Core collapse supernovae are dominated by energy transport from neutrinos. Therefore, some supernova properties could depend on symmetries and features of the standard model weak interactions. The cross section for neutrino capture is larger than that for antineutrino capture by one term of order the neutrino energy over the nucleon mass. This reduces the ratio of neutrons to protons in the $\nu$-driven wind above a protoneutron star by approximately 20% and may significantly hinder r-process nucleosynthesis.

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Core collapse supernovae are perhaps the only present day large systems dominated by the weak interaction. They are so dense that photons and charged particles diffuse very slowly. Therefore energy transport is by neutrinos (and convection).

We believe it may be useful to try and relate some supernova properties to the symmetries and features of the standard model weak interaction. Parity violation in a strong magnetic field could lead to an asymmetry of the explosion [1]. Indeed, supernovae explode with a dipole asymmetry of order one percent in order to produce the very high ‘recoil’ velocities observed for neutron stars [2]. However, calculating the expected asymmetry from P violation has proved complicated. Although explicit calculations have yielded somewhat small asymmetries [3–5] it is still possible that more efficient mechanisms will be found.

In this letter we calculate some effects from the difference between neutrino and antineutrino interactions. In Quantum Electrodynamics the cross section for $e^-p$ is equal to that for $e^+p$ scattering (to lowest order in $\alpha$). In contrast, the standard model has $\bar{\nu}$-nucleon cross sections systematically smaller than $\nu$-nucleon cross sections.

However at the low $\nu$ energies in supernovae, time reversal symmetry limits the difference between $\nu$ and $\bar{\nu}$ cross sections. Time reversal can relate $\nu-N$ elastic scattering and $\bar{\nu}-N$ where the nucleon scatters from final momentum $p_f$ to initial momentum $p_i$. If the nucleon does not recoil then the $\nu$ and $\bar{\nu}$ cross sections are equal. Thus the difference between $\nu$ and $\bar{\nu}$ cross sections are expected to be of recoil order $E/M$ where $E$ is the neutrino energy and $M$ the nucleon mass. This ratio is relatively small in supernovae. However the coefficient multiplying $E/M$ involves the large weak magnetic moment of the nucleon (see below).

The standard model has larger $\nu$ cross sections than those for $\bar{\nu}$. For neutral currents, this leads to a longer mean free path for $\bar{\nu}_x$ compared to $\nu_x$ (with $x=\mu$ or $\tau$). Thus even though $\nu_x$ and $\bar{\nu}_x$ are produced in pairs, the antineutrinos escape faster leaving the star neutrino rich. The muon and tau number for the protoneutron star in a supernova could be of order $10^{54}$ [6]. Supernovae may be the only known systems with large $\mu$ and or $\tau$ number. For charged currents, the interaction difference can change the equilibrium ratio of neutrons to protons and may have important implications for nucleosynthesis. We discuss this below. To our knowledge, all previous work on nucleosynthesis in supernovae assumed equal $\nu$ and $\bar{\nu}$ interactions (aside from the n-p mass difference).

The neutrino driven wind outside of a protoneutron star is an attractive site for r-process nucleosynthesis [7]. Here nuclei rapidly capture neutrons from a low density medium to produce heavy elements [8]. This requires, as a bare minimum, that the initial material have more neutrons than protons. The ratio of neutrons to protons $n/p$ in the wind depends on the rates for the two reactions:

$$\nu_e + n \rightarrow p + e^-,$$  

(1a)

$$\bar{\nu}_e + p \rightarrow n + e^+. $$  

(1b)

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1We expect the difference for charged current interactions to be of the same order if one can neglect the neutron-proton mass difference.
The standard model cross sections for Eqs. (1a,1b) to order $E/M$ are,

$$\sigma = \frac{G^2 \cos^2 \theta_c}{\pi} (1 + 3g_a^2)E_c^2[1 - \gamma \frac{E}{M} \pm \delta \frac{E}{M}], \quad (2)$$

with $G$ the Fermi constant (and $\theta_c$ the Cabbibo angle), $E_c = E \pm \Delta$ the energy of the charged lepton and $\Delta = 1.293$ MeV is the neutron-proton mass difference. The plus sign is for Eq. (1a) and the minus sign for Eq. (1b). We use $g_a \approx 1.26$.

Equation (2) neglects small corrections involving the electron mass and coulomb effects (see for example [3]) while the finite nucleon size only enters at order $(E/M)^2$.

We refer to the $\gamma$ term as a recoil correction. It is the same for $\nu$ and $\bar{\nu}$.

$$\gamma = (2 + 10g_a^2)/(1 + 3g_a^2) \approx 3.10 \quad (3)$$

Finally, the $\delta$ term involves the interference of vector $(1 + 2F_2^\nu)$ and axial $(g_a)$ currents. This violates P, which by CP invariance also violates C. This increases the $\nu$ and decreases the $\bar{\nu}$ cross section.

$$\delta = 4g_a(1 + 2F_2^\nu)/(1 + 3g_a^2) \approx 4.12 \quad (4)$$

Here $F_2$ is the isovector anomalous moment of the nucleon. (This is the weak magnetism contribution.)

We average Eq. (2) over the $\nu_e$ spectrum to get,

$$<\sigma>_\nu = \frac{G^2 \cos^2 \theta_c}{\pi} (1 + 3g_a^2)\bar{\epsilon}^i [1 + \frac{2\Delta}{\bar{\epsilon}} + a_0 \frac{\Delta^2}{\bar{\epsilon}^2}] [1 + (\delta - \gamma) a_2 \frac{\bar{\epsilon}}{M}], \quad (5)$$

for Eq. (1a). Here the mean energy $\epsilon$ is defined as,

$$\epsilon = <E^2>/<E>, \quad (6)$$

and $a_2$ is a shape factor $a_2 = <E^3>/<E>/<E^2>^2$. Finally $a_0 = <E^2>/<E^2>$ and $<E^i>$ are the ith energy moments of the $\nu_e$ spectrum. Note, $\epsilon \approx 1.2 <E>$.

Likewise, averaging over the $\bar{\nu}_e$ spectrum for Eq. (1b) gives,

$$<\sigma>_{\bar{\nu}} = \frac{G^2 \cos^2 \theta_c}{\pi} (1 + 3g_a^2)\bar{\epsilon}^i [1 - \frac{2\Delta}{\bar{\epsilon}} + a_0 \frac{\Delta^2}{\bar{\epsilon}^2}] [1 - (\delta - \gamma) a_2 \frac{\bar{\epsilon}}{M}], \quad (7)$$

with the mean antineutrino energy $\bar{\epsilon} = <\bar{E}^2>/<\bar{E}>$ and $<\bar{E}^i>$ the ith moment of the $\bar{\nu}_e$ spectrum. We assume similar shape factors $a_2$ and $a_0$ for $\nu_e$ and $\nu_e$. The shape factor $a_2 = 1.23 (1.15)$ for a Fermi Dirac distribution with chemical potential $\mu = \eta T_{\nu}$ and temperature $T_\nu$ for $\eta = 0 (3.5)$. See for example [10]. For simplicity we adopt $a_2 = a_0 = 1.2$.

The equilibrium electron fraction per baryon $Y_e$ (which is equal to the proton fraction assuming charge neutrality) is simply related to the rate $\lambda$ for Eq. (1b) divided by the rate $\lambda$ for Eq. (1a).

$$Y_e = (1 + \frac{\bar{\lambda}}{\lambda})^{-1} \quad (8a)$$

This assumes the neutrino capture rates dominate those for other reactions. Reference [11] contains some discussion of the small corrections from $\epsilon^{\pm}$ capture. The ratio $n/p$ is,

$$\frac{n}{p} = \frac{1}{Y_e} - 1. \quad (8b)$$

Taking the ratio of Eq. (7) to Eq. (5) gives,

$$Y_e = \left(1 + \frac{L_{\bar{\nu}_e} \bar{\epsilon}}{L_{\nu_e} \epsilon QC}\right)^{-1}. \quad (9)$$

\[2\] Note in principle, there is another correction to Eq. (2) from the thermal motion of the nucleons. This is of order $T/M$ and increases both the $\nu$ and $\bar{\nu}$ cross sections. However we assume the temperature in the wind $T$ is much less than the neutrino sphere temperature [12] and neglect this term.

\[3\] The coefficient $a_0$ only makes a very small contribution and our results are insensitive to its value.
Here $L_{\nu_e}$ ($L_{\bar{\nu}_e}$) is the $\nu_e$ ($\bar{\nu}_e$) luminosity, $Q$ is the correction from the reaction Q value,

\begin{equation}
Q = \frac{1 - 2\Delta + a_0 \Delta^2}{1 + 2\Delta + a_0 \Delta^2},
\end{equation}

and the C violating term, Eq. (4), contributes the factor $C$,

\begin{equation}
C = \frac{1 - (\delta + \gamma)a_2 \frac{\xi}{\mu}}{1 + (\delta - \gamma)a_2 \frac{\xi}{\mu}}.
\end{equation}

Note, the recoil term $\gamma$ makes a small but nonzero contribution to Eq. (11) because the $\nu$ and $\bar{\nu}$ energies are different. Simply evaluating Eq. (11) for typical parameters yields $C \approx 0.8$. Thus, the difference between $\nu$ and $\bar{\nu}$ interactions reduces the equilibrium n/p ratio by approximately 20%. This is a major result of the present paper and will be discussed below.

Figure 1 shows the values of $\epsilon$ and $\bar{\epsilon}$ necessary for $Y_e = 0.5$. We assume equal luminosities $L_{\nu_e} = L_{\bar{\nu}_e}$. The region to the upper left is neutron rich and to the lower right proton rich. The conditions for $Y_e = 0.5$, assuming $C = 1$ in Eq. (9), are indicated by the dotted line. Including $C$ shifts the conditions for $Y_e = 0.5$ to the solid line. Thus the difference in $\nu$ and $\bar{\nu}$ interactions converts the region between the solid and dotted lines from neutron rich to proton rich.

We also show in Fig. 1 the values of $\epsilon$ and $\bar{\epsilon}$ from a supernova simulation by J.R. Wilson as reported in ref. [12]. The symbols show how the mean energies evolve with time. As the protoneutron star becomes more neutron rich, the opacity for $\bar{\nu}_e$ decreases because there are fewer protons. This allows the $\bar{\nu}_e$ to escape from deeper inside the hot protoneutron star. Therefore $\bar{\epsilon}$ increases with time. Without $C$ the wind starts out with $Y_e \approx 0.5$ and then becomes neutron rich. With $C$ the wind starts out proton rich and ends up with $Y_e \approx 0.5$. If $L_{\bar{\nu}_e} \approx L_{\nu_e}$ the wind is never significantly neutron rich. If $L_{\bar{\nu}_e} \approx 1.1L_{\nu_e}$ the wind will end slightly neutron rich. However, the n/p ratio is still 20% lower with $C$ than without. For example, if $Y_e$ drops as low as 0.42 in a model without $C$ it will only drop to approximately 0.48 when the difference between $\nu$ and $\bar{\nu}$ interactions is included. With this increase in $Y_e$, it is very unlikely that successful r-process nucleosynthesis can take place in the wind of this or similar models.

Note, we are being slightly inconsistent to include the $E/M$ term in Eqs. (2,4) for the neutrino absorption while it is not included in the simulation used for $\epsilon$ and $\bar{\epsilon}$. Indeed this term could change the location of the neutrino spheres and slightly increase $\bar{\epsilon}$ and decrease $\epsilon$. This could cancel a small part of the effect on the n/p ratio. However, our preliminary estimates suggest this change in the spectrum is very small. Including the term in a full simulation would be useful [13]. For completeness we give a C violating term for neutrino-electron scattering NES which may be useful for calculating differences between the $\nu_e$ and $\bar{\nu}_e$ spectrum.

The total cross section $\sigma_c$ for NES (see ref. [14] for example) is expanded in powers of $E/E_F$ where $E_F$ is the electron Fermi energy. To order $(E/E_F)^2$,

\begin{equation}
\sigma_c \approx \frac{G^2 E^2}{\pi} \left( c_v^2 + c_a^2 \right) \frac{E}{5E_F} \left( 1 \pm \delta_c \frac{E}{E_F} \right),
\end{equation}

with $\delta_c = 4c_v c_a / 3(c_v^2 + c_a^2)$ and the plus sign is for $\nu$ and the minus sign for $\bar{\nu}$. The couplings are $c_v = 2 \sin^2 \theta_W \pm 1/2$ and $c_a = \pm 1/2$. Here the plus sign is for $\nu_e$ and the minus sign for $\nu_x$. The C violating coefficient $\delta_c = 0.55$ for $\nu_e$ and $\approx 0.1$ for $\nu_x$. Although this term is nominally of larger order, $E/E_F$ for NES than $E/M$ for nucleon scattering, the coefficient is smaller $\delta_c \ll \delta$. Therefore we do not expect large differences from NES (except perhaps at low densities).

With the approximately 20% reduction in n/p from the difference between $\nu$ and $\bar{\nu}$ interactions, there appears to be very serious problems with r-process nucleosynthesis in the wind of present supernova models. In addition to the initial lack of neutrons, one has to overcome the effects of neutrino interactions during the assembly of $\alpha$ particles and during the r-process itself [13]. These further limit the available neutrons per seed nucleus. Thus, it is unlikely that present wind models will produce a successful r-process. Of course, the wind in supernovae may not be the r-process site, although this may be unappealing (see for example [15,16]). If the wind is not the site, one must look for alternative environments.

However, the effects of neutrino interactions may be very general. The only requirement is that energy transport from neutrinos plays some role in helping material out of a deep gravitational well. Given this, it is quite likely that the n/p ratio will be determined by the relative rates of Eqs. (1a,1b). Therefore differences in $\nu$ and $\bar{\nu}$ interactions may be important for just about any nucleosynthesis site that involves neutrinos. Indeed, Haxton et al. [17] claim the abundance of isotopes produced by neutrino spallation imply significant neutrino fluences during the r-process.
If the $\nu$-driven wind is the r-process site, it is very likely, present models of the neutrino radiation in supernovae are incomplete. The high values of $Y_e$ make it almost impossible to have a successful r-process by only changing matter properties, such as the entropy. The neutrino fluxes will (almost assuredly) need to be changed. Changes in the astrophysics used in the simulations or new neutrino physics such as neutrino oscillations [18] could change $\bar{\epsilon}$, $\epsilon$ and or the luminosities and lead to a more neutron rich wind. The oscillations of more energetic $\bar{\nu}_x$ with $\bar{\nu}_e$ could increase $\bar{\epsilon}$. However, we have some information on the $\bar{\nu}_e$ spectrum from SN1987a [19]. Thus one can not increase $\bar{\epsilon}$ without limit. Indeed if anything, the Kamiokande data suggest a lower $\epsilon$. Any model which tries to solve r-process nucleosynthesis problems by increasing $\bar{\epsilon}$ should first check consistency with SN1987a observations [20]. Alternative modifications could include oscillations of $\nu_e$ to a sterile neutrino or a lowering of $\epsilon$. (However, we know of no model which lowers $\epsilon$.) Whatever the modification of the neutrino fluxes, one will still need to include the differences between $\nu$ and $\bar{\nu}$ interactions in order to accurately calculate n/p.

In conclusion, supernovae are one of the few large systems dominated by energy transport from weakly interacting neutrinos. Therefore, some supernova properties may depend on symmetries and features of the standard model weak interactions. The cross section for neutrino capture is larger than that for antineutrino capture by a term of order the neutrino energy over the nucleon mass. This difference between neutrino and antineutrino interactions reduces the ratio of neutrons to protons in the $\nu$-driven wind above a protoneutron star by approximately 20 % and may significantly hinder r-process nucleosynthesis.

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FIG. 1. Mean antineutrino energy $\bar{\epsilon}$ vs mean neutrino energy $\epsilon$, see Eq. (6). The solid line indicates an equilibrium electron fraction $Y_e = 0.5$ including the difference between $\nu$ and $\bar{\nu}$ interactions, $C$ term in Eqs. (9,11), while the dotted line shows $Y_e = 0.5$ without this term. The symbols are the mean energies of a simulation by J.R. Wilson as reported in ref. [12] for the indicated times in seconds after collapse. The $\nu$-driven wind is neutron rich in the upper left of the figure and proton rich in the lower right. The region between the dotted and solid lines is converted from neutron rich to proton rich by the $C$ term.