Pixel-based correction for Charge Transfer Inefficiency in the Hubble Space Telescope Advanced Camera for Surveys

Richard Massey\textsuperscript{1}\textsuperscript{⋆}, Chris Stoughton\textsuperscript{2}, Alexie Leauthaud\textsuperscript{3}, Jason Rhodes\textsuperscript{4,5},
Anton Koekemoer\textsuperscript{6}, Richard Ellis\textsuperscript{5} and Edgar Shaghoulian\textsuperscript{7}

\textsuperscript{1}Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, U.K.
\textsuperscript{2}Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, U.S.A.
\textsuperscript{3}Physics Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, U.S.A.
\textsuperscript{4}Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, U.S.A.
\textsuperscript{5}California Institute of Technology, 1200 East California Boulevard, Pasadena, CA 91125, U.S.A.
\textsuperscript{6}Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, U.S.A.
\textsuperscript{7}Stanford University, Physics building, 382 Via Pueblo Mall, Stanford, CA 94305, U.S.A.

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ABSTRACT
Charge Transfer Inefficiency (CTI) due to radiation damage above the Earth’s atmosphere creates spurious trailing in Hubble Space Telescope (HST) images. Radiation damage also creates unrelated warm pixels – but these happen to be perfect for measuring CTI. We model CTI in the Advanced Camera for Surveys (ACS)/Wide Field Channel (WFC) and construct a physically motivated correction scheme. This operates on raw data, rather than secondary science products, by returning individual electrons to pixels from which they were unintentionally dragged during readout. We apply our correction to images from the HST COSMOS survey, successfully reducing the CTI trails by a factor of $\sim 30$ everywhere in the CCD and at all flux levels. We quantify changes in galaxy photometry, astrometry and shape. The remarkable 97% level of correction is more than sufficient to enable a (forthcoming) reanalysis of downstream science products, and the collection of larger surveys.

Key words: space vehicles: instruments — instrumentation: detectors — methods: data analysis

1 INTRODUCTION
Charge-Coupled Device (CCD) imaging detectors convert incident photons into electrons. The electrons are stored within a silicon substrate, in a pixellated grid of electrostatic potential wells that gradually fill up during an exposure. At the end of the exposure, the electrons are shuffled, row by row, to a readout register at the edge of the device. They are then counted and converted into a digital signal.

Above the Earth’s atmosphere, a continuous bombardment of high energy particles makes a harsh environment for sensitive electronic equipment. Protons, neutrons, high-energy electrons and heavy ions create bulk damage by colliding with and displacing atoms from the silicon lattice. The dislodged atoms can come to rest in the interstitial space, and the vacancies left behind move about the lattice until they combine with interstitial impurities, such as phosphorous, oxygen or another vacancy (Janesick et al. 2001). Such defects degrade a CCD’s ability to shuffle electrons, known as its Charge Transfer Efficiency (CTE; Charge Transfer Inefficiency $\text{CTI}=1-\text{CTE}$). Electrons can become temporarily trapped in the local potential, then released after a delay that depends upon the properties of the lattice and impurities, and the operating temperature of the device (Shockley & Read 1952; Hall 1952). If a few electrons are trapped during CCD readout, and held while other electrons are moved along, they are released as a spurious trail (see figure 1B). Regions of the image furthest from the readout register are worst affected, because electrons starting there encounter the most charge traps during their journey across the device.

Charge Transfer Inefficiency gets worse over time, as cumulative radiation damage creates more charge traps. By January 2007, when the Hubble Space Telescope (HST) Advanced Camera for Surveys (ACS) encountered an electronic failure, the instrument had been in operation on-orbit for almost 5 years and its Scientific Imaging Technologies (SITe) ST002A CCDs were already significantly affected by CTI. Similarly, the Wide Field and Planetary Camera 2 (WFPC2) instrument spent 16 years on-orbit. By the time...
it was decommissioned, its truly severe CTI created trailing that was readily visible on every exposure.

Different astronomical measurements are hindered by different species of charge traps. Charge traps with a characteristic release time of a few CCD clock cycles move electrons by a few pixels, creating short trails that alter the apparent position (astrometry) and shape (morphology) of faint objects, thus also lowering their measured brightness (photometry). Several parametric schemes have been developed to estimate and thus correct all these effects at a catalogue level. Fitting formulae have been approximated, as a function of objects’ detector position, date of observation and flux, for incorrect astrometry with WFPC2 (Cawley et al. 2002), photometry with ACS (Riess 2003), morphology with STIS (Rhodes et al. 2004) and ACS (Rhodes et al. 2007), and others. Although such schemes provide first order mean correction for a large number of objects, they are inaccurate for high precision measurement of individual objects. For example, no account is made for the screening of faint objects by bright objects slightly closer to the readout register (which pre-fill charge traps) or, with extended sources, for the dependence upon object size, radial profile, and shape (see discussion in Riess 2000 and Rhodes et al. 2009).

More precise removal of CTI trailing requires manipulation of the raw pixel data, using software to move electrons back to their original locations. Since CCD readout is the last stage of data acquisition, this should be the first stage of a final data reduction pipeline (e.g. CALACS, Pavlovsky et al. 2006). Even this will never achieve precision at the level of single electrons, since charge trapping and release are stochastic, quantum mechanical effects. However, such software has been demonstrated to be much more accurate than parametric solutions for the correction of HST STIS imaging (Bristow 2003a; Piatek et al. 2003, 2006 & 2007) and spectroscopy (Bristow 2003d), and Chandra ACIS spectroscopy (Grant et al. 2004). This approach can also provide physically motivated model parameters, rather than an ad-hoc fitting function to arbitrary parameters.

In this paper, we develop an empirical but physically motivated, pixel-based CTI correction scheme for ACS/WFC imaging. We concentrate on traps with release times of a few CCD clock cycles, but our algorithm is sufficiently general to be able to incorporate additional species of traps with longer release times if necessary. In §2 we build a model of the readout process by measuring the rate at which the CCDs’ potential wells are filled by electrons, and the properties of charge traps within those wells. In §3 we describe software to implement this readout model, and use the iterative approach of Bristow (2003a) to reverse the process. In §4 we apply this to real data and obtain better measurements of galaxy photometry, astrometry and ellipticity in the HST COSMOS survey. In §5 we discuss possible improvements to our algorithm, and summarise our work.

2 MODELLING THE ACS/WFC CCDS

2.1 Data and dates

We primarily base our analysis on the “raw” data obtained during HST cycles 12 and 13 for the COSMOS survey (HST-GO-09822, P.I.: N. Scoville). These are 2368 uniform, extra-galactic exposures of 507 seconds each (Scoville et al. 2007; Koekemoer et al. 2007). For convenience, we first split each exposure into the four quadrants read out through different amplifiers, rotating and flipping them to orient the readout in the same direction for all. We then multiply the image by the calibrated gain (which varies slightly between amplifiers) and subtract the corresponding superbias images created by STScI. This reverts the images to units of electrons, in approximately their configuration on the CCD immediately prior to readout (but note uncertainty about the imperfectly modeled injection of charge by warm pixels during readout in §2.2).
We have also tested our CTE correction method in a less systematic manner on imaging from cycle 14 (HST-GO-10496, P.I.: S. Perlmutter). This extends the analysis to nearly January 2007, when ACS failed. While this smaller survey provided sufficient data to verify an extrapolation of our correction parameters, such non-uniform observational strategies proved less useful for tracking the gradual CTE degradation.

Our model could also be extrapolated to correct observations obtained after servicing mission 4. However, this would be a large extrapolation, and the trap density did not necessarily continue to rise at the same rate while the instrument was offline. We would therefore recommend new calibrations as soon as sufficient images become available.

2.2 Warm pixels

A detector’s CTE can be measured using First Pixel Response (FPR) tests, Extended Pixel Edge Response (EPER) tests, or $^{55}$Fe radiation \cite{Janesiek}. FPR and EPER tests involve uniformly illuminating a CCD and measuring deviations from that uniformity in the few pixels nearest the readout register and (virtual) pixels obtained by continuing to clock charge past the end of the CCD. In addition, a radioactive iron source can be used to create a series of $\delta$-functions on the CCD, all of a fixed energy level. Deficiencies in the observed number of electrons in some events, or deviations in their shape, reveal the presence of charge traps between the event and the readout electronics.

A continuous programme of FPR and EPER monitoring has been carried out on the ACS/WFC detectors since launch \cite{HST-ACS}. Radiation testing was performed prior to launch, but is not possible on-orbit in HST.

After experimenting on all three types of data acquired in a laboratory environment, we decided that the parameters needed for our model are most easily extracted from radiation tests. Although in-flight $^{55}$Fe data are unavailable, cosmic ray damage has also created randomly distributed “warm pixels” that suffice instead \cite{Biretta}. Warm pixels are short circuits in the CCD electrostatic potentials used to collect charge, and continuously inject spurious charge into the device. During a long exposure, warm pixels create single-pixel $\delta$-functions. Since the rate of charge injection varies between warm pixels, these $\delta$-functions have a range of amplitudes. In some ways, this is actually more convenient than an $^{55}$Fe source.

To identify warm pixels within each image $I$, we locate 2D local maxima, after unsharp masking to eliminate extended objects (whether they be resolved galaxies or merely a Point Spread Function). One such warm pixel is labeled $p_0$ in figure 1 and contains $n_0 = I(p_0)$ electrons. This number has been only slightly changed during readout. We measure the trail

$$T_i(n_0) \equiv I(p_i) - I(p_{i-1}),$$

where $p_{i+1}$ are pixels defined relative to the location of the warm pixel $p_0$ in figure 1 and $i$ is an integer from $i = 1...9$. Note that we could have replaced pixels with negative indices by a locally-determined background level. Our better scheme is robust to the accidental inclusion of any extended sources (or cosmic rays covering more than one pixel) in our catalogue. Although such individual artefacts have a shape, they should not have a preferred direction on average, and therefore should not bias our measurement of $T_i$. Indeed, we could even have intentionally measured the trailing behind extended sources. However, in practice, averaging out their intrinsic radial profile adds more noise than the extra numbers of pixels add signal.

At the very faint end of our warm pixel catalogue, which we push down to $n_e = 100$ electrons so that it only just exceeds the background level, our algorithm often finds noise peaks instead of true warm pixels. If the noise were due to photon shot noise, these pixels would be ideal for our purposes, but it is more often read noise – which is added after clocking and therefore untrailed. To prevent the dilution of our trail measurements, we discard pixels that are not flagged as warm in at least half of the exposures. In the faintest two bins, this process removes 85% and 50% of the catalogue, but quickly becomes negligible for brighter pixels. True warm pixels persist through many exposures.

We avoid “hot” pixels containing more than 76230 electrons, which is $\sim 90\%$ of the full well depth. Saturated pixels bleed charge into nearby pixels on the same column, interfering with the observed shape of the CTE trail.

Note that warm pixels also inject a low level of charge into a column of pixels during readout. This is a relatively small complication: the ratio of charge in this line to that in the $\delta$-function is equal to the ratio of the clock speed to the total exposure time. The data can be corrected along all but the worst-affected columns by subtracting superbias frames (produced from a zero-second exposure but normal readout). We subtract bias frames before anything else, since we are trying to obtain data as it was immediately before readout. However, this is a simplification because (some of) the spurious charge was also present during readout, when the CTE trails were created. A more complete CCD model might also measure the “temperature” of each warm pixel, then incorporate a continuous injection of charge during readout. This would be most important for measurements of the density of charge trap species with long release times, whose extended trail is partly degenerate with a constant line of charge injection (as opposed to charge re-emitted from short timescale traps for which the original source can be readily identified). Even more worryingly, there is (unexplained) temporal structure in the injection of charge to some warm pixels that is not necessarily reproduced in the superbias frames. We therefore decided to incorporate the few percent of columns containing the warmest pixels in the CALACS bad pixel mask. This is more severe than excluding hot pixels, since the entire rest of the column is masked.

2.3 Model assumptions and parameters

To emulate the process of CCD readout, we first need to model the collection of electrons within a pixel’s potential well. In theory, the expected locations of a given number of electrons can be calculated from a model of the CCD, by solving Poisson’s equation within the appropriate applied potential. \cite{Hardy} and \cite{Seabroke} find that, for typical CCDs, electrons are contained within a well-defined volume – outside which their density falls rapidly to zero. Adopting this result, we
shall regard a given number of electrons as filling a three dimensional pixel up to a specific height.

We then need to model the charge traps. We assume that none are filled during integration. However, following e.g. [Grant et al. 2004] (but not [Bristow 2003a]), we simplify the problem of charge capture as soon as readout begins by assuming that unoccupied traps instantaneously capture any electrons in their vicinity (i.e. all traps inside the well-defined volume get filled). In reality, there is an exponential decay time needed for traps to capture an electron, but this is typically much shorter than the electron dwell time within one CCD clock cycle [Hardy, Murowinski & Deen 1998]. The rapid and efficient capture of electrons, even in locations where the electron density is low, further delineates the quantifiable volume of a pixel occupied by a given number of electrons. It also means that any charge traps physically located near the bottom of a pixel (below the ambient background level) reach the steady state of being permanently occupied, even during readout. In our model such traps never affect an image, so we can never probe them; but then, we never need to.

Each filled trap releases its electron after a time determined by \( \tau \), the e-folding time of an exponential decay (Shockley & Read 1952; Hall 1952). If the trap is inside the volume currently occupied by electrons, it will immediately capture a new one; if it is above, the electron will be pulled down and added back to the cloud (which thus expands and may even be exposed to new charge traps). We concentrate on species of traps with capture times \( \tau \) of a few CCD clock cycles (3212 \( \mu \)s in the parallel direction; Sirianni et al. 2004). Traps with long release times affect the leading edge of the first bright object in a CCD column, but then reach a full steady state (at least while the rest of the object passes by) and only minimally affect its photometry, astrometry and morphology [Rhodes et al. 2004]. For a fixed clock speed, \( \tau \) can be conveniently expressed in units of the number of pixels that have passed since capture.

If the charge traps are evenly distributed, we need measure only the mean density of each species. In 24 we indeed find the distribution of traps to be uniform in the \( y \) direction; we have no reason to suggest that the distribution is not similarly uniform in the other directions. In addition, our approach will internally parametrize out any smooth density gradients in the vertical direction.

We assume that each charge trap always captures a single electron. Real traps might be able to hold several, or occasionally capture none. If they hold several, and especially if the trap densities are low, simultaneous release of many electrons could produce correlated spikes in the trails. Trapping mechanisms are not yet sufficiently understood to know whether electrons are released singly or in groups (C. Bebek, priv. commun.). We therefore simply assume that any multiple (or partial) occupancy of traps is equivalently modelled by an the effective density of single-electron traps.

Finally, we ignore CCDs’ three-phase clocking cycle, by which a cloud of electrons is transferred from one pixel to the next [Janesick et al. 2001]. The transition period exposes the electron cloud to additional regions of silicon, and additional charge traps. However, with our assumption of instantaneous charge capture, extra charge traps in regions between pixels can again be modeled to first order as an increase in the effective trap density. These traps are a fraction of a pixel nearer to (or further from) the readout register, but horizontal translations of the exponential release curve are also degenerate with a change in normalisation. The only remaining finesse is that the width (and therefore the height) of the electron cloud changes during the three-phase clocking cycle, making the cloud dwell temporarily at different heights within the silicon and therefore being exposed to a different number of charge traps. We ignore this second-order effect, but incorporating a model of the full, three-phase clock cycle might be a useful exercise for future work.

Our model features the following free parameters:

- number of (relevant) species of charge trap,
- characteristic release time of each species of charge trap,
- density of each species of charge trap

(or a functional form for their distribution if they are not uniformly located throughout the CCD), plus the

- height occupied by a cloud of electrons in a pixel’s potential well (i.e. the number of charge traps they are exposed to), as a function of the number of electrons.

As we shall now demonstrate, warm pixels enable us to measure all of these quantities from on-orbit science exposures.
2.4 The characteristic release times of different species of charge traps

To obtain maximum signal to noise, we begin by studying the brightest warm pixels, furthest from the readout register and in data obtained near the chronological end of the COSMOS survey. We select 20 images taken on 15 May 2005, 1171 days after the launch of ACS, and measure the mean trail $T$ around all warm pixels between 1634 and 2039 pixels from the readout register, and containing between 3234 and 76230 electrons. The mean trail is shown in figure 2.

Using Levenberg-Marquardt least-squares minimization, we find that the mean trail is well-fit by a sum of two exponentials,

$$T_i = A_1 e^{-i/\tau_1} + A_2 e^{-i/\tau_2},$$

where $A_1 = 327\pm6 e^{-}$, $\tau_1 = 10.4\pm3.2$ pixels, $A_2 = 108\pm3 e^{-}$ and $\tau_2 = 0.88\pm0.2$ pixels. At the $-83^\circ$C operating temperature of ACS/WFC, these correspond precisely with the release times of trap species found by Hopkinson (2001) in a Marconi CCD47-20 n-channel detector at 0.34 eV and 0.31 eV below the conduction band. The origin of these particular traps is unknown, but they have been speculated to be E-centres (Phosphorous-Vacancy complexes) with impurities of either carbon (Hopkinson 1991) or hydrogen (Tokuda & Itô 2000). Pure E-centre charge traps at 0.44 eV exist in both the Marconi and ACS/WFC CCDs, with a characteristic release time of $\tau_0 \sim 300$ pixels in the Marconi. All other charge traps in the Marconi CCD have $\tau < 0.01$ pixels (Hopkinson 2001), so this 1-to-1 correspondence is unlikely to be coincidence.

However, allowing a third exponential prevents our fit from converging, with positive $A_3$ but $\tau_3$ iterating towards infinity. Fitting slowly decaying exponentials is a standard problem (e.g. Cover 2008), because they are nearly degenerate with a constant. A line of charge above a warm pixel could represent either a species of trap with a very long release time or contamination from continued charge injection could represent either a species of trap with a very long release time or contamination from continued charge injection.

2.5 Height of pixel occupied by electrons and the density of charge traps

The total number of electrons that have been displaced from a packet originally containing $n_e$ electrons is the integral under the curve

$$\sum_{1}^{\infty} T_i = \frac{A_1}{e^{1/\tau_1} - 1} + \frac{A_2}{e^{1/\tau_2} - 1} \equiv n_q,$$

where $A_1 = 327\pm6 e^{-}$, $\tau_1 = 10.4\pm3.2$ pixels, $A_2 = 108\pm3 e^{-}$ and $\tau_2 = 0.88\pm0.2$ pixels. At the $-83^\circ$C operating temperature of ACS/WFC, these correspond precisely with the release times of trap species found by Hopkinson (2001) in a Marconi CCD47-20 n-channel detector at 0.34 eV and 0.31 eV below the conduction band. The origin of these particular traps is unknown, but they have been speculated to be E-centres (Phosphorous-Vacancy complexes) with impurities of either carbon (Hopkinson 1991) or hydrogen (Tokuda & Itô 2000). Pure E-centre charge traps at 0.44 eV exist in both the Marconi and ACS/WFC CCDs, with a characteristic release time of $\tau_0 \sim 300$ pixels in the Marconi. All other charge traps in the Marconi CCD have $\tau < 0.01$ pixels (Hopkinson 2001), so this 1-to-1 correspondence is unlikely to be coincidence.

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Since we are assuming that any traps surrounded by free electrons are instantly filled, this number of trapped electrons equals the (effective) number of charge traps in the volume $V$ traversed by electrons between their original pixel and the readout register.

The volume $V$ is the product of the distance $y$ to the readout register, the width of a pixel, and the height within a pixel filled by $n_e$ electrons, above a background level $b$. Warm pixels are available with a range of $y$ and $n_e$ and in images with a range of backgrounds $b$ (this varies around $51\pm9$ e$^-$ due to the angle between the telescope and the sun: Leauthaud et al. 2007). By counting the number of electrons trailed behind a variety of warm pixels, we directly measure $n_q(y, n_e, b)$. Note that the interpretation of electrons filling pixels to a given height is instructive but not necessary: this function is precisely what we require.

Figure 3 shows the mean CTI trail behind warm pixels at varying distances from the readout register and of varying warmth. The top-right panel reproduces figure 2. The other panels show trails behind different pixels within the same set of 20 images. Although a busy plot, it has been simplified for illustration. For the real analysis, the number of bins was doubled in both $y$ and $n_e$ directions, and an additional dimension, with five bins of varying background level, has been marginalised over here. Towards the left hand side, the trails may be systematically underestimated due to the inclusion in our catalogue of read noise peaks. However, the efforts in 2.2 to exclude them have kept this small, as demonstrated by the still-recovered shape of the trails. Each trail is fitted with double exponential decays (2), the integral under which gives the total number of trapped electrons, and hence the number of exposed traps $n_q$.

Figure 4 shows the total number of charge traps exposed to each set of warm pixels for which a panel was allocated in figure 3. Solid lines join the data points measured at constant distances from the readout register (rows in figure 3). These data are well modelled by

$$n_q = \rho_q V = \rho_q \left[ \left( \frac{\max\{n_e - d, 0\}}{w} \right)^\alpha - \left( \frac{\max\{b - d, 0\}}{w} \right)^\alpha \right] y^\beta,$$

where $\alpha$, $\beta$, $d$ and $\rho_q$ are to be measured. Parameter $w$ is the full well depth, which we assume to be fixed at 84700 e$^-$ even though it varies by a few thousand electrons across the device (Gilliland 2004), and $d$ is the depth of the supplementary buried channel or “notch” in the CCDs, which we fit. The notch is a small region at the bottom of a potential well used to gather electrons when only a small number are present, much like a narrow channel cut into the bed of a water drainage canal. In our model, the notch has zero volume, so it merely adjusts the starting point of the power-law increase in height governed by $\alpha$.

We again use Levenberg-Marquardt least-squares minimization to fit the free parameters. We find that $\beta = 1.01 \pm 0.01$, consistent with a uniformly increasing number of traps with increasing distance from the readout register. Since we would expect this behavior, we explicitly fix $\beta$.

$^2$ These values are chosen so that more bins can be added later, equally spaced in distance from the readout register and log($n_e$).

$^3$ There is no reason why we would expect this to be an integer.

$^4$ This assumes that the height of an electron cloud changes by only a small amount during readout; for further discussion, see Rhodes et al. (2009).
Figure 3. CTI trails behind a range of warm pixels, containing different amounts of flux (increasing left to right) and at greater distances from the readout register (increasing bottom to top). Figure 2 is reproduced exactly in the top right panel. The smooth dotted lines show fits of double exponential function 2 assuming a fixed ratio of the densities of different species of charge trap, and hence the same shaped curve in each case. Open circles depict negative values. These measurements can be used to probe the distribution of charge traps on the CCD, and the rate at which a pixel's potential well fills up as a function of the incident flux, thereby exposing electrons to additional traps. The mean distances from the readout register for each row, and the flux levels for each column, are provided in figure 4.
number of traps increases linearly with that distance, implying
pixels from the readout register (bottom to top). The cumula
tive
that the traps are uniformly distributed throughout the CCD.

\[ \beta \equiv 1 \] and then obtain best-fit values of \( \alpha = 0.576 \pm 0.013, \]
df = 96.5 \pm 2.0 \text{ e}^- \) and \( \rho_q = 0.544 \pm 0.008 \text{ pixel}^{-1} \), split be-
tween the two species of charge trap in a ratio of 3.0:1. Note
that these errors do not include the uncertainty propagated
from the measurement of the charge trap release times and
density ratio.

\[ \sum_{n} \text{number of traps exposed between a pair and the readout register} \]

\[ \text{Number of electrons } n_e \text{ in a pixel} \]

\[ \text{Total number of traps } n_q \text{ exposed between a warm pixel and the readout register} \]

\[ \text{Number of electrons } n_e \text{ in a pixel} \]

\[ \text{Total number of charge traps for which } \] 

\[ \text{Number of electrons } n_e \text{ in a pixel} \]

\[ \text{Figure 4. The total number of charge traps } n_q \text{ exposed between a warm pixel and the readout register, as a function of the number of electrons } n_e \text{ in that pixel. If traps are uniformly distributed, this is proportional to the height within a pixel filled by } n_e \text{ electrons. Error bars are underestimated, because they do not include contributions propagated from uncertainty in } \tau. \] The solid lines
join data from warm pixels 212, 618, 1024, 1430 and 1836 \pm 203
pixels from the readout register (bottom to top). The cumulative
number of traps increases linearly with that distance, implying
that the traps are uniformly distributed throughout the CCD.
The dashed curves show a model in which the electrons occupy
zero volume within the CCD notch (the depth of which is a free
parameter), and increase as a power law of the number of elec-
trons above it.

\[ \text{Figure 5. The accumulation of charge traps in the ACS/WFC} \]

\[ \text{CCDs between 9 September 2003 and the end of 2005. The data}
\]

\[ \text{points and left hand axis show measurements of the total den-
}

\[ \text{sity of charge traps, split between the two species in a ratio of}

\[ \text{approximately 3.0:1. Horizontal error bars show the total t}

\[ \text{ime during which it was necessary to average data to obtain sufficient}

\[ \text{signal to noise. The histogram and right hand axis show the nu}

\[ \text{mber of exposures taken as part of the COSMOS survey, which we}

\[ \text{analysed to obtain these figures. The dotted vertical lines show}

\[ \text{the times when the ACS CCDs were annealed (these are also}

\[ \text{listed in table A1).} \]

\[ \text{3.1 Forward operation} \]

\[ \text{We now describe a software imitation of the hardware CCD}
\]

\[ \text{readout, using the model developed in \([2]\). Just like the}

\[ \text{real version on HST, our software uses a fairly simple al-
}

\[ \text{gorithm to shuffle charge to an adjacent pixel, and this is}

\[ \text{repeated many times to complete a full readout. For con-
}

\[ \text{venience, our code has a user interface written in IDL, but}

\[ \text{for speed, the core is written in Java. Even so, the software}

\[ \text{algorithm is much slower than an on-board hardware read-
}

\[ \text{out, taking 25 minutes per 4096 \times 4096 ACS/WFC image on}

\[ \text{a single 2 GHz processor, compared to 2.5 minutes on HST}
\]

\[ \text{Pavlovsky et al. 2006). However, it is trivially parallelisable} \]
for multiple exposures, and need ever be run only once on any given exposure.

Since the mean density of charge traps $\rho_q$ is known, but their exact locations are not, traps are scattered at random 3D locations within the model silicon wafer. Importantly, each trap is allocated a particular vertical height in some detector pixel. The occupancy of each will be continuously monitored. Electrons are then added into the pixel array, in the numbers we think they were at the end of an exposure.

A cloud of $n_e$ electrons is assumed to fill a pixel to a fractional height $\left(\max\{n_e - d_i, 0\}/w\right)^n$. Any charge traps inside the occupied volume immediately capture one electron. All free electrons are then transferred instantaneously to their adjacent pixel (or to the serial register for the last pixel in parallel array, or to the preamplifier from the last pixel of the serial register). As discussed in [2] we do not attempt to model the three-phase clock cycle of a real CCD. Finally, electrons inside traps are allowed to decay with probability $1 - e^{-1/\tau}$. Released electrons are returned to the free electron pool, and the process is repeated.

Shot noise in this model created ragged trails behind faint objects and amplified background noise fluctuations. We introduce two numerical schemes to ameliorate this. Firstly, to counter the random release or non-release of electrons from charge traps, our model traps are allowed to contain fractional numbers of electrons. In this scheme, each electron is gradually released in a floating-point exponential trail, until the trap is refilled to a whole electron by the passing of a new bright pixel (removing only a fractional number of electrons from the free pool). After readout, the number of electrons in each pixel is rounded back to an integer value for storage. Secondly, to counter the necessarily random locations of model traps, we can run the code multiple times with different random seeds and average the results. To save CPU time, this is equivalently implemented by increasing the density of traps by a factor of 3 (or any other number specified as an optional input parameter), each of which is only allowed to contain a third of an electron. This most improves the sampling of traps in the thin slice of the CCD above the image background level that profoundly affect faint sources. With both schemes, the change in background noise level during readout becomes less than 1%.

### 3.2 Reverse operation

Moving electrons back to where they belong, and thus removing the CTI trails, requires the image mapping to be inverted. This is not analytically possible. However, the trail representing a small perturbation around the true image, so it can be achieved using the forward algorithm, via an iterative approach. Following Bristow & Alexov (2002), Bristow (2003a) and Bristow, Kerber & Rosal (2003), Table 1 describes a way to obtain an image that, after a software readout, reproduces the data actually downloaded from HST.

| Iteration | Equation | Result |
|-----------|----------|--------|
| True image | $I$ | Not available |
| Downloaded from HST | $I + \delta$ | (A) |
| After one extra readout | $I + 2\delta + \delta^2$ | (B) |
| (A)+(A)−(B) | $I - \delta^2$ | (C) |
| After another readout | $I + \delta - \delta^2 - \delta^3$ | (D) |
| (A)+(C)−(D) | $I + \delta^3$ | (E) |
| After another readout | $I + \delta + \delta^3 + \delta^4$ | (F) |
| (A)+(E)−(F) | $I - \delta^4$ | etc |

Table 1. Iterative method to remove CTI trailing, using only a forward algorithm that adds trailing. The true image $I$ is desired but not available. Only a version with a (small amount of) trailing, $I + \delta$, can be obtained from the telescope. However, by running that image through a software version of the readout process, and subtracting the difference, deviations from the true image can be reduced to $O(\delta^2)$. Successive iterations further reduce the trails until an image is produced that, when “read out”, reproduces data arbitrarily close to those obtained from the telescope. This is the corrected image.

### 3.3 Additional features of the software

Our CCD readout software has several additional features that we did not find necessary for CTI correction in ACS/WFC, but which may be useful for other instruments. We document these features here for completeness and future reference.

Although we used only two species of charge trap, the code is capable of handling many (currently, up to four) different species, each with a density $\rho_q$ and characteristic release time $\tau$. With additional measurements of flux loss due to long-lifetime charge traps, this could be used to more accurately correct photometric measurements of astronomical sources.

The code can shuffle electrons both in the parallel direction and along a serial register. In this mode, the execution time is doubled. Charge traps in the serial register, with characteristic release times similar to the serial clock speed, can create trails at $90^\circ$ to those described by [1]. However, it is well established (e.g. [Riess 2003], [Mutchler & Sirianni 2003]) that the serial CTE in the ACS/WFC CCDs is very high, and that trailing is almost exclusively in the parallel direction. Indeed, we attempted to measure serial CTI trails but found a signal consistent with zero. We therefore neglected this entire mechanism.

By default, the code places model charge traps randomly within the model CCD. These are not at the same locations as the charge traps in the real CCD, which introduces additional noise during the correction. This is ameliorated by increasing the density of traps but decreasing their capacity as described in [3]. However, radiation-hardened detectors in future cameras (e.g. [Dawson et al. 2008]) may contain sufficiently few traps for this noise to become significant. If their individual locations could be measured on orbit, for example via pocket pumping, the code can place model traps at the correct locations.
4 TESTING THE CORRECTION

4.1 Warm pixel trails

Figure 7 shows the residual CTI trails measured from corrected versions of the images used for figure 6. Note the reduction in the level of all trails by factor of about 30. This reduction is matched or bettered during all four of the times (the first, sixth, seventh and thirteenth data points in figure 5) when many COSMOS exposures happened to be obtained during a single day, and a more precise measurement could be made. This demonstrates that we have achieved an approximately \((1 - 1/30) = 97\%\) precision in our trapping and readout model, at all locations on the CCD array and for traps at all heights within the CCD.

If the images are instead corrected using two iterations of the method in table 1 the data points in figure 7 move only within their error bars, justifying our use of only one iteration for the sake of speed.

4.2 Galaxy photometry measurement

We now stack the corrected images to create 577 “sci” science frames, following the pipeline of Koekemoer et al. (2007), which makes use of the Drizzle/Multidrizzle software (Fruchter & Hook 2002; Koekemoer et al. 2002). There are four exposures per pointing, dithered almost along a straight line (two exposures separated by 6.1″ at an angle of 91.2° from the coordinate frame and another pair, separated by 3.1″ from the first, at an angle of 85.4°). These are transformed into a projection of the pixel array onto the sky, with oversampled 0.03″ pixels that are aligned with the instrument coordinate frame to keep the two readout registers at the top and bottom. The dithers complicate the issue of how far each galaxy is from the readout register, because it was in different areas of the focal plane in different exposures. To roughly divide the catalogue into objects at final positions \((x, y)\) that have traversed different numbers of charge traps, we calculate

\[
n_{\text{transfers}} = \max \{0, 2048 - |6046 - 1.67y - 0.0614x|\}
\]

and separately consider objects with values of \(n_{\text{transfers}}\) in the ranges 0–475, 475–950, 950–1425, 1435–1900 and 1900–2048. The final bin also includes most of the area of the oversampled images in which only data was available from only three of the four exposures, due to the gap between the CCDs.

Figure 6. Top: The HST ACS/WFC image from figure 3 after CTI correction, in units of electrons. Bottom: Difference image.
Figure 7. Measurement of the CTI trails from the same images used in figure 5 after correction. The y axes are logarithmic; open circles and dotted (rather than dashed) lines show negative values.
Figure 8. Changes in galaxy photometry during CTI correction, as a function of the measured magnitude and detection S/N in the corrected images. A fractional change is calculated as \((\text{old} - \text{new})/\text{new}\). The five lines connect subsets of galaxies between approximately 0–475, 475–950, 950–1425, 1435–1900 and 1900–2048 transfers from the readout register; the photometry is measured on stacked exposures and the exact number of transfers varies slightly between dithers. The points show the median value in the top panel, since the distribution of points is highly skewed, and the mean value in the bottom panel, which is less so. Error bars show 68% confidence limits on the mean.

Figure 9. Change in galaxy astrometry during CTI correction, in the direction of the readout registers. Readout registers are located at opposite edges of the array, with pixels from each half being read out at their nearest register. Solid lines connect subsamples of galaxies between magnitude limits of \(F814W = 17–19\), \(19–21\), \(21–23\), \(23–25\) and \(25–27\). For the sake of clarity in this figure, the points show medians in each bin. The error bars show 68% confidence limits, and the mean values behave consistently around these bounds, but noise renders the overall behaviour less clear. Note that faint galaxies are also typically smaller than bright galaxies, unlike the point sources used in previous studies.

4.3 Galaxy astrometry measurement

The change in galaxies’ positions during CTI correction is shown in figure 9. The spurious shift is \(2.42 \pm 0.17 \times 10^{-6}\) per (0.05") pixel transfer for galaxies between magnitudes 17 and 19, and \(0.57 \pm 0.24\)" per transfer for 25th to 27th magnitude galaxies.

The distance depends as expected upon distance from the readout register, but is initially surprising as a function of galaxy flux. Using very similar code, Bristow (2004) found relatively large volume of silicon and are exposed to a greater number of charge traps.

Photometric measurements can also be expressed in terms of the galaxies’ detection signal to noise. The interpretation for galaxies with S/N < 20 is complicated by two issues. Firstly, a selection bias in which sources whose detection signal to noise increased greatly were not included in the pre-correction catalogue. Secondly, the correction is a form of unsharp masking, and noise in adjacent pixels becomes less correlated during correction. As discussed in Rhodes et al. (2009), this can actually lower the detection S/N of faint sources.
short release times (Rhodes faint ones. Astrometry is most affected by charge traps with
However, bright resolved sources also tend to be larger than
own code also moves a given object less if its flux is increased.
bright point sources to be less affected than faint ones. Our
mean values within bins and 68% confidence limits.
Solid lines connect the same samples as in figure 8. Points sho w
Figure 10. Change in galaxy FWHM size during CTI correction.

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own code also moves a given object less if its flux is increased.
However, bright resolved sources also tend to be larger than
faint ones. Astrometry is most affected by charge traps with

Figure 11. Component of mean galaxy ellipticities in the
direction of the readout direction (e cos 2θ, where e is the ellipticity
and θ is the angle between the major axis and the line joining op-posite sides of the CCD) before and after CTI correction. Values
are shown after correction for convolution with the PSF, whose
shape is imprinted on the galaxies’, but this correction increases
the scatter. Solid lines connect subsamples of galaxies within mag-nitude limits of F814W = 22–23, 23–24, 24–25, 25–26 and 26–27.
Points show mean values in bins and 68% confidence limits.

4.4 Galaxy shape measurement

Figure 10 shows the change in galaxies’ full width at half
maximum size during CTI correction, as measured by Source Extractor (Bertin & Arnouts 1996). The CTI trails had spu-riously enlarged objects, which are shrunk slightly when the
flux in the trails is pushed back into the core. The effect in-creases as expected with the number of charge transfers, and is
most pronounced in faint galaxies, which are intrinsically
smallest, and also encounter the largest number of charge
traps per electron during readout.

Galaxy ellipticities are a particularly interesting case
because the true value of ellipticity is known, at least sta-tistically. If there is no preferred direction in the Universe,
galaxies cannot preferentially align in any direction and their
mean ellipticity ought to be zero. However, the CTI trails
coherently elongate galaxies in the readout direction.

In practice, high precision measurements of galaxy
shapes are made difficult because they are imprinted with the shape of the telescope’s PSF. Furthermore, the
ACS/WFC PSF changes during orbit, as HST expands or
contracts due to thermal shifts. Most inconveniently, the
predominant PSF patterns happen to align themselves with the
readout direction and also tend to align galaxies in that direction (Rhodes et al. 2007). Fortunately, a great
deal of software has been developed for effectively “de-
convolving” galaxy ellipticities from the PSF, in order to
measure weak gravitational lensing. We use the algorithm
by Rhodes, Refregier & Groth (2000), as implemented by
Leauthaud et al. (2007) and Massey et al. (2007a), to re-
cover the underlying galaxy ellipticities.

The top panel of figure 11 shows how galaxies appear
spuriously elongated in the direction of the readout regis-
ters before CTE correction. The faintest galaxies are worst
affected, although the elongation is also seen in the bright-
est galaxies. As shown in Rhodes et al. (2004), the effect
is also worst for galaxies intrinsically elongated perpen-
dicular to the readout direction. Neither such effect is modelled
by the parametric correction of Rhodes et al. (2007). The
mean spurious ellipticity of galaxies in the faintest two bins
increases as approximately (24.3 ± 2.9) × 10−6 per (0.05″
pixel) transfer and (7.7 ± 2.0) × 10−6 per transfer. The noise
is predominantly due to the process of PSF deconvolution.
After CTI correction, as shown in the bottom panel, this
effect has been successfully reduced to (0.2 ± 3.1) × 10−6 and
(2.2 ± 2.1) × 10−6. The mean shear is similarly reduced by an
order of magnitude from (1.5 ± 0.1)% to (0.2 ± 0.2)% in the

Figure 10. Change in galaxy FWHM size during CTI correction.
Solid lines connect the same samples as in figure 8. Points show
mean values within bins and 68% confidence limits.
faintest bin, and from \((7.4 \pm 1.1) \times 10^{-6}\) to \((0.9 \pm 1.1) \times 10^{-6}\) overall.

5 DISCUSSION & CONCLUSIONS

We have corrected Charge Transfer Inefficiency trailing in ACS/WFC images by more than an order of magnitude. As demonstrated by the improvement of HST COSMOS survey images between figures 3 and 4 and in figure 11 we have achieved a 90-99% precision in our model, at all locations on the CCD array and for traps at all heights within the CCD. Our method works at the pixel level, moving individual electrons back to locations from which they were dragged during readout, and model parameters correspond to real, physical properties of the device. This is far better than the ad-hoc, parametric correction described by [Rhodes et al. (2007)]. Since the trails are created during the final step of data acquisition, the correction should be applied as the first step in data reduction and analysis.

As suggested by Biretta & Kozhurina-Platais (2005), warm pixels in the detectors were successfully used to measure device properties including the charge trap density and release times. The ubiquitous distribution of warm pixels allows the locations of charge traps to be continuously measured across the CCD, using in-orbit science data. Exceeding earlier expectations, however, the range of flux levels in warm pixels can also be used to obtain the 3D height of the traps, and the rate at which electrons fill up the volume of a pixel’s potential well. All of these parameters are needed to correct CTI trailing, and it is remarkable that they can all be measured using a separate consequence of the radiation damage. To improve on this with future large surveys will probably require laboratory-based measurements of the CCD and charge trap characteristics before launch (and possibly after retrieval), or the integration of on-board electronics to allow in-flight tests such as pocket pumping.

We find two significant species of charge traps that affect the astrometry and morphology of astronomical objects, with characteristic release times \(\tau = 10.4 \pm 3.2\) pixels and \(\tau = 0.88 \pm 0.2\) pixels. These accumulated in a ratio of approximately 3:1 and, by the end of HST cycle 13, there was slightly more than one in every other pixel, uniformly distributed throughout the CCD. An electron beginning 2048 pixels away from the readout register therefore encounters a large number of traps during readout. The (random) locations of traps in our model are inevitably different to those in the real CCD. This adds shot noise during correction that is particularly noticeable in the structure of the image background. We have eliminated half of the shot noise by using repeated images of standard stellar fields, such as the STScI internal CTE calibration programme. Alternatively, once the uniform distribution of other charge trap species has been established, they could also be measured from EPER or FPR tests. If their density is measured after servicing mission 4, our software can easily handle additional species of charge traps, and thus improve the simultaneous correction of object photometry, astrometry and morphology.

We measure a functional form describing the rate at which electrons fill up the volume of a pixel’s potential well. Below a “notch” depth of \(96.5 \pm 2.0\) e\(^{-}\), they occupy negligible volume; additional electrons overflow and the cloud expands with a power law of index \(\alpha = 0.576 \pm 0.013\). This volume determines the number of traps encountered by the cloud, as it is moved through the CCD substrate towards the readout register. Note that, in order to cope with the huge data volume, we developed the model stepwise: fixing the first measured parameters before fitting the later ones. Since errors on some measurements were not carried through, these later errors are likely to be underestimated.

Our pixel filling model neatly illustrates why CTI trailing is a non-linear function of object flux, and why it can be reduced by pre- or post-flashing an image to increase the background level. During readout, the large cloud of electrons within a bright source has to navigate more charge traps that the small cloud within a faint source – but the ratio of exposed traps to electrons falls as \(n_d/n_e \approx n_e^{\alpha-1}\). Since \(\alpha < 1\), the fractional amount of charge that gets trailed behind a faint source is greater than that behind a bright source. In addition, a fat zero reduces the total amount of trailing behind all sources by moving all pixels to the right in figure 4. For faint sources of fixed flux, the total number of exposed traps is proportional to \(dn_d/dn_e\), which decreases as \(\sim (b-d)^{\alpha-1}\).

The two biggest simplifications in our model are reducing the three-phase clocking cycle to a single shift, and not explicitly treating the continuous injection of charge from warm pixels during readout. The latter would be particularly important if this method were required for measurements of charge traps with long release times. Improvements in these areas would be interesting directions for future studies.

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REFERENCES

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Table A1. Dates and times when the ACS detectors were annealed during HST cycles 12 and 13.

| Date       | Time   |
|------------|--------|
| 9 Sep 2003 | 08:34:58 |
| 12 Oct 2003| 07:09:36 |
| 6 Nov 2003 | 15:11:06 |
| 1 Dec 2003 | 15:48:13 |
| 4 Jan 2004 | 09:01:30 |
| 30 Jan 2004| 01:08:26 |
| 4 Mar 2004 | 01:21:04 |
| 27 Mar 2004| 00:59:38 |
| 25 Apr 2004| 21:31:00 |
| 22 May 2004| 07:34:57 |
| 18 Jun 2004| 04:36:37 |
| 6 Jul 2004 | 16:28:51 |
| 14 Jul 2004| 16:52:05 |
| 10 Aug 2004| 04:41:59 |
| 8 Sep 2004 | 06:01:41 |
| 8 Oct 2004 | 09:05:45 |
| 5 Nov 2004 | 19:42:53 |
| 2 Dec 2004 | 14:11:50 |
| 30 Dec 2004| 12:40:00 |
| 29 Jan 2005| 11:00:00 |
| 12 Feb 2005| 09:04:52 |
| 4 Mar 2005 | 22:55:30 |
| 24 Mar 2005| 09:12:24 |
| 19 Apr 2005| 07:33:57 |
| 20 May 2005| 05:00:00 |
| 12 Jun 2005| 22:35:22 |
| 16 Jul 2005| 07:58:20 |
| 11 Aug 2005| 20:37:49 |
| 9 Sep 2005 | 08:45:56 |
| 8 Oct 2005 | 10:06:47 |
| 3 Nov 2005 | 15:20:00 |
| 25 Nov 2005| 08:37:56 |
| 31 Dec 2005| 04:30:00 |

APPENDIX A: CCD ANNEAL DATES