A New Doublet-Triplet Splitting Mechanism for Supersymmetric SO(10) and Implications for Fermion Masses

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We present a new mechanism for doublet-triplet splitting in supersymmetric SO(10) models using a missing vev pattern which is different from the one used in the currently popular Dimopoulos-Wilczek method. In our method, the doublets in a $16, 16$ pair are the ones split from the rest of the multiplet and are then mixed with the doublets from one or two $10$'s giving rise to the doublets $H_u$ and $H_d$ of the standard model. This approach provides a natural way to understand why top quark is so much heavier than the bottom quark. It also enables us to generate both hierarchical and nonhierarchical pattern for neutrino masses, the latter being of interest if neutrino is the hot component of the dark matter of the universe. We construct a simple, realistic model based on this idea. The model uses only simple representations, has no unwanted flat directions and maintains coupling constant unification as a prediction.

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I. INTRODUCTION

Supersymmetry appears to be the simplest way to provide a satisfactory resolution of two of the outstanding puzzles of the standard model: (i) the Higgs mass problem and (ii) the origin of electroweak symmetry breaking. The minimal supersymmetric extension of the standard model (MSSM) which embodies both these features has rightly been the focus of intense activity in the past decade. An additional advantage of the MSSM particle content is that it automatically leads to the unification of the gauge couplings near $2 \times 10^{16}$ GeV raising the hope that the high energy theory of particles and forces may indeed be a supersymmetric grand unified theory based on some simple group. One of the currently favoured groups for grand unification is the SO(10) group which provides a natural way to incorporate the neutrino masses. This will be the subject of this letter.

A key problem of all SUSY GUTs is how to split the weak MSSM doublets from the color triplet fields that accompany them as part of the representation of the GUT symmetry. This is essential for constructing realistic SUSY GUTs since both coupling constant unification and suppression of proton decay require that the MSSM doublets $H_u$ and $H_d$ be at the weak scale whereas the triplets which mediate proton decay must have GUT scale mass. This is the famous doublet-triplet splitting problem (DTS).

The most popular mechanism for implementing the DTS in SO(10) SUSY GUTs is the missing vev pattern advocated by Dimopoulos and Wilczek (DW) and applied to realistic models by Babu and Barr. Further analysis and improved applications of this idea have been carried out in Ref. Key observation underlying this procedure is that the vev pattern of the 45-dim. Higgs multiplet, $A$ of the following form i.e. $< A > = \ell_2 \otimes \text{Diag}(a, a, a, 0, 0)$ leads to one pair of massless standard model doublets for each of the two 10-dim Higgs fields that couples to it. After giving mass to two of these four doublets the low energy spectrum is the MSSM.

In this paper we present an alternative to the Dimopoulos-Wilczek mechanism for doublet triplet splitting in SO(10). The idea behind our method is to use missing vev patterns to split the doublets in a $16, 16$ pair (denoted by $P$ and $\bar{P}$) from the rest of the multiplet. These doublets are then mixed with the doublets from a pair of 10's so that the $H_u$ and $H_d$ of the standard model emerge as linear combinations of the 16's and the 10's. This approach provides a natural way to understand why the top quark is so much heavier than the bottom quark and has all the ingredients needed to generate a realistic mass and mixing pattern for the charged fermions. We also point out how this new doublet-triplet splitting mechanism can be used to generate nonhierarchical mass pattern for neutrinos using what is called the type II seesaw mechanism. This is to be contrasted with the usual seesaw (type I) mechanism which leads to strong hierarchy among the neutrino masses. If the neutrinos are to constitute the hot dark matter of the universe, then such a nonhierarchical pattern is clearly needed. It is our hope that the observations made in this paper will open up new ways to build realistic SO(10) models incorporating desired neutrino mass patterns.

The missing vev patterns required for splitting the doublets in $P$ and $\bar{P}$ from the rest of the multiplets are:

(i) a 45 dimensional Higgs field $A$ with a vev pattern complimentary to the Dimopoulos-Wilczek type i.e. $< A > = \ell_2 \otimes \text{Diag}(0, 0, 0, b, b)$ that couples to $P$ and $\bar{P}$ as $P\bar{P}$.

(ii) another 16, 16 pair $C$ and $\bar{C}$ with vevs along the right handed neutrino direction that couple to $P$ and $\bar{P}$ as $C\bar{P} + \bar{C}P$ where $A$ is another 45 which does not have any vacuum expectation value.

In order to understand how this works it is best to decompose the representations according to their transformation properties under $SU(4)_C \times SU(2)_L \times SU(2)_R$. Then
Now the vev of $A$ is along the $(1,1,3)$ direction and hence the coupling $PA\bar{P}$ makes all the fields in $(\bar{4},1,2)$ and $(4,1,2)$ of $P$ and $\bar{P}$ massive. Since the vevs of $C$ and $\bar{C}$ are along the right handed neutrino direction the only coupling which could potentially give mass to the $(4,2,1)$ of $P$ is of the form

$$<4,1,2>_C(6,2,2)_{\bar{A}}(4,2,1)_P$$

However because the 6 of $SU(4)$ is antisymmetric in its indices this fails to give mass to the lone $SU(2)_L$ doublet in $P$ which is uncharged under $SU(3)_C$. This doublet has the same quantum numbers as $H_d$ of the standard model. In exactly the same manner a doublet in $\bar{P}$ which has the same quantum numbers as $H_u$ is left massless. It is then a straightforward matter to to mix the doublets in a pair of 10s with those in the 16s so that the light MSSM doublets emerge as linear combinations of the 10s and the 16s.

In order to illuminate our our main ideas we construct an SO(10) model with the following field content in the Higgs sector: one 54 (S); two 45’s ($\bar{A}, \bar{A}$), three pairs of $16 \oplus \bar{16}$ (denoted by $C \oplus \bar{C}, P \oplus \bar{P}$ and $D \oplus \bar{D}$), two 10’s ($H_{1,2}$) and three singlets ($T, T', Y$). By analysing the supersymmetry preserving conditions at the GUT scale we will show that there is a vacuum where only $S, A, C \oplus \bar{C}, T, T'$ have vevs with the new missing vev pattern for the $A$. We then show using the rest of the multiplets that one can implement our new doublet triplet splitting suggestion; the resulting MSSM doublets then lead to a pattern of fermion masses that is free of difficulties such as vanishing CKM angle or bad quark lepton mass relations.

II. THE SYMMETRY BREAKING SECTOR

Let us split the superpotential for our example into three pieces: $W = W_1 + W_2 + W_3$; $W_1$ being responsible for breaking SO(10) down to the MSSM and giving the desired SUSY invariant pattern of vevs, $W_2$ causing the doublet triplet splitting and $W_3$ giving rise to the fermion masses and mixings. Let us first discuss $W_1$:

$$W_1 = \lambda S^3 + M_1 S^2 A + Y(C\bar{C} - M_2^2) + M_3 A^2 + C(A + T)D + C(A + T')\bar{D} + M_0 \bar{D}$$

We have scaled some dimensionless couplings in the above eqn. to 1 for simplicity. Note that the remaining spinors and the adjoint have no role at this stage since they can be self consistently assumed to have zero vev. Let us now assume the ground state form for the various Higgs fields to be as follows: $<A> = i\tau_2 \otimes Diag(a_1, a_2, a_3, a_4, a_5)$, $<S> = 1 \otimes Diag(s_1, s_2, s_3, s_4, s_5)$, $<C_{\nu}> = <\bar{C}_{\nu}> = v_R$ and $<T> = <T'> = 0$. The various F-term vanishings then imply the following equations:

$$3\lambda s_1^2 + 2M_1 s_1 + a_1^2 + \gamma = 0$$

where $\gamma$ denotes the Lagrange multiplier needed to guarantee the tracelessness condition of the 54-plet field. Next we have from $F_A = 0$,

$$a_4 (M_3 + s_4) = 0$$

The vanishing conditions for the other F-terms imply $\Sigma_i a_i = -<T> = -<T'>$, $<D> = <\bar{D}> = 0$, $<Y> = 0$ and $v_R = M_2$. Let us therefore analyze the Eq. (4) and (5). Note first that Eq. (5) implies that either $a_i = 0$ or $s_i = -M_3$. This implies that there is always one solution for which $a_4, 5 \neq 0$ and $s_4 = s_5 = -M_3$ with $a_{1,2,3} = 0$ and $s_{1,2,3}$ arbitrary. Eq. (2) then implies that $s_\alpha (\alpha = 1, 2, 3)$ satisfy the equation

$$3\lambda s_\alpha^2 + 2M_1 s_\alpha + \gamma = 0$$

We can now choose $s_{1,2,3}$ all equal as a solution of the above equation. The tracelessness condition on $S$ then implies that $s_1 = 2M_3/3$. Then equation (6) determines $\gamma$ in terms of $M_1$ and $M_3$. Once $\gamma$ is known we can use equation (4) to determine $a_4, 5 \neq 0$. Hence all the vevs have been determined and we have established that the desired new missing vev pattern is a consistent solution of all vanishing F-term equations.
III. DOUBLET-TRIPLET SPLITTING AND THE MSSM DOUBLETS:

In order to establish that our new vev pattern indeed leads to a useful pattern of doublet-triplet splitting, let us write down $W_2$:

$$W_2 = f_1 P A P + f_2 C A P + f_3 C A P + h_1 C P H_1 + h_2 C P H_2 + M_4 H_1^2 + M_5 H_2^2 + M_6 A^2$$  \hspace{1cm} (7)

Since the all the fields in above equation other than $C, \bar{C}$ and $A$ have zero vev, we can easily write down the doublet mass matrix which is represented schematically as:

$$M_D = \begin{pmatrix} \bar{P}_u & H_{1u} & H_{2u} \end{pmatrix} \begin{pmatrix} 0 & <\bar{C}> & 0 \\ <C> & 0 & M_4 \\ 0 & M_5 & 0 \end{pmatrix} \begin{pmatrix} P_d \\ H_{2d} \\ H_{1d} \end{pmatrix}$$  \hspace{1cm} (8)

where the rows and columns denote the $SU(2)_L$ doublets in the $10$’s and the spinors. From this equation we see that the above matrix has two zero eigenstates: $H_u \equiv c_1 \bar{P}_u + s_1 H_{2u}$ and $H_d \equiv c_2 P_d + s_2 H_{1d}$. The remaining four doublets pick up GUT scale mass.

Turning to the color triplet mass matrix, let us think in the $SU(5)$ submultiplet language. Note that since $C$ and $\bar{C}$ acquire vevs along the $SU(5)$ singlet direction, the $f_{2,3}$ couplings gives masses to the $SU(5)$ $10$ and $\bar{10}$ multiplets in the spinors by combining them with the appropriate $SU(5)$ $10$ and $\bar{10}$ fields from the $A$. Thus we only have to consider the mass matrix for the triplets from the $SU(5)$ submultiplet $5$ and $\bar{5}$’s from the spinors and the $H_{1,2}$. The resulting mass matrix for them can be written schematically as:

$$M_T = \begin{pmatrix} \xi_1 & \xi_2 & \xi_P \end{pmatrix} \begin{pmatrix} M_4 & 0 & <\bar{C}> \\ 0 & M_5 & 0 \\ 0 & <C> & <A> \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_P \end{pmatrix}$$  \hspace{1cm} (9)

Clearly this matrix has no zero eigenvalues implying that all the color triplet states are now massive and at the GUT scale. Thus doublet triplet splitting has been successfully implemented. Since $H_u$ and $H_d$ emerge from different multiplets of $SO(10)$, realistic up and down mass matrices can be constructed in a straightforward manner.

While this the most general scenario for doublet-triplet splitting, variations on this theme can be constructed in particular applications. Below we present one such case which helps in a natural understanding of why the top quark is so much heavier than the bottom quark. In the subsequent section we show how the same model can provide us with a type II seesaw realization which can generate a nonhierarchical pattern for neutrino masses.

IV. UNDERSTANDING THE TOP-BOTTOM MASS HIERARCHY

In the $SO(10)$ models quark lepton masses arise from two kinds of terms: (i) renormalizable terms involving the $10$ Higgses of the form $\psi\psi H$ where the $\psi_a$ denotes the matter field spinor for generation $a = 1, 2, 3$ and (ii) nonrenormalizable terms of several different kinds of which the one of our interest are $\psi\psi\bar{P}C$ or $\psi\psi\bar{P}C$ provided the MSSM doublets contain the doublet pieces from the corresponding $H, P, \bar{P}$ after doublet-triplet splitting. Since the nonrenormalizable terms are suppressed by some higher mass scale such as the string scale or the Planck scale, their contribution to the quark lepton masses are naturally suppressed by a factor $M_U/M_{\ell}$ or $M_U/M_P$ which are respectively of order $1/20$ or $1/100$ compared to the mass terms that arise from the renormalizable terms. It is then clear that if the $H_d$ of MSSM consists solely of the doublet in the spinor Higgs $P$ whereas the $H_u$ consists of the doublet in $10$ with or without a contribution from $\bar{P}$, then the bottom quark mass is automatically suppressed compared to the top quark.

The particular appeal of this idea for the present doublet triplet splitting scenario is that our primary light doublet comes from the spinor Higgs fields and therefore all we have to do is to mix in $10$ component in the light $H_u$ that to start with consists only of the doublet from the $16$. This is easily achieved if in the model discussed in the previous section we drop the $H_1$ field which means that the doublet mass matrix at the GUT scale looks like

$$M_D = \begin{pmatrix} \bar{P}_u & H_{2u} \end{pmatrix} \begin{pmatrix} 0 & <\bar{C}> \\ 0 & M_5 \end{pmatrix} \begin{pmatrix} P_d \\ H_{2d} \end{pmatrix}$$  \hspace{1cm} (10)

The light MSSM doublets are then given by $H_d \equiv P_d$ and $H_u \equiv \alpha H_{2u} + \beta \bar{P}_u$. This is the desired combination that naturally explains the top-bottom mass hierarchy as explained in the previous paragraph. Clearly, by appropriate
choice of the various renormalizable and nonrenormalizable terms, we can obtain a realistic pattern of masses and mixings.

Two comments are in order here: (i) the model outlined above is more economical in the sense that we use only one 10 rather than two commonly used when the Dimopoulos-Wilczek pattern is used; (ii) furthermore in models that use DW pattern, it highly nontrivial to have the $H_d$ purely as coming from the 16-Higgs field, whereas in our case it is relatively straightforward.

V. TYPE II SEESAW FORMULA AND NONHIERARCHICAL NEUTRINO MASSES

Let us begin by reminding the reader that in SO(10) models where the seesaw mechanism is implemented via the 126 Higgs fields, the neutrino mass has two contributions: one which depends quadratically on the quark masses and a second one that depends quadratically on the electroweak scale. When the first one alone appears, we will call it type I seesaw formula. When both appear, we will call it type II seesaw formula\[3\]. Since the second one is not related to the quark or lepton masses, it is in general nonhierarchical. Furthermore, the new contribution depends on the magnitude of the vev of the $SU(2)_L$ triplet field (to be denoted $\Delta_L$) in the 126 representation, which in turn depends on the mass of the $\Delta_L$. If we interested in a contribution to $m_\nu$, of order of an eV, one would expect a mass of the $\Delta_L$ to be of order $10^{12}$ GeV or so. This requires an intermediate scale unification.

On the other hand, when there are no 126 representations in the theory as is apparently the case in string derived models\[3\] small neutrino masses arise via the usual seesaw mechanism (type I) once we include nonrenormalizable terms of the form $\psi C/M$ which will generate the mass matrix of the right handed Majorana neutrinos with arbitrary texture. To the best of our knowledge, this was considered to be the only contribution to the neutrino masses leading to belief that the neutrino masses in such models will be necessarily hierarchical.

We however find that if the MSSM doublet $H_u$ contains a piece from the 16 Higgs field, a new contribution to the light neutrino mass matrix can arise from the nonrenormalizable terms of the form $\psi P \bar{\psi} / M$. This then leads to the type II seesaw formula and hence in our case depending on the model, we can have either a hierarchical or a nonhierarchical mass pattern for neutrinos.

The magnitude of the nonhierarchical contribution depends on what is chosen for $M$. If we choose $M = M_U$ as would be the case in the minimal model, the nonhierarchical contribution can be $\sim 10^{-3}$ eV. They can be larger if $M$ is a smaller mass than the GUT scale. The latter situation can happen for example in a model which has an extra singlets $Z_1$ and a superpotential of the form:

$$W_4 = \psi \bar{P} Z_1 + \lambda' Z_1^2 \bar{C} C / M_{Pl}$$

(11)

After integrating out the $Z_1$ field we get a value for the scale $M = \lambda' v_P^2 / M_{Pl}$ and is thus smaller than $M_U$ by at least a couple of orders of magnitudes or so. They can therefore easily lead to new contributions to the neutrino masses of order of an eV, as needed, say for the hot dark matter component of the universe. A further implication is that in this model the unification is still a single stage type (i.e. no intermediate scales).

A very simple modification of Eq. 11 can also lead to a maximal mixing between the $\nu_\mu$ and $\nu_\tau$. For instance if we choose $W_4 = MT \bar{T} P + \bar{P} \psi_3 \bar{P}$, then this leads to maximal mixing between the $\nu_\mu$ and $\nu_\tau$. The splittings can then arise out of the usual seesaw contributions. Of course to solve the solar neutrino problem, one must invoke the sterile neutrino.

To summarize this section, even though the SO(10) model discussed in this paper does not have 126 Higgs fields, one can have significant new contributions to neutrino masses that alter the hierarchical pattern expected from the usual type I seesaw formula. The origin of these new contribution is linked to the new way of implementing the doublet-triplet splitting advocated in this paper.

In conclusion, we have presented a new mechanism for doublet-triplet splitting in the supersymmetric SO(10) grand unification using a missing vev pattern that is different from the one currently most popular. We have provided an explicit realization of this scheme using only renormalizable terms so that no unwanted states with intermediate scale masses arise and unification of couplings is maintained. We also show how the model can provide a natural explanation of the top bottom mass hierarchy as well as a nonhierarchical contribution to neutrino masses.

This work is supported by the National Science Foundation under grant no. PHY-9802551.

Note added in proof: After this paper was submitted for publication, a paper by G. Dvali and S. Pokorski\[10\] was brought to our attention. This paper has some similarity to certain aspects of our work in that it uses 45 vev patterns similar to ours and uses the doublets in 16 as low energy doublets. However, the ways to understand top bottom splitting is completely different. The remarks in our paper about neutrino mass are also new.
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