INSPIRALING DOUBLE COMPACT OBJECT DETECTION AND LENSING RATE – FORECAST FOR DECIGO AND B-DECIGO

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ABSTRACT

Emergence of gravitational wave (GW) astronomy revived the interest in exploring the low frequency GW spectrum inaccessible from the ground. Satellite GW observatory DECIGO in its original configuration and the currently proposed smaller scale B-DECIGO are aimed to cover deci-Hertz part of the GW spectrum, which fills the gap between LISA milli-Hertz and deca- to kilo-Hertz range probed by ground-based detectors. In this paper we forecast the detection rates of inspiraling double compact objects (DCOs) and the unresolved confusion noise from these sources in DECIGO and B-DECIGO. In the context of DECIGO we use, for the first time, the population synthesis intrinsic inspiral rates of NS-NS, BH-NS and BH-BH systems. We also estimate the expected gravitational lensing rates of such sources for DECIGO and B-DECIGO. The result is that yearly detection of resolvable DCOs inspirals for the DECIGO is of order of $O(10^3)$ for NS-NS, $O(10^4)$ for BH-NS and $O(10^5)$ for BH-BH systems, while for a much smaller scale B-DECIGO they are about $O(10)$ for NS-NS, $O(10^2)$ for BH-NS and $O(10^3)$ for BH-BH systems. Taking into account that considerable part of these events would be detectable by ground-based GW observatories the significance of DECIGO/B-DECIGO could be substantial. Due to contamination by unresolved sources, both DECIGO and B-DECIGO will not be able to register lensed NS-NS or BH-NS systems, but the lensed BH-BH systems could be observed at the rate of about 50 per year in DECIGO. Smaller scale B-DECIGO will be able to detect a few lensed BH-BH systems per year. We also address the question of the magnification bias in the GW event catalogs of DECIGO and B-DECIGO.

Keywords: gravitational lensing, gravitational waves: sources

1. INTRODUCTION

First laboratory detections of gravitational waves (GWs) on Earth (Abbott et al. 2016) opened up a new branch of science – GW astronomy. Continuing efforts of LIGO/Virgo team (with now completed O1, O2 and O3 scientific runs) brought numerous detections of binary black hole (BH-BH) mergers (Abbott et al. 2019a), probably the first mixed black hole - neutron star (BH-NS) merger Abbott et al. (2020a) and the first detection of binary neutron star (NS-NS) coalescence (Abbott et al. 2017a). The NS-NS merger was accompanied by identification of its electromagnetic (EM) counterpart and its afterglow was followed up at different EM wavelengths (Goldstein et al. 2017; Coulter et al. 2017). This observation moved multimessenger astronomy to the next level. Besides the tests of general relativity and modified gravity theories (Abbott et al. 2019b), strong bounds on the speed of GWs (Abbott et al. 2017b), GW astronomy has proven that (almost) all classes of double compact objects (DCOs): NS-NS, BH-NS, BH-BH really exist in Nature. The event GW190425 is marginally compatible with NS-NS merger, so the existence of BH-NS systems still needs to be empirically proven. It is likely that this will happen soon.

Successful operation of ground-based interferometric detectors revived the interest in broadening the GW spectrum to lower frequencies (lower than 1 Hz) fundamentally inaccessible from the ground due to irremovable seismic noise. In particular, LISA was proposed (Amaro-Seoane et al. 2017) as a new generation space mission with a robust strain sensitivity level at frequencies between 0.1 mHz and 100 mHz over a science lifetime of at least 4 years. The technology behind LISA, an ESA-led mission expected to be launched by 2034, has been recently tested by the LISA Pathfinder experiment with outstanding results (Armano et al. 2016). The remaining decihertz band is the target of the DECihertz Interferometer Gravitational wave Observatory (DECIGO), a planned Japanese space-borne GW detector (Kawamura et al. 2019). Original DECIGO (Seto et al. 2001) or currently proposed smaller-scale version B-DECIGO (Sato et al. 2017) are aimed to cover the low-frequency band extending from the mHz to 100 Hz range. DECIGO would be able to detect inspiraling DCO — main targets to LIGO/Virgo, KAGRA or next generation ET, long time (weeks to years) before they enter the hectohertz band accessible from the ground. Moreover, the overlap with ground based GW detectors sensitivity bands is very advantageous since the joint detection with DECIGO and ground based detectors (e.g. ET) would greatly improve the parameter estimation of the binaries.
Reach of the DECIGO will be considerably higher than the next generation of ground based interferometric detectors, like the ET. With much higher volume probed, one may expect that non-negligible number of GW signals from coalescing DCOs would be gravitationally lensed. Gravitational lensing statistics and magnification cross-sections by galaxies has been discussed e.g. in (Zhu & Wu 1997; Zhu 1998). Rates of GW lensing for LIGO/Virgo have been calculated in Ng et al. (2018) and for the ET in Piórkowska et al. (2013); Biesiada et al. (2014); Ding et al. (2015); Li et al. (2018); Oguri (2019); Yang et al. (2019), respectively. The robust prediction is that the third generation of GW interferometric detectors would yield 50 - 100 lensed GW events per year. Strongly lensed GW signals, observed together with their electromagnetic (EM) counterparts, have been demonstrated to enhance our understanding regarding fundamental physics (Fan et al. 2017; Collett & Bacon 2017; Cao et al. 2019), dark matter (Liao et al. 2018), and cosmology (Sereno et al. 2010, 2011; Taylor & Gair 2012; Liao et al. 2017; Wei & Wu 2017). Gravitational lensing of GWs has been widely discussed concerning defraction effects in lensed GW events (Liao et al. 2019; Takahashi & Nakamura 2003; Nakamura 1998), the waveform distortion caused by the gravitational lensing (Cao et al. 2014), the influence on the statistical signatures of black hole mergers (Dai et al. 2017).

In this paper we make predictions concerning detection rates of inspiraling DCO systems, confusion noise regarding unresolved DCO systems and finally gravitational lensing of GW signals detectable by the DECIGO.

2. GW BACKGROUND NOISE FROM DCO SYSTEMS

We start with the assessment of the background noise created by unresolved DCO systems. In next sections we will discuss the detection rate of such inspiraling systems and their lensing rate by background galaxies. In all these considerations one needs the detector’s sensitivity curve and intrinsic DCO merger rates as input. We will discuss these issues below.

2.1. DECIGO sensitivity

DECIGO will be composed of four units of detectors. Each unit is planned to contain three drag-free spacecrafts to form a nearly regular triangle. These four units will be tilted 60° inwards relative to the ecliptic plane to keep its arm lengths nearly constant, and move around the sun with the orbital period of 1 yr. Centers of the triangular configuration of the units will form an equilateral triangle. The fourth one will be anchored to one of the units rotated 180° to form a Star of David configuration. Details concerning the DECIGO design could be found in Yagi & Seto (2011).

For the reference design parameters of DECIGO, one can prove (Yagi & Seto 2011) that a single triangular detector unit is equivalent to two L-shaped interferometers rotated by 45°. Their noises are uncorrelated and the noise spectrum of such an effective L-shaped DECIGO is given by:

\[ S_h(f) = 10^{-48} \times \left[ 7.05 \left( 1 + \frac{f^2}{f_p^2} \right) + 4.80 \times 10^{-3} \times \frac{f^{-4}}{1 + \left( \frac{f}{f_p} \right)^2 + 5.33 \times 10^{-4} f^{-4} } \right] \text{Hz}^{-1} \]  

(1)

where \( f_p = 7.36 \text{ Hz} \).

The original DECIGO mission concept was proposed in 2001 by Seto et al. (2001). What now seems more realistic to be commissioned in the near future is a scaled smaller project called B-DECIGO. It will consist of three satellites in a 100 km equilateral triangle, having sun-synchronous dusk-dawn circular orbits 2000 km above the Earth Sato et al. (2017); Nakamura et al. (2016). With B-DECIGO operating, we will soon probe the decihertz window for the first time, completing the full gravitational spectrum. Therefore we extend our predictions to the B-DECIGO. We use the noise power spectrum density \( S_h(f) \) for B-DECIGO proposed by Nakamura et al. (2016); Isoyama et al. (2018):

\[ S_h(f) = 10^{-48} \times \left[ 4.040 + 6.399 \times 10^{-2} f^{-4} + 6.399 \times 10^{-3} f^2 \right] \text{Hz}^{-1} \]  

(2)

2.2. DCO merger rate from the population synthesis

Previous predictions concerning DECIGO (Yagi & Seto 2011; Yagi 2013) used an analytical approximation of the NS-NS merger rate as a function of redshift up to redshift \( z = 5 \). Moreover, they did not assess the BH-BH or BH-NS rates precisely. In this paper, we use the values of the intrinsic inspiral rates \( \dot{n}_q(z) \) forecasted by Dominik et al. (2013) for each type of DCO at redshift slices spanning the range of \( z \in [0.04; 17] \). These data, available at https://www.syntheticuniverse.org have been used in our previous papers (Biesiada et al. 2014; Ding et al. 2015), where one can find more details about them. For the current purpose it will be sufficient to recall the following facts. To account for the varied chemical composition of the Universe, they performed the cosmological calculations for two scenarios of galactic metallicity evolution, called “low–end” and “high–end”, respectively. Essentially, they modeled metallicity (in fact oxygen abundance, assumed to be correlated with metallicity) by empirical function of galaxy mass (derived from Schechter type distribution). The normalization factor of this relation is redshift dependent, which can be heuristically modeled by a relation, whose coefficient choice (in fact, it’s a bit more complicated - see Dominik et al. (2013)) lead to two distinct metallicity evolution profiles. The first one called “high–end” predicts a median value of metallicity of 1.5 \( Z_\odot \) at \( z \sim 0 \). Another one, called “low–end” yields a median metallicity value of 0.8 \( Z_\odot \). Concerning binary system evolution from the ZAMS to the final formation of DCO binary system (NS-NS, NS-BH or BH-BH) we will consider four scenarios: standard, optimistic common envelope, delayed SN explosion and high BH natal kicks as specified in Dominik et al. (2013), where the reader is referred to for more details. In previous papers cited above, the median masses: 1.2\( M_\odot \) for NS-NS, 3.2\( M_\odot \) for BH-NS, and 6.7\( M_\odot \) for BH-BH.
were used according to Dominik et al. (2012). However, these were values obtained under the assumption of solar metallicity of initial binary systems. Such scenario was a right guess before the first detections of GWs. Now, the data collected with the LIGO/Virgo detectors demonstrated that observed chirp masses (in particular of BH-BH systems) are much higher. Hence one is forced to change the aforementioned assumption. Moreover, as we will see further on, probability density of DCO inspirals (observable by DECIGO) peak at \( z=2 \), when the metallicity was significantly sub-solar. Therefore, guided by the real data gathered so far we will adopt different values - according to Abbott et al. (2019a). We will assume the median value of BH-BH systems chirp masses reported in their Table III. Since the data on BH-NS systems is more scarce, we will take the value of Abbott et al. (2020b).

Hence, the following chirp masses will be adopted by us as representative of DCO inspiraling systems: \( 1.2 \, M_\odot \) for NS-NS, \( 6.09 \, M_\odot \) for BH-NS, and \( 24.5 \, M_\odot \) for BH-BH. In order to comply with the assumptions underlying population synthesis simulation, we assume flat ΛCDM cosmology with \( H_0 = 70 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) and \( \Omega_m = 0.3 \).

2.3. GW background from unresolved DCO systems

Magnitude of a stochastic GW background is usually characterized by its fractional energy density per logarithmic frequency interval:

\[
\Omega_{GW} = \frac{1}{\rho_{cr}} \frac{d\rho_{GW}}{d\ln f}
\]

where \( \rho_{cr} = 3H_0^2/8\pi G \) is the critical energy density of the Universe. Let us note, that in the context of a confusion noise due to unresolved sources, the distinction between background and foreground is not clear-cut. We will call it background.

According to Phinney (2001) one can conveniently calculate the energy density parameter \( \Omega_{GW}^{DCO} \) corresponding to the unresolved signals from the DCO systems, as

\[
\Omega_{GW}^{DCO} = \frac{8\sigma_5^{5/3}}{9e^2 H_0^2} (GM)^{5/3} f^{2/3} \int_0^\infty \frac{n_s(z)}{1+z} 4^{3/2} H(z) \, dz
\]

where \( n_s(z) \) is the DCO merger rate per proper time per comoving volume at redshift \( z \), and the Hubble parameter \( H(z) \) is given by \( H(z)^2 = H_0^2[\Omega_m(1+z)^3 + 1 - \Omega_m] \). Calculating numerically the integral in Eq.(4) one obtains:

\[
\Omega_{GW}^{DCO} = \Omega_{GW}^{DCO} \left( \frac{H_0}{10 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}} \right)^{-3} \times \left( \frac{M}{M_{DCO}} \right)^{5/3} \left( \frac{f}{1 \, \text{Hz}} \right)^{2/3}
\]

where \( M_{DCO} \) is the median value of the DCO considered, i.e. \( (M_{NSNS}; M_{BHNS}; M_{BHBH}) = (1.22; 6.09; 24.5) \, M_\odot \) and values of \( \Omega_0^{DCO} \) coefficients are reported in Table 1.

In order to compare with the detector's noise power spectrum the normalized energy density \( \Omega_{GW}^{DCO} \) should be expressed as the total sky-averaged GW spectrum:

\[
S_h^{GW;DCO} = \frac{4}{\pi} f^{-3} \rho_{cr} \Omega_{GW}^{DCO}.
\]

The background spectrum of three DCO populations imposed on the DECIGO sensitivity curve is shown in Figure 1. Let us note that B-DECIGO will be much less contaminated from the unresolved DCO systems, yet the events like those detected by LIGO/Virgo will be detectable enabling their discovery and study long before they will enter the ground-based detectors sensitivity band. As discussed in details in Isoyama et al. (2018), time to coalescence can be estimated as: \( t_c = 1.03 \times 10^6 \, \text{s} \ (M_{2}/30.1 \, M_\odot)^{-5/3} (f/0.1 \, \text{Hz})^{-8/3} \), which means that GW150914 and GW170817 could have been visible in (B-)DECIGO band for \( \sim 10 \) days and \( \sim 7 \) yrs prior to coalescence with large numbers of GW cycles \( (10^5 \text{ and } 10^7 \), respectively).

![Figure 1](link to image)

**Figure 1.** Confusion noise due to DCO systems (NS-NS, BH-NS and BH-BH) superimposed on the noise spectrum density for DECIGO and B-DECIGO. Inspiral rates correspond to the standard scenario and “low-end” metallicity evolution. The effective squared spectrum density corresponding to GW150914 and GW170817 events has been also included.

3. DETECTION RATE FOR UNLENSED EVENTS

Matched filtering signal-to-noise ratio for a single detector reads (Taylor & Gair 2012):

\[
\rho = 8\Theta \frac{r_0}{d_L(z_s)} \left( \frac{M_{2}}{1.2 \, M_\odot} \right)^{5/6} \sqrt{\zeta(f_{max})}
\]

where \( d_L \) is the luminosity distance to the source, \( \Theta \) is the orientation factor capturing part of sensitivity pattern due to (usually non-optimal) random relative orientation of DCO system with respect to the detector (more details below). Zeta parameter capturing the overlap between the signal and the detector’s sensitivity band width is defined as:

\[
\zeta(f_{max}) = \frac{1}{x_{7/3} f_{max}} \int_0^{2f_{max}} df \frac{(\pi M_0)^2}{(\pi f M_0)^{7/3} \bar{S}_h(f)}
\]

where \( 2f_{max} \) is the wave frequency at which the inspiral detection template ends and \( x_{7/3} = (\pi M_0)^{-1/3} f_{7/3} \) (see below for definition of \( f_{7/3} \)). Calculating \( \zeta \) factor with the above definition one obtains \( \zeta(f_{max}) \approx 1 \) for all DCO systems considered. However, the confusion noise of unresolved systems will influence our ability to detect inspiraling DCO systems. Therefore, we should use the
modified noise spectrum \( S_h(f) + \frac{S_{GW,DCO}(f)}{f} \) (system by system, and scenario by scenario). There are two ways to do that, both leading to the same results. First, is the take the detector’s characteristic distance \( r_0 \) (defined below) and hence \( x \) calculated from the detector’s noise \( S_h(f) \) but modified noise in the calculation of \( \zeta(f_{\max}(x)) \) (in \( x \) is not modified). In this case \( \zeta \) factors are considerably smaller. Second approach is to consider noise spectrum modified by DCO confusion noise and regard different detector’s characteristic distances \( r_0 \) (system by system and scenario by scenario). Then \( \zeta(f_{\max}) \approx 1 \) is valid. By \( r_0 \) we denote the detector’s characteristic distance parameter, which can be estimated according to:

\[
\frac{r_0^2}{0} = \frac{5}{192\pi^{4/3}} \left( \frac{3}{20} \right)^{5/3} \left( \frac{GM_{\odot}}{c^3} \right)^{5/3} f_{T/3}
\]

where:

\[
f_{T/3} = \int_0^{f_{T/3}} S_h(f)^{-1} df
\]

Using DECIGO sensitivity Eq.(1) one gets:

\[
r_0 = 6709 \text{ Mpc}
\]

Similarly, from the B-DECIGO sensitivity given by Eq.(2) one obtains \( r_0 = 535 \text{ Mpc} \), meaning that B-DECIGO will be able to probe about a 1000 times smaller volume than DECIGO.

The orientation factor \( \Theta \) is defined as

\[
\Theta = 2[F^2 + (1 + \cos^2 \theta) - 4F_x^2 \cos^2 \theta]^{1/2}
\]

where \( F_x = \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi \) and \( F_x = \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi \) are the interferometer strain responses to different polarizations of gravitational wave. The angles \( (\theta, \phi) \) describe orientation (polar angles) of direction to the source with respect to the detector plane, \( (\psi, t) \) are the angles describing DCO orbit orientation with respect to the plane tangent to the celestial sphere at source location (so called polarization angle and inclination). The above formulae for \( F_{+/x} \) are for just one L-shaped detector \( F_{I+/x} \), for the second one it would be \( F_{I+/-x} = F_{I+/x}(\theta, \phi - \pi/4, \psi, t) \).

Probability distribution for \( \Theta \) calculated under assumption of uncorrelated orientation angles \( (\theta, \phi, \psi, t) \) is known to be of the following form (Finn 1996):

\[
\begin{align*}
P_0(\Theta) &= 5\Theta(4 - \Theta)^{3/2} / 256, \quad \text{if } 0 < \Theta < 4 \\
&= 0, \quad \text{otherwise}
\end{align*}
\]

The yearly detection rate of DCO sources originating at redshift \( z_s \) and producing the signal with SNR exceeding the detector’s threshold \( \rho_0 = 8 \) (assumption made in previous DECIGO studies (Yagi & Seto 2011; Isiyama et al. 2018)) can be expressed as:

\[
\dot{N}(> \rho_0 | z_s) = \int_0^{z_s} \frac{d\dot{N}(> \rho_0)}{dz} dz
\]

where

\[
\frac{d\dot{N}(> \rho_0)}{dz} = 4\pi \left( \frac{c}{H_0} \right)^3 \frac{\dot{n}_0(z_s) \nu^2(z_s)}{1 + z_s E(z_s)} C_0(x(z_s, \rho_0))
\]

\( \dot{n}_0(z_s) \) denotes intrinsic coalescence rate at redshift \( z_s \), \( C_0(x) = \int_0^\infty P_0(\Theta) d\Theta \) and \( x(z, \rho) = \frac{\rho}{(1 + z)^{1/6} c \nu(z)} \).

Throughout this paper we use the values of inspiral rates \( \dot{n}_0(z_s) \) obtained with StarTrack evolutionary code. The results are summarized in Table 2. Probability density of DCO inspiral events as a function of redshift is shown in Fig 2. One can see that even though B-DECIGO is considerably smaller scale enterprize, it would be able to register tens of resolvable NS-NS inspirals and hundred thousands of resolvable BH-BH inspirals per year.

4. Lensed GW Signals Statistics

We assume that the population of lenses comprise only elliptical galaxies modeled as singular isothermal spheres (SIS). This assumption is supported by galaxy strong lensing studies (Koopmans et al. 2009). Einstein radius, which is a characteristic angular scale of separation between images in the SIS model can be expressed as:

\[
\theta_E = 4\pi \left( \frac{2}{\pi} \right) \frac{d_A(z_s)}{d_s} \frac{\sigma(z_s)}{H(z_s)}
\]

where \( \sigma(z_s) \) is the velocity dispersion of stars in lensing galaxy, \( d_A(z_s) \) and \( d_s \) are angular diameter distances between the lens and the source and to the source, respectively (Cao et al. 2012a, 2015). Calculations are simplified if one expresses the angular distance of the image from the center of the lens \( \theta \) and the angular position of the source \( \beta \) as dimensionless parameters: \( x = \frac{\theta}{\theta_E} \) and \( y = \frac{\beta}{\theta_E} \). Then the necessary condition for strong lensing (appearance of multiple images) is \( y < 1 \). Brighter \( I_+ \) and fainter image \( I_- \) form at locations \( x = \pm 1 \) and \( y \) with magnifications:

\[
\mu_\pm = \frac{1}{y} \pm 1
\]

Gravitationally lensed GW signal would come from these two images with appropriate relative time delay \( \Delta t = \Delta t_0(\sigma, z_s, z_s) \) (for details see (Piórkowska et al. 2013; Biesiada et al. 2014)) and with different amplitudes:

\[
h_\pm = \sqrt{\mu_+} \quad h(t) = \sqrt{\frac{1}{y}} \pm 1 \quad h(t)
\]

where \( h(t) \) denotes the intrinsic amplitude (i.e. the one which would have been observed without lensing). In order to observe lensed image \( I_+ \) or \( I_- \) the detected SNR \( \rho_\pm \) must exceed the threshold for detection \( \rho_0 = 8 \). This happens, if the misalignment of the source with respect to the optical axis of the lens satisfies:

\[
y_\pm \leq y_\pm,\max = \left( \frac{8}{\rho_{\text{intr.}}} \right)^2 + 1
\]

where \( \rho_{\text{intr.}} \) is the intrinsic SNR of the (unlensed) source.

Consequently, the cross section for lensing is (see e.g. (Piórkowska et al. 2013)):

\[
S_{cr,\pm}(\sigma, z_l, z_s, \rho) = 16\pi^3 \left( \frac{\sigma}{c} \right)^4 \left( \frac{\dot{n}_0}{\nu} \right)^2 y_\pm,\max^2
\]

and the optical depth for lensing leading to magnifications of \( I_+ \) and \( I_- \) images above the threshold reads:

\[
\tau_\pm(z_s, \rho) = \frac{1}{4\pi} \int_0^{z_s} dz_l \int_0^\infty ds \frac{4\pi}{\sigma} \left( \frac{c}{H_0} \right)^3 \times \frac{\dot{n}_0(z_s) \nu^2(z_s)}{1 + z_s E(z_s)} C_0(x(z_s, \rho_0))
\]

\[
\frac{\dot{n}_0(z_s) \nu^2(z_s)}{1 + z_s E(z_s)} C_0(x(z_s, \rho_0)) \frac{\rho^2}{E(z_l)} S_{cr,\pm}(\sigma, z_l, z_s) \frac{dn}{d\sigma}.
\]
Double compact object detection and lensing

Table 1
Numerical factors $\Omega_0^{\text{DCO}}$ in the Eq.(5) for different classes of DCO systems under different evolutionary scenarios, assuming “low-end” and “high-end” metallicity evolution.

| Evolutionary scenario | standard | optimistic | delayed SN | high BH kicks |
|-----------------------|----------|------------|------------|---------------|
| NS-NS                 |          |            |            |               |
| low-end metallicity   | 1.33     | 10.26      | 1.51       | 1.34          |
| high-end metallicity  | 2.17     | 9.95       | 2.44       | 2.14          |
| BH-NS                 |          |            |            |               |
| low-end metallicity   | 6.94     | 18.79      | 3.74       | 0.68          |
| high-end metallicity  | 4.10     | 14.30      | 2.22       | 0.48          |
| BH-BH                 |          |            |            |               |
| low-end metallicity   | 554.58   | 2983.01    | 427.78     | 20.88         |
| high-end metallicity  | 368.68   | 2026.12    | 262.41     | 13.65         |

Table 2
Yearly detection rate of inspiraling DCOs of different classes under different evolutionary scenarios, assuming “low-end” and “high-end” metallicity evolution. Predictions for the DECIGO and B-DECIGO. Yearly detection rates of resolvable DCO systems are reported.

| Evolutionary scenario | Yearly detection rate for DECIGO | Yearly detection rate for B-DECIGO |
|-----------------------|---------------------------------|-----------------------------------|
|                       | standard | optimistic CE | delayed SN | high BH kicks | standard | optimistic CE | delayed SN | high BH kicks |
| NS-NS                 |          |            |            |               |          |            |            |               |
| low-end metallicity   | 233.1    | 119.       | 335.5      | 3054.4        |
| high-end metallicity  | 439.6    | 203.9      | 707.3      | 8807.7        |
| BH-NS                 |          |            |            |               |          |            |            |               |
| low-end metallicity   | 2688.9   | 1239.5     | 1836.8     | 1877.6        |
| high-end metallicity  | 2000     | 1314.6     | 1614.5     | 1613.7        |
| BH-BH                 |          |            |            |               |          |            |            |               |
| low-end metallicity   | 207755.2 | 384698.    | 178991.7   | 20125.8       |
| high-end metallicity  | 166436   | 145583.5   | 15379.5    | 15397.5       |
| TOTAL                 |          |            |            |               |          |            |            |               |
| low-end metallicity   | 210677.2 | 380565.5   | 181165.8   | 25057.8       |
| high-end metallicity  | 168875.6 | 147905.3   | 25800.9    | 25800.9       |

We model the velocity dispersion distribution in the population of lensing galaxies as a modified Schechter function

$$dn/d\sigma = n_*(\sigma/\sigma_*)^\alpha \exp \left( - (\sigma/\sigma_*)^\beta \right) (\sigma/\sigma_*)^{-\frac{1}{2}}$$

with parameters $n_*, \sigma_*, \alpha$ and $\beta$ taken after Choi, Park & Vogeley (2007). The choice of this particular model, despite the existence of more recent data on velocity dispersion distribution functions is motivated by its best representing the pure elliptical galaxy population in agreement with our model assumption (Biesiada et al. 2014; Cao et al. 2012b).

Under the above assumptions, total optical depth for lensing is:

$$\tau_\pm = \frac{16}{30} \pi^4 \left( \frac{c}{H_0} \right)^3 \bar{r}_s^3 \left( \frac{\sigma_0}{c} \right)^4 \frac{\Gamma}{\Gamma/\beta} \left( \frac{\sigma_0}{\beta} \right)^\beta y_{\pm,\text{max}}.$$  \hspace{1cm} (15)

The above formula is valid only in the case of a continuous search. If instead the survey has a finite duration $T_{\text{survey}}$ some of the events, i.e. those whose signals come
near the beginning or the end of the survey, would be lost because of lensing time delay. In other words we would register the signal from just one image and cannot tell because of lensing time delay. In other words we would register the signal from just one image and cannot tell in fact the event was lensed. Finite duty cycle of the detector influences the optical depth $\tau_{\pm}$. Namely, it should be corrected as:

$$\tau_{\pm}^{\Delta t_{\pm}} = \tau \left[ 1 - \frac{1}{7} \left( \frac{\alpha + 8}{\beta} \right) \frac{\Delta t_{\pm}}{T_{\text{surv}}} \right]$$

(16)

where $\Delta t_{\pm} = \frac{32\pi^2}{H_0^2} \tau_s \left( \frac{\tau_s}{\tau} \right)^4 \Delta t_{\pm,\text{max}}$ (for detailed calculations see Piórkowska et al. (2013)). This correction is particularly important here, because the DECIGO and B-DECIGO missions are planned for $T_{\text{surv}} = 4 \text{ yr}$. We present predictions for the first year of operation and for the total 4 yrs of nominal duration. Moreover, we conservatively assume that the lensed system is intrinsically loud enough to exceed the detector’s threshold $\rho_0 = 8$ and we assume that the fainter image exceeds the threshold. This means that we only consider $y_{\text{max}} = y_{-,\text{max}} = 0.5$, according to Eq. (12).

Cumulative yearly detection of lensed events up to the

Figure 2. Probability density of DCO inspiral events as a function of redshift for the DECIGO. Different colors refer to different scenarios: red – standard, blue – OCE, green – delayed SN, magenta – high BH kicks. Solid line corresponds to the high-end metallicity evolution, dashed line – low-end metallicity.

Figure 3. Differential lensing rate $\frac{1}{N_{\text{surv}}} \frac{dN_{\text{lensed}}}{dz}$ (as a function of source redshift) of DCO inspiraling binaries for different evolutionary scenarios (solid lines and dashed lines corresponds respectively to high-end and low-end metallicity evolution). Predictions for the DECIGO 4 years operation.
Double compact object detection and lensing

source redshift $z_s$ can be calculated as:

$$N_{lensed}(z_s) = \int_0^{z_s} \tau_{\Delta t}(z_s, y_{max}, T_{\text{sure}}) \frac{dN(>\rho_0)}{dz} dz.$$  

(17)

The results for both DECIGO and B-DECIGO are shown in Table 3. In Fig. 3 normalized differential lensing rate (as a function of redshift) is displayed. One can verify that DECIGO in its original design would be able to register $5 - 6$ strongly lensed NS-NS (or BH-NS) systems, if the contamination from unresolved sources is neglected. These systems could be accompanied by the EM counterpart during their final merger phase detectable by ground based interferometric detectors. Hence, they are the most promising lensed sources enabling e.g. precise cosmological inference (Liao et al. 2017) or probing dark matter substructure in lensing galaxies (Liao et al. 2018).

However, contamination by unresolved systems (predominantly BH-BH systems) make the chances of detecting lensed NS-NS or BH-NS binaries negligible. On the other hand tens of lensed inspirals could be seen from the BH-BH binary systems both in DECIGO and B-DECIGO.

5. CONCLUSIONS

In this paper we have made the predictions of yearly detection rates of GW signals from the DCO inspirals detectable in the future dechertz space-borne detectors DECIGO and its smaller scale version B-DECIGO. All previous papers concerning DECIGO used only analytical estimates of NS-NS merger rates up to $z = 5$. In our calculations, we have used for the first time the StarTrack population synthesis results concerning intrinsic merger rates at different redshifts of distinct classes of DCO systems: NS-NS, BH-NS and BH-BH binaries. Now, when more accurate estimates of merger rates from the LIGO/Virgo data are available (Abbott et al. 2019a) one can make a comparison between StarTrack predictions and real data estimates (Hou et al. 2020b). It turns out that “low-end” metallicity predictions are in agreement with those based on LIGO/Virgo derived merger rates.

We have also estimated the stochastic noise levels due to unresolved DCO systems. The conclusion is that DECIGO would be significantly contaminated by unresolved BH-BH binary systems, while the level of this stochastic noise component is not so relevant for B-DECIGO. Concerning the DCO yearly detection rate its order of magnitude ranges from $10^2$ for NS-NS, $10^3$ for BH-NS systems to $10^6$ for BH-BH systems in DECIGO. Respective rates for the B-DECIGO are 10 (NS-NS), $10^2$ (BH-NS) and $10^5$ (BH-BH).

The detector’s distance parameter $r_0$ for the DECIGO is about 4 times bigger than for the ET, meaning that DECIGO will probe 64 times bigger volume than the next generation of ground based detectors. Hence, we addressed the issue of gravitationally lensed DCO inspirals observable by the DECIGO. For the completeness of discussion we considered B-DECIGO as well, even though B-DECIGO’s $r_0$ is about 3 times smaller than for the ET. Our basic assessment was performed under assumption that the GW source is intrinsically loud (i.e. would be detectable without lensing). The result is that due to contamination of unresolved systems, either DECIGO or B-DECIGO will not be able to register any lensed NS-NS or BH-NS inspirals. However, they could register up to $O(10)$ lensed BH-BH inspirals.

In the appendix we enrich our discussion relaxing the $\rho_{\text{intr.}} \geq 8$ assumption and consider intrinsically faint signals, i.e. those being detectable exclusively due to lensing magnification. Such inclusion of lensing magnification could significantly enlarge the statistics of lensed events with intrinsic SNR $\rho_{\text{intr.}} \geq 8$ for both DECIGO and B-DECIGO. The appendix also addresses the question of the magnification bias in the full inspiral GW event catalogs of DECIGO and B-DECIGO. One finds out that the magnification bias is of the order of $10^{-3} - 10^{-4}$, which means that the cosmological inferences drawn from these catalogs would not be affected very much.

Let us stress that DECIGO and B-DECIGO would be able to register inspiral signals from the DCOs weeks or years (the higher redshifted chip mass the shorter time of passing through DECIGO band) before they enter the high frequency band of ground-based detectors to finally end their lives in energetic mergers. Having in mind this circumstance and the benefits that are expected from registering lensed GW one would expect that DECIGO’s detections of lensed GW signals could trigger a concerted efforts for searching strong lenses in the EM domain at possible locations suggested by the DECIGO. This would be very profitable in many aspects, in particular for identifying the host galaxy before the merger and despite of no bright EM counterpart as it was the case to the BH-BH mergers registered so far. In the light of considerable rate of DCO inspiral signals detectable by the DECIGO, one should be concerned how to distinguish gravitationally lensed signals. This issue deserves separate studies on simulated mock catalogs of signals. It is usually expected that lensed signals would have the same temporal behaviour (frequency and its rate of change) differing only by amplitude due to magnification and come to the detector after some delay (Dai et al. 2017; Ng et al. 2018; Biesiada et al. 2014). This is a very reasonable expectation for lensed signals from the final merger phase. However, unlike the ground-based detector where one registers a transient event, DECIGO would observe lensed events in an adiabatic inspiral phase as two (or more) unresolved images, likely producing interference patterns in the waveforms (Hou et al. 2020a). Let us note that interference beat patterns are interesting on their own. This topic is besides the scope of the present paper which focused only on the statistics of GW lensing and is the subject of a separate study Hou et al. (2020b).

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Gravitational lensing magnifies the lensed images of the source. Therefore, one can relax the assumption that sources should be intrinsically loud enough to be detected without lensing and treat their signal to noise parameter $\rho$ as a free one. In such a case one would get estimates for the population of DCO systems detectable exclusively due to gravitational lensing. In other words this would provide forecasts for magnification bias in the catalog of lensed DCO inspirals. We will follow the strategy used in Ding et al. (2015) in the case of the ET. Therefore, instead of Eq. (11) we have to start with the differential inspiral rate per redshift and per SNR parameter $\rho$:

$$\frac{\partial^2 \dot{N}}{\partial z \partial \rho} = 4\pi \left( \frac{c}{H_0} \right)^3 \frac{\eta_0(z_s) \xi^2(z_s)}{1 + z_s E(z_s)} P_\Theta(x(z_s, \rho)) x(z_s, \rho) \frac{\partial^2 \dot{N}}{\partial z \partial \rho} \rho \quad (18)$$

then one should calculate cross-sections $S_{\text{crit},z}$ and optical depths $\tau_{\pm}$ for lensing separately for each image $I_+$ or $I_-$.

The final result will be the lensing rate of intrinsically faint ($\rho < \rho_0$) DCOs having $I_+$ or $I_-$ images magnified above the threshold $\rho_0$:

$$\dot{N}_{\text{lensed},\pm} = \int_0^{z_{\text{max}}} d z_s \int_0^{\rho_0} \tau_{\text{crit},\pm}(z_s, \rho) \frac{\partial^2 \dot{N}}{\partial z \partial \rho} d \rho \quad (19)$$

or differential lensing rates with respect to $\rho$ or $z_s$ respectively:

$$\frac{d\dot{N}_{\text{lensed},\pm}}{d \rho} = \int_0^{z_{\text{max}}} \tau_{\text{crit},\pm}(z_s, \rho) \frac{\partial^2 \dot{N}}{\partial z \partial \rho} d z_s \quad (20)$$

$$\frac{d\dot{N}_{\text{lensed},\pm}}{d z_s} = \int_0^{\rho_0} \tau_{\text{crit},\pm}(z_s, \rho) \frac{\partial^2 \dot{N}}{\partial z \partial \rho} d \rho. \quad (21)$$

Table 4 contains the results of predicted intrinsically faint lensed GW events for which the the $I_-$ image is magnified above threshold $\rho_0 = 8$ for DECIGO and B-DECIGO. As one can see, inclusion of lensing magnification of intrinsically faint events would significantly enlarge the statistics of lensed events with intrinsic SNR $\rho_{\text{inetr}} > 8$ (see Tab. 3). This conclusion is true both for DECIGO and B-DECIGO.

The normalized differential yearly detection rates $\frac{1}{\dot{N}} \frac{\partial \dot{N}}{\partial \rho}$ v.s. the source redshift $z_s$. Upper figure corresponds to $\rho_{\text{inetr}} < 8$ for the $I_+$ image. The lower one corresponds to the $I_-$ image including both $\rho_{\text{inetr}} < 8$ and $\rho_{\text{inetr}} > 8$. Low-end metallicity galaxy evolution and standard model of DCO formation are assumed.

Figure 5 shows the normalized differential yearly detection rates $\frac{1}{\dot{N}} \frac{\partial \dot{N}}{\partial \rho}$ v.s. the source redshift $z_s$. The top panel illustrates lensed faint systems with $\rho_{\text{inetr}} < 8$ with the $I_+$ image magnified above the threshold. In other words, both images will be registered by DECIGO or B-DECIGO. In the lower panel differential detection rate is shown for systems of both $\rho_{\text{inetr}} < 8$ and $\rho_{\text{inetr}} > 8$, i.e. for the total catalog of lensed GW events of DECIGO or B-DECIGO. For the sake of transparency only standard scenario of DCO formation with “low-end” metallicity evolution is
Table 3
Expected numbers of lensed GW events from inspiraling DCOs of different classes under different evolutionary scenarios, assuming “low-end” and “high-end” metallicity evolution. Predictions for the DECIGO and B-DECIGO under assumption of $T_{\text{surv}} = 4$ yrs.

| Evolutionary scenario | DECIGO  | B-DECIGO |
|-----------------------|---------|----------|
|                       | standard | optimistic CE | delayed SN | high BH kicks |
| NS-NS                 |         |             |           |               |
| low-end metallicity   | 0.02    | 0.03       | 0.03      | 0.64          |
| high-end metallicity  | 0.06    | 0.05       | 0.10      | 0.98          |
| BH-NS                 |         |             |           |               |
| low-end metallicity   | 0.83    | 0.38       | 0.49      | 0.13          |
| high-end metallicity  | 0.86    | 0.48       | 0.52      | 0.12          |
| BH-BH                 |         |             |           |               |
| low-end metallicity   | 26.6    | 66.1       | 21.5      | 0.62          |
| high-end metallicity  | 21.1    | 66.1       | 16.3      | 0.45          |
| TOTAL                 |         |             |           |               |
| low-end metallicity   | 27.5    | 66.5       | 22.0      | 1.39          |
| high-end metallicity  | 22.0    | 66.6       | 16.9      | 1.55          |

Table 4
Expected numbers of lensed GW events observed by DECIGO and B-DECIGO with $\rho_{\text{intr}} < 8$ for which the $I_-$ image is magnified above threshold $\rho_0 = 8$. We assumed the survey duration $T_{\text{surv}} = 4$ yrs. Nomenclature of DCO formation scenarios and galaxy metallicity evolution follows that of Dominik et al. (2013).

| Evolutionary scenario | DECIGO  | B-DECIGO |
|-----------------------|---------|----------|
|                       | standard | optimistic CE | delayed SN | high BH kicks |
| NS-NS                 |         |             |           |               |
| low-end metallicity   | 0.002   | 0.003      | 0.002     | 0.01          |
| high-end metallicity  | 0.004   | 0.004      | 0.005     | 0.02          |
| BH-NS                 |         |             |           |               |
| low-end metallicity   | 0.122   | 0.054      | 0.08      | 0.06          |
| high-end metallicity  | 0.112   | 0.064      | 0.08      | 0.04          |
| BH-BH                 |         |             |           |               |
| low-end metallicity   | 24.3    | 31.5       | 22.0      | 2.39          |
| high-end metallicity  | 22.4    | 34.0       | 20.1      | 1.98          |
| TOTAL                 |         |             |           |               |
| low-end metallicity   | 24.4    | 31.6       | 22.1      | 2.5           |
| high-end metallicity  | 22.5    | 34.6       | 20.2      | 2.0           |
Expected numbers of lensed GW events observed by DECIGO and B-DECIGO with $\rho_{\text{intr}} < 8$ for which the $I_4$ image is magnified above threshold $\rho_0 = 8$. Other assumptions and terminology – like in Table 4.

### DECIGO

| Evolutionary scenario | standard | optimistic CE | delayed SN | high BH kicks |
|-----------------------|----------|---------------|------------|--------------|
| NS-NS                 |          |               |            |              |
| low-end metallicity   | 0.04     | 0.04          | 0.067      | 2.31         |
| high-end metallicity  | 0.14     | 0.061         | 0.254      | 4.08         |
| BH-NS                 |          |               |            |              |
| low-end metallicity   | 3.94     | 1.30          | 2.43       | 0.68         |
| high-end metallicity  | 4.16     | 1.72          | 2.60       | 0.630        |
| BH-BH                 |          |               |            |              |
| low-end metallicity   | 135.6    | 335.7         | 109.4      | 3.11         |
| high-end metallicity  | 107.4    | 333.0         | 82.6       | 2.29         |
| TOTAL                 |          |               |            |              |
| low-end metallicity   | 139.6    | 348.7         | 111.9      | 6.1          |
| high-end metallicity  | 111.7    | 334.8         | 85.4       | 7.0          |

### B-DECIGO

| Evolutionary scenario | standard | optimistic CE | delayed SN | high BH kicks |
|-----------------------|----------|---------------|------------|--------------|
| NS-NS                 |          |               |            |              |
| low-end metallicity   | 0.002    | 0.003         | 0.003      | 0.017        |
| high-end metallicity  | 0.005    | 0.005         | 0.008      | 0.032        |
| BH-NS                 |          |               |            |              |
| low-end metallicity   | 0.33     | 0.11          | 0.22       | 0.23         |
| high-end metallicity  | 0.29     | 0.13          | 0.22       | 0.17         |
| BH-BH                 |          |               |            |              |
| low-end metallicity   | 120.0    | 141.0         | 109.4      | 12.1         |
| high-end metallicity  | 111.4    | 158.1         | 100.7      | 10.0         |
| TOTAL                 |          |               |            |              |
| low-end metallicity   | 120.3    | 141.1         | 109.6      | 12.3         |
| high-end metallicity  | 111.7    | 158.2         | 100.9      | 10.2         |

Figure 6. Probability densities of DCO inspiral events yearly rate to be detected by DECIGO (left panel) and B-DECIGO (bright panel) during their survey duty cycles $T_{\text{surv}} = 4$ years. The solid curves are for the total catalogue of lensed and non-lensed systems, and the dashed curves are for the non-lensed ones. Note the logarithmic scale used in this figure.
shown. Detector’s operation period of $T_{\text{surv}} = 4 \text{ years}$ is assumed. From these figures, one infers that the faint sources can be used to probe higher redshifts, and these higher redshift sources would thus contaminate the future catalog of gravitationally lensed GW events. This is expected on the ground of general idea of how the magnification bias works. Since DECIGO’s characteristic radius $r_0$ is much larger than that of B-DECIGO, the former configuration is more suitable to probe high redshift sources, as shown for Fig. 5.

Besides the magnification bias on the lensed GW events, the magnification bias at the level of the full DCO inspiral events catalogue can also be obtained. For this end, one may calculate the detection rate of the intrinsically faint events whose $I_\ell$ image is magnified above the threshold. Table 5 shows the predictions for DECIGO and B-DECIGO. One can clearly see the increase in the detection rates, since $y_{+\ell, \text{max}} > y_{-\ell, \text{max}}$. Comparing this table with Table 2, one finds out that the magnification bias at the level of the resolvable inspiral DECIGO or B-DECIGO event catalogs would be of the order of $10^{-3} - 10^{-4}$ depending on the DCO population. This means that the cosmological inferences drawn from this catalog would not be affected very much.

One can demonstrate this effect by plotting together probability density of yearly detection rate of non-lensed sources and total prediction, which is shown in Fig. 6 for DECIGO and B-DECIGO. The upper panel of Fig. 6 shows that the magnification bias is negligible for all three types of the DCO sources in the case of DE-CIGO. For B-DECIGO, the magnification bias is negligibly small for NS-NS and BH-NS binaries, while for BH-BH binaries, it is barely noticeable, according to the lower panel of Fig. 6.

REFERENCES

Abbott, B. P., et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), 2016, PRL, 116, 061102
Abbott, B. P., et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), 2017a, PRL, 119, 161101
Abbott, B. P., et al. (The LIGO Scientific Collaboration and Virgo Collaboration, FermiGamma-ray Burst Monitor, and INTEGRAL) 2017b, ApJL, 848, L13
Abbott, B. P., et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), 2019a, PRX, 9, 031040
Abbott, B. P., et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), 2019b, PRD, 100, 104036
Abbott, B. P., et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), 2020a, ApJL, 892, L3
Abbott, B. P., et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), 2020b, ApJL, 896, L44
Amaro-Seoane, P., et al. 2017, arXiv:1702.00786
Armano, M., et al. 2016, PRL, 116, 231101

Belczynski, K., Kalogera, V., Bulik, T. 2002, ApJ, 572, 407
Biesiada, M., Ding, X., Piorkowska, A., Zhu, Z.-H. 2014, JCAP, 10, 080
Cao, S., Fan, Y., Biesiada, M., Godlowski, W., & Zhu, Z.-H. 2012, JCAP, 03, 016
Cao, S., Covone, G., & Zhu, Z.-H. 2012, ApJ, 755, 31
Cao, S., Biesiada, M., Gavazzi, R., Piorkowska, A., & Zhu, Z.-H. 2015, ApJ, 806, 185
Cao, S., et al. 2019, NatSR, 9, 11608
Cao, Z., Li, L.-F., Wang, Y. 2014, PRD, 90, 062003
Choi, Y.-Y., Park, C., & Vogelezang, M. S., 2007, ApJ, 658, 884
Collett, T. E., Bacon, D. 2017, PRL, 118, 091101
Coulter, D. A., et al. 2017, Science, 358, 1556
Dai, L., Vennumadhav, T., Sigurdson, K. 2017, PRD, 95, 044011
Ding, X., Biesiada, M., & Zhu, Z.-H. 2015, JCAP, 12, 006
Dominik, M., et al. 2012, ApJ, 759, 52
Dominik, M., et al. 2013, ApJL, 779, 72
Fan, X.-L., Liao, K., Biesiada, M., Piorkowska-Kurpas, A., & Zhu, Z.-H. 2017, PRL, 118, 091102
Finn, L. S. 1996, PRD, 53, 2878
Goldstein, A., et al. 2017, ApJL, 848, L14
Hou, S., Li, P., Yu, H., Biesiada, M., Fan, X.-L., Kawamura, S., Zhu, Z.-H., “Lensing rates of gravitational wave signals displaying beat patterns detectable by DECIGO and B-DECIGO” 2020, PRD accepted [arXiv:2009.08116]
Isoyama, S., Nakano, H., Nakamura T. 2018, Prog. Theor. Exp. Phys. 073E01
Kawamura, S., et al. 2019, IJMPD, 28, 1845001
Koopmans, L. V. E., et al. 2009, ApJL, 703, L51
Li, S. S., Mao, S., Zhao, Y., & Lu, Y. 2018, MNRAS, 476, 2220
Liao K., Fan X.-L., Ding X.-H., Biesiada M., Zhu Z.-H. 2017, Nat. Commun., 8, 1148
Liao, K., Ding, X., Biesiada, M., Fan, X.-L., Zhu, Z.-H. 2018, ApJ, 867, 69
Liao, K., Biesiada, M., & Fan, X.-L. 2019, ApJ, 875, 139
Nakamura T., 1998, PRL, 80, 1138
Nakamura T. et al., 2016, Prog. Theor. Exp. Phys. 093E01
[arXiv:1607.00897]
Ng, K. K. Y., Wong, K. W. K., Broadhurst, T., & Li, T. G. F. 2018, PRD, 97, 023012
Oguri, M. 2018, MNRAS, 480, 3842
Phinney, E. S. 2001, arXiv:0108028
Piorkowska, A., Biesiada, M., Zhu, Z.-H. 2011, JCAP, 10, 022
Sato S., et al. 2017, J. Phys.: Conf. Ser. 840, 012010
Sereno, M., Sesana, A., Bleuler, A., Jetzer, P., Volonteri, M., & Begelman, M. C. 2010, PRL, 105, 251101
Sereno, M., Jetzer, P., Sesana, A., & Volonteri, M. 2011, MNRAS, 415, 2773
Seto, N., Kawamura, S., & Nakamura T. 2001, PRL, 87, 221103
Souther, S., Li, P., Yu, H., Biesiada, M., Fan, X.-L., Kawamura, S., Zhu, Z.-H., “Lensing rates of gravitational wave signals displaying beat patterns detectable by DECIGO and B-DECIGO” 2020, PRD accepted [arXiv:2009.08116]
Takahashi, R. & Nakamura T. 2003, ApJ, 595, 1039
Taylor, S. R. & Gair, J. R. 2012, PRD, 86, 023502
Wei, J-J. & Wu, X-F. 2017, MNRAS, 472, 2906
Yagi, K. & Seto, N. 2011, PRD, 83, 044011
Yagi, K. 2013, IJMPD, 22, 1341013
Yang, L., Ding, X., Biesiada, M., Liao, K., Zhu, Z.-H. 2019, ApJ, 874, 139
Zhu, Z.-H. & Wu, X.-P. 1997, A&A, 324, 483
Zhu, Z.-H. 1998, A&A, 338, 777