Positive and Negative Goos–Hänchen Shifts and Negative Phase–Velocity Mediums (alias Left–Handed Materials)

Akhlesh Lakhtakia, CATMAS—Computational & Theoretical Materials Science Group, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, PA 16802–6812, USA.
E–mail: AXL4@psu.edu

On total reflection from a half–space filled with an isotropic, homogeneous, weakly dissipative, dielectric–magnetic medium with negative phase–velocity (NPV) characteristics, it is shown here that a linearly polarized beam can experience either a negative or a positive Goos–Hänchen shift. The sign of the shift depends on the polarization state of the beam as well as on the signs of the real parts of the permittivity and the permeability of the NPV medium.

1 Introduction

This communication addresses the topic of Goos–Hänchen shifts of beams at the specularly flat interface of two isotropic dielectric–magnetic mediums, one of which displays negative phase–velocity (NPV) characteristics and the other displays positive phase–velocity (PPV) characteristics. While positive phase velocities are commonly encountered [1], negative phase velocities are permitted by the structure of the Maxwell postulates and several instances have been recorded [2].

Negative Phase Velocity: Isotropic dielectric–magnetic materials with negative phase velocity — i.e., phase velocity opposed in direction to the time–averaged Poynting vector — have attracted much attention of late [3]. Over three decades ago, Veselago [4] suggested many unusual properties of materials with negative real permittivity and negative real permeability at a certain frequency, including inverse refraction, negative radiation pressure, and inverse Doppler effect. But his suggestion was completely speculative until a breakthrough was announced by Smith et al. [5] in 2000. Many names have been proposed for these materials [3] — of which the most inappropriate is left–handed material. The least ambiguous of all extant names is NPV material. Two recent publications are recommended for ongoing developments on NPV materials [3, 6].

Goos–Hänchen Shift: If a beam of light were to impinge on a planar interface with an optically rarer dielectric material and total reflection were to occur, Newton had conjectured that the reflected beam would be displaced forward by a distance d parallel to the interface [7]; see Figure 1. Performing an ingenious experiment some two centuries later, Goos and Hänchen were able to prove Newton correct [8, 9]. Since then, these shifts have been estimated as well as measured for planar interfaces between several different pairs of homogeneous materials [10, 11]. Although Figure 1 shows a positive shift, negative shifts are also possible [12] — without violation of causality — as shown in Figure 2. The significance of Goos–Hänchen shifts has grown with the emergence of near–field optical microscopy and lithography [13].

The vast majority of publications on Goos–Hänchen shifts deal with dielectric mediums. The role of permeability appears to have been largely ignored. But per-
meability is virtually of the same status as permittivity for NPV materials.

Reversal of the signs of the real parts of both the permittivity and the permeability of the optically rarer medium has been proved to result in the reversal of the sign (and, therefore, the direction) of the Goos–Hänchen shift \[14\]. Furthermore, if the optically rarer medium has negative real permittivity and negative real permeability, the Goos–Hänchen shifts of both perpendicularly and parallel polarized beams have been shown to be negative \[15\]. However, a NPV material need not have both of those constitutive quantities as negative \[16\], which prompted the research reported here.

2 Theoretical preliminaries

Consider the planar interface of two homogeneous mediums labeled \(a\) and \(b\), with respective relative permittivities \(\varepsilon_{a,b} = \varepsilon'_{a,b} + i\varepsilon''_{a,b}\) and relative permeabilities \(\mu_{a,b} = \mu'_{a,b} + i\mu''_{a,b}\) at the angular frequency \(\omega\) of interest. Medium \(a\) is supposed to be nondissipative (i.e., \(\varepsilon''_{a} = 0\) and \(\mu''_{a} = 0\)), while dissipation in medium \(b\) is assumed to be very small (i.e., \(\varepsilon''_{b} \ll \varepsilon'_{b}\) and \(\mu''_{b} \ll \mu'_{b}\)) for the sake of simplicity. The incident and the reflected beams lie in medium \(a\), and an \(\exp(-i\omega t)\) time-dependence is implicit.
The planewave reflection coefficients for this situation are given by

\[ r_\parallel = \frac{1 - (\alpha_a/\alpha_b)(\epsilon_b/\epsilon_a)}{1 + (\alpha_a/\alpha_b)(\epsilon_b/\epsilon_a)} \]  

(1)

for parallel polarization, and

\[ r_\perp = \frac{1 - (\alpha_a/\alpha_b)(\mu_b/\mu_a)}{1 + (\alpha_a/\alpha_b)(\mu_b/\mu_a)} \]  

(2)

for perpendicular polarization. In these equations, \( \alpha_{a,b} = k_0 \sqrt{\epsilon_{a,b} \mu_{a,b} - \epsilon_a \mu_a \sin^2 \theta_{inc}} \) involve the free–space wavenumber \( k_0 \) and the angle of incidence \( \theta_{inc} \in [0, \pi/2) \). We must choose \( \text{Im} [\alpha_b] \geq 0 \), as befits any passive medium.

The phase velocity vector opposes the direction of the time–averaged Poynting vector in medium \( b \), whenever the inequality

\[ (|\epsilon_b| - \epsilon'_b) (|\mu_b| - \mu'_b) > \epsilon''_b \mu''_b \]  

(4)

holds [16]. Thus, the simultaneous satisfaction of both \( \epsilon'_b < 0 \) and \( \mu'_b < 0 \) is a sufficient, but not necessary, requirement for the phase velocity to be negative. In other words, both \( \epsilon'_b \) and \( \mu'_b \) do not have to be negative for the phase velocity to be negative.

### 3 Analysis and conclusions

When investigating Goos–Hänchen shifts, it is best to write the reflection coefficients as

\[ r_{\parallel,\perp} = |r_{\parallel,\perp}| \exp(i\varphi_{\parallel,\perp}) . \]  

(5)

As dissipation in medium \( b \) is assumed here to be very small, the reflection coefficients are essentially of unit amplitude when \( \theta_{inc} \) exceeds the critical angle. Artmann [17] showed that Goos–Hänchen shifts can then be be estimated as

\[ d_{\parallel} = -\frac{\partial \varphi_{\parallel}}{\partial \kappa}, \quad d_{\perp} = -\frac{\partial \varphi_{\perp}}{\partial \kappa}, \]  

(6)

where \( \kappa = k_0 \sqrt{\epsilon'_a \mu'_a \sin \theta_{inc}} \). The foregoing expressions are not adequate when \( \theta_{inc} \) is very close to either the critical angle or \( \pi/2 \), but suffice for the present purpose.

With \( \epsilon_{a,b}, \mu_{a,b} \) and \( \alpha_a \) real–valued, \( \alpha_b \) is purely imaginary when the angle of incidence exceeds the critical angle. Then, the expressions

\[ \varphi_{\parallel} = -2 \tan^{-1} \left( \frac{\alpha_a \epsilon_b}{|\epsilon_b| \epsilon_a} \right) \]  

(7)

and

\[ \varphi_{\perp} = \pi - 2 \tan^{-1} \left( \frac{\alpha_a \mu_b}{|\mu_b| \mu_a} \right) \]  

(8)
follow from (1) and (2). Reversal of the sign of \( \epsilon_b \) would certainly affect \( |\alpha_b| \), but a more significant effect shall be on the sign of \( \varphi_b \parallel \) (in contrast to \( \varphi_b \perp \)) and therefore on the sign of \( d_b \parallel \). Likewise, reversal of the sign of \( \mu_b \) would change the signs of \( \varphi_b \perp \) and \( d_b \perp \). This understanding should hold also when medium \( b \) is weakly dissipative (i.e., \( \epsilon''_b \ll \epsilon'_b \) and \( \mu''_b \ll \mu'_b \)).

We can conclude the following: Let the medium of incidence and reflection be nondissipative and of the PPV type, and the refracting medium be optically rarer as well as of the NPV type with weak dissipation. When the conditions for total reflection prevail, the Artmann expressions (6) suggest that negative Goos–Hänchen shifts are possible for

- only parallel–polarized beams if the real part of the permittivity of the NPV medium is negative,
- only perpendicularly polarized beams if the real part of the permeability of the NPV medium is negative, and
- both parallel– and perpendicularly polarized beams if the real parts of both the permittivity and the permeability of the NPV medium are negative.

References

[1] Chen, H.C.: Theory of Electromagnetic Waves. Fairfax, VA, USA: TechBooks, 1993.
[2] Pendry, J.B.: Introduction. Opt. Exp. 11 (2003), 639.
[3] Lakhtakia, A.; McCall, M.W.; Weiglhofer, W.S.: Brief overview of recent developments on negative phase–velocity mediums (alias left–handed materials). Arch. Elektr. Über. 56 (2002), 407–410.
[4] Veselago, V.G.: The electrodynamics of substances with simultaneously negative values of \( \epsilon \) and \( \mu \). Sov. Phys. Usp. 10 (1968), 509–514.
[5] Smith, D.R.; Padilla, W.J.; Vier, D.C.; Nemat–Nasser, S.C.; Schultz, S.: Composite medium with simultaneously negative permeability and permittivity. Phys. Rev. Lett. 84 (2000), 4184–4187.
[6] The April 7, 2003 issue of Optics Express, available to any netizen at http://www.opticsexpress.org/issue.cfm?issue_id=186, is focused on NPV materials.
[7] Lotsch, H.K.V.: Beam displacement at total reflection: The Goos–Hänchen effect, I. Optik 32 (1970), 116–137.
[8] Goos, F.; Hänchen, H.: Ein neuer und fundamentaler Versuch zur Totalreflexion. Ann. Phys. Lpz. 1 (1947), 333–346.
[9] Goos, F.; Lindberg–Hänchen, H.: Neumessung des Strahlversetzungseffektes bei Totalreflexion. Ann. Phys. Lpz. 5 (1949), 251–252.

[10] Haibel, A.; Nimtz, G.; Stahlhofen, A.A.: Frustrated total reflection: the double– prism revisited. Phys. Rev. E 63 (2001), 047601.

[11] Depine, R.A.; Bonomo, N.E.: Goos–Hänchen lateral shift for Gaussian beams reflected at achiral–chiral interfaces. Optik 103 (1996), 37–41.

[12] Aničin, B.A.; Fazlić, R.; Koprić, M.: Theoretical evidence for negative Goos–Häenchen shifts. J. Phys. A: Math. Gen. 11 (1978), 1657–1662.

[13] de Fornel, F.: Evanescent Waves. Berlin: Springer, 2001.

[14] Lakhtakia, A.: On planewave remittances and Goos–Hänchen shifts of planar slabs with negative real permittivity and permeability. Electromagnetics 23 (2003), 71–75.

[15] Berman, P.R.: Goos–Hänchen shift in negatively refractive media. Phys. Rev. E 66 (2002), 067603.

[16] McCall, M.W.; Lakhtakia, A.; Weiglhofer, W.S.: The negative index of refraction demystified. Eur. J. Phys. 23 (2002), 353–359.

[17] Artmann, K.: Berechnung der Seitenversetzung des totalreflektierten Strahles. Ann. Phys. Lpz. 2 (1948), 87–102.