Fast and Low-Memory Deep Neural Networks Using Binary Matrix Factorization
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Abstract—Despite the outstanding performance of deep neural networks in different applications, they remain computationally expensive and require a great amount of memory. This motivates more research on reducing the resources needed for implementing such networks. An efficient approach addressed for this purpose is matrix factorization, which has been shown to be effective on different networks. In this paper, we design a training algorithm which utilizes binary matrix factorization and show its efficiency in reducing the required resources in deep neural networks. In effect, this technique leads to the fast and practical implementation of such networks.

Index Terms—Deep neural networks, computation reduction, network compression, memory reduction.

I. INTRODUCTION

The remarkable success of Deep Neural Networks (DNNs) has made them the go-to choice for tackling many problems in different fields. Convolutional Neural Networks (CNN) [1]–[3] and Long Short-Term Memory (LSTM) networks [4], [5] are the most common types of networks used for visual recognition and language modeling tasks. The success of DNN can be attributed to over-parametrization, which leads to computational and memory complexity. Accordingly, different methods are employed to alleviate these burdens and make state-of-the-art networks to be more practical.

Early network compression methods are based on removing parameters with insignificant contribution to the network, commonly referred to as “pruning” [6], [7], [9], [10]. While effective in reducing the number of parameters, the performance of these methods is largely dependent on the initial stages of pruning, meaning that the hyper-parameters play a great role in the success of pruning methods. Another strategy reported is based on reducing the number of parameters by applying $l_1$ regularization to the network parameters [8].

Reducing the precision of network parameters is another effective way of mitigating the computation and memory costs of deep neural networks. Training neural networks is usually done in single-precision (FP32) arithmetic. However, half-precision (FP16) arithmetic can also be utilized without losing much accuracy [39]. A combination of both FP32 and FP16 parameters can be also used to improve the performance at the cost of reducing compression. While using mixed training does not change the number of Floating-Point Operations (FLOPs) in a network, it reduces the computational cost of each operation [40].

Binary Neural Networks (BNN) are a special type of lower precision networks that exhibit low computational and memory costs. The main goal of these methods is to restrain the values of the network parameters to +1 and -1. This is commonly utilized in CNNs, where convolution operation accounts for a large portion of total operations. The most important challenge in these methods is the training stage, for which different approaches have been addressed. In [30], a constraint is imposed on the value of gradients and then trains the network. [31] combines binary convolutions and residual training to increase the performance of the network. To compensate for the accuracy loss caused by using binary parameters, other methods have been proposed by focusing on: minimizing the quantization error [32], [33], modifying and improving the loss function [34], [35], [36], and reducing the gradient estimation error [37], [38]. While these methods seem to increase the performance of the networks, there still exists a considerable gap between BNNs and real-valued CNNs.

Another recent line of research focuses on exploiting the representation redundancy of neural networks to reduce the number of parameters. As such, matrix and tensor factorization methods have been effective in different structures, especially in CNN and Recurrent Neural Networks (RNN) [11]–[15]. One of the most recent methods uses the multiplication of sparse matrices as a substitute for the original matrices [17]. Applying this method yields sparse networks whose accuracy is very close to that of the original, over-parametrized networks. While matrix factorization and pruning methods are distinctly different methods, both attempt to compress DNNs by the means of sparsification.

In this paper, we propose the "Fast and Low-Memory Deep Neural Networks" method (FLM-DNN) by implementing binary matrix factorization (BMF) [18], [19] in DNNs, which leads to significant reduction in computational complexity and memory requirements.

The paper is organized as follows. In section II the proposed method is introduced and explained. In section III, the performance metrics adopted to compare the performance of different methods are introduced. Section IV is dedicated to simulations and experiments, which are conducted on a wide variety of DNNs.

II. PROPOSED METHOD

A. Notation

Similar to [17], we use $\Theta$, $\Theta_d$, and $\Theta_r$ to show the index set of all weight matrices, the weight matrices to be factorized, and the weight matrices to be left unfactorized, respectively. The forward mapping of a network denoted by $F(y|x)$ is
B. Training Binary-Factorized Deep Neural Networks

Matrix factorization in DNNs can be used to train a network with sign-factorized weights matrices first, and then convolution kernels, that is, where $W_k = Z_k R_k$, $Z_k \in \{0,1\}^{n \times r}$, $R_k \in \mathbb{R}^{r \times m}$. 

The main goal of this work is to represent DNNs with binary-factorized weight matrices or convolution kernels, that is

\[
W_k = Z_k R_k, \quad Z_k \in \{0,1\}^{n \times r}, \quad R_k \in \mathbb{R}^{r \times m}, \quad (4)
\]

where $Z_k$ and $R_k$ are referred to as the binary and loading matrices, respectively. We will show in Section III that the use of binary weight matrices and real-valued parameters in the DNN will significantly reduce the memory and computational costs. However, a major question that may arise is whether binary factorization can theoretically preserve the representational ability of the corresponding DNNs. To answer this question, we briefly refer to the following theorems which detailed proofs are found in [18], [19].

**Theorem 1.** Matrix $B \in \mathbb{R}^{n \times m}$ admits a trivial sign component decomposition with inner dimension $r = n$, as long as matrix $S \in \{\pm 1\}^{n \times n}$ is non-singular. Furthermore, a minimal binary factorization can be found where $r = \text{rank}(B)$. In the cases of minimal decomposition, sign decomposition is unique.

**Theorem 2.** Assume matrix $B \in \mathbb{R}^{n \times m}$ admits a sign factorization as $B = SR$, where $S \in \{\pm 1\}^{n \times r}$ and $R \in \mathbb{R}^{r \times m}$. Then, matrix $C = \frac{1}{2}(B + E)$ admits a binary decomposition as $C = ZR$ where $Z \in \{0,1\}^{n \times r}$ and $E$ is a matrix of ones with appropriate dimensions. Not only loading matrices for both matrices $B$ and $C$ are equal, but also there exists the mapping

\[
F : \{0,1\}^{n \times r} \rightarrow \{\pm 1\}^{n \times r}; \quad F : Z \rightarrow 2Z - E
\]

between the sign and binary matrices $S$ and $Z$.

Thus, to represent a DNN with factorized weights matrices, according to (4), (i) is modified as

\[
\{W_i\} = \arg \max_{\{W_i\}} \mathbb{E}_{x,y} \left[ \log \left( p \left( \{W_i\}, x, y \right) \right) \right], \quad (5)
\]

subject to $W_i = Z_i R_i$, $Z_i \in \{0,1\}^{n \times r}$, $R_i \in \mathbb{R}^{r \times m}$.

It is immediately obvious that training a network using (5) is very challenging due to the fact that the imposed binary constraint on matrices $Z_i$ is not convex. Instead, an equivalent convex problem can be defined as follows, which enables us to train a network with sign-factorized weights first, and then map the sign matrices to binary matrices:

\[
\{W_i\} = \arg \max_{\{W_i\}} \mathbb{E}_{x,y} \left[ \log \left( p \left( \{W_i\}, x, y \right) \right) \right], \quad (6)
\]

subject to $W_i = S_i A_i$,

\[
-1 \leq s_{jk} \leq 1, \quad \|S\|_{F}^{2} = n \times r \quad \text{for} \quad \forall S \in \{S\}_i.
\]

According to (6), as long as weight matrix entries remain within $[-1, 1]$, and the Frobenius norm of each sign weight...
The number of parameters in the networks used to experiment in this work, the number of FLOPs is calculated in a similar way to multi-layer perceptron networks. In recurrent networks such as vanilla RNN (Recurrent Neural Network), GRU (Gated Recurrent Unit), and LSTM (Long Short-Term Memory) wherein the dominant operation is matrix-vector factorization and vector addition, the number of FLOPs is calculated in a similar way to multi-layer perceptron networks.

When matrix factorization is applied to a layer, the number of FLOPs is obtained as follows \cite{17}:

- When a weight matrix $W \in \mathbb{R}^{n \times m}$ between two fully-connected layers is factorized into two dense and real-valued matrices $A \in \mathbb{R}^{n \times r}$ and $B \in \mathbb{R}^{r \times m}$, it is modified to

$$
FLOPs = 2 \times \left( \frac{nr}{t_A} + \frac{nr}{t_B} \right),
$$

where $t_A$ and $t_B$ are the compression rates of $A$ and $B$, respectively. When the latter matrices are not sparse, their compression rates are set to 1.

- When a convolutional kernel is factorized into two kernels, it is calculated by dividing the number of FLOPs in new kernels by their respective compression rates. Convolutional layer factorization is performed according to \cite{41}.

Lastly, when binary representation is used along with sparse factorization, the number of additions and multiplications in the network should be calculated separately and added.

The number of parameters in the networks used to experiment on here can be calculated using the following formulae.

\begin{align}
\text{Total Memory} &= \text{(number of real-valued params.)} \times 32 \\
&\quad + \text{the number of binary params.}.
\end{align}
• In a dense fully-connected layer, the number of parameters equals 
  \((m \times n) \times d \times k\) 
where \(k\) and \(d\) are the number of filters in a layer and 
its previous layer, respectively, and \(m \times n\) is the shape of 
the kernels in the current layer.

• In a dense convolutional layer, the number of parameters is 
  \(((m \times n) \times d) + 1) \times k\)

A vanilla RNN having three layers with \(m\) input, \(n\) output, 
and \(h\) hidden units, the number of parameters, including 
two bias vectors, at each time stamp is equal to 
\(h^2 + nh + mh + n + h\).

Similarly, a GRU network with \(m\) input and \(n\) output units 
has \(3(n^2 + nm + 2n)\) parameters. Finally, in an LSTM 
network with the same number of inputs and outputs, the 
total number of parameters is \(4(n^2 + nm + n)\).

IV. EXPERIMENTS

In this section, the effectiveness of FLM-DNN algorithm is 
evaluated on a wide variety of networks and some commonly 
used datasets, and its performance is compared with other 
state-of-the-art network compression methods.

A. Setup and Datasets

We experimented on nine different networks including 
LeNet-300-100, LeNet-5 Caffe [25], a VGG-like network [26], 
ResNet164, ResNet50, Vanilla RNN, two LSTM networks, 
and one GRU network. The datasets are differently selected 
according to the task and network. Overall, we use five datasets 
in the experiments including: MNIST [21], CIFAR-10 [22], 
CIFAR-100 [22], ImageNet [1], and Penntreebank (PTB) [24].

To introduce, the MNIST consists of 70000 labeled gray images, 
CIFAR-10 and CIFAR-100 consist of 60000 color images with 
10 and 100 classes, respectively, ImageNet is an annotated 
color image data set of 14,197,122 images. Also, the Penn 
Treebank (PTB) project selected 2,499 stories from 
a three year Wall Street Journal (WSJ) collection of 
98,732 stories for syntactic annotation, which is widely used 
for Natural Language Processing (NLP) applications. All the data 
sets are split according to the respective original references.

For Multi-Layer Perceptron (MLP) networks, LeNet-300- 
100 is used, which is a four-layer fully-connected network 
with two hidden layers containing 300 and 100 neurons, 
respectively. The output layer consists of 10 neurons. Also, 
the MNIST is applied for training and evaluating the network.

The values and settings provided here will remain the same 
throughout the rest of the experiments, unless stated otherwise. 
Adam [27] is used as the optimizer, and the number of epochs 
for MLP training is set to 300. Since norm regularization 
methods only make the weights approach zero, a threshold 
value \(\epsilon\) is adopted. This value is \(exp(-4)\) for perceptron layers, 
\(exp(-6)\) for convolutional layers, and \(exp(-4)\) for recurrent 
units. (Similar to [20],) We adopt memory consumption and 
the number of FLOPs needed in a single forward pass stage 
as metrics to evaluate and compare the performance of our 
proposed method with a few other state-of-the-art DNN com-
pression methods.

B. Validation of Training Algorithm

In this section, we design an experiment to empirically 
evaluate the effectiveness of the FLM-DNN training method 
to approximate a given matrix \(W \in \mathbb{R}^{n \times m}\) with rank \(r\) in 
an MLP network with only one factorized layer. To do so, 
consider a neural network with \(m\) input and \(n\) output neurons 
and one hidden layer. Furthermore, assume the hidden and 
output layers have identity activation function \(\sigma(X) = x\). 
Then, using binary factorization according to (3) and (4), 
the relationship between the input vector \(x \in \mathbb{R}^n\) and output 
vector \(\hat{y} \in \mathbb{R}^r\) can be written as 
\[
\hat{y} = Z_rR_x
\]
where \(Z_r \in \{0, 1\}^{n \times r}\) and \(R_x \in \mathbb{R}^{r \times m}\). Also, assume that 
we utilize this network for a linear regression problem. Therefore, 
the loss function over a mini-batch \(\{x_{n'}, y_{n'}\}\) of the training data 
with \(N'\) data points is defined as 
\[
L(\hat{y}, y) = \frac{1}{N'} \sum_{n'=0}^{N'-1} \|\hat{y}_{n'} - y_{n'}\|^2_2 
\]
(11)

Then, we can design the data set in a way that the loss function 
in (11) achieves its minimum value for \(Z_r, R_x = W\).

**Proposition 1.** For a set of linearly independent vectors 
\(\{x_{n'}\}\), by setting \(y_{n'} = Wx_{n'}\), the loss function in (11) 
achieves its minimum for \(Z_r, R_x = W\).

**Proof.** Computing the gradient of \(L(\hat{y}, y)\) with respect to \(y\) yields 
\[
\nabla_y L(\hat{y}, y) = -\frac{2}{N'} \sum_{n'=0}^{N'-1} (Z_rR_x x_{n'} - y_{n'}) ,
\]
(12)

and setting \(y_{n'} = Wx_{n'}\) reduces \(\nabla_y L(\hat{y}, y)\) to zero. As 
\(L(\hat{y}, y)\) is a convex function in \(y\), this solution achieves the 
global minimum. This concludes the proof.

We conduct Monte-Carlo simulations to verify Proposition 
1 and the representational capability of FLM-DNN. In each 
iteration, \(Z \in \{0, 1\}^{n \times m}\) and \(R \in \mathbb{R}^{r \times m}\) are generated 
randomly using Bernoulli and Gaussian distributions, respectively. 
Next, matrix \(W = ZR\) is constructed. Then, \(2^{18}\) input vectors 
\(\{x_i\}\) are randomly drawn from a Gaussian distribution, and 
the output vectors \(\{y_i\} = \{Wx_i\}\) are calculated. Finally, the 
corresponding input and output vectors are paired together as 
\(\{(x_i, y_i)\}\) and the network is trained using Algorithm [1]. 
This process is repeated 20 times, and the mean results are reported 
in Tables [1] and [11]. First, we investigate the construction error

\[\text{https://github.com/geifmany/cifar-vgg.git}\]
of matrices with fixed rank and varying sizes. The results are shown in Table I in which the reconstruction error (RE) is defined as

$$RE = \frac{\|W - W_*\|_F}{\|W\|_F}$$

(13)

for reconstructed matrix $W_*$. It is observed that for low-rank matrices, the FLM-DNN training algorithm approximates $W$ very accurately, in the sense that a reconstruction error of 0.01 causes almost indistinguishable differences visually, as will be demonstrated in Section IV-C.

The second experiment focuses on the case where the dimensions of matrix $W$ are fixed to $n = 300$ and $m = 150$, but its rank varies. The results of this experiment, as demonstrated in Table II and Fig. 2, suggests that as the rank of $W$ increases, the reconstruction error increases as well. This is due to the combinatorial and discrete nature of binary matrix factorization, meaning that as the matrix rank increases, the number of independent binary vectors that should be correctly found by the network increases as well. In other words, this may lead to an increasing number of incorrectly identified binary vectors, which leads to increasing reconstruction errors.

C. Image Reconstruction with Autoencoders

In the first practical experiment, we employ the FLM-DNN in image reconstruction using an autoencoder network, which consists of 7 hidden layers with 256, 128, 64, 10, 64, 128, and 256 neurons. The dataset used is MNIST, and the output layer has 10 neurons and utilizes softmax activation.

And 256 neurons. The dataset used is MNIST, and the output consists of 7 hidden layers with 256, 128, 64, 10, 64, 128, and 256 neurons. The dataset used is MNIST, and the output layer has 10 neurons and utilizes softmax activation.

Next, we investigate the effect of factorizing different layers on accuracy and computational complexity in LeNet-300-100 network. The results are compared with the other state-of-the-art methods, such as the progressive pruning [6], Dynamic Network Surgery (DNS) [14], and Sparse Variational Dropout (SVD) [16]. The layer-wise scaling regularization factors are selected as $[0.00001, 0.000025, 0.00015]$. Similar to [17], a notation like "1-0-0" means that the first weight matrix is factorized, and the others are not. Table III shows the results.

D. Experiments on Multi-Layer Perceptron Networks

To evaluate the proposed FLM-DNN in MLP networks, we investigate the accuracy and computational complexity of different algorithms in the LeNet-300-100 network [21]. The results are compared with the other state-of-the-art methods, such as the progressive pruning [6], Dynamic Network Surgery (DNS) [14], and Sparse Variational Dropout (SVD) [16]. The layer-wise scaling regularization factors are selected as $[0.00001, 0.000025, 0.00015]$. Similar to [17], a notation like "1-0-0" means that the first weight matrix is factorized, and the others are not. Table IV shows the results.

Our next experiment investigates the effect of factorizing different layers on accuracy and computational complexity in LeNet-300-100 network. The results are compared in Table IV. Next, we investigate the effect of changing the common dimension of LeNet-300-100 network on the accuracy and computational complexity of algorithms. In this experiment, only the first layer of the network is factorized. The results are demonstrated in Table V.

The following conclusions can be drawn from the results shown in Tables IV, V, and VI: DO YOU MEAN THESE TABLES???. (WE SHOULD TALK???)

1) FLM-DNN competes with the other state-of-the-art network compression methods in terms of accuracy, but with a lower computational cost.
2) As demonstrated in Table V configurations in which the first layer of the LeNet-300-100 network is factorized are more compressed than others. Furthermore, factorizing the weight matrix of the second layer leads to more compression than the third layer. Since the layers of the LeNet-300-100 network decrease in size as we move toward the output layer, this finding suggests that factorizing larger layers in an MLP network lead to more memory and computation reduction.

3) Reducing the rank of the factorized weight matrix will not deteriorate the performance of the network, unless the dimension is chosen very small. Furthermore, reducing the dimension does not necessarily lead to more compression, since it reduces the number of parameters in the weight matrix of a layer as well as the rank of that matrix, meaning that the achievable compression rate for that weight matrix is also reduced.

### TABLE IV
**Comparison of different methods in terms of memory usage, FLOPs, and error rate (ER).**

| Method          | Memory (KBits) | FLOPs   | ER (%) |
|-----------------|---------------|---------|--------|
| Original [21]   | 8518          | 5.32e5  | 1.64   |
| Progressive [6] | 696.83        | 4.36e4  | 1.59   |
| DNS [14]        | 151.63        | 9.48e3  | 1.99   |
| SVD [10]        | 127.51        | 7.97e3  | 1.92   |
| SMF (1-0-0-1) [17]| 122.14       | 7.63e3  | 1.83   |
| FLM-DNN (1-0-0-1)| 86.52        | 7.1e3   | 1.85   |

### E. Experiments on CNNs

We have considered four CNN networks for our experiments. The first one is LeNet-5 Caffe [25], one of the simplest CNNs consisting of an input layer, four convolutional layers, two pooling layers, and two fully-connected layers with 50 and 10 neurons, respectively. The training and evaluation dataset is MNIST. The next network is a simplified version of the VGG network [28], which shall be referred to as “VGG-Like” [26], for which we utilize the CIFAR-10 dataset. Furthermore, the two residual networks ResNet164 and ResNet50 [3] are tested and ImageNet and CIFAR-100 datasets are used for training, respectively. Since most of the LeNet-5 parameters lie in the fully-connected layers, only those layers are factorized. The common dimension of factorization is \( p = 400 \), and \( l_1 \) regularization is utilized.

From Table VII one can see that while FLM-DNN reduces the memory consumption about 17%, the number of FLOPs almost stays the same. The reason is that, despite the fact that a huge portion of the parameters lie in the first fully-connected layer, it is the second deep convolutional layer that takes up
the biggest part of the calculations. Fortunately, convolutional layers can be factorized in a similar manner to Fig. 1 by using horizontal and vertical filters to approximate the original filter [41] as shown in Fig. 4. We apply FLM-DNN algorithm to the method proposed in [41] to compress the convolutional layers in VGG-like and two residual networks.

Given a convolutional layer that includes \( n \times c \) filters with a common spatial size of \( 3 \times 3 \), one can factorize them into two low-rank groups of \( n \times p \) and \( p \times c \) layers with the spatial sizes of \( 3 \times 1 \) and \( 1 \times 3 \), forming horizontal and vertical filters, respectively. In the VGG-like network, the last three convolutional layers (\( n = c = 512 \)) along with the first fully-connected layer (\( m = n = 512 \)) are factorized. The learning rate for training this network is 0.0005. The norm regularization factor is set to \( \lambda_0 = 0.0006 \). Table VIII demonstrates the effectiveness of FLM-DNN in a VGG-like network. As observed, by applying binary convolutional factorization, both memory consumption and the number of calculations are considerably reduced. The reason is that by binary-factorizing the last convolutional layer, the number of FLOPs are significantly reduced, which was not the case in LeNet-5 network.

The same convolutional layer factorization method is applied to ResNet-50 and ResNet164 residual networks, which consist of residual "blocks" that are repeated consecutively to form larger blocks, or "modules". More information can be found in [3] on how different structures of residual networks are formed. Here, we only factorize the convolution layers in the last module of ResNet-50 and ResNet-164. The results are shown in Table IX. Here, similar to the VGG-Like network, factorizing the layers using FLM-DNN method has led to significant reduction in FLOPs and memory.

F. Experiments on RNN, GRU, and LSTM Networks

First, a vanilla RNN [42] is trained using MNIST dataset. The rows of images are fed into the network sequentially. The vanilla RNN has three parametrized input-to-hidden, hidden-to-hidden, and hidden-to-output layers. However, we only

![Fig. 4. Factorizing a convolutional layer into two layers with a lower rank. The original kernel is divided into horizontal and vertical layers [41].](image)

TABLE VII

| Network          | Method | Memory (Mb) | FLOPs  | ER (%) |
|------------------|--------|-------------|--------|--------|
| LeNet-5 Caffe    | Original [25] | 13.21 | 4.58e6 | 0.8    |
|                  | SMF [17]            | 0.048 | 3.0e5  | 0.88   |
|                  | FLM-DNN             | 0.040 | 2.99e5 | 0.90   |

TABLE VIII

| Network            | Method | Memory (Mb) | FLOPs  | ER (%) |
|--------------------|--------|-------------|--------|--------|
| VGG-like           | Original [69] | 765.19 | 3.69e8 | 0.90   |
|                    | SMF [17] | 104.83 | 8.76e6 | 0.88   |
|                    | FLM-DNN | 85.91  | 7.05e6 | 0.90   |

TABLE IX

| Network          | Method | Memory (Mb) | FLOPs  | ER (%) |
|------------------|--------|-------------|--------|--------|
| ResNet50         | Original [3] | 733.03 | 3.8e9  | 23.51  |
|                  | SMF [17] | 109.73 | 6.5e7  | 24.65  |
|                  | FLM-DNN | 88.88  | 5.54e7 | 24.82  |
| ResNet164        | Original [3] | 54.08  | 2.6e8  | 23.71  |
|                  | SMF [17] | 13.93  | 7.6e7  | 23.91  |
|                  | FLM-DNN | 11.74  | 6.53e7 | 23.99  |

TABLE X

| Network          | Method | Memory (Mb) | FLOPs  | ER (%) |
|------------------|--------|-------------|--------|--------|
| Vanilla RNN      | Original [22] | 2.65  | 4.62e6 | 1.67   |
|                  | SMF [17] | 0.03137 | 1.954e3 | 1.58  |
|                  | FLM-DNN | 0.01883 | 1.421e3 | 1.60  |

TABLE XI

| Network          | Method | Memory (Mb) | FLOPs  | ER (%) |
|------------------|--------|-------------|--------|--------|
| GRU               | Original [43] | 7.11   | 1.77e7 | 0.85   |
|                  | SMF [17] | 0.0327 | 6.21e4 | 1.18   |
|                  | FLM-DNN | 0.01933 | 5.49e4 | 1.20   |

Words are fed into the network, which predicts the sequential words in each sentence. "Perplexity" is the evaluation metric we use in this scenario [29]. In this experiment, two LSTM networks consisting of ten layers including one embedding, four input-to-hidden, four hidden-to-hidden, and one hidden-to-output layer are considered. Two LSTM cells are used in each network. What distinguishes these networks is the number of inputs and representation dimensions, which are 650 and 1500 for each network. \( L_1 \) norm regularization is adopted and all layers except the hidden-to-output layers are factorized.

V. Conclusion

In this paper, we proposed the FLM-DNN method by implementing binary factorization of weight matrices in deep neural networks. Due to the properties of binary factorization,
such as high representational capability and incorporating binary parameters as well as real-valued ones. FLM-DNN demonstrates an impressive ability to reduce the computational cost of deep neural networks. Most importantly, we showed that along with this merit, the network preserves its accuracy. To evaluate FLM-DNN, ten different networks with different structures were used. The effectiveness of this algorithm was shown in comparison to some state-of-the-art methods such as Sparse Matrix Factorization, Progressive Pruning, Variational Dropout, and Dynamic Network Surgery. Also, the results suggested that applying binary factorization to weight matrices that belong to more computationally expensive layers leads to more compression and resource efficiency. This was especially highlighted in ResNet and LSTM networks, where an almost 20% reduction in the required memory and a 15% reduction in the number of FLOPs were achieved.

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