Study on Similarity of Spur Gears Based on Dimensional Analysis Method

Chengli Zhu1,*, Xiaofang Wang1

1 Department of Industrial Engineering, Tsinghua University, Beijing, China

*Corresponding author e-mail: chenglizhu888@163.com

Abstract. In the process of gear development, bench test is an important and necessary method to study and examine the dynamic performance of the developed gears. But, it is very difficult to do bench test when the developed gear size is too big or too small. Such as for the big and heavy gears, it is not convenient to be moved and measured. And it will take a long time and consume much more materials and energy if we use the full-size gear to do bench test. On the other hand, too small gear size also makes the installation and measurements are very difficult in the bench test process. Sometimes, because of the unsuitable gear size, it is difficult to do bench test with the existing test equipment. Model test method can help us to solve this problem. Model test method allow us to scale-up or scale-down the size of the gear to get the similar gear model according to the similarity principle, and use it to do bench test instead of the full-size gear, and then apply the test results and rules of the model gear on the full-size gear. In order to do similar gear model test, the similar principle between the model and original gears must be established. In this paper, the similarity principle of spur gear was analysed in the respects of geometry, mechanics and kinematics with dimensional method based on the similarity theory. It provides a theoretical basis for gear model test method.

1. Introduction

Gear is one of the most important components in many mechanical products. The gear technology is developing in the direction of heavy load, light vibration and lower noise, small size, light weight, energy saving and high efficiency with the increasing performance requirements of high-tech mechanical products, such as vehicles, airplanes, high-speed trains and modern warships in recent years [1]. In addition, the gear structure is also becoming more and more complex, and its working environment is becoming even worse. The complexity of the gear structures makes that the gear products must be tested before they are used in a mechanical system. Only through the bench test and verification, the reliability and dynamic performance of the developed gears can be ensured to meet the real requirements [2].
The gear bench test process is a long and tedious process. The real gear is usually very big and heavy. The full-size gear is not convenient to be moved and measured, and it is difficult to be tested on the bench test equipment. To do bench test using the full-size gear, it not only consumes much more materials and costs, but also takes longer time. On the other hand, it is difficult to get the satisfied results [3]. Sometimes, it is very difficult to effectively simulate the real high-speed and high-power working conditions due to the limitations of the bench test system.

The model test method allows us to scale-up or scale-down the real size of the gear to get the similar model according to the similarity principle, and use it to do bench test instead of the full-size gear, and then apply the test results and rules of the model gear on the full-size gear. Compared with the full-size gear test, the similarity model test is much more flexible in selection of the gear size and materials. And it is easy to control the test parameters and test conditions [4]. Moreover, in the model test process, the minor influence factors can be eliminated or reduced. At the same time, it is easy to collect the data and get better results.

The model test method was studied and used in the fields of hydromechanics, vibration acoustics and mechanical engineering in recent years. In order to simplify the test process of the ship, D Vassalos, et al. [5] studied the similarity model test of the ship with dimensional analysis method. In his study, the size scaling law of the ship was deduced and the accuracy and limitation conditions of the model test were studied based on the similarity theory. Ramu et al. [6] established a cantilever beam scaling model of different materials according to the similarity requirements for free vibration response test. The similar relationship between the model and the prototype was studied, which provided an effective method for the free vibration model test of cantilever beam. Xi-Chen L et al. [7] applied similarity theory on the research of piles in civil engineering. Shi Qiyin et al. [8] make an airport tower model to do aeroelastic wind tunnel experiment when he studied the structure of an airport tower, and applied the test results to the full-size airport tower. Shude Ji et al. [9] established the similar principle for welding residual stress according the heat conduction theory and similarity theory, and used the similar model test results to predict the residual stress distribution of large-scale weld assembly. Dingyi et al. [10] make a similar model for a gas turbine engine to study the similarity rule of some gas turbine engine parameters, such as speed and flow, and proposed the correction method for similar the transformation of these parameters.

The study and application of the similarity for model test greatly promote the development of similarity theory, which is much helpful to simplify the engineering test or experiment procedures.

In the gear model test process, we can use the similarity principle to scale-up or scale-down the gear size to get the gear similar model, and use it to do bench test instead of the full-size gear, and then apply the results to the full-size gears according to the similarity theory. It will not only bring us great convenience and flexibility in the design, development and test of gears, but also can make the simulation of actual working conditions much easier. Because the gear structure and movement are very complex, there are few studies on the gear similarity principle according to the recent literatures. In order to do gear model test, it is very necessary to study the similar requirements of mechanical system and establish the gear similarity principles.

In this paper, the similarity principle of spur gear was analyzed in the respects of geometry, mechanics and kinematics with dimensional method based on the similarity theory. It provides a theoretical basis for the gear model test method.
2. Similarity principle and dimensional analysis

2.1. Similarity theory

Similarity refers to the similar relationship in the nature and functions between things or physical phenomena. It is a means to study the objective laws between things or physical phenomena using the similarity features. There are three similarity theorems, which form a relatively complete similarity theory system. These similarity theorems are the theoretical basis of model test and the other applications of similarity [11,12]. They determine the basic properties and the similarity criterion equations of similar phenomena. On the other hand, it defines the necessary and sufficient conditions for the similarity of physical phenomena. Therefore, they are the theoretical basis of the model test.

If two physical phenomena have the equal similarity criteria and similar single-value, they can be considered two similar phenomena. On the contrary, if they only have the equal similarity criteria, they cannot be considered similar phenomena. The third similarity theorem is the basis of model test. Only all of the similar conditions in the third theorem are satisfied, the model can be considered completely similar with the prototype. Many practical engineering problems are very complex, and it is very difficult to determine the single-value similar conditions. Therefore, it is difficult to meet all the similarity requirements in geometry, material, load, boundary conditions and working environment, when we establish the similarity principle of two physical phenomena. Sometimes, we can only choose some decisive physical variables and ignore some minor influence factors according to our experiences.

2.2. Dimensional analysis method

In the process of studying similarity between two physical phenomena and deriving the similarity criterion equations, it is usually done by means of dimensional analysis. The physical variables that describe a physical phenomenon are composed of some fundamental quantities, which can form other physical quantities. These fundamental quantities are called dimensions. For the same kind of physical quantity, no matter what kind of measurement unit is selected, as long as the size is certain, the relative value between them is a constant. Dimensional analysis is an effective method to analyze the relationship of a physical phenomenon variables. The dimensional variables can be converted to nondimensional quantities using dimensional analysis method. After that, we can get the nondimensional quantity equations. Then, the similar principle between two similar physical phenomena can be established [13]. Rayleigh firstly used the dimensional analysis to study the similarity of two physical phenomena and established a first scientific similarity model. After that, some researchers continually improve and develop the dimensional analysis method, which had become an important method to study similar phenomena.

According to the second similarity theorem, if a physical phenomenon can be described by an equation, which contains $n$ dimensional quantities, and there are $r$ dimensional quantities are dimension independent quantities, then the physical phenomenon can be described by another equation which has $n-r$ dimensionless quantities. Both the Rayleigh method and the π theorem are all well-known dimensional analysis methods [14].

In general, there are three dimension independent physical quantities can be selected in geometry, kinematics and dynamics respectively, i.e. $r=3$. This is helpful to meet the requirement of dimension independent. In the process of determining the dimensionless π items, $r$ foundermental dimension independent quantities should be selected from $n$ quantities that are used to describe a physical phenomenon. If the dimensions of the other physical quantities can be expressed by the products of the
powers of \( r \) fundamental dimensions, the ratios of the quantities can be considered dimensionless \( \pi \) terms. After that, according to the second similarity theorem, the results of model test can be arranged into the similarity criterion equations, the results and the data of model test can be applied to the prototype [15].

3. Similarity analysis of spur gears

The similarity of an engineering mechanical system is mainly determined by three kinds of similarities: geometrical similarity, kinematic similarity and dynamic similarity [15]. The gear work and transmit torque under a certain time, space and load environment. It is a typical engineering mechanics system. So, it follows the general laws of kinematics and dynamics. When we establish a similarity principle for a spur gear transmission system, it needs to study the geometrical similarity, kinematic similarity and dynamic similarity of a spur gear transmission. In order to make gear’s velocity and acceleration similar, similarity in geometry and force should be guaranteed, and the similarity in boundary and initial conditions, such as load characteristics, initial speed, lubrication, environmental temperature, service conditions, is needed.

3.1. Analysis of geometrical similarity for spur gear

The surface of a standard external involute spur gear is shown in figure 1. Actually, a spur gear has 5 basic geometric parameters: tooth number \( z \), modulus \( m \), pressure Angle \( \alpha \), addendum coefficient \( h^*_a \), clearance coefficient \( c^* \). Because the pressure angles at different points of the involute tooth profile have different values, for convenience of design and manufacture, the pressure angles on the reference circle are standardized in China. Usually, its value is 20º. Addendum is the product of modulus multiplied by addendum coefficient, while dedendum is the product of modulus multiplied by the sum of addendum coefficient and clearance coefficient. In China, for normal gears, when \( m \geq 1\text{mm} \), \( h^*_a = 1 \), \( c^* = 0.25 \); When \( m < 1\text{mm} \), \( h^*_a = 1 \), \( c^* = 0.35 \). And the other geometrical parameters can be calculated by the basic parameters [16]. Geometric parameters, dimensions, and geometric similarity ratio of involute external meshing spur gear are shown in table 1.

![Figure 1. Dimensions of spur gear](image)
Table 1. The geometric similarity ratio of standard spur gear

| Physical quantity | Symbol | Calculation formula | Dimension | Similarity ratio |
|-------------------|--------|---------------------|-----------|------------------|
| Modulus           | $m$    | —                   | $L$       | $L_r$            |
| Pressure angle    | $a$    | —                   | —         | 1                |
| Tooth number      | $z$    | —                   | —         | 1                |
| Reference diameter| $d$    | $d_i = m z_i$       | $L$       | $L_r$            |
| Base diameter     | $d_b$  | $d_c \cos a$        | $L$       | $L_r$            |
| Addendum          | $h_a$  | $h_a = h^* m$       | $L$       | $L_r$            |
| Dedendum          | $h_f$  | $h_f = (h^* + c') m$| $L$       | $L_r$            |
| Tooth thickness   | $s$    | $s_f = \pi m / 2$   | $L$       | $L_r$            |
| Pitch             | $p_t$  | $p_t = \pi m$       | $L$       | $L_r$            |
| Facewidth         | $B$    | —                   | $L$       | $L_r$            |
| Transmission ratio| $i$    | $i = z_2/z_1$       | —         | 1                |
| Centre distance   | $a$    | $a = m(z_1 + z_2)/2$| $L$       | $L_r$            |

Notes: $L_r$ is the geometric similarity ratio.

It can be seen from the above analysis that geometric similarity of two spur gears should have the same similarity ratio in modulus, reference diameter, facewidth and the other parameters, which is measured in the unit of length.

The contact ratio of a spur gear $\varepsilon_a$ can be written as:

$$\varepsilon_a = \frac{1}{2\pi} \left[ z_1 \left( \tan \alpha_{a1} - \tan \alpha' \right) + z_2 \left( \tan \alpha_{a2} - \tan \alpha' \right) \right]$$  \hspace{1cm} (1)

Where, $z_1$ is the tooth number of pinion; $z_2$ is the tooth number of wheel; $\alpha'$ is the actual engaging angle, for standard spur gear and standard installation $\alpha' = \alpha = 20^\circ$. The $\alpha_{a1}$ and $\alpha_{a2}$ can be calculated by:

$$\alpha_{a1} = \arccos \frac{z_1 \cos \alpha}{z_1 + 2h^*}$$  \hspace{1cm} (2)

$$\alpha_{a2} = \arccos \frac{z_2 \cos \alpha}{z_2 + 2h^*}$$  \hspace{1cm} (3)

The parameters determining the contact ratio of spur gear are tooth number and engaging angle. If there are two spur gear pairs, their corresponding wheel and pinion has the same tooth number and engaging angle, and they must have the same contact ratio. It means that if two spur gear pairs are similar in geometric, their contact ratio similar ratio is 1.

3.2. Analysis of dynamic similarity for spur gear

1) Force similarity of spur gear teeth

Force analysis of a standard external spur gear is shown in Figure 2. The force $F_n$ applied perpendicular to the tooth surface of a spur gear can be decomposed into vertical peripheral force $F_i$ and radial force $F_r$, and the calculation formula of them are the follows:
\[ F_t = \frac{2T}{d} \]  
\[ F_r = \frac{F_t}{\tan \alpha} = \frac{2T}{d \tan \alpha} \]  
\[ F_n = \frac{F_t}{\cos \alpha} = \frac{2T}{d \cos \alpha} \]

Where, \( T \) is the torque transmitted by the gear; \( d \) is the reference diameter; \( \alpha \) is the pressure angle.

The force dimensional analysis of a standard external spur is shown in table 2.

![Figure 2. Force analysis of standard external spur gear](image)

| Physical Quantity and Corresponding Dimensions of Normal Force of Spur Gear Tooth |
|-----------------|-----|-----|-----|
| \( F \)        | 1   | 0   | 1   |
| \( d \)        | 0   | 1   | 1   |
| \( T \)        | 0   | 1   | 0   |
| \( \alpha \)   | 0   | 1   | 0   |

There are two dimension independent quantities. So there are \( 4-2 = 2 \) dimensionless \( \pi \) terms. If \( d \) and \( T \) are chosen as dimensionless quantities, the following dimensionless \( \pi \) terms can be obtained:

\[ \pi_1 = \frac{F_T}{T}, \quad \pi_2 = \alpha \]

According \( \pi \) theorem (the second similarity theorem), \( \pi_m = \pi_p \) \((i = 1, 2)\), where \( m \) subscript means it belongs to the model and \( p \) subscript means it belongs to the prototype. Thus there are:

\[ \frac{F_m}{F_p} = \frac{d_p}{d_m}, \quad \frac{T_m}{T_p} = \frac{1}{L_r}, \quad \frac{T_m}{T_p} = \frac{L_r}{T_p} \]

\[ \alpha_m = \alpha_p \]

Where, \( L_r \) is the geometrical similarity ratio of the spur gear.

If the force of two similar standard spur gears has the same value, i.e. \( F_m = F_p \), then, \( T_m/T_p = L_r \). It means that the ratio of torque transmitted by them has the same value of that of their geometric.

2) Tooth contact stress similarity analysis of spur gear

The tooth contact stress of a spur gear generally refers to the contact stress at pitch point. For standard spur gears, the contact stress can be written as follow [17]:
where, the “+” is for the external gear, and the “-” is for the internal gear; \( K \) is the load coefficient, \( d \) is the reference circle diameter; \( T \) is the transmission torque; \( b \) is the tooth width; \( \mu \) is the ratio of tooth number; \( Z_{H} \) is the pitch point area coefficient, when the pressure angle \( \alpha = 20^\circ \), \( Z_{H} = 2.5 \); \( Z_{E} \) is the elastic coefficient of the gear material, which represents the influence of the material elastic modulus \( E \) and Poisson's ratio \( \nu \), the express of it is:

\[
Z_{E} = \sqrt{\frac{1}{\pi}} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)
\]

Where, \( E_1 \) is the elastic modulus of the driving gear material; \( E_2 \) is the elastic modulus of the driven gear material; \( \nu_1 \) is the Poisson's ratio of the driving gear material; \( \nu_2 \) is the Poisson's ratio of the driven gear material.

The physical variables that determine the contact stress of a spur gear are: the input torque \( T \); the pitch circle diameter \( d \); tooth width \( b \); the ratio of tooth number \( \mu \); the pressure angle \( \alpha \); the Possion’s ratio \( \nu \) and elastic modulus \( E \) of gear material. Their dimensions are shown in table.3.

| Table 3. Physical quantities and dimensions of spur gear tooth contact stress |
|-----------------|--|--|--|--|--|--|--|
| \( \sigma_H \) | \( T \) | \( d \) | \( b \) | \( \mu \) | \( A \) | \( \nu \) | \( E \) |
| \( F \) | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| \( L \) | -2 | 1 | 1 | 1 | 0 | 0 | 0 | -2 |

From table.3, it can be seen that the number of independent dimensions is 2, so there are 8-2=6 dimensionless \( \pi \) terms. If \( d \) and \( E \) are selected as the foundermental dimensions, the dimensionless \( \pi \) terms can be written as:

\[
\pi_1 = \frac{\sigma_H}{E}, \quad \pi_2 = \frac{T}{Ed^3}, \quad \pi_3 = \frac{b}{d}, \quad \pi_4 = u, \quad \pi_5 = \alpha, \quad \pi_6 = \nu
\]

According to \( \pi \) theorem, it can be obtain: \( \pi_{im} = \pi_{ip} \) (i = 1, 2, ..., 6), the subscript \( m \) means they belong to the model and subscribe \( p \) means they belong to the prototype.

\[
\frac{\sigma_{Hm}}{\sigma_{Hp}} = \frac{E_m}{E_p}
\]

\[
\frac{T_m}{T_p} = \frac{E_m}{E_p} \frac{d_m^3}{d_p^3} = \frac{E_m}{E_p} \frac{L_r^3}{L_r^3}
\]

\[
\frac{b_m}{b_p} = \frac{d_m}{d_p} = L_r
\]

\[
u_m = \nu_p, \quad \alpha_m = \alpha_p, \quad \nu_m = \nu_p
\]
If we make two similar spur gears have the same contact stress, it can be get from equations (14) and (15) that $E_m = E_p$, $T_m/T_p = L_r^3$. And the elastic modulus and Possion’s ratio of the gear materials, tooth number, contact ratio, pressure angle must have the same value.

3) Tooth root bending stress similarity analysis of spur gear

Because the bending moment of at the root of the spur gear tooth has the largest value. At the same time, there is a stress concentration at this area. The bending fatigue strength at the spur gear tooth root radius is weakest. Therefore, the tooth bending fatigue stress of the external spur gear is calculated by the following equation [18]:

$$\sigma_F = \frac{2KT}{bdm}Y_{fa}Y_{sa}Y_c$$

(17)

Where, $K$ is the load coefficient; $T$ is the transmission torque; $b$ is the tooth width; $m$ is the modulus; $Y_{fa}$ is the tooth shape coefficient; $Y_{sa}$ is the stress correction coefficient; $Y_c$ is the contact ratio coefficient.

It can be known from equation (17) that the tooth root bending stress of spur gear is mainly determined by the physical quantities of torque, tooth width, reference circle diameter, modulus, and tooth number. Their dimensions are shown in table 4.

| Table 4. Physical quantities and dimensions of spur gear tooth bending stress |
|------------------|---|---|---|---|---|
| $\sigma$ | $T$ | $d$ | $b$ | $m$ | $z$ |
| $F$ | 1 | 1 | 0 | 0 | 0 |
| $L$ | -2 | 1 | 1 | 1 | 0 |

As shown in table 4, there are 2 dimension independent physical variables, and the number of dimensionless $\pi$ terms is 6-2=4. $T$ and $d$ can be selected as dimension independent foundermental variables, the dimensionless $\pi$ terms can be written as:

$$\pi_1 = \frac{\sigma_F d^3}{T}, \quad \pi_2 = \frac{b}{d}, \quad \pi_3 = \frac{m}{d}, \quad \pi_4 = z$$

(18)

According to the secondary similarity theorem, there are: $\pi_i = \pi_i$ (i = 1,...,4). So, the similar ration between model and prototype can be written as:

$$\frac{\sigma_{Fm}}{\sigma_{Fp}} = \frac{T_m}{T_p} \frac{d_p^3}{d_m^3} \frac{b_m}{d_p} \frac{m_m}{d_m} = L_r, \frac{d_m}{d_p} = L_r, \frac{m_m}{m_p} = L_r, \frac{b_m}{d_p} = L_r$$

(19)

If we make the tooth root bending stress of two similar spur gears has the same value, it means that:

$$\sigma_{Fm} = \sigma_{Fp}$$

(20)

Substitute the equation (20) into equation (19), it can be get:

$$\frac{T_m}{T_p} = \frac{d_m^3}{d_p^3} = L_r^3$$

(21)

It can be known from equations (19) and (21) that, for similar spur gears, in order to make their root bending stress has the same value, their tooth width and pitch circle diameter should have the same value as geometric ratio, their number of teeth, contact ratio also should have the same value,
and the ratio of torque transmitted by their must equal to the cube of the geometric similarity ratio value.

3.3. Similarity of gear velocity

The linear velocity \( V_n \) of any point \( n \) on the spur gear can be written as:

\[
V_n = r_n \omega
\]  

(22)

Where \( r_n \) is the distance from point \( n \) to the centre of rotation \( O \), and \( \omega \) is the speed of the gear. The dimensionless \( \pi \) term of the gear line speed is:

\[
\pi = \frac{V_n}{r_n \omega}
\]  

(23)

If the line speed of two gears is similar, \( \pi_m = \pi_p \), then:

\[
\frac{V_{nm}}{r_{nm} \omega_m} = \frac{V_{np}}{r_{np} \omega_p}
\]  

(24)

and:

\[
\frac{V_{nm}}{V_{np}} = \frac{r_{nm} \omega_m}{r_{np} \omega_p} = L_r L_{\omega}
\]  

(25)

Where, \( L_r \) is the geometric similarity ratio of spur gear, \( L_{\omega} \) is the angular velocity of spur gear.

From equation (25) it can be known that making two similar gears have the same line velocity needs their ratio of angular velocity is \( 1/L_r \).

4. Conclusion

Based on the study of similarity principles, this paper analyzes the similar relationship in the respects of geometrical, mechanical, tooth surface contact stress, root bending stress for external spur gear with dimensional analysis method. The investment results can be summarized as follows:

If two spur gears have the same similarity ratio in modulus, reference diameter, facewidth and the other parameters which are measured in the unit of length, they are similar in geometry.

For geometric similar spur gears, in order to make their root bending stress and contact stress has the same value, their materials should have similar performance, i.e. the elastic modulus and Possion’s ratio of their materials should have the same value. At the same time, the ratio of their transmitted torque should equal to the cube of the geometric similarity ratio.

If we want to make two geometric similar gears have the same line velocity, it needs the value of their angular velocity ratio equal to the reciprocal of the geometric similarity ratio.

The analyzing of geometrical similarity, kinematic similarity and dynamic similarity is the basis of other similarity analyzing for the spur gears. The similarity relationship of these basic variables can be used to deduce other similarities and define the model test conditions for gear transmission. It provides theoretical basis for gear model test.

5. Acknowledgments

This work was financially supported by Tsinghua University Laboratory Innovation fund.
References

[1] Wang Y Q., He Z C. Effect of impact load on transient elastohydrodynamic lubrication of spur gears[J]. Advanced Materials Research, 2011, 317(2):548-551.

[2] Shen Y, Tong J. Technology of Gear Shaping of Face Gear and Experimental Studies[C]// Digital Manufacturing and Automation (ICDMA), 2010 International Conference on. IEEE Computer Society, 2011.

[3] Lin C, Hou Y J, Zeng Q L, et al. Study and Experimental Analysis of Oval Bevel Gear Transmission[J]. Applied Mechanics and Materials, 2011, 121-126:2263-2267.

[4] Li M. Probabilistic Life Prediction and Bending Fatigue Test for Gear[J]. Journal of Mechanical Engineering, 2017, 53(18).

[5] Vassalos D. Physical modelling and similitude of marine structures[J]. Ocean Engineering, 1998, 26(2):111-123.

[6] Murugan Ramu, V. Prabhu Raja, P. R. Thyla. Establishment of structural similitude for elastic models and validation of scaling laws[J]. KSCE Journal of Civil Engineering, 2013, 17(1).

[7] Xi-Chen L, Lei N, Bin Q I. Similarity in the Simulation Test for Bearing Capacity of Pile Foundation[J]. Journal of Jiling University, 2005, 35(4):491-495.

[8] QiYin Shi, Dongsheng, Du, Peibin Li, et al. Wind-induced Vibration Similarity Theory Analysis to the Model of New Toll Tower Structure on Bering Airport with Aero Elastic Model Test[J]. Special Structures, 2004, 21(1): 47-9.

[9] Shude Ji, Liguo Zhang, Yafan Li, et al. Establishment of relation about welding residual stress between simulative component and practical component on basis of similitude principles[J]. Transactions of The China Welding Institution, 2010, 31(6):61-64.

[10] Yi Ding, Xianghua Huang and Tianhong Zhang. The Research of Gas Turbine Performance Modeling Based on Theory of Similarity[J]. Journal of Aerospace Power, 2004, 19(5): 689-94.

[11] Chuanyuan W. Exploration and study on similarity theory[J]. Journal of Systems Engineering & Electronics, 1992, 3(1):9-20.

[12] Rushton J P, Russell R J. Genetic similarity theory: a reply to Mealey and new evidence[J]. Behavior Genetics, 1985, 15(6):575-582.

[13] Qing-Ming Tan, Dimensional analysis: with case studies in mechanics[M]. Springer Science & Business Media, Springer-Verlag Berlin Heidelberg 2011.

[14] EVANS J H. Dimensional Analysis and the Buckingham Pi Theorem [J]. American Journal of Physics, 1972, 40(12): 1815-22.

[15] Shehadeh M, Shennawy Y, El-Gamal H. Similitude and scaling of large structural elements: Case study[J]. Alexandria Engineering Journal, 2015, 54(2):147-154.

[16] Zhonghe Ye, Zhaohui Lan and M.R. Smith. Mechanisms and Machine Theory[M]. Higher Education Press, 2006-01.

[17] Jelaska D. 4. Elements of Cylindrical Gear Drive Design[J]. 2012.

[18] Peter R. N. Childs, Mechanical Design Engineering Handbook[M]. Butterworth-Heinemann, 2014.