Nonlinear Meissner effect in Nb$_3$Sn coplanar resonators

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Superconducting Radio Frequency (SRF) Cavities

- Nb cavities are operated at 2K
- Nb cavities are approaching the intrinsic limit of the material
- Look for alternative materials for further improvement
- Nb$_3$Sn is the most promising alternative so far
**Nb$_3$Sn**

- Critical Temperature $\sim$ 18 K
  - Increase the operating temperature from 2K to $\sim$ 4K while maintaining high $Q_0$
  - Reduces the cryogenic operation cost and the size of the plant
  - Opens up the possibility for the operation with LHe free cryocooler
- High superheating field
  - Cavity can sustain a higher magnetic field than Niobium
  - Higher maximum accelerating field up to 100MV/m, twice that of Nb
- Current state-of-the-art Nb$_3$Sn cavities can reach $\sim$20 MV/m
Nb$_3$Sn thin film study

• Study the response of Nb$_3$Sn thin-film resonator as a function of magnetic field

• Film was grown by Chris Sundahl from the University of Wisconsin-Madison.

• Co-sputtered using Sn and Nb Targets

C. Sundahl, et al., Sci. Rep. 11, 7770 (2021).
$T_C \approx 17.2 \text{ K}$

Residual Resistance Ratio $\frac{R_s(300 \text{ K})}{R_s(18 \text{ K})} \approx 3.2$
Meissner effect (Linear)

\[ \mathbf{H}(z) = H_0 e^{-z/\lambda} \]
\[ \mathbf{J}(z) = \frac{H_0}{\lambda} e^{-z/\lambda} \]

Two-fluid model: \( n(T) = n_s(T) + n_n(T) \)

\[ \frac{d\mathbf{J}_s}{dt} = \frac{e^2 n_s}{m} \mathbf{E} \quad \text{First London Equation} \]

\[ \lambda^2 \nabla \mathbf{H} - \mathbf{H} = 0 \quad \text{Second London Equation} \]

(Using \( \nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{B} \) and \( \nabla \times \mathbf{H} = \mathbf{J}_s \))

\[ \lambda(T) = \left( \frac{m}{e^2 n_s(T) \mu_0} \right)^{1/2} \]
Nonlinear Meissner Effect (NLME)

\[
J = -\frac{A}{\mu_0 \lambda^2}, \quad \lambda = \left(\frac{m}{e^2 n_s \mu_0}\right)^{1/2}
\]

- Any dependence on \( \lambda \) on the current density results in the **nonlinear Meissner effect**

- **Possible Causes:**
  - Pair breaking effect in which the strong field reduces \( n_s \)
  - Weakly coupled grain boundaries and local non-stoichiometry on the surface cause an additional increase in \( \lambda \)

- Important for \( Nb_3Sn \) coated SRF cavity because it can introduce nonlinear response to surface impedance.
Probing NLME

• We probed NLME by measuring the reactive part of the surface impedance \( Z = R_S + iX_S \)

• Reactive part \( \rightarrow \) Kinetic Inductance

\[
\frac{1}{2} L_k I^2 = \frac{1}{2} \int \mu \lambda^2 |\mathbf{J}|^2 dS
\]

• For a rectangular strip with thickness \( t \ll \lambda \) and width \( w \)

\[
L_k = \frac{\mu \lambda^2}{wt}
\]

• Resonant frequency of the resonator:

\[
f_r = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{(L_g + L_k)C}}
\]
Nb3Sn Coplanar Waveguide Resonator

• Transmission line resonator pattern on a thin film Nb$_3$Sn
• Measure the resonant frequency as a function of the field to probe the change in kinetic inductance
• Study the dependence on the field orientation
  • B $\parallel$ z : parallel to the rf current
  • B $\parallel$ y : perpendicular to the rf current
Sample Design

- $w = 14.90 \, \mu m, \, s = 8.86 \, \mu m$
- Total length: 24.6mm (2.236GHz)
- Deposited on 10mm x 10mm $Al_2O_3$
- Using $t = 50nm \ll \lambda$ film increases the onset of penetration of the magnetic fluxes in the material
Vortex state

- In type II superconductor, when $B > B_{c1}$ it becomes energetically favorable for the flux to penetrate the material as vortices.
- $n_s$ drops to zero at the core
- We want to avoid vortices to probe NLME

$$B_{c1} = \frac{2\phi_0}{\pi t^2} \ln \left( \frac{t}{\xi} \right) \approx 1.2 \text{ T}$$

(Asserted 2x bulk value)
Probe Station

Superconducting magnet and Sample stage

Helmholtz coils

Sample stage shield

Lid

probe 1a

probe 2a

probe 1b

probe 2b

Hexapod
Probe Station Sample stage

Top view of the sample stage
Experimental Procedure

• Cool down the sample to 7K
• Measure the resonant frequency $f_r$ as a function of $T$ to calculate $\lambda(T)$
• Align the field to be parallel to the surface of the film
• Measure the shift in $f_r(B)$ as a function of the field in two orientations: $B \parallel z$ and $B \parallel y$.
• Repeat for higher temperatures up to 12K.
Resonant frequency vs Temperature

\[ f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \sqrt{(L_g + L_k(T))C}} \]

\[ \frac{\partial f(T)}{f_0} = \frac{f(T) - f(7K)}{f(7K)} = \frac{\sqrt{L_g + L_k(7K)}}{\sqrt{L_g + L_k(T)}} - 1 \]

\[ L_k \approx \frac{\mu_0 \lambda^2(T)}{wt} \approx \frac{\mu_0}{wt} \frac{\lambda(0)^2}{1 - \left(\frac{T}{T_c}\right)^4} \]

Fit gives \( \lambda(0) \)
Resonant frequency vs Temperature

- $\lambda(0) = 353 \text{ nm}$
- Much larger than clean stoichiometric Nb$_3$Sn (90nm)
  - Nonstoichiometric inclusions
  - Grain boundaries
- Grain boundaries can facilitate a favorable condition for field penetration.
Field Alignment for the NLME measurements

• Eliminate contribution of the perpendicular field due to misalignment of the field orientation

• $B_\perp$ wraps around the edge and is enhanced

• Need to minimize the influence of fluxes by aligning the field parallel to the plane of the sample.
Steps for aligning the field

• Steps in aligning the field:
  1. Measure $f_{0i}$ at $B = 0$
  2. Ramp the field to 60 mT and then back down to 0 mT
  3. Measure $f_{0a}$ at $B = 0$ again
  4. Adjust the angle $\zeta$ until minimum of $(f_{0a} - f_{0i})/f_0$ is found
Field Alignment Results

- Frequency shift is minimized at $\zeta = -3.8^\circ$
- After aligning the field, measure $f_R(B)$ for temperature between 7K – 12K
Results of $f_r(B)$

- $f_r(B)$ decreases nearly linearly above 30-40 mT
- Slope in $B \parallel z$ is almost twice as large as that of $B \parallel y$

(Parallel to rf current)

$B \parallel z$

(Perpendicular to rf current)

$B \parallel y$
Contributions to NLME

• Pair breaking effect in which the current reduces $n_s$

• Weakly coupled grain boundaries and local non-stoichiometry on the surface cause an additional increase in $\lambda$
NLME Contribution from Pair Breaking

• The nonlinear current:

\[
J = -\frac{A}{\mu_0 \lambda^2} \left( 1 - \alpha(T) \frac{A^2}{A_c^2} \right)
\]

\[
A_y = xB \cos \varphi \quad A_z = xB \sin \varphi + \delta A_\omega
\]

Only \( A_z \) component is coupled with the weak rf field
NLME Contribution from Pair Breaking

\[ \lambda(B)^2 = \lambda^2(0) \left[ 1 + \frac{1}{3} \left( \frac{2\pi \xi t B}{\phi_0} \right)^2 \left( \frac{1}{4} + \frac{2 \sin^2 \varphi}{4 + \omega^2 \tau^2} \right) \right] \]

\[ \frac{\delta f}{f} = -\frac{1}{6} \left( \frac{\pi t \xi B}{\phi_0} \right)^2 \left[ 1 + \frac{2 \sin^2 \varphi}{1 + (\omega \tau / 2)^2} \right] \]

• Pair-breaking NLME correction is quadratic in B
• \( \sin^2 \varphi \) angular dependence
  • Maximum when \( B \) is perpendicular to the rf current (\( B \parallel y \))
NLME Contribution from Pair Breaking

- $f_r(B)$ decreases nearly linearly above 30-40mT (not quadratic)
- Slope in $B \parallel z$ is almost twice as large as that of $B \parallel y$ (opposite)

\[
\frac{\delta f}{f} = -\frac{1}{6} \left( \frac{\pi t \xi B}{\phi_0} \right)^2 \left[ 1 + \frac{2 \sin^2 \varphi}{1 + (\omega \tau / 2)^2} \right]
\]

At $B = 100$ mT, $\frac{\delta f}{f} \approx 4 \times 10^{-4}$ for $\varphi = \frac{\pi}{2}$

This is much smaller than what we measured.
Grain Boundary Contribution

- A significant contribution to $L_k$ can come from grain boundaries
- Sn depletion at grain boundaries acts as weak Josephson junctions.

\[ J = J_c \sin \theta \]

$J_c$: Maximum current density across the barrier

\[ \theta = \Phi_2 - \Phi_1 + \frac{2\pi}{\phi_0} \int A \cdot dl \]
Josephson Kinetic Inductance

• The grain boundary gives rise to additional Kinetic Inductance term

\[ L_j = \frac{\phi_0}{2\pi I_c \cos \theta} \]

• The field dependence of \( L_j(B) \) is determined by \( \theta \) induced by the external field on the grain boundary.

• Low critical current can significantly increase \( L_j \) contribution
Grain Boundary Contributions

- Calculate average $< \cos \theta(B) >$ for randomly distributed grain boundary

$$L_j(B) = \frac{\phi_0}{2\pi I_c <\cos \theta(B)>}$$

$$\frac{\delta f(B)}{f_0(0)} = \frac{\sqrt{L_g + L_j(0)}}{\sqrt{L_g + L_j(B)}} - 1$$

$L_1 \lesssim t \approx 50 \text{ nm}$

$L_2 \sim 0.1 - 1 \mu m$
Numerical Calculation of GB Contributions

(a) $B \parallel z$

(b) $B \parallel y$
GB Contribution

• In general, both $n_n$ and $n_s$ can tunnel through the grain boundaries.
• Model the impedance across the grain boundary using a circuit model with $\langle L_j \rangle$ and $R_{GB}$ in parallel

$$Z_j(B) = \left( \frac{1}{R_{GB}} + \frac{1}{i\omega \langle L_j \rangle} \right)^{-1}$$

$$R_j(B) \sim \frac{\omega^2}{\omega_t l_2 R_{GB}} \langle L_j \rangle^2 \propto B^2$$
Conclusion

• The nonlinear Meissner effect on a thin Nb$_3$Sn film was studied in the cryogenic probe station.

• Aligning the field parallel to the film and using the thin film $t \ll \lambda$ extends the field onset of the penetration of the flux.

• The results showed the linear dependence of the frequency with field, indicating that the weakly coupled grain boundaries can have a more significant contribution compared to the pair breaking Meissner current in a polycrystalline Nb$_3$Sn film.

• These results give an insight into nonlinear electrodynamic response to the field for a polycrystalline Nb$_3$Sn which could have additional field-dependent resistance in the SRF cavity.