Joint Access Point Selection and Interference Cancellation for Cell-Free Massive MIMO

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Abstract

Cell-Free Massive MIMO is a highly promising approach to enhance network capacity by moving a large number of distributed access points (AP) closer to mobile users while utilizing simple matched filtering and conjugate beamforming. Recent work on centralized implementation of this approach using minimum mean-squared-error (MMSE) receiver shows significant capacity increase, but at the cost of high computational complexity which may be impractical for beyond 5G systems. We propose a significantly lower complexity alternative where the central processing unit (CPU) exploits readily available knowledge of users channel estimates to perform joint process of combining selected strongest AP signals for each user and canceling the sum of interfering users estimates. We provide complexity and sum spectral efficiency (SE) analyses and illustrative examples to answer why this approach yields higher performances compared with the other alternatives. Extensive numerical results are then provided to confirm substantial capacity and complexity gains while outperforming MMSE using fewer APs.

Index Terms

Cell-Free Massive MIMO system, High capacity, Interference cancellation, Low complexity, Small cells

I. INTRODUCTION

Recently, cell free (CF) massive MIMO has received widespread attention as being one of the most promising approaches to enhance the user experience of mobile users for beyond 5G technologies by deploying a large number of distributed antennas or APs closer to mobile users. It benefits from gains afforded by such large antenna arrays while using simple matched filter (MF) combining and conjugate beamforming for maintaining low complexity [1]-[4]. User centric (UC) approach in [2] and more recently an effective gain-based AP selection [3] have been proposed to select only a subset of APs to reduce computations and backhaul signalling compared with the total APs selection in [1].

The schemes [1]-[3] utilizing traditional interference ignorant MF detection method cannot achieve full capacity gains of large array distributed antenna APs. So minimum-mean-squared-error (MMSE) receivers with interference suppression capabilities at the CPU is investigated in [2] to show substantial capacity gains while ignoring computational load aspects. The MMSE schemes require inversion of large matrices, and hence requiring significantly

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higher computational efforts to afford the gains in capacity. Interference cancellation (IC) is a lower complexity alternative that don't involve matrix inversions and has been studied extensively in code division multiple access (CDMA) detection problems [7], [8]. It can achieve comparable performance to zero forcing (ZF) and MMSE receivers and hence is more appealing in this context.

With this background, we propose a new low complexity and high capacity alternative of joint access point selection and interference cancellation at the CPU to benefit from the advantages of the both. More specifically, it a) combines signals only from the strongest APs to generate better initial data estimates and b) within the same process, cancels sum of all interfering user estimates from the raw data to refine desired users data estimates in multiple stages referred to here as Joint AP Selection and Interference Cancellation (JAPSIC) to meet the above objectives. We verify the gains in signal-to-interference noise ratio (SINR) and hence sum spectral efficiency (SE) of JAPSIC over the MF [1], UC [2] and the MMSE schemes [5] analytically and using numerical simulations. We also detail savings in computational efforts, backhaul overheads to justify its implementation.

Notations: Bold faces lowercase letters \( \mathbf{x} \) denote column vectors; boldface uppercase letters \( \mathbf{X} \) denote matrices. The superscripts \( \{.,\}^T \) and \( \{.,\}^H \) denote transpose and conjugate transpose, respectively; \( \mathbf{0}_N \) denotes a row vector of size \( N \) consisting of all zeros, \( \mathbf{I}_M \) denotes an identity matrix of size \( M \times M \). The operator \( \mathbb{E}\{x\} \) denotes expectation with respect to \( \{x\} \); and \( \text{size}\{.,\} \) denotes the cardinality of the input data.

II. System Model

We consider an urban CF wireless environment with \( K \) single antenna mobile users and \( M \) distributed single antenna APs that are connected via backhaul links to the CPU where all channel estimation and decoding of users data is performed. While we focus this letter to analyze the uplink performances, we envisage that extension of the proposed methods to the downlink is conceivable using precoding/pre-cancellation based on e.g. the uplink-downlink duality principle [6]. We consider a centralized setup similar to [5] consisting of a) training phase of pilot transmission from the mobile users to the APs to allow channel estimation at the CPU and b) uplink data transmission phase from all mobile users to all APs generating complex raw data statistics that are sent to the CPU for final decoding.

A. Propagation Model

The propagation model used here is based on 3GPP Urban Microcell model that captures the essence of typical dense urban environment better than the three-slope path loss model [5]. For a typical carrier frequency of 2 GHz, this gives coefficients \( \beta_{k,m} \), in dB, capturing large scale fading and shadowing effects as follows: \( \beta_{k,m} = -30.5 - 36.7 \log_{10} \left( \frac{d_{k,m}}{1m} \right) + F_{k,m} \), where \( d_{k,m} \) is the distance between user \( k \) and AP \( m \) and \( F_{k,m} \) is the shadow fading loss with distribution \( \mathcal{N}(0, 4^2) \). Mean values for shadowing correlation between APs and users are assumed 0 if they are spaced \( > 50 \) m apart and \( 4^2 2^{-\delta_{k,i}/9m} \) otherwise, where \( \delta_{k,i} \) is the distance between users \( k \) and \( i \). Short-term fading terms \( h_{k,m} \) are assumed flat across the coherence bandwidth and static during each channel realization with coherence time of \( \tau_c \) and follow Rayleigh distribution \( \mathcal{CN}(0, 1) \). The complex channel \( g_{k,m} \) between
user \( k \) and AP \( m \) incorporating large scale fading, shadowing and short term fading can then be written as:
\[
g_{k,m} = \beta_{k,m}^{1/2} h_{k,m}, \quad k = 1, 2, \ldots, K; \ m = 1, 2, \ldots, M.
\]

B. Uplink Training and Channel Estimation Model

In this phase, all \( K \) users simultaneously transmit pilot sequences of length \( \tau_p \) to APs, that are forwarded to the CPU for estimating all the channels. We assume \( \tau_p \) orthogonal pilot sequences are used and high loading scenario of \( \tau_p << K \) so multiple users in a set \( \mathcal{P}_k \subset \{1, 2, \ldots, K\} \), share the same sequence with \( \rho = \text{size}\{\mathcal{P}_k\} \) and MMSE channel estimation is used. Therefore, our results include effects of pilot contamination that have direct impact on SINR and SE results. The estimates will improve when \( \tau_p \geq K \) as each user is assigned unique pilot, but this will reduce SE as will be shown in (5). A channel estimate obtained at the CPU for user \( k \) at AP \( m \), \( \hat{g}_{k,m} \), can be modelled as:
\[
\hat{g}_{k,m} = g_{k,m} + c_{k,m}, \quad \text{where} \ c_{k,m} \text{ is a channel estimation error that is uncorrelated with the channel and is assumed i.i.d. } \mathcal{CN}(0, \sigma^2_c).
\]

C. Uplink Data Transmission

In this phase lasting \( \tau_c - \tau_p \) symbols, all \( K \) users transmit their data \( s_k, k = 1, 2, \ldots, K \) over their respective channels \( g_{k,m} \) to give each AP \( m = 1, 2, \ldots, M \) their received signals:
\[
y_m = \sum_{k=1}^{K} \sqrt{p_k} g_{k,m} s_k + \nu_m.
\]

The data estimation process at the CPU involves specification of \( \mathbf{w}_k(n) \) depending on the receiver methods used, including manipulations of intermediate soft estimates to arrive as close as possible to the original data \( \hat{s}_k(n) \rightarrow s_k(n), \forall k, \forall n \).

III. PROPOSED JAPSIC RECEIVER

The process for obtaining \( k_{th} \) users data estimate \( \hat{s}_k(n), k = 1, 2, \ldots, K \) at the \( n_{th} \) symbol period, \( n = 1, 2, \ldots, N \), involves taking the vector \( \mathbf{y}(n) \) and multiplying it with a combining vector \( \mathbf{w}_k(n) \) as follows:
\[
\hat{s}_k(n) = \mathbf{w}_k(n) \mathbf{y}(n).
\]

A. Existing CF Massive MIMO Schemes

With the CF scheme using MF decoding, \( m_{th} \) an AP sends soft data estimate that it obtains by multiplying the raw data with the conjugate of local channel estimates i.e. \( w_{k,m} = g_{k,m}^* \) while ignoring the presence of multiuser interference (MUI): \( y_{k,m} = w_{k,m} y_m \). The CPU receives \( M \) such estimates to generate final data estimate.
\[ \hat{s}_k = \sum_{m=1}^{M} y_{k,m}. \]

The UC approach \[2\] is obtained by processing subset of \( M_u \) APs instead of all APs. The MMSE schemes suppress multi user interference (MUI) and noise to give better performance than MF \[5\]. This involves inverting channel correlation \( G \), estimation error \( C \) and noise estimation \( \sigma^2 I_M \) matrices of sizes \( K \times M \), \( K \times M \) and \( M \times M \) to minimize the mean squared error of data \( \mathbb{E}\{|\hat{s}_k(n) - s_k(n)|\}^2 \). The combining vector \[5\], dropping \((n)\) notation for simplicity here, can be written as:

\[
    w_k = p_k \left( \sum_{i=1}^{K} p_i \left( \hat{g}_i \hat{g}_i^H + C_i \right) + \sigma^2 I_M \right)^{-1} \hat{g}_k^H. \tag{3}
\]

MMSE-SIC enhances upon the MMSE by successively decoding and cancelling strongest users before decoding weaker users.

### B. Proposed JAPSIC Algorithms

Under this scheme, the CPU collects raw data \( y \) at the initial stage \( 0 \) and exploits the already available knowledge of all users channel estimates e.g. \( g_k \), to generate soft data \( y_k^l \) to refine the users soft data estimate \( \hat{s}_k^l \) in multiple stages \( l = 1, 2, \ldots, L \). It entails taking the estimate from the previous stage \( \hat{s}_k^{l-1} \) and subtracting sum of all interfering users estimates \( \Psi_k^l \) from \( y \) involving minimal number of complex multiplications. This gives the JAPSIC a big advantage over MMSE on computational efforts, which becomes pronounced in highly mobile environments. Channel coherence time in such case is much shorter, requiring frequent calculation of complex vectors e.g. 3GPP LTE Rel. 8 specifies that up to 2 pilots are inserted every 7 OFDM symbols \[9\]. Two variants are detailed here JAPSIC based on channel-power threshold referred to as JAPSIC \( \theta \), and JAPSIC based on \( M_u \) subset of APs with strongest signals referred to as JAPSIC \( M_u \).

1) **JAPSIC \( \theta \) Algorithm:** This variant utilizes a threshold value \( \theta \) that is compared against each users estimated channel power at each AP \( |g_{k,m}|^2 \), to use as a measure to qualify an APs raw data for processing and cancellation in subsequent stages. The algorithm steps are shown in Table I.

2) **JAPSIC \( M_u \) Algorithm:** This variant differs from the former in how APs are selected for each users data decoding in that, instead of using selecting any APs meeting a given channel power threshold \( |g_{k,m}|^2 > \theta \), a fixed number of \( M_u \leq M \) APs with strongest channel power is selected using the users channel estimates vectors from Phase I. The full algorithm is given in Table II.

### IV. Sum SE and Complexity Analyses

We start with derivation of SINR for the JAPSIC process at the final \( L_{th} \) stage after \( L - 1 \) iterations of refining desired user data, leaving residual MUI which consists of noisy leftover MUI after cancelling the reconstructed interference signals from the total MUI and the total noise component. Note that linear interference cancellation methods that do not use hard decision in each stage such as one described here, refine data estimates without...
TABLE I
JAPSIC $\theta$ ALGORITHM FOR DATA ESTIMATION

1) Set a channel-power threshold $= \theta, \forall k, \forall m; k = 1, 2, .., K; m = 1, 2, .., M$
2) For each channel coherence block $n = 1 : N,$
3) For each user, $k = 1 : K,$
4) Calculate indices vector $\iota_k(n)$ by evaluating the threshold:
   $\mu_k(n) = \text{size}\{\iota_k(n)\} \leq M, \forall k,$
5) For each stage of detection, $l = 0, 1, 2, .., L,$
   a) Obtain combining vector and raw data for the user by selecting
      the subset with indices $\iota_k(n)$ from $w_k(n)$:
      $w_{\iota_k(n)} = \hat{g}^H_{\iota_k(n)}$
   b) Obtain the JAPSIC cancellation vector, $\Psi_k(n),$ by summing
      all interfering users signals:
      $\Psi_k(n) = \sum_{i,i \neq k} \hat{g}_{\iota_{ik}(n)}^H \hat{s}_{i}^{l-1}(n)$
   c) Update data statistics for user $k; y_{\iota_k(n)}^{l} = y_{\iota_k(n)} - \Psi_k(n)$
   d) Obtain a soft data estimate for the $k_{th}$ user
      $\hat{s}_k(n)$, using
      signal statistics from all APs, $\hat{s}_k(n) = w_{\iota_k(n)} y_{\iota_k(n)}^{l}$
6) End $k,$ and end $n.$
7) Calculate mean number of APs selected $M = \frac{1}{KNN} \sum_{k=1}^{K} \sum_{n=1}^{N} \mu_k(n)$

TABLE II
JAPSIC $M_u$ ALGORITHM FOR DATA ESTIMATION

1) Set the number of APs to select $= M_u, \forall k, k = 1, 2, .., K, M_u \leq M$
2) For each channel coherence block $n = 1, 2, .., N,$
3) For each user, $k = 1, 2, .., K,$
4) Initialize a vector of indices to be assigned to $M_u$ selected APs from
   all $M$ APs, $\iota_k(n) = \mathbf{0}_{M_u}; M_u = \text{size}\{\iota_k(n)\} \leq M, \forall k$
5) Derive the indices vector $\iota_k(n),$ by sorting all APs’ channel powers
   $|g_k(n)|^2$ in descending order and picking only the first $M_u$ APs
   indices
6) Use the steps 5 and 6 as in Table I.

causing error propagation and close to optimal decoding can be achieved with e.g. $L = 10$ \cite{8}. For the sake of
gaining insight of the results, we use JAPSIC $M_u$ as example to obtain mean SINR for user $k, \Gamma_k$ as follows:

$$\Gamma_k = \frac{p_k \sum_{m \in \mathcal{I}_k} |g^\dagger_{k,m}g_{k,m}|^2}{\sum_{i=1,i \neq k}^K p_i \sum_{m \in \mathcal{I}_k} |g_{k,m}g_{i,m} - \hat{g}_{k,m}^\dagger \hat{g}_{i,m}|^2 + \sigma_n^2 \sum_{m \in \mathcal{I}_k} |g_{k,m}|^2},$$

(4)

where the numerator is formed by collecting energies from the desired user from the subset of $M_u \leq M$ APs. The denominator is formed of residual MUI and thermal noise from $M_u$ APs, likewise. Finally, the achievable sum SE $\Upsilon_{sum}$ is obtained by summing SEs of all $K$ users using their expected SINR values over all channel realizations:

$$\Upsilon_{sum} = \sum_{k=1}^K \left(1 - \frac{\tau_c}{\tau_c}\right) E\left\{\log_2(1 + \Gamma_k)\right\}.$$  

(5)

Next we compare the computational efforts of JAPSIC with the others in terms of complex multiplication/division operations while ignoring additions and subtraction terms in Table III. Most schemes include matched filtering with $KM$ operations at the initial stage. The JAPSIC $\theta$ scheme adds modest demand of $K M_u$ multiplications per stage for $L-1$ stages to reconstruct and cancel MUI estimates. JAPSIC $M_u$ requires ranking of channel powers of all APs, adding further $M \log_2(M)$ computations for selecting $M_u$ APs. MMSE schemes require inversion of matrices of size $M \times M$ for each user, leading to $\mathcal{O}(K^3)$ multiplications. MMSE-SIC demands $\approx K/2$ times the efforts of MMSE. To give some numbers: for $K = 40$, and $M = 100$, we find computations required for: MMSE-SIC $= 11284000$, and MMSE $= 568000$ operations while for JAPSIC $\theta = 40000$, and JAPSIC $M_u = 40660$ assuming $M_u = M = M/2$, and $L = 10$. The MF \cite{1} uses $KM = 4000$ operations. With UC using $M_u = 50$, this reduces to 2600.

In terms of backhaul signalling efforts, JAPSIC requires $M$ APs to send total of $\tau_c M$ complex scalars to the CPU every coherence period. No channel correlation matrices needed to be known at the CPU unlike in \cite{5}, thus saving signalling efforts to send further $KM/2$ complex scalars.

It is not difficult to see from (4) that when $M_u$ is sufficiently large, the JAPSIC receivers collect large part of useful signals while also substantially reducing the MUI to give higher SINR and SE compared with MF and UC. Proving superiority of the JAPSIC over the MMSE is not that straightforward, but we investigate here using a simple analysis followed by an example to compare the differences. A ratio of sum of residual MUIs of the JAPSIC over the MMSE $\Phi_k, k = 1, 2, \ldots, K$, can be used to highlight the points in a compact form as follows:

$$\Phi_k = \frac{\sum_{i=1,i \neq k}^K \sum_{m \in \mathcal{I}_k} |g_{k,m}g_{i,m} - \hat{g}_{k,m}^\dagger \hat{g}_{i,m}|^2}{\sum_{i=1,i \neq k}^K \sum_{m=1}^M \left(1 - \frac{|g_{k,m}g_{i,m}|^2}{|\hat{g}_{k,m}^\dagger \hat{g}_{i,m}|^2 + \sigma_n^2}\right)}.$$  

(6)

To test this, let’s assume $M = 2$ and $K = 2$ with $\sigma_n^2 = 0.01$ while $g_{1,1} = -1.1 + 0j$ and $g_{2,1} = 0.9 + 0j$ for AP 1; but we get $\hat{g}_{1,1} = -1.15 + 0j$ and $\hat{g}_{2,1} = 0.92 + 0j$, respectively. However, say for AP 2, channel gains are weaker with $g_{1,2} = 0.1 + 0j$ and $g_{2,2} = -0.2 + 0j$ but we get $\hat{g}_{1,2} = 0.17 + 0j$ and $\hat{g}_{2,2} = -0.22 + 0j$, respectively. Plugging them into (6) gives residual MUI of 0.005 for JAPSIC, and -0.140 for MMSE at AP 1, but significantly lower value of 0.0003 for JAPSIC at the AP2 compared with 0.720 of MMSE which suffers from enhancement of errors. This example shows that in typical urban wireless environments with likely non-zero channel estimation
TABLE III

COMPARISON OF COMPUTATIONAL EFFORTS OF DIFFERENT CELL-FREE MASSIVE MIMO SCHEMES FOR EACH DETECTION CYCLE

| Scheme       | Initial Filtering, Post Processing of Signal Vectors                                      |
|--------------|-------------------------------------------------------------------------------------------|
| MMSE [5]     | $K M + K((M^2 + KM) + M)$                                                                 |
| MMSE-SIC [5] | $K M + K/2\{K((M^2 + KM) + M)\}$                                                           |
| JAPSIC $\theta$ | $K M + (L - 1)2KM$                                                                          |
| JAPSIC $M_u$ | $K M + (L - 1)2KM_u + M \log_2(M)$                                                         |
| UC [2]       | $KM_u + M \log_2(M)$                                                                       |
| MF [1]       | $KM$                                                                                       |

errors, JAPSIC can have lower sum residual MUI and hence the higher SINR if channel gains at some APs are low.

V. NUMERICAL RESULTS

For further comparisons, we use simulations assuming the following setup. We take an 1 km $\times$ 1 km area with $K$ users randomly distributed within the area and $M = 100$ single antenna APs. All users transmit with power $p_k = 100$ mW, carrier center frequency is 2 GHz and system bandwidth 20 MHz, thermal noise power is $-174$ dBm/Hz and noise figure at APs of 5 dB, $\tau_c = 200$, $\rho = 4$ and $\tau_p = K/\rho$.

Figure 1 shows the cumulative distribution function (CDF) graphs of the sum SEs achieved by the JAPSIC schemes against the MF [1], UC [2], MMSE [5] and MMSE-SIC [5] with $K = 40$ and $M = 100$ under the same centralized system setup for fair comparisons. As expected, JAPSIC schemes give much higher sum SE as $\theta/M_u$ is lowered/increased. With this change, the JAPSIC algorithms pick more APs with stronger channels to give better estimates of users data and this knowledge is aptly exploited in multiple stages to cancel MUI and refine all users data estimates. JAPSIC schemes with $\theta = 0.01/M_u = 10$ far outperform MF/UC, but are still inferior to MMSE; note however that the modest increase of complexity of JAPSIC may still be justifiable. With $\theta = 0.0001/M_u = 50$, JAPSIC schemes outperform MMSE as well as MMSE-SIC while achieving almost the full interference cancellation (F-IC) with about half the APs. To give some numbers, at the 50% likely SE points, the JAPSIC with $M_u = 50$ gives 220 bits/s/Hz compared with 199 for MMSE and 214 for MMSE-SIC.

Figure 2 continues by showing the sum SEs of JAPSIC compared with other schemes at a lighter load of $K = 16$ for the same setup. As expected, the sum SEs of the JAPSIC increase with the increase/decrease in $M_u/\theta$ and they outperform MF and UC in all cases considered. When $M_u = 50/\theta = 0.0001$, JAPSIC schemes generally outperform MMSE and almost reach the sum SEs of F-IC and MMSE-SIC. This shows that with a careful receiver design, it is actually possible to achieve high capacity without processing large number of AP signals. Comparing with the Figure 1 results, it appears that JAPSIC outperform MMSE schemes more in higher $K/M$ conditions.

VI. CONCLUSION

We demonstrated a new low complexity and high capacity approach called JAPSIC that employs joint process of selective combining of AP signals and multistage interference cancellation as an attractive alternative to MMSE.
Fig. 1. Comparison of CDF of sum spectral efficiencies for the proposed JAPSIC schemes against other CF schemes for $K = 40, M = 100$; where JAPSIC $M_u$ with $M_u = \{10, 50\}$ and JAPSIC $\theta$ with $\theta = \{0.01, 0.0001\}$ giving $M = \{10.55, 43.20\}$, respectively, are used.

Based on CF massive MIMO. With analyses and numerical results, we verified substantial gains both in terms of spectral and computational efficiencies that merits the scheme proposed. For example, at a computational demand of just 0.3% of the MMSE-SIC, it can achieve higher sum SE of 220 bits/s/Hz compared with 214 and 199 for the MMSE and almost double the MF that achieves only 112 bits/s/Hz. For the future work, it will be interesting to investigate group collaborative methods for pilot sharing by multiple users to address the capacity loss due to the pilot contamination problem.

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Fig. 2. Comparison of CDF of sum spectral efficiencies for the proposed JAPSIC schemes, with $M_u = \{10, 50\}$ and $\theta = \{0.01, 0.0001\}$ against other CF schemes, $K = 16$, $M = 100$.

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