LHC phenomenology of supersymmetric models beyond the MSSM

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Abstract. We discuss various phenomenological aspects of supersymmetric models beyond the MSSM. A particular focus is on models which can correctly explain neutrino data and the possibilities of LHC to identify the underlying scenario.

1. Introduction

Supersymmetric extensions of the standard model (SM) are promising candidates for new physics at the TeV scale [1, 2] as the solve several short-comings of the Standard Model (SM). The Minimal Supersymmetric Standard Model (MSSM) solves the hierarchy problem of the SM [3], leads to a unification of the gauge couplings [4, 5] and introduces several candidates for dark matter depending on how SUSY is broken [6, 7]. Its phenomenology with respect to present and future colliders has been widely explored, see e.g. [8, 9].

However, similarly as the SM it needs additional ingredients to explain neutrino data, e.g. by either incooperating heavy new particles giving rise to tiny neutrino masses via the seesaw mechanism [10] or via the braking of R-parity [11]. Moreover, a new problem arises in the MSSM not present in the SM: the superpotential contains a parameter with dimension mass, namely the so called $\mu$ parameter which gives masses to the Higgs bosons and higgsinos. From a purely theoretical point of view, the value of this parameter is expected to be either of the order of the GUT/Planck scale or exactly zero, if it is protected by a symmetry. For phenomenological aspects, however, it has to be of the order of the scale of electroweak symmetry breaking (EWSB) and it has to be non-zero to be consistent with experimental data. This discrepancy is the so called $\mu$-problem of the MSSM [12].

In this paper we discuss various supersymmetric models addressing at least one of these two topics focusing on features of their phenomenology which can be tested at the LHC and which differ from the usual MSSM phenomenology. We will take as main guideline the requirement that neutrino data are correctly explained. We will first discuss briefly the case of Dirac neutrinos and afterwards discuss models leading to Majorana neutrinos, both in the context of conserved R-parity. In the third part we cover models with broken R-parity and their phenomenology at the LHC.

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2. Dirac neutrinos

Technically the easiest way to obtain masses for neutrinos is by introducing a new Yukawa coupling similar to the ones for the SM fermions. In this case the MSSM has to be extended by additional right handed neutrinos, which are gauge singlets with respect to the SM gauge group, and the superpotential of the MSSM

\[ W_{\text{MSSM}} = \hat{H}_d \hat{L}_c \hat{E}^c + \hat{H}_d \hat{Q}_c \hat{D}^c + \hat{H}_u \hat{Q}_c \hat{U}^c - \mu \hat{H}_u \hat{H}_d \] (1)

has to be extended by the term

\[ W_{\nu^c} = \hat{H}_u \hat{L}_c \hat{\nu}^c. \] (2)

Here \( \hat{H}_d, \hat{H}_u, \hat{Q}, \hat{D}, \hat{U}, \hat{L} \) and \( \hat{E}^c \) are the MSSM superfields containing the Higgs bosons, quarks, leptons and their supersymmetric partners. In addition the superfield \( \hat{\nu}^c \) for the right-handed neutrino and the right-sneutrino has been introduced. The corresponding Yukawa coupling \( Y_{\nu^c} \) has to be tiny, of the order \( 10^{-12} \) and smaller, to explain correctly neutrino masses in the sub-eV range as required by data [13]. This in turn implies that one is essentially left with the usual MSSM phenomenology, except for the case where the right-sneutrino is the lightest supersymmetric particle (LSP): in this case all decays are as in the MSSM down to the next to lightest supersymmetric particle (NLSP) which eventually decays into the LSP. The corresponding width of the last step is proportional to \( |Y_{\nu^c}|^2 \) and thus rather small implying at the LHC decay lengths ranging from \( O(cm) \) up to \( O(km) \) [14]. Consequently the signal of such a scenario is very two long-lived particles in each event which can even appear as stable particles in a typical detector of high energy collider experiments. Detailed studies have been performed for the cases of a stau [15] NLSP and a stop [16] NLSP demonstrating that LHC should be able to identify such scenarios.

3. Majorana neutrinos via the seesaw mechanism

A possibility to obtain tiny neutrino masses while having at the same time sizable Yukawa couplings are seesaw scenarios where very heavy new particles are postulated inducing the so-called Weinberg operator [17, 18]

\[ \frac{f_{\alpha\beta}}{\Lambda} (H_u L_\alpha)(H_u L_\beta) \Rightarrow (m^\nu)_{\alpha\beta} = \frac{f_{\alpha\beta} v^2}{2\Lambda} \] (3)

when integrating out the heavy degrees of freedom. Here \( \Lambda \) is a measure of the scale for these new particles and \( f_{\alpha\beta} \) is usually a combination of different Yukawa couplings. One can show, that at tree-level only three possibilities exist to realize such scenarios [19]. Type-I is the well-known case of the exchange of a heavy fermionic singlet usually denoted \( \nu_R \) [10, 20, 21, 22]. Type-II corresponds to the exchange of a scalar \( SU(2) \) triplet [23, 24]. In seesaw type-III one adds (at least two) fermionic \( SU(2) \) triplets to the field content of the SM [25]. In the last two models particles charged under the SM gauge group are added and which correspond to incomplete \( SU(5) \) representations. As they would destroy the nice feature of gauge coupling unification one often adds at the seesaw scale(s) additional particles to obtain complete \( SU(5) \) representations to maintain the feature of gauge coupling unification. A detailed discussion including the embedding in \( SU(5) \) models can be found in e.g. in ref. [26].

Below we present only the various superpotential for briefness. In addition there will also be the corresponding soft SUSY terms which, however, reduce at the electroweak scale to the MSSM ones once the states with masses at the seesaw scale(s) are integrated out and, thus, are not discussed further. These additional terms only contribute to threshold corrections at the seesaw scale(s) and their effect is negligible if one requires universal boundary conditions for all soft terms [27]. Here we will assume common soft SUSY breaking at the GUT-scale...
To specify the spectrum at the electroweak scale: a common gaugino mass $M_{1/2}$, a common scalar mass $m_0$ and the trilinear coupling $A_0$ which gets multiplied by the corresponding Yukawa couplings to obtain the trilinear couplings in the soft SUSY breaking Lagrangian. In addition the sign of the $\mu$ parameter is fixed as well as $\tan \beta = v_u/v_d$ at the electroweak scale where $v_d$ and $v_u$ are the the vacuum expectation values (vevs) of the neutral component of $H_d$ and $H_u$, respectively.

3.1. Supersymmetric seesaw type-I

In this class of models one postulates very heavy right-handed neutrinos yielding the following superpotential below the GUT-scale:

$$W_I = W_{MSSM} + W_{\nu^c} + \frac{1}{2} \d^c M_R \d^c$$

For the neutrino mass matrix one obtains the well-known formula

$$m_\nu = -\frac{v_u^2}{2} Y_T M^{-1} \d \nu.$$  

(5)

Being complex symmetric, the light Majorana neutrino mass matrix in eq. (5) is diagonalized by a unitary $3 \times 3$ matrix $U$

$$m_\nu = U^T \cdot m_\nu \cdot U.$$  

(6)

Inverting the seesaw equation eq. (5) allows to express $Y_\nu$ as

$$Y_\nu = \sqrt{2} \frac{i}{v_u} \sqrt{M_R} \cdot R \cdot \sqrt{m_\nu} \cdot U^\dagger,$$

(7)

where the $\hat{m}_\nu$ and $\hat{M}_R$ are diagonal matrices containing the corresponding eigenvalues. $R$ is in general a complex orthogonal matrix. We assume $R = 1$ implying that $Y_\nu$ contains only “diagonal” products $\sqrt{M_i m_i}$.

3.2. Supersymmetric seesaw type-II

In seesaw models of type II one adds a scalar $SU(2)$ triplet $T$ to generate neutrino masses. As this triplet carries also hypercharge one has to embed it in a 15-plet of $SU(5)$ which has under $SU(3) \times SU_L(2) \times U(1)_Y$ the following decomposition [29]

$$15 = S + T + Z$$

$$S \sim (6,1,-\frac{2}{3}), \quad T \sim (1,3,1), \quad Z \sim (3,2,\frac{1}{6}).$$  

(8)

In supersymmetric models one adds a pair 15 and $\overline{15}$ to avoid anomalies below the GUT-scale. Below the GUT scale in the $SU(5)$-broken phase the superpotential reads

$$W_{II} = W_{MSSM} + \frac{1}{\sqrt{2}} (Y_T \tilde{T}_1 \tilde{L} + Y_S \tilde{D}^c \tilde{S}_1 \tilde{D}^c) + Y_Z \tilde{D}^c \tilde{Z}_1 \tilde{L}$$

$$+ \frac{1}{\sqrt{2}} (\lambda_1 \tilde{H}_d \tilde{T}_1 \tilde{H}_d + \lambda_2 \tilde{H}_u \tilde{T}_2 \tilde{H}_u) + M_T \tilde{T}_1 \tilde{T}_2 + M_Z \tilde{Z}_1 \tilde{Z}_2 + M_S \tilde{S}_1 \tilde{S}_2$$  

(9)

where the states with index 1 (2) belong the 15-plet ($\overline{15}$-plet). The second term in eq. (9) is responsible for the generation of the neutrino masses yielding

$$m_\nu = -\frac{v_u^2}{2} \frac{\lambda_2}{M_T} Y_T.$$  

(10)
Note that

$$\hat{Y}_T = U^T \cdot Y_T \cdot U$$

(11)
i.e. \(Y_T\) is diagonalized by the same matrix as \(m_\nu\).

In addition there are the couplings \(Y_S\) and \(Y_Z\), which in principle are not determined by any low-energy data. In the calculation of lepton flavour violating observables both Yukawa couplings, \(Y_T\) and \(Y_Z\), contribute. Having a GUT model in mind we require for the numerical discussion later the \(SU(5)\) boundary conditions \(Y_T = Y_S = Y_Z\) at the GUT scale. As long as \(M_Z \sim M_S \sim M_T \sim M_{15}\) gauge coupling unification will be maintained. The equality need not be exact for a successful unification. In our numerical studies we have taken into account the different running of these mass parameters but we decouple them all at the scale \(M_T(M_T)\) because the differences are small.

3.3. Supersymmetric seesaw type-III

In the case of a seesaw model type III one needs new fermions \(\Sigma\) at the high scale being in the adjoint representation of \(SU(2)\). They have to be embedded in a 24-plet to obtain a complete \(SU(5)\) representation which decomposes under \(SU(3) \times SU_L(2) \times SU_Y(1)\) as

$$24_m = (1, 1, 0) + (8, 1, 0) + (1, 3, 0) + (3, 2, -5/6) + (3^*, 2, 5/6)$$

(12)

\(\bar{B}_M + \bar{G}_M + \bar{W}_M + \bar{X}_M + \bar{\Sigma}_M\)

The fermionic components of \((1, 1, 0)\) and \((1, 3, 0)\) have exactly the same quantum numbers as \(\nu^c\) and \(\Sigma\). Thus, the \(24_m\) always produces a combination of the type-I and type-III seesaw. In the \(SU(5)\) broken phase the superpotential reads

$$W_{III} = W_{MSSM} + \hat{H}_u(\sqrt{2}\hat{W}_M Y_N - \sqrt{3/10} \hat{B}_M Y_B)\hat{\tilde{L}} + \hat{H}_u\hat{\tilde{X}}_M Y_X \hat{D}_c$$

$$+ \frac{1}{2} \hat{B}_M M_B \hat{\tilde{B}}_M + \frac{1}{2} \hat{G}_M M_G \hat{\tilde{G}}_M + \frac{1}{2} \hat{W}_M M_W \hat{\tilde{W}}_M + \hat{\tilde{X}}_M M_X \hat{\tilde{X}}_M$$

(13)

As above we use at the GUT scale the boundary condition \(Y_N = Y_B = Y_X\) and \(M_B = M_G = M_W = M_X\). Integrating out the heavy fields yields the following formula for the neutrino masses at the low scale:

$$m_\nu = -v_\nu^2 \left(\frac{3}{10} Y_B^T M_B^{-1} Y_B + \frac{1}{2} Y_W^T M_W^{-1} Y_W\right).$$

(14)

The boundary conditions at \(M_G\) imply that at the seesaw scale(s) one still has \(M_B \simeq M_W\) and \(Y_B \simeq Y_W\) so that one can write in a good approximation

$$m_\nu = -v_\nu^2 \frac{4}{5} Y_W^T M_W^{-1} Y_W$$

(15)

and one can use the corresponding decomposition for \(Y_W\) as discussed in section 3.1.

3.4. Effect of the heavy particles on the MSSM spectrum

The appearance of charged particles at scales between the electroweak scale and the GUT scale leads to changes in the beta functions of the gauge couplings [29, 30]. In the MSSM the corresponding values at 1-loop level are \((b_1, b_2, b_3) = (33/5, 1, -3)\). In case of one 15-plet the additional contribution is \(\Delta b_i = 7/2\) whereas in case of 24-plet it is \(\Delta b_i = 5\). This results in case of type II in a total shift of \(\Delta b_i = 7\) for the minimal model and in case of type III in \(\Delta b_i = 15\) assuming 3 generations of 24-plets. This does not only change the evolution of the gauge couplings but also the evolution of the gaugino and scalar mass parameters with profound
Figure 1. Mass parameters at $Q = 1$ TeV versus the seesaw scale for fixed high scale parameters $m_0 = M_{1/2} = 1$ TeV, $A_0 = 0$, $\tan \beta = 10$ and $\mu > 0$. The full lines correspond to seesaw type I, the dashed ones to type II and the dash-dotted ones to type III. In all cases a degenerate spectrum of the seesaw particles has been assumed.

Implications on the spectrum [30, 31]. Additional effects on the spectrum of the scalars can be present if some of the Yukawa couplings get large [31, 32, 33]. In figure 1 we exemplify this by showing the values of selected mass parameters at $Q = 1$ TeV versus the seesaw scale for fixed high scale parameters $m_0 = M_{1/2} = 1$ TeV, $\tan \beta = 10$ and $A_0 = 0$ where we have assumed that the additional Yukawa couplings are small. As expected, the effects in case of models of type II and III are larger the smaller the corresponding seesaw-scale is. The scalar mass parameters shown are of the first generation and, thus, the results are nearly independent of $\tan \beta$ and $A_0$.

Note that in all three model types the ratio of the gaugino mass parameters is nearly the same as in the usual mSUGRA scenarios but the ratios and differences of the sfermion mass parameters change [30, 31]. One can form four 'invariants' where at least at the 1-loop level the dependence on $M_{1/2}$ and $m_0$ is rather weak, e.g. $(m_{\tilde{E}}^2 - m_{\tilde{E}}^0)/M_1^2$, $(m_{\tilde{Q}}^2 - m_{\tilde{E}}^0)/M_1^2$, $(m_{\tilde{D}}^2 - m_{\tilde{L}}^0)/M_1^2$ and $(m_{\tilde{Q}}^2 - m_{\tilde{U}}^0)/M_1^2$. Here one could replace $M_1$ by any of the other two gaugino masses which simply would amount in an overall rescaling. In figure 2 we show these 'invariants' in the leading-log approximation at 1-loop order to demonstrate the principal behaviour for seesaw type II with a pair of 15-plets and seesaw type III with three 24-plets. Due to the larger change in the beta-coefficient the effect is more pronounced in case of type III models. From this one concludes that in principle one has a handle to get information on the seesaw scale for given assumptions on the underlying neutrino mass model and assuming universal boundary conditions at the GUT scale. For the type-I, i.e. singlets only, of course $\Delta b_i = 0$ and no change with respect to mSUGRA are expected. A detailed discussion can be found e.g. in [31].

3.5. Lepton flavour violation in the slepton sector and signals at the LHC

Additional effects beside the ones on the spectrum discussed above can occur due to lepton flavour mixing entries in the sleptons mass matrix which are induced due to RGE effects by the additional Yukawa couplings. From a one-step integration of the RGEs one gets, assuming mSUGRA boundary conditions, a first rough estimate for the lepton flavour violating entries in the slepton mass parameters:

$$m_{L,ij}^2 \approx - \frac{a_k}{8\pi^2} \left( 3m_0^2 + A_0^2 \right) \left( y^{k,\dagger} Y y^{k} \right)_{ij}$$

(16)
Figure 2. Four different “invariant” combinations of soft masses versus the mass of the $15$-plet or $24$-plet, $M_{15} = M_{24}$. The calculation is at 1-loop order in the leading-log approximation. The lines running faster up towards smaller $M$ are for type-III seesaw, the lower ones are for type-II seesaw.

\[ A_{l,ij} \simeq -\frac{3a_k}{16\pi^2} A_0 \left( Y_E Y^{k,\dagger}_i Y_N^j \right) \delta_{ij} \]  

for $i \neq j$ in the basis where $Y_E$ is diagonal and $L_{ij} = \log(M_G/M_i)\delta_{ij}$. The corresponding coefficient $a_k$ for seesaw type $k$ are: $a_I = 1$, $a_{II} = 6$ and $a_{III} = 9/5$. All models have in common that the predict negligible flavour violation for the $R$-sleptons $m_{\tilde{E}_{ij}} \approx 0$. Sizable entries for the these parameters would be a clear hint for a left-right symmetric extension of the MSSM [35].

These flavour mixing entries induce on the one hand rare lepton decays, e.g. $\mu \rightarrow e\gamma$, and on the other hand lepton flavour violating decays of sleptons and neutralinos. As a typical example we show in fig. 3 the flavour violating decays of the heavier stau for degenerate right-handed neutrinos, $M_0 = 90$ GeV, $M_{1/2} = 400$ GeV, $A_0 = 0$ GeV, $\tan \beta = 10$, $\mu > 0$, from ref. [34].

Figure 3. Flavour violating decays of the heavier stau for degenerate right-handed neutrinos, $M_0 = 90$ GeV, $M_{1/2} = 400$ GeV, $A_0 = 0$ GeV, $\tan \beta = 10$, $\mu > 0$, from ref. [34].

4. R-parity violation

We now turn to the possibility that low-energy supersymmetry itself may provide the origin of neutrino mass [37, 11], for a review see Ref. [38]. Usually one assumes that R-parity, defined as $(-1)^{3B+L+2S}$, is an exact symmetry under which all superpartners are odd and SM particles even. However the terms that break R-parity are allowed by supersymmetry as well as the SM gauge invariance. Expressed as superfields, they have the form $\hat{L}H_u$, $\hat{L}\hat{E}^c$, $\hat{Q}\hat{L}\hat{D}^c$ and $\hat{U}^c\hat{D}^c\hat{E}$. 
Figure 4. Cross section $\sigma(pp \rightarrow \tilde{\chi}^0_2) \times BR(\tilde{\chi}^0_2 \rightarrow \sum_{i,j} \tilde{l}_i \tilde{l}_j \rightarrow \mu^+ \tau^- \tilde{\chi}^0_1)$ as a function of $M_{1/2}$ for various values of $m_0$, $A_0 = 0$, $\tan \beta = 10$, $\mu > 0$. Left side, seesaw type I with degenerate $\nu_R$, right side seesaw type II with $\lambda_1 = 0.02$, $\lambda_2 = 0.5$; from ref. [36].

If all four terms are present proton decay becomes very rapid. This problem is circumvented by simply forbidding the last term, e.g. by using baryon triality or a similar symmetry [39, 40]. The remaining three terms break lepton number explicitly implying that a combination of tree and loop diagrams in these models can lead to realistic neutrino masses and mixings. From the point of view of collider physics, there is an important implication of LSP decay as at least some of its decay properties are correlated to neutrino physics since the same couplings governing neutrino physics also lead to visible decays of the LSP.

We will exemplify this by focusing on bilinear $R_P$ breaking, for discussion of tri-linear $R_p$ see for example [41, 42] and for the so-called $\mu$νSSM see for example [43]. The absence of tri-linear terms could be explained, for example, if bilinear $R$-parity breaking is the effective low-energy limit of some spontaneous $R_p$ model, see below.

4.1. Explicit bilinear $R$-parity violation

The superpotential of the bilinear $R_P$ model can be written as

$$W = W_{MSSM} + \epsilon_i \tilde{L}_i \tilde{H}_u.$$  \hspace{1cm} (18)

In addition, one must include bilinear $R_P$ soft supersymmetry breaking terms

$$V_{soft} = \epsilon_i B_i \tilde{L}_i H_u + V_{soft}^{MSSM}. $$ \hspace{1cm} (19)

These terms induce mixings between the MSSM Higgs bosons and the left scalar neutrinos which in consequence also obtain vevs $v_i$ once electro-weak symmetry is broken. Usually one trades the $B_i$ by the $v_i$ using the corresponding tad-pole equations which are a consequence of eq. (19) as the connections to neutrino physics become more apparent.

The effective neutrino mass matrix at tree-level can then be cast into a very simple form

$$m_{\nu,ij} = -\frac{m_\gamma}{4 \det(M_{\chi_0})} \Lambda_i \Lambda_j$$ \hspace{1cm} (20)

The “photino” mass parameter is defined as $m_\gamma = g^2 M_1 + g'^2 M_2$, $\det(M_{\chi_0})$ is the determinant of the $(4,4)$ neutralino mass matrix and $\Lambda_i \equiv \epsilon_i v_i + v_i \mu$ are the “alignment parameters”.

Due to the projective nature of eq. (20) the other two neutrino masses are generated only at 1-loop order. Generally the most important contributions come from loops with sbottoms and
Figure 5. Ratio of semi-leptonic branching ratios, $BR(\chi^0_1 \rightarrow \mu q \bar{q})$ over $BR(\chi^0_1 \rightarrow \tau q \bar{q})$ as a function of the atmospheric neutrino angle calculated within bilinear $R_p/SUSY$, see ref. [49].

Figure 6. Branching ratio $BR(\mu \rightarrow e J)$ versus visible lightest neutralino decay. $\mu \rightarrow e J$ and $\chi^0_1 \rightarrow J \nu$ are correlated, see ref. [50].

staus [44, 45]. However, there exist also parameter regions in the general $R_p$ MSSM where the sneutrino-anti-neutrino loop gives a sizeable contribution [46, 47, 48]. One finds that in order to explain the observed neutrino mixing angles one requires certain relations among the $R_p$ parameters to be satisfied [44], e.g. the maximal atmospheric angles requires $\Lambda_\mu \simeq \Lambda_\tau$.

Once R-parity is broken the LSP decays. The decays of a neutralino LSP have been studied in [51, 49]. Decay lengths for the neutralino are approximately fixed once the neutrino masses are fitted to experimental data. Typical lengths range from tens of cm for very light neutralinos to sub-millimeter for neutralinos of several hundred GeV [49]. One of the most exciting aspects of bilinear $R_p$, however, is the fact that once neutrino angles are fitted to the values required [13] by the neutrino oscillation data, the ratios of LSP decay branching ratios are fixed and correlate with the observed neutrino mixing angles, as illustrated for example in fig. 5. Measurements at the LHC should allow to test this prediction, if signals of SUSY are found [52].

Within $R_p/SUSY$ any supersymmetric particle can be the LSP. It has been shown that within bilinear $R_p$ correlations between the measured neutrino angles and ratios of LSP decays can be found for all LSP candidates [53, 54, 55]. Thus, it is possible to exclude the minimal bilinear $R_p$ model experimentally at the LHC.

4.2. Spontaneous R-parity violation

In spontaneous R-parity violation models [56, 37, 57] R-parity violation results from the minimization of the Higgs potential through nonzero sneutrino vacuum expectation values. If lepton number is ungauged this implies the existence of a Nambu-Goldstone boson - the majoron. However, a doublet majoron is ruled out by LEP measurements of the $Z$ width [58]. Hence, viable spontaneous R-parity breaking models must be characterized by two types of sneutrino vevs, those of right and left sneutrinos, singlets and doublets under SU(3) $\otimes$ SU(2) $\otimes$ U(1)$_Y$ respectively [59, 60]. These obey the “vev-seesaw” relation $v_{ULVR} \sim Y_\nu m^2_W$, where $Y_\nu$ is the small Yukawa coupling that governs the strength of R–parity violation [59, 60].

In this case the majoron is so weakly coupled that bounds from LEP and astrophysics [61] are easily satisfied. For example, the superpotential of [59] can be written as

$$W = \tilde{H}_u \hat{Q} Y_u \hat{U} + \tilde{H}_d \hat{Q} Y_d \hat{D} + \tilde{H}_u \hat{L} Y_e \tilde{E} + \tilde{H}_u \tilde{L} \nu \tilde{c} - h_0 \tilde{H}_d \tilde{H}_u \tilde{\phi} + h \tilde{\phi} \tilde{c} \tilde{S} + \frac{\lambda}{3!} \tilde{\phi}^3. \quad (21)$$
The first three terms are the usual MSSM Yukawa terms. The terms coupling the lepton doublets to $\tilde{\nu}^c$ fix lepton number. The coupling of the field $\Phi$ with the Higgs doublets generates an effective $\mu$-term a lâ the Next to Minimal Supersymmetric Standard Model (NMSSM) [62, 63, 64]. Note, that $v_R = \langle \tilde{\nu}^c \rangle \neq 0$ generates effective bilinear terms $\epsilon_i = Y_{\ell i}^iv_R$ and that $v_R$, $v_S$ and $v_{L_1}$ violate lepton number and R-parity spontaneously. Neutrino oscillation data enforce $v_{L_1}^2 \ll v_R^2$ and $v_{L_1}^2 \ll v_S^2 + v_{L_1}^2$ implying the majoron is mainly a singlet in this model [65, 66].

The existence of the majoron affects the phenomenology at colliders mainly in two ways: (i) the lightest Higgs can decay invisibly into two majorons [65, 66]. (ii) Also the decays of the lightest neutralino are affected, since the new decay channel $\chi_1^0 \rightarrow J_{\nu}$ is invisible at colliders. In ref. [67, 68] it has been shown that this mode can be close to 100% for certain parameter combinations implying that a large luminosity at the LHC might be required to detect R-parity violation. However, as can be seen in fig. 6 $\chi_1^0 \rightarrow J_{\nu}$ is correlated with the decay $\mu \rightarrow J_{\tau}$, thus allowing to probe for a complementary part of parameter space [50] with low energy experiments.

Spontaneous R-parity violation can also be obtained by enlarging the gauge group [69, 70, 71, 72, 73, 74, 75]. In this case the majoron would be the longitudinal component of an additional neutral heavy vector boson which can be produced in s-channel processes at the LHC.

4.3. LHC studies

In the following we mainly focus on mSUGRA models which are augmented by bilinear R-parity breaking parameters at the electroweak scale implying that one has eleven free parameters, namely $m_0$, $M_{1/2}$, $\tan\beta$, $\sin(\mu)$, $A_0$, $\epsilon_i$, and $\lambda_i$. In order to fit current neutrino oscillation data, the effective strength of R-parity violation must be small. This implies that supersymmetric particle spectra are expected to be the same as in the conventional R-parity conserving model, and that processes involving single production of SUSY states [76] are negligible at the LHC and that the only differences occur due to the decays of the LSP.

In these scenarios the lightest neutralino is the LSP and its main decay channels are $\chi_1^0 \rightarrow \nu\ell^+\ell^-$ with $\ell = e, \mu$ denoted by $\ell\ell$; $\chi_1^0 \rightarrow \nu\tau^+\tau^-$, called $\tau\tau$; $\chi_1^0 \rightarrow \tau\nu\ell$, called $\tau\ell$. $\chi_1^0 \rightarrow vqq$ denoted $jj$; $\chi_1^0 \rightarrow \tau q\bar{q}$, called $\tau jj$; $\chi_1^0 \rightarrow \ell q\bar{q}$, called $\ell jj$; $\chi_1^0 \rightarrow \nu bb$, which we denote by $bb$; $\chi_1^0 \rightarrow \nu vv$, which we denote by $bb$; $\chi_1^0 \rightarrow \nu vv$.

In ref. [77] a comparison has been performed between the reach of LHC for R-parity violating SUSY using the same cuts as for R-parity conserving models [78]. The main topologies are: Inclusive jets and missing transverse momentum; zero lepton, jets and missing transverse momentum; one lepton, jets and missing transverse momentum; same sign lepton pair, jets and missing transverse momentum; three–leptons, jets and missing transverse momentum; multi-leptons, jets and missing transverse momentum. Due to the reduced missing energy the all-inclusive channel will have a reduced reach in the parameter space. However, the decays of the neutralino increase the multiplicities of the multi-lepton channel. As an example we display in Fig. 7 the LHC reach in the three– and multi–lepton channels with/without R–parity conservation for an integrated luminosity of 100 fb$^{-1}$. These constitute the best standard channels for the discovery of bilinear R-parity violation.

The sizable decay length of the neutralino is quite useful as this topology has little, if any, background expected at the LHC. This feature has been exploited in ref. [77, 79], where a comparison has been performed between the reach of LHC for R-parity violating SUSY including explicitly the displaced vertex topologies. In figure 8 we present the displaced vertex reach in the $m_0$-$M_{1/2}$ plane for $\tan\beta = 10$, $\mu > 0$, $A_0 = -100$ GeV. As one can see form this figure, the LHC will be able to look for the displaced vertex signal in the $m_{1/2}\sim800$ (1000) GeV for a large range of $m_0$ values and an integrated luminosity of 10 (100) fb$^{-1}$. Notice that the reach in this channel is rather independent of $m_0$. However, this signal disappears in the region where the stau is the LSP due to its rather short lifetime.
Figure 7. LHC discovery potential in the three lepton channel (top panel) and the multi-lepton one (bottom panel) for $A_0 = -100$ GeV, $\tan \beta = 10$, $\mu > 0$ and an integrated luminosity of 100 fb$^{-1}$, from ref. [77].

Next we discuss a tantalizing possibility, namely a double discovery at the LHC: (i) find evidence for supersymmetry, and (ii) uncover the Higgs boson. There are regions in parameter space where $\tilde{\chi}^0_1$ may have a sizeable branching ratio up to 22% into the channel $\nu h^0$ where $h^0$ is the lightest Higgs boson [80]. This would lead to displaced vertices containing two b-jets as a characteristic signature for Higgs production at the LHC [80]. The displaced vertex signal implies that also LHCb will have good sensitivity for such scenarios in particular in case of final states containing muons such as $\tilde{\chi}^0_1 \rightarrow \nu \mu^+ \mu^-$. Figure 9 demonstrates that the ATLAS and CMS experiments will be able to look for the signal up to $M_{1/2} \sim 700$ (900) GeV for a LHC integrated luminosity of 10 (100) fb$^{-1}$. The hatched region in Fig. 9 indicates the LHCb reach for 10 fb$^{-1}$. Due to the strong cut on the pseudo-rapidity required by this experiment the reach for 2 fb$^{-1}$ is severely depleted and only a small region of the parameter space is covered.

4.4. Trilinear $R$-parity breaking

Bilinear $R$-parity violation is essentially equivalent to tri-linear $R$-parity breaking with the superpotential

$$W_{tri} = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k$$

(22)

where the tri-linear couplings have the following structures

$$\lambda_{ijk} \simeq \frac{\epsilon_i}{\mu} Y^j_3^k, \quad \lambda'_{ijk} \simeq \frac{\epsilon_i}{\mu} Y^j_3^d$$

(23)

Obviously the phenomenology will be very similar if tri-linear $R$-parity violation is close to this structure. In the case of significant deviations from this structure is realized, one gets new interesting signatures. For example there exists light stau LSP scenarios where $\tilde{\tau}_1$ decays dominantly via 4-body decays such as $\tilde{\tau}_1 \rightarrow \tau^- \mu^+ \mu^- \bar{u} \bar{d}$ with long lifetimes leading to displaced vertices [81]. Another interesting signals are the resonant production of sleptons as discussed in [82, 83] or associated production of single sleptons with $t$-quarks [84].
Figure 8. Discovery reach for displaced vertices channel in the $m_0 - M_{1/2}$ plane for $\tan \beta = 10$, $\mu > 0$, $A_0 = -100$ GeV. The stars (squares) stand for points where there are more than 5 displaced vertex signal events for an integrated luminosity of 10 (100) $fb^{-1}$. The marked grey (green) area on the left upper corner is the region where the stau is the LSP, from ref. [77].

Figure 9. LHC reach for Higgs search in the $m_0 - M_{1/2}$ plane for $\tan \beta = 10$, $A_0 = -100$ GeV, and $\mu > 0$. The yellow stars (blue squares) show the reach for an integrated luminosity of 10 (100) $fb^{-1}$ and the hatched region the reach of LHCb for an integrated luminosity of 10 $fb^{-1}$. The (yellow) shaded region in the bottom is excluded by direct LEP searches, while the (red) upper–left area represents the region with a stau as LSP; the black lines delimit different regimes of LSP decay length, from ref. [80].

An interesting question is to which extent one can measure deviations from the hierarchical structure above, e.g. the coupling $\lambda_{211}'$ can still be of order 0.1. It has been shown in [85] that in such a case one LHC will be able to measure such couplings of such a strength with an accuracy of about 10%.

5. Summary
We have discussed aspects of various extensions of the MSSM taking the explanation of neutrino data as guide-line. As a first model the case has been considered where one adds a Dirac mass term for the neutrinos with tiny Yukawa couplings. In this case one obtains the usual MSSM phenomenology except for the case where the right-sneutrino is the LSP because in this case the NLSP will be very long-lived and one gets as signature quasi-stable particles at the LHC.

In case of seesaw models one finds that in case of type II and type III models the spectrum can be quite different when compared to the usual mSUGRA scenarios as a consequence of additional charged particles between the seesaw scale(s) and the GUT scale. These differences are the larger the lower the seesaw scale and affect in particular the masses of the sfermions. One can study four ratios at the LHC which can give information on the GUT scale if one assumes universal boundary conditions for the soft SUSY breaking parameters at the GUT scale. Moreover, there are parameter regions in all seesaw models where lepton flavour violating signals can be found in SUSY cascade decays. However, for this in general a rather high luminosity in the order of 100 $fb^{-1}$ or higher is required.
Supersymmetric models also offer the possibility to explain neutrino data via R-parity violation. Here we have mainly focused on models where R-parity is broken by bilinear terms as this is sufficient to explain neutrino data and to explore the relevant LHC phenomenology. Moreover, this class of models can be obtained as effective model in case of spontaneous R-parity breaking. Bilinear R-parity breaking implies correlations between neutrino data and decays properties of the LSP, e.g. in case of an neutralino LSP the ratio $BR(\tilde{\chi}_0^0 \to W^−τ^+)/BR(\tilde{\chi}_0^0 \to W^−μ^+)$ $\simeq$ tan²$θ_{\text{atm}}$ where $θ_{\text{atm}}$ is the mixing angle related to the the atmospheric neutrino sector. In addition one finds very often that the LSP life time is sufficiently small to produce a displaced vertex in a typical collider experiments.

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