Photoproduction of $\ell = 1$ Baryons: Quark Model versus Large $N_c$

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Abstract

We consider the electromagnetic decays of the orbitally-excited SU(6) 70-plet baryons, to compare the predictions of the naive quark model with those of large-$N_c$ QCD. The helicity amplitudes measured in $N^*$ photoproduction are computed in a large-$N_c$ effective field theory, based on a Hartree approximation, and the amplitudes are fit to the current experimental data. Our results indicate that the success of the naive quark model predictions cannot be explained by large-$N_c$ arguments alone. This is consistent with the conclusions of an earlier study of the $N^* \rightarrow N\pi$ decays, that utilized the same approach.
1 Introduction

That the constituent quark model can work remains something of a mystery. A simple
realization of the model treats baryon states as spin-flavor SU(6) wave functions and
treats interactions that lead to the decay or production of the states as single-quark
operators. A group theoretical strategy can be adopted in connection with this model, so
that instead of calculating specific dynamics, one just finds the structure of the hopefully
few allowed one-body operators that can contribute, and then fits the coefficients of
these operators to a subset or all of the data and attempts to make predictions. One
might think that there would be large corrections to this type of model, for example
via multiquark strong interactions that could not simply be absorbed into the binding
potential.

Recently, it has been noted that large-$N_c$ quantum chromodynamics (QCD) can
be used to show that corrections to the simplest operators are in a number of cases
suppressed by powers of $1/N_c$. These cases include analyses of operators that contribute
to the axial current matrix elements and the masses of the SU(6) ground state baryons,
and to operators that would modify the SU(6) wave functions even for low spin states
$[1, 2, 3, 4]$. However, a different large-$N_c$ result has been noted in the decays of the negative
parity P-wave baryons, the $70$-plet of SU(6), into ground state baryons plus pions. Here
one may write down all the independent one-quark and multiquark operators that are
allowed $[5]$. One finds that, while some of the two-body operators are suppressed by a
power of $1/N_c$ for low spin states, a well-defined subset of the two-body operators are
unsuppressed relative to the one-body operators in the large $N_c$ limit. The reason for the
lack of suppression is clear. The two-body operators themselves contain a suppression by
a factor of $1/N_c$, but coherent effects in the matrix elements can give a compensatory
factor of $N_c$. There is still something of a mystery, however. An explicit numerical
analysis of the decays of the $70$-plet into a pion plus a member of the $56$-plet, the
ground state, showed that the coefficients of the two-body operators are small.

In this paper, we shall investigate the radiative decays of the $70$-plet, or equivalently
their radiative production. In particular we will be interested in the guidance that we
can get from large-$N_c$ QCD regarding which operators give the leading contributions to
the decay matrix elements.

The study of the radiative $70$-plet decays using SU(6) wave functions and just one-
body operators is long since textbook material $[7]$. There are 3 one-body operators, and
hence 3 coefficients that may be used to describe the 19 independent decay amplitudes of the 70-plet into nucleons. The simple description, as we shall review below, works reasonably well for most decays.

One might expect sizable corrections from two-body operators to the simple picture. Again, two-body operators are operators where a second quark is interacting with the quark that absorbs the photon, and doing so in ways that cannot be just incorporated into the binding interaction. The two-body operators involve strong interaction corrections to a simple photon interaction, and are not in a regime where one would expect the corrections to be perturbative. They could be big.

In terms of the large-$N_c$ limit, since only one quark is excited in the 70-plet, it is elementary to show that matrix elements of the one-body operators are of order 1 in the $N_c$ expansion. Now consider the matrix elements of the two-body operators. The two-quark operators themselves are of order $1/N_c$, including factors of the QCD coupling in with the operator. However, because of the many possible choices for the second quark when a matrix element is taken, there is a possible $O(N_c)$ coherence factor. This leads to the result that certain two-quark operators are also overall $O(1)$ in the large $N_c$ expansion. So in any analysis of the 70-plet decays it would seem an oversight to exclude the two-body operators. We should test the significance of the two-quark operators by including them in fits to data, and see if their presence can be shown. (We shall find that the reverse is true.)

Before describing our work in more detail, let us make two comments. One is that some of the two-body operator matrix elements are indeed suppressed by $1/N_c$. This can happen if they are proportional to quark spin, in which case they add up to the total quark spin of the state, which for low spin states does not compensate for the $O(1/N_c)$ size of the operator before the matrix element is taken. The other is that spin-spin interaction effects can, for high spin states, build up and significantly modify the SU($2f$) form of the wave function. The effect is small for low spin states. Hence if the $\Delta$ can be treated as a low spin state, which a spin-$3/2$ state surely is in the context of large $N_c$, the SU($2f$) wave functions should be good for it. Extrapolating to $N_c = 3$, we shall treat the $\Delta$ as a good SU(6) state, but shall also keep in mind that this may be less well justified for the $\Delta$ than for the nucleon.

In Section 2 we explain the applications of the high $N_c$ limit to radiative decays of the excited baryons and give other information about our calculation, Section 3 gives our numerical results, and Section 4 gives our conclusions.


2 Framework

Formalism. Our basic approach is the same as that of Refs. [1, 5]. We assume that we can represent the large-$N_c$ baryon states in the tensor product space of the spin and flavor indices of the $N_c$ valence quarks. Our states therefore have the same spin, flavor, and orbital angular momentum structure as representations of nonrelativistic SU(6)×O(3). If we were to consider the limit where the quarks are heavy compared to $\Lambda_{QCD}$, this assumption clearly would be correct, since the nonrelativistic quark model description of the baryon states can be then derived from QCD. The spin-flavor states with the same spatial wave functions are degenerate in the limit of large quark masses, and form complete SU(6)×O(3) multiplets. Although it appears that this description would break down completely for baryons containing quarks that are much lighter than $\Lambda_{QCD}$, one must keep in mind that the splittings between states of low spin within a given multiplet are not only suppressed by $1/m_q$, where $m_q$ is the quark mass, but also by $1/N_c$. Thus, in the large-$N_c$ limit, we expect that the states with lowest spin within each spin-flavor multiplet should be well described in the same tensor product space that is appropriate for large $m_q$. We only need assume that there is no discontinuity in the description of the baryon states as we gradually lower the quark mass.

The approximate symmetry described above is an unusual one, in that overall it is badly broken within every multiplet. However, since the dimension of each multiplet grows with $N_c$, while the symmetry breaking effects scale as $1/N_c$, it is at least possible to make reliable predictions at the low-spin end of each multiplet. As we describe below, we will ultimately work with $N_c = 3$ baryon wave functions, where the distinction between the “top” and “bottom” of a multiplet is not completely clear. To avoid any ambiguity, we choose to work with complete spin-flavor representations, keeping in mind that our results may be less reliable for the states of highest spin.

Assuming this basis, we can write the Hartree-Fock interaction Hamiltonian as

$$H_{int} = \sum_{n=1}^{N_c} \sum_{\{\alpha_1, \ldots, \alpha_n\} \subset \{1, \ldots, N_c\}} \int d^3r_{\alpha_1} \times \cdots \times d^3r_{\alpha_n} \Phi(r_{\alpha_1})_{\alpha_1}^\dagger \otimes \cdots \otimes \Phi(r_{\alpha_n})_{\alpha_n}^\dagger \mathcal{O}(r_{\alpha_1}, \ldots, r_{\alpha_n}) \Psi(r_{\alpha_1})_{\alpha_1} \otimes \cdots \otimes \Psi(r_{\alpha_{n-1}})_{\alpha_{n-1}} \otimes \Psi^*(r_{\alpha_n})_{\alpha_n} \tag{2.1}$$

where $\mathcal{O}$ is any quark operator of interest. The $\Phi$ and $\Psi$ factors are the individual quark wave functions in the 56 and 70-plet baryons, respectively. The asterisk on the $n^{th}$ $\Psi$ wave function indicates that it is the one corresponding to the orbitally-excited quark. Notice that Eq. (2.1) sums over all possible multiquark interactions involving $n$ quarks,
and then over all possible values of $n$. By Witten’s power counting arguments, we expect a general $n$-body interaction to scale as $1/N_c^{n-1}$ [3]. Since a transition between the 70 and 56-plet baryons always involves the deexcitation of the orbitally-excited quark, the lowest-order operators will be one-body, acting only on the excited quark line. Higher-body operators will involve the orbitally-excited quark as well as one or more of the other valence quarks, and be suppressed by powers of $1/N_c$.

The most useful way of thinking about the interaction Hamiltonian in Eq. (2.1) is in terms of its spin and flavor transformation properties. Assuming $f$ quark flavors, each large-$N_c$ baryon state can be viewed as a $(2f)^{N_c}$ dimensional vector, while $H_{int}$ can be thought of a $(2f)^{N_c} \times (2f)^{N_c}$ matrix acting in this space. While we cannot explicitly evaluate this matrix (by computing the Hartree wave functions and performing the integration in Eq. (2.1)) we can determine the form of $H$, up to unknown coefficients, from symmetry considerations. The argument goes as follows:

Since the wave functions $\Phi$ and $\Psi$ are computed from the portion of the Hartree potential that is zeroth order in spin and flavor symmetry breaking, these wave functions are spin-flavor independent and spherically symmetric. Thus, they are proportional to identity matrices in the spin-flavor space, and can be replaced by ordinary functions

$$\Phi(r) \rightarrow \phi(r), \quad \Psi(r) \rightarrow \psi(r) = \phi(r),$$

where $r = |r|$ is the radial coordinate. The last equality follows from the observation that the Hartree potential generated by order $N_c$ quarks should remain unaffected by the excitation of a single quark. The wave function of the excited quark, on the other hand, has the form

$$\Psi_*(r) = f(r)Y_{\ell=1,m}(\theta, \varphi) = \sqrt{\frac{3}{4\pi}} f(r) (\hat{r} \cdot \varepsilon_m)$$

where $f(r)$ is a spherically-symmetric function, and $Y$ is a spherical harmonic; the $\ell = 1$ polarization vectors $\varepsilon_m$ are given by

$$\varepsilon_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ -i \\ 0 \end{pmatrix}, \quad \varepsilon_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \varepsilon_{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}.$$
zeroth-order part of Hartree Hamiltonian. Notice that this corresponds to a different $SU(2f)$ symmetry for every quark line. We then write down the most general set of operators transforming in the same way, out of the spin and flavor generators, $\sigma^a$ and $\lambda^a$, the polarization vector $\varepsilon_m$, and the momentum $k$ of the light decay product. We discuss the construction of these operators in greater detail below. We may evaluate any matrix elements of interest by letting the operators act on the tensor representations of the baryon states; the undetermined coefficients can then be fit to the corresponding experimental data.

What makes this approach nontrivial is that certain types of operators that are subleading in $1/N_c$ may nonetheless have matrix elements that are as important as those of the leading operators. For example, the matrix element of a two-body operator that contributes to a 70-plet baryon decay necessarily involves a spin-flavor matrix acting on the excited quark line, as well as a spin-flavor matrix acting on a nonexcited quark line, summed over the $N_c - 1$ ground state quarks. If this sum is coherent, it will scale as $N_c$, and the matrix element of this two-body operator will be of the same order as those of the leading one-body operators. Thus, by determining what types of sums are potentially coherent on the low-spin baryon states, we can restrict the set of operators that appear in the low-energy effective theory. Since the one-body operators are often identified with the naive quark model predictions, our large $N_c$ arguments imply that there are additional operators, with distinct spin-flavor transformation properties, that may be of equal importance in describing the experimental data. A fit to the data will then determine whether this is actually the case. The different spin-flavor matrices and their properties were summarized in Ref. [5], and we provide them again for convenience:

- Constant terms – these always add coherently when summed over quark lines, but have no nontrivial spin-flavor structure. Thus, these terms are irrelevant.
- Spin terms - These involve the sum
  \[ \sum_{\text{quarks}} \sigma^j_{\alpha} \]  
  where $\sigma^j$ is a pauli matrix. These are of order one on the low spin states [1], and are thus associated with a factor of $1/N_c$ relative to the leading terms.
- Flavor terms – These involve the sum
  \[ \sum_{\text{quarks}} \lambda^a_{\alpha} \]  
  where $\lambda^a$ is a Gell-Mann matrix, for flavor $SU(3)$. These terms can add coherently, depending on the flavor quantum numbers of the large $N_c$ baryon state.
Below, we will use this approach to write down the operators relevant in computing
the $N \rightarrow N^*$ helicity amplitudes. After we have determined the set of operators whose
matrix elements are potentially of leading order, we will evaluate the matrix elements
using the baryon wave functions corresponding to $N_c = 3$. This allows us to avoid
any ambiguity in determining which states in a large-$N_c$ baryon multiplet should be
identified with the physical states observed in nature. Note that this feature makes our
approach different from Refs. [3, 4], where a reasonable embedding of the physical
baryon states within the large-$N_c$ multiplet is assumed.

Operators. Since we are interested in the $N^*N\gamma$ coupling, we have at the quark level
the QED vertex,

$$\bar{\tau}Q\gamma^{\mu}qA_\mu \quad (2.7)$$

where the charge $Q$ is a matrix in SU(3) flavor space, $Q = \text{diag}(2/3, -1/3, -1/3)$. Thus,
at lowest order in flavor symmetry breaking, our operators will involve one factor of the
matrix $Q$, acting on the flavor indices of the baryon states. Other quark-level operators
arise when the operator above acts on one quark line, with gluon exchange involving
other lines. From the start, we will work in Coulomb gauge, where $A^0 = 0$ and $\nabla \cdot A = 0$. Although gauge invariance will not be manifest, this choice is physically equivalent to
any other, and greatly simplifies the operator analysis.

The set of interactions that we consider in this paper include the one-body operators

$$a_1 Q_\alpha \tilde{\epsilon}_m \cdot \vec{A} \quad ,$$

$$ib_1 Q_\alpha \tilde{\epsilon}_m \cdot \vec{\nabla}(\vec{\sigma}_s \cdot \vec{\nabla} \times \vec{A}) \quad ,$$

$$ib_2 Q_\alpha \vec{\sigma}_s \cdot \vec{\nabla}(\tilde{\epsilon}_m \cdot \vec{\nabla} \times \vec{A}) \quad ,$$

and the two-body operators,

$$ic_1 \left( \sum_{\alpha \neq \ast} Q_\alpha \right) (\vec{\sigma}_s \times \tilde{\epsilon}_m \cdot \vec{A}) \quad (2.11)$$

$$c_2 \left( \sum_{\alpha \neq \ast} Q_\alpha \vec{\sigma}_s \right) \cdot \tilde{\epsilon}_m (\vec{\sigma}_s \cdot \vec{A}) \quad (2.12)$$

$$c_3 \left( \sum_{\alpha \neq \ast} Q_\alpha \vec{\sigma}_s \right) \cdot \vec{\sigma}_s (\tilde{\epsilon}_m \cdot \vec{A}) \quad (2.13)$$

$$c_4 (\vec{\sigma}_s \cdot \tilde{\epsilon}_m) \left( \sum_{\alpha \neq \ast} Q_\alpha \vec{\sigma}_x \right) \cdot \vec{A} \quad (2.14)$$

$$id_1 \left( \sum_{\alpha \neq \ast} Q_\alpha \right) \tilde{\epsilon}_m \cdot \vec{\nabla}(\vec{\sigma}_s \cdot \vec{\nabla} \times \vec{A}) \quad (2.15)$$
\[ d_2 \left( \sum_{\alpha \neq \ast} Q_{\alpha} \bar{\sigma}_\alpha \right) \cdot \bar{\nabla} (\bar{\varepsilon}_m \cdot \bar{\nabla}) (\bar{\sigma}_\ast \cdot \bar{A}) \]  

(2.16)

\[ d_3 \left( \sum_{\alpha \neq \ast} Q_{\alpha} \bar{\sigma}_\alpha \right) \cdot \bar{\nabla} (\bar{\sigma}_\ast \cdot \bar{\nabla}) (\bar{\varepsilon}_m \cdot \bar{A}) \]  

(2.17)

\[ d_4 \left[ \left( \sum_{\alpha \neq \ast} Q_{\alpha} \bar{\sigma}_\alpha \right) \times \bar{\sigma}_\ast \cdot \bar{\nabla} \right] \left( \bar{\varepsilon}_m \cdot \bar{\nabla} \times \bar{A} \right). \]  

(2.18)

where the asterisk denotes the excited quark line. We assume that the derivatives in these operators are suppressed by the scale \( \Lambda_{QCD} \), which we have left implicit, for

\[ \begin{array}{c}
A_{3/2}^p(\Delta_{3/2}(1700))
A_{1/2}^p(\Delta_{3/2}(1700))
A_{1/2}^p(\Delta_{1/2}(1620))
A_{3/2}^p(N_{3/2}(1675))
A_{3/2}^n(N_{3/2}(1675))
A_{1/2}^p(N_{3/2}(1675))
A_{1/2}^n(N_{3/2}(1675))
A_{3/2}^p(N_{3/2}(1700))
A_{3/2}^n(N_{3/2}(1700))
A_{1/2}^p(N_{3/2}(1700))
A_{1/2}^n(N_{3/2}(1700))
A_{1/2}^p(N_{1/2}(1650))
A_{1/2}^n(N_{1/2}(1650))
A_{3/2}^p(N_{3/2}(1520))
A_{3/2}^n(N_{3/2}(1520))
A_{1/2}^p(N_{3/2}(1520))
A_{1/2}^n(N_{3/2}(1520))
A_{1/2}^p(N_{1/2}(1535))
A_{1/2}^n(N_{1/2}(1535))
\end{array} \]

Figure 1: Results of a fit to the one-body operators and mixing angles. The fit corresponds to the parameter set \( a_1 = 0.615, b_1 = -0.294, b_2 = -0.297, \theta_{N1} = 0.597 \) and \( \theta_{N3} = 3.060 \). The resulting numerical values of the helicity amplitudes are also given in Table 2.
notational convenience. The fact that we find only three linearly independent one-body operators is consistent with a long-known quark model result. For example, in Ref. \[7\] the helicity amplitudes that we consider are computed by assuming the photon couples to the current $J$, where $J_+ = AL_+ + BS_+ + CS_+L_+$. Here $S_+$ and $L_+$ represent the quark spin and orbital angular momentum raising and lowering operators, while $A$, $B$ and $C$ are SU(6)-invariant coefficients. These are related to the coefficients of our one-body operators above by

\[ a_1 = \sqrt{3/2}A, \quad b_1 = -\sqrt{3/2}B, \quad \text{and} \quad b_2 = -\sqrt{3/2}C^*. \]

As we describe below, the other operators we present completely span the space of possible two-body interactions. A three-body operator would involve two sums over the $\alpha \neq *$ quark lines; at lowest order in $Q$, at least one of these sums would be of the form $\sum \sigma_\alpha$, which is subleading in $1/N_c$, by the rules described earlier in this section. Note that we can generate additional operators if we include pions in our theory, since the pion-quark-quark coupling involves the spin-flavor structure $\sigma^j \lambda^a$, and can yield a coherent effect when summed over $N_c - 1$ quark lines. However, in our framework such couplings match onto pion-baryon interactions in the low-energy effective theory, and thus any new spin-flavor operators arise as only as loop effects, which should be small. This interpretation is consistent with our numerical results, which indicate that our operator list above is sufficient to give an excellent description of the data.

The helicity amplitudes $A_{1/2}$ and $A_{3/2}$ which we would like to study are defined via the photoproduction differential cross section of the $N^*$ baryons. There are 19 independent helicity amplitudes (not connected by isospin) for 70-plet production off $p$ or $n$ targets. If on the other hand we would like to think about the problem in the $N^*$ rest frame, the $N^* \rightarrow N\gamma$ partial decay widths are proportional to the sum of these squared amplitudes. For a complete discussion we refer the reader to Ref. \[8\], and references therein. The definitions that we require for our analysis are

\[ A_{1/2} = K \xi \langle N^*, s_z = \frac{1}{2}; \gamma, \varepsilon_{+1}|H_{int}|N, s_z = -\frac{1}{2} \rangle \]

\[ A_{3/2} = K \xi \langle N^*, s_z = \frac{3}{2}; \gamma, \varepsilon_{+1}|H_{int}|N, s_z = \frac{1}{2} \rangle , \]

where $s_z$ is the $z$ component spin in the $N^*$ rest frame, and the $\varepsilon_{+m}$ shown indicates the photon polarization. $K$ is a kinematical factor, given by $[4\pi\alpha m_{N^*}^2/(m_{N^*} - m_N^2)]^{1/2}$, and the states have the standard nonrelativistic normalization. The factor $\xi$ is the sign of the $N^* \rightarrow N\pi$ amplitude by which the $N^*$ is detected in photoproduction experiments. This

*See the comment after Eq. (3.2), regarding the factors of momentum in our operators.
takes into account the proper sign convention that experimenters use when they extract the helicity amplitudes from their data, and also renders our results independent of the sign conventions of the states. We evaluate the matrix elements in Eqs. (2.19) and (2.20) by allowing the our one- and two-body operators to act on tensor representations of the baryon states. It is important to point out that we have restricted our set of two-body interactions to include only those operators whose matrix elements in (2.19) and (2.20) are linearly independent over the set of nonstrange 70-plet states. In this subspace, we find that the combination of our large-$N_c$ power counting rules, and the constraint of conservation of angular momentum restricts the number of two-body interactions to 11; we then found 3 additional, nontrivial operator relations that reduced this set to the 8 operators shown above\footnote{Our original set of 11 two-body operators included $c_5 \sum_{\alpha \neq \ast} Q_\alpha \varepsilon_m \cdot \vec{A}$, $c_6 \sum_{\alpha \neq \ast} Q_\alpha \vec{\sigma}_\alpha \times \varepsilon_m \cdot \vec{A}$, and $i d_5 \varepsilon_m \cdot \nabla (\sum_{\alpha \neq \ast} Q_\alpha \vec{\sigma}_\alpha \cdot \nabla) \times \vec{A}$. However we found that the matrix elements of these operators were related to those given in the text by $c_5 = -a_1$, $c_6 = -b_1 + b_2$ and $d_5 = -b_1$.}

The actual computation was done using code written in MAPLE\footnote{The matrix elements of the one-body operators are easy to do by hand; the code is helpful in evaluating the two-body operators.}, and fits to the data were performed using standard FORTRAN minimizations routines.

3 Results

In this section we present our fits to the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ of the nonstrange $\ell = 1$ baryons. We take our experimental data points and error bars from the 1996 Review of Particle Properties [9]. Our possible set of free parameters includes the coefficients of the one- and two-body operators, defined in the previous section, as well as two mixing angles: $\theta_{N1}$ and $\theta_{N3}$. These mixing angles are necessary to specify the spin-1/2 and spin-3/2 nucleon mass eigenstates,

$$
\begin{bmatrix}
N(1535) \\
N(1650)
\end{bmatrix} = \begin{bmatrix}
\cos \theta_{N1} & \sin \theta_{N1} \\
-\sin \theta_{N1} & \cos \theta_{N1}
\end{bmatrix}
\begin{bmatrix}
N_{11} \\
N_{31}
\end{bmatrix}
$$

and

$$
\begin{bmatrix}
N(1520) \\
N(1700)
\end{bmatrix} = \begin{bmatrix}
\cos \theta_{N3} & \sin \theta_{N3} \\
-\sin \theta_{N3} & \cos \theta_{N3}
\end{bmatrix}
\begin{bmatrix}
N_{13} \\
N_{33}
\end{bmatrix}
$$

where $N_{ij}$ represents a state with total quark spin $i/2$ and total angular momentum $j/2$. Note that we absorb any factors of momentum/$\Lambda_{QCD}$ that appear in the operators.
Figure 2: Results of the fit including the one-body operators and operator $c_3$, with mixing angles fixed. The fit corresponds to the parameter set $a_1 = 0.816$, $b_1 = -0.299$, $b_2 = -0.308$, $c_3 = -0.072$, $\theta_{N1} = 0.610$ and $\theta_{N3} = 3.040$; see also Table 3.

into our definition of the fit coefficients. This is consistent since we are extracting the leading order contributions to the amplitudes in $1/N_c$, and in this limit each SU(6) multiplet is not split. We do take into account the actual mass spectrum of the states when evaluating the kinematical factor $K$. We will first show that a fit retaining only the one-body operators, and allowing the mixing angles to vary, provides a reasonable (though by no means perfect) description of the experimental data. We will find that the most likely values of the mixing angles obtained in this fit are in excellent agreement with the results of Ref. [5], which were obtained by a completely independent analysis of the $N^* \to N\pi$ decays. We will then show that inclusion of the two-body operators does not significantly improve the fit, so that our conclusions are qualitatively similar.
to those described in Ref. [5]. Finally, we will show that we can achieve a drastic improvement in the fit involving the one-body operators alone by discarding a single experimental data point. The improvement is so dramatic, we are left with the suspicion that one of the helicity amplitudes is affected by physics that we do not include in our analysis.

![Amplitudes (GeV$^{-1/2}$)](image)

Figure 3: Results of the fit including all the one- and two-body operators given in Section 2, with mixing angles fixed. The fit corresponds to the parameter set $a_1 = 0.808$, $b_1 = -0.192$, $b_2 = -0.498$, $c_1 = -0.087$, $c_2 = -0.036$, $c_3 = -0.077$, $c_4 = 0.049$, $d_1 = 0.077$, $d_2 = 0.009$, $d_3 = 0.013$, $d_4 = 0.012$, $\theta_{N1} = 0.610$ and $\theta_{N3} = 3.040$; see also Table II.

Fig. 1 shows the fit obtained by retaining the three one-body operators, allowing their coefficients as well as the mixing angles $\theta_{N1}$ and $\theta_{N3}$ to vary. The $\chi^2$ for this fit
is 52.9, with the greatest contributions coming from the amplitudes $A_{1/2}(\Delta^+(1620))$

![Graph]

Figure 4: Results of the fit to the one-body operators and mixing angles, with $A_{1/2}(\Delta^+(1620))$ omitted from the $\chi^2$. The fit corresponds to the parameter set $a_1 = 0.642$, $b_1 = -0.268$, $b_2 = -0.297$, $\theta_{N1} = 0.591$ and $\theta_{N3} = 3.031$; see also Table 5.

$(\Delta \chi^2 = 27.5)$ and $A_{1/2}(N^0(1520))$ $(\Delta \chi^2 = 7.7)$. What is perhaps most striking about this fit is the favored values of the mixing angles, $\theta_{N1} = 0.60 \pm 0.16$ and $\theta_{N3} = 3.06 \pm 0.11$. In Ref. [5] the same mixing angles were extracted from a large-$N_c$ analysis of $N^* \rightarrow N\pi$ decays, and the values $\theta_{N1} = 0.61 \pm 0.09$ and $\theta_{N3} = 3.04 \pm 0.15$ were obtained. Thus, the two analyses are in complete agreement. (Furthermore, both are consistent with the mixing angles found in earlier studies of the decays of baryon resonances that are not in the context of large-$N_c$ [10].) Therefore, it is justified to fix the mixing angles at the values extracted from the fit to the pion decays, and explore how the helicity
amplitude fit improves as we introduce the two-body operators.

We can begin exploring the effects of the two-body operators by introducing them one at a time. To be precise, we now do a series of fits in which we include the three one-body operators as well as a single two-body operator, with the mixing angles fixed at their preferred values. The change in $\chi^2$ gives some indication of the impact of the given two-body operator on the quality of the fit, or equivalently, on the two problematic amplitudes $A_{1/2}(\Delta^+(1620))$ and $A_{1/2}(N^0(1520))$. This approach can be misleading if the inclusion of more than one two-body operator leads to near cancellations between their matrix elements in some of the helicity amplitudes, but not in others. Such a situation can arise accidentally, as a consequence of the particular choice of operator basis. However, we will also consider a fit that includes all the one- and two-body operators, and verify that our conclusions remain unchanged.

The resulting $\chi^2$ values for fits including the three one-body operators and a single two-body operator are shown in Table 1. The only operator whose inclusion had a non-negligible impact on the quality of the fit was operator $c_3$; the corresponding predictions for the helicity amplitudes are displayed in Fig. 2. While there is a slight improvement in the agreement between theory and data, the two problematic amplitudes still remain.

A fit that includes all the one- and two-body operators is shown in Fig. 3. Although we have added 8 operators in addition to those of the fit in Fig. 1, the $\chi^2$ only improves to 28.87, with the greatest contributions coming from the amplitudes $A_{1/2}(\Delta^+(1620))$ ($\Delta \chi^2 = 7.39$) and $A_{1/2}(N^0(1650))$ ($\Delta \chi^2 = 7.13$). Thus, the $\chi^2$ per degree of freedom in our purely one-body fit, 3.8, reduces to 3.6 when we include all of the remaining operators. While we might have guessed, given the arguments in Section 2, that at least some of the two-body operators would have a significant impact on the goodness of the fit, what we have found is that none of them play a crucial role in describing the experimental data.

| Two-body Operator | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|-------------------|------|------|------|------|------|------|------|------|
| $\chi^2$         | 52.85| 52.04| 39.21| 52.91| 48.38| 52.81| 52.23| 52.59|

Table 1: $\chi^2$ for fits including the three one-body operators, and a single two-body operator. Mixing angles are fixed at $\theta_{N_1} = 0.61$ and $\theta_{N_3} = 3.04$, discussed in the text. For comparison, the fit with the one-body operators alone had $\chi^2 = 52.97$. 
The fit in Fig. 4 involves the same assumptions as in Fig. 1, except that we now omit the contribution of $A_{1/2}(\Delta^+(1620))$ to the total $\chi^2$. The resulting fit has $\chi^2 = 21.45$, or 1.65 per degree of freedom. If we allow for the possibility that there is something wrong with this experimental datum, then one can argue that the result from the one-body fit alone is good enough to obviate consideration of the higher-body operators.

4 Conclusions

What we have found by studying the electromagnetic couplings of the $\ell = 1$ baryons are results that are qualitatively similar to those found in the analysis of the $N^* \rightarrow N\pi$ decays in Ref. [5]. The one-body interactions appear to give a good description of the observed phenomenology, better than one would naively expect from large-$N_c$ arguments presented in Section 2. As the authors of Ref. [5] pointed out, this does not necessarily imply that these large-$N_c$ arguments are wrong, but could indicate that the two-body operators in the example we’ve considered have small coefficients by chance. The difficulty with this explanation is that it seems less probable, given the total number of two-body operators that have been evaluated in Ref. [5] and here. Thus, we conclude in the same way as Ref. [5], by noting that perhaps there is something more to the success of the naive quark model than large-$N_c$ alone.

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A Appendix

In the tables below we present the numerical results corresponding to Figures 1 through 4 in the text, as well as the experimental data.

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Parameters: \[ a_1 = 0.615 \pm 0.029 \quad b_1 = -0.294 \pm 0.039 \quad b_2 = -0.297 \pm 0.035 \]
\[ \theta_{N1} = 0.597 \pm 0.160 \quad \theta_{N3} = 3.060 \pm 0.110 \]

|          | \( A_{1/2}^p \) | \( A_{3/2}^p \) | \( A_{1/2}^n \) | \( A_{3/2}^n \) |
|----------|-----------------|-----------------|-----------------|-----------------|
| \( \Delta(1700) \) | 0.084           | 0.083           | -               | -               |
| \( \Delta(1620) \) | 0.085           | -               | -               | -               |
| \( N(1675) \) | 0               | 0               | -0.035          | -0.050          |
| \( N(1700) \) | -0.002          | 0.012           | 0.015           | -0.019          |
| \( N(1650) \) | 0.047           | -               | -0.047          | -               |
| \( N(1520) \) | -0.028          | 0.164           | -0.035          | -0.127          |
| \( N(1535) \) | 0.074           | -               | -0.074          | -               |

Table 2: Fit displayed in Figure 1, with \( \chi^2 = 52.92 \). The amplitudes are in units of GeV\(^{-1/2} \). In this fit, the mixing angles were free to vary.

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Table 3: Fit displayed in Figure 2, with $\chi^2 = 39.21$. The amplitudes are in units of GeV$^{-1/2}$.

| Parameters: | $a_1 = 0.816 \pm 0.061$ | $b_1 = -0.299 \pm 0.038$ | $b_2 = -0.308 \pm 0.032$ |
| $c_3 = -0.072 \pm 0.020$ | $\theta_{N1} = 0.610$ (fixed) | $\theta_{N3} = 3.04$ (fixed) |
| A$^p_{1/2}$ | A$^p_{3/2}$ | A$^n_{1/2}$ | A$^n_{3/2}$ |
| Δ(1700) | 0.070 | 0.057 | - | - |
| Δ(1620) | 0.063 | - | - | - |
| N(1675) | 0 | 0 | -0.036 | -0.051 |
| N(1700) | -0.003 | 0.015 | 0.014 | -0.023 |
| N(1650) | 0.046 | - | -0.058 | - |
| N(1520) | -0.032 | 0.163 | -0.048 | -0.151 |
| N(1535) | 0.070 | - | -0.088 | - |

Table 4: Fit displayed in Figure 3, with $\chi^2 = 28.87$. The amplitudes are in units of GeV$^{-1/2}$.

| Parameters: | $a_1 = 0.808 \pm 0.063$ | $b_1 = -0.192 \pm 0.118$ | $b_2 = -0.498 \pm 0.115$ |
| $c_1 = -0.087 \pm 0.047$ | $c_2 = -0.036 \pm 0.044$ | $c_3 = -0.077 \pm 0.026$ |
| $c_4 = 0.049 \pm 0.041$ | $d_1 = 0.077 \pm 0.036$ | $d_2 = 0.009 \pm 0.035$ |
| $d_3 = 0.013 \pm 0.032$ | $d_4 = 0.012 \pm 0.034$ | $\theta_{N1} = 0.610$ (fixed) |
| $\theta_{N3} = 3.04$ (fixed) |
| A$^p_{1/2}$ | A$^p_{3/2}$ | A$^n_{1/2}$ | A$^n_{3/2}$ |
| Δ(1700) | 0.077 | 0.061 | - | - |
| Δ(1620) | 0.057 | - | - | - |
| N(1675) | 0.016 | 0.011 | -0.049 | -0.062 |
| N(1700) | -0.012 | 0.015 | 0.041 | 0.009 |
| N(1650) | 0.038 | - | -0.071 | - |
| N(1520) | -0.025 | 0.162 | -0.045 | -0.150 |
| N(1535) | 0.074 | - | -0.087 | - |
Parameters: \[ a_1 = 0.648 \pm 0.030 \quad b_1 = -0.278 \pm 0.039 \quad b_2 = -0.294 \pm 0.037 \]
\[ \theta_{N1} = 0.603 \pm 0.158 \quad \theta_{N3} = 3.027 \pm 0.143 \]

|       | \( A_{1/2}^p \) | \( A_{3/2}^p \) | \( A_{1/2}^n \) | \( A_{3/2}^n \) |
|-------|-----------------|-----------------|-----------------|-----------------|
| \( \Delta(1700) \) | 0.086           | 0.089           | -               | -               |
| \( \Delta(1620) \) | 0.090           | -               | -               | -               |
| \( N(1675) \)     | 0               | 0               | -0.034          | -0.048          |
| \( N(1700) \)     | -0.002          | 0.017           | 0.014           | -0.022          |
| \( N(1650) \)     | 0.048           | -               | -0.050          | -               |
| \( N(1520) \)     | -0.021          | 0.169           | -0.040          | -0.133          |
| \( N(1535) \)     | 0.076           | -               | -0.077          | -               |

Table 5: Fit displayed in Figure 4, with \( \chi^2 = 22.92 \). The amplitudes are in units of GeV\(^{-1/2}\). The difference between this fit and that of Figure 1/Table 2 is the data point for \( A_{1/2}^p(\Delta(1620) \to p\gamma) \) was not used in the fit, although the value resulting from the fit parameters is shown above.

|       | \( A_{1/2}^p \) | \( A_{3/2}^p \) | \( A_{1/2}^n \) | \( A_{3/2}^n \) |
|-------|-----------------|-----------------|-----------------|-----------------|
| \( \Delta(1700) \) | 0.104 \( \pm 0.015 \) | 0.085 \( \pm 0.022 \) | -               | -               |
| \( \Delta(1620) \) | 0.027 \( \pm 0.011 \) | -               | -               | -               |
| \( N(1675) \)     | 0.019 \( \pm 0.008 \) | 0.015 \( \pm 0.009 \) | -0.043 \( \pm 0.012 \) | -0.058 \( \pm 0.013 \) |
| \( N(1700) \)     | -0.018 \( \pm 0.013 \) | -0.002 \( \pm 0.024 \) | 0.000 \( \pm 0.050 \) | -0.003 \( \pm 0.044 \) |
| \( N(1650) \)     | 0.053 \( \pm 0.016 \) | -               | -0.015 \( \pm 0.021 \) | -               |
| \( N(1520) \)     | -0.024 \( \pm 0.009 \) | 0.166 \( \pm 0.005 \) | -0.059 \( \pm 0.009 \) | -0.139 \( \pm 0.011 \) |
| \( N(1535) \)     | 0.070 \( \pm 0.012 \) | -               | -0.046 \( \pm 0.027 \) | -               |

Table 6: Experimental data, from Ref. [9], in units of GeV\(^{-1/2}\).