Application of spinning electrodynamic tether system in changing system orbital parameters

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Abstract. Spinning electrodynamic tether system is known to be promising in many applications, including space debris removal, transportation missions, changing orbital parameters of spacecrafts and so on. Feasibility of spinning electrodynamic tether system for changing system orbital parameters is analysed in this paper. The main advantage of spinning electrodynamic tether system is the stability of its movement under the action of perturbations. However, when the conductive tether spins in a magnetic field, the Lorentz force constantly changes its direction, which leads to the need to develop special control methods that ensure desired change in system orbital parameters. In this paper, control laws are proposed for changing the semi-major axis and the eccentricity of the orbit, which take the change of tether spatial position into account. To change the semi-major axis, the current is regulated by the cosine law. To change the eccentricity of the orbit, the onboard power system provides a given amount of current when the system is near the apogee and perigee of the orbit. The combination of executing both manoeuvres at the same time is also studied. The numerical results show that the proposed laws enable spinning electrodynamic tether system to change the parameters of the orbit to the desired values.

1. Introduction

In modern time, one of the focuses is to reduce the cost of space missions with any feasible ways, while traditional rocket launching is one of the most expensive engineering projects [1]. Tether satellite systems (TSS) provide good solution to reduce the cost of space missions [2]. In combination with reusable launch vehicles (RLV), TSS can be used for launching payloads from the Earth or transferring payloads between orbits [1], for removing space debris, for studying the Earth’s atmosphere and magnetic field, etc. In addition, TSS is a reusable structure with a rather short re-preparation time compared to RLV. Therefore, payload transportation is one of the most promising applications for TSS [3].

After years of research, momentum exchange tether systems (MET) is considered as the best solution for payload transportation and even for interplanetary missions, and the spinning tether system (STS) provides the best ability and stability among all types of METs [3-5]. In the last decade, electrodynamic tether system (EDT) has received much focus, since it provides a propellantless solution to orbital maneuvers [6]. EDT is widely used in removing space debris [7,8]. The
combination with EDT and MET has also been proposed by various scientists to provide propellantless orbital maneuvering, including delivering payloads to a given orbit, or re-enter payload into Earth [8,9].

Due to the condition of system stability, traditional EDTs need to regulate current to keep tether around equilibrium position, which significantly limit the capabilities of the system. However, spinning electrodynamic tether system (SEDT) successfully avoids such restriction, because tether is always spinning and such equilibrium position simply does not exist. In addition, with centrifugal force produced by spinning motion of tether, the tether deformation due to the Ampere force (Lorentz force) is counteracted into insignificant level, and tether shape is near to the straight line. For missions like stabilizing orbital parameters or transporting payloads, SEDT can provide the largest velocity increment $\Delta V$ among all types of MET. Therefore, the combination of MET (in particular STS) and EDT is considered to be an ideal choice for large-scale missions of payload transportation [1, 10].

Though seems promising, application of SEDT requires extensive research before it becomes mature. Momentum exchange during payload tossing will decrease semi-major axis and increase eccentricity of SEDT orbit. Therefore, one of the key processes of payload transportation is to restore orbital parameters of SEDT after payload tossing, in order to prepare for another transportation [1, 11]. However, as tether rotates, the Lorentz (Ampere) force acting on the conducting tether constantly changes its direction, which requires special current control laws to change orbit parameters to the expected values [10].

Motivated by this, this paper mainly focuses on the problem of changing (restoring) orbital parameters with SEDT. The main focus of the work is the analysis of stability and the control of the orbital motion of SEDT. In particular, the problems of changing semi-major axis and eccentricity of SEDT orbit are considered. The law of current control is proposed on the basis of the cosine law, in order to increase (or decrease) the semi-major axis of the orbit. To change eccentricity, one control law is proposed, which defines acting area of current near the apogee or perigee of the orbit. For payload transportation, SEDT will drop its orbital height due to momentum exchange, which results in increasing of eccentricity and decreasing of semi-major axis. Therefore, the combination method is also proposed for the case that requires changing eccentricity and semi-major axis at the same time. Effectiveness of the proposed methods and closed-loop controller is validated by numerical results.

2. Mathematical model of tether system

2.1. Dynamic model of SEDT motion

In this paper, mass center of SEDT is assumed to be revolving along unperturbed Keplerian orbits. End-body (sub-satellite) and mother spacecraft are considered as mass points $m_1,m_2$. Only Ampere force and gravitational force are considered in describing motion of SEDT mass center. Newtonian gravitational field is adopted to describe gravitational force. The motion of SEDT could be divided into the orbital motion of the system mass center and the spinning motion of tether. Coordinate systems to describe motion of SEDT are shown in figure 1: 1. Geocentric equatorial coordinate $O_{eq}X_{eq} Y_{eq} Z_{eq}$ (fixed coordinate). The origin of coordinate is fixed at the center of Earth, axis $O_{eq}X$ points to the vernal equinox. $O_{eq}Z$ is parallel to the rotational axis of Earth, and $O_{eq}Y$ is defined by the right-hand rule; 2. Geocentric orbital coordinate $OXYZ$ : axis $OZ$ points to the perigee or the ascending node of SEDT orbit. $OZ$ is perpendicular to the orbit plane and is directed along the angular velocity vector of the SEDT mass center, and $OY$ is complemented by the right-hand rule; 3. Orbital moving coordinate $Cxyz$ (right-hand coordinate): The origin of coordinate is fixed at the mass center of SEDT ($C$), axis $Cx$ is directed along the radius vector of SEDT mass center, axis $Cz$ is parallel to the axis $OZ$, $Cy$ is defined by the right-hand rule; 4. Tether-connected coordinate(moving coordinate) $C_{T}x_{T} y_{T} z_{T}$: The origin of coordinate is fixed at the center of SEDT $C$, $C_{T}x$ is directed along the stretched tether from the SEDT mass center to the end-body $m_1$. In-plane angle $\theta$ defines the angular motion of SEDT relative to its mass center in the orbital plane, out-of-plane angle $\phi$ defines
the angular oscillations of SEDT outside the orbital plane. The in-plane angle $\theta$ determines the motion of the SEDT in the orbital plane, and out-of-plane angle $\phi$ determines the angular oscillations of the system outside the orbital plane.

Orbital motion of SEDT mass center could be described by the following orbital elements: $A$ - semi-major axis, $e$ - eccentricity, $\Omega$ - right ascension of ascending node, $\vartheta$ - true anomaly, $i$ - orbital inclination.

![Figure 1. Coordinates of SEDT.](image)

If mass of tether is much smaller than the total mass of SEDT, the motion of SEDT is described by the following equations [12]:

$$\frac{dA}{dt} = 2\nu \sqrt{\frac{A^3}{K(1-q^2-k^2)}} \left[ a_s \sin u - k \cos u + a_i \right]$$

$$\frac{dq}{dt} = \sqrt{\frac{p}{K}} \left[ a_s \sin u + a_i \left( 1 + \frac{1}{\nu} \right) \cos u + \frac{1}{\nu} (qa_i + ka_u \cot u \sin u) \right]$$

$$\frac{dk}{dt} = \sqrt{\frac{p}{K}} \left[ -a_s \cos u + a_i \left( 1 + \frac{1}{\nu} \right) \sin u + \frac{1}{\nu} (ka_i - qa_u \cot u \sin u) \right]$$

$$\frac{di}{dt} = \nu \left( \frac{p}{K} \cos u \right)$$

$$\frac{d\Omega}{dt} = a_u \left( \frac{p}{K} \sin i \right)$$

$$\frac{du}{dt} = \frac{1}{\nu} \left( \frac{p}{K} v^3 \frac{K}{p^2} - a_u \cot u \sin u \right)$$

$$\dot{r} - r \left[ \dot{\varphi}^2 + \left( \dot{\theta} + \omega \right)^2 \right] \cos^2 \varphi + \nu^2 \omega^2 \left( 3 \cos^2 \theta \cos^2 \varphi - 1 \right) = \frac{Q_1}{m_e}$$

$$\dot{\theta} + \omega + 2 \left( \dot{\theta} + \omega \right) \left( \frac{r}{r - \varphi} \tan \varphi \right) + 1.5 \nu^{-1} \omega^2 \sin 2\theta = \frac{Q_2}{m_e \nu^2 \cos^2 \varphi}$$

$$\dot{\varphi} + 2 \dot{\varphi} r / r + \left[ 0.5 \left( \dot{\theta} + \omega \right)^2 \right] + 1.5 \nu^{-1} \omega^2 \cos^2 \theta \right] \sin 2\varphi = \frac{Q_3}{m_e \nu^2}$$

where: $r$ - distance between tether tips (tether length); $u = \vartheta - \omega_z$, $\omega_z$ - argument of perigee, $e = \sqrt{q^2 + k^2}$; $\nu = 1 + q \cos(\vartheta) + k \sin(\vartheta)$; $p = A \left( 1 - q^2 - k^2 \right)$, $\mu$ - Earth gravitational constant; $a_s, a_i, a_u$ - components of the perturbing acceleration from the Ampere force along the axis $Cx, Cy, Cz$ correspondingly; $\omega = \dot{\varphi} = \left( \mu / p^3 \right)^{\frac{1}{2}} \nu^2$, $\dot{\omega} = \dot{\varphi} = -2 \mu \nu^2 \nu \sin \vartheta / p^3$; $m_e = (m_1 m_2) / (m_1 + m_2)$.

Generalized forces $Q_1, Q_2, Q_3$ and projections of perturbed acceleration $a_s, a_i, a_u$ are dependent from Ampere force, which have the forms [12]:

$$\left[ \begin{array}{c} a_s \\ a_i \\ a_u \end{array} \right] = q \left[ \begin{array}{c} q \cos(\vartheta) + k \sin(\vartheta) \\ 1 + q \cos(\vartheta) + k \sin(\vartheta) \\ 0 \end{array} \right]$$
\[ Q_1 = \frac{1}{2} B_1 |r| \left( \cot \psi \cos^2 \varphi + \psi^{-1} \sin^2 \varphi \right) \]
\[ Q_2 = \cos \varphi \left( B_x \cos \varphi - B_z \sin \varphi \cos \theta - B_z \sin \varphi \sin \theta \right) \Delta r \]
\[ Q_3 = Q_3^{(1)} + Q_3^{(2)} \]
\[ Q_3^{(1)} = 0.5 B_1 |r|^2 \sin \varphi \cos \varphi \left( \cot \psi - \psi^{-1} \right) \]
\[ Q_3^{(2)} = -\left( B_y \cos \theta - B_z \sin \theta \right) \Delta r \]
\[ a_s = \frac{m_1 + m_2}{I r \left( B_x \cos \varphi \sin \theta - B_z \sin \varphi \right)} \]
\[ a_t = \frac{m_1 + m_2}{I r \left( B_y \cos \varphi \cos \theta - B_z \cos \varphi \sin \theta \right)} \]
\[ a_w = \frac{I r \left( B_y \cos \varphi \cos \theta - B_z \cos \varphi \sin \theta \right)}{m_1 + m_2} \]

where \( I \) is the current, its positive direction is directed from end-body \( (m_1) \) to the spacecraft \( (m_2) \); \( \psi \) is the angle which describes the deformation of tether (figure 2); \( \Delta = 0.5 r \left( m_2 - m_1 \right) / \left( m_1 + m_2 \right) \) is distance from the action center of Ampere forces to the SEDT mass center (algebraic quantity); \( B_x, B_y, B_z \) are the projection of magnetic induction in axis \( Cx, Cy, Cz \) correspondingly.

![Figure 2](image1.png)

**Figure 2.** The position of the deformed tether relative to the coordinate system \( Cxyz \).

In equation (2), it is assumed that the shape of weightless tether is a part of an arc \([12]\). The tether deformation has a significant effect on SEDT motion when tether oscillates or spins slowly. However, the deformation is reduced due to the counteraction of centrifugal force. Therefore, if SEDT spins rapidly, tether deformation is usually neglected: \( Q_1 = T, Q_3^{(1)} = 0, \dot{r} = \dot{r} \equiv 0, r \equiv L_T, T \) is the natural tension force along tether.

2.2. **Model of Earth magnetic field**

Tilted dipole is used for the model of Earth magnetic field. It assumes that the Earth magnetic field is formed by a huge magnet dipole, which has inclination of \( i_d = 11^\circ 34' \) relative to the rotational axis of Earth \([10,12]\). This model could be considered as the first order approximation of the International Geomagnetic Reference Field \([13]\). As shown in figure 3, the magnetic dipole rotates with Earth, its spatial distribution could be treated as a cone relative to the rotational axis of the Earth, which is described by two angles \( \alpha, \gamma \):

![Figure 3](image2.png)

**Figure 3.** Earth magnetic dipole.
Angles $\alpha, \gamma$ are calculated as:

$$\alpha = \Omega t, \quad \gamma = i_d$$

where $\Omega$ is the rotational velocity of Earth, $t$ is the current time. The projections of the magnetic induction vector on coordinate $OXYZ$ are described from the following expressions:

$$B_x = -2B_0[\cos \gamma \sin^2 u + \cos \alpha \sin \gamma (\cos \Omega u \cos u - \cos \Omega u \sin u)] + \sin \gamma \sin \alpha (\cos u \sin \Omega u + \cos \Omega u \sin u)$$

$$B_y = B_0[\cos u \cos \gamma \sin u - \cos \alpha \sin \gamma (\cos \Omega u \sin u + \cos \Omega u \sin u)]$$

$$B_z = B_0[\cos \gamma \cos \alpha \sin \sin \Omega u - \sin \gamma \cos \Omega u \sin \alpha \sin t]$$

3. Changing orbital parameters with SEDT

3.1. Method of changing semi-major axis

To change semi-major axis, the Ampere force must be directed parallel to the speed vector of SEDT orbit (in the same or opposite direction). Thus, the magnitude of the current should depend on the spatial position of the system. Figure 4 shows the direction of the current and the Ampere force relative to the coordinate system $Cxyz$. The direction of the Ampere force is determined by the vector product $F = I(\tau \times B)$, where $\tau$ is the unit vector of current direction.

Here $C$ is the SEDT mass center, $D$ is the action point of the Ampere force, $F_y, F_z$ are the projections of the Ampere force along the direction of the orbital motion. To increase the value of semi-major axis (raising orbital height), a relay control law is proposed:

$$I = \begin{cases} I_{max}, & \cos \theta \leq 0 \\ -I_{max}, & \cos \theta > 0 \end{cases}$$

where $I_{max}$ is the maximum available current.

The sign of current in the control method (6) should be changed if it is necessary to reduce the semi-major axis (decreasing orbital height). When using control method (6), the direction of the current changes twice during each period of spinning motion. This control method changes the semi-major axis approximately linearly, and the SEDT spins with a near-constant angular velocity. The eccentricity of the orbit also oscillates with respect to its initial value, which is shown in numerical results.

3.2. Method of changing eccentricity

To change the eccentricity of SEDT orbit, the following control method is proposed:

$$I = \begin{cases} I_{max}, & \text{if } \cos \theta < 0 \cap (\beta_1 < \theta < \beta_2) \\ -I_{max}, & \text{if } \cos \theta > 0 \cap (\beta_1 < \theta < \beta_2) \\ 0, & \text{otherwise} \end{cases}$$
where $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4$ are the values of true anomalies, which determine the active range of Ampere force. Intervals $\vartheta_1 < \vartheta < \vartheta_2$ and $\vartheta_3 < \vartheta < \vartheta_4$ are located near the apogee or perigee.

In accordance with the control (7), the current turned on only near the apogee or perigee, therefore the total mission time will be much longer than the case of changing semi-major axis.

3.3. Method of joint maneuver

In practical space missions, eccentricity and semi-major axis are often changed at the same time. Therefore, it is necessary to consider changing eccentricity and semi-major axis simultaneously. Numerical results indicate that changing semi-major axis does not significantly influence eccentricity, while changing eccentricity will cause slow but continuous change of semi-major axis. Therefore, we can regulate current to firstly change eccentricity, and then change semi-major axis. If the goal is to decrease eccentricity and increase semi-major axis, control law of current has the form:

$$I = \begin{cases} I_{\text{max}}, & \text{if } \cos \theta < 0 \cap (\text{phase 1 or 2}) \\ -I_{\text{max}}, & \text{if } \cos \theta > 0 \cap (\text{phase 1 or 2}) \\ 0, & \text{otherwise} \end{cases}$$

(8)

where phases 1 and 2 are the corresponding phases to change eccentricity and semi-major axis. For decreasing eccentricity and increasing semi-major axis, the conditions are defined as:

- phase 1: $(t \leq t') \cap \{ (\vartheta_1 < \vartheta < \vartheta_2) \cup (\vartheta_3 < \vartheta < \vartheta_4) \} \cap (e \geq e_i)$
- phase 2: $(t \geq t') \cap (A \leq A_i)$

here $t'$ is the switching time from changing eccentricity into changing semi-major axis. $e_i$, $A_i$ are the desired eccentricity and semi-major axis. The process of changing eccentricity will help increase part of semi-major axis, and thus save time and energy for the process of semi-major axis.

4. Numerical result of SEDT controlled motion

Following shared parameters are used to simulate motion of SEDT in all 3 cases:

| Parameters | Values |
|------------|--------|
| $m_1, m_2$ | 30 kg, 6000 kg |
| $r$ | 3 km |
| $\theta_0, \theta_0'$ | 0, 0.01 rad/s |
| $\varphi_0, \varphi_0'$ | 0, 0 |
| $I_{\text{max}}$ | 5A |
| $\vartheta_0, \omega_{\vartheta_0}$ | 0, 0 |
| $i_0$ | 11.5 deg |

4.1. Changing semi-major axis

Here it is assumed that SEDT revolves in the initial orbit with the semi-major axis $A_0 = 6885$ km and eccentricity $e_0 = 0.002$. It is necessary to increase the semi-major axis of the orbit by 100 km, that is, $A_f = 6985$ km. With control method (6), numerical result is shown in Figures 5-6.

Figures 5-6 show the first 3 hours of maneuver, and the full increment of semi-major axis (100 km) takes about 419 hours (17 days). It is noteworthy that the discrete change in the magnitude of the current has the period of 6 minutes (spinning velocity is 0.01 rad/s). The semi-major axis increases linearly, the eccentricity and angular spinning velocity ($\dot{\vartheta}$) oscillate around its initial values. The tension force of tether also oscillates around its initial value (about 11 N). The angle $\varphi$ oscillates within about 2 deg. The inclination is reduced by 0.04 degrees for 17 days. Thus, when using control
(6), the eccentricity of the orbit, the angular velocity $\dot{\theta}$, and the orbital inclination do not change significantly.

Figure 5. Characteristics of SEDT motion when changing semi-major axis.

Figure 6. The current in tether when changing semi-major axis.

4.2. Changing eccentricity

Here it is assumed that the SEDT revolves in the initial orbit with the semi-major axis $A_0 = 6940 km$ and eccentricity $e_0 = 0.01$. It is required to change the eccentricity to the value $e_f = 0.002$. The current acts near the apogee of the orbit, and $\vartheta_1 = \vartheta_4 = 0.9\pi, \vartheta_2 = \vartheta_3 = 1.1\pi$. The trajectories of SEDT using the control (7) are shown in Figures 7-8.

Figure 7. Characteristics of SEDT motion when changing eccentricity.
Figures 7-8 show the first 7 hours of maneuver, it takes approximately 2400 hours (100 days) to reach the desired eccentricity. Such a sufficiently large period of time is due to the fact that the Ampere force acts only for a short period of time on each period. The semi-major axis of SEDT orbit increases with approximately 57 km. Other characteristics of SEDT orbit are changed as follows: the angular velocity $\dot{\theta}$ oscillates relative to its initial value; tether tension oscillates relative to its average value (11 N), the orbital inclination decreases by 0.05 degree. It should be noted that the angular oscillations $\varphi$ increases to 20 degrees. This phenomenon requires a detailed study in order to minimize such perturbation.

4.3. Joint maneuver of changing eccentricity and semi-major axis simultaneously
Here it is assumed that the SEDT revolves in the initial orbit with orbital parameters in table 2. The current is regulated according to equation (8). The trajectories of SEDT are shown in Figures 9-10.

Table 2. Parameters of SEDT and initial conditions for joint maneuver.

| Parameters                          | Values              |
|-------------------------------------|---------------------|
| Initial and desired eccentricity    | $e_0 = 0.01, e_f = 0.002$ |
| Initial and desired semi-major axis | $A_0 = 6940km, A_f = 7040km$ |
| Acting range of current (Phase I)   | $\vartheta_1 = \vartheta_3 = 0.9\pi, \vartheta_2 = \vartheta_4 = 1.1\pi$ |
| Switching time                      | $t' = 8734270s(2426h)$ |

Figure 9. Characteristics of SEDT motion when changing eccentricity and semi-major axis at the same time.
Figures 9-10 show the whole trajectories of maneuver, while the detailed trajectories are similar to the parts 4.1 and 4.2. It takes approximately 2616 hours (109 days) to reach the desired orbit. Most part of time is spent on the first phase of changing eccentricity (2426h), while semi-major axis increases about 57km in this phase. As a result, the second phase of changing semi-major axis takes about 190h (8days), which is 9 days shorter than changing semi-major axis only. Therefore, the joint maneuver saves about 9 days compared to change two orbital parameters separately. Angular velocity $\dot{\theta}$ oscillates relative to its initial value the orbital inclination decreases insignificantly. It should be noted that the angular oscillations $\phi$ also increases to about 20 degrees. Although it does not influence stability of the whole process, it is beyond the acceptable level. Thruster can be introduced in closed-loop controller to stabilize out-of-plane motion.

5. Conclusion
Based on the analysis we can draw the following conclusions:

1. Spinning electrodynamic tether systems can be used to change the orbital parameters, which is necessary for payload transportation with tethers. Such a problem can be solved by controlling the magnitude and direction of current in conductive tethers.

2. The proposed control methods make it possible to change the semi-major axis and the eccentricity of system orbit. In the proposed case of this paper, the angular velocity of spinning tether and the tension force oscillate relative to its average values. Two orbital parameters can be changed simultaneously according to the practical need.

3. Amplitude of out-of-plane oscillation slowly accumulates with time. It is necessary to introduce corresponding controller in closed-loop controller.

4. The paper does not address the issues of optimization of the considered orbital maneuvers, which require further study.

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