ISOMETRIES OF IDEAL LATTICES
AND HYPERKÄHLER MANIFOLDS

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Abstract. We prove that there exists a holomorphic symplectic manifold
deformation equivalent to the Hilbert scheme of two points on a K3 surface
that admits a non-symplectic automorphism of order 23, that is the maximal
possible prime order in this deformation family. The proof uses the theory of
ideal lattices in cyclotomic fields.

1. Introduction

The study of automorphisms on deformation families of hyperkähler manifolds
is a recent and very active field of research. One of the main objectives in the recent
published papers concerns the classification of prime order automorphisms: fixed
locus, moduli spaces and deformations. We refer for instance to [5, 13, 17] and
references therein for a more complete picture. The purpose of this paper is to
answer a question of [6] concerning the existence of automorphisms of order 23.

Let \( X \) be an irreducible holomorphic symplectic manifold. Its second cohomology
\( H^2(X, \mathbb{Z}) \) is an integral lattice for the Beauville–Bogomolov–Fujiki quadratic
form [4]. Let \( f \) be a holomorphic automorphism of \( X \) of prime order \( p \) acting
non-symplectically: \( f \) acts on \( H^{2,0}(X) \) by multiplication by a primitive \( p \)-th root
of the unity. Such automorphisms can exist only when \( X \) is projective. It follows
that the invariant lattice \( T(f) \subset H^2(X, \mathbb{Z}) \) is a primitive sublattice of the Néron–
Severi group \( \text{NS}(X) \) and consequently the characteristic polynomial of the action
of \( f \) on the transcendental lattice \( \text{Trans}(X) \) is a multiple \( k \) of the \( p \)-th cyclotomic
polynomial \( \Phi_p \). Thus \( k\varphi(p) = \text{rank}_\mathbb{Z} \text{Trans}(X) \) and in particular
\[
\varphi(p) \leq b_2(X) - \rho(X),
\]
where \( \varphi \) is the Euler function and \( \rho(X) = \text{rank}_\mathbb{Z} \text{NS}(X) \) is the Picard number of \( X \).
Assume that \( X \) is in the deformation class of the Hilbert scheme of two points on
a projective K3 surface (an IHS-K3[2] for short). Since \( b_2(X) = 23 \), the maximal
order for \( f \) is \( p = 23 \) and this can happen only when \( \rho(X) = 1 \). The main result of
this paper is:

**Theorem 1.1.** There exists an IHS-K3[2] with a non-symplectic automorphism of
order 23.

We show in §3 that the Néron–Severi group of \( X \) has rank one, generated by
an ample line bundle of square 46 with respect to the Beauville–Bogomolov–Fujiki

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quadratic form: up to now there does not exist any geometric construction of such an IHS-$K3^{[2]}$ (see [16]). We emphasize that such an automorphism can not exist on the Hilbert scheme of two points on a K3 surface since this has Picard number two.

The strategy of the proof consists in constructing an isometry of order 23 of the lattice $E_8^{[2]} \oplus U^{[3]} \oplus (-2)$ with the required properties (Corollary 5.4) and then to use the surjectivity of the period map and the global Torelli theorem to construct the variety with its automorphism (Theorem 6.1). Assuming that such an automorphism does exist, the invariant lattice $T$ and its orthogonal complement $S$ are uniquely determined up to isometry so the main step (Proposition 5.3) consists in constructing an order 23 isometry on the lattice $S$: we obtain it by proving that the lattice $S$ can be realized as an ideal lattice in the 23rd cyclotomic field, using results of Bayer–Fluckiger [11,2,3].

2. PRELIMINARIES ON LATTICE

A lattice $L$ is a free $\mathbb{Z}$-module equipped with a nondegenerate symmetric bilinear form $\langle \cdot, \cdot \rangle_L$ with integer values. Its dual lattice is $L^\vee := \text{Hom}_\mathbb{Z}(L, \mathbb{Z})$. It can be also described as follows:

$$L^\vee \cong \{ x \in L \otimes \mathbb{Q} \mid \langle x, v \rangle_L \in \mathbb{Z} \ \forall v \in L \}.$$ 

Clearly $L$ is a sublattice of $L^\vee$ of the same rank, so the discriminant group $A_L := L^\vee / L$ is a finite abelian group whose order is denoted $d_L$ and called the discriminant of $L$. In a basis $(e_i)$ of $L$, for the Gram matrix $M := ((e_i, e_j))_{i,j}$ one has $d_L = \vert \det(M) \vert$.

A lattice $L$ is called even if $\langle x, x \rangle \in 2\mathbb{Z}$ for all $x \in L$. In this case the bilinear form induces a quadratic form $q_L : A_L \rightarrow \mathbb{Q}/2\mathbb{Z}$. Denoting by $(s_{(+)}, s_{(-)})$ the signature of $L \otimes \mathbb{R}$, the triple of invariants $(s_{(+)}, s_{(-)}, q_L)$ characterizes the genus of the even lattice $L$ (see [5] and references therein).

A lattice $L$ is called unimodular if $A_L = \{ 0 \}$. A sublattice $M \subset L$ is called primitive if $L/M$ is a free $\mathbb{Z}$-module. If $L$ is unimodular and $M \subset L$ is a primitive sublattice, then $M$ and its orthogonal $M^\perp$ in $L$ have isomorphic discriminant groups and $q_M = -q_{M^\perp}$.

Let $p$ be a prime number. A lattice $L$ is called $p$-elementary if $A_L \cong \left( \frac{\mathbb{Z}}{p^a} \right)^{\oplus \alpha}$ for some non-negative integer $a$ (also called the length $\ell(A_L)$ of $A$). We write $\frac{\mathbb{Z}}{p^a} (\alpha)$, $\alpha \in \mathbb{Q}/2\mathbb{Z}$ to denote that the quadratic form $q_L$ takes value $\alpha$ on the generator of the $\frac{\mathbb{Z}}{p^a}$ component of the discriminant group. Recall that an even indefinite $p$-elementary lattice of rank $r \geq 3$ with $p \geq 3$ is uniquely determined by its signature and discriminant form (see [5, Theorem 2.2]).

3. BASIC RESULTS ON NON-SYMPLECTIC AUTOMORPHISMS

From now on, we assume that $X$ is an IHS-$K3^{[2]}$ with a non-symplectic automorphism $f$ of prime order $3 \leq p \leq 23$. The lattice $H^2(X, \mathbb{Z})$ has signature $(3,19)$ and is isometric to $L := E_8^{[2]} \oplus U^{[3]} \oplus (-2)$, where $U$ is the unique even unimodular hyperbolic lattice of rank two and $E_8$ is the negative definite lattice associated to the corresponding Dynkin diagram. We restate in this special case some results of Boissière–Nieper-Wisskirchen–Sarti [6]: the case $p = 23$ was left apart since it requires different arguments due to the fact that the ring of integers of the 23rd cyclotomic field is not a PID, but some basic facts extend easily.
The automorphism $f$ induces an isometry $g := f^*$ on $H^2(X, \mathbb{Z})$. We denote by \( G = \langle g \rangle \) the group generated by $g$ and we put
\[
\tau := g - 1 \in \mathbb{Z}[G], \quad \sigma := 1 + g + \cdots + g^{p-1} \in \mathbb{Z}[G].
\]
One has $T(f) = \ker(\tau) \cap H^2(X, \mathbb{Z})$ and we define $S(f) := \ker(\sigma) \cap H^2(X, \mathbb{Z})$.

Denote by $\Phi_p \in \mathbb{Q}[X]$ the $p$-th cyclotomic polynomial. Consider the cyclotomic field $K = \mathbb{Q}[X]/(\Phi_p) \cong \mathbb{Q}[\zeta_p]$ with ring of algebraic integers $O_K \cong \mathbb{Z}[\zeta_p]$ (here $\zeta_p = X \mod \Phi_p$ should not be considered as a complex number). The $G$-module structure of $K$ is defined by $g \cdot x = \zeta_p x$ for $x \in K$. For any fractional ideal $I$ in $K$, and $\alpha \in I$, we denote by $(I, \alpha)$ the module $I \oplus \mathbb{Z}$ whose $G$-module structure is defined by $g \cdot (x, k) = (\zeta_p x + k\alpha, k)$. By a theorem of Diederichsen–Reiner \cite{DiederichsenReiner}, $H^2(X, \mathbb{Z})$ is isomorphic as a $\mathbb{Z}[G]$-module to a direct sum:
\[
(A_1, a_1) \oplus \cdots \oplus (A_r, a_r) \oplus A_{r+1} \oplus \cdots A_{r+s} \oplus Y
\]
for some $r, s \in \mathbb{N}$, where $A_i$ are fractional ideal in $K$, $a_i \in A_i$ are such that $a_i \notin (\zeta_p - 1)A_i$ and $Y$ is a free $\mathbb{Z}$-module of finite rank on which $G$ acts trivially.

**Lemma 3.1.** The quotient $\frac{H^2(X, \mathbb{Z})}{T(f) \oplus S(f)}$ is a $p$-torsion module.

**Proof.** First we observe that $T(f) \cap S(f) = 0$ since $H^2(X, \mathbb{Z})$ has no $p$-torsion. It is clear that $Y \subseteq T(f)$ and $O_K \subseteq S(f)$. Let $A = \sum_i O_K\alpha_i$ be a fractional ideal of $K$, with $\alpha_i \in K$. Clearly $A \subseteq S(f)$. In any term $(A, a) = A \oplus \mathbb{Z}$, denoting $v := (0, 1)$ in this decomposition, we show that $pv \in T(f) \oplus S(f)$. One has $\tau(pv) = (pa, 0)$. Write $a = \sum_i x_i\alpha_i$ with $x_i \in O_K$. Since $O_K/(\zeta_p - 1) = \mathbb{Z}/p\mathbb{Z}$, there exists $z_i \in O_K$ such that $px_i = (\zeta_p - 1)z_i$. Hence $\tau(pv) = ((\zeta_p - 1)z, 0)$ with $z := \sum_i z_i\alpha_i \in A$. Now $\tau((z, 0)) = ((\zeta_p - 1)z, 0)$ hence $\tau((pv - (z, 0)) = 0$ and $\sigma((z, 0)) = 0$ so finally $pv = (pv - (z, 0)) + (z, 0) \in T(f) \oplus S(f)$.

We define $a_f \in \mathbb{N}$ such that
\[
\frac{H^2(X, \mathbb{Z})}{T(f) \oplus S(f)} \cong \left( \frac{\mathbb{Z}}{p\mathbb{Z}} \right)^{\oplus a_f}.
\]
By definition, $S(f)$ is a torsion-free $O_K$-module for the action $\zeta_p \cdot x = g(x)$ for all $x \in S(f)$, hence $S(f)_{\mathbb{Q}} := S(f) \otimes_{\mathbb{Z}} \mathbb{Q}$ is a $K$-vector space. It follows that there exists $m_f \in \mathbb{N}^+$ such that
\[
\text{rank}_\mathbb{Z} S(f)_{\mathbb{Q}} = \dim_\mathbb{Q} S(f)_{\mathbb{Q}} = (p - 1)m_f.
\]
It is easy to check that $S(f)$ is the orthogonal complement of $T(f)$ in the lattice $H^2(X, \mathbb{Z})$ (see \cite{Günter} Lemma 6.1). By a similar argument as in \cite{Günter} Lemma 6.5 one deduces from Lemma 3.1 that the invariant lattice $T(f)$ has signature $(1, 22 - (p - 1)m_f)$ and discriminant
\[
A_{T(f)} \cong \left( \frac{\mathbb{Z}}{2\mathbb{Z}} \right)^{\oplus a_f} \oplus \left( \frac{\mathbb{Z}}{p\mathbb{Z}} \right)^{\oplus a_f}
\]
and that $S(f)$ has signature $(2, (p - 1)m_f - 2)$ and discriminant
\[
A_{S(f)} \cong \left( \frac{\mathbb{Z}}{p\mathbb{Z}} \right)^{\oplus a_f}.
\]
If $p \geq 3$, as explained in \cite{Günter} proof of Lemma 6.5 the action of $G$ on $A_{S(f)}$ is trivial. Since $f$ acts non-symplectically one has $\text{Trans}(X) \subset S(f)$ and $\text{rank}_\mathbb{Z} \text{Trans}(X) \geq p - 1$. In particular, if $m_f = 1$ this forces $\text{Trans}(X) = S(f)$ and consequently...
\[ NS(X) = T(f). \] Since 1 is not an eigenvalue of \( f^*|_{S(f)} \) the characteristic polynomial of \( f^*|_{S(f)} \) is \( \Phi_p \).

All the possible isometry classes for the lattices \( T(f) \) and \( S(f) \) have been classified in [7] when \( 2 \leq p \leq 19 \) (only partially for \( p = 5 \)) by using the previous properties (for \( p = 2 \) the situation is a bit different), the Lefschetz fixed point formula and a relation between the cohomology modulo \( p \) of the fixed locus and the integers \( a_f, m_f \) obtained using Smith theory methods [6].

If \( p = 23 \) the only possibility is that \( m_f = 1 \), \( T(f) \) has signature \((1,0)\) and \( S(f) \) has signature \((2,20)\). The case \( a_f = 0 \) is impossible by Milnor’s theorem since there exists no even unimodular lattice with signature \((2,20)\).

Since \( T(f) \) has rank one, so \( A_{T(f)} \cong \frac{\mathbb{Z}}{p\mathbb{Z}} \) and finally \( T(f) \) is isometric to the lattice \((46)\). By results of Nikulin and Rudakov–Shafarevich [15, 18] \( S(f) \) splits as a direct sum \( U \oplus W \) where \( W \) is hyperbolic and \( 23 \)-elementary of signature \((1,19)\), \( AW \cong \frac{\mathbb{Z}^{22}}{p\mathbb{Z}} \) and \( W \) is unique up to isometry. It follows that \( S(f) \) is uniquely determined and we deduce that \( S(f) \) is isometric to the lattice \( U^{\oplus 2} \oplus E_8^{\oplus 2} \oplus K_{23} \) where \( K_{23} := \begin{pmatrix} -12 & 1 \\ 1 & -2 \end{pmatrix} \). As a consequence, if there exists an \( \text{IHS-K3}^{[2]} \), say \( X \), with a non-symplectic automorphism \( f \) of order 23, then necessarily it has \( \rho(X) = 1, \ NS(X) = T(f) = (46) \) and \( \text{Trans}(X) = S(f) = E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23} \). Such a variety does not belong to any of the known families: Hilbert schemes of points or moduli spaces of semi-stable sheaves on projective K3 surfaces have Picard number greater than 2, Fano varieties of lines on cubic fourfolds are polarized by a class of square 6 (see [3] § 5.5.2 and references therein) and similarly the degree of the polarisation is 2 for double covers of EPW sextics, it is 38 for the sums of powers of general cubics of Iliev–Ranestad and it is 22 for the varieties of Debarre–Voisin (see [16] §0).

4. Ideal lattices in cyclotomic fields

The relation between automorphisms of lattices with given characteristic polynomials and ideals in cyclotomic fields has been studied by many authors, in particular Bayer-Fluckiger [1, 2, 3] and Gross–McMullen [10]. We recall here some results that are needed in the sequel.

Assume that \( p \) is an odd prime number. Recall that \( K = \mathbb{Q}(\zeta_p) \) denotes the cyclotomic field with ring of algebraic integers \( O_K = \mathbb{Z}[\zeta_p] \). We denote respectively by \( \text{Tr}_{K/\mathbb{Q}} \) and \( N_{K/\mathbb{Q}} \) the trace and the norm maps. The complex conjugation on \( K \) is defined as the \( \mathbb{Q} \)-linear involution \( K \to K, x \mapsto \overline{x} \) such that \( \overline{\zeta_i^p} = \zeta_i^{p^{-1}} \) for all \( i \). We denote by \( F \subset K \) the real subfield of \( K \), that is
\[ F := \{ x \in K \mid \overline{x} = x \}. \]
Denoting \( \mu_p := \zeta_p + \zeta_p^{p^{-1}} \) one has \( F = \mathbb{Q}(\mu_p) \). The \( \mathbb{Q} \)-linear pairing
\[ (-,-)_K : K \times K \to \mathbb{Q}, \quad (x, y) \mapsto \text{Tr}_{K/\mathbb{Q}}(xy) \]
is non-degenerate and has determinant \( D_K := p^{p-2} \) in the basis \((1, \zeta_p, \ldots, \zeta_p^{p-2})\).

Let \( (S, (-,-)_S) \) be an integral even lattice of rank \( p - 1 \), signature \((s_+, s_-)\) and discriminant \( d_S \). Assume that \( S \) admits a non trivial isometry \( \varphi \) of order \( p \). Its characteristic polynomial is then \( \Phi_p \) so \( S \) admits a natural structure of \( O_K \)-module defined by \( \zeta_p \cdot x = \varphi(x) \) for all \( x \in S \). For dimensional reasons \( S_\mathbb{Q} := S \otimes_{\mathbb{Z}} \mathbb{Q} \) is isomorphic to \( K \) so the inclusion \( S \hookrightarrow S_\mathbb{Q} \cong K \) identifies the lattice \( S \) with an
Since the bilinear form on $S$ is symmetric one has $\langle x, y \rangle_S = \langle y, x \rangle_S$ for all $x, y \in S$.

If $I \subset K$ is a fractional ideal and $\alpha \in F$, we denote by $I_\alpha$ the ideal lattice whose bilinear form is $\langle x, y \rangle_\alpha := \text{Tr}(\alpha x \overline{y})$. Some of the main invariants of the lattice $I_\alpha$ correspond to properties of $\alpha$ that we explain now.

Recall that the norm of $I$ is defined as $N(I) := |\det(\psi)|$ where $\psi : K \rightarrow K$ is any $\mathbb{Q}$-linear automorphism such that $\psi(O_K) = I$. By a direct computation one finds that the discriminant $d_{I_\alpha}$ of $I_\alpha$ satisfies the relation:

$$d_{I_\alpha} = N(I)^2 |N_{K/\mathbb{Q}}(\alpha)| D_K.$$  

Observe that since $\alpha \in F$ one has

$$N_{K/\mathbb{Q}}(\alpha) = N_{F/\mathbb{Q}}(N_{K/F}(\alpha)) = N_{F/\mathbb{Q}}(\alpha^2) = N_{F/\mathbb{Q}}(\alpha)^2.$$  

We recall that the codifferent $O_\alpha^\vee$ of $K$ is defined by

$$O_\alpha^\vee := \{ x \in K \mid \forall y \in O_K, \text{Tr}_{K/\mathbb{Q}}(xy) \in \mathbb{Z} \}.$$  

If $I_\alpha$ is an integral lattice, for any $x, y \in I$ one has $\text{Tr}_{K/\mathbb{Q}}(\alpha x \overline{y}) \in \mathbb{Z}$ so $\alpha x \overline{y} \in O_\alpha^\vee$. The integrality of $I_\alpha$ is thus equivalent to the condition:

$$\alpha x \overline{y} \in O_\alpha^\vee \quad \forall x, y \in I.$$  

Note that if $\alpha$ satisfies the above property the lattice $I_\alpha$ is automatically even:

Putting $\gamma := \frac{(-1)^{p-1}/2}{\zeta_p}$, since $\gamma + \overline{\gamma} = 1$ one has for any $x \in I$

$$\langle x, x \rangle_I = \text{Tr}_{K/\mathbb{Q}}(\alpha x \overline{y}) = \text{Tr}_{K/\mathbb{Q}}((\gamma + \overline{\gamma}) \alpha x \overline{y}) = \text{Tr}_{K/\mathbb{Q}}(\gamma \alpha x \overline{y}) \in 2\mathbb{Z}$$

since $\alpha x \overline{y} \in O_\alpha^\vee$ by assumption.

The field $K$ admits $p - 1$ complex embeddings defined by $\zeta_p \mapsto e^{2\pi ik/p}$, $1 \leq k \leq p - 1$ that induce real embeddings of $F$. We denote by $t$ the number of these real embeddings such that $\alpha$ is negative. One can show that the lattice $I_\alpha$ has signature

$$(p - 1 - 2t, 2t).$$

This is a special case of [1] Proposition 2.2, we recall the argument for convenience. First observe that $K$ is a quadratic extension of $F$ with minimal polynomial $X^2 - \mu p X + 1 \in F[X]$. Denoting $\theta := \zeta_p^2 + \zeta_p^{-2} - 2$ one has thus $K \cong F(\sqrt{\theta})$. Each complex embedding of $K$ induces a real embedding $v : F \rightarrow \mathbb{R}$ such that $v(\theta) < 0$.

It follows that $K \otimes_{\mathbb{Q}} \mathbb{R}$ decomposes in a direct sum

$$K \otimes_{\mathbb{Q}} \mathbb{R} = \bigoplus_{v : F \rightarrow \mathbb{R}} \mathbb{R} \left(\sqrt{v(\theta)}\right)$$

where the sum runs over all real embeddings of $F$. Each factor is isomorphic to $\mathbb{C}$ and the complex conjugation on $K$ induces the usual complex conjugation on each factor $\mathbb{C}$. On each factor, the form $\langle -, - \rangle_\alpha$ computed in the $\mathbb{R}$-basis $(1, \sqrt{v(\theta)})$ is
diag(2v(\alpha), -2v(\alpha)v(\theta)) so it has signature (2, 0) if v(\alpha) > 0 and signature (0, 2) if v(\alpha) < 0. The result follows.

5. Construction of isometries of lattices

We want to determine if a given integral even p-elementary lattice \( S \) of rank \( p - 1 \) with fixed signature and discriminant form admits an isometry of order \( p \) whose characteristic polynomial is the cyclotomic polynomial \( \Phi_p \). By the results of Section 4, one first has to find an element \( \alpha \in F \) satisfying conditions (1), (2), (3).

Example 5.1. Assume that \( p = 5 \) and \( S = U \oplus H_5 \) with \( H_5 = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \). The lattice \( S \) is 5-elementary with \( d_S = 5 \), it has signature (2, 2) and discriminant form \( A_S = \mathbb{Z} \oplus \mathbb{Z}(\sqrt{5}) \). In [5, Table 2] this case is denoted by \((p, m, a) = (5, 1, 1)\). In order to recover this lattice as an ideal lattice, since \( O_K \) is a PID we take \( I = \beta O_K \) for some \( \beta \in K \). Equation (1) writes:

\[
N_{K/\mathbb{Q}}(\beta)^2 N_{F/\mathbb{Q}}(\alpha)^2 = \frac{1}{5^2}.
\]

Assuming that \( \beta = 1 \), we run a basic computer search program to determine \( \alpha \in F \) that satisfies all the needed conditions. Taking \( \alpha = \frac{1}{5}(3\mu_5 + 4) \), in the basis \((1, \zeta_5, \zeta_5^2, \zeta_5^3)\) of \( I \) the bilinear form writes

\[
\begin{pmatrix}
2 & 1 & -2 & -2 \\
1 & 2 & 1 & -2 \\
-2 & 1 & 2 & 1 \\
-2 & -2 & 1 & 2
\end{pmatrix}
\]

so condition (2) is satisfied and it is easy to check that this lattice has signature (2, 2) and discriminant form \( \mathbb{Z} \oplus \mathbb{Z}(\sqrt{5}) \). As mentioned above, these invariants characterize the lattice \( U \oplus H_5 \) up to isometry. By construction, the order 5 isometry of this lattice, written in this basis, is the companion matrix of \( \Phi_5 \):

\[
\begin{pmatrix}
0 & 0 & 0 & -1 \\
1 & 0 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & -1
\end{pmatrix}
\]

Example 5.2. Assume that \( p = 13 \) and that \( S = U^{\oplus 2} \oplus E_8 \). The lattice \( S \) is unimodular of signature (2, 10). In [5, Table 5] this case is denoted by \((p, m, a) = (13, 1, 0)\). If \( S \) admits an order 13 isometry it induces an identification \( S = \beta O_K \) for some \( \beta \in K \). Equation (1) writes:

\[
N_{K/\mathbb{Q}}(\beta)^2 N_{F/\mathbb{Q}}(\alpha)^2 = \frac{1}{1311}.
\]

It is clear that this equation has no solution, so this lattice does not admit an isometry whose characteristic polynomial is \( \Phi_{13} \). This answers a question left open in [5, Theorem 7.1]: this case cannot be realized by a non-symplectic automorphism of order 13 on an IHS-\( K^3 \).[2]

We assume now that \( p = 23 \) and we consider the lattice \( S := E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23} \). It is 23-elementary with \( d_S = 23 \), signature (2, 20) and discriminant form \( A_S = \mathbb{Z} \oplus \mathbb{Z}(\sqrt{23}) \). 

Proposition 5.3. The lattice $U \oplus^2 \oplus E_8^\oplus$ admits an isometry of order 23 which acts trivially on the discriminant group $A_S$.

Proof. We apply the strategy developed above. Taking $I = O_K$, equation (11) writes:

$$N_{F/\mathbb{Q}}(\alpha) = \frac{1}{23^{10}}.$$ 

The software MAGMA [?] provides only one solution to this equation:

$$\alpha_0 := \frac{1}{23}(-\mu_{23}^7 + \mu_{23}^6 + 7\mu_{23}^5 - 6\mu_{23}^4 - 14\mu_{23}^3 + 9\mu_{23}^2 + 7\mu_{23} - 2).$$

This element $\alpha_0$ does not satisfy all the needed solutions so we look for an element $\alpha = \alpha_0 \cdot \varepsilon$ with $\varepsilon \in O_F^\times$: this has the same norm but letting $\varepsilon$ vary this will produce lattices with different signature and discriminant form. By the Dirichlet unit theorem, the group of units $O_F^\times$ is the product of the finite cyclic group of roots of unity of $F$ with a free abelian group of rank 10. A computation with the software SAGE [19] shows that $O_F^\times \cong \mathbb{Z}/2 \times \mathbb{Z}^{10}$ (the only roots of unity in $F$ are $\pm 1$) where the free part is generated by the following fundamental units:

$$\begin{align*}
e_1 & := \mu_{23}^2 - 4\mu_{23}^2 + 2 \\
e_2 & := \mu_{23}^8 - 8\mu_{23}^6 + 20\mu_{23}^4 - 16\mu_{23}^2 + 2 \\
e_3 & := \mu_{23}^7 - 7\mu_{23}^5 + 14\mu_{23}^3 - 7\mu_{23} \\
e_4 & := \mu_{23}^2 - 2 \\
e_5 & := \mu_{23}^9 - 9\mu_{23}^7 + 27\mu_{23}^5 - 30\mu_{23}^3 + 9\mu_{23} \\
e_6 & := \mu_{23}^5 - 8\mu_{23}^4 + 20\mu_{23}^3 - 16\mu_{23}^2 + \mu_{23}^2 + 2\mu_{23} - 1 \\
e_7 & := \mu_{23} \\
e_8 & := \mu_{23}^7 + \mu_{23}^5 - 6\mu_{23}^3 - 5\mu_{23}^2 + 10\mu_{23} + 6\mu_{23}^2 - 4\mu_{23} - 1 \\
e_9 & := \mu_{23}^8 + \mu_{23}^7 - 8\mu_{23}^6 - 7\mu_{23}^5 + 20\mu_{23}^4 + 14\mu_{23}^3 - 16\mu_{23}^2 - 7\mu_{23} + 3 \\
e_{10} & := \mu_{23}^7 + \mu_{23}^5 - 8\mu_{23}^4 - 7\mu_{23}^3 + 20\mu_{23}^2 + 14\mu_{23}^1 - 16\mu_{23} - 6\mu_{23}^2 + 3\mu_{23} - 1 
\end{align*}$$

We consider an element $\alpha \in F$ given as follows:

$$\alpha = \alpha_0\varepsilon_0^\nu_1 \cdots \varepsilon_{10}^\nu_{10}$$

for some $\nu_i \in \mathbb{Z}$ with $\varepsilon_0 \in \{-1, 1\}$. By running a basic computer search program we find that the choice $\varepsilon_0 = 1$, $\nu = (2, 1, 2, 2, 0, 1, 1, 2, 1, 0)$ satisfies all the needed conditions: in the basis $(1, \zeta_{23}, \ldots, \zeta_{23}^{22})$ of $I$ the bilinear form is the matrix given in Appendix A. It is easy to check that this lattice has signature $(2, 20)$ and discriminant form $\frac{Z_{122}}{252} \langle 44 \rangle$. As already mentioned, these invariants characterize the lattice $U \oplus^2 \oplus E_8^\oplus \oplus K_{23}$ up to isometry. By construction, the order 23 isometry of this lattice, written in this basis, is the companion matrix of the polynomial $\Phi_{23}$ and a direct computation shows that that this isometry acts trivially on the discriminant group. \qed

Corollary 5.4. The lattice $L := E_8^\oplus \oplus U^\oplus \oplus (-2)$ admits an order 23 isometry whose invariant lattice is isometric to $T := \langle 46 \rangle$ and such that the orthogonal complement of $T$ in $L$ is isometric to $S := E_8^\oplus \oplus U^\oplus \oplus K_{23}$. 
Proof: Denoting $T = \mathbb{Z}t$ with $t^2 = 46$, the discriminant group $A_T$ is generated by $	au := t/46 \in T^*$ with $\tau^2 = 1/46$. We denote by $\sigma \in S^+\tau$ a generator of $A_S$ such that $\sigma^2 = 44/23$. The vector $2\sigma + 4\tau \in (S \oplus T)^*$ is isotropic in $A_{S \oplus T}$ so it defines an even overlattice

$$M := S \oplus T \oplus (2\sigma + 4\tau)\mathbb{Z} \subset (S \oplus T)^*.$$ 

Consider the quotient $H := \frac{M}{S \oplus T} \subset A_{S \oplus T}$. One computes that $H^+ / H$ is generated by the class $23\tau$ with $(23\tau)^2 = 3/2 \in \mathbb{Q}/2\mathbb{Z}$. Since $A_M \cong H^+ / H$ (see [15]) we conclude that $M$ is an even lattice of signature $(3, 20)$ and discriminant form $A_M = \frac{Z}{2\mathbb{Z}} \langle \frac{1}{2} \rangle$. By [14, Theorem 2.2] these invariants characterize $M$ up to isometry, so $M$ is isomorphic to $S$. It follows directly from the construction that $S$ is the orthogonal of $T$ in $M$.

Let $\varphi$ be the isometry of order $23$ on $S$ constructed in Proposition 5.3. Since $\varphi$ acts trivially on $A_S$, the isometry $\varphi \oplus id$ of $S \oplus T$ extends to an isometry on $L$ with the required properties. □

Remark 5.5. In the lattice $L$, denoting by $(e, f)$ a basis on one of the factors isometric to the lattice $U$ and by $\delta$ a generator of the factor isometric to $(-2)$ it is easy to see that an explicit embedding of $T$ in $L$ whose orthogonal is isometric to $S$ is given by $t \mapsto 2e + 12f + \delta$. Applying Nikulin’s results on primitive embeddings and decomposition of lattices (see [3] and references therein) one can see that $T$ admits up to isometry a second embedding in $L$ whose orthogonal complement is isometric to $E_8^{\oplus 2} \oplus U \oplus (-2) \oplus (2) \oplus K_{23}$, an explicit embedding being given by $t \mapsto e + 23f$.

6. An IHS-K3$[2]$ with a Non-Symplectic Automorphism of Order 23

Theorem 6.1. There exists an IHS-K3$[2]$ with a non-symplectic automorphism of order 23. This variety $X$ and its automorphism $f$ have the following properties:

\begin{enumerate}
  \item $\rho(X) = 1$, $\text{NS}(X) \cong \langle 46 \rangle$ and $\text{Trans}(X) \cong E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$;
  \item $T(f) = \text{NS}(X)$ and $S(f) = \text{Trans}(X)$.
\end{enumerate}

Proof. The proof is an application of the surjectivity of the period map and of the global Torelli theorem for IHS manifolds.

Construction of the variety. Let $M_0^0_L$ be a connected component of the moduli space of pairs $(X, \eta)$ where $X$ is an IHS-K3$[2]$ and $\eta: H^2(X, \mathbb{Z}) \to L$ is an isometry. The period domain is

$$\Omega_L := \{\omega \in \mathbb{P}(L\mathcal{C}) \mid \langle \omega, \omega \rangle_L = 0, \langle \omega, \overline{\omega} \rangle_L > 0\}.$$ 

Recall that the period map $P: M_0^0_L \to \Omega_L$ defined by $P((X, \eta)) = \eta(H^{2, 0}(X))$ is surjective [11, Theorem 8.1].

Consider as in Corollary 5.4 the embedding of $T = \langle 46 \rangle$ in the lattice $L$ whose orthogonal complement is $S = E_8^{\oplus 2} \oplus U^{\oplus 2} \oplus K_{23}$, with the isometry $\varphi$ of order 23 acting trivially on $T$. We denote by $\omega$ a generator of the one-dimensional eigenspace of $S_\mathbb{C}$ corresponding to the eigenvalue $\xi := e^{2\pi i / 2}$. Recall that by construction $S$ is identified with the ring of integers $O_K$ of the cyclotomic field $K = \mathbb{Q}(\zeta_{23})$ so that $\omega \in S_\mathbb{C} = K \otimes \mathbb{C}$ with basis $(1, \zeta_{23}, \ldots, \zeta_{24})$. In this basis, the isometry $\varphi$ acts by the companion matrix of the 23rd cyclotomic polynomial and it is easy to check
that up to a multiplicative constant one has
\[ \omega = \sum_{i=0}^{21} \left( \sum_{j=0}^{21-i} \xi_j \right) \zeta_{23}^i. \]

Since \( \varphi(\omega) = \xi \omega \) one has \( \langle \omega, \omega \rangle_L = 0 \). Denoting by \( \text{Tr}_{K'/Q} : K_C \to \mathbb{C} \) the \( \mathbb{C} \)-linear extension of the trace, one has
\[ \langle \omega, \varpi \rangle_L = \langle \omega, \varpi \rangle_S = \text{Tr}_{K'/Q}(\omega \alpha) \]
where \( \alpha \) is given in the proof of Proposition 5.3. An explicit computation shows that \( \text{Tr}_{K'/Q}(\omega \alpha) > 0 \), so \( \omega \in \Omega_L \). By surjectivity of the period map, there exists \( (X, \eta) \in M^0_L \) such that \( \eta(H^{2,0}(X)) = \omega \). Then
\[ \eta(\text{NS}(X)) = \{ \lambda \in L \mid \langle \lambda, \omega \rangle_L = 0 \} \supset T. \]

Let us show that \( \eta(\text{NS}(X)) = T \). For this, we show that there is no element \( \lambda \in S \) with the property that \( \langle \lambda, \omega \rangle_S = 0 \). In the basis \( (1, \zeta_{23}, \ldots, \zeta_{23}^{21}) \), denoting
\[ \Xi := (\zeta_{23}^{21}, \ldots, \zeta_1, 1) \]
and \( J := \begin{pmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 1 \end{pmatrix} \) one has by definition \( \omega = J \Xi \). Denote by \( M \) the matrix of the lattice \( S \) in the basis \( (1, \zeta_{23}, \ldots, \zeta_{23}^{21}) \) (see Appendix A). For any \( \lambda \in S \), since \( S = O_K = \mathbb{Z}[\zeta_{23}] \) the element \( \lambda \) can be identified with a column vector with integer coordinates. Then
\[ \langle \lambda, \omega \rangle_S = \lambda^\top M \omega = \lambda^\top M J \Xi. \]
If \( \lambda^\top M J \Xi = 0 \), since \( \lambda^\top M J \) has integer coordinates and since the coordinates of \( \Xi \) are linearly independent over \( \mathbb{Q} \) it follows that \( \lambda^\top M J = 0 \). But the matrix \( M J \) is invertible, so \( \lambda = 0 \). This proves that \( \text{NS}(X) \cong T \) and in particular \( X \) is projective [11, Theorem 3.11].

Construction of the automorphism. The isometry \( \varphi \) preserves the space \( H^{2,0}(X) = \mathbb{C} \omega \) so it is a Hodge isometry. Denoting by \( q_X \) the Beauville-Bogomolov-Fujiki quadratic form on \( H^2(X, \mathbb{Z}) \), the positive cone \( C_X \) is defined as the connected component of the cone
\[ \{ x \in H^2(X, \mathbb{Z}) \mid q_X(x) > 0 \} \]
that contains the Kähler cone. By Markman [12, Lemma 9.2] the group of monodromy operators of \( H^2(X, \mathbb{Z}) \) is equal to the group of isometries of \( H^2(X, \mathbb{Z}) \) preserving the positive cone \( C_X \). Here the generator of \( \text{NS}(X) \cong T \) is an ample class so it lives in the Kähler cone and since \( \text{NS}(X) \) is invariant by \( \varphi \) the cone \( C_X \) is preserved, so \( \varphi \) is a monodromy operator that leaves invariant a Kähler class. By the Global Torelli Theorem of Markman–Verbitsky [12, Theorem 1.3] there exists an automorphism \( f \) of \( X \) such that \( f^* = \varphi \) on \( H^2(X, \mathbb{Z}) \). Since the natural map \( \text{Aut}(X) \to O(H^2(X, \mathbb{Z})) \) is injective (see for instance [13, Lemma 1.2] and references therein), \( f \) is an order 23 non-symplectic automorphism of \( X \). \( \square \)

Remark 6.2. Since \( \text{NS}(X) = \langle 46 \rangle \), it follows from [12, Theorem 2.2] that \( (X, \eta) \) is the only Hausdorff point in the fiber \( P^{-1}(\omega) \) of the period map so this variety with its automorphism of order 23 is unique, although it belongs to a 20-dimensional family of IHS-K3\(^{[2]}\) polarized by a class of square 46.
Remark 6.3. The same method can be used to produce order 23 automorphisms on deformations of $K3^{[n]}$ with $n \geq 3$, under some arithmetic conditions on $n$. 
Here is the matrix of the bilinear form on the lattice $\mathbb{E}_8^2 \oplus \mathbb{K}_{23}$, written in a basis such that the order 23 isometry of this lattice is the companion matrix of the cyclotomic polynomial $\Phi_{23}$:

\[
\begin{pmatrix}
-2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 \\
3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 \\
0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 \\
3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 \\
0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 & 3 \\
0 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 \\
1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 \\
0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 \\
-1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 & 0 \\
-2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
-2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
-3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
-3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
-2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
-2 & -4 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
-1 & -2 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
1 & 0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -2 & -1 & 0 & 1 & 2 & 0 & 3 & 0 & 3 & -2 & 3 & 0 & 3 & 0 & 2 & 1 & 0 & -1 & -2 & -3 & -3 & -2 & -1 & 0 & 1 & 2 \\
\end{pmatrix}
\]
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ISOMETRIES OF IDEAL LATTICES AND HYPERKÄHLER MANIFOLDS

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