1. Introduction

Betavoltaic effect refers to the electric power production by a p-n junction bombarded by beta-particles that ionize the semiconductor material. Among the advantages of beta-batteries are their long service duration, amounting to years or even decades, and the possibility to use in the hard-to-reach areas. Betavoltaics and photovoltaics are related disciplines. In both cases, electric power results from the separation of electron–hole pairs produced by beta-electrons or photons by a p-n junction in the presence of a load in the circuit. In comparison to photovoltaics, publications in the field of the basic principles and applications of betavoltaic elements have been less numerous initially (see, e.g., [1–7]), but started to attract the attention of the researchers in the recent years [8–16].

The main task in betavoltaic design is the choice of a beta-source/semiconductor combination, which should meet certain requirements. In particular, the beta-particles produced by the source must be absorbed efficiently by the semiconductor. Within the semiconductor, the diffusion length of the electron–hole pairs generated by the beta-flux should be large enough to allow them to reach the p-n junction with as little losses as possible. Because only the relatively low-energy beta-electrons are utilized effectively (with energies varying between 5 and 70 keV) for the realistic semiconductor thicknesses, three main beta-sources are presently employed in betavoltaic applications: Tritium $^3$H, Nickel $^{63}$Ni, and Promethium $^{147}$Pm [8]. The respective mean energies of the electrons produced by these sources are 5.7, 18, and 62 keV.
The efficiency, $\eta$, of a betavoltaic converter is proportional to the collection coefficient, $Q$, of the electron–hole pairs generated by the beta-flux. In [2, 3], $Q$ was calculated under the assumption that the generation function of electron–hole pairs by a beta-flux is $g(x) \propto \exp(-\alpha x)$, where $\alpha$ is the effective absorption coefficient. This Ansatz, analogous to the Beer-Lambert law for the absorption of light, is an approximation, see [15, 16] for its discussion. In reality, the generation function initially increases and exhibits a maximum at some distance $x_m$ from the surface [13–15, 17] (see the next section for more details). This implies that this exponential approximation is adequate starting from some $x$-value greater than $x_m$. The emergence of the maximum in the $g(x)$ curve is due to the fact that, initially, the primary electrons pass through the semiconductor with only weak scattering. The dead layer thickness $x_m$ increases with the energy of the incident beta-electrons [13–15]. For GaAs, $x_m$ is in the range 0.1–1 $\mu$m [17].

Although the works [2, 3] do report analytical expressions for $Q$ (obtained under the assumption of the absence of the dead layer), the values of $Q = 1$ and 0.7 were used in the calculations of beta-conversion efficiency [3, 8]. While the value $Q = 1$ corresponds to the limiting conversion efficiency [3, 8]. The corresponding diffusion coefficient, $D$, is obtained under the assumption of the absence of the dead layer, the values of $\tau_{SR}$ are usually short, and are in the range of $10^{-9}–10^{-7}$ s. We use the so obtained collection coefficient to derive the expression for the realistically attainable beta-conversion efficiency $\eta$ of various combinations of beta-sources and direct-bandgap semiconductors. When calculating the efficiency, we focus on GaAs as a typical example. We show that decreasing $\tau_{SR}$ and increasing the dead layer thickness leads to a strong reduction of $Q$ below 1, and to the corresponding reduction of the beta-conversion efficiency.

2. Analysis of the collection coefficient

The physical picture adopted in this work is as follows. The incident beta-particles generate excess electron–hole pairs by ionizing the semiconductor. The excess electron and hole densities are practically the same, and are much smaller than the intrinsic charge carrier density. Due to the Coulomb attraction between the charge carriers of opposite signs, the excess charge carriers diffuse towards the p-n junction together in the process known as ambipolar diffusion, as explained, e.g. in chapter 10.2 of [18]. The corresponding diffusion coefficient, which is common for the excess charge carriers of both signs, is a weighted sum of the respective diffusion coefficients for electrons and holes. In the limit where the excess charge carrier density is much smaller than the majority charge carrier density, the ambipolar diffusion coefficient becomes equal the minority carriers’ diffusion coefficient. For brevity, we omit the word ‘ambipolar’ in the rest of this paper, but keep in mind the ambipolar character of the diffusion process.

![Figure 1. Schematic illustration of a p-n junction of thickness](image)

As for the generation function of the excess electron–hole pairs, it is a well-known problem in the design of semiconductor diode particle detectors that there exists a relatively thick dead layer right under the front surface, where the device is insensitive to the incident radiation, see [19] and section 10.5.1 of [20]. This dead layer, first suggested by Klein [21], owes its existence to the fact that primary beta-electrons initially travel straight into the target down to a certain distance without significant scattering. This results in a pronounced peak of the generation function, as observed, e.g. in Monte-Carlo simulations of a betavoltaic element [13]. In the open-circuit regime, typical for cathodoluminescence experiments [17] and semiconductor detector design [19, 20], the dead layer also manifests itself in the increased surface recombination rate as compared to the bulk semiconductor, resulting in poor collection of the electron–hole pairs produced in the dead layer.

We assume that the electron–hole pairs are generated only weakly within the dead layer, $x < x_m$, while for $x > x_m$, the generation function has the form $g(x) = I_0 \exp(-\alpha(x-x_m))$, where $I_0$ is the electron–hole pair generation rate in the $x_m$-plane, and $\alpha^{-1}$ is the characteristic decay length. Furthermore, we assume that $d_p < x_m$ and $S_d \ll D/L$, $d_p$ being the junction depth, $S_d$ the recombination rate on the back surface of the base, and $L$ and $D$ the diffusion length and coefficient of the excess electron–hole pairs generated in the base region. The sketch of our structure is summarized in figure 1. Note that we adopt the terminology from the photovoltaics literature [22] and refer to the different regions of the diode as ‘emitter’ and ‘base’.

Apart from the SRH mechanism with the lifetime $\tau_{SR}$, the electron–hole pairs in GaAs also recombine radiatively; the characteristic time of this process is $\tau = (BN_d)^{-1}$, where $B$ is the radiative recombination coefficient, and $N_d$ is the base doping concentration. Therefore, the diffusion length in the quasineutral base region can be written as...
with \( \eta_b = (\chi_{\text{SR}}^{-1} + \tau_r^{-1})^{-1} \) being the effective lifetme in the neutral base region.

Continuity equation for the excess concentration of the electron–hole pairs, \( \Delta p_1 \), within the dead layer (i.e. for \( x < x_m \), region 1), where generation is negligible, has the form

\[
\frac{d^2 \Delta p_1}{dx^2} - \frac{\Delta p_1}{L^2} = 0 ,
\]

(2)

In the rest of the semiconductor \( (x > x_m \), region 2), the continuity equation for the excess electron–hole pair density, \( \Delta p_2 \), is

\[
\frac{d^2 \Delta p_2}{dx^2} - \frac{\Delta p_2}{L^2} = -\alpha h e^{-\alpha(x-x_m)} .
\]

(3)

The equations (2) and (3) are supplemented by the boundary conditions

\[
\begin{align*}
\Delta p_1(x = d_p) &= 0 , \\
\frac{d \Delta p_1}{dx}(x = d) &= 0 , \\
\Delta p_1(x = x_m) &= \Delta p_2(x = x_m) , \\
\frac{d \Delta p_1}{dx}(x = x_m) &= \frac{d \Delta p_2}{dx}(x = x_m) .
\end{align*}
\]

(4)

The first condition reflects the fact that the electron–hole pairs are separated at the junction depth. The second one indicates the absence of surface recombination at the back of the base. The remaining two expressions are the usual continuity conditions for \( \Delta p(x) \) and \( d \Delta p(x)/dx \) at \( x = x_m \). The collection coefficient is then defined as the ratio of the current at the junction depth, \( d_p \), to the pair generation rate in the plane of highest generation at \( x = x_m \):

\[
Q = \frac{D}{I_0} \frac{d \Delta p_1}{dx}(x = d_p) .
\]

(5)

The solution of (2) and (3) that satisfies the first two conditions (4) can be written as

\[
\begin{align*}
\Delta p_1(x) &= C \sinh \frac{x - d_p}{L} , \\
\Delta p_2(x) &= C' \cosh \frac{x - d}{L} \\
&\quad + A(e^{-\alpha(x-x_m)} - \beta e^{-\alpha L}) , \\
A &= \frac{\alpha I_0 L^2}{D(1 - \alpha^2 L^2)} , \quad \beta = \alpha L \exp\left(\frac{1}{L} - \alpha \right) .
\end{align*}
\]

(6)

with constants \( C, C' \) to be determined from the remaining two conditions (4). This procedure yields:

\[
Q = \alpha L \frac{\alpha L \left( \cosh \frac{d - x_m}{L} - e^{-\alpha(d - x_m)} \right) - \sinh \frac{d - x_m}{L} }{[(\alpha L)^2 - 1] \cosh \frac{d - d_p}{L}} .
\]

(7)

If \( d - x_m \gg L \) and \( \alpha(d - x_m) \gg 1 \), this expression simplifies to

\[
Q = \frac{\alpha L}{1 + \alpha L} e^{\alpha d_p / L} .
\]

(8)

Figure 2. (a) Collection coefficient, \( Q \), as a function of the diffusion length, \( L \), for different absorption coefficients, \( \alpha \), in the limit \( \alpha(d - x_m) \gg 1, d - x_m \gg L \), see (8). The values used, \( \alpha = 10^3, 6 \cdot 10^3, \) and \( 6 \cdot 10^2 \text{ cm}^{-1} \), approximately correspond to the respective mean beta-energies of 5.7 keV \( ({}^{3}{}_{7}\text{H-source}) \), 20 keV \( (\text{\textsuperscript{60}\text{Ni-source})} \), and 60 keV \( (\text{\textsuperscript{147}\text{Pm-source})} \) for the GaAs-based p-n junction [23]. The dashed curves are calculated for different dead layer thicknesses, \( x_m \), and \( d_p = 10^{-3} \text{ cm} \). The solid curves are from the standard relation \( Q = \alpha L/(1 + \alpha L) \), valid in the absence of the dead layer. (b) Collection coefficient (8) for different junction depth values for \( x_m = 10^{-5} \text{ cm} \) and \( \alpha = 10^3 \text{ cm}^{-1} \), corresponding to the beta-particle energy of about 5.7 keV in the \( {}^{3}{}_{7}\text{H}/\text{GaAs} \) combination.

Figure 2 shows the dependence of the collection coefficient \( Q \) on the diffusion length from (8). As seen in this figure, the strongest reduction of \( Q \) due to the presence of the dead layer is for the case of the \( {}^{3}{}_{7}\text{H} \) beta-source. The smallest discrepancy in the \( Q \)-values obtained with and without taking into account the dead layer is found for the curves corresponding to \( \alpha = 6 \cdot 10^2 \text{ cm}^{-1} \), realized in the case of the \( \text{\textsuperscript{147}\text{Pm-source})} \). In this case, to obtain \( Q \approx 1/2 \), one would need the diffusion length \( L \approx 35 \mu \text{m} \). The values \( Q \approx 1 \) can be achieved only in Si p-n junctions with long minority carrier lifetimes [24].

In figure 2(b), the junction depth was varied at a fixed electron energy (and thus constant \( \alpha \) ) and dead layer thickness. As seen in this figure, the collection coefficient increases not only upon increasing \( L \), but also upon approaching the junction depth. This effect is especially important for small diffusion length \( L \).

A further conclusion from figure 2 is that collection of the electron–hole pairs generated by the electron flux will be quite efficient in the case when the diffusion length exceeds the dead layer thickness, \( L > x_m \). An alternative way to increase \( Q \) is to use deeper junctions with \( d_p \approx x_m \).
Let us first assume that the GaAs p-n junction base is of D. Then, for \( B \approx 2 \cdot 10^{-10} \text{ cm}^3 \text{ s}^{-1} \), \( N_d \approx 10^{17} \text{ cm}^{-3} \), and lifetimes \( \tau_{SR} = 10^{-9}, 10^{-8}, \) and \( 10^{-7} \) s, diffusion length \( L \) has the respective values of 2.2, 6.45, and 12.9 \( \mu \text{m} \).

The space-charge (or depletion) region influences the collection coefficient. However, these figures exceed the width of the space-charge region by at least an order of magnitude. Indeed, the space-charge region width can be estimated [18, 28] as \( w \approx \sqrt{2\kappa qT \ln(N_d^2 p_i^2)} \approx 0.1 \mu\text{m} \), where \( \kappa \) is the dielectric constant of GaAs, and \( n_i \approx 1.8 \cdot 10^6 \text{ cm}^{-3} \) the intrinsic carrier density. For this reason, we can neglect the influence of the space-charge region on the transport properties of the excess charge carriers.

Figure 3(a) shows the dependence of the collection coefficient, \( Q \), of a pair \( ^3\text{H}/\text{GaAs} \) as a function of the junction depth for different Shockley–Reed lifetimes and element thicknesses for the case of the (a) p-type base and (b) n-type base.

Let us now analyze the collection coefficient for the \( ^{147}\text{Pm}/\text{GaAs} \) pair. In this case, according to [23], \( \alpha \approx 600 \text{ cm}^{-1} \), i.e. excess electron-hole density decays much more slowly than in the \( ^3\text{H}/\text{GaAs} \) case. For this the inequality \( \alpha L \gg 1 \) is always satisfied even for the shortest lifetime of \( 10^{-9} \) s. In contrast, for \( ^{147}\text{Pm} \) source, \( \alpha L = 1.5 \) for \( L = 25 \mu\text{m} \), while \( \alpha L = 0.06 \) for \( L = 1 \mu\text{m} \), so that \( Q \) is always notably smaller than 1.
But this is not the only reason for the reduction of $Q$ in realistic $^{147}\text{Pm}/\text{GaAs}$ structures. When manufacturing solar cells based on the direct-bandgap semiconductors, such as GaAs, full thicknesses of p-n junctions are chosen rather small (of the order of a few $\mu$m). Such structures were used in [9]. In contrast, for the $^{147}\text{Pm}/\text{GaAs}$ pair used in betavoltaics, the situation might be very different, especially for large values of $L$. In this case, the product $\alpha d$ will be small, so that for full absorption of beta-flux much thicker p-n junctions are required compared to those typically used in photovoltaics.

Shown in figure 4 is the collection coefficient as a function of $d_p$ for a $^{147}\text{Pm}/\text{GaAs}$ pair calculated for different lifetimes $\tau_{SR}$ and junction thicknesses $d$ of 10 and 100 $\mu$m. In this case, according to [17], $x_m = 3 \times 10^{-4}$ cm$^3$ s$^{-1}$. Panels (a) and (b) correspond to the cases of p- and n-base conduction types, respectively. As seen in the figure, rather high values of $Q > 0.4$ for the $^{147}\text{Pm}/\text{GaAs}$ pair can be achieved only for the junction thickness $d \approx 100 \, \mu$m. Also, collection coefficient decreases dramatically as $\tau_{SR}$ decreases.

It should be noted that similar results for the attainable $Q$ are expected for other direct-bandgap A$_3$B$_3$ semiconductors, in particular, the ones based on the three-component compounds.

3. Open-circuit voltage analysis

When estimating the limiting efficiency value [3, 11], we used the Shockley–Queisser approach [27], in which not only the current density, but also the open-circuit voltage, $V_{OC}$, is assumed to be maximal. Therefore, our next task is to calculate the open-circuit voltage, $V_{OC}$, with realistic values of $\tau_{SR}$.

It is given by the standard expression

$$V_{OC} = \frac{k_BT}{q} \ln \left(\frac{N_d \Delta p^*}{n_i^2}\right),$$

where $\Delta p^* = \Delta p(x = d_p + w)$ is the excess minority carrier density in the base at the boundary between the space-charge region of thickness $w$ and the quasineutral region, $N_d$ is the equilibrium density of the majority carriers in the quasineutral base region, and $n_i$ is the intrinsic charge carrier density. It is related to the effective densities of states in the conduction and valence bands, $N_c$ and $N_v$, as

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2k_BT}\right).$$

We assume that both $d_p$ and $w$ are much smaller than the diffusion length $L$. This allows us to approximate

$$\Delta p(x = 0) \approx \Delta p^*.$$  

Such an approximation introduces a negligible error into $V_{OC}$ from (9) in view of its logarithmic dependence on $\Delta p^*$.

We will assume that recombination dominates in the quasineutral base region and in the space-charge region. Then, $V_{OC}$ can be obtained using the approach from [26]. Taking into account the generation-recombination processes, we first write the continuity equation for the excess carrier density supplemented by the boundary conditions:

$$\frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{L^2} - r(x) \Delta p(x) + g(x) = 0,$$

$$\frac{d\Delta p}{dx}(x = d) = 0,$$

$$D \frac{d\Delta p}{dx}(x = 0) = S_0 \Delta p^*,$$$$

where the third term describes recombination processes in the space-charge region of the abrupt junction, and the last one corresponds to the beta-induced generation. The first boundary condition is consistent with our assumption $S_0 \ll D/L$ from the beginning of the previous section, and the second one is responsible for recombination effects in the $x = d_p + w$ plane.

Integration of the continuity equation results in the balance equation for the generation-recombination currents, according to which the current density for electronic excitation is proportional to the integral of the generation term,

$$J_3 = q \int_0^d dx \frac{\Delta p(x)}{t_b} + q(S_0 + R_{SC}) \Delta p^*,$$$$

where $q$ is the elementary charge. The right-hand side in (13) is responsible for the recombination in the bulk and on the front side of the emitter and within the space-charge region. The space-charge region recombination rate is given by [28]

$$R_{SC}(\Delta p^*) = \frac{L_0}{\sqrt{2 \tau_{SR}}} \int_{y_m}^{-1} dy \, N_d (1 - y + e^y)^{-1/2} \times \left[ N_p e^y + n_e e^{E_r - \phi_0} + b \left(\frac{n_i^2}{N_d} + \Delta p^*\right)e^{-y} + bn_i e^{E_r - \phi_0} \right]^{-1},$$

where $b = q \sigma_p / \sigma_h$ is the ratio of the capture cross-sections of holes and electrons by a recombination level, $E_r$ is the recombination level energy measured from the middle of the bandgap, $y_m$ is the dimensionless potential at the p-n boundary, $L_0$ is the Debye length.

To evaluate the first integral in (13), we have employed the following approximative procedure. First, we write the solution of the continuity equation (12) as a sum of homogeneous and inhomogeneous parts,

$$\Delta p(x) = \frac{e^{-x/L} + e^{\pm 2x/d_L} - \Delta p^* - \Delta p_h(x)}{1 + e^{2x/d_L}},$$

where the homogeneous term satisfies the first boundary condition in (12) and gives the value $\Delta p(x = 0) = \Delta p^*$. The inhomogeneous contribution $\Delta p_h(x)$, with $\Delta p_h(x = 0) = 0$, is notably different from zero only within a relatively thin layer below the front surface of the emitter, where the generation-recombination processes take place. Therefore, the contribution to the integral of the second term can be neglected in comparison to the integral of the homogeneous term, allowing us to write

$$\int_0^d dx \Delta p(x) \approx \Delta p^* L \tanh(d/L).$$
This approximation should produce a negligible error in $V_{\text{OC}}$ in view of its logarithmic dependence on $\Delta p^*$. Substitution of this result into (9) taking into account that $L^2 = D\tau_L$ yields

$$J_\beta = q\Delta p^* \left[ \frac{D}{L} \tanh \left( \frac{d}{L} \right) + S_0 + R_{\text{SC}}(\Delta p^*) \right].$$

The current density $J_\beta$ is inversely proportional to the energy required to create one electron-hole pair, $\varepsilon$, which is approximately related to the bandgap $E_g$ as [29]

$$\varepsilon = 2.8 E_g + 0.5 \text{ eV}.$$  

Denoting the current density in the case of Si ($E_g = 1.12 \text{ eV}$) by $J_0$, the current density in the case of arbitrary bandgap can be approximated as

$$J_\beta = J_0 Q \cdot 3.64 \text{ eV}/\varepsilon.$$  

We note that, usually, $J_0$ is in the $1 \times 10^2 \mu\text{A cm}^{-2}$ range [3]. The value of $\Delta p^*$ found from (16) should be substituted into (9) to obtain the open-circuit voltage $V_{\text{OC}}$.

Figure 5 shows the dependence of $V_{\text{OC}}$ of a GaAs-based p-n junction on the base doping level, $N_d$, neglecting the surface recombination, that is, $S_0 \approx 0$. As seen in figure 5, $V_{\text{OC}}$ increases with $N_d$. On the one hand, the values of $V_{\text{OC}}$ for the pair 3H/GaAs is notably smaller than in the solar cells [26], because the beta-produced current densities are at least two order of magnitude smaller than the short-circuit current densities in photovoltaic cells. On the other hand, the open-circuit voltages in figure 5 exceed the values obtained experimentally in [9]. The reason is that, in [9], the current density $J_0$ was of the order of $1 \mu\text{A cm}^{-2}$, whereas in our calculations, we have taken $J_0 = 10 \mu\text{A cm}^{-2}$. If the values $J_0 = 1 \mu\text{A cm}^{-2}$, $N_d = 5 \cdot 10^{16} \text{ cm}^{-3}$, and $\tau_{SR} = 10^{-9} \text{ s}$ are used, we obtain $V_{\text{OC}} = 0.44 \text{ V}$, which practically coincides with the value given in [9].

## 4. Refined calculation of the limiting betaconversion efficiency

According to Olsen [3], the efficiency of a betavoltaic element, $\eta$, is

$$\eta = \eta_0 \eta_C \eta_S,$$

where

$$\eta_0 = N_v/N_0$$

is the fraction of beta-flux that reaches the semiconductor,

$$\eta_C = (1 - r) Q$$

is the coupling efficiency, given by the product of absorption probability of a beta-particle ($r$ is the electron reflection coefficient from the semiconductor surface) and collection efficiency $Q$ of electron-hole pairs, and, finally, the semiconductor efficiency

$$\eta_S = q V_{\text{OC}} FF/\varepsilon,$$

where $q$ is the elementary charge, $V_{\text{OC}}$ is the open-circuit voltage, $FF$ is the fill factor, $\varepsilon$ is the energy necessary to generate one electron-hole pair from (17).

Let us obtain $V_{\text{OC}}$ within the Shockley–Queisser approximation, where $\tau_{SR} \to \infty$, $S_0$ and $R_{\text{SC}} \to 0$, and the only recombination mechanism present is radiative recombination, characterised by the coefficient $B$. In this case, $V_{\text{OC}}^{\text{lim}}$ can be found analytically from (13) and (16):

$$V_{\text{OC}}^{\text{lim}} = \frac{k_B T}{q} \ln \frac{J_0}{q B d n_i^2},$$

The fill factor can be found using the expression from [3]

$$FF = [V_{\text{OC}} - \ln(V_{\text{OC}} + 0.72)]/(V_{\text{OC}} + 1),$$

where $V_{\text{OC}} = V_{\text{OC}}/k_B T$.

To calculate the limiting beta-conversion efficiency, we take $Q = 1$, $r = 0$, $\eta_0 = 1$, corresponding to the bidirectional source in the terminology of [3]. In this case

$$\eta_{\text{lim}} = q V_{\text{OC}}^{\text{lim}} FF_{\text{lim}} / 2.8 E_g + 0.5,$$

where $V_{\text{OC}}^{\text{lim}}$ is given by (23).

When calculating $\eta_{\text{lim}}$, several issues may arise. First, the parameters $N_v$, $N_0$, and $B$ are material-specific in every semiconductor. Second, when evaluating $V_{\text{OC}}^{\text{lim}}$ and $FF_{\text{lim}}$, Olsen had used, for each source, concrete current density $J_0$ of the order of $10^2 \mu\text{A cm}^{-2}$ for 147Pm and $1 \mu\text{A cm}^{-2}$ for 3H. Finally, $V_{\text{OC}}^{\text{lim}}$ depends on the p-n junction thickness $d$. Therefore, all parameters in (25) must be specified. Since such key parameters as $B$, $N_v$, and $N_0$ are known only for concrete semiconductors.
and concrete bandgap values $E_g$, in the best-case scenario, the dependence $\eta_{\text{lim}}(E_g)$ can be found as a set of support points for the known semiconductors with different $E_g$. Fitting this with a smooth curve might not be accurate enough.

In this work, we calculated $\eta_{\text{lim}}$ only for the case of GaAs using (25). For $B = 2 \cdot 10^{-10}$ cm$^3$ s$^{-1}$ and $d = 10 \mu$m gives for $J_0 = 10^7$ μA cm$^{-2}$, the value $\eta_{\text{lim}} \approx 17\%$, and for $J_0 = 1$ μA cm$^{-2}$, $\eta_{\text{lim}} \approx 14\%$. Note that the values of $\eta_{\text{lim}}$ obtained here notably exceed the ones obtained by Olsen in [3, 8]. In the rest of this work, we will use the values obtained for the $^{147}$Pm/GaAs and $^3$H/GaAs combinations, respectively.

5. Calculation of the attainable betaconversion efficiency

Figure 6 shows the attainable efficiency as a function of junction depth for different Schokley-Reed lifetimes for the case of the (a) p-type base and (b) n-type base.

$\eta = (4–7)\%$. In these works, a $^3$H-source was used with the $A_3B_8$-based semiconductors. But, as evident from the figures shown, the possibilities of increasing the efficiency of $^3$H/$A_3B_8$ betaconversion are far from being exhausted.

Shown in figure 7 is the attainable beta-efficiency (26) as a function of $d_p$ for $^{147}$Pm/GaAs pair with $\eta_{\text{lim}} = 17\%$. The $\tau_{\text{SR}}$ values used were $10^{-6}$, $10^{-5}$, and $10^{-7}$ s, and GaAs thicknesses were 10 and 100 μm. Figure 7(a) and (b) correspond to the p- and n-types of the base conductivity. As seen in this figure, $\eta$ reduces rather strongly as $\tau_{\text{SR}}$ is decreased. For the highest $\tau_{\text{SR}} = 10^{-7}$ s, $\eta$ decreases with decreasing $d$. The highest efficiency attainable, $\eta = 7.25\%$, is achieved for $\tau_{\text{SR}} = 10^{-7}$ s and $d = 100 \mu$m, and the lowest value of 0.51% is realized for $\tau_{\text{SR}} = 10^{-9}$ s and $d = 10 \mu$m.

Thus, we conclude that a $^{147}$Pm/GaAs-based betaconverter is not as efficient as a $^3$H/GaAs-based one. Perhaps, the very small efficiency of the $^{147}$Pm/GaAs battery obtained in [5] is due to the small thickness of GaAs and small lifetime $\tau_{\text{SR}}$. The same applies also to the cases when, instead of GaAs, other direct-bandgap semiconductors are used.

6. Conclusions

Our analysis, focusing on the attainable collection coefficient $\eta$ and open-circuit voltage values $V_{\text{OC}}$, has revealed the following features of current collection of the GaAs-based beta-elements.

Efficient collection of the electron–hole pairs generated by a beta-flux can be achieved when the diffusion length

- \begin{align*}
\eta &= \eta_{\text{lim}} Q V_{\text{OC}} \\
V_{\text{OC}} &= \frac{h c}{e} \left( \frac{2 m_e}{\hbar^2} \right)^{1/2} E_g^{3/2}
\end{align*}

Figure 6. Beta-conversion efficiency of a $^3$H/GaAs pair as a function of junction depth for different Schokley-Reed lifetimes for the case of the (a) p-type base and (b) n-type base.

Figure 7. Beta-conversion efficiency of a $^{147}$Pm/GaAs element versus junction depth for different Schokley-Reed lifetimes and element thicknesses for the (a) p-type base and (b) n-type base.
exceeds the dead layer thickness, \( L > x_m \). An alternative way to increase collection coefficient is to use deep junctions, for which \( d_p \approx x_m \).

Additional mechanisms responsible for the reduction of current generated by beta-electrons are possible, leading to smaller betavoltaic efficiency. They may be due, for instance, to the strong absorption of the beta-electrons by auxiliary layers of a betavoltaic element.

Using the Shockley–Queisser approximation, we have derived the limiting betavoltaic efficiency, \( \eta_{\text{lim}}(E_g) \). Our analysis has shown that, because the main parameters affecting the efficiency are very different for different semiconductors, the \( \eta_{\text{lim}}(E_g) \) curve can be build as a set of support points for semiconductors with different bandgaps, and not as a smooth curve.

\(^{147}\)Pm beta-source performs more poorly than \(^3\)H-source, because the electron–hole pair generation depth in the case of \(^{147}\)Pm-source is large, whereas the diffusion length of GaAs is small. Therefore, the majority of electron–hole pairs generated in the base recombine before reaching the p-n junction.

In the case of \(^3\)H-source, the picture is different. The collection coefficient is rather high, because of the small generation depth of electron-hole pairs. Therefore, the realistic betavoltaic efficiency for the \(^3\)H/GaAs pair will be rather high for relevant parameters (lifetimes and diffusion coefficients) of the semiconductor.

Similar results are expected also in the case, when other direct-bandgap semiconductors are used instead of GaAs.

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