THE IRON $K_{\alpha}$-LINE DIAGNOSTICS OF A ROTATIONAL BLACK HOLE METRIC

ALEXANDER F. ZAKHAROV
Institute of Theoretical and Experimental Physics, Moscow 117259, Russia; Astro Space Centre of Lebedev Physics Institute, Moscow, Russia, E-mail: zakharov@vitep1.itep.ru

SERGE V. REPIN
Space Research Institute, Moscow, 117810, Russia. E-mail: repin@mx.iki.rssi.ru

The original idea to show the spacetime geometry using few geodesics was developed by Johnson and Ruffini (1974). We used this idea to interpret the observational data for rotating BH’s. We developed the imitation approach to simulate a propagation of radiation near BH’s. An important problem for this approach is the diagnostics of a black hole metric using X-ray observational data of the iron $K_{\alpha}$-line. Observations of Seyfert galaxies in X-ray region reveal the broad emission lines in their spectra, which can arise in inner parts of accretion disks, where the effects of General Relativity (GR) must be counted. A spectrum of a solitary emission line (the $K_{\alpha}$-line of iron, for example) of a hot spot in Kerr accretion disk is simulated, depending on the radial coordinate $r$ and the angular momentum $a = J/M$ of a black hole, under the assumption of an equatorial circular motion of a hot spot. Using results of numerical simulations it is shown that the characteristic two-peak line profile with the sharp edges arises at a large distance, (about $r \approx (3-10)r_g$). The inner regions emit the line, which is observed with one maximum and extremely broad red wing. High accuracy future spectral observations, being carried out, could detect the angular momentum $a$ of the black hole. We analyzed the different parameters of problems on the observable shape of this line and discussed some possible kinds of these shapes. The total number of geodesics is about $10^9$ (to simulate possible shapes of the $K_{\alpha}$-line), so the number is great enough, especially in comparison with few geodesics in the original paper by Johnson and Ruffini (1974).

It is clear that an analysis of geodesics gives the direct way to investigate a metric. In particular, Johnson and Ruffini calculated geodesics near a Kerr black hole. These geodesics were very beautiful and they were used to create the known ICRA emblem. However, there is a problem to recognize such shapes of these geodesics near black holes, because practically we use something like photon geodesics to look at the geodesics near black holes, but the photon geodesics are bent by a very complicated way, so it is extremely difficult to reconstruct the shapes of geodesics near black holes. Below we will discuss how to extract an information about a metric from an analysis of geodesics around black holes.

The general status of black holes described in a number of papers (see, for example the following papers and references therein). As it was emphasized in these reviews the most solid evidence for an existence of black holes comes from observations of some Seyfert galaxies because we need a strong gravitational field approximation to interpret these observational data, so probably we observe manifestations radiation processes from the vicinity of the black hole horizon (these regions are located inside the Schwarzschild black hole horizon, but outside the...
Kerr black hole horizon, thus we should conclude that we have manifestations of rotational black holes.

Recent observations of Seyfert galaxies in X-ray band reveal the existence of wide iron $K_\alpha$ line (6.4 keV) in their spectra along with a number of other weaker lines (Ne X, Si XIII-XIV, S XIV-XVI, Ar XVII-XVIII, Ca XIX, etc). The line width corresponds to the velocity of the matter motion of tens of thousands kilometers per second, reaching the maximum value $v \approx 80000 - 100000$ km/s for the galaxy MCG-6-30-15 and $v \approx 48000$ km/s for MCG-5-23-16. In some cases the line has characteristic two-peak profile with a high “blue” maximum and the low “red” one and the long red wing, which gradually drops to the background level.

To simulate these shapes of the spectral lines we choose a minimal number of assumptions. We used the numerical approach based on the method, described earlier.

Many astrophysical processes, where the great energy release is observed, are assumed to be connected with the black holes. Because the main part of the astronomical objects, such as the stars and galaxies, possesses the proper rotation, then there are no doubts that the black holes, both stellar and supermassive, possess the intrinsic proper rotation too. Therefore we consider an emission of monoenergetic quanta near a Kerr black hole.

The large amount of observational data requires its comprehension, theoretical simulation and interpretation. The numerical simulations of the accretion disk spectrum under GR assumptions has been reported in the paper. The observational manifestations of GR effects are considered in X-ray binaries. Different physical models of the origin of a broad emission iron $K_\alpha$ line in the nuclei of Seyfert galaxies are analyzed in the papers.

The stationary black holes are described by the Kerr metric:

$$ds^2 = -\frac{\Delta}{\rho^2} (dt - a \sin^2 \theta d\phi)^2 + \frac{\sin^2 \theta}{\rho^2} \left[ (r^2 + a^2) d\phi - a dt \right]^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2, \quad (1)$$

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad (2)$$

$$\Delta = r^2 - 2Mr + a^2. \quad (3)$$

The photons trajectories can be described by the standard equations of geodesics:

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{kl} \frac{dx^k}{d\lambda} \frac{dx^l}{d\lambda} = 0, \quad (4)$$

where $\Gamma^i_{kl}$ are the Christoffel symbols. The equations geodesics however can be simplified if we will use the complete set of the first integrals which were found by Carter: $E = p_t$ is the particle energy at infinity, $L_z = p_\phi$ is $z$-component of its angular momentum, $m = p_i p^i$ is the particle mass and $Q$ is the Carter’s separation constant:

$$Q = p_\theta^2 + \cos^2 \theta \left[ a^2 (m^2 - E^2) + L_z^2 / \sin^2 \theta \right]. \quad (5)$$
As shown by Zakharov, the equations of photon motion can be reduced to

\[
\frac{dt'}{d\sigma} = -a \left( a \sin^2 \theta - \xi \right) + \frac{r^2 + a^2}{\Delta} \left( r^2 + a^2 - \xi a \right),
\]

\[
\frac{dr}{d\sigma} = r_1,
\]

\[
\frac{dr_1}{d\sigma} = 2r^3 + \left( a^2 - \xi^2 - \eta \right) r + \left( a - \xi \right) + \eta,
\]

\[
\frac{d\theta_1}{d\sigma} = \cos \theta \left( \frac{\xi^2}{\sin^2 \theta} - a^2 \sin \theta \right),
\]

\[
\frac{d\phi}{d\sigma} = -\left( a - \frac{\xi}{\sin^2 \theta} \right) + \frac{a}{\Delta} \left( r^2 + a^2 - \xi a \right),
\]

where \( \eta = Q/M^2 E^2 \) and \( \xi = L_z/ME \) are the Chandrasekhar’s constants, which should be derived from the initial conditions in the disk plane; \( r \) and \( a \) are the appropriate dimensionless radial variable and constant of rotation respectively. The system (6)-(11) has also two integrals,

\[
\epsilon_1 \equiv r_1^2 - r^4 - \left( a^2 - \xi^2 - \eta \right) r^2 - 2 \left( a - \xi \right) r + a^2 \eta = 0,
\]

\[
\epsilon_2 \equiv \theta_1^2 - \eta - \cos^2 \theta \left( a^2 - \frac{\xi^2}{\sin^2 \theta} \right) = 0,
\]

which can be used for the precision control. This method differs from the approach which was developed in papers.

First, one should define the Chandrasekhar’s constants for each quantum and then integrate the system (6)-(11) to either the infinity or the events horizon, depending on the constants values.

We assume that the hot ring emits quanta which are distributed by isotropic way in its local frame. The simulations are based on the trajectory classification, depending on the Chandrasekhar’s constants.

The simulated spectrum of a hot ring for \( a = 0.9, \theta = 60^\circ \) and different radius values is shown in Fig. 1. The proper quantum energy (in co-moving frame) is set to unity. The observer at infinity registers the characteristic two-peak profile, where the “blue” peak is higher than the “red” one and the center is shifted to the left. Some spectrum ”oscillations” near its minimum is explained by pure statistical reasons and has no the physical nature.

As far as the radius diminishes the spectrum is enhanced, i.e. increases the residual between the maximum and minimum quanta energy, registered by a distant observer. For example, for \( a = 0.9, \ r = 1.2 \ r_g \) and \( \theta = 60^\circ \), where \( r_g \) has its standard form \( r_g = 2kM/c^2 \), i.e. in the vicinity of the marginally stable orbit, the quanta, flown out to the distant observer, may differ 5 times in their energy. The red maximum decreases its height with diminishing the radius and at \( r < 2 \ r_g \) is almost undistinguished. It is interesting to note that the spectrum has very sharp edges, both red and blue. Thus, for \( a = 0.9, \ r = 3 \ r_g, \theta = 60^\circ \) the distant observer
Figure 1. Spectrum of the hot ring for $a = 0.9$, $\theta = 60^\circ$ and different radial coordinates. The marginally stable orbit lays at $r = 1.16 \, r_g$.

has registered 1433 quanta of 20417 emitted by isotropic way; 127 of them ($\approx 9\%$) drop to the interval $1.184 < E < 1.202$ (blue maximum) and 43 quanta drop to $0.525 < E < 0.533$ (red maximum), whereas no one quantum has the energy $E < 0.518$ or $E > 1.236$.

A spectrum of a hot ring for $a = 0.9$, $r = 1.5 \, r_g$ and different $\theta$ values is shown on Fig. 2. The spectrum for $\theta = 60^\circ$ and the same $a$ and $r$ values is included in Fig. 1 and should be added to the current figure too. As it follows from the figure, the spectrum critically depends on the disk inclination angle. For large $\theta$ values, when the line of sight slips almost along the disk plane, the spectrum is strongly stretched, its red maximum is essentially absent, but the blue one appears narrow and very high. The red wing is strongly stretched because of the Doppler effect, so that the observer registers the quanta with 5 times energy difference. As far as the $\theta$ angle diminishes the spectrum grows narrow and changes the shape: its red maximum first appears and then gradually increases its height. At $\theta = 0^\circ$ both maxima merge to each other and the spectrum looks like the $\delta$-function. It is evident because all the points of the emitting ring are equal in their conditions with respect to the observer. The frequency of registered quanta in that case is 2 time lower than the frequency of the emitted ones. A fall in frequency consists here in two effects, acting in the same direction: the transversal Doppler effect and the gravitational red shift.

The strong variability of Seyfert galaxies in X-ray does not contradict the assumption, that we observe the emission of the hot rings from the inner region of accretion disk, which can decay or grow dim, going towards a horizon as time
passes. The spectrum dynamics is understood qualitatively by reference to Fig. 1 considered sequentially from top to bottom.

To analyze an influence of a disk width on the shapes of the line we consider the case of a wide accretion disk and it was shown that the shape of the spectral line retains its type with two peaks (see Fig. 3). It is noted that the inner parts give the essential contribution into red wing of spectrum.

It is known that the standard disk models (like, for example, Shakura – Sunyaev and Novikov – Thorne disk models) hardly ever could be used to describe temperature distributions in accretion disks of Seyfert galaxies, however to show an influence of a temperature distribution on the spectral line shapes we use the standard disk model as a template. Fig. 4 demonstrates the shape of emitted monochromatic line in Schwarzschild black hole field with temperature distributed according to $\alpha$-disk model.

Details of computations and a full list of references could be found in papers. An application of such approach to estimate magnetic fields in AGNs and microquasars is described in details.

Acknowledgements

We would like to thank the Organizers of the Xth ICRA Workshop and especially prof. V. Gurzadyan for the kind and warm hospitality in Rome and Pescara. AFZ would like to thank Dipartimento di Fisica Universita di Lecce and INFN, Sezione di Lecce where the final version of the paper was prepared. This work was supported in part by Russian Foundation for Basic Research (project N 00-02-16108).
References

1. M. Johnson and R. Ruffini, Phys. Rev. D 10, 2324 (1974).
2. E.P. Liang, Phys. Rep. 302, 67 (1998).
3. A.F. Zakharov, in Proc. of the XXIII Workshop on High Energy Physics and Field Theory, p. 169, IHEP, Protvino, 2000.
4. I.D. Novikov and V.P. Frolov, Physics – Uspekhi 44, 291 (2001).
5. A.C. Fabian et al., Mon. Not. Roy. Astron. Soc. 277, L11 (1995).
6. Y. Tanaka et al., Nature 375, 659 (1995).
7. K. Nandra et al., Astrophys. J. 476, 70 (1997).
8. K. Nandra et al., Astrophys. J. 477, 602 (1997).
9. A. Malizia et al., Astrophys. J. Suppl. 113, 311 (1997).
10. R.M. Sambruna et al., Astrophys. J. 495, 749 (1998).
11. A.C. Fabian, in Relativistic Astrophysics, 20th Texas Symposium on Relativistic Astrophysics Austin, Texas, 10-15 December 2000, edited by J. Craig Wheeler and Hugo Martel, pp. 643, American Institute of Physics, AIP proceedings, 586, Melville, New York, 2001.
12. K.A. Weawer, J.H. Krolik and E.A. Pier, Astrophys. J. 498, 213 (1998).
13. T. Yaqoob et al., Astrophys. J. 490, L25 (1997).
14. A.F. Zakharov, Soviet Astronomy 35, 30 (1991).
15. A.F. Zakharov, Preprint MPA 755 (1993).
16. A.F. Zakharov, Mon. Not. Roy. Astron. Soc. 269, 283 (1994).
17. A.F. Zakharov, in Annals of the New York Academy of Sciences, 17th Texas Symposium on Relativistic Astrophysics and Cosmology, edited by H. Böhringer, G.E. Morfill and J.E. Trümper, 759, p. 550, The New York Academy of Sciences (1995).
18. A.F. Zakharov and S.V. Repin, Astronomy Reports 43, 705 (1999).
19. A.F. Zakharov and S.V. Repin, Astronomy Reports 46, 360 (2002).
20. B.C. Bromley, K. Chen and W.A. Miller, Astrophys. J. 475, 57 (1997).
21. W. Cui, S.N. Zhang and W. Chen, Astrophys. J. 257, 63 (1998).
22. J.W. Sulentic et al., Astrophys. J. 501, 54 (1998).
23. C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation, W.H. Freeman, San Francisco, 1973.
24. B. Carter, Phys. Rev. 174, 1559 (1968).
25. S. Chandrasekhar, Mathematical Theory of Black Holes, Clarendon Press, Oxford, 1983.
26. C.T. Cunningham, Astrophys. J. 202, 788 (1975).
27. C.T. Cunningham and J.M. Bardeen, Astrophys. J. 183, 237 (1973).
28. V. Karas, D. Vokrouhlický and A.G. Polnarev, Mon. Not. Royal Astron. Soc. 259, 569 (1992).
29. K.P. Rauch and R.D. Blandford, Caltech Preprint GRP-334 (1993).
30. A.F. Zakharov, Sov. Phys. – Journal of Experimental and Theoretical Physics, 64, 1 (1986).
31. A.F. Zakharov, Sov. Phys. – Journal of Experimental and Theoretical Physics 68, 217 (1989).
32. A.F. Zakharov and S.V. Repin, in Proc. of the Eleven Workshop on Gen-
eral Relativity and Gravitation in Japan, edited by J. Koga, T. Nakamura, K. Maeda, K. Tomita, p. 68, Waseda University, Tokyo, 2002.

33. N.I. Shakura and R.A. Sunyaev, *Astron. & Astrophys.* **24**, 337 (1973).

34. A.F. Zakharov and S.V. Repin, *Advances in Space Research* (accepted).

35. A.F. Zakharov, N.S. Kardashev, V.N. Lukash and S.V. Repin, *Mon. Not. Royal Astron. Soc.* (accepted); [astro-ph/0212008](http://arxiv.org/abs/astro-ph/0212008).
Figure 3. The spectral line shapes for different $\theta$ angles. The emitting region is the wide ring and its inner boundary is the last stable orbit (for rotational parameter $a = 0.9$ this $r$-value is equal to $r = 1.16 r_g$), its outer boundary corresponds to $r = 10 r_g$. 
Figure 4. The spectral line shapes corresponding to an accretion disk with outer and inner radii ($r_{\text{in}} = 3r_g$ and $r_{\text{out}} = 10r_g$) in Schwarzschild black hole field for different position angles of a distant observer. The temperature is distributed according to the Shakura–Sunyaev model.