Why understanding neutrino interactions is important for oscillation physics

\[
U = \begin{pmatrix}
    C_{13}C_{12} & C_{13}S_{12} & S_{13}e^{-i\delta} \\
    -C_{23}S_{12} - S_{13}S_{23}C_{12}e^{i\delta} & C_{23}C_{12} - S_{13}S_{23}S_{12}e^{i\delta} & C_{13}S_{23} \\
    S_{23}S_{12} - S_{13}C_{23}C_{12}e^{i\delta} & -S_{23}C_{12} - S_{13}C_{23}S_{12}e^{i\delta} & C_{13}C_{23}
\end{pmatrix}
\]

Introduction * Cross-Sections * Effects on Experiments

Chris Walter / Duke University
NuInt07 FNAL / May 30th 2007
What's the current picture of neutrino oscillations?

\[ P_{F_1F_2} = \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E} \]

\[ \Delta m^2_{\text{atm}} \sim 10^{-3} \text{eV}^2 \]

\[ \Delta m^2_{\text{sun}} \sim 10^{-5} \text{eV}^2 \]

\[ \Delta m^2_{12} + \Delta m^2_{23} = \Delta m^2_{13} \]

Three neutrinos allow two mass differences!

\[ C_{xy} = \cos(\theta_{xy}) \quad S_{xy} = \sin(\theta_{xy}) \]

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[
U =
\begin{pmatrix}
1 & 0 & 0 \\
0 & C_{23} & S_{23} \\
0 & -S_{23} & C_{23}
\end{pmatrix}
\begin{pmatrix}
C_{13} & 0 & S_{13} e^{i\delta} \\
0 & 1 & 0 \\
-S_{13} e^{i\delta} & 0 & C_{13}
\end{pmatrix}
\begin{pmatrix}
C_{12} & S_{12} & 0 \\
-S_{12} & C_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

muons “disappear” Can make electrons “appear” electrons “disappear”
What can we learn with oscillation experiments?

- What is the mass hierarchy?
- What are precise values of $\Delta m^2$, $\sin^2 \theta_{23}$
- Is $\theta_{13}$ non zero?
- Is CP violated?

Important issues in $\nu$ oscillation experiments.

- **We always measure $\nu$ flux * cross-section**
- We measure with a detector which has different efficiencies for different cross-section channels.
- We need to be able to predict (or measure) both the non-oscillated spectra and interpret distortions.
- Starting with MINOS, the statistics from long-base line oscillation experiments are becoming comparable with the systematic errors caused by nuclear effects.
“Seeing” Neutrinos

Production

This $\mu$ is stopped

Interaction

Threshold for producing each lepton:
- $E_{\nu_e} > 0.15$ MeV
- $E_{\nu_\mu} > 110$ MeV
- $E_{\nu_\tau} > 3500$ MeV
Different neutrino experiments see different proportions of cross-sections

- For ~300km baselines, $\Delta m^2 \sim 3 \times 10^{-3}$, The flux is maximally oscillated $\sim 1$ GeV
- Scales with baseline: 750 km $\sim 2.5$ GeV
There are still things we don't understand!
Example: Low Q2 effect

Even after correcting for what we know about at low momentum transfers the data is suppressed relative to the MC.

Seen in K2K(kton,scifi, scibar) and Miniboone.

CC QE candidates @ Miniboone

PRELIMINARY
Monte Carlo error bars from:
neutrino $\sigma$,
light extinction,
& light scattering length uncertainties
Using Near and Far Detectors.

**Number:**

$N_1$ Events

Detector 1

$N_2 = \alpha \cdot N_1 \cdot \text{Ratio}(2/1)$

Detector 2

Function of Flux, X-sec, efficiencies.

*cancel for identical detectors and fluxes*

We use a detector near to the beam to measure the number and energy spectra of the produced neutrinos.

Then we predict how many we should see based on what we measured and the divergence of the beam.

*Often not good enough!* There are differences in flux and efficiencies.
Two types of physics analysis: disappearance and appearance

\(\nu_e\) appearance: determine \(\theta_{13}\)

\(\nu_\mu\) disappearance: determine \(\theta_{23}\) and \(\Delta m^2_{23}\)

For appearance three main types of background:

*intrinsic \(\nu_e\), misidentified \(\pi^0\), mis-identified charged \(\mu\).

Must control to the few per mill level
Important x-secs to know

**Quasi-Elastic**
- V-A
- Llewellyn-Smith 1972
- Must set MA(QE) Value

**Single Pion**
- Resonance production
- Rein & Seghal 1981
- Many Other Models + MA(1π)

**Coherent Pion**
- Rein & Seghal/?
- Suppress CC production?

**Deep Inelastic Scattering**
- GRV 94 parton distribution
- Bodek/Yang 2001

**Nuclear Effects**
- Fermi motion
- Pauli blocking
- Nuclear

Note: also will need comparable x-secs for anti-neutrinos for future experiments.

![Diagram showing Normal and Inverted Hierarchy for NOνA (θ₁₃ near CHOOZ bound)]
WC $E_{\nu}$ Reconstruction (assuming QE)

In Water Cherenkov detectors not every particle is above Cherenkov threshold.

Luckily, in a Quasi-Elastic reaction, even if only the muon is visible we can reconstruct the neutrino energy!

If the interaction is non Quasi-Elastic then the reconstructed energy will be incorrect.

$$E_{\nu} = \frac{m_N E_\mu - m_\mu^2 / 2}{m_N - E_\mu + p_\mu \cos (\Theta_\mu)}$$

$m_N = \text{Neutron mass}$

$E_\mu = \text{Muon energy}$

$m_\mu = \text{Muon mass}$

$p_\mu = \text{Muon momentum}$

$\Theta_\mu = \text{Muon angle wrt beam}$
Non-QE interactions and $E_{\nu}$

Reconstruction

Example: K2K Flux MC

Non-QE reconstructs at low-energy in the oscillation dip!
In a calorimetric detector all of the particles are visible and you add up all of the visible energy to get the neutrino energy.

\[ E_\nu = E_\mu + E_{\text{shower}} \]

Nucleus absorbs and re-scatters pions. If they “disappear” so does the energy.
Effect of absorption:

MINOS uses a hadronic energy scale uncertainty of ~ 10%: The 2nd largest systematic error on the $\Delta m^2$.

Also very important: Hadronic shower shapes for NC vs. CC separation.
Another kind of x-sec bkg: Opera

Main backgrounds:
- charm decays
- large angle $\mu$ scattering
- hadron re-interactions

Short Decay:
Look for a kink from the decay of the tau in an emulsion block.
Quasi-Elastic Cross Section

\[
\frac{d \sigma_{QE}}{dQ^2} = \frac{M^2 G^2 \cos^2(\theta_c)}{8 \pi E_\nu^2} \left[ A(Q^2) - B(Q^2)(s-u) + C(Q^2)(s-u)^2 \right]
\]

- \(A=4\left(\frac{m^2}{4M^2} + \tau\right)[(1+\tau)|F_A|^2-(1-\tau)|F_V^1|^2+\tau(1-\tau)|\xi F_V^2|^2+4\tau \xi \text{Re}F_V^1 F_V^2]-m^2/4M^2(|F_V^1+\xi F_V^2|^2+|F_A+2F_p|^2-4(1+\tau)F_p^2)]\)
- \(B=-4\tau \text{Re}F_A^*(F_V^1+\xi F_V^2)\)
- \(C=4(|F_A|^2+|F_V^1|^2+\tau|\xi F_V^2|^2)\)

Where \((s-u)=4ME_\nu-Q^2-M_\mu\), \(\tau = Q^2/4M^2\), \(\xi = u_p - u_n\),

\(F_p\) is the pseudo scalar form factor, and \(F_A\) is the axial vector form factor.

- The vector form factors:
  - \(F_V^1 = (G_{Ep} - G_{En} - \tau(G_{Mp} - G_{Mn}))/(1+\tau)\)
  - \(\xi F_V^2 = (G_{Mp} - G_{Mn} - G_{Ep} + G_{En})/(1+\tau)\)
Neutrino Interactions

From EM Scattering:

\[ G_{EP}(Q^2=0) = 1 \quad G_{EN}(Q^2=0) = 0 \]
\[ G_{MP}(Q^2=0) = 2.79 \quad G_{MN}(Q^2=0) = -1.91 \]
\[ G^p_v(Q^2) = \frac{G^p_M(Q^2)}{2.79} = \frac{G^n_M(Q^2)}{-1.91} = G_{dipole}(Q^2) = \left(1 + \frac{Q^2}{0.71 \text{GeV}/c^2} \right)^{-2} \]

**Charged Current**

\[ J^{1+i/2}_a = V^{1+i/2}_a - A^{1+i/2}_a \]
\[ <p(p') | J^{CC}_a | n(p) > =< p(p') | V^{1+i/2}_a - A^{1+i/2}_a | n(p) > \]
\[ <p(p') | V^{1+i/2}_a | n(p) > = \bar{u}(p') \left[ \gamma_\alpha F_1^v(Q^2) + \frac{i}{2M} \sigma_{\alpha\beta} q^\beta F_2^v(Q^2) \right]u(p) \]
\[ <p(p') | A^{1+i/2}_a | n(p) > = \bar{u}(p') \left[ \gamma_\alpha \gamma_5 F_A(Q^2) + q_a F_p(Q^2) \right]u(p) \]

\[ F_A(Q^2) = \frac{F_A(0)}{\left(1 + Q^2/M_A^2 \right)^2}, \text{ with } F_A(0) = -1.2617 \pm 0.0035 \]
\[ F_p(Q^2) = \frac{2MF_A(Q^2)}{m^2 + Q^2} \]

Changing the axial mass changes the shape and the normalization.
Nucleon Vector Form Factors

- The simple dipole fit is only good to ~10-20%. New SLAC/JLAB e-p/e-D data shows that vector form factors must be updated.

- New parameters from P.E. Bosted, "Empirical fit to nucleon electromagnetic form factors, Phys Rev C, V 51, 409, '95
  (Also E.J.Brash et al,PRC65,051001,2002)

| Form Factor | Old       | New                                                 |
|-------------|-----------|-----------------------------------------------------|
| $G_{En}$    | 0         | $-1.25 \mu_n D \tau/(1+18.3 \tau)$                  |
| $G_{Ep}$    | D         | $1/(1+1.14Q+3.01Q^2+.02Q^3+1.20Q^4+.32Q^5)$         |
| $G_{Mn}$    | $\mu_n D$ | $\mu_n/(1-1.74Q+9.29Q^2-7.63Q^3+4.63Q^4)$          |
| $G_{Mp}$    | $\mu_p D$ | $\mu_p/(1+.14Q+3.01Q^2+.02Q^3+1.20Q^4+.32Q^5)$     |
| $F_p$       | 0         | $2M^2F_A(Q^2)/(m_{\pi}^2+Q^2)$                      |

- New cross section is smaller at low $Q^2$, and larger at higher $Q^2$
- $\sim \pm 5\%$ overall difference in $d\sigma_{QE}/dQ^2$
- Changes $M_A$ fit value by .05
Resonant and coherent pion production

Neutral current production of $\pi$ is largest background for $\nu_e$ appearance!

70% NC pions
preliminary

Charge current coherent seems to be suppressed/non-existent.

New theoretical work has addressed this.
\[ F_2(x) = \sum_i e_i^2 \left( x q_i(x) + x \bar{q}_i(x) \right) \]

\[ F_2(x) = \frac{Q^2}{Q^2 + 0.188} F_2(x_w) \]

where \( x_w = x \left( Q^2 + 0.624 \right) / \left( Q^2 + 1.735x \right) \).

Dashed: GRV94

Red: Bodek-Yang

This correction is significant at low \( Q^2 \) region.
Pauli Exclusion Effect

Nuclear effects are large in the low $Q^2$ region, where the cross section is large.

**Quasi-elastic**

$$E_\nu = 1.3 \text{ GeV}, \quad k_F = 220 \text{ MeV/c}$$

If $P < k_F$, suppressed.

**Δ production**

- 10-15% suppression
- At low $Q^2$
- Total 3% reduction
Models beyond the Fermi-gas model
Spectral Function Calculation or Local Density Approximation
(Pandharipande@nuint01, Benhar, Nakamura, Gallagher@nuint02)

Spectral Functions $P(p,E)$ for various nuclei, eg. $^{16}\text{O}$, are estimated by Benhar et al. using $e$-$\text{N}$ data.

$P(p,E)$ : Probability that the target nucleon has momentum $p$ and binding energy $E$. 
Lepton energy in quasi-elastic $\nu$-N interaction
Comparison of **Fermi Gas model** and **Spectral Function Calculation**

- Spectral function gives high energy tail.
- Shift at a level of 10 MeV may exist.
- We can test this using electron scattering data.
- This shift effects the energy of the outgoing lepton and can effect the calculated $\Delta m^2$.

Benhar, Gallagher, Nakamura@nuin02
The effect of proton-rescattering

No rescattering

With rescattering

Proton loses momentum!

No second track is recorded!

\[ \text{Mom. of P exiting from Nucleus (MeV/C)} \]

\[ \langle \text{Momentum Diff} \rangle = 92 \text{ MeV/C} \]

NEUT study with K2K beam
Nuclear PDFs and the effect on the neutrino xsec

It seems that the presence of other nucleons in heavy nuclei can effect the DIS cross-section
Use in Experiments:
Need to treat on a equal footing with detector related errors

Example: systematic errors in SK atmospheric analysis

- $M_A$ value in quasi-elastic and single pion
- Quasi-elastic scattering (Difference between two models)
- Quasi-elastic scattering (cross section normalization)
- Single pion production (cross section normalization)
- Multi pion production (With and without B-Y correction)
- Multi pion production (cross section normalization)
- Coherent pion production (with and without coherent CC suppr.)
- NC/CC ratio (Vary ratio of x-secs)
- Nuclear effect in $^{16}$O (Change nuclear mean/free path)
- Pion Energy Spectrum (Difference between NEUT and NUANCE)
- CC Tau Production (cross section normalization)

$$
\chi^2 = \sum_{n=1}^{370} \left[ 2 \left( N_{exp}^n \left( 1 + \sum_{i=1}^{45} f_i^n \cdot \epsilon_i \right) - N_{obs}^n \right) + 2 N_{obs}^n \ln \left( \frac{N_{obs}^n}{N_{exp}^n \left( 1 + \sum_{i=1}^{45} f_i^n \cdot \epsilon_i \right)} \right) \right]
+ \sum_{i=1}^{43} \left( \frac{\epsilon_i}{\sigma_i} \right)^2
$$

- $N_{obs}$ Number of observed events in $n$-th bin
- $N_{exp}$ Number of expected events in $n$-th bin
- $\epsilon_i$ $i$-th systematic error term
- $f_i^n$ Systematic error coefficient
- $\sigma_i$ 1 sigma value of systematic error

(11 of about 70 systematic errors)
Where Do We Come From? What Are We? Where Are We Going?

NuInt05  
Miniboone Anti-nu  
Scibar @ miniboone  
T2K 280M  
T2K 2KM WC+LAr  
Minerva

NuInt04  
T2K Near Detector  
Ideas for future LaR  
T2K near detectors  
Minerva  
Finesse

NuInt02  
Finesse  
NUMI on axis  
NUMI off axis  
Lar with Acc.

NuInt01  
Scibar Proposal  
JLAB overview  
NUMI scatt. expt  
“small ICARUS” det.

Paul Gauguin 1897-98

I showed this slide at NuInt05 showing the experimental efforts in this area: Since NuInt01 we made a good start at our work. Let’s step back here and understand what we have done, and what we still have to do!
Conclusions

- If you want to understand neutrino oscillations you need to understand what happens when a neutrino interacts inside of a nucleus.
  - There are important effects both in the nucleus and in the nucleon.
  - Different sorts of detectors are sensitive to different neutrino interaction effects.
- We are now entering the phase where the errors introduced by these effects on measured neutrino oscillation parameters will be of the same size as the statistical errors of experiments.