A Note on Quantum States and Observables in Psychological Measurements

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Abstract

The problem considered is how to map the concepts of Quantum Theory (QT) to elements of a psychological experiment. The QT concepts are “measurement,” “state,” and “observable”. The elements of a psychological experiment are trial, stimulus, instructions, questions, and responses.

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1 Introduction

This note can be viewed as an extensive commentary to [KBDB].

Quantum Theory (QT) operates with observables and states. The problem we consider here is how to map these concepts to those describing a psychological experiment.

On a very general level, QM accounts for the probability distributions of measurement results using two kinds of entities, called observables $A$ and states $\psi$ (of the system on which the measurements are made). We assume that measurements are performed in a series of consecutive trials numbered $1, 2, \ldots$. In each trial $t$ the experimenter decides what measurement to make (e.g., what question to ask), and this amounts to choosing an observable $A$. The formulas are

$$\Pr[v(A) = v \text{ in trial } t \mid \text{measurements in trials } 1, \ldots, t - 1] = F(\psi(t), A, v). \quad (1)$$

$$\psi(t+1) = G(\psi(t), A, v). \quad (2)$$

$$\psi(t) \Delta = H(\psi(t+1), \Delta). \quad (3)$$

In a psychological experiment we basic constituents of trials are instructions/questions $q$ (specifying, among other things, the allowable responses), and stimuli $s$.

2 [KBDB] Approach

In the approach adopted in [KBDB] stimuli and questions together, $(q, s)$, determine observables. The state before the experiment is generally undetermined, although influenced by the general instructions. The state before every other trial is determined by (2) and (3). The pair $(q, s)$ constitutes an input to the system.

For instance, a detection experiment in psychophysics is traditionally considered as involving a single question $(q = \text{"does the stimulus have the target property, yes or no?"})$ and two stimuli, the “empty” one $s = a$ and the target one $s = b$. We assume in [KBDB] that the input $(q, s)$ in each trial uniquely determines the observable $S$. Since only $s$ varies, we can denote the observables by $A$ (corresponding to $a$) and $B$ (corresponding to $b$), with two values each. But, of course, if the question were different (e.g., $q = \text{"does the stimulus have the target property, yes or no or uncertain?"}$), the observables would be different (although there will still be two of them). The psychophysical analysis of such an experiment consists in identifying the hit-rate and false-alarm-rate functions (conditioned on the previous stimuli and responses)

$$\Pr[v(A) = 1 \text{ in trial } t \mid \text{measurements in trials } 1, \ldots, t - 1] = F(\psi(t), A, 1),$$

$$\Pr[v(B) = 1 \text{ in trial } t \mid \text{measurements in trials } 1, \ldots, t - 1] = F(\psi(t), B, 1). \quad (4)$$
The learning (or sequential-effect) aspect of such analysis consists in identifying the function

\[ \psi^{(t+1)} = G \left( \psi^{(t)}, S, v \right), \quad S \in \{A, B\}, \quad v \in \{0, 1\}, \]

combined with the “pure” inter-trial dynamics \[ \text{(3)}. \]

By contrast, an opinion-polling experiment is usually considered as involving several questions \( q \) and no sensory stimuli. Thus, in one of Moore’s polls we have \( q = a = “Is \text{ Bill Clinton} \text{ honest} \), yes or no?”, and \( q = b = “Is \text{ Al Gore} \text{ honest} \), yes or no?”\). The inputs therefore are \( a \) and \( b \), and the corresponding observables are \( A, B \), with two values each. The analysis, formally, is precisely the same as above:

\[
\begin{align*}
\Pr [v(A) = 1 \text{ in trial } t | \text{ measurements in trials } 1, \ldots, t - 1] &= F(\psi^{(t)}, A, 1), \\
\Pr [v(B) = 1 \text{ in trial } t | \text{ measurements in trials } 1, \ldots, t - 1] &= F(\psi^{(t)}, B, 1).
\end{align*}
\]

3 [BB] Approach

In this approach one strictly distinguishes questions from stimuli and assume that questions are mapped into observables while stimuli are mapped into states. So, in the detection experiment (with a single question \( q \) and two stimuli \( a, b \)) there is a single observable \( Q \) and two states \( \psi_a \) and \( \psi_b \):

\[
\begin{align*}
\Pr[v(Q) = 1 \text{ in trial } t \text{ with } a | \text{ measurements in trials } 1, \ldots, t - 1] &= F(\psi_a, Q, 1), \\
\Pr[v(Q) = 1 \text{ in trial } t \text{ with } b | \text{ measurements in trials } 1, \ldots, t - 1] &= F(\psi_b, Q, 1).
\end{align*}
\]

The dynamics of the states here, \( \text{(2)-(3)} \), are irrelevant, because whatever the transformation \( \psi'_s = G(\psi_s, Q, 1) \), the next state will be reset by the next stimulus into \( \psi_a \) or \( \psi_b \).

In the opinion-polling experiment the two approaches coincide, because the input there consists of a question only.

4 Comparison

Consider the situation when, in the detection paradigm, a stimulus \( a \) is repeated in trials 1 and 2. In the [KBDB] approach we have

\[
\Pr [v(A) = 1 \text{ in trial } 1 | v(A) = 1] = F(\psi^{(1)}, A, 1),
\]

where \( \psi^{(1)} \) is the initial state in which the participant is at the start of trial 1 (due to her “preparation” by the pre-experiment experience and by general instructions). The state then is transformed into

\[
\psi^{(2)} = G(\psi^{(1)}, A, 1),
\]

and then into

\[
\psi^{(2)}_\Delta = H(\psi^{(2)}, \Delta)
\]

in the interval \( \Delta \) between the two trials. The next (conditional) probability of responding 1 is

\[
\Pr [v(A) = 1 \text{ in trial } 2 | v(A) = 1 \text{ in trial } 1] = F(\psi^{(2)}_\Delta, A, 1).
\]

Clearly, the relationship between \( F(\psi^{(1)}, A, 1) \) and \( F(\psi^{(2)}_\Delta, A, 1) \) is complex. In particular, they need not coincide.

In the [BB] approach, however, the situation is much simpler. We have in the first trial

\[
\Pr [v(Q) = 1 \text{ in trial } 1 \text{ with } a] = F(\psi_A, Q, 1),
\]

and then, irrespective of how \( \psi_A^{(1)} \) transforms as a result of this measurement and between the two trials, in the next trial we have

\[
\Pr [v(Q) = 1 \text{ in trial } 2 \text{ with } a | v(Q) = 1 \text{ in trial } 1 \text{ with } a] = F(\psi_A, Q, 1).
\]

We see that the two probabilities must coincide. This is definitely not what happens empirically (see [ACK]).

The [BB] approach therefore has to be modified. Thus, one might assume that stimulus \( s \) determines the state \( \psi_s \) not uniquely, but depending on the previous state too:

\[
\psi'_s = K(s, \psi).
\]

This could save the approach, but would introduce a mechanism other than described by the QT generalizations \( \text{(1)-(2)-(3)}. \)
5 Logical Problems With the [BB] Approach

Even if by means of some extraneous to QT considerations one could make the [BB] approach work, it would still encounter the logical difficulty: it is not clear how to distinguish stimuli from questions.

Thus, in the opinion polling (say, in the Moore’s Clinton-Gore one [M]), suppose that the respondents are first instructed “We will show you a picture of a well known politician: tell us whether you trust him/her, yes or no.” This would amount to a single question $Q$. Then the pictures of Clinton and of Gore would amount to two stimuli $a$, $b$. Intuitively, the results of this procedural modification need not dramatically change the outcomes.

And in the [KBDB] approach it does not: the input is still essentially the same, consisting of a question specifying allowable responses and of the variable identifier of the question’s target (the spoken or written word “Clinton” is not much different from Clinton’s photograph).

In the [BB] approach, however, the procedural modification in question would amount to change from dealing with two observables and with states varying according to projection-evolution rules to dealing with a single observable and with states forced by the photographs.

6 Conclusion

Our analysis shows that [BB] is more problematic than [KBDB]. No doubt, [BB] can be modified in many ways, but it seems that [KBDB] is a more straightforward and general application of QT, unifying psychophysical, opinion polling, and quantum physical considerations.

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References

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1 The argument presented in this section was suggested to me by Harald Atmanspacher in a conversation we had in April 2014.