Can R-parity violating supersymmetry be seen in long baseline beta-beam experiments?

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ABSTRACT

Long baseline oscillation experiments may well emerge as test beds for neutrino interactions as are present in R-parity violating supersymmetry. We show that flavour diagonal (FDNC) and flavour changing (FCNC) neutral currents arising therefrom prominently impact a neutrino $\beta$-beam experiment with the source at CERN and the detector at the proposed India-based Neutrino Observatory. These interactions may preclude any improvement of the present limit on $\theta_{13}$ and cloud the hierarchy determination unless the upper bounds on $R$ couplings, particularly $\lambda'$, become significantly tighter. If $R$ interactions are independently established then from the event rate a lower bound on $\theta_{13}$ may be set. We show that there is scope to see a clear signal of non-standard FCNC and FDNC interactions, particularly in the inverted hierarchy scenario and also sometimes for the normal hierarchy. In favourable cases, it may be possible to set lower and upper bounds on $\lambda'$ couplings. FCNC and FDNC interactions due to $\lambda$ type $R$ couplings are unimportant.

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I Introduction

Neutrino physics, a bit player on the physics stage in yesteryears, has now donned a central role. Various experiments on atmospheric \[1\], solar \[2\], reactor \[3\], and long baseline neutrinos \[4\] not only indicate oscillation but also pin down most neutrino mass and mixing parameters – two mass-square differences and two mixing angles. The best-fit \[5\] values with 3$\sigma$ error from atmospheric neutrinos are $|\Delta m^2_{23}| \simeq 2.12^{+1.09}_{-0.81} \times 10^{-3}$ eV$^2$, $\theta_{23} \simeq 45.0^{+10.55}_{-9.33}^\circ$ and from solar neutrinos $\Delta m^2_{12} \simeq 7.9^{+1.0}_{-0.8} \times 10^{-5}$ eV$^2$, $\theta_{12} \simeq 33.21^{+4.85}_{-4.55}^\circ$. Here $\Delta m^2_{ij} = m_j^2 - m_i^2$. The sign of $\Delta m^2_{23}$ is yet unknown and the neutrino mass spectrum will be referred here as normal (inverted) hierarchical if it is positive (negative). Using reactor antineutrinos \[5, 3, 6\], an upper bound has been set on the third mixing angle at 3$\sigma$ level as $\sin^2 \theta_{13} < 0.044$ resulting in $\theta_{13} < 12.1^\circ$. The phase $\delta$ in the neutrino mixing matrix is not known.

Research in this area is now poised to move into the precision regime. Can we use upcoming neutrino experiments to probe non-standard interactions like $R$ supersymmetry? If present, can they become spoilers in attempts to further sharpen the neutrino properties? We attempt to address these issues with a specific experiment as our laboratory.

Long baseline neutrino oscillation experiments using neutrino factories \[7\] and $\beta$-beams \[8, 9, 10, 11\] hold promise of refining our knowledge of $\theta_{13}$, $\delta$, and the sign of $\Delta m^2_{23}$. A possible experiment in this category would use neutrinos from a $\beta$-beam from CERN to the proposed India-based Neutrino Observatory \[12\] (INO), a baseline of $\sim 7152$ Km. This is the set-up which we consider here. For such an experiment the $\beta$-beam is required to be boosted to high $\gamma$.

Interaction of neutrinos with matter affect long baseline experiments and this becomes more prominent at higher values of $\theta_{13}$. Various authors \[13\] have considered this effect for atmospheric neutrinos.

Apart from the electroweak effects, there may well be non-standard interactions leading to flavour changing as well as flavour diagonal neutral currents (FCNC and FDNC). Here we have in mind interactions with quarks and leptons involving an initial and a final neutrino. If there is no change in the neutrino flavour – as, for example, in $Z^0$ exchange – this is classified as an FDNC process, while it would be FCNC otherwise. R-parity violating supersymmetric models (RPVSM) \[14\], which have such interactions already built-in\(^1\), will be the main focus of our work. Very recently a model in which couplings associated with FCNC and FDNC can be quite a bit higher than permitted in RPVSM has also been considered \[15, 16\]. Naturally, here the matter effect will be further enhanced. However, as RPVSM is a well-studied, renormalizable model which can satisfy all phenomenological constraints currently available, we shall restrict our main analysis only to it and shall make qualitative remarks about the other model, for which our results can be easily extended.

Consequences of FCNC and FDNC for solar and atmospheric neutrinos \[17, 18\], and neutrino factory experiments \[19\] have been looked into. Our focus is on $\beta$-beam experiments, particularly over a long distance (7152 Km) baseline. Our analysis encompasses both normal and inverted hierarchies and we also incorporate all relevant trilinear R-parity violating couplings leading to FCNC and FDNC. Huber et al \[20\] have a somewhat similar analysis using neutrino beams obtained from muon decays.

The very long baseline from CERN to INO will capture a significant matter effect and offers a scope to signal non-standard interactions. We examine whether the presence of $R$

\(^1\)e.g. through squark ($\lambda'$-type couplings) or slepton ($\lambda$-type couplings) exchange.
interactions will come in the way of constraining the mixing angle $\theta_{13}$ or unraveling the neutrino mass hierarchy. The possibility to obtain bounds on some R-parity violating couplings is also probed.

II $\beta$-beams

A beta-beam \cite{8}, is a pure, intense, collimated beam of electron neutrinos or their antiparticles produced via the beta decay of completely ionized, accelerated radioactive ions circulating in a storage ring \cite{21}. It has been proposed to produce $\nu_e$ beams through the decay of highly accelerated $^{18}$Ne ions and $\bar{\nu}_e$ from $^6$He \cite{21}. More recently, $^8$B and $^8$Li \cite{22} with much larger end-point energy have been suggested as alternate sources since these ions can yield higher energy $\nu_e$ and $\bar{\nu}_e$ respectively, with lower values of the Lorentz boost $\gamma$. It may be possible to have both beams in the same ring, an arrangement which will result in a $\nu_e$ as well as a $\bar{\nu}_e$ beam pointing towards a distant target. In such a set-up the ratio between the boost factors of the two ions is fixed by the $e/m$ ratios of the ions. Here, we will present our results with the $^8$B ion ($Q = 13.92$ MeV and $t_{1/2} = 0.77s$) taking $\gamma = 350$. As we show, $\gamma \sim 350$ may permit a distinction between matter effects due to Standard Model interactions and those from R-parity violating supersymmetry (SUSY). Details of the neutrino flux from such a $\beta$-beam can be found in \cite{11}.

Using the CERN-SPS at its maximum power, it will be possible to access $\gamma \sim 250^2$ \cite{23}. For a medium $\gamma \sim 500$, a refurbished SPS with super-conducting magnets or an acceleration technique utilizing the LHC \cite{23, 24, 25} would be required. In the low-$\gamma$ configuration, $1.1 \times 10^{18}$ useful decays per year can be obtained with $^{18}$Ne ions \cite{26, 27}. We have used this same luminosity for $^8$B and higher $\gamma$ \cite{28}. This issue is being examined in an on-going dedicated machine study at CERN.

III R-parity violating Supersymmetry

In supersymmetric theories \cite{14}, gauge invariance does not imply baryon number (B) and lepton number (L) conservation and, in general, R-parity (defined as $R = (-1)^{3B+L+2S}$ where $S$ is the spin) is violated. To maintain consistency with non-observation of fast proton decay etc., in the R-parity violating Minimal Supersymmetric Standard Model (imposing baryon number conservation) one may consider the following superpotential

$$W_k = \sum_{i,j,k} \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \mu_i L_i H_u,$$  

(suppressing colour and $SU(2)$ indices) where $i, j, k$ are generation indices. Here $L_i$ and $Q_i$ are $SU(2)$-doublet lepton and quark superfields respectively; $E_i$, $D_i$ denote the right-handed $SU(2)$-singlet charged lepton and down-type quark superfields respectively; $H_u$ is the Higgs superfield which gives masses to up-type quarks. Particularly, $\lambda_{ijk}$ is antisymmetric under the interchange of the first two generation indices. We assume that the bilinear terms have been rotated away with appropriate redefinition of superfields and focus on the two trilinear $L$-violating terms with $\lambda$ and $\lambda'$ couplings. Expanding those in standard four-component

$^{2}\gamma = 250$ yields too few events in this experiment for the extraction of interesting physics.
Dirac notation, the quark-neutrino interaction lagrangian can be written as:

\[ \mathcal{L}_\nu = \lambda'_{ijk} [ \bar{d}_i^L \bar{R}_j^k \nu^i_L + (\bar{d}_i^R)^*(\bar{\nu}_i^L)^c d^k_L] + h.c. \quad (2) \]

Above, the sfermion fields are characterized by the tilde sign. The charged lepton interacts with the neutrino via

\[ \mathcal{L}_\nu = \frac{1}{2} \lambda_{ijk} [ \bar{e}_i^L \bar{R}_j^k \nu^i_L + (\bar{e}_i^R)^*(\bar{\nu}_i^L)^c e^j_L] + (i \leftrightarrow j)] + h.c, \quad (3) \]

In what follows, we shall consider these couplings as real but will entertain both positive and negative values. The interactions of neutrinos with electrons and $d$-quarks in matter induce transitions

(i) $\nu_i + d \rightarrow \nu_j + d$ and (ii) $\nu_i + e \rightarrow \nu_j + e.$ (i) is possible through $\lambda'$ couplings via squark exchange for all $i, j$ and through $Z$ exchange for $i = j$ while (ii) can proceed via $W$ and $Z$ exchange for $i = j$, as well as through $\lambda$ couplings via slepton exchange for all $i, j$.

### IV Neutrino oscillation probabilities & matter effect

In a three neutrino framework, the neutrino flavour states \( |\nu_\alpha\rangle, \alpha = e, \mu, \tau \), are related to the neutrino mass eigenstates \( |\nu_i\rangle, i = 1, 2, 3 \), with masses \( m_i \):

\[ |\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle, \quad (4) \]

where \( U \) is a \( 3 \times 3 \) unitary matrix which can be expressed as:

\[ U = V_{23}V_{13}V_{12}, \quad (5) \]

where

\[
V_{23} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}; \quad V_{13} = \begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}; \quad V_{12} = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}. \quad (6)
\]

\( c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} \) and \( \delta \) denotes the CP violating (Dirac) phase. There may be a diagonal phase matrix on the right containing two more Majorana phases. These are not considered below. Apart from some qualitative remarks, we present our result considering \( \delta = 0 \) corresponding to the CP conserving case. In the mass basis of neutrinos

\[ M^2 = \text{diag}(m_1^2, m_2^2, m_3^2) = U^\dagger M_\nu^+ M_\nu U, \quad (7) \]

where \( M_\nu \) is the neutrino mass matrix in the flavour basis and \( m_1, m_2, \) and \( m_3 \) correspond to masses of three neutrinos in the ascending order of magnitudes respectively for the normal hierarchy. If \( m_2 > m_3 \), we have an inverted hierarchy.

The neutrino flavour eigenstates evolve in time as:

\[ i \frac{d}{dt} \begin{pmatrix}
\nu_e(t) \\
\nu_\mu(t) \\
\nu_\tau(t)
\end{pmatrix} = H \begin{pmatrix}
\nu_e(t) \\
\nu_\mu(t) \\
\nu_\tau(t)
\end{pmatrix}, \quad (8) \]
where
\[ H = E \times 1_{3\times3} + U \left( \frac{M^2}{2E} \right) U^\dagger + R. \] (9)

Here \( E \) is the neutrino energy while \( 1_{3\times3} \) is the identity matrix. \( R \) is a \( 3 \times 3 \) matrix reflecting the matter effect, absent for propagation in vacuum.

\[ R_{ij} = R_{ij}(SM) + R_{ij}(\lambda') + R_{ij}(\lambda). \] (10)

Specifically,
\[ R_{ij}(SM) = \sqrt{2} G_F n_e \delta_{ij} (i, j = 1) + \frac{G_F n_e}{\sqrt{2}} \delta_{ij}, \] (11)
\[ R_{ij}(\lambda') = \sum_m \left( \frac{\lambda'_{im1} \lambda_{jm1}}{4m^2(d_m)} n_d + \frac{\lambda'_{im1} \lambda_{jm1}}{4m^2(d_m)} n_d \right), \] (12)
\[ R_{ij}(\lambda) = \sum_{k \neq i, j} \frac{\lambda_{ik1} \lambda_{jk1}}{4m^2(l_k^+)} n_e + \sum_n \frac{\lambda_{i1n} \lambda_{j1n}}{4m^2(l_n^+)} n_e, \] (13)

where \( G_F \) is the Fermi constant, \( n_e, n_n, \) and \( n_d \), respectively, are the electron, neutron, and down-quark densities in earth matter. Note that \( R \) is a symmetric matrix and also that antineutrinos will have an overall opposite sign for \( R_{ij} \). Assuming earth matter to be isoscalar, \( n_e = n_p = n_n \) and \( n_d = 3n_e \). The current bounds on the \( R \) couplings \[14\] imply that the \( \lambda' \) induced contributions to \( R_{11}, R_{12} \) and \( R_{13} \) are several orders less than \( \sqrt{2} G_F n_e \). We neglect those terms in our analysis. The upper bounds on all couplings in \( R_{ij}(\lambda) \) are also very tight \[14\] in comparison to \( \sqrt{2} G_F n_e \) and their effect will be discussed later. So, first we consider, in addition to the Standard Model contribution, only

\[
R_{23} = R_{32} = \frac{n_d}{4m^2(d_m)} (\lambda'_{21m} \lambda_{31m} + \lambda'_{21m} \lambda_{31m}), \\
R_{22} = \frac{n_d}{4m^2(d_m)} (\lambda'_{21m}^2 + \lambda_{21m}^2), \quad R_{33} = \frac{n_d}{4m^2(d_m)} (\lambda_{31m}^2 + \lambda_{31m}^2),
\] (14)

which are comparable to \( \sqrt{2} G_F n_e \). One can see from eq. (14) that \( R_{23} \neq 0 \) implies both \( R_{22} \) and \( R_{33} \) are non-zero\(^3\).

The current bounds on the relevant couplings are as follows \[14\]:

\[
|\lambda'_{221}| < 0.18; \quad |\lambda'_{21m}| < 0.06; \quad |\lambda_{331}| < 0.58; \quad |\lambda_{321}| < 0.52; \quad |\lambda'_{31m}| < 0.12,
\] (15)

for down squark mass \( m_{\tilde{d}} = 100 \, \text{GeV} \). The chosen limits on \( \lambda_{21m} \) and \( \lambda'_{31m} \) do not conflict with the ratio \( R_{\tau\pi} = \Gamma(\tau \to \pi \nu_\tau)/\Gamma(\tau \to \mu \nu_\mu) \) \[14\]. However, the recently published BELLE bound on the mode \( \tau \to \mu \pi^0 \) \[29\] tightly constrains precisely those products of the \( \lambda' \) couplings which enter in \( R_{23} = R_{32} \) in eq. (14). It has been shown that \( |\lambda'_{21m} \lambda'_{31m}| \) and \( |\lambda'_{21m} \lambda_{31m}| \) both must be \( < 1.8 \times 10^{-3} (\frac{\tilde{m}}{100\,\text{GeV}})^2 \) \[30\]. This effectively makes \( R_{23} \) negligible for our purposes.

In general, it is cumbersome to write an analytical form of the probability of neutrino oscillation in the three-flavour scenario with matter effects. However, under certain reasonable approximations it is somewhat tractable. Firstly, a constant matrix can be extracted from \( M^2 \) in eqs. (7) and (9). Also for the energies and baselines under consideration, \(^3\)However, in other models \[15\] this may not be the case.
Under this approximation, the $V_{12}$ part of $U$ drops out from eq. (9). With these modifications, the effective mass squared matrix is:

\[
\frac{\tilde{M}^2}{2E} = H - (E + \frac{G_F n_e}{\sqrt{2}})I_{3\times3}. 
\]  

(16)

In the special case where $R_{22} = R_{33}$, if one uses the best-fit value of the vacuum mixing angle $\theta_{23} = \pi/4$ then the neutrino mass squared eigenvalues are:

\[
\left(\frac{\tilde{M}^2}{2E}\right)_{22} = R_{22} - R_{23}, \quad \left(\frac{\tilde{M}^2}{2E}\right)_{13} = \frac{1}{2} \left(\Delta m^2_{13} + R_{11} + R_{22} + R_{23} \mp A\right),
\]  

(17)

where

\[
A = \left[\left(\frac{\Delta m^2_{13}}{2E}\right)^2 + (-R_{11} + R_{22} + R_{23})^2 - 2\frac{\Delta m^2_{13}}{2E} \cos 2\theta_{13} (R_{11} - R_{22} - R_{23}\right)\right]^{1/2}. 
\]  

(18)

The matter induced neutrino mixing matrix is given by

\[
U^m = \begin{pmatrix}
U^m_{11} & 0 & U^m_{13} \\
N_1 & -\frac{1}{\sqrt{2}} & N_3 \\
N_1 & \frac{1}{\sqrt{2}} & N_3 
\end{pmatrix}. 
\]  

(19)

Here

\[
N_{1,3} = \left[\left(\frac{\Delta m^2_{13}}{2E}\right)^2 + 2\left(-R_{11} + R_{22} + R_{23} + \frac{\Delta m^2_{13} \cos 2\theta_{13} \pm A}{2}\right)^2 \mp A\right]^{-1/2}, 
\]  

(20)

where $N_1$ ($N_3$) corresponds to $+$ ($-$) sign in the above expression. Neglecting the $CP$ phase in the standard parametrization of $U^m$, one may write $U^m_{13} = \sin \theta_{13}^m$ and $U^m_{23} = \sin \theta_{23}^m \cos \theta_{13}^m$. From eq. (19) it follows that $\theta_{23}^m = \theta_{23}$, the vacuum mixing angle. $\theta_{13}^m$, on the other hand, changes from its vacuum value and it is $\pi/4$ for

\[
R_{11} - R_{22} - R_{23} = \frac{\Delta m^2_{13}}{2E} \cos 2\theta_{13}. 
\]  

(21)

In the absence of non-standard interactions, $R_{22} = R_{23} = R_{33} = 0$ and $R_{11} = \sqrt{2}G_F n_e$, this is the well-known condition for matter induced maximal mixing. Since in eq. (19) $U^m_{12} = 0$, in the $\nu_e$ to $\nu_\mu$ oscillation probability the terms involving $(\tilde{M}^2_2 - \tilde{M}^2_1)$ and $(\tilde{M}^2_3 - \tilde{M}^2_2)$ will not survive and we get:

\[
P_{\nu_e \rightarrow \nu_\mu} = 4 \left(\frac{U^m_{13}}{U^m_{23}}\right)^2 \left(\frac{U^m_{23}}{U^m_{13}}\right)^2 \sin^2 (1.27 A L), 
\]  

(22)

where $E$, $\Delta m^2_{13}$ and $L$ are expressed in GeV, eV$^2$, and Km, respectively. This expression is also valid for antineutrinos. Using eqs. (17) and (19) one can easily obtain the oscillation probabilities for other channels.

We use the above analytical formulation as a cross-check on our numerical results. For example, Fig. 1 which shows the variation of $P_{\nu_e \rightarrow \nu_\mu}$ as a function of the energy, is obtained using the full matter-induced three-flavour neutrino propagation including non-standard
interactions. The range of energy is chosen in line with the discussions in the rest of the paper. The probability falls with decreasing $\theta_{13}$ and, for illustration, we have chosen a value in the middle of its permitted range. The purpose of Fig. 1 is twofold: (a) to show how the distinguishability between the normal and inverted hierarchies may get blurred by the RPVSM interactions, and (b) how irrespective of the hierarchy chosen by Nature the results may be completely altered by the presence of these interactions. Each panel of Fig. 1 has three curves: the solid line (only electroweak interactions), dot-dashed line (in addition, $R_{33}$ gets a non-zero RPVSM contribution), and dashed line ($R_{22} = R_{33}$ are nonzero, in addition to the electroweak contribution). Only in the last case is the analytical formula we have presented above applicable. We find excellent agreement. Two aspects of the results are worth pointing out.

First, in the absence of non-Standard interactions, for an inverted hierarchy the resonance condition eq. (21) is not satisfied and the oscillation probability is negligible (right panel solid line). This could be altered prominently by the RPVSM interactions (dot-dashed curve) so that the distinguishability between the two hierarchy scenarios may well get marred by $R_SUSY$.

Secondly, for the normal hierarchy, it is seen that the peak in the probability may shift to a different energy in the presence of the RPVSM interactions. This is because the condition for maximal mixing in eq. (21) is affected by the $R$ interactions. For the inverted hierarchy, the oscillation probability is considerably enhanced for some energies. Thus, physics expectations for both hierarchies will get affected by RPVSM.

In the following section we dwell on the full impact of this physics on a long baseline $\beta$-beam experiment.

V Results

We consider a long baseline experiment with a $\nu_e$ $\beta$-beam source. $\beta$-beams producing $\bar{\nu}_e$ are also very much under consideration. Broadly speaking, the results obtained for a $\nu_e$...
beam with a normal (inverted) hierarchy are similar to that with a \( \bar{\nu}_e \) beam for an inverted (normal) hierarchy but details do differ.

The average energy of the \( \nu_e \) beam in the lab frame is \( \langle E \rangle = 2\gamma E_{\text{cms}} \), where \( E_{\text{cms}} \) is the mean center-of-mass neutrino energy. With \( \gamma = 350 \) and \( Q = 13.92 \) MeV for an \(^8\text{B}\) source, \( \langle E \rangle \approx 5 \) GeV. The proposed ICAL detector at INO \(^{12}\) consists of magnetized iron slabs with glass resistive plate chambers as interleaved active detector elements. We present results for a 50 Kt iron detector with energy threshold 2 GeV. As signature of \( \nu_e \to \nu_\mu \) oscillation, prompt muons will appear\(^4\). Their track reconstruction will give the direction and energy of the incoming neutrino. ICAL has good charge identification efficiency (~95%) and a good energy resolution ~10% above 2 GeV. Details about the detector and neutrino nucleon cross sections may be found in \(^{11}\). For the cross section we include contributions from quasi-elastic, single pion, and deep inelastic channels. For our chosen high threshold (2 GeV), the contribution from the deep inelastic channel is relatively large. For the CERN-INO baseline, the averaged matter density is 4.21 g cm\(^{-3}\). We use best-fit values of vacuum neutrino mixing parameters as mentioned in the Introduction. All the presented results are based on a five-year ICAL data sample\(^5\).

At the production and detection levels, FCNC and FDNC effects can change the spectrum and detection cross sections by a small (~0.1%) amount but this would not alter the conclusions. At the source and detector, they may also mimic the oscillation signal itself, but these effects are tiny\(^6\) (~\( \mathcal{O}(10^{-14}) \)). Here we discuss how FCNC and FDNC may significantly modify the propagation of neutrinos through matter over large distances.

V.1 Extraction of \( \theta_{13} \) and determination of hierarchy

If neutrinos have only Standard Model interactions then the expected number of muon events is fixed\(^7\) for a particular value of \( \theta_{13} \) with either normal or inverted hierarchy as may be seen from the solid lines in Fig. 2. The vast difference for the alternate hierarchies picks out such long baseline experiments as good laboratories for addressing this open question of the neutrino mass spectrum.

If non-standard interactions are present then, depending on their coupling strength, the picture can change dramatically. In Fig. 2 the shaded region corresponds to the allowed values when SUSY FCNC and FDNC interactions are at play. It is obtained by letting the \( \lambda' \) couplings\(^8\) vary over their entire allowed range – both positive and negative – given in eq. \(^{15}\), subject to the further constraints on particular products.

It is seen that to a significant extent the distinguishability of the two hierarchies is obstructed by the \( R \) interactions unless the number of events is more than about 60. Also, the one-to-one correspondence is lost between \( \theta_{13} \) and the number of events and, at best, a lower bound can now be placed on \( \theta_{13} \) from the observed number. Of course, if the neutrino mass hierarchy is known from other experiments, then this lower bound can be strengthened, especially for the inverted hierarchy.

\(^4\)The \( \nu_e \to \nu_\tau \to \tau \to \mu \) route is suppressed by phase space for \( \tau \) production and the branching ratio for the decay. It contributes at a few per cent level to the signal.

\(^5\)Backgrounds can be eliminated by imposing directionality cuts. The detector is assumed to be of perfect efficiency.

\(^6\)This is due to the very tight constraints from \( \mu \to e \) transition limits in atoms \(^{31}\).

\(^7\)Recall we assume that, but for \( \theta_{13} \) and the mass hierarchy, the other neutrino mass and mixing parameters are known.

\(^8\)In fact, we have chosen the subscript \( m \) in the \( \lambda' \) couplings in eq. \(^{14}\) to be any one of 1, 2, or 3.
Figure 2: Number of muon events for normal and inverted mass hierarchies as a function of $\sin^2 2\theta_{13}$ for a five-year ICAL run. The solid lines correspond to the absence of any non-Standard neutrino interaction. The shaded area is covered if the $\lambda'$ couplings are varied over their entire allowed range.

It is also noteworthy that for some values of $\lambda'$-couplings there may be more events than can be expected from the Standard Model interactions, no matter what the value of $\theta_{13}$. Thus, observation of more than 161 (5) events for the normal (inverted) hierarchy would be a clear signal of new physics.

V.2 Constraining $\lambda'$

If $\theta_{13}$ is determined from other experiments then it will be easier to look for non-standard signals from this $\beta$ beam experiment. However, even if the precise value remains unknown at the time, considering the upper bound on $\theta_{13}$ one may tighten the constraints on the $\lambda'$ couplings. Fig. 2 reflects the overall sensitivity of the event rate to the $\mathcal{R}$ interactions obtained by letting all RPVSM couplings vary over their entire allowed ranges. In this subsection, we want to be more specific and ask how the event rate depends on any chosen $\lambda'$ coupling.

At the outset, it may be worth recalling that the BELLE bound on $\tau \rightarrow \mu \pi^0$ [29] severely limits the products $\lambda'_{2m1} \lambda'_{3m1}$ and $\lambda'_{21m} \lambda'_{31m}$. Thus, $R_{23}$ can be dropped in the effective neutrino mass matrix eq. (9). If only $\lambda'_{2m1}$ and/or $\lambda'_{21m}$ ($\lambda'_{3m1}$ and/or $\lambda'_{31m}$) is non-zero, then $R_{22}$ ($R_{33}$) alone receives an RPVSM contribution. Both $R_{22}$ and $R_{33}$ can be simultaneously non-zero if $\lambda'_{21m}$ and $\lambda'_{3m1}$ (or $\lambda'_{2m1}$ and $\lambda'_{31m}$) are non-zero at the same time.

In the light of this, we consider the situation where only of the above $\mathcal{R}$ coupling is non-zero. In such an event, only one of $R_{22}$, $R_{33}$ is non-zero. The dependence of the number of events on a non-zero $\lambda'_{331}$ or $\lambda'_{2m1}$, for a chosen $\sin^2 2\theta_{13}$, can be seen from Fig. 3. In this figure, we use the fact that if only one of these $\mathcal{R}$ couplings is non-zero, it appears in the results through $|\lambda'|$. For the normal hierarchy, the curves for $\lambda'_{2m1}$, for $m = 2, 3$, are terminated at the maximum allowed value of 0.18. Fig. 3 can also be used for $\lambda'_{321}$, $\lambda'_{31m}$ and $\lambda'_{21m}$, bearing in mind their different upper bounds. For the inverted hierarchy, the number of events is small for $\lambda'_{2m1}$ and $\lambda'_{21m}$ and insensitive to the magnitude of the coupling. These are not shown. It is seen that for the normal hierarchy there is a good chance to determine
Figure 3: The number of events as a function of a coupling $|\lambda^'|$, present singly, for the normal (left panel) and inverted (right panel) hierarchies. The thick (thin) lines are for $|\lambda_{331}^'|$ ($|\lambda_{2m1}^'|$, $m=2,3$). The chosen $\sin^2 2\theta_{13}$ are indicated next to the curves.

the $R$ couplings from the number of events. In fact, if the number of events is less than about 50 there is a disallowed region for $|\lambda^'|$, while for larger numbers there is only an upper bound. For the inverted hierarchy, more than about five events will set a lower bound on the coupling.

V.3 Effect of $\lambda$

The $\lambda$ couplings which can contribute in eq. (13) have strong existing bounds [14] and their contribution to $R$ is rather small in comparison to $\sqrt{2} G_F n_e$. Among them, the bounds $\lambda_{121} < 0.05$ and $\lambda_{321} < 0.07$ for $m_1 = 100$ GeV are relatively less stringent [14]. We show their very modest impact in Fig. 4. It is clear from this figure that (a) the $\lambda$-type couplings cannot seriously deter the extraction of $\theta_{13}$ or the determination of the neutrino mass hierarchy, and (b) when $\theta_{13}$ is known in future it will still not be possible to constrain these couplings through long baseline experiments.

VI Conclusions

R-parity violating supersymmetry is among several extensions of the Standard Model crying out for experimental verification. The model has flavour diagonal and flavour changing neutral currents which can affect neutrino masses and mixing and can leave their imprints in long baseline experiments. This is the focus of this work.

We consider a $\beta$-beam experiment with the source at CERN and the detector at INO. We find that the $R$ interactions may obstruct a clean extraction of the mixing angle $\theta_{13}$ or determination of the mass hierarchy unless the bounds on the $\lambda^'$ couplings are tightened.
On the other hand, one might be able to see a clean signal of new physics. Here, the long baseline comes as a boon over experiments like MINOS which cover shorter distances. Two experiments of these contrasting types, taken together, can expose the presence of a non-standard interaction like RPVSM.

There are other non-standard models [15] where four-fermion neutrino couplings with greater strength have been invoked. The signals we consider will be much enhanced in such cases.

Our results are presented for the $CP$ conserving case. As $\theta_{13}$ is small, the $CP$ violating effect is expected to be suppressed. We have checked this for the Standard Model, where the ‘magic’ nature of the baseline [32] also plays a role.

Finally, in this paper we have restricted ourselves to a $\beta$-beam neutrino source. Much the same could be done for antineutrinos as well; then the signs of all terms in $R$ – see eq. (9) – will be reversed. It follows from eq. (21) that $\theta_{13}$ can then be maximal only for the inverted hierarchy and as such more events are expected here than in the normal hierarchy. Broadly, results similar to the ones presented here with neutrinos can be obtained with antineutrinos if normal hierarchy is replaced by inverted hierarchy and vice-versa.

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