Value Driven Representation for Human-in-the-Loop Reinforcement Learning

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ABSTRACT
Interactive adaptive systems powered by Reinforcement Learning (RL) have many potential applications, such as intelligent tutoring systems. In such systems there is typically an external human system designer that is creating, monitoring and modifying the interactive adaptive system, trying to improve its performance on the target outcomes. In this paper we focus on algorithmic foundation of how to help the system designer choose the set of sensors or features to define the observation space used by reinforcement learning agent. We present an algorithm, value driven representation (VDR), that can iteratively and adaptively augment the observation space of a reinforcement learning agent so that it is sufficient to capture a (near) optimal policy. To do so we introduce a new method to optimistically estimate the value of a policy using offline simulated Monte Carlo rollouts. We evaluate the performance of our approach on standard RL benchmarks with simulated humans and demonstrate significant improvement over prior baselines.

CCS CONCEPTS
- Human-centered computing → Human computer interaction (HCI).

KEYWORDS
Reinforcement Learning, Human-in-the-Loop

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1 INTRODUCTION
Interactive adaptive systems powered by reinforcement learning can improve over time and have many potential applications. These include intelligent tutoring systems that improve their teaching as they instruct more students, smart home devices that adjust temperature settings in response to weather and human preferences, and mobile wellness applications that support chronic care management. In such cases the system is learning a good decision policy – a mapping from a current observation (of a student, of a home, of a person and their context) to action (what pedagogical activity to propose, how to set the temperature, what health nudge to suggest) in order to maximize overall outcomes over time (how much student has learned, minimize energy, maximize total activity).

In such systems there is typically an external system designer that is creating, monitoring and modifying the interactive adaptive system, trying to improve its performance on the target outcomes (Figure 1). A key question is how interactive adaptive systems can better provide input back to their system designers to help support the designer in improving the performance. In this paper we focus on one particular direction for this, how to help the system designer choose the set of sensors or features to define the observation space used by the interactive system to make decisions. For example, does the tutoring system only log student responses to problems, or is a webcam also used to detect frustration levels?

The observation space specification implicitly constrains the decision policy class the interactive system can optimize over. Here we consider the case when the interactive system itself can monitor its own performance, and propose to the system designer some potential modifications to the observation space that it expects might yield improved performance. The human designer can then choose to augment the system with additional features and/or sensors that the adaptive system can then use going forward. For example, the system may recognize that there are number of observations which are currently identical (for example, same performance on a set of problems) where selecting the action (next problem) does not seem to yield the best outcomes, and ask the human if there might be an additional feature/sensor that could be used to distinguish such currently aliased observations. This set of observations can provide
information to the human designer about what types of features/sensors might be useful to add to the system, like a webcam and emotion classifier to detect frustration.

In this short paper we focus on the algorithmic foundations of this idea, providing a proof of concept in simulated domains. Precisely, we present an algorithm, value driven representation (VDR), that can iteratively augment the observation space of a reinforcement learning agent (such as an interactive adaptive system), if the algorithm estimates that the resulting augmented observation space could yield an improved policy performance in the real environment. VDR can be applied both to situations where additional features could be added later in a demand driven way (from a human system designer), or when the full set of features is known in advance but there are computational, performance and interpretability benefits to starting with a more compact representation.

Our approach starts with a coarse observation space (a small minimal set of features). Note that this small subset may only be known because these are the initial features seem to be relevant by a human system designer or due to cost or other constraints. We assume the algorithm is part of an adaptive interactive system that is acting in a Markov decision process (MDP), but the set of initial features provided may be a small subset of the features needed to satisfy the Markov assumption. We can view the coarse observation space as a state abstraction. A goal of our algorithm is to be able to augment the observation space in order to reach the minimal set of features sufficient to make the same optimal decisions as would be possible with the full (unknown) set of features (formally known as optimal $\pi^*$-irrelevance abstraction [10]).

Our algorithm proceeds by proposing splits of existing observations that look identical under the current set of features. The key contribution of our work is to estimate the potential value of policies with new augmented observation space using old data without making the Markov assumption. Our algorithm for doing so is inspired by Upper Confidence Trees [9], a popular Monte Carlo Tree Search method, but adapted to focus on decision policy evaluation and, more importantly, does not require an MPD dynamics and reward so that it can be run using old data.

We evaluate the performance of our approach on several simulation domains where the true Markovian state space is known and demonstrate significant improvement over prior baselines. While a key next step is to try this out with a human designer in the loop, this is an encouraging step of the potential benefit of adaptively adjusting the feature representation.

2 RELATED WORKS
Reinforcement Learning in non-Markovian observation space has been long studied. UTree [14] is a history-based method that uses tree-based representations of the value function and splits observation based on local gain and predictive power. Predictive state representation (PSR) [1, 8, 11] is another history-based method that tries to find the sufficient statistics from history to represent a notion of state. In contrast, our algorithm focuses on utility gain (gain in the value of a policy) rather than the predictive power of the state representation.

Feature RL [7] is a framework that defines a mapping between history to states such that state representation becomes Markovian, and then uses general RL algorithms to solve the proposed MDP. A brief summary of FRL can be found in [3]. The main difference of our work with this line of research is that our agent does not seek a Markovian representation and finds the policy in a possibly non-Markovian observation space. Many other related works are based on the AIXI agent [6] a formal mathematical solution to the general RL agent, e.g. MC-AIXI-CTW [17, 23]; however, in these methods, the policy representation is not explicit and the agent needs to run UCT at every step.

Our work strongly relates to the state aggregation/abstraction literature [2, 19, 20, 22]. However, our work differentiates itself with those in the way that our algorithm starts learning in a small observation space that is often non-Markovian and then trying to augment the observation space to learn the optimal policy, similar to some Bayesian methods like iPOMDP [4] which learns a POMDP while growing the state space.

3 PROBLEM SETUP
We consider human-in-the-loop reinforcement learning, where a human system designer can modify the observation space definition used by a reinforcement learning agent, such as when a designer can modify the observation space of a RL intelligent tutoring system interacting students. More precisely, we assume the RL agent is acting in an episodic Markov decision process $M = (S, A, T, R, \gamma)$, where $S$ is a finite set of states (such as a student’s current state of learning), $A$ is a finite set of actions, and $T$ is a dynamics model that specifies $p(s' | s, a)$ – the probability of transitioning to state $s'$ after taking action $a$ in state $s$ (for example, the probability the student will not understand 1 digit addition, do a problem on addition, and transition to a new state $s'$ in which the student understands addition.) $R$ specifies the reward $r(s, a)$ received by taking action $a$ in state $s$: e.g., high reward when a student takes a test and passes it. $\gamma$ is a discount factor that weighs immediate vs future reward.

We assume the state space $S$ (such as the true internal state of the student) is only indirectly observable by the agent through sensors that provide the observation space $O$ (e.g., the agent can observe if the student got a problem correct). There is a many-to-one deterministic mapping from states to observations, and therefore the observation space can be viewed as an aggregated state space. We denote the aggregated states $s$ under observation $o$ as $S_o$.

A decision policy $\pi$ for the RL agent is a stochastic mapping from states to actions. The state-action value of a policy $Q^\pi(s, a)$ is the expected discounted sum of rewards the RL agent would obtain by taking action $a$ from state $s$ and then following the policy. In RL the dynamics and reward model are unknown.

The agent is provided with an initial observation space $O_0$ that can be modified by the human system designer. The goal is for the interactive reinforcement learning agent to, together with the designer, find the smallest observation space that yields the maximal expected reward policy such that the resulting policy matches the performance of the best policy under the (unknown) MDP.

4 ALGORITHM
We present a novel human-in-the-loop RL algorithm, Value Driven Representation (VDR). VDR involves two key components. First, VDR performs optimistic reinforcement learning given the current
observation space specification (e.g., it tries to optimize a decision policy for teaching a student given the current available features that distinguish student learning states) (Section 4.1). Second, VDR evaluates potential augmentations of the existing observation space that might enable a better decision policy and proposes to a human designer to split the observation that is evaluated to be most beneficial (e.g., for the human to provide another feature, like frustration, that can be used to refine an observation from "solved problem 1" to "solved problem 1 with frustration" and "solved problem 1 without frustration") (Section 4.2). Note in this initial work we only simulate input human experts and leave a human user study to later work.

Algorithm 1 VDR

1: \( O \leftarrow O_0, D \leftarrow [] \) // data
2: for each episode do
3: \((\pi, V^\pi) \leftarrow \text{optimistic Off-Policy Policy Optimization (D)}\)
4: Execute \( \pi \) for 1 episode and collect trajectory \( t, D \leftarrow D \cup t \)
5: for all observations \( o_t \in D \) do
6: Propose augmentation of \( o_t \rightarrow (o'_t, a'_t) \) // section 3.2
7: \( D' \leftarrow \text{relabel } o_t \text{ in the data } D \text{ with } o'_t, a'_t \)
8: \((\pi^*_{\text{o}}), V^*_{\text{o}}) \leftarrow \text{Off-Policy Policy Optimization (D')} // \text{section 3.1}
9: end for
10: \((\text{score}_{\text{best}}), o_{\text{best}}) \leftarrow \max D_{KL}(\pi^*_{\text{o}} || \pi(V^*_{\text{o}} - V^\pi))\)
11: if \( \text{score}_{\text{best}} \geq c \) then
12: \( O \leftarrow O \cup (o'_{\text{best}}, o_{\text{best}}) \setminus o_{\text{best}} \)
13: end if
14: end for

4.1 Off-Policy Policy Optimization

We first consider how an RL agent should act given the current policy \( \pi \) (Algorithm 1) for a particular node previously (e.g., at step \( d \)), we terminate the simulated roll-out and use a model free Monte Carlo estimate of the current observation-action pair Q-value \( Q(o_d, a_d) \) at the leaf node, computed by averaging over all returns obtained after observing \( (o, a) \) tuple. Our complete Off Policy Tree Evaluation (OPTE) is shown in algorithm 2 and 3, which takes the average of \( N \) simulations.

\[ V^\pi = \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{l-1} r_t + y^d Q(o_d, a_d) \right) \]

Algorithm 2 OPTE(\( \pi \), graph, \( N \), opt)

Input: \( \pi \): evaluation policy, graph: trajectory tree, \( N \): number of simulations, opt: optimism flag
1: for \( i = 1 : N \) do
2: node \( \leftarrow \text{graph}.\text{initialNode}()\)
3: \( o \leftarrow \text{node}.\text{observation}(); R(i) \leftarrow []; \text{done} \leftarrow \text{False} \)
4: while not done do
5: node, \( r \), done \( \leftarrow \text{nextNode}(\pi(o), \text{opt})\)
6: \( R(i).\text{append}(r); o \leftarrow \text{node}.\text{observation}()\)
7: end while
8: end for
9: return \( \frac{1}{N} \sum_i \sum_t y^t R(i) \)

Algorithm 3 next(node, a, opt)

Input: node in trajectory graph, a: action, opt: optimism flag
1: \( C \leftarrow 1 \)
2: counts \( \leftarrow \text{node}.\text{getTransitionCounts}(a)\)
3: counts.\text{append}(C) // Unobserved outcome
4: model \( \leftarrow \text{MLE(counts)}\)
5: nextNode \( \leftarrow \text{model}(a)\)
6: if nextNode is -1 then
7: // Sample a new potential outcome
8: \( o \leftarrow \text{node}.\text{OBSERVATION}(); \text{done} \leftarrow \text{True} \)
9: if opt == True then
10: \( r \leftarrow \text{graph}.Q(o, a) + \frac{r_{\text{max}}}{1+y} \sqrt{\frac{\log(n)}{n(o,a)}} \)
11: else
12: \( r \leftarrow \text{graph}.Q(o, a) \)
13: end if
14: else
15: \( r, \text{done} \leftarrow \text{nextNode}.\text{Info}()\)
16: end if
17: return \( (\text{nextNode}, r, \text{done})\)

Additionally, in order to encourage strategic exploration, we propose a method that is inspired by optimism under uncertainty approaches. Precisely, we take the OPTE algorithm (Algorithm 2) described above and add in a reward bonus to the \( Q(o, a) \) used at the tree leaves. Similar to upper confidence bound RL algorithms [18] we use \( Q(o, a) + \frac{r_{\text{max}}}{1+y} \sqrt{\frac{\log(n)}{n(o,a)}} \), where \( r_{\text{max}} \) is the maximum reward and \( n(o, a) \) are visitation counts of an observation and an observation-action pair. This approach is a minor modification of OPTE, which can be computed by setting the input opt to True in algorithm 3. Off-Policy Tree Optimization (OPTO) can be done using any policy optimization method combined with using simulated roll-outs on the trajectory tree \( T \).

4.2 Observation Augmentation

The initial input observation space may be insufficient to achieve a high performance policy: for example, it may be crucial to change
the policy depending on whether a student is frustrated after com-
pleting problem 1 correctly, yet initially this distinction may be
lacking in the observation space. We now propose how the RL sys-
tem can itself try to identify which observation refinements might
yield an improved performance (policy value) if a human designer
could provide a feature that distinguished between observations
that are currently aliased.

To do this, at every episode the algorithm creates and scores
\[ \{O_i\} \] new potential observation spaces. Each observation space
\[ O_i \] is derived from taking observation \[ o_i \] in the current \[ O_{current} \]
observation space, splitting it into two new observations \[ o_i^1 \] and \[ o_i^2 \],
and adding these two new observations to all of the other non-split
observations \[ o_k \notin O_{current} \] \[ (\{o_i^1, o_i^2, \forall k \notin I \} \}.

Splitting a particular observation \[ o_i \] into two is performed by
executing Expectation Maximization (EM) [16] on the existing col-
lected trajectories to hypothesize 2 potential latent observations
with different dynamics and/or rewards models. Given the EM
learned parameters for a observation split \[ (o_i \rightarrow o_i^1 \text{ or } o_i^2) \], the Viterbi algorithm [5] can be used to relabel prior
trajectories, turning all instances of observation \[ o_i \] into \[ o_i^1 \] or \[ o_i^2 \].

Using the relabeled trajectories \( D' \), we can build a new trajectory
tree \( T' \) and perform off-policy policy optimization using OPTO to
estimate the value of the best policy \( V^{T_{+1}} \) for the modified space.

The objective is to present an augmentation to the system de-
signer if 1) splitting an observation yields an optimal policy with
a higher value than the existing best policy for the observation
space, and 2) if the best policy for the augmented observation
( evaluated using OPTO) is sufficiently different than the previous
best policy, measured by the KL-divergence of two policies. Pre-
cisely, define \[ score(i) = D_{KL}(\pi^i || \pi^*) \] \[ (V^i - V^{*}) \] where \( D_{KL} \)
is the KL-divergence, \[ \pi^i \] is the optimal policy after splitting ob-
ervation \[ o_i \]. Our agent proposes an observation augmentation
\[ i = \arg \max_i score(i) \] to the system designer if \( \max_i score(i) \geq c \)
exceeds a threshold \( c \).

5 SIMULATION RESULTS

This human-in-the-loop RL system is designed ultimately to be used
for helping humans and RL agents best work together to achieve
a good representation that can be used to quickly identify good
policies. However, as an initial proof of concept, we first conduct
simulated experiments where a simulated human designer will
agree a proposed observation refinement is beneficial if \( c = 0.25 \)
and where the revised associated observation is split as follows.
Assuming MDP states \( s \in S_0 \) was clustered under observation \[ o \],
we assign state \( s \) to observation \[ O^1 \] \[ (or \ O^2) \] if more than 50 percent
of the time \( s \) was assigned to \[ O^1 \] \[ (or \ O^2) \] by the Viterbi algorithm.

We simulate our VDR algorithm in two existing RL tasks. One
is a navigation task CheeseMaze [23] inspired by a robot needing
to learn how to reach a destination: the robot requires particular
features in order to be able to learn and represent the optimal policy.

The second is mountain car [21], where a car on a hill must reach
the goal position up the right side of the hill (figure 3a).

The goal is to see if our method will propose the necessary
augmentation to the simulated human to learn the optimal policy.
Additionally, we are interested to evaluate how fast our algorithm
can learn starting from a coarse observation space, when repre-
senting the optimal policy does not require full Markovian state
space.

5.1 Cheese Maze

Cheese Maze was used as a benchmark environment in [23]; for
details refer to figure 2a. We set the maximum length of each episode
to 20, and consider an augmentation every 5 episodes. We com-
pare to MC-AIXI-CTW that outperformed other history based and
feature RL method including UTree [14, 23].

As shown in Figure 2b VDR outperforms MC-AIXI-CTW and
finds the optimal policy in fewer number of episodes. VDR finds the
optimal policy by splitting observation 1 (see figure 2a) into two
(1a, 1b) that requires different action for representing the optimal
policy in center and left side of the maze.

5.2 Mountain Car

We considered mountain car where the agent always starts at the
same location and velocity \( x_0 = -0.5, v_0 = 0.03 \). In all simulations

Figure 2: Cheese Maze, (a) Environment, agent has 4 actions:
move up, down, left, right and receives a reward of -10 if
it hits the wall, -1 for moving to an empty state and 10 for
getting to the cheese state which also teleports the agent
to the initial state (bottom left). Numbers in each cell indicate
the initial observation space, and split of observation 1 into
(1a, 1b). (b) Comparison of our algorithm to MC-AIXI-CTW

Figure 3: Mountain Car, (a) Environment. (b) Comparison
of VDR with Q-learning, tile coding and DQN. (c) Observation
space augmentation: each color represents distinct observa-
tion and the observation space is shown superimposed on
the underlying 8x8 grid.
the initial starting position and velocity is fixed for that entire process. We set the maximum episode length to 500 and consider an observation augmentation every 20 episodes. We treat the underlying true state space as a discrete 8x8 grid, though the true space is best modeled continuously. We compared our algorithm with Q-learning with ε-greedy exploration on 20x20 grid (location and velocity). Additionally we compared to DQN [15] (with two hidden layers of size 64) and tile coding with 2 tilings.

Figure 3b shows the result for \(v_0 = 0.03\), where VDR can learn the optimal policy in 100 episodes with only 3 augmentations: this is enough to represent the optimal policy. Our experiments with other initial velocity (\(v_0 = 0.05\)) showed the same results. Figure 3c shows the observation augmentation. Starting with only two observation shown in figure 3c superimposed on the underlying 8x8 grid, VDR find the optimal policy after three splits.

6 FUTURE WORK AND CONCLUSION

Adaptive interactive reinforcement learning systems often benefit from a human-in-the-loop system designer that can modify the state or action space of the RL system to improve performance. Given the potential set of modifications, guidance from the RL system to the human designer could be helpful. In this short paper we propose a way for a RL system to proactively propose augmentations to its observation representation that may enable improved performance. Our simulations suggest the potential benefit of this approach in small domains. Exploring the scalability of this approach and testing it with real humans in the loop are clear interesting next steps.

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A DETAILS OF VDR ALGORITHM

In this section we present some details of VDR algorithm (algorithm 1). Section A.1 describes the details of off-policy policy optimization, section A.2 describes the details of augmenting the observation space and section A.3 presents a running example of a trajectory tree.

A.1 Off-Policy Policy Optimization

A natural question is how can we update the policy parameters when using OPTO? By using the trajectory tree \(\mathcal{T}\) as a simulator, any policy optimization method can be applied by performing rollouts on the trajectory tree \(\mathcal{T}\) to achieve a (near) optimal stochastic policy. For example, if the policy is parameterized by \(\theta\), one can use REINFORCE [24] to update the parameters using \(\theta_{t+1} = \theta_t + \alpha \nabla J(\theta)\), where \(\nabla J(\theta)\) is

\[
\nabla J(\theta) = \mathbb{E}_T [G_T \nabla_\theta \pi(a_t | s; \theta)] / \pi(\hat{a}_t | s; \hat{\theta})
\]

Where \(G_T\) is the discounted return of an episode and the expectation can be calculated with a sample episode on the trajectory tree \(\mathcal{T}\), as described in algorithm 4.

Algorithm 4 REINFORCE on \(\mathcal{T}\)

1. repeat
2. Generate an episode with \(\pi_\theta\) on \(\mathcal{T}\)
3. for \(t = 0, \ldots, T-1\) do
4. \(G_t \leftarrow \text{return from step } t\)
5. \(\theta \leftarrow \theta + \alpha \gamma^t G_t \nabla_\theta \log \pi(a_t | s_t; \theta)\)
6. end for
7. until timeout
A.2 Splitting

In order to split (augment) an observation \( o \in O \), we notice that:

1) A predicted improvement in the value could arise due to more accurate estimates of transition probabilities and rewards. Therefore a split is only done if the best policy for the augmented observation space is sufficiently different from the best policy before the split. In order to compare the two policies, we augment the policy before split \( \pi_{old} \) to \( \pi'_{old} \) by setting \( \forall a \in A : \pi_{old}(o, a) = \pi'_{old}(a_1, a) = \pi_{old}^*(a_2, a) \), and the same policy for all other observations.

2) Observation augmentation will turn one observation into two, and by definition indicates that at least one observation will now have less counts in observed trajectories compared to when that observation was not refined. This reduced data will generally increase the variance of the computed estimated values of the possible refined observation spaces. Therefore we use a bootstrap procedure on the all old data to compute \( B \) estimates of the refined observation values. In deciding whether to split, we compare the potential benefit to the estimated standard deviation across bootstrap estimates of the value of the new proposed observation splits as

\[
\text{score}(i) = \frac{D_{KL}(\pi'_{old}||\pi_i)\cdot \text{avg}(V_{\pi_i}) - V_{\pi_{old}}}{\text{std}(V_{\pi_{old}})}
\]

Where \( D_{KL} \) is the KL-divergence. We split if score exceeds an input threshold. This is a heuristic estimate of a significance test (Z-score) for whether the algorithm is confident that the new split representation will outperform the prior. Pseudo code for the complete splitting procedure is shown in Algorithm 5.

**Algorithm 5 Split(\( \pi_{old} \)-graph, \( B = 10 \), \( N = 100 \))**

1. \( u_{old} \leftarrow \text{OPTE}(\pi_{old}, \text{graph}, N, \text{False}) \)
2. \( \pi'_{old} \leftarrow \text{augment the policy } \pi_{old} \)
3. for \( o_j \in O \) do
4. \( o_1^j, o_2^j \leftarrow o_j \text{ // use EM to split observation} \)
5. \( O_j \leftarrow (O \setminus o_j) \cup \{o_1^j, o_2^j\} \)
6. Construct \( \text{graph}_{ij} \) for \( O_j \)
7. \( \pi_j = \max_{\pi} \text{OPTE}(\pi, \text{graph}_{ij}, N, \text{False}) \)
8. for \( b = 1 : B \) do
9. Construct \( \text{graph}_b \) for bootstrap \( b, O_j \)
10. \( V_{\pi}(b) = \text{OPTE}(\pi_j, \text{graph}_b, N, \text{False}) \)
11. end for
12. \( \text{splitScore}(j) = D_{KL}(\pi_{old}||\pi_j) \cdot \text{avg}(V_{\pi_j}) - u_{old} \)
13. end for
14. if \( \text{max(splitScore)} > \text{thresh} \) then
15. \( J \leftarrow \text{argmax}_{j} \text{splitScore} \)
16. \( O \leftarrow O_j \)
17. end if

A.3 Example of Trajectory Tree

In order to make trajectory tree \( T \) clear, here is a concrete example of an environment and a trajectory associated with it.

We use a 3-state deterministic Markov decision process introduced by [13] (see Figure 4 for full details). Consider that at the start the initial observation space \( O_0 \) aliases all 3 states into a single observation \( o_1 \). Also assume that each episode lasts for 3 time steps, and an agent has acted in this decision process for 2 episodes where it only has access to the observations space (and not the true states). Let this initial data be as defined in Table 1 which displays both the true latent states \( (S) \) and the observations \( (O_0) \) available to the agent.

Figure 4: 3-state MDP introduced by [13]. Agent can take an action right \((a_1)\) or left \((a_2)\) from any T-intersection, agent is always facing the wall at any intersection and starts at \( s_3 \). Action from \( s_1 \) and \( s_2 \) immediately teleports to state \( s_1 \). Immediate reward is sum of two components: some negative reward for walking plus reward through reward \((R)\) or punishment \((P)\) gate.

Here there is only 1 observation and only 1 potential split. While EM is a procedure only guaranteed to yield a local optima, in this case one such optima would be that states \( s_1 \) and \( s_2 \) remain aliased to a single observation \( o_1^1 \) but state \( s_3 \) is distinguished and represented as observation \( o_2^1 \), as illustrated in Table 1. Given this split, Figure 5 shows the trajectory tree \( T \) obtained by data shown in table 1.

|Trajectory 1| | Trajectory 2|
|---|---|---|
|\( S \)| \( (s_1, a_1, 0.7) \) | \( (s_1, a_1, 0.7) \) |
|\( O_0 \)| \( (s_1, a_1, 0.7) \) | \( (s_1, a_1, 0.7) \) |
|\( O_1 \)| \( (o_1^1, a_1, 0.7) \) | \( (o_1^2, a_1, 0.7) \) |

Table 1: 2 episodes of 3 time steps from 3-state MDP. \( S \) denotes the Markov sequence of latent states, actions and rewards, \( O_0 \) shows the sequence observed to the agent given an observation space that aliases all the states, \( O_1 \) shows the sequence observed to the agent after the split of \( o_1 \) into \( o_1^1 \) and \( o_1^2 \).

B ASYMPTOTIC ANALYSIS

In this section we analyze the asymptotic behaviour of our proposed algorithm, under the assumptions of infinite data. This assumption allows us to study the asymptotic behaviour, however as shown in experiments, our empirical evaluation shows the desired effect with limited data. By this assumption we can have an accurate estimate of \( V_{\pi} \) using \( \text{OPTE} \) by setting \( C = 1 \) and \( \text{opt} \) to \( \text{False} \).
Figure 5: Trajectory Tree $T$ of 2 episodes for 3-state MDP after 1 observation split, for the data shown in Table 1. Actions are represented by edges which include the count $n$ of the number of times the action has been taken given its ancestor nodes, and the rewards obtained during those experiences. Nodes are labeled with observations. Here as the dynamics are deterministic, each action goes to a single next observation, but more generally there can be trajectory branching for each action.

**Lemma B.1.** Let $M$ be a Markov decision process, and $T$ be a trajectory tree generated by infinite data gathered using a policy $\pi$ s.t. $\forall o, a : \pi(o, a) > 0$. Then $\forall \pi : O \times A \rightarrow [0, 1], V_\pi^T \xrightarrow{i.p.} V_\pi^M$. Where $V_\pi^T$ and $V_\pi^M$ are values of the policies evaluated in trajectory tree $T$ and MDP $M$, respectively.

**Proof.** This follows by the fact that with infinite data evaluating the policy using $T$ doesn’t require bootstrapping $Q$ values at the leaf, and all the history based transition probabilities and rewards converges in probability to their real values by the law of large numbers. As a result value of the policy: $\sum_{t=1}^{\infty} P(t) G(t)$ converges in probability to its value in MDP $M$, where $t \sim \pi$ are all the trajectories generated by policy $\pi$, $P(t)$ is the probability of trajectory under policy $\pi$, and $G(t)$ is the return of the trajectory.

However, in the case of limited data the accuracy of $V_\pi^T$ depends on the accuracy of the model free $Q(o, a)$ estimates.

**Theorem B.2.** Let $M$ and $\hat{M}$ be Markov decision processes over the same action space $A$ and state spaces $S, \hat{S}$, respectively. Where $\hat{S} = S \setminus s_0 \cup \{s_0^1, s_0^2\}$ such that $s_0^1, s_0^2$ are the split of state $s_0$. Let $\pi^* : S \rightarrow A$ be the optimal policy in $M$. Then, $\exists \pi' : \hat{S} \rightarrow A$ such that $V_{\pi'}^\pi = V_{\pi'}^M$ where $\pi'(s_0^1) = \pi'(s_0^2) = \pi^*(s_0)$, and $\forall \pi' \neq \pi : V_{\pi'}^M \leq V_{\pi'}^M$.

**Proof.** Since both $M$ and $\hat{M}$ are MDP, W.L.G we assume that all polices are deterministic. Setting $\pi'(s_0^1) = \pi'(s_0^2) = \pi^*(s_0)$ will trivially retrieve the optimal policy of $M$ with the same value $V_{\pi^*}^M = V_{\pi^*}^M$. Now consider two cases when:

1. $\pi'(s_0^1) = \pi'(s_0^2)$: then $\pi'$ is also a policy in $M$ with $\pi'(s_0^1) = \pi'(s_0^2) = \pi^*(s_0)$, where $V_{\pi'}^M \leq V_{\pi^*}^M = V_{\pi^*}^M$ since $\pi^*$ is the optimal policy in $M$.

2. $\pi'(s_0^1) \neq \pi'(s_0^2)$: W.L.G assume $V_{\pi'}^\pi(s_0^1) \geq V_{\pi'}^\pi(s_0^2)$, we show that the policy can be improved by setting $\pi'(s_0^1)$ equal to $\pi'(s_0^2)$. Then followed by case 1, $V_{\pi'}^M \leq V_{\pi^*}^M = V_{\pi^*}^M$.

$Q_{\pi'}^\pi(s_0^1, \pi'(s_0^2))$

$= R(s_0^1, \pi'(s_0^2)) + \sum_{s' \in \hat{S}} p(s'|s_0^1, \pi'(s_0^2)) V_{\pi'}^\pi(s')$

$= R(s_0^2, \pi'(s_0^2)) + \sum_{s' \in \hat{S}} p(s'|s_0^2, \pi'(s_0^2)) V_{\pi'}^\pi(s')$

$= Q_{\pi'}^\pi(s_0^2, \pi'(s_0^2)) = V_{\pi'}^\pi(s_0^2) \geq V_{\pi'}^\pi(s_0^1) = Q_{\pi'}^\pi(s_0^1, \pi'(s_0^1))$

The second equality comes from the fact that in Markovian representation, state is sufficient to determine the transition probabilities and rewards so,

$\forall s, a : P(s|s_0^1, a) = P(s|s_0^2, a)$

$a \in \hat{A}$: $R(s_0^1, a) = R(s_0^2, a)$

$\square$

Theorem B.2 states that, by Lemma B.1, our algorithm will not split a Markov representation further. However, in the case of limited data and using policy optimization to find the optimal policy, we might not get an accurate estimate of the value, or find the deterministic policy that is optimal in MDP.

The $\pi^*$-irrelevance abstraction [10] is a state abstraction of an MDP that, in every cluster, the optimal action is the same. The following lemma states that our algorithm will not split an observation space that is a $\pi^*$-irrelevance abstraction.

**Lemma B.3.** If the observation space $O$ is $\pi^*$-irrelevance abstraction of an MDP $M$, our algorithm will not split further.

**Proof.** (sketch) Based on theorem B.2, the optimal policy is the optimal policy in Markovian state representation, and the optimal policy in $\pi^*$-irrelevance abstraction has the same value as the optimal policy for $M$. Thus, there does not exist a split that yields higher return and our algorithm will not split further. $\square$