Effects of non-uniform magnetic field on the spin current of strongly dissipatively driven XXZ spin chains

Kang Hao Lee, 1 Vinitha Balachandran, 1 Ryan Tan, 2 Chu Guo, 3 and Dario Poletti 1, 2, 3

1 Science and Math Cluster, Singapore University of Technology and Design, 8 Somapah Road, 487372 Singapore
2 Engineering Product Development Pillar, Singapore University of Technology and Design, 8 Somapah Road, 487372 Singapore
3 Henan Key Laboratory of Quantum Information and Cryptography, Zhengzhou 450004, China

(Dated: February 3, 2020)

In XXZ chains, spin transport can be significantly suppressed when the anisotropy in the chain and the bias of the dissipative driving are large enough. This phenomenon of negative differential conductance is caused by the formation of two oppositely polarized ferromagnetic domains at the edges of the chain. Here we consider the effect of a non-uniform, or disordered, magnetic field. We show that, while in general disorder reduces the current, in the regime of negative differential conductance disorder can enhance currents. This effect is limited and more pronounced in shorter spin chains. We consider different types of disorder distributions: uniform, algebraic and dichotomous. Analyzing in detail the effects of dichotomous disorder we find a resonant behavior with peaks of currents related to avoided crossings in the energy spectrum of the spin chain. Furthermore, by studying the effect of all possible configurations of the magnetic field, we find a configuration which results in a very strong spin current rectification, thus turning the spin chain into an effective diode.

I. INTRODUCTION

Quantum spin systems exhibit rich transport properties. For instance, tuning the interactions in the system, spin transport can change from ballistic to diffusive [1–3]. One effect that is particularly relevant for our work is the emergence of negative differential conductance (NDC), that is the phenomenon whereby the spin current decreases as the bias imposed by the spin baths increases [4, 5]. Such an apparently counterintuitive phenomenon is due to the fact that the interplay between the dissipative driving and the interactions in the system result in the formation of ferromagnetic domains at the edges of the chain, which significantly suppress the spin current. The effect can be so strong that the spin chain becomes an insulator. The addition of dephasing, which typically suppresses the current, results in an enhancement of the current by reducing the ferromagnetic domains [6]. Here, instead, we investigate how disorder affects the NDC.

It should be pointed out that disorder can strongly affect the transport properties of a system. A disordered non-interacting spin system is an Anderson insulator, with exponentially localized states [7]. In presence of disorder and interactions, a system may be many-body localized [8, 9]. It has also been presented that, before the many-body localization transition, transport may be sub-diffusive [10–12]. Hence, generically, disorder slows down or impedes transport. In the context of negative differential conductance, and in particular of the ferromagnetic domains that cause it, disorder could weaken the order in the domains and thus allow larger spin currents.

Hence, here we study a boundary driven spin chain in the NDC regime with an external disordered magnetic field. In order to obtain more generic conclusions we consider three different types of disorder realizations: (i) uniformly distributed, (ii) algebraically distributed and (iii) dichotomous, i.e. the disorder only takes two possible values. Regardless of the nature of disorder, we find that the NDC regime is robust against disorder, although we find a reduction of NDC and a corresponding enhancement of current in presence of strong external dissipative bias. We show that this is due to the reduction of the ferromagnetic domains. We point out that the current enhancement is limited in size, and it is more prominent for shorter spin chains. For the particular case of dichotomous disorder, we find that the current is enhanced at some particular magnitudes of disorder, which are due to the interplay between the dissipative boundary driving and the energy level structure of the spin chain. A more detailed study of the effect of different dichotomous realizations of the magnetic field show that two configurations, such that the magnetic field is in one direction in half of the chain and in the other direction for the other half, strongly enhance or even more strongly suppress the ferromagnetic domains. This results, respectively, in the smallest or largest spin currents between all the possible dichotomous realizations of the magnetic field. Since these two configurations are mirror-symmetric, we study the rectification properties of a strongly interacting chain with this external magnetic field, and we show that spin current rectification can be of the order of 108. This adds significantly to the recent results on rectification in spin chains with no disorder, [13–14], with disorder [15] or with external fields at the edges [16].

The manuscript is organized as follows: in Sec. II we describe our model, and in Sec. III we discuss our results. Last, in Sec. IV we draw our conclusions.
II. MODEL

We consider an XXZ spin chain of length \( L \) with the following Hamiltonian

\[
\hat{H} = \sum_{i=1}^{L-1} 2J(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_i^- \hat{\sigma}_{i+1}^+) + J_{zz}\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \sum_{i=1}^{L} h_i \hat{\sigma}_i^z, \tag{1}
\]

where \( \hat{\sigma}_i^\pm \) are the raising and lowering operators acting on site \( i \) and \( \hat{\sigma}_i^z \) is a Pauli spin matrix. \( J \) and \( J_{zz} \) denote the tunneling strength and magnitude of the nearest neighbor interaction respectively. In the following, the interplay between tunneling and interaction will be parameterized by the anisotropy defined as \( \Delta = J_{zz}/J \). We use \( h_i \) for the local field.

The chain is coupled to two spin baths at the edges and we model the evolution via a GKSL master equation

\[
\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + \sum_{n=1}^{4} \Gamma_n \hat{\rho} \Gamma_n^\dagger - \frac{1}{2} \sum_{n=1}^{4} \{ \Gamma_n^\dagger \Gamma_n, \hat{\rho} \}, \tag{2}
\]

where the \( \Gamma_n \) are the jump operators given by

\[
\Gamma_1 = \sqrt{\gamma \mu_L} \hat{\sigma}_i^+, \quad \Gamma_2 = \sqrt{\gamma (1 - \mu_L)} \hat{\sigma}_i^-, \tag{3}
\]

\[
\Gamma_3 = \sqrt{\gamma \mu_R} \hat{\sigma}_i^-, \quad \Gamma_4 = \sqrt{\gamma (1 - \mu_R)} \hat{\sigma}_i^+. \tag{4}
\]

Here \( \gamma \) describes the system-reservoir coupling strength and \( \mu_L (\mu_R) \) is the left (right) dissipation bias. We choose a symmetric driving at the boundaries, i.e. \( \mu_{L,R} = (1 \pm \mu)/2 \). Thus, \( \mu \equiv \mu_R - \mu_L \in [0,1] \) is the dissipative boundary driving bias due to the reservoirs. In the limiting case when \( \mu = 1 \) so that \( \mu_L = 0 \) and \( \mu_R = 1 \), the left reservoir tries to impose spins up to spins down conversions, while the right reservoir would do the opposite, only converting spins down to spins up.

The non-equilibrium steady state (NESS) of the system, \( \hat{\rho}_{ss} \), is computed by setting the time derivative to zero in Eq. (2) and using exact diagonalization. The spin current \( \mathcal{J} \) is defined by the continuity equation for local magnetisation \( \hat{\sigma}_i^z \), so that \( \dot{\mathcal{J}} = \text{Tr}(\hat{j}_i \hat{\rho}_{ss}) \), where \( \hat{j}_i = 4iJ(\hat{\sigma}_i^- \hat{\sigma}_{i+1}^+ - \hat{\sigma}_i^+ \hat{\sigma}_{i+1}^-)/\hbar \). In the following we work in units for which \( J \) and \( h \) are 1.

To broaden the scope of our results, we consider three different distributions that have markedly different characteristics. We consider a dichotomous distribution, for which, on each site, the local field \( h_i \) can take only the two discrete values \( \pm h \) with equal probabilities. For this distribution the average is 0 and the variance is \( h^2 \). We also use a uniform distribution for which the local field can take any value in the interval \([-b,b]\), which has the zero mean and variance of \( b^2/3 \). In addition, we consider an algebraic distribution which has unbounded support and power-law tails, i.e. \( p(x) = A/((cx)^4 + 1) \) with \( A = \sqrt{2c}/\pi \) which has 0 mean and variance of \( 1/c^2 \). In order to compare fairly the three distributions, i.e. so that their first two moments are identical, we set \( b = \sqrt{3}h, c = 1/h \). In our simulations we consider up to 5000 disorder realizations for the uniform and the algebraic distributions, while for dichotomous noise we use all \( 2^L \) possible disorder realizations.

III. RESULTS

![Figure 1](image.png)

Figure 1: (a-f) Spin current \( \mathcal{J} \) versus driving strength \( \mu \) for different magnitudes of disorder, \( h = 0 \) (solid line), \( h = 0.5 \) (dashed line), \( h = 1.5 \) (dotted line) and \( h = 4 \) (dot-dashed line). Panels (a,b) are for uniformly distributed disorder, (c,d) for dichotomous disorder and (e,f) for algebraically distributed disorder. Panels (b,d,f) are enlargements of the dashed regions in (a,c,e). (g) Current \( \mathcal{J} \) as a function of \( h/\Delta \) at \( \mu = 1 \) for different types of disorder, uniformly distributed disorder (green dashed line), for dichotomous disorder (blue dotted line) and for algebraically distributed disorder (red dashed line). For both the uniform and the algebraic distributions we have averaged over 5000 realizations. The errorbars indicate the standard error. Other common parameters are anisotropy \( \Delta = 4 \), system-reservoir coupling \( \gamma = 1 \) and system size \( L = 4 \).

To analyse the effect of disorder on the transport properties in the NDC regime, in Fig.1(a-f) we plot the steady-state spin current \( \mathcal{J} \) versus bias \( \mu \). In Fig.1(a,b) we consider the disorder to be uniformly distributed, in Fig.1(c,d) dichotomous disorder and in Fig.1(e,f) the algebraic distribution. In the absence of disorder (black solid line), there is a strong negative differential conductance, which significantly reduces the current for larger values of \( \mu \). The origin of NDC can be traced back to
the formation of ferromagnetic domains at the two edges of the chain at large biases \cite{4,5}. The remaining lines in Figs.1(a-f) correspond to different magnitudes of disorder. The presence of disorder has two effects: for small to intermediate values of the bias \( \mu \), disorder reduces the current, however, for \( \mu \approx 1 \) the current for the disordered case can be larger than that of the clean case. This is shown more clearly in Figs.1(b,d,f) which are an enlargement of the regions in the dashed boxes of, respectively, panels (a,c,e). While the NDC effect is reduced, it seems that even for these small systems, the NDC is generically robust. We are thus set to analyze the current at the maximum bias \( \mu = 1 \) for different types and magnitudes of the disordered magnetic field. In Fig.1(g) we plot the current \( J \) as a function of \( h/\Delta \) for \( \mu = 1 \) and for different types of disorder: dichotomous noise (blue dotted line), uniformly distributed (green dashed line) and algebraically distributed (red dashed line). Here we observe more clearly that increasing disorder can, deeply in the regime of NDC, lead to significant increase of spin current. We also notice a resonant response for the dichotomous noise which we will study in more detail later.

Figure 2: Spin magnetization profiles \( \langle \sigma_z^i \rangle \) for the clean case without disorder (black continuous line), dichotomous disorder (blue dotted line), uniform distribution disorder (green dashed line), and algebraic distribution (red dot-dashed line). Panels (a) and (b) correspond to disorder strength \( h = 2 \) and panels (c) and (d) correspond to disorder strength \( h = 4 \). Panels (a) and (c) are for a system size of \( L = 6 \) and panels (b) and (d) are for a system size of \( L = 8 \). For both the uniform and the algebraic distributions we have averaged over 5000 realizations. The errorbars indicate the standard error. Common parameters in all panels are \( \Delta = 4 \), \( \gamma = 1 \) and \( \mu = 1 \).

To investigate the response of the ferromagnetic domains to disorder, we study the magnetization profile in the chain at \( \mu = 1 \) in Fig. 2. Here panels (a,c) are for \( L = 6 \), and panels (b,d) for \( L = 8 \), while a weaker disorder \( h = 2 \) is considered for panels (a,b), and a larger one, \( h = 4 \), for panels (c,d). In clean chains (continuous line), two oppositely polarized magnetic domains are present, thus inhibiting the current. The results of the disordered chains are depicted by the other lines, following the representation of Fig.1(g). In the presence of disorder we find that the size of domains are reduced favoring transport across the chain, and resulting in larger currents compared to the chain without disorder. However we also note that even for large disorder, ferromagnetic domains, although reduced, persist near the edges. From Figs.1 and 2 we notice that for small and large disorder strengths, the two disorder distributions give very similar results.

In Fig.1(g), for the dichotomous disorder case, we observed a non-monotonous behavior of the current as a function of the magnitude of disorder \( h \). We thus study this phenomenon more in detail in Fig.3. Here we plot the current versus disorder strength \( h/\Delta \) at maximal driving \( \mu = 1 \) for different system sizes \( L \), and for different anisotropies \( \Delta \). In particular we present the cases for \( L = 4 \) in panel (a), \( L = 6 \) in panel (b), and \( L = 8 \) in panel (c), while the solid continuous lines corresponds to \( \Delta = 2 \), the red dashed line to \( \Delta = 4 \), the green dash-dotted line to \( \Delta = 6 \), and the black dotted line to \( \Delta = 8 \). Larger anisotropies result in more defined peaks, hence to a large relative increase of the current compared to the case without disorder \( h = 0 \), although overall the current is reduced when the anisotropy’s magnitude is
greater. For larger system sizes the number of peaks increases which, as we will show later, is due to the more complex structure of the system Hamiltonian.

Figure 4: Spin current $J$ as a function of disorder strength $h$ for system sizes $L = 4$ (a), $L = 6$ (b) and $L = 8$ (c). Different lines correspond to each of the $2^L$ configurations of dichotomous disorder. We highlight two magnetic field realizations from the disorder distribution: with the red dot-dashed line we show the current for a field which is $h$ for the first half of the chain, and $-h$ for the second half of the chain, which we refer to as $(+, ..., +, - ,..., -)$, and with the blue dashed line the realization in which the field is $-h$ in the first half of the chain and $h$ in the second half $(+, ..., - ,+ ,..., +)$. The average of all disorder realizations, which is the equivalent to the dichotomous disorder distribution plot in Fig. 3, is shown as thick pink continuous line. Common parameters are $\Delta = 4$, $\gamma = 1$ and $\mu = 1$. Peaks of the red-dotted line in panel (a) are signalled by a think black dashed line.

Note that dichotomous distribution is equivalent to the average of $2^L$ possible disorder configurations with local field $h_i$ as either $+h$ or $-h$. To gain better understanding, we plot the current contribution from different configurations in Fig. 4. In particular we highlight with a red-dotted line the curve corresponding generally to the disorder configuration which results in the largest current, and with the blue-dashed line the curve corresponding to the smallest current, while the curves corresponding to all the other disorder configurations are thin and grey. With the pink continuous line we show the average of all the possible disorder configurations. In the three panels we present the cases with $L = 4$, panel (a), $L = 6$, panel (b) and $L = 8$, panel (c). In all the panels of Fig. 4 we observe that the disorder configuration corresponding to the largest current is such that the local magnetic field is positive in the first half of the chain, and negative in the second half, which we indicate by $(+, +, - ,..., -)$, $(+, +, +, - ,..., -)$ and $(+, +, +, - ,..., -)$ respectively in panels (a-c). On the contrary, the configuration corresponding to the smallest current is such that the local magnetic field is negative in the first half of the chain and positive in the second half, i.e. $(-, - ,+ ,..., +)$.

We now analyze in more detail the origin of the peaks in the current vs disorder strengths in Fig. 3 and Fig. 4 for the dichotomous disorder. The peaks occur at values of $h \approx \Delta/2$ and $\Delta$, which hints that this could be due to the energy level structure of the system Hamiltonian. So here we consider two things: (i) that in the strong NDC regime without external magnetic field, the steady state is well approximated by a product state with half spins up and half spins down $|↓↓↑↑\rangle$, and (ii) that the magnetic field configuration which more strongly affects NDC is such that the magnetic field points up for the first half of the chain and down for the other half (thus going against the ferromagnetic domains). We thus study the energy spectrum of the Hamiltonian as a function of the disorder strength for a magnetic field configuration $(+, ..., +, - ,..., -)$. In Fig. 5 we show the different eigenenergies $E_s$ for $\Delta = 4$ and system size $L = 4$. For each disorder magnitude $h$ we plot a point to indicate the value of the energy. We also use a different color for each point depending on the overlap of the corresponding eigenvector $|\psi_s\rangle$ with the state $|↓↓↑↑\rangle$, i.e. $|\langle\psi_s | ↓↓↑↑\rangle|^2$. With vertical dashed lines we show the position of the peaks of current for system size $L = 4$, as taken from Fig. 4. In this way we can indeed show that the avoided crossings correspond with the maxima of the current.

In Fig. 4 we noted that two particular configurations of the magnetic field correspond to the largest and the smallest currents. This is due to the fact that the configuration $(-, ..., - ,+ ,..., +)$ favors the formation of the magnetic domains which inhibit transport, thus
ensuring a very small current, while the configuration 
(+, ..., +, −, ..., −) is the one which best disfavours
the formation of the ferromagnetic domains, and thus signifi-
cantly facilitate transport. Interestingly, one can prepare
a chain with an external magnetic field which is in one
direction for half the chain and in the other direction for
the other half, and then one can invert the bias from
the baths. This would result in very different currents
in one bias, which we call forward bias current \( J_f \), as
compared to the other, which we refer to as the reverse
bias \( J_r \). To quantify the rectification of this
spin chain we calculate \( \mathcal{R} = J_f / J_r \) which we plot in Fig.
6 as a function of \( \hbar / \Delta \) for different system sizes (blue
continuous line for \( L = 8 \), red dashed line for \( L = 6 \)
and black dotted line for \( L = 4 \)) and for the largest bias
\( \mu = 1 \). The spin chain with this configuration of the
magnetic field results in an excellent spin current diode,
with rectification as high as \( 10^8 \) for a chain of length
\( L = 8 \). Moreover, the rectification increases significantly
with the system size, and the forward bias current is, as
shown in Fig 4, sizeable, with peaks of the order of \( 10^{-2} \).
Thus we have shown that applying this particular con-
figuration of external magnetic field (+, ..., +, −, ..., −)
in the negative differential conductance regime results in
an excellent spin diode.

IV. CONCLUSION

We have studied the interplay between interactions and
a non-uniform magnetic field in the negative differential
conductance regime of a spin chain. We have used different
types of disordered fields and found that, in general,
disorder can reduce the negative differential conductance,
although this effect may not be large, especially for larger
spin chains. When using dichotomous disorder we have observed that current can be larger for some particular
magnitudes of disorder, an effect related to the energy
levels structure of the spin chain. We have also studied
the role of each different configuration of the magnetic
field and found two configurations which give the largest
and the smallest currents. Since these two configurations
are mirror symmetric, a spin chain with such magnetic
field profile behaves as a highly performing spin current
rectifier.

In future works we could consider the role of longer-
rang interactions, as the induced frustration could sig-
ificantly affect the steady state. Another possible di-
rection is that of considering the performance of the spin
chain with the rectifying magnetic field for heat currents.

Acknowledgments

D.P. acknowledges support from the Ministry of Edu-
cation of Singapore AcRF MOE Tier-II (Project No.
MOE2016-T2-1-065). This work was partially performed
at and supported by the MPI-PKS Advanced Study
Group “Open quantum systems far from equilibrium”.
The computational work for this article was performed on
resources of the National Supercomputing Centre, Singa-
apore [21].

[1] T. Prosen, Open XXZ Spin Chain: Nonequilibrium
Steady State and a Strict Bound on Ballistic Transport,
Physical Review Letters 106, 217206 (2011).
[2] M. Znidarić, Spin Transport in a One-Dimensional
Anisotropic Heisenberg Model, Physical Review Letters
106, 220601 (2011).
[3] T. Prosen, Exact Nonequilibrium Steady State of a
Strongly Driven Open XXZ Chain, Physical Review Let-
ters 107, 137201 (2011).
[4] G. Benenti, G. Casati, T. Prosen, D. Rossini, and
M. nidari, Charge and spin transport in strongly corre-
lated one-dimensional quantum systems driven far from
equilibrium, Physical Review B 80, 035110 (2009).
[5] G. Benenti, G. Casati, T. Prosen, and D. Rossini,
Negative differential conductivity in far-from-equilibrium
quantum spin chains, EPL (Europhysics Letters) 85,
37001 (2009).
[6] J. J. Mendoza-Arenas, T. Grujic, D. Jaksch, and S. R.
Clark, Dephasing enhanced transport in nonequilibrium
strongly correlated quantum systems, Phys. Rev. B 87,
235130 (2013).
[7] P. W. Anderson, Absence of Diffusion in Certain Random
Lattices, Physical Review 109, 1492 (1958).
[8] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Metalins-
sulator transition in a weakly interacting many-electron
system with localized single-particle states, Annals of
Physics 321, 1126 (2006).
[9] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Col-
loquium: Many-body localization, thermalization, and entanglement, Reviews of Modern Physics 91, 021001 (2019).

[10] R. Nandkishore and D. Huse, Annu. Rev. Condens. Matter Phys. A 6, 15 (2015).

[11] J. Bardarson, F. Pollmann, U. Schneider, and S. Sondhi, Ann. Phys., 529 (2017).

[12] F. Alet and N. Laflorencie, Compte Rendus Physique 19, 498 (2018).

[13] V. Balachandran, G. Benenti, E. Pereira, G. Casati, and D. Poletti, Perfect Diode in Quantum Spin Chains, Physical Review Letters 120, 10.1103/PhysRevLett.120.200603 (2018).

[14] V. Balachandran, G. Benenti, E. Pereira, G. Casati, and D. Poletti, Heat current rectification in segmented $xxz$ chains, Phys. Rev. E 99, 032136 (2019).

[15] V. Balachandran, S. R. Clark, J. Goold, and D. Poletti, Energy current rectification and mobility edges, Phys. Rev. Lett. 123, 020603 (2019).

[16] L. Arrachea, G. S. Lozano, and A. A. Aligia, Thermal transport in one-dimensional spin heterostructures, Phys. Rev. B 80, 014425 (2009).

[17] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of Nlevel systems, Journal of Mathematical Physics 17, 821 (1976).

[18] G. Lindblad, On the generators of quantum dynamical semigroups, Communications in Mathematical Physics 48, 119 (1976).

[19] M. Terraneo, M. Peyrard, and G. Casati, Controlling the energy flow in nonlinear lattices: A model for a thermal rectifier, Phys. Rev. Lett. 88, 094302 (2002).

[20] N. Li, J. Ren, L. Wang, G. Zhang, P. Hänggi, and B. Li, Colloquium: Phononics: Manipulating heat flow with electronic analogs and beyond, Rev. Mod. Phys. 84, 1045 (2012).

[21] www.nscc.sg.