An effective Lagrangian for the continuous transition in an extended Kondo lattice model

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We propose an effective Lagrangian for the continuous transition from the heavy fermion metal to the antiferromagnetic metal in an extended Kondo lattice model. Based on the slave-boson representation we introduce an additional new order parameter associated with difference of the chemical potential between conduction electrons \(c_{i\sigma}\) and local spinons \(f_{i\alpha}\). This order parameter allows pseudospin construction \(T_{ix} = \frac{1}{2}(\langle c_{i\sigma}f_{i\alpha} + f_{i\alpha}^\dagger c_{i\sigma}\rangle - \langle f_{i\alpha}c_{i\sigma} - f_{i\alpha}^\dagger c_{i\sigma}\rangle)\), \(T_{iz} = \frac{1}{2}(\langle c_{i\sigma}f_{i\alpha} - f_{i\alpha}^\dagger c_{i\sigma}\rangle)\), where \(T_{ix} = T_{iz}\) corresponds to the usual hybridization order parameter in the slave-boson representation of the Kondo lattice model. The resulting effective action is shown to be an anisotropic pseudospin model with a Landau damping term for the screened-unscreened (XY–Ising) phase transition. To describe the emergence of antiferromagnetic order in the unscreened (Ising) phase, we phenomenologically introduce the antiferromagnetic Heisenberg model for the localized spins, where the effective coupling strength is given by \(J_{eff} = J\langle (T_{ix})^2 \rangle\). This ad-hoc construction allows the continuous transition from the heavy fermion phase to the antiferromagnetic phase because breakdown of Kondo screening \((\langle T_{ix} \rangle = 0)\) causes effective exchange interactions between unscreened local moments.

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I. INTRODUCTION

In the Landau-Ginzburg-Wilson (LGW) theoretical framework a continuous phase transition is generically forbidden between two different orders characterized by different symmetry breaking if multi-critical points resulting from fine tuning of the couplings in the LGW theory are not taken into account. Recently, Berry phase mechanism was proposed for generic continuous transitions in quantum antiferromagnets and Bose-Mott insulators.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\) If the disorder parameter in one phase transforms as the order parameter in the other phase, the continuous transition between the two phases is naturally expected to occur. However, this situation does not happen generally. Condensation of skyrmions in the antiferromagnetic phase or vortices in the superconducting phase kills the ordered phase. The resulting disordered state would be a paramagnetic phase or Bose-Mott insulating phase, preserving all possible symmetries. But, the presence of Berry phase can break the symmetries associated with lattice translations or rotations since the Berry phase plays the role of effective potentials in the disorder parameters, causing the lattice-symmetry breaking. Critical field theories were also proposed, and in some cases “exotic” quantum criticality is expected to appear, described by gauge-interacting deconfined particles carrying fractional quantum numbers.

Some heavy fermion systems also exhibit the LGW forbidden continuous transition from the heavy fermion metal to the antiferromagnetic metal, where the heavy fermion phase is understood by hybridization between conduction electrons and local moments while the antiferromagnetic state is described by the magnetic long range order of localized spins.\(^6\) The former is associated with an internal U(1) gauge symmetry like superconductivity, and the latter, the spin rotational symmetry. In such heavy fermion systems the Berry phase mechanism of the magnetic and superconducting systems does not seem to work because the disorder parameter in one phase does not transform as the order parameter in the other phase even in the presence of Berry phase.

Unfortunately, the mechanism of the continuous transition is still far from consensus. In the slave-boson formulation the heavy fermion phase naturally appears via the hybridization order parameter while the antiferromagnetic phase arises from the antiferromagnetic order parameter of localized spins.\(^7\)\(^8\) Because these two order parameters are associated with different symmetry breaking, the continuous transition is generally forbidden in the LGW framework, as discussed above. In the slave-fermion representation the antiferromagnetic order naturally arises via the condensation of bosonic spinons while the heavy fermion phase is described by new emergent spinless fermions instead of some bosonic order parameters.\(^9\)\(^10\) But, the strong Kondo coupling regime is not easy to explain in this framework. On the other hand, in the Bose-Fermi Kondo model the continuous transition is expected to arise in low dimensions since divergences of the local and global spin susceptibilities should occur at the same time.\(^11\) But, in three dimensions the divergences in both spin susceptibilities would not happen simultaneously.

In the present paper we revisit the slave-boson formulation of the Kondo lattice model, and propose an effective Lagrangian for the continuous transition from the heavy fermion metal to the antiferromagnetic metal. We introduce an additional new order parameter associated with difference of the chemical potential between conduction electrons \(c_{i\sigma}\) and local spinons \(f_{i\alpha}\), extending the usual slave-boson construction. This order parameter
allows pseudospin construction \( T_{ix} = \frac{1}{2}(c_{i\alpha}^\dagger f_{i\alpha} + f_{i\alpha}^\dagger c_{i\alpha}) \), \( T_{iy} = -\frac{i}{2}(c_{i\alpha}^\dagger f_{i\alpha} - f_{i\alpha}^\dagger c_{i\alpha}) \), and \( T_{iz} = \frac{1}{2}(c_{i\alpha}^\dagger c_{i\alpha} - f_{i\alpha}^\dagger f_{i\alpha}) \), where \( T_{iz} = T_{ix} \pm iT_{iy} \) corresponds to the usual hybridization order parameter in the slave-boson representation of the Kondo lattice model. The resulting effective action is shown to be an anisotropic pseudospin model with a Landau damping term for the screened-unscreened (XY–Ising) phase transition. To describe the emergence of antiferromagnetic order in the unscreened (Ising) phase, we phenomenologically introduce the antiferromagnetic Heisenberg model for the localized spins, where the effective coupling strength is given by \( J_{eff} = J\langle T_{iz}\rangle^2 \). This ad-hoc construction allows the continuous transition from the heavy fermion phase to the antiferromagnetic phase because breakdown of Kondo screening \((\langle T_{ix}\rangle = 0\) and \(\langle T_{iz}\rangle \neq 0\)) causes effective exchange interactions between unscreened local moments. We argue that our effective theory results in discontinuous volume change of the Fermi surface across the transition. Furthermore, we propose a critical field theory, and discuss critical behaviors near the quantum critical point.

II. DISCUSSION ON RESULTS

We start from discussion of results in order to get a clear physical picture. The effective action is proposed to be

\[
S_{eff} = \frac{1}{\beta} \sum_{q,\Omega} (\xi_{\perp}^{-2} + q^2 + \frac{\gamma |\Omega|}{q}) T_+(q,i\Omega) T_-(q,i\Omega) \\
+ \frac{1}{\beta} \sum_{q,\Omega} (\xi_{\parallel}^{-2} + q^2 + \frac{\gamma |\Omega|}{q}) T_z(q,i\Omega) T_z(-q,-i\Omega) \\
+ J|\langle T_{iz}\rangle|^2 \sum_{ij} S_i \cdot S_j.
\]

(1)

Here \( T_{ix} = T_{ix} + iT_{iy} \), \( T_{ix} = T_{ix} - iT_{iy} \), and \( T_{iz} \) are the pseudospin fields, considered as

\[
T_{ix} = \frac{1}{2}(c_{i\alpha}^\dagger f_{i\alpha} + f_{i\alpha}^\dagger c_{i\alpha}), \\
T_{iy} = -\frac{i}{2}(c_{i\alpha}^\dagger f_{i\alpha} - f_{i\alpha}^\dagger c_{i\alpha}), \\
T_{iz} = \frac{1}{2}(c_{i\alpha}^\dagger c_{i\alpha} - f_{i\alpha}^\dagger f_{i\alpha}),
\]

and \( S_i \) the local spin fields, given by

\[
S_i = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{i\beta},
\]

where \( c_{i\alpha} \) is a conduction electron, and \( f_{i\alpha} \) a spinon for localized spins in the slave-boson representation. One can find that the expectation value of the pseudospin raising operator given by \( \langle T_{ix}\rangle = \langle c_{i\alpha}^\dagger f_{i\alpha}\rangle \) is nothing but the hybridization order parameter in the slave-boson formulation. Thus, introduction of the 3-component pseudospin variable naturally extends the slave-boson formulation. The \( T_{iz} \) operator represents difference of the chemical potential between the conduction electrons and local spinons, considered to be another measure for the Kondo screening. It is important to notice that introduction of the \( T_{iz} \) operator completes the commutation relation \( [T_{ix}, T_{iz}] = \pm \delta_{ij} T_{iz} \), resulting in the right uncertainty relation.

\( \xi_{\perp} \) and \( \xi_{\parallel} \) are the pseudospin correlation lengths associated with the XY and Ising orders respectively, obtained to be \( \xi_{\perp} = \xi_{\parallel} \) in the SU(2) symmetric case. Physically, \( \xi_{\parallel} \) corresponds to the Kondo screening length, given by \( \xi_{\parallel}^{-2} \sim 1 - JK D \), thus diverging at the quantum critical point \( J_K^c = 1/D \). Here \( D \) is the density of fermion states. We remark one crucial assumption \( \xi_{\perp}^{-2} \cdot \xi_{\parallel}^{-2} \leq 0 \) in this paper. This ad-hoc construction allows the continuous transition between the XY and Ising orders of the pseudospin fields through the quantum critical point \( J_K = J_K^c \) satisfying \( \xi_{\perp}^{-2} \cdot \xi_{\parallel}^{-2} \to 0 \).

In the case of \( J_K > J_K^c \) (\( \xi_{\perp}^{-2} > 0 \) and \( \xi_{\parallel}^{-2} < 0 \)) the XY order appears to describe the heavy fermion phase, characterized by \( \langle T_{iz}\rangle = 0 \) and \( \langle T_{ix}\rangle = 0 \). This means that there exist no unscreened local magnetic moments owing to the Kondo hybridization, and the vacuum expectation value of the localized spins vanishes, \( \langle S_i \rangle = 0 \). In the case of \( J_K < J_K^c \) (\( \xi_{\perp}^{-2} < 0 \) and \( \xi_{\parallel}^{-2} > 0 \)) the Ising order arises to imply incomplete Kondo screening, thus the heavy fermion phase disappears, described by \( \langle T_{ix}\rangle = 0 \) and \( \langle T_{iz}\rangle = 0 \). As a result, unscreened local magnetic moments occur due to the chemical potential difference, causing effective exchange interactions \( J|\langle T_{iz}\rangle|^2 \) between the local moments. This mechanism is reflected in the antiferromagnetic Heisenberg model of localized spins, resulting in the antiferromagnetic long range order \( \langle S_i \rangle \neq 0 \) in the unscreened (Ising) phase. Because the effective exchange interactions depend on the vacuum expectation value of the \( T_{iz} \) operator, the antiferromagnetic order should start from the quantum critical point. As a result, the second order transition from the heavy fermion phase to the antiferromagnetic one is truly achieved. Unfortunately, we failed the derivation of these effective exchange couplings. This ad-hoc construction is another important assumption in this paper.

The XY and Ising orders become degenerate at the quantum critical point because of \( \xi_{\perp}^{-2} \cdot \xi_{\parallel}^{-2} \to 0 \), indicating the emergence of a new symmetry that is absent away from the quantum critical point. The emerging symmetry is SU(2) pseudospin symmetry larger than U(1) in the XY phase and Z(2) in the Ising state. This enlarged symmetry plays an important role in critical dynamics of pseudospins with the frequency-dependent terms in Eq. (1) known as Landau damping terms with a damping coefficient \( \gamma \), resulting from low energy fermion excitations near a Fermi surface. This will be discussed later.

It should be noted that our effective action exhibits a discontinuous change in the Fermi surface volume. In the Kondo screened phase a large Fermi surface is expected
owing to the Kondo hybridization $\langle T_+ \rangle \neq 0$. On the other hand, in the decoupled phase a small Fermi surface would appear due to $\langle T_+ \rangle = 0$. Although the XY to Ising transition is continuous, discontinuity in the volume change of the Fermi surface should be observed right at the critical point since the large Fermi surface is sustained as long as the Kondo hybridization exists.

A schematic phase diagram is shown in Fig. 1, where HF denotes the heavy fermion phase, AF the antiferromagnetic phase, and QC the quantum critical regime. Since HF and AF are described by the XY and Ising orders in terms of the pseudospin fields approximately, one might expect that their finite temperature transitions fall into the Berezinskii-Kosterlitz-Thouless (BKT) transition in two dimensions and Ising one, respectively. Precisely speaking, this guess is not correct because the Landau damping term modifies the transition nature apparently, as will be discussed later. Because there is no finite temperature transition in the SU(2) symmetric model owing to the Mermin-Wagner-Hohenberg-Coleman (MWHC) theorem, both the XY and Ising transition lines meet and vanish at the SU(2) symmetric quantum critical point, i.e. $T_{XY} = T_{Ising} \to 0$.

**III. DERIVATION OF THE EFFECTIVE LAGRANGIAN**

We derive an effective pseudospin action from the Kondo lattice model

\[
H = H_c + H_m + H_K,
\]

\[
H_c = - t \sum_{ij} c_{i\alpha}^\dagger c_{j\alpha},
\]

\[
H_m = J \sum_{ij} S_i \cdot S_j,
\]

\[
H_K = J_K \sum_i S_i \cdot c_{i\alpha}^\dagger \sigma_{\alpha\beta} c_{i\beta},
\]

where $J_K$ is the Kondo coupling between the conduction electrons and local spins, and $J$ the antiferromagnetic coupling between the local spins.

Using the slave-boson representation, one can obtain a one-body effective Lagrangian in terms of the conduction electrons and spinons coupled to order parameters.

\[
Z = \int Dc_{i\alpha} Df_{i\alpha} D\psi_{i\alpha} e^{- \int dt L},
\]

\[
L = \sum_i c_{i\alpha}^\dagger (\partial_t - \mu_c) c_{i\alpha} - \frac{1}{2} \sum_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_i f_{i\alpha}^\dagger f_{i\alpha} - \chi_0 \sum_{ij} f_{i\alpha}^\dagger f_{j\alpha} + \sum_{ij} \frac{\chi_0^2}{J} - \sum_i (b_i^\dagger b_i + H.c.) + \sum_i \frac{|b_i|^2}{J_K}.
\]

Here $\mu_c$ is the chemical potential of conduction electrons, and $\mu_f$ that of local spinons. $b_i$ is called holon, representing hybridization between the conduction electron and spinon. $\chi_0$ is the hopping order parameter of the spinons, assumed to be spatially homogeneous. Mean field values of the order parameters are given by

\[
b_i = J_K \langle f_{i\alpha}^\dagger f_{i\alpha} \rangle,
\]

\[
\chi_0 = \frac{J}{2} \langle f_{i\alpha}^\dagger f_{j\alpha} \rangle.
\]

Here we consider the case of half filling for the spinons, $\langle f_{i\alpha}^\dagger f_{i\alpha} \rangle = \langle f_{i\alpha}^\dagger f_{i\alpha} \rangle$.

As mentioned in the introduction, the slave-boson effective Lagrangian is difficult to describe the continuous transition to the antiferromagnetic long range order in the mean field level. In this respect an extension of Eq. (3) is indispensable. It is convenient to express Eq. (3) in terms of the "Nambu" spinor

\[
\psi_{i\alpha} = \begin{pmatrix} \psi_{i1\alpha} \\ \psi_{i2\alpha} \end{pmatrix} = \begin{pmatrix} c_{i\alpha} \\ f_{i\alpha} \end{pmatrix}.
\]

Inserting this into Eq. (3), we obtain

\[
L_{eff} = \sum_i \psi_{i\alpha}^\dagger (\partial_t - \mu_n) \psi_{i\alpha} - \sum_{ij} t_n \psi_{i\alpha}^\dagger \psi_{j\alpha} - \sum_i \psi_{i\alpha}^\dagger T_{i+} \psi_{i\alpha} - \sum_i \psi_{i\alpha}^\dagger T_{i-} \psi_{i\alpha} + \sum_i T_{i+} T_{i-} J_K
\]

with $\mu_n = \left( \begin{array}{c} \mu_c \\ -\mu_f \end{array} \right)$ and $t_n = \left( \begin{array}{c} t _e \\ \chi _0 \end{array} \right)$. Here $T_{i+}$ corresponds to the holon field $b_i$.

The $T_{i+}$ and $T_{i-}$ operators can be considered as the raising and lowering operators of the pseudospin operators if we introduce the $z$-component pseudospin field $T_{iz}$.

Consider the following term

\[
L_z = -U_K \sum_i (c_{i\alpha}^\dagger c_{i\alpha} - f_{i\alpha}^\dagger f_{i\alpha})^2 - U_K \sum_i (\psi_{i\alpha}^\dagger \tau_{nm} \psi_{i\alpha})^2.
\]

FIG. 1: A schematic phase diagram in the effective action of the Kondo lattice model
These local interactions compete with the Kondo couplings because this term favors unscreened local moments. Performing the Hubbard-Stratonovich transformation, and adding the resulting Lagrangian

\[ L_z = -\sum_i \psi_{ina}^{\dagger} T_{iz} \tau_{nm} \psi_{jma} + \sum_i \frac{T_{iz}^2}{4U_K} \]

into Eq. (5), where \( T_{iz} \) is the \( z \)-component pseudospin variable, the effective Lagrangian Eq. (5) can be naturally generalized into

\[ L_{\text{eff}} = \sum_i \psi_{ina}^{\dagger} (\partial_{\tau} - \mu_n) \psi_{ina} - \sum_i t_n \psi_{ina}^{\dagger} \psi_{jma} - \sum_i \psi_{ina}^{\dagger} T_i \cdot \tau_{nm} \psi_{jma} + \sum_i \left( \frac{T_{i+T_{i-}}}{4J_K} + \frac{T_{iz}^2}{4U_K} \right), \]

(7)

where \( T_{iz(y)} \) was replaced with \((1/2) T_{iz(y)}\). Here the pseudospin variable is obtained to be in the mean field level

\[ \frac{T_{ix}}{4J_K} = \frac{1}{2} \left( \psi_{ina}^{\dagger} c_{ia} + f_{ia}^{\dagger} c_{ia} \right) = \frac{1}{2} \left( \psi_{ina}^{\dagger} \tau_{nm} \psi_{jma} \right), \]
\[ \frac{T_{iy}}{4J_K} = -\frac{i}{2} \left( \psi_{ina}^{\dagger} f_{ia} - f_{ia}^{\dagger} c_{ia} \right) = \frac{1}{2} \left( \psi_{ina}^{\dagger} \tau_{nm} \psi_{jma} \right), \]
\[ \frac{T_{iz}}{4U_K} = \frac{1}{2} \left( \psi_{ina}^{\dagger} c_{ia} - f_{ia}^{\dagger} f_{ia} \right) = \frac{1}{2} \left( \psi_{ina}^{\dagger} \tau_{nm} \psi_{jma} \right). \]

(8)

The chemical potential term in Eq. (7) can be written as

\[ -\mu_n \psi_{ina}^{\dagger} \psi_{ina} = -\mu \psi_{ina}^{\dagger} \psi_{ina} - \mu_z \psi_{ina}^{\dagger} \tau_{nm} \psi_{jma} \]

with \( \mu = (\mu_c + \mu_f)/2 \) and \( \mu_z = (\mu_c - \mu_f)/2 \). Shifting the \( z \)-component pseudospin as \( T_{iz} \to T_{iz} - \mu_z \), we obtain the effective Lagrangian

\[ L_{\text{eff}} = \sum_i \psi_{ina}^{\dagger} (\partial_{\tau} - \mu) \psi_{ina} - \sum_i t_n \psi_{ina}^{\dagger} \psi_{jma} - \sum_i \psi_{ina}^{\dagger} T_i \cdot \tau_{nm} \psi_{jma} + \sum_i \left( \frac{T_{i+T_{i-}}}{4J_K} + \frac{(T_{iz} - \mu_z)^2}{4U_K} \right). \]

(9)

A. Strong coupling approach

In the strong coupling limit we consider the \( CP^1 \) representation for the pseudospin variable

\[ T_i \cdot \tau_{nm} = M_K U_{in} \tau_{pq} U_{jm}, \]
\[ U_{inm} = \left( \begin{array}{cc} z_{i1} & z_{i2} \\ z_{i2} & z_{i1} \end{array} \right), \]

(10)

where \( U_{inm} \) is a SU(2) matrix field in terms of a complex boson field \( z_{in} \) with the pseudospin index \( n = 1, 2 \), and \( M_K \) amplitude of the pseudospin. Inserting Eq. (9) into Eq. (8), and performing the gauge transformation

\[ \psi_{ina} = U_{inm} \psi_{jma}, \]

we obtain

\[ L_{\text{eff}} = \sum_i \psi_{ika}^{\dagger} \left( (\partial_{\tau} - \mu) \delta_{km} - M_K \tau_{km} \right) U_{inm} \psi_{jma} \]
\[ - \sum_{ij} t_n \psi_{ika}^{\dagger} U_{ikn} U_{jnm} \psi_{jma} - \sum_i M_K \psi_{ika}^{\dagger} \tau_{km} \psi_{jma}. \]

(11)

Integrating out the spinor fields \( \psi_i \), and expanding the resulting logarithmic term for \( U^{\dagger}_i \partial_{\tau} U_i \) and \( U^{\dagger}_i U_j \), we obtain

\[ S_{\text{eff}} = -\text{tr} \ln \left( (\partial_{\tau} - \mu) \delta_{km} - M_K \tau_{km} \right) \]
\[ + U_{ikn}^{\dagger} \partial_{\tau} U_{inm} - t_n U_{ikn}^{\dagger} U_{jnm} \]
\[ \approx \sum_i \text{tr} \left[ G_0 (U^{\dagger}_i \partial_{\tau} U_i) \right] \]
\[ + \frac{1}{2} \sum_i \text{tr} \left[ G_0 U^{\dagger}_i t U_{j} G_0 U^{\dagger}_j t U_i \right], \]

(12)

where \( G_0 = - (\partial_{\tau} - \mu - M_K \tau_{z})^{-1} \) is the single particle propagator. Notice that the spinor propagator depends on only time or frequency, not position or momentum, implying that the kinetic energy of electrons and spinons is ignored owing to strong hybridization. This means that the strong coupling expansion justified in the limit of \( t << M_K \) can be applied to an insulating phase of electrons and spinons. In other words, the assumed ground state of spinors is an insulating phase in this expansion scheme. This will be more clarified later. The first term leads to Berry phase while the second results in an exchange interaction term, both well evaluated in Refs. 3 12. One difference from the previous studies\textsuperscript{3} is the presence of the chemical potential term. But, this does not result in any important modification.

The resulting effective action is obtained to be

\[ S_{\text{eff}} = iS \sum_i \omega(\{T_{i}(\tau)\}) + \int_0^\beta d\tau H_{\text{eff}}, \]
\[ H_{\text{eff}} = I_K \sum_{ij} \left( T_{iz} T_{jx} + T_{iy} T_{jy} \right) + V_K \sum_{ij} T_{iz} T_{jz}, \]

(13)
where $S$ is the pseudospin value, given by $S = 1/2$. It is interesting that the effective Hamiltonian for the Kondo screening is given by the antiferromagnetic pseudospin model in the strong coupling limit. The first term in $S_{\text{eff}}$ is a Berry phase term of the pseudospin field. $I_K$ and $V_K$ are the XY and Ising exchange couplings, given by $I_K = V_K = 2t^2/M_K$ for the SU(2) symmetric case. Here the anisotropy in the hopping integrals is neglected for simplicity, discussed later in more detail.

Eq. (13) shows a continuous transition from the XYZ to the Ising one at the quantum critical point $V_K/I_K = 1$. If $1/J_K$ is replaced with $V_K/I_K$ in Fig. 1, the antiferromagnetic pseudospin model exhibits a similar phase diagram with Fig. 1. In the limit of $V_K/I_K << 1$ a Kondo insulator is expected while in the opposite limit an antiferromagnetic insulator would result. The finite temperature transitions are simply given by the BKT transition in the two dimensional XY phase of $V_K/I_K < 1$ and the Ising one in the Ising state of $V_K/I_K > 1$, respectively. These transition lines should meet and vanish at the quantum critical point owing to the enlarged SU(2) pseudospin symmetry (MWHC theorem).

B. Weak coupling approach

In the weak coupling approach valid in the $t >> M_K$ limit, one obtains the following effective action after integrating out the spinor fields $\psi_{\alpha \sigma}$ in Eq. (8)

$$S_{\text{eff}} = -t r \ln \left[ \partial_\tau - \mu - t_{ij} - T_i \cdot \tau \right].$$

Note the difference between Eq. (12) and Eq. (14). The main difference is that the spinor propagator in Eq. (14) includes the momentum (position) dependence, implying that the assumed ground state is a noninteracting Fermi gas, thus metal. In this respect the weak coupling approach can be applied to a metallic phase.

Expanding the logarithmic action for the pseudospin field, one finds

$$S_{\text{eff}} = \frac{1}{\beta} \sum_{q, \Omega} \left( \chi_{\perp}^{-1}(q, i\Omega) T_+(q, i\Omega) T_-(q, i\Omega) ight)$$

$$+ \chi_{\perp}^{-1}(q, i\Omega) T_z(q, i\Omega) T_z(-q, -i\Omega)$$

$$+ u \int_0^\beta d\tau d^2r [\mathbf{T}(r, \tau) \cdot \mathbf{T}(r, \tau)]^2.$$ (15)

Here $\chi_{\perp}^{-1}(q, i\Omega)$ and $\chi_{\perp}^{-1}(q, i\Omega)$ are transverse and longitudinal pseudospin susceptibilities, and $u$ a local interaction parameter of the pseudospin fields. The pseudospin susceptibility is evaluated in the noninteracting fermion ensemble, thus given by

$$\chi_{\perp}^{-1}(q, i\Omega) = \xi_{\perp}^{-2} + \gamma |\Omega|/q,$$

$$\chi_{\perp}^{-1}(q, i\Omega) = \xi_{\perp}^{-2} + \gamma |\Omega|/q.$$ (16)

Here $\xi_{\perp}$ is the Kondo screening length, obtained to be $\xi_{\perp}^{-2} \sim 1 - J_K D$, where $D$ is the density of states of spinors. It diverges at the quantum critical point $J_K = 1/D$. $\xi_{\perp}$ is another pseudospin correlation length associated with the Ising order. In the SU(2) symmetric case one obtains $\xi_{\perp} = \xi_{||}$. As mentioned before, the following relation $\xi_{||}^{-2} \leq 0$ is an important assumption in this paper. This assumption is parallel to the pseudospin anisotropy in Eq. (13). This ad-hoc construction allows the continuous transition between the XY and Ising orders of the pseudospin fields. The presence of the Landau damping term confirms that this approach is based on the metallic phase since it originates from low energy fermion excitations near the Fermi surface.

Eq. (15) is nothing but the Hertz-Millis theory for the magnetic quantum phase transition in itinerant electron systems. Recently, it was claimed that the Hertz-Millis theory may not be applied because the Landau expansion in Eq. (14) can result in a non-analytic correction to the spin susceptibility instead of the normal $q^2$ contribution beyond the Eliashberg approximation. In this paper we limit our consideration within the Eliashberg theory, thus do not allow the non-analytic correction to the pseudospin susceptibility.

The above effective action does not include spin degrees of freedom, thus not allowing antiferromagnetic order. To describe the antiferromagnetic transition, we introduce the antiferromagnetic Heisenberg Hamiltonian for the localized spins

$$H_{\text{AF}} = J_{\text{eff}} \sum_{ij} S_i \cdot S_j,$$ (17)

where $J_{\text{eff}}$ is an effective coupling strength. Because the antiferromagnetic long range order should start from the transition point $J_K = J_{K}^c$ associated with the Kondo screening, it is natural to set $J_{\text{eff}} = J |\langle T_z \rangle|^2$ with the bare coupling strength $J$. Remember that the $T_{ij}$ operator leads to the difference of chemical potential between the conduction electrons and local spinors. In the case of $J_K < J_{K}^c$ the Kondo hybridization vanishes, resulting in incomplete Kondo screening owing to the chemical potential difference. As a result, unscreened local magnetic moments appear, generating the effective exchange interactions. The presence of local moments is a necessary condition for the antiferromagnetic long range order. Unfortunately, we cannot derive this effective coupling constant as mentioned before. However, this simple construction results in the continuous transition from the heavy fermion phase to the antiferromagnetic one in a very simple way.

C. Hubbard model for Kondo phenomena

In this section we give an alternative view for the pseudospin model as an effective action. Consider the follow-
ing Lagrangian
\[
L_{\text{eff}} = \sum_i \bar{\psi}_{i\alpha}^\dagger (\partial_\tau - \mu_n) \psi_{i\alpha} - \sum_{ij} t_{nj} \bar{\psi}_{j\alpha}^\dagger \psi_{i\alpha} - \sum_{ij} t_{ni} \bar{\psi}_{i\alpha}^\dagger \psi_{j\alpha}
+ U \sum_{i} N_{i+} N_{i-},
\] (18)

where the spinor \( \psi_{i\alpha} \), the hopping integral \( t_{nj} \), and the chemical potential \( \mu_n \) are defined in Eq. (4) and Eq. (5), respectively. The Hubbard interaction consists of \( N_{i+} = \bar{\psi}_{i\alpha}^\dagger \psi_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha} \) and \( N_{i-} = \bar{\psi}_{i\alpha}^\dagger \psi_{i\alpha} = f_{i\alpha}^\dagger f_{i\alpha} \), indicating that local repulsive interactions work between the conduction electrons and spinons. As well known, the Hubbard interaction can be decomposed into "charge" and "spin" channels in the following way

\[\langle \psi_{i\alpha}^\dagger \psi_{i\alpha} \rangle \psi_{i\alpha}^\dagger \psi_{i\alpha} = 1/4 \left| \psi_{i\alpha}^\dagger \right|^2 - 1/4 \left[ \left( \bar{\Omega}_i \cdot \vec{\tau} \right)_{nm} \psi_{n\alpha} \right]^2, \quad (19)\]

where \( \vec{\Omega}_i \) is a unit spin vector.

Performing the Hubbard-Stratonovich transformation, we obtain

\[
L_{\text{eff}} = \sum_i \bar{\psi}_{i\alpha}^\dagger (\partial_\tau - \mu_n) \psi_{i\alpha} - \sum_{ij} t_{nj} \bar{\psi}_{j\alpha}^\dagger \psi_{i\alpha} - \sum_{ij} t_{ni} \bar{\psi}_{i\alpha}^\dagger \psi_{j\alpha}
+ \sum_i \left( \frac{1}{U} \Delta_{i\alpha}^2 - i \Delta_{i\alpha} \psi_{i\alpha}^\dagger \psi_{i\alpha} \right)
+ \sum_i \left( \frac{1}{U} M_i^2 - M_i \cdot \bar{\psi}_{i\alpha}^\dagger \psi_{i\alpha} \right),
\] (20)

where \( \Delta_{i\alpha} \) is an effective potential for total number density, and \( M_i \) an effective magnetic field for the pseudospin \( \bar{\psi}_{i\alpha}^\dagger \psi_{i\alpha} \), corresponding to \( T_i \). Eq. (20) is essentially the same as Eq. (8) if we introduce the pseudospin anisotropy and ignore the charge channel. This implies that the Hubbard model Eq. (18) can be considered as an effective model for the Kondo phenomenon.

The strong coupling limit \( t_n << U \) in Eq. (18) results in the antiferromagnetic Heisenberg model Eq. (13) in terms of the pseudospin fields. Since the strong coupling approach is based on an insulating phase of spinors, this approach can be applied to the Kondo insulators, as discussed before. On the other hand, in the weak coupling limit \( t_n >> U \) Eq. (18) describes an itinerant spinor system. Thus, both ferromagnetic and antiferromagnetic pseudospin instabilities can occur near the Fermi surface. In this paper we do not take into account Fermi surface nesting, thus only consider a ferromagnetic pseudospin state.

**IV. QUANTUM CRITICAL POINT**

We investigate critical pseudospin dynamics near the quantum critical point. The effective action would be written in terms of the spinor field \( \psi_{i\alpha} \) interacting via critical ferromagnetic pseudospin fluctuations \( T_i \), given by

\[
S_{\text{eff}} = \frac{1}{\beta} \sum_{q,\Omega} \left( \chi_{\perp}^{-1}(q, i\Omega) T_+(q, i\Omega) T_-(q, i\Omega) \right)
+ \chi_{\parallel}^{-1}(q, i\Omega) T_z(q, i\Omega) T_z(-q, -i\Omega) + \int_0^\beta d\tau d^2 r (\mathbf{T} \cdot \mathbf{T})^2
+ \int_0^\beta d\tau d^2 r \left[ \psi_{i\alpha}^\dagger (\partial_\tau - \mu) \psi_{i\alpha} + \frac{1}{2m_n} |\nabla \psi_{i\alpha} |^2 \right]
- \frac{1}{n_{\psi_{i\alpha}}} \mathbf{T} \cdot \vec{\tau}_{nm} \psi_{i\alpha},
\] (21)

where \( m_n \sim t_n^{-1} \) is the bare band mass of spinors. The antiferromagnetic interaction term may be ignored since its coupling strength vanishes at the quantum critical point. However, we admit that it is difficult to justify the ignorance of antiferromagnetic spin fluctuations. In this paper we investigate the role of critical Kondo fluctuations in critical phenomena.

We conjecture that \( SU_p(2) \) pseudospin symmetry appears at the critical point. Note that the \( SU_p(2) \) symmetry does not exist in the microscopic model, the Kondo lattice Hamiltonian since the chemical potential and hopping integral of conduction electrons differ from those of spinons. At the quantum critical point the chemical potential of spinons is the same as that of conduction electrons. One uncertainty is about the hopping integral \( t_n \) at the critical point. We conjecture that the hopping integral of conduction electrons is the same as that of spinons at the quantum critical point. Even if this conjecture is not correct, the difference of hopping integrals between the conduction electrons and local spinons would not cause severe differences for critical phenomena, compared to those of the \( SU_p(2) \) symmetric case. As a result, \( SO(4) \sim SU_p(2) \otimes SU_s(2) \) symmetry is expected to emerge at the quantum critical point, where \( SU_s(2) \) is the spin rotation symmetry. In terms of the symmetry language the heavy fermion phase is understood as breaking the \( U_p(1) \) symmetry but preserving the \( SU_s(2) \) symmetry while the antiferromagnetic phase is associated with breaking the \( Z_p(2) \) symmetry and the \( SU_s(2) \) symmetry. Unfortunately, Eq. (21) cannot describe the \( SU_s(2) \) symmetry breaking, as mentioned before. To summarize, the critical action for the Kondo screening is the same as that describing the ferromagnetic quantum criticality.

One important point for the ferromagnetic quantum criticality is the Landau damping term in Eq. (16). The propagator of ferromagnetic pseudospin fluctuations is given by \( \chi_{\parallel}(q, i\Omega) = \chi_{||}(q, i\Omega) = 1/(q^2 + \gamma|\Omega|^2) \) at the quantum critical point \( \xi_\parallel = \xi_\parallel \to \infty \). As shown in the pseudospin susceptibility, the dispersion of ferromagnetic pseudospin fluctuations is given by \( \Omega \sim q^3 \), thus the dynamical critical exponent \( z \) is obtained to be \( z = 3 \). Since the effective dimensionality of the system is \( D_{\text{eff}} = d + z \) with \( d \) spatial dimensions, it is given by \( D_{\text{eff}} = 5 \) in \( d = 2 \) and \( D_{\text{eff}} = 6 \) in \( d = 3 \). Considering that the upper critical dimension is \( D_{\text{up}} = 4 \) in the O(3) vector model, the ferromagnetic quantum critical point
is above the upper critical dimension, thus described by the Gaussian theory for pseudospin fluctuations. It is valuable to note that the $z = 3$ Gaussian critical theory is found in the context of U(1) spin liquid with a Fermi surface.\[16\]

The spinor self-energy is given by

$$\Sigma(k, i\omega) = \frac{1}{\beta} \sum_{\Omega} \int \frac{d^d q}{(2\pi)^d} G(k + q, i\omega + i\Omega) \chi_{\perp}(q, i\Omega),$$

where $G(k, i\omega) = 1/(i\omega - \mu - \epsilon_k)$ is the spinor propagator with its dispersion $\epsilon_k$. Scattering with $z = 3$ critical fluctuations is well known to give $\Sigma'' \sim T^{2/3}$ in $d = 2$ and $\Sigma'' \sim T$ in $d = 3.\[8]$ where $\Sigma'' \sim T$ is the imaginary part of the self-energy. Remember $\Sigma'' \sim T^2$ in the Fermi liquid. In a similar expression the scattering rate $\tau^{-1}$ associated with the dc conductivity is obtained to be $\tau^{-1} \sim T^{4/3}$ in $d = 2$ and $\tau^{-1} \sim T^{5/3}$ in $d = 3.\[17]$ The specific heat $C_v$ in the quantum critical regime can be obtained from $C_v \sim \partial^2 F_s / \partial^2 T$, where $F_s$ is the singular contribution of the free energy due to Gaussian fluctuations of the critical pseudospin modes, given by

$$F_s = \beta^{-1} \sum \Omega \int \frac{d^d q}{(2\pi)^d} \ln \chi_{\perp}(q, i\Omega).$$

The specific heat is obtained to be $C_v \sim T^{2/3}$ in $d = 2$ and $C_v \sim T\ln T$ in $d = 3.\[8]$ Thus, $C_v / T$ diverges in the $T \to 0$ limit at the quantum critical point.

Since the contribution of spin excitations in local moments is not taken into account in the present critical theory, we cannot explain the $\Omega / T$ scaling with an anomalous exponent in the magnetic susceptibility.\[6] This remains an important future work in this direction.

V. SUMMARY

Introducing the new order parameter associated with the chemical potential difference between the conduction electrons and local spinons, we proposed an effective theory in terms of the pseudospin field for the continuous transition from the heavy fermion phase to the antiferromagnetic phase in the slave-boson representation of the extended Kondo lattice Hamiltonian. Our pseudospin construction shows an abrupt change in the Fermi surface volume across the transition. Furthermore, we argued that the quantum critical point in the present critical theory ignoring antiferromagnetic spin fluctuations of the localized spins is understood in the context of the ferromagnetic quantum criticality of itinerant electron systems.

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