The 0$^{-+}$ and 0$^{++}$ glueballs
and the multiplets including them

Michał Majewski
Department of Theoretical Physics II, University of Lodz
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Abstract

The properties of the 0$^{++}$ and 0$^{-+}$ meson multiplets are discussed. Quoted are the 0$^{++}$ and 0$^{-+}$ glueball masses determined from data fit.

1 Introduction

The existence of glueballs is predicted by QCD, but to day none is definitively established. The main problem is their identification.

There are predictions of lattice QCD simulation [1] for the lowest mass glueball of given signature $J^{PC}$. For the 0$^{++}$ and 0$^{-+}$ glueballs they are: $m_{0^{++}} \simeq 1.5$GeV and $m_{0^{-+}} = 2.3 \pm 2.5$GeV.

There are also experimental statements. At present, the $f_0(1500)$ and $\eta(1405)$ mesons are allowed as glueball dominated mixtures with isoscalar $q\bar{q}$ states [2]. The data confirm lattice prediction for the 0$^{++}$, but they drastically disagree with the prediction for the 0$^{-+}$ glueball. This makes a trouble as people trust the lattice predictions.

Of course, we may look for another field theoretical approach to describe the data. Such description already exists. Faddeev, Niemi and Widner proposed recently a topological model of the glueball as closed flux tube [3]. The model predicts degeneracy of the 0$^{++}$ and 0$^{-+}$ glueball masses and admits the region 1.3 \pm 1.5$GeV, where they are really observed.

However, there is also a problem of exploiting data. Both the $f_0(1500)$ and $\eta(1405)$ have been chosen as glueball candidates from among a few isoscalar mesons on a basis of qualitative information about their production and decay (e.g., "gluon rich environment", or "flavor independence"). Although the predictive and verification power of such procedure is not high, it is the only generally accepted method of the glueball identification. However, such a procedure exploits only part of the accessible data which can be used.

Overpopulation of a nonet is an important signal of the glueball. The glueball should mix with the isoscalar nonet states. All properties of the decuplet which arises this way, including mixing between three isoscalar states, can be described entirely by the masses. Hence, the data on the masses can be used to determine parameters of the glueball.

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†e-mail: m.majewski@merlin.phys.uni.lodz.pl
2 Exotic commutators and master equations

We assume that the following set of exotic commutators vanishes [4]:

\[
[T_\alpha, \frac{d^j T_\beta}{d^j t}] = 0, \quad (j = 1, 2, 3, \ldots)
\]

where \(T\)s are \(SU(3)_F\) generators, \(t\) is time and \((\alpha, \beta)\) is an exotic combination of indices; that means that \(T_\alpha, T_\beta\) are chosen such that operator \([T_\alpha, T_\beta]\) does not belong to the octet representation. These equations are basic for the model of exotic commutators (ECM). They can be transformed [4] into the system of \textit{master equations} (ME):

\[
\langle z_8 \mid (\hat{m}_c^2)^j \mid z_8 \rangle = \frac{1}{3} a_c^j + \frac{2}{3} b_c^j, \quad (j = 1, 2, 3, \ldots)
\]

where \(\hat{m}_c^2 = \hat{m}^2 - i\hat{m}\hat{\Gamma}\) is complex-mass squared operator and \(\hat{m}\) and \(\hat{\Gamma}\) are hermitean and commute. The operator \(\hat{m}_c^2\) can be diagonalized and has orthogonal eigenfunctions. \(|z_8\rangle\) is the isoscalar octet state, \(a_c\) is the isovector particle mass squared, \(b_c = 2K_c - a_c\) and \(K_c\) is the mass squared of the isospinor particle.

The substitution \(|z_8\rangle = \Sigma l_i|z_i\rangle (\Sigma|l_i|^2 = 1)\), where \(|z_i\rangle\) are isoscalar physical states (i=1,2 – for the nonet and i=1,2,3 – for the decuplet) transforms ME into a system of linear equations with respect to octet contents \(|l_i|^2\):

\[
\Sigma|l_i|^2 z_i^j = \frac{1}{3} a_c^j + \frac{2}{3} b_c^j, \quad (j = 0, 1, 2, 3, \ldots)
\]

where the equation for \(j = 0\) takes into account the normalization of \(l_i\)s. To find \(|l_i|^2\) we need at least two equations – for the nonet and three – for the decuplet. Any additional equation must comply with the solution and thus sets requirement on the masses. We can choose the number of equations for the nonet and decuplet such as to have just one complex mass formula. The \(|l_i|^2\)s being the solution of the ME are expressed by the complex masses, but they must be: 1\(^{\text{th}}\) real, 2\(^{\text{nd}}\) positive.

1\(^{\text{st}}\). The condition imposes a linear dependence between the widths and masses (straight \textbf{flavor stitch line (FSL)}) – the same for the nonet and its decuplet extension. The dependence reduces the solution \(|l_i|^2\) (not quoted here) and the complex mass formulae to the form of the real mass meson multiplets:

\[
(x_1 - a)(x_2 - a) + 2(x_1 - b)(x_2 - b) = 0 \quad \text{for the nonet}
\]

and

\[
(x_1 - a)(x_2 - a)(x_3 - a) + 2(x_1 - b)(x_2 - b)(x_3 - b) = 0 \quad \text{for the decuplet.}
\]

Here \(x_i\) are the masses squared of the isoscalar particles \(z_i\).

2\(^{\text{nd}}\). This requirement, together with the mass formula, defines the \textbf{mass ordering rule (MOR)} as another condition for the existence of the multiplet:

\(x_1 < a < x_2 < b\) or \(a < x_1 < b < x_2\) – for the nonet and

\(x_1 < a < x_2 < x_3 < b\) – for the decuplet.

Rectlinearity of the FSL follows from the flavor symmetry, but the slope \(k_s\) of the line is not determined by ME – it can be found only from data fit. The determination is not always possible, but in all cases where it can be done the slope is almost the same: \(k_s \simeq -0.5\).

\(^{1}\)Note that there are three kinds of the nonets: Gell-Mann – Okubo (GMO), Schwinger (S) and Ideal Mixing (I) [6]. The ME system giving one mass formula points out the S nonet.
However, if the mass of a particle is smaller of about 1.5GeV its \((m,\Gamma)\) coordinates may not comply with FSL, because the decay may be suppressed by some additional "kinematical" (non-flavor) mechanism \([6]\). The point with such coordinates lies below the FSL. The solutions of ME include also relations between the imaginary parts of the complex masses. They have the form of the mass formulae but are satisfied by data worse than FSL.

The definition of the multiplet is based entirely on the mass formula, hence it is independent of the "kinematical" breaking of the widths. Also the mixing matrix of the decuplet isoscalar states does not depend on the widths; it is determined by the masses via solution \(|d_i|^2\) and is real, even for complex masses.

### 3 Spectra of multiplets

The independence of the definition of the multiplet on the widths proves very important for the question of nature of the \(0^{++}\) mesons in the 1GeV region. The width argument against their \(q\bar{q}\) structure is not valid and there is no need for introducing the four quark states for them \([5]\). Strong suppression of the \(f_0(980)\) decay is just the manifestation of some ("kinematical") suppression mechanism. Hence, the scalar mesons from the 1GeV region belong to the common \(q\bar{q}\) nonet, but the energy dependence of the phases \(\delta_{f=0}^I\) and \(\delta_{f=1}^I\) does not reflect the properties of the flavor interaction.

The \(0^{++}\) nonet includes \([5]\) (\(f_0(1710)\) is pointed out by the S mass formula): \(a_0(980),\ K_0(1460),\ f_0(980),\ f_0(1710)\); (the MOR is: \(x_1 < a < x_2 < b\)) Other \(0^{++}\) mesons (not included into the nonet) constitute the decuplet: \(a_0(1460),\ K_0(1950),\ f_0(1370),\ f_0(1500),\ f_0(2200)/f_0(2330)\), where \(f_0(1500)\) is the glueball candidate \([8]\). The mass domains of these multiplets overlap, but there is no mixing between their states.

There exists one more meson observed below the mass scales of these two multiplets – the \(\sigma(600)\) one. This meson cannot join the nonet to form a decuplet (compare MORs of the nonet and decuplet).

Also the \(0^{-+}\) mesons form a nonet and decuplet. Observe, that the old nonet \(\pi,\ K,\ \eta,\ \eta'\) is the only known GMO one. The decuplet comprises \([7]\): \(\pi(1300),\ K(1450),\ \eta(1295),\ \eta(1405),\ \eta(1475)\), where \(\eta(1405)\) is recognized as glueball candidate \([2],[9]\).

Hence, both the \(0^{++}\) and the \(0^{-+}\) mesons form the same sequences of multiplets. Some of the masses are not exactly known, but this does not disturb the general picture. Notice, that not only the sequences of the multiplets are similar - also the inner structures of the decuplets are:

- the physical mesons \(f_0(1500)\) and \(\eta(1405)\) which are dominated by glueball states are settled down just between the remaining isoscalar mesons dominated by \(N\) and \(S\) quark states,
- both decuplets are built up of the excited \(q\bar{q}\) states, hence both glueballs mix with the excited \((q\bar{q})_{isoscalar}\) states.

The latter property suggests affinity of the glueball with the excited states. This is suggested especially by the mixing of the \(0^{++}\) glueball; its mass belongs to the region where the nonet ground states and the decuplet excited states are overlapping, but the glueball prefers mixing just with the excited \(q\bar{q}\) states.

The lack of data does not allow us to extend this comparison to higher masses.
The $0^{-+}$ and $0^{++}$ mesons form the parity related multiplets (nonets and decuplets). The sequences of these multiplets differ only due to existence of the scalar meson $\sigma(600)$ which has no pseudoscalar counterpart. But the nature of this meson is a matter of discussion. Several authors suggest that its nature is different from the nature of other mesons. By abandoning the $\sigma(600)$ we find that $0^{-+}$ and $0^{++}$ mesons form parity related spectra of flavor multiplets.

4 Glueballs

The masses of the glueballs are: $m_{G^{-+}} = 1.369 \text{GeV}$ and $m_{G^{++}} = 1.497 \text{GeV}$. Approximately $m_{G^{++}} - m_{G^{-+}} = m_\pi$; in any case they satisfy $m_{G^{-+}} < m_{G^{++}}$. Clearly, the value of $m_{G^{-+}}$ supports the prediction of the closed flux-tube model and is far from the lattice QCD prediction.

5 Conclusions

1. The mesons $f_0(1500)$ and $\eta(1405)$ can be understood as glueball dominated.
2. The nonet and decuplet states with the same $J^{PC}$ do not mix.
3. The glueballs have affinity to the excited $q\bar{q}$ states.
4. The mesons $0^{++}$ and $0^{-+}$ form parallel (parity related) spectra of multiplets.
5. Perhaps the $\sigma(600)$ meson does not belong to the flavor spectrum of $0^{++}$ multiplets.

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