In this talk, some aspects of duality symmetries are presented. In the last few years, duality symmetries in field theory and in string theory have been studied extensively. Let us list some of them: 

- **Electric-Magnetic Dualities**: strong-weak coupling duality which relates apparently different field theories under the interchange of electric degrees of freedom with magnetic degrees of freedom (for a review, see \[1\], \[2\]).

- **T-Duality**: target space duality in string theory relates different string backgrounds which are physically equivalent (for a review, see \[3\]).

- **S-Duality**: strong-weak coupling duality in string theory (for a review, see \[4\]).

- **U-Duality**: intertwines T-duality and S-duality \[5\].

- **String-String Dualities**: relate different string theories to each other (for a review, see \[6\]).

- **World-Sheet ↔ Target-Space Dualities**: relate a theory on a world-sheet and target-space \((\Sigma, T)\) to a theory on a different world-sheet and a different target-space \((\tilde{\Sigma}, \tilde{T})\). \[7\].

The new results in this talk are based on work in ref. \[8\]. We present duality symmetries in 4D, Abelian gauge theories which involve also the (Euclidean, compact) space-time \(M^4\). Such dualities – rather mysterious from the 4D point of view – are better understood as geometrical symmetries of theories in higher dimensions, compactified to \(M^4\) on some internal space.

Explicitly, we find dualities which relate a pair \((M^4, \tau)\) to a different pair \((\tilde{M}^4, \tilde{\tau})\), where \(\tau\) is the complex coupling-constant matrix of a \(U(1)^r\) gauge theory on \(M^4\), and \(\tilde{\tau}\) is the dual coupling-constant matrix of a \(U(1)^{\tilde{r}}\) gauge theory on \(\tilde{M}^4\).

Some of the dualities considered can be understood as the consequence of string dualities, in the limit where gravity is decoupled. A simple example is the string-string triality which relates the heterotic string compactified on \(T^4 \times T^2\) to Type II strings on \(K^3 \times T^2\). In the low-energy effective field theory, at the limit \(M_{planck} \to \infty\), this triality becomes part of the electric-magnetic duality group of an \(N = 4, U(1)^4\) supersymmetric Yang-Mills (YM) theory on \(\mathbb{R}^{3,1}\).

In the rest of the talk, we continue by touring more examples which involve also space-time:

1. **S ↔ U Duality in N = 4 YM Theory and Self-Dual Superstring in 6D**: consider \(SU(2)\), \(N = 4\) YM theory on \(M^4 = S^3_\beta \times S^1_\beta \times T^2_\beta\), where \(\beta\) is the inverse temperature, \(R\) is the radius of a circle, and \(T^2_\beta\) is a 2-torus with complex structure \(U\). Let \(S\) denote the complex gauge coupling. The partition function at large scalar VEVs was computed in \[9\]. It turns out that not only is it invariant under \(SL(2,\mathbb{Z})\), S-duality and the geometrical \(SL(2,\mathbb{Z})\) transformations acting on the complex structure, but also under the transfor-
2. **Compactification of a 2-Form Theory in 6D and Triality.** by compactifying a theory of a self-dual 2-form in 6D on $T^6_0 \times T^6_2$, we obtain a triality symmetry under the permutations of $S,T$ and $U$. In a certain large volume limit, the partition function is identical to the classical part of the one-loop partition function of a 2D sigma-model with a $T^2$ target-space. In string theory, this triality—observed sometime ago—is rather mysterious, because $T$ is the complex structure of the world-sheet torus, while $U$ and $S$ are the complex structure and the Kähler structure of the target-space torus, respectively. However, for the 2-form theory on $T^2_0 \times T^2_2 \times T^2_3$, this triality is the geometrical symmetry permuting the three 2-tori.

3. **Compactifications of a 2-Form Theory in 6D and More Dualities:** a generalization of the previous example is to compactify a self-dual 2-form theory in 6D on $\Sigma_g \times T^6_3 \times \bar{\Sigma}_g\bar{\Sigma}$, where $\Sigma_g$ and $\bar{\Sigma}_g\bar{\Sigma}$ are (different) Riemann surfaces with genus $g$ and $\bar{\Sigma}_g\bar{\Sigma}$, respectively. The partition function is identical to the classical partition function of a 2D sigma-model with world-sheet $\Sigma_g$ (alternatively, $\bar{\Sigma}_g\bar{\Sigma}$) and target-space $T^{2g}$ (alternatively, $T^{2g}$) whose metric and antisymmetric background are parametrized by $S$ and the period matrix of $\Sigma_g$ (alternatively, by the period matrix of $\Sigma_g$). We find a world-sheet $\leftrightarrow$ target-space duality between the pairs

$$\{\Sigma_g, T^{2g}(\bar{\Sigma}_g)\} \leftrightarrow \{\bar{\Sigma}_g, T^{2g}(\Sigma_g)\}.$$ 

This duality is a manifest consequence of the geometrical symmetry interchanging $\Sigma_g \leftrightarrow \bar{\Sigma}_g\bar{\Sigma}$ in the compactified 2-form theory. Similar world-sheet $\leftrightarrow$ target-space dualities were observed in\[. These can be understood as the $\Sigma_g \leftrightarrow \bar{\Sigma}_g\bar{\Sigma}$ geometrical symmetries of a compactified 6D 2-form theory, which instead of self-duality obey other conditions\[.

4. **(M^4, \tau) \leftrightarrow (\tilde{M}^4, \tilde{\tau}) Duality and Compactifications of a Self-Dual 4-Form Theory in 10D:** a reduction of a 10D self-dual 4-form theory to 4D on $M^4 \times T^2_3$ gives a 4D, $U(1)^{b_2}$ gauge theory, where $M^4$ is a 4-manifold with $b_1(M^4) = 0$; $b_1, b_2$ are the Betti numbers of $M^4$. Upon further compactification on $M^4 \times T^2_3 \times M^4$ we find a manifest symmetry under the interchange $M^4 \leftrightarrow \tilde{M}^4$. This translates into a rather “puzzling” duality of the 4D gauge theory:

$$\{M^4, \tau(M^4)\} \leftrightarrow \{\tilde{M}^4, \tilde{\tau}(M^4)\},$$

where $\tau(M^4)$ is the gauge-coupling matrix given in terms of the geometrical data of $M^4$, and $\tilde{\tau}(M^4)$ is the dual gauge-coupling matrix given in terms of the geometrical data of $M^4$. This duality relates a theory on space-time manifold $M^4$ and coupling-constant matrix $\tau$ to a theory with a different 4-manifold and a different gauge-coupling matrix. This construction sometimes works for more than the partition function of an Abelian gauge theory. For $M^4 = K^3$ and $M^4 = \tilde{K}^3$, this construction can be embedded in an $N = 4$ compactification of type IIB string on $K^3 \times T^2_3 \times \tilde{K}^3$.

To summarize:

- Duality groups of Abelian gauge theories on 4-manifolds and their reduction to 2D were considered.
- The duality groups include elements that relate different space-times in addition to relating different gauge-coupling matrices.
- We interpreted such dualities as geometrical symmetries of compactified theories in higher dimensions.
- In particular, we considered compactifications of a (self-dual) 2-form in 6D, and of a self-dual 4-form in 10D.
- Relations with a (conjectured) self-dual superstring in 6D and with Type IIB superstring were mentioned.
• There are many more duality symmetries of the classical partition sum of free $U(1)^r$ gauge theories on $M^4$ which were not discussed here.

• To recover all symmetries of $4D$ gauge theories, and their possible geometrical origin from higher dimensions, is a problem which may shed more light on the non-perturbative dynamics of gauge theories and strings.

• Here we considered aspects of this problem in some simple, yet probably significant cases.

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