Comments on “Quantum theory cannot consistently describe the use of itself”

Liang Chen¹ and Ye-Qi Zhang¹

¹Mathematics and Physics Department, North China Electric Power University, Beijing, 102206, China

Recently, a delicately designed Gedankenexperiment was proposed to check the self-consistence of quantum theory in the description of the agents who are using this theory. It was demonstrated that the quantum theory is inconsistent. Here a critical improvement is presented, which can lead to a consistent explanation of the Gedankenexperiment by using quantum theory.

We first give a brief introduction to the Gedankenexperiment proposed in Ref. [1]. Suppose that F, F, W, and W are four agents in the experiment, which may take many rounds. Each round of the experiment is carried out as follows. There is a random number generator based on the measurement of a quantum system R, whose initial state is \( |\text{init}\rangle_R = \sqrt{1/3}|\text{heads}\rangle_R + \sqrt{2/3}|\text{tails}\rangle_R \). For each round of the experiment, Agent F invokes \( |\text{init}\rangle_R \) to agent F, else if she got \( |\text{tails}\rangle_R \), she sends \( |\text{ok}\rangle \) to agent W. Then agent F measures spin S in the orthogonal basis \( \{|\uparrow\rangle_S, |\downarrow\rangle_S\} \). Next, agent W makes a measurement with respect to a basis containing the vector \( |\text{heads}\rangle_{\text{L}} = \sqrt{1/2} (|\uparrow\rangle_{\text{L}} - |\downarrow\rangle_{\text{L}}) \) on the lab L. L contains the agent F and the quantum system R. Both L and L (introduced in the following context) remain isolated during the experiment unless the protocol explicitly prescribes measurements applied to them. \( |\text{heads}\rangle_{\text{R}} \) and \( |\text{tails}\rangle_{\text{R}} \) are the orthogonal states corresponding to agent F’s measurement results \( |\text{heads}\rangle_{\text{R}} \) and \( |\text{tails}\rangle_{\text{R}} \), respectively. If agent W got the result \( |\text{ok}\rangle \), \( \mathbf{w} = |\text{ok}\rangle \), otherwise, \( \mathbf{w} = |\text{fails}\rangle \).

Then agent W makes an announcement to agent W about his result. Then, agent W makes his own measurement with respect to a basis containing the vector \( |\text{ok}\rangle_{\text{L}} = \sqrt{1/2} (|\downarrow\rangle_{\text{L}} - |\uparrow\rangle_{\text{L}}) \) on the lab L. L contains agent F and spin S at this stage. \( |\downarrow\rangle_{\text{L}} \) and \( |\uparrow\rangle_{\text{L}} \) are two orthogonal vectors corresponding to agent F’s measurement results \( |\downarrow\rangle_{\text{S}} \) and \( |\uparrow\rangle_{\text{S}} \), respectively. If agent W got the result \( |\text{ok}\rangle \), \( w = |\text{ok}\rangle \), otherwise, \( w = |\text{fails}\rangle \). Here ends one round of the experiment.

Analysing the results, one can find that the probability to get the result \( (w, w) = (|\text{ok}\rangle, |\text{ok}\rangle) \) is 1/12 when one use the standard quantum theory to describe L and L. However, if the agents make some logical reasoning based on their own measurement results at hand and three assumptions containing quantum theory, as shown in Ref. [1], agent W will never get the result \( w = |\text{ok}\rangle \) if he knows \( \mathbf{w} = |\text{ok}\rangle \), i.e., the probability to get the result \( (\mathbf{w}, w) = (|\text{ok}\rangle, |\text{ok}\rangle) \) is zero. Hence, the analyzation arrives at a contradiction. The other two assumptions, except quantum theory, seem to be reasonable, so the two authors conclude that quantum theory cannot consistently describe the use of itself.

In this work, we show that there exists a critical improvement of the above logical reasoning. We assume that the three assumptions (Q), (S) and (C) given in Ref. [1] are correct, and use them in the following derivation. The key issue in the following derivation is that, for each round of the Gedankenexperiment, agent W uses quantum theory and actually had already acquired some information of lab L before he did the measurement on it. Hence, W should make conclusion at some time point based on the conclusions made by other agents at the same time point.

Let us first see how quantum theory works in the perspective of W. Suppose that at one round of the experiment, agent W gets the result \( \mathbf{w} = |\text{ok}\rangle \). Agent W can make the statement that ‘I am certain that F knows that \( z = +\frac{1}{2} \) at time n:11’, as shown in Table 3 of Ref. [1]. After receiving the announcement from agent W that \( \mathbf{w} = |\text{ok}\rangle \) at time n:21, agent W can also make the statement that ‘I am certain that F knows that \( z = +\frac{1}{2} \) at time n:11’ by using the same derivation as agent W has used, i.e., using assumptions (Q) and (C). Furthermore, agent W can make the statement that ‘I am certain that \( z = +\frac{1}{2} \) at time n:11’ according to assumption (C). In addition, according to Eq. (6) in Ref. [1], the measurement of W at time n:20 does not change this certainty between time n:11 and n:30. So, at the time before the measurement with respect to a basis containing the vector \( |\text{ok}\rangle_{\text{L}} \), agent W has already got some information of state \( |\text{ok}\rangle_{\text{L}} \), he can confirm that \( |\text{ok}\rangle_{\text{L}} = |\uparrow\rangle_{\text{L}} \). Suppose that at time n:30, agent W did not make the measurement with respect to a basis containing the vector \( |\text{ok}\rangle_{\text{L}} \), he made the measurement in the orthogonal basis \( \{|\uparrow\rangle_{\text{L}}, |\downarrow\rangle_{\text{L}}\} \), he would get the result \( z = +\frac{1}{2} \) definitely according to his information \( \mathbf{w} = |\text{ok}\rangle \) and his knowledges (Q), (C), (S). So, when he makes the measurement with respect to a basis containing the vector \( |\text{ok}\rangle_{\text{L}} \), he has a probability \( |\text{ok}\rangle_{\text{L}} = |\uparrow\rangle_{\text{L}} \) to get the result \( w = |\text{ok}\rangle \).

Now we reanalyse the probability related to the measurement result \( (\mathbf{w}, w) = (|\text{ok}\rangle, |\text{ok}\rangle) \) in the logical reasoning way. Obtaining this measurement result depends on four conditions step by step, \( r = \text{tails} \), \( z = +\frac{1}{2} \), \( w = |\text{ok}\rangle \), and \( w = |\text{ok}\rangle \). Their conditional probabilities are 2/3, 1/2, 1/2 and 1/2, respectively. So we get \( \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{12} \), which is quantitatively consistent with the quantum theory result, Eq. (7) in Ref. [1]. Therefore, the contradiction proposed in Ref. [1] is inexistential.

In order to see how the logical reasoning in Ref. [1] breaks down, we focus on the inference from statements
quantum states after the operations

| initialization of quantum system R | quantum states after the operations |
|-----------------------------------|------------------------------------|
| $\bar{F}$ measures R, sets the spin S | \[
\begin{align*}
\sqrt{\frac{3}{4}}|\text{heads}\rangle_R + \sqrt{\frac{3}{4}}|\text{tails}\rangle_R \\
\sqrt{\frac{3}{4}}|\text{heads}\rangle_S + \sqrt{\frac{3}{4}}|\text{tails}\rangle_S \\
\sqrt{\frac{3}{4}}|\text{heads}\rangle_L + \sqrt{\frac{3}{4}}|\text{tails}\rangle_L \\
\sqrt{\frac{3}{4}}|\text{heads}\rangle + \sqrt{\frac{3}{4}}|\text{tails}\rangle \\
\end{align*}
\] |
| $\bar{F}$ sends spin S to $F$ | \[
\begin{align*}
|\text{heads}\rangle_S + |\text{tails}\rangle_S \\
|\text{heads}\rangle_L + |\text{tails}\rangle_L \\
|\text{heads}\rangle + |\text{tails}\rangle \\
\end{align*}
\] |
| $F$ measures S | \[
\begin{align*}
\sqrt{\frac{3}{4}}|\text{heads}\rangle_L + \sqrt{\frac{3}{4}}|\text{tails}\rangle_L \\
\frac{1}{\sqrt{2}}|\text{heads}\rangle + \frac{1}{\sqrt{2}}|\text{tails}\rangle \\
\frac{1}{\sqrt{2}}|\text{heads}\rangle + \frac{1}{\sqrt{2}}|\text{tails}\rangle \\
\end{align*}
\] |
| $\bar{W}$ measures lab $L$ | \[
\begin{align*}
\sqrt{\frac{3}{4}}|\text{heads}\rangle_L + \sqrt{\frac{3}{4}}|\text{tails}\rangle_L \\
\frac{1}{\sqrt{2}}|\text{heads}\rangle + \frac{1}{\sqrt{2}}|\text{tails}\rangle \\
\end{align*}
\] |
| $W$ measures lab $L$ | \[
\begin{align*}
\frac{1}{\sqrt{2}}|\text{heads}\rangle + \frac{1}{\sqrt{2}}|\text{tails}\rangle \\
\end{align*}
\] |

$F^{n:00}$ and $F^{n:01}$ to $F^{n:02}$, where the conclusion $w = \text{fails}$ appears for the first time (see the discussion below Eq. (4) and Table (3) in Ref. [1]). This inference is conditional, and depends on the fact that the quantum state $\sqrt{1/2} (|\text{heads}\rangle_L + |\text{tails}\rangle_L)$ cannot be changed before agent $W$'s measurement on lab $L$ at time $n:30$. However, the measurement of agent $\bar{W}$ on lab $L$ at time $n:20$ breaks this condition. This is because labs $L$ and $L$ are quantum entangled to each other by the spin $S$ sent from agent $F$ to agent $F$, any measurement on lab $L$ will induce a collapse of both quantum states $|\text{heads}\rangle_L$ and $|\text{tails}\rangle_L$ according to quantum theory. For completeness, in Table I, we list the time evaluation of quantum states for each step of the experimental procedure given in Box 1 of Ref. [1]; these results are calculated by $W$ using the standard quantum theory.

Based on the above discussion, our logical reasoning can be expressed explicitly as an Improved Assumption (C):

**Improved Assumption (C)**

Suppose that agent $A$ has established that:

Statement $A^{(i)}$: "I am certain that agent $A'$, at the same time point as I make deduction, upon reasoning within the same theory as the one I am using, is certain that $x = \xi$ at time $t$.

Then agent $A$ can conclude that:

Statement $A^{(ii)}$: "I am certain that $x = \xi$ at time $t$.''

In order to see more clearly that this Improved Assumption (C) is consistence with Assumptions (Q) and (S), we would like to give some illustration below. The flow chart of the whole process is plotted in Picture 1. We introduce here notations like $\mathcal{F}(t_2)F(t_2)\bar{W}(t_3)W(t_3)$ to represent a deduction by $W$ at $t_3$ through the conclusion by $\bar{W}$ at $t_3$, through the conclusion by $\mathcal{F}$ at $t_2$, through the conclusion by $\mathcal{F}$ at $t_2$. We will see it is crucial and reasonable to take account of these time points when someone deduces through another agent’s conclusion by $\mathcal{F}$ (t_2)F(t_2)\bar{W}(t_3)W(t_3)$.

In the original paper, when $w = \text{ok}$, the deduction $\mathcal{F}(t_2)F(t_2)\bar{W}(t_3)W(t_3)$ made by agent $W$ at $t_3$ is "At $t_3$, I am certain that, $\bar{W}$ is certain at $t_3$ that, $F$ is certain at $t_2$ that, $\mathcal{F}$ is certain at $t_1$ that $R = \text{tails}$ and $w = \text{ok}$ will not happen". Actually, we would like to point out this is one deduction of 9 total deductions $W$ could make at $t_3$, as shown in Table II. We have shown, see Table I, $W$’s deduction $\mathcal{F}(t_3)F(t_3)\bar{W}(t_3)W(t_3)$ made at $t_3$ is "at $t_3$, I am certain that, $\bar{W}$ is certain at $t_3$ that, $F$ is certain at $t_2$ that, $\mathcal{F}$ is certain at $t_1$ that $R = \text{tails}$ and $w = \text{ok}$ will happen with some probability”.

At first glance, the two deductions made by agent $W$ at $t_3$ are contradict to each other. However, they are not, for it is natural for someone ($\bar{W}$) to think another agent ($F$) has different opinions on an event at different time points. Contradiction will occur when $W$ use the original assumption (C). The conclusion by agent $W$ at $t_3$ (w = ok will not happen) deduced from $\mathcal{F}(t_3)F(t_3)\bar{W}(t_3)W(t_3)$ is in contradict with that (w = ok will happen with some probability) deduced from $\mathcal{F}(t_3)F(t_3)\bar{W}(t_3)W(t_3)$.

**TABLE II. Pathways of W’s deduction through $\mathcal{W}$, $F$, and $\mathcal{F}$, there is 9 different combinations of time points.**

| subject | pathway | $W$ | $F$ | $\mathcal{F}$ |
|---------|---------|-----|-----|-------------|
| $W(t_3)$ | $t_1$ | $t_2$ | $t_3$ |

**TABLE III. Opinions of the entanglement between $L$ and $L$ to different quantum theory users.**

| time | quantum users | $\bar{F}$ | $F$ | $\mathcal{F}$ |
|-----|--------------|-----|-----|-------------|
| $t_3$ | $N$ | $Y$ | $Y$ |
| $t_2$ | $N$ | $N$ | $Y$ |
| $t_1$ | $N$ | $N$ | $N$ |
show that it should deduced logically from our Improved Assumption \((C)\) (same time point deduction) to be in consistent with nowadays quantum theory, in which the quantum theory used by someone does not involve oneself, but involves someone else.

Returning to the Gedankenexperiment, as shown in Table \(\text{III}\), the opinions of whether entanglement between labs \(L\) and \(\overline{L}\), and whether the measurement on lab \(L\) will influence lab \(L\), are different for agents \(F\), \(\overline{F}\), and \(W\). But these opinions are subjective and consistence to the users who use nowadays quantum theory themselves. It should be noted that \(W\) is the subject who make the deduction to predict the probability of event \(w = \text{ok}\). And meanwhile \(W\) is the user who uses nowadays quantum theory in the deduction, hence he will treat his world except himself as quantum systems. That is to say, agent \(W\) think \(F\), \(\overline{F}\), and \(\overline{W}\) are all quantum and can be influenced by measurement within quantum theory. The same reasoning holds for agent \(\overline{W}\) when he deals with \(F\) and \(\overline{F}\), and so on. Specifically, to agent \(\overline{W}\)’s opinion, the states of \(F\) are changed by measurements, in other words, if \(\overline{W}\) accept standard quantum theory, he must deduce a conclusion at \(t_3\) based on the conclusion made by \(F\) at the same time \(t_3\), i.e., based on the Improved Assumption \((C)\).

The argument indicates that, with our Improved Assumption \((C)\), quantum theory can still consistently describe the use of itself.

\textbf{Acknowledgements—}This work was supported by NSFC under Grant Nos. 11504106, 11805065, 11247308, and 11447167, the Fundamental Research Funds for the Central Universities under Grant Nos. 2018MS049 and 2018MS056.

[1] Frauchiger, D. & Renner, R. Quantum theory cannot consistently describe the use of itself, \textit{Nat. Commun.} \textbf{9}, 3711 (2018).