Mathematical model of high-precision reproduction of alternating current

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Abstract
The optimal solution to the problem of metrological assurance in the field of production and operation of the precision measuring instruments of alternating current in Ukraine is to create a high-precision measurement standard. The theoretical principles of creating such measurement standards were outlined. A method of comparing alternating current with equivalent direct current using the precision thermal converter and shunts was selected to construct the measurement standard. Since the reproduction of alternating current unit must be realized by comparison with equivalent direct current, the main sources of measurement uncertainty of this unit is a combination of measurement uncertainty of both a thermoelectric converter and a precision shunt. As the main result of this work, a mathematical model was obtained for the AC reproduction process, taking into account the contribution of precision meters of both the output voltage of the resistance measure and the output signal of the thermal converter.

Keywords: measurement standard; alternating current unit; AC/DC transfer; thermal converter; precision shunt; measurement uncertainty.

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1. Introduction
Precision electrical instruments, including digital ammeters, electricity meters (meters having measuring circuits, both alternating current (AC) and voltage), are widely used in the power industry and must have proven accuracy. A significant amount of electricity produced in Ukraine is annually exported for the billions of Ukrainian hryvnia. Loss of accuracy of electricity meters can result in an annual multimillion-hryvnia loss for the national economy. The reference instruments are used to verify metrological characteristics. Insufficient level of precision of metrological assurance hinders the development of national production of measuring equipment and testing equipment for AC, and also makes it impossible to recognize these products at the world level. The most accurate means of metrological assurance of measuring instruments are the state primary measurement standards. In [1] the problem of the lack of a legally approved measurement standard of alternating current unit was pointed out and the direction of its solution was suggested.

All of the world’s existing measurement standards of AC unit use the following measurement methods: AC/DC comparison method with a precision thermoelectric converter and shunts, method of reproducing an alternating current unit using multifunctional calibrators or precision meters with well-defined and studied metrological characteristics as well as the method of determining the magnitude of the current calculating by Ohm’s law for too small currents [2]. Given the level of national scientific, technical and economic development, it is advisable to use as a guideline the experience of operating the national measurement standards of countries with similar indicators of the overall development of the country, such as Poland, the Russian Federation, Turkey or Hungary. In all of the countries listed above, the method of constructing the high-precision measurement standards is based on the method of thermoelectric AC/DC comparison using thermal converters and shunts [3, 4].

The purpose of the paper is to highlight the results in creating the high-precision measurement standard of AC unit, in particular: to outline the theoretical basis for the operation of the measurement standard, to derive a mathematical model for AC unit reproduction.

2. Theoretical basis for creating a high-precision measurement standard of alternating current unit
Unlike direct current (DC), the amplitude of the alternating signals, that is, the power or voltage of the AC, periodically changes the polarity, and their instantaneous value is not a constant in the interval between the change in polarity. Each instantaneous AC value has a specific amplitude and polarity. DC and AC
are equivalent in the sense of producing an identical average power dissipated on pure active resistance. Average power is dissipated as a heat which is measured in turn by a sensor that has a direct dependence of the output value on temperature, and this output provides a direct voltage signal proportional to the input temperature. The devices of such a kind are called thermoelectric converters.

As the main elements of the created measurement standard should be a precision thermal comparator and a precision shunt, it is necessary to consider the peculiarities of their design and application. The transition from AC to DC based on the use of thermoelectric converters to measure the average power produced by AC on an active resistance for comparison with the value produced by an equivalent DC on the same resistance [5]. The comparison of AC and DC is most often done using a thermoelectric converter which measures the AC and DC voltage drop along the current shunt.

The sensing element of a thermal converter is a thermal element, that is, a short heating conductor with an electrically isolated thermocouple [6]. Furthermore, both elements are placed in a glass evacuated shell. Alternately, DC and AC input signals are applied to the input of the thermal converter, causing the current of a corresponding kind to flow through the heating element, resulting in a temperature rise that is reflected in the thermocouple output signal.

The maximum value of the current measured by the current thermal converter is determined by the cross-section of the heater which is directly connected to the circuit where the measured current flows. The range of the currents that can be measured by the thermal converter is from milliamperes to tens of amperes.

The maximum frequency of the measured current depends on the cross-section of the heater and its length and at the minimum size reaches hundreds of megahertz. The main advantage of the thermoelectric converters based on thermocouples is the independence of such device readout from the shape of the input signal curve, as a consequence, the high accuracy of comparing DC and AC values. The disadvantages include the non-linear dependence of the output signal and the inadmissibility of overloading the input circuit of the thermal converter. Unlike the previous type of thermal converters, root-mean-square (RMS) sensors have almost linear dependence of the output signal on the input signal.

A precision thermal comparator, which is part of the created measurement standard, namely Fluke 792A, contains an RMS sensor in its design. The current through the input circuit of such a thermal comparator should not exceed 6 mA [7]. To extend the range of measured current to the required specifications, a set of precision A40 shunts is used, and the output voltage of A40 shunt must be supplied to the input of the Fluke 792A thermal comparator.

An elemental shunt is a low-ohm resistor and it performs the function of converting the electric current into the voltage [8]. The main disadvantage of the shunt is the lack of galvanic isolation between the circuit in which the measured current flows and the chain of information processing. The shunts are made of materials with high resistivity and low-temperature coefficient (constantan, manganin, nickel, etc.) so that the increase of the shunt temperature during the current flow in it does not cause changes in the output thermo-EMF of the thermocouple. Uncertainty of the measuring shunt depends on the parameters of surrounding air (most of all a temperature), a presence of a reactive component of resistance, a stray capacitance. The coaxial A40 shunts, manufactured by Fluke, are specially designed to reduce shunt inductance to low values [8].

The created measurement standard includes the elements needed to determine the equivalent value of DC, namely, the measure of electrical resistance to the DC and the precision meter of its output voltage. Their influence on the result of the reproduction of the AC unit goes to negligible values compared with the influence of the main elements discussed above, that is, precision shunts and a thermal comparator whose total contribution is an order of magnitude higher.

The measurement cycle typically lasts from three to six minutes and consists of several steps of applying AC and DC in turns [9]. This minimizes the influence of thermoelectric effects in the middle of thermal converters and drifts during measurement. It is necessary to control the value of the output signal of the reference thermal comparator Fluke 792A from the complement of the created measurement standard, which has a negligible drift value compared with a thermal converter that obtains the transmitted size of AC unit. If the unit size is transmitted to a precision meter or current source, this will eliminate the five-minute drift of these measuring instruments.

3. Derivation of mathematical model for alternating current unit reproduction

The AC unit can be disseminated in several ways through measuring instruments (precision AC sources, precision AC meters, precision thermoelectric converters) calibrated using the national measurement standard of AC/DC current transfer. The smallest measurement uncertainty can be achieved with the use of thermal converters. However, the absolute value of the AC unit is not disseminated from the national measurement standard of AC/DC current transfer, but the relative difference between the values of AC and DC is determined. The AC/DC current transfer difference has an expression:

$$\delta_{X}^{AC-DC} = \left| \frac{I_{AC}}{I_{DC}} - 1 \right| E_{AC} = E_{DC}$$  \hspace{1cm} (1)
Mathematical model of high-precision reproduction of alternating current

where \( I_{AC} \) is an average value of AC through an input circuit of a thermal converter;

\( I_{DC} \) is an average value of DC through an input circuit of a thermal converter.

To reproduce the alternating current unit, the circuit shown in Fig. must be implemented.

During the flow of AC \( I_{AC} \) through a precision shunt with an electrical resistance to AC \( Z_{SH} \) (an impedance), a voltage drop \( U_{SH1} \) occurs on this resistance. The alternating current is determined by the expression:

\[
I_{AC} = U_{SH1}/Z_{SH}.
\]

Measuring circuit for alternating current unit reproduction
\( Z_{SH} \) is a precision shunt; \( TC \) is a reference thermal comparator; \( PV1, PV2 \) are the precision direct voltage meters; \( R_m \) is a standard resistance

Since \( U_{SH1} \) is the input signal for the Fluke 792A thermal comparator, bearing in mind the linear dependence of the output signal, one can obtain expression:

\[
I_{AC} = K_{AC} \cdot U_1/Z_{SH},
\]

where \( K_{AC} \) is a characteristic conversion coefficient of the Fluke 792A thermal comparator for AC input voltage; \( U_1 \) is an output DC voltage of the Fluke 792A thermal comparator when AC flows in the current circuit.

The output signal of the Fluke 792A thermal comparator is DC voltage, and it should be measured by a precision DC meter (\( PV1 \)). The correction of the measurement of the Fluke 792A output signals in the case of AC and DC flows can be considered the same since the voltage will not be significantly different.

Depending on the frequency of the input signal, the characteristic coefficient \( K_{AC} \) has a slight difference due to the design feature of the Fluke 792A thermal comparator (there are semiconductors, active and reactive elements that affect the precision of conversion to some extent).

Taking into account the AC/DC transfer difference of the Fluke 792A thermal comparator, expression (3) can be represented as follows:

\[
I_{AC} = K_{DC} \cdot (1+\delta_{TC}) \cdot U_1/Z_{SH},
\]

where \( K_{DC} \) is a characteristic conversion coefficient of the Fluke 792A thermal comparator for DC input voltage; \( \delta_{TC} \) is the AC/DC transfer difference of the Fluke 792A thermal comparator.

In the case of a DC flow \( I_{DC} \) in the scheme of Fig. through a precision shunt with active electrical resistance \( R_{SH} \), a voltage drop \( U_{SH2} \) occurs on this resistance. The direct current is determined by the expression:

\[
I_{DC} = U_{SH2}/R_{SH}.
\]

Since \( U_{SH2} \) is the input signal for the Fluke 792A thermal comparator, bearing in mind the linear dependence of the output signal, one can obtain from the expression (5):

\[
I_{DC} = K_{DC} \cdot U_2/R_{SH},
\]

\( U_2 \) is an output DC voltage of the Fluke 792A thermal comparator when DC flows in the current circuit.

The internationally agreed definition of an error of a transfer from AC to DC, i.e., AC/DC transfer difference, is the relative difference between AC and DC, provided that the output thermo-EMFs are equal.
in both cases. Taking into account expression (1) for determining the current AC/DC transfer difference, the Fluke 792A output signal by expression (6) (taking into account the AC/DC transfer difference of A40 shunt) can be represented as follows:

\[ I_{DC} = K_{DC} \cdot \frac{U_2}{Z_{SH}} \cdot (1 + \delta_{SH}). \] (7)

When DC flows through both a precision shunt and a standard resistance measure in series, the signal in proportion to the current is transmitted to the PV1 and PV2 meter input.

The process of reproducing an alternating current unit can be described by a system of equations:

\[
\begin{align*}
(U_+ + U_-)/R_M &= (U_{dc+} + U_{dc-})/K_{DC} \cdot R_{SH}; \\
I_{AC} &= U_{ac}/K_{AC} \cdot Z_{SH},
\end{align*}
\] (8)

Continuing the transformation of the system (9), given the unequal coefficient K:

\[
\begin{align*}
K_{DC} &= R_M \cdot (U_{dc+} + U_{dc-})/R_{SH} \cdot (U_+ + U_-); \\
I_{AC} &= U_{ac} \cdot (1 + \delta_{TC}) \cdot (1 + \delta_{SH})/K_{DC} \cdot R_{SH}.
\end{align*}
\] (10)

Substituting \( K_{dc} \) to determine the current in the lower member of a system (10), we obtain:

\[ I_{AC} = (1 + \delta_{TC}) \cdot (1 + \delta_{SH}) \cdot U_{ac} \cdot (U_+ + U_-)/R_M \cdot (U_{dc+} + U_{dc-}). \] (11)

From the obtained expression (11), it can be seen that the value of AC is calculated by Ohm’s law based on the values of electrical resistance of the measure and the readout of the meter PV2.

The AC/DC transfer difference of both the reference thermal comparator Fluke 792A and the shunt A40 should also be taken into account when reproducing the AC unit. These AC/DC transfer difference values depend on the ratio between the characteristic coefficients of the Fluke 792A thermal comparator and the ratio between the active and reactive components of the A40 shunt resistance (i.e., the frequency dependence of the output signals). The values of the voltage readout of PV1 indirectly influence on the value of current obtained since they should be the same in equation (1).

4. Conclusion

The main elements of a high-precision measurement standard of alternating current unit are a precision thermal converter, a precision shunt, precision DC voltage meters, a standard measure of electrical resistance. The determination of the magnitude of both the electrical resistance and the electric DC voltage is ensured with great accuracy to date.

Since this cannot be said about alternating current, the main source of the measurement uncertainty in the reproduction of this physical quantity is the AC/DC transfer differences of both precision thermal converter and precision shunt. The AC/DC transfer difference of a precision thermal converter depends on the relationship between the DC voltage conversion coefficient and the AC voltage conversion coefficient depending on a frequency.

The AC/DC transfer difference of a precision shunt depends on the relationship between the active resistance and the impedance depending on a frequency of current in a circuit.

A mathematical model for alternating current reproduction has been derived based on the analysis which allowed to identify the main influencing factors that could distort the measurement results that are:

- the values of AC/DC transfer difference of both precision thermal converter and precision shunt;
- the resistance value of the standard resistance measure;
- the value of the measured voltage at the output terminals of resistance measure;
- the offset between these values of voltage due to the flow of direct current in different directions;
- the difference between the output signals of thermal comparator due to the flow of the current of different polarity;
- the difference between the output signals of thermal comparator depending on the frequency.
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Annotation

The optimal solution to the problem of metrological support in the production and operation of precision alternating current measurement devices involves the creation of a high-precision standard. The theoretical foundations for creating such standards were established. The method of comparison of alternating current with an equivalent constant current using a precision thermoprocessor and shunt was chosen for the construction of the standard. Since the reproduction of the unit of alternating current must be carried out by comparison with an equivalent constant current, the main source of measurement uncertainty is the combination of uncertainty of the thermoelectric converter measurement and precision shunt.

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