The Zeno and anti-Zeno effects: studying modified decay rates for spin-boson models with both strong and weak system-environment couplings

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In this paper, we look into what happens to a quantum system under repeated measurements if system evolution is removed before each measurement is performed. Beginning with investigating a single two-level system coupled to two independent baths of harmonic oscillators, we move to replacing it with a large collection of such systems, thereby invoking the large spin-boson model. Whereas each of our two-level systems interacts strongly with one of the aforementioned baths, it interacts weakly with the other. A polaron transformation is used to make it possible for the problem in the strong coupling regime to be treated with perturbation theory. We find that the case involving a single two-level system exhibits qualitative and quantitative differences from the case involving a collection of them; however, the general effects of strong and weak couplings turn out to be the same as those in the presence of system evolution, something which allows us to establish that system evolution has no practical bearing on any of these effects.

I. INTRODUCTION

We start with the paradigmatic spin-boson model [2], but we consider the presence of both strong and weak system-environment couplings. The Hamiltonian describing the problem is the following:

\[ H^{(0)}_L = \frac{\epsilon}{2} \sigma_z + \frac{\Delta}{2} \sigma_x + \sum_k \omega_k b_k^\dagger b_k + \sum_k \alpha_k c_k^\dagger c_k + \sigma_z \sum_k \left( g_k^* b_k + g_k b_k^\dagger \right) + \sigma_x \sum_k \left( f_k^* c_k + f_k c_k^\dagger \right). \]

Here, \( \frac{\epsilon}{2} \sigma_z + \frac{\Delta}{2} \sigma_x \) is the system Hamiltonian, \( \sum_k \omega_k b_k^\dagger b_k + \sum_k \alpha_k c_k^\dagger c_k \) is the environment Hamiltonian, and \( \sigma_z \sum_k \left( g_k^* b_k + g_k b_k^\dagger \right) + \sigma_x \sum_k \left( f_k^* c_k + f_k c_k^\dagger \right) \) gives the system-environment interaction. \( \epsilon \) characterizes the energies of the system energy eigenstates, \( \Delta \) is the tunneling amplitude, and \( \omega_k \) and \( \alpha_k \) are the frequencies of harmonic oscillators in the two harmonic oscillator baths interacting with the system. \( b_k^\dagger b_k \) and \( c_k^\dagger c_k \) are the annihilation/creation operators of the first and second baths, respectively, and \( \sigma_z \) and \( \sigma_x \) are the standard Pauli operators, and we set \( \hbar \) equal to 1 throughout. Superscript (0) denotes the first version of our Hamiltonian, subscript \( L \) denotes the lab frame, and henceforth, we use the following definitions: \( H_{S1} = \frac{\epsilon}{2} \sigma_z \), \( H_{S2} = \frac{\Delta}{2} \sigma_x \), \( H_{B1} = \sum_k \omega_k b_k^\dagger b_k \), \( H_{B2} = \sum_k \alpha_k c_k^\dagger c_k \), \( V_{C1} = \sigma_z \sum_k \left( g_k^* b_k + g_k b_k^\dagger \right) \), and \( V_{C2} = \sigma_x \sum_k \left( f_k^* c_k + f_k c_k^\dagger \right) \).

As per our Hamiltonian, a spin-1/2 (two-level) system interacts with an environment comprised of two independent baths of harmonic oscillators. Whereas one bath interacts with the \( z \) component of the spin of the system, the other interacts with the \( x \) component. All through this paper, we assume that interaction, or coupling, with the \( z \) component is strong and that interaction, or coupling, with the \( x \) component is weak, something which implies that \( |g_k| > |f_k| \) and \( |f_k| < 1 \).

If the system is prepared in the state \(|\uparrow\rangle\), then it evolves both due to the tunneling term \( \frac{\epsilon}{2} \sigma_z \) and due to the system-environment couplings. However, interested in changes in the system state stemming from the system-environment interactions only, we remove the evolution due to our system Hamiltonian, \( H_{S1} + H_{S2} \), before performing any measurement to check the state of the system [3]. The effective decay rate obtained as a result is what we call the modified decay rate, and we investigate how it depends on the strengths of the strong and weak system-environment couplings.

We apply this very treatment to the problem generalizing our spin-boson model to \( N_S \) two-level systems, all of which interact with the aforementioned environment of harmonic oscillator baths in the same way as the single two-level system described above; the only difference is that we prepare our system in a coherent spin state rather than \(|\uparrow\rangle\) this time. Also known as the large spin-boson model [4], this problem is described by the following Hamiltonian:

\[ H^{(1)}_L = \epsilon J_z + \Delta J_x + \sum_k \omega_k b_k^\dagger b_k + \sum_k \alpha_k c_k^\dagger c_k + 2J_z \sum_k \left( g_k^* b_k + g_k b_k^\dagger \right) + 2J_x \sum_k \left( f_k^* c_k + f_k c_k^\dagger \right), \]

where \( J_z \), \( J_y \), and \( J_x \) are the usual angular momentum operators obeying the commutation relations \([J_k, J_l] = i\epsilon_{klm}J_m\); superscript (1) denotes the second version of our Hamiltonian; and all other symbols have the meanings ascribed to them in the paragraph follow-
ing Eq. (1). Moreover, for this problem, we use the following definitions henceforth: \( H'_{S1} = \epsilon J_z, \) \( H'_{S2} = \Delta J_z, \) \( H'_{B1} = \sum_k \omega_k b_k^\dagger b_k, \) \( H'_{B2} = \sum_k \alpha_k c_k^\dagger c_k, \) \( V'_{C1} = J_z \sum_k \left( g_k b_k + g_k b_k^\dagger \right), \) and \( V'_{C2} = J_z \sum_k \left( f_k c_k + f_k c_k^\dagger \right). \)

II. RESULTS

A. Spin-boson model

1. System density matrix

In this case, our Hamiltonian is \( H_L^{(0)} \) (Eq. (1)), and we use a polaron transformation defined by \( U_P = e^{\chi S_0^z/2} \), where \( \chi = \sum_k \left( \frac{2\alpha_k b_k^\dagger b_k}{\omega_k} - \frac{2\alpha_k b_k b_k^\dagger}{\omega_k} \right) \), to transform it [5]. In the polaron frame then, the Hamiltonian is

\[
H^{(0)} = \frac{\epsilon}{2} \sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sum_k \alpha_k c_k^\dagger c_k + \frac{\Delta}{2} + \sum_k \left( f_k c_k + f_k c_k^\dagger \right) \left( \sigma_+ e^\chi + \sigma_- e^{-\chi} \right). \tag{3}
\]

Since system-environment coupling in the polaron frame is weak, the initial system-environment state could be written as \( \rho^{(0)}(0) = \rho_\beta^{(0)}(0) \otimes \rho_B^{(0)}(0) \), where \( \rho_\beta^{(0)}(0) = |1\rangle \langle 1|, \) \( \rho_B^{(0)}(0) = \frac{e^{-\beta H_B}}{Z_B} \) with \( Z_B = \text{Tr}_B \left( e^{-\beta H_B} \right), \) and \( \rho_B^{(0)}(0) = \frac{e^{-\beta H_{B2}}}{Z_{B2}} \) with \( Z_{B2} = \text{Tr}_{B2} \left( e^{-\beta H_{B2}} \right). \) Using time-dependent perturbation theory [6], we find that

\[
\rho_S^{(0)}(\tau) = \text{Tr}_P \left[ \rho_\beta^{(0)}(0) \right] + \text{Tr}_B \left( \rho_B^{(0)}(0) \right) e^{i H_S^{(0)}(\tau) - i H^{(0)}(\tau)} e^{-i H^{(0)}(\tau)}, \tag{4}
\]

This matrix \( \rho_S^{(0)}(\tau) \), and it happens to be

\[
\text{Tr}_{B1,B2} \left( e^{i H_S^{(0)}(\tau) - i H^{(0)}(\tau)} \rho^{(0)}(0) e^{i H^{(0)}(\tau) - i H^{(0)}(\tau)} \right),
\]

where \( H_S^{(0)} = \frac{\epsilon}{2} \sigma_z + \frac{\Delta}{2} \left( \sigma_+ e^\chi + \sigma_- e^{-\chi} \right). \) We note that \( e^{i H^{(0)}(\tau)} \) and \( e^{-i H^{(0)}(\tau)} \) remove the evolution due to the system Hamiltonian before a measurement is performed. Since we assume that the tunneling amplitude and the system-environment coupling in the polaron frame are small, we could expand both \( e^{-i H_S^{(0)}(\tau)} \) and \( e^{-i H^{(0)}(\tau)} \) into perturbation series. Keeping terms to second order then, we find that \( \rho_S^{(0)}(\tau) \) is the sum of \( \rho_\beta^{(0)}(\tau) \) and some additional terms. It could easily be shown that most of these additional terms contribute nothing to the modified decay rate. The terms that need to be worked out, however,
are

\[
\begin{align*}
\text{Tr}_{B1,B2} & \left( U^{(0)}(\tau) A^{(0)}_1 \rho^{(0)}(0) U^{(0)*}(\tau) A^{(0)}_{SP1} \right), \\
\text{Tr}_{B1,B2} & \left( A^{(0)}_{SP1} U^{(0)}(\tau) \rho^{(0)}(0) A^{(0)}_1 U^{(0)}(\tau) \right), \\
\text{Tr}_{B1,B2} & \left( A^{(0)}_{SP1} U^{(0)}(\tau) \rho^{(0)}(0) U^{(0)*}(\tau) A^{(0)}_{SP1} \right), \\
\text{Tr}_{B1,B2} & \left( U^{(0)}(\tau) A^{(1d)}_1 \rho^{(0)}(0) U^{(0)*}(\tau) A^{(0)}_{SP1} \right), \\
\end{align*}
\]

and

\[
\begin{align*}
\text{Tr}_{B1,B2} & \left( A^{(0)}_{SP1} U^{(0)}(\tau) \rho^{(0)}(0) A^{(1d)}_{1d} U^{(0)*}(\tau) \right)
\end{align*}
\]

with \( U^{(0)}(\tau) = U^{(0)}(\tau) U^{(0)}(\tau) U^{(0)}(\tau) \), \( A^{(0)}_1 = -\frac{\lambda}{2} \sum_\mu \int_0^\tau dt \bar{F}_\mu(t) \otimes \bar{B}_\mu(t) \), and \( A^{(0)}_{SP1} = -\frac{\lambda}{2} \sum_\mu \int_0^\tau dt \bar{F}_\mu(t) \otimes B_\mu \).

2. Survival probability and modified decay rate

At time \( t = 0 \), we prepare our system in the state \(|\uparrow\rangle\), where \( \sigma_z |\uparrow\rangle = |\uparrow\rangle \), and we subsequently perform a measurement after every interval of duration \( \tau \) to check if the system state is still \(|\uparrow\rangle\). The evolution due to the system Hamiltonian is removed before each measurement is performed so that the survival probability obtained could be used to calculate the modified decay rate. The required survival probability then is \( s^{(0)}_n(\tau) = 1 - \langle |\psi^{(0)}_n(\tau) |\uparrow\rangle \). Using our expression for \( \rho^{(0)}(\tau) \) (Eq. (4))—together with the additional terms we computed—to calculate \( \rho^{(0)}(\tau) \), we obtain

\[
s^{(0)}_n(\tau) = 1 - 2 \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}_{12}(t_1, t_2) K^{(0)}_{12}(t_1, t_2) e^{i\epsilon(t_1-t_2)} \right)
- \frac{\Delta^2}{2} \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}_{12}(t_1, t_2) e^{i\epsilon(t_1-t_2)} \right)
- \frac{\Delta^2}{4} \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}_{12}(0, 0) e^{i\epsilon(t_1-t_2)}
+ \frac{\Delta^2}{2} \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}_{12}(\tau, t_1) e^{i\epsilon(t_2-t_1)} \right)

= 1 - 2 \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}(t_1-t_2) K^{(0)}(t_1-t_2) e^{i\epsilon(t_1-t_2)} \right)
- \frac{\Delta^2}{2} \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}(t_1-t_2) e^{i\epsilon(t_1-t_2)} \right)
- \frac{\Delta^2}{4} \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}(0, 0) e^{i\epsilon(t_1-t_2)}
+ \frac{\Delta^2}{2} \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}(\tau-t_1) e^{i\epsilon(t_2-t_1)} \right)

= 1 - 2 \Re \left[ \int_0^\tau dt_1 \int_0^{t_1} dt_2 \epsilon^{(t_1-t_2)} + \frac{\Delta^2}{4} C^{(0)}(t_1) e^{i\epsilon t_1} \right]
- \frac{\Delta^2}{4} \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}(0, 0) e^{i\epsilon(t_1-t_2)}
+ \frac{\Delta^2}{2} \Re \left( \int_0^\tau dt_1 \int_0^{t_1} dt_2 C^{(0)}(t_1) e^{i\epsilon(t_1-t_2)} \right).
\]
frame is weak, we could neglect the build-up of correlations between the system and the environment and write
the survival probability at time $t = N\tau$ as $S_n^{(0)}(t = N\tau) = \left(\frac{s_n^{(0)}(\tau)}{s_0^{(0)}}\right)^N \equiv e^{-\Gamma_n^{(0)}(\tau)N\tau}$, thereby defining
the modified decay rate $\Gamma_n^{(0)}(\tau)$. It follows that $\Gamma_n^{(0)}(\tau) = -\frac{1}{2} \ln s_n^{(0)}(\tau)$ [1]. Using the expression we derived for $s_n^{(0)}(\tau)$ (Eq. (5)), we can work out the following expression for $\Gamma_n^{(0)}(\tau)$:

\[
\frac{2}{\tau} \int_0^\tau dt \int_0^t dt' e^{-\Phi_{R1}(\tau)} \cos(\epsilon t' - \Phi_{I1}(t')) \Phi_{R2}(t') + \frac{2}{\tau} \int_0^\tau dt \int_0^t dt' e^{-\Phi_{R1}(\tau)} \sin(\epsilon t' - \Phi_{I1}(t')) \Phi_{I2}(t') + \frac{\Delta^2}{2\tau} \sin^2 \left(\frac{\epsilon\tau}{2}\right) e^{-\Phi_{R1}(\tau) - 2\Phi_{I1}(\tau)} - \frac{\Delta^2}{2\tau} \sin \left(\frac{\epsilon\tau}{2}\right) \int_0^\tau dt e^{-\Phi_{R1}(t)} \cos \left[\epsilon \left(t - \frac{\tau}{2}\right) - \Phi_{I1}(t)\right].
\]

We now plot $\Gamma_n^{(0)}(\tau)$ against $\tau$. To do so, we model the spectral densities as $J(\omega) = G\omega^\alpha \omega_1^2 e^{-\omega/\omega_c}$ and $H(\omega) = F\omega^r \omega_1^2 e^{-\omega/\omega_c}$, where $G$ and $F$ are dimension-
less parameters characterizing the system-environment coupling strengths, $\omega_c$ and $\epsilon_c$ are cut-off frequencies, and $s$ and $r$ are Ohmicity parameters [7]. Whereas $G$
corresponds to strong coupling, $F$ corresponds to the weak one; therefore, in our plots, we keep $G$ greater than $F$. To be particular, we work at zero temperature and look at the Ohmic case for each of the spectral densities ($s = 1$
and $r = 1$). Doing so gives $\Phi_{R1}(t) = 2G \ln(1 + \omega_c^2 t^2)$, $\Phi_{I1}(t) = 4G \tan^{-1}(\omega_c t)$, $\Phi_{R2}(t) = F \frac{\alpha_c^2(1 - \alpha_c^2 t^2)}{(1 + \alpha_c^2 t^2)^2}$, and $\Phi_{I2}(t) = 2F \frac{\alpha_c^2 t}{(1 + \alpha_c^2 t^2)^2}$, allowing us to write our expression for $\Gamma_n^{(0)}(\tau)$ as

\[
\frac{2F}{\tau} \int_0^\tau dt \int_0^t dt' \frac{\alpha_c^2(1 - \alpha_c^2 t^2)}{(1 + \alpha_c^2 t^2)^2} \cos(\epsilon t' - 4G \tan^{-1}(\omega_c t')) + \frac{4F}{\tau} \int_0^\tau dt \int_0^t dt' \frac{\alpha_c^2 t}{(1 + \alpha_c^2 t^2)^2} \frac{1}{(1 + \omega_c^2 t^2)^2} \sin(\epsilon t' - 4G \tan^{-1}(\omega_c t')) + \frac{2}{\tau} \int_0^\tau dt \int_0^t dt' \frac{\Delta^2}{4} \cos(\epsilon t' - 4G \tan^{-1}(\omega_c t')) + \frac{\Delta^2}{2\tau} \sin^2 \left(\frac{\epsilon\tau}{2}\right) e^{-\Phi_{R1}(\tau) - 2\Phi_{I1}(\tau)} - \frac{\Delta^2}{2\tau} \sin \left(\frac{\epsilon\tau}{2}\right) \int_0^\tau dt e^{-\Phi_{R1}(t)} \cos \left[\epsilon \left(t - \frac{\tau}{2}\right) - \Phi_{I1}(t)\right].
\]

The integrals above could be worked out numerically, and results (graphs of $\Gamma_n^{(0)}(\tau)$ against $\tau$) are shown in
Fig. 1 for different values of the system-environment coupling strengths, $G$ and $F$. What is absolutely clear is that
despite having removed the system Hamiltonian evolution, we obtain the results of Ref. [1]: increasing the
strong coupling strength leads to a decrease in the decay rate (Fig. 1a) whereas increasing the weak coupling
strength leads to an increase (Fig. 1b). Also, just as Ref. [1] illustrates, whereas increasing the weak coupling
strength does not change the qualitative behavior of the Zeno/anti-Zeno transition, increasing the strong coupling
strength does have an effect, namely that it causes the transition to occur at smaller values of $\tau$. Clearly, system
evolution has no bearing on the general effects of strong and weak system-environment couplings.
B. Large spin-boson model

1. System density matrix

The Hamiltonian to be considered for this case is $H_L^{(1)}$ (Eq. (2)), and we again use a polaron transformation to transform it, the transformation being the same as that used in section II A 1. The Hamiltonian in the polaron frame is thus

$$H^{(1)} = \epsilon J_z + \sum_k \omega_k b_k^\dagger b_k + \sum_k \alpha_k c_k^\dagger c_k - \kappa J_z^2$$

$$+ \left[ \frac{\Delta}{2} + \sum_k \left( f_k^c c_k + f_k^t c_k^\dagger \right) \right] (J_z e^x + J_z e^{-x}),$$

(6)

where $\kappa = 4 \sum_k \frac{|n_k|^2}{\omega_k}$.

Since the system-environment coupling in the polaron frame is weak again, we could write the initial system-environment state as $\rho^{(1)}(0) = \rho_S^{(1)}(0) \otimes \rho_B^{(1)}(0) \otimes \rho_B^{(2)}(0)$, where

$$\rho_S^{(1)}(0) = |j\rangle \langle j|, \quad \rho_B^{(1)}(0) = e^{-\beta H_B^{(1)}} \text{ with } Z_{B1}^\prime = \text{Tr}_{B1} e^{-\beta H_B^{(1)}}.$$ 

and $\rho_B^{(2)}(0) = e^{-\beta H_B^{(2)}}$ with $Z_{B2} = \text{Tr}_{B2} e^{-\beta H_B^{(2)}}$. $|j\rangle$ represents a spin coherent state with $j = N_S/2$, where $N_S$, as said in section I, is the number of two-level systems we work with in our large spin-boson model. Using time-dependent perturbation theory then, we find that

$$\rho_S^{(1)}(\tau) = U_S^{(1)}(\tau) \left[ \rho_S^{(1)}(0) + \frac{\beta}{\kappa} \sum_{\mu} \int_0^\tau dt_1 \left[ \rho_S^{(1)}(0), \bar{F}_\mu^{(1)}(t_1) \right] \left\langle \bar{B}_\mu^{(1)}(t_1) \right\rangle_{B1} \left\langle \bar{J}_\mu^{(1)}(t_1) \right\rangle_{B2} \right] U_S^{(1)}(\tau).$$

(7)

Here, $F_1^{(1)} = J_+$, $F_2^{(1)} = J_-$, $B_1^{(1)} = X$, $B_2^{(1)} = X^\dagger$, $J_1^{(1)} = \sum_k \left( f_k^c c_k + f_k^t c_k^\dagger \right)$, and $J_2^{(1)} = \sum_k \left( f_k^c c_k + f_k^t c_k^\dagger \right)$. Their time-evolved counterparts happen to be $\bar{F}_\mu^{(1)}(t) = U_S^{(1)}(t) F_\mu^{(1)} U_S^{(1)\dagger}(t)$ with $U_S^{(1)}(t) = e^{-i(H^{(1)} - \kappa J_z^2) t}$, $\bar{B}_\mu^{(1)}(t) = U_B^{(1)}(t) B_\mu^{(1)} U_B^{(1)\dagger}(t)$ with $U_B^{(1)}(t) = e^{-iH_B t}$, and $\bar{J}_\mu^{(1)}(t) = U_B^{(1)}(t) J_\mu^{(1)} U_B^{(1)\dagger}(t)$ with $U_B^{(1)}(t) = e^{-iH_B t}$. Finally, $\langle \ldots \rangle_B$ stands for $\text{Tr}_{B} \left[ \rho_B^{(1)}(\ldots) \right]$, environment correlation functions are defined as $C_{\mu\nu}^{(1)}(t_1, t_2) = \left\langle \bar{B}_\mu^{(1)}(t_1) \bar{B}_\nu^{(1)}(t_2) \right\rangle_{B1}$ and $K_{\mu\nu}^{(1)}(t_1, t_2) = \left\langle \bar{J}_\mu^{(1)}(t_1) \bar{J}_\nu^{(1)}(t_2) \right\rangle_{B2}$, and h.c. denotes the Hermitian conjugate.

Since we want the modified decay rate, however, we compute the system density matrix with the system Hamiltonian evolution removed. Calling this matrix $\rho_{Sn}^{(1)}(\tau)$ in the polaron frame, we find it to be

$$\text{Tr}_{B1, B2} \left( e^{iH_S^{(1)} \tau} e^{-iH^{(1)} \tau} \rho_S^{(1)}(0) e^{iH^{(1)} \tau} e^{-iH_S^{(1)} \tau} \right),$$

where $H^{(1)} = \epsilon J_z + \frac{\Delta}{2} (J_+ X + J_- X^\dagger)$. It could easily be noted that $e^{iH_S^{(1)} \rho \tau}$ and $e^{-iH_S^{(1)} \rho \tau}$ remove the evolution due to the system Hamiltonian before a measurement is performed. Assuming that the tunneling amplitude and the system-environment coupling in the polaron frame are small, we expand both $e^{-iH_S^{(1)} \rho \tau}$ and $e^{-iH^{(1)} \tau}$ into perturbation series, and keeping terms to second order only, we find that $\rho_{Sn}^{(1)}(\tau)$ is the sum of $\rho_S^{(1)}(\tau)$ and some additional terms. As before, we could show that most of these additional terms do not contribute anything to the modified decay rate. The terms that need to be worked
out, however, are

\[
\text{Tr}_{B_1B_2} \left( A^{(1)}_{S\rho_1} U^{(1)}(\tau) A^{(1)}_{\rho_1} (0) U^{(1)}(\tau) \right),
\]

\[
\text{Tr}_{B_1B_2} \left( A^{(1)}_{S\rho_2} U^{(1)}(\tau) A^{(1)}_{\rho_2} (0) U^{(1)}(\tau) \right),
\]

\[
\text{Tr}_{B_1B_2} \left( U^{(1)}(\tau) \rho^{(1)}(0) U^{(1)}(\tau) A^{(1)}_{S\rho_2} \right),
\]

\[
\text{Tr}_{B_1B_2} \left( U^{(1)}(\tau) \rho^{(1)}(0) A^{(1)}_{\rho_1} U^{(1)}(\tau) A^{(1)}_{S\rho_1} \right),
\]

\[
\text{Tr}_{B_1B_2} \left( U^{(1)}(\tau) \rho^{(1)}(0) A^{(1)}_{\rho_2} U^{(1)}(\tau) A^{(1)}_{S\rho_1} \right),
\]

\[
\text{Tr}_{B_1B_2} \left( U^{(1)}(\tau) \rho^{(1)}(0) A^{(1)}_{\rho_1} U^{(1)}(\tau) A^{(1)}_{S\rho_2} \right),
\]

and

\[
\text{Tr}_{B_1B_2} \left( U^{(1)}(\tau) \rho^{(1)}(0) U^{(1)}(\tau) A^{(1)}_{S\rho_2} \right).
\]

with

\[
U^{(1)}(\tau) = U^{(1)}(\tau) U^{(1)}(\tau) U^{(1)}(\tau),
\]

\[
A^{(1)}_{\rho_1} = -\frac{1}{\hbar} \sum_{\mu} \int_{0}^{\tau} dt \vec{F}_{\mu}(t) \otimes \vec{B}_{\mu}(t),
\]

\[
A^{(1)}_{\rho_2} = -\frac{1}{\hbar} \sum_{\mu} \int_{0}^{\tau} dt \vec{F}_{\mu}(t) \otimes B_{\mu},
\]

\[
A^{(1)}_{S\rho_1} = -\frac{1}{\hbar} \sum_{\mu} \int_{0}^{\tau} dt \vec{F}_{\mu}(t) \otimes \vec{B}_{\mu}(t),
\]

\[
A^{(1)}_{S\rho_2} = -\frac{1}{\hbar} \sum_{\mu} \int_{0}^{\tau} dt \vec{F}_{\mu}(t) \otimes B_{\mu}.
\]

\[2. \text{ Survival probability and modified decay rate}\]

We prepare our system in the state \( |j\rangle \), where \( J_{\rho} |j\rangle = j |j\rangle \), at time \( t = 0 \) and perform a measurement after every interval of duration \( \tau \) to check if the system state is still \( |j\rangle \). The system Hamiltonian evolution is removed before every measurement so that the survival probability obtained corresponds to the modified decay rate. The required survival probability is thus \( s^{(1)}_{\rho_{\text{Sn}}}(\tau) = \langle j | \rho^{(1)}_{\text{Sn}}(\tau) | j \rangle \), where \( \rho^{(1)}_{\text{Sn}} \) is just the sum of \( \rho^{(1)}(\tau) \) (Eq. (7)) and the additional terms we computed. We hence get

\[s^{(1)}_{\rho_{\text{Sn}}}(\tau) = 1 - 4 \text{Re} \left( \int_{0}^{\tau} dt \int_{0}^{\tau} dt C_{12}^{(1)}(t_1, t_2) C_{12}^{(1)}(t_1, t_2) e^{i(t_1-t_2)/2} + \sum_{\kappa=1,2} \int_{0}^{\tau} dt \int_{0}^{\tau} dt C_{12}^{(1)}(t_1, t_2) e^{i(t_1-t_2) + \kappa(1-2j)(t_1-t_2)} \right)
\]

where \( C_{12}^{(1)}(t_1, t_2) = C_{12}^{(1)}(t_1, t_2) \) and \( K_{12}^{(1)}(t_1, t_2) = K_{12}^{(1)}(t_1, t_2) \), and in the last step, we change variables via \( t' = t_1 - t_2 \) and \( t = t_1 \) in the first two integrals and via \( t' = \tau - t_1 \) and \( t = t_2 \) in the fourth one. Calculating the correlation functions \( C^{(1)}(t) \) and \( K^{(1)}(t) \), we find that \( C^{(1)}(t) = e^{-\Phi_{\text{Sn}}(t)} e^{-\Phi_{\text{Sn}}(t)} \)
with \( \Phi_{R1}(t) = \int_0^\infty d\omega J(\omega) \left( \frac{d}{d\omega} \cos(\omega t) \right) \coth \left( \frac{\omega}{2T} \right) \) and \( \Phi_{I1}(t) = \int_0^\infty d\omega J(\omega) \left( \frac{\sin(\omega t)}{\omega^2} \right) \) and that \( K^{(1)}(t) = \Phi_{R2}(t) - \mathcal{I} \Phi_{I2}(t) \) with \( \Phi_{R2}(t) = \int_0^\infty d\alpha H(\alpha) \cos(\alpha t) \coth \left( \frac{\alpha}{2T} \right) \) and \( \Phi_{I2}(t) = \int_0^\infty d\alpha H(\alpha) \sin(\alpha t) \), where the spectral density has been introduced as \( \sum_k |g_k|^2 (\ldots) \to \int_0^\infty d\omega J(\omega)(\ldots) \) and \( \sum_k |f_k|^2 (\ldots) \to \int_0^\infty d\alpha H(\alpha)(\ldots) \).

Again, since system-environment coupling in the polaron frame is weak, we could ignore the correlations building between the system and the environment and follow the reasoning in section II.A.2 to show that the modified decay rate, \( \Gamma_n^{(1)}(\tau) \), is

\[
\frac{4j}{\tau} \int_0^\tau dt \int_0^t dt' e^{-\Phi_{R1}(t') \cos(D_1(t'))} \Phi_{R2}(t') \\
+ \frac{4j}{\tau} \int_0^\tau dt \int_0^t dt' e^{-\Phi_{R1}(t') \sin(D_1(t'))} \Phi_{I2}(t') \\
+ \frac{\Delta^2}{\tau} \int_0^\tau dt \int_0^t dt' e^{-\Phi_{R1}(t') \cos(D_1(t'))} \\
+ \frac{\Delta^2}{\tau} (2j) \frac{1}{\epsilon^2} \sin^2 \left( \frac{\epsilon t}{2} \right) e^{-\Phi_{R1}(0) - \Phi_{I1}(0)} \\
- \frac{\Delta^2}{\tau} (2j) \frac{1}{\epsilon} \sin \left( \frac{\epsilon t}{2} \right) \int_0^\tau dt' e^{-\Phi_{R1}(t') \cos(D_2(t))},
\]

where \( D_1(t) = \epsilon t + \kappa(1 - 2j) t - \Phi_{I1}(t) \) and \( D_2(t) = -\kappa(2j - 1) t + \epsilon(t - \tau/2) - \Phi_{I1}(t) \).

To plot \( \Gamma_n^{(1)}(\tau) \) against \( \tau \), we model the spectral densities the same way as before and look at the Ohmic case for each of them. Working at zero temperature then allows us to write our expression for \( \Gamma_n^{(1)}(\tau) \) as

\[
\frac{4Fj}{\tau} \int_0^\tau dt \int_0^t dt' \frac{\alpha_2^2(1 - \alpha_2^2 t^2) \cos(D_1(t'))}{(1 + \alpha_2^2 t^2)^2(1 + \omega_c^2 t^2)^2} \\
+ \frac{8Fj}{\tau} \int_0^\tau dt \int_0^t dt' \frac{\alpha_2 t' \sin(D_1(t'))}{(1 + \alpha_2^2 t^2)^2(1 + \omega_c^2 t^2)^2} \\
+ \frac{\Delta^2}{\tau} \int_0^\tau dt \int_0^t dt' \frac{\cos(D_1(t'))}{(1 + \omega_c^2 t^2)^2} \\
+ \frac{\Delta^2}{\tau} (2j) \frac{1}{\epsilon^2} \sin^2 \left( \frac{\epsilon t}{2} \right) \int_0^\tau dt' \frac{\cos(D_2(t))}{(1 + \omega_c^2 t^2)^2}.
\]

The integrals could again be worked out numerically. Results are shown in Fig. 2 for different values of the system-environment coupling strengths, \( G \) and \( F \), and they are precisely what one would expect them to be if the system Hamiltonian evolution were kept: increasing the weak coupling strength increases the decay rate (Fig. 2b) whereas increasing the strong coupling strength decreases it generally (Fig. 2a). Also, as is evident, a change in the weak coupling strength has no effect on the qualitative behavior of the Zeno/anti-Zeno transitions. A change in the strong coupling strength, however, does have an effect; as before, increasing it causes the transitions to occur at smaller values of \( \tau \). Once again, since we obtain these very results even if the system Hamiltonian evolution is kept, we conclude that the system evolution has no practical bearing on any of them.

FIG. 2: Variation of the modified decay rate for the large spin-boson model with both strong and weak system-environment coupling strengths. (a) Graph of \( \Gamma_{(1)}(\tau) \) (at zero temperature) against \( \tau \) when \( G = 0.4 \) (solid, blue curve), \( G = 0.8 \) (dashed, orange curve), and \( G = 1.5 \) (dot-dashed, green curve). Here, we have used an Ohmic environment \( (\tau) = 1 \) with \( \epsilon = 1, \alpha_c = 1, \Delta = 0.05, \) and \( j = 3. \) The initial state is \( |j\rangle \). (b) Graph of \( \Gamma_{(1)}(\tau) \) (at zero temperature) against \( \tau \) when \( F = 0.03 \) (solid, blue curve), \( F = 0.05 \) (dashed, orange curve), and \( F = 0.1 \) (dot-dashed, green curve). Here, we have again used an Ohmic environment \( (s = 1 \) and \( r = 1 \) with \( G = 1.5, \epsilon = 1, \omega_c = 1, \alpha_c = 1, \Delta = 0.05, \) and \( j = 3. \) The initial state is still \( |j\rangle \). \( \hbar \) is equal to 1 throughout.
III. DISCUSSION

Our results clearly demonstrate that removing the evolution effected by the system Hamiltonian just before measurements are performed does not change the general effects of strong and weak system-environment couplings. We expand the previous work done in this area by considering the presence of both strong and weak couplings, and we successfully show not only that independent coupling baths produce independent effects but also that this independence remains intact even after the system evolution is removed, an observation hinting at the deeper independence of independent strong and weak system-environment couplings.

Although we apply our strategy to spin-boson and large spin-boson models only, our work establishes a guide for the implementation of our strategy to other such instances of open quantum systems as well, thereby adding to our knowledge of how multiple system-environment couplings could be treated to study the quantum Zeno and anti-Zeno effects.

Given the ubiquity of open quantum systems, our work is highly relevant, for it could be appended to any investigation of Zeno and anti-Zeno effects in these systems. Just to highlight its scope, we propose that our work could be used to expand investigations of the Unruh effect and Unruh-DeWitt detectors, it could be applied to the studies of nonselective projective measurements, and it could even be employed in analyses of the Zeno and anti-Zeno effects in quantum field theory [8–10]. Hence, our work—though simple—has pretty extensive applications.

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