THE COSMIC DENSITY OF MASSIVE BLACK HOLES FROM GALAXY VELOCITY DISPERSIONS

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ABSTRACT

Supermassive black holes are thought to be relics of quasars, and their numbers and masses are therefore related to the quasar luminosity function and its evolution with redshift. We have used the relationship between black hole mass and bulge velocity dispersion (the $M_{\bullet} - \sigma$ relation) to make an improved estimate of the mass density and mass spectrum of supermassive black holes. Uncertainties in the $M_{\bullet} - \sigma$ relation have little effect on the mass density. We find a mass density of $(4.8 \pm 1.6) h^3 \times 10^5 M_\odot$ Mpc$^{-3}$. Some of the variance in published density estimates comes from the use of different values of the Hubble constant.

Key words: galaxies: general — galaxies: nuclei

1. INTRODUCTION

It is now fairly well established that most or all bulges contain supermassive black holes (Richstone et al. 1998; Kormendy & Gebhardt 2002; Magorrian et al. 1998). Within the last 2 years, it has become clear that the mass of the black hole (BH) is related to the velocity dispersion of the bulge of the host galaxy. This relationship has less scatter than a previously noted relationship between the BH mass and the bulge light (Gebhardt et al. 2000; Ferrarese & Merritt 2000; Kormendy & Gebhardt 2002). It has also been known for some time that the masses and cosmic mass density of BHs crudely correspond to the masses and numbers predicted by the luminous output of quasars, under the assumption that they are powered by accreting BHs with radiative efficiencies near 10%. Published estimates of the BH mass density predicted by quasars are around $2 \times 10^5 M_\odot$ Mpc$^{-3}$ (Chokshi & Turner 1992; Soltan 1982; Salucci et al. 1999), while estimates from the X-ray background tend to be larger. Using the X-ray background, Salucci et al. (1999) obtain an estimate of $3-5 \times 10^5 M_\odot$ Mpc$^{-3}$, while Fabian & Iwasawa (1999) estimate the mass density to be $6-9 \times 10^5 M_\odot$ Mpc$^{-3}$.

It is useful to note that the above density estimates are independent of the Hubble constant or cosmological model (so long as it is general relativistic, isotropic, and homogeneous) because they can be cast in terms of the surface brightness of quasar light on the sky and hence determine its local energy density. Since the mass density of the radiating objects declines as the universe expands as $(1+z)^3$ and the photon energy of their radiation declines as $(1+z)^4$, the only cosmological correction connecting the observed radiation to the local BH density is $(1+z)$, where $z$ is the redshift of the epoch of the emission of the radiation.

On the other hand, the local density of supermassive BHs is obtained by multiplying the density of galaxies by the typical BH mass per galaxy. Kormendy & Richstone (1995) used the BH mass–bulge luminosity relationship, together with the Loveday et al. (1992) estimate of the luminosity density in galaxies, to estimate the local density of BHs; for a Hubble constant of 80 km s$^{-1}$ Mpc$^{-1}$, this is $1 \times 10^6 M_\odot$ Mpc$^{-3}$. More recently, Merritt & Ferrarese (2001a) estimated this same quantity using the newer $M_{\bullet} - \sigma$ relation. Their approach was to use the $M_{\bullet} - \sigma$ relation to calibrate a mass–luminosity relation and then multiply by the galaxy luminosity function.

In this paper, we take an alternate approach. We use multiple estimates of the luminosity functions of galaxies for different Hubble types with estimates of bulge luminosity as a function of Hubble type, together with observed relationships between luminosity and velocity dispersions or rotation velocity, to construct a “velocity function” (the number of galaxies per magnitude per unit volume) for bulges as a function of Hubble type. This function can then be used, together with the $M_{\bullet} - \sigma$ relation, to estimate BH mass functions. Our basic result using this procedure is that the mean mass density of supermassive BHs is $(4.8 \pm 1.6) h^3 \times 10^5 M_\odot$ Mpc$^{-3}$. After we completed much of the analysis in this paper, we became aware of an estimate by Yu & Tremaine (2002) based on the Sloan Digital Sky Survey (SDSS) that gives an estimate of $\rho_\bullet = (2.5 \pm 0.4) h_{100}^2 55 \times 10^5 M_\odot$ Mpc$^{-3}$. By assuming that $h = 0.65$, as in their paper, our results are comparable with theirs. The rather surprising conclusions about the radiative efficiency of quasars derived by Yu & Tremaine (2002) from their low value of $\rho_\bullet$ illustrate the importance of an improved estimate of this parameter.

2. METHOD

The basic assumption underlying this note is that the bulge velocity dispersion of a galaxy can be used to predict the mass of its supermassive BH. Under that assumption, the best way to proceed would be to directly observe velocity dispersions of a volume–limited sample of galaxy bulges and to integrate over the implied distribution. Such a catalogue might well exist in the future,1 but at present we adopt a simpler expedience. We use a power law (Faber & Jackson 1976) to convert various luminosity functions to a “dispersion function” and the $M_{\bullet} - \sigma$ relation for BHs to convert the dispersion function to a BH mass function.

In general, if there are $N$ galaxies per cubic megaparsec brighter than a luminosity $L$, then the luminosity function is

$$dN/dL = \Phi(L) \, .$$

This can be directly converted to the BH mass function by using the derivative of the relationship between luminosity and the velocity dispersion ($\sigma$) and the BH mass ($M_{\bullet}$) and $\sigma$

1 After starting this project, the early release data for the SDSS became available, but we pursued this calculation from completely independent data.
for bulges. Without any loss of generality, it can be written as

\[ \Theta(M_*) = \frac{dN}{dM_*} = \Phi(L) \left( \frac{dL}{d\sigma} \right) \left( \frac{dM_*}{d\sigma} \right)^{-1} . \]  

(2)

Since in what follows we will rely on luminosity functions due to Marzke et al. (1994a), we adopt his parameterization of the luminosity function of different types of galaxies as Schechter functions and adopt power laws for the two relationships. To simplify the algebra, we normalize luminosity, dispersion, and BH mass by their values at the fiducial luminosity of the Schechter function \(L_*\) as follows:

\[ \tilde{L} = L/L_* , \quad \tilde{\sigma} = \sigma/\sigma_* , \quad \tilde{M}_* = \frac{M_*}{M_*} . \]

(3)

We then can write the Schechter parameterization and the relationships between luminosity and dispersion and \(M_*\) as

\[ \frac{dN}{dL} = \Phi(L) = \left( \frac{\phi_*}{L_*} \right) \tilde{L}^\alpha e^{-\tilde{L}} , \]

(6)

\[ \tilde{L} = \tilde{\sigma}^\gamma , \]

(7)

\[ \tilde{M}_* = \frac{M_*}{M_*} = \tilde{\sigma}_*^\gamma e^{-\tilde{M}_*} , \]

(8)

with \(\phi_*\) in Mpc\(^{-3}\). Substituting these equations into equation (2) gives the dispersion function as

\[ \frac{dN}{d\sigma} = \left( \frac{\phi_*}{\sigma_*} \right) \tilde{\sigma}^{\beta-1} e^{-\tilde{\sigma}} \]  

(9)

with \(\beta = n(\alpha + 1)\), and the mass function for supermassive BHs as

\[ \frac{dN}{dM_*} = \Theta(M_*) = \theta_* M_*^{\gamma-1} e^{-\tilde{M}_*} , \]

(10)

with \(\theta_* = \epsilon \phi_*\) and \(\gamma = \beta/\lambda = n(\alpha + 1)/\lambda\).

Unless explicitly parametrized by \(h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})\), all calculations were done assuming \(H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}\).

3. RESULTS

In order to determine the number density as a function of dispersion, the Schechter function fits, valid for \(M_Z \geq 21.5\) (corresponding to \(\sigma_* = 400 \text{ km s}^{-1}\) and \(M_* = 1.8 \times 10^9 M_\odot\) for an elliptical galaxy), obtained by Marzke et al. (1994a) were modified using the corresponding bulge-to-total galaxy luminosity ratios, which were calculated from the data given in Simien & de Vaucouleurs (1986) for the appropriate galaxy range, as given in Table 1.

**TABLE 1**

| Galaxy Type | \(M_*\) | \(\phi_*\) (10\(^{-3}\) Mpc\(^{-3}\)) | \(\alpha\) | T Types | \(\Delta M\) |
|-------------|--------|-------------------------------|------|--------|----------|
| Elliptical   | −19.23 | 1.5 ± 0.4                    | −0.85| N/A    | 0        |
| S0           | −18.74 | 7.6 ± 2.0                    | −0.94| 3−0    | 0.64 ± 0.30 |
| Sa−Sb        | −18.72 | 8.7 ± 2.2                    | −0.58| 1−4    | 1.46 ± 0.56 |
| Sc−Sd        | −18.81 | 4.4 ± 1.1                    | −0.96| 5−7    | 2.86 ± 0.59 |

To produce a fit as a function of the bulge-only luminosity for the spiral galaxies:

\[ \frac{dN}{dL_{\text{bulge}}} = \Phi(L_{\text{bulge}}) = \left( \frac{h^2 \phi_*}{L_{\text{bulge}}^a} \right) L_{\text{bulge}}^{a-1} e^{-L_{\text{bulge}}} , \]

(11)

\[ L_{\text{bulge}} = L_{\text{total}}/L_{\text{bulge}} . \]

(12)

A power-law relation of the form

\[ L = \sigma^p , \]

(13)

where \(L\) refers to the total luminosity ratio for the elliptical galaxies and to the bulge-only luminosity ratio for the spiral galaxies, may be used to convert the function of luminosity (eqs. [6] and [11]) to one of dispersion such that

\[ \frac{dN}{d\sigma} = h^2 \left( \frac{\phi_*}{\sigma_*} \right) \sigma^{p-1} e^{-\sigma^p} , \]

(14)

with \(p = n(\alpha + 1)\).

To obtain a value for the parameter \(\sigma_*\) for each galaxy type, the parameter \(M_*\) from the Marzke et al. (1994a) Schechter function fits are converted to the equivalent dispersion, \(\sigma_*\), using a relationship between the magnitude and dispersion given by de Vaucouleurs & Olson (1982). The “best” relationship between magnitude and dispersion, selected from several in this paper, was determined by Gonzalez et al. (2000) to be

\[-(M_{B_T} + 5 \log h) = a + b(\log \sigma - 2.3) , \]

(15)

with \(a = 19.71 \pm 0.08\) and \(b = 7.7 \pm 0.7\). This results in a slope for the power-law relation (eq. [13]) of \(n = b/2.5 = 3.08 \pm 0.28\). However, this equation is in the \(M_{B_T}\) magnitude system, and so must be converted into the Zwicky magnitude system in order to be combined with the Marzke et al. (1994a) Schechter function fitting parameters. Despite concerns that there may not be a linear relationship between these two magnitude systems (Marzke, Huchra, & Geller 1994b), an approximate conversion was adopted by combining the conversion given in Shanks et al. (1984),

\[ M_{B_T} = M_Z - 0.45 , \]

(16)

with that given in Gonzalez et al. (2000),

\[ M_{B_T} \approx M_{B_T} + 0.06 , \]

(17)

to obtain

\[ M_{B_T} \approx M_Z - 0.51 , \]

(18)

which allows the magnitude-dispersion relation to be written as

\[-(M_Z + 0.51 + \Delta M) = a + b(\log \sigma - 2.3) + 5 \log h . \]

(19)

This relation may then be combined with the Simien & de Vaucouleurs (1986) magnitude correction for the bulge-to-total galaxy magnitude ratio, \(\Delta M\), so that only the \(M_{\text{bulge}}\) is used to calculate the dispersion for the spiral galaxies, such that

\[-(M_Z - 0.51 + \Delta M) = a + b(\log \sigma - 2.3) + 5 \log h . \]

(20)

Hence, the velocity dispersion, \(\sigma_*\), may be expressed as

\[ \sigma_* = 199.5 \times 10^\lambda , \]

(21)
with
\[ \chi = (-M_{Z,*} + 0.51 - a - 5 \log h - \Delta M)^{-1}. \]  
(22)

The number density as a function of dispersion (eq. [14]) may then be combined with the relation between the dispersion and the BH mass given by Tremaine et al. (2002), corrected from \( h = 0.8 \) (to \( h = 1 \)) for consistency,
\[ \mathcal{M}_* = h^{-1} 0.8 \times 10^{5.13 \pm 0.06} (\sigma/200)^{3} \]  
(23)

(\( \mathcal{M}_* \) is defined as the BH mass times the distance in Mpc, corrected for dispersion). To obtain the final equation for the number density function as a function of BH mass:
\[ dN/dM_* = \Theta(M_*) = h^2 \theta_* \mathcal{M}_*^{-1} \mathcal{M}_*^{-1} e^{-M_*}, \]  
(24)

again with \( \theta_* = c \phi_*, \epsilon = n/\lambda = 0.8 \pm 0.1, \gamma = \beta/\lambda, \) and
\[ \mathcal{M}_* = h^{-1} (0.8 \times 10^{5.13 \pm 0.06} 10^\lambda). \]  
(25)

This method of calculating \( \mathcal{M}_* \), rather than directly applying a relationship between \( \mathcal{M}_* L_{\text{bulge}} \), was used because Gebhardt et al. (2000) indicate that there is a tighter correlation between \( \mathcal{M}_* \) and \( L_{\text{bulge}} \) than between \( \mathcal{M}_* \) and \( L_{\text{bulge}} \).

Using equation (24) for the number density, an expression for the total mass density of BHs may be obtained by integrating the function
\[ \rho = \int \mathcal{M}_* \Theta(M_*)dM_*, \]  
(26)

from an assumed minimum BH mass to an infinite BH mass for each galaxy type, and then summing the results to obtain an estimate for the total mass density of BHs. The calculation was done adopting a minimum BH mass limit of \( 10^6 M_\odot \), which results in a total mass density for all galaxy types of
\[ \rho = (4.8 \pm 1.6) h^2 \times 10^5 M_\odot \text{ Mpc}^{-3}. \]  
(28)

The results of this calculation are given in Table 2, along with the number density function fitting parameters for each galaxy type. The uncertainties and average values given for each of these parameters were derived using a Monte Carlo analysis, with the largest source of uncertainty stemming from the bulge–total galaxy luminosity ratios given in Table 1. Figures 1–4 show histograms of the resulting densities from the Monte Carlo calculation, which illustrate that the median value for the density tends to be slightly lower than the average value, but still well within the stated uncertainties of the density, with a relatively symmetric distribution of densities.

The calculation of the mass density was then redone using exactly the same method, but using completely different galaxy luminosity functions obtained from Madgwick et al. (2002). In Madgwick et al. (2002), the galaxies were divided into subsets based on spectral type, \( \eta \), rather than by Hubble T-type. The parameter \( \eta \) is determined based on the absorption and emission line strengths in the galaxy spectrum. In order to apply the method developed in this paper, it was necessary to convert the spectral types into T-types so that the appropriate bulge–total luminosity conversions could be applied. This was done by using the data given in Figure 4 of Madgwick et al. (2002), which showed the correlation between \( \eta \) and morphological type, and assuming a relatively even distribution of \( \eta \) over T-types. For the luminosity function designated as Type 1 (\( \eta < -1.4 \)), an assumption was made that galaxies were evenly divided between elliptical, S0, and Sa galaxies for the purposes of this calculation.

The resulting mass density obtained using the Madgwick et al. (2002) luminosity functions is
\[ \rho = (6.9 \pm 1.4) h^2 \times 10^5 M_\odot \text{ Mpc}^{-3}, \]  
(29)

with the results for the individual spectral types given in Table 3. This is slightly larger than the result obtained using the Marzke functions; however, it is strongly dependent on the assumptions made regarding the division into morpho-

TABLE 2

| Galaxy Type   | \( M_{*,*} \) (10^6 M_\odot) | \( \theta_* \) (10^-3 Mpc^-3) | \( \gamma \) | \( \bar{\rho} \) (10^5 M_\odot Mpc^-3) |
|---------------|-----------------------------|-----------------------------|------------|-----------------------------|
| Elliptical... | 11.2 ± 1.9                  | 1.2 ± 0.3                   | 0.12 ± 0.01 | 1.5 ± 0.5                   |
| S0............. | 3.1 ± 1.3                   | 5.9 ± 1.6                   | 0.046 ± 0.006 | 2.1 ± 1.1                   |
| Sa–Sb.......... | 1.3 ± 1.0                   | 6.7 ± 1.8                   | 0.32 ± 0.04 | 1.1 ± 1.0                   |
| Sc–Sd........... | 0.29 ± 0.26                 | 3.4 ± 0.9                   | 0.031 ± 0.004 | 0.1 ± 0.1                   |

Note: For all galaxy types \( \epsilon = n/\lambda = 0.8 ± 0.1 \).

TABLE 3

| Madgwick Galaxy Type | Assumed T-Type(s) | \( \Delta M \) | \( M_{*,*} \) (10^6 M_\odot) | \( \bar{\rho} \) (10^5 M_\odot Mpc^-3) |
|----------------------|-------------------|---------------|-----------------------------|-----------------------------|
| Type 1 (\( \eta < -1.4 \)) | Elliptical         | 0             | 10.0 ± 1.8                  | 3.0 ± 0.6                   |
| Type 1 (\( \eta < -1.4 \)) | −3−0              | 0.64 ± 0.30   | 4.9 ± 2.0                  | 1.5 ± 0.6                   |
| Type 1 (\( \eta < -1.4 \)) | 1−2               | 1.19 ± 0.51   | 2.7 ± 1.7                  | 0.81 ± 0.50                 |
| Type 2 (−1.4 ≤ \( \eta < 1.1 \)) | 1−5              | 1.58 ± 0.60   | 1.8 ± 1.3                  | 1.1 ± 0.9                   |
| Type 3 (1.1 ≤ \( \eta < 3.5 \)) | 3−5              | 1.85 ± 0.59   | 0.86 ± 0.70                | 0.37 ± 0.36                 |
| Type 4 (\( \eta ≥ 3.5 \)) | 5                 | 2.47 ± 0.45   | 0.37 ± 0.26                | 0.07 ± 0.07                 |
logical type, particularly of the Madgwick et al. (2002) type 1 galaxies. If it is assumed that all of the type 1 galaxies are elliptical, the resulting mass density becomes
\[ (10.5 / 2.0) h^2 \times 10^5 M_\odot \text{ Mpc}^{-3}; \]
if it is assumed that they are all S0 galaxies, then the mass density becomes
\[ (6.0 / 2.0) h^2 \times 10^5 M_\odot \text{ Mpc}^{-3}; \]
and if it is assumed that they are all Sa galaxies, then the mass density is calculated to be
\[ (4.0 / 1.8) h^2 \times 10^5 M_\odot \text{ Mpc}^{-3}. \]
Considering the uncertainties in using the Madgwick et al. (2002) spectral type classifications for the purpose of applying a bulge-to-disk luminosity correction, these results are consistent with those obtained using Marzke et al. (1994a). Also of importance is the fact that Madgwick et al. (2002) specifically take into account issues of Malmquist bias when determining their results.
luminosity functions. Marzke et al. (1994) make no mention of this. Therefore, it appears that this possible omission will not have a significant impact on the final density of BHs.

4. DISCUSSION

In Table 2, as expected, the elliptical galaxies have the largest central BH mass, \( M_\bullet \), values, while later type galaxies have increasingly smaller \( M_\bullet \) values, corresponding to their smaller relative bulge sizes. Plots of the number function (Fig. 5) and the mass function (Fig. 6) illustrate that the contribution from the S0 galaxies dominates at low \( M_\bullet \) while the elliptical galaxies dominate for high \( M_\bullet \) values. Overall, the strongest contribution to the total mass density is from S0 galaxies, with important contributions from the elliptical and Sa–Sb galaxies but a small contribution from the Sc–Sd galaxies.

The total cosmic mass density of BHs of \((4.8 \pm 1.6) h^2 \times 10^5 M_\odot \text{Mpc}^{-3}\) obtained from this calculation is relatively independent of several of the adopted parameters. As illustrated by Figure 7, decreasing the lower limit of integration in the calculation of the mass density does not significantly change the resulting mass density estimate. This is because the BHs located in small bulge galaxies are, themselves, small, and there are not enough small bulge galaxies to make up in number for this lack in mass. This is further illustrated by the flat slope in the plot of the mass function (Fig. 6). Similarly, altering the value of \( \lambda \), the power-law index, in the relationship between the BH mass, \( M_\bullet \), and the velocity dispersion, \( \sigma \) (eqs. [5] and [8]), has a small effect on the final density. This is because most of the contribution is in the range of \( 10^5 – 10^7 M_\odot \)—the same region in which the \( M_\bullet \) for this relation is located, and in which the relationship is determined best. A perturbation in the power-law index for masses near this mass will have little effect on the final answer. Changing the zero point, however, would have a roughly linear effect on the resulting mass density, although for the currently accepted relationships, the differences in the zero point are small. Redoing the calculation using the best estimate Merritt & Ferrarese (2001b) relation stating that

\[
M_\bullet = (1.48 \pm 0.24) \times 10^8 (\sigma/200)^{(4.65 \pm 0.48)}
\]

instead of the Tremaine et al. (2002) relation (eq. [24]), pro-
duces a result of \((4.3 \pm 1.6) \times 10^5 \, M_\odot \, \text{Mpc}^{-3}\), which agrees well with our previous estimate. This estimate for the mass density also agrees, within the limits of the uncertainty, with that given in the Merritt & Ferrarese (2001b) paper of \(\rho_0 \approx 3 \times 10^3 \, M_\odot \, \text{Mpc}^{-3}\).

The parameters with the most significant impact on the resulting density, other than the \(M-\sigma\) relationship from de Vaucouleurs & Olson (1982) and the Schechter functions from Marzke et al. (1994a), are the assumptions about the morphological distributions of galaxies and the resulting bulge-to-disk luminosity corrections, adopted from Simien & de Vaucouleurs (1986). This is illustrated by the results obtained using the galaxy luminosity functions from Madgwick et al. (2002) instead of Marzke et al. (1994a). By assigning a different morphological distribution to the Madgwick et al. (2002) type 1 luminosity function galaxies, the resulting mass density varies by a factor of approximately 2.5. Given this uncertainty in the mass density resulting from the calculation using the Madgwick et al. (2002) data, we are more confident in our estimates derived from the Marzke et al. (1994a) luminosity functions.

The Schechter function fits may be less applicable in the low BH mass regime, particularly for \(M_\bullet < 10^8 \, M_\odot\), and to a lesser degree for galaxies with \(M_\bullet < 10^7 \, M_\odot\). While there is information on BHs down to \(10^6 \, M_\odot\) in our own galaxy and M32, the Schechter function fits may not correctly apply to elliptical galaxies, may include two types of galaxies within one classification, or may mix exponential and de Vaucouleurs–law galaxies together. We assume that while the Schechter function fits for elliptical galaxies may be dominated by dwarf elliptical galaxies at low luminosities, the same \(M_\bullet-\sigma\) relationship is still applicable; in our own galaxy and a small number of other galaxies with exponential profiles and measured BH masses, this relationship appears to hold true.

It must be noted that the resulting mass density of BHs may be an underestimate due to a possible bias in the relationship between the BH mass and the galaxy velocity dispersion (pointed out to us by Scott Tremaine). This bias relates to the fact that for any given velocity dispersion, there is a range of BH masses, and this distribution may not be symmetrical about the ridgeline. We have investigated this possible bias by assuming a log-normal distribution of BH masses, with a dispersion of 0.15 in the log space, and recomputing the BH mass density. This results in a mass density of \(\rho_0 = (5.0 \pm 2.1) \times 10^5 \, M_\odot \, \text{Mpc}^{-3}\), which is 4% larger than our original estimate.

It is often useful to have a simple approximation available for crude calculations. While the number functions we have derived are not of the Schechter form, we can find approximating functions of the form

\[
dN/dM_\bullet = c \cdot M_\bullet^{-\alpha} \cdot e^{-\lambda \cdot M_\bullet}. \tag{31}
\]

For the number function based on the Marzke et al. (1994) data, the roll off at high mass is too gradual to be well approximated by any function of the form (31). We illustrate a fair fit in Figure 8. This fit, which works best near \(M_\bullet \approx 10^8 \, M_\odot\), where the information on the numbers of BHs is the most extensive, has the parameters \((c, M_\bullet, \alpha) = (1.3 \times 10^{-10} \, M_\odot^{-1} \, \text{Mpc}^{-3}, 7.9 \times 10^7 \, M_\odot, 0.95)\). We can find a better fit to the number function produced by the Madgwick et al. (2002) luminosity function. It is shown in Figure 9. The parameters for this fit are \((c, M_\bullet, \alpha) = (3.2 \times 10^{-11} \, M_\odot^{-1} \, \text{Mpc}^{-3}, 1.3 \times 10^8 \, M_\odot, 1.25)\).

The recent paper by Yu & Tremaine (2002) uses luminosity and velocity dispersion functions from early-type galaxies obtained as part of the SDSS to perform a similar
calculation. They obtain a result of \((2.5 \pm 0.4)h^{2}_{0.65} \times 10^{2} \, M_{\odot} \, \text{Mpc}^{-3}\). If we take \(h = 0.65\) to compare our result with Yu & Tremaine (2002), we obtain \(\rho_{\bullet} = (2.0 \pm 0.7) \times 10^{5} \, M_{\odot} \, \text{Mpc}^{-3}\), which is in reasonable agreement with their result, particularly when the possible 4% bias is taken into account.

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