A NEXT-TO-LEADING ORDER CALCULATION OF HADRONIC THREE
JET PRODUCTION∗

WILLIAM B. KILGORE
Physics Department, Building 510A, Brookhaven National Laboratory, Upton, NY 11973,
USA
E-mail: kilgore@bnl.gov

WALTER T. GIELE
Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, IL 60510,
USA
E-mail: giele@fnal.gov

We present results of a next-to-leading order calculation of three jet production at hadron colliders.
This calculation will have many applications. In addition to computing such three-jet observables as
spectra, mass distributions, this calculation permits the first next-to-leading order studies at hadron
colliders of jet and event shape variables.

1 Introduction

One of the difficulties in interpreting experimental results is in assessing the uncertainty
to be associated with the theoretical calculation. This is particularly true in QCD where
the coupling is quite strong and one expects higher order corrections to be significant.

Typically, one characterizes theoretical uncertainty by the dependence on the renormalization scale \( \mu \). Since we don’t actually
know how to choose \( \mu \) or even a range of \( \mu \), the uncertainty associated with scale dependence is somewhat arbitrary. One motivation
for performing next-to-leading order (NLO) calculations is to reduce the scale dependence
associated with the calculation.

However, this is not the only benefit of an NLO calculation. There are times when the LO calculation is a bad estimator of the
physical process. It may be that leading order kinematics artificially forbids the most
important physical process. It could also be
that the NLO corrections are simply large. Even if the overall NLO correction is rela-
tively small, there may be regions of phase space, where NLO corrections are large. It is
only in those regions of phase space where the NLO corrections are well behaved (as determined by the ratio of the NLO to LO terms)
that one has confidence in the reliability of the calculation and can begin to believe the
uncertainty estimated from scale dependence and it is only when one has a reliable estimate
of the theoretical uncertainty that comparisons to experiment are meaningful.

2 Methods

The NLO three jet calculation consists of two parts: two to three parton processes at one-
loop (\( gg \rightarrow ggg \), \( q\bar{q} \rightarrow ggg \), \( gq \rightarrow Qg \), \( Qq \rightarrow Qg \),
and processes related to these by crossing symmetry) and two to four parton processes
(\( gg \rightarrow gggg \), \( q\bar{q} \rightarrow gggg \), \( gq \rightarrow Qgg \), \( Qq \rightarrow QgQ \), \( \bar{Q}g \rightarrow Q\bar{Q}Q \), \( q\bar{Q} \rightarrow QQ \bar{Q} \), \( Q\bar{g} \rightarrow Q\bar{Q}g \),
and the crossed processes) computed at tree-level. Both of these contributions are infrared singular; only the sum
of the two is infrared finite and meaning-
ful. The Kinoshita-Lee-Nauenberg theorem guarantees that the infrared singularities of
the one-loop processes cancel those of the real emission processes for sufficiently inclu-

∗Talk presented by W.B.K. at the XXXth International Conference on High Energy Physics, Osaka,
Japan, July 27 – August 2, 2000.
sive observables.

In order to implement the kinematic cuts necessary to compare a calculation to experimental data one must compute the cross section numerically. Thus, we must find a numerically safe way of canceling the singularities. The method we use is the “subtraction improved” phase space slicing method.

3 Applications

The next-to-leading order calculation of three jet production will have a wide array of phenomenological applications.

3.1 Measurement of $\alpha_s$

It should be possible to extract a purely hadronic measurement of $\alpha_s$. One possibility for such a measurement would be a comparison of the three jet to two jet event rate. Since both processes are sensitive to all possible initial states at tree-level, a next-to-leading order comparison should be relatively free of bias from the parton distributions. Because the measurement will be simultaneously performed over a wide range of energy scales, the running of $\alpha_s$ can be used to constrain the fits and enhance the precision of the combined measurement.

It has also been suggested that $\alpha_s$ can be determined from Dalitz distributions of three jet events.

3.2 Study Jet Clustering Algorithms

Because there are up to four partons in the final state, as many as three partons can end up in a single jet. This makes the three jet calculation sensitive to the details of jet clustering algorithms. This sort of study in pure gluon production uncovered an infrared sensitivity in the commonly used iterative cone algorithms.

3.3 Study Jet Structure and Shape

Because there can be three partons clustered into a single jet, this calculation will allow truly next-to-leading order studies of the energy distribution in jets. Studies of jet production in deep inelastic scattering show that the next-to-leading order correction for this variable is substantial and agrees rather well with experimental measurements.

3.4 Study Event Shape Variables

There has been a long history of studying event shape variables like Thrust at $e^+e^-$ colliders. These measurements challenge the ability of perturbative QCD to describe the data and provide a means (other than event rate) of obtaining a precise measurement of $\alpha_s$. It will be interesting to see if one can make a meaningful study of such variables at hadron colliders.

3.5 Study Backgrounds to New Physics

Models of physics beyond the standard model typically involve massive states that can generate three jet signals either by associated production or by decay into three jets. To identify such signals, one must understand the pure QCD contribution to three jet production and look for deviations from the expected distributions.

4 Results

At present we have results for some of the most basic event distributions: the transverse energy spectrum and the angular distributions. The transverse energy distribution and its scale dependence has been shown in other conference proceedings. We find that the next-to-leading order correction to the total rate is very small. The scale dependence of the NLO calculation, however, is a factor of two smaller than that of the LO calculation. The combination of a small NLO
Figure 1. Distribution of events in $\cos \theta^*$, where $\theta^*$ is the angle between the leading jet and the beam axis in the three jet center of momentum. The next-to-leading order results are shown as points and the leading order results as solid lines.

Figure 2. Distribution of events in $\psi_3$, where $\psi_3$ is the angle between the plane formed by leading jet and the beam axis and the plane formed by the second and third leading jets in the three jet center of momentum. The next-to-leading order results are shown as points and the leading order results as solid lines.
correction and reduced scale dependence indicates that we are obtaining a reliable calculation of three jet production.

We also have results for the angular distributions of the three jet events. Shown below are the distributions in $\cos \theta^*$, where $\theta^*$ is the angle between the leading jet and the beam axis in the three jet center of momentum, and $\psi_3$, where $\psi_3$ is the angle between the plane formed by leading jet and the beam axis and the plane formed by the second and third leading jets in the three jet center of momentum.\[4\]

Again, the NLO correction is quite small. The important feature, however, is that the NLO results are more reliable than the LO results. Such distributions are particularly important for identifying (or eliminating) signals of new physics. It was the fact that the angular distributions of dijet events looked like QCD that eliminated the more exotic explanations of the famous high $E_T$ anomaly in the one jet inclusive distribution.\[15,16\]

Acknowledgments

This work was supported by the US Department of Energy under grant DE-AC02-98CH10886.

References

1. Z. Bern, L. Dixon, D.A. Kosower, Phys. Rev. Lett. 70, 2677 (1993) [hep-ph/9302280].
2. Z. Bern, L. Dixon and D.A. Kosower, Nucl. Phys. B 437, 259 (1995) [hep-ph/9409393].
3. Z. Kunszt, A. Signer and Z. Trócsányi, Phys. Lett. B 336, 529 (1994) [hep-ph/9405386].
4. T. Kinoshita, J. Math. Phys. 3, 650 (1962); T.D. Lee and M. Nauenberg, Phys. Rev. 133, 1549 (1964).
5. W.T. Giele and E.W.N Glover, Phys. Rev. D 46, 1980 (1992).
6. W.T. Giele, E.W.N. Glover and D.A. Kosower, Nucl. Phys. B 403, 633 (1993) [hep-ph/9302225].
7. W.B. Kilgore and W.T. Giele, Phys. Rev. D 55, 7183 (1997).
8. B. Abbott et al., The D0 Collaboration, Fermilab Preprint FERMILAB-PUB-00-218-E [hep-ex/0009012].
9. W.T. Giele, E.W.N. Glover and J. Yu, Phys. Rev. D 53, 120 (1996) [hep-ph/9506442].
10. A. Brandl et al., The CDF Collaboration, Proceedings of the XXXVth Rencontres de Moriond, Les Arcs, France, March 18–25, 2000, FERMILAB-CONF-00-170-E.
11. F. Abe et al., The CDF Collaboration, Phys. Rev. Lett. 70, 713 (1993); S. Abachi et al., The D0 Collaboration, Phys. Lett. B 357, 500 (1995); W.T. Giele, E.W.N. Glover and D.A. Kosower, Phys. Rev. D 57, 1878 (1998) [hep-ph/9706210]; M.H. Seymour, Nucl. Phys. B 513, 269 (1998) [hep-ph/9707338].
12. N. Kauer, L Reina, J. Repond, and D. Zeppenfeld, PLB 460, 189 (1999) [hep-ph/9904500].
13. W.B. Kilgore and W.T. Giele, Proceedings of the XXXVth Rencontres de Moriond: QCD and High Energy Hadronic Interactions, Les Arcs, France, March 18–25, 2000 [hep-ph/0009176].
14. F. Abe et al., The CDF Collaboration, Phys. Rev. D 54, 4221 (1996) [hep-ex/9605004]; S. Abachi et al., The D0 Collaboration, Phys. Rev. D 53, 6000 (1996) [hep-ex/9509005].
15. F. Abe et al., The CDF Collaboration, Phys. Rev. Lett. 77, 5336 (1996), PRL 78, 4307 (1997) [hep-ex/9609011].
16. B. Abbott et al., The D0 Collaboration, Phys. Rev. Lett. 80, 666 (1998) [hep-ex/9707016].