Recent Theoretical Developments in
CP Violation in the $B$ System

Robert Fleischer
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract
After a brief review of the present status of the standard methods to extract CKM phases from CP-violating effects in non-leptonic $B$-decays, an overview of recent theoretical developments in this field is given, including extractions of $\gamma$ from $B \rightarrow \pi K$ and $B_{s(d)} \rightarrow J/\psi K_S$ decays, a simultaneous determination of $\beta$ and $\gamma$, which is provided by the modes $B_d \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$, and extractions of CKM phases from angular distributions of certain $B_{d,s}$ decays, such as $B_d \rightarrow J/\psi \rho^0$ and $B_s \rightarrow J/\psi \phi$.

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1Robert.Fleischer@cern.ch
1 Setting the Stage

CP violation is one of the central and fundamental phenomena in modern particle physics, providing a very fertile testing ground for the Standard Model. In this respect, the \( B \)-meson system plays an outstanding role, which is also reflected in the tremendous experimental effort put in the preparations to explore \( B \) physics. The BaBar (SLAC) and BELLE (KEK) detectors have already seen their first events – which manifests the beginning of the \( B \)-factory era in particle physics – and CLEO-III (Cornell), HERA-B (DESY) and CDF-II (Fermilab) will start taking data in the near future. Although the physics potential of these experiments is very promising, it may well be that the “definite” answer in the search for new physics will be left for second-generation \( B \)-physics experiments at hadron machines, such as LHCb (CERN) or BTeV (Fermilab), which offer, among other things, very exciting ways of using \( B_s \) decays.

Within the framework of the Standard Model, CP violation is closely related to the Cabibbo–Kobayashi–Maskawa (CKM) matrix \[1\], connecting the electroweak eigenstates of the down, strange and bottom quarks with their mass eigenstates. As far as CP violation is concerned, the central feature is that – in addition to three generalized Cabibbo-type angles – also a complex phase is needed in the three-generation case to parametrize the CKM matrix. This complex phase is the origin of CP violation within the Standard Model. Concerning tests of the CKM picture of CP violation, the central targets are the unitarity triangles of the CKM matrix. The unitarity of the CKM matrix, which is described by

\[ \hat{V}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}}^\dagger = \hat{1} = \hat{V}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}}^\dagger, \]  

leads to a set of 12 equations, consisting of 6 normalization relations and 6 orthogonality relations. The latter can be represented as 6 triangles in the complex plane, all having the same area \[2\]. However, in only two of them, all three sides are of comparable magnitude \( \mathcal{O}(\lambda^3) \), while in the remaining ones, one side is suppressed relative to the others by \( \mathcal{O}(\lambda^3) \) or \( \mathcal{O}(\lambda^4) \), where \( \lambda \equiv |V_{us}| = 0.22 \) denotes the Wolfenstein parameter \[3\]. The orthogonality relations describing the non-squashed triangles are given as follows:

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]  \[ V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0. \]
The two non-squashed triangles agree at leading order in the Wolfenstein expansion ($O(\lambda^3)$), so that we actually have to deal with a single triangle at this order, which is usually referred to as “the” unitarity triangle of the CKM matrix [4]. However, in the era of second-generation experiments, starting around 2005, we will have to take into account the next-to-leading order terms of the Wolfenstein expansion, and will have to distinguish between the unitarity triangles described by (2) and (3), which are illustrated in Fig. 1. Here, $\overline{\rho}$ and $\overline{\eta}$ are related to the Wolfenstein parameters $\rho$ and $\eta$ through

$$\overline{\rho} \equiv \left(1 - \lambda^2/2\right) \rho, \quad \overline{\eta} \equiv \left(1 - \lambda^2/2\right) \eta,$$

and the angle $\delta\gamma = \lambda^2 \eta$ in Fig. 1(b) measures the CP-violating weak $B^0_s - \overline{B}^0_s$ mixing phase, as we will see in Subsection 2.1.

The outline of this paper is as follows: in Section 2, the standard methods to extract CKM phases from CP-violating effects in non-leptonic $B$ decays are reviewed briefly in the light of recent theoretical and experimental results. In Section 3, we then focus on new theoretical developments in this field, including extractions of $\gamma$ from $B \to \pi K$ and $B_{s(d)} \to J/\psi K_S$ decays, a simultaneous determination of $\beta$ and $\gamma$, which is provided by the modes $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$, and extractions of CKM phases and hadronic parameters from angular distributions of certain $B_{d,s}$ decays, such as $B_d \to J/\psi \rho^0$ and $B_s \to J/\psi \phi$. Finally, in Section 4 we summarize the conclusions and give a brief outlook.
2 A Brief Look at the Standard Methods to Extract CKM Phases

In order to determine the angles of the unitarity triangles shown in Fig. 1 and to test the Standard-Model description of CP violation, the major role is played by non-leptonic $B$ decays, which can be divided into three decay classes: decays receiving both “tree” and “penguin” contributions, pure “tree” decays, and pure “penguin” decays. There are two types of penguin topologies: gluonic (QCD) and electroweak (EW) penguins, which are related to strong and electroweak interactions, respectively. Because of the large top-quark mass, also EW penguins play an important role in several processes [6]. An outstanding tool to extract CKM phases is provided by CP-violating effects in non-leptonic decays of neutral $B$-mesons.

2.1 CP Violation in Neutral $B$ Decays

A particularly simple and interesting situation arises if we restrict ourselves to decays of neutral $B_q$-mesons ($q \in \{d, s\}$) into CP self-conjugate final states $|f\rangle$, satisfying the relation $(CP)|f\rangle = \pm |f\rangle$. In this case, the corresponding time-dependent CP asymmetry can be expressed as

$$a_{\text{CP}}(t) = \frac{\Gamma(B_q^0(t) \rightarrow f) - \Gamma(B_{\bar{q}}^0(t) \rightarrow f)}{\Gamma(B_q^0(t) \rightarrow f) + \Gamma(B_{\bar{q}}^0(t) \rightarrow f)} = 2e^{-\Gamma_q t} \left[ A_{\text{dir}}^\text{CP} \left( \frac{1 - |\xi_{q/f}|^2}{1 + |\xi_{q/f}|^2} \right) \cos(\Delta M_q t) + A_{\text{mix}}^\text{ind} \left( \frac{2 \text{Im} \xi_{q/f}}{1 + |\xi_{q/f}|^2} \right) \sin(\Delta M_q t) \right],$$

where $\Delta M_q = M_{H_q} - M_{L_q}$ denotes the mass difference between the $B_q$ mass eigenstates, and $\Gamma_{H,L}^{(q)}$ are the corresponding decay widths, with $\Gamma_q \equiv (\Gamma_{H}^{(q)} + \Gamma_{L}^{(q)}) / 2$. In Eq. (5), we have separated the “direct” from the “mixing-induced” CP-violating contributions, which are described by

$$A_{\text{CP}}^{\text{dir}}(B_q \rightarrow f) \equiv \frac{1 - |\xi_{f}^{(q)}|^2}{1 + |\xi_{f}^{(q)}|^2} \quad \text{and} \quad A_{\text{CP}}^{\text{mix-ind}}(B_q \rightarrow f) \equiv \frac{2 \text{Im} \xi_{f}^{(q)}}{1 + |\xi_{f}^{(q)}|^2},$$

respectively. Here direct CP violation refers to CP-violating effects arising directly in the corresponding decay amplitudes, whereas mixing-induced CP
violation is due to interference effects between $B^0_q - \bar{B}^0_q$ mixing and decay processes. Whereas the width difference $\Delta \Gamma_q \equiv \Gamma_{H}^{(q)} - \Gamma_{L}^{(q)}$ is negligibly small in the $B_q$ system, it may be sizeable in the $B_s$ system \cite{7,8}, thereby providing the observable

$$A_{\Delta \Gamma}(B_q \to f) \equiv \frac{2 \text{Re} \xi_f^{(q)}}{1 + |\xi_f^{(q)}|^2}, \quad (7)$$

which is not independent from $A_{\text{dir}}^{\text{CP}}(B_q \to f)$ and $A_{\text{mix}}^{\text{CP}}(B_q \to f)$:

$$\left[A_{\text{dir}}^{\text{CP}}(B_s \to f)\right]^2 + \left[A_{\text{mix}}^{\text{CP}}(B_s \to f)\right]^2 + \left[A_{\Delta \Gamma}(B_s \to f)\right]^2 = 1. \quad (8)$$

Essentially all the information needed to evaluate the CP asymmetry $\xi_f^{(q)}$ is included in the following quantity:

$$\xi_f^{(q)} = \mp e^{-i \phi_q} \frac{A(\bar{B}_q^0 \to f)}{A(B_q^0 \to f)} = \mp e^{-i \phi_q} \frac{\sum_{j=u,c} V_{jr}^* V_{jb} \mathcal{M}_{jr}}{\sum_{j=u,c} V_{jr}^* V_{jb}^* \mathcal{M}_{jr}}, \quad (9)$$

where the $\mathcal{M}_{jr}$ denote hadronic matrix elements of certain four-quark operators, $r \in \{d, s\}$ distinguishes between $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transitions, and

$$\phi_q = \begin{cases} +2 \beta & (q = d) \\ -2 \delta \gamma & (q = s) \end{cases} \quad (10)$$

is the weak $B^0_q - \bar{B}^0_q$ mixing phase. In general, the observable $\xi_f^{(q)}$ suffers from hadronic uncertainties, which are due to the hadronic matrix elements $\mathcal{M}_{jr}$. However, if the decay $B_q \to f$ is dominated by a single CKM amplitude, the corresponding matrix elements cancel, and $\xi_f^{(q)}$ takes the simple form

$$\xi_f^{(q)} = \mp \exp \left[-i \left(\phi_q - \phi_D^{(f)}\right)\right], \quad (11)$$

where $\phi_D^{(f)}$ is a weak decay phase, which is given by

$$\phi_D^{(f)} = \begin{cases} -2 \gamma & \text{for dominant } \bar{b} \to \bar{u} \bar{u} \bar{r} \text{ CKM amplitudes}, \\ 0 & \text{for dominant } \bar{b} \to \bar{c} \bar{c} \bar{r} \text{ CKM amplitudes}. \end{cases} \quad (12)$$
2.2 The “Gold-Plated” Mode $B_d \to J/\psi K_S$

Probably the most important application of the formalism discussed in the previous subsection is the decay $B_d \to J/\psi K_S$, which is dominated by the $b \to c s \bar{c} \bar{s}$ CKM amplitude [6], implying

$$A_{\text{mix-ind}}(B_d \to J/\psi K_S) = + \sin(-2\beta - 0) \quad .$$

(13)

Since (11) applies with excellent accuracy to $B_d \to J/\psi K_S$ – the point is that penguins enter essentially with the same weak phase as the leading tree contribution, as is discussed in more detail in Subsection 3.2 – it is referred to as the “gold-plated” mode to determine the CKM angle $\beta$ [9]. Strictly speaking, mixing-induced CP violation in $B_d \to J/\psi K_S$ probes $\sin(2\beta + \phi_K)$, where $\phi_K$ is related to the CP-violating weak $K^0 - \bar{K}^0$ mixing phase. Similar modifications of (11) and of the corresponding CP asymmetries must also be performed for other final-state configurations containing $K_S$- or $K_L$-mesons. However, $\phi_K$ is negligibly small in the Standard Model, and – owing to the small value of the CP-violating parameter $\varepsilon_K$ of the neutral kaon system – can only be affected by very contrived models of new physics [10].

First attempts to measure $\sin(2\beta)$ through the CP asymmetry (13) have recently been performed by the OPAL and CDF collaborations [11]:

$$\sin(2\beta) = \begin{cases} 3.2^{+1.8}_{-2.0} \pm 0.5 \quad (\text{OPAL Collaboration}), \\ 0.79^{+0.41}_{-0.44} \quad (\text{CDF Collaboration}). \end{cases}$$

(14)

Although the experimental uncertainties are very large, it is interesting to note that these results favour the Standard-Model expectation of a positive value of $\sin(2\beta)$. In the $B$-factory era, an experimental uncertainty of $\Delta \sin(2\beta)|_{\text{exp}} = 0.08$ seems to be achievable, whereas second-generation experiments of the LHC era aim at $\Delta \sin(2\beta)|_{\text{exp}} = O(0.01)$.  

Another important implication of the Standard Model, which is interesting for the search of new physics, is the following relation:

$$A_{\text{dir}}(B_d \to J/\psi K_S) \approx 0 \approx A_{\text{CP}}(B^\pm \to J/\psi K^\pm) \quad .$$

(15)

In view of the tremendous accuracy that can be achieved in the LHC era, it is an important issue to investigate the theoretical accuracy of (13) and (15). A very interesting channel in this respect is $B_s \to J/\psi K_S$ [12], allowing us to extract $\gamma$ and to control the – presumably very small – penguin uncertainties in the determination of $\beta$ from the CP-violating effects in $B_d \to J/\psi K_S$. We shall come back to this strategy in Subsection 3.2.
2.3 The Decay $B_d \rightarrow \pi^+\pi^-$

If this mode would not receive penguin contributions, its mixing-induced CP asymmetry would allow a measurement of $\sin(2\alpha)$:

$$
A_{\text{mix-ind}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-) = -\sin[-(2\beta + 2\gamma)] = -\sin(2\alpha). \quad (16)
$$

However, this relation is strongly affected by penguin effects, which were analysed by many authors [13, 14]. There are various methods on the market to control the corresponding hadronic uncertainties. Unfortunately, these strategies are usually rather challenging from an experimental point of view.

The best known approach was proposed by Gronau and London [15]. It makes use of the $SU(2)$ isospin relation

$$
\sqrt{2} A(B^+ \rightarrow \pi^+\pi^0) = A(B^0_d \rightarrow \pi^+\pi^-) + \sqrt{2} A(B^0_d \rightarrow \pi^0\pi^0) \quad (17)
$$

and of its CP-conjugate, which can be represented in the complex plane as two triangles. The sides of these triangles can be determined through the corresponding branching ratios, while their relative orientation can be fixed by measuring the CP-violating observable $A_{\text{mix-ind}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-)$ [1]. Following these lines, it is in principle possible to take into account the QCD penguin effects in the extraction of $\alpha$. It should be noted that EW penguins cannot be controlled with the help of this isospin strategy. However, their effect is expected to be rather small, and – as was pointed out recently [16, 17] – can be included through an additional theoretical input. Unfortunately, the Gronau–London approach suffers from an experimental problem, since the measurement of BR($B_d \rightarrow \pi^0\pi^0$), which is expected to be at most of $O(10^{-6})$, is very difficult. However, upper bounds on the CP-averaged $B_d \rightarrow \pi^0\pi^0$ branching ratio may already be useful to put upper bounds on the QCD penguin uncertainty that affects the determination of $\alpha$ [14, 18].

Alternative methods to control the penguin uncertainties in the extraction of $\alpha$ from $B_d \rightarrow \pi^+\pi^-$ are very desirable. An important one for the asymmetric $e^+e^-$ B-factories is provided by $B \rightarrow \rho\pi$ modes [19]. Here the isospin triangle relations are replaced by pentagonal relations, and the corresponding approach is rather complicated. As we will see in Subsection 3.3, an interesting strategy for second-generation $B$-physics experiments at hadron machines to make use of the CP-violating observables of $B_d \rightarrow \pi^+\pi^-$ is offered by the mode $B_s \rightarrow K^+K^-$, allowing a simultaneous determination of $\beta$ and $\gamma$ without any assumptions about penguin topologies [20].
The observation of \( B_d \to \pi^+\pi^- \) has very recently been announced by the CLEO collaboration, with a branching ratio of \( 0.47^{+0.18}_{-0.15} \pm 0.13 \) [21]. Other CLEO results on \( B \to \pi K \) modes (see Subsection 3.1) indicate that QCD penguins play in fact an important role, and that we definitely have to worry about them in the extraction of \( \alpha \) from \( B_d \to \pi^+\pi^- \). Needless to note that also a better theoretical understanding of the hadronization dynamics of \( B_d \to \pi^+\pi^- \) would be very helpful in this respect. In a recent paper [22], an interesting step towards this goal was performed.

### 2.4 Extracting \( 2\beta + \gamma \) from \( B_d \to D^{(*)}\pm\pi^\mp \) Decays

The final states of the pure “tree” decays \( B_d \to D^{(*)}\pm\pi^\mp \) are not CP eigenstates. However, as can be seen in Fig. 2, \( B_d^0 \) and \( B_d^{0*} \)-mesons may both decay into the \( D^{(*)}\pm\pi^- \) final state, thereby leading to interference effects between \( B_d^0 - B_d^{0*} \) mixing and decay processes. Consequently, the time-dependent decay rates for initially, i.e. at time \( t = 0 \), present \( B_d^0 \) or \( B_d^{0*} \)-mesons decaying into the final state \( f \equiv D^{(*)}\pm\pi^- \) allow us to determine the observable \[ \xi_f^{(d)} = -e^{-i\phi_d} \frac{A(B_d^{0*} \to f)}{A(B_d^0 \to f)} = -e^{-i(\phi_d + \gamma)} \frac{1}{\lambda^2 R_b} \frac{M_f}{M_f}, \] where \( \phi_d \) is the mixing phase and \( R_b \) is the branching ratio.

Whereas those corresponding to \( \bar{f} \equiv D^{(*)-}\pi^+ \) allow us to extract

\[ \xi_{\bar{f}}^{(d)} = -e^{-i\phi_d} \frac{A(B_d^{0*} \to \bar{f})}{A(B_d^0 \to \bar{f})} = -e^{-i(\phi_d + \gamma)} \lambda^2 R_b \frac{M_f}{M_f}. \]
Here \( R_b \equiv |V_{ub}/(\lambda V_{cb})| = 0.41 \pm 0.07 \) is the usual CKM factor, and

\[
\begin{align*}
\overline{M}_f & \equiv \left \langle f \right \vert \mathcal{O}_1(\mu)C_1(\mu) + \mathcal{O}_2(\mu)C_2(\mu) \left \vert B_d^0 \right \rangle \\
M_T & \equiv \left \langle T \right \vert O_1(\mu)C_1(\mu) + O_2(\mu)C_2(\mu) \left \vert B_d^0 \right \rangle 
\end{align*}
\tag{20}
\]

are hadronic matrix elements of the following current–current operators:

\[
\begin{align*}
\mathcal{O}_1 & = (\bar{d}_\alpha u_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A}, \\
\mathcal{O}_2 & = (\bar{d}_\alpha c_\beta)_{V-A} (\bar{\epsilon}_\beta b_\alpha)_{V-A}, \quad O_1 = (\bar{d}_\alpha c_\beta)_{V-A} (\bar{\epsilon}_\beta b_\alpha)_{V-A}.
\end{align*}
\tag{22}
\]

The observables \( \xi_f^{(d)} \) and \( \xi_f^{(d)} \) allow a theoretically clean extraction of the weak phase \( \phi_d + \gamma \) \[23\], as the hadronic matrix elements \( \overline{M}_f \) and \( M_T \) cancel in

\[
\xi_f^{(d)} \times \xi_f^{(d)} = e^{-2i(\phi_d + \gamma)}.
\tag{23}
\]

Since the \( B_d^0 - B_d^0 \) mixing phase \( \phi_d, \) i.e. \( 2\beta, \) can be determined rather straightforwardly with the help of the “gold-plated” mode \( B_d \rightarrow J/\psi K_S, \) we may extract the CKM angle \( \gamma \) from \[23\]. As the \( \bar{b} \rightarrow \bar{u} \) quark-level transition in Fig. \[3\] is doubly Cabibbo-suppressed by \( \lambda^2 R_b \approx 0.02 \) with respect to the \( b \rightarrow c \) transition, the interference effects are tiny. However, the branching ratios are large (\( \mathcal{O}(10^{-3}) \)), and the \( D(\ast)^{\pm} \pi^{\mp} \) states can be reconstructed with a good efficiency and modest backgrounds. Consequently, \( B_d \rightarrow D(\ast)^{\pm} \pi^{\mp} \) decays offer an interesting strategy to determine \( \gamma \) \[24\]. For the most optimistic scenario, an accuracy of \( \gamma \) at the level of \( 4^\circ \) may be achievable at LHCb after 5 years of taking data.

2.5 The “El Dorado” for Hadron Machines: \( B_s \) System

Since the e\(^+\)–e\(^-\) \( B \)-factories operating at the \( \Upsilon(4S) \) resonance will not be in a position to explore the \( B_s \) system, it is of particular interest for hadron machines. There are important differences to the \( B_d \) system:

- Within the Standard Model, a large \( B_s^0 - B_s^0 \) mixing parameter \( x_s \equiv \Delta M_s/\Gamma_s = \mathcal{O}(20) \) is expected, whereas the mixing phase \( \phi_s = -2\lambda^2\eta \) is expected to be very small.

- There may be a sizeable width difference \( \Delta \Gamma_s \equiv \Gamma^{(s)}_H - \Gamma^{(s)}_L \); the most recent theoretical analysis gives \( \Delta \Gamma_s/\Gamma_s = \mathcal{O}(10\%) \).
There is an interesting correlation between $\Delta \Gamma_s$ and $\Delta M_s$:

$$\frac{\Delta \Gamma_s}{\Gamma_s} \approx -\frac{3\pi}{2S(x_t)} \frac{m_b^2}{M_W^2} \frac{\Delta M_s}{\Gamma_s},$$

(24)

where $S(x_t)$ denotes one of the well-known Inami–Lim functions. The present experimental lower limit on $\Delta M_s$ is given by $\Delta M_s > 12.4 \text{ ps}^{-1}$ (95% C.L.). Interestingly, this lower bound already puts constraints on the allowed region for the apex of the unitarity triangle shown in Fig. 1(a). A detailed discussion of this feature can be found, for instance, in [25].

It is also interesting to note that the non-vanishing width difference $\Delta \Gamma_s$ may allow studies of CP-violating effects in “untagged” $B_s$ rates [7, 26]:

$$\Gamma[f(t)] = \Gamma(B^0_s(t) \to f) + \Gamma(B^{0\ast}_s(t) \to f) \propto R_L e^{-\Gamma_L^{(s)} t} + R_H e^{-\Gamma_H^{(s)} t},$$

(25)

where there are no rapid oscillatory $\Delta M_s t$ terms present. Studies of such untagged rates, allowing us to extract the observable $A_{\Delta \Gamma}$ introduced in (7) through

$$A_{\Delta \Gamma} = \frac{R_H - R_L}{R_H + R_L},$$

(26)

are more promising than “tagged” rates in terms of efficiency, acceptance and purity. Let us next have a brief look at the $B_s$ benchmark modes to extract CKM phases.

2.5.1 $B_s \to D^\mp_s K^\mp$

These decays, which receive only contributions from tree-diagram-like topologies, are the $B_s$ counterparts of the $B_d \to D^{(*)\pm} \pi^\mp$ modes discussed in Subsection 2.4 and probe the CKM combination $\gamma - 2\delta \gamma$ instead of $\gamma + 2\beta$ in a theoretically clean way [27]. Since one decay path is only suppressed by $R_b \approx 0.41$, and is not doubly Cabibbo-suppressed by $\lambda^2 R_b$, as in $B_d \to D^{(*)\pm} \pi^\mp$, the interference effects in $B_s \to D^\pm_s K^\mp$ are much larger.

2.5.2 $B_s \to J/\psi \phi$

The decay $B_s \to J/\psi[\to l^+ l^-] \phi[\to K^+ K^-]$ is the $B_s$ counterpart of the “gold-plated” mode $B_d \to J/\psi K_S$. The observables of the angular distribution of its decay products provide interesting strategies to extract the $B^0_s - \bar{B}^0_s$
mixing parameters $\Delta M_s$ and $\Delta \Gamma_s$, as well as the CP-violating weak mixing phase $\phi_s \equiv -2\delta \gamma$ \[28\]. Because of $\delta \gamma = \lambda^2 \eta$, this phase would allow us to extract the Wolfenstein parameter $\eta$. However, since $\delta \gamma = O(0.02)$ is tiny within the Standard Model, its extraction from the $B_s \to J/\psi \phi$ angular distribution may well be sizeably affected by penguin topologies. These uncertainties, which are an important issue for second-generation $B$-physics experiments at hadron machines, can be controlled with the help of the decay $B_d \to J/\psi \rho^0$ \[29\], as is discussed in more detail in Subsection 3.4.

Since the CP-violating effects in $B_s \to J/\psi \phi$ are very small in the Standard Model, they provide an interesting probe for new physics \[10\]. In the case of $B_s \to J/\psi \phi$, the preferred mechanism for new physics to manifest itself in the corresponding observables are CP-violating new-physics contributions to $B^0_s \to \overline{B}^0_s$ mixing. In various scenarios for new physics, for example in the left–right-symmetric model with spontaneous CP violation \[30\], there are in fact large contributions to the $B^0_s \to \overline{B}^0_s$ mixing phase.

Because of its very favourable experimental signature, studies of $B_s \to J/\psi \phi$ are not only promising for dedicated second-generation $B$-physics experiments, such as LHCb or BTeV, but also for ATLAS and CMS \[31\].

### 2.6 CP Violation in Charged $B$ Decays

Since there are no mixing effects present in the charged $B$-meson system, non-vanishing CP asymmetries of the kind

$$A_{CP} = \frac{\Gamma(B^+ \to f) - \Gamma(B^- \to f)}{\Gamma(B^+ \to f) + \Gamma(B^- \to f)}$$ \[27\]

would give us unambiguous evidence for “direct” CP violation in the $B$ system, which has recently been demonstrated in the kaon system by the new experimental results of the KTeV (Fermilab) and NA48 (CERN) collaborations for $\text{Re}(\varepsilon'/\varepsilon)$ \[32\].

The CP asymmetries \[27\] arise from the interference between decay amplitudes with both different CP-violating weak and different CP-conserving strong phases. In the Standard Model, the weak phases are related to the phases of the CKM matrix elements, whereas the strong phases are induced by final-state-interaction processes. In general, the strong phases introduce severe theoretical uncertainties into the calculation of $A_{CP}$, thereby destroying the clean relation to the CP-violating weak phases. However, there is an
important tool to overcome these problems, which is provided by *amplitude relations* between certain non-leptonic $B$ decays. There are two kinds of such relations:

- Exact relations: $B \to DK$ (pioneered by Gronau and Wyler [33]).
- Approximate relations, based on flavour-symmetry arguments and certain plausible dynamical assumptions: $B \to \pi K, \pi\pi, K\bar{K}$ (pioneered by Gronau, Hernández, London and Rosner [34, 35]).

Unfortunately, the $B \to DK$ approach, which allows a *theoretically clean* determination of $\gamma$, involves amplitude triangles that are expected to be very squashed. Moreover, we have to deal with additional experimental problems [36], so that this approach is very challenging from a practical point of view. More refined variants were proposed in [36]. Let us note that the colour-allowed decay $B^- \to D^0K^-$ was observed by CLEO in 1998 [37]. The flavour-symmetry relations between the $B \to \pi K, \pi\pi, K\bar{K}$ decay amplitudes have received considerable attention in the literature during the last couple of years and led to interesting strategies to probe the CKM angle $\gamma$, which are the subject of the following subsection.

### 3 A Closer Look at New Strategies to Extract CKM Phases

#### 3.1 Extracting $\gamma$ from $B \to \pi K$ Decays

In order to obtain direct information on $\gamma$ in an experimentally feasible way, $B \to \pi K$ decays seem very promising. Fortunately, experimental data on these modes are now starting to become available. In 1997, the CLEO collaboration reported the first results on the decays $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$; in the following year, the first observation of $B^\pm \to \pi^0 K^\pm$ was announced. So far, only results for CP-averaged branching ratios have been reported, with values at the $10^{-5}$ level and large experimental uncertainties [38]. However, already such CP-averaged branching ratios may lead to highly non-trivial constraints on $\gamma$ [39]. So far, the following three combinations of $B \to \pi K$ decays were considered in the literature: $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ [39–41], $B^\pm \to \pi^\pm K$ and $B^\pm \to \pi^0 K^\pm$ [16, 34, 42], as well as the combination of the neutral decays $B_d \to \pi^0 K$ and $B_d \to \pi^\mp K^\pm$ [10].
3.1.1 The $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ Strategy

Within the framework of the Standard Model, the most important contributions to these decays originate from QCD penguin topologies. Making use of the $SU(2)$ isospin symmetry of strong interactions, we obtain

$$A(B^+ \to \pi^+ K^0) \equiv P, \quad A(B^0_d \to \pi^- K^+) = - \left[ P + T + P_{ew}^C \right], \quad (28)$$

where

$$T \equiv |T| e^{i\delta_T} e^{i\gamma} \quad \text{and} \quad P_{ew}^C \equiv - |P_{ew}^C| e^{i\delta_{ew}^C} \quad (29)$$

are due to tree-diagram-like topologies and EW penguins, respectively. The label “C” reminds us that only “colour-suppressed” EW penguin topologies contribute to $P_{ew}^C$. Making use of the unitarity of the CKM matrix and applying the Wolfenstein parametrization, generalized to include non-leading terms in $\lambda$ [5], we obtain [13]

$$P \equiv A(B^+ \to \pi^+ K^0) = - \left( 1 - \frac{\lambda^2}{2} \right) \lambda^2 A \left[ 1 + \rho e^{i\theta} e^{i\gamma} \right] P_{tc}, \quad (30)$$

where

$$\rho e^{i\theta} = \frac{\lambda^2 R_b}{1 - \lambda^2/2} \left[ 1 - \left( \frac{P_{uc} + A}{P_{tc}} \right) \right]. \quad (31)$$

Here $P_{tc} \equiv |P_{tc}| e^{i\delta_{tc}}$ and $P_{uc}$ describe differences of penguin topologies with internal top- and charm-quark and up- and charm-quark exchanges, respectively, and $A$ is due to annihilation topologies. It is important to note that $\rho$ is strongly CKM-suppressed by $\lambda^2 R_b \approx 0.02$. In the parametrization of the $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ observables, it turns out to be useful to introduce

$$r \equiv \frac{|T|}{\sqrt{\langle |P|^2 \rangle}}, \quad \epsilon_C \equiv \frac{|P_{ew}^C|}{\sqrt{\langle |P|^2 \rangle}}, \quad (32)$$

with $\langle |P|^2 \rangle \equiv \langle |P|^2 \rangle + \langle \overline{P}^2 \rangle/2$, as well as the strong phase differences

$$\delta \equiv \delta_T - \delta_{tc}, \quad \Delta_C \equiv \delta_{ew}^C - \delta_{tc}. \quad (33)$$

In addition to the ratio

$$R \equiv \frac{BR(B^0_d \to \pi^- K^+) + BR(B^0_d \to \pi^+ K^-)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)} \quad (34)$$
of CP-averaged branching ratios, also the “pseudo-asymmetry”

$$A_0 \equiv \frac{\text{BR}(B_d^0 \to \pi^- K^+) - \text{BR}(B_d^0 \to \pi^+ K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0)}$$  \hspace{1cm} (35)$$

plays an important role in the probing of $\gamma$. Explicit expressions for $R$ and $A_0$ in terms of the parameters specified above are given in [43].

So far, the only available result from the CLEO collaboration is for $R$:

$$R = 1.0 \pm 0.4,$$  \hspace{1cm} (36)$$

and no CP-violating effects have been reported. However, if in addition to $R$ also the pseudo-asymmetry $A_0$ can be measured, it is possible to eliminate the strong phase $\delta$ in the expression for $R$, and to fix contours in the $\gamma-r$ plane [43]. These contours, which are illustrated in Fig. 3, correspond to the mathematical implementation of a simple triangle construction [40]. In order to determine $\gamma$, the quantity $r$, i.e. the magnitude of the “tree” amplitude $T$, has to be fixed. At this stage, a certain model dependence enters. Since the properly defined amplitude $T$ does not receive contributions only from colour-allowed “tree” topologies, but also from penguin and annihilation processes [43, 44], it may be sizeably shifted from its “factorized” value. Consequently, estimates of the uncertainty of $r$ using the factorization hypothesis, yielding typically $\Delta r = \mathcal{O}(10\%)$, may be too optimistic.
Figure 4: Rescattering process contributing to $B^+ \to \pi^+ K^0$.

Interestingly, it is possible to derive bounds on $\gamma$ that do not depend on $r$ at all [39]. To this end, we eliminate again $\delta$ in $R$ through $A_0$. If we now treat $r$ as a “free” variable, we find that $R$ takes the minimal value [43]

$$R_{\text{min}} = \kappa \sin^2 \gamma + \frac{1}{\kappa} \left( \frac{A_0}{2 \sin \gamma} \right)^2 \geq \kappa \sin^2 \gamma,$$

(37)

where

$$\kappa = \frac{1}{w^2} \left[ 1 + 2 (\epsilon_C w) \cos \Delta + (\epsilon_C w)^2 \right],$$

(38)

with $w = \sqrt{1 + 2 \rho \cos \theta \cos \gamma + \rho^2}$. The inequality in (37) arises if we keep both $r$ and $\delta$ as free parameters [39]. An allowed range for $\gamma$ is related to $R_{\text{min}}$, since values of $\gamma$ implying $R_{\text{exp}} < R_{\text{min}}$ are excluded. In particular, $A_0 \neq 0$ would allow us to exclude a certain range of $\gamma$ around $0^\circ$ or $180^\circ$, whereas a measured value of $R < 1$ would exclude a certain range around $90^\circ$, which would be of great phenomenological importance. The first results reported by CLEO in 1997 gave $R = 0.65 \pm 0.40$, whereas the most recent update is that given in [39]. If we are willing to fix the parameter $r$, significantly stronger constraints on $\gamma$ can be obtained from $R$ [10, 17]. In particular, these constraints require only $R \neq 1$ and are also effective for $R > 1$.

The theoretical accuracy of the strategies to probe $\gamma$ with the decays $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ is limited both by rescattering processes of the kind $B^+ \to \{\pi^0 K^+, \pi^0 K^{*-}, \ldots\} \to \pi^+ K^0$ [45, 46], which are illustrated in Fig. 4, and by “colour-suppressed” EW penguin contributions [11, 40]. In Eq. (37), these effects are described by the parameter $\kappa$. If they are neglected, we have $\kappa = 1$. The rescattering effects, which may lead to values of $\rho = \mathcal{O}(0.1)$, can be controlled in the contours in the $\gamma$–$r$ plane and the
constraints on $\gamma$ related to (37) through experimental data on $B^{\pm} \rightarrow K^{\pm}K$ decays, which are the $U$-spin counterparts of $B^{\pm} \rightarrow \pi^{\pm}K$ [43, 47]. Another important indicator for large rescattering effects is provided by $B_d \rightarrow K^{+}K^{-}$ modes, for which there already exist stronger experimental constraints [48].

An improved description of the EW penguins is possible if we use the general expressions for the corresponding four-quark operators, and perform appropriate Fierz transformations. Following these lines [43, 46], we obtain

$$q_C e^{i\omega_C} \equiv \frac{\epsilon_C}{r} e^{i(\Delta_C - \delta)} = 0.66 \times \left[ \frac{0.41}{R_0} \right] \times a_C e^{i\omega_C}, \quad (39)$$

where $a_C e^{i\omega_C} = a_2^{\text{eff}} / a_1^{\text{eff}}$ is the ratio of certain generalized “colour factors”. Experimental data on $B \rightarrow D^{(*)}\pi$ decays imply $a_2 / a_1 = \mathcal{O}(0.25)$. However, “colour suppression” in $B \rightarrow \pi K$ modes may in principle be different from that in $B \rightarrow D^{(*)}\pi$ decays, in particular in the presence of large rescattering effects [10]. A first step to fix the hadronic parameter $a_C e^{i\omega_C}$ experimentally is provided by the mode $B^+ \rightarrow \pi^+\pi^0$ [13]; interesting constraints were derived in [17, 49]. For a detailed discussion of the impact of rescattering and EW penguin effects on the strategies to probe $\gamma$ with $B^{\pm} \rightarrow \pi^{\pm}K$ and $B_d \rightarrow \pi^{\pm}K^{\pm}$ decays, the reader is referred to [16, 44, 47].

### 3.1.2 The Charged $B^{\pm} \rightarrow \pi^{\pm}K$, $B^{\pm} \rightarrow \pi^{0}K^{\pm}$ Strategy

Several years ago, Gronau, Rosner and London proposed an interesting $SU(3)$ strategy to determine $\gamma$ with the help of the charged decays $B^{\pm} \rightarrow \pi^{\pm}K$, $\pi^{0}K^{\pm}$, $\pi^{0}\pi^{\pm}$ [31]. However, as was pointed out by Deshpande and He [50], this elegant approach is unfortunately spoiled by EW penguins, which play an important role in several non-leptonic $B$-meson decays because of the large top-quark mass [51]. Recently, this approach was resurrected by Neu- bert and Rosner [12], who pointed out that the EW penguin contributions can be controlled in this case by using only the general expressions for the corresponding four-quark operators, appropriate Fierz transformations, and the $SU(3)$ flavour symmetry (see also [40]). Since a more detailed presentation of these strategies can be found in the contribution by D. Pirjol to these proceedings, we will just have a brief look at their most interesting features.

In the case of $B^+ \rightarrow \pi^+K^0$, $\pi^0K^+$, the $SU(2)$ isospin symmetry implies

$$A(B^+ \rightarrow \pi^+K^0) + \sqrt{2} A(B^+ \rightarrow \pi^0K^+) = - [(T + C) + P_{\text{ew}}]. \quad (40)$$
The phase structure of this relation, which has no \( I = 1/2 \) piece, is completely analogous to the \( B^+ \rightarrow \pi^+ K^0, B_d^0 \rightarrow \pi^- K^+ \) case (see (28)):

\[
T + C = |T + C| e^{i\delta_{T+C}} e^{i\gamma}, \quad P_{\text{ew}} = - |P_{\text{ew}}| e^{i\delta_{\text{ew}}}. \tag{41}
\]

In order to probe \( \gamma \), it is useful to introduce the following observables \[16\]:

\[
R_c \equiv 2 \left[ \frac{BR(B^+ \rightarrow \pi^0 K^+) + BR(B^- \rightarrow \pi^0 K^-)}{BR(B^+ \rightarrow \pi^+ K^0) + BR(B^- \rightarrow \pi^- K^0)} \right], \tag{42}
\]

\[
A_{0}^c \equiv 2 \left[ \frac{BR(B^+ \rightarrow \pi^0 K^+) - BR(B^- \rightarrow \pi^0 K^-)}{BR(B^+ \rightarrow \pi^+ K^0) + BR(B^- \rightarrow \pi^- K^0)} \right], \tag{43}
\]

which correspond to \( R \) and \( A_0 \); their general expressions can be obtained from those for \( R \) and \( A_0 \) by making the following replacements:

\[
r \rightarrow r_c \equiv \frac{|T + C|}{\sqrt{\langle |P|^2 \rangle}}, \quad \delta \rightarrow \delta_c \equiv \delta_{T+C} - \delta_{tc}, \quad P_{\text{ew}}^C \rightarrow P_{\text{ew}}. \tag{44}
\]

The measurement of \( R_c \) and \( A_{0}^c \) allows us to fix contours in the \( \gamma-r_c \) plane, in complete analogy to the \( B^\pm \rightarrow \pi^\pm K, B_d \rightarrow \pi^\mp K^\pm \) strategy. However, the charged \( B \rightarrow \pi K \) approach has interesting advantages from a theoretical point of view. First, the \( SU(3) \) symmetry allows us to fix \( r_c \propto |T + C| \) \[34\]:

\[
T + C \approx - \sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} A(B^+ \rightarrow \pi^+ \pi^0), \tag{45}
\]

where \( r_c \) thus determined is – in contrast to \( r \) – not affected by rescattering effects. Second, in the strict \( SU(3) \) limit, we have \[12\]

\[
q e^{i\omega} \equiv \left| \frac{P_{\text{ew}}}{T + C} \right| e^{i(\delta_{\text{ew}} - \delta_{T+C})} = 0.66 \times \left[ \frac{0.41}{R_b} \right], \tag{46}
\]

which does not – in contrast to (39) – involve a hadronic parameter.

The contours in the \( \gamma-r_c \) plane may be affected – in analogy to the \( B^\pm \rightarrow \pi^\pm K, B_d \rightarrow \pi^\mp K^\pm \) case – by rescattering effects \[16\]. They can be taken into account with the help of additional data \[13, 17, 52\]. The major theoretical advantage of the \( B^+ \rightarrow \pi^+ K^0, \pi^0 K^+ \) strategy with respect to \( B^\pm \rightarrow \pi^\pm K, B_d \rightarrow \pi^\mp K^\pm \) is that \( r_c \) and \( P_{\text{ew}}/(T + C) \) can be fixed by using only \( SU(3) \) arguments. Consequently, the theoretical accuracy is mainly limited by non-factorizable \( SU(3) \)-breaking effects.
Let us finally note that the observable $R_c$ – the present CLEO result is $R_c = 2.1 \pm 1.1$ – may also imply interesting constraints on $\gamma$ \cite{12}. These bounds, which are conceptually similar to \cite{33}, are related to the extremal values of $R_c$ that arise if we keep the strong phase $\delta_c$ as an “unknown”, free parameter. As the resulting general expression is rather complicated \cite{16}, let us expand it in $r_c$ \cite{12}. If we keep only the leading-order terms and make use of the $SU(3)$ relation (46), we obtain

$$R_c^{\text{ext}} \Big|_{\delta_c} = 1 \pm 2 r_c | \cos \gamma - q |.$$

Interestingly, there are no terms of $O(\rho)$ present in this expression, i.e. rescattering effects do not enter at this level \cite{12}. However, final-state-interaction processes may still have a sizeable impact on the bounds on $\gamma$ arising from the charged $B \rightarrow \pi K$ decays. Several strategies to control these uncertainties were considered in the recent literature \cite{16, 52}.

### 3.1.3 The Neutral $B_d \rightarrow \pi^0 K$, $B_d \rightarrow \pi^\mp K^\pm$ Strategy

At first sight, the strategies to probe $\gamma$ that are provided by the observables of the neutral decays $B_d \rightarrow \pi^0 K$, $\pi^\mp K^\pm$ are completely analogous to the charged $B^{\pm} \rightarrow \pi^{\pm} K$, $\pi^0 K^\pm$ case \cite{16}, as the corresponding decay amplitudes satisfy a similar isospin relation (see (40)). However, if we require that the neutral kaon be observed as a $K_S$, we have an additional observable at our disposal, which is due to “mixing-induced” CP violation in $B_d \rightarrow \pi^0 K_S$ and allows us to take into account the rescattering effects in the extraction of $\gamma$ \cite{16}. To this end, time-dependent measurements are required. The theoretical accuracy of the neutral strategy is only limited by non-factorizable $SU(3)$-breaking corrections, which affect $|T + C|$ and $P_{ew}$.

### 3.1.4 Some Thoughts about New Physics

Since $B_d^0 - \bar{B}_d^0$ mixing ($q \in \{d, s\}$) is a “rare” flavour-changing neutral-current (FCNC) process, it is very likely that it is significantly affected by new physics, which may act upon the mixing parameters $\Delta M_q$ and $\Delta \Gamma_q$ as well as on the CP-violating mixing phase $\phi_q$. Important examples for such scenarios of new physics are non-minimal SUSY models, left–right-symmetric models, models with extended Higgs sectors, four generations, or $Z$-mediated
Figure 5: The allowed region in the $R_c - A_c^0$ plane, characterizing the $B^\pm \to \pi^\pm K$, $\pi^0 K^\pm$ system in the Standard Model: (a) $0.18 \leq r_c \leq 0.30$, $q = 0.63$; (b) $r_c = 0.24$, $0.48 \leq q \leq 0.78$. Rescattering effects are neglected.

FCNCs [53]. Since $B_d \to J/\psi K_S$ and $B_s \to J/\psi \phi$ – the benchmark modes to measure $\phi_d$ and $\phi_s$ – are governed by current–current, i.e. “tree”, processes, new physics is expected to affect their decay amplitudes in a minor way. Consequently, these modes still measure $\phi_d$ and $\phi_s$.

In the clean strategies to measure $\gamma$ with the help of pure “tree” decays, such as $B \to DK$, $B_d \to D^{(*)\pm} \pi^\mp$ or $B_s \to D^\pm K^\mp$, new physics is also expected to play a very minor role. These strategies therefore provide a “reference” value for $\gamma$. Since, on the other hand, the $B \to \pi K$ strategies to determine $\gamma$ rely on the interference between tree and penguin contributions, discrepancies with the “reference” value for $\gamma$ may well show up in the presence of new physics. If we are lucky, we may even get immediate indications for new physics from $B \to \pi K$ decays [54], as the Standard Model predicts interesting correlations between the corresponding observables that are shown in Figs. 5 and 6. Here the dotted regions correspond to the present CLEO results for $R_c$ and $R$. A future measurement of observables lying significantly outside the allowed regions shown in these figures would immediately indicate the presence of new physics. Although the experimental uncertainties are still too large for us to draw definite conclusions, it is interesting to note that the present central value of $R_c = 2.1$ is not favoured by the Standard Model (see Fig. 5). Moreover, if future measurements should stabilize at such a large value, there would essentially be no space left for $A_c^0$. These features should be compared with the situation in Fig. 6. The strategies discussed in the following subsections are also well suited to search for new physics.
Figure 6: The allowed region in the $R-A_0$ plane, characterizing the $B^\pm \to \pi^\pm K$, $B_d \to \pi^\pm K^\pm$ system within the Standard Model for $0.16 \leq r \leq 0.26$, $q_C e^{i\omega_C} = 0.66 \times 0.25$. Rescattering effects are neglected.

### 3.2 Extracting $\gamma$ from $B_{s(d)} \to J/\psi K_S$

As we have already noted in Subsection 2.2, the “gold-plated” mode $B_d \to J/\psi K_S$ plays an outstanding role in the determination of the CP-violating weak $B^0_d - \overline{B}^0_d$ mixing phase $\phi_d$, i.e. of the CKM angle $\beta$. In this subsection, we will have a closer look at $B_s \to J/\psi K_S$, which is related to $B_d \to J/\psi K_S$ by interchanging all down and strange quarks, as can be seen in Fig. 7.

Making use of the unitarity of the CKM matrix and applying the Wolfenstein parametrization \[3\], generalized to include non-leading terms in $\lambda$ \[5\], we obtain \[12\]

$$A(B^0_d \to J/\psi K_S) = \left(1 - \frac{\lambda^2}{2}\right) A' \left[1 + \left(\frac{\lambda^2}{1 - \lambda^2}\right) a' e^{i\theta' e^{i\gamma}}\right], \quad (48)$$

where

$$A' \equiv \lambda^2 A \left(A'^{cc}_{cc} + A'^{ct}_{pen}\right), \quad (49)$$

with $A'^{ct}_{pen} \equiv A'^{ct}_{pen} - A'^{ct}_{pen}$, and

$$a' e^{i\theta'} \equiv R_b \left(1 - \frac{\lambda^2}{2}\right) \left(\frac{A'_{ut}^{pen}}{A'^{ct}_{cc} + A'^{ct}_{pen}}\right). \quad (50)$$

The amplitudes $A'^{ct}_{cc}$ and $A'^{ct}_{pen}$ ($q \in \{u, c, t\}$) describe the current–current, i.e. “tree”, and penguin processes in Fig. 7, and $A'^{ct}_{pen}$ is defined in analogy
Figure 7: Feynman diagrams contributing to $B_{d(s)} \to J/\psi K_S$. The dashed lines in the penguin topology represent a colour-singlet exchange.

To $A'_{\text{pen}}$. On the other hand, the $B_s^0 \to J/\psi K_S$ decay amplitude can be parametrized as follows:

$$A(B_s^0 \to J/\psi K_S) = -\lambda A \left[ 1 - a e^{i\theta} e^{i\gamma} \right],$$

(51)

where

$$A \equiv \lambda^2 A \left( A_{cc}^c + A_{\text{pen}}^{ct} \right)$$

(52)

and

$$a e^{i\theta} \equiv R_b \left( 1 - \frac{\lambda^2}{2} \right) \left( \frac{A_{\text{ut}}^{ct}}{A_{cc}^c + A_{\text{pen}}^{ct}} \right)$$

(53)

correspond to (49) and (50), respectively. It should be emphasized that (48) and (51) rely only on the unitarity of the CKM matrix. In particular, these Standard-Model parametrizations of the $B_{d(s)}^0 \to J/\psi K_S$ decay amplitudes also take into account final-state-interaction effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges [44].

If we compare (48) and (51) with each other, we observe that the quantity $a'e^{i\theta'}$ is doubly Cabibbo-suppressed in the $B_d^0 \to J/\psi K_S$ decay amplitude (15), whereas $ae^{i\theta}$ enters in the $B_s^0 \to J/\psi K_S$ amplitude (51) in a Cabibbo-allowed way. Consequently, there may be sizeable CP-violating effects in $B_s \to J/\psi K_S$. As was pointed out in [12], the $U$-spin flavour
symmetry of strong interactions allows us to extract $\gamma$, as well as interesting hadronic quantities, from the CP asymmetries $A^\text{dir}_{\text{CP}}(B_s \to J/\psi K_S)$, $A^\text{mix}_{\text{CP}}(B_s \to J/\psi K_S)$ and the CP-averaged $B_{d(s)} \to J/\psi K_S$ branching ratios. The theoretical accuracy of this approach is only limited by $U$-spin-breaking corrections, and there are no problems due to final-state-interaction effects. As an interesting by-product, this strategy allows us to take into account the presumably very small penguin contributions in the determination of $\phi_d = 2\beta$ from $B_d \to J/\psi K_S$, which is an important issue in view of the impressive accuracy that can be achieved in the LHC era. Moreover, we have an interesting relation between the direct $B_{s(d)} \to J/\psi K_S$ CP asymmetries and the corresponding CP-averaged branching ratios:

$$\frac{A^\text{dir}_{\text{CP}}(B_d \to J/\psi K_S)}{A^\text{dir}_{\text{CP}}(B_s \to J/\psi K_S)} \approx - \frac{\text{BR}(B_s \to J/\psi K_S)}{\text{BR}(B_d \to J/\psi K_S)}.$$  (54)

The experimental feasibility of the extraction of $\gamma$ sketched above depends strongly on the size of the penguin effects in $B_s \to J/\psi K_S$, which are very hard to estimate. A similar strategy is provided by $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ decays. For a detailed discussion, the reader is referred to [12].

3.3 Extracting $\beta$ and $\gamma$ from $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$

In this subsection, a new way of making use of the CP-violating observables of the decay $B_d \to \pi^+\pi^-$ is discussed [20]: combining them with those of $B_s \to K^+K^-$, the $U$-spin counterpart of $B_d \to \pi^+\pi^-$, a simultaneous determination of $\phi_d = 2\beta$ and $\gamma$ becomes possible. This approach is not affected by any penguin topologies – it rather makes use of them – and does not rely on certain “plausible” dynamical or model-dependent assumptions. Moreover, final-state-interaction effects, which led to considerable attention in the recent literature in the context of the determination of $\gamma$ from $B \to \pi K$ decays (see Subsection 3.1), do not lead to any problems, and the theoretical accuracy is only limited by $U$-spin-breaking effects. This strategy, which is furthermore very promising to search for indications of new physics [54], is conceptually quite similar to the extraction of $\gamma$ from $B_{s(d)} \to J/\psi K_S$ discussed in the previous subsection. However, it appears to be more favourable in view of the $U$-spin-breaking effects and the experimental feasibility.
Figure 8: Feynman diagrams contributing to $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$. 

The leading-order Feynman diagrams contributing to $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$ are shown in Fig. 8. If we make use of the unitarity of the CKM matrix and apply the Wolfenstein parametrization [3], generalized to include non-leading terms in $\lambda$ [5], the $B_d^0 \to \pi^+ \pi^-$ decay amplitude can be expressed as follows [20]:

$$A(B_d^0 \to \pi^+ \pi^-) = e^{i\gamma} \left(1 - \frac{\lambda^2}{2}\right) C \left[1 - d e^{i\theta} e^{-i\gamma}\right], \quad (55)$$

where

$$C \equiv \lambda^3 A R_b \left(A'^u_{cc} + A'^{ut}_{pen}\right), \quad (56)$$

with $A'^{ut}_{pen} \equiv A'^u_{pen} - A'^t_{pen}$, and

$$d e^{i\theta} \equiv \frac{1}{(1 - \lambda^2/2) R_b} \left(\frac{A'^{ct}_{pen}}{A'^u_{cc} + A'^{ut}_{pen}}\right). \quad (57)$$

In analogy to (55), we obtain for the $B_s^0 \to K^+ K^-$ decay amplitude

$$A(B_s^0 \to K^+ K^-) = e^{i\gamma} \lambda C' \left[1 + \left(1 - \frac{\lambda^2}{\lambda^2}\right) d' e^{i\theta'} e^{-i\gamma}\right], \quad (58)$$

where

$$C' \equiv \lambda^3 A R_b \left(A'^{u'}_{cc} + A'^{ut'}_{pen}\right) \quad (59)$$
and
d' e^{i\theta'} \equiv \frac{1}{(1 - \lambda^2/2)R_b} \left( \frac{A^\text{up}_{\text{pen}}}{A^\text{cc}_{\text{up}} + A^\text{up}_{\text{pen}}} \right)

(60)
correspond to (56) and (57), respectively. The general expressions for the
$B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ observables (6) and (7) in terms of the
parameters specified above can be found in [20].

As can be seen in Fig. 8, $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to
each other by interchanging all down and strange quarks. Consequently, the
$U$-spin flavour symmetry of strong interactions implies

d' = d \quad \text{and} \quad \theta' = \theta.

(61)

If we assume that the $B^0_s - \overline{B^0_s}$ mixing phase $\phi_s$ is negligibly small, or that
it is fixed through $B_s \to J/\psi \phi$, the four CP-violating observables provided
by $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ depend – in the strict $U$-spin limit –
on the four “unknowns” $d, \theta, d' = 2\beta$ and $\gamma$. We have therefore sufficient
observables at our disposal to extract these quantities simultaneously. In
order to determine $\gamma$, it suffices to consider $A^\text{mix}_{\text{CP}}(B_s \to K^+K^-)$ and the
direct CP asymmetries $A^\text{dir}_{\text{CP}}(B_s \to K^+K^-)$, $A^\text{dir}_{\text{CP}}(B_d \to \pi^+\pi^-)$. If we make
use, in addition, of $A^\text{mix}_{\text{CP}}(B_d \to \pi^+\pi^-)$, $\phi_d$ can be determined as well. The
formulae to implement this approach in a mathematical way are given in [20].

If we use the $B^0_d - \overline{B^0_d}$ mixing phase as an input, there is a different way of
combining $A^\text{dir}_{\text{CP}}(B_d \to \pi^+\pi^-)$, $A^\text{mix}_{\text{CP}}(B_d \to \pi^+\pi^-)$ with $A^\text{dir}_{\text{CP}}(B_s \to K^+K^-)$,
$A^\text{mix}_{\text{CP}}(B_s \to K^+K^-)$. The point is that these observables allow us to fix
contours in the $\gamma - d$ and $\gamma' - d'$ planes as functions of the $B^0_d - \overline{B^0_d}$ and $B^0_s - \overline{B^0_s}$
mixing phases in a theoretically clean way. In Fig. [4], these contours are shown
for a specific example [20]:

\[
\begin{align*}
A^\text{dir}_{\text{CP}}(B_d \to \pi^+\pi^-) & = +24\%, & A^\text{mix}_{\text{CP}}(B_d \to \pi^+\pi^-) & = +4.4\%, \\
A^\text{dir}_{\text{CP}}(B_s \to K^+K^-) & = -17\%, & A^\text{mix}_{\text{CP}}(B_s \to K^+K^-) & = -28\%. \\
\end{align*}
\]

(62)
corresponding to the input parameters $d = d' = 0.3, \theta = \theta' = 210^o, \phi_s = 0,$
$\phi_d = 53^o$ and $\gamma = 76^o$. In order to extract $\gamma$ and the hadronic parameters $d,$
$\theta, \theta'$ with the help of these contours, the $U$-spin relation $d' = d$ is sufficient.
The intersection of the contours shown in Fig. [4] yields a twofold solution for
$\gamma$, given by $51^o$ and our input value of $76^o$. The resolution of this ambiguity
is discussed in [20]. A first experimental feasibility study for LHCb, using
Figure 9: The contours in the $\gamma-d'$ planes fixed through the $B_d \rightarrow \pi^+\pi^-$, $B_s \rightarrow K^+K^-$ observables for a specific example discussed in the text.

the set of observables given in (62), gave an uncertainty of $\Delta\gamma|_{\text{exp}} = 2.3^\circ$ for five years of data taking and looks very promising [55].

It should be emphasized that the theoretical accuracy of $\gamma$ and of the hadronic parameters $d, \theta$ and $\theta'$ is only limited by $U$-spin-breaking effects. In particular, it is not affected by any final-state-interaction or penguin effects.

A first consistency check is provided by $\theta = \theta'$. Moreover, we may determine the normalization factors $C$ and $C'$ of the $B_d^0 \rightarrow \pi^+\pi^-$ and $B_s^0 \rightarrow K^+K^-$ decay amplitudes (see (55) and (58)) with the help of the corresponding CP-averaged branching ratios. Comparing them with the “factorized” result

$$\left| \frac{C'}{C}_{\text{fact}} \right| = \frac{f_K}{f_\pi} \frac{F_{B_s K} (M_K^2, 0^+)}{F_{B_d \pi} (M_\pi^2, 0^+)} \left( \frac{M_{B_d}^2 - M_K^2}{M_{B_s}^2 - M_K^2} \right),$$

we have another interesting probe for $U$-spin-breaking effects. Interestingly, $d'e^{i\theta'} = de^{i\theta}$ is not affected by $U$-spin-breaking corrections within a certain model-dependent approach (a modernized version of the “Bander–Silverman–Soni mechanism” [50]), making use – among other things – of the “factor-
ization” hypothesis to estimate the relevant hadronic matrix elements [20]. Although this approach seems to be rather simplified and may be affected by non-factorizable effects, it strengthens our confidence into the $U$-spin relations used for the extraction of $\beta$ and $\gamma$ from the decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$. The strategy discussed in this subsection is very promising for second-generation $B$-physics experiments at hadron machines, where the physics potential of the $B_s$ system can be fully exploited. At the asymmetric $e^+e^-$ $B$-factories operating at the $\Upsilon(4S)$ resonance, BaBar and BELLE, which have already seen the first events, this is unfortunately not possible. However, there is also a variant of the strategy to determine $\gamma$, where $B_d \to \pi^\pm K^\mp$ is used instead of $B_s \to K^+K^-$ [20]. This approach has the advantage that all required time-dependent measurements can in principle be performed at the asymmetric $e^+e^-$ machines. On the other hand, it relies – in addition to the $SU(3)$ flavour symmetry – on the smallness of certain “exchange” and “penguin annihilation” topologies, which may be enhanced by final-state-interaction effects. Consequently, its theoretical accuracy cannot compete with the “second-generation” $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ approach, which is not affected by such problems.

### 3.4 Extracting CKM Phases and Hadronic Parameters from Angular Distributions of $B_{d,s}$ Decays

A very interesting laboratory to explore CP violation and the hadronization dynamics of non-leptonic $B$ decays is provided by quasi-two-body modes $B_q \to X_1X_2$ of neutral $B_q$-mesons, where both $X_1$ and $X_2$ carry spin and continue to decay through CP-conserving interactions [26, 57]. In this case, the time-dependent angular distribution of the decay products of $X_1$ and $X_2$ provides valuable information. For an initially, i.e. at time $t = 0$, present $B_q^0$-meson, it can be written as

$$f(\Theta, \Phi, \Psi; t) = \sum_k O^{(k)}(t)g^{(k)}(\Theta, \Phi, \Psi),$$

where we have denoted the angles describing the kinematics of the decay products of $X_1$ and $X_2$ generically by $\Theta$, $\Phi$ and $\Psi$. There are two different kinds of observables $O^{(k)}(t)$, describing the time evolution of the angular
distribution (64): observables $|A_f(t)|^2$, corresponding to “ordinary” decay rates, and interference terms of the type

$$\text{Re}[A_f^*(t)A_f(t)], \quad \text{Im}[A_f^*(t)A_f(t)],$$

(65)

where the amplitudes $A_f(t)$ correspond to a given final-state configuration $[X_1 X_2]_f$. In comparison with strategies using $B_q \to P_1 P_2$ decays into two pseudoscalar mesons, the angular distributions of the $B_q \to X_1 X_2$ modes provide many more cross-checks and allow, in certain cases, the resolution of discrete ambiguities, which usually affect the extraction of CKM phases. The latter feature is due to the observables (65).

In a recent paper [29], I presented the general formalism to extract CKM phases and hadronic parameters from the time-dependent angular distributions (64) of certain $B_q \to X_1 X_2$ decays, taking also into account penguin contributions. If we fix the mixing phase $\phi_q$ separately, it is possible to determine a CP-violating weak phase $\omega$, which is usually given by the angles of the unitarity triangle shown in Fig. 1(a), and interesting hadronic quantities as a function of a single hadronic parameter (this feature is also discussed in another recent paper [58]). If we determine this parameter, for instance, by comparing $B_q \to X_1 X_2$ with an $SU(3)$-related mode, all remaining parameters, including $\omega$, can be extracted. If we are willing to make more extensive use of flavour-symmetry arguments, it is in principle possible to determine the $B_0^d - \bar{B}_0^d$ mixing phase $\phi_d$ as well. As the technical details of this approach are rather involved, let us just have a brief look at some of its applications.

### 3.4.1 $B_d \to J/\psi \, \rho^0$ and $B_s \to J/\psi \, \phi$

The structure of the decay amplitudes of these modes is very similar to the ones of $B_s \to J/\psi \, K_S$ and $B_d \to J/\psi \, K_S$ discussed in Subsection 3.2. They can be related to each other through $SU(3)$ and certain dynamical arguments, involving “exchange” and “penguin annihilation” topologies, and allow the extraction of the $B_0^d - \bar{B}_0^d$ mixing phase $\phi_d = 2\beta$. Because of the interference effects leading to the observables (65), both $\sin \phi_d$ and $\cos \phi_d$ can be determined, thereby allowing us to fix $\phi_d$ unambiguously. As we have seen above, this phase is an important input for several strategies to determine $\gamma$. For alternative methods to resolve the twofold ambiguity arising in the extraction of $\phi_d$ from $A_{\text{CP-ind}}^{\text{mix}}(B_d \to J/\psi \, K_S) = -\sin \phi_d$, the reader is referred to [58].
Should the penguin effects in \( B_d \rightarrow J/\psi \rho^0 \) be sizeable, \( \gamma \) can be determined as well. As an interesting by-product, this strategy allows us to take into account the penguin effects in the extraction of the \( B_s^0 - \bar{B}_s^0 \) mixing phase from \( B_s \rightarrow J/\psi \phi \), which is an important issue for the LHC era. Moreover, valuable insights into \( SU(3) \)-breaking effects can be obtained.

3.4.2 \( B_d \rightarrow \rho^+ \rho^- \) and \( B_s \rightarrow K^{*+} K^{*-} \)

The structure of the decay amplitudes of these transitions is completely analogous to the ones of \( B_d \rightarrow \pi^+ \pi^- \) and \( B_s \rightarrow K^+ K^- \) discussed in Subsection 3.3. They can be related to each other through \( U \)-spin arguments, thereby allowing the extraction of \( \gamma \) and of the \( B_d^0 - \bar{B}_d^0 \) and \( B_s^0 - \bar{B}_s^0 \) mixing phases. In contrast to the \( B_d \rightarrow \pi^+ \pi^- \), \( B_s \rightarrow K^+ K^- \) strategy, both mixing phases can in principle be determined, and many more cross-checks of interesting \( U \)-spin relations can be performed.

3.4.3 \( B_d \rightarrow K^{*0} \bar{K}^{*0} \) and \( B_s \rightarrow K^{*0} \bar{K}^{*0} \)

These decays are also \( U \)-spin counterparts and allow the simultaneous extraction of \( \gamma, \phi_d \) and \( \phi_s \). As they are pure penguin-induced modes, they are very sensitive to new physics. A particular parametrization of the \( B_d \rightarrow K^{*0} \bar{K}^{*0} \) decay amplitude allows us to probe also the weak phase \( \phi \equiv \phi_d - 2\beta \). Within the Standard Model, we have \( \phi = 0 \). However, this relation may well be affected by new physics, and represents an interesting test of the Standard-Model description of CP violation. Therefore it would be very important to determine this combination of CKM phases experimentally. The observables of the \( B_d \rightarrow K^{*0} [\rightarrow \pi^- K^+] \bar{K}^{*0} [\rightarrow \pi^+ K^-] \) angular distribution may provide an important step towards this goal.

Since the formalism presented in [29], which we have sketched in this subsection, is very general, it can be applied to many other decays. Detailed studies are required to explore which channels are most promising from an experimental point of view.
4 Conclusions and Outlook

In conclusion, we have seen that the phenomenology of non-leptonic decays of $B$-mesons is very rich and provides a fertile testing ground for the Standard-Model description of CP violation. Research has been very active in this field over the last couple of years, and we have discussed some of the most recent theoretical developments, including determinations of $\gamma$ from $B \to \pi K$ and $B_{s(d)} \to J/\psi K_S$ decays, an extraction of $\beta$ and $\gamma$, which is offered by $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$, and a general approach to extract CKM phases and hadronic parameters from angular distributions of certain non-leptonic decays of $B_{d,s}$-mesons. In these new strategies, a strong emphasis was given to the $B_s$ system, which has a very powerful physics potential and is of particular interest for $B$-physics experiments at hadron machines.

The $B$-factory era in particle physics has just started, as the BaBar and BELLE detectors have recently observed their first events. In the near future, CLEO-III, HERA-B and CDF-II will also start taking data, and the first results will certainly be very exciting. However, in order to establish the presence of physics beyond the Standard Model, it may well be that we have to wait for second-generation $B$-physics experiments at hadron machines such as LHCb or BTeV, which are expected to start operation around 2005. Hopefully, these experiments will bring several unexpected results, leading to an exciting and fruitful interaction between theorists and experimentalists!

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