Wobbling excitations and tilted rotation in $^{163}$Lu

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Using a microscopic self-consistent model, we analyse wobbling excitations built upon the rotational band in $^{163}$Lu, which is identified with a rotation of a triaxial, strongly deformed shape. We find that the presence of pairing correlations substantially affects the energy of the wobbling excitations. Our calculations predict an onset of a tilted rotation at a critical rotational frequency where the energy of the wobbling excitations approaches zero.

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The study of rotational and vibrational excitations is a major source of our understanding of a nuclear dynamics at low energy. For instance, the concept of deformation of a nuclear shape, created by an effective nuclear potential, is well established. The analysis of interplay of a nuclear shape and orientation of an angular momentum led to various important discoveries. Recently, rotating non-axially deformed nuclei attract a considerable experimental attention. For a long time, experimental data did not provide an irrefutable proof for existence of the non-axiality. The modern generation of detectors opens new avenues to study nuclear spectra and, in particular, low-lying excitations near yrast line with high precision. In fact, the non-axial deformation gives rise to a new type of dynamics involving the orientation degree of freedom. This includes the chiral rotation and wobbling excitations.

As a rule, low-lying rotational bands are well described within the cranking model with a principal axis rotation. If a nuclear shape becomes non-axial, one expects low-lying vibrational excitations built on top of such a rotational band. These excitations, called the wobbling excitations by analogy with a classical wobbling motion, are created by a fluctuation of the angular momentum direction around one of the principal axes of a deformed nucleus. Having in mind that nuclei can rotate also about a tilted axis, one may suggest for non-axial shapes a possible transition from a wobbling mode with a rotational axis fluctuating around one principal axis towards a tilted rotation with a fixed rotational axis lying in a plane in between two principal axes.

We recall that the wobbling excitations in rotating nuclei were suggested in Ref. [1] and analysed first within the microscopic approach in Ref. [2]. However, the first experimental evidence of such excitations is reported only recently [3, 4, 5].

The interest in wobbling excitations was sparked by the discovery of a particular excited rotational band above the rotational band in $^{163}$Lu [5], which is associated with a triaxial, strongly deformed (TSD) nuclear shape. The lowest band is called TSD1 band, while the excited one is denoted as TSD2 band. In accordance with the rates of the observed inter-band $E2$ transitions the structure of the TSD2 band is identified with wobbling excitations. Later, in the same nucleus a second rotational band was found, which was interpreted as a two phonon wobbling band [6]. The fact that the two phonon band has less then twice the excitation energy of the one phonon band has been seen as a sign of non-harmonic vibrations. The theoretical description of those band structures was done in terms of the phenomenological, particle plus rotor model [14, 15]. At the same time, a non self-consistent, microscopic analysis (within a mean field plus random phase approximation approach) were performed in Refs. [10] and [17]. In Ref. [17] it was concluded that the pairing correlations do not affect the wobbling phonon, and this should be considered as a specific feature of such excitations. On the other hand, it was also found that the wobbling motion is very sensitive to a single-particle alignment. It is well known, however, that the alignment decreases the pairing correlations. Therefore, the question arises about the validity of this conclusion. One of our goals is to clarify this issue in a calculation based on the cranking+random phase approximation (called hereafter CRPA), with and without the pairing interaction and where we take care of the self-consistency. Following the analysis of Ref. [5], we also aim to study a possible transition to a tilted mean field solution at the point where the RPA solution goes to a zero energy.

We start with a pairing+Q̃Q Hamiltonian [18]:

$$
\hat{H} = \sum_k \epsilon_k c_k^\dagger c_k - \frac{K}{2} \hat{Q} \cdot \hat{Q} - \sum_{\tau=n,p} G_{\tau} \hat{P}_{\tau} \hat{P}_{\tau}.
$$ (1)
Here $\epsilon_k$ are the single-particle energies of the spherical modified oscillator Hamiltonian $\hat{h}_{\text{sph}}$

$$\hat{h}_{\text{sph}} = \frac{\hat{p}^2}{2M} + \frac{M}{2}\omega_0^2\hat{\tau}^2 - \hbar\omega_0\hat{\kappa}\left[2\mathbf{s} + \hat{\mu}\left(\mathbf{I}^2 - \langle \mathbf{I}^2 \rangle_N\right)\right].$$  \hspace{1cm} (2)

The operators $\hat{c}_k^\dagger$ ($\hat{c}_k$) are fermion creation (annihilation) operators with the suffix $k(l)$ labelling a complete set of quantum numbers. The parameters $\hat{\kappa}$ and $\hat{\mu}$ are standard ones \cite{19}. The quadrupole residual interaction $\hat{\epsilon}_Q = \hat{\epsilon}_0 + \hat{\epsilon}_2 \hat{\tau}^2$, is a sum over the five components of the quadrupole operators built up from $\hat{r}^2\hat{y}_2^0$ and the linear combinations $\hat{r}^2(\hat{y}_{2m} \pm \hat{y}_{2-m})$ for $m = \pm(1,2)$. The quadrupole operators are defined as $\hat{Q}_m = \sum_{kl\tau} q_{m,kl} \hat{c}_{k\tau}^\dagger \hat{c}_{l\tau}$, $(m = 0, \pm 1, \pm 2)$ where $q_{m,kl} = (k|Q_m|l)$ and $\tau = \pm 1$ distinguishes neutrons and protons, respectively. The pairing operator $\hat{P}$ has a usual form $\hat{P}_\tau^m = \sum_{k>\sigma} \hat{c}_{k\tau}^\dagger \hat{c}_{k\tau}$. The index $\bar{k}$ refers to the time conjugated state.

To study rotational properties of the system we perform the Legendre transformation into the rotating frame

$$\hat{H}' = \hat{H} - \hat{\omega} \cdot \hat{J}$$

where $\hat{J} = (\hat{j}_x, \hat{j}_y, \hat{j}_z)$ is the angular momentum operator and $\hat{\omega} = \omega (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ is the rotational angular frequency vector. Here, $\omega$ is the magnitude of the rotational frequency and $\theta$ and $\varphi$ are Euler (tilt) angles of the cranking direction. It should be pointed out that we are not restricted to the signature symmetry used in the approach of Refs. \cite{14} and \cite{17}.

The mean field part of $\hat{H}'$, Eq. (3), can be written as

$$\hat{h}_{\text{MF}}' = \hat{h}_0 - \hat{\omega} \cdot \hat{J} - \frac{2}{3}\hbar\omega_0 \epsilon_2 \left(\hat{Q}_0 \cos \gamma - \hat{Q}_2 \sin \gamma\right) - \sum_\tau \Delta_\tau \left(\hat{P}_\tau^0 + \hat{P}_\tau\right) - \lambda_\tau \hat{N}_\tau$$

where $\Delta_\tau$ are the pairing field strengths and $\lambda_\tau$ are constraints for the particle numbers. Further, $\epsilon_2$ and $\gamma$ denote the deformation parameters in the intrinsic frame of reference defined by the self-consistency conditions: $\kappa \langle \hat{Q}_0 \rangle = \frac{\hbar^2}{2}\omega_0 \epsilon_2 \cos \gamma$, $\kappa \langle \hat{Q}_2 \rangle = -\frac{\hbar^2}{2}\omega_0 \epsilon_2 \sin \gamma$, $\langle \hat{Q}_{\pm 1, -2} \rangle = 0$ (see details in Ref. \cite{2}). Here $< ... >$ means the mean field value. Using the Bogoliubov transformation such as $\hat{c}_k^\dagger = \sum_k U_{\bar{k}k} \hat{c}_{\bar{k}}^\dagger + V_{\bar{k}k} \hat{c}_{\bar{k}}$, we obtain Hartree-Bogoliubov equations that are solved for the TSD1 band in $^{163}$Lu, with and without the pairing field. The major shells $N = 4 \rightarrow 6$ are considered for both protons and neutrons. We used $\Delta N = 0$ quadrupole interaction only. This type of interaction provides quite reasonable results with regard to equilibrium deformations in the mean field calculations (see also discussion in Ref. \cite{24}). Further, this configuration space is sufficient for studying the pairing effects on low-lying excitations. The pairing strength is adjusted to reproduce the global fit of the odd-even mass differences for the ground state of even-even nuclei \cite{21}. The proton pair field is reduced by 20% due to the odd proton. The quadrupole interaction strength is fitted to reproduce the ground state deformation obtained with Nilsson-Strutinsky calculations. This is done separately with and without pairing. The self-consistent solution $\Phi' = \langle \Omega \rangle$ (shortly denoted as $\langle \Omega \rangle$), corresponds to the minimum of the energy surface $E'(\epsilon_2, \gamma, \Delta_\tau, \omega, \theta, \varphi) = \langle \hat{H}' \rangle$. This implies that the equilibrium values for all these parameters change as a function of the rotational frequency $\omega$. Note, that our vacuum states $\langle \Omega \rangle$ are rotating odd-A particle configurations, similar to the ones used in Refs. \cite{16} and \cite{17}.

For $\hbar \omega < 0.5$ MeV we find a principal axis rotation as the lowest solution. At $\hbar \omega \approx 0.25 (0.4)$ MeV the proton (neutron) pair field disappears due to the gradual breaking of quasi-particle pairs (see Fig 1). We also find a increasing $\gamma$-deformation and a slowly changing $\epsilon_2$-deformation along the band, see Fig 2. At small rotational frequencies strong proton and neutron pair fields affect the deformation. In contrast to the unpaired case, the calculations with the pairing forces predict a small $\gamma$-deformation at low rotational frequencies. Since the triaxial minimum is quite shallow in the unpaired calculations, the presence of the pair field is enough to almost restore the axial symmetry. With the increase of the rotational frequency the equilibrium deformations manifest a triaxially, strongly deformed shape of $^{163}$Lu.

Once the self-consistent mean cranking solutions are found, we apply the quasi-boson approximation in standard way \cite{18} in order to construct the vibrational wobbling excitations by the RPA approach. The particularities of this method for the rotational case can be found, e.g., in Ref. \cite{22}. The CRPA Hamiltonian is diagonalised by solving the equations of motion. As a result, we obtain the determinant of the secular equations (see details in Ref. \cite{22}) which is solved numerically.

The RPA equations have several spurious solutions related to the symmetries, i.e., the rotational invariance...
and the particle number conservation, broken in the mean field calculations. If the mean field problem is solved with a high accuracy, the spurious solutions connected with operators $\hat{J}_x$ and $\hat{N}_z$ will appear at zero energy and the solution connected with the operator $\hat{J}^+ \sim \hat{J}_x - i \hat{J}_y$ will appear at the rotational frequency. Thus, the spurious solutions are completely decoupled from the physical solutions. In contrast to the approach of Ref. 17, we obtain the RPA solutions related to the wobbling excitations from the full RPA determinant. In Ref. 17 this determinant is reduced to a simple dispersion equation for the wobbling excitations, which is valid, if and only if all spurious solutions are separated from the physical solutions. Accordingly, the numerical analysis of this dispersion equation alone is not a warrant for a decoupling of the physical wobbling excitations from the spurious solutions. Due to the admixture of the spurious modes such an analysis would not be fully reliable.

In Fig. 3 we compare our results of the wobbling excitations with experimental data 5,8. Our results without the pairing correlations are similar to the ones obtained in Ref. 16. The calculations predict an almost constant wobbling excitation energy up to $\hbar \omega \approx 0.4$ MeV. Above this value we obtain a rapid decrease of the wobbling excitations, which leads to transition into a stable tilted solution. In contrast to the conclusion of Ref. 17, the pairing interaction dramatically changes the results at small rotational frequencies. The wobbling excitation energy is substantially larger at small rotational frequencies, while it is decreasing with the increase of the rotational frequency. The rate of the reduction as a function of the rotational frequency is slightly faster than it is seen in the experiment. After the collapse of both pair fields at $\hbar \omega \approx 0.4$ MeV, the paired and unpaired calculations predict similar results. Note, that for zero pair gap even though pairing vibrations are still present in RPA they do no longer mix with the quadrupole vibrations. Large values of the wobbling excitation energy at small rotational frequencies are brought about by strong pairing fields that reduce $\gamma$-deformation. These results confirm the prediction 5 with regard to the $\gamma$ vibrational mode of the negative signature. According to Ref. 6, the increase (decrease) of the rotational frequency (the pair field) transforms the negative signature $\gamma$ vibrational states with odd spins to the low-lying wobbling excitations with the increase of the triaxiality.

It is a well known problem of cranked mean field calculations that the collapse of the static pair field at high rotational frequency happens too abrupt. This fact may explain why the reduction of the wobbling excitation energy comes too fast in our calculations compared to the experimental data. As a consequence, we also obtain a transition to a tilted solution at a relatively low rotational frequency. This discrepancy could be resolved probably by introducing a self-consistent treatment with particle number projection.

The ratio of the inter-band to intra-band $E2$ transitions for the lower part of the wobbling band is shown in Fig. 4. We use an effective charge of 0.5 (1.5) for neutrons (protons) to compensate the restricted size of our configuration space. One could observe a reasonable agreement with the experimental data, when the pairing correlations are included into the calculations, even though the calculated ratio is too large. Without the pairing correlations, we obtain a ratio which is smaller than is seen in the experiment. For comparison, we included the results from Ref. 16 which are also smaller.
than the experimental data.

Summarising, the self-consistent treatment of the wobbling excitations in the CRPA provides a good description of the experimental data for $^{163}$Lu. We conclude that the pairing interactions change the energy of the wobbling phonon, at least, indirectly by changing the $\gamma$-deformation. In addition, our calculations indicate the onset of a tilted rotation in $^{163}$Lu above a critical rotational frequency. The disappearance of the energy splitting between the two signature partner bands is one possible indication for a transition to a tilted rotational regime. Another indication would be that the two bands are connected via mixed $M1$ and $E2 \Delta I = 1$ transitions.

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