Application of the tuning algorithm with the least squares approximation to the suboptimal control algorithm for integrating objects

V F Kuzishchin¹, E I Merzlikina² and Hoang Van Va³

Department of Automated Control Systems for Thermal Processes, Moscow Power Engineering Institute – National Research University, Moscow, Krasnokazarmennaya 14, Russia, 111250.

Email: ¹KuzishchinV@yandex.ru, ²MerzlikinaYI@mpei.ru, ³Hoangvatdh@gmail.com

Abstract. The problem of PID and PI-algorithms tuning by means of the approximation by the least square method of the frequency response of a linear algorithm to the sub-optimal algorithm is considered. The advantage of the method is that the parameter values are obtained through one cycle of calculation. Recommendations how to choose the parameters of the least square method taking into consideration the plant dynamics are given. The parameters mentioned are the time constant of the filter, the approximation frequency range and the correction coefficient for the time delay parameter. The problem is considered for integrating plants for some practical cases (the level control system in a boiler drum). The transfer function of the suboptimal algorithm is determined relating to the disturbance that acts in the point of the control impact input, it is typical for thermal plants. In the recommendations it is taken into consideration that the overregulation for the transient process when the setpoint is changed is also limited. In order to compare the results the systems under consideration are also calculated by the classical method with the limited frequency oscillation index. The results given in the paper can be used by specialists dealing with tuning systems with the integrating plants.

1. Introduction

Automatic control systems (ACS) with integrating objects (IO) with a transport delay are widely used in the industry, for example, ACS of the level in boiler drums and so on. Tuning of such systems is interesting because of some special features of the IO. This paper considers ACS with IO and PI and PID-algorithms. The main purpose is to analyze the efficiency of the controller tuning by the method of the least squares (further MLS) approximation to the sub-optimal algorithm [1]. The main advantage of the MLS is that the tuning parameters are calculated using simple formulae without iterations and it is convenient from the practical point of view. For comparison the same calculations are carried out by the familiar method [2].

The characteristics of the suboptimal algorithm is determined when the object is influenced by disturbance $\lambda$ (see figure 1) which is typical for the objects under consideration. The transient process with little oscillation is acceptable (the permissible value of the frequency oscillation index $M=1.2$). In order to use MLS it is necessary to define the smoothing filter time constant $T_f = k_{Tf}\tau$, the correction coefficient
for the dynamical error $k_i$ and the frequency range of approximation $\Delta \Omega$. IOs are not considered in [1], so it is necessary to obtain recommendations how to choose the parameters mentioned.

![Figure 1: Structure of ACS](image)

The transfer function of the integrating objects used in the article is

$$W_{ob}(s) = \frac{\exp(-s \cdot \tau)}{T_1 \cdot s \cdot (T_2 \cdot s + 1)}.$$  \hspace{1cm} (1)

The values of the IO parameters $T_1$, $T_2$ and $\tau$ are taken from [3] for the following variants

- the level in the low pressure drum of the heat-recovery boiler: $T_1 = 555$ s, $T_2 = 0$, $\tau = 20$ s; \hspace{1cm} (2)
- the level in the high-pressure drum of the heat-recovery boiler: $T_1 = 100$ s, $T_2 = 30$ s, $\tau = 5$ s. \hspace{1cm} (3)

Let us consider MLS-approximation of the frequency response (FR) of the PI and PID-algorithms to the suboptimal algorithm FR when disturbance $\lambda(t)$ influences the object.

### 2. Tuning of the controller when disturbance $\lambda(t)$ influences the object

According to [1], the optimal transient process coincides to the step response of the IO $h(t)$ as far as $t = 2\tau$, then the optimal system process value $y(t)$ instantaneously decreases to zero and in the suboptimal system $y(t)$ decreases along some curve. It is difficult to provide the dynamical error $y_{din} = h(2\tau)$ because it is very small. In order to soften the demands to the dynamical error the correction coefficient $k_i > 1$ is introduced, thus the dynamical error must be $y_{din} = h(2\tau)$. The transient processes for this case are given in figure 2.

![Figure 2: The step responses of the object and the suboptimal system: 1 - the object; 2 - the optimal closed system; 3 - the suboptimal system for $t = 2\tau$; 4 - the suboptimal system for $t = 2k\tau$.](image)

The Laplas transform for the object step response is:

$$h_0(s) = \frac{1}{s} \cdot W_{ob}(s) = \left( \sum_{i=0}^{n} \frac{c_i}{s + a_i} \right) \cdot e^{-s \cdot \tau}. \hspace{1cm} (4)$$

For the object with transfer function (1) the Laplas transform for the step response is:

$$h_0(s) = \frac{c_0 + c_1}{s} + \frac{c_2}{s + a}, \quad e^{-s \cdot \tau}, \text{ где } c_0 = \frac{T_2}{\tau_1}; \quad c_1 = \frac{1}{\tau_1}; \quad c_2 = -c_{0}; \quad a = \frac{1}{\tau_2}. \hspace{1cm} (5)$$

The object step response is:

$$h_0(t) = \left( c_0 + c_1 \cdot (t - \tau) + c_2 \cdot e^{-\alpha \cdot (t - \tau)} \right) \cdot 1(t - \tau). \hspace{1cm} (6)$$

When $t = 2k \cdot \tau$ the formula is the following:

$$h_{02k}(t) = \left( b_{0k} + b_{1k} \cdot (t - 2 \cdot k \cdot \tau) + b_{2k} \cdot e^{-\alpha \cdot (t - 2k \cdot \tau)} \right) \cdot 1(t - 2 \cdot k \cdot \tau), \hspace{1cm} (7)$$

where $b_{0k} = c_0 + c_1 \cdot (2k - 1); \quad b_{1k} = c_1; \quad b_{2k} = c_2 \cdot e^{-\alpha \cdot (2k - 1)}$.

The step response $h_{02k}(t)$ of the suboptimal closed system is:

$$h_{02}(t) = h_0(t) - h_{02k}(t). \hspace{1cm} (8)$$
For the first order smoothing filter with the time constant $T_f$, the step response $h_d(t)$ when $t=2\cdot k_i \cdot \tau$ is:

$$h_d(t) = (h_{02}(2\cdot k_i \cdot t) \cdot e^{-\alpha_f (t-2\cdot k_i \cdot \tau)}) \cdot 1(t-2\cdot k_i \cdot \tau),$$

where $a_i = 1/T_f$.

(9)

The step response of the closed suboptimal system $\bar{h}_d(t)$ with the filter is:

$$\bar{h}_d(t) = h_0(t) - h_{02}(t) - h_d(t).$$

(10)

The step responses of the suboptimal system with IO (1) according to (9) are shown in figure 2.

The transfer function of the suboptimal ACS with object (1) may be written as

$$W_{\alpha y k}(s) = W_{ob}(s) + \left[\frac{h_{02k}(s)}{s+\alpha_f} - \left(\frac{b_{0k}}{s} + \frac{b_{2k}}{s+\alpha}\right)\right] \cdot e^{-2\cdot s \cdot k \cdot \tau}.$$  

(11)

Then the transfer functions of the suboptimal algorithm (12) and the open system (13) are

$$W_{r,sub}(s) = \frac{1}{W_{\alpha y k}(s)} - \frac{1}{W_{ob}(s)}.$$  

(12)

$$W_{r}(s) = \frac{W_{ob}(s)}{W_{\alpha y k}(s)} - 1.$$  

(13)

Let us consider tuning of the PI and PID-algorithm and choosing the filter time constant, the correction coefficient $k_t$ and the frequency range of approximation $\Delta \Omega$ taking into account the demands mentioned above. The series of the object (1) parameter values are taken from [3,4]:

- series 1: $\tau = [10, 20, 75], T_1 = [61.4, 555, 17.9]$ when $T_2=0, \beta_1 = \tau/T_1 = [0.16, 0.04, 4.2]$,
- series 2: $\tau = [5, 20, 7], T_1=[100, 333, 286], T_2 = [30, 60, 5], \beta_2 = \tau/T_2 = [0.17, 0.33, 1.4]$.

As a result of the calculations mentioned above the dependencies of coefficients $k_{T_1}$ and $k_t$ from the IO properties are obtained. For the ACS with the IO with the parameters from series 1 and the PI-algorithm the coefficients are $k_{T_1} = 30, k_t = 1.3$, where $k_{T_1} = T_1/\tau$. For the ACS with the PID-algorithm the coefficients are $k_{T_1} = 15, k_t = 1.1$. The dependencies for the ACS with object (2) and the PI-algorithm are shown in figure 3, where $\beta_1 = \tau_2/T_2$, $k_{T_1} = T_1/\tau$.

![Figure 3. Dependencies for the ACS with the PI-algorithm and the object from series 2: a) $k_{T_1} = f(\beta_2)$; b) $k_t = f(\beta_2)$](image1)

The approximating formulae for the ACS with the PI-algorithm and the object from series 2 are:

$$k_{T_1} = \frac{2.4 \cdot 4}{0.22 \cdot \beta_2 - 0.023},$$

(16)

$$k_t = \frac{0.68}{0.15 \cdot \beta_2^2 - 0.07 \cdot \beta_2 + 0.14},$$

(17)

Figure 4 shows the like curves for the ACS with the PID-algorithm and the IO from series 2. The approximating formulae for the ACS with the PID-algorithm and the IO from series 2 are:

$$k_{T_1} = \frac{2.4 \cdot 4}{0.22 \cdot \beta_2 - 0.023},$$

(16)

$$k_t = \frac{0.68}{0.15 \cdot \beta_2^2 - 0.07 \cdot \beta_2 + 0.14}.$$  

(17)

Let us consider the approximation range. Figure 5a shows the curves for the FR of the suboptimal and PID-algorithms for the IO from series 1 when the MLS used for calculation. The frequency range must be in the region where the open system FR does not surround point (-1; j0), and $\Omega_2$ must correspond to the point where the open system FR is tangent to the M-circle (Figure 5b). Thence the formulae for the non-dimensional frequency $\Omega_2 = f(\beta_1)$ are obtained, where $\Omega_2 = \omega_2 \cdot T_2; \beta_1 = \nu T_1$ (see table 1).
Dependencies for the ACS with the object from series 2: a) \( k_{Tf} = f(\beta_2) \); b) \( k_i = f(\beta_2) \)

The approximating formulae for the ACS with the PI and PID and the object from series 1 are:

\[
\Omega_2 = \frac{0.23}{0.4 \beta_1 + 0.003} \quad \text{PI-algorithm},
\]

\[
\Omega_2 = \frac{0.23}{0.4 \beta_1 - 0.007} \quad \text{PID-algorithm}.
\]

Table 1: The values of \( \Omega_2 \) and \( \beta_1 \) for the PI- and PID-algorithms (the object from series 1):

|           | PI-algorithm | PID-algorithm |
|-----------|--------------|---------------|
| \( \beta_1 \) | 0.16 0.04 4.2 | 0.16 0.04 4.2 |
| \( \Omega_2 \) | 3.07 11.11 0.12 | 3.68 22.22 0.18 |

The approximating formulae for the ACS with the PI and PID and the object from series 1 are:

\[
\Omega_2 = \frac{0.23}{0.4 \beta_1 + 0.003} \quad \text{(18)};
\]

\[
\Omega_2 = \frac{0.23}{0.4 \beta_1 - 0.007} \quad \text{(19)}.
\]

The frequency range can be determined as

\[
\Delta \Omega = [\Omega_1; \Omega_2],
\]

where for the object from series 1 \( \Omega_2 = T \omega_2 \) and for the object from series 2 \( \Omega_2 = T_2 \omega_2; \Omega_1 = \Omega_2/5 \).

Using formulae (14)-(22) it is possible to calculate the values of the parameters \( k_{Tf}, k_i \) and \( \Delta \Omega \).

Using calculated \( k_{Tf}, k_i \) and \( \Delta \Omega \) and the suboptimal algorithm transfer function (12) the parameters of the PI and PID-algorithms can be obtained by MLS [1]:

Figure 4. Dependencies for the ACS with the PID-algorithm and the object from series 2: a) \( k_{Tf} = f(\beta_2) \); b) \( k_i = f(\beta_2) \)

Figure 5. FR for the ACS with the object from the series 1: a) for the algorithms: 1-sub-optimal, 2 - PID; b) the open ACS: 1 - sub-optimal; 2 - with PID; 3 - M-circle.

Figure 6. Dependencies of \( \Omega_2 \) from \( \beta_2 \) for the object from series 2: a) PI-algorithm; b) PID-algorithm.
tuned by the traditional method, 2
algorithm and the
parameters
ACS with the IO
transient
from series 2
and parameters
T_f = 100 s, \tau_2 = 30 s, \tau_3 = 5 s are given in
Table 2.

Table 2: PI-algorithm parameters and values of k_{Ti}, k_i and \Delta\Omega

| Object | Method | Tuning parameters | \( k_{Ti}, k_i, \Delta\Omega \) |
|--------|--------|-------------------|-----------------------------|
| Series 1 | Traditional (M=1.2) | 16.6 305 | - - - |
|        | MLS | 18.03 802.6 | 30 1.3 [2.22, 11.11] |
| Series 2 | Traditional (M=1.2) | 2.3 700 | - - - |
|        | MLS | 2.43 767.7 | 140 7.5 [0.05, 0.24] |

The transient processes in the ACS with the IO from series 1 and the PI-algorithm are given in figure 7, for the IO from series 2 - in figure 8. Curve 1 is for the ACS tuned by the traditional method, 2 - for MLS.

Figure 7. Transient processes in the ACS with the PI-algorithm and the object from series 1: a) - the setpoint is changed; b) - the step disturbance influences the object.

Figure 8. Transient processes in the ACS with the PI-algorithm and the object from series 2: a) - the setpoint is changed; b) - the step disturbance influences the object.

The parameters for the PID-algorithm are given in table 3, the transient processes in the ACS with the PID-algorithm and the IO from series 1 are given in figure 9 and from series 2 - in figure 10. Curve 1 is for the ACS tuned by the traditional method, 2 - by the MLS where \( k_{Ti} = 180, k_i = 5 \) and 3 - \( k_{Ti} = 90, k_i = 8 \).

Table 3: PID-algorithm parameters and values \( k_{Ti}, k_i, \Delta\Omega \)

| Object | Method | Tuning parameters | \( k_{Ti}, k_i, \Delta\Omega \) |
|--------|--------|-------------------|-----------------------------|
| Series 1 | Traditional (M=1.2) | 22.5 174.8 6.1 | - - - |

\[
S = \sum \frac{1}{n} \left[ \text{Re}(W_{ms}(\omega_i)) - C_i \right]^2 + \left[ \text{Im}(W_{ms}(\omega_i)) - \left( -\frac{1}{\omega_i} + C_i \omega_i \right) \right]^2 \rightarrow \min.
\]

where \( C_1 = K_p; C_2 = K_p/T_i; C_3 = K_p\cdot T_d \). For the PI-algorithm \( C_3 = 0 \), more formulae are given in [1], they allow to tune the controller algorithm through one cycle. The results of the PI-algorithm tuning for the ACS with the IO from series 1 and parameters \( T = 555 \text{ s}, \tau_1 = 20 \text{ s} \) and with the IO from series 2 and parameters \( T = 100 \text{ s}, \tau_2 = 30 \text{ s}, \tau_3 = 5 \text{ s} \) are given in table 2.
| MLS | 24.5 | 32.1 | 8.2 | 15 | 1.1 | [4.44, 22.2] |
|-----|------|------|-----|----|-----|------------|
| Series 2 | Traditional (M=1.2) | 3.7 | 218.3 | 14.1 | - | - | - |
| | MLS | 4.79 | 1021 | 10.1 | 180 | 5 | [0.08, 0.4] |

**Figure 9.** Transient processes in the ACS with the PID-algorithm and the object from series 1: a) - the setpoint is changed; b) - the step disturbance influences the object.

**Figure 10.** Transient processes in the ACS with the PID-algorithm and the object from series 2: a) - the setpoint is changed; b) - the step disturbance influences the object.

### 3. Conclusion

Recommendations how to determine the MLS parameters ($\Delta \Omega$, $k_T$ and $k_v$) allowing to tune the PI and PID-algorithms trough one cycle are obtained on the basis of the calculations for the ACS with the IO.

The quality of the transient processes for the ACS with the IOs and PI-algorithm tuned by the MLS using these recommendations is close to the one for the traditional method.

The quality of the transient processes for the ACS with the PID-algorithm is more or less the same for both cases when the setpoint is changed. But the case when the disturbance influences the IO is more important and here the quality of the process in the system tuned by the MLS is considerably lower.

The materials of the paper can be used by the specialists tuning the ACS with the IOs.

### References

[1] Pikina G A and Burtseva 2014 Y S. Non-search Tuning of the Linear Algorithms on the Minimum of the Square Criterion, *Thermal Engineering* (vol. 3) pp 23-27

[2] Rotach V Y 2008 *Automatic Control Theory* (Moscow: MPEI Publishing House) p 396

[3] Zhigunov V V 2017 *Research and Introduction of the Automatic Control System for the Integration Objects in the Thermal Power Engineering* (Moscow MPEI)

[4] Mazurov V V 2003 Automatic Controllers in Control Systems and Their Tuning. Part 1. Industrial Control Objects, *Components and Technologies* (vol. 4) pp 154-157