Distinguishing Indirect Signatures of New Physics at the NLC: $Z'$ Versus $R$-Parity Violation

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Abstract

$R$-parity violation and extensions of the Standard Model gauge structure offer two non-minimal realizations of supersymmetry at low energies that can lead to similar new physics signatures at existing and future colliders. We discuss techniques that can be employed at the NLC below direct production threshold to distinguish these two new physics scenarios.

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1 Introduction

While the Standard Model (SM) is in relatively good agreement with all precision electroweak data[1], it leaves too many unanswered questions that will somehow need to be addressed by new physics at or above the electroweak scale. Supersymmetry (SUSY), in the guise of the Minimal Supersymmetric Standard Model (MSSM), provides a potential starting point for the exploration of this new physics; however, while the MSSM provides a simplified framework in which to work, most authors would agree that the MSSM is itself inadequate due to the very large number of free parameters it contains. Furthermore, the MSSM cannot be the whole story of low-energy SUSY since, on its own, it does not explain how SUSY is broken or why the scale of this breaking is of order $\sim 1$ TeV. In going beyond the MSSM there are many possible paths to follow. In this paper we discuss two of the simplest of these scenarios: an extension of the SM gauge group by an additional $U(1)$ factor broken near the TeV scale and $R$-parity violation, both of which are well-motivated by string theory. Although these two alternatives would appear to have little in common, we will see below that they can lead to similar phenomenology at present and future colliders and may be easily confused in certain regions of the parameter space for each class of model.

Unlike the case of Grand Unified Theories (GUT), where any additional $U(1)$’s may break at any arbitrary scale below $M_{GUT}$, perturbative string models with gravity mediated SUSY breaking are known to predict an assortment of new gauge bosons with masses of order 1 TeV, as well as the existence of other exotic matter states with comparable masses[2]. Such models lead one to expect that the existence of a $Z'$ at mass scales which will be accessible at Run II of the Tevatron or at future colliders is quite natural. Similarly, the case for potential $R$-parity violation is also easily demonstrated and appears to be just as natural as not. As is well known, the conventional gauge symmetries of the supersymmetric extension of the SM allow for the existence of additional terms in the superpotential that violate either
Baryon($B$) and/or Lepton($L$) number. One quickly realizes that simultaneous existence of such terms leads to rapid proton decay. These phenomenologically dangerous terms can be written as

$$W_R = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k + \epsilon_i L_i H,$$

where $i, j, k$ are family indices and symmetry demands that $i < j (j < k)$ in the terms proportional to $\lambda (\lambda' \lambda'')$ Yukawa couplings. In the MSSM, the imposition of the discrete symmetry of $R$-parity removes by brute force all of these ‘undesirable’ couplings from the superpotential. However, it is easy to construct alternative discrete symmetries which may arise from strings that allow for the existence of either the $L$- or $B$-violating terms in $W_R$ (but not both kinds) and are just as likely to exist as $R$-parity itself. (Interestingly, at least some, if not all, of these dangerous couplings in $W_R$ may be removed from the superpotential if the SM fields also carry an additional set of $U(1)$ quantum numbers.) As far as we know there exists no strong theoretical reason to favor the MSSM realization over such $R$-parity violating scenarios. Since only $B$- or $L$-violating terms survive when this new symmetry is present the proton now remains stable in these models. Consequently, various low-energy phenomena then provide the only significant constraints on the Yukawa couplings $\lambda, \lambda'$ and $\lambda''$. For example, constraints on the trilinear $LLE^c$ couplings are typically of order $\lambda \sim 0.05(m/100\text{GeV})$, where $m$ is the mass of the exchanged sfermion. In what follows we will be interested in $\tilde{\nu}$ masses in the TeV range so that Yukawa couplings not much less than unity can be phenomenologically viable.

If $R$-parity is violated much of the conventional wisdom associated with the phenomenology of the MSSM goes by the wayside, e.g., the LSP (now not necessarily a neutralino!) is unstable and sparticles may now be produced singly. In particular, it is possible that the exchange of sparticles can significantly modify SM processes and may even be produced as $s$-channel resonances, appearing as bumps in cross sections if they are
kinematically accessible. Below threshold, these new spin-0 exchanges may make their presence known via indirect effects on cross sections and other observables even when they occur in the $t$- or $u$-channels\[8\]. Here we will address the question of whether the effects of the exchange of such particles can be differentiated from those conventionally associated with a $Z'$. (Recall the expectation that at linear colliders such as the NLC, the effects of a $Z'$ with a mass in the several TeV range will appear as deviations from the SM values for observables associated with the processes $e^+ e^- \rightarrow f \bar{f}$.)

In many cases it will be quite straightforward to differentiate these two alternative sources of new physics. For example, if a new resonance is actually produced and is found to dominantly decay to SUSY partners, including gauginos, or violate lepton number, we will know immediately that the new particle is most probably a sfermion with couplings that result from $R$-parity violation. If, on the other hand, such a particle were to be produced at a lepton or hadron collider and dominantly decay to SM fields, the angular distribution of the final state products, either leptons or jets, would conclusively tell us\[6\] the spin of the resonance given sufficient statistics, \textit{i.e.}, several hundred events. We will not be concerned with this scenario below.

The situation becomes far more uncertain, however, when below threshold exchanges are involved and the existence of the interaction produced by the new particle is uncovered \textit{only} through its modification of cross sections and asymmetries for SM processes. As an example, both a leptophobic $Z'$ and a squark coupling via the $B$-violating $U^c D^c D^c$ term in $W_R$ can alter the angular distribution of dijets via an $s$-channel exchange below threshold at the Tevatron. It is \textit{not} so obvious that these two scenarios can be easily, if at all, distinguished by a detailed analysis of these deviations.

Since we are concerned here with NLC physics we will by necessity limit our attention solely to the trilinear $L$-violating terms in the superpotential. If only the $LLE^c$ terms are
present it is clear that only the observables associated with leptonic processes will be affected by the exchange of $\tilde{\nu}$'s in the s- or t-channels or both and no input into the analysis from hadron collider experiments is possible. On the other hand, if $LQD^c$ terms are also present then the $Q = -1/3$ final states at linear colliders will also potentially be affected by $\tilde{\nu}$ exchange. Simultaneously a $\tilde{\nu}$ resonance may show up at a hadron collider in the Drell-Yan or dijet channels if kinematically allowed and the Yukawa couplings to first generation down-type quarks is sizeable. In the analysis below we will consider for simplicity only the former situation; the extension of our analysis to the more general case involving final state quarks is quite straightforward. This implies that we will be directly comparing the s-channel exchange of an essentially hadrophobic $Z'$ with $\tilde{\nu}$ exchanges.

How does a generic $Z'$ couple to leptons? In most GUT-type models, $Z'$ couplings are both flavor diagonal and universal, i.e., generation independent. However, it is easy to construct more generalized models [9] where the $Z'$ couplings remain flavor diagonal but are rendered generation-dependent. It is this specific class of $Z'$ models which we will consider below since they mimic $\tilde{\nu}$'s most closely. Thus, while observing different deviations in the $e^+e^-, \mu^+\mu^-$ and $\tau^+\tau^-$ processes might be considered a unique $R$-parity violating signature, we see here that this need not be generally true, i.e., universality violation is not necessarily a smoking gun signal for $R$-parity violation.

The conventional approach in analyzing $R$-parity violating phenomenology is to consider the case where only one or two of the Yukawa couplings in $W_R$ can be significantly large at a time [5, 6]. If we follow this approach we can immediately write down which reactions are modified by s- or t-channel $\tilde{\nu}$ exchanges for a given non-zero $\lambda$ or pair of $\lambda$'s at the NLC. For simplicity, any small mass splittings between sneutrinos and anti-sneutrinos will be ignored [10] in this analysis. In the case when only one non-zero Yukawa coupling is present, Table 1 informs us that $\tilde{\nu}$'s may contribute to either $e^+e^- \rightarrow \mu^+\mu^-$ or $\tau^+\tau^-$ via t-channel
| Reaction          | Yukawa Coupling | Exchange(s) |
|-------------------|-----------------|-------------|
| $e^+e^- \rightarrow e^+e^-$ | $\lambda_{121}$ | $\tilde{\nu}_\mu(s,t)$ |
|                   | $\lambda_{131}$ | $\tilde{\nu}_\tau(s,t)$ |
| $e^+e^- \rightarrow \mu^+\mu^-$ | $\lambda_{121}$ | $\tilde{\nu}_e(t)$ |
|                   | $\lambda_{122}$ | $\tilde{\nu}_\mu(t)$ |
|                   | $\lambda_{132}$ | $\tilde{\nu}_\tau(t)$ |
|                   | $\lambda_{231}$ | $\tilde{\nu}_\tau(t)$ |
| $e^+e^- \rightarrow \tau^+\tau^-$ | $\lambda_{123}$ | $\tilde{\nu}_\mu(t)$ |
|                   | $\lambda_{131}$ | $\tilde{\nu}_e(t)$ |
|                   | $\lambda_{133}$ | $\tilde{\nu}_\tau(t)$ |
|                   | $\lambda_{231}$ | $\tilde{\nu}_\mu(t)$ |

Table 1: Reactions that can be mediated by $\tilde{\nu}$'s if only one Yukawa coupling in the $LLE^c$ term of the superpotential is large. The $s$ and/or $t$ in the right hand column labels the exchange channel.
exchange while \( e^+e^- \rightarrow e^+e^- \) receives both \( s \)- and \( t \)-channel contributions. Note that if the \( \lambda_{121}, \lambda_{131} \) or \( \lambda_{231} \) are non-zero, \( \tilde{\nu} \) exchange of different flavors can contribute to deviations in more than one final state. Table 2 shows us that if two Yukawas are simultaneously large, most final states are lepton family number violating, \( e.g., e^+e^- \rightarrow e^+\tau^- \). In such cases, the separation of the \( Z' \) and \( R \)-parity violation scenarios would again be straightforward since it is very unlikely that a TeV mass \( Z' \) would have large lepton family number violating couplings. However, we also see from this Table that if only the product of Yukawas \( \lambda_{121}\lambda_{233} \) or \( \lambda_{131}\lambda_{232} \) is non-zero then \( s \)-channel \( \tilde{\nu} \) exchange would contribute to the \( \tau^+\tau^- \) or \( \mu^+\mu^- \) final state, respectively. Putting this together with the results of Table 1 we see that if either of these two products of Yukawa couplings is non-zero all possible leptonic final states may receive contributions from \( R \)-parity violating \( \tilde{\nu} \) exchanges. We now turn to a study of these various cases.

The organization of this paper is as follows. In Section 2 of this paper we consider the case where the \( \tilde{\nu} \) is exchanged in the \( t \)-channel leading to modifications in the reactions \( e^+e^- \rightarrow \mu^+\mu^- \) and/or \( \tau^+\tau^- \). \( s \)-channel exchange is discussed in Section 3 and Bhabha scattering in Section 4. Our summary and conclusions can be found in Section 5. We note that although we have only considered the case of \( R \)-parity exchanges in the \( s \)- and/or \( t \)-channels in this paper the analysis we follow can be easily adapted to other possible scalar (or higher spin) exchanges.

## 2 \( t \)-channel \( \tilde{\nu} \) Exchange

In this section we will compare and contrast the \( s \)-channel \( Z' \) contribution to \( e^+e^- \rightarrow \mu^+\mu^- \) or \( \tau^+\tau^- \) with that of a \( \tilde{\nu} \) in the \( t \)-channel. To be specific, in the numerical analysis that follows we will consider a 1 TeV NLC with an integrated luminosity of 150 \( fb^{-1} \). The
\[ \lambda_{121} \lambda_{122} \quad e\mu \quad \tilde{\nu}_\mu(s, t) \]
\[ \lambda_{121} \lambda_{123} \quad e\tau \quad \tilde{\nu}_\mu(s, t) \]
\[ \lambda_{121} \lambda_{231} \quad e\tau \quad \tilde{\nu}_\mu(s) \]
\[ \lambda_{121} \lambda_{232} \quad \mu\tau \quad \tilde{\nu}_\mu(s) \]
\[ \lambda_{121} \lambda_{233} \quad \tau\tau \quad \tilde{\nu}_\mu(s) \]
\[ \lambda_{122} \lambda_{123} \quad \mu\tau \quad \tilde{\nu}_\mu(t) \]
\[ \lambda_{131} \lambda_{132} \quad e\mu \quad \tilde{\nu}_\tau(s, t) \]
\[ \lambda_{131} \lambda_{133} \quad e\tau \quad \tilde{\nu}_\tau(s, t) \]
\[ \lambda_{131} \lambda_{231} \quad e\mu \quad \tilde{\nu}_\tau(s) \]
\[ \lambda_{131} \lambda_{232} \quad \mu\mu \quad \tilde{\nu}_\tau(s) \]
\[ \lambda_{131} \lambda_{233} \quad \mu\tau \quad \tilde{\nu}_\tau(s) \]
\[ \lambda_{132} \lambda_{133} \quad \mu\tau \quad \tilde{\nu}_\tau(t) \]

Table 2: $e^+e^-$ final states that can result from $\tilde{\nu}$ exchange in the $s$- and/or $t$-channels if two Yukawa couplings in the $LLE^c$ term of the superpotential are simultaneously non-zero.
extension to other colliders with different center of mass energies and integrated luminosities is straightforward and can be partially obtained through a simple scaling relations\[11\]. With this luminosity almost all errors will be statistically dominated. Following\[6\] the notational conventions of Kalinowski et al., the differential cross section for the process $e^+e^- \rightarrow f\bar{f}$, where $f = \mu$ or $\tau$, allowing for possible $t$-channel $\tilde{\nu}$ or $s$-channel $Z'$ exchange, can be written as

$$\frac{d\sigma}{dz} = \frac{\pi \alpha^2}{8s} \left[ (1+z)^2 \left\{ \frac{1}{2} |f^s_{LR}|^2 + \frac{1}{2} |f^s_{RL}|^2 \right\} + (1-z)^2 \left\{ \frac{1}{2} |f^s_{LL}|^2 + \frac{1}{2} |f^s_{RR}|^2 \right\} \right], \quad (2)$$

where $z = \cos \theta$, the angle with respect to the $e^-$ beam and

$$f^s_{LR} = 1 + P_Z (g^{e}_L)^2 \oplus P_{Z'} (g'^e_L g'^{\bar{f}}_L) \oplus 0,$$

$$f^s_{RL} = 1 + P_Z (g^{e}_R)^2 \oplus P_{Z'} (g'^e_R g'^{\bar{f}}_R) \oplus 0,$$

$$f^s_{LL} = 1 + P_Z g^e_L g^e_R \oplus P_{Z'} (g'^e_L g'^{\bar{f}}_L) \oplus \frac{1}{2} C_{\tilde{\nu}} P^t_{\tilde{\nu}},$$

$$f^s_{RR} = 1 + P_Z g^e_R g^e_L \oplus P_{Z'} (g'^e_R g'^{\bar{f}}_L) \oplus \frac{1}{2} C_{\tilde{\nu}} P^t_{\tilde{\nu}}, \quad (3)$$

where $P_{Z,Z'} = s/(s - M_{Z,Z'}^2 + i M_{Z,Z'} \Gamma_{Z,Z'}) \simeq s/(s - M_{Z,Z'}^2)$ provides an adequate approximation when $M_Z^2 \ll s \ll M_{Z'}^2$, $P^t_{\tilde{\nu}} = s/(t - m_{\tilde{\nu}}^2)$ with $t = -s(1-z)/2$, $C_{\tilde{\nu}} = \lambda^2/4\pi\alpha$, with $\lambda$ being the relevant Yukawa coupling from the superpotential, and the $Z$ and $Z'$ gauge couplings are normalized such that $g^e_L = c(\frac{1}{2} + x)$ and $g^e_R = cx$ with $x = \sin^2 \theta_w$ and $c = \{\sqrt{2}G_F M_Z^2/\pi\alpha\}^{1/2}$. By ‘$\oplus$’ in the equation above we mean that we may choose either term, $i.e.$, the term after the first $\oplus$ corresponds to a potential $Z'$ contribution while that after the second $\oplus$ arises due to $t$-channel $\tilde{\nu}$ exchange. In addition, we note that the parameter $P$ in the expression above represents the polarization of the incoming electron beam, which we take to be 90% in our analysis below (although it’s specific value will not
be too important as we will soon see). This single beam polarization allows us to construct a $z$-dependent Left-Right Asymmetry, $A_{LR}(z)$:

$$A_{LR}(z) = \frac{(1 + z)^2 \{ |f^s_{LR}|^2 - |f^s_{RL}|^2 \} + (1 - z)^2 \{ |f^s_{LL}|^2 - |f^s_{RR}|^2 \}}{(1 + z)^2 \{ |f^s_{LR}|^2 + |f^s_{RL}|^2 \} + (1 - z)^2 \{ |f^s_{LL}|^2 + |f^s_{RR}|^2 \}}.$$  \hspace{1cm} (4)

Figure 1: Indirect search reach for $t$-channel exchanged $\tilde{\nu}$'s as a function of their mass from the process $e^+e^- \rightarrow \mu^+\mu^-$ or $\tau^+\tau^-$ at a 1 TeV NLC with an integrated luminosity of $L = 150 \text{ fb}^{-1}$ including the effects of initial state radiation. The discovery region lies below the curve.

For a $Z'$ or $\tilde{\nu}$ with fixed couplings the first question one must address is the search reach for either particle assuming that only one of the $\mu^+\mu^-$ or $\tau^+\tau^-$ final states is affected.
In the $Z'$ case, this result is essentially already documented\[12\]; for typical coupling strengths the search reach for a $Z'$ is $(4.5 - 7)\sqrt{s}$ with the lower end of the range being the most relevant in our case due to the fact that only leptonic observables of a given flavor are now employed to set the limit. A similar analysis following an identical approach leads to Fig.1 which shows the corresponding reach for $\tilde{\nu}$ exchange in the $t$-channel. As in the $Z'$ case, for a fixed coupling strength we examine the deviations in the binned distributions for both the conventional production cross section as well as $A_{LR}(z)$ as functions of the $\tilde{\nu}$ mass accounting for both statistical and systematic errors after angular acceptance cuts of $10^{\circ}$ are imposed. Lepton identification efficiencies of 100\% are assumed for all three generations. The dominant systematic errors in the case of lepton final states are those associated with uncertainties in the machine luminosity and the beam polarization which we take from Ref.\[12\]. As we lower the $\tilde{\nu}$ mass from some initially very large value, the new physics effects become sufficiently large in comparison to the anticipated errors that the discovery of some type of new physics can be claimed. For more details of this procedure see Ref.\[12\]. It is important to remember that these search reaches are only telling us that new physics beyond the SM is definitely present but not what its nature may be. It is clear that only for a somewhat lighter $Z'$ or $\tilde{\nu}$ would sufficient statistics be available to differentiate the two new physics sources.

The angular distribution and $A_{LR}(z)$ provide us with potential tools to attack this problem. Unfortunately, $A_{LR}(z)$ and/or the angular averaged quantity, $A_{LR}$, is numerically small at $\sqrt{s} = 1$ TeV and relatively poorly determined with integrated luminosities of 150 $fb^{-1}$. For example, in the SM one finds $A_{LR} = (6.31 \pm 1.06)\%$ assuming only statistical errors. In the numerical examples we will consider below, $A_{LR}$ is found to vary by no more than $\sim 0.5\sigma$ from this SM value and is thus not a good discriminator between $Z'$ and $\tilde{\nu}$ exchanges. This leaves us solely with the angular distribution with which to work and we will thus neglect the effects associated with single beam polarization in what follows. We
note, however, that if $\tilde{\nu}$ exchange were to modify hadronic final states via the $LQD^c$ term in the superpotential we would find a significantly larger and much more useful value of $A_{LR}$ for those states.

![Figure 2: Binned angular distribution for the process $e^+e^- \rightarrow \mu^+\mu^-$ or $\tau^+\tau^-$ at a 1 TeV NLC in the SM (histogram) and for the case where a 3 TeV $\tilde{\nu}$ with $\lambda = 0.5$ exchanged in the $t$-channel also contributes. The errors are statistical only and represent an integrated luminosity of $L = 150 \, fb^{-1}$. Initial state radiation has been included.

At first glance one would think that these two new physics models are easily separable since the exchanges are in distinct channels. This is true provided we are reasonably sensitive to the $t$-dependent part of the $\tilde{\nu}$ propagator which would certainly not be the case if we were
in the contact interaction limit, i.e., $s, |t| \ll M_{Z', \tilde{\nu}}^2$. (As we will see below, this parameter space region is quite large.) How does $Z'$ and $\tilde{\nu}$ exchange influence the angular distributions? Fig. 2 shows the bin-integrated angular distribution for the $R$-parity violating case assuming $\lambda = 0.5$ and a $\tilde{\nu}$ mass of 3 TeV in comparison to that for the SM. Here we see the general feature that at large positive $z$ the two distributions completely agree but the $\tilde{\nu}$ exchange causes a depletion of events with negative $z$. We note from the figure that this depletion is clearly statistically meaningful. This result will hold for all interesting mass and coupling values and thus we learn that if an increase of the angular distribution is observed for negative $z$ the new physics that accounts for it cannot arise from $R$-parity violation and may be attributable to a $Z'$.

In the $Z'$ case assuming a fixed gauged boson mass, we have four couplings that we can freely vary, i.e., $g_{L,R}^{et,fr}$. For simplicity we will assume that all these couplings have the same magnitude (but we strongly emphasize that this need not be the case), i.e., $|g_{L,R}^{et,fr}| = 0.3c$, and in this case the four possible relative sign combinations can lead to quite different angular distributions as shown in Fig. 3. Here we see that depending on the choice of relative signs, the $Z'$ exchange can lead to positive or negative modifications in the distribution in both the positive and negative ranges of $z$. Clearly if these four couplings were allowed to vary freely almost any reasonable shift in the distribution could be obtainable. We would thus expect that some choice of $Z'$ couplings could be made to completely simulate the $\tilde{\nu}$ signal.

How would the analysis then proceed? The exact form of the angular distribution given above suggests the following approach: once deviations in the distribution are observed a two parameter fit of the data could be performed to a trial distribution of the form

$$\frac{d\sigma}{dz} \sim A(1 + z)^2 + B(1 - z)^2,$$

(5)

12
Figure 3: Same as the previous figure but now including a 3 TeV \( Z' \) exchange in the s-channel. The magnitude of all \( Z' \) couplings is taken to be the same value, i.e., \( |g_{e,f}^{\alpha,\beta}| = 0.3c \), for purposes of demonstration. In the top panel, the relative signs of \( (g_L^e, g_L^f, g_R^e, g_R^f) \) are chosen to be \((+,−,+,(+))\) for the upper[lower] series of data points, while in the bottom panel they correspond to the choices \((+,−,+,(+))\) for the upper[lower] series, respectively.

Figure 4: 95% CL fits to the values of $A$ and $B$ for the data generated with $\bar{\nu}$ exchange (dashed region) and for the data generated for the four possible choices of $Z'$ couplings (dots). The SM result is represented by the square in the center of the figure while the diamonds are the locations of the best fits.
where from the exact expression above we see that \( A \sim |f^*_{LR}|^2 + |f^*_{RL}|^2 \) and \( B \sim |f^*_{LL}|^2 + |f^*_{RR}|^2 \).

A fit to this distribution may isolate whether the new physics occurs in the value of coefficient \( A, B, \) or both. In the SM and \( Z' \) cases both \( A \) and \( B \) are constants, but \( B \) picks up an additional \( z \) dependence in the case of \( \tilde{\nu} \) exchange. If this additional \( z \) dependence is strong, \( i.e., \) we are not in the contact interaction limit, then the \( \chi^2 \) of the fit assuming a constant \( B \) in the case of \( \tilde{\nu} \) exchange will be poor. Let us consider the ‘data’ as shown in Figs. 2 and 3 as input into this analysis for purposes of demonstration; the result of the fitting procedure for these sample cases is shown in Fig.4. Here we see that all five sets of ‘data’ lie quite a distance from the SM point clearly indicating the presence of new physics at a high confidence level. In the case of \( \tilde{\nu} \) exchange we see that the value of \( A \) arising from the fit is in excellent agreement with the expectations of the SM, while in the \( Z' \) case the values of both \( A \) and \( B \) have been altered. Note that all five allowed regions are statistically well separated from each other. Furthermore, in all cases the resulting confidence level (CL) of the fits are very good indicating no special sensitivity to any variation in the value of \( B \) with \( z \) for \( \tilde{\nu} \) exchange. (Numerically, we find the bin-averaged value of \( B \) to vary between 0.546 and 0.518 as we go from large negative to large positive \( z \).) Given the distribution of the \( Z' \) results one can imagine that a suitably chosen conspiratorial set of values for the couplings \( g^{e,f}_{L,R} \) could lead to a substantial overlap with the extracted \( \tilde{\nu} \) coupling region in which case the two new physics sources would not be distinguishable. Except for this conspiratorial region, however, it would appear that the fits to the angular distribution do provide a technique to separate these two SM extensions.

As discussed above, when the value of \( \lambda/m_{\tilde{\nu}} \) becomes sufficiently large it will become apparent that the fit with a constant \( B \) will no longer provide a good fit. Exactly when does this happen? To address this question we vary both \( \lambda \) and the \( \tilde{\nu} \) mass and perform a multitude of fits assuming that \( A \) and \( B \) are constant and obtain the confidence level of
Figure 5: Average confidence level of the best fit to the parameters $A$ and $B$ as a function of the $\tilde{\nu}$ mass in the case of $t$-channel $\tilde{\nu}$ exchange for various values of the Yukawa coupling $\lambda$ in the range 0.3 to 1.0 in steps of 0.1 from top left to lower right.
the best fit for each case. The result of this analysis is shown in Fig.5. In this figure we see that typically one finds that this type of fit begins to fail in a qualitative way when \( \lambda/m_{\tilde{\nu}} \geq 0.5 \text{ TeV}^{-1} \). For much smaller values of this parameter, as in the sample case above, the data will be insensitive to the nature of the \( t \)-channel exchange and we will be living in the contact interaction limit of parameter space. How does this bound scale with the collider energy? Since the \( t \)-channel \( \tilde{\nu} \) exchange interferes directly with the SM contribution, assuming that most of the error is statistical in origin, we expect the bound on the ratio \( \lambda/m_{\tilde{\nu}} \) to roughly scale as \( \sim (L \cdot s)^{-\frac{1}{4}} \), where \( L \) in the integrated luminosity and \( s \) in the machine center of mass energy.

3 \( s \)-channel \( \tilde{\nu} \) Exchange

When a \( Z' \) or \( \tilde{\nu} \) are exchanged in the \( s \)-channel, the general form of the cross section with a polarized electron beam can be written as:

\[
\frac{d\sigma}{dz} = \frac{\pi \alpha^2}{8s} \left\{ (1 + z)^2 \left\{ \frac{1 + P}{2} |f_{LR}^s|^2 + \frac{1 - P}{2} |f_{RL}^s|^2 \right\} \right. \\
+ \left. (1 - z)^2 \left\{ \frac{1 + P}{2} |f_{LL}^s|^2 + \frac{1 - P}{2} |f_{RR}^s|^2 \right\} + 4 \left\{ \frac{1 + P}{2} |f_{LL}^t|^2 + \frac{1 - P}{2} |f_{RR}^t|^2 \right\} \right\}, \quad (6)
\]

where \( f_{ij}^s \) are obtainable above and

\[
\begin{align*}
    f_{LL}^t & = f_{RR}^t = 0 \quad (Z') , \\
    f_{LL}^t & = f_{RR}^t = \frac{1}{2} C_{\tilde{\nu}} P_{\tilde{\nu}}^s \quad (\tilde{\nu}) ,
\end{align*}
\]

(7)

with \( P_{\tilde{\nu}}^s = s/(s - m_{\tilde{\nu}}^2 + im_{\tilde{\nu}} \Gamma_{\tilde{\nu}}) \simeq s/(s - m_{\tilde{\nu}}^2) \) in the same limit as employed above. Our first step here is to determine the search reach for a \( \tilde{\nu} \) being exchanged in the \( s \)-channel. Our standard analysis yields the results shown in Fig.6; note that the search reach for a
fixed value of $\lambda_0$ is somewhat larger in the $t$-channel than in the $s$-channel but generally comparable in magnitude. Note that here $\lambda_0^2 = \lambda_1\lambda_2$, with $\lambda_{e,f}$ being the values of the Yukawa couplings for the $\tilde{\nu}$ to initial state electrons and the fermion $f$ in the final state.

Figure 6: Same as Fig. 1 but now for $s$-channel $\tilde{\nu}$ exchange. Here $\lambda_0^2$ equals the product of the relevant Yukawa couplings in the superpotential. The typical region excluded by low energy data is that below the dashed curve in the lower right hand corner.

As before a short analysis demonstrates that single beam polarization will not help distinguish these two new physics models due the small value of the resulting asymmetry, so we set $P = 0$ and again examine the angular distribution. First, we note that when a $\tilde{\nu}$ is exchanged in the $s$-channel the angular distribution picks up a constant, i.e., $z$-independent
with $A, B$ given as before and here $C \sim 2[C_\nu P_\nu]^2$. As expected, when the value of the constant $C$ is sufficiently large it will become apparent that the resulting fit which assumes that only $A$ and $B$ are present is no longer valid due to an increase in $\chi^2$ and a lower confidence level. However, for moderate coupling strengths we find that it is possible to adjust the values of $A$ and $B$ to mask the contributions of the $C$ term. In Fig.7 we show the CL obtained by performing a large number of fits to the parameters $A$ and $B$ for different values of both $\lambda_0$ and the $\tilde{\nu}$ mass from generating ‘data’ samples via Monte Carlo. For small $\lambda_0$’s or large masses, as in the above example, we see that the CL of the fit is always quite good. In the opposite limit, the fit fails and the CL is quite small. Typically, we see that the fit begins to fail qualitatively when $\lambda_0/m_{\tilde{\nu}} \geq 0.25 - 0.30$ TeV$^{-1}$. This reach in coupling-mass parameter space is not very good and so we seek other observables with which to extend our reach.

In the case where a $\tau$ pair is being produced in the final state we can employ a clever idea used by Bar-Shalom, Eilam and Soni(BES) in a somewhat different context. If the $\tau$ spins can be analyzed, a spin-spin correlation can be formed which is sensitive to the spin of any new particle exchanged in the $s$-channel. Integrating over all production angles, this quantity can be written as an asymmetry:

$$B_{zz} = \frac{|f^{s}_{LR}|^2 + |f^{s}_{LR}|^2 + |f^{s}_{LR}|^2 - \frac{3}{4}(|f^{t}_{LL}|^2 + |f^{t}_{RR}|^2)}{|f^{s}_{LR}|^2 + |f^{s}_{LR}|^2 + |f^{s}_{LR}|^2 + |f^{s}_{LR}|^2 + \frac{3}{4}(|f^{t}_{LL}|^2 + |f^{t}_{RR}|^2)},$$

where we see immediately that for the case of the SM or a $Z'$ one obtains $B_{zz} = 1$ whereas a $\tilde{\nu}$ exchange in the $s$-channel will force this observable to smaller, even negative values. In Fig.8 we display the value of the asymmetry $B_{zz}$ as a function of the $\tilde{\nu}$ mass for several
Figure 7: Average confidence level of the best fit to the parameters $A$ and $B$ as a function of the $\tilde{\nu}$ mass in the case of $s$-channel $\tilde{\nu}$ exchange for various values of the Yukawa coupling $\lambda_0$ in the range 0.3 to 1.0 in steps of 0.1 from top left to lower right.
values of $\lambda_0$. Even if the efficiency for making this spin-spin correlation measurement is only 50%, the anticipated statistical error on this quantity will be of order 1% since there are about 9000 $\tau$-pairs in the data sample. Thus a value of $B_{zz}$ below $\simeq 0.95$ would provide a very strong indication that there is a scalar exchange in the $s$-channel. From the figure we see that this implies that the parameter space region $\lambda_0/m_{\tilde{\nu}} \geq 0.15 - 0.20 \, TeV^{-1}$ would certainly be probed by such measurements. Unfortunately, this technique does not help us in the case of a corresponding $t$-channel exchange.

Figure 8: Double $\tau$ spin asymmetry at a 1 TeV NLC as a function of the $\tilde{\nu}$ mass for different values of the Yukawa coupling $\lambda_0$. From left to right, $\lambda_0$ varies from 0.3 to 1.0 in steps of 0.1 as in the previous figure. In the case of either the SM or a $Z'$, $B_{zz} = 1$.
It is apparent that for non-τ pair final states we cannot use this trick. While we have already observed that single beam polarization is not useful, if both initial beams can be polarized more observables can be investigated. In this case, integration over $z$ gives the following expression for the cross section with two polarized beams:

$$\sigma(P_1, P_2) \sim [LL]\{|f_{LL}^s|^2 + |f_{LR}^s|^2\} + [RR]\{|f_{RL}^s|^2 + |f_{RR}^s|^2\} + \frac{3}{4}[LR]\{|f_{LL}^t|^2 + |f_{RR}^t|^2\}, \quad (10)$$

where we have employed the notation

$$[LL] = \frac{1}{4}[1 + P_1 + P_2 + P_1P_2],$$

$$[RR] = \frac{1}{4}[1 - P_1 - P_2 + P_1P_2],$$

$$[LR] = \frac{1}{2}[1 - P_1P_2], \quad (11)$$

with $P_{1,2}$ being the polarizations of the incoming electron and positron beam respectively. From these cross sections a double polarization asymmetry can be obtained:

$$A_{\text{double}} = \frac{\sigma(+,+) + \sigma(-,-) - \sigma(-,+) - \sigma(+,-)}{\sigma(+,+) + \sigma(-,-) + \sigma(-,+) + \sigma(+,-)}. \quad (12)$$

Let us assume that $P_1 = P_{e^-} = 0.90$ while $P_2 = P_{e^+} = 0.65$ as given in Ref.\[13\]; we then calculate $A_{\text{double}}$ readily and obtain a value of 0.585 for both the SM and when a $Z'$ is present. However, as in the case of $B_{zz}$, the presence of $\tilde{\nu}$ exchange in the $s$-channel can lead to significantly smaller values of $A_{\text{double}}$. It is interesting to note that this double polarization asymmetry would not have helped in the case of $t$-channel $\tilde{\nu}$ exchange since it and the $Z'$ contribute to the same amplitudes.
Figure 9: The double polarization asymmetry, $A_{\text{double}}$ as a function of the $\tilde{\nu}$ mass at a 1 TeV NLC for different choices of $\lambda_0$. From left to right, $\lambda_0$ varies from 0.3 to 1.0 in steps of 0.1. The dotted curve corresponds to the value obtained for both the SM and in the case of a $Z'$. 

23
Fig. 9 shows the set of results obtained for $A_{\text{double}}$ in this case as a function of the mass of the $\tilde{\nu}$ assuming various values for the Yukawa coupling $\lambda_0$. Since the statistical error on $A_{\text{double}}$ is again expected to be somewhat less than 1% for our assumed integrated luminosity (a value of 0.86% is obtained for the SM) it is clear that for values of $\lambda_0/m_{\tilde{\nu}} \geq 0.15 - 0.20\,\text{TeV}^{-1}$, a statistically significant signal for scalar $s$-channel exchange will be observed. This reach is quite comparable to that obtained using the double spin asymmetry technique discussed above and is superior to that found by an examination of the angular distribution alone. How does this bound scale with the collider energy? Since the $s$–channel $\tilde{\nu}$ exchange does not directly interfere with the SM contribution, assuming that most of the error is statistical in origin, we expect the bound on the ratio $\lambda/m_{\tilde{\nu}}$ to roughly scale as $\sim (L \cdot s^3)^{-1/8}$, where $L$ in the integrated luminosity and $s$ in the machine center of mass energy.

4 Bhabha Scattering

Bhabha scattering represents the most difficult case of the ones we have considered since $\gamma$ and $Z$ exchanges are already present in both the $s$- and $t$-channels in the SM and in fact the $t$-channel $\gamma$ pole dominates. Allowing for $s$- and $t$-channel $Z'$ or $\tilde{\nu}$ exchange for the case where both electron and positron beams are polarized, the differential cross section can be written as

$$\frac{d\sigma}{dz} = \frac{\pi \alpha^2}{8s} \left[(1 + z)^2 \left\{ [LL]|f_{LR}^s|^2 + [RR]|f_{RL}^s|^2 + [LL]|f_{LR}^t|^2 + [RR]|f_{RL}^t|^2 \right\} + 2[LK]|f_{LR}^s f_{LR}^t| + 2[RR]|f_{RL}^s f_{RL}^t] \right)$$

$$+ (1 - z)^2 \left\{ [LL]|f_{LL}^s|^2 + [RR]|f_{RR}^s|^2 \right\} + 2[LK] \left\{ |f_{LL}^t|^2 + |f_{RR}^t|^2 \right\}, \quad (13)$$
where the $f_{ij}^s$ can be obtained from the expressions above and

$$
\begin{align*}
    f_{LR}^t &= \frac{s}{t} + P_Z^t(g_L^e)^2 \oplus P_{Z'}^t(g_L'^e)^2 \oplus 0, \\
    f_{RL}^t &= \frac{s}{t} + P_Z^t(g_R^e)^2 \oplus P_{Z'}^t(g_R'^e)^2 \oplus 0, \\
    f_{LL}^t &= \frac{s}{t} + P_Z^t g_L^e g_R^e \oplus P_{Z'}^t g_L'^e g_R'^e \oplus \frac{1}{2} C_{\tilde{\nu}} P_{\tilde{\nu}}, \\
    f_{RR}^t &= \frac{s}{t} + P_Z^t g_L^e g_R^e \oplus P_{Z'}^t g_L'^e g_R'^e \oplus \frac{1}{2} C_{\tilde{\nu}} P_{\tilde{\nu}},
\end{align*}
$$

with $P_{Z,Z'}^t = s/(t - M_{Z,Z'}^2)$. The search reaches for a $Z'$ or $\tilde{\nu}$ in this channel are found to be very comparable to that of the case of $s$-channel exchange discussed above.

To examine this cross section in any detail, angular cuts are necessary due to the photon pole in the forward direction. We first employ a weak cut of $|z| < 0.985$, corresponding to $\theta \geq 10^\circ$, which is motivated by detector requirements[3]. This has little effect in the backward direction and leaves an enormous rate in the forward direction. To further tame the cross section in this direction we strengthen this cut to $z < 0.95$ to remove more of the photon pole. The result of this procedure for the SM and for the case of a 3 TeV $\tilde{\nu}$ with $\lambda = 0.5$ is shown in Fig.10 for a $\sqrt{s} = 1$ TeV NLC assuming unpolarized beams and an integrated luminosity of $150 \ fb^{-1}$. As one might expect, the distribution in the far forward direction is overwhelmingly dominated by the photon pole and hence there is no signal for new physics there even with the large statistics available. In the backwards direction, the $\tilde{\nu}$ exchange is seen to lead to a characteristic and statistically significant increase in the cross section above that predicted by the SM. Since $\tilde{\nu}$ exchange can only increase the cross section in the backward region, any observed decrease in the cross section may be attributable to a $Z'$. As can be seen in Fig.11, when the product of $Z'$ couplings $g_L^e g_R' > (<) 0$, the resulting cross section is seen to increase(decrease) in this case.
Figure 10: Same as Fig. 2 but now for the case of Bhabha scattering. Angular cuts as described in the text have been employed to render the cross section finite in the forward direction.
Figure 11: Same as the previous figure but now for a 3 TeV $Z'$ in comparison to the SM. The upper(lower) set of data points corresponds to $g_L^{\ell'} = g_R^{\ell'} = 0.5c$($g_L^{\ell'} = -g_R^{\ell'} = 0.5c$).
From this discussion it is clear that using the Bhabha scattering angular distribution alone it will be possible to easily distinguish new physics in the form of a $\tilde{\nu}$ from a $Z'$ when $g'_{L} g'_{R} < 0$. When the product of $Z'$ couplings have the opposite sign we need to use an additional observable. One immediate possibility is to employ $A_{\text{double}}$ as defined above in the case that both initial beams are polarized. However, due to the dominance of the photon pole at $z = 1$ we limit ourselves to events with $z < 0$; for the SM this corresponds to about 7000 events when the integrated luminosity is 150 $fb^{-1}$ at a 1 TeV NLC after ISR and gives $A_{\text{double}}(SM) = -0.273 \pm 0.011$, obtained by taking $P_{1} = 0.90$ and $P_{2} = 0.65$ as in the discussion above. A scan of the $\lambda$ and $\tilde{\nu}$ mass parameter space leads us to the observation that $\tilde{\nu}$ exchange always decreases the value of the asymmetry from that obtained in the case of the SM. $Z'$ exchange also modifies the value of this asymmetry; unfortunately we find that for $g'_{L} g'_{R} > 0$, $A_{\text{double}}$ also decreases as it does for the case of $\tilde{\nu}$ exchange. Thus $A_{\text{double}}$ does not help us resolve this potential ambiguity in the case of Bhabha scattering.

5 Discussion and Conclusion

In this paper we have considered the problem of how to distinguish two potential new physics scenarios from each other below the threshold for direct production of new particles at the NLC: $R$-parity violation and a extension of the SM gauge group by an additional $U(1)$ factor. Both kinds of new physics can lead to qualitatively similar alterations in SM cross sections, angular distributions and various asymmetries but differ in detail. These detailed differences provide the key to the two major weapons that are useful in accomplishing our task: (i) the angular distribution of the final state fermion and (ii) an asymmetry formed by polarizing both beams in the initial state, $A_{\text{double}}$. The traditional asymmetry, $A_{LR}$, formed when only a
single beam is polarized, was shown not to be useful for the case of purely leptonic processes we considered, but will be useful in an extension of the analysis to hadronic final states. This same analysis employed above can be easily extended to other new physics scenarios which involve the exchange on new particles\cite{E5} as in the case of massive graviton exchange in theories with compactified dimensions.

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References

[1] For a recent review of the electroweak data, see M. Grünwald and D. Karlen, talks given at the XXIX International Conference on High Energy Physic, Vancouver B.C, July 23-28, 1998; see also J.Erler and P. Langacker, hep-ph/9809352 and J.H. Field, hep-ph/9809292 and hep-ph/9810288.

[2] M. Cvetic et al., Phys. Rev. D56, 2861 (1997).

[3] For a review, see H. Dreiner, hep-ph/9707435, to be published in Perspectives on Supersymmetry, ed. by G.L. Kane. See also P. Roy, hep-ph/9712520.

[4] See, for example, T.G. Rizzo, Phys. Rev. D46, 5102 (1992) and Ref.3.

[5] See, for example, V. Barger, G.F. Giudice and T. Han, Phys. Rev. D40, 2987 (1989); D. Choudhury and P. Roy, Phys. Lett. B387, 153 (1996); G. Bhattacharyya, hep-ph/9709393; G. Bhattacharyya and P.B. Pal, Phys. Lett. B439, 81 (1998) and hep-ph/9809493; I. Hinchliffe and T. Kaeding, Phys. Rev. D47, 279 (1993); A.Y. Smirnov and F. Vissani, Phys. Lett. B380, 317 (1996).

[6] J. Kalinowski et al., Phys. Lett. B406, 314 (1997) and Phys. Lett. B414, 297 (1997); J.Erler, J.L. Feng and N. Polonsky, Phys. Rev. Lett. 78, 3063 (1997); B.C. Allanach et al., Phys. Lett. B420, 307 (1998); S.Bar-Shalom, G.Eilam and A. Soni, hep-ph/9804333 and Phys. Rev. Lett. 80, 4629 (1998); J.L. Feng, J.F. Gunion and T. Han, Phys. Rev. D58, 071701 (1998).

[7] J.L. Hewett and T.G. Rizzo, hep-ph/9809525 and SLAC-PUB-7999.

[8] See J.L. Hewett and T.G. Rizzo, Phys. Rev. D56, 5709 (1997) and Phys. Rev. D58, 055005 (1998) and references therein.
[9] See, for example, R. Foot, G.C. Joshi and H. Lew, Phys. Rev. D40, 2487 (1989); X.G. He, G.C. Joshi, H. Lew and R.R. Volkas, talk given at the XXV International Conference on High Energy Physic, Singapore, August 2-8, 1990; T.P. Cheng and L.F. Li, Phys. Rev. Lett. 38, 381 (1977); E. Ma, Phys. Lett. B433, 74 (1998); E. Ma, D.P. Roy and U. Sarkar, hep-ph/9810309; E. Ma and D.P. Roy, hep-ph/9811266.

[10] For a general discussion of $\bar{\nu}$ properties in $R$-parity violating models, see Y. Grossman and H.E. Haber, hep-ph/9810530.

[11] For a discussion of approximate scaling rules for $Z'$ searches, see A. Leike, hep-ph/9805494.

[12] See for example, T.G. Rizzo, hep-ph/9609248, in New Directions for High High Energy Physics, Proceedings of the 1996 DPF/DPB Summer Study on High Energy Physics, eds. D.G Cassel, L.T. Gennari and R.H. Siemann, Snowmass, CO 1996.

[13] The possibility of polarizing both the electron and positron beams at the NLC has been discussed in the Zeroth Order Design Report for the Next Linear Collider, hep-ex/9605011, Physics and Technology of the Next Linear Collider, eds. D. Burke and M. Peskin, 1996.

[14] See for example, F. Cuypers, hep-ph/9611336, in New Directions for High High Energy Physics, Proceedings of the 1996 DPF/DPB Summer Study on High Energy Physics, eds. D.G Cassel, L.T. Gennari and R.H. Siemann, Snowmass, CO 1996.

[15] J.L. Hewett, hep-ph/9811356.