Automated Decisional Model for Optimum Economic Order Quantity Determination Using Price Regressive Rates

M M ROŞU¹, C I TARBĂ² and C NEAGU³,

¹,²,³ POLITEHNICA University of Bucharest, Splaiul Independenţei, no. 313, building CB, sector 6, Bucharest, Romania

e-mail: magdalena.rosu@upb.ro

Abstract. The current models for inventory management are complementary, but together they offer a large pallet of elements for solving complex problems of companies when wanting to establish the optimum economic order quantity for unfinished products, row of materials, goods etc. The main objective of this paper is to elaborate an automated decisional model for the calculus of the economic order quantity taking into account the price regressive rates for the total order quantity. This model has two main objectives: first, to determine the periodicity when to be done the order n or the quantity order q; second, to determine the levels of stock: lighting control, security stock etc. In this way we can provide the answer to two fundamental questions: How much must be ordered? When to Order? In the current practice, the business relationships with its suppliers are based on regressive rates for price. This means that suppliers may grant discounts, from a certain level of quantities ordered. Thus, the unit price of the products is a variable which depends on the order size. So, the most important element for choosing the optimum for the economic order quantity is the total cost for ordering and this cost depends on the following elements: the medium price per units, the stock cost, the ordering cost etc.

1. Introduction

For a company who wants to reduce production costs and the stock generated costs, the inventory management is a necessary condition but it is not enough. The inventory management must be harmonised with the business strategy, with the financial management, with the information management and with the other traditional disciplines for management but it depends significantly on human resource factors that are responsible for the forecast on medium and large term.

The development of the decisional model for determining the optimal amount of supply involves the analysis of the entire supply system [1], Figure 1, within a company with all the three main subsystems which make up the forecasting, the tracking and the launch.

The forecast subsystem focuses on specifying the conditions under which orders should be launched (times and quantities). It is very important to know at what point the orders should be launched and in the same time it is necessary to provide for the use of each item. For detecting any changes in rates of use, the forecasts are drawn up relatively frequently, usually once a month or twice a month.

¹ magdalena.rosu@upb.ro or magdalena.rosu@upb.ro
The tracking subsystem focuses on the inventory control so as to verify its actual situation and to justify the ordering. If the satisfactory stocks still cover the needs, the order is not launched. Tracking the stocks should be rigorously given the permanent consumption of stock. It is not uncommon that in addition to tracking the operations conducted weekly or even twice a week, some items control the situation after each stock release from the warehouse.

The launching orders subsystem focuses on establishing the order quantities, taking into account the changing consumption and costs. The strategy for launching the orders must ensure a minimum level of expenditure for the supply and storage.

![Supply System Diagram](image)

**Figure 1. Supply system [1]**

2. **Model for the Inventory Optimisation**
This model follows two main objectives: the first one is to calculate an optimum relative for time (n) and the second one is to calculate the economic order quantity (Qo). This is the way to answer the following questions: „how much to order?” and „when is it necessary to order?”. This model is based on the Wilson model which is a determinist model relying on the following elements: constant demand, linear variation of use, equal time periods for demand, the order launching at start of each time period, quick supply and continuum process. By taking into account all these elements, the model follows the minimum costs for the inventory.
So, the optimum quantity, \( q_0 \), will be calculated with the formulas below taking into account the price for different order quantities [1], [3]:

For the first price interval, \( q_{o1} \), it is calculated by formula (1):

\[
q_{o1} = \sqrt{\frac{2 \cdot D \cdot c}{p_1 \cdot h}}
\]  

(1)

where:
- \( D \) is total quantity for demand;
- \( c \) – the inventory cost;
- \( p \) – the price per item;
- \( h \) – the rate for hold cost.

For the second price, the interval \( q_{o2} \), it is calculated by formula (2). In this case, it will be taken into account the medium price between the first and second price interval and the limit point for price, \( L \):

\[
q_{o2} = \sqrt{\frac{2 \cdot D \left[ L_1(p_1 - p_2) + c \right]}{p_2 \cdot h}}
\]

(2)

For the third price, the interval \( q_{o3} \) is calculated by formula (3):

\[
q_{o3} = \sqrt{\frac{2 \cdot D \left[L_1(p_1 - p_2) + L_2(p_2 - p_3) + c \right]}{p_3 \cdot h}}
\]

(3)

If we have an \( n \) price interval for price, the optimum quantity will be calculated by formula (4):

\[
q_{on} = \sqrt{\frac{2 \cdot D \left[L_1(p_1 - p_2) + L_2(p_2 - p_3) + ... + L_{n-1}(p_{n-1} - p_n) + c \right]}{p_n \cdot h}}
\]

(4)

The total cost for demand, \( TC \), will be calculated taking into account the medium price per unit, MPU, between the adjacent price interval and the quantity. For the calculus of the total cost for the demand in the first price interval, there will be used the procurement price. For the second price interval, the MPU is calculated with formula (5) shown below:

\[
MPU_1 = \frac{L_1(p_1 - p_2)}{q} + p_2
\]

(5)

For the third price interval, the MPU is calculated with formula (6) shown below:

\[
MPU_2 = \frac{L_1(p_1 - p_2) + L_2(p_2 - p_3)}{q} + p_3
\]

(6)

For the \( n \) price interval, the MPU is calculated with formula (7) shown below:

\[
MPU_{n-1} = \frac{L_1(p_1 - p_2) + L_2(p_2 - p_3) + ... + L_{n-1}(p_{n-1} - p_n)}{q} + p_3
\]

(7)

The total cost for the demand will be determined by means of the formula below, but taking into account the medium price per unit [1], [3], [4].

\[
TC(q) = D \cdot MPU + \frac{D}{q} \cdot c + \frac{1}{2} MPU \cdot q \cdot h
\]

(8)

For this application the input data is shown in Table 1. These are total quantities for the demand \( (D) \), the price \( (p) \) for procurement and the limits for price \( (L) \), the ordering cost \( (c) \) and the rate for the hold cost \( (h) \). For running the automated model there will be established the following input data:
Table 1. Input data for the automated decisional model of the economic order quantities determination

| Input data                          | Notation | Values |
|------------------------------------|----------|--------|
| Total quantity for demand [items]  | D        |        |
| Ordering cost [units]              | c        |        |
| Rate for hold cost [%]             | h        |        |
| First interval for price [units/item] | p₁      |        |
| minimum quantity [items]           | L₀       |        |
| maximum quantity [items]           | L₁       |        |
| Second interval for price [units/item] | p₂      |        |
| minimum quantity [items]           | L₁+1     |        |
| maximum quantity [items]           | L₂       |        |
| Third interval for price [units/item] | p₃      |        |
| minimum quantity [items]           | L₂+1     |        |
| maximum quantity [items]           | L₃       |        |

The application model was developed with Microsoft Excel and for the generated random values of the quantity and for the calculation of the economic order quantity, Q₀, there was written a code in Microsoft Visual Basic for the Application using specific functions and methods for doing this calculus.

The data from Figure 2 is obtained by running the application model for the automated calculation of the economic order quantities. For each price interval there will be calculated the minimum point of the costs curve by means of the formulas 1, 2, 3 and it is easy to show if these values are in the established price interval. For our example, only the first value of the optimum quantity is the price interval.

For calculating the total cost for some different quantities, as shown in Figure 3, the model will analyse different aleatory values of quantity between imposed limits of price. For the random
quantities generated by the model there will be calculated the medium price for the interval of the supply price and then the total cost (TC) will be generated.

All these values will be obtained while running the model by pushing the grey button of the application. The economic order quantity, Qo, will be displayed below the grey button, as shown in Figure 3. The value of the economic order quantity must be the divisor of the total demand and must have the minimum total cost of the ordering. The optimum relative of time, n, is the result of dividing the total demand to the economic order quantity. This value will be displayed in the same cell with the economic order quantity.

![Figure 3. Automated Model for the Calculus of the Economic Order Quantity Qo](image)

3. Results and Discussions

The model generated a report with the economic order quantity (Qo) and with the optimum relative time (n) for the supply, as shown in Figure 3. All the data is automatically stored in a database, at each model iteration.

The graph of the total cost for the supply depending on the quantity ordered, is shown in Figure 4. This graph is generated automatically by the model after each iteration of the model.

It is easy to show on the graph where the minimum cost for the supply is but the model will generate the exact amount for the supply and the number of the time relative for the supply.

For this model there can be added other features to generate reports using the average unit price for calculating the optimal economic order quantity compared to other methods for determining the optimal amount of the supply such as using the uniform price method reduction. These features could provide better tracking of the supply process and of the manufacturing process.
In the future there will be developed a model which will make an analysis of both known methods for determining the economic order quantity, firstly by uniformly reducing the price of the supply and secondly by using the regressive price of the supply orders.

![Total Cost Graph](image)

**Figure 4. Total Cost Graph**

4. Conclusions
The automated model tool presented enables the optimisation of the economic order quantity, \( Q_0 \) for the supply system management by meeting the critical criterion such as the order in time and the minimum of the stock level. By using this model tool, the supply process is simplified and made accessible to all levels of personnel from the supply department in the case of using the regressive rate for the procurement price.

The model above can be used for different total quantities and for many price intervals and it also is very useful for production process planning.

Future developments of this model will include more features from the production process planning, such as the calculus of the economic batch size which many times depends on the total quantity of the unfinished products demands. Other supplementary data, instructions or even messages, while running the model, can be added if there is a requirement for it.

5. References
[1] C. Neagu, *Ingineria şi Managementul Producţiei*, Editura Didactică şi Pedagogică, 2005, Bucuresti.
[2] C. Neagu, M. G. Catană, E. Nițu, M. M. Roșu, *Ingineria si managementul productiei. Aplicatii*. BREN, Bucharest.
[3] S. Nahmias, *Production and Operation Analysis*, Sixth Edition, McGraw Hill Education, New York, 2013.
[4] R. D. Reid., N. Sanders, *Operation Management an integrated approach*, 5th ed., Willey, New York, 2013, Inventory Management, pp 451-504.
[5] S. N. Chapman, *The Fundamentals of Production Planning and Control*, Prentice Hall, 2005