Theory of shot noise in space-charge limited diffusive conduction regime

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As is well known, the fluctuations from a stable stationary nonequilibrium state are described by a linearized nonhomogeneous Boltzmann-Langevin equation. The stationary state itself may be described by a nonlinear Boltzmann equation. The ways of its linearization sometimes seem to be not unique. We argue that there is actually a unique way to obtain a linear equation for the fluctuations. In the present paper we treat as an example an analytical theory of nonequilibrium shot noise in a diffusive conductor under the space charge limited regime. Our approach is compared with that of Schomerus, Mishchenko and Beenakker [Phys. Rev. B 60, 5839 (1999)]. We find some difference between the present theory and the approach of their paper and discuss a possible origin of the difference. We believe that it is related to the fundamentals of the theory of fluctuation phenomena in a nonequilibrium electron gas.

I. INTRODUCTION

The present paper is devoted to the theory of shot noise in space-charge limited diffusive conduction regime. The motivation of the present paper can be formulated as follows. It is well known that the fluctuations from a stable stationary nonequilibrium state are described by a linearized nonhomogeneous Boltzmann-Langevin equation (see, for instance, [1–7]). At the same time, the stationary state itself is described by a nonlinear Boltzmann equation. There are instances where the ways of linearization of the nonlinear Boltzmann equation seem to be not unique. We believe, however, that in each such case there is a unique way to obtain the linearized Boltzmann equation for the fluctuations and we give general considerations to find out this way and indicate it for the particular case treated in the present paper.

We work out a theory of nonequilibrium shot noise in a nondegenerate diffusive conductor under space charge limited regime. This regime is extensively discussed in the literature (see, for instance, Refs. [8,9]). The current noise under such a regime has been studied recently by Monte Carlo simulation by González et al. [10]. Quite recently the noise under the same conditions has been studied analytically by Schomerus, Mishchenko...
and Beenakker [11]. Their general finding was that due to the Coulomb correlation between the electrons the shot noise is reduced below the classical Poisson value. Both the authors of Ref. [10] and of Ref. [11] came to the conclusion that under certain conditions the suppression factor in the nondegenerate 3D case can be close to \(1/3\).

Later on Nagaev [12] has shown on a special example that unlike the \(1/3\) noise reduction in degenerate systems, the noise suppression by the Coulomb interaction is nonuniversal in nondegenerate systems. The noise suppression in such systems may depend on the details of electron scattering.

We agree with the conclusion [10,11] that the reduction of the shot noise power in nondegenerate diffusive conductors can be sometimes close to the value \(1/3\) predicted theoretically for a three-dimensional (3D) degenerate electron gas. As we have mentioned above, we also come to some conclusions that may prove important for the general theory of fluctuations in nonequilibrium systems. As is well known, the fluctuation phenomena in nonequilibrium stable systems are described by a linearized Boltzmann equation. We would like to use the example analyzed in detail in the present paper to show that one should be careful performing the linearization. In particular, there is a difference between the analytical procedures used in Ref. [11] and in the present paper for calculation of the shot noise power. We discuss the origin of such difference and its implications. As the point leading to the discrepancy is very subtle it demands a rather detailed analysis that we perform in the present paper partly repeating the calculations of Ref. [11] with some modifications. Our starting point is the Boltzmann equation that is formulated for description of the stationary state and will be applied for analysis of the fluctuations.

II. BOLTZMANN EQUATIONS

We will consider the simplest model, exploited in Ref. [11], for the diffusion-controlled and space charge limited transport. As the starting point we use the Boltzmann equation in the presence of electric field

\[
\left( \frac{\partial}{\partial t} + \mathcal{J}_p \right) f_p = 0, \tag{2.1}
\]

\[
\mathcal{J}_p f_p \equiv \left( v \frac{\partial}{\partial x} + eE \frac{\partial}{\partial p} + I_p \right) f_p. \tag{2.2}
\]

Here we have introduced the collision integral \(I_p\) describing the electron scattering:

\[
I_p f_p = \sum_{p'} \left( W_{pp'} f_{p'} - W_{pp} f_p \right), \tag{2.3}
\]
(we deal with the nondegenerate statistics, so that $f_p \ll 1$).

Splitting the distribution function into the even and odd parts with respect to $p$ one gets
$$f_p^\pm = \frac{1}{2} (f_p \pm f_{-p}).$$

We assume that the collision operator acting on the even (odd) part of the distribution function gives an even (odd) function. This may be due either to the central symmetry of the crystal itself and the scatterers or to the possibility to use the Born approximation for calculation of the scattering probability. The first split equation is
$$\frac{\partial f_p^-}{\partial t} + \mathbf{v} \frac{\partial f_p^+}{\partial \mathbf{r}} + eE \frac{\partial f_p^+}{\partial \mathbf{p}} = -I_p f_p^-.$$  \hspace{1cm} (2.4)

Being interested in relatively small frequencies of fluctuations $\omega \tau_p \ll 1$ where $\tau_p$ is the characteristic value of $I_p^{-1}$ we can neglect the time derivative and express $f_p^-$ as
$$f_p^- = -I_p^{-1} \left( \mathbf{v} \frac{\partial f_p^+}{\partial \mathbf{r}} + eE \frac{\partial f_p^+}{\partial \mathbf{p}} \right).$$ \hspace{1cm} (2.5)

Inserting this expression into the second split equation for $f_p^+ \simeq f(\varepsilon, \mathbf{r}, t)$ and averaging over the surface of constant energy in the quasimomentum space we arrive at
$$\nu(\varepsilon) \frac{\partial f^+}{\partial \varepsilon} - \left( \frac{\partial}{\partial \varepsilon} + eE_\alpha \frac{\partial}{\partial \varepsilon} \right) \nu(\varepsilon) D_{\alpha\beta}(\varepsilon) \left( \frac{\partial}{\partial \varepsilon_{\beta}} + eE_{\beta} \frac{\partial}{\partial \varepsilon} \right) f = -\sum_p \delta(\varepsilon - \varepsilon_p) I_p^{(inel)} f$$ \hspace{1cm} (2.6)

where the term on the right-hand side describes the inelastic collisions while the density of states $\nu(\varepsilon)$ and the diffusion tensor $D_{\alpha\beta}(\varepsilon)$ are defined as
$$\nu(\varepsilon) D_{\alpha\beta}(\varepsilon) = \sum_p \delta(\varepsilon - \varepsilon_p) v_\alpha I_p^{-1} v_\beta, \hspace{0.5cm} \nu(\varepsilon) = \sum_p \delta(\varepsilon - \varepsilon_p). \hspace{1cm} (2.7)$$

The electric field obeys the Poisson equation
$$\kappa \nabla \cdot \mathbf{E} = 4\pi e \left[ n(\mathbf{r}, t) - n^{eq} \right], \hspace{0.5cm} n(\mathbf{r}, t) = \int_0^\infty d\varepsilon \nu(\varepsilon) f(\varepsilon, \mathbf{r}, t),$$ \hspace{1cm} (2.8)

where $\kappa$ is the dielectric susceptibility, $n^{eq}$ is the equilibrium concentration (equal to the concentration of donors). In what follows we neglect it as compared with the nonequilibrium concentration $n$.

The part of the distribution function contributing to the current consists of two terms proportional to the spatial and energy derivatives of $f(\varepsilon, \mathbf{r}, t)$ respectively
$$j_\alpha = e \sum_p \mathbf{v} f_p^- = -e \nu(\varepsilon) D_{\alpha\beta}(\varepsilon) \left( \frac{\partial}{\partial \varepsilon_{\beta}} + eE_{\beta} \frac{\partial}{\partial \varepsilon} \right) f.$$ \hspace{1cm} (2.9)
Let us consider the case \( D\tau_\varepsilon \gg L^2 \), where \( L \) is the sample length, \( \tau_\varepsilon \) is the energy relaxation time (of the order of \( [I_p^{(inel)}]^{-1} \)). Then we can omit the term on the right-hand side of Eq. (2.6) which describes the energy relaxation. Under the same conditions we get the following Boltzmann equation for the fluctuations of the distribution function (we remind that we consider here low frequency fluctuations with \( \omega \ll I_p \approx 1/\tau_p \) where \( \tau_p \) is the characteristic time of elastic collisions)

\[
\left( \frac{\partial}{\partial x_\alpha} + eE_\alpha \frac{\partial}{\partial \varepsilon} \right) \delta j^\alpha_\omega + e\delta E^\alpha_\omega \frac{\partial}{\partial \varepsilon} j_\alpha = ey_\omega(\varepsilon, x) \tag{2.10}
\]

where

\[
\delta j^\alpha_\omega = e \sum_p v_\alpha \delta f^\alpha_p - e\nu(\varepsilon)D_{\alpha\beta}(\varepsilon) \left( \left[ \frac{\partial}{\partial x_\beta} + eE_\beta \frac{\partial}{\partial \varepsilon} \right] \delta f_\omega + e\delta E^\beta_\omega \frac{\partial}{\partial \varepsilon} f_\omega \right) \tag{2.11}
\]

and where the source of the current fluctuations \( g^\alpha_\omega \) is related to the Langevin forces \( y^\omega_p \) as

\[
g^\alpha_\omega = e \sum_p \delta(\varepsilon - \varepsilon_p)v_\alpha I_p^{-1}y^\omega_p, \tag{2.12}
\]

\[
y_\omega(\varepsilon, x) = \sum_p \delta(\varepsilon - \varepsilon_p)y^\omega_p = 0. \tag{2.13}
\]

The last equality is a consequence of elasticity of scattering that leads to the particle conservation within the surface of constant energy in the quasimomentum space.

The correlation function of the Langevin forces is well known [7]

\[
<y_p(r)y_{p'}(r')>_\omega = (J_p + J_{p'})\delta_{rr'}\delta_{pp'}f_p. \tag{2.14}
\]

Integrating Eq.(2.10) over \( \varepsilon \) we obtain the continuity equation

\[
A \frac{d}{dx} \int_0^\infty \varepsilon \delta j_\omega(\varepsilon, x) = \frac{d}{dx} \delta J_\omega(x) = 0, \tag{2.15}
\]

which implies that the low frequency current fluctuations are spatially homogeneous.

III. DISTRIBUTION FUNCTION.

Consider a semiconductor with a uniform cross section \( A \) connecting two identical metallic electrodes. The sample’s length \( L \) is assumed to be much bigger than the elastic scattering length \( l \) and much smaller than the inelastic one. We use the 1D versions of the Boltzmann equations describing evolution of the distribution function along the dc current direction.
To obtain the stationary solution of Eq. (2.6) in the accepted approximation we rewrite it in the form
\[
\left( \frac{\partial}{\partial x} + eE \frac{\partial}{\partial \varepsilon} \right) j(\varepsilon, x) = \delta(x)j(\varepsilon). \tag{3.1}
\]
Here we assume that the current density at \( x = 0 \), \( j(\varepsilon) \), does not vanish only for \( \varepsilon > 0 \). In the absence of tunneling \( j(\varepsilon) \) at the contact \( x = 0 \) should have the property
\[
j(\varepsilon) \to 0 \quad \text{for} \quad T \to 0, \tag{3.2}
\]
\( T \) being the temperature. This condition should be valid, irrespective to whether one has a Schottky barrier or an Ohmic contact. The total current \( J \) given by Eq. (3.3) below should have of course the same property.

The solution of Eq. (3.1) is a function of the total energy \( \mathcal{E} \)
\[
\mathcal{E} = \varepsilon + U(x),
\]
where \( U(x) = e\varphi(x) - e\varphi(0) \). It can be found employing, for instance, the inverse differential operator
\[
\frac{1}{\partial_x} \Phi(x) = \int_0^x d\xi \Phi(\xi).
\]
One has
\[
j(\varepsilon, x) = \frac{1}{\partial_x + eE(x)\partial_\varepsilon} \delta(x)j(\varepsilon) = \exp[e\varphi(x)\partial_\varepsilon] \frac{1}{\partial_x} \exp[-e\varphi(x)\partial_\varepsilon] \delta(x)j(\varepsilon) = j(\mathcal{E})
\]
and \( j(\varepsilon, x) \) has at a given \( x \) nonzero values only if \( \varepsilon > -U(x) \), \( (\mathcal{E} \geq 0) \). The total current through the sample is
\[
J = A \int_0^\infty d\varepsilon \, j(\varepsilon, x) = A \int_{-U(x)}^\infty d\varepsilon \, j[\varepsilon + U(x)] = A \int_0^\infty d\mathcal{E} \, j(\mathcal{E}). \tag{3.3}
\]
Now, from Eq. (2.9) we get
\[
f(\varepsilon, x) = -\frac{1}{\partial_x + eE(x)\partial_\varepsilon} \frac{j(\varepsilon, x)}{e\lambda(\varepsilon)} + f[\varepsilon + U(x)], \tag{3.4}
\]
or
\[
f(\varepsilon, x) = -j[\mathcal{E}] \int_0^x d\xi \frac{1}{e\lambda(\mathcal{E} - U(\xi))} + f[\mathcal{E}] \tag{3.5}
\]
where \( \lambda(\varepsilon) \equiv \nu(\varepsilon)D(\varepsilon) \). We have taken into account the boundary condition at the source. Equation (3.3) can be rewritten as
\[ f[\mathcal{E} - U(x), x] = \frac{\int_0^x \frac{d\xi}{\lambda}[\mathcal{E} - U(\xi)] + f(\mathcal{E}) \int_x^L \frac{d\xi}{\lambda}[\mathcal{E} - U(\xi)]}{\int_0^L \frac{d\xi}{\lambda}[\mathcal{E} - U(\xi)]} \]  \tag{3.6}

where \( j(\varepsilon) \) is expressed through the difference of the distribution function at \( x = 0 \) and \( x = L \)

\[ j(\mathcal{E}) \int_0^L \frac{dx}{e\lambda[\mathcal{E} - U(x)]} = f(\mathcal{E}) - f[\mathcal{E} - U(L)]. \]  \tag{3.7}

An advantage of the form we have chosen for Eq. (3.6) is its physical transparency. The first term on the right-hand side gives the contribution from the right boundary while the second one gives the contribution from the left boundary. The solution clearly demonstrates that the thermally exited carriers injected from the contact at \( x = L \) make negligible contribution to the distribution function \( f[\mathcal{E} - U(x), x] \) since \( f(\mathcal{E}) \gg f[\mathcal{E} - U(L)] \) (\( \mathcal{E} \geq 0 \)), for the parameter \( |U(L)|/k_B T \) is assumed to be large. Neglecting this term in our solution of Eq. (3.6) we come to the solution already obtained in [11] by assuming absorbing boundary conditions at the current drain.

**IV. FIELD DISTRIBUTION**

We use the Poisson equation to determine the selfconsistent electric field that can be expressed through the obtained distribution function. We consider such values of \( x \) that \( x > x_\tau \), where

\[ -U(x_\tau) \gg \mathcal{E} \sim k_B T \]

\[ -\frac{\kappa}{4\pi e^2} \frac{d^2U}{dx^2} = \int_0^\infty d\mathcal{E} \nu[\mathcal{E} - U(x)] \]

\[ \times f[\mathcal{E} - U(x), x] = \int_0^\infty d\mathcal{E} \nu[\mathcal{E} - U(x)] j(\mathcal{E}) \int_x^L \frac{d\xi}{e\lambda[\mathcal{E} - U(\xi)]} \]

\[ \simeq \nu[-U(x)] \frac{J}{eA} \int_x^L \frac{d\xi}{\lambda[-U(\xi)]} \]  \tag{4.1}

Finally we get

\[ -\frac{\kappa}{4\pi e^2} \frac{1}{\nu[-U(x)]} \frac{d^2U}{dx^2} = \frac{J}{eA} \int_x^L \frac{d\xi}{\lambda[-U(\xi)]} \]  \tag{4.2}

Let us check that for large \( x \) this equation is consistent with the requirement of a uniform total current. Assuming \( \nu(\varepsilon) = \nu_0 \varepsilon^{d/2 - 1} \) and \( D(\varepsilon) = D_0 \varepsilon^{s+1} \) we integrate Eq. (2.9) over the transverse coordinates and energy.
\[
\frac{J}{A} = -e \frac{d}{dx} \int_0^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x) + eD_0\kappa(d+2s) \frac{16\pi}{[\varepsilon-U(x)]^s} \frac{d}{dx} E^2(x)
\]

(4.3)

We integrate by parts the second term and take into account that at \( x > \varepsilon \) we can neglect \( \mathcal{E} \) as compared to \( |U(x)| \) and use the Poisson equation (2.8). The first term in Eq. (4.3) can be simplified in the same way [note that due to Eq. (3.5) the distribution function \( f(\varepsilon, x) \) has nonzero values only for \( \varepsilon > -U(x) \) ]

\[
\int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x) = \int_0^\infty d\varepsilon \nu(\varepsilon - U(x)) D(\varepsilon - U(x)) f(\varepsilon - U(x), x)
\]

\[
= D[-U(x)] \int_0^\infty d\varepsilon \nu(\varepsilon - U(x)) f(\varepsilon - U(x), x) = D[-U(x)] \frac{\kappa}{4\pi \varepsilon d} E
\]

(4.4)

Here in the second equality we have taken into account that \( \varepsilon \ll |U(x)| \). Inserting Eq. (4.4) into Eq. (4.3) we get the following simplified equation

\[
\frac{4\pi|J|}{D_0\kappa A} = \frac{d}{dx} \left[ -U^{s+1} \frac{dE}{dx} \right] + |e| \frac{2s + d}{4} \frac{dE^2}{dx}
\]

(4.5)

It can be used to verify the self-consistency of our approach. Indeed, multiplying Eq. (4.2) by \( U^{s+d/2} \) and taking the derivative we arrive at Eq. (4.3) that has been obtained from the equation for the current. A dimensionless version of Eq. (4.5) is

\[
\chi^s \left( \frac{d - 2}{2} \chi'' - \chi' \chi''' \right) = 1
\]

(4.6)

where the dimensionless potential \( \chi \) is related to \( \varphi \) by

\[
\varphi = \left( \frac{4\pi|J|L^3}{D_0\kappa A |e|^{s+1}} \right)^{1/(s+2)} \chi(x/L).
\]

(4.7)

V. CURRENT AND FIELD FLUCTUATIONS

In what follows we consider the particular cases \( s = 0, \quad D(\varepsilon) = D_0\varepsilon; \quad s = -1/2, \quad D(\varepsilon) = D_0\varepsilon^{1/2}; \quad \) and \( s = 1/2, \quad D(\varepsilon) = D_0\varepsilon^{3/2} \). We begin with investigation of the energy-independent-scattering-time case \( s = 0 \). This case can be related to the scattering of electrons by the neutral impurities, such as hydrogen-like shallow donor and acceptor states. The scattering is analogous to the scattering of electron by a hydrogen atom \(^{[13]}\) (with the effective Bohr radius \( a_B \)). The scattering cross-section turns out to be about \( 2\pi\hbar/(pa_B) \) times larger than the geometrical cross-section \( \pi a_B^2 \) (that would result in an energy-independent scattering time).

In the case of defects with deep energy levels we encounter a short-range scattering potential with the scattering length about atomic length. The scattering cross-section
does not depend on the energy. As a result, the scattering rate is proportional to the electron density of states $\varepsilon^{1/2}$ and the diffusion coefficient $v^2\tau$ is proportional to $\varepsilon^{1/2}$, i.e. $s = -1/2$. (This is one of the main scattering mechanisms in metals since the scattering length there is determined by the radius of screening which is of the order of interatomic distance.) The case $s = -1/2$ (that, in particular, describes elastic scattering by acoustic phonons) and $s = 1/2$ will be discussed at the end of this section.

A. Energy independent scattering time

Integrating Eq. (2.11) over $\varepsilon$, we get

$$\frac{1}{A}(\delta J_\omega - G_\omega) = -e \frac{d}{dx} \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) \delta f_\omega(\varepsilon, x) + \frac{eD_0d\kappa}{8\pi} \frac{d}{dx} E(x) \delta E_\omega(x)$$  (5.1)

Note that the Fourier transform of the current fluctuations $\delta J_\omega$ due to Eq.(2.10) is spatially homogeneous. Here $G_\omega$ is the current fluctuations source integrated over the energy and transverse coordinates

$$G_\omega(x) = \int_0^\infty d\varepsilon \int d\mathbf{r}_\perp g_\omega(\varepsilon, \mathbf{r})$$  (5.2)

$$<G(x)G(x')>_\omega = e^2 \int_0^\infty d\varepsilon \int_0^\infty d\varepsilon' \sum_{pp'} \delta(\varepsilon - \varepsilon_p) \delta(\varepsilon' - \varepsilon_{p'}) v_x v_{x'}$$

$$\times \frac{1}{|p|} \frac{1}{|p'|} \int d\mathbf{r}_\perp d\mathbf{r}'_\perp <y_p y_{p'}>_\omega$$  (5.3)

The odd (with respect to $p \rightarrow -p$) part of the distribution function vanishes after one inserts it into the correlation function (2.14) of the Langevin forces and subsequent integration over $p$ and $p'$. As a result, we are left with the integral of the even function

$$<G(x)G(x')>_\omega = \delta_{xx'} <G^2(x)>_\omega, \quad (5.4)$$

$$\langle G^2(x) \rangle_\omega = 2e^2 A \int_0^\infty d\varepsilon f(\varepsilon, x) \sum_p \delta(\varepsilon - \varepsilon_p) v_x \frac{1}{|p|} v_x = 2e^2 A \int_0^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x). \quad (5.5)$$

The second term on the right hand side of Eq. (5.1) can be simplified in the same way as Eq. (4.4)

$$\int_0^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) \delta f_\omega(\varepsilon, x) = D(-U(x)) \frac{\kappa}{4\pi e} \frac{d}{dx} \delta E_\omega$$  (5.6)

and finally we get the equation for $\delta E_\omega$
\[
\frac{d}{dx} \left( U(x) \frac{d}{dx} \delta E_\omega(x) \right) + e \frac{d}{dx} E(x) \delta E_\omega(x) = \frac{4\pi}{AD_0\kappa} (\delta J_\omega - G_\omega). \tag{5.7}
\]

In order to justify the simplification in Eq. (5.6) we will show that \(\delta f_\omega(\varepsilon, x)\) is also a function acquiring nonzero values only at \(\varepsilon > -U(x)\). Indeed, from Eq. (2.10) and Eq. (2.11) one can get the following solutions

\[\delta j_\omega[\varepsilon - U(x), x] = \delta U_\omega(x) \frac{\partial}{\partial \varepsilon} j(\varepsilon) + \Delta j(\varepsilon)_\omega, \tag{5.8}\]

\[\delta f_\omega[\varepsilon - U(x), x] = \int_x^L d\xi \frac{\partial}{\partial \varepsilon} f[\varepsilon - U(\xi), \xi] - \int_x^L d\xi \frac{\partial}{\partial \varepsilon} f_\omega[\varepsilon - U(\xi), \xi] - \delta j_\omega[\varepsilon - U(\xi), \xi] \tag{5.9}\]

which show that \(\delta f\) has the above mentioned property. Here \(\Delta j(\varepsilon)\) are the fluctuations of the current at the left boundary \(x = 0\). The fluctuations of the distribution function \(\Delta f(\varepsilon)\) at the right boundary are assumed to be zero. If we assume \(\lambda(\varepsilon)\) to be a constant (not depending on the energy), taking into account Eqs. (5.8) and (5.9) and the equation \(\delta f_\omega(\varepsilon, 0) = 0\) we immediately arrive at the result obtained by Nagaev

\[\Delta J = \frac{1}{L} \int_0^L dx \int d\varepsilon g[\varepsilon - U(x), x]. \tag{5.10}\]

### B. Comparison with approach of Ref. [4]

Now we embark on setting forth the crucial point of the paper. Eq. (5.7) does not coincide with the equation for the field fluctuations obtained in [11] by direct linearization of Eq. (4.3) for \(s = 0\)

\[\frac{d}{dx} \left[ \delta U_\omega(x) \frac{d}{dx} E(x) \right] + \frac{d}{dx} \left[ U(x) \frac{d}{dx} \delta E_\omega(x) \right] + e \frac{d}{dx} E(x) \delta E_\omega(x) = \frac{4\pi}{AD_0\kappa} (\delta J_\omega - G_\omega). \tag{5.11}\]

It is necessary to understand the origin of this discrepancy.

First, we adopt for the time being the scheme of Ref [11] and reconsider Eq. (4.3) for the current

\[\frac{J}{A} = -e \frac{d}{dx} \int_{-U(x)}^{\infty} d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x) + \frac{3}{2} D_0 e^2 E(x) \int_{-U(x)}^{\infty} d\varepsilon \nu(\varepsilon) f(\varepsilon, x). \tag{5.12}\]
For the total current (the d.c. current plus fluctuations) the equation reads

\[
\frac{J + \delta J - G}{A} = -e \frac{d}{dx} \int_{-U(x) - \delta U(x)}^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) [f(\varepsilon, x) + \delta f(\varepsilon, x)] + \frac{3}{2} D_0 e^2 [E(x) + \delta E(x)] \int_{-U(x) - \delta U(x)}^\infty d\varepsilon \nu(\varepsilon) [f(\varepsilon, x) + \delta f(\varepsilon, x)]
\]

(5.13)

Taking into account Eq. (5.12) we get the following linearized equation

\[
\frac{\delta J - G}{A} = -e \frac{d}{dx} \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) \delta f(\varepsilon, x) + \frac{3}{2} D_0 e^2 E(x) \left\{ \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) \delta f(\varepsilon, x) + \delta U \frac{\delta}{\delta U(x)} \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) f(\varepsilon, x) \right\} + \frac{3}{2} D_0 e^2 \delta E(x) \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) f(\varepsilon, x) - e \frac{d}{dx} \delta U \frac{\delta}{\delta U(x)} \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x).
\]

(5.14)

If one linearized the Poisson equation in the spirit of Ref. [11] one would see that the term in the curly brackets in Eq. (5.14) would coincide with \((\kappa/4\pi e)(d\delta E/dx)\), so that

\[
\frac{\kappa}{4\pi e} \frac{d\delta E}{dx} = \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) \delta f(\varepsilon, x) + \delta U \frac{\delta}{\delta U(x)} \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) f(\varepsilon, x).
\]

(5.15)

Simplifying the first, third and fourth terms on the right-hand side of Eq. (5.14) with the help of Eq. (5.6) and inserting instead of the term in the curly brackets \((\kappa/4\pi e)(d\delta E/dx)\) we arrive at

\[
\frac{\delta J - G}{A} = e \frac{d}{dx} \left( D_0 U(x) \frac{\kappa}{4\pi} \frac{d\delta E}{dx} \right) + \frac{3}{2} D_0 e \frac{\kappa}{4\pi} \frac{d}{dx} E \delta E + e D_0 \frac{\kappa}{4\pi} \frac{d}{dx} \delta U \left[ U \frac{dE}{dx} \right]
\]

(5.16)

One can see that the last term on the right-hand side of this equation coincides with the first term on the left-hand side of Eq. (5.11). To avoid confusion note that we believe Eq. (5.13) to be also wrong. We have written it here only for the sake of detailed comparison with the approach of Ref. [11]. We believe that the correct Poisson equation for the fluctuation field is

\[
\frac{\kappa}{4\pi e} \frac{d\delta E}{dx} = \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) \delta f(\varepsilon, x).
\]

(5.17)

Now we will add in Eq. (4.3) for the d.c. current the terms that actually vanish as they are proportional to the integrals of the distribution function over \(\varepsilon\) with the upper limit \(-U(x)\) whereas the distribution function \(f(\varepsilon, x) = 0\) for \(\varepsilon < -U(x)\). The point is that when we calculate the fluctuations by replacement \(U(x) \rightarrow U(x) + \delta U(x)\) they will give a nonvanishing result. We have

\[
\frac{J}{A} = -e \frac{d}{dx} \int_0^{-U(x)} d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x) - e \frac{d}{dx} \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x) + \frac{3}{2} D_0 e^2 E(x) \int_0^{-U(x)} d\varepsilon \nu(\varepsilon) f(\varepsilon, x) + \frac{3}{2} D_0 e^2 E(x) \int_{-U(x)}^\infty d\varepsilon \nu(\varepsilon) f(\varepsilon, x).
\]

(5.18)
Rewriting this equation for the total current we get

\[
\frac{J + \delta J - G}{A} = -e \frac{d}{dx} \int_{-U(x)-\delta U}^{U(x)} d\varepsilon \nu(\varepsilon) D(\varepsilon)[f(\varepsilon, x) + \delta f(\varepsilon, x)] \\
-\frac{e}{2} \int_{-U(x)-\delta U(x)}^{U(x)} d\varepsilon \nu(\varepsilon) D(\varepsilon)[f(\varepsilon, x) + \delta f(\varepsilon, x)] \\
+ \frac{3}{2} D_0 e^2 [E(x) + \delta E(x)] \int_{-U(x)-\delta U(x)}^{U(x)} d\varepsilon \nu(\varepsilon) [f(\varepsilon, x) + \delta f(\varepsilon, x)] \\
+ \frac{3}{2} D_0 e^2 [E(x) + \delta E(x)] \int_{-U(x)-\delta U(x)}^{U(x)} d\varepsilon \nu(\varepsilon) [f(\varepsilon, x) + \delta f(\varepsilon, x)].
\] (5.19)

Linearizing this equation and using relations like

\[
\frac{\delta U}{\delta U(x)} \int_{-U(x)}^{\infty} d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x) = - \int_{-U(x)}^{-U(x)-\delta U(x)} d\varepsilon \nu(\varepsilon) D(\varepsilon) f(\varepsilon, x)
\] (5.20)

we arrive at Eq. (5.1) that has been derived above. We see that the cancellation of the linear in \( \delta U \) contributions in Eq. (5.19) is due to the terms that vanish in the equation for the d.c. current but should be taken into account when one considers fluctuations. This is why the linearization of Eq. (4.5) leads to Eq. (5.11) that we believe to be wrong as it does not take into account all the sources of fluctuation or, in other words, all the terms in Eq. (5.18) containing \( U(x) \).

The solution of Eq. (5.7) with boundary conditions

\[
E(x)\delta E_\omega(x) \bigg|_{x \to 0} \rightarrow 0,
\]

\[
U(x) \frac{d}{dx} \delta E_\omega(x) \bigg|_{x \to 0} \rightarrow 0
\]

is

\[
- \frac{AD_0\kappa}{4\pi} \delta E_\omega(x) = U^{d/2}(x) \left[ C + \int_0^x \frac{d\xi}{U^{d/2+1}(\xi)} \int_0^\xi d\eta (\delta J_\omega - G(\eta)\omega) \right]
\] (5.22)

where \( C \) is an integration constant. Requiring a nonfluctuating applied voltage

\[
\int_0^L dx \delta E_\omega = 0
\]

we get from Eq.(5.22) that the constant is

\[
C = \int_0^L dx \left( \frac{\psi(x)}{\psi(L)} - 1 \right) \frac{1}{U^{d/2+1}(x)} \int_0^x d\xi (\delta J_\omega - G_\omega(\xi)),
\] (5.23)

where

\[
\psi(x) = \int_0^x d\xi U^{d/2}(\xi).
\] (5.24)

Now we require at the right boundary
\[
\frac{d}{dx} \delta E_\omega(x) \bigg|_{x = L} = 0 \quad (5.25)
\]
and get
\[
\delta J_\omega = \frac{1}{Z} \int_0^L dx \Pi(x) G_\omega(x) \quad (5.26)
\]
where
\[
Z = L + \frac{dU''(L)U^{d/2}(L)}{2\psi(L)} \int_0^L dx \frac{x\psi(x)}{U^{d/2+1}(x)}, \quad (5.27)
\]
\[
\Pi(x) = 1 + \frac{dU''(L)U^{d/2}(L)}{2\psi(L)} \int_x^L d\xi \frac{\psi(\xi)}{U^{d/2+1}(\xi)}. \quad (5.28)
\]
Then the noise power \( P \) is
\[
P = \frac{2}{Z^2} \int_0^L dx \Pi^2(x) < G^2(x) >_\omega. \quad (5.29)
\]
Since, according to Eq. (5.3)
\[
< G^2(x) >_\omega = 2e^2 A \int_0^\infty d\nu(\frac{\nu}{2}) D(\nu) f(\nu, x) = 2e^2 AD_0U(x) \frac{\kappa}{4\pi} \frac{d^2U}{dx^2}. \quad (5.30)
\]
Finally we arrive at
\[
P = \frac{4AD_0\kappa}{4\pi Z^2} \int_0^L dx \Pi^2(x)U(x) \frac{d^2U}{dx^2}. \quad (5.31)
\]
The potential distribution can be found following method of Ref. [11], i.e. solving Eq. (4.5) with boundary condition Eq. (4.2) at \( x = L \). Using Eqs. (5.24), (5.27), (5.28) and (5.31) we calculate the suppression factor \( P/P_{\text{Poisson}} \). For physically relevant different values of the dimensionality \( d \) we get
\[
P/P_{\text{Poisson}} = \begin{cases} 
0.3188 & \text{for } d = 3, \\
0.4512 & \text{for } d = 2, \\
0.682 & \text{for } d = 1.
\end{cases} \quad (5.32)
\]
Thus, in this particular case our results differ from those calculated in Refs. [11] both analytically (that, in our opinion, is of principal importance) and numerically (although in this particular case the difference is not great). Naturally, there is essentially no difference with the results calculated within an ensemble Monte Carlo scheme in Ref. [10].
C. Energy dependent scattering time

Here we calculate the noise power for $s = \pm 1/2$ and $d = 3$. The equation for the fluctuations is

$$-\frac{4\pi}{\kappa D_0 A} (\delta J_\omega - G_\omega) = \frac{d}{dx} \left[ (U)^{s+1} \frac{d \delta E_\omega}{dx} \right] - e \frac{2s + d}{2} (-U)^s \frac{d}{dx} (E \delta E_\omega).$$  \hfill (5.33)

Introducing the dimensionless potential $\chi$ given by Eq. (4.7) and the fluctuation of the field $\Delta E$

$$\delta E(x) = \frac{1}{L} \left( \frac{4\pi |J| L^3}{\kappa D_0 A |\epsilon|^{s+1}} \right)^{1/(s+2)} \Delta E \left( \frac{x}{L} \right),$$  \hfill (5.34)

one can rewrite Eq. (5.33) as

$$\Delta E'' + \left( 1 - \frac{d}{2} \right) \frac{\chi'}{\chi} \Delta E' - \left( s + \frac{d}{2} \right) \frac{\chi''}{\chi} \Delta E = \frac{1}{\chi^{s+1}} \frac{(G - \delta J)}{|J|}. \hfill (5.35)$$

We assume $s = -1/2$, $d = 3$ and get

$$\Delta E'' - \frac{1}{2} \frac{\chi'}{\chi} \Delta E' - \frac{\chi''}{\chi} \Delta E = \chi^{-1/2} \frac{(G - \delta J)}{|J|}. \hfill (5.36)$$

This equation differs from that derived in Ref. [11] while equation for the potential $\chi$ coincides with

$$\frac{1}{2\chi^{1/2}} \chi' \chi'' - \chi^{1/2} \chi''' = 1. \hfill (5.37)$$

To calculate the Green’s function of Eq. (5.36) we need the function $\psi_1(x)$ obeying the homogeneous equation

$$\psi'' - \frac{1}{2} \frac{\chi'}{\chi} \psi' - \frac{\chi''}{\chi} \psi = 0$$  \hfill (5.38)

and satisfying the boundary condition $\psi_1'(x=0) = 0$. The second function $\psi_2$ obeying the boundary condition $\psi_2'(x=L) = 0$ can be expressed through the functions $\chi$ and $\psi_1$

$$\psi_2(x) = -\psi_1 \left[ \frac{\chi^{1/2}(1)}{\psi_1(1) \psi_1'(1)} + \int_x^L d\xi \frac{\chi^{1/2}(\xi)}{\psi_1^2(\xi)} \right]. \hfill (5.39)$$

The solution of Eq. (5.36) can be written using the Green’s function

$$G(x, x') = \frac{1}{\chi^{1/2}(x')} [\theta(x - x') \psi_1(x') \psi_2(x) + \theta(x' - x) \psi_1(x) \psi_2(x')] \hfill (5.40)$$
as
\[
\Delta E = \int_0^1 dx' G(x, x') \frac{(G(x') - \delta J)}{\chi^{1/2}(x') |J|}.
\]  
(5.41)

Requiring a nonfluctuating applied voltage we get
\[
\delta J = \frac{1}{Z} \int_0^1 dx \frac{G(x)}{\chi(x)} \Pi(x)
\]  
(5.42)

where
\[
\Pi(x) = \psi_1(x) \int_x^1 d\xi \psi_2(\xi) + \psi_2(x) \int_0^x d\xi \psi_1(\xi),
\]  
(5.43)

\[
Z = \int_0^1 dx \frac{\Pi(x)}{\chi(x)}.
\]  
(5.44)

Expressing the correlation function \( <G^2(x)> \) through \( \chi \) we get for the shot noise power reduction factor
\[
\frac{P}{P_{\text{Poisson}}} = \frac{2}{Z^2} \int_0^1 dx \frac{\chi''(x)}{\chi^{3/2}(x)} \Pi^2(x).
\]  
(5.45)

We determine potential \( \chi \) following Ref. [11] and find numerically \( \psi_1 \) from Eq. (5.38). Now the functions \( \psi_2, \Pi \) and the constant \( Z \) can be found from Eqs. (5.39), (5.43), and (5.44). The reduction factor can be evaluated as
\[
P/P_{\text{Poisson}} = 0.4257
\]  
(5.46)

which is about 10% larger than the result obtained in Ref. [11]. As indicated by González, González, Mateos, Pardo, Reggiani, Bulashenko, and Rubí [10], they obtained for \( s = -1/2 \) as a result of numerical simulation
\[
P/P_{\text{Poisson}} = 0.42 - 0.44.
\]  
(5.47)

One can see that this interval is noticeably nearer to the value given by Eq. (5.46) than the result of Ref. [11].

In the case \( s = 1/2 \) the reduction factor can be evaluated as
\[
P/P_{\text{Poisson}} = 0.1974.
\]  
(5.48)

that is slightly smaller than the result of Ref. [10].
VI. CONCLUSION

In summary, we have developed an analytical theory of shot noise in a diffusive conductor under the space charge limited regime. We find differences between the present theory and the approach developed earlier and indicate a possible origin of the difference.

Several conclusive remarks. The calculated nonequilibrium shot noise power in a non-degenerate diffusive semiconductor for two types of physically relevant elastic scattering mechanisms turned out to be very close to the ones obtained in numerical simulations by the authors of Ref. [10]. The computed noise suppression factor $P/P_{\text{Poisson}}$ for the case of an energy-independent scattering time is also rather close to the analytical results obtained earlier by Schomerus et al. [11]. However, for an energy-dependent scattering the numerical difference between our results and the ones of Ref. [11] is considerable.

Let us clarify once more the point as to why the authors of Ref. [11] arrived at the equations that differ from ours. As an example we take the Poisson equation. According to Ref. [11] one could write

$$n = \int_{-U(x)}^{\infty} d\varepsilon \nu(\varepsilon) f(\varepsilon, x),$$

where $n, U$ are the exact total concentration and potential energy, $f$ is the total distribution function (the mean value plus the fluctuating part). Linearization of this equation leads to equations of Ref. [11]. The authors of Ref. [11] could have argued that since the voltages in the reservoirs do not fluctuate and $U$ is set to zero at the left boundary and since the total energy $E = \varepsilon + U$ is and remains positive therefore the total distribution function is zero for $\varepsilon < -U$.

Our point is that one cannot justify Eq. (6.1) for the total values of these variables including the stationary and fluctuating parts. This is readily seen from the fact that the fluctuating part of the distribution function itself depends implicitly on the mean value of the distribution function through the correlation function. One should bear in mind that an equation involving both the mean and the fluctuating quantities must be regarded symbolically. Indeed, such an equation is in fact equivalent to two equations: one for the mean values and the other for the fluctuating part. Regarded literally it can lead to confusion. For instance, analyzing the equation

$$\underline{n} + \delta n = \int_{-U-\delta U}^{-U} d\varepsilon \nu(\varepsilon) (\overline{f} + \delta f)$$

one could have come to a wrong conclusion that the mean value $\underline{n}$ depends on such an average as $\overline{U\delta f}$.

A few words about the boundary conditions for the potential. The boundary conditions used are not applicable within the length $R_V = \sqrt{\kappa V/4\pi en(0)}$ near the electrodes. As the
nonequilibrium noise power is a bulk property [mark, for instance, integration over the
coordinate in Eq. (5.45)] this approximation is justified since we assume that the sample’s
length $L$ is much bigger than $R_V$.

Being interested in analysis of the fluctuation phenomena in the simplest situation
of the space-charge limited diffusive conduction regime, we have not taken into account
the electron-electron collisions. Meanwhile, such collisions can bring about additional
electron-electron correlation [7] which one should consider treating a more general case.

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