Universal conductance for the Anderson Model

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Abstract. We discuss the thermal dependence of the zero-bias electrical conductance for a quantum dot embedded in a quantum wire, or side-coupled to it. In the Kondo regime, the temperature-dependent conductances map linearly onto the conductance for the symmetric Anderson Hamiltonian. The mapping fits accurately numerical renormalization-group results for the conductance in each geometry. In the side-coupled geometry, the conductance is markedly affected by a gate potential applied to the wire; in the embedded geometry, it is not.

1. Introduction
Since the single-electron transistor (SET) was developed [1], nanostructured devices have been extensively studied. Along with a wealth of findings, puzzles calling for firm theoretical results have surfaced [2–8]. In response to them, we have combined analytical and numerical methods to study the conductance for the Anderson model of a quantum dot embedded in a wire or side-coupled to it. In the Kondo regime, each geometry leads to a mapping between the temperature-dependent zero-bias conductance and the universal function for the conductance of the (particle-hole) symmetric Anderson Hamiltonian [9]. Here, we compare the mappings with numerical renormalization-group (NRG) [10] data for the conductance.

Our analysis contrasts the two devices in Fig. 1. The spin-degenerate Anderson Hamiltonian

\[ H_A = \sum_k \epsilon_k c_k^\dagger c_k + V (f_d^\dagger c_d + H. c.) + \epsilon_d c_d^\dagger c_d + Un_d n_d + V_w f_d^\dagger f_0 \]  

(1)
describes both. Here, the \( N \) energies \( \epsilon_k \) form a structureless half-filled band, with width \( 2D \) and density of states \( N \rho = N/2D \). The shorthand \( f_0 \equiv (1/\sqrt{N}) \) denotes the Wannier orbital closest to the dot. The gate voltage \( V_d \) controls the dot energy \( \epsilon_d \), the coupling \( V \) broadens the dot level \( c_d \), and the potential \( V_w \) renormalizes the resulting width \( \Gamma = \pi \rho V^2 \) to \( \Gamma_w = \Gamma/[1+(\pi \rho V_w)^2] \). In the embedded geometry, each \( \epsilon_k \) is an even combination of the degenerate conduction operators in the two wires; the odd combinations are decoupled from the dot and need not be considered. With \( V_w = 2\epsilon_d + U = 0 \), Eq. (1) defines the (particle-hole) symmetric Anderson Hamiltonian, whose ground-state dot occupation is \( n_d = 1 \). Although the Hamiltonians are identical, the conductances through the two devices are different: in the \( T \)-shaped (SET) geometry, the conductance is controlled by the spectral density for the operator \( f_0 \) (\( \epsilon_d \)) [9; 10].

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Figure 1. Quantum dots coupled to quantum wires in the (a) SET and (b) T-shaped configurations. The arrows indicate conduction. $V_d$ and $V_w$ are the dot and wire gate potentials.

Figure 2. Universal conductances for the symmetric SET (solid) and T-shaped (dashed) devices. Inset: NRG data for $V_w = 0$, $U = 3D$, $\epsilon_d = -0.3D$, and $\Gamma = 0.1D$, for the SET [□, fitted by Eq. (2a)] and T-shaped [△, Eq. (2b)] devices.

For $V_w = 0$, the transport properties of Eq. (1) are well understood [11; 12]. In the embedded geometry of Fig. 1(a), at high temperatures the Coulomb blockade bars conduction. At low $T$, in the Kondo regime, i.e., for gate potentials $V_d$ favoring odd dot occupations, the Kondo screening of the dot moment hybridizes the dot orbital to the wire electrons and lifts the blockade. In the side-coupled geometry of Fig 1(b), the conduction path bypasses the dot. While at high $T$ the conductance is ballistic, at low $T$ the Kondo cloud blocks conduction through the wire.

Opposite trends are therefore expected from the two devices: monotonically decreasing conductances $G(T)$ for the SET; monotonically increasing ones for the T-shaped device. Illustrations appear in Fig. 2, whose main plot shows that, relative to the $G = G_2/2$ line, where $G_2 = 2e^2/h$ is the conductance quantum, the conductance through the T-shaped device mirrors the SET conductance. For the symmetric Hamiltonian in the Kondo regime (solid line), the SET conductance is universal, i.e., plotted as a function of the temperature scaled by the Kondo temperature $T_K$, it is model-parameter independent [10; 13]. Likewise the conductance through the T-shaped device is universal: $G_{T-sh}(T/T_K) = G_2 - G_{SET}(T/T_K)$ [9].

For asymmetric Hamiltonians with $V_w = 0$, the SET conductance also mirrors the conductance through the T-shaped device. The inset in Fig. 2 displays an example. The SET (T-shaped) conductance now deviates significantly from $G_{SET}^S$ (from $G_2 - G_{SET}^S$): at low temperatures, in particular, it fails to reach $G_2$ (to vanish). Nonetheless, the two curves are symmetric, $G_{T-sh}(T) = G_2 - G_{SET}(T)$. Moreover, as the solid lines suggest and Ref. [9] shows, they map onto the universal conductance $G_{SET}^S$.

The mappings are linear. Let $\delta$ be the ground-state phase shift of the conduction electrons, and $\delta_w = -\arctan(\pi \rho W)$, the wire phase shift for $V = 0$. In the Kondo regime, the mathematical argument in Ref. [9], an algebraic derivation too long to be reproduced here, leads to twin expressions for the temperature-dependent conductances through the two devices in Fig. 1:

$$G_{SET}\left(\frac{T}{T_K}\right) - \frac{G_2}{2} = -\left(G_{SET}^S\left(\frac{T}{T_K}\right) - \frac{G_2}{2}\right) \cos 2(\delta - \delta_w) \quad (2a)$$

$$G_{T-sh}\left(\frac{T}{T_K}\right) - \frac{G_2}{2} = \left(G_{SET}^S\left(\frac{T}{T_K}\right) - \frac{G_2}{2}\right) \cos(2\delta) \quad (2b)$$
For the symmetric Hamiltonian, $\delta_w = 0$, $\delta = \pi/2$, and Eqs. (2) reduce to the main plot in Fig. 2. More generally, with error $O(\Gamma_w/U)$, the Schrieffer-Wolff transformation [14] shows that the phase-shift at $T \gg T_K$ is $\approx \delta_w$. The Friedel sum rule [15] then yields $\delta \approx \delta_w - \pi/2$, and it follows from Eq. (2a) that $G_{\text{SET}} \approx G_{\text{SET}}^\pi$.

The conductance $G_{\text{T-sh}}(T/T_K)$ in Eq. (2b), by contrast, is sensitive to changes in $V_w$. For $V_w \approx 0$, $\delta \approx \pi/2$, and $G_{\text{T-sh}} \approx G_2 - G_{\text{SET}}$. For $|V_w| \to \infty$, on the other hand, $\delta \approx \pi$, and $G_{\text{T-sh}} \approx G_{\text{SET}}^\pi$. For intermediate $V_w$, with $\delta \approx \pi/4$, $G_{\text{T-sh}}(T)$ is flat.

The right-hand sides of Eqs. (2) are different because different charges control the conductances through the two devices. In the $T$-shaped device, the charge $n_0$ piled in the wire controls conduction, because it obstructs the current. According to Friedel’s sum rule [15], $\pi n_0 = 2\delta$, which is the argument of the trigonometric function in Eq. (2b). In the SET, the Kondo charge $n_K$ is in control, for it promotes conduction across the dot. The Kondo charge is the wire charge $n_0$ minus the charge $2\delta_w/\pi$ induced by the wire gate potential, i. e., $\pi n_K = 2(\delta - \delta_w)/\pi$, equal to the argument of the cosine in Eq. (2a).

Finally, we present NRG results for $G_{\text{SET}}(T)$ and $G_{\text{T-sh}}(T)$. To calculate the conductances, we first evaluate analytically the integrals over energy relating $G_{\text{SET}}(T)$ and $G_{\text{T-sh}}(T)$ to the spectral densities for $c_d$ and $f_0$, respectively [9; 10]. The resulting sums are only slightly more difficult to compute numerically than thermodynamical averages. For the SET, e. g., we find

$$G_{\text{SET}}(T) = G_2 \frac{\pi \beta \Gamma_w}{Z} \sum_{mn} \frac{|\langle m| c_d |n \rangle|^2}{e^{\beta E_m} + e^{\beta E_n}},$$

where $Z$ is the partition function, and $m$ and $n$ label eigenstates of the model Hamiltonian.

For details of the numerical computation, see [16]. In summary, to compute $G_{\text{SET}}(T)$, we first diagonalize $H_A$ iteratively with $\Lambda = 6$ for two choices (1 and 0.5) of the auxiliary discretization parameter $\varepsilon$ [10]. In each iteration, we keep up to 4000 states. Averaged over the two $\varepsilon$’s, the right-hand side of Eq. (3) yields $G_{\text{SET}}(T)$ with absolute deviations inferior to 0.001 $e^2/h$.

Figure 3 compares NRG data for the SET conductance with Eq. (2). Except for the minor deviations at $k_B T > 10^{-2}D$, due to the $O(k_B T/D)$ contribution of irrelevant operators, the
agreement is excellent. The five curves were computed for the same asymmetric Hamiltonian, with wire potentials ranging from \( \rho V_w = 0 \) to \( \rho V_w = 0.7 \). With \( \rho V_w = 0 \), the relatively large dot width \( \Gamma = |\epsilon_d|/2 \) places the Hamiltonian at the outskirts of the Kondo regime: \( k_B T_K = 2 \times 10^{-2} D \) and \( \delta = 0.39\pi \). As \( \rho V_w \) grows, the shrinking effective width \( \Gamma_w = \Gamma/(1 + \pi \rho V_w^2) \) drives the model into the Kondo regime: \( T_K \) diminishes, the difference \( \delta - \delta_w \) approaches \( \pi/2 \) (for \( \rho W = 0.7 \), e.g., \( \delta - \delta_w = 0.49\pi \)) and the conductance approaches the solid curve in Fig. 2.

The conductance through the T-shaped device is qualitatively different. For \( V_w = 0 \), as in the inset in Fig. 2, the conductance is the mirror image of the circles in Fig. 3. As \( V_w \) grows, so does the phase shift \( \delta \), and the temperature dependence of the conductance is reversed, from steeply increasing to nearly flat to steeply decreasing. For large \( V_w \), \( \delta_w \) approaches \( \pi/2 \), and the Friedel sum rule forces \( \delta \to 0 \). In compliance with Eq. (2), the open diamonds lie close to \( G_{\text{SET}} \).

In brief, while the SET conductance is insensitive to the wire gate potential \( V_w \), the thermal dependence of the conductance through the T-shaped device flips, from monotonically increasing to monotonically decreasing, as \( V_w \) grows. Confirmed by the NRG data in Fig. 4, this remarkable consequence of Eq. (2) agrees qualitatively with the experimental observation reported in Ref. [7]. As Ref. [9] shows, moreover, Eq. (2) reproduces quantitatively the data of Sato et al. [5].

Acknowledgments
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