Boosting Sharing Economy: Social Welfare or Revenue Driven?

Zhixuan Fang
IIIS, Tsinghua University, Beijing, China
fzx13@mails.tsinghua.edu.cn

Longbo Huang
IIIS, Tsinghua University, Beijing, China
longbohuang@tsinghua.edu.cn

ABSTRACT
Product sharing over online platforms, or sharing economy, has become a growing trend and has the potential to mitigate many social problems such as wasted products, idle resources, road congestions, and even greenhouse gas emissions. Despite its quick and successful development so far, there has been a lack of clear understanding about who is a better candidate for boosting sharing economy: government-like organizations who care about social welfare, or profit-driven entities who mainly focus on revenue?

To investigate this problem, we propose a game-theoretic model among users participating in a sharing platform and analyze the social welfare under different platform pricing strategies. Specifically, we derive tight bounds of social welfare loss of the revenue maximization policy compared to social welfare maximization, and show that revenue maximization leads to more sharing supply and ensures a better quality-of-service to users. We further conduct case studies and show the advantage of sharing over a common platform compared to separated sharing groups. Our numerical results based on real data also show that, when the sharing demand/supply ratio is large, the revenue maximizing policy also achieves optimal social welfare.

To the best of our knowledge, we are the first to study social welfare with general user utility functions in sharing economy, and to compare the performance of different pricing policies.

1. INTRODUCTION
Along with the rapid recent developments of internet and mobile technologies, sharing economy [1] (also known as collaborative consumption) has emerged as an efficient way to utilize available resources. For example, Airbnb [30] for sharing rooms, Uber [9] and Lyft [2] for car sharing, and many bike sharing systems around the world served as part of city public transportation [29]. In fact, sharing economy attracts great amount of capital, which supports dozens of startups to chase the trend, as the revenue in the industry may reach $3.5 billion this year according to Forbes [13]. Meanwhile, governments and many other non-profit organizations are eager to help develop sharing economy since it can “address social and economic challenges” [18], quoted from the British government’s response to the sharing economy.

Among many factors that facilitates the great success of sharing economy, a very important one is that it enables efficient usage of many resources that are essentially wasted: According to [27], the average time in use for cars in US and Europe is 8% and the average used time of an electric drill over its lifetime is 6-13 minutes by [6]. Data from European Environment Agency [11] also shows that the average number of passengers per car is 1.58 in UK and 1.42 in Germany. One can enumerate many other kinds of resources remain idle in daily life: empty rooms, idle computing resources, etc. Due to the large quantity of these under-utilized resources, even a small improvement in utilization can result in great societal benefits. Indeed, UK parliament [29] estimates that increasing the average utilization of cars to 2 passengers per car would save 17 billion tonnes of CO2. This is exactly the great potential of sharing economy.

Due to its importance, there have been many works discussing the definition, motivation, and development of sharing economy, e.g., [6], [17]. There are also concrete case studies, e.g., [23] on car sharing, phone minute sharing and bike sharing, and [30] on the impact of Airbnb on the hotel industry. Different from these prior works that mainly focus on qualitative assessments, in this paper, we try to provide an analytical framework for modeling sharing economy and for understanding how it can be effectively boosted. Specifically, we consider a general product-sharing system with a set of product owners and renters. The sharing platform regulates sharing supply and demand by choosing an appropriate market price. Product owners and renters then decide on their own whether to share or rent, and determine their usage level. Product owners receive monetary payments sharing their products, but spend maintenance costs, and all users obtain utility from using the products. Our goal is to understand the overall social welfare of the system and the quality-of-service (QoS) in sharing under two most common operation modes: social-welfare maximization, e.g., run by non-profit organizations, or revenue maximization, e.g., run by companies like Uber. This general framework captures various important aspects of sharing economy including user incentive, system efficiency, and QoS, a metrics that reflects both users’ satisfaction and resource utilization.

Most prior works on sharing have focused on peer-to-peer (P2P) networks. For instance, [24] discusses the social wel-
Among the thriving sharing industries, we take two common cases as examples. The first one is car sharing. Companies like Uber, Zipcar, RelayRides provide car sharing services. Zipcar provides their own cars for sharing, while the other two match car owners and renters via online platform. In the case of Uber, car owners put their cars online for share when they want to, while renters applied for cars through Uber’s platform, and rents are charged by distances or duration time of use. The other example is house sharing: hosts put their available rooms online and guests can book them via online platform, e.g., Airbnb.

The same pattern of these online sharing is that, product owner shares his products by putting some portion of them (part of the product or part of the usage time) available online, and he can use what is left of the product by himself. Owner’s problem is to decide how much amount of product to put online for share, e.g., a driver may share 100% usage time of his car when the renting price is really high, or 0% if the profit is lower than his cost. He earns income by doing so, but also pays a maintenance cost, and can face competitions in getting renters due to other owners’ sharing actions. As for renters, they pay as they rent the products.

3. MODEL

3.1 Owners and Renters

Consider a product sharing system that consists of two groups of users: product owners denoted by the set $\mathcal{O}$, and renters denoted by $\mathcal{R}$, with $N_O = |\mathcal{O}|, N_R = |\mathcal{R}|$ denoting the numbers of owners and renters.

For each user $i \in \mathcal{O} \cup \mathcal{R}$, we normalize his maximum usage of the product to be 1, e.g., product usage frequency. Then, we use $u_i \in [0, 1]$ to denote one’s usage of the product, e.g., fraction of time one uses his own/rented car. If user $i$ is a product owner, we also use $s_i \in [0, 1]$ to denote the level at which he shares the product, e.g., drives it as a Uber car. Note that we always have $u_i + s_i \in [0, 1]$.

Note that with $u_i, s_i \in [0, 1]$, our model can be viewed as considering the average usage over a period of time. This is different from using a realtime supply and demand model, and facilitates the quantification of long-term sharing behavior of users in the market.

3.2 Sharing Demand and Supply

We assume that sharing takes place over a platform, e.g., Uber. In this platform, renters pay the market renting price for using cars from owners. The market price $P$ is set by the sharing platform. Depending on their own usage preferences, desire to share products, and the renting price, users choose their own $u_i$ and $s_i$ ($s_i$ for product owners only) values, which result in different demand and supply conditions in the system.

We denote the total sharing supply $S(P)$ and demand $D(P)$ (renting demand) of the market under price $P$ as the sum of owners’ sharing levels and the sum of renters’ usage levels, respectively. That is,

$$S(P) = \sum_{i \in \mathcal{O}} s_i(P), \quad D(P) = \sum_{i \in \mathcal{R}} u_i(P).$$

3.3 User Utility

For each user $i \in \mathcal{O} \cup \mathcal{R}$, we denote $U_i(u_i)$ his self-use utility of using the product, e.g., personal satisfaction. The
The expected delay for renters is equal to \(1\), the "effective price" \(P\) (after the sharing price is set). As for renters, they have to pay the renting price \(P\) per unit usage, e.g., gasoline, tolls, or housekeeping. The term \(\min \{D(P)/S(P), 1\}\) is introduced to captured the fact that in practice, it is typical that a certain percentage of owner's sharing levels will not generate any income, especially when the total demand is less than the supply. For instance, the time a Uber driver spends driving around waiting for customer's order, or the time an Airbnb room provider spends keeping his apartment empty. In other word, \(\min \{D(P)/S(P), 1\}\) can be treated as the probability that an owner meets a customer. Interestingly, the term \(\min \{D(P)/S(P), 1\}\) in (2) is a also nature indicator of quality-of-service (QoS) in sharing: a lower \(D(P)/S(P)\) value implies a higher service quality to customers, as it reflects a lower difficulty level for renters to rent a product. Indeed, if we intuitively imagine that the system operates according to an \(M/M/1\) queueing model, with \(D(P)\) being the arrival rate (demand) and \(S(P)\) being the service rate (supply), then the expected delay for renters is equal to \(1/(S(P) - D(P))\). As a result, a lower \(D(P)/S(P)\) value indicates a smaller service delay. That is to say, the item \(P \min \{D(P)/S(P), 1\}\) can also be seen as the QoS-weighted market price, i.e., when the service quality is low (\(D(P)/S(P)\) is close to 1), the "effective price" \(P \min \{D(P)/S(P), 1\}\) seen by owners becomes relatively high, and this attracts more supplies. Note that the denominator of \(S(P)\) includes every owner's contribution in sharing. Due to this coupling among sharing owners, the problem becomes indeed a game among users (after the sharing price \(P\) is set). As for renters, they have to pay the renting price \(P\) per unit usage for using the product. Hence, the overall utility a renter obtains with product usage \(u_i\) is given by

\[
W_i(u_i) = U_i(u_i) - u_i P. \tag{3}
\]

The fraction of time the server is busy in the \(M/M/1\) queue is exactly \(D(P)/S(P)\). Therefore renter \(i\) will choose his demand independently according to his own overall utility function. When total demand exceeds total supply, some renters may not be able to get access to products and thus obtain zero utility. There are three reasons that we use independent demand decision instead of putting a similar "discount factor" such as \(\min \{S(P)/D(P), 1\}\) in renter's utility functions to capture the fraction of time a renter actually gets served:

- **Renters’ independence.** Product owner can choose their self-use amount and sharing amount based on the expected payoff when he put his product online for share, while renters in real world usually apply for shared product on demand, which is independently decided, similar independent setting can be found in \([8]\).
- **Short term behavior.** Product owner can be seen as long term participators in the sharing platform who value long term expected payoff. By contrast, renters can be seen as “one shot” comers who only care about whether they can get access to the product “this time”, e.g., one may immediately try taxi or subway if he can not get served through Uber, therefore setting his utility gained from sharing platform to be zero is more meaningful to be a long term average.

- **most importantly, using \([8]\) does not lose any generality, as we will prove that in the cases of interests, i.e., revenue maximization and social welfare maximization, supply always exceeds demand. That is, even we add a factor \(\min \{S(P)/D(P), 1\}\) in renters' utility function, we will still have \(\min \{S(P)/D(P), 1\} = 1\) in the two cases we discuss in this paper.

\[3.4 \quad \text{User’s Response and Platform’s Policy}\]

Given the utility functions (3) and (2), under a market price \(P\), each owner chooses his usage and amount of share by maximizing his utility, i.e., choosing \(u_i^*(P)\) and \(s_i^*(P)\) by solving the following optimization problem:

\[
\max_{u_i, s_i} \quad W_i(u_i, s_i) \tag{4}
\]

\[
s.t. \quad u_i \geq 0, \quad s_i \geq 0, \quad s_i + u_i \leq 1
\]

and each renter choose \(u_i^*(P)\) similarly by maximizing his utility (5).

\[3.5 \quad \text{System Objective}\]

Under a given market price \(P\), we say that the market reaches a Nash equilibrium when each owner is executing his best response action, i.e., the solution of problem (4). The platform’s objective is to find an optimal market price to maximize certain given objective function when the market is at a Nash equilibrium (if it exists). For example, a maximum social welfare policy maximizes user’s social welfare at the Nash equilibrium.

In this paper, we focus on two important objectives that are commonly studied, revenue maximization and social welfare maximization. They represent two dominant modes under which sharing economy can be operated, i.e., government or non-profit organization driven, and revenue-driven. We are interested in understanding how the system behaves under these two objectives.

\[3.6 \quad \text{Remarks}\]
A few remarks are in place for our model. First of all, the setting of fixed identities for owners and renters is based on the assumption that users’ identities do not change frequently. Second, only assuming concavity of the utility functions makes our framework and results very general. In fact, most of the main results in this paper, e.g., Theorem 3 are derived based only on this assumption and are independent of the form and parameters of each individual’s utility function.

4. SHARING ANALYSIS

In this section, we present our results. To facilitate reading, all detailed proofs are presented in the appendices and additional technical report [4].

4.1 Market Equilibrium

We first establish the existence of market equilibrium under any fixed sharing price. This result is critical, as the existence allows subsequent studies about the market performance.

**Theorem 1.** Nash equilibrium exists among all users in \( \mathcal{O} \) for any given market price \( P \).

The theorem is proved by showing that each owner’s utility function is continuous in everyone’s strategy and is concave in his own strategy. Note that according to [3], renters decide their levels of demand to the platform independently, considering only their own utilities and the market price. That is, demand request is fixed given price \( P \), and the overall payoff is \( P \) if they get served. However, it can happen that under certain equilibria (determined by \( P \)), the resulting total supply is less than the total demand, i.e., \( S(P) < D(P) \). In this case the actual utility a renter gets in practice is different from that determined by [3], as he is not even served. Fortunately, in two cases of interest in this paper, we show that this will never happen, i.e., for both revenue and social welfare maximization, we have \( S(P) \geq D(P) \) at the equilibria.

4.2 Social Welfare and Revenue Maximization

In this section, we look at the aggregate social welfare under two different market price mechanisms that correspond to two types of platform providers, i.e., government-like organizations who care about maximizing total social welfare, and companies who are after maximum revenue. Our goal is to understand the differences of user behavior and system performance under these two modes.

We start with **social welfare maximization**. To this end, we define the aggregate social welfare as follows:

\[
\tilde{W}(P) = \sum_{i \in \mathcal{O}} W_i(u_i^*, s_i^*) + \sum_{i \in \mathcal{R}'} W(u_i^*)
\]

Here \( \mathcal{R}' \) denotes the set of renters who are actually served (when \( D(P) > S(P) \), some renters may not be able to meet an owner). Notice here that the welfare is defined over all possible uniform price policies. We adopt this definition to avoid unfair comparisons with welfare maximization policies with non-uniform prices, i.e., different charges can be set to different users in the system, e.g., [23]. Maximum social welfare policy is to maximize \( \tilde{W}(P) \) by choosing optimal \( P \).

The **revenue maximization** case is to maximize the volume of trade defined as follows:

\[
V(P) = P \min \{ D(P), S(P) \}.
\]

The adoption of this objective is motivated by the fact that in many sharing systems, the platform obtains a transaction fee from each successful business event, e.g., Uber charges its drivers 20% commission fee [19]. Thus, maximizing the trading volume is equivalent to maximizing the company revenue.

We first present a few propositions to describe some basic features of demand and supply in the market. They also serve as preliminaries for later theorems.

**Proposition 1.** The total demand \( D(P) \) is non-increasing in the market renting price \( P \) and total demand eventually becomes 0 when \( P \) is high enough.

Proposition 1 is intuitive, since renters’ decisions are independent and are guided by objective [3]. Thus, as the price increases, each renter will cut down his demand.

**Proposition 2.** There exists some \( 0 < P_c \leq P_{upper} \), such that (i) \( S(P) \) is non-decreasing when \( P \leq P_c \), (ii) \( S(P) \geq D(P) \), when \( P \in [P_c, P_{upper}] \), and (iii) \( S(P) = 0 \), when \( P \geq P_{upper} \).

Some intuition about \( P_{upper} \) and \( P_c \) in Proposition 2 is in place. First, if we increase \( P \) from zero, supply will continue to increase as long as supply is less than demand. However, as supply increases, it eventually exceeds demand and competition among owners becomes more and more intense. Then when the price is higher than some threshold price \( P_{upper} \), resulting in \( S(P_{upper}) = c \), no owner will share any more since the profit can not even cover the cost. Second, the price \( P_c \) is the lowest market clearing price: \( P_c = \min \{ P | D(P) = S(P) \} \). However, establishing its existence requires showing the continuity of \( S(P) \) at the demand-supply intersection point and is nontrivial.

From Proposition 2 we see that when the market renting price is low such that \( D(P) > S(P) \), some renters may not be able to rent a product, complicating the analysis of the aggregate social welfare. Fortunately, we prove in the following proposition that both the social welfare maximizer and revenue maximizer guarantee that the supply exceeds demand, i.e., \( S(P) \geq D(P) \).

**Proposition 3.** Let \( P_{sw} \) denote the market price which maximizes social welfare [3] and \( P_r \) denote the market price which maximizes revenue [6]. We have \( S(P_{sw}) \geq D(P_{sw}) \) and \( S(P_{sw}) \geq D(P_{sw}) \).

Proposition 3 guarantees that renters have enough supply at \( P_r \) and \( P_{sw} \). Therefore, all renters will get served. Hence, \( \mathcal{R}' = \mathcal{R} \) in [3], and that \( V(P) = P \cdot D(P) \) at \( P_r \) and \( P_{sw} \).

After the above propositions, we now have our first main theorem.

**Theorem 2.** We have following results with respect to \( P_{sw} \) and \( P_r \):

- **lowest market clearing price** \( P' \) maximizes aggregate social welfare, i.e., \( P_{sw} = P' \), and...
- Maximum revenue price is higher than maximum social welfare price: \( P_r \geq P_{sw} \).
- The total sharing supply under revenue maximization is no less than that under social welfare maximization, i.e., \( S(P_r) \geq S(P_{sw}) \), where \( P_r \) and \( P_{sw} \) are the revenue maximizing price and the welfare maximizing price, respectively.

It is interesting to note that revenue maximization oriented sharing actually encourages more supply from owners. The reason is that the pursuit of trading volume is consistent with owners’ interests. Thus, this objective increases the form of trading volume, therefore higher trading volume magnifies “effective price”, which stimulates supply. Another interesting point is that a higher sharing amount does not imply a higher social welfare. Take car sharing as an example, a higher sharing amount at higher price may lead to more empty running cars and result in lower utility for everyone.

Despite different prices and supply under these two cases, the following theorem bounds the social welfare loss under the maximum revenue policy.

**Theorem 3.** The social welfare gap between the maximum social welfare policy and a maximum revenue policy is bounded by:

\[
0 \leq \bar{W}(P_{sw}) - \bar{W}(P_r) \leq P_r [D(P_{sw}) - D(P_r)] \frac{S(P_{sw})}{S(P_r)}.
\]

Moreover, the bounds are tight.

Here \( \bar{W}(P_r) \) denotes the aggregate social welfare under the revenue-maximizing price \( P_r \). Theorem 3 states that maximum revenue policies are “good enough,” in the sense that their social welfare losses are bounded. One important feature of the gap is that it can be easily evaluated by third-party organizations who are interested, e.g., government, without the necessity to know user’s private utility functions, i.e., it only depends on the total demand and supply under prices \( P_r \) and \( P_{sw} \). The tightness result will be demonstrated by examples in section 6.3.

Our next main result is an immediate corollary of Theorem 3. We decide to make it a theorem due to its importance.

**Theorem 4.** The QoS at \( P_r \) is higher than the QoS at \( P_{sw} \), i.e., \( \frac{D(P_{sw})}{S(P_{sw})} \geq \frac{D(P_r)}{S(P_r)} \).

Theorem 4 shows an interesting result that, in addition to providing a bounded loss in social welfare, the revenue maximizing approach also guarantees a better QoS for customers.

### 4.3 Robustness

In this subsection, we investigate the robustness of our results to the heterogeneity of users. Specifically, we assume that each owner \( i \) may have different costs for sharing and personal usage, denoted by \( c_{i,s} \) and \( c_{i,u} \), respectively, and that users can have different preferences on sharing, e.g., the effective price seen by an altruist maybe \( P_r (\frac{D(P)}{S(P)}) + \epsilon_i, \epsilon_i > 0 \), which is always higher than \( P_r (\frac{D(P)}{S(P)}) \). Similarly, \( \epsilon_i < 0 \)

![Figure 1: Since \( P_r \) will only be chosen in the interval where supply is no less than demand, under the setting in Section 6.3, setting price at \( P_r (\epsilon = 0.5) \) can obtain more revenue than \( P_r (\epsilon = 0) \).](image)

Figure 1: Since \( P_r \) will only be chosen in the interval where supply is no less than demand, under the setting in Section 6.3, setting price at \( P_r (\epsilon = 0.5) \) can obtain more revenue than \( P_r (\epsilon = 0) \),

means that a user is not very willing to share. Therefore, an owner’s utility function becomes:

\[
W_i(u_i, s_i) = U_i(u_i) + P (\min \{\frac{D(P)}{S(P)} , 1\} + \epsilon_i)s_i - c_{i,u}u_i - c_{i,s}s_i.
\]

In this case, we have the following result indicating model’s robustness:

**Theorem 5.** Under the owner’s utility of (7), though \( P_r \), \( P_{sw} \) and supply may be different, all theorems above still hold.

Theorem 5 shows that our results indeed hold for a more general class of scenarios, where (i) owners can have different usage costs (captured by \( c_{i,u} \)), due to product diversity, e.g., SUVs may cost more gasoline than sedans, (ii) owners can have different endurance levels to the wear and tear costs due to renter’s use, or the moral hazard because of renter’s potential abusive use (captured by \( c_{i,s} \)), as considered in [20] and [7], (iii) owners have different preferences towards sharing (captured by \( \epsilon_i \)), i.e., being revenue-driven or altruistic.

A positive \( \epsilon \) for product owners can help lower \( P_r \) since supply altruists provide more supply. Note that this is crucial for platform to obtain higher revenue, as shown in Fig. 1 (with 200 renter and 120 owners, see 6.3 for detail settings), since \( P_r \) will only be chosen in the interval where supply is no less than demand, \( P_r (\epsilon = 0) = P_r (\epsilon = 0) = 13 \) and \( P_r (\epsilon = 0.5) = P_r (\epsilon = 0) = 12 \), here \( P_r \) is the market clearing price. That is, according to the revenue curve in this example, a positive \( \epsilon \) can increase supply and help platform gain more revenue.

### 5. Case Study

In this section, we present two concrete cases to demonstrate our results. Specifically, we consider the linear usage utility and quadratic usage utility. These two scenarios are commonly adopted in social welfare analysis, see [7] [16]. Though linear function is not strictly concave, we find that our results also fit well here. Moreover, we show that in these two cases, the bound in Theorem 3 is tight.

Theorem 3 shows that revenue maximization oriented sharing actually encourages more supply from owners. The reason is that the pursuit of trading volume is consistent with owners’ interests. Thus, this objective increases the form of trading volume, therefore higher trading volume magnifies “effective price”, which stimulates supply. Another interesting point is that a higher sharing amount does not imply a higher social welfare. Take car sharing as an example, a higher sharing amount at higher price may lead to more empty running cars and result in lower utility for everyone.

**CASE STUDY**

In this section, we present two concrete cases to demonstrate our results. Specifically, we consider the linear usage utility and quadratic usage utility. These two scenarios are commonly adopted in social welfare analysis, see [7] [16]. Though linear function is not strictly concave, we find that our results also fit well here. Moreover, we show that in these two cases, the bound in Theorem 3 is tight.
5.1 Linear Usage Payoff

Consider the case when each user’s payoff is linear, where \( \alpha_i \) is the private value of user \( i \).

\[
U_i(u_i) = \alpha_i u_i, \quad \forall i \in \Omega, R
\] (8)

In this case, owners’ best response is choosing either to share his car all the time (\( s_i = 1 \)) or to use his car all the time (\( s_i = 0 \)), depending on whether his private value \( \alpha_i \) is lower or higher than the market price. Similarly, renter \( i \) will choose to request either full use of a car (\( u_i = 1 \)) or zero demand (\( u_i = 0 \)), depending on whether his private value \( \alpha_i \) is higher or lower than the market price.

Without loss of generality, suppose no users have the same private value. We label all users, including owners and renters, ranked and labeled by their private value:

\[
\alpha_1 > \alpha_2 > ... > \alpha_{N_R + N_O}
\] (9)

5.1.1 Algorithm of Allocation

We now give a concrete maximum social welfare policy.

**Proposition 4.** The maximum social welfare price is set such that users (including owners and renters) of the first \( N_O \) highest private value \( \alpha_i \) get full use of the product (\( u_i = 1 \)).

Note that this is the best possible arrangement for maximizing social welfare, since all users with higher private values get access to products. In this case, demand equals supply and none of the products are wasted, e.g., no cars are running empty.

Below we also check the performance of the bound we give in Theorem 5. Specifically, we show that the bound given in Theorem 5 is tight between maximum social welfare and maximum revenue policies under linear usage payoff.

We consider the following concrete example in car sharing. Suppose \( N_R = N_O = 3 \). Each owner’s and renter’s private value is shown in Table 1.

Table 1: Example 1 of social welfare gap bound’s tightness

| Ranking of private value | Owners | Renters |
|--------------------------|--------|---------|
| 1                        | 5      | 4 + \( \epsilon \) > 4 |
| 2                        | 2      | 1.5     |
| 3                        | 1      | 0.5     |

From Proposition 4 let \( c = 0.01 \), the maximum social welfare price is \( P_{sw} \in (1, 5, 2) \), letting renter \( r_1 \) use owner \( o_1 \)’s car and owners \( o_1, o_2 \) use their own cars. The trade volume is \( 1 \cdot P_{sw} \in (1, 5, 2) \) and the social welfare is \( 11 + \epsilon - 3c \). Meanwhile, the maximum revenue price \( P_r \) will be \( 4 + \epsilon \) such that only renter \( r_1 \) can rent a car and owners \( o_2, o_3 \) share their cars all the time. The trade volume is \( 4 + \epsilon \) and the social welfare is \( 9 + \epsilon - 3c \).

Thus, the social welfare gap between these two policies is \( W(P_{sw}) - W(P_r) = 2 \), and the bound given by Theorem 5 is \( W(P_{sw}) - W(P_r) \leq P_r [D(P_{sw}) - D(P_r) \frac{S(P_{sw})}{S(P_r)}] = 2 + \frac{\epsilon}{2} \). Since \( \epsilon \) is infinitely small, the bound is tight.

5.1.2 Understanding the Social Welfare Loss Bound

We further interpret the welfare loss bound in the linear utility case. First, we rewrite the welfare loss as follows.

\[
\tilde{W}(P_{sw}) = P_r [D(P_{sw}) - D(P_r) \frac{S(P_{sw})}{S(P_r)}]
\]

The first item of \( (10) \) is an upper bound for the utility loss of renters, since \( D(P_{sw}) - D(P_r) \) is exactly the number of renters who request for a full use of cars at \( P_{sw} \) but demand nothing at \( P_r \) and \( P_r \) is the upper bound of these renters’ private values. Similarly, the second item is the upper bound of the utility loss of owners since \( S(P_{sw}) - S(P_r) \) is exactly the number of owners who provide zero sharing at \( P_{sw} \) but share all their products at \( P_r \) under risk of meeting no renters, and \( P_r \frac{D(P_r)}{S(P_r)} \) is the upper bound of these owners’ private values.

5.1.3 Power of Aggregation

Here we also discuss the platform’s contribution, by comparing the system to the one without the platform. In that case, users are separated into many groups, e.g., people live nearby group together to share with each other. Such sharing groups are difficult to grow since they are mainly based on location, or relationship etc. Let \( J \) denote the set of all isolated sharing groups.

We optimistically suppose each small sharing group can reach a contract price which maximizes the social welfare inside the group. We will show that even in this case, maximizing social welfare among all users over the platform achieves higher social welfare. To do so, let the maximum social welfare price of small group \( J \in J \) be \( P_J \). Then, the total social welfare can be computed by the sum of each isolated sharing group’s utility, each of which is at its maximum social welfare price, i.e., \( \tilde{W}_{iso} = \sum_{i \in J} \sum_{u \in i} W_i(P_J) \).

The contribution of a sharing platform is to bring different people into one whole group so that all renters can access all owner’s products. The following theorem guarantees that a platform can achieve a higher social welfare compared to having multiple small isolated sharing groups.

**Theorem 6.** When users’ utility functions are linear, sharing over a common platform achieves a higher social welfare than having isolated sharing groups, i.e., \( W(P_{sw}) \geq \tilde{W}_{iso} \)

This theorem is straightforward since the policy showed in Proposition 4 gives the best possible social welfare, and separated groups can only degenerate the performance. For example, consider users showed in Table 1 again. If users are divided into two groups, group 1 includes \{\( o_1, o_2, r_1 \}\) and group 2 includes \{\( o_3, r_2, r_3 \}\), maximum social welfare of group 1 and group 2 will be \( 9 + \epsilon - 2c \) and \( 1 - c \), which is less than the social welfare showed in 5.1.1.

5.2 Quadratic Usage Payoff

In this subsection, we study the case when each user’s usage utility is a quadratic function:

\[
U_i(u_i) = -a_i u_i^2 + b_i u_i, \quad a_i, b_i > 0, \quad \forall i \in \Omega, R
\] (11)

Under quadratic usage payoff, each renter’s best response demand is \( \frac{b_i - P}{2a_i} \), which means the total demand is a descending piecewise linear function.

Similar to the linear utility case, we present an example where the bound given in Theorem 5 is tight. Consider the following example. Let \( N_O = N_R = 1 \) and their \( a_i, b_i \) parameters are given in Table 2.

As proved in Proposition 6, we only consider the case when \( S(P) \geq D(P) \). Under this setting, renter’s demand
Table 2: Example 2 of social welfare gap’s tightness

| Identity | \(U_i(u_i)\) |
|----------|--------------|
| owner \(o\) | \(-u_o^2 + 2u_o\) |
| renter \(r\) | \(-u_r^2 + u_r\) |

will be \(\frac{1-P}{2}\). As for the owner, he will have no desire to provide supply more than the demand since there is no competition. Thus, demand equals supply. Let \(c = 0\), the owner’s best responses will be to fully utilize the product, i.e., \(u_o + s_o = 1\). Hence, the owner’s utility function is \(W_o = -u_o^2 + 2u_o + P\min\{D, s_o\}\) and \(u_o + s_o = 1\). Therefore, owner’s best response will always be choosing \(s_o = D\) and \(u_o = 1 - s_o\).

A revenue maximizer will maximize the volume of trade, i.e., \(PD(P) = P(1-P)\), which yields the optimal revenue price \(P_r = \frac{1}{2}\). A social welfare maximizer, instead, will maximize the total social welfare:

\[
W = W_o + W_r = -(1 - \frac{1}{2}P)^2 + 2(1 - \frac{1}{2}P) + P\frac{1-P}{2} + \frac{(1-P)^2}{4},
\]

which also gives \(P_{sw} = \frac{1}{2}\).

As a result, both policies use the same price, which leads to the same demand and supply. The bound given in Theorem 3 is \(P_r[D(P_{sw}) - D(P_r)] \frac{8(P_{sw})}{3(P_r)} = 0\), which is tight in this example.

6. EXPERIMENT

In this section, we conduct numerical experiments to validate the results in this paper, on both generated data and real data. In addition, we also discuss some insights delivered by simulations.

6.1 Simulation Setup

In [6.2] we conduct simulation on generated data with quadratic usage payoff. A few different parameter settings are examined. All user usage utilities \(U_i(u_i)\) are set to be quadratic functions here, i.e., \(U_i(u_i) = -a_iu_i^2 + b_iu_i\) with \(a_i, b_i > 0\). This is because quadratic utilities provide both tractability and representativeness, since quadratic functions can be seen as second order approximations for concave functions under Taylor expansion. Specifically, each user’s \(a_i, b_i\) are uniformly chosen from \((0.1, 1.2)\) and \((0, 1)\).

In [6.3], we use real data from DiDi [5], the biggest car sharing platform in China. Data includes transaction records on the first three weeks in January, 2016, inside a Chinese city, including driver ID, passenger ID, starting district ID, destination district ID and fee on each ride. We choose to use data on Jan 8th randomly, which includes 399938 order records.

In searching Nash Equilibrium, we use best response iteration, which turns out to be a fast converging algorithm under our setting.

6.2 Results on Quadratic Usage Payoff

6.2.1 Supply-Demand Relationship

Here we consider the tightness of social welfare loss bound (Theorem 3) under different supply-demand relationship. That is, we set number of owner to be 100 and compare the social welfare gaps from numerical results and theoretical bounds under different renter numbers.

It is clearly showed in Fig.2 that when the number of renters increases, the social welfare gap decreases to zero. There are two important implications that we want to point out: (i) with the growth of renter population, the theoretical bound not only provides an upper bound of actual social welfare loss, but also an accurate estimation, (ii) when the renter population is a few times more than owners, which is common in real world scenarios, maximum revenue policy also achieves maximum social welfare.

Intuitively, this is because when the resources become scarce, a maximum social welfare policy will first satisfy the needs of renters with higher utilities, which is the same as what a maximum revenue policy will do. In other words, when resources are scarce, a revenue maximization platform run by company simultaneously guarantees maximum social welfare.

6.2.2 \(P_{sw}\) and \(P_r\)

Here we compare \(P_{sw}\) and \(P_r\) under different settings, e.g., different cost or different degree of scarcity of the products. From Fig.3(a), \(P_{sw}\) and \(P_r\) are stable when \(c\) is low, but along with the cost increment, \(P_{sw}\) increases quickly towards \(P_r\), and eventually becomes the same when \(c\) is high enough. Compare the results in Fig.3.3 and Fig.3(a) if the cost can be chosen properly, say \(c = 0.3\), one can both keep a low price and low wastage rate.

Different degree of product scarcity also influences \(P_{sw}\) and \(P_r\). When resources become scarce, as show in Fig.3(b), \(P_{sw}\) rapidly increases towards \(P_r\), that is because when there is insufficient supply, a maximum social welfare policy also needs to guarantee the usage of renters with higher utilities, which is the same as what a maximum revenue policy will do. Therefore, it is reasonable that when supply is abundant (Fig.3(c)), \(P_{sw}\) becomes much lower than \(P_r\); a maximum social welfare policy wants to let more people get access to idle resources while a maximum revenue policy still focuses on high value clients.

6.3 Results on Real Data

We also conduct experiment based on real data from DiDi. Since data are transaction records on Jan 8, 2016, we use the statistical volume of transaction at different prices to represent renters’ demand under different prices. Total demand curve is fitted to exponential function \(D(P) = ae^{-\beta P}\) with
and supply under different cost, and Fig.6(b) is drawn under price to maximize revenue.

We examine some practical cases: if the minimum charge fee is $10 (similar situation happens if we compare $c = 0.1$ and $c = 0.3$ in Fig.6(b)), we also find that higher cost significantly suppresses sharing, e.g., drivers will choose their sharing level more conservatively if they have to pay more cost when their cars are running empty waiting for customers’ order. This shows that a properly chosen cost $c$ can help reduce redundant supply while maintaining an acceptable QoS. This is crucial to many social issues such as greenhouse gas emission, road congestion, because it is feasible to implement such a scheme: government can carefully choose a proper $c$, e.g., gasoline price, instead of designing very complex mechanisms.

6.3.3 How can DiDi do Better?

After showing the experiment results, we can also derive the possible optimal revenue that the platform can get. Since trade volume is defined as $O = \alpha N_{t}$ and by Proposition 3 $P' = 1/\beta = 12$ is the price which maximizes $PD(P)$, therefore maximizes revenue. The platform obtains maximum revenue when they set the price such that average charge for each ride is $P'$ plus minimum charge, under the condi-

![Figure 5: Under exponential form of real demand, when supply is not abundant, maximizing revenue is actually maximizing social welfare at the same time.](image)

Figure 5: Under exponential form of real demand, when supply is not abundant, maximizing revenue is actually maximizing social welfare at the same time. quadratic usage payoff given in 6.2 for comparison. These two settings actually shows similar results. As showed in both two figures, cost $c$ is the dominant factor in determining the lowest price to enable sharing, i.e., there will be no sharing when price is lower than $c$.

In Fig.6(a) if we compare the supply curves of $c = 5$ and $c = 10$ (similar situation happens if we compare $c = 0.1$ and $c = 0.3$ in Fig.6(b)), we also find that higher cost significantly suppresses sharing, e.g., drivers will choose their sharing level more conservatively if they have to pay more cost when their cars are running empty waiting for customers’ order. This shows that a properly chosen cost $c$ can help reduce redundant supply while maintaining an acceptable QoS. This is crucial to many social issues such as greenhouse gas emission, road congestion, because it is feasible to implement such a scheme: government can carefully choose a proper $c$, e.g., gasoline price, instead of designing very complex mechanisms.

6.3.3 How can DiDi do Better?

After showing the experiment results, we can also derive the possible optimal revenue that the platform can get. Since trade volume is defined as $O = \alpha N_{t}$ and by Proposition 3 $P' = 1/\beta = 12$ is the price which maximizes $PD(P)$, therefore maximizes revenue. The platform obtains maximum revenue when they set the price such that average charge for each ride is $P'$ plus minimum charge, under the condi-

![Figure 4: Histogram of transaction volume and fitted exponential form demand curve.](image)

Figure 4: Histogram of transaction volume and fitted exponential form demand curve.

R-square 0.9991, where $\alpha = 19190, \beta = 0.0832$. Fig

If we let each renter’s $U_i(u_i)$ take the following form,

$$U_i(u_i) = \frac{1}{3} (u_i + u_i \log(\frac{\alpha}{N_{t}}) - u_i \log(u_i),$$

and plugging it into (3), we will have exactly the exponential form of total demand mentioned above. Since (12) are based on statistical results from real data, with out loss of generality, we also assume all product owners have the same form of usage utility as (12). To balance the computation time and credibility, we scale $\alpha$ and $N_{t}$ to $\alpha = N_{t} = 1919$ in computation, then each renter’s demand will be $e^{-\beta P} \in [0, 1]$ and total demand will be $N_{t} e^{-\beta P} = \alpha e^{-\beta P}$. Since DiDi has a minimum charge on each ride (at 10 RMB), we can reset the zero price point at the minimum charge fee.

6.3.1 Social Welfare Loss Under Exponential Demand

Here we want to see the social welfare loss when DiDi is maximizing revenue under exponential form of demand function listed above. We examine some practical cases: when supply are less or slightly more than demand ($N_{o} = 100, 500, 1000, 2000, 2500, 3000$ while $N_{t} = 1919$). Fig.5 shows that when supply is not abundant, DiDi is actually achieving maximum social welfare simultaneously when it sets the price to maximize revenue.

6.3.2 The Role of Cost

We study the role of cost $c$ here. Fig.6(a) shows demand and supply under different cost, and Fig.6(b) is drawn under zero price point at the minimum charge fee.
actual transaction, we can calculate the average transaction price that \( S(P') \geq D(P') \). Meanwhile, since all data are from actual transaction, we can calculate the average transaction price by:

\[
P = \frac{\int_0^{\infty} P e^{-\beta P} \, dp}{\int_0^{\infty} e^{-\beta P} \, dp} = \frac{1}{\beta}
\]

which is exactly the optimal price mentioned above. That is, DiDi has already accurately chosen their price which could generate best possible revenue. Nevertheless, the platform still need to make sure there are enough supply at \( P' \), otherwise \( P' \) will not be the correct maximum revenue price according to Proposition 3.

Given 1919 renter and 1500 owners, we know from Fig. 3 that the revenue maximization price will be 12.9, which is not at the best optimal revenue price \( P' = 12 \) due to the lack of supply. Actually, among all the 395938 orders in order in the data we study, there are only 43648 car driver IDs, while number of customer ID is 237800. In another word, shortage of supply is now the biggest obstacle for sharing platform to obtain best possible revenue. There are many ways to augment supply, e.g., Uber adopt “surge price” to raises the price at the area where supply is scarce in the hope of attracting more drivers.

However, under some circumstance where price is fixed, recruiting more altruistic product owners will be a better choice, as illustrated in Fig 4. Fig 6(c) shows the impact of recruiting some actively sharing product owners, under the setting of 1919 renter and 1500 owners: with only a slightly increase of product owner’s enthusiasm (\( \epsilon = 0.05 \)), actual \( P' \) can be set to the best possible revenue maximization price \( P' = 12 \) so that platform will gain optimal revenue. Meanwhile, from social welfare maximizer’s perspective, altruistic product owners’ participation will also lower \( P_{sw} \) and therefore increase total social welfare.

7. CONCLUSION

In this paper we consider the fundamental problem about how to efficiently boost sharing economy. Based on a game-theoretic model, we derive various tight bounds of social welfare loss of the maximum revenue policy compared to maximum social welfare pricing policy. We also show that the revenue maximization ensures higher sharing supply level and ensures better quality-of-service to users in the sharing system. Our numerical results also demonstrate that as the renter/owner ratio increases, revenue optimal policy is also welfare optimal. Our results provide novel insights and guidances on how to boost sharing economy towards maximizing social welfare by commercial power who chases maximum revenue.

8. REFERENCES

[1] The rise of the sharing economy. *Economist*, 2013.
[2] Lyft. [https://www.lyft.com/](https://www.lyft.com/), 2015.
[3] Uber. [https://www.uber.com](https://www.uber.com), 2015.
[4] Additional technical report of this paper. [https://www.dropbox.com/s/bxcagx9gh40ar16/mobihoc.pdf](https://www.dropbox.com/s/bxcagx9gh40ar16/mobihoc.pdf), 2016.
[5] Didi research. [http://research.xiaojukeji.com/competition/main.action?competitionId=DiTech2016](http://research.xiaojukeji.com/competition/main.action?competitionId=DiTech2016), 2015.
[6] R. Belk. You are what you can access: Sharing and collaborative consumption online. *Journal of Business Research*, 67(8):1595–1600, 2014.
[7] S. Benjaafar, G. C. Kong, X. Li, and C. Courcoubetis. Peer-to-peer product sharing: Implications for ownership, usage and social welfare in the sharing economy. *Available at SSRN: 2669823*, 2015.
[8] G. P. Cachon, K. M. Daniels, and R. Lobel. The role of surge pricing on a service platform with self-scheduling capacity. *Available at SSRN*, 2015.
[9] L. Chen, A. Mislove, and C. Wilson. Peeking beneath the hood of uber. In *Proceedings of the 2015 ACM Conference on Internet Measurement Conference*, pages 495–508. ACM, 2015.
[10] G. Debreu. A social equilibrium existence theorem. *Proceedings of the National Academy of Sciences of the United States of America*, 38(10):886, 1952.
[11] EEA. Occupancy rates of passenger vehicles. [http://www.eea.europa.eu/data-and-maps/indicators/occupancy-rates-of-passenger-vehicles/occupancy-rates-of-passenger-vehicle](http://www.eea.europa.eu/data-and-maps/indicators/occupancy-rates-of-passenger-vehicles/occupancy-rates-of-passenger-vehicle), 2015.
[12] K. Fan. Fixed-point and minimax theorems in locally convex topological linear spaces. *Proceedings of the National Academy of Sciences of the United States of America*, 38(2):121, 1952.
[13] T. Geron. Airbnb and the unstoppable rise of the share economy. [http://www.forbes.com/sites/tomiogeron/2013/01/23/airbnb-and-the-unstoppable-rise-of-the-share-economy/](http://www.forbes.com/sites/tomiogeron/2013/01/23/airbnb-and-the-unstoppable-rise-of-the-share-economy/), 2013.
9. APPENDIX

Proofs of Theorem 1 and Proposition 4 are in additional technical report due the space limitation.

9.1 Proof of Theorem 1

Proof. Let \( S = (s_1, s_2, ..., s_{N_o}) \), with each component being each owner’s share level, and \( U = (u_1, u_2, ..., u_{N_u+N_o}) \), with each component being each user’s usage.

First we notice that an owner’s strategy space is \( (u, s) \in \{ u \geq 0, s \geq 0, u + s \leq 1 \} \) and a renter’s action space is \( u \in [0,1] \), both of which are compact convex sets.

Then, for renter \( i \), it is clear that his utility function shown in \( (u, s) \) is continuous in \( S \) and \( U \), and concave in \( u_i \). As for the owners, consider Equation [2]. The total demand \( D \) is fixed as long as \( P \) is set, as it only involves renters. The total supply is the sum of \( s_i \). Thus, owner’s utility function is continuous in \( \{ s_1, s_2, ..., s_{i-1}, s_{i+1}, ..., s_{N_o}\} \), \( u_i \).

Depending on \( D(P) \) and others’ supply \( \sum_{j \neq i} s_j \), there are three cases:

- \( D(P) < \sum_{j \neq i} s_j \): the utility function for \( i \) becomes \( W_i(u_i) = U_i(u_i) + P(D(P))s_i - cu_i - cs_i \) and we know that \( P(D(P))s_i \) is concave in \( s_i \).
- \( D(P) > \sum_{j \neq i} s_j + 1 \): the utility function for \( i \) becomes \( W_i(u_i) = U_i(u_i) + P(s_i) - cu_i - cs_i \) which is also concave in \( s_i \).
- \( \sum_{j \neq i} s_j < D(P) < \sum_{j \neq i} s_j + 1 \), the utility function for \( i \) becomes

\[
W_i(u_i) = \begin{cases} 
U_i(u_i) + P(D(P))s_i - cu_i - cs_i & D(p) \leq S(P) \\
U_i(u_i) + Ps_i - cu_i - cs_i & D(P) > S(P) 
\end{cases}
\]

and it is continuous. It is concave in \( s_i \) since each piece is concave in \( s_i \) and at the intersection point of these two pieces, the left derivative \( P - c \) is greater than the right derivative \( P(D(P))S(P) - s_i \) = \( c \).

Hence, owner \( i \)’s utility function in above three cases are always continuous and concave to its own strategy \((u_i, s_i)\). According to [11, 10, 22], a pure strategy Nash equilibrium exists since each user’s strategy space is convex and compact, and the utility function of each user is concave in his strategy; and continuous in \( S \) and \( U \).
For any renter $i$, by Assumption 4 there exists $m_i$ such that $U_i(u_i) < m_iu_i$, $u_i \in [0, 1]$. Define $P_{\text{nodemand}} = \max\{m_i\}$. Then, every renter has zero demand when $P \geq P_{\text{nodemand}}$. □

### 9.3 Proof of Proposition 2

**Proof.** Due to Assumption 4 each renter’s demand $u_i^*(P)$ is continuous in $P$, so $D(P)$ is continuous in $P$. Let $P_{\text{upper}}$ be the price satisfying the equation $P_{\text{upper}}D(P_{\text{upper}}) = c$, which means even only one owner provides production sharing with no competition, he can not make positive profit. It is clear that no owner will share products at price beyond $P_{\text{upper}}$. We start by proving (ii) of the proposition, and then (i) and (iii)

(ii) We first prove that there exists $P' < P_{\text{upper}}$ such that $D(P') = S(P')$. By Proposition 1 total demand decreases to 0 when market price exceeds $P_{\text{upper}}$. Thus, there must exists some price $P'$ such that $D(P_\star) > S(P_\star)$ for $P_\star = P' - \delta$ and $D(P_+)$ for $P_+ = P' + \delta$, where $\delta > 0$ is infinite small. We only need to show that $D(P') = S(P')$, i.e., $S(P')$ is continuous at the intersection point $P'$. Suppose the opposite that $S(P_\star) - S(P_\star) = \Delta > 0$, where $\Delta$ is some positive constant. Then, there must exists some owner $i$, such that $s_i^*(P_\star) - s_i^*(P_\star) = \Delta > 0$, and the effective price at $P_\star$ is $P_\star = P_\star - \Delta$. Since $\delta > 0$, i share more at $P_\star$ while the effective price is lower, which leads to contradiction. We can also show that $P' < P_{\text{upper}}$. That is because since $S(P) = D(P) > 0$, we must have $P' < P_{\text{upper}}$.

Now we show that $D(P) \leq S(P)$ for $P \geq P'$. Consider owner $i$’s utility function at any $P_0 \geq P'$ satisfying $D(P_0) \leq S(P_0)$, e.g., $P_0 = P'$ (we have $D(P_0) = S(P_0)$):

$$W_i(u_i) = U_i(u_i) + P_i\delta_i - c_u - c_s,$$

and the best response is denoted by $(u_i^*(P_0), s_i^*(P_0))$. Assume for contradiction that for some $P_1 > P_0$ we have $D(P_1) > S(P_1)$.

Then, owner $i$’s utility function becomes:

$$W_i(u_i) = U_i(u_i) + P_i\delta_i - c_u - c_s,$$

where the best response is $(u_i^*(P_1), s_i^*(P_1))$. Compare equation (14) and (15), since $P_1 > P_0 \frac{D(P_0)}{S(P_0)}$, we must have:

$$U_i(u_i^*(P_0)) + P_1\delta_i^*(P_0) - c_u^*(P_0) - c_s^*(P_0) \geq U_i(u_i^*(P_1)) + P_0\delta_i^*(P_1) - c_u^*(P_1) - c_s^*(P_1)$$

and

$$U_i(u_i^*(P_1)) + P_1\delta_i^*(P_1) - c_u^*(P_1) - c_s^*(P_1) \geq U_i(u_i^*(P_0)) + P_0\delta_i^*(P_0) - c_u^*(P_0) - c_s^*(P_0)$$

from (17) we have:

$$P_i\delta_i^*(P_0) - P_i\delta_i^*(P_1) \geq U_i(u_i^*(P_0)) - U_i(u_i^*(P_1)) - c_s^*(P_0) + c_u^*(P_0) - c_u^*(P_1) + c_s^*(P_1),$$

plugging (16) into it we have:

$$P_i\delta_i^*(P_0) - P_i\delta_i^*(P_1) \geq P_0\delta_i^*(P_1) - P_1\delta_i^*(P_0)$$

since $P_1 > P_0 \frac{D(P_0)}{S(P_0)} > 0$, we have:

$$s_i^*(P_1) - s_i^*(P_0) \geq 0.$$

(19)

Since $i$ is arbitrary, we have $S(P_1) \geq S(P_0)$, and $S(P_1) \geq S(P_0) \geq D(P_0) = \geq D(P_1)$ by Proposition 1 which leads to contradiction.

Therefore let $P'$ be the smallest market clearing price, $P' = \min\{P_+ | D(P_+) = S(P_+)\}$, we have $S(P) \geq D(P)$, $\forall P \geq P'$.

(i) Similarly we know $S(P)$ is nondecreasing when $P \leq P'$, since the utility function at $P < P'$ for each owner is:

$$W_i(u_i) = U_i(u_i) + P_i\delta_i - c_u - c_s,$$

and each one’s $s_i$ is nondecreasing when $P$ increases.

(iii) It has been proved at the beginning. □

### 9.4 Proof of Proposition 3

**Proof.** (i) Consider the case when the price is $P_0$ and $D(P_0) > S(P_0)$. There exists some renters who are not able to rent products since available products are not enough. Let $R_0$ denote the set of renters who have successfully rented a product and $R/R_0$ denotes the other renters. We know that $W_i(u_i) \geq 0, \forall i \in R_0$ and $W_i(u_i) = 0, \forall i \notin R_0, i \in R$. According to Proposition 2 there exists some market clearing price $P' > P_0$ such that $D(P') = S(P')$.

For owners, we must have each owner’s social welfare $W_i(u_i) = U_i(u_i) + P_i\delta_i - c_u - c_s$, increases compared to the case when price is $P_0$.

For renters in $R/R_0$, each renter gains non-negative social welfare increment. For renters in $R_0$, each renter $i$ suffers utility lost not exceeding $(P' - P_0)u_i^*(P_0)$, with $u_i^*(P_0)$ being his best response usage at price $P_0$. Therefore total social welfare lost for group $R_0$ will be less than $(P' - P_0)S(P_0)$. However for the owners who have served users in $R_0$, they have at least $(P' - P_0)S(P_0)$ utility increment at price $P'$ due to the price increase even if they do not augments supplies.

To sum up, the social welfare at lowest market clearing price $P'$, is higher than the social welfare at any price satisfying $D(P') > S(P')$, i.e., $P_{sw} \geq P'$. According to Proposition 2 we always have $S(P_{sw}) \geq D(P_{sw})$.

(ii) According to Proposition 2 let $P'$ be the lowest price when $D(P') = S(S')$. Consider any price $P_0 < P'$ and we have $D(P_0) > S(P_0)$. The volume of trade will be $P_0S(P_0)$. According to Proposition 3 there exists some $P_1 > P'$ such that $S(P_1) = D(P_1)$ and the volume of trade will be $P_1D(P_1) = P_1S(P_1)$ and we have $P_1D(P_1) > P_0S(P_0)$. □

### 9.5 Proof of Theorem 2

(1) We prove the first bullet in Theorem 2 here. Suppose for some $P_1 > P'$, we must have $\sum_{i \in R} W_i(u_i) = \sum_{i \in R} W_i(u_i) < -((P_1 - P')D(P_1)), which is the additional payment paid by renters for demand $D(P_1)$ after price changed to $P_1$. As for the renters, they have earned additional payment $A$ from renters:

$$A = P_1D(P_1) - P'D(P') = P_1D(P_1) - P'(D'(P') - D(P_1))$$

where the first term in (20) is exactly the lower bound of renters’ loss, and we only need to prove that owners can not completely retrieve the loss in the second term by choosing their own usage. For owner $i$, we have:

$$U_i(u_i^*(P')) + P'D(P') \frac{S_i'(P') - c_u'(P') - c_s'(P')}{S'(P')}$$

$$\geq U_i(u_i^*(P_1)) + P'D(P') \frac{S_i'(P_1) - c_u'(P_1) - c_s'(P_1)}{S'(P_1)}$$

(21)
where \( \triangle s_i = s_i^*(P_1) - s_i^*(P') \). Thus, 
\[
U(u_i^*(P_1)) - cu_i^*(P_1) = s_i^*(P_1) - U(u_i^*(P')) + cu_i^*(P') + cs_i^*(P') 
\]
\[
\leq P'D(P') - \frac{P'_D(P')}{{S(P')}^2} s_i^*(P_1) - \frac{P_D(P_1)}{S(P')} s_i^*(P_1) 
\]
summarize (22) over all owners, we have: 
\[
\sum_{i \in O} [U(u_i^*(P_1)) - cu_i^*(P_1) - cs_i^*(P_1)] - U(u_i^*(P')) + cu_i^*(P') + cs_i^*(P') 
\]
\[
< P'D(P') - \sum_{i \in O} \frac{P'D(P')}{S(P')} s_i^*(P_1) - \sum_{i \in O} P_D(P_1) 
\]
\[
= P'D(P') - \sum_{i \in O} \frac{P'D(P')}{S(P')} s_i^*(P_1) + \frac{P_D(P_1)}{S(P')} s_i^*(P_1) 
\]
\[
= P'D(P') - \sum_{i \in O} P_D(P_1) \tag{23} 
\]
If \( S(P_1) > S(P') \), we show that for any owner \( i \), \( \triangle s_i \geq 0 \). Suppose the contradiction that there exists some \( i \) such that \( \triangle s_i > S(P_1) - S(P') \), while some \( j \) with \( \triangle s_j < 0 \). Therefore, the effective price seen by \( j \) will be lower at \( P_1 \), i.e., \( \frac{P_D(P_1)}{S(P_1)} \leq \frac{P_D(P_1)}{S(P')} \), otherwise \( j \) can always increase its sharing by arbitrarily small amount beyond \( s_i^*(P') \) to gain higher utility. Hence, we know that \( i \) also see the decreased effective price and decrease his sharing supply, which leads to contradiction. Thus, we have: 
\[
P'D(P') - \sum_{i \in O} \frac{P'D(P')}{S(P')} s_i^*(P_1) < P'D(P') - \sum_{i \in O} \frac{P'D(P')}{S(P')} s_i^*(P_1) 
\]
\[
= S(P') + \triangle s_i \tag{24} 
\]
Similarly, if \( S(P_1) < S(P') \), we will have \( \triangle s_i \leq 0 \) for all owner \( i \). Since both the supply and effective price decrease at \( P_1 \), we have \( \frac{P'D(P_1)}{S(P')} > \frac{P_D(P_1)}{S(P_1)} \). So the owner will bear the payment loss \( P'D(P') - P_D(P_1) \), which can be showed similarly as above to be irretrievable by changing self-use:

\[
\sum_{i \in O} [U(u_i^*(P_1)) - cu_i^*(P_1) - cs_i^*(P_1)] - U(u_i^*(P')) + cu_i^*(P') + cs_i^*(P') 
\]
\[
< P'D(P') - \sum_{i \in O} \frac{P'D(P')}{S(P')} s_i^*(P_1) - \sum_{i \in O} P_D(P_1) 
\]
\[
< P'D(P') - P_D(P_1) \tag{25} 
\]
(2) By Proposition 3 and results above, we must have 
\( P_r \geq P_{sw} \).

(3) The volume of trade can be represented as \( P_r D(P_r) \) and \( P_{sw} D(P_{sw}) \) by Proposition 3. Since \( P_r \) maximizes the volume of trade, we know that \( P_r D(P_r) \geq P_{sw} D(P_{sw}) \).

We prove by contradiction, suppose that \( S(P_r) < S(P_{sw}) \). Let the best response of owner \( i \) at \( P_r \) and \( P_{sw} \) be \( s_i^*(P_r) \) and \( s_i^*(P_{sw}) \). Since \( S(P_r) < S(P_{sw}) \), without loss of generality, we can assume \( s_i^*(P_r) < s_i^*(P_{sw}) \) for some \( i \). Then, as far as \( i \) can see, the “effective” price \( \frac{P_r D(P_r)}{S(P_r)} \) is greater than the “effective” price \( \frac{P_{sw} D(P_{sw})}{S(P_{sw})} \). Under such circumstances, \( i \) will find that he can always increase its sharing supply, at least by positive \( \delta \to 0 \), to increase his utility, as long as the total effective price be still higher than \( \frac{P_{sw} D(P_{sw})}{S(P_{sw})} \), since \( i \)’s best response at \( \frac{P_{sw} D(P_{sw})}{S(P_{sw})} \) is \( s_i^*(P_{sw}) \) and \( i \) can actually share more if price is higher. This contradicts the fact that \( i \) is at Nash equilibrium and \( s_i^*(P_r) \) is his best response at \( P_r \).

9.6 Proof of Theorem 3

Proof. The first inequality is trivial. From Proposition 3 we know that total supply exceeds total demand at \( P_{sw} \) and \( P_r \), which means each renters with positive demand can get served.

Denote \( W^*_i(P) \) as \( i \)’s optimal utility and \( u_i^*(P), s_i^*(P) \) for \( i \in O \) as his best response at price \( P \). Then, \( u_i^*(P), s_i^*(P) \) and \( u_i^*(P_{sw}), s_i^*(P_{sw}) \) are \( i \)’s best response usages and share levels for \( i \in O \) at price \( P_r \) and \( P_{sw} \). The total demand and total supply at price \( P_r \) and \( P_{sw} \) are given by:

\[
D(P_r) = \sum_{i \in R} u_i^*(P_r), \quad S(P_r) = \sum_{i \in O} s_i^*(P_r) 
\]
\[
D(P_{sw}) = \sum_{i \in R} u_i^*(P_{sw}), \quad S(P_{sw}) = \sum_{i \in O} s_i^*(P_{sw}) \tag{26} 
\]

We first consider renters’ social welfare. Renter \( i \)’s utility at some arbitrary price \( P \) is \( W_i(u_i) = U_i(u_i) - P u_i \). Since \( u_i^*(P_r) \) yields maximum utility of \( i \) at price \( P_r \), we must have:

\[
W_i(u_i(P_{sw})) - P_r u_i(P_{sw}) \leq W_i(u_i(P_r)) - P_r u_i(P_r) \tag{27} 
\]

Plugging (27) into the gap of all renters’ social welfare:

\[
\sum_{i \in R} W_i(P_{sw}) - \sum_{i \in R} W_i(P_r) = \sum_{i \in R} [U_i(u_i^*(P_{sw})) - P_{sw} u_i^*(P_{sw}) - (U_i(u_i^*(P_r))) - P_r u_i^*(P_r)] \leq (P_r u_i^*(P_{sw}) - P_r u_i^*(P_r)) - P_r u_i^*(P_{sw}) + P_r u_i^*(P_r) = (P_r - P_{sw}) D(P_{sw}) \tag{28} 
\]

After bounding the renters’ social welfare, we now take a look at the owners. Since total demand is less than total supply, owner \( i \)’s utility function is

\[
W_i(u_i) = U_i(u_i) + \frac{P D(P)}{S(P)} s_i - cu_i - cs_i \tag{29} 
\]

Owner group’s utility gap between two prices will be:

\[
\sum_{i \in O} [W_i(u_i^*(P_{sw})) - W_i^*(P_r)] 
\]
\[
= \sum_{i \in O} [U_i(u_i^*(P_{sw})) - U_i^*(u_i^*(P_r))] 
\]
\[
+ \sum_{i \in O} [-c(u_i^*(P_{sw}) + s_i^*(P_{sw})) + c(u_i^*(P_r) + s_i^*(P_r))] 
\]
\[
+ P_{sw} \frac{D(P_{sw})}{S(P_{sw})} - P_r \frac{D(P_r)}{S(P_r)} \tag{30} 
\]

Since \( u_i^*(P_r), s_i^*(P_r) \) is \( i \)’s best response at \( P_r \), we must have for all owner \( i \):

\[
U_i(u_i^*(P_r)) + \frac{P}{S(P_r)} s_i^*(P_r) - cu_i^*(P_r) - cs_i^*(P_r) \geq U_i(u_i^*(P_{sw})) + P \frac{D(P)}{S(P)} s_i^*(P_{sw}) - s_i^*(P_r) + s_i^*(P_{sw}) \tag{31} 
\]
Note that
\[
\sum_{i \in O} \left[ P_i \frac{D(P_r)}{S(P_r)} - s^*_i(P_r) + s^*_i(P_{sw}) - P_i \frac{D(P_r)}{S(P_r)} s^*_i(P_{sw}) \right] = P_i D(P_r) s^*_i(P_r) \sum_{i \in O} \left[ \frac{1}{S(P_r) - s^*_i(P_r) + s^*_i(P_{sw}) - s^*_i(P_{sw})} \right] = P_i D(P_r) s^*_i(P_r) \frac{1}{S(P_r) - s^*_i(P_r) + s^*_i(P_{sw}) - s^*_i(P_{sw})} \\
\geq P_i D(P_r) s^*_i(P_r) \frac{1}{S(P_r) - s^*_i(P_r) + s^*_i(P_{sw}) - s^*_i(P_{sw})} \\
\geq 0
\]
(32)

We have used Theorem 2 to obtain last inequality of (32). Plugging (32) into (31), we have:
\[
\sum_{i \in O} \left[ U_i(u^*_i(P_{sw})) - U_i(u^*_i(P_r)) \right] \leq P_i D(P_r) s^*_i(P_r) - c_i u^*_i(P_{sw}) + c_i s^*_i(P_{sw}) - c_i s^*_i(P_{sw}).
\]
(33)

Plugging (33) into (30), we have:
\[
\sum_{i \in O} W^*_i(P_{sw}) - W^*_i(P_r) \leq P_i D(P_r) s^*_i(P_r) - P_i D(P_r) s^*_i(P_r) + s^*_i(P_{sw}) + P_{sw} D(P_{sw}) S(P_{sw}) - P_r D(P_r) S(P_r).
\]
(34)

Adding (34) and (28) together we have proved the theorem.

9.7 Proof of Theorem 5

It can be verified that the proofs are the same for Theorem 1 and 2. We only need to show the differences in proving Theorem 3. Specifically, the renter’s utility is the same as in (28), and owner’s utility gap becomes:
\[
\sum_{i \in O} W^*_i(P_{sw}) - W^*_i(P_r) = \sum_{i \in O} \left[ U_i(u^*_i(P_{sw})) - c_i u^*_i(P_{sw}) - c_i s^*_i(P_{sw}) - U_i(u^*_i(P_r)) + c_i u^*_i(P_r) + c_i s^*_i(P_r) + P_{sw} D(P_{sw}) S(P_{sw}) + c_i s^*_i(P_{sw}) - P_r D(P_r) S(P_r) \right]
\]
(35)

Similarly to (31), since $u^*_i(P_r)$ and $s^*_i(P_r)$ are the best responses at $P_r$, we have:
\[
U_i(u^*_i(P_r)) + P_r \left( \frac{D(P_r)}{S(P_r)} + \epsilon_i \right) s^*_i(P_r) - c_i u^*_i(P_r) - c_i s^*_i(P_r) \geq U_i(u^*_i(P_{sw})) + P_{sw} \left( \frac{D(P_{sw})}{S(P_{sw})} - s^*_i(P_{sw}) + \epsilon_i \right) s^*_i(P_{sw}) - c_i u^*_i(P_{sw}) - c_i s^*_i(P_{sw})
\]
(36)

Plugging (36) into (35), and use the similar method showed in (32), we have:
\[
\sum_{i \in O} W^*_i(P_{sw}) - W^*_i(P_r)
\]
\[
\leq \sum_{i \in O} \left[ P_i \frac{D(P_{sw})}{S(P_{sw})} + \epsilon_i \right] s^*_i(P_{sw}) - P_r \left( \frac{D(P_r)}{S(P_r)} + \epsilon_i \right) s^*_i(P_r) + P_r \left( \frac{D(P_r)}{S(P_r)} + \epsilon_i \right) s^*_i(P_{sw}) - P_{sw} D(P_{sw}) S(P_{sw}) - P_r D(P_r) S(P_r).
\]
(37)

which is the same as (34). Therefore we have proved Theorem 3 still holds under modified owner utility of 2 and Theorem 4 again comes as a natural corollary of Theorem 3. So far we have proved Theorem 4.

9.8 Proof of Proposition 4

Since the social welfare under the scheme stated in the proposition is obviously the highest possible one, we only need to show that there exists such a scheme. As showed in 7, we can set the market renting price $P_{sw} \in (\alpha_{NO}, \alpha_{NO}, \epsilon)$ such that renters of first $D(P_{sw})$ highest $\alpha_i$ get access to cars provided from owners of first $S(P_{sw})$ lowest $\alpha_i$, while the other owners fully use their own car. We know that at $P_{sw}$, demand equals supply, since $S(P_{sw}) = N_O - (N_O - D(P_{sw})) = D(P_{sw})$ (here $(N_O - D(P_{sw}))$ denotes the number of owners whose private value is higher than $P_{sw}$). Under this circumstance, we achieve maximum social welfare as stated in the proposition.