A New Class of Four-Dimensional $\mathcal{N} = 1$ Supergravity with Non-minimal Derivative Couplings

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ABSTRACT: In the $\mathcal{N} = 1$ four-dimensional new-minimal supergravity framework, we supersymmetrise the coupling of the scalar kinetic term to the Einstein tensor. This coupling, although introduces a non-minimal derivative interaction of curvature to matter, it does not introduce harmful higher-derivatives. For this construction, we employ off-shell chiral and real linear multiplets. Physical scalars are accommodated in the chiral multiplet whereas curvature resides in a linear one.

KEYWORDS: nonminimal derivative coupling, supergravity
1 Introduction

The most generic theory propagating a massless spin-2 and a scalar degree of freedom is not General Relativity minimally coupled to a scalar field (GRM). Indeed, Horndeski [1] proved that tensor-scalar theories with only second order differential equations are not restricted to GRM. Up to quadratic terms in matter fields and in four-dimensions, Horndeski showed that the most generic theories propagating a massless spin-2 and a spin-0 are

\[
\mathcal{L} = \mathcal{L}_{\text{GRM}} \pm \frac{1}{M_I^2} \mathcal{L}_I \pm \frac{1}{M_{II}^2} \mathcal{L}_{II} + \xi \mathcal{L}_{III},
\]

where

\[
\mathcal{L}_{\text{GRM}} = \frac{1}{2} \left[ M_P^2 R - \partial_\mu \phi \partial^\mu \phi \right],
\]

\[
\mathcal{L}_I = (M_I^2 \phi + \phi^2) R_{\text{GB}},
\]

\[
\mathcal{L}_{II} = G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,
\]

\[
\mathcal{L}_{III} = (M_{III}^2 \phi + \phi^2) R.
\]

and

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \quad R_{\text{GB}}^2 = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2.
\]
are the Einstein and Gauss-Bonnet tensors, respectively, $M_{(I,II)}, M_{\phi}^{I,II}$ are mass scales, $\xi$ a constant and finally $M_P$ is the Planck constant. That $\mathcal{L}_I$ leads to second order evolution equation follows easily from the fact that the Gauss-Bonnet combination is a total derivative in four-dimensions and it is linear in second order derivatives. Instead, $\mathcal{L}_{II}$ leads to second order equations as, in Hamiltonian ADM formalism [2], $G_{II}$ and $G_{III}$ contain only first time derivatives, since $G_{II}$ and $G_{III}$ are the Hamiltonian and momentum constraints.

While the supersymmetrization of $\mathcal{L}_I$ has been worked out in [3, 4] and $\mathcal{L}_{III}$ for the $\mathcal{N} = 1$ case in an arbitrary Jordan frame in [5], to our knowledge, the supersymmetric theory containing $\mathcal{L}_{II}$ was never found. It is the purpose of this work to construct the supersymmetric version of $\mathcal{L}_{II}$.

Apart from the obvious interest of studying the most generic supersymmetric theories avoiding Ostrogradski (higher derivatives) instabilities [6, 7], we note that the interaction (1.4) effectively describe part of the cubic graviton-dilaton-dilaton vertex in heterotic superstring theory and therefore appear in the low-energy 10D heterotic string effective action [8].\footnote{However, it should also be noted that this term has not been found in the heterotic quartic effective supergravity action constructed in [9].} Moreover, it has also been shown in [10], that there exists a field redefinition up to $\alpha'$ corrections, such as to generate the terms $\mathcal{L}_I, \mathcal{L}_{II}$ out of a stringy effective action.

From a more phenomenological point of view, the theory $\mathcal{L}_{II}$ plays a fundamental role in the so called “Gravitationally Enhanced Friction” (GEF) mechanism developed in [11–15]. There, thanks to the GEF, any steep (or not) scalar potential, can in principle produce a cosmic inflation for (relatively) small mass scale $M_{II}$. This is due to an enhanced friction produced by the Universe expansion acting on the (slow) rolling scalar field. Obviously then, the supersymmetrization of the GEF may notably enlarge the possibilities to find inflationary scenarios in supergravity and/or string theory. An additional motivation for studying supergravities with higher derivative terms, is related to the well known fact that they appear in the effective field theory action for the massless states of the superstring theory, after integrating out all superstring massive states.

In order to supersymmetrize $\mathcal{L}_{II}$ we will need to use the new-minimal supergravity framework of [16–19]. In fact, it has been already shown in [20] that, in the context of the standard (old) minimal-supergravity, the interaction (1.4) does not emerge and extra degrees of freedom appear.

All efforts to build higher-derivative supergravities in 4D are based on off-shell formulations. The latter are drastically different from the on-shell ones and, most importantly, they are not unique. This also happens in global supersymmetry where there are more than one off-shell formulations of an on-shell theory. We may recall for example the $\mathcal{N} = 1$ 4D theory where a scalar and a pseudoscalar may be completed off-shell by an auxiliary scalar field resulting in a chiral multiplet. Replacing the pseudoscalar by an antisymmetric two-form, a linear multiplet arises. In this case, there is no need of extra auxiliary fields as the off-shell degrees of freedom of an antisymmetric form field are more than those of a scalar. These degree of freedom are the exact number needed to complete the off-shell content of the linear multiplet. On-shell, of course, the two multiplets are the same.
This feature persists also in local supersymmetry where at least for the minimal \( \mathcal{N} = 1 \) 4D supergravity we are interested in, many off-shell formulations exist. The reason is that \( \mathcal{N} = 1 \) superfields carry highly reducible supersymmetry multiplets and additional constraints should be implemented for their truncation. Then the constraints together with the torsion and Bianchi identities are used to solve for the independent fields. As there are various ways implementing this procedure, there are also various off-shell formulations. Known examples are the off-shell supergravity formulation based on the 12 + 12 multiplet [21, 22] and the new minimal 12 + 12 multiplet [16–19]. There are also other non-minimal formulations like the one based on the non-minimally 20 + 20 [23–25] or 16 + 16 [26, 27] multiplets. Nevertheless, these formulations may be considered reducible in the sense that they can be mapped to the minimal \( \mathcal{N} = 1 \) supergravities coupled with extra multiplets. What is important to know though, is that it has been proven [28] that when no higher derivative terms are present, the off-shell formulations of minimal supergravities are equivalent. For example old-minimal and new minimal supergravities at the two-derivative level are connected by a duality transformation, where the chiral compensator of the former is mapped to a linear compensator of the latter. When higher derivatives are present, the duality transformation does not work any more due to derivatives of the compensator and the two formulations are \textit{not equivalent}. This is the key to find the supersymmetrization of \( \mathcal{L}_{II} \), as we shall see.

2 New Minimal \( \mathcal{N} = 1 \) 4D Supergravity

The simplest example of \( \mathcal{N} = 1 \) four-dimensional Poincaré supergravity is based on 12 bosonic and 12 fermionic off-shell degrees of freedom. These can be arranged into a multiplet in two ways. In the first one, the gravitational multiplet consists of

\[
e^{a}_{\mu}, \quad \psi_{\mu}, \quad b_{\mu}, \quad M
\]

and describes the dynamics of the so-called old minimal (standard) supergravity. Here, \( e^{a}_{\mu} \) is the vierbein, \( \psi_{\mu} \) is the gravitino, \( b_{\mu} \) is a vector, \( M \) a scalar and latin indices spans over the tangent space. As usual the vierbein should be used to convert tangent space indices \((a, b, ...)\) to world space indices \((\mu, \nu, ...)\) and throughout this work the tangent space metric is mostly plus (more on conventions can be found in the appendix).

In the new minimal supergravity instead, the multiplet consists of the vierbein \( e^{a}_{\mu} \) and its supersymmetric partner, the gravitino \( \psi^{a}_{\mu} \). In order to implement supersymmetry off-shell and the propagation of the physical degrees of freedom only, one has to also add auxiliary fields, as in the old minimal supergravity. However, in this case, the auxiliary fields are no longer a vector and a scalar but a 2-form \( B_{\mu\nu} \) with gauge invariance (B-gauge)

\[
\delta B_{\mu\nu} = \partial_{\mu} \xi_{\nu} - \partial_{\nu} \xi_{\mu},
\]

and a gauge vector \( A_{\mu} \) with associated R gauge invariance

\[
\delta A_{\mu} = \partial_{\mu} \phi.
\]
Thus, to wrap it up, the off-shell new minimal supergravity is based on the gravitational multiplet

\[ e^a_\mu, \quad \psi_\mu, \quad A_\mu, \quad B_{\mu\nu}. \]  

(2.4)

For more specific details on the structure of this theory the reader should consult [29].

It has been argued that the natural superspace geometry for four-dimensional \( \mathcal{N} = 1 \) heterotic superstring corresponds to the new minimal formulation of the \( \mathcal{N} = 1 \) supergravity [30–32]. This can be understood in terms of supervertices and superspace Bianchi identities of the vertex multiplets. In addition, it seems that there is a deep connection between the \( U(1) \) gauge symmetry of \( A_\mu \) in (2.3) above with the \( U(1) \) Kac-Moody symmetry of the \( \mathcal{N} = 2 \) superconformal algebra of the underlying superconformal theory.

This \( R \) symmetry is however anomalous (actually it is a mixed superconformal-Weyl-\( U(1) \) anomaly [33]). Nevertheless, by using the Green-Schwarz mechanism, the symmetry is restored at one loop thanks to the introduction of a matter linear multiplet together with supersymmetric Lorentz and Chern-Simons terms [34, 35].

In the new minimal supergravity, there exist three sets of chiral and Lorentz connections

\[ \omega^{\pm}_{abc} = \omega_{abc} \pm H_{abc}, \]
\[ A^{\pm}_\mu = A_\mu - H_\mu, \]
\[ A^{-}_\mu = A_\mu - 3H_\mu, \]  

(2.5)

where the following notation has been used

\[ H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}, \]
\[ + \frac{i}{8} \bar{\psi}_\mu \gamma_\nu \psi_\lambda + \frac{i}{8} \bar{\psi}_\nu \gamma_\lambda \psi_\mu + \frac{i}{8} \bar{\psi}_\lambda \gamma_\mu \psi_\nu, \]
\[ H^\mu = -\frac{1}{3!} \epsilon^{\mu\nu\kappa\lambda} H_{\nu\kappa\lambda}. \]  

(2.6)

The covariant derivatives in this formulation are therefore defined as

\[ D = d + \delta_L(\omega_{ab}) + \delta_A(A), \]
\[ D^\pm = d + \delta_L(\omega^{\pm}_{ab}) + \delta_A(A^{\pm}), \]  

(2.7)

with

\[ \delta_A(\phi) \Phi = i n \phi \Phi, \]
\[ \delta_L(\Lambda) \Phi = \frac{i}{2} S_{ab} \Lambda^{ab} \Phi, \]
\[ \omega^{\pm}_{ab} = \omega^{\pm}_{a\mu} dx^\mu, \quad A^{\pm}_\mu = A^{\pm}_{a\mu} dx^\mu. \]  

(2.8)

For the gravitino, for example, we have \( S_{ab} = \sigma_{ab}/2 \) and \( n = -\gamma_5/2 \). Here \( \delta_A(\phi) \), \( \delta_L(\Lambda) \) denote the \( U(1) \) R-symmetry and Lorentz transformations with parameters \( \phi \) and \( \Lambda \), respectively. Supercovariant derivatives \( \hat{D} \) are defined as usual and it should be noted for future reference that \( \hat{D}_a H_b = \hat{D}_a H_b \) and for any neutral vector \( \hat{D}_a V^a = \hat{D}_a V^a \).
The transformations of the supergravity multiplet fields under supersymmetry are [17, 18, 29]

\[
\delta e^a_\mu = \frac{i}{2} \gamma^a \psi_\mu ,
\]
\[
\delta \psi_\mu = - D^+_\mu \epsilon ,
\]
\[
\delta B_{\mu \nu} = \frac{i}{4} \bar{\epsilon} \gamma_{[\mu} \psi_{\nu]} ,
\]
\[
\delta A_{\mu}^- = \frac{i}{4} \bar{\epsilon} \gamma_\mu \gamma^5 \sigma^{ab} \psi_{ab} ,
\]

(2.9)

these transformations form an algebra along with general coordinate, Lorentz, chiral and B-gauge transformations. The supersymmetry parameter \( \epsilon \) transforms as \( \delta A \epsilon = -(i\gamma_5/2)\phi \epsilon \) under chiral transformations so that in two component notation \( \psi_\mu, \epsilon, \theta \) have chiral weight \( \frac{1}{2} \) and \( \bar{\psi}_\mu, \bar{\epsilon}, \bar{\theta} \) have chiral weight \( -\frac{1}{2} \). The chiral weight of the other components follows by these rules. The gravitino curvature used in (2.9) is defined in the Appendix A. The superspace derivatives are defined in the usual way [29, 36, 37] and the very structure of the new minimal supergravity is incarnated in their commutation and anti-commutation relations

\[
\{ \nabla, \bar{\nabla} \} = 2 i \nabla^- ,
\]
\[
[\nabla^-_a, \nabla^-_b] = \gamma_a \left( \frac{1}{2} T^{bc} S_{bc} + T_n - i \gamma_5 \bar{\nabla} \right) ,
\]
\[
[\nabla^-_a, \nabla^+_b] = i \frac{2}{S} S^{cd} R_{cdab} + i m F_{ab}^- - 2 E_{abc} \gamma^d \nabla^-_a + \frac{1}{2} \bar{T}_{ab} \nabla .
\]

(2.10)

Here \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the field strength of the gauge field \( A_\mu \), \( E_{abc} = -\epsilon_{abcd} E^d \) and the superfields \( E_a, T_{ab} \) and \( T \) will be defined in a moment.

3 Multiplets

A general multiplet of new minimal supergravity is

\[
V = (C, \chi, H, K, V_a, \lambda, D) .
\]

(3.1)

It is specified by the spin and the chiral weight

\[
\delta_L C = \frac{i}{2} \Lambda_{ab} S^{ab} C ,
\]
\[
\delta_A C = i m \phi C
\]

(3.2)

(3.3)
of its lowest component $C$, respectively. Frequently the two real scalars $H, K$ are traded for a complex $H + iK$ one. The supersymmetry transformations of this multiplet are

$$\delta C = -\frac{1}{2} \bar{\epsilon} \chi,$$

$$\delta \chi = \frac{1}{2} \left\{ i \hat{\nabla} - \frac{\gamma_5}{2} \{ H - \gamma_5 K \} \right\} \epsilon(-)^F,$$

$$\delta (H \pm iK) = -i \epsilon \frac{1}{2} \left\{ \gamma_5 \lambda - \frac{\gamma_5}{2} \gamma \right\} \epsilon(-)^F = - i \bar{\epsilon} \{ \gamma_5 \lambda + \hat{\nabla} \chi - 2 i \gamma_5 H \chi - i \lambda C \},$$

$$\delta V_a = - \frac{i}{2} \bar{\epsilon} \left\{ \gamma_a \lambda - \gamma_5 \hat{D}_a \chi - i \gamma_a H \chi \right\} + i \left\{ \frac{i}{4} \sigma_{ab} \hat{P}^{ab} + i \gamma_5 \hat{D} \right\} \epsilon(-)^F - \frac{i}{2} \bar{\epsilon} \{ \gamma_5 \lambda \},$$

$$\delta \lambda = - \left\{ \frac{i}{4} \sigma_{ab} \hat{P}^{ab} + i \gamma_5 \hat{D} \right\} \epsilon(-)^F - \frac{i}{2} \bar{\epsilon} \{ \gamma_5 \lambda \},$$

$$\delta D = - \frac{1}{2} \bar{\epsilon} \left\{ \hat{D} - \frac{1}{2} \hat{\nabla} C \right\} \epsilon(-)^F.$$

We have used the following definitions

$$\xi = \frac{i}{2} \psi_{ab} S^{ab} - i \gamma_5 \gamma \cdot r n,$$

$$\Delta = - \frac{i}{2} \hat{P}^{+} S^{ab} - \frac{i}{2} \hat{R} - n,$$

$$\hat{P}_{ab} = \hat{D}_a V_b - \hat{D}_b V_a - 2 H_{abc} V^d + \frac{i}{2} \bar{\psi}_{ab} \gamma_5 \chi,$$

and the factor $(-)^F$ accounts for the Fermi or Bose statistics of the first component. Note that $\xi$ and $\Delta$ only involve the spin and chiral generators of the first component. The properties of the general multiplet can be encoded in the following superfield representation

$$V = C - \bar{\theta} \chi - \frac{1}{2} \bar{\theta} \left\{ H - i \gamma_5 K + \gamma_5 V \right\} \theta,$$

$$+ i(\bar{\theta} \theta) \left\{ \gamma_5 \lambda + \frac{1}{2} \hat{D} \chi - \frac{3 i}{2} \gamma_5 H \chi - i \lambda C \right\} + \frac{1}{4} (\bar{\theta} \theta)^2 \left( D + \frac{1}{2} \hat{\nabla} C \right).$$

Constraint multiplets may be obtained by imposing appropriate constraints on the general multiplet $V$. Known representations include complex vector and real vector multiplets, gauge and chiral multiplets and, linear and real linear multiplets. We will discuss below the chiral and real linear multiplets as they are involved in our discussion.

### 3.1 Chiral Multiplet

A chiral multiplet $\Phi(A, \chi, F)$ is defined by the constraint $\nabla_\alpha \Phi = 0$ and its embedding in the general multiplet is given by

$$V(\Phi) = (A, \chi, F, -iF, -i \hat{D}_a A, -i \xi A, -i \Delta A).$$

The transformation rules are

$$\delta A = \frac{1}{2} \bar{\epsilon} \chi,$$

$$(-)^F \delta \chi = i \hat{D} \bar{\epsilon} A + F \epsilon,$$

$$\delta F = \frac{1}{2} \bar{\epsilon} (i \hat{D} + 2 \hat{H}) \chi + \bar{\epsilon} \xi A.$$
and its chiral superfield representation is

$$\Phi = A + \theta \chi + \theta^2 F.$$  \hspace{1cm} (3.9)

Up to field redefinitions one can always define the components of a superfield by projections. A common projection which we use throughout this work is \cite{36, 37}

$$\Phi| = A, \quad \nabla_\alpha \Phi| = \chi_\alpha, \quad -\frac{1}{4} \nabla^2 \Phi| = F,$$  \hspace{1cm} (3.10)

where $\nabla^2 \equiv \nabla^\alpha \nabla_\alpha$ and similarly $\bar{\nabla}^2 \equiv \bar{\nabla}^\dot{\alpha} \bar{\nabla}_{\dot{\alpha}}$. Moreover, from an arbitrary multiplet $V$ of weight $n$, one can form a chiral multiplet with weight $n + 1$ by the chiral projection operator

$$\Pi(V) = -\frac{1}{4} \bar{\nabla}^2 V,$$  \hspace{1cm} (3.11)

with components

$$\Pi(V)| = \bar{F}, \quad \nabla_\alpha \Pi(V)| = i(\bar{D} \bar{\chi} + 2i H \bar{\chi} - \lambda - i \xi C)_\alpha, \quad -\frac{1}{4} \nabla^2 \Pi(V)| = \frac{1}{2} \left\{ D - i(\hat{D}^- - 2i H) : (V + i \hat{D}^- C) + i \Delta C + \frac{i}{2} \bar{\psi}_{ab} \sigma^{ab} \bar{\chi} + 2 \bar{\xi} \bar{\chi} \right\}.$$  \hspace{1cm} (3.12)

### 3.2 Real Linear Multiplet

A real linear multiplet is defined by the constraints

$$L = L^*, \quad \nabla^2 L = \bar{\nabla}^2 L = 0$$  \hspace{1cm} (3.13)

and has zero chiral weight. The independent components of this multiplet are $C, \chi, V_a$, and the embedding in the general multiplet is

$$V = (C, \chi, 0, 0, V_a, \lambda, D),$$  \hspace{1cm} (3.14)

where the highest $\lambda$ and $D$ components depend on the lower ones

$$\lambda = -\gamma_5 \bar{D}^- \chi + 2i H \bar{\chi} - \frac{i}{2} \gamma_5 \bar{\psi}_{cd} S^{cd} C,$$

$$D = -\square^- C + 2H : V + \frac{i}{2} \bar{\psi}_{ab} (S_{ab} + \frac{\sigma_{ab}}{2}) \chi.$$  

Again one can define its independent components by projection as

$$L| = C, \quad \nabla_\alpha L| = \chi_\alpha, \quad \bar{\nabla}_\dot{\alpha} L| = \bar{\chi}_{\dot{\alpha}}, \quad -\frac{1}{2} |\nabla_\beta, \bar{\nabla}_{\dot{\beta}} | L| = V_{\beta\dot{\beta}}.$$  \hspace{1cm} (3.15)

It should be noted for future reference that one may define a new multiplet by acting with a superspace derivative on the general multiplet.
3.3 Curvature Multiplets

The gravitational curvature multiplets of this theory are the Einstein multiplet, $E_a$, and the Riemann multiplet, $T^a_{\alpha \beta}$. The irreducible pieces of the Riemann multiplet are the scalar curvature multiplet, $T^a_{\alpha}$, and the Weyl multiplet, $W^a_{\alpha \beta}$. The Einstein multiplet is a real linear multiplet (with chiral weight zero), which means that

$$E_a = E_a^*, \quad \nabla^2 E_a = \bar{\nabla}^2 E_a = 0,$$

(3.16)

and moreover, it satisfies the Bianchi identity

$$\nabla_a E^a = 0,$$

(3.17)

a property that only appears in the new minimal supergravity and it is of crucial importance for our results. Indeed, one can see that the independent components of the Einstein multiplet contain the Einstein tensor as the highest component. Specifically

$$E_a = \left( H_a, i\gamma_5 r^a, \frac{1}{2}(\hat{G}^+_a - \hat{F}^+_a) \right),$$

(3.18)

where $\hat{G}^+_a - \hat{F}^+_a = \hat{G}_{ab} - \hat{F}_{ab} - g_{ab}H_aH^d - 2H_aH_b$ with $\hat{F}^+_a$ the supercovariant dual of the field strength defined as $\hat{F}^+_{\mu \nu} = \frac{1}{2}\epsilon_{\mu \nu \kappa \lambda}F^{\kappa \lambda}$. Moreover, $r^a$ is the Rarita-Schwinger operator and $\hat{G}_{ab}$ is the supercovariant Einstein tensor [29]. The Riemann multiplet is chiral ($\nabla_a T^a_{\alpha \beta} = 0$) with components

$$T^a_{\alpha \beta} = \psi^a_{\alpha \beta} - \left( \frac{i}{2}\sigma^a_{\alpha \beta} \hat{\mathcal{R}}^+_{\alpha \beta} + i\hat{F}^+_{ab} \right) \theta + i\hat{D}^- \bar{\psi}^a_{\alpha \beta} \theta^2.$$

(3.19)

The rest curvature multiplets are defined as

$$T^a = \frac{1}{2}\sigma^a_{\alpha \beta} T^a_{\alpha \beta},$$

$$W^a_{\alpha \beta} = \frac{1}{24} (3\sigma^a_{\alpha \beta} \sigma^a_{\alpha \beta} + \sigma^a_{\alpha \beta} \sigma^a_{\alpha \beta}) T^{cd},$$

that is the scalar curvature multiplet and Weyl multiplet respectively. Finally, there also exists the gauge multiplet of the supersymmetry algebra, namely

$$V^a_{\alpha} = \left( A^a_{\mu} - \gamma_5 \gamma^\cdot r, -\frac{1}{2} \hat{\mathcal{R}}^- \right),$$

(3.20)

with $\hat{\mathcal{R}}^- = \hat{\mathcal{R}} + 6H_aH^a$, which we will use in the following.

4 Supersymmetric Actions

Chiral multiplets with chiral weight $n = 1$ can be used to form invariant actions by the $F$-density formula [18]

$$[\Sigma]_F = e \left\{ F + \frac{i}{2} \chi \sigma \cdot \bar{\psi} + \frac{i}{2} A \bar{\psi}^a \sigma^a \psi^b \right\}.$$

(4.1)
In superfield notation this can be written as

\[ [\Sigma]_F = \int d^2 \theta \mathcal{E} \Sigma, \quad (4.2) \]

with

\[ \mathcal{E} = e \left( 1 - i \theta \sigma \cdot \bar{\psi} + \frac{i}{2} \theta^2 \bar{\psi}^a \bar{\sigma}_{ab} \bar{\psi}^b \right). \quad (4.3) \]

The restriction \( n = 1 \) follows as \( d\theta \) has \( n = -\frac{1}{2} \) (\( d\theta \) has \( n = \frac{1}{2} \)). Furthermore, one can also build invariant actions from a multiplet with chiral weight zero, using the \( D \)-density formula

\[ [V]_D = e \left\{ D - \frac{\bar{\psi} \cdot \gamma_5 \lambda}{2} + \left( V_{\mu} + \frac{i}{2} \bar{\psi}_\mu \gamma_5 \lambda \right) \varepsilon^{\mu\rho\lambda} \partial_\nu B_{\rho\lambda} \right\} + \text{surface terms}. \quad (4.4) \]

We mention here that the \( F \) and \( D \) density formulas are related by \( [V]_D = 2 \Pi(V) F + \text{surface terms} \).

The action for Poincaré supergravity is obtained by the Fayet-Iliopoulos term of the chiral gauge multiplet (3.20) and reads

\[ \frac{1}{\kappa^2} L_{sugra}^{\text{on-shell}} = \frac{1}{\kappa^2} [V_p]_D = \frac{1}{\kappa^2} e \left( \frac{1}{2} R + \bar{\psi}^a r_a + 2 A_\mu H^\mu - 3 H_\mu H^\mu \right) + \text{surface terms}. \quad (4.5) \]

Variation of the action (4.5) with respect to \( A_\mu \) and \( B_{\mu\nu} \) gives

\[ H_\mu = 0 = \varepsilon^{\mu\nu\rho\sigma} \partial_\nu A_\rho \cdot A_\sigma. \quad (4.6) \]

Thus the vector \( H_\mu \) vanish and \( A_\mu \) reduces to a pure gauge and can therefore be set to zero by a gauge transformation. Finally then, the on-shell action of the new-minimal supergravity turns out to be

\[ S_{sugra}^{\text{on-shell}} = \frac{1}{\kappa^2} \int d^4 x e \left( \frac{1}{2} R + \bar{\psi}^a r_a \right), \quad (4.7) \]

which matches the on-shell \( \mathcal{N} = 1 \) old minimal supergravity [38, 39].

### 4.1 Non-Minimal Derivative Couplings

In order to construct non-minimal derivative couplings, we will introduce a chiral superfield \( \Phi \) with chiral weight \( n = 0 \). Since the kinetic term of a general chiral superfield is given by the \( F \)-term density formula (4.2), we will have in our case

\[ \mathcal{L}_{\text{kin}}^{(0)} = \int d^2 \theta \mathcal{E} \Phi \left[ -\frac{1}{4} \nabla^2 \Phi \right] + \text{h.c.}, \quad (4.8) \]

where \(-\frac{1}{4} \nabla^2\) is the chiral projection operator for the new minimal supergravity. In component form, and recalling that \( \Phi \) has a zero chiral weight \( n = 0 \), the bosonic part of the Lagrangian (4.8) is found to be

\[ \mathcal{L}_{\text{kin}}^{(0)} = 2e A^2 A^* + 2e FF^* - 2ie H^c (A \partial_c A^* - A^* \partial_c A). \quad (4.9) \]
We should couple now the chiral multiplet $\Phi$ to some curvature multiplet in order to get the the desired non-minimal coupling (1.4). As both $\Phi$ and $E_a$ have zero chiral weight, the term $\Phi^\dagger E^a \nabla_a^\dagger \Phi$ is a general superfield with zero chiral weight as well. Therefore $\nabla^2 [\Phi^\dagger E^a \nabla_a \Phi]$ is a chiral superfield with chiral weight $n = 1$ and thus the superspace Lagrangian

$$\mathcal{L}^{(0)}_{int} = \int d^2 \Theta \left\{ -\frac{i}{4} \nabla^2 \left[ \Phi^\dagger E^a \nabla_a \Phi \right] \right\} + \text{h.c.} \quad \text{(4.10)}$$

is supersymmetric. Now, (4.10) can be expanded as

$$\mathcal{L}^{(0)}_{int} = \frac{i}{16} e \nabla^2 \nabla^2 \left[ \Phi^\dagger E^a \nabla_a \Phi \right] + \text{h.c.} = A + B + C, \quad \text{(4.11)}$$

where

$$A = \frac{i}{16} e \left[ \left( \nabla^2 \nabla^2 \Phi^\dagger \right) E^a \nabla_a \Phi \right] + \text{h.c.},$$

$$B = \frac{i}{16} e \left[ \left( \nabla^2 \Phi^\dagger \right) E^a \left( \nabla^2 \nabla_a \Phi \right) \right] + \text{h.c.},$$

$$C = \frac{i}{16} e \left[ 4 \left( \nabla_\gamma \nabla_\delta \Phi^\dagger \right) \left( \nabla^\gamma \nabla^\delta E^a \right) \left( \nabla_a \Phi \right) \right] + \text{h.c.} \quad \text{(4.12)}$$

Keeping only bosonic fields, after a straightforward calculation we find

$$A = 2eH^b \partial_b A^* H^a \partial_a A + i e \Box A^* H^a \partial_a A + \text{h.c.}$$

$$B = -\frac{i}{4} e F^* H^a \left( 8iF_{H^a} - 4D_a^\dagger F \right) + \text{h.c.}$$

$$C = \frac{1}{2} e \partial^d A^* \partial^d A \left( G_{dc} - \eta_{dc} H^a H_a - 2H_d H_c \right) + i e \partial_b A^* \partial_c A D^b H^c + \text{h.c.} \quad \text{(4.13)}$$

In the above formulas we used that $D_a^\dagger F = \partial_a F - i A_a^* F$ with $A_a^* = A_a - 3H_a$, since $F$ has a chiral weight $n_F = -1$. Additionally, in the above derivation one should use the helpful splitting $\nabla^\gamma \nabla^\delta E^a = \frac{1}{2} \left\{ \nabla^\gamma, \nabla^\delta \right\} E^a + \frac{1}{2} \left\{ \nabla^\gamma, \nabla^\delta \right\} E^a$.

We see that the desired nonminimal derivative coupling with the Einstein tensor indeed appears in $C$. Thus, the bosonic part of the interaction reads

$$\mathcal{L}^{(0)}_{int} = e G^{ab} \partial_a A \partial_b A^* + 2e FF^* H^a A_a - 2e FF^* H^a H_a + i e H^a \left( F^* \partial_a F - F \partial_a F^* \right)$$

$$- e \partial_b A \partial^b A^* H^a A_a - 2e H^a \partial_a H^a \partial_b A^* - i e H_c \left( \partial_b A^* D^c \partial^b A - \partial_b A D^c \partial^b A^* \right) \quad \text{(4.14)}$$

In summary, assembling the Lagrangians (4.5, 4.8, 4.10) we find that the bosonic sector of the theory is

$$\mathcal{L}_0 = \frac{1}{\kappa^2} \mathcal{L}_{sagra} + \frac{1}{2} \mathcal{L}_{kin} + w^2 \mathcal{L}_{int}$$

$$= \frac{1}{\kappa^2} \left[ \frac{1}{2} e \mathcal{R} + 2eH^a A_a - 3eH^a H_a \right]$$

$$+ e A \Box A^* + e FF^* - i e H^c \left( A \partial^c A^* + A^* \partial^c A \right)$$

$$+ e H^a \left( F^* \partial_a F - F \partial_a F^* \right) - e \partial_b A \partial^b A^* H_a H^a$$

$$+ 2e H^a \partial_a H^b \partial_b A^* - i e H_c \left( \partial_b A^* D^c \partial^b A - \partial_b A D^c \partial^b A^* \right) \quad \text{(4.15)}$$
where we have introduced the dimensionful parameter $w^2 = \pm M_{11}^{-2}$ and $\kappa^2 = M_{P}^{-2}$.

We may now integrate out the auxiliary fields to find the on-shell action. For $w^2 > 0$ we may define

$$V^a = A^a \left(1 + \kappa^2 w^2 F F^* \right) + \frac{\kappa^2}{2} \left(i A^a \partial^a A - i A^a \partial^a A^* - i w^2 F \partial^a F^* + i w^2 F^* \partial^a F - i w^2 \partial_b A^a D^b \partial^b A^* + i w^2 \partial_b A^a D^b \partial^b A^* \right),$$  \hspace{1cm} (4.16)

in terms of which (4.15) is written as

$$e^{-1} L_0 = \frac{1}{\kappa^2} \left[ \frac{1}{2} \mathcal{R} + 2 V^a H_a - 3 H^a H_a \right] + A \square A^* + FF^* + w^2 \left[ G^{ab} \partial_b A^a \partial_a A - 2 F F^* H^a H_a - \partial_b A^a \partial^b A^* H_a H^a + 2 H^a \partial_b A^a H^b \partial_a A^* \right].$$  \hspace{1cm} (4.17)

It is important to notice here that since $A, F$ have chiral weights $n = 0, -1$, respectively, $V_\mu$ transforms under the $U(1)$ symmetry as it should, i.e.,

$$\delta V_\mu = \partial_\mu \phi$$

and thus it is physically equivalent to $A_\mu$.

To find the on-shell action, we should eliminate the auxiliary fields $V_\mu, B_{\mu \nu}, F$. This can be done exactly in the same way as in the pure supergravity case (4.5) where we find $V_\mu = H_\mu = 0$. Similarly, the elimination of the auxiliary $F$ of the chiral superfield is straightforward and the bosonic part of the supersymmetric Lagrangian (4.10) turns out to be

$$e^{-1} L_0 = \frac{1}{2\kappa^2} \mathcal{R} + A \square A^* + w^2 G^{\mu \nu} \partial_\mu A^a \partial_\nu A^*.$$

(4.19)

There is a difference when $w^2 < 0$. Variation with respect to $A_a$ gives the following equation

$$\left( \frac{1}{\kappa^2} + w^2 F F^* \right) H_a = 0.$$  \hspace{1cm} (4.20)

For $w^2 > 0$ the only solution is $H_a = 0$ and we may define $V^a$ in (4.16) as described above. However, for $w^2 < 0$, there are two solutions: i) a supersymmetric solution $H_a = 0$ and ii) a non-supersymmetric one $FF^* = \frac{1}{\kappa^2 w^2}$. For the supersymmetric solution, we arrive at the bosonic part (4.19) of our supersymmetric theory. On the other hand for the non-supersymmetric solution, $A^a$ cannot anymore be traded for $V^a$. Moreover, it generates a cosmological constant as expected, introducing at the same time higher derivatives. Indeed, in this case, the last term of (4.15) would not vanish leading to harmful higher-derivative interactions.

The properties of the theory (4.19) have been studied in [40, 41]. In particular, in [41] the scalar $A$ has been dubbed as the *Slotheon* for the reason that, generically, for a given
kinetic energy, its time derivative is smaller than the same calculated for a canonical scalar field. This again proves the usefulness of this theory for Inflation, where, in order to get an accelerated expansion of the primordial Universe, the scalar field should have a very small time derivative. In [41] it has also been proven that spherically symmetric Black Holes cannot have slotheonic hairs and, finally, it has been conjectured that this theory does not violate the no-hair theorem generically.

We should note that the Lagrangian (4.10) can easily be generalized to describe more general non-minimal couplings of the form $V(A, A^*)G^{\mu\nu}\partial_\mu A \partial_\nu A^*$. Indeed, we may employ a holomorphic function $W(\Phi)$ as follows

$$L(W) = \int d^2 \Theta E \left\{ -\frac{i}{4} \nabla^2 \left[ \bar{W}(\Phi^\dagger) E^a \nabla_a W(\Phi) \right] \right\} + \text{h.c.} \ . \ (4.21)$$

The computation of (4.21) goes straightforward as in the previous case and the result, after combining with (4.5,4.8,4.10) and by doing an appropriate shifting of the $U(1)$ vector, turns out to be

$$e^{-1}L(W) = \frac{1}{\kappa^2} \left[ \frac{1}{2} \mathcal{R} + 2V^a H_a - 3H^a H_a \right] + A^a A^* + FF^*$$

$$+ w^2 \left| \frac{\partial W}{\partial A} \right|^2 \left( G^{ab} \partial_b A^* \partial_a A - 2 FF^* H^a H_a \right.$$

$$- \partial_b A \partial^b A^* H^a + 2 H^a H^b \partial_a A \partial_b A^* \right) , \ (4.22)$$

where $W$ is the lowest component of $W$.

Again, field equations for $V^\mu$ and $H^\mu$ force the latter to vanish and the former to be a pure gauge. With this in mind, the bosonic part of the Lagrangian, after elimination of the auxiliary fields is

$$e^{-1}L(W) = \frac{1}{2\kappa^2} \mathcal{R} + A^a A^* + w^2 \left| \frac{\partial W}{\partial A} \right|^2 G^{\mu\nu} \partial_\mu A^* \partial_\nu A \ . \ (4.23)$$

An obvious question concerns possible potential terms. Due to the requirement of R-invariance, one cannot use the F-density formula (4.2) to write general Lagrangians, unless the F-density has a total chiral weight of $n = 1$. For the neutral chiral multiplet we have used to construct our theory, it is not possible to write an R-symmetric potential term, unless new chiral fields are introduced. However, one can introduce explicit soft supersymmetry breaking terms of the form $m^2 A A^*$, as potential for the neutral scalar.

A second question is why the neutral $n = 0$ prescription in (4.10) is fundamental to avoid higher-derivatives. An R-charged multiplet with $n \neq 0$ would give charge to the scalar $A$. In this case, $A$ would be minimally coupled to the $U(1)$ gauge field $A_\mu$ inducing quadratic terms for the gauge field. Moreover, kinetic terms for both $A_\mu$ and $H_\mu$ will appear. In this case, $A_\mu$ and $H_\mu$ could not be eliminated algebraically anymore. Specifically, the equation for $A_\mu$ would read $H_\mu \sim \partial_\mu A + \ldots$. It is then clear that the elimination of $H_\mu$ would produce quartic derivatives of the scalar $A$ and consequently a higher derivative theory from, for example, the last term of (4.15). Therefore, only for a neutral $n = 0$ chiral field a theory with no harmful higher derivatives can be obtained. However, for completeness, in
Appendix B we present the bosonic sector of a general R-charged chiral multiplet of chiral weight $n$.

Finally, we note that in the fermionic sector of the theory, among the various fermionic interactions that arise, the term

$$L_\chi = -w^2 e^{-\frac{i}{4} \chi^a \tilde{G}^{ab} \sigma_{b\alpha\dot{\alpha}} \tilde{D}_a \tilde{\chi}^{\dot{\alpha}} - i w^2 e\sigma^d D_d A^a \chi \bar{\chi}^a}.$$  \hspace{1cm} (4.24)

is the direct supersymmetric partner of the Einstein coupling in (4.19) needed to cancel scalar supersymmetry variations of $L_{II}$. The first term in (4.24) was for first time introduced in non supersymmetric models in [42]. In [42] it has been shown that each time couplings of the form (4.24) or (4.19) are introduced, dependently upon the scale $w$, fields get dynamically localized around domain walls.

5 Conclusions

General Relativity (GR) minimally coupled to scalars is not the most generic tensor-scalar theory propagating only a massless spin-2 and a spin-0 field. In fact, non-minimally coupled theories of curvatures to matter fields can also be constructed with these properties. Such theories, which do not produce any higher derivatives in the equation of motion and, at the same time, maintain the GR constrains able to reduce the graviton degree of freedom to only two polarization (in four dimension), are found in [1]. These theories have non-derivative and derivative couplings of matter to curvatures.

The supergravity extensions of the non-derivative coupled theories such as $L_{III}$ has been already constructed in the literature [5]. However, non-minimal derivative coupled supergravities to matter fields, without extra propagating modes, are restricted to the Gauss-Bonnet interactions $L_I$. Here we focused on the supersymmetrization of the non-minimal derivative coupled Lagrangian, $L_{II}$. This was achieved in the framework of new-minimal supergravity by employing a chiral multiplet and the linear curvature multiplet. The use of the linear multiplet is in fact crucial as, a construction of higher dimensional derivative operators in the (old-minimal) $N=1$ supergravity context, would only lead to a theory with extra (higher-derivatives) propagating degrees of freedom [20].

A theory described by (4.19) or, more generically, (4.23), may have many phenomenological interesting properties. The first one is that, each time a domain wall is present in the theory, dependently upon the scale $w$, the scalar field gets dynamically localized around the domain wall itself [42]. In fact, one may consider $L_{II}$ as a field theoretical realization of the quasi-localization mechanism of [43]. A second, perhaps more important, phenomenological aspect is related to Inflation. Whenever the background Einstein tensor is larger than the mass scale $w^{-2}$, no matter what potential is driving $A$, Inflation is naturally produced without exceeding the perturbative cut-off scale of the theory, which is below the Planck scale as it should be for a ghost-free theory [44]. This is due to an enhanced gravitational friction acting on the evolving scalar field and sourced by the Universe expansion itself [11–15]. We therefore believe that the supersymmetrization of the $L_{II}$ might open new possibilities for exploring inflation in supergravity/string theory.
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A Conventions

Throughout the work we use a Minkowski metric with signature (-,+,+,+), and the fully antisymmetric tensor is taken as $\varepsilon_{0123} = +1$. The Dirac matrix conventions are $\{\gamma_a, \gamma_b\} = -2g_{ab}$, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, while we use $\sigma_{ab} = \frac{1}{2}[\gamma_a, \gamma_b]$, and $\bar{\psi} = \psi^\dagger C$.

In a Majorana representation $C = \gamma^0$ and the Majorana condition is $\psi = \psi^\dagger$. The two-component spinor formalism is derived from the following chiral representation of the Dirac matrices,

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma_a = \begin{pmatrix} 0 & \sigma_a \\ \bar{\sigma}_a & 0 \end{pmatrix},$$

$$\sigma_a = (1, \vec{\sigma}), \quad \bar{\sigma}_a = (1, -\vec{\sigma}),$$

$$\psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}_\dot{\alpha} \end{pmatrix}, \quad \bar{\psi} = (-\psi^\alpha, -\bar{\psi}^\dot{\alpha}). \quad (A.1)$$

The gravitino curvature is given by

$$\psi_{\mu\nu} = D^+_\mu \psi_\nu - D^+_\nu \psi_\mu, \quad \psi_{ab} = e^a_\mu e^b_\nu \psi_{\mu\nu} \quad (A.2)$$

and the Rarita-Schwinger operator is

$$r^a = \frac{1}{4}\gamma_5\gamma_b \varepsilon^{bade} \psi_{de}. \quad (A.3)$$

Finally, the Ricci scalar, the Ricci tensor and the Riemann curvature are given by

$$\mathcal{R} = \eta^{ca} R_{ca},$$

$$R_{ca} = R_{amb} e^m_c e^n_c,$$

$$R_{amb} = \partial_a \omega^b_{ma} - \partial_b \omega^b_{na} + \omega^c_{ma} \omega^b_{nc} - \omega^c_{na} \omega^b_{mc}. \quad (A.4)$$
B Lagrangian for non-zero chiral weight

The bosonic part of the Lagrangian for a general chiral weight \( n \) reads:

\[
e^{-1} \mathcal{L}_n = \frac{1}{\kappa^2} \left[ \frac{1}{2} \mathcal{R} + 2H^a A_a - 3H^a H_a \right] + A\Box^{-} A^* + FF^* - \frac{1}{2} n A A^* (\mathcal{R} + 6H^a H_a) - iH^c (A D^c_\alpha A^* - A^* D^\alpha A) + n^2 \left\{ iH^b \left[ \Box^{-} A^* D^\beta_b A - \Box^{-} A D^\beta_b A^* \right] + i\frac{1}{2} n H^b (\mathcal{R} + 6H^a H_a) (A D^\beta_b A^* - A^* D^\beta_b A) + 4H^c D^\beta c A^* H^b D^\beta b A + iD^d A^* D^\alpha d A \left( D^d H^a - D^a H^d \right) + D^d A^* D^\alpha d A \left( G^{da} - g^{da} H^b H_b - 2H^d H^a \right) + iH^a \left( F^\alpha d A^* F - F^\alpha d F^a \right) + 4FF^* H^a H_a + \frac{i}{2} n H^d R \left( AD^\alpha d A^* - A^* D^\alpha d A \right) + i n F^\alpha d H_a (A D^\alpha d A^* - A^* D^\alpha d A) + in H_a (D^\alpha d H^a) e^{\beta d a} (AD^\beta d A^* - A^* D^\beta d A) - \frac{1}{4} n H_a * F_{l b} e^{\beta d a} (AD^\beta d A^* + A^* D^\beta d A) - n H^l (D^l H^d) (AD^\alpha d A^* + A^* D^\alpha d A) + n H^b (D^d H_a) (AD^\alpha d A^* + A^* D^\alpha d A) - \frac{1}{4} n A A^* (\mathcal{R} + 6H^a H_a)^2 + \frac{1}{2} n A A^* * F_{d e} * F^f c - n A A^* e^{\beta d a} * F_{k a d} D^f d H^c + n A A^* (D^\beta d H^a) \left( D^\alpha d H^c - D^c H^d \right) \right\} . \quad (B.1)
\]

It is clear that, for \( n \neq 0 \), the vector \( A_a \) of field strength \( F^{ab} \) becomes dynamical and therefore, as discussed in the text, cannot be removed by a gauge transformation.

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