Jewels on a wall ring
(Sine-Gordon kinks on a domain wall ring)

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Abstract

We construct a stable domain wall ring with lump beads on it in a baby Skyrme model with a potential consisting of two terms linear and quadratic in fields.
I. INTRODUCTION

Vortices and domain walls are topological solitons present in various physical systems from field theory [1] and cosmological models [2] to condensed matter systems [3]. In contrast to instantons and monopoles which have been studied extensively in high energy physics and mathematics, vortices and domain walls have not been paid much attention thus far in those fields. However they play essential roles in condensed matter systems such as superconductors, superfluids, magnetism, quantum Hall states, nematic liquids, optics, and so on. The coexistence of these two kinds of solitons can also happen in various condensed matter systems; a Bloch line in a Bloch wall in magnetism [4], half-quantized vortices inside a chiral domain wall in chiral $p$-wave superconductors [5], and a Mermin-Ho vortex within a domain wall in $^3$He superfluid (see Fig. 16.9 of Ref. [3]). Further examples can be found in the limit of infinitely heavy domain walls: Josephson vortices within an insulator in Josephson junctions of two superconductors [6] and Josephson vortices in high-$T_c$ superconductors with multi-layered structures [7] and in two coupled Bose-Einstein condensates [8], where the insulators or inter-layers can be regarded as (heavy) domain walls. In all these cases, vortices become sine-Gordon solitons once absorbed into a domain wall. A field theoretical model of the coexistence of domain walls and vortices as Josephson vortices was given recently [9] in order to explain a previously known relation between vortices and sine-Gordon solitons [10]. (They are the lowest dimensional example of “matryoshka Skyrmions” [11].) Slightly different field theoretical models admitting the coexistence of domain walls and vortices were also considered before [12].

In these cases, vortices are all absorbed into a domain wall. In this sense, there seem to be no freely moving vortices in the bulk outside the domain wall. However, a question arises. What happens if one makes a closed loop of the domain wall? If a vortex is absorbed into a closed domain line, it may be regarded as a freely moving vortex, apart from its stability. In fact, recently such configurations of a domain wall ring with vortices on it have been theoretically proposed in condensed matter systems, such as chiral $p$-wave superconductors [2] and multi-gap superconductors [13]. Motivated by these works, in this paper, we propose a field theoretical model admitting a stable domain wall ring with vortices absorbed in it.

We consider an $O(3)$ nonlinear sigma model on the target space $S^2$ in $d = 2 + 1$ dimensions, described by a unit three-vector of scalar fields $n(x) = (n_1(x), n_2(x), n_3(x))$ with the
constraint $n^2 = 1$, which is equivalent to a $CP^1$ model. The $O(3)$ model admits lumps or sigma model instantons [14] as a relative of vortices. The $CP^1$ model with a potential term admitting two discrete vacua is known as the massive $CP^1$ model, which can be made supersymmetric with additional fermions [15] and admits a Bogomol’nyi-Prasad-Sommerfield domain wall solution interpolating the two discrete vacua [16, 17]. In the presence of a potential term, lumps are unstable to shrink in general. Instead, if one gives them a linear time-dependence on their $U(1)$ moduli, they become stable Q-lumps [18]. If we consider a four derivative (Skyrme) term, the lumps are stabilized to become baby Skyrmions [19].

A closed domain line or a wall ring is nothing but a lump if the $U(1)$ modulus of the domain wall is twisted along the ring [20]. This twisted domain wall ring is unstable to shrink unless one puts linear time-dependence on the $U(1)$ modulus or adds the Skyrme term, as denoted above. More precisely, the originally proposed baby Skyrme model has the potential term $V = m^2(1 - n_3)$ [19], which admits the unique vacuum and does not admit a domain wall. A new baby Skyrme model proposed later [21, 22] has the potential $V = m^2(1 - n_3)(1 + n_3)$ admitting two discrete vacua $n_3 = \pm 1$ and a domain wall solution interpolating between these two vacua [22, 23], as the case without a Skyrme term [16, 17]. In this model, a baby Skyrmion is in fact in the shape of a domain wall ring. In this paper, we consider both types of the potential terms $V = \beta^2 n_1 + m^2(1 - n_3)(1 + n_3)$ in the regime $\beta \ll m$. In magnetism, this potential term appears in Heisenberg ferromagnets with two easy axes. Such an $O(3)$ sigma model without the Skyrme term was studied recently to consider a vortex absorbed into a straight domain wall [9], but it does not admit a stable domain wall ring. Here we consider the Skyrme term to stabilize a domain wall ring. We numerically construct domain wall rings with one, two and three vortices (lumps), which have the topological lump charges $k = 1, 2, 3$, respectively, looking like jewels on a ring. These vortices are sine-Gordon kinks on the domain wall ring. We find that lumps are placed with the same distance from each other because of repulsions among them.

This paper is organized as follows. After our model is given in Sec. III, we give a numerical solution of a twisted domain line in the absence of the term $\beta^2 n_1$ in the potential in Sec. III. It carries the topological lump charge and is nothing but a baby Skyrmion. In Sec. IV, we give numerical solutions of a lump within a straight domain wall in the models with the term $\beta^2 n_1$ in the potential without [9] and with the Skyrme term. Then, in Sec. V, we give numerical solutions of domain wall rings with one, two and three lumps. Section VI is
devoted to a summary and discussion.

II. THE MODEL

We consider an $O(3)$ sigma model in $d = 2 + 1$ dimensions described by a three vector of scalar fields $\mathbf{n}(x) = (n_1(x), n_2(x), n_3(x))$ with a constraint $\mathbf{n} \cdot \mathbf{n} = 1$. The Lagrangian of our model is given by

$$L = \frac{1}{2} \partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n} - L_4(\mathbf{n}) - V(\mathbf{n}),$$

(1)

with $\mu = 0, 1, 2$. Here, the four derivative (baby Skyrme) term is expressed as

$$L_4(\mathbf{n}) = \kappa [\mathbf{n} \cdot (\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})]^2 = \kappa (\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})^2,$$

(2)

and the potential term is given by

$$V(\mathbf{n}) = m^2(1 - n_3^2) + \beta^2 n_1.$$  

(3)

The potential with $m = 0$ was originally considered in Ref. [19], and the one with $\beta = 0$ was proposed later in Refs. [21, 22]. The choice of our potential is physically quite natural, since it is known as the Heisenberg ferromagnet with anisotropy with two easy axes.

With introducing the projective coordinate $u (\in \mathbb{C})$ of $\mathbb{C}P^1$ by

$$n_i = \phi^i \sigma_i \phi, \quad \phi^T = (1, u) / \sqrt{1 + |u|^2},$$

(4)

the Lagrangian (1) can be rewritten in the form of the $\mathbb{C}P^1$ model with potential terms, given by

$$L = 2 \frac{\partial_\mu u^* \partial^\mu u}{(1 + |u|^2)^2} - 8\kappa \frac{(\partial_\mu u^* \partial^\mu u)^2 - |\partial_\mu u \partial^\mu u|^2}{(1 + |u|^2)^4} - V$$

(5)

$$V = m^2(1 - D_3^2) + \beta^2 D_1 = m^2 g_{uu^*} |\partial_u D_3|^2 + \beta^2 D_1,$$

(6)

$$D_3 \equiv \frac{1 - |u|^2}{1 + |u|^2} = n_3, \quad D_1 \equiv \frac{u + u^*}{1 + |u|^2} = n_1.$$ 

(7)

Here, $g_{uu^*} = 1/(1 + |u|^2)^2$ is the Kähler (Fubini-Study) metric of $\mathbb{C}P^1$, $g^{uu^*} = (1 + |u|^2)^2$ is its inverse, and $D_i = n_i$ are called the moment maps (or the Killing potentials) of the $SU(2)$ isometry generated by $\sigma_i$, respectively. With $\beta = 0$ and $\kappa = 0$, this model is known as the massive $\mathbb{C}P^1$ model with the potential term of the norm of the Killing vector $\partial_u D_3$ corresponding to the isometry generated by $\sigma_3$, which is a truncated version of a supersymmetric sigma model with eight supercharges [13, 16].
III. TWISTED CLOSED DOMAIN WALL

For a while, let us ignore the four derivative baby Skyrme term ($\kappa = 0$) in the Lagrangian in Eq. (1). Let us consider the potential in Eq. (3) with $\beta = 0$. It admits two discrete vacua $n_3 = \pm 1$. A domain wall or an anti-domain wall solution interpolating these two vacua is given by [11, 16, 17]

$$\theta(x^1) = 2 \arctan \exp(\pm \sqrt{2} m(x^1 - X)), \quad 0 \leq \theta \leq \pi,$$

$$n_1 = \cos \alpha \sin \theta(x^1), \quad n_2 = \sin \alpha \sin \theta(x^1), \quad n_3 = \cos \theta(x^1), \quad (8)$$

with a phase modulus $\alpha$ ($0 \leq \alpha < 2\pi$) and the translational modulus $X \in \mathbb{R}$ of the domain wall. The moduli $\alpha$ and $X$ can be regarded as Nambu-Goldstone modes corresponding to $U(1)$ and translational symmetries spontaneously broken down in the vicinity of the domain wall, respectively. A domain wall solution in the presence of the baby Skyrme term was studied in Refs. [22, 23].

A loop of the domain wall carries a lump charge if the $U(1)$ modulus $\alpha$ winds along the wall loop [20]. The topological charge of the lump $\pi_2(S^2) \simeq \mathbb{Z}$ is given by

$$k = \frac{1}{4\pi} \int d^2x \mathbf{n} \cdot (\partial_1 \mathbf{n} \times \partial_2 \mathbf{n}) = \frac{1}{4\pi} \int d^2x \varepsilon_{ijk} n_i \partial_1 n_j \partial_2 n_k$$

$$= \frac{1}{2\pi} \int d^2x \frac{i(\partial_1 u^* \partial_2 u - \partial_2 u^* \partial_1 u)}{(1 + |u|^2)^2}. \quad (9)$$

However, a twisted closed wall line is unstable to shrink. It can be stabilized in the presence of the baby Skyrme term, which results in a baby Skyrmion. We construct a numerical solution of a twisted domain wall ring with the unit lump charge ($k = 1$) by a relaxation method; see Fig. 1. One can see that the topological lump charge as well as the energy density is uniformly distributed along the ring.

In the context of magnetism, this configuration is called a bubble domain [4].

IV. SINE-GORDON KINKS ON A STRAIGHT DOMAIN WALL

With promoting the moduli to fields $\alpha(t, x^2)$ and $X(t, x^2)$ on the domain wall world-volume $(t, x^2)$, the effective theory of the domain wall can be constructed by the moduli approximation [24]. It is a free field theory of $\alpha(t, x^2)$ and $X(t, x^2)$ or a sigma model on $\mathbb{R} \times S^1$. The $U(1)$ symmetry is explicitly broken when $\beta$ is taken into account in the potential.
FIG. 1: A twisted domain wall ring as a baby Skyrmion. (a) The textures $n(x)$. The color of each arrow shows the value of $n_3$. (b) The total energy density $E \equiv (\partial_2 n \cdot \partial^2 n)/2 + L_4(n) + V(n)$ ($a = 1, 2$). (c) The topological lump charge density $c \equiv \{n \cdot (\partial_1 n \times \partial_2 n)\}/(4\pi)$. As numerical parameters, we fix $\kappa = 0.02$ and $m^2 = 20000$, and plot the values in the region $-0.29 \leq x^{1,2} \leq 0.29$.

Then, a potential term is induced on the domain wall effective action and it becomes the sine-Gordon model [9]. A sine-Gordon kink in the wall effective theory corresponds to a lump in the bulk [9], in which the topological lump charge $k$ coincides with the topological charge $k$ of sine-Gordon kinks.

In the left column of Fig. 2 we give a numerical solution of one sine-Gordon kink on the domain wall by using the relaxation method. In (a), we plot our solutions $n_i(x)$ by arrows. In (b), we plot the energy contribution from the term $\beta^2 n_1$ in the potential, in order to show sine-Gordon kinks. The total energy density is plot in (c). The lump charge density given in the integrand of Eq. (9) is distributed around the sine-Gordon kink as seen in (d). In the right column of Fig. 2 we give a numerical solution of the same configuration in the presence of the Skyrme term. One can see that the size of sine-Gordon kink becomes wider due to the Skyrme term.

Multiple sine-Gordon kinks repel each other, and such a configuration cannot be static on the straight domain wall. However, they can be stabilized once the domain wall is closed as demonstrated in the next section.
FIG. 2: (a) The textures $n(x)$. The color each arrow shows the value of $n_3$. (b) The energy densities $\mathcal{E}_2 \equiv \beta^2 n_1$. (c) The total energy densities $\mathcal{E}$. (d) The topological charge densities $c$. The left and right columns correspond to the cases without and with the Skyrme term, respectively.

The numerical box satisfies the periodic boundary condition in the vertical ($x^2$) direction, i.e., $n_i(x^1, x^2 + L) = n_i(x^1, x^2)$. As numerical parameters, we fix $L = 0.5$, $m^2 = 8000$ and $\beta^2 = 800$ for both left and right figures, and $\kappa = 0.002$ for right figures, and plot the values in the region $-0.12 < x^1 < -0.12$ and $0 < x^2 < 0.5$. 


V. JEWELS ON A DOMAIN WALL RING

Next we make a closed loop of a domain wall with sine-Gordon kinks on it. In Fig. 3, we show our numerical results by using a relaxation method. We constructed configurations with the topological lump charge \( k = 1, 2, 3 \). In (a), we plot our solutions \( n_i(x) \) by arrows. In (b), we plot the energy contribution from the term \( \beta^2 n_1 \) in the potential, in order to show sine-Gordon kinks. One clearly finds that sine-Gordon kinks are separated from each other with the same distance for \( k = 2, 3 \). This is because they repel each other. In (c), we plot the total energy of each configuration. In (d), we plot the topological lump charge density [the integrand of Eq. (9)]. One can see that the topological charge density is distributed on the wall ring and has peaks at the sine-Gordon kinks.

VI. SUMMARY AND DISCUSSION

We have constructed stable configurations of sine-Gordon kinks on a domain wall ring in a baby Skyrme model with the two potential terms linear and quadratic in fields. The number of the sine-Gordon kinks coincides with the topological lump charge.

Similar configurations of a wall ring with vortices on it are present in condensed matter systems, such as multi-gap superconductors \([13]\) and chiral \( p \)-wave superconductors \([5]\) in which a four derivative term is not needed. Our present work was motivated by these works.

Our model can be promoted to a \( U(1) \) gauge theory coupled with two complex scalar fields \( \phi_1(x) \) and \( \phi_2(x) \), in which lumps are replaced with semi-local vortices. In this case, the term \( \beta^2 n_1 \) is reproduced from the Josephson term \( \beta^2 \phi_1^* \phi_2 + \text{c.c.} \). Then, the model becomes close to exotic superconductors considered in \([5, 13]\), if we replace the Josephson term by \( \beta^2 (\phi_1^* \phi_2)^2 + \text{c.c.} \). In this case, one vortex is decomposed into two fractional vortices once absorbed into a domain wall. However, we still need four derivative term in scalar fields for the stability of wall rings.

If we promote our configuration linearly in \( d = 3 + 1 \) dimensions, it becomes a tube with domain lines along it. It can be regarded as some exotic cosmic strings which may have some impacts on cosmology. For instance, it is a very nontrivial question whether two of such strings reconnect each other when they collide, because they have internal structures. It may be one of the interesting future directions.
FIG. 3: (a) The textures $n(x)$. The color of each arrow shows the value of $n_3$. (b) The energies $E_2$. (c) The total energies $E$. (d) The topological charge densities $c$. The topological charges are $k = 1, 2, 3$ from left to right. As numerical parameters, we fix $\kappa = 0.02$, $m^2 = 20000$ and $\beta^2 = 2000$, and plot the values in the region $-0.29 \leq x^{1,2} \leq 0.29$. 
In Ref. [26], a configuration of a sine-Gordon kink on a domain wall was embedded into the 2 + 1 dimensional world-volume of a non-Abelian vortex [27] in $d = 4 + 1$ dimensions. In this case, the sine-Gordon kinks correspond to lumps in the vortex world-volume [26] and to instantons in the bulk [28, 29], while the domain wall in the vortex world-volume corresponds to a monopole string in the bulk [28, 29]. The configuration is an instanton confined by two monopole strings attached from both sides [26]. Similarly to this, if one embeds our solution in this paper, it becomes instanton beads on a closed monopole string, as illustrated in Fig. 4. For this configuration to be stabilized, one needs higher derivative corrections to the vortex effective action [31, 32].

We have studied a massive $\mathbb{C}P^1$ model. One can extend it to massive $\mathbb{C}P^n$ model which admits $n - 1$ parallel domain walls [33]. It is an open question whether one can construct multiple $n - 1$ rings with lump beads on them in this model. Also, one can discuss non-Abelian domain walls [34] in the massive Grassmannian sigma model [35] or in non-Abelian gauge theories.

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