Measurement method for the nuclear anapole moment of laser trapped alkali atoms

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Abstract

Weak interactions within a nucleus generate a nuclear spin dependent, parity violating electromagnetic moment, the anapole moment. We analyze a method to measure the nuclear anapole moment through the electric dipole transition it induces between hyperfine states of the ground level. The method requires tight confinement of the atoms to position them at the anti-node of a standing wave Fabry Perot cavity driving the anapole-induced micro-wave E1 transition. We explore the necessary limits in the number of atoms, excitation fields, trap type, interrogation method, and systematic tests necessary for such measurements in francium, the heaviest alkali.

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Zel’dovich postulated in 1957 that the weak interactions between nucleons would generate a parity violating, time reversal conserving moment called the anapole moment \cite{1}. Flambaum and Khriplovich calculated the effect it would have in atoms \cite{2}. Experiments in thallium gave a limit for its value \cite{3}, and it was measured for the first time with an accuracy of 14\% through the hyperfine dependence of atomic parity nonconservation (PNC) in cesium \cite{4, 5}.

We present in this paper a measurement strategy of the nuclear anapole moment by direct excitation of the microwave electric dipole (E1) transition between the ground hyperfine levels in a chain of isotopes of an alkali atom. Alkali atoms are the best understood quantitatively in their electronic properties associates with PNC. The precision of the Cs PNC experiments has required more detailed studies of the nuclear structure \cite{6}. Measurements over a chain of isotopes offer the advantage that they can focus on the differences appearing as the number of neutrons changes. This task has been accomplished well by theory (see for example Ref. \cite{7}) for the hyperfine anomaly measurements in Fr.

Current plans at the Isotope Separator and Accelerator (ISAC) at TRIUMF, in Vancouver Canada, should provide access to all the neutron deficient long lived isotopes of Fr with lifetimes above 30 s and to a similar number of the neutron rich isotopes, a sufficient variety to give a difference in number of neutrons of more than 10. The expected production rates should be at least two orders of magnitude larger than those obtained at Stony Brook, the leading place for study of Fr \cite{8}.

A measurement of the anapole moment in a chain of isotopes will provide information about neutral weak currents in the nucleus. The measurements can also give information on the nuclear structure and its changes as the number of neutrons increases \cite{9, 10}.

The E1 transition between hyperfine levels is parity forbidden, but becomes allowed by the anapole induced mixing of levels of opposite parity. The general approach has been suggested in the past \cite{11, 12, 13, 14, 15, 16, 17, 18}. We would place many atoms inside a microwave Fabry-Perot cavity and hold them in a blue detuned dipole trap. The atoms would interact with the microwave field and with a Raman field generated by a pair of laser beams, in the presence of a static magnetic field. We would confine the atoms to the node (anti-node) of the magnetic (electric) microwave field to drive only an E1 transition.
between hyperfine levels. The atoms would start in the lower hyperfine level, with the signal proportional to the population of atoms in the upper hyperfine level after the excitation. The interference with a Raman transition would give a signal linear in the E1 transition.

Recent work related to time-reversal invariance tests in atomic traps [19, 20], points to the many potential advantages of combining traps with tests of fundamental symmetries, but also highlights the potential systematic errors present in such measurements, making a careful evaluation of the method prior to its implementation necessary. We focus our study primarily on isotopes of francium, the heaviest of the alkali atoms [8], in an optical dipole trap, where the effect is expected to be large.

The organization of the paper is as follows: Section II gives the theoretical background for the nuclear anapole moment, section III explains the proposed measurement method, section IV presents an analysis of noise sources and systematic effects, and section V contains the conclusions.

II. THEORETICAL BACKGROUND

The exchange of weak neutral currents between electrons and nucleons constitute the main source of parity violating atomic transitions. The currents are of two kinds, depending on whether the electron or the nucleon enters as the axial vector current. The corresponding terms in the Hamiltonian differ on their dependence on the nuclear spin. The part independent of the nuclear spin is generally the dominant contribution in atomic PNC. This is not the case for the present work, where we consider transitions between hyperfine levels of the ground state, and the contribution from the nuclear spin independent part is zero [21]. The Hamiltonian for the spin dependent part in the shell model with a single valence nucleon of unpaired spin is given by [22]

\[ H = \frac{G}{\sqrt{2}} \frac{KI \cdot \alpha}{I(I + 1)} \kappa_i \delta(r), \]  

(1)

where \( G = 10^{-5} \) m\(_p\)\(^{-2}\) is the Fermi constant, m\(_p\) is the proton mass, \( K = (I+1/2)(-1)^{l+1/2-l} \), \( l \) is the nucleon orbital angular momentum, I is the nuclear spin, \( \alpha \) are Dirac matrices, and \( \kappa_i \) is the interaction constant, with \( i = p, n \) for a proton or a neutron. The terms proportional to the anomalous magnetic moment of the nucleons and the electrons have been neglected.

The interaction constant is given by [22]
\[ \kappa_i = \kappa_{a,i} = \frac{K - 1/2}{K} \kappa_{2,i} + \frac{I + 1}{K} \kappa_{Q_W}, \]  

(2)

with \( \kappa_{2,p} = -\kappa_{2,n} = -1.25(1 - 4 \sin^2 \theta_W)/2 \), corresponding to the tree level approximation, with \( \sin^2 \theta_W \sim 0.23 \) the Weinberg angle. Equation 2 has two corrections, \( \kappa_{a,i} \) the effective constant of the anapole moment, and \( \kappa_{Q_W} \) that is generated by the nuclear spin independent part of the electron nucleon interaction together with the hyperfine interaction. The three parts of this interaction constant can be traced to different ways in which the weak interacting vector boson \( Z^0 \) appears in the Feynman diagrams. The first one (the anapole) correspond to vertex corrections due to weak hadronic interactions on the nuclear side of the electromagnetic interaction coupled to the electron through a virtual photon. The second one takes the direct effect of a \( Z^0 \) exchange between the electron vector current and the nuclear axial current. The last one is the simultaneous exchange of a \( Z^0 \) and a photon modifying the hyperfine interaction. Flambaum and Murray showed that \( \kappa_{Q_W} = -\frac{1}{3} \left( \frac{Q_N}{A} \right) \mu_N \frac{\alpha A^{2/3}}{m_p r_0} \)

(3)

where \( \alpha \) is the fine structure constant, \( \mu_i \) and \( \mu_N \) are the magnetic moments of the external nucleon and of the nucleus respectively, \( r_0 = 1.2 \text{ fm} \), \( A \) is the atomic mass number, \( Q_W \) is the weak charge, and \( g_i \) gives the strength of the weak nucleon-nucleus potential with \( g_p \sim 4 \) for a proton and \( 0.2 < g_n < 1 \) for a neutron.\[23\] The anapole moment is the dominant contribution to the interaction in heavy atoms, for example, \( \kappa_{a,p}/\kappa_{Q_W} \simeq 15 \) for \( ^{209}\text{Fr} \). We will assume from now on that \( \kappa_i = \kappa_{a,i} \).

A. The anapole moment

The anapole moment of a nucleus is a parity non-conserving (PNC), time reversal conserving moment that arises from weak interactions between the nucleons (see the review by Haxton and Wieman \[24\].) It can be detected in a PNC electron-nucleus interaction, and reveals itself in the spin dependent part of the PNC interaction. Wood et al. \[4, 5\] measured the anapole moment of \( ^{133}\text{Cs} \) by extracting the dependence of atomic PNC on the hyperfine levels involved.
The anapole moment classically is defined by (see for example [25])

\[
a = -\pi \int d^3r r^2 J(r),
\]

(4)

with \(J\) the electromagnetic current density [26]. The nuclear anapole moment in francium arises mainly from the weak interaction between the valence nucleons and the core. Flambaum, Khriplovich and Sushkov [2] estimate the anapole moment from Eq. 4 for a single valence nucleon to be

\[
a = \frac{G e}{e\sqrt{2} j (j+1)} \kappa_{a,i} \vec{j} = C_{an}^i \vec{j},
\]

(5)

where \(j\) is the nucleon angular momentum and \(e\) the electron charge. The calculation assumes a homogeneous nuclear density, and a core with zero angular momentum, leaving the valence nucleon carrying all the angular momentum.

The measurement of the anapole moment gives information on the weak nucleon-nucleon interactions. A measurement of the anapole moment in a chain of isotopes would provide a separation of the anapole moment due to the valence proton or neutron.

**B. Calculations of the anapole moment of francium isotopes**

We use Eqs. 3 and 5 to estimate the anapole moment of five light francium isotopes with radioactive lifetimes longer than one minute [7]. The unpaired valence proton generates the anapole moment in even-neutron isotopes, whereas in the odd-neutron isotopes both the unpaired valence proton and neutron participate. Francium has an unpaired \(h_{9/2}\) proton for all the isotopes and a \(f_{5/2}\) neutron for the odd-neutron isotopes around \(210\)Fr. The protonic and neutronic contributions add vectorially to generate the anapole moment:

\[
a = \frac{C_{p}^i \vec{j}_p \cdot \vec{I} + C_{n}^i \vec{j}_n \cdot \vec{I}}{I^2} = \frac{G}{e\sqrt{2} I(I+1)} \kappa \vec{I},
\]

(6)

with \(C_{i}^an \vec{j}_i\) the anapole moment for a single valence nucleon \(i\) (proton or neutron) as given by Eq. 5 (\(j_p = 9/2, j_n = 5/2\)). Equation 6 defines the coupling strength of the total anapole moment (\(\kappa_a\)) resulting from adding the valence proton and neutron. Figure 1 shows the predicted values of \(\kappa_a\) for a string of francium isotopes [7] using \(g_n = 1\).
FIG. 1: Anapole moment effective constant for different isotopes of francium.

C. Perturbation theory

The anapole moment induces a small mixing of electronic states of opposite parity. The effect of the anapole moment Hamiltonian on the ground state hyperfine levels according to first order perturbation theory is

$$|sFm⟩ = |sFm⟩ + \sum_{F'm'} \frac{⟨pF'm'|H_a|sFm⟩}{E_{p'} - E_s} |pF'm⟩,$$

where $E_p$ and $E_s$ are the energies of the $p$, and $s$ states respectively, and

$$H_a = |e|\alpha \cdot a \delta(r),$$

is the anapole moment Hamiltonian from Eq. 1, with $a$ the anapole moment from Eq. 5. The matrix element in Eq. 7 gives

$$⟨pF'm'\mid H_a \mid sFm⟩ = i \frac{ξZ^2R}{(\varrho_s \varrho_p)^{3/2}} \frac{2γ + 1 (I + 1/2)κ_α Ry}{3/I(I + 1)} \times (F(F + 1) - I(I + 1) - 3/4)δ_{F,F'}δ_{m,m'};$$

with $ξ = Gm_e^2α^2/\sqrt{2π} = 3.651 \times 10^{-17}$, $m_e$ the electron mass, $Z$ the atomic number, $\varrho_s$ and $\varrho_p$ the effective principal quantum number for the $s$ and $p$ electronic states, $γ = \sqrt{(J + 1/2)^2 - Z^2α^2}$, $J$ the electron total angular momentum, and $Ry$ the Rydberg. The relativistic enhancement factor $R$ is given by

$$R = 4(a_0/2Zr_0)^22γ/Γ^2(2γ + 1),$$

with $a_0$ the Bohr radius, and $r_0 = r_0A^{1/3}$. 
The anapole moment mixes only states with the same $F$ and $m$, and the mixing grows as $Z^{8/3} R$. For the $^{209}\text{Fr}$ ground state, we obtain

$$|sFm\rangle = |sFm\rangle - i \frac{5.9 \times 10^{-13}}{9 \times 10^{-13}} \kappa_a \times (F(F + 1) - 25.5)|pFm\rangle.$$  

(11)

The mixing coefficient is imaginary due to time reversal symmetry. In practice, the mixing would be measured through the $E1$ transition amplitude $A_{E1}$ (Eq. 16) it induces between two hyperfine levels. The effect in francium is 11 times larger than in cesium [27].

**III. PROPOSED MEASUREMENT STRATEGY**

High efficiency magneto-optical traps (MOT) for francium atoms on line with an accelerator have been demonstrated [28]. Their performance and reliability matches the needs of the current proposed measurement strategy. Atoms captured on a first trap would then be transferred to a second MOT in a separated chamber. We would load the atoms into a dipole trap located at the electric field anti-node of a standing wave in a microwave Fabry-Perot cavity. We would optically pump them into a single Zeeman sublevel, and prepare a coherent superposition of the hyperfine ground levels with a Raman pulse of amplitude $A_R$ and duration $t_R$. Simultaneously we would drive the $E1$ transition of amplitude $A_{E1}$ with the cavity microwave field, and measure the population in the upper hyperfine level (normalized to the total number of atoms $N$) using a cycling transition. The population in the upper hyperfine level at the end of each sequence would be

$$\Xi_{\pm} = N|c_e|^2 = N \sin^2 \left(\frac{(A_R \pm A_{E1}) t_R}{2 \hbar}\right),$$  

(12)

where $c_e$ is the upper hyperfine level amplitude. The sign depends on the handedness of the coordinate system defined by the external fields, as explained in the next section. The signal for the measurement:

$$S = \Xi_+ - \Xi_- = N \sin \left(\frac{A_R t_R}{\hbar}\right) \sin \left(\frac{A_{E1} t_R}{\hbar}\right) \approx N \sin \left(\frac{A_R t_R}{\hbar}\right) \left(\frac{A_{E1} t_R}{\hbar}\right),$$  

(13)
FIG. 2: Schematic setup of the proposed apparatus. The microwave cavity axis is along the $y$-axis. The microwave electric field inside the cavity oscillates along the $x$-axis. The two Raman laser beams are polarized along the $x$-axis and $z$-axis, respectively. The microwave magnetic field and the static magnetic field are both directed along the $z$-axis. A dipole trap (not shown) holds the atoms at the origin that coincides with an anti-node of the microwave electric field.

would be the difference between populations in the upper hyperfine level for both handedness. The last step assumes a small $A_{E1}$, the quantity proportional to the anapole moment constant $\kappa_a$.

A. Apparatus setup

Figure 2 shows a diagram of the proposed apparatus. The atoms would be placed inside a microwave Fabry-Perot cavity at the electric field anti-node, confined in a blue detuned dipole trap to a volume with 10 $\mu$m length along the cavity axis, and a 1 mm diameter in the radial dimension. Observation of the electric dipole ($E1$) microwave transitions would be done through an interference method and extraction of the signal would require repeating the excitation varying the coordinate system.

Preparation of the atoms in a particular Zeeman sublevel of the lower hyperfine level $|F_1, m_1\rangle$ in an applied static magnetic field $\mathbf{B} = B_0\hat{z}$ would be necessary. A resonant standing-wave microwave electric field $\mathbf{E}(t) = E \cos(2\pi \nu_m t + \psi) \cos(k_m y)\hat{x}$ would excite the atoms to a particular Zeeman sublevel in the upper hyperfine level $|F_2, m_2\rangle$. The microwave
magnetic field \( \mathbf{M} \) would be aligned along \( \mathbf{B} \), and it is \( \pi/2 \) out of phase (for a perfect standing wave) with \( \mathbf{E} \) so that \( \mathbf{M}(t) = M \sin(2\pi \nu_m t + \psi) \sin(k_m y) \mathbf{\hat{z}} \), with \( M = E \) in cgs units.

The Raman transition would include two plane-wave optical fields, \( \mathbf{E}_{R1}(t) = E_{R1} \cos(\omega_R t + \phi_R) \mathbf{\hat{x}} \) and \( \mathbf{E}_{R2}(t) = E_{R2} \cos((\omega_R + 2\pi \nu_m) t + \phi_R) \mathbf{\hat{z}} \), phase locked to the microwave field. The Raman carrier frequency \( \omega_R \) would be tuned sufficiently far from optical resonance that only the vector part of the Raman transition amplitude \( (\mathbf{V} \propto i \mathbf{E}_{R1} \times \mathbf{E}_{R2}) \) would be non-negligible \[29\]; that is, we ignore the tensor part of the Raman amplitude.

**B. Observable and reversals**

The various electric and magnetic fields of the apparatus would define a coordinate system related to the measured rate \( \Xi_\pm \). The transition rate \( \Xi_\pm \) depends on three vectors: The polarization of the E1 transition (\( \mathbf{E} \)), the polarization of the Raman transition (\( \mathbf{V} \)), and the static magnetic field \( \mathbf{B} \) that provides an axis for the spins of the nuclei. We combine these three vectors to produce the time reversal preserving pseudo scalar \( i(\mathbf{E} \times (\mathbf{E}_{R1} \times \mathbf{E}_{R2}) \cdot \mathbf{B}) \), proportional to the measured quantity.

A single reversal of any of the fields in the above pseudo scalar changes the sign of the interference term of \( \Xi_\pm \). We then would have the following reversals:

1. - Magnetic field reversal (\( \beta \) reversal).
2. - A shift of \( \pi \) in the relative phase between the E1 and the Raman fields (\( s \) reversal).

The Zeeman sublevels reverse with the magnetic field. The state preparation has to be inverted in order to reach the correct Zeeman sublevel, meaning that \( \sigma^+ \) light goes into \( \sigma^- \) and vice versa. The magnitude of the static magnetic field and the microwave cavity frequency remain unchanged for this reversal.

**C. Apparatus requirements**

1. **Magnetic field**

   We would drive E1 transitions between two particular Zeeman sublevels, \( |F_1, m_1\rangle \rightarrow |F_2, m_2\rangle \) in different hyperfine levels of the ground state. While the frequencies of the exciting fields can be well controlled, the energy difference of the Zeeman states is determined primarily by the static magnetic field.
TABLE I: Parameters of the five relevant francium isotopes: \( \tau s_{1/2} \) state \([30, 31]\), Zeeman sublevels \( m_1, m_2 \), and their energy separation \( \nu_m \) at the static magnetic field \( B_0 \) used in the proposed measurement.

| Isotope | Spin | Hfs (MHz) | \( m_1 \) | \( m_2 \) | \( B_0 \) (Gauss) | \( \nu_m \) (MHz) |
|---------|------|----------|--------|--------|----------------|----------------|
| 208     | 7    | 49880.3  | 0.5    | 1.5    | 2386.5         | 49433          |
| 209     | 9/2  | 43033.5  | 0      | -1     | 1553.0         | 42816          |
| 210     | 6    | 46768.2  | 0.5    | 1.5    | 2586.4         | 46208          |
| 211     | 9/2  | 43569.5  | 0      | -1     | 1572.3         | 43349          |
| 212     | 5    | 49853.1  | 0.5    | 1.5    | 3265.7         | 49015          |

The experimental design should minimize the sensitivity to magnetic field fluctuations. The energy difference between two levels passes through a minimum at the static magnetic field \( B_0 \), and depends quadratically on the magnetic field around that point. We would use the Zeeman sublevels that give the smallest quadratic dependence. Table II lists the Zeeman sublevels and magnetic fields selected for different francium isotopes. The experiment would work between the \( |F_1, m_1\rangle \) and \( |F_2, m_2\rangle \) levels and also between the \( |F_1, m_2\rangle \) and \( |F_2, m_1\rangle \) levels, interchanging \( m_1 \) and \( m_2 \). The operating point of the static magnetic field and the frequency of the microwave cavity would have to be corrected slightly because of the nuclear spin contribution. The state preparation would also change to start in the appropriate level. The change of \( m_1 \) (\( m_2 \)) for \( m_2 \) (\( m_1 \)) does not work as a reversal because of the difference in transition amplitude, but it can still be useful as a consistency check.

The frequency for the \( F = 4, m = 0 \) to the \( F = 5, m = -1 \) transition in \(^{209}\text{Fr}\), expanded around the critical field \( B_0 = 1553 \) Gauss, is

\[
\nu_m = 42.816 \times 10^9 + 90(B - B_0)^2 \text{Hz,} \tag{14}
\]

with \( B \) in Gauss. Control of the magnetic field to 0.06 Gauss (three parts in \( 10^5 \)) reduces the frequency noise due to magnetic field fluctuations down to \( \Delta \nu_m \sim 0.3 \) Hz.

The experiment would take place in a large magnetic field whereas the state preparation and detection occur in a small magnetic field. The transition between both regimes should be done adiabatically. The time scale is determined by the precession time in a small magnetic field, resulting in a magnetic field ramp duration of hundreds of microseconds.
2. The microwave cavity

The francium hyperfine separation requires a Fabry-Perot microwave cavity operating at around 45 GHz (wavelength $\lambda_m \sim 0.66$ cm) in a Fabry-Perot configuration; for example a cavity with a mirror separation of $d \sim 20\lambda_m \sim 13$ cm and a mirror radius of $r_m = 3.5$ cm. These parameters combine to minimize diffraction losses as the Fresnel number $F_N > 1$, where $F_N = r_m^2/\lambda_m d$ [32].

The quality factor ($Q$) of the cavity is

$$Q = \frac{d}{2\varsigma},$$

where $\varsigma$ is the skin depth and is equal to $\sqrt{2/\omega\mu_0\sigma}$ with $\mu_0$ the magnetic constant and $\sigma$ the conductivity ($5.8 \times 10^7 \Omega^{-1}\text{m}^{-1}$ for copper at room temperature). The conductivity limited quality factor is $Q = 1.9 \times 10^5$. It is possible to couple 58 mW into the cavity with current available technology, which would give an electric field of 476 V/cm to drive the E1 transition.

The E1 transition amplitude for $^{209}$Fr between the initial hyperfine level ($\tilde{F}$) $F = 4, m = 0$ to the final hyperfine level ($\tilde{F}$) $F = 5, m = -1$ with a static magnetic field of 1553 Gauss (see Table I) is

$$A_{E1}/\hbar = \langle \tilde{F} | -eE \cdot r\tilde{n}/\hbar \rangle = 0.01i \left[ \frac{E}{476\text{V/cm}} \right] \left[ \frac{\kappa_a}{0.45} \right] \text{rad/s.}$$

A more accurate result can be obtained with the use of many-body perturbation theory [27, 33, 34].

A 1 cm cavity waist would cover the atoms in the 1 mm diameter 10 $\mu$m length trap, and radius of curvature of $R_m = 9.9$ cm for the cavity mirrors ensure a stable cavity, since $(1 - (d/2R_m))^2 < 1$. The curvature of the wave fronts could create a gradient of polarization of the microwave field smaller than $3 \times 10^{-5}$ rad cm$^{-1}$ over the volume of the trap. We show later that this rotation is within acceptable ranges.

The field inside the cavity can be decomposed into a standing wave and a travelling wave. The presence of the travelling wave generates M1 transitions despite the location of the atoms at the node of the standing wave magnetic field. Significant reduction of the
amplitude of intra-cavity travelling waves comes with a symmetrical arrangement of identical antennas, one on each mirror. Antennas give a high coupling efficiency into the cavity as compared to a slit or a grating. The electric field inside the cavity is given by

\[ E = e^{-i\omega_m t} \left( \frac{1}{1 - r_2 e^{2i k_m d}} \right) \times \left[ E_1 t_1 \left( e^{ik_m z} - r_2 e^{i k_m d} e^{-ik_m z} \right) + E_2 t_2 \left( e^{-ik_m z} - r_1 e^{i k_m d} e^{ik_m z} \right) \right], \tag{17} \]

where \( r \) is the reflectivity, \( t \) the transmissivity, \( k \) is the wave-vector of the microwave field, \( d \) the separation between the mirrors, and the sub indices 1 and 2 refer to the two mirrors. The first (second) term is the field generated by antenna 1 (2). The expression is the sum of two waves, one travelling to the right and the other to the left. The difference in amplitude between these two contributions results in a travelling wave. The ratio of travelling to standing wave assuming a symmetrical cavity, that is \( r_1 = r_2 = r \) and \( t_1 = t_2 = t \), is

\[ \mathcal{R}_{T/S} = \left( \frac{i \vartheta}{4} + \frac{E_1 - E_2}{4E_1} \right) \left( i(1 - r) + k_m \Delta d \right), \tag{18} \]

with \( \vartheta \) the phase mismatch from both antennas and \( \Delta d \) the deviation of the cavity mirrors separation from the ideal position. Assuming \( \vartheta = 0 \) and control of the amplitude from each antenna to 1%, the position of the mirrors to 0.1 \( \mu m \) and taking \( 1 - r = 3.6 \times 10^{-4} \) (consistent with the Q factor computed above), we obtain \( \mathcal{R}_{T/S} = (3 + 9i) \times 10^{-7} \).

3. Dipole trap

We choose a far-detuned dipole trap to contain the atoms for the duration of the measurement since the perturbations introduced by it are small and measurable. A variety of different geometries have been proposed over the years. These include red-detuned traps based on focused beams, and blue-detuned traps with hollow beams (see Refs. 37, 38 for reviews of recent work.)

The trap would confine the atoms within 10 \( \mu m \) around the microwave electric field anti-node and 1 mm diameter in the radial dimension. The region of confinement would be smaller than the microwave wavelength (Lamb-Dicke regime), so Doppler broadening becomes negligible.
The AC Stark shift ($\Delta E$), which produces the restoring force of the dipole trap, displaces the two hyperfine levels of ground state in the same direction but not by the same amount. The differential shift changes the resonant frequency for the cavity-driven E1 transition used in the anapole moment measurement. The change in the hyperfine separation for a detuning ($\delta = w - w_e$) larger than the hyperfine splitting ($\Delta_{HFS}$) is approximately equal to ($\Delta_{HFS}/\delta$)$\Delta E$ \cite{39}. The shift reduces considerably using a blue detuned far off resonance trap (FORT) at 532 nm.

The dipole trap in combination with the cavity field may generate a multiphoton transition. There are four vectors available for that transition: $\mathbf{E}_{1D}$, $\mathbf{M}_{1D}$ the dipole trap electric and magnetic fields, $\mathbf{E}$ the microwave electric field and $\mathbf{B}$ the static magnetic field. The parity and time reversal conserving observables created with combinations of the above vectors that produce a resonant transition $((\mathbf{E}_{1D} \cdot \mathbf{E})(\mathbf{M}_{1D} \cdot \mathbf{B}), (\mathbf{E}_{1D} \cdot \mathbf{B})(\mathbf{M}_{1D} \cdot \mathbf{E}), (\mathbf{E}_{1D} \times \mathbf{E}) \cdot (\mathbf{M}_{1D} \times \mathbf{B}), (\mathbf{E}_{1D} \times \mathbf{B}) \cdot (\mathbf{M}_{1D} \times \mathbf{E}), (\mathbf{E}_{1D} \times \mathbf{M}_{1D}) \cdot (\mathbf{E} \times \mathbf{B})$, and $i(\mathbf{E}_{1D} \times \mathbf{E}) \cdot \mathbf{M}_{1D}$), give a negligible contribution if the trap laser propagates along $\mathbf{B}$.

4. $M_1$ transition

The dominant transition between the two hyperfine states is a magnetic dipole $M_1$ transition. The magnetic component of the microwave field could drive $M_1$ transitions. A microwave magnetic field polarized along the $x$ axis would have the same signature as a parity violating signal. The $M_1$ transition amplitude ($A_{M1}$) between the levels of interest is given by

$$A_{M1}/\hbar = \langle \vec{f}|(-e/2m_e)(\mathbf{J} + \mathbf{S}) \cdot \mathbf{M}_{1}\rangle/\hbar$$

$$= 7.8 \times 10^6 \left[ \frac{M}{1.6 \text{ Gauss}} \right] \text{ rad/s}, \quad (19)$$

doing for the maximum expected microwave magnetic field in the Fabry-Perot cavity. The ratio of the E1 transition (Eq. [10]) to the M1 transition is $|A_{E1}/A_{M1}| \sim 1 \times 10^{-9}$. The success of the measurement depends on reducing and understanding this transition. We propose to suppress it in three ways.

First (see Fig. [b]), we would place the atoms at the magnetic field node (electric field...
anti-node) of the microwave cavity. The magnitude of the microwave magnetic field at the edges of the atomic trap is reduced by a factor $\mathcal{R} = \sin(2\pi d_t/\lambda_m)$, with $d_t = 10 \mu m$ the length of the trap along the cavity axis. The reduction factor at 45 GHz is $\mathcal{R} = 4.8 \times 10^{-3}$.

Second (see Fig. 3[b]), we would direct the polarization of the M1 field to be along the $z$-axis (Fig. 2). The non-resonant M1 transitions in this case would be of the type $\Delta m = 0$. The static magnetic field ($B_0$) would split the Zeeman sublevels of the two hyperfine levels, and the microwave field would be resonant for the $|\Delta m| = 1$ E1 transitions (the microwave electric field would be polarized along the $x$ axis.) The alignment imperfections give a suppression factor equal to $\sin(\phi) \sim \phi \sim 10^{-3}$ rad, the angle of the microwave magnetic field polarization with respect to the $z$ axis.

Third (see Fig. 3[c]), the atoms in the dipole trap would oscillate around the microwave magnetic field node. An atom crossing the node would see a microwave magnetic field...
pointing in the opposite direction. The change in position effectively would flip the phase of the magnetic field that the atom sees, and would reverse the evolution generated by the M1 transition. The dynamical suppression only takes place if the frequency of oscillation ($\zeta$) of the atoms inside the trap is larger than the Rabi frequency of the M1 transition and is given by $(1/\sqrt{N})\Omega_{M1}/\zeta$. The frequency of oscillation along the cavity axis for the proposed geometry would be $\zeta/2\pi \sim 300$ Hz.

Taken together, the three suppression mechanisms would reduce the expected M1 transition amplitude to $A_{M1s}/\hbar = 1.9 \times 10^{-5}$ rad/s for $10^6$ atoms. This is 500 times smaller than the amplitude for the E1 transition.

D. Signal to noise ratio

The magnitude of the signal from Eq. \[13\] reaches a maximum for a Raman transition amplitude of $A_R = (2n + 1)\pi/2$ with $t_R = 1$. The measurement of the upper hyperfine state population collapses the state of each atom into one of the two hyperfine levels. The collapse distributes the atoms binomially between the two hyperfine levels and leads to an uncertainty in the measured excited state fraction called projection noise $N_P$ \[40\]. The projection noise is given by

$$N_P = \sqrt{N}|c_e|^2(1 - |c_e|^2). \quad (20)$$

The projection noise vanishes when all the atoms are in one of the hyperfine levels, but in those cases the noise is dominated by other sources, such as the photon shot noise.

The signal to noise ratio for a projection noise limited measurement is

$$\frac{S}{N_P} = 2\frac{A_{E1}t_R}{\hbar}\sqrt{N}. \quad (21)$$

Taking $A_{E1}$ from Eq. \[16\], $t_R = 1$ s, and integrating over $10^4$ cycles, we would reach a 3\% measurement with only 300 atoms.

The high-efficiency MOT that we developed at Stony Brook, with production rates around $10^6$ s\(^{-1}\), captures in excess of $10^5$ francium atoms \[28\]. We expect to trap $10^6$ atoms after transferring them to an ultra-high vacuum environment. In this case, Eq. \[21\] predicts a signal to noise ratio of 20 in 1 s. Higher francium production rates could be obtained at
other facilities, such as ISAC at TRIUMF, where an actinite target could deliver in excess of $10^8$ atoms per second of a single isotope.

While measurements in francium benefit from a large $A_{E1}$, large atomic samples of other alkalis are easily prepared. We could obtain the same signal to noise ratio in a cesium sample with 100 times more atoms and the same strength-driving field. While the fundamental signal to noise ratio indicates the inherent trade-offs between different alkali species, technical noise, specific to the instruments dedicated to the measurement, must also be considered. For a discussion of technical noise in the cesium PNC Boulder experiment see Ref. [4].

IV. NOISE AND SYSTEMATIC EFFECTS

The measurement of the anapole moment would come from determining the population transferred from the lower to the upper hyperfine level by the application of the Raman and microwave fields. Both of these fields (or any other stray field) are characterized by a field amplitude, frequency (or detuning), and interaction time. The total transition amplitude for a common detuning ($\delta$) and interaction time ($t_R$) is:

$$A = (A_{R1} + A_{E11} + A_1) + i(A_{R2} + A_{E12} + A_2),$$

where $A_{R1, R2}$ are the real and imaginary components of the Raman amplitude, $A_{E11, E12}$ the corresponding for the $E1$ transition amplitude and $A_{1, 2}$ are the real and imaginary parts of any other transition present such as an $M1$ transition.

Table II shows the phase of the transitions for given field polarizations, with their transformation under magnetic field reversal assuming all the excitation fields are in phase. We control the phase difference ($\psi$) between the Raman field and the cavity $E1$ field. Varying $\psi$ introduces an additional factor of $e^{i\psi}$ on the $E1$ transition amplitude while the Raman transition remains unchanged. The standing wave $M1$ field inside of the cavity is $90^\circ$ out of phase with the $E1$ field, which gives a factor of $ie^{i\psi}$ for the $M1$ transition. If instead the $M1$ field corresponds to a traveling wave, then it is in phase with the $E1$ field.

The Raman field would be polarized along the $y$ axis so that $A_{R1} = A_{Ry}$, and the $E1$ transition polarized along the $x$ axis so that $A_{E11} = iA_{E1x}$ (or $\psi = \pi/2$). These two amplitudes would interfere since both are in phase and only one (the $E1$) changes sign under magnetic field reversal as shown in Table II. Expanding Eq. (22) for large $A_{Ry}$ compared to
TABLE II: Phase ($P = A/|A|$) of the relevant transition amplitudes for the initial state $F_1 = 4$, $m_1$ and final state $F_2 = 5$, $m_2$ and polarized along the specified axis. For this table all the fields have the same phase (equal to 0). $P_{Rx}$ represents the Raman transition with one vector along the $y$ axis and the other along the $z$ axis, such that their cross product points along the $x$ axis. $\beta$ represents the static magnetic field reversal together with a sign change on the Zeeman sublevel $m$.

| Reversal | $m_1$ | $m_2$ | $P_{E1x}$ | $P_{M1x}$ | $P_{M1y}$ | $P_{Rx}$ | $P_{Ry}$ |
|----------|-------|-------|-----------|-----------|-----------|----------|----------|
| Normal   | 0     | -1    | $i$       | 1         | $i$       | $i$      | 1        |
| $\beta$  | 0     | 1     | $-i$      | -1        | $i$       | $-i$     | 1        |

the detuning and other amplitudes, we obtain

$$\Xi/N \approx \sin^2 \left(\frac{A_{Ry}t_R}{2\hbar}\right) + \frac{1}{2} \sin \left(\frac{A_{Ry}t_R}{\hbar}\right) \left(\frac{A_{ef}t_R}{\hbar}\right),$$  \hspace{1cm} (23)

with,

$$A_{ef} = \left(iA_{Ex} + A_1 + \frac{\hbar^2\delta^2}{2A_{Ry}} + \frac{1}{A_{Ry}}(A_{Rx} + A_2)^2\right).$$  \hspace{1cm} (24)

$A_{ef}$ contains the signal ($A_{Ex}$) and noise ($A_1$, $A_2$, $A_{Rx}$ and $\delta$) terms. We can use this expression to set limits in the different experimental parameters and identify the corresponding observable. Expanding the last term in Eq. \(23\) for small $t_R$ gives

$$\frac{t_R^2}{2\hbar^2}A_{Ry}A_{ef}.$$  \hspace{1cm} (25)

The first term in $A_{ef}$ is proportional to $iA_{Ry}A_{E1x}$, which corresponds to the PNC signal $i(E \times (E_{R1} \times E_{R2}) \cdot B)$.

The amplitudes of interest are the Raman amplitudes $A_{Rx,Ry}$, the E1 amplitude $A_{E1x}$, a M1 transition that is in phase with the E1 field $A_{Mix,Miy}$ and an M1 transition that is $\pi/2$ out of phase with the E1 field $A_{Mox,Moy}$. As an example, if the standing wave magnetic field inside of the cavity is tilted towards the $x$ axis it generates an amplitude $A_{Mox}$ since this field is out of phase with the E1 field. The M1 amplitudes are included in Eq. \(24\) as $A_1$ or $A_2$ depending on their phase relation to $A_{Ry}$.

The relevant values for the relative phase ($\psi$) between the E1 and the Raman transition are multiples of $\pi/2$. First we study the case with $\psi = 0, \pi$. This does not correspond to
the PNC measurement since the E1 and Raman transitions are out of phase and do not interfere. The signal obtained with this configuration is still helpful in the evaluation of unwanted contributions. We can rewrite $A_{ef}$ from Eq. 24 using Table II and ignoring the detuning ($\delta$) as

$$
A_{ef} = \frac{1}{A_Ry} \left[ (A_{Mox}^2 - A_{Miy}^2 - A_{Rx}^2) + s(iA_{Ry}A_{Moy} - 2iA_{Rx}A_{Mox}) + \beta(-2iA_{Miy}A_{Mox}) + s\beta(A_{Ry}A_{Mix} - 2A_{Miy}A_{Rx}) \right],
$$

(26)

with $s = 1, -1$ when $\psi = 0, \pi$ respectively and $\beta = 1, -1$ depending if we have the normal experiment or we apply a magnetic field reversal. With $\psi = \pi/2, -\pi/2$ instead we get

$$
A_{ef} = \frac{1}{A_Ry} \left[ (A_{Mix}^2 - A_{Rx}^2 - A_{Moy}^2) + s(iA_{Ry}A_{Miy} + 2iA_{Rx}A_{Mix}) - \beta(-2iA_{Moy}A_{Mix}) + s\beta(2A_{Rx}A_{Moy} - A_{Ry}A_{Mox} + iA_{Ry}A_{E1x}) \right],
$$

(27)

where now $s = 1, -1$ when $\psi = \pi/2, -\pi/2$ respectively. This corresponds to the experimental condition for the PNC measurement. The PNC signal is contained in the last term, and it changes sign under both reversals. Equations 26 and 27 show how reversals can be used to isolate the PNC signal.

We divide the analysis of the different experimental parameters into three parts: Systematic effects that include terms that mimic the PNC signal and that are contained in the last parenthesis of Eq. 27, line broadening mechanisms, which contain all other terms and that average to zero after an infinite number of cycles, and calibration errors that modify the value of the extracted constants on the PNC signal.

### A. Line broadening mechanisms

We start with terms that do not change under both reversals. They include the detuning term from Eq. 24 and all the terms in Eq. 27 except for the last parenthesis. We present the requirements to achieve a precision of 3% in the measurement after $10^4$ repetitions. Each noise amplitude has to be controlled to $3A_{E1}$
We could reduce the effect of some noise terms by increasing $A_{Ry}$ (see Eq. [24]). We would take $A_{Ry}$ to be exactly equal to a $(2n + 1)\pi/2$ pulse, and include any deviation from this value into $A_1$. We would control the Raman pulse to 0.025% in one second with shot noise limited detection. This would limit the maximum value for the Raman pulse to $A_{Ry}/\hbar = 121$ rad/s or $n = 38$. We now proceed to analyze the spurious terms in Eq. [27] that contaminate the signal.

1. $\hbar^2 \delta^2/2A_{Ry}$

The detuning can have its origin in poor frequency control on the microwave or Raman fields, or changes in the external fields that shift the energy levels. The detuning would have to be controlled to $\delta = 2.7$ rad/s. The required accuracy for the microwave field frequency is one part in $10^{11}$.

Control of the static magnetic field $B_0$ to a fractional stability of $5 \times 10^{-5}$ would keep the detuning under control.

The presence of an M1 transition produces an AC shift of the levels. The value of the maximum shift is $\sim 3$ mHz, which is negligible.

The atoms in the trap occupy different vibrational levels. Transitions between different vibrational levels are suppressed for a sufficiently far detuned trap. Each vibrational level has slightly different resonance frequency that leads to broadening of the signal and loss of coherence.

Coherence times as long as 4.4 s have been measured for atoms in a blue detuned trap [41]. The main source for decoherence was the distribution of Stark shifts felt by the atoms. We expect a coherence time 16 times smaller in francium than in Ref. [41] using a laser at 532 nm because of the difference in hyperfine splitting and detuning. The dephasing grows slowly in time and can be reversed with the use of an “echo” technique. The atoms would spend approximately half of the time in each hyperfine level with a Raman transition amplitude $A_R = (2n + 1)\pi/2$ for large $n$. It is necessary in that case to keep the coherence for a time approximately equal to $t_R/n$, with $t_R$ the duration of the experiment. We would need a coherence time of 26 ms for $n = 38$ to have an interaction time of 1 s. This is below the expected 300 ms coherence time.

The average differential Stark shift seen by the atoms would be approximately equal
to $kT(\Delta_{HFS}/\delta)/\hbar = 6.3$ Hz. The effect of the time varying detuning generated by the oscillations in the trap is similar to a steady state detuning of the same magnitude, and can be compensated by adjusting the microwave frequency. We must control the power of the trap laser to 7%.

2. $A_{Rx}^2/A_{Ry}$

This term appears due to a bad polarization alignment of the Raman field. Control of the polarization of the Raman field to one part in $10^3$ would be necessary to suppress this term.

3. $(A_{Mix}^2, A_{Moy}^2, A_{Rx}A_{Mix}, A_{Moy}A_{Mix})/A_{Ry}$

These terms are multiplied by a small number and their contribution becomes negligible. For example, $A_{Rx}A_{Mix}/A_{Ry}$ has the small factor $A_{Rx}/A_{Ry}$ appearing due to polarization misalignment in the Raman beams.

4. $A_{Ry}A_{Miy}/A_{Ry} = A_{Miy}$

This is the dominant term that depends on the M1 transition. The M1 field appears due to imperfections in the microwave cavity field that create a traveling wave component that may be in or out of phase with the E1 transition.

Eq. 18 gives the amplitude of the traveling wave expected in our setup. The traveling wave is polarized along the $z$ axis, so we can include the polarization suppression factor of $10^{-3}$. Combining these two numbers with the amplitude for the M1 transition we get an amplitude of $0.25 A_{E1}$ out of phase with the E1 transition, and an in phase amplitude of $0.75 A_{E1}$.

The relative phase between both antennas ($\vartheta$) can be adjusted by minimizing the M1 contribution when the static magnetic field ($B$) is tilted slightly. The antennas phase mismatch contribution remains controlled for $\vartheta < 0.01$ rad.
B. Systematic effects

The systematic effects include the terms in the last parenthesis in Eq. 27. They change sign under both $s$ and $\beta$ reversals just as the PNC signal. The constraints for these terms are stronger since they do not average to zero. Their contribution must be below $0.03 A_{E1}$ to reach a 3% measurement. We proceed to analyze each one of these terms.

1. $A_{Rx} A_{Moy}/A_{Ry}$

This term appears because of a combination of misalignment of the Raman field and misalignment of the microwave field or imperfections in the microwave cavity. It corresponds to the observable $M \times (E_{R1} \times E_{R2}) \cdot B$. This term is reduced by the Raman misalignment ($A_{Rx}/A_{Ry}$) and its contribution would become negligible.

2. $A_{Ry} A_{Mox}/A_{Ry} = A_{Mox}$

This term has the same origin as the previous one, but its contribution is considerably larger since it is not suppressed by the Raman misalignment. It gives the limiting factor in the precision of the measurement and its control depends completely on the suppression mechanisms.

The cavity mirrors may have some birefringence, which generate a microwave magnetic field $x$-axis component. The microwaves make roughly 1000 reflections in the cavity. We need a polarization rotation smaller than $10^{-3}$ rad or a rotation per reflection smaller than $10^{-6}$ rad to keep the M1 suppression unchanged. The constraint for a 3% measurement is 14 times smaller.

The atomic sample would have to be precisely held at the node of the microwave magnetic field. The maximum displacement we can tolerate is $3 \times 10^{-11}$ m for a 3% measurement.

C. Calibration errors and requirements on theoretical calculations

The PNC signal (Eq. 13) would give directly the $A_{E1}$ amplitude since the uncertainty in the Raman amplitude is negligible. $A_{E1}$ is the product of the microwave electric field and the matrix element. The microwave electric field amplitude has to be known to 3%.
The electric field could be measured by tilting the magnetic field and inducing an M1 transition. The extraction of information about the weak interaction from an experimental measurement requires theoretical input [24, 42]. The quality of the electronic wave functions is the most important. The accuracy of the matrix elements has to be comparable to that of the experiment. The effective constant of the anapole moment $\kappa_a$ is obtained after subtracting the other two contributions to $\kappa_i$ (Eq. 2). Johnson et al. show that the other contributions for the case of Fr amount to a few percent [27]. The anapole moment of the even-neutron isotopes comes only from the unpaired proton, while the odd-neutron isotopes contain contributions from the unpaired proton and neutron. A measurement of the anapole moment to better than 10% would give an initial separation of both contributions [22].

D. Other sources of fluctuations

The microwave magnetic field would generate transitions to other levels of the type $\Delta m = 0$, which are non-resonant at the proposed magnetic field (detuning $\sim 0.4$ GHz). Nevertheless, these transitions will have to be taken into account in a detailed analysis of the data.

Stray electric fields produce Stark induced transitions that mimic the PNC signal. A stray electric field of 13 V/cm in the $z$ direction would generate a transition amplitude equal to the parity violating signal. Stray fields large enough to be a problem are unlikely to occur and can be ignored [29].

Gradients induce higher order multipole transitions, such as an E2 transition. Fortunately, these higher order transitions between the two hyperfine ground levels are strongly suppressed. Table III summarizes the results of the analysis of noise and systematic effects.

V. CONCLUSION

The anapole moment provides a unique probe of weak hadronic interactions. In particular it is sensitive to weak long-range meson exchange interactions, and consequently allows a measurement of weak neutral currents in the nucleus. This is not the case in high-energy experiments where the weak contribution must be separated from the strong and electromagnetic contributions that are much larger. We have presented the analysis of a proposed
TABLE III: Fractional stability required for a 3% measurement. The observable associated with each constraint is also included.

| Observable Constraint | Set value       | Stability       |
|-----------------------|-----------------|-----------------|
| $A_{E1}$              | Microwave amplitude | 476 V/cm      | 0.03            |
| $A_{Ry}$              | Raman amplitude  | 121 rad/s       | $2.5 \times 10^{-4}$ |
| $(h\delta)^2$         | Microwave frequency | 45 GHz        | $10^{-11}$      |
|                       | Dipole trap Stark shift | 6.3 Hz        | 0.07            |
|                       | DC Magnetic field | 1500 Gauss      | $4.7 \times 10^{-5}$ |
| $A_{Rx}$              | Raman polarization | 0 rad         | $10^{-3}$ rad   |
| $A_{Miy}$             | Mirror separation | 13 cm         | $7.7 \times 10^{-7}$ |
|                       | Antenna power    | 57 mW          | 0.02            |
|                       | Antenna phase    | 0 rad          | 0.01 rad        |
| $A_{Mox}$             | Mirror birefringence | 0 rad        | $1 \times 10^{-4}$ rad |
|                       | Trap displacement | 0 m            | $3 \times 10^{-11}$ m |

measurement strategy of the nuclear spin dependent part of the PNC interaction, dominated by the anapole moment. While the proposed measurement method can be extended to other alkali atoms, a series of measurements in a chain of francium isotopes allows the separation of the proton and neutron contributions to the anapole moment.

As noted by Fortson et al. [9, 10] studies of atomic parity non conservation give information on the nuclear physics. The nuclear weak interaction at low energies is often parameterized by a series of coupling constants, either with a meson exchange formalism, the so-called DDH parametrization [43], or more recently with effective field theories (EFT) [44]. A program of measurements of the anapole moment in a chain of francium isotopes will contribute significantly to constraint some of the DDH parameters, which together with the EFT program will provide a model independent input for theoretical analysis of low energy weak interaction constants. It is important to note that the measurement of the anapole moment of an even and an odd isotope of francium give almost orthogonal bands in the meson coupling parameter space. This is subject to the assumption that the anapole moment is carried mainly by the last nucleons [22], but as shown by the measurements of the hy-
perfine anomaly [7], this is a reasonable assumption. These measurements will significantly contribute to deepen our understanding of the nuclear structure.

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[1] Y. B. Zel’dovich, Sov. Phys.-JETP 6, 1184 (1958).
[2] V. V. Flambaum, I. B. Khriplovich, and O. P. Sushkov, Phys. Lett. B 146, 367 (1984).
[3] P. A. Vetter, D. M. Meekhof, P. K. Majumder, S. K. Lamoreaux, and E. N. Fortson, Phys. Rev. Lett. 74, 2658 (1995).
[4] C. S. Wood, S. C. Bennett, J. L. Roberts, D. Cho, and C. E. Wieman, Can. J. Phys. 77, 7 (1999).
[5] C. S. Wood, S. C. Bennett, D. Cho, B. P. Masterson, J. L. Roberts, C. E. Tanner, and C. E. Wieman, Science 275, 1759 (1997).
[6] A. Derevianko and S. G. Porsev, Phys. Rev. A 65, 052115 (2002).
[7] J. S. Grossman, L. A. Orozco, M. R. Pearson, J. E. Simsarian, G. D. Sprouse, and W. Z. Zhao, Phys. Rev. Lett. 83, 935 (1999).
[8] E. Gomez, L. A. Orozco, and G. D. Sprouse, Rep. Prog. Phys. 66, 79 (2006).
[9] E. N. Fortson and Y. Pang and L. Wilets, Phys. Rev. Lett. 65, 2857 (1990).
[10] S. J. Pollock and E. N. Fortson and L. Wilets, Phys. Rev. C 46, 2587 (1992).
[11] C. E. Loving and P. G. H. Sandars, J. Phys. B 10, 2755 (1977).
[12] V. G. Gorshkov, V. F. Ezhov, M. G. Kozlov, and A. I. Mikhailov, Sov. J. Nucl. Phys. 48, 867 (1988).
[13] D. Budker, in Physics Beyond the Standard Model, edited by P. Herczug, C. M. Hoffman, and H. V. Klapdor-Klinkrothaus (World Scientific, Singapore, 1998).
[14] V. E. Balakin and S. I. Kozhemyachenko, JETP Lett. 31, 297 (1980).
[15] V. N. Novikov and I. B. Khriplovich, JETP Lett. 22, 74 (1975).
[16] E. A. Hinds and V. W. Hughes, Phys. Lett. B 67, 487 (1977).
[17] E. G. Adelberger, T. A. Trainor, E. N. Fortson, T. E. Chupp, D. Holmgren, M. Z. Iqbal, and H. E. Swanson, Nuc. Instr. and Meth. 179, 181 (1981).
[18] N. Fortson, Phys. Rev. Lett. 70, 2383 (1993).
[19] M. V. Romalis and E. N. Fortson, Phys. Rev. A 59, 4547 (1999).
[20] C. Chin, V. Leiber, V. Vuletic, A. J. Kerman, and S. Chu, Phys. Rev. A 63, 033401 (2001).
[21] E. J. Angstmann, T. H. Dinh and V. V. Flambaum, Phys. Rev. A 72, 052108 (2005).
[22] V. V. Flambaum and D. W. Murray, Phys. Rev. C 56, 1641 (1997).
[23] I. B. Khriplovich, Parity Non-Conservation in Atomic Phenomena (Gordon and Breach, New York, 1991).
[24] W. C. Haxton and C. E. Wieman, Annu. Rev. Nucl. Part. Sci. 51, 261 (2001).
[25] R. R. Lewis, Phys. Rev. A 48, 4107 (1993).
[26] The anapole moment operator is given by $\hat{a} = (\pi e/m) [\mu (r \times \sigma) - (q/2)(pr^2 + r^2p)]$, see for example Ref. [21]. The anapole interaction for a spin-1/2 system is given by the Lagrangian $\mathcal{L}_a = (a/M^2)\bar{\psi}(x)\gamma_\mu \gamma_5 \psi \partial_\mu F^{\mu\nu}$, see for example, J. Erler and M. J. Ramsey-Musolf, Prog. Nucl. Part. Phys. 54, 351 (2005).
[27] W. R. Johnson, M. S. Safronova, and U. I. Safronova, Phys. Rev. A 67, 062106 (2003).
[28] S. Aubin, E. Gomez, L. A. Orozco, and G. D. Sprouse, Rev. Sci. Instrum. 74, 4342 (2003).
[29] D. DeMille and M. G. Kozlov, [arXiv:physics/9801034] (1998).
[30] A. Coc, C. Thibault, F. Touchard, H. T. Duong, P. Juncar, S. Liberman, J. Pinard, J. Lermé, J. L. Vialle, S. Büttgenbach, A. C. Mueller, A. Pesnelle, and the ISOLDE Collaboration, Phys. Lett. B 163B, 66 (1985).
[31] A. Coc, C. Thibault, F. Touchard, H. T. Duong, P. Juncar, S. Liberman, J. Pinard, M. Carre, J. Lermé, J. L. Vialle, S. Büttgenbach, A. C. Mueller, A. Pesnelle, and the ISOLDE Collaboration, Nucl. Phys. A 468, 1 (1987).
[32] S. Ramo, J. F. Whinnery, and T. V. Duzer, Fields and Waves in Communication Electronics (John Wiley and Sons, New York, 1993).
[33] S. G. Porsev and M. G. Kozlov, Phys. Rev. A 64, 064101 (2001).
[34] C. Bouchiat and C. A. Piketty, Phys. Lett. B 269, 195 (1991).
[35] U. Harbarth, J. Kowalski, R. Neumann, S. Noehte, K. Scheffzek, and G. Z. Putlitz, J. Phys. E. Sci. Instrum. 20, 409 (1987).
[36] J. W. Dees, and A. P. Sheppard, IEEE Trans. Inst. Meas. 14, 52 (1965).
[37] V. I. Balykin, V. G. Minogin, and V. S. Letokhov, Rep. Prog. Phys. 63, 1429 (2000).

[38] N. Friedman, A. Kaplan, and N. Davidson, Advances in Atomic, Molecular, and Optical Physics 48, 99 (2002).

[39] A. Kaplan, M. F. Andersen, and N. Davidson, Phys. Rev. A 66, 045401 (2002).

[40] W. M. Itano, J. C. Bergquist, J. J. Bollinger, J. M. Gilligan, D. J. Heinzen, F. L. Moore, M. G. Raizen, and D. J. Wineland, Phys. Rev. A 47, 3554 (1993).

[41] N. Davidson, H. J. Lee, C. S. Adams, M. Kasevich, and S. Chu, Phys. Rev. Lett. 74, 1311 (1995).

[42] J. S. M. Ginges and V. V. Flambaum”, Phys. Rep. 397, 63 (2004).

[43] B. Desplanques and J. F. Donoghue and B. R. Holstein, Ann. Phys. (NY) 124, 449 (1980).

[44] S. Zhu and C. M. Maekawa and B. R. Holstein and M. J. Ramsey-Musolf and U. van Kolck, Nucl. Phys. A 748, 435 (2005).