Annihilation Poles for Form Factors in XXZ Model

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Abstract

The annihilation poles for the form factors in XXZ model are studied using vertex operators introduced in [1]. An annihilation pole is the property of form factors according to which the residue of the 2n-particle form factor in such a pole can be expressed through linear combination of the 2n−2-particle form factors. To prove this property we use the bosonization of the vertex operators in XXZ model which was invented in [2].

1 Introduction

Recently much effort was spent for the further development of the “non-conformal” models in quantum field theory bypassing the standard approach to such models as Bethe ansatz [1, 3]. The first successful consideration of quantum massive integrable models (such as sine-Gordon, su(2)-invariant Thirring and o(3) nonlinear σ-models) was invented by F. Smirnov on the early days of development of quantum inverse scattering method [4]. He found the system of axioms which should be satisfied by the form factors of the local operators of the model in order to ensure the locality of the field operators. These axioms are:

Axiom 1. The form factor $f(\beta_1, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_n}$ has the symmetry property

$$f(\beta_1, \ldots, \beta_i, \beta_{i+1}, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_i, \varepsilon_{i+1}, \ldots, \varepsilon_n} \epsilon_{\varepsilon_i, \varepsilon_{i+1}}^{\varepsilon_i'}(\beta_i - \beta_{i+1})$$

$$= f(\beta_1, \ldots, \beta_{i+1}, \beta_i, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_{i+1}, \varepsilon_i', \ldots, \varepsilon_n}$$  

(1)

Axiom 2. Form factor $f(\beta_1, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_n}$ satisfies the equation

$$f(\beta_1, \ldots, \beta_n + 2\pi i)_{\varepsilon_1, \ldots, \varepsilon_n} = f(\beta_n, \beta_1, \ldots, \beta_{n-1})_{\varepsilon_n \varepsilon_1, \ldots, \varepsilon_{n-1}}$$  

(2)

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**Axiom 3.** If the spectrum of the model does not contain the particles of the different kinds or bound states the form factor $f(\beta_1, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_n}$ has only singularities at the points $\beta_i = \beta_j + \pi i$, $i > j$ and these singularities are first order poles with residues

$$2\pi i \text{ res}_{\beta_n=\beta_{n-1}+\pi i} f(\beta_1, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_n} = f(\beta_1, \ldots, \beta_{n-2})_{\varepsilon_1, \ldots, \varepsilon_n} \delta_{\varepsilon_n, -\varepsilon_{n-1}}$$

$$\times \left( \delta_{\varepsilon_1, 1} \cdots \delta_{\varepsilon_{n-2}, 1} S_{\varepsilon_1, \varepsilon_2}^{\varepsilon_n, 1} (\beta_{n-1} - \beta_1) \cdots S_{\varepsilon_{n-3}, \varepsilon_{n-2}}^{\varepsilon_n, 1} (\beta_{n-1} - \beta_{n-2}) \right).$$

(3)

By the form factor $f(\beta_1, \ldots, \beta_n)_{\varepsilon_1, \ldots, \varepsilon_n}$ we mean here the vacuum expectation value of local operator and $n$ operators which create the particles with rapidities $\beta_i$ and internal symmetry indexes $\varepsilon_i$ and satisfy the Zamolodchikov-Faddeev algebra with $S$-matrix $S_{\varepsilon_1, \varepsilon_2}^{\varepsilon_n, 1} (\beta_1 - \beta_2)$. The third axiom is important for proving the fact that two local operators commute on a space-like interval [3]. Recently it was understood [5] that combination of first two axioms is in fact the Yangian version of the deformed Knizhnik-Zamolodchikov (KZ) equation introduced by Frenkel and Reshetikhin [6].

The axioms (1)–(3) appeared in consideration the relativistic integrable models. It is well known that one of this models, namely $su(2)$-invariant Thirring model, can be obtained as scaling continuum limit of the XXZ lattice model. The creation and annihilation operators of the latter model depends on variable $z$ which parametrizes the momentum of the spin waves (particles) in XXZ model [1]. We can introduce the “rapidity” $\beta$ which is related to $z$ by

$$z = \exp \left( -2\beta \frac{\ln(-q)}{\pi i} \right)$$

(4)

and varies in the limits $\pi^2 / \ln (-q) < \beta < -\pi^2 / \ln (-q)$, $-1 < q < 0$. When $q \to -1 + 0$ the range of $\beta$ increases to $-\infty < \beta < \infty$ and the parameter $\beta$ indeed turns out into the rapidity for the particle in $su(2)$-invariant Thirring model. The energy and momentum

$$\epsilon(\beta) = -(q - q^{-1}) z \frac{d}{dz} \ln e^{-i\varphi(\beta)}, \quad e^{-i\varphi(\beta)} = \frac{1}{\sqrt{z}} \frac{\Theta_{q^4}(q z)}{\Theta_{q^4}(q z^{-1})}$$

(5)

of the spin waves in this scaling limit turns into relativistic energy and momentum of the continuum theory.

The question we are trying to answer in this paper is whether the form factors of XXZ model satisfy the analogous axioms as the form factors of the $su(2)$-invariant Thirring model do. We will see that these form factors indeed satisfy these axioms enhanced by the fourth one.

**Axiom 4.** The form factors of the XXZ model are single valued functions with respect to parameters $z$'s [4].

From another point of view the XXZ model can be considered as deformation of XXX model. Since the form factor description is related to the particle content of the model considered, the properties of the form factors in XXZ model are interesting
from the point of view presented in [7] as a certain regularization of the particle picture in XXX model [8].

Key steps in understanding the role of \( q \)-deformed KZ equation in quantum integrable models was made in the wonderful papers [1, 2]. The XXZ model with Hamiltonian

\[
H_{\text{XXZ}} = -\frac{1}{2} \sum_{k=-\infty}^{\infty} \left( \sigma_{k+1}^x \sigma_k^x + \sigma_{k+1}^y \sigma_k^y + \Delta \sigma_{k+1}^z \sigma_k^z \right)
\]

in anti-ferromagnetic regime was considered there. Formally this Hamiltonian act on the infinite tensor product \( V^{\otimes \infty} = \cdots \otimes V \otimes V \otimes \cdots \) of the two-dimensional spaces \( V = \mathbb{C}^2 = v_+ \oplus v_- \), where the quantum affine algebra \( U_q(\hat{\mathfrak{sl}}(2)) \) act via iterated coproduct. However these actions are defined only formally and the main problem was how define theory free from the divergences. In [1] the main idea was to replace the formal object \( V^{\otimes \infty} \) by the level 0 \( U_q(\hat{\mathfrak{sl}}(2)) \)-module (\( \Delta = (q+q^{-1})/2, -1 < q < 0 \))

\[
\mathcal{F}_{\lambda, \mu} = V(\lambda) \otimes V(\mu)^{\ast a} \simeq \text{Hom}(V(\lambda), V(\mu)),
\]

where \( V(\lambda), \lambda = \Lambda_0, \Lambda_1 \) are the level 1 highest weight \( U_q(\hat{\mathfrak{sl}}(2)) \)-module. [1]

In order to connect the naive picture of \( V^{\otimes \infty} \) with these representation theoretical objects the embedding of the module \( V(\Lambda_i) \) to the half-infinite tensor product \( \cdots \otimes V \) was used. This can be done by iterating the vertex operators (VO)

\[
\Phi_{\Lambda_i}^{A_1-i}(z) : V(\Lambda_i) \to V(\Lambda_{1-i}) \otimes V_z,
\]

\[
\Phi_{\Lambda_i}^{A_1-i}(v)(z)(v) = \Phi_{\Lambda_i}^{A_1-i}(z)(v) \otimes v_+ + \Phi_{\Lambda_i}^{A_1-i}(z)(v) \otimes v_-.
\]

The above vertex operators are called type I and allow someone to construct the local operator in the model [2] (see (14) below). Obviously besides these VO it is possible to define type II VO

\[
\Phi_{\Lambda_i}^{V A_1-i}(z) : V(\Lambda_i) \to V_z \otimes V(\Lambda_{1-i}),
\]

\[
\Phi_{\Lambda_i}^{V A_1-i}(z)(v) = v_+ \otimes \Phi_{\Lambda_i}^{V A_1-i}(z)(v) + v_- \otimes \Phi_{\Lambda_i}^{V A_1-i}(z)(v)
\]

which are responsible for the particle content of the theory. Both sets of VO are defined uniquely by the requirement that they intertwine the corresponding \( U_q(\hat{\mathfrak{sl}}(2)) \)-modules and possess the normalization

\[
\Phi_{\Lambda_i}^{A_1-i}(z)(u_{\Lambda_i}) = (u_{\Lambda_{1-i}}) \otimes v_{\varepsilon_i} + \cdots,
\]

\[
\Phi_{\Lambda_i}^{A_1-i}(z)(u_{\Lambda_i}) = v_{\varepsilon_i} \otimes (u_{\Lambda_{1-i}}) + \cdots, \quad \varepsilon_0 = -, \quad \varepsilon_1 = +.
\]

Another important object that appeared in [1] is the notion of the invariant inner product on the space \( \mathcal{F}_{\lambda, \mu} \). For any vectors \( f, g \in \mathcal{F}_{\lambda, \mu} \) and \( x \in U_q(\hat{\mathfrak{sl}}(2)) \) the scalar product

\[
\langle f \mid g \rangle = \text{tr}_{V(\Lambda_i)}(q^{-2p} f \circ g)
\]

\( ^1 \) The \( V(\mu)^{\ast a} \) means that \( U_q(\hat{\mathfrak{sl}}(2)) \)-module structure is defined in the dual module via antipode. See [1] for the precise treatment.
obviously satisfy the relation \( \langle fx | g \rangle = \langle f | xg \rangle \), where \( \rho = \Lambda_0 + \Lambda_1 \).

By the form factor in XXZ model we will understand the trace over the module \( V(\Lambda_i) \) of the composition of the local operator \( (\mathbb{1}) \) and the type II vertex operators

\[
\text{tr}_{V(\Lambda_i)} \left( q^{-2\rho} \mathcal{L}^{(i)}(u_1, \ldots, u_n) \Phi_{\mu_2m}^{(1-i)}(z_{2m}) \ldots \Phi_{\mu_1}^{(i)}(z_1) \right),
\]

where the operators \( \Phi_{\epsilon}^{(i)}(z) \) are given in the principal picture that differ from those of \( (\mathcal{1}) \) by normalization factor

\[
\Phi_{\epsilon}^{(i)}(z) = z^{-i/2+(1+\epsilon)/4} \Phi_{\Lambda_i}^{1-V}(z), \quad \Phi_{\epsilon}^{(i)}(z) = z^{-i/2+(1+\epsilon)/4} \Phi_{\Lambda_i}^{V^{1-i}}(z).
\]

Because of the commutation relations

\[
\Phi_{\epsilon_2}^{(1-i)}(z_2) \Phi_{\epsilon_1}^{(i)}(z_1) = -R_{\epsilon_2 \epsilon_1}^{\epsilon_2 \epsilon_1} \left( \frac{z_2}{z_1} \right) \Phi_{\epsilon_1}^{(1-i)}(z_1) \Phi_{\epsilon_2}^{(i)}(z_2),
\]

\[
q^{-2\rho} \Phi_{+}^{(i)}(z) = \Phi_{+}^{(i)}(zq^4)q^{-2\rho},
\]

where trigonometric \( R \)-matrix is given by

\[
R_{12}(z) = r(z) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b(z) & c(z) & 0 \\ 0 & c(z) & b(z) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b(z) = \frac{(1 - z)q}{1 - q^2z},
\]

\[
r(z) = \frac{1}{\sqrt{z(q^2z)_\infty}} (q^2z^z(q^2z^{-1})_\infty, \quad c(z) = \frac{(1 - q^2z^{1/2})}{1 - q^2z}
\]

and trace property, the form factor \( (\mathcal{1}) \) satisfy the axioms \( (\mathcal{1}) \) and \( (\mathcal{2}) \) by definition. The goal of the present paper is to demonstrate that the form factor of the XXZ model satisfy also the third axiom. To do that we will use bosonization formulas for the vertex operators invented in \( (\mathcal{2}) \).

The main result of the paper can be summarized as follows. The form factor \( (\mathcal{1}) \) has only simple poles at the points \( q^{-2}z_i/z_j = 1, \ i < j \) with residue\(^2\)

\[
\text{res}_{q^{-2}z_i/z_j = 1} \text{tr}_{V(\Lambda_i)} \left( q^{-2\rho} \mathcal{L}^{(i)}(u_1, \ldots, u_n) \Phi_{\mu_2m}^{(1-i)}(z_{2m}) \ldots \Phi_{\mu_2}^{(1-i)}(z_2) \Phi_{\mu_1}^{(i)}(z_1) \right)
\]

\[
= -\frac{(q^2)_\infty^\alpha}{(q^3)_\infty^\beta} \delta_{\mu_1}^{\mu_2} \left( \delta_{\mu_2m}^{\mu_2} \ldots \delta_{\mu_3}^{\mu_3} \delta_{\mu_2}^{\mu_2} - R_{\mu_2m \mu_2m-3}^{\mu_2m \mu_2m-3} (z_{2m}/z_2) \ldots R_{\mu_3 \mu_2 \mu_2}^{\mu_3 \mu_2 \mu_2} (z_3/z_2) \right) \times
\]

\[
\times \text{tr}_{V(\Lambda_i)} \left( q^{-2\rho} \mathcal{L}^{(i)}(u_1, \ldots, u_n) \Phi_{\mu_2m}^{(1-i)}(z_{2m}) \ldots \Phi_{\mu_3}^{(i)}(z_3) \right)
\]

where \( \mathcal{L}^{(i)}(u_1, \ldots, u_n) \) is the local operator acting on \( V(\Lambda_i) \).

The meaning of \( (\mathcal{1}) \) is two-fold. First, it demonstrates that the axioms \( (\mathcal{1}) \), \( (\mathcal{2}) \) and \( (\mathcal{3}) \) have, in a sense, invariant meaning independent of the massive integrable models considered. Second, using this result it is possible to prove an analog of the locality theorem as it was done in \( (\mathcal{3}) \) for relativistic integrable models.

The paper organized as follows. In sect. 2 we recall the bosonization formulas for the type I and type II VO operators following \( (\mathcal{4}) \). The sect. 3 is devoted to the explicit calculation of the residue of the four-particle form factor. In sect. 4 we discuss the “locality” theorem for the XXZ model.

\(^2\)We define \( \text{res}_{z=1} f(z)/(z-1) = f(1) \).
2 Bosonization of type I and type II vertex operators

Bosonization of the VO is based on the Drinfeld’s new realization of deformed affine algebra \( U_q'(\hat{\mathfrak{sl}}(2)) \) and the construction of the level one irreducible highest weight representations of this algebra as appeared in the work by I. Frenkel and N. Jing. Following [9], the associative algebra \( U_q'(\hat{\mathfrak{sl}}(2)) \) is generated by the symbols

\[
 x_k^\pm, \ a_n, \ \gamma^{\pm 1/2}, \ K \mid k, n \in \mathbb{Z}, n \neq 0
\]

satisfying the relations

\[
 [\gamma, \text{everything}] = 0, \\
 [a_n, a_m] = \delta_{n,-m} \left[ \frac{2n}{n} \gamma^n - \gamma^{-n} / q - q^{-1} \right], \ [a_n, K] = 0, \\
 [a_n, x_m^\pm] = \pm \left[ \frac{2n}{n} \gamma^{\pm |n|/2} x_{n+m}^\pm \right], \ K x_m^\pm K^{-1} = q^{\pm 2} x_m^\pm, \\
 x_{n+1}^\pm x_n^\pm = q^{\pm 2} x_m^\pm x_{m+1}^\pm - x_{m+1}^\pm x_m^\pm, \\
 [x_n^+, x_m^-] = \frac{1}{q - q^{-1}} (\gamma^{(n-m)/2} \psi_{n+m} - \gamma^{(m-n)/2} \phi_{n+m}),
\]

where \( |n| = (q^n - q^{-n})/(q - q^{-1}) \) is a quantum number and \( \psi_n, \ \phi_n, n \in \mathbb{Z}_{\geq 0} \) are defined as follows

\[
 \psi(z) = \sum_{n=0}^{\infty} \psi_n z^{-n} = K \exp \left( (q - q^{-1}) \sum_{n=1}^{\infty} a_n z^{-n} \right), \\
 \phi(z) = \sum_{n=0}^{\infty} \phi_n z^n = K^{-1} \exp \left( -(q - q^{-1}) \sum_{n=1}^{\infty} a_n z^n \right).
\]

The Chevalley generators of \( U_q'(\hat{\mathfrak{sl}}(2)) \) can be expressed via new generators (18) by relations

\[
t_0 = \gamma K^{-1}, \ t_1 = K, \ e_1 = x_0^+, \ f_1 = x_0^-, \ e_0 = x_1^- K^{-1}, \ f_0 = K x_{-1}^+.
\]

Let us recall the results of the paper [10]. The space \( W \) considered there, is the direct product of linear span of all possible finite monomials \( \prod_{j_k \geq \cdots \geq j_1 > 0} a_{-j_k} \cdots a_{-j_1} \) and the group algebra of the weight lattice of the algebra \( \mathfrak{sl}(2) \) \( (e^{n \alpha} \mid n \in \frac{1}{2} \mathbb{Z}) \). The space \( W \) becomes \( U_q'(\hat{\mathfrak{sl}}(2)) \)-module if the action of the operators \( a_n, K, \gamma \) on this space is defined as follows

\[
a_n = \text{the left multiplication by} \ a_n \otimes 1 \ \text{for} \ n < 0 \\
= [a_n, \cdot] \otimes 1 \ \text{for} \ n > 0 \\
e^{n_1 \alpha}(a_{-j_k} \cdots a_{-j_1} \otimes e^{n_2 \alpha}) = a_{-j_k} \cdots a_{-j_1} \otimes e^{(n_1 + n_2) \alpha} \\
K(a_{-j_k} \cdots a_{-j_1} \otimes e^{n_1 \alpha}) = q^{2n_1} a_{-j_k} \cdots a_{-j_1} \otimes e^{n_1 \alpha} \\
\gamma(a_{-j_k} \cdots a_{-j_1} \otimes e^{n_1 \alpha}) = q a_{-j_k} \cdots a_{-j_1} \otimes e^{n_1 \alpha}
\]
with the action of the generators $x_n^\pm$ given by the generating functions

$$X^+(\xi) = \sum_{n \in \mathbb{Z}} x_n^+ \xi^{-n-1} = \exp \left( \sum_{n=1}^{\infty} \frac{q^{-n/2}\xi^n a_{-n}}{[n]} \right) \exp \left( - \sum_{n=1}^{\infty} \frac{q^{-n/2}\xi^{-n} a_n}{[n]} \right) e^{\alpha/2} \xi^{-\alpha/2}$$

$$X^-(\xi) = \sum_{n \in \mathbb{Z}} x_n^- \xi^{-n-1} = \exp \left( - \sum_{n=1}^{\infty} \frac{q^{n/2}\xi^n a_{-n}}{[n]} \right) \exp \left( \sum_{n=1}^{\infty} \frac{q^{n/2}\xi^{-n} a_n}{[n]} \right) e^{-\alpha/2} \xi^{\alpha/2}$$

(21)

(22)

The submodules which are the linear spans of the elements $a_{-j} \cdots a_{-j_1} \otimes e^{n\alpha}$ and $a_{-j_1} \cdots a_{-j} \otimes e^{(n+1/2)\alpha}$, $n \in \mathbb{Z}$ are isomorphic to the level one irreducible highest weight modules $V(\Lambda_0)$ and $V(\Lambda_0)$ with the highest weight vectors $u_{\Lambda_0} = 1 \otimes 1$ and $u_{\Lambda_1} = 1 \otimes e^{\alpha/2}$, respectively.

Now we are in position to write down the bosonized expression for the VO. Following [2, 3], we can use the partially known information about comultiplication of the generators $x_n^\pm$ and $a_n$, $n \in \mathbb{Z}$ [11].

$$\Phi^i_+(z) = \Phi_+(z)(-q^3)^{-i/2}(-qz)^{1/2}, \quad \Phi^i_-(z) = \Phi_-(z)(-q^3)^{-i/2}(-q)^{1/2}$$

(23)

$$\Psi^i_+(z) = \Psi_+(z)(-q^3)^{i/2}z^{1/2}, \quad \Psi^i_-(z) = \Psi_-(z)(-q^3)^{i/2}$$

(24)

$$\Phi_+(z) = \exp \left( - \sum_{n=1}^{\infty} \frac{q^{-n/2}z^n a_{-n}}{[2n]} \right) \exp \left( \sum_{n=1}^{\infty} \frac{q^{-3n/2}z^{-n} a_n}{[2n]} \right) e^{-\alpha/2} (-qz)^{-\alpha/2}$$

(25)

$$\Psi_-(u) = \exp \left( \sum_{n=1}^{\infty} \frac{q^{n/2}u^n a_{-n}}{[2n]} \right) \exp \left( - \sum_{n=1}^{\infty} \frac{q^{5n/2}u^{-n} a_n}{[2n]} \right) e^{\alpha/2} (-q^3z)^{\alpha/2}$$

(26)

Only $+$ and $-$ components of the type II and type I VO respectively can be found in simple form of exponential functions of bosons. The operators $\Psi^i_+(z)$ and $\Phi^i_+(z)$ can be determined from the condition that they intertwine the action of operators $x_0^+$ and $x_0^-$

$$\Psi^i_+(z) = [\Psi^i_-(z), x_0^-], \quad \Phi^i_+(z) = [\Phi^i_-(z), x_0^+]$$

(27)

where $[X, Y] = XY - qYX$ is the $q$-commutator.

In what follows we will use also the formulas

$$q^{-2\rho} \Phi_+(z) = q^2 \Phi_+(zq^4)q^{-2\rho}$$

(28)

$$[\Phi_+(z), X^+(\xi)] = \frac{z(q^2 - 1)}{\xi - z} \Phi_+(z) X^+(\xi) = \frac{qz(q^2 - 1)}{\xi - q^2z} X^+(\xi) \Phi_+(z)$$

(29)

$$\Phi^i_+(z) = \oint_{C_1} d\xi \Phi^i_+(z) X^+(\xi) - q \oint_{C_2} d\xi X^+(\xi) \Phi^i_+(z) = \oint_{C} d\xi [\Phi^i_+(z) X^+(\xi)]_q$$

(30)

where the contour $C$ is such that points $\ldots, q^4z, z$ are inside and points $q^2z, q^{-2}z, \ldots$ are outside of the contour. Formula (30) can be obtained by considering the residue of the two-particle form factor of identity operator which is zero by definition.

The bosonization of the type I VO was considered in this paper only but it is straightforward exercise to write down the bosonized expressions for the type II VO. The key formula that should be used in both calculation is how bosons $a_n$ act on the elements $v_+z^n$ and $v_-z^n$ of a level 0 $U_q(\mathfrak{sl}(2))$-module $V_z$. 

6
3 Calculation the residue of the four-particle form factor

Our goal is to consider the residue at the point \( q^{-1}z_1/z_2 = 1 \) of the four-particle form factor

\[
\text{tr}_{V(A_i)} \left( q^{-2\rho} L^{(i)}(u_1, \ldots, u_n) \Phi^{(1-i)}_{\mu_4}(z_4) \Phi^{(i)}_{\mu_3}(z_3) \Phi^{(1-i)}_{\mu_2}(z_2) \Phi^{(i)}_{\mu_1}(z_1) \right) \\
= \left\langle \Phi^{(1-i)}_{\mu_4}(z_4) \Phi^{(i)}_{\mu_3}(z_3) \Phi^{(1-i)}_{\mu_2}(z_2) \Phi^{(i)}_{\mu_1}(z_1) \right\rangle_i
\]

(31)

where \( L^{(i)}(u_1, \ldots, u_n) \) is the local operator. In order to define this operator we have to consider besides the operators \( \tilde{\Phi}^{A_{1-i}}_{\Lambda_i}(z) \), the operators \( \tilde{\Phi}^{A_{1-i}}_{\Lambda_i, V}(z) \)

\[
\tilde{\Phi}^{A_{1-i}}_{\Lambda_i, V}(z) : V(\Lambda_i) \otimes V_z \rightarrow V(\Lambda_1-i), \\
\tilde{\Phi}^{A_{1-i}}_{\Lambda_i, V}(z)(v \otimes v_\pm) = \tilde{\Phi}^{A_{1-i}}_{\Lambda_i, V}(z) (v).
\]

(32)

and

\[
\tilde{\Phi}^{A_{1-i}}_{\Lambda_i, V}(z) = (-q)^{i+(e-1)/2} \tilde{\Phi}^{A_{1-i}}_{\Lambda_i, V}(z/q^2).
\]

(33)

Using (3) and (32) the operator \( L^{(i)} \) can be written in the form (up to normalization factor)

\[
L^{(i)}(u_1, \ldots, u_n) \sim \Psi^{(i+1)}_{\mu_1}(u_1/q^2) \circ \cdots \circ \Psi^{(i+n)}_{\mu_n}(u_n/q^2) \\
\circ (\text{id}_{V(A_{1+n})} \otimes E_{\mu_1}^{\mu_2} \otimes \cdots \otimes E_{\mu_1}^{\mu_n}) \\
\circ \Psi^{(i+n-1)}_{\mu_1}(u_n) \circ \cdots \circ \Psi^{(i)}_{\mu_1}(u_1).
\]

(34)

and \( \Psi^{(i)}_{\mu_k}(u_k) = z^{-i/2+\mu_k-1/4} \Phi^{A_{1-i}}_{\Lambda_k, \mu_k}(u_k), k = 1, \ldots, 2n \), are type I VO in the principal picture (13). \( 2 \times 2 \) matrices \( E_i, i = 1, \ldots, n \) belong to End \( V \).

We will see below that it is sufficient to calculate residue of the form factor (31) at the point \( q^{-2}z_2/z_3 = 1 \), when \( \mu_4 = +, \mu_3 = +, \mu_2 = -, \mu_1 = - \). All other possibilities can be obtained from this particular form factor using the symmetry properties (14) and

\[
\Psi^{(i-1)}(u) \Phi^{(i)}(z) = \Phi^{(i-1)}(z) \Psi^{(i)}(u) \sqrt{u} \Theta_{q^4}(qz/u) \Theta_{q^4}(qu/z),
\]

\[
\Theta_{q^4}(z) = (z)_\infty (q^4 z^{-1})_\infty (q^4)_\infty.
\]

(35)

It follows immediately from (33) that local operator \( L^{(i)}(u_1, \ldots, u_n) \) commute with any type II VO

\[
[L^{(i)}(u_1, \ldots, u_n), \Phi^{(i)}(z)] = 0.
\]

(36)

After substitution (31) to the form factor

\[
\left\langle \Phi^{(1-i)}_{++}(z_4) \Phi^{(i)}_{++}(z_3) \Phi^{(1-i)}_{--}(z_2) \Phi^{(i)}_{--}(z_1) \right\rangle_i
\]

(37)

one can see that in the limit \( \sqrt{z_2} \rightarrow -q \sqrt{z_3} \) the contour \( C \) has pinchings, either between points \( z_2 \) and \( q^2 z_3 \), where integrand expression has poles defined by (29) or
between points $q^2z_2$ and $q^4z_3$. The last pole appears as a result of taking trace in (31).

To calculate (31) we write this form factor

$$
\langle \Phi^1_{+}(z_4)\Phi^j_{+}(z_3)\Phi^{1-i}_{+}(z_2)\Phi^{i}_{+}(z_1) \rangle_i = \\
= \langle \Phi_{+}(z_4)\Phi_{+}(z_3)\Phi_{-}(z_2)\Phi_{-}(z_1) \rangle_i (-q)^{-1}(z_4z_3)^{1/2}
$$

as sum of four integrals

$$(q)^{-1}(z_4z_3)^{1/2}$$

\[ \int_{C_3} d\xi_2 \int_{C_1} d\xi_1 \langle \Phi_{+}(z_4)\Phi_{+}(z_3)\Phi_{+}(z_2)X^+(\xi_2)\Phi_{+}(z_1)X^+(\xi_1) \rangle_i \\
-q \int_{C_2} d\xi_2 \int_{C_1} d\xi_1 \langle \Phi_{+}(z_4)\Phi_{+}(z_3)X^+(\xi_2)\Phi_{+}(z_2)\Phi_{+}(z_1)X^+(\xi_1) \rangle_i \\
-q \int_{C_1} d\xi_2 \int_{C_2} d\xi_1 \langle \Phi_{+}(z_4)\Phi_{+}(z_3)\Phi_{+}(z_2)X^+(\xi_2)X^+(\xi_1)\Phi_{+}(z_1) \rangle_i \\
+q^2 \int_{C_2} d\xi_2 \int_{C_1} d\xi_1 \langle \Phi_{+}(z_4)\Phi_{+}(z_3)X^+(\xi_2)\Phi_{+}(z_2)X^+(\xi_1)\Phi_{+}(z_1) \rangle_i \]

where the contours in these integrals are specified by the position of the vertex operators $\Phi_{+}(z)$ with respect to positions of the currents $X^+(\xi)$. Each integral has double pinching in the limit $\sqrt{z_2} \to -q\sqrt{z_3}$. It means that the residue (31) will be equal to the sum of eight contour integrals with integrand expressions which are traces of the following products of the vertex operators and currents.

$$
-q^{-1}\sqrt{z_4z_3} \left\langle \Phi_{+}(z_4)\Phi_{+}(z_2)X^+(\xi_2)\Phi_{+}(q^4z_3)\Phi_{+}(z_1)X^+(\xi_1) \right\rangle_i \\
xq^2r \left( \frac{z_4}{z_3} \right) r \left( \frac{z_1}{q^4z_3} \right) \frac{(\xi_1 - q^6z_3)}{q(\xi_1 - q^4z_3)}
$$

$$
-q^{-1}\sqrt{z_4z_3} \left\langle \Phi_{+}(z_4)X^+(\xi_2)\Phi_{+}(z_1)\Phi_{+}(z_2)X^+(\xi_1)\Phi_{+}(q^4z_3) \right\rangle_i \\
xq^2r \left( \frac{z_4}{z_3} \right) r \left( \frac{z_2}{z_1} \right) \frac{q(\xi_2 - z_2)}{(\xi_2 - q^2z_2)}
$$

$$
\sqrt{z_4z_3} \left\langle \Phi_{+}(z_4)\Phi_{+}(z_3)X^+(\xi_2)\Phi_{+}(z_2)\Phi_{+}(z_1)X^+(\xi_1) \right\rangle_i
$$

$$
\sqrt{z_4z_3} \left\langle \Phi_{+}(z_4)X^+(\xi_2)\Phi_{+}(z_1)\Phi_{+}(z_2)X^+(\xi_1)\Phi_{+}(q^4z_3) \right\rangle_i \\
xq^2r \left( \frac{z_4}{z_3} \right) r \left( \frac{z_2}{z_1} \right)
$$

$$
\sqrt{z_4z_3} \left\langle \Phi_{+}(z_4)\Phi_{+}(z_2)X^+(\xi_2)\Phi_{+}(q^4z_3)X^+(\xi_1)\Phi_{+}(z_1) \right\rangle_i \\
xq^2r \left( \frac{z_4}{z_3} \right) r \left( \frac{z_1}{q^4z_3} \right) \frac{(\xi_1 - q^6z_3)}{q(\xi_1 - q^4z_3)}
$$
\[ \sqrt{z_3 z_4} \left\langle \Phi_+(z_4) X^+(\xi_2) \Phi_+(z_2) X^+(\xi_1) \Phi_+(q^4 z_3) \Phi_+(z_1) \right\rangle_i \]
\[ \times q^2 r \left( \frac{z_4}{z_3} \right) r \left( \frac{z_1}{q^4 z_3} \right) q(\xi_2 - z_2) \]
\[ - q \sqrt{z_3 z_4} \left\langle \Phi_+(z_4) \Phi_+(z_3) X^+(\xi_2) \Phi_+(z_2) X^+(\xi_1) \Phi_+(z_1) \right\rangle_i \]  
\[ - q \sqrt{z_3 z_4} \left\langle \Phi_+(z_4) X^+(\xi_2) \Phi_+(z_2) X^+(\xi_1) \Phi_+(q^4 z_3) \Phi_+(z_1) \right\rangle_i \]
\[ \times q^2 r \left( \frac{z_4}{z_3} \right) r \left( \frac{z_1}{q^4 z_3} \right) \]  
(45)

(46)

(47)

We used (29), (14) and (36) in order to arrange the vertex operators in necessary order. The underlined combinations of the vertex operators in (40)-(47) mean that we will consider the pinching of the integral over \( \xi_1 \) or \( \xi_2 \) in the corresponding summand to calculate the residue of the form factor (35).

Note that if instead the local operator \( \mathcal{L} \) we will consider the form factor of the product of type I vertex operators, then in the terms where we interchanged the positions of the operators \( \mathcal{L} \) and \( \Phi_+(z_3) \) the product of momentum of quasi-particles appears
\[ \prod_{k=1}^{2n} \frac{u_k \Theta_{q^k} (q z_3 / u_k)}{z_3 \Theta_{q^k} (q u_k / z_3)} \]  
(48)
due to (33).

Let us calculate the residue of the trace in (42)
\[ \text{Res}_{q^{-2}z_2/z_3=1} \sqrt{z_3 z_4} \times \]
\[ \times \oint_{C_2} d\xi_2 \oint_{C_1} d\xi_1 \left\langle \Phi_+(z_4) \Phi_+(z_3) X^+(\xi_2) \Phi_+(z_2) \Phi_+(z_1) X^+(\xi_1) \right\rangle_i \]
\[ = q^{-1} \left( \frac{q^4}{q^4} \right)^{1/2} \oint_{C_1} d\xi_1 \left\langle \Phi_+(z_4) \Phi_+(z_1) X^+(\xi_1) \right\rangle_i. \]  
(49)

To obtain (41) the operator identity\footnote{Note that besides the identity (60) we can write down the identities which involve the operators \( X^-(z), \phi(z) \) and \( \psi(z) \). Namely: \( \Phi_-(z) X^-(q^4 z) \Psi_-(q^2 z): = 1 \), \( \Phi_-(z) X^-(q^2 z) \Psi_-(q^2 z): = \phi^{-1} (q^{7/2} z) \psi(q^{3/2} z) \). It is interesting question whether it is possible to solve these identities for the operators \( X^\pm(z), \phi(z), \psi(z) \).}
\[ :\Phi_+(z_3) X^+(q^2 z_3) \Phi_+(q^2 z_3): = 1 \]  
(50)

was used. Similarly, (46) yields
\[ - \left( \frac{q^2}{q^4} \right)^{1/2} \oint_{C_2} d\xi_1 \left\langle \Phi_+(z_4) X^+(\xi_1) \Phi_+(z_1) \right\rangle_i, \]  
(51)
So the total contribution of (42) and (46) to the residue is
\[ - \frac{(q^2)^\infty}{(q^4)^\infty} \text{tr}_V(\Lambda_i) \left( q^{-2p} L_{\epsilon_1, \ldots, \epsilon_n}^{(i)} \Phi_{(1-i)}^+(z_4) \Phi_{(i)}^-(z_1) \right). \]  
(52)

This partial result is related to the local composition formula for the type II vertex operators
\[ \Phi_{(1-i)}^+(z_3) \circ \Phi_{(i)}^-(z_2) = \frac{1}{1 - q^{-2} \frac{z_2}{z_3}} g \delta_{\epsilon_2, -\epsilon_3} \text{id} + O(1), \]  
(53)
which can be easily obtained by considering any matrix element of the composition \( \Phi_{(1-i)}^+(z_3) \circ \Phi_{(i)}^-(z_2) \) and using commutation relation (29) and the formula
\[ \Phi_+(z_3)\Phi_+(z_2) = \sqrt{-qz_3} \frac{(z_2/z_3)^\infty}{(q^2z_2/z_3)^\infty} :\Phi_+(z_3)\Phi_+(z_2):. \]

But the formula (53) cannot be used directly under the trace \( \text{tr}_V(\Lambda_i)(q^{-2p} \cdot \cdot \cdot) \) in calculation of the residue, because the additional poles appear after taking the trace.

To calculate the summarized contribution to the residue at the point \( \sqrt{z_2} = -q\sqrt{z_3} \) of the rest six combinations of the vertex operators we proceed as follows. First, we interchange the positions of the vertex operators \( \Phi_+(z_1) \) and \( X^+(\xi_1) \) in (40) using (28). Second, we calculate the residue
\[ \text{res}_{q^{-1}z_2/z_3 = 1} q^{-1} \sqrt{z_3z_4} \times \]
\[ \times \oint_{C_1} d\xi_2 \oint_{C} d\xi_1 \left( \Phi_+(z_4) \Phi_+(z_2) X^+(\xi_2)\Phi_+(q^4z_3)X^+(\xi_1)\Phi_+(z_1) \right)_i \]
\[ = -q^{-3}(-qz_4)^{1/2} g \oint_{C_1} d\xi_1 \left( \Phi_+(z_4)X^+(\xi_1)\Phi_+(z_1) \right)_i \]
using the operator identity (50). Furthermore from the identity
\[ r \left( \frac{z_2}{z_1} \right)_{\sqrt{z_2} = -q\sqrt{z_3}} = - \frac{(q^4z_3 - z_1)}{q(z_1 - q^2z_3)} r \left( \frac{z_1}{q^4z_3} \right) \]  
(54)
follows that the sum of the rational functions which appeared in these six combinations (we renamed somewhere the integration variables)
\[ \frac{(\xi_2 - q^4z_3)}{(\xi_2 - q^4z_3)(\xi_2 - q^2z_1)} - \frac{(\xi_2 - q^2z_3)}{(\xi_2 - q^2z_3)(z_1 - q^2z_3)} + \]
\[ + \frac{(q^4z_3 - z_1)}{(z_1 - q^2z_3)} - \frac{(\xi_2 - q^6z_3)}{(\xi_2 - q^6z_3)(\xi_2 - q^4z_3)} - \frac{q^2(\xi_2 - q^2z_3)}{(\xi_2 - q^4z_3)} + q^2 \]  
(55)
is equal to
\[ \frac{(z_1 - q^4z_3)}{q(z_1 - q^2z_3)} \frac{qz_1(q^2 - 1)}{(\xi_2 - q^2z_1)}. \]
Using the crossing and symmetry relations for the $R$-matrix formula (56) can be rewritten as follows (we neglect the inessential factor $-g$ in (58))

$$\res_{q^2 z_2/z_3 = 1} \left\langle \Phi_+^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \Phi_-^{(1-i)}(z_2) \Phi_-^{(i)}(z_1) \right\rangle \bigg|_i = (1 - R_{++}(z_2 q^4/z_4) R_{--}(z_2/z_1)) \left\langle \Phi_+^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \right\rangle \bigg|_i. \quad (58)$$

It is convenient to write down this residue in a different form. First, we move the operator $\Phi_-^{(i)}(z_1)$ on the left hand sides in both form factors in (58) using (14) and the trace property and second, rename the points $z$'s as follows $z_4 \to z_3$, $z_3 \to z_2$, $z_2 \to z_1$, $z_1 \to q^4 z_4$. Then instead of (58) we obtain

$$\res_{q^2 z_1/z_2 = 1} \left\langle \Phi_-^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \Phi_+^{(1-i)}(z_2) \Phi_+^{(i)}(z_1) \right\rangle \bigg|_i = (1 - R_{++}(z_4/z_2) R_{++}(z_3/z_2)) \left\langle \Phi_-^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \right\rangle \bigg|_i. \quad (59)$$

To calculate the residue

$$\res_{q^2 z_1/z_2 = 1} \left\langle \Phi_+^{(1-i)}(z_4) \Phi_-^{(i)}(z_3) \Phi_+^{(1-i)}(z_2) \Phi_-^{(i)}(z_1) \right\rangle \bigg|_i = (1 - R_{++}(z_4/z_2) R_{--}(z_3/z_2)) \left\langle \Phi_+^{(1-i)}(z_4) \Phi_-^{(i)}(z_3) \right\rangle \bigg|_i - R_{++}(z_4/z_2) R_{++}(z_3/z_2) \left\langle \Phi_-^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \right\rangle \bigg|_i \quad (60)$$

we start from $\left\langle \Phi_-^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \Phi_+^{(1-i)}(z_2) \Phi_-^{(i)}(z_1) \right\rangle \bigg|_i$ and interchange the positions of the operators $\Phi_-^{(1-i)}(z_4) \Phi_+^{(i)}(z_3)$ using commutation relation (14). We obtain two form factors such that the residue of one we can calculate using (59) and the other, residue of which we are interesting in. We arrive to the above relation (60) after taking the residue in both sides of the equation obtained and using again the commutation relation (14).

Now we are in position to calculate the residue of the following form factor

$$\res_{q^2 z_1/z_2 = 1} \left\langle \Phi_+^{(1-i)}(z_4) \Phi_+^{(i)}(z_3) \Phi_-^{(1-i)}(z_2) \Phi_-^{(i)}(z_1) \right\rangle \bigg|_i. \quad (61)$$

To do that we start from the residue (58) and move the operators $\Phi_+^{(i)}(z_3) \Phi_-^{(1-i)}(z_2)$ to the right to obtain two form factors of the type

$$\left\langle \Phi_+^{(1-i)}(z_4) \Phi_+^{(i)}(z_1) \Phi_-^{(1-i)}(z_3) \Phi_-^{(i)}(z_2) \right\rangle \bigg|_i,$n

$$\left\langle \Phi_+^{(1-i)}(z_4) \Phi_-^{(i)}(z_1) \Phi_+^{(1-i)}(z_3) \Phi_-^{(i)}(z_2) \right\rangle \bigg|_i.$$
The first form factor is exactly what we are looking for, while the residue of the second one was already calculated in (60) up to renaming of the variables $z$’s. Again by using the crossing and symmetry relations for the elements of $R$-matrix we obtain

$$
\text{res}_{q^{-2}z_1/z_2=1} \left\langle \Phi^{(1-i)}_{4}(z_4)\Phi^{(i)}_{3}(z_3)\Phi^{(1-i)}_{2}(z_2)\Phi^{(i)}_{1}(z_1) \right\rangle_i = -R^{++}_{++}(z_4/z_2)R^{--}_{+-}(z_3/z_2) \left\langle \Phi^{(1-i)}_{4}(z_4)\Phi^{(i)}_{3}(z_3) \right\rangle_i
- R^{+-}_{++}(z_4/z_2)R^{--}_{+-}(z_3/z_2) \left\langle \Phi^{(1-i)}_{4}(z_4)\Phi^{(i)}_{3}(z_3) \right\rangle_i.
$$

(62)

Similar arguments reproduce residues for all other possible combinations of $+$ and $-$ such that the formulas (59), (60) and (62) and analogous to them can be written in the compact form

$$
\text{res}_{q^{-2}z_1/z_2=1} \left\langle \Phi^{(1-i)}_{\mu_4}\Phi^{(i)}_{\mu_3}(z_3)\Phi^{(1-i)}_{\mu_2}(z_2)\Phi^{(i)}_{\mu_1}(z_1) \right\rangle_i = -g\delta^{\mu_4 \mu_1} \times
\times \left( S^{\mu_4}_{\mu_3}R_{\mu_3 \mu_2}(z_4/z_2)R_{\mu_2 \mu_1}(z_3/z_2) \right) \left\langle \Phi^{(1-i)}_{\mu_4}(z_4)\Phi^{(i)}_{\mu_3}(z_3) \right\rangle_i.
$$

(63)

Looking at (63) we can formulate the following rule. The residues of multi-particle form factor basically consist of two terms. One term appears after we move the left operator from the pair that will be contracted to the right until it meets the other operator and then use the local formula (53). To obtain the other term in residue we have to move the same operator to the left and then around all operators under the trace using the trace property. We continue moving to the left until it meets the second contracted operator which will be now on the left hand side from the moving one. Then we again use the local formula (53). We would like to remark here that this rule is only the recipe to write down the answer for the residue but not a method to calculate.

In order to prove this general rule we have to consider trace $\text{tr}_{V(A_1)}(x^{-\rho/2} \cdots)$ for generic value of $x$ and note that when $x \to q^4$ the trace (63) has two closely located poles with respect to variable $z_1/z_2$. One at the point $z_1/z_2 = q^2$ and another at the point $z_1/z_2 = q^{-2}x$. So the residue (63) will be equal to the sum of residues at these two different point. To calculate the residue at the point $z_1/z_2 = q^2$ we can now use the local formula (53) because there are no additional pinchings of the integrals when $x \neq q^4$. To obtain the residue at the point $z_1/z_2 = q^{-2}x$ using the same formula (53) we have to move the left operator around the all operators to produce the necessary shift in the parameter $z_2$ and then again use (53). In this way we obtain the $R$-matrix factors in second summand in (63). After that we can put $x = q^4$ and arrive to the general formula (17).

4 “Locality” theorem for the XXZ model

As we have already mentioned the type II vertex operators are responsible for the particle picture of the XXZ model. Let us briefly recall the particle picture of the XXZ model following [1].
It was argued there that the Fock space structure for the Hamiltonian $H_{XXZ}$ in the anti-ferromagnetic regime is

$$\mathcal{F} = \left[ \bigoplus_{n=0}^{\infty} \int \cdots \int V_{z_n} \otimes \cdots \otimes V_{z_1} \frac{dz_n}{2\pi i z_n} \cdots \frac{dz_1}{2\pi i z_1} \right]_{\text{symm}} \cdot$$

(64)

In order to embed the space

$$V_{z_n} \otimes \cdots \otimes V_{z_1} \to \mathcal{F}_{\mu,\lambda}$$

(65)

one has to consider the vertex operators of the following type

$$\tilde{\Phi}_{V\Lambda_i}^v(z) : V \otimes V(\Lambda_i) \to V(\Lambda_{1-i}), \quad \Phi^v_\pm(z) = \tilde{\Phi}_{V\Lambda_i}^v(z)(v_\pm \otimes v).$$

(66)

The $\pm$-components of the operator $\Phi^v(z)$ are

$$\Phi_+^v(z) = -q^{-1}\Phi_{V\Lambda_i}^v(q^2z), \quad \Phi_-^v(z) = \Phi_{V\Lambda_i}^v(q^2z)$$

(67)

when they act on the module $V(\Lambda_i)$. The $n$-particle state can be defined as

$$\Phi^v_{\epsilon_1}(z_1) \cdots \Phi^v_{\epsilon_n}(z_n) \otimes \text{id})|\text{vac}\rangle, \quad (68)$$

where $|\text{vac}\rangle_\lambda$ is identity element from $\mathcal{F}_{\lambda,\lambda}$. The generalized form factor of the local operator $\mathcal{L}$ between $m$ in-particles and $k$ out-particles up to normalization constant is given by the trace ([4], sect. 7.2)

$$\text{tr}_{V(\lambda)} \left( q^{-2\rho} \Phi_{\epsilon_1}(z_1) \cdots \Phi_{\epsilon_k}(z_k) \mathcal{L} \Phi^v_{\epsilon_m}(z'_m) \cdots \Phi^v_{\epsilon_1}(z'_1) \right) = F(z_1 \ldots z_k | z'_m \ldots z'_1)_{\epsilon_1 \ldots \epsilon_k ; \epsilon'_m \ldots \epsilon'_1}. \quad (69)$$

Using the rule formulated at the end of the previous section we can write down the residue at the point $z_j / z'_i = 1$ of the form factor (69) as follows (we again omit the inessential factor $-(q^2)_{\infty} / (q^4)_{\infty}$ in this formula)

$$\text{res}_{z_j / z'_i = 1} F(z_1 \ldots z_j \ldots z_k | z'_m \ldots z'_i \ldots z'_1) \quad \begin{aligned}
\quad & = R_{jj+1} \left( \frac{z_j}{z_{j+1}} \right) \cdots R_{jk} \left( \frac{z_j}{z_k} \right) F(z_1 \ldots \hat{z}_j \ldots z_k | z'_m \ldots z'_i \ldots z'_1) \\
& \times I_{ji} R_{i+1i} \left( \frac{z'_i}{z'_{i+1}} \right) \cdots R_{mi} \left( \frac{z'_m}{z'_i} \right) \\
& - R_{j-1j} \left( \frac{z_{j-1}}{z_j} \right) \cdots R_{1j} \left( \frac{z_1}{z_j} \right) F(z_1 \ldots \hat{z}_j \ldots z_k | z'_m \ldots z'_i \ldots z'_1) \\
& \times I_{ji} R_{i+1i} \left( \frac{z'_i}{z'_{i+1}} \right) \cdots R_{i1} \left( \frac{z'_i}{z'_1} \right),
\end{aligned} \quad (70)$$

where we use the tensor notation and $F(z_1 \ldots \hat{z}_j \ldots z_k | z'_m \ldots z'_i \ldots z'_1)$ means that the $i$th in-particle and $j$th out-particle are omitted. The operator $I_{ij}$ is the identification
operator of the $i$th and $j$th internal spaces. Note that the formula (70) is exactly the same as in (3) (formula (26)).

Because of this coincidence we can immediately expand the Smirnov’s approach for the proving the locality theorem to the case of XXZ model. As result we arrive to the following statement.

*The commutativity of two local operators in XXZ model which are separated on the lattice is equivalent to the statement that the form factors of these local operators satisfy the axioms (4)–(5) and 4.*

In relativistic integrable model the proving the locality is based apart from the axioms (1)–(3) on the fact that the function $\exp(ip(\beta)x)$ is a quickly decreasing when $\text{Re}\beta \to \pm\infty$ on the “physical sheet” $0 < \text{Im}\beta < \pi$ for $x > 0$ and $p(\beta) = m\text{sh}\beta$ is the momentum of the particle. The statement that should be utilized in the proving the analogous statement in XXZ model is that eigenvalue of the translation operator $T$ on the state created by the operator $\Phi^*(z)$ (see sect. 7.2 in [1])

$$T\Phi^*(z)T^{-1} = \frac{1}{\sqrt{z}}\frac{\Theta_{q^4}(qz)}{\Theta_{q^4}(qz^{-1})}\Phi^*(z)$$

has no poles in the “physical sheet” $1 < |z| < q^{-2}$.

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