THE SUNYAEV-ZELDOVICH EFFECT: SIMULATIONS AND OBSERVATIONS

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ABSTRACT

The Sunyaev-Zeldovich effect (SZ effect) is a complete probe of ionized baryons, the majority of which are likely hiding in the intergalactic medium. We ran a \(512^3\) cold dark matter simulation using a moving mesh hydro code to compute the statistics of the thermal and kinetic SZ effect such as the power spectra and measures of non-Gaussianity. The thermal SZ power spectrum has a very broad peak at multipole \(l \sim 2000-10^4\) with temperature fluctuations \(\Delta T \sim 15\ \mu\text{K}\). The power spectrum is consistent with available observations and suggests a high \(\sigma_8 \simeq 1.0\) and a possible role for nongravitational heating. The non-Gaussianity is significant and increases the cosmic variance of the power spectrum by a factor of \(~5\) for \(l < 6000\). We explore optimal drift-scan survey strategies for the Array for Microwave Background Anisotropy and their dependence on cosmology. For SZ power spectrum estimation, we find that the optimal sky coverage for 1000 hr of integration time is several hundred square degrees. One achieves an accuracy better than 40% in the SZ measurement of the power spectrum and an accuracy better than 20% in the cross-correlation with Sloan galaxies for 2000 < \(l\) < 5000. For cluster searches, the optimal scan rate is around 280 hr deg\(^{-2}\) with a cluster detection rate of one every 7 hr, allowing for a false positive rate of 20% and better than 30% accuracy in the cluster SZ distribution function measurement.

Subject headings: cosmic microwave background — cosmology: observations — cosmology: theory — galaxies: clusters: general — large-scale structure of universe

On-line material: color figures

1. INTRODUCTION

Big bang nucleosynthesis and cosmic microwave background (CMB) experiments such as BOOMERANG (Netterfield et al. 2002) and the Degree Angular Scale Interferometer (Pryke et al. 2002) predict that ordinary baryonic matter accounts for about 5% of the total matter in the universe. However, the directly observed components, such as stars, interstellar medium, and intracluster gas, account for only a few percent of this baryon budget, while more than 90% of baryons have escaped direct detection (Persic & Salucci 1992; Fukugita, Hogan, & Peebles 1998).

This is known as the missing baryon problem. The missing baryons are believed to be in the form of the intergalactic medium (IGM) and have been difficult to detect directly. To understand their state, such as density, temperature, peculiar velocity, and metallicity, stands as a major challenge to both observation and theory and is crucial to understanding the thermal history of the universe and galaxy formation.

The IGM has various direct observable tracers: (1) Neutral hydrogen absorbs the background light of quasars and produces the Ly\(\alpha\) forest. (2) Ionized electrons have thermal and peculiar motions and are capable of scattering CMB photons and generate secondary CMB temperature fluctuations, which are known as the thermal and kinetic Sunyaev-Zeldovich effects (SZ effect), respectively. Their precision measurements are becoming routinely available with the devoted CMB experiments such as the Array for Microwave Background Anisotropy (AMIBA\(^1\)) and the Sunyaev-Zeldovich array (SZA). (3) Ionized electrons and protons interact with each other and emit X-rays through thermal bremsstrahlung and contribute to the soft X-ray (0.5–2 keV) background (XRB). Several other tracers have been proposed, including X-ray absorption techniques (Perna & Loeb 1998), but they depend strongly on chemical compositions and would be difficult to associate with direct IGM properties. These tracers probe different IGM phases and help to extract the IGM state. For example, the XRB flux upper limits place constraints on the gas clumpiness and suggest a potentially strong role for feedback (Pen 1999).

Among these tracers, the SZ effect is a particularly powerful IGM probe for the following reasons. (1) It provides a complete sample of the intergalactic medium. All free electrons participate in Thomson scattering and contribute to the SZ effect. The Thomson optical depth from the epoch of reionization \(z \sim 10\) to the present is \(\tau \sim 0.1\), which means that about 10% of CMB photons have been scattered by electrons. Since the number of CMB photons is much larger than the baryon number and the ionization fraction of our universe is larger than 99%, most baryons contribute to the SZ effect. Compton scattering does not depend on redshift and is not affected by distance or the expansion of the uni-

\(^{1}\) See http://www.asiaa.sinica.edu.tw/amiba.
verse. Thus, the SZ effect can probe the distant universe. Furthermore, the SZ effect does not strongly depend on gas density and probes a large dynamic range of baryon fluctuations. In contrast, the X-ray emission measure depends on density squared and primarily probes the densest IGM regions at low redshift. The Lyα forest probes the neutral IGM, which in ionization equilibrium depends also on the square of the baryon density. The neutral fractions account for only a tiny fraction of total baryons. Extrapolations over many orders of magnitude are required to understand the bulk of the baryons. (2) It is straightforward to understand. The simple dependence of the SZ effect on the ionized gas pressure or momentum does not put as stringent a requirement on simulation resolution, nor require as accurate an understanding of the gas state such as metallicity, temperature, velocity, and ionization equilibrium, as X-rays and Lyα properties do. Pressure is the total thermal energy content of the gas, and finite-volume flux-conservative simulations are particularly amenable to its modeling. It is also easier to model analytically. (3) It has strong observational potential. In this paper we will primarily consider the thermal SZ effect, which is easier to observe. Its unique dependence on frequency allows us in principle to disentangle the SZ effect from the contamination of the primary CMB and other noise sources (Cooray, Hu, & Tegmark 2000). Hereafter, if not otherwise specified, the SZ effect always means the thermal SZ effect.

In SZ observations, all redshifts are entangled and smear some key information of the intervening IGM: its spatial distribution and time evolution. These properties around redshift \( z \sim 1 \) are sensitive to many cosmological parameters and thus potentially promising to break degeneracies in cosmologies from the primary CMB experiments. One can hope to resolve the equation of state of the dark energy. We are currently performing simulations for different cosmologies, which are degenerate in the primary CMB, to test the potential of such a procedure (P. J. Zhang, U. L. Pen, & U. Seljak 2002, in preparation). If combined with other observations, the SZ effect will become even more powerful. For example, with the aid of a galaxy photometric redshift survey, the evolution of the three-dimensional gas pressure power spectrum and its cross-correlation with the galaxy distribution can be extracted (Zhang & Pen 2001).

The above analysis depends on a detailed quantitative theoretical understanding of the SZ statistics. Analytical approaches considered in the past strongly depend on various assumptions. The Press-Schechter approach as adopted by Cole & Kaiser (1988), Makino & Suto (1993), Atrio-Barandela & Mucket (1999), Komatsu & Kitayama (1999), Cooray (2000), and Molnar & Birkshaw (2000) requires ad hoc models for the gas profile in halos, whose shape and evolution are uncertain. The hierarchical method as proposed by Zhang & Pen (2001) strongly depends on the gas–dark matter correlation, which is also poorly understood. These estimates can be significantly improved by high-resolution simulations. However, past simulation results (Scaramella, Cen, & Ostriker 1993; da Silva et al. 2000; Refregier et al. 2000; Seljak, Burwell, & Pen 2001; Springel, White, & Hernquist 2001) disagreed on both the amplitude and the shape of the SZ power spectrum (see Springel et al. 2001 for a recent discussion). Simulation resolution may play a key role in this discrepancy since the SZ effect is sensitive to small structures, as discussed by Seljak et al. (2001). Differences between code algorithms may also cause part of the discrepancy. To address these problems, we ran the largest adaptive SZ simulation to date, a direct 512\(^3\) moving mesh hydro (MMH) code (Pen 1998a) simulation, to reinvestigate the SZ statistics. For a better understanding of numerical and code issues in the SZ effect, our group is carrying out a series of simulations with identical cosmological parameters and initial conditions but using different codes with different resolutions (Codes Comparison Program 2002\(^2\)).

Apart from simulation issues, a problem that has received little attention is analysis strategies for SZ data. Several interferometric arrays such as AMIBA and SZA are under construction, but detailed data analysis models are still in their infancy. With our simulated SZ maps, we can estimate the sensitivity and accuracy of SZ observations. Given a sky scan rate, how accurately can the SZ power spectrum be measured? How many clusters can be found? How accurately can the SZ decrement be determined for individual clusters? What are optimal strategies for these measurements? In this paper, we will take AMIBA as our target to address these questions.

This paper is organized as follows: in § 2, we describe the SZ effect, its statistics, and our method of analyzing these statistics. We then present our simulation results of these statistics and possible constraints from observations (§ 3). In § 4 we simulate AMIBA drift-scan observations to estimate the accuracy of AMIBA measurement of these statistics. We conclude in § 5.

2. FORMULATION AND BACKGROUND REVIEW

The thermal SZ effect causes CMB temperature and intensity fluctuations in the sky. At a given position given by a unit vector \( \hat{n} \) pointing from the earth to some point on the sky, the fractional change in the CMB intensity \( \delta I/I \) depends on both the observing frequency \( \nu \) and the integral of the gas pressure along the line of sight (Zeldovich & Sunyaev 1969):

\[
\frac{\delta I}{I} = -2y(\hat{n})S_T[x(\nu)].
\]

For the CMB intensity, the spectral dependence for inverse Compton scattering off nonrelativistic electrons is described by

\[
S_T(x) = x e^x / (e^x - 1)(2 - x/[2\tanh(x/2)]),
\]

where \( x \equiv h\nu/(k_B T_{\text{CMB}}) = \nu/57 \text{ GHz}. \) The corresponding fractional change in CMB temperature \( \Theta(\hat{n}) \equiv \Delta T_{\text{CMB}}(\hat{n})/T_{\text{CMB}} \) is then

\[
\Theta(\hat{n}, \nu) = -2y(\hat{n})S_T[x(\nu)] = -2y(2 - x/[2\tanh(x/2)]).
\]

In the Rayleigh-Jeans limit (\( x \ll 1 \)), \( S_T(x) = S_R(x) = 1 \). \( S_T(x) \) reaches the highest response, \( S_R(x) \approx 1.6 \) at \( \nu \approx 100 \text{ GHz} \), which is well matched to the AMIBA frequency range of 80–100 GHz. \( S_T(x) \) monotonously decreases with frequency. We show \( S_T(x) \) and \( S_R(x) \) in Figure 1, where the frequency responses of various experiments that already have the CMB observation data at small angular scales (\( \ell > 2000 \)) such as the Australia Telescope Compact Array (ATCA; Subrahmanyan et al. 2000), Berkeley-Illinois-

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\(^2\) TreeSPH: J. Dubinski; Gasoline code: M. Ruetalo & J. Wasley; Gadget code: T. Woo & J. Wu; MMH code: P. J. Zhang & U. L. Pen, collectively known as the Codes Comparison Program 2002, in preparation.
The SZ effect contains contributions from all redshifts, and it is challenging to recover the smeared redshift information. We have shown that cross-correlating the SZ effect with a galaxy photometric redshift survey, we can infer the redshift resolved IGM pressure–galaxy cross-correlation and the IGM pressure autocorrelation (Zhang & Pen 2001). This method is robust but does not capture all the information in the SZ observation. In this paper, we utilize the one-point distribution function (PDF) of the $y$-parameter and the distribution of peaks of $y$ to extract more information.

Since smoothing is always present for a real experiment with a finite beam, we calculate the statistics of the $y$-parameter smoothed on a given angular scale $\theta$. If we are interested in virialized objects, for example, clusters and groups of galaxies, we would expect them to be peaks in the smoothed or filtered $y$-maps. $N(y > y_p)$, the cumulative distribution function (CDF) of peaks with smoothed $y$-parameter bigger than a certain value $y_p$, is the raw observable. The $y_p$ CDF is the SZ analog to a luminosity function. If we choose a top-hat window so that the observation cone is large enough to include an entire object such as a cluster and is small enough that typically no more than one such object can be found along each cone, then when the cone is centered at the center of each object, a peak $y \equiv y_p$ appears in the smoothed map. This $y_p$ is directly related to the total gas mass and temperature of the individual object. Assuming a halo to be isothermal, the total mass $M$ of a halo is related to the gas temperature $T$ by $M/M_8 = (T/T_8)^{3/2}$. $M_8 = 1.8 \times 10^{14} (\Omega_0/0.3) h^{-1} M_\odot$ is the mass contained in a $8 h^{-1}$ Mpc sphere of the universe of mean density today, which is roughly the mass scale of clusters. $T_8(z)$ is the corresponding temperature of a halo with mass $M_8$ at redshift $z$. We then obtain

$$y_p = \frac{M_8 f_{\text{g}} \sigma_T k_B T_8}{\mu m_{\text{H}} d_4^2 \Delta \Omega m_e^2} \left(\frac{M}{M_8}\right)^{5/3}.$$  

(4)

Here $\Delta \Omega$ is the solid angle of the cone, $f_{\text{g}}$ is the gas fraction of halos, and $d_4$ is the angular diameter distance. For clusters and groups, the typical angular size at $z = 1$ is about $1'$.

For the present cluster number density $n(T > 2 \text{ keV}) \sim 10^{-5} h^{-3} \text{ Mpc}^3$ (Pen 1998b), the average number of clusters in a cone with angular radius $\theta \approx 20'$ projected to $z = 2$ is about one, allowing for the evolution of cluster number density. Thus, the size of the smoothing scale for the peak analysis should be between these two scales. In this case, the $N(y > y_p)$ is just the number of halos with $y > y_p$, which is given by

$$N(y > y_p) = \int_0^{r_{\text{re}}} \frac{\chi^2}{\sqrt{1 - K r^2}} \int_{M(y_p, z)}^{\infty} \frac{dn}{dM} dM.$$  

(5)

$dn/dM(M, z)$ is the halo comoving number density distribution function and is well described by the Press-Schechter formalism (Press & Schechter 1974); $\chi$, $r$, and $K$ are the comoving distance, radial coordinate, and curvature of the universe, respectively. The subscript “re” means the reionization epoch. $M(y_p, z)$ is the mass of the halo with smoothed $y = y_p$ given by equation (4). Equation (5) has two applications. First, given an SZ survey and the best-
strained cosmological parameters as determined by CMB experiments, Type Ia supernovae, weak lensing, etc., and an SZ survey, the only unknown variable in equation (5) is $T_b(z)/f_g(z)$, which then is uniquely determined. $T_b$ is robustly predicted since it is mainly determined by $M_b$ through hydrostatic equilibrium and has only weak dependence on the thermal history. For example, comparing the cluster temperature function as inferred from simulations with the Press-Schechter formalism, Pen (1998b) found that $T_b = 4.9(1+z)f_g(z)$ for a $\Lambda$ cold dark matter (CDM) universe. The above relation is sufficient to extract the evolution of the gas fraction $f_g$, which is very sensitive to the thermal history. For example, nongravitational energy injection decreases $f_g$. Second, the number of clusters strongly depends on cosmology, as characterized by $dn/dM$. Given a good understanding of cluster temperature and gas fraction, equation (5) allows one to constrain cosmological parameters. In combination with the measurement of cluster redshifts, this method is more sensitive (Weller, Battye, & Kneissl 2001). In § 4, we will study the statistics of these peaks for maps filtered in optimal ways to measure clusters of galaxies.

With the measurement of the cluster SZ temperature distortion and follow-up X-ray observations of cluster X-ray flux $F_X$ and X-ray temperature $T$, cosmological parameters can be constrained. For a cluster with electron number density $n_e$, proper size $L$, and angular size $\theta$, $F_X \propto n_e^2 \Lambda(T) L^2/d_L^2$ and $\gamma \propto n_e L/T$. Here $\Lambda(T)$ is the X-ray emissivity temperature dependence. Then the luminosity distance $d_L(z) \propto (y^2/F_X)(\Lambda(T)/T^2) |\theta/(1+z)^2|$. The only uncertainties in this relation are the intracluster gas profile and metallicity, which affects the X-ray emissivity. These properties could be modeled and are potentially observable. In this sense, the SZ effect can be used as a cosmological distance indicator (Silk & White 1978; Barbosa et al. 1996; Mason, Myers, & Readhead 2001; Fox & Pen 2001). Since dark energy dominates at low redshift where the SZ observation and X-ray observation of clusters are relatively straightforward, SZ clusters are a potential probe to constrain the equation of state for the dark energy.

3. SIMULATIONS

In this section, we describe our SZ simulation used to investigate the above statistics. We will also use our simulation to provide SZ maps for our estimation of the sensitivity of upcoming SZ experiments (§ 4) and its effect on data analysis strategies. We used an MMH code (Pen 1998a). It features a full curvilinear total-variation–diminishing (TVD) hydro code with a curvilinear particle mesh (PM) N-body code on a moving coordinate system. The full Euler equations are solved in an explicit flux-conservative form using a second-order TVD scheme. The curvilinear coordinates used in the code are derived from a gradient of the Cartesian coordinate system. If $x^i$ are the Cartesian coordinates, the curvilinear coordinates are $\xi^i = x^i + \partial_i \phi(\xi)$. The transformation is completely specified by the single potential field $\phi(\xi, t)$. The potential deformation maintains a very regular grid structure in high-density regions. The gravity and grid deformation equations are solved using a hierarchical multigrid algorithm for linear elliptic equations. These are solved in linear time and are asymptotically faster than the fast Fourier transform (FFT) gravity solver. At the same time, adaptive dynamic resolution is achieved. During the evolution any one constraint can be satisfied by the grid. In our case, we follow the mass field such that the mass per unit grid cell remains approximately constant. This gives all the dynamic range advantages of smooth particle hydro combined with the speed and high resolution of grid algorithms. The explicit time integration limits the time step by the Courant condition. To achieve a reasonable run time, we limit the compression factor to a factor of 5 in length, corresponding to a factor of 125 in density. Most SZ contributions arise below such densities, giving diminishing returns if higher compression factors are used.

The parameters we adopted in our 512$^3$ simulation are $\Omega_0 = 0.37$, $\Omega_\Lambda = 0.63$, $\Omega_b = 0.05$, $h = 0.7$, $\sigma_8 = 1.0$, power spectrum index $n = 1$, box size $L = 100 h^{-1}$ Mpc, and smallest grid spacing 40 $h^{-1}$ kpc. The simulation used 30 Gbyte memory and took about 3 weeks ($\sim 1500$ steps) on a 32-processor shared-memory Alpha GS 320 at the Canadian Institute for Theoretical Astrophysics using Open MP parallelization directives. During the simulation we store two-dimensional projections through the three-dimensional box at every light-crossing time through the box. The projections are made alternatively in the $x$-, $y$-, and $z$-directions to minimize the repetition of the same structures in the projection. We store projections of thermal SZ, kinetic SZ, and gas and dark matter densities. For the thermal SZ, we store $2\Delta y = 0.05$ of $P_{\kappa L}$ as given by equation (3). Our two-dimensional maps are stored on 2048$^2$ grids. As tested by Seljak et al. (2001), this preserves all the information down to the finest grid spacing. After the simulation, we stack the SZ sectional maps separated by the width of the simulation box, randomly choosing the center of each section and randomly rotating and flipping each section. The periodic boundary condition guarantees that there are no discontinuities in any of the maps. We then add these sections onto a map of constant angular size. Using different random seeds for the alignments and rotations, we make 40 maps of width 1:19 to calculate the SZ statistics. The mean $y$-param-

3.1. The SZ Power Spectrum

The power spectrum in the Rayleigh-Jeans regime averaged over 40 maps is shown in Figure 4. The thermal SZ power spectrum is fairly flat, with a broad peak at $l \sim 2000–10^4$ and a typical fluctuation amplitude $\Theta \sim 5.5 \times 10^{-6}$. It begins to dominate over the primary CMB at $l \sim 2000$. The shape of the kinetic SZ power spectrum is similar to the thermal one, but the amplitude is about 30 times smaller.

In comparison with available CMB observations, our thermal SZ power spectrum is well below the upper limit (95% confidence) of ATCA (Subrahmanyan et al. 2000), SUZIE (Church et al. 1997), and VLA (Partridge et al. 1997). C$\ell$ at $l \sim 2000$ is consistent with recent indications from the CBI experiment (Mason et al. 2001; Sievers et al. 2001) and may suggest a high $\sigma_8 \approx 1.0$. However, our result is higher than the BIMA $1 \sigma$ result, although it is consistent with the upper limit (95% confidence) of the BIMA result.
The SZ map clearly shows the structures with positive \( T \) regions is moving away. \( T \) regions is approaching us, and that in the negative \( T \) regions is moving away. 

See the electronic edition of the Journal for a color version of this figure.

Published theoretical predictions differ a lot in both amplitude and shape (see Springel et al. 2001 for a review). Our power spectrum has a higher amplitude than all previous predictions. It is also significantly flatter at the range 2000 < \( l \) < 15,000. The difference in amplitude can be explained by the strong dependence of the SZ effect on cosmological parameters, especially \( \sigma_8 \). One expects \( y \propto \Omega_{bh} \sigma_8^2 \) and \( C_l \propto (\Omega_{bh})^2 \sigma_8^2 \) as predicted by various authors (Komatsu & Kitayama 1999; Seljak et al. 2001; Zhang & Pen 2001). For example, Seljak et al. (2001) used the same MMH code with lower resolution (256 \(^3\)) to simulate a universe with \( \sigma_8 = 0.8 \). This \( \sigma_8 \) difference accounts for a factor of 1.6–2.0 difference in the mean \( y \) estimation and 3.8–7.5 difference in the \( C_l \) estimation. After accounting for these effects, our power spectrum is consistent with theirs at small angular scales (\( l > 3000 \)). However, the difference in shape cannot be explained in this way. For example, even after accounting for the effect of cosmological parameters, our power spectrum is still much larger at large angular scales (\( l \sim 1000 \)) than that of Seljak, Burwell, & Pen (2001) (at \( l \sim 1000, \sim 2 \) times larger). This flatness behavior may be a manifestation of the resolution effect.

We notice that there are numerous high-\( y \) regions of arcminute or subarcminute scales in SZ maps as seen in Figure 2. To quantify this phenomenon, we show the dependence of \( N_\theta(y > y_p) \) on smoothing angular size \( \theta \) (Fig. 5). \( N_\theta(y > y_p) \) is a measure of the number of structures with angular size larger than or comparable to \( \theta \). Figure 5 shows that when the smoothing scale increases from 0.5 to 1', \( N(y > y_p) \) drops significantly in high-\( y \) regions. This behavior suggests the existence of numerous subarcminute, high-\( y \) structures in SZ maps. Higher resolution reveals more such structures. Increasing the number of these objects increases the amplitude of the power spectrum while making the power spectrum flatter around the peak. The first effect is obvious, and the second one can be explained by the Press-Schechter picture. The SZ power spectrum is dominated by the halo gas-pressure profile \( f_P(r) \) at all interesting angular scales.

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(Dawson et al. 2001). We will further discuss these issues below.

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Fig. 2.—One typical thermal SZ map in our MMH code simulation. The cosmology is a \( \Lambda \)CDM with \( \Omega_b = 0.37, \Omega_{\Lambda} = 0.63, \Omega_m = 0.05, h = 0.7, \) and \( \sigma_8 = 1.0 \). The map size is 1.19 \( \times \) 1.19. The gray scale represents the SZ temperature fluctuation in the Rayleigh-Jeans regime \( \Delta T/T = -2y \). We have omitted the negative sign. The SZ map clearly shows the structures with angular scale \( \sim 1^\prime \), which is the typical scale of clusters and groups. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 4.—Thermal and kinetic SZ effect power spectra in our simulation. They are averaged over 40 maps. Dashed lines are the corresponding 1 \( \sigma \) upper limit and lower limit of the mean power spectrum, respectively. Both effects peak at \( l \sim 10^4 \). For the thermal SZ effect, the power spectrum is almost flat in the range of 2000 \( \leq l \leq 15,000 \). As a comparison, we show the BIMA result (Dawson et al. 2001) under a Gaussian assumption. The comparable amplitude between theory and observation puts strict constraints on the understanding of the SZ effect. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 3.—Kinetic SZ map in the same simulation at the same field of view as in Fig. 2. The amplitude of temperature fluctuation of the kinetic SZ effect is generally 1 order of magnitude lower than the thermal SZ effect. Gas in the positive \( \Delta T/T \) regions is approaching us, and that in the negative \( \Delta T/T \) regions is moving away. [See the electronic edition of the Journal for a color version of this figure.]
Fig. 5.—\( N(y > y_p) = \int_{y_p}^{\infty} n(y) dy \) is the number of peaks with Compton parameter \( y \) bigger than \( y_p \). The result is averaged over 40 maps. The smoothing function we adopt is the top-hat window function with radius \( \theta \).

\( N(y > y_p) \) as a function of \( \theta \) is a direct measurement of the number of structures with scale larger than \( \theta \). The quick drop of \( N(y > y_p) \) from \( \theta = 0.5' \) to \( \theta = 1' \) shows that there are a lot of arcminute-scale structures. We believe that these structures with large \( y \) are responsible for the divergence of SZ simulations toward high resolution. [See the electronic edition of the Journal for a color version of this figure.]

scales (Komatsu & Kitayama 1999). For a singular isothermal sphere (SIS), \( \chi_p(r) = \frac{n_e(r)k_B T(r)}{\rho(r)} \propto n_e(r) \propto r^{-2} \). Its projection along the line of sight then has a \( \theta^{-1} \) radial dependence, which produces an angular autocorrelation function of shape \( \ln(\theta) \). Thus, the resulting power spectrum is nearly flat. A more accurate way to see the flatness behavior is to adopt Limber’s equation. The three-dimensional gas-pressure power spectrum \( P_p(k) \propto \delta_g^2(k) \propto k^{-2} \), where \( \delta_g(k) \) is the Fourier transform of the gas-pressure profile \( \chi_p(r) \). From Limber’s equation, \( C_l \propto \int P_p(l/\chi(z), z)f(z)dz/2\pi l^2 \), thus, \( l(l+1)C_l/(2\pi) \propto l^0 \). Here \( f(z) \) is the redshift dependence of the SZ effect. The halo mass function also plays a role for the flat power spectrum. Since the SIS profile does not apply to the core of halos, a given cluster is no longer SIS at scales smaller than its core size. Smaller clusters take over and extend the flat power spectrum. The power spectrum in our simulation clearly shows this flatness and suggests the role of these subarcminute halos. These halos also explain the discrepancy between our simulation and analytical predictions. \( 1' \) corresponds to the comoving size \( \lesssim 0.8 \cdot h^{-1} \text{Mpc} \) for \( z < 1 \) where the dominant SZ contribution comes from. This physical size corresponds to groups of galaxies. Current predictions from the Press-Schechter picture assume a lower mass limit cutoff corresponding to the mass scale of groups. In the hierarchical method, the gas window function is a free parameter, which has an implicit dependence on this lower mass cutoff (they can be related by the gas density dispersion). The cutoff for contributions from groups of galaxies in analytical models results in a smaller \( y \) and power spectrum. If only gravitational heating is included, high-mass halos and low-mass halos should have comparable gas fractions.

The Press-Schechter formalism predicts many more halos toward the low-mass end, so one expects pure gravity simulations to have increasing power spectra with increasing resolution. Nongravitational heating avoids such a divergence. As described in the halo model of Pen (1999), nongravitational heating has two effects. First, the energy injection makes the halo gas less clumpy. Then the contribution to the power spectrum at smaller angular scales decreases relative to larger angular scales. This could explain the slight differences between our simulation and CBI and BIMA results. Second, the nongravitational energy injection increases the thermal energy of the gas. For halos with mass lower than some threshold, the gravity cannot hold gas and most of the gas is ejected from these halos, as must have been the case for galactic-size halos. This provides a reasonable lower mass limit cutoff in the Press-Schechter picture. Thus, the amplitude and shape of the SZ power spectrum are a sensitive measure of the nongravitational energy injection. To obtain a better understanding of the SZ effect, other physical processes such as radiative cooling need to be considered. Since radiative cooling through thermal bremsstrahlung is a \( \rho^3 \) process, it becomes relevant only at scales \( \lesssim 100 \text{ kpc} \), which corresponds to \( l \gg 10^4 \). These angular scales are not observable by any planned experiments, so we neglect the discussion of radiative cooling in this paper. Our current simulation has reached the resolution needed to see the contribution from small halos, and the predicted SZ amplitude is already near the observed values. We expected to be able to observe the effects of nongravitational heating from galaxy winds and other sources in the upcoming experiments.

In order to solve the discrepancy problem completely, differences between codes must be considered. Our group is currently running different codes with identical initial conditions, identical cosmological parameters, and various resolutions from 64\(^3\) to 512\(^3\) (Codes Comparison Program 2002).

Our prediction is consistent with the BIMA 95% confidence result. However, the comparison between observations and theoretical predictions needs further investigation. In the simulation part, as we discuss above, the detailed normalization depends on \( \sigma_8 \), nongravitational heating, and resolution effects. Observations will put strong constraints on these aspects. In the observational part, the BIMA result needs to be reconsidered. It covers the sky where no strong SZ temperature distortion due to known galaxy clusters exists. Since clusters contribute a significant fraction to the SZ power spectrum, the BIMA result may be smaller than the statistical mean value. Furthermore, the error of the BIMA result is estimated under the Gaussian assumption, but as we will see below, the SZ effect is highly non-Gaussian around the BIMA central multipole \( l = 5530 \). The strong non-Gaussianity increases the intrinsic error of the SZ power spectrum measurement. According to Figure 6, the error in the power spectrum measurement caused by the SZ effect is about 3 times larger than the corresponding Gaussian case.

Nonetheless, these results demonstrate the convergence between theory and observation. In the near future, routine and accurate measurement of the SZ effect will be possible in random fields. It will put a stronger requirement on our theoretical understanding of the SZ effect and allow us to study the effects of nongravitational heating, radiative cooling, and the thermal history of the IGM.
3.2. The SZ Non-Gaussianity

In contrast to the primary CMB, the SZ effect is non-Gaussian, arising from the nonlinearity of the intervening gas. This non-Gaussianity affects the error analysis and may help to separate the SZ effect from the primary CMB in observation. To quantify these effects, we smooth SZ maps using a top-hat window of radius $\theta$ and measure the kurtosis of the smoothed $y$. The kurtosis $\theta_4 \equiv (y - \bar{y})^4/\sigma_4^4 - 3$ is generally a function of smoothing scale. For a Gaussian signal, $\theta_4 = 0$. Figure 6 shows that $\theta_4 \approx 200$ at small scales as $\theta \to 0$. This result is consistent with the prediction from our hierarchical model approach of the SZ effect (Zhang & Pen 2001). We predicted, at small angular scales, $\theta_4 \approx S_4\sigma_4^2(z \sim 1) \approx 200$. Here $S_4 \equiv \langle \delta^4 \rangle/\langle \delta^2 \rangle^2 \approx 40$ is a hierarchical model coefficient (Scoccimarro & Frieman 1999) and $\sigma_4(z \sim 1) \approx 5$ is the gas density dispersion at $z \sim 1$. At large angular scales $\theta \sim 20\', \theta_4 > 1$ and reflects the strong non-Gaussianity at this scale. It means that, at this angular scale, the dominant contribution to the $y$-parameter is from highly nonlinear regions and there is still strong correlation at angular scales down to $l \sim 1000$, as can be seen from the SZ power spectrum.

The kurtosis in multipole space $a_4 \equiv \langle |a|^4 \rangle / \langle |a|^2 \rangle^2 - 3$. As usual, $a \equiv a_{lm}$ defined by $\Theta(n) \equiv \sum_{lm} a_{lm} Y_l^m(n)$ are multipole modes of $\Theta$ and $C_l \equiv \sum_{l} \langle |a_{lm}|^2 \rangle / (2l + 1) \equiv \langle |a|^2 \rangle$. Here $Y_l^m$ are the spherical harmonics. \(\langle \ldots \rangle\) is averaged over all $m = -l, \ldots, l$ and all maps. We calculate $a_4$ indirectly through the map-map variance $\sigma_M(l)$ of $C_l$, which is related to $a_4$ by $\sigma_M(l) = C_l/[(2l + 1)\Delta l_{\text{map}}]^{1/2}$. $\Delta l_{\text{map}}$ is the fractional sky coverage of each map, which reflects the cosmic variance, and $\Delta l = 2\pi/\Delta l_{\text{map}}$ is the $l$ bin size in our grid map with $\Delta l_{\text{map}} = 1.19$. The factor $2l + 1$ arises because $C_l$ is averaged over $2l + 1$ independent $a_{lm}$ modes. In our calculation, $C_l$ of each map is obtained using FFT under the flat-sky approximation. Then we obtain the map-map variance $\sigma_M(l)$. One might expect $a_4$ to have similar behavior to $\theta_4$ at corresponding scales, for example, at small angular scales (large $l$), $a_4 > 2$. However, Figure 6 shows the opposite: $a_4 \sim 50$ around $l \sim 1000$, rises slowly until $l \sim 3000$, and approaches Gaussian ($a_4 = 0$) quickly after that. In fact, $\theta_4$ and $a_4$ cannot be directly compared. The multipole modes are nonlocal and averaged over many patches. If these patches are spatially uncorrelated, then the central limit theorem ensures that the multipole modes are Gaussian independent of the actual non-Gaussianity of the patches. From this viewpoint, we can estimate the relation between $a_4$ and $\theta_4$. $Y_l^m(n)$ are quickly fluctuating functions with period $\Delta \theta \sim 2\pi/l$. Two signals separated by a distance larger than $4\Delta \theta$ will have effectively no correlation. Thus, for an $l$-mode, $a_{lm}$ is approximately the sum of $N \sim \sigma_2/(4\Delta \theta)^2$ uncorrelated patches. The non-Gaussianity of each patch is roughly characterized by $\theta_4$. Then, $a_4 \approx \theta_4/N$, which may explain the behavior of $a_4$.

The shape of $a_4(l)$ is similar to the shape of the pressure bias $b_\rho(k, z)$ (Fig. 2 in Zhang & Pen 2001) defined by $b_\rho^2(k, z) \equiv P_\rho(k, z)/P_{\Sigma}(k, z)$, where $P_\rho$ and $P_{\Sigma}$ are the corresponding gas-pressure power spectrum and dark matter density power spectrum. This similarity suggests a common origin. In the hierarchical approach, $a_4(l)$ is related to $b_\rho^2(k, z)$ and we expect $a_4(l)$ to keep a similar shape but with smaller amplitude variation than $b_\rho^2$, as illustrated by the behavior of $a_4(l)$ in our simulation and $b_\rho^2(k, z)$ in our analytical model.

We further show the $y$ PDF (Fig. 7) as a function of smoothing scale. The distribution of $\log_{10} y$ is roughly Gaussian, especially for large smoothing scales, as sug-
4. SIMULATED OBSERVATIONS OF THE SZ EFFECT

In real SZ observations, instrumental noise and primary CMB cause additional errors in the SZ statistics such as the power spectrum and peak number counts. We need to estimate these effects to derive optimal observing strategies to measure these statistics in the presence of noise. With these observational errors, our methods (§ 2) of extracting three-dimensional gas information are limited and we must check their feasibility. In this section, we take AMIBA as our target to address these issues.

4.1. AMIBA

AMIBA is a 19 element interferometer with 1.2 m dishes. All dishes are closely packed in three concentric rings. It operates at $\nu_{\text{center}} = 90$ GHz with $\Delta \nu = 16$ GHz, system noise $T_{\text{sys}} = 100$ K, and system efficiency $\eta = 0.7$. At this frequency, $\Theta \approx -1.6\gamma$ with $S_T(90\text{GHz}) \approx 0.8$. The goal of this experiment is to image maps of the CMB sky with arc-minute resolution. We consider observations with fixed integration time and aim at finding the optimal sky area $\Omega$ and sky fractional coverage $f_{\text{sky}} = \Omega/4\pi$ for a given statistics.

For closely packed interferometers observing such weak signals, the ground fringe can be a major source of noise. To eliminate the ground fringe, AMIBA plans to drift scan: the telescope is parked while the sky drifts by. This ensures that the ground fringe remains constant with time. The mean value of each fringe is then subtracted from the scan, cleanly eliminating the ground. A field is mosaicked by a series of adjacent scans, and the most uniform coverage is achieved by incrementally offsetting the pointing center on each scan to yield a finely sampled two-dimensional map. The raw output of the experiment is correlations, two for each baseline, polarization, and frequency channel, corresponding to the real and imaginary correlator outputs. We can think of each of these outputs as corresponding to an image of the sky filtered through some anisotropic beam. As a first step, we can combine degenerate baselines and polarizations, reducing the 171 baselines to 30 nondegenerate baselines. Since the CMB is expected not to be significantly unpolarized, we can combine the two polarization channels, leading to 60 raw maps per frequency channel. These maps can be merged optimally into one global map by convolving each map with its own beam, scaling each map to the same noise level, and co-adding these maps, resulting in a “clean” map. Each of the constituent maps had explicitly white noise, so the noise statistics of the summed map are easily computable. The clean map can be deconvolved by the natural beam, resulting in a “natural” map. This natural map is an image of the sky convolved with the natural beam of the telescope plus white noise. The angle-averaged natural beam is shown in Figure 9. The CMB intensity fluctuation $\delta I_c$ measured by AMIBA has three components: the primary CMB, the SZ effect, and the instrumental noise. It is related to the temperature fluctuation by

$$\frac{\delta I_c}{T_c^{\text{CMB}}} = \frac{x \exp(x)}{\exp(x) - 1} \left[ \left( \frac{\delta T}{T} \right)^{\text{CMB}} + \left( \frac{\delta T}{T} \right)^{\text{SZ}} + \left( \frac{\delta T}{T} \right)^{\text{RJ}} \right]. \tag{6}$$

Here $(\delta T/T)_{\text{CMB}}$ and $(\delta T/T)_{\text{SZ}}$ are the corresponding temperature fluctuations seen through the AMIBA natural beam; $x = h\nu/k_B T_{\text{CMB}}$. We have chosen the normalization of the beam such that the noise power spectrum is white, with

$$C_N = 4\pi T_{\text{sys}}^2 f_{\text{sky}}/(2\Delta \nu \eta^2)/T_{\text{RJ}}^2. \tag{7}$$

$T_{\text{RJ}} = T_{\text{CMB}}^2 x \exp(x)/[\exp(x) - 1] = 2.22$ K is the Rayleigh-Jeans equivalent CMB temperature at $\nu = 90$ GHz. The factor of 2 is due to the two polarizations of the AMIBA experiment. For a single dish of infinite aperture, with a single pixel detector, the window would be identically one: the scanned image is just the CMB distribution on the sky. Since AMIBA has many detectors, one can combine them either to lower the noise or to boost the signal. We have chosen to use the equivalent noise of a single-pixel sin-
gle-dish experiment and normalized the beam accordingly. We calculate the natural beam $W_N(l)$ by equations (17) and (46) of U.-L. Pen, K.-W. Ng, M. J. Kesteven, & B. Sault (2002). The natural beam in multipole space and real space are shown in Figure 9. The natural beam has an FWHM of 2'. It peaks at $l_p \approx 2\pi \lambda / D_1$. Here $\lambda \approx 3.3 \text{ mm}$ is the AMIBA operating wavelength and $D_1$ is the distance of the $i$th baselines. The first peak $l_{\text{peak}} \approx 2273$ corresponds to the shortest baseline $D_1 = 1.2 \text{ m}$. At this angular scale, the sum of all the baselines improves throughput by a factor of almost 3 ($\epsilon \approx 2.77$). This is analogous to having a 9 pixel detector on a single dish. More detailed definitions of the beams and strategies are described in detail in U.-L. Pen, K.-W. Ng, M. J. Kesteven, & B. Sault (2002).

In our simulated pipeline, we add primary CMB map fluctuations to our simulated sky maps using CMBFAST-generated (Seljak & Zaldarriaga 1996) power spectra with the same cosmological parameters except for the use of COBE normalization for $\sigma_8$, which is slightly different from our simulation value. As we will show below, the primary CMB is not the main source of noise for the SZ power spectrum measurement at AMIBA angular scales ($l > 2000$) and is negligible in SZ cluster searches. Thus, the effect of this inconsistent $\sigma_8$ is insignificant in our analysis. Adding SZ maps and CMB maps, we obtain simulated sky maps (Fig. 8). In these maps, SZ structures, especially those caused by diffuse IGM, are superposed with the primary CMB. We then convolve these maps with the natural beam. The beam function decreases quickly to zero toward large angular scales where the primary CMB dominates, so it efficiently filters most primary CMB structures larger than this scale. We then add the noise given by equation (7) to this map. The normalization in the beaming and filtering (described below) process is arbitrary. We choose the normalization in such a way that the power spectrum of the map at the scale of the peak response does not change after beaming or filtering. It corresponds to normalizing the global maximum of the beam function to be unity. Under such normalization, when we add instrumental noise to the simulated CMB+SZ map, the noise is depressed by a normalization factor $\epsilon \approx 2.77$. Our 1'9 map has 2048 pixels, so the white instrumental noise dominates on small scales. For a 20 hr deg$^-2$ survey, the dispersion of the noise temperature fluctuation $\sigma_N = T_{\text{sys}}/(2Nu_{\text{pixel}})^{1/2}(\eta T_{\text{RJ}}) \sim 0.002 \eta y f_{\text{pixel}}$ is the observing time for each pixel. Thus, all signals are hidden under the instrumental noise. One needs further filtering to obtain an image that is not overwhelmed by the small-scale noise. Ignoring the CMB fluctuations, we know that point sources have the shape of the beam. A point source–optimized search would convolve the natural map with the shape of the beam, and peaks in this map correspond to the maximum likelihood locations of point sources. We can think of the CMB as a further source of noise and filter that away as well. In this case, the filter depends on the ratio of CMB and noise amplitudes, i.e., the actual integration time. One can also optimize a filter for a structure of a known intrinsic shape. Since clusters are approximately isothermal, a better filter would be one matched to the extended isothermal nature, where objects in the natural map have the shape of an isothermal sphere convolved with the natural beam. Combining all these considerations, the optimal filter for noise cleaning is given by $W_F(l) = W_F(l) W_N(l)/(1 + W_N(l) C_{CMB}/C_N)$ (Fig. 9). $W_F(l)$ is the Fourier transform of the source intrinsic shape $W_F(r)$. For point sources, $W_F(l) = 1$. For clusters, on scales smaller than the cluster virial radius and larger than the core radius, $W_F(l) \propto l^{-1}$ ($\theta$ is the angular distance to the cluster center). Then $W_F(l) \propto l^{-1}$. Since the corresponding angular size of the core radius is much smaller than the beam size, the above approximation is sufficiently accurate. We have shown the filter with a 280 hr deg$^{-2}$ scan rate in Figure 9. It has an FWHM of 2'. In multipole space, it drops to near zero at angular scales $l \sim 1500$ and $l > 9000$ and peaks at cluster scales $l \sim 3000$. Thus, it is efficient at filtering away the residual primary CMB and the instrumental noise while amplifying SZ signals. We tried different sky scan rates to find the optimal survey strategy. We show final resulting maps in Figure 10 with a 280 hr deg$^{-2}$ scan rate. With the simulations we have the luxury of seeing maps with noise (right panels of Fig. 10) and without noise (left panels of Fig. 10). We discuss the simulated AMIBA measurement of the SZ power spectrum and cluster searching in the next two subsections.

4.2. The SZ Power Spectrum in the Simulated AMIBA Experiment

One of the key goals of AMIBA is to measure the SZ power spectrum. As discussed in § 3, it is a sensitive measure of the gas thermal history. Furthermore, combining the cross-correlation with photometric galaxy surveys, an SZ survey can measure the IGM pressure power spectrum as a function of redshift. This gives us access to the evolution of the IGM state. For the purpose of the SZ power spectrum estimation, the two noise sources, the primary CMB and the thermal instrument noise, are both Gaussian. The intrinsic

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Fig. 8.—Combined SZ+CMB map. At large angular scales, the primary CMB dominates and smears the structure of the thermal SZ effect. [See the electronic edition of the Journal for a color version of this figure.]

4 http://www.cita.utoronto.ca/~pen/download/Amiba/drift.ps.gz.
SZ variance causes further error. The combined error for the power spectrum estimation is

$$
\Delta C_l^{SZ} = \sqrt{a_4 C_{SZ}^2(l) + 2 \left[ C_{SZ}(l) + C_{CMB}^2 + C_N^2 / W_N^2 \right]^2 / (2l + 1) \Delta f_{\text{sky}}}
$$

(8)

If the SZ effect is Gaussian ($a_4 = 0$), we recover the usual expression of the error. We take a large bin width $\Delta l = 1/4$ to estimate the error. Since the SZ power spectrum is nearly flat in the range $l \sim 2000$–15,000, this choice of bin size does not lose significant information. At large angular scales ($l < 1500$), the error from primary CMB dominates, and at small scales ($l > 5000$), the instrumental noise dominates.

Fig. 9.—Natural beam and optimal filter functions of AMIBA in (left) multipole and (right) real space. The main goal of the beaming and filtering is to filter away the primary CMB, which dominates at large scale ($l \lesssim 1000$), and instrumental noise at small scale ($l \rightarrow \infty$). This goal is clearly illustrated in the large and small $l$ behavior of these functions. The optimal filter depends on the noise amplitude. We show the case for an AMIBA scan rate of 280 hr deg$^{-2}$. This filter has a $\sim 2'$ FWHM. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 10.—Filtered maps (left) without noise and (right) with noise of AMIBA 390 hr observing. Many peaks in the total map are not real signal peaks, while some peaks in the clean map disappear in the total map. For AMIBA’s frequency, $\Theta = -1.6 \mu$. For clarity, we plot $-\Theta$. [See the electronic edition of the Journal for a color version of this figure.]
In the intermediate scales, the intrinsic error of the SZ effect dominates. Different errors have different dependences on the sky coverage. When we increase $f_{\text{sky}}$ while fixing the integration time, the errors from the primary CMB and SZ effect decrease but the one from the instrumental noise increases, so there exists an optimal sky coverage for a given integration time. Since the errors from the primary CMB and SZ effect both scale as $f_{\text{sky}}^{-1/2}$ and the instrumental noise scales as $f_{\text{sky}}$, the minimum error is obtained when $(a_4 + 2)C_{SZ}^2 + 2C_{CMB}^2 + 4C_{SZ}C_{CMB} = 2C_N^2/W_N^2$, which gives the optimal sky coverage. For 1000 hr of observing, several hundred square degree sky coverage is nearly optimal (Fig. 12). Figure 11 shows that AMIBA is able to measure the SZ power spectrum with an accuracy of ~40% for $l$ between 2000 and 5000 in 1000 hr observing of 100 deg$^2$ of sky. The strong dependence of the SZ effect on $\sigma_8$ strongly affects our error estimation. A smaller $\sigma_8$ reduces the signal and signal-to-noise ratio. In order to keep the signal-to-noise ratio, smaller sky coverage is needed to reduce the noise. The dependence of the optimal sky coverage $f_{\text{sky}}^{\text{opt}}$ on $\sigma_8$ is shown in Figure 12. In this estimation, we have assumed the most extreme dependence of the SZ effect on $\sigma_8$, namely, $C_{SZ}(l) \propto \sigma_8^4$.

If we cross-correlate the observed SZ effect with the Sloan Digital Sky Survey (SDSS; 2002), we can extract the underlying three-dimensional gas-pressure power spectrum and pressure-galaxy cross-correlation. To test its feasibility, we estimate the error in the angular cross-correlation measurement by

$$\frac{\Delta C_{SZ, \alpha}(l)}{C_{SZ, \alpha}(l)} = \frac{\sqrt{1 + r^{-2}(1 + C_{\text{CMB}}^2/C_{SZ}^2 + C_{SZ}^N/C_{SZ}^2)(1 + C_N^2/C^2)}}{(2l + 1)f_{\text{sky}}^{-1/2}}.$$ 

SDSS will cover $f_{\text{sky}}^{\text{SDSS}} = \frac{1}{4}$ of the sky and will detect $N_G \approx 5 \times 10^7$ galaxies with photometry in five bands. The

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Fig. 11.—Errors in the AMIBA measurement of the SZ power spectrum (left panel) and cross-correlation (right panel) with SDSS galaxies. The SZ power spectrum is 0.82 times the one in Fig. 4 since at the AMIBA operating frequency $\nu = 90$ GHz, $\Theta = -1.6\gamma$ in contrast to $\Theta = -2\gamma$ in the Rayleigh-Jeans regime. The thick solid error bar is the Gaussian variance; the thin solid error bar is the actual variance calculated from a 40 map ensemble; and the dotted error bar is the total error including primary CMB and instrumental noise. For a 1000 hr survey of a 100 deg$^2$ area, the accuracy in the SZ power spectrum measurement is about 40% at the range $2000 < l < 5000$. In the same survey, the measured cross-correlation has about 20% accuracy at similar $l$-range. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 12.—Dependence of the optimal sky coverage in the SZ power spectrum measurement on $\sigma_8$ in a 1000 hr AMIBA survey. Our simulation used $\sigma_8 = 1.0$. The $\sigma_8 = 0.9$ and $\sigma_8 = 0.8$ cases are estimated assuming the strongest dependence of $C_l$ on $\sigma_8$: $C_l \propto \sigma_8^4$. [See the electronic edition of the Journal for a color version of this figure.]
Poison noise in SDSS has the power spectrum $C_0^{SZ} = 4 \pi f_{SDSS}^2 / N_G$. We assume a linear bias between galaxy number overdensity and dark matter overdensity to calculate the galaxy surface density power spectrum $C_0^G(l)$. Since SDSS is flux-limited, we take the galaxy selection function $dn/dz = 3z^2/2(z_m/1.412)^3 \exp[-(1.412z/z_m)^{1/2}]$ (Baugh & Efstathiou 1993) with an SDSS fit $z_m = 0.33$ (Dodelson et al. 2002). The galaxy-SZ power spectrum $C_{SZG}^G(l)$ is estimated by the SZ-galaxy cross-correlation coefficient $r \equiv C_{SZG}^G / (C_{SZ}C^G)^{1/2}$. We choose $r = 0.7$ (Zhang & Pen 2001) as predicted by the hierarchical model. We show the error in Figure 11. The larger the sky coverage, the smaller the error. With the optimal scan rate for SZ power spectrum measurement, the accuracy in the cross-correlation measurement is about 20%.

4.3. AMIBA Cluster Search

Another important goal of AMIBA is searching for clusters. In the power spectrum measurement, the observable is the direct sum of signal and noise, so the noise contribution can be subtracted linearly in the power spectrum. The only net effect is an increase in the error bars, which is important only at very small angular scales or very large scales. However, when peaks in a map are counted, effects of noise are much more complicated and are not easily interpreted. Noise introduces false peaks in the observed SZ maps, changes the value of real peaks, shifts the peak positions, and even makes real peaks disappear. Thus, noise affects the measurement of cluster counts, richness, and positions. Thus, we would want a much longer integration time for the purpose of a cluster search. We count peaks in the filtered maps with and without noise and quantify these effects as follows. (1) We estimate the accuracy of $y$-measurements by calculating the $y$-dispersion of noise $\sigma_y^N$. (2) We distinguish real peaks from false peaks by comparing the position and amplitude of each peak in the clean SZ maps without noise and in the total maps with noise (Fig. 13). One FWHM is roughly the size of noise structures after filtering and corresponds to the maximum position shift noise can exert on real peaks. $2\sigma_y^N$ is roughly the maximum peak amplitude change that noise can cause. Thus, if a peak in a total map whose distance to the nearest peak in the corresponding clean SZ map is less than one FWHM of the filter and its $y$ value is in the $2\sigma_y^N$ range of the real value, we classify it as real. (3) We estimate the accuracy of $y$ peak CDF by comparing clean SZ maps, noise maps, and total maps (Fig. 14). For signal peaks with $y \gg \sigma_y^N$, the signal peaks remain to be peaks in the total map. Noise mainly changes the amplitude of the $i$th real signal from $y_i$ to $y_p = y_i + y^N$ in the total map. Here $y^N$ is the value of the noise and $y_p$ is the value in the total map. Since noise is Gaussian, we know the distribution of $y^N$, which can be described by $P(y, \sigma_y^N)$, the probability for the Gaussian noise with dispersion $\sigma_y^N$ to have a value bigger than $y$. Then we can relate the CDF of peaks in clean maps to the one of total maps by

$$N_{tot}(y > y_p) = \sum_{i=1}^{N} P(y_p - y_i, \sigma_y^N). \quad (10)$$

This relation gives a good fit in the $y \gg \sigma_y^N$ regime. For signals with $y \sim \sigma_y^N$, noise changes both $y$-value and positions of some peaks, makes some peaks disappear, and introduces a large fraction of false peaks. The sum of $N_{tot}(y > y_p)$ in equation (10) and $N_{noise}(y > y_p)$ considers the effect that noise introduces false peaks, and we expect that it would give a good fit in the region where $y \sim 2\sigma_y^N$. We show the modeled CDF of peaks in total maps following the above procedures in Figure 15. The result is well fitted in the range $y > 2\sigma_y^N$ with better than 30% accuracy.

The optimal survey should find clusters as quickly as possible and measure the $y$-parameter and peak CDF as accurately as possible. We find that the optimal scan rate for...
cluster searching is about 280 hr deg$^{-2}$. At this rate, the rate of cluster detection is about one every 7 hr allowing for a false positive rate of 20%. The measured cluster CDF $N(y > y_p)$ is accurate to 30% up to $N \approx 60$ deg$^{-2}$. We recall that the optimal scan rate for SZ power spectrum measurement is about 10 hr deg$^{-2}$. At this rate, the cluster detection rate is about one cluster every 60 hr. We can consider the effect of cosmology on our estimation. A smaller $\sigma_8$ reduces signals and therefore the cluster detection rate. We assume the extremest dependence of the $y$-parameter on $\sigma_8$, namely, $y \propto \sigma_8^2$ as predicted by Zhang & Pen (2001). For $\sigma_8 = 0.9$, the detection rate decreases by a factor of 4. However, the optimal scan rate remains approximately the same.

At this optimal scan rate, in a 1000 hr AMIBA survey, several hundred clusters can be found. Comparing the SZ cluster counts to Press-Schechter predictions or X-ray surveys (§2) allows us to reconstruct the thermal history of the IGM and nongravitational heating, which is expected to arise from galaxy formation feedback (Pen 1999).

5. CONCLUSION

We have performed the largest high-resolution SZ simulations to date and analyzed the results. We found further increases in the power spectrum relative to previous simulations and significantly more small-scale structures. This trend is confirmed in Press-Schechter estimates and suggests nominally increasing power spectra. The small-structure behavior may not be a robust prediction, since nongravitational effects will significantly modify those scales. We have examined the skewness and kurtosis on the sky maps, found a strong non-Gaussianity on subdegree scales, and confirmed the lognormal distribution found in previous studies. The Gaussian estimates of power spectrum sample variances are less severely affected, because of the averaging of many patches by each Fourier mode. However, its effect on the power spectrum error analysis is significant.

We simulated SZ observations with AMIBA and analyzed the sensitivity for different scan rates. In a 1000 hr survey, the optimal strategy for power spectrum estimations is to scan several hundred square degrees. The SZ angular power spectrum measured in such a survey can be determined to an accuracy of $\sim 40\%$ over a range of $2000 \leq l \leq 5000$. A cross-correlation with SDSS should allow an accuracy of 20% in the cross-correlation measurement, which suggests that the time-resolved measurement of the pressure power spectrum is then possible. This scan rate results in a low detection rate of clusters of galaxies, approximately one per 60 hr with a false positive rate of 20%. For the purpose of cluster search, the optimal scan rate is around 280 hr deg$^{-2}$, which could find one cluster every 7 hr, with an accuracy of 30% in the cluster $y$-measurement and 30% in the cluster CDF up to $N(y > y_p) \approx 60$ deg$^{-2}$.

The predicted SZ power spectrum is consistent with recent indications from the CBI experiment (Mason et al. 2001; Sievers et al. 2001) and the BIMA upper limit (95% confidence). However, it is higher than the BIMA $1\sigma$ result. This may be a first indication of IGM nongravitational feedback. Future blank-sky surveys with data analysis considering actual SZ non-Gaussianity will provide us with a quantitative understanding of the thermal history of the universe.

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REFERENCES

Atrio-Barandela, F., & Mucket, J. 1999, ApJ, 515, 465
Barbosa, D., Bartlett, J. G., Blanchard, A., & Oukbir, J. 1996, A&A, 314, 13
Baugh, C. M., & Efstathiou, G. 1993, MNRAS, 265, 145
Church, S. E., Ganga, K. M., Ade, P. A. R., Holzapfel, W. L., Mauskopf, P. D., Willbanks, T. M., & Lange, A. E. 1997, ApJ, 484, 523
Cole, S., & Kaiser, N. 1988, MNRAS, 233, 637
Cooray, A. 2000, Phys. Rev. D, 62, 103506
Cooray, A., Hu, W., & Tegmark, M. 2000, ApJ, 540, 1
da Silva, A., Barbosa, D., Liddle, A. R., & Thomas, P. A. 2000, MNRAS, 317, 37
Dawson, K. S., Holzapfel, W. L., Carlstrom, J. E., Joy, M., LaRoque, S. J., & Reese, E. 2001, ApJ, 553, L1
Dodelson, S., et al. 2002, ApJ, 572, 140
Fixsen, D. J., Cheng, E. S., Gales, J. M., Mather, J. C., Shafer, R. A., & Wright, E. L. 1996, ApJ, 473, 576
Fox, D., & Pen, U. L. 2001, preprint (astro-ph/0103311)
Fukugita, M., Hogan, C. J., & Peebles, P. J. E. 1998, ApJ, 503, 518
Komatsu, E., & Kitayama, T. 1999, ApJ, 526, L1
Makino, N., & Suto, Y. 1993, ApJ, 405, 1
Mason, B., & CBI Collaboration. 2001, BAAS, 199, 3401
Mason, B. S., Myers, S. T., & Readhead, A. C. S. 2001, ApJ, 555, L11
Molnar, S. M., & Birkinshaw, M. 2000, ApJ, 537, 542
Netterfield, B., et al. 2002, ApJ, 571, 604
Partridge, R. B., Richards, E. A., Fomalont, E. B., Kellermann, K. I., & Windhorst, R. A. 1997, ApJ, 483, 38
Pen, U. L. 1998a, ApJS, 115, 19
———. 1998b, ApJ, 498, 60
———. 1999, ApJ, 510, L1
Perna, R., & Loeb, A. 1998, ApJ, 503, L135
Persic, M., & Salucci, P. 1992, MNRAS, 258, 14P
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Pryke, C., Halverson, N. W., Leitch, E. M., Kovac, J., Carlstrom, J. E., Holzapfel, W. L., & Dragovan, M. 2002, ApJ, 568, 46
Refregier, A., Komatsu, E., Spergel, D. N., & Pen, U.-L. 2000, Phys. Rev. D, 61, 123001
Scaramella, R., Cen, R., & Ostriker, J. 1993, ApJ, 416, 399
Scoccimarro, R., & Frieman, J. 1999, ApJ, 520, 35
Seljak, U., Burwell, J., & Pen, U. 2001, Phys. Rev. D, 63, 063001
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Sievers, J., et al. 2001, BAAS, 199, 3402
Silk, J., & White, S. 1978, ApJ, 226, L103
Springel, V., White, M., & Hernquist, L. 2001, ApJ, 549, 681
Subrahmanyan, R., et al. 2000, MNRAS, 315, 808
Weller, J., Battye, R., & Kneissl, R. 2001, preprint (astro-ph/0110353)
Zeldovich, Y. B., & Sunyaev, R. 1969, Ap&SS, 4, 301
Zhang, P. J., & Pen, U. L. 2001, ApJ, 549, 18