Efficient distribution of toy products using ant colony optimization algorithm

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Abstract. CV Atham Toys (CVAT) produces wooden toys and furniture, comprises 13 small and medium industries. CVAT always attempt to deliver customer orders on time but delivery costs are high. This is because of inadequate infrastructure such that delivery routes are long, car maintenance costs are high, while fuel subsidy by the government is still temporary. This study seeks to minimize the cost of product distribution based on the shortest route using one of five Ant Colony Optimization (ACO) algorithms to solve the Vehicle Routing Problem (VRP). This study concludes that the best of the five is the Ant Colony System (ACS) algorithm. The best route in 1st week gave a total distance of 124.11 km at a cost of Rp 66,703.75. The 2nd week route gave a total distance of 132.27 km at a cost of Rp 71,095.13. The 3rd week best route gave a total distance of 122.70 km with a cost of Rp 65,951.25. While the 4th week gave a total distance of 132.27 km at a cost of Rp 74,083.63. Prior to this study there was no effort to calculate these figures.

Keywords: Ant Colony Optimization, Vehicle Routing Problem, wooden toys, delivery route.

1. Introduction

CV. Atham Toys (CVAT) is a small and medium industry (IKM) company engaged in the production and distribution of wooden toys for children. The company was founded by Mr. Thamrin in 2009. The company operates in Ciledug, Tangerang area. In this study, the researchers compared two of the five types of Ant Colony Optimization (ACO) algorithms namely the Ant System (AS) and Ant Colony System (ACS) to determine the best algorithm in order to minimize the distribution costs in CVAT. Currently CVAT plans its delivery routes by determining the nearest travels intuitively, but the selected route is not optimal and the cost is high. With this study CVAT can plan the minimum cost delivery path. Figure 1 shows the spread of the distributors (numbered 1-13) for Jakarta and Tangerang areas, and the central warehouse (point 0) of CVAT. CVAT has more distributors in other provinces like West Java and Central Java. The main motive for this study is to find the best route to deliver the goods such that CVAT can minimize costs and use the saving for other more productive activities.

The following are assumptions and contraints used for this study:

1. The schedule is done only for distributors in Tangerang regional area,
2. Delivery activities are done within one business day,
3. One liter of gasoline can be used to travel as far as 12 km,
4. The types of toys for delivery are always available in the warehouse.
2. Methods

The method used for this study is based on the mathematical formula of ACO.

2.1. Ant Colony Optimization (ACO)

ACO is inspired by the behavior of ants in determining the shortest path to finding and collecting food. The remainder of the pheromone left by the ants on each route is a communication medium for ants to exchange information [1]. ACO offers five types of algorithms: Ant System (AS), Elitist Ant System (EAS), Rank Based Ant System (ASRank), Min-Max Ant System (MMAS), and Ant Colony System (ACS). This study is conducted in the following steps:

2.1.1. Tour Construction

In this rule the ant will explore nodes that have never been visited. This is the probability of the m ants at point i going to point j or other.

\[
P_{ij} = \begin{cases} 
\frac{\tau_{ij}^{\alpha} \eta_{ij}^{\beta} \prod_{k \in F_k} (1 - q_k)}{\sum_{u \in F_k} \tau_{ui}^{\alpha} \eta_{ui}^{\beta} \prod_{k \in F_k} (1 - q_k)} & \text{if } h \in F_k(i) \\
0 & \text{if } h \notin F_k(i) \text{ or other} \\
\end{cases} 
\] (1)

At this stage an ant is positioned at node i to select city j with an implementation rule called pseudorandom proportional rule, as follows:

\[
\text{arg max}_{u \in F_k(r) \{|(\tau(i,j)\mid(\eta(i,j))\beta\} \text{if } q \leq q_0 \text{ (exploitation)}} = S, \text{ otherwise (exploration)} 
\] (2)

where:
\(\tau\) = pheromone value
\(\eta\) = Inverse distance d, worth of 1 / \(d_{ij}\)
\(F_k\) = A collection of towns to be visited by ants in the city i
\(\alpha\) = Pheromone intensity control constant \((\alpha > 0)\)
\(\beta\) = Visibility control constants \((\beta > 0)\)
\(\rho\) = Pheromone evaporation coefficient \((0 \leq \rho \leq 1)\)
\(q\) = Random numbers are uniformly distributed, ranging from 0 to 1
\(q_0\) = Cumulative probability \((0 \leq q_0 \leq 1)\)
\(j\) = Random variables are selected based on the Pij’s opportunity distribution
When an ant in town i must choose a city \( j \) to pass by giving an example of random value \( 0 \leq q \leq 1 \). If \( q \leq q_0 \), then the best side \( j \) will be selected (exploitation).

With the pseudorandom proportional rule, the ant will follow a route with larger pheromone. Global renewal takes place after all the ants have completed their journey [2],[3]. The pheromone level is updated according to following formula:

\[
\tau_{ij}^{new} = \rho \tau_{ij} + \sum_{k}^{n} \Delta \tau_{ij} \tag{3}
\]

Where \( \Delta \tau_{ij} \) is a change of pheromone value between nodes by ants based on the following equation:

\[
\Delta \tau(i,j) = \begin{cases} 
\frac{Q}{L_k} & \text{if } (i,j) \in \text{best global travel} \\
0 & \text{otherwise}
\end{cases} \tag{4}
\]

2.1.2. Mathematical Formula

The objective formula for obtaining the shortest distance can be written as follows:

\[
FT = Z_{min} = \sum_{k=1}^{n} \sum_{i=0}^{\nu_{ij}} \sum_{j=1}^{15} d_{ij} x_{ij}^k \tag{5}
\]

The purpose of making this model to minimize the distance that must be taken by the vehicle when distributing.

Constraints:

\[
\sum_{k=0}^{n} \sum_{j=1}^{15} x_{ij}^k = 1 \quad k = 2,3,4 \tag{6}
\]

Constraint (6) To ensure that distributors are visited only 1 time.

\[
\sum_{l=0}^{n} \left( q_i \sum_{j=1}^{15} x_{ij}^l \right) \leq W \quad W = 24 \text{ koli} \tag{7}
\]

Constraint (7) to ensure that the load of goods transported is less than equal to the maximum capacity of the vehicle.

\[
\sum_{l=1}^{15} x_{0ij}^k = 1 \tag{8}
\]

Constraint (8) is a vehicle only departs once from the warehouse or depot.

\[
\sum_{l=0}^{n} x_{il}^k = (n + 1) \tag{9}
\]

Constraint (9) is the vehicle used also must end in the depot.

\[
\sum_{l=1}^{15} x_{il}^k - \sum_{j=1}^{15} x_{ij}^k = 0 \tag{10}
\]

Constraints (10) of the same vehicle will arrive and depart from the customers served.

\[q_j = 0 \rightarrow x_{ij}^k = 0 \tag{11}\]

Constraint (11) if the demand for goods in the city i does not exist or is zero then the city is not selected in the travel route.

Where:

- \( Z \) = Minimum distance gained
- \( k \) = The number of fleets of vehicles
- \( i = 1, 2, \ldots 15 \)
- \( j = 1, 2, \ldots 15 \)
\( V_i \) = Vertex city \( i \)  
\( V_j \) = Vertex city \( j \)  
\( q_i \) = Demand for goods in \( i \)  
\( X_{kj}^i \) = Has a value of 1 if a vehicle \( k \) runs from distributor \( i \) to distributor \( j \), and has value 0 otherwise.  
n = Number of cities / distributors  
\( W \) = Maximum capacity of vehicle  
\( d_{ij} \) = Distance from city \( i \) to city \( j \)

2.2. Flowchart of the method

Figure 2 display the flowchart of study.

The ACS program is created with the Java programming language using the Netbeans IDE 8.1 Platform. The following is a description of the ACS program.

Identification and determination of parameter values \( \alpha, \beta, \rho, \) and \( m \), the researchers used the Taguchi Design of Experiment (DOE) method [4].
3. Execution of experiment and discussion of results

3.1. Basic data from CVAT
Distribution operational activities of CVAT starts at 08.00 am until 17.00 pm. The fuel cost is Rp 6,500/liter. Vehicle capacity for 1 route of 24 carton consisting of 15 units of toys. Distribution cost in 1 day delivered by CVAT is Rp 150,000. Vehicles currently owned by CVAT is 1 unit of Grand Max Blind Van 1300cc car. CVAT may have to hire a second car.

The study was only conducted on 15 distributors in Tangerang area. Table 1 shows the coordinates of their locations. Table 2 shows the distributors orders in October 2016. Toys must be transported in units of cartons 1 carton can take 15 units of toys. Number of less than 1 is rounded up.

On the calculation of sending unit sales order CVAT October 2016 is divided into 4 weeks: Week1 (October 3-9), Week2 (October 10-14), Week3 (October 17-21), and Week4 (October 24-28). The table shows that the largest order is in week 4 with a total order of 38 cartons or equal to 491 toy units.

Table 1. Coordinate Distributor CVAT in Tangerang Region

| No | Distributor's Name     | X (Easting) | Y (Northing) |
|----|------------------------|-------------|--------------|
| 0  | Gudang CV. Atham       | 705.79963   | 9306.62807   |
| 1  | Popplay BSD            | 686.029439  | 9302.64486   |
| 2  | Popplay VMM            | 683.82541   | 9306.91772   |
| 3  | Popplay GS             | 684.49476   | 9303.2274    |
| 4  | Popplay AS             | 682.719023  | 9309.94893   |
| 5  | Popplay Cipondoh       | 686.624012  | 9312.22912   |
| 6  | Popplay Ciledug        | 690.451186  | 9311.06506   |
| 7  | Popplay Kelapa Dua     | 678.20562   | 9310.3712    |
| 8  | Popplay Taman Royal    | 683.811962  | 9316.33106   |
| 9  | DS Toys                | 689.029877  | 9311.76542   |
| 10 | Rosaria                | 686.70899   | 9307.35121   |
| 11 | Jessica                | 681.85277   | 9303.70999   |
| 12 | Silvia                 | 684.244129  | 9304.19808   |
| 13 | Susan Prelix           | 679.498932  | 9309.67848   |
| 14 | Toys Palace            | 684.921529  | 9309.43017   |
| 15 | Trixie                 | 684.250123  | 9304.20392   |

Table 2. CVAT Distributor Orders in Tangerang Region in October 2016

| No | Distributor's name | Week1 | Week2 | Week3 | Week4 |
|----|--------------------|-------|-------|-------|-------|
|    | Toys Carton        | Toys Carton | Toys Carton | Toys Carton | Toys Carton |
| 1  | Popplay BSD        | 17    | 2     | 20    | 2     | 16    | 2     | 20    | 2     |
| 2  | Popplay VMM        | 28    | 2     | 35    | 3     | 20    | 2     | 21    | 2     |
| 3  | Popplay GS         | 17    | 2     | 27    | 2     | 18    | 2     | 25    | 2     |
| 4  | Popplay AS         | 30    | 2     | 20    | 2     | 20    | 2     | 30    | 2     |
| 5  | Popplay Cipondoh   | 20    | 2     | 18    | 2     | 15    | 1     | 25    | 2     |
| 6  | Popplay Ciledug    | 16    | 2     | 15    | 1     | 18    | 2     | 35    | 3     |
| 7  | Popplay Kelapa Dua | 35    | 3     | 21    | 2     | 11    | 1     | 28    | 2     |
| 8  | Popplay Taman Royal| 22    | 2     | 25    | 2     | 18    | 2     | 16    | 2     |
| 9  | DS Toys            | 0     | 0     | 0     | 0     | 0     | 0     | 28    | 2     |
| 10 | Rosaria            | 121   | 9     | 43    | 3     | 49    | 4     | 109   | 8     |
| 11 | Jessica            | 36    | 3     | 78    | 6     | 70    | 5     | 53    | 4     |
| 12 | Silvia             | 0     | 0     | 0     | 0     | 17    | 2     | 0     | 0     |
| 13 | Susan Prelix       | 0     | 0     | 9     | 9     | 0     | 0     | 0     | 0     |
| 14 | Toys Palace        | 0     | 0     | 0     | 0     | 0     | 0     | 101   | 7     |
| 15 | Trixie             | 0     | 0     | 0     | 0     | 117   | 8     | 0     | 0     |
|    | TOTAL              | 342   | 29    | 311   | 34    | 389   | 33    | 491   | 38    |

Both tables 1 and 2 are used as inputs for the developed ACS-VRP program. Table 1 provided the distance matrix, while table 2 provided the demands for each distributor.
3.2. Data processing and calculations.

3.2.1. Program Formulation for Ant Colony System Algorithm (ACS) for VRP

Amongst Ant Colony Optimization (ACO) algorithm family, ACS algorithm is the more optimal algorithm compared to Ant System algorithm (AS). ACS algorithm can produce the best tour in faster time compared to AS. In addition, ACS was proven to be able to solve bigger problems (in terms of numbers of cities) compared to AS. The number of cities that ACS can test is ≥ 30 cities, while for the AS the number of cities that can be tested is <30 cities [5].

Based on calculations conducted in this study, the results of the calculation algorithm AS result of the smallest total distance is equal to 63.90 km. While using the total ACS algorithm the smallest distance obtained is 60.47 km. So it can be analyzed that on the same problem calculation by using ACS algorithm yield more optimal result compared to using AS. The formulation of an ACS program for solving VRP problems using Java programming languages can be analyzed that, the use of the program can facilitate the completion of the calculation. This is because the greater the number of distributors calculated then the time required for completion will be longer [2].

3.2.2. Parameter Value Determination Using Design of Experiment (DoE)

In this study 4 parameters are defined. α is the parameter for ant pheromone intensity trace controller, β is visibility control parameter, ρ is pheromone evaporation coefficient, and m is the number of ants. The combination of these factors can influence the determination of the best route and the total route followed by the ants. To facilitate testing, the researchers defined three levels of each factor, i.e. low, medium, and high. In order to get accurate results, the researcher conducted 9 experiments (Pn) and each combination was tested 3 times. Using the Minitab 16 software, Table 3 shows the combination of the four parameters.

| Experiment (Pn) | Level | α | β | ρ | m |
|----------------|-------|---|---|---|---|
| 1              | 1     | 1 | 2 | 0.1 | 2 |
| 2              | 1     | 2 | 1 | 4  | 0.45 | 3 |
| 3              | 1     | 3 | 3 | 6  | 0.9  | 4 |
| 4              | 2     | 1 | 2 | 2  | 0.45 | 4 |
| 5              | 2     | 2 | 3 | 4  | 0.9  | 2 |
| 6              | 2     | 3 | 1 | 6  | 0.45 | 3 |
| 7              | 3     | 1 | 3 | 2  | 0.9  | 3 |
| 8              | 3     | 2 | 1 | 4  | 0.1  | 4 |
| 9              | 3     | 3 | 2 | 6  | 0.45 | 2 |

Figure 3 presents the output results of the Taguchi method in Table 3. The Minitab 16 program results initially as the number of known factors with the Factors description and the number of combinations of parameters represented by the description of Runs. Symbols P1, P2, etc are the number of experiments conducted by researchers. So the experiments performed have a signal to noise value (S/N) and means shown in Figure 3. Parameters with the smallest means of each factor are selected.

From the experimental results of determining the parameters for the VRP settlement, the parameters with the smallest means are obtained, i.e. the parameter levels are: α at level 1, β at level 3, ρ at level 1, and m at level 1.

To verify the correctness of the mathematical formulation, the researchers run the program using different combinations of parameters. This is shown in Table 4. With parameter values (1) of α = 1, β = 6, ρ = 0.1; and m = 2, the total distance solution produced is 120.84 km. While the parameter values (2) of α = 3, β = 4, ρ = 0.9; and m = 4, the total distance solution produced is 130.83 km. This is consistent with an experiment with the Taguchi method, wherein the higher value of the selected parameters results in a higher total distance.
parameter using the Taguchi method produces the least total distance, i.e. alternative (1). $\alpha$ is the parameter for ant pheromone intensity trace controller = 1, $\beta$ is visibility control parameter = 6, $\rho$ is pheromone evaporation coefficient = 0.1, and $m$ is the number of cars = 2.

| No | $\alpha$ | $\beta$ | $\rho$ | $m$ | Total Distance (km) |
|----|---|---|---|---|------------------|
| 1  | 1  | 6  | 0.1 | 2  | 120.837          |
| 2  | 2  | 2  | 0.45| 3  | 127.799          |
| 3  | 3  | 4  | 0.9 | 4  | 130.825          |

| 5/30/2017 9:36:12 PM |

Welcome to Minitab, press F1 for help.

Results for: Worksheet 2

Taguchi Design

Taguchi Orthogonal Array Design

L9(3**4)

Factors: 4  
Runs: 9

Columns of L9(3**4) Array

1 2 3 4

Taguchi Analysis: P1, P2, P3 versus Alpha, Beta, Rho, Semut

Response Table for Signal to Noise Ratios

Nominal is best (10^Log10(Year^2/s^2))

| Level | Alpha | Beta | Rho | Semut |
|-------|-------|------|-----|-------|
| 1     | 14.44 | 22.92| 13.00| 35.00 |
| 2     | 26.50 | 35.58| 20.79| 21.84 |
| 3     | 43.68 | 43.59| 29.06| 23.57 |

Response Table for Means

| Level | Alpha | Beta | Rho | Semut |
|-------|-------|------|-----|-------|
| 1     | 131.0 | 137.7| 131.6| 133.5 |
| 2     | 132.0 | 144.8| 137.1| 139.8 |
| 3     | 132.6 | 132.1| 127.7| 133.1 |

Calculation of Total Best Routes and Cost Using Developed Program

Table 4. Parameter Combination Results

### Table 4. Parameter Combination Results

| No | $\alpha$ | $\beta$ | $\rho$ | $m$ | Total Distance (km) |
|----|---|---|---|---|------------------|
| 1  | 1  | 6  | 0.1 | 2  | 120.837          |
| 2  | 2  | 2  | 0.45| 3  | 127.799          |
| 3  | 3  | 4  | 0.9 | 4  | 130.825          |

**Figure 3.** Output of Taguchi Method in Determining Parameter Experiment with Minitab 16

#### Table 5. Results Calculation of Total Best Routes Using Programs

| Week | 1st route | Cost (Rp) | 2nd route | Total distance (km) | Total Cost (Rp) | Efficiency (%) |
|------|-----------|-----------|-----------|---------------------|----------------|----------------|
| 1    | 0-9-6-5-14-4-13-7-12-15-1-3-11-2-8-0 | 46,171 | 0-10-0 | 20,533 | 124.11 | 66,704 | 56 |
| 2    | 0-1-3-12-15-2-14-4-13-7-9-5-6-0 | 36,878 | 0-8-10-11-0 | 34,217 | 132.27 | 71,095 | 53 |
| 3    | 0-6-9-5-14-4-13-7-8-2-15-12-3-1-0 | 39,458 | 0-10-11-0 | 26,493 | 122.70 | 65,951 | 56 |
| 4    | 0-5-9-6-10-14-4-2-1-0 | 34,830 | 0-13-7-8-11-15-0 | 39,254 | 137.83 | 74,084 | 51 |

From the interviews with resource persons, the current daily distribution cost incurred by the CVAT is Rp 150,000. Table 5 shows the result of route calculation in October 2016. The shortest route is indicated for week 3, with a total distance of 122.70 km, and efficiency of 56%.

### 4. Conclusions

Of the five types of ACO algorithms to solve CVAT problem, the researchers concluded that the Ant System (AS) algorithm is better than the other algorithms. Due to the selection process the next point
becomes easier based on pheromone updates on the edge which is the best tour. The number of ants used in the US and ACS should be more than one. ACS algorithm can solve more problems that is $\geq 30$ cities.

The parameters designed in the ACO algorithm are the value of the pheromone instability control constant ($\alpha$), the visibility control constant ($\beta$), the pheromone evaporation coefficient ($\rho$), and the number of ants ($m$). The found optimal parameters are $\alpha = 1$, $\beta = 3$, $\rho = 1$, and $m = 1$.

The optimal route path and minimum cost for CVAT for October 2016 are as follows. For the 1st week total distance of 124.11 km at a cost of Rp 66,703.70, and efficiency = 56%. At week 2nd the total distance is 132.27 km at a cost Rp 71,095.13, and efficiency = 53%. In the 3rd week the total distance is 122.70 km at a cost Rp 65,951.25, and efficiency = 56%. At week 4th the total distance of 137.83 km with a cost of Rp 74,083.63, and 51% efficiency.

The optimum number of vehicles owned by CVAT to support its distribution activities is 2 four-wheeled vehicles with a capacity of 24 cartons.

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