Effects of Jet Opening Angle and Velocity Structure on Blazar Parameters

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Abstract. We had earlier shown that for a constant velocity jet the discrepancy between the low speeds ($\beta$) indicated by VLBI knot motions and the high Doppler factors ($\delta$) inferred from emission of TeV photons could be reconciled if ultrarelativistic jets possessed modest opening angles. Here we evaluate the (flux-weighted) viewing angles of the jet and the apparent $\beta$ and $\delta$ values of the radio knots on parsec scales. The influence of the jet opening angle on these radio knot parameters are found for the usually considered types of relativistic nuclear jets: those with uniform bulk speeds and those where the bulk Lorentz factor of the flow decreases with distance from the jet axis, known as “spine–sheath” flows. For both types of jet velocity structures the expectation value of the jet orientation angle at first falls dramatically with increases in the (central) jet Lorentz factor, but for extremely relativistic jets it levels off at a fraction of the opening angle. The effective values of the apparent speeds and Doppler factors of the knots always decline substantially with increasing jet opening angle. The rarity of highly superluminal parsec-scale radio components in TeV blazars can be understood if their jets are both highly relativistic and intrinsically weaker, so probably less well collimated, than the jets in ordinary blazars.

1. Introduction

While Very Long Baseline Interferometry (VLBI) monitoring of the radio knots in blazar jets has found several sources containing knots with apparent speeds $v_{\text{app}}$ in excess of 25$c$, the typical speeds for blazars known to emit the highest energy $\gamma$-ray photons (TeV blazars) are found to be much more modest, with $v_{\text{app}} < 5c$, and their radio knots are actually often subluminal (e.g., [Piner & Edwards 2004]). But very large bulk Lorentz factors in these parsec-scale blazar jets ($\Gamma > 30$) are inferred from TeV flux variations (e.g., [Krawczynski, Coppi & Aharonian 2002]). Ultra-relativistic bulk Lorentz factors have also been inferred from the intraday radio variability of some blazars (e.g., [Macquart & de Bruyn 2006] and from VLBI measurements of the brightness temperatures of several blazar nuclei (e.g., [Horiuchi et al. 2004]). This large difference between the estimates of $\Gamma$ derived from these observations led some authors to postulate a dramatic jet deceleration between sub-parsec and parsec scales (e.g., [Georganopoulos & Kazanas 2002]).
Another approach to this contradiction has been to invoke a “spine–sheath” configuration for the jets; the fast spine close to the jet axis yields the \(\gamma\)-ray emission and the surrounding slower moving sheath produces the radio emission (e.g., Ghisellini, Tavecchio, & Chiaberge 2005).

We recently undertook an analytical study which showed that the modest apparent speeds of the knots of blazars, which are mostly unresolved by VLBI, can be reconciled with the extremely relativistic bulk motion inferred for TeV, and some other, blazars, if one considers a modest full opening angle (\(\omega \sim 5^\circ - 10^\circ\)) for the parsec-scale jets (Gopal-Krishna, Dhurde, & Wiita 2004, hereafter, Paper1). We also showed that the actual viewing angles, \(\theta\), of such conical jets from the line-of-sight can be substantially larger than those commonly inferred (e.g., Jorstad et al. 2005) by combining the flux variability and the VLBI proper motion data (Gopal-Krishna, Wiita, & Dhurde 2006, Paper II). Direct support for the assumption of conical parsec-scale jets comes from the VLBI imaging of the nuclear jets in the nearest two radio galaxies, M87 and Centaurus A.

Here we summarize the predictions for two widely discussed jet forms (i.e., those with uniform \(\Gamma\) and those with velocities decreasing away from the jet axis, the spine–sheath types) for how three key parameters of the VLBI radio knots in blazar jets, the apparent speed (\(\beta_{\text{app}}\)), Doppler factor (\(\delta\)), and viewing angle (\(\theta\)), would be influenced by the jet opening angle. The details of this work have recently appeared in Gopal-Krishna et al. (2007a), and kinematical diagrams of \(\delta\) against \(\beta\) for conical jets are in Gopal-Krishna, Sircar, & Dhurde (2007b).

2. Finding Apparent Parameters for Conical Jets

We approximate a radio knot with a circular disk-shaped region of uniform intrinsic synchrotron emissivity, and thus uniform surface brightness, presumably arising from a shock in the jet. We define the angle between the axis of the jet and the line-of-sight from the stationary core of the blazar to be \(\theta\) and the velocity of the knot along the jet to be \(\vec{\beta} \equiv \vec{v}/c\). Jets with finite opening angles require more general forms of the usual relations: \(\delta = \left[\Gamma (1 - \vec{\beta} \cdot \hat{n})\right]^{-1}\), where \(\Gamma = (1 - \beta^2)^{-1/2}\), \(\hat{n}\) is the unit vector along the sight line, and \(\vec{v}_{\text{app}} = \vec{v} \times \hat{n} / (1 - \vec{\beta} \cdot \hat{n})\). The values of \(\delta\) and \(\vec{v}_{\text{app}}\) arising from each \(d\Omega\) patch vary when \(\omega \neq 0\). We generalize those results by noting that the magnitude of the bulk velocity, \(c\beta\), might also vary as a function of the angular distance (\(r\)) of that patch from the jet axis.

The effective values, \(\beta_{\text{eff}}\) and \(\delta_{\text{eff}}\), for a knot, as would be determined from VLBI observations unable to resolve the width of the jet (i.e., knot) are:

\[
S_{\text{obs}} = \int_{\Omega} \delta'(\Omega') S_{\text{em}}(\Omega') d\Omega' = A(\theta) S_{\text{em}}; \tag{1}
\]

\[
\delta_{\text{eff}} = A^{1/2}(\theta); \quad \beta_{\text{app,eff}} = \frac{1}{S_{\text{obs}}} \int_{\Omega} \frac{\vec{\beta}(\Omega') \delta'(\Omega') S_{\text{em}}(\Omega') d\Omega'}. \tag{2}
\]

Here, \(S_{\text{em}}\) and \(S_{\text{obs}}\) are the emitted and the (Doppler boosted) observed flux densities, respectively, \(\Omega' = \vec{\beta} \cdot \hat{n} / \beta\) denotes the location on the knot given by the direction cosine between the local velocity and the line of sight to the nucleus, \(\Omega\) is the entire solid angle subtended by the knot, \(A(\theta)\) is the flux boosting

\(\left[\text{Gopal-Krishna, Tavecchio, & Chiaberge 2005}\right]\)
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factor averaged over the radio knot’s cross-section, and \( p \) is usually taken as 3, appropriate for a compact discrete source with flat spectral index. The viewing angle, \( \theta \), to the jet axis measures the angular offset of the circular radio disk’s center from the direction of the AGN core. The probability distribution of the viewing angle, \( P(\theta) \), is used to compute the expectation value of \( \theta \) for any particular combination of \( \Gamma, \omega, p \) and \( q \), with the last parameter defined below.

The shock is taken to cover the jet’s full cross-section with uniform intrinsic emissivity for all cases. As there is no well supported form for the gradient in the literature, we adopt an exponential approximation, \( \Gamma(r) = \Gamma_0 e^{-2rq/\omega} \), where \( r \) is the angular separation from the centre of the radio knot. We evaluated results for such stratified jets, taking two values for \( q \) (1 and 2). The minimum \( \Gamma_0 \) allowed is such that the corresponding \( \Gamma(r = \omega/2) > 1 \).

The probability of finding a jet at a viewing angle \( \theta \), in a flux-limited sample, is given approximately by (Paper I; Cohen 1989):

\[
P(\theta) d\theta \propto \sin \theta A_{\text{eff}}^{3/2}(\theta) d\theta,
\]

where the exponent 3/2 is the typical slope of integral source counts at centimetre wavelengths and \( A_{\text{eff}} \) was taken from the present computations (Eq. 1). This was numerically evaluated from \( \theta = 0^\circ \) to \( 90^\circ \) and then used to compute the expectation values \( \langle \theta \rangle \) for different combinations of \( \Gamma_0, \omega, p \) and \( q \). The numerical computations were made for both uniform \( \Gamma \) jets \( (q = 0) \) and for the stratified jets \( (q = 1 \text{ and } 2) \).

3. Discussion

Fig. 1 shows a steep initial decline of \( \langle \theta \rangle \) with increasing \( \Gamma_0 \), for all values of \( \omega \) or \( q \). This decline continues until the regime of ultra-relativistic jets \( (\Gamma_0 > 30) \) is reached, when \( \langle \theta \rangle \) becomes essentially independent of \( \Gamma_0 \). The near constancy of \( \langle \theta \rangle \) for extremely relativistic jets is particularly striking for the conical jets having a uniform \( \Gamma \) and at least a moderate opening angle \( (\omega \sim 5^\circ – 10^\circ) \). It is clear that in a typical radio flux-limited sample of blazars, a larger opening angle of an ultra-relativistic jet would correspond to a considerably larger \( \langle \theta \rangle \); hence a milder foreshortening due to projection, compared to that obtained from the assumption of a pencil jet \( (\omega = 0) \), is typically expected (see Paper II).

We now discuss the influence of \( \omega \) on the effective apparent speed of a radio knot, \( c\beta_{\text{eff}} \), and its effective Doppler factor, \( \delta_{\text{eff}} \) (Eqs. 2). The decline of \( \beta_{\text{eff}} \) with \( \omega \) is sharper for the knots associated with jets of higher \( \Gamma \) (for both uniform and stratified types). On the other hand, for well collimated jets \( (i.e., \omega < 0.5^\circ) \), \( \beta_{\text{eff}} \) for a uniform \( \Gamma \) jet would typically be 1.5 to 2 times higher than if \( q = 1 \), and between 2 and 4 times higher than if \( q = 2 \), cases. Thus, the fastest spine component of the jet flow, which is near the jet axis, would be substantially concealed in the VLBI measurements.

Note that the much sharper fall of \( \beta_{\text{eff}} \) with \( \omega \) found for the uniform \( \Gamma \) case insures that already for modest jet opening angles, the value for \( \beta_{\text{eff}} \) of such jets would drop below the corresponding values for both of the spine–sheath models. While for well collimated ultra-relativistic jets \( \beta_{\text{eff}} \) is more strongly suppressed (relative to uniform \( \Gamma \)) when \( \Gamma \) is more peaked towards the axis \( (i.e., q = 2) \), the opposite is found for the jets having a significant opening angle \( (\omega \sim 2^\circ \text{ to } 10^\circ) \), with the exact cross-over value of \( \omega \) depending on \( \Gamma_0 \) and \( p \). In summary, if the conical jet is moderately wide \( (\omega \sim 10^\circ) \), the measured apparent speed...
Figure 1. Expectation values of the viewing angle, \(\langle \theta \rangle\), against the jet (central) Lorentz factor, \(\Gamma_0\), for uniform (top panel) and transversely structured (middle and lower panels) jets. The left panels correspond to \(p = 2\) and the right to \(p = 3\). Solid, dashed and dot-dashed lines are for \(\omega = 10^\circ, 5^\circ,\) and \(1^\circ\), respectively. Reproduced by permission from Gopal-Krishna et al. (2007a); ©Royal Astronomical Society.
of the VLBI knots typically remain under 10° for stratified jets as well as for uniform velocity jets, even if Γ₀ were extremely large (∼100).

Consider the ω-dependence of the δ_eff for the different transverse velocity distributions. We find that for cases of very good collimation (ω ≤ 0.5°), the uniform Γ jets would typically have 2 to 4 times larger δ_eff compared to the stratified jets, implying roughly an order-of-magnitude stronger Doppler boost. As expected for stratified jets, a sharper spine–sheath contrast (i.e., a larger q) leads to a lower δ_eff. A significant reduction in δ_eff with ω is the typical expectation for both kinds of jets, the dependence being stronger for extremely relativistic jets, particularly the uniform Γ type. Extremely relativistic jets of both uniform and stratified Γ types end up with comparable δ_eff values, once ω exceeds about 5°. These still very high values of δ_eff mean that rapid variability in TeV γ-ray emission is to be expected in any of our ultrarelativistic models as the variability timescale is proportional to δ⁻¹_eff.

But why is the Lorentz factor dichotomy so striking only for TeV blazars (e.g., Piner & Edwards 2004)? All blazars show two correlated peaks in their spectral energy distributions and are now usually classified by the frequency at which the lower frequency (synchrotron) peak occurs. According to the popular scheme unifying high energy peaked (HBL) and low energy peaked blazars (LBL) (e.g., Fossati et al. 1998), TeV emission, which is an HBL characteristic, would be more common among lower luminosity blazars.

On kpc-scales extragalactic jets tend to be less well collimated for lower luminosity sources. If this correlation holds on sub-pc scales, then the connection between HBL properties and intrinsically weaker jets would mean that the jet opening angle should be larger for TeV emitting jets. This implies that the probability of detecting TeV blazars would be enhanced, since its effective beaming angle is actually more like the jet opening angle. Also, the wider jet would mean a bigger reduction in the apparent velocity of the radio knot. A key implication is that the combination of these factors is probably responsible for the surprising preference of TeV blazars for possessing VLBI knots displaying slower motions.

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