Radiative Effects in Gauge Models with Homogeneous Condensate and Curved Space-Time

V. Ch. Zhukovsky, V. V. Khudyakov, I. V. Mamsurov
Physical Faculty, Moscow State University,
119899, Moscow, Russia

Abstract

Different models with nonabelian homogeneous condensate fields are considered in the one-loop approximation. Effective action in a model of gluodynamics in curved space is calculated. Free energy and its minimum in a (2+1)-dimensional model of QCD are investigated. Photon polarization operator (PO) is obtained.
1 Introduction

Despite considerable efforts that have been undertaken in the recent years there is no consistent theory of the QCD vacuum state at present. Quark and gluon condensates were for the first time investigated in the framework of the sum-rule method by M. Shifman et al. and then basing upon some simple models of the gluon background field (instantons, Savvidy vacuum). Nonperturbative approaches were developed also by Dosch and Simonov (cluster expansion of the vacuum field method). Active investigation of various radiative effects in non-Abelian external fields has been initiated in this connection.

On the other hand many problems of elementary particle physics have thermodynamical aspects. Relevant to this are problems of phase transitions in gauge theories at finite temperature and non-zero chemical potential, the question of possible stabilization of the classical Yang-Mills field configurations on account of temperature effects, and a number of other problems. The chromomagnetic field configuration formed by nonabelian potentials turned out to be stable with respect to the decay into pairs of real particles. In this case substantially nonperturbative dependence of such objects, as the photon PO, on the vacuum field was obtained.

Recently much attention has been paid to the effects of topology in forming the vacuum condensate, as well as to the non-analytic dependence of the observables on the intensity of the gluon condensate. The study of radiative effects in the 3-dimensional theories, carried out recently by the MSU group demonstrated importance of the so called Chern-Simons topological term for regularization of infrared divergences in calculating higher loop diagrams even in the case when background fields are present.

2 One-loop effective action for gauge fields in curved space-time

We investigate here a model of chromomagnetic vacuum in the space-time continuum of nonzero curvature. Let us consider SU(2) gluodynamics in $S^2 \times R^2$ space-time, which is a direct product of a plane and a sphere of radius $\rho$. Generating functional can be written in the form

$$Z[A, j] = \int da^a_{\mu} d\chi d\chi \exp \left\{ - \int d^4 x \sqrt{g} (L + ja) \right\},$$

where potential $A^a_{\mu} = A^a_{\mu}$ is the background, $a^a_{\mu}$ are quantum fluctuations, $L = (F^a_{\mu\nu})^2/4 + (D^a_{\mu} a^a_{\mu})^2/(2\xi) + \chi_a (D^2)_{ab} \chi_b$. Here $F^a_{\mu\nu} = \nabla_{\mu} A^a_{\nu} - \nabla_{\nu} A^a_{\mu} - ig(T^a)^{bc} A^b_{\mu} A^c_{\nu}$ is the field tensor, $D^a_{\mu} = \delta^a_{\mu} \nabla_{\mu} - ig(T^c)^{ab} A^c_{\mu}$ is the background derivative and $\chi_a$ are ghost fields. Using $Z = \exp(\Gamma)$, one obtains effective action $\Gamma$ in the one-loop approximation $\Gamma^{(1)}[A] = 1/2 \Tr \ln(\Theta^{ab}_{\mu\nu} - \Theta^{ab}_{\mu\nu})$ with the first and second terms as gluon and ghost contributions respectively.

A model condensate abelian field can be chosen in the form $A^a_{\mu} = n^a A^a_{\mu}$, $F^a_{\mu\nu} = n^a F^a_{\mu\nu}$, $n^a$ are eigenvalues of matrix $n^c T^c$. Then, for $\xi = 1$, upon diagonalizing $\Theta^{ab}_{\mu\nu}$ one obtains $\Theta^{a}_{\mu\nu} = -g_{\mu\nu}(\nabla_\lambda - ig\nu^\lambda A^c_\lambda)^2 + ig\nu^\rho F^c_{\mu\nu} - R_{\mu\nu}$. Here $\nu^a = \{1, 1, 0\}$ ($a = 1, 2, 3$) are eigenvalues of matrix $n^c T^c$. 


Consider large values of parameter $\chi^a = \omega^a \rho^2 \gg 1$ ($\omega^a = gH|\nu^a|$). Define $p = [\sqrt{\chi}]$ as the integer part, $\varepsilon = \sqrt{\chi} - \rho$ as the fractional part. Then in homogeneous chromomagnetic background field $H$ we obtain:

$$\text{Im} \Gamma^{(1)}[A] = -\frac{\Omega}{8\pi} (gH)^2(1 - \frac{2}{p}\alpha); \quad \alpha = \begin{cases} \varepsilon, & \varepsilon < 1/2 \\ 1 - \varepsilon, & \varepsilon > 1/2 \end{cases},$$

where $\Omega$ is the 4-volume. Since $2\alpha/\rho < 1$, we have $\text{Im} \Gamma^{(1)} \neq 0$. After regularization the real part of the effective action with account for the tachyonic mode contribution reads:

$$\Gamma^R^{(1)}[A] = \frac{\Omega}{4\pi^2} (gH)^2 \left( \frac{11}{12} \left( \ln \frac{gH}{\rho^2} - \frac{1}{2} \right) - \frac{\gamma - 1}{p} \right),$$

where $\gamma$ is the Euler’s constant.

For small values of $\chi$ one obtains $\text{Im} \Gamma^{(1)}[A] = -\Omega/(4\pi^4)$, and

$$\text{Re} \Gamma^{(1)}[A] = \frac{\Omega}{8\pi^2} (gH)^2 \left( \frac{1}{\rho^2 gH} \ln \frac{\rho^2}{gH} - \frac{23}{12} \ln \frac{gH}{\rho^2} - \ln \rho^2 gH \right).$$

It should be emphasized that the imaginary part of the effective action, $\text{Im} \Gamma^{(1)}$, never disappears in this model and, in contrast to [15], no stabilization occurs.

### 3 Free energy in (2+1)-dimensional SU(2) model of QCD with vacuum condensate

Consider an SU(2) model of QCD in (2+1)-dimensional space-time at finite temperature. The one-loop euclidean effective action

$$\Gamma^{(1)} = -\frac{1}{2} \int \frac{dq_4}{2\pi} \sum_r \ln(q_4^2 + \varepsilon_r^2(G)) + \sum_{j=1}^{N_f} \int \frac{dp_4}{2\pi} \sum_k \ln(p_4^2 + \varepsilon_k^2(Q_j))$$

is expressed in terms of one-particle boson $\varepsilon_r(G)$ and fermion $\varepsilon_k(Q_j)$ spectra with quarks of color $a$ and flavor $j = \Gamma, N_f$. Introducing finite temperature $T = 1/\beta$ and generating functional $Z$ at $T \neq 0$ in the conventional way, we define the effective potential as $V = -T \ln Z/L^2 = \Gamma^{(1)} L^{-3}$. Here $L^3 = \beta L^2$ stands for the of 3-dimensional space-time volume. Separating the background field energy density $V^{(0)} = (\mathcal{T}^\mu_{\mu})^2/4$ and the one-loop quantum correction $v = v^G + v^Q$, where $v^G$ and $v^Q$ are the quark and gluon contributions respectively, we have $V = V^{(0)} + v$. Uniform vacuum condensate fields can be defined by gauge potential $\mathcal{T}^\mu = \delta_{\mu 2} \delta_{\alpha 3} H x_1 + \delta_{\mu 4} \delta_{\alpha 3} A_0 = \delta_{\alpha 3} A_\mu$. Chromomagnetic field $H = \text{const}$ and $A_0$-condensate are directed along third color axis. The gluon energy spectrum with only physical degrees of freedom taken into account (for zero chemical potential) is $\varepsilon(G) = \sqrt{2gH(n - 1/2) + gA_0 - i \epsilon}$, $n = 0, 2, 3, 4, \ldots$; $\epsilon > 0$. Here $n = 0$ corresponds to the tachyonic mode.

Substituting the energy spectrum in (5) with account for the degeneracy of the energy spectrum we obtain the gluon contribution to the effective potential, which is periodical in $gA_0$ with period $2\pi/\beta$. Only those gauge transformations are admissible, for which $\mathbb{Z}_2$ symmetry is conserved $A_0 \rightarrow A_0' = A_0 + 2\pi n/\beta g$ ($n \in \mathbb{Z}$). It should be noted that $v^G$ is real for
\[ \sqrt{gH} < gA_0 < 2\pi T - \sqrt{gH}. \] At \( gA_0 = \sqrt{gH} \) the effective potential has a singularity due to the tachyonic mode contribution. The divergence is cured by account for radiative correction \( \Delta \epsilon \sim \alpha_s = g^2/4\pi \) in the tachyonic energy \( \varepsilon_{\text{tach}}^2 = -gH - 2i\alpha_s\sqrt{gH}. \) Temperature contribution can be separated as \( v^G = v^G_{T=0} + v^G_{\xi}. \) Here \( v^G_{T=0} \) coincides with Trottier’s result [19]. The effective potential reaches its extremum (minimum) value at \( \sqrt{gH_0} \approx 0.218 g^2. \) For \( H \to 0 \) the temperature part reads

\[ v^G_{T=0} \bigg|_{H=0} \to -\frac{1}{\pi \beta^3} \left( \zeta(3) + \frac{(\beta gA_0)^2}{2} \left[ \ln(\beta gA_0) - \frac{3}{2} \right] \right). \] (6)

In [19] condensate \( H \) was found to evaporate for \( T > T_{cr} \). However, the tachyonic mode was neglected there. For convenience we introduce dimensionless variables \( x = \beta \sqrt{gH}, y = \beta gA_0. \) Then the form of \( V \) is demonstrated by the function \( U(x, y) = x^4(T/g^2)^4 + u(x, y). \) For \( T < T_{cr} \sim g^2 \) the temperature contribution is small and \( U(x, y) \) attains a global minimum at \( x = x_{\min} \) and \( y = y_{\min}. \) The second kind phase transitions in \( U(x, y) \) occur with \( y_{\min} = \pi \) and \( y_{\min} = 0 \) interchanging. For \( T > T_{cr} \) the values of \( y_{\min} \) and \( x_{\min} \) decrease from \( \pi \) to 0 with growing temperature.

Polyakov loop is defined as \( \mathcal{P} = T \exp \left[ i \int_0^\beta dtA_0^a \lambda^a / 2 \right] \). If in the fundamental representation \( \text{Tr}_F(\mathcal{P}) = 0 \), then there exists a confinement phase. In our case \( \text{Tr}_F(\mathcal{P}) = 2 \cos(\beta gA_0/2). \) Therefore \( \text{Tr}_F(\mathcal{P}) = 0 \) is satisfied at \( \beta gA_0 = \pi. \)

The quark contribution to the effective potential possesses a trivial global minimum only. However, the quark contribution is much smaller than the gluon one. Thus, quarks do not change the qualitative result given by gluons.

We emphasize that the tachyonic modes are responsible for instability, signaled by \( \text{Im} \, \nu \neq 0 \). Higher order corrections help to cure the singularity at \( gA_0 = \sqrt{gH} \), which is due to zero modes in the energy spectrum. For \( T < T_{cr} \) there arise a set of regions of confinement and deconfinement. This is explained by the oscillating contribution of tachyonic modes to the effective potential.

### 4 Photon PO in U(1)×SU(2) model with non-abelian vacuum condensate

Spinor electrodynamics of “quarks” with charge \( e \) and mass \( m \) interacting with \( \text{SU}(2) \) gauge field is described by the Lagrangian

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu} + \bar{\psi}(i\gamma^\mu(D_\mu - m))\psi + e\bar{\psi}\gamma_\mu A_\mu \psi. \] (7)

Here \( G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + f^{abc} G^b_\mu G^c_\nu \) and \( D_\mu = \partial_\mu - ig^2 G_\mu. \) Photon PO in the momentum representation can be written in the one-loop approximation as

\[ \Pi_{\mu\nu}(q, q') = ie^2 \delta^4(q + q') \int d^4p \, \text{tr} \left[ \gamma_\mu S(p + \frac{q}{2}) \gamma_\nu S(p - \frac{q}{2}) \right]. \] (8)

Here \( S(P) = 1/(\gamma P - m) \) is the quark Green’s function, \( P_\mu = p_\mu + G_\mu. \) Performing UV finite integration in (8) the explicit expression for PO has been obtained. For \( q \to 0, \) in the lowest
order in $G_{\mu}$ the antisymmetric part of PO takes the form $\Pi^A_{\mu\nu} = \frac{5}{6\pi^2} \frac{e^2}{m^2} \text{tr} [G_{\mu}G_{\nu}(q_\alpha G^\alpha)]$. In the case of the non-abelian spherically symmetric condensate $G_1^1 = G_2^2 = G_3^3 = \sqrt{\lambda}$, $G_0^0 = 0$, $H_i^3 = \delta_i^\lambda$ ($i = 1, 2, 3$) the photon topological mass $\Theta_{\text{ind}}(\lambda) = \Pi^A(0)$ (with $\Pi^A_{\mu\nu}(q) = i\epsilon_{\mu\nu\alpha\beta} q^\alpha \Pi^A(q^2)$) takes the value $\Theta_{\text{ind}}(\lambda) = \frac{5}{24\pi^2} \frac{e^2}{m^2} \lambda^{3/2}$. The nonzero antisymmetric part of PO leads to the effect of rotation of the photon polarization plane.

For large Euclidean momentum $Q^2 = -q^2$ one can improve the well known perturbative expression for symmetric part of PO by accounting for its nonanalytical dependence on the background field. To this end the effective quark mass is introduced: $m^2_\ast = m^2 + \frac{3\lambda}{4}$. Upon subtracting vacuum contribution one obtains a pure background field contribution $\Pi^S_{\mu\nu}(q) = e^2/(8\pi^2)(Q^2g_{\mu\nu} - Q_\mu Q_\nu)\Pi(Q)$, where

$$\Pi(Q) = -4\frac{m^4}{Q^4} \left( \ln \left( \frac{m^2}{m^2_\ast} \right) + \frac{3\lambda}{4m^2} - \frac{\lambda^2}{32m^4} \right).$$

(9)

This result can be applied to describe radiative corrections to deep inelastic lepton-hadron scattering $l(p) + h(P) \rightarrow l(p') + X(P')$ with consideration for the model uniform chromomagnetic vacuum field. Let $q = p - p'$ stand for the momentum transferred. Then the amplitude of this process is calculated to be $T = T_0[1 - \alpha(Q)\Pi(Q^2)/(2\pi)]$, where $\alpha(Q)^{-1} = \alpha^{-1} - 1/(3\pi) \ln (Q^2/m^2)$ is the fine-structure constant improved by account for the renormalization group, $T_0$ is the tree level amplitude. Corresponding correction to the cross-section is included in the expression $d\sigma = d\sigma_0(1 - \alpha(Q)\Pi(Q^2)/\pi)$.

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