance; 
\(\Psi_f\) represents the permanent magnet flux;

According to Equation (1) we can have another equation:

\[
\begin{align*}
\frac{di_d}{dt} &= -\frac{R_s}{L_s} i_d + W_i e_q + \frac{U_d}{L_s} \\
\frac{di_q}{dt} &= -\frac{R_s}{L_s} i_q - W_i e_d + \frac{U_q}{L_s} - \frac{W_e \Psi_f}{L_s}
\end{align*}
\]

(2)

Then matrix form transferred from above equations is listed as bellow,

\[
\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \frac{R_s}{L_s} & W_e \\ -W_e & \frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} U_d + \frac{R_s \Psi_f}{L_s} \\ U_q \end{bmatrix}
\]

(3)

We define the new variables \(i_d^* = i_d + \frac{\Psi_f}{L_s}, i_q^* = i_q, U_d^* = U_d + \frac{R_s \Psi_f}{L_s}, U_q^* = U_q\), and bring these new variables into Equation (3), the new form is (4) and we call this equation reference model.

\[
\frac{d}{dt} \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} = \begin{bmatrix} \frac{R_s}{L_s} & W_e \\ -W_e & \frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} U_d^* \\ U_q^* \end{bmatrix}
\]

(4)
As we can see, the first matrix in right side contains the rotor’s electrical speed value, that is exactly we are going to estimate, therefore we replace $W_e$ with estimated value $\hat{W}_e$, then the whole matrix equation will be changed into Equation (5), this equation is named adjusted model.

$$\frac{d}{dt}\begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_s} & \frac{R}{L_s} \\ -\frac{L_s}{R} & -\frac{L_s}{R} \end{bmatrix} \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} + \frac{1}{L_s} \begin{bmatrix} U_d^* \\ U_q^* \end{bmatrix}$$

(5)

Subtract adjusted model from reference model, the error equation can be listed as bellow:

$$\frac{d}{dt}\begin{bmatrix} e_d^* \\ e_q^* \end{bmatrix} = \begin{bmatrix} -R & -R \\ L_s & L_s \\ -\frac{L_s}{R} & -\frac{L_s}{R} \end{bmatrix} \begin{bmatrix} e_d^* \\ e_q^* \end{bmatrix} - (\hat{W}_e - W_e) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix}$$

(6)

Where $e_d = i_d^* - i_d^\wedge$, $e_q = i_q^* - i_q^\wedge$, $A = \begin{bmatrix} -\frac{R}{L_s} & \frac{R}{L_s} \\ -\frac{L_s}{R} & -\frac{L_s}{R} \end{bmatrix}$, $W = (\hat{W}_e - W_e)Ji_q^\wedge$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $i_q^\wedge = \begin{bmatrix} i_d^\wedge \\ i_q^\wedge \end{bmatrix}$, and the simplified form of above equation is

$$\frac{de}{dt} = Ae - W.$$

In order to use Popov super stability theory, The Popov Equivalent feedback system is illustrated as Figure 1, as we can see $V = De$, the matrix $D$ is a gain matrix, which transfers the error value $e$ to the input of the nonlinear time-variant system’s input, in order to calculate simply we let $D$ be unit matrix, then $V = e = \begin{bmatrix} e_d \\ e_q \end{bmatrix}$, and assume that $W$ is output of the nonlinear time-variant system.

![Figure 1. Equivalent feedback system.](image)

According to the Popov super stability theory [7, 8], it must satisfy two conditions so that the MARS will be stable which means $\lim_{t \to \infty} e(t) = 0$.

1. The transfer function of feedforward $G(s) = D(sI - A)^{-1}$ is a strictly positive matrix;
2. Define $\eta(0,t_i) = \int_0^t W dt$, and make sure $\eta(0,t_i) \geq -r^2$, $\forall t_i > 0$, where $r$ is a limited positive number.
There are many approaches to prove the first condition easily, as to second condition, we take \( V \) and \( W \) into above equation. Then we can get Equation (7), without loss of generality, the adaptive law is chosen as PI controller where 
\[
\hat{W}_e = \int_0^t f_1 \, dt + f_2 + \hat{W}_e(0),
\]
assume that the SPMSM is started from standstill so we let \( \hat{W}_e(0) = 0 \), as for now the mission is to find proper \( f_1 \) and \( f_2 \) to meet Equation (7). Using backstepping method, finally the equation of \( \hat{W}_e \) can be achieved as Equation (8).

\[
\eta(0,t) = \int_0^t e^\tau (\hat{W}_e-W_e) \, dt \geq -r^2
\]

\[
\hat{W}_e = \int_0^t K_i (i_q^* - i_q^* - i_d^* + i_d^*) \, dt + K_p (i_q^* - i_q^* - i_d^* + i_d^*)
\]

Where \( K_i \) and \( K_p \) are the PI controller’s gain. Replacing \( i_d^*i_q^* \) with \( i_d i_q \) and, we can get (9).

\[
\hat{W}_e = \int_0^t K_i (i_d - i_d - i_q - i_q) \, dt + K_p (i_d - i_d - i_q - i_q)
\]

In above equation, \( i_d i_q \) are the output of reference model, \( i_d^*i_q^* \) are the output of adjusted model. After gaining the estimated speed value, the rotor position can be calculated by integration 
\[
\theta_e = \int_0^t \hat{W}_e \, dt
\]

and the whole MARS frame is illustrated as Figure 2.

![Figure 2. MARS scheme.](image)

3. Parameter identification

The Equation (9) contains \( L_s \) which is SPMSM’s inductance, obviously if \( L_s \) is changed the estimated rotor speed will not converge on the true value. Given above statement, it’s important to identify the inductance online when using MARS sensorless control. In this paper we use Recursive Least Square method which is widely used method in parameters identification [9, 10].

Assume that a system state can be expressed by following equation.

\[
y = \theta_1 x_1 + \theta_2 x_2 + ... + \theta_n x_n
\]
\[
\begin{bmatrix}
y_1 & x_1 & x_2 & \ldots & x_n 
\end{bmatrix}
\]
represents observed value, \([\theta_1 \ \theta_2 \ldots \theta_n]\) contains parameter need be estimated.

After \(m\) times observes, we can get a linear equation set (11).

\[
\begin{align*}
y(1) &= \theta_1 x_1(1) + \theta_2 x_2(1) + \ldots + \theta_n x_n(1) \\
y(2) &= \theta_1 x_1(2) + \theta_2 x_2(2) + \ldots + \theta_n x_n(2) \\
\vdots \\
y(m) &= \theta_1 x_1(m) + \theta_2 x_2(m) + \ldots + \theta_n x_n(m)
\end{align*}
\]
(11)

Let \(Y = X \theta\) (12)

In consideration of the existence of measure error and model error, we define the error variable \(E = Y - X \theta\), \(E = \begin{bmatrix} E_1 & E_2 & \ldots & E_n \end{bmatrix}^T\), the principle of Least Square method is to find proper estimated parameter to get the least error sum of squares. The calculation of error sum of squares is as follow:

\[
F = \sum_{i=1}^{n} E_i^2 = E^T E = (Y - X \theta)^T (Y - X \theta)
\]
(13)

Let derivative of \(F\) with respect of \(\theta\) equal to zero, the estimated parameter can be expressed as follow:

\[
\hat{\theta} = (X^T X)^{-1} X^T Y
\]
(14)

However, if the new observations come, we need combine the new state value with the later data to recalculate Equation (14). This will increase computation and reduce efficiency without doubt and to overcome the above flaw, Recursive Least Square method based on Least Square method use iteration method is proposed.

For convenience’s sake, we subscript Equation (12) with \(m\):

\[
Y_m = X_m \theta
\]
(15)

If we get a new set of data, the subscript will be \((m+1)\), and new observer equation is \(y(m+1) = \theta x_1(m+1) + \theta_2 x_2(m+1) + \ldots + \theta_n x_n(m+1)\). Combination this new equation with Equation (14) can be written as bellow:

\[
\begin{align*}
Y_{m+1} &= X_{m+1} \theta \\
\text{Where } Y_{m+1} &= \begin{bmatrix} y(1) \\
y(2) \\
\vdots \\
y(m) \\
y(m+1) \end{bmatrix}, X_{m+1} &= \begin{bmatrix} x_1(1) x_2(1) \ldots x_n(1) \\
x_1(2) x_2(2) \ldots x_n(2) \\
\vdots \\
x_1(m) x_2(m) \ldots x_n(m) \\
x_1(m+1) x_2(m+1) \ldots x_n(m+1) \end{bmatrix}
\end{align*}
\]

repeat derivative calculation, the updated parameter is listed below:

\[
\hat{\theta}(m+1) = (X_{m+1}^T X_{m+1})^{-1} X_{m+1}^T Y_{m+1}
\]
(16)
In order to avoid the complicated computation, we define two matrices:
\[
\begin{align*}
P(m) &= (X_m^T X_m)^{-1} \\
P(m+1) &= (X_{m+1}^T X_{m+1})^{-1}
\end{align*}
\]

According to matrix identity Theorem (17) where \(A, A + BC, E + CA^{-1}B\) are non-singular matrix and \(E\) is identity matrix, we can get following Equation (18).
\[
P(m+1) = \left[P^{-1}(m) + X(m+1)X^T(m+1)\right]^{-1} = P(m) - P(m)X(m+1)\left[1 + X^T(m+1)P(m)X(m+1)\right]^{-1} X^T(m+1)P(m)
\]

Considering Equations (16) and (18), finally the results are listed below:
\[
\begin{align*}
\hat{\theta}(m+1) &= \hat{\theta}(m) + r(m+1)P(m)X(m+1) \\
y(m+1) &= X^T(m+1)\hat{\theta}(m) \\
P(m+1) &= P(m) - r(m+1)P(m)X(m+1) \\
X^T(m+1)P(m) &= 1 / \left[1 + X^T(m+1)P(m)X(m+1)\right]
\end{align*}
\]

For the purpose of weighting the new data more than old data, we introduce a forgotten factor, rewrite \(F = \sum_{i=1}^{m} \lambda^{m-i} E_i^2\), \(0 < \lambda \leq 1\), that means new data has more influence on parameters. Adding into Equation (19), the new form is expressed as follow:
\[
\begin{align*}
\hat{\theta}(m+1) &= \hat{\theta}(m) + r(m+1)P(m)X(m+1) \\
y(m+1) &= X^T(m+1)\hat{\theta}(m) \\
P(m+1) &= \frac{1}{\lambda + X^T(m+1)P(m)X(m+1)} \left[P(m) - r(m+1)P(m)X(m+1)\right] \\
r(m+1) &= \frac{1}{\lambda + X^T(m+1)P(m)X(m+1)}
\end{align*}
\]

As we can see, when \(\lambda = 1\), Recursive Least Square method will turn into Least Square method in this paper. \(\lambda\) is chosen as 0.9.

According to Equation (1), we can get a parameter estimated equation:
\[
\begin{bmatrix}
i_q \\
di_q/dt \\
i_d W_e \\
L_e
\end{bmatrix}
\begin{bmatrix}
R \\
\Psi_f W_e
\end{bmatrix}
= U_q - \Psi_f W_e
\]

Where \(i_q, i_d, W_e, U_q\) are as observed value, and take that into (20) can finally get the parameter we want.

4. Simulation results
In order to verify the feasibility of the MARS speed sensorless control and RLS inductance identification, the simulation’s model has been built in Simulink. The whole control frame is shown in Figure 3, and speed reference is started at 100rad/s and step up to 150rad/s at 0.2s with 10 loads.
Figure 3. System simulink picture.

From Figure 4 we can see that the estimated speed value and rotor position by MARS has a big error, also has little chartering without inductance identification, in another word, MARS method does rely on the accuracy of parameter. Obviously, these estimated values can not be used as feedback of SPMSM control frame.

Figure 4. MARS without inductance identification.
Figure 5 is result of inductance identification where real value is three times less than nominal value. From the picture, we can conclude that the identified parameter can converge to the real value at 0.1s.

![Figure 5. Inductance identification from RLS.](image)

Figure 6 shows us the effect of MARS with inductance identification. The consequent shows us that estimated speed value and rotor position is quite close to the real value compared with non-RLS-MARS.

![Figure 6. MARS with RLS.](image)

When using the above estimated speed value rotor position value as feedback, meanwhile taking inductance value into MPC basic control method, phase current is shown in Figure 7. Obviously at beginning inasmuch as the parameter doesn't converge to the true value, phase current has little chattering phenomenon, not long after the current is changed back to the normal sine wave. In other words, the control strategy proposed in this paper can be well operated under the inductance mismatch.
5. Summary
From the simulation’s results, the effectiveness of the proposed control strategy in this paper has been proved. In the circumstances of inductance mismatch, the combination of MARS with RLS can complete close-loop control well even though performance at beginning is not perfect, hence there is still a lot of efforts to against this drawback, also how to identify the parameter better in dynamic process is another research point. Optimize algorithm or switch method in prophase is a main question which should be pay a lot of attention in the future.

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