Combinatorial framework for planning in geological exploration

Mark Sh. Levin

The paper describes combinatorial framework for planning of geological exploration for oil-gas fields. The suggested scheme of the geological exploration involves the following stages: (1) building of special 4-layer tree-like model (layer of geological exploration): productive layer, group of productive layers, oil-gas field, oil-gas region (or group of the fields); (2) generations of local design (exploration) alternatives for each low-layer geological objects: conservation, additional search, independent utilization, joint utilization; (3) multicriteria (i.e., multi-attribute) assessment of the design (exploration) alternatives and their interrelation (compatibility) and mapping if the obtained vector estimates into integrated ordinal scale; (4) hierarchical design (‘bottom-up’) of composite exploration plans for each oil-gas field; (5) integration of the plans into region plans; and (6) aggregation of the region plans into a general exploration plan. Stages 2, 3, 4, and 5 are based on hierarchical multicriteria morphological design (HMMD) method (assessment of ranking of alternatives, selection and composition of alternatives into composite alternatives). The composition problem is based on morphological clique model. Aggregation of the obtained modular alternatives (stage 6) is based on detection of a alternatives ‘kernel’ and its extension by addition of elements (multiple choice model). In addition, the usage of multiset estimates for alternatives is described as well. The alternative estimates are based on expert judgment. The suggested combinatorial planning methodology is illustrated by numerical examples for geological exploration of Yamal peninsula.

Keywords: combinatorial modeling, planning, geological exploration, oil-gas field, combinatorial optimization, morphological analysis, multicriteria decision making, heuristic, multiset

Contents

1 Introduction 2

2 Description of framework parts 3

2.1 Hierarchical morphological design 3

2.1.1 Hierarchical system model 3

2.1.2 Ordinal estimates of system components 4

2.1.3 Multiset based estimates of system components 5

2.2 Aggregation of modular solutions 5

3 Geological exploration 6

3.1 Problem formulation 6

3.2 Examples for oil-gas fields 7

3.3 Exploration plan for region 11

3.4 Aggregation of solutions 12

3.5 Example of multiset estimates based synthesis 13

4 Conclusion 13
1. Introduction

In recent decades significance of mineral resources and their geological exploring has been increased. This paper is focusing on combinatorial planning of geological exploration for oil and gas fields. The suggested framework is the following (Fig. 1):

Stage 1. An analysis of the initial applied problem and a preliminary its structuring (for example, partitioning the problem into parts, generation of basic requirements/criteria).

Stage 2. Designing a special four-layer tree-like model (as a multi-layer model of geological objects): (i) productive stratum (reservoir), (ii) group of productive stratum (reservoirs), (iii) oil and gas field, and (iv) group of oil and gas fields (region).

Stage 3. Generation for each bottom-layer geological object a set of local design alternatives (for geological exploration) DAs: (a) conservation, (b) appraisal work, (c) independent production time, (d) joint independent production time,

Stage 4. Multicriteria assessment of DAs and their interconnection (compatibility IC) and mapping the obtained vector estimates into an ordinal scale.

Stage 5. Hierarchical (bottom-up) composition of composite exploration plans for each field.

Stage 6. Integration of the obtained plans for the fields into region plans.

Stage 7. Aggregation of the obtained region plans (solutions).

Stage 2, 3, 4, 5, 6 are based on hierarchical morphological multicriteria design (HMMD) method (ranking of DAs, selection and composition of DAs) [3,4,5,9]. Stage 7 consists in aggregation of the obtained modular solutions (detection of a 'kernel' of the obtained solutions and its extension by some additional solution elements) [6,9]. An example of combinatorial solution based on multiset estimates of DAs is described as well. All stages above are based on expert judgment. Note, combinatorial approach for selection of optimal geological actions based on knapsack-like model was described in [2].

The suggested combinatorial framework is illustrated by a numerical example for Yamal peninsula. The preliminary compressed material was published in [10]. Mainly, initial information is based on handbook [11] and expert judgment of Vladimir I. Poroskun. A preliminary Russian version of the material is contained in [8].
2. Description of framework parts

Here brief descriptions of the suggested framework parts are presented (as a simplified introduction for readers).

2.1. Hierarchical morphological design

Hierarchical morphological multicriteria design (HMMMD) method has been described in several publications [3,4,5,9].

2.1.1. Hierarchical system model

In HMMMD a special hierarchical (tree-like) model of the analyzed system 'morphological tree' is used: (i) tree-like system model, (ii) set of leaf node as bottom-layer components (parts) of the systems, (iii) set of design alternatives (DAs) for each bottom system component, (iv) ranking of DAs for each each bottom system component (to obtain an ordinal estimate/priority for each DA), (v) ordinal estimates of compatibility between DAs of neighbor system components.

2.1.2. Ordinal estimates of system components

Here the basic version of HMMMD (ordinal estimated of DAs) is briefly described. The following is assumed: (1) system quality is considered a two-component estimate: quality of components and quality of their compatibility; (2) monotone criteria for the system and its parts are considered; (3) an ordinal scale is used for quality of system components (i.e., local solutions); and (4) an ordinal scale is used for quality of system component compatibility. The following designations are used: (a) priorities (ordinal estimates) for design alternatives (DAs): \( \eta_1 = 1, \ldots, \eta_l \), \( \eta_1 \) corresponds the the best quality level; (b) ordinal compatibility for pair of DAs: \( w = 0, \ldots, \nu \), \( w \) corresponds to impossible (the worst) quality level.

Let \( S \) be a composite solution (DA) consisting of \( m \) parts. The synthesis problem is based on morphological clique model:

Find composite system \( S = S(1) \ast \ldots \ast S(i) \ast \ldots \ast S(m) \), consisting of parts/components (i.e., local DAs) (one representative \( S(i) \) for \( i \)-th system component \( i = 1, \ldots, m \)) with non-zero compatibility estimates between the selected pair of DAs.

The poset of the system quality for composite solution \( S \) is based on vector: \( N(S) = (w(S); e(S)) \), where \( w(S) \) corresponds to minimum of compatibility estimates for DAs pair in \( S \), \( e(S) = (\eta_1, \eta_1, \ldots, \eta_l) \), where \( \eta_i \) corresponds to the number of local DAs at quality level \( i \) in \( S \). Two-criteria optimization model is:

\[
\max e(S), \quad \max w(S), \quad s.t. \quad w(S) \geq 1.
\]

As a result, non-dominated by \( N(S) \) (Pareto-efficient) composite solutions are search for. The model belongs to class of NP-hard problems. The evident solving scheme involves two stages:

Stage 1. Building of all admissible solutions (composite DAs);

Stage 2. Selection of Pareto-efficient solutions.

Two algorithms can be used for the problem [3,4,5]: (1) directed enumeration of solutions (start solution(s) corresponds to the best quality estimate(s)), search (2) dynamic programming based method (series construction of admissible solutions for system parts). Note, in the case of a small degree of the system tree-based model (for example, [3...7]) algorithmic complexity of the first algorithm is sufficiently small. An illustrative example of tree-component system \( S = H \ast B \ast V \) is depicted in Fig. 2 (priorities of DAs are shown in parentheses). The following solutions can be considered:

(a) \( S_1 = H_1 \ast B_1 \ast V_2 \), \( N(S_1) = (3; 1, 1, 1) \);
(b) \( S_2 = H_2 \ast B_3 \ast V_2 \), \( N(S_2) = (2; 2, 1, 0) \);
(c) \( S_3 = H_3 \ast B_2 \ast V_1 \), \( N(S_3) = (1; 3, 0, 0) \).
Fig. 2. Illustrative example of combinatorial synthesis

Fig. 3 illustrates the system quality poset without taking into account component compatibility. Here poset parameters are: \( m = 3, \ l = 3 \). The general system quality poset based on \( N(S) (w = 1, 2, 3) \) is depicted in Fig. 4. This poset consists of three posets from Fig. 3.

2.1.3. Multiset based estimates of system components

Fundamentals of multiset theory are presented in [112]. Interval estimates based on multisets and their applications in combinatorial synthesis have been suggested in [79]. Here the following basic scale is used: \( [1, 2, ..., l] \) (\( 1 > 2 > 3 > ... \)). Interval estimate \( e \) for object (alternative) \( A \) by scale \( [1, l] \) is (position representation): \( e(A) = (\eta_1, ..., \eta_i, ..., \eta_i) \), where \( \eta_i \) corresponds to the number of elements at the quality level \( i \) (\( i = 1, l \)). The following conditions are assumed:

\textit{Condition 1:} \( \sum_{i=1}^{l} \eta_i = \eta \) (or \( |e(A)| = \eta \)).
\textit{Condition 2:} \( \eta_i > 0 \) \& \( \eta_{i+2} > 0 \) \( \implies \eta_{i+1} > 0 \) (\( i = 1, l - 2 \)).

Presentation of the estimate as multiset is:
\[ e(A) = \{1, \eta_1, 2, \eta_2, 3, \eta_3, ..., l, \eta_l\}. \]

The number of multisets for fixed value of element numbers \( \eta \) is called coefficient of multiset or multiset number:
\[ \mu^l, \eta = \frac{l(l+1)(l+2)...(l+\eta-1)}{\eta!}. \]
This number corresponds to possible number of estimates or cardinality (without taking into account condition 2). In the case of condition 2, the number of the estimates is decreased. In \[P^{1,3,4}\], the following designations for assessment problems based on the interval multiset estimates are suggested: \(P^{1,3,4}\).

In the numerical example the following assessment problem is used \(P^{3,4}\). Clearly, the basic version of HMMD is based on assessment problem \(P^{1,3,4}\).

An integrated multiset estimate is described as follows. There are \(n\) initial estimates:

\[e_1 = (\eta_1^1, \ldots, \eta_n^1), \ldots, e^\kappa = (\eta_1^\kappa, \ldots, \eta_n^\kappa), \ldots, e^n = (\eta_1^n, \ldots, \eta_n^n).\]

The integrated multiset estimate is:

\[e^l = (\eta_1^l, \ldots, \eta_n^l), \quad \eta_i^l = \frac{1}{n} \sum_{\kappa=1}^n \eta_i^\kappa \quad \forall l = 1, \ldots, n.\]

The following operation is used: \(\cup\) \(e^l = e^1 \cup \ldots \cup e^n\).

The vector proximity between two multiset estimates \(e(A_1), e(A_2)\) is:

\[\delta(e(A_1), e(A_2)) = (\delta^-(A_1, A_2), \delta^+(A_1, A_2)),\]

where (i) \(\delta^-\) corresponds to the number of one-step changes (modifications) of quality element \(\iota+1\) into quality element \(\iota\) (\(\iota = 1, \ldots, l-1\)) (this is improvement); (ii) \(\delta^+\) corresponds to the number of one-step changes (modifications) of quality element \(\iota+1\) into quality element \(\iota\) (\(\iota = 1, \ldots, l-1\)) (this is decreasing of quality). This description corresponds to modification as editing of object (alternative) \(A_1\) into alternative \(A_2\). In addition, the following is assumed: \(|\delta(e(A_1), e(A_2))| = \max\{|\delta^-(A_1, A_2)|, |\delta^+(A_1, A_2)|\}\).

Further, aggregation of estimate (as searching for a median) is examined. There are a set of estimates (as a set of objects/alternatives):

\[\hat{E} = \{e_1, \ldots, e_n\},\]

the set of possible estimates is \(\hat{D} (\hat{E} \subseteq \hat{D})\). Aggregation estimate as generalized median is \[6,9\]:

\[M^g = \arg \min_{X \in \hat{D}} \bigcup_{\kappa=1}^n |\delta(e(X), e_\kappa)|.\]

Thus, combinatorial synthesis problem based on multiset estimates of DAs is the following:

\[\max e(S) = M^g = \arg \min_{X \in \hat{D}} \bigcup_{\kappa=1}^n |\delta(e(X), e_\kappa)|, \quad \max w(S), \quad s.t. \quad w(S) \geq 1.\]

2.2. Aggregation of modular solutions

Basic aggregation strategies for modular solutions are considered in \[6,9\]. Let \(\bar{S} = \{S_1, \ldots, S^n\}\) be a set of initial modular solutions. A general aggregation strategy is targeted to searching for consensus/median solution \(S^M\) (this is generalized median) for the initial solutions \(\bar{S} = \{S_1, \ldots, S^n\}\):

\[S^M = \arg \min_{X \in \bar{S}} \left(\sum_{i=1}^n \rho(X, S^i)\right),\]

where \(\rho(X, Y)\) is a proximity between two solutions \(X, Y \in \bar{S}\). This problem (searching for the generalized median) is often NP-hard. It may be reasonable to use simplified (approximate) strategies, for example: (a) selection of solution from the set of initial solutions (i.e., set median), (b) extension strategy, (c) compressed strategy. The last two strategies are as follows:

1. Extension strategy. 1.1. design of a ‘kernel’ for the initial solutions (substructure or an extended substructure), 1.2. generation of some additional elements for possible inclusion into the ‘kernel’, 1.3. selection of the additional elements while taking into account their ‘profit’ and resource requirements (e.g., cost) (here basic knapsack problem can be used).

2. Compression strategy. 2.1. design a super structure for the initial solutions, 2.2. generation of the superstructure elements as possible candidates for deletion, 2.3. selection of the elements for deletion from the superstructure while taking into account their ‘profit’ resource requirements (e.g., cost) (here knapsack problem with minimization of the objective function can be used).

In the paper, extension aggregation strategy is used.
3. Geological exploration

In this section, combinatorial planning of geological exploration is examined as a numerical example for oil and gas fields of Yamal peninsula [11] (Fig. 5). The general plan involves five parts: $S = A^1 \ast A^2 \ast A^3 \ast A^4 \ast A^5$, where $A^1$ corresponds to field Kharosovey, $A^2$ corresponds to field Arkticheskoe, $A^3$ corresponds to field Neitinskoe, $A^4$ corresponds to Kruzensternskoe, $A^5$ corresponds to field Bovanenkovskoe.

Fig. 5. Oil-gas fields (Yamal region)

The solving scheme consists of two stages:

Stage 1. Hierarchical combinatorial construction of the exploration plan for oil and gas fields (here only two oil and gas fields are described).

Stage 2. Composition of the general exploration plan for region.

3.1. Problem formulation

The following four-layer hierarchy of geological objects is considered: (a) productive stratum (reservoir) (bottom hierarchical level); (b) bore hole as a group of productive stratum (reservoirs); (c) oil and gas field; (d) group of oil and gas fields (region). The following assessment parameters (attributes) are used:

1. parameter of reservoir existence (‘3’ corresponds to existence of reservoir, ‘2’ corresponds to prospective geological position (horizon) in the traprock);
2. cover of thickness, m;
3. type of fluid, i.e., classification factor: (i) gas, (ii) gas and condensate (condensed fluid), (iii) oil;
4. volume of geological reserves or resources (gas - million cubic meters, oil - thousand tonnes);
5. production rate of work wellsite (cubic meters per 24 hours);
6. complexity of geological situation (‘1’ - simple, ‘2’ - complex, ‘3’ - very complex);
7. reliability (risk) to obtain the results ([0...100]);
8. validity (adequacy) of assessment of geological reserves (i.e., oil/gas/condensate in place, probable reserves) ($C_1 - 20\%, C_2 - 50\%, C_3 - 80\%$, etc.);
9. proximity to technological base (gas-oil pipeline, km).

Eight DAs are examined for each geological object (as stratum) (Table 1) (the corresponding bottom index is used for the designation).

| Notation | Content of geological exploration |
|----------|---------------------------------|
| 1.       | $X_1$ conservation              |
| 2.       | $X_2$ appraisal work            |
| 3.       | $X_3$ independent production time (gas) |
| 4.       | $X_4$ independent production time (oil) |
| 5.       | $X_5$ independent production time (oil and gas) |
| 6.       | $X_6$ joint independent production time (gas) |
| 7.       | $X_7$ joint independent production time (oil) |
| 8.       | $X_8$ joint independent production time (oil and gas) |

Further, a subset of alternative actions (DAs) for each geological object (stratum) at bottom layer of the system model (i.e., productive stratum) is selected (from initial eight basic DAs) (expert judgment). This a preliminary selection at the bottom layer of the problem. At the next step, the selected DAs are
used as a basis for composition of composite DAs for more higher layer of the problem (i.e., for group of geological objects as bore holes, and for fields)

Each strategy component (geological object, group of objects, strategy) is noted by symbol (the level of effectiveness of priority \( \iota \) is pointed out for each components in parentheses). It is assumed that experts have their skills for the following: (1) selection of DAs for each geological object, (2) ranking of DAs for each geological object, (3) assessment of compatibility among DAs (by an ordinal scale).

The illustrative hierarchical model of oil and gas field is depicted in Fig. 6.

3.2. Examples for oil-gas fields

The modular exploration strategy for field Arkticheskoe is shown in Fig. 7. Table 2 contains compatibility factors for the strategy elements (here 'C5+v' corresponds to level of hydrocarbon in gas as 'C5' and more).

![Fig. 6. Illustration for hierarchical model of field](image)

![Fig. 7. Strategy for field Arkticheskoe](image)
Table 2. Compatibility factors for DAs pair

| DA & DA          | Factors                                      |
|------------------|----------------------------------------------|
| 1. TP 14 E & TP 14A F | Geological reserves, proximity, 'C5+v'    |
| 2. TP 14 E & TP 15 G | Geological reserves, proximity, 'C5+v'    |
| 3. TP 14 E & TP 17 J | Geological reserves, proximity              |
| 4. TP 14 E & TP 18 I | Geological reserves, proximity              |
| 5. TP 14A F & TP 15 G | Geological reserves, proximity              |
| 6. TP 14A F & TP 17 J | Geological reserves, proximity              |
| 7. TP 14A F & T 18 I | Proximity                                    |
| 8. TP 15 G & TP 17 J | Geological reserves, proximity, 'C5+v'    |
| 9. TP 15 G & TP 18 I | Geological reserves, proximity, 'C5+v'    |
| 10. TP 17 J & TP 18 I | Geological reserves, proximity, 'C5+v'   |
| 11. TP 24 P & NP 3 Q | Proximity, 'C5+v'                            |

Compatibility estimates between DAs (expert judgment) are contained in Table 3 and Table 4. Composite DAs for group of geological objects are obtained for TP 14 - TP 18 (W), TP 24 - NP3 (D). Thus, 6 versions of exploration strategy (field Arkticheskoe) are obtained:

\[
A_1^2 = W_1 \ast D_1 \ast B_3(1), \quad A_2^2 = W_2 \ast D_1 \ast B_3(1), \quad A_3^2 = W_1 \ast D_2 \ast B_3(1), \\
A_4^2 = W_2 \ast D_2 \ast B_3(1), \quad A_5^2 = W_3 \ast D_1 \ast B_3(1), \quad A_6^2 = W_3 \ast D_2 \ast B_3(1).
\]

Table 6 contains some examples of bottlenecks and possible improvement operations. Fig. 8 depicts quality of composite DAs for component W.

The exploration strategy for field Kruzensternskoe is shown in Fig. 9. Table 7 contains compatibility factors for strategy elements. The compatibility estimates among DAs (expert judgment) are presented in Table 8, Table 9. Composite DAs for B and H are presented in Table 10. The obtained two solutions for field Kruzensternskoe are: \(A_1^4 = B_1 \ast H_1, \quad A_2^4 = B_2 \ast H_2\). Fig. 10 illustrates quality of composite solutions for component H.

Table 3. Compatibility for DAs (groups TP 14 - TP 18, part W)

|     | F2 | F6 | G2 | G3 | G6 | J2 | J6 | I3 | I6 |
|-----|----|----|----|----|----|----|----|----|----|
| E2  | 2  | 3  | 2  | 3  | 4  | 1  | 0  | 2  | 3  | 3  |
| E3  | 3  | 4  | 2  | 4  | 4  | 1  | 2  | 3  | 2  | 2  |
| E6  | 3  | 4  | 3  | 4  | 4  | 1  | 4  | 3  | 3  | 4  |
| F2  | 2  | 3  | 4  | 2  | 3  | 2  | 3  | 2  | 3  | 2  |
| F6  | 1  | 3  | 4  | 3  | 3  | 2  | 3  | 1  |    |    |
| G2  | 2  | 3  | 2  | 1  | 3  |    |    |    |    |    |
| G3  | 2  | 4  | 2  | 3  | 1  |    |    |    |    |    |
| G6  | 2  | 4  | 2  | 3  | 1  |    |    |    |    |    |
| J2  | 2  | 1  | 1  |    |    |    |    |    |    |    |
| J6  | 1  | 3  | 4  |    |    |    |    |    |    |    |

Table 4. Compatibility for DAs (groups TP 24 - NP 3, part D)

|     | Q2 | Q5 |
|-----|----|----|
| P2  | 2  | 3  |
| P3  | 3  | 4  |

Table 5. Composite DAs

| Intermediate composite DAs | N   |
|-----------------------------|-----|
| D1 = P3 \ast Q5            | 4:1, 1, 0 |
| D2 = P3 \ast Q2            | 3:2, 0, 0 |
| W1 = E5 \ast F6 \ast G6 \ast J6 \ast I6 | 4:2, 3, 0 |
| W2 = E5 \ast F6 \ast G3 \ast J6 \ast I3  | 2:5, 0, 0 |
| W3 = E5 \ast F6 \ast G3 \ast J6 \ast I3  | 3:4, 1, 0 |
Table 6. Bottlenecks and improvement operations

| Intermediate composite DAs | Bottlenecks: DAs | IC | Improvement operation w/r |
|---------------------------|-----------------|----|--------------------------|
| 1. \(D_1 = P_3 \times Q_5\) | \(Q_5\) | 2 \(\Rightarrow\) 1 |
| 2. \(D_2 = P_3 \times Q_2\) | \((P_3, Q_2)\) | 3 \(\Rightarrow\) 1 |
| 3. \(W_1 = E_6 \times F_6 \times G_6 \times J_6 \times I_6\) | \(E_6\) | 2 \(\Rightarrow\) 1 |
| 4. \(W_1 = E_6 \times F_6 \times G_6 \times J_6 \times I_6\) | \(G_6\) | 2 \(\Rightarrow\) 1 |
| 5. \(W_1 = E_6 \times F_6 \times G_6 \times J_6 \times I_6\) | \(I_6\) | 2 \(\Rightarrow\) 1 |
| 6. \(W_3 = E_6 \times F_6 \times G_3 \times J_6 \times I_3\) | \(E_6\) | 2 \(\Rightarrow\) 1 |

Fig. 8. System quality poset for \(N(W)\)

Strategy \(A^4 = B \times H\)

\[A^1_1 = B_1 \times H_1\]
\[A^2_2 = B_1 \times H_2\]

PK 1 - PK 11

PK 12 - TP 11

\(B = E \times F \times G \times J\)
\(B_1 = E_3 \times F_3 \times G_3 \times J_6\)

\(H = K \times L \times V \times O \times P\)
\(H_1 = K_6 \times L_6 \times V_5 \times O_3 \times P_6\)
\(H_2 = K_6 \times L_6 \times V_5 \times O_3 \times P_2\)

PK 12

TP 1-2

TP 5-5A

TP 10

TP 11

| PK 12 | TP 1-2 | TP 5-5A | TP 10 | TP 11 |
|-------|--------|---------|-------|-------|
| \(K\) | \(L\)  | \(V\)   | \(O\) | \(P\) |
| \(K_2(3)\) | \(L_2(2)\) | \(V_2(2)\) | \(O_2(2)\) | \(P_2(1)\) |
| \(K_6(1)\) | \(L_6(1)\) | \(V_5(1)\) | \(O_3(1)\) | \(P_6(2)\) |

Fig. 9. Strategy for field Kruzensternskoe
Table 7. Compatibility factors for DAs pair (part B)

| DA & DA | Factors                        |
|---------|--------------------------------|
| 1.      | PK 1-4 E & PK 9 F              |
| 2.      | PK 1-4 E & PK 10 G             |
| 3.      | PK 1-4 E & PK 11 J             |
| 4.      | PK 9 F & PK 10 G               |
| 5.      | PK 9 F & PK 11 J               |
| 6.      | PK 10 G & PK 11 J              |
| 7.      | PK 12 K & TP 1-2 L             |
| 8.      | PK 12 K & TP 5-5A V            |
| 9.      | PK 12 K & TP 10 O              |
| 10.     | PK 12 K & TP 11 P              |
| 11.     | TP 1-2 L & TP 5-5A V           |
| 12.     | TP 1-2 L & TP 10 O             |
| 13.     | TP 1-2 L & TP 11 P             |
| 14.     | TP 5-5A V & TP 10 O            |
| 15.     | TP 5-5A V & TP 11 P            |
| 16.     | TP 10 O & TP 11 P              |

Table 8. Compatibility for DAs (groups PK 1 - PK 11, part B)

|   | $F_2$ | $F_3$ | $G_2$ | $G_3$ | $G_6$ | $J_2$ | $J_3$ | $J_6$ |
|---|-------|-------|-------|-------|-------|-------|-------|-------|
| $E_2$ | 2     | 1     | 2     | 1     | 2     | 1     | 2     |       |
| $E_3$ | 4     | 3     | 4     | 3     | 1     | 4     | 3     | 1     |
| $E_6$ | 1     | 4     | 1     | 4     | 2     | 1     | 3     | 4     |
| $F_2$ | 3     | 4     | 2     | 3     | 4     | 2     |       |       |
| $F_3$ | 3     | 4     | 4     | 3     | 4     |       |       |       |
| $G_2$ |       |       |       |       |       | 3     | 4     | 4     |
| $G_3$ |       |       |       |       |       | 4     | 4     | 4     |
| $G_6$ |       |       |       |       |       | 3     | 4     | 4     |

Table 9. Compatibility for DAs (groups PK 12 - TP 11, part H)

|   | $L_2$ | $L_6$ | $V_2$ | $V_5$ | $O_2$ | $O_3$ | $P_2$ | $P_6$ |
|---|-------|-------|-------|-------|-------|-------|-------|-------|
| $K_2$ | 4     | 3     | 2     | 4     | 3     | 1     | 4     | 3     |
| $K_6$ | 1     | 4     | 3     | 4     | 3     | 4     | 3     | 4     |
| $L_2$ | 2     | 3     | 4     | 2     | 3     | 4     |       |       |
| $L_6$ | 2     | 4     | 4     | 3     | 4     |       |       |       |
| $V_2$ | 4     | 4     |       | 2     | 3     |       |       |       |
| $V_5$ | 3     | 4     | 3     | 4     |       |       |       |       |
| $O_2$ |       |       | 4     | 4     |       |       |       |       |
| $O_3$ |       |       |       |       |       | 3     | 4     |       |

Table 10. Intermediate composite DAs

| Composite DAs | $N$ |
|---------------|-----|
| $B_1 = E_3 \times F_3 \times G_3 \times J_3$ | 3; 4, 0, 0 |
| $H_1 = K_6 \times L_6 \times V_5 \times O_3 \times P_6$ | 4; 4, 1, 0 |
| $H_2 = K_6 \times L_6 \times V_5 \times O_3 \times P_2$ | 3; 5, 0, 0 |
3.3. Exploration plan for region

Thus, the following composite strategy for region is obtained (Fig. 11):
0. General composite strategy \( S = A^1 \times A^2 \times A^3 \times A^4 \times A^5 \)
1. Strategy for oil-gas field Kharosovey: \( A^1_1 \).
2. Strategy for oil-gas field Arkticheskoe: \( A^2_1, A^2_2, A^2_3, A^2_4, A^2_5 \).
3. Strategy for oil-gas field Neitinskoe: \( A^3_1 \).
4. Strategy for oil-gas field Kruzensternskoe: \( A^4_1, A^4_2 \).
5. Strategy for oil-gas field Bovanenkovskoe: \( A^5_1, A^5_2 \).

Finally, 24 composite exploration strategies for the region are (without compatibility analysis):
\[
\begin{align*}
S_1 &= A^1_1 \times A^2_1 \times A^3_1 \times A^4_1 \times A^5_1, \\
S_2 &= A^1_1 \times A^2_1 \times A^3_1 \times A^4_2 \times A^5_1, \\
S_3 &= A^1_1 \times A^2_1 \times A^3_2 \times A^4_1 \times A^5_1, \\
S_4 &= A^1_1 \times A^2_2 \times A^3_1 \times A^4_1 \times A^5_1, \\
S_5 &= A^1_1 \times A^2_2 \times A^3_2 \times A^4_1 \times A^5_1, \\
S_6 &= A^1_1 \times A^2_2 \times A^3_2 \times A^4_2 \times A^5_1, \\
S_7 &= A^1_1 \times A^2_2 \times A^3_2 \times A^4_3 \times A^5_1, \\
S_8 &= A^1_1 \times A^2_2 \times A^3_2 \times A^4_3 \times A^5_1, \\
S_9 &= A^1_1 \times A^2_3 \times A^3_1 \times A^4_1 \times A^5_1, \\
S_{10} &= A^1_1 \times A^2_3 \times A^3_1 \times A^4_2 \times A^5_1, \\
S_{11} &= A^1_1 \times A^2_3 \times A^3_1 \times A^4_3 \times A^5_1, \\
S_{12} &= A^1_1 \times A^2_3 \times A^3_2 \times A^4_1 \times A^5_1, \\
S_{13} &= A^1_1 \times A^2_3 \times A^3_2 \times A^4_2 \times A^5_1, \\
S_{14} &= A^1_1 \times A^2_3 \times A^3_2 \times A^4_3 \times A^5_1, \\
S_{15} &= A^1_1 \times A^2_3 \times A^3_3 \times A^4_1 \times A^5_1, \\
S_{16} &= A^1_1 \times A^2_3 \times A^3_3 \times A^4_2 \times A^5_1, \\
S_{17} &= A^1_1 \times A^2_3 \times A^3_3 \times A^4_3 \times A^5_1, \\
S_{18} &= A^1_1 \times A^2_4 \times A^3_1 \times A^4_1 \times A^5_1, \\
S_{19} &= A^1_1 \times A^2_4 \times A^3_1 \times A^4_2 \times A^5_1, \\
S_{20} &= A^1_1 \times A^2_4 \times A^3_1 \times A^4_3 \times A^5_1, \\
S_{21} &= A^1_1 \times A^2_4 \times A^3_2 \times A^4_1 \times A^5_1, \\
S_{22} &= A^1_1 \times A^2_4 \times A^3_2 \times A^4_2 \times A^5_1, \\
S_{23} &= A^1_1 \times A^2_4 \times A^3_2 \times A^4_3 \times A^5_1, \\
S_{24} &= A^1_1 \times A^2_5 \times A^3_1 \times A^4_1 \times A^5_1.
\end{align*}
\]
Now an additional analysis of the obtained strategies can be considered to design the best final strategy (e.g., multicriteria analysis and selection, expert judgment). On the other hand, the final strategy can be build by aggregation of the obtained solutions.

3.4. Aggregation of solutions

In the considered example, there are 24 solutions (previous section): \( S_1, \ldots, S_{24} \). The substructure of the solutions is shown in Fig. 12. This structure is used as a 'kernel' for an extension process. The superstructure is shown in Fig. 13.

Table 11 contains design alternatives for extension of the 'kernel' including their estimates (ordinal scales are used, expert judgment).

| \( \kappa \) | Versions | Binary variable | Cost | Profit |
|---|---|---|---|---|
| 1. | \( A_1^1 \) | \( x_{11} \) | 4 | 4 |
| 2. | \( A_2^1 \) | \( x_{12} \) | 6 | 6 |
| 3. | \( A_3^1 \) | \( x_{13} \) | 3 | 2 |
| 4. | \( A_1^2 \) | \( x_{14} \) | 3 | 3 |
| 5. | \( A_2^2 \) | \( x_{15} \) | 4 | 3 |
| 6. | \( A_3^2 \) | \( x_{16} \) | 5 | 3 |
| 7. | \( A_4^2 \) | \( x_{21} \) | 3 | 4 |
| 8. | \( A_5^2 \) | \( x_{22} \) | 3 | 3 |
| 9. | \( A_6^2 \) | \( x_{31} \) | 3 | 3 |
| 10. | \( A_7^2 \) | \( x_{32} \) | 4 | 4 |

It is assumed, the DAs are compatible. The aggregation problem (extension strategy) is based on multiple choice problem:

\[
\max \sum_{i=1}^{3} \sum_{j=1}^{q_i} c_{ij} x_{ij} \quad \text{s.t.} \quad \sum_{i=1}^{3} \sum_{j=1}^{q_i} a_{ij} x_{ij} \leq b, \quad \sum_{j=1}^{q_i} x_{ij} = 1 \quad \forall i = 1, 3, \quad x_{ij} \in \{0, 1\}.
\]

In this model, \( q_1 = 6, q_2 = 2, q_3 = 2 \). By the usage of a greedy algorithm (i.e., linear ordering of elements by \( c_i / a_i \)) the following solutions are obtained for four versions of constraints:

1. \( b^1 = 9: \ (x_{14} = 10, x_{21} = 1, x_{31} = 1) \); \( S_{b^1}^{agg} = A_1^1 \times A_2^1 \times A_3^1 \times A_4^1 \times A_5^1 = R_1 \times P_3 \times D_2 \times Q_4 \times U_1 \times Z_1 \times Y_2 \times 0 \times 1; \)

2. \( b^2 = 10: \ (x_{14} = 1, x_{21} = 1, x_{32} = 1) \); \( S_{b^2}^{agg} = A_1^1 \times A_2^2 \times A_3^1 \times A_4^1 \times A_5^1 = R_3 \times P_3 \times D_2 \times Q_4 \times U_1 \times Z_1 \times Y_2 \times 0 \times 1; \)

3. \( b^3 = 11: \ (x_{11} = 1, x_{21} = 1, x_{31} = 1) \); \( S_{b^3}^{agg} = A_1^1 \times A_2^1 \times A_3^1 \times A_4^1 \times A_5^1 = R_3 \times P_3 \times D_2 \times Q_4 \times U_1 \times Z_1 \times Y_2 \times 0 \times 1; \)

4. \( b^4 = 11: \ (x_{12} = 1, x_{22} = 1, x_{31} = 1) \); \( S_{b^4}^{agg} = A_1^1 \times A_2^2 \times A_3^1 \times A_4^1 \times A_5^1 = R_4 \times P_3 \times D_2 \times Q_4 \times U_1 \times Z_1 \times Y_2 \times 0 \times 1; \)
3.5. Example of multiset estimates based synthesis

A scale based on multiset estimates (as a poset) for the used assessment problem $P_{3,4}^3$ is depicted in Fig. 14. The illustrative numerical example is based on multiset estimates for Arkticheskoe oil-gas field (Fig. 15). Multiset estimates for local DAs are shown in Fig. 15 (in parentheses). Compatibility estimates from Table 3 are used. Two solutions are considered:

$W_1^M = E_6 \ast F_6 \ast G_6 \ast J_6 \ast I_6$, \hspace{1cm} $N(W_1^M) = (w(W_1^M); e(W_1^M)) = (4; 1, 3, 0)$;

$W_2^M = E_6 \ast F_6 \ast G_3 \ast J_6 \ast I_3$, \hspace{1cm} $N(W_2^M) = (w(W_2^M); e(W_2^M)) = (3; 3, 1, 0)$.

Estimates $e(W_1^M) = (1, 3, 0)$, $e(W_2^M) = (3, 1, 0)$ are medians for estimates of the corresponding components.

Fig. 14. Estimates for assessment problem $P_{3,4}^3$.

Fig. 15. Arkticheskoe oil-gas field (multiset estimates).

4. Conclusion

This paper describes a hierarchical approach to combinatorial planning of geological exploration. The approach is based on the following: (a) expert judgment; (b) planning consists in bottom-up selection and composition of local solutions (design/exploration alternatives DAs) into composite solutions at the
higher layer of the plan hierarchy; (c) aggregation of the obtained plans (solutions) is considered as an extension of a 'kernel' of the preliminary obtained solution versions. The approach is illustrated by a numerical example as oil and gas geological planning for Yamal peninsula. It may be reasonable to consider the following future directions: (1) examination of multistage exploration strategies; (2) study of combinatorial evolution models for oil and gas field(s); (3) using the suggested framework in education.

REFERENCES

1. D.E. Knuth, The Art of Computer Programming. Vol. 2, Seminumerical Algorithms. Addison Wesley, Reading, 1998.
2. A.A. Koltun, O.Y. Pershin, A.M. Ponomarev, Models and algorithms for the selection of optimal set of geotechnical arrangements on the oil fields. Autom. and Rem. Contr., 66(8), 35–45, 2005.
3. Levin M.Sh. Hierarchical Morphological Multicriteria Design of Decomposable Systems. Concur. Eng.: Res. and Appl., 4(2), 111–117, 1996.
4. M.Sh. Levin, Combinatorial Engineering of Decomposable Systems. Springer, 1998.
5. M.Sh. Levin, Composite Systems Decisions. Springer, 2006.
6. M.Sh. Levin, Aggregation of composite soutions: strategies, models, examples. Electr. prepr., 72 p., Nov. 29, 2011. http://arxiv.org/abs/1111.6983 [cs.SE]
7. M.Sh. Levin, Multiset estimates and combinatorial synthesis. Electr. prepr., 30 p., May 9, 2012. http://arxiv.org/abs/1205.2046 [cs.SY]
8. M.Sh. Levin, Decision Support Technology for Modular Systems. Electr. book, Moscow, 2013 (in Russian). [http://www.mslevin.iitp.ru/Levin-bk-Nov2013-071.pdf]
9. M.Sh. Levin, Modular System Design and Evaluation. Springer, 2015.
10. M.Sh. Levin, V.I. Poroskun, Combinatorial planning of oil-gas exploration. In: Geological exploration and utilization of natural resources. Issue 4, Moscow, Ministry of Nature Resources, Private Company ‘Geoinformmark’, pp. 58–63, 1997 (in Russian).
11. S.P. Maximov (ed), Oil-gas Field in USSR. Handbook. Moscow, Nedra Publ. Hause, 1979 (in Russian).
12. R.R. Yager, On the theory of bags. Int. J. of General Systems, 23–37, 1986.

Author address:
Mark Sh. Levin, Inst. for Information Transmission Problem, Russian Acadademy of Sciences [http://www.mslevin.iitp.ru/] email: mslevin@acm.org