Numerical simulation of a bulb turbine with an eddy-preserving limiter scheme

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Abstract. The eddy-preserving limiter scheme has been demonstrated to outperform the conventional van Albada limiter for Monotone Upstream-centered Schemes for Conservation Laws (MUSCL) for vortical flows. It reduces the dissipation by inactivating the conventional van Albada limiter in the interpolations of the velocity components on the swirl plane of the vortex. In this work, we extend the limiter for the interpolation of pressure, since a minimum pressure often exists along the axis of a free vortex. The conventional van Albada limiter is inactivated in the interpolation of pressure at smooth extrema. A 3D inviscid vortex advection case is employed to demonstrate the effects of the novel scheme. The extended eddy-preserving limiter scheme has been demonstrated to be able to further minimize the dissipation of the vortex than the original eddy-preserving limiter scheme. Finally, the scheme is applied to the BulbT test case and the numerical results are compared against experimental data.

1. Introduction
The draft tube is an important component of a hydro turbine. The role of the draft tube is to decelerate the flow from the turbine runner and convert its dynamic pressure into a rise of static pressure [1]. At part load conditions, the energy is not fully utilized by the runner-shaft-rotor, resulting in high residual swirl when the flow exits the runner. The swirling flow with high angular momentum enters the draft tube and encounters the lower momentum fluid in the inlet conical zone. The shear layer between them gives rise to a large helical precessing vortex, commonly referred to as a vortex rope. Due to the adverse pressure gradient and hydraulic instabilities present in the draft tube, the vortex breaks down and a reverse flow occurs, further complicating the flow phenomena. As a consequence, at part load conditions, the turbine experiences severe low frequency and large amplitude pressure fluctuations induced by the vortex rope in the draft tube. The pressure fluctuations will not only cause variations in the power output, but also endanger other hydro turbine components, especially when its frequency approaches the natural frequency of the turbine [2]. Thus, knowledge of the dynamic load on the turbine is a major concern of the hydraulic community, and the simulation of flows in draft tubes has attracted attention during the past two decades.

In order to accurately capture the flow features, efforts have been made by employing more advanced turbulence models, improving the grid quality and extending the computational domain to the complete hydro turbine. Apart from turbulence models and grid effects, discretization schemes also play a crucial role in the simulation of the vortex rope. With the vast adoption of commercial software, the role of numerical schemes have unfortunately been
removed. It is known that the dissipation error is characterized by a loss of wave amplitude, while the dispersion error is characterized by a phase difference between the numerical and analytical solution. Since the frequency of the vortex rope was well predicted while the pressure pulsation amplitudes were underestimated in most previous studies [1, 3, 4, 5], the inherent dissipation of the numerical schemes requires a careful reexamination.

The eddy-preserving limiter scheme [6] was designed to improve the ability of capturing vortical flow features based on the Monotonic Upstream-centered Scheme for Conservation Laws (MUSCL) scheme. The eddy-preserving limiter scheme has been implemented for turbulent flows in hydraulic turbines and some preliminary results have been presented [7]. As suggested in [6], the application of a similar limiter for the interpolation of other flow variables might further enhance the vortex resolution. In this paper, we extend the eddy-preserving limiter scheme by modifying the limiting algorithm for the interpolation of pressure, since a minimum pressure often exists along the axis of a free vortex. At smooth extrema of the pressure, the limiting is unnecessary and thus the conventional van Albada limiter is inactivated to reduce the dissipation. The smooth region of pressure is detected by comparing the neighbouring second difference of pressure, a criterion proposed by Huynh [8]. The effects of the original and extended eddy-preserving limiter scheme are demonstrated through a 3D inviscid vortex advection case, and then applied to the simulation of the BulbT test case.

2. Methodology
2.1. Governing Equations
Consider the three-dimensional compressible Navier-Stokes equations using Einstein notation,

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}_i}{\partial x_i} - \frac{\partial \mathbf{F}_{vi}}{\partial x_i} = 0,$$

(1)

where the state vector $\mathbf{W}$, the inviscid fluxes $\mathbf{F}_i$, and the viscous fluxes $\mathbf{F}_{vi}$ are given by,

$$\mathbf{W} = \begin{pmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{pmatrix}, \quad \mathbf{F}_i = \begin{pmatrix} \rho u_i \\ \rho u_1 u_i + p \delta_{i1} \\ \rho u_2 u_i + p \delta_{i2} \\ \rho u_3 u_i + p \delta_{i3} \\ \rho E u_i + p u_i \end{pmatrix}, \quad \mathbf{F}_{vi} = \begin{pmatrix} 0 \\ \tau_{ij} \delta_{i1} \\ \tau_{ij} \delta_{i2} \\ \tau_{ij} \delta_{i3} \\ u_j \tau_{ij} + \kappa \frac{\partial E}{\partial x_i} \end{pmatrix}.$$  

(2)

In the above equations, $\rho, p$ and $T$ are the density, pressure, and temperature respectively. The specific total energy is denoted by $E$ and $\mathbf{u} = [u_1, u_2, u_3]^T$ is the velocity vector. $\tau_{ij}$ is the viscous stress and $\delta_{ij}$ is the Kronecker delta function. The governing equations are solved with an in-house second-order finite-volume approach and the k-ω SST turbulence model.

2.2. Equation of state
To simulate incompressible hydro-turbine flows, the stiffened gas law is employed as the equation of state, which is given by,

$$p = (\gamma - 1)\rho e - \gamma p_\infty,$$

(3)

and the speed of sound is computed by,

$$c = \sqrt{\frac{\gamma p + p_\infty}{\rho}}.$$  

(4)

The specific heat ratio, $\gamma$ and $p_\infty$ are two parameters used to define liquids with different properties. The stiffened gas equation of state can provide a reasonable approximation for
Table 1. Parameters for stiffened gas equation of state used by various authors.

| Authors                  | γ    | $p_\infty$ ($\times 10^8$ Pa) | Sound Speed (m/s) |
|-------------------------|------|--------------------------------|-------------------|
| Goncalves & Patella [9] | 1.01 | 0.1211                         | 110.7 (artificial)|
| Chang & Liou [10]       | 1.932| 11.645                          | 1482              |
| Le Métayer et al. [11]  | 2.35 | 10.0                            | 1533              |
| Paillere et al. [12]    | 2.8  | 8.5                             | 1543              |
| Barberon & Helluy [13]  | 3.0  | 8.533                           | 1600              |
| Saurel & Abgrall [14]   | 4.4  | 6.0                             | 1626              |
| Shyue [15]              | 7.0  | 3.0                             | 1449              |
| Luo et al. [16]         | 7.0  | 3.0975                          | 1459              |
| Gallouët et al. [17]    | 7.15 | 3.0                            | 1465              |
| Present paper           | 7.15 | 3.1                            | 1489              |

For water, values for $\gamma$ and $p_\infty$ used by various authors are listed in Table 1. In this paper, to respect the physical speed of sound in water at 20°C, $\gamma$ and $p_\infty$ are chosen to be 7.15 and $3.1 \times 10^8$ Pa respectively.

2.3. Low-speed preconditioning

As a consequence of the large speed of sound, the Mach number is typically very low which makes the system difficult to converge. To alleviate this problem, a low-speed preconditioner [18] is implemented in this work.

$$\mathbf{P}^{-1} \frac{\partial \mathbf{W}}{\partial t} + \mathbf{R} (\mathbf{W}) = 0. \quad (5)$$

The preconditioner for the entropy variables $\mathbf{W}_0 = [p, u, v, w, S]^T$ is

$$\mathbf{P}_0 = \begin{pmatrix} \beta^2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

Since the conservative variables are used in the present work, the preconditioner $\mathbf{P}$ for the conservative variables $\mathbf{W} = [\rho, \rho u, \rho v, \rho w, \rho E]^T$ can be computed by,

$$\mathbf{P} = \frac{\partial \mathbf{W}}{\partial \mathbf{W}_0} \mathbf{P}_0^0 \frac{\partial \mathbf{W}_0}{\partial \mathbf{W}}. \quad (7)$$

For optimal preconditioning, $\beta^2$ should be proportional to the square of the local Mach number $M$, a reference Mach number $M_{\text{ref}}$ is used to avoid singularities at stagnation points. Then the value of $\beta^2$ is modified to account for the viscous effects,

$$\beta^2_{\text{inv}} = K_1 M^2 + K_2 M^2_{\text{ref}}, \quad (8)$$

$$\beta^2_{\text{vis}} = (1 + 2 \mu \frac{1}{\Delta x^3}) \beta^2_{\text{inv}}, \quad (9)$$

where $\mu$ is the cell Reynolds number, $K_1$ and $K_2$ are constants and set to be 1.0 in this paper. The final value for $\beta^2$ must be less or equal to unity, $\beta^2 = \min(\beta^2_{\text{vis}}, 1.0)$. When $\beta^2$ equals unity, $\mathbf{P}$ is an identity matrix and the preconditioner is switched off.
2.4. Monotonic Upstream-centered Scheme for Conservation Laws (MUSCL)

In the MUSCL scheme, the primitive variables $\mathbf{V} = [\rho, u_1, u_2, u_3, p]^T$ are interpolated at the interfaces of cell $i$ and cell $i + 1$ using,

$$\mathbf{V}_{i+\frac{1}{2}}^L = \mathbf{V}_i + \frac{\Phi_i}{4}[(1 - \kappa)\Delta_{i+\frac{1}{2}}^u \mathbf{V} + (1 + \kappa)\Delta_{i+\frac{1}{2}}^c \mathbf{V}], \quad (10)$$

$$\mathbf{V}_{i+\frac{1}{2}}^R = \mathbf{V}_{i+1} - \frac{\Phi_{i+1}}{4}[(1 - \kappa)\Delta_{i+\frac{1}{2}}^u \mathbf{V} + (1 + \kappa)\Delta_{i+\frac{1}{2}}^c \mathbf{V}], \quad (11)$$

where $\Phi_i$ and $\Phi_{i+1}$ are slope limiters, and $-1 \leq \kappa \leq 1$. The upwind and central increments are computed by,

$$\Delta_{i-\frac{1}{2}}^u \mathbf{V} = \mathbf{V}_i - \mathbf{V}_{i-1}, \quad \Delta_{i+\frac{1}{2}}^c \mathbf{V} = \mathbf{V}_{i+1} - \mathbf{V}_i. \quad (12)$$

$$\Delta_{i+\frac{1}{2}}^\kappa \mathbf{V} = \mathbf{V}_{i+1} - \mathbf{V}_i. \quad (13)$$

In our baseline MUSCL scheme, $\kappa = \frac{1}{3}$ and the van Albada limiter is employed,

$$\Phi(R) = \begin{cases} \frac{2R}{R + 1}, & \text{if } R > 0, \\ 0, & \text{if } R \leq 0, \end{cases} \quad \text{where } R_i = \frac{\mathbf{V}_{i+1} - \mathbf{V}_i}{\mathbf{V}_i - \mathbf{V}_{i-1}}. \quad (14)$$

When $\kappa$ is further increased, the artificial dissipation decreases. In the limit, where $\kappa = 1$, a purely central but unstable convective flux calculation scheme is obtained.

2.5. Eddy-preserving limiter scheme

The concept of the eddy-preserving limiter is to prevent the slope limiter to be activated (or to use a less dissipative limiter) during the reconstruction of the velocity component along the tangential direction of a vortex. To identify vortical flow regions, the enhanced swirling strength criterion of Chakraborty et al. [19] is employed, whereby in a vortical region, the velocity gradient tensor $\nabla \mathbf{v}$ possesses a conjugate pair of complex eigenvalues,

$$\sigma(\nabla \mathbf{v}) = \{\lambda_r, \lambda_{cr} + i\lambda_{ci}, \lambda_{cr} - i\lambda_{ci}\}, \quad \{|\lambda_{ci}| > \epsilon, \} \quad (15)$$

where $\epsilon$ is a small positive real number. A local measure for the compactness of the vortical motion is added to further limit the vortical regions to areas where the following condition is satisfied,

$$-\zeta \leq \frac{\lambda_{cr}}{\lambda_{ci}} \leq \delta, \quad (16)$$

where $\zeta$ and $\delta$ are positive thresholds for verifying the compactness of the vortex. The velocity gradient tensor can be decomposed into the following form,

$$\nabla \mathbf{v} = \begin{bmatrix} \hat{u}_r & \hat{u}_{cr} & \hat{u}_{ci} \end{bmatrix} \begin{bmatrix} \lambda_r & 0 & 0 \\ 0 & \lambda_{cr} & \lambda_{ci} \frac{|u_{ci}|}{|u_{cr}|} \\ 0 & -\lambda_{ci} \frac{|u_{ci}|}{|u_{cr}|} & \lambda_{cr} \end{bmatrix} \begin{bmatrix} \hat{u}_r & \hat{u}_{cr} & \hat{u}_{ci} \end{bmatrix}^{-1}, \quad (17)$$

where $\hat{u}_r$, $\hat{u}_{cr}$, and $\hat{u}_{ci}$ are normalized eigenvectors of the velocity gradient tensor. The mapping and transformation matrix from the original Cartesian system $S_0$ to the local vortex system $S_{\omega}$ spanned by $\hat{u}_r$, $\hat{u}_{cr}$, and $\hat{u}_{ci}$, are given by,

$$\hat{\mathbf{M}} : S_0 \mapsto S_{\omega}, \quad \hat{\mathbf{M}} = \begin{bmatrix} \hat{u}_r & \hat{u}_{cr} & \hat{u}_{ci} \end{bmatrix}^{-1}. \quad (18)$$

The algorithm for the eddy-preserving limiting procedure is described below:
1. Calculate eigenvalues of the velocity gradient tensor $\nabla v$. If the velocity gradient tensor has only real eigenvalues, then exit and employ the conventional van Albada limiter.
2. Verify the compactness of the vortical motion using Eqn. (16). If the flow lacks vortical compactness, then exit and employ the conventional van Albada limiter.
3. Calculate eigenvectors of the velocity gradient tensor $\nabla v$, and define the transformation matrix $[M]$.
4. Transform velocity components into the vortex system $S_\omega$.
5. In the axial direction, reconstruct variables using the conventional van Albada limiter with $\kappa = \frac{1}{3}$.
6. In the swirl plane, reconstruct variables using a higher $\kappa$ which leads to less artificial dissipation and inactivate the van Albada limiter by setting $\Phi_i = \Phi_{i+1} = 1$.
7. Transform interpolated velocity components back to the original system $S_0$ and evaluate the fluxes.

2.6. Extended eddy-preserving limiter scheme
To extend the eddy-preserving limiter, the van Albada limiter for the interpolation of pressure is switched off at smooth extrema. By setting $\Phi_i = \Phi_{i+1} = 1$, the interpolation for pressure at smooth extrema is given by,

$$
\begin{align*}
 p_{i+\frac{1}{2}}^L &= p_i + \frac{1}{4}[(1-\kappa)\Delta^u_{i+\frac{1}{2}}p + (1+\kappa)\Delta^c_{i+\frac{1}{2}}p], \\
 p_{i+\frac{1}{2}}^R &= p_{i+1} - \frac{1}{4}[(1-\kappa)\Delta^u_{i+\frac{1}{2}}p + (1+\kappa)\Delta^c_{i+\frac{1}{2}}p].
\end{align*}
$$

(19) (20)

The smooth extrema is detected based on a criterion proposed by Huynh [8]. If the solution is smooth at the extremum, then the second difference in neighbouring cells are comparable. Define the second difference of $p$,

$$
\Delta^2 p = p_{i-1} - 2p_i + p_{i+1}.
$$

(21)

If the following conditions are satisfied,

$$
\begin{align*}
4 \leq \frac{\Delta^2_{i-1}p}{\Delta^2 p} &\leq 5, \\
4 \leq \frac{\Delta^2_{i+1}p}{\Delta^2 p} &\leq 5,
\end{align*}
$$

(22) (23)
then the extremum is considered to be smooth, and the limiting is unnecessary. As remarked by Huynh [8], the factors $\frac{4}{5}$ and $\frac{2}{3}$ can be relaxed to $\frac{2}{3}$ and $\frac{2}{5}$ for the case of linear convection.

3. Numerical Results

3.1. 3D inviscid vortex advection

An inviscid vortex advection case is designed to demonstrate the effects of the eddy-preserving limiter schemes. The direction of advection is perpendicular to the vortex surface, as illustrated in figure 2. The flow field is initialized as an isentropic vortex superimposed by a uniform flow, where $u_\infty = 0$, $v_\infty = 0$ and $w_\infty = 1$. The flow variables are computed by:

$$
\rho = \left[ 1 - \frac{(\gamma - 1)\beta^2}{8\gamma\pi^2} e^{1-r^2} \right]^{1/(\gamma-1)},
$$

$$
u = u_\infty + \delta u = -\frac{\beta y}{2\pi} e^{(1-r^2)/2},
$$

$$
v = v_\infty + \delta v = \frac{\beta x}{2\pi} e^{(1-r^2)/2},
$$

$$
w = w_\infty,
$$

$$
p = \rho^\gamma,
$$

where the centre of the vortex is located at $(x, y) = (0, 0)$, $r = \sqrt{x^2 + y^2}$ represents the distance to the centre of the vortex, and $\beta$ is the vortex strength and set to a value of 5.

The geometry of the computational domain is a rectangular box, where $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, and $0 \leq z \leq 12$. Three set of grids, i.e. the coarse grid where $\Delta x = \Delta y = \Delta z = 0.5$, the medium grid where $\Delta x = \Delta y = \Delta z = 0.25$, and the fine grid where $\Delta x = \Delta y = \Delta z = 0.125$ are employed. The time step is set to be constant $\Delta t = 0.04$ and the final time is $t = 10$.

The test cases are simulated with the baseline MUSCL scheme, the original and the extended eddy-preserving limiter scheme. The computed results are denoted by “MUSCL”, “EDDY”, and “EDDY-P” respectively.

![Figure 3. Velocity profiles at $x = 0$. (MUSCL: •; EDDY: ○; EDDY-P: -; Exact: --.)](image)

The profiles of the $x$ component of velocity and pressure at $x = 0$ and the final time $t = 10$ are shown in figure 3. The profiles computed by the EDDY and EDDY-P schemes are similar, which means the modification for the interpolation of pressure has little impact on the velocity profiles. The predictions of both the EDDY and EDDY-P schemes agree better against the exact solutions than the prediction of the MUSCL scheme, due to their lower numerical dissipations.
Figure 4. Pressure profiles at $x = 0$. (MUSCL: $\rightarrow$; EDDY: $\cdot\cdot\cdot$; EDDY-P: $\cdot\cdot$; Exact: $\cdot\cdot\cdot\cdot$)

The pressure profiles at $x = 0$ and the final time $t = 10$ are shown in figure 4. The EDDY scheme outperforms the MUSCL scheme, while the EDDY-P scheme preserves the pressure minimum even better than the EDDY scheme, which shows that the modification for the interpolation of pressure is able to further reduce the dissipation.

### 3.2. BulbT test case

The BulbT project was initiated at the “Laboratoire de Machines Hydrauliques” (LAMH) of Laval University, and aimed at investigating the flow phenomena in a bulb turbine [20]. The turbine model consists of an intake, a bulb and a draft tube, as illustrated in figure 5. The experiments were conducted at five Operating Points (OP) as shown in figure 6. OP2 is the one closest to the best efficiency point, and the flow phenomenon is less complex compared to other OPs. Therefore, in this work, OP2 is chosen for validating the developed framework.

Figure 5. Turbine model of BulbT [21].

Figure 6. Operating points of BulbT [21].

The grid has 14 million cells and only consists of the hub, the draft tube and an extension as shown in figure 7. No-slip boundary condition is applied to all solid walls on the rotating hub and the stationary draft tube, while slip boundary condition is imposed on the solid walls of the extension. The inlet profile is extracted from a numerical simulation conducted with the complete turbine. At the outlet, a zero pressure boundary condition is imposed. The numerical simulations were performed with the baseline MUSCL scheme, the original and extended eddy-preserving limiter scheme. The dissipation of all three schemes were scaled down with a factor of $\alpha = 0.375$ in computing the convective flux, $F_{i+\frac{1}{2}} = \frac{1}{2}(F(W_{i+\frac{1}{2}}^L) + F(W_{i+\frac{1}{2}}^R) - \alpha P^{-1}(W_{i+\frac{1}{2}}^R - W_{i+\frac{1}{2}}^L))$, where $\lambda$ is the spectral radius of the preconditioned Jacobian.
Figure 7. Geometry of the grid.

Figure 8. Location of 4BY0 [21].

Figure 9. Axial velocity profiles at 4BY0 (a) Full view (b) Zoom-in view. (MUSCL: ▲; EDDY: ●; EDDY-P: ▼; EXP Az0: □; EXP Az180: ○.)

In the experiment, the axial and circumferential velocity profiles were measured by the Laser Doppler Velocimetry (LDV) at 4BY0, which is the green line labeled as Axis B in figure 8. As shown in figure 9 and 10, in general, the velocity profiles computed by all schemes are almost identical, and agree well against the experimental data. However, a zoom-in view in figure 9(b) and 10(b) shows some differences in the central region. In the experiment, there is a backflow region in the centre. The MUSCL scheme shows poor predictions of the backflow region on all grids, while the EDDY and EDDY-P schemes can resolve the backflow region, and improve towards the experimental data. As for the axial velocity, it first decelerates from the centre point gently to a local maximum, which represents the forced vortex induced by the rotating hub. Then it decelerates to zero, and the flow rotational direction is reversed. The circumferential velocity accelerates in the new direction to a local maximum and then decelerates. A shear flow region is formed at approximately \( r/D_{ref} = \pm 0.068 \). The EDDY and EDDY-P schemes outperform the MUSCL scheme significantly in predicting the locations as numerical results computed by the three schemes are denoted by “MUSCL”, “EDDY”, and “EDDY-P” respectively.
The results for the turbulent kinetic energy (TKE) are plotted in figure 11. There are two peaks in the experiment, which corresponds to the shear flows at approximately $r/D_{ref} = \pm 0.068$. Overall, the TKE profiles were underpredicted by all schemes in most of the region. Incorrect TKE peaks appear in the central region, which are mainly due to the deviation of the computed velocity profiles. Compared to the MUSCL scheme, the EDDY and EDDY-P schemes predict slightly higher values at the two side peaks and lower values in the central core region, which leads to better agreement against the experimental data.

The pressure profiles at 4BY0 are compared in figure 12. The numerical result computed
by the EDDY-P scheme on the 50M grid is employed as a reference, due to the lack of an experimental distribution of pressure across the cross-section of the draft tube, and denoted by “EDDY-P-50M”. The EDDY-P scheme was used as the reference for two reasons; first, as the grid is refined, all schemes tend towards the results of the EDDY-P scheme on the 50M grid; and second, the EDDY-P pressure distribution seems more invariant to the grid refinement. Due to the mispredicted gradients of the circumferencial velocity near the central region, MUSCL produces an incorrect peak in the pressure profile. In contrast, the EDDY and EDDY-P schemes produced a flat profile in the central region similar to the result of EDDY-50M. Moreover, compared to the prediction of EDDY, the prediction of EDDY-P is closer to the reference profile EDDY-P-50M, which shows that the dissipation of the scheme is further reduced. In summary, the EDDY and EDDY-P schemes produced better pressure profiles than the MUSCL scheme, and the EDDY-P is less dissipative than EDDY as expected.

The comparisons demonstrate that the original and extended eddy-preserving limiter schemes are capable of producing better predictions for velocity, TKE and pressure profiles than the baseline MUSCL scheme, and the extended eddy-preserving limiter scheme has a lower dissipation for the prediction of pressure than the original eddy-preserving limiter scheme.

4. Conclusions
The eddy-preserving limiter scheme has been extended by employing a new limiting algorithm for the interpolation of pressure. The conventional van Albada limiter is inactivated in the interpolation of pressure at smooth extrema. Numerical simulations of a 3D inviscid vortex advection and the BulbT test case were performed. The extended eddy-preserving limiter scheme has been demonstrated to further minimize the dissipation for vortices over the original eddy-preserving limiter scheme.

Acknowledgements
The authors would like to thank the participants to the Consortium on Hydraulic Machines for their support and contribution to this research project: GE Renewable Energy, Andritz Hydro Canada Inc., Hydro-Québec, Laval University, NRCan, and Voith Hydro Inc. The authors would also like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC).
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