False vacuum decay in a brane world cosmological model

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March 24, 2022

Abstract

The false vacuum decay in a brane world model is studied in this work. We investigate the vacuum decay via the Coleman-de Luccia instanton, derive explicit approximative expressions for the Coleman-de Luccia instanton which is close to a Hawking-Moss instanton and compare the results with those already obtained within Einstein’s theory of relativity.

Keywords: false vacuum decay, brane world, Coleman-de Luccia instanton.

1 Introduction

The total energy-density of our universe is very close to the critical value corresponding to the Friedmann-Robertson-Walker metrics of the flat type. Numerically this means that the total matter-density parameter has the value in the thin range around 1: $\Omega = 1.02 \pm 0.04$. There are two possible interpretations of this fact: the universe is exactly flat; or the early evolution forced the universe to evolve to the present state which is very close to the flat Friedmann-Robertson-Walker metric, however the geometry may be both open or closed. There are models within open inflationary universe scenario, e.g. [1], [2], [3] that can successfully lead to an acceptable present-day universe. These models are built on Einstein’s theory of relativity in four dimensional space-time. There are also models of ”creation of an infinite universe within a finite bubble” based on modifications of the Einstein’s relativity theory like the model presented in [5] where the false vacuum decay via Coleman-de Luccia (CdL) instanton and subsequent second phase of inflation within the nucleated bubble is studied in the context

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of Jordan-Brans-Dicke theory.

The progress in the superstring theory during last years has forced the cosmologists to consider the extra dimensions in various models of the universe evolution e.g. [6] and [7]. Especially, the gravity-scalar instantons have been considered in [8]. The creation of an open or closed universe within the brane world scenario has been considered in [9]. The physical and geometrical discussion of the semiclassical instability of the Randall-Sundrum brane world resulting in the vacuum decay via instantons is done in [10], [11].

In this paper we consider the decay of the false vacuum of a scalar field (inflaton) confined to a four dimensional brane in a five dimensional brane world model. We analyze the Euclidean cosmological equations in the Randall-Sundrum type II scenario [4] that are supposed to describe semiclassically the false vacuum decay. We are inspired by the work done by del Campo, Herrera and Saavedra [12] in which the authors investigate the possibility of realization of the open inflation scenario in the brane world models including the existence of the Coleman-de Luccia instanton providing the false vacuum decay and subsequent inflation within created open universe. The authors of the cited paper are interested in a specially chosen theory (the self interaction $V$ of the scalar field). They investigate the model only for fixed parameters of the self-interaction and obtain both CdL instanton and plausible evolution after tunneling. Our aim is to study the CdL instantons for arbitrary potentials and compare the properties of CdL instantons in the standard Einstein’s gravity in four dimensional space-time with those from our brane world. A similar problem has been recently considered in [13], where the authors study the vacuum decay on the brane within the thin-wall approximation. Unlike the authors of the paper [13] we are interested in another problem that can be solved analytically, namely the CdL instanton(s) of the first order close to a Hawking-Moss instanton.

The paper is organized as follows: in section 2 we briefly review the basic fact about the CdL instantons in four dimensional de Sitter space-time to be able to compare them with the results of this paper. In section three we present the formulation of the instanton equations in considered brane world model and some consequences of these equations are discussed. Perturbative computation of the first order CdL instanton in our brane world model is presented in section four, and finally the main quantity characterizing the instanton - its action - is computed in the fifth section.

2 False vacuum decay via CdL instanton in four dimensional space-time within Einstein’s relativity theory

CdL instanton introduced in [14] describes false vacuum decay in a de Sitter universe within the semiclassical approximation. If $V = V(\Phi)$ is the
effective potential for the scalar field, this CdL instanton can be introduced as the \(O(4)\)-symmetric and finite-action solution of the Euclidean version of the Einstein equations. The \(O(4)\)-symmetry means that the scalar field \(\Phi\) lives on a (squeezed) four-sphere with the metric
\[
\text{d}s^2 = \text{d}\tau^2 + a^2(\tau) \left[ \text{d}\chi^2 + \sin^2(\chi)\text{d}\Omega_2^2 \right],
\]
with \(\tau \in [0, \tau_f]\). The action (Euclidean version of the Einstein-Hilbert action) reads
\[
S = 2\pi^2 \int_0^{\tau_f} \left[ \left( \frac{1}{2}(\Phi')^2 + V \right) a^2 - \frac{1}{C}(a'(a')^2 + 1) \right] \text{d}\tau,
\]
where \(C = 8\pi/3\) and the prime denotes the derivative with respect to \(\tau\). Varying the action (2) we obtain the (Euclidean) equations of motion for the scale parameter and the inflaton:
\[
a''(\tau) = -C \left( (\Phi')^2 + V \right) a, \quad \Phi'' + 3a'a' + \Phi' - \partial_\Phi V = 0.
\]
The local energy conservation law and the requirement of finiteness of the action impose the boundary conditions on the functions \(a\) and \(\Phi\):
\[
a(0) = 0, \quad a'(0) = 1, \quad \Phi'(0) = \Phi'((\tau_f)) = 0,
\]
where \(\tau_f > 0\) is to be determined from \(a(\tau_f) = 0\). The action (2) of a CdL instanton can be considerably simplified by using the equations of motion (3):
\[
S = -\frac{4\pi^2}{3C} \int_0^{\tau_f} a \text{d}\tau.
\]
After Coleman and de Luccia have proposed the idea of the description of the vacuum decay in [14], many authors have studied the system (3) and (4). Coleman and de Luccia themselves have found the solution of the instanton equations in the thin-wall limit. We suppose the effective potential \(V\) is non-negative function with two non-degenerate minima. The top of the potential barrier is reached at the point we denote by \(\Phi_M\).
Furthermore, we denote: \(V_M \equiv V(\Phi_M), V''_M \equiv \partial_\Phi^2 V(\Phi_M), H^2_M = CV_M\) and \(H^2(\Phi) = CV(\Phi)\). The properties of \(V\) in the neighborhood of \(\Phi_M\) are crucial for the existence of CdL instanton. Motivated by the earlier works [15] and [16] the authors of the papers [17], [18] and [19] have achieved information about the solution(s) of the instanton equations interesting for us:
- if \(-V''_M/H^2_M > 4\) then the CdL instanton exists
- if a CdL instanton exist then \(-V''(\Phi)/H^2(\Phi) > 4\) somewhere in the barrier
• if \(-V''_M/H_M^2 > l(l + 3)\), where \(l\) is an arbitrary integer, then CdL instanton crossing in the \(\Phi\)-direction \(l\)-times the top of the barrier (\(l\)th order CdL instanton) exists

• if \(-V''_M/H_M^2 \to l(l + 3)\) we have the explicit approximative formulas for the instanton and its action, [18]

3 False vacuum decay on a brane - elementary discussion of the instanton equations

The dynamics of the scalar field confined to a four dimensional brane in a five dimensional bulk in our model is defined by the action

\[ S = M_5^3 \int d^5x \sqrt{|G|} \left[ R(5) - 2\Lambda(5) \right] - \int d^4x \sqrt{|g|} L_\Phi \]  

(5)

with the matter term

\[ L_\Phi = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi), \]

where \(R(5)\) is the scalar curvature of the five-dimensional bulk metric \(G\), \(M_5\) and \(\Lambda(5)\) stand for the five-dimensional Planck mass and cosmological constant, respectively. These quantities are related to the effective four-dimensional cosmological constant \(\Lambda(4)\), the brane tension \(\sigma\) and the four-dimensional Planck mass \(M(4)\) by the relations

\[ \Lambda(4) = \frac{4\pi}{M_5^3} \left( \Lambda(5) + \frac{4\pi}{3M_5^2} \sigma^2 \right), \]

\[ M(4) = \left( \frac{3}{4\pi} \right)^{1/2} \frac{M_5^3}{\sigma^{1/2}}. \]  

(6)

We will use the units where \(M(4) = 1\). Following the works [20] and [12] we come at the Euclidean equations of motion for the inflaton \(\Phi\) on the brane and the induced metric (under the assumption of \(O(4)\)-symmetry which involves the line element of the form \(ds^2 = d\tau^2 + a^2(\tau)[d\chi^2 + \sin^2(\chi)d\Omega_2^2]\)):

\[ a'' = -C \left\{ (\Phi')^2 + V + \frac{1}{8\sigma} \left[ (5(\Phi')^2 + 2V)(-\Phi')^2 + 2V \right] \right\} a, \]

\[ \Phi'' + 3\frac{a'}{a} \Phi' - \partial_\Phi V = 0. \]  

(7)

The functions \(a\) and \(\Phi\) must obey the boundary conditions [41]. We see that in the \(\sigma \to +\infty\) limit we recover the standard general-relativistic equation for the scale parameter \(a\) [44], the equation for \(\Phi''\) remains unchanged with respect to the Einstein’s relativity theory. If we assume that \((\bar{a}, \bar{\Phi})\) is a
CdL instanton with $\Phi(0) = \Phi_i$ and $\Phi(\tau_f) = \Phi_f$, then we can write for $\tau \to 0^+$:

$$\ddot{\Phi} + \frac{3}{\tau} \dot{\Phi} - \partial_\Phi V(\Phi_i) = 0 \Rightarrow \dot{\Phi} = \Phi_i + \frac{\partial_\Phi V(\Phi_i)}{8} \tau^2,$$  \hspace{1cm} (8)

and for $\tau \to \tau_f$:

$$\ddot{\Phi} + \frac{3}{\tau} \dot{\Phi} - \partial_\Phi V(\Phi_f) = 0 \Rightarrow \dot{\Phi} = \Phi_f + \frac{\partial_\Phi V(\Phi_f)}{8} (\tau_f - \tau)^2. \hspace{1cm} (9)$$

Under the assumption that $a$ is a concave function (surely guaranteed by the positivity of the term proportional to $1/\sigma$ in the equation for $a''$) we deduce from eqs. (8) and (9) that the CdL instanton (in $\Phi$-direction) must cross the value $\Phi_M$ once at least. The asymptotic of the non-instanton solution (we can say - solutions with "randomly" chosen initial value of $\Phi$) of the system of equation (7) can be found in the same way as in Einstein’s general relativity.

We are interested in solutions which are close to the so-called Hawking-Moss (HM) instanton \[21\] that describes the false vacuum tunneling as a process in which the inflaton “jumps” (within a horizon-size domain) at the top ($\Phi_M$) of the potential $V$. The HM instanton is the $O(5)$-symmetric (constant $\Phi$) solution of the system (7):

$$\Phi = \Phi_M, \quad a = \hat{H}_M^{-1} \sin \left( \hat{H}_M \tau \right) , \hspace{1cm} (10)$$

where $\hat{H}_M$ is a modification of the Hubble parameter $H_M$ introduced in the previous section. $\hat{H}_M$ is determined inserting the proposed solution into the first equation of (7). One easily obtains

$$\hat{H}_M^2 = \frac{8\pi}{3} \left( V_M + \frac{V_M^2}{2\sigma} \right) = H_M^2 \left( 1 + \frac{V_M}{2\sigma} \right).$$

We can study the CdL instantons close to this HM instanton in the following way. We insert the expression for $a$ from (10) into the equation for $\Phi''$, linearize the term $\partial_\Phi V$ and using new variables: $x = \hat{H}_M \tau$ and $y = \Phi - \Phi_M$ we obtain

$$\frac{d^2 y}{dx^2} + 3 \cot(x) \frac{dy}{dx} - \frac{V_M''}{\hat{H}_M^2} y = 0 ,$$

or transforming the independently variable $x$ to: $z = \cos(\hat{H}_M \tau)$ we get the standard hypergeometric equation:

$$\left( 1 - z^2 \right) \frac{d^2 y}{dz^2} - 4z \frac{dy}{dz} - \frac{V_M''}{\hat{H}_M^2} y = 0 .$$

The boundary conditions (4) restrict possible values of the parameter $\frac{V_M''}{\hat{H}_M^2}$ to the eigenvalues of the Laplace-Beltrami operator on $S^4$:

$$-\frac{V_M''}{\hat{H}_M^2} = l(l+3), \quad l \in \{0, 1, 2, \ldots \} \hspace{1cm} (11)$$
and the solutions $y = y_l$ read for odd $l$:

$$y_l = c_l z _ 2 {}_2 F_1 \left( \frac{1-l}{2}, 2 + \frac{l}{2}, \frac{3}{2}, z^2 \right)$$  \hspace{1cm} (12)$$

and for even $l$:

$$y_l = c_l _ 2 {}_2 F_1 \left( \frac{3+l}{2}, -\frac{l}{2}, \frac{1}{2}, z^2 \right),$$  \hspace{1cm} (13)$$

where $c_l$ are arbitrary constants and $2 F_1$ stands for nondegenerate hypergeometric function. (In fact, the hypergeometric functions with special arguments according \[12\], \[13\] reduce to the Gegenbauer polynomials in the variable $z$. However, we will not need this explicitly.) The function $y_0$ correspond to the HM instanton and the functions $y_l$ approximate the $l$th order CdL instanton in its $\Phi$-direction. The restriction \[11\] is formally the same as in the case of four dimensional space-time with $H_M$ changed to $\hat{H}_M$. This change means that we have a new parameter (except the old one $-V_M''/H_M^2$) which value is crucial for the existence of the CdL instanton, namely $V_M/\sigma$. It is obvious that for $V_M/\sigma \ll 1$ the theory of vacuum decay on our brane reduces to the theory of vacuum decay in four-dimensional space-time. The first-order CdL instanton plays the most important role in the Einstein’s theory of gravity \[16\], \[18\], therefore we write down explicitly:

$$y_1 = kz = k \cos(x),$$  \hspace{1cm} (14)$$

with $k$ - the amplitude of the inflaton during its Euclidean evolution. In the next we will be interested in the first-order CdL instanton only.

## 4 The first order CdL instanton - perturbative approach

The idea of our analysis of the system of equations \[7\] is to expand all the relevant quantities entering these equations (and the boundary conditions \[11\]) into the powers of the $\Phi$ amplitude $k$, see \[14\]. This means explicitly that the following formulas are of our interest:

$$y(x) = \sum_{n=0}^{\infty} k^n u_n(x), \quad a(x) = \hat{H}_M^{-1} \sum_{k=0}^{\infty} k^n v_n(x),$$

$$-\frac{V_M''}{H_M^2} = 4 + \sum_{n=1}^{\infty} k^n \Delta_n,$$

together with the Taylor expansion of the potential $V$ around its local maximum. It can be useful to write down the form of the system of linear equations by which we replace equations \[7\]:

$$u''_n + 3 \cot(x) u'_n + 4 u_n = U_n, \quad v''_n + v_n = V_n \sin(x),$$
where the source-terms $U_n$ and $V_n$ are to be computed order-by-order. We know, from the previous section, that

$$u_0 = 0, \quad v_0(x) = \sin(x), \quad u_1(x) = \cos(x).$$

The boundary conditions imposed on $a$ and $\Phi$ require that for all $n \geq 1$ we have $v(0) = v'(0) = 0$. The right-end point $x_f$ at which the derivative of $y$ has to vanish is determined by $a(x_f) = 0$, therefore we have also the expansion of this quantity:

$$x_f = \pi + \sum_{n=1}^{\infty} k^n x^{(n)}_f.$$  \hspace{1cm} (15)

The fact that the potential $V$ is supposed to have the local maximum at $\Phi_M$ implies that $v_1 = 0$ and $x^{(1)}_f = 0$. The contribution of the second order in $k$ to the scale factor $a$ is nonzero, and reads explicitly

$$v_2 = \frac{1}{32} \left\{ \left[ C + \frac{15 - 9C}{4\sigma} V_M \right] 4x \cos(x) + \left[ 5C + \frac{51C - 45}{4\sigma} V_M \right] \sin(x) - \left[ 3C + 5(1 + C) \frac{V_M}{4\sigma} \right] \sin(3x) \right\}. \hspace{1cm} (15)$$

Having this results we come at the shift of the right-end point $x_f$

$$x^{(2)}_f = -\frac{\pi}{8} \left[ C + \frac{3V_M}{4\sigma} (5 - 3C) \right]. \hspace{1cm} (16)$$

In the limit that $\sigma \to \infty$ this quantity is negative but for a finite value $V_M/\sigma$ it can be both negative or positive and it vanishes at $V_M/\sigma \approx 0.555$. Careful and a little bit tedious computation shows that $\Delta_1 = 0$ and that $u_2$ obeys equation

$$u''_2 + 3 \cot(x) u'_2 + 4u_2 = \frac{1}{2} H^2 M^2 \frac{V'''}{H^2}.$$

The right-end point is still $\pi$, i.e. we seek for the solution(s) for which $u'_2(0) = u'_2(\pi) = 0$. This determines $u_2$ as follows

$$u_2 = \frac{1}{24} \frac{V'''}{H^2 M} \left[ 1 - 2 \cos^2(x) \right]. \hspace{1cm} (17)$$

We continue with time-consuming computations without any extra-idea and derive the equation for $v_3$ we do not write down. However, the explicit formula for $v_3$ is

$$v_3 = -\frac{V'''}{288 H^2 M} \left\{ 2C \left[ -2 \sin(2x) + \sin(4x) \right] + \frac{V_M}{\sigma} \left[ -16(C - 1) \sin(x) + 2(3C - 5) \sin(2x) + (C + 1) \sin(4x) \right] \right\}. \hspace{1cm} (18)$$
We have divided the expression for $v_3$ into two parts: the first one does not contain the brane tension $\sigma$ and represents the contribution coming from the Einstein’s general relativity and the second one is connected with the brane-tension containing terms of the action (5). Finally, it remains to find the function $u_3$ to determine the amplitude of $\Phi$. First of all we should realize that the shift of the right-end point (16) as well as the term $k^2\Delta_2$ from the expansion of $-V''_M/\tilde{H}_M^2$ enter the equation for $u_3$. This fact allows for determining a relation between $k$ and $-V''''_M/\tilde{H}_M^2$ as it is shown bellow. To keep the range of the argument of function $u_3$ equal to $[0, \pi]$ we pass from the independent variable $x$ to $w = (1 - k^2x_f^{(2)}/\pi)x$. For the simplicity we introduce the notation

$E = C + \frac{15 - 9C}{4\sigma}V_M$,  \quad F = 5C + \frac{51C - 45}{4\sigma}V_M,$

$G = 3C + 5(1 + C)\frac{V_M}{4\sigma}.$

Within this notation we can derive straightforwardly differential equation for $u_3$:

$$u''_3 + 3\cot(w)u'_3 + 4u_3 = A\cos(w) + B\cos^3(w), \quad (19)$$

where

$$A = \frac{5}{8}C + \frac{3}{8}E + \frac{3}{4}G + \frac{1}{24}(\frac{V''_M}{\tilde{H}_M^2})^2 - \Delta_2,$$

$$B = -\frac{3}{4}G - \frac{1}{12}(\frac{V''_M}{\tilde{H}_M^2})^2 + \frac{1}{6}(\frac{V'''_M}{\tilde{H}_M^2}).$$

The solution is: $u_3 = \beta\cos^3(w)$, where the coefficient $\beta$ has to satisfy two conditions:

$$6\beta = A, \quad -14\beta = B.$$

However, the fixation of $\beta$ is, at the moment, only supplementary for us because the coefficient $A$ contains also $\Delta_2$. Eliminating $\beta$ from the previous system of equations we obtain the expression for $\Delta_2$ (we do not write down) and subsequently we get $k^2$ as the function of $-4 - V''''_M/\tilde{H}_M^2$:

$$k^2 = -7 \left(4 + \frac{V''_M}{\tilde{H}_M^2}\right) \left\{16C + \frac{1}{24}(\frac{V''_M}{\tilde{H}_M^2})^2 + \frac{1}{32}\frac{V_M}{\sigma}(435 - 69C)\right\}^{-1}. \quad (20)$$

Assuming $4 + V''_M/\tilde{H}_M^2 < 0$ we need positive sign of the denominator in eq. (20). This sign can be changed, for given values of $V_M$ and $\sigma$, only due to $V'''_M/\tilde{H}_M^2$. We introduce the critical value of $V'''_M/\tilde{H}_M^2$

$$\zeta_c = \frac{1}{16}\frac{V_M}{\sigma}(69C - 435) - 32C - \frac{1}{24}(\frac{V'''_M}{\tilde{H}_M^2})^2. \quad (21)$$

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at which the mentioned denominator vanishes, and we have the first-order CdL instanton with inflaton amplitude given by the formula (20) in the two cases:

- for \( V_{M}''''/\dot{H}_{M}^{2} > \zeta_{c} \) as \( 4 + V_{M}''/\dot{H}_{M}^{2} \to 0^{−} \),
- for \( V_{M}''''/\dot{H}_{M}^{2} < \zeta_{c} \) as \( 4 + V_{M}''/\dot{H}_{M}^{2} \to 0^{+} \).

The first term in (21) is positive (if \( V_{M} \) and \( \sigma \) are positive). This causes the difference with respect to the situation when vacuum decays in four dimensional space-time within Einstein’s general relativity because now \( V_{M}''''/\dot{H}_{M}^{2} \) can be both positive and less than \( \zeta_{c} \) (In the first paper of refs. [19] it is argued that the negative value of \( V_{M}'''' \) is not like). Let us mention that the situation when \( −V_{M}''/\dot{H}_{M}^{2} > 4 \) and \( V_{M}''''/\dot{H}_{M}^{2} < \zeta_{c} \) does not mean automatically a kind of stabilization of the false vacuum because we still can have some CdL instanton with large amplitude in \( \Phi \) that is not included in our previous analysis and moreover the false vacuum can decay also via HM instanton. However, a consideration of such an instanton would involve some class of non-perturbative analysis of the system (7) or the numerical analysis, if we have a concrete potential \( V \) and a brane tension \( \sigma \).

5 The action of the first-order CdL instanton

The crucial quantity that tells us how probable is the vacuum decay via the CdL (or HM) instanton is its action. Our task is to find the approximative formula for the action of the first-order CdL instanton we investigated in previous section. In [12] the authors have also considered the action of the CdL instanton. They were interested in the action within the thin-wall approximation that correspond with their example of the instanton. Following [12] we can write the action

\[
S = 2\pi^{2} \int_{0}^{\tau_{f}} d\tau \left[ a^{3} \left( \frac{1}{2}(\Phi')^{2} + V \right) + \frac{a^{3}}{2\sigma} \left( \frac{1}{2}(\Phi')^{2} + V \right) \right] + \frac{1}{C} \left( a^{2}a'' + a(a')^{2} - a \right).
\]

Using the equation of motion (7) we rewrite the action into a much simpler form

\[
S = -\frac{4\pi^{2}}{3C} \int_{0}^{\tau_{f}} a d\tau - \frac{\pi^{2}}{\sigma} \int_{0}^{\tau_{f}} a^{3}(\Phi')^{4} d\tau = \left( 22 \right) \quad (22)
\]

\[
-\frac{4\pi^{2}}{3C} S^{(I)} - \frac{\pi^{2}}{\sigma} S^{(II)}. \quad (23)
\]

The structure of the \( S^{(I)} \) term is the same as in the case of Einstein’s relativity theory. To avoid a confusion we must mention that brane-tension
is included in this term throughout $a$. We can easily find the action of the
HM instanton

$$S_{HM} = -\frac{\pi}{H^2_M} = -\frac{3}{8V_M} \frac{V_M}{1 + \frac{V_M}{2\sigma}}.$$ (24)

We see that the action of the HM instanton in our brane world model is for
fixed $V_M$ and any given (positive) $\sigma$ less than the action of corresponding
HM instanton in Einstein’s relativity theory. Now we can compute the
action of the first-order CdL instanton. Up to the second order in $k$ one has

$$S^{(I)} = \hat{H}_M^{-2} \left\{ \int_0^{\pi + k^2 x_f^{(2)}} v_0 dx + k^2 \int_0^\pi v_2 dx \right\} =$$

$$\hat{H}_M^{-2} \left[ 2 + k^2 \frac{4C - 5V_M}{3\sigma} \right]$$

and $S^{(II)}$ does not contribute because it is of the order $k^4$ at most. Putting
these results together we obtain the difference between the actions of our
first-order CdL instanton and related HM instanton

$$S_{CdL} - S_{HM} = -\frac{4\pi^2}{3C} \frac{4C - 5V_M}{3H_M^2} \frac{1}{\sigma} k^2.$$ (25)

First of all: this difference vanishes as $\sigma \to \infty$ as it must be because in
Einstein’s relativity theory the difference between the actions of our CdL
instanton and related HM instanton is of the fourth-order in $k$ as it is shown in [18]. For positive $k^2$ the difference (25) is negative (this coincides
with the general-relativistic result [18]), this means that the vacuum decay
via our CdL instanton is more probable than the decay via HM instanton.
The fact that the difference of the actions (25) is of the $k^2$ order unlike
the general-relativistic case where it is of the order $k^4$ means that the
false vacuum decay rate in our brane world model is higher than for the
conventional gravity. The same is shown for the false vacuum decay rate in
another extremal case when the thin-wall approximation can be used [13].

6 Summary

We have compared, from the point of view of the semiclassical description
of the false vacuum decay, the systems of differential equations (3) and (7)
defining the CdL instantons in the standard Einstein’s relativity theory
and in the brane world model. We have been interested in the situation
when the effective curvature of the potential at its top is close to the cri-
tical value 4. In this situation we were able to obtain explicit perturbative
formulas for the instanton, and mainly for its actions that is the relevant
quantity describing the instanton as ”mediator” of the vacuum decay. This
allows for comparing our CdL instanton with always existing Hawking -
Moss instanton. We have concluded that it is the first order CdL instanton
that is preferred (to the corresponding Hawking - Moss instanton) in the considered region of parameters of the potential. A kind of lower symmetry of the instanton equations with respect to the situation in general relativity causes that the difference between actions of mentioned instantons is proportional to the square of inflaton amplitude in the instanton rather than to the fourth power of that amplitude. Physically, this difference of the actions is inversely proportional to the string tension ($V_M/\sigma$ is the parameter controlling the string tension influence) and therefore in the limit $V_M/\sigma \to 0$ we recover the results of the general relativity.

Acknowledgement

This work was supported by the Slovak Scientific and Educational Grant Agency, project no. 1/0250/03.

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