A Simple Three-Dimensional Laser Interferometer Gravitational-Wave Detector

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ABSTRACT

The Gravitational wave (GW) has opened a new window to the universe beyond the electromagnetic spectrum. Since 2015, dozens of GW events have been caught by ground based GW detectors through laser interferometry. However, both the ground based and future space-based detectors are two-dimensional, which have very limited directional response to the GW. Here we propose a simple three-dimensional (3-D) laser interferometer in the shape of a regular triangular pyramid. Such a 3-D detector can provide much stronger direction constraints for GW sources and wider directional response than the two-dimensional ones, which is more efficient in the research jointing GW and electromagnetic emission. The most sensitive band of such a detector is around kilo-Hz, which is suitable for measuring the post-merger signal from the binary neutron stars. Such a detection will be of importance in the restriction of equation of state of the neutron stars.

1. INTRODUCTION

Gravitational waves produced by the dynamic acceleration of celestial objects are the direct predictions of the Einsteins General Theory of Relativity, which make people have access to the universe beyond the electromagnetic spectrum and deepen people’s understanding of the dynamic nature of the universe. In recent decades, many detectors have been proposed and constructed, including the ground-based and the space-based detectors for various wavelength of gravitational waves (Adhikari 2014). And the basic principle behind every gravitational wave detector is to measure the relative displacement of the freely falling bodies.

Currently, most activities of GW detection on the ground are in the high frequency band (10Hz-100kHz), performed by the long arm laser interferometers, such as TAMA 300 m arm length interferometer (Ando et al. 2001), the GEO 600 m interferometer (Lück et al. 2006), and the kilometer size laser-interferometric GW detectors like Advance LIGO (4 km arm length) (LIGO Scientific Collaboration et al. 2015), Advance VIRGO (3 km arm length) (Acernese et al. 2014), and the following ET (10 km arm length) (Punturo et al. 2010). The space-based interferometers under construction correspond to low frequency band (100mHz -100mHz), such as eLISA/LISA (Oliver et al. 2011), Big Bang Observer (Crowder & Cornish 2005), and TIANQIN (Luo et al. 2016) et al. For the very-low-frequency band (300 pHz -100 nHz), the major detections are supported by the pulsar timing arrays (PTAs), such as EPTA (Kramer & Champion 2013), PPTA (Hobbs 2013), NANOGrav (McLaughlin 2013), as well as IPTA (Hobbs et al. 2010).

Up to now, interferometer detectors on the ground are all two-dimensional, and most of them are Michelson interferometers. This kind of detectors has quite limited sensitivity for specific directions, namely, blind directions, which is the main reason why the Advance VIRGO failed to detect the GW170817 event (Abbott et al. 2017). Under such circumstances, a 3-D laser interferometer is expected, which can be realized by the configuration of a regular triangular pyramid, with three arms as shown in Fig. 1.

![Sketchy configuration of the 3-D interferometer.](image)

**Figure 1.** Sketchy configuration of the 3-D interferometer. Left panel: configuration for three arms and the platform; the three arms are in a shape of regular triangular pyramid. Right panel: configuration for one arm.
2. STRUCTURE

The 3-D detector is made up of two Michelson interferometers, one in X-Y plane and the other in Y-Z plane as shown in Fig. 2. In such a design, the Y arm is perpendicular to the X arm and the Z arm respectively. A Fabry-Perot resonant cavity in each arm is used to enhance the phase shift produced by an arm length change. The power recycling system and signal recycling system are also included in order to strengthen the light power inside the interferometer and to widen the arm cavity bandwidth for the signal sidebands.

Why do two arms, Y₁ and Y₂, lie in the same Y-axis? Because it enables two “independent” interferences, X arm with Y₁ arm; and Z arm with Y₂ arm respectively. And since Y₁ and Y₂ correspond to two beams of light from the same source, as shown in Fig. 2, light from Y₁ and Y₂ are in phase. Consequently, a 3-D interference can be realized by two “independent” interferences.

All components in the detector can be mounted in an ultra-high vacuum system on the seismically isolated platform, so that noise environment of each arm is identical. Such a detector allows one more dimensional information and accordingly more stringent constraints on parameters of GWs than the conventional ground-based laser interferometer.

![Figure 2. Optical configuration of the 3-D interferometer.](image)

ITM: input test mass; ETM: end test mass; PRM: power recycling mirror; SRM: signal recycling mirror; PD: photodetector.

3. BENEFIT

Such a 3-D laser interferometer will greatly improve the pattern functions (Forward 1978), thereby, less uncertainty on the direction of a GW source and wider direction response can be achieved. As the interferometer has different sensitivity to GWs from different directions, it is necessary to derive the pattern functions of the detector.

In Fig. 3, the geometry is illustrated with the frame \((x, y, z)\). The arms of the interferometer are along the \(x\), \(y\) and \(z\) axes. Another reference frame \((x', y', z')\) is shown in this figure, in which \(z'\) axis represents the propagation direction of GW. Meanwhile, a polarization angle of \(\psi = 0\) is assumed, so that the polarizations \(h_+\) and \(h_\times\) can be determined by the \(x', y'\) axes. Then in the frame \((x', y', z')\) GW has the form,

\[
h' = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

The frame \((x', y', z')\) can be achieved through the rotation of frame \((x, y, z)\), with a rotation matrix of

\[
R = \begin{pmatrix}
    \cos \phi & -\sin \phi & 0 \\
    \sin \phi & \cos \phi & 0 \\
    0 & 0 & 1
\end{pmatrix}
\]

Then the GW in frame \((x, y, z)\) is obtained by \(h_{ij} = R_{ik} R_{lj} h'_{kl}\). As a result, the relative phase shifts between the \(x\) and \(y\) arms as well as between the \(y\) and \(z\) arms can be given,

\[
\frac{1}{2}(h_{xx} - h_{yy}) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi h_+ - \sin 2\phi \cos \theta h_\times
\]

\[
\frac{1}{2}(h_{yy} - h_{zz}) = \frac{1}{2}(\sin^2 \phi - \cos^2 \phi \cos^2 \theta + \sin^2 \theta) h_+ + \frac{1}{2} \sin 2\phi \cos \theta h_\times
\]

The pattern functions are obtained:

for \(X\) and \(Y\) arms,

\[
F_+(\theta, \phi) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi,
\]

\[
F_\times(\theta, \phi) = -\cos \theta \sin 2\phi.
\]

for \(Y\) and \(Z\) arms,

\[
F_+(\theta, \phi) = \frac{1}{2}(\sin^2 \phi - \cos^2 \phi \cos^2 \theta + \sin^2 \theta),
\]

\[
F_\times(\theta, \phi) = \frac{1}{2} \sin 2\phi \cos \theta.
\]

Comparing the Eq. (5) to (8), one can find that such a 3-D interferometer mainly improves the detector response in (+) polarization. As is shown in Fig. 4, the
Figure 3. The geometry used to compute the detector pattern functions. The arms of the interferometer are along the $x$, $y$ and $z$ axes, while the $z'$ axis stands for the direction of GW propagation.

detector’s response for (+) polarized GW in $X$ direction is enhanced. And the sub-interferometer on $X$-$Y$ plan and $Y$-$Z$ plan have different pattern functions for (+) polarized GW. Consequently, the positional uncertainty for GW sources can be reduced by the correlation of Eq. (3) and (4).

Figure 4. Interferometer on $X$-$Y$ plan response for (+) polarization [left]; interferometer on $Y$-$Z$ plan response for (+) polarization [middle], and total response for (+) polarization [right]. Color indicates increasing sensitivity from indigo to red.

4. SENSITIVITY ESTIMATION

The level of total noise determines the weakest GW signals detectable. As a matter of fact, the detector will suffer from several fundamental noises, including quantum noise (Braginski˘i & Vorontsov 1975), thermal noise (Saulson 1990), seismic noise (Aki & Richards 2002), and gravity gradient noise (Saulson 1984) et al. Here we estimate the sensitivity limit of the detector based on idealized parameters which can come true in the future.

Referring other laser interferometer detectors, the parameters of the 3-D is assumed as follows. The length of each arm is 10 meters, the mass of each mirror 200 kg, the loss angle of the coating $5 \times 10^{-5}$, the loss angle of the substrate $5 \times 10^{-9}$, the temperature 20 K. Moreover, each mirror is suspended by the quadruple pendulum with the resonant frequency of 10 Hz and the loss angle of $10^{-9}$. Besides, a Fabry-Perot cavity with a fineness of 1000 is placed in each arm, and the laser power at the beam splitter is assumed to be 10 kW. In such cases, the corresponding noise estimation can be obtained. As shown by the black curve in Fig. 5, the total noise level of the 3-D detector at frequency of kilo-Hz is close to the designed sensitivity of aLIGO at the frequency of hundreds Hz (LIGO Scientific Collaboration et al. 2015).

Figure 5. Estimated sensitivity of the proposed 3-D interferometer.

5. DISSCUSSION

The advantage of such a 3-D configuration is capable of detecting GW from more directions sensitively. Moreover, the positional uncertainty of GW sources is reduced by the contrast of response between two sub-interferometers.

Furthermore, the whole system is mounted on the seismically isolated platform, and each mirror is suspended by pendulum system. Such design can effectively reduce the seismic noise to $10^{-23}/\sqrt{\text{Hz}}$ at the frequency beyond 30 Hz. And the feature of regular triangular pyramid can maintain the same sensitivity limit of the two sub-interferometers. Thus, the differences of the pattern functions are the only reason responsible for the divergence between the signals observed by two sub-interferometers.

However, it is difficult to build such a 3-D detector at a large scale. The arm is 10 meters in our design. In such a case, the quantum noise dominates the
sensitivity limit of the detector, because the standard quantum limit depends on the interferometer parameters,
\[ S_{SQL}^{1/2}(f) = \frac{1}{\pi f L} \sqrt{\frac{8 \hbar}{M}}, \]
where \( L \) and \( M \) are one arm length and one mirror mass.

With the sensitivity estimated before, the observation target of the detector is merger signals from the compact binaries, especially post-merger signals from the binary neutron stars, which spans from 1 to several kHz. Undoubtedly, the detection of the peak frequencies of the post-merger stage can play an important role in constraining the equation of state of neutron star (Chatziioannou et al. 2017; Bose et al. 2018; Torres-Rivas et al. 2019).

6. CONCLUSION

We propose a 3-D interferometer GW detector, with two Michelson interferometers setting in a regular triangular pyramid. This detector aims to probe signals from merging compact binaries, particularly signals at the post-merger stage of binary neutron stars. The detector can sensitively detect GWs from more directions, in which the direction of the GW sources can be constrained by comparing the response of two sub-interferometers. Moreover, such a 3-D detector is mounted on the seismically isolated platform, which makes it more convenient for seismic isolation and vacuum processing. Hence it will be economical to arrange multiple 3-D detectors at different locations, then more detailed behavior of GW is expected. And combining with electromagnetic signals, our understanding of fundamental physics will be enhanced.

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