Generalized type IIB supergravity equations and non-Abelian classical $r$-matrices

Domenico Orlando$^1$, Susanne Reffert$^1$, Jun-ichi Sakamoto$^2$ and Kentaroh Yoshida$^2$

$^1$ Institute for Theoretical Physics, Albert Einstein Center for Fundamental Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland
$^2$ Department of Physics, Kyoto University, Kitashirakawa Oiwake-cho, Kyoto 606-8502, Japan

E-mail: sreffert@itp.unibe.ch

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Abstract
We study Yang–Baxter deformations of the $\text{AdS}_5 \times S^5$ superstring with non-
Abelian classical $r$-matrices which satisfy the homogeneous classical Yang–
Baxter equation. By performing a supercoset construction, we can get
deformed $\text{AdS}_5 \times S^5$ backgrounds. While this is a new area of research, the
current understanding is that Abelian classical $r$-matrices give rise to solutions
of type IIB supergravity, while non-Abelian classical $r$-matrices lead to
solutions of the generalized supergravity equations. We examine here some
examples of non-Abelian classical $r$-matrices and derive the associated
backgrounds explicitly. All of the resulting backgrounds satisfy the general-
ized equations. For some of them, we derive ‘$T$-dualized’ backgrounds by
adding a linear coordinate dependence to the dilaton and show that these
satisfy the usual type IIB supergravity equations. Remarkably, some of the ‘$T$-
dualized’ backgrounds are locally identical to undeformed $\text{AdS}_5 \times S^5$ after an
appropriate coordinate transformation, but this seems not to be generally
the case.

Keywords: Yang–Baxter deformations, generalized supergravity equations,
non-Abelian classical $r$-matrices, $T$-duality, integrability

1. Introduction
Yang–Baxter deformations provide a systematic way to study integrable deformations of
nonlinear sigma models in two-dimensions. This method was invented by Klimcik for
principal chiral models based on the modified classical Yang–Baxter equation (mCYBE) [1].
It was generalized to the symmetric coset case in [2] and the homogeneous classical Yang–Baxter equation (CYBE) case in [3].

It is very interesting to consider some applications of Yang–Baxter deformations in the context of the AdS/CFT correspondence [7]. The Green–Schwarz action of type IIB superstring on $AdS_5 \times S^5$ was constructed by Metsaev and Tseytlin in terms of the supercoset [8]

$$\frac{PSU(2, 2|4)}{SO(1, 4) \times SO(5)}$$

(1.1)

This supercoset has a $\mathbb{Z}_4$-grading, generalizing the $\mathbb{Z}_2$-grading of the symmetric cosets. Hence the $AdS_5 \times S^5$ superstring exhibits classical integrability in the sense of kinematic integrability [9] (for nice reviews, see [10–12]).

The first application of Yang–Baxter deformation to the $AdS_5 \times S^5$ superstring was carried out by Deluc et al [13]. The classical action was constructed with a classical $r$-matrix of Drinfeld–Jimbo type [14] and the associated symmetry algebra is a $q$-deformation of $su(2, 2|4)$. This specific example is often called $\eta$-deformation. The metric and NS–NS two-form were subsequently computed by performing a coset construction for the bosonic element [15]. After that, some works were done towards constructing the full background by directly solving the equations of motion of type IIB supergravity [16, 17]. This strategy was preferred simply because of the technical difficulty of the supercoset construction. In the end, Arutyunov, Borsato and Frolov accomplished doing the supercoset construction and the full background was determined [18] (for a good review, see [19]). A remarkable point is that the resulting background does not satisfy the equations of motion of type IIB supergravity.

However, the situation is not hopeless. The ten-dimensional background is related to the full solution of the usual type IIB supergravity via $T$-dualities up to the linear dependence of the dilaton [20]. The world-sheet theory is not Weyl invariant but still scale invariant, i.e., the world-sheet beta function does not vanish but is given by a total derivative. From the viewpoint of the spacetime, this result indicates that the on-shell condition of type IIB supergravity is weakened to a generalized set of the equations. In fact, the generalized equations of type IIB supergravity were proposed in [21] and the background obtained in [18] is a solution of the generalized equations. For a more general argument on the relation between the $\kappa$-symmetry of the $\eta$-deformed $AdS_5 \times S^5$ superstring and the generalized equations, see [22].

Other examples of Yang–Baxter deformations of the $AdS_5 \times S^5$ superstring are based on the homogeneous CYBE [23]. In comparison to the mCYBE case, there are some advantages. One of them is that one can perform partial deformations of $AdS_5 \times S^5$, i.e., only $AdS_5$ or only $S^5$ can be deformed. This class of Yang–Baxter deformations includes well-known examples such as gravity duals of non-commutative gauge theories [24, 25], the gamma-deformations of $S^5$ [26, 27], and Schrödinger spacetimes [28]. In a series of works [30–36], the associated classical $r$-matrices were identified with these backgrounds. In a remarkable advance, the supercoset construction has been performed in [43] and now the full background can be obtained for arbitrary classical $r$-matrices. For Abelian classical $r$-matrices, the R–R sector and dilaton have been confirmed for well-known backgrounds and it seems likely that Yang–Baxter deformations work well [43]. However, for non-Abelian classical $r$-matrices,
the associated backgrounds do not in general satisfy the equations of motion of type IIB supergravity [43, 44].

An interesting question here is whether deformed backgrounds for non-Abelian classical $r$-matrices satisfy the generalized equations of motion or not. This question can be answered if the deformed string theory is supposed to be the canonical action of the Green–Schwarz string [22]. This assumption however has to be confirmed by a separate analysis and it is still an open problem. A confirmation was also given in [44] based on scaling limits of the $\eta$-deformed $\text{AdS}_5 \times S^5$. It may also be intriguing to study ‘$T$-dualized’ backgrounds related to non-Abelian classical $r$-matrices and consider their physical interpretation. We use quotation marks for $T$-dualities involving solutions to the generalized SUGRA equations, where the usual Buscher rules have to be supplemented with a prescription for the behavior of the dilaton [21].

In this paper, we consider Yang–Baxter deformations of the $\text{AdS}_5 \times S^5$ superstring with the homogeneous CYBE and focus on some examples of non-Abelian classical $r$-matrix. We derive the associated backgrounds explicitly and show that all of the resulting backgrounds satisfy the generalized equations. For some of them, ‘$T$-dualized’ backgrounds are derived by adding a linear coordinate dependence to the dilaton and these satisfy the usual type IIB supergravity equations. Remarkably, some of the ‘$T$-dualized’ backgrounds are locally $T$-dual to the undeformed $\text{AdS}_5 \times S^5$ via appropriate coordinate transformations, though it does not seem that this is always the case. At least in cases where the undeformed $\text{AdS}_5 \times S^5$ background can be reproduced, the classical integrability of the ‘$T$-dualized’ background is manifest.

This paper is organized as follows. Section 2 introduces Yang–Baxter deformations of the $\text{AdS}_5 \times S^5$ superstring based on the homogeneous CYBE and an outline of the supercoset construction. In section 3, we briefly introduce the generalized type IIB supergravity equations of motion. In section 4, we study six examples of non-Abelian classical $r$-matrices and the associated full backgrounds. For four of them, ‘$T$-dualized’ backgrounds, which are solutions of the usual type IIB supergravity, are found. Then, three of the ‘$T$-dualized’ backgrounds are shown to be locally equivalent to the undeformed $\text{AdS}_5 \times S^5$. Section 5 is devoted to conclusion and discussion.

2. Yang–Baxter deformed $\text{AdS}_5 \times S^5$ backgrounds

In this section, we shall give a short summary of Yang–Baxter deformations of the $\text{AdS}_5 \times S^5$ superstring based on the homogeneous CYBE and an outline of the supercoset construction.

2.1. The deformed string action

The action of the deformed system is given by

$$ S = -\frac{\sqrt{\lambda_c}}{4} \int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma (\gamma^{ab} - \epsilon^{ab}) \text{STr} \left[ A_a d \circ \frac{1}{1 - \eta R_g \circ d} (A_b) \right], \quad (2.1) $$

where the left-invariant one-form $A_a$ is defined as

$$ A_a \equiv -g^{-1} \partial_a g, \quad g \in SU(2, 2|4) \quad (2.2) $$

with the world-sheet index $a = (\tau, \sigma)$. We are in conformal gauge and the world-sheet metric takes the diagonal form $\gamma^{ab} = \text{diag}(-1, +1)$. Hence there is no coupling of the dilaton to the
world-sheet scalar curvature. The anti-symmetric tensor $\epsilon^{ab}$ is normalized as $\epsilon^{rr} = +1$. The constant $\lambda_c$ is the usual ‘t Hooft coupling. The deformation is measured by a constant parameter $\eta$. When $\eta = 0$, the undeformed $\text{AdS}_5 \times S^5$ action [8] is reproduced.

An important quantity here is the chain of operations $R^c$ defined as

$$R^c(X) \equiv g^{-1}R(gXg^{-1})g, \quad \forall X \in \mathfrak{su}(2, 2|4),$$

where the linear operator $R : \mathfrak{su}(2, 2|4) \to \mathfrak{su}(2, 2|4)$ is a solution of the homogeneous CYBE:

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = 0.$$  

This $R$-operator is related to the skew-symmetric classical $r$-matrix in the tensorial notation through the following formula:

$$R(X) = \text{STr}_2[r(1 \otimes X)] = \sum_i (a_i \text{STr}[b_i X] - b_i \text{STr}[a_i X]).$$

Here $r$ is written as

$$r = \sum_i a_i \otimes b_i \equiv \sum_i (a_i \otimes b_i - b_i \otimes a_i) \quad \text{with} \quad a_i, b_i \in \mathfrak{su}(2, 2|4).$$

The projection operator $d$ is a linear combination of projectors defined as

$$d \equiv P_1 + 2P_2 - P_3,$$

where $P_\ell$ ($\ell = 0, 1, 2, 3$) are projectors to the $\mathbb{Z}_4$-graded components of $\mathfrak{su}(2, 2|4)$. In particular, $P_0(\mathfrak{su}(2, 2|4))$ is a local symmetry of the classical action, $\mathfrak{so}(1, 4) \oplus \mathfrak{so}(5)$. The numerical coefficients in (2.7) are fixed by requiring kappa-symmetry [8, 23].

### 2.2. A parametrization of the group element

The classical action (2.1) is written in terms of a group element $g \in SU(2, 2|4)$ and a coordinate system can be introduced by fixing an explicit parametrization.

First of all, the group element $g$ is represented by the product of the bosonic and fermionic elements $g_b$ and $g_f$ as follows:

$$g = g_b g_f.$$  

For our later convenience, let us parametrize the bosonic element $g_b$ as

$$g_b = g_b^{\text{AdS}_5} g_b^{S^5},$$

$$g_b^{\text{AdS}_5} = \exp[x^i P_0 + x^1 P_1 + x^2 P_2 + x^3 P_3 \exp((\log z)D)],$$

$$g_b^{S^5} = \exp\left[\frac{i}{2} (\phi_1 h_1 + \phi_2 h_2 + \phi_3 h_3)\right] \exp[\xi J_{\phi\phi}] \exp[-ir P_3].$$

Here, $P_\mu$ ($\mu = 0, \ldots, 3$) and $D$ are translations and dilatation $D$ in the four-dimensional conformal algebra $\mathfrak{su}(2, 2)$, and $h_i$ ($i = 1, 2, 3$) are the Cartan generators of $\mathfrak{su}(4)$. For the other generators and the details of the notation, see [43]. The coordinates $x^\mu$ and $z$ describe the Poincaré $\text{AdS}_5$, and $r$, $\xi$, $\phi_i$ parametrize the round five-sphere. The resulting metric is given by $ds^2 = ds^2_{\text{AdS}_5} + ds^2_{S^5}$, where

$$ds^2_{\text{AdS}_5} = -\frac{(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{z^2} + \frac{dz^2}{z^2},$$

$$ds^2_{S^5} = dr^2 + \sin^2 r d\xi^2 + \cos^2 \xi (\sin^2 r d\phi^2_1 + \sin^2 r \sin^2 \xi d\phi^2_2 + \cos^2 \xi d\phi^2_3).$$
Then the fermionic group element \( g_f \) is generated by the supercharges \( Q^I \) as follows:

\[
q_f = \exp(Q^I \theta_I), \quad Q^I \theta_I \equiv (Q^I)_{\alpha\dot{\alpha}} (\theta_{\alpha\dot{\alpha}}) (l = 1, 2; \, \alpha, \dot{\alpha} = 1, \ldots, 4).
\]

Here, \( \theta_{\alpha\dot{\alpha}} \) are Grassmann-odd coordinates and correspond to a couple of 16-component Majorana–Weyl spinors satisfying the Majorana condition (see [43] for details).

The coset construction for the bosonic element \( g_b \) is relatively straightforward and the metric and NS–NS two-form of the corresponding deformed geometries are well-studied. However, the supercoset construction including the fermionic element \( g_f \) is quite complicated and until very recently, the R–R sector and dilaton have not been studied.

Eventually, the supercoset construction has been carried out concretely for the \( \eta \)-deformed AdS\(_5\) × S\(_5\) superstring [18]. Following this pioneering work, it has been generalized to the homogeneous CYBE case [43]. In the following, we will give an outline of the supercoset construction.

### 2.3. An outline of the supercoset construction

In order to understand the geometries corresponding to the integrable deformations, we need to express the deformed action in equation (2.1) in terms of the component fields of type IIB supergravity. In other words, we need to put it in the canonical form

\[
S = \int \int_{-\infty}^{\infty} dt d\sigma \left[ \gamma_{ab} \widetilde{G}_{MN} \delta_a X^M \delta_b X^N - \epsilon^{ab} B_{MN} \partial_a X^M \partial_b X^N \right]
- \frac{\sqrt{\lambda}}{2} \int \Theta (\gamma^{ab} \delta^{IJ} - \epsilon^{ab} \sigma^I_5) \bar{\epsilon}^m_a \Gamma_m D_b \Theta_K + \mathcal{O} \theta^4,
\]

where the spinorial covariant derivative \( \widetilde{D} \) is defined as [45]

\[
\widetilde{D}_a \equiv \delta^{IJ} \left( \partial_a - \frac{1}{4} \delta^{mn} \Gamma_m \right) + \frac{1}{8} \epsilon^{IJ} \bar{\epsilon}^m_a H_{nmp} \gamma^{np}
- \frac{1}{8} \epsilon^{IJ} \gamma^p F_p + \frac{1}{3!} \sigma^I_5 \gamma^{pq} F_{pq} + \frac{1}{2 \cdot 5!} \epsilon^{IJ \gamma^{pqrs} F_{pqrs}} \right] \bar{\epsilon}^m_a \Gamma_m.
\]

Here, \( \bar{\epsilon}^m_a \) is the vielbein for the deformed metric, and \( \Theta \) is a 32-component spinor composed of \( \theta_I \).

Although it is quite difficult to rewrite the action at all orders in \( \theta \), the second-order result is enough for our purposes. This expansion simplifies the analysis that remains nevertheless quite involved. For the technical details, see [18, 43].

### 3. The generalized type IIB supergravity equations

One of the main problems in the study of integrable deformations is to determine if the corresponding ten-dimensional field content can be understood as a string theory background.

In the case of the \( \eta \)-deformation, the ten-dimensional background can be obtained via the supercoset construction outlined above and one can see that the fields do not satisfy the type IIB equations of motion [18]. In fact, as was shown in [21], the worldsheet theory is not Weyl invariant, but only scale invariant: the corresponding beta function does not vanish but is a total derivative. This fact weakens the usual on-shell construction of type IIB supergravity

\[\text{Note here that it is assumed that the deformed system can be regarded as the canonical Green–Schwarz string. Rigorously speaking, this identification has to be confirmed by other arguments.}\]
and has lead to the definition of generalized type IIB equations [21]. Note that as of now, this is a purely on-shell construction for which no ten-dimensional effective action has been found yet.

Interestingly, in recent work [22], it has been shown that the $\kappa$-symmetry of the Green–Schwarz superstring generally leads to the generalized equations of type IIB supergravity, while in the older literature [47], it was shown that the on-shell condition of the usual type IIB supergravity leads to the kappa-invariance of the Green–Schwarz string action.

The generalized type IIB supergravity equations [21] are

\[
R_{MN} - \frac{1}{4} H_{MKL} H_N^{KL} - T_{MN} + D_M X_N + D_N X_M = 0, \quad (3.1)
\]

\[
\frac{1}{2} D^K H_{KMN} + \frac{1}{2} F^K F_{KMN} + \frac{1}{12} F_{MNKL} F^{KLP} X^K H_{KMN} + D_M X_N - D_N X_M,
\]

\[
R - \frac{1}{12} H^2 + 4 D_M X^M - 4 X_M X^M = 0, \quad (3.3)
\]

\[
D^M F_M - Z^M F_M - \frac{1}{6} H^{MNPQ} F_{MNPQ} = 0, \quad I^M F_M = 0, \quad (3.4)
\]

\[
D^K F_{KMN} - Z^K F_{KMN} - \frac{1}{6} H^{KPOQ} F_{KPOQMN} - (I \wedge F_I)_{MN} = 0, \quad (3.5)
\]

\[
D^K F_{KMNPO} - Z^K F_{KMNPO} + \frac{1}{36} (MNPQRSTUVW) H^{RST} F^{UVW} - (I \wedge F_I)_{MNPO} = 0. \quad (3.6)
\]

Here $M,N = 0,1,\ldots,9$. The meaning of these equations is summarized as follows:

- The first equation (3.1) is for the metric in the string frame $G_{MN}$. The matter contribution $T_{MN}$ is given by

\[
T_{MN} \equiv \frac{1}{2} F_M F_N + \frac{1}{4} F_{MKL} F_N^{KL} + \frac{1}{4 \times 4!} F_{MPQRS} F_N^{PQRS} - \frac{1}{4} G_{MN} (F_K F^K + \frac{1}{6} F_{PQR} F^{PQR}). \quad (3.7)
\]

Here $F_M, F_{MNK}, F_{MNPQ}$ are the rescaled R–R field strengths

\[
F_{n_1n_2\ldots} = e^\Phi F_{n_1n_2\ldots}, \quad (3.8)
\]

where $\Phi$ is the dilaton whose motion is described by (3.3).

- The second equation (3.2) is for the field strength $H_{MNK}$ of the NS–NS two-form.

- The fourth, fifth and sixth equations (3.4)–(3.6) are for the R–R one-form, three-form and five-form field strengths.

The Bianchi identities for the R–R field strengths are also generalized as

\[
(d F_1 - Z \wedge F_1)_{MN} - I^K F_{MKN} = 0, \quad (3.9)
\]

\[
(d F_3 - Z \wedge F_3 + H_3 \wedge F_1)_{MNPO} - I^K F_{MNPQK} = 0, \quad (3.10)
\]

\footnote{For an earlier argument on the generalized equations at the linear order level, see [46].}
Together with the standard type IIB fields, the equations (3.1)–(3.6) involve the three new vector fields $X$, $I$ and $Z$. Let us consider them in detail. In fact, only two of them are independent as the vector $X$ is expressed as

$$X_M \equiv I_M + Z_M.$$  

(3.12)

$I$ and $Z$ satisfy the following relations:

$$D_M I_N + D_N I_M = 0, \quad D_M Z_N - D_N Z_M + I^K H_{KMN} = 0, \quad I^M Z_M = 0. \tag{3.13}$$

The first equation of (3.13) is the Killing vector equation. Assuming that $I_M$ is chosen such that the Lie derivative vanishes

$$(\mathcal{L}_I B)_{MN} = I^K \partial_K B_{MN} + B_{KN} \partial_M I^K - B_{KM} \partial_N I^K = 0, \tag{3.14}$$

the second equation of (3.13) can be solved by

$$Z_M = \partial_M \Phi - B_{MN} I^N. \tag{3.15}$$

Thus $Z$ can be regarded as a generalization of the dilaton gradient $\partial_M \Phi$. In particular, when $I$ vanishes, $Z_M$ becomes $\partial_M \Phi$ and the generalized equations (3.1)–(3.6) are reduced to the usual type IIB supergravity equations.

As mentioned at the beginning of this section, the generalized equations (3.1)–(3.6) were found in the study of the $\eta$-deformed $\textit{AdS}_5 \times S^5$ superstring. However, they also appear for Yang–Baxter deformations with non-Abelian classical $r$-matrices satisfying the homogeneous CYBE, as we will show in the next section.

4. Examples of non-Abelian classical $r$-matrices

In this section, we will consider six examples of non-Abelian classical $r$-matrices satisfying the homogeneous CYBE. The supercoset construction is performed explicitly by following [43] and it is shown that all of the resulting backgrounds satisfy the generalized type IIB supergravity equations proposed in [21]. For four of the examples, we derive ‘$T$-dualized’ backgrounds which are solutions of the usual type IIB supergravity. Then for three of them, we show that the backgrounds are $T$-dual to the undeformed $\textit{AdS}_5 \times S^5$ after an appropriate coordinate transformation.

We will concentrate on deformations of the $\textit{AdS}_5$ part and basically follow the notation and conventions for the $\text{su}(2, 2)$ generators in [43] to represent classical $r$-matrices.

4.1. $r = P_1 \wedge D$

As a first example, let us consider the following non-Abelian classical $r$-matrix:

$$r = \frac{1}{2} P_1 \wedge D. \tag{4.1}$$

Here $P_\mu$, $\mu = 0, 1, 2, 3$ are the generators of translations in the four-dimensional Poincaré algebra. The generator $D$ represents the dilatation. This is a solution of the homogeneous CYBE which was already used to study a Yang–Baxter deformation of four-dimensional Minkowski spacetime [38].

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By performing the supercoset construction [43], the associated background is found to be

\[ ds^2 = z^2 \left[ \frac{1}{z^2} + \frac{d\xi^2 + 2\eta^2}{z^2 + \eta^2 (t^2 + z^2)} + \frac{r^2}{z^2} \left( -d\phi^2 + \cosh^2 \phi d\theta^2 \right) + dz^2, \right. \\
B_2 = \eta \frac{Idt \wedge dx + z dz \wedge dx^1}{z^4 + \eta^2 (t^2 + z^2)}, \\
F_3 = -\frac{4\eta \eta^2 \cosh^2 \phi}{z^4} \left[ dt \wedge d\phi \wedge d\phi - \frac{r}{z} d\phi \wedge dz \wedge dz \right], \\
F_5 = 4z^4 \left[ z^4 + \eta^2 (t^2 + z^2)^2 \omega_{AdS_5} + \omega_{s^3} \right], \\
\Phi = \frac{1}{2} \log \left[ \frac{z^4}{z^4 + \eta^2 (t^2 + z^2)^2} \right], \]

(4.2)

where we have rewritten the four-dimensional Cartesian coordinates as:

\[ x^0 = t \sinh \phi, \quad x^2 = t \cosh \phi \cos \theta, \quad x^3 = t \cosh \phi \sin \theta. \]

(4.3)

Note here that the \( \phi \) direction has the time-like signature. These fields do not satisfy the equations of motion of type IIB supergravity, but solve the generalized equations of section 3 when supplemented with the following vectors:

\[ I = \frac{\eta z^2}{z^4 + \eta^2 (t^2 + z^2)} dx^1, \quad Z = -\frac{2\eta \eta^2}{z^4 + \eta^2 (t^2 + z^2)} \left( \frac{1}{z} dt - \frac{1}{z} dz \right), \]

(4.4)

Let us now perform \( T \)-dualities for the deformed background (4.2). Following [21], the extra fields are traded for a linear term in the dual dilaton. \( T \)-dualising along the \( x^1 \) and \( \phi_3 \) directions, we find:

\[ ds^2 = z^2 \left( dx^1 \right)^2 + \frac{1}{z^2} \left[ (dt - \eta dx^1)^2 + (dz - \eta \xi dx^1)^2 - r^2 d\phi^2 + \cosh^2 \phi d\theta^2 \right] + dr^2 + \sin^2 r d\xi^2 + \cos^2 r \sin^2 \xi \sin^2 r d\phi_1^2 + \sin^2 r \sin^2 \xi d\phi_2^2 + \frac{d\phi_2^2}{\cos^2 r}, \]

\[ \mathcal{F}_5 = \frac{4r^2 \cosh \phi}{z^4 \cos r} \left( dt - \eta dx^1 \right) \wedge (dz - \eta \xi dx^1) \wedge d\phi \wedge d\phi \wedge d\phi_3 + 2z \sin^2 r \sin 2\xi dx^1 \wedge dr \wedge d\xi \wedge d\phi_1 \wedge d\phi_2, \]

\[ \Phi = -\eta x + \log \left[ \frac{z}{\cos r} \right]. \]

Remarkably, this is a solution of the usual type IIB supergravity equations rather than the generalized ones. Note, however, that the dilaton has acquired a linear dependence on \( x^1 \). This means that \( \partial_1 \) is not an isometry and, as it is, the background cannot be dualized in the directions \( (x^1, \phi_3) \) to go back to the initial frame. This result is very similar to the fact that the Hoare–Tseytlin solution [20] is ‘\( T \)-dual’ to the \( \eta \)-deformed background. Indeed, we have followed the same strategy as in [20].

The ‘\( T \)-dualized’ background in equation (4.5) is a solution to the standard type IIB equations and has a remarkable property: it is \emph{locally equivalent} to undeformed \( AdS_5 \times S^5 \).

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9 The metric and NS–NS two-form were computed in [36].
Let us first perform the following change of coordinates:

\[ t = \eta \phi^1, \quad z = \eta \bar{z}^1, \quad x^1 = \frac{1}{\eta} \log(\eta \bar{z}^1). \]  

(4.6)

Note that the new coordinate system does not cover all of spacetime: the new coordinate \( x^1 \) has to be restricted to be positive (negative) when \( \eta > 0 \) (\( \eta < 0 \)). The signature of \( \eta \) is fixed when we have chosen the deformation. This change of coordinates achieves the following points:

- it diagonalizes the metric;
- it absorbs the \( x^1 \)-dependence of the dilaton into the \( \bar{z} \) variable, such that \( \partial_1 \) is now a symmetry of the full background.

Explicitly, we find

\[
\begin{align*}
\mathrm{d}s^2 &= \bar{z}^2 (\mathrm{d}\bar{z}^1)^2 + \frac{1}{\bar{z}^2} [\mathrm{d}\rho^2 + \mathrm{d}\bar{z}^2 - \rho^2 \, \mathrm{d}\phi^2 + \rho^2 \cosh^2 \phi \, \mathrm{d}\theta^2] \\
&\quad + \rho^2 \left[ \frac{}{} + \sin^2 r \, \mathrm{d}\xi^2 + \cos^2 \xi \sin^2 r \, \mathrm{d}\phi_1^2 + \sin^2 r \sin^2 \xi \, \mathrm{d}\phi_2^2 + \frac{\mathrm{d}\phi_3^2}{\cos^2 r} \right], \\
\mathcal{F}_5 &= \frac{4\rho^2 \cosh \phi}{\bar{z}^2 \cos r} \rho \wedge \mathrm{d}\bar{z} \wedge \mathrm{d}\theta \wedge \mathrm{d}\phi \wedge \mathrm{d}\phi_3 \\
&\quad + 2 \bar{z} \sin^2 r \sin 2\xi \mathrm{d}\bar{z}^1 \wedge \mathrm{d}r \wedge \mathrm{d}\xi \wedge \mathrm{d}\phi_1 \wedge \mathrm{d}\phi_2, \\
\Phi &= \log \left[ \frac{\bar{z}}{\cos r} \right].
\end{align*}
\]

(4.7)

Now we can perform again the two standard T-dualities along \( \bar{z}^1 \) and \( \phi_3 \) to find, as advertised above, \textit{undeformed} \( \text{AdS}_5 \times S^5 \).

Let us stop for a moment to summarize what we have done. We have started with a Yang–Baxter deformation of \( \text{AdS}_5 \) described by the non-Abelian \( r \)-matrix (equation (4.1)). Using the supercoset construction outlined in section 2, we have found the corresponding ten-dimensional background (equation (4.2)) that does not satisfy the usual type IIB equations but is a solution to the generalized equations described in section 3. Then we have 'T-dualized' this background using the rules of [21] to find a new background (equation (4.5)) which solves the standard supergravity equations, but whose dilaton depends linearly on one of the T-dual variables. Finally, we have observed that after a change of variables, this last background is nothing else than the 'T-dual' of the standard undeformed \( \text{AdS}_5 \times S^5 \). This result points to the fact that the Yang–Baxter deformation with the classical \( r \)-matrix in equation (4.1) can be interpreted as an integrable twist, just like in the case of Abelian classical \( r \)-matrices (see for example [27, 48–50]). It might also suggest a relation between a gravity dual of the omega-background [51] and a non-Abelian classical \( r \)-matrix.

4.2. \( r = P_0 \wedge D \)

Our next example is the classical \( r \)-matrix

\[ r = \frac{1}{2} P_0 \wedge D, \]

(4.8)

\(^{10}\) The usual Poincaré coordinates are found using the same change of coordinates as in equation (4.3).
where $P_0$ is the generator of time translations. This is a solution of the homogeneous CYBE which was originally utilized to study a Yang–Baxter deformation of 4D Minkowski spacetime [38].

By performing the supercoset construction [43], the associated background is found to be

$$\begin{aligned}
dx^2 &= \frac{z^2[-(dx^0)^2 + dz^2 + d\rho^2] - \eta^2(d\rho - \rho z^{-1}dz)^2 + \rho^2(d\theta^2 + \sin^2\theta d\phi^2)}{z^4 - \eta^2(z^2 + \rho^2)} + \frac{\rho^2(d\theta^2 + \sin^2\theta d\phi^2)}{z^2} + dz_S^2, \\
B_2 &= \eta \frac{zd\rho \wedge dz + \rho d\rho \wedge d\rho}{z^4 - \eta^2(z^2 + \rho^2)}, \\
F_3 &= \frac{4\eta}{z^4} \left[ \rho^2 \sin\theta d\rho \wedge d\theta \wedge d\phi - \frac{\rho^3 \sin\theta}{z} d\theta \wedge d\phi \wedge dz \right], \\
F_3 &= \frac{4}{z^4} \left[ \frac{z^4}{z^4 - \eta^2(z^2 + \rho^2)} \omega_{\text{AdS}_5} + \omega_3 \right], \\
\Phi &= \frac{1}{2} \log \left[ \frac{z^4}{z^4 - \eta^2(z^2 + \rho^2)} \right],
\end{aligned}$$

(4.9)

where the Cartesian coordinates of the four-dimensional Minkowski spacetime are

$$\begin{aligned}
x^1 &= \rho \sin\theta \cos\phi, \\
x^2 &= \rho \sin\theta \sin\phi, \\
x^3 &= \rho \cos\theta.
\end{aligned}$$

(4.10)

In this case we see that with the following two vectors

$$\begin{aligned}
I &= \frac{-\eta z^2}{z^4 - \eta^2(\rho^2 + z^2)} dx^0, \\
Z &= \frac{2\eta \rho}{z^4 - \eta^2(\rho^2 + z^2)} (d\rho - \frac{\rho}{z} dz),
\end{aligned}$$

(4.11)

the above background becomes a solution of the generalized supergravity equations.

Our next task is to perform 'T-dualities' for the background (4.9) in the $x^0$ and $\phi_3$ directions. We find a solution of the standard type IIB supergravity equations\(^ \text{12} \) with a dilaton that depends linearly on $x^0$:

$$\begin{aligned}
dx^2 &= -z^2(dx^0)^2 + \frac{1}{z^2} [(d\rho + \eta \rho dx^0)^2 + (dz + \eta dx^0)^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2] \\
&\quad + d\tau^2 + \sin^2 \tau d\xi^2 + \cos^2 \tau \sin^2 \tau d\phi_1^2 + 2 \sin^2 \tau \sin^2 \xi d\phi_2^2 + \frac{d\phi_3^2}{\cos^2 \tau}, \\
\mathcal{F}_5 &= -\frac{4i\rho^2 \sin\theta}{z^4 \cos \tau} (d\rho + \eta \rho dx^0) \wedge (dz + \eta dx^0) \wedge d\theta \wedge d\phi \wedge d\phi_3 \\
&\quad + 2iz \sin \tau \sin 2\xi dx^0 \wedge d\tau \wedge d\xi \wedge d\phi_1 \wedge d\phi_2, \\
\Phi &= \eta x^0 + \log \left[ \frac{z}{\cos \tau} \right]
\end{aligned}$$

(4.12)

Finally, let us show that the 'T-dualized' background (4.12) is again equivalent to the undeformed $\text{AdS}_5 \times S^5$. First of all, we perform the following coordinate transformations:

\(^{11}\) This background was studied in [36], but only the metric and NS–NS two-form were computed therein.

\(^{12}\) Having performed a time-like T-duality we necessarily find a purely imaginary five-form flux.
Note here that the new coordinate $x^0$ is restricted to be positive (negative) when $\eta > 0$ ($\eta < 0$). Just like in the previous case, the metric is diagonal and the dilaton does not depend anymore on $x^0$, such that $\partial_0$ is an isometry:

$$\begin{aligned}
\text{d}s^2 &= -\xi^2 (\text{d}x^0)^2 + \frac{1}{\xi^2} [\text{d}\theta^2 + \text{d}z^2 + \text{d}r^2 + \text{d}\phi_1^2 + \text{d}\phi_2^2 + \text{d}\phi_3^2 - \frac{\text{d}\phi_3^2}{\cos^2 r}]

&+ \text{d}r^2 + \sin^2 r \text{d}\xi^2 + \cos^2 \xi \sin^2 r \text{d}\phi_1^2 + \sin^2 r \sin^2 \xi \text{d}\phi_2^2 + \frac{\text{d}\phi_3^2}{\cos^2 r},

F &= \log \left[ \frac{z}{\cos r} \right].
\end{aligned}$$

Again, by performing two $T$-dualities along the $x^0$ and $\phi_3$ directions, we go back to the undeformed $\text{AdS}_5 \times S^5$ background.

**4.3. $r = (P_0 - P_3) \wedge (D - L_{03})$**

Let us consider now the classical $r$-matrix

$$r = \frac{1}{2\sqrt{2}} [P_0 - P_3] \wedge (D - L_{03}),$$

where $L_{03}$ is the generator of the Lorentz rotation in the plane $(x^0, x^3)$.

Performing the supercoset construction [43], we obtain the corresponding background:

$$\begin{aligned}
\text{d}s^2 &= -2 \text{d}x^+ \text{d}x^- + \rho^2 \frac{\text{d}\rho^2}{z^2} + \rho^2 \text{d}\theta^2 + \frac{\text{d}z^2}{z^2} - \eta^2 \left[ \rho^2 \frac{1}{z^2} + \frac{1}{\rho z^4} \right] (\text{d}x^+)^2 + \text{d}x_3^2,

B_2 &= \left[ -\frac{\rho^2 \text{d}x^+ \wedge \rho \text{d}\rho + \rho \rho' \text{d}\rho \wedge \text{d}z + \rho^2 \text{d}x^+ \wedge \rho \text{d}\rho} {z^4} \right],

F_3 &= 4\eta \left[ -\frac{\rho^2 \text{d}x^+ \wedge \text{d}\theta \wedge \text{d}z + \rho \rho' \text{d}\rho \wedge \text{d}\theta}{z^4} \right],

F_5 &= 4(\omega_{\text{AdS}}, + \omega'_{\text{AdS}}),

\Phi &= \Phi_0 \text{ (constant)},
\end{aligned}$$

where the Cartesian coordinates of the four-dimensional Minkowski spacetime $x^\mu$ are

$$x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^3), \quad x^3 = \rho \cos \theta, \quad x^2 = \rho \sin \theta.$$  \hspace{1cm} (4.17)

This background is a solution of the generalized supergravity equations\(^{13}\) when supplemented by the vectors $I$ and $Z$:

$$I = I_M \text{d}x^M = -\frac{2\eta}{z^2} \text{d}x^+,

Z_M = 0 = B_{MN} I^N.$$  \hspace{1cm} (4.18)

\(^{13}\) This result is also supported in [44] based on a scaling argument of the $\eta$-deformed $\text{AdS}_5 \times S^5$.\footnote{This result is also supported in [44] based on a scaling argument of the $\eta$-deformed $\text{AdS}_5 \times S^5$.}
Let us perform four ‘$T$-dualities’ along the $x^+, x^-, \phi_1$ and $\phi_2$ directions\textsuperscript{14}. The resulting background is given by

$$\begin{align*}
dx^2 &= -2\varepsilon^2 dx^+ dx^- + \frac{(d\rho - \eta d\rho dx^-)^2 + \rho^2 d\theta^2 + (dz - \eta d\rho dx^-)^2}{\varepsilon^2} \\
&\quad + dr^2 + \sin^2 r d\xi^2 + \frac{d\phi_1^2}{\cos^2 \xi \sin^2 r} + \frac{d\phi_2^2}{\sin^2 r \sin^2 \xi} + \cos^2 r \, d\phi_3^2, \\
\mathcal{F}_5 &= \frac{4i\rho}{\varepsilon^3 \sin \xi \cos \xi \sin^2 r} \left(d\rho - \eta d\rho dx^- \right) \wedge \, d\theta \wedge (dz - \eta d\rho dx^-) \wedge d\phi_1 \wedge d\phi_2 \\
&\quad + 4i\varepsilon^2 \sin r \cos r \, dx^+ \wedge dx^- \wedge dr \wedge d\xi \wedge d\phi_3, \\
\Phi &= -2\eta x^- + \log \left[ \frac{\varepsilon^2}{\sin^2 r \sin \xi \cos \xi} \right],
\end{align*}$$

(4.19)

where the other components are zero.

The ‘$T$-dualized’ background in equation (4.19) is a solution to the standard type IIB equations and is again locally equivalent to undeformed $\text{AdS}_5 \times S^5$. Let us first change the coordinates as follows:

$$x^- = \frac{1}{2\eta} \log(x^-), \quad \rho = \tilde{\rho} \sqrt{x^-}, \quad z = \xi \sqrt{x^-}.$$  

(4.20)

Explicitly, we find

$$\begin{align*}
dx^2 &= -2\varepsilon^2 dx^+ dx^- + \frac{d\rho^2 + \tilde{\rho}^2 d\theta^2 + dz^2}{\varepsilon^2} \\
&\quad + dr^2 + \sin^2 r d\xi^2 + \frac{d\phi_1^2}{\cos^2 \xi \sin^2 r} + \frac{d\phi_2^2}{\sin^2 r \sin^2 \xi} + \cos^2 r \, d\phi_3^2, \\
\mathcal{F}_5 &= \frac{4i\tilde{\rho}}{\varepsilon^3 \sin \xi \cos \xi \sin^2 r} d\rho \wedge d\theta \wedge dz \wedge d\phi_1 \wedge d\phi_2 \\
&\quad + 4i\varepsilon^2 \sin r \cos r \, dx^+ \wedge dx^- \wedge dr \wedge d\xi \wedge d\phi_3, \\
\Phi &= \log \left[ \frac{\varepsilon^2}{\sin^2 r \sin \xi \cos \xi} \right].
\end{align*}$$

(4.21)

Now, rewriting the light-like coordinates in terms of the Cartesian coordinates as

$$x^+ \equiv \frac{1}{\sqrt{2}}(x^0 + x^3), \quad x^- \equiv \frac{1}{\sqrt{2}}(x^0 - x^3),$$

(4.22)

and performing four $T$-dualities along $\tilde{x}^0, \tilde{x}^3, \phi_1$ and $\phi_2$, we reproduce the undeformed $\text{AdS}_5 \times S^5$ background.

4.3.1. Mixing of Abelian and non-Abelian classical $r$-matrices. This example admits a generalization, obtained by mixing Abelian and non-Abelian classical $r$-matrices:

\textsuperscript{14} To perform the $T$-dualities in the two light-like directions one can equivalently pass to Cartesian coordinates $(x^0, x^3)$. $T$-dualize in these and finally introduce light-like combinations for the $T$-dual variables.
When \( a_2 = 0 \), the classical \( r \)-matrix reduces to the one described above; when \( a_1 = 0 \), the \( r \)-matrix becomes Abelian and the associated background is the Hubeny–Rangamani–Ross solution of \([52]\), as shown in \([43]\).

In \([43]\) it was shown that with a supercoset construction, one finds the following ten-dimensional background

\[
ds^2 = \frac{-2dx^+dx^- + d\rho^2 + \rho^2d\theta^2 + dz^2}{z^2} - \eta^2 \left[ (a_1^2 + a_2^2) \rho^2 \left( \frac{\rho^2}{z^4} + \frac{a_1^2}{z^4} \right) (dx^+)^2 + dz^2 \right],
\]

\( B_2 = \frac{\eta}{z^3}dx^+ \wedge [a_1(\rho d\rho + zdz) - a_2\rho^2d\theta], \)

\( F_3 = \frac{4i\eta}{z^5}dx^+ \wedge [a_1(zd\rho - \rho dz) \wedge d\theta + a_2d\rho \wedge dz], \)

\( F_3 = 4(\omega_{\text{AdS}_4} + \omega_{\text{AdS}_5}). \)

\( \Phi = \Phi_0 \) (constant).

(4.24)

This background is still a solution of the generalized equations with the two vectors \( I \) and \( Z \) given by

\( I = \frac{-2\eta a_1}{z^2}dx^+, \quad Z_M = 0. \)

(4.25)

In the special case \( a_1 = 0 \), the above background reduces to a solution of standard type IIB supergravity.

Let us next take four ‘\( T \)-dualities’ along the \( x^+, x^- , \phi_1 \) and \( \phi_2 \) directions. Then we can obtain a solution of the usual type IIB supergravity as

\[
ds^2 = -2z^2dx^+dx^- + \left( \frac{d\rho - \eta a_1 \rho dz}{z^2} + \frac{d\theta + \eta a_2 dz}{z^2} \right)^2 + \frac{dz}{z^2} + \frac{d\phi_1^2}{\sin^2 \xi \sin^2 \rho} + \frac{d\phi_2^2}{\sin^2 \xi \cos^2 \rho} + \cos^2 \rho \frac{d\phi_3^2}{z^2},
\]

\( F_5 = \frac{4i\rho}{z^3 \sin \xi \cos \xi \sin^2 \rho} (d\rho - \eta a_1 \rho dz) \wedge (d\theta + \eta a_2 dz) \wedge (dz - \eta a_1 dz) \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\xi \wedge d\phi_3, \)

\( \Phi = \frac{-2\eta a_1 x^- + \log \left( \frac{z^2}{\sin^2 \rho \sin \xi \cos \xi} \right).} {2 \eta a_1} \)

(4.26)

where the other components are zero. It is easy to see that this is just a twist of the previous solution (in equation (4.21)) and in fact there is a change of variables

\( \rho = \rho e^{\eta a_1 x^+}, \quad z = z e^{\eta a_1 x^-}, \quad \theta = \theta - \eta a_2 x^- , \quad x^- = \frac{1}{2 \eta a_1} \log(2\eta a_1 \tilde{x}^-). \)

(4.27)
that maps this background to the same local form:
\[
\begin{align*}
\text{d}s^2 &= -2\tilde{z}^2\text{d}x^+\text{d}x^- + \frac{\text{d}\tilde{\rho}^2 + \tilde{\rho}^2 \text{d}\tilde{\theta}^2 + \text{d}\tilde{z}^2}{\tilde{z}^2} \\
&+ \text{d}r^2 + \sin^2 r \text{d}\xi^2 + \frac{\text{d}\phi_1^2}{\sin^2 \xi \sin^2 r} + \frac{\text{d}\phi_2^2}{\sin^2 r \sin^2 \xi} + \cos^2 r \text{d}\phi_3^2,
\end{align*}
\]
\[
\mathcal{F}_5 = \frac{4i\tilde{\rho}}{\tilde{z}^3 \sin \xi \cos \xi \sin^2 r} \text{d}\tilde{\phi} \wedge \text{d}\tilde{\theta} \wedge \text{d}\tilde{z} \wedge \text{d}\phi_1 \wedge \text{d}\phi_2 \\
&+ \frac{4iz^2 \sin r \cos r \text{d}x^+ \wedge \text{d}x^- \wedge \text{d}r \wedge \text{d}\xi \wedge \text{d}\phi_3},
\]
\[
\Phi = \log \left[ \frac{z^2}{\sin^2 r \sin \xi \cos \xi} \right].
\]
which is a \(T\)-dual of the undeformed \(\text{AdS}_5 \times S^5\) background.

4.4. \( r = (P_0 - P_3) \wedge D \)

Let us now consider the non-Abelian classical \( r\)-matrix given by
\[
r = \frac{1}{2\sqrt{2}} (P_0 - P_3) \wedge D,
\]
which is another solution of the homogeneous CYBE.

Using the supercoset construction \([43]\), the associated background is found to be\(^{15}\)
\[
\begin{align*}
\text{d}s^2 &= \frac{1}{z^4 - \eta^2(x^+)^2} \left[ z^2(-2\text{d}x^+\text{d}x^- + \text{d}z^2) + 2\eta^2z^{-2}x^+\rho \text{d}x^+ \text{d}\rho - \eta^2z^{-2}\rho^2(\text{d}x^+)^2 \\
&- \eta^2(\text{d}x^+ + x^+z^{-1}\text{d}z)^2 \right] + \frac{\text{d}\rho^2 + \rho^2 \text{d}\theta^2 + \text{d}z^2}{z^2} + \text{d}s^2, \\
B_2 &= \eta \frac{\text{d}x^+ \wedge (\eta \text{d}z + \rho \text{d}\rho - x^+ \text{d}x^-)}{z^4 - \eta^2(x^+)^2},
\]
\[
\begin{align*}
F_3 &= 4\eta \frac{\rho}{z^3} \left[ \frac{\rho}{z} \text{d}x^+ \wedge \text{d}\theta \wedge \text{d}z + \text{d}x^+ \wedge \text{d}\rho \wedge \text{d}\theta - x^+ \frac{1}{z} \text{d}\rho \wedge \text{d}\theta \wedge \text{d}z \right], \\
F_5 &= 4 \left[ \frac{z^4}{z^4 - \eta^2(x^+)^2} \omega_{\text{AdS}_5} + \omega_{S^5} \right],
\]
\[
\Phi = \frac{1}{2} \log \left[ \frac{z^4}{z^4 - \eta^2(x^+)^2} \right].
\]

Here the following new coordinates have been introduced:
\[
\begin{align*}
x^0 = \frac{x^+ + x^-}{\sqrt{2}}, \\
x^3 = \frac{x^+ - x^-}{\sqrt{2}}, \\
x^4 = \rho \cos \theta, \\
x^5 = \rho \sin \theta.
\end{align*}
\]
This background satisfies the generalized SUGRA equations with the two vectors \(I\) and \(Z\) given by

\(^{15}\) The metric and NS–NS two-form were computed in \([36]\).
As of now, we have not found an appropriate $T$-dual frame in which this background is a solution to the standard type IIB equations with a linear dilaton. It would be interesting to understand if this means that we need a different realization of a scale (non-Weyl) invariant worldsheet theory, different from the one discussed in [21].

4.5. The light-like $\kappa$-Poincaré $r$-matrix

Let us next consider the following non-Abelian classical $r$-matrix:

$$r = \frac{1}{2\sqrt{2}} [ (L_{01} - L_{31}) \wedge P_1 + (L_{02} - L_{32}) \wedge P_2 + L_{03} \wedge (-P_0 + P_3) ].$$

This is a solution of the homogeneous CYBE which was employed to study a light-like deformation of the four-dimensional Poincaré algebra [53] and was also utilized to study a Yang–Baxter deformation of four-dimensional Minkowski spacetime [39].

By performing the supercoset construction [43], the associated background is determined to be

$$\begin{align*}
\text{dx}^2 &= \frac{1}{z^4 - \eta^2(x^+)^2} \left[ \eta^2 z^{-2} \rho \right. \\
&\quad \left. + 2 \eta^2 x^{-2} + \eta^2 z^{-2} \rho^2 \right] + \frac{d\rho^2 + \rho^2 d\theta^2}{z^2} + \text{d}x^2,
\end{align*}$$

$$B_2 = \eta \frac{\text{dx}^+ \wedge (\rho \text{d}\rho - x^+ \text{d}x^-)}{z^4 - \eta^2(x^+)^2},$$

$$F_3 = 4\eta \frac{\rho}{z} (\rho \text{d}x^+ \wedge \text{d}\theta \wedge \text{d}z - x^+ \text{d}\rho \wedge \text{d}\theta \wedge \text{d}z),$$

$$F_5 = 4 \left[ \frac{z^4}{z^4 - \eta^2(x^+)^2} \omega_{\text{AdS}_5} + \omega_s^5 \right],$$

$$\Phi = \frac{1}{2} \log \left[ \frac{z^4}{z^4 - \eta^2(x^+)^2} \right].$$

(4.34)

Here the coordinate system (4.31) has been used. This background does not satisfy the equations of the usual type IIB supergravity, but the generalized equations with the two vectors $I$ and $Z$ given by

$$I = \frac{3\eta z^2}{z^4 - \eta^2(x^+)^2} \text{d}x^+, \quad Z = -\frac{2\eta^2 x^+}{z^4 - \eta^2(x^+)^2} \text{d}x^+ - \frac{2\eta^2(x^+)^2}{z(z^4 - \eta^2(x^+)^2)} \text{d}z.$$

(4.35)

This example is quite close to the one discussed in the previous subsection. Also in this case, a $T$-duality to a solution of the standard type IIB equations is as of now missing.

16 The metric and NS–NS two form were computed in [36].
4.6. A scaling limit of the Drinfeld–Jimbo r-matrix

Our last example is the classical r-matrix

\[ r = -D \wedge P_0 - L_{0\mu} \wedge P^\mu - L_{12} \wedge P_2 - L_{13} \wedge P_3, \]  

(4.36)

It was originally studied in [44] in relation to a scaling limit of the classical r-matrix of Drinfeld–Jimbo type.

By performing the supercoset construction\(^{17}\), the full background is determined to be\(^{18}\)

\[
\begin{align*}
\text{ds}^2 &= \frac{-(dx^0)^2 + dz^2}{z^2 - 4\eta^2} + \frac{z^2[(dx^1)^2 + d\rho^2]}{z^4 + 4\eta^2\rho^2} + \frac{\rho^2 d\theta^2}{z^2} + \text{d}s^2_{\mathfrak{s}^7}, \\
B_2 &= \frac{2\eta}{z(z^2 - 4\eta^2)}dx^0 \wedge dz + \frac{2\eta \rho}{z^4 + 4\eta^2\rho^2}dx^1 \wedge d\rho, \\
F_1 &= \frac{16\eta^2\rho^2}{z^4}d\theta, \\
F_3 &= \frac{8\eta\rho^2}{z^3(z^2 - 4\eta^2)^2}dx^0 \wedge d\theta \wedge dz + \frac{8\eta \rho}{z^4 + 4\eta^2\rho^2}dx^1 \wedge d\theta, \\
F_3 &= 4 \left[ \frac{z^6}{(z^2 - 4\eta^2)(z^4 + 4\eta^2\rho^2)} \omega_{\text{ads}_5} + \omega_{\text{s}^2} \right], \\
\Phi &= \frac{1}{2} \log \left[ \frac{z^6}{(z^2 - 4\eta^2)(z^4 + 4\eta^2\rho^2)} \right].
\end{align*}
\]

(4.37)

Again, this background does not satisfy the usual type IIB supergravity equations, but becomes a solution of the generalized equations by taking \(I\) and \(Z\) as

\[
I = -\frac{8\eta dx^0}{z^2 - 4\eta^2} + \frac{4\eta^2 dx^1}{z^4 + 4\eta^2\rho^2}, \quad Z = \left[ \frac{2(z^2 + 2\eta^2)}{z(z^2 - 4\eta^2)} - \frac{2z^3}{z^4 + 4\eta^2\rho^2} \right]dz
\]

\[+ \frac{4\eta^2 d\rho}{z^4 + 4\eta^2\rho^2}. \]  

(4.38)

Performing ‘\(T\)-dualities’ for all of the \(U(1)\) directions, we can obtain a solution of the standard type IIB supergravity:

\[
\begin{align*}
\text{ds}^2 &= -z^2(dx^0)^2 + z^2(dx^1)^2 + \frac{(d\rho + 2\eta dx^0)^2 + (dz + 2\eta z dx^0)^2}{z^2} + \frac{z^2 d\theta^2}{\rho^2} \\
&+ \text{dr}^2 + \sin^2 r \text{d}\xi^2 + \frac{d\phi_1^2}{\sin^2 r \cos^2 \xi} + \frac{d\phi_2^2}{\sin^2 r \sin^2 \xi} + \frac{d\phi_3^2}{\cos^2 r}, \\
F_5 &= -\frac{-4i}{z^2 \sin^2 r \cos r \sin \xi \cos \xi} (d\rho + 2\eta \rho dx^0) \wedge (dz + 2\eta z dx^0) \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \\
&+ 4i \frac{z^3}{\rho} \sin r dx^0 \wedge dx^1 \wedge d\theta \wedge dr \wedge d\xi, \\
\Phi &= 8\eta \times^0 - 4\eta \times^1 + \log \left[ \frac{z^3}{\rho \sin^2 r \cos r \sin \xi \cos \xi} \right].
\end{align*}
\]

(4.39)

where the other components are zero.

\(^{17}\) In this case, a non-trivial axion is turned on and hence the supercoset construction procedure in [43] has to be extended by letting \(U\) include \(\lambda^{\mu} \lambda^{\nu} \Gamma_{\mu\nu\rho}\). For details, see [54].

\(^{18}\) The metric and NS–NS two-form were computed in [44] without the total derivative term in \(B_2\).
Just like in the first examples that we have discussed, there is a simple change of coordinates
\[ \rho = \tilde{\rho} \, e^{-2\eta \, x^i}, \quad z = \tilde{z} \, e^{-2\eta \, x^0}, \]  
that diagonalizes the metric:
\[
d s^2 = e^{-4\eta \, z^2} \left[ -(d\theta)^2 + (dx^1)^2 \right] + \frac{e^{4\eta \,(x^0 - x^i)}}{\tilde{z}^2} \frac{d\tilde{\rho}^2 + d\tilde{z}^2}{\tilde{\rho}^2} + e^{-4\eta \,(x^0 - x^i)} \tilde{z}^2 d\theta^2 \\
+ dr^2 + \sin^2 r \, d\xi^2 + \frac{d\phi_1^2}{\sin^2 r \cos^2 \xi} + \frac{d\phi_2^2}{\sin^2 r \sin^2 \xi} + \frac{d\phi_3^2}{\cos^2 r},
\]
\[ \mathcal{F}_5 = \frac{-4i e^{2\eta \,(x^0 - x^i)}}{\tilde{z}^2 \sin^2 r \cos r \sin \xi \cos \xi} \, d\rho \wedge d\xi \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 + 4i \frac{e^{-2\eta \,(3x^0 - x^i)}}{\tilde{\rho}} \, \sin r \, dx^0 \wedge dx^1 \wedge d\theta \wedge dr \wedge d\xi, \]
\[ \Phi = 2\eta \,(x^0 - x^i) + \log \left[ \frac{\tilde{z}^3}{\tilde{\rho} \sin^2 r \cos r \sin \xi \cos \xi} \right]. \]
In this case, however, the linear dependence of the dilaton on the $T$-dual variables remains.

It may be helpful to recall the result for the case of 3D Schrödinger spacetime [6]. An affine symmetry algebra is given by a twisted Yangian, which is called an exotic symmetry in [6], and can be mapped to the standard Yangian by undoing the integrable twist. Then, however, the target spacetime is not mapped to the undeformed AdS$_3$. The resulting geometry is described by a dipole-like coordinate system and hence it is very close to, but not identical to AdS$_3$.

4.6.1. The case without the total derivative of $B_2$ It would be interesting to study also the case without the total derivative term of $B_2$ in (4.37). Then the 'T-dualized' background is different from the one of equation (4.39) and the resulting background is given by
\[
d s^2 = -(z^2 - 4\eta^2) \, (dx^0)^2 + \tilde{z}^2 \, (dx^1)^2 + \frac{(d\rho + 2\eta \, dx^1)^2}{\tilde{z}^2} + \frac{d\tilde{z}^2}{\tilde{z}^2 - 4\eta^2} + \frac{z^2 d\theta^2}{\rho^2} \\
+ dr^2 + \sin^2 r \, d\xi^2 + \frac{d\phi_1^2}{\sin^2 r \cos^2 \xi} + \frac{d\phi_2^2}{\sin^2 r \sin^2 \xi} + \frac{d\phi_3^2}{\cos^2 r},
\]
\[ \mathcal{F}_5 = \frac{-4i}{\tilde{z}^2 \sin^2 r \cos r \sin \xi \cos \xi} \, (d\rho + 2\eta \, \rho dx^1) \times \left( \frac{z^2 d\xi}{z^2 - 4\eta^2} + 2\eta \, z \, dx^0 \right) \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 + 4i \frac{z^3}{\tilde{\rho}} \, \sin r \left( dx^0 - \frac{2\eta \, dz}{z(z^2 - 4\eta^2)} \right) \wedge dx^1 \wedge d\theta \wedge dr \wedge d\xi, \]
\[ \Phi = -8\eta \, x^0 - 4\eta \, x^1 + \log \left[ \frac{(z^2 - 4\eta^2)^2}{\rho \tilde{z} \sin^2 r \cos r \sin \xi \cos \xi} \right]. \]
This background is a solution of the usual type IIB supergravity, and agrees with the one obtained in [44] after fixing some typos19.

Now it is natural to ask whether (4.39) and (4.42) are equivalent or not, and if so, whether this equivalence holds locally or globally. The local equivalence can be shown explicitly by using the coordinate transformations20

\[
x^0 \rightarrow -x^0 + \frac{1}{4\eta} \left[ \frac{z^2 - 4\eta^2}{z^2} \right] \quad z > 2\eta, \\
x^0 \rightarrow -x^0 + \frac{1}{4\eta} \left[ \frac{4\eta^2 - z^2}{z^2} \right] \quad z < 2\eta.
\]

(4.43)

For the case of the global equivalence, some subtleties arise. The background (4.39) is regular while the one (4.42) has a coordinate singularity at \( z = 2\eta \) and so are the coordinate transformations. Moreover, two types of time directions have to be introduced. Due to these observations, a more involved analysis is necessary in order to argue the global equivalence.

5. Conclusion and discussion

In this paper we have studied the relation between non-Abelian classical \( r \)-matrices and the generalized type IIB equations. We have discussed several examples of non-Abelian classical \( r \)-matrices and derived the corresponding ten-dimensional backgrounds by performing a supercoset construction.

Our main result is that these backgrounds do not satisfy the standard type IIB equations of motion but a set of generalized equations, corresponding to a scale-invariant—as opposed to Weyl-invariant—worldsheet theory. This is consistent with the scaling result of [44], and the general result of [22] under the assumptions that the deformed string action is the canonical Green–Schwarz string. For some of these examples, we were able to perform a \( T \)-duality transformation leading to a new solution of the standard IIB equations, where the dilaton depends linearly on some of the \( T \)-dual variables. For some of these examples, we have even been able to find a change of variables showing that the linear-dilaton backgrounds are locally \( T \)-dual to undeformed \( dS_5 \times S^5 \). This fact makes the classical integrability of these examples manifest.

We have however not been able to find appropriate \( T \)-dual frames for all of the examples. At this point, it is unclear whether this is only due to technical issues or whether there is an underlying physical reason for this. It would therefore be of use to explore more general classical \( r \)-matrices, as well as lower-dimensional cases, such as deformations of \( dS_2 \times S^2 \) or \( dS_1 \times S^1 \) [55, 56], and the relationship between our construction and the \( \lambda \) deformation of [57–60].

We find that a criterion for distinguishing classes of non-Abelian classical \( r \)-matrices is of importance. A possible criterion is whether a given non-Abelian classical \( r \)-matrix is constructed as a twist of the Drinfeld–Jimbo \( r \)-matrix or not (for details, see section 3.1 of [23]). This direction would be useful for the classification of non-Abelian classical \( r \)-matrices.

Gaining a deeper understanding of the underlying algebraic structure giving rise to our findings is essential, as it might throw light on the fundamental mathematical underpinnings of the gravity/CYBE correspondence.

19 We would like to thank Ben Hoare and Stijn van Tongeren for clarifying this point.
20 The authors would like to thank Ben Hoare and Stijn van Tongeren for pointing out the coordinate transformations.
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