Abstract

The recent measurement of the muon anomalous magnetic dipole moment shows $2.6\sigma$ deviation of $a_\mu$ from the standard model prediction which can be explained by a chargino-sneutrino loop correction in the supersymmetric models. In this paper we consider extra U(1) models where the $\mu$ parameter is radiatively generated. This model predicts the sign of $\mu$ is positive in wide parameter regions. But even the $2\sigma$ constraint causes a serious contradiction to the experimental bound of the extra neutral gauge boson mass. Although a minimal supergravity scenario is ruled out, a very small window is remained as an allowed region for a no-scale model with non-universal gaugino masses.
1 Introduction

Supersymmetry is the most simplest solution of the gauge hierarchy problem in the standard model (SM), which is one of the main subjects of present particle physics [1]. The minimal supersymmetric extension of the SM (MSSM) predicts a desirable gauge coupling unification and may be the remnant of more fundamental theory, i.e. GUT or string theory. Another favorable feature of the MSSM is radiative electroweak symmetry breaking (EWSB). Large top-Yukawa coupling induces negative Higgs squared mass through the quantum correction at a low energy scale, which explains naturally why the electroweak symmetry is broken. Although the MSSM is the most promising extension of the SM, there remain some theoretical problems. The famous one is known as the $\mu$-problem. The MSSM has a supersymmetric mass term $\mu H_1 H_2$ and $\mu$ must be $O(M_Z)$ to cause an appropriate scale of EWSB. However the size of the $\mu$ scale is naively expected to be $O(M_{GUT})$ which is the fundamental energy scale of the MSSM, so we cannot explain why $\mu$ is so small. It is natural to consider the origin of the $\mu$ scale as some result of supersymmetry breaking. Otherwise, even if we have understood the smallness of the $\mu$ scale, there would remain another problem why soft breaking mass parameters and the $\mu$ scale highly degenerate.

Recently, the new measurement of the muon anomalous magnetic dipole moment (AMDM) $a_{\mu}^{\text{exp}} = 11659202(14) \times 10^{-10}$ shows that it is 2.6$\sigma$ away from the standard-model prediction $a_{\mu}^{\text{SM}}$ [2, 3]. It is interesting to consider this deviation as a new physics effect, especially supersymmetry (SUSY). The required SUSY contribution is

$$a_{\mu}^{\text{SUSY}} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 43(16) \times 10^{-10},$$

which has a positive sign. In the case of the MSSM the dominant contribution to $a_{\mu}^{\text{SUSY}}$ comes from chargino-sneutrino loop diagrams [4, 5]. In a small chargino mixing approximation, a chargino contribution is given by

$$a_{\mu}^{\chi^{\pm}\tilde{\nu}} \sim \frac{3g_2^2 m_\mu M_2 \tan \beta}{16\pi^2 m_{\tilde{\nu}}^2 (M_2^2 - \mu^2)} \left[ f(M_2^2/m_{\tilde{\nu}}^2) - f(\mu^2/m_{\tilde{\nu}}^2) \right],$$

where $\tan \beta$ is a ratio of the vacuum expectation values (VEVs) of $H_1$ and $H_2$. Since $f(x)$ is a simply increasing function, $a_{\mu}^{\chi^{\pm}\tilde{\nu}}$ is positive for positive $M_2\mu$. The Brookhaven E821 experiment implies the sign of $\mu$ must be same as $M_2$.

It is very interesting to examine the model which can dynamically generate both the appropriate size and the sign of $\mu$ in taking account of $a_{\mu}^{\text{SUSY}}$ [7]. One possibility of such a model is an introduction of a SM gauge singlet field $S$ which replaces $\mu H_1 H_2$ by a Yukawa type coupling $\lambda S H_1 H_2$ in the MSSM superpotential. If the field $S$ develops a VEV of order 1 TeV as a result of the negative squared mass of $S$ ($m_S^2 < 0$), a weak scale value of $\mu$ is dynamically generated through $\mu = \lambda < S >$. There are two scenarios to stabilize the VEV of $S$. One is to add a cubic term $\kappa S^3$ to superpotential. This term breaks PQ symmetry.
and prohibits the appearance of a problematic massless axion. To forbid the fundamental \( \mu \)-term, we must introduce discrete symmetry. This model is well known as the NMSSM. Another one is to extend a gauge symmetry of the SM. If the field \( S \) has a nontrivial charge of the extra gauge symmetry, the potential of a scalar component of \( S \) is stabilized by a D-term which comes from this extra gauge multiplet. In this case, the massless axion does not appear because of the Higgs mechanism. The most simplest extension of the gauge structure is to add an extra \( U(1) \) symmetry. In this paper we consider this extra \( U(1) \) model. The extra \( U(1) \) symmetry forbids the appearance of \( \mu H_1 H_2 \) in the superpotential without causing the domain wall problem unlike the NMSSM. In order to introduce the extra \( U(1) \) symmetry, additional chiral fermions are needed for anomaly cancellation. Here, we confine our attention to superstring inspired \( E_6 \) models and embed the MSSM matter multiplets in a 27 representation of \( E_6 \).

In this paper we estimate the muon AMDM taking account of a constraint of the extra neutral gauge boson mass in the correct vacuum. The correct vacuum is determined as the radiatively induced minimum of the effective potential in the suitable parameter space. In this approach we use the one-loop effective potential and solve the relevant renormalization group equations (RGEs) numerically. In sec.2 we give a short introduction of the extra \( U(1) \) model and sec.3 and sec.4 are devoted to numerical analysis of the muon AMDM.

2 Extra \( U(1) \) models

In this section we define a \( \mu \)-problem solvable extra \( U(1) \) model. The extra matter contents are determined so as to complete a 27 representation of \( E_6 \), which are listed in Table 1. \( E_6 \) is a rank 6 group and has two extra \( U(1) \)s in addition to the SM gauge group as its subgroup. It is decomposed as

\[
SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi \subset E_6.
\]

At a TeV region only one of two independent linear combinations of \( U(1)_\psi \) and \( U(1)_\chi \) are assumed to remain unbroken and be broken only by the VEV of \( S \). In this paper only two extra \( U(1) \) models are considered. They are known as the \( \eta \) model and the \( \xi \) model and are defined by

\[
Q_i = Q_\psi \cos \theta_i + Q_\chi \sin \theta_i,
\]

\[
\tan \theta_\eta = -\sqrt{\frac{3}{5}},
\]

\[
\tan \theta_{\xi \pm} = \frac{1}{\sqrt{15}}.
\]

The matter contents are given by

\[
[3(Q, \bar{U}, \bar{D}, L, \bar{E}) + (H_1, H_2)]_{\text{MSSM}} + 3(g, \bar{g}) + 2(H_1, H_2) + 3(S) + 3(N),
\]

which can be derived from three 27s of \( E_6 \) as is shown in Table 1. This set satisfies the anomaly free conditions. Unfortunately this matter multiplet spoils the successful gauge
Table 1: The charge assignment of extra U(1)s which are derived from $E_6$. These charges are normalized as $\sum_{i \in 27} Q_i^2 = 5$ [11].

coupling unification in the MSSM. To preserve the unification we must add extra chiral multiplets to these in the form of vector representation

$$(H^a_1) + (\bar{H}^a_2),$$

where $a=1$ or 2 and $\bar{H}^a_2$ comes from 27 of $E_6$. At least in the sector of $SU(3)_c \times SU(2)_L \times U(1)_Y$ these matter contents are the same as [MSSM$+$ 3($5$+$\bar{5}$)] where 5 and $\bar{5}$ are the representations of the usual SU(5).

The superpotential of the extra U(1) model is defined by

$$W = h_i Q H_2 U + h_6 Q H_1 \bar{D} + h_7 L H_2 \bar{E} + \sum_{i=1}^{3} k_i S g_i \bar{g}_i + \lambda_6 S H_1 H_2 + h_9 L H_2 \bar{N}$$

(6)

$$+ \lambda_6 g Q Q + \lambda_7 g \bar{U} \bar{D}$$

(7)

$$+ \lambda_8 g \bar{E} \bar{U} + \lambda_9 g L Q + \lambda_{10} g \bar{D} \bar{N}$$

(8)

where $g_i$ and $\bar{g}_i$ stand for the extra color triplet chiral superfields. We neglect the first and the second generation Yukawa couplings except for the one including $g$ and $\bar{g}$ in Eq.(6). Since the coexistence of Eq.(7) and Eq.(8) induces rapid proton decay at an unacceptable rate, only one of them can exist. The model with Eq.(7) and the model with Eq.(8) correspond to diquark and leptoquark model, respectively. Generally these couplings $\lambda_i$ are stringently constrained by electroweak rare processes [4].

The existence of multi-generation extra fields brings an ambiguity in Eq.(6). The coupling $\lambda$ and $k$ can have generation indices for extra fields such as $S$, $H_1$, $H_2$, $g$ and $\bar{g}$. On this point we make the following assumption, for simplicity. For $S^i$ and $H^i_{1,2}$, only one generation of them can have Yukawa type couplings and get the VEVs. In this case another two generations remain massless after symmetry breaking, which is not phenomenologically acceptable. But even tiny Yukawa couplings $\lambda_{ij} \sim O(10^{-2})$ can generate enough size of higgsino mass through $\lambda_{ij} S H^i_1 H^j_2$ and such small couplings do not affect our RGE analysis and are safely neglected. On the other hand, the fermion components of the remaining $S$ which donot couple to the usual Higgs boson $H^i_{1,2}$ cannot
have a tree level mass, which is problematic. Since the extra colored singlets \((g_i, \bar{g}_i)\) have a diagonal coupling to \(S\) as \(k_i S g_i \bar{g}_i\), their fermion components can get mass through this coupling. This cubic term plays a crucial role in the breaking of the extra \(U(1)\) symmetry by driving \(m^2_S\) to negative \([10, 11]\). For the extra chiral multiplets \((H^4_1, \bar{H}^4_1)\), we must introduce a supersymmetric mass term such as \(W_\mu = \tilde{\mu} H^4_1 \bar{H}^4_1\). If the mass scale of \(\tilde{\mu}\) is an appropriate intermediate scale, the problem of discrepancy between \(M_{GUT} \sim 2 \times 10^{16}\) GeV and \(M_{string} \sim 4 \times 10^{17}\) GeV can be solved. However we are not concerned with this problem and assume simply \(\tilde{\mu}\) is at the weak scale. As this term does not play an essential role in our analysis, it will be omitted below.

In this model, there are three new contributions to the muon AMDM. The first one is a \(Z'\) exchange and the second one is a leptoquark \((g, \bar{g})\) exchange. The extra neutral gauge boson gives a small contribution \([4]\)

\[
a_\mu^{Z'} \sim O(1) \times 10^{-11} \left( \frac{m^2_Z}{m^2_{Z'}} \right),
\]

which is difficult to be observed for the collider constraint of \(m_{Z'}\) \((m_{Z'} > 600\) GeV). The leptoquark contribution is argued in \([12, 13]\). For the large Yukawa couplings \(\lambda_{8,9} \sim O(10^{-1})\), this gives a sizable contribution to the muon AMDM. In this paper we do not consider this effect, for simplicity. The third contribution comes from the extra \(U(1)\) gaugino and the fermion partner of \(S\) and we include them in the neutralino contributions (see appendix A).

![Diagram](image)

Figure 1: The \(Z'\) and the leptoquark exchange diagrams contribute to the muon AMDM.

\(^1\)If we add \(\lambda_k S^k H_1 H_2\) to Eq.(6), the fermion components of \(S^k\) become massive through the mixing with the usual higgsino. But \(\lambda_k\) must be very large \((\sim O(1))\) to generate a phenomenologically acceptable mass.
3 Minimal supergravity case

Soft supersymmetry breaking parameters are introduced as

$$\mathcal{L}_{\text{soft}} = -\sum_i m_{\phi_i}^2 |\phi_i|^2 - \left( \sum_a \frac{1}{2} M_a \lambda_a \lambda_a^c + h.c. \right)$$

$$+ \left( A_\lambda S H_1 H_2 + A_k S g g + A_t Q H_2 \bar{U} + A_b Q H_1 \bar{D} + A_r L H_1 \bar{E} + h.c. \right), \quad (10)$$

where the first two terms are mass terms of the scalar component $\phi_i$ of each chiral supermultiplet and of gauginos $\lambda_a$. The last term is a scalar trilinear coupling. We use the same notation for the scalar component as the one of the chiral superfield. In the minimal supergravity scenario, the values of soft breaking parameters at the GUT scale are given by

$$m_{\bar{Q}}^2 = m_{\bar{U}}^2 = \cdots = m_S^2 = m_0^2,$$

$$a_t = a_b = \cdots = a_k = A_0,$$

$$M_Y = M_X = M_2 = M_3 = M_4,$$

where $A_i = y_i a_i$. It is not so easy to find the phenomenologically favorable potential minimum under completely universal soft breaking parameters because the $\mu$ is not a free parameter unlike the MSSM. To solve the potential minimum condition exactly, we allow the non-universality in the region $0.9 < m_i/m_0 < 1.1$ among soft supersymmetry breaking masses of Higgs scalars. Such a small non-universality does not change the mass spectrum significantly and may give almost the same result as a perfectly universal case.

In our considering models the tree level scalar potential including the soft supersymmetry breaking terms can be written as

$$V^{(0)} = \frac{1}{8} (g_Y^2 + g_X^2)(|H_1|^2 - |H_2|^2)^2 + (|\lambda S H_1|^2 + |\lambda S H_2|^2 + |\lambda H_1 H_2|^2)$$

$$+ m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_S^2 |S|^2 - (a_\lambda \lambda S H_1 H_2 + h.c.)$$

$$+ \frac{1}{2} g_X^2 (Q_1 |H_1|^2 + Q_2 |H_2|^2 + Q_S |S|^2)^2, \quad (11)$$

where $Q_1$, $Q_2$, and $Q_S$ are the extra U(1) charges of $H_1$, $H_2$ and $S$, respectively. The third line is a D-term contribution of the extra U(1) and $g_X$ stands for its gauge coupling constant. If we replace $\lambda S$ and $a_\lambda$ with $\mu$ and $B$ and put $g_X = 0$, this potential becomes the same as the one of the MSSM. Since we can take $< H_1 >$ and $< H_2 >$ positive without loss of generality, it is obvious that the global minimum of the scalar potential is in a positive $a_\lambda \lambda S$ region.

The RGEs of the soft breaking parameters are given in appendix B. The sign of $a_\lambda$ and $\mu$ are always positive in natural parameter region at the GUT scale for reproducing a realistic vacuum, i.e. unbroken $SU(3)_c \times U(1)_{em}$. The reason for this is as follows. In the present extra U(1) model, the $\beta$-function of $SU(3)_c$ gauge coupling is equal to zero, so it always takes a large value from $M_Z$ to $M_{\text{GUT}}$ unlike the MSSM. The stronger
SU(3)$_c$ gauge coupling makes gluino contribution dominant in RGEs of $a_t$ and $a_k$ and drives them to negative. Then the negative $a_t$ and $a_k$ change the sign of $\beta$-function of $a_\lambda$ negative which forces $a_\lambda$ take positive before the electroweak scale is reached [15]. Finally, the potential minimum condition forces $\mu$ to take same sign as $a_\lambda$ and the gluino mass. However, even if we take $A_0$ to be dominant in the RGEs, this scenario does not change. As long as $A_0$ is not much larger, the dominant $a_{t,k}$ contributions drive themselves to-zeros, and the gluino dominant condition is again satisfied. To conclude, the present extra U(1) models predict the positive $\mu$ and the positive $a_\mu^{SUSY}$.

Potential minimum condition for Eq.(11) can be written as,

\[ m_1^2 = -\frac{1}{4}(g_Y^2 + g_2^2)(v_1^2 - v_2^2) - g_X^2 Q_1 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S u^2) - \lambda^2 (u^2 + v_2^2) + \lambda a_\lambda u \frac{v_2}{v_1}, \]
\[ m_2^2 = \frac{1}{4}(g_Y^2 + g_2^2)(v_1^2 - v_2^2) - g_X^2 Q_2 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S u^2) - \lambda^2 (u^2 + v_1^2) + \lambda a_\lambda u \frac{v_1}{v_2}, \]
\[ m_S^2 = -g_X^2 Q_S (Q_1 v_1^2 + Q_2 v_2^2 + Q_S u^2) - \lambda^2 (v_1^2 + v_2^2) + \lambda a_\lambda \frac{v_1 v_2}{u}, \]

(12)

where $v_1$, $v_2$ and $u$ are the VEVs of $H_1$, $H_2$ and $S$, respectively. In the extra U(1) models the value of $u$ can be constrained from below by the experimental bounds on the mass of this extra U(1) gauge boson and its mixing with the ordinary $Z^0$, so that we must assume $u \gg v_1, v_2$. The experimental constraint of the extra U(1) gauge boson mass is discussed in [14]. In this paper we donot consider the detail structure of the $Z - Z'$ mixing, for simplicity. The third line of Eq.(12) determines the VEV of $S$ such as

\[ u \sim \sqrt{-m_S^2/g_X^2 Q_S^2}, \]

(13)

and the second line determines the weak scale as

\[ - (\lambda^2 + Q_2 Q_S g_X^2) u^2 - m_2^2 \sim \frac{1}{2} m_Z^2, \]

(14)

where the large $\tan \beta$ approximation should be understood. This condition constrains the allowed range of $\lambda(M_S)$ and $\mu_{eff}$ severely. The first line of Eq.(12) is written as

\[ \lambda a_\lambda \tan \beta \sim m_1^2 + (\lambda^2 + g_X^2 Q_1 Q_S) u^2, \]

(15)

which can be consistent with the large $\tan \beta$ solution as far as $m_1^2 \gg m_2^2$ is satisfied. This condition makes it difficult to realize the large $\tan \beta$ solution. From the point of view of the RGE analysis, the large $\tan \beta$ makes the low energy values of $m_1^2$ and $m_2^2$ degenerate due to the same RGE evolution [15]. However, the degeneracy between $m_1^2$ and $m_2^2$ makes $v_2/v_1$ small at the scalar potential minimum. In this way, the moderate $\tan \beta$ solution is favored for the extra U(1) models (Too large $\tan \beta$ solution is disfavored).

For more precise estimation, we must take account of the radiative correction to the potential, as it may make a sizable contribution mainly due to the heavy stops. It is well-known that the one-loop contribution to the effective potential can be written as

\[ V^{(1)} = \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \left( \ln \frac{M^2}{\Lambda^2} - \frac{3}{2} \right), \]

(16)
where $\mathcal{M}^2$ is a matrix of the squared mass of the fields contributing to the one-loop correction and $\Lambda$ is a renormalization point which is taken as $M_S (=1\text{ TeV})$. In the case of the MSSM, this one-loop correction is dominated by top and stop contributions because of their large Yukawa coupling and the other fields are irrelevant. In the study of the extra U(1) models $k$ is rather large and then we should also take account of the effect on $\mathcal{M}^2$ from the extra colored chiral superfields $g$ and $\bar{g}$. Mass matrices of these and another sparticles are given in appendix C.

Taking account of experimental constraints, we get phenomenologically allowed regions of the parameter space, which are given in the form of mass bounds as [16],

\begin{align}
& m_{H^\pm} \geq 69 \text{ GeV}, \quad m_{\chi^\pm} \geq 72 \text{ GeV}, \quad m_{\tilde{t}^\pm} \geq 86 \text{ GeV}, \\
& M_3 \geq 180 \text{ GeV}, \quad m_{Z'} \geq 600 \text{ GeV}, \quad m_{H^0} \geq 114 \text{ GeV}, \\
& m_{\tilde{\tau}^\pm} \geq 81 \text{ GeV}, \quad m_{\tilde{\chi}^\pm} \geq 75 \text{ GeV}, \quad m_{\tilde{g},\tilde{g}} \geq 220 \text{ GeV}, \quad (17)
\end{align}

where the mass of $Z'$ boson is written by

\begin{equation}
\begin{aligned}
m^2_{Z'} = 2g_X^2(Q_1^2v_1^2 + Q_2^2v_2^2 + Q_S^2u^2) \sim -2m_{S^2}.
\end{aligned}
\end{equation}

The explicit formulas of the masses of neutral and charged Higgs bosons are given in our previous work [11]. These mass spectra are mainly governed by $m_0$ and $M_{1/2}$ and are highly correlated each other. If the mass bounds of $Z'$ boson and charginos are satisfied, the other mass bounds become trivial [2]. Large $\mu_{\text{eff}}$ makes a charged Higgs boson heavy and large soft breaking parameters make sparticles heavy. Both are immediate results of the heavy $Z'$. If the chargino mass bound is satisfied, we get

\begin{equation}
\begin{aligned}
M_2 = \frac{\alpha_2}{\alpha_3} M_3 \sim 0.3 M_3 > 72 \text{ GeV}
\end{aligned}
\end{equation}

from the gaugino mass unification relation:

\begin{equation}
\begin{aligned}
M_3 \alpha_3 = M_2 \alpha_2 = k_Y M_1 \alpha_Y,
\end{aligned}
\end{equation}

where $k_Y$ is Kac-Moody level of $U(1)_Y$, then the gluino mass bound is trivially satisfied.

For dimensionless parameters we investigate the parameter region such as

\begin{equation}
\begin{aligned}
0.4 > \lambda(M_S) > 0.2, \quad 0.7 > k(M_S) > 0.4, \quad \tan \beta = (10, 20),
\end{aligned}
\end{equation}

where the value of $\tan \beta$ is given at $M_Z$. At $M_Z$ we convert the running masses of tau and bottom to the tau and bottom Yukawa couplings by

\begin{align}
y_b(M_Z) &= \tilde{m}_b(M_Z)/v_1(M_Z) = 2.92 \text{ GeV}/v(M_Z) \cos \beta, \\
y_\tau(M_Z) &= \tilde{m}_\tau(M_Z)/v_1(M_Z) = 1.74 \text{ GeV}/v(M_Z) \cos \beta,
\end{align}

where $v = 175 \text{ GeV}$. Using these initial conditions of gauge and Yukawa couplings they are given at $M_Z$ by [17]

\begin{equation}
\begin{aligned}
\alpha_Y(M_Z) &= 0.01698,
\end{aligned}
\end{equation}

\(^2\text{In all the parameter space, the lightest chargino is always wino-like chargino because of the large }\mu\text{ parameter.}\)
\[ \alpha_2(M_Z) = 0.03364, \]
\[ \alpha_3(M_Z) = 0.118, \]
\[ y_b(M_Z) = 0.0166 / \cos \beta, \]
\[ y_\tau(M_Z) = 0.0099 / \cos \beta. \]

We run them from \( M_Z \) to \( M_{\text{top}} \) with the RGEs for the SM. At \( M_{\text{top}} \), we define the top-Yukawa coupling by

\[ y_t(M_{\text{top}}) v_2(M_{\text{top}}) = \hat{m}_{\text{pole}} (1 + \frac{4}{3\pi} \alpha_3(M_{\text{top}}))^{-1}, \tag{21} \]

where \( \hat{m}_{\text{pole}} = M_{\text{top}} = 175 \text{ GeV} \). Finally we run them from \( M_{\text{top}} \) to \( M_S \) with the RGEs for the two Higgs doublet model \[18\],

\[ (2\pi) \frac{\alpha_Y}{dt} = 7 \alpha_Y, \tag{22} \]
\[ (2\pi) \frac{\alpha_2}{dt} = -3 \alpha_2, \tag{23} \]
\[ (2\pi) \frac{\alpha_3}{dt} = -7 \alpha_3, \tag{24} \]
\[ (2\pi) \frac{Y_t}{dt} = Y_t [\frac{9}{2} Y_t + \frac{1}{2} Y_b - \frac{17}{12} \alpha_Y - \frac{9}{4} \alpha_2 - 8 \alpha_3], \tag{25} \]
\[ (2\pi) \frac{Y_b}{dt} = Y_b [\frac{9}{2} Y_b + \frac{1}{2} Y_t + \frac{1}{2} Y_\tau - \frac{5}{12} \alpha_Y - \frac{9}{4} \alpha_2 - 8 \alpha_3], \tag{26} \]
\[ (2\pi) \frac{Y_\tau}{dt} = Y_\tau [3 Y_b + \frac{5}{2} Y_\tau - \frac{15}{4} \alpha_Y - \frac{9}{4} \alpha_2], \tag{27} \]
\[ (2\pi) \frac{v_1}{dt} = \frac{1}{2} v_1 [\frac{3}{4} \alpha_Y + \frac{9}{4} \alpha_2 - 3 Y_b - Y_\tau], \tag{28} \]
\[ (2\pi) \frac{v_2}{dt} = \frac{1}{2} v_2 [\frac{3}{4} \alpha_Y + \frac{9}{4} \alpha_2 - 3 Y_t], \tag{29} \]

from which we get initial conditions of dimensionless couplings at \( M_S \). For soft supersymmetry breaking parameters we give them at \( M_{\text{GUT}} \) by

\[ m_0 = 0.6, \quad 0.18 < M_1^2 < 0.36, \quad -1.2 < A_0 < 1.2, \tag{30} \]

where these are given by a TeV unit and the gluino mass lower bound is taken account of previously\[3\]. Out of these parameter regions it is difficult to satisfy the potential minimum condition. For fixed values of the Yukawa couplings it is difficult to satisfy the potential minimum condition. For fixed values of the Yukawa couplings, the parameter set \((m_0, A_0, M_1^2)\) has the only one degree of freedom since they are imposed by two constraints from Eq.(12). So the SUSY breaking scale is represented by only one of them, we take it as \( m_0 \). In order to improve the one-loop effective potential we use two-loop RGEs for Yukawa and gauge coupling constants and soft scalar masses and one-loop ones for A-parameters and gaugino masses from \( M_{\text{GUT}} \) to supersymmetry breaking scale \( M_S \) \[19\].

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\[3\]In our considering extra U(1) models, the \( \beta \)-function of \( M_3 \) equals to zero so that \( M_3 \) is constant.
We evaluate the chargino and neutralino contributions to the muon AMDM based on the known formula for the MSSM as given in appendix A. The results are shown in Fig.2. With universal soft breaking terms, the allowed region never enters inside the 1σ bound $27 < a_{\mu}^{SUSY} < 59 \times 10^{-10}$ either for the $\eta$ model or the $\xi^-$ model. This does not change even in the case where the large $\tan\beta$ enhancement exists. Although there are new contributions from an extra U(1) gaugino and a new singlet fermion $\tilde{S}$, since neutralino contributions are always small due to small mixing angle of smuon eigenstates, they do not play essential role. The main obstacle of inducing the large $a_{\mu}^{SUSY}$ is due to the chargino mass lower bound because the potential minimum condition favors the small gluino mass. It is shown in Fig.3 that the chargino mass constraint is stronger than the one of extra neutral gauge boson mass. In the case of larger $m_0$, sneutrino becomes heavier and suppresses $a_{\mu}^{SUSY}$ more strongly. On the other hand, in the smaller $m_0$ case, the chargino mass bound excludes the wider region of the parameter space. In conclusion, the minimal supergravity scenario is ruled out by the muon AMDM constraint at the 1σ level if we take account of only the chargino and neutralino loop effects. The 2σ bound $11 < a_{\mu}^{SUSY} < 76 \times 10^{-10}$ gives the upper mass bound of extra neutral gauge boson about 600 GeV for the $\eta$ model. However, the $\xi$ model is excluded even at the 2σ level.

Because of the above argument, it is interesting to consider the case without the gaugino mass universality in order to escape from the chargino mass constraint. But if we allow the gaugino mass non-universality, there is no reason why the non-universality of soft scalar masses and scalar trilinear couplings are forbidden. Although such a general non-universality case is interesting, in that case we must take care of the FCNC constraints and must invoke some FCNC suppression mechanism, which is out of our present scope.
Figure 3: Allowed regions of the extra U(1) models in (λ, k) plane, where (A) is for tan β = 10 and (B) is for tan β = 20. The square satisfies both sparticle and Higgs mass constraints given in Eq.(17) except for the chargino and Z’ boson mass bounds. The small circle satisfies $m_{Z'} > 600 \text{ GeV}$ and the large square satisfies $m_{\chi^\pm} > 72 \text{ GeV}$.

4 No-scale boundary condition with non-universal gaugino masses

In order to escape from the FCNC consideration, here we choose the no-scale type boundary condition ($m_0 = A_0 = 0$) with the non-universal gaugino masses [20, 21],

$$0.18 < M_{30} < 0.36, \quad 0.25 < M_{20} < 1.2, \quad 0.25 < M_{Y0} = M_{X0} < 1.2,$$

where we allow the non-universality among Higgs soft scalar masses in the region $|m_i| < 100 \text{ GeV}$ as argued in previous section. It is well known that there is a dangerous $U(1)_{em}$
breaking minimum due to the tachyonic slepton mass in some parameter space of the no-scale model. In the case of the extra U(1) models, the large D-term contribution from an extra U(1) gauge multiplet is important. In the \( \eta \) model, \( \tilde{b}_R, \tilde{\tau}_L, \tilde{\nu}_L, \tilde{g} \) and \( \tilde{g}^c \) get negative squared mass contribution from a D-term. On the other hand, only \( \tilde{g} \) and \( \tilde{g}^c \) get negative squared mass contribution in the \( \xi^- \) model. But in the both models, a right handed stau gets a large positive squared mass contribution unlikely in the case of the MSSM. Taking account of the RGE evolution effect, the values of soft scalar masses at\( h \) handed stau gets a large positive squared mass contribution unlikely in the case of the \( \beta \) model. In the case of the extra U(1) model the tree level lightest Higgs boson mass at Higgs mass, we need a large stop loop contribution, but it is difficult for the no-scale model. In order to induce such a large massless as noticed previously, it is rather easy to make a LSP neutral.

\[ m_{Q}^2 \sim 0.26M_{20}^2 + 1.90M_{30}^2 - 0.07M_{20}M_{30} - 0.01M_{Y0}M_{30} - 0.01M_{X0}M_{30}, \]
\[ m_{U}^2 \sim 0.03M_{Y0}^2 - 0.12M_{20}^2 + 1.41M_{30}^2 - 0.14M_{20}M_{30} - 0.02M_{Y0}M_{30} - 0.02M_{X0}M_{30}, \]
\[ m_{D}^2 \sim 0.01M_{Y0}^2 + 2.37M_{30}^2, \]
\[ m_{L}^2 \sim 0.03M_{Y0}^2 + 0.33M_{20}^2, \]
\[ m_{E}^2 \sim 0.12M_{Y0}^2 + 0.01M_{X0}^2, \]
\[ m_{N}^2 \sim 0.08M_{X0}^2, \]
\[ m_{g}^2 \sim 0.01M_{Y0}^2 + 2.15M_{30}^2, \]
\[ m_{g}^2 \sim 0.01M_{Y0}^2 + 2.15M_{30}^2 + 0.04M_{X0}^2, \]

where we took \( \tan \beta = 10, \ k = 0.6 \) and \( \lambda = 0.3 \) and used the \( \eta \)-model RGE given in appendix B. Because of the large gluino mass contribution there is no problem against the color breaking minimum, so the color and charge conservation conditions are always satisfied in the \( \xi^- \) model. However, it is not always the case for the \( \eta \) model because of the negative D-term contribution to the slepton mass. From the other point of view, this might be seen as a chance for the \( \eta \) model to enhance \( \alpha_{\mu}^{SUSY} \) by the light sneutrino.

Another problem in the no-scale model is the charged LSP [21]. In the extra U(1) models, since there is another serious problem the superpartners of \( S_{(1)} \) and \( S_{(2)} \) are massless as noticed previously, it is rather easy to make a LSP neutral.

Recently the experimental lower bound of Higgs boson mass is raised to about 113.5 GeV, this constraint is nontrivial for the no-scale model. In order to induce such a large Higgs mass, we need a large stop loop contribution, but it is difficult for the no-scale model. In the case of the extra U(1) model the tree level lightest Higgs boson mass at the large \( \tan \beta \) limit is given by

\[ m_h^2 \sim m_Z^2(1 + \frac{g_X^2Q_2^2}{g_Y^2 + g_2^2}) \sim m_Z^2(1 + Q_2^2 \sin^2 \theta_W), \quad (32) \]

where \( m_h \) becomes about 100 GeV in the \( \eta \) model and the large loop correction is not necessarily required as compared to the MSSM. The typical range of the lightest Higgs boson mass is \( 115 \sim 120 \) GeV for the \( \xi \) model and \( 120 \sim 130 \) GeV for the \( \eta \) model. If we include 2-loop contributions to the Higgs mass, they give negative contributions by few GeV and the result of our analysis may change drastically in the \( \xi \) model.

The results of numerical analysis are given in Fig.4 for the \( \eta \) model and the \( \xi^- \) model. In the case of the no-scale model \( m_{Z'} \) becomes significantly small and the allowed region
is limited in a very narrow range in the \((a_{\mu}^{SUSY}, m_{Z'})\) plane. But the muon AMDM constraint never excludes all the parameter space. As was expected, the values of \(a_{\mu}^{SUSY}\) and \(m_{Z'}\) are highly correlated. In order to satisfy the potential minimum conditions, the values of \(a_{\lambda}\) is very important. In the large \(\tan \beta\) region, Eq.(15) requires the small \(a_{\lambda}\). The \(a_{\lambda}\) is given by

\[
a_{\lambda} = -0.02M_{Y0} - 0.33M_{20} + 1.34M_{30} - 0.04M_{X0},
\]

which needs a large cancellation between \(M_{20}\) and \(M_{30}\) to make \(a_{\lambda}\) small. Due to such a fine tuning structure of the scalar potential, in the large \(\tan \beta\) case, the phenomenologically allowed region shrinks significantly into the \((\lambda, k)\) plane as shown in Fig.3. In this way \(M_{20}\) is never used for tuning of the sneutrino mass, the enhancement of \(a_{\mu}^{SUSY}\) due to the small sneutrino mass does not occur. So \(a_{\mu}^{SUSY}\) depends only on the typical supersymmetry breaking scale i.e. \(m_{Z'}\).\(^4\)

As shown in Fig.4, both the \(Z'\) mass bound \(m_{Z'} > 600\ \text{GeV}\) and the muon AMDM \(2\sigma\) bound are satisfied only in a very narrow range of the \((a_{\mu}, m_{Z'})\) plane for \(\tan \beta = 20\).

Figure 4: Same as Fig.2 for the no-scale model.

Since the no-scale condition is too strong to satisfy the experimental bound of \(m_{Z'}\), we allow to add the universal scalar mass \(m_{0} = 200, 400\ \text{GeV}\) without asking its origin. Here we allow the non-universality of Higgs soft scalar masses as \(|m_{i} - m_{0}| < 50\ \text{GeV}\). In this case \(m_{Z'}\) becomes large enough but the muon AMDM \(1\sigma\) bound excludes almost all parameter region that has been allowed if we would not take account of this new constraint. It is obvious from Fig.4 and Fig.5 that the larger the \(m_{0}\) is the weaker the

\(^4\)In the extra U(1) models, since two parameters \(\mu\) and \(m_{0}\) in \(a_{\mu}^{\pm}\) are strongly correlated to \(m_{Z'}\), it is expected that the same results are obtained in more general non-universal case, because it is difficult to realize \(\mu \ll m_{Z'}\) and \(m_{0} \ll m_{Z'}\) simultaneously.
correlation between $a_\mu$ and $m_{Z'}$ is. The reason is that the soft scalar mass dominates over the D-term contributions in the mass formula of sneutrino (see appendix C). The dominating soft scalar mass weakens the correlation between $m_{Z'}$ and $m_\tilde{\nu}$. For $\tan \beta = 20$, the $2\sigma$ bound gives the upper bound of extra neutral boson mass ($m_{Z'}^{2\sigma}$) about 600 (A), 800 (B) and 900 GeV (C) in the $\eta$ model and about 700 (A), 800 (B) and 850 GeV (C) in the $\xi$ model. Although it is shown that the larger $m_0$ makes $m_{Z'}^{2\sigma}$ larger, the value of $m_{Z'}^{2\sigma}$ seems to be saturated around 900 GeV in both models.

Figure 5: The same ones as Fig.3 except for (B) is for $m_0 = 200$ GeV and (C) is for $m_0 = 400$ GeV.
5 Summary

We have estimated the allowed region of the extra neutral boson mass in taking account of the new measurement of the muon AMDM in the $\mu$-problem solvable extra U(1) model. We focussed our attention only on the chargino-sneutrino and neutralino-smuon contributions to $a_{\mu}^{\text{SUSY}}$. Another exotic contribution from leptoquark exchange was neglected. Although such a loop contribution needs the enhancement either due to the large $\tan \beta$ or the light sneutrino as in the case of the MSSM in order to explain the muon AMDM data, the allowed parameter region of the extra U(1) model shrinks significantly as compared to the MSSM. The reason is that the large $\tan \beta$ solution is allowed only in a very narrow region of the ($\lambda$, $k$) plane and the light sneutrino solution contradicts the phenomenological constraint on the extra neutral gauge boson mass or the chargino mass.

In the case of the minimal supergravity, in order to preserve the hierarchy such as $u \gg v_{1,2}$ and $v_2 \gg v_1$ against the quantum correction, the light gluino solution is favored. Due to the gaugino mass universality, this requires lighter chargino so that the chargino mass lower bound is more stringent. Because of these obstacles, the minimal supergravity scenario is excluded by the muon AMDM constraint.

In the case of the non-universal gaugino mass, the chargino mass constraint disappears in the large $M_{20}$ region. Thus the small window is still remained for a no-scale model. Because of the strong correlation between $m_{Z'}$ and $a_{\mu}^{\text{SUSY}}$ in the extra U(1) models, we can expect to get a new information on the upper bound of $m_{Z'}$ from the further improvement of the ($g - 2$)$_\mu$ measurement.

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A Notations

In this paper, we use the following superpotential:

$$ W = -y_t (t_L t_R H_2^0 - t_R b_L H_2^+) - y_b (b_L b_R H_1^0 - b_R t_L H_1^-) - k S g \tilde{g} $$

$$ - y_t (\tau_L \tau_R H_1^0 - \tau_R \nu_L H_1^-) + \lambda S (H_1^0 H_2^0 - H_1^- H_2^+), $$

and the soft SUSY breaking terms:

$$ \mathcal{L}_{\text{soft}} = \int d\theta^2 \theta^2 (aW + \frac{1}{2} MW^a W_\alpha) $$

$$ = \frac{1}{2} M_Y \lambda_Y \lambda_Y + \frac{1}{2} M_2 \lambda_2^{(a)} \lambda_2^{(a)} + \frac{1}{2} M_X \lambda_X \lambda_X $$

$$ - y_t a_t (H_2^0 \tilde{t}_L - H_2^+ \tilde{t}_R \tilde{b}_L) - y_b a_b (H_1^0 b_R \tilde{b}_L - H_1^- b_R \tilde{t}_L) $$

$$ - y_{\tau} a_{\tau} (H_1^0 \tilde{\tau}_R \tilde{\tau}_L - H_1^- \tilde{\tau}_R \tilde{\nu}_\tau) + \lambda a_s (H_1^0 H_2^0 - H_1^- H_2^+) $$

$$ - k a_{\kappa} S g \tilde{g} g^c + h.c. $$

(34)
In this notation, any term in the RGEs of $a_i$ apper with same sign. The muon-chargino and the muon-neutralino interactions are described by

$$\mathcal{L}_\mu = y_\mu [\mu_L \bar{\mu}_R \tilde{H}_1^0 + \mu_R (\tilde{H}_1^0 \bar{\mu}_L - \bar{\nu}_L \tilde{H}_1^-)]$$

$$= \frac{1}{\sqrt{2}} (\bar{\nu}^*_L, \tilde{\nu}^*_L) \left( \begin{array}{c} g_2 \lambda_2^{(3)} - g_Y \lambda_Y + 2g_X Q_L \lambda_X \\ g_2 \lambda_2^{(1)} + i \lambda_2^{(2)} \\ -g_2 \lambda_2^{(3)} - g_Y \lambda_Y + 2g_X Q_L \lambda_X \end{array} \right) \left( \begin{array}{c} \mu_L \\ i \lambda_2^{(1)} - i \lambda_2^{(2)} \\ \mu_L \end{array} \right)$$

and the chargino and the neutralino mass terms are given by

$$\mathcal{L}_M = -\lambda [u(\tilde{H}_1^0 \tilde{H}_2^0 - \tilde{H}_1^- \tilde{H}_2^-) + v_1 \tilde{S} \tilde{H}_2^0 + v_2 \tilde{S} \tilde{H}_1^0]$$

$$= \frac{1}{\sqrt{2}} (v_1, 0) \left( \begin{array}{ccc} g_2 \lambda_2^{(3)} - g_Y \lambda_Y + 2g_X Q_L \lambda_X & g_2 \lambda_2^{(1)} - i \lambda_2^{(2)} \\ g_2 \lambda_2^{(1)} + i \lambda_2^{(2)} & -g_2 \lambda_2^{(3)} - g_Y \lambda_Y + 2g_X Q_L \lambda_X \end{array} \right) \chi_0^0$$

where the six components of the neutralino $\chi^0$ and the chargino $\tilde{w}^\pm$ are defined as

$$\chi^{0T} = (\lambda_Y, \lambda_2^{(3)}, \lambda_X, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S})$$

$$\tilde{w}^\pm = -\frac{\lambda^{(1)}_2 + i \lambda^{(2)}_2}{\sqrt{2}}.$$
where $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$. We can diagonalize the mass matrices $M_{\chi^\pm}, M_{\chi^0}$ and $M^2_{\tilde{\mu}}$ by unitary matrix $U_{\chi^\pm}, U_{\chi^0}$ and $U_{\tilde{\mu}}$ as

\[
(U_{\chi^\pm}^\dagger M_{\chi^0} U_{\chi^0}) = m_{\chi^0 \chi} \delta_{XY} \quad (X, Y = 1 - 6),
\]

\[
(U_{\chi^\pm}^\dagger M_{\chi^0} U_{\chi^0}) = m_{\chi^\pm \chi} \delta_{XY} \quad (X, Y = 1, 2),
\]

\[
(U_{\tilde{\mu}}^\dagger M_{\tilde{\mu}} U_{\tilde{\mu}}) = m_{\tilde{\muA}}^2 \delta_{AB} \quad (A, B = 1, 2),
\]

respectively. In this base the muon-chargino and the muon-neutralino interaction terms are rewritten as

\[
\mathcal{L}_{int} = \sum_{AX} \bar{\mu}(N_{AX}^L P_L + N_{AX}^R P_R)\chi^0_X \bar{\muA}
\]

\[
+ \sum_{X} \bar{\mu}(C_X^L P_L + C_X^R P_R)\chi^+ \bar{\nu} h.c.,
\]

\[
C_X^L = y_\mu (U_X -)_{2X},
\]

\[
C_X^R = -g_2(U_X^+)_{1X},
\]

\[
N_{AX}^L = -y_\mu (U_X)_{1X}(U_{\tilde{\mu}})_{LA} - \sqrt{2}[g_Y (U_{\chi^0})_{1X} + Q_E g_X (U_{\chi^0})_{3X}](U_{\tilde{\mu}})_{RA},
\]

\[
N_{AX}^R = \left[\frac{g_2}{\sqrt{2}}(U_X^0)_{2X} + \frac{g_Y}{\sqrt{2}}(U_{\chi^0})_{1X} - \sqrt{2}Q_L g_X (U_{\chi^0})_{3X}](U_{\tilde{\mu}})_{LA}
\]

\[
- y_\mu (U_X)_{1X}(U_{\tilde{\mu}})_{RA},
\]

where we redefined the superfield as $H_1^- \rightarrow -H_1^-$ and $\mu_L \rightarrow -\mu_L$ to use the same notation as $\bar{\mu}$ except for smuon mass mixing.

The neutralino-smuon loop contribution is

\[
\Delta a_{\tilde{\mu}}^0 = \frac{m_\mu}{16\pi^2} \sum_{AX} \left(-\frac{m_\mu}{6m_{\tilde{\muA}}^2(1 - x_{AX})^4}(|N_{AX}^L|^2 + |N_{AX}^R|^2)
\right.

\[
\times (1 - 6x_{AX} + 3x_{AX}^2 + 2x_{AX}^3 - 6x_{AX}^2 \log x_{AX})
\]

\[
- \frac{m_{\chi^0 \chi}}{m_{\tilde{\muA}}^2(1 - x_{AX})^3} N_{AX}^LN_{AX}^R (1 - x_{AX}^2 + 2x_{AX} \log x_{AX})],
\]

\[
x_{AX} = \frac{m_{\chi^0 \chi}}{m_{\tilde{\muA}}^2},
\]

and the chargino-sneutrino loop contribution is

\[
\Delta a_{\tilde{\mu}}^\pm = \frac{m_\mu}{16\pi^2} \sum_{X} \left[\frac{m_\mu}{3m_{\tilde{\mu}}^2(1 - x_X)^4}(|C_X^L|^2 + |C_X^R|^2)
\right.

\[
\times (1 + \frac{3}{2}x_X - 3x_X^2 + \frac{x_X^3}{2} + 3x_X^2 \log x_X)
\]

\[
- \frac{3m_{\chi^\pm \chi}}{m_{\tilde{\mu}}^2(1 - x_X)^3} C_X^LC_X^R (1 - \frac{4}{3}x_X + \frac{1}{3}x_X^2 + \frac{2}{3} \log x_X)],
\]

\[
x_X = \frac{m_{\chi^\pm \chi}}{m_{\tilde{\mu}}^2}.
\]
B The RGEs

In our notation, the renormalization group equations of soft breaking terms in the $\eta$ model are given by

\[
(2\pi) \frac{a_t}{dt} = 6Y_{t}a_{t} + Y_{b}a_{b} + Y_{\lambda}a_{\lambda} + \frac{16}{3} \alpha_{3}M_{3} + 3\alpha_{2}M_{2} + \frac{13}{9} \alpha_{Y}M_{Y} + \frac{4}{3} \alpha_{X}M_{X},
\]
\[
(2\pi) \frac{a_{b}}{dt} = 6Y_{b}a_{b} + Y_{t}a_{t} + Y_{\tau}a_{\tau} + Y_{\lambda}a_{\lambda} + \frac{16}{3} \alpha_{3}M_{3} + 3\alpha_{2}M_{2} + \frac{7}{9} \alpha_{Y}M_{Y} + \frac{1}{3} \alpha_{X}M_{X},
\]
\[
(2\pi) \frac{a_{\tau}}{dt} = 4Y_{\tau}a_{\tau} + 3Y_{b}a_{b} + Y_{\lambda}a_{\lambda} + 3\alpha_{2}M_{2} + 3\alpha_{Y}M_{Y} + \frac{1}{9} \alpha_{X}M_{X},
\]
\[
(2\pi) \frac{a_{\lambda}}{dt} = 3Y_{t}a_{t} + 3Y_{b}a_{b} + Y_{\tau}a_{\tau} + 9Y_{k}a_{k} + 4Y_{\lambda}a_{\lambda} + 3\alpha_{2}M_{2} + \alpha_{Y}M_{Y} + \frac{7}{3} \alpha_{X}M_{X},
\]
\[
(2\pi) \frac{a_{k}}{dt} = 11Y_{k}a_{k} + 2Y_{\lambda}a_{\lambda} + \frac{16}{3} \alpha_{3}M_{3} + \frac{4}{9} \alpha_{Y}M_{Y} + \frac{7}{3} \alpha_{X}M_{X},
\]

where $\alpha = \frac{g^2}{4\pi}$ and $Y = \frac{g^2}{4\pi}$. For the soft scalar masses they are expressed as

\[
(2\pi) \frac{m_{Q(3)}}{dt} = Y_t M_t^2 + Y_b M_b^2 - 3\alpha_2 M_2^2 - \frac{16}{3} \alpha_3 M_3^2 - \frac{1}{9} \alpha_Y M_Y^2 - \frac{4}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{U(3)}}{dt} = 2Y_t M_t^2 - \frac{16}{3} \alpha_3 M_3^2 - \frac{1}{9} \alpha_Y M_Y^2 - \frac{4}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{D(3)}}{dt} = 2Y_b M_b^2 - \frac{16}{3} \alpha_3 M_3^2 - \frac{1}{9} \alpha_Y M_Y^2 - \frac{4}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{E(3)}}{dt} = Y_r M_r^2 - 3\alpha_2 M_2^2 - \frac{1}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{H(3)}}{dt} = 2Y_r M_r^2 - 4\alpha_Y M_Y^2 - \frac{4}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{N(3)}}{dt} = -\frac{25}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{H_3(3)}}{dt} = 3Y_b M_b^2 + Y_\lambda M_\lambda^2 - 3\alpha_2 M_2^2 - \alpha_Y M_Y^2 - \frac{1}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{H_2(3)}}{dt} = 3Y_t M_t^2 + Y_r M_r^2 + Y_\lambda M_\lambda^2 - 3\alpha_2 M_2^2 - \alpha_Y M_Y^2 - \frac{16}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{g(3)}}{dt} = Y_k M_k^2 - \frac{4}{9} \alpha_Y M_Y^2 - \frac{16}{3} \alpha_3 M_3^2 - \frac{16}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{g_2(3)}}{dt} = Y_k M_k^2 - \frac{4}{9} \alpha_Y M_Y^2 - \frac{16}{3} \alpha_3 M_3^2 - \frac{1}{9} \alpha_X M_X^2,
\]
\[
(2\pi) \frac{m_{S(3)}}{dt} = 9Y_k M_k^2 + 2Y_\lambda M_\lambda^2 - \frac{25}{9} \alpha_X M_X^2,
\]

where we omitted the two-loop contributions, for simplicity, and

\[ M_t^2 = m_{Q(3)}^2 + m_{U(3)}^2 + m_{H_2(3)}^2 + a_t^2, \]
\begin{align*}
M_b^2 &= m_{Q_{(3)}}^2 + m_{D_{(3)}}^2 + m_{H_{1(3)}}^2 + a_b^2, \\
M_\tau^2 &= m_{L_{(3)}}^2 + m_{E_{(3)}}^2 + m_{H_{2(3)}}^2 + a_\tau^2, \\
M_\lambda^2 &= m_{H_{1(3)}}^2 + m_{H_{2(3)}}^2 + m_{S_{(3)}}^2 + a_\lambda^2, \\
M_k^2 &= m_g^2 + m_{\tilde{g}}^2 + m_{\tilde{S}_{(3)}}^2 + a_k^2,
\end{align*}

and \( m_{g(1)}^2 = m_{g(2)}^2 = m_{g(3)}^2 = m_g^2 \) is assumed.

\section{Sfermion spectrums}

In our notation, a sfermion mass matrix is given by

\begin{align*}
M_t^2 &= \begin{pmatrix}
m_{t_{L}}^2 + y_t v_2^2 & y_t v_2 (a_t - \lambda u \cot \beta) \\
y_t v_2 (a_t - \lambda u \cot \beta) & m_{t_{R}}^2 + y_t v_2^2
\end{pmatrix}, \\
M_b^2 &= \begin{pmatrix}
m_{b_{L}}^2 + y_b v_1^2 & y_b v_1 (a_b - \lambda u \tan \beta) \\
y_b v_1 (a_b - \lambda u \tan \beta) & m_{b_{R}}^2 + y_b v_1^2
\end{pmatrix}, \\
M_\tau^2 &= \begin{pmatrix}
m_{\tau_{L}}^2 + y_\tau v_2^2 & y_\tau v_1 (a_\tau - \lambda u \tan \beta) \\
y_\tau v_1 (a_\tau - \lambda u \tan \beta) & m_{\tau_{R}}^2 + y_\tau v_1^2
\end{pmatrix}, \\
M_g^2 &= \begin{pmatrix}
m_g^2 + k^2 u^2 & a_k k u - \lambda k v_1 v_2 \\
a_k k u - \lambda k v_1 v_2 & m_{\tilde{g}}^2 + k^2 u^2
\end{pmatrix}, \\
M_{\tilde{\nu}_e}^2 &= m_{\tilde{\nu}_e}^2,
\end{align*}

where

\begin{align*}
m_{t_{L}}^2 &= m_{Q_{3}}^2 + \frac{3 g_3^2 - g_Y^2}{12} (v_1^2 - v_2^2) - \frac{1}{3} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{t_{R}}^2 &= m_{D_{3}}^2 + \frac{g_Y^2}{3} (v_1^2 - v_2^2) - \frac{1}{3} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{b_{L}}^2 &= m_{Q_{3}}^2 - \frac{3 g_3^2 + g_Y^2}{12} (v_1^2 - v_2^2) - \frac{1}{3} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{b_{R}}^2 &= m_{D_{3}}^2 - \frac{g_Y^2}{6} (v_1^2 - v_2^2) + \frac{1}{6} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{\tau_{L}}^2 &= m_{L_{3}}^2 + \frac{g_3^2 + g_Y^2}{4} (v_1^2 - v_2^2) + \frac{1}{6} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{\tau_{R}}^2 &= m_{E_{3}}^2 - \frac{g_Y^2}{2} (v_1^2 - v_2^2) - \frac{1}{3} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{\tilde{\nu}_e}^2 &= m_{L_{3}}^2 + \frac{g_3^2 + g_Y^2}{4} (v_1^2 - v_2^2) + \frac{1}{6} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_g^2 &= m_g^2 + \frac{g_Y^2}{6} (v_1^2 - v_2^2) + \frac{2}{3} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2), \\
m_{\tilde{g}}^2 &= m_{\tilde{g}}^2 + \frac{g_Y^2}{6} (v_1^2 - v_2^2) + \frac{1}{6} g_X (\frac{1}{6} v_1^2 + \frac{2}{3} v_2^2 - \frac{5}{6} u^2),
\end{align*}
The mass spectra of the extra U(1) models with \( \tan \beta = 20 \) and \( A_0 = 0 \) (GeV). The typical sparticle spectra are given in Table 2.

| \( m_0 \) | 0(\( \eta \)) | 200(\( \eta \)) | 400(\( \eta \)) | 0(\( \xi \)) | 200(\( \xi \)) | 400(\( \xi \)) |
|------|------|------|------|------|------|------|
| \( k \) | 0.64 | 0.64 | 0.62 | 0.66 | 0.66 | 0.62 |
| \( \lambda \) | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 |
| \( M_{30} \) | 220 | 220 | 220 | 220 | 220 | 220 |
| \( M_{20} \) | 800 | 800 | 750 | 800 | 750 | 700 |
| \( M_{Y0} \) | 900 | 800 | 500 | 900 | 1050 | 700 |

- \( Z' \) | 599.5 | 641.2 | 724.0 | 605.5 | 664.3 | 721.4 |
- \( \tilde{t}_+ \) | 601.2 | 631.6 | 696.8 | 568.9 | 584.0 | 624.4 |
- \( \tilde{t}_- \) | 218.3 | 271.6 | 396.0 | 143.8 | 254.5 | 350.1 |
- \( \tilde{b}_+ \) | 558.1 | 592.3 | 664.3 | 527.2 | 546.6 | 624.4 |
- \( \tilde{b}_- \) | 264.7 | 315.9 | 443.1 | 432.5 | 493.7 | 585.1 |
- \( \tilde{\tau}_+ \) | 447.6 | 478.1 | 544.4 | 566.5 | 602.8 | 665.6 |
- \( \tilde{\tau}_- \) | 409.9 | 440.3 | 528.9 | 355.0 | 455.5 | 505.6 |
- \( \tilde{\nu}_\tau \) | 440.7 | 471.7 | 538.7 | 561.2 | 597.8 | 661.0 |
- \( \tilde{g}_+ \) | 1071.3 | 1138.7 | 1256.1 | 939.8 | 1029.9 | 1084.7 |
- \( \tilde{g}_- \) | 729.5 | 803.4 | 922.2 | 602.9 | 687.5 | 736.1 |
- \( \chi_1^\pm \) | 255.3 | 257.4 | 244.1 | 246.7 | 236.7 | 224.0 |
- \( \chi_2^\pm \) | 467.4 | 496.3 | 554.5 | 400.7 | 429.7 | 460.4 |
- \( \tilde{h}_0 \) | 125.7 | 125.7 | 125.7 | 115.5 | 116.5 | 116.4 |
- \( \tilde{h}^\pm \) | 442.1 | 502.9 | 661.6 | 309.0 | 380.9 | 455.9 |

so the mass eigenvalues are given by,

\[
m_{\tilde{t}_\pm}^2 = \frac{1}{2}(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2) + y_t^2 v_2^2 \pm \sqrt{\frac{1}{4}(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)^2 + y_t^2 v_2^2 (a_t - \lambda u \cot \beta)^2},
\]
\[
m_{\tilde{b}_\pm}^2 = \frac{1}{2}(m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2) + y_b^2 v_1^2 \pm \sqrt{\frac{1}{4}(m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2)^2 + y_b^2 v_1^2 (a_b - \lambda u \tan \beta)^2},
\]
\[
m_{\tilde{\tau}_\pm}^2 = \frac{1}{2}(m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2) + y_\tau^2 v_1^2 \pm \sqrt{\frac{1}{4}(m_{\tilde{\tau}_L}^2 - m_{\tilde{\tau}_R}^2)^2 + y_\tau^2 v_1^2 (a_\tau - \lambda u \tan \beta)^2},
\]
\[
m_{\tilde{g}_\pm}^2 = \frac{1}{2}(m_{\tilde{g}}^2 + m_{\tilde{\tau}_R}^2) + k^2 u^2 \pm \sqrt{\frac{1}{4}(m_{\tilde{g}}^2 - m_{\tilde{\tau}_R}^2)^2 + (a_k k u - \lambda k v_1 v_2)^2}.
\]

The smuon has the same mass matrix structure as the stau. However the muon Yukawa coupling is very small and the off-diagonal element of the smuon mass matrix is negligible. The typical sparticle spectra are given in Table 2.
References

[1] H.P. Nilles, *Supersymmetry, Supergravity and Particle Physics* Phys. Rep. 110(1984)1.

[2] Muon (g-2) Collaboration, *Precise measurement of the positive muon anomalous magnetic moment* Phys. Rev. Lett. 86(2001)2227 [hep-ex/0102017].

[3] A. Czarnecki and W. J. Marciano, *The Muon Anomalous magnetic Moment: A Harbinger For “New Physics”* Phys. Rev. D64(2001)013014 [hep-ph/0102122].

[4] D. A. Kosower, L. M. Krauss and N. Sakai, *Low energy supergravity and the anomalous magnetic moment of the muon* Phys. Lett. B133(1983)305.

[5] J. L. Lopez, D. V. Nanopoulos and X. Wu, Wang, *Large (g − 2)µ in SU(5) × U(1) supergravity models* Phys. Rev. D49(1994)366.

[6] T. Moroi, *Muon Anomalous Magnetic Dipole Moment in the Minimal Supersymmetric Standard Model* Phys. Rev. D53(1996)6565, Phys. Rev. D56(1997)4424E.

[7] S. P. Martin, *Predictions of the Sign of µ from supersymmetry breaking models* hep-ph/0106280.

[8] J. L. Hewett and T. G. Rizzo, *Low-energy phenomenology of superstring-inspired E6 models* Phys. Rep. 183(1989)193, F. Zwirner, *Phenomenological Aspects of E6 Superstring-Inspired Models* Int. J. Mod. Phys. A3(1988)49, M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi and N. Siberg *Superstring Model Building* Nucl. Phys. B259(1985)549.

[9] B. A. Campbell, J. Ellis, K. Enqvist, M. K. Gaillard and V. Nanopoulos, *Superstring Models Challenged by Rare Processes* Int. J. Mod. Phys. A2(1987)831.

[10] D. Suematsu and Y. Yamagishi, *Radiative Symmetry Breaking in a Supersymmetric Model with an Extra U(1)* Int. J. Mod. Phys. A10(1995)4521.

[11] Y. Daikoku and D. Suematsu, *Mass bound of the lightest neutral Higgs scalar in the extra U(1) models* Phys. Rev. D62(2000)095006 [hep-ph/0003205].

[12] D. A. Morris, *Potentially large contributions to the muon anomalous magnetic moment from weak-isosinglet squarks in E6 superstring models* Phys. Rev. D37(1988)2012, K. Cheung, *Muon Anomalous Magnetic Moment and Leptoquark Solutions* Phys. Rev. D64(2001)033001 [hep-ph/0102238].

[13] Y. Daikoku, in preparation.
[14] Gi-Chol Cho, K.Hagiwara and Y.Umeda, *Z' bosons in supersymmetric $E_6$ models confront electroweak data* Nucl. Phys. **B531**(1998)65 [hep-ph/9805448], Nucl. Phys. **B555**(1999)651E, M.Cvetič, D.Demir, J.R.Espinosa, L.Everett and P.Langacker, *Electroweak breaking and the $\mu$ problem in supergravity models with an additional $U(1)$* Phys. Rev. **D56**(1997)2861.

[15] J.L.Feng, K.T.Matchev and T.Moroi, *Focus points and naturalness in supersymmetry* Phys. Rev. **D61**(2000)075005 [hep-ph/9909334], J.L.Feng and K.T.Matchev, *Focus Point Supersymmetry: Proton Decay, Flavor and CP Violation, and the Higgs Boson Mass* Phys. Rev. **D63**(2001)095003 [hep-ph/0011356].

[16] Particle Data Group, Eur.Phys.J. **C15**(2000)1.

[17] D.M.Pierce, J.A.Bagger, K.T.Matchev, and Ren-Jie Zhang, *Precision corrections in the minimal supersymmetric standard model* Nucl. Phys. **B491**(1997)3.

[18] C.R.Das and M.K.Parida, *New Formulas and Predictions for Running Fermion Masses at Higher Scales in SM, 2HDM, and MSSM* Eur.Phys.J. **C20**(2001)121 [hep-ph/0010004], D.A.Demir, *Two-Higgs doublet models from TeV-scale supersymmetric extra $U(1)$ models* Phys. Rev. **D59**(1999)0105002 [hep-ph/9809358].

[19] S.P.Martin and M.T.Vaughn, *Two-loop renormalization group equations for soft supersymmetry-breaking couplings* Phys. Rev. **D50**(1994)2282, Y.Yamada, *Two-loop renormalization group equations for soft supersymmetry-breaking scalar interactions: Supergaph method* Phys. Rev. **D50**(1994)3537. S.P.Martin, *A Supersymmetry Primer* hep-ph/9709356.

[20] A.B.Lahanas and D.V.Nanopoulos, *The Road to No-Scale Supergravity* Phys. Rep. **145**(1987)1.

[21] S.Komine and M.Yamaguchi, *No-Scale Scenario with Non-Universal Gaugino Masses* Phys. Rev. **D63**(2001)035005 [hep-ph/0007327], S.Komine, T.Moroi and M.Yamaguchi, *No-scale Scenarios in the Light of New Measurement of Muon Anomalous Magnetic Moment* Phys. Lett. **B507**(2001)224 [hep-ph/0103182].

[22] D.Suematsu, *Neutralino decay in the $\mu$-problem solvable extra $U(1)$ models* Phys. Rev. **D57**(1998)1738 [hep-ph/9708413].