QUARK AND GLUON SIVERS FUNCTIONS∗

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The physics of hadron single transverse spin asymmetries is discussed. Possible measurements of both the quark and gluon Sivers functions are proposed.

1. Introduction

In the usual QCD factorization formalism, a collinear approximation for the partonic intrinsic motion is used, and therefore the total inclusive hadronic cross section is written as a convolution of a hard elementary partonic cross section with distribution and fragmentation functions in which the transverse motion of the partons has been integrated. Nevertheless, the intrinsic quark and gluon transverse momenta are important, because for one thing they provide corrections to the collinear approximation, and moreover they are essential in order to explain single spin asymmetries (SSA) within a generalized transverse momentum dependent QCD factorization formalism. In this case the cross section for an inclusive process $AB \to CX$ is written as:

$$d\sigma = \sum_{abc} f_{a/A}(x_a, k_{1a}) \otimes f_{b/B}(x_b, k_{1b}) \otimes d\sigma^{ab\to c...}_{(x_a, x_b, k_{1a}, k_{1b})} \otimes D_{C/c}(z, k_{1C})$$

In dealing with SSAs, the most important transverse momentum dependent functions are the Sivers distribution function $f_{1T}^{a} = f_{a/p_T}^{+}(x_a, \vec{k}_{1a}) - f_{a/p_L}^{+}(x_a, \vec{k}_{1a})$, which gives the probability distribution of finding unpolarized quarks inside a transversely polarized proton, and the Collins fragmentation function $H_{1}^{+}$, which gives the probability of unpolarized hadrons

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coming from the fragmentation of a transversely polarized quark. In several processes both functions can contribute.

2. Quark Sivers Function

The Sivers function is proportional to the T-odd correlation \( \vec{S}_\perp \cdot (\vec{P} \times \vec{k}_\perp) \), and therefore contains an azimuthal asymmetry with respect to the direction of the hadron momentum \( \vec{P} \). In 1993 Collins gave a proof about the vanishing of this function\(^1\), but which was shown later on by Collins himself to be incorrect.

In order to understand the physics that is present in the Sivers function let us consider a specific process: semi-inclusive deep inelastic lepton scattering \( \ell p^\uparrow \rightarrow \ell' \pi X \). In the target rest frame, single-spin correlations correspond to the T-odd triple product \( i\vec{S}_p \cdot \vec{p}_\pi \times \vec{q} \), where the phase \( i \) is required by time-reversal invariance. The differential cross section thus has an azimuthal asymmetry proportional to \( |\vec{p}_\pi||\vec{q}|\sin \theta_{\pi} \sin \phi \) where \( \phi \) is the angle between the plane containing the photon and pion and the plane containing the photon and proton polarization vector \( \vec{S}_p \). In a general frame, the azimuthal asymmetry has the invariant form \( \frac{i}{4!} \epsilon_{\mu \nu \sigma \tau} P^\mu S^\nu_p p^\sigma_\pi q^\tau \) where the polarization four-vector of the proton satisfies \( S^2_p = -1 \) and \( S_p \cdot P = 0 \).

In general we can express the SSA as:

\[
SSA = \frac{\langle \uparrow \uparrow \rangle - \langle \downarrow \downarrow \rangle}{\langle \uparrow \uparrow \rangle + \langle \downarrow \downarrow \rangle},
\]

where the transverse spin basis is related to the helicity basis by \( |\uparrow / \downarrow \rangle = \frac{1}{\sqrt{2}} (|+\rangle \pm i |\rangle) \), which means that the SSA can be written as:

\[
\frac{2\text{Im} \langle + | - \rangle}{\langle + | + \rangle + \langle - | - \rangle}
\]

Therefore in order to produce a correlation involving a transversely-polarized proton, there are two necessary conditions: (1) There must be two proton spin amplitudes \( M[J^z_p = \pm \frac{1}{2}] \rightarrow F \) with \( J^z_p = \pm \frac{1}{2} \) which couple to the same final-state \( |F> \); and (2) The two amplitudes must have different, complex phases. The correlation is proportional to \( \text{Im}(M[J^z_p = +\frac{1}{2}]^* M[J^z_p = -\frac{1}{2}]) \).

As a result, we can reach the following conclusions: (1) The analysis of single-spin asymmetries requires an understanding of QCD at the amplitude level. (2) It also provides a handle on the proton angular momentum. Since we need the interference of two amplitudes which have different proton spin \( J^z_p = \pm \frac{1}{2} \) but couple to the same final-state, the orbital angular momentum...
of the two proton wavefunctions must differ by $\Delta L^z = 1$. The anomalous magnetic moment for the proton is also proportional to the interference of amplitudes $M[\gamma^* p(J^z_p) \to F]$ with $J^z_p = \pm \frac{1}{2}$ which couple to the same final-state $|F\rangle$. (3) Since we need an Imaginary part, the SSA cannot come from tree level diagrams.

Final state interactions clearly fit into this picture. If a target is stable, its light-front wavefunction must be real. Thus the only source of a nonzero complex phase in lepton production in the light-front frame are final-state interactions. The rescattering corrections from final-state exchange of gauge particles produce Coulomb-like complex phases which, however, depend on the proton spin.

In Ref. 2 the single-spin asymmetry in semi-inclusive electroproduction $\gamma^* p \to HX$, induced by final-state interactions, was calculated in a model of a spin-1/2 proton of mass $M$ with charged spin-1/2 and spin-0 constituents of mass $m$ and $\lambda$, respectively, as in the QCD-motivated quark-diquark model of a nucleon. The basic electroproduction reaction is then $\gamma^* p \to q(q\bar{q})_0$.

There it was shown that the final-state interactions from gluon exchange between the outgoing quark and the target spectator system leads to single-spin asymmetries in deep inelastic lepton-proton scattering at leading twist in perturbative QCD; i.e., the rescattering corrections are not power-law suppressed at large photon virtuality $Q^2$ at fixed $x_{bj}$. The azimuthal single-spin asymmetry transverse to the photon-to-pion production plane decreases as $\alpha_s(r^2_\perp)x_{bj}M_{r\perp}|\ln r^2_\perp/r^2_\perp|$ for large $r_\perp$, where $r_\perp$ is the magnitude of the momentum of the current quark jet relative to the virtual photon direction. The fall-off in $r^2_\perp$ instead of $Q^2$ compensates for the dimension of the $\bar{q}-q$-gluon correlation. The mass $M$ of the physical proton mass appears here since it determines the ratio of the $L^z=1$ and $L^z=0$ matrix elements. This is the same type of physics that gives shadowing and antishadowing effects in both electromagnetic and weak deep inelastic scattering in nuclei3.

A related analysis also predicts that the initial-state interactions from gluon exchange between the incoming quark and the target spectator system lead to leading-twist single-spin asymmetries in the Drell-Yan process $H_1 H_2^* \to \ell^+ \ell^- X^{4,5}$. These final- and initial-state interactions can be identified as the path-ordered exponentials which are required by gauge invariance and which augment the basic light-front wavefunctions of hadrons4,5. Both pictures, final and initial state interactions and different gauge links, lead to the conclusion that the Sivers function is not really universal, but changes
sign between SSAs in deep inelastic scattering and Drell-Yan processes.

3. How to obtain the Sivers function?

There have been many theoretical and experimental analysis about ways in which to separate the Collins and Sivers effects. Probably the simplest is to use the SSAs which can be measured in weak interaction processes. For example, consider charged current neutrino semi-inclusive deep inelastic scattering, where a hadron (pion) is measured in the final state. In this case, the transversity distribution cannot contribute to the cross section since the produced quark from the weak interaction of the $W$ boson is always left-handed. On the other hand, in the final-state interaction picture the SSA in charged and neutral current weak interactions will also be present, just as in the electromagnetic case. Thus these weak interaction processes will clearly distinguish the underlying physical mechanisms which produce target single-spin asymmetries.

Let us see this in more detail. The quark distribution in the proton is described by a correlation matrix:

$$\Phi^{\alpha\beta}(x, p_\perp) = \frac{d^2\xi}{(2\pi)^3} e^{ip \cdot \xi} < P, S | \bar{\psi}^\beta(0) \psi^\alpha(\xi) | P, S > |_{\xi^+ = 0}, \quad (4)$$

where $x = p^+/P^+$. The correlation matrix $\Phi$ is parameterized in terms of the transverse momentum dependent quark distribution functions:

$$\Phi(x, p_\perp) = \frac{1}{2} \left[ f_1^{\perp} \gamma_5 + f_1^{TT} \gamma_5 \sigma_{\mu \nu} n^\mu p_\perp^\nu S_\perp^\alpha \frac{1}{M} + g_1^{\perp} \gamma_5 \right] + h_1^{\perp} \sigma_{\mu \nu} n^\mu p_\perp^\nu + h_1^{\perp} \sigma_{\mu \nu} n^\mu p_\perp^\nu + h_1^{\perp} \sigma_{\mu \nu} n^\mu p_\perp^\nu,$$$$

where the distribution functions have arguments $x$ and $p_\perp$, and $n^\mu = (n^+, n^-, n_\perp) = (0, 2, 0_\perp)$.

Similar expressions and parametrization can be obtained for the quark fragmentation correlation matrix $\Delta^{\alpha\beta}(z, k_\perp)$.
3.1. Electromagnetic case

The hadronic tensor of the leptoproduction by the electromagnetic interaction in leading order in $1/Q$ is given by

$$2MW^\mu\nu(q, P, P_h) = \int d^2p_\perp d^2k_\perp \delta^2(p_\perp + q_\perp - k_\perp)$$

$$\times \frac{1}{4} \text{Tr} \left[ \Phi(x_B, p_\perp) \gamma^\mu \Delta(z_h, k_\perp) \gamma^\nu \right]$$

$$+ \left( q \leftrightarrow -q, \mu \leftrightarrow \nu \right),$$

where $x_B = Q^2/2P \cdot q$ and $z_h = P \cdot P_h / P \cdot q$. The momentum $q_\perp$ is the transverse momentum of the exchanged photon in the frame where $P$ and $P_h$ do not have transverse momenta.

The single-spin asymmetry (SSA) in semi-inclusive deep inelastic scattering (SIDIS) $e^+ e^- \rightarrow e' \pi X$, which is given by the correlation $S_\pi \cdot \bar{q} \times p_\pi$, is obtained from (6). As mentioned before, for the electromagnetic interaction there are two mechanisms for this SSA: $h_1 H^\perp$ and $f_1^T D_1$ (Collins and Sivers effects), where $h_1$ is the transversity distribution and $D_1$ the unpolarized quark fragmentation function.

We can also consider the SSA of $e^+ e^- \rightarrow \gamma^* \rightarrow \pi \Lambda \rightarrow \pi \Lambda X$. The $\Lambda$ reveals its polarization via its decay $\Lambda \rightarrow p\pi^-$. The spin of the $\Lambda$ is normal to the decay plane. Thus we can look for a SSA through the T-odd correlation $\epsilon_{\mu\nu\rho\sigma} S_\Lambda^P p_\Lambda^\mu q_\pi^\rho v_\pi^\sigma$. This is related by crossing to SIDIS on a $\Lambda$ target.

3.2. Charged weak current case

**Charged currents:** Let us consider the SSA in the charged current (CC) weak interaction process $\nu p^\perp \rightarrow \ell \pi X$. For the CC weak interaction, the trace in (6) becomes

$$\text{Tr} \left[ \Phi_\mu P_L \Delta_\nu P_L \right] = \text{Tr} \left[ \Phi P_R \gamma^\mu P_L \Delta P_R \gamma^\nu P_L \right] = \text{Tr} \left[ \Phi_{\text{CC}} \gamma^\mu \Delta_{\text{CC}} \gamma^\nu \right],$$

where $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, and

$$\Phi_{\text{CC}} \equiv P_L \Phi P_R, \quad \Delta_{\text{CC}} \equiv P_L \Delta P_R.$$
from the Sivers FSI mechanism $f^\perp_{1T}D_1$ in leading order in $1/Q$; in contrast, both the Collins $h_1H^\perp_1$ and Sivers $f^\perp_{1T}D_1$ mechanisms contribute to SSAs for the electromagnetic and neutral current (NC) weak interactions.

In a similar way, we can also consider the SSAs of the processes $\pi p \leftrightarrow (or \ pp \leftrightarrow) WX \rightarrow \ell \nu X$. If $y$ denotes the $W$ rapidity, it can be shown that the region $y \sim -1$ is very sensitive to the antiquark Sivers functions, whereas the region $y \sim +1$ is sensitive to the quark Sivers functions. Furthermore, it turns out that the SSA for $W^+$ gives information about the Sivers $u$ quark distribution in the region $y \rightarrow 1$ and about the Sivers $\bar{d}$ in the region $y \rightarrow -1$. Something similar happens for the $W^-$ SSA (interchanging $u$ and $d$). Therefore the measurement of the SSAs $A^{W\pm}$ is a practical way to separate the $u$ and $d$ quarks Sivers functions and their corresponding antiquark distributions $\bar{u}$ and $\bar{d}$.

Neutral currents: Let us now consider the SSA in the neutral current weak interaction process $\nu p \leftrightarrow \nu pX$. For the NC weak interaction, the interaction vertex of $Z$-f-f is given by $(-ie/\sin\theta_W\cos\theta_W)(c_L P_L + c_R P_R)$ with the weak isospin-dependent coefficients $c_{L,R} = \frac{I_3^W - Q}{2}$. Explicit values of $c_L, c_R$ are given by $c_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$, $c_R = -\frac{2}{3}\sin^2\theta_W$ for $u, c, t$ quarks, and $c_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$, $c_R = \frac{1}{3}\sin^2\theta_W$ for $d, s, b$ quarks.

The trace in (6) becomes

$$a \, \text{Tr}\left[\gamma^\mu(c_L P_L + c_R P_R)\gamma^\nu(c_L P_L + c_R P_R)\right] = a \, \text{Tr}\left[\Phi_{NC}\gamma^\mu\gamma^\nu\right],$$

where $a = 1/\sin^2\theta_W \cos^2\theta_W$ and

$$\Phi_{NC} = (c_L P_L + c_R P_R)\Phi(c_L P_R + c_R P_L).$$

In this case, for the Sivers effect we find that the SSA is given by that of the electromagnetic case with $f^\perp_{1T}D_1$ replaced by

$$a \, \frac{c_L^2 + c_R^2}{2} \, f^\perp_{1T}D_1.$$ (11)

However, $f_1$ is also weighted by the same factor $a (c_L^2 + c_R^2)/2$. Therefore, the SSA from the final-state interaction mechanism in the NC weak interaction is the same as that in the electromagnetic interaction. This can be confirmed in the simple quark-diquark model.

For the $h_1H^\perp_1$ mechanism, we find that the SSA is given by that of the electromagnetic case with $(h_1H^\perp_1)/(f_1D_1)$ replaced by

$$\frac{2c_L c_R}{c_L^2 + c_R^2} \, \frac{h_1H^\perp_1}{f^\perp_{1T}D_1}. (12)$$
That is, the SSAs are modified by the quark weak isospin-dependent factor \( \frac{2c_Lc_R}{c_L^2 + c_R^2} \) in comparison with the electromagnetic case. The same factor appears in the linear \( \cos \theta \) forward-backward asymmetry in the \( e^+e^- \rightarrow Z \rightarrow q\bar{q} \) reaction.

The SSA of the Drell-Yan processes at the \( Z_0 \), such as \( \pi p \uparrow \) (or \( pp \uparrow \)) \( \rightarrow ZX \rightarrow \ell^+\ell^-X \), can arise from the \( h_1h_1^\perp \) and \( f_1^\perp D_1 \) mechanisms. We can also consider the SSA of the \( e^+e^- \) annihilation processes such as \( e^+e^- \rightarrow Z \rightarrow \pi\Lambda^\perp X \), which can arise from the \( H_1H_1^\perp \) and \( D_1^\perp D_1 \) mechanisms\(^{10}\). The SSAs of these processes have the same situation as those of the above SIDIS case. The initial/final-state interaction mechanisms have the same formulas as the electromagnetic case, whereas the Collins mechanisms are weighted by the quark weak isospin-dependent factor \( \frac{2c_Lc_R}{c_L^2 + c_R^2} \) present in (12).

4. Gluon Sivers function

The gluon Sivers function was mentioned for the first time in Ref. 9, and recently it was also considered in jet correlations\(^{11}\) and in \( D \) meson production\(^{12}\) in \( p^\uparrow p \) collisions.

The direct photon production in \( pp \) collisions can provide a clear test of short-distance dynamics as predicted by perturbative QCD, because the photon originates in the hard scattering subprocess and does not fragment, which immediately means that the Collins effect is **not** present\(^{13}\). This process is very sensitive to the gluon structure function, since it is dominated by the quark-gluon Compton subprocess in a large photon transverse momentum range. Prompt-photon production, \( pp(p\bar{p}) \rightarrow \gamma X \), has been a useful tool for the determination of the unpolarized gluon density and it is considered one of the most reliable reactions for extracting information on the polarization of the gluon in the nucleon\(^{14}\).

It turns out that both Sivers functions for quarks and gluons are involved in the SSA for direct photon production \( A^\gamma(s,x_F) \), and therefore it is necessary to identify a kinematic region where the gluon Sivers function dominates. The cross section contains two terms, the first one involves a product of the quark Sivers function at the light-cone momentum variable \( x_b \) and the transverse momentum dependent unpolarized gluon distribution at light-cone momentum \( x_a \), while the second involves a product of the gluon Sivers function at \( x_b \) times the transverse momentum dependent unpolarized quark distribution at \( x_a \). Thus it is necessary to determine the range of integration over \( x_a \) and to study the relative magnitude of \( x_a \).
and $x_b$. As an example, taking $\sqrt{s} = 200$ GeV and $p_T = 20$ GeV, we find that the minimum value of $x_a$, $x_{\text{min}} \approx x_F$ in the region $x_F > 0.3$. On the other hand, we can also see that when $x_a$ is integrated over the range $[x_{\text{min}}, 1]$, the main contribution comes from the low $x_b$ values. Therefore, when we look at the large $x_F$ region, where $x_a$ is large but $x_b$ is small, the asymmetry can be approximately expressed as

$$A^\gamma(s,x_F) = \frac{\langle \Delta N G \rangle}{\langle G \rangle},$$

where $\langle \Delta N G \rangle$ and $\langle G \rangle$ mean the corresponding values over an appropriate integrating range.

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