A Description of Rotating Multicharged Black Holes
in terms of Branes and Antibranes

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ABSTRACT

We describe rotating multicharge black holes as stacks of intersecting branes and antibranes together with massless gases on them. Assuming the energies of the gases to be equal, we find that their angular momentum parameters, corresponding to black hole rotations, are also equal. The entropy $S$ of this model is given by $S = XS_{sg}$ where $S_{sg}$ is the supergravity entropy. One can obtain $X = 1$ under an assumption which violates conservation of energy. We show that $X = 1$ can also be obtained if one assumes that there is only one single gas, which is some sort of superposition of the gases mentioned above, and that the brane tensions are reduced by a factor of four. In this interpretation, energy is conserved and the unusual assumption that energies, not temperatures, of the gases are equal becomes superfluous.
1 Introduction

Enormous progress has been made in understanding the microscopic origin of entropy and Hawking radiation of extremal and near extremal black holes using various brane configurations in string or M theory and low energy excitations on them [1]. Despite a variety of attempts [2], a similar level of understanding of non extremal black holes, in particular Schwarzschild black holes, is lacking.

A few years ago, Danielsson, Güijosa and Kruczenski (DGK) proposed a field theoretic model for non extremal black holes [3]. Their description is valid far from extremality, thus also for Schwarzschild black holes. They considered stacks of D3, M2, or M5 branes and antibranes together with massless gases on them. This field theoretic model has been generalised to other single charge black holes with or without rotation [4, 5, 6], described as stacks of Dp branes and antibranes together with massless gases on them; and, also to multicharge black holes with no rotation [7, 8, 9], described as stacks of intersecting branes and antibranes [10] together with $2^K$ types of massless gases on them where $K$ is the number of charges.

The gases are characterised by their energies $E_1, E_2, E_3, \ldots, E_{2^K}$. Their entropies are obtained from the near extremal limit of the corresponding supergravity solutions. Assuming the gases to have equal energies, instead of equal temperatures which is usually the case, and following the analysis of [3], it has been found that non rotating black hole entropy $S$ in the field theoretic model is identical, up to a constant numerical factor, to the entropy $S_{sg}$ in the supergravity description. That is, $S = X S_{sg}$ where the ‘deficit factor’ $X$ is a constant that depends on $K$ and the number of transverse dimensions.

Furthermore, as noted in [3], one can obtain $X = 1$ for non rotating black holes under a further assumption about the energies of the gases which, however, violates conservation of energy. This violation, as noted in [3], is perhaps due to the binding energy of branes and antibranes not being taken into account. Alternately, one can obtain $X = 1$ with no violation of conservation of energy if, as shown in [7, 9], one assumes that the available energy is all taken by one single gas, which is some sort of superposition of $2^K$ possible types of gases, and further assumes that its entropy is an average of the $2^K$ gas entropies and that all the brane tensions are reduced by a factor of four. The unusual assumption that energies, not temperatures, of the gases are equal then becomes superfluous.
Consider black holes with rotation. In the case of single charge rotating black holes, the gases are also characterised by angular momentum parameters $l_j$ where the subscript $j$ denotes possible rotations in the transverse space. Assuming the energy and the parameters $l_j$ to be the same for the gases on branes and antibranes, and with a further assumption about the energies of the gases which violates conservation of energy, it is shown in [4, 5] that the field theoretic and supergravity entropies agree, namely $S = S_{sg}$. However, without the last assumption above, one obtains $S = XS_{sg}$ where the deficit factor $X$ is not constant but depends on black hole parameters; it becomes constant when rotation is absent.

In this paper we study rotating multicharge black holes, described as stacks of intersecting branes and antibranes together with $2^K$ types of gases. The gases are also characterised by angular momentum parameters which correspond to black hole rotations and are, in general, different for each gas.

We assume that the energies of the gases are equal and that the angular momentum parameters are different. Following the methods of [3, 4, 5], we then find that the angular momentum parameters are also equal for all the $2^K$ types of gases. We also find that the field theoretic and supergravity entropies agree, namely $S = S_{sg}$, under a further assumption about the energies of the gases which violate conservation of energy. Without this assumption, one obtains $S = XS_{sg}$ where the deficit factor $X$ is not constant but depends on black hole parameters; it becomes constant when rotation is absent. These results are similar to those for single charge rotating black holes [4, 5], but are now shown to be valid for multicharge black holes also.

The result that the deficit factor $X$ is not constant when rotation is present makes the field theoretic model less appealing. The assumption under which one obtains $X = 1$ is not satisfactory since it violates conservation of energy. This violation is perhaps due to the neglect of binding energies [5], but the details of the binding energies are not sufficiently well known to verify this idea. Note also the necessity of the unusual assumption that energies, not temperatures, of the gases are equal in obtaining this result.

In contrast, the deficit factor $X = 1$ was obtained in [7, 9] for non rotating multicharge black holes without the assumptions mentioned above. In this paper, we show that $X = 1$ can be obtained similarly for rotating multicharge black holes also.

This paper is arranged as follows. In section 2 we give relevant results for rotating multicharge black holes from supergravity description. In sections 3.1 and 3.2 we describe the field theoretic model for rotating single and
multicharge black holes. In section 4 we show that \( X = 1 \) can be obtained without the assumptions mentioned earlier. In section 5 we conclude with a brief summary and a few issues for further study.

2 Supergravity description of multicharge rotating black holes

In string theory, spinning p-branes describe rotating black holes. Spinning intersecting p-branes describe multicharge rotating black holes. Supergravity solutions for various intersecting branes have been studied [1]. An algorithm for finding explicit solutions for multicharge black holes is given in [11].

Supergravity expressions for mass, angular momenta, and entropy of single charge rotating black holes may be found in [12]. For two charge cases, it may be found in [13]. In general, the supergravity expressions for mass \( M_{sg} \), angular momenta \( J_{j,sg} \), and entropy \( S_{sg} \), of \( K \)-charge rotating black holes can be written as

\[
M_{sg} = b \left( 2\lambda\mu + \sum_{i=1}^{K} \sqrt{Q_{i,sg}^2 + \mu^2} \right)
\]

\[
J_{j,sg} = \frac{2b}{n} l_{j,sg} (2\mu)^{2-K} \prod_{i=1}^{K} \left( \sqrt{\mu^2 + Q_{i,sg}^2} + \mu \right)^{\frac{1}{2}}
\]

\[
S_{sg} = \frac{4\pi b}{n} r_{H,sg} (2\mu)^{2-K} \prod_{i=1}^{K} \left( \sqrt{\mu^2 + Q_{i,sg}^2} + \mu \right)^{\frac{1}{2}}
\]

where \( r_{H,sg} \) is the radius of horizon which is given by the equation

\[
r_{H,sg}^{n} \prod_{j} \left( 1 + \frac{l_{j,sg}^2}{r_{H,sg}^2} \right) = 2\mu.
\]

In the above expressions, \( \mu \) is a non extremality parameter, \( Q_{i,sg} \) are the charges, \( l_{j,sg} \) are the rotation parameters, \( \lambda = \frac{n+1}{n} - \frac{K}{2} \), and \( b = \frac{\omega_{n+1} V_p}{(2\pi)^{n} g_{5}^2} \) where \( \omega_{n+1} \) is the area of an unit \((n + 1)\)-dimensional sphere, \( V_p \) is the volume of the \( p \)-dimensional compact space, and \( n = 7 - p \). Here and in the following, the subscripts \( i = 1, 2, \cdots, K \) always refer to the charges and the subscripts \( j = 1, 2, \cdots, \left[ \frac{n+2}{2} \right] \) always refer to the rotations in the \((n + 3)\)-dimensional transverse space.
Note that \( \frac{2 \pi J_{j sg}}{S_{sg}} = \frac{I_{j sg}}{r_{H sg}} \). Define \( \rho_{sg} \) by the equation

\[
\rho_{sg}^n = \frac{r_{H sg}}{2 \mu} = \prod_j \left( 1 + \frac{l_{j sg}^2}{r_{H sg}^2} \right)^{-1}.
\]

Then, \( S_{sg} \) can be written as

\[
S_{sg} = \rho_{sg} S_{0 sg}, \quad S_{0 sg} = \frac{4 \pi b}{n} (2 \mu)^{\lambda} \prod_{i=1}^{K} \left( \sqrt{\mu^2 + Q_{i sg}^2} + \mu \right)^{\frac{1}{2}}
\]

where \( S_{0 sg} \) is the entropy in the non rotating case, namely when \( l_{j sg} = 0 \).

The charges \( Q_{i sg} \) can also be parametrised in terms of \( \phi_i \) as follows:

\[
Q_{i sg} = \mu \sinh 2\phi_i, \quad i = 1, 2, \ldots, K.
\]

The mass, angular momenta, and entropy are then given by

\[
M_{sg} = b \mu \left( 2 \lambda + \sum_{i=1}^{K} \cosh 2\phi_i \right)
\]

\[
J_{j sg} = \frac{4b \mu}{n} l_{j sg} \prod_{i=1}^{K} \cosh \phi_i
\]

\[
S_{sg} = \frac{8 \pi b \mu}{n} r_{H sg} \prod_{i=1}^{K} \cosh \phi_i.
\]

In the no-rotation and extremal limit, where \( l_{j sg} = 0, \mu \to 0 \) and \( \phi_i \to \infty \) such that \( Q_{i sg} \) remain finite, the mass becomes \( M_{sg ext} = \sum_i bQ_{i sg} \). In string theory, non rotating extremal black holes are identical to stacks of intersecting p–branes. Let such a stack consist of \( N_i \) number of \( i^{th} \) type of branes with tension \( \tau_i \) and volume \( V_i \). The total brane mass is then \( M = \sum_i N_i \tau_i V_i \). Identifying these two masses then gives the relation between the charges \( Q_{i sg} \) and the number \( N_i \) of \( i^{th} \) type of branes: \( N_i = \frac{bQ_{i sg}}{\tau_i V_i} \).

A little calculation enables one to get near extremal values of mass, angular momenta, and entropy. To the leading order in \( \mu \), they are given by

\[
M_{sg} = \sum_{i=1}^{K} bQ_{i sg} + E
\]

\[
J_{j sg} = \frac{2b}{n} l_{j sg} \left( \frac{E}{\lambda b} \right)^{\frac{2-K}{2}} \prod_{i} \sqrt{Q_{i sg}}
\]

\[
S_{sg} = \frac{4 \pi b}{n} r_{H sg} \left( \frac{E}{\lambda b} \right)^{\frac{2-K}{2}} \prod_{i} \sqrt{Q_{i sg}}
\]
where $E = 2b\lambda\mu$ is the energy above extremality, and $r_{Hsg}$ is given by

$$r_{Hsg}^n \prod \left(1 + \frac{l_j^2}{r_{Hsg}^2}\right) = \frac{E}{\lambda b}.$$  \hspace{1cm} (12)

## 3 Field theoretic description of rotating black holes

### 3.1 Single charge rotating black holes

Description of single charge rotating black holes in the field theoretic model was given in [4, 5]. Following them we consider, as in non rotating case [3], stacks of branes and antibranes together with a massless gas on each stack. So the model consists of (i) $N$ spinning branes, (ii) $\bar{N}$ spinning antibranes, and (iii) two gases of massless excitations on branes and antibranes. The masses of branes and antibranes are $\tau V_p N$ and $\tau V_p \bar{N}$, and their charges $q$ and $\bar{q}$ are $\frac{\tau V_p N}{b}$ and $\frac{\tau V_p \bar{N}}{b}$, $\tau$ being brane tension and $V_p$ being brane volume.

Upto now we considered a system very similar to one considered in [3]. Together with the above set up, this time we also consider two sets of extra parameters $l_j$ and $\bar{l}_j$ which are the charges corresponding to the symmetries of the rotational space. These ‘charges’, which are analogous to R-symmetry charges in the case of D3 branes, characterise rotation of black hole. We will write these charges as angular momentum parameters.

This system is very similar to one given in [4, 5]. Unlike in these works, however, we assume here that different gases have different angular momentum parameters, i.e. that $l_j$ and $\bar{l}_j$ are different. Here, in the field theoretic model, near extremal non rotating black holes are described as stacks of branes and antibranes. We assume that branes and antibranes do not interact with each other. The total mass, angular momenta, and entropy will then be additive. Also, these quantities are assumed to be given, as in [3], by the supergravity expressions in the near extremal regime, namely by equations (9) – (12).

Moreover, the energies of the gas on the branes and antibranes are assumed to be equal, namely $E = \bar{E}$. Note that, normally, subsystems of any given system all have the same temperature. Hence, normally, temperature of branes and antibranes should have been set equal. But here, instead of temperature, one has to assume that energies are equal; otherwise field the-
oretic and the supergravity entropies do not match. For now, we make this assumption and continue our analysis, although no physical mechanism is known which enforces equality of energies, instead of temperatures, among the subsystems. Later in the paper, we will see how this assumption becomes superfluous.

With $E = \bar{E}$, the total mass $M$, charge $Q$, angular momenta $J_j$, and entropy $S$ in the field theoretic model here will be

$$M = b(q + \bar{q}) + 2E \quad (13)$$

$$Q = q - \bar{q} \quad (14)$$

$$J_j = \frac{2b}{n\sqrt{b\lambda}} (l_j \sqrt{q} + \bar{l}_j \sqrt{\bar{q}}) \sqrt{E} \quad (15)$$

$$S = \frac{4\pi b}{n\sqrt{b\lambda}} (r_H \sqrt{q} + \bar{r}_H \sqrt{\bar{q}}) \sqrt{E} \quad (16)$$

where $\lambda = \frac{n+1}{n} - \frac{K}{2} = \frac{n+2}{2n}$ for the single charge case $K = 1$. In the above expressions, $r_H$ and $\bar{r}_H$ are functions of $(E, \{l_j\})$ and $(E, \{\bar{l}_j\})$ respectively and are given by

$$r_H^n \prod_j \left(1 + \frac{l^2_j}{r_H^2}\right) = \bar{r}_H^n \prod_j \left(1 + \frac{\bar{l}^2_j}{r_H^2}\right) = \frac{E}{\lambda b} \quad . (17)$$

This is our field theoretic model for single charge rotating black holes, with the entropy $S$ given by equation (16). The parameters $q, \bar{q}, l_j, \bar{l}_j,$ and $E$ are arbitrary subject only to the constraints that the quantities $M, Q$ and $J_j$ are fixed.

This system is dynamical because more and more brane antibrane pairs can be created taking energy from massless gases, or annihilated giving energy to them; moreover $l_j$ and $\bar{l}_j$ can flow from one gas to another. The system reaches equilibrium in a state where entropy is maximum for given $M, Q,$ and $J_j$. So, to obtain the equilibrium state, we maximise $S$ subject to the constraints that $M, Q$ and $J_j$ given in equations (13) – (15) are held fixed.

These constraints are incorporated by Lagrange multiplier method. Hence we maximise the function $\mathcal{F}(q, \bar{q}, l_j, \bar{l}_j, E)$ defined by

$$\mathcal{F} = S + A_j \left\{ \frac{2b}{n\sqrt{b\lambda}} (l_j \sqrt{q} + \bar{l}_j \sqrt{\bar{q}}) \sqrt{E} - J_j \right\}$$

$$+ \left\{ B(q - \bar{q} - Q) + C (b(q + \bar{q}) + 2E - M) \right\}$$

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where \(A_j, B\) and \(C\) are Lagrange multipliers. Varying \(\mathcal{F}\) with respect to \(q, \bar{q}, l_j, \bar{l}_j,\) and \(E,\) we have that \(d\mathcal{F} = 0\) at the maximum. Hence,

\[
0 = 2 \left\{ \frac{\partial r_H}{\partial l_j} + \frac{A_j}{2\pi} \right\} \sqrt{qE} \, dl_j + 2 \left\{ \frac{\partial \bar{r}_H}{\partial \bar{l}_j} + \frac{A_j}{2\pi} \right\} \sqrt{\bar{q}E} \, d\bar{l}_j
\]

\[
+ \left( \frac{\partial S}{\partial q} + \frac{b}{n\sqrt{b\lambda}} A_j l_j \sqrt{\frac{E}{q}} + B + bC \right) dq
\]

\[
+ \left( \frac{\partial S}{\partial \bar{q}} + \frac{b}{n\sqrt{b\lambda}} A_j \bar{l}_j \sqrt{\frac{E}{\bar{q}}} - B + bC \right) d\bar{q}
\]

\[
+ \left\{ \frac{\partial S}{\partial E} + \frac{A_j b}{n\sqrt{b\lambda}} (l_j\sqrt{q} + \bar{l}_j\sqrt{\bar{q}}) \frac{1}{\sqrt{E}} + 2C \right\} dE
\]

which implies that the coefficients of \(dl_j, d\bar{l}_j, dq, d\bar{q}\) and \(dE\) must vanish. Equating the coefficients of \(dl_j\) and \(d\bar{l}_j\) to zero, we get

\[
\frac{\partial}{\partial l_j} r_H = \frac{\partial}{\partial \bar{l}_j} \bar{r}_H. \quad (18)
\]

Using equations (17) for \(r_H\) and \(\bar{r}_H,\) one gets

\[
\frac{\partial r_H}{\partial l_k} = -\frac{2l_k r_H}{r_H^2 + l_k^2} \left[ n - \sum_j \frac{l_j^2}{r_H^2 + l_j^2} \right]^{-1} \quad (19)
\]

\[
\frac{\partial \bar{r}_H}{\partial \bar{l}_k} = -\frac{2\bar{l}_k \bar{r}_H}{\bar{r}_H^2 + \bar{l}_k^2} \left[ n - \sum_j \frac{l_j^2}{\bar{r}_H^2 + l_j^2} \right]^{-1}. \quad (20)
\]

In the above equations, \(r_H\) and \(\bar{r}_H\) are functions of \(l_j\) and \(\bar{l}_j\) respectively. Substituting them in equation (18) then leads to a complicated equation. Equation (18), however, always admits one solution given by \(l_j = \bar{l}_j\) and, hence, \(r_H = \bar{r}_H.\) In the following, we take this to be the solution, which is always present. Also, as will be seen below, this solution leads to the correct form of entropy.

With \(l_j = \bar{l}_j\) and \(r_H = \bar{r}_H,\) the angular momenta \(J_j\) and entropy \(S\) now become

\[
J_j = \frac{2b}{n\sqrt{b\lambda}} l_j (\sqrt{q} + \sqrt{\bar{q}}) \sqrt{E} \quad (21)
\]

\[
S = \frac{4\pi b}{n\sqrt{b\lambda}} r_H (\sqrt{q} + \sqrt{\bar{q}}) \sqrt{E}. \quad (22)
\]
Note that we now get \( \frac{l_j}{r_H} = \frac{2\pi J_j}{S} \) since \( l_j = \tilde{l}_j \) and \( r_H = \tilde{r}_H \). Also, just as in the supergravity case, define \( \rho \) by the equation

\[
\rho^n = \frac{b\lambda r_H^n}{E} = \prod_j \left(1 + \frac{l_j^2}{r_H^2}\right)^{-1}
\]

so that \( S \) can be written as

\[
S = \rho S_0, \quad S_0 = \frac{4\pi b}{n} (\lambda b)^{-\lambda} (\sqrt{q} + \sqrt{\bar{q}}) E^\lambda
\]

where \( S_0 \) is the entropy in the non rotating case, namely when \( l_j = 0 \).

One should next solve the equations obtained by setting to zero the coefficients of \( dq, d\bar{q}, \) and \( dE \) in the expression \( dF = 0 \). But it is much easier, and equivalent, to use the results \( l_j = \tilde{l}_j \) and \( r_H = \tilde{r}_H \) in the expression for the entropy \( S \) and maximise the resultant expression with respect to the remaining variables \( q, \bar{q}, \) and \( E \), subject to the two remaining constraints \( q - \bar{q} = Q \) and \( b(q + \bar{q}) + 2E = M \). Thus, using the Lagrange multipliers \( B \) and \( C \), we start from the equation

\[
d \{ S + B(q - \bar{q} - Q) + C (b(q + \bar{q}) + 2E - M) \} = 0
\]

which implies that

\[
\left( \frac{\partial S}{\partial q} + B + bC \right) dq + \left( \frac{\partial S}{\partial \bar{q}} - B + bC \right) d\bar{q} + \left( \frac{\partial S}{\partial E} + 2C \right) dE = 0 .
\]

The coefficients of \( dq, d\bar{q} \) and \( dE \) must vanish. So we get three equations. Eliminating \( B \) and \( C \) from them we get

\[
\frac{\partial S}{\partial q} + \frac{\partial S}{\partial \bar{q}} - b \frac{\partial S}{\partial E} = 0 . \tag{24}
\]

Since \( \frac{l_j}{r_H} = \frac{2\pi J_j}{S} \) and \( S = \rho S_0 \), equation \( (23) \) for \( \rho \) can be written, following [5], as an implicit function of \( J_j \) and \( S_0 \) as follows:

\[
\rho^n = \prod_j \left(1 + \left(\frac{2\pi J_j}{\rho S_0}\right)^{2}\right)^{-1} . \tag{25}
\]

Then, \( \partial S \) is given by \( \frac{\partial S}{\partial q} = \left(S_0 \frac{\partial \rho}{\partial S_0} + \rho \right) \frac{\partial S_0}{\partial q} \). The partial derivatives \( \frac{\partial S}{\partial q} \) and \( \frac{\partial S}{\partial E} \) can be similarly expressed. Putting them back in equation \( (24) \), we find

\[
\left(S_0 \frac{\partial \rho}{\partial S_0} + \rho \right) \left\{ \frac{E^\lambda}{2\sqrt{q}} + \frac{E^\lambda}{2\sqrt{\bar{q}}} - \lambda b(\sqrt{q} + \sqrt{\bar{q}}) E^{\lambda-1} \right\} = 0 . \tag{26}
\]
Using equation (25) for \( \rho(S_0) \) one can show that
\[
S_0 \frac{\partial \rho}{\partial S_0} + \rho \neq 0.
\]
So from equation (26) one then finds \( E = 2 \lambda b \sqrt{q \bar{q}} \).

Let \( q = \frac{m}{2} e^{2\theta} \) and \( \bar{q} = \frac{m}{2} e^{-2\theta} \). Then \( E = \lambda bm \) and \( M, Q, J, \) and \( S \) of the system are given, using equations (13) – (16), by
\[
\begin{align*}
M &= bm \left(2 \lambda + \cosh 2\theta\right) \\
Q &= m \sinh 2\theta \\
J_j &= \frac{4bm}{n} \frac{l_j}{\sqrt{2}} \cosh \theta \\
S &= \frac{8\pi bm}{n} \frac{r_H}{\sqrt{2}} \cosh \theta
\end{align*}
\]
where \( r_H \) is given implicitly by the equation
\[
r_H^n \prod_j \left(1 + \frac{l_j^2}{r_H^2}\right) = m.
\]

Now we compare the above expressions with the supergravity ones. Setting \( M = M_{sg}, Q = Q_{sg}, \) and \( J_j = J_{j,sg} \) gives \( m = \mu, \theta = \phi, \) and \( l_j = \sqrt{2} l_{j,sg} \). We then have that
\[
S(M, Q, J_j) = X S_{sg}(M, Q, J_j), \quad X = \frac{1}{\sqrt{2}} \frac{r_H}{r_{H,sg}}.
\]

Thus, the field theoretic entropy \( S \) differs from the supergravity entropy \( S_{sg} \) by a ‘deficit’ factor \( X \) given above.

An implicit equation for \( X \) can be obtained easily. Using equation (4) with \( \mu = m, l_{j,sg}^2 = \frac{l_j^2}{2} \), and \( r_{H,sg}^2 = \frac{r_H^2}{2X^2} \), and equation \( r_H^n \prod_j \left(1 + \frac{l_j^2}{r_H^2}\right) = m \) for \( r_H \), it follows that \( X \) is given implicitly by the equation
\[
X^n = 2^{-\frac{n+2}{2n}} \prod_j \frac{r_H^2 + X^2l_j^2}{r_H^2 + l_j^2}
\]
and, hence, that the factor \( X \) depends non trivially on \( l_j \) and \( r_H \), thus on black hole parameters \( l_j \) and \( m \). However, in the non rotating case, \( l_j = 0 \) and the deficit factor \( X \) reduces to just a numerical constant, namely \( X = 2^{-\lambda} \) where \( \lambda = \frac{n+1}{2n} \), see [3, 5, 6].

Equations (27) – (30) are obtained from solving the variational equation \( dF = 0 \). However, to show that the entropy \( S \) is a maximum, one also has to show that second order variations of \( F \) satisfy appropriate conditions. This can be shown for simple cases but the calculations become tedious for general cases. Hence, following [3 – 9], we assume in this paper that the solutions obtained from solving \( dF = 0 \) corresponds to the maximum of entropy.
3.2 Multicharge rotating black holes

Multicharge black holes are described by stacks of intersecting branes and antibranes. For $K$ charge black holes, we have $K$ types of brane antibrane pairs. There will be $2^K$ types of gases, which can be understood as follows. The $K$ types of branes, and antibranes, are numbered as $1, 2, \ldots, K$, and $\bar{1}, \bar{2}, \ldots, \bar{K}$. $K$ charge black holes can be thought of as obtained by taking, for example, the stack $1, \bar{2}, 3, \ldots, K$ and its anti stack $\bar{1}, 2, \bar{3}, \ldots, \bar{K}$, with a pair of gases on these two stacks. Clearly, there are $2^K - 1$ such ways of constructing $K$ charge black holes and, consequently, the system is to be thought of as containing a total of $2^K$ types of gases on $2^K$ types of stacks.

We assume, as in [3] – [9], that each type of gas has same energy $E$. This assumption is analogous to assuming $E = \bar{E}$ in the single charge case, and is necessary to obtain the entropy which matches that in supergravity case. Then, the total mass $M$ and the charges $Q_i$, $i = 1, 2, \ldots, K$, are given by

$$M = b \sum_{i=1}^{K} (q_i + \bar{q}_i) + 2^K E$$

$$Q_i = q_i - \bar{q}_i .$$

Each type of gas has different set of angular momentum parameters, i.e. $I^{th}$ type of gas has parameters $l^I_j$ where $I = 1, 2, 3, \ldots, 2^K$. For example, in single charge case, $K = 1$ and there are two types of gases, with the parameters denoted as $l^1_j = l_j$ and $l^2_j = \bar{l}_j$. Since angular momenta and entropy of gases are assumed to be additive, total angular momenta $J_j$ and total entropy $S$ will be

$$J_j = \frac{2b}{n} \left( \frac{E}{\lambda b} \right)^{1 - \frac{K}{2}} \sum_{I=1}^{2^K} l^I_j \prod_{i=1}^{K} \left[ \sqrt{\tilde{q}_i} \right]^I$$

$$S = \frac{4\pi b}{n} \left( \frac{E}{\lambda b} \right)^{1 - \frac{K}{2}} \sum_{I=1}^{2^K} r^I_H \prod_{i=1}^{K} \left[ \sqrt{\tilde{q}_i} \right]^I$$

where $\lambda = \frac{n+1}{n} - \frac{K}{2}$, $\tilde{q}_i$ is either $q_i$ or $\bar{q}_i$ depending on the stack, and $r^I_H$ which are now functions of $(E, \{l^I_j\})$ are given by

$$(r^I_H)^n \prod_{j} \left( 1 + \frac{(l^I_j)^2}{(r^I_H)^2} \right) = \frac{E}{\lambda b} .$$
This is our field theoretic model for K charge rotating black holes, with the entropy $S$ given by equation (35). The parameters $q_i$, $\bar{q}_i$, $l_j^I$, and $E$ are arbitrary subject only to the constraints that the quantities $M$, $Q_i$ and $J_j$ are fixed.

This system is dynamical because more and more brane antibrane pairs can be created taking energy from massless gases, or annihilated giving energy to them; moreover $l_j^I$ can flow from one gas to another. The system reaches equilibrium in a state where entropy is maximum for given $M$, $Q_i$ and $J_j$. So, to obtain the equilibrium state, we maximise $S$ subject to the constraints that $M$, $Q_i$ and $J_j$ given in equations (32) – (34) are held fixed.

These constraints are incorporated by Lagrange multiplier method. Hence we maximise the function $F(q_i, \bar{q}_i, l_j^I, E)$ defined by

$$F = S + A_j \left( \sum_I \frac{2b}{n} l_j^I \left( \frac{E}{\lambda b} \right)^{1-K^2} \sqrt{\prod \tilde{q}_i} - J_j \right)$$

$$+ \sum_i B_i(q_i - \bar{q}_i - Q_i) + C \left( b \sum_i (q_i + \bar{q}_i) + 2^K E - M \right)$$

where $A_j$, $B_i$ and $C$ are Lagrange multipliers. Varying $F$ with respect to $q_i$, $\bar{q}_i$, $l_j^I$, and $E$, we have that $dF = 0$ at the maximum. Hence,

$$0 = \left( \frac{E}{\lambda b} \right)^{1-K^2} \sum_I \sqrt{\prod \tilde{q}_i} \left( \frac{\partial r_H^I}{\partial l_j^I} + \frac{A_j}{2\pi} \right) d l_j^I$$

$$+ \sum_i \left( \frac{\partial S}{\partial q_i} + A_j \frac{\partial}{\partial \bar{q}_i} \sum_I \frac{2b}{n} l_j^I \left( \frac{E}{\lambda b} \right)^{1-K^2} \sqrt{\prod \tilde{q}_i} - B_i + Cb \right) dq_i$$

$$+ \sum_i \left( \frac{\partial S}{\partial \bar{q}_i} + A_j \frac{\partial}{\partial q_i} \sum_I \frac{2b}{n} l_j^I \left( \frac{E}{\lambda b} \right)^{1-K^2} \sqrt{\prod \tilde{q}_i} + B_i + Cb \right) d\bar{q}_i$$

$$+ \left( \frac{\partial S}{\partial E} + A_j \frac{\partial}{\partial E} \sum_I \frac{2b}{n} l_j^I \left( \frac{E}{\lambda b} \right)^{1-K^2} \sqrt{\prod \tilde{q}_i} + 2^K C \right) dE$$

which implies that the coefficients of each $dl_j^I$, $dq_i$, $d\bar{q}_i$ and $dE$ must vanish. Equating the coefficients of each $l_j^I$, we get

$$\frac{\partial r_H^I}{\partial l_j^I} = - \frac{A_j}{2\pi} \quad \forall I .$$
The functional dependence of \( r_I^H \) on \( l_I^j \) are same for all \( I \). So, just as in the single charge case, the above \( 2^K \) equations always admit one solution where

\[
l_I^1 = l_I^2 = l_I^3 = \cdots = l_I^{2^K} \equiv l_I
\]

and, hence, \( r_I^H \equiv r_H \) are also the same for all \( I \). Such a solution is always present and, as will be seen below, it leads to the correct form of entropy. With this solution, the angular momenta \( J_j \) and entropy \( S \) now become

\[
J_j = \frac{2b}{n} l_j \left( \frac{E}{\lambda b} \right)^{1-\frac{K}{2}} \prod_{i=1}^{K} \left( \sqrt{q_i} + \sqrt{\bar{q}_i} \right)
\]

\[
S = \frac{4\pi b}{n} r_H \left( \frac{E}{\lambda b} \right)^{1-\frac{K}{2}} \prod_{i=1}^{K} \left( \sqrt{q_i} + \sqrt{\bar{q}_i} \right)
\]

where we have used the identity

\[
\sum_{i=1}^{2^K} \prod_{i=1}^{K} \left[ \sqrt{q_i} \right]^I = \prod_{i=1}^{K} \left( \sqrt{q_i} + \sqrt{\bar{q}_i} \right).
\]

Also, as in the single charge case, the entropy \( S \) can be written as

\[
S = \rho S_0, \quad S_0 = \frac{4\pi b}{n} (b\lambda)^{-\lambda} \prod_{i=1}^{K} \left( \sqrt{q_i} + \sqrt{\bar{q}_i} \right) E^\lambda
\]

where \( \rho \) is defined in equation (23) and \( S_0 \) is the entropy in the non rotating case, namely when \( l_j = 0 \).

Proceeding as in the single charge case, we use the above results in the expression for the entropy \( S \) and maximise the resultant expression with respect to \( q_i, \bar{q}_i \), and \( E \), subject to the constraints \( q_i - \bar{q}_i = Q_i \) and \( \sum_i b(q_i + \bar{q}_i) + 2^K E = M \). Thus, using the Lagrange multipliers \( B_i \) and \( C \), we start from the equation

\[
d \left\{ S + \sum_i B_i(q_i - \bar{q}_i - Q_i) + C \left( \sum_i b(q_i + \bar{q}_i) + 2^K E - M \right) \right\} = 0
\]

which implies that

\[
\sum_i \left( \frac{\partial S}{\partial q_i} + B_i + Cb \right) dq_i + \sum_i \left( \frac{\partial S}{\partial \bar{q}_i} - B_i + Cb \right) d\bar{q}_i + \left( \frac{\partial S}{\partial E} + 2^K C \right) dE = 0
\]
which, in turn, implies that
\[
\frac{\partial S}{\partial q_i} + \frac{\partial S}{\partial \bar{q}_i} - 2^{1-K} b \frac{\partial S}{\partial E} = 0.
\] (39)

Using \(S = \rho S_0\), with \(\rho\) taken as a function of \(S_0\), one now obtains
\[
\left( S_0 \frac{\partial \rho}{\partial S_0} + \rho \right) \left\{ \frac{E^\lambda}{2\sqrt{q_i}} + \frac{E^\lambda}{2\sqrt{\bar{q}_i}} - 2^{1-K} \lambda b E^{\lambda-1} \right\} = 0.
\]

One can show that \(S_0 \frac{\partial \rho}{\partial S_0} + \rho \neq 0\). It then follows that \(2^{K} E = 4\lambda b \sqrt{q_i \bar{q}_i}\).

Let \(q_i = \frac{m}{2} e^{2\theta_i}\) and \(\bar{q}_i = \frac{m}{2} e^{-2\theta_i}\). Then \(E = 2^{1-K} \lambda bm\) and \(M, Q_i, J_j,\) and \(S\) of the system are given, using equations (32), (33), (37), and (38), by
\[
M = bm \left( 2\lambda + \sum_{i=1}^{K} \cosh 2\theta_i \right)
\]
\[
Q_i = m \sinh 2\theta_i
\]
\[
J_j = 2 \frac{K(K-2)}{2} \frac{4bm}{n} l_j \prod_{i=1}^{K} \cosh \theta_i
\]
\[
S = 2 \frac{K(K-2)}{2} \frac{8\pi bm}{n} r_H \prod_{i=1}^{K} \cosh \theta_i
\]

where \(r_H\) is given implicitly by the equation \(r_H^n \prod_j \left( 1 + \frac{l_j^2}{r_H^2} \right) = 2^{1-K} m\).

Now we compare the above expressions with the supergravity ones, see footnote 1. Setting \(M = M_{sg}, Q_i = Q_{i\,sg},\) and \(J_j = J_{j\,sg}\) gives \(m = \mu, \theta_i = \phi_i,\) and \(l_j = 2 \frac{K(K-2)}{2} l_{j\,sg}\). We then have that
\[
S(M, Q_i, J_j) = X S_{sg}(M, Q_i, J_j), \quad X = 2 \frac{K(K-2)}{2} \frac{r_H}{r_{H\,sg}}.
\] (44)

Thus, the field theoretic entropy \(S\) differs from the supergravity entropy \(S_{sg}\) by a ‘deficit’ factor \(X\) given above.

An implicit equation for \(X\) can be obtained easily. Using equation (44) with \(\mu = m, l_j^2 = 2^{K(K-2)} l_{j\,sg}^2,\) and \(r_{H\,sg}^2 = 2^{K(K-2)} \frac{r_H^2}{X^2}\), and equation \(r_H^n \prod_j \left( 1 + \frac{l_j^2}{r_H^2} \right) = 2^{1-K} m\) for \(r_H,\) it follows that \(X\) is given implicitly by the equation
\[
X^n = 2^{-(n+1-\frac{4K}{2})K} \prod_j \left( \frac{r_H^2 + X^2 l_j^2}{r_H^2 + l_j^2} \right)
\]

Thus, the field theoretic entropy \(S\) differs from the supergravity entropy \(S_{sg}\) by a ‘deficit’ factor \(X\) given above.
and, hence, that the factor $X$ depends non trivially on $l_j$ and $r_H$, thus on black hole parameters $l_j$ and $m$. However, in the non rotating case, $l_j = 0$ and the deficit factor $X$ reduces to just a numerical constant, namely $X = 2^{-\lambda K}$ where $\lambda = \frac{n+1}{n} - \frac{K}{2}$, see [7, 9].

4 The deficit factor $X$

As seen above, the deficit factor $X$ is just a numerical constant in the non rotating case. Then, the field theoretic entropy differs from the supergravity one by just a numerical factor. However, with rotation present, the entropies differ by a factor which now depends on black hole parameters. If the deficit factor is allowed to depend on the black hole parameters then, very likely, any model can be argued to reproduce black hole entropy upto such a deficit factor. The field theoretic model of DGK then becomes less appealing.

In the field theoretic model of DGK, one can obtain $X = 1$ if one assumes that each of the $2^K$ types of gases has an energy $= 2^K E$. This has been shown in [3, 4, 5] for the single charge case with or without rotation and, as will be seen below, is true for the multicharge case also with or without rotation.

Obviously, however, this assumption violates the conservation of energy. As pointed out in [5], this violation is perhaps due to the neglect of binding energies in the field theoretic model. But the details of the binding energies are not sufficiently well known and, hence, it is difficult at present to verify this idea.

Note also that, in the field theoretic model, the energies of the gases are to be assumed equal. This is unusual since, in such systems, it is the temperatures which must be equal. No physical mechanism is known which can enforce such an equality of energies, instead of temperatures.

For the multicharge black holes with no rotation, it is shown in [7, 9] that $X = 1$ can also be obtained if one assumes that the available energy is all taken by one single gas and, further, that all the brane tensions are reduced by a factor of four. This single gas may perhaps be thought of as some sort of superposition of $2^K$ possible types of gases, and as “living” equally likely on any of the $2^K$ possible stacks. Its entropy is an average of the entropies it has when on these stacks. Clearly, in this approach, there is no violation of conservation of energy. Also, the unusual assumption that energies, not temperatures, of the $2^K$ types of gases are equal becomes superfluous.

We will now show that $X = 1$ can be obtained similarly even when
rotation is present. The angular momenta of the single gas is to be taken as an average of the angular momenta it has when on $2^K$ possible stacks.

For this purpose, we introduce a set of parameters $\alpha, \chi, \sigma,$ and $\epsilon$ in the expressions for mass, charges, angular momenta, and entropy as follows:

\begin{align*}
M &= \alpha b \sum_{i=1}^{K} (q_i + \bar{q}_i) + 2^K E = \alpha \sum_{i=1}^{K} \tau_i V_i (N_i + \bar{N}_i) + 2^K E \quad (45) \\
Q_i &= \alpha (q_i - \bar{q}_i) = \frac{\alpha \tau_i V_i}{b} (N_i - \bar{N}_i) \quad (46) \\
J_j &= \chi \frac{2b}{n} l_j \left( \frac{\epsilon E}{\lambda b} \right)^{1-K \frac{2}{2}} \prod_{i=1}^{K} (\sqrt{q_i} + \sqrt{\bar{q}_i}) \quad (47) \\
S &= \sigma \frac{4\pi b}{n} r_H \left( \frac{\epsilon E}{\lambda b} \right)^{1-K \frac{2}{8}} \prod_{i=1}^{K} (\sqrt{q_i} + \sqrt{\bar{q}_i}) \quad . \quad (48)
\end{align*}

where we have set $l_j^I = l_j$ and $r_H^I = r_H$ for $I = 1, 2, 3, \cdots, 2^K$, and used the relation between the numbers and the charges of branes. The expression for $r_H$ is given by

\begin{equation}
\frac{r_H^n}{\prod_{j} \left( 1 + \frac{l_j^2}{r_H^2} \right)} = \frac{\epsilon E}{\lambda b} . \quad (49)
\end{equation}

Analysing this system as before, one finds $E = 2^{1-K} \alpha \lambda bm$ and

\begin{align*}
M &= \alpha b m \left( 2\lambda + \sum_{i=1}^{K} \cosh 2\theta_i \right) \\
Q_i &= \alpha m \sinh 2\theta_i
\end{align*}

where we have set $q_i = \frac{m}{2} e^{2\theta_i}$ and $\bar{q}_i = \frac{m}{2} e^{-2\theta_i}$ . Setting $M = M_{sg}$ and $Q_i = Q_{i, sg}$ gives $\alpha m = \mu$ and $\theta_i = \phi_i$. The angular momenta and entropy then become

\begin{align*}
J_j &= \chi \frac{1}{\alpha b} \left( \frac{2^K}{\epsilon} \right)^{\frac{K-2}{2}} \frac{4b \mu}{n} l_j \prod_{i=1}^{K} \cosh \theta_i \\
S &= \sigma \frac{1}{\alpha b} \left( \frac{2^K}{\epsilon} \right)^{\frac{K-2}{2}} \frac{8\pi b \mu}{n} r_H \prod_{i=1}^{K} \cosh \theta_i
\end{align*}

where $r_H$ is given implicitly by the equation

\begin{equation}
\frac{r_H^n}{\prod_{j} \left( 1 + \frac{l_j^2}{r_H^2} \right)} = \epsilon 2^{1-K} \mu . \quad (50)
\end{equation}
Setting \( J_j = J_{j\, sg} \) gives \( l_j = \frac{\alpha K}{\chi} \left( \frac{2^K}{\epsilon} \right)^{\frac{K-2}{2}} J_{j\, sg} \). We then have that

\[
S(M, Q_i, J_j) = X S_{sg}(M, Q_i, J_j), \quad X = \frac{\sigma}{\alpha K} \left( \frac{2^K}{\epsilon} \right)^{\frac{K-2}{2}} \frac{r_H}{r_{H_{sg}}}. \quad (51)
\]

Consider now the deficit factor \( X \) given above. Let \( \sigma = \alpha = \chi = 1 \), which corresponds to the model of section 3.2. Then \( \epsilon = 2^K \) will give \( l_j = l_{j\, sg} \) and, hence, \( r_H = r_{H_{sg}} \) as follows from equations (50) and (4). One then obtains \( X = 1 \). However, since \( \epsilon = 2^K \), this means that energy of each of the \( 2^K \) types of gases is \( 2^K E \). This method of obtaining \( X = 1 \) is similar to that in [3, 4, 5], and violates conservation of energy. As we have just shown, it is also applicable in the multicharge case with rotation.

However, \( X = 1 \) can also be obtained if one chooses \( \epsilon = 2^K \) and \( \sigma = \chi = \alpha \frac{K}{2^K} \). Then, again, \( r_H = r_{H_{sg}} \) as follows from equations (50) and (4). If one further chooses \( \sigma = \frac{1}{2^K} \) and \( \chi = \frac{1}{2^K} \) then this choice of values for \( \epsilon, \sigma \) and \( \chi \) admits the following interpretation: There is only a single gas which may perhaps be thought of as some sort of superposition of \( 2^K \) possible types of gases. This single gas has all the available energy \( 2^K E \), and “lives” equally likely on any of the \( 2^K \) possible stacks. Hence its entropy and angular momenta are the averages of their \( 2^K \) possible values, as signified by the choices \( \sigma = \chi = \frac{1}{2^K} \), and equations (47) and (48) for \( J_j \) and \( S \).

But this implies that \( \alpha = \sigma \frac{K}{2^K} = \frac{1}{4} \). This may be taken to mean, see equations (15) and (46), that brane tensions are effectively normalised by this factor – namely, they are all reduced by a factor of four. This method of obtaining \( X = 1 \) is similar to that in [7, 9]. We have now shown that it is valid even when rotation is present. However, neither the details of the superposition mentioned above nor the physical reason for normalising brane tensions is clear to us at present.

5 Conclusion

We now summarise our results briefly and mention a few issues that may be studied further.

We generalised the field theoretic model of DGK to multicharge black holes with rotation. They are described as stacks of intersecting branes and antibranes with \( 2^K \) types of gases on them which are characterised by energies and angular momentum parameters. Assuming the energies of the gases to
be equal, and following the methods of [3, 4, 5], we found that the angular
momentum parameters for the gases must also be equal. We found that the
field theoretic and the supergravity entropies are related by $S = X S_{sg}$ where,
in the presence of rotation, the deficit factor $X$ is not constant but depends
on black hole parameters.

The deficit factor $X$ not being constant makes the field theoretic model
less appealing. One can obtain $X = 1$ with a further assumption which,
however, violates conservation of energy. This is perhaps due to the neglect
of binding energies but the details of the binding energies are not sufficiently
well known to verify this idea. Also, the assumption that the energies, not
temperatures, of the gases are equal is unusual.

We showed that $X = 1$ and, hence, $S = S_{sg}$ can be obtained, as in [7, 9],
for rotating multicharge black holes also. The physical interpretation of the
field theoretic model is also similar. In particular, the assumptions mentioned
above are superfluous and are not needed in this interpretation.

However, this interpretation involves a single gas, thought of as some sort
of superposition of $2^K$ types of gases, and also a reduction of brane tensions
by a factor of four. We do not understand the nature of the superposition or
the reason for the reduction of brane tensions. It is important to understand
these aspects.

It is also important to understand if and how the field theoretic descrip-
tion of non extremal black holes connects up with the string/M theoretic
description of extremal and near extremal ones [1]. This may help in un-
derstanding the Hawking radiation of non extremal black holes in terms of
the field theoretic models used here. See [3, 5] for some discussions on these
issues.

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