Specification Inference from Demonstrations

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Abstract

Learning from expert demonstrations has received a lot of attention in artificial intelligence and machine learning. The goal is to infer the underlying reward function that an agent is optimizing given a set of observations of the agent’s behavior over time in a variety of circumstances, the system state trajectories, and a plant model specifying the evolution of the system state for different agent’s actions. The system is often modeled as a Markov decision process (Puterman 2014), that is, the next state depends only on the current state and agent’s action, and the agent’s choice of action depends only on the current state. While the former is a Markovian assumption on the evolution of system state, the later assumes that the target reward function is itself Markovian. In this work, we explore learning a class of non-Markovian reward functions, known in the formal methods literature as specifications. These specifications offer better composition, transferability, and interpretability. We then show that inferring the specification can be done efficiently without unrolling the transition system. We demonstrate on a 2-d grid world example.

Introduction

Learning from expert demonstrations has received a lot of attention in artificial intelligence and machine learning. The goal is to infer the underlying reward function that an agent is optimizing given a set of observations of the agent’s behavior over time in a variety of circumstances, the system state trajectories, and a plant model specifying the evolution of the system state for different agent’s actions. The system is often modeled as a Markov decision process (Puterman 2014), that is, the next state depends only on the current state and agent’s action, and the agent’s choice of action depends only on the current state. While the former is a Markovian assumption on the evolution of system state, the later assumes that the target reward function is itself Markovian. We assume that the system evolution is provided as a probabilistic automata (Rabin 1963) and thus, the state evolution is Markovian but the reward function can depend on the history of states. The MDP formulation is very general and theoretically, any reward function can be made Markovian by extending the state to record all relevant events; for example, if reward function depends on the last ten states; one can construct an MDP from the probabilistic automata where each state of the MDP corresponds to a sequence of ten states of the automata. But practically, this can lead to potentially exponential blow-up in the size of the states. In this paper, we directly address the problem of learning non-Markovian rewards without explicit construction of MDPs.

Our work is also motivated by the goal of learning an interpretable specification of an agent’s intent from observations of its behaviors. Interpretability would allow examination and analysis of learned specification before being used in high-assurance systems such as autonomous cars or industrial robots. Thus, we choose to use logical specifications such as linear temporal logic (Vardi 1996) to describe the agent’s goals and actions. The use of logical language to learn and express the specification instead of the traditional numerical reward function also facilitates composability and transferability. For example, reward functions learned from demonstrations of surveillance and those of tracking a mobile target can not be easily composed to infer ideal reward of an autonomous sentry. But if we instead infer logical specifications for these different tasks, standard frameworks such as the theory of contracts (Benveniste et al. 2007) and relational interfaces (Tripakis et al. 2009) can be conveniently used to compose these specifications. Logical specifications are also more readily transferred to new environments/tasks. For example, specification learned for surveillance on a slower vehicle can be transferred to a faster vehicle with some parametric adjustment which would require much fewer number of demonstrations.

Related Work Our work is related to literature in two fields: inverse reinforcement learning, and formal specification mining. Kalman proposed the problem of inverse optimal control (Kalman 1964) and the inverse reinforcement learning (IRL) problem was first formulated in (Ng, Russell, and others 2000). In IRL, also called apprenticeship learning, the agent observes its states resulting from a demonstration that is assumed to be arising from an optimal policy. It tries to learn a reward function mapping states to real-valued rewards such that the demonstrated policy is better than all alternatives with respect to this reward function. The learned reward in (Ng, Russell, and others 2000) tries to maximize the difference between the observed policy and the next best policy. This idea is further extended in (Ratliff,
Bagnell, and Zinkevich 2006) which proposes max-margin learning of optimal reward. Another extension is proposed in (Abbeel and Ng 2004) where the purpose is not to recover the actual reward function, but to find a reward function that leads to a policy equivalent to the observed demonstration, measured by the total reward collected by following that policy. They propose a strategy of matching feature expectations which is both necessary and sufficient to achieve the same performance as the agent if the reward function is linear in those features. In (Ziebart et al. 2008), authors consider the scenario where the demonstrations are sub-optimal and mixture of policies are required to match feature expectation. They resolve the ambiguity in choosing the distribution by using the principle of maximum entropy (Jaynes 1957), resulting in a stochastic policy which is no more committed to any particular behavior than what is required for matching feature expectations. This approach based on maximum entropy has many applications and extensions. For example, extensions have been proposed to complex reward functions modeled as Gaussian processes (Levine, Popovic, and Kolton 2011) and neural networks (Finn, Levine, and Abbeel 2016). (Nguyen, Low, and Jaillet 2015) considers a sequence of changing reward function and (Choi and Kim 2011) considers partially observable Markov Decision Processes. Our approach is an extension of the maximum entropy based approach where the reward function is no longer Markovian, but instead it corresponds to a logical specification that depends on history of bounded length. This work can also be seen as a new unsupervised technique for formal specification mining from black box agents. Previous works have tuned parametric specifications using only positive examples (Asarin et al. 2011), inferred the specification that minimizes the mis-classification rate given positive and negative examples via optimization (Kong et al. 2014) or decision trees (Bombara et al. 2016). Recently, the community has begun adapting clustering techniques either to directly work within the parameter space (Vazquez-Chanlatte et al. 2017) or as a post processing step from hierarchical clustering dendograms (Bombara and Belta 2017). By comparison, while our technique also learns from unlabeled positive examples from black box agents, our technique takes into account the stochastic environment the agent is operating in.

**Contributions** The key idea in this work is that the choice of optimal action is not Markovian in many applications, but instead depends on a bounded history of previous states and actions. In these cases, an optimal policy may be better captured using a logical specification such as linear temporal logic (Pnueli 1977) than the Markovian reward functions traditionally used in inverse reinforcement learning. Further, learning logical specification instead of numerical rewards facilitates interpretability and transferability by allowing human-examination of learned specification, and easy transfer to a related but different scenario where only parameters of the learnt specification needs to be tweaked. This is particularly important for high-assurance multitask applications such as robotics and autonomous systems. For such applications, we present an efficient algorithm to learn logical specifications given a set of demonstrations.

## Background and Setup

### Transition Systems and Policies

We shall assume our Teacher works within the confines of some stochastic environment model. Our stochastic environment model is simply a Markov Decision Process without an implicit reward function on states. To emphasize this, we simply refer to it as a Probabilistic Automata:

**Definition 1** (Probabilistic Automata). Let \( M = (S, s_0, A, \delta) \) be a Probabilistic Automata where \( S \) is the set of states, \( s_0 \in S \) is the starting state, \( A \) is the set of actions, and \( \delta : S \times A \times S \rightarrow [0, 1] \) specifies the transition probability of going from \( s \) to \( s' \) given action \( a \), i.e. \( \delta(s, a, s') = P(s' | s, a) \). For simplicity, we assume \( M \) does not deadlock\(^3\).

**Example 1** \((n \times n \text{ Grid World})\). Imagine a robot moving on a 2-d grid some fixed height above the ground. We assume that initially, the robot starts in some arbitrary cell on the grid. If the robot moves off the grid, it falls and breaks. Formally, let \( S_G = \{0, 1, \ldots, n-1\}^2 \) be the grid states and fallen = \( \{-1, 0, \ldots, n-1\}^2 \setminus S_G \) denote the states in which the robot has fallen off the platform. Let \( S_0 \) be the start state from which any action transitions to a grid cell with uniform probability. The set of states is \( S = S_G \cup \text{fallen} \cup \{s_0\} \). Let \( A = \{\pm y, \pm y, \pm x \pm y\} \) denote attempting to move all 8 cardinal directions, where \( x = (1, 0) \) and \( y = (0, 1) \). Finally, let \( p \in [0, 1] \) be the probability that the robot slips, uniformly transitioning to a neighboring cell, \( \text{Neighbor}(s) = \{s + a, \forall a \in A\} \). The transition probabilities are given in the following table:

| \( s \)      | \( s' \)      | \( P(s' | s, a) \) |
|------------|--------------|-----------------|
| \( = s_0 \) | \( \in S_G \) | \( s \) \( n^{-2} \) |
| \( \in \text{fallen} \) | \( = s \) | \( 1 \) |
| \( \in S_G \) \( \in \text{Neighbors}(s) \) | \( = s + a \) | \( p \) |
| \( \in S_G \) \( \in \text{Neighbors}(s) \) | \( = s + a \) | \( 1 - p \) |

**Policies** A policy determines which action to take next. In general, policies can be stochastic and depend on the sequence of previous states seen (called the history). Formally,

**Definition 2** (\( \tau \)-Language of \( M \)). The \( \tau \)-language of \( M \), \( \mathcal{L}[\tau, M] \subseteq (S \times A)^\tau \) is the set of all i/o sequences (also called behaviours, histories, traces, and demonstrations) of length \( \tau \in \mathbb{N} \) generatable by \( M \).

When we need to separately refer to the state sequence and the action sequence, we adopt the notation \( \vec{s} \times \vec{a} \in \mathcal{L}[\tau, M] \) to denote the sequence of states, \( \vec{s} \), and the sequence of actions, \( \vec{a} \), in an i/o sequence.

**Definition 3** (Policies). A policy is a collection of distributions over actions indexed by the history:

\[
\pi : \bigcup_{\tau \in \mathbb{N}} \mathcal{L}[\tau, M] \times A \rightarrow [0, 1] \tag{0.1}
\]

We will denote the collection of all policies by \( \Pi \).

We illustrate with an example policy that inherently depends on the history.

\(^3\)That is, for each state action pair, there is a possible next state.
Example 2. Let $M$ be a grid world transition system. Let the policy be the map
\[(y, +\hat{x}) \mapsto 1 \text{ iff } |y| \text{ is prime}
\[(y, -\hat{x}) \mapsto 1 \text{ iff } |y| \text{ is not prime}
\] (0.2)
In other words the robot moves left or right depending on whether $|y|$ is prime.

Each policy induces a distribution over traces.

Definition 4 (Trace Distributions). Given a policy $\pi$, a transition system $M$, and a length $\tau \in \mathbb{N}$, the induced distribution $(M \circ_\tau \pi) : (S \times A)^\tau \rightarrow [0, 1]$ over all traces of length $\tau$ is defined by:
\[(M \circ_\tau \pi)(x) \overset{\text{def}}{=} \prod_{i=0}^{\tau-1} \delta(s_i, a, s_{i+1})\pi(x[i]; a) \] (0.3)
where $x[i]$ denotes the $i$ length prefix of $x = s \times \hat{a}$.

If $\tau$ is obvious from context or not important, we will often just write $M \circ \pi$.

Specifications Next, we develop machinery to specify which traces are desirable and which traces are undesirable. That is, the set of traces that satisfy some abstract property. For simplicity, in this work we stick to properties that are decidable by a fixed step $\tau \in \mathbb{N}$ and leave general specifications over infinite sequences as future work. For example, we do not allow “eventually event A will occur”, but do allow “event A will occur before step 20”.

Definition 5 (Bounded Specification). Given a set of states $S$, a set of actions $A$, and a trace length $\tau \in \mathbb{N}$, a Bounded Specification $\phi \subseteq (S \times A)^\tau$ is the set of desired behaviors. We will denote $\emptyset$ and $(S \times A)^\tau$ as $\bot$ and $\top$ resp. Further, we define $\neg \phi = (S \times A)^\tau \setminus \phi$.

The scope, an upper bound on the length of the trace required to guarantee satisfaction, of a Bounded Specification over $\tau$ length traces is said to be $\tau$.

Example 3. Recall our gridworld from Ex 1. Given a predicate $p$ and two positive numbers, $a, r$
\[p(a, r, p) \overset{\text{def}}{=} \{x, \exists t \in [0, \tau] \forall t' \in [t, t+a], p(x(t'))\} \] (0.4)
The interpretation is that within $\tau$ time steps, the robot should make predicate $p$ true for a consecutive time steps. Now suppose on the North West and South West corners of the grid there are two experiments that each need to be run within 2 units (per dimension) of the corners for 4 time steps. Using the predicates:
\[p_1^\tau((x, y)) \overset{\text{def}}{=} 0 \leq (x, y) \leq r \] (0.5)
\[p_2^\tau((x, y)) \overset{\text{def}}{=} n - 1 - r \leq (x, y) \leq n - 1 \]
The specification can be encoded as:
\[\phi^* = p(a = 4, r = 2, p_2) \cap p(a = 4, r = 2, p_2) \] (0.6)
Notice that this specification can be easily transferred between related transition systems. For example, even if our grid world had a hole carved into the center or was embedded in $\mathbb{R}^2$, the specification would still be well defined (and have the same semantics).

To discuss the event that a specification is satisfied we define:

\[J_\phi(\pi) \overset{\text{def}}{=} P(\text{SAT}_\phi \mid M \circ \pi) \] (0.7)
We shall assume that $J_\phi(\pi)$ is always known, either empirically, called $\bar{J}_\phi$, or through some other means.

Resolving Ambiguities

Before formally defining our learning problem, observe that knowing $J_\phi(\pi)$ does not uniquely define the teacher’s policy. The following example illustrates this:

Example 4. Consider a 4 state system with 3 actions, $S = \{1, 2, 3, 4\}$, $A = \{a, b, c\}$, $s_0 = 1$ and $Pr(2 \mid 1, a) = 1$, $Pr(3 \mid 1, b) = 1$, $Pr(4 \mid 1, c) = 1$ with 2, 3, 4 being sink nodes, see Figure 1. Let the specification $\phi$ be: Never enter state 4. Now $\pi$ gives $J_\phi = 1$ iff $\pi(1, c) = 0$ (i.e., probability of taking action $c$ at $s = 1$ is 0). Notice however that $\pi(1, a)$ and $\pi(1, b)$ are only constrained then to sum to 1, and hence there are infinitely-many choices for optimal $\pi$.

Thus, we assume that this ambiguity is resolved via some constraint on $\Pi$. This leads us to the following definition:

Definition 8 (Teacher Model). A Teacher $T$ is a map $M, \phi \mapsto \pi_\phi$, that associates with each transition system/specification pair a unique policy.

Informally, for each environment and specification, a teacher has a well defined policy for giving demonstrations. We are now ready to give our formal problem statement.

Formal Problem Statement

Definition 9 (Specification Inference from Demonstrations). Formally, our learning problem is a Maximum a Posteriori Probability (MAP) inference problem:
1. Let $M = (S, s_0, A, \delta)$ be a stochastic transition system.
2. Let $T$ be a Teacher Model.
3. Let $Y$ be a sequence of demonstrations drawn from $M \circ_\tau T(M, \varphi)$ for some unknown specification $\varphi$.
4. Let $\Phi \subseteq 2^{(S \times A)^\tau}$ be a family of Bounded Specifications.
Our goal is to find the most probable $\phi^* \in \Phi$:
\[\phi^* = \arg \max_{\phi \in \Phi} P(\phi | Y, M \circ T(M, \varphi)) \] (0.8)
Reduction to IRL. We briefly remark that it is possible to reduce Problem 0.8 to an Inverse Reinforcement Learning Problem. Suppose the scope of the bounded specifications is $\tau$ and we have a transition system $M = (S, s_0, A, \delta)$. First, we unroll our transition system $\tau$ steps to create a new transition system, $M'$. Namely, each state in $M'$ corresponds to a time step $i + 1$ and an element of $(S \times A)$ corresponding to the history of states/actions seen by step $i$. Notice that $M'$ is a tree rooted at $s_0$. Now to each state attach a reward $R: S \to \{0, 1\}$, such that non-leaf states have reward $0$ and leaf states have reward either $0$ or $1$. Next we lift rewards to traces using the following discounted sum of rewards:

$$ R(\tilde{s}) = \sum_{i \in \mathbb{N}} \frac{1}{2^i} R(s_i) $$

Finally, observe that (1) There is only $1$ unique path from $s_0$ to a given leaf node and (2) Reaching the leaf nodes requires $\tau$ steps, and thus the satisfaction is decided. Consider the specification:

$$ \phi = \{ x \in L[\tau, M] : R(x) \neq 0 \} $$

Since there’s a bijection between leaf nodes and paths, $\phi$ can encode any Bounded Specification. Nevertheless, as discussed in the introduction, this reduction suffers from an exponential blow up and lacks transferability. As such, in the sequel we seek an alternative approach.

Distribution given Simplifying Assumptions

We now derive an analytic form for $P(\phi, M \circ T)$ under the following $3$ assumptions.

Uniform Prior over $\Phi$ We assume that the prior of $\phi \in \Phi$ is uniform, reducing our MAP instance to a Maximum Likelihood Estimation.

$$ P(\phi | Y) = P(Y | \phi) $$

Demonstrations have length $\tau$ We assume that all demonstrations have length $\tau = \text{scope}(\phi)$. This means that each trace in the demonstrations either satisfies or doesn’t satisfy $\phi$, enabling the following indicator function:

**Definition 10** (Satisfaction Indicator).

$$ [x \in \phi] \overset{\text{def}}{=} 1 \text{ if } x \in \phi \text{ else } 0 $$

We note that $J_\phi$ is exactly the mean of this indicator.

**Lemma 1.** Let $C$ be a distribution over $\tau$ length traces. Let $\phi$ be a Bounded Specification with scope $\tau$.

$$ P(\text{SAT}_{\phi} | C) = \mathbb{E}_{x \sim C} ([x \in \phi]) $$

**Proof.**

$$ P(\text{SAT}_{\phi} | C) = \sum_{x \in \phi} [x \in \phi] P(x \in \phi | x \sim C) $$

$$ = \mathbb{E}_{x \sim C} ([x \in \phi]) $$

Thus, the empirical rate of satisfaction, constrains the mean of $[x \in \phi]$ to be $J_\phi$.

The Principle of Maximum Entropy Let $\Pi^*$ denote the set of policies that match the empirical satisfaction rate. Recall from Example 4 that $\Pi^*$ may be highly non-unique. To resolve such ambiguity, we take a page from (Ziebart et al. 2008) and apply the principle of maximum entropy. There are many justifications often given for using this principle, but here we merely remark that in maximizing the entropy, we minimize the bias encoded in the distribution while maintaining consistency with other observations. Functionally, this means weighting traces by their “riskiness” and satisfaction. This enables resilience to imperfect demonstrations and the possibility that the class of specifications we are searching over does not contain the true specification.

**Example 5.** Remembering our $4$ state example with $\phi = \text{never enter state } 4$, the principle of maximum entropy tightens the requirement that $P(a|1) + P(b|1) = 1$ to $P(a|1) = 1/2$ and $P(b|1) = 1/2$.  

Now, given a specification $\phi$ and an observed satisfaction rate $J_\phi$, what is the resulting distribution over traces?

**Theorem 1.** Let $M = (S, s_0, A, \delta)$ be a Transition system, $\phi$ be a Bounded Specification with length $\tau$ and $y = \tilde{s} \times \tilde{a} = L[\tau, M]$ be a $\tau$ length i/o trace. If $T$ asserts Maximum Entropy with satisfaction rate $J_\phi$, then

$$ P(y | \phi, M \circ T) = G(y) \cdot \begin{cases} \frac{J_\phi}{G_\phi} & y \in \phi \\ \frac{J_{\phi^c}}{G_{\phi^c}} & y \notin \phi \end{cases} $$

Where $G_\phi$ is the probability that a random action sequence, (chosen uniformly) will satisfy $\phi$.  

$$ G_\phi \overset{\text{def}}{=} P(\text{SAT}_{\phi} | M, \tilde{a} \sim \text{Uniform}(A^\tau)) $$

**Proof.** The probability of seeing a trace $x = \tilde{s} \times \tilde{a}$ given $M$ and $T[\phi, M, T]$ is (by definition):

$$ P(x | \phi) = \prod_{i=0}^{\tau-1} \delta(s_i, a_i, s_{i+1}) T(s_{i+1} | a_i, T) $$

Now recall that $[x \in \phi]$ is a function with mean $J_\phi$ over the random variable $x$. In such cases, the principle of maximum entropy distribution requires high values of $[x \in \phi]$ to be exponentially more probable (Jaynes 1957).  

Since the transition probabilities are fixed (with our only degree of freedom being $T$), Eq. 0.17 becomes:

$$ P(x | \phi) = \frac{[x \in \phi]}{Z} \prod_{s', s, a, \tilde{s}, \tilde{a}} \delta(s, a, s') $$

For some $c, Z \in \mathbb{R}_+$ such that the mean of Eq. 0.18 is $\bar{J}_\phi$ and the sum over all $x$ is $1$. Now, define $w_x$ to be the probability of $\tilde{s}$ given $\tilde{a}$. Namely, $w_{\tilde{s} \times \tilde{a}} \overset{\text{def}}{=} \prod_{i=0}^{\tau-1} \delta(s_i, a_i, s_{i+1})$. Further, the sum of all $w_x$ for $x$ in some specification is defined as $W_\phi \overset{\text{def}}{=} \sum_{x \in \phi} w_x$. Applying the aforementioned constraints on $Z$ and $c$ yields:

$$ \bar{J}_\phi = \sum_{x \in \phi} [x \in \phi] P(x | \phi) = c \frac{W_\phi}{Z} $$

$$ Z = c \sum_{x \in \phi} w_x + \sum_{x \notin \phi} w_x = c W_\phi + W_{\phi^c} $$

$^2$Thus, $G_{(\chi)}$ is read as the probability of seeing trace $x$ given a random demonstration.
Combining yields, \( Z = W_{\neg \phi}/(J_{\neg \phi}) = W_{\neg \phi}/(1 - J_{\phi}) \). If \( x \not\in \phi \), then \( c^{[x \in \phi]} = 1 \). Plugging into Eq 0.18:

\[
P(x|\phi, M \circ T, x \not\in \phi) = J_{\neg \phi}/W_{\neg \phi} \tag{0.20}
\]

If \( x \in \phi \) (implying \( W_{\phi} \neq 0 \)) then \( c = JZ/W_{\phi} \). Thus,

\[
P(x|\phi, M \circ T, x \in \phi) = J_{\phi}/W_{\phi} \tag{0.21}
\]

Finally, observe that \( G_{\phi} = W_{\phi}/W_{\top} \). Substituting and factoring yields Eq 0.15.

**Corollary 1.** Let \( Y : [0 . . . n] \rightarrow L[\tau, M] \) be sequence of \( n \) demonstrations drawn i.i.d. from \( M \circ T \). The likelihood of \( Y \) is:

\[
P(Y|\phi, M \circ T) = \gamma_{\gamma} \left( \frac{J_{\phi}}{G_{\phi}} \right)^{N_{\phi}} \left( \frac{J_{\neg \phi}}{G_{\neg \phi}} \right)^{N_{\neg \phi}} \tag{0.22}
\]

where \( \gamma_{\gamma} \equiv \prod_{x \in \phi} G_{\{x\}}, N_{\phi} \equiv \sum_{i=0}^{n} [Y(i) \in \phi] \). 34

**Remark 1.** Note that via excluded middle \( J_{\phi} = (J_{\phi} + J_{\neg \phi}) = 1 \) and \( G_{\neg \phi} + G_{\phi} = 1 \). Further, \( J_{\phi} = N_{\phi}/|Y| \). Thus, when changing \( \phi \), there are only two degrees of freedom.

Notice that when comparing specifications, the probabilities of the demonstrations do not play any direct role, but rather are implicitly encoded in \( J_{\phi} \). 35

### Lattices of Specifications

We now seek an algorithm that exploits the inherent structure in Eq 0.22 to approximate the most likely specification given a set of demonstrations. Note that in general, computing a gradient over this likelihood will be difficult because it relies on having gradients for \( G_{\phi} \) and \( J_{\phi} \), which in general depend on the structure of \( \Phi \). We note that in practice \( J_{\phi} \) is easy to compute. For example, if \( \phi \) is a propositional logic expression (as in Bounded Temporal Logic), the evaluation is simply a combinatorial circuit evaluation, linear in the circuit’s depth (and thus linear in the trace length, state size). \( G_{\phi} \) on the other hand may often be expensive to compute. This is because analytic solutions are often not known for complex transition system/specification pairs. Thus, one often resorts to probabilistic model checking (Kwiatkowska, Norman, and Parker 2011), Monte Carlo, or weighted model counting (Chavira and Darwiche 2008). In our examples, we will use Monte Carlo, as it requires no additional development, and is easy to implement via a uniformly random policy. As such queries are comparatively expensive, and because \( \Phi \) can be very large, it is crucial to attempt to minimize the number of \( G_{\phi} \) and \( J_{\phi} \) queries. We quickly remark that the gradient of \( G_{\phi} \) and \( J_{\phi} \) must respect set inclusion. This motivates exploring \( \Phi \) that are organized into a lattice.

36By definition we take \((\ldots)^{0} = 1 \).

37We have suppressed a multinomial coefficient required if any two demonstrations are the same. However, since this term will not change as \( \phi \) varies, one can simply absorb this coefficient into \( \gamma_{\gamma} \).

38Indeed notice that the probability that \( N_{\phi} \) of the demonstrations satisfy \( \phi \) and \( N_{\neg \phi} = 1 - |Y_{\phi}| \) traces don’t forms a Geometric distribution as expected given fixed \( J_{\phi} \) (Cover and Thomas 2012). This distribution could also be used to infer \( \phi \), but the lack of explicit \( G_{\phi} \) dependence doesn’t enable reasoning about the environment’s effect on \( J_{\phi} \).

39For notational convenience we write \( J_{\phi} \) and \( J_{\neg \phi} \) instead of \( J_{\{\phi\}} \) and \( J_{\{\neg \phi\}} \) respectively.

40For the unit box, curves with non-negative slopes are chains and maximal antichains are hyper-surfaces that partition the unit-box into two regions. Due to the embedding, we shall sometimes be a bit sloppy when distinguishing the lattice over the unit box from our lattice over specifications. Next, we develop an algorithm to exploit this structure to approximately solve the problem given in Def 9. Further, for technical reasons we assume that any maximal antichain, \( B \), of \( \mathbb{L} \) is upper and lower bounded by finite antichains. Formally, let \( (\preceq) \) to sets as

\[
B \preceq B' \iff \forall b, b' \in B \times B'. \ b \preceq b' \tag{0.24}
\]

Our assumption is then that for all antichains, \( B \), there exists finite antichains \( B, B' \) such that, \( B \preceq B \preceq B' \).

### Organizing \( \Phi \) into a Lattice

Namely, we shall assume \( \Phi \) forms a lattice \( \mathbb{L} \equiv (\Phi, \preceq) \) such that \( \phi \preceq \phi' \iff \phi \subseteq \phi' \).

**Example 6.** Consider the case where \( \Phi \) is a finite set of unrelated specifications. Then by adjoining \( \top \) and \( \bot \) to \( \Phi \) one creates a lattice where \( \phi \preceq \phi' \) iff \( \phi' = \top \) or \( \phi = \bot \).

Let us quickly review some basic Lattice Theory.

**Definition 11** (Ascending Chains). Given a lattice \( \mathbb{L} = (A, \preceq) \), an ascending chain (or just chain) is a sequence of elements of \( A \) ordered by \( \preceq \).

**Definition 12** (Antichains). Given a lattice \( \mathbb{L} = (A, \preceq) \), an antichain is a set of incomparable elements. An antichain \( B \) is said to be maximal if all elements of \( A \setminus B \) are comparable to \( B \).

**Definition 13.** Given two antichains \( B \) and \( B' \) of lattice \( \mathbb{L} \), denote the set of specifications between \( B \) and \( B' \) as \( \mathbb{L}[B : B'] \).

Next we specialize to a fairly general lattice, the unit box, \([0, 1]^{n}\) with the standard product ordering. For many lattices of interest, there will often be a trivial embedding into \(([0, 1]^{n}, \preceq) \). Put another way, we shall assume that \( \Phi \) is parameterized by \( n \) variables each ranging from \([0, 1] \). Formally, if we denote our parameterization by \( \psi : [0, 1]^{n} \rightarrow \Phi \), then:

\[
\forall \theta, \theta' \in [0, 1]^{n} . \ \theta \preceq \theta' \iff \psi(\theta) \preceq \psi(\theta') \tag{0.23}
\]

**Example 7.** Consider generalizing Ex 3 to take 2 parameters: \( \phi(a, r) = \vee(\phi(\varepsilon, p_{1}) \cap \phi(a, r, p_{2})) \). Note, that \( \phi \) grows as a decreases and \( \psi \) increases. Further, one can simply scale and round to embed into \([0, 1]^{n}\).
Likelihood on Ascending Chains

To understand how \( P(Y|\phi) \) changes as \( \phi \) changes, first, observe the following lemma:

**Lemma 2.** Let \( Y \) be a set of demonstrations. Along any chain in \( L = (\Phi, \leq) \), \( N_0 \) can only change \( |Y| + 1 \) times.

*Proof.* Follows directly from \( N_0 = |\phi \cap Y| \) monotonically increasing with \((\leq)\) and \( N_0 \) only taking on \(|Y| + 1 \) values.  

**Corollary 2.** There are \(|Y| + 2 \) maximal antichains bounding the constant regions of \( N_0 \).

Call these collections of maximal antichains \( B \) and let \( B_k \in B \) denote the maximal antichain in \( B \) separating \( N_0 = k - 1 \) and \( N_0 = k \) (where \( B_1 = \emptyset_1 \) and \( B_T = B_{|Y|+1} \) if necessary). In summary, \( B \) segments the unit box into regions of constant \( \bar{J}_\phi \) and \( N_0 \). Recalling that \( \gamma_Y \) is only a function of the demonstrations/environment, within \( L[B_k : B_{k+1}] \) the changes in likelihood only occur due to changes in \( G_\phi \). Notice that \( G_\phi \) is strictly increasing on ascending chains.

**Lemma 3.** \( \phi' \subseteq \phi \implies G_{\phi'} \leq G_\phi \)

*Proof.* \( G_\phi = (W_{\phi'} + \sum_{x \in \phi_\setminus\phi'} w_x)/W_T \). Further, \( w_x \geq 0 \). Thus, \( G_{\phi'} \leq G_\phi \).

Next, define \( f(G_\phi, Y) \defeq P(Y|\phi)/\gamma_Y \) which is simultaneously maximized with the likelihood of the demonstrations. We now give a lemma describing how \( f \) changes as \( G_\phi \) changes.

**Lemma 4.** Let \( B, B' \) be successive antichains in \( B \):  
\[
\delta_{G_\phi} f \geq 0 \iff \forall \phi \in L[B : B'] : G_\phi \geq \bar{J}_\phi \quad (0.25)
\]

*Proof.* Observe that if for both antichains \( N_0 = |Y| \), then \( f = G_\phi^{-1}[Y] \) and \( \bar{J}_\phi = 1 \geq G_\phi \). Note that \( \delta_{G_\phi} f \leq 0 \) as required. Similarly if for both antichains \( N_0 = 0 \), then \( f = (1 - G_\phi)^{-1}[Y] \) and \( \delta_{G_\phi} f \geq 0 \) as required. Otherwise,  
\[
\delta_{G_\phi} f = f|G_\phi [Y|G_\phi \geq N_0 \mid G_\phi (1 - G_\phi) \quad (0.26)
\]

Note that \( f|G_\phi \) and \( G_\phi(1 - G_\phi) \) are positive, thus \( \delta_{G_\phi} f \geq 0 \iff |Y|G_\phi - N_0 \geq 0 \). Rearranging and substituting \( \bar{J} \) yields Eq 0.25.

**Lemma 5.** \( G_\phi = J_\phi \) if the likelihood is a minima.

*Proof.* Recall that along chains in \( L, G_\phi \) monotonically increases. The sign of \( \delta_{G_\phi} f \) is determined by the sign of \( G_\phi - \bar{J}_\phi \). Thus, as \( G_\phi \) increases the sign of \( \delta_{G_\phi} f \) goes from negative to positive. Further, if any other anti-chain, \( B \) contained a minima, it must be the case the between the \( G_\phi = J_\phi \) and \( B \) the sign of the gradient changed. But this is impossible via Lemma 0.25.

Further, from these lemmas it follows that a global maxima must lie on \( B \).

**Theorem 2.** If \( \phi^* \in \Phi \) maximizes \( f \), then \( \exists \phi \in \cup B \) such that \( f(\phi) = f(\phi^*) \).

*Proof.* By Lems 3 and 4

Maximal Likelihood Criteria and Regions

Now by assumption each maximal antichain in \( B_N \) can be upper and lower bounded by finite antichains. For the unit-box lattice, (Maler 2017) gives an iterative algorithm for over approximating such antichains as a union of rectangles described below. While in the general case this algorithm is exponential in dimension \( n \) (both the number of rectangles and time), many examples (particularly axis aligned boundaries) have much better performance. Let \( R \) denote the set of hyper-rectangles in \([0,1]^n\).

**Definition 14** \( (OM_i(pri, \mathcal{O})) \). Let \( \mathcal{O} : [0,1]^n \to \{\text{true}, \text{false}\} \) be a monotone threshold function. Namely if we define false \( \leq \text{true} \), then \( \forall \theta_1, \theta_2 \in [0,1]^n : \)

\[
\theta_1 \leq \theta_2 \implies \mathcal{O}(\theta_1) \leq \mathcal{O}(\theta_2) \quad (0.27)
\]

and let pri : \( R \to \mathbb{R} \) denote a rectangle’s priority. Let \( B \) denote the antichain separating true and false. \( OM_i(pri, \mathcal{O}) \) is a series of rectangle collections with the following properties.

\[
OM_i(pri, \mathcal{O}) = \{[0,1]^n\}
\]

\[
OM_i(pri, \mathcal{O}) \subset OM_{i+1}(pri, \mathcal{O})
\]

\[
B \subset \bigcup OM_i(pri, \mathcal{O})
\]

Further, rectangles are refined in order of priority pri.

Defining \( \mathcal{O}_k(pri, \mathcal{O}) \) to be \( |\mathcal{Y}(\phi_k(\theta))| \geq k \), gives a threshold function that will approximate \( B_k \) via a set of rectangles. Thus, one can then construct \( \mathcal{B} \) and \( \mathcal{B}^\prime \) from the inf and sup of each rectangle resp. Next notice that for fine enough approximation, \( B_k, B_k, \mathcal{B}_k \), one can determine if \( B_k \) contains (local) maxima.

**Lemma 6.** Let \( B_k \in B \) be the \( N_\phi = k \) maximal antichain and let \( \mathcal{B}_k \) and \( \mathcal{B}_k^\prime \) be lower and upper bounding antichains s.t. \( B_{k-1} \leq B_k \leq \mathcal{B}_k \leq \mathcal{B}_{k+1} \). If \( B_k \) contains a local maxima iff \( \forall \phi, \phi' \in L[B_k, B_k^{\prime}] \times L[B_k, B_k^{\prime}] : \)

\[
\bar{J}_\phi \leq G_{\phi'} \leq G_\phi \leq J_\phi \quad (0.29)
\]

*Proof.* By Lem 4 and Thm 2.

Figure 3: On the left a cartoon of antichains segmenting unit square into regions of constant \( N = N_0 \). On the right, an illustration of the rectangle approximations of Oded's of the \( \mathcal{N}_0 \geq 1 \) threshold antichain. \( \mathcal{B}_1^\prime \) is all the bottom left corners and \( \mathcal{B}_1^\prime \) is all the top right corners.
Let denote by $B_{\text{max}}$ the antichains in $B$ that contain local maxima. Next, we describe how to bound the maximum $f$ of $B_k \in B_{\text{max}}$.

**Lemma 7** (f bound). Let $B$ be a maximal antichain in $B_{\text{max}}$ separating $N_\phi = k$ and $k'$. Further Let $\overline{B}, B$ be upper and lower bounding antichains of $B$ resp. Define:

$$f_k(B) \overset{\text{def}}{=} \max_{\phi \in \overline{B}} (f(G_\phi, k)) \quad (0.30)$$

Then,

$$f_k(\overline{B}) \leq f_k(B) \leq f_k(B) \quad (0.31)$$

And,

$$f_{k'}(\overline{B}) \leq f_{k'}(B) \leq f_{k'}(\overline{B}) \quad (0.32)$$

**Proof.** By Lems 4 and 6.

As we won’t know $B_{\text{max}}$ initially, we will need to only trust this bound after the condition for Lem 6 is satisfied.

**Algorithm** This inspires the following algorithm:

1. $\forall B_k$, use $OM_i$ with oracle $O_k$ and priority proportional to infinity norm of the rectangle diagonals to approximate $B_k$ the largest priority is less than $\epsilon_1$.
2. Call the set of all bounding rectangles $R_i$.
3. If any $r', r \in R_i$ are equal and have $\inf(r) = \sup(r)$, remove the rectangle with smaller $N_\phi$.
4. If any $r', r \in R_i$ still overlap, $\epsilon_1 \mapsto \epsilon_1/2$ and repeat.
5. Define $\mu(r)$ as the midpoint of the $f$ bound.
6. Define $\Delta(r)$ as the size of the $f$ bound in $r$.
7. $\forall r \in R_i$, compute $\mu(r)$ and $\Delta(r)$
8. Partition $R_i$ into sets $A_+$ and $A_-$ where Lem 6 is and isn’t satisfied resp.
9. If the the lowest upperbound in $A_+$ is less than the largest upperbound in $A_-, \epsilon_1 \mapsto \epsilon_1/2$ and repeat.
10. If the rectangle with the largest $\mu(r)$ has $\Delta(r) \leq \epsilon_2$, then return the sup of the rectangle.
11. Otherwise $\epsilon_1 \mapsto \epsilon_1/2$ and repeat.

Before concluding, we discuss some experiments done on our grid world example for $n = 2$ and $n = 8$. Again, let $M$ be the probabilistic transition system from Ex 1. For the $n = 2$ case the slip probability was set to $1/5$ and for the $n = 8$ case the slip probability was $1/20$. All demonstrations had length $\tau = 18$. Our specification class, $\Phi$ will again be the parametric specification from Ex 7. Parameter $a$ ranged from $[0, 4]$ and $r$ ranged from $[0, \frac{n}{2}]$. We embedded this lattice into the unit box via $a \mapsto \text{round}(a/4)$ and $r \mapsto \text{round}(2r/n)$. $\epsilon_1 = \epsilon_2 = 0.001$. We found that such a tight $\epsilon_1$ resulted in exactly recovering the original discrete lattice. Computing $J_\phi$ required a linear pass over the traces counting how many consecutive steps were spent in the target regions. $G_\phi$ was computed by implementing a random controller, simulating $1000$ runs and then computing $J_\phi$ on the simulations. For the $n = 2$ case, we created a controller for $r = 0, a = 0$. For the $n = 8$ case we created a controller for $r = 2, a = 3$. $n = 2$ had 1000 demonstrations, $n = 8$ had 2797 demonstrations.

Table 4 summarizes the results. Crucially, notice that the specifications with larger $J_\phi$ are not necessarily picked. In particular, in the $n = 2$ case, $a = 0, r = 1$ is the only specification that captures all demonstrations, but because it is a tautology, it is given a low likelihood. As the primary bottleneck appears to be computing $G_\phi$, we believe this system will scale to larger settings given more efficient implementations.

### Conclusion and Future Work

In this work we explored learning (from demonstrations) non-Markovian reward structures corresponding to logical specifications. While it is possible to reduce this inference problem to Inverse Reinforcement Learning, doing so removes transferability, interpretability, ease of composition, and requires unrolling the transition system. We derived an analytic form of distributions given a specification using the principle of maximum entropy. We then specialized to the case where the specifications under consideration are organized into a lattice that respects subset inclusion. In our experiments, we found that we were able to recover the specification intended by a hand crafted policy. Further, we found that our procedure was robust to environment induced failures, which stands in contrast to learning from positive (and negative) examples without an environment model. Future work includes more complex experiments, infinite scope specifications, learning across multiple environments, and adapting to the continuous setting.

| $a$ | $r$ | $J_\phi$ | $G_\phi$ | log($f$) | $a$ | $r$ | $J_\phi$ | $G_\phi$ | log($f$) |
|-----|-----|---------|---------|---------|-----|-----|---------|---------|---------|
| 0   | 0   | 0.581   | 0.107   | 665     | 3   | 2   | 0.854   | 0.0003  | 18226   |
| 2   | 1   | 0.415   | 0.141   | 224     | 0   | 1   | 0.959   | 0.0013  | 17334   |
| 1   | 1   | 0.674   | 0.375   | 183     | 2   | 2   | 0.951   | 0.0012  | 17334   |
| 3   | 1   | 0.17    | 0.0527  | 89      | 1   | 2   | 0.959   | 0.0047  | 13888   |
| 4   | 1   | 0.066   | 0.0198  | 34      | 4   | 3   | 0.942   | 0.0005  | 13434   |
| $\geq$ | 0  | 1.0     | 1.0     | 1.0     | 3   | 3   | 0.955   | 0.012   | 11331   |

*When implementing the above algorithm there are several optimizations one can perform, such as rectangle pruning based on the maximization criteria and parallelization of $OM_i, G_\phi, J_\phi, \mu$, and $\delta$.

*Randomly generated 4 digit number.
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