Super-Planckian excursions of the inflaton and quantum corrections

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Models of inflation with super-Planckian excursion seem well in agreement with the recent observations of B-mode polarization in the cosmic microwave background (CMB) radiation by the BICEP2 data. In this note, we highlight the challenges faced by such models from ultraviolet (UV) completion. In particular, we will discuss radiative corrections to the inflaton Lagrangian and to the gravitational sector. We will emphasize why we would require an UV complete theory of gravity to tackle some of the issues for the super-Planckian excursion. In particular, we will highlight how higher derivative terms in the inflaton and gravity sectors cause problems from non-locality and ghosts, if considered order by order, and thus prompt us to take into account infinite series of such terms. We will also stress how the presence of a scale of new physics below the Planck scale would make some of the UV related problems more compelling and invalidate some of the remedies that have been proposed in the literature. Finally, we will briefly speculate on possible ways of curing some of the challenges.

I. INTRODUCTION

The discovery of B-mode in the polarization of the CMB (cosmic microwave background) radiation at large angular scales by the BICEP2 team [1] has made a very strong case for the inflationary paradigm [2,3]. The signal is very well explained in terms of the primordial gravitational waves being stretched during inflation. The same stretching of the modes, also acting on scalar (matter) fluctuations, lead to the CMB temperature anisotropy, as observed by WMAP [5] and Planck [6].

The amplitude of these tensor fluctuations is usually expressed by the ratio of the scalar and tensor power spectra, $P_\zeta$ and $P_T$, dubbed tensor-to-scalar ratio,

$$0.15 \leq r(k) = \frac{P_T(k)}{P_\zeta(k)} \leq 0.27,$$

at the pivot scale, $k_p = 0.002$ Mpc$^{-1}$ [1]. The amplitudes of the matter power spectrum $P_\zeta \sim 2.1 \times 10^{-9}$ denotes the amplitude of the CMB temperature anisotropy [1]. For a slow-roll dominated inflation, one can extract the potential energy of the inflationary vacuum:

$$P_T = \frac{H_{inf}^2}{\pi^2 M_p^2} \approx \frac{2V_{inf}}{3\pi M_p^2} \sim 4.2 \times 10^{-10},$$

where $H_{inf} \approx (V_{inf}/3M_p^2)$ is the Hubble expansion rate during inflation, $V_{inf}$ is the inflationary potential and $M_p \sim 2.44 \times 10^{18}$ GeV is the reduced Planck mass. This suggests that the inflationary potential perhaps comes from the physics very close to the Grand Unified Theory scale, i.e., $V_{inf}^{1/4} \sim 2 \times 10^{16}$ GeV. This is also the first evidence of physical scale beyond the Standard Model and the scale of gravity at $M_p$.

$$V \sim V_0 + \frac{m^2}{2} \phi^2 + \cdots$$

In fact, the simplest potential, also known as Chaotic inflationary potential, for a single scalar field [2,7]

$$V \sim V_0 + \frac{m^2}{2} \phi^2 + \cdots$$

matches the current observations of CMB data extremely well, i.e., the amplitude of the CMB temperature anisotropies power spectrum, its tilt, and the tensor-to-scalar ratio at $r \sim 0.16$ [1]. The field $\phi$ could be either fundamental or composite [3]. The potential in Eq. (3) yields an exponential inflation within slow roll regime for $m_\phi \sim 10^{13}$ GeV, with a Hubble rate $H_{inf} \sim 10^{14}$ GeV. In Eq. (3), the ellipses stand for higher order terms that would be present in the potential because of quantum corrections, and are assumed to be negligible in order for the model to work.

Assuming such monotonic behaviour of the potential, one can derive an upper bound on the value of $r$, known as the Lyth bound [3]:

$$r = 16\epsilon \leq 0.003 \left(\frac{50}{N}\right)^2 \left(\frac{\Delta \phi}{M_p}\right)^2.$$

where $\Delta \phi$ denotes the field excursion during inflation, lasted $N$ e-folds. For the observed $r \sim 0.2$, one obtains $\Delta \phi \sim 10 M_p$. Here $\epsilon = (M_p^2/2)(V'/V)^2$ is the slow roll parameter, and the prime denotes the derivative of the potential w.r.t. $\phi$.

Both the slow-roll condition and the Lyth bound impose super-Planckian field excursions for $\phi$ during inflation. The challenge is to understand the physical consequences of this super-Planckian VEV of inflation, see [9].

Indeed this is a well-known problem [7,10], although the validity of the effective picture of inflation for the potential in Eq. (3) is justified, since

$$V_{inf}^{1/4} \ll M_p,$$

during the slow roll inflation, the problem arises for the higher order corrections, due to the fact that the field

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1 Neglecting the subleading part due to tensor fluctuations.
value of the inflaton, i.e. \( \phi \sim 10 M_p \), exceeds the Planck scale during inflation. This suggests that the effective field theory treatment itself breaks down at the verge of super-Planckian inflation.

In this note, we argue that the quantum corrections will eventually require considering higher-order terms in the Lagrangian, and in particular certain classes of such corrections would spoil the inflationary potential obtained when the energy density is dominated simply by the quadratic term in Eq. (3).

The article is organized as follows: we discuss the corrections to the Lagrangian of the inflaton field, their magnitude, and the relevance of having different scales of high-energy physics in section II. We address some relevant arguments proposed in the literature to deal with these issues in section II.C. We then focus on typically neglected corrections, affecting the kinetic terms for the inflaton and the purely gravitational sector, respectively, in sections II and IV. Finally, in section V we speculate on some possible remedies to the problems that we have pointed out.

II. HIGHER ORDER QUANTUM CORRECTIONS

A. Presence of multi-scales

The higher order terms in the inflaton potential in Eq. (3) would derive from the coupling of the inflaton to heavy fields in the underlying UV-complete theory. Their presence follows naturally from the facts, that

- the existing inflation models are not UV complete.
- the inflaton, being a gauge singlet order parameter, would couple to many degrees of freedom in the complete theory, both hidden and Standard Model (SM) ones, see [3].
- gravitational couplings are universal, therefore one has to construct an effective field theory argument based on the inflaton and its interactions with other degrees of freedom, while taking into account gravitational corrections.

One important point in our discussion is the role that different energy scales would play. In fact, the question of effective theories in relation with super-Planckian field excursions should not be dealt with only in terms of \( M_p \).

One particular example of this, is (super)string theory [1], which we will often cite as it is a well-developed candidate for a UV complete theory. Here, typically one would have a series of important scales: principally a) the compactification/Kaluza-Klein scale \( M_s \), b) the string scale \( M_s \), and c) the supersymmetry breaking scale \( M_{susy} \).

The four-dimensional Planck scale is a derived quantity related to \( M_s \) and \( M_p \), and in the most well established models the scales are ordered as \( M_c \leq M_s \leq M_p \).

B. Radiative corrections due to inflaton couplings

For the sake of simplicity and generality, let us consider a generic scale of new physics \( M_I \leq M_p \), and we will discuss cases, when \( M_I \approx M_p \). In this section, we will focus on the corrections arising from integrating out fields coupled to the inflaton, in section IV we will discuss how higher order correction do arise also from the purely gravitational sector.

Generically for a SM gauge singlet inflaton, the inflaton can couple to various species of fermions and gauge fields:

\[
V \sim \sum_i g_i \phi \bar{\psi}_i \psi_i, \quad V \sim \sum_i g'_i \phi F_{\mu \nu}^i F^{i \mu \nu}, \quad (6)
\]

and, of course, gravity. Here, \( g_i, \ g'_i \) are appropriate couplings.

Such interactions of the inflaton to the matter field would lead to a host of corrections both to the potential and to the kinetic terms for the inflaton:

\[
\delta \mathcal{L} \sim \sum_n \frac{n \phi^n}{M_I^{n-4}} + \sum_{n,m} \left( \nabla \phi \right)^2 \frac{m \phi^m}{M_I^{m-4}} + \cdots, \quad (7)
\]

where \( (\nabla \phi)^2 = g^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi \), \( \nabla \) being the covariant derivative for the metric \( g_{\mu \nu} \), and \( \cdots \) indicates terms where more than one derivative act on the inflaton field.

The operators are weighted by the scale \( M_I \) at which the new physics appears, with \( \lambda_n, \ d_n \sim \mathcal{O}(1) \) for \( g_i, \ g'_i \sim \mathcal{O}(1) \).

The corrections to the kinetic terms have been somewhat neglected in the literature regarding super-Planckian field excursion, see for example [10, 12–14], disregarding the rather distinctive issues that they would introduce as compared to the corrections to the potential. In fact, they too play an essential role as we will discuss in particular in sections III and IV.

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2 In this work, we will focus on the quantum radiative corrections in the inflaton Lagrangian, while neglecting any possible thermal corrections.

3 To be precise: corrections arising from graviton loops will always be weighted by \( M_p \), while those coming from heavy fields will be suppressed by the scale \( M_I \) relevant for those fields, where \( M_I \leq M_p \).

4 Models of higher-derivative inflations have been discussed, such as k-inflation, Galileon models, Horndeski-like theories [13, 16], but in those cases the Lagrangians are truncated at a certain order, or have specific ad-hoc structures that eliminate the issues which we will point out in the following. These models are not UV complete, and also face the open problem of UV completion and robustness of the inflationary predictions against corrections.
In the case of string theory, the non-renormalizable operators in Eq. (7) come from higher dimensional and string loops, and the corrections are suppressed by the scale of heavy degrees of freedom and the string coupling $g_s$. In this setup, the problem of destabilizing the effective four-dimensional picture for super-Planckian field excursion becomes particularly relevant.

C. Issues concerning radiative corrections and proposed remedies

In this section we will discuss the validity of some relevant remedies proposed to cope with the issue of the higher order corrections to the inflaton action in presence of super-Planckian field excursion. In the following sections III and IV, we will further focus on important unsolved issues that have been typically overlooked in the literature.

1. Small numbers

One evident possibility for reconciling with the current CMB observations is to require small couplings of the inflaton, i.e. $g, g' \lesssim 10^{-3}$ or so, so that the fine-tuning would lead to sufficiently small $\lambda_n, d_n$ in Eq. (4). For example, to maintain the flatness of the potential generating the right amplitude of scalar and tensor perturbations, one would require, $g^4 \ll 10^{-12}$ for the $\phi^4$ term.

This is all right, and one can systematically make the higher order non-renormalizable couplings small order by order, although the fine tuning might become very strong if the inflaton $\phi$ has super-Planckian excursion and $M_f < M_p$. However, this raises the question why and how nature would generate these small numbers, leading to the anthropic arguments, which we will not resort to in this discussion.

5 We will not discuss light moduli and multifield scenarios in this paper.

6 There has been a discussion of a somewhat technically natural suppression of the couplings of light fields in some string theory setups, but one still has to resort to the full UV-complete analysis, which is lacking. Moreover, in explicit calculations, the corrections to kinetic terms do not appear to be tunable. Also, other light fields could arise besides the inflaton, which would spoil the cosmological predictions as strict constraints arise from the isocurvature perturbations. Finally, building up models in agreement with the BICEP2 results in those scenarios has been shown to be difficult (sometimes leading again to anthropic arguments to motivate the choice of compactifications and the mass spectra).

2. The case when $M_f = M_p$

For the specific case, when $M_f = M_p$, it has been argued that Eq. (7), obtained by integrating out heavy fields using their standard propagators, would simply not be valid, because of black hole formation.

Indeed, one might be able to argue that according to the Einstein’s theory, if the inflaton has a super-Planckian VEV with $g \sim O(1)$, the coupled fermions and/or gauge fields would be forming a blackhole of mass $m_\psi, A_n \sim g(\phi) \sim 10 M_p$ with Planckian-sized Schwarzschild’s radius $r_h \sim (10^9/8\pi M_p)$. Therefore, one might argue, as in Ref. [21], that the inflaton is virtually coupled to a sea of back holes, and therefore the effective correction should have a Boltzmann suppression, and enter the potential as

$$V \supset \mathcal{N} e^{-8 \Omega_i (\phi, \nabla \phi, \ldots) \frac{M^4}{M^4_f}} S = \frac{g^2 \phi^2}{M^2_p}. \quad (8)$$

for all possible operators $\mathcal{O}_i$, with dimensions $\Delta_i$, salvaging the perturbative expansion.

However, a couple of points arise. The first one is that this argument relies on the fact that there is no scale of new physics before $M_p$, the scale of black hole formation [21]. To assume the validity of simply the Standard Model and General Relativity right up to $M_p$ is a rather strong assumption.

An interesting observation concerns the issue of back reaction when $M_f \approx M_p$. Just from the above arguments, the universe at the onset of inflation would be filled with black holes. The number of such black holes can be estimated by counting the number of degrees of freedom that the inflaton interacts with. For example, if the inflaton couples universally to $n_\psi$ fermion species, then the total energy density stored in the sea of black holes for $O(1)$ couplings would be roughly of the order of $\sim (n_\psi M_p)^4 \sim (10 n_\psi g M_p)^4$, for $m_\psi \sim g(\phi)$ and $\langle \phi \rangle \sim 10 M_p$. This energy density should be less than the inflaton energy density, which requires, taking into account BICEP2 results, $(10 n_\psi g M_p)^4 \lesssim (m_\psi^2 \phi^2)^4 \sim 10^{64}$ (GeV)$^4$, or

$$n_\psi \times g \lesssim 10^{-3}. \quad (9)$$

In fact, the constraint from this counting argument leads to a value of the coupling similar to the fine tuning required to make the quantum corrections sufficiently small. Indeed, the $g\phi\nabla\phi$ interactions would yield a $\lambda \phi^4$ term, which would be negligible for $\lambda \lesssim 10^{-12}$, in turn requiring $g \lesssim 10^{-3}$.

Another related question is whether a semiclassical treatment of gravity would be valid or not at $M_p$ during inflation - the required condition is that the total energy density is sub-Planckian. Usually, Eq. (3) is claimed to guarantee this. However, Eq. (3) does not take into account the full series of corrections to potential and kinetic terms, see Eq. (7). To ensure that the total energy density is sub-Planckian in presence of super-Planckian field excursions would require again fine tuning.
One can also decide to abandon the semi-classical approach to gravity, but at that point one would require a full non-perturbative formulation of gravity, which we sorely lack at this moment. Even in string theory, we do not have a full understanding of all orders α’ and loop corrections (we will come back to this point in the following).

3. Resumming the potential

The non-renormalizable operators in the series in Eq. (7) generically come with different signs, hence the properties of their resummation may not be evident from the individual terms. For example, in the cases of purely gravitational corrections (graviton loops) it has been shown that the corrections to the potential can be resummed and the expression is weighted by the inflaton potential and its derivative w.r.t. the inflaton field φ, i.e. $V(\phi)$, $V''(\phi)$ [22]. These parameters are then considered under slow roll conditions, which demand that $V'(\phi) \ll M_P^2$, $V(\phi) < M_P^2$, and so on, see for instance [7].

However, there are a couple of points which arise:

- the very foundation of the argument is based on the validity of the slow roll conditions, and the universe has begun inflating. However, inflation is not guaranteed at the first point, if we consider corrections such as Eq. (7), contrary to what is being generally argued, see sections III IV.

- it is not guaranteed that the sum of the other corrections to the potential, beside the purely gravitational ones, lead to the same sort of rearrangement in the final result and leaves intact the predictions arising from the lowest-order terms. Even more remarkably, for what concerns kinetic terms correction that is not the case, due to the appearance of ghosts and singularities (see sections III IV).

There is one case where the corrections would be expected to be naturally weighted by the tree-level potential, giving rise to a suppression of higher terms during inflation: if the tree-level potential softly breaks an underlying symmetry such as shift symmetry, and the inflaton is a pseudo Nambu-Goldstone boson.

In that case higher corrections are also proportional to the symmetry breaking terms, and, when these are small, the corrections can be under control [23 24]. However, to avoid the breaking of continuous global symmetry operated by gravity, these shift-symmetries should be related to underlying short-scale fundamental symmetries such as gauge ones [25 27], and the corrections to the inflaton kinetic term are not guaranteed to be negligible, see [25] and section V.

4. Stationary points in the potential

One the other hand, one can always entertain a slightly different approach: could the corrected potential lead to inflation thanks to stationary or inflection-like points, where the higher derivatives of the potential with respect to φ vanish?

Examples of inflection-point and saddle-point inflation where respectively $V''(\phi_0) = 0$ and $V''(\phi_0) = V''(\phi_0) = 0$ have been studied in [28] and [29].

For instance, one may consider a potential of the type

$$V = |f(\phi)|^2, \quad f(\phi) \equiv \sum_{n>1} \lambda_n \phi^{3n-1} M_p^{3n-3}, \quad (10)$$

where we assume the canonical kinetic term for φ for the time being.

The values of $\lambda_n$ could be kept arbitrary at the time being, but soon we will see that the cosmological observations would start putting constraints on the lowest order coefficient. For $n_{\text{max}} = 1$, the potential is renormalizable, and yields $\lambda_1^3 f^4$.

Potential of type Eq. (10) can be analyzed in the context of catastrophe theory. The highest order term is called the catastrophe germ and the coefficients of the lower orders are called control parameter, dictating at which field values the critical and/or inflection points are located, see Refs [24 30].

For example, let us consider the case with $n_{\text{max}} = 3$. Then

$$V'(\phi) = f'(\phi)f^*(\phi) + h.c.$$ is zero at the points: $\phi = (0, a^{1/3}, b^{1/3}) M_p$, where $a + b = -5 \lambda_2/8 \lambda_3$ and $ab = \lambda_1/4 \lambda_3$. These solutions exist for any values of $\lambda_1$, $\lambda_2$, $\lambda_3$ for φ being complex [29]. In fact this argument goes beyond $n = 3$, and beyond the specific case of saddle points, i.e. those where $V'' = V''' = 0$. In fact, as shown in Ref. [29], one can find the roots where $n - 1$ derivatives of the potential vanish. This is a consequence of the fact that there are $n - 1$ complex roots, $a_i$, of

$$f'(\phi) \propto \phi \prod_{i=1}^{n-1} [f(\phi/M_p)^3 - a_i]. \quad (11)$$

When two roots coincide, one gets inflection-points, and for higher degenerate points one requires more roots to coincide.

However, a simple analysis of $n_{\text{max}} = 3$ would illustrate that one still needs the lowest order coupling to be very tiny. Indeed, the inflection point is given by:

$$\phi = \phi_0 \exp(i \pi/3, i \pi, i 5\pi/3), \quad \phi_0 = \left( \frac{5 \lambda_2}{16 \lambda_3} \right)^{1/3} M_p, \quad (12)$$

In principle, one can accommodate some of the coefficients to have large values, such as $\lambda_2 \sim 10^{12}$ and $\lambda_3 \sim 10^9$, or $\lambda_2 \sim \mathcal{O}(1)$ and $\lambda_3 \sim 10^{-3}$, such that $\phi_0 \sim$
\(O(10) M_p\). However, to explain the current CMB perturbations the amplitude of the temperature anisotropy, which is given by \(\delta T \sim V''(\phi_0)N_{CMB}^2/30\pi H_{inf}\), where \(H_{inf} \approx (V(\phi_0)/3M_p^2)^{1/2}\) and \(N_{CMB} \sim 50 - 60\) e-foldings, would require

\[
\lambda_1 \left( \frac{16 \lambda_1}{5 \lambda_2} \right)^{1/3} \leq 10^{-8}.
\]

One can see that irrespective of \(\lambda_2\) and \(\lambda_3\), if one requires \(\phi_0 \sim O(10 M_p)\), one inevitably requires a small coupling, i.e. \(\lambda_1 \leq 10^{-8}\) \(\cite{22}\).

In conclusion, also driving inflation at special points does not help in fine tuning the couplings or the self interactions of the inflaton. One still needs at least some of the couplings to be very small. Note also that so far we have always assumed canonical kinetic term for illustration, but this is a point that we will argue to be incorrect in section IV.

III. THE IMPORTANCE OF HIGHER-DERIVATIVE TERMS

In this and the next section, we will focus on the corrections to the kinetic terms in the inflaton and in the gravitational sector of the theory. In the literature for super-Planckian inflation these terms have often been neglected, by invoking a series of arguments. Instead, we are now going to discuss their relevance.

A first argument to discard the terms involving derivative of the inflaton, both spatial and temporal, is that chaotic inflation can only occur if their contribution to the energy density is smaller than the Planck scale. It is acknowledged that naturally derivative terms do scale like the Planck mass (typically it would be \(\phi \sim O(10 M_p)\), \(\dot{\phi} \sim O(M_p^2)\), \(\ddot{\phi} \sim O(M_p^4)\) \(
\cdots\)), but it has been argued that:

- it makes sense to speak of (semiclassical) inflation only in inflating patches, where those terms have to be small (and the probability for such patches do exist is variably estimated), see \(\cite{7} \cite{31}\).

- often there is an attractor solution that drives the inflaton to a regime where its derivatives are small \(\cite{31} \cite{32}\) (in this case the probability of having an inflationary patch is order 1).

However, these two arguments are not guaranteed to hold at all when higher order terms are included. The attractor mechanism, indeed, need to be re-analyzed taking into account the effects of higher derivative terms in the field equations. Let us remind that typically these terms will lead to non-locality and stability issues \(\cite{33}\), and they can also introduce extra states, such as ghost-like ones that violates unitary and/or make the vacuum unstable. Even abandoning the attractor solution argument, and invoking only the random generation of an inflationary patch, the probability of generation of such patch needs to be estimated considering the quantum corrected action for superPlanckian field excursions.

One particularly relevant implication of higher derivative terms is that they would generally force to consider a complete series of such terms \(\cite{7}\). In fact, a rather generic feature of covariant "finite-order" higher derivative theories is the presence of new ghost states. Let us consider a simple scalar field theory model of the form:

\[
S = \int d^4x \left[ \mathcal{G}(\Box)\phi - V_{int}(\phi) \right], \quad \Box = g_{\mu\nu} \nabla_\mu \nabla_\nu (14)
\]

where \(\mathcal{G}\) is a finite polynomial function. In this case one can always write \(\mathcal{G}\) as:

\[
\mathcal{G}(p^2) \propto (p^2 + m_1^2)(p^2 + m_2^2) \cdots (p^2 + m_n^2).\]

In order for the theory to be non-tachyonic, all the \(m_i^2\) have to be positive real, and we are using the metric convention \(+ - \cdots\). Moreover, if there are at least two discrete single poles (say \(m_1 \neq m_2\)), then at least one of them is ghost like, i.e. one of the residues has to be negative \(\cite{34}\):

\[
\frac{1}{(p^2 + m_1^2)(p^2 + m_2^2)} \sim \frac{1}{p^2 + m_1^2} - \frac{1}{p^2 + m_2^2} \quad (16)
\]

A double pole can be represented as the convergence of two simple poles with opposite residues, and suffers from similar problems \(\cite{33} \cite{35}\). Similar arguments follow for higher order poles, rendering higher derivative theories of the form Eq. (14) inconsistent.

These considerations prompt to take into account a whole series of perturbations, which could be ghost-free. Examples of the Lagrangian they could originate have been studied in the context of string field theory and related contexts, see \(\cite{36} \cite{38}\). For instance, in the so-called p-adic string action \(\cite{39}\), see also \(\cite{37}\), one has

\[
\mathcal{L} \sim \frac{M_s^4}{g_p^2} \left[ -\frac{1}{2} \phi e^{-\frac{m_p}{M_s^2}} \phi + \frac{\phi^{p+1}}{p+1} \right], \quad (17)
\]

where \(g_p^{-2} = g_s^{-2}(p^2/p - 1)\) and \(m_p^2 = 2M_s^2/\ln p\), and one can evade some of the problems mentioned above, because the action is intrinsically non-perturbative in nature. In this case the propagator is modified in such a way that it does not contain any poles \(\cite{8}\). In other words there are no physical ghosts states around the true vacuum. On the other hand, the kinetic term does modify the UV behaviour of the theory, in particular this

\footnote{For the case of Horndeski-like theories, see footnote 4. We recall here that such theories are not UV complete, and so face the challenges of preserving their properties against radiative corrections.}

\footnote{The propagator has a form of an entire function, which does not have any poles other than the essential singularities at the infinities.}
example has been studied in the context of inflationary
cosmology in Ref. \[39\].

Certainly, inflationary slow roll conditions are now
modified and one has to find a full solution to the action. Modifications akin to Eq. (17) have been proposed in the gravitational sector, altering the UV properties of gravity, see Refs. \[34\] \[40\]. We will now turn to this point and study the consequences.

IV. HIGHER-DERIVATIVE CORRECTIONS FROM GRAVITY

A super-Planckian field excursion and the necessity to
take into account near-Planck-scale physics prompt to
deal with the corrections also in the purely gravitational
sector of the theory. Let us illustrate the case at the
lowest order, up to $O(h_{\mu\nu}^2)$, where $h_{\mu\nu}$ is a small perturbation around the Minkowski metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$. We would expect higher derivative corrections of the type:

$$L_{\text{gr}} \sim \frac{R}{2} + R F_1 \left( \frac{\Box}{M_f^2} \right) R + R_{\mu\nu} F_2 \left( \frac{\Box}{M_f^2} \right) R^{\mu\nu} + R_{\mu\nu\lambda\sigma} F_3 \left( \frac{\Box}{M_f^2} \right) R^{\mu\nu\lambda\sigma} + \ldots \quad (18)$$

where,

$$F_i \left( \frac{\Box}{M_f^2} \right) = \sum_{n \geq 0} f_i, n \Box^n, \quad \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu.$$

(19)

In the case of string theory such corrections would descend from higher-dimensional and string dynamics, which yield $g_s^2$ and $\alpha' = M_s^{-2}$ corrections.

Such terms, if considered order by order, would yield ghosts, as shown in Ref. \[34\] \[40\], similar to the example we have demonstrated in our simple scalar field theory.

The higher derivative corrections in the gravity sector would modify the graviton propagator by introducing extra propagating states, which would be ghost-like. The only way one can tame the issue of ghosts is to make sure that there must not be any extra pole other than that of the massless graviton. Therefore, any modification in the graviton propagator should be such that it not only recovers the normal GR in the infrared, but should not violate unitarity at all. This would eventually add constraints on $F_i$: they cannot be arbitrary, and their form should be such that the unitarity and covariance are maintained all the way up to the $M_s$ scale. This problem has been studied extensively in Ref. \[40\] \[41\], where the form of $F_i$ have been constrained appropriately to make sure that the only propagating degree of freedom remains the massless graviton around the flat space time \[40\]. The modified kinetic terms have then to be entire functions (except for the massless graviton pole).

For a particular choice of $F_i$, the gravitational sector behaves like being asymptotically free, i.e. the gravity becomes weakened at the UV \[40\] \[41\]:

$$F_3 = 0, \quad F_1 \left( \frac{\Box}{M_f^2} \right) = \frac{e^{-\Box/M_f^2} - 1}{\Box} = -\frac{F_2 \left( \frac{\Box}{M_f^2} \right)}{2}. \quad (20)$$

Of course these conditions change if we discuss physics in different space time backgrounds, as in case of deSitter or anti-deSitter backgrounds. During inflation, we are indeed interested in the de Sitter case, and the above constraints have to be revisited accordingly.

However, the points are that:

- one would have to consider complete series of higher derivative corrections

- higher derivative corrections in the gravitational sector induces corrections into the scalar degree of freedom. For instance, let us first concentrate on a sub-class of the above action, which was first considered in Ref. \[34\]:

$$L \sim R + R \sum_i c_i \Box_i R. \quad (21)$$

This action is actually equivalent to a scalar-tensor action of the form \[34\]:

$$L \sim \Phi R + \psi \sum_{i=1}^{\infty} c_i \Box_i \psi - \psi(\Phi - 1) - c_0 \psi^2, \quad (22)$$

the equivalence can be seen by $\psi = R$ and $\Phi = 1$.

Incidentally, this action, Eq. (21), has been studied extensively in the context of cosmology. It yields a non-singular bouncing cosmology with the scale factor:

$$a(t) = \cosh \lambda t, \quad (23)$$

where $\lambda \sim \sqrt{\rho_\phi/M_p^2}$, with $\rho_\phi \sim$ const, which signifies an era of super-inflation as pointed out in Ref. \[42\]. The important lesson for us is to keep in mind that the higher derivative terms in the gravitational sector tend to ameliorate the UV properties of Einstein’s general relativity.

The necessity to take into considerations also the gravitational corrections is also important in the context of string theory. In fact, string theory can resum all $\alpha' = M_s^{-2}$ corrections in certain cases, leading for example to the DBI action \[43\] (which has been used in the inflationary models of \[18\]). However, this action does not include string loops, which are those accounting for the higher-order gravitational corrections. At this moment there is no UV complete formulation of string theory capable of taking into account all $g_s$ and $M_s$ corrections, and the tools coming from on-perturbative approaches such as gauge/gravity duality are not sufficiently developed either.
V. POSSIBLE REMEDIES

Given some of the challenges we have posed for super-Planckian inflation, it is important to suggest some solutions to the problem, which would be able at the same time to explain the cosmological observations such as the temperature anisotropy in the CMB and the BICEP2 data.

1. Sub-Planckian field excursion

**Assisted inflation:** It has been known for some time that inflation could be driven collectively by $N$ independent copies of the inflaton field, as suggested in inflation [14], and its generalization when specific cross-couplings were introduced [15]. The simplest choice will be to take then $N$ copies of $m^2 \phi^2$ potential. Similar constructions utilizing axion field have been studied in “N-flation” [27].

One of the virtues of such $N$ copies is that inflation can now occur at sub-Planckian VEVs as pointed out in Refs. [40], and in Ref. [47]. However, in order to explain the temperature anisotropy one would require large number of fields: $N \sim 10^3-10^4$. This models can also account for $r \sim 0.16$, yielding predictions similar to a chaotic inflation model with super-Planckian VEVs.

In [27], this mechanism employing $N$ copies of inflation has been studied in the string theory context with axionic fields enjoying shift symmetry. One advantage of having $N$ fields is that inflation can occur with the decay constants $f_i$ of the axions below $M_p$, which solves the problems of being able to engineer $f > M_p$, as would be needed in single field models, see [18]. One drawback is that the large number of fields can enhance the radiative corrections.

**Single field, inflection-point inflation:** As an alternative, one may imagine that $M_f = M_p$, and realize inflation via some inflection-point inflation [28, 29].

We have briefly discussed such a model in the context of super-Planckian excursion, but the initial motivation for these models were to drive inflation within sub-Planckian field VEVs. In principle it is possible to obtain large observable $r \sim 0.2$ with sub-Planckian excursion of the inflaton field if there exists an inflection point as pointed out in Refs. [40]. One of the advantage is that both the energy density and the VEV of the inflaton remains below the cut-off for the validity of an effective field theory.

However, in presence of a new fundamental scale $M_f < M_p$ and sufficiently large field excursions, one has to revise the model taking into account higher derivative corrections in the inflaton and gravity sector.

2. Asymptotically Free Gravity: non-singular bounce/loitering universe:

There are hints that string theory motivated $\alpha'$ corrections in the gravitational sector might lead to ameliorating the ultraviolet aspects of gravity [50], and Refs. [34, 40]. The gravity can be weakened at scale close to the string scale, $M_s < M_p$, in such a way that it can resolve the singularity arising in the mini Schwarzschild’s black hole (linearized solutions of Eq. (18)). In such a case, even if the inflaton VEV exceeds that of $M_p$, one would not form Planckian size black holes [40], through inflaton coupling to the matter fields. Furthermore, such a non-perturbative formulation of gravity can also yield a non-singular bouncing cosmology [44]. The sub-class of action shown in Eq. (21) also yield a non-singular bouncing cosmology [44], whose perturbations around the bounce solution have been found to be stable [51].

One interesting point when weakening gravity is the possibility to realize a prolong phase of *loitering universe* with a string gas domination [52, 55]. All these models do deviate from inflation to generate density perturbations for the structure formation, and they still need a mechanism to stretch the perturbations on super-Hubble scales.

3. Shift-symmetry from an underlying fundamental symmetry:

As we have mentioned, a shift symmetry of the inflaton field would prevent the appearance of many higher-order terms. This approach has been employed in a number of inflationary models and reconsidered after BICEP2 results, see for example [23, 27, 54, 60]. In particular, if the shift symmetry descends from an underlying short-scale fundamental symmetry (such as it is the case in certain inflationary realisation where it derives from higher-dimensional gauge symmetries), it would not be broken by gravitational effects, see [23, 27]. In the chaotic inflationary scenario the shift symmetry is of course broken already by the $m^2 \phi^2$ term, but this would be just a soft breaking term [23], so that the higher order terms would still be weighted by the soft breaking coefficients.

However, these models still present serious issues to provide predictions in agreement with the observations (for example, some of them need to accommodate for dimensionful parameters larger than $M_p$, or suffer from poor control of string loop corrections, or resort to anthropic arguments applied to the landscape of string flux compactifications to have certain hierarchies among scales or to tune certain coefficients, see the above references). Together with these issues, one should also investigate the corrections to the inflaton kinetic term. In fact, higher derivative kinetic terms do preserve the shift-symmetry, and would appear naturally from $g_s$ and $\alpha'$ corrections.
In conclusion, super-Planckian field excursions of the inflaton would provide us with an important handle on very high-energy physics. In particular, we have pointed out that if the fundamental scale of nature is such that \( M_f \leq M_p \), one would be forced to consider quantum corrections to the inflaton potential and to the kinetic term of the inflaton field. Both these corrections are non-trivial, but in particular the higher derivative corrections to the inflaton kinetic terms lead to ghosts and one has to sum these higher derivative operators to all orders to avoid the ghost problem.

Excluding an anthropically motivated fine tuning of the inflaton coupling to matter, we have outlined some promising approaches, which, interestingly, appear to invoke specific features of the UV theory, in terms of symmetries or UV aspects of gravity. This further stresses the relevance of fully understanding the physical implication of the picture of super-Planckian field excursions in the context of high-energy physics.

Acknowledgments

AM is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1. The work of DC is supported by the Belgian National “Fond de la Recherche Scientifique” F.R.S.-F.N.R.S. with a contract “chargé de recherche”. AM would like to thank Tirthabir Biswas and Masahide Yamaguchi for clarifying some of the relevant issues.

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