The Design of A Rheological Model for Semisolid Metal Slurries

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Abstract. An improved rheological model is obtained to perfect the formula of agglomeration, deagglomeration and stacking mode of particles based on a previous model with the experimental data and statistical mechanics. It is believed that the time evolution of the microstructure determines the rheological behaviour of semisolid metal (SSM), and the effective solid volume fraction connect them perfectly. The new model can describe the time-dependent and time-independent rheological behaviour under a uniform theory. Then, the study on AlSi4Mg2 alloy shows that the predicted apparent viscosity agrees well with the experimental results. The correlation between the apparent viscosity and its microstructure is also identified. This verifies the reliability of the present model.

1. Introduction
The advantage of semisolid metal (SSM) processing originates from its unique rheological behaviour. Following the step of Spencer et al. [1], this technique has attracted a lot of scientists attentions to find the correlation between the rheology and the processing of SSM [2-9]. In the theoretical studies, basing on the power law model built by Joly et al. [6], some theoretical models have been constructed. Among them, it needs to be mentioned that the CF model [7] has explained the rheological behaviour of the SSMS reasonably. However, this model still has its own deficiencies which needs to be improved. In this work, the CF model [7] is improved to be a more reliable model (ICF) on the basis of experimental results and statistical theory.

2. ICF model
In the CF model [7], a semisolid metal (SSM) slurry is regarded as a kind of suspension, where spherical solid particles are suspended in it. In a shear field, there is a competing dynamical process between solid particles and their agglomerate, which results in larger agglomerate (agglomeration) or smaller agglomerate (deagglomeration) under the viscous force. It is thought that the external flow rheological properties of SSM slurry is determined uniquely by its internal degree of agglomeration.

2.1. Time evolution of microstructure
For a given SSM slurry, there are \(N_0\) particles dispersed in the liquid matrix. To simplify the calculation, the solid particle is considered as a sphere (d represents its diameter), the volume is assumed to be 1. The total number of the solid particles in the SSM is given by

\[
N_0 = \frac{6\Phi}{\pi d^3} \tag{1}
\]

where, the \(\Phi\) is solid volume fraction. It can also be expressed as [7]
\[ N_0 = \sum_{i=1}^{\infty} iC_i(t) \]  
(2)

where, \( C_i(t) \) represents the sum of i-mers. Thus, the sum of agglomerates \( M(t) \) is given by

\[ M(t) = \sum_{i=1}^{\infty} C_i(t) \]  
(3)

Subsequently, the average agglomerate size in the slurry, \( n(t) \), is

\[ n(t) = \frac{N_t}{M(t)} \]  
(4)

It can be seen from equation (4), the expression of \( C_k(t) \) should be deduced firstly to get \( n(t) \). The \( C_k(t) \) is described as

\[
\frac{dC_k}{dt} = \frac{1}{2} \left[ \sum_{i=1}^{k-1} K_{i\rightarrow k-i} S_{i\rightarrow k-i} C_i C_{k-i} - \sum_{j=1}^{k} K_{k\rightarrow j} S_j C_k C_j - \frac{1}{2} \sum_{i=1}^{k-1} F_{i\rightarrow k-i} C_k + \sum_{j=1}^{k} F_{k\rightarrow j} C_{k+j} \right]
\]  
(5)

where, \( K \) terms represent the collision rate between agglomerates, \( F \) terms represent the deagglomeration rate, \( S \) terms is the sticking probability. During colliding, the agglomeration rate between the solid particles or agglomerates can be expressed as [7]

\[ A_{ij} = K_{ij} S_{ij} = \left[ \alpha_i (i+j) \Phi + \alpha_2 \hat{\gamma} \right] \left( d_1 \hat{\gamma} + d_2 \right) \frac{c}{c + \hat{\gamma}} = \left[ \alpha_i (i+j) \Phi + \alpha_2 \hat{\gamma} \right] K_o \]  
(6)

where, \( \alpha_i \) and \( \alpha_2 \) are material constants. It should be noted that \( \alpha_c = -0.002 \) for SnPb15 alloy [7], which is meaningless. By analyzing the equation (1), the first term in the square brackets can represents all the size effects. Thus, in this work, the agglomeration rate is formulated by

\[ A_{ij} = K_{ij} S_{ij} = (i+j) \Phi (d_1 \hat{\gamma} + d_2) \frac{c}{c + \hat{\gamma}} = (i+j) \Phi K_a \]  
(7)

As for the deagglomeration rate, its expression is [7]

\[ F_{ij} = K_{dij} = d_3 \hat{\gamma} \]  
(8)

It is in agreement with the experimental finding [8] that the deagglomeration rate is proportional to the shear rate. However, the \( F_{ij} = 0 \) when the shear rate is zero. That means there is only the agglomeration but no deagglomeration at this time according to equation (8). In fact, the agglomeration and deagglomeration exist simultaneously at any time. In a shear field, the agglomeration process is dominant when the shear rate is smaller, conversely, the deagglomeration process is dominant when the shear rate is larger. Therefore, in this work, the deagglomeration rate is given by

\[ F_{ij} = K_d = d_4 \hat{\gamma} \]  
(9)

where \( d_4 \) is a material constant. Combining equation (3)-equation (5), equation (7) and equation (9), \( n(t) \) is derived as

\[
\frac{1}{n(t)} = \frac{1}{n_c} + \left( \frac{1}{n_c} - \frac{1}{n_0} \right) e^{-\lambda t}
\]  
(10)

where, \( n_0 \) is chosen according to the given alloy system, the other parameters are

\[
n_c = 1 + \frac{2N_0 \Phi K_a}{K_d} = 1 + \frac{12\Phi^2 K_a}{\pi d^3 K_d}
\]  
(11)
\[ \lambda = \frac{6 \alpha \Phi^2 K_w}{\pi d^3} + \frac{1}{2} K_d \]  

(12)

2.2. Effective solid volume fraction
The effective solid volume fraction is [7]

\[ \Phi_{\text{eff}} = \left( 1 + \frac{B}{n_e} \right) \Phi \]  

(13)

where, \( B = 6r_{ol}/\pi d^3 \), it is assumed to be \( B = (n-1)A \), where \( A \) is only related to \( \Phi \) [7]. As a matter of fact, the packing of the solid particles in SSM slurry may be relevant to both its internal properties (such as the shape, the size) and its external conditions (such as shear rate, time). The experimental results of Perez et al. [9] indicate that the volume of entrapped liquid in an agglomerate increases when the number of solid particles increases, that is, \( B \) is related to \( n \). This illustrates that both the shear rate and the solid volume fraction can change the volume of the entrapped liquid in an agglomerate by changing the number of solid particles. Based on the above consideration, \( B \) is expressed as

\[ B = f_1 n_e^{f_2} + f_3 \]  

(14)

where, \( f_1, f_2, f_3 \) are the material constants. Subsequently, the effective solid volume fraction is

\[ \Phi_{\text{eff}} = \left( 1 + \frac{B}{n_e} \right) \Phi = \left( 1 + \frac{f_1 n_e^{f_2} + f_3}{n_e} \right) \Phi \]  

(15)

2.3. Apparent viscosity
According to the Dougherty and Krieger’s equation [10], the apparent viscosity is given by

\[ \eta = \frac{\eta_0}{(1 - \Phi_{\text{eff}})^{2.5}} \]  

(16)

Thus, the constitutive equation are

\[ \tau = \eta(n, \dot{\gamma}) \dot{\gamma} \]  

(17)

\[ \frac{dn}{dt} = g(n, \dot{\gamma}) \]  

(18)

3. Application of ICF model to the AlSi4Mg2 Alloy
It needs to determine the parameters before applying the ICF model to the AlSi4Mg2 Alloy. Here, the experimental results of Zhou et al. [9] are used to fit the parameters. The fitting parameters are listed in Table 1.

| Parameter | Value |
|-----------|-------|
| \( f_1 \) | 0.9   |
| \( f_2 \) | 1.01  |
| \( f_3 \) | -0.9  |
| \( d_1 \) | 1.413 |
| \( d_2 \) (s) | 0.01 |
| \( d_3 \) | 0.12  |
| \( d_4 \) (s) | 0.2  |
| \( c \) (s) | 0.129 |
Now, the calculated viscosity and the experimental data[9] are plotted in Fig.1. As shown in Fig.1, the present prediction agrees with the experimental findings[9]. Fig.2 may explain this variation in Fig.1. Specifically, as shown in Fig.2, the increasement of the solid volume fraction results in the variation of the packing mode and the agglomerate size, which increases the effective solid volume fraction and the apparent viscosity, respectively. This can be identified from the derivation of equation(13)-equation (16).

4. Conclusions
The ICF model is proposed to improve the limits of CF model by refining the expression of the agglomerate rate, the deagglomerate and the packing mode. The new model can describe the pseudoplastic behaviour and the thixotropic behaviour simultaneously. It also discloses the nature of the rheology of SSMS, which origins its microstructure and the effective solid volume fraction is the bridge between them. Subsequently, the rheological behaviour of AlSi4Mg2 alloy is studied on the new model. It shows that the calculated apparent viscosity is in agreement with the experimental data. There is a close relationship between the rheological behaviour and its microstructure. This indicates the ICF model is reliable to predict the rheological behaviour of AlSi4Mg2 alloy.

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