The lowest order inelastic QED processes at polarized photon-electron high energy collisions

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The compact expressions for cross sections of photoproduct ion of a pair of charged particles $e^+, e^-; \mu^+, \mu^-; \pi^+, \pi^-$ as well as the double Compton scattering process are given. The explicit analytic expressions for the case of polarized photon and the initial electron in the kinematics when all the particles can be considered as a massless ones are presented. The photon polarization is described in the terms of Stokes parameters.

A process of charged pair photoproduction on electron is usually used as a polarimeter process. A lot of attention was paid, for instance, to a trident production process by a linearly polarized photon (with $e^-e^-e^+$ final state) where the differential distribution over the recoil electron can be used to arrange the left-right azimuthal asymmetry of an order of 14% (see for instance [1] and the references therein).

Linear $e^+e^-$ high energy colliders (planned to be arranged [2]) provide a possibility (using the backward laser Compton scattering) to obtain the photon-electron as well as photon-photon colliding beams. The problem of calibration as well as the problem of important QED background are to be taken into account for this kind of colliders. The QED processes at $\gamma e^\pm$ colliders are relevant for this purposes. For colliding high energy electron-photon beams with detections of large-angle emitted final particles the totally differential cross sections can be useful. For our knowledge the relevant formulae are absent in the literature, which was the motivation of our paper. It is organized as follows. First, using the standard methods
we calculate the chiral amplitudes of processes

\[ \gamma(k, \lambda_\gamma) + e^-(p, \lambda_e) \rightarrow e^-(p', \lambda_e) + \gamma(k_1, \lambda_1), \]  

(1)

\[ \gamma(k, \lambda_\gamma) + e^-(p, \lambda_e) \rightarrow e^-(p', \lambda') + a(q_-, \lambda_-) + \bar{a}(q_+, \lambda_+), \ a = e^-, \mu^-, \]  

(2)

\[ \gamma(k, \lambda_\gamma) + e^-(p, \lambda_e) \rightarrow e^-(p', \lambda') + \pi^-(q_-) + \pi^+(q_+), \]  

(3)

\[ \gamma(k, \lambda_\gamma) + e^-(p, \lambda_e) \rightarrow e^-(p', \lambda') + \gamma(k_1, \lambda_1) + \gamma(k_2, \lambda_2), \]  

(4)

with the parameters \( \lambda_i = \pm 1 \) describe the chiral states of particles. We consider the experimental setup, when the polarization of final particles is not measured. Using the explicit form of chiral amplitudes of the production processes mentioned above, we construct the chiral matrix. Then, converting it with photon density matrix, we obtain the cross sections of the photoproduction processes for arbitrary polarization state of the initial photon.

In Appendix we give some details of the calculations.

We consider the kinematics when all the 4-vector scalar products defined by

\[ s = 2p.p', \quad s_1 = 2q_-q_+, \quad t = -2p.q_-, \quad t_1 = -2p'.q_+, \]  

(5)

\[ u = -2p.q_+ , \quad u_1 = -2p'.q_-, \quad \chi = 2k.p, \quad \chi' = 2k'.p, \]  

\[ \chi_{\pm} = 2k.q_{\pm}, \quad \chi_j = 2k_j.p, \quad \chi'_j = 2k_j.p', \quad j = 1, 2, \]  

are large compared with the electron, pion and muon masses squared

\[ s \sim s_1 \sim -t \sim -t_1 \sim -u \sim -u_1 \sim \chi_j \sim \chi'_j \gg m_{\mu}^2, \]  

(6)

\[ p^2 = p'^2 = q^2_{\pm} = k^2 = k_j^2 = 0. \]

These kinematical invariants are not independent ones. Some relations can be obtained using the conservation law and mass shell conditions

\[ s_1 + t + u = s + t_1 + u_1, \quad \chi = s_1 - t_1 - u_1, \quad \chi' = -s_1 - t - u, \]  

(7)

\[ \chi_+ = u_1 + s - t, \quad \chi_- = t_1 + s - u, \quad 2k_1k_2 = \chi - \chi' - s. \]

Photon polarization 4-vectors with the definite chirality \( \lambda = \pm 1, \varepsilon_\mu^\lambda(k), \hat{\varepsilon}_\mu^\lambda = \gamma^\mu\varepsilon_\mu^\lambda \) can be
put in 5 different representations

\[ \hat{\varepsilon}_p^\lambda(k_j) = N_p(k_i)[p \hat{p} \hat{k}_i \omega_{-\lambda} - \hat{k}_i \hat{p} \hat{p} \omega_{\lambda}], \quad k_j = k, k_1, k_2, \]

\[ \hat{\varepsilon}_q^\lambda(k) = N_q[\hat{q} \hat{q} \hat{k} \omega_{-\lambda} - \hat{k} \hat{q} \hat{q} \omega_{\lambda}], \]

\[ \hat{\varepsilon}_{11}^\lambda(k) = N_{11}[p \hat{p} \hat{k} \omega_{-\lambda} - \hat{k} \hat{p} \hat{p} \omega_{\lambda}], \]

\[ \hat{\varepsilon}_s^\lambda(k) = N_s[\hat{q} \hat{q} \hat{k} \omega_{-\lambda} - \hat{k} \hat{q} \hat{q} \omega_{\lambda}], \]

\[ \hat{\varepsilon}_\pi^\lambda(k) = N_\pi[k q_{-\mu} - k q_{\mu} + i \lambda \varepsilon_{\mu \alpha \beta} q^\alpha q^\beta k^\gamma], \]

with

\[ \omega_{\pm} = (1/2)(1 + \lambda \gamma_5), \quad \omega_{\pm}^2 = \omega_{\pm} \pm \omega_{\mp} = 0, \]

\[ N = N_p(k) = [s \chi \chi'/2]^{-1/2}, \quad N_q = [s_1 \chi + \chi - 2/2]^{-1/2}, \quad N_{11} = [-t_1 \chi \chi'/2]^{-1/2}, \]

\[ N_{1,2} = N_p(k_{1,2}) = [s \chi_{1,2} \chi'_{1,2}/2]^{-1/2}, \quad N_t = [-t \chi \chi'/2]^{-1/2}. \]

Here we omit the terms of type \( \hat{k}, k \mu \) in the right-hand side of equations for \( \varepsilon(k) \), which are irrelevant due to gauge conditions. These vectors obey the conditions

\[ \varepsilon_p^\lambda(k_j) = \varepsilon_p^\lambda e^{i \lambda \varphi_j} + c_j k, \quad e^{i \lambda \varphi_j} = -(1/4) Tr \hat{\varepsilon}_p^\lambda \hat{\varepsilon}_j^{-\lambda}, \]

\[ \varepsilon_1^\lambda \varepsilon_2^\lambda = 0 \quad (i \neq j, \ i = j), \quad \varepsilon_1^\lambda \varepsilon_1^{-\lambda} = -1 \quad (i = j). \]

The quantity \( c_i \) can be safely omitted due to gauge invariance of the amplitude. We will systematically omit such terms. Performing the calculations of chiral amplitudes we use \( \varepsilon_p^\lambda \) and transform it to the relevant form considering the definite Feynman amplitudes. Note as well the useful relation

\[ sN_p \hat{\varepsilon}_p^\lambda + s_1 N_q \hat{\varepsilon}_q^\lambda = -[tN_{11} \hat{\varepsilon}_1^\lambda + t_1 N_t \hat{\varepsilon}_2^\lambda]. \]

The relation (11) can be written in the form

\[ [s_{\pm}] = -e^{\pm i \varphi_2} [t_{\pm}], \quad [s_{\pm}] = sN_p + s_1 N_q e^{\pm i \varphi_2}, \]

\[ [t_{\pm}] = N_{11} t_1 + tN_{11} e^{\pm i \varphi_{12}}, \quad \varepsilon_2^\lambda = \varepsilon_1^\lambda e^{i \lambda \varphi_{12}}. \]

Following one can obtain (see Appendix)

\[ [t_+][t_-] = [s_+][s_-] = \frac{W}{2} = r_s + r_s + r_t + r_{t_1} - r_u - r_{u_1}, \]

\[ W = -\left( \frac{q + q_1}{k.q_+} - \frac{q - q_1}{k.q_-} + \frac{p + p_1}{k.p} - \frac{p - p_1}{k.p} \right)^2, \]

\[ r_s = \frac{2s}{\chi \chi'}, \quad r_s = \frac{2s_1}{\chi + \chi'}, \quad r_t = \frac{-2t}{\chi \chi'}, \quad r_{t_1} = \frac{-2t_1}{\chi' \chi'}, \quad r_u = \frac{-2u}{\chi \chi'}, \quad r_{u_1} = \frac{-2u_1}{\chi' \chi'}. \]
Chiral states of leptons are defined by

\[ u^\lambda(p) = \omega_\lambda u(p), \quad \bar{u}(p)^\lambda = \bar{u}(p)\omega_{-\lambda}, \]
\[ v^\lambda(p) = \omega_{-\lambda} v(p), \quad \bar{v}(p)^\lambda = \bar{v}(p)\omega_\lambda, \]
\[ u^\lambda(p)\bar{u}^\lambda(p) = \omega_\lambda \hat{p}, \quad v^\lambda(p)\bar{v}^\lambda(p) = \omega_{-\lambda} \hat{p}. \] (14)

The spin density matrix for the initial electron in the ultrarelativistic case has the form

\[ u(p, a)\bar{u}(p, a) = \frac{1}{2} \hat{p}[1 - \gamma_5(\lambda + \hat{\lambda}_\perp)], \quad \lambda^2 + |\lambda_\perp|^2 \leq 1, \quad -1 < \lambda < 1. \] (15)

The effect of transversal polarization of the fermion \( \lambda_\perp \) is negligible (the relevant contribution is suppressed by a factor \( m/\sqrt{\chi} \) compared to the case of longitudinal polarization of fermions). Cross section for the incomplete polarization of the initial electron will have the form

\[ d\sigma = \zeta_+ d\sigma_+ + \zeta_- d\sigma_-, \quad \zeta_{\pm} = \frac{1}{2} (1 \pm \lambda) \] (16)

and \( d\sigma_\pm \) corresponds to the case of complete longitudinal polarization of the initial electron described by the pure chiral state. Below we suggest that the initial electron belongs to the pure chiral state.

The chiral amplitudes \( M_{\lambda,\lambda}^{\lambda,\lambda'} \) for processes of a charged pair of lepton production, \( M_{\lambda,\lambda}^{\lambda'} \) for a pair of charged pion production and \( M_{\lambda,\lambda}^{\lambda_1\lambda_2\lambda'} \) for a two photon-electron final states are defined as a usual matrix element calculated with chiral states of photons and leptons.

The matrix elements in the lower order of perturbation theory are determined by tree-type Feynman diagrams.

The matrix element for muon pair production has the form \( M_{\mu^+\mu^-} = M_1 + M_2 \), with

\[ M_1 = -\frac{1}{s} \bar{u}(q_-) \left[ \hat{q}_- \gamma_\eta - \hat{k}_- \frac{\gamma_\eta - \hat{q}_+ + \hat{k}_+}{\chi_+} \right] v(q_+) \bar{u}(p') \gamma_\eta u(p), \] (17)
\[ M_2 = \frac{1}{s_1} \bar{u}(q_-) \gamma_\sigma v(q_+) \bar{u}(p') \left[ \hat{q}_+ - \hat{k}_+ \frac{\gamma_\sigma - \hat{q}_- + \hat{k}_-}{\chi_-} \right] u(p).

For the \( e^+e^- \) pair production the identity of electrons must be taken into account which result in the appearance of two additional terms

\[ M_{e^+e^-} = M_1 + M_2 - \tilde{M}_1 - \tilde{M}_2, \] (18)
with

\[
M_1 = \frac{1}{t} \bar{u}(q_-) \gamma_\rho u(p) \bar{u}(p') \left[ \gamma^\rho \gamma^\lambda - \frac{1}{m^2} \gamma^\rho - \frac{1}{m^2} \gamma^\lambda \right] v(q_+),
\]

\[
M_2 = -\frac{1}{t_1} \bar{u}(q_-) \left[ \gamma^\rho \gamma^\lambda - \frac{1}{m^2} \gamma^\rho - \frac{1}{m^2} \gamma^\lambda \right] u(p) \bar{u}(p') \gamma_\eta v(q_+).
\]

For the \(\pi^+\pi^-\) pair we have \(M_{\pi^+\pi^-} = M_3 + M_4\) with

\[
M_3 = -\frac{1}{s} \bar{u}(p') \gamma_\eta u(p)
\]

\[
\times \left[ -2(q_- - q_+ - k) \eta \frac{\hat{q}_- \gamma^\eta}{\chi_-} + 2(q_- - q_+ + k) \eta \frac{\hat{q}_+ \gamma^\eta}{\chi_+} - 2e_\eta \right],
\]

\[
M_4 = \frac{1}{s_1} (q_- - q_+) \sigma \bar{u}(p') \left[ \frac{\gamma_\sigma \hat{p} + \hat{k}}{\chi} + \frac{\bar{\gamma} \gamma^\eta}{\chi} \right] u(p).
\]

Using these expressions we calculate the chiral amplitudes of the processes.

The matrix elements squared summed over the spin states of the final particles have the form of the conversion of the chiral matrix with the photon density matrix

\[
\sum |M|^2 = \frac{1}{2} \text{Tr} \left( \begin{array}{cc} m_{11} & m_{12} \\ m_{21} & m_{22} \end{array} \right) \left( \begin{array}{cc} 1 + \xi_2 & i\xi_1 - \xi_3 \\ -i\xi_1 - \xi_3 & 1 - \xi_2 \end{array} \right),
\]

with the vector of photon polarization \(\vec{\xi}\) parameterized by Stokes parameters fulfilling the condition \(\xi_1^2 + \xi_2^2 + \xi_3^2 \leq 1\).

The matrix elements of the chiral matrix \(m_{ij}\) are constructed from the chiral amplitudes of the process \(M^{\lambda_-\lambda_+\lambda'}_{\lambda_-\lambda_+}\) as

\[
m_{11} = \sum_{\lambda_-\lambda_+\lambda'} |M^{\lambda_-\lambda_+\lambda'}_{++}|^2, \quad m_{22} = \sum_{\lambda_-\lambda_+\lambda'} |M^{\lambda_-\lambda_+\lambda'}_{+-}|^2,
\]

\[
m_{12} = \sum_{\lambda_-\lambda_+\lambda'} M^{\lambda_-\lambda_+\lambda'}_{++} (M^{\lambda_-\lambda_+\lambda'}_{+-})^*, \quad m_{21} = m_{12}^*.
\]

We put here only half of all chiral amplitudes which correspond to \(\lambda_+ = +1\). The other half can be obtained from these ones by a space parity operation (replacement \(\omega_\lambda\) by \(\omega_{-\lambda}\) and \([s_+]\) by \([s_-]\)).

For the process of Compton scattering the matrix element \(M^{\lambda_-\lambda_+}_{\lambda_-}\) has the form (here and further we omit the factor \(i(4\pi\alpha)^{n/2}\))

\[
M^{++}_{+} = sN_1N\bar{u}(p')\hat{k}\omega_+ u(p), \quad M^{+-}_{+} = sN_1N\bar{u}(p')\hat{k}\omega_+ u(p).
\]
For the muon pair production for \( M_{\lambda_{\nu\lambda}}^{+\lambda} \) we have

\[
M_{++}^{++} = \frac{-2u}{s_1}[s_+][q] \frac{\bar{u}(q_-)\gamma_{\lambda}^+ + \gamma_{\nu} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

\[
M_{++}^{+-} = \frac{2u}{s}[s_-] \frac{\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

\[
M_{+-}^{++} = \frac{-2t_1}{s_1}[s_+] \frac{\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

\[
M_{+-}^{+-} = \frac{-2}{s_1}[s_-] \frac{\bar{u}(q_-)^2 p^\nu \gamma_{\lambda} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

For the case of the \( e^+e^- \) pair production

\[
M_{++}^{+-} = \frac{2s_1}{s}[s_+] \frac{\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

\[
M_{++}^{++} = \frac{-2}{s_1}[s_-] \frac{\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

\[
M_{++}^{+-} = \frac{-2}{s_1}[s_-] \frac{\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} u(p)}{\bar{v}(q_+)\gamma_{\lambda} u(p')},
\]

\[
M_{++}^{++} = 2[s_1][A_+ - s s_1][u(p')\gamma_{\lambda}^+ \gamma_{\nu} u(p)]\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} \gamma_{\lambda} u(p'),
\]

\[
M_{++}^{++} = 2[s_1][A_+ - s s_1][u(p')\gamma_{\lambda}^+ \gamma_{\nu} u(p)]\bar{u}(q_-)\gamma_{\lambda}^+ \gamma_{\nu} \gamma_{\lambda} u(p'),
\]

with \( A_{\pm} = \text{Tr} \gamma_{\lambda}^+ \gamma_{\nu} \gamma_{\lambda} \pm \).

For process of charged pion pair production \( M_{\lambda_{\nu\lambda}}^{+\lambda} \) we have

\[
M_{++}^{++} = \frac{2}{s_1}[s_+] \frac{u(p')^2(\bar{u} q_+ - t q_+ t q_+ \gamma_{\lambda} u(p)},
\]

\[
M_{++}^{++} = \frac{2}{s_1}[s_-] \frac{u(p')^2(\bar{u} q_+ - t q_+ t q_+ \gamma_{\lambda} u(p)}.
\]

For the case of double Compton scattering chiral amplitudes \( M_{\lambda_{\nu\lambda}}^{+\lambda} \) we have

\[
M_{++}^{++} = -s^2 \gamma_{\lambda_1} N_1 N_2 N \bar{u}(p') \hat{k}_{\lambda_2} u(p),
\]

\[
M_{++}^{++} = -s^2 \gamma_{\lambda_1} N_1 N_2 N \bar{u}(p') \hat{k}_{\lambda_2} u(p),
\]

\[
M_{++}^{++} = -s^2 \gamma_{\lambda_1} N_1 N_2 N \bar{u}(p') \hat{k}_{\lambda_2} u(p),
\]

\[
M_{++}^{++} = -s^2 \gamma_{\lambda_1} N_1 N_2 N \bar{u}(p') \hat{k}_{\lambda_2} u(p),
\]

\[
M_{++}^{++} = -s^2 \gamma_{\lambda_1} N_1 N_2 N \bar{u}(p') \hat{k}_{\lambda_2} u(p),
\]

\[
M_{++}^{++} = -s^2 \gamma_{\lambda_1} N_1 N_2 N \bar{u}(p') \hat{k}_{\lambda_2} u(p),
\]
The remaining amplitudes are equal to zero. We put here some useful relations

\[
\left| \frac{\bar{u}(q_-) \hat{q}_+ \omega_+ u(p)}{\bar{v}(q_+)} \right|^2 = \frac{s_1}{s}, \quad \left| \frac{\bar{u}(q_-) \hat{p}_1' \omega_+ u(p)}{\bar{v}(q_+)} \right|^2 = \frac{s}{s_1},
\]

\[
\left| \frac{\bar{u}(q_-) \hat{q}_+ \hat{p}_1' \omega_- u(p)}{\bar{v}(q_+)} \right|^2 = ss_1, \quad \left| \frac{\bar{u}(q_-) \hat{p}_1' \hat{q}_- \omega_- u(p)}{\bar{v}(q_+)} \right|^2 = ss_1 t^2,
\]

\[
\left| \frac{\bar{u}(q_-) \hat{p}_1' \omega_+ v(q_+)}{\bar{u}(p) \omega_- u(p')} \right|^2 = tt_1, \quad \left| \frac{\bar{u}(q_-) \hat{q}_+ \hat{p}_1' \hat{q}_- \omega_- v(q_+)}{\bar{u}(p) \omega_- u(p')} \right|^2 = s_1^2 t t_1,
\]

\[
\left| \bar{u}(p') \hat{q}_+ \omega_+ u(p) \bar{u}(q_-) \hat{p}_1' \omega_+ v(q_+) \right|^2 = u^2 t t_1,
\]

\[
\left| u(p') v_+ \omega_- u(p) \bar{u}(q_-) \hat{p}_1' \omega_+ v(q_+) \right|^2 = u_1^2 t t_1,
\]

\[
\left| \bar{u}(p') (u \hat{q}_- - t \hat{q}_+ \omega_+ u(p)) \right|^2 = ss_1 u t,
\]

\[
\left| \bar{u}(p') (u_1 \hat{q}_+ - t_1 \hat{q}_- \omega_+ u(p)) \right|^2 = ss_1 t u_1,
\]

\[
\left| \bar{u}(p') \hat{k} \omega_+ u(p) \right|^2 = \chi \chi', \quad \left| \bar{u}(p') \hat{k} \omega_+ u(p) \right|^2 = \chi_1 \chi_1',
\]

\[
\left| \bar{u}(p') \hat{k} \omega_+ u(p) \right|^2 = \chi_2 \chi_2'.
\]

Besides we have \( |A_\pm|^2 = |\bar{u}(q_+) \hat{p}_1' \hat{q}_- \omega_\pm u(q_+)|^2 = ss_1 t t_1 \) and \( |A_\pm - ss_1|^2 = ss_1 uu_1. \)

For the case of muon pair production we have

\[
m_{11} = 4[s_+] [s_-] \frac{u^2 + t^2}{ss_1}, \quad m_{22} = 4[s_+] [s_-] \frac{u_1^2 + t_1^2}{ss_1},
\]

\[
m_{12} = - \frac{4[s_+]^2}{(ss_1)^2} \left[ (ss_1)^2 + (tt_1)^2 + (uu_1)^2 - 2tt_1 uu_1 - ss_1(tt_1 + uu_1) \right.
\]

\[
\left. + 4i(tt_1 - uu_1) A \right],
\]

where \( A = \varepsilon_{\mu \nu \rho \sigma} q_\mu q_\nu p_\rho p_\sigma. \)

For the electron pair production we have

\[
m_{11} = 4[s_+] [s_-] \frac{t_1 t + u_1^3 + u_1^3 s}{ss_1 t t_1}, \quad m_{22} = 4[s_+] [s_-] \frac{t_1^2 u + u_1^3 u + u_1^3 s}{ss_1 t t_1},
\]

\[
m_{12} = \frac{4[s_+]^2}{(ss_1 t t_1)^2} \left[ (uu_1)^2 - (ss_1)^2 - (tt_1)^2 \right] \frac{1}{2} ((uu_1)^2 + (ss_1)^2 + (tt_1)^2
\]

\[
- 2uu_1(ss_1 + tt_1)) + 2iA(uu_1 - ss_1 - tt_1) \right].
\]

For pions one gets

\[
m_{11} = [s_+] [s_-] tu, \quad m_{22} = [s_+] [s_-] t u_1,
\]

\[
m_{12} = \frac{[s_+]^2}{ss_1} \left[ \frac{1}{2} (uu_1 - tt_1)^2 - ss_1(uu_1 + tt_1) + 2i(tt_1 - uu_1) A \right].
\]
The explicit expression for \([s_+]^2\) is given in Appendix.

For the double Compton scattering we have

\[
m_{11} = \frac{8s}{D} \left[ \chi_1^3 \chi_1^1 + \chi_2^3 \chi_2^2 + \chi_1^3 \chi_2^1 + \chi_1^1 \chi_2^3 + \chi_1^3 \chi_2^2 \right],
\]

\[
m_{22} = \frac{8s}{D} \left[ \chi_1^3 \chi_1 + \chi_2^3 \chi_2 \right],
\]

\[
m_{12} = \frac{8s}{D} \left\{ \frac{1}{2} \left( \chi_1 \chi_2 + \chi_2 \chi_1 \right) \right\} \left[ \chi_1 \chi_2 + \chi_2 \chi_1 + s(s + \chi - \chi') \right]
\]

\[+ 2iB(\chi_2 \chi'_1 - \chi_1 \chi'_2) \right\},
\]

with \(D = \chi \chi' \chi_1 \chi_2 \chi_2 \) and \(B = \varepsilon_{\mu \nu \rho \sigma} k_1^\mu k_1^\nu p^\rho p^\sigma\).

The differential cross section of the Compton scattering process (1) has the form

\[
d\sigma_{\gamma^- e^- \rightarrow \gamma^+ e^+} = \frac{\alpha^2}{4\lambda} \left[ \frac{\chi^2 + \chi'^2}{\chi \chi'} + \xi_2 \lambda_\epsilon \frac{\chi^2 - \chi'^2}{\chi \chi'} \right] dO_\gamma.
\]

(33)

The cross section of processes (2, 3) has the form

\[
\frac{d\sigma}{d\Gamma} = \frac{\alpha^3}{2\pi^2 \chi} \left[ m_{11} + m_{22} + \xi_2 \lambda_\epsilon (m_{11} - m_{22}) - 2\xi_3 \text{Re}(m_{12}) + 2\xi_4 \text{Im}(m_{12}) \right],
\]

\[
d\Gamma = \frac{d^3 p'}{\epsilon'} \frac{d^3 q_+}{\epsilon} \frac{d^3 q_-}{\epsilon} \delta^4(p + k - p' - q_+ - q_-)
\]

(34)

The cross section of processes (4) has the form

\[
\frac{d\sigma}{d\Gamma} = \frac{1}{2!} \frac{\alpha^3 s}{2\pi^2 \chi D} \left[ \chi \chi' (\chi^2 + \chi'^2) + \chi_1 \chi_1' (\chi_1^2 + \chi_1'^2) + \chi_2 \chi_2' (\chi_2^2 + \chi_2'^2) \right]
\]

\[+ 4\xi_1 B(\chi_1 \chi'_2 - \chi_2 \chi'_1) - \xi_3 (\chi_1 \chi'_2 + \chi_2 \chi'_1)(\chi \chi' - \chi_1 \chi'_1 - \chi_2 \chi'_2)
\]

\[+ \lambda_\epsilon \xi_2 \chi \chi' (\chi^2 - \chi'^2) + \chi_1 \chi_1' (\chi_1^2 - \chi_1'^2) + \chi_2 \chi_2' (\chi_2^2 - \chi_2'^2) \right],
\]

\[
d\Gamma = \frac{d^3 p'}{\epsilon'} \frac{d^3 k_1}{\epsilon_1} \frac{d^3 k_2}{\epsilon_2} \delta^4(p + k - p' - k_1 - k_2),
\]

(35)

where the multiplier \(1/2!\) takes into account the identity of photons in the final state.

As a conclusion we note that the unpolarized part of the cross section dominates over the polarized one, providing the positivity of cross sections. Asymmetries (the ratios of the polarized part to the unpolarized one) in general kinematics are the quantities of an order of unity.

**Appendix**

It is convenient to write down \([s_+]\) in the form

\[
[s_+] = \frac{1}{2 \sqrt{r_s}} \left[ 2r_s - r_u - r_{u1} + r_t + r_{t1} - 8i \frac{(\chi_+ + \chi_-) A}{\chi \chi' + \chi} \right].
\]

(36)
Then the quantity \([s_+]^2\) which enters into the cross sections of processes (2, 3) has the form

\[
[s_+]^2 = T_1 - iT_2A, \tag{37}
\]

\[
T_1 = r_s - r_{s1} + r_t + r_{t1} - r_u - r_{u1} + \frac{1}{2r_s}(r_u + r_{u1} - r_t - r_{t1})^2,
\]

\[
T_2 = \frac{4(\chi_+ + \chi_-)}{\chi_+\chi_-\chi' r_s} [2r_s - r_u - r_{u1} + r_t + r_{t1}]k^\mu q^\nu p^\rho p'^\sigma.
\]

To check the expression \([s_+]^2[s_-]^2 = \frac{W^2}{4}\) we used the relation

\[
(\chi_+ + \chi_-)A = \varepsilon_{\mu\nu\rho\sigma}(\chi_+k^\mu q^\nu p^\rho p'^\sigma + \chi_-q^\mu k^\nu p^\rho p'^\sigma).
\]

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