Response of open two-band systems to a momentum-carrying single-mode quantized field

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As a new quantum state, topological insulators have become the focus of condensed matter and material science. The open system research of topological insulators has aroused the interest of many researchers. Recently, many aspects, especially experimental aspects, have been developed rapidly, such as prediction and discovery of many novel quantum effects and applications of topological properties of new materials, but the theoretical research is slightly tough. In this paper, we study the response of topological insulator driven by momentum-carrying single-mode field. We solve the ground state of the system after the addition of a single mode light field with adjustable photon momentum. Specifically, We show that from the analytical solution of hall conductance compared with the closed system, there is an extra correction term, and hall conductance can no longer be expressed in terms of the chern number or the weighted sum of the chern number. Furthermore, the topological properties are analyzed and discussed through the results of different instance with their illustration. Such as, the phase transition point of topological phase is robust to the environment, and the system still has topological phase transition. It is expected to be realized or controlled by experiments, and our observations may contribute to its application and extension in condensed matter physics and quantum statistical physics.

I. INTRODUCTION

In 1879, American physicist Edwin.H.Hall found that when there is a current in a two-dimensional conductor, if a magnetic field perpendicular to the current direction is superimposed on the conductor, voltage will be generated at both ends of the conductor along the direction perpendicular to the current and magnetic field, which is called Hall effect[1]. In 1980, under the experimental conditions of low temperature, strong magnetic field, a German physicist Klaus von Klitzing et al.observed two-dimensional electron gas Hall conductivity value is $\frac{e^2}{h}$ integer times, and this is obviously different from the classic Hall conductance, it showed ladder-like changes. This macroscopic scale new quantum phenomenon, known as the quantum Hall effect (QHE) [2]. And, researchers found that it was almost impervious to detailed changes in the shape, size and purity of the material studied. The insensitivity of Hall conductance to details indicates that there may be some invariants in the material. Subsequently, topological invariants were first proposed by D.J.Thouless et al [3], and be called Chern number [4].

Through further study, Edwin.H.Hall found that the Hall effect in ferromagnetic materials is stronger than that in non-magnetic conductors, and it can be observed in ferromagnetic conductors even without applying magnetic field [5], which is called anomalous Hall effect. Topological insulators (TIs) were theoretically predicted to exist and have been experimentally discovered in [6–8], they are materials that have a bulk electronic band gap like an ordinary insulator but have protected conducting topological states(edge states) on their surface. In the last decades, these topological materials have gained many interests of scientific community for their unique properties such as quantized conductivities, dissipationless transport and edge states physics [9, 10]. Although the exploration of topological phases of matter have become a major topics at the frontiers of the condensed matter physics, the behavior of TIs subjected to dissipative dynamics has been barely explored. This leads to a lack of capability to discuss issues such as their robustness against decoherence, which is crucial in applications of these materials in quantum information processing.
and spintronics. Most recently, the study of topological states was extended to non-unitary systems, going a step further beyond the Hamiltonian ground-state scenario. The topological invariant was first derived by Thouless et al. [3], which provides a characterization of fermionic time-reversal-broken (TRB) topological order in two spatial dimensions. This is done in such a way that the Hall conductivity is written in terms of the topological invariant (or the Chern number), which by linear response theory is related to an adiabatic change of the Hamiltonian in momentum space. For instance, the topological order might be caused by dissipation [11], the TIs’ different responses to single-mode quantized field [12], the existence of topological phase is discovered in two-dimensional atomic lattice gas [13], the topological magnon bands of ferromagnetic Shastry-Sutherland lattices [14], the Bloch oscillations of topological edge modes [15], etc. The Hall conductivity given by the Kubo formula is a linear response of the quantum transverse transport to a weak electric field [16]. Moreover, the study of TIs has been extended to optical conductivity [17–21].

Topologically non-trivial phases of matter [22] have been at the forefront of condensed matter physics during the last several decades, since the discovery of the integer and fractional quantum Hall effects [2, 23]. The more recently discovered topological insulators, superconductors [24, 25], and semimetals [26] have opened the door to novel exciting possibilities, e.g., topologically protected quantum computing [27]. The idea of employing the engineered bath interactions scheme to stabilize a topologically ordered ground state has already received some attention in recent years [28–34].

In this paper, we study the response of topological insulator driven by momentum-carrying single-mode field. We solve the ground state of the system after the addition of a single mode light field with adjustable photon momentum. Specifically, we show that from the analytical solution of hall conductance compared with the closed system, there is an extra correction term, and hall conductance can no longer be expressed in terms of the Chern number or the weighted sum of the Chern number. Furthermore, the topological properties are analyzed and discussed through the results of different instance with their illustration. Such as, the phase transition point of topological phase is robust to the environment, and the system still has topological phase transition. It is expected to be realized or controlled by experiments, and our observations may contribute to its application and extension in condensed matter physics and quantum statistical physics.

II. THE EFFECTIVE HAMILTONIAN

We now introduce the model for the two-band 2D model, we use a simple taching model describing a single particle Bloch Hamitonian

\[ \hat{H}_0 = \sum_k c_k^\dagger (\mathbf{d}(\mathbf{k}) \cdot \sigma) c_k \] (1)

where \( c_k^\dagger = (c_{k\uparrow}^\dagger, c_{k\downarrow}^\dagger) \) and \( \sigma = (\sigma^x, \sigma^y, \sigma^z) \) is the vector of Pauli matrices, we take \( \mathbf{d}(\mathbf{k}) = (\sin(ak_x), \sin(ak_y), u + \cos(ak_x) + \cos(ak_y)) \), such that Eq.(1) describes QWZ model for a two-dimensional Chern insulator [35, 36]. The model has a single parameter \( u \), and the Chern number of the model is determined by the value of \( u \) as the material parameter. \( \mathbf{d}(\mathbf{k}) \) are the momentum-dependent coefficients which describe the spin-orbit interactions and exchange interaction of magnetic impurities and describes the hybridization between bands [36]. The models we construct in this chapter describe the so-called Quantum Anomalous Hall Effect. Here we have the same connection between edge states and bulk Chern number as in the Quantum Hall Effect.

The corresponding eigenstates of \( \hat{H}_0 \) take

\[ |\varepsilon_+\rangle = \cos \frac{\theta}{2} e^{-i\phi} |\uparrow\rangle + \sin \frac{\theta}{2} |\downarrow\rangle \]

\[ |\varepsilon_-\rangle = -\sin \frac{\theta}{2} e^{-i\phi} |\uparrow\rangle + \cos \frac{\theta}{2} |\downarrow\rangle \] (2)

where \( \cos \theta = d_z/|\mathbf{d}| \) and \( \tan \phi = d_y/d_x |\varepsilon_+\rangle \) and \( |\varepsilon_-\rangle \) represent the energy of the upper and the lower band, respectively.

We focus on a single momentum-dependently light fields with photon momentum \( \mathbf{q} \), which consistent with frequency \( \omega_\mathbf{q} = \nu |\mathbf{q}| \), and the Hamiltonian of the momentum-dependently light field

\[ \hat{H}_l = \hbar \omega_\mathbf{q} \hat{b}_\mathbf{q}^\dagger \hat{b}_\mathbf{q} \] (3)

where \( \nu \) is the speed of light. \( \hat{b}_\mathbf{q}^\dagger \) and \( \hat{b}_\mathbf{q} \) are creation and annihilation operators of the momentum of photon \( q \). For the electron-photon coupling we take the following

\[ \hat{H}_r = \sum_{\mathbf{r}} \hat{c}_{\mathbf{r},\nu}^\dagger [g^\ell \sigma_{\nu,\nu}^\ell \hat{b}_{\mathbf{r}}^\dagger + g_\ell \sigma_{\nu,\nu}^\ell \hat{b}_{\mathbf{r}}] \hat{c}_{\mathbf{r},\nu'} \] (4)
FIG. 1: The bulk dispersion relation of the total model, for various values of photon momentum $q$ as indicated in the plots. The parameters are: a) $q = 0$, b) $q = 0.5$, c) $q = 1$, d) $q = 2$. $k_x$ and $k_y$ take the value from $-\pi$ to $\pi$ and traverse the entire Brillouin zone.

where $g_i$ denote the coupling constants of electron-photon. The coordinate $r$ is confined to the 2D plane. Now we can get the two-dimensional momentum space Hamiltonian by Fourier transform Eq. (4) as shown in appendix A.

$$
\hat{H}_I = \sum_k g_i^* \hat{c}_{k+q}^\dagger \hat{c}_k + g_1 \hat{b}_q^\dagger \hat{b}_q + \sum_k g_i \hat{c}_{k+q}^\dagger \hat{b}_q \hat{c}_k
$$

Notice that we only focus on a single-mode light field, so we remove the momentum of the photon from the summation sign. We can easily found that it annihilate an electron whose momentum position on $k+q$ and create an electron whose momentum position on $k$. Where $\{\uparrow, \downarrow\}$ denote the pseudospin bound to the momentum position.

In the following, we obtain the total effective Hamiltonian of the system

$$
\hat{H} = \hat{H}_0 + \hat{H}_I = |d| \tau_z + \hbar \omega \hat{b}_q^\dagger \hat{b}_q + \sum_k \begin{pmatrix}
\varepsilon_k^+ + \varepsilon_{k+q}^- & -g_2 \varepsilon_k^- \\
-g_2^* \varepsilon_{k+q}^+ & \varepsilon_k^+
\end{pmatrix}
$$

where $g_1^* = g_1^* \cos \frac{\theta}{2} \cos \frac{\phi_1}{2} e^{-i\phi_1}$ and $g_2^* = g_2^* \sin \frac{\theta}{2} \sin \frac{\phi_1}{2} e^{-i\phi_1}$. The difference between Eq.(4) and Eq.(6) is that we hide the spin information in the band here.

Next, we investigate the characteristics of the ground state of the system under the coupling of different spin modes and the topological properties of the system.

III. THE GROUND STATE OF THE WHOLE SYSTEM

Obviously, the effective Hamiltonian can be solved exactly, and firstly we choose a group of the basis vectors (just consider one photon in the system) \{|\varepsilon_k^+ \otimes |0\rangle\}, \{|\varepsilon_{k+q}^+ \otimes |0\rangle\}, \{|\varepsilon_k^- \otimes |1\rangle\}, \{|\varepsilon_{k+q}^- \otimes |1\rangle\} to expand the Hamiltonian into a matrix form

$$
\begin{bmatrix}
|d(k)| + \hbar \omega & 0 & 0 & g_2^2 \\
0 & |d(k+q)| & g_1 & 0 \\
0 & g_1 & -|d(k)| + \hbar \omega & 0 \\
g_2 & 0 & 0 & -|d(k+q)|
\end{bmatrix}
$$

(7)
The ground state of the system that can be obtained by solving the eigenvalue of Eq. (7), the ground state is

$$|E⟩ = a |\varepsilon^{k+q}⟩ \otimes |\{0\}\rangle + b |\varepsilon^{k}⟩ \otimes |\{1\}\rangle$$

(8)

where $a$ and $b$ are normalization constant. $|\{0\}\rangle$ is Vacuum state. Hence, the 3D graph drawn according to energy eigenvalues is shown in Fig. 1. The variation of energy band under different parameters is described. For simplicity, we just consider the momentum of the photon along the $x$ direction. $a) q = 0$, $b) q = 0.5$, $c) q = 1$, $d) q = 2$ represent the energy spectrum at different photon momentum. When the photon momentum $q$ is zero, the system has double merger degree, as the momentum of photon increases, the original two bands begin to split into four, it middle two bands will reach the situation where overlapping.

We choose periodic boundary conditions along the $y$ direction, and open boundary conditions in the $x$ direction. Because the strip is translation invariant along the edge, the wavenumber $k_y$ is a good quantum number and $k_y$ take the value from $-\pi$ to $\pi$ and traverse the entire Brillouin zone. We show the marginal position probability distribution of the $N$th energy eigenstate. This state can be an edge state on the right edge (b), on the left edge (c), or a bulk state (d).

IV. HALL CONDUCTANCE

In this section, we consider the electric field is introduced through the vector potential, which changes the Hamiltonian to be time-dependent. Formally, we use the Pierels substitution,

$$k \rightarrow k - \frac{qA}{\hbar} = k - \frac{eEt}{\hbar}$$

(9)

This gives rise to a hall current proportional and perpendicular to the electric field by Kubo formula [16]. The velocity operator and the current density along $y$ direction, therefore the Hall Conductance is given by [38].

$$\sigma_{xy}^m = \frac{i e^2}{2\pi \hbar} \int dk^2 \nabla \times \langle u_m(k) | \nabla_k | u_m(k) \rangle$$

(10)
The Chern number could describe the topological property of the ground-state wave function, and then it is a measurable physical quantity [39]. Substituting the ground $|E\rangle$ into Eq. (10)

$$\sigma_{xy} = \frac{ie^2}{2\pi\hbar} \int dk^2 \nabla \times \langle E | \nabla_k | E \rangle$$

$$= \frac{ie^2}{2\pi\hbar} \int dk^2 \left\{ \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} + \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} - \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} + \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} \right\}$$

$$+ \left( -ia \right) \cos^2 \frac{\theta}{2} \left( \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} - \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} \right) + ia \cos^2 \frac{\theta}{2} \left( \frac{\partial a}{\partial k_x} \frac{\partial a}{\partial k_y} - \frac{\partial a}{\partial k_x} \frac{\partial a}{\partial k_y} \right)$$

$$+ \left( -ib \right) \sin^2 \frac{\theta}{2} \left( \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} - \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} \right) + ib \sin^2 \frac{\theta}{2} \left( \frac{\partial b}{\partial k_x} \frac{\partial b}{\partial k_y} - \frac{\partial b}{\partial k_x} \frac{\partial b}{\partial k_y} \right)$$

$$+ \left[ a^* \left( \frac{\partial \langle \xi^k_+ \rangle}{\partial k_x} - \frac{\partial \langle \xi^k_+ \rangle}{\partial k_y} \right) - b^* \left( \frac{\partial \langle \xi^k_+ \rangle}{\partial k_x} - \frac{\partial \langle \xi^k_+ \rangle}{\partial k_y} \right) \right]$$

$$= \int dk^2 \Omega_+$$

where the zeroth-order of $\sigma_H$ take

$$\sigma^{(0)}_H = \frac{ie^2}{2\pi\hbar} \int dk^2 \Omega_+$$

where the first-order of $\sigma_H$ take

$$\sigma^{(1)}_H = \frac{ie^2}{2\pi\hbar} \int dk^2 \left\{ \left( a^* \Omega_+ - b^* \Omega_- \right) \right\}$$

$$+ \left( \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} + \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} - \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} + \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} \right)$$

$$- ia \cos^2 \frac{\theta}{2} \left( \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} - \frac{\partial a^*}{\partial k_x} \frac{\partial a}{\partial k_y} \right)$$

$$+ ia \cos^2 \frac{\theta}{2} \left( \frac{\partial a}{\partial k_x} \frac{\partial a}{\partial k_y} - \frac{\partial a}{\partial k_x} \frac{\partial a}{\partial k_y} \right)$$

$$- ib \sin^2 \frac{\theta}{2} \left( \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} - \frac{\partial b^*}{\partial k_x} \frac{\partial b}{\partial k_y} \right)$$

$$+ ib \sin^2 \frac{\theta}{2} \left( \frac{\partial b}{\partial k_x} \frac{\partial b}{\partial k_y} - \frac{\partial b}{\partial k_x} \frac{\partial b}{\partial k_y} \right)$$

Notice that the Berry curvature of the lower band is defined by

$$\Omega_- = \left( \frac{\partial \langle \xi^k_- \rangle}{\partial k_x} \cdot \frac{\partial \langle \xi^k_- \rangle}{\partial k_y} - \frac{\partial \langle \xi^k_- \rangle}{\partial k_y} \cdot \frac{\partial \langle \xi^k_- \rangle}{\partial k_x} \right)$$

and the the Berry curvature of the lower band is defined by

$$\Omega_+ = \left( \frac{\partial \langle \xi^{k+q}_+ \rangle}{\partial k_x} \cdot \frac{\partial \langle \xi^{k+q}_+ \rangle}{\partial k_y} - \frac{\partial \langle \xi^{k+q}_+ \rangle}{\partial k_y} \cdot \frac{\partial \langle \xi^{k+q}_+ \rangle}{\partial k_x} \right)$$

The parameter $\nu$ shifts the whole space, we can easy find that the trend of changes of Chern number. For closed system, the Chern number is $C = 0$ for $|\nu| > 2$ and with $C = -1$ for $-2 < \nu < 0$ and $C = 1$ for $0 < \nu < 2$. In Fig. 3, the features found that while a small momentum $q$, the platform of the Hall conductance will not be an integer, but as the $q$ momentum increases gradually, the hall dot guide of the system will gradually approach the value shown on the platform. By analyzing the analytical results, we can find that when momentum increases gradually, the first order part of the hall point derivative of the system tends to zero. The photon momentum takes a fixed value $q = 0.5$
as in Fig. 4. Energy spectrum in the case with $u = 1$ contact that the Chern number of the system close to 1. While $u = 1.9$ close to 2, the middle two bands of the energy spectrum overlap at this point, the number of systems in this case is undefinable strictly. The middle two bands in the band are separated again at $u = 3$. The topological phase transition of the system can be considered to happen at $u = 1$.

The Chern number for 2D systems is an example of a topological invariant for energy bands in a setting where no symmetries constrain the form of the Hamiltonian. The field of topological insulators arose from the realization that, when one constrains the Hamiltonian to have additional symmetries, new topological invariants emerge in various spatial dimensions. These paper describe a TR symmetry-breaking system. The Hall conduction of the system is quantized and it is determined by the first Chern number of the Berry phase gauge field in the BZ [3].

V. CONCLUSION

In summary, we have introduced the response of a two-band system to a quantized single-mode field with photon momentum. We study the response of topological insulator driven by momentum-carrying sigle-mode field. We solve the ground state of the system after the addition of a single mode light field with adjustable photon momentum. Specifically, We show that from the analytical solution of hall conductance compared with the closed system, there is an extra correction term, and hall conductance can no longer be expressed in terms of the chern number or the weighted sum of the chern number. Furthermore, the topological properties are analyzed and discussed through the results of different instance with their illustration. Such as, the phase transition point of topological phase is robust to the environment, and the system still has topological phase transition. In addition, We analyzed the energy spectrum and edge state density under the open boundary condition, compared with the analytical results of Hall point derivations, it is found that topological properties (phase transition points) still exist in the system, and the time inversion symmetry of the system is broken. It is expected to be realized or controlled by experiments, and our
observations may contribute to its application and extension in condensed matter physics and quantum statistical 
physics.

As is known to all that the change of the ground state topological structure is accompanied with a topological phase transition. As mentioned in the celebrated paper by D.J.Thouless, the Hall conductance can be probed, and the Chern number usually remains invariant with the change of Hamiltonian. Then, the Chern number has a discontinuous jump, which indicates a topological phase transition in the ground state [40].

From the picture of the specific model, the phase transition point of the Hall conductance is still robust with the environmental inhibition. Topological phase transition still occurs in the open system, so we can use the quantum Hall conductance to explore the topological properties of topological quantum states in the system. As a supplement and correction, we explore a new linear response of open system with the external field. It would be interesting and important to explore the higher order of the system with light or the electric field. Although our starting point is from a circumstance introduced by a phenomenally dissipator, the general results of this paper should also be applicable to other field described by various master equations. We believe that the observation we get makes the topological materials almost immune to the influence of the quantum field, and then supports its application into quantum optics and condensed matter physics.

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Appendix A: THE HAMILTONIAN MATRIX H

In this Appendix, We start from the Hamiltonian matrix defined in Eq.(4). We choose the Fourier transform as

\[
\begin{align*}
\hat{c}_{\mathbf{r},\nu}^\dagger &\sim \sum_{\mathbf{k}} \hat{c}_{\mathbf{k},\nu} e^{i\mathbf{k}\cdot\mathbf{r}} \\
\hat{b}_{\mathbf{r},\nu}^\dagger &\sim \sum_{\mathbf{k}} \hat{b}_{\mathbf{k},\nu} e^{i\mathbf{k}\cdot\mathbf{r}} \\
\hat{c}_{\mathbf{r},\nu} &\sim \sum_{\mathbf{k}} \hat{c}_{\mathbf{k},\nu} e^{-i\mathbf{k}\cdot\mathbf{r}} \\
\hat{b}_{\mathbf{r},\nu} &\sim \sum_{\mathbf{k}} \hat{b}_{\mathbf{k},\nu} e^{-i\mathbf{k}\cdot\mathbf{r}}
\end{align*}
\]  

(A1)

We obtain the total Hamiltonian of the system

\[
\hat{H} = \hat{H}_0 + \hat{H}_I = |\mathbf{d}|\tau_z + hv_l |\mathbf{q}| \hat{b}_q^\dagger \hat{b}_q + g_1^* \hat{b}_1^\dagger \left[ \cos \frac{\theta_1}{2} \sin \frac{\theta}{2} e^{-i\phi_1} |\varepsilon_+^k\rangle \langle \varepsilon_+^{k+q}| + \cos \frac{\theta}{2} \cos \frac{\theta_1}{2} e^{-i\phi_1} |\varepsilon_-^k\rangle \langle \varepsilon_-^{k+q}| \right. \\
- \sin \frac{\theta}{2} \sin \frac{\theta_1}{2} e^{-i\phi_1} |\varepsilon_-^k\rangle \langle \varepsilon_+^{k+q}| - \sin \frac{\theta_1}{2} \cos \frac{\theta}{2} e^{-i\phi_1} |\varepsilon_-^k\rangle \langle \varepsilon_-^{k+q}| \\
+ g_1 \hat{b}^\dagger |\cos \frac{\theta_1}{2} \sin \frac{\theta}{2} e^{-i\phi_1} |\varepsilon_+^k\rangle \langle \varepsilon_+^{k+q}| + \cos \frac{\theta}{2} \cos \frac{\theta_1}{2} e^{i\phi_1} |\varepsilon_-^k\rangle \langle \varepsilon_-^{k+q}| \langle \varepsilon_-^k| \\
- \sin \frac{\theta}{2} \sin \frac{\theta_1}{2} e^{i\phi_1} |\varepsilon_-^k\rangle \langle \varepsilon_+^{k+q}| - \sin \frac{\theta_1}{2} \cos \frac{\theta}{2} e^{i\phi_1} |\varepsilon_-^{k+q}\rangle \langle \varepsilon_-^k| \Biggr] 
\]  

(A2)

We define \(g_1^* \equiv g_1^* \cos \frac{\theta}{2} \cos \frac{\theta_1}{2} e^{-i\phi_1}, g_2^* \equiv g_1^* \sin \frac{\theta}{2} \sin \frac{\theta_1}{2} e^{-i\phi_1}\), we assume that transitions at different momentum positions in the same energy band are substantially detuned. We have

\[
\hat{H} = \hat{H}_0 + \hat{H}_I = |\mathbf{d}|\tau_z + hv_l |\mathbf{q}| \hat{b}_q^\dagger \hat{b}_q + \hat{b}_1^\dagger |g_1^* \varepsilon_-^k\rangle \langle \varepsilon_-^{k+q}| + g_1 \hat{b}^\dagger |\varepsilon_+^k\rangle \langle \varepsilon_-^{k+q}| \langle \varepsilon_-^k| \]  

(A3)
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