Cosmic-Ray Convection–Diffusion Anisotropy

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Abstract

Under nonuniform convection, the distribution of diffusive particles can exhibit dipole and quadrupole anisotropy induced by the fluid inertial and shear force, respectively. These convection-related anisotropies, unlike the Compton–Getting effect, typically increase with the cosmic-ray (CR) energy, and are thus candidate contributors for the CR anisotropy. In consideration of the inertial effect, CR observational data can be used to set an upper limit on the average acceleration of the local interstellar medium in the equatorial plane to be on the order of 100 $\mu m s^{-2}$. Using Oort constants, the quadrupole anisotropy above 200 TeV may be modeled with the shear effect arising from the Galactic differential rotation.

Unified Astronomy Thesaurus concepts: High energy astrophysics (739); Particle astrophysics (96); Cosmic rays (329); Galactic cosmic rays (567); Cosmic ray astronomy (324); Interstellar medium (847); Interstellar dynamics (839); Interstellar plasma (851); Intergalactic gas (812)

1. Introduction

The arrival direction of cosmic rays (CRs) has been studied with a long history. Observations show that CRs have a power-law spectrum with a highly isotropic angular distribution, whose relative amplitude of anisotropy is on the order of 0.1% at TeV energies. The anisotropy mainly increases with the CR energy, while observations in recent years reveal a local decrease in about 10–100 TeV, and correspondingly a flip of the dipole anisotropy phase roughly from the Galactic anticenter to center direction (Aartsen et al. 2016; Amenomori et al. 2017; Bartoli et al. 2018). For ultrahigh-energy CRs (UHECRs), i.e., those above EeV, the anisotropy seems to increase to the order of 1%, with the dipole phase returning to the anticenter direction (Aab et al. 2017; Abbasi et al. 2020).

It is recognized that the CR anisotropy mainly arises from diffusion rather than convection. Reference is generally made to the point that a standard diffusion model of particles just predicts a dipole anisotropy proportional to the diffusion coefficient, which typically increases with the particle energy. On the contrary, the dipole anisotropy induced by uniform convection, i.e., the Compton–Getting (CG; Compton & Getting 1935) effect, is energy-independent for a power-law spectrum of particles, and is therefore inconsistent with the CR observation.

Obviously, the above argument is not based on a complete description of the convection–diffusion picture, which can only be studied in the concept of the general fluid rest frame with the fluctuation-relaxation theory. Earl et al. (1988) proposed the Bhatnagar–Gross–Krook (BGK) approximation of such a problem to study the CR viscosity. The fluctuational anisotropy obtained in the work includes additional effects from nonuniform convection, which are typically proportional to the diffusion coefficient, implying that they have the potential to explain the CR observation. However, these effects seem to be overlooked in the CR anisotropy problem. This paper provides some further discussion about such effects in combination with the observation.

2. Multipole Expansion

Generally, the multipole expansion of the relative anisotropy can be written as

$$\frac{\Delta f}{f} = -D \cdot \frac{p}{p} + Q_{ij} \frac{p^i p^j}{2p^2},$$

(1)

where $D$ and $Q_{ij}$ are the dipole and quadrupole moment with respect to the observer’s LOS vector $-p/p$, respectively, and $p$ is the particle momentum.

In a scattering system without external forces, the anisotropic fluctuation $\Delta f$ (Equation (5) in Earl et al. 1988) of the particle phase-space distribution function in a local reference frame in which the scattering centers are at rest satisfies the multipole expansion, with

$$D = \tau \left(v \nabla \ln f - \frac{a}{v} \frac{\partial \ln f}{\partial \ln p}\right),$$

(2)

$$Q_{ij} = 2\tau S_{ij} \frac{\partial \ln f}{\partial \ln p},$$

(3)

where $f$ is the isotropic distribution function, $v$ is the particle speed, $\tau$ is the scattering relaxation time, and

$$a = \dot{u},$$

(4)

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i}\right) - \frac{\delta_{ij}}{3} \nabla \cdot u$$

(5)

are the fluid acceleration and shear-rate tensor, respectively, with $u$ the nonrelativistic flow velocity, $x^i$ the $i$th spatial coordinate, and $\delta_{ij}$ the Kronecker delta. As seen, the above anisotropies do not depend directly on $u$ itself, but on its derivative. Via an inertial transformation, one obtains the CG anisotropy with the dipole moment $(u/v)\partial \ln f/\partial \ln p$ for
\( u \ll v \) (Ahlers & Mertsch 2017), which is determined directly by \( u \). For CRs, \( u \) may be considered primarily as the Alfvén velocity.

A brief interpretation to the convection–diffusion approximation can be as follows. In the rest frame of a scattering center, the scattered particle “forgets” its initial state of motion in the scattering time. In a collection of the scattering centers, this may be translated to some extent as “a given anisotropy \( \Delta f \) can completely be relaxed in a time of \( \tau \) only in the fluid rest frame.” This reference frame is generally noninertial, with the inertial force \(-ap/\nu\) and shear-restoring force \(-\nabla p/\nu^2\) acting on a particle, in response to the change of \( u \). A pure scattering system can be considered to be free of external forces. To isotropize the distribution in the time \( \tau \), \( \Delta f \) should be the distributional increment caused by the time reversal motion, i.e., by changes of phase-space coordinates in a time of \(-\tau\), during which Liouville’s theorem is valid. Thus if \( \tau \) is small, one can write \( \Delta f \approx -\tau (\nu \cdot \nabla f + \dot{p} \cdot \partial f/\partial p) \). This directly yields Equation (1) when \( \dot{p} \) represents the inertial plus shear force. The spatial gradient term corresponds to the standard diffuse anisotropy, while for convenience we shall refer to that associated with the inertial and shear effect as the inertial and shear anisotropy, respectively. Note that \( \Delta f \) can consistently be observed under the fluctuation-relaxation equilibrium.

### 3. Inertial Anisotropy

For a microscopic steady flow, the magnitude of the inertial anisotropy in Equation (2) is generally \( O(\nu/u) \) times smaller than that of the shear anisotropy of Equation (3). For turbulence, the shear as well as CG effect will vanish after ensemble averaging over some scales on which the flow is fully stochastic, i.e., \( \langle u \rangle = 0 \), while the inertial effect can exist as \( \langle a \rangle = \langle u \cdot \nabla u \rangle \neq 0 \). If the fluid is further incompressible, i.e., \( \nabla \cdot u = 0 \), one has \( \langle a \rangle = 2u \nabla u/3 \). That is to say, over sufficiently large (spacetime) scales, the average anisotropy associated with the background flow should be dominated by the inertial effect, with the dipole direction roughly along \( \nabla u \) for \( \partial f/\partial p < 0 \). CR observations do include some average effects due to time integration; thus anisotropies on \( O(u/\nu) \) may be suppressed to some extent in the data.

In general, \( \tau \) is an increasing function of \( p \), and is related to the diffusion coefficient \( \kappa \) in Parker’s transport equation by \( \kappa = \pi \tau^2/3 \) (Earl et al. 1988). Thus not only diffusion but also nonuniform convection is eligible for modeling the CR anisotropy, which mainly increases with the particle energy. The inertial term in Equation (1) can be written as

\[
-\frac{\nu + 2}{|\nu + 2|} \mathcal{A}_1 \cos \theta_1, \tag{6}
\]

where \( \nu = -2 - \partial \ln f/\partial \ln p \) is the (ultrarelativistic) energy spectral index, \( \theta_1 \) is the angle between \( p \) and \( a \), and

\[
\mathcal{A}_1 = \frac{T_0}{v} |\nu + 2| \approx 5.6 \times 10^{-4} \left( \frac{c}{v} \right)^3 \times \frac{\kappa}{10^{29} \text{ cm}^2 \text{ s}^{-1}} \frac{a}{100 \mu m \text{ s}^{-2}} |\nu + 2| \tag{7}
\]

is the first-harmonic amplitude with \( c \) the speed of light. On this order of magnitude, if there is any observable contribution to the CR anisotropy by the inertial effect, it is likely to be related to inner structures of the local interstellar medium (ISM). For comparison, we emphasize that the acceleration of the local standard of rest on Galactic scales is only about 0.2 nm s\(^{-2}\) (Klioner et al. 2021). Figure 1 is the schematic view of the inertial anisotropy.

Within a classical fluid concept in which the flow property is independent of the particle energy, the inertial effect cannot solely explain the CR anisotropy at all energies, nor can the typical scenario of pure diffusion on Galactic scales (e.g., the leaky-box model) provide a complete explanation. It has been shown that the observed amplitude dip and phase flip of the TeV dipole anisotropy may be ascribed to a superposition effect of some source components (Ahlers 2016; Qiao et al. 2019; Zhang et al. 2022). Since the diffuse component is presumably essential for the dipole moment, it seems difficult to strictly quantify the CR inertial effect. Nevertheless, one can still estimate an upper limit of the local ISM acceleration by considering that the anisotropy produced by Equation (7) should not much exceed the observed value. On the other hand, a significant inertial effect on the CR anisotropy observationally requires a minimum acceleration. In conclusion, for the inertial anisotropy (\( \nu \sim 3 \)) to overlap the amplitude data in Figure 2, we have

\[
1 \mu m s^{-2} \lesssim \frac{\kappa (\text{TeV})}{10^{29} \text{ cm}^2 \text{ s}^{-1}} a \cos \delta_1 \lesssim 100 \mu m s^{-2}, \tag{8}
\]

where \( \delta_1 \) is the decl. of the dipole direction. This constrains the average acceleration only in the equatorial plane, because here we consider only R.A. projected anisotropy data, which are most reported by existing observations. Such data can be seen as fitting results of the decl.-averaged, R.A.-projected relative fluctuation

\[
\psi = \frac{1}{\sin \delta_{up} - \sin \delta_{low}} \int_{\delta_{low}}^{\delta_{up}} \frac{\Delta f}{f} \cos \delta d\delta, \tag{9}
\]
where $\delta_{\text{low}}$ and $\delta_{\text{up}}$ are the lower- and upper-limit decl. of the observatory’s time-integrated field of view (Ahlers & Mertsch 2017). For the dipole component, one has $\psi_1 = A_1 \cos (\alpha - \phi_1)$, where $\alpha$ is the R.A. of the LOS direction, and the phase $\phi_1$ is equal to the R.A. of the dipole direction $\alpha_1$. For our crude estimation, it may be enough to assume a full-sky scan with $A_1 = \pi A_0 \cos \delta_1/4$.

Although it is possible to neutralize an overproduced inertial anisotropy via introducing diffuse components with specified dipole directions, the upper limit in Equation (8) makes sense for physical simplicity. Say, if the CR anisotropy below 10 TeV is ascribed to the inertial effect, the data above 10 TeV are difficult to be explained. Moreover, as there are indications that the dipole anisotropy around EeV has similar flattening property than that around 100 TeV, which needs to be further verified by future precise observations of UHECRs, all data below EeV are unlikely to have a simple explanation by an inertial effect with energy-independent $a$. However, for turbulence, the flow property depends on the scale of view. Since this scale must be related to the Larmor radius, $a$ can have dependence on the particle energy. It is then possible to model the anisotropy decrease via a competition effect of low- and high-energy flows with opposite directions of $a$. The observed anisotropy then can be used to probe the properties of the turbulent flow.

4. Shear Anisotropy

As we know, any deformation of a continuous medium can be decomposed into an isotropic part, i.e., an expansion (or compression) characterized by the strain-rate tensor $\delta_{ij} \nabla \cdot \mathbf{u}/3$, and an anisotropic constant-volume part known as a pure shear with the strain rate $S_{ij}$ (Landau & Lifshitz 1959). Interestingly, the shear anisotropy of the microscopic distribution corresponds exactly to that of the macroscopic medium via Equation (3), while the isotropic deformation affects only the isotropic part of the particle distribution through the adiabatic process.

As a traceless symmetric tensor, $S_{ij}$ generally contains five independent parameters, and can (locally) be diagonalized via a rotation transformation to the eigenbasis. The diagonalized $S_{ij}$ is expressed as a sum of shear-rate tensors that represent simple extensions (or shortening) along the eigenvectors, with the parameters being three Euler angles (determining directions of the eigenvectors) and two of the eigenvalues. A simple extension can be obtained from a 1D flow, i.e., $\mathbf{u} = u(z) \hat{z}$, where $u_z$ spatially depends only on $z$, and $\hat{z}$ denotes the unit vector in the $z$-direction. In this system, $S_{ij}$ contains three independent parameters, i.e., two of the Euler angles (determining $\hat{z}$) and one of the eigenvalues. Then the quadrupole term in Equation (1) can be described via the Legendre polynomial

$$-\frac{4}{3} S_{zz} \frac{\nu + 2}{|S_{zz}|} |\nu + 2| A_2 P_2 (\cos \theta_2),$$

where $\theta_2$ is the angle between $\mathbf{p}$ and $\hat{z}$, and

$$A_2 = \frac{3}{4} \frac{\tau |S_{zz}| |\nu + 2|}{|S_{zz}|} \frac{4 \times 10^{-4} \left(\frac{c}{v}\right)^2}{\frac{10^{29} \text{cm}^2 \text{s}^{-1}}{10 \text{ Myr}^{-1}}} \frac{\kappa}{5},$$

is the second-harmonic amplitude. Figure 3 is the schematic view of the simple extension, showing that the particle distribution tends to increase in shortening directions of the fluid element (if $\nu > -2$). For a steady state, the bulk acceleration is

$$u_z \frac{\partial u_z}{\partial z} = \frac{3}{2} u_z S_{zz} \approx 4.8 \frac{u_z}{10 \text{ km s}^{-1}} \frac{S_{zz}}{10 \text{ Myr}^{-1}} \text{nm s}^{-2}.$$  

This means that the quadrupole anisotropy, compared with the dipole, is much more easily induced by a nonuniform flow. The presence of such a quadrupole effect in CR data depends on the interpretation of observations. As mentioned previously, the shear anisotropy may be important only if regularity of the flow survives after ensemble averaging. Nevertheless, there is likely to be a significant effect of the ISM shear on the CR anisotropy, as the characteristic acceleration producing the observed quadrupole amplitude is only around nm s$^{-2}$.
To compare with observations over a broad energy range, it is better to study the R.A. projected anisotropy. Considering a full-sky scan for simplicity, following Equation (9) the projection of any quadrupole fluctuation onto the equatorial plane is

\[ \psi_2 = \frac{1}{4} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left( \begin{array}{ccc} \cos \delta \cos \alpha & \cos \delta \sin \alpha & \sin \delta \\ \cos \delta \sin \alpha & \cos \delta \cos \alpha & \cos \delta \\
\sin \delta & \cos \delta & \sin \delta \end{array} \right) \left( \begin{array}{ccc} Q_{\xi \xi} & Q_{\xi \eta} & Q_{\xi \zeta} \\ Q_{\eta \xi} & Q_{\eta \eta} & Q_{\eta \zeta} \\ Q_{\zeta \xi} & Q_{\zeta \eta} & Q_{\zeta \zeta} \end{array} \right) \cos \delta d\delta d\alpha \\
= \frac{1}{3} \left[ \frac{Q_{\xi \xi} - Q_{\eta \eta}}{2} \cos(2\alpha) + Q_{\xi \eta} \sin(2\alpha) \right] \\
= A_2 \cos(2(\alpha - \phi_2)), \tag{13} \]

where \( \xi, \eta, \) and \( \zeta \) are equatorial rectangular coordinates, with \( \xi \) and \( \zeta \) denoting the vernal-equinox and north-polar direction, respectively. That is to say, ideally the quadrupole effect on the equatorial plane can always be described via second Fourier harmonics. To express \( A_2 \) and \( \phi_2 \) in terms of eigenvalues of \( Q_{ij} \) (with the eigenvectors in \( x-, y-, z\)-directions), we rewrite the equatorial representation of \( Q_{ij} \) as

\[ R_{\xi}^\phi R_{\phi}^\zeta R_{\eta}^\gamma \left( \begin{array}{ccc} Q_{xx} & 0 & 0 \\ 0 & Q_{yy} & 0 \\ 0 & 0 & Q_{zz} \end{array} \right) R_{\xi}^\phi R_{\phi}^\zeta R_{\eta}^\gamma, \tag{14} \]

where \( R_{ij}^\phi \) denotes the rotation matrix rotating a vector by an angle of \( \phi \) about the \( i \) axis, \( \alpha_2 \) and \( \delta_2 \) are the R.A. and decl. of \( \zeta \), respectively, and \( \phi \) is the azimuthal parameter in the \( x-y \) plane.

After some calculations, we find

\[
A_2 \begin{bmatrix} \cos(2\phi_2) \\ \sin(2\phi_2) \end{bmatrix} = \frac{R_{\phi}^\phi}{2} \begin{bmatrix} q_{xx} - q_{yy} \left( 1 - \cos^2 \delta_2 \right) \cos(2\phi_2) + \frac{q_{zz}}{2} \cos^2 \delta_2 \\ \frac{q_{xx} - q_{yy}}{3} \sin \delta_2 \sin(2\phi_2) \end{bmatrix}, \tag{15} \]

where \( R_{\phi} \) is the 2D rotation matrix.

In particular, for a 1D flow with \( q_{xx} = q_{yy} \), one has \( A_2 = 2A_2 \cos^2 \delta_2/3 \), \( \phi_2 = \alpha_2 + n\pi \) if \( q_{zz} > 0 \), and \( \phi_2 = \alpha_2 + (n + 1/2)\pi \) if \( q_{zz} < 0 \), where \( n \) is an arbitrary integer. We may naively make use of this simple formalism, just as we did to obtain Equation (8), to give an order-of-magnitude estimate of the ISM shear rate required to produce the CR anisotropy shown in Figure 4. The data suggest that

\[
0.1 \text{ Myr}^{-1} \lesssim \frac{\kappa(\text{TeV})}{10^{30} \text{ cm}^2/\text{s}} |S_{zz}| \cos^2 \delta_2 \lesssim 10 \text{ Myr}^{-1}, \tag{16} \]

and there is also an amplitude dip and phase flip for the second-harmonic signal around 100 TeV. Note that we have shifted some of the \( \phi_2 \) data reported by original literature by \( \pm \pi \) to obtain the two-phase-like shape, based on the fact that all values of \( \phi_2 + n\pi \) are equivalent. Despite the lack of observations, the available data show no strong indication for a local decrease of \( A_2 \) at ultrahigh energies, in agreement with constant \( \phi_2 \) in the same energy range. Only then it is possible to
explain the R.A. projected quadrupole anisotropy above 200 TeV with an energy-independent fluid shear, e.g., the 1D flow with \( \alpha_2 \sim 0^\circ \) and \( Q_{zz} > 0 \). The phase flip around 100 TeV may indicate a transition between two quadrupole anisotropy regimes. A possible scenario is that low-energy CRs are largely affected by the local interstellar magnetic field (LIMF), whose turbulence gives rise to multipole anisotropies (Ahlers et al. 2014). For a \( \mu G \) LIMF, if its coherence length is 0.1 pc, 100 TeV should be the critical energy above which protons are no longer trapped by the LIMF. Intuitively, it is also possible that the phase flip arises from a competition effect of energy-dependent flows, or of magnetic fields averaged over small and large scales.

A few observations also report detailed parameters of \( Q_{ij} \) from fits directly to the CR anisotropy sky map. As far as we know, above 200 TeV such data have only been reported by the Pierre Auger Observatory and Telescope Array at EeV energies (Abbasi et al. 2014, 2018). The reported eigenvalues, i.e., \( \lambda_+, \lambda_0, \) and \( \lambda_0 (\lambda \equiv Q/2) \) shown in Figure 4, reveal an interesting feature: \( \lambda_+ \sim -\lambda_0 \gg |\lambda_0| \). This motivates us to set \( Q_{zz} = 2\lambda_0 = 0 \), which indicates \( A_2 \leq 2\lambda_+ /3 \) according to Equation (15). Since the \( A_2 \) and \( \lambda_+ \) data appear to be comparable, \( \delta_2 \) should be far from \( 0^\circ \) if \( \varphi = (\alpha + 1/2)\pi /2 \). The angular parameters are unreported in Abreu et al. (2012), while above 10 EeV they can be found in Aab et al. (2014). Aab et al. (2018) reported all independent components of \( Q_{ij} \) above 4 EeV in equatorial rectangular coordinates. However, these reconstruction results are still not statistically significant.

In the shear anisotropy model, the Auger quadrupole data imply \( S_{XY} = 0 \), which however does not represent a (quasi) 1D flow. Instead, the simplest configuration may be an incompressible simple-shear flow, i.e., \( u = u_\nu(X)\hat{\nu} \) with the following diagonalization of \( S_{ij} \),

\[
\begin{pmatrix}
0 & S_{XY} \\
S_{XY} & 0
\end{pmatrix}
R^\nu = R^\nu \begin{pmatrix}
S_{XY} & 0 \\
0 & -S_{XY}
\end{pmatrix}.
\]

where

\[
\frac{1}{2} \frac{\partial u_\nu}{\partial X} = |S_{XY}| = \frac{\lambda_+}{\tau|\nu + 2|}
\approx 19(v/c)^2 10^{33} \text{ cm}^2 \text{ s}^{-1} 0.01 \text{ Gyr}^{-1}.
\]

For suitable values of \( \kappa, \nu, \) and \( \lambda_+ \), this rate of shear can be comparable with the Oort constant \( A \), which measures the local azimuthal shear rate in the Galactic plane, and is significantly greater than the radial shear rate \( C \) and divergence rate \( K \). We thus naturally speculate that the azimuthal shear due to the Galactic differential rotation has significant effects on the CR quadrupole anisotropy. The weak CG effect does indicate a Galactic corotation of CRs (Amenomori et al. 2006), which implies high electric conductivity such that the magnetic field is frozen into the rotating plasma fluid.

Assuming vertical homogeneity of the local ISM with respect to the Galactic plane (i.e., \( \partial u_\nu /\partial z = \nabla u_\nu = 0 \)), the “realistic” average \( S_{ij} \) in the Galactic coordinate system (with \( \hat{X} \) and \( \hat{Y} \) directed at the Galactic 0° and 90° longitude, respectively) is expressed as follows in terms of the Oort constants (Li et al. 2019),

\[
\begin{pmatrix}
K/3 + C & A & 0 \\
A & K/3 - C & 0 \\
0 & 0 & -2K/3
\end{pmatrix} \approx \begin{pmatrix}
-3.3 & 15 & 0 \\
15 & 2.2 & 0 \\
0 & 0 & 1.1
\end{pmatrix} \text{ Gyr}^{-1}.
\]

This can directly be used to calculate the CR shear anisotropy. Eliminating all line-of-sight (LOS)-independent and dipole terms in Equation (1), the relative quadrupole intensity with excess-deficit symmetry is obtained. The left-hand panel of Figure 5 shows the predicted shear anisotropy sky map, whose
full-sky R.A. projection gives

\[ A_2 \approx 0.003 \frac{\kappa}{10^{33} \text{ cm}^2 \text{s}^{-1}} \tag{20} \]

and \( \phi_2 \approx 28^\circ \). The right-hand panel of Figure 5 shows the most similar intensity profile within uncertainties, to the left-hand panel, derived with reconstructed \( Q_{ij} \) in Aab et al. (2018), with the parameters adopted given in Table 1. For \( \kappa \) (5 EeV) \( \sim \) \( 10^{33} \text{ cm}^2 \text{s}^{-1} \) that is roughly consistent with some theoretical expectations of UHECR propagation (Giacinti et al. 2012), the two maps are highly coincident. It should be noted that within uncertainties the Auger profile can also differ remarkably from the model prediction. On the other hand, the inertial anisotropy calculated from Equation (7) using the Oort constants gives \( A_0 \) (5 EeV) \( \sim \) \( 10^{-5} \), which can be neglected compared with the observed CR dipole anisotropy. However, the true average inertial effect may not be so weak as it depends on the variance of turbulence.

Given the large mean free path \( 3 \kappa \) (5 EeV) \( / \nu \sim \) \( 30 \) kpc, the fluid description of UHECRs in fact makes sense on intergalactic scales. The coincidence of the Oort and Auger anisotropy may imply a corotation of the intergalactic medium with the Galactic disk. Recent observations of X-ray absorption have inferred that within about 100 kpc from the Galactic center (GC), the hot gaseous halo spins at comparable velocities as the disk (Hodges-Kluck et al. 2016). As \( \tau \) (5 EeV) \( \sim \) \( 100 \) kyr \( \ll \) \( 70 \) Myr \( \approx \) \( 1/A \), the UHECR relaxation is still rapid in comparison with the quasi-static shear flow.

After all, the Oort shear rate of Equation (19) is derived typically from stellar and ISM populations within a heliocentric distance of a few kiloparsecs (Bovy 2017). Such "disk" hydrodynamics should be established under \( \kappa \lesssim \) \( \times 10^{32} \text{ cm}^2 \text{s}^{-1} \), which constrains \( A_2 \lesssim \times 3 \times 10^{-4} \) according to Equation (20). Also given the change of \( \phi_2 \) around 200 TeV, the Oort anisotropy model is better to be compared with PeV CR data, yet detailed results of \( Q_{ij} \) at such energies have not been reported in the literature. Nevertheless, the R.A. projected data in Figure 4 imply \( A_2 \) (PeV) \( \sim \) \( 3 \times 10^{-4} \). We may then define that the Oort anisotropy is important if \( \kappa \) (PeV) \( \gtrsim \) \( \times 10^{31} \text{ cm}^2 \text{s}^{-1} \), which can be reached via \( \partial \ln \kappa \) (TeV–PeV) \( / \partial \ln p \gtrsim \) \( 1/2 \), provided that \( \kappa \) (TeV) \( \sim \) \( \times 10^{20} \text{ cm}^2 \text{s}^{-1} \) as inferred from recent studies of AMS-02 data (Di Mauro et al. 2020). The Oort anisotropy dominates the PeV range if the validity of Equation (19) can be extended to intergalactic hydrodynamics with \( \kappa \) (PeV) \( \sim \) \( 10^{32} \text{ cm}^2 \text{s}^{-1} \). In that case, the post-knee quadrupole anisotropy may be an indirect evidence of dark halo spin.

### Table 1

| Reconstructed \( Q_{ij} \) in 4–8 EeV Reported by Aab et al. (2018), and That Used in the Right-hand Panel of Figure 5 | \( Q_{ij} \) | \( Q_{ij} - Q_{me} \) | \( Q_{ij} - Q_{e} \) |
|---|---|---|---|
| Aab et al. (2018) | 0.011 | 0.007 | 0.0097 |
| Figure 5 Right-hand Panel | 0.004 ± 0.015 | 0.007 | |
| | -0.02 ± 0.019 | -0.001 | |
| | -0.005 ± 0.019 | 0.0093 | |

5. Summary

In this paper, based on the BGK description of the convection–diffusion system, we discuss the effect of nonuniform convection on the CR anisotropy. Assuming that relaxation of particle trajectories can only be observed in the fluid rest frame, the inertial and shear-restoring force give rise to dipole and quadrupole anisotropy in the particle distribution, respectively. Unlike the CG effect, these two convection-related anisotropies typically increase with the CR energy, and are thus eligible for modeling the CR observation. After looking into the data, we conclude:

1. There is no explanation of the CR anisotropy at all energies simply by energy-independent nonuniform convection;
2. The decrease of the dipole anisotropy in 10–100 TeV implies an upper limit about 100 \( \mu \text{m s}^{-2} \) for the local ISM average acceleration in the equatorial plane;
3. The quadrupole anisotropy above 200 TeV may be relevant to the shear effect due to the Galactic differential rotation (including halo spin), which can partially be characterized with Oort constants.
We hope that future experiments such as LHAASO (Ma et al. 2022) can provide more precise measurements and comprehensive analyses of CR multipole anisotropies above PeV, which are essential for further validation of the model.

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