Cooling of a $\Lambda$-type three-level atom in a high finesse optical cavity

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Abstract. A theoretical study is carried out for the cavity cooling of a $\Lambda$-type three level atom in a high-finesse optical cavity with a weakly driven field. Analytical expressions for the friction, diffusion coefficients and the equilibrium temperatures are obtained by using the Heisenberg equations, then they are calculated numerically and shown graphically as a function of controlling parameters. For a suitable choice of these parameters, the dynamics of the cavity field interaction with the $\Lambda$-type three-level atom introduces a sisyphus cooling mechanism yielding lower temperatures below the Doppler limit and allowing larger cooling rate, avoiding the problems induced by spontaneous emission.

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1 Introduction

The technical developments of laser cooling and trapping greatly promote atom manipulation, and have opened a new period in cavity QED experimentation. New cavity cooling mechanism[1,2], has been proposed for a single atom strongly coupled to a high-finesse optical cavity, and the cooling mechanism results in an extended storage time and improved localization of atoms. An estimate shows that the observed cooling rate is at least five times large than the cooling rate which can be achieved by free-space cooling methods for comparable excitation of the atom[2]. Compared to all conventional laser cooling methods[3,4,5], cavity cooling does not rely on repeated cycles of optical pumping and spontaneous emission of photons by the atom, but on the escape of photons from the cavity. Moreover, as there is no requirement in cavity cooling for a closed multilevel system, it is an attractive approach for creating ultracold molecules[6,7]. Recently, cold atoms and molecules[8,9,10,11] are now the basis of many new areas of fundamental physics and technology, and are the central to investigation of the Bose-Einstein condensates[12,13], and quantum information processing[14], etc.

A single atom coupled to a single mode of an optical cavity is the archetype model of dissipative electrodynamics. As is well known, the atom-cavity system has two important characteristics, the first is the cavity mirrors confining the light, which can lead to significantly modified of atom by the cavity field. On the contrary, the back action of the atom on the intra-cavity field is also notable; The second is the cavity mirrors offering an extra loss channel, which is responsible for a new dissipative and can efficiently damp atomic motion[12,15,16]. It is well known that the cavity cooling depends on the optical forces, as well as on the dissipation properties. This motivates us to investigate these forces and dissipation properties in detail. These forces are quite substantial, which can be viewed as the dipole force and friction force. However, dissipation inevitably leads to momentum diffusion, which will heat the atom. In our paper, there are two major contributions to the momentum diffusion. One is the random momentum transfer of absorbed and emitted photons, the second referred to the momentum diffusion is due to the fluctuations of the dipole force. Finally, we can obtain the equilibrium temperatures when the contributions of the friction and heating cancel.

Even though a two-level atom model is usually used to analyze the cavity cooling in a high-finesse optical cavity[17,18,19,20,21], the nature of a three-level atom-cavity system is quantitatively different from that of a two-level atom system. For a there-level atom in the optical cavity, there are more controlling parameters, namely, two detunings, two Rabi frequencies, cavity detunings, etc. Moreover, the extended level structure also provides flexibility. Because of these reasons, it is of great interest to extend the investigation on the optical forces and momentum diffusion on the three-level atom in a high-finesse optical cavity. Very recently, we have studied the friction force of a $V$-type three-level atoms in a high-Q cavity[22]. Here, we will concentrate our attention on the variation of the
friction, diffusion coefficients and the equilibrium temperature of a $\Lambda$-type three-level atom in high-finesse cavity, then the full analytical expressions of the friction, diffusion coefficients and the equilibrium temperature are obtained. For a suitable choice of the parameters, we can obtain lower equilibrium temperatures than the Doppler limit. For simplicity, we will only consider one-dimensional situation.

The paper is organized as follows. In Section 2 we will give the full analytical solutions for the optical forces and friction coefficient by using Heisenberg equations. According to quantum regression theorem we obtain the momentum diffusion and get the equilibrium temperature in Section 3. Section 4 we study the variations of the friction, diffusion coefficients and the equilibrium temperature with detunings, coupling strengths, etc. The friction, diffusion coefficients and the equilibrium temperature are calculated numerically and shown graphically as a function of controlling parameters. Some concluding remarks are presented in Section 5.

2 Dipole force and friction force in the $\Lambda$-type three-level atom

Consider a $\Lambda$-type three-level atom strongly coupled to a single mode of the electromagnetic field contained in a high-finesse optical cavity with the cavity photons decay at a rate $\kappa$. The schematic sketch of the basic system is given in Fig. 1. The upper level is $|3\rangle$ (energy $0$), while the two lower levels are $|2\rangle$ (energy $-\hbar\omega_{32}$) and $|1\rangle$ (energy $-\hbar\omega_{31}$). The transition between $|3\rangle$ and level $|2\rangle (|1\rangle)$ is mediated by frequency $\omega_{32} (\omega_{31})$. The coupling strengths are denoted as $g_1(x) = g_0 \cos(k_1 x)$ and $g_2(x) = g_0 \cos(k_2 x)$, respectively, $k_1$, $k_2$ are the wave vector. The transition between states $|2\rangle$ and $|1\rangle$ is forbidden. The state $|3\rangle$ decays radiatively into $|1\rangle$ ($|2\rangle$) at rate $\Gamma_1$ ($\Gamma_2$). In general, the system is pumped along the cavity axis by a coherent laser field of frequency $\omega_p$ and effective amplitude $\eta$. The master equation describing the model in Fig. 1 is given in the rotating-wave approximation and in a frame rotating with the pump frequency $\omega_p$ as [20]

$$\dot{\rho} = -\frac{i}{\hbar}[H_{ac}, \rho] - \frac{i}{\hbar}[H_p, \rho] + L_{\kappa \rho} + L_{\gamma \rho},$$

where

$$H_{ac} = -\hbar \Delta_c a^\dagger a + \hbar \Delta_1 A_{11} + \hbar \Delta_2 A_{23} + \hbar g_1(x)(a^\dagger A_{13} + A_{31} a) + \hbar g_2(x)(a^\dagger A_{23} + A_{32} a)$$

$$H_p = i\hbar \eta (a^\dagger - a),$$

$$L_{\kappa \rho} = \kappa (2 a \rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a),$$

$$L_{\gamma \rho} = \Gamma_1 (2 A_{13} \rho A_{31} - \rho A_{31} A_{13} - A_{31} A_{13} \rho) + \Gamma_2 (2 A_{23} \rho A_{32} - \rho A_{32} A_{23} - A_{32} A_{23} \rho)$$

with the atomic detunings $\Delta_1 = \omega_p - \omega_{31}$ and $\Delta_2 = \omega_p - \omega_{32}$, and cavity detuning $\Delta_c = \omega_p - \omega_c$. The term $H_{ac}$ represents the energy of the atomic system and the interaction of the atom with the cavity field, the term $H_p$ is the pump hamiltonian. $L_{\kappa \rho}$ and $L_{\gamma \rho}$ describe the decay of the resonator mode and the atomic damping to background modes, respectively. Instead of solving the master equation one can equivalently deal with the Heisenberg equations for atomic and cavity filed mode operators. Considering the spontaneous emission and cavity decay, the Heisenberg equations about the atomic coupling to the vacuum field can be obtained [23].

$$\dot{a} = i \Delta_c a - i g_1(x) A_{13} - i g_2(x) A_{23} - \kappa a + \eta,$$

$$\dot{A}_{13} = i \Delta_1 A_{13} + i g_1(x) (A_{33} - A_{13}) a - \Gamma_1 A_{13},$$

$$\dot{A}_{23} = i \Delta_2 A_{23} + i g_2(x) (A_{32} - A_{23}) a - \Gamma_2 A_{23},$$

where the atomic operator $A_{ij} = |i\rangle \langle j|$ satisfies the commutator relation $[A_{ij}, A_{kl}] = \delta_{il} \delta_{jk} - \delta_{ik} \delta_{jl}$. In order to linearize Eq. (3) to Eq. (5), we assume that the cavity is very weakly driven then the approximation of small saturation and weak excitation of an atom can be fitted [24-28]. In this case there is at most one photon in the cavity and we assume that $A_{11} + A_{22} + A_{33} = 1$. For simplicity, we neglect the population of the excited state and assume $\langle A_{11} \rangle = 0$ and $\langle A_{22} \rangle = 1 - \langle A_{22} \rangle$. Thus,

$$\langle (A_{33} - A_{13}) a \rangle = -F(a),$$

$$\langle (A_{33} - A_{22}) a \rangle = -(1 - F(a)).$$

Then the time evolution of the expectation values can be written as

$$\langle \dot{a} \rangle = \left( i \Delta_c \langle a \rangle - i g_1(x) \langle A_{13} \rangle - i g_2(x) \langle A_{23} \rangle - \kappa \langle a \rangle + \eta \right),$$

$$\langle \dot{A}_{13} \rangle = \left( i \Delta_1 \langle A_{13} \rangle - i g_1(x) \langle a \rangle - \Gamma_1 \langle A_{13} \rangle \right),$$

$$\langle \dot{A}_{23} \rangle = \left( i \Delta_2 \langle A_{23} \rangle - i (1 - F) g_2(x) \langle a \rangle - \Gamma_2 \langle A_{23} \rangle \right).$$

With the following definitions:

$$I = \begin{pmatrix} \eta & 0 \\ 0 & 0 \end{pmatrix},$$

$$Y = \begin{pmatrix} a & A_{13} \\ A_{23} & 0 \end{pmatrix},$$

$$Z = \begin{pmatrix} i \Delta_c - \kappa - i g_1(x) - i g_2(x) & 0 \\ -i(1 - F) g_2(x) & i \Delta_1 - \Gamma_1 \end{pmatrix},$$

$$\dot{\rho} = \frac{i}{\hbar}[H_{ac}, \rho] - \frac{i}{\hbar}[H_p, \rho] + L_{\kappa \rho} + L_{\gamma \rho},$$

$$\rho = Y \rho \ Y^\dagger - \frac{1}{\hbar^2} \{ Z \rho, Z^\dagger \}.$$
the system of linear differential equations (7) can be written in a compact matrix notation

$$
\langle \dot{Y} \rangle = Z(Y) + I_0.
$$

(9)

The steady state solution of this system of linear differential equations is given by

$$
\begin{align*}
(A_{13})_0 &= \frac{ig_1(x)F(\alpha)}{(i\Delta_1 - \Gamma_1)}, \\
(A_{21})_0 &= \frac{ig_2(x)(1 - F)(\alpha)}{(i\Delta_2 - \Gamma_2)}.
\end{align*}
$$

(10)

Inserting Eqs. (10) into the amplitude of the field we find

$$
\langle \alpha \rangle = (i\Delta_e - \kappa - F\gamma_1(x) - iFU_1(x) - (1 - F)\gamma_2(x) - i(1 - F)U_2(x))\langle \alpha \rangle + \eta,
$$

(12)

where

$$
\begin{align*}
U_1(x) &= \Delta_1 \frac{g_1^2(x)}{\Delta_2 + \Gamma_1}, U_2(x) = \Delta_2 \frac{g_2^2(x)}{\Delta_2 + \Gamma_2}, \\
\gamma_1(x) &= \Gamma_1 \frac{g_1^2(x)}{\Delta_2 + \Gamma_1}, \gamma_2(x) = \Gamma_2 \frac{g_2^2(x)}{\Delta_2 + \Gamma_2}.
\end{align*}
$$

(13)

The dipole force acting on the rest A-type three-level atom is obtained,

$$
\mathbf{F}(x) = \mathbf{P} = \frac{i}{\hbar}[\mathbf{H}, \mathbf{P}] = -\hbar \nabla g_1(x)(a^\dagger A_{13} + A_{31}a) - \hbar \nabla g_2(x)(a^\dagger A_{23} + A_{23}a),
$$

(14)

and inserting Eqs. (12) yields

$$
\langle \mathbf{F}(x) \rangle_0 = -\hbar \eta^2 \left( \frac{F(2 + F)(\Delta_2^2 + \Gamma_1^2)\Delta_1(\nabla g_1(x))^2}{\text{det}(Z)^2} + \frac{(1 - F)(3 - F)(\Delta_2^2 + \Gamma_1^2)\Delta_2(\nabla g_2(x))^2}{\text{det}(Z)^2} \right),
$$

(15)

where \( \text{det}(Z) \) is the determinant of \( Z \), which is given by

$$
\text{det}(Z) = (i\Delta_e - \kappa)(i\Delta_1 - \Gamma_1)(i\Delta_2 - \Gamma_2) + Fg_1^2(x)(i\Delta_2 - \Gamma_2) + (1 - F)g_2^2(x)(i\Delta_1 - \Gamma_1).
$$

(16)

The expression is the dipole force. It is straightforward to find the expression for the friction force, the linear velocity dependence of the force for small velocities \( (kv << \kappa) \) due to the cavity dynamics. Use Eq. (12) in power of \( v \) to obtain the operator expectation values to first order [20],

$$
\langle a \rangle_1 = \frac{v \cdot \nabla \langle a \rangle_0}{\Theta},
$$

(17)

where \( \Theta = (i\Delta_e - \kappa - F\gamma_1(x) - iFU_1(x) - (1 - F)\gamma_2(x) - i(1 - F)U_2(x))\langle a \rangle \).

Thus the expectation value of the force operator in first order of the velocity \( v \),

$$
\langle \mathbf{F}(x) \rangle_1 = -\hbar \left( a_1^\dagger a_0 + a_0^\dagger a_1 \right) + \langle \alpha \rangle_1 \langle a \rangle_0 + \langle a \rangle_0 \langle a \rangle_0^\dagger \langle \mathbf{F}(1 - F)U_1(x) + (1 - F)(3 - F)U_2(x) \rangle
\equiv -\beta v,
$$

(18)

\( \beta \) is called friction coefficient. Inserting Equation (17) into an expansion of the force Equation (18). Finally one obtains the friction force in the high-finesse optical cavity,

$$
\langle \mathbf{F}(x) \rangle_1 = -\frac{8\hbar v \eta^2 (\Delta_2^2 + \Gamma_1^2)(\Delta_2^2 + \Gamma_1^2)}{\text{det}(Z)^2},
$$

(19)

where

$$
\begin{align*}
A_{11} &= g_2^2(x)(\Delta_2^2 + \Gamma_1^2)\Delta_1(\nabla g_1(x))^2(1 - F)g_1^2(x)/(1 - F)^2 A_{11} + 2F(1 - F)B_{11} + FC_{11}, \\
A_{22} &= g_2^2(x)(\Delta_2^2 + \Gamma_1^2)\Delta_2(\nabla g_2(x))^2(1 - F)g_2^2(x)/(1 - F)^2 A_{22} + 2(1 - F)B_{22} + (1 - F)C_{22}, \\
B_{11} &= g_2^2(x)(\Delta_2^2 + \Gamma_1^2)(\Delta_1^2 + \Gamma_1^2)(\nabla g_1(x))^2(1 - F)g_1^2(x) + k(\Delta_1 \Delta_2 + \Gamma_1 \Gamma_2 - \Delta_1 \Gamma_2 \Delta_2) + \Delta_1 \Gamma_2 \Delta_2, \\
C_{11} &= (\Delta_2^2 + \Gamma_1^2)^2(2Fg_1^2(x) + (2F^2g_1^2(x) - 2\kappa \Delta_1 \Delta_2 + \Gamma_1 \kappa_2 (\Delta_2^2 + \Gamma_1^2)), \\
C_{22} &= (\Delta_2^2 + \Gamma_1^2)^2((1 - F)^2g_2^2(x) + (2(1 - F)^2g_2^2(x) - 2\kappa \Delta_2 \Delta_1 + \Gamma_2 \kappa_2 (\Delta_2^2 + \Gamma_1^2)).
\end{align*}
$$

(20)

The total force can be written as

$$
\langle \mathbf{F}(x) \rangle_1 = \langle \mathbf{F}(x) \rangle_0 + \langle \mathbf{F}(x) \rangle_1 = \langle \mathbf{F}(x) \rangle_0 - \beta v,
$$

(21)

The equation (21) describes the total force of the A-type three-level atom within the high-finesse cavity, the first term is dipole force, which can be found from the steady state quantum average of the atom-cavity field interaction. The second term is the friction force. When the friction coefficient is positive, corresponding to cooling, while the friction coefficient is negative, leading to heating. The value of the friction coefficient can significantly modify the cooling process, so it is necessary to investigate the friction coefficient.

3 THE DIFFUSION COEFFICIENT AND THE EQUILIBRIUM TEMPERATURE

In the previous section we have obtained the friction coefficient, \( \beta > 0 \), the atom can be cooled. However, momentum diffusion counteracts this cooling and prohibits that the atom completely stops at rest. In our paper, there are two major contributions to the momentum diffusion. The first referred to the momentum diffusion is due to the fluctuations of the dipole force. The other one is the random moment transfer of absorbed and emitted photons.
We first calculate the momentum diffusion coefficient due to the fluctuations of the dipole force. The dipole force operator reads
\[
    F(x) = -\hbar \nabla g_1(x)(a_\dag A_{13} + A_{31} a) \\
    - \hbar \nabla g_2(x)(a_\dag A_{23} + A_{32} a).
\]

(22)

For simplified calculation, we can derive equations for the expectation values of the normally ordered operator products, which yields
\[
    \langle \mathbf{X} \rangle = (\Pi)_{9 \times 9}(\mathbf{X}) + \eta(\Sigma),
\]

(23)

where the several matrices \((M_j)_{3 \times 3}(i, j = 1, 2, 3)\) are given by
\[
    (M_{11})_{3 \times 3} = \begin{bmatrix}
        - (\kappa + I_1) & 0 & - (\Delta_1 - \Delta_e) \\
        0 & - (\kappa + I_2) & 0 \\
        \Delta_1 - \Delta_e & 0 & - (\kappa + I_1)
    \end{bmatrix},
\]

\[
    (M_{12})_{3 \times 3} = \begin{bmatrix}
        0 & 0 & - g_2(x) \\
        0 & 0 & g_1(x) \\
        0 & 0 & - g_1(x)
    \end{bmatrix},
\]

\[
    (M_{13})_{3 \times 3} = \begin{bmatrix}
        0 & 0 & - i g_2(x) \\
        0 & 0 & i g_1(x) \\
        0 & 0 & 0
    \end{bmatrix},
\]

\[
    (M_{21})_{3 \times 3} = \begin{bmatrix}
        0 & \Delta_2 - \Delta_e & 0 \\
        0 & 0 & - g_1(x) \\
        0 & 0 & - g_1(x)
    \end{bmatrix},
\]

\[
    (M_{22})_{3 \times 3} = \begin{bmatrix}
        - (\kappa + I_1) - 2 g_2(x) (1 - F) & 0 \\
        - g_2(x) & - 2 \kappa & 0 \\
        0 & - 2 I_1 & 0
    \end{bmatrix},
\]

\[
    (M_{23})_{3 \times 3} = \begin{bmatrix}
        2 g_2(x) g_1(x) & 0 \\
        0 & g_1(x) & 0 \\
        0 & 0 & 0
    \end{bmatrix},
\]

\[
    (M_{31})_{3 \times 3} = \begin{bmatrix}
        0 & 0 & - g_2(x) \\
        0 & 0 & - g_2(x)
    \end{bmatrix},
\]

\[
    (M_{32})_{3 \times 3} = \begin{bmatrix}
        - g_2(x) (1 - F) & 0 \\
        - g_1(x) F & 0 \\
        0 & 0
    \end{bmatrix},
\]

\[
    (M_{33})_{3 \times 3} = \begin{bmatrix}
        -2 I_2 & 0 & 0 \\
        - (I_1 + I_2) & - i(\Delta_1 - \Delta_2) & 0 \\
        0 & - i(\Delta_1 - \Delta_2) & - (I_1 + I_2)
    \end{bmatrix}.
\]

(25)

and the variable \(X\) is defined as
\[
    X = \begin{bmatrix}
        X_1 \\
        X_2 \\
        X_3 \\
        X_4 \\
        X_5 \\
        X_6 \\
        X_7 \\
        X_8 \\
        X_9
    \end{bmatrix} = \begin{bmatrix}
        a_\dag A_{13} + A_{31} a \\
        a_\dag A_{23} + A_{32} a \\
        - i(a_\dag A_{13} - A_{31} a) \\
        - i(a_\dag A_{23} - A_{32} a) \\
        a^t a \\
        A_{31} A_{13} \\
        A_{32} A_{23} \\
        A_{31} A_{23} + A_{32} A_{13} \\
        A_{31} A_{23} - A_{32} A_{13}
    \end{bmatrix}.
\]

(26)

\[
    \Sigma = \begin{bmatrix}
        A_{13} + A_{31} \\
        A_{23} + A_{32} \\
        - i(A_{13} - A_{31}) \\
        - i(A_{23} - A_{32}) \\
        a + a^t \\
        0 \\
        0 \\
        0 \\
        0
    \end{bmatrix}.
\]

(27)

According to the normally ordered operators, the dipole force can be expressed as
\[
    F(x) = -\hbar \nabla g_1(x) X_1 - \hbar \nabla g_2(x) X_2.
\]

(28)

The part of the diffusion due to the dipole fluctuation can be written as
\[
    D_{dp} = \hbar^2 (\nabla g_1(x))^2 Re \int_0^\infty dt \langle \delta X_1(0) \delta X_1(t) \rangle \\
    + \hbar^2 (\nabla g_2(x))^2 Re \int_0^\infty dt \langle \delta X_2(0) \delta X_2(t) \rangle.
\]

(29)

The Eq. (23) can be written as a homogeneous one for \(\langle \tilde{X} \rangle = \langle X \rangle - X_0\), where \(X_0\) is the steady-state value of \(\langle X \rangle\). So we get
\[
    \frac{\partial \langle \tilde{X} \rangle}{\partial \tau} = (\Pi)_{9 \times 9}(\langle \tilde{X} \rangle),
\]

(30)

Due to the relation,
\[
    \langle \delta a_\mu \delta a_\nu \rangle = \langle a_\nu a_\mu \rangle - \langle a_\nu \rangle \langle a_\mu \rangle,
\]

(31)

We can determine the correlation and get the correlation function \(\langle \delta X_1(0) \delta X_1(t) \rangle\) and \(\langle \delta X_2(0) \delta X_2(t) \rangle\) from the quantum regression theorem,
\[
    \frac{\partial \langle \tilde{X}(0) \tilde{X}(\tau) \rangle}{\partial \tau} = (\Pi)_{9 \times 9}(\langle \tilde{X}(0) \tilde{X}(\tau) \rangle),
\]

(32)
we obtain
\[
\int_0^\infty \langle \delta X(0) \delta X(t) \rangle d\tau = -(\Pi_{1,0}^{-1}(\mathcal{X}(0)) = \Xi, \tag{33}
\]

The diffusion coefficient can be reduced to
\[
D_{dp} = \hbar^2 (\nabla g_1(x))^2 \Xi_{11} + \hbar^2 (\nabla g_2(x))^2 \Xi_{22}. \tag{34}
\]

The second contribution to the diffusion is the random momentum transfer of absorbed and emitted photons, the recoil contributes to the total diffusion by
\[
D_s = \hbar^2 k^2 \frac{g_1^2(x) F_1^2 \Gamma_1 (\Delta_2^2 + \Gamma_2^2)}{|det(Z)|^2} + \hbar^2 (1 - F)^2 \frac{g_2^2(x) (\Delta_2^2 + \Gamma_2^2)}{|det(Z)|^2}. \tag{35}
\]

The total diffusion can be obtained
\[
D = D_{dp} + D_s. \tag{36}
\]

Hence, we can obtain the equilibrium temperature,
\[
k_B T = \frac{D}{\beta}. \tag{37}
\]

Using the Heisenberg equations and the quantum regression theorem we obtain the full analytical expressions for the friction, diffusion coefficients and the equilibrium temperature. Because we have many more parameters available to control the cooling process in the \( \Lambda \)-type three-level atomic system, it is necessary to study the variation of the friction, diffusion coefficients and the equilibrium temperature with these parameters.

### 4 Numerical calculation

Using the theory described in the previous paragraphs, we have given a detailed analysis of the cooling process of the \( \Lambda \)-type three-level atom in the high-finesse optical cavity. For simplicity, we will only consider one-dimensional situation, and assume \( K_1 = K_2 = k, \Gamma_1 = \Gamma_2 = \Gamma \) in the following. Our results are represented in scaled quantities: the parameters \( \Delta_\lambda, \Delta_1, \Delta_2, g_0, \Gamma \) and \( \eta \) are divided by \( \kappa \). The friction, diffusion coefficients and the equilibrium temperature are calculated numerically and shown graphically. For a suitable choice of the controllable parameters, the equilibrium temperature can be cooled down below the Doppler limit. Fig. 2 shows that the average of friction, diffusion coefficients and the equilibrium temperature along the cavity axis over a length of one wavelength are plotted as functions of the cavity detuning \( \Delta_\lambda \). (a) the average of the friction coefficient, (b) the average of the diffusion coefficient, (c) the equilibrium temperature. The parameters are set to \( g_0 = 8\kappa, \Delta_1 = 8\kappa, \Delta_2 = 7\kappa, \Gamma = 1.4\kappa \) and \( \eta = 1.5\kappa \).

**Fig. 2.** The average of the friction, diffusion coefficients and the equilibrium temperature along the cavity axis over a length of one wavelength are plotted as functions of the cavity detuning \( \Delta_\lambda \). (a) The average of the friction coefficient, (b) the average of the diffusion coefficient, (c) the equilibrium temperature. The parameters are set to \( g_0 = 8\kappa, \Delta_1 = 8\kappa, \Delta_2 = 7\kappa, \Gamma = 1.4\kappa \) and \( \eta = 1.5\kappa \).
Fig. 3. (a) the average of the friction coefficient, (b) the average of the cavity diffusion coefficient (c) the equilibrium temperature along the cavity axis over a length of $\lambda$ are plotted as functions of atomic detunings $\Delta_1$ and $\Delta_2$ for $\Delta_0 = 0$. The other parameters are set to $g_0 = 8\kappa$, $\Gamma = 1.4\kappa$ and $\eta = 1.5\kappa$.

Fig. 4. The dependence of (a) the average of friction, (b) the average of diffusion coefficients and (c) the equilibrium temperature on the coupling strengths $g_0 = 4\kappa$, $g_0 = 8\kappa$, $g_0 = 12\kappa$. The other parameters are set to $\Delta_1 = 8\kappa$, $\Delta_2 = 7\kappa$, $\eta = 1.5\kappa$, $\Gamma = 1.4\kappa$.

diffusion coefficients significantly. For a suitable choice of the parameters, we can obtain the low equilibrium temperature, which can be seen from the Fig. 4(c). According to Fig. 3 we give a especial case for a fixed values of $\Delta_1$ and $\Delta_2$. On one hand, we consider the dependence of the average of friction, diffusion coefficients and equilibrium temperature on the coupling strengths, on the other hand, we investigate the average of friction, diffusion coefficients and equilibrium temperature with the pumping strengths.

Firstly, we demonstrate the dependence of the average of friction, diffusion coefficients and equilibrium temperature on the coupling strengths $g_0 = 4\kappa$, $g_0 = 8\kappa$, $g_0 = 12\kappa$ in Fig. 4. The other parameters are set to $\Delta_1 = 8\kappa$, $\Delta_2 = 7\kappa$, $\eta = 1.5\kappa$, $\Gamma = 1.4\kappa$. It can found that the friction and diffusion coefficients are enhanced, while the equilibrium temperature is reduced with increase of the coupling strengths. As is well known, in the strongly coupled regime, a strong friction force exists, there is also a strong heating rate. For a better understanding of the strong friction force, the interpretation can use the dressed states and sisyphus cooling mechanism. For the A-type three-level atom coupled with the cavity, the eigenvalues corre-
Fig. 5. Energies of the A-type three-level atom within cavity system (dressed states) as a function of the axial position for (a) \(g_0 = 12\kappa\), (b) \(g_0 = 8\kappa\).

The energies correspond to the energy are \(E_1\), \(E_2\), and \(E_3\) respectively (\(\hbar \equiv 1\)).

\[
E_1 = \omega_p - \frac{(2\Delta_2 - \Delta_1)}{3} + \sqrt{q - \sqrt{q^2 + p^2}} + \sqrt{q + \sqrt{q^2 + p^2}},
\]
\[
E_2 = \omega_p - \frac{(2\Delta_2 - \Delta_1)}{3} - \frac{1}{2}(\sqrt{q - \sqrt{q^2 + p^2}} + \sqrt{q + \sqrt{q^2 + p^2}})
- \frac{\sqrt{3}}{2}(\sqrt{q - \sqrt{q^2 + p^2}} - \sqrt{q + \sqrt{q^2 + p^2}}),
\]
\[
E_3 = \omega_p - \frac{(2\Delta_2 - \Delta_1)}{3} - \frac{1}{2}(\sqrt{q - \sqrt{q^2 + p^2}} + \sqrt{q + \sqrt{q^2 + p^2}})
+ \frac{\sqrt{3}}{2}(\sqrt{q - \sqrt{q^2 + p^2}} - \sqrt{q + \sqrt{q^2 + p^2}}),
\] (38)

Where

\[
p = -\frac{(2\Delta_2 - \Delta_1)}{3}, \quad q = \frac{(2\Delta_2 - \Delta_1)}{3},
\]

Fig. 6. The dependence of (a) the average of friction, (b) the average of diffusion coefficients and (c) the equilibrium temperature on the pumping strengths \(\eta = 0.5\kappa\), \(\eta = 1.0\kappa\), \(\eta = 1.5\kappa\). The other parameters are set to \(\Delta_1 = 8\kappa, \Delta_2 = 7\kappa, g_0 = 8\kappa, \Gamma = 1.4\kappa\).

\[
\begin{align*}
&+ \frac{(2\Delta_2 - \Delta_1)(g_1^2(x) + g_2^2(x) + 4\Delta_2(\Delta_2 - \Delta_1))}{24} \\
&+ \frac{g_1^2(x)(\Delta_2 - \Delta_1)}{8}.
\end{align*}
\] (39)

Here \(g_1(x)\) and \(g_2(x)\) are the coupling strength, \(\Delta_1 = \omega_p - \omega_{11}\) and \(\Delta_2 = \omega_p - \omega_{22}\) are the atomic detunings. The ground state energy is \(E_0\). Eigenvalues \(E_1\), \(E_2\), and \(E_3\) as a function of axial position are shown in Fig. 5. In this atom-cavity system, cavity cooling arise when the system is excited near a minimum of one of the dressed states, the system will remain in this state while the atom moves further and thus the system energy will vary according to corresponding eigenvalues. For the A-type three-level atom system, if being excited in the potential valley of the dressed state, the stronger of the coupling strength, the steeper hills for the A-type three-level atom has to climb, which can be seen in Fig. 5(a) and in Fig. 5(b). Hence, friction coefficient become stronger with the bigger coupling strength. On the contrary, the momentum diffusion can be affected by the fluctuations of atomic dipole coupled to the cavity field, because the A-type three-level atom ex-
experience a stronger force with the increase of the coupling strength, momentum diffusion coefficient will be enhanced by increase of the coupling strength. However, for a suitable choice of parameters, the atomic cooling temperature can be down to lower than the Doppler temperature limit $T_D$, which can be seen from the Fig. 11(c).

Moreover, we illustrate the dependence of the average of friction, diffusion coefficients and equilibrium temperature on the pumping field strengths $\eta = 0.5\kappa, \eta = 1.0\kappa, \eta = 1.5\kappa$ in Fig. 4. The other parameters are set to $\Delta_1 = 8\kappa, \Delta_2 = 7\kappa, g_0 = 8\kappa, \Gamma = 1.4\kappa$. It can be found that with the increase of the pumping strengths, the friction and diffusion coefficients are enhanced, while the equilibrium temperature is reduced. This is due to the friction and diffusion coefficients are proportion to quadratic of the pumping field, while the equilibrium temperature is proportion to $\hbar\kappa/k_B$, we can obtain the final temperature lower than the Doppler temperature limit.

5 CONCLUSION

In this paper, using the master equation and the Heisenberg equations, we have derived the analytical solution of the dipole force and friction force for the $A$-type three-level atom in the high-finesse cavity. The friction coefficient can significant affect the cooling process, however, momentum diffusion counteracts this cooling and prohibit that the atom completely stops at rest. In our paper, there are two major contributions to the momentum diffusion. One is the random momentum transfer of absorbed and emitted photons, the second referred to the momentum diffusion is due to the fluctuations of the dipole force. According to quantum regression theorem, we obtain the diffusion coefficient and the equilibrium temperature. The friction, diffusion coefficients and the equilibrium temperature are calculated numerically and shown graphically as a function of controlling parameters. For a suitable choice of parameters, the atomic cooling temperature can be down to lower than the Doppler temperature limit.

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