On tachyon and sub-quantum phantom cosmologies

Pedro F. González-Díaz.
Colina de los Chopos, Instituto de Matemáticas y Física Fundamental
Consejo Superior de Investigaciones Científicas
Serrano 121, 28006 Madrid, SPAIN

June 12, 2004

Abstract

This paper deals with dark and phantom energy in the tachyon and sub-quantum models for dark energy. We obtain that the simplest condition for such a regime to occur in these scenarios is that the scalar field be Wick rotated to imaginary values which correspond to an axionic field classically. By introducing analytical expressions for the scale factor or the Hubble parameter that satisfy all constraint equations of the used models we show that such models describe universes which develop a big rip singularity in the finite future.

1 Introduction

The increasing dropping of detailed analysis leading to quite an ample observationally acceptable parameter space beyond the cosmological-constant barrier [1] is opening the really intriguing possibility that the universe is currently dominated by what is dubbed as phantom energy [2]. Phantom energy has rather weird properties which include [3]: an energy density increasing with time, naively unphysical superluminal speed of sound, violation of the dominant energy condition which would eventually allow existence of inflating wormholes and ringholes [4], and ultimately emergence of a doomsday singularity in the finite future which is known as the big rip [5].

The regime for phantom energy takes place for state equation parameters \( \omega = p/\rho < -1 \) and has been shown to occur in all current dark-energy models. However, whereas the big rip singularity is allowed to happen in quintessence [5] and k-essence [6] models, it is no longer present in models based on generalized Chaplygin-gas equations of state [7,8] having the form \( P = -A/\rho^n \), with \( A \) and \( n \) being constants. No discussion has been so far made nevertheless on the occurrence of phantom energy and big rip in the other major contender model for dark energy: the tachyon matter scenario of Padmanabhan et al. [9] or its sub-quantum generalization [10]. Abramo and Finelli have in fact used [11] a Born-Infeld Lagrangian with a power-law potential and recovered a nice dark-energy behaviour, but did not considered the negative kinetic terms which appear to characterize phantom energy.
While defining a phantom energy regime in such scenarios appears to be rather straightforward, it is quite more difficult to obtain an associated expression for the scale factor of the accelerating universe which allows us to see whether or not a big rip singularity may occur. Under the assumption of a constant parameter for the equation of state, we derive in this report a rather general solution for the scale factor of a universe dominated by tachyon matter, and show that, quite similarly to as it happens in current quintessence models, that solution predicts the occurrence of a big rip singularity. In the case that a sub-quantum potential is added to the theory, we also obtain the same result, though in this case the scale factor differs from the expression obtained for current quintessence and “classical” tachyon fields by terms which generally depend on the sub-quantum potential, and the regime of phantom energy is restricted by the smallness of the field kinetic term which may even make such a regime to vanish.

The paper can be outlined as follows. In Sec. II we discuss a rather general solution for the scale factor which satisfies all requirements and constraints imposed by tachyon theory. The phantom regime for such a solution is then investigated in Sec. III, where it is seen that it shares all funny properties of quintessential phantom energy. The sub-quantum generalization of the tachyon theory is also considered in some detail in Sec. IV. In this case it is shown that the accelerating expansion will depend on the value of the sub-quantum potential, and that the extend of the phantom regime inversely depends on the value taken by the field kinetic energy. Results are summarized and briefly discussed in Sec. V.

2 Tachyon model for dark energy

For the most favored cosmological spatially flat scenario, the Friedmann equations read

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G (\rho + 3P)}{3}, \]  

(2.1)

where \( \rho = \rho_{NR} + \rho_R + \rho_\phi \) is the energy density for, respectively, non-relativistic, relativistic and tachyon matter, and \( P \) is the corresponding pressure. We shall restrict ourselves to consider a description of the current cosmic situation where it is assumed that the tachyon component largely dominates and therefore we shall disregard in what follows the non-relativistic and relativistic components of the matter density and pressure. For the tachyon field \( \phi \) we have [9]

\[ \rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad P_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}, \]  

(2.2)

in which \( V(\phi) \) is the tachyon potential energy. Assuming an equation of state \( P_\phi = \omega_\phi \rho_\phi \) for the tachyon matter, we then deduce that

\[ \omega_\phi = \dot{\phi}^2 - 1. \]  

(2.3)

Finally, the equation of motion for \( \phi \) is

\[ \ddot{\phi} + (1 - \dot{\phi}^2) \left[ 3H \dot{\phi} + \frac{1}{V(\phi)} \frac{dV(\phi)}{d\phi} \right] = 0. \]  

(2.4)
We shall show next that there exists a general solution for the scale factor \( a(t) \) in this tachyon-field scenario which has exactly the same dependence on time as that in the general solution for a pure quintessence scalar field, and hence we also show that such a scenario admit the existence of a tachyon phantom field which leads to a singularity in finite time. In fact, a recipe has been provided by Padmanabhan himself [9] according to which, given the explicit form for the scale factor \( a(t) \), a complete specification of the full \( \phi \)-field theory can be achieved by using the following relations:

\[
\frac{\dot{\rho}}{\rho} = 2 \frac{\dot{H}}{H} \tag{2.5}
\]

\[
\dot{\phi} = \left( -\frac{2}{3} \frac{\dot{H}}{H^2} \right)^{1/2} \tag{2.6}
\]

\[
V = \frac{3H^2}{8\pi G} \left( 1 + 2 \frac{\dot{H}}{3H^2} \right)^{1/2} \tag{2.7}
\]

Our task then is to choose a general expression for \( a(t) \) which simultaneously satisfies relations (5), (6) and (7), together with the Friedmann equations (1) and the equation of motion for the tachyon field (4) which be able to match the accelerating behaviour of the current universe and implies a physically reasonable and suitably motivated field potential. If we assume a linear time-dependence of the tachyon field \( \phi \) and hence constancy of parameter \( \omega \), then it is not difficult to check that a general form of such an expression for \( a(t) \) can be written as

\[
a(t) = \left[ a_0^3 \left( 1 + \omega \phi \right)/2 + \frac{3}{2} \left( 1 + \omega \phi \right) t \right]^{2/[3(1+\omega \phi)]}, \tag{2.8}
\]

where \( a_0 \) is the initial value of the scale factor. We note that this solution describes an accelerating universe in the interval \(-1/3 > \omega \phi > -1\). At the extreme point \( \omega \phi = -1/3 \), \( a(t) \) describes a universe whose size increases just as \( t \), such as it should be expected. It is worth realizing that by simply trivially re-scaling the time parameter, solution (8) turns out to be nothing but the scale factor that represents the most general solution for the case of a quintessence scalar field for a constant equation of state [4]. On the other hand, for a scale factor (8) the tachyon field and potential are given by

\[
\phi = \phi_0 + \sqrt{1 + \omega \phi} \ t \tag{2.9}
\]

\[
V(\phi) = \frac{3\sqrt{-\omega \phi}}{8\pi G} \left[ a_0^3 \left( 1 + \omega \phi \right)/2 + \frac{3}{2} \sqrt{1 + \omega \phi} (\phi - \phi_0) \right]^2. \tag{2.10}
\]

We note that as \( \phi \to \infty \) this potential reasonably vanishes after taking the form already considered by Padmanabhan and others [9]. As \( \phi \to \phi_0 \) at \( t = 0 \) \( V(\phi) \) tends to a finite constant value, so clearly separating from the unphysical behaviour of the potential considered by Padmanabhan and compatible with what can be supported by string theories [9]. We regard therefore potential (11) as
being physically reasonable. Finally, we obtain for the speed of sound

$$c_s^2 = \frac{\dot{P}_\phi}{\rho_\phi} = \omega_\phi. \quad (2.11)$$

For the accelerating-expansion regime, we see thus that the speed of sound becomes imaginary, a case which could imply a collapsing of the tachyon stuff that can still be circumvented however [12].

### 3 Takyon phantom cosmology

The phantom energy regime will be characterized by values of the state equation parameter such that \(\omega_\phi < -1\) and a consequent violation of the dominant energy condition, i.e.

$$P_\phi + \rho_\phi = \frac{V(\phi)\dot{\phi}^2}{\sqrt{1 - \dot{\phi}^2}} < 0. \quad (3.12)$$

Such a regime can be obtained by simply Wick rotating the tachyon field so that \(\phi \rightarrow i\Phi\), with which the field \(\Phi\) can be viewed as an axion tachyon field [13], as the scale factor \(a(t)\) and the field potential \(V(\Phi)\) keep being positive and given respectively by

$$a(t) = \left[ a_0^{-3(|\omega_\phi|-1)/2} - \frac{3}{2} (|\omega_\phi| - 1) t \right]^{-2/[3(|\omega_\phi|-1)]}, \quad (3.13)$$

which accounts for a big rip singularity at finite future time

$$t_* = \frac{2}{3(|\omega_\phi|-1)a_0^{3(|\omega_\phi|-1)/2}}, \quad (3.14)$$

and

$$V(\Phi) = \frac{3\sqrt{|\omega_\phi|}}{8\pi G \left[ a_0^{-3(|\omega_\phi|-1)/2} - \frac{3}{2} \sqrt{|\omega_\phi|} - 1(\Phi - \Phi_0) \right]^2}, \quad (3.15)$$

with \(\Phi_0 \rightarrow -i\phi_0\). We note that both this potential and the phantom tachyon energy density,

$$\rho_\Phi = \frac{3}{8\pi G \left[ a_0^{-3(|\omega|-1)/2} - \frac{3}{2} (|\omega| - 1)t \right]^2}, \quad (3.16)$$

increase with time up to blowing up at \(t = t_*\), to steadily decrease toward zero thereafter. Thus, the tachyon model for dark energy contains a regime for phantom energy which preserves all the weird properties shown by this in current quintessence and k-essence scenarios; i.e. superluminal speed of sound, increasing energy density, violation of dominant energy condition and a big rip singularity.
4 Sub-quantum phantom cosmology

If we extend next the concept of tachyon dark energy to include also a sub-quantum potential $V_{SQ}$, then the Lagrangian for the system can be generalized to read [11]:

$$L = -V(\phi)E(x, k), \quad (4.17)$$

where $E(x, k)$ is the elliptic integral of the second kind [14], with

$$x \equiv x(\phi) = \arcsin \sqrt{1 - \dot{\phi}^2}, \quad k \equiv k(\phi) = \sqrt{1 - \frac{V_{SQ}^2(V(\phi))}{V(\phi)^2}}, \quad (4.18)$$

in case that we consider a FRW spacetime. We note that in the limit $V_{SQ} \to 0$, the above Lagrangian reduces to the simple expression $L = -V(\phi)\sqrt{1 - \dot{\phi}^2}$ which is the Lagrangian of the tachyon theory discussed in Secs. II and III. Defining as the pressure and energy density,

$$p_\phi = -V(\phi)E(x, k) \quad (4.19)$$

$$\rho_\phi = \frac{V(\phi)\sqrt{1 - \frac{\Delta V^2(1 - \dot{\phi}^2)}{V(\phi)^2}} \dot{\phi}}{\sqrt{1 - \dot{\phi}^2}} + V(\phi)E(x, k), \quad (4.20)$$

where $\Delta V^2 = V(\phi)^2 - V_{SQ}^2$, we get for the sub-quantum model again relation (5), together with

$$\frac{\sqrt{1 - \frac{\Delta V^2(1 - \dot{\phi}^2)}{V(\phi)^2}} \dot{\phi}}{\sqrt{1 - \dot{\phi}^2}} + E(x, k)\sqrt{1 - \dot{\phi}^2} = -\frac{2\dot{H}}{3H^2}, \quad (4.21)$$

and

$$V(t) = -\left[ \frac{2\dot{H}}{8\pi G} - \dot{\phi}^2V_{SQ}^2 \right]^{1/2} \sqrt{1 - \dot{\phi}^2}. \quad (4.22)$$

Now, if an expression like (8) is taken for the scale factor, then $V(t)$ would not vanish in the limit that $t \to \infty$ (actually $V$ becomes an imaginary finite constant at that limit), a behaviour which cannot be accepted as reasonable. Therefore, a form as that is given by Eq. (8) appears here as being not suitable for the scale factor. A Hubble parameter which satisfies however the requirement that $V(t) \to 0$ as $t \to \infty$ is

$$H = -\frac{1}{6(1 + \omega)} \left\{ \sigma(\phi, V_{SQ}, t) + 2\pi G\dot{\phi}V_{SQ} \ln \left[ \frac{\sigma(\phi, V_{SQ}, t) - 4\pi G\dot{\phi}V_{SQ}}{\sigma(\phi, V_{SQ}, t) + 4\pi G\dot{\phi}V_{SQ}} \right] \right\}, \quad (4.23)$$

with

$$\sigma(\phi, V_{SQ}, t) = \sqrt{(4\pi G\dot{\phi}V_{SQ})^2 + \frac{9(1 + \omega)^2}{a_0^3(1 + \omega)^2 + \frac{3}{2}(1 + \omega)t^4}} \quad (4.24)$$
Even though we have not been able to integrate Eq. (23) so that a closed-form expression for the scale factor could be obtained, we are already prepared to check whether or not the tachyon theory equipped with a sub-quantum potential can show a big rip singularity.

From Eqs. (19) and (20) we can in general obtain

$$\omega = -\frac{E(x, k)V(\phi)\sqrt{1 - \dot{\phi}^2}}{\sqrt{\Delta V^2 \dot{\phi}^2 + V_{SQ}^2 \dot{\phi} + E(x, k)V(\phi)\sqrt{1 - \dot{\phi}^2}}} \quad (4.25)$$

$$p_\phi + \rho_\phi = V(\phi)\frac{\sqrt{\Delta V^2 \dot{\phi}^2 + V_{SQ}^2 \dot{\phi}}}{V(\phi)\sqrt{1 - \dot{\phi}^2}} \quad (4.26)$$

It follows from these two expressions that a regime with phantom behaviour showing weird properties similar to those found in the phantom tachyon model can also be achieved by Wick rotating $\phi \rightarrow i\Phi$, if and only if

$$\Delta V^2 \dot{\Phi}^2 > V_{SQ}^2$$

and

$$\sqrt{\Delta V^2 \dot{\Phi}^2 - V_{SQ}^2 \dot{\Phi}} < V(\Phi)E(x, k)\sqrt{1 + \dot{\Phi}^2},$$

in which now $\Delta V^2 = V(\Phi)^2 - V_{SQ}^2$, $x = \sinh^{-1} \sqrt{1 + \dot{\Phi}^2}$, and $k = \sqrt{1 - \frac{V_{SQ}^2}{V(\Phi)^2}}$.

Note that such a regime will only be possible for reasonably large values of $|\dot{\Phi}|$.

On the other hand, by closely inspecting Eqs. (23) and (24) it is not difficult to convince oneself that in the regime where $\omega < -1$ both the scale factor

$$a = \exp \left( \int H dt \right)$$

(which in fact describes an accelerating universe for $\omega > -1$) and the potential (22) blow up at the finite time $t_*$ if $\omega < -1$, so indicating the presence of the big rip singularity in the sub-quantum phantom domain. We have thus shown that the presence of phantom energy in the sub-quantum cosmological model also leads to a big rip doomsday.

5 Conclusions and further comments

By using tachyon-like theories whose starting Lagrangians generally are inspired by string theories, we have investigated the properties of general FRW cosmological solutions that are fully consistent with the whole dynamical structure of the theories. Assuming a general equation of state $p = \omega \rho$, such solutions describe accelerating universes in the interval $-1/3 > \omega > -1$ and, among other weird properties characterizing the phantom regime, all show a big rip singularity in finite time when $\omega < -1$. If in deriving the Lagrangian we begin with a relativistic mechanical momentum-energy quantum relation interpretable in terms of a sub-quantum potential, then the existence of a phantom regime will depend on how much slowly-varying the scalar field is allowed to be. Thus, whenever the
kinetic term for the field is smaller or comparable with the sub-quantum potential then the phantom regime cannot exist in this kind of tachyon theory.

A potential problem with the tachyon models considered in this paper stems from the imaginary value of the sound speed. In fact a value $c_s^2 < 0$ implies occurrence of instability on scales below the Jeans limit for scalar field fluctuations [15] which grow therefore exponentially. Whereas when these models are considered as pure cosmic vacuum components this could in fact be regarded as an actual problem, if the tachyon models are viewed as the sum of two components [9], one describing the negative pressure vacuum stuff and the other describing the dust-like cold dark matter contributing $\Omega_m \sim 0.35$ and clustering gravitationally at small scales, the occurrence of instabilities arising from a negative value for $c_s^2$ might instead be regarded as a way to explain some recent observations in galaxies and superclusters that concern gravitationally collapsed objects such as supermassive black holes and dark-matter galactic halos. That possibility deserves further consideration and is in fact being the subject of a more thorough research by the author.

acknowledgements
The author thanks Carmen L. Sigüenza for useful discussions and Yun-Song Piao and Fabio Finelli for valuable correspondence. This work was supported by DG-ICYT under Research Project BMF2002-03758.

References

1 A.C. Baccigalupi, A. Balbi, S. Matarrase, F. Perrotta and N. Vittorio, Phys. Rev. D65, 063520 (2002); M. Melchiorri, L. Mersini, C.J. Odman and M. Tradden, Phys. Rev. D68, 043509 (2003); M. Doupias, A. Riazuelo, Y. Zolnierowski and A. Blanchard, Astron. Astrophys. 405, 409 (2003); L. Tonry el al., Astrophys. J. 594, 1 (2003); J.S. Alcaniz, astro-ph/0312424 .

2R.R. Caldwell, Phys. Lett. B545, 23 (2002).

3 B. McInnes, JHEP 0208, 029 (2002); G.W. Gibbons, hep-th/0302199; A.E. Schulz and M.J. White, Phys. Rev. D64, 043514 (2001); J.G. Hao and X. Z. Li, Phys. Rev. D67, 107303 (2003); S. Nojiri and S.D. Odintsov, Phys. Lett.B562, 147 (2003); B565, 1 (2003); B571, 1 (2003); P.Singh, M. Sami and N. Dadhich, Phys. Rev. D68, 023522 (2003); J.G. Hao and X.Z. Li, Phys. Rev. D68, 043501; 083514 (2003); X.Z. Li and J.G. Hao, hep-th/0303093, Phys. Rev. D (in press); M.P. Dabrowski, T. Stachowiak and M. Szydlowski, Phys. Rev. D68, 067301 (2003); M. Szydlowski, W. Zaja and A. Krawiec, astro-ph/0401293; E. Elizalde and J. Quiroga H., Mod. Phys. Lett. A19, 29 (2004); V.B. Johri, astro-ph/0311293; L.P. Chimento and R. Lazkoz, Phys. Rev. Lett. 91, 211301 (2003); H.Q. Lu, hep-th/0312082; M. Sami, A. Toporensky, gr-qc/0312009; R. Naboulsi, gr-qc/0303007, Class. Quan. Grav (in press); X.H. Meng and P. Wang, hep-ph/0311070; H. Stefancic, astro-ph/0310904, Phys. Lett. B (in press) ; astro-ph/0312484; D.J. Liu and X.Z. Li, Phys. Rev. D68, 067301 (2003); A. Yurov, astro-ph/0305019; Y.S. Piao and E. Zhou, Phys. Rev. D68, 083515 (2003); Y.S. Piao and Y.Z. Zhang,
astro-ph/0401231; H.Q. Lu, hep-th/0312082; M. Szydlowski, W. Czaja and A. Krawiec, astro-ph/0401293; J.M. Aguirregabiria, L.P. Cimento and R. Lazkoz, gr-qc/0403157; J. Cepa, astro-ph/0403616, Astron. Astrophys. (in press); V. Faraoni, gr-qc/0404078, Phys. Rev. D. (in press); Z.-K. Guo, Y.-S. Piao and Y.-Z. Zhang, astro-ph/0404154; E. Elizalde, S. Nojiri and S. Odintsov, hep-th/0405034; Y.-H. Wei and Y. Tian, gr-qc/0405038; F. Piazza and S. Tsujikawa, hep-th/0405054; S. Nojiri and S.D. Odintsov, hep-th/0405078.

5 R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003); J.D. Barrow, Class. Quant. Grav. 21, L79 (2004).

6 J.D. Barrow, G.J. Galloway and F.J. Tipler, Mon. Not. R. Astron. Soc. 223, 835 (1986).

4 P.F. González-Díaz, Phys. Rev. D68, 084016 (2003).

5 R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).

6 P.F. González-Díaz, Phys. Lett. B586, 1 (2004).

7 A. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B511, 265 (2001); N. Bilic, G.B. Tupper and R. Viollier, Phys. Lett. B535, 17 (2001); M.C. Bento, O. Bertolami and A.A. Sen, Phys. Rev. D66, 043507

8 P.F. González-Díaz, Phys. Rev. D68, 021303(R) (2003); M. Bouhmadi and J.A. Jiménez-Madrid, astro-ph/0404540.

9 T. Padmanabhan, Phys. Rev. D66, 021301 (2002); T. Padmanabhan and T.R. Choudhury, Phys. Rev. D66, 081301 (2002); J.S. Bagla, H.K. Jassal, T. Padmanabhan, Phys. Rev. D67, 063504 (2003)

10 P.F. González-Díaz, Phys. Rev. D69, 103512 (2004).

11 L.R.W. Abramo and F. Finelli, Phys. Lett. B575, 165 (2003).

12 T. Padmanabhan, Structure Formation in the Universe (Cambridge University Press, Cambridge, UK, 1993).

13 P.F. González-Díaz, Phys. Rev. D69, 063522 (2004). Report-no. IMAFF-RCA-03-08

14 M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions (Dover, New York, USA, 1965).

15 D. Carturan and F. Finelli, Phys. Rev. D68, 103501 (2003); H.B. Sandvik, M. Tegmark, M. Zaldarriaga and I. Waga, astro-ph/0212114.