Reflection characteristics of EM waves over homogeneous Earth model

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Received on July 10th, 1980

Abstract

Theoretical analysis have been carried out to investigate the effect of subsurface electrical parameters such as resistivity and dielectric constant; and angle of incidence on reflection coefficients for vertically and horizontally polarized electromagnetic waves. It is shown that the reflection coefficients for vertically and horizontally polarized electromagnetic waves are strongly dependent on wave frequency and angle of incidence. At small angles of incidence the $R_v$ and $R_h$ is seen to go through a minimum value which is seen to differ from one material to the other. Reflection characteristics of electromagnetic waves discussed in this paper can be helpful in the accurate interpretation of geophysical data.

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RIASSUNTO

Sono state eseguite analisi teoriche per indagare sull'effetto dei parametri elettrici di sottosuperficie, come la resistività, la costante dielettrica e l'angolo di incidenza sui coefficienti di riflessione per onde elettromagnetiche polarizzate verticalmente e orizzontalmente. Si nota che i coefficienti di riflessione per onde elettromagnetiche polarizzate verticalmente e orizzontalmente, dipendono strettamente dalla frequenza dell'onda e dall'angolo di incidenza.

A piccoli angoli d'incidenza $R_v$ e $R_h$ passano attraverso un valore minimo che si nota differire da un materiale all'altro.

Le caratteristiche della riflessione di onde elettromagnetiche di cui si è trattato in questa nota, possono essere di aiuto per una più accurata interpretazione di dati geofisici.

INTRODUCTION

The electromagnetic wave propagation and interaction with various types of earth's material has been extensively studied. The reflection and attenuation of electromagnetic waves from the earth's subsurface is cumulatively affected by the subsurface material and their electrical properties. The studies on electromagnetic field attenuation and change of reflection coefficient with frequency, polarization and angle of incidence play an important role in revealing the subsurface electrical properties (Shivprasad and Stotz, 1972; Kozaki, 1970; Wait, 1971; Philippe, 1973; Lytle, 1974; Suzuki et al., 1975; Evans, 1977; Singh and Singh, 1979) employing transmitter receiver systems with large separations. These features of received electromagnetic waves are primarily controlled by the dielectric constant ($\varepsilon$) and conductivity ($\sigma$) of the subsurface layer.

In this paper, computations have been carried out to see the effect of polarization and angle of incidence for general value of dielectric constant and conductivity, of subsurface constituents.
THEORY OF SURFACE WAVE PROPAGATION

We consider the electromagnetic wave propagation through an interface separating the earth's atmosphere from the earth's surface. The plane electromagnetic wave propagating through a medium can be represented as superposition of two waves (Schmucker and Jankowski, 1972):

i) Transverse electric mode wave,
ii) Transverse magnetic mode waves.

The electromagnetic wave propagation of these waves at low and high frequencies are significantly different. The electromagnetic wave propagation at high and low frequencies are significantly different. The reflection coefficient at low frequencies is such that the direct and subsurface reflected waves cancel out and propagation is primarily as surface waves. However, in the high frequency electromagnetic waves, the magnitude of the signal primarily depends on the relative phase of the direct and ground reflected waves reaching the receiving site. The working formula for the amplitude of the reflection coefficient is written as (Jordan and Balmain, 1969).

\[
R_h = \frac{\sin \theta_i + \left\{ (\varepsilon_r - \cos^2 \theta_i)^2 + (\sigma/\varepsilon_r \omega)^2 \right\}^{\frac{1}{4}}}{\sin \theta_i - \left\{ (\varepsilon_r - \cos^2 \theta_i)^2 + (\sigma/\varepsilon_r \omega)^2 \right\}^{\frac{1}{4}}} \quad [1]
\]

and

\[
R_v = \frac{\left\{ \varepsilon_v \sin \theta_i + \left( \frac{\sigma}{\varepsilon_v \omega} \right)^2 \sin^2 \theta_i \right\}^{\frac{1}{2}} + \left\{ (\varepsilon_v - \cos^2 \theta_i)^2 + \left( \frac{\sigma}{\varepsilon_v \omega} \right)^2 \right\}^{\frac{1}{4}}}{\left\{ (\varepsilon_v - \cos^2 \theta_i)^2 + \left( \frac{\sigma}{\varepsilon_v \omega} \right)^2 \right\}^{\frac{1}{4}} + \left\{ \varepsilon_v \sin \theta_i + \left( \frac{\sigma}{\varepsilon_v \omega} \right)^2 \sin^2 \theta_i \right\}^{\frac{1}{2}}} \quad [2]
\]
RESULTS AND DISCUSSIONS

The polarization dependent reflection coefficients are governed by the composition of the subsurface material and change with changing frequency of the electromagnetic wave and their angle of incidence as depicted by equations [1] and [2]. The frequency dependence of $R_h$ and $R_v$ for some preminent constituents of the earth's subsurface are shown in Figures (la, b, c, d, e). Frequency dependence values of dielectric constant and conductivity of soil, shale, dolomite, gabbro and quartzite have been taken from (Scott...
Fig. 1a

Reflection Coefficient

Shale
- \( R_H \)
- \( R_V \)

Fig. 1b

Reflection Coefficient

Dolomite
- \( R_H \)
- \( R_V \)

Fig. 1c

Frequency in Hz
et. al., 1967; Katsube and Collett, 1976). The reflection coefficient is nearly one at low frequencies and is seen to decrease rather drastically with increasing frequency. The reflection coefficients \( R_i \) and \( R_v \) for soil, shale, dolomite, gabbro and quartzite show similar variations with frequency although their magnitudes are different. The \( R_i \) variation depicted by the solid curve shows minimum variation at 30° and a systematic increase in variation with increasing angle of incidence 60° and 90°. However, in the case of \( R_v \), the maximum variation is seen at 30° and the variation is seen to decrease with increasing angle of incidence 60° and 90°. The angle of incidence considerably influences \( R_i \) and \( R_v \) in the higher frequency range. Computational results to study the variation of \( R_v \) with angle of incidence at two frequencies 10⁴ and 10⁶ Hz are presented in Fig. 2. The magnitude of \( R_v \) at 10⁴ Hz is seen to increase with the increase in angle of incidence for soil, shale, gabbro, dolomite and quartzite. However, at a higher frequency e.g. 10⁶ Hz, the reflection coefficient \( R_v \) is seen to behave differently. It decreases with the increase in the angle of incidence till attains a minimum, whereafter it increases again. At a given frequency the angle for minimum \( R_v \) is seen to change for soil, shale, gabbro, dolomite and quartzite.

Further computations of \( R_v \) and \( R_i \) with the angle of incidence for dielectric constant values 3, 81 and resistivity values 10⁴, 10⁵ ohm meter at two frequencies 10⁴ and 10⁵ Hz have been carried out and are presented in Fig. 3. At 10⁴ Hz, \( R_i \) is seen to decrease slowly with the angle of incidence. The increase in resistivity from 10⁴ ohm meter to 10⁵ ohm meter decrease the magnitude of \( R_i \). The change of dielectric value from 3 to 81 does not influence the \( R_i \) values for 10⁴ ohm meter resistivity value whereas for 10⁵ ohm meter, its magnitude is lowered slightly with increase in dielectric constant. The behaviour of \( R_v \) variation at 10⁵ Hz is different from that of \( R_i \). It decreases, with the increase in angle of incidence, till it attains a minimum, whereafter it increases to attain a saturation value. Here again increase in resistivity, decreases the magnitude of \( R_v \). For higher dielectric constant value and the same resistivity, the minimum occurs at smaller angle of incidence and its magnitude is lower as compared to that for lower dielectric constant. After the minimum, a
Fig. 2 - Variation of $R_r$ with angle of incidence for soil, shale dolomite, gabbro and quartzite.
Fig. 3 - Variation of $R_v$ and $R_h$ for $\varepsilon_r = 3.81$ and $\rho = 10^4 \Omega \cdot m$ with angle of incidence.
cross over and reversal in the magnitude takes place, but the values are closed to each other. The characteristic features of \( R_v \) and \( R_n \), variations discussed above are retained at 10\(^5\) Hz but the variation in magnitude is large for \( R_n \), whereas for \( R_v \), the minimum magnitude is more or less or equal for all cases.

**Concluding Remark**

The sharp variation in \( R_v \) with angle of incidence and appearance of a minimum at a specific frequency are remarkable features which control the magnitude of the received signal at the receiving site. Therefore, the measurement of wave amplitude seem to provide a potential probing tool for the study of surface features of the earth's crustal terrain. The large separation between the transmitter and receiver is essential to measure the angular variation of reflection coefficients for each of these polarizations. This method of electromagnetic exploration is widely used by professional prospecting agencies and it is capable of mapping the crustal structure in extended areas.

**Acknowledgements**

One of us (RPS) is thankful to CSIR, New Delhi for the award of Senior Research Fellowship. We are grateful to Dr. J. Singh, Visiting Professor for his keen interest in this work.
REFERENCES

Evans, D. D., 1977 - IEEE Trans. Antennas Propagation, AP-25, p. 490.

Jordan, E. C., Balmain, K. G., 1969 - Electromagnetic Waves and Radiating Systems. Prentice Hall of India, New Delhi.

Katsube, T. J., Collett, L. S., 1976 - In: Physics and Chemistry of Minerals and Rocks, edited by R.G.K. Strens, John Wiley and Sons, U.K., p. 279.

Kozuki, S., 1970 - IEETE Trans. Antennas Propagation, AP-18, p. 138.

Lytle, R. J., 1974 - IEEE Trans. Geoscience Elect., GE-12, p. 81.

Philippe, A. E., 1973 - IEEE Trans. Antennas Propagation, AP-21, p. 234

Schmucker, U., Jankowski, J., 1972 - « Tectonophysics », 13, (1:4), p. 233.

Scott, J. H., Carroll, R. D., Cunningham, D. R., 1967 - « J. Geophysical Research », 73, p. 5101.

Shivprasad, K., Stotz, K. C., 1972 - IEEE Trans Antennas and Propagation, p. 161.

Singh, Ramesh P., Singh, Rishi N., 1979 - Proceeding Indian Academy of Science, 88A, Pt. II, 3, p. 209.

Suzuki, T., Ogawa, E., Fujio, H., 1975 - IEEE Trans. Antennas Propagation, AP-23, p. 127.

Wait, J. R., 1971 - Electromagnetic probing in geophysics. « The Golem Press », Boulder, Colorado.