A $w$ phantom transition at $z_t < 0.1$ as a resolution of the Hubble tension

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A rapid transition of the dark energy equation of state parameter $w$ at a transition redshift $z_t < 0.1$ from $w \approx -1$ at $z > z_t$ to $w < -1$ at $z < z_t$ can lead to a higher value of the Hubble constant while closely mimicking a Planck18/ΛCDM form of the comoving distance $r(z) = \int_0^z \frac{dt'}{H(t')}$ for $z > z_t$. Such a Late $w$ Phantom Transition ($LwPT$) avoids the discontinuity of $H(z)$ suggested in previous studies and thus does not require a step in the Pantheon Hubble diagram which is strongly constrained. We demonstrate that such an ultra low $z$ abrupt feature of $w(z)$ provides a better fit to cosmological data compared to smooth late time deformations of $H(z)$ that also address the Hubble tension. The strongly present day phantom dark energy behavior implied by this class of models hints towards a rapid approach of a Big Rip singularity which for $z_t = 0.02$ will rip the universe in less than 3.5 billion years. Early hints of such effect may be observable in the dynamics of the nearest and largest bound systems (e.g. Virgo structures). The $LwPT$ can be generically induced by a phantom scalar field frozen by Hubble friction mimicking the cosmological constant and currently entering its ghost instability phase as Hubble friction decreases below the field dynamical scale.

I. INTRODUCTION

The cosmological comoving distance to redshift $z$ defined in a flat universe as $r(z) = \int_0^z \frac{dt'}{H(t')}$ where $H(z)$ is the Hubble expansion rate has been constrained to a level of about 2% using standard candles (SnIa calibrated with Cepheid stars [1–3] and Red Giant stars [4, 5] or megamasers in accretion disks [6]), a standard ruler (the sound horizon at last scattering calibrated using the CMB anisotropy spectrum [7, 8] and/or Big Bang Nucleosynthesis (BBN) [9]), strong gravitational lensing [10, 11] and gravitational waves [12, 13]. The comoving distance determined using calibrated standard candles at $z < 0.23$ is offset with the comoving distance determined using the sound horizon standard ruler at $z > 0.23$, by about 9% which corresponds to a tension of about 5σ. In the context of a Planck18/ΛCDM form of $H(z)$ this mismatch of $r(z)$ becomes realized as a mismatch of the values of the Hubble constant determined by the two methods [14, 15].

Despite of intense efforts [16–19] it has not been possible to reliably identify systematic errors of the calibrators used in the context of the two methods. For example parallax data from Gaia have recently confirmed [20, 21] the calibration of Cepheid stars while attempts to recalibrate the sound horizon assuming e.g. neutrino self interactions [22], early dark energy [23–29] or modified gravity [30] or modified gravity have either failed to significantly reduce the tension level [30, 31] or produced new tensions with other cosmological data [32, 33] (including growth rate from weak lensing [34–37] and peculiar velocities [38–43]). It therefore becomes increasingly probable that the mismatch in the high-z -low-z form of $r(z)$ is indeed a physical effect that will require deformation of $H(z)$ from its Planck18/ΛCDM form.

Attempts to consider smooth deformations of $H(z)$ [44–50] at $z \approx O(1)$ have been successful in matching $r(\bar{z}_{rec})$ with $r(z = 0)$ but have been unable to match the value of $r(z \approx O(1))$ which is strongly constrained by BAO and SnIa data to be close to the form indicated by Planck18/ΛCDM.

A remaining possibility is that of an abrupt deformation of $H(z)$ at $z_t \lesssim 0.1$ ($H(z)$ transition). Such a deformation has been considered in previous studies [51–53] as a discontinuity of $H(z)$ occurring at $z_t < 0.1$. It was shown however, that if such a feature occurs at $z_t < 0.01$ i.e. below the redshift where Hubble flow starts, it would be undetectable by standard candles [52] and thus it would not be able to justify the measured decreased value of $r(z)$ at low $z$. On the other hand, if it occurred at $0.01 < z_t < 0.1$ with the proper amplitude to reduce $r(z)$ to the required level, it would have to produce a step-like feature in the SnIa Hubble diagram with amplitude $\Delta m = 0.2$. A discontinuity with such an amplitude is inconsistent with the Pantheon data. It is therefore clear that even though the existence of a feature in the form of $H(z)$ at $z_t < 0.1$ is likely, this feature would need to be smoother than a discontinuity. Such a feature could occur at redshifts as low as $z_t \approx 0.02$ without significantly affecting the locally determined value of $H_0$ [53]. In the present analysis we propose such a feature in the form of a discontinuity of the equation dark energy of state parameter $w(z) \equiv \frac{p_{de}(z)}{\rho_{de}(z)}$ rather than a discontinuity of $H(z)$ [54, 55].

In particular, we consider a transition of $w(z)$ as

$$w(z) = -1 + \Delta w \Theta(z_t - z) \quad (1.1)$$

where $\Theta$ is the Heaviside step function. The equation of state parameter determines the gravitational properties and the evolution of dark energy density $\rho_{de}$. From energy momentum conservation $d(\rho_{de}a^3) = -p_{de}d(a^3)$ it is
easy to show that the evolution of dark energy density is obtained as

$$
\rho_{de}(z) = \rho_{de}(z_p) \int_{z_p}^{z} \frac{dz'}{1+z'} (1+w(z')) = \rho_{de}(z_p) \left( \frac{1+z}{1+z_p} \right)^{3(1+w)}
$$

where in the last equality a constant $w$ was assumed and $z_p$ is a pivot redshift which may be assumed equal to the present time or equal to the transition time $z_t$. Eqs. (1.1) and (1.2) imply a continuous Hubble expansion rate $h(z) \equiv H(z)/100 \, \text{km}/(\text{sec} \, \text{Mpc})$ of the form

$$
\begin{align*}
\Omega_m(z)^2 &\equiv \omega_m(1+z)^3 + \omega_r(1+z)^4 + (h^2 - \omega_m - \omega_r) \left( \frac{1+z}{1+z_t} \right)^3 \Delta w, & z < z_t \\
\Omega_m(z)^2 &\equiv \omega_m(1+z)^3 + \omega_r(1+z)^4 + (h^2 - \omega_m - \omega_r), & z > z_t
\end{align*}
$$

where $\omega_m \equiv \Omega_{0m} h^2$, $\omega_r \equiv \Omega_{0r} h^2$ are the matter and radiation density parameters assumed fixed to their Planck18/ΛCDM values in the next section and $h$ is a parameter distinct from the rescaled measurable Hubble parameter $h(z = 0)$\(^1\). In what follows we assume $0.01 < z_t < 1$ and define $h_{local} \equiv 0.74$ and $h_{CMB} \equiv 0.674$ which correspond to the Hubble constant values obtained with local standard candle measurements of $r(z)$ and sound horizon standard ruler measurements (in the context of Planck18/ΛCDM) respectively.

In the context of the above Late w Phantom Transition ($LwPT$) model the following interesting questions emerge:

- What is the functional form of $\Delta w(z_t)$ so that $h_w(z = 0) = h_{local}$ as implied by local measurements while maintaining the required Planck18/ΛCDM form of $r(z)$ for $z \gg z_t$?
- How closely does the $LwPT$ model reproduce the form of the Planck18/ΛCDM comoving distance $r(z)$ for $z > z_t$? How does this form of $r(z)$ compare with the corresponding form of the $H(z)$ transition?
- How does the quality of fit of the $LwPT$ model to cosmological data (CMB, SNeIa, BAO and SH0ES) compare with the corresponding quality of fit of typical models that utilize smooth deformations of $H(z)$ to address the $H_0$ tension?
- What are the favored values of $\Delta w$ and what are the implications for the future evolution of the universe?

\(^1\) The parameter $h$ would be equal to the measured rescaled Hubble parameter $h_w(z = 0)$ in the limit $z_t \to 0$.

In the present analysis we address the above questions. The structure of this paper is the following: In the next section we investigate analytically the ability of the $LwPT$ model (1.3) to reproduce the Planck18/ΛCDM form of the comoving distance for $z > z_t$ while keeping $h_w(z = 0) = 0.74$. We also identify the values $\Delta w(z_t)$ that achieve this goal using an analytical approach. In section III we use cosmological data (CMB, SNeIa, BAO and SH0ES) to identify the best fit $w_0, w_n, \Omega_{0m}$ parameter values for various transition redshifts $z_t$ and identify the improvement of the quality of fit as $z_t$ decreases down to the minimum acceptable value $z_t \simeq 0.02$. We also compare this quality of fit to the data with the Planck18/ΛCDM model (without the SH0ES datapoint) and with a typical smooth $H(z)$ deformation model ($wCDM$) that is designed to address the Hubble tension. Finally in section IV we summarize the main results of our analysis and discuss the implications of these results for the future evolution of the universe if this model is indeed realized in nature. We also discuss possible future extensions of this analysis.

**II. THE COSMOLOGICAL COMOVING DISTANCE IN THE $LwPT$ MODEL**

In order to fix the parameters $\omega_m, \omega_r, h$ and $\Delta w$ in the $LwPT$ ansatz (1.3) we impose the following conditions:

- It should reproduce the comoving distance corresponding to Planck18/ΛCDM $r_\Lambda$ for $z > z_t$ where

$$
\begin{align*}
\Delta w(z) &\equiv \int_0^{z} \frac{dz'}{\omega_m(1+z')^3 + \omega_r(1+z')^4 + (h^2 - \omega_m - \omega_r)} \left( \frac{1+z}{1+z_t} \right)^3 \\
&\quad \text{for } z > z_t
\end{align*}
$$

where $\omega_m \equiv \Omega_{0m} h^2 = 0.143$, $\omega_r \equiv \Omega_{0r} h^2 = 4.64 \times 10^{-5}$ and $h = h_{CMB} = 0.674$. 

FIG. 2. Satisfy by construction two necessary conditions where $\delta h$ are assumed fixed to their Planck18/ΛCDM best fit values. Since we consider $z_t < 0.1 \ll 1$ it is straightforward to obtain an upper bound for the relative difference

$$\frac{\Delta r}{r}(z) \equiv \frac{r_w(z) - r_L(z)}{r_L(z)} < \frac{h_{\text{local}} - h_{\text{CMB}}}{h_{\text{CMB}}} \simeq 0.1 \quad (2.3)$$

where $h_{\text{local}} = h_{\text{CMB}}$ and $\omega_m$, $\omega_r$ are assumed fixed to their Planck18/ΛCDM best fit values. The fixed $w (w\text{CDM})$ smooth $H(z)$ deformation model is defined as

$$h_{wL}(z)^2 \equiv \omega_m(1 + z)^3 + \omega_r(1 + z)^4 + (h^2 - \omega_m - \omega_r)(1 + z)^{(1+w)} \quad (2.6)$$

These conditions along with the fact that we fix the parameters $\omega_m$ and $\omega_r$ to their best fit ΛCDM values secure the fact that all three models produce the same CMB anisotropy spectrum as Planck18/ΛCDM while at the same time they predict a Hubble parameter equal to its locally measured value $h(z = 0) = h_{\text{local}}$. However, the three models do not approach the Planck18/ΛCDM comoving distance $r_L(z)$ with the same efficiency as $z$ increases. As is clearly seen in Fig. 2, the $LwPT$ model with both $z_t = 0.1$ and $z_t = 0.05$ approaches $r_L(z)$
FIG. 2. The function \( f(z) = \frac{z}{r(z)} \) where \( r(z) \) is the comoving distance to redshift \( z \) for the cosmological models Planck18/ΛCDM (black continuous line), \( wCDM \) with \( w = -1.2 \) (magenta dotted line), \( H(z) \) transition (2.5) with \( z_t = 0.05 \) and \( \Delta w = -2.78 \) as indicated by eq. (2.4) (green dashed line) and \( LwPT \) with \( z_t = 0.1 \) and \( w(z < z_t) = -1 - \Delta w = -1.91 \) as indicated by eq. (2.4) (blue continuous line). Notice that even though all three models approach \( r_{\Lambda}(z) \) asymptotically, the two \( LwPT \) models remain closest to the Planck18/ΛCDM comoving distance \( r_{\Lambda}(z) \) while at the same time they are consistent with the local measurement of the Hubble constant since \( h_w(z = 0) = 0.74 \).

faster than the other two models. Since Planck18/ΛCDM provides an excellent fit to most geometric cosmological probes at \( z > 0.1 \) it is anticipated that \( LwPT \) will produce a better fit to cosmological data than the smooth deformations of \( H(z) \) like \( wCDM \) or the discontinuous \( H(z) \) transition model which produces an unnatural step in \( r(z) \) and moves away from \( r_{\Lambda}(z) \) for \( z < z_t \) as \( z \) increases. This improved quality of fit is also demonstrated in the next section.

III. FITTING \( LwPT \) TO COSMOLOGICAL DATA AND COMPARISON WITH \( wCDM \)

In this section we use a wide range of cosmological data to estimate the quality of fit and the best fit parameter values of three representative cosmological models:

- The \( LwPT \) class of models defined by a Hubble expansion rate similar to that of eq. (1.3). Here we remove the constraint \( w > -1 \) for \( z > z_t \) as well as the constraint \( \omega_m = 0.143 \). Thus the model is now allowed to have three free parameters for each fixed value of \( z_t \): \( w_>, w_< = w_> + \Delta w \) and \( \omega_m \). However, as discussed below, the additional free parameters end up constrained by the data very close to the values considered fixed in the previous section. The constraint \( h(z = 0) = h_{local} \) is imposed as a prior in the analysis.

- The \( wCDM \) model defined in (2.6) with two free parameters: \( w \) and \( \omega_m \). The constraint \( h(z = 0) = h_{local} \) is imposed as a prior in the analysis.

- The ΛCDM model defined by (2.6) with \( w = -1 \). No constraint for \( h(z = 0) \) is imposed on this model in order to maximize the quality of fit to the data and use the model as a benchmark for comparison with the other models that address the \( H_0 \) tension. Thus we use the term uACDM (“unconstrained”) to denote it. It is considered as a baseline to compute residuals of \( \chi^2 \) to compare the other two representative models. Its best fit parameter values \( (\Omega_m = 0.312 \pm 0.006, H_0 = 67.579 \pm 0.397) \) in the context of the dataset we use are almost identical with Planck18/ΛCDM.

We use the following data to identify the quality of fit of these models:

- Planck18/ΛCDM
- \( LwPT \) (\( z_t = 0.1 \))
- \( LwPT \) (\( z_t = 0.05 \))
- \( \delta H_0 \)
- \( wCDM \)
• The Pantheon SNIa dataset [56] consisting of 1048 distance modulus datapoints in the redshift range \(z \in [0.01, 2.3]\).

• A compilation of 9 BAO datapoints in the redshift range \(z \in [0.1, 2.34]\). The compilation is shown in the Appendix.

• The latest Planck18/ΛCDM CMB distance prior data (shift parameter \(R [57]\) and the acoustic scale \(l_a [58]\)). These are highly constraining datapoints based on the observation of the sound horizon standard ruler at the last scattering surface \(z \approx 1100\). The covariance matrix of these datapoints and their values are shown in the Appendix.

• A compilation of 41 Cosmic Chronometer (CC) datapoints in the redshift range \(z \in [0.1, 2.36]\). These datapoints are shown in the Appendix and have much less constraining power than the other data we use.

Using these data (total of 1100 datapoints) we used the maximum likelihood method [59] to minimize the total \(\chi^2\) defined as

\[
\chi^2 = \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CC}} + \chi^2_{\text{Panth}}
\]

(3.1)

and calculate the residual \(\Delta \chi^2\) with respect to the uCDM model for the \(LwPT\) class (as a function of \(z_t\)) and for \(wCDM\). Since the CMB data are the most constraining, we have found the anticipated best fits \(\omega_m \approx 0.143\) and \(w = -1.22\) for \(wCDM\) (see Ref. [44] for a detailed analysis of these results).

![FIG. 3. The residuals \(\Delta \chi^2\) plotted against the values of the transition redshift \(z_t\) for the \(LwPT\) (blue dots) and \(wCDM\) (black dotted line) with \(w = -1.22\). The \(LwPT\) model seems to achieve a significantly better fit for small \(z_t\) values.](image)

In Fig. 3 we show the residuals \(\Delta \chi^2\) for the best fit \(LwPT\) models as a function of \(z_t\) (blue points) and the corresponding residual \(\Delta \chi^2\) for the best fit \(wCDM\) model (horizontal black line). The rapid improvement of the fit compared to \(wCDM\) for the \(LwPT\) models as \(z_t\) decreases below \(z_t \approx 0.15\) is clear. The best fit parameter values for \(w_\leq (z < z_t)\) and \(w_\geq (z > z_t)\) are shown in Table I. In parenthesis next to each \(w_\leq\) best fit we show the predicted value in the context of the analysis of the previous section (eq. (2.4)) which assumes \(w_\geq = -1\).

| \(z_t\) | \(\Delta \chi^2\) | \(\Omega_m\) | \(w_\leq (z < z_t)\) | \(w_\geq (z > z_t)\) |
|-------|-----------------|----------|-----------------|-----------------|
| 0.005 | -1.9            | 0.2609   | -18.44 (−18.4)  | -1.095          |
| 0.01  | 0.8             | 0.2608   | -9.93 (−9.7)    | -1.001          |
| 0.02  | 9.7             | 0.2607   | -5.28 (−5.3)    | -1.011          |
| 0.04  | 23.1            | 0.2606   | -2.93 (−3.2)    | -1.037          |
| 0.05  | 27.6            | 0.2607   | -2.48 (−2.8)    | -1.049          |
| 0.06  | 31.3            | 0.2607   | -2.19 (−2.5)    | -1.059          |
| 0.08  | 37.9            | 0.2608   | -1.81 (−2.1)    | -1.085          |
| 0.1   | 43.3            | 0.2611   | -1.58 (−1.9)    | -1.115          |
| 0.2   | 50.1            | 0.2622   | -1.22 (−1.4)    | -1.230          |

The forms of the comoving Hubble parameter \(H(z)/(1 + z)\) for two \(LwPT\) models with \(z_t < 0.1\), the best fit \(wCDM\) and \(uCDM\) are shown in Fig. 4. This figure demonstrates the efficiency of \(LwPT\) in mimicking the best fit \(uCDM\) model (which is almost identical with Planck18/ΛCDM) while at the same time addressing the Hubble tension by reaching \(h(z = 0) = h_{\text{local}}\) in a continuous manner. On the other hand the smoother approach of \(wCDM\) is much less efficient in mimicking Planck18/ΛCDM and the price it pays for this inability is a much worse quality of fit compared to \(LwPT\) as shown in Fig. 3.
The difficulty of the smooth $H(z)$ deformation models that address the Hubble tension in fitting the BAO and SnIa data is also demonstrated in Fig. 5 where we show the BAO and SnIa data (residuals from the best fit $\Lambda$CDM) along with the best fit residuals for the $w$CDM and $LwPT$ models.

The right panel of Fig. 5 indicates that the $LwPT$ model with $z_t = 0.02$ which can resolve the Hubble tension, closely mimics the apparent magnitudes of $u$CDM for $z > z_t$ but for $z < z_t$ it predicts a small reduction of the residual apparent magnitudes. The question therefore to address is the following: Is there a hint for such a statistically significant reduction of the measured absolute magnitudes in redshifts close to the transition redshift $z_t \simeq 0.02$? Interestingly, this is indeed the case!

The left panel of Fig. 6 shows the form of $\Delta m_N(z)$ in redshifts $z \leq z_t$. Since the points are standardized and ignoring their correlations, we expect that the $1\sigma$ region will approximately correspond to $\sigma \simeq 1/\sqrt{N} \simeq 0.08$ which is also indicated in Fig. 6 up to the $3\sigma$ level. Interesting features of the binned Pantheon data have been identified in previous studies[17, 18]. Related to such features is a clear abrupt drop of the moving average of the standardized residuals from the $+2\sigma$ region to the $-3\sigma$ region and beyond clearly seen in the left panel of Fig. 6. The deepest part of this drop is at a redshift of about 0.02. This is precisely the type of signature anticipated in the context of the $LwPT$ model. Once we consider the residuals with respect not to the best fit $\Lambda$CDM but to the best fit $LwPT$ model with $z_t = 0.02$, this peculiar feature disappears (Fig. 6 right panel). In addition, the standard deviation of the points of the moving average of residuals decreases by about 20% (from 0.1 to 0.8) while their mean value shown in Fig. 6 drops sharply from 0.03 to 0.001. This is also a hint that the best fit $LwPT$ with $z_t = 0.02$ is a more natural pivot model than the best fit $\Lambda$CDM.

**IV. CONCLUSION-DISCUSSION-OUTLOOK**

We have demonstrated using both an analytical approach and a fit to cosmological data that a Late dark energy equation of state $w$ (Phantom Transition ($LwPT$) from $w_+ = -1 \ (z > z_t)$ to $w_- < -1 \ (z < z_t)$ at transition redshift $z_t \in \{0.01, 0.1\}$ can lead to a resolution of the
Hubble tension in a more efficient manner than smooth deformations of the Hubble tension and other types of late time transitions (the Hubble expansion rate transition). The required type of transition is a phantom transition with \( w(z_t) < 0 \) for \( z < z_t \). The moving average statistic of the standardized residual Pantheon absolute magnitude SNIa data indicated the presence of a peculiar feature at \( z < 0.1 \) which is consistent with the anticipated signatures of the \( \Lambda \)CDM model.

There is a wide range of physical models that can reproduce the \( \Lambda \)CDM. Such models include the following:

- The most natural model that can induce a \( \Lambda \)CDM involves a phantom scalar field initially frozen at \( \phi = \phi_0 \) due to cosmic friction close to the zero point of its potential which could be assumed to be of the form \( V(\phi) = s \phi^n \). Such a field would initially have a dark energy equation of state \( w = -1 \) mimicking a cosmological constant. Once Hubble friction becomes smaller than the field mass scale, the field becomes free to roll up its potential (phantom fields move up their potential in contrast to quintessence fields \([60, 61]\)) and develops a rapidly changing equation of state parameter \( w < -1 \). Thus the universe enters a ghost instability phase which will end in a Big Rip singularity in less than a Hubble time. Such a scenario for the simple (but also generic) case of linear potential \((n = 1)\) has been investigated in Ref. \([60]\). For a general phantom potential we anticipate a redshift dependence of the equation of state \( w(z) = w_\infty(z) \) after the transition \((z < z_t)\). In fact the phantom field potential could be reconstructed by demanding a form of \( w_\infty(z) \) that further optimizes the quality of fit to the low \( z \) data or by simply demanding that \( w_\infty \) is constant.

- A scalar-tensor modified gravity theory field initially frozen due to Hubble friction, mimicking general relativity and a cosmological constant. Once Hubble friction becomes smaller than the field mass scale, the field becomes free to roll down its potential inducing deviations from general relativity on cosmological scales and a phantom departure from the cosmological constant. Note that scalar tensor theories can induce phantom behavior without instabilities in contrast to a simple minimally coupled scalar field \([62]\).

The detailed investigation of the above described dynamical scalar field evolution that can reproduce the \( \Lambda \)CDM is an interesting extension of the present analysis.

If the phantom \( \Lambda \)CDM is realized in Nature it would imply the existence of a rapidly approaching Big Rip singularity \([64, 65]\) which may be avoided due to quantum effects \([66]\). Given the value of \( w_\infty \) which emerges at approximately the present time \( t_0 \), it is straightforward to calculate the time \( t_* \) of the Big Rip singularity assuming that \( w = w_\infty < -1 \) at the present time \( t_0 \). The result is \([65]\)

\[
\frac{t_*}{t_0} = \frac{w_\infty}{1 + w_\infty} \tag{4.1}
\]

For example for \( z_t = 0.02 \) we have \( w_\infty \simeq -5 \) which implies that the universe will end in a Big Rip singularity in less than 3.5 billion years (for \( t_0 = 13.8 \times 10^9 \) yrs). This implies that there may be observational effects of such coming singularity on the largest bound systems like the Virgo cluster, the Coma Cluster or the Virgo supercluster. A detailed investigation of the observational effects
on bound systems of the \textit{LwPT} is an interesting extension of the present analysis.

Another interesting extension of the present analysis is the detailed comparison of the quality of fit of the \textit{LwPT} models with a variety of other smooth $H(z)$ deformation models addressing the Hubble tension using also full CMB spectrum data and possibly other cosmological data sensitive to the dynamics of galaxies in clusters and superclusters.

**Numerical Analysis Files:** The numerical files for the reproduction of the figures can be found in [67].

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**Appendix A: Data Used in the Analysis**

The covariance matrix which corresponds to the latest Planck18/ΛCDM CMB distance prior data (shift parameter $R$ and the acoustic scale $l_a$), for a flat universe has the following form [58]

\[
C_{ij} = 10^{-8} \begin{pmatrix}
1598.9554 & 17112.007 \\
17112.007 & 811208.45
\end{pmatrix}
\]

where the corresponding Planck18/ΛCDM values for $R$ and $l_a$ are presented in Table II. Furthermore, we present the full dataset of the BAO and CC likelihoods used in the Mathematica analysis in Tables III and IV respectively.

**Table II:** The CMB Distance Prior data for a flat Universe used in our analysis.

| Index | CMB Observable | CMB Value   | Reference |
|-------|----------------|-------------|-----------|
| 1     | $R$            | 1.74963     | [58]      |
| 2     | $l_a$          | 301.80845   | [58]      |

**Table III:** The BAO data that have been used in the analysis along with the corresponding references.

| Index | $z$  | $D_A/r_s$ (Mpc) | $D_H/r_s$ (km/sec Mpc) | $D_V/r_s$ (Mpc) | Ref. |
|-------|------|----------------|------------------------|----------------|------|
| 1     | 0.106| -              | -                      | 2.98 ± 0.13    | [68] |
| 2     | 0.44 | -              | -                      | 13.69 ± 5.82   | [69] |
| 3     | 0.6  | -              | -                      | 13.77 ± 3.11   | [69] |
| 4     | 0.73 | -              | -                      | 16.89 ± 5.28   | [69] |
| 5     | 2.34 | 11.28 ± 0.65   | -                      | -              | [70] |
| 6     | 2.34 | -              | 9.18 ± 0.28            | -              | [70] |
| 7     | 0.15 | -              | -                      | 4.465 ± 0.168  | [71] |
| 8     | 0.32 | -              | -                      | 8.62 ± 0.15    | [72] |
| 9     | 0.57 | -              | -                      | 13.7 ± 0.12    | [72] |

**Table IV:** The Cosmic Chronometer data that have been used in the analysis.

| Index | $z$   | $H(z)$ (km/sec Mpc) | Ref. |
|-------|-------|---------------------|------|
| 1     | 0.09  | 69 ± 12             | [73] |
| 2     | 0.17  | 83 ± 8              | [74] |
| 3     | 0.179 | 75 ± 4              | [75] |
| 4     | 0.199 | 75 ± 5              | [75] |
| 5     | 0.27  | 77 ± 14             | [74] |
| 6     | 0.352 | 83 ± 14             | [75] |
| 7     | 0.3802| 83 ± 13.5           | [76] |
| 8     | 0.4   | 95 ± 17             | [74] |
| 9     | 0.4004| 77 ± 10.2           | [76] |
|   | Value       | Error       | Reference                        |
|---|-------------|-------------|----------------------------------|
| 10| 0.4247      | 87.1 ± 11.2 | [76]                             |
| 11| 0.4497      | 92.8 ± 12.9 | [76]                             |
| 12| 0.4783      | 80.9 ± 9   | [76]                             |
| 13| 0.48        | 97 ± 62    | [77]                             |
| 14| 0.593       | 104 ± 13   | [75]                             |
| 15| 0.68        | 92 ± 8     | [75]                             |
| 16| 0.781       | 105 ± 12   | [75]                             |
| 17| 0.875       | 125 ± 17   | [78]                             |
| 18| 0.88        | 90 ± 40    | [77]                             |
| 19| 0.9         | 117 ± 23   | [74]                             |
| 20| 1.037       | 154 ± 20   | [75]                             |
| 21| 1.3         | 168 ± 17   | [74]                             |
| 22| 1.363       | 160 ± 33.6 | [79]                             |
| 23| 1.43        | 177 ± 18   | [74]                             |
| 24| 1.53        | 140 ± 14   | [74]                             |
| 25| 1.75        | 202 ± 40   | [74]                             |
| 26| 1.965       | 186.5 ± 50.4 | [79]                       |
| 27| 0.35        | 82.7 ± 8.4 | [80]                             |
| 28| 0.44        | 82.6 ± 7.8 | [81]                             |
| 29| 0.57        | 96.8 ± 3.4 | [72]                             |
| 30| 0.6         | 87.9 ± 6.1 | [81]                             |
| 31| 0.73        | 97.3 ± 7   | [81]                             |
| 32| 2.34        | 222 ± 7    | [82]                             |
| 33| 0.67        | 69 ± 19.6  | [78]                             |
| 34| 0.12        | 68.6 ± 26.2 | [78]                       |
| 35| 0.2         | 72.9 ± 29.6 | [78]                       |
| 36| 0.24        | 79.69 ± 2.65 | [83]                    |
| 37| 0.28        | 88.8 ± 36.6 | [78]                     |
| 38| 0.43        | 86.45 ± 3.68 | [83]                       |
| 39| 0.57        | 92.4 ± 4.5 | [84]                             |
| 40| 2.3         | 224 ± 8    | [85]                             |
| 41| 2.36        | 226 ± 8    | [86]                             |

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