Bidyon as an electromagnetic model for charged particle with spin

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Abstract

A general model of nonlinear electrodynamics with dyon singularities is considered. We consider the field configuration having two dyon singularities with identical electric and opposite magnetic charges and we name it bidyon. We investigate the sum of two dyon solutions as an initial approximation to the bidyon solution. We consider the case when the velocities of the dyons have equal modules and opposite directions on a common line. It is shown that the associated field configuration has a constant full angular momentum which is independent of distance between the dyons and their speed. This property permits a consideration of this bidyon configuration as an electromagnetic model for charged particle with spin. We discuss the possible electrodynamic world with oscillating bidyons as particles.
A hypothetical particle having both electric and magnetic charges is said to be dyon \[1\]. An electromagnetic field configuration with \(N\) point dyons satisfies the following two differential conditions:

\[
\begin{align*}
\text{Div} \mathbf{D} &= 4\pi j^0 \\
\text{Div} \mathbf{B} &= 4\pi \bar{j}^0 
\end{align*}
\]  

(1)

where \(\text{Div} \mathbf{D} \equiv \partial_i D^i\), \(\overline{\partial}_\mu \equiv \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^\mu} \sqrt{|g|}\), \(\partial^0 \equiv \sqrt{|g|} \det(g_{\mu\nu})\), the Latin indices take the values 1, 2, 3, the Greek ones take the value 0, 1, 2, 3, \(g_{\mu\nu}\) is a metric of space-time coordinate system,

\[
\begin{align*}
j^0 &\equiv \frac{1}{\sqrt{|g|}} \sum_{n=1}^{N} d \delta(x - \vec{a}) \\
\bar{j}^0 &\equiv \frac{1}{\sqrt{|g|}} \sum_{n=1}^{N} b \delta(x - \vec{a})
\end{align*}
\]  

(2)

\(d\) is an electric charge and \(b\) is a magnetic one for \(n\)-th singular point, \(\vec{a} = \vec{a}(x^0)\) is a trajectory of it.

Here we use the definition for three-dimensional \(\delta\)-function which is suitable for discontinuous functions \(f(x)\):

\[
\int_{\vec{\Omega}} f(x) \delta(x - \vec{a}) (dx)^3 \equiv \lim_{\vec{\sigma} \to 0} \frac{1}{|\vec{\sigma}|} \int_{\vec{\sigma}} f(x) d\vec{\sigma}, \quad |\vec{\sigma}| \equiv \int d\vec{\sigma},
\]

(3)

where \(\vec{\Omega}\) is a region of three-dimensional space including the point \(x = \vec{a}\), \(\vec{\sigma}\) is a closed surface enclosing this point, \(d\vec{\sigma}\) is an area element of the surface \(\vec{\sigma}\), \(|\vec{\sigma}|\) is an area of the whole surface \(\vec{\sigma}\).

Eqs. (1) are the part of Maxwell system of equations in any space-time coordinate system with a metric \(g_{\mu\nu}\) (see also [2]).

To have a natural interaction between the dyons (see [2, 3]) we must take the fields \(\mathbf{D}\) and \(\mathbf{B}\) satisfying some nonlinear Maxwell equations that may be written in the following general form:

\[
\begin{align*}
\partial_0 \mathbf{D} - \text{Rot} \mathbf{H} &= -4\pi j \\
\overline{\partial}_0 \mathbf{B} + \text{Rot} \mathbf{E} &= -4\pi \bar{j}
\end{align*}
\]  

(4)
where $$(\text{Rot}E)^i \equiv -\varepsilon^{0ijk} \partial E_k / \partial x^j$$, $\varepsilon^{0123} = -|g|^{-1/2}$, $\varepsilon_{0123} = |g|^{1/2}$,

$$E_i = \frac{\partial \mathcal{H}}{\partial D^i}$$, \quad $$H_i = \frac{\partial \mathcal{H}}{\partial B^i}$$, \quad $$\mathcal{H} = \mathcal{H}(D,B)$$ \quad (5)

$$j \equiv \frac{1}{\sqrt{|g|}} \sum_{n=1}^N \mathbf{n} \cdot \mathbf{V} \delta(x - \mathbf{a}_n), \quad \bar{j} \equiv \frac{1}{\sqrt{|g|}} \sum_{n=1}^N \mathbf{n} \cdot \mathbf{V} \delta(x - \mathbf{a}_n), \quad \mathbf{V} \equiv \frac{d\mathbf{a}_n}{dx^0}. \quad (6)$$

According to Eqs. (5) we have some dependencies $E = E(D,B)$ and $H = H(D,B)$ (see also [4, 3]). If $D, B$ appears only quadratically in $H$ then we have a linear electrodynamics but in general case the function $H(D,B)$ defines a nonlinear electrodynamic model. In this approach the vector fields $D, B$ play the role of unknown functions for system of equation (4) with additional differential conditions (1). This representation is best suitable for an investigation of the interaction between the dyons. From the fields $E, B$ satisfying equations (1), (4) we can obtain an appropriate electromagnetic potential. In the case of the dyon singularity of electromagnetic field a space part of the four-potential has a line singularity [3].

The singular currents (2), (6) must satisfy to the following condition [3]:

$$F_{\mu\nu} j^\nu - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} f^{\sigma\rho} j^\nu = 0 \quad ,$$

where $F_{00} = E_i$, $F_{ij} = \varepsilon_{0ijk} B^k$, $f^{0i} = D^i$, $f^{ij} = \varepsilon_{0ijk} H_k$.

Using Eqs. (1), (4), (5), (7) we can check directly the following differential conservation laws for energy-momentum tensor (in Cartesian coordinate systems):

$$\frac{\partial \mathcal{H}}{\partial x^0} = -\frac{\partial}{\partial x^j} \left( \varepsilon^{ipq} E_p H_q \right), \quad (8)$$

$$\frac{\partial \mathcal{P}_i}{\partial x^0} = -\frac{\partial}{\partial x^j} \left[ \delta_i^j (D \cdot E + B \cdot H - \mathcal{H}) - \left( D^j E_i + B^j H_i \right) \right], \quad (9)$$

where $\mathcal{P}_i \equiv \varepsilon_{ipq} D^p B^q$ or $\mathcal{P} \equiv D \times B$ ($\varepsilon_{123} = \varepsilon^{123} = 1$).

From (8) and (9) we easily obtain that the full energy-momentum$^1$

$$\mathcal{E} = \frac{1}{4\pi} \int \mathcal{H} \, (dx)^3 \quad , \quad \mathcal{P} = \frac{1}{4\pi} \int \mathcal{P} \, (dx)^3 \quad , \quad (10)$$

$^1$Note, here we take the function $\mathcal{H}$ such that $\mathcal{H} = 0$ for $D = B = 0$. This is distinction from the designation which is used in the article [3] for Born-Infeld electrodynamics.
and the vector of full angular momentum

\[ \mathbf{M} = \frac{1}{4\pi} \int (\mathbf{x} \times \mathbf{P}) \, (dx)^3 \]  

are conserved on time, i.e. \( \frac{d\mathcal{E}}{dx^0} = \frac{d\mathbf{P}}{dx^0} = \frac{d\mathbf{M}}{dx^0} = 0. \)

Let us consider a solution of system (4), (1) having two dyon singularities with identical electric and opposite magnetic charges: \( \frac{1}{d} = \frac{2}{d}, \frac{1}{b} = -\frac{2}{b}. \) We name this solution bidyon. Let us consider the case when the velocities of the singularities have equal absolute values and opposite directions on a common line. At first we use a cylindrical coordinate system \( \{z, \rho, \varphi\} \) such that the dyon singularities are on the axis \( z. \) This configuration is shown in Fig. 1, where \( d = \pm \bar{d}, b = \pm \bar{b} \) and \( \bar{d}, \bar{b} \) are some positive constants.

![Figure 1: Disposition of the two dyons in the cylindrical coordinate system.](image)

We can search the solution by some iterative procedure and we can take a sum of two moving dyon solutions as initial approximation. That is we consider the following initial approximation to the bidyon solution:

\[ \mathbf{D}^{(0)} = \frac{1}{D} + \frac{2}{D}, \quad \mathbf{B}^{(0)} = \frac{1}{B} + \frac{2}{B}. \]  

(12)
Here we consider the dyon solutions with constant velocity. For z- and ρ-components of these solutions (see [3]) we have the following expressions:

$$ \begin{align*}
\frac{1}{D_z} \frac{dz}{d} &= \frac{1}{B_z} = \frac{1}{\sqrt{1-V^2}} \frac{z}{r^3}, \\
\frac{1}{D_\rho} \frac{d\rho}{d} &= \frac{1}{B_\rho} = \frac{1}{\sqrt{1-V^2}} \frac{\rho}{r^3}, \\
\frac{2}{D_z} \frac{dz}{d} &= \frac{2}{B_z} = \frac{1}{\sqrt{1-V^2}} \frac{z-a}{r^3}, \\
\frac{2}{D_\rho} \frac{d\rho}{d} &= \frac{2}{B_\rho} = \frac{1}{\sqrt{1-V^2}} \frac{\rho}{r^3},
\end{align*} $$

(13)

where $V \equiv \frac{da}{dx}, \ r = \sqrt{(z' + a')^2 + \rho^2}, \ r' = \sqrt{(z' - a')^2 + \rho^2}, \ z' = \frac{z}{\sqrt{1-V^2}}, \ a' = \frac{a}{\sqrt{1-V^2}}.$

Forms of ϕ-components for the vector fields $D, B, D', B'$ depend on forms of the functions $E = E(D, B), H = H(D, B)$ but forms (13) for ρ- and z-components are independent of the specific model’s nonlinearity. The ϕ-components equals zero when $V = 0$. The lines of the vector fields $D^{(0)}$ and $B^{(0)}$ in $z\rho$-plane for $V = 0$ are shown in Fig. 2.

Now let us calculate the vector of full angular momentum $M$ (11) for field configuration (12) with (13). Because of a symmetry property of the element of integration into (11), for our case we have $M_\rho = M_\varphi = 0$ and

$$ M_z = \frac{1}{4\pi} \int P^{(0)}_\varphi \rho \ d\rho d\varphi. $$

(14)

Using (13) we can easily obtain the following expression:

$$ P^{(0)}_\varphi = \frac{4 a b d \rho}{r^3} \frac{1}{r^3} \frac{1}{1 - V^2}. $$

(15)

Substituting (13) into (14) and introducing new variables of integration we obtain
Figure 2: Lines of the fields $\mathbf{D}^{(0)}$ (discontinuous lines) and $\mathbf{B}^{(0)}$ (continuous lines).

\[
M_z = \frac{bd}{\pi (1-V^2)} \int \frac{a \rho^3}{\frac{1}{3} \frac{2}{3}} dz d\rho d\varphi
\]

\[
= \frac{bd}{\pi} \int \frac{a' \rho^3}{\frac{1}{3} \frac{2}{3}} dz' d\rho d\varphi
\]

\[
= \frac{bd}{\pi} \int \left(\frac{\rho''}{\frac{1}{3} \frac{2}{3}}\right)^3 dz'' d\rho'' d\varphi
\]

(16)  
(17)  
(18)

where $z'' \equiv z'/a'$, $\rho'' \equiv \rho/a'$,

\[
\frac{1}{3} \frac{2}{3} \equiv \sqrt{(z'' + 1)^2 + (\rho'')^2}, \quad \frac{2}{3} \equiv \sqrt{(z'' - 1)^2 + (\rho'')^2}.
\]

As we see, in first change $\text{(16)} \rightarrow \text{(17)}$ the dependence on speed is canceled. In second change $\text{(17)} \rightarrow \text{(18)}$ the dependence on $a'$ is canceled. Thus we obtain that the full angular momentum for field configuration $\text{(12), (13)}$ is independent of dyon’s speed ($V$) and distance between the dyons ($2a$)!

For calculation $\text{(18)}$ we introduce the variables of integration $\{\xi, \zeta, \varphi\}$ that appropriate to the bispherical coordinate system with unit parameter characterizing positions of focal points:

\[
z'' = \frac{\sinh \xi}{\cosh \xi - \cos \zeta}, \quad \rho'' = \frac{\sin \zeta}{\cosh \xi - \cos \zeta}
\]

(19)
The bispherical element of value has the form
\[(\rho'' d\zeta'' d\rho'' d\varphi) = \frac{\sin \zeta d\xi d\zeta d\varphi}{(\cosh \xi - \cos \zeta)^3} \] (20)

We have also that
\[\frac{1}{r''} = \frac{\sqrt{2} \exp (\xi/2)}{\sqrt{\cosh \xi - \cos \zeta}}, \quad \frac{2}{r''} = \frac{\sqrt{2} \exp (-\xi/2)}{\sqrt{\cosh \xi - \cos \zeta}} \] (21)

Substituting (19), (20), (21) into (18) and introducing the variable \(z = \cos \zeta\), we obtain
\[M_z = \frac{b d}{8 \pi} \int_\pi d\varphi \int_{-\infty}^\infty d\xi \int_1^{-1} \frac{(1 - z^2)}{(\cosh \xi - z)^2} dz \] (22)

Making firstly the integration over \(z\) and \(\xi\) in the finite limits \([-\bar{\xi}, \bar{\xi}]\), we obtain
\[\int_{-\infty}^\infty d\xi \int_1^{-1} \frac{(1 - z^2)}{(\cosh \xi - z)^2} dz = \lim_{\xi \to \infty} \left[ 4 \left( \ln \frac{\cosh \bar{\xi} + 1}{\cosh \xi - 1} \right) \sinh \bar{\xi} \right] = 8 \] (23)

Thus we have
\[M_z = 2 bd \] (24)

Of course, the full angular momentum for an appropriate exact solution is conserved. This implies that we may have internal movements of the singularities, which don’t change the full angular momentum. Thus here we have verified that our choice of two moving dyons (12), (13) as the initial approximation is appropriate.

It is evident that the full momentum for field configuration (12), (13) is zero. To verify satisfaction of the conservation law for full energy, we must take a concrete function \(H(D, B)\). That is, we must investigate the concrete nonlinear electrodynamic model. In this case the condition \(E = \text{const}\) can be used for defining a trajectory \(a(x^0, E)\) of the dyons in the initial approximation. This problem was investigated for Born-Infeld electrodynamics [3] and it was shown that the initial field configuration may behave as nonlinear oscillator. (A wave part of the dyon solutions connected with acceleration of the singular points was not included to the initial approximation.)
The field configuration (12), (13) looks like charged particle with spin. The charge of this particle is $2 \tilde{d}$ and its spin is equal to $|M_z|$. We may set $2 \tilde{d} = e$, where $e$ is the absolute value of the electron charge, and $|M_z| = \hbar/2$. In this case we have

$$\bar{b} e = \frac{\hbar}{2} \implies \bar{b} = \frac{e}{2} \frac{\hbar}{e^2} = \frac{1}{2} \frac{e}{\alpha} \implies \frac{\bar{d}}{\bar{b}} = \bar{\alpha} \; ,$$

where $\bar{\alpha} = e^2/\hbar \approx 1/137$ is the fine structure constant.

The field configuration (13) is considered here as initial approximation to unknown exact bidyon solution. This initial approximation does not include a wave part of the bidyon solution. For periodical bidyon solution this wave part must have the form of some standing wave localized near dyon singularities. Because this problem has the boundary conditions (which follow from (7), see also [3]) in two (moving) points (in which the dyon singularities are at a current instant of time), it is possible that the bidyon solution has some discrete set of allowable frequencies.

With the help of Lorentz transformation, from the rest oscillating bidyon we can obtain an appropriate moving bidyon solution. It is evident that the moving oscillating bidyon has both particle and wave properties.

The field model under consideration allows existence for great number of the dyon singularities with charges $\bar{n}\tilde{d}$ and $\bar{n}\bar{b}$. We may assume that there is some kind invariance of the theory, such that

$$\bar{n}\tilde{d} = \pm \tilde{d} \; , \quad \bar{n}\bar{b} = \pm \bar{b} \; .$$

For the suitable dimensional system we can take $\tilde{d} = 1$ or $\bar{b} = 1$. Thus we have the relation $\tilde{d}/\bar{b}$ as the single dimensionless constant of the theory. We can set that this relation equals the fine structure constant $\bar{\alpha}$ (25).

We can fancy a world constructed from the great number of the bidyon-type field configurations as particles. Particles with full angular momentum divisible by $\hbar/2$ may be constructed from some number of the bidyons. Because of $\tilde{d}/\bar{b} = \bar{\alpha} \ll 1$, we may build a perturbation theory with the fine structure constant $\bar{\alpha}$ as small parameter, for some aspects of the mathematical model of this world. In this case we will have an analogy with the procedure of perturbation theory in quantum electrodynamics.

As a result we can assume that there is some correlation between the bidyon solution of a nonlinear electrodynamic model and leptons.
References

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