Physical modeling of axisymmetric hydrofracturing by plastic material injection in elastic medium

I.V. Kolykhalov

Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
E-mail: ikolykhalov@mail.ru

Abstract. The article describes the experimental and numerical investigation of hydraulic fracture propagation under injection of a plastic material near the free surface and the surface loaded by a die block to simulate the effect of an open fracture in the course of the multiple hydrofracturing. The experimental and calculated data are compared.

1. Introduction
Hydrofracturing is a technological process designed to disintegrate rocks. In hydrofracturing a viscous fluid or a plastic material is pumped into a natural or artificial fracture thus inducing growth of a fracture up to a wanted size. The process under consideration provides a better opportunity to extract a mineral deposit in a safe and ecofriendly manner without mine construction. The directed hydrofracturing became popular in mining of tight roof, coal seam degasing, sulfide ore leaching, rock plugging, blast-free extraction of precious crystal-bearing rock blocks, and production of construction stone [1, 2]. The latest novel application of hydrofracturing is intensification of oil and gas production from low-permeability reservoirs [3, 4]. Concept of the process runs as follows: a series of fractures is initiated perpendicularly to a borehole with a preset interval in order to increase maximally an area of hydrocarbons intake. The maximum performance of the process is gained when induced fractures are of disk-type and distance between them is equal to radius of drainage area. This condition is not always fulfilled, as induced fractures can be curved in shape and intercrossing. Geometry of induced fractures is appreciably influenced by presence of earlier artificial or natural fracture, free surfaces, fluid pumping modes, and properties of a working fluid. Theoretical studies on hydrofracture development nearby earlier available fractures are reported in [5, 6], where the algorithm for calculation of a path of successive evolution of co-axial fractures is developed for an axisymmetric statement of the problem. In the present paper the researcher models propagation of an axisymmetric fracture in hydrofracturing with a plastic material in the vicinity of earlier available open fracture and free surface. The aim of the present studies is to check compliance of experimental fracture parameters with numerical calculated results.

2. Experimental procedure
Physical modeling was carried out on blocks made of polymethylacrylate (organic glass) of 230×160×106 mm and 170×170×36 mm in size. An available open hydrofracture was modeled by
surface loading with a round ellipsoid-shape print press with displaced center $U = 0.1 \text{ mm}$ (small semi-axis) and diameter of 100 mm (large semi-axis). The print was pressed and fixed with a preset force with bolts of 6 mm in diameter on the maximum-size side of the test specimen (Figure 1a).

![Figure 1](image)

**Figure 1.** Schematic of experiments on (a) development of closely located hydrofractures and (b) external view of an induced fracture shape.

To imitate a production well, a hole of 10 mm in diameter was made on the opposite side along axis of the available fracture. A narrow incipient axisymmetric fracture imitating a transverse well was cut in the bottom of the hole in distance $h$ from the available fracture. The fracture diameter with account for well’s diameter was 14 mm. Next, a proppant agent was fed by means of a screw auger to the incipient fracture at flowrate of 0.5–0.7 cm³/min. The fracture developed at the near-equilibrium mode up to the moment until projection of its radius on a plane of the available fracture reached 50–60 mm. The proppant agent was plasticine OST 6-15-1525-86 applied at temperature 24°C. A series of tests was carried out in different distances $h$ between a growing fracture and the available one (Figure 1b). To study the effect of free surface on parameters of growing fracture the tests were performed under the described procedure with no print press loading of the test specimen in different distances to free surface.

### 3. Numerical calculations

Numerical softwares developed by discontinuous component methods were used to calculate fracture growth [7]. Computation algorithm is described in details in [5, 6]. The experimental problem statement was: three co-axial parallel disk-type fractures locate normally to axis $z$ in an elastic space infinitely compressed under principal stresses (Figure 2). The first fracture has radius $Q_0$ and boundary conditions on its edges $\sigma_n = 0, \tau_s = 0$. This fracture imitates stress-free surface of a block. The second and third fractures have radii $R_0$ and $r$ and represent the earlier available and incipient fractures, respectively. Distance $H$ is between free surface and the available fracture and $h$ is between the incipient and available fractures. Boundary conditions on edges of the available fracture are prescribed by ellipsoid opening equal to $2U_0$ in its center. Pressure in the incipient fracture is assumed $p(s, t)$, where $S$ is coordinate along the length of the fracture line in meridian cross-section, $t$ is time.
Development of the fracture was computed in near-equilibrium mode. The working fluid flow is described by the law of the plastic material deformation. The solution to material motion along the fracture was obtained by AM Linkov’s hypothesis on proportionality of effective normal stresses acting on fracture edges relative to their normal displacements [8]. Considering this hypothesis the set of equations of the plastic material flow is:

\[ p(s, t) = \frac{\pi E}{4(1-\nu^2)} \frac{U(s, t)}{R(t)}, \]

\[ \frac{\partial p(s, t)}{\partial s} = -\frac{\tau_0}{U(s, t)}, \]

\[ q_0 = \frac{dV}{dR} \cdot \dot{R}(t), \tag{1} \]

where \( E \) is elasticity modulus of a medium; \( \nu \) is Poisson ratio; \( U(s, t) \) is semi-opening of a fracture; \( R \) is length of fracture line in meridian section; \( R_\gamma \) is a length of a fracture section filled with a plastic material; \( \tau_0 \) is yield limit of a plastic material; \( V \) is fracture volume; \( q_0 \) is plastic material consumption.

From first two equations (1) for calculation of \( p(s, t) \) we obtain differential equation:

\[ \frac{\partial p(s, t)}{\partial s} = -\frac{\pi \tau_0 E}{4(1-\nu^2)} \frac{1}{p(s, t)R(t)}, \tag{2} \]

which solution should meet condition \( p(0, t) = P_0(t) \). \( P_0(t) \) is pressure in the center of the fracture where the equilibrium fracture growth condition is fulfilled; \( dW/dS = 2\gamma \) is Griffith’s energy fracture criterion, where \( dW/dS \) is intensity of liberated elastic energy; \( 2\gamma \) is failure viscosity. In terms of boundary conditions in the center of the fracture the analytical solution is:

\[ p(s, t) = \sqrt{P_0^2 - \frac{\pi \tau_0 E}{2(1-\nu^2)} \frac{s}{R(t)}}. \tag{3} \]

In numerical calculations the following parameters were used: \( E = 3.3 \cdot 10^3 \) MPa, \( \nu = 0.33 \), \( R_0 = 50 \) mm, \( 2U_0 = 0.2 \) mm, failure viscosity \( 2\gamma = 2.7 \cdot 10^{-4} \) MPa \( \cdot \) m, corresponding to the critical coefficient of stress intensity \( K_{IC} = 1 \) MPa \( \cdot \) m\(^{0.5} \), \( \tau_0 = 0.01 \) MPa.

4. Analysis of the experimental results

![Figure 2. Initial position of fractures in cross-section plane \( y = 0 \)](image-url)
In Figures 3a and 4 the fractures are produced experimentally and calculated numerically for different distances $H$ and $h$. Solid lines correspond to tracks, calculated numerically; lines with triangle marks are experimental fractures; dashed lines are for calculated paths with the use of a perfect fluid as a working fluid, that is $p(s,t) = p(t)$. Points denote free surface. It is obvious that shapes of fractures calculated with the use of a plastic material exhibit less curvature and better compliance with experimental curves. As well as its curvature tends to lower with increase in distance from a developing fracture to a print press and from free surface.

![Figure 3](image1.png)

**Figure 3.** Experimental results on the specimen of $H = 36$ mm: (a) paths of axisymmetric fracture, developing from the initial spot in distance $h = 18$ from the available fracture with opening in the center $2U_0 = 0.2$ mm; (b) relationship of a length of fracture section stuffed with a plastic material versus its length.

As a rule, a delay of the working fluid front from fracture edge is observed at small values of normal stresses on the fracture contour from the side of the external compression field. In Figure 3b relationship of a length of a section filled with a plastic material $R_\gamma$ versus fracture length $R$ is shown. Dashed line is for the case when the working fluid fills completely fracture ($R_\gamma(t) = R(t)$), solid line is for numerical calculations, marks are for experimental data. Divergence between numerical and experimental data does not exceed 15%.

![Figure 4](image2.png)

**Figure 4.** Paths of axisymmetric fractures induced in distance $h = 53$ mm and $h = 65$ mm from the available one in a specimen of $H = 106$ mm
Let consider development of the fracture in the vicinity of only one free surface. Figure 5a demonstrates the growth paths of axisymmetric fractures obtained experimentally (triangle marks) and by numerical computation (solid lines) nearby free surface. Free surface coincides with abscissa. In Figure 5b solid and dashed lines present relations of calculated volumes of the fracture and a plastic material versus fracture length. The experimental volume of plasticine pumped into a fracture is denoted by markers.

![Figure 5.](image)

**Figure 5.** (a) Paths of axisymmetric fractures initiated in distance of 15 and 25 mm from free surface in a test specimen of $H = 106$ mm; (b) relationship of a volume of pumped plastic material (dashed line) and a volume of a fracture versus fracture length in the growth stage in distance 15 mm from free surface.

Comparison of theoretical and experimental results, reported in Figures 3–5 reveals that computation scheme described in [5, 6] enables to describe clearly parameters of fractures initiated in hydrofracturing transverse to a borehole.

5. Conclusion

The schemes are developed for experimental and numerical modeling of growth of axisymmetric fractures transverse to hydrofracturing borehole with the use of a plastic material in an active zone containing an earlier available disc-type fracture and free surface. Series of experiments were undertaken in different distances between fractures and distances to free surface. The comparative analysis of parameters of fractures produced experimentally with theoretical calculations showed their good compliance.

References

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