Weak gravitational lensing has great potential to probe dark matter, neutrinos, dark energy, and gravity at cosmological scales (Refregier 2003; Albrecht et al. 2006; Hoekstra & Jain 2008; Munshi et al. 2008; Weinberg et al. 2013). All these great applications rely on accurate weak lensing measurements. Cosmic shear lensing induced coherent distortion in galaxy shapes can achieve a sub-percent statistical error in weak lensing measurements. It is therefore a major driver of the science for massive cosmological surveys such as the DES, Euclid, HSC, LSST, and SKA radio surveys. However, cosmic shear suffers from a variety of systematic errors such as photometric redshift errors and galaxy intrinsic alignment (for reviews, refer to LSST Science Collaboration et al. 2009; LSST Dark Energy Science Collaboration 2012; Troxel & Ishak 2014). Tremendous efforts have been put to understand and correct these systematic errors.

One systematic error that has received intensive scrutiny is shear estimation error. It is often conveniently decomposed into a multiplicative error and an additive error (Heymans et al. 2006). Stage IV projects such as Euclid and LSST put a stringent requirement of controlling multiplicative error to \(\sim 0.2\%-0.5\%\) (Huterer et al. 2006; Cropper et al. 2013; Massey et al. 2013). A series of blind community challenges of massive scales have been carried out over the last decade (STEP1: Heymans et al. 2006; STEP2: Massey et al. 2007; GREAT08: Bridle et al. 2010; GREAT10: Kitcing et al. 2012; GREAT3: Mandelbaum et al. 2014). The latest GREAT3 challenge shows that control over multiplicative error/bias has been significantly improved. For mock catalogs, various shear estimation methods can control multiplicative error to 1%, or even 0.1% when a point-spread function (PSF) is given (Mandelbaum et al. 2014). Further improvement may still be expected by refining existing shear estimation methods or from emerging new methods such as the recently proposed Fourier-space method (Zhang 2008, 2010, 2011; Zhang & Komatsu 2011; Zhang et al. 2013).

Nevertheless, the performance of shear estimation methods depends on many factors such as galaxy size, flux signal-to-noise ratio (S/N), morphology, selection criteria, weighting scheme, and the accuracy of PSF interpolation (e.g., GREAT3: Mandelbaum et al. 2014). Given the unprecedented variety of galaxies at \(z \sim 0-4\) of stage IV surveys, one must keep caution on whether these shear estimation methods can achieve the accuracy estimated from simulated lensed galaxies. It would then be safer to design and apply independent diagnostics of multiplicative bias based on real data in a model-independent manner. If multiplicative bias is detected by such diagnostics, it can then be consequently calibrated. Vallinotto et al. (2011) proposed to calibrate the multiplicative error against the lensing magnification in galaxy flux and size, and demonstrated the potential of such diagnostics. In this paper, we propose an alternative method, that is, calibrating multiplicative error by shear magnification of Type Ia supernovae (SNe Ia).

Supernova (SN) flux is magnified by gravitational lensing by a factor \(\mu = 1/(1 - \kappa) - \gamma^2\). Here, \(\mu, \kappa, \gamma\) are the lensing magnification/amplification, convergence, and shear, respectively. \(\gamma^2 = \gamma_\ell^2 + \gamma_s^2\), where \(\gamma_\ell, \gamma_s\) are the intrinsic shear on elliptical and circular galaxies, respectively. In the weak lensing regime, the measured flux fluctuation of SNe Ia (after standardization) is \(\delta_F = \mu - 1 + \delta_F^{\text{int}}\). Here \(\delta_F^{\text{int}}\) is the intrinsic flux fluctuation of SNe Ia. On one hand, this lensing magnification contaminates the Hubble diagram and degrades cosmological constraints from SNe Ia distance–redshift measurement (Kantowski et al. 1995; Frieman 1996; Holz 1998; Dalal et al. 2003; Cooray et al. 2006b). On the other hand, it provides an independent measure of weak lensing through the lensing induced flux fluctuation (Metcalf 1999; Hamana & Futamase 2000; Ménard & Dalal 2005; Cooray et al. 2006a; Dodelson & Vallinotto 2006; Zentner & Bhattacharya 2009; Amendola et al. 2014; Ben-Dayan & Kalaydzhyan 2014;
Fedeli & Moscardini 2014), and is therefore a useful source of information. Existing data already allows for marginal detection of lensing magnification in SN flux (Kronborg et al. 2010; Betoule et al. 2014; Castro & Quintin 2014). With orders of magnitude of more $z \sim 1$ SNe Ia expected in future surveys, precision lensing measurements through SN magnification is very promising.

Lensing measured in this way is free of the multiplicative error troubling cosmic shear measurements. The measured cosmic shear $\gamma_{\nu \nu} = 1,2$ can be conveniently parametrized as (Heymans et al. 2006)

$$\gamma_{\nu \nu}^{\text{obs}} = (1 + m)\gamma_i + c_i + \gamma_{\nu \nu}^{\text{int}},$$

(1)

with an extra term $\gamma_{\nu \nu}^{\text{int}}$ arising from the intrinsic galaxy shape noise. For the moment we approximate the measured $\gamma/(1 - \kappa)$ (reduced shear) as shear $\gamma$. The neglected complexity will be discussed in Section 4. $\gamma_{\nu \nu}^{\text{int}}$ in general has a dominant component of no spatial correlation, and a spatially correlated component (galaxy intrinsic alignment), $m_i$ is the multiplicative error/bias and $c_i$ is the additive error. The two $m_i$ can differ form each other (e.g., Mandelbaum et al. 2014).

For brevity, we will work with $m$ defined with respect to cosmic shear E-mode ($\kappa$).

The two data sets (cosmic shear and SNe Ia magnification) both measure the same weak lensing, but with different prefactors (e.g., $1 + m$). Therefore combining the two data sets we can measure $m$ without assumptions on the true lensing signal. Furthermore, if the two data sets are located in the same cosmic volume, cosmic variance of the weak lensing field will be eliminated and will not degrade the constraint on $m$.

The major obstacle in this approach is the low number density of SNe Ia and therefore heavy shot noise. Later in this paper we will show that at least half a million SNe Ia are required to diagnose $|m| \lesssim 1\%$, the minimum requirement for stage IV weak lensing surveys (Huterer et al. 2006). Surveys of a million SNe Ia with well measured light curves are highly ambitious. Nevertheless, surveys of such scale can be accessible by telescopes such as the LSST telescope and have been proposed (Zhan et al. 2008; LSST Dark Energy Science Collaboration 2012). Cosmological benefits of such surveys will be many-folds, besides the luminosity distance measurement and peculiar velocity measurement. The large scale structure of these SNe Ia allows for measurement of baryon acoustic oscillation, which can significantly improve cosmological constraints from weak lensing alone (Zhan et al. 2008).

The proposed diagnosis of $m$ is a new bonus of such SN survey.

To quantify the capability of diagnosing and calibrating multiplicative bias with SN magnification, we adopt the baseline survey of SNe Ia as the D2k survey proposed in Zhan et al. (2008). This proposed five year survey over 2000 deg$^2$ by the LSST telescope will result in about 2 million SNe Ia with well measured light curves. Among them, 0.7 million locate at $0.8 < z < 1.2$, one of the primary target redshift bins for precision weak lensing measurement. For LSST cosmic shear, we assume a total of 3 billion galaxies over 20,000 deg$^2$, with a normalized redshift distribution $n_g(z) = z^2 \exp(-z/z_*)/(2z_*^3)$ (Huterer et al. 2006; Zhan & Knox 2006) and $z_* = 0.4$. The median redshift is 2.675 which is 1.07. The forecasted constraint on multiplicative error calibration is sensitive to SN survey parameters, but is very insensitive to cosmic shear survey parameters.

This paper is organized as follows. We discuss two implementations (M1 and M2) of diagnosing and calibrating multiplicative error combining cosmic shear and SN magnification in Sections 2 and 3, respectively. We discuss and conclude in Section 4. Some technical details of calculation are presented in the appendix.

2. METHOD ONE

Method one only uses the two measurements, namely cosmic shear and SN magnification, to calibrate multiplicative error. For theoretical estimation of the expected S/N, it is much more convenient to work in Fourier space than in real space. The observable will be $\delta_\ell (\ell)$ and $\gamma_{\nu \nu}^{\text{obs}}(\ell)$, $\ell$ is an independent multipole mode. Since we are only able to measure lensing magnification of SNe Ia at $\ell \lesssim 1000$ (Figure 1), we can treat the lensing field as Gaussian. The corresponding Fisher matrix is (Tegmark 1997)

$$F_{\alpha \beta} = \sum_{\ell} \frac{1}{2} \text{Tr} \left[ C^{-1}(\ell) C_{\alpha \beta}(\ell) C^{-1}(\ell) C_{\beta \alpha}(\ell) \right]$$

(2)

Here, $\gamma_{\alpha \beta}(\ell) \equiv \partial / \partial \lambda_{\alpha \beta}(\ell)$ and $\lambda_{\alpha \beta}(\ell)$ is the $\alpha (\beta)$ th parameter to be constrained. $C(\ell)$ is the covariance matrix for the given $\ell$ mode,

$$C_{\mu \nu} = \left( \begin{array}{cc} C^\mu(\ell) + N^\mu & \gamma(\ell) \gamma^\dagger(\ell) \\ (1 + m) C^\gamma(\ell) & (1 + m)^2 C^\gamma(\ell) + N^\gamma \end{array} \right).$$

Here $C^\mu$, $C^\gamma$, and $C^\gamma$ are the angular power spectra of $\mu$, $\gamma$ (E-mode), and their cross power spectrum, respectively. $N^\ell = 4\pi f_{\text{sky}} T_{\text{SN}}^2 / N_{\text{SN}}$ is the noise power spectrum in SNe Ia magnification measurement. $f_{\text{sky}}$ is the fractional sky coverage of overlapping SN survey and cosmic shear survey. For the D2k survey, $f_{\text{sky}} = 2000 / (4 \times 180^2 / \pi) = 4.8\%$. Figure 1. Lensing power spectrum (solid line) and corresponding noises (dash lines). The source redshift is $0.8 < z < 1.2$. Shot noises, from large to small, are $N^\mu$, $N^\gamma$ in SN magnification, $N^\gamma$ in cosmic shear, and $N^\gamma$ in the weighted galaxy distribution, respectively. Forecast of shot noise targets at the proposed D2k survey of 0.72 million SNe Ia (Zhan et al. 2008) with the LSST telescope.
\[ \sigma_F \equiv \sqrt{\langle \sigma_F^2 \rangle} \] is the rms dispersion of flux of standardized SNe Ia. \( N_{SN} \) is the total number of SNe Ia in the given sky area and in the given redshift bin. \( N\gamma = 4\pi f_{sky} \sigma_\gamma^2 / N_e \) is the noise power spectrum in cosmic shear measurements, \( \sigma_\gamma \) is the rms ellipticity, and \( N_e \) is the total number of galaxies in the given sky area and redshift bin for cosmic shear measurements. We will take the approximation \( \mu \approx 1 + 2\kappa \). On the other hand, the E-mode shear \( \gamma_E = \kappa \). Therefore \( C'' = 4C\gamma \) and \( C^{(\gamma)} = 2C\gamma \).

In numerical evaluation of \( \sigma_m \) throughout the paper, we adopt \( \sigma_F = 0.1 \) and \( \sigma_\gamma = 0.3 \). \( \sigma_F \) quoted here is solely the intrinsic scatter \( \sigma_{F,\text{int}} \). In reality, it should include that induced by photo-z error. Photo-z error of \( \sigma_\gamma \) increases SN scatter to \( \sigma_{F,\text{int}} \approx \sigma_F^2 [1 + a(\sigma_F / \sigma_{\text{int}})^2] \), with \( a = 2(d\ln D_L(z)/(dz))^2 = 2.5 \) at \( z = 1 \). For \( \sigma_{F,\text{int}} = 0.1 \) and \( \sigma_\gamma = 0.01(1 + z) \) (Zhan et al. 2008), we have \( \sigma_\gamma \approx 1.1\sigma_{F,\text{int}} = 0.11 \). We then conclude that including photo-z error does not significantly change our forecast. We focus on redshift bin \( 0.8 < z < 1.2 \), in which \( 0.72 \times 10^{9} \) SNe Ia can be observed by the proposed D2k survey. Figure 1 plots \( C\gamma \), \( N\gamma /4 \) and \( N\gamma \) for \( 0.8 < z < 1.2 \). The lensing power spectrum is calculated using the Limber integral, in which the nonlinear matter power spectrum is calculated using the halofit model (Smith et al. 2003). Due to sparse SN samples, each single lensing multipole mode is overwhelmed by shot noise at \( \ell > 80 \). However, with \( 2\Delta\ell f_{sky} \) modes for each bin of width \( \Delta\ell \), we can beat down shot noise by a factor of \( \sqrt{2\Delta\ell f_{sky}} \). Therefore we can still measure the lensing power spectrum through SN magnification with \( S/N \geq 3 \) at \( \ell \sim 1000 \) for \( \Delta\ell /\ell = 0.1 \).

In the Fisher matrix analysis, we combine all \( n \) independent \( \ell \) modes, which we label as \( \ell_i \) (\( i = 1, \ldots, n \)). We take the unknown parameters to be \( \lambda \equiv (\lambda_0, \lambda_1, \ldots, \lambda_n) = (m, C\gamma (\ell_1), \ldots, C\gamma (\ell_n)) \) with \( \lambda_0 = m \). The Fisher matrix \( F_{\lambda,\lambda} \) is calculated and inverted in the appendix. We find that the error in \( m \) is

\[ \sigma_m \approx \left( \frac{2\ell_d f_{sky}}{C\gamma (N\gamma /4 + N\gamma ) + N\gamma N\gamma /4} \right)^{1/2} \].

Immediately we find a fundamental lower limit for the \( m \) calibration,

\[ \sigma_m > \sigma_{m,\text{min}} = \frac{\sigma_F}{2\sigma_\gamma} N_{\text{m,SN}}^{-1/2} \approx 5 \times 10^{-3} \left( \frac{\sigma_F}{0.1} \right) \left( \frac{0.01}{\sigma_\gamma} \right) \left( \frac{N_{\text{m,SN}}}{10^6} \right)^{-1/2} \).

Notice that \( \sigma_\gamma^2 \equiv \langle \kappa^2 \rangle = \int (\ell^2 C\gamma (\ell)/(2\pi)) d\ell /\ell \). This fundamental lower limit corresponds to the limit that all other sources of statistical errors vanish and the only one left is shot noise in SN magnification.

This limit can only be achieved under the condition \( N\gamma \ll N\mu \) and \( N\gamma \ll \ell C\gamma \). The first condition is usually satisfied (Figure 1) since the galaxy population is much denser than the SN population. For example, the number density of cosmic shear galaxies in a LSST-like survey at \( 0.8 < z < 1.2 \) is 600 times higher than that of SNe Ia even for an ambitious D2k survey, resulting in \( N\gamma \sim 0.1N\mu/4 \). It is for this reason that the constraint on the multiplicative error \( m \) is limited by SN survey configurations.

On the other hand, the second condition \( N\gamma \ll \ell C\gamma \) is violated at \( \ell \gtrsim 500 \) (Figure 1), reflecting non-negligible shot noise per multipole mode in cosmic shear measurement. Therefore in reality we are not to reach the limit \( \sigma_{m,\text{min}} \).

Figure 2 shows \( \sigma_m \) as a function of \( N_{\text{m,SN}} \). We find that the actual \( \sigma_m \sim 3\sigma_{m,\text{min}} \) (Figure 2). Nevertheless, the calibration accuracy on \( m \) can reach \( \sigma_m = 8 \times 10^{-3} \) for \( N_{\text{m,SN}} = 0.72 \times 10^6 \) SNe Ia expected in the D2k survey. This constraint is close to the requirement of 0.5% in \( m \) for LSST (Huterer et al. 2006) and is therefore encouraging. Constraints of \( m \) for other redshift bins are shown in Table 1.

Are there possibilities to further improve constraint on \( m \)? Figure 3 shows \( (S/N)^2 \), the constraining power per logarithmic \( \ell \) bin defined through

\[ \left( \frac{S}{N} \right)^2 = \int \left( \frac{S}{N} \right)^2 d\ell /\ell \] for the method discussed in this section (M1), the constraining power peaks at \( \ell \sim 1000 \) (Figure 3). Contribution from \( \ell > 1000 \) is suppressed, since \( N\gamma \gg \ell C\gamma \) at \( \ell > 1000 \).
(Figure 1). Since most contribution comes from relatively large scale $\ell \lesssim 10^3$, the Gaussian approximation adopted through the Fisher matrix estimation is valid. However, Figure 1 shows that the lensing signal peaks at $\ell \sim 3000$, so there are rooms for further improvement. We present the second method (M2) to do so.

3. METHOD TWO

Method two combines the galaxy distribution available in the same survey with cosmic shear and SN magnification to improve the constraints on multiplicative error. For a survey like LSST, the highest S/N measurement is for galaxy clustering (Figure 1), the next is cosmic shear, and the lowest for SN magnification. Therefore, we can utilize the galaxy–SN magnification cross correlation and galaxy–cosmic shear cross correlation to improve the magnification and cosmic shear measurements. Combining the two cross correlations allows better determination of $m$.

The galaxy surface overdensity is

$$\delta \Sigma_g = \frac{\int_0^\infty \delta n_g(z) W_g(z)dz}{\int_0^\infty n_g(z)W_g(z)dz}. \quad (7)$$

$n_g(z)$ is the mean galaxy redshift distribution and the redshift integral is over the given redshift bin. Since galaxies in LSST have photo-$z$ information, we can apply a redshift dependent weighting $W_g(z)$ to improve the measurement accuracy of $m$.

We then have two measures of cross power spectra. $\hat{C}_{gg}$ is the measured SN magnification–galaxy overdensity cross power spectrum. $\hat{C}_{gg}$ is the measured cosmic shear–galaxy overdensity cross power spectrum. The “hat” on top of corresponding property (e.g., $\hat{C}$) denotes the measured quantity with measurement errors. We expect that $C_{gg} = (1 + m)C_{gg}/2$. Therefore, we can estimate $m$ combining the two measurements $\hat{C}_{gg}$ and $\hat{C}_{gg}$,

$$\hat{m} = 1 - \frac{\hat{C}_{gg}(\ell)/2}{\hat{C}_{gg}(\ell)}.$$

The requirement here is that the bin size $\Delta \ell$ is sufficiently large so the error in $\hat{C}_{gg}(\ell)$ is small ($\delta C_{gg} \ll C_{gg}$). The expectation value $\langle \hat{m} \rangle = m + O(m^2)$. Since $\left| m \right| \ll 1$, the above estimator is virtually free of systematic bias. When taking the ratio, cosmic variance in the galaxy–shear correlation cancels that in the galaxy–magnification correlation because the two share an identical cosmic volume and hence identical cosmic variance. Constraint on $m$ from a single $\ell$ bin, assuming Gaussianity, is

$$\sigma_m^2(\ell) = \frac{\langle C_g(\ell) + N_{gw} \rangle \langle N^g/4 + N^\gamma \rangle}{2\ell \Delta l_{sky} C_{gg}^2(\ell)/4} = \frac{1}{2\ell \Delta l_{sky} r^2} \left( \frac{C_g + N_{gw}}{C_g} \right) \left( \frac{N^g/4 + N^\gamma}{C_g/4} \right). \quad (9)$$

Here, $N_{gw} = \frac{4\pi\Delta l_{sky}}{N_g^W}$ is the shot noise in the weighted galaxy clustering. $N_g^W = N_{gw}/\langle W_g \rangle$ is the weighted galaxy number in the given cosmic volume, and $\langle W_g \rangle = \int n_g(z) W_g(z)dz/\int n_g(z)dz$. Finally we will combine all multipole bins to constraint $m$,

$$\sigma_m = \left[ \sum \sigma_m^2(\ell) \right]^{-1/2}. \quad (10)$$

In Equation (9), $r \equiv \sqrt{C_{gg}/C_g C_g}$ is the cross correlation coefficient between the weighted galaxy distribution and lensing. An important step to reduce the calibration error is to increase $r$. Due to the large amount of galaxies and strong clustering between them, $C_g \gg N_{gw}$ at $\ell \lesssim 10^3$. We then have the luxury to weigh these galaxies to increase $r$. Since we have (photometric) redshift information of the galaxies and an accurate measurement of galaxy bias, we can exert a nearly optimal weighting to galaxies such that their mean redshift distribution matches that of the lensing kernel. The weighting is

$$W_g(z) = \frac{W_L(z)H_0}{n_g(z)b_g(z)H(z)}. \quad (11)$$

Here, $W_L$ is the lensing kernel defined through

$$\kappa = \int \delta \mu W_L(z)\frac{dx}{c/H_0}. \quad (12)$$

$W_L(z) = \langle W_L(z, z_s) \rangle$ is the lensing kernel averaged over the source galaxy distribution. $W_L(z, z_s)$ is the lensing kernel of a single source redshift $z_s$ (e.g., Refregier 2003)

$$W_L(z, z_s) = \begin{cases} \frac{3}{2} \Omega_m (1+z) \frac{\chi(z)}{c/H_0} & \text{if } z < z_s, \\ 0 & \text{if } z \geq z_s. \end{cases}$$

After this weighting, we expect $r \approx 1$. Under this limit, the weighted galaxy angular power spectrum $C_g = C_{gg} = C_g$. The requirement to achieve $r = 1$ is negligible stochasticity in galaxy bias. Galaxy stochasticity will bring $r < 1$ and therefore

\begin{align*}
\text{Figure 3. Relative contribution per logarithmic $\ell$ bin for $0.8 < z < 1.2$ of the proposed D2k SN survey, for the two methods, respectively.}
\end{align*}
degrade the constraint on \( m \). Figure 3 shows that most constraining power comes from \( \ell \lesssim 2000 \), where stochasticity in galaxy distribution is insignificant. Therefore we only expect modest degradation in the constraint of \( m \) caused by stochasticity. Given complexities in modeling galaxy stochasticity, we will simply ignore it in this paper and only caution that \( \sigma_m \) of method two can be slightly underestimated.

Under the condition \( C^g \gg N^{ew} \), we can prove that
\[
\sigma_m = \sigma_{m,\text{min}}. \]
However, Figure 1 shows that even this condition breaks at \( \ell \gtrsim 1000 \), and even worse, \( C^g \lesssim N^{ew} \) at \( \ell \gtrsim 2000 \). Therefore the actual constraint on \( m \) is poorer (\( \sigma_m > \sigma_{m,\text{min}} \)). Numerical results shown in Figure 2 find \( \sigma_m \sim 1.5\sigma_{m,\text{min}} \) over a wide range \( 10^4 < N_{\text{SN}} < 10^6 \). Therefore, method two can deliver a factor of two better constraint on \( m \), with respect to method one (\( 3\sigma_{m,\text{min}} \sim 1.5\sigma_{m,\text{min}} \)). For \( 0.8 < z < 1.2 \), the constraining power peaks at \( \ell \sim 2000 \), so method two utilizes more lensing information than method one, whose constraining power peaks at \( \ell \sim 1000 \) (Figure 3). This explains why method two works better than method one.

With \( 0.72 \times 10^6 \) SNe Ia at \( 0.8 < z < 1.2 \) that the proposed D2k survey can measure, method two can achieve \( \sigma_m = 5 \times 10^{-3} \). This basically meets the requirement for LSST (Huterer et al. 2006). Applying method two to other redshift bins also turns out excellent constraints on \( m \) (0.4\%–0.9\%, Table 1). For this purpose, adding an SN survey like the proposed D2k to LSST would be highly beneficial.1

4. DISCUSSION

Method two is superior to method one in many aspects. The constraints on \( m \) of various redshift bins are shown in Table 1. \( \sigma_m \) of method two is usually a factor of 1.5–2 smaller than that of method one, showing that method two is superior in statistical error. Furthermore, method two is unbiased to the presence of galaxy intrinsic alignment and additive error in cosmic shear measurement. Galaxy intrinsic alignment has negligible contamination to the galaxy-lensing cross correlation measurement, since the weighted galaxy distribution has vanishing weighting within the source redshift bin. Additive error is expected to be uncorrelated with the large scale structure and therefore is not expected to bias the galaxy-lensing cross correlation measurement. For these reasons, the measured \( m \) using method two is insensitive to neither contamination of galaxy intrinsic alignment nor additive error in shear estimation.

In contrast, method one is susceptible to galaxy intrinsic alignment and additive error. The magnification auto power spectrum contributes little to constraining multiplicative error since it suffers from a much larger measurement error (e.g., Figure 1). Therefore, method one basically interprets the relative difference between the measured cosmic shear power spectrum and magnification–cosmic shear cross power spectrum as multiplicative error. However, galaxy intrinsic alignment and additive error contaminate the cosmic shear power spectrum in Equation (3), and therefore can cause fake diagnosis of multiplicative error.

This potential problem in method one can be rendered into a valuable measurement of galaxy intrinsic alignment/additive error, with the help of method two. Basically, method two determines \( m \) by the ratio of galaxy–cosmic shear cross correlation and galaxy–magnification cross correlation, free of intrinsic alignment/additive error. Method one measures the combination of \( m \) and intrinsic alignment/additive error through the ratio of magnification–cosmic shear cross correlation and cosmic shear auto correlation. With the measured \( m \) from method two, we can isolate the combined effect of galaxy intrinsic alignment and additive error. Therefore, by combining method one and method two one can, in principle, measure galaxy intrinsic alignment/additive error and multiplicative error simultaneously.

So far we have demonstrated the potential of SN magnification in calibrating multiplicative error in cosmic shear measurement. There are a number of caveats in the proposed calibration. One is the underlying assumption \( C' = 4C' \). In reality, \( \mu = 1 + 2\kappa + 3\kappa^2 + \gamma^2 \cdots \). For cosmic shear we actually measure the reduced shear \( g = \gamma/(1 - \kappa) = \gamma + \gamma\kappa + \cdots \). These high order terms lead to \( C' \neq 4C' \) and the induced difference in the two properties is on the order of \( \sigma^2 \sim 10^{-3} \). With the presence of these high order terms, measuring \( m \) will rely on modeling these terms and therefore rely on cosmology. These complexities can be incorporated by simultaneously fitting \( m \) and cosmological parameters, which determine these high order terms.

Another potential problem is dust extinction by intergalactic gray dust (Corasaniti 2006). Such extinction causes little reddening and therefore can not be efficiently corrected by conventional reddening recipes. The induced flux fluctuation is spatially correlated and therefore biases lensing measurements from SN magnification (Zhang & Corasaniti 2007). It is a potential problem for calibrating multiplicative error with SN magnification. It is also a potential problem for lensing measurements based on galaxy flux fluctuations (e.g., Schmidt et al. 2012) and multiplicative error calibration with galaxy flux fluctuations (Vallinotto et al. 2011). Fortunately, in principle, this problem can be alleviated. Gray dust extinction induces galaxy number density fluctuation, which differs from that induced by lensing (Yang et al. 2015). Therefore we can infer the dust extinction through galaxy clustering and eliminate it in SN magnification. Nevertheless, given large uncertainty in our understanding of intergalactic gray dust, it is an important open question to pay attention to.

Error in photometry calibration can also potentially cause problem. Its calibration error can be spatially correlated. It affects both method one and method two. For method one, it directly alters SN flux and biases the power spectrum measurement by lensing magnification. It also affects the number of galaxies in bins of observed magnitude. It then induces a correlation between the SN flux fluctuation and galaxy number overdensity. Thus it can bias the calibration of \( m \) using method two.2 The approach proposed in Vallinotto et al. (2011) by calibrating multiplicative error with the lensing induced size bias is free of both the gray dust extinction problem and the photometry calibration problem. So it provides an independent and highly complementary approach to diagnose and calibrate multiplicative error.

SNe Ia are highly complementary to other cosmological probes. They not only contribute as standard candles and valuable measures of weak lensing, but they have already

---

1 Euclid requires \(|m| \leq 0.2\% \) (Cropper et al. 2013; Massey et al. 2013). It would require a dedicated SN survey (if any), with twice as many SNe Ia as the D2k survey, to meet the Euclid requirement.

2 We thank the anonymous referee for pointing out this issue.
provided robust measurements of peculiar velocity at $z \lesssim 0.05$ (Bonvin et al. 2006; Haugboelle et al. 2007; Watkins & Feldman 2007; Dai et al. 2011) and will in the future even to $z \sim 0.5$ (Zhang & Chen 2008). With millions of SNe Ia, they can be used as tracers of large scale structures to measure baryon acoustic oscillation (Zhan et al. 2008). Our work adds a new application of SNe Ia and a new reason to include a survey of millions of SNe Ia by the LSST telescope or other weak lensing facilities.

I thank Jun Zhang and Hu Zhan for helpful discussions. This work was supported by the National Science Foundation of China (grant Nos. 11025316, 11320101002, and 11433001), the National Basic Research Program of China (973 Program 2015CB857001), the Strategic Priority Research Program “The Emergence of Cosmological Structures” of the Chinese Academy of Sciences (grant No. XDB09000000), and the Key Laboratory grant from the Office of Science and Technology, Shanghai Municipal Government (No. 11DZ2260700).

APPENDIX
DERIVING THE CONSTRAINT ON MULTIPlicative ERROR USING METHOD ONE

The Fisher matrix (Equation (2)) can be decomposed into four blocks,

$$ F = \begin{pmatrix} M & E \\ G & H \end{pmatrix}. $$

$$ M = \sum_{i=1}^{n} \text{Tr} \left[ C^{-1}(\ell_i)C_{0}(\ell_i)C^{-1}(\ell_i)C_{0}(\ell_i) \right] = \sum_{i} M_{i}, $$

$$ E = G^T \text{ is a } 1 \times n \text{ matrix, with components } E_{i} \equiv F_{00} = \frac{1}{2} \text{Tr} \left[ C^{-1}(\ell_i)C_{0}(\ell_i)C^{-1}(\ell_i)C_{i}(\ell_i) \right]. $$

$$ H_{ii} = \frac{1}{2} \text{Tr} \left[ C^{-1}(\ell_i)C_{i}(\ell_i)C^{-1}(\ell_i)C_{i}(\ell_i) \right]. $$

The inversion of $F$ can be done by block operation,

$$ \left(F^{-1}\right)_{00} = \left(M - EH^{-1}G\right)^{-1}. $$

Since $H$ is diagonal, we have

$$ M - EH^{-1}G = \sum_{i=1}^{n} \left[ M_{i} - E_{i}H_{ii}^{-1}G_{i} \right]. $$

The error in $m$ is then

$$ \sigma_{m} = \left[ F^{-1/2}_{00} \right]^{-1/2} \left[ \sum_{i=1}^{n} \left( M_{i} - E_{i}H_{ii}^{-1}G_{i} \right) \right]^{-1/2}. $$

From Equation (3), we can do the matrix inversion analytically to obtain $C^{-1}$. We then plug the expression of $C, C^{-1}$, and $C_{i}$ into the above equations. Finally we obtain

$$ \sigma_{m} = \left[ \sum_{i} \left( C^{-1}(\ell_i) \left( 1 + m \right)^{2} N^{i} / 4 + N^{i} \right) + N^{i} \right]^{-1/2}. $$

The sum over independent modes ($\sum_{i}$) can be replaced by the integral in the continuum limit. Finally we obtain

$$ \sigma_{m} \approx \left[ \int \frac{2\ell d\ell d\theta d\phi}{C^{\gamma}(\ell \left( N^{i} / 4 + N^{i} \right) + N^{i})} \right]^{-1/2}. $$

This is the most important result for method one, and is used in numerical evaluations shown in Figures 2 and 3 and Table 1.

REFERENCES

Albrecht, A., Bernstein, G., Cahn, R., et al. 2006, arXiv:astro-ph/0609591
Amendola, L., Marrin, V., & Quartin, M. 2014, MNRAS, 414, 520
Ben-Dayan, I., & Kalaydzhyan, T. 2014, PhRvD, 90, 083509
Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 586, A22
Bonvin, C., Durrer, R., & Kunz, M. 2006, PhRvL, 96, 191302
Bridle, S., Balan, S. T., Bethge, M., et al. 2010, MNRAS, 405, 2044
Castro, T., & Quartin, M. 2014, MNRAS, 443, L6
Cooray, A., Holz, D. E., & Huterer, D. 2006a, ApJ, 637, L77
Cooray, A., Huterer, D., & Holz, D. E. 2006b, PhRvD, 96, 021301
Corasaniti, P. S. 2006, MNRAS, 372, 191
Cropp, M., Hoekstra, H., Kitching, T., et al. 2013, MNRAS, 431, 3103
Dai, D.-C., Kinney, W. H., Stojkovic, D., et al. 2011, JCAP, 4, 15
Dalal, N., Holz, D. E., Chen, X., & Frieman, J. A. 2003, ApJ, 585, L11
Dodelson, S., & Vallinotto, A. 2006, PhRvD, 74, 063515
Fedeli, C., & Moscardini, L. 2014, MNRAS, 442, 2659
Frieman, J. A. 1996, ComAp, 18, 323
Hamana, T., & Fujimura, T. 2000, ApJ, 534, 29
Haugboelle, T., Hannestad, S., Thomsen, B., et al. 2007, ApJ, 661, 650
Heymans, C., van Waerbeke, L., Bacon, D., et al. 2006, MNRAS, 368, 1323
Hoekstra, H., & Jain, B. 2008, ARNPS, 38, 99
Holz, D. E. 1998, ApJ, 506, L1
Huterer, D., Takada, M., Bernstein, G., & Jain, B. 2006, MNRAS, 366, 101
Kantowski, R., Vaughan, T., & Branch, D. 1995, ApJ, 447, 35
Kitching, T. D., Balan, S. T., Bridle, S., et al. 2012, MNRAS, 423, 3163
Kronberg, T., Hardin, D., Guy, J., et al. 2010, A&A, 514, A44
LSST Dark Energy Science Collaboration 2012, arXiv:1211.0310
LSST Science Collaboration Abell, P. A., Allison, J., et al. 2009, arXiv:0912.2021
Mandelbaum, R., Rowe, B., Armstrong, R., et al. 2014, arXiv:1412.1825
Massey, R., Heymans, C., Bergé, J., et al. 2007, MNRAS, 376, 13
Massey, R., Hoekstra, H., Kitching, T., et al. 2013, MNRAS, 429, 661
Ménard, B., & Dalal, N. 2005, MNRAS, 358, 101
Metcalf, R. B. 1999, MNRAS, 305, 746
Munshi, D., Valageas, P., van Waerbeke, L., & Heavens, A. 2008, PhR, 462, 67
Regier, A. 2003, ARA&A, 41, 645
Schmidt, F., Leauthaud, A., Massey, R., et al. 2012, ApJ, 744, L22
Smith, R. E., Peacock, J. A., Jenkins, A., et al. 2003, MNRAS, 341, 1311
Tegmark, M. 1997, PhRvL, 79, 3806
Troxel, M. A., & Ishak, M. 2014, arXiv:1407.6990
Vallinotto, A., Dodelson, S., & Zhang, P. 2011, PhRvD, 84, 103004
Watkins, R., & Feldman, H. A. 2007, MNRAS, 379, 343
Weinberg, D. H., Mortonson, M. J., Eisenstein, D. J., et al. 2013, PhR, 530, 87
Yang, X., Zhang, P., Zhang, J., & Yu, Y. 2015, MNRAS, 447, 345
Zentner, A. R., & Bhattacharya, S. 2009, ApJ, 693, 1543
Zhan, H., & Knox, L. 2006, ApJ, 644, 663
Zhan, H., Wang, L., Pinto, P., & Tyson, J. A. 2008, ApJL, 675, L1
Zhang, J. 2008, MNRAS, 383, 113
Zhang, J. 2010, MNRAS, 403, 673
Zhang, J. 2011, JCAP, 11, 41
Zhang, P., & Chen, X. 2008, PhRvD, 78, 023006
Zhang, P., & Corasaniti, P. S. 2007, ApJ, 657, 71
Zhang, J., & Komatsu, E. 2011, MNRAS, 414, 1047
Zhang, J., Luo, W., & Foucaud, S. 2013, arXiv:1312.5514