The Schrödinger Wave Functional and S-branes

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ABSTRACT: In this paper we will consider the minisuperspace approach to S-branes dynamics in the Schrödinger picture description. Time-evolution of vacuum wave functional for quantum field theory on S-brane is studied. Open string pair production is calculated. The analysis of density matrix for mixed states is also performed.

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1. Introduction

In [1] new string theory ingredients called S (spacelike)-branes were introduced. Whereas ordinary D-branes can be realised as timelike kinks and vortices of the tachyon field [2, 3, 4], spacelike defects can be defined as spacelike kinks and vortices in the background of a time-dependent tachyon condensation process called rolling tachyon [1, 5, 6, 7]. These S-branes can be thought of as the creation and subsequent decay of an unstable brane. This process has recently attracted much attention [1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47]. S-branes have also been extensively studied in supergravity approach with potentially interesting cosmological applications [50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72].

Very nice worldsheet construction of S-branes was given in the classical $g_s = 0$ limit by A. Sen [5, 6, 7] where he introduced class of models in bosonic string theory obtained by perturbing the flat space $c = 26$ CFT with the exactly marginal deformation

$$S_{\text{bond}} = \lambda \int d\tau \cosh X^0(\tau),$$  \hspace{1cm} (1.1)

where $X^0$ is time coordinate, $t$ is a coordinate on the worldsheet boundary and $\lambda$ is a free parameter in the range $0 \geq \lambda \leq \frac{1}{2}$. This is family of exact solutions of classical

\footnote{We work in units $\alpha' = 1$.}
open string theory whose space-time interpretation is that of an unstable brane being created at a time $X^0 \sim -\tau$ and decaying at a time $X^0 \sim \tau$ with $\tau = -\log(\sin(\pi \lambda))$.

As was shown in [8, 9, 11] this time-dependent process of tachyon condensation that in fact defines S-brane has many intriguing properties. In particular, it is known that in time-dependent backgrounds there is in general no preferred vacuum and particle production is unavoidable. In [10] the open string vacua on S-brane were studied. It was shown that for (1.1) there is open string pair production with a strength characterised by the Hagedorn temperature $T_H = \frac{1}{4\pi |g_s|}$. As was argued [8, 9, 11] this temperature arises from the periodicity of the boundary interaction (1.1) in imaginary time $^3$.

In [8, 9, 10] S-brane vacua and their properties were studied in minisuperspace approximation where all open string modes acquire exponentially growing masses from tachyon background. The result is open string pair production. It was shown in [8] that for any nonzero $g_s$ this production is typically divergent. The divergent density of open strings with exponentially growing masses will couple strongly to closed strings suggesting that S-brane density is quickly released into closed strings.

The minisuperspace description of S-branes studied in [8, 9, 10] was performed in the Heisenberg picture description of quantum field theory where we treat quantum fields as Heisenberg operators that are time-dependent while quantum states do not evolve. As companion to this approach we will analyse the minisuperspace approach of S-brane using the Schrödinger formulation of quantum field theory. The Schrödinger picture of quantum field theory provides a simple and intuitive description of vacuum states in quantum field theory in situations where the background metric is time-dependent or in case where some parameters of quantum field theory depend on time. The Schrödinger picture characterises vacuum states explicitly by simple wave functional specified single, possibly time-dependent, kernel function satisfying a differential equation with prescribed boundary conditions. This makes no reference to the assumed spectrum of excited states and so circumvents the difficulties of the conventional canonical description of a vacuum as a ”no-particle” state with respect to the creation and annihilation operators defined by a particular mode decomposition of the field, an approach which is not well suited to the time-dependent problems as for example minisuperspace description of S-branes.

The wave functional that defines the vacuum state satisfies a functional Schrödinger equation describing its time evolution. We will explicitly construct Gaussian vacuum state functional for quantum field theory on half S-brane and on S-brane. We choose the initial condition in such a way that the vacuum wave functional approaches standard Gaussian flat space functional in the far past $t \to -\infty$ in case of half S-brane, or that approaches Minkowski flat space vacuum functional for $t \to 0$ in case of S-brane.

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3The appearance of Hagedorn temperature signals a breakdown of string perturbation theory. To avoid this problems we can work at the limit of vanishing string coupling constant $g_s = 0$. 

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Then we will study the aspect of particle production during time evolutions of these vacuum states. Next we turn to the S-brane thermodynamics. We will construct the density matrix for mixed states. Since the rolling tachyon background is time-dependent process the S-branes are highly time-dependent configurations. However temperature is an equilibrium (or at best adiabatic) concept, so it is usually does not to make sense to put a time-dependent configuration at finite temperature. The standard procedure applied to the half S-brane is to assume that at time $t_0 = -\infty$ the initial density matrix is thermal with the temperature $T = 1/\beta$. It is only in this initial state that the notion of temperature is meaningful. As the system departs from equilibrium one cannot define a thermodynamic temperature. We will construct these mixed states that approach standard thermal vacuum at past infinity. Following [10] we will show that there is sort of mixed states with temperature $T = 1/(2\pi n)$ that retain thermal periodicity at all times. For full S-brane we will proceed in the same way however we will demand that density matrix approaches standard thermal density matrix at $t_0 = 0$ when in the limit of $\lambda \to 0$ the interaction can be neglected and we have ordinary unstable D-brane.

This paper is organised as follows. In the next section (2) we review minisuperspace analysis of S-branes, following [10]. We will demonstrate on an example of half S-brane the fact that particle creation is natural process during unstable D-brane decay. In section (3) we will study the Schrödinger picture description of quantum field theory on half S-brane. We will explicitly construct the vacuum wave functional that approaches standard Minkowski vacuum wave functional at the asymptotic past $t \to -\infty$. We will calculate the number of particle produced during time evolution of this state and confirm the result [10] that open string particle production is natural process during D-brane decay. In section (4) we will apply Schrödinger picture description to the full S-brane. In section (5) we will study S-brane thermodynamics. We will construct density matrix for mixed states in Schrödinger approach for half S-brane and for full S-brane. We will also calculate the equal-time correlators in these mixed states. And finally in conclusion (6) we outline our results given in this paper.

2. Review of the minisuperspace approach

In this section we present a short review of superspace approach to the study of S-brane dynamics [3, 8, 10]. We will closely follow these papers.

We wish to understand the dynamics of the open string worldsheet theory with a time-dependent tachyon

$$ S = -\frac{1}{4\pi} \int_{\Sigma_2} d^2\sigma \partial^a X^\mu \partial_a X^\mu + \int_{\partial\Sigma_2} d\tau m^2 (X^0) . $$

(2.1)

For the open bosonic string $m^2 = T$ where $T$ is the spacetime tachyon, while for the open superstring $m^2 \sim T^2$ after integrating out worldsheet fermions. We use the
symbol $m^2$ to denote the interaction because the coupling (among other effects) im-
parts a mass to the open string states. We consider three interesting cases described 
by the marginal interactions

$$m^2_+(X^0) = \lambda \frac{e^{X^0}}{2}$$  \hspace{1cm} (2.2)

$$m^2_-(X^0) = \lambda \frac{e^{-X^0}}{2}$$ \hspace{1cm} (2.3)

$$m^2_s(X^0) = l \cosh X^0$$ \hspace{1cm} (2.4)

The first case $m^2_+$ describes the process of brane decay, in which an unstable brane 
decays via tachyon condensation. The second case describes the time-reverse process 
of brane creation, in which an unstable brane emerges from the vacuum. The final case describes an S-brane, which is the process of brane creation followed by brane 
decay. Brane decay can be thought of as the future (past) half of an S-brane, i.e. 
as the limiting case where the middle of the S-brane is pushed into the infinite past 
(future).

We will study S-brane dynamics using the minisuperspace analysis in which the 
effect of the interaction is simply to give a time-dependent shift to the masses of 
all the open string states. In the minisuperspace approximation only the zero-mode 
dependence of the interaction $m^2(X^0)$ is considered. In this case we can plug in 
the usual mode solution for the free open string with oscillator number $N$ to get an 
effective action for the zero modes

$$S = \int d\tau \left( -\frac{1}{4} \dot{x}^\mu \dot{x}_\mu + (N - 1) + 2m^2(X^0) \right).$$ \hspace{1cm} (2.5)

This is the action of a point particle with a time dependent mass. Here $x^\mu(\tau)$ is the 
zero mode part of $X^\mu(\sigma, \tau)$, and the second term in is an effective contribution from 
the oscillators, including the usual normal ordering constant. From upper action we 
can write down the Klein-Gordon equation for the open string wave function $\phi(t, x)$,

$$\left( \partial^\mu \partial_\mu - 2m^2(t) - (N - 1) \right) \phi(t, x) = 0,$$ \hspace{1cm} (2.6)

where $(t, x)$ are the spacetime coordinates corresponding to the worldsheet fields 
$(X^0, X^i)$. This is the equation of motion for a scalar field with time-dependent mass. 
At this point, we should make a few remarks about field theories with time-dependent mass $^4$. Time translation invariance has been broken, so energy is not conserved and 
there is no preferred set of positive frequency modes. This is a familiar circumstance 
in the study of quantum field theories in time-dependent backgrounds which leads to 
particle creation. The probability current $j_\mu = i(\phi^\ast \partial_\mu \phi - \partial_\mu \phi \phi^\ast)$ is still conserved, allowing us to define the Klein-Gordon inner product

$$\langle f | g \rangle = i \int_S d\Sigma^\mu (f^\ast \partial_\mu g - \partial_\mu f^\ast g)$$ \hspace{1cm} (2.7)

$^4$For review of quantum field theory in curved spacetime, see [73, 74, 75].
where $\Sigma$ is a spacelike slice. This norm does not depend on the choice of $\Sigma$ if $f$ and $g$ solve the wave equation. Normalised positive frequency modes are chosen to have $\langle f|f \rangle = 1$. Negative frequency modes are complex conjugates of positive frequency modes, with $\langle f^*|f \rangle = -1$. There is a set raising and lowering operators associated to each choice of mode decomposition – these operators obey the usual oscillator algebra if the corresponding modes are normalised with respect to (2.7). We also define a vacuum state associated to each mode decomposition – it is the state annihilated by the corresponding lowering operators. To illustrate this idea we give an example of the half S-brane corresponding to the decay of an unstable D-brane. In other words, we will consider previous equation with the mass term

$$m^2_+(t) = \frac{\lambda}{2}e^t . \quad (2.8)$$

Since in the far past $t \to -\infty$ the mass term $m^2_+(-\infty) = 0$, it is natural to take all open string modes in their usual ground state. In other words, we define $|\text{in}\rangle$ vacuum as a vacuum with no particle present. Expanding field in plane waves

$$\phi_p(x) \equiv \phi_p(t,x) = e^{ipx}u_p(t) \quad (2.9)$$

the wave equation becomes

$$(\partial_t^2 + \lambda e^t + \omega_p^2)u_p = 0 \ , \ \omega_p^2 = p^2 + N - 1 \equiv p^2 + m^2 . \quad (2.10)$$

This is a form of Bessel’s equation. It has normalised, positive frequency solutions

$$u_p^{\text{in}} = \lambda \omega_p \frac{\Gamma(1 - 2i\omega_p)}{\sqrt{2\omega_p}} J_{-2i\omega_p}(2\sqrt{\lambda e^{t/2}}) , \quad (2.11)$$

where superscript $\text{in, out}$ and 0 on a wave function denotes solutions that are purely positive frequency when $t \to -\infty, t \to 0$ or $t = 0$. These solutions have been chosen because they approach flat positive frequency plane waves in the far past $t \to -\infty$

$$u_p^{\text{in}} \sim \frac{1}{\sqrt{2\omega_p}} e^{-i\omega_p t} . \quad (2.12)$$

We will also consider the wave functions

$$u_p^{\text{out}} = \sqrt{\frac{\pi}{2}} (i e^{2\pi \omega_p})^{-1/2} H_{-2i\omega_p}^{(2)}(2\sqrt{\lambda e^{t/2}}) , \quad (2.13)$$

that are purely positive frequency in the far future $t \to \infty$

$$u_p^{\text{out}} \sim \frac{\lambda^{-1/4}}{\sqrt{2}} \exp \left\{ -t/4 - 2i \sqrt{\lambda e^{t/2}} \right\} . \quad (2.14)$$

Using these solutions we can define vacuum state in general time-dependent background. In the canonical framework we perform an expansion of scalar field in terms
of modes that are solutions of free wave equation and that have positive and negative norm with respect Klein-Gordon product. Then operators that come with positive modes are regarded as lowering operators that annihilate vacuum state and operators that come with negative norm modes are rising operators. More precisely, we have an expansion

$$\phi(x) = \sum_p a_{in,p} \phi_{in}^p(x) + a_{in,p}^\dagger \phi_{in}^{*p}(x),$$  \hspace{1cm} (2.15)

where $\phi_{in}^p(x) = e^{ipx}u_{in}^p(t)$ and $u_{in}^p(t)$ are given in (2.11). Then $|in\rangle$ vacuum state is defined as the state that is annihilated by all $a_{in,p}$

$$a_{in,p} |in\rangle = 0.$$  \hspace{1cm} (2.16)

In the same way we can write

$$\phi(x) = \sum_p a_{out,p} \phi_{out}^p(x) + a_{out,p}^\dagger \phi_{out}^{*p}(x),$$  \hspace{1cm} (2.17)

where $\phi_{out}^p(x) = e^{ipx}u_{out}^p(t)$ and $u_{out}^p(t)$ are given in (2.13). Now we define $|out\rangle$ vacuum state as the state that is annihilated by all $a_{out,p}$

$$a_{out,p} |out\rangle = 0.$$  \hspace{1cm} (2.18)

Generally $u_{out}^p$ and $u_{in}^p$ are related by celebrated Bogolubov transformations

$$u_{out}^p = A_p u_{in}^p + B_p u_{in}^{*p},$$  \hspace{1cm} (2.19)

where

$$A_p = e^{2\pi \omega_p + \pi i/2} B_p^* \sqrt{\frac{\lambda^{-i\omega_p}}{\sinh 2\pi \omega_p \Gamma(1 - 2i\omega_p)}}.$$  \hspace{1cm} (2.20)

These coefficients obey unitarity relation $|A_p|^2 - |B_p|^2 = 1$. From (2.19) we get the relation between in and out creation and annihilation operators

$$a_{in,p} = A_p a_{out,p} + B_p a_{out,p}^\dagger.$$  \hspace{1cm} (2.21)

From this, the condition $a_{in,p} |in\rangle = 0$ implies that $|in\rangle$ is squeezed state

$$|in\rangle = \prod_p (1 - |\gamma_p|^2)^{1/4} \exp \left\{ -\frac{1}{2} \gamma_p (a_{out,p}^\dagger)^2 \right\} |out\rangle, \gamma_p = B_p^*/A_p.$$  \hspace{1cm} (2.22)

Physically this means that particles are produced during brane decay: if we start in a state with no particles at $t \to -\infty$, there will be many particles at time $t \to \infty$ with the density of particles with momentum $p$

$$N_p = |B_p|^2 = \frac{1}{e^{\omega_p/T_H} - 1}, T_H = \frac{1}{4\pi}.$$  \hspace{1cm} (2.23)
Despite the fact that $|\text{in}\rangle$ is pure state the particle density at far future is the same as thermal density of particles at temperature $T_H = 1/4\pi$. In string units, $T_H$ is the Hagedorn temperature. As was argued in [8, 10, 15, 21] the appearance of the Hagedorn temperature signals a breakdown of string perturbation theory. To find mechanisms which cuts off this divergence is interesting problem which we will not try to solve in this paper. To avoid this problem we will work, following [10] at $g_s = 0$.

In the next section we will formulate the Schrödinger picture description of the quantum field theory on S-brane which is especially useful for the study of quantum field theory in time-dependent background.

### 3. QFT in Schrödinger picture

In this section we introduce Schrödinger formalism for QFT on half S-brane. We follow mainly [76, 77, 78, 79]. We begin with an action for massive scalar field which describes open string modes on Sp-brane in the minisuperspace approach\(^5\)

$$S = \int dtdx L = -\frac{1}{2} \int dtdx \left( -\partial_t \phi \partial_t \phi + \eta^{ab} \partial_a \phi \partial_b \phi + m^2(t) \phi^2 \right).$$  \hfill (3.1)

The canonical momentum conjugate to $\phi(t, x)$ is

$$\pi(t, x) = \frac{\delta L}{\delta \partial_t \phi(t, x)} = \partial_t \phi(t, x)$$ \hfill (3.2)

and the Hamiltonian

$$H = \int dx (\pi \partial_t \phi - L) = \frac{1}{2} \int dx \left( \pi^2 + \eta^{ab} \partial_a \phi \partial_b \phi + m^2 \phi \right).$$ \hfill (3.3)

The system can be canonically quantized by treating the fields as operators and imposing appropriate commutation relations. This involves choice of a foliation of a spacetime in a succession of spacelike hypersurfaces. We choose these to be the hypersurfaces of fixed $t$ and impose equal-time commutation relations

$$[\hat{\phi}(x, t), \hat{\pi}(y, t)] = i\delta(x - y), [\hat{\phi}(x, t), \hat{\phi}(y, t)] = [\hat{\pi}(x, t), \hat{\pi}(y, t)] = 0.$$ \hfill (3.4)

In the Schrödinger picture we take the basis vector of the state vector space to be the eigenstate of the field operator $\hat{\phi}(t, x)$ on a fixed $t$ hypersurface, with eigenvalues $\phi(x)$

$$\hat{\phi}(t, x) |\phi(x), t\rangle = \phi(x) |\phi(x), t\rangle.$$ \hfill (3.5)

Notice that the set of field eigenvalues $\phi(x)$ is independent of the value of $t$ labelling the hypersurface. In this picture, the quantum states are explicit functions of time

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\(^5\)Our convention is $\eta_{\mu\nu} = \text{diag}(-1, \ldots, 1)$, $\mu, \nu = 0, \ldots, p + 1$, $a, b, c, \ldots = 1, \ldots, p + 1$. We also denote $x^0 = t$ and $x = (x^1, \ldots, x^{p+1})$.\]
and are represented by wave functionals $\Psi[\phi(\mathbf{x}), t]$. Operators $\hat{O}(\hat{\pi}, \hat{\phi})$ acting on these states may be represented by

$$\hat{O}(\hat{\pi}(\mathbf{x}), \hat{\phi}(\mathbf{x})) = \mathcal{O} \left( -i \frac{\delta}{\delta \phi(\mathbf{x})}, \phi(\mathbf{x}) \right).$$

(3.6)

The Schrödinger equation which governs the time evolution of the wave functional is

$$i \frac{\partial \Psi[\phi, t]}{\partial t} = H \left( -i \frac{\delta}{\delta \phi(\mathbf{x})}, \phi(\mathbf{x}) \right) \Psi[\phi, t] = \frac{1}{2} \int d\mathbf{x} \left[ -\frac{\delta^2}{\delta \phi^2} + \eta_{ab} \partial_a \phi \partial_b \phi + m^2(t) \phi^2 \right] \Psi[\phi, t].$$

(3.7)

To solve this equation, we make the ansatz, that up to time-dependent phase, the vacuum functional is simple Gaussian. We therefore write

$$\Psi_0[\phi, t] = N_0(t) \exp \left\{ -\frac{1}{2} \int d\mathbf{x} \mathbf{y}(\mathbf{x}) G(\mathbf{x}, \mathbf{y}, t) \phi(\mathbf{y}) \right\} = N_0(t) \psi_0(\phi, t).$$

(3.8)

After inserting (3.8) into (3.7) and comparing the terms of order $\phi^0$ we get

$$i \frac{\partial N_0(t)}{\partial t} = \frac{N_0(t)}{2} \int d\mathbf{z} G(\mathbf{z}, \mathbf{z}, t).$$

(3.9)

Comparing terms of order $\phi^2$ we obtain

$$i \frac{\partial G(\mathbf{x}, \mathbf{y}, t)}{\partial t} = \int d\mathbf{z} G(\mathbf{z}, \mathbf{x}, t) G(\mathbf{y}, \mathbf{z}, t) - \left( \eta_{ab} \partial_a \phi \partial_b \phi + m^2(t) \right) \delta(\mathbf{x}, \mathbf{y}).$$

(3.10)

Since the spatial sections on S-branes are flat it is natural to perform a Fourier transformation on the space dependence of the field configuration

$$\phi(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{p+1}} e^{i \mathbf{k} \cdot \mathbf{x}} \alpha(\mathbf{k}).$$

(3.11)

Similarly we can define $\delta / \delta \alpha(\mathbf{k})$ as

$$\frac{\delta}{\delta \phi(\mathbf{x})} = \int \frac{d\mathbf{k}}{(2\pi)^{p+1}} e^{i \mathbf{k} \cdot \mathbf{x}} \frac{\delta}{\delta \alpha(\mathbf{k})}. \quad \quad \quad (3.12)$$

Reality of $\phi$ implies $\alpha^*(\mathbf{k}) = \alpha(-\mathbf{k})$. By definition the functional derivative obeys

$$\frac{\delta \phi(\mathbf{x})}{\delta \phi(\mathbf{x}')} = \delta(\mathbf{x} - \mathbf{x}')$$

(3.13)

which implies

$$\frac{\delta \alpha(\mathbf{k})}{\delta \alpha(\mathbf{k}')} = (2\pi)^{p+1} \delta(\mathbf{k} + \mathbf{k}') \quad \quad \quad (3.14)$$
Functional integrals in the k-space variables require extra care because of the constraint \( \alpha^*(k) = \alpha(-k) \). The path integral measure \( \int d\phi \) is rewritten as

\[
\int d\alpha(k) d\alpha^*(k) \delta(\alpha(-k) - \alpha^*(k)) .
\] (3.15)

In the expression which results, it is simplest to perform all partial functional derivatives before carrying out the functional integrations. Then \( \alpha^*(k) \) integration will simply kill the \( \delta \) function and replace \( \alpha^*(k) \) integration by \( \alpha(-k) \). In \( k \)-space, the Hamiltonian is

\[
H = \frac{1}{2} \int \frac{dk}{(2\pi)^{p+1}} \left[ -\frac{\delta^2}{\delta \alpha(k) \delta \alpha(-k)} + \Omega_k^2(t) \alpha(k) \alpha(-k) \right] ,
\] (3.16)

where

\[
\Omega_k^2(t) = (m^2(t) - m^2) + \omega_k^2 , \omega_k^2 = k^2 + m^2 .
\] (3.17)

As expected in a free theory, a complete decoupling of modes has been achieved by the Fourier transformation. In fact, for each \( k \), the integrand in (3.16) represents a harmonic oscillator with the time dependent frequency \( \Omega_k^2(t) \). After performing the Fourier transformation for the kernel \( G(x, y, t) \)

\[
G(x, y, t) = \int \frac{dk}{(2\pi)^{p+1}} e^{ik(x-y)} \tilde{G}(k, t)
\] (3.18)

the kernel equation (3.10) reduces to

\[
i \frac{\partial \tilde{G}(k, t)}{\partial t} = \tilde{G}^2(k, t) - \Omega_k^2(t)
\] (3.19)

that can be solved with the ansatz

\[
\tilde{G}(k, t) = -i \frac{\dot{\psi}_k(t)}{\psi_k(t)} , \dot{f} = \frac{df(t)}{dt} ,
\] (3.20)

where \( \psi_k(t) \) obeys

\[
\ddot{\psi}_k + \Omega_k^2(t) \dot{\psi}_k = 0 .
\] (3.21)

Now the vacuum state functional has the form

\[
\Psi_0[\phi, t] = N_0(t) \exp \left( i \int \frac{dk}{(2\pi)^{p+1}} \alpha(-k) \frac{\dot{\psi}_k(t)}{\psi_k(t)} \alpha(k) \right) ,
\]

where

\[
\dot{N}_0(t) = -i \frac{N_0(t)}{2} V \int \frac{dk}{(2\pi)^{p+1}} \tilde{G}(k, t) \Rightarrow N_0(t) = N e^{-i \int^t dt' E_0(t')}
\] (3.23)

and

\[
E_0(t) = \frac{1}{2} V \int \frac{dk}{(2\pi)^{p+1}} \tilde{G}(k, t) .
\] (3.24)

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In the previous expression $V$ is volume of spatial section on $D(p+1)$-brane and $N$ is time-independent normalisation constant. It is important to stress that (3.21) is differential equation of the second order so that its general solution is linear superposition of two independent solutions and hence it is parameterised by two independent constants. On the other hand $\psi_k$ appears in $\tilde{G}_k$ in combination $\dot{\psi}_k \psi_k$ so that the overall normalisation factor is unimportant. As a result the vacuum functional contains one free parameter that is usually determined by initial conditions we impose on the vacuum wave functional.

Now we apply the Schrödinger picture description reviewed above to the case of the minisuperspace description of half S-brane. As we have seen in the previous section this approach leads to the equation of motion with time-dependent mass and consequently the time-dependent frequency that appears in (3.21) is equal to

$$\Omega^2_k(t) = \omega^2_k + \lambda e^t.$$  \hspace{1cm} (3.25)

The general solution of (3.21) is

$$\psi_k(t) = A_k \psi_{\text{in}}^k(t) + B_k \psi_{\text{in}}^{\star k}(t),$$  \hspace{1cm} (3.26)

where

$$\psi_{\text{in}}^k(t) = \frac{\lambda \Gamma(1 - 2i\omega_k)}{\sqrt{2\omega_k}} J_{2i\omega_k} (2\sqrt{\lambda} e^{t/2}).$$  \hspace{1cm} (3.27)

To fix the free parameter in the vacuum wave functional we demand that in the asymptotic past $t \to -\infty$ $\Psi_0[\phi, t]$ approaches the usual positive frequency Minkowski vacuum state

$$\psi_0(\phi, -\infty) = \exp \left( -\frac{1}{2} \int \frac{dk}{(2\pi)^{p+1}} |\alpha(k)|^2 \omega_k \right).$$  \hspace{1cm} (3.28)

This initial condition implies that $A_k = 0, B_k = 1$. We denote the vacuum state functional for QFT on half S-brane that approaches Minkowski vacuum in asymptotic past as $\Psi_{\text{in}}[\phi, t]$.

Now we would like to examine the behaviour of $\Psi_{\text{in}}[\phi, t]$ in the asymptotic future $t \to \infty$. It is convenient to express the kernel $\tilde{G}(k, t) = -i \frac{\dot{\psi}_k^{\text{in} \star}(t)}{\psi_k^{\text{in}}(t)}$ in terms of the solution of (3.10) $\psi_{\text{out}}^k$

$$\psi_{\text{out}}^k(t) = \sqrt{\frac{\pi}{2}} (ie^{2\pi \omega_k})^{-1/2} H_{2i\omega_k} (2\sqrt{\lambda} e^{t/2}),$$  \hspace{1cm} (3.29)

with simple asymptotic behaviour for $t \to \infty$

$$\psi_{\text{out}}^k \sim \frac{\lambda^{-1/4}}{\sqrt{2}} \exp \left( -t/4 - 2i\sqrt{\lambda} e^{t/2} \right).$$  \hspace{1cm} (3.30)

The relation between $\psi_{\text{in}}$ and $\psi_{\text{out}}$ modes is given by Bogolubov transformation

$$\psi_{\text{out}}^k = A_k \psi_{\text{in}}^k + B_k \psi_{\text{in}}^{\star k},$$

$$\psi_{\text{in}}^{\star k} = A_k \psi_{\text{out}}^{\star k} - B_k^{\star} \psi_{\text{out}}^k,$$  \hspace{1cm} (3.31)
where $A_k$, $B_k$ are Bogolubov coefficients

$$A_k = e^{2\pi \omega_k + \pi i/2} B_k^* = \sqrt{\omega_k \pi} e^{\pi \omega_k - \pi i/4} \left( \frac{\lambda - i \omega_k}{\sinh 2\pi \omega_k \Gamma(1 - 2i \omega_k)} \right).$$  \hspace{1cm} (3.32)

In time-dependent theory we are usually interested in the number of particles that are created during the time evolution of given state. Let us introduce following operators

$$A(k, t) = \frac{1}{\sqrt{2 \Omega_k(t)}} \left[ \frac{\delta}{\delta \alpha(k)} + \Omega_k(t) \alpha(k) \right]$$

$$A^\dagger(k, t) = \frac{1}{\sqrt{2 \Omega_k(t)}} \left[ -\frac{\delta}{\delta \alpha(-k)} + \Omega_k(t) \alpha(-k) \right]$$

so that the Hamiltonian can be written as

$$H(t) = \int \frac{dk}{(2\pi)^{p+1}} \Omega_k(t) \left[ A^\dagger(k, t) A(k, t) + \frac{V_{p+1}}{2} \right], V_{p+1} = (2\pi)^{p+1} \delta_k(0),$$

where

$$A^\dagger(k, t) A(k, t) = \frac{1}{2\Omega_k(t)} \left[ -\frac{\delta^2}{\delta \alpha(-k) \alpha(k)} + \Omega_k^2(t)\alpha(k)\alpha(-k) - \Omega_k(t)(2\pi)^{p+1}\delta_k(0) \right].$$

Now (3.34) can be interpreted as Hamiltonian for collection of harmonic oscillators with time-dependent frequencies $\Omega_k(t)$. Then we see that it is natural to define an operator of number of particles with momentum $k$ at time $t$ as

$$N_k(t) = A^\dagger(k, t) A(k, t).$$

We would like to calculate its expectation value for the vacuum wave functional $\Psi^{in}[\phi, t]$. Generally, the vacuum expectation value of any arbitrary operator is defined by

$$\langle \mathcal{O} \rangle = \langle \Psi_0(t) | \mathcal{O} | \Psi_0(t) \rangle = \int d\alpha(k) \Psi_0^* \mathcal{O} \Psi_0.$$ 

(3.37)

Evaluation of vacuum expectation values is easily performed by introducing the source term in the vacuum probability density as follows

$$|\Psi_0^2[j]| \equiv |\Psi_0^2| \exp \left[ \int \frac{dk}{(2\pi)^{p+1}} \alpha(k) j(k) \right],$$

(3.38)

where the vacuum probability density is

$$|\Psi_0^2 = \Psi_0^*[\alpha, t] \Psi_0[\alpha, t] = N_0^2 \exp \left\{ -\frac{1}{2} \int \frac{dk}{(2\pi)^{p+1}} 2\alpha(-k) G_R(k) \alpha(k) \right\}.$$  \hspace{1cm} (3.39)
Then we get
\[
\langle \alpha(k)|\alpha(k') \rangle = \left. \frac{\delta^2}{\delta j(k)\delta j(k')} \langle \Psi_0|\Psi_0 \rangle \right|_{j=0} = |N_0(t)|^2 \times
\]
\[
\times \frac{\delta^2}{\delta j(k)\delta j(k')} \int d\alpha \exp \left[ -\frac{1}{2} \int \frac{dk''}{(2\pi)^{p+1}} \left( 2\tilde{G}_R(k'', t)|\alpha(k'')|^2 - 2j(k'')\alpha(k'') \right) \right]_{j=0} = \frac{\delta^2}{\delta j(k)\delta j(k')} \exp \left[ \frac{1}{2} \int \frac{dk''}{(2\pi)^{p+1}} \left( \frac{2\pi)^{p+1}j(k'')j(-k'')}{2\tilde{G}_R(k'')} \right] \right|_{j=0} = \frac{(2\pi)^{p+1}\delta(k+k')}{2\tilde{G}_R(k', t)}.
\]

(3.40)

In order to evaluate
\[
\left\langle \frac{\delta^2}{\delta \alpha(k)\delta \alpha(k')} \right\rangle
\]

(3.41)

it is convenient to use integration by parts to write
\[
\Psi_0^\ast \frac{\delta^2}{\delta \alpha(k)\delta \alpha(k')} \Psi_0 = -\frac{\delta}{\delta \alpha(k')} \Psi_0^\ast \frac{\delta}{\delta \alpha(k')} \Psi_0 = -\tilde{G}^\ast(k', t)\alpha(k')\tilde{G}(k, t)\alpha(k),
\]

(3.42)

where we have neglected the surface term and we also use the fact that
\[
\frac{\delta}{\delta \alpha(k)} \Psi_0 = -\tilde{G}(k, t)\alpha(k)\Psi_0.
\]

(3.43)

From (3.42) and from (3.40) we get
\[
\left\langle \frac{\delta^2}{\delta \alpha(k)\delta \alpha(k')} \right\rangle = -\frac{\tilde{G}^\ast(k', t)\tilde{G}(k, t)}{2\tilde{G}_R(k, t)}(2\pi)^{p+1}\delta(k+k').
\]

(3.44)

Then the vacuum expectation value of the number of particles with momentum \( k \) is equal to
\[
\langle N(k, t) \rangle = \left\langle \frac{1}{2\Omega_k} \left[ -\frac{\delta^2}{\delta \alpha(-k)\alpha(k)} + \Omega_k^2(t)\alpha(k)\alpha(-k) - \Omega_k(t)(2\pi)^{p+1}\delta_k(0) \right] \right\rangle = \left(2\pi)^{p+1}\delta_k(0) \right) \left( \frac{\tilde{G}^\ast(k, t)\tilde{G}(k, t)}{2\tilde{G}_R(k, t)} + \frac{\Omega_k^2(t)}{2\tilde{G}_R(k', t)} - \Omega_k(t) \right) = (2\pi)^{p+1}\delta_k(0) \left( \frac{\Omega_k(t) - \tilde{G}(k, t)}{2\Omega_k(t)(\tilde{G}(k, t) + \tilde{G}^\ast(k, t))} \right).
\]

(3.45)

Since the factor \((2\pi)^{p+1}\delta_k(0)\) is equal to the volume of spatial section of \( D(p+1) \)-brane it is convenient to define the vacuum expectation value of the spatial density of the number of particle with momentum \( k \) as
\[
\langle \bar{N}_k(t) \rangle \equiv \frac{\langle N_k(t) \rangle}{V} = \frac{\Omega_k(t) - \tilde{G}(k, t)}{2\Omega_k(t)(\tilde{G}(k, t) + \tilde{G}^\ast(k, t))}.
\]

(3.46)
Let us calculate $\langle N_k(t) \rangle$ for the vacuum state functional $\Psi^{in}[\phi, t]$ in the limit $t \to \infty$. Using

$$\frac{\dot{\psi}_k^{in}}{\psi_k^{in}} = \frac{A_k \dot{\psi}_k^{out} - B_k^* \dot{\psi}_k^{out}}{A_k \psi_k^{out} - B_k^* \psi_k^{out}}$$

(3.47)

and the asymptotic behaviour of $\psi_k^{out}$ for $t \to \infty$

$$\dot{\psi}_k^{out} \sim -i\sqrt{\lambda} e^{t/2} \psi_k^{out}, \text{ for } t \to \infty$$

(3.48)

we obtain an asymptotic form of the kernel $\tilde{G}^{in}(k, t)$

$$\tilde{G}^{in}(k, t) = -\frac{\dot{\psi}_k^{in}}{\psi_k^{in}} = \sqrt{\lambda} e^{t/2} \left(1 + \gamma_k e^{-4\sqrt{\lambda} e^{t/2}}\right) \left(1 - \gamma_k e^{-4\sqrt{\lambda} e^{t/2}}\right)$$

(3.49)

for $t \to \infty$, where

$$\gamma_k = \frac{B_k^*}{A_k}. \quad (3.50)$$

For $t \to \infty$ we also have $\Omega_k = \sqrt{\lambda} e^{t/2}$ and consequently the vacuum expectation value of the density particles with momentum $k$ on half S-brane at far future is equal to

$$\langle N_k(\infty) \rangle = \frac{|\gamma_k|^2}{1 - |\gamma_k|^2} = |B_k|^2. \quad (3.51)$$

Now using the fact that

$$B_k^* = \sqrt{\omega_k \pi} e^{-\pi \omega_k} e^{-\frac{3\pi i}{4}} \frac{\lambda^{-i \omega_k}}{\sinh 2\pi \omega_k \Gamma(1 - 2i \omega_k)},$$

$$|\Gamma(1 + 2i \omega_k)|^2 = \frac{2\pi \omega_k}{\sinh(2\pi \omega_k)},$$

$$|B_k|^2 = \frac{e^{-2\pi \omega_k}}{2 \sinh 2\pi \omega_k} = \frac{1}{e^{\frac{\omega_k}{T}}} - 1, \quad T_H = \frac{1}{4\pi}, \quad (3.52)$$

we obtain the final result

$$\langle N_k(t) \rangle = \frac{1}{e^{\frac{\omega_k}{T}}} - 1, \quad T_H = \frac{1}{4\pi}. \quad (3.53)$$

We see that even if $\Psi^{in}[\phi, t]$ is pure state the density of particles with momentum $k$ at far future is the same as the density of particles in the thermal state at the temperature $T_H = \frac{1}{4\pi}$. According to [8] this means that the branes tries to produce open strings at the Hagedorn temperature although the final state is pure state. The fact that this "temperature" is so high means that $g_s$ corrections are important even for $g_s \to 0$. As was stressed in previous section, we will follow [10] and we will not consider these in our description.
4. Full S-brane

In this section we formulate Schrödinger picture description of the quantum field theory on the full S-brane. In fact, the analysis is almost the same as in the previous section. The only difference is that modes $\psi_k$ in (3.20) obey following differential equation

$$\partial^2_k \psi_k + 2\lambda \cosh t \psi_k + \omega_k^2 \psi_k \equiv \left( \partial^2_t + \Omega_k^2(t) \right) \psi_k = 0 .$$  \hspace{1cm} (4.1)

This equation was studied in the context of S-brane dynamics in [10]. According to [10] solution $\psi_k^{in}(t)$ that approaches positive frequency mode at far past and $\psi_k^{out}(t)$ that approaches positive frequency modes at the far future are given

$$\psi_k^{in}(t) = \sqrt{\frac{\pi}{2}} (ie^{2\pi\tilde{\omega}_k})^{1/2} H^{(1)}(-2i\tilde{\omega}_k, -t/2) ,$$

$$\psi_k^{out}(t) = \sqrt{\frac{\pi}{2}} (ie^{2\pi\tilde{\omega}_k})^{-1/2} H^{(2)}(-2i\tilde{\omega}_k, t/2) .$$  \hspace{1cm} (4.2)

The explicit form of the functions $H^{(1)}$, $H^{(2)}$ can be found in [10]. For our purposes it is sufficient to know their asymptotic behaviour

$$H^{(1)}(-2i\tilde{\omega}_k, t/2) \to \frac{\lambda^{-1/4}}{\sqrt{\pi}} e^{-\pi\tilde{\omega}_k} \exp \left( -\frac{t}{4} + 2i\sqrt{\lambda e^{t/2}} - i\frac{\pi}{4} \right) , \text{ as } t \to \infty ,$$

$$H^{(2)}(-2i\tilde{\omega}_k, t/2) \to \frac{\lambda^{-1/4}}{\sqrt{\pi}} e^{\pi\tilde{\omega}_k} \exp \left( -\frac{t}{4} + 2i\sqrt{\lambda e^{t/2}} - i\frac{\pi}{4} \right) , \text{ as } t \to \infty .$$  \hspace{1cm} (4.3)

The frequency $\tilde{\omega}_k$ is function of $\omega_k$ and $\lambda$ which for small $\lambda$ can be written as

$$\omega_k^2 = \tilde{\omega}_k^2 + \frac{2\lambda^2}{4\tilde{\omega}_k^2 + 1} + \frac{(20\tilde{\omega}_k^2 - 7)\lambda^4}{2(4\tilde{\omega}_k^2 + 1)^3(\tilde{\omega}_k^2 + 1)} + \ldots .$$  \hspace{1cm} (4.4)

The relation between in and out modes is

$$\psi_k^{in}(t) = \frac{1}{2 \sinh 2\pi \tilde{\omega}_k} \left[ i \left( e^{2\pi\tilde{\omega}_k} \xi_k - e^{-2\pi\tilde{\omega}_k} \right) \psi_k^{out}(t) + \left( \xi_k - \frac{1}{\xi_k} \right) \psi_k^{out*}(t) \right] =$$

$$= \alpha_k \psi_k^{out}(t) + \beta_k \psi_k^{out*}(t) ,$$  \hspace{1cm} (4.5)

where $\alpha_k$, $\beta_k$ are the Bogolubov coefficients

$$\alpha_k = \frac{i}{2 \sinh 2\pi \tilde{\omega}_k} \left( e^{2\pi\tilde{\omega}_k} \xi_k - e^{-2\pi\tilde{\omega}_k} \right) , \beta_k = \frac{1}{2 \sinh 2\pi \tilde{\omega}_k} \left( \xi_k - \frac{1}{\xi_k} \right) .$$  \hspace{1cm} (4.6)

Although the dependence of $\tilde{\omega}_k$, $\xi_k$ on $\omega_k$, $\lambda$ is in general quite complicated, it can be shown that Bogolubov coefficients satisfy the unitarity relation $|\alpha_k|^2 - |\beta_k|^2 = 1$. In the case of large $\omega_k$ and $\lambda \ll \omega_k$ we have

$$\tilde{\omega}_k = \omega_k[1 + O(\lambda^2/\omega_k^4)] , \xi_k = \frac{\Gamma(1 - 2i\omega_k)}{\Gamma(1 + 2i\omega_k)} e^{2i\omega_k}[1 + O(\lambda^2/\omega_k^2)]$$  \hspace{1cm} (4.7)
and the Bogolubov coefficients are
\[ \alpha_k = \frac{\sin(\theta_k + 2\pi i \omega_k)}{\sinh 2\omega_k}, \beta_k = -i \frac{\sin \theta_k}{\sinh 2\pi \omega_k}, e^{i\theta_k} = \lambda^{-2i\omega_k} \frac{\Gamma(1 + 2i\omega_k)}{\Gamma(1 - 2i\omega_k)}. \] (4.8)

We will be mainly interested in the case when \( \lambda \to 0 \) which has nice physical interpretation. For such a \( \lambda \) there is a long region around \( t = 0 \) of duration \( \ln \lambda \) in which the interaction can be neglected and we just have ordinary unstable brane. There is then a natural vacuum state \( \Psi^E[\phi, t] \) which is defined by the requirement that there are no particles at \( t = 0 \). It is associated with the function \( \psi_k \) appearing in the kernel \( \tilde{G}(k, t) = -i \frac{\psi_k}{\psi_k^*} \)

\[ \psi^0_k(t) = \sqrt{\frac{4\pi \xi_k}{\sinh 2\pi \omega_k}} J(-2i\tilde{\omega}_k, t/2). \] (4.9)

Using the relation
\[ J = \frac{1}{2}[H^{(1)} + H^{(2)}] \] (4.10)
\[ \psi^0_k \] can be expressed in terms of \( \psi^{in}_k \) as
\[ \psi^0_k(t) = a_k \psi^{in}_k(t) - b^*_k \psi^{in*}_k(t), \] (4.11)

where
\[ a_k = \frac{e^{\pi \tilde{\omega}_k - i\frac{\pi}{4}}}{\sqrt{2\xi_k \sinh 2\pi \omega_k}}, b_k = -e^{-\pi \tilde{\omega}_k - i\frac{\pi}{4}} \sqrt{\frac{\xi_k}{2 \sinh 2\pi \omega_k}}. \] (4.12)

We can easily confirm that the vacuum wave functional \( \Psi^E[\phi, t] \) does not contain particles at \( t = 0 \) which implies that it is correct vacuum state for unstable D-brane at \( t = 0 \). In fact, for \( \lambda \to 0 \) we have
\[ \Omega_k(0) = \sqrt{\omega_k^2 + \lambda^2} \sim \omega_k(1 + \lambda^2/\omega_k^2) \] (4.13)
while the kernel \( \tilde{G}^E(k, t) \) around \( t = 0 \) is equal to
\[ \tilde{G}(k, t) = -i \frac{\psi^0_k(t)}{\psi^*_k(t)} \sim \tilde{\omega}_k \sim \omega_k \] (4.14)
and hence the vacuum state functional \( \Psi^E[\phi, t] \) at \( t = 0 \) has the form
\[ \Psi^E[\phi, 0] = N \exp \left[ -\frac{1}{2} \int \frac{d\mathbf{k}}{(2\pi)^{p+1}} \alpha(-\mathbf{k}) \omega_k \alpha(\mathbf{k}) \right] \] (4.15)
which is standard Minkowski vacuum wave functional. Following calculations presented in the previous section it is easy to see that particle excitations are absent in the state (14.15).

We can also calculate the number of particles created during the time evolution of the Euclidean vacuum state \( \Psi^E[\phi, t] \). In fact, the calculation is the same as in case
of half S-brane studied in the previous section so that the vacuum expectation value of density of particles with momentum $k$ in the state $\Psi^E$ at the asymptotic future is equal to

$$\langle N_k(\infty) \rangle = \frac{|\gamma_{0\rightarrow out,k}|^2}{1 - |\gamma_{0\rightarrow out,k}|^2} = |b_k|^2 . \quad (4.16)$$

As in case of half S-brane we see that the vacuum state $\Psi^E[\phi, t]$ despite the fact that it is pure state it is populated in far future by particles with the particle density corresponding to the thermal state with the temperature $T_H = \frac{1}{4\pi}$. In the same way we can calculate the vacuum expectation value of particle density in the asymptotic past and we again obtain the result that the particle occupation number is thermal with the temperature $T_H = 1/4\pi$.

So far we have studied the pure states of quantum field theory on S-brane. In the next section we will consider more general approach when we will construct mixed states in the Schrödinger picture description.

5. S-brane thermodynamics in Schrödinger picture

In this section we will formulate the Schrödinger picture description of S-brane thermodynamics. When the initial state of a system is a pure state, described by definite wave functional $\Psi$, the time-dependent Schrödinger equation determines uniquely its time evolution. This situation was studied in the previous two sections. However we can consider more general case when initially at time $t_0$ the system is in mixed state, described by a functional density matrix $\hat{\rho}(t_0)$. In the Schrödinger picture, when operators $\hat{\phi}(x), \hat{\pi}(x)$ are time independent while states change with time, the Liouville equation that determines the time evolution of density matrix $\hat{\rho}(t)$, reads

$$\frac{i}{\hbar} \frac{\partial \hat{\rho}(t)}{\partial t} = [\hat{H}(t), \hat{\rho}(t)] . \quad (5.1)$$

A formal solution of this equation may be written in terms of the time evolution operator

$$\hat{U}(t, t') = P \exp \left[ -i \int_{t'}^t dt'' \hat{H}(t'') \right] \quad (5.2)$$

in the form

$$\hat{\rho}(t) = \hat{U}(t, t_0) \hat{\rho}(t_0) \hat{U}^{-1}(t, t_0) . \quad (5.3)$$

In (5.2) the symbol $P$ means path ordering. The quantity $\hat{\rho}(t_0)$ determines the initial state of the system. Usual choice is to let the initial state be one of local thermodynamic equilibrium at a temperature $T = 1/\beta$. The unnormalised initial density matrix then takes the form

$$\hat{\rho}(t_0) = \exp \left( -\beta \hat{H}(t_0) \right) . \quad (5.4)$$
Now the ensemble averages of operators are given by the expression
\[
\langle \hat{O}(t) \rangle = \frac{\text{Tr}[\hat{\rho}(t)\hat{O}]}{\text{Tr}\hat{\rho}(t)} = \frac{\text{Tr}[\hat{\rho}(t_0)\hat{U}(t_0, t')\hat{U}(t', t)\hat{O}\hat{U}(t, t_0)]}{\text{Tr}\hat{\rho}(t_0)},
\]
(5.5)
where we have used cyclicity of the trace and we have also inserted the identity \( \hat{U}(t, t')\hat{U}(t', t) \). Since the spatial sections are flat we perform Fourier transform of \( \phi(x) = \int \frac{dk}{(2\pi)^{d+1}} e^{ikx} \) and consider the density matrix in the form
\[
\rho(\phi_1, \phi_2, t) = \prod_k \rho_k(\alpha_1(k), \alpha_2(k), t),
\]
\[
\rho(\alpha_1(k), \alpha_2(k), t) = N_k(t) \exp \left\{ \frac{1}{2} A_k(t) \alpha_1(k) \alpha_1(-k) + \frac{1}{2} B_k(t) \alpha_2(k) \alpha_2(-k) \right\},
\]
(5.6)
where \( B_k(t) \) is real as follows from the Hermicity of the density matrix \( \rho(\phi_1, \phi_2) = \rho^*(\phi_2, \phi_1) \). Now the equations that determine time evolution of coefficients \( A_k(t), B_k(t) \), follow from (5.1) that in k-space has the form
\[
i \frac{\partial \rho(\alpha_1, \alpha_2, t)}{\partial t} = \int \frac{dk}{(2\pi)^{d+1}} \left[ -\frac{1}{2} \left( \frac{\delta^2}{\delta \alpha_1(k)\alpha_1(-k)} - \frac{\delta^2}{\delta \alpha_2(k)\alpha_2(-k)} \right) + \frac{\Omega_k^2(t)}{2} \left( \alpha_1(k) \alpha_1(-k) - \alpha_2(k) \alpha_2(-k) \right) \right] \rho(\alpha_1, \alpha_2, t).
\]
(5.7)
Since the modes do not mix the equations for kernels in the density matrix are obtained by comparing the powers of \( \alpha \) on both sides of the above equation which gives following equations for coefficients \( A_k, B_k \)
\[
\frac{\dot{N}_k}{N_k} = -i (A_k - A_k^*),
\]
\[
\dot{A}_k = -i \left[ A_k^2 - B_k^2 + \Omega_k^2 \right],
\]
\[
\dot{B}_k = -i B_k (A_k - A_k^*).
\]
(5.8)
One can show that
\[
\frac{d}{dt} \left( \frac{B_k(t)}{A_{kR}(t)} \right) = 0,
\]
\[
\frac{d}{dt} \left( \frac{N_k(t)}{\sqrt{A_{kR}(t) + B_k(t)}} \right) = 0,
\]
(5.9)
where \( A_{kR} = \frac{1}{2}(A_k + A_k^*) \), \( A_{kI} = \frac{1}{2i}(A_k - A_k^*) \). The relations (5.9) suggest that we can take
\[
A_{kR}(t) = C_k A_{kR}(t) = D_k B_k(t), A_{kI}(t) = A_{kI}(t),
\]
where the constants \( C_k, D_k \) depend on choice of initial conditions. We can also define following quantity \( A_k = A_{kR} + iA_{kI} \). From (5.8) immediately follows that \( A_k \) obeys equation
\[
\dot{A}_k(t) = -i [A_k^2(t) - \Omega_k^2(t)].
\]
This equation can be linearised as
\[
\dot{A}_k(t) = -i \frac{\dot{f}_k(t)}{f_k(t)},
\]
where \( f_k \) obeys
\[
\ddot{f}_k(t) + \Omega_k^2(t)f_k(t) = 0.
\]
In the previous equation \( \Omega_k^2(t) = \omega_k^2 + \lambda e^t \) for half S-brane and \( \Omega_k^2(t) = \omega_k^2 + 2\lambda \cosh t \) for full S-brane and hence the modes \( f_k(t) \) are the same as \( \psi_k(t) \) given in previous sections. From the known functions \( f_k(t) \) we immediately get
\[
A_{kR}(t) = \frac{1}{2}(A_k(t) + A_k^*(t)) = \frac{1}{C_k |f_k|^2},
\]
\[
A_{kI} = \frac{1}{2i}(A_k - A_k^*) = -\frac{1}{2} \frac{\dot{f}_k f_k^* + \dot{f}_k^* f_k}{|f_k|^2},
\]
\[
B_k(t) = \frac{1}{D_k |f_k|^2}.
\]
(5.14)

Now we are ready to calculate the equal-time Green functions
\[
G(x, y, t) = \text{Tr} \hat{\rho}(t)\hat{\phi}(x)\hat{\phi}(y) = \int d\phi \rho(\phi, \phi)\phi(x)\phi(y) =
\]
\[
= \int \frac{dk}{(2\pi)^{p+1}} \frac{dk'}{(2\pi)^{p+1}} e^{i(kx+k'y)} \langle \alpha(k)\alpha(k') \rangle,
\]
where
\[
\langle \alpha(k)\alpha(k') \rangle = \int d\alpha \rho(\alpha, \alpha)\alpha(k)\alpha(k').
\]
(5.15)

As in section (3) we define
\[
\rho(J, \alpha, \alpha) \equiv \rho(\alpha, \alpha) \exp \left( \int \frac{dk}{(2\pi)^{p+1}} J(k)\alpha(k) \right).
\]
(5.16)

Since the diagonal form of the density matrix is equal to
\[
\rho(\alpha, \alpha, t) = \prod_k \rho_k(\alpha, \alpha, t) = \exp \left[ - \int \frac{dk}{(2\pi)^{p+1}} (A_{kR} + B_{kR}) \alpha(k)\alpha(-k) \right]
\]
(5.17)
we can easily calculate two-point function (5.16)

\[
\langle \alpha(k)\alpha(k') \rangle = \frac{\delta^2(\delta J(k)\delta J(k'))}{\delta J(\delta J(k))} \int d\alpha N(t) \exp \left\{ -\frac{1}{2} \int \frac{dk}{(2\pi)^p} \left[ 2(A_{kR} + B_{kR})|\alpha(k)|^2 + 2J(k)\alpha(k) \right] \right. \\
+ \left. \frac{\delta^2}{\delta J(\delta J(k'))} \exp \left[ \frac{1}{2} \int \frac{dk''}{(2\pi)^p} \frac{(2\pi)^{p+1}J(k'')J(-k'')} {2(A_{kR} + B_{kR})} \right] \right. \\
= (2\pi)^{p+1} \frac{\delta(k + k')}{2(A_{kR}(t) + B_{kR}(t))} = (2\pi)^{p+1} \frac{\delta|f_k(t)|^2}{2} \frac{C_kD_k}{C_k + D_k}.
\]

(5.19)

We choose the constants \(C_k, D_k\) in such a way so that the density matrix \(\hat{\rho}(t_0)\) at time \(t_0\) reduces to the thermal density matrix at the temperature \(T = 1/\beta\). This can be achieved with these constants

\[
C_k = 2 \tanh(\beta\omega_k), \\
D_k = -2 \sinh(\beta\omega_k), \\
f_k(t_0) = \frac{1}{\sqrt{2\omega_k}}, \quad f_k(t_0) = -i\omega_k f_k, \\
N_k(t_0) = \left[ \frac{\omega_k}{\pi} \tanh \left( \frac{\beta\omega_k}{2} \right) \right]
\]

(5.20)

and consequently the equal time two point function is

\[
\langle \alpha(k)\alpha(k') \rangle = (2\pi)^{p+1} \frac{\delta(k + k')}{2} \frac{|f_k(t)|^2}{\coth(\frac{\beta\omega_k}{2})}.
\]

(5.21)

In the same way we obtain

\[
\left\langle \frac{\delta^2}{\delta \alpha_1(k)\delta \alpha_2(k')} \right\rangle = (2\pi)^{p+1} \frac{\delta(k + k')}{2} \frac{A_k(t)A_k(t) - B_k^2(t)}{2(A_{kR}(t) + B_k(t))}.
\]

(5.22)

As it is clear from previous sections it is natural to take \(t_0\) to be equal to \(-\infty\) for half S-brane, while for full S-brane \(t_0 = 0\). At time \(t = t_0\) we have

\[
\left\langle \frac{\delta^2}{\delta \alpha(k)\delta \alpha(k')} \right\rangle = \left(2\pi\right)^{p+1} \delta(k + k') \frac{1}{2\omega_k} \coth \left( \frac{\beta\omega_k}{2} \right), \\
\left\langle \frac{\delta^2}{\delta \alpha(k)\delta \alpha(k')} \right\rangle = -\omega_k^2 \left(2\pi\right)^{p+1} \delta(k + k') \frac{\omega_k}{2} \coth \left( \frac{\beta\omega_k}{2} \right).
\]

(5.23)

It is easy to see that the density of particles with momentum \(k\) at time \(t_0\) is equal to

\[
N_k(t_0) = \frac{1}{V} \text{Tr} \hat{N}_k \hat{\rho}(t_0) = \frac{1}{\exp(\beta\omega_k) - 1}.
\]

(5.24)
as it should be for a system at thermal state with temperature $T = 1/\beta$.

Above we have constructed mixed states for S-branes that approach standard thermal states at far past in case of half S-brane, or at $t = 0$ in case of full S-brane. These mixed states can be defined for any initial temperature $T = 1/\beta$. However as was shown in [10] for temperatures $T = \frac{1}{2\pi n}$ these mixed states have interesting property that Green functions evaluated in these states retain their thermal periodicity for any time $t$. To see this, we firstly show that the evolution operator (5.2) on half S-brane obeys

$$
\dot{U}(t + 2\pi n, t) = P \exp \left[ -i \int_t^{t + 2\pi n} dt' \dot{H}(t') \right] = 
= P \exp \left[ -i \int_t^{t + 2\pi n} dt' \left( \dot{H}(t_0) + (m^2(t') - m^2) \right) \right] = 
= \exp \left[ 2\pi n \dot{H}(t_0) \right],
$$

(5.25)

where we have used

$$
\int_t^{t + 2\pi n} dt' (m^2(t') - m^2) = i\lambda e^{i\int_0^{2\pi n} dy \int_0^t d\gamma y} = \lambda e^{i2\pi n - 1} = 0 .
$$

(5.26)

It is clear that for full S-brane the integral $\int_t^{t + 2\pi n} dt' (m^2(t) - m_0^2)$ is zero as well. And finally, $\dot{H}(t_0)$ in (5.23) is the time-independent Hamiltonian of the free scalar field

$$
\dot{H}(t_0) = \int d\mathbf{x} \left[ \hat{\pi}^2 + \delta^{ab} \partial_a \hat{\phi} \partial_b \hat{\phi} + m^2 \hat{\phi}^2 \right] .
$$

(5.27)

Using (5.27) we immediately get

$$
\dot{U}(t_0 + 2\pi n, t_0) = \exp \left[ 2\pi n \dot{H}(t_0) \right] = \dot{\rho}_n(t_0)^{-1}
$$

(5.28)

and also

$$
U(t_0 - 2\pi n, t_0) = \exp \left[ -2\pi n \dot{H}(t_0) \right] = \dot{\rho}_n(t_0) ,
$$

(5.29)

where $\dot{\rho}_n(t_0)$ is initial density matrix at temperature $T = \frac{1}{2\pi n}$. For our next purposes it will be useful to rewrite the equal-time two point function in terms of Heisenberg picture operators

$$
G(\mathbf{x}, \mathbf{y}, t) = \text{Tr} \dot{\rho}(t) \hat{\phi}(\mathbf{x}) \hat{\phi}(\mathbf{y}) = \text{Tr} \dot{U}(t, t_0) \dot{\rho}(t_0) \dot{U}^{-1}(t, t_0) \hat{\phi}(\mathbf{x}) \dot{U}(t, t_0) \times \dot{U}^{-1}(t, t_0) \hat{\phi}(\mathbf{y}) \dot{U}(t, t_0) = \text{Tr} \dot{\rho}_H \hat{\phi}_H(\mathbf{x}, t) \hat{\phi}_H(\mathbf{y}, t) ,
$$

(5.30)

where

$$
\hat{\phi}_H(\mathbf{x}, t) = \dot{U}^{-1}(t, t_0) \hat{\phi}(\mathbf{x}) \dot{U}(t, t_0) = \dot{U}_H(t, t_0) \hat{\phi}(\mathbf{x}) \dot{U}_H^{-1}(t, t_0) ,
$$

$$
\dot{\rho}_H = \dot{\rho}(t_0) , \dot{U}_H(t, t_0) \equiv P \exp \left[ i \int_{t_0}^t dt' \dot{H}(t') \right].
$$

(5.31)
Now we generalise the equal-time two point function written in Heisenberg representation (5.30) to the two point functions evaluated at different times $t, t'$

$$G(x, t, x', t') = \text{Tr} \hat{\rho}_H \hat{\phi}_H(x', t') \hat{\phi}_H(x, t).$$

(5.32)

Using the fact that

$$\hat{U}_H(t + 2\pi i n, t_0) = \hat{U}_H(t + 2\pi i n, t) \hat{U}_H(t, t_0) = \exp \left[ 2\pi n \hat{H}(t_0) \right] \hat{U}_H(t, t_0) = \hat{\rho}_H \hat{U}_H(t, t_0)$$

(5.33)

we can show that (5.30) retain its thermal periodicity at all times

$$G_n(x, t + 2\pi i n, x', t') = \text{Tr} \hat{\rho}_H \hat{\phi}_H(x', t') \hat{\rho}_H \hat{\phi}(x, t) \hat{\rho}_H^{-1} =$$

$$= \text{Tr} \hat{\rho}_H \hat{\phi}(x, t) \hat{\phi}_H(x', t') = G(x', t', x, t)$$

(5.34)

and also

$$G(x, t, x', t' - 2\pi i n) = \text{Tr} \hat{\rho}_H \hat{\phi}_H^{-1}(x', t') \hat{\rho}_H \hat{\phi}_H(x, t) =$$

$$= \text{Tr} \hat{\rho}_H \hat{\phi}_H(x, t) \hat{\phi}_H(x', t') = G(x', t', x, t)$$

(5.35)

so that we see that $G_n(x, t, x', t')$ retain their thermal periodicity for finite $t, t'$. In other words, we can say that among the mixed states on half S-brane that can be defined for any temperature there are mixed states with the temperature $T_n = \frac{1}{2\pi n}$\footnote{More precisely, in the exact open string theory description of S-brane the condition of locality in boundary conformal field theory forces us to consider the temperatures $T_n$ only [10, 13].} that reduce to usual thermal vacuum at past infinity while retaining thermal periodicity at all times. The same result is valid for the full S-brane as well. Such thermal states approximate the quantum states of open strings on an S-brane created from incoming closed string excitations.

6. Conclusion

In this paper we have formulated the quantum field theory that describes S-branes in minisuperspace approach in the language of Schrödinger functional formalism. As was shown in [10] the quantum field theory on unstable D-brane in the presence of rolling tachyon is similar to the quantum field theory in time-dependent background. In particular, the specification of vacuum state on unstable D-brane in rolling tachyon background is unambiguous and leads to the open strings production during the D-brane decay [10].
In this paper we studied the quantum field theory, that arises in minisuperspace approach to S-brane dynamics, in the Schrödinger formalism. The reason for this choice is to avoid the description of the vacuum as the "no-particle" state in a Fock space of states generated by the creation operators of particles defined with respect to a particular mode decomposition of quantum field. In particular we tried to avoid "particle interpretation" of the out vacua in case of half S-brane where according to standard interpretation of the D-brane decay there should not exist any open string perturbative modes.

We have also studied S-brane thermodynamics in Schrödinger picture. We have explicitly construct the density matrix for mixed states on half S-brane that approach standard thermal mixed states at past infinity. We have shown, according to [10] that for temperatures \( T = \frac{1}{2\pi n} \) these mixed states retain their thermal periodicity for all times. We have seen that these mixed states are natural states for quantum field theory on S-brane and presumably define low energy effective theory on S-brane.

To summarise, we mean that the Schrödinger formalism for minisuperspace description of S-brane dynamics gives very natural and intuitive picture of the vacuum state of open string modes during unstable D-brane decay. We also hope that the Schrödinger picture description could be useful in the minisuperspace approach to the rolling tachyon in closed string theory [40, 48, 49].

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