THE BULGE-HALO CONNECTION IN GALAXIES: A PHYSICAL INTERPRETATION OF THE \( V_c - \sigma_0 \) RELATION

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ABSTRACT

We explore the dependence of the ratio of a galaxy’s circular velocity, \( V_c \), to its central velocity dispersion, \( \sigma_0 \), on morphology, or equivalently light concentration, for rotationally and pressure supported galaxies. Such a dependence is expected if light traces the mass. Over the full range of galaxy types, masses and brightnesses, and assuming that the gas velocity traces the circular velocity, we find that galaxies obey the relation \[ \log V_c = (0.74 \pm 0.07) \log \sigma_0 + (0.80 \pm 0.15). \] (1)

The realisation that elliptical and spiral galaxies appear to obey the same \( V_c - \sigma_0 \) relation\(^1\) prompted P05 to suggest that the central velocity dispersion of a galaxy is independent of morphological type for a given dark matter halo. Assuming that mass traces light, the opposite is however expected on first dynamical principles.

We can write the Jeans equation for spherical, or nearly spherical, self-gravitating systems (e.g. WKS79; Binney & Tremaine 1987, Eq. 4-55; see also Dekel et al. 2005):

\[ V_c^2(r) = [\alpha(r) + \gamma(r) - 2\beta(r)]\sigma^2(r) \] (2)

where \( \alpha \) is the logarithmic derivative of the stellar density profile \( \nu, \gamma = -d\ln \sigma^2/d\ln r \), and \( \beta = 1 - \sigma^2/\sigma^2_c \) is the anisotropy parameter. We can rewrite Eq. (2) as:

\[ \frac{V_{c,\text{out}}}{\sigma_{r,\text{in}}} = \frac{V_{c,\text{out}}}{V_{c,\text{in}}} [\alpha_{\text{in}} + \gamma_{\text{in}} - 2\beta_{\text{in}}]^{1/2} \] (3)

where the indices “in/out” refer to quantities measured in the inner and outer parts of the galaxy.

The ratio \( V_c/\sigma_0 \sim V_{c,\text{out}}/\sigma_{r,\text{in}} \) depends strongly on the shape of the underlying rotation curve, through \( V_{c,\text{out}}/V_{c,\text{in}} \), which

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1 Not to be confused with the \( V_c/\sigma_0 \) diagnostic of Kormendy & Illingworth (1982; also Binney & Tremaine Fig. 4-6) for elliptical galaxies which uses different (mass-weighted) velocity measurements; Kormendy & Illingworth measure \( V_c \) in the bulge (e.g. at the bulge effective radius) whereas our \( \sigma_0 \) represent a measurement in the galaxy’s outskirts at or near the peak of the rotation curve.
is itself directly related to morphology or concentration (e.g. Kent 1987). And while a dependence of $\alpha, \beta,$ and $\gamma$ on concentration is also expected on first principles, a full derivation is thwarted by the non-trivial covariances between these terms (see below) and the limited data to allow accurate decompositions of $V_c/\sigma_0$ in terms of Jeans’ equation parameters. Still, we verify empirically in this Letter, in line with WKS79 but in contrast with P05 and other reports of a linear fit between $V_c$ and $\sigma_0$, the dependence of the $V_c/\sigma_0$ ratio on concentration.

The simultaneous fitting of $V_c(r)/\sigma(r)$, and the light profile, $L(r)$, in order to fully constrain galaxy dynamics has thus far been attempted mostly for dwarf galaxies which are dark matter dominated at nearly all radii and where concerns about stellar populations are lessened (e.g. Dutton et al. 2005). The dynamical mapping for dwarf galaxies was explored by many as a potential route to solving the so-called “satellite overproduction” problem in $\Lambda$CDM structure formation models since the same physics to predict galaxy satellite distributions and internal dynamics is at play (e.g. Moore et al. 1999; Stoehr et al. 2002; Kazantzidis et al. 2004; Mashchenko, Sills, & Couchman 2006). These models involve significant simplifications and solutions are not unique; a major limitation is the degree of velocity dispersion anisotropy which remains poorly constrained with current observations; there indeed exists a well-known degeneracy between velocity anisotropy and mass distribution given only rotation and velocity dispersion data (e.g. Gerhard et al. 1998), as well as significant degeneracy in the $V_{\text{max}}-R_{\text{max}}$ values of the host halo that reproduce observed $\sigma(r)$ and surface brightness profiles (Bullock 2006; priv. comm.) Furthermore dynamical studies of dwarf galaxies have often assumed $V_c \simeq \sqrt{2}\sigma_0$, the expectation for an isothermal sphere (Binney & Tremaine 1987; Eq. 4-127b).

We show in this Letter that this assumption is unlikely.

While the mass modeling of dwarf, and more massive, galaxies in the context of $\Lambda$CDM cosmogony still falls short of providing a clear mapping of $V_c(r)/\sigma(r)$ with galaxy types, progress can still be achieved by considering specific parameters such as the maximum circular velocity $V_c$, the central velocity dispersion $\sigma_0=\sqrt{c^2\sigma(r)<r_{\text{eff}}}>$, and their relation across the full range of galaxy masses.

Below we use and expand upon existing data bases to demonstrate the dependence of $V_c/\sigma_0$ on the galaxy light concentration. We attempt to bolster this result in §4 using self-consistent dynamical models. We conclude with a discussion of the significance of this result for studies of galaxy dynamics and formation models.

2. THE DATA

We have compiled data from the literature which includes measurements or modeling of $V_c$ and $\sigma_0$ for spiral, lenticular, and elliptical galaxies. We have first built upon the compilation of $V_c$ and $\sigma_0$ by P05 from the heterogeneous data bases of F02 and B03. P05 added data of their own for a total of 40 high surface brightness (HSB) galaxies of types S0/a to Scd and 8 low surface brightness (LSB) galaxies of types Sa to Sc. These 48 barred and unbarred spiral galaxies were selected to have well-defined flat rotation curves. Following F02, P05’s galaxy list includes 20 elliptical galaxies whose $V_c$ was inferred by non-parametric dynamical modeling of the absorption line features and surface brightness profiles (Kronawitter et al. 2000 [K00]; Gerhard et al. 2001; see also Gerhard et al. 1998 and Cappellari et al. 2006). To these, P05 added 5 elliptical galaxies with inner gaseous disks whose $V_c$ could be measured via H I line widths. We have expanded P05’s galaxy compilation with the independent sample of 66 spiral galaxies of all Hubble types with measurements of $V_c$ and $\sigma_0$ by Prugniel, Maubon, & Simien (2001 [PMS01]). These authors measured central stellar velocity dispersions from absorption features of galaxy spectra and maximum circular velocities were derived from H I line widths\(^2\). Bedregal et al. (2006; B06) also compiled heterogeneous kinematic data for 51 S0 galaxies that we shall use; the stellar kinematics to estimate $V_c$ were all corrected for asymmetric drift following Neistein et al. 1999. Finally, in order to map the lower range of galaxy masses and brightnesses, we have also considered the dynamical measurements for 8 Local Group dwarf irregular (dI) galaxies with $V_c \gtrsim 10$ km s\(^{-1}\) by Mateo (1998) and Woo et al. (2006).

Our heterogeneous sample includes 154 HSB/LSB spiral, 54 lenticular, 24 elliptical, and 11 dwarf irregular galaxies with measured $\sigma_0$ and either measured or inferred $V_c$. Our compilation, including new concentrations below, is available at www.astro.queensu.ca/~courteau/data/VSigmaC28.txt.

Based on an overlap of 10 galaxies between F02 and PMS01, we estimate a systematic difference of 24 km s\(^{-1}\) for $V_c$ and 9 km s\(^{-1}\) for $\sigma_0$. There is no overlap between the Southern hemisphere galaxies compiled by B03 and the samples of PMS01, F02, or B06. Our sub-samples are all collections of heterogeneous data and it is not clear, due to the small sample sizes, whether any bias exists among the data sets. We make the assumption that our compiled data for $V_c$ and $\sigma_0$ are uniformly calibrated and can be inter-compared. As supporting evidence we note that our main conclusions hold whether we analyze the individual samples or the full data compilation. One must also keep in mind that $\sigma_0$’s in spiral galaxies may be polluted by disk stars and thus represent a lower limit. We also make the assumption that the gas velocity is an accurate tracer of the circular velocity, i.e. $V_c = V_{\text{rot}}$.

Fig. 1 shows the distribution of $V_c$ and $\sigma_0$ for our full sample. A well-defined though broad $V_c - \sigma_0$ relation seems to hold for galaxies with $\sigma_0 \gtrsim 80$ km s\(^{-1}\) and $V_c \gtrsim 220$ km s\(^{-1}\); this contrasts with the linear baryonic Tully-Fisher relation of disk galaxies down to $V_c \sim 90$ km s\(^{-1}\) (McGaugh 2005). For smaller galaxies, rotational and dispersion estimates are less certain due to relatively more prominent gas turbulence, velocity anisotropy and measurement errors.

The morphological dependence of the scatter of the $V_c - \sigma_0$ relation for spiral galaxies is obvious. For a given $V_c$, or total luminosity, early-type spirals have a higher $\sigma_0$ than later type ones (WKS79). For the sample of 21 ellipticals reported in P05, K00 find that the anisotropy parameter $\beta \lesssim 0.3$ at $R_e/2$ and most of their galaxies exhibit near-isotropy near the center. This sample is dominated by E0-E1 ellipticals and we expect their $V_c - \sigma_0$ distribution, as for any bright galaxy, to scatter about the isothermal $V_c \simeq \sqrt{2}\sigma_0$ line, as we see in Fig. 1. The possible dependence of $V_c - \sigma_0$ on morphology for elliptical galaxies is discussed in the next section.

Note how LSB galaxies finely delineate the upper envelope of bright spirals in the $V_c - \sigma_0$ distribution on account of their relatively small bulges, or low concentrations. P05 viewed the distributions of LSB and HSB spirals as two distinct $V_c - \sigma_0$ relations but fit HSB spiral and elliptical galaxies with a common linear $V_c - \sigma_0$ relation (Eq. 1); this statement however negates the $V_c - \sigma_0$ dependence on galaxy concentration that we expect for galaxies of all types. We quantify this relation\(^2\) Provided suitable measurements, HI line widths and the circular velocities trace the same dynamics (Courteau 1997).
The comparison of $V_c - \sigma_0$ for ellipticals with HI disks requires care as the inner disks within flattened ellipticals are embedded in a complex triaxial potential of halo stars and likely dark matter particles, unlike pure spiral disks which may revolve in a more spherical cloud of dark matter and for which the observed radial velocity $V_{\text{rot}}$ is assumed to trace the mass. For the E1-2 (Sy1) galaxy NGC 4278, as reported in P05, $V_c(\text{HI}) = 326 \pm 40$ while $V_c(\text{model}) = 416 \pm 13$. The lack of luster agreement makes for a rather ambiguous interpretation of $V_c(\text{HI})$. The exact meaning of $V_c$ in this context will require extensive modeling and new data.

3. LIGHT CONCENTRATION

To assess the dependence of $V_c/\sigma_0$ on galaxy structure, we use the galaxy light concentration (Kent 1987):

$$C_{28} = 5 \log(r_{50}/r_{20}),$$

where the radii are measured at 80% and 20% of the total light extrapolated to infinity.

We compute concentrations for SLOAN Digital Sky Survey (York et al. 2000, [SDSS]) i-band images of 32 spiral, 12 elliptical, and 33 lenticular galaxies in our sample. We have verified that the concentrations extracted from SDSS multi-band galaxy profiles compare well with those from deep surface brightness profiles (e.g. Courteau 1996; Courteau et al. 2000). The relation between $V_c/\sigma_0$ and $C_{28}$ is shown in Fig. 2. A fit to the data yields the relation:

$$\log(V_c/\sigma_0) = 0.63 - 0.11C_{28}$$

The lenticular (brown open circles) and elliptical galaxies (black dots) in Fig. 2 show signs of a match with the $V_c - \sigma_0 - C_{28}$ relation of spirals, as expected theoretically, but firm confirmation that Eq. 5 applies to galaxies of all Hubble types will require new data for lower luminosity spheroids. The measurement and modelling of $\sigma(r)$ and $V_c(r)$ for elliptical galaxies, much like dwarf systems (§1), is complex and suffers from limited radial coverage and relatively large error bars. Indeed a large scale effort (multiple galaxies) has only been attempted for bright spheroids so far (e.g. K00) and should be interpreted with care. More duplication and measurements of new systems by many separate teams is direly needed.

That spheroidal galaxies, via model construction, may trace a similar $V_c - \sigma_0 - C_{28}$ relation as disk galaxies would suggest that (i) the mass distributions in disks and spheroids are self-similar, and (ii) the central velocity dispersions are a reflection of similar environments in galaxy’s central regions such that Eq. 2 applies across the full Hubble sequence. Some of the scatter for disk galaxies in Fig. may be ascribed to disk star contamination in $\sigma_0’s$; however we have verified with simulations described below that this effect is small (< 5%) at least for Milky Way-type galaxies with pressure-supported bulges.

For spiral disks, the galaxy light concentration correlates well with the bulge-to-total (B/T) light ratio. For the spiral galaxies in our sample, we find $V_c/\sigma_0 = 2(1 - B/T)$. For pure exponential disks, $B/T \to 0, C_{28} \sim 2.8$ and both expressions for $V_c/\sigma_0$ based on $B/T$ or $C_{28}$ yield $V_c/\sigma_0 \sim 2$.

3 See MacArthur et al. (2003) for details and caveats about the parametric and non-parametric computations of $B/T$. Our measurement of $B/T$ is determined from bulge-to-disk decompositions of the galaxy color gradients.

The light grey circles are from self-consistent equilibrium models of disk galaxies by Widrow & Dubinski (2005) that we describe below.

4. DISCUSSION

We have demonstrated the dependence of $V_c/\sigma_0$ on galaxy concentration (Eq. 5), as expected theoretically (Eq. 4). We now explore the $V_c - \sigma_0 - C_{28}$ relation in the context of dynamical galaxy models.

We have used the self-consistent equilibrium galaxy models of Widrow & Dubinski (2005 [WD05]) to generate a suite of stable galaxy models for a wide range of disk masses and sizes, bulge and disk mass-to-light ratios, and dark halo masses. The dynamical models, which include an exponential disk with anisotropic velocity dispersion, a Hernquist bulge, and an NFW dark halo, are constructed directly from phase-space distribution functions that self-consistently solve the collisionless-Boltzmann and Poisson equations.

A line-of-sight central velocity dispersion and maximum circular velocity of the disk are measured to mimic actual observations. For each WD05 model, we calculate the maximum circular rotation speed, $V_c$, between $0.5R_{25}$ and 4 disk scale lengths. $R_{25}$ is the radius at which the surface brightness equals 25 mag arcsec$^{-2}$. We also calculate the line-of-sight velocity dispersion $\sigma_0$ within the effective radius of the bulge; aperture effects are minimal (just as observed with real data (F06)). The galaxy light concentration $C_{28}$ is calculated by summing up all the particles out to the edge of the galaxy and using suitable bulge and disk mass-to-light ratios.

We use a random selection of WD05 models constrained to match the size-luminosity and velocity-luminosity relations of galaxies and to obey $0.3 < V_{\text{disk}}/V_{\text{tot}} < 0.85$ (McGaugh 2005; Courteau et al. 2006; Dutton et al. 2006).

The light grey circles in Fig. are the result of 950 dynamically stable models. The model boundaries correspond to $C_{28} = 2.8$ for pure exponential (Freeman Type I) disks and $V_c = \sqrt{2}\sigma_0$ for isothermal systems. Real data can have $C_{28} < 2.8$ for Freeman Type II galaxies or systems with strong spiral arms (which WD05 simulations do not reproduce). It is intriguing that the measurements for many spheroidal (E+S0) galaxies have $V_c < \sqrt{2}\sigma_0$. While this departure from theoretical expectations is likely due to anisotropic velocity ellipsoids and/or the strength of the central cusp, more data on spheroid systems and independent data analyses in addition to model refinements are required to verify the observed trend.

As we expect from first principles and verify with the WD05 simulations, though limited as they may be, the distribution of stellar $M/L$ ratios follows roughly the $V_c - \sigma_0 - C_{28}$ line as we show in Fig. 2. A direct correlation between $C_{28}$ and stellar $(M/L)$ must exist but its calibration also awaits new data (i.e. stellar $M/L$’s measured from multi-band brightness profiles and accurate stellar population models.).

The $V_c - \sigma_0 - C_{28}$ relation (Eq. 5) is seemingly a product of the RL and VL relations in galaxies and dynamical considerations. A precise interpretation remains however beyond the range of our analytical models as a detailed understanding of galaxy formation and dynamics is still lacking (e.g. WD05; Dutton et al. 2006). For instance, the disk/spheroid-halo connection in galaxies and details about dissipational processes (e.g. cooling and feedback, adiabatic expansion or compression of the halo) and dynamical effects (e.g. angular momentum transfer).
that are currently ill-constrained. As well, the galaxy concentration parameter is likely a reflection of different mass accretion histories which defy accurate modeling at present. From an empirical standpoint, the scatter of scaling relations that involve dynamical parameters, (e.g. Faber & Jackson 1976 or Tully & Fisher 1977) should clearly be reduced by virtue of Eq. (5). We propose that the $V_c - \sigma - C_{28}$ relation be used as a new constraint for galaxy formation and dynamical models.

It is unfortunate that in 30 years since WKS79 the number of galaxies with measured central velocity dispersions and circular velocities has only grown from a few dozen to a couple hundred. The situation is especially dire for elliptical galaxies. It is of utmost interest for the study of galaxy formation and evolution, especially in the wake of large imaging surveys such as SDSS and UKIDSS (Hewett et al. 2006), that this predicament be remedied with all-sky, high spectral resolution ($R > 5000$) 2D spectroscopic surveys of thousands of galaxies of all Hubble types. These resolved velocity measurements will impact dramatically the study of galaxy scaling relations (e.g. fundamental plane analyses), the mass function of galaxies, and tests of baryonic physics to map the coupling between compact massive objects, bulges, disks and halos.

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FIG. 1.— Maximum circular velocity, $V_c$, versus central velocity dispersion, $\sigma_0$, for 154 spiral, 24 elliptical, 54 lenticular, and 11 dwarf irregular galaxies. It is assumed that gas velocities in spiral galaxies trace circular motion. The spirals systems show a clear dependence on morphological type such that, for a given dark matter halo ($V_c$), earlier-type galaxies have a higher $\sigma_0$. Massive galaxies scatter about the $V_c = \sqrt{2} \sigma_0$ line for isothermal stellar systems, with significant departures from the isothermal expectation for systems with $V_c < 200$ km s$^{-1}$.
Fig. 2.— Ratio of maximum circular velocity to bulge velocity dispersion versus concentration ratio for 32 spiral, 12 elliptical, and 33 lenticular galaxies whose surface brightness profiles could be extracted from SDSS images. The point types are the same as in Fig. 1. The galaxy concentration \( C_{28} = 5 \log(r_80/r_{20}) \) uses radii at 20% and 80% of the total galaxy light extrapolated to infinity. The light grey filled circles represent self-consistent equilibrium galaxy models by Widrow & Dubinski (2005) that satisfy the size-luminosity and velocity-luminosity relations of galaxies. The models extending to higher concentrations likely result from assuming (highly concentrated) NFW halos. The data-model comparison is described in §4. The dashed-line is a linear fit through the data for galaxies with measured concentrations. The solid arrow depicts the trend of increasing \( M_\ast/L \) in the \( V_c - \sigma_0 - C_{28} \) plane based on theoretical expectations.

\[
\log(V_c/\sigma_0) = 0.63 - 0.11C_{28}
\]