Fractional Sliding Mode Control for Nonlinear Aerospace Systems

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Abstract. This paper focuses on the study of fractional order terminal sliding mode control for nonlinear aerospace systems. Firstly, a novel fractional order integral terminal sliding mode control (FO-I-TSMC) method is proposed for the control of first order nonlinear system. FO-I-TSMC has three attractive advantages: i) Non-singular control law; ii) Elimination of the reaching phase; iii) Calculable finite convergence time. Furthermore, theory analysis is presented to reveal the potential advantages of the FO-I-TSMC method over its integer order counterparts. Secondly, a novel fractional order derivative integral-TSMC (FO-DI-TSMC) method is presented to deal with second order nonlinear system. Finally, FO-DI-TSMC is extended to deal with a general class of higher order control system. Simulation results are given to verify the effectiveness of the proposed methods.

1. Introduction

Aerospace systems, such as satellite, spacecraft, unmanned aerial vehicles, are highly nonlinear systems. High performance aerospace control system should own the features of high tracking response, small overshoot and strong anti-disturbance ability. Conventional PID controller has already been applied in aerospace control system because of its clear functionality, simplicity and ease of implementation [1]. Nevertheless, due to the nonlinearities, unmeasured disturbances and parameters variations, traditional PID controller cannot ensure a sufficiently high control performance. Therefore, various methods of nonlinear control theory, such as predictive control, robust control and fractional order control [2-4], have been proposed for the aerospace system. Among these advanced control strategies, the sliding mode control is regarded as a powerful method to improve the control performance. The main advantage of sliding mode control is that the control system can exhibit strong robustness properties with respect to both internal parameter variations and external unmeasured disturbances.

The sliding mode control usually consists of two steps: 1) design a sliding surface that ensures the desired error dynamics of the system; 2) drive the system to reach and stay on the specified sliding surface. In conventional sliding mode control, linear sliding surface is adopted to describe the desired performance of the control system. Although the convergence rate of these methods sometimes can be very fast, the system state can only converge to zero in infinite time.

Therefore, in order to further improve the control performance of the closed loop system, nonlinear sliding surfaces are presented to accomplish finite time error convergence. Venkataraman and Gulati
firstly proposed the terminal sliding mode control (TSMC) with a nonlinear sliding surface. Since then, TSMC has attracted a considerable attention both from an academic and industrial point of view. Meanwhile, it has been successfully applied in the servo drive system [5]. TSMC is of interest because the system with finite time convergence can demonstrate very nice characteristics such as higher precision and better robustness. Nevertheless, there exists an intrinsic singular problem in conventional TSMC because of utilizing nonlinear sliding surface. This may result in the instability of the control system.

Fractional calculus is a generalization of the traditional integration and differentiation to the non-integer order [6]. Recently, fractional order controllers have drawn an increasing attention. The extra degrees of freedom from adopting the fractional order made it possible to further improve the control performance compared with integer order controller. Fractional order sliding mode control is one of those fractional order control strategies and it has gained much interest from the control community. In fact, fractional order theory has already been incorporated into the conventional sliding mode control, conventional TSMC and non-singular TSMC methods. For instance, by introducing fractional order dynamic into the traditional sliding surface, BiTao Zhang et al. proposed a novel fractional order sliding mode control strategy for the permanent magnet synchronous motor [7]. Sara Dadras et al. combined the fractional calculus with TSMC to formulate a new fractional order TSMC method for a class of dynamical systems with uncertainty [8]. However, to the best of our knowledge, there is no report on adopting the fractional calculus in the design procedure of I-TSMC and DI-TSMC methods.

Therefore, designing fractional order I-TSMC and DI-TSMC methods still remains an open and challenging problem. Motivated by the previous work, to further improve the tracking response and robustness of the PMLSM servo system, this paper will incorporate the fractional calculus into the design process of I-TSMC and DI-TSMC methods. The detail contributions are as follows:

1) A novel fractional order I-TSMC (FO-I-TSMC) method is proposed for the first order nonlinear system. FO-I-TSMC has three attractive advantages: i) Non-singular control law; ii) Elimination of the reaching phase; iii) Calculable finite convergence time.

2) A novel fractional order DI-TSMC (FO-DI-TSMC) method is presented to deal with second order nonlinear aerospace system.

3) Due to aerospace systems are often high order nonlinear systems. FO-DI-TSMC is further extended to deal with a general class of higher order control system.

2. Preliminaries and definitions

2.1. Fractional calculus

In this section, we will introduce some basic definitions for fractional calculus. Fractional calculus generalizes the ordinary integration and derivation to the non-integer order operators \( ^{\alpha}D_t^\lambda \). It is given by:

\[
^{\alpha}D_t^\lambda = \begin{cases} 
\frac{d^\lambda}{dt^\lambda} & \text{if } \lambda > 0 \\
1 & \text{if } \lambda = 0 \\
\int_{t_0}^t (dt)^{\lambda-1} & \text{if } \lambda < 0
\end{cases}
\]

where \( \lambda \in \mathbb{R} \) is the fractional order.

There are several definitions for the fractional integration and derivation. The following two definitions will be used in our proposed control method.

**Definition 1** [6]. The Riemann-Liouville fractional integration of a function \( f(t) \) is defined as:

\[
^{\alpha}D_t^{-\lambda} f(t) = \frac{1}{\Gamma(-\lambda)} \int_{t_0}^{t} (t-\tau)^{-\lambda-1} f(\tau) d\tau
\]

The fractional derivation of is defined as
\[
\dot{\eta}_b D^\alpha_t f(t) = \frac{d^m}{d\tau^m} \left[ \frac{1}{\Gamma(m-\lambda)} \int_0^t \frac{f(t)}{(t-\tau)^{\lambda-m+\alpha}} d\tau \right]
\]

**Definition 2** [6]. The Grunwald-Letnikov fractional integration and derivation of a function \( f(t) \) is defined as:

\[
\dot{\eta}_b D^\lambda_t f(t) = \lim_{h \to 0} \frac{1}{h} \left[ (t-ih)^{\lambda} \sum_{i=0}^{[t-ih]/h} \frac{(-1)^i}{i!} \right] f(t)
\]

where \( h \) represents the sampling time, \([t-ih]/h\] is the rounding function and

\[
\left( \lambda \right)_i = \frac{\lambda(\lambda-1)\cdots(\lambda-i+1)}{i!}
\]

**Remark 1.** Note that for the number of addends in the above equation becomes enormously large. This cannot be implemented in practice. Using the short memory principle in Ref. [6], the Grunwald-Letnikov definition can be approximated as

\[
\dot{\eta}_b D^\lambda_t f(t) \approx \dot{\eta}_b D^\lambda_{t-L} f(t)
\]

where \( L \) is the memory length.

2.2. **Control objectives**

We will consider the control problem for the following three classes of nonlinear systems, i.e., first order, second order and high order nonlinear systems. These nonlinear systems can describe a wide class of aerospace nonlinear systems, such as satellite, spacecraft, unmanned aerial vehicles, servo drive systems etc.

1) First order nonlinear systems

\[
\dot{e}_v(t) = f_v(e_v) + g_v(e_v)\dot{e}_v + d(t)
\]

where \( e_v \) is the velocity tracking error, \( f \) and \( g \) are nonlinear systems, \( d \) is disturbance.

2) Second order nonlinear systems

\[
\ddot{e}_\theta(t) = f_\theta(e_\theta) + g_\theta(e_\theta)\dot{e}_\theta + d(t)
\]

where \( e_\theta \) is the position tracking error, \( f \) and \( g \) are nonlinear systems, \( d \) is disturbance.

3) High order nonlinear systems

\[
x^{(n)}(t) = f(x) + \ddot{g}(x)u + \ddot{d}(t)
\]

where \( x \) represents the state, \( f \) and \( g \) are nonlinear systems, \( d \) is disturbance.

Considering the tracking reference, he above equation can be written as

\[
e^{(n)}(t) = x^{(n)} - f(x, -e) - \ddot{g}(x, -e)u - \ddot{d}(t)
\]

where \( e \) is the tracking error.

The control objective is to design a controller \( u \) such that the tracking error is minimized.

3. **Fractional order integral terminal sliding mode control**

We will consider the sliding mode control of first order system. The sliding mode control for the first order system can be adopted for the speed control of some aerospace system. Next, we will present the FO-I-TSMC method. FO-I-TSMC consists of two different types, sign and fractional integral sliding mode.

3.1. **Type I: fractional-order sign integral sliding mode**

The fractional-order sign integral sliding surface is designed as:

\[
s(t) = e(t) + \alpha e_\tau(t)
\]

\[
\dot{e}_\tau(t) = P(e(t)) = \begin{cases} 
\dot{e}_\tau(t) = \dot{\eta}_b D^{\lambda}_{t}[\text{sgn}(e(t))] & \text{if } e(t) \neq 0 \\
0 & \text{if } e(t) = 0
\end{cases}
\]

where \( \alpha > 0 \), \( 0 < \lambda < 2 \) and we also have

\[
e_\tau(0) = -e(0)/\alpha.
\]
By adopting the above definition, we have the following results:

**Theorem 1.** The first order system, i.e. the system described by (1) is in the sliding surface all the time, i.e. \( s(t) = 0, \forall t \geq 0 \), if the following controller is applied

\[
    u(t) = g(e)^{-1}[-f(e) - \alpha P(e(t)) - \eta \text{sgn}(s(t))]
\]

where \( \eta > \delta \geq |d(t)| \).

When using the above control action, the convergence time of the system will be finite.

**Theorem 2.** Suppose the above control law is adopted, the tracking error \( e \) will converge to zero in finite time

\[
    T_s = \left( \frac{e(0) |\Gamma(\lambda + 1)|^{\mu_\lambda}}{\alpha} \right)^{1/\mu_\lambda}.
\]

**3.2. Type II: fractional-order fractional integral sliding mode**

The fractional-order sign integral sliding surface is designed as:

\[
    s(t) = e(t) + \alpha e_i(t)
\]

\[
    \dot{e}_i(t) = Q(e(t)) = \begin{cases} \alpha D_{t}^{\lambda} [e(t)^{q/p}] & \text{if } e(t) \neq 0 \\ 0 & \text{if } e(t) = 0 \end{cases}
\]

where \( \alpha > 0 \), \( 0 < \lambda < 2 \) and we also have

\[
    e_i(0) = -e(0) / \alpha.
\]

By adopting the above definition, we have the following results:

**Theorem 3.** The first order system, i.e. the system described by (1) is in the sliding surface all the time, i.e. \( s(t) = 0, \forall t \geq 0 \), if the following controller is applied

\[
    u(t) = g(e)^{-1}[-f(e) - \alpha Q(e(t)) - \eta \text{sgn}(s(t))]
\]

where \( \eta > \delta \geq |d(t)| \).

The convergence time of the system will also be finite.

**Theorem 2.** Suppose the above control law is adopted, the tracking error \( e \) will converge to zero in finite time

\[
    T_s = \left( \frac{e(0) |\Gamma(\lambda + 1)|^{\mu_\lambda}}{\alpha(1 - q/p)} \right)^{1/\mu_\lambda}.
\]

**Remark 4.** The proposed FO-I-TSMC methods have some nice features which are similar to the IO-I-TSMC [12]. 1) According to Theorems 1 and 3, it can be concluded that there is no reaching phase and the control system is always kept in the sliding surface. This indicates that the servo system could have better robustness to parameter uncertainties and faster response. 2) The tracking error can converge to zero in finite time. Moreover, the convergence time can be calculated easily based on Theorems 2 and 4. 3) We can see that the FO-I-TSMC methods do not have singular problems compared with the conventional TSMC. 4) The gain only depends on the bound of the uncertainty. Therefore, the gain can be adjusted by using some adaptation mechanism such as neural networks or gradient descent methods.

**4. Fractional order derivative-integral terminal sliding mode control**

As can be seen, the results in Section 3 are only suitable for the first order system. Also the speed loop of the aerospace nonlinear system is typically a first order system. For the control of position loop or other higher order system, the FO-I-TSMC cannot be applied. Therefore, in this section the fractional order derivative-integral TSMC method (FO-DI-TSMC) is proposed to deal with the output tracking problem for higher order system. First, we will consider the control of second order system. Next, we will consider a general class of high order system.

**4.1. FO-DI-TSMC for a class of second order system**

FO-DI-TSMC for second order system is in fact a combination of the non-singular fractional order TSMC in Refs. [10-11] and our proposed FO-I-TSMC. The fractional-order sign integral sliding surface is defined as:
\[ e_{n}(t) = \left[ a D_{t}^{\alpha} \dot{e}_{n}(t) \right]^{\rho/n} + \beta_{n}e_{n}(t) \]

\[ s(t) = e_{n}(t) + \alpha e_{1}(t) \]

where

\[ e_{1}(0) = -e_{n}(0) / \alpha . \]

According to the above definition, the sliding surface can be expressed as:

\[ \dot{s}(t) = \frac{p_{1}}{q_{1}} \left[ a D_{t}^{\alpha} \dot{e}_{n}(t) \right]^{\rho/n} \cdot a D_{t}^{\gamma} \dot{e}_{n}(t) + \psi \]

\[ = \frac{p_{1}}{q_{1}} \left[ a D_{t}^{\alpha} \dot{e}_{n}(t) \right]^{\rho/n} \cdot a D_{t}^{\gamma} \left[ f(e) + g(\dot{e})u + d(t) \right] + \psi \]

where

\[ \psi = \beta_{n}\dot{e}_{n} + \alpha \dot{e}_{1} \]

Let

\[ F_{x} = \begin{cases} \left[ a D_{t}^{\alpha} \dot{e}_{n}(t) \right]^{\rho/n} & \text{if } \left[ a D_{t}^{\alpha} \dot{e}_{n}(t) \right]^{\rho/n} \geq \varepsilon \\ \varepsilon & \text{if } \left[ a D_{t}^{\alpha} \dot{e}_{n}(t) \right]^{\rho/n} < \varepsilon \end{cases} \]

Then we have

\[ |(F_{x} - [a D_{t}^{\alpha} \dot{e}_{n}(t)]^{\rho/n})F_{x}^{-1}| \leq \varphi < 1 \]

Using the above result, we have

**Theorem 5.** The overall system is in the sliding mode by Definition 5 all the time, i.e. \( s(t) = 0, \forall t \geq 0, \)

**if the following controller is applied**

\[ u(t) = g(e)^{-1}[-f(e) \]

\[ - a D_{t}^{\gamma} \left[ K_{q} \left| \frac{q_{1}}{p_{1}} F_{x}^{-1} \right| + \eta \left| \frac{q_{1}}{p_{1}} F_{x}^{-1} \right| \right] \]

**Theorem 6.** Suppose the above control law is adopted, and \( \dot{e}_{1}(t) \) is designed as \( P(e(t)) \) the tracking error \( e \) will converge to zero in finite time

\[ T_{y} = \frac{e(0) \Gamma(\lambda+1)}{\alpha} e^{\gamma} + \frac{e(t_{1})^{\rho/n}}{\beta_{1}(1-q/p)} \]

\[ T_{y} \leq \frac{e(0) \Gamma(\lambda+1)}{\alpha} e^{\gamma} + \frac{2e(t_{1})}{e(t_{1})} \]

**Theorem 7.** Suppose the above control law is adopted, and \( \dot{e}_{1}(t) \) is designed as \( Q(e(t)) \) the tracking error \( e \) will converge to zero in finite time

\[ T_{y} = \frac{e(0) \Gamma(\lambda+1)}{\alpha(1-q/p)} e^{\gamma} + \frac{e(t_{1})^{\rho/n}}{\beta_{1}(1-q/p)} \]

\[ T_{y} \leq \frac{e(0) \Gamma(\lambda+1)}{\alpha(1-q/p)} e^{\gamma} + \frac{2e(t_{1})}{e(t_{1})} \]

5. **FO-DI-TSMC for a class of high order system**

In this subsection, we will consider the control of a general high order system described by (3). To begin with, we will introduce the fractional order derivative-integral sliding surface.

The fractional-order sign integral sliding surface is defined as:

\[ e_{n}(t) = [\dot{e}_{n}(t)]^{\rho/n} + \beta_{n}e_{n}(t) \]

\[ e_{n}(t) = [\dot{e}_{n}(t)]^{\rho/n} + \beta_{n}e_{n}(t) \]

\[ \cdots \]

\[ e_{n+1}(t) = [\dot{e}_{n+1}(t)]^{\rho/n} + \beta_{n+1}e_{n+1}(t) \]

\[ s(t) = e_{n+1}(t) + \alpha e_{1}(t) \]

\[ e_{1}(0) = -e_{n+1}(0) / \alpha \]
From the above definition, we can derive

\[ \dot{s}(t) = \dot{\varepsilon}_n + \alpha \dot{\varepsilon}_s, \]

\[ = \frac{p_2}{q_2} [e_n^{n+1} \varepsilon_n^{n+1} + \beta e_n^{n+1} + \alpha \dot{\varepsilon}_s] \]

\[ = \frac{p_2}{q_2} [e_n^{n+1} \varepsilon_n^{n+1} + \beta e_n^{n+1} + \alpha \dot{\varepsilon}_s], \]

\[ \phi = \frac{p_2}{q_2} [e_n^{n+1} \varepsilon_n^{n+1} + \beta e_n^{n+1} + \alpha \dot{\varepsilon}_s], \]

\[ = \frac{p_2}{q_2} [e_n^{n+1} \varepsilon_n^{n+1} + \beta e_n^{n+1} + \alpha \dot{\varepsilon}_s]. \]

Then, we have the following results

**Theorem 8.** The overall system is in the sliding mode by Definition 6 all the time, i.e., \( s(t) \equiv 0, \forall t \geq 0 \), if the following controller is applied

\[ u(t) = g(e)^{-1}(-f(e)) \]

\[ -K_x |\psi| \left( \prod_{i=1}^{n} q_i F^{-1} \right) sgn(s(t)) - \eta sgn(s(t)). \]

Meanwhile, the convergence time is given by

**Theorem 9.** Suppose the above control law is adopted and \( \dot{\varepsilon}_s (t) \) is designed as \( P(e_{n+1} (t)), \) then the tracking error \( e(t) \) will converge to zero in finite time

\[ T_e = \frac{\left| e(0) \right|}{\alpha (\frac{1}{q} + 1)} \sum_{i=1}^{n} \left| e_{n+1} (t_i) \right|^{\frac{q}{p}}. \]

**Theorem 10.** Suppose the above control law is adopted and \( \dot{\varepsilon}_s (t) \) is designed as \( Q(e_{n+1} (t)), \) then the tracking error \( e(t) \) will converge to zero in finite time

\[ T_e = \frac{\left| e(0) \right|}{\alpha (1 - q / p)} \sum_{i=1}^{n} \left| e_{n+1} (t_i) \right|^{\frac{q}{p}}. \]

6 Simulation

In this section, simulations are conducted to verify the effectiveness of the proposed FO-I-TSMC and FO-DI-TSMC methods. The proposed methods are mainly adopted to control the servo motor system, which has been widely applied in aerospace engineering. The simulation environment is the MATLAB 2014a. The model of the motor is identified from a real experimental platform shown in Fig. 1.

First, we will focus on the speed control of the PMLSM system. A step speed reference is exerted on the servo system. Type I IO-I-TSMC and FO-I-TSMC are adopted. The control parameters are set as: \( \alpha = 1, \eta = 1 \). The tracking responses are shown in Fig. 2. It can be seen that the rising time of FO-I-TSMC with \( 0 < \lambda < 1 \) is the shortest. FO-I-TSMC with \( \lambda > 1 \) has a sluggish tracking response. Therefore, it can be concluded that smaller \( \lambda \) will result in faster tracking. Meanwhile, it can also be seen from the partial enlarged view in Fig. 1 that in the steady state FO-I-TSMC with \( \lambda > 1 \) has the smallest chattering. However, FO-I-TSMC with \( 0 < \lambda < 1 \) has the largest. Hence, we can conclude that a larger \( \lambda \) can reduce the chattering of the system.

Fig. 3 also shows the variation of \( P(e(t)) \). It can be seen that for IO-I-TSMC \( |P(e(t))| \) is always equal to one. Nevertheless, for FO-I-TSMC, \( |P(e(t))| \) for \( 0 < \lambda < 1 \) which indicates a faster tracking, and \( |P(e(t))| < 1 \) for \( \lambda > 1 \) which implies a smaller overshoot and chattering.
In order to obtain a faster tracking response and smaller chattering, one can use a time varying fractional order I-TSMC. A larger $\lambda$ may be needed for the tracking process and a smaller $\lambda$ for the steady state. We select $\lambda = 0.8$ when $|e(t)| \geq 0.01$ and $\lambda = 1.4$ when $|e(t)| < 0.01$. The tracking response of the time varying fractional order I-TSMC is illustrated in Fig. 4. It can be seen that compared with the IO-I-TSMC, it not only has faster tracking response but also a smaller chattering.

Figure 1. Platform of servo system.

Figure 2. Step responses for Type I I-TSMC.

Figure 3. Variation of $P(e(t))$ for step response.
7. Conclusions
In this paper, we propose two kinds of fractional order terminal sliding mode control methods, i.e., FO-I-TSMC and FO-DI-TSMC, to improve the control performance of some aerospace nonlinear system. FO-I-TSMC can be applied in the speed loop of the servo system. Theory analysis and simulation results all verify the effectiveness of FO-I-TSMC over its integer order counterparts. FO-DI-TSMC can be applied in the position loop of the servo system.

8. References
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