Cosmological Parameters from a re-analysis of the WMAP 7 year low resolution maps

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ABSTRACT

Cosmological parameters from WMAP 7 year data are re-analyzed by substituting a pixel-based likelihood estimator to the one delivered publicly by the WMAP team. Our pixel based estimator handles exactly intensity and polarization in a joint manner, allowing to use low-resolution maps and noise covariance matrices in $T, Q, U$ at the same resolution, which in this work is 3.6°. We describe the features and the performances of the code implementing our pixel-based likelihood estimator. We perform a battery of tests on the application of our pixel based likelihood routine to WMAP publicly available low resolution foreground cleaned products, in combination with the WMAP high-$\ell$ likelihood, reporting the differences on cosmological parameters evaluated by the full WMAP likelihood public package. The differences are not only due to the treatment of polarization, but also to the marginalization over monopole and dipole uncertainties present in the WMAP pixel likelihood code for temperature. The credible central value for the cosmological parameters change below the 1σ level with respect to the evaluation by the full WMAP 7 year likelihood code, with the largest difference in a shift to smaller values of the scalar spectral index $n_S$.

Key words: Cosmology: cosmic microwave background, cosmological parameters

1 INTRODUCTION

The anisotropy pattern of the cosmic microwave background (CMB) is a treasure for understanding the constituents of our Universe and how it evolved from the Big Bang. Under the assumption of isotropy and Gaussianity of CMB fluctuations, the power spectra of intensity and polarization anisotropies include all the compressed information on our Universe through the determination of the cosmological parameters. There has been a tremendous improvement in the estimate of cosmological parameters driven by the increasingly better quality of CMB data, mainly due to the full sky observations in temperature and polarization by the Wilkinson Microwave Anisotropy Probe (WMAP), (see Larson et al. (2010); Komatsu et al. (2010) and references therein) and to the small angular scales measurements by QUBD in polarization (Brown et al. 2004), by the South Pole Telescope (Lueker et al. 2010; Keisler et al. 2011; Reichardt et al. 2012) and the Atacama Cosmology Telescope (Das et al. 2011; Dunkley et al. 2011) in temperature. PLANCK will lead to a drastic improvement of CMB full sky maps in temperature and polarization, leading to an eagerly expected improvement in cosmological parameters with uncertainties at the percent level (Planck Collaboration 2005).

A joint likelihood analysis in temperature and polarization is one of the accepted methods in securing the scientific expectations of observational achievements in terms of cosmological parameters. Although the likelihood could be written exactly in the map domain under the Gaussian hypothesis, its computation is almost prohibitive already at the resolution of 2 degrees, whereas cosmological information is encoded in the temperature and polarization power spectra up to the angular scales of the order of few arcminutes, where the Silk damping suppress the CMB primary anisotropy spectrum. It is now commonly accepted to use an hybrid approach which combines a pixel approach at low resolution with an approximated likelihood based on power spectrum estimates at high multipoles (see Bond, Jaffe and Knox (2000); Verde et al. (2003); Hamimeche and Lewis (2008) for some of these approximations).

Since the three year release of the full polarization information, the WMAP team adopted such a hybrid

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scheme approach, which has been suggested independently in Efstathiou (2004), Slosar, Seljak and Makarov (2004), O’Dwyer et al. (2004), Efstathiou (2006). At a first appearance of the three year data, the WMAP team adopted a pixel approach on HEALPIX (Gorski et al. 2005) resolution $N_{\text{side}} = 8^3$ temperature and polarization maps, and considered the high-$\ell$ approximated likelihood to start at $\ell = 13$ in temperature and $\ell = 24$ in polarization and temperature-polarization cross-correlation for the determination of cosmological parameters in Spergel et al. (2007). The WMAP team treats separately temperature and polarization as explained in Page et al. (2007) and Hinshaw et al. (2007), by using the approximation that the noise in temperature is negligible. As a consequence, the WMAP likelihood code includes either $(Q, U)$ and the temperature-polarization cross-correlation in the same sub-matrix. It was then shown by Eriksen et al. (2007) that by increasing the resolution of the temperature map to HEALPIX $N_{\text{side}} = 16$ and therefore the multipole of transition to high-$\ell$ approximated likelihood in temperature from $\ell = 12$ to $\ell = 30$, the mean value for the scalar spectral index $n_s$ shifted to higher values by a 0.4 $\sigma$. The asymmetric handling of the low-resolution temperature map at $N_{\text{side}} = 16$ and polarization at $N_{\text{side}} = 8$, became the final treatment of the three year data release. This low-$\ell$ likelihood aspect in the WMAP hybrid approach has not changed since the final release of the WMAP 3 year data to the current WMAP 7 year one.

In this paper we wish to perform an alternative determination of the cosmological parameters from WMAP 7 public data, substituting the WMAP low-$\ell$ likelihood approach with a pixel based likelihood code which treats $T, Q, U$ at the same HEALPIX resolution $N_{\text{side}} = 16$ connected to the standard WMAP high-$\ell$ package. In this analysis we therefore increase the resolution of polarization products digested by the pixel based likelihood from $N_{\text{side}} = 8$ to $N_{\text{side}} = 16$, in analogy with what done by Eriksen et al. (2007) for temperature only. The WMAP 7 year foreground cleaned $(Q, U)$ maps, covariance matrices and masks at the resolution $N_{\text{side}} = 16$ are also publicly available at http://lambda.gsfc.nasa.gov; therefore, all data used in this paper are made available by the WMAP team.

The paper is organized as follows. In Section II we briefly describe the WMAP hybrid approach to the likelihood, with particular care to the low multipole part. In Section III we describe our pixel approach, implemented in the BoPix code. We then present in Section IV the cosmological parameters obtained by using our alternative pixel approach in place of the WMAP one for an $\Lambda$CDM scenario. In Section V we extend our investigations to other cosmological models. In Section VI we draw our conclusions.

2 A BRIEF DESCRIPTION OF THE WMAP HYBRID LIKELIHOOD ANALYSIS

In the map domain, the likelihood as function of the cosmological parameters $\{\theta\}$

$$\mathcal{L}(d|\theta) = \frac{1}{(2\pi)^{1/2} |\mathbf{C}|^{1/2}} \exp \left[ -\frac{1}{2} d^T \mathbf{C}^{-1} d \right]$$

(1)

where the data, $d = s + n$, is a CMB fully polarized map, considered as a vector combining $T$, $Q$ and $U$ foreground reduced maps, the sum of signal $s$ and noise $n$; the quantity $\mathbf{C} = \mathbf{S} + \mathbf{N}$ is the total covariance matrix, the sum of the CMB signal covariance matrix $\mathbf{S}(\theta)$, and the noise matrix $\mathbf{N}$. The signal covariance matrix is constructed by the power spectra $C_{\ell}^{XY}$, where $X, Y$ are any of $T, E, B$ (Zaldarriaga and Seljak 1997) as given in Tegmark and de Oliveira-Costa (2001): if not otherwise stated, the sum over multipoles starts from $\ell = 2$.

The WMAP low-$\ell$ likelihood is described in the Appendix of Page et al. (2007) and we report here the essentials. The WMAP approach is based on the assumption to ignore the noise in temperature, which leads to a simplification of the likelihood, useful from the numerical computation perspective. By assuming that the noise in temperature is negligible at low multipoles, the WMAP approach consists in rewriting Eq. (1) as:

$$\mathcal{L}(d|\theta) \approx \exp \left[ -\frac{1}{2} \sum_{\ell \geq 2} C_{\ell}^{QQ} \sum_{m=-\ell}^{\ell} a_{\ell m}^{TT}(Y_{2\ell m}^+ - Y_{2\ell m}^-) \right]$$

(2)

where $C_{\ell}^{QQ}$ is the temperature signal sub-matrix, the new polarization data vector is $d_p = \tilde{d}_p + \tilde{n}_p$, with $\tilde{d}_p = (\tilde{Q}, \tilde{U})$ given by

$$\tilde{Q} \equiv Q - \frac{1}{2} \sum_{\ell=2}^{\ell_P} \sum_{m=-\ell}^\ell a_{\ell m}^{TT}(Y_{2\ell m}^+ - Y_{2\ell m}^-),$$

(3)

$$\tilde{U} \equiv U - \frac{1}{2} \sum_{\ell=2}^{\ell_P} \sum_{m=-\ell}^\ell a_{\ell m}^{TT}(Y_{2\ell m}^+ - Y_{2\ell m}^-),$$

(4)

with $\tilde{S}(\tilde{N}_p)$ the signal (noise) covariance matrix for the new polarization vector $d_p$, $\ell_P = 23$.

According to Page et al. (2007), Eq. (1) and Eq. (2) are mathematically equivalent when the temperature noise is ignored. With this assumption, the new form, Eq. (4), allows the WMAP approach to factorize the likelihood of temperature and polarization, with the information in their cross-correlation, $C_{\ell}^{TE}$, retained in the polarization sub-matrix. As already mentioned in the introduction, temperature is considered at the HEALPIX resolution $N_{\text{side}} = 16$ and smoothed with a Gaussian beam of 9.1285, whereas polarization is considered at $N_{\text{side}} = 8$ and not smoothed. The range of multipoles used in the polarization sub-matrix is up to the Nyquist limit at $N_{\text{side}} = 8$, i.e. $\ell_P = 23$. Two computation options are available for the temperature likelihood. Gibbs sampling (Jewell, Levin and Anderson 2004, Wandelt, Larson and Lakshminarayanan 2004) with a range of multipole considered up to $N_{\text{side}} = 8$.

NOTE: The number of pixels in a map is given by $N_{\text{pix}} = 12N_{\text{side}}^2$, i.e. 768 for $N_{\text{side}} = 8$ and 3072 for $N_{\text{side}} = 16$. © 2010 RAS, MNRAS 000, 000–000.
to $\ell_T = 32$ and direct pixel evaluation, with $\ell_T = 30$\(^2\). All the computations by the WMAP low-$\ell$ likelihood reported here are performed with the option ifore=2 for temperature (we have checked that differences are minimal with respect to the alternative options ifore=0 and 1) and without considering marginalization over foreground uncertainties in polarization.

The high-$\ell$ likelihood, described in Larson et al. (2010) and in Verde et al. (2003), has been updated to beam/point sources uncertainties through the various subsequent WMAP releases Hinshaw et al. (2005, 2007, 2009). The high-$\ell$ TT likelihood takes into account multipoles from $\ell = 31$ ($\ell = 33$) when connected with the pixel (Gibbs) likelihood evaluation of the low resolution temperature data up to $\ell = 1200$; the high-$\ell$ TE (and TB when used) likelihood takes into account multipoles from $\ell = 24$ (Page et al. 2007) to $\ell = 800$. High-$\ell$ EE and BB data have not used so far in the various releases of the WMAP likelihood code.

4 DATA SET FOR BOPIX

We use the temperature ILC map smoothed at 9.1285 degrees and reconstructed at HealPix (Gorski et al. 2005) resolution $N_{\text{side}} = 16$, the foreground cleaned (unsmoothed) low resolution maps and the noise covariance matrix in $(Q,U)$ publicly available at the LAMBDA website \url{http://lambda.gsfc.nasa.gov/} for the frequency channels $K_a$ (23 GHz), $Q$ (41 GHz) and $V$ (61 GHz) as considered by Larson et al. (2010) for the low $\ell$ analysis. These frequency channels have been co-added by inverse noise covariance weighting accordingly to the WMAP team (Jarosik et al. 2007).

$$d_{\text{pol}} = c_{\text{pol}}(c_{K_a}^{-1}d_{K_a} + c_{Q}^{-1}d_Q + c_{V}^{-1}d_V).$$

where $d_i$, $c_i$ are the foreground reduced polarization maps and covariances, respectively (for $i=K_a, Q$ and $V$). The total foreground reduced inverse noise covariance matrix is therefore:

$$c_{\text{pol}}^{-1} = c_{K_a}^{-1} + c_{Q}^{-1} + c_{V}^{-1}. \tag{6}$$

This polarization data set has been extended to temperature considering the ILC map with an extra noise term, as suggested in Dunkley et al. (2009). We have therefore added to the temperature map a random noise realization with variance of $\sigma_{TT}^2 = 1\mu K^2$ and consistently, the noise covariance matrix for TT is taken to be diagonal with variance equal to $1\mu K^2$. The total noise covariance $N$ for WMAP 7 yr data is therefore:

$$N = \left( \begin{array}{cc} \sigma_{TT}^2 & 0 \\ 0 & c_{\text{pol}} \end{array} \right).$$

Let us note that this prescription of the noise in the temperature ILC map added to mitigate the uncertainties due to foreground cleaning violates the assumption that the noise in temperature is vanishing, used to obtain Eqs. (4,5) from Eq. (1).

Two masks are considered: KQ85y7 for T and P06 for $(Q, U)$. Monopole and dipole have been subtracted from the observed ILC map through the HealPix routine REMOVE-DIPOLE (Gorski et al. 2003). The same data set has been used for the WMAP 7 yr power spectrum re-analysis by the Quadratic Maximum Likelihood (QML) estimator BolPol in Gruppuso et al. (2011) (similar data set for WMAP 5 yr data were previously used in Gruppuso et al. (2009); Paci et al. (2010)).

5 COSMOLOGICAL PARAMETERS EXTRACTION

We use CosmoMC (Lewis and Bridle 2002) in order to compute the Bayesian probability distribution of model parameters. The pivot scale of the primordial scalar and tensor

\(^2\) The temperature signal covariance matrix is constructed with multipoles up to $\ell = 64$, but from $\ell = 31$ to 64 the $C^{TT}_\ell$ are not varied, but fixed to those of a fiducial cosmology.
power spectra was set to $k_s = 0.017$ Mpc$^{-1}$, as recommended by Cortes, Liddle and Mukherjee (2009). We vary the physical baryon density $\Omega_B h^2$, the physical cold dark matter density $\Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling $\theta$, the reionisation optical depth $\tau$, the amplitude and spectral index of curvature perturbations $n_s$ and $\log_{10}[10^{10}A_s]$. We assume a flat universe, and so the cosmological constant for each model is given by the combination $\Omega_M = 1 - \Omega_B - \Omega_c$. We set the CMB temperature $T_{\text{CMB}} = 2.725$ K (Mather et al. 1999) and the primordial helium fraction to $y_{\text{He}} = 0.24$. We assume three neutrinos with a negligible mass. In order to fit WMAP data, we use the lensed CMB and we follow the method implemented in CosmoMC consisting in varying a nuisance parameter $A_{SZ}$ which accounts for the unknown amplitude of the thermal SZ contribution to the small-scale CMB data points assuming the model of Komatsu and Seljak (2002). We use CAMB (Lewis, Challinor and Lasenby 2000) with accuracy setting of 1. We sample the posterior using the Metropolis-Hastings algorithm (Hastings 1970) at a temperature $T = 1$, generating four parallel chains and imposing a conservative Gelman-Rubin convergence criterion (Gelman and Rubin 1992) of $R < 1 < 0.005$.

With the settings specified above we extract cosmological parameters with the WMAP likelihood code (version v4p1) available at [http://lambda.gsfc.nasa.gov/](http://lambda.gsfc.nasa.gov/) as benchmarks. We prefer to not quote the estimates for the cosmological parameters performed by the WMAP team since the conventions and the CAMB version might differ from those used by Larson et al. (2010) and Komatsu et al. (2010).

We then extract cosmological parameters by substituting the WMAP $\ell$ likelihood approach with BoPix. In doing this we implicitly use the WMAP inputs in polarization at $N_{\text{side}} = 16$ as described in Section III and not those contained in the WMAP likelihood routine publicly available. Since temperature and polarization are treated at the same resolution by BoPix, we include the WMAP high $\ell$ likelihood starting at $\ell = 31$ both in temperature and temperature-polarization cross-correlation when using BoPix, unless otherwise stated. Unless otherwise stated, in BoPix we vary the $C_l$ up to $\ell = 30$ and we use the publicly available file test_clb_v4.dat as a fiducial power spectrum to complete the full covariance at low resolution from $\ell = 31$ to $\ell = 64$, as done for temperature only by the WMAP pixel likelihood.

We find small differences in the estimate of the cosmological parameters by substituting BoPix to the WMAP $\ell$ likelihood, as reported in Table I. The main difference between the estimate of the cosmological parameters derived by our alternative low-$\ell$ likelihood code and the one obtained with the WMAP approach is in the spectral index $n_s$: we obtain a value for $n_s$ which is 0.856$ lower than the WMAP one. This change would lead to quantitative differences in the evidence against the Harrison-Zeldovich of the WMAP 7 yr data. However, also the other directly sampled cosmological parameters differ from the WMAP estimate in about 0.5$, pointing towards values higher for the physical CDM abundance $\Omega_c h^2$ and the amplitude of scalar perturbations $A_S$ and smaller for the baryon physical content $\Omega_B h^2$ and optical depth $\tau$. As a derived parameters, we have a higher value for the matter content $\Omega_M$ and $\sigma_8$, smaller for the present Hubble rate $H_0$. We show more details about these different estimates in the two-dimensional plots of Fig. 1. These differences seems robust to the change in the multipole transition to the high likelihood approximation and to the change of the fiducial model to complete the covariance at low resolution. Special mention should be made for the case in which we do not consider $C_T = C_P$, but we adopt the same $\ell_T = 30$ and $\ell_P = 23$ adopted by the WMAP team, but with BoPix for low resolution: the differences with respect to the estimates by the full WMAP $\ell$ likelihood are slightly smaller than in the case of $\ell_T = \ell_P = 30$, as can be seen in Table I. This means that differences we find are not fully due to the different threshold multipoles for polarization adopted in the two low-$\ell$ likelihood approaches. No appreciable differences are noticed by constructing the signal covariance matrix up to $3N_{\text{side}}$ instead up to $4N_{\text{side}}$. This can be understood since this different prescription in constructing the signal covariance matrix is damped by the Gaussian smoothing in intensity and is much below the noise in polarization.

We have performed a further test excluding $A_{SZ}$, just for code comparison. We find a smaller discrepancy between the estimates for the cosmological parameters and the best-fits from the two likelihood approaches when the nuisance parameter $A_{SZ}$ is omitted (i.e. fixed to zero). This additional foreground parameter $A_{SZ}$ is not well constrained by WMAP, but it contributes to the shape of the final likelihood and to the marginalized values of the parameters (shifting slightly the value of $n_s$, for instance). We have checked that the different realizations of the $\mu$K rms noise added to the ILC temperature map in the WMAP and BoPix likelihood lead to much smaller differences than those reported.

Most of these small differences reported in the estimate of the cosmological parameters interfere destructively because of the cosmic confusion (Efstathiou and Bond 1998) and the best-fits $C_l$ from the two likelihood analysis agree.
Re-analysis of WMAP 7 year cosmological parameters

Table 1. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters estimated by the WMAP 7 year full likelihood (second and third column) and by the BoPix plus WMAP 7 year high $\ell$ likelihood for different transition multipoles $\ell_T = \ell_P$ (fourth, fifth and sixth column), for $\ell_T \neq \ell_P$ and different fiducial theoretical power spectrum to complete the signal covariance matrix in BoPix (last column). Below the thick line analogous mean values and bounds are presented for derived parameters.

| Parameter | WMAP 7 likelihood | WMAP 7 high $\ell$ likelihood | $\ell_T = \ell_P = 30$ | $\ell_T = \ell_P = 24$ | $\ell_T = \ell_P = 36$ | $\ell_T = 30$ | $\ell_P = 23$ | Different Fiducial |
|-----------|------------------|-------------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\Omega_M h^2$ | $2.250 \pm 0.056$ | $2.252^{+0.057}_{-0.056}$ | $2.213 \pm 0.055$ | $2.215 \pm 0.055$ | $2.224^{+0.057}_{-0.054}$ | $2.213^{+0.055}_{-0.054}$ | $2.114 \pm 0.055$ | $2.121 \pm 0.058$ |
| $\Omega_b h^2$ | $0.1114^{+0.0054}_{-0.0053}$ | $0.1114 \pm 0.0055$ | $0.1145^{+0.0055}_{-0.0056}$ | $0.1142 \pm 0.0055$ | $0.1152^{+0.0055}_{-0.0054}$ | $0.1145^{+0.0056}_{-0.0057}$ | $0.1144 \pm 0.0056$ |
| $\tau$ | $0.089 \pm 0.015$ | $0.089^{+0.014}_{-0.013}$ | $0.085^{+0.014}_{-0.013}$ | $0.085^{+0.015}_{-0.014}$ | $0.085^{+0.014}_{-0.013}$ | $0.085^{+0.015}_{-0.014}$ | $0.085 \pm 0.015$ |
| $n_s$ | $0.968^{+0.014}_{-0.013}$ | $0.968^{+0.014}_{-0.013}$ | $0.956 \pm 0.014$ | $0.957^{+0.014}_{-0.013}$ | $0.954^{+0.014}_{-0.013}$ | $0.955 \pm 0.014$ | $0.956^{+0.014}_{-0.013}$ |
| $\log(10^{10} A_s)$ | $3.116^{+0.053}_{-0.052}$ | $3.116 \pm 0.053$ | $3.130 \pm 0.033$ | $3.126 \pm 0.052$ | $3.133^{+0.051}_{-0.052}$ | $3.129 \pm 0.052$ | $3.128 \pm 0.033$ |

We present the CMB bestfit $C_\ell$ in temperature and lensing (the latter not entering in the likelihood evaluation) obtained by BoPix in combination with the WMAP 7 high-$\ell$ likelihood in comparison with those obtained by the full WMAP 7 likelihood in Fig. 2. The difference in the best-fit $C_\ell$ in temperature is consistent with the different central values for the cosmological parameters displayed in Table 1. Note how the relative difference in the lensing is slightly larger than the one in temperature and does not decrease at high multipoles. Differences in polarization and temperature-polarization cross-correlation are smaller than the ones shown here. We have checked that the best-fit $C_\ell$ obtained in this work by the full WMAP 7 likelihood has $\Delta(-2\log W_{\text{MAP}}) = -7.42$ with respect to the reference WMAP 7 test_xls_v4.dat; the best-fit $C_\ell$ obtained in this work by BoPix in combination with the high-$\ell$ WMAP 7 likelihood provides a better fit, with $\Delta(-2\log W_{\text{MAP}}) = -7.75$ with respect to the reference WMAP 7 test_xls_v4.dat.

We have then tested BoPix against the WMAP likelihood within the same range of multipole, i.e. up to $\ell = 30$: BoPix has been run on the low-resolution WMAP 7 yr $N_{\text{side}} = 16$ products varying $C_\ell^{TT}$, $C_\ell^{EE}$, $C_\ell^{TE}$ up to $\ell = 30$ and compared to the likelihood obtained by the WMAP 7 yr pixel based routine plus the high-$\ell$ modeling for TE from $\ell = 24$ to $\ell = 30$. In this way we subtract the same high-$\ell$ likelihood information from hybrid runs presented in Table 1. By assuming $\Omega_M h^2 = 0.02246$, $\Omega_b h^2 = 0.1117$ and sound horizon $\theta = 1.03965$, we obtain results quite consistent with the hybrid ones: a slight smaller value in the estimate of $\tau$ and $n_S$ and a larger one for $A_S$, as shown in Fig. 3.

As already mentioned, one important aspect of the WMAP 7 yr low-$\ell$ likelihood is to use two different resolution for temperature and polarization; the polarization information at HEALPix resolution $N_{\text{side}} = 8$ is used up to the Nyquist multipole, i.e. $\ell_P = 23$. We run the two low-$\ell$ likelihoods with $\ell_T = \ell_P = 16$ to make sure that the differences are not due mainly to a mismatch in the polarization data sets. As reported in Table 2, the differences in the estimates of the parameters decrease, as expected, but do not disappear.

Another important difference between BoPix and the

Table 2. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters with a transition in the hybrid likelihood at $\ell = 16$. The results of the WMAP 7 year full likelihood (BoPix plus WMAP 7 year high $\ell$ likelihood) are reported in the left (right) column. Below the thick line analogous mean values and bounds are presented for derived parameters.

| Parameter | WMAP 7 likelihood | WMAP 7 high $\ell$ likelihood |
|-----------|------------------|-------------------------------|
| $\Omega_M h^2$ | $2.246 \pm 0.057$ | $2.231^{+0.057}_{-0.054}$ |
| $\Omega_b h^2$ | $0.1119 \pm 0.0055$ | $0.1113 \pm 0.0067$ |
| $\tau$ | $0.088 \pm 0.015$ | $0.087 \pm 0.015$ |
| $n_s$ | $0.967^{+0.013}_{-0.014}$ | $0.962^{+0.014}_{-0.015}$ |
| $\log(10^{10} A_s)$ | $3.118 \pm 0.033$ | $3.117 \pm 0.033$ |

WMAP 7 yr likelihood routine is the treatment of monopole and dipole for the temperature map. In the ILC temperature map with the additional noise of $1 \mu K$ rms used in BoPix, the monopole and dipole in the masked sky are removed; no monopole and dipole terms are considered in the construction of the covariance matrix. The WMAP 7 yr temperature pixel routine instead does not subtract the monopole and dipole in the masked sky; in the observed sky with the KQ85y7 mask, the ILC temperature map has an offset of $-0.07 \mu K$ and a dipole $C_3 = 4.6 \mu K^2$. To take into account monopole and dipole residuals, the full sky signal covariance matrix is modified according to Slosar, Seljak and Makarov (2004):

$$S(\theta) \rightarrow S(\theta) + \lambda \left( \frac{P_0}{4\pi} + \frac{3}{4\pi} P_1 \right)^2$$

where $P_0(\cos \theta) = 1$ and $P_1(\cos \theta) = \cos \theta$ are the Legendre polynomials associated to monopole and dipole, respectively. The fixed amplitude of the monopole and dipole terms is taken to be equal to the quadrupole of the fiducial $\Lambda CDM$ model, i.e. $\lambda = 1262 \mu K^2$. The subtraction of monopole and dipole in the masked ILC map has a little impact on the esti-
Figure 2. Comparison of the best-fit $\ell(\ell+1)C_\ell^{TT}/(2\pi)$ and $\ell^2(\ell+1)^2C_\ell^{\phi\phi}/(2\pi)$ obtained by BoPix in combination with the WMAP 7 high-$\ell$ likelihood (solid) vs. the WMAP 7 full likelihood (dashed) is shown in the first and third panel from above. To make the difference more visible, the relative difference between the $C_\ell$ bestfits in temperature and lensing potential are shown in the second and fourth panels, respectively. Note that the differences are well within the cosmic variance.

Figure 3. Marginalized one-dimensional probabilities for $\tau$, $n_s$ and $\log[10^{10} A_s]$ as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high $\ell$ likelihood (black lines). See text for further details.

Figure 4. Marginalized one-dimensional probabilities for cosmological parameters as estimated by the WMAP 7 year full likelihood for $\lambda = 1262 \mu K^2$ (black line), $\lambda = 12.62 \mu K^2$ (red line), $\lambda = 1.262 \mu K^2$ (blue line), $\lambda = 0.168 \mu K^2$ (purple line). The green line is obtained with the WMAP by removing monopole and dipole in the masked sky and setting $\lambda = 0$. The black vertical lines are the mean values obtained by BoPix in combination with the WMAP 7 yr high $\ell$ likelihood listed in the fourth column of Table 1, which agree with the central values of the posteriors in purple.

mate of cosmological parameters. Cosmological parameters instead have a strong dependence on the amplitude $\lambda$ of the monopole and dipole terms which contribute to the signal covariance matrix, as shown in Fig. 4. The results obtained by subtracting monopole and dipole in the ILC temperature map used by the WMAP 7 yr temperature pixel likelihood routine and setting $\lambda = 0$ in the construction of the temperature covariance matrix do not match with those obtained by BoPix, as shown in Fig. 4. Viceversa, by tuning the amplitude of the monopole and dipole term to $0.17 \mu K^2$ the results of the WMAP 7 yr likelihood routine agrees with those by BoPix. We conclude that part, but not all, of the discrepancy between BoPix and WMAP 7 yr likelihood is due to the monopole and dipole marginalization in Eq. 7.
6 OTHER EXTENDED COSMOLOGICAL MODELS

We now consider few cosmological models beyond the $\Lambda$CDM model which can be constrained by WMAP 7 year data only. We consider only the baseline $\ell_{\text{trans}} = 30$ and all the other settings consistently with the previous section, unless otherwise stated.

Gravitational Waves.

We consider all inflationary models which can be described by the primordial perturbation parameters consisting of the scalar amplitude and spectral index ($A_s$, $n_s$), and the tensor-to-scalar ratio $r$. In canonical single-field inflation, in the slow-roll limit, the tensor spectrum shape is not independent of the scalar one. We will consider a tensor spectrum with a tilt $n_T = -r/8$, as predicted for canonical single-field inflation at first-order in slow-roll.

Our marginalised 68%-credible interval for the scalar spectral index is given by $n_s = 0.974^{+0.203}_{-0.219}$, half a sigma redder than the result we obtain by the full WMAP 7 year likelihood $0.987 \pm 0.020$.

At 95% confidence level, our result for the tensor-to-scalar ratio is $r < 0.36$, fully consistent with the result we obtain from the full WMAP 7 year likelihood, i.e. $r < 0.34$. Let us note that, differently from the WMAP low-$\ell$ likelihood code, BoPix include BB polarization in the construction of the covariance at low resolution. Estimates of the cosmological parameters including tensor modes are compared in Table 3. The differences in the $(n_s, r)$ are shown in Fig. 5 and are mainly due to a shift of the constraints at smaller values for $n_s$, as occurs for the standard $\Lambda$CDM model discussed in the previous section. Theoretical predictions of few popular inflationary models (including reheating uncertainties where appropriate) are displayed. One of the phenomenological differences from the different constraints would be a minor tension for a massless self-interacting inflation model with WMAP 7 year data only (see Komatsu et al. (2010); Finelli et al. (2011) as examples for a higher tension of the $\lambda\phi^4$ potential with observations when additional cosmological data sets are added to WMAP).

Running of the scalar spectral index.

In this subsection we consider the variation of the scalar spectral index with wavelength, i.e. we allow $n_{\text{run}}$ to vary in the range $[0.2, 0.2]$. Our marginalised 95%-credible interval for the scalar spectral index is given by $-0.065 < n_{\text{run}} < 0.042$, which has to be compared with the result we obtain by the full WMAP 7 year likelihood $-0.074 < n_{\text{run}} < 0.030$. The results, shown in Table 4 and Fig. 6, are both consistent with the hypothesis of no wavelength dependence of the scalar spectral index.

Neutrino Mass.

In this subsection we constrain the total mass of neutrinos $\sum m_{\nu} = 9\Omega_{\nu}h^2$ eV, allowing to vary the fraction of massive neutrino energy density relative to the total dark matter one $f_{\nu} = \Omega_{\nu}/\Omega_{DM}$. At 95% confidence level, our result for the fraction of massive neutrinos is $f_{\nu} < 0.113$, whereas we obtain $f_{\nu} < 0.094$ from the full WMAP 7 year likelihood. The resulting neutrino mass bound at 95% confidence level is $\sum m_{\nu} < 1.4$ eV, compared to 1.1 eV obtained.

Table 3. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters including the tensor-to-scalar ratio estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high $\ell$ likelihood (right column). For the tensor-to-scalar ratio $r$ the 95%-credible upper bound is quoted. Below the thick line analogous mean values and bounds are presented for derived parameters.

| Parameter | WMAP 7 likelihood | WMAP 7 high $\ell$ likelihood |
|-----------|-------------------|-----------------------------|
| $100 \, \Omega_b h^2$ | $2.307^{+0.071}_{-0.072}$ | $2.270 \pm 0.073$ |
| $\Omega_c h^2$ | $0.1073 \pm 0.0063$ | $0.1095^{+0.0067}_{-0.0066}$ |
| $\tau$ | $0.091^{+0.015}_{-0.014}$ | $0.087^{+0.015}_{-0.014}$ |
| $n_s$ | $0.987 \pm 0.020$ | $0.977 \pm 0.021$ |
| $\log [10^{10} A_s]$ | $3.093 \pm 0.038$ | $3.102 \pm 0.039$ |
| $r$ | $< 0.34$ | $< 0.36$ |
| $n_{\text{run}}$ | $0.246^{+0.031}_{-0.032}$ | $0.262^{+0.035}_{-0.036}$ |
| $H_0$ | $73.2 \pm 3.2$ | $71.6^{+3.2}_{-3.3}$ |
| $\sigma_8$ | $0.797 \pm 0.033$ | $0.805 \pm 0.033$ |

Table 4. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters including the running of the scalar spectral index $n_{\text{run}}$ estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high $\ell$ likelihood (right column). For the running of the scalar spectral index $n_{\text{run}}$ the 95%-credible upper bound is quoted.

| Parameter | WMAP 7 likelihood | WMAP 7 high $\ell$ likelihood |
|-----------|-------------------|-----------------------------|
| $100 \, \Omega_b h^2$ | $2.198^{+0.074}_{-0.072}$ | $2.184 \pm 0.081$ |
| $\Omega_c h^2$ | $0.1167^{+0.0082}_{-0.0015}$ | $0.1175^{+0.0083}_{-0.0084}$ |
| $\tau$ | $0.091^{+0.016}_{-0.016}$ | $0.087 \pm 0.015$ |
| $n_s$ | $0.961 \pm 0.016$ | $0.953^{+0.015}_{-0.016}$ |
| $\log [10^{10} A_s]$ | $3.154 \pm 0.054$ | $3.151^{+0.054}_{-0.054}$ |
| $n_{\text{run}}$ | $-0.074 < n_{\text{run}} < 0.030$ | $-0.065 < n_{\text{run}} < 0.042$ |
| $\Omega_M$ | $0.303 \pm 0.049$ | $0.310^{+0.050}_{-0.051}$ |
| $H_0$ | $68.2^{+3.7}_{-3.6}$ | $67.5^{+3.8}_{-3.7}$ |
| $\sigma_8$ | $0.820^{+0.033}_{-0.032}$ | $0.826^{+0.033}_{-0.032}$ |
Figure 6. Marginalized 68% and 95%-credible contours for $(n_s, n_{run})$ as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high $\ell$ likelihood (black lines).

Table 5. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters including the total mass of the neutrinos estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high $\ell$ likelihood (right column). For the total mass of the neutrinos $\sum m_\nu$ the 95%-credible upper bound is quoted.

| Parameter | WMAP 7 likelihood | BoPix plus WMAP 7 high $\ell$ likelihood |
|-----------|-------------------|----------------------------------------|
| $100 \Omega_b h^2$ | 2.219$^{+0.062}_{-0.060}$ | 2.174$^{\pm0.061}$ |
| $\Omega_c h^2$ | 0.117$^{+0.0071}_{-0.0075}$ | 0.1226$^{+0.0084}_{-0.0090}$ |
| $\tau$ | 0.087$^{+0.014}_{-0.015}$ | 0.082$^{\pm0.014}$ |
| $n_s$ | 0.960$^{+0.016}_{-0.017}$ | 0.945$^{+0.016}_{-0.017}$ |
| $\log\left[10^{10}A_s\right]$ | 3.120$^{\pm0.032}$ | 3.134$^{\pm0.033}$ |
| $f_r$ | < 0.994 | < 0.113 |
| $\Omega_M$ | 0.329$^{+0.057}_{-0.056}$ | 0.374$^{+0.075}_{-0.072}$ |
| $H_0$ | 65.7$^{+4.5}_{-4.6}$ | 62.8$^{+4.6}_{-4.6}$ |
| $\sigma_8$ | 0.712$^{+0.073}_{-0.074}$ | 0.695$^{+0.087}_{-0.083}$ |
| $\sum m_\nu$ | < 1.1 eV | < 1.4 eV |

Figure 7. Marginalized 68% and 95%-credible contours for $(\sum m_\nu, \Omega_M h^2)$ (left panel) and $(n_s, \sum m_\nu)$ (right panel) as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high $\ell$ likelihood (black lines).

The above formulae are valid when the primordial B-mode polarization is negligible, which is assumed in this paper. We have therefore sampled $\alpha$ in radians with a flat prior $[-0.5,0.5]$ plus the other six cosmological parameters of the $\Lambda$CDM model by inserting Eqs. (8). As shown in Table 6 our marginalised 68% (95%)-credible interval for $\alpha$ is $\alpha = -1.3^{+0.7}_{-0.7} +0.6^{+2.3}_{-2.3}$ in agreement with the full WMAP 7 year likelihood result which we find $\alpha = -1.4^{+0.8}_{-0.7} +0.6^{+2.4}_{-2.3}$. Either the result using BoPix or the one based on the full WMAP 7 year likelihood are consistent with vanishing cosmological birefringence at 95% CL, just by assuming the statistical uncertainty, and the agreement increases by using the systematic uncertainty, which is estimated as 1°.4 by the WMAP team [Komatsu et al. 2010].

Since the weight of the high-$\ell$ TB likelihood plays a relevant role in these constraints we have also considered the case in which this is not taken into account. Such setting which emphasizes the role of polarization on large angular scales would be relevant to show clearly the potential differences between BoPix and the WMAP pixel likelihood code. On using only low resolution products to constrain cosmological birefringence, by using BoPix on $N_{side} = 16$ resolution $Q,U$ maps and matrices we obtain $\alpha = -4^{+0.2}_{-0.8} +1^{+0.5}_{-0.5}$, still in agreement with the values we find by the WMAP 7 likelihood on $N_{side} = 8$ resolution $Q,U$ maps and matrices $\alpha = -4^{+0.2}_{-0.8} +1^{+0.5}_{-0.5}$. Although with larger uncertainties, our results agree with vanishing cosmological birefringence at 95% CL, without invoking systematic uncertainties. Note also that our result agrees with the analysis on large angular scales by [Gruppuso et al. 2012], where much tighter constraints are given probably because all the cosmological parameters except $\alpha$ are kept fixed.

\[ C_{EE,obs} = C_T^{EE} \cos^2(2\alpha), \]
\[ C_{BB,obs} = C_T^{BB} \sin^2(2\alpha), \]
\[ C_{EB,obs} = \frac{1}{2} C_T^{EE} \sin(4\alpha), \]
\[ C_{T,obs} = C_T^{BB} \sin(2\alpha). \]
We have performed an alternative estimate of the cosmological parameters from WMAP 7 year public data, by substituting the WMAP 7 low-$\ell$ likelihood with a pixel likelihood code which treats $(T, Q, U)$ at the same resolution without any approximation. We have used this code at the HEALPIX resolution $N_{\text{side}} = 16$ on foreground cleaned public data, therefore increasing the resolution of the pixel based polarization products used in our extraction of the cosmological parameters with respect to the WMAP standard one. We have consistently increased the transition multipole from $\ell = 24$ to $\ell = 31$ for the high-$\ell$ WMAP 7 year temperature-polarization cross-correlation likelihood and included the marginalization over the nuisance parameter $A_{SZ}$.

With this setting we have found estimates for the cosmological parameters consistent with those obtained by the full WMAP 7 year likelihood package, although for some parameters the differences are of half $\sigma$ or more. These differences between the two low-$\ell$ likelihood treatments we find are larger than the WMAP 7 yr likelihood uncertainties from tests on simulations reported in Larson et al. (2010); however, we need to keep in mind that our differences between two likelihood treatments are reported for real data, with WMAP 7 year beam/point source corrections and various marginalizations taken fully into account, differently from the simulation analysis performed in Larson et al. (2010). The difference between the two best-fit $C_{\ell}^{TT}$ for LCDM found by the two alternative likelihood treatments show a maximum of 4% around at $\ell \sim 10$ and oscillate with an amplitude below 1% for $\ell > 100$. A 5% percent difference is found in the two best-fits for the lensing power spectrum, whereas smaller differences are found for temperature-polarization cross-correlation and polarization power spectra. We have shown how part of the discrepancy, but not all, can be ascribed to the monopole/dipole marginalization used in the WMAP temperature likelihood and described in Slosar, Seliak and Makarov (2004).

On restricting to the $\Lambda$CDM model the most important difference is for the scalar spectral index $n_S$, which decrease to 0.956 from the value 0.968 we obtain with the full WMAP 7 yr likelihood code, i.e. a decrease of 0.86 $\sigma$. This different value for $n_S$ would increase the evidence against the Harrison-Zeldovich spectrum from WMAP 7 yr data. This difference for $n_S$ is consistent with the one between the two best-fit $C_{\ell}$ and depend only partially from the threshold multipole from which the high-$\ell$ TE likelihood starts. Other previous alternative likelihood treatments also reported the most important discrepancy for the scalar spectral index (Eriksen et al. 2007; Rudjord et al. 2009). A smaller value for $n_S$ with respect to the estimate by the full WMAP 7 year likelihood code, always within 1 $\sigma$, is then seen in all the extension of $\Lambda$CDM considered here. No major changes are found for the $95\%$ credible intervals for the tensor to scalar ratio and for the running of the scalar spectral index. A slight degradation has been found for the $95\%$ credible interval on the neutrino mass. The case of cosmological birefringence has been taken as a sensitive test for the two alternative likelihoods, whose most relevant difference is the treatment of polarization on large scales. A slight difference on the posterior of the polarization angle $\alpha$ has been found.

4 We have checked that either the difference between the two best-fit $C_{\ell}$ or between the estimates of the cosmological parameters decrease when the nuisance parameter $A_{SZ}$ is set to zero in both alternative likelihood treatments. The net effect of the variation of this foreground parameter, which is unconstrained by the data, is to increase the differences between the estimates of the cosmological parameters from the two likelihood treatments for the $\Lambda$CDM model.
when only low resolution data are used, whereas the results are fully consistent when the high-\(\ell\) TB data are added to both likelihoods.

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