Modeling and Simulation Method of Flight Trajectory for Ship-To-Air Missile Based on Vector Rotation

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Abstract. Aiming at trajectory modeling for ship-to-air missile, a novel modeling method based on vector rotation is proposed, which can be used under different conditions of ship-to-air missile launching scheme. Compared with traditional Euler modeling method and quaternion modeling method, its advantage lies in simple principle and extensive practicality. The implementation method is adopted as mixed programming of C language and Matlab language, which combines both simplicity of Matlab developing environment to display 3D graph and efficiency of C language. Addition, using dynamic link library to implement trajectory calculation is easy to extend. The simulation results verify the effectiveness and practicality of the proposed method.

1. Introduction

Trajectory modeling and simulation plays an important role in the field of ballistic design and simulation. Currently, there are three main methods of trajectory modeling: (1) Based on the mathematical model of missile system, the missile motion equations are established for ballistic calculation. The complete mathematical model of the missile system includes: the target motion model; the projectile body model (kinetic model of centroid motion, kinematic model of centroid motion, kinetic model of motion around the center of mass, kinematics model of motion around the center of mass); control relation model (guidance loop model and attitude control loop model) and relative kinematic model. The established missile motion equations are nonlinear differential equations with variable coefficients, which are computationally intensive and complex; (2) Ignore the control relation, retain the missile kinematics equation and adopt a simplified dynamic model[1]. The dynamic equation is constrained by such conditions as acceleration and overload, that is, dynamic response is determined by maneuvering performance limits. The velocity of centroid motion is obtained by simplifying the dynamic equation, while the position and attitude of the missile are solved by the kinematics equation; (3) According to the characteristics of each stage of missile flight, a universal solution model of missile position is established. The method can be used to simulate the trajectory’s certain stage of a missile, the trajectory in different situations and the trajectory of different types of missiles by setting the parameters of each stage of missile flight. In this paper, the third method is used to model the ship-to-air missile trajectory according to the missile model, the specific missile parameters, ballistic characteristics and guiding law.

In the process of trajectory modeling, the literature [2] only simulates the trajectory in the two-dimensional plane, which could not reflect the real situation of the missile flight. The proportional guidance method is usually used in the homing guidance of the ship (ground)-to-air missile. When calculating the kill zone based on the trajectory simulation, the proportional guidance is decomposed into a vertical direction and a horizontal direction in the literature [3]. In the prediction model of the
aerial target’s hit point, the literature [4] also decomposes the missile motions into vertical plane or horizontal plane. In order to realize the three-dimensional trajectory, the literature [5] does not adopt the thought of decomposition, but establishes the simulation model of three-dimensional trajectory based on the vector idea. However, the description of its concrete implementation is not clear.

In the traditional trajectory modeling method, the attitude angle of the missile is described by the Euler angles, which is not suitable for the trajectory modeling of vertically-launched missiles. The quaternion method can be used to solve this problem[6]. After trajectory modeling, the numerical integration method is usually adopted, including Euler method, Runge-Kutta method and Adams method. The integration step size may be a definite step or a variable step. Similar to Euler method, the literature [7] gives the difference equation of proportional guidance method when calculating the simulation trajectory, for seeking the discrete three-dimensional coordinates of the missile at intervals. The realization of the solution and display after trajectory modeling is usually implemented by Matlab language programming, which is not engineering and real-time.

To overcome the above shortcomings, a new modeling method of flight trajectory for ship-to-air missile based on vector rotation is put forward for the trajectory simulation of ship-to-air missile with different types of launching schemes. Compared with Euler method and Quaternion method, this method is more practical and understandable.

2. Simulation condition

2.1. Simulation Target Analysis
The Russian SA-N-6 vertically launched ship-to-air missile weapon system is taken as an example to conduct trajectory modeling. The characteristics of the missile is maximum speed of 1800 m/s, the average speed of 1100 m/s, high trajectory, maximum distance of 150 km, with 85° quasi vertical cold launch mode. After the launcher ejects the missile for 1.5s, the missile main engine ignites and begins to control the missile turn. When the desired course angle and trajectory inclination are reached, the turn phase ends. At the same time, intermediate guidance begins with semi-active command. After the missile seeker captures the target signal, the intermediate command guidance is converted into the terminal guidance [8].

According to the above description, the trajectory of the missile is divided into four phases: the free trajectory phase (launch phase), the initial guidance phase (the turn phase), the intermediate guidance phase and the terminal guidance phase. Because both of them are guided by proportional guidance, the intermediate guidance and terminal guidance can be regarded as a phase in the trajectory simulation, which is the guidance phase. Therefore, this paper divides the trajectory into three phases to conduct modeling and simulation respectively, that is, the free trajectory phase, the turn phase and the guidance phase.

2.2. Trajectory Simulation Hypothesis
The missile’s complete flight control system, projectile body dynamics and kinematics model are very complex, but we are mainly concerned with the change of the missile’s center of mass. Therefore, the missile is regarded as a maneuverable particle, and the kinematics and dynamical equations of the missile’s center of mass motion are studied to determine the coordinate position of the center of mass in each moment. The dynamics equations are simplified and analyzed from an overload point of view, and we assumed that the gravitational acceleration does not vary with altitude.

In the modeling and simulation, the geodetic coordinate system is adopted, and the initial position of the missiles at the launch time of the ship-to-air missile is taken as the origin of coordinates.

2.3. Target Motion Model
Suppose the target to do uniform linear motion. If the coordinate of the current target is \((X'_t, Y'_t, Z'_t)\), the flying speed is \(V'_t\), the course angle is \(\psi'_t\), and the velocity vector inclination is \(\theta'_t\), then the target
coordinate at the next time is

\[ X_{i}^{t+\Delta t} = X_{i}^{t} + V_{x, i}^{t} \Delta t \]
\[ Y_{i}^{t+\Delta t} = Y_{i}^{t} + V_{y, i}^{t} \Delta t \]
\[ Z_{i}^{t+\Delta t} = Z_{i}^{t} + V_{z, i}^{t} \Delta t \]  \hspace{1cm} (1)

Where the flight velocity vector direction of the target is

\[ I_{x, i}^{t} = \cos \theta_{i} \cos \psi_{i}, \quad I_{y, i}^{t} = \sin \theta_{i}, \quad I_{z, i}^{t} = \cos \theta_{i} \sin \psi_{i} \]  \hspace{1cm} (2)

3. Modeling of flight trajectory for ship-to-air missile based on vector rotation

3.1. The Free Trajectory Phase

(1) The velocity vector’s direction of the missile remains unchanged, given by

\[ I_{w, x, m}^{t} = \cos \theta_{w, x, m}^{t} \cdot \cos \psi_{w, x, m}^{t}, \quad I_{w, y, m}^{t} = \sin \theta_{w, x, m}^{t}, \quad I_{w, z, m}^{t} = \cos \theta_{w, z, m}^{t} \cdot \sin \psi_{w, z, m}^{t} \]  \hspace{1cm} (3)

Where \( \theta_{w, x, m}^{t} \) is the ballistic inclination of the missile launching time, \( \psi_{w, x, m}^{t} \) is the ballistic deflection of the missile launching time.

(2) The coordinates of the missile at the next moment can be expressed by

\[ X_{m}^{t+\Delta t} = X_{m}^{t} + V_{x, m}^{t} \cdot I_{w, x, m}^{t} \Delta t, \quad Y_{m}^{t+\Delta t} = Y_{m}^{t} + V_{y, m}^{t} \cdot I_{w, y, m}^{t} \Delta t, \quad Z_{m}^{t+\Delta t} = Z_{m}^{t} + V_{z, m}^{t} \cdot I_{w, z, m}^{t} \Delta t \]  \hspace{1cm} (4)

Where \( V_{m}^{t} \) is the velocity of the missile, which could be obtained by fitting and sampling the maximum velocity and average velocity.

(3) The end of the free trajectory phase is controlled by the time variable \( t \), i.e., \( 0 < t \leq 1.5s \).

3.2. The Turn Phase

(1) The velocity vector direction \( (I_{z, x, m}^{t}, I_{z, y, m}^{t}, I_{z, z, m}^{t}) \) of the missile at the beginning is the velocity vector direction at the end of the free trajectory phase.

(2) The change rate of velocity vector is calculated by

\[ \theta_{w, z, m}^{t} = k_{v} \cdot V_{m}^{t}, \quad k_{v} = \frac{n_{z, w} \cdot \frac{G}{V_{max}}}{n_{w, z}} \]  \hspace{1cm} (5)

Where \( n_{z, w} \) is the available overload of the missile; \( k_{v} \) is the adjustment coefficient; \( G \) is the acceleration of gravity.

(3) The change direction of the velocity vector in the turn is determined by the cross product direction of the direction of the missile velocity vector at the beginning of the turn phase and the missile velocity vector determined by the initial bookbinding parameters, that is

\[ I_{z, x, m}^{t} = \cos \theta_{z, x, m}^{t} \cdot \cos \psi_{z, x, m}^{t}, \quad I_{z, y, m}^{t} = \sin \theta_{z, y, m}^{t}, \quad I_{z, z, m}^{t} = \cos \theta_{z, z, m}^{t} \cdot \sin \psi_{z, z, m}^{t} \]  \hspace{1cm} (6)

Where \( \theta_{z, x, m}^{t} \) is the trajectory inclination and \( \psi_{z, x, m}^{t} \) is the trajectory declination binding at the end of the turn.

(4) The velocity vector of the missile at the next moment is

\[ (I_{z, x, m}^{t+\Delta t}, I_{z, y, m}^{t+\Delta t}, I_{z, z, m}^{t+\Delta t}) = R \cdot (I_{z, x, m}^{t}, I_{z, y, m}^{t}, I_{z, z, m}^{t}) \]  \hspace{1cm} (7)

Where the rotation matrix \( R \) is determined by the change rate and change direction of the velocity vector in the turn phase.

(5) Coordinates at the next moment are

\[ X_{m}^{t+\Delta t} = X_{m}^{t} + V_{x, m}^{t} \cdot I_{z, x, m}^{t} \Delta t, \quad Y_{m}^{t+\Delta t} = Y_{m}^{t} + V_{y, m}^{t} \cdot I_{z, y, m}^{t} \Delta t, \quad Z_{m}^{t+\Delta t} = Z_{m}^{t} + V_{z, m}^{t} \cdot I_{z, z, m}^{t} \Delta t \]  \hspace{1cm} (8)

(6) Suppose the angle between the velocity vector’s direction of the missile \( (I_{z, x, m}^{t}, I_{z, y, m}^{t}, I_{z, z, m}^{t}) \) and the missile velocity vector’s direction determined by the initial bookbinding parameters
\[ (I'_{dzc_{m}}, I'_{dly_{m}}, I'_{dzc_{m}}) \text{ is } \alpha'_{m}. \] When \( \alpha'_{m} \) meets the following conditions, the turn phase end

\[ \alpha'_{m} = \arccos(I'_{dx_{m}} \cdot I'_{dx_{m}} + I'_{dly_{m}} \cdot I'_{dly_{m}} + I'_{dzc_{m}} \cdot I'_{dzc_{m}}) \]  \tag{9} 

When \( \alpha'_{m} \) is less than or equal to \( \alpha^{*} \), stop calculation, where \( \alpha^{*} \) is the precision requirement that control the end of turn.

### 3.3. The Guidance Phase

The guidance phase adopts the proportional guidance law, that is, the angular velocity of the missile velocity vector \( \psi_{m} \) is proportional to the angular velocity of the target \( \varphi_{m} \), which is as follows

\[ \frac{d\varphi_{m}}{dt} = K_n \frac{dq_{m}}{dt} \]  \tag{10} 

Where \( K_n \) is the proportional guidance coefficient. In the proportional guidance, the relative movement of missile and airplane is shown in Figure 1.

![Figure 1](image-url)

**Figure 1.** The relative movement of missile and airplane under the proportional guidance law

According to the proportional guidance law, the solution of the trajectory model of the guidance phase is summarized:

1. At the beginning, the angle between the target-missile line of sight direction and the missile velocity direction is

\[ \beta_{m,x_{m}} = \arccos(I'_{x_{m}} \cdot I'_{dx_{m}} + I'_{y_{m}} \cdot I'_{dly_{m}} + I'_{z_{m}} \cdot I'_{dzc_{m}}) \]  \tag{11} 

Where the target-missile line of sight direction is

\[ I'_{x_{m}} = X'_{i} - X'_{m}, \quad I'_{y_{m}} = Y'_{i} - Y'_{m}, \quad I'_{z_{m}} = Z'_{i} - Z'_{m} \]  \tag{12} 

After normalization of Eq.(12), the following equation is obtained.

\[ I'_{x_{m}} = \frac{I'_{x_{m}}}{\sqrt{I'_{x_{m}}^2 + I'_{y_{m}}^2 + I'_{z_{m}}^2}}, \quad I'_{y_{m}} = \frac{I'_{y_{m}}}{\sqrt{I'_{x_{m}}^2 + I'_{y_{m}}^2 + I'_{z_{m}}^2}}, \quad I'_{z_{m}} = \frac{I'_{z_{m}}}{\sqrt{I'_{x_{m}}^2 + I'_{y_{m}}^2 + I'_{z_{m}}^2}} \]  \tag{13} 

The velocity vector direction of the missile \( (I'_{dx_{m}}, I'_{dly_{m}}, I'_{dzc_{m}}) \) at the beginning is the velocity vector direction at the end of the turn phase.

2. At the beginning, the angle between the target-missile line of sight direction and the target velocity direction is

\[ \beta_{m,y_{i}} = \arccos(I'_{x_{m}} \cdot I'_{x_{i}} + I'_{y_{m}} \cdot I'_{y_{i}} + I'_{z_{m}} \cdot I'_{z_{i}}) \]  \tag{14} 

Where \( (I'_{x_{i}}, I'_{y_{i}}, I'_{z_{i}}) \) is the target velocity vector direction.

3. The angular velocity of target-missile line of sight direction at the beginning is
\[ q_m^j = \frac{\dot{r}_{mt}^j}{r_{mt}^j} \]  

(15)

Where the missile-target distance is \( r_{mt}^j = \sqrt{(X_{mt}^j - X_m)^2 + (Y_{mt}^j - Y_m)^2 + (Z_{mt}^j - Z_m)^2} \) and the missile-target distance rate is \( r_{mt}^j = V_t \cdot \sin \beta_{m,v_t} - V_m \cdot \sin \beta_{m,v_u} \).

(4) The change rate of the missile velocity vector direction at the beginning is

\[ \dot{q}_m^i = K_n q_m^i \]

(16)

(5) The change direction of the velocity vector at the beginning is determined by the cross product direction of the direction of the missile velocity vector and the missile velocity vector at the beginning.

(6) The velocity vector of the missile at the next moment is the same as step 4 in the turn phase.

(7) Missile coordinates at the next moment are the same as step 5 in the turn phase.

(8) The ending condition of the guidance phase is \( r_{mt}^{j+M} \leq r_{mt}^j \) and \( r_{mt}^{j+M} \leq R \), where \( r_{mt}^{j+M} = r_{mt}^j + r_{mt}^{\Delta t} \), \( R \) is the ship-to-air missile’s kill radius.

4. Simulation implementation method

According to the needs of the project, C and Matlab languages are used to realize the ballistic solution and display. The solution part is realized by dynamic link library in VC6.0 integrated development environment, and the display part is three-dimensionally demonstrated in Matlab7.1 environment. The main advantage of this method is the combination of the convenience of 3D display in Matlab environment and the high efficiency of C language program.

In the trajectory modeling process, when the velocity vector is rotated from the current position to another location, two key parameters need to be known: rotation angle and rotation direction. The solution of these two parameters has been given in the trajectory modeling section. In the case of these two parameters known, the following rotation matrix can be used to realize the vector rotation:

\[
R = \begin{bmatrix}
x^2 + (1 - x^2) \cos \alpha & xy(1 - \cos \alpha) - z \sin \alpha & xz(1 - \cos \alpha) + y \sin \alpha \\
yx(1 - \cos \alpha) + z \sin \alpha & y^2 + (1 - y^2) \cos \alpha & yz(1 - \cos \alpha) - x \sin \alpha \\
zx(1 - \cos \alpha) - y \sin \alpha & zy(1 - \cos \alpha) + x \sin \alpha & z^2 + (1 - z^2) \cos \alpha
\end{bmatrix}
\]

(17)

Where \((x, y, z)\) is the rotation direction of normalization, and \(\alpha\) is the angle of rotation. The rotation matrix multiplies the velocity vector right to get the new velocity vector.

5. Simulation result

During the simulation, at the beginning of launching, the ship-to-air missile trajectory angle is 85°, the ballistic deflection angle is 10°. The turn phase adjustment coefficient is \( k_i = 1.6 \), the missile overload is \( K_n = 1.6 \), the guidance phase proportion coefficient is \( K_a = 1.6 \), and the simulation step is 0.1s.

Let the target parameters at the missile launch time as follows: target direction \( B_t = 45° \), the target slant range \( R_t = 100km \), the target height \( H_t = 10km \), the target flying straight and level at a constant speed, the target speed \( V_t = 500m/s \). Figure 2(a) and (b) are the ballistic trajectories of the ship-to-air missiles at the target course \( C_t = 180° \) and \( C_t = 270° \) respectively.

The following conclusions can be obtained through the simulation results: (1) In Figure 2(a), the shortest distance between the missile and the target is about 26m. In Figure 2(b), the shortest distance between the missile and the target is about 41m. If the kill radius is 50m, the ship-to-air missile can
effectively destroy the target; (2) It can realize the combat simulation of ship-to-air missile and the aircraft target in different battlefield environment by changing the target or the missile parameters; (3) In this paper, the trajectory simulation of SA-N-16 ship-to-air missiles is carried out. But the simulation ideas and steps are universal, which can be used in ballistic simulation of other ship-to-air missile weapon systems and can also be widely applied to ballistic simulation of anti-ship missile weapon systems.

![Flight trajectory of ship-to-air missile SA-N-6](image)

Figure 2. The simulation result of flight trajectory of ship-to-air missile SA-N-6

6. Conclusion
In this paper, a new method of ship-to-air missile trajectory modeling is proposed based on vector rotation, which can be effectively applied to different types of ship-to-air missile trajectory simulation. The method of mixed programming of C language and Matlab language is adopted in the concrete realization of ballistic calculation and display. It takes advantage of the convenience of 3D display in Matlab environment and the efficiency of C language design program, and uses dynamic linking Library technology for ballistic calculations with application scalability. The simulation results verify the effectiveness and practicability of the proposed method. In the ballistic modeling process, some reasonable and simplified assumptions are made. Given the known aerodynamic forces, aerodynamic moments, thrusts, standard atmospheric parameters, mass changes, etc., more realistic ballistic trajectories can be obtained, coupled with environmental considerations including wind, flow, atmospheric media and the electromagnetic environment, etc., as well as the impact of the target’s mobility, more real countermeasure scene can be simulated.

References
[1] Ku H., Kang F. J., Hang L., et al. 2006. Modeling and simulation technology of typical anti-ship missile motion. Journal of System Simulation, 18, 8 (Aug. 2006), 2067-2069.
[2] Li P., Hu Y.. 2010. Modeling and Simulation of typical Motion of Anti-ship Missile. Computer Simulation, 27, 5(May. 2010), 46-48.
[3] Zhao J. J., Li W. B., Sang D. Y., et al. 2010. Killing Zone calculation based on trajectory Simulation. Ordnance Industry Automation, 29, 6(Jun. 2010), 8-10.
[4] Wang J., Zhou L., Lei H. M.. 2009. Prediction model and algorithm on hit point of ground to air missile and aerial target. Journal of System Simulation, 21, 1(Jan. 2009), 80-83.
[5] Ma Q. D., Ma L. T., Zhong Z. T.. 2008. Research on 3-Dimensional trajectory simulation of proportional guidance ship-to-air missile. Missiles and Space Vehicles, 3, (Mar. 2008), 15-18.
[6] Fang Y., Sun S. C., Liu F. 2010. Research on trajectory simulation of thermal vertical launching ship-to-air missile. Ship Electronic Engineering, 30, 11(Nov. 2010), 110-113.
[7] Gao S.. 2003. Simulation on missile ideal trajectory under proportional guidance. Computer Engineering and Design, 24, 8(Aug. 2003), 66-68.
[8] Zhang Y. L.. 1997. Naval Ship-to-Air Missile Weapon Manual. Weapons Industry Press, Beijing.