The \( \pi\sigma \)-Axial Exchange Current

K. Tsushima\(^1,3\) and D.O. Riska\(^2\)

\(^1\) Institute for Theoretical Physics, University of Tuebingen, D72076 Tuebingen, Germany  
\(^2\) Department of Physics, SF-00014 University of Helsinki, Finland

Abstract

The axial exchange current operator that arises from the coupling of the axial field to the pion and effective scalar field in nuclei is constructed. The spatial dependence of this exchange current operator may be determined directly from the nucleon-nucleon interaction. This \( \pi\sigma \)-axial exchange current contributes about 5\% to the quenching of the nucleon axial current coupling constant \( g_A^{GT} \) in heavy nuclei.

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The value of the effective axial current coupling constant $g_A^{GT}$ of nucleons in nuclei is considerably smaller than the corresponding free nucleon value [1,2]. Shell model analyses [3] as well as studies of giant Gamow-Teller resonances [4] indicate that it is quenched by about 30% already in sd-shell nuclei. This quenching effect is conventionally ascribed to the effect of $\Delta_{33}$-excitations [5,6] and core-polarization [1]. The short range exchange currents that are associated with the axial current coupling through intermediate nucleon-antinucleon pairs to the short range parts of the nucleon-nucleon interaction have been found to be insignificant [7].

The absolute magnitude of both the corrections that are associated with intermediate $\Delta_{33}$-excitation mechanisms and the second order configuration mechanisms remain somewhat uncertain as they are sensitive to the short range components and especially to the strength of the nucleon-nucleon tensor interaction. The large core polarization correction obtained with a strong model for the nucleon-nucleon tensor interaction in ref. [1] appears to explain most of the empirically found quenching of $g_A^{GT}$ by itself. We here point out that the axial exchange current that arises from the coupling of the axial field to the pion and the effective scalar field in nuclei ("the $\sigma$-meson") represents another mechanism that contributes a small but still significant amount to the quenching of the value for $g_A^{GT}$ in nuclei (Fig. 1). We demonstrate that the corresponding axial exchange current operator can be constructed directly from the model for the nucleon-nucleon interaction, and thus involves no parameters that do not appear in the interaction model. Using realistic phenomenological models for the interaction we estimate that the $\pi\sigma$-mechanism alone leads to a quenching of about 5% of $g_A^{GT}$. The presence of this additional quenching mechanism suggests that the previous estimates for the core polarization correction in ref. [1] need to be revised downwards by employment of a weaker model for the tensor force.

It proves convenient to base the derivation of the effective $\pi\sigma$ axial exchange current on the modified $\sigma$-model [8,9]

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1,$$  \hspace{1cm} (1)

where $\mathcal{L}_0$ is the Lagrangian density for the usual linear $\sigma$-model.
\[ L_0 = \bar{\psi} [i\gamma^\mu \partial_\mu - g(\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5)] \psi + \frac{1}{2}[(\partial_\mu \vec{\pi})^2 + (\partial_\mu \sigma)^2] + \frac{1}{2}m_0^2(\vec{\pi}^2 + \sigma^2) - \frac{1}{4}\lambda^2(\vec{\pi}^2 + \sigma^2)^2, \]

(2)

and \( L_1 \) is the chirally symmetric term

\[ L_1 = C\{\bar{\psi} \gamma_\mu \frac{\vec{\pi}}{2} \psi \cdot (\vec{\pi} \times \partial^\mu \vec{\pi}) + \bar{\psi} \gamma_\mu \gamma_5 \frac{\vec{\pi}}{2} \psi \cdot [\vec{\pi} \partial^\mu \sigma - \sigma \partial^\mu \vec{\pi}]\}, \]

(3)

which is introduced so as to allow \( g_A \) to differ from 1 at the tree level. We here treat the \( \sigma \)-field as an effective representation of the isoscalar component of the interacting \( \pi \pi \) system.

The Lagrangian (1) implies that the vacuum breaks chiral symmetry spontaneously (\( m_0^2 > 0 \)). In the vacuum \( \langle \sigma \rangle = f_\pi \) (the pion decay constant). The dynamical isoscalar \( \sigma \)-field is defined as \( \sigma' = \sigma - f_\pi \). We here view the field \( \sigma' \) as an effective representation of the isoscalar part of the interacting \( \pi \pi \) system.

The parameters \( g \) and \( C \) in (2) and (3) are determined as [9]

\[ g_A = 1 + Cf_\pi^2 = 1.262, \]

(4a)

\[ g = G/g_A, \]

(4b)

where \( G \) is the usual \( \pi NN \) (pseudoscalar) coupling constant (\( \approx 13.5 \)).

The effective \( \pi NN \) and \( \sigma NN \) couplings that are implied by the Lagrangian model (1) are

\[ L_{\pi NN} \simeq -iG\bar{\psi} \vec{\pi} \cdot \vec{\pi} \gamma_5 \psi, \]

(5a)

\[ L_{\sigma NN} \simeq \frac{G}{g_A} \bar{\psi} \sigma' \psi. \]

(5b)

The \( \pi \sigma \) axial vector coupling is determined by the axial current operator [9]:

\[ A^{\dagger}_\mu = g_A\bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^i}{2} \psi + \pi^i \partial_\mu \sigma' - \sigma' \partial_\mu \pi^i - f_\pi \partial_\mu \pi^i. \]

(6)
Using these expressions the $\pi\sigma$-axial exchange current operator is found to be

$$
\vec{A}_i = \frac{G^2}{m_Ng_A}(\vec{k}_1 - \vec{k}_2)\left\{\vec{\sigma}^2 \cdot \vec{k}_2 \tau_i^2 \left(\frac{m^2_\pi}{k_2^2 + m^2_\pi}(k_1^2 + m^2_\sigma) - \frac{m^2_{\pi}}{k_1^2 + m^2_\sigma}(k_2^2 + m^2_\sigma)\right)\right\}. 
$$

Here $m_N$ is the nucleon, $m_\pi$ the pion and $m_\sigma$ the mass of the effective scalar meson. The momentum variables $\vec{k}_1$ and $\vec{k}_2$ denote the momentum transfers to nucleons 1 and 2. Note that the corresponding charge operator is proportional to the difference between the energies of the two mesons, both of which are small, and hence that operator is insignificant in comparison to the axial exchange charge operators associated with both pion [10] and short range exchange [11] mechanisms.

To assess the importance of the $\pi\sigma$-exchange current (7) we shall reduce it to the form of a correction to the effective single nucleon axial current operator:

$$
\vec{A}_i = -g_A(1 - \delta_A)\vec{\sigma} \cdot \vec{\sigma} - \frac{g_P(1 - \delta_P)}{2m_Nm_\mu}\vec{q} \cdot \vec{q} \cdot \vec{\tau}_i^2.
$$

Here $m_\mu$ is the muon mass. The terms 1 in the factors $(1 - \delta_A)$ and $(1 - \delta_P)$ represent the current of a free nucleon and the terms $\delta_A$ and $\delta_P$ represent the exchange current contribution. Note that the induced pseudoscalar coupling constant $g_P$ is related to $g_A$ as

$$
g_P = -\frac{2m_Nm_\mu}{q^2 + m^2_\pi}g_A
$$

for free nucleons.

The reduction of the $\pi\sigma$-axial exchange current to an effective single nucleon current operator leads to a direct term and an exchange term. Since $\vec{q} = \vec{k}_1 + \vec{k}_2$ the direct term is readily found to contribute only to the induced pseudoscalar current, the magnitude of the contribution being
\[ \delta_P = \frac{G^2}{g_A^2 m_N m_\sigma^2} \rho, \]  
\[ \text{(10)} \]

where \( \rho \) is the nucleon density. This result was first obtained in ref. [9] as the nuclear renormalization correction to the pion decay constant. The other mechanisms that contribute to \( \delta_P \) have been discussed recently in ref. [12].

The exchange term leads to a contribution to \( \delta_A \). Taking \( \vec{q} = 0 \) (i.e. \( \vec{k}_1 = -\vec{k}_2 \)) we find, using the Fermi gas model, this contribution to be

\[ \delta_A = \frac{G^2}{g_A^2 m_N m_\sigma^2} \rho I, \]  
\[ \text{(11)} \]

where \( I \) is the integral

\[ I = 4x_\sigma^2 \int_0^1 dx \frac{x^4(1-x)}{(x^2 + x_\pi^2)(x^2 + x_\sigma^2)} \]

\[ = 2x_\sigma^2 + \frac{4x_\sigma^2}{x_\sigma^2 - x_\pi^2} \{ x_\pi^3 \arctan \frac{1}{x_\pi} - x_\sigma^3 \arctan \frac{1}{x_\sigma} \}

- \frac{x_\pi^4}{2} \log(1 + \frac{1}{x_\pi^2}) + \frac{x_\sigma^4}{2} \log(1 + \frac{1}{x_\sigma^2}) \}. \]  
\[ \text{(12)} \]

with

\[ x_\pi \equiv m_\pi/2k_F, \quad x_\sigma \equiv m_\sigma/2k_F, \]  
\[ \text{(13)} \]

where \( k_F \) is the Fermi momentum. In the limit of a zero pion and an infinite \( \sigma \)-meson mass the integral \( I \) takes the value 1/3. To obtain an estimate for the first factor in (11) we take \( m_\sigma \approx m_N \) and \( \rho = 2k_F^3/3\pi^2 \), with \( k_F = 1.40 \text{fm}^{-1} \), which is the value for normal nuclear matter. This qualitative estimate of the \( \pi\sigma \) contribution to the quenching of \( g_A \) yields the value \( \delta_A \approx 0.066 \). This value is quite close to the value calculated below using a realistic boson exchange potential model and taking into account the corresponding short range correlations.

In order to reduce the model dependence of the \( \pi\sigma \) axial exchange current operator (7) we note that it can be expressed in terms of the effective isospin
independent scalar $v^+_S$ and isospin dependent pseudoscalar $v^-_P$ components of the nucleon-nucleon interaction as

$$\vec{A}_i = -\frac{g_A}{m_N G^2} (\vec{k}_1 - \vec{k}_2) \{ v^+_S(k_1) v^-_P(k_2) \vec{\sigma} \cdot \vec{k}_2 \tau_i^2 \\
- v^+_S(\vec{k}_2) v^-_P(\vec{k}_1) \vec{\sigma}^1 \cdot \vec{k}_1 \tau_i^1 \}.$$  \hspace{1cm} (13)

Here $v^+_S(\vec{k})$ and $v^-_P(\vec{k})$ are the Fourier transforms of the scalar and pseudoscalar potential components of the nucleon-nucleon interaction defined as in [13]. Here we have replaced the factors $G^2/(k^2 + m^2)$ in (7) with $v^-_P(k)$ and the factors $(k^2 + m^2)^{-1}$ by $-g^2_A v^+_S(k)/G^2$. This interpretation of the meson propagators in (7) as the corresponding components of the nucleon-nucleon interaction is natural and allows us to determine the short range behaviour of the exchange current operator directly from the nucleon-nucleon model without the need to introduce ad hoc cut off parameters. With this expression for the $\pi\sigma$-exchange current the contribution to the quenching factor $\delta_A$ (11) is replaced by the expression

$$\delta_A = -\frac{16k^2_F}{m_N G^2 \rho} \int_0^1 dx x^4 (1 - x) v^-_P(2k_F x) v^+_S(2k_F x).$$ \hspace{1cm} (14)

Note that all terms in this expression with the exception of $m_N$ and $k_F$ are completely determined by the nucleon-nucleon interaction model.

If we construct the potential components $v^-_P$ and $v^+_S$ using the parametrized Paris potential [14] we obtain the value $-3.60 fm^4$ for the integral in (14) and correspondingly at nuclear matter density the value $\delta_A = 0.024$. Using the Bonn potential model [15] we find the value $-8.98 fm^4$ for the integral in (14) and thus the value $\delta_A = 0.060$ at nuclear matter density. These values are further reduced somewhat by short range correlations: if the spin averaged correlation function that is obtained by constructing the $G$-matrix for these two potential models is taken into account in the integrals in (14) the numerical values for $\delta_A$ are reduced to $\delta_A = 0.023$ (Paris) and 0.055 (Bonn) respectively.

The very small value for $\delta_A$ that is obtained with the Paris potential model is a consequence of the anomalous repulsive behaviour of the effective isospin independent scalar potential component at short distances in that
model [13]. We therefore view the larger value obtained with the Bonn potential to be the more realistic one. The conclusion is then that the effective $\pi\sigma$ axial exchange current causes a $\approx 5\%$ quenching of the axial current coupling constant $g_{GT}^A$ at nuclear matter densities. This is somewhat larger than the corresponding contribution to the quenching caused by the $\pi\rho$ exchange mechanism [17].

If the 5% contribution of the $\pi\sigma$-axial exchange current to the quenching of $g_{GT}^A$ is combined with the 20-23% quenching due to the $\Delta_{33}$ excitation, $\pi\rho$ exchange and short range exchange currents obtained in ref. [7] the predicted net exchange current caused quenching of $g_{GT}^A$ becomes 25-28% at nuclear matter density. If this number is combined with the large quenching (40%) estimated to arise from second order configuration mixing in ref. [1] the predicted quenching of $g_{GT}^A$ is about 2 times too large. As the latter effect was calculated in ref. [1] on the basis of the Hamada-Johnston potential [18], which has a very strong isospin dependent tensor component, it would be important that this calculation be redone with a modern weaker tensor potential in order to obtain a quantitative estimate for a weaker core polarization contribution. Another reason to believe that the second order configuration mixing contribution to $g_{GT}^A$ in ref. [1] may be unrealistically large is the fact that the effective tensor force in the medium is also somewhat weakened by a core polarization induced screening mechanism [19].

The $\pi\sigma$ axial exchange current operator (7), (13) will also contribute to the exchange current correction to the transition rate for $\beta$-decay of $^3H$. To obtain an estimate for the magnitude of this contribution one may compare the expression (7) for the $\pi\sigma$ axial exchange current operator to the corresponding one for the $\pi\rho$ axial exchange current [20]:

\begin{equation}
\end{equation}
\[
\vec{A}_i(\pi\rho) = -g_A \frac{g_\rho^2}{m_N} \frac{\vec{\sigma}^2 \cdot \vec{k}_2}{(m_\rho^2 + k_1^2)(m_\pi^2 + k_2^2)} (\vec{\tau}^1 \times \vec{\tau}^2)_i. \\
\{(1 + \kappa)\vec{\sigma} \times \vec{k}_1 - i(\vec{p}_{1}' + \vec{p}_{1}^\prime)\} + (1 \leftrightarrow 2).
\]

Here \(g_\rho\) is the \(\rho NN\) vector coupling constant, \(\kappa\) the \(\rho NN\) tensor coupling and \(m_\rho\) is the \(\rho\)-meson mass. The symbol \((1 \leftrightarrow 2)\) stands for a term in which all particle coordinates are exchanged. The contribution of this operator to the \(\beta\)-decay rate of \(^3H\) has been found to be only about 0.7% in a calculation based on several realistic wavefunctions [20].

The local part of the \(\pi\rho\) axial exchange current operator is very similar in form to that of the \(\pi\sigma\) axial exchange current operator (7). Moreover the numerical values of the coefficients in front are also of similar order of magnitude: \(G^2/g_A \simeq 145\) versus \(g_A g_\rho^2 (1 + \kappa) \simeq 60\) (with \(g_\rho^2/4\pi = 0.5, \kappa = 6.6\)). As, however, only the component of (7) that is antisymmetric in isospin contributes to the \(\beta\)-decay of \(^3H\) the factor 145 above should be divided by 2 before the comparison, and thus the numerical coefficients in the \(\pi\sigma\) and \(\sigma\rho\) axial exchange current operators differ by only about 20%. Finally the mass of the "effective" \(\sigma\)-meson in the nucleon-nucleon interaction is close to that of the \(\rho\)-meson. Although the matrix elements of the two operators between the \(^3H\) and \(^3He\) are not identical it is nevertheless evident from this comparison that the contribution of the \(\pi\sigma\) axial exchange current to the rate for \(\beta\)-decay of \(^3H\) cannot be much larger than that of the \(\pi\rho\) axial exchange current (which is very small). A quantitative calculation of that reaction would nevertheless be important, as this mechanism will also contribute to the solar neutrino reactions \(pp \rightarrow d + e^+ + \nu_e\) and \(p + ^3He \rightarrow ^4He + e^+ + \nu_e\). As in the latter reaction the \(\pi\rho\) exchange current operator has found to be significant, we expect the \(\pi\sigma\) axial exchange current operator to be of equal significance [20].
1. A. Arima et al., Adv. Nucl. Phys. 18 (1987) 1
2. F. Osterfeld, Rev. Mod. Phys. 64 (1992) 491
3. B.A. Brown and B.H. Wildenthal, Phys. Rev. C28 (1983) 2397
4. C. Gaarde et al., Nucl. Phys. A334 (1980) 24
5. M. Rho, Nucl. Phys. A231 (1974) 493
6. K. Ohta and M. Wakamatsu, Nucl. Phys. A234 (1974) 445
7. K. Tsushima and D.O. Riska, Nucl. Phys. A549 (1992) 313
8. B.W. Lee, Chiral Dynamics, Gordon and Breach, New York (1972)
9. E.Kh. Akhmedov, Nucl. Phys. A500 (1989) 596
10. K. Kubodera, J. Delorme and M. Rho, Phys. Rev. Lett. 40 (1978) 755
11. M. Kirchbach, D.O. Riska and K. Tsushima, Nucl. Phys. A542 (1992) 616
12. M. Kirchbach and D.O. Riska, The Effective Induced Pseudoscalar Coupling Constant, Preprint HU-TFT-93-46 (1993)
13. P.G. Blunden and D.O. Riska, Nucl. Phys. A536 (1992) 697
14. M. Lacombe et al., Phys. Rev. C21 (1980) 861
15. R. Machleidt, K. Holinde and Ch. Elster, Phys. Repts 149 (1987) 1
16. M. Hjorth-Jensen, M. Kirchbach, D.O. Riska and K. Tsushima, Nucl. Phys. A563 (1993) 525
17. K. Tsushima, D.O. Riska and P.G. Blunden, Nucl. Phys. A559 (1993) 543
18. T. Hamada and I.D. Johnston, Nucl. Phys. 34 (1962) 382
19. K. Nakayama, Phys. Lett. 165B (1985) 239
20. J. Carlson, D.O. Riska, R. Schiavilla and R.-B. Wiringa, Phys. Rev. C44 (1991) 619

Figure Caption

Fig. 1. Diagrammatic representation of the $\pi\sigma$ axial exchange current operator
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