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Calculation of two-size particles filtration in a porous medium

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Abstract. Filtration describes a variety of the construction complex problems: strengthening loose soil to create a solid foundation, the movement of groundwater with solid impurities near underground structures, and many others. A model of two-sized deep bed filtration particles moving with different velocities in a porous medium with three-size pores is considered. The competition of pores and various size particles for deposit formation is modeled. Solutions are constructed at the porous medium inlet and on the concentrations front of the fast particles. For constant filtration coefficients, a global exact solution is obtained. Numerical calculation illustrates the evolution of the filtration process.

1. Introduction

The transportation and retention of solid particles by the fluid flow describe strengthening loose soil to create a solid foundation, the construction of underground and hydraulic structures, and so on. The filtration problems have wide applications in the construction complex \([1, 2]\). The migration of groundwater solid impurities, the displacement of the grout, pumped into lose ground is described by various mathematical filtration models of a suspension in a porous medium \([3, 4]\). Exact and asymptotic solutions for some models are obtained \([5–7]\). In the general case, analytical solutions are unknown, and the problem is solved numerically \([8–10]\).

The paper considers a filtration model for two-size particles \(d_1, d_2\) \((d_1 > d_2)\) moving in a fluid flow with different velocities. If the sizes of particles and pores are of the same order, the main cause of deposit formation is the geometric mechanism of particle capture: the particles freely pass through large pores and get stuck in the pore throats, smaller than the particle diameter \([11]\). Assume that the porous medium has pores of three various diameters \(D_1, D_2, D_3\), and \(D_1 > d_1 > D_2 > d_2 > D_3\). All particles pass freely through the pores \(D_1\) and get stuck at the inlet of the pores \(D_3\). Large particles \(d_1\) get stuck in the pores \(D_2\), and small particles \(d_2\) pass through them unhindered (Figure 1).

The flow rate increases with the cross section of the pore. If large particles pass only through large pores \(D_1\), their velocity \(\alpha_1\) is greater than the average speed \(\alpha_2\) of small particles \(d_2\) passing through the big pores \(D_1\) and medium pores \(D_2\). The filtration of two-size particles moving with different velocities was studied in \([12]\). However, there are no medium pores in this model, which substantially simplify the structure of the porous medium and the particles-pores interaction.

A mathematical model of the filtration problem for two-size particles moving with different velocities in a homogeneous porous medium is constructed in Section 2. In Section 3, local solutions
are obtained on the concentrations front of the fast particles and at the porous medium inlet. In Section 4, a global solution is constructed for constant filtration coefficients. The results of numerical calculations are given in Section 5. Conclusions in Section 6 finalize the paper.

Figure 1. Transport and retention of 2-size particles.

2. Mathematical model
The one-dimensional filtration problem of a two-size particles suspension in a homogeneous porous medium is considered in the domain

\[ \Omega = \{(x,t) : 0 < x < 1, t > 0 \} . \]

For each size of the particle \( d_i \), \( i = 1, 2 \), the suspended and retained particles concentrations \( C_i \) and \( S_i \) satisfy the mass balance equation

\[ \frac{\partial C_i}{\partial t} + \alpha_i \frac{\partial C_i}{\partial x} + \frac{\partial S_i}{\partial t} = 0 , \]

and the kinetic equation of deposit growth rate

\[ \frac{\partial S_i}{\partial t} = \Lambda_i C_i, \quad i = 1, 2 . \]

Here \( \alpha_i \) are the particle velocities, \( 0 < \alpha_2 < \alpha_1 \); \( \Lambda_i (S) \) are the filtration coefficients, depending on the deposit concentration.

A suspension of constant concentration is injected into the porous medium inlet; at the initial time, the porous medium does not contain suspended and retained particles. The corresponding boundary and initial conditions have the form

\[ C_i \big|_{t=0} = p_i, \quad p_i > 0 ; \]

\[ C_i \big|_{x=0} = 0; \quad S_i \big|_{x=0} = 0; \quad i = 1, 2 . \]

Consider the transportation and retention of particles in the pores in detail, the deposit \( S_2 \) of small particles \( d_2 \) is formed only in small pores \( D_3 \) (Figure 1). The deposit \( S_1 \) of large particles \( d_1 \) is distributed between the pores \( D_3 \) and \( D_2 \):

\[ S_1 = S_1^3 + S_1^2 . \]

The deposit in medium pores \( D_2 \)

\[ S^2 = S_1^2 ; \]

the total deposit in small pores of diameter \( D_3 \)
\[ S^3 = S^3_1 + S^3_2. \]  
(6)

When all small and medium pores are clogged, the pore deposits \( S^2, S^3 \) reach its maximum values

\[ S_{\text{max}}' = \lim_{t \to \infty} S^i; \quad i = 2, 3. \]  
(7)

If the porous medium is homogeneous, the limit deposits \( S_{\text{max}}' \) do not depend on the coordinate \( x \). The maximum limit deposit

\[ S_{\text{max}} = S_{\text{max}}^2 + S_{\text{max}}^3. \]  
(8)

The maximum deposits (7), (8) are assumed to be known, since they are determined by the concentrations of 3-size pores in a porous medium.

The deposit growth rate depends on the number of free pores that can be blocked by particles. We assume that the filtration coefficients are linear. It follows from (5) - (8) that for a porous medium with 3-size pores equations (2) take the form

\[
\frac{\partial S^2_i}{\partial t} = \lambda_1^2 (S_{\text{max}}^2 - S^2_i)C_i; \quad \frac{\partial S^3_i}{\partial t} = \lambda_1^3 (S_{\text{max}}^3 - S^3_i - S_i)C_i; \quad \frac{\partial S^3_i}{\partial t} = \lambda_2^3 (S_{\text{max}}^3 - S^3_i - S_i)C_i. \]
(9)

The concentration fronts of fast and slow particles \( t = x / \alpha_i, \quad i = 1, 2 \) divide \( \Omega \) into 3 subdomains

\[
\Omega_0 = \{0 < x < 1, \quad t < x / \alpha_1\}; \quad \Omega_i = \{0 < x < 1, \quad x / \alpha_i < t < x / \alpha_2\}; \quad \Omega_2 = \{0 < x < 1, \quad t > x / \alpha_2\}. 
\]

In the domain \( \Omega_0 \), the porous medium is empty; in \( \Omega_i \) only suspended and retained fast particles \( d_1 \) are present; there are all types of particles \( d_1, d_2 \) in the domain \( \Omega_2 \) (Figure 2).

![Figure 2: 2-size particles in a porous medium.](image)

The solutions \( C_i(x,t) \) are discontinuous on the concentrations front \( t = x / \alpha_i, \quad i = 1, 2 \); the solutions \( S^2_i, S^3_i, S_2 \) are continuous in the domain \( \Omega \).

3. **Local solutions**

3.1. **The solution on the concentrations front of the fast particles**

The solution on the concentrations front \( t = x / \alpha_i \); \( C_2 = 0; \quad S_i = S_2 = 0 \) (see Figure 2).

Substitution of (5), (9) into equation (1) for \( i = 1 \) gives

\[
\frac{\partial C_1}{\partial t} + \alpha_1 \frac{\partial C_1}{\partial x} + \Lambda C_1 = 0, \quad \Lambda = \lambda_1^2 (S_{\text{max}}^2) + \lambda_1^3 (S_{\text{max}}^3). \]
(10)
The solution of equation (10) with the condition (3)
\[ C_i(x,t) |_{t=x/\alpha_i} = p_i \exp(-\Delta x / \alpha_i). \] (11)

3.2. The solution at the porous medium inlet

According to condition (3), at the porous medium inlet \( x = 0 \) equations (9) have the form
\[ \frac{\partial S_i}{\partial t} = \lambda_i^1 (S_{\text{max}} - S_i^2) p_i; \quad \frac{\partial S_i^1}{\partial t} = \lambda_i^2 (S_{\text{max}} - S_i^1 - S_i^2) p_i; \quad \frac{\partial S_i^2}{\partial t} = \lambda_i^3 (S_{\text{max}} - S_i^3 - S_i^2) p_i. \] (12)

The solution of the first equation (12) with the condition (4)
\[ \int_0^{S_i^1(0,\cdot)} \frac{dS}{\lambda_i^2 (S_{\text{max}} - S)} = p_i t. \] (13)

Addition of the second and third equations (12) and the use of the notation (6) yields
\[ \frac{\partial S^3}{\partial t} = \lambda(S_{\text{max}} - S^3), \quad \lambda = \lambda_1^3 p_1 + \lambda_2^3 p_2. \] (14)

The solution of equation (14) with the condition (4)
\[ \int_0^{S^3(0,\cdot)} \frac{dS}{\lambda(S_{\text{max}} - S)} = t. \] (15)

Using (15), the solutions of the second and third equations (12) are obtained
\[ S_i^1(0,t) = p_i \int_0^t \lambda_i^3 \left( S_{\text{max}}^3 - S^3(0,t) \right) dt; \quad S_i^2(0,t) = p_i \int_0^t \lambda_i^2 \left( S_{\text{max}}^2 - S^3(0,t) \right) dt. \] (16)

4. Exact solution for constant coefficients

For constant filtration coefficients the system (1), (2) takes the form
- in the domain \( \Omega_1 \):
\[ \frac{\partial C_i}{\partial t} + \alpha_i \frac{\partial C_i}{\partial x} + \frac{\partial S_i}{\partial t} = 0; \quad \frac{\partial S_i^2}{\partial t} = \lambda_i^1 C_i; \quad \frac{\partial S_i^3}{\partial t} = \lambda_i^2 C_i; \quad S_i = S_i^1 + S_i^2; \] (17)
- in the domain \( \Omega_2 \):
\[ \frac{\partial C_i}{\partial t} + \alpha_i \frac{\partial C_i}{\partial x} + \frac{\partial S_i}{\partial t} = 0; \quad \frac{\partial S_i^2}{\partial t} = \lambda_i^1 C_i; \quad \frac{\partial S_i^1}{\partial t} = \lambda_i^2 C_i; \quad S_i = S_i^1 + S_i^2; \] (18)

with the conditions (3) and conditions on the concentrations fronts
\[ C_i |_{x=0} = p_i; \quad S_i |_{x=x_{i+1}} = 0; \quad i = 1, 2. \] (19)

The solution of the system in the domain \( \Omega \) is given below (Figure 2).
In the domain \( \Omega_1 \):
\[ C_i(x,t) = 0; \quad S_i(x,t) = 0; \quad i = 1, 2. \] (20)

In the domain \( \Omega_2 \):
\[ C_i(x,t) = p_i e^{-\lambda_i x / \alpha_i}, \quad S_i^1(x,t) = \lambda_i^3 p_i e^{-\lambda_i x / \alpha_i} (t - x / \alpha_i); \quad S_i^2(x,t) = \lambda_i^2 p_i e^{-\lambda_i x / \alpha_i} (t - x / \alpha_i); \quad C_2(x,t) = 0; \quad S_2(x,t) = 0. \] (21)

In the domain \( \Omega_2 \):
\[ C_i(x,t) = p_i e^{-\lambda_i x / \alpha_i}, \quad C_2(x,t) = p_2 e^{-\lambda_2 x / \alpha_2}; \quad S_i^1(x,t) = \lambda_i^3 p_i e^{-\lambda_i x / \alpha_i} (t - x / \alpha_i); \quad S_i^2(x,t) = \lambda_i^2 p_i e^{-\lambda_i x / \alpha_i} (t - x / \alpha_i); \quad \Lambda = \lambda_2^2 + \lambda_1^3. \] (22)

5. Numerical modeling

To calculate the solution (20) - (22), the following parameters are selected:
\[ \alpha_1 = 1; \quad \alpha_2 = 0.5; \quad p_1 = p_2 = 1; \quad \lambda_1^3 = 0.5; \quad \lambda_2^3 = 1; \quad \lambda_1^2 = 1.5 \]

In Figures 3-5, graphs of suspended and retained particles concentrations are presented for \( t = 0.5; 1; 5 \). The graphs of \( C_i, S_i^1 \) are drawn by a dashed line, \( C_2, S_2 - \) by a solid line, and \( S_i^2 - \) by a dotted line.

**Figure 3.** Concentrations Suspended \( C_i \mid_{t=0.5}; \quad C_i \mid_{t=0.5} \) (a); Retained \( S_i^1 \mid_{t=0.5}; \quad S_i^2 \mid_{t=0.5} \); \( S_2 \mid_{t=0.5} \) (b).

**Figure 4.** Concentrations Suspended \( C_i \mid_{t=1}; \quad C_i \mid_{t=1} \) (a); Retained \( S_i^1 \mid_{t=1}; \quad S_i^2 \mid_{t=1}; \quad S_2 \mid_{t=1} \) (b).

In Figures 3-5, the suspended and retained particles concentrations are zero before its front and are positive behind the front. The large particles concentrations front moves faster than the small particles front. Graphs 3-5 b show that the formation of large and small particles deposit occurs most intensively at the porous medium inlet; the suspended and retained concentrations of both particles types decrease with increasing distance \( x \) to the inlet.
Figure 5. Concentrations Suspended $C_1|_{t^*,2}$; $C_2|_{t^*,2}$ (a); Retained $S_1^1|_{t^*,2}$; $S_1^2|_{t^*,2}$; $S_2^2|_{t^*,2}$ (b).

6. Conclusion
The filtration problem of a suspension with two-size particles in a porous medium with three-size pores is considered. A mathematical model describing the competitions of large and small particles for blocking small pores and of small and medium pores for large particles retention is constructed.

It is shown if large and small particles move with different velocities, two concentrations fronts propagate in a porous medium. Before the fast particles front, the porous medium is empty. Only large suspended and retained particles are present between the fast and slow particles fronts. There are particles of all types behind the slow particles front.

Local exact solutions are constructed on the characteristics of the system, at the porous medium inlet and on the concentrations front of the fast particles. For constant filtration coefficients, a global solution is obtained.

Exact solutions give way to construct the asymptotics, which depends explicitly on the parameters of the problem, allowing fine-tuning of the experiments and production processes [13]. This is the subject of another study.

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