The effect of noncommutativity of the conservation laws on the development of thermodynamical and gas dynamical instability (The method of skew-symmetric differential forms)
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In the works by the author[1,2] it has been shown that the conservation laws for material media (the conservation laws for energy, linear momentum, angular momentum, and mass, that establish a balance between the variation of a physical quantity and the corresponding external action), turn out to be noncommutative. The noncommutativity of the conservation laws that leads to an emergence of internal forces and an appearance of the nonequilibrium is a cause of development of instability in material media (material systems).

These results were obtained with the help of the mathematical apparatus of skew-symmetric differential forms. To study the causes of developing instability it is necessary to inspect the evolutionary relation obtained from equations of the balance conservation laws and to analyze the differential form commutator that enters into this relation.

In the present work the method of skew-symmetric differential forms has been applied in study of the thermodynamical and gas dynamical systems.

It was shown that the principle of thermodynamics follows from two conservation laws, namely, the balance conservation laws for energy and linear momentum. In this case the second principle of thermodynamics, from which the state function (which specifies a state of the thermodynamical system) is obtained, follows from the first principle.

A mechanism of development of instability in gas dynamical systems is described and there are explained such processes as emergence of waves, vortices, turbulent pulsations and so on.

1 Analysis of principles of thermodynamics

The thermodynamics is based on the first and second principles of thermodynamics that were introduced as postulates [3].

Let us show that the first principle of thermodynamics follows from the balance conservation laws for energy and linear momentum and is valid for the case when the heat influx is the only external action. The mathematical representation of the first principle of thermodynamics appears to be a nonidentical relation, and this points out that the balance conservation laws are noncommutative and the state of thermodynamic system is nonequilibrium.

The second principle of thermodynamics with the equality sign is obtained from the first principle under realization of the integrability condition when a role of the integrating factor plays the inverse temperature, and this corresponds to a locally equilibrium state of the system when the temperature appeared to be realized. The second principle of thermodynamics with the inequality sign takes into account an availability of some actions other than the heat influx.
As it is well known, the first principle of thermodynamics can be presented in the form
\[ dE + \delta w = \delta Q \]
where \( dE \) is the change of energy of the thermodynamic system, \( \delta w \) is the work done by the system (this means that \( \delta w \) is expressed in terms of the system parameters), \( \delta Q \) is an amount of the heat put into the system (i.e., the external action onto the system). Since the term \( \delta w \) can be expressed in terms of the system parameters and specifies a real (rather than virtual) change, it can be designated by \( dw \), and hence, the first principle of thermodynamics takes the form
\[ dE + dw = \delta Q \] (1.1)

What is the difference between the first principle of thermodynamics and the balance conservation laws?

For thermodynamic system the balance conservation law of energy can be written as
\[ dE = \delta Q + \delta G \] (1.2)
where by \( \delta G \) we designate energetic actions with the exception of heat influx. For thermodynamic system the balance conservation law for linear momentum (a change of linear momentum of the system in its dependence on the force mechanical action onto the system) can be written as
\[ dw = \delta W \] (1.3)
Here \( \delta W \) stands for the force (mechanical) action onto the system (for example, an external compression of the system, an influence of boundaries and so on).

If to combine relations (1.2) and (1.3), one can obtain the relation
\[ dE + dw = \delta Q + \delta G + \delta W \] (1.4)

By comparing relation (1.4) that follows from the balance conservation laws for energy and linear momentum and relation (1.1), one can see that they coincide if the heat influx is the only external action onto the thermodynamic system (\( \delta W = 0 \) and \( \delta G = 0 \)).

Thus, the first principle of thermodynamics follows from the balance conservation laws for energy and linear momentum. (This is analogous to the evolutionary relation for the thermodynamic system [1]).

The significance of the first principle of thermodynamics, (as well as that of the evolutionary equation), consists in that it clarifies a nature of interactions of two balance conservation laws (rather than it only corresponds to the conservation law for energy).

Since \( \delta Q \) is not a differential (closed form), relation (1.1) which corresponds to the first principle of thermodynamics, as well as the evolutionary relation, appears to be a nonidentical nonintegrable relation. This points to a noncommutativity of the balance conservation laws (for energy and linear momentum) and to a nonequilibrium state of the thermodynamic system. Although the form
\( dE + p\,dV \) consists of differentials, in the general case without the integrating factor it is not a differential (closed exterior differential forms) because of that its terms depend on different variables, namely, the first term is determined by variables that specifies the internal structure of elements, and the second term depends on variables that specify an interaction between elements, for example, the pressure. The commutator of the form \( dE + p\,dV \) is nonzero. This points to an availability of internal forces and the nonequilibrium state of the thermodynamical system.

As it follows from the analysis of the evolutionary relation \([1]\), the transition to a locally equilibrium state must correspond to realization of the additional condition (i.e. the integrability condition). If this condition be satisfied, from the nonidentical evolutionary relation, which corresponds to the first principle of thermodynamics, it follows an identical relation. It is an identical relation that corresponds to the second principle of thermodynamics.

Let us consider the case when the work performed by the system is carried out through the compression. Then \( dw = p\,dV \) (here \( p \) is the pressure and \( V \) is the volume) and \( dE + dw = dE + p\,dV \). As it is known, the form \( dE + p\,dV \) can become a differential if there is the integrating factor \( \theta \) (a quantity which depends only on the characteristics of the system), where \( 1/\theta = pV/R \) is called the temperature \( T \) \([3]\). In this case the form \( (dE + p\,dV)/T \) turns out to be a differential (interior) of some quantity that refereed to as entropy \( S \):

\[
(\frac{dE + p\,dV}{T}) = dS \tag{1.5}
\]

If the integrating factor \( \theta = 1/T \) has been realized, that is, relation (1.5) proves to be satisfied, from relation (1.1), which corresponds to the first principle of thermodynamics, it follows

\[
dS = \delta Q/T \tag{1.6}
\]

This is just the second principle of thermodynamics for reversible processes. It takes place when the heat influx is the only action onto the system. If in addition to the heat influx the system experiences a certain mechanical action \( \delta W \) (for example, an influence of boundaries), then according to relation (1.4) from relation (1.5) we obtain

\[
dS = (\frac{dE + p\,dV}{T}) = (\frac{\delta Q + \delta W + \delta G}{T}) \tag{1.7}
\]

from which it follows

\[
dS > \frac{\delta Q}{T} \tag{1.8}
\]

that corresponds to the second principle of thermodynamics for irreversible processes.

Relations (1.6), (1.8) that can be written as

\[
dS \geq \frac{\delta Q}{T}, \tag{1.9}
\]

express the second principle of thermodynamics. (It is well to bear in mind that the differentials in relations (1.5), (1.6), (1.8), (1.9) are not total differentials.
They are satisfied only with the presence of the integrating factor, namely, the temperature, which depends on the system parameters.

Thus, the first principle of thermodynamics follows from the balance conservation laws for energy and linear momentum, and the second principle of thermodynamics follows from the first one. The second principle of thermodynamics with the equality sign follows from the first principle under the fulfillment of the condition of integrability, i.e. a realization of the integrating factor (the inverse temperature). (This corresponds to the transition from a nonequilibrium state to a locally equilibrium state. Phase transitions, the origin of fluctuations, etc are examples of such transitions). In this case entropy proves to be the state function. And the second principle of thermodynamics with the inequality sign takes into account an availability in actual processes other actions besides the heat influx. For this case entropy is a functional.

In the case examined above a differential of entropy (rather than entropy itself) becomes a closed form. In this case entropy manifests itself as the thermodynamic potential, namely, the function of state. To the pseudostructure there corresponds the state equation that determines the temperature dependence on the thermodynamic variables.

For entropy to be a closed form itself (a conservative quantity), one more condition must be realized. Such a condition could be the realization of the integrating direction, an example of that is the speed of sound: \( a^2 = \frac{\partial p}{\partial \rho} = \frac{\gamma p}{\rho} \). In this case it is valid the equality \( ds = d\left(\frac{p}{\rho^\lambda}\right) = 0 \) from which it follows that entropy \( s = \frac{p}{\rho^\lambda} = \text{const} \) is a closed form (of zero degree). (However it does not mean that a state of the gaseous system is identically isoentropic. Entropy is constant only along the integrating direction (for example, on the adiabatic curve or on the front of the sound wave), whereas in the direction normal to the integrating direction the normal derivative of entropy has a break).

Under realization of the integrating direction the transition from the variables \( E, V \) to the variables \( p, \rho \) is a degenerate transform.

It worth underline that both temperature and the speed of sound are not continuous thermodynamic variables. They are variables that are realized in the thermodynamic processes if the thermodynamic system has any degrees of freedom. One can see the analogy between the inverse temperature and the speed of sound: the inverse temperature is the integrating factor and the speed of sound is the integrating direction. (Notice that in actual processes a total state of the thermodynamic system is nonequilibrium one and the commutator of the form \( dE + pdV \) is nonzero. A quantity that is described by a commutator and acts as an internal force can grow. Prigogine [4] defined this as the "production of the excess entropy". It is the increase of the internal force that is perceived as the growth of entropy in the irreversible processes).

A closed static system, if left to its own devices, can tend to a state of total thermodynamic equilibrium. This corresponds to tending the system functional to its asymptotic maximum. In the dynamical system the tending of the system to a state of total thermodynamic equilibrium can be violated by dynamical processes and transitions to a state of local equilibrium.
2 The development of the gas dynamic instability

To study a development of instability it is necessary to analyze the balance conservation laws.

For example, we take the simplest gas dynamical system, namely, a flow of ideal (inviscous, heat nonconductive) gas [5].

Assume that gas is a thermodynamic system in the state of local equilibrium (whenever the gas dynamic system itself may be in nonequilibrium state), that is, it is satisfied the relation [3]

\[ Tds = de + pdV \] (2.1)

where \( T, p \) and \( V \) are the temperature, the pressure and the gas volume, \( s, e \) are entropy and internal energy per unit volume.

Let us introduce two frames of reference: an inertial one that is not connected with the material system and an accompanying frame of reference that is connected with the manifold formed by the trajectories of the material system elements. (Both Euler’s and Lagrange’s systems of coordinates can be examples of such frames).

In the inertial frame of reference the Euler equations are the balance conservation laws for energy, linear momentum and mass of ideal gas [5].

The equation of the balance conservation law of energy for ideal gas can be written as

\[ \frac{Dh}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = 0 \] (2.2)

where \( \frac{D}{Dt} \) is the total derivative with respect to time (if to designate the spatial coordinates by \( x_i \) and the velocity components by \( u_i \), \( \frac{D}{Dt} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i} \)). Here \( \rho = 1/V \) and \( h \) are respectively the mass and the enthalpy densities of the gas.

Expressing enthalpy in terms of internal energy \( e \) with the help of formula \( h = e + p/\rho \) and using relation (2.1) the balance conservation law equation can be put to the form

\[ \frac{Ds}{Dt} = 0 \] (2.3)

And respectively, the equation of the balance conservation law for linear momentum can be presented as [5,6]

\[ \text{grad} s = (\text{grad} h_0 + \mathbf{U} \times \text{rot} \mathbf{U} - \mathbf{F} + \frac{\partial \mathbf{U}}{\partial t})/T \] (2.4)

where \( \mathbf{U} \) is the velocity of the gas particle, \( h_0 = (\mathbf{U} \cdot \mathbf{U})/2 + h \), \( \mathbf{F} \) is the mass force. The operator \( \text{grad} \) in this equation is defined only in the plane normal to the trajectory. [Here it was tolerated a certain incorrectness. Equations (2.3), (2.4) are written in different forms. This is connected with difficulties when deriving these equations themselves. However, this incorrectness will not effect on results of the qualitative analysis of the evolutionary relation obtained from these equations.]
Since the total derivative with respect to time is that along the trajectory, in the accompanying frame of reference equations (2.3) and (2.4) take the form:

\[
\frac{\partial s}{\partial \xi^1} = 0 \quad (2.5)
\]

\[
\frac{\partial s}{\partial \xi^\nu} = A_{\nu}, \quad \nu = 2, ...
\]

where \(\xi^1\) is the coordinate along the trajectory, \(\partial s/\partial \xi^\nu\) is the left-hand side of equation (2.4), and \(A_{\nu}\) is obtained from the right-hand side of relation (2.4).

\{In the common case when gas is nonideal equation (2.3) can be written in the form\}

\[
\frac{\partial s}{\partial \xi^1} = A_1 \quad (2.7)
\]

where \(A_1\) is an expression that depends on the energetic actions. In the case of ideal gas \(A_1 = 0\) and equation (2.7) transforms into (2.5). In the case of the viscous heat-conductive gas described by a set of the Navier-Stokes equations, in the inertial frame of reference the expression \(A_1\) can be written as [5]

\[
A_1 = \frac{1}{\rho} \frac{\partial}{\partial x_i} \left( -\frac{q_i}{T} \right) - \frac{q_i}{\rho T} \frac{\partial T}{\partial x_i} + \frac{\tau_{ki}}{\rho} \frac{\partial u_k}{\partial x_i} \quad (2.8)
\]

Here \(q_i\) is the heat flux, \(\tau_{ki}\) is the viscous stress tensor. In the case of reacting gas extra terms connected with the chemical nonequilibrium are added [5].

Equations (2.5) and (2.6) can be convoluted into the equation

\[
ds = \omega \quad (2.9)
\]

where \(\omega = A_{\mu} d\xi^\mu\) is the first degree differential form (here \(\mu = 1, \nu\)).

Relation (2.9) is an evolutionary relation for gas dynamic system (in the case of local thermodynamic equilibrium). Here \(\psi = s\). {It worth notice that in the evolutionary relation for thermodynamic system the dependence of entropy on thermodynamic variables is investigated (see relation (2.1)), whereas in the evolutionary relation for gas dynamic system the entropy dependence on the space-time variables is considered}.

If relation (2.9) appears to be identical one (if the form \(\omega\) be the closed form, and hence it is a differential), one can obtain a differential of entropy \(s\) and find entropy as a function of space-time coordinates. It is entropy that will be the gas dynamic function of state. {It should underline once again that entropy as a thermodynamic function of state is not gas dynamic function of state}. The availability of the gas dynamic function of state would point to the equilibrium state of the gas dynamic system. If relation (2.9) be not identical (this takes place for actual processes), then from this relation the differential of entropy \(s\) cannot be defined. This will point to an absence of the gas dynamic function of state and nonequilibrium state of the system. Such nonequilibrium is a cause of the development of instability.
Since the nonequilibrium is produced by internal forces that are described by the commutator of the form $\omega$, it becomes evident that a cause of the gas dynamic instability is something that contributes into the commutator of the form $\omega$. Without accounting for terms that are connected with a deformation of the manifold formed by the trajectories the commutator can be written as

$$K_{1\nu} = \frac{\partial A_\nu}{\partial \xi^1} - \frac{\partial A_1}{\partial \xi^\nu} \quad (2.10)$$

From the analysis of the expression $A_\nu$ and with taking into account that $A_1 = 0$ one can see that terms that are related to the multiple connectedness of the flow domain (the second term of equation (2.4)), the nonpotentiality of the external forces (the third term in (2.4)) and the nonstationarity of the flow (the forth term in (2.4)) contribute into the commutator. (In the general case the terms connected with transport phenomena and physical and chemical processes will contribute into the commutator (see equation (2.8))

One can see that the development of instability is caused by not a simply connectedness of the flow domain, nonpotential external (for each local domain of the gas dynamic system) forces, a nonstationarity of the flow, transport phenomena. (In common case on the gas dynamic instability it will effect the thermodynamic, chemical, oscillatory, rotational, translational nonequilibrium).

All these factors lead to emergence of internal forces, that is, to nonequilibrium and to development of various types of instability. (It can be noted that for the case of ideal gas Lagrange derived a condition of the eddy-free stable flow. This condition is as follows: the domain must be simple connected one, forces must be potential, the flow must be stationary. One can see, that under fulfillment of these conditions there are no terms that contribute into the commutator).

And yet for every type of instability one can find an appropriate term giving contribution into the evolutionary form commutator, which is responsible for this type of instability. Thus, there is an unambiguous connection between the type of instability and the terms that contribute into the evolutionary form commutator in the evolutionary relation. (In the general case one has to consider the evolutionary relations that correspond to the balance conservation laws for angular momentum and mass as well).

Whether the gas dynamic system can get rid of the internal force and transfer into the locally equilibrium state? (The mechanism of evolutionary processes in material media and the transition from the nonequilibrium state to the locally equilibrium state is presented in detail in works [1,2]).

The locally equilibrium state corresponds the state differential that is a closed form. The transition from evolutionary differential form $\omega$ to closed form, that would correspond to the transition from the nonequilibrium state of the system to the locally equilibrium state, is possible only as the degenerate transform, i.e. the transform that does not conserve the differential. The evolutionary differential form $\omega$, involved into evolutionary relation, is an unclosed one for real processes. The commutator, and hence the differential, of this form
is nonzero. The locally equilibrium state corresponds the state differential that is a closed form. The differential of the closed form is zero.

To the degenerate transform it must correspond a vanishing of some functional expressions. Such functional expressions may be Jacobians, determinants, the Poisson brackets, residues and others. It is obvious that the condition of degenerate transform has to be due to the gas dynamic system properties. This may be, for example, the availability of any degrees of freedom in the gas dynamic system.

If the transformation is degenerate, from the unclosed evolutionary form it can be obtained a differential form closed on some structure (pseudostructure). The differential of this form equals zero. That is, it is realized the transition

\[ d\omega \neq 0 \rightarrow (\text{degenerate transform}) \rightarrow d\pi \omega = 0, \ d\pi^* \omega = 0 \]

Here \( \pi \omega \) is the dual form (the equation of the pseudostructure \( \pi \) [1]).

(The degenerate transformation is realized as a transition from the accompanying noninertial coordinate system to the locally inertial that).

On the pseudostructure \( \pi \) evolutionary relation (2.9) transforms into the relation

\[ d\pi s = \omega_\pi \]

which proves to be the identical relation. Indeed, since the form \( \omega_\pi \) is a closed one, on the pseudostructure it turns out to be a differential of some differential form.

The identical relation (2.10) obtained from the nonidentical evolutionary relation under degenerate transform integrates the state differential and the closed (inexact) exterior differential form [1]. The availability of the state differential indicates that the material system state becomes a locally equilibrium state (that is, the local domain of the system under consideration changes into the equilibrium state). The availability of the exterior closed on the pseudostructure differential form means that the physical structure is present. This shows that the transition of material system into the locally equilibrium state is accompanied by the origination of physical structures.

As it has been shown in works [1,2], in the material system origination of a physical structure reveals as a new measurable and observable formation that spontaneously arises in the material system. In the physical process this formation is spontaneously extracted from the local domain of the material system and so it allows the local domain of material system to get rid of an internal force and come into a locally equilibrium state.

The gas dynamic formations that correspond to these physical structures are shocks, shock waves, turbulent pulsations and so on.

It is evident that the characteristics of the formation (intensity, vorticity, absolute and relative speeds of propagation of the formation) are determined by the evolutionary form and its commutator and by the material system characteristics.

In works [1,2] it has been shown a connection of characteristics of the formation originated with characteristics of the evolutionary forms, the evolutionary
form commutators obtained from closed forms, and the material system characteristics. In the present work we shall not fix our attention on these problems.

Additional degrees of freedom are realized as the condition of the degenerate transform, namely, vanishing of determinants, Jacobians of transforms, etc. These conditions specify the integral surfaces (pseudostructures): the characteristics (the determinant of coefficients at the normal derivatives vanishes), the singular points (Jacobian is equal to zero), the envelopes of characteristics of the Euler equations and so on. Under passing throughout the integral surfaces the gas dynamic functions or their derivatives suffer breaks. Below we present the expressions for calculation of such breaks of derivatives in the direction normal to characteristics (and to trajectories).

The extraction of some formation from the local domain of the system is accompanied by emergence of the break surfaces (contact breaks). These breaks are connected with trajectories of the material system elements.

Let us analyze which types of instability and what gas dynamic formation can originate under given external action.

1). Shock, break of diaphragm and others. The instability originates because of nonstationarity. The last term in equation (2.4) gives a contribution into the commutator. In the case of ideal gas whose flow is described by equations of the hyperbolic type the transition to the locally equilibrium state is possible on the characteristics and their envelopes. The corresponding structures are weak shocks and shock waves.

2). Flow of ideal (inviscous, heat nonconductive) gas around bodies Action of nonpotential forces. The instability develops because of the multiple connectedness of the flow domain and a nonpotentiality of the body forces. The contribution into the commutator comes from the second and third terms of the right-hand side of equation (2.4). Since the gas is ideal one and \( \partial s/\partial \xi^1 = A_1 = 0 \), that is, there is no contribution into the each fluid particle, an instability of convective type develops. For \( U > a \) (\( U \) is the velocity of the gas particle, \( a \) is the speed of sound) a set of equations of the balance conservation laws belongs to the hyperbolic type and hence the transition to the locally equilibrium state is possible on the characteristics and their envelopes. The corresponding structures are weak shocks and shock waves.

3. Boundary layer. The instability originates due to the multiple connectedness of the domain and the transport phenomena (an effect of viscosity and thermal conductivity). Contributions into the commutator produce the second term in the right-hand side of equation (2.4) and the second and third terms in expression (2.8). The transition to the locally equilibrium state is allowed at singular points. because in this case \( \partial s/\partial \xi^3 = A_3 \neq 0 \), that is, the external exposure acts onto the gas particle separately, the development of instability and the transitions to the locally equilibrium state are allowed only in an individual fluid particle. Hence, the structures emerged behave as pulsations. These are the turbulent pulsations.
It is commonly believed that the instability is an emergence of any structures in the gas dynamic flow. From this viewpoint the laminar boundary layer is regarded as stable one, whereas the turbulent layer regarded as unstable layer. However the laminar boundary layer cannot be regarded as a stable one because of the fact that due to the not simple connectedness of the flow domain and the transport processes the instability already develops although any structures do not originate. In the turbulent boundary layer the emergence of pulsations is the transition to the locally equilibrium state, and the pulsations themselves are local formations. The other matter, due to the global nonequilibrium the locally equilibrium state is broken up and the pulsations weaken.

Studying the instability on the basis of the analysis of entropy behavior was carried out in the works by Prigogine and co-authors [7]. In that works entropy was considered as the thermodynamic function of state (though its behavior along the trajectory was analyzed). By means of such state function one can trace the development (in gas fluxes) of the hydrodynamic instability only. To investigate the gas dynamic instability it is necessary to consider entropy as the gas dynamic state function, i.e. as a function of the space-time coordinates. Whereas for studying the thermodynamic instability one has to analyze the commutator constructed by the mixed derivatives of entropy with respect to the thermodynamic variables, for studying the gas dynamic instability it is necessary to analyze the commutators constructed by the mixed derivatives of entropy with respect to the space-time coordinates.

In conclusion it should be said a little about modelling instable flows. As it is known, some authors tried to account for the development of instability by means of improving the equations modelling the balance conservation laws (for example, by introducing the high-order moments) or by introducing additional equations. However, such attempts give no satisfactory results. To describe the nonequilibrium flow and the emergence of the gas dynamic structures (waves, vortices, turbulent pulsations) one must add the evolutionary relation obtained from the balance conservation law equations to the balance conservation law equations. Under numerical modelling the gas flows one has to trace for the transition from the evolutionary nonidentical relation to the identical relation (for the transition from an evolutionary unclosed form to an exterior closed form), and this will point to the emergence of a certain physical structure.

Below we present an example of calculating the breaks of derivatives of the gas dynamic functions that are necessary for numerical analysis of gas dynamic flows.

**Breaks of normal derivatives on characteristics and trajectories**

While studying the effects connected with the origination of the vorticity one can notice a certain specifics of numerical solving the Euler equations [5]. This
may be demonstrated by the following example. Assume, that the initial conditions correspond to the isentropic flow, that is, entropy is the same along all trajectories. For ideal gas under consideration entropy conserves along the trajectory. From this it follows that entropy has to conserve during all time of flow. However, in actual cases (unsteady flow, flow along a body, heterogeneous medium) the derivative of entropy along the direction normal to trajectory suffers the break. Thus we have that, from one side, entropy (function) must be constant and, from other hand, its derivatives suffer the breaks. This contradiction is resolved with taking into account the fact that the break of derivative is compensated by changing the stream function or bending the trajectory. It is this effect that must be accounted for in the process of numerical calculation. In particular, when calculation the one-dimensional nonstationary nonisoentropic flow of gas the conditions on the characteristics includes the derivative of entropy with respect to the coordinate normal to trajectory (in space of two variables, namely, time and coordinate). To calculate this derivative one must know the break of derivative of entropy. This can be obtained from the relations that connects the breaks of derivatives of the gas dynamic functions.

These relations are found from the dynamic conditions of a consistency of the Euler equations. In paper [8] the dynamical conditions of consistency of the Euler equations for the case $p = f(\rho)$ were considered.

In the present work in a similar manner it is analyzed the case when $p = f(\rho, s)$, where $s$ is the entropy, and the relations that connect the breaks of derivatives of the functions describing the particle velocity, the sound speed, and entropy are obtained. These relations enable one to carry out numerical calculations of the nonisoentropic gas flows.

The scheme of obtaining these relations for one-dimensional nonstationary equations is the following. At the beginning, the Euler equations are written down. Then the equations for characteristics and the conditions on characteristics are derived. Kinematical conditions of consistency [8] that mutually connect the breaks of derivatives of the gas dynamic functions are written down. These conditions are substituted into the Euler equations. As a result, a homogeneous set of equations for the breaks of derivatives of the functions desired is obtained. On the characteristic surface the determinant of this set equals zero, and from this it is found the nontrivial solution for the breaks of derivatives of the functions desired in their dependence on a value of one of others.

If to take $U$ (the gas velocity), $a$(the sound speed), and $s$, the following relations are obtained:

1) In the direction normal to the trajectory the derivatives of the sound speed and entropy suffer breaks (the derivative of velocity does not suffer a break). These breaks are connected between them by the relation:

$$\left[ \frac{\partial a}{\partial \eta_1} \right] = \left[ \frac{\partial s}{\partial \eta_1} \right] \frac{a}{2\gamma s}$$

where $\eta_1$ is the direction normal to the trajectory, $\gamma$ is the Poisson constant.

2) In the direction normal to the characteristics the derivatives of the gas velocity and the speed of sound suffer breaks (the derivative of entropy does not
These breaks are connected between them by the relation:

$$\left[ \frac{\partial u}{\partial \eta^+} \right] = \pm \left[ \frac{\partial a}{\partial \eta^-} \right] \frac{2}{\gamma - 1}$$

where $\eta^+,$ $\eta^-$ are the directions normal to the corresponding characteristics.

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