Mapping Poincaré cosmology to Horndeski theory for emergent dark energy

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The ten-parameter, quadratic Poincaré gauge theory of gravity is a plausible alternative to general relativity (GR). We show that the rich background cosmology of the gauge theory is described by a non-canonical bi-scalar-tensor theory in the Jordan frame: the metrical analogue. This provides a unified framework for future investigation by the broader community. Just as the tetrad (or vierbein) of PGT\textsuperscript{q,+} is in some sense the square root of \( g_{\mu\nu} \), the MA contains a non-canonical kinetic term of the form \( \sqrt{X^{\phi\psi}} \), where \( X^{\phi\psi} \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \psi \). Such fields are known in cosmology as Cuscutons [28]: they provide a rich phenomenology [29], but are naturally challenging to motivate (see e.g. EFT applications in Horava–Lifshitz gravity [30]). We will show that teleparallelism [31] has an Einstein–Hilbert MA, while the MA of Einstein–Cartan–Kibble–Sciama (ECKS) theory [32] is a pure Cuscuton.

We will show that Class \( ^2A^* \) of PGT\textsuperscript{q,+} retains the DR of Class \( ^3C^* \), while the 0\textsuperscript{-} mass generates dark energy (DE). The Cuscuton tends to ‘stall’ the cosmology in a state equivalent to ΛCDM. With relevance to the Hubble tension and cosmological constant problem [33, 34], our results build the case for further careful scrutiny of the underlying theory. We use natural units \( c \equiv h \equiv 1 \), reduced Planck mass \( m_p^2 \equiv \kappa^{-1} \) and signature (+, −, −, −).

Metric theories. – The generalised galileon, more commonly known as Horndeski theory [35], is the most general \( \phi \cdot g_{\mu\nu} \) coupling with maximally second-order field equations – a precaution against ghosts given by Ostrogradsky’s theorem. The generalised bi-galileon [36] introduces a second scalar \( \psi \) and is known not to be the most general second-order bi-scalar-tensor theory [37], but follows a simple prescription and is also often called Horndeski theory. The generality of the bi-galileon is provided by six arbitrary \( G \)-functions. Of these, it suits our needs to discard \( G^0_3 \), \( G^0_5 \) and \( G^\psi_5 \) (adopting the usual notation [38]) for a total Lagrangian

\[
L_T = G_2(\phi, \psi; X^{\phi\phi}, X^{\psi\psi}) + G_4(\phi, \psi) R + L_{\text{m}}(\Phi; g). 
\]

Note that \( G_2 \) couples \( \partial \phi \) and \( \partial \psi \) to \( g_{\mu\nu} \), and \( G_4 \) non-minimally couples \( \phi \) and \( \psi \) to \( \partial g \) and \( \partial^2 g \) via the Ricci scalar \( R \approx R^\mu_{\mu\nu} \), where the Riemann tensor is

\[
R^{\nu}_{\alpha\beta\gamma} \equiv 2(\partial_\beta \Gamma^\gamma_{\alpha\mu} + \Gamma^\lambda_{\alpha[\mu} \Gamma^\nu_{\beta]\lambda}),
\]

and the Levi-Civita connection \( \Gamma^\gamma_{\mu\nu} \) is of the form \( \partial g \). As with GR, one cannot formally fit the whole standard

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**Introduction.** – Candidate discrepancies between the cosmic concordance model (ΛCDM) [1, 2] and observation [3–9] have fuelled interest in modifications to general relativity (GR). In order to bypass Lovelock’s theorem, scalar-tensor theories couple various scalar fields \( \phi \) to the metric \( g_{\mu\nu} \) on a curved spacetime \( \mathcal{M} \) [10]. This approach is prevalent in effective field theory (EFT) extensions to GR, and even used to model inflation within ΛCDM [11]. Scalar-tensor theories are tractable and widely studied, and in this sense they are self-motivating.

The ten-parameter, quadratic Poincaré gauge theory of gravity (PGT\textsuperscript{q,+}) of Kibble [12], Utiyama [13] and Sciama [14] and others posits no scalar field, but augments the Einstein–Cartan–Kibble–Sciama (ECKS) theory as an \( \text{ECKS} \) theory reproduces GR up to a dark radiation component, and produce their own dark energy.

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*Prepared for submission to Phys. Rev. Lett.*
model (SM) into the matter Lagrangian $L_m(\Phi;g)$. This is an elementary but occasionally overlooked limitation of metric theories: the matter fields $\Phi$ must be tensorial representations of $GL(4,\mathbb{R})$, and are thus bosonic. Note also that while $\phi$ and $\psi$ are historically termed \textit{galileons}, the covariantisation of the theory with respect to $g_{\mu\nu}$ breaks the Galilean shift symmetry. In exchange, (1) acquires diffeomorphism invariance and (like GR) may be interpreted as a geometric $\text{R}^{1,3}$ gauge theory.

\textbf{Tetrad theories.} – Various other geometric gauge theories have been proposed. Promotion of the proper, or-thochronous Lorentz rotations to a local symmetry yields the Poincaré gauge theory (PGT) of $\text{R}^{1,3} \times \text{SO}^+(1,3)$. The geometric interpretation of PGT replaces $\mathcal{M}$ with a spacetime of Riemann–Cartan type in order to accommodate torsion. The modern picture [39–41] is perhaps more commensurate with physics in assuming a flat metric $\gamma_{\mu\nu}$ on Minkowski spacetime $\mathcal{M}$. Translations are gauged by the field $h_a^\mu$ and its inverse $b^a_\mu$, where $h_a^\mu b^a_\nu \equiv \delta^\mu_\nu$ and $h_a^\mu b^a_\mu \equiv \delta_c^a$. The Roman indices refer to an anholonomic, Lorentzian basis. Lorentz rotations are gauged by the field $A^a_{\mu} \equiv A^a_{\mu}$. The fields $h_a^\mu$ and $A^a_{\mu}$ can be geometrically interpreted as the tetrad and spin connection. They provide two field strengths

$$R^a_{\phantom{a}cd} = 2h_{\phantom{\mu}\nu}^\mu d^\nu (\partial_{[\mu}A_{\nu]}^{ab} + A_{\alpha\mu}^{ab} \epsilon^{\alpha\beta\gamma\delta} h_{\beta\gamma}^{\nu} + A_{\alpha\mu}^{ab} \epsilon^{\alpha\beta\gamma\delta} h_{\beta\gamma}^{\nu})$$

(3a)

$$T^a_{\phantom{a}bc} = 2b_{\phantom{\mu}\nu}^\mu h_{\nu}^{\nu} (\partial_{[\mu}A_{\nu]}^{ab} + A_{\alpha\mu}^{ab} \epsilon^{\alpha\beta\gamma\delta} h_{\beta\gamma}^{\nu} + A_{\alpha\mu}^{ab} \epsilon^{\alpha\beta\gamma\delta} h_{\beta\gamma}^{\nu})$$

(3b)

which are referred to as Riemann and torsion tensors, but which confer no geometry to $\mathcal{M}$. The Ricci tensor $R^a_{\phantom{a}bc} \equiv R^{abc}_{\phantom{a}bc}$, Ricci scalar $\mathcal{R} \equiv R^a_{\phantom{a}a}$, and torsion contraction $T_a \equiv T^a_{\phantom{a}ab}$ are then used to construct the most general total Lagrangian up to quadratic order in the field strengths and invariant under particle inversions

$$L_T = -\frac{1}{2}m_p^2 \epsilon_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} + \alpha_1 R^2 + \alpha_2 R_{\alpha\beta} R^{\alpha\beta} + \alpha_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_4 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_5 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_6 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_7 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_8 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_9 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + \alpha_{10} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

(4)

This general theory is termed PGT$^{q,+}$, and is parameterised by ten dimensionless coupling constants. Note that the remaining \textit{fermionic} fields $\Psi$ of the SM are now permitted in $L_m(\Phi;\Psi;h,A)$ as representations of SL(2, $\mathbb{C}$), which universally covers SO$^+(1,3)$. The Maxwell-like terms in (4) are motivated by analogy to the Yang-Mills structure of the SM: since Eqs. (3a) and (3b) are at lower order than (2), maximally second-order field equations are guaranteed by construction.

\textbf{Scale-invariance.} – Pushing the SM analogy further, one considers scale-invariance. This pertains to local conformal (or Weyl) transformations

$$g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}, \quad \phi \mapsto \Omega^{-1} \phi, \quad \psi \mapsto \Omega^{-1} \psi$$

(5a)

$$h^a_\mu \mapsto \Omega h^a_\mu, \quad A^a_{\mu} \mapsto A^a_{\mu}$$

(5b)

The Lagrangia (1) and (4) are scale-invariant if they transform with weight $-4$, which cancels with the measure $\sqrt{|g|}$, or $h^{-1} \equiv \text{det} b^{-1}_a$. A scale-invariant PGT$^{q,+}$ has $\alpha_0 = \beta_1 = \beta_2 = \beta_3 = 0$, which eliminates the explicit mass scale $m_p$. Conventionally, $\phi$ and $\psi$ have weight $-1$ [42] and $A^a_{\mu}$ has weight 0 [41]. As a slight aside, an inhomogeneously rescaling $A^a_{\mu}$ was recently used in an \textit{extension} of Weyl gauge theory (eWGT) [41]. Quite unlike PGT, eWGT is scale-invariant by construction. However, when expressed in terms of scale-invariant variables [43–45], the quadratic, parity-preserving version (eWGT$^{q,+}$) was shown to be dynamically equivalent to PGT$^{q,+}$ under the SCP [21]. At this level, PGT$^{q,+}$ and eWGT$^{q,+}$ differ only through a scale-dependent interpretation of the coupling constants. We will briefly return to eWGT$^{q,+}$ in closing.

\textbf{The full metrical analogue.} – We will now construct an instance of (1) which mimics (4) under the spatially-flat SCP. Adopting dimensionful Cartesian coordinates on $\mathcal{M}$, the flat FLRW metric has interval

$$ds^2 = dt^2 - a^2 dx^2.$$  

(6)

The dimensionless scale factor $a$ provides the Hubble number $H = \partial_t a/a$. Under conformal transformations of the form (5a), the form of (6) is always preserved by implicit combination with the diffeomorphism

$$dt \mapsto \Omega^{-1} dt, \quad H \mapsto \Omega^{-1}(H - \partial_t \Omega).$$

(7)

Analogous Cartesian coordinates $\gamma_{\mu\nu} = \eta_{\mu\nu}$, assumed to transform according to (7) under Weyl rescalings of the form (5b), then allow us to equate component values $g_{\mu\nu} \equiv \eta_{\mu\nu} h_{\mu\nu}$ and $g_{\mu\nu} \equiv \eta_{\mu\nu} b_{\mu} b^\nu$. Our ‘analogue equality’ flags the notational abuse of incompatible tangent spaces. The torsion tensor on $\mathcal{M}$ is restricted by the SCP to the scalar $U$ and pseudoscalar $Q$, which are the $0^+$ and $0^-$ modes [46–48]

$$T^a_{\phantom{a}bc} = \delta^a_0 \left( 2 \frac{U}{\mathcal{R}} - \text{h} \text{b} \text{i} \text{d} \text{l} - Q \epsilon^a_{\phantom{a}d\text{b}c} \right).$$

(8)

These are homogeneous cosmological fields in the same sense as $\phi$ and $\psi$, inviting the analogue of torsion on $\mathcal{M}$

$$\phi \equiv \frac{\mathcal{R}}{2} U - 2H, \quad \psi \equiv Q.$$  

(9)

Related constructions are used in [21, 49, 50] for algebraic convenience. In our case we see that (9) corrects the inhomogeneous rescaling of $T^a_{\phantom{a}bc}$, endowing the galileons with a weight of $-1$. Thus, all relations in (5a) are reconciled with those in (5b). Finally, we tacitly convert matter fermions into bosons so as to preserve the stress-energy tensor $2(\frac{\delta}{\delta g})_{\mu\nu} [\sqrt{|g|} L_m(\Phi;g)] \rightarrow \eta_{\alpha\beta} \eta^{\rho\lambda} (\frac{\delta}{\delta g})_{\mu\nu} [h^{-1} L_m(\Phi;\Psi;h,A)],$ see e.g. [51]. The spin tensor $\frac{\delta}{\delta g}_{\mu\nu} [h^{-1} L_m(\Phi;\Psi;h,A)]$ is neglected.

At this point we are ready to derive the specific $G_2$ and $G_4$ which facilitate (4). Throughout the PGT$^{q,+}$ equations, the nine Maxwell-like couplings appear exclusively
in five linear combinations under the SCP
\[
\begin{align*}
\sigma_1 &\equiv \frac{3}{2} \alpha_1 + \frac{1}{2} \alpha_2 + \frac{1}{4} \alpha_3 + \frac{1}{2} \alpha_5 - \frac{1}{2} \alpha_6, \\
\sigma_2 &\equiv \frac{3}{2} \alpha_1 + \frac{1}{2} \alpha_2 + \frac{1}{2} \alpha_3 + \frac{3}{2} \alpha_4 - \frac{1}{4} \alpha_5 + \frac{1}{2} \alpha_6, \\
\sigma_3 &\equiv \frac{3}{2} \alpha_1 + \frac{1}{2} \alpha_2 + \frac{1}{2} \alpha_3 + \frac{1}{2} \alpha_4 + \frac{1}{2} \alpha_5 + \frac{1}{2} \alpha_6, \\
v_1 &\equiv -2 \beta_1 + 2 \beta_2, \quad v_2 \equiv 2 \beta_1 + \beta_2 + 3 \beta_3.
\end{align*}
\]
These physical couplings are insensitive to e.g. a Gauss–Bonnet variation $4\delta \alpha_1 = -\delta \alpha_3 = 4\delta \alpha_6$, which is topological in $D \leq 4$. A naive ansatz restricts to polynomial G-functions, but inspection of the minisuperspace Lagrangian of (1) and (4) reveals that this is only viable up to surface terms if $\alpha_0 + v_2 = \sigma_1 = 0$ [52]. These constraints eliminate terms of first order in $\partial_t \phi$ and $H$ from the field equations. Such terms are non-canonical, but can be included (and the constraints removed) through the action by extending (1) to $L_T \rightarrow L_T + \Delta L_T$, where
\[
\Delta L_T = (G_6^\alpha(\phi, \psi) \partial_\alpha \phi + G_6^\alpha(\phi, \psi) \partial_\alpha \psi) B^\mu + m_p (m_p^2 - B_p B^\mu) \chi,
\]
with two new G-functions. The neutral vector $B^\mu$ and scalar $\chi$ may be thought of as gravitational spurions: they constrain the theory by singling out a preferred timelike vector under the SCP without breaking general covariance in the action [53]. The spurions are generally non-dynamical and are integrated out directly such that (11) merely renormalises $G_2$. Writing out the final G-functions explicitly, the full MA of (4) is
\[
L_T = \left( \frac{1}{2} m_p^2 v_2 + \sigma_3 \phi^2 + \frac{1}{2} (\sigma_3 - \sigma_2) \psi^2 \right) R
+ 12 \sigma_3 X^{\phi \psi} + 6 (\sigma_3 - \sigma_2) X^{\psi \psi} + \sqrt{|J_p J^p|}
+ \frac{3}{2} \sigma_2 \phi^4 - 3 \sigma_2 \phi^2 \psi^2 + \frac{3}{2} \sigma_3 \psi^4
+ \frac{3}{2} m_p^2 (\alpha_0 + v_2) \phi^2 - 3 m_p^2 (\alpha_0 - 4 v_1) \psi^2
+ L_m(\Phi, g),
\]
subject to $J_p \equiv 0$ and $m_p < 0$, the admixture of $T$ in (13) leads to $R \rightarrow \text{Cuscuton}$ contributions in (14) which exactly cancel in the $g_{\mu \nu}$ equation. However, true teleparallelism, with $\beta = 1$ and $\alpha_0 = 0$ is also equivalent to GR if curvature vanishes identically [31, 39, 59]. The constraint $R^{ab} c_{cd} = 0$ is properly imposed via Lagrange multiplier fields [39], but in practice this just restricts $A^a_b$ to a pure gauge (the Weitzenböck connection) and fixes $\phi \equiv \psi = 0$. By (9) we will then have $Q = 0$ and $U = \mp 3H$. Since the Cuscuton is now eliminated, $T$ is represented purely by $R$, and the equivalence to GR is immediate.

First impressions. – Noting in what follows that $\sqrt{|J_p J^p|}$ carries an implicit factor of $\text{sgn}(J^p)$ for continuity [54], a straightforward calculation confirms that (12) and (4) are dynamically coincident under the spatially-flat SCP. In this Letter we will not consider inhomogeneous applications, e.g. to acoustic stability. Various features of the MA are already apparent at the Lagrangian level. Since $G_4$ is not constant, $\phi$ and $\psi$ are non-minimally coupled to $R$, thus the MA has been unwittingly but naturally constructed in the Jordan conformal frame (JF). It will prove convenient later to transform to the Einstein frame (EF), but since the EF derives its meaning from the artificial context of the MA, we cannot take it to be physical. Equivalently, to work at the usual level of the PGT$^a$ equations is to work in the JF of the MA and know no better. While counter-intuitive, we find this picture to be unavoidable [55]. A scale-invariant PGT$^a$ sets $\alpha_0 = v_1 = v_2 = 0$, reducing the MA to a manifestly conformal field theory [42]. In our minimal formulation, this restricts to a pure radiation cosmology (see e.g. [50]), but we note that various Higgs-like scale symmetry-breaking extensions to the gauge theory have been proposed [56–58].

Application to established theories. – Before addressing the novel theories, we will analyse some ‘conventional’ PGT$^a$’s with non-dynamical $A^a_b$. Consider the representative two-parameter theory
\[
L_T = -\frac{1}{2} m_p^2 \alpha_0 R + \frac{1}{2} \frac{1}{2} m_p^2 \beta T + L_m(\Phi, \Psi; h, A),
\]
i.e. a linear combination of $R$ and the teleparallel term $T \equiv \frac{1}{4} T_{abc} T^{abc} + \frac{1}{2} T_{abc} T^{bac} - T_n T^n$, with the MA
\[
L_T = -\frac{1}{2} m_p^2 \beta R + m_p^2 (\beta - \alpha_0) \sqrt{|X^{\phi \psi}|}
- \frac{3}{4} m_p^2 \phi^2 + \frac{3}{4} m_p^2 \psi^2 + L_m(\Phi, g).
\]
We see that the MA is a linear combination of $R$, a quadratic Cuscuton $\phi$ with equation of motion $\phi = -2H$, and a non-dynamical mass which sets $\psi = 0$. By (9) we will have $U = Q = 0$. As a general principle, the Cuscuton is a non-dynamical constraint field, and preserves the form of the usual Friedmann equations of GR that follow from $R$. This can be seen by substituting $\phi$ into the $g_{\mu \nu}$ equation of (14) [29]. ECKS theory is equivalent to GR when the spin tensor vanishes, and is defined by $\alpha_0 = 1$ and $\beta = 0$ in (13) [32]. Remarkably, this eliminates $R$ from (14) entirely, so that $R$ is represented purely by the Cuscuton. If $\beta \neq 0$, the admixture of $T$ in (13) leads to $R \rightarrow \text{Cuscuton}$ contributions in (14) which exactly cancel in the $g_{\mu \nu}$ equation. However, true teleparallelism, with $\beta = 1$ and $\alpha_0 = 0$ is also equivalent to GR if curvature vanishes identically [31, 39, 59]. The constraint $R^{ab} c_{cd} = 0$ is properly imposed via Lagrange multiplier fields [39], but in practice this just restricts $A^a_b$ to a pure gauge (the Weitzenböck connection) and fixes $\phi \equiv \psi = 0$. By (9) we will then have $Q = 0$ and $U = \mp 3H$. Since the Cuscuton is now eliminated, $T$ is represented purely by $R$, and the equivalence to GR is immediate.

Application to novel theories. – The cases in [17] are defined by linear constraints on the ten PGT$^a$’s parameters. These constraints structurally alter the saturated propagator, obtained by inverting the linearised, matter-free Lagrangian in (4). The SCP groups the cases into classes as shown in Fig. 1. The constraint $\alpha_0 = 0$ marks a complete break with ECKS theory: one is left only with quadratic invariants which have no EFT interpretation as loop corrections to the PGT Ricci scalar $R$. The further constraint $\sigma_3 = 0$ then triggers the $k$-screening mechanism, in which the physical spatial curvature $k \in \{ \pm 1, 0 \}$ is eliminated from the PGT$^a$ equations: a hyperspherical, hyperbolic or simply flat choice of universe does not affect the background dynamics [21]. The description of such classes as offered by the MA is thus not limited by our earlier assumption of spatial flatness in (6).
We consider Class $2A^*$, defined by the further constraint $\sigma_2 = \sigma_1$ (note that Class $3C^*$ will always be the special case $\nu_1 = 0$). We next set $\sigma_1 < 0$ (no ghost) and $\nu_1 < 0$ (no tachyon); these unitarity conditions are listed in [17]. They may also be read off from (12) near the vacuum $R = \phi = \psi = 0$, once the defining constraints are imposed. We finally take a third condition $\nu_2 < 0$ by analogy to the Einstein-Hilbert Lagrangian, although this is not listed in [17]. A conformal transformation $\Omega^2$ takes the MA of Class $2A^*$ into the EF. Following the conventions of e.g. Brans–Dicke theory [60], we then partly reconcanalise the MA through two new fields $\zeta(\phi, \psi)$ and $\xi(\psi)$ [61]

\[ L_T = -\frac{1}{2}m_p^2R + X\xi + m_p^2W(\xi)^3\sqrt{|X\xi|} - V(\xi) + \frac{1}{4}m_p^2W(\xi)^4 + L_m(\Phi, \xi; g), \quad (15a) \]

\[ V(\xi) \equiv -\frac{4m_1}{3\sigma_1\nu_2}m_p^4(1 + \frac{1}{8}W(\xi)^2)(1 + \frac{1}{2}W(\xi)^2), \quad (15b) \]

\[ W(\xi) \equiv \sqrt{3\cosh(\sqrt{2/3\xi/m_p}) - 5}. \quad (15c) \]

Noting that $\Omega^2 = -\frac{4}{3\sigma_1}(1 + \frac{1}{8}W^2)$, it seems natural in what follows to take $\nu_2 = -4/3$, and this choice will be justified in stages. The ‘conformal shift’ $W$ now measures the degree to which the physical $\Phi$ has strayed from the EF, and so mediates any $\xi$-$\Phi$ coupling. Note that $W$ also weights the field $\zeta$, which is a Cuscuton. The field $\zeta$ is canonical, and in moving from Class $3C^*$ to Class $2A^*$ it acquires a potential $V$. Note that $V$ traces back to the mass of $\psi$, which in turn corresponds to the massive $0^-$ D.o.F in Fig. 1. By inspection, $V$ must act as a (quintessence) DE source, since $\nu_1/\sigma_1 > 0$. In the final sections we will make the nature of this DE more concrete, using the $\zeta$ equation of (15) as a heuristic guide

\[ W^2(\sqrt{2\partial_W\partial_\xi} + \sqrt{2WH} - W^2\zeta) = 0. \quad (16) \]

**Negative, screened dark energy.** – By analogy to (14), suppose that the Cuscuton obeys $\zeta \propto H$, which was its ‘minimally-coupled’ behaviour. This is possible if the last two terms in (16) cancel, whereupon the decay of $\zeta$ stalls above the natural vacuum of $V$ at constant conformal shift $W \to \sqrt{2H/\zeta}$. This solution has the following utility. Accelerated expansion is difficult to drive with a negative bare cosmological constant $\Lambda_b < 0$ in many gravitational theories. This can make them hard to reconcile with attractive, more fundamental the-

in many gravitational theories. This can make them hard to reconcile with attractive, more fundamental the-

\[ \frac{\partial_\xi^2}{p^2\partial_\xi^2} \equiv m_p^2(\partial_\xi)^2/6H^2 and \quad y^2 \equiv V_T/3m_p^2H^2 \] \n
\[ \nu_1 \to -3/2 \]

The stability of the CS should be verified for all cosmological fluids in $\Lambda$CDM, but the earlier dynamical systems approach is impractical in this case. Such fluids may be caracterised by linear equasions of state (E.o.S) $\rho = \omega\dot{\gamma}$,
A cosmological constant \( \Lambda \) reflects the universe towards the de Sitter attractor \( B \) in the inflationary region \( q < 0 \), where it feels a positive effective cosmological constant \( \Lambda = 0.1 m_p^{-2} \). The Einstein frame deceleration parameter is \( 1 + q = -\dot{H}/H^2 \). Dimensionless Hamiltonian coordinates \( y \) and \( x \) describe the 0–torsional mode, here interpreted as a canonical inflaton. Dimensionless phase velocity naturally reflects elapsing Hubble-times.

which dilute away as \( \rho \propto a^{-3(1+w)} \). For any such dominant fluid, a perturbation around the de Sitter attractor \( B \) is equivalent to adding an effective fluid \( \rho \mapsto \rho + \rho_{\text{eff}} \) to GR. The effective E.o.S parameter tracks the dominant \( w \) according to

\[
w_{\text{eff}}(w) = \frac{1}{2}(w+1) - \frac{1}{2} \sqrt{9w^2 + 3},
\]

which is valid for both Class \( 3^C* \) and Class \( 2^A* \). The effective fluid becomes increasingly sub-dominant (and the CS is stable) when \( w_{\text{eff}}(w) > w \); the only exception in our universe \( (w \in [-1, 1/3]) \) is co-dominant DR, since \( w_{\text{eff}}(1/3) = 1/3 \). The possible utility of this DR in shrinking the sound horizon at recombination (and raising the early-universe inference of \( h \)) is discussed in [21]. Finally in the late universe, a stalled \( V \) readily gives the effective cosmological constant

\[
\Lambda = \Lambda_b + \frac{m_p}{\sigma_1}m_p^2,
\]

with no a priori assumption about \( \text{sgn}(\Lambda_b) \).

**Discussion.** – We constructed a scalar-tensor theory which lays bare the rich background cosmology of PGT\(^{\alpha+, \beta+} \). Our approach invites inflationary applications in the early universe, and extension to Weyssenhoff fluids [68]. In this Letter we focussed on late-universe DE in recently proposed, superficially healthy cases of PGT\(^{\alpha+, \beta+} \).

As expressed in (18), the proposed DE still does not address the 'strong' cosmological constant problem [33, 34]. Let us assume \( \Lambda_0 = 0 \). CMB-inference fixes \( \Lambda = 7.15 \pm 0.19 \times 10^{-121} \text{ m}_p^{-2} \) [9], with some (slight) shift expected from any DR we may choose to add [69, 70].

The requisite \( v_1/\sigma_1 \sim 10^{-121} \) then reveals an apparent hierarchy. The hierarchy may be removed by invoking some vacuum \( \Lambda_b \sim -m_p^2 \), but only at the cost of exquisite fine-tuning. Bearing this in mind, we tentatively observe that the hierarchy problem appears less severe in the scale-invariant eWGT counterpart [41]. The \( \sim 4.1 \text{ Gpc} \) Hubble horizon endows specific physical eWGT\(^{\alpha+, \beta+} \) couplings with a natural length scale, and should give an emergent \( \Lambda \) consistent with the non-gravitating vacuum \( \Lambda_0 = 0 \). This builds the case for a future extension of the systematic analysis in [16, 17, 71] to eWGT\(^{\alpha+, \beta+} \), whose propagator is currently unexplored.

In a conservative summary, Class \( 2^A* \) of PGT\(^{\alpha+, \beta+} \) not only matches the GR background, but can provide dark radiation and (hierarchical) dark energy. Unlike GR [72], the perturbative renormalisability of this unitary theory is not precluded by a simple power counting [16, 17]. The 0–torsional mode must survive averaging over homogeneous comoving scales of \( \gtrsim 300 \text{ h}^{-1} \text{ Mpc} \) [73, 74]. This mode has yet to be constrained, even in an Earth-based laboratory [75–77], and its strength is not separable here from the \( \sigma_1 \) or \( v_1 \) couplings. Indeed, the expansion history only determines \( v_2 \) and \( v_1/\sigma_1 \), which translate to the two freedoms in Lovelock’s theorem.

**Acknowledgments.** – We are grateful to Fernando Quevedo for essential comments following the original presentation of this work at the DAMTP GR Seminar Series on 14th February 2020, and David Tong, Miguel Zumalacárregui and Sunny Vagnozzi for helpful correspondence. We also thank Amel Durakovic for many valuable discussions and suggestions which improved the manuscript. W. E. V. B. is supported by the Science and Technology Facilities Council – STFC under Grant ST/R504671/1, and W. J. H. by a Gonville and Caius Research Fellowship.


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