GMPE-consistent hard-rock site adjustment factors for Western North America

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Abstract
Empirical ground-motion prediction equations (GMPEs) such as the Next Generation Attenuation-West2 (NGA-West2) GMPEs are limited in the number of recordings on hard-rock stations used to develop the models. Therefore, the site response scaling in the GMPEs cannot be reliably extrapolated to hard-rock conditions. The state of practice for the development of hard-rock adjustment factors involves the use of analytical methods that typically assign small values to the small-strain damping parameter ($\kappa_0$) for hard-rock sites resulting in large scaling factors at short periods. Alternatively, the hard-rock scaling factors developed in Ktenidou and Abrahamson (KA16) based on empirical ground-motion data are used. These empirical factors, developed for a broad rock site category, show that the average hard-rock scaling factors observed in ground-motion data are small in amplitude contrary to the large factors typically obtained from analytical studies. The empirically derived KA16 factors also suffer from limitations due to the relatively small number of rock sites in the data set and do not distinguish between different hard-rock conditions. To address the shortcomings in the current state of practice, we present a methodology to develop linear site adjustment factors to adjust the NGA-West2 GMPEs from $V_{S30}$ of 760 m/s to target hard-rock site conditions with $V_{S30}$ ranging from 1000 to 2200 m/s. These factors are analytically derived using the inverse random vibration theory (IRVT) approach of Al Atik et al. but with inputs constrained using the empirical KA16 factors and normalized to the scaling of the NGA-West2 GMPEs for $V_{S30}$ of 1000 m/s. The proposed factors merge the results of the NGA-West2 site response scaling for $V_{S30} \leq 1000$ m/s with the KA16 hard-rock category factors to produce a site factor model that is a continuous function of $V_{S30}$. The epistemic uncertainty of these factors is evaluated.

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Introduction
Modern ground-motion prediction equations (GMPEs) such as the Next Generation Attenuation-West2 (NGA-West2) GMPEs characterize site response as a continuous function of the time-averaged shear-wave velocity over the top 30 m of the site profile ($V_{S30}$). Other parameters, such as the depth to a shear-wave velocity of 1.0 or 2.5 km/s horizon ($Z_{1.0}$ and $Z_{2.5}$), are used to characterize the long-period site amplification due to deep basin effects. Nonlinear site response is modeled in the GMPEs as a function of $V_{S30}$ and the median spectral acceleration or peak ground acceleration on rock. The histogram of the number of recording stations in the different $V_{S30}$ bins used in the development of the Abrahamson et al. (2014) (ASK14) GMPE is shown in Figure 1. This figure shows that the number of recording stations with $V_{S30} > 1000$ m/s is limited to 33 stations out of a total of 3642 stations in the data set. Similarly, only 15 of the stations in the ASK14 data set have $V_{S30} > 1200$ m/s. The $V_{S30}$ dependence of the site factors is modeled in the GMPEs as a linear function of ln($V_{S30}$). With the sampling of $V_{S30}$ in the NGA-West2 data set shown in Figure 1, the coefficient for ln($V_{S30}$) is constrained by empirical ground-motion data for $V_{S30}$ values between 200 and 800 m/s. For hard-rock sites with $V_{S30} > 1000$ m/s, the site factor is based on an extrapolation of this slope to high $V_{S30}$ values with little empirical constraints. To reflect the limited hard-rock data, some GMPEs limit the application of the site factor at high $V_{S30}$ values (e.g. 1500 m/s).

The development of hard-rock site adjustment factors has been the subject of numerous studies over the past 30 years. A literature review generally reveals two broad methods, analytical and empirical, used for the development of hard-rock factors. On the analytical side, some of these methods involve the hybrid empirical approach of Campbell (2003), the broadband method of Silva et al. (1996), and the $V_S$-$\kappa_0$ adjustment method of Al Atik et al. (2014). These analytical methods require characterization of the host and the target hard-rock site conditions in terms of $V_S$ profiles and $\kappa_0$ values, which is a difficult task involving a large degree of uncertainty. Hard-rock factors derived using analytical methods tend to be large (factors $> 2$) at high frequencies as a result of the small $\kappa_0$ values typically assigned to hard-rock sites. More recently, empirically derived hard-rock factors have been presented in the literature (Kotha et al., 2017; Ktenidou and Abrahamson, 2016; Laurendeau et al., 2018). These factors are based on analyses of the ground motion recorded on hard-rock sites and provide average adjustment factors that are not site-specific. Empirically derived average hard-rock factors are generally smaller in amplitude at high frequencies compared to those obtained from analytical studies with small $\kappa_0$ values typically assigned for hard-rock conditions. While the use of empirical hard-rock factors avoids the issues involved in the characterization of target site conditions, these factors suffer from limitations due to the small subsets of ground motion recorded on hard-rock sites and, as a result, are mainly applicable to a broad rock site category. Bard et al. (2020) discusses the different available approaches for hard-rock ground-motion prediction. A detailed review of the empirical hard-rock factors of Ktenidou and Abrahamson (2016) is presented here due to their relevance to this study.
Empirical hard-rock adjustment factors were developed in Ktenidou and Abrahamson (2016) (KA16) to adjust GMPEs from $V_{S30}$ of 760 m/s to average hard-rock conditions based on the Next Generation Attenuation-East Project (NGA-East) and the BCHydro British Columbia ground-motion data sets. These data sets are described in Ktenidou and Abrahamson (2016). Hard-rock adjustment factors were developed using the average of total ground-motion residuals with $V_{S30}$ greater than 1000 m/s relative to median ground-motion predictions for $V_{S30}$ of 760 m/s. For the NGA-East data set, the residuals were computed relative to an interim NGA-East GMPE using a $V_{S30}$ of 760 m/s. For the BCHydro data set, the residuals were computed relative to the Chiou and Youngs (2014) (CY14) GMPE with $V_{S30}$ of 760 m/s. The KA16 scaling factors account for differences in the $V_{S}$ profiles and the small-strain damping between the reference site condition in the GMPEs with $V_{S30}$ of 760 m/s and an average hard-rock site in the NGA-East and the BCHydro data sets. The $\kappa_0$ parameter is often assumed to represent the effect of the small-strain damping. The two hard-rock scaling models proposed by KA16 are shown in Figure 2: model 2 is based on the residual analysis of the BCHydro data set, whereas model 1 is based on the weighted average of the scaling obtained from the BCHydro and the NGA-East data sets. KA16 indicate that the average $V_{S30}$ values for the BCHydro and the NGA-East data used to develop the models are 2380 and 1975 m/s, respectively. The KA16 models shown in Figure 2 were developed as an interim measure to provide an alternative to the typically large scale factors computed using analytical methods for hard-rock sites. These models indicate that observed ground-motion data on hard-rock sites, on average, do not show the large scale factors at short periods typically found in analytical studies that assign small $\kappa_0$ values, on the order of 0.006 s, for hard-rock sites based on Hashash et al. (2014).

Figure 1. Histogram of the number of stations in different $V_{S30}$ bins in the ASK14 data set.
The KA16 hard-rock adjustment factors suffer from shortcomings stemming from the poor characterization of site conditions at the recordings stations in the NGA-East and BCHydro data sets and the low sample rates at some seismic stations that limit the usable frequency band. Most of the hard-rock stations in the NGA-East data set do not have measured $V_{S30}$ values, and the NGA-East ground motions suffer from frequency-bandwidth limitations affecting data quality at low and high frequencies. The limited velocity information was one of the motivations for the recent Electric Power Research Institute (EPRI, 2021) study. Similarly, all rock stations in the BCHydro data set are classified based on surface geology (Ktenidou and Abrahamson, 2016). Errors in the $V_{S30}$ estimates for the hard-rock sites could lead to misclassification of the sites and affect the resulting scale factors and average $V_{S30}$ values. Another limitation of the KA16 factors is that they are not a continuous function of $V_{S30}$ and instead apply to a broad hard-rock site class with $V_{S30}$ = 1000 m/s. This is a result of the limited recordings on hard-rock sites in the empirical data sets.

In summary, the current state of practice for the development of site adjustment factors for hard-rock site conditions involves three different approaches: (1) use of $V_S$-$k_0$ correction factors developed using analytical methods with assigned target $k_0$ values and $V_S$ profiles; (2) use of empirical hard-rock factors such as the KA16 factors which are developed for a broad category of hard-rock sites; and (3) use of the $V_{S30}$ scaling in the NGA-West2 GMPEs extrapolated to hard-rock $V_{S30}$ despite the limited empirical data on hard-rock site conditions available to constrain the $V_{S30}$ scaling and the lack of explicit $k_0$ scaling in the GMPEs. To address the shortcomings in the current state of practice discussed in this section, we present a methodology to develop hard-rock site factors to adjust the NGA-West2 GMPEs from $V_{S30}$ of 760 m/s to target hard-rock site conditions with $V_{S30}$ ranging from 1000 to 2200 m/s. These factors are analytically derived using the inverse random vibration theory (IRVT) approach of Al Atik et al. (2014) but constrained using the empirical KA16 factors and also normalized to the NGA-West2 site factors for $V_{S30}$ of 1000 m/s. These

![Figure 2. KA16 hard-rock scaling factors relative to $V_{S30}$ 760 m/s. PSA HR/PSA 760 m/s refers to the ratio of spectral acceleration on hard rock to that $V_{S30}$ of 760 m/s.](image)
empirical constraints allow for the calibration of the hard-rock properties in the analytical study so that they are consistent with the observed ground-motion scaling for these sites. The proposed factors merge the results of the NGA-West2 $V_{s30}$ scaling for $V_{s30} < 1000$ m/s with the KA16 hard-rock category factors to produce a model for the site factors that is a continuous function of $V_{s30}$ and consistent with the empirical hard-rock factors.

Our approach starts with an evaluation of the average hard-rock site conditions representative of the KA16 scaling factors by inverting for the average $V_{s30}$, $V_s$ profile, and $\kappa_0$ implicit in the KA16 models. Next, the target site conditions ($V_s(z)$ profile and $\kappa_0$) are defined for a range of hard-rock site conditions with $V_{s30}$ of 1000 to 2200 m/s based on a literature review of hard-rock $V_s$ profiles and $\kappa_0$ estimates in Western North America (WNA). The target site parameters are adjusted using empirical constraints, and hard-rock site adjustment factors are derived and presented for a suite of 13 target $V_{s30}$ values between 1000 and 2200 m/s. The site factors presented in this article extend the NGA-West2 linear site response scaling to hard-rock site conditions with target hard-rock sites defined based on WNA average rock site properties.

**$V_s$ profile and $\kappa_0$ inversion for the KA16 models**

The use of the KA16 models to constrain the analytical hard-rock site adjustment factors requires an evaluation of the implied average site conditions representative of these scaling factors. Because a large number of the stations used in the KA16 analysis did not have measured $V_{s30}$ and $\kappa_0$ values, we use the $\kappa_0$ and $V_s$ profile inversion methodology of Al Atik and Abrahamson (2021) to invert for representative $V_{s30}$, $V_s(z)$ profile, and $\kappa_0$ for average site conditions implied by the KA16 hard-rock scaling factors.

The inversion is performed using the CY14 GMPE because KA16 model 2 is based on the residual analysis of the BCHydro data set with respect to CY14 and because the spectral shape of the CY14 GMPE generally falls in the center of the range of spectral shapes from the NGA-West2 GMPEs. The first step involves converting the KA16 hard-rock scaling factors from pseudospectral acceleration (PSA) domain to Fourier amplitude

![Figure 3. CY14 IRVT-based Fourier amplitude spectra for $V_{s30} = 760$ m/s (solid lines) and for spectra corrected to hard-rock conditions (dashed lines) using KA16 model 1 (a) and model 2 (b).](image)
spectra (FAS) domain. As such, CY14 median response spectra for strike-slip scenarios with magnitudes 5, 6, and 7, a distance of 5, 10, and 20 km, and $V_{S30}$ of 760 m/s are computed. These scenarios are selected to capture the short-distance $\kappa_0$ scaling from a range of hazard-significant magnitudes. Nonlinear site response is not included in the calculation of the CY14 median response spectra. The CY14 median response spectra are then corrected to hard-rock conditions by multiplying them with the KA16 model 1 and model 2 factors. Next, the IRVT approach of Al Atik et al. (2014) is used to convert the GMPE’s response spectra for $V_{S30} = 760$ m/s and the spectra corrected to hard-rock conditions into corresponding FAS. Duration estimates for the different scenarios are calculated using estimates of source and path durations with generic Western US (WUS) parameters based on Campbell (2003). The peak factor of Vanmarcke (1975) is used in the IRVT method. The FAS for the scenarios with $V_{S30} = 760$ m/s and those corrected to hard-rock conditions are presented in Figure 3. These IRVT-based FAS show a change in their spectral shape at a frequency of about 50 Hz. This is likely due to saturation effects in the IRVT process discussed in Al Atik et al. (2014). For the hard-rock FAS, the sharp change observed around 50 Hz is also caused by the sharp changes in the KA16 factors in the same frequency range, particularly for KA16 model 2.

For each earthquake scenario, the ratio of FAS for the hard-rock site condition relative to FAS for $V_{S30} = 760$ m/s is computed. Figure 4 presents these ratios for each of the nine scenarios considered for model 1 and model 2 along with the average of the ratios over the nine scenarios. These ratios approximate the FAS linear site factors for hard rock relative to the reference site condition with $V_{S30}$ of 760 m/s. These relative site factors represent the differences in the $V_S$ profile and $\kappa_0$ scaling between the average hard-rock site implied by the KA16 models and the average site condition for CY14 for $V_{S30}$ of 760 m/s. The FAS ratios are stable over all nine scenarios for frequencies up to about 20–30 Hz as shown in Figure 4. For frequencies greater than 20 Hz, the FAS ratios start diverging due to potential saturation effects in the IRVT-derived FAS discussed in Al Atik et al. (2014) but are still relatively stable for frequencies between 20 and 30 Hz. The average relative site factors

Figure 4. Hard-rock site factors in FAS domain relative to $V_{S30} = 760$ m/s for a suite of scenarios (solid lines) and average relative site factors over all scenarios (dashed lines) for KA16 model 1 (a) and model 2 (b).
are smoothed as shown in Figure 5. These average relative FAS site factors are considered reliable for frequencies between 0.6 and 30 Hz. The upper limit is imposed to avoid potential saturation effects in the IRVT-based FAS, and the lower limit is based on the KA16 factors being constrained by data for frequencies beyond 0.6 Hz.

The next step involves converting the hard-rock site factors relative to 760 m/s to total site factors (relative to \(V_S\) and density at the source depth). As such, we use the CY14-compatible \(V_S\) profile and \(k_0\) of Al Atik and Abrahamson (2021) as representative of the reference site condition with \(V_{S30}\) of 760 m/s. The reference \(V_S\) profile and corresponding quarter-wavelength (QWL) linear site amplification for CY14 for \(V_{S30}\) of 760 m/s are shown in Figure 6. The QWL site amplification is computed according to Boore (2003) with a zero angle of incidence (vertical incidence) and source \(V_S\) and density set at 3.5 km/s and 2.75 g/cm\(^3\), respectively. \(k_0\) for CY14 for \(V_{S30}\) of 760 m/s is 0.039 s. The choice of the angle of incidence and its impact on the QWL site amplifications were evaluated by Boore (2013), who found a small reduction of about 15\% in site amplification when using an angle of incidence of 45\(^\circ\) compared with vertical incidence. The impact of the angle of incidence on the results presented in this study is therefore expected to be small and is further reduced by using a zero angle of incidence throughout this study consistent with its use in the development of the CY14-compatible \(V_S\) profile (Al Atik and Abrahamson, 2021) and for the target hard-rock sites presented later on in this article. The total FAS site factors for the average hard-rock site condition representative of the KA16 models 1 and 2 are obtained by multiplying the relative hard-rock site factors with the site factors (site amplification and \(k_0\) high-frequency attenuation) of the CY14 reference site condition and are shown in Figure 7.

**Inversion of KA16 model 1**

The total linear site factors represent the combined effects of the linear site amplification of the \(V_S\) profile and the attenuation due to damping, parameterized by \(k_0\). To reduce the
trade-off between the $V_S$ profile and $\kappa_0$ at high frequencies, we assume that the depth dependence of the $V_S$ profile follows a power law (e.g. $a \cdot z^b$). With this assumption, we use an analytical solution for the combined effects of the site amplification of the $V_S$ profile in the top 30 m and the $\kappa_0$ attenuation given the $V_{S30}$ value. The methodology is described in Al Atik and Abrahamson (2021).

Figure 6. (a) Host (CY14 Vs760) $V_S$ profile based on Al Atik and Abrahamson (2021) and (b) corresponding QWL linear site amplification.

Figure 7. Total FAS site factors for the average hard-rock site conditions representative of the KA16 models. Dashed red vertical lines indicate the reliable frequency range (0.6–30 Hz).
Using the total linear site factors for KA16 model 1 shown in Figure 7, the inversion is performed to estimate the average $k_0$ and $V_S$ profile representative of the average hard-rock site condition in the model. A zero angle of incidence and a source $V_S$ and density of 3.5 km/s and 2.75 g/cm$^3$, respectively, are used in the inversion. The density–$V_S$ relationship used in Al Atik and Abrahamson (2021) is used in this inversion. Because $V_{330}$ is unknown for the KA16 models, the inversion is performed to estimate $V_{330}$ as well as for different assumed $V_{330}$ values. Using the frequency range of 10–20 Hz (10 Hz roughly corresponds to the frequency associated with QWL amplification for the top 30 m of the profile and 20 Hz was chosen to avoid the unreliable higher frequencies in the IRVT-based FAS), $k_0$, $V_{330}$, and the $V_S$ profile in the top 30 m are estimated analytically by fitting the total linear site factors in the 10–20 Hz frequency range assuming that the top 30 m of the $V_S$ profile follows a power law function. The estimated $k_0$ and $V_{330}$ are 0.032 s and 1300 m/s, respectively.

The high-frequency fit is compared to the total site factors for frequencies $>10$ Hz in Figure 8a. For frequencies $<10$ Hz, the fit is constrained to be equal to the initial site factors. For frequencies $>10$ Hz, the fit deviates slightly from the initial site factors (pink curve in Figure 8a). The fit to the site factors is divided by the $k_0$ operator to obtain the linear site amplification function due only to the $V_S$ profile, which is subsequently smoothed as shown in Figure 8b. The inverse QWL approach outlined in Al Atik and Abrahamson (2021) is then applied to invert for the $V_S$ profile working from high to low frequencies of the site amplification and solving for the shallow to deep layers of the profile. The inverted $V_S$ profile, which is subsequently smoothed, is shown by the pink curve in Figure 8c. Because linear site amplifications are considered reliable for frequencies $>0.6$ Hz, the $V_S$ profile could only be inverted to a depth of 1.06 km. A comparison of the initial relative site factors of KA16 model 1 to those obtained using the inversion results is shown in Figure 8d. This plot shows that the inverted $V_S$ profile and $k_0$ representative of the hard-rock condition for KA16 model 1 used along with the reference $V_S$ profile and $k_0$ for CY14 at $V_{330}$ of 760 m/s can approximate reasonably well the initial relative site factors of KA16 model 1 for frequencies up to 30 Hz.

Next, the inversion of KA16 model 1 described in this section is repeated using different assumed $V_{330}$ values instead of inverting for $V_{330}$ as shown above. This sensitivity analysis allows for a more robust estimation of $k_0$ from the high-frequency site factors as well as an evaluation of the range of $V_{330}$ and $k_0$ values that can fit the hard-rock site factors of KA16 model 1 relative to the reference site condition with $V_{330}$ of 760 m/s. Assumed $V_{330}$ values of 1500, 1700, and 1975 m/s are used in this sensitivity analysis. The value of 1975 m/s is used because it represents the average $V_{330}$ of the NGA-East hard-rock data used in KA16. A comparison of the initial relative site factors of KA16 model 1 to those obtained using the inversion for the derived and assumed $V_{330}$ values is shown in Figure 9. The inversion results for the different assumed $V_{330}$ values indicate that, as the assumed $V_{330}$ increases, the derived $k_0$ value decreases and the slope of the inverted $V_S$ profile in the top 30 m becomes less steep approaching a single constant layer.

The sum-of-squared errors (SSEs) between the inversion-based relative site factors and the initial site factors in the frequency range of 0.6–30 Hz are calculated and listed in the plots of Figure 9. An evaluation of the SSE values for the different inversion cases as well as the corresponding shapes of the inverted $V_S$ profiles indicates that the assumed $V_{330}$ of 1975 m/s does not represent the average hard-rock site conditions of KA16 model 1. The average $V_{330}$ of 1975 m/s obtained using the NGA-East hard-rock data in KA16 is likely biased high due to a large number of stations with estimated or assigned $V_{330}$ values. As a
result, we conclude that, within the context of the QWL approach used in these inversions and the related assumptions made, a $V_{S30}$ of 1300 m/s (with a range of 1300–1500 m/s) and $\kappa_0$ of 0.032 s (with a range of 0.03–0.032 s) are representative of the average site conditions of KA16 hard-rock model 1.

**Inversion of KA16 model 2**

An inversion approach similar to that described in the previous section is applied to estimate the average hard-rock site characteristics representative of KA16 model 2. The first inversion case is performed to estimate $V_{S30}$ along with $\kappa_0$ for the total site factors of KA16 model 2 for the high-frequency range of 12–25 Hz shown in Figure 7. The frequency range of 12 to 25 Hz is chosen to capture the smaller-$\kappa_0$ scaling expected for this model while staying below the high-frequency limit of 30 Hz. The inversion for KA16 model 2 results

**Figure 8.** Inversion results for KA16 model 1: (a) Hard-rock site factors and high-frequency fit to estimate $\kappa_0$ and $V_{S30}$, (b) site amplification function obtained by dividing the fitted site factors by the $\kappa_0$ operator, (c) inverted $V_S$ profile and smoothed, and (d) comparison of the hard-rock site factors relative to $V_{S30}$ of 760 m/s obtained from the inversion (calculated) to the initial relative site factors.
in an average $V_{S30}$ estimate of 1600 m/s and $\kappa_0$ of 0.025 s. We note that, for KA16 model 2, the inverted $V_{S30}$ value is sensitive to the frequency range used to fit the site factors with the analytical function that assumes that the top 30 m of the $V_S$ profile can be approximated with a power law function. Moreover, the inversion of KA16 model 2 generally required more smoothing than that of model 1 due to the shape of the KA16 model 2 hard-rock factors with bigger jumps in the site factors in the high-frequency range and less smooth transitions.

Next, KA16 model 2 is inverted using different assumed $V_{S30}$ values of 1500, 1700, 1850, 2000, and 2380 m/s. $V_{S30}$ of 2380 m/s is reported in KA16 as the average $V_{S30}$ of the BCHydro data used to derive model 2 scaling factors. Inverted $\kappa_0$ values and calculated SSE are included in the plots.

Figure 9. Comparison of the KA16 model 1 hard-rock site factors relative to $V_{S30}$ of 760 m/s to the relative site factors obtained from the inversions for the cases of (a) derived $V_{S30}$ and assumed $V_{S30}$ values of (b) 1500, (c) 1700, and (d) 1975 m/s. Derived $\kappa_0$ values and calculated SSE are included in the plots.
2000 m/s. Based on a qualitative evaluation of the inversion results as well as the SSE values for the different cases, we conclude that the inversion results for $V_{30}$ of 1700 m/s (range of 1600–1850 m/s) and $k_0$ of 0.024 s (range of 0.022–0.025 s) best represent the average hard-rock site conditions of KA16 model 2. The best-case inversion results for KA16 model 2 in terms of $k_0$ fit, site amplification, and inverted $V_S$ profile are shown in Figure 10.

Discussion of $V_s$ profile and $V_s$ inversions of KA16 models

The inverted $V_S$ profiles and $k_0$ values presented in this section are representative of the average hard-rock site conditions of KA16 models 1 and 2 within the context of the QWL method used in the inversion and the assumptions employed to solve for the multiple unknowns in this process. These assumptions are related to the assigned half-space $V_S$ and density values, density–$V_S$ relationship, vertical angle of incidence, smooth $V_S$ profiles, and the representation of the top 30 m of the $V_S$ profile with a power law function. While these assumptions are reasonable, they do introduce a level of uncertainty in the resulting inverted $V_S$ profiles and $k_0$ values. Moreover, due to the frequency limitations of the KA16 hard-rock factors and their jagged appearance, the inverted $V_S$ profiles are limited in their depth range.

Boore (2013) compared site amplifications calculated using the QWL method to those obtained from theoretical simulations of wave propagation in layered media accounting for the constructive and destructive interference of all reverberations in the layers (full resonant (FR) method). For velocity models made up of gradients, Boore (2013) found that the QWL method systematically underestimates the theoretical FR site amplification over a wide frequency range. This underestimation can be on the order of 20%. Based on that, the QWL-based inversion can potentially underestimate the derived $V_S$ profiles compared to those expected from the FR method for the same site amplification. The use of the QWL method in the inversion is, however, consistent with the approach used to develop analytical site adjustment factors presented in the next section. Therefore, we consider the inverted profiles and $k_0$ values presented in this section as appropriate for use with the QWL method to represent the average hard-rock site conditions of the KA16 factors. We use these inverted profiles and $k_0$ values to constrain the inputs to the analytical calculations of the hard-rock factors.

### Table 1. Results of the inversion for KA16 model 2 for the different cases analyzed

| Case       | $V_{30}$ (m/s) | Inverted $k_0$ (s) | SSE (0.6–30 Hz) |
|------------|---------------|--------------------|----------------|
| Inverted   | 1602          | 0.025              | 0.325          |
| Assumed    | 1500          | 0.026              | 0.371          |
| Assumed    | 1700          | 0.024              | 0.350          |
| Assumed    | 1850          | 0.022              | 0.414          |
| Assumed    | 2000          | 0.021              | 0.477          |
| Assumed    | 2380          | 0.019              | 0.769          |

SSE: sum-of-squared error.
Development of GMPE-consistent analytical hard-rock site adjustment factors

The inversion of the KA16 empirical hard-rock factors indicates that these factors can be used to scale response spectra from a reference $V_{S30}$ of 760 m/s to the target $V_{S30}$ of about 1300 (model 1) or 1700 m/s (model 2). To develop rock site adjustment factors that are a continuous function of $V_{S30}$ between 1000 and 2200 m/s, we use the analytical IRVT method of Al Atik et al. (2014) with empirical constraints based on the KA16 scaling factors for $V_{S30}$ of 1300 and 1700 m/s and the NGA-West2 scaling factors for $V_{S30}$ of 1000 m/s. Because the spectral shape for CY14 generally lies in the center of the range of spectral shapes of the NGA-West2 GMPEs, we develop the rock scaling factors using the CY14 GMPE and assume the resulting factors are applicable to the other NGA-West2 GMPEs.

Figure 10. Inversion results for KA16 model 2: (a) Hard-rock site factors and high-frequency fit to estimate $k_0$ for an assumed $V_{S30}$ of 1700 m/s, (b) site amplification function obtained by dividing the fitted site factors by the $k_0$ operator, (c) inverted $V_S$ profile and smoothed, and (d) comparison of the hard-rock site factors relative to $V_{S30}$ of 760 m/s obtained from the inversion (calculated) to the initial relative site factors.

Development of GMPE-consistent analytical hard-rock site adjustment factors

The inversion of the KA16 empirical hard-rock factors indicates that these factors can be used to scale response spectra from a reference $V_{S30}$ of 760 m/s to the target $V_{S30}$ of about 1300 (model 1) or 1700 m/s (model 2). To develop rock site adjustment factors that are a continuous function of $V_{S30}$ between 1000 and 2200 m/s, we use the analytical IRVT method of Al Atik et al. (2014) with empirical constraints based on the KA16 scaling factors for $V_{S30}$ of 1300 and 1700 m/s and the NGA-West2 scaling factors for $V_{S30}$ of 1000 m/s. Because the spectral shape for CY14 generally lies in the center of the range of spectral shapes of the NGA-West2 GMPEs, we develop the rock scaling factors using the CY14 GMPE and assume the resulting factors are applicable to the other NGA-West2 GMPEs.
The development of analytical site adjustment factors requires the definition of host and target site conditions in terms of $V_s$ profiles and $k_0$ values. For the host site condition, the $V_s$ profile and $k_0$ value of 0.039 s inverted for CY14 for $V_{S30}$ of 760 m/s in Al Atik and Abrahamson (2021) are used. Thirteen target site conditions are defined having $V_{S30}$ ranging between 1000 and 2200 m/s. The $V_s$ profiles for the target sites are obtained using Boore (2016) based on a $V_{S30}$-based interpolation between generic WUS and Eastern US profiles with $V_{S30}$ of 618 and 2780 m/s, respectively. The host CY14 $V_s$ profile along with the target $V_s$ profiles and their corresponding QWL site amplifications are presented in Figure 11. There is a significant difference between the host $V_s$ profile for CY14 for $V_{S30}$ of 760 m/s and the target profile for $V_{S30}$ of 1000 m/s. This difference is due to CY14 having a relatively high $V_s$ scaling from 1000 to 760 m/s resulting in higher site amplification, particularly at long periods and softer $V_s$ profile for $V_{S30}$ of 760 m/s compared to the target profile $V_{S30}$ of 1000 m/s. These effects are discussed in Al Atik and Abrahamson (2021). In the section “implementation,” the derived site factors are normalized with respect to the average of the site factors from the NGA-West2 GMPEs for $V_{S30} = 1000$ m/s, so this difference in the $V_s$ profiles seen for the CY14 model is not applied to all GMPEs.

**Target $k_0$**

The high-frequency attenuation parameter $k$ was first introduced by Anderson and Hough (1984) to measure the observed decay in acceleration spectral amplitude at high frequencies. Since its introduction, the physics behind this parameter has been subject to debate. Numerous papers in the literature attempt to explain the physical phenomena that cause the observed high-frequency spectral decay parameterized with $k$ (Ktenidou et al., 2014, 2015; Purvance and Anderson, 2003; Van Houtte et al., 2011). These studies note the source, path, and site contributions to $k$. While the source contributions are generally small (Van Houtte et al., 2011), the path contribution to $k$ is distance-dependent and is the result of anelastic attenuation. The site component of $k$, referred to as zero-distance kappa

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![Figure 11](image-url)
or $k_0$, is assumed to be the result of damping and scattering effects underneath the site. The estimation of $k_0$ is an important aspect of site characterization in site response analyses and the development of site adjustment factors.

The estimation of site-specific $k_0$ is a complex process that often involves a large degree of uncertainty and trade-offs. Several approaches exist for the estimation of $k_0$ and are presented in Ktenidou et al. (2014). The origins of $k_0$ and the relationship between the observed high-frequency attenuation in FAS ($k_0$ scaling) and the low-strain damping at a site are subject of ongoing debate. The current paradigm assumes that $k_0$, estimated with the source, path, and site effects removed, is due only to damping at the site (EPRI, 1993). As a result, a low $k_0$ implies low damping that must lead to an increase in the high-frequency ground motion. Hard-rock to soft-rock site factors of 2–3 at the frequency range of 20–40 Hz are common (Biro and Renault, 2012). When the current paradigm was established in the 1990s, there were only four hard-rock recordings with low $k_0$ values and they were consistent with the large factors of 2–3 amplification for hard-rock sites relative to soft-rock sites. The current data sets for hard-rock sites are much larger with over 100 recordings, and they do not show the large site factors at high frequencies that are predicted for hard-rock sites with $k_0$ in the 0.006-s range (Ktenidou and Abrahamson, 2016; Laurendeau et al., 2018). This indicates that estimated $k_0$ values are not just the result of damping; they also reflect the errors in the assumed source, path, and site effects on the slope of the FAS used to estimate $k_0$. Negative $k_0$ values estimated for some sites are indicative of such errors. Ktenidou et al. (2021) discuss this apparent lack of spectral decay in the high frequencies of some recordings that can be explained by trade-offs between damping and site amplification at these frequencies. These errors and trade-offs often result in underestimated $k_0$ values for hard-rock sites.

To avoid the common tendency for underestimating $k_0$, we use target $k_0$ values that are consistent with the observed ground-motion scaling at high frequencies for rock site conditions. By using the amplitude of the ground motion and not just the high-frequency slope of the FAS, the $k_0$ values can be interpreted as effects of damping and used in the traditional $k_0$ scaling methodology. We note that our resulting target $k_0$ values are not site-specific; they are average values that can be expected for hard-rock sites with different $V_{S30}$ values. We also account for the uncertainty in the average $k_0$ value for a rock site condition as described below.

For this study, target $k_0$ values are estimated based on a review of Silva and Darragh (1995) with additional empirical constraints. Silva and Darragh (1995) analyzed 49 rock sites in WNA and 22 rock sites in Eastern North America (ENA). Table 5–3 of Silva and Darragh (1995) lists the median and range of $k_0$ values for average site conditions in WNA and ENA. It indicates that average $k_0$ values for WNA rock site conditions are not small and are larger than those for ENA. Silva and Darragh (1995) interpreted $k_0$ as the result of damping in the top 1–2 km below the site and proposed two Q models ($Q = g^3 V_S$) with $g = 0.007$ and 0.029 s/m for soft-rock and hard-rock sites, respectively. Their soft-rock and hard-rock sites are representative WNA and ENA generic $V_S$ profiles, respectively, and are shown in Figure 11a.

For each of the 13 target $V_S$ profiles in this study, we estimate $k_0$ by summing up the damping in the profile layers over the top 1 and 2 km of the profile as shown in Equation 1 (Campbell, 2009) and Equation 2 (Silva and Darragh, 1995). We do not explicitly include scattering effects in our estimates of $k_0$ as these effects are expected to be small for the smooth target $V_S$ profiles used in this study. Moreover, these effects are implicit in the
Silva and Darragh (1995) Q models and will be accounted for by constraining target $k_0$ to be consistent with average scaling observed in empirical data. Two profile depths are used to capture the uncertainty in the total depth of the profile contributing to damping. Two alternative Q models are used: a linear Q model with $g = 0.007 \text{ s/m}$ representative of WNA soft-rock condition and a bilinear Q model with $g$ of 0.007 s/m for the profile layers with $V_S < 2700 \text{ m/s}$ and 0.029 s/m for larger $V_S$. This results in a total of four $k_0$ estimates for each target $V_S$ profile. The alternative target $k_0$ estimates as a function of $V_{30}$ vs $k_0$ are shown in Figure 12a and are compared to empirical $k_0$ estimates inferred from ground-motion data. Empirical $k_0$ estimates shown in Figure 12 are based on $k_0$ estimates for the four NGA-West2 GMPEs in Al Atik and Abrahamson (2021) for $V_{30}$ of 760 and 1000 m/s and on $k_0$ and $V_{30}$ inverted for the KA16 models. The upper estimates of target $k_0$ values for this study shown in Figure 12 are the result of using $g = 0.007 \text{ s/m}$ and a profile depth of 2 km contributing to damping while the lower estimates are the result of the bilinear Q model with a profile depth of 1 km contributing to damping.

\begin{align*}
\kappa_0 &= \sum_i \frac{H_i}{V^2_{S,i}Q_i} \\
Q &= \gamma \times V_S
\end{align*}

The target $\kappa_0$ values shown in Figure 12a have a similar trend with $V_{30}$ as the empirical $\kappa_0$ estimates but with the average target $\kappa_0$ values falling below the average empirical $\kappa_0$ estimates. This indicates an underestimation of the target $\kappa_0$ values compared to the empirical data. Because this study uses CY14 to develop analytical hard-rock site adjustment factors, we constrain the average target $\kappa_0$ for $V_{30} = 1000 \text{ m/s}$ to match that of CY14 (0.0345 s). As a result, the target $\kappa_0$ values are scaled up by a constant factor and

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure12}
\caption{(a) Comparison of target $\kappa_0$ values as a function of $V_{30}$ to $\kappa_0$ inferred from empirical ground motion data and (b) scaled target $\kappa_0$ values such that their average matches CY14 $\kappa_0$ at $V_{30}$ of 1000 m/s. The upper estimates of target $\kappa_0$ are the result of using $\gamma = 0.007 \text{ s/m}$ and a profile depth of 2 km contributing to damping while the lower estimates are the result of the bilinear Q model with a profile depth of 1 km contributing to damping.}
\end{figure}
the adjusted target \( k_0 \) values are shown in Figure 12b. The trend of the empirical \( k_0 \) values as a function of \( V_{S30} \) is still different from that of the scaled target \( k_0 \) values for this study. Our ultimate goal is not to match the exact empirical \( k_0 \) values but to have a good match between the analytical and the empirical rock site adjustment factors. Our objective is to match the hard-rock scaling observed in empirical data reflecting the combined effects of \( k_0 \) and \( V_S \) profile scaling. The upper estimates of the scaled target \( k_0 \) are within the range of \( k_0 \) values for WNA rock from Silva and Darragh (1995) and are considered reasonable. Table 2 lists the four \( k_0 \) values for the different target \( V_S \) profiles along with their average and standard deviation.

### Table 2. Target \( k_0 \) values used in the development of the analytical rock site adjustment factors. The \( k_0 \) values were scaled such that their average matches CY14 at \( V_{S30} \) of 1000 m/s. The geometric mean of the alternative target \( k_0 \) values and their standard deviation are listed for each target \( V_{S30} \).

| \( V_{S30} \) (m/s) | Alternative \( k_0 \) (s) | Avg. \( k_0 \) (s) | \( \sigma \) (LN unit) |
|------------------|--------------------------|-----------------|------------------|
|                  | 1                        | 2               | 3               | 4               |
| 1100             | 0.0296                   | 0.0462          | 0.0235          | 0.0276          | 0.0307 | 0.289 |
| 1200             | 0.0275                   | 0.0436          | 0.0206          | 0.0245          | 0.0279 | 0.322 |
| 1300             | 0.0258                   | 0.0416          | 0.0182          | 0.0221          | 0.0256 | 0.353 |
| 1400             | 0.0245                   | 0.0399          | 0.0162          | 0.0200          | 0.0237 | 0.387 |
| 1500             | 0.0233                   | 0.0385          | 0.0144          | 0.0182          | 0.0220 | 0.420 |
| 1600             | 0.0223                   | 0.0373          | 0.0129          | 0.0165          | 0.0205 | 0.458 |
| 1700             | 0.0215                   | 0.0363          | 0.0116          | 0.0153          | 0.0193 | 0.490 |
| 1800             | 0.0208                   | 0.0354          | 0.0106          | 0.0142          | 0.0182 | 0.521 |
| 1900             | 0.0202                   | 0.0346          | 0.0095          | 0.0131          | 0.0172 | 0.560 |
| 2000             | 0.0196                   | 0.0339          | 0.0088          | 0.0124          | 0.0164 | 0.584 |
| 2100             | 0.0192                   | 0.0333          | 0.0082          | 0.0117          | 0.0157 | 0.611 |
| 2200             | 0.0187                   | 0.0327          | 0.0075          | 0.0110          | 0.0150 | 0.640 |

Hard-rock site adjustment factors

For each of the target \( V_{S30} \) values ranging from 1000 to 2200 m/s, four sets of adjustment factors are developed using the IRVT approach of Al Atik et al. (2014) corresponding to the four target \( k_0 \) values listed in Table 2. Strike-slip earthquake scenarios with magnitudes 5, 6, and 7, distances of 5, 10, and 20 km, and \( V_{S30} \) of 760 m/s are used in the IRVT approach. CY14 median response spectra are computed for the nine scenarios considered for the linear site response. These response spectra are converted into compatible FAS using the IRVT approach as described in the previous sections. Then, each FAS is scaled to adjust for the differences in the linear site amplification and \( k_0 \) scaling between the host and target \( V_S \) profiles and \( k_0 \) values. The \( V_S-k_0 \) scaled FAS are then converted into a \( V_S-k_0 \) scaled response spectra using random vibration theory. The \( V_S-k_0 \) scaling factors are calculated as the ratio of the scaled response spectra to the initial GMPE response spectra and averaged over the nine scenarios considered.

For each target \( V_S \) profile, four sets of \( V_S-k_0 \) scaling factors are computed corresponding to the four target \( k_0 \) values. Average \( V_S-k_0 \) scaling factors are derived assuming equal weights for the four target \( k_0 \) values. The \( V_S-k_0 \) scaling factors for the individual target \( k_0 \) values as well as the average scaling factors for \( V_{S30} \) of 1700 m/s are compared to the empirical hard-rock factors of KA16 in Figure 13. There is good agreement between the average analytical factors for \( V_{S30} \) of 1700 m/s and the KA16 model 2 factors which have a representative \( V_{S30} \) of about 1700 m/s.
The set of average analytical hard-rock adjustment factors for the range of $V_{S30}$ of 1000–2200 m/s are compared to the CY14 empirical site factors for $V_{S30}$ of 1000 m/s and the KA16 hard-rock factors in Figure 14. While some mismatch can be observed in Figure 14 between the analytical factors for $V_{S30}$ of 1300 m/s and the KA16 model 1 factors, there is good agreement between the analytical hard-rock factors for $V_{S30}$ of 1000 m/s and the corresponding CY14 site factors for frequencies less than 20 Hz and between the analytical factors for $V_{S30}$ of 1700 m/s and the KA16 model 2 factors for frequencies of 15–30 Hz. At frequencies less than 15 Hz, the agreement between the analytical factors for $V_{S30}$ of 1300 and 1700 m/s and the KA16 factors is worse due to the limited number of hard-rock sites in the empirical data set and the reduction of the number of available recordings at long periods affecting the derived KA16 factors as well as the unsmooth shape of these factors. For frequencies less than 0.6 Hz, the observed disagreement between the analytical and the KA16 factors is due to the KA16 factors being completely unconstrained with empirical data in this frequency range. Despite these noted limitations, we conclude that, on average, the analytical hard-rock factors are reasonable based on their comparison with empirical scaling for rock site conditions (CY14 for $V_{S30} = 1000$ m/s and KA16 factors).

**Implementation**

The hard-rock site adjustment factors derived in this study are used to extrapolate the average NGA-West2 empirical site factors to hard-rock conditions in a relative sense to ensure a smooth transition in the scaling factors to hard-rock sites. As such, the ratios of hard-rock analytical factors relative to those for $V_{S30}$ of 1000 m/s are used to model the scaling of the hard-rock site factors. These ratios are then applied to the empirical site factors for $V_{S30}$ of 1000 m/s relative to reference $V_{S30} = 760$ m/s. This normalization of the
analytical site factors allows the site factors from the analytical modeling to be centered on the GMPEs which provides a smooth scaling from soft-rock to hard-rock site conditions.

The empirical linear site factors for $V_{S30}$ of 1000 m/s are obtained by averaging the ratio of median response spectra for $V_{S30}$ of 1000 m/s relative to 760 m/s for four NGA-West2 GMPEs (Abrahamson et al., 2014; Boore et al., 2014; Campbell and Bozorgnia, 2014; Chiou and Youngs, 2014). The resulting rock-site adjustment factors are shown in Figure 15 and are included as an Electronic Appendix to this article. The average empirical linear site factors of the NGA-West2 GMPEs for $V_{S30}$ of 680–1000 m/s relative to the reference 760 m/s are also shown in Figure 15. The GMPEs nonlinear site response is not included in the calculation of the average empirical site factors. The proposed approach results in a smooth extrapolation of the average empirical site factors from the GMPEs to hard-rock conditions based on the analytical factors described in this article.

The linear $V_S$ scaling of the NGA-West2 GMPEs relative to $V_{S30}$ of 760 m/s and extrapolated to hard-rock conditions is shown in Figure 16. This figure also shows the average of the scaling from the four NGA-West2 GMPEs and the hard-rock scaling proposed in this study. Comparisons of the linear $V_S$ scaling are shown for frequencies of 0.2, 1, 5, and 25 Hz. These comparisons indicate that, for $V_{S30}$ values > 1000 m/s, linear $V_S$ scaling varies among the NGA-West2 GMPEs reflecting the different hard-rock extrapolation constraints imposed in the models. The extrapolated hard-rock scaling in the NGA-West2 GMPEs is unconstrained with empirical data for hard-rock conditions and is, therefore, unreliable for application to hard-rock sites. For 25 Hz (Figure 16a), there is a large difference between the hard-rock site factors from this study and those from the NGA-W2 GMPEs with the proposed site factors increasing with $V_{S30}$. This difference reflects the lack

![Figure 14. Analytical hard-rock site adjustment factors for target $V_{S30}$ of 1000–2200 m/s relative to $V_{S30}$ of 760 m/s compared to the CY14 site factors for $V_{S30} = 1000$ m/s and the KA16 hard-rock site adjustment factors. KA16 model 1 has an inverted best estimate $V_{S30}$ of 1300 m/s and $k_0$ of 0.032 s. KA16 model 2 has an inverted best estimate $V_{S30}$ of 1700 m/s and $k_0$ of 0.024 s.](image)
of $k_0$ scaling in the GMPEs. At short periods, the NGA-West2 linear $V_S$ scaling should not be extrapolated to hard-rock sites and the factors presented in this article should be used instead.

The average hard-rock adjustment factors from this study, presented in Figure 15 and included as an electronic appendix to this article, can be applied to correct the average median ground motion predicted by the NGA-West2 GMPEs with $V_{S30}$ of 760 m/s to a hard-rock site with $V_{S30}$ between 1000 and 2200 m/s. To be consistent with the assumption in this study, any nonlinear site response should be disabled when calculating the NGA-West2 ground-motion predictions for $V_{S30}$ of 760 m/s before applying the hard-rock adjustment factors. For target $V_{S30}$ values not explicitly listed in the electronic appendix, hard-rock factors can be obtained using linear interpolation on a log-log scale of the provided factors for the neighboring $V_{S30}$ values. For hard-rock sites with a qualitative assessment of site conditions, hard-rock adjustment factors for a range of target $V_{S30}$ values can be enveloped to estimate the median hard-rock adjustment factors.

**Epistemic uncertainty of hard-rock factors**

To evaluate the epistemic uncertainty in the hard-rock adjustment factors, we examine the site-to-site variability ($\phi_{S2S}$) in the NGA-West2 GMPEs for soil versus rock sites. Following the terminology and notation of Al Atik et al. (2010), $\phi_{S2S}$ represents the site-to-site variability within a site class. When systematic site-specific effects are removed from ground-motion residuals, $\phi_{S2S}$ represents the epistemic uncertainty of the site terms. For this study, site terms were obtained using a mixed-effects regression on the within-event residuals of the NGA-West2 GMPEs with the station term as the random effect and using earthquakes with magnitude $\geq 5$ and stations with a minimum of three recordings as

![Figure 15. Proposed linear site adjustment factors for $V_{S30} = 680–2200$ m/s relative to 760 m/s. Solid and dashed lines show empirical and analytical factors, respectively.](image-url)
described in Al Atik (2015). Ground-motion data with magnitude $< 5$ are not used in this analysis to reduce the dependence of linear site factors on earthquake magnitude. This effect was examined in Stafford et al. (2017) and was found to be most pronounced at short periods and for small magnitude scenarios. Soil sites in the NGA-West2 database are classified with $V_{S30} \leq 680$ m/s, while rock sites have $V_{S30} \geq 680$ m/s. Site terms for each NGA-West2 GMPE are divided into these two site categories and the resulting $\phi_{S2S}$ are computed as the standard deviation of the site terms in each category.

$\phi_{S2S}$ for soil and rock sites obtained using the residuals of ASK14, Boore et al. (2014) (BSSA14) and CY14 for magnitude $\geq 5$ were examined and the comparison using the CY14 residuals is shown in Figure 17. We note that $\phi_{S2S}$ for Campbell and Bozorgnia

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**Figure 16.** Linear $V_S$ scaling factors relative to $V_{S30}$ of 760 m/s for the NGA-West2 GMPEs extrapolated to hard-rock conditions compared to hard-rock scaling factors from this study for frequencies of 25 (a), 5 (b), 1 (c), and 0.2 Hz (d).
(2014) (CB14) is not included in this analysis due to the limited data set for CB14 as a result of restricting the residuals to magnitudes $\geq 5$ and stations with more than three recordings. This impacted the stability of the $\phi_{S2S}$ estimates for CB14. The large error bars in Figure 17 reflect the smaller subset of stations with $V_{S30} \geq 680$ m/s compared to the number of softer sites in the NGA-West2 data set. For example, using the CY14 residuals, the average $V_{S30}$ is about 390 m/s for soil sites and 830 m/s for rock sites. The comparison of $\phi_{S2S}$ for soil and rock sites indicates that the NGA-West2 $\phi_{S2S}$ values are generally comparable for the two site groups at high frequencies as shown in Figure 17. At periods greater than 1 s, $\phi_{S2S}$ values for rock sites are lower than those for soil sites; however, the subsets of data for rock sites are very limited in a number of stations for frequencies $< 0.25$ Hz. We conclude that, for hazard-significant scenarios with magnitudes $\geq 5$, $\phi_{S2S}$ obtained from the NGA-West2 residuals for all $V_{S30}$ can be used to estimate $\phi_{S2S}$ for hard-rock sites with modifications associated with the expected spectral shapes of site variability for hard-rock site conditions. These spectral shape modifications, described in the next paragraphs, are broadly similar to those made in Carlton and Abrahamson (2014) for the adjustment of the correlation coefficients for hard-rock sites.

The average $\phi_{S2S}$ obtained using the site terms of ASK14, BSSA14, and CY14 for magnitude $\geq 5$ and for all $V_{S30}$ values is shown in Figure 18a. The peak in $\phi_{S2S}$ at frequency 5–10 Hz is likely related to the variability of the resonance frequency of shallow layers for soil and soft-rock sites. For hard-rock sites, this peak is expected to be shifted to higher frequencies reflecting the variability in kappa scaling for hard-rock conditions. Due to the limited number of hard-rock stations in the NGA-West2 data set, we examine this effect using $\phi_{S2S}$ obtained from a residual analysis of Japanese crustal ground-motion data recorded on the ground surface and at depth in boreholes.

Figure 17. Site-to-site variability ($\phi_{S2S}$) of CY14 for soil sites with $V_{S30} < 680$ m/s and rock sites with $V_{S30} \geq 680$ m/s using data with magnitude $\geq 5$. Error bars show one standard error around the $\phi_{S2S}$ estimates.
The Japanese data set consists of ground-motion recordings from active crustal earthquakes recorded on the KiK-net stations. The data set is comprised of 13,735 six-component (three at the surface and three at the borehole) ground-motion recordings from 679 active crustal earthquakes recorded at 643 stations. A description of the Japanese data set can be found in Dawood (2014) and a discussion of the residual analysis using this data set is presented in Goulet et al. (2018).

The abundance of ground-motion recordings in the Japanese data set on the ground surface and at depth provides a unique opportunity to examine the $f_{S2S}$ trends for soil site conditions at the surface compared to rock site conditions in the borehole. Figure 18b presents a comparison of $f_{S2S}$ for the surface and borehole Japanese data with magnitude $\geq 5$. Borehole $f_{S2S}$ values obtained using stations with $V_S \geq 1000$ m/s are also shown. Figure 18b shows a shift in the peak of $f_{S2S}$ to higher frequencies for the rock borehole data compared to the surface data. As a result, we correct the average $f_{S2S}$ for NGA-West2 to follow the high-frequency scaling of the Japanese borehole $f_{S2S}$ for frequencies $> 2.5$ Hz. This frequency of 2.5 Hz is chosen based on the observed start of the high-frequency peak in $f_{S2S}$ for the NGA-West2 data set that we aim to shift to higher frequencies. For frequencies less than 2.5 Hz, the $f_{S2S}$ shape is based on the NGA-West2 data.

The resulting proposed $f_{S2S}$ model for use for hard-rock sites is shown in Figure 18a and listed in Table 3. To construct this model, we compute the $f_{S2S}$ spectral shape for the Japanese borehole data with magnitude $\geq 5$ and $V_S \geq 1000$ m/s between 2.5 and 100 Hz normalized by the value at 100 Hz. Similarly, we compute the $f_{S2S}$ spectral shape for the NGA-West2 data between 0.1 and 2.5 Hz normalized by the value of 2.5 Hz. The high-frequency shape of $f_{S2S}$ from the Japanese borehole data is multiplied by $f_{S2S}$ of the NGA-West2 data at 100 Hz to obtain the proposed $f_{S2S}$ model between 2.5 and 100 Hz. The resulting $f_{S2S}$ at 2.5 Hz is then multiplied by the low-frequency $f_{S2S}$ spectral shape from the NGA-West2 data to obtain the proposed $f_{S2S}$ model for frequencies less than 2.5 Hz.

For the two frequency ranges of 0.1–2.5 and 2.5–100 Hz, $f_{S2S}$ is denoted $f_{S2S-LF}$ and $f_{S2S-HF}$, respectively. For the high-frequency range, $f_{S2S-HF}$ is given by:

![Figure 18. (a) Average $f_{S2S}$ based on the NGA-West2 residuals and proposed $f_{S2S}$ model for hard-rock sites adjusted at high frequencies and (b) $f_{S2S}$ for the Japanese surface and borehole data.](image-url)
in which $\phi_{S2S-NGA}(f)$ is from the NGA-W2 data for all $V_{S30}$ values.

This proposed $\phi_{S2S}$ model can be used to characterize the epistemic uncertainty of the average rock-site adjustment factors presented in this study if additional site-specific information is not available to constrain the epistemic uncertainty of the site factors. For hard-rock adjustments of the NGA-West2 GMPEs using the ergodic aleatory variability model, the standard deviation models in the NGA-West2 GMPEs, calculated for $V_{S30}$ of 760 m/s without including effects of nonlinear site response, could be used for hard-rock sites. The use of the ergodic NGA-West2 sigma models is likely conservative for some frequency ranges and might not capture the expected peaks in the variability for hard-rock sites. Alternatively, an estimate of the ergodic sigma for hard-rock sites can be computed using the $\phi_{S2S}$ model proposed in this study along with NGA-West2 $\tau$ models for the between-event standard deviation and published single-station within-event standard deviation models for WUS (Al Atik, 2015). We note that the $\phi_{S2S}$ model proposed in this study for hard-rock site adjustment factors is a simplified model based on adjusting the NGA-West2 $\phi_{S2S}$. A more detailed study of the ground-motion variability and its components for hard-rock sites is warranted.

### Table 3. Proposed site-to-site variability ($\phi_{S2S}$) model

| Frequency (Hz) | Period (s) | $\phi_{S2S}$ (LN unit) |
|---------------|-----------|------------------------|
| 100.00        | 0.010     | 0.3110                 |
| 50.00         | 0.020     | 0.3110                 |
| 33.33         | 0.030     | 0.3275                 |
| 10.00         | 0.050     | 0.3901                 |
| 13.33         | 0.075     | 0.3894                 |
| 10.00         | 0.100     | 0.3627                 |
| 6.67          | 0.150     | 0.3308                 |
| 5.00          | 0.200     | 0.3182                 |
| 4.00          | 0.250     | 0.3182                 |
| 3.33          | 0.300     | 0.3182                 |
| 2.50          | 0.400     | 0.3182                 |
| 2.00          | 0.500     | 0.3312                 |
| 1.33          | 0.750     | 0.3446                 |
| 1.00          | 1.000     | 0.3739                 |
| 0.67          | 1.500     | 0.4001                 |
| 0.50          | 2.000     | 0.4185                 |
| 0.33          | 3.000     | 0.4232                 |
| 0.25          | 4.000     | 0.4065                 |
| 0.20          | 5.000     | 0.3965                 |
| 0.13          | 7.500     | 0.3480                 |
| 0.10          | 10.000    | 0.2877                 |

\[
\phi_{S2S-HF}(f) = \frac{\phi_{S2S-J}(f)}{\phi_{S2S-J}(f = 100 \text{ Hz})} \phi_{S2S-NGA}(f = 100 \text{ Hz})
\]  

\[
\phi_{S2S-LF}(f) = \frac{\phi_{S2S-NGA}(f)}{\phi_{S2S-NGA}(f = 2.5 \text{ Hz})} \phi_{S2S-HF}(f = 2.5 \text{ Hz})
\]

in which $\phi_{S2S-J}(f)$ is from the Japanese borehole data for rock sites.

For the low frequencies, $\phi_{S2S-LF}$ is given by:
Conclusion and discussion

Hard-rock adjustment factors are derived to adjust the NGA-West2 GMPEs from their average host site conditions with $V_{S30}$ of 760 m/s to target sites with $V_{S30}$ ranging from 1000 to 2200 m/s. These analytical factors are obtained using the IRVT approach (Al Atik et al., 2014) and are generally consistent with empirical scaling observed in ground-motion data. These factors can be applied to adjust median NGA-West2 ground motions at $V_{S30}$ of 760 m/s to hard-rock conditions and can be assumed to have the same overall site-to-site variability inherent in the NGA-West2 GMPEs with modifications associated with the expected spectral shape of site variability for hard-rock site conditions.

The site adjustment factors developed in this study are computed using generic $V_S$ profiles and $\kappa_0$ values that are representative of average site response in WUS for rock site conditions. The KA16 scaling factors obtained using ENA and BCHydro data are used as empirical constraints for this study because of the scarcity of empirical data on hard-rock sites in WUS and because KA16 showed that average hard-rock scaling in ENA is comparable to what would be expected for WUS sites.

The proposed hard-rock factors are intended for use at sites with limited site characterization such as sites with measured or estimated $V_{S30}$ or sites with qualitative assessment of site condition. For hard-rock sites with site-specific measurements of $V_S$ profiles extending below the shallow 20–30 m of the profile and site-specific estimates of $\kappa_0$, the hard-rock adjustment factors presented here are not recommended to be used. For such sites, site-specific adjustments should be developed following a characterization of the target site-specific conditions in terms of best estimates and uncertainty of $V_S$ profiles and $\kappa_0$. As discussed in this article, caution should be exercised in the characterization of site-specific $\kappa_0$ to avoid common issues and trade-offs resulting in the underestimation of site-specific $\kappa_0$ values and the overprediction of high-frequency ground motion for the target site. Similarly, the epistemic uncertainty in site factors from a site-specific study can be constrained based on the availability of site-specific data. The $\phi_{S30}$ model developed in this study based on the variability of site terms from all rock sites in a database is expected to be larger than the epistemic uncertainty in the site factors from a site-specific study. While this is true on average, it may not be true for every site.

Declaration of conflicting interests

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Data and resources

The pyrvt program used to perform the IRVT and RVT calculations (Kottke, 2020). An Excel file containing the hard rock adjustment factors for the NGA-West2 GMPEs is included as a supplemental material.
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**Supplemental material**
Supplemental material for this article is available online.

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