CANONICAL SEESAW MECHANISM IN ELECTRO-WEAK $SU(4)_L \otimes U(1)_Y$ MODELS

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Abstract

In this paper we prove that the canonical seesaw mechanism can naturally be implemented in a particular class of electro-weak $SU(4)_L \otimes U(1)_Y$ gauge models. The resulting neutrino mass spectrum is determined by just tuning a unique free parameter $a$ within the algebraical method of solving gauge models with high symmetries. All the Standard Model phenomenology is preserved, being unaffected by the new physics occuring at a high breaking scale $m \sim 10^{11}$ GeV.

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1 Introduction

One of the main reasons driving the search for various extensions of the Standard Model (SM) [1] - [3] - which has been established as a gauge theory based on the local group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ undergoing in its electro-weak sector a spontaneous symmetry breakdown (SSB) up to the electromagnetic universal $U(1)_{em}$ stems in the recently unfolded neutrino phenomenology [4]. At the SM level, it is well known that left-handed neutrinos are massless in all orders of perturbations and there is no need at all for right-handed neutrinos. Therefore, no mixing occurs in the lepton sector, in contrast with the quark sector. Observational collaborations such as SuperKamiokande [5] [6], K2K [7], SNO [8], KamLAND [9], LSND [10] and others have definitely proved within the last decade that neutrinos oscillate and, consequently, they must carry non-zero masses. Although the mass spectrum in the neutrino sector exhibits certain features such as the mass splitting ratio $r_\Delta = \Delta m^2_\odot / \Delta m^2_{atm} \sim 0.03$ and particular mixing angles ($\theta_\odot \simeq 34^\circ$ and $\theta_\odot \simeq 45^\circ$, along with $\theta_{13} \simeq 0$ in the mixing matrix), the absolute mass hierarchy has not been determined yet. What we only know at present is that it lies most likely in the $eV$ region.

However, the theoretical devices designed to face such a reality (see Refs. [11] - [13] for excellent reviews on neutrino mass issue) mainly included two distinct purposes - (i) radiative mechanisms (initially proposed by Zee [14]) and (ii) various types of see-saw [15] - [18] - in order to obtain viable predictions for the massive neutrino
sector. Notwithstanding, these approaches seem more efficient in some exten-
sions of the SM, since models such as $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y$ (3-3-1) introduced and
developed by Frampton, Pisano and Pleitez [19], Frampton [20], Long [21] and others 
[22] - [33] and $SU(3)_C \otimes SU(4)_L \otimes U(1)_Y$ (3-4-1) [34] - [48] have emerged in the 
literature. Neutrino masses generated through radiative patterns in 3-3-1 models can 
be found in Refs. [49] - [56], while the way see-saw mechanisms work in those models is exploited in Refs. [57] - [61].

Here we are concerned with the well-known see-saw mechanism [15] - [17]) worked 
out in the particular electro-weak $SU(4)_L \otimes U(1)_Y$ model without exotic electric 
charges [44] - [47]. As a matter of fact, this efficient mathematical procedure calls 
for both left-handed and right-handed neutrinos. Since they can naturally be embedded 
in lepton multiplets of the 3-4-1 model, there is no need for supplemental ingredients 
like a new small parameter as in Ref. [59].

The paper is organized as follows: Sec. 2 reviews the main features of the $SU(3)_C \otimes 
SU(4)_L \otimes U(1)_Y$ gauge model without exotic electric charges [44] - [47]. As a matter of fact, this efficient mathematical procedure calls 
for both left-handed and right-handed neutrinos. Since they can naturally be embedded 
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like a new small parameter as in Ref. [59].

The paper is organized as follows: Sec. 2 reviews the main features of the $SU(3)_C \otimes 
SU(4)_L \otimes U(1)_Y$ gauge model without exotic electric charges treated within the framework of the general algebraical method proposed by Cotăescu [62]. Sec. 3 deals with the fermion mass issue with a special emphasize on the neutrino Yukawa sector in the 
3-4-1 model of interest. Our conclusions are sketched in the last section (Sec. 4) where 
also some numerical estimates are given.

2 $SU(4)_L \otimes U(1)_Y$ model without exotic charges

All the details of the of solving gauge models with high symmetries undergoing in their 
electro-weak a SSB can be found in the paper of Cotăescu [62]. Note that it was al-
ready succesfully applied by the author in the case of 3-3-1 gauge models in a series of 
papers [63] - [67] which accomodated in a natural way the neutrino phenomenology. In 
each of these cases, the method itself led to viable predictions regarding the boson mass 
spectrum and currents (both charged and neutral), recovering all the SM phenomenol-
ogy. This goal was simply achieved by introducing a particular metric in the scalar 
sector that finally offered a framework with a single free parameter to be tuned. The 
same procedure was exploited in recent papers dealing with the 3-4-1 models [43, 47] 
with a remarkable succes with regard to the boson mass spectrum and coupling coef-
ficients for the charged and neutral currents (at least in the no-exotic-electric-charges 
class of 3-4-1 models [47]). In this latter case we go further by introducing a special 
arrangement in the Higgs sector leading to a natural see-saw context in the neutrino 
sector.

In this sector we briefly present the particle content of the 3-4-1 model of interest 
here, namely Model A in Ref. [40]. For the $SU(4)_L$ group the 3 diagonal generators 
are defined as: $D_1 = T_3 = \frac{1}{2} \text{Diag}(1, -1, 0, 0), D_2 = T_8 = \frac{1}{2 \sqrt{2}} \text{Diag}(1, 1, -2, 0)$, 
and $D_3 = T_{15} = \frac{1}{2 \sqrt{6}} \text{Diag}(1, 1, 1, -3)$ respectively. The irreducible representations 
(irreps) with respect to the gauge group of the theory are denoted by $(n_{\text{color}}, n_{\rho}, y_{\rho}, \chi_{\rho})$ 
while the versor assignment needed in the general method [62] stands as $\nu_1 = 0, 
\nu_2 = 0, \nu_3 = -1$. The parameter matrix [62] in the scalar sector is taken as $\eta^2 = 
(1 - \eta_{0}^2) \text{Diag} \left(1 - c, c - a, \frac{1}{2} a + b, \frac{1}{2} a - b \right)$ in order to fulfil the condition $Tr(\eta^2) =$
1−η2 in the general method. At the same time, one assumes the condition \( e = g \sin \theta_W \) established in the SM and the relation between \( \theta_W \) and \( \theta \) (introduced by the method itself in order to separate the electromagnetic field in a general Weinberg transformation - see Sec. 5 in Ref. [62]) yields \( \sin \theta = \sqrt{\frac{2}{3}} \sin \theta_W \). Under these circumstances, the coupling matching was inferred [40] on algebraical reasons: \( \frac{g'}{g} = \frac{\sin \theta_W}{\sqrt{1 - \frac{2}{3} \sin^2 \theta_W}} \) (where, obviously, \( g \) is the \( SU(4)_L \) coupling and \( g' \) is the \( U(1)_{em} \) coupling).

2.1 Fermion content
The fermion sector of the Model A [40] consists of the following representations:

**Lepton families**

\[
\begin{pmatrix}
N'_{\alpha} \\
N_{\alpha} \\
\nu_{\alpha} \\
e_{\alpha}
\end{pmatrix}_{L} \sim (1, 4^*, -1/4) \quad (e_{\alpha L})^c \sim (1, 1, 1)
\]

2.2 Boson sector
The boson sector is determined by the standard generators \( T_a \) of the \( su(4) \) algebra. In this basis, the gauge fields are \( A_{\mu} \) and \( A_{\mu}^0 \in su(4) \), that is

\[
A_{\mu} = \frac{1}{2}
\begin{pmatrix}
D^1_{\mu} & \sqrt{2}Y_{\mu} & \sqrt{2}X'_{\mu} & \sqrt{2}X''_{\mu} \\
\sqrt{2}Y^*_{\mu} & D^2_{\mu} & \sqrt{2}K_{\mu} & \sqrt{2}K'_{\mu} \\
\sqrt{2}X''^*_{\mu} & \sqrt{2}K''_{\mu} & D^3_{\mu} & \sqrt{2}W_{\mu} \\
\sqrt{2}X''_{\mu} & \sqrt{2}K''^*_{\mu} & \sqrt{2}W^*_{\mu} & D^4_{\mu}
\end{pmatrix},
\]

with \( \alpha = 1, 2, 3 \) and \( i = 1, 2 \).

With this assignment the fermion families cancel the axial anomalies by just an interplay between them, although each family remains anomalous by itself.
with \( D_\mu^1 = A_\mu^1 + A_\mu^8 / \sqrt{3} + A_\mu^{15} / \sqrt{6} \), \( D_\mu^2 = -A_\mu^3 + A_\mu^8 / \sqrt{3} + A_\mu^{15} / \sqrt{6} \), \( D_\mu^3 = -2A_\mu^8 / \sqrt{3} + A_\mu^{15} / \sqrt{6} \), \( D_\mu^4 = -3A_\mu^{15} / \sqrt{6} \) as diagonal bosons. Apart from the charged Weinberg bosons \((W^\pm)\), there are two new charged bosons, \(K^0, K'^0\), while \(X^0, X'^0\) and \(Y^0\) are new neutral bosons, but distinct from the diagonal ones \((Z, Z', Z'')\) plus the massless \(A_{em}\).

### 2.3 Minimal Higgs mechanism

The general method assumes also a particular minimal Higgs mechanism (mHM) based on a special parametrization in the scalar sector, such that the \(n\) Higgs multiplets \(\phi^{(1)}, \phi^{(2)}, \ldots, \phi^{(n)}\) satisfy the orthogonality condition \(\phi^{(i)+} \delta^{(j)} = \delta^{ij}\), in order to eliminate the unwanted Goldstone bosons that could survive the SSB. \(\phi\) is a gauge-invariant real scalar field while the Higgs multiplets \(\phi^{(i)}\) transform according to the irreps \((1, n, y^{(i)})\) whose characters \(y^{(i)}\) are arbitrary numbers that can be organized into the diagonal matrix \(Y = \text{Diag}(y^{(1)}, y^{(2)}, \ldots, y^{(n)})\). The Higgs sector needs, in our approach, a parameter matrix

\[
\eta = \text{Diag}(\eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(n)})
\]

with the property \(Tr(\eta^2) = 1 - \eta_{00}^2\). It will play the role of the metric in the kinetic part of the Higgs Lagrangian density \((L_d)\) which reads

\[
L_H = \frac{1}{2} \eta_{00}^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \sum_{i=1}^{n} \left( \phi^{(i)} \right)^2 \left( D_\mu \phi^{(i)} \right)^+ \left( D_\mu \phi^{(i)} \right) - V(\phi)
\]

where \(D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - ig(A_\mu + y^{(i)} A_\mu^0) \phi^{(i)}\) are the covariant derivatives of the model and \(V(\phi)\) is the scalar potential generating the SSB of the gauge symmetry. This is assumed to have an absolute minimum for \(\phi = \langle \phi \rangle \neq 0\) that is, \(\phi = \langle \phi \rangle + \sigma\) where \(\sigma\) is the unique surviving physical Higgs field. Therefore, one can always define the unitary gauge where the Higgs multiplets, \(\phi^{(i)}\) have the components \(\hat{\phi}^{(i)}_k = \delta_{ik} \phi = \delta_{ik} \langle \phi \rangle + \sigma\).

The masses of both the neutral and charged bosons depend on the choice of the matrix \(\eta\) whose components are free parameters. Here it is convenient to assume the following matrix

\[
\eta^2 = (1 - \eta_{00}^2) \text{Diag}\left(1 - c, c - a, \frac{1}{2} a + b, \frac{1}{2} a - b\right),
\]

where, for the moment, \(a, b\) and \(c\) are arbitrary non-vanishing real parameters. Obviously, \(a, b, c \in [0, 1]\), \(a \in (0, c)\) and \(b \in (-a, +a)\).

With this assignment - for all the details the reader is referred to Ref [47] - after some algebra exploiting the mass relation from SM \(m_Z^2 = m_W^2 (W)/\cos^2 \theta_W\) (equivalent with \(\text{Det} \left| M^2 - \frac{m_W^2}{\cos^2 \theta_W} \right| = 0\) and enforcing some physical arguments in the above presented 3-4-1 model regarding the decoupling of the heaviest \(Z''\) as the symmetry is broken to \(SU(3)\) (equivalent with \(c = (1 + a)/2\), one obtains a one-parameter mass scale (by working out the relation \(b = \frac{1}{a} \tan^2 \theta_W\)) (see Sec. 2.4).
It is worth noting that the parameter matrix now becomes

$$\eta^2 = (1 - \eta_0^2) \text{Diag} \left( \frac{1 - a}{2}, \frac{1 - a}{2}, -\frac{a}{2}(1 + \tan^2 \theta_W), -\frac{a}{2}(1 - \tan^2 \theta_W) \right), \quad (9)$$

while the 4 scalar 4-plets of the Higgs sector are represented by $\phi^{(1)}, \phi^{(2)}, \phi^{(3)} \sim (1, 4, 1/4)$ and $\phi^{(4)} \sim (1, 4, -3/4)$. They can be re-defined as $\phi^{(i)} \rightarrow \eta^{(i)} \phi^{(i)}$ without altering the physical content (as one can see in Sec. 3).

### 2.4 Boson mass spectrum

With the following notation $m^2 = g^2 \langle \phi \rangle^2 (1 - \eta_0^2)/4$ the masses of the physical bosons stand

$$m^2(W) = m^2 a, \quad (10)$$
$$m^2(X) = m^2 a \left( \frac{1 + \tan^2 \theta_W}{2} \right), \quad (11)$$
$$m^2(X') = m^2 a \left( \frac{1 - \tan^2 \theta_W}{2} \right), \quad (12)$$
$$m^2(K) = m^2 a \left( \frac{1 + \tan^2 \theta_W}{2} \right), \quad (13)$$
$$m^2(K') = m^2 a \left( \frac{1 - \tan^2 \theta_W}{2} \right), \quad (14)$$
$$m^2(Y) = m^2(1 - a), \quad (15)$$
$$m^2(Z) = m^2 a / \cos^2 \theta_W, \quad (16)$$
$$m^2(Z') = m^2 \frac{\cos^4 \theta_W - a \sin^4 \theta_W}{\cos^2 \theta_W (2 - 3 \sin^2 \theta_W)}, \quad (17)$$
$$m^2(Z'') = m^2(1 - a). \quad (18)$$

One can observe that the above mass scale is just a matter of tuning the parameter $a$ in accordance with the possible values for $\langle \phi \rangle$.

### 2.5 Neutral charges

Now one can compute all the charges for the fermion representations in model A with respect to the neutral bosons ($Z, Z', Z''$), since the general Weinberg transformation (gWt) is determined by the matrix
approach, it reads

Now, let us inspect the gauge-invariant $L_d$ of the Yukawa sector for leptons. In our

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The couplings are listed in the following Table.

where the conditions $\phi_1 = 0, \phi_2 = 0$, $\nu_3 = -1$ by:

$$Q^\nu(Z^\nu) = g \left[ D_1^\nu \omega_{1,1} + D_2^\nu \omega_{2,1} + \left( \frac{D_3^\nu \cos \theta + g'_{\nu_e}}{g} \sin \theta \right) \omega_{3,1} \right],$$

where the conditions $\frac{g'}{g} = \frac{\sin \theta_W}{\sqrt{1 - \frac{3}{2} \sin^2 \theta_W}}$ and $\sin \theta = \sqrt{\frac{3}{2}} \sin \theta_W$ have to be inserted.

The couplings are listed in the following Table.

The following 3 terms in Eq. (21) - when boosting to the unitary gauge - will
depend develop the following matrix

$$\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3\sqrt{1 - \sin^2 \theta_W}}} & \frac{\sqrt{2 - 3\sin^2 \theta_W}}{\sqrt{3\sqrt{1 - \sin^2 \theta_W}}} \\ 0 & -\frac{\sqrt{2 - 3\sin^2 \theta_W}}{\sqrt{3\sqrt{1 - \sin^2 \theta_W}}} & \frac{1}{\sqrt{3\sqrt{1 - \sin^2 \theta_W}}} \end{pmatrix}. \quad (19)$$

They will be expressed (assuming the above versor assignment $\nu_1 = 0, \nu_2 = 0,

$$L_Y^{\text{lep}} = G_{\alpha\beta} f^{c}_{\alpha\beta} \left( \phi^{(4)} e^{c}_{\alpha L} + S_{R} f^{c}_{\beta L} + S_{D} f^{c}_{\beta L} + S'_{D} f^{c}_{\beta L} \right) + H.c. \quad (21)$$

where $S$ matrices are defined as follows $S_R = \phi^{-1}(\phi^{(1)} \otimes \phi^{(2)} \otimes \phi^{(3)} \otimes \phi^{(4)}) \sim (1, 10, 1/2), S_D = \phi^{-1}(\phi^{(2)} \otimes \phi^{(3)} \otimes \phi^{(4)} \otimes \phi^{(1)}) \sim (1, 10, 1/2), S'_D = \phi^{-1}(\phi^{(1)} \otimes \phi^{(3)} \otimes \phi^{(4)} \otimes \phi^{(2)}) \sim (1, 10, 1/2)$.

After the SSB the first term in Eq. (21) supplies the masses for all the charged

leptons: $m(e) = A \langle \phi^{(4)} \rangle$, $m(\mu) = B \langle \phi^{(4)} \rangle$, $m(\tau) = C \langle \phi^{(4)} \rangle$. Obviously, $A = G_{11}, B = G_{22}, C = G_{33}$.

The following 3 terms in Eq. (21) - when boosting to the unitary gauge - will contribute to the mass of the neutrinos if the first two positions in the lepton 4-plet gain a particular semnification. A very strange - but meaningful outcome! - occurs for $N_{\alpha L}$ and $N'_{\alpha L}$ when inspecting the Table containing the fermion couplings to the neutral currents. As one expects, they do not couple to the SM $Z$ boson, while their couplings to $Z'$ are identical. Regarding their couplings to $Z''$ they are identical up to a sign. This state of affairs entitles us to consider that these neutral fermions could well be interpreted as 3 flavors of right-handed neutrinos and their correspondig charge conjugates, in the manner $N_{\alpha L} \equiv \nu_{\alpha R}$ and $N'_{\alpha L} \equiv (\nu_{\alpha R})^c$. With this identification one can easily observe that the Yukawa $L_Y$ (21) leads (after the SSB) straightforwardly to the canonical see-saw terms in the neutrino sector

$$L_Y'' = L_Y^D(a) + L_Y^{\alpha}(a) + L_Y^R(a), \quad (22)$$

which develop the following matrix
Table 1: Coupling coefficients of the neutral currents in 3-4-1 model

| Particle/Coupling (e/$\sin 2\theta_W$) | $Z \rightarrow ff$ | $Z' \rightarrow ff$ | $Z'' \rightarrow ff$ |
|---------------------------------------|------------------|------------------|------------------|
| $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ | 1                | $\frac{1-3 \sin^2 \theta_W}{2\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $e_L, \mu_L, \tau_L$                  | $2 \sin^2 \theta_W$ | $\frac{1-3 \sin^2 \theta_W}{2\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $N_{eL}, N_{\mu L}, N_{\tau L}$      | 0                | $-\frac{3 \cos^2 \theta_W}{2\sqrt{2-3 \sin^2 \theta_W}}$ | $\cos \theta_W$ |
| $N'_{eL}, N'_{\mu L}, N'_{\tau L}$   | 0                | $-\frac{3 \cos^2 \theta_W}{2\sqrt{2-3 \sin^2 \theta_W}}$ | $-\cos \theta_W$ |
| $e_R, \mu_R, \tau_R$                  | $2 \sin^2 \theta_W$ | $-\frac{2 \sin^2 \theta_W}{\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $u_L, c_L$                            | $1 - \frac{4}{3} \sin^2 \theta_W$ | $\frac{2-3 \cos^2 \theta_W}{2\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $d_L, s_L$                            | $-1 + \frac{2}{3} \sin^2 \theta_W$ | $\frac{2-3 \cos^2 \theta_W}{2\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $t_L$                                 | $1 - \frac{4}{3} \sin^2 \theta_W$ | $\frac{2+3 \cos^2 \theta_W}{6\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $b_L$                                 | $-1 + \frac{2}{3} \sin^2 \theta_W$ | $\frac{2+3 \cos^2 \theta_W}{6\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $u_R, c_R, t_R, U_{1R}, U'_{iR}$      | $-\frac{4}{3} \sin^2 \theta_W$ | $\frac{4 \sin^2 \theta_W}{3\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $d_R, s_R, b_R, D_{1R}, D'_{iR}$      | $+\frac{2}{3} \sin^2 \theta_W$ | $-\frac{2 \sin^2 \theta_W}{3\sqrt{2-3 \sin^2 \theta_W}}$ | 0                |
| $D_{1L}, D_{2L}$                      | $\frac{2}{3} \sin^2 \theta_W$ | $\frac{5-3 \sin^2 \theta_W}{6\sqrt{2-3 \sin^2 \theta_W}}$ | $-\cos \theta_W$ |
| $D'_{1L}, D'_{2L}$                    | $\frac{2}{3} \sin^2 \theta_W$ | $\frac{5-3 \sin^2 \theta_W}{6\sqrt{2-3 \sin^2 \theta_W}}$ | $\cos \theta_W$ |
| $U_{3L}$                              | $-\frac{4}{3} \sin^2 \theta_W$ | $-\frac{14 \sin^2 \theta_W}{6\sqrt{2-3 \sin^2 \theta_W}}$ | $\cos \theta_W$ |
| $U'_{3L}$                             | $-\frac{4}{3} \sin^2 \theta_W$ | $-\frac{14 \sin^2 \theta_W}{6\sqrt{2-3 \sin^2 \theta_W}}$ | $-\cos \theta_W$ |


\[ M_{\alpha\beta}^{M+D} = G_{\alpha\beta} \begin{pmatrix} m_R & m^T_D \\ m_D & 0 \end{pmatrix} \]  

(23)

since the specific Dirac Yukawa Ld stands as \[ \mathcal{L}_V^D = -m_D \bar{\psi}^c \psi + H.c. \] and the Majorana mass term as \[ \mathcal{L}_V^M = -\frac{1}{2}m_M \bar{\psi}^c \psi + H.c. \]

From this point on, our parametrization of the scalar sector plays a crucial role in working out the see-saw mechanism. Assuming the parameter outcome (9) and the re-definition of the scalar fields presented at the end of Sec. 2.3, one obtains:

\[ M_{\alpha\beta} = G_{\alpha\beta} \begin{pmatrix} 2(1-a) & \sqrt{a(1-a)(1+\tan^2 \theta_W)} \\ \sqrt{a(1-a)(1+\tan^2 \theta_W)} & 0 \end{pmatrix} \langle \phi \rangle. \]  

(24)

If the most suitable case requires the parameter \( a \to 0 \), the above see-saw mechanism exhibits the eigenvalue-matrix:

\[ M(\nu_L) = \frac{1}{2} a (1+\tan^2 \theta_W) \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} \langle \phi \rangle, \]  

(25)

for the left handed-neutrinos, and

\[ M(\nu_R) = 2(1-a) \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} \langle \phi \rangle, \]  

(26)

for their right-handed partners. The Yukawa couplings in the above expressions are \( A = G_{ee} \), \( B = G_{\mu\mu} \), \( C = G_{\tau\tau} \), \( D = G_{e\mu} \), \( E = G_{e\tau} \), \( F = G_{\mu\tau} \) in our notation, and they should disappear by solving an appropriate set of equations for different mixing angles choices.

The physical neutrino mass issue can be addressed if we consider first neutrino mixing (for details see Refs. [11] - [13]). The unitary mixing matrix \( U \) (with \( U^T U = 1 \)) links the gauge-flavor basis to the physical basis of massive neutrinos in the manner:

\[ \nu_{\alpha L}(x) = \sum_{i=1}^{3} U_{\alpha i} \nu_{i L}(x) \]  

(27)

where \( \alpha = e, \mu, \nu \) (corresponding to neutrino gauge eigenstates), and \( i = 1, 2, 3 \) (corresponding to massive physical neutrinos with masses \( m_i \)). The mixing matrix \( U \) that diagonalizes the mass matrix \( U^T M U = m_{ij}\delta_{ij} \) has in the standard parametrization the form:

\[ U = \begin{pmatrix} c_2 c_3 & s_2 c_3 & s_3 e^{-i\delta} \\ c_2 s_3 + c_3 s_2 e^{i\delta} & -s_2 s_3 c_1 + c_3 s_1 e^{i\delta} & c_3 s_1 \\ -c_2 s_3 + c_3 s_2 e^{-i\delta} & s_2 s_3 c_1 + c_3 s_1 e^{-i\delta} & c_3 c_1 \end{pmatrix} \]  

(28)
where the substitutions $\sin \theta_{23} = s_1$, $\sin \theta_{12} = s_2$, $\sin \theta_{13} = s_3$, $\cos \theta_{23} = c_1$, $\cos \theta_{12} = c_2$, $\cos \theta_{13} = c_3$ for the mixing angles have been made, and $\delta$ is the CP phase. Bearing in mind that $TrM(\nu_L) = \sum_i m_i$ and phenomenological values $m_i$ of neutrino masses are severely limited to few $eV$, one obtains: $\sum_i m_i = \frac{1}{2}a(1 + \tan^2 \theta_W) \langle \phi \rangle (A + B + C)$. That is

$$TrM(\nu_L) = \frac{1}{\sqrt{2}} \left( \frac{1 + \tan^2 \theta_W}{\sqrt{1 - \tan^2 \theta_W}} \right) m(\tau) \left[ 1 + \frac{m(\mu)}{m(\tau)} + \frac{m(e)}{m(\tau)} \right] \sqrt{a} \quad (29)$$

With its good approximation:

$$\sum_i m_i \approx \frac{1}{\sqrt{2}} \left( \frac{1 + \tan^2 \theta_W}{\sqrt{1 - \tan^2 \theta_W}} \right) m(\tau) \sqrt{a} \quad (30)$$

where we neglected the small ratios $m(\mu)/m(\tau) \sim 0.05$ and $m(e)/m(\tau) \sim 0.0002$ in Eq. (29) and exploited $m(\tau) = C\sqrt{\frac{\alpha(1 - \tan^2 \theta_W)}{\sqrt{2}} \langle \phi \rangle}$. With this result (taking into account the PDG results [68]) one can estimate the range of the free parameter $a$ in order to match the observed tiny masses ($\sim 1eV$) in the left-handed neutrino spectrum. It has to be $a \sim 0.25 \times 10^{-18}$ corresponding to a mass scale $m \sim 1.6 \times 10^{11}$GeV. (The latter was inferred from Eq. (10) in order to ensure $m(W) = 80.4$GeV. Under these circumstances, right-handed neutrinos must exhibit masses in the range $\sim 1.2 \times 10^{10}$GeV, unaccessible yet to a direct observation.

Regarding the mixing angles, the neutrino mass hierarchy (normal, inverted or degenerate) and its splitting, or possible additional symmetries (such as $L_e - L_\mu - L_\tau$) for a neutrino mass matrix including diagonal entries that are proportional to the charged lepton masses was treated in Ref. [69]. Those results are suitable for the model of interest in this paper, since our diagonal entries here exhibit the same proportionality.

We mention also that some new bosons ($Y$, $Z'$ and $Z''$) gain masses at the $10^{11}$GeV level. But this is not a contradiction, since our point of departure in our analysis consisted in decoupling of the heavier neutral boson ($Z''$) from its two companions ($Z$ and $Z'$).

### 4 Conclusions

In conclusion, in this paper we have worked out the neutrino mass issue in a 3-4-1 electro-weak model without exotic electric charges, proving that the canonical see-saw mechanism can naturally arise - without resorting to any supplemental ingredients! - by just exploiting the general method of treating gauge models with high symmetries. This assumes a geometrical approach - given by a proper parameter set in the Higgs sector - combined with the redefinition of the scalar multiplets and a particular gauge fixing (we work in unitary gauge from the very beginning for some needed scalar 10-plets - i.e. matrices $S$ in Eq. (21) constructed as tensor products out of the existing 4-plets). This procedure leads straightforwardly to the one-parameter ($a$) see-saw mechanism giving the right order of magnitude for the left-handed neutrinos $\sim eV$ when the mass
scale of the whole model lies in the range \( m \sim 10^{11}\text{GeV} \). The SM phenomenology is not disturbed by this mathematical approach, since all the masses and couplings of the SM particles - namely, leptons, quarks, and bosons \( W, Z \) plus the massless \( A_{em} \) - computed through our method come out at their established values.

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