Abstract

The dependence of the subprocess cross section for dijet production at fixed transverse momentum on the (large) rapidity separation $\Delta y$ of the dijets can be used to test for ‘BFKL physics’, i.e. the presence of higher-order ($\alpha_s \log \Delta y)^n$ contributions. Unfortunately in practice these subtle effects are masked by the additional, stronger dependence arising from the parton distributions. We propose a simple ratio test using two different collider energies in which the parton distribution dependence largely cancels. Such a ratio does distinguish between the ‘asymptotic’ analytic BFKL and lowest-order QCD predictions. However, when subasymptotic effects from overall energy-momentum conservation are included, the BFKL predictions for present collider energies change qualitatively. In particular, the real gluon emission contributions to the BFKL cross section are suppressed. The proposed ratio therefore provides an interesting laboratory for studying the interplay of leading-logarithm and kinematic effects.
Quantum Chromodynamics has been very successful in describing jet production in high energy collider experiments. Perturbative QCD at fixed order in the strong coupling constant $\alpha_s$ provides sufficient predictive power for a wide variety of high energy phenomena. However in some regions of phase space, large logarithms can multiply the coupling, spoiling the good behavior of fixed-order expansions. In certain cases these large logarithms can be resummed, as in the Balitsky, Fadin, Kuraev and Lipatov (BFKL) equation [1], and the predictive power of the theory is then in principle restored. The latter statement should be subject to experimental test, since the predictions of BFKL are distinct from those of fixed-order QCD.

Being asymptotic, the regions for which BFKL resummation is presumed to be necessary have until recently been beyond the reach of experiments, and therefore the predictions of BFKL have been difficult to test. The last few years, however, have seen experiments at the HERA $ep$ collider and the Fermilab Tevatron that approach the kinematic regimes relevant to BFKL. At HERA the relevant regime is small parton momentum fraction $x$, where BFKL predicts a sharp rise in the structure function $F_2$ as $x$ decreases. Further discussion can be found in Refs. [2, 3] and references therein; here we are concerned with BFKL physics at hadron colliders.

At the Tevatron $\bar{p}p$ collider, and at hadron colliders in general, BFKL resummation applies to dijet production when the rapidity separation $\Delta$ of the two jets is large, $\Delta \gg 1$, while fixed-order QCD should be adequate for $\Delta = O(1)$. BFKL predicts, among other things, a rise in the dijet subprocess cross section $\hat{\sigma}$ as $\Delta$ increases [4], in contrast to the $\hat{\sigma} \to$ constant behavior expected at lowest order. In practice such behavior can be difficult to observe because $\hat{\sigma}$ gets folded in with parton distribution functions (pdfs), which decrease with $\Delta$ much more rapidly than the subprocess cross section increases. In particular, the BFKL rise with $\Delta$ gets killed by kinematic constraints. The challenge is to find measurable quantities in dijet production that are insensitive to the pdfs, but that retain the distinctive behavior characteristic of BFKL resummation. Some possibilities have been discussed in Refs. [5, 6, 7, 8, 9, 10], including the azimuthal decorrelation of the two jets: the multiple emission of soft gluons between the leading jets predicted by BFKL leads to a stronger decorrelation than does fixed-order QCD, and the prediction is relatively insensitive to the pdfs.

Another possibility is to look for the increase in $\hat{\sigma}$ with $\Delta$ by considering different collision energies [4]. The idea is to choose $\Delta$’s that correspond to the same parton momentum fractions at different energies so that the pdf dependence is the same for both, thereby allowing the $\Delta$ dependence to be extracted. The DØ collaboration has in fact recently reported the results of a preliminary study of dijet cross section ratios at fixed parton momentum fraction [11].

In this paper we investigate the dependence of the dijet cross section on collision energies in leading-order QCD and in the BFKL approach. We define a cross section ratio at two collider energies in which the pdf dependence largely cancels, and in which the predictions of leading-order QCD and ‘naive’ BFKL can be distinguished. (By naive BFKL, we mean the predictions obtained by resumming leading-logarithm $(\alpha_s \Delta)^n$ contributions arising from the emission of soft real and virtual gluon contributions in the absence of overall kinematic constraints.) However, one difficulty with the BFKL approach is that even for the largest $\Delta$ values which are accessible at present, subasymptotic effects, for example from energy-
momentum conservation, are likely to be important. This has led to ‘improved’ BFKL dijet cross section calculations, in which some of these effects are taken into account. In this study we will use the BFKL Monte Carlo approach developed in Ref. [10] (see also [9]). We find that at present collider energies the predictions of naive BFKL are indeed substantially modified when subasymptotic kinematic effects are included.

Our calculations will therefore give us a sequence of predictions — full leading-order QCD, asymptotic leading-order QCD, naive BFKL, improved BFKL — to confront experiment. In what follows we develop each of these approximations in turn.

We begin with the lowest-order QCD inclusive two-jet production cross section as a function of the two jet rapidities $y_1, y_2$ and their common transverse momentum $p_T$. For simplicity we will consider the symmetric situation where the two rapidities are equal and opposite. Setting $y_1 = - y_2 = \frac{1}{2} \Delta$ gives for the differential cross section

$$
\frac{d\sigma}{dy_1 dy_2 dp_T^2} \bigg|_{y_1 = - y_2 = \frac{1}{2} \Delta} = \frac{1}{256\pi p_T^4 \cosh^4\left(\frac{1}{2} \Delta\right)} \sum_{a,b,c,d=q,g} x f_a(x, \mu^2) x f_b(x, \mu^2) \sum |M(ab \rightarrow cd)|^2,
$$

(1)

where the subprocess matrix elements are functions only of the rapidity difference $\Delta$. In the limit of large $\Delta$ the $gg, qg$ and $qq$ subprocess matrix elements squared become equal, up to overall color factors of $C^2_A, C_A C_F$ and $C_F^2$ respectively. This defines the effective subprocess approximation to Eq. (1):

$$
\frac{d\sigma}{dy_1 dy_2 dp_T^2} \bigg|_{y_1 = - y_2 = \frac{1}{2} \Delta} \simeq \frac{[xG(x, \mu^2)]^2}{256\pi p_T^4 \cosh^4\left(\frac{1}{2} \Delta\right)} \sum |M(gg \rightarrow gg)|^2,
$$

(3)

where

$$
G = g + 4 \frac{q + \bar{q}}{9}.
$$

(4)

In an experiment (and in the BFKL formalism to be discussed below), we are interested in events with jets above some transverse momentum threshold $P_T$. Integrating over the jet transverse momentum $p_T > P_T$ then gives

$$
\frac{d\sigma}{dy_1 dy_2 dp_T^2} \bigg|_{y_1 = - y_2 = \frac{1}{2} \Delta} \simeq \frac{\sum |M(gg \rightarrow gg)|^2}{256\pi \cosh^4\left(\frac{1}{2} \Delta\right)} \frac{1}{P_T^2} \int_{1}^{X^{-2}} \frac{du^2}{u^4} \left[xG(x, \mu^2)\right]^2 |_{x = X u},
$$

(5)

with $X = p_T(\text{min})/p_T(\text{max}) = 2 P_T \cosh\left(\frac{1}{2} \Delta\right)/\sqrt{s}$. In the asymptotic limit where $\Delta$ is large, the $\Delta$ dependence in the prefactor disappears so that

$$
\frac{\sum |M(gg \rightarrow gg)|^2}{256\pi \cosh^4\left(\frac{1}{2} \Delta\right)} \rightarrow \frac{1}{2} \pi C_A^2 \alpha_s^2,
$$

(6)
and hence
\[
\frac{d\sigma}{dy_1dy_2}\bigg|_{y_1=-y_2=\frac{1}{2}\Delta} \approx \frac{\pi C_A^2 \alpha_s^2}{2P_T^2} \int_1^{X^{-2}} \frac{du^2}{u^4} \left[ xG(x,\mu^2) \right]^2_{x=Xu} \equiv \hat{\sigma}_0 \mathcal{F}(X,\mu^2) \quad (7)
\]
where
\[
\hat{\sigma}_0 = \frac{\pi C_A^2 \alpha_s^2}{2P_T^2} \quad (8)
\]
and \( \mathcal{F} \) contains the integration over the parton distribution functions. Equation (7) defines the asymptotic QCD LO cross section to which we will compare the BFKL predictions below.

Fig. 1 shows the QCD LO dijet cross sections calculated using (i) the 2 \( \rightarrow \) 2 matrix elements (Eq. (1)), (ii) the effective subprocess approximation (Eq. (5)), and (iii) the asymptotic form of the effective subprocess approximation (Eq. (7)), for two \( p\bar{p} \) collider energies \( \sqrt{s} = 630, \ 1800 \text{ GeV} \). The jet \( p_T \) threshold is \( p_T = 20 \text{ GeV/c} \), the partons are CTEQ4L with \( \mu = p_T \) \cite{12}, and \( \alpha_s \) is evaluated in leading order also at scale \( \mu = p_T \) (the numerical value is 0.170). The effective subprocess approximation is seen to reproduce the exact result to within a few per cent over the entire \( \Delta \) range. The asymptotic form is approached at large \( \Delta \), as expected.

At higher orders in QCD perturbation theory the subprocess cross section receives large logarithmic corrections from soft gluon emission in the rapidity interval between the two leading jets. For fixed coupling \( \alpha_s \) we have
\[
\hat{\sigma} = \hat{\sigma}_0 \left[ 1 + \sum_{n\geq1} a_n (\alpha_s \Delta)^n + \ldots \right], \quad (9)
\]
with \( \hat{\sigma}_0 \) as defined above. The BFKL formalism to be discussed below resums the logarithms and in the leading-logarithm, fixed-coupling ‘naive’ approximation gives a subprocess cross section which is not constant but grows asymptotically with \( \Delta \): \( \hat{\sigma} \sim \exp(\lambda \Delta) \) with \( \lambda = 4C_A \ln 2\alpha_s/\pi \approx 0.5 \).

At fixed \( \sqrt{s} \) and minimum transverse momentum \( p_T \), both \( \hat{\sigma} \) and \( \mathcal{F} \) depend on \( \Delta \). In fact because of the shape of the parton distributions the latter quantity decreases rapidly with increasing \( \Delta \) and vanishes at the kinematic limit \( \Delta = 2 \cosh^{-1}(\sqrt{2s}/2p_T) \), as in Fig. 1. It is therefore difficult to observe the relatively slow rise with \( \Delta \) of \( \hat{\sigma} \) (see for example Fig. 8 of Ref. \cite{4}). The original idea of Ref. \cite{4} was to increase the collider energy \( \sqrt{s} \) as \( \Delta \) increases such that \( X \) and therefore \( \mathcal{F} \) remains fixed. Any observed rise in the cross section could then only arise from higher-order contributions to \( \hat{\sigma} \).

If two collider energies are available (\( \sqrt{s_1} \) and \( \sqrt{s_2} \) say) one can make use of this idea by comparing the dijet cross sections at two rapidity separations (\( \Delta_1 \) and \( \Delta_2 \) for which the asymptotic leading-order cross sections are equal. Specifically, if for a given \( \Delta_1 \) we define \( \Delta_2 \) such that
\[
\frac{\cosh(\frac{1}{2}\Delta_1)}{\cosh(\frac{1}{2}\Delta_2)} = \frac{\sqrt{s_1}}{\sqrt{s_2}} \quad (10)
\]
then the cross sections defined by Eq. (7) will be equal, i.e. \( d\sigma(\sqrt{s_1}, \Delta_1) = d\sigma(\sqrt{s_2}, \Delta_2) \). Using for \( \sqrt{s_1} \) and \( \sqrt{s_2} \), respectively, the two Tevatron collider energies 630 and 1800 GeV, we have the following values for \( \Delta_1 \) and \( \Delta_2 \):
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$\Delta_1(\sqrt{s_1} = 630 \text{ GeV})$ & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 6.8 \\
\hline
$\Delta_2(\sqrt{s_2} = 1800 \text{ GeV})$ & 3.42 & 3.68 & 4.33 & 5.19 & 6.13 & 7.11 & 8.10 & 8.90 \\
\hline
\end{tabular}

Note that for large $\Delta_1$ Eq. (11) reduces to $\Delta_2 = \Delta_1 + \ln(s_2/s_1) = \Delta_1 + 2.1$, as can be seen in the table.

The equality of the asymptotic cross sections at lowest order for the two $\Delta$’s leads us to define a cross section ratio

$$R_{12} = \frac{d\sigma(\sqrt{s_1}, \Delta_1)}{d\sigma(\sqrt{s_2}, \Delta_2)} \quad (11)$$

as a function of $\Delta_1$, with $\Delta_2$ given by Eq. (10). This ratio has the advantage of being directly accessible experimentally, since it depends on the rapidity difference between the two ‘outside’ jets, a quantity more easily measured than for example the parton momentum fractions themselves. By construction, $R_{12} = 1$ in asymptotic LO QCD, but notice that the ratio is not equal to unity for the leading-order cross section at subasymptotic rapidity separations calculated using exact matrix elements, or using the effective subprocess approximation. This is because the prefactor multiplying the integral in for example Eq. (5) only becomes independent of $\Delta$ at large $\Delta$. This is illustrated in Fig. 2, which shows $R_{12}$ as a function of $\Delta_1$ for $\sqrt{s_1} = 630 \text{ GeV}$ and $\sqrt{s_2} = 1800 \text{ GeV}$, using the parameters of Fig. 1. The effective subprocess approximation curve is very close to the exact curve and is not shown. The ratio approaches unity at large $\Delta_1$, as expected. Not surprisingly, the ‘exact matrix element’ ratio is insensitive to the choice of parton distributions. The dashed line shows the ratio obtained using GRV94LO distributions \[13\]. The curves are almost identical. Similarly, the scale choice dependence is also very weak. For example, using the more natural ‘running’ choice $\mu = p_T$ instead of $\mu = P_T$ in the parton distributions and in $\alpha_s$ has a negligible effect on the ratios.

The BFKL formalism resums the leading-logarithm (1) higher-order corrections which arise from multiple emission of real and virtual gluons in the rapidity interval between the two leading jets. The theoretical details of how this is achieved can be found elsewhere (see for example Ref. \[15\]) and will not be repeated here.

In fact the naive BFKL prediction, in which for example subasymptotic effects from overall energy momentum conservation are ignored, has an analytic representation which leads to a simple expression for $R_{12}$ in the large $\Delta_1$ limit. If the coupling $\alpha_s$ is assumed to be constant, all kinematic constraints are ignored, and only the leading logarithms are resummed, then the result is

$$\hat{\sigma} = \hat{\sigma}_0 \, C_0 \left( \frac{\alpha_s C_A}{\pi} \Delta \right),$$

with

$$C_0(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{dz}{z^2 + \frac{1}{4}} e^{2t\chi_0(z)},$$

$$\chi_0(z) = \text{Re} \left[ \psi(1) - \psi(1/2 + iz) \right].$$

\[1\] In fact, sub-leading logarithms ($\alpha_s^2\Delta^{n-1}$ etc.) can also be resummed in principle \[14\]. In practice, however, only the leading contributions are known at present in a form which is phenomenologically useful.
The asymptotic (large $t$, equivalently large $\Delta$) behavior is

\[ C_0(t) \sim \frac{1}{\sqrt{2\pi 7\zeta(3)t}} e^{4\log^2 t}. \]  

(14)

It follows immediately that the naive BFKL prediction for the ratio $R_{12}$ at large rapidity separations is

\[ R_{12} = \frac{C_0 \left( \frac{\alpha_s C_A}{\pi} \Delta_1 \right)}{C_0 \left( \frac{\alpha_s C_A}{\pi} \Delta_2 \right)}. \]  

(15)

The prediction is shown in Fig. 2. Note that now $R_{12}$ is well below the QCD LO curves, and in fact at first sight this appears to be a primary test of the presence of higher-order ‘BFKL-like’ corrections to the dijet cross section. The formalism is of course only supposed to be valid for large $\Delta_1$ (which implies large $\Delta_2$ also) and so we only show the predictions for $\Delta_1 > 2$, which is roughly where the leading-order prediction begins to approach its asymptotic limit $R_{12} = 1$. The asymptotic large $\Delta_1$ behavior of the ratio is readily obtained from Eqs. (14) and (15):

\[ R_{12} \rightarrow \left( \frac{s_1}{s_2} \right)^{\lambda}, \quad \lambda = \frac{\alpha_s C_A}{\pi} 4 \log 2. \]  

(16)

One interesting aspect of this result is that one is tempted to use the experimentally measurable quantity $K = \log(R_{12})/\log(s_1/s_2)$ to determine the effective BFKL exponent $\lambda$, i.e.

\[ K|_{\text{LO asymp.}} = 0, \]

\[ K|_{\text{naive BFKL}} \rightarrow \frac{\alpha_s C_A}{\pi} 4 \log 2 = 0.45, \]  

(17)

where the numerical value is obtained from using $\alpha_s$ evaluated at $\mu = p_T = 20$ GeV, as in Fig. 2. However it is clear from the figure that the asymptotic behavior is attained only very slowly. For this choice of parameters we expect $R_{12} \rightarrow 0.39$ as $\Delta_1 \rightarrow \infty$, but we see that the ratio has only decreased to $R_{12} = 0.45$ at the kinematic limit $\Delta_1 = 6.9$. Furthermore, as we shall see below, an improved BFKL calculation does not lead to such a simple prediction.

So far we have considered the predictions of naive BFKL, which can be written analytically but at the cost of making several assumptions which do not hold in experimental situations. In particular, the analytic solutions result from integrating over arbitrarily large values of the transverse momenta of emitted gluons, and the analytic phase space integrations at the subprocess level prevents a proper incorporation of parton distributions. In addition, $\alpha_s$ is assumed to be fixed. The BFKL approach can be improved and these assumptions avoided by recasting the BFKL cross section as an event generator \cite{9, 10} using an iterated solution to the BFKL equation. Kinematic constraints and the running of $\alpha_s$ can then be included. The effect is to restrict the growth of $\hat{\sigma}$ with increasing $\Delta$ compared to naive BFKL, since the emission of relatively large transverse momentum gluons, which give a positive contribution to the resummed cross section, is suppressed. Details of the calculation and application to the azimuthal decorrelation can be found in \cite{10}. 

5
Here we repeat the calculation of the cross section and ratios $R_{12}$ using the Monte Carlo of Ref. [10] to get an ‘improved’ BFKL prediction. One feature of the MC calculation that will be relevant to our results is worth mentioning. The MC solution to the BFKL equation is obtained by separating the contributions from real gluon emission into ‘resolved’ and ‘unresolved’ contributions, the idea being that below a certain energy scale $\mu_0$, emitted real gluons are not detectable in practice. The contribution from unresolved real gluons is combined with the virtual gluon contribution to give an overall suppressing form factor. The differential cross section takes the form, for fixed $\alpha_s$,

$$
\frac{d\hat{\sigma}_{gg}}{d^2p_{T1}d^2p_{T2}d\Delta y} = \frac{\alpha_s^2 C_A}{p_{T1}^2 p_{T2}^2} \left[ \frac{\mu_0^2}{p_{T1}^2} \right] \alpha_s C_A \Delta / \pi \mathcal{R}(\vec{p}_{T1}, \vec{p}_{T2}, \Delta y), \quad (18)
$$

and $\mathcal{R}$ can be written as a sum over resolved real gluon emissions:

$$
\mathcal{R}(\vec{p}_{T1}, \vec{p}_{T2}, \Delta) = \sum_{n=0}^{\infty} \mathcal{R}^{(n)}(\vec{p}_{T1}, \vec{p}_{T2}, \Delta), \quad (19)
$$

with $\mathcal{R}^{(0)} = \delta(\vec{p}_{T1} + \vec{p}_{T2})$ and $\mathcal{R}^{(n)}$ for $n > 0$ given in [10]. The point is that even for $n = 0$ the form factor $\left[ \frac{\mu_0^2}{p_{T1}^2} \right] \alpha_s C_A \Delta / \pi$ appears in the cross section and represents a suppression of the probability that no resolvable gluons will be emitted. The inclusion of resolvable gluon contributions ($n \geq 1$) counteracts this suppression and removes the dependence on $\mu_0$.

Fig. 3 shows the dijet cross section as a function of $\Delta$ for the naive BFKL and improved BFKL MC cases at the two collision energies, with $P_T = 20$ GeV. Asymptotic QCD LO is also shown for reference. There are several features worth noting here. First, the naive BFKL cross section (dashed curve) is always largest, because it includes the analytic subprocess cross section given in Eq. (12), which allows emission of any number of gluons with arbitrarily large energies. When exact kinematics for entire events are included in both the subprocess cross section and the parton densities, as in the BFKL MC (solid curve), there is a dramatic suppression of the total cross section. In fact the suppression is so strong that it drives the BFKL MC cross section below that for asymptotic QCD LO. The reason is due to simple kinematics: the QCD LO cross section contains only two jets, but the BFKL MC cross section also includes additional jets, each of which increases the subprocess center-of-mass energy and elicits a corresponding price in parton densities. In the naive BFKL calculation, the contribution to the subprocess energy from additional jets is ignored and their net effect is to combine with the virtual gluons to increase the subprocess cross section. Interestingly, the effect is compounded by the fact that the (naive) BFKL increase of $\hat{\sigma}$ only starts at $\mathcal{O}(\alpha_s^2)$ in perturbation theory, i.e. $C_0 = 1 + \mathcal{O}(\alpha_s^2)$ in Eq. (13). The $\mathcal{O}(\alpha_s^4)$ correction is zero because the real and virtual gluon contributions exactly cancel. However the effect of the pdfs is to suppress only the real contributions, giving an overall negative correction at this order.

\footnote{We show the fixed $\alpha_s$ expression for simplicity; see [10] for the corresponding running $\alpha_s$ expression.}

\footnote{The running of $\alpha_s$, which we include, also contributes to the suppression, but it has a much smaller effect than kinematics.}
The ratio $R_{12}$ as calculated in the improved BFKL MC is shown in Fig. 4, again with the naive BFKL and asymptotic QCD LO predictions. Here the effects of kinematic suppression and the parton distributions are quite dramatic. The BFKL MC curve does not fall between the naive BFKL and QCD LO curves as one might expect. Instead, it is everywhere greater than or equal to the QCD LO curve, and in particular, $R_{12}$ is everywhere greater than 1.

This behavior can be understood as follows. We can represent $R_{12}$ schematically as

$$R_{12} \sim \frac{1 + \alpha_s \Delta_1 C_1 + \ldots}{1 + \alpha_s \Delta_2 C_2 + \ldots},$$

(20)

where the $C_i$ are coefficients modified by the effects of the kinematic suppression. By the arguments given above, we expect $C_i < 0$. In the formal\footnote{We do not of course expect the BFKL formalism to apply for small rapidity separations; here our aim is simply to explain the behavior of the predictions of the BFKL MC calculation in Fig. 4.} limit $\Delta_1 \to 0$, the nonleading terms in the numerator vanish, the denominator is $< 1$ and hence $R_{12} > 1$. On the other hand, in the limit $\Delta_1 \to \Delta_{\text{max}}$, there is no phase space available for any gluon emission, hence $C_i \to 0$ and $R_{12} \to 1$. The observation of a ‘BFKL effect’ would require a region of phase space where $\Delta_i \gg 1$ but $C_i \simeq 1$. Increasing the collider energy at fixed transverse momentum threshold would eventually lead to this situation. In such a region we would expect to see $R_{12} < 1$, as predicted by the naive BFKL calculation shown in Fig. 4. Finally, why does the BFKL MC curve in Fig. 4 rise again at large $\Delta_1$, instead of approaching 1 as simple kinematics would suggest? The origin of this behavior arises in the form factor (see Eq. (18)) which suppresses the probability of no real (i.e. resolved) gluon emission in the rapidity interval between the dijets. As $\Delta$ increases towards its maximum allowed value there is an effective upper (kinematic) limit on the transverse momentum of each emitted gluon. This in turn leads to a suppressing form factor with the ‘artificial’ parameter $\mu_0$ replaced by a physical parameter determined by kinematics and the pdfs. In other words, the all-orders leading-log contribution from virtual gluon emission that is built into the overall form factor is not completely compensated by corresponding real emission because the kinematics do not allow it. The kinematic suppression increases with $\Delta$, hence $\Delta_2 > \Delta_1$ implies $R_{12} > 1$.

The arguments above illustrate the general behavior of the cross section ratio – starting out at a value greater than 1, falling and then increasing again at large $\Delta$ – as predicted by the ‘improved’ BFKL Monte Carlo calculation. The details, however – how far the ratio falls (and in particular whether it falls below unity as predicted in the naive calculation) and for how long – depend on the specifics of the experimental configuration, such as the collider energies and the transverse momentum threshold. In this study we have restricted ourselves to parameter values which are currently accessible at the Tevatron. Clearly it would be desirable to have more than the two Tevatron $p\bar{p}$ energies available for such a measurement. In principle, the LHC at 14 TeV and RHIC in $pp$ mode at 500 GeV would expand the range of $\Delta$. In practice, one is limited by $\Delta_{\text{max}}$ at the lower energy. Furthermore, comparing jet measurements at different machines with different detectors can be challenging, and ideally one would like to construct ratios of measurements made at a single machine in a single detector, so that some systematic uncertainties would cancel. For example, running the LHC at 10 and 14 TeV would make such a measurement possible, although ideally the two energies should be more different. The essential point is that higher collider energies...
appear to be necessary in order to prevent the BFKL effects being swamped by kinematical constraints.

In summary, we have shown that the effects of the increase in the dijet subprocess cross section predicted in the naive BFKL approach can in principle be detected by measuring the ratio of cross sections at energies and rapidities chosen such that the asymptotic QCD LO ratio is equal to 1. Forming the ratio minimizes the effects of the parton distribution functions. However an improved BFKL calculation which includes the subasymptotic effects of the conservation of energy and momentum gives a result for the ratio that is qualitatively different from that of naive BFKL. In particular, for dijet production at the two Tevatron energies, the improved BFKL calculation gives either agreement with lowest-order QCD or deviations opposite to those predicted by naive BFKL. If our arguments about the kinematic suppression of higher-order real gluon emission are correct, then it would appear that a fixed-order perturbation theory approach would be a more appropriate calculational framework for the cross section ratio at present collider energies than all-orders resummation. Therefore, an important next step is to perform a calculation of $R_{12}$ in exact next-to-leading-order QCD, to determine whether it provides a satisfactory description of the data.

Finally we emphasise that this study has concentrated on the cross section ratio only. It may well be that other quantities are better suited to revealing underlying BFKL effects. Indeed, we note that the azimuthal decorrelation of the dijet pair $\pi_1 \pi_2$, as calculated in the naive BFKL approach, is already non-zero at $\mathcal{O}(\alpha_s)$ \[^4]\, and that its effects are not completely swamped by kinematics in an improved BFKL calculation \[^10]\.

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Figure 1: The dependence of the leading-order \((2 \to 2)\) dijet cross sections on the dijet rapidity separation. The partons are the CTEQ4L set [12]. The three curves at each collider energy use: (i) exact matrix elements (solid lines), (ii) the effective subprocess approximation (dashed lines), and (iii) the asymptotic \((\Delta \gg 1)\) form of the latter (dotted lines).
Figure 2: The ratio $R_{12}$ of the dijet cross sections at the two collider energies $\sqrt{s_1} = 630 \text{ GeV}$ and $\sqrt{s_2} = 1800 \text{ GeV}$, as defined in the text. The curves are: (i) the exact leading-order ($2 \rightarrow 2$) predictions using CTEQ4L (solid curve) and GRV94LO partons (dashed curve), both with $\mu = P_T = 20 \text{ GeV}$, and (ii) the ‘naive BFKL’ prediction (dot-dashed curve). Note that the asymptotic leading-order prediction is $R_{12} = 1$. 

$y_1 = -y_2 = \Delta_i/2$  
$p_T > 20 \text{ GeV/c}$
Figure 3: The dependence of the BFKL and asymptotic QCD leading-order dijet cross sections on the dijet rapidity separation. The pdfs are the CTEQ4L set [12]. The three curves at each collider energy use: (i) ‘improved’ BFKL MC (solid lines), (ii) ‘naive’ BFKL (dashed lines), and (iii) the asymptotic ($\Delta \gg 1$) form of QCD leading order (dotted lines).
Figure 4: The ratio $R_{12}$ of the dijet cross sections at the two collider energies $\sqrt{s_1} = 630$ GeV and $\sqrt{s_2} = 1800$ GeV, as defined in the text. The curves are: (i) the ‘improved’ BFKL MC predictions using CTEQ4L [12] pdfs (solid curve), with $\mu = P_T = 20$ GeV, (ii) the ‘naive’ BFKL prediction (dashed curve), and (iii) the asymptotic leading-order prediction (dotted curve) $R_{12} = 1$. 

$y_1 = -y_2 = \Delta_i/2$

$p_T > 20$ GeV/c