STRUCTURES OF GRAVITATIONAL VACUUM AND THEIR ROLE IN THE UNIVERSE

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The production of gravitational vacuum defects and their contribution in energy density of the Universe are discussed. These topological microstructures could be produced as the result of defect creation of the Universe from ”nothing” as well as the result of the first relativistic phase transition. They must be isotropically distributed on background of the expanding Universe. After Universe inflation these microdefects smoothed, stretched and broke up. Parts of them have survived and now they are perceived as the structures of Λ-term (quintessence) and unclustered dark matter. It is shown that for phenomenological description of vacuum topological defects of different dimensions (worm-holes, micromembranes, microstrings and monopoles) the parametrizational noninvariant members of Wheeler -DeWitt equation can be used. The mathematical illustration of these processes may be the spontaneous breaking of local Lorentz-invariance of quasi-classical equations of gravity. In addition, 3-dimensional topological defects revalues Λ-term.

INTRODUCTION

Previously cosmology of gravitational vacuum was not practically discussed although the influence of gravity on a vacuum was considered[1]. On the other hand cosmology of other vacua is very often discussed [2]. To be more exact we can say that we shall consider structures of a gravitational vacuum condensate. Among fundamental interactions gravity plays the central role as the determinant of the space-time structure and as the arena of physical reality [3]. The question is to find the internal structure of gravitational vacuum starting from the quantum regime. The quantum regime of gravity has not been satisfactorily explained although many approaches have been done [4].

We present some analogy between known vacuum structures and hypothetical structures of gravitational vacuum (as it is known condensates of the quark-gluon type consist of topological structures - instantons). The general representation about topological defects is: 3-dimensional topological structures (D = 3) - worm holes; 2-dimensional topological structures (D = 2) -membranes; 1-dimensional topological structures (D = 1) -strings;
point defects (singularities) ($D = 0$) - gas of topological monopoles. The full theory of vacuum defects is absent although we understand that the presence of defects breaks the symmetry of a system. The difficult question is to know the loss of which symmetry in quantum theory of gravity applies to the presence of topological defects of gravitational vacuum. Here it is necessary to use the experience of vacuum physics.

Even now formulation of the problem of the microscopic description of gravitational properties of vacuum is absent. From general considerations we may propose that strong fluctuations of topology could take place on the Planck scale. Probably in these fluctuations stable structures averaging characteristics of which are constant or slowly changing in time may exist. We have taken into account that among possible parametrization-noninvariant potentials of Wheeler-DeWitt equation are ones topological properties of which can be used for macroscopic description of the gas of topological defects (wormholes, micromembranes, microstrings and monopoles). In our opinion, these circumstances allow us to propose a hypothesis on which the future theory of quantum gravity and the problem of quantum topological structures and parametrization-noninvariant of Wheeler-DeWitt equation will be solved in combination. In the frame of this hypothesis based on mathematical observations and analogies we suggest the mathematical apparatus of systematic modelling of the superspace metric and the effective potential permitting us to choose structures which are concerned with a cosmological point of view.

Thus we discuss gravitational vacuum in which energy-momentum characteristics depends on the radius of a closed Universe. Our arguments are only heuristic:

1. this is the simplest mathematical model;
2. our model is synthesized with general property of Wheeler-DeWitt equation for a closed isotropic Universe. Gauge invariance of classic theory of gravity has an additional aspect - the parametrization-al invariance on the relative choice of variables on which are put gauge conditions. In classic theory a parametrization and a gauge are single operations the division of which on separate steps is conditional. In quantum geometrodynamics (QGD) the situation is different - the basic equations of QGD are gauge invariant but parametrization is gauge noninvariant [5]. Physical consequences of the parametrization noninvariance are not general. Authors [6] have suggested to reject this problem and to
fix the choice of gauge variables by the concrete way;
3. also a conjecture argument has arisen after the introduction in cosmology of quintessence:
a gravitation vacuum condensate may be considered as a possible factor of the breaking
of local vacuum Lorentz-invariance.
Of course gravitational vacuum condensate (as well as and its structures) may arise af-
after the first relativistic phase transition [7] but in this article we remain in the frame of
Wheeler-DeWitt conceptions on which phase transitions are absent.

Generally speaking the nature of dependence of vacuum energy from time is good
unknown and the mathematical simulation has discussion character. Although compens-
sation mechanisms have probably taken place (vacuum energy may be decreased by jumps
in the result of negative contributions during relativistic phase transitions [8]). Models
of quintessence [9] appeal mainly to classical fields (scalar one as example). We shall
show that in quasiclassical approach quintessence can be simulated using mathematical
structures arising naturally in Wheeler-DeWitt quantum geometrodynamics.

BASIC STATEMENTS

Probably the Universe creation is a quantum geometrodynamical transition from
”nothing” (the state of ”nothing” has geometry of zero volume (a=0)) in the state of a
closed 3-space of small sizes having some particles and fields. That is the closed isotropic
Universe was created through tunneling process with a metric:

\[ ds^2 = N^2 dt^2 - a^2(t) dl^2 \]  (1)

\[ dl^2 = d\rho^2 + \sin^2 \rho (d\Theta^2 + \sin^2 \Theta d\Phi^2) \]  (2)

here \( N = \sqrt{g_{00}} \) is a gauge variable which is necessary to fix before the solution of
Einstein equations and a tensor energy-momentum(TEM): \( T^\mu_\nu = diag(\epsilon, -p, -p, -p) \)
For simplisity we have restricted by the consideration only 3-space, although a geometrody-
namical transition in the state of a closed space of large dimensions may be also possible.
As known in the modern epoch, vacuum energy has overcome the curvature (it is a
dominant component of the Universe) and now the Universe expands accelerationally:
\( \Omega_\Lambda \sim 0.7; \Omega_m \sim 0.3 \) and no contradiction that the Universe was created closed. Besides
vacuum energy (if it was positive) was decreasing by jumps because of negative contributions during its relativistic phase transitions [8]. On the first stage we work with the classic theory in which energy-carriers are described by the hydrodynamical TEM and which is locally Lorentz-invariant (generally covariant) and which has the standard $\Lambda$-term. For a closed Universe Einstein equations are:

\[ R^0_0 - \frac{1}{2} R = 3\left(\frac{\dot{a}^2}{N^2 a^2} + \frac{1}{a^2}\right) = \omega(\epsilon_s + \Lambda_0) \]  

\[ R^k_i - \frac{\delta^k_i}{2} R = \delta^k_i \left[ \frac{1}{N^2} \left(2\frac{\ddot{a}}{a} - \frac{\dot{N}\dot{a}}{Na} + \frac{\dot{a}^2}{a^2} \right) + \frac{1}{a^2} \right] = \omega\dot{\delta}^k_i (-p_s + \Lambda_0) \]  

\[ -R = 6\left[ \frac{1}{N^2} \left(\frac{\dot{a}}{a} - \frac{\dot{N}\dot{a}}{Na} + \frac{\dot{a}^2}{a^2} \right) + \frac{1}{a^2} \right] = \omega(\epsilon_s - 3p_s + 4\Lambda_0) \]  

Quantum theory of a closed Universe - quantum geometrodynamics[10] is based on the Wheeler idea about a superspace. This idea includes the manifold of all possible geometries of 3-space, matter and field configurations in which the Universe wave function is defined. Before quantization of classical theory, it is necessary to impart the form of Lagrange and Hamilton theory with couples (we are able to quantify only Hamilton theory but the construction of Hamilton theory precedes the construction of Lagrange one). Also evidently that two variables must be in Lagrange formulation of the theory: dynamical variable $a(t)$ relating to an equation of motion and some Lagrange multiplier $\lambda = \lambda(t)$ relating to an equation of couple. From a infinite multiplicity of variants of the inserting of a Lagrange multiplier we choose the concrete variant related with quantum geometrodynamics. In quantum geometrodynamics the task arises which has not classical analogy: it is necessary to formulate the procedure of operators ordering on a generalized momentum and a coordinate. It is assumed that the procedure of ordering must be based on the covariance principle of Wheeler-DeWitt equation in Wheeler superspace. The metric of superspace $\gamma(a)$ together with a Lagrange multiplier are inserted for this conception.

The explicit form of function $\gamma(a)$ is not fixed but this conception allows us to understand in which terms the problem of parametrizational invariance is formulated. The
The abovementioned program is carried out if: \( N = \frac{\lambda a}{\gamma(a)} \) where \( \lambda \) is a Lagrange multiplier, \( \gamma(a) \) is the metric of a superspace. Then we have:

\[
3 \left( \frac{\gamma^2 \dot{a}^2}{\lambda^2 a^4} + \frac{1}{a^2} \right) = \kappa \left[ \epsilon_s(a) + \Lambda_0 \right]
\]  

(6)

\[
\frac{\gamma^2}{\lambda^2 a^2} \left( \frac{\ddot{a}}{a} - 2 \frac{\dot{a}}{\lambda a} + 2 \frac{\dot{a}}{\gamma a} - 3 \frac{\dot{a}^2}{a^2} \right) - \frac{1}{a^2} = \kappa \left[ \epsilon_s(a) + a \cdot \frac{d\epsilon_s(a)}{da} + \Lambda_0 \right]
\]  

(7)

The system of equations which is mathematically equivalent to these is obtained by the variational procedure from some effective action. The gravitational part of this action is the known expression written for a isotropic Universe.

\[
S_g = \int \left( \frac{1}{2\kappa} R + \Lambda_0 \right) \sqrt{-g} dt^4
\]  

(8)

Over this expression some standard operations are necessary to perform: transformation of it in a quadratic form on generalized velocity \( \dot{a} \) by excluding the total derivative; integration on volume of a closed Universe \( V = 2\pi^2 a^3 \); inserting of parametrizational of time. The effective action of matter and radiation is added to the received result using Rubakov-Lapchinsky recept [11]. This recept is very simple: the energy density of matter \( \epsilon_s(a) \) depending on radius of the Universe in an effective Lagrangian and as \( \Lambda \)-term has the status of an effective potential energy. Therefore, for receiving of right expression it is necessary to do a replacement \( \Lambda_0 \) to \( \Lambda_0 + \epsilon_s(a) \). The final expression for effective action is:

\[
S_{\gamma \{a, \lambda\}} = \int L_{\gamma(a, \lambda)} dt, \quad L_{\gamma(a, \lambda)} = \frac{6\pi^2}{\kappa \epsilon} \frac{1}{\lambda} \gamma(a) \dot{a}^2 - \frac{\lambda a U(a)}{\gamma(a)}
\]  

(9)

where

\[
U(a) = \frac{6\pi^2}{\kappa \epsilon} a - 2\pi^2 a^3 [\epsilon_s(a) + \Lambda_0]
\]

is a total effective potential energy accounting topology of a closed Universe, standard \( \Lambda \)-term and matter. The variation of action on \( \lambda(t) \) gives equation:

\[
\frac{\delta S_{\gamma \{a, \lambda\}}}{\delta \lambda} = \frac{\partial L_{\gamma(a, \lambda)}}{\partial \lambda} = -\frac{2\pi^2}{\kappa \epsilon \gamma} \left( \frac{3\gamma^2 \dot{a}^2}{\lambda^2} + 3a^2 - \kappa a^4 [\epsilon_s(a) + \Lambda_0] \right) = 0
\]  

(10)
which is mathematically equivalent to the equation of the couple. The variation on
dynamical variable $a(t)$ gives the Lagrange equation:

$$\frac{\delta S_\gamma}{\delta a} = -\frac{d}{dt} \frac{\partial L_\gamma}{\partial \dot{a}} + \frac{\partial L_\gamma}{\partial a} = 0$$  \hspace{1cm} (11)

After some transformations of the Lagrange equation we have:

$$\frac{d}{dt} \frac{\partial L_\gamma}{\partial a} - \frac{\partial L_\gamma}{\partial a} = \frac{6\pi^2 \lambda}{ae} \left\{ \frac{\gamma^2}{\lambda^2} \left( 2\ddot{a} + 2\dot{\gamma} \dot{a} - 2\dot{\lambda} \dot{a} - 3\dot{a}^2 \right) - a - \frac{\alpha \lambda a^3}{3} (\epsilon_s(a) + a \frac{d\epsilon_s(a)}{da} + \Lambda_0) + J \right\} = 0$$  \hspace{1cm} (12)

where

$$J = \frac{2\pi^2 \lambda}{ae} \frac{d}{da} \left( \frac{3\gamma^2 \dot{a}^2}{\lambda^2} + 3a^2 - \alpha a^4 \right)$$

These equations produce a total system of the Lagrange model. Of course, these
equations must be considered in combination. It is easy to see that for $J = 0$ the last
equation is mathematically equivalent to the combination of Einstein’s equations which
have been written before. Thus we have proved that this model gives us the Lagrange
method of the description of an isotropic Universe, energy-carriers of which are setted by
functions of the scale factor $\epsilon_s(a), \ p_s(a)$. In classic theory this result has a methodical
character only (Einstein’s equations in the initial form are more convenient to work in
classic theory). However we want to transfer these to the quantum geometrodynamics
in which Hamilton formulation is necessary. Hamilton model can be built on basis of
Lagrange one. Note, that introducing of function $\gamma(a)$ is the operation of parametrization.
The index $\gamma$ shows that action and Lagrangian correspond to the definite parametrization.
A Hamiltonian of our system is built on standard rules:

$$H \Phi_s = 0$$  \hspace{1cm} (13)

$$H = P\dot{a} - L = \lambda (\frac{\alpha}{24\pi^2} \frac{1}{\gamma} P^2 + aU(a))$$  \hspace{1cm} (14)
where \( P = \frac{\partial L}{\partial \dot{a}} = \frac{12\pi^2}{\alpha} \gamma \dot{a} \) is a generalized momentum. Note that there are also the parametrization problems of Wheeler-DeWitt theory. The first problem of the commutation connection is given by the operator \( p \) with accuracy to \( \dot{p} = -i\hbar \frac{\partial}{\partial a} + f(a) \). The second problem is the ordering of operators in the Hamiltonian (this one is created for any nontrivial function \( \gamma(a) \)). The partial solution of these problems is proposed in the frame of hypothesis of covariant differentiation in a curved space. In the frame of this hypothesis Wheeler-DeWitt equation has the view:

\[
-\frac{\alpha \hbar^2}{24\pi^2} \frac{1}{\sqrt{\gamma}} \frac{d}{da} \frac{1}{\sqrt{\gamma}} \frac{d\Phi_s(a)}{da} + \frac{1}{\gamma^2} \left[ \frac{6\pi^2}{\alpha} a^2 - 2\pi^2 a^4 (\epsilon_s(a) + \Lambda_0) \right] \Phi_s(a) = 0
\]

Here we have introduced the quantum index \( (s) \) numerating quantum states of matter and vacuum. The wave function of the Universe satisfies the condition:

\[
\int_0^\infty \sqrt{\gamma} da \Phi_s^*(a) \Phi_{s'}^*(a) = \delta(s - s')
\]

where \( \delta(s - s') \) is the delta function or the discrete \( \delta \) symbol in dependence on the concrete properties of Wheeler-DeWitt equation solutions. Unfortunately the hypothesis of covariant differentiation does not totally decide parametrizational problems of the theory. For a more evidential effect of the parametrizational invariance the multiplicative redefinition of wave function is necessary:

\[
\Phi_s(a) = \gamma^{1/4} \Psi_s(a)
\]

and Wheeler-DeWitt equation is rewritten in the form:

\[
(\frac{\alpha \hbar}{12\pi^2})^2 \frac{d^2 \psi_s}{da^2} + [a^2 - \frac{a^4}{3}(\alpha \epsilon_s(a) + \alpha \Lambda_0 + \alpha \epsilon_{GVC}(a))] \Psi_s(a) = 0
\]

where

\[
\epsilon_{GVC}(a) = \frac{\alpha \hbar^2}{192\pi^2} \frac{1}{a^4} (\mu'' - \frac{1}{4} (\mu')^2)
\]

Here we meant \( ' \) as a derivative of parametric function \( \mu(a) = \ln \gamma(a) \) on the scale factor. All parametrizational noninvariant effects are collected in the function \( \epsilon_{GVC}(a) \), which we have named the density of energy of gravitational vacuum (gravitational vacuum condensate \( (GVC) \)). As it is easy to see that parametrizational noninvariant effects are
clean quantum ones, \( \epsilon_{GVC} \sim \hbar^2 \). The parametrization noninvariant contributions have not the physical generalization if we do not know the physical nature of their creation. These contributions have arised from nonconservation of classical symmetry on quantum level. The experience of modern quantum field theory speaks that a vacuum state rebuilds when a symmetry is broken. For this reason we have named parametrizational noninvariant contributions as the density of GVC energy.

However general symmetric arguments do not have the clear physical connection to vacuum energy. In this situation the examples from QCD are useful. In quantum theory classical conform and chiral symmetries do not conserve resulting in the appearence of a quark-gluon condensate. Probably concrete vacuum topological structures exist in a gravitation vacuum and they are the consequence of the parametrizational noninvariance of quantum geometrodynamics. From general considerations it is evident that presence in space-time of topological defects makes it impossible for any continuous transformations of coordinates and time. That is, concrete properties of defects permitting us to determine the parametrization of gauge variables.

**COSMOLOGICAL APPLICATION**

In cosmology this means that properties of topological microscopic defects inside space on average are isotropic and homogeneous (isotropization in brane gas cosmology is also a natural consequence of the dynamics [12]. They are contained in the function \( \mu(a) \). We propose that all topological quantum defects with \( D \geq 1 \) have the typical Planck size. On this reason breaking up defects with a change of their number in variable volume \( V = a^3(t) \) must take place. From simple consideration the number of defects in this volume are:

\[
N_D \sim \left( \frac{a}{l_{pl}} \right)^D, \quad l_{pl} = (G\hbar)^{1/2} = \frac{(a\hbar)^{1/2}}{\sqrt{8\pi}}, \quad \text{here } c = 1.
\]

In accordance with these representations we wait untill the energy density of the system of topological defects contains a constant part corresponding to worm-holes and also members of type \( 1/a^3; 1/a^2; 1/a \) corresponding to gas of point defects, micromembranes and microstrings. Besides, the function \( \epsilon_{GVC}(a) \) must contain additional members describing interactions of microdefects between each other. Accounted representations correspond to the next choice of function \( \mu(a) \):
\[ \mu(a) = c_0 \ln a + c_1 a + \frac{1}{2} c_2 a^2 + \frac{1}{3} c_3 a^3, \quad c_i = \text{const} \]  

(19)

After this it get easy:

\[
\Lambda_0 + \epsilon_{GVC}(a) = \Lambda_0 - \frac{\alpha \hbar^2}{768 \pi^2 c_3^2} + \frac{\alpha \hbar^2}{192 \pi^2} \left[ -\frac{1}{2} c_2 c_3 \frac{1}{a} - \left(\frac{c_2^2}{4} + \frac{1}{2} c_1 c_3\right) \frac{1}{a^2} + \right.

\left. + \left(2 c_3 - \frac{1}{2} c_1 c_2 - \frac{1}{2} c_0 c_3\right) \frac{1}{a^3} + \left(c_2 - \frac{1}{4} c_1^2 - \frac{1}{2} c_0 c_2\right) \frac{1}{a^4} - \frac{1}{2} c_0 c_1 \frac{1}{a^5} - (c_0 + \frac{c_2^2}{4}) \frac{1}{a^6} \right] 
\]

(20)

The last three members can be interpreted as energy of gravitational interaction of defects between each other but their discussion is not the case since quasiclassical dynamics is only right in a region of large \(a\). Note also that 3-dimensional topological defects revalues \(\Lambda\)-term. Observed value of \(\Lambda\)-term is:

\[
\Lambda = \Lambda_0 - \frac{\alpha \hbar^2}{768 \pi^2 c_3^2} 
\]

(21)

Probably the term \(\frac{1}{a^3}\) may be like to dark matter (DM). This gives the limitation on parameters of function \(\mu(a)\):

\[
\frac{1}{3} l_p^4 \left[ 2 c_3 \left(1 - \frac{c_0}{a}\right) - \frac{1}{2} c_1 c_2 \right] = \alpha M 
\]

(22)

where \(M\) is a mass in volume \(a^3\). As known Wheeler-DeWitt quantum geometrodynamics is the extrapolation of quantum-theoretical conceptions on the scale of the Universe as the whole. The initial state of the Universe from QGD point of view was located in the region of small values of the scale factor in a minisuperspace. From classical point of view the initial state of the Universe is a structureless singular state. Here we must postulate the defect creation of the Universe if it was born from “nothing”. After the release of defects, probably in our Universe, the stage of quick expansion (inflation) has taken place. In the result defects are smoothed, stretched and broken up. Some defects have left and perceived now as \(\Lambda\)-term (quintessence) and unclustered DM. Note again, that topological defects of gravitational vacuum may also be produced after the first relativistic phase transition, but according to the ideas of Wheeler-DeWitt, in the frame of which we are, phase transitions are absent. More fully physics of topological defects arising during phase transitions is discussed by T.Kibble [13].
CONCLUSION

We have taken into account that among parametrizational noninvariant potentials of Wheeler-DeWitt equation are ones which have macroscopic properties suitable to macroscopic description of a gas of topological defects (worm-holes, micromembranes, microstrings and monopoles). This circumstance allows us to propose that in the future theory of quantum gravity, the problem of topological structures of gravitational vacuum and parametrizational noninvariance of Wheeler-DeWitt equation will be solved jointly (authors [14] even attempted to parametrize an equation of state of dark energy). Also we have shown that quasi-classical corrections (which are proportional $\sim \hbar^2$) are completely defined by a superspace metric (that is $\gamma(a)$). This means that these corrections in the theory of gravity are not entirely defined by physics of the 4-dim space-time. In the frame of quantum geometrodynamics a part of the corrections having an influence on the evolution of the Universe in 4-dim is defined by physics of a superspace (that is they come from another level). If we interpret these corrections as defects then this means that defects appear as the result of the interaction of universes in this superspace. The development of these ideas may be realized in the frame of tertiary quantization. Besides, it has been noted that the property of Lorentz invariance attributed to 4-manifold joints 1-dim time and 3-dim space. Here 3-dim defects (worm-holes) give the contribution in the Lorentz-invariant $\Lambda$-term). Quantum topological defects with $D=0,1,2$ give Lorentz-noninvariant contributions in vacuum TEM. Thus we emphasize that topological defects of gravitational vacuum are quantum structures produced at the Planck epoch of the evolution of the Universe (Lorentz invariance at the Planck scale must probably be modified [15]). Topological defects of the gravitational vacuum shall be included in the composition of $\Lambda$-term (quintessence) and unclustered dark matter. Probably, this data allows us to improve our understanding of the content of the Universe's main components.

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