The First and Second Homotopy Groups of a Homogeneous Space of a Complex Linear Algebraic Group

Let $X$ be a homogeneous space of a connected linear algebraic group $G$ defined over the field of complex numbers $\mathbb{C}$. Let $x \in X(\mathbb{C})$ be a point. We denote by $H$ the stabilizer of $x$ in $G$. When $H$ is connected, we compute the topological fundamental group $\pi_{1}^{\text{top}}(X(\mathbb{C}), x)$. Moreover, we compute the second homotopy group $\pi_{2}^{\text{top}}(X(\mathbb{C}), x)$.

Keywords: Fundamental group, second homotopy group, homogeneous space, linear algebraic group.

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