On the Injection Energy Distribution of Ultra-High-Energy Cosmic Rays

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Abstract

We investigate the injection spectrum of ultra-high-energy (> $10^{15}$ eV) cosmic rays under the hypotheses that (1) these cosmic rays are protons and (2) the sources of these cosmic rays are extra-galactic and are homogeneously distributed in space, although they may have had a different strength in the past; furthermore, we assume that we are not unusually close to any individual source(s). The most puzzling aspect of the observed ultra-high-energy cosmic ray spectrum is the apparent nonexistence of a “Greisen cut-off” at about $10^{19.8}$ eV. Such a cut-off would be expected due to rapid energy loss from photopion production caused by interactions with the microwave background. We show that this fact could be naturally explained if most (or all) of the cosmic rays presently observed above about $10^{19.6}$ eV were initially injected with energy above the Greisen cut-off. However, we find that the injection of cosmic rays above the Greisen cut-off cannot account for the observed flux below about $10^{19.6}$ eV unless the injection rate of these particles was enormously higher in the past, as would be the case if the injection resulted from the decay of an ultra-massive particle with lifetime of order $10^9$ yr.
Even with such a rapid source evolution, the observed cosmic ray spectrum below about $10^{18.5}$ eV cannot be explained by injection of particles above the Greisen cut-off in the distant past. However, we show that a $1/E^3$ injection spectrum can account for the observed spectrum below $10^{18.5}$ eV, with the steepening observed by the Fly’s Eye group between $10^{17.6}$ eV and $10^{18.5}$ eV being very naturally explained by $e^+e^-$ production effects. This latter fact lends support to the hypothesis that the cosmic rays in this energy regime are protons. However, due to $e^+e^-$ production effects, a $1/E^3$ injection spectrum cannot account for the observed flux above about $10^{18.5}$ eV.
I. INTRODUCTION

The existence of ultra-high-energy cosmic ray particles is certainly one of the most remarkable phenomena observed in nature. Their existence at energies above about $10^{19.8} \text{ eV}$\(^1\) is particularly mysterious since, if the cosmic rays are protons, photopion production caused by interactions with the microwave background should result in a rapid loss of energy and consequent depletion of the observed flux of these particles \([3,4]\). Similar serious difficulties in accounting for the existence of such cosmic rays occur under the various alternative hypotheses concerning the nature of the cosmic ray particles; see, e.g., the discussions in \([3]\) and \([6]\).

It seems clear that the explanation of the existence of the highest energy cosmic rays will require some unconventional ideas, or, at least, the extrapolation of conventional ideas to unconventional extremes. In the absence of a reliable theoretical framework, it is difficult to make arguments concerning the plausibility of various hypotheses. Furthermore, the experimental data on the highest energy cosmic rays suffers from poor statistics as well as from significant uncertainties in energy determinations, so there are very few “facts” that can be pinned down with complete certainty.

Nevertheless, under suitable hypotheses about the nature of the cosmic ray particles and the distribution of their sources in space and time, it is possible, in principle, to say a great deal about the energy distribution which the cosmic rays must have possessed at the time they were injected into the universe, since the energy loss rate of cosmic rays is governed by well established physics. In this paper, we shall investigate the injection energy distribution of cosmic rays of present energy $> 10^{15} \text{ eV}$ under the hypotheses that

1. these cosmic rays are protons \(^1\)

\(^1\)It should be noted, however, that some (model dependent) evidence was reported in \([7]\) that the composition of the the cosmic rays below $10^{18.5} \text{ eV}$ tends toward heavy nuclei.
2. the sources of these cosmic rays are extra-galactic and are homogeneously distributed in space. Furthermore, we are not unusually close to any individual source(s) so that the cosmic ray flux we see is representative of that occurring elsewhere in the universe.

In our analysis we allow for the possibility that the sources may have been more numerous (or less numerous) in the past but we assume that the shape of the injection energy spectrum of the sources does not vary with time. However, except for the decaying particle model considered at the end of Section III, source evolution will not play an important role in any of our arguments. Note that our assumptions automatically give rise to a homogeneous, isotropic cosmic ray population. Consequently, under our hypotheses, the possible presence of an intergalactic magnetic field will have no effect upon the cosmic ray spectrum.

Even under these assumptions we cannot, in principle, uniquely determine the injection spectrum, since the presently observed spectrum depends upon both the injection spectrum and the time history of the sources. Furthermore, as already mentioned above, the detailed structure of the observed spectrum at the highest energies is quite uncertain, so even if a mathematical inversion could be done, it probably would not have much significance. Nevertheless, we shall see that some quite general, nontrivial constraints on the injection spectrum and source evolution can be obtained.

Under our two assumptions above, the following two things can be concluded immediately about the cosmic ray injection spectrum: (a) In the energy range from about $10^{15}$ eV to about $10^{17.6}$ eV, the observed spectrum is well fit by a $1/E^3$ power law. In this energy range, the only significant energy loss mechanism for protons is cosmological redshift. Since redshift takes a power law spectrum (injected at any initial time) to a power law of the same power, we can conclude that in this “low energy” regime, the injection spectrum of the protons follows a $1/E^3$ power law. (b) Above about $10^{19.8}$ eV in the present universe, protons rapidly lose energy due to photopion production reactions with the microwave background. Nevertheless, cosmic rays are observed above this “Greisen cut-off”. Indeed, the observations of two events well above $10^{20}$ eV have been reported recently [1,2]. Thus, assuming, as above,
that these cosmic rays are protons, we can conclude that, in the present universe, protons are being injected with energies above the Greisen cut-off at a rate that is directly calculable from the observed flux and the photopion production energy loss rate (see Section II).

The injection rate of above-Greisen-cut-off protons obtained from (b) lies far above extrapolations of the $1/E^3$ injection spectrum deduced from (a). Thus, there is no reason to expect that the “low energy” (i.e., between $10^{15}$ eV and $10^{17.6}$ eV) and highest energy (i.e., $>10^{19.8}$ eV) cosmic rays are produced by a common mechanism. Hence, we shall presume that there are (at least) the following two independent sources of cosmic rays (though we do not exclude the possibility that these sources correspond to the same physical objects or phenomena): (i) A “low energy” source which injects protons with a $1/E^3$ power law. (ii) A “high energy” source which injects protons with energies above the Greisen cut-off. Our main goal in this paper is to investigate how far one can go toward explaining the entire high energy cosmic ray spectrum by (simple extrapolations of) these two sources.

Our main results are the following. First, in Section III, we investigate the contributions of the high energy source. Specifically, we compute the cosmic ray spectrum which would be observed if protons with energies above the Greisen cut-off are injected at a rate corresponding to the presently observed above-Greisen-cut-off flux, and if this rate does not vary with time. We find that the predicted spectrum is quite compatible with the observed spectrum at energies down to about $10^{19.6}$ eV. This implies that, under our two assumptions above, many—and, quite possibly, all—of the cosmic rays with energies above $10^{19.6}$ eV were initially injected with energy above the Greisen cut-off. However, the predicted spectrum below $10^{19.6}$ eV lies well below the observed spectrum. We cannot significantly improve the predicted spectrum by assuming that the high energy source injects protons with a power law spectrum (rather than injecting all the protons at energies above the Greisen cut-off), since a hard injection spectrum (say, $1/E^2$ or harder) also gives poor agreement below $10^{19.6}$ eV, whereas a soft injection spectrum (say, $1/E^2$ or softer) produces a dramatic Greisen cut-off. However, we show that good agreement with the observed spectrum down to an energy of about $10^{18.5}$ eV would be obtained if the injection rate of protons per comoving volume was
significantly higher in the past. Indeed, to get good agreement it is necessary for the high energy source to be about 200 times stronger at a redshift $z = 1/2$. This is much too rapid an increase in the injection rate to plausibly attribute to increased activity of active galactic nuclei or other possible astrophysical sources of cosmic rays. However, if the injection of above-Greisen-cut-off protons results from the decay of an ultra-massive particle, the required source strengthening would correspond to a lifetime of about $10^9$ yr. The existence of such a particle with this lifetime and with the mass and abundance required to produce the highest energy cosmic rays appears to be compatible with known constraints, though remarkably close to the limit obtained from the gamma ray background [8].

Second, in Section IV, we investigate the contributions of the low energy source to the observed cosmic ray spectrum. Specifically, we analyze the extent to which an extrapolation of the $1/E^3$ injection spectrum to arbitrarily high energies could account for observed spectrum above $10^{15}$ eV. We show that the steepening in the spectrum observed by the Fly’s Eye group [4] between about $10^{17.6}$ eV and $10^{18.5}$ eV is naturally accounted for by $e^+e^-$ production effects. However, we find that the cosmic ray flux between $10^{18.5}$ eV and $10^{19.6}$ eV is significantly depleted by these effects, so a $1/E^3$ (or softer) injection spectrum cannot explain the observed spectrum above about $10^{18.5}$ eV.

In Section II we describe the methods and approximations used to calculate the relationship between the injected and observed cosmic ray spectra. The contribution to the observed cosmic ray spectrum of protons initially injected with energy above the Greisen cut-off is analyzed in Section III and the contribution from protons initially injected with a $1/E^3$ spectrum is analyzed in Section IV. Our conclusions are summarized in Section V.

II. METHODS

There are three potentially significant causes of energy loss for ultra-high energy cosmic ray protons propagating in intergalactic space: (1) Cosmological redshift, (2) $e^+e^-$ pair production scattering with the cosmic microwave background radiation (CMBR), and (3) pion
production scattering with the CMBR. These effects have been calculated previously by many authors [3,4,10–15] under various approximations and computational schemes. The purpose of this section is to describe our calculation of these effects in some detail.

The energy loss due to cosmological redshift is a continuous process, which is governed by a simple differential equation. The energy loss due to $e^+e^-$ pair production is a stochastic process, but the pair production cross-section is high and the fractional energy lost per scattering event is very small (on the order of $2m_e/m_p \simeq 10^{-3}$ at threshold and decreasing thereafter). Hence, a continuous, mean energy loss approximation should be excellent for treating the effects of process (2), as we have verified by comparing the results of our calculations to the Monte Carlo calculations of Yoshida and Teshima [14] (see subsection II C below). On the other hand, for process (3), the fraction of energy lost per pion production event is 0.13 at threshold and rises thereafter [3], so, in general, statistical fluctuations are of importance for calculating the effects of pion production scattering. However, in this paper, we restrict consideration to spatially homogeneous and temporally continuous injection, which provides a smoothing that mimics the smoothing provided by the statistical fluctuations. Consequently, in our case, the continuous, mean energy loss approximation [12] should be adequate for treating process (3), as we have verified by comparing our results with a Monte Carlo calculation provided to us by F. Aharonian (see below). We now describe in more detail the formulas we used to calculate the energy loss rate of cosmic ray protons.

A. Energy loss processes

1. Cosmological redshift

We take our cosmological model to be a standard, matter-dominated, Robertson-Walker universe, with no cosmological constant. As we shall see below, for the source time evolutions we consider, the contribution of cosmic rays injected at redshift, $z$, greater than about 1/2 will not be of great importance. Hence, our results should not be very sensitive to the
precise value of the closure parameter, $\Omega$, and we shall simply set $\Omega = 1$, corresponding to a spatially flat universe with scale factor $a(t)$ given by

$$\frac{a(t)}{a(t_0)} = \left(\frac{t}{t_0}\right)^{2/3}. \tag{1}$$

Our calculations are somewhat sensitive to the assumed value of Hubble’s constant. For most of our calculations, we assumed that the universe is 15 billion years old ($t_0 = 4.7 \times 10^{17}$ s), corresponding to a “low” value of Hubble’s constant ($H_0 = 43.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$). However, we also recalculated most effects using a Hubble constant of 75 km s$^{-1}$ Mpc$^{-1}$ in order to verify that our conclusions did not depend sensitively on $H_0$.

The energy of a relativistic particle propagating through the universe scales inversely with $a(t)$, so we have an energy loss rate due to cosmological redshift given by

$$\left(\frac{dE_p}{dt}\right)_{rs} = -\frac{2E^3}{3t}, \tag{2}$$

In particular, it should be noted that the attenuation length for cosmological redshift is independent of energy.

2. Pair production scattering

Calculations of proton energy loss due to pair production scattering with the CMBR have been performed by Blumenthal [10], by Berezinsky and Grigor’eva [12], and others. We were unaware of this prior work when we began our investigations, and derived our mean energy loss formula independently. Since our formula differs slightly from that of other authors, we present the derivation of it here.

In the “lab frame” (i.e., the isotropy frame of the CMBR), the photons of the CMBR have a number density per energy per solid angle per volume given by the standard Planck formula

$$n(E) dE \sin \theta \, d\theta \, d\phi \, dV = \frac{1}{(2\pi hc)^3} \frac{2E^2}{e^{E/kT} - 1} dE \, \sin \theta \, d\theta \, d\phi \, dV \tag{3}$$
In the rest frame of a proton travelling with velocity parameter $\beta$ in the $+z$ direction, this distribution corresponds to
\[
\bar{n}(\bar{E}, \bar{\theta}) \, d\bar{E} \, \sin \bar{\theta} \, d\bar{\theta} \, d\bar{\phi} \, d\bar{V} = \frac{1}{(2\pi \hbar c)^3} \frac{2\bar{E}^2}{\exp \left[ \frac{\bar{E}}{kT} (1 + \beta \cos \bar{\theta}) \right] - 1} \, d\bar{E} \, \sin \bar{\theta} \, d\bar{\theta} \, d\bar{\phi} \, d\bar{V}.
\]  
(4)

where the bars denote the corresponding variables in the proton frame. Since we are only interested in ultra-relativistic protons, this distribution will be very sharply peaked about $\bar{\theta} = \pi$, i.e., essentially all of the blackbody photons will be incident head-on in the proton frame. Integrating over the angles, we obtain $f(\bar{E})$, a number density per energy per volume in the proton frame,
\[
f(\bar{E}) \, d\bar{E} \, d\bar{V} = \frac{4\pi}{(2\pi \hbar c)^3} \frac{(kT)^2 \xi}{\beta} \ln \left( \frac{1 - \exp[-\xi(1 - \beta)^{-1}]}{1 - \exp[-\xi(1 + \beta)^{-1}]} \right) \, d\bar{E} \, d\bar{V},
\]  
(5)

where $\xi = \bar{E}/\gamma kT$. Finally, we take the ultra-relativistic limit ($\beta \to 1$), giving us
\[
f(\bar{E}) = \frac{4\pi}{(2\pi \hbar c)^3} (kT)^2 \xi \ln \left( \frac{1}{1 - e^{-\xi/2}} \right).
\]  
(6)

We now drop the bars on the proton frame variables and denote the energy of an incident photon as $E_\gamma$. In the limits that $\beta \approx 1$ but $E_\gamma \ll m_p c^2$, the energy loss of the proton in the lab frame due to a single scattering event is given by
\[
E_{\text{loss}} = \gamma p_p c \cos \theta,
\]  
(7)

where $p_p$ is the recoil momentum of the proton in the proton frame and $\theta$ is the recoil angle.

The probability distributions of these recoil variables as a function of $E_\gamma$ were provided by a second order QED calculation given by Jost et al. [16]. They obtained the following expression for the differential cross-section as a function of the recoil variables (see their Eq. (47))
\[
\frac{d\sigma}{dQ d\eta} = \frac{\alpha r^2}{\epsilon^2} \frac{1}{Q^2} \left\{ \ln \frac{1 - \omega}{1 + \omega} \left[ \left(1 - \frac{\epsilon^2}{\eta^2}\right) \left(1 - \frac{1}{4\eta^2} + \frac{1}{2\eta Q} - \frac{1}{8Q^2}\right) - \frac{Q}{\eta} + \frac{Q^2}{2\eta^2} \right] + \frac{\epsilon^2}{2\eta^4} \right\} + \omega \left[ \left(1 - \frac{\epsilon^2}{\eta^2}\right) \left(1 - \frac{1}{4\eta^2} + \frac{1}{2\eta Q} \right) + \frac{1}{\eta^2} \left(1 - \frac{2\epsilon^2}{\eta^2}\right) \left(1 - 2Q\eta + Q^2\right) \right]
\]  
(8)

where $\epsilon = E_\gamma/2m_e c^2$, $Q = p_p/2m_e c$, $\eta = \epsilon \cos \theta$, and $\omega = [1 - 1/(2Q\eta - Q^2)]^{1/2}$. In terms of these $\epsilon$, $Q$, and $\eta$ variables, our single event lab frame energy loss is given by
\[ E_{\text{loss}} = \frac{2m_e c^2 \gamma}{\epsilon} Q \eta. \]  

(9)

Therefore, at a given photon energy \( \epsilon \) we obtain a mean energy loss by integrating \( E_{\text{loss}} \) against the differential cross section and then dividing by total cross section.

\[
\langle E_{\text{loss}} \rangle = \frac{2m_e c^2 \gamma}{\epsilon} \int_1^{\infty} d\eta \int_{\eta-\sqrt{\eta^2-1}}^{\eta+\sqrt{\eta^2-1}} dQ \left( Q \eta \frac{d\sigma}{dQd\eta} \right)
\]  

(10)

where the limits of the integrals are set by kinematic constraints.

Finally, to calculate \( (dE_p/dt)_{e^+e^-} \), we multiply our number density \( f(E_\gamma) \) by \( c \) to produce a flux of photons per photon energy and integrate against \( \langle E_{\text{loss}} \rangle \sigma \), remembering to insert a factor of \( 1/\gamma \) to convert from an event rate in the proton frame to an event rate in the lab frame:

\[
\left( \frac{dE_p}{dt} \right)_{e^+e^-} = -c \int_2^{\infty} \langle E_{\text{loss}} \rangle \sigma f \; dE_\gamma.
\]  

(11)

Our energy loss formula differs slightly from that of Blumenthal [10] in that we used the exact expression for a highly blueshifted black body distribution, and it also differs slightly from that of Berezinsky and Grigor’eva [12] in that they made some approximations to the formula for the cross-section. However, we obtained excellent agreement with the results of both Blumenthal and Berezinsky and Grigor’eva when we evaluated the energy loss rate numerically for a range of proton energies at CMBR temperature \( T = T_0 = 2.73 \) K.

3. Pion production scattering

A formula for the energy loss due to photopion production can be obtained in parallel with our above derivation for the energy loss due to \( e^+e^- \) production. However, analytic expressions for the differential cross section as a function of recoil momentum are unavailable for pion production scattering, so one is forced to rely on experimental data. In our calculations, we used the approximations due to Berezinsky and Grigor’eva [12], which make use of the fact that, at the proton energies of relevance here, the photopion production predominantly occurs near threshold. Hence, it should be a reasonable approximation to treat
the photopion production as being isotropic in the rest frame of the proton, and to treat
the total cross-section as a linearly rising function of energy, i.e., \( \sigma(E) = (E - \varepsilon_{th}) \sigma' \). This
yields the following simple formula for the energy loss rate, \( \frac{dE_p}{dt}_\pi \), due to photopion
production in terms of the single parameter \( \sigma' \) (cf. Eq. (8) of [12])

\[
\left( \frac{dE_p}{dt} \right)_\pi = \frac{2(kT)^3 \sigma' \varepsilon_0^2 \gamma}{\pi^2 c^4 \hbar^3} \exp\left( -\frac{\varepsilon_0}{2\gamma kT} \right)
\]

(12)

where \( \varepsilon_0 = m_\pi (1 + m_\pi/m_p) \). Berezinsky and Grigor’eva used \( \sigma' = 6.8 \times 10^{-36} \text{ cm}^2/\text{eV} \) for
their calculations, but we used \( \sigma' = 3.45 \times 10^{-36} \text{ cm}^2/\text{eV} \) in order to more closely approximate
the cross section data given by Hikasa et al. [17].

As a check on the validity of this approximation we plotted the attenuation length as a
function of proton energy at \( T = T_0 \) through the range where pion production dominates
(see Figure 1) and obtained excellent agreement with the Monte Carlo results of Yoshida
and Teshima [14] up to a proton energy of \( 10^{21} \text{ eV} \).

**B. Calculation of the present spectrum of cosmic rays**

The present energy spectrum of cosmic ray protons is determined from the injection rate
of protons per comoving volume per energy per time, \( I(t, E) \), by integrating the mean energy
loss equation

\[
\frac{dE_p}{dt} = \left( \frac{dE_p}{dt} \right)_{\text{rs}} + \left( \frac{dE_p}{dt} \right)_{e^+e^-} + \left( \frac{dE_p}{dt} \right)_{\pi},
\]

(13)

where \( E_p(t) \) is the energy of the proton. The expressions for the various terms on the right
side were given in the previous subsection. The information contained in this equation is
most usefully encoded by expressing the initial energy, \( E' \), in terms of the energy today, \( E \),
and the injection time, \( t \); that is, by finding \( E'(E, t) \). This we did by numerically solving (13)
with the boundary condition \( E_p(t_0) = E \).

The calculations of this paper assume that the sources of cosmic rays are homogeneously
distributed throughout the universe. In addition, we assume that \( I(t, E) \) is of the form
\( I(t, E) = h(t)g(E) \), so that the only time dependence in the injection spectrum is the overall rate of injection. (This would be the case if the nature of the sources of the cosmic-rays did not change but the number of such sources did.) By definition, the total number of protons per comoving volume element, \( dn \), injected between times \( t \) and \( t + dt \) with energies between \( E' \) and \( E' + dE' \) is

\[
\frac{dn}{dt} = h(t)g(E') \, dt \, dE'.
\] (14)

Hence, the present number density spectrum of cosmic rays arising from those injected between times \( t \) and \( t + dt \) is given by

\[
\frac{dn}{dE} = h(t)g(E') \frac{\partial E'}{\partial E} \, dE \, dt,
\] (15)

where \( E'(E, t) \) is obtained from integration of (13) as described above. This expression can then be integrated over all times to give the comoving number density of protons today. Since these protons are distributed homogeneously and isotropically and are all travelling at approximately \( c \), the differential flux per energy per time per steradian today, \( J(E) \), is given by

\[
J(E) = \frac{c}{4\pi} \frac{dn}{dE} = \frac{c}{4\pi} \int_{t_0}^t h(t)g(E') \frac{\partial E'}{\partial E} \, dt.
\] (16)

C. Numerical methods

All of our numerical calculations were performed using Mathematica on a Silicon Graphics workstation. In particular, the differential equation (13) was integrated using Mathematica’s NDSolve routine for a large number of injection energies and times to produce a discrete version of the map \( E'(E, t) \). The contribution from each time was computed by calculating the derivative \( \partial E'/\partial E \) point by point, and then the integral (16) was approximated by simply adding these contributions multiplied by the appropriate \( \Delta t \) values.

As a check on all of the above, as well as our energy loss formula, we set the parameters of our injection function \( I(E, t) \) to match those considered by Yoshida and Teshima in
their Monte Carlo calculations [14] and attempted to reproduce curves 1, 2 and 4 of their Figure 5 (For this calculation we used their value of $H_0 = 75\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$). Agreement in all three cases was generally better than 10% at all energies, although somewhat larger discrepancies occurred at energies corresponding to crossover points of the energy loss processes. We believe that the main source of the small discrepancies in our calculations was their neglect of multiple pair-production scattering in a single time step. As a check on the validity of our mean energy loss approximation in the high energy region where pion production dominates, we compared our results to Monte Carlo calculations provided to us by F. Aharonian (using the code of Aharonian and Cronin [15]) for monoenergetic injection of protons at $10^{21}\,\text{eV}$. We obtained good agreement everywhere between the injection energy and approximately $10^{20}\,\text{eV}$, where pair production effects and evolution of the CMBR, both neglected in the code of Aharonian and Cronin, begin to become important.

III. CONTRIBUTION TO THE COSMIC RAY SPECTRUM OF PROTONS INJECTED WITH ENERGIES ABOVE THE GREISEN CUT-OFF

The attenuation length, $L(E)$, for a particle with velocity $v \simeq c$ is defined by the equation

$$L = -cE\left(\frac{dE}{dt}\right)^{-1}. \quad (17)$$

The attenuation length for cosmic ray protons in the present universe as determined from Eq. (13) is plotted in Figure 1. Note that for proton energies below about $10^{17.8}\,\text{eV}$, cosmological redshift is the only significant source of energy loss in the present universe. For

2The two calculations differ in the region above $10^{20.2}\,\text{eV}$ in the following respects: In the Monte Carlo calculation, about 10% of the particles above $10^{20.2}\,\text{eV}$ are not scattered and remain at $10^{21}\,\text{eV}$. In our mean energy loss calculation, these particles are, of course, distributed continuously, and our spectrum tracks the Monte Carlo spectrum very closely but differs in overall normalization by being roughly 25% higher.
energies between $10^{17.8}\,\text{eV}$ and $10^{19.6}\,\text{eV}$, the dominant source of energy loss is $e^+e^-$ pair production, whereas photopion production dominates above $10^{19.6}\,\text{eV}$. By $10^{19.8}\,\text{eV}$, the attenuation length is about an order of magnitude smaller than the Hubble radius, and is dropping rapidly with increasing energy. Thus, photopion production should significantly deplete the cosmic ray proton population at energies of $10^{19.8}\,\text{eV}$ and higher, although how dramatic a “cut-off” one obtains will depend, to some degree, on the assumed form of the injection energy spectrum. For definiteness in our terminology, we will refer to the energy $10^{19.8}\,\text{eV}$ as the “Greisen cut-off” energy [3,4] (This is consistent with the definition used by Berezinsky and Grigor’eva [12]). Note that in prior epochs, the microwave background was at a higher temperature, and the corresponding “Greisen cut-off” energy is thereby redshifted to roughly the value $(10^{19.8}\,\text{eV})/(1+z)$, where $z$ denotes the redshift factor. Since the photon density increases in the past as $(1+z)^3$ but the Hubble radius decreases only as $(1+z)$, in prior epochs, the attenuation length for a proton with energy above the “redshifted Greisen cut-off” was an even smaller fraction of the Hubble radius than it is in the present universe.

Despite the prediction of an effective cut-off in the observed cosmic ray spectrum, cosmic rays have been observed with energies well in excess of $10^{20}\,\text{eV}$ [1,2]. One possible explanation for this fact is that these cosmic rays are not protons. We shall not consider this possibility in this paper. If they are protons, then they must come from a nearby source. It is possible that we are unusually close to such a source (i.e., the distance from us to the source is significantly less than the average distance between sources); in that case, the spectrum of protons we observe above the Greisen cut-off would not be representative of the flux occurring in other regions of the universe. Again, we shall not consider this possibility here, but will assume that the cosmic ray sources are homogeneously distributed in space and that we are not unusually close to any single source. The presence of a cosmic ray flux above the Greisen cut-off then implies a corresponding injection rate of above-Greisen-cut-off protons throughout the present universe. If such protons also were injected at prior epochs in the evolution of the universe, they will contribute to the presently observed cosmic ray
spectrum at energies below the Greisen cut-off. The purpose of this section is to calculate this contribution to the cosmic ray spectrum under various hypotheses about the strength of the sources in the past.

If the cosmic ray spectrum above $10^{19.5} \text{ eV}$ were accurately known, the comparison of the observed and predicted spectra in this energy range (particularly near $10^{19.8} \text{ eV}$) would provide a great deal of quantitative information, which likely would be sufficient to confirm or rule out models in which the cosmic rays are protons. Unfortunately, the data above $10^{19.5} \text{ eV}$ suffers from very poor statistics as well as significant random and systematic uncertainties in energy determinations. For this reason, we shall not attempt to interpret any nuances in the data in this energy range reported by the various groups, and merely view the data as indicating that the differential energy spectrum of cosmic rays above $10^{19.5} \text{ eV}$ appears to be compatible with a $1/E^3$ fall-off, which continues up to and beyond $10^{20} \text{ eV}$, without any dramatic break.

We do not know the initial energy, $E_0$, at which the above-Greisen-cut-off cosmic rays are injected into the universe. However, the contribution to the present cosmic ray spectrum of cosmic ray protons injected at energy $E_0 \gg 10^{19.8} \text{ eV}$ is largely independent of $E_0$. To see this, consider, first, the cosmic ray protons which have present energy $10^{19.8} \text{ eV} < E < E_0$. These protons must have been injected within a small fraction of the Hubble time ago, so the temperature change of the microwave background can be neglected in calculating their energy loss. In this energy range, the observed flux between $E$ and $E + dE$ should be proportional to the amount of time the proton spends in this energy interval, which, in turn, is proportional to $L(dE/E)$, where $L$ was defined in Equation 17. Thus, the spectral shape of this high energy portion of the present energy spectrum should be independent of $E_0$—provided, of course, that we restrict attention to energies $E < E_0$. On the other hand, the cosmic ray protons which have present energy less than about $10^{19.5} \text{ eV}$ were injected at least 2 Gyr ago, which is roughly 50 attenuation times for a proton at $10^{21} \text{ eV}$ in the present universe. To a good approximation, the energy loss due to photopion production for these protons can be treated as causing an instantaneous decrease of their energy to the
redshifted Greisen cut-off energy, and, thus, the energy they have today will be essentially independent of $E_0$. Consequently, the predicted spectrum below about $10^{19.5}$ eV also does not depend upon $E_0$. Although the choice of $E_0$ could have a small effect on the present spectrum between about $10^{19.5}$ eV and $10^{19.8}$ eV, this is not significant enough to concern us here.

Figure 2 shows the predicted present energy spectrum of cosmic rays arising from the injection into the universe of protons with initial energy $E_0 = 10^{21}$ eV at a rate per comoving volume which does not vary with time. Note that the spectrum shown in Figure 2 is the “hardest” possible spectrum compatible with the assumptions that the cosmic rays are protons and their sources are homogeneously distributed in space and time. By the arguments of the previous paragraph, essentially the same energy distribution would result from any injection spectrum such that most of the protons have initial energy much greater than $10^{19.8}$ eV. (An example of such a differential injection spectrum is a power law $E^{-\gamma}$ with $\gamma$ near 1 (or smaller) and a cut-off (if any) taken to be well above $10^{19.8}$ eV.) As can be seen from Figure 2, the predicted spectrum is remarkably compatible with the observed spectrum at energies above $10^{19.6}$ eV (i.e. roughly a $1/E^3$ spectrum), but lies significantly below a $1/E^3$ spectrum at lower energies. From these facts, we can immediately draw the following two key conclusions:

1. The injection of protons at energies above the Greisen cut-off at a steady rate compatible with the observed above-Greisen-cut-off cosmic ray flux could plausibly account for most (or even all) of the cosmic rays presently observed with energies above $10^{19.6}$ eV. In any case, there is very little “room” for injection of additional cosmic rays at energies between, say, $10^{19.6}$ eV and $10^{20}$ eV.

2. A steady injection of protons at energies above the Greisen cut-off cannot account for the observed cosmic ray flux below about $10^{19.6}$ eV.

The above calculations and conclusions refer to a source of cosmic ray protons which injects the protons only at energies well above the Greisen cut-off. However, it would seem
more reasonable to assume that a realistic source would inject protons with a distribution of energies that extends to below the Greisen cut-off energy. If so, one may ask whether such a source could plausibly account for cosmic rays with energy below \(10^{19.6}\) eV as well. However, this does not appear to be at all likely: In order to significantly improve the agreement between predicted and observed spectra at energies below \(10^{19.6}\) eV, one would need a differential injection spectrum at these energies that is “softer” than \(E^{-2}\). On the other hand, in order to maintain the absence of a dramatic drop in the predicted flux above the Greisen cut-off energy, one needs an injection spectrum considerably “harder” than \(E^{-2}\) at energies above \(10^{19.8}\) eV. Thus, unless nature has contrived to put in a break in the injection spectrum very close to the Greisen cut-off, one cannot simultaneously account for the observed flux below \(10^{19.6}\) eV and the absence of a dramatic drop in the predicted flux above \(10^{19.8}\) eV.

In the next section, we will argue against the possibility that the cosmic ray flux between \(10^{18.5}\) eV and \(10^{19.6}\) eV can be understood as a continuation of the \(1/E^3\) spectrum observed at lower energies. For the remainder of this section, we shall investigate the extent to which the presently observed cosmic ray flux below \(10^{19.6}\) eV could be explained by injection of protons at initial energy above the Greisen cut-off by sources which were more numerous in the past. We will model the comoving source density by an exponential time dependence, i.e., we will take the injection rate of above-Greisen-cut-off protons to vary with time as \(h(t) = \exp(-t/\tau)\), as would be appropriate if the source were an unstable, decaying particle produced in the big bang; exponential time dependence is also used to model quasar activity [18]. However, we would expect the results for a power law time dependence (presumably appropriate for the decay of particles produced by cosmic strings) to be qualitatively similar (except for effects occurring in the very early universe if one extrapolates the power law dependence all the way back to the big bang).

Figure 3 shows the results of our attempt to fit the Fly’s Eye stereo energy spectrum data [7] down to as low an energy as possible. Note that the fit is very poor for the three highest energy points between \(10^{19.4}\) eV and \(10^{19.6}\) eV, but the statistics in this energy
region are very poor (two or three events per bin) and this “dip” is not observed by other groups (see, e.g., the graphs in [4]). The optimal choice of $\tau$ appears to be $\tau \simeq t_0/12 = 1.25$ Gyr. Smaller values of $\tau$ (stronger evolutions) would produce a larger “bump” at about $10^{19.8}$ eV due to enhanced injection of particles at earlier times. Larger values of $\tau$ (weaker evolutions) tend to produce spectra which more closely approximate a “flat” $1/E^3$ spectrum above $10^{19.6}$ eV, but fail to fit the data in the lower energy regions, falling off dramatically somewhere between $10^{18.5}$ eV and $10^{19.6}$ eV. In any case, even with strong evolution it does not appear possible to fit the data below the bottom of the Fly’s Eye “dip” at $10^{18.5}$ eV. Finally, note that our predicted spectrum has a minimum (when plotted in this manner) slightly above $10^{20}$ eV, which could be compatible with the existence of a “gap” in the cosmic ray spectrum hinted at by the observational data.

Our calculations show that the present cosmic ray proton energy of $10^{18.5}$ eV corresponds to an above-Greisen-cutoff injection at redshift $z \simeq 1/2$. (Even for a Hubble constant of $H_0 = 75$ km s$^{-1}$ Mpc$^{-1}$ the injection would have occurred at $z \simeq .6$.) Note that it follows that in any model where the cosmic rays are protons, all cosmic rays with present energy $\geq 10^{18.5}$ eV must have been injected quite recently (namely at $z \leq 1/2$). In the case of the model above, the optimal choice of $\tau$ corresponds to an injection rate at $z = 1/2$ which is $\sim 200$ times greater than the present injection rate. This is much too rapid a change to plausibly result from the evolution of any ordinary astrophysical source.

However, one possible candidate for the source of ultra-high-energy cosmic rays is an unstable relic particle. As will be seen below, in order to avoid making an unacceptably large contribution to the $\gamma$-ray background, it will be necessary for this particle to decay efficiently into protons. Hence, we will assume that this particle is a baryon and produces one proton per decay. Since the particle has a comoving number density, $n_X$, which varies with time as $n_X = n_0 \exp(-t/\tau)$, the proton injection rate per unit volume today, $r_0$, is given by

$$r_0 = -\frac{dn_X}{dt} \bigg|_{t_0} = \frac{n_0}{\tau} e^{-t_0/\tau}.$$  

(18)
Using the values $r_0 = 5 \times 10^{-46} \text{ cm}^{-3}$ and $\tau = t_0/12 = 4 \times 10^{16} \text{ s}$ which were used to fit the data in Figure 3, we obtain

$$n_0 = 3 \times 10^{-24} \text{ cm}^{-3}. \quad (19)$$

This corresponds to a present number density of $2 \times 10^{-29} \text{ cm}^{-3}$.

The highest energy cosmic ray ever observed had an energy of $3 \times 10^{20} \text{ eV}$ [1]. This would imply a lower limit on the mass of the decaying particle of order $m_X = 10^{21} \text{ eV}$. Choosing this value of $m_X$, we find that in the present universe $m_X n_X = 2 \times 10^{-8} \text{ eV cm}^{-3}$, which is roughly $10^{-11}$ of the critical density. Thus, such a hypothetical particle would make a negligible contribution to the mass density of the present universe. Even at time $t \sim \tau$ (when a large fraction of the particles had not yet decayed), the contribution to the mass density of the universe would be only of order $10^{-6}$ of the critical density, which is too small to have an influence on the dynamics of the universe, though possibly large enough to produce astrophysically interesting effects.

The existence of such a decaying particle would produce some potentially observable consequences, and, hence, observation may be used to set bounds on its mass, lifetime, and abundance. The most relevant bound arises from a consideration of the effect of the decaying particle on the $\gamma$-ray background. The high-energy cosmic rays produced by the decay of the relic particle interact with the thermal background radiation, producing $e^+e^-$ pairs. These pairs subsequently inverse-Compton scatter on photons in the background radiation, producing more high-energy photons, many of which themselves then pair-produce on the thermal background. This electromagnetic cascade continues until the highest energy photons drop below the threshold for pair-production. A well-defined prediction is thus obtained for the $\gamma$-ray spectrum resulting from this cascade, and a bound on the mass and abundance of the decaying particle (which depends only weakly on its lifetime for the range of lifetimes relevant here) results from the requirement that the flux of $\gamma$-rays from the cascade not exceed the observed flux. The bound derived by Ellis et al. [8] arises from observation.
of the gamma ray flux at 170 MeV and yields the limit $m_X n_0 < 4 \times 10^{-3} \text{eV cm}^{-3}$. This is remarkably close to the value of $m_X n_0 = 3 \times 10^{-3} \text{eV cm}^{-3}$ obtained using the mass and abundance given above. However, it should be noted that our calculated value of $n_0$ is very sensitive to the assumed value of $\tau$. A lifetime of $t_0/10$ rather than $t_0/12$ would still provide an acceptable fit to the Fly’s Eye Data, but would result in a value of the number density an order of magnitude below the observational limit. Thus it appears that the above relic particle decay model is compatible with observational constraints. However, these constraints would appear to require the particle to be a baryon of mass not much larger (or smaller) than $10^{21} \text{eV}$, which decays in a “clean” manner.

IV. CONTRIBUTION TO THE COSMIC RAY SPECTRUM OF PROTONS INJECTED WITH A $1/E^3$ SPECTRUM

We have shown in the previous section that injection of protons above the Greisen cut-off cannot account for the observed cosmic ray spectrum below about $10^{18.5} \text{eV}$ even with strong source evolution. The observed spectrum has a $1/E^3$ energy dependence in the region below $10^{17.6} \text{eV}$ extending down to $10^{15} \text{eV}$. In this energy regime, cosmological redshift dominates and therefore the power index of the injection spectrum should be preserved. Hence, it is natural to ask to what extent the observed spectrum can be explained by a $1/E^3$ injection spectrum.

The statistics of the available data are much better in this low energy region, and the following details of the observed spectrum are worth noting. The differential spectrum reported by the Fly’s Eye group [7] has a $1/E^3$ dependence up to approximately $10^{17.6} \text{eV}, \hspace{1cm} ^3$It should be possible to strengthen this bound (by roughly a factor of 5) using the more recent data analyzed in [9], since this data extends to higher $\gamma$-ray energies (though with larger uncertainties). However, it undoubtedly will be necessary to undertake a much more complete and accurate analysis of the predicted spectrum in order to set any firm bounds.
but it then steepens in the region between $10^{17.6}$ eV and $10^{18.5}$ eV (the Fly’s Eye group reports a power index of $-3.27$) and then flattens between $10^{18.5}$ eV and $10^{19.6}$ eV (Fly’s Eye reports a power index of $-2.75$). When plotted in the usual way ($J[E] \times E^3$) this results in the so-called “dip” in the differential spectrum with its minimum at $10^{18.5}$ eV.

In Figure 4 we plot the spectrum which would result from protons being injected with a $1/E^3$ spectrum and no source evolution, normalized to the Fly’s Eye data at $10^{17.3}$ eV. It should be noted that, apart from this overall normalization, there are no free parameters in our plot (although small adjustments to the curve could be produced by varying the Hubble constant or considering evolutionary effects [see below]). The agreement with the Fly’s Eye data up to about $10^{18.5}$ eV is very good, but diverges strongly from the data above $10^{18.5}$ eV. Two conclusions can be drawn from this result. First, the steepening of the observed spectrum is very naturally explained by the depletion of the region above $10^{17.6}$ eV by pair production effects. This can be viewed as supporting our hypothesis that the cosmic rays in this energy regime are protons of extragalactic origin. Second, this depletion produces a steepening of the spectrum which persists well beyond the bottom of the “dip” at $10^{18.5}$ eV, producing the strong disagreement with the data above this energy noted above.

This depletion is unavoidable under the hypothesis that the cosmic rays are protons of extragalactic origin, so, under this hypothesis, a $1/E^3$ (or softer) injection spectrum spectrum with no source evolution cannot explain the observed cosmic ray flux above $10^{18.5}$ eV.

In order to show the possible effects of source evolution on this spectrum, we plot in Figure 5 the relative contributions of several epochs spaced uniformly in time. The integration which produces the predicted spectrum today in the case of no source evolution is equivalent to simply summing these curves. (However, in our calculations, we used a much finer time spacing.) Therefore, by weighting these curves appropriately, one can predict the shape of the spectrum for any source evolution model. It can be seen from Figure 5 that unless the source evolution is extremely strong the contributions from redshifts $z > 1$ will be insignificant at energies above $10^{17.8}$ eV. It is also clear from Fig. 5 that any source evolution function which is stronger in the past will simply produce a steepening which is sharper and
which begins at a lower energy than is seen in the case of no evolution above. Finally it can be seen (as we have verified by detailed calculations) that no choice of evolutionary model can provide a good fit to the Fly’s Eye data at energies above \(10^{18.5}\) eV.

V. CONCLUSIONS

We have shown that, under our hypotheses that the cosmic rays are protons of extragalactic origin and that their sources are homogeneously distributed in space, a \(1/E^3\) injection spectrum plausibly accounts for the observed cosmic ray spectrum up to \(10^{18.5}\) eV but cannot account for the observed flux at higher energies. We have also shown, under the same hypotheses, that the injection of protons with energy above the Greisen cut-off at the rate needed to account for the presently observed above-Greisen-cutoff flux could plausibly account for the observed spectrum down to an energy of \(10^{19.6}\) eV. However, we have demonstrated that, in order to explain the observed flux between \(10^{18.5}\) eV and \(10^{19.6}\) eV under our hypotheses, one must postulate either that (A) the above-Greisen-cutoff source was very much stronger in the past, (B) that the injection spectrum of the low energy source becomes considerably harder than \(1/E^3\) above \(10^{18.5}\) eV, or (C) that there is yet another source of cosmic rays injecting particles between \(10^{18.5}\) eV and \(10^{19.6}\) eV. The decay of an ultra-massive particle with a lifetime of order \(10^9\) yr would provide an apparently viable mechanism for possibility (A). However, we leave it to the reader’s judgement to determine if this fact should be viewed as evidence in favor of the existence of such a particle or as a demonstration of the lengths to which one must go in order to account for the observed high energy cosmic ray spectrum under our above assumptions.

Unless there is a major theoretical breakthrough in our understanding of the physical mechanisms underlying the sources of cosmic rays it is likely that further advances in our understanding of ultra-high-energy cosmic rays will require better observational data. In particular, a much more precise determination of the energy spectrum in the vicinity of \(10^{19.8}\) eV should provide a stringent test of any model in which the cosmic rays are protons.
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FIGURES

FIG. 1. The attenuation length for a proton travelling through intergalactic space in the present universe. The energy loss is due to pion production scattering, pair production scattering, and cosmological redshift.

FIG. 2. The differential energy spectrum of cosmic rays resulting from monoenergetic injection at $10^{21}$ eV with no source evolution. The overall normalization of this curve was chosen to correspond to the same present-day injection rate as in Figure 3 below.

FIG. 3. The differential energy spectrum of cosmic rays due to monoenergetic injection of protons at $10^{21}$ eV with source evolution given by $\exp(-t/\tau)$ with $\tau = t_0/12$ (solid line) as compared to the spectrum reported by the Fly’s Eye group (points). The normalization of the curve corresponds to a present day injection rate of $5 \times 10^{-46}$ cm$^{-3}$ s$^{-1}$.

FIG. 4. The differential spectrum of cosmic rays due to a $1/E^3$ injection spectrum of protons with no source evolution (solid line) as compared to the spectrum reported by the Fly’s Eye group (points).

FIG. 5. The relative contributions to the predicted spectrum from several epochs under the assumption of a $1/E^3$ injection spectrum and no source evolution. (The overall normalization is chosen in order to make the contribution from the present universe equal to unity.) The labels on the curve give the redshift value of the epoch.
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