Effect of Axial Deformation on Elastic Properties of Irregular Honeycomb Structure

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Abstract
Irregular honeycomb structures occur abundantly in nature and in man-made products, and are an active area of research. In this paper, according to the optimization of regular honeycomb structures, two types of irregular honeycomb structures with both positive and negative Poisson’s ratios are presented. The elastic properties of irregular honeycombs with varying structure angles were investigated through a combination of material mechanics and structural mechanics methods, in which the axial deformation of the rods was considered. The numerical results show that axial deformation has a significant influence on the elastic properties of irregular honeycomb structures. The elastic properties of the structure can be considered by the enclosed area of the unit structure, the shape of the unit structure, and the elastic properties of the original materials. The elastic properties considering the axial deformation of rods studied in this study can provide a reference for other scholars.

Keywords: Axial deformation, Elastic modulus, Positive Poisson’s ratio, Negative Poisson’s ratio, Irregular honeycombs

1 Introduction
Honeycomb structures have many excellent characteristics, such as a higher elastic modulus [1], higher shear modulus [2], and higher energy absorption [3–6]. Therefore, negative Poisson’s ratio materials are widely used in aerospace [7, 8], automobiles, ships [9, 10], and other fields. Zhang et al. [11] studied a hierarchical regular hexagonal honeycomb structure. Mukhopadhyay and Adhikari [12] found that the elastic modulus of irregular honeycombs was highly influenced by the structural irregularity in auxetic honeycombs. Yang and Deng [13] reviewed the development of materials and structures with a negative Poisson’s ratio and the prospect development of porous materials. Lan et al. [14] analyzed a thin-walled honeycomb structure and investigated the effects of its structural and material parameters. Upreti et al. [15] studied honeycomb sandwich composites with a hexagonal honeycomb core and found that deformation decreased with increasing face sheet thickness. Thus, honeycomb structures have always been of profound interest to the research community.

Compared with traditional regular structures, irregular structures are more widely used in engineering applications owing to their better compressibility [16, 17] and higher buckling [18, 19]. Other important research areas related to the study of different honeycomb structures are their thermal and acoustic properties [20–23]. Therefore, much research has been carried out to predict the elastic properties of irregular honeycombs [24, 25]. Deng and Yang [26] investigated the behaviors of functionally graded structures in three different types of elastic moduli. In the numerical results, it was found that Poisson’s ratio exhibits appreciable effects on bearing capacity, which indicates that static properties can be improved by optimal design of cell shape material distribution and computational methodology. Hu et al. [27] studied the Poisson’s ratio of re-entrant honeycombs using numerical and structural methods. Huang et al. [28] presented a novel zero in-plane Poisson’s ratio honeycomb designed for large out-of-plane deformations and studied the relationship between in-plane stiffness and geometric parameters. Wang et al. [29] analyzed the
elastic modulus and Poisson's ratio of the re-entrant auxetic cellular structure in two principal directions. Bubert et al. [30] fabricated a skin supported by an accordion honeycomb and analyzed the in-plane equivalent elastic moduli in two directions without discussing the modulus in the third direction. Li et al. [31] studied the dynamic crushing response of irregular honeycomb structures and found that the propagation velocity of the stress wave is different in different honeycomb structures. Liu et al. [32] analyzed the effect of Poisson's ratios on the crashworthiness of in-plane honeycombs. Therefore, honeycomb structures [33], especially irregular structures [34], have wide research prospects.

In this study, the equivalent elastic modulus and Poisson's ratio of an irregular honeycomb in the \(\sigma_1\) and \(\sigma_2\) directions are derived using Castigliano's second theorem, and the internal bending moment, axial deformation, and Poisson's ratio of the original material are considered. Finally, the results in this paper are verified with the results reported in Ref. [24], and the effects of different structural shapes and axial deformation of rods on the elastic properties are analyzed. The results show that when considering the axial deformation, the absolute value of Poisson's ratio is lower than that without considering the axial deformation, and the variation in the structural shape has a slight influence on the elastic properties.

2 Irregular Honeycomb Structure

Figure 1 shows two types of regular structures: (a) and (b) are regular honeycomb structures with both positive and negative Poisson's ratios. Figure 2 shows two types of irregular structures: (a) and (b) are irregular honeycomb structures with both positive and negative Poisson's ratios.

3 Irregular Honeycomb Structures with Positive Poisson's Ratio

Figure 3 shows irregular honeycomb structures loaded in two different directions, where \(\sigma_1\) and \(\sigma_2\) are uniformly distributed loads in two mutually perpendicular directions.

3.1 Elastic Modulus in the \(\sigma_1\) Direction

Figure 4 shows the structural parameters and force analysis of rod \(AB\), where \(b\) is the thickness of the unit structure, \(t\) is the depth of the unit structure, \(l_1\) and \(h\) are the lengths of the inclined cell walls with inclination angle \(\theta\) and the length of the vertical rod, respectively. The moment \(M_1\) of rod \(AB\) can be expressed as

\[
M_1 = \frac{P_1 l_1 \sin \theta_1}{2},
\]

where \(P_1 = \sigma (h + l_1 \sin \theta_1) b\) is the force in the \(\sigma_1\) direction.

From the standard beam theory [33], the deflection \(\delta_{AB}\) of rod \(AB\) can be expressed as

\[
\delta_{AB} = \frac{p_1 l_1^3 \sin \theta_1}{12EI}.
\]

Axial force \(F'\) along rod \(AB\) is

\[
F' = p_1 \cos \theta_1.
\]

Axial deformation \(\Delta l\) can be expressed as

\[
\Delta l = \frac{F'L}{EA} = \frac{p_1 \cos \theta_1 l_1}{Ebt},
\]
where $E$ is the elastic moduli of the original materials, and the total deformation $\delta_1$ of rod $AB$ along the $\sigma_1$ direction is

$$\delta_1 = \delta_{AB}\sin \theta_1 + \Delta l \cos \theta_1 = \frac{p_1 l_1^3 \sin^2 \theta_1}{12EI} + \frac{p_1 \cos^2 \theta_1 l_1}{Ebt}.$$

Similarly, the total deformation $\delta_2$ of rod $BC$ along the $\sigma_1$ direction can be expressed as follows:

$$\delta_2 = \delta_{BC}\sin \theta_2 + \Delta l \cos \theta_2 = \frac{p_2 l_2^3 \sin^2 \theta_2}{12EI} + \frac{p_1 \cos^2 \theta_2 l_2}{Ebt}.$$

Figure 2  Irregular structures

Figure 3  Applied tensile stresses in the (a) $\sigma_1$ and (b) $\sigma_2$ directions
Combing Eqs. (2)–(6), the strain \( \varepsilon_1 \) parallel to the \( \sigma_1 \) direction is given by
\[
\varepsilon_1 = \frac{\delta_1 + \delta_2}{l_1 \cos \theta_1 + l_2 \cos \theta_2} = \frac{\sigma \gamma}{l_1 \cos \theta_1 + l_2 \cos \theta_2},
\]
where
\[
\gamma = b \left[ (h + l_1 \sin \theta_1) \left( \frac{l_1^3 \sin^2 \theta_1}{12EI} + \frac{\cos^2 \theta_1 l_1}{Eb_t} \right) \right. \\
+ (h + l_2 \sin \theta_2) \left( \frac{l_2^3 \sin^2 \theta_2}{12EI} + \frac{\cos^2 \theta_2 l_2}{Eb_t} \right) \right].
\]
Thus, the elastic modulus \( E_{1U} \) in the \( \sigma_1 \) direction can be expressed as follows:
\[
E_{1U} = \frac{(l_1 \cos \theta_1 + l_2 \cos \theta_2)}{\gamma}.
\]

### 3.2 Elastic Modulus in the \( \sigma_2 \) Direction
To derive the expression of the transverse elastic modulus, stress \( \sigma_2 \) is applied, as shown in Figure 3(b). Figure 5 shows that the deflection of rod BD consists of two parts: bending deformation and rotational deformation. The bending deformation \( \delta_{2vb} \) caused by the moment \( M_1 \) in the \( \sigma_2 \) direction can be expressed as
\[
\delta_{2vb} = \left( \frac{w \cos \alpha \left( \frac{l_1}{3EI} \right)^3}{3EI} \right) \cos \alpha,
\]
where
\[
\begin{align*}
\frac{w}{\text{EI}} &= \sigma_2 (l_1 \cos \theta_1 + l_2 \cos \theta_2) b, \\
I &= \frac{bt^3}{12}, \\
M_1 &= w s_1 \cot \alpha.
\end{align*}
\]
Because the rotation angles of the three rods connected to point B are identical, the rotation angle \( \phi \) of joint B can be written as
\[
\phi = \frac{M_1 l_1}{l_1 + l_2} \frac{l_1}{6EI}.
\]
Thus, the deformation \( \delta_{2vr} \) of the cell wall with an inclination angle \( \alpha \) in the \( \sigma_2 \) direction is given by:
\[ \delta_{2v} = \phi \left( \frac{s_1}{\sin \alpha} \right) \cos \alpha. \]  
\[ \delta_{2v} = \frac{w \cos \alpha \left( \frac{s_1}{\sin \alpha} \right)^3}{3EI} \cos \alpha + \frac{M_1 l_1 l_1}{l_1 + \frac{l_2}{6EI}} \frac{s_1}{\sin \alpha} \cos \alpha, \]  
\[ \delta_{v2FH} = \frac{w \cos \beta \left( \frac{s_2}{\sin \beta} \right)^3}{3EI} \cos \beta + \frac{M_1 l_4 l_4}{l_3 + \frac{l_4}{6EI}} \frac{s_2}{\sin \beta} \cos \beta. \]  

Now, the axial deformations of rod BD and rod FH in the \( \sigma_2 \) direction can be expressed as

\[ \Delta l_{2BD} = \frac{w s_1}{Ebt} \sin \alpha, \]  
\[ \Delta l_{2FH} = \frac{w s_2}{Ebt} \sin \beta. \]

The deflections \( \delta_{vBD} \) and \( \delta_{vGF} \) of rods AB and GF (Figure 6) in the \( \sigma_2 \) direction can be expressed as

\[ \delta_{vAB} = \frac{\left( \frac{l_1 w}{l_1 + l_2} \cos \theta_1 \right) s_1^3}{12EI} \cos \theta_1, \]  
\[ \delta_{vGF} = \frac{\left( \frac{l_4 w}{l_3 + l_4} \cos \theta_4 \right) s_2^3}{12EI} \cos \theta_4. \]

The axial deformation of rods AB and GF in the \( \sigma_2 \) direction can be expressed as

\[ \Delta l'_{2} = \frac{\left( \frac{l_1 w}{l_1 + l_2} \sin \theta_1 \right)}{Ebt} \sin \theta_1, \]  
\[ \Delta l'_{3} = \frac{\left( \frac{l_4 w}{l_3 + l_4} \sin \theta_4 \right)}{Ebt} \sin \theta_4. \]

Thus, the total deformation \( \delta_2 \) of the structure in the \( \sigma_2 \) direction can be expressed as

\[ \delta_2 = \delta_{v2BD} + \delta_{v2FH} + \Delta l_{2BD} + \Delta l_{2FH} + \delta_{vAB} + \delta_{vGF} + \Delta l'_{2} + \Delta l'_{3}, \]  

while Eq. (22) can be rewritten as follows:

\[ \delta_2 = w \varphi, \]

where

\[ \varphi = \left[ \left( \frac{\cos \alpha \left( \frac{s_1}{\sin \alpha} \right)^3}{3EI} \cos \alpha + \frac{s_1}{l_1 + \frac{l_2}{6EI}} \frac{\cos \alpha}{\sin \alpha} \right) \cos \alpha + \left( \frac{\cos \beta \left( \frac{s_2}{\sin \beta} \right)^3}{3EI} \cos \beta + \frac{s_2}{l_3 + \frac{l_4}{6EI}} \frac{\cos \beta}{\sin \beta} \right) \cos \beta + \frac{s_1}{Ebt} \sin \alpha + \frac{s_2}{Ebt} \sin \beta \right. \]

\[ + \left. \left( \frac{\frac{l_1 w}{l_1 + l_2} \cos \theta_1}{12EI} \right)^3 \sin \theta_1 + \frac{l_4 w}{l_3 + l_4} \cos \theta_4 + \frac{l_4 w}{l_3 + l_4} \sin \theta_4 \right]. \]

Strain \( \varepsilon_2 \) in the \( \sigma_2 \) direction can be obtained as

\[ \varepsilon_2 = \frac{\delta_2}{h + s_1 + s_2 + l_1 \sin \theta_1 + l_4 \sin \theta_4} \]

\[ = \frac{w \varphi}{h + s_1 + s_2 + l_1 \sin \theta_1 + l_4 \sin \theta_4}, \]

where \( w = \sigma_2 (l_1 \cos \theta_1 + l_2 \cos \beta) \); thus, the elastic modulus in the \( \sigma_2 \) direction of the structure can be expressed as
\[ E_{2U} = \frac{\sigma_2}{\varepsilon_2} = \frac{h + s_1 + s_2 + l_1 \sin \theta_1 + l_4 \sin \theta_4}{(l_1 \cos \theta_1 + l_2 \cos \theta_2) b \varphi}. \] (26)

### 3.3 Poisson’s Ratio \( \nu_{12} \)

Poisson’s ratios were calculated by taking the negative ratios of strains normal to and parallel to the loading direction. Poisson’s ratio \( \nu_{12} \) of the unit structure can be defined as

\[ \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}, \] (27)

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are strains in the \( \sigma_1 \) and \( \sigma_2 \) directions, respectively, due to the load in the \( \sigma_1 \) direction. In addition, \( \varepsilon_1 \) can be obtained from Eq. (7) and \( \varepsilon_2 \) can be expressed as

\[ \varepsilon_2 = -\frac{\delta'_1 + \delta'_2}{h + s_1 + s_2 + l_1 \sin \theta_1 + l_4 \sin \theta_4}, \] (28)

where

\[
\begin{aligned}
\begin{cases}
\delta'_1 = \delta_{AB} \cos \theta_1 + \Delta l \sin \theta_1 = \frac{p_1 l_3 \sin \theta_1 \cos \theta_1}{2EI} + \frac{p_1 l_3 \sin \theta_1 \cos \theta_1}{12EI}, \\
\delta'_2 = \delta_{GF} \cos \theta_4 + \Delta l \sin \theta_4 = \frac{p_2 l_4 \sin \theta_4 \cos \theta_4}{2EI} + \frac{p_2 l_4 \sin \theta_4 \cos \theta_4}{12EI},
\end{cases}
\end{aligned}
\] (29)

The Poisson’s ratio of a structure in the \( \sigma_1 \) direction can be expressed as

\[ \nu_{12} = \frac{(\delta'_1 + \delta'_2)(l_1 \cos \theta_1 + l_2 \cos \theta_2)}{(\delta_1 + \delta_2)(h + s_1 + s_2 + l_2 \sin \theta_2 + l_3 \sin \theta_3)}. \] (30)

### 3.4 Poisson’s Ratio \( \nu_{21} \)

Poisson’s ratio of a structure for loading in the \( \sigma_2 \) direction can be expressed as

\[ \nu_{21} = -\frac{\varepsilon'_1}{\varepsilon_2}, \] (31)

where \( \varepsilon'_1 \) and \( \varepsilon'_2 \) are the strains in the \( \sigma_1 \) and \( \sigma_2 \) directions, respectively. \( \varepsilon'_2 \) can be obtained from Eq. (23) as

\[ \varepsilon'_2 = \frac{w \varphi}{h + s_1 + s_2 + l_1 \sin \theta_1 + l_4 \sin \theta_4}, \] (32)

where \( w = \sigma_2 (l_1 \cos \theta_1 + l_2 \cos \theta_2) b \), and \( \varepsilon'_1 \) can be obtained as

\[ \varepsilon'_1 = -\frac{\delta_{vAB1} + \delta_{vBC1}}{l_1 \cos \theta_1 + l_2 \cos \theta_2} = -\frac{w \varphi'}{l_1 \cos \theta_1 + l_2 \cos \theta_2}, \] (33)

where \( \delta_{vAB1} \) and \( \delta_{vBC1} \) are the deformations in the \( \sigma_1 \) direction due to the load in the \( \sigma_2 \) direction.

The Poisson’s ratio of a structure in the \( \sigma_1 \) direction can be expressed as

\[ \psi' = \left( \frac{l_1}{l_1 + l_2} \cos \theta_1 \right) \frac{l_3^3}{12EI} \sin \theta_1 + \left( \frac{l_2}{l_1 + l_2} \cos \theta_2 \right) \frac{l_3^3}{12EI} \sin \theta_2 \\
+ \frac{l_1}{l_1 + l_2} \sin \theta_1 \frac{Ebt}{Ebt} \cos \theta_1 + \frac{l_2}{l_1 + l_2} \sin \theta_2 \frac{Ebt}{Ebt} \cos \theta_2. \] (34)

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**Figure 7** Tensile stresses along the (a) \( \sigma_1 \) and (b) \( \sigma_2 \) directions
Thus, Poisson’s ratio $\nu_{21}$ of a structure in the $\sigma_2$ direction can be expressed as

$$\nu_{21} = \frac{\varphi'(h + s_1 + s_2 + l_2 \sin \theta_2 + l_3 \sin \theta_1)}{\varphi(l_1 \cos \theta_1 + l_2 \cos \theta_2)}.$$  \hfill (35)

### 4 Irregular Honeycomb Structure with Negative Poisson’s Ratio

Figure 7 shows the irregular honeycomb structures loaded in two different directions. $\sigma_1$ and $\sigma_2$ are uniformly distributed loads in two mutually perpendicular directions.

#### 4.1 Elastic Modulus in $\sigma_1$ Direction

Figure 8 shows the force analysis of rods $AB$ and $GF$ in the $\sigma_1$ and $\sigma_2$ directions, respectively. $E'_1$ is the elastic modulus of an irregular structure with a negative Poisson’s ratio and has the same value as an irregular structure with a positive Poisson’s ratio. The elastic modulus $E'_1$ in the $\sigma_1$ direction can be obtained as described in Section 3.1:

$$E'_1 = \frac{\left( l_1 \cos \theta_1 + l_2 \cos \theta_2 \right)}{\gamma},$$  \hfill (36)

where

$$\gamma = b \left[ (h - l_1 \sin \theta_1) \left( \frac{l_1^3 \sin^2 \theta_1}{12EI} + \frac{\cos^2 \theta_1 l_1}{Eb} \right) ight] + (h - l_2 \sin \theta_2) \left( \frac{l_2^3 \sin^2 \theta_2}{12EI} + \frac{\cos^2 \theta_2 l_2}{Eb} \right).$$  \hfill (37)

#### 4.2 Elastic Modulus in the $\sigma_2$ Direction

The elastic modulus of structures with a negative Poisson’s ratio can be obtained as described in Section 3.2. Thus, the strain $\varepsilon_{21}$ in the $\sigma_2$ direction can be obtained as follows:

$$\varepsilon_{21} = \frac{\delta}{h + s_1 + s_2 - l_1 \sin \theta_1 - l_4 \sin \theta_4},$$  \hfill (38)

where $w = \sigma_2(l_1 \cos \theta_1 + l_2 \cos \theta_2)b$.

Thus, the elastic modulus of $E'_2$ in the $\sigma_2$ direction can be expressed as

$$E'_2 = \frac{\sigma_{21}}{\varepsilon_{21}} = \frac{h + s_1 + s_2 - l_1 \sin \theta_1 - l_4 \sin \theta_4}{(l_1 \cos \theta_1 + l_2 \cos \theta_2)b},$$  \hfill (39)

where

$$\varphi = \left[ \left( \frac{\cos \alpha \left( \frac{a_1}{3EI} \right)^3}{3EI} \right) \cos \alpha + \frac{s_1 \cos a_1}{l_1 + l_2} \frac{l_1}{6E} \frac{s_1}{\sin \alpha} \right] \cos \alpha + \left( \frac{\cos \beta \left( \frac{s_1}{3EI} \right)^3}{3EI} \right) \cos \beta + \frac{s_1 \cos a_4}{l_3 + l_4} \frac{l_4}{6E} \frac{s_2}{\sin \beta} \cos \beta + \frac{s_1}{Ebt} \sin \alpha + \frac{s_2}{Ebt} \sin \beta + \frac{\frac{4}{Ebt} \cos \theta_1}{12EI} \frac{l_1^3}{\cos \theta_1} + \frac{\frac{4}{Ebt} \sin \theta_1}{12EI} \frac{l_1}{\cos \theta_1} + \frac{\frac{4}{Ebt} \cos \theta_4}{12EI} \frac{l_4^3}{\cos \theta_4} + \frac{\frac{4}{Ebt} \sin \theta_4}{12EI} \frac{l_4}{\sin \theta_4}. \right.$$

$$\left. + \frac{\frac{4}{Ebt} \cos \theta_1}{12EI} \frac{l_1^3}{\cos \theta_1} + \frac{\frac{4}{Ebt} \sin \theta_1}{12EI} \frac{l_1}{\cos \theta_1} + \frac{\frac{4}{Ebt} \cos \theta_4}{12EI} \frac{l_4^3}{\cos \theta_4} + \frac{\frac{4}{Ebt} \sin \theta_4}{12EI} \frac{l_4}{\sin \theta_4}. \right)$$  \hfill (40)
4.3 Poisson’s Ratio $v'_{12}$

To derive Poisson’s ratio for an irregular structure, the mechanics formula is calculated to obtain Poisson’s ratio in the $\sigma_1$ direction:

$$v'_{12} = -\frac{\varepsilon''_2}{\varepsilon'_1},$$

(41)

where $\varepsilon''_1$ and $\varepsilon''_2$ are strains in the $\sigma_1$ and $\sigma_2$ directions, respectively, due to the load in the $\sigma_1$ direction. $v'_{12}$ is Poisson’s ratio in the $\sigma_1$ direction, $\varepsilon''_1$ can be obtained from Eq. (7), and $\varepsilon''_2$ can be expressed as

$$\varepsilon''_2 = \frac{\delta''_1 + \delta''_2}{h + s_1 + s_2 - l_2 \sin \theta_2 - l_3 \sin \theta_3},$$

(42)

where $\delta''_1$ and $\delta''_2$ can be expressed as

$$\left\{\begin{array}{l}
\delta''_1 = \delta_{AB} \cos \theta_1 + \Delta l \sin \theta_1 = \frac{p_1 l_1^2 \sin \theta_1 \cos \theta_1}{12EI} + \frac{p_1 l_1 \sin \theta_1 \cos \theta_1}{Ebt}, \\
\delta''_2 = \delta_{GF} \cos \theta_4 + \Delta l \sin \theta_4 = \frac{p_2 l_3^3 \sin \theta_4 \cos \theta_4}{12EI} + \frac{p_2 l_3 \sin \theta_4 \cos \theta_4}{Ebt}.
\end{array}\right.$$  

(43)

Thus, Poisson’s ratio $v'_{12}$ of a structure in the $\sigma_1$ direction can be expressed as

$$v'_{12} = -\frac{(\delta''_1 + \delta''_2)(l_1 \cos \theta_1 + l_2 \cos \theta_2)}{(\delta_1 + \delta_2)(h + s_1 + s_2 - l_2 \sin \theta_2 - l_3 \sin \theta_3)},$$

(44)

where

$$\varphi' = \frac{\left(\frac{l_1}{l_1 + l_2} \cos \theta_1\right) l_1^3}{12EI} \sin \theta_1 + \frac{\left(\frac{l_2}{l_1 + l_2} \cos \theta_2\right) l_2^3}{12EI} \sin \theta_2 + \frac{l_1}{l_1 + l_2} \sin \theta_1 + \frac{l_2}{l_1 + l_2} \sin \theta_2 \cos \theta_2.$$  

(46)

4.4 Poisson’s Ratio $v'_{21}$

Poisson’s ratio $v'_{21}$ of a unit structure in the $\sigma_2$ direction can be obtained from Section 3.4, as follows:

$$v'_{21} = \frac{\varphi'(h + s_1 + s_2 - l_2 \sin \theta_2 - l_3 \sin \theta_3)}{\varphi(l_1 \cos \theta_1 + l_2 \cos \theta_2)},$$

(45)

5 Results and Discussions

In this study, the geometric configuration of the unit structure with a fixed enclosed area is defined, as shown in Figure 9.
Figure 9 shows two structures: a positive and a negative Poisson’s ratio structure, which are analyzed with the same enclosed area, where $b = 0.01$ m, $t = 0.01$ m, the original material is aluminum, and the elastic modulus is $70 \times 10^9$ Pa. To obtain the elastic properties, the enclosed area of the structure is fixed in this study, so the variation in $\theta_1$ can only influence the structure shape.

According to the structure in Figure 9, when the value of $\theta_1$ is 30°, 34.3°, 40.9°, 45°, and 49.1°, $\theta_2$ becomes 90°, 75°, 60°, 53.8°, and 49.1°, respectively, and the irregular structure becomes symmetrical when $\theta_1 = \theta_2 = 49.1^\circ$, which has the same structure as the regular honeycomb structure.

5.1 Elastic Properties of Structure with Positive Poisson’s Ratio

The equivalent elastic modulus of the positive Poisson’s ratio structure due to changes in $\theta_1$ is shown in Figure 10, where $E_1$ is the equivalent elastic modulus in the $\sigma_1$ direction, $E_y$ is the equivalent elastic modulus of the regular structure obtained from Ref. [24], and $E_{1x}$ is the equivalent elastic modulus without considering axial deformation of rods. It can be seen that when $\theta_1 = \theta_2 = 49.1^\circ$ (structure becomes regular), the equivalent elastic modulus reaches the maximum value, and $E_{1x}$ has the same value as $E_x$. Because the structures are identical when $\theta_1 = \theta_2 = 49.1^\circ$, the correctness of the results obtained in this study is verified. When the axial deformation of the rods is considered, the elastic modulus in the $\sigma_1$ direction is lower than that without considering the axial deformation of the rods. When $\theta_1$ changes, the values of the elastic modulus of the regular structure are higher than those of the irregular structure.

Figure 11 shows the equivalent elastic modulus of the positive Poisson’s ratio structure due to changes in $\theta_1$ in the $\sigma_2$ direction. $E_y$ is the equivalent elastic modulus of the regular structure obtained from Ref. [24]. $E_{2y}$ is the equivalent elastic modulus calculated without considering the axial deformation in the $\sigma_2$ direction. When the $\theta_1$ angle is 49.1° (structure becomes a regular structure), the equivalent elastic modulus reaches the maximum value and $E_{2y}$ has the same value with $E_y$; this proves the validity of the calculation results of this study. With the increase of structural irregularity, differences between $E_y$ and $E_{2y}$ decrease gradually. It can be seen that structure shape and axial deformation have a significant influence on elastic modulus.

Poisson’s ratio in the $\sigma_1$ direction with variation in $\theta_1$ is shown in Figure 12. $v_x$ is the Poisson’s ratio of the regular structure obtained from Ref. [24]. $v_{12x}$ is the Poisson’s ratio without considering axial deformation in the $\sigma_1$ direction. When the $\theta_1$ angle is 49.1° (structure becomes regular), $v_{12x}$ has the same value as $v_x$, and the results obtained from Ref. [24] are the same as the values obtained in this study; this verifies the correctness of the obtained results. It can be seen that when $\theta_1$ is lower than 49.1°, Poisson’s ratio gradually decreases. Axial deformation has a significant influence on Poisson’s ratio, and

Figure 13 Poisson’s ratio with varying $\theta_1$ angle in the $\sigma_2$ direction

Figure 14 Elastic modulus $E'_1$ with varying $\theta_1$ angle in the $\sigma_1$ direction
when the $\theta_1$ angle is 49.1°, the Poisson's ratio of the structures reaches the minimum value.

Figure 13 shows the variation of $v_{21}$ with different structural shapes in the $\sigma_2$ direction. $v_y$ is the Poisson's ratio of the regular structure obtained from Ref. [24]. $v_{21y}$ is the Poisson's ratio without considering the axial deformation in the $\sigma_2$ direction. When the $\theta_1$ angle is 49.1° (structure becomes regular), $v_{21y}$ and $v_y$ have the same value; this verifies the validity of this study. It can be observed that cell angle $\theta_1$ influences $v_{21}$. Poisson's ratio considering axial deformation is lower than that without considering axial deformation, and positive Poisson's ratio structures reach the maximum value in 49.1°.

5.2 Elastic Properties of Structure with Negative Poisson's Ratio

The equivalent elastic modulus of the positive Poisson's ratio structure due to changes in $\theta_1$ is shown in Figure 14, where $E_{\sigma_1}$ is the equivalent elastic modulus of the regular structure obtained from Ref. [24], $E_{\sigma_1}^{'}$ is the equivalent elastic modulus of the irregular structure without considering axial deformation, and $E_{\sigma_1}^{''}$ is the equivalent elastic modulus of an irregular structure considering axial deformation. It can be seen that $E_{\sigma_1}^{''}$ and $E_{\sigma_1}$ have the same value when $\theta_1 = \theta_2 = 49.1^\circ$, which verifies the correctness of the results in this study, and the equivalent elastic modulus of the structures reaches the maximum value at $\theta_1 = 49.1^\circ$. When the $\theta_1$ angle changes, the elastic modulus decreases with an increase in structural irregularity.

Figure 15 shows the equivalent elastic modulus of the negative Poisson's ratio structure due to changes in $\theta_1$ in the $\sigma_2$ direction. $E_{\sigma_2}$ is the equivalent elastic modulus of the regular structure obtained from Ref. [24], $E_{\sigma_2}^{'}$ is the equivalent elastic modulus calculated without considering axial deformation in the $\sigma_2$ direction. When the $\theta_1$ angle is 49.1°, the equivalent elastic modulus reaches the maximum value and $E_{\sigma_2}^{''}$ is the same value with the result reported in Ref. [24]; this verifies the correctness of the results obtained in this study. The elastic modulus considering axial deformation is lower than that without considering axial deformation, and the difference between them reaches the maximum value when $\theta_1 = 49.1^\circ$. Whether or not the axial deformation of the rod is considered has a great influence on the equivalent elastic modulus.

The Poisson's ratio of the negative Poisson's ratio structure in the $\sigma_1$ direction is shown in Figure 16. $v_{\sigma_1}^{'}$ is the Poisson's ratio of the regular structure obtained from Ref. [24], $v_{\sigma_1}^{''}$ is the Poisson's ratio without considering the axial deformation in the $\sigma_1$ direction. $v_{\sigma_1}^{'}$ is the Poisson's ratio considering the axial deformation in the $\sigma_1$ direction. When $\theta_1 = \theta_2 = 49.1^\circ$, the Poisson's ratios of the structure reaches the maximum value and $v_{\sigma_1}^{''}$ has the
same value as $v_{12}'$, verifying the correctness of the results obtained in this study. The consideration of axial deformation has a negligible influence on Poisson’s ratio in the $\sigma_1$ direction. When the $\theta_1$ angle is lower than 49.1°, $v_{12}'$ increases with an increase in the $\theta_1$ angle, and when $\theta_1$ is higher than 49.1°, $v_{12}'$ decreases with an increase in the $\theta_1$ angle.

Figure 17 shows the variation in $v_{21}$ with different structural shapes in the $\sigma_2$ direction. $v_{21}'$ is the Poisson’s ratio of the regular structure obtained from Ref. [24], $v_{12}'$ is the Poisson’s ratio without considering the axial deformation in the $\sigma_2$ direction. When the cell angle $\theta_1$ is 49.1°, $v_{21}'$ and $v_{12}'$ reach their minimum values. The value of $v_{12}'$ without considering axial deformation is the same as the results of Ref. [24] at an angle of 49.1°. Figure 17 also shows that the Poisson’s ratio considering axial deformation is higher than that without considering axial deformation, and the difference between them reaches the maximum value when $\theta_1 = 49.1°$.

Figure 18 shows the equivalent elastic moduli of the irregular structures in the $\sigma_1$ and $\sigma_2$ directions. $E_{11}, E_{12}, E_2, E_{1y}, E_{1x}, E_{1z}, E_4$, and $E_{2y}$ are the equivalent elastic moduli obtained from Figures 10, 11, 12, 13, 14, 15, 16, and 17. It can be seen that the equivalent elastic moduli in the $\sigma_2$ direction is higher than those in the $\sigma_1$ direction. When $\theta_1 = \theta_2 = 49.1°$, the equivalent elastic moduli in the $\sigma_2$ direction reach the maximum value, and $E_{1y}$ is the same value with $E_{1y}$. The axial deformation of rods has a significant influence on the equivalent elastic moduli. The equivalent elastic moduli in the $\sigma_1$ direction have insignificant variation when $\theta_1$ varies.

Figure 19 shows the Poisson’s ratio of the irregular structures in the $\sigma_1$ and $\sigma_2$ directions. $v_{12}, v_{12x}, v_{21}, v_{21y}, v_{12}'$, $v_{12x}'$, $v_{21}'$, and $v_{21y}'$ are the equivalent elastic moduli obtained from Figures 10, 11, 12, 13, 14, 15, 16, and 17. Notably, the value of $v_{21y}'$ is higher than the rest. The absolute value of the Poisson’s ratio in two structures reaches the maximum value. Moreover, $v_{12}, v_{12x}$, and $v_{12}'$, $v_{12x}'$ have similar values, respectively. Whether or not the axial deformation is considered has little influence on the Poisson’s ratio of the structure.

6 Conclusions
In this study, two types of irregular honeycomb structures were studied using material mechanics and structural mechanics methods. Considering axial deformation of rods, the elastic properties of irregular structures with different structure shapes were studied. Compared with the results reported in Ref. [24], the results were in good agreement.

The results show that when the enclosed area of the irregular honeycomb structure is fixed, the equivalent elastic modulus and Poisson’s ratio of the structure will vary with varying structure shape. The $\theta_1$ angle has a significant influence on the equivalent elastic modulus. When the structure is regular, the absolute value of the elastic modulus and Poisson’s ratio in the $\sigma_2$ direction reaches the maximum value, and the Poisson’s ratio in the $\sigma_1$ direction reaches the minimum value. The elastic properties of the structure considering axial deformation are higher than those without considering the axial deformation. Therefore, the elastic properties of irregular structures can be achieved by the axial deformation of the rods, structural shapes, and original materials.

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Authors’ contributions
NW and QD were in charge of the whole analysis; NW and QD wrote the manuscript. All authors read and approved the final manuscript.

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Competing Interests
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