Bimodal parametric excitation of a micro-ring gyroscope

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Received: 10 July 2020
Accepted: 11 November 2020
DOI: 10.1002/pamm.202000153

Parametric excitation in vibratory systems is known well over hundred years. Recently it is also being exploited in Micro-electromechanical Systems (MEMS). One example of these are micro-ring gyroscopes, which are used in the automotive, the navigation and in the space industries. The sensitivity of such gyroscopes can be impaired by signal noise induced through parasitic capacitance between drive and sense electrodes. Applying parametric excitation and exploiting parametric amplification recently was shown to open new promising paths. On the theoretical side of parametric vibrations, recently it was found that the simultaneous parametric excitations in two coordinates or more with phase difference can lead to very interesting phenomena. The micro-ring gyroscope is a good example of making use of parametric excitation and amplification to solve some of the problems in the modern MEMS. This work aims at modelling a micro-ring gyroscope with two simultaneous phase lagged parametric excitations. The equations of motion can be derived using Hamilton’s principle, both for an inextensible as well as for the extensible ring, and in the fully symmetric cases the eigenvalue problem can be solved analytically. The completely symmetric problem, leads to two eigenfunctions for any given eigenfrequency, except for the fundamental ones. The axial symmetry is in general destroyed by the way in which the vibrating ring is elastically supported by discrete elastic supports. Instead of truly double eigenfrequencies, the problem then has pairs of two closely spaced eigenfrequencies. In the simplest case, the ring can then be described by two generalized coordinates. Making use of direct excitation of each of the modes, as well as parametric excitation, makes it possible to exploit parametric resonance and parametric amplification. This has been applied before for the micro-ring gyroscope, however not for the case of parametric excitation in two coordinates and with phase lag between the two parametric excitations. Taking into account this phase lag as an additional parameter, opens several new ways of sensing the angular velocity of the moving base of the micro-ring gyroscope.

The investigation of parametric excitations in MEMS was first introduced by Rugar and Grütter [1]. But in MEMS gyroscopes parametric excitations were first considered by Oropesa-Ramos et al [2] for the conventional comb-gyroscopes and by Gallacher et al [3] for micro-ring gyroscopes. An overview on the effects of having two coupled parametric excitation in general two degrees of freedom (i.e. bimodal) systems is given in Mettler [4]. Cesari [5] in a special case found that instability can be caused at all excitation frequencies under some conditions and named this as total instability. Recently Karev et al [6] included the effect of circulatory and gyroscopic effects to the study of the problem. In the present contribution the case of having bimodal parametric excitation in micro-ring gyroscopes is investigated.

For the sake of deriving micro-ring gyroscope’s governing equation of motion, Let the displacement of an infinitesimal ring element \( P \) be \( u(t, \theta) \) and \( v(t, \theta) \) in polar coordinates (s. Fig. 1). The ring’s geometry is described by its radius \( R \), cross section area \( A \), and second moment of area \( I_z \), while having a density \( \rho \) and Young’s modulus \( E \). Functioning as a gyroscope, it is attached to a rotating frame with constant rate of \( \Omega \), through elastic supports, whose coefficients are \( k_u \) and \( k_v \). In addition, an external parametric radial force \( F_p \) is distributed on its circumference. The application of parametric excitation aims at amplifying the ring’s first mode shape. Coriolis force is then responsible for transferring energy from the first mode shape to the second one [2]. Since the second mode is only driven by Coriolis force (gyroscopic terms) at that frequency, then the reference frame rotation rate \( \Omega_z \) can be deduced. Hence, the vibration of the second mode senses the rotation of the gyroscope’s frame.

By substituting in Hamilton’s principle \( \int_{t_1}^{t_2} \delta [T - U + W_{nc}] dt = 0 \) for the kinetic and potential energies and for the non conservative work we get the partial differential equation (PDE) of the system as

\[
\frac{EI_z}{\rho AR^4} \left( \frac{\partial^2 v}{\partial \theta^2} + 2 \frac{\partial^4 v}{\partial \theta^4} + \frac{\partial^6 v}{\partial \theta^6} \right) + \left( \frac{\partial^2}{\partial \theta^2} \left( \frac{\partial^2 v}{\partial \theta^2} - v \right) - \Omega_z^2 \left( \frac{\partial^2 v}{\partial \theta^2} - v \right) - 4\Omega_z \frac{\partial v}{\partial \theta} \frac{\partial}{\partial \theta} \right) + \frac{1}{\rho A} \left( k_u - F_p(t) \right) \frac{\partial^2 v}{\partial \theta^2} - k_v v = 0. \tag{1}
\]

where \( v \) is continuous and differentiable up to the 3\(^{rd} \) degree in addition to having the periodicity condition \( v(0, t) = v(2\pi, t) \). As a first step, we solve this partial differential equation (PDE) for the autonomous self-adjoint system, i.e. without external or gyroscopic forces, in order to get the system natural frequencies and vibration modes.

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A solution in the form $v(t, \theta) = \Phi(\theta)q(t)$ is sought. The PDE is found to be separable under this ansatz into two ordinary differential equations (ODEs) in time and angle $\theta$. Both ODEs are solved giving the following general solution for a free elastic inextensible micro-ring

$$v(t, \theta) = (K + Lt) + \sum_{n=1}^{\infty} \left( S_n \sin(n\theta) + C_n \cos(n\theta) \right) e^{i\omega_n t}, \quad \omega_n^2 = \frac{Elz(n^2 - 1)^2}{AR^3\rho(n^2 + 1)}.$$  \hspace{1cm} (2)

An external parametric excitation can be applied by placing circumferentially distributed electrodes, where each electrode together with the opposing ring section act as a capacitor [3]. The electric field for infinitesimal ring section is defined as $dE = [(ebR)/(2d)]V^2(t)d\theta$, where $e$ is the electric permittivity, $b$ is the capacitor’s thickness, $d$ is the separating distance and $V(t)$ is the potential difference between the two electrodes.

The virtual work done by the electrostatic field is then calculated to be $\delta W = (ebV^2)/(2d^2) \int_{0}^{2\pi} [1 + (2u)/(d_0)] \delta uRe^2 \delta\theta$. The corresponding generalized force coefficient $F_p$ in the radial direction is thus found to be $F_p(t) = (ebV^2)/(d^2)$, where $d = d_0 - u_r$.

Galerkin’s method is then used to discretize the PDE (1) using the ansatz $v(\theta, t) = \sum_{i=1}^{N} \phi_i(\theta)q_i(t)$, where $\phi_i(\theta)$ is a Fourier series after excluding the rigid body modes, i.e. when $\omega_n = 0$. Then the system is reduced by $v \simeq \sin(2\theta)q_1(t) + \cos(2\theta)q_2(t)$ to its first degenerate vibration mode, where $n = 2$. By inserting this ansatz and following Galerkin’s procedure we obtain the following 2 DoF system for the first vibration eigenfrequency

$$\ddot{q} + \begin{pmatrix} \delta_{11} & 0 \\ 0 & \delta_{22} \end{pmatrix} \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix} + \begin{pmatrix} 0 & \gamma \\ -\gamma & 0 \end{pmatrix} \dot{q} + \begin{pmatrix} \kappa_{11} & 0 \\ 0 & \kappa_{22} \end{pmatrix} + \frac{ERbV_2(t)}{d_0^2} \begin{pmatrix} V_{12}^2(t) \\ V_{21}^2(t) \end{pmatrix} \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$  \hspace{1cm} (3)

where the gyroscopic term is $\gamma = \frac{\pi}{2}\Omega_p$, stiffness terms are $\kappa_{ii} = (\omega^2 - \Omega^2 + k_{ii})$, in which $k_{ii}$ is the elastic support’s modal stiffness, and the modal damping is $\delta_{ii}$. In (3) we see a fully populated parametric excitation matrix, which has 4 components of time-dependent voltages $V_{ij}$, which all has the same voltage amplitude of $V_p$ and frequency $\Omega_p$. The diagonal terms can be achieved by the excitation of each mode shape independently, whereas the off-diagonal coupling terms are obtained by a feedback control electronic circuit.

A new method is proposed in this paper for exciting the micro-gyroscopes, where $V_{12}$ and $V_{21}$ are phase shifted by $-\pi/2$ as discussed in [6]. Thereby a significant amplification is obtained at the non-resonant frequencies in addition to common amplification obtained at the parametric combination frequencies [1–3]. This is can be shown in Fig. 2, where by increasing voltage amplitude $V_p$ the system’s maximum eigenvalue $\lambda_{max}$ shows destabilization in frequency bands between combination resonances, which complies with the phenomenon of “total stability” discussed above. These values were obtained by the semi-analytical normal form method (s. Fig. 2, 3) and verified numerically using Floquet method (s. Fig. 3).

Acknowledgements This work is supported by the ‘Excellence Initiative’ of the German Federal and State Governments and the Graduate School of Computational Engineering at Technical University of Darmstadt.

Open access funding enabled and organized by Projekt DEAL.

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