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Reynolds dependence of third-order velocity structure functions

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We study the experimental dependence of the third-order velocity structure function on the Taylor based Reynolds number, obtained in different flow types over the range $72 \leq R_\lambda \leq 2260$. As expected, when the Reynolds number is increasing, the third-order velocity structure functions (plotted in a compensated way) converge very slowly to a possible $-4/5$ plateau value according to the Kolmogorov 41 theory. Actually, each of these normalized third-order functions exhibits a maximum, at a scale close to the Taylor microscale $\lambda$. In this Brief Communication, we show that experimental data are in good agreement with the recent predictions of Qian and Lundgren. We also suggest that, from an experimental point of view, a log-similar plot suits very well to study carefully the behavior of the third-order velocity structure functions with the flow Reynolds number.

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In Fig. 1, no plateau is observed, the third-order velocity structure function does not strictly behave as a power law, even in the middle of the inertial range of one of the highest Reynolds number flow never investigated. Note that the accuracy of measurements does not affect the shapes of the $S_3(r)$ (and cannot lessen this finding), it only affects the level value of the $S_3(r)$ which drastically depends on the squares of the velocity gradients calculated at the Kolmogorov scale $\eta$. As expected, these maxima continuously converge to the horizontal line 4/5 as $R_\lambda$ is increasing. This monotonous evolution suggests that data have been obtained with a rather good accuracy despite the difficulty to measure the small scale velocity gradients. The lack of a true "−4/5 plateau" has been studied in detail by Danaila et al.\textsuperscript{16} in the case of a grid turbulence. From the complete form of the Kolmogorov equation involving both the viscous and the nonstationary terms, this Brief Communication shows how the latter term acts as a negative production term at large scale. The authors showed that experimental data verify the Kolmogorov equation if one accounts for the nonstationarity of the second-order moments. The aim here is not to study this interesting finding in others types of flow, but rather to compare behaviors of the functions $S_3(r)$ towards the flow type and the Reynolds number value.

A way to highlight the gap between $[S_3(r)]_{\text{max}}$ and 4/5 studied by Qian\textsuperscript{8,9} and Lundgren,\textsuperscript{10,11} is to plot the quantity $(1-(5S_3(r)/4))R_{\lambda}^{-3/2}$ versus the separation $r$. Such a stringent diagram is shown on Fig. 2, where the separation $r$ has been rescaled with the Taylor microscale $\lambda$ (and not with $\eta$ as in Fig. 1). We observe (as in Ref. 5) that all the minima of the experimental curves (corresponding to $[S_3(r)]_{\text{max}}$) occur at a unique scale close to $\lambda$. To our knowledge, there is no physical explanation of this feature. Even though the minima values agree (more or less) with the previous theoretical predictions $[\ln(8.45)=2.13]$, these curves diverge each other both at large and small scales.

In fact, due to the existence of the two scales $\ell_0$ and $\eta$, the problem is that neither $r/\ell_0$ nor $r/\eta$ can be considered as the good variable. A characteristic scale in turbulence is not identified by its ratio to $\ell_0$ but by the way this ratio scales with the Reynolds number. If this scale is almost equal to the

| Grid turbulence | Jet | ONERA |
|-----------------|-----|-------|
| $R_\lambda$     | 72  | 144   | 350  | 695   | 2260 |
| $U$ (m/s)       | 4.95| 19.6  | 1.68 | 6.3   | 20.75|
| $u'$ (m/s)      | 0.17| 0.64  | 0.44 | 1.55  | 1.58 |
| $\eta$ (mm)     | 0.4 | 0.15  | 0.33 | 0.14  | 0.30 |
| $\lambda$ (cm)  | 1.02| 0.35  | 1.23 | 0.67  | 2.80 |
| $L$ (cm)        | 4   | 4     | 20   | 20    | ≈410 |
| $\langle \epsilon \rangle$ (m$^3$/s)$^3$ | 0.0093 | 7.7  | 0.29 | 11.7  | 0.95 |
| $10^3 \nu$ (m$^2$/s) | 1.51 | 1.55  | 1.52 | 1.53  | 2    |

FIG. 1. $S_3(r)$ in lin–log coordinates.
Taylor scale \( \lambda \), while its ratio with \( \ell_0 \) scales like \( \frac{R}{\lambda}^{3/2} \), it must be identified with \( \eta \) and not \( \lambda \). In this context, as recognized in previous studies,\(^{12,17} \) the good variable is of kind \( \ln(\ell_0/\eta) \), in such a way that characteristic scales having the same behavior with \( R \) converge towards the same abscissa when \( R \lambda \) tends to infinity. Faster convergence is obtained using \( \ln(R_*/R_\phi) \) with \( R_\phi = 28 \)\(^{12,17} \) which ensures that \( \ln(\ell_0/\eta) \) remains close to \( \frac{1}{3} \ln(R_*/R_\phi) \) down to moderate \( R_\lambda \).

We then propose such a plot. As we want to emphasize the logarithmic ordinates by the same factor \( \ln(\ell_0/\eta) \), we take the abcissa \( \beta \ln(\ell_0/\eta) \) with \( \beta = 1/\ln(R_*/R_\phi) \). To maintain the scaling laws, we have to divide the logarithmic ordinates by the same factor \( \ln(R_*/R_\phi) \).

We thus take as the ordinate \( \beta \ln(1-5S_3(r)/4) \). In such coordinates, Fig. 3 shows the experimental curves (the same as those of Fig. 2), while Fig. 4 displays the results obtained from Eq. (4).

While none of the two plots show perfect merging, this presentation has numerous advantages. First, the ordinate \( -\mu \) of the minimum gives the Reynolds dependence of the maximum of \( S_3(r) \) that we define as \( [S_3(r)]_{\text{max}} = 4/5 - A(R_*/R_\phi)^{-\mu} \), analogously to the predicted Eqs. (3) and (5). Second, the slopes close to zero ordinate give the power law dependences of the outer scale \( (\sim \ell_0) \) and inner scale \( (\sim \eta) \) corrections to the \( -4/5 \) law as expressed in Eqs. (2) and (4). Last, the distance between the two points of intersection of each curve with the zero ordinate is bounded by \( R_\lambda \approx 3/[3\ln(R_*/28)] \). The different data converge towards the infinite Reynolds curve is characteristic of the coefficients of these laws. For instance, it can be seen that choosing \( R_\phi = 28 \) gives \( A \) close to 1, both for Eqs. (3) and (5) and for real data. Indeed, this choice comes from an independent work (cf. Ref. 12) and this coincidence has to be remarked. Figure 4 shows how curves degenerate in two straight lines when \( R_\lambda \to \infty \). The position of the maximum of \( S_3 \), its dependence with \( R_\lambda \), the power law dependences of the correction to \( S_3 \), and even their coefficients, all are probably linked. For instance, if we write, as Lundgren did \( 1 - (5S_3/4) = (R_*/R_\phi)^{-\alpha}(A_0(r/\lambda)^{10} + A_1(r/\lambda)^9) \), we must have \( \gamma_0 - \gamma_1 = 2 \) as one of the terms comes from the integral of \( \langle \delta u(r)^2 \rangle \), and the second from its derivative. Then, to have the maximum \( [S_3(r)]_{\text{max}} \) at an abscissa \( r \) close to \( \lambda \), even at moderate \( R_\lambda \), we must have \( A_0 = 2/3 \) and \( A_1 = 1/3 \) if \( \gamma_1 = -4/3 \).

Further remarks can be discussed in relation to the questions addressed in the beginning of this paper.

The two slopes plotted in Fig. 3 (equal to \( -4/3 \) and \( 2/3 \) according to the theoretical predictions) seem to be reached only for \( R_\lambda > 1000 \) (note that an error on \( \beta \) does not affect these slopes). On the large scale side, the two highest Reynolds data behave as nearly as \( \sim R^{2.5} \) even though they correspond to two very different flows. Moreover, on the large
scale side of experimental curves, we observe that the correction slope [noted \(m\) in Eq. (2)] never exceeds the value \(2/3\), whatever the flow type. In fact, the different power input types (grid turbulence, jet) are hardly noticeable, contrary to the theoretical expectations.

With regard to the accuracy of the Reynolds number determination, such coordinates clearly reveal that in Ref. 15 data, \(R_L\) has been grossly overestimated. This underestimates \(\beta\) and causes the diagram to shrink.

Other differences between the theoretical predictions and the real data are within the experimental uncertainty. This was to be expected, in particular, the intermittency effect cannot probably be detected in such a diagram, as the following calculus shows.

In his matched asymptotic expansion, Lundgren derives an outer and an inner expression for the second moment \(\langle \delta u(r)^2 \rangle\) \((R_L = R_L^0/15)\) such as \(\langle \delta u(r)^2 \rangle = U^2 b_{(0)}^i (x_0)\) with \(x_0 = r/L\), and \(\langle \delta u(r)^2 \rangle = U^2 R_L^{-1/2} b_{(i)}^i (x_i)\) with \(x_i = r/\eta\) \(= R_L^{3/4} x_0\).

Now, remarking that the von Karman equation in the outer (respectively, inner) variables has \(R_L^{-1}\) (respectively, \(R_L^{-1/2}\)) as small parameter, he writes \(b_{(0)}^i = \sum b_{(0)}^{i+1} + \cdots\) and \(b_{(i)}^i = \sum b_{(i)}^{i+1} + \cdots\).

But, as remarked by Barenblatt,\(^1\) incomplete self-similarity can result in the absence of limit for infinite \(R_L\) for one of these functions. The general case writes \(b_{(i)}^i = R_L^{-\delta} b_{(i)}^{i+1} + \cdots\).

Then, Eq. (27) of Lundgren, which ensures the matching of the two expressions, becomes \(U^2 R_L^{-1/2} R_L^{-\delta} b_{(i)}^{i+1} (x_0 R_L^{3/4}) = U^2 b_{(i)}^{i+1} (x_0)\) and thus \(b_{(i)}^{i+1} = C K^{2(1 + 2\delta)/3}\) and \(b_{(i)}^i = C K^{1 + 2\delta)/3}\).

By the comparison between these two latest equations and the result of intermittency studies, the value of \(\delta\) should be of order \(\delta = 0.02\). The two exponents \(\gamma_0\) and \(\gamma_i\) (which are the limit slopes in the log-similar diagram) are \(\gamma_0 = 2/3 + 4\delta/3\) and \(\gamma_i = -4/3 + 4\delta/3\). The minimum occurs at \(\beta \ln(\rho/\lambda) = \delta 2\) and its value is \(-2/3 - \delta\). The very small value of \(\delta\) explains why the difference with the Lundgren prediction is not visible.

In summary, the present experimental data confirm the very slow convergence of \(S_3(\tau)\) towards an asymptotic regime, whatever the flow type and/or the Reynolds number. Also, the pertinent scale of \([S_3(\tau)]_{\text{max}}\) seems to be close to the Taylor microscale but with a different Reynolds scaling. From an experimental point of view, the log-similarity plot suits to account for the detailed behavior of \(S_3(\tau)\) in the whole inertial range when the Reynolds number tends to infinity. The question of intermittency is not relevant here and must be studied by other means.

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