A note on “Traversable wormholes in Einstein-Dirac-Maxwell theory”

S. V. Bolokhov, a,1 K. A. Bronnikov, a,b,c,2 Serguey Krasnikov, d and M. V. Skvortsova a,3

a Peoples’ Friendship University of Russia (RUDN University), ul. Mikhukho-Maklaya 6, Moscow 117198, Russia
b Center for Gravitation and Fundamental Metrology, VNIIMS, Ozyornaya ul. 46, Moscow 119361, Russia
c National Research Nuclear University “MEPhI”, Kashirskoe sh. 31, Moscow 115409, Russia
d Central Astronomical Observatory at Pulkovo, St. Petersburg, 196140, Russia

In their Letter [Phys. Rev. Lett. 126, 101102 (2021); arXiv: 2010.07317], J. L. Blázquez-Salcedo, C. Knoll and E. Radu have constructed a very interesting class of wormhole solutions in general relativity (GR), supported by a pair of classical charged spinor fields obeying the Dirac equation. The main new feature of these solutions is that such Dirac spinor fields can possess exotic properties, necessary for the existence of static wormhole configurations in GR. The present note contains a few remarks clarifying some points concerning this approach.

1. The authors of [1] insist that their spinor fields are particle wave functions, which entails their one-particle normalization and the corresponding values of the solution parameters.

As shown by Fig. 2 in [1], the curvature scalars at the throat are of the order of unity in Planck units (the Ricci and Kretschmann scalars are \( \approx 0.1 \) and \( \approx 6 \), respectively). This suggests — as no fine tuning is mentioned in the text — that vacuum polarization inducing geometric corrections to the Einstein equations are of the same order of magnitude as the their left-hand sides themselves. Thus the metric solving the Einstein equations generally does not solve the actual equations of motion.

On the other hand, a direct \( n \)-particle generalization of the present solutions is impossible due to the Pauli exclusion principle, so the problem should be attacked anew.

2. The authors claim that if \( \nu' \neq 0 \) at the throat (where \( 2\nu = \ln(-g_{tt}) \)) requires the presence of extra matter forming a thin shell at the throat \( r = 0 \). It is not true: in fact, \( \nu'(0) \neq 0 \) simply means that the wormhole must be nonsymmetric relative to the throat, while a thin shell is only required if one postulates an unnecessary \( Z_2 \) symmetry with respect to \( r = 0 \). A search for nonsymmetric wormhole solutions can be one of possible interesting extensions of the authors’ study. Examples of other non-\( Z_2 \)-symmetric spherical wormholes are well known.

3. It is asserted in [1] that the solutions obtained numerically are \( Z_2 \)-symmetric. Meanwhile, after Eq. (8) we read that “\( Q_N \) is the Noether charge of a spinor (or number of particles),” and then, after the unnumbered expression for \( Q_N \), we read that “Similar relations hold for the \( r < 0 \) region, with mass, electric charge and Noether charge changing sign.” This raises some objections: (i) it looks too strange to have a negative particle number, not to mention that a normalizing integral should cover the whole volume, \( r \in \mathbb{R} \), and the particle number should pertain to the whole configuration; (ii) \( Z_2 \) symmetry implies equal ADM masses on both ends (as is confirmed by the curve for \( g_{tt} \) in Fig. 2), then how to understand a “mass changing sign”?

4. The metric (5), obtained as an exact solution with massless spinors, has been previously discussed as a possible metric in a brane world supported by a bulk-induced tidal stress-energy tensor [23].

---

1 e-mail: boloh@rambler.ru
2 e-mail: kb20@yandex.ru
3 e-mail: milenas577@mail.ru
5. The words in the abstract, that the wormhole solutions are obtained “without needing any form of exotic matter,” look misleading since exotic matter (by definition, matter violating the Null Energy Condition) is quite necessary at a wormhole throat [4]. It would be better to say that Dirac spinor fields become exotic matter under certain circumstances.

We conclude that the authors of [1] have made an interesting contribution to wormhole physics, but hope that our remarks clarify some important points concerning their approach.

It is worth mentioning that the idea of using spinor fields to construct traversable wormholes in four dimensions was studied in a number of earlier works, for instance, by J. Maldacena et al. [5].

Since the present note was written and submitted (April 2021), a few relevant papers have appeared [6-9]. In particular, J. L. Blázquez-Salcedo et al. [6] presented a more detailed description of the wormhole solution under discussion, including an investigation of the domain of existence for the solutions. R. Konoplya et al. [7,8] obtained and discussed other wormhole configurations supported by a pair of spinor fields, mentioning that changing the sign of the fermionic charge density at the throat can hardly be understood as a physically realistic scenario. Lastly, in the recent paper by D. Danielson et al. [9] it is argued that the wormhole solution presented in [1] suffers from a failure of the Maxwell and Dirac fields to satisfy the necessary matching conditions.

References

[1] Jose Luis Blázquez-Salcedo, Christian Knoll, and Eugen Radu, “Traversable wormholes in Einstein-Dirac-Maxwell theory,” Phys. Rev. Lett. 126, 101102 (2021).

[2] K. A. Bronnikov and S.-W. Kim, “Possible wormholes in a brane world,” Phys. Rev. D 67, 064027 (2003).

[3] K. A. Bronnikov, V. N. Melnikov, and H. Dehnen, “On a general class of brane-world black holes,” Phys. Rev. D 68, 024025 (2003).

[4] M. S. Morris, K. S. Thorne, and U. Yurtsever, “Wormholes, time machines, and the Weak Energy Condition,” Phys. Rev. Lett. 61, 1446 (1988).

[5] Juan Maldacena, Alexey Milekhin, and Fedor Popov, “Traversable wormholes in four dimensions,” arXiv: 1807.04726.

[6] Jose Luis Blázquez-Salcedo, Christian Knoll, and Eugen Radu, “Einstein-Dirac-Maxwell wormholes: ansatz, construction and properties of symmetric solutions,” arXiv: 2108.12187.

[7] R. A. Konoplya and A. Zhidenko, “Traversable wormholes in General Relativity without exotic matter,” arXiv: 2106.05034.

[8] M. S. Churilova, R. A. Konoplya, Z. Stuchlik, and A. Zhidenko, “Wormholes without exotic matter: quasinormal modes, echoes and shadows,” arXiv: 2107.05977.

[9] Daine L. Danielson, Gautam Satishchandran, Robert M. Wald, and Robert J. Weinbaum, “The Blázquez-Salcedo, Knoll, and Radu wormholes are not solutions to the Einstein-Dirac-Maxwell equations,” arXiv: 2108.13361.