Optical properties of 1D photonic crystals based on multiple-quantum-well structures

M. V. Eremenchouk, L. I. Deych, and A. A. Lisyansky
Physics Department, Queens College, City University of New York, Flushing, New York 11367, USA

A general approach to the analysis of optical properties of photonic crystals based on multiple-quantum-well structures is developed. The effect of the polarization state and a non-perpendicular incidence of the electromagnetic wave is taken into account by introduction of an effective excitonic susceptibility and an effective optical width of the quantum wells. This approach is applied to consideration of optical properties of structures with a pre-engineered break of the translational symmetry. It is shown, in particular, that a layer with different exciton frequency placed at the middle of an MQW structure leads to appearance of a resonance suppression of the reflection.

PACS numbers: 72.15.Rn,05.40.-a,42.25.Dd,71.55.Jv

I. INTRODUCTION

Structures with spatially modulated dielectric properties (photonic crystals) attract an ever growing interest since the first papers where they were considered. This interest is caused by unique opportunities that such structures provide, to affect, in a controllable way, fundamental microscopic processes of light-matter interaction through a modification of macroscopic geometric characteristics of the structures. This makes such structures of obvious interest not only for fundamental physics but also for applications. Most of the works devoted to photonic crystals considered structures made of materials with a frequency independent dielectric constant. Recently, however, a new class of structures, which can be described as resonant or optically active photonic crystals, has attracted particular attention. In these structures periodic modulation of the dielectric constant is accompanied by the presence of internal excitations of constituent materials resonantly interacting with light within a certain frequency region, and resulting in a strong frequency dispersion of constituent dielectric constants. Extreme cases of such structures are so called optical lattices, in which well localized resonant elements are periodically distributed through the medium with a uniform dielectric constant. Originally, the concept of optical lattices referred to structures formed by cold atoms but it was also applied to a special kind of multiple quantum well structure (MQW), which were considered as a semiconductor analog of a one-dimensional optical lattice.

MQW is a periodic multilayer structure built of two semiconductor materials, for instance, GaAs and Al_xGa_{1-x}As, in which electrons and holes are confined in narrower layers of a material with a smaller band gap (quantum wells) separated by wider layers of a semiconductor with larger band-gap (barriers). In this case, the role of dipole active resonant excitations is played by excitons confined to respective quantum wells, and if the width of the barriers is large enough, excitons from different quantum wells do not interact directly. They, however, still can interact through their common radiation field, and in this sense they are similar to atomic optical lattices. This analogy is, however, exact only if one can neglect a difference in refractive indexes of wells and barriers. This approximation was widely used in most papers devoted to long-period MQW structures, in which the period of the structure is comparable with the wavelength of exciton radiation. Of special interest are so called Bragg structures, in which the excitonic wavelength is in Bragg resonance with the periodicity, and which are characterized by a significantly enhanced radiative coupling between quantum well excitons. As a result of this coupling, light propagates through such a structure in the form exciton-polaritons, whose dispersion law is characterized by two branches with a band-gap between them. The width of this stop band is significantly enhanced compared to off-Bragg structures, and this is what makes such structures of particular interest for applications.

In realistic MQW structures, however, dielectric constants of the wells and barriers are not equal to each other, and the presence of resonant optical excitations is accompanied by a periodic modulation of the background dielectric constant. These structures, therefore, represent a special case of one-dimensional resonant photonic crystals, optical properties of which are characterized by an interplay between interface reflections and resonant light-exciton interaction. The effects of the refractive index contrast on the optical properties of MQW structures have not been, of course, overlooked in previous studies. In particular, a modification of the Bragg condition and reflection spectra at normal incidence of Bragg MQWs in the presence of the contrast have been discussed. The effects of the dielectric mismatch on optical properties of single quantum wells or an MQW structure embedded in a dielectric environment was also taken into account. However, a complete analytical theory of MQW based photonic crystals has not yet been developed. While optical spectra of any given MQW based structure can be easily obtained numerically, this is not sufficient when one needs to design a structure with predetermined optical properties, which is a key element in utilizing these structures for optoelectronic applications. The main difficulty of this task is the presence of a large number of experimental parameters such as...
an angle of incidence, a polarization state, indices of refraction, widths of the barriers and the quantum wells, etc., which are in a complicated way related to spectral characteristics of a structure. In order to resolve this difficulty, one needs a general effective analytical approach that would facilitate establishing relationships between material parameters and spectral properties of MQW based structures for an arbitrary angle of incidence and polarization state of incoming light. In the present paper we develop such a method and apply it to a case of MQW structure with an intentionally broken periodicity (an MQW structure with a “defect”). The method is based on a transfer matrix approach and consists of two steps. In the first step we show that a quantum well embedded in a dielectric environment can be described in exactly the same way as a quantum well in vacuum by introducing an effective excitonic susceptibility and an effective optical width of the quantum well layer. In the second step we establish relations between these effective quantum well characteristics and parameters of a total transfer matrix, describing propagation of light throughout an entire structure. The method is rather general and can be applied to a great variety of different MQW structures with light of an arbitrary polarization, incident at an arbitrary angle. In order to demonstrate the power of our approach, we consider reflection spectra of an MQW structure in which a central well is replaced with a well having a different resonant frequency. Such structures have been considered previously in a number of papers\cite{23,24,25} in the optical lattice approximation. Here we show that the presence of the refractive index contrast does not destroy the remarkable reflection properties of such structures, confirming, therefore, their potential for optoelectronic applications.

II. A SINGLE QUANTUM WELL IN A DIELECTRIC ENVIRONMENT

Propagation of the electromagnetic wave in structures under discussion is governed by the Maxwell equation

$$\nabla \times \nabla E = \frac{\omega^2}{c^2}[\epsilon_\infty(z)E + 4\pi P_{exc}],$$

with modulated background dielectric permeability, $\epsilon_\infty(z)$, which is assumed to take values $n_b^2$ and $n_w^2$ in the barriers and the quantum wells materials respectively. For the sake of concreteness we assume hereafter that $n_w > n_b$, unless otherwise explicitly specified. $P_{exc}$ is the excitonic contribution to the polarization and is defined by

$$P_{exc} = \chi(\omega) \int \Phi(z)\Phi(z')E(z')dz',$$

where $\Phi(z)$ is the exciton envelope function. Here we have restricted ourselves by taking into account 1s heavy-hole excitons only and have neglected the in-plane dispersion of the excitons. The frequency dependence of the excitonic susceptibility is described by

$$\chi(\omega) = \frac{\alpha}{\omega_0 - \omega - i\gamma},$$

where $\omega_0$ is the exciton resonance frequency, $\gamma$ is the non-radiative decay rate of the exciton, $\alpha = \epsilon_\omega \omega_{LT} a_B^2 \omega_0^2/4c^2$, $\omega_{LT}$ is the exciton longitudinal-transverse splitting and $a_B$ is the bulk exciton Bohr’s radius.

Due to the absence of an overlap of the exciton wave functions localized in different quantum wells and the linearity of the Maxwell equations, the propagation of the electromagnetic wave along the structure can be effectively described by a transfer-matrix. Using the usual Maxwell boundary conditions the transfer matrix through one period of the structure in the basis of incoming and outgoing plane waves can be written in the form

$$T = T_b^{1/2}T_{wb}T_wT_{wb}T_b^{1/2},$$

where

$$T_b^{1/2} = \begin{pmatrix} e^{i\phi_b/2} & 0 \\ 0 & e^{-i\phi_b/2} \end{pmatrix}$$

is the transfer matrix through the halves of the barriers surrounding the quantum well. Here $\phi_b = \omega n_b d_b \cos \theta_b/c$ with $d_b$ being the width of the barrier and $\theta_b$ being an angle between the wave vector $k$ inside the barrier and the direction of the $z$-axis, $\hat{e}_z$.

The scattering of the electromagnetic wave at the interface between the quantum well and the barrier caused by the mismatch of the indices of refraction of their materials is described by

$$T_{bw} = T_{wb}^{-1} = T_p(\rho) = \frac{1}{1 + \rho} \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

where $\rho$ is the Fresnel reflection coefficient. The interface scattering depends upon both the angle of incidence of the wave and its polarization state. These effects are effectively described by Fresnel coefficients (see, e.g. Ref. \cite{26}) $\rho_s$ and $\rho_p$,

$$\rho_s = \frac{n_w \cos \theta_w - n_b \cos \theta_b}{n_w \cos \theta_w + n_b \cos \theta_b},$$

$$\rho_p = \frac{n_w \cos \theta_b - n_b \cos \theta_w}{n_w \cos \theta_b + n_b \cos \theta_w}$$

for $s$ ($E \perp (k, \hat{e}_z)$) and $p$ ($E \parallel (k, \hat{e}_z)$) polarizations respectively. The angular dependence of these coefficients upon the angle of incidence measured inside the barrier is schematically shown in Fig. \ref{fig:angle_dependence}.

Finally,

$$T_w = \begin{pmatrix} e^{i\phi_w} (1 - iS) \\ iS \end{pmatrix} \begin{pmatrix} e^{-i\phi_w} & -iS \\ iS & e^{i\phi_w} (1 + iS) \end{pmatrix}$$


is the transfer matrix through the quantum well. Here $\phi_w = \omega n w d w \cos \theta_w/c$, where $d_w$ is the width of the quantum well and $\theta_w$ is the angle between $k$ inside the quantum well and $\mathbf{e}_z$. The excitonic contribution to the scattering of the light is described by the function

$$S = \frac{\Gamma_0}{\omega - \omega_0 + i\gamma},$$

which we will call the excitonic susceptibility in what follows. The radiative decay rate, $\Gamma_0$, at normal incidence is determined by

$$\Gamma_0 = \frac{1}{2} \pi \omega_L T_1^3 \left( \int \Phi(z) \cos k z dz \right)^2.$$  

\hspace{1cm} (9)

For oblique incidence the radiative decay rates are renormalized in different ways for different polarizations. For $p$-polarization in addition to this renormalization it is also necessary to take into account a possible splitting of $Z$- and $L$-exciton modes\cite{29,30,31} that gives rise to a two-pole form of $S$. However, in materials with the zinc-blend structure, the $Z$-mode of the heavy-hole excitons is optically inactive, and one can describe angle dependencies of the radiative decay rate for $s$- and $p$-polarizations respectively by simple expressions

$$\Gamma_0^{(s)} = \Gamma_0/\cos \theta_w, \quad \Gamma_0^{(p)} = \Gamma_0 \cos \theta_w.$$  

\hspace{1cm} (10)

Thus, propagation of light in the structures under consideration depends upon a number of natural parameters such as Fresnel coefficients, exciton frequencies and radiative decay rate, and optical widths, which (with the exception of $\omega_0$) depend upon the angle of incidence of the wave and its polarization state.

Our next step will be to simplify the presentation of the total transfer matrix through the period of the structure in such a way that makes the relations between the elements of the transfer matrix and the natural parameters of the structure more apparent. The most complicated part of the transfer matrix is the product $T_{bw} T_w T_{wb}$, which describes the reflection of the wave from the interface and its interaction with quantum well excitons. We simplify it by noting that this product can be presented as $T_{bw} T_w T_{wb} = T_w$, where $T_w$ has the same form as $T_w$, Eq. (8), but with renormalized parameters

$$T_w = T_{bw} T_w T_{wb} = \left( e^{i\phi_w (1 - i\tilde{S})} \right) \left( i\tilde{S} \right) e^{i\phi_w (1 + i\tilde{S})},$$  

\hspace{1cm} (12)

where the effective excitonic susceptibility, $\tilde{S}$, and the phase shift, $\phi_w$, are defined as

$$\tilde{S} = S \frac{1 + \rho^2 - 2\rho \cos \phi_w}{1 - \rho^2} + 2 \rho \frac{\sin \phi_w}{1 - \rho^2},$$

$$e^{i(\phi_w - \tilde{\phi}_w)} = \frac{1 - \rho e^{-i\phi_w}}{1 - \rho e^{i\phi_w}}.$$  

\hspace{1cm} (13)

Here $\rho$ denotes the Fresnel coefficient for the wave of a respective polarization. Taking into account the diagonal form of the transfer matrix through the barrier $T_1$, one can see that the total transfer matrix through the period of the structure again has the form of a single quantum well transfer matrix and is determined by $\tilde{S}$, where the phase $\tilde{\phi}_w$ is replaced by a total phase $\phi = \phi_b + \tilde{\phi}_w$.

Thus, we have shown that the propagation of the wave in MQW based photonic crystals can be described in terms of properties of a respective optical lattice with renormalized parameters. The renormalization of the phase is the simplest one: the expression for $\tilde{\phi}_w$ can be rewritten as

$$\tilde{\phi}_w = \phi_w \frac{1 + \rho}{1 - \rho}$$  

\hspace{1cm} (14)

provided that the change of phase $\phi_w$ across the well is much smaller than $2\pi$, which is usually true for long period MQW structures. Hence, one of the effects of the index of refraction contrast is reduced to a simple renormalization of the optical width of the quantum well.

The effective susceptibility, $\tilde{S}$, consists of two terms. One of them has a singularity at the exciton frequency while the second remains smooth in a wide frequency region. The relation between these terms depends upon the frequency and near the exciton resonance the second term is negligibly small. The frequency region where the nonsingular addition to the effective susceptibility can be neglected is determined by

$$|\omega - \omega_0| < \omega_{\text{min}} = \frac{\Delta \Gamma}{2 \Delta \rho c} \frac{1 - \rho}{1 + \rho},$$  

\hspace{1cm} (15)

where $\Delta \Gamma = \sqrt{2T_1 \omega_0/\pi}$ is the half-width of the forbidden gap in a Bragg MQW without a mismatch of the indices of refraction, and $\Delta \rho c = 2\omega_0 \sin(|\phi_w(\omega_0)|) / \pi(1 - \rho^2)$ is the half-width of the forbidden gap in a passive photonic crystal characterized by the same value of the mismatch calculated with the assumption of the narrowness of the
gap in comparison with the position of the center of the gap, $\omega_r$.

It follows from Eq. (15) that the resonant part of the effective susceptibility, Eq. (18), makes the main contribution over the entire frequency region of exciton polaritons stop band when

$$\Delta \Gamma > 2 \Delta_{PC}. \quad (16)$$

In realistic multiple GaAs/Al$_x$Ga$_{1-x}$As structures the values $\rho < 0.03$ of the Fresnel coefficient at normal incidence$^{38}$ and $\phi_w(\omega_0) \sim 0.1\pi$ can be considered as typical, so both quantities $\Delta_F$ and $\Delta_{PC}$ are of the same order of magnitude $\sim 10^{-2}$ eV. Therefore, in principle, both signs of the inequality in Eq. (16) are possible. Here, however, we consider the simplest case when one can neglect the non-singular term in $\tilde{S}$ and approximate the latter by

$$\tilde{S} = S \frac{1-\rho}{1+\rho}. \quad (17)$$

In this approximation, the effect of the contrast is reduced to an additional modification of the radiative decay rate (or, consequently, the oscillator strength). Depending upon the value of the Fresnel coefficient one can observe either an enhancement (when $\rho < 0$) or a reduction (when $\rho > 0$) of exciton radiative recombination. Since usually $n_w > n_b$, the Fresnel coefficient for the normal incidence is positive and therefore the oscillator strength is diminished compared to the case of the absence of the contrast. When the angle of incidence increases, in addition to different dependences of the Fresnel coefficients corresponding to different polarization states following from Eqs. (17), it is necessary to take into account direct modification of the oscillator strength given by Eqs. (11). Fig. 2 shows the dependence of the factor modifying the radiative decay rate upon the angle of incidence.

III. OPTICAL PROPERTIES OF MQW STRUCTURES

The reduced description presented in the previous section works well within a frequency region satisfying the inequality given in Eq. (15). For a single quantum well (or even for short MQW structures) and for angles not too close to the angle of total internal reflection, the condition (15) is fulfilled for the entire frequency region of interest, which, in this case, is of the order of magnitude of $\Gamma_0$. Therefore, under these circumstances the mismatch of the indices of refraction can be treated perturbatively, so that all results neglecting the mismatch remain valid up to renormalization of the oscillator strength. However, when the number of quantum wells in an MQW structure increases, the frequency region affected by excitons widens almost linearly$^{32}$ and, therefore, the regular contribution to the effective susceptibility becomes important, and more detailed analysis is necessary.

![Fig. 2: Change of the effective radiative decay rates $\Gamma_s$ (solid line) and $\Gamma_p$ (dotted line) with the angle of incidence for a single quantum well.](attachment:image.png)

Keeping in mind subsequent application to more complicated structures, it is convenient to introduce a special representation for a transfer matrix

$$T(\theta, \beta) = \begin{pmatrix} \cos \theta - i \sin \theta \cosh \beta & -i \sin \theta \sinh \beta \\ i \sin \theta \sinh \beta & \cos \theta + i \sin \theta \cosh \beta \end{pmatrix}, \quad (18)$$

where the parameters of the representation, $\theta$ and $\beta$, are related to the “material” parameters $S$ and $\phi_w$ entering the transfer matrix by

$$\cos \theta = \text{Tr} T/2 = \cos \phi + S \sin \phi, \quad \coth \beta = \cos \phi - S^{-1} \sin \phi. \quad (19)$$

This representation is valid for an arbitrary system that possesses a mirror symmetry with respect to a plane passing through the middle of the structure. It can be easily derived taking into account the equality of the determinant of the matrix to one, and the circumstance that the mirror symmetry requires off-diagonal elements to be imaginary. Due to the general character of the representations (15), the material parameters entering Eq. (14) can be either the parameters of a single quantum well, Eq. (8), or the effective parameters $S$ and $\phi$, Eqs. (12) and (13), of a barrier-well sandwich, or even parameters characterizing the entire MQW structure as long as the latter possess the mirror symmetry.

Using this representation we can introduce the following transformation rule for transfer matrices

$$T_H(\psi) T(\theta, \beta) T_H^{-1}(\psi) = T(\theta, \beta + 2\psi), \quad (20)$$

where matrix $T_H$ describes a hyperbolic rotation with a dilation and has the form of

$$T_H(\psi) = e^\psi \begin{pmatrix} \cosh \psi & -\sinh \psi \\ -\sinh \psi & \cosh \psi \end{pmatrix}. \quad (21)$$
This transformation rule can be used, for instance, for diagonalization of transfer matrices, which can be achieved by choosing parameter \( \psi = -\beta/2 \). Matrix \( T_H \) can be turned into matrix \( T_{bw} \), Eq. (4), which describes propagation of waves through interface between two media with different refraction coefficients by using the following relation between \( \psi \) and the Fresnel parameter \( \rho \):

\[
\rho = -\tanh(\psi) \quad \text{(a detailed discussion of a relation between interface scattering and the hyperbolic rotation can be found in Ref. 33)}.
\]

This means that the transformation, Eq. (20), can be either used to describe the interface between two different structures, or in order to present any type of non-diagonality of the transfer-matrix as resulting from some effective interface. With the help of Eq. (20), any symmetric multilayer structure can be replaced by a uniform slab with the width given by \( \theta \) and the index of refraction determined by \( \psi \). Therefore, it can be used to describe structures which are more complicated than a simple three layer barrier-well sandwich considered in the previous section. For instance, using Eq. (20) we can immediately derive an expression for the reflection coefficient \( r \) of a sequence of identical blocks described by \( T(\theta, \beta) \):

\[
T_N = T(\theta, \beta)^N = T(N\theta, \beta).
\]

Because the reflection from the structure described by the transfer matrix \( T \) given in the basis of incoming and outgoing waves is

\[
r = -\frac{T_{21}}{T_{22}},
\]

for a structure containing \( N \) blocks we have

\[
r_N = -\frac{i \sinh \beta}{\cot N\theta + i \cosh \beta}.
\]

Eq. (21) in the case of \( \Gamma_0 = 0 \) reproduces the result well-known for a passive multilayer structure.4-7,35

To find a relation between the quantities entering this expression and the elements of the transfer matrix through the period of the structure it is enough to multiply both the numerator and the denominator by \( \sin \theta \) and to use Eq. (19). If each block is characterized by an effective susceptibility \( \tilde{S} \) and the phase \( \phi = \phi_b + \phi_w \) then we obtain for the reflection coefficient

\[
r_N = \frac{i \tilde{S}}{\cot(N\lambda) \sin \lambda + i(S \cos \phi - \sin \phi)}.
\]

We analyze the reflection coefficient for not too long structures, such that \( N \ll N_c = \lambda^{-1} \), which appear naturally in most applications of MQW structures. In this approximation, \( \cot(N\lambda) \sin \lambda \approx N \) and the reflection can be written directly in terms of the material parameters

\[
r_N = \frac{i N \tilde{S}}{1 + i N(S \cos \phi - \sin \phi)}.
\]

The amplitude of reflection, \( |r|^2 \), has the typical form shown in Fig. 3. It is characterized by a strong reflection band around the exciton frequency, which is a manifestation of the strong resonant exciton-light interaction. It is interesting to note that \( |r_N|^2 \) goes to 0 at \( \omega = \omega_0 - \omega_{\text{min}} \) [see Eq. (20)] that is at the frequency where \( \tilde{S} = 0 \). The reflection has an asymmetric form since it is a sum of two terms: one of them is even with respect to the frequency \( \omega_0 \) and the second is odd. The latter is due to nonzero mismatch and in the approximation used above does not vanish at infinity. Both these terms have a typical width

\[
\delta = \frac{N\Gamma_0(1-\rho)^2 \cos \phi_+}{(1-\rho^2)^2 + N^2(\sin \phi_+ - \rho^2 \sin \phi_-)^2},
\]

where \( \phi_{\pm} = \phi_b \pm \phi_w \). It should be noted that the actual optical width of the quantum well determined by \( \phi_w \) enters the definition of \( \phi_\pm \), rather than the modified \( \phi_w \). We are interested in maximizing exciton related effects in the reflection spectra of our structures, which means designing structures with as large a width \( \delta \) as possible. One can see from Eq. (24) that the width demonstrates essentially non-monotonous dependence upon the number of quantum wells in the structure: it grows linearly for small \( N \), but starts decreasing as \( N^2 \) for larger \( N \). If the coefficient in front of the \( N^2 \)-term in the denominator of Eq. (24) vanishes, then the linear growth of \( \delta \) would go unchecked as long as \( N \) does not exceed \( N_c \). Thus, the condition for maximizing the excitonic effects in the reflection spectrum can be formulated as

\[
\rho^2 = \frac{\sin \phi_+(\omega_0)/ \sin \phi_-(\omega_0)}{}.
\]
If one neglects the mismatch of the indices of refraction, this equation takes a well known form of a condition for the Bragg resonance between the period of the structure and the exciton frequency. This effect was studied in regular passive one-dimensional photonic crystals and in Bragg MQWs in the optical lattice approximation. It was found, in particular, that the defect affects the spectrum most strongly when it is placed in the middle of the structure. In this case, the system demonstrates the mirror symmetry and the results of the previous sections can be used. Indeed, for a structure ABA built of blocks A and B described by the transfer matrices with parameters $\theta_A, \beta_B$ and $\beta_A, \theta_B$, one has

$$T(\theta_A, \beta_A)T(\theta_B, \beta_B)T(\theta_A, \beta_A) = T(\theta, \beta).$$

That is, the whole structure can be described by the matrix with

$$\cos \theta = \cos \varphi_+ \cosh^2 \delta \beta - \cos \varphi_- \sinh^2 \delta \beta,$$

$$\coth(\beta - \beta_A) = \cos(2\theta_1) \cosh\delta \beta + \frac{\sin \varphi_+}{\sin \theta_2 \sinh(2\delta \beta)},$$

where $\varphi_+ = 2\theta_1 \pm \theta_2$ and $\delta \beta = (\beta_2 - \beta_1)/2$. Applying now relations one can express the result in terms of parameters $S$ and $\phi$, and use the results for reflection described above.

In some particular cases, however, the problem of scattering of light can be solved without resorting to the transformation rule. Let us consider a situation when the block $B$ is a single quantum well surrounded by barriers so it can be described by the matrix similar with parameters $S_d$ and $\phi_d$. Let the block $A$ be an MQW structure described by $\theta$ and $\beta$. Thus, the transfer matrix through the whole structure is

$$T = T(\theta, \beta)T_\rho(\rho)T(S_d, \phi)T_\rho^{-1}(\rho')T(\theta, \beta),$$

where $T_\rho(\rho)$ takes into account a possible mismatch of the indices of refraction of the defect layer and the host and, $\rho$ is the corresponding Fresnel coefficient.

The transfer matrix can be simplified in several steps. First, as has been described before, we can treat $\beta$ as an addition to the mismatch noting that

$$T_H^{-1}(\beta/2)T_\rho(\rho) = T_\rho(\tilde{\rho}),$$

where $\tilde{\rho} = (\rho + \rho')/(1 + \rho \rho')$ and $\rho' = \tanh(\beta/2)$. Then, similar to Eq. we can introduce effective quantities $S$ and $\phi$

$$\tilde{S} = S_d \frac{1 + \tilde{\rho}^2 - 2\tilde{\rho} \cos \phi}{1 - \tilde{\rho}^2} + 2\tilde{\rho} \frac{\sin \phi}{1 - \tilde{\rho}^2},$$

$$e^{i(\phi - \varphi)} = \frac{1 - \tilde{\rho} e^{-i\phi}}{1 - \tilde{\rho} e^{i\phi}}.$$
$T$ takes the form \( \tilde{T} \), i.e. $T = T(\tilde{S}, \tilde{\phi})$ with

$$
\tilde{S} = \frac{1 + \rho^2 + 2\rho' \cos(\tilde{\phi} + 2\theta)}{1 - \rho^2} - 2\rho \frac{\sin(\tilde{\phi} + 2\theta)}{1 - \rho^2},
$$

$$
e^{i(\tilde{\phi} - \tilde{\phi} - 2\theta)} = \frac{1 + \rho' e^{-i\tilde{\phi} - 2i\theta}}{1 + \rho' e^{i\tilde{\phi} + 2i\theta}}. \tag{36}
$$

These expressions together with Eq. (24) give a complete solution of the problem of propagation of light through the MQW structure with an arbitrary defect in the middle.

One can consider several particular types of defects. An example is a well with the exciton frequency different from the frequencies of all other wells, an $\Omega$-defect. Another possible example could be a defect element with the width of the barriers different from the rest of the structure. It is interesting to note that a standard optical structure. It is interesting to note that a standard optical

for the reflection coefficient also allows for analyzing the position of the maximum of reflection, and its maximum magnitude, but the resulting expressions turn out to be too cumbersome and we do not present them here.

The formalism presented in this paper allows one to take into account effects of homogeneous and inhomogeneous broadening because the main results obtained do not use a particular form of the excitonic susceptibility. For example, the reflection coefficient of a structure with an inhomogeneous broadening can be obtained in the effective medium approximation by using Eq. (24) with $\tilde{S}$ instead of $S$ and with inhomogeneously broadened $S_{h,d}$ in Eq. (38):

$$
S_{h,d} = \int d\omega_0 f_{h,d}(\omega_0) \frac{\Gamma_0}{\omega - \omega_0 + i\gamma}. \tag{39}
$$

where $f_{h,d}$ are the distribution functions of the exciton frequencies in the host and defect layers.
respectivey. However, as has been discussed in Ref. [24] if \( \omega_R \) is far enough from \( \omega_d \), i.e. if \( |\omega_R - \omega_d| \gg \Delta \), where \( \Delta \) is the inhomogeneous broadening, the effect of the latter is negligible and the magnitude of the reflection is determined by

\[
|\hat{S}| \approx \frac{\pi \gamma (2M + 1)}{4M \omega_d \Delta_w^2} \left[ \Delta_R^2 (2M + 1)^2 + 16M (2M - 1) \Delta_{PC} \Delta_w \right]
\]

(40)

and is small provided the smallness of the homogeneous broadening, \( \gamma \ll \omega_d \), and \( \Delta_w \gtrsim \Delta_R \gtrsim \Delta_{PC} \).

A typical form of the reflection in the vicinity of this resonant drop has been discussed in Refs. [24,25]. Here we would like to note that for multiple defect structures composed of several blocks \( \Lambda \) considered above the reflection can be qualitatively described by Eq. (40) where \( N \) is understood as the number of such blocks. This expression shows that increasing \( N \) can be interpreted as increasing an effective radiative decay rate. This qualitatively explains the results of numerical calculations in [25] where it was found that the maximal value of the reflection increases with \( N \) while the value of the contrast remains about the same.

\[\text{V. CONCLUSION}\]

In the present paper, the general problem of light propagation in an MQW based photonic crystal characterized by both spatial modulation of the dielectric constant, and dipole active exciton states in quantum wells, is considered. It is shown that the mismatch of indices of refraction between barriers and wells can be taken into account by introduction of an effective excitonic susceptibility and an effective optical width of the quantum wells. The effective susceptibility has two terms, one of which is almost independent of frequency, while the second is resonant in nature. For short enough MQW structures (or in the vicinity of the exciton frequency) the regular term can be neglected and the effect of the mismatch of the indices of refraction reduces to a modification of the excitonic oscillator strength. In a general case, the reflection spectrum becomes essentially asymmetric and non-trivially dependent upon the number of quantum wells in the structure. It is shown that in order to obtain the strongest exciton induced reflection band the structure must satisfy a certain resonance condition. This is a Bragg resonance condition between the period of the MQW structures and the wavelength of the electromagnetic wave. The latter has to be calculated from a dispersion law for a structure with the spatially modulated reflection index.

The developed approach is applied to analysis of the reflection spectrum of a structure with an intentionally introduced defect element, which breaks the translational symmetry of a system. More detailed analysis is carried out for a special kind of defect, characterized by the presence of a layer in the middle of the structure with a different frequency of the exciton resonance. It is shown that the main characteristics of the reflection spectrum of such structures obtained in the absence of the refraction index contrast survive in the presence of the additional interface reflections. In particular, a significant decrease of the reflection takes place even in the presence of the contrast, which is important for possible applications of such structures.

\[\text{REFERENCES}\]

1. E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987).
2. S. John, Phys. Rev. Lett. 58, 2486 (1987).
3. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, Photonic Crystals: Molding the Flow of Light (Princeton Univ. Press, 1995).
4. A. Yariv and P. Yeh, Optical waves in crystals, (Wiley, New Jersey, 2003).
5. V. Kuzmiak, A. A. Maradudin, and A. R. McGurn, Phys. Rev. B 55, 4298 (1997).
6. L.I. Deych, D. Livdian, and A.A. Lisiansky, Phys. Rev. E. 57, 7254 (1998).
7. S. Nojima, Phys. Rev. B 61, 9940 (2000).
8. L. Pilozzi, and A. D’Andrea, Phys. Rev. B 61, 4771 (2000).
9. A. Yu. Sivachenko, M. E. Raikh, and Z. V. Vardeny, Phys. Rev. A 64, 013809 (2001).
10. M. Lonar, T. Yoshie, A. Scherer, P. Gogna, and Y. Qiu, Appl. Phys. Lett. 81, 2680 (2002); M. Lonar, T. Yoshie, K. Okamoto, Y. Qiu, J. Vučković, and A. Scherer, IEICE Trans. Electron. E87-C, 291 (2004).
11. A. Christ, S. G. Tikhodeev, N. A. Gippius, J. Kuhl, and H. Giessen, Phys. Rev. Lett. 89, 183901 (2003).
12. K.C. Huang, P. Bienstman, J. D. Joannopoulos, K. A. Nelson, and S. Fan, Phys. Rev. B 68, 075209 (2003).
13. K.C. Huang, P. Bienstman, J. D. Joannopoulos, K. A. Nelson, and S. Fan, Phys. Rev. Lett. 90, 196402 (2003).
14. I. H. Deutsch, R. J. C. Spreeuw, S. L. Rolston, and W. D. Phillips, Phys. Rev. B 52, 1394 (1995).
15. E.L. Ivchenko, and G.E. Pikus, Superlattices and other heterostructures: symmetry and optical phenomena (Springer, New York, 1997).
16. E. L. Ivchenko, Fiz. Tverd. Tela (St.-Petersburg) 33, 2388 (1991) [Phys. Solid State 33, 1344 (1991)].
17. E. L. Ivchenko, A. I. Nesvizhinskii, and S. Jorda, Fiz. Tverd. Tela (St.-Petersburg) 36, 2118 (1994) [Phys. Solid State 36, 1156 (1994)].
18. E. L. Ivchenko, V. P. Kochershko, A. V. Platonov, D. R. Yakovlev, A. Waag, W. Ossau, and G. Landwehr, Fiz. Tverd. Tela (St.-Petersburg) 39, 2072 (1997) [Phys. Solid State 39, 1852 (1997)].
19. E. L. Ivchenko, M. M. Voronov, M. V. Erementchouk, I. I. Deych, and A. A. Lisiansky, to appear in Phys. Rev. B 70, (2004).
20. A. V. Kavokin and M. A. Kaliteevski, Solid State Commun. 95, 859 (1995).
21. F. Tassone, F. Bassani, and L. C. Andreani, Phys. Rev. B 45, 6023 (1992).
D. Ammerlahn, B. Grote, S. W. Koch, J. Kuhl, M. Hübner, R. Hey, and K. Ploog, Phys. Rev. B, 61, 2000.

23 L. I. Deych, A. Yamilov, and A. A. Lisyansky, Phys. Rev. B 64, 075321 (2001).

24 L. I. Deych, M. V. Erementchouk, A. A. Lisyansky, Phys. Rev. B 69, 75308 (2004).

25 L. I. Deych, M. V. Erementchouk, and A. A. Lisyansky, Appl. Phys. Lett. 83, 4562 (2003).

26 M. Born and E. Wolf, Principles of Optics, 7th edition (Cambridge:Cambridge University Press), 1999.

27 A. Yariv, P. Yeh, and C.-S. Hong, J. Opt. Soc. Am. 67, 423 (1977).

28 J. M. Benedickson, J. P. Dowling, and M. Scalora, Phys. Rev. E 53, 4107 (1996).

29 L. C. Andreani, F. Tassone, and F. Bassani, Solid State Commun. 77, 641 (1991).

30 D. S. Citrin, Phys. Rev. B 47, 3832 (1993).

31 D. S. Citrin, Phys. Rev. B 50, 5497 (1994).

32 S. Adachi, J. Appl. Phys. 58, R1 (1985).

33 J. J. Monzon, L. L. Sanchez-Soto, Eur. J. Phys 23, 1 (2002).

34 M. Hübner, J. P. Primeas, C. Ell, P. Brick, E. S. Lee, G. Khitrova, H. M. Gibbs, S. W. Koch, Phys. Rev. Lett. 83, 2841 (1999); J. P. Primeas, C. Ell, E. S. Lee, G. Khitrova, H. M. Gibbs, S. W. Koch, Phys. Rev. B 61, 13863 (2000).

35 R. P. Stanley, R. Houdré, U. Oesterle, M. Ilegems, and C. Weisbuch, Phys. Rev. A 48, 2246 (1993).

36 N. Liu, Phys. Rev. B 55, 4097 (1997).

37 A. Figotin, and V. Gorentsveig, Phys. Rev. B. 58, 180 (1998).

38 D. Felbacq, J. Phys. A: Math. Gen. 33, 7137 (2000).

39 R. Ozaki, Y. Matsuhisa, M. Ozaki, and K. Yoshino, Appl. Phys. Lett. 84, 1844 (2004).

40 L. I. Deych, and A. A. Lisyansky, Phys. Lett. A 243, 156 (1998).

41 G. Malpuech, A. Kavokin, G. Panzarini, Phys. Rev B 60, 16788 (1999).

42 L. C. Andreani, G. Panzarini, A. V. Kavokin, and M. R. Vladimirova, Phys. Rev. B 57, 4670 (1998).