The $\pi N \rightarrow e^+e^-N$ reaction close to the vector meson production threshold

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Abstract. The $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ reaction cross sections are calculated below and in the vicinity of the vector meson ($\rho^0$, $\omega$) production threshold. These processes are largely responsible for the emission of $e^+e^-$ pairs in pion-nucleus reactions and contribute to the dilepton spectra observed in relativistic heavy ion collisions. They are dominated by the decay of low-lying baryon resonances into vector meson-nucleon channels. The vector mesons materialize subsequently into $e^+e^-$ pairs. Using $\pi N \rightarrow \rho^0N$ and $\pi N \rightarrow \omega N$ amplitudes calculated in the center of mass energy interval $1.4 < \sqrt{s} < 1.8$ GeV, we compute the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ reaction cross sections in these kinematics. Below the vector meson production threshold, the $\rho^0 - \omega$ interference in the $e^+e^-$ channel appears largely destructive for the $\pi^-p \rightarrow e^+e^-n$ cross section and constructive for the $\pi^+n \rightarrow e^+e^-p$ cross section. The pion beam and the HADES detector at GSI offer a unique possibility to measure these effects. Such data would provide strong constraints on the coupling of vector meson-nucleon channels to low-lying baryon resonances.

Keywords: Vector meson production; Baryon resonances; Dileptons

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1. Introduction

The study of the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ processes described in this work aims at gaining understanding of the $\pi N \rightarrow \rho^0N$ and $\pi N \rightarrow \omega N$ scattering amplitudes for center of mass energies close and below the vector meson production threshold ($1.5 < \sqrt{s} < 1.8$ GeV). There are well-known baryon resonances in this energy range, which contribute to the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ scattering

\begin{itemize}
  \item This talk is based on the work published in Ref. [1].
\end{itemize}
amplitudes through their coupling to the $\pi N$, $\rho^0 N$ and $\omega N$ channels. These amplitudes involve in addition significant non-resonant processes. The phenomenological $\rho N N^*$ and $\omega N N^*$ coupling strengths needed to understand the data related to the $\pi N \rightarrow \rho^0 N$ and $\pi N \rightarrow \omega N$ amplitudes are pivotal quantities for baryon structure studies [2].

The exclusive observation of neutral vector mesons through their $e^+e^-$ decay presents definite advantages over their observation through final states involving pions. Firstly, there are no competing processes, such as $\pi \Delta$ production which leads to the same $\pi\pi N$ final state and impairs consequently the identification of the $\rho$-meson in that channel. Secondly, both the $\rho^0$- and $\omega$-mesons decay into the $e^+e^-$ channel. This leads to a quantum interference pattern which is expected to reflect sensitively the structure and relative sign of the $\pi N \rightarrow \rho^0 N$ and $\pi N \rightarrow \omega N$ scattering amplitudes.

A proper understanding of the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ reactions appears also as a necessary step towards a detailed interpretation of the production of lepton pairs off nuclei induced by charged pions. Such reactions would be particularly sensitive to the propagation of $\omega$-mesons in nuclei [3].

In Section 2, we present the relativistic coupled-channel model [2] used to describe the $\pi N \rightarrow e^+e^-N$ reaction for $e^+e^-$ pair invariant masses ranging from $\sim 0.4$ to $\sim 0.8$ GeV. Assuming Vector Meson Dominance for the electromagnetic current [4], the $\pi N \rightarrow \rho^0 N$ and $\pi N \rightarrow \omega N$ amplitudes are the basic quantities entering the calculation of the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ cross sections in the Vector Meson Dominance model. Our numerical results for these cross sections are displayed in Section 3. We discuss the $\rho^0 - \omega$ quantum interference pattern in the $e^+e^-$ spectrum for both the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ reactions. We conclude briefly in Section 4.

2. Calculation of the $\pi^-p \rightarrow e^+e^-n$ and $\pi^+n \rightarrow e^+e^-p$ cross sections close to the vector meson production threshold

We describe the $\pi N \rightarrow e^+e^-N$ reaction for $e^+e^-$ pair invariant masses ranging from $\sim 0.4$ to $\sim 0.8$ GeV. Assuming Vector Meson Dominance for the electromagnetic current [4], the $\pi N \rightarrow \rho^0 N$ and $\pi N \rightarrow \omega N$ amplitudes are the basic quantities entering the calculation of the $\pi N \rightarrow e^+e^-N$ cross section. This assumption is illustrated in Fig. 1, where we show the diagrams contributing to the $\pi^-p \rightarrow e^+e^-n$ process.

We use the $\pi N \rightarrow \rho^0 N$ and $\pi N \rightarrow \omega N$ amplitudes obtained in the recent relativistic and unitary coupled-channel approach to meson-nucleon scattering of Ref. [2]. The available data on pion-nucleon elastic and inelastic scattering and on meson photoproduction off nucleon targets are fitted in the energy window $1.4 < \sqrt{s} < 1.8$ GeV, using an effective Lagrangian with quasi-local two-body meson-baryon interactions and a generalized form of Vector Meson Dominance to describe the coupling of vector mesons to real photons. The scheme comprises the $\pi N$, $\pi \Delta$, $\rho N$, $\omega N$, $K\Lambda$, $K\Sigma$ and $\eta N$ hadronic channels. The coupling constants entering the effective Lagrangian are parameters which are adjusted to reproduce the data. In view of the kinematics, only s-wave scattering in the $\rho N$ and $\omega N$ channels is
The $\pi N \rightarrow e^+ e^- N$ reaction

Fig. 1. Diagrams contributing to the $\pi^- p \rightarrow e^+ e^- n$ amplitude with intermediate $\rho^0$- and $\omega$-mesons.

included, restricting $\pi N$ and $\pi \Delta$ scattering to s- and d-waves. The pion-nucleon resonances in the $S_{11}$, $S_{31}$, $D_{13}$ and $D_{33}$ partial waves are generated dynamically by solving Bethe-Salpeter equations \[2\]. In the $\rho^0 N$- and $\omega N$-channels, the restriction to s-wave scattering means that the model applies to situations where the vector meson is basically at rest with respect to the scattered nucleon ($\sqrt{s} \simeq M_N + M_V$).

This assumption implies that the range of validity of the present calculation is limited to $e^+ e^-$ pairs with invariant masses $m_{e^+ e^-}$ close to $\sqrt{s}$ and to values of $\sqrt{s}$ below and very close to threshold i.e. $1.5 < \sqrt{s} \leq 1.75$ GeV.

The $\pi N \rightarrow \rho N$ amplitude has isospin 1/2 and isospin 3/2 components while the $\pi N \rightarrow \omega N$ amplitude selects the isospin 1/2 channel. Both amplitudes have spin 1/2 and spin 3/2 parts.

The invariant transition matrix elements for the $\pi N \rightarrow \rho N$ and $\pi N \rightarrow \omega N$ reactions are given by

$$\langle \rho^j(\overline{q}) N(\overline{p}) | T | \pi^i(q) N(p) \rangle = (2\pi)^4 \delta^4(q + p - \overline{q} - \overline{p}) \overline{\pi}(\overline{q}) \epsilon^\mu(\overline{q}) T^{ij}_{(\pi N \rightarrow \rho N) \mu} u(p),$$

$$\langle \omega(\overline{q}) N(\overline{p}) | T | \pi^i(q) N(p) \rangle = (2\pi)^4 \delta^4(q + p - \overline{q} - \overline{p}) \overline{\pi}(\overline{p}) \epsilon^\mu(\overline{q}) T^{ij}_{(\pi N \rightarrow \omega N) \mu} u(p),$$

where $T^{ij}_{(\pi N \rightarrow \rho N) \mu}$ and $T^{ij}_{(\pi N \rightarrow \omega N) \mu}$ are functions of the three kinematic variables $w = p + q = \overline{p} + \overline{q}$ ($\sqrt{w^2} = \sqrt{s}$), $q$ and $\overline{q}$. These scattering amplitudes can be
decomposed into isospin invariant components as

\[ T_{ij}^{(\pi N \rightarrow \rho N)\mu}(q, q; w) = \sum_I T_{ij}^{(I)}(\pi N \rightarrow \rho N)\mu (q, q; w) P_{(\rho)}^{ij}, \]  

\[ T_{ij}^{(\pi N \rightarrow \omega N)\mu}(q, q; w) = \sum_I T_{(\pi N \rightarrow \omega N)\mu}^{(I)} (q, q; w) P_{(\omega)}^{ij}, \]  

in which \( P_{(\rho)}^{ij} \) and \( P_{(\omega)}^{ij} \) are isospin projectors [1]. The isospin invariant amplitudes can be expanded further into components of total angular momentum,

\[ T_{ij}^{(\pi N \rightarrow V N)\mu}(q, q; w) = M_{\pi N \rightarrow V N}^{(J=\frac{3}{2})}(s) Y_{(J=\frac{1}{2})\mu}(q, q; w) + M_{\pi N \rightarrow V N}^{(J=\frac{1}{2})}(s) Y_{(J=\frac{3}{2})\mu}(q, q; w). \]

\( V \) stands for \( \rho \) or \( \omega \) and \( Y_{(J=\frac{1}{2})\mu}(q, q; w) \) and \( Y_{(J=\frac{3}{2})\mu}(q, q; w) \) are relativistic angular momentum projectors [1].

The \( \pi N \rightarrow \rho N \) and \( \pi N \rightarrow \omega N \) amplitudes in the \( S_{11}, S_{31}, D_{13} \) and \( D_{33} \) channels obtained in Ref. [2] are displayed in Figs. 2 and 3. The quantities shown are the amplitudes \( M_{\pi N \rightarrow \rho N}^{(J)} \) and \( M_{\pi N \rightarrow \omega N}^{(J)} \) defined by Eq. (5), which depend only on the center of mass energy \( \sqrt{s} \).

![Fig. 2. Real and imaginary parts of the \( \pi N \rightarrow \rho^0 N \) amplitudes in the pion-nucleon \( S_{11}, S_{31}, D_{13} \) and \( D_{33} \) partial waves [1].](image)

The coupling to subthreshold resonances is clearly exhibited in these pictures. In the \( S_{11} \) channel, the \( N(1535) \) and the \( N(1650) \) resonances lead to peak structures.
in the imaginary parts of the amplitudes. The pion-induced $\omega$ production amplitudes in the $D_{13}$ channel reflect the strong coupling of the $N(1520)$ resonance to the $\omega N$ channel. The $\pi^-p \rightarrow \rho^0n$ and $\pi^-p \rightarrow \omega n$ amplitudes are obtained from the

\[
M_{\pi^-p \rightarrow \rho^0n} = \frac{\sqrt{2}}{3} M_{\pi N \rightarrow \rho N}^{(1/2,J)} + \frac{\sqrt{2}}{3} M_{\pi N \rightarrow \rho N}^{(3/2,J)},
\]

(6)

\[
M_{\pi^-p \rightarrow \omega n} = \sqrt{\frac{2}{3}} M_{\pi N \rightarrow \omega N}^{(1/2,J)}.
\]

(7)

Similarly the $\pi^+n \rightarrow \rho^0p$ and $\pi^+n \rightarrow \omega p$ amplitudes are given by

\[
M_{\pi^+n \rightarrow \rho^0p} = \frac{\sqrt{2}}{3} M_{\pi N \rightarrow \rho N}^{(1/2,J)} - \frac{\sqrt{2}}{3} M_{\pi N \rightarrow \rho N}^{(3/2,J)},
\]

(8)

\[
M_{\pi^+n \rightarrow \omega p} = \sqrt{\frac{2}{3}} M_{\pi N \rightarrow \omega N}^{(1/2,J)}.
\]

(9)
The phases of the isospin coefficients appearing in Eqs. (1) and (3) play a crucial role in determining the $\rho^0 - \omega$ interference in the $\pi^- p \rightarrow e^+ e^- n$ and $\pi^+ n \rightarrow e^+ e^- p$ reaction cross sections. The real and imaginary parts of the $\pi^- p \rightarrow \omega n$ and of the $\pi^+ n \rightarrow \omega p$ amplitudes are the same and mostly positive. In contrast, the $\pi^- p \rightarrow \rho^0 n$ and $\pi^+ n \rightarrow \rho^0 p$ amplitudes have opposite signs. The $\pi^- p \rightarrow \rho^0 n$ amplitudes are predominantly negative and will therefore interfere destructively with the $\pi^- p \rightarrow \omega n$ amplitudes. The $\pi^+ n \rightarrow \rho^0 p$ and $\pi^+ n \rightarrow \omega p$ amplitudes have the same sign over a large $\sqrt{s}$ interval, leading to a constructive interference.

The $\pi^- p \rightarrow e^+ e^- n$ and $\pi^+ n \rightarrow e^+ e^- p$ cross sections are calculated from the $\pi^- p \rightarrow \rho^0 n$, $\pi^- p \rightarrow \omega n$, $\pi^+ n \rightarrow \rho^0 p$ and $\pi^+ n \rightarrow \omega p$ amplitudes, assuming Vector Meson Dominance of the electromagnetic current [4, 5]. This assumption can be enforced in the effective Lagrangian by introducing vector meson-photon interaction terms of the form,

$$L^{\gamma V}_{\gamma} = \frac{f_{\rho}}{2M^{2}_{\rho}} F^{\mu \nu}_{\rho} \rho^{\mu \nu} + \frac{f_{\omega}}{2M^{2}_{\omega}} F^{\mu \nu}_{\omega} \omega^{\mu \nu}, \quad (10)$$

where the photon and vector meson field tensors are defined by

$$F^{\mu \nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}, \quad (11)$$

$$V^{\mu \nu} = \partial^{\mu} V^{\nu} - \partial^{\nu} V^{\mu}. \quad (12)$$

In equation (10), $M_{\rho}$ and $M_{\omega}$ are the $\rho$- and $\omega$-masses and $f_{\rho}$ and $f_{\omega}$ are dimensional coupling constants. Their magnitude can be determined from the $e^+ e^-$ partial decay widths of the $\rho$- and $\omega$-mesons to be [6]

$$|f_{\rho}| = 0.036 \text{ GeV}^2, \quad (13)$$

$$|f_{\omega}| = 0.011 \text{ GeV}^2. \quad (14)$$

The relative sign of $f_{\rho}$ and $f_{\omega}$ is fixed by vector meson photoproduction amplitudes [2]. We assume that the phase correlation between isoscalar and isovector currents is identical for real and virtual photons as in Sakurai’s realization of the Vector Meson Dominance assumption [4, 5]. With the conventions used in this paper, both $f_{\rho}$ and $f_{\omega}$ are positive.

3. Numerical results

With the $\pi N \rightarrow \rho N$ and $\pi N \rightarrow \omega N$ amplitudes and the Vector Meson Dominance assumption discussed in Section 2, we have calculated the differential cross section $\frac{d\sigma}{d\omega}$ for the $\pi^- p \rightarrow e^+ e^- n$ and $\pi^+ n \rightarrow e^+ e^- p$ reactions. The magnitude of the 4-vector $\text{7}$ is the invariant mass $m_{e^+ e^-}$ of the $e^+ e^-$ pair. We refer to [7] for calculational details.
The differential cross sections for the \( \pi^-p \rightarrow e^+e^-n \) and the \( \pi^+n \rightarrow e^+e^-p \) reactions are computed for values of the total center of mass energy \( \sqrt{s} \) ranging from 1.5 GeV up to 1.75 GeV. We explore the dependence of the \( \rho^0 - \omega \) interference pattern in the \( e^+e^- \) channel on \( \sqrt{s} \) in this energy range, in particular in the vicinity of the \( \omega \)-meson production threshold (\( \sqrt{s}=1.72 \) GeV). We illustrate our results below threshold by displaying in Figs. 4 and 5 the differential cross sections for the \( \pi^-p \rightarrow e^+e^-n \) and the \( \pi^+n \rightarrow e^+e^-p \) reactions at \( \sqrt{s}=1.5 \) GeV, where the N(1520) and N(1535) baryon resonances play a dominant role. These figures show very clearly the isospin effects discussed in Section 2. For the two reactions, the \( \omega \) and \( \rho^0 \) contributions to the cross section are the same. The \( \rho^0-\omega \) interference for the \( \pi^-p \rightarrow e^+e^-n \) reaction and constructive for the \( \pi^+n \rightarrow e^+e^-p \) process. Consequently, the \( \pi^-p \rightarrow e^+e^-n \) differential cross section is extremely small in the range of invariant masses considered in this calculation. In contrast, the constructive \( \rho^0-\omega \) interference for the \( \pi^+n \rightarrow e^+e^-p \) reaction leads to a sizeable differential cross section. This is a very striking prediction, linked to the resonant structure of the scattering amplitudes \( M^{1/2}_{\pi N \rightarrow V N} \) and \( M^{3/2}_{\pi N \rightarrow V N} \). Data on differential cross sections for the \( \pi^-p \rightarrow e^+e^-n \) and \( \pi^+n \rightarrow e^+e^-p \) reactions at \( \sqrt{s}=1.5 \) GeV would be very useful for making progress in the understanding of the couplings of both the N(1520) and N(1535) baryon resonances to the vector meson-nucleon channels.

![Differential cross section for the \( \pi^-p \rightarrow e^+e^-n \) reaction at \( \sqrt{s}=1.5 \) GeV as function of the invariant mass of the \( e^+e^- \) pair. The \( \rho^0 \) and the \( \omega \) contributions are indicated by short-dashed and dotted lines respectively. The long-dashed line shows the \( \rho^0 - \omega \) interference. The solid line is the sum of the three contributions.](image-url)
Fig. 5. Differential cross section for the $\pi^+ n \rightarrow e^+ e^- p$ reaction at $\sqrt{s}=1.5$ GeV as function of the invariant mass of the $e^+ e^-$ pair. The $\rho^0$ and the $\omega$ contributions are indicated by short-dashed and dotted lines respectively. The long-dashed line shows the $\rho^0 - \omega$ interference. The solid line is the sum of the three contributions.

The differential cross sections for the $\pi^- p \rightarrow e^+ e^- n$ and $\pi^+ n \rightarrow e^+ e^- p$ reactions below threshold have been calculated also at $\sqrt{s}=1.55$, 1.60, 1.65 and 1.70 GeV [1]. The cross sections vary smoothly with the total center of mass energy. They exhibit the features discussed for $\sqrt{s}=1.5$ GeV, reflecting however dynamics associated with higher-lying resonances. Just below threshold ($\sqrt{s}=1.70$ GeV), the $\omega$-contribution begins to increase, while the general features of the $e^+ e^-$ production in the two isospin channels remain the same.

The interference pattern changes drastically above the $\omega$-meson threshold. Figs. 6 and 7 show the $\pi^- p \rightarrow e^+ e^- n$ and $\pi^+ n \rightarrow e^+ e^- p$ differential cross sections at $\sqrt{s}=1.75$ GeV. At this energy, the differential cross sections for the $\pi^- p \rightarrow e^+ e^- n$ and $\pi^+ n \rightarrow e^+ e^- p$ reactions are completely dominated by the $\omega$-contribution. The magnitudes of the cross sections for the two reactions are comparable. The $\rho^0 - \omega$ interference is still destructive in the $\pi^- p \rightarrow e^+ e^- n$ channel and constructive in the $\pi^+ n \rightarrow e^+ e^- p$ channel, albeit very small. In both reactions, crossing the $\omega$-production threshold leads to a sharp increase in the cross section, by two orders of magnitude in the $\pi^- p \rightarrow e^+ e^- n$ channel and by one order of magnitude in the $\pi^+ n \rightarrow e^+ e^- p$ channel.
4. Conclusion

We have computed the $e^+e^-$ pair invariant mass distributions for the $\pi^- p \rightarrow e^+e^- n$ and $\pi^+ n \rightarrow e^+e^- p$ reactions below and close to the vector meson production threshold. We took as input the $\pi N \rightarrow \rho^0 N$ and $\pi N \rightarrow \omega N$ amplitudes obtained in a recent relativistic and unitary coupled-channel approach to meson-nucleon scattering [2]. Using the Vector Meson Dominance assumption, we have shown that the differential cross sections for the $\pi^- p \rightarrow e^+e^- n$ and $\pi^+ n \rightarrow e^+e^- p$ reactions below the $\omega$-threshold are very sensitive to the coupling of low-lying baryon resonances to vector meson-nucleon final states contributing to the $\rho^0$- and $\omega$-meson production amplitudes. We find that the $\rho^0 - \omega$ interference is destructive in the $\pi^- p \rightarrow e^+e^- n$ channel and constructive in the $\pi^+ n \rightarrow e^+e^- p$ channel (see also Ref. [7]). We predict a very small cross section for the $\pi^- p \rightarrow e^+e^- n$ reaction below threshold and a sizeable cross section for the $\pi^+ n \rightarrow e^+e^- p$ reaction in this energy range. Above the $\omega$-meson production threshold, both cross sections are comparable and much larger.

The magnitude of the $\pi^- p \rightarrow e^+e^- n$ and $\pi^+ n \rightarrow e^+e^- p$ differential cross sections below the $\omega$-threshold depends strongly on the structure and dynamics of baryon resonances. These reactions deserve experimental studies. Such a programme could be carried at GSI (Darmstadt) using the available pion beam and the HADES spectrometer [8]. These measurements would provide a necessary step towards the understanding of $e^+e^-$ pair production in pion-nucleus reactions and in general significant constraints on the propagation of vector mesons in the nuclear medium.
Fig. 7. Differential cross section for the $\pi^+ n \to e^+ e^- p$ reaction at $\sqrt{s}=1.75$ GeV as function of the invariant mass of the $e^+ e^-$ pair.

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