Stability of Universe Model Coupled with Phantom and Tachyon Fields

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Abstract
In this paper, we study phase space analysis of locally rotationally symmetric Bianchi type I universe model by taking different linear combinations for the interactions between scalar field models and dark matter. An autonomous system of equations is established by defining normalized dimensionless variables. In order to investigate stability of the system, we evaluate corresponding critical points for different values of the parameters. We also evaluate power-law scale factor whose behavior shows different cosmological phases. The dynamical analysis indicates a matter dominated epoch ultimately followed by a late accelerated expansion phase. It is found that all the critical points indicate accelerated expansion of the universe for tachyon coupled field. We conclude that negative values of $m$ provide more stable future attractors as compared to its positive values.

Keywords: Phase space analysis; Bianchi type I universe.

PACS: 04.20.-q; 95.36.+x; 98.80.-k.

1 Introduction
Recent observations (type Ia supernova, large scale structure and cosmic microwave background radiation (CMBR)) suggest that our universe is expanding at an accelerating rate [1]. These observational probes indicate two

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cosmic phases, i.e., the cosmos phase before radiation and ultimately the current cosmic era. Many substantial attempts have been taken to explore the facts behind the current cosmic acceleration. Some mysterious source of unusual anti-gravitational force, known as dark energy (DE), was proposed by physicists while trying to examine the formation of galaxies in cosmic scenario. It is an exotic energy constituent having large negative pressure which dominates over the matter content of cosmos. This energy is supposed to be responsible for the current cosmic expansion.

There have been several proposals of DE to study its ambiguous nature. The cosmological constant ($\Lambda$) is considered as the simplest candidate but its characterization has two well-known problems, i.e., fine-tuning and cosmic coincidence. There are various alternative dynamical models which can be taken as a substitute of $\Lambda$ like quintessence [2], phantom model [3], tachyon field [4] and k-essence [5] that also predict cosmic expansion. The generalization of simple barotropic equation of state (EoS) to more exotic forms like Chaplygin gas [6] and its modification [7] also correspond to DE candidates.

The concept of introducing scalar field, with an EoS parameter other than $-1$, has played a remarkable role to interpret the universe evolution due to its progressive implementation in various cosmological problems like cosmic acceleration and cosmic coincidence problem. The scalar field models can also predict the early inflationary cosmic era. We can choose scalar field models (in particular, phantom and tachyon) as a dynamical DE candidate interacting with DM by interchanging energy between them which may solve the coincidence problem. Researchers have paid an extensive attention to the tachyon cosmology where the tachyon is basically attributed by string theory [8, 9]. Gibbons discussed cosmological influence of the tachyon rolling down to its ground phase [10]. The universe model undergoes accelerated expansion as the tachyon field rolls down [9]. A tachyonic matter may yield inflation at early era and ultimately some new form of DM at late times [11]. A phantom field was also presented as an alternative of DE which constitutes large negative pressure with EoS parameter $w < -1$ and plays an important role for accelerated expansion of the universe [12]. One of the significant features of the phantom model is that the universe will end with a big-rip (future singularity). The interaction of DE (phantom or tachyon) and DM describes energy flow between the components such that no component remains conserved separately. It is also demonstrated that an interaction between the components may alleviate the coincidence problem [13].

A phase space describes all possible states (position and momentum) as-
associated with each point of the system. The analysis of viable stable late-time attractors has remarkable significance for different cosmological models. This provides dynamical behavior of a cosmological model by minimizing complexity of the equations. It is useful to study different patterns of evolution by transforming the system of equations to an autonomous one. This investigates the influence of initial data on stability of any system by checking whether the system remains stable for a long time \[14\]. The stability of different universe models via phase space helps to explore their qualitative features.

Copeland et al. \[15\] discussed phase space analysis of inflationary models which was unable to solve density problem. Guo et al. \[16\] studied stability of FRW universe model filled with barotropic fluid as well as phantom scalar field and found that phantom dominated solution is a stable late-time attractor. Guo et al. \[17\] analyzed phase space analysis of interacting phantom energy with DM. Yang and Gao \[18\] explored phase space analysis for k-essence cosmology and found that stability of critical points play a substantial role for the cosmic evolution. Xiao and Zhu \[19\] investigated stability of FRW universe model in loop quantum gravity by using phase space analysis along with barotropic fluid and positive field potential. Acquaviva and Beesham \[20\] discussed this analysis for FRW model and found that nonlinear viscous model describes possibility of current cosmic expansion. Recently, we have studied the impact of nonlinear electrodynamics on stability of accelerated expansion of FRW universe model \[21\]. Shahalam et al. \[22\] presented dynamical analysis of coupled phantom and tachyon fields by taking linear combinations of the coupling for FRW universe model.

Bianchi universe models have widely been discussed in literature to study expected primordial anisotropy and some large angle anomalies detected by CMBR which yield violation of statistical isotropy of cosmos \[23\]. Belinski and Khalatnikov \[24\] studied phase plane technique for Bianchi type I (BI) model under the influence of shear and bulk viscosity. Coley and Dunn \[25\] used phase plane approach to study dynamical behavior of Bianchi type V model containing a viscous fluid. Burd and Coley \[26\] investigated the effects of shear as well as bulk viscosity on the stability of Bianchi universe models. Goliath and Ellis \[27\] discussed dynamical evolution of Bianchi universe model via phase space by including \(\Lambda\). Sharif and Waheed \[28\] explored phase space analysis of locally rotationally symmetric (LRS) BI universe for chameleon scalar field in Brans-Dicke gravity. Chaubey and Raushan \[29\] studied phase space analysis of LRS BI model in the presence
of scalar field.

This paper investigates stability of LRS BI universe by taking linear interactions of phantom and tachyon fields coupled with DM via phase space analysis. The plan of the paper is as follows. In section 2, we provide some basic formalism for evolution equations. An autonomous system of equations is developed by introducing normalized dimensionless variables. Section 3 deals with dynamical analysis of interacting phantom energy and DM by taking three different forms of interactions. We discuss phase space analysis of tachyon field coupled with DM in section 4. Section 5 deals with the formulation of power-law scale factor. Finally, we conclude our results in the last section.

2 General Equations

Bianchi universe models are the simplest extensions of FRW universe by adding anisotropic effects. It has been observed that some large angle anomalies in CMBR tend to violate the statistical isotropy of present cosmic models [23]. In this context, homogeneous anisotropic universe models under plane symmetric background has substantial role to understand these anomalies. The LRS BI model with anisotropic effects is defined by the line element

\[ ds^2 = -dt^2 + a(t)dx^2 + b(t)(dy^2 + dz^2), \]

where \( a(t) \) and \( b(t) \) represent the cosmic expansion radii. We can define the mean Hubble parameter as

\[ H = \frac{1}{3} [H_1 + H_2] = \frac{1}{3} \left[ \frac{\dot{a}}{a} + \frac{2\dot{b}}{b} \right] = \frac{1}{3} \left( \frac{\dot{v}}{v} \right), \]

where \( H_1 = \frac{\dot{a}}{a}, \ H_2 = \frac{\dot{b}}{b} \) are directional Hubble parameters. For a spatially homogeneous spacetime, the normal congruence to homogeneous expansion leads to a constant ratio, i.e., the expansion and shear scalars are proportional to each other. We assume a power-law relation \( a = b^m, \ m \neq 0, 1 \), where \( m \) is a constant anisotropic parameter which differentiates the expansion along \( x \) and \( y \) directions and represents the deviation of anisotropic universe model from isotropic. We define the average Hubble expansion by a relationship between mean and directional Hubble parameters as

\[ H_1 = mH_2 = \left( \frac{3m}{m + 2} \right) H. \]
Collins [30] studied physical consequences of this assumption by taking perfect fluid and barotropic EoS in a general way. Bennett et al. [31] proposed small scale deviation from perfect isotropy of the order $10^{-5}$ which was later confirmed by high resolution WMAP data. Many other authors have also used this condition in literature [32].

The cosmic fluid is considered by coupling phantom field and matter. We consider that these two components interact through the interaction term $Q$ such that the conservation of energy yield

\begin{align}
\dot{\sigma}_m + 3(\sigma_m + p_m)H &= Q, \\
\dot{\sigma}_\phi + 3(\sigma_\phi + p_\phi)H &= -Q, \\
\dot{\sigma} + 3(\sigma + p)H &= 0,
\end{align}

where dot represents derivative with respect to time, $\sigma = \sigma_m + \sigma_\phi$, $p = p_m + p_\phi$, $\sigma_m$, $\sigma_\phi$, $p_m$ and $p_\phi$ correspond to energy densities and pressures of matter and phantom energy, respectively. It is noted that the sign of interaction term denotes the transfer of energy between two components. For $Q > 0$, the energy flows from phantom to matter while $Q < 0$ corresponds to vice versa. The interaction term gives an additional degree of freedom which can be restricted by the constant energy density ratio at late times. It has always been interesting to study cosmological consequences of these interactions by considering their several forms [33]. It is clear from the above conservation equations that $Q = Q(H, \sigma_m, \sigma_\phi)$. The constraint and Raychaudhuri equations obtained from the field equations are given by

\begin{align}
H^2 &= \frac{(m + 2)^2}{9(2m + 1)}(\sigma_m + \sigma_\phi), \\
0 &= \left(\frac{6}{m + 2}\right)\dot{H} + \frac{27}{(m + 2)^2}H^2 + p_\phi,
\end{align}

where $\sigma_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$. Due to many arbitrary parameters, it seems difficult to find analytical solution of the evolution equation.

For this purpose, we define the following normalized dimensionless quantities [17, 34]

\begin{align}
\mu &= \frac{(m + 2)\dot{\phi}}{\sqrt{6(2m + 1)}H}, \quad \nu = \frac{(m + 2)\sqrt{V}}{\sqrt{3(2m + 1)}H}, \quad \lambda = -\frac{V'}{V},
\end{align}
that direct the evolution equations into an autonomous system. Differentiating $\mu$ and $\nu$ with respect to $N = \frac{m+2}{3m} \ln a$, we have

$$\mu' = \frac{(m+2)}{3m} \mu \left[ \frac{\ddot{\phi}}{H \dot{\phi}} - \frac{\dot{H}}{H^2} \right],$$

(10)

$$\nu' = -\frac{(m+2)}{3m} \nu \left[ \sqrt{6} \mu \lambda + \frac{\dot{H}}{H^2} \right].$$

(11)

For an exponential potential, Raychaudhuri and conservation equations in terms of these dimensionless quantities become

$$\frac{\dot{H}}{H^2} = -\frac{1}{2(m+2)} \left[ 9 + (2m+1)^2 \{ \mu^2 - \nu^2 \} \right],$$

(12)

$$\frac{\ddot{\phi}}{H \dot{\phi}} = 3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda \nu^2}{\mu} + \frac{Q}{H \dot{\phi}^2},$$

(13)

where $\lambda$ is taken as a constant. We can write from the constraint equation

$$\Omega_\phi = \frac{(m+2)^2 \sigma_\phi}{9(2m+1) H^2} = \frac{2m+1}{3} [ - \mu^2 + \nu^2 ].$$

(14)

The effective EoS for the cosmic fluid and phantom field are given by

$$w_{eff} = -1 - \frac{2 \dot{H}}{3H^2}, \quad w_\phi = \frac{w_{eff}}{\Omega_\phi}. \quad (15)$$

## 3 Dynamics of Interacting Phantom Energy

This section deals with stability of LRS BI model through phase space analysis by taking interaction between phantom energy and matter. We consider scalar field models for dynamical analysis to study whether we can alleviate the ambiguities like fine-tuning as well as cosmic coincidence arising from the consistency of $\Lambda$ with the recent cosmic observations. In order to find critical points $\{\mu, \nu\}$, we need to solve the dynamical system of Eqs.(10) and (11) by imposing the condition $\mu' = \nu' = 0$. The stability of LRS BI universe model will be discussed according to the nature of critical points and the corresponding eigenvalues. In the following, we consider three different forms of interactions between phantom field and matter.
3.1 Coupling $Q = \alpha \dot{\sigma}_m$

Firstly, we take a model of interaction $Q = \alpha \dot{\sigma}_m$ for cosmos where both phantom field as well as DM are present [22]. Different forms of coupling have been discussed in literature which are proportional to the time derivative of their energy densities [17, 35]. Equation (13), in terms of this coupling, turns out to be

$$\frac{\ddot{\phi}}{H\dot{\phi}} = 3 - \sqrt{\frac{3}{2}} \frac{2m + 1}{m + 2} \frac{\lambda \nu^2}{\mu} - \frac{3\alpha \Omega_m}{2(1 - \alpha)\mu},$$

(16)

where $\Omega_m = 1 - \Omega_\phi$. The corresponding autonomous system of equations reduces to

$$\mu' = \frac{(m + 2)}{3m} \mu \left[ 3 - \sqrt{\frac{3}{2}} \frac{2m + 1}{m + 2} \frac{\lambda \nu^2}{\mu} - \frac{3\alpha \Omega_m}{2(1 - \alpha)\mu^2} + \frac{1}{2(m + 2)} \right] \left(9 + (2m + 1)^2(\mu^2 - \nu^2)\right),$$

(17)

$$\nu' = -\frac{(m + 2)}{3m} \nu \left[ \sqrt{6}\mu \lambda - \frac{1}{2(m + 2)} \left(9 + (2m + 1)^2(\mu^2 - \nu^2)\right)\right].$$

(18)

The eigenvalues can be determined by the Jacobian matrix

$$A = \left( \begin{array}{cc} \frac{\partial f}{\partial \mu} & \frac{\partial f}{\partial \nu} \\ \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial \nu} \end{array} \right),$$

(19)

where suffix 0 gives the values at critical points $(\mu_c, \nu_c)$. The critical point is called a source (respectively, a sink) if both eigenvalues consist of positive (respectively, negative) real parts. The real parts of the eigenvalues having opposite signs correspond to a saddle point of the system. We evaluate the following critical points in this case. For $P_1 = (\mu_c, \nu_c) = \left(-\frac{1}{\sqrt{6}\lambda} \left\{ \frac{6 - \alpha(2m + 7)}{2(1 - \alpha)} \right\}, 0\right)$, the eigenvalues of Jacobian matrix are given by

$$\eta_1 = \frac{(m + 2)}{3m} \left[ 3 - \frac{3\alpha}{2(1 - \alpha)} \left\{ \frac{24(1 - \alpha)^2\lambda^2}{[6 - \alpha(2m + 1)]^2} - \frac{2m + 1}{3} \right\} + \frac{1}{2(m + 2)} \left\{9 + (2m + 1)^2[6 - \alpha(2m + 7)]^2 \right\} \right],$$

(20)

$$\eta_2 = -\frac{(m + 2)}{3m} \left[ \frac{\alpha(2m + 7) - 6}{2(1 - \alpha)} - \frac{1}{2(m + 2)} \left\{9 \right\} \right].$$
We are interested to study the impact of parameters $m$ and $\alpha$ on the stability of critical points in the presence of scalar field model. We plot the dynamical behavior of critical points for $Q = \alpha \dot{\sigma}_m$ by taking different values of $\alpha$ and $m$ as shown in Figure 1. In these numerical plots, we observe that the eigenvalues are positive indicating the point $P_1$ as an unstable past attractor for $m > 0$ in the physical phase space except for $\alpha = 1, m = -2$ at which the system becomes undetermined. For $m < 0$, this point becomes stable future attractor. The dynamical analysis shows a matter dominated era ultimately followed by a late accelerated expansion phase of the universe.

For $P_2 = \left(\frac{\sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2}}{(2m+1)^2}, 0\right)$, the corresponding eigenvalues are

$$\eta_1 = \frac{(m+2)}{3m} \left[3 - \frac{3\alpha}{2(1-\alpha)} \left\{ \frac{(2m+1)^4}{(m+2)^2} \right\} \sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2} \right]$$

$$\eta_2 = -\frac{(m+2)}{3m} \sqrt{6}(m+2)\lambda \left\{ \frac{\sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2}}{(2m+1)^2} \right\} - \frac{1}{2(m+2)}$$

$$\times \left\{ 9 + (2m+1)^2(m+2) \right\} \left\{ \frac{\sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2}}{2m+1} \right\}^2, \quad (22)$$

$$\eta_3 = \frac{(m+2)}{3m} \left[3 - \frac{3\alpha}{2(1-\alpha)} \left\{ \frac{(2m+1)^4}{(m+2)^2} \right\} \sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2} \right]$$

$$\eta_4 = -\frac{(m+2)}{3m} \sqrt{6}(m+2)\lambda \left\{ \frac{\sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2}}{(2m+1)^2} \right\} - \frac{1}{2(m+2)}$$

$$\times \left\{ 9 + (2m+1)^2(m+2) \right\} \left\{ \frac{\sqrt{6\lambda} \pm \sqrt{6\lambda^2 - \left(\frac{3(2m+1)}{m+2}\right)^2}}{2m+1} \right\}^2, \quad (23)$$
Figure 1: Plots for the phase plane evolution of phantom coupled universe model with $Q = \alpha \dot{\sigma}_m$ and $\lambda = 2$. 
This point corresponds to unstable past attractor without accelerated expansion for \( m > 0 \). By taking negative values of parameter \( m \), this point becomes stable future attractor or a saddle point depending on values of \( \alpha \). It is noted that the stable point undergoes accelerated expansion as \( q < 0 \).

For \( P_3 = \left( \frac{\sqrt{3(2m+7)+\alpha(2m^2-m-19)}+\sqrt{3(2m+7)+\alpha(2m^2-m-19)^2-\alpha(\alpha-1)(m+2)(2m+1)}}{2(\alpha-1)(2m+1)}, 0 \right) \), we have

\[
\begin{align*}
\eta_1 &= \frac{(m+2)}{3m} \left[ 3 - \frac{3\alpha}{2(1-\alpha)} \left\{ \frac{2(\alpha-1)(2m+1)}{3(2m+7)+\alpha(2m^2-m-19)+\xi} \right. \right. \\
&\quad - \frac{2m+1}{3} \left. \right\} + \frac{1}{2(m+2)} \left\{ 9 + \frac{3(2m+1)^2}{2(\alpha-1)(2m+1)} [3(2m+7) \\
&\quad + \alpha(2m^2-m-19)+\xi] \right\},
\end{align*}
\]

(24)

\[
\begin{align*}
\eta_2 &= -\frac{(m+2)}{3m} \left[ \sqrt{3\lambda} \frac{\sqrt{3(2m+7)+\alpha(2m^2-m-19)+\xi}}{(\alpha-1)(2m+1)} \\
&\quad - \frac{1}{2(m+2)} \left\{ 9 + \frac{(2m+1)^2}{2(\alpha-1)(2m+1)} [3(2m+7)\alpha(2m^2-m-19) \\
&\quad + \xi] \right\},
\end{align*}
\]

(25)

where \( \xi = \sqrt{[3(2m+7)+\alpha(2m^2-m-19)]^2-\alpha(\alpha-1)(m+2)(2m+1)} \).

We find the same behavior of this point for positive values of \( m \). This point is also a stable future attractor for \( m < 0 \) showing accelerated expanding universe model. The effective potential for the cosmic fluid is given by

\[
w_{eff} = -1 + \frac{1}{m+2} [9 + (2m+1)(\mu^2-\nu^2)].
\]

(26)

The effective EoS parameter and deceleration parameter are given by

\[
\begin{align*}
w_\phi &= \frac{1}{\mu^2+\nu^2} \left[-1 + \frac{1}{m+2} \{9 + (2m+1)(\mu^2-\nu^2)\} \right],
\end{align*}
\]

(27)

\[
\begin{align*}
q &= -1 + \frac{1}{m+2} [9 + (2m+1)(\mu^2-\nu^2)].
\end{align*}
\]

(28)

It is mentioned here that points \( P_1 \) and \( P_2 \) undergo decelerated expansion while the point \( P_3 \) is a stable future attractor that lies in accelerated expanding phase of the universe as \( q < 0 \) and \( \omega_\phi < -1 \). The summary of the results for evolution as well as stability of LRS BI model coupled with phantom energy and matter is given in Table 1.
Table 1: Stability Analysis for the Phantom Coupled System with \( Q = \alpha \dot{\sigma}_m \).

| Ranges of \( \alpha \) and \( m \) for Critical Points | Stability | Acceleration |
|--------------------------------------------------------|-----------|--------------|
| \( P_1 \)                                              |           |              |
| \( \alpha > 0, \ m > 0 (\alpha \neq 1) \)             | Unstable  | No           |
| \( \alpha < 0, \ m > 0 \)                              | Unstable  | No           |
| \( \alpha < 0, \ m < 0 (m \neq -2) \)                 | Stable    | No           |
| \( \alpha > 0, \ m < 0 \)                              | Stable    | No           |
| \( P_2 \)                                              |           |              |
| \( \alpha > 0, \ m > 0 (\alpha \neq 1) \)             | Unstable  | No           |
| \( \alpha < 0, \ m > 0 \)                              | Unstable/Saddle | No |
| \( \alpha < 0, \ m < 0 (m \neq -0.5, -2) \)            | Stable/Saddle | Yes          |
| \( \alpha > 0, \ m < 0 \)                              | Stable/Saddle | Yes          |
| \( P_3 \)                                              |           |              |
| \( \alpha > 0, \ m > 0 (\alpha \neq 1) \)             | Unstable/Saddle | No |
| \( \alpha < 0, \ m > 0 \)                              | Saddle    | No           |
| \( \alpha < 0, \ m < 0 (m \neq -0.5, -2) \)            | Stable    | Yes          |
| \( \alpha > 0, \ m < 0 \)                              | Stable    | No           |

3.2 Coupling \( Q = \beta \dot{\sigma}_\phi \)

For this coupling, Eq. (13) takes the form

\[
\frac{\ddot{\phi}}{H\dot{\phi}} = 3 - \sqrt{3}\frac{2m + 1}{2} \frac{\lambda \nu^2}{m + 2} \frac{\mu}{1 + \beta} - \frac{3\beta}{1 + \beta}.
\] (29)

The autonomous system of equations becomes

\[
\mu' = \frac{(m + 2)}{3m} \mu \left[ 3 - \sqrt{3} \frac{2m + 1}{2} \frac{\lambda \nu^2}{m + 2} \frac{\mu}{1 - \beta} + \frac{1}{2(m + 2)} \right] \times \{9 + (2m + 1)^2(\mu^2 - \nu^2)\},
\] (30)

\[
\nu' = -\frac{(m + 2)}{3m} \nu \left[ \sqrt{6} \mu \lambda - \frac{1}{2(m + 2)} \{9 + (2m + 1)^2(\mu^2 - \nu^2)\} \right],
\] (31)

We follow the same procedure to find the critical points. For \( P_1 = (0, 0) \), we have

\[
\eta_1 = \frac{3}{2m} + \frac{m + 2}{m(1 + \beta)}, \quad \eta_2 = \frac{3}{2m}.
\] (32)
This point shows a varying behavior for different values of parameters $\beta$ and $m$. For $\alpha = 0.8, -0.2$, we find that point $P_1$ is unstable/saddle node by taking only positive values of $m$ and $\lambda = 2$ (Figure 2). For $\beta = -1$, the eigenvalues become undetermined, hence we neglect it. We observe that negative values of $m$ show a stable future attractor. It is mentioned here that point $P_1$ undergoes decelerated cosmic expansion since $q < 0$ for all choices of parameters.

For $P_2 = \left( \frac{1}{2m+1} \sqrt{\frac{3[3\beta-(2m+1)]}{1+\beta}}, 0 \right)$, the eigenvalues are given by

$$\eta_1 = \frac{m + 2}{m(1 + \beta)} + \frac{3[1 + 4\beta - 2(m + 1)]}{2m(1 + \beta)}, \quad (33)$$

$$\eta_2 = -\frac{\sqrt{2}(m + 2)\lambda}{m(2m + 1)} \sqrt{\frac{3\beta - (2m + 1)}{1 + \beta}} + \frac{3 + 6\beta - 2(m + 1)}{2m(1 + \beta)}. \quad (34)$$

This point shows opposite behavior as compared to the previous point. Here all choices of $m$ and $\alpha$ give stable nodes except $m > 0$ and $\alpha > 0$ that correspond to unstable node. In this case, the universe is in decelerated expansion phase for all values of $m$. For $P_3 = \left( -\frac{1}{2m+1} \sqrt{\frac{3[3\beta-(2m+1)]}{1+\beta}}, 0 \right)$, the corresponding eigenvalues yield

$$\eta_1 = \frac{m + 2}{m(1 + \beta)} + \frac{3[1 + 4\beta - 2(m + 1)]}{2m(1 + \beta)}, \quad (35)$$

$$\eta_2 = \frac{\sqrt{2}(m + 2)\lambda}{\sqrt{3m(2m + 1)}} \sqrt{\frac{3[3\beta - (2m + 1)]}{1 + \beta}} + \frac{3 + 6\beta - 2(m + 1)}{2m(1 + \beta)}. \quad (36)$$

We find that point $P_3$ is an unstable past attractor for all values of $\alpha$ and $m$ except for $-0.9 < \beta < -0.1$ at which it behaves as a stable node. For $P_4 = (0, \pm \frac{3}{2m+1})$, the eigenvalues are

$$\eta_1 = \frac{m + 2}{m(1 + \beta)}, \quad \eta_2 = \frac{m + 2}{m(1 + \beta)}. \quad (37)$$

This point is also an unstable past attractor for positive values of $m$. It is noted that for $Q = \beta \dot{\sigma}_\phi$, all points lie in a region of decelerated expansion. A general dynamical analysis is given in Table 2.
Figure 2: Plots for the phase plane evolution of phantom coupled universe model with $Q = \beta \dot{\sigma}_\phi$ and $\lambda = 2$. 
Table 2: Stability Analysis for the Phantom Coupled System with $Q = \beta \dot{\sigma}_\phi$.

| Ranges of $\beta$ and $m$ for Critical Points | Stability | Acceleration |
|---------------------------------------------|-----------|--------------|
| $P_1$                                       |           |              |
| $\beta > 0, m > 0$                          | Unstable  | No           |
| $\beta < 0, m > 0, \beta \neq -1$          | Unstable/Saddle | No           |
| $\beta < 0, m < 0$                          | Stable    | No           |
| $\beta > 0, m < 0$                          | Stable    | No           |
| $P_2$                                       |           |              |
| $\beta > 0, m > 0$                          | Unstable  | No           |
| $\beta < 0, m > 0, \beta \neq -1$          | Stable    | No           |
| $\beta < 0, m < 0, m \neq -0.5$             | Stable    | No           |
| $\beta > 0, m < 0$                          | Stable    | No           |
| $P_3$                                       |           |              |
| $\beta > 0, m > 0$                          | Unstable  | No           |
| $\beta < 0, m > 0, \beta \neq -1$          | Stable/Unstable | No           |
| $\beta < 0, m < 0, m \neq -0.5$             | Unstable  | No           |
| $\beta > 0, m < 0$                          | Stable for $-0.9 < \beta < -0.1$ | No           |
| $P_4$                                       |           |              |
| $\beta > 0, m > 0$                          | Unstable  | No           |
| $\beta < 0, m > 0, \beta \neq -1$          | Unstable  | No           |
| $\beta < 0, m < 0$                          | Stable    | No           |
| $\beta > 0, m < 0$                          | Stable    | No           |

3.3 Coupling $Q = \gamma (\dot{\sigma}_m + \dot{\sigma}_\phi)$

Here we consider the coupling as a linear combination of $\dot{\sigma}_m$ and $\dot{\sigma}_\phi$ for which Eq. (13) becomes

$$\frac{\ddot{\phi}}{H \dot{\phi}} = 3 - \sqrt{\frac{3}{2} \frac{2m + 1}{m + 2}} \frac{\lambda \nu^2}{\mu} - \frac{3\gamma \Omega_m}{2(1 - \gamma)\mu^2} - \frac{3\gamma}{1 + \gamma}. $$

(38)

The evolution and conservation equations yield

$$\mu' = \frac{(m + 2)}{3m} \mu \left[ 3 - \sqrt{\frac{3}{2} \frac{2m + 1}{m + 2}} \frac{\lambda \nu^2}{\mu} - \frac{3\gamma \Omega_m}{2(1 - \gamma)\mu^2} - \frac{3\gamma}{1 + \gamma} + \frac{1}{2(m + 2)} \right] \times \{9 + (2m + 1)^2(\mu^2 - \nu^2)\}, $$

(39)
\[ \nu' = -\frac{(m+2)}{3m} \nu \left[ \sqrt{6}\mu\lambda - \frac{1}{2(m+2)} \left( 9 + (2m+1)^2 (\mu^2 - \nu^2) \right) \right]. \quad (40) \]

For \( P_1 = \left( \frac{\sqrt{6}\lambda + \sqrt{6}\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2}{(2m+1)^2/2(m+2)}, 0 \right) \), the corresponding eigenvalues are

\[ \eta_1 = \frac{(m+2)}{3m} \left[ 3 - \left\{ \frac{(2m+1)^4}{4(m+2)^2} \left\{ \sqrt{6}\lambda + \sqrt{6}\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2 \right\} \right. \\
- \frac{2m+1}{3} \left\{ \frac{3\gamma}{2(1-\gamma)} - \frac{3\gamma}{1+\gamma} + \frac{1}{2(m+2)} \left( 9 + 12(m+2)^2 \right. \right. \right. \\
- \frac{\sqrt{6}\lambda + \sqrt{6}\lambda^2 \left( \frac{3(2m+1)}{m+2} \right)^2}{2m+1} \left\} \right\} \right], \quad (41) \]

\[ \eta_2 = -\frac{(m+2)}{3m} \left[ 2\sqrt{6}(m+2)\lambda \left\{ \frac{6\lambda + \sqrt{6}\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2}{(2m+1)^2} \right\} \right. \right. \\
- \frac{1}{2m+1} \left\{ 9 + 4(m+2)^2 \left\{ \frac{6\lambda + \sqrt{6}\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2}{(2m+1)} \right\} \right\}. \quad (42) \]

In this case, the nature of eigenvalues indicates unstable nodes for \( m > 0 \) with all choices of \( \gamma \) except for \( \gamma = 1, -1 \) (Figure 3). We find both eigenvalues negative for \( m = -0.2 \) showing stable attractors. All the choices of parameters \( m \) and \( \gamma \) show decelerated expanding universe as \( q > 0 \). The summary of respective results is shown in Table 3.

For \( P_2 = \left( \frac{\sqrt{6}\lambda + \sqrt{6}\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2}{(2m+1)^2/2(m+2)}, 0 \right) \), the corresponding eigenvalues are
Figure 3: Plots for the phase plane evolution of phantom coupled universe model with \( Q = \gamma (\dot{\sigma}_m + \dot{\sigma}_\phi) \) and \( \lambda = 2 \).
Table 3: Stability Analysis for the Phantom Coupled System with $Q = \gamma(\dot{\sigma}_m + \dot{\sigma}_\phi)$.

| Ranges of $\gamma$ and $m$ for Critical Points | Stability | Acceleration |
|-----------------------------------------------|-----------|--------------|
| $P_1$                                         |           |              |
| $\gamma > 0$, $m > 0$, $\gamma \neq 1$        | Unstable  | No           |
| $\gamma < 0$, $m > 0$, $\gamma \neq -1$       | Unstable  | No           |
| $\gamma < 0$, $m < 0$, $m \neq -0.5, -2$      | Stable for $m = -0.2$ | No |
| $\gamma > 0$, $m < 0$                         | Stable for $m = -0.2$ | No |
| $P_2$                                         |           |              |
| $\gamma > 0$, $m > 0$, $\gamma \neq 1$        | Unstable  | No           |
| $\gamma < 0$, $m > 0$, $\gamma \neq -1$       | Unstable/Saddle | No |
| $\gamma < 0$, $m < 0$, $m \neq -0.5, -2$      | Saddle    | No           |
| $\gamma > 0$, $m < 0$                         | Saddle    | No           |
| $P_3$                                         |           |              |
| $\gamma > 0$, $m > 0$, $\gamma \neq 1$        | Unstable/Saddle | No |
| $\gamma < 0$, $m > 0$, $\gamma \neq -1$       | Unstable/Saddle | No |
| $\gamma < 0$, $m < 0$, $m \neq -2$            | Stable    | Yes          |
| $\gamma > 0$, $m < 0$                         | Stable    | Yes          |

given as

\[
\eta_1 = \frac{(m+2)}{3m} \left[ \frac{3}{2(1-\gamma)} - \frac{3\gamma}{1+\gamma} + \frac{1}{2(m+2)} \left( 9 + 12(m+2)^2 \right) \right]
- \frac{2m+1}{3} \left[ 2(1-\gamma) - \frac{3\gamma}{1+\gamma} + \frac{1}{2(m+2)} \left( 9 + 12(m+2)^2 \right) \right],
\]

\[
\eta_2 = -\frac{(m+2)}{3m} \left[ 2\sqrt{6}(m+2)\lambda \left\{ \frac{6\lambda - \sqrt{6\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2}}{(2m+1)^2} \right\} \right]
\]

$\lambda$
\[ - \frac{1}{2m+1} \left\{ 9 + 4(m+2)^2 \left\{ \frac{6\lambda - \sqrt{6\lambda^2 - \left( \frac{3(2m+1)}{m+2} \right)^2}}{2m+1} \right\}^2 \right\}, \]

which corresponds to either unstable or saddle node that lies in matter dominated era for all choices of different parameters. For \( P_3 = \left( \frac{\xi_1}{\sqrt{2}}, 0 \right) \), the eigenvalues are

\[ \eta_1 = \frac{(m+2)}{3m} \left[ 3 - \frac{3\gamma}{2(1-\gamma)} \left\{ \frac{2}{\xi_1^2} - \frac{2m+1}{3} \right\} - \frac{3\gamma}{1+\gamma} \right] + \frac{1}{2(m+2)} \left\{ 9 + \frac{3(2m+1)^2\xi_1^2}{2} \right\}, \]

\[ \eta_2 = -\frac{(m+2)}{3m} \left[ \sqrt{3\lambda\xi_1} - \frac{1}{2(m+2)} \left( 9 + \frac{(2m+1)^2\xi_1^2}{2} \right) \right], \]

where

\[ \tilde{\xi}_1 = \sqrt{\frac{3(2m+3) + \gamma(2m^2 - m - 10) + \gamma^2(2m^2 - 5m - 25)}{(2m+1)^2(\gamma^2 - 1)}} + \tilde{\xi}_2, \]

\[ \tilde{\xi}_2 = \sqrt{\frac{-12\gamma(\gamma + 1)(\gamma^2 - 1)(m + 2)(2m+1)^2 + 39 - 5\gamma(5\gamma + 2) + \tilde{\xi}_3^2}{2}} \]

\[ \tilde{\xi}_3 = 2m^2\gamma(\gamma + 1) + m(5\gamma^2 - \gamma + 6)^2. \]

The nature of eigenvalues as well as trajectories show that point \( P_3 \) is unstable past attractor in deceleration region for \( m > 0 \) with \( \gamma \neq 1, -1 \). This point becomes a stable global attractor for negative values of \( m \) except for \( m = -0.5, -2 \) that give undetermined eigenvalues. In this case, \( q < 0 \) and \( \omega_\phi < -1 \) showing accelerated expansion of the universe.

## 4 Coupled Tachyon Dynamics

Now we discuss phase space analysis of the universe model by taking a tachyon coupled cosmic component. The conservation equations are

\[ \dot{\sigma}_m + 3(\sigma_m + p_m)H = Q, \]
\[ \dot{\sigma}_\phi + 3(\sigma_\phi + p_\phi)H = -Q, \]  

(48)

where \( \sigma_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \) and \( p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2} \). The evolution equations yield

\[ H^2 = \frac{(m + 2)^2}{9(2m + 1)} \left[ \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + \sigma_m \right], \]  

(49)

\[ \frac{\ddot{\phi}}{1 - \dot{\phi}^2} = -\left[ 3\dot{H} + \frac{V'(\phi)}{V(\phi)} + \frac{Q}{V(\phi)} \sqrt{1 - \dot{\phi}^2} \right]. \]  

(50)

We introduce the following dimensionless parameters

\[ \mu = \frac{(m + 2)\dot{\phi}}{2m + 1}, \quad \nu = \frac{(m + 2)\sqrt{V}}{\sqrt{3(2m + 1)H}}, \quad \lambda = -\frac{V'}{V \sqrt{V}}, \]  

(51)

such that the autonomous system of equations takes the form

\[ \mu' = \frac{(m + 2)^2}{3m(2m + 1)} \frac{\ddot{\phi}_\mu}{H\phi}, \]  

\[ \nu' = \frac{(2m + 1)\lambda \mu^2}{2\sqrt{3m(m + 2)}} - \frac{(m + 2)\nu \dot{H}}{3m H^2}. \]  

(53)

We take inverse square potential with constant parameter \( \lambda \). In this case, we consider the only coupling \( Q = \beta \dot{\sigma}_\phi \) for which Eqs.\((49)\) and \((50)\) give

\[ \frac{\dot{H}}{H^2} = \frac{(2m + 1)^2 \nu^2}{2(m + 2)} \sqrt{1 - \left( \frac{2m + 1}{m + 2} \right)^2 \mu^2 - \frac{9}{2(m + 2)}}, \]  

(54)

\[ \frac{\ddot{\phi}}{\phi H} = \left[ 1 - \left( \frac{2m + 1}{m + 2} \right)^2 \mu^2 \right] \left[ \frac{\sqrt{3} \nu \lambda}{\mu} + \frac{3\beta}{1 + \beta} - 3 \right]. \]  

(55)

The effective EoS and deceleration parameters are given by

\[ w_{eff} = -1 - \frac{1}{3(m + 2)} \left[ (2m + 1)^2 \nu^2 \sqrt{1 - \left( \frac{2m + 1}{m + 2} \right)^2 \mu^2 - 9} \right], \]  

(56)
Figure 4: Plots for the phase plane evolution of tachyon coupled universe model with $Q = \beta \dot{\sigma} \phi$ and $\lambda = 2$. 
\[ q = -1 - \frac{1}{2(m+2)} \left[ (2m+1)^2 \nu^2 \sqrt{1 - \left( \frac{2m+1}{m+2} \right)^2 \mu^2 - 9} \right]. \tag{57} \]

The critical points and their corresponding eigenvalues for tachyon coupled field are given as follows. For \( P_1 = (0,0) \), we have

\[ \eta_1 = -\frac{(m+2)^2}{m(2m+1)(1+\beta)}, \quad \eta_2 = \frac{3}{2m}. \tag{58} \]

In case of tachyon coupled field, source and sink can be observed according to the sign of eigenvalues. We investigate stability of critical points corresponding to different values of \( m \) and other parameters. The cosmic portrait includes a matter dominated epoch ultimately followed by a late accelerated expansion phase. We find that point \( P_1 \) is saddle/unstable node for positive values of \( m \) (Figure 4). This point becomes global stable node for \( m < 0 \) \((m \neq -0.5, \beta \neq -1\) showing accelerated expansion of the universe model as \( q < 0 \). The summary of the obtained results is given in Table 4.

### Table 4: Stability Analysis for the Tachyon Coupled System with \( Q = \beta \dot{\sigma}_\phi \).

| Ranges of \( \beta \) and \( m \) for Critical Points | Stability | Acceleration |
|-----------------------------------------------------|-----------|-------------|
| \( P_1 \)                |           |             |
| \( \beta > 0, \ m > 0, \ \beta \neq -1 \)        | Saddle    | Yes         |
| \( \beta < 0, \ m > 0 \)        | Unstable  | Yes         |
| \( \beta < 0, \ m < 0, \ m \neq -0.5 \)          | Stable    | Yes         |
| \( \beta > 0, \ m < 0 \)        | Stable    | Yes         |
| \( P_2 \)                |           |             |
| \( \beta > 0, \ m > 0 \)        | Unstable  | Yes         |
| \( \beta < 0, \ m > 0, \ \beta \neq -1 \)        | Unstable  | Yes         |
| \( \beta < 0, \ m < 0, \ m \neq -0.5 \)          | Saddle    | Yes         |
| \( \beta > 0, \ m < 0 \)        | Saddle    | Yes         |
| \( P_3 \)                |           |             |
| \( \beta > 0, \ m > 0 \)        | Stable    | Yes         |
| \( \beta < 0, \ m > 0, \ \beta \neq -1 \)        | Stable    | Yes         |
| \( \beta < 0, \ m < 0, \ m \neq -0.5 \)          | Saddle/Unstable | Yes         |
| \( \beta > 0, \ m < 0 \)        | Saddle/Unstable | Yes         |
For $P_2 = (\pm \frac{m+2}{2m+1}, 0)$, the eigenvalues become
\[ \eta_1 = \frac{2(m+2)^2}{m(2m+1)(1+\beta)}, \quad \eta_2 = \frac{3}{2m}, \] (59)
which correspond to unstable nodes for positive values of parameter $m$ lying in accelerated expanding phase of cosmos. For $m < 0$, we have stable global attractors undergoing accelerated expansion of the universe. When $P_3 = (0, \pm \frac{3}{2m+1})$, the eigenvalues are given by
\[ \eta_1 = -\frac{(m+2)^2}{m(2m+1)(1+\beta)}, \quad \eta_2 = -\frac{3}{m}. \] (60)
In this case, the nature of eigenvalues indicate stable future attractor for $\beta > 0$ and $m > 0$ which undergoes accelerated expansion of the universe as $q < 0$. For $\beta < 0$ and $m > 0$, the point $P_3$ is stable except for $\beta = -1$. In this case, $q = -1$ and $w_{eff} = -1$ which indicate de Sitter phase of the universe. It is found that the respective point corresponds to saddle/unstable nodes for $\beta < 0$, $m < 0$ ($m \neq -0.5$) indicating accelerated expansion ($q < 0$). For $\beta > 0$, $m < 0$, it also shows saddle/unstable node which corresponds to de Sitter ($q = -1, w_{eff} = -1$) phase of cosmos.

5 Power-Law Scale Factor

In this section, we discuss the power-law behavior of the scale factor by applying some assumptions corresponding to both phantom as well as tachyon coupled fields. In this context, we integrate Eq. (12) which leads to
\[ \dot{\Theta} = -\frac{1}{6(m+2)}[9 + (2m+1)^2(\mu^2 - \nu^2)]\Theta^2, \] (61)
where $\Theta = 3H$ is the expansion scalar. For $\Theta \neq 0$, we determine power-law scale factor whenever $9 + (2m+1)^2(\mu^2 - \nu^2) \neq 0$. We find the corresponding generic critical point by solving $\dot{\Theta} = \frac{\dot{a}}{a} + \frac{2\dot{b}}{b}$ for $a(t)$ and $b(t)$ as
\[ b^{m+2} = b_0^{m+2}(t - t_0)^{\frac{6(m+2)}{9 + (2m+1)^2(\mu^2 - \nu^2)}}. \] (62)
It is noticed that behavior of the term “$9 + (2m+1)^2(\mu^2 - \nu^2)$” is quite important to assess different cosmological phases. If $9 + (2m+1)^2(\mu^2 -$
\( \nu^2 \) = 0, it gives exponential expansion of the cosmological model. Also, \( 9 + (2m+1)^2(\mu^2 - \nu^2) \gtrless 0 \) corresponds to accelerated expansion or contraction of the universe, respectively. Figure 5 shows different cosmological phases for power-law scale factor, where blue and gray regions correspond to contraction and accelerated expansion of the universe model, respectively. It is found that the region for decelerated expansion tends to increase by increasing \( m \). For \( m < 0 \), there exists gray region only which shows that the universe model undergoes accelerated expansion.

In case of tachyon coupled fluid, Eq.(54) yields

\[
\dot{\Theta} = -\frac{1}{6(m+2)} \left[ (2m+1)^2\nu^2 \sqrt{1 - \left( \frac{2m+1}{m+2} \right)^2 \mu^2 - \frac{9}{2(m+2)}} \right] \Theta^2. \quad (63)
\]

For \( \Theta \neq 0 \), we again evaluate power-law scale factor if \( (2m+1)^2\nu^2 \sqrt{1 - \left( \frac{2m+1}{m+2} \right)^2 \mu^2 - \frac{9}{2(m+2)}} \neq 0 \). The generic critical point is found by solving \( \Theta = \frac{\dot{a}}{a} + \frac{2b}{b} \) as

\[
b^{(m+2)} = b_0^{(m+2)} \left[ (2m+1)^2 \nu^2 \sqrt{1 - \left( \frac{2m+1}{m+2} \right)^2 \mu^2 - \frac{9}{2(m+2)}} \right]. \quad (64)
\]

We explore different cosmological phases according to \( (2m+1)^2\nu^2 \sqrt{1 - \left( \frac{2m+1}{m+2} \right)^2 \mu^2 - \frac{9}{2(m+2)} \gtrless 0 \). In contrast to the phantom coupled matter, we find different results for tachyon coupled field. We observe that the region for decelerated expansion decreases by increasing \( m \) while \( m < 0 \) shows contraction region only which means that the universe model undergoes decelerated expansion for negative values of \( m \) (Figure 6).

6 Summary

This work is devoted to discuss phase space analysis of LRS BI universe model by taking a coupling between scalar field models and DM. An autonomous system of equations has been developed by defining normalized dimensionless variables which plays a remarkable role to study the stability of dynamical system. We have evaluated the corresponding critical points for different values of the parameters. We have also calculated eigenvalues characterizing these critical points and investigated the impact of \( m \) on their stability in
Figure 5: Plots of qualitative phase space analysis for power-law scale factor with phantom coupled matter. Blue and gray regions indicate contraction and accelerated expansion of the universe model, respectively.
Figure 6: Plots of qualitative phase space analysis for power-law scale factor with tachyon coupled matter.
the presence of phantom and tachyon fields. We summarize our results as follows.

Firstly, we have discussed stability of the universe dominated by the coupling of phantom energy and DM through their eigenvalues corresponding to different values of $m$ (Figures 1-3). We have considered three different linear combinations for the coupling constant. For $Q = \alpha \dot{\sigma}_m$, we have found an unstable matter dominated state undergoing decelerated expansion for all points with $m > 0$ and various choices of $\alpha$ (Figures 1). The dynamical analysis shows a matter dominated era ultimately followed by a late accelerated expansion phase of the universe. For $m < 0$, all the points become stable future attractor undergoing accelerated expansion as $q < 0$ except the point $P_1$ which lies in decelerated expanding region. In this case, the results for points $P_2$ and $P_3$ do not solve the coincidence problem which is well consistent with the results for FRW universe model [22, 36].

For $Q = \beta \dot{\sigma}_\phi$, all the eigenvalues and trajectories show unstable nodes for positive values of $m$ which become stable for $m < 0$ corresponding to different choices of $\alpha$ (Figure 2). These points lie in non-accelerating phase of the universe as $q > 0$ for all choices of parameters $m$ and $\beta$ which may alleviate coincidence problem as compared to [22]. For the coupling $Q = \gamma (\dot{\sigma}_m + \dot{\sigma}_\phi)$, we have found unstable past attractor for positive values of $m$. When $m < 0$, stable node is observed for point $P_1$ while point $P_2$ corresponds to saddle node undergoing decelerated expansion (Figure 3). In this case, point $P_3$ shows stable future attractor in accelerating phase as compared to FRW universe model [22]. It is worth mentioning here that all the critical points for the couplings $Q = \beta \dot{\sigma}_\phi$ and $Q = \gamma (\dot{\sigma}_m + \dot{\sigma}_\phi)$ indicate decelerated expanding universe except point $P_3$.

Secondly, we have studied stability of the universe model by taking interaction between tachyon field and DM. In this case, we consider $Q = \beta \dot{\sigma}_\phi$ only. The cosmic portrait shows a matter dominated epoch ultimately followed by a late accelerated expansion phase (Figure 4). For $m > 0$, we have found unstable/saddle node for points $P_1$ and $P_2$ while point $P_3$ gives stable future attractor which undergoes an accelerated expansion. For $m < 0$, the point $P_1$ corresponds to stable node showing accelerated expansion of the universe while the remaining points give saddle/unstable node that corresponds to de Sitter phase of the universe. We note that all the points show accelerated expansion of the universe for tachyon coupled field. We conclude that negative values of $m$ enhance stability of the universe model as compared to its positive values.
Finally, we have studied the behavior of power-law scale factor corresponding to different values of $m$. The power-law scale factor indicates various phases of evolution (accelerated or exponential expansion) for the respective universe model as shown in Figures 5 and 6. For phantom coupled matter, it is found that the region for decelerated expansion gets larger by increasing $m$ while $m < 0$ corresponds to accelerated expansion of cosmos. In case of tachyon coupled field, the contraction region decreases by increasing $m$ while the gray region becomes larger. Also, $m < 0$ shows only blue region which corresponds to the decelerated expanding universe model.

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