Developing an Extenics-Based Model for Evaluating Bus Transit System

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Evaluating bus transit performance periodically is a key step to improve service quality and system efficiency. An extenics-based model is developed with the real-world data. The performance indices employed for the evaluation are identified. The dependent function applied for measuring the correlation between the bus transit and the system performance is formulated. The hybrid weights associated with the indices are determined by subjective weights through analytic hierarchy process (AHP) and objective weights through the entropy method. The proposed extenics model is applied for evaluating a bus transit system of a medium-sized city in China. The model outcomes are informative, while the suggestions corresponding to the identified weakness are concluded for future planning and operation.

1. Introduction

Urbanization and motorization improve the mobility and convenience of traveling, albeit sometimes hurt the quality of human’s life caused by traffic congestion, environmental pollution (i.e., air quality deterioration, and noise), and energy dependence. A recent study [1] indicated that the ownership of private vehicles in China has exceeded 200 million in 2018, which increased 11.13% compared to that in the year before. Promoting public transportation, including enhancing the existing systems and developing new ones, seems a consensus to mitigate the associated negative impacts. To stimulate ridership by improving transit efficiency and elevating service quality, the overall reduction in energy consumption and vehicle emission may be expected.

In the past decade, many studies focused on bus transit evaluation [2–14] from the selection of performance indices to the development of evaluation methods. However, there are limitations of previous methods. Some of those focused on one/few aspects of bus transit, and the results cannot offer an overall system performance, while others evaluated bus transit at a system level but output limited details. It is desirable to develop a sound method which can offer more insights of the evaluated system to the stakeholders (i.e., government sectors, public transportation suppliers, and users). The extenics theory [15] seems a candidate approach to assess the performance of bus transit through a dependent function, which can measure the correlation between performance indices and the holistic system performance. However, the dependent function is critical to the accuracy of the assessment result. Previous extenics-based models have a weakness with respect to dependent function without considering index type (cost/efficiency-based), which may lead the evaluation results to less precise. In general, for a cost-based (efficiency-based) type index, the larger the index value is, the worse (better) the index is. When the index type is not considered during the calculation of dependent function, for a specific index, different values may have the same dependent degree, which indicates the correlation between the index and the system performance. Thus, it is necessary to consider the type of evaluation index when conducting the evaluation.

This study aims to develop an extenics-based model for evaluating bus transit systems, which may be applied to specific performance indices as well as the holistic system performance. The proposed extenics-based model is
significantly improved from previous ones by employing cost-based and efficiency-based indices while developing a new dependent function. From the results of the evaluation of a real-world bus transit in Taiyuan city, the weak indicators are identified and potential alternatives to improve the holistic system performance and sustainable operation are suggested.

2. Literature Review

Developing an effective and efficient bus transit is one of the viable ways to mitigate the impacts of urban transport problems [16–18]. Zhang et al. [17] explored the significant impact of bus stop design on various modes of transport. To alleviate bus bunching and improve service reliability, Liu et al. [18] proposed a model for vehicle holding control to meet bus schedule. Simulated results demonstrate that the model is applicable for optimizing bus operation. Many studies evaluated bus transit focusing on different aspects and performance indices with various methods, which are summarized in Table 1.

| Authors               | Method                                      | Focused evaluation                      | Indices                                                                 |
|-----------------------|---------------------------------------------|-----------------------------------------|-------------------------------------------------------------------------|
| Nordfjørn and Rundmo  | Descriptive statistics and SEM              | Safety                                  | Security, injury probability and severity, worry, demand for risk mitigation |
| Bryniarska and Zakowska | Marketing survey                           | Transfer hubs                           | Spatial compactness, visibility, additional facilities for transfer      |
| Mavi et al. [7]       | Stepwise weight assessment method           | BRT performance                         | Economic, environmental, social, risk, and safety criteria              |
| Chen et al. [8]       | DEA                                         | Accessibility                           | Service coverage, bus stop density, service frequency, and route diversity |
| Zhang et al. [9]      | Structural entropy-TOPSIS model             | Priority of system development          | Overall development, infrastructure, service level, policy support       |
| Barabino and Francesco [12] | Customer satisfaction survey and secret shopper survey | Service quality                      | Waiting time, space on board, and vehicle cleanliness                   |
| Wei et al. [10]       | DEA and spatial optimization models         | Service quality                         | Operation time, fleet size, operation mileage                            |
| Zhang et al. [11]     | Entropy theory and DEA                     | Service quality                         | Fleet size, labors, subsidy, operation revenue, passenger satisfaction   |
| Hu [13]               | Extenics method                             | System performance                      | Network capacity, operation cost and revenue, level of service, sustainable development |
| Hu et al. [14]        | Extenics method                             | System performance                      | Network capacity, operation cost and revenue, environmental impact, service capacity, level of service, passenger satisfaction |
| Zou et al. [6]        | AHP and fuzzy assessment                   | System performance                      | Infrastructure, service operation, IT application, sustainable development, government support, social benefit |

Based on survey data, Nordfjørn and Rundmo [2] conducted a quantitative assessment through the descriptive statistics and structural equation modelling (SEM) to investigate users’ risk evaluation on public transit considering respondents’ experience as they were exposed to security issues, such as harassment and bullying. The descriptive statistics were used to disclose sample characteristics, and SEM was used to predict users’ preference to use public transit modes. Bryniarska and Zakowska [3] assessed the performance of transfer hubs, including tram/bus stops, considering the indices related to spatial compactness, visibility, and additional facilities. Based on a marketing survey, the index values were obtained, but it was hard to give a comprehensive evaluation result at the system level. To conduct a comprehensive evaluation, Kittelson & Associates, Inc. [4] suggested using basic capacity concepts and a quantitative classification method to assess transit quality of service. The quantitative classification method was developed based on the empirical formula aiming at the North American practice. Barabino et al. [5] selected transit service-related indices through an integrated approach, which begins with a long list of key quality indicators (KQI) identification, then defines the properties of KQI, involves experts to provide judgments, evaluates and adjusts the long list, and finally proposes the most suitable KQI.

Some studies were conducted to evaluate the holistic or local performance of public transit with mathematical models [6–12]. During the evaluation process, the index weight is a critical component leading to the evaluation results. The index weighting methods can be classified into subjective, objective, and hybrid. The evaluation results with subjective weighting can reflect decision-maker’s expectation, but the results may be distanced from the reality due to high subjectivity. The objective weighting has a strong theoretical meaning, but the evaluation results may deviate from decision-makers’ expectation since the weights are determined without considering subjective conditions [11]. In comparison, the hybrid weighting seems promising because it may improve the limitations of subjective and objective weighting methods.

Zou et al. [6] evaluated the bus transit system performance with a method which integrated analytic hierarchy process (AHP) and fuzzy assessment. Mavi et al. [7] analyzed bus rapid transit (BRT) performance using a stepwise weight assessment method, where the weights associated with
evaluation indices were given by experts in transit industry. Chen et al. [8] investigated the accessibility of bus transit with data envelopment analysis (DEA), and the index weights were determined by data comparison and aggregation. Zhang et al. [9] explored the relation between the service quality of bus transit and organizational forms, in which the index weights used in DEA were determined by the entropy theory. Wei et al. [10] evaluate the service quality of bus transit, and the index weights used in DEA were determined by an optimization model which maximized the service coverage for disadvantaged customers (i.e., the elderly, children, noncar households, poor people, unemployed, disabled, and non-white population) and the efficiency of public transit operation. Considering the expert opinions with hybrid weighting, Zhanget al. [11] analyzed bus transit system priority with structural entropy—technique for order preference by similarity to an ideal solution (TOPSIS) model based on hybrid weighting.

However, the DEA-based methods merely give the relative efficiencies of decision-making units instead of absolute efficiencies, and TOPSIS is difficult to produce the ideal solutions, while the rest methods mentioned above mainly present an evaluation result of bus transit system with limited information, such as “Good,” “Poor,” and “Very Poor”. Barabino and Di Francesco [12] measured the performance of transit service quality by percentage values, making it possible to evaluate the impact of service quality on passengers from both the system user and supplier sides. Furthermore, the extenics method can be applied to evaluate the bus transit performance comprehensively (including specific performance indices and the holistic system performance), which integrates qualitative and quantitative analyses [19], to describe the relationship between the matter characteristics and the matter itself. The core of extenics is matter-element theory, extension set theory, and extension logic, in which the logical cells are matter element, affair element, and relation element. Matter-element theory is used to describe subjects, while extension set theory is to quantify the relation between the real variable and the interval. Extenics methods have been popularly applied in engineering practices, such as building energy conservation assessment [20], community home-care service evaluation [21], cylinder’s precision-stability analysis [22], smart village planning [23], robot motion control [24], and industrial designing [25].

For evaluating the performance of bus transit including the infrastructure capacity and service quality, few studies assessed bus transit with the extenics theory. Hu [13] and Hu et al. [14] evaluated bus transit by classifying the system performance into five levels. A dependent function was developed, which is a core component with the extenics method to calculate the correlation between the holistic system performance and evaluation indices. However, the dependent function suffered some limitations which may distance the evaluation results from the reality as mentioned before.

This study aims to develop an extenics model for evaluating bus transit performance, explore the primary operational problems or issues, and suggest alternatives to improve the operation and planning of bus transit. To cope with the limitations in the previous studies discussed earlier, the dependent function formulated in this study will enable the proposed model producing more accurate and detailed results.

3. Model Development

To effectively evaluate bus transit performance, the proposed extenics-based model is developed and discussed in this section. To cope with the limitations of previous extenics methods, the dependent function is formulated considering cost-based and efficiency-based indices and index weights are determined considering the limitations of subjective and objective weightings. For calculating the index weights, a hybrid weighting which integrates the objective and subjective weightings is used.

The flowchart of the model development is presented in Figure 1. The first step is to define matter element, which consists of evaluation object, characteristics, and associated values. Given the defined matter element, the performance levels are determined next, where indices are classified into classical and section domains for calculating the dependent function in the following step. After determining the hybrid index weights considering subjective and objective aspects, the extenics model is ready for evaluating the system performance.

3.1. Matter-Element Definition. According to the matter-element theory, matter element R is generally expressed by an ordered ternary R = (M, C, X), where M is evaluation object, C consists of characteristics of M, and X is a set of values associated with C. In this study, M is the holistic performance of the study bus transit, C is a vector of performance indices c, (i = 1, 2, . . . , m), and X is a vector of index values xi, (i = 1, 2, . . . , m) associated with ci.

For each ci, a section domain and associated a set of classical domains are denoted as xiL and xiR, respectively, where i is the level of system performance varying from 1 to 5. Thus, xiL = [aiL, biL] is the classical domain of index i with level l, where aiL and biL are the lower and upper bounds of the domain, and xiR = [aiR, biR].

3.2. Performance Levels and Associated Indices Description. Suppose that a bus transit system consists of Q bus stops, S bus routes, and T buses. The 18 indices selected for bus transit are based on suggestions of previous studies [13, 26–30], experts’ opinions, and available data provided by the agency. The indices including number of bus renewal factor, safety factor, and accident rate are suggested by PRC Ministry of Housing and Urban-Rural Development [26, 27]. Circuitry, stop spacing, transfer passenger factor, transfer distance, and peak-hour travel time are suggested by PRC Ministry of Construction [28]. Route density, service coverage, number of standard buses, exclusive busway factor, punctuality, average speed, and peak utilization are suggested by PRC Ministry of Transport [29]. Route repetition and travel expense factor are suggested by Hu [13], and
air-conditioned bus rate is suggested by Tan and Guo [30]. It is worth noting that these 18 indices are evaluated based on a holistic system consisting all bus routes.

As shown in Table 2, the proposed performance evaluation system is organized into four hierarchies. The 1st hierarchy represents the goal, which is to evaluate the performance of bus transit. The 2nd hierarchy consists of two main indicators: capacity and service quality. The former indicates the carrying capacity of the study bus transit, while the latter one reflects the service capacity to meet the passenger travel demand. The 3rd hierarchy includes several subindicators. Basic capacity and transportation capacity are associated with capacity, while convenience, punctuality, mobility, safety, comfort, and cost are associated with service quality. It is worth noting that basic capacity focuses on bus transit network design, while transportation capacity focuses on service capacity of bus operation. Convenience concerns transfer time and transfer distance for passengers. Punctuality focuses on the on-time operation, while mobility focuses on passenger travel time. Safety is regarded with passenger traveling and vehicle operation. Comfort relates to the environment inside the vehicle, while cost concerns ticket fare and annual income.

Finally, there are 18 indices identified and placed in the lowest hierarchy. Route density, route repetition, circuity, and service coverage are with basic capacity. Number of standard buses, exclusive busway factor, bus renewal factor, and stop spacing are with transportation capacity. Transfer passenger factor and transfer distance are with convenience. Punctuality is with punctuality. Average speed and peak-hour travel time are with mobility. Safety factor and accident rate are with safety. Peak utilization and air-conditioned bus rate are with comfort, and finally travel expense factor is with cost. Given the index system structure and data availability, the 18 indices listed in the lowest hierarchy are employed to characterize the overall system performance of the study bus transit. The definitions and types of indices are also given in Table 2. Note that all of the indices represent the performance of the holistic system instead of specific routes. For example, punctuality is defined as the number of on-time bus trips divided by all bus trips for the study system.

These performance indices are classified into five levels (e.g., Excellent, Good, Moderate, Poor, and Very Poor for levels 1 through 5, respectively). The ranges of performance levels are shown in Table 3, which were determined based on relevant industry regulations [26–29] for a medium-size city [31] such as Taiyuan. According to Code for Urban Road Traffic Planning and Design [28], route density in the city center should be 3–4 km/km², which are thus defined as levels 1 through 5 for [3.5, 4], [3, 3.5), [2.5, 3), [1.5, 2.5), and [0, 1.5), respectively. It is worth noting that these five ranges are also the classical domains subjected to each system performance level for route density, and the corresponding section domain is [0, 4].

3.3. The Dependent Function Formulation. Previous extenics methods did not distinguish the index type, and as a result, similar evaluation objectives cannot be evaluated in a comparable way. For example, in the evaluation of bus transit punctuality, if the score range of [90%, 100%) is defined as the evaluation level of "Excellent," then the dependent degree of the bus transit punctuality with 92% points is the same with that of the bus transit punctuality with 98% points between their scores with "Excellent" level. In fact, the bus transit punctuality with higher score usually seems more excellent than that with lower score. In the proposed model, the dependent function for measuring the correlation between system performance level and evaluation index is formulated as follows:

Figure 1: The flowchart of the model development.
where \( x_i \) is the value of index \( i \); \( k_i(x_i) \) is the dependent degree of index \( i \) of system performance level \( l \); \( \rho(x_i, x_{li}) \) is the distance between \( x_i \) and interval \( x_{li} = [a_{li}, b_{li}] \); and \( \rho(x_i, x_{Li}) \) is the distance between \( x_i \) and interval \( x_{Li} = [a_{Li}, b_{Li}] \). For index route density, as shown in Table 3, \( x_{11} = [3.5, 4] \) and \( x_{L1} = [0, 4] \).

It is worth noting that equation (1) is formulated to improve the accuracy of the previous extenics model by considering index type. As shown in equation (1), when taking the index type into consideration, the dependent degree increases with the index value for efficiency-based type while decreases for the cost-based type. Moreover, the dependent function can be expressed as a real number instead of a range of \([0, 1]\), and thus, more information will be provided [20].

3.4. Index Weight Determination. AHP determines index weights based on expert scoring, which is applied to determine subjective index weights. On the other hand, the entropy method determines weights based on index information, which is applied to determine objective index weights. The hybrid weights derived from subjective and objective weights for the evaluation indices are discussed next.

3.4.1. Subjective Weights with AHP. AHP is a common multicriteria decision-making approach, which evaluates indices through dividing the complicated system into a hierarchical structure of elements, calculating the local weights of indices at the same hierarchy with respect to the higher hierarchy and the final subjective weights. For the sake of space, the steps of using AHP to determine the weights of indices in the lowest hierarchy within sub-indicator basic capacity depending on the main indicator capacity shown in Table 2 are discussed as follows:

Step 1: pairwise comparison matrix establishment based on expert scoring

In general, experts play an important role in this procedure to provide judgments. Taking the professional background into consideration, the experts who are familiar with bus transit system design, operation, and management are invited to conduct the pairwise comparison among the indices. There are four indices in the 4th hierarchy within basic capacity depending on the main indicator capacity. Firstly, experts are asked to conduct the 9-scale pairwise comparison to quantify the local weights of any pair among four indices based on expert scoring. In particular, scales 1 to 9 mean that compared with the latter index, the former index is equally important to extremely important. Each expert is required to provide his/her judgment, and the average values are regarded as the final results. Establish the pairwise comparison matrix based on expert scoring as follows:

\[
U = \begin{pmatrix}
\alpha_{ij} \end{pmatrix}_{4 \times 4},
\]
### Table 2: Evaluation indices and definitions of bus transit.

| Goal | Main indicator | Subindicator | Index | Definition (units) | Type |
|------|----------------|--------------|-------|--------------------|------|
| Urban bus transit system performance evaluation | Route density | Route density | Bus route length per unit area of service coverage (km/km<sup>2</sup>) | Efficiency-based |
| | Route repetition | Route repetition | Total route length divided by total link length | Cost-based |
| | Basic capacity | Circuitry | Actual route length divided by straight-line distance between the start and end stations | Cost-based |
| | Capacity | Service coverage | Bus stop service area divided by bus network service area (%) | Efficiency-based |
| | | Number of standard buses | Number of standard buses per 10,000 population | Efficiency-based |
| | Transportation capacity | Exclusive busway factor | Exclusive busway length divided by total bus route length (%) | Efficiency-based |
| | | Bus renewal factor | Actual updated buses divided by scheduled buses (%) | Efficiency-based |
| | | Stop spacing | Bus route length divided by the number of stops (m) | Cost-based |
| | | Transfer passenger factor | Number passengers divided by passengers without transfer | Cost-based |
| | | Transfer distance | Avg. transfer walking distance per transfer passenger (m) | Cost-based |
| | Convenience | Punctuality | On-time bus trips divided by all bus trips (%) | Efficiency-based |
| | | Average speed | Route distance divided by travel time (km/h) | Efficiency-based |
| | Service quality | Peak-hour travel time | Avg. one-way travel time during peak hours (min) | Cost-based |
| | | Safety factor | Annual operation mileage divided by bus accidents (10,000 km) | Efficiency-based |
| | Safety | Accident rate | Annual traffic accidents per 10,000 motor vehicles | Cost-based |
| | Comfort | Peak utilization | Maximum demand divided by service capacity in the peak hours (%) | Cost-based |
| | | Air-conditioned bus rate | Percentage of air-conditioned buses | Efficiency-based |
| | Cost | Travel expense factor | Annual cost for purchasing ticket divided by annual income (%) | Efficiency-based |

### Table 3: Performance levels and associated indices under classical and section domains.

| Index | Classical domains | Section domain |
|-------|-------------------|----------------|
| Route density | [3.5, 4] | [3, 3.5] | [2.5, 3] | [1.5, 2.5] | [0, 1.5] | [0, 4] |
| Route repetition | [0, 1.5] | (1.5, 2] | (2, 2.5] | (2.5, 3] | (3, 4] | [0, 4] |
| Circuitry | [0, 1] | (1, 1.4] | (1.4, 1.8] | (1.8, 2.2] | (2.2, 3] | [0, 3] |
| Service coverage | (85, 100] | (70, 85] | (65, 70] | (50, 65] | [0, 50] | [0, 100] |
| Number of standard buses | (10, 15] | (8, 10] | (6, 8] | (4, 6] | [0, 4] | [0, 15] |
| Exclusive busway factor | (30, 40] | (20, 30] | (10, 20] | (5, 10] | [0, 5] | [0, 40] |
| Bus renewal factor | (75, 100] | (65, 75] | (50, 65] | (35, 50] | [0, 35] | [0, 100] |
| Stop spacing | (0, 350] | (350, 500] | (500, 700] | (700, 900] | [900, 1000] | [0, 1000] |
| Transfer passenger factor | [1, 1.5] | (1.5, 2] | [2, 2.5] | [2.5, 3] | [3, 3.5] | [1, 3.5] |
| Transfer distance | [0, 200] | (200, 300] | (300, 400] | (400, 500] | (500, 600] | [0, 600] |
| Punctuality | (90, 100] | (75, 90] | (65, 75] | (45, 60] | [0, 45] | [0, 100] |
| Average speed | [30, 40] | [25, 30] | [20, 25] | (15, 20] | [10, 15] | [0, 40] |
| Peak-hour travel time | [0, 20] | (20, 30] | (30, 40] | (40, 50] | (50, 60] | [0, 60] |
| Safety factor | [125, 130] | (100, 125] | (75, 100] | (50, 75] | [0, 50] | [0, 130] |
| Accident rate | [0, 20] | (20, 40] | (40, 60] | (60, 80] | (80, 100] | [0, 400] |
| Peak utilization | [0, 60] | (60, 70] | (70, 80] | (80, 90] | (90, 100] | [0, 100] |
| Air-conditioned bus rate | (85, 100] | (70, 85] | (55, 70] | (40, 55] | [0, 40] | [0, 100] |
| Travel expense factor | [0, 5] | (5, 10] | (10, 20] | (20, 30] | (30, 50] | [0, 50] |
Step 2: local weight determination
Based on comparison matrix $U$, the local weights of index $i$ with respect to subindicator basic capacity denoted as $w_{ijmg}$ can be obtained through calculating and normalizing the eigenvector of matrix $U$, as shown in the following equation:

$$w_{ijmg} = \left( \frac{\prod_{j=1}^{4} U_{ijmg}}{\sum_{i=1}^{4} \prod_{j=1}^{4} U_{ijmg}} \right)^{1/4}. \quad (5)$$

Step 3: consistency check
Consistency ratio (CR) is used for consistency check of comparison matrix $U$ to ensure the consistency of subjective judgment. It is defined as follows:

$$\text{CR} = \frac{\text{CI}}{\text{RI}}. \quad (6)$$

$$\text{CI} = \frac{\lambda_{\text{max}} - 4}{(4 - 1)}, \quad (7)$$

where CI is the consistency index, as presented in equation (7), $\lambda_{\text{max}}$ is the largest eigenvalue of comparison matrix $U$. RI is the random index, indicating the average value of consistencies of random matrices with the same order, which is a set of standard criteria generated by a random method [20]. The introduction of RI can avoid the defect that CI increases with comparison matrix order. When CR < 0.1, it is considered that the consistency of the comparison matrix is acceptable, and otherwise, the comparison matrix needs to be revised, i.e., the pairwise comparison should be carried out again to avoid inconsistency in logic and steps 1 to 3 need to be re-conducted.

Step 4: final subjective weights determination
Similarly, the local weight of subindicator basic capacity with respect to the main indicator capacity denoted as $w_{mg}$ and the local weight of main indicator capacity with respect to the final goal denoted as $w_{g}$ can be calculated according to steps 1–3. The final subjective weight of index $i$ within basic capacity depending on the main indicator capacity can be expressed as follows:

$$w_i = w_{ijmg} w_{mg} w_g. \quad (8)$$

The rest of indices weights can be calculated in the same way from steps 1 to 4.

3.4.2. Objective Weights with the Entropy Method. The entropy method calculates the evaluation index weights according to the information included in the observation values of each index. Suppose there are $m$ evaluation indices and $n$ alternatives to be evaluated, the data matrix of indices can be written as $A = (x_{ij})_{mn}$ where $x_{ij}$ indicates index $i$ in alternative $j$. The steps of using the entropy method to determine the objective weights are discussed as follows:

Step 1: data preparation
Let $x'_{ij}$ be the processed data of $x_{ij}$, to avoid the logarithm is insignificant when calculating the corresponding entropy in the following steps, the data are prepared as follows:

$$x'_{ij} = \begin{cases} \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}}, & i = 1, 2, \ldots, m, x_i \text{ is efficiency-based type}, \\ \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}, & i = 1, 2, \ldots, m, x_i \text{ is cost-based type}. \end{cases} \quad (9)$$

It should be noted that for convenience, the processed data $x'_{ij}$ will still be denoted as $x_{ij}$ in the following steps.

Step 2: calculate the variation coefficient
The variation coefficient is an indicator that reflects the dispersion degree of measured data. According to the entropy theory, for index $i$, the larger the variation coefficient denoted as $g_i$ (formulated as equation (10)), the more important index $i$ is, the more weight will be given:

$$g_i = 1 + k \sum_{j=1}^{n} P_{ij} \ln P_{ij}, \quad (10)$$

where $k$ is a constant in entropy, determined by the number of performance indices $m$. Note that $k$ is the inverse of $\ln m$, $P_{ij}$ is the ratio of index $i$ in alternative $j$ to the sum of the same index values of all alternatives $n$.

Thus,

$$P_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}}. \quad (11)$$
Greatly. Thus, it is necessary to take the distribution of another performance level, while these levels are different dependent degrees of one performance level is close to that of evaluation performance level.

3.4.3. Hybrid Weights. The hybrid weight of index i denoted as $W_i$ is determined based on a linear synthesis of subjective and objective weights determined by AHP and the entropy method, respectively. Thus, $W_i = \alpha w_i + (1-\alpha)w_i'$, (13)

where $\alpha$ is the adjustment coefficient, varying between 0 and 1, which can be determined by the evaluators representing government sectors and public transportation agencies. When $\alpha$ is less than 0.5, it indicates that the index weights are dominated by objective weights. However, if $\alpha$ is greater than 0.5, it indicates that the index weights lean on subjective weights.

3.5. The Proposed Extensics Model. The synthesis dependent degree of system performance level $l$, denoted as $K_l(M)$, is the weighted sum of $K_i(x_i)$. Thus, $K_l(M) = \sum_{i=1}^{n} W_i K_i(x_i)$. (14)

Specifically, a positive $K_l(M)$ indicates that the performance of evaluation object $M$ is suitable for level $l$ and the degree of conformity is positively correlated, while a negative $K_l(M)$ means that $M$ is not suitable for corresponding evaluation performance level.

On the other hand, there is a possibility that the synthesis dependent degree of one performance level is close to that of another performance level, while these levels are different greatly. Thus, it is necessary to take the distribution of $K_l(M)$ into consideration. Since the range of $K_l(M)$ is extended to real axis, it is necessary to normalize $K_l(M)$ into a value between zero and one as follows:

$$
\mathcal{K}_l(M) = \frac{K_l(M) - \min\{K_l(M)\}}{\max\{K_l(M)\} - \min\{K_l(M)\}}
$$

(15)

Specifically, when $K_l(M) = \min\{K_l(M)\}$, $\mathcal{K}_l(M) = 0$; when $K_l(M) = \max\{K_l(M)\}$, $\mathcal{K}_l(M) = 1$; otherwise, $\mathcal{K}_l(M) \in (0, 1)$.

Thus, based on the level evaluation method [19], the overall performance level index denoted as $l^*$ is formulated as follows:

$$
l^* = \frac{\sum_{l=1}^{5} l \times \mathcal{K}_l(M)}{\sum_{l=1}^{5} \mathcal{K}_l(M)}
$$

(16)

where $l^*$ varies between 1 and 5 indicating the degree of closeness to the system performance level. For example, $l^* = 3.4$ suggests that the system performance level is between level 3 and level 4 but closer to level 3.

4. Case Study

This case study is conducted to assess a real-world bus transit with the developed extensics model. The study system is located in Taiyuan, a typical medium-size city in Shanxi Province, China, which has 2,407 bus stops, 197 bus routes, and 2,253 buses in 2016. The data used for index calculation were obtained from various sources including Taiyuan Statistical Yearbook, Taiyuan Public Security Bureau, and Taiyuan Transportation Bureau. The system evaluation committee consists of twenty experts/representatives from universities/research institutes, enterprises, and government agencies. The system evaluation committee plays a critical role in both index selection and subjective weights determination with AHP.

The values of the 18 indices listed in Table 2 are determined by processing the data and shown in Table 4. The definitions of classical and section domains given in Table 3, the associated dependent degrees determined by equation (1), and the index weights determined by equation (13) are summarized in Table 5. To determine the hybrid weights, the adjustment coefficient $\alpha$ is 0.50. The synthesis dependent degrees, which indicate the overall system performance, are
determined by equation (14) for five performance levels. The overall performance index, which indicates the distribution of synthesis dependent degrees, is determined by equations (15) and (16). The results show that the maximum value among five synthesis dependent degrees is 0.04, corresponding to performance level 3. The overall performance level index $I^*$ is 3.47.

In Table 5, the level associated with a positive dependent degree suggests the suitable performance level for each index. For example, there are two indices falling in level 1 (Excellent) and six indices falling in level 5 (Very Poor). According to the maximum synthesis dependent degree denoted as $K_2(M)$, it indicates the overall performance of the study bus transit lies on level 3 (Moderate). And that positive value 0.04 suggests that the difference between the reality and the evaluation results is relatively small since the degree of conformity is positively correlated. The overall performance level index $I^*$ means the performance level is between 3 (Moderate) and 4 (Poor) but leans to 3, indicating there is great room for bus transit improvement in study city. Furthermore, it is calculated that the performance level for the main indicator capacity is at level 5 and that for service quality is at level 3. Namely, to improve the bus transit performance level in study city, effort on capacity is more important. Considering the index weights shown in Table 5, we found that number of standard buses (10.19) is on the excellent level in the capacity category, while average transfer distance (128.13 meters) outperforms other indices in the category of service quality. The weights of route density (0.12), service coverage (0.07), and stop spacing (0.07) are relatively higher than others, which should be focused to improve service quality. For comparison, the synthesis dependent degrees are calculated based on the previous extenics model for which the dependent function did not consider index type and is different from what developed in this study. The maximum synthesis dependent degree is $K_2(M) = -0.24$ and $I^*$ is calculated as 2.99. As the results show, the bus transit system in Taiyuan belongs to "Good." But taking $I^*$ into consideration, it indicates the system performance level of bus transit in Taiyuan is between "Good" and "Moderate" but closer to "Moderate." Meanwhile, the negative value of $K_2(M)$ means the bus transit system performance in Taiyuan is not suitable for "Good" in fact. It can be found that the proposed extenics model and the previous extenics model give the different system performance levels for bus transit in Taiyuan. The reason behind this phenomenon is because that the dependent degree of the previous extenics model for the evaluation indices has not considered the index type as we mentioned before.

Furthermore, given that the fuzzy-based model is well accepted (e.g., [32, 33]), it is employed to analyze this case problem as a comparison to verify the acceptability of the proposed extenics model. Aiming at the selected 18 indices with five performance levels, the evaluation matrix $E$ is determined by calculating the membership degrees of all indices:

$$E = \begin{bmatrix} 0.07 & 0.07 & 0.20 & 0.60 & 0.07 \\ 0.00 & 0.07 & 0.13 & 0.27 & 0.53 \\ 0.07 & 0.13 & 0.60 & 0.13 & 0.07 \\ 0.00 & 0.07 & 0.13 & 0.27 & 0.53 \\ 0.73 & 0.13 & 0.07 & 0.07 & 0.00 \\ 0.20 & 0.67 & 0.07 & 0.07 & 0.00 \\ 0.00 & 0.07 & 0.13 & 0.20 & 0.60 \\ 0.00 & 0.27 & 0.53 & 0.20 & 0.00 \\ 0.00 & 0.20 & 0.60 & 0.13 & 0.07 \\ 0.73 & 0.20 & 0.07 & 0.00 & 0.00 \\ 0.00 & 0.13 & 0.67 & 0.20 & 0.00 \\ 0.00 & 0.07 & 0.13 & 0.60 & 0.20 \\ 0.13 & 0.67 & 0.13 & 0.07 & 0.00 \\ 0.00 & 0.00 & 0.13 & 0.20 & 0.67 \\ 0.00 & 0.00 & 0.13 & 0.27 & 0.60 \\ 0.00 & 0.07 & 0.13 & 0.67 & 0.13 \\ 0.00 & 0.00 & 0.07 & 0.27 & 0.67 \\ 0.00 & 0.13 & 0.73 & 0.13 & 0.00 \end{bmatrix}. \quad (17)$$

The weight vector $W$ is composed of index weights shown in Table 5. Based on the calculated evaluation matrix and index weights, the results of the fuzzy-based model are determined by the following equation:

$$B = W \times E = [0.08, 0.16, 0.33, 0.26, 0.17]. \quad (18)$$

As a result, according to the principle of maximum membership degree, it suggests that the study bus transit is at the level of "Moderate," which is consistent with the evaluation results based on the proposed extenics model. However, the evaluation results based on the fuzzy-based model only offer a final performance level and unable to illustrate detailed information as those suggested by the proposed extenics model. The comparative results for different models are summarized in Table 6.

The results of the proposed model suggested the performance of the study bus transit system is "Moderate" with the evaluation index of 3.47. In addition, the maximum synthesis dependent degree (0.04) is positive, which indicates that the system performance is suitable for "Moderate" level. With the previous extenics model on the other hand, the maximum synthesis dependent degree (−0.24) is negative, which indicates that the system performance is not suitable for "Good" level. As shown in Table 6, the proposed model can produce detailed evaluation results while indicating the degree of closeness to the
system performance, but the outcomes from fuzzy-based model can only offer the performance lever without detailed information.

5. Conclusions

Evaluating bus transit system periodically is a critical step for sustainable system planning, operation, and management. This study developed an extenics-based model for evaluating the performance of a real-world bus transit system, in which the dependent function is properly formulated. The comparison of results shown in Tables 5 and 6 suggests that the proposed model outperforms fuzzy-based and previous extenics models because the weakness of the study system can be properly identified for improving system operation and future planning.

As summarized in Table 1, there are a considerable part of studies evaluating one/few aspects of bus transit [2–5, 7–12]. This study focuses on the bus transit system performance including capacity and service quality. Compared with the studies for system performance evaluation [6, 13, 14], this study supplements three other indices (stop spacing, transfer distance, and air-conditioned bus rate) in capacity and service quality considering the national standards, experts’ opinions, and data availability.

After evaluating Taiyuan bus transit, we found that route density, service coverage, and stop spacing shall be enhanced for improving capacity, while punctuality, travel expense factor, transfer passenger factor, accident rate, and average speed shall be focused for improving service quality.

To improve the overall system performance, the model results suggest that new bus routes shall be offered to cover the areas with less accessibility. For existing routes, the number and locations of bus stops shall be optimized or at least justified to reduce stop spacing with spatiotemporal passenger travel demand [34–36]. For example, the stop spacing shall be reduced from 690 meters to 300–400 meters as advised by Levinson [16] to improve accessibility.

For improving service quality, it is critical to provide dynamic bus arrival time information [37–40], promote bus priority treatments (e.g., transit signal priority, bus lanes, and exclusive busway), facilitate passenger boarding and alighting (e.g., adoption of low-floor buses and new automatic fare collection techniques), and increase service reliability (e.g., punctuality) [41] and travel speed (e.g., integrated express and local bus service) [42, 43]. Besides, optimizing fare structure and policy [44, 45] for students, elderly, and handicapped passengers may reduce travel expense factor and stimulate the ridership.

Reducing the transfer passenger factor would be effective to further improve travel service, such as optimizing route network to yield the least number of transfers [46, 47]. Meanwhile, solution to reduce accidents requires efforts on improving roadway design (e.g., exclusive busways, sidewalks, pedestrian islands, and bus turnout or laybys) and enterprise management (e.g., establishing performance standards for drivers and administrative staff, adopting the competitive employment form) [16].

### Table 5: Dependent degrees associated with each index.

| Main indicator   | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 | Weight |
|------------------|---------|---------|---------|---------|---------|--------|
|                   |         |         |         |         |         |        |
| **Capacity**      |         |         |         |         |         |        |
| Route density (km/km²) | −0.48   | −0.39   | −0.27   | **0.32** | −0.15   | 0.12   |
| Number of standard bases | **0.04** | −0.04   | −0.31   | −0.47   | −0.56   | 0.07   |
| Exclusive busway factor (%) | −0.25   | **0.50** | −0.25   | −0.50   | −0.57   | 0.07   |
| Service coverage (%) | −0.44   | −0.32   | −0.27   | −0.05   | **0.95** | 0.07   |
| Stop spacing (m) | −0.52   | −0.38   | **0.05** | −0.03   | −0.40   | 0.05   |
| Bus renewal factor (%) | −0.82   | −0.79   | −0.73   | −0.61   | **0.39** | 0.04   |
| Route repetition  | −0.63   | −0.54   | −0.39   | −0.08   | **0.92** | 0.03   |
| Circuity          | −0.23   | −0.01   | **0.95** | −0.21   | −0.36   | 0.01   |
| **Service quality** |         |         |         |         |         |        |
| Punctuality (%)   | −0.39   | −0.06   | **0.89** | −0.33   | −0.52   | 0.12   |
| Travel expense factor (%) | −0.40   | −0.26   | **0.46** | −0.23   | −0.49   | 0.11   |
| Transfer passenger factor | −0.33   | 0.00    | **1.00** | −0.33   | −0.50   | 0.10   |
| Accident rate     | −0.98   | 0.98    | −0.98   | −0.98   | **0.02** | 0.07   |
| Average speed (km/h) | −0.42   | −0.30   | −0.13   | **0.50** | −0.13   | 0.06   |
| Air-conditioned bus rate (%) | −0.79   | −0.74   | −0.67   | −0.55   | **0.46** | 0.03   |
| Peak utilization (%) | −0.48   | −0.30   | **0.10** | −0.05   | −0.34   | 0.02   |
| Peak-hour travel time (min) | −0.24   | **0.06** | −0.02   | −0.27   | −0.41   | 0.02   |
| Safety factor (10,000 km) | −0.90   | −0.87   | −0.83   | −0.74   | **0.26** | 0.01   |
| Transfer distance (m) | **0.36** | −0.36   | −0.57   | −0.68   | −0.74   | 0.00   |
| **Synthesis dependent degree K(M)** | −0.45   | −0.26   | **0.04** | −0.23   | −0.19   |        |

Note: numbers with boldface indicate the maximum dependent degree.

### Table 6: Results derived from different models.

| Model                  | Proposed extenics model | Previous extenics model | Fuzzy-based model |
|------------------------|-------------------------|-------------------------|-------------------|
| Evaluation performance level | Moderate               | Good                    | Moderate          |
| Maximum synthesis dependent degree | 0.04                   | −0.24                   |                   |
| Evaluation level index  | 3.47                    | 2.99                    |                   |
the government agencies shall initiate an effective policy to ensure fast, healthy, and sustainable development of public transit [48, 49].

It is worth noting that the data used for bus transit system evaluation are mainly from city government without considering users' opinions. As an immediate extension of this study, transit users will be interviewed, so that the perceived service quality may be considered in the evaluation process. Other factors such as the performance indices related to environmental protection and economic development will be all incorporated into a future model.

Abbreviations

| AHP | Analytic hierarchy process |
| BRT | Bus rapid transit |
| CR | Consistency ratio |
| CI | Consistency index |
| DEA | Data envelopment analysis |
| KQI | Key quality indicators |
| RI | Random index |
| SEM | Structural equation modelling |
| TOPSIS | Technique for order preference by similarity to an ideal solution |

Symbols

| $a_i$ | Lower bound of interval $x_{ij}$ |
| $A$ | Matrix of performance indices |
| $b_{ij}$ | Upper bound of interval $x_{ij}$ |
| $c_i$ | Description of index $i$ |
| $C$ | Vector of performance indices |
| $E$ | Evaluation matrix for fuzzy-based model |
| $g_i$ | Variation coefficient |
| $h$ | System performance level corresponding to $\max\{K_i(M)\}$ |
| $k$ | Constant in entropy |
| $k_i(x_i)$ | Dependent degree of index $i$ of system performance level $l$ |
| $K_i(M)$ | Synthesis dependent degree of system performance level $l$ |
| $K_i^*(M)$ | Weight of system performance level $l$ |
| $l_r$ | Level of system performance varying from 1 to 5 |
| $m$ | Number of performance indices |
| $M$ | Holistic performance of the study bus transit |
| $n$ | Number of alternatives for the same index |
| $P_{ij}$ | Ratio of index $i$ in alternative $j$ to the sum of the same index values |
| $Q$ | Number of bus stops in bus transit system |
| $R$ | Matter element |
| $S$ | Number of bus routes in bus transit system |
| $T$ | Number of buses in bus transit system |
| $u_{ij}$ | Scale of index $i$ comparing with index $j$ within subindicator basic capacity |
| $U$ | Pairwise comparison matrix |
| $w_{ij}$ | Local weights of index $i$ with respect to subindicator basic capacity |

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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