Study of a two-spring electromagnetic harvester

N P Georgiev¹ and R P Raichev²

¹Department of Electrical Engineering, Technical University of Sofia, Branch Plovdiv, 25Tsanko Dystabanov Street, Plovdiv, Bulgaria
²Department of Mechanical Engineering, Technical University of Sofia, Branch Plovdiv, 25Tsanko Dystabanov Street, Plovdiv, Bulgaria

¹nikola.georgiev@tu-plovdiv.bg
²rpraichev@tu-plovdiv.bg

Abstract. The paper studies an electromagnetic harvester, consisting of a П-shaped plate with 4 magnets, placed on two springs, rigidly fixed at their low end. A coil lies between the magnets. Modeling by means of ANSYS has been done and the horizontal deviation of the mechanical system has been obtained. The magnetic field of the electromagnetic harvester has been modeled by Femm and its electromotive force has been defined. Experimental studies have been conducted for validating the developed model.

1. Introduction
Magnetic harvesters are more and more frequently used for converting mechanical energy into electrical and thus they replace batteries and low-consumption power-supplying electronic devices [1, 2]. Two types of motion of the magnet with respect to the coil are normally used. In the first case the magnet, which is most frequently placed in the air gap of the coil, moves in parallel with it [3], while in the second its movement is vertical in relation to the coil [4].

The present paper studies a two-spring electromagnetic harvester with two different types of mass in the form of two П-shaped steel plates, with 4 magnets each. The harvester has a rigidly fixed coil, around which two pairs of rare-earth magnets move in parallel. Modeling by means of ANSYS has been done and the horizontal deviation of the mechanical system has been obtained. With the help of Femm 4.2 the magnetic field has been modeled and the magnetic induction has been determined. The magnetic fluxes through the winding and the induced e.m.f. of the harvester have been calculated. The voltages and the powers have been both calculated and measured at direct-current mode.

2. Exposition
Figure 1 presents the construction of the two-spring electromagnetic harvester. The following notations have been used: 1 – base; 2- steel springs; 3 – П-shaped steel plate; 4 - rare-earth magnets; and 5 – a coil. The springs are with wire diameter \(d_s=0.8\) mm, diameter of the spring \(D_s=6.7\) mm and length of the spring \(l_s=20\) mm. The П-shaped steel plates are two in number, with the same dimensions and with 4 permanent magnets fixed to each of them. The magnets on the first plate are with dimensions 20x10x2 mm and weight \(m_1=15\) grams, while the second plate is with magnets, measuring 20x10x3 mm and weighing \(m_2=22\) grams. The diameter of the coil is \(D_c=20\) mm, its thickness - \(l_c=10\) mm, number of turns \(N=800\), and wire diameter \(d_c=0.1\) mm.

The trajectory of the plates with the magnets in the considered electromagnetic harvester is an arch and thus a non-linear mechanical oscillating system is obtained.
When a force, changing in a sinusoidal way, with a resonant frequency is applied, the mechanical system, consisting of a mass (Π-shaped plate with permanent magnets) and two springs starts oscillating in parallel with the coil. Thus the magnetic flux goes differently through the coil and creates an alternating magnetic field, whereas alternating e.m.f. is induced.

The harvesters under consideration are non-linear mechanical oscillating systems. During their simulations by ANSYS R19.1 the following parameters have been taken into account: the way of fixing, the influence of the force of weight on the Π-shaped plates with permanent magnets, as well as the mechanical characteristics of the used springs. During the process of modelling the horizontal deviation $x$ of the mechanical system, consisting of a mass (Π-shaped plate with permanent magnets) and two springs, has been obtained, Figure 2.

![Figure 1. Two-spring electromagnetic harvester](image1.png)

![Figure 2. Simulation of the mechanical deviation of a harvester](image2.png)

![Figure 3. Distribution of the magnetic field for a small-mass harvester](image3.png)

![Figure 4. Distribution of the magnetic field for a big-mass harvester](image4.png)

![Figure 5. Densities of the magnetic induction for a harvester with small springs](image5.png)

![Figure 6. Densities of the magnetic inductions for a harvester with big springs](image6.png)
The distribution of the magnetic field in the electromagnetic harvester is obtained by means of Femm 4.2. Figure 3 illustrates the distribution of the magnetic induction at maximum horizontal deviation for a small-mass harvester, while Figure 4 gives the picture for a harvester with a big mass. Figure 5 shows the densities of the magnetic induction along the horizontal \( x \) - \( B_{cx} \) and the vertical \( y \) - \( B_{cy} \), as well as the volume of the winding \( V_c \) for a small-mass harvester, and Figure 6 shows the same picture for the harvester with a big mass. The maximum and the minimum magnetic induction \( (B'_{cymax} \text{ and } B'_{cymin}) \) along \( y \) are calculated with the help of \( B_{cy} \) and \( V_c \) for the two types of mass.

\[
B'_{cymax} = \frac{B_{cymax}}{V_c} \quad B'_{cymin} = \frac{B_{cymin}}{V_c}
\]

(1)

Thus the difference between the maximum and the minimum magnetic induction \( \Delta B \) through the coil can be obtained

\[
\Delta B = B'_{cymax} - B'_{cymin}
\]

(2)

![Figure 7. Horizontal deviation at a sinusoidal force](image)

The horizontal deviation at a force, changing in a sinusoidal way, with a resonant frequency \( \omega \), is also sinusoidal, Figure 7

\[
x(t) = X_m \sin \omega t
\]

(3)

The horizontal oscillation velocity is equal to

\[
v(t) = \frac{dx(t)}{dt} = \omega X_m \sin (\omega t + \frac{\pi}{2})
\]

(4)

The magnetic flux can be expressed by the change in the cross-section of the coil \( A \), through which the difference between the maximum and the minimum magnetic induction \( \Delta B \) goes

\[
\Phi(t) = \Delta B \frac{dA(t)}{dt}
\]

(5)

The diameter of the coil \( D_c \) is multiplied by a coefficient 0.785 due to the transformation from a circle to a square because of Femm 4.2

\[
D_c = 0.785 D'_c
\]

(6)

The change of the cross-section \( A \) in the course of time is equal to the changing horizontal deviation \( x(t) \) multiplied by the diameter \( D_c \)

\[
\frac{dA(t)}{dt} = D_c x(t)
\]

(7)

From (5) and (7) for the magnetic flux through the coil it is obtained
The induced e.m.f. in the winding of the electromagnetic harvester is
\[ \Phi(t) = \Delta B \, D_c \, x(t) \quad (8) \]

The induced e.m.f. in the winding is given by
\[ e(t) = -N \frac{d\Phi(t)}{dt} \quad (9) \]

From (8) and (9) for the induced e.m.f. at idle run mode it is obtained
\[ e(t) = -N \Delta B \, D_c \, \frac{d x(t)}{dt} \quad (10) \]

The amplitude of the induced e.m.f. in the winding is in function of the amplitude of the oscillation velocity \( \omega X_m \)
\[ E_m = N \Delta B \, D_c \, \omega X_m \quad (11) \]

The circular frequency of the forced oscillations can be expressed by means of the frequency \( f \), Figure 7
\[ \omega = 2\pi f \quad (12) \]

From (11) and (12) for the amplitude of the induced e.m.f. in the winding at resonance it is obtained
\[ E_m = N \Delta B \, D_c \, 2\pi f \, X_m \quad (13) \]

Thus with the help of the calculated by Femm 4.2 magnetic induction and the given resonant frequency, number of turns, diameter of the coil and horizontal deviation, the amplitude of the induced e.m.f. in the winding can be calculated at idle run for the corresponding resonant frequency.

Figure 8 shows the equivalent circuit of the electromagnetic harvester with a doubling voltage amplifier at active load. By \( R_c \) and \( L_c \) the active resistance and the inductance of the winding of the electromagnetic harvester are denoted, by \( e_c(t) \) - the induced e.m.f., and by \( R_L \) – the active load resistance. The rectified voltage on the load resistance is denoted by \( U_L \).

The active power at direct-current mode is calculated using the amplitude of the induced e.m.f. in the winding at idle run and at resonance, as well as with the help of the voltage on the germanium diode \( U_D \) and the parameters of the equivalent circuit (14).

\[ P_L = \frac{1}{R_L} \left( \frac{E_m R_L}{\left( \frac{R_C}{b} + R_L \right)^2 + \left( \omega L_c \right)^2} - U_D \right)^2 \quad (14) \]
In the obtained expression \( b \) is the coefficient of attenuation, defined by means of the logarithmic decrement of attenuation \( \delta \) [5], where \( T \) is the period of oscillation and \( E_m(t) \) - the amplitude of the measured e.m.f.

\[
\begin{align*}
  b &= 2m\delta, \\
  \delta &= \frac{1}{T} \ln \frac{E_m(t)}{E_m(t+T)}
\end{align*}
\]  

(15)  

(16)

3. Experimental studies

Figure 9 presents the characteristics of the relationship between the rectified voltage at idle run and the frequency of the small mass harvester \((m_1=15 \text{ grams})\), while Figure 10 gives them for a big mass \((m_2=22 \text{ grams})\).

![Figure 9](image1.png)

**Figure 9.** Rectified voltage at idle run as a function of the frequency of the small mass harvester

![Figure 10](image2.png)

**Figure 10.** Rectified voltage at idle run as a function of the frequency of a big mass harvester
It can be seen from the characteristics that with the increase in the mass, the resonant frequency of the harvesters decreases, while the maximum voltage at idle run increases. Figure 11 shows the characteristics of both the measured and the obtained during modeling direct-current resistances of a small-mass harvester, while Figure 12 gives the same characteristic for the big-mass harvester at load resistances from 0 to 100 kΩ. The voltages of a big-mass harvester are three times greater than the ones of a small-mass harvester for the entire studied range of resistances.

**Figure 11.** Rectified voltage as a function of the load resistance for a harvester with a small mass.

**Figure 12.** Rectified voltage as a function of the load resistance for a harvester with a big mass.

Figure 13 shows the characteristics of the measured and the obtained by modeling direct-current powers for a small-mass harvester, while Figure 14 gives the characteristics for a harvester with a big mass. It can be seen from the characteristics that with the increase in the active load the output powers increase and reach a maximum at 50 kΩ. With the increase in the mass both the measured and the
obtained by modeling powers considerably increase. The powers of a big-mass harvester are ten times greater than the ones, of a harvester with a small mass for the entire studied range of load resistances.

Figure 13. Active power in function of the load resistance of a harvester with a small mass

Figure 14. Active power in function of the load resistance of a harvester with a big mass

Table 1 presents the maximum voltages and powers for the corresponding resonant frequencies of the considered electromagnetic harvester with the two types of mass.

Table 1. Measured voltages, powers and resonant frequencies for a harvester at two types of mass

| Mass (gr) | Maximum voltage (V) | Maximum power (μW) | Resonant frequency (Hz) |
|-----------|---------------------|--------------------|-------------------------|
| 15        | 0.99                | 11.3               | 7                       |
| 22        | 3.12                | 105.8              | 6                       |
4. Conclusions

Theoretical derivations, programming simulations and experimental studies of an electromagnetic harvester with a mechanical system, consisting of a mass (H-shaped plate with permanent magnets), two springs and a fixed coil, have been carried out. The harvester under consideration has two types of mass – a smaller and a bigger one.

Modeling of the two-spring electromagnetic harvester for the two types of mass has been done by ANSYS R19.1 and the maximum horizontal deviation $X_m$ of the mechanical system has been obtained. The distribution of the magnetic field in the electromagnetic harvester has been defined by means of Femm 4.2. Modeling has been performed, by means of which an expression for the induced e.m.f. has been derived. An equivalent electric circuit of the electromagnetic harvester has been drawn and an expression for the direct-current power at active load has been found. From the frequency characteristics it can be seen that with the increase in the concentrated mass the resonant frequencies of the harvester decrease, while the rectified voltages increase. Both the measured and the obtained by modeling maximum powers increase with the increase in the mass. When the active load increases, the output powers increase and reach a maximum at 50 kΩ.

The voltages of the big-mass harvester are three times greater than the ones, of the harvester with a small mass for the entire studied range of load resistances. The powers of a big-mass harvester are ten times greater than the powers of the harvester with a small mass for the entire range of load resistances. Consequently, a harvester with a big mass has a considerably better output electrical parameters and is more appropriate for practical use.

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