Viscous dissipation effect on Williamson nanofluid over stretching/shrinking wedge with thermal radiation and chemical reaction

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Abstract
This paper scrutinizes the effect of viscous dissipation on unsteady two-dimensional boundary layer flow of Williamson nanofluid over a stretching/Shrinking wedge. To express the boundary condition in concentration problem the passive control concept used. The governing PDEs are converted to ODEs by means of a similarity transformation before being solved numerically by finite difference scheme called Keller-Box method. The equations were numerically solved by using Matlab software 2013a. The characteristics of parameters such as wedge angle, unsteadiness, Williamson, slip, Brownian motion, thermophoresis, chemical reaction parameters, Prandtl number, Biot-number, Eckert number and Lewis number on velocity, concentration and temperature profiles and skin friction coefficient, Nusselt number and Sherwood number are presented in graphs and tables. The result of the study designates that the velocity profiles increased with an upsurge of wedge angle, unsteady parameter and suction parameter while it is diminished with an increase of Williamson and injection parameter. The temperature profiles upsurges with the distended Williamson parameter, Biot number and injection parameter, while it is declined for large values of wedge angle, unsteady and suction parameter. With an increase of Williamson, unsteady and suction parameter the concentration profiles upsurges, while it is decreased with an increase of wedge angle and injection parameter. The numerical results are compared with available literature and obtained a good agreement.

Nomenclature

| Symbol | Description                  |
|--------|------------------------------|
| A      | Suction/Injection parameter  |
| A₁     | First Rivlin-Ericson tensor  |
| B₁     | Biot Number                  |
| c      | Volumetric volume expansion coefficient |
| Cₙ     | Skin friction coefficient    |
| DB     | Brownian diffusion coefficient |
| DT     | Thermophoretic diffusion coefficient |
| Eₙ     | Eckert number                |
| f      | Dimensionless stream function |
| hₙ     | Heat transfer coefficient    |
| I      | Identity vector              |
| k      | Thermal conductivity        |
| K⁺     | Absorption constant         |
| K      | Chemical reaction parameter |
| Symbol | Definition |
|--------|------------|
| $Le$  | Lewis number |
| $Nb$  | Brownian motion parameter |
| $Nt$  | Thermophoresis parameter |
| $Nu_x$ | Nusselt number |
| $p$  | Pressure |
| $Pr$  | Prandtl number |
| $qm$  | Mass flux of the nanofluid |
| $q_{sw}$ | Heat flux of the nanofluid |
| $R$  | Radiation parameter |
| $Re$  | Reynolds number |
| $S$  | Cauchy stress tensor |
| $Sh_x$  | Sherwood number |
| $T$  | Temperature |
| $T_{\infty}$  | Free stream temperature |
| $u_c$  | Free stream velocity |
| $u_w$  | The velocity of stretching/shrinking wedge |
| $(u,v)$  | Velocity components in $x$ and $y$ coordinate |

Greek symbols:

| Symbol | Definition |
|--------|------------|
| $\psi$  | Stream function |
| $\theta$  | Dimensionless temperature |
| $\eta$  | Similarity variable |
| $\mu_0$  | Limiting viscosities at zero |
| $\mu_\infty$  | Limiting viscosities at infinite shear rate |
| $\beta$  | Wedge angle parameter |
| $\alpha$  | Thermal diffusivity of the nanofluid |
| $\phi$  | Dimensionless concentration function |
| $(\rho c)_f$  | Heat capacity of the fluid |
| $(\rho c)_p$  | Heat capacity of the nanoparticles |
| $\rho_p$  | Density of nanoparticles |
| $\xi$  | The parameter defined by $(\rho c)_p/(\rho c)_f$ |
| $\tau$  | Extra stress tensor |
| $\gamma$  | Stretching parameter |
| $\Gamma$  | Positive time constant |
| $\zeta$  | The ratio of viscosities |
| $\sigma^*$  | Stefan-Boltzman constant |
| $\pi$  | Second invariant strain tensor |
| $i$  | Slip Coefficient |
| $\lambda_1$  | Unsteadiness parameter |
| $\lambda_2$  | Williamson parameter |
| $\tau_{sw}$  | Skin friction or shear stress |
| $\rho$  | Density |
| $\nu$  | Kinematic viscosity |

1. Introduction

Due to its applications the flow of fluid in a boundary adjacent to the wedge has great attention to the researchers and Engineers. Some of its applications are geothermal systems, polymer processes, crude oil extraction, cooling or heating of films/sheets, etc. Firstly the steady laminar flow above a wedge was investigated by Fankner and
many researchers have investigated the delivered many successful sources of information about the behavior of incompressible boundary layers. Later Skan\cite{1}, Alam et al \cite{2}, Sattar \cite{3}, Bharathi Devi and Gangadhar \cite{4}, Mohammad et al \cite{5} and Ali and Alim \cite{6} have studied Falkner–Skan flow over a wedge with different boundary conditions and with various parameters. Their study shows that with an increasing pressure gradient parameter the magnitude velocity increases. Moreover, unsteady MHD heat and mass transfer boundary layer flow along a wedge with different dimensionless parameter were inspected by Radiah Bte Mohamad \cite{7}, Kandasamy \cite{8}, Rahman \cite{9}, Alam et al \cite{10}, Muhamin et al \cite{11}, Ali et al \cite{12}, Rahman et al \cite{13} and Mohd Rijal Ilias et al \cite{14}. Their results indicate that unsteadiness significantly control the flow, heat transfer and mass transfer of non-Newtonian nanofluids. Furthermore, in the presence of variable viscosity dual solutions of a boundary layer problem for MHD nanofluids through a permeable wedge was studied by Xiaoqin Xu and Shumei Chen \cite{15}. Furthermore, Subba Reddy Gorla and Mahesh Kumari \cite{16} have investigated mixed convective heat transfer of boundary layer flow along a vertical porous wedge with a nanofluid. The heat transfer of MHD nanofluid flow along a wedge with viscous effects of embedded in Porous Media were examined by Wubshet Ibrahim and Ayele Tulu \cite{17}. In most industries nowadays the importance of non-Newtonian fluids dominates the Newtonian fluids. The rheological properties of non-Newtonian fluids cannot be illuminated by the classical Naiver-Stokes equations. Also no single model is sufficient to describe non-Newtonian fluids characteristics. To overcome this difficulty abundant models have come into being. The rheological models that were projected were Williamson, Cross, Ellis, power law, Carreau fluid model, etc Typical of a non-Newtonian fluid model with shear drawing property is Williamson fluid model and was first projected by Williamson \cite{18}. Nadeem et al \cite{19} studied the two dimensional flow of Williamson fluid above stretching sheet. Moreover, Nadeem and Hussain \cite{20} have scrutinized the impacts of nanoparticles on Williamson fluid over a stretching sheet. Furthermore, the inspection of Williamson fluid has been carried out by investigators such as, Abegunrin and Animasaun \cite{21}, Kumar et al \cite{22}, Nagendra et al \cite{23}, Wubshet Ibrahim and Dachasa Gamachu \cite{24}, Hashim et al \cite{25}, Aamir Hamid et al \cite{26} and Talha \cite{27} past different physical geometry like radially stretching surface, static/moving wedge and stretching sheet.

Choi and Eastman \cite{28} introduced nanofluid, which is a mixture of nanoparticles and the base fluid such as water, ethylene glycol, and propylene glycol. The most important properties of nanofluid are to increase effective thermal heat conductivity and heat transfer coefficient. Therefore, many researchers were attracted to this fact. Accordingly, Gireesha et al \cite{29}, Sulochana et al \cite{30} and Mahanthesh et al \cite{31} investigated the heat and mass transfer characteristics in nanofluids in three dimensions and they found that in the presence of the nanoparticles the effective thermal conductivity of the fluid upsurges appreciably and consequently improves the heat transfer characteristics. Furthermore, unsteady MHD nanofluid flow over a slendering stretching surface with slip effects was studied by Ramana Reddy et al \cite{32}. The result indicates that with an increase of unsteadiness parameter, the heat and mass transfer coefficient are increased.

The influence of radiation on flow and heat transfer is significant in the framework of space technology and processes involving high temperature. Some of the application of thermal radiation are in the design of nuclear power plant, fire propagation, plume dynamics, materials processing, rocket propulsion, missiles, aircraft, space technology, hypersonic fights, gas turbines and etc. The application and the effect of radiations on fluid flow was firstly investigated by Plum et al \cite{33}. Later, Anuradha and Yeegamai \cite{34}, Majeed et al \cite{35}, Abdul Maleque \cite{36} and Sajid \cite{37} investigated the effect of thermal radiation and activation energy on boundary layer flow of nanofluids on different surfaces. Moreover, the influence of thermal radiation on hydromagnetic boundary layer flow of Williamson fluid with Ohmic dissipation was examined by Hayat et al \cite{38}. From their result it can be seen that when the values of Weissenberg number, magnetic parameter, radiation parameter and the Eckert number Ec, rises the heat transfer rate reduces. Furthermore, Hameed Khan et al \cite{39}, Abdul Samad Khan et al \cite{40} and Sameh et al \cite{41} have studied the impact of thermal radiation on boundary layer flow non-Newtonian Maxwell nanofluids with heat generation/absorption. Still further, the enhancement of thermal radiation on hydromagnetic Casson fluid flow over a stretched cylinder in the presence of suspension of liquidparticles was inspected by Ramesh et al \cite{42}. Rajput and Gaurav kumar \cite{43} examined the effect of thermal radiation on heat and mass transfer of MHD flow through an oscillating inclined plate. The effect of chemical reaction and thermal radiation on free convective heat and mass transfer of MHD fluid flow from over stretching surface embedded in a saturated Porous medium was examined by Rashad et al \cite{44}. The main aim of this study is to investigate the Effect of Viscous Dissipation on Unsteady Williamson Nanofluid over Stretching/Shrinking Wedge with Thermal Radiation and Chemical reaction. The novelty of this paper are (i) inclusion of viscous dissipation on energy equation, since it has significant impact on heat transfer; especially for high-velocity flows, fluids with a moderate Prandtl number, highly viscous flows at moderate velocities and moderate velocities with small wall-to-fluid temperature difference. (ii) consideration of Falkner–Skan flow of Williamson fluid over stretching/shrinking wedge. It palys a vital role in geothermal
systems, polymer processes, crude oil extraction, cooling or heating of films/sheet. The novel governing partial differential equation of momentum, energy and concentration are reduced to non linear ordinary differential equation and then solved by implicit finite difference method known as Keller box. In addition, graphically the impact of different physical parameters such as Suction/Injection, Stretching/Shrinking, Biot number, Williamson parameter, wedge angle parameter, unsteady parameter, Frandtl number, Lewis number, Thermophoresis parameter, Brownian motion parameter, Eckert number, radiation parameter and chemical reaction parameter on velocity, temperature and concentration profiles scrutinized. Moreover, in tabular the numerical values of volume friction, mass and heat transfer rate were examined.

2. Mathematical formulation

Unsteady two-dimensional incompressible laminar boundary flow of non-Newtonian Williamson nanofluids over stretching/shrinking wedge is considered. In this study we consider $u_w(x)$ the velocity of stretching/shrinking wedge, $u_e(x)$ free steam velocity, $T_\infty$ ambient temperature, $T_w$ stretching/shrinking wedge temperature, $C_w$ nanoparticle at stretching/shrinking wedge and $C_\infty$ ambient nanoparticle. The physical coordinate system is chosen with $x$ along the surface of the wedge and is measured from the origin and $y$ is the coordinate normal to the surface (see figure 1). It is noted that the wedge is stretching when $u_w(x) > 0$ whereas shrinking when $u_w(x) > 0$.

For an incompressible Williamson fluid the continuity and momentum equation are given by

$$ \nabla \cdot V = 0, $$

$$ \rho \frac{dV}{dt} = \nabla S + \rho b, $$

where $\rho$ is the density, $V$ is the velocity vector, $S$ is the Cauchy stress tensor, $b$ represents the specific body force vector and $d/dt$ represents the material time derivative. The essential equations for Williamson fluid model are specified as follows:

$$ S = -pI + \tau A_1, $$

$$ \tau = \left[ \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma \dot{\gamma}} \right], $$

where $p$ is pressure, $I$ is identity vector, $\tau$ is extra stress tensor, $\mu_0$ and $\mu_\infty$ are the limiting viscosities at zero and at infinite shear rate respectively, $\Gamma > 0$ is the time constant, $A_1$ is the first Rivlin-Ericson tensor and $\dot{\gamma}$ is defined as follows:

$$ \dot{\gamma} = \frac{\pi}{\sqrt{2}} , \pi = \text{trace}(A_1^\gamma), $$

where $\pi$ is the second invariant strain tensor. Here we consider the case in which $\mu_\infty = 0$ and $\Gamma \dot{\gamma} < 1$. Then we obtain
\[ \tau = \mu_0 \left[ \zeta + \frac{1 - \zeta}{1 - \Gamma \gamma} \right]. \]  

(6)

Here \( \zeta = \frac{\mu_0}{\mu_f} \) is the ratio of viscosities.

The components of stress tensor are

\[ \tau_{xx} = 2\mu_0(1 - \Gamma \gamma) \frac{\partial u}{\partial x}, \]  

(7)

\[ \tau_{xy} = \tau_{yx} = \mu_0(1 - \Gamma \gamma) \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \]  

(8)

\[ \tau_{yy} = 2\mu_0(1 - \Gamma \gamma) \frac{\partial v}{\partial y}. \]  

(9)

In the absence of body force the governing equations can be described in Cartesian system as listed below (See Aamir Hamid et al [28]).

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  

(10)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + v \frac{\partial^2 u}{\partial y^2} \left[ \zeta + (1 - \zeta) \left( 1 - \Gamma \frac{\partial u}{\partial y} \right)^{-1} \right] + u \frac{\partial v}{\partial x}, \]  

(11)

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \zeta \left\{ \frac{D_b \frac{\partial C}{\partial y}}{\frac{\partial T}{\partial y}} + \frac{D_T}{T_{\infty}} \left( \frac{\partial T}{\partial y} \right)^2 \right\} - \frac{1}{\rho_f \mu_f} \frac{\partial q_r}{\partial y} + \nu \left( \frac{\partial u}{\partial y} \right)^2, \]  

(12)

\[ \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_b \frac{\partial^2 C}{\partial y^2}}{\frac{\partial T}{\partial y}} + \frac{D_c \frac{\partial^2 T}{\partial y^2}}{\frac{\partial T}{\partial y}} = K_r (C - C_{\infty}). \]  

(13)

The appropriate boundary conditions are (See Aamir Hamid et al [27])

\[ u = \gamma u_e + U_{\text{slip}}, \quad v = v_\infty, \quad -k \frac{\partial T}{\partial y} = h_f (T_f - T), \quad D_b \frac{\partial C}{\partial y} + D_T \frac{\partial T}{T_{\infty}} = 0, \quad \text{at } y = 0, \]  

(14)

\[ u \rightarrow u_e(x), \quad T \rightarrow T_{\infty}, \quad C \rightarrow C_{\infty}, \quad \text{as } y \rightarrow \infty. \]  

(15)

\( U_{\text{slip}} \) is a velocity slip for Williamson fluid model and defined as (See Aamir Hamid et al [28]).

\[ U_{\text{slip}} = \frac{1}{L} \frac{\partial u}{\partial y} \left[ \zeta + (1 - \zeta) \left( 1 - \Gamma \frac{\partial u}{\partial y} \right)^{-1} \right], \]  

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) directions respectively, \( \rho_f \) is the density of the base fluid, \( \alpha_m = \frac{k}{\nu_\infty} \) is the thermal diffusivity, \( \nu \) is the kinematic viscosity of the fluid, \( k \) is the thermal conductivity of the fluid, \( D_b \) is the Brownian diffusion coefficient, \( D_T \) is the thermophoretic diffusion coefficient, \( \xi = \frac{\nu c_p}{\lambda c_v} \) is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, \( c \) is the volumetric volume expansion coefficient and \( \rho_p \) is the density of the particles, \( L \) is the velocity slip length with dimension of length, \( h_f \) is heat transfer slip, \( T_f \) is the temperature of the fluid \( v_\infty \) is suction velocity at the wall. We can write the thermal radiation using Rosseland approximation for heat flux \( q_r \) as follows (See Hayat et al [40] and Ramesh et al [44])

\[ q_r = -\frac{4\sigma^* T^4}{3k^* \frac{\partial T}{\partial y}}, \]  

(16)

where \( \sigma^* \) is the Stefan-Boltzman constant, \( k^* \) is the absorption constant. Assuming the temperature difference within the flow such that \( T^4 \) may be expanded in a Taylor series about \( T_{\infty} \) and neglecting higher orders. We get \( T^4 = 4TT_{\infty}^3 - 3T_{\infty}^4 \).

Hence

\[ \frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_{\infty}^3}{3k^* \frac{\partial T}{\partial y}^2}. \]  

(17)

In order to obtain similarity solution for equations (10)–(15) assume the free steam velocity for the wedge (See Sattar [5] and Rahman et al [15]).
\[ u_x(x, t) = \frac{\nu x^n}{\delta x^{m+1}} \]  

(18)

where \( \delta = \delta(t) \) is time dependent length scale (See Sattar [5] and Rahman et al [11]), \( m \) is a positive constant related to the wedge angle which is \( m = \frac{\beta}{2 - \beta} \) (0 \( \leq m \leq 1 \)) and \( \beta \) is Hartree pressure gradient parameter which resembles to \( \beta = \frac{\beta}{2} \) for a total angle \( \Omega \) of the wedge. The constant moving parameter \( \gamma \) is defined as \( \gamma = \frac{\partial \omega}{\partial y} \), so that \( \gamma > 0 \) corresponds to stretching wedge, \( \gamma < 0 \) corresponds to a shrinking wedge, and \( \gamma = 0 \) corresponds to flat plate.

In order to solve equations (11)–(13) subjected boundary condition (14) and (15) we introduce similarity transformations as follows (See Sattar [5] and Rahman et al [11]):

\[
\psi = \nu \sqrt{\frac{2x^{m+1}}{(m+1)\delta x^{m+1}}} f(\eta), \quad \theta(\eta) = \frac{(T-T_x)}{(T_w-T_x)}, \quad \phi(\eta) = \frac{(C-C_w)}{(C_w-C_x)}, \quad \eta = \gamma \sqrt{\frac{(m+1)x^{m-1}}{2\delta x^{m+1}}}.
\]  

(19)

We choose the stream function \( \psi(x, y) \) such that

\[
\frac{\partial \psi}{\partial y} = u, \quad \text{and} \quad -\frac{\partial \psi}{\partial x} = v.
\]  

(20)

Equations (11)–(13) are transformed into the non-dimensional ordinary differential equation by applying the similarity transformation equation (18) as follows:

\[
f''(\zeta) + (1 - \zeta)(1 - \lambda_2 f')^{-1} + \lambda_2 f''(1 - \zeta)(1 - \lambda_2 f')^{-2} + f'' + \beta(1 - f') = 0,
\]  

(21)

\[
\left[ 1 + \frac{4}{3} R \right] \theta'' + Pr \left[ \theta'' + N b \phi \theta' + N t \theta'^2 + E c f'' + \lambda_1 \eta \theta' \right] = 0,
\]  

(22)

\[
\phi'' + \frac{N t}{N b} \theta'' - K L \phi + \left( \rho C_w \right) \theta' = 0.
\]  

(23)

With corresponding boundary conditions

\[
f(\eta) = A, \quad f'(\eta) = \gamma + \nu f''(\eta) [\zeta (1 - \zeta)(1 - \lambda_2 f'(\eta))^{-1}],
\]  

(24)

\[
\theta'(\eta) = \beta \theta(\eta) - 1, \quad \text{and} \quad \phi'(\eta) + \nu \lambda_1 \eta \theta' = 0, \quad \text{at} \quad \eta = 0,
\]  

(25)

\[
f'(\eta) \rightarrow 1, \quad \theta'(\eta) \rightarrow 0, \quad \phi'(\eta) \rightarrow 0, \quad \text{as} \quad \eta \rightarrow \infty.
\]  

(26)

where \( f' \) is dimensionless velocity, \( \theta \) is dimensionless temperature, \( \phi \) is dimensionless concentration and \( \eta \) is the similarity variable. The prime denotes differentiation with respect to \( \eta \). The overall governing parameters are defined as: \( \lambda_2 = \sqrt{\frac{(m-1)}{(m+1)^3}} \nu \) is non-Newtonian Williamson parameter, \( \beta = \frac{\beta}{2} \) is the wedge angle parameter that corresponds to \( \Omega = \beta \pi \), \( \lambda_n = \frac{n^m}{\nu (m+1)^{1/2}} \) is unsteadiness parameter, \( R = \frac{4\pi T_0 \nu}{\kappa x} \) is thermal radiation parameter, \( k = \alpha \frac{\partial \rho C_w}{\partial y} \) is thermal conductivity, \( Pr = \frac{\nu}{\alpha} \) is Prandtl number, \( N b = \frac{(\partial \rho C_w)}{\lambda_1} \) is Brownian motion parameter, \( N t = \frac{(\partial \rho C_w)}{\lambda_1} \) is thermophoresis parameter, \( Le = \frac{\nu}{D_0} \) is Lewis number, \( K = \frac{2K_0 \nu^{m-1}}{(m+1)^{2m+1}} \) is chemical reaction parameter, \( Ec = \frac{\nu^2}{\gamma (\nu - 1)} \) is Eckert number, \( \iota = L \sqrt{\frac{(m+1)x^{m-1}}{2\delta x^{m+1}}} \) is the slip parameter, \( Bi = \sqrt{\frac{2}{\nu (m+1)^{1/2}} \frac{\nu \lambda_1}{\kappa x}} \) Biot number or surface convention parameter and \( A = \sqrt{\frac{2}{\nu (m+1)^{1/2}} \frac{\nu \lambda_1}{\kappa x}} \) is the wall mass transfer coefficient.

The skin friction \( C_f \), local Nusselt number \( N u_x \) and the Sherwood number \( S h_x \) are the important physical quantities of interest in this problem which are defined as

\[
C_f = \frac{2 \tau_w}{\rho u_0^2}, \quad N u_x = \frac{\lambda_1 u_{xy}}{k(T_w - T_m)}, \quad S h_x = \frac{\lambda_1 u_{xy}}{D_0 (C_w - C_x)},
\]  

(26)

here \( \tau_w = \rho_0 \left( \zeta (1 - \zeta)(1 - \frac{\partial \alpha}{\partial y})^{-1} \right) \frac{\partial \alpha}{\partial y} \) at \( \gamma = 0 \) is the surface shear stress, \( q_w = -k \left( \frac{\partial T}{\partial y} \right) \gamma = 0 \) is the surface heat flux and \( \alpha_m = -D_0 \left( \frac{\partial C}{\partial y} \right) \gamma = 0 \) is the surface mass flux.

\[
C_f \frac{R e_x}{\alpha} = \frac{1}{\sqrt{2 - \beta}} f''(0)[\zeta (1 - \zeta)(1 - \lambda_2 f'(0))^{-1}],
\]  

(27)

\[
= \frac{1}{\sqrt{2 - \beta}} \left( 1 + \frac{4}{3} R \right) \theta'(0) = N u_x R e x ^ {1/2}, \quad -\phi(0) = S h_x R e x ^ {1/2},
\]

where \( R e_x \) is local Reynolds number.
3. Solution methodology

3.1. Keller box method

The transmuted ordinary differential equations (15), (16) and (17) subject to boundary conditions (18) and (19) are solved numerically using an implicit finite difference method (Keller box) in combination with the Newton linearization techniques. The key features of this method are:

1. Only to some extent more arithmetic to solve than the Crank-Nicolson method.
2. Second-order correctness with arbitrary (non-uniform) x and y spacing’s.
3. Tolerates very speedy x variations.
4. Tolerates easy programming of the solution of hefty numbers of coupled equation.

Four steps are considered to solve an equation by this method.

1. Reduce the equation or equations to a first-order system.
2. Using central differences write difference equations.
3. Linearize the resulting algebraic equations (if they are nonlinear), and write them in matrix-vector form.
4. Solve the linear system by the block-tridiagonal-elimination method.

We consider the net rectangle in the $x - \eta$ plane shown in figure 2 and the net points defined as below

\[
\begin{align*}
x^0 &= 0 \\
x^i &= x^{i-1} + k_i & i &= 1, 2, 3, \ldots, I \\
\eta^0 &= 0 \\
\eta_j &= \eta_{j-1} + h_j & j &= 1, 2, 3, \ldots, J \\
\eta_j &= \eta_\infty
\end{align*}
\]

where $k_i$ is the $\Delta x$-spacing and $h_j$ is the $\Delta \eta$-spacing. Here $i$ and $j$ are the sequence of numbers that indicate the coordinate location, not tensor indices or exponents.

Since only first derivatives appear in the governing equations, centered differences and two-point averages can be constructed involving only the four corner nodal values of the 'box'. For example, if $g$ represents any of the dependent variables then

\[
\begin{align*}
\left[ g_{i-\frac{1}{2}}^{j} \right] &= 0.5(g_{i-1}^{j} + g_{i}^{j}) \\
\left[ g_{i+\frac{1}{2}}^{j} \right] &= 0.5(g_{i-\frac{1}{2}}^{j} + g_{i+\frac{1}{2}}^{j}) \\
\left[ \frac{\partial g}{\partial \eta} \right]_{i-\frac{1}{2}} &= 0.5 \left( \left[ \frac{\partial g}{\partial \eta} \right]_{j-\frac{1}{2}} + \left[ \frac{\partial g}{\partial \eta} \right]_{j+\frac{1}{2}} \right) \\
\left[ \frac{\partial g}{\partial \eta} \right]_{j-\frac{1}{2}} &= 0.5 \left( \left[ g_{i+\frac{1}{2}}^{j} - g_{i-\frac{1}{2}}^{j} \right] / (\eta_j - \eta_{j-1}) \right)
\end{align*}
\]

Figure 2. Net rectangle for difference approximations.
Table 1. Comparison of current result of $-f''(0)$ and $-\theta'(0)$ with Watanaba and Wubshet.

| m        | Watanaba [47] | Wubshet [19] | Present Result | Watanaba | Wubshet | Present Result |
|----------|----------------|---------------|----------------|-----------|----------|----------------|
| 0.0000   | 0.46960        | 0.46960       | 0.46964        | 0.42016   | 0.42016  | 0.420152       |
| 0.0141   | 0.50461        | 0.50461       | 0.504476       | 0.42578   | 0.42578  | 0.425656       |
| 0.0435   | 0.56898        | 0.56898       | 0.568979       | 0.43548   | 0.43548  | 0.435469       |
| 0.0909   | 0.65498        | 0.65498       | 0.655006       | 0.44730   | 0.44730  | 0.447302       |
| 0.1429   | 0.73200        | 0.73200       | 0.732004       | 0.45693   | 0.45694  | 0.456933       |
| 0.2000   | 0.80213        | 0.80213       | 0.802130       | 0.45503   | 0.45503  | 0.455026       |
| 0.3333   | 0.92765        | 0.92765       | 0.927653       | 0.47814   | 0.47814  | 0.485139       |

4. Result and Discussion

By taking $Bi = 2.0, \ Nt = 0.1, \ K = 0.1 = R = \epsilon = Ec = \gamma = \lambda_1 = \lambda_2, \ \beta = 0.5, \ Pr = 1.0, \ and \ Le = 1.0$ numerical calculations are carried out unless otherwise specified. The results are revealed in a graphical and in tabular form for different values of parameters such as wedge angle parameter ($\beta$), unsteadiness parameter ($\lambda_1$), Williamson parameter ($\lambda_2$), Lewis number ($Le$), Brownian motion parameter ($Nt$), thermophoresis parameter ($\gamma$), Prandtl number ($Pr$), slip parameter ($\delta$), Eckert number ($Ec$), Radiation parameter ($R$), Chemical reaction parameter ($K$) and Biot number ($Bi$) on the flow, heat and mass transfer characteristics. The results are compared with those of others authors Watanaba [47] and Wubshet and Ayele [19] and found to be in good agreement as shown in table 1. This ensures the validity and accuracy of the present work. Also, various values of the skin friction, rate of heat transfer and rate of nanoparticle concentration are presented in table 2 for different values of $\beta, \ \lambda_1, \ \lambda_2, \ A, \ and \ Ec$. From table 2 it is observed that as value of wedge $\beta$ and suction $A$ parameter increases the skin friction, Local Nusselt number and local Sherwood number are increased. Moreover, from this table we see that $-\theta'(0)$ and $-\phi'(0)$ increases with an increase of unsteady parameter $\lambda_1$ but opposite results happened for $-f''(0)$. Furthermore, when the values of Williamson parameter increase the skin friction coefficient upsurges whereas the local Nusselt number and Sherwood number declines. The table also indicates that skin friction coefficient remains constant with an increase of Eckert number but local Nusselt number rises and local Sherwood number declines.

Figure 3 revealed that the impact of the slip parameter on the velocity profiles within the boundary layer. From the figure it can be observed that the velocity profiles increase with a growing in values of slip parameter. The horizontal line which approaches to one boundary layer thickness decreases as the thermophoresis parameter increases.

The boundary layer thickness of velocity profiles converges to 1 with Watanaba and Wubshet. The results are revealed in a graphical and in tabular form for different values of parameters such as wedge angle parameter ($\beta$), unsteadiness parameter ($\lambda_1$), Williamson parameter ($\lambda_2$), Lewis number ($Le$), Brownian motion parameter ($Nt$), thermophoresis parameter ($\gamma$), Prandtl number ($Pr$), slip parameter ($\delta$), Eckert number ($Ec$), Radiation parameter ($R$), Chemical reaction parameter ($K$) and Biot number ($Bi$) on the flow, heat and mass transfer characteristics. The results are compared with those of others authors Watanaba [47] and Wubshet and Ayele [19] and found to be in good agreement as shown in table 1. This ensures the validity and accuracy of the present work. Also, various values of the skin friction, rate of heat transfer and rate of nanoparticle concentration are presented in table 2 for different values of $\beta, \ \lambda_1, \ \lambda_2, \ A, \ and \ Ec$. From table 2 it is observed that as value of wedge $\beta$ and suction $A$ parameter increases the skin friction, Local Nusselt number and local Sherwood number are increased. Moreover, from this table we see that $-\theta'(0)$ and $-\phi'(0)$ increases with an increase of unsteady parameter $\lambda_1$ but opposite results happened for $-f''(0)$. Furthermore, when the values of Williamson parameter increase the skin friction coefficient upsurges whereas the local Nusselt number and Sherwood number declines. The table also indicates that skin friction coefficient remains constant with an increase of Eckert number but local Nusselt number rises and local Sherwood number declines.

Figure 3 revealed that the impact of the slip parameter on the velocity profiles within the boundary layer. From the figure it can be observed that the velocity profiles increase with a growing in values of slip parameter. The horizontal line which approaches to one (converges to 1) is an asymptote to the graph as $\eta$ tends to infinity ($\infty$). The boundary layer thickness of velocity profile is an increasing function for slip parameter ($\delta$). The boundary layer thickness decreases as the fluid velocity increases.

The influence of the stretching parameter $\gamma > 0$ on the velocity profile is displayed in figure 4. From the figure it can be observed that the velocity profiles within the boundary layer intensified with the rising values of the stretching parameter. Moreover, the velocity boundary layer thickness increases with an increasing stretching parameter. Figure 5 displayed that the effect of shrinking parameter on the velocity profile. The figure reveals that the velocity boundary layer width declines when the shrinking parameter $\gamma$ rises from $-0.01$ to $-0.1$.

Figures 6–11 displayed the impact of suction parameter ($A$) and injection parameter ($-A$) on velocity profiles, temperature profiles and concentration profiles. The figure 6 describes that the fluid velocity within the boundary layer upsurges whereas the thickness of the boundary layer reduces with the growing values of the suction parameter ($A$). This is because the removal of the decelerated fluid particles through the porous wedge surface reduces the growth of the boundary layer thickness. Figure 7 displays that the fluid velocity within the boundary layer diminishes with the growing in the values of injection parameter ($-A$). Figure 8 displays when the values of suction parameter ($A$) upsurges the thermal boundary layer thickness decreases. Figure 9 demonstrates that as the values of injection parameter ($-A$) increases from $-0.01$ to $-0.6$ temperature of the fluid within the boundary layer increases. Figure 10 and 11 reveals that the graph of concentration for different values suction parameter ($A$) and injection parameter ($-A$) respectively. The graphs show that when the
values of suction parameter \((A)\) raises the concentration profiles within boundary layer increase whereas with an increase of injection parameter \((-A)\) the concentration declines.

The effects of unsteady parameter \(\lambda_1\) on velocity, temperature and concentration profiles are displayed through figures 12–14. From the graphs we can see that as the values of unsteady parameter \(\lambda_1\) upsurges the concentration and velocity profiles are rises whereas the temperature profiles is declines within boundary layer. Figures 15–17 illustrates that the influence of Williamson parameter on velocity, concentration and temperature profiles. The graphs demonstrate that when the values of non-Newtonian Williamson parameter \(\lambda_2\) grows the concentration and temperature increases but opposite result is obtained for velocity within the boundary layer.

For different values of Prandtl number \((Pr)\) the graph of temperature and concentration is displayed through figures 18–19. From figure 18 it is found that as the values of \(Pr\) rises the temperature at the wedge surface decreases and the thermal boundary layer width shrinks. This shows that a fluid with higher \(Pr\) has relatively low thermal conductivity, which results in heat conduction and there by the thermal boundary layer.

| \(\beta\) | \(\lambda_1\) | \(\lambda_2\) | \(A\) | \(Ec\) | \(-f'(0)\) | \(-\theta'(0)\) | \(-\phi'(0)\) |
|---|---|---|---|---|---|---|---|
| 0.2 | 0.078400 | 0.535463 | 0.722917 |
| 0.4 | 0.241352 | 0.564698 | 0.772358 |
| 0.6 | 0.368578 | 0.575692 | 0.835371 |
| 0.8 | 0.469464 | 0.589358 | 0.916694 |
| 1.0 | 0.551879 | 0.591315 | 1.024316 |
| 0.0 | 0.392314 | 0.426145 | 0.589306 |
| 0.01 | 0.365288 | 0.432299 | 0.597823 |
| 0.02 | 0.337470 | 0.438380 | 0.606393 |
| 0.03 | 0.308686 | 0.444387 | 0.615013 |
| 0.04 | 0.279943 | 0.450319 | 0.623681 |
| 0.1 | 0.308668 | 0.444387 | 0.615013 |
| 0.5 | 0.322775 | 0.443652 | 0.614005 |
| 1.0 | 0.345883 | 0.442548 | 0.612500 |
| 1.5 | 0.382273 | 0.441085 | 0.610521 |
| 2.0 | 0.490683 | 0.438461 | 0.606843 |
| 0.01 | 0.308869 | 0.497249 | 0.702157 |
| 0.03 | 0.311218 | 0.501663 | 0.709569 |
| 0.05 | 0.313564 | 0.506072 | 0.717042 |
| 0.07 | 0.315905 | 0.510477 | 0.724578 |
| 0.09 | 0.318241 | 0.514877 | 0.732176 |
| 1.0 | 0.490683 | 0.514635 | 0.531957 |
| 2.0 | 0.490683 | 0.536659 | 0.514055 |
| 3.0 | 0.490683 | 0.546884 | 0.496152 |
| 4.0 | 0.490683 | 0.562711 | 0.478246 |
| 5.0 | 0.490683 | 0.578740 | 0.460339 |
thickness, and temperature drops. The concentration profile within the boundary layer increases with an increasing the value of Prandtl number ($Pr$) was displayed in figure 19. Near the wall of the wedge, the concentration growths with a growth in Pr and at some point away from the wall it starts falling before coming to a point far away from where it becomes to a stable position. The effect Eckert number ($Ec$) (viscous dissipation parameter) on temperature is plotted in figure 20. It is found that the temperature profile is increased with an increase of Eckert number ($Ec$). This is due to the fact that heat energy is stored in the liquid due to the frictional heating. The impact of rising Eckert number is to increase the temperature at any point. Figure 21 is plotted to indicate the effect of radiation parameter ($R$) on temperature profiles. It is observed that the temperature profile
decreases for increasing values of $R$. This is due to the fact that an increase in the radiation parameter $R$ leads to decrease in the boundary layer thickness and enhances the heat transfer rate.

A ratio of the hot fluid side convection resistance to the cold fluid side convection resistance on a surface is the heat convection parameter (Biot number). From this fact we observed that when the values of heat

![Figure 7. Velocity graph for different values of Injection parameter.](image7)

![Figure 8. Temperature graph for different values of Suction parameter.](image8)

![Figure 9. Temperature graph for different values of Injection parameter.](image9)
convection parameter upsurge the hot fluid side convection resistance reduces and as a result the surface temperature rises. Accordingly, figure 22 reveals that with an increase of Biot number the temperature profiles within the boundary layer raises. The impact of heat convection parameter on concentration profiles is plotted in figure 23. As the values of heat convection parameter raises the concentration profiles upsurges within boundary layer.

In figures 24–26 the effect of wedge angle parameter was plotted. From figure 24 we observed that as the value wedge angle parameter raises the velocity increased within boundary layer. This displays that velocity
boundary layer width reduced with an increase of wedge angle parameter ($\beta$). Figures 25 and 26 illustrates that with an increment of wedge angle parameter ($\beta$) the profiles of temperature and concentration decreases. The temperature and concentration graphs with various values of Brownian motion ($Nb$) are depicted in figures 27–28. Figure 27 indicates that when Brownian motion ($Nb$) parameter increases, the movement of nanoparticles from the hot surface to cold surface is happened and ambient fluid occurred. Due to this the
temperature and thermal boundary layer thickness rises. From figure 28 we see that when Brownian motion (\(\text{Nb}\)) upsurges volume fraction of nanoparticles within the boundary layer upsurges. It is interesting to note that Brownian motion of the nanoparticles at the molecular and nanoscale levels is a key mechanism in governing their thermal behavior. In nanofluid system, due to the size of the nanoparticles, Brownian motion affects the
heat transfer properties. As the particle size scale approaches to the nano-meter scale, Brownian motion on the surrounding liquids play an important role in heat transfer. The influence of Thermophoresis on concentration and temperature was displayed in figures 29–30. The graphs indicated that when the value of Thermophoresis increases both temperature and concentration profiles and boundary layer thickness are increased. This is
Figure 22. Temperature graph for different values of Biot Number.

Figure 23. Concentration graph for different values of Biot Number.

Figure 24. Velocity graph for different values of Wedge angle parameter.
Figure 25. Temperature graph for different values of Wedge angle parameter.

Figure 26. Concentration graph for different values of Wedge angle parameter.

Figure 27. Temperature graph for different values of Brownian motion parameter.
because of the fact that temperature difference between surface and ambient growths for larger thermophoresis parameter and hence the fluid temperature accelerates for both cases.

Figure 31 displays the effects of Chemical reaction parameter on the nanoparticles volume fraction profile. From this figure we have observed that increasing the value of the chemical reaction reduces the concentration
profile in the boundary layer. The physical reason of this phenomenon is negative chemical reaction declines the concentration boundary layer thickness and upsurges the mass transfer. The concentration curves for different values of Lewis number ($Le$) have been shown in figure 32. From the definition of Lewis number, it is clear that an increase in Lewis number ($Le$) may be because of a larger thermal diffusivity of the fluid for a constant mass diffusivity. This causes an increase of the flow within the boundary layer. Therefore, the graph reveals that when Lewis number ($Le$) increases the concentration upsurges at the wedge wall and at some point away from the wall it starts falling before coming to a stable position.

5. Conclusions

This study presents a mathematical model of unsteady boundary layer flow of Williamson nanofluid past a stretching/Shrinking permeable wedge with thermal radiation, viscous dissipation and chemical reaction. The study extended the previous work of Wubshet and Ayele [19] by taking into consideration the effects of unsteadiness parameter, Williamson fluid, chemical reaction, slip effect and the passively controlled boundary condition. Fluid suction/injection is imposed on the wedge surface. The governing unsteady non-linear ordinary differential equations (ODEs) are reduced to a set of a non-linear differential equations (DEs) by introducing appropriate similarity transformations and then solved numerically by using Keller Box scheme with MATLAB software (R2013a) for different values of the parameters. The obtained numerical results are in good agreement with the previously published data in limiting condition. The numerical results of velocity, temperature and concentration for the dimensionless parameters are presented graphically. As well as the numerical values of local skin friction, local Nusselt number and local Sherwood number are presented in tabular form. Depending on these the result of the study is summarized as follows:
1. Increasing suction \( A \) and unsteady \( \lambda_2 \) parameters raises both the velocity and concentration profiles near the surface of the wedge but it diminishes the thermal boundary layer thickness.

2. Both temperature and concentration profiles are declining function within the boundary layer but the velocity boundary layer thickness enhanced for raising the values of \( \beta \) parameter.

3. For growing the values of \( Bi \) and \( Nt \) the thermal boundary layer thickness and the concentration profiles growths near the wedge surface.

4. The temperature profile declines within the boundary layer and the concentration profile enhanced near the wedge surface for rising values of \( Pr \).

5. When the values of thermal radiation and Eckert number rises, the temperature profiles are amplified.

6. As the values of chemical reaction upsurges the concentration profiles diminishes but the concentration profiles grows with a rise of Lewis number.

7. For increasing the values of \( Nb \) both the temperature and concentration profiles are increased.

8. The skin friction coefficient rises with growing values of \( \lambda_2 \), \( \beta \) and \( A \). But it declines for enlargement values of \( \lambda_1 \).

9. The heat transfer rate \((-\theta'(0))\) and the mass transfer rate \((-\phi'(0))\) enhances with rising values of \( \beta \), \( A \) and \( \lambda_1 \).

10. The heat transfer rate is improved for rising values of \( Ec \) but it declines for rising values of \( \lambda_2 \).

11. The mass transfer rate reduces for an increase of \( \lambda_2 \) and \( Ec \).

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**References**

[1] Falkner V M and Skan S W 1931 Some Approximate Solutions of Boundary Layer Equations *Philosophical Magazine and Journal of Science*. 12 865–96

[2] Seddeek M A, Afify A A and Al- Hanaya A M 2009 Similarity solutions for a steady MHD falkner–skan flow and heat transfer over a wedge considering the effects of variable viscosity and thermal conductivity *An International Journal Applications and Applied Mathematics*. 4 301–13

[3] Martin M J and Boyd I D 2009 Falkner–Skan flow over a wedge with slip boundary conditions *American Institute of Aeronautics and Astronautics*

[4] Alam M S, Ali M, Alim M A, Haque Munshi M J and Uddin Chowdhury M Z 2017 Solution of falkner–skan unsteady MHD boundary layer flow and heat transfer past a moving porous wedge in a nanofluid *Procedia Engineering*. 194 414–20

[5] Sattar M A 2011 A local similarity transformation for the unsteady two-dimensional hydrodynamic boundary layer equations of a flow past a wedge *Int. J. Appl. Math. and Mech.* 7 15–28

[6] Devi B M and Gangadhar K 2015 Effect of viscous dissipation on Falkner–skan boundary layer flow past a Wedge through a porous medium with slips boundary condition *International Journal of Engineering Inventions*. 4 21–35

[7] Mohammadi F, Hosseini M M, Debghahn A and Maalek Ghaini F M 2012 Numerical solutions of falkner–skan equation with heat transfer *Studies in Nonlinear Sciences*. 3 86–93

[8] Ali M and Alim M A 2018 Boundary layer analysis in nanofluid flow past a permeable moving wedge in presence of magnetic field by using falkner–skan model *Int. J. of Applied Mechanics and Engineering*. 23 1005–13

[9] Mohammadi F, Kandasamy R and Mohaimin I 2013 Enhance of heat transfer on unsteady Hiemenz flow of nanofluid over a porous wedge with heat source sink due to solar energy radiation with variable stream condition *Heat Mass Transfer*. 231 1–9

[10] Kandasamy R, Mohaimin I and Rosmila A J 2014 The performance evaluation of unsteady MHD non-Darcy nanofluid flow over a porous wedge due to renewable (solar) energy *Renewable Energy*. 64 1–9

[11] Rahman A T M M, Alam M S, Chowdhury M K and Rahman M M 2011 Unsteady two-dimensional convective heat and mass transfer flow along a wedge with thermophoresis *International Conference on Mechanical Engineering*. 13

[12] Alam M S, Islam T and Rahman M M 2015 Unsteady hydromagnetic forced convective heat transfer of a micropolar fluid along a porous wedge with convective surface boundary condition *Int. J. Heat Technol.* 33

[13] Mohaimin I, Kandasamy R and Khamis Azme B 2013 and Rosalan Rozaini. In *Influence of thermophoresis particle deposition and chemical reaction on unsteady non-Darcy MHD mixed convective flow over a porous wedge in the presence of temperature dependent viscosity* *Mech. Sci.* 27 1545–55

[14] Ali M, Alim M A, Nasrin R, Alam M S and Haque Munshi M J 2017 Similarity solution of unsteady MHD boundary layer flow and heat transfer past a moving wedge in a nanofluid using the buongiorno model *Procedia Engineering*. 194 407–13

[15] Rahman A T M M, Alam M S, Alim M A and Chowdhury M K 2013 Unsteady MHD forced convective heat and mass transfer flow along a wedge with variable electric conductivity and thermophoresis *Procedia Engineering*. 56 531–7

[16] IlIAS M R, Ismail N S, Siah M, Esah W S and Hussain C 2018 Unsteady aligned MHD boundary layer flow of a magnetic nanofluid over a wedge *International Journal of Civil Engineering and Technology (IJCIET)* 9 794–810
Xu X and Chen S 2017 Dual solutions of a boundary layer problem for MHD nanofluids through a permeable wedge with variable viscosity. *Xu and Chen Boundary Value Problems*. 2017:174

Goria R S R and Kumari M 2012 Mixed convective boundary layer flow over a vertical wedge embedded in a porous medium saturated with a nanofluid. *Nano Engineering and Nano Systems*. 225:55–66

Ibrahim W and Turu A 2019 Magnetohydrodynamic (MHD) boundary layer flow past a wedge with heat transfer and viscous effects of nanofluid embedded in porous media. *Mathematical Problem in Engineering*. (https://doi.org/10.1155/2019/4507852)

Williamson R V 1929 The flow of pseudo plastic materials. *Ind. Eng. Chem. Res.* 21:1108

Nadeem S, Hussain S T and Lee C 2013 Flow of a Williamson fluid over a stretching sheet. *Brazilian Journal of Chemistry Engineering*. 30:519–25

Nadeem S and Hussain S T 2014 Flow and heat transfer analysis of Williamson nanofluid. *App. Nanosci* 4:1005–12

Abegunnin O A and Animasun I L 2017 Motion of Williamson fluid over an upper horizontal surface of a paraboloid of revolution due to partial slip and buoyancy. *Defect and Diffusion Forum*. 378:16–27

Kumaran G, Sandeep N and Vijayaraghavan R 2017 Melting heat transfer in magnetohydrodynamic radiative Williamson fluid flow with non-uniform heat source. *Materials Science and Engineering*. 263:062022

Nagendra N, Amanulla C H and Sudhakar Reddy M 2018 Slip effects on MHD flow of a Williamson fluid from an isothermal sphere. *AMSE Journals*. 86:782–807

Ibrahim W and Gamachu D 2019 Nonlinear convection flow of Williamson nanofluid past a radially stretching surface. *AIP Adv.* 9:085026

Hashim A, Hamid and Khan M 2019 Multiple solutions for MHD transient flow of Williamson nanofluids with convective heat transport. *Taiwan Inst. Chem. Eng.* 103:126–37

Hamid A, Khan M and Alishamroni A S 2019 Non-linear radiation and chemical reaction effects on slip flow of Williamson nanofluid due to a static/moving wedge. *Applied Nano Science*. (https://doi.org/10.1007/s13204-019-01172-5)

Talha M A, Osman Gani M and Ferdows M 2018 Numerical solution of steady Magnetohydrodynamic boundary layer flow due to Gyrotactic microorganisms for Williamson nanofluid over stretched surface in the presence of exponential internal heat generation in *International Journal of Mathematical and Computational Sciences*. 12

Choi S U and Eastman J A 1995 Enhancing thermal conductivity of fluids with nanoparticles, presented at ASME International Mechanical Engineering Congress and Exposition. *1217* (No. ANL/MSD/CP-84938; CONF-951135-29) Argonne National Lab., IL (United States)

Gireesh B J, Ganesh Kumar K, Ramesh G K and Prasannamukumara B C 2018 Nonlinear convective heat and mass transfer of Oldroyd-B nanofluid over a stretching sheet in the presence of uniform heat source. *Sink Results in Physics*. 9:1555–63

Sulochana C, Ashwinkumar G P and Sandeep N 2016 Similarity solution of 3D Casson nanofluid flow over a stretching sheet with convective boundary conditions. *Journal of the Nigerian Mathematical Society*. 35:128–41

Mahanthesh B, Gireesh B J and Goria R S R 2016 Nonlinear radiative heat transfer in MHD three-dimensional flow of water based nanofluid over a non-linearly stretching sheet with convective boundary condition. *Journal of the Nigerian Mathematical Society*. 35:178–98

Ramana Reddy J V, Sugunamma V and Sandeep N 2018 Thermophoresis and Brownian motion effects on unsteady MHD nanofluid flow over a slendering stretching surface with slip effects. *Alexandria Engineering Journal*. 57:2465–73

Plum O A, Huenfeld J S and Eschbach E J 1981 The effect of cross flow and radiation on natural convection from vertical heated surfaces in saturated porous media. *Proc. of the AIAA*. pp 23–5

Anuradha S and Yegammai M 2017 MHD radiative boundary layer flow of nanofluid past a vertical plate with effects of binary chemical reaction and activation energy. *Global Journal of Pure and Applied Mathematics*. 13:6377–92

Majeed A, Zeeshan A and Noori F M 2019 Numerical study of darcy-forchheimer model with activation energy subject to chemically reactive species and momentum slip of order two. *AIP Adv.* 9:045035

Kh A M 2013 Effects of exothermic/endothermic chemical reactions with arrenius activation energy on MHD free convection and mass transfer flow in presence of thermal radiation. *Journal of Thermodynamics*. 69:2516

Sajid T, Sagheer M, Hussain S and Bilal M 2018 Darcy–forchheimer flow of maxwell nanofluid flow with nonlinear thermal radiation and activation energy. *AIP Adv.* 8:035102

Hayat T, Shafiq A and Alsaedi A 2016 Hydromagnetic boundary layer flow of Williamson fluid in the presence of thermal radiation and ohmic dissipation. *Alexandria Engineering Journal*. (https://doi.org/10.1016/j.aej.2016.06.004)

Kh A M, Haneef M, Shah Z, Islam S, Khan W and Muhammad S 2018 The combined magneto hydrodynamic and electric field effect on an unsteady maxwell nanofluid flow over a stretching surface under the influence of variable heat and thermal radiation. *Appl. Sci*. 8:1–17

Kh A M, Nie Y and Shah Z 2019 Impact of thermal radiation and heat source/sink on MHD time-dependent thin-film flow of oldroyd-B, Maxwell, and Jeffery fluids over a stretching surface. J. Phys. C 7:1–18

Ahmed S E, Mohamed R A, Aly A E M and Soliman M S 2019 Magnetohydrodynamic maxwell nanofluids flow over a stretching surface through a porous medium with the effects of non-linear thermal radiation, convective boundary conditions and heat generation/absorption. *International Journal of Mechanical, Industrial and Aerospace Sciences*. 13

Ramesh G K, Ganesh Kumar K, Shethad S A and Gireesh B J 2017 Enhancement of radiation on hydromagnetic Casson fluid flow towards a stretched cylinder with suspension of liquid-particles. *Can. J. Phys.* 1555:2017–0307. R1:1–24

Rajput U S and Kumar G 2017 Radiation effect on MHD flow past an oscillating inclined plate with heat and mass transfer. *Journal of Mathematics and Informatics*. 09

Rashad A M, Modather M, Abdou M and Ali Chamkha. 2011 MHD free convective heat and mass transfer of a chemically-reacting fluid from radius stretching surface embedded in a saturated porous medium. *Int. J. Chem. Reactor Eng*. 9:1–15

Watanabe T 1990 Thermal boundary layer over a wedge with uniform suction and injection in forced flow. *Acta Mech*. 83:119–26