Non-perturbative Effects in Generalized Schwinger Models

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Abstract

We analyze generalizations of the Schwinger model, with more massless fermions and more vector fields. These have conformal sectors that survive at long distances. We find that in addition to local operators that go to “unparticle operators” with non-zero anomalous dimensions at long distances, some of these models contain local operators like the $\bar{\psi}_L \psi_R$ operator in the Schwinger model which go to 0-dimension operators at long distances. These operators have calculable vacuum expectation values (up to phases). Cluster decomposition applied to correlation functions involving these operators yields nontrivial and calculable nonperturbative constraints on correlation functions. One dramatic consequence of these constraints is “conformal coalescence” in which linear combinations of short distance operators can disappear from the long-distance theory. But cluster decomposition for these operators gives constraints in the full theory, not just in the long-distance limit, and the basic mechanism does not require the presence of a non-trivial conformal sector. If similar operators exist in 3+1 dimensions, they could provide an interesting means to study the breakdown of anomalous symmetries.
The Schwinger model \cite{1} of the electrodynamics of a massless fermion in 1+1 dimensions is a completely solvable quantum field theory that exhibits some of the interesting features of QCD: confinement \cite{2, 3}, an anomalous chiral $U(1)$ \cite{5}; instantons and $\theta$ vacua \cite{6, 7, 8}; non-zero vacuum expectation values (VEVs) of composite operators \cite{9, 10, 11}. In this note, we consider generalizations of the Schwinger model with more massless fermions or more gauge fields or both. These models can be solved exactly if all the gauge couplings commute. We developed the tools to study these models in \cite{12} where we analyzed the most general Sommerfield models \cite{13} from which we can construct the generalized Schwinger models.

In the Schwinger model, not only are there composite operators that have non-zero vacuum expectation values, but the magnitude of the vacuum expectation value is exactly calculable \cite{9}. We will find similar phenomena in several infinite classes of generalized Schwinger models. But unlike the Schwinger model, in which the calculable VEV is an interesting curiosity without important physical consequences, these generalized models have VEVs that produce a variety of non-perturbative physical effects.

Zdops

To think about generalizations of the Schwinger model, it is useful to consider them as limits of Sommerfield models, with massless fermions with commuting couplings to one or more vector bosons, but with bare masses for the vector bosons \cite{12}. All such Sommerfield models are examples of Bank-Zaks models \cite{14} with weak coupling at short distances and conformal invariance with non-trivial anomalous dimensions at long distances. We define what we mean by “generalized Schwinger models” as the limits of the gauge invariant sectors of Sommerfield models as the vector boson masses go to zero. The results of \cite{12} for gauge invariant operators for a model with vector bosons $A^j_\mu$ for $j = 1$ to $n_A$ with diagonal vector couplings $e_{j\alpha}$ to fermions $\psi_\alpha$, for $\alpha = 1$ to $n_F$ are that in gauge invariant combinations of LH fermions $\psi_{\alpha 1}$ and RH fermions $\psi_{\alpha 2}$, the Lagrangian fields can be replaced by products of free fields

$$\psi_{\alpha 1} \rightarrow e^{-i(\sum_j (e_{j\alpha}/m_j)([C]_j-[B]_j))} \psi_{\alpha 1} \quad \psi_{\alpha 2} \rightarrow e^{i(\sum_j (e_{j\alpha}/m_j)([C]_j-[B]_j))} \psi_{\alpha 2}$$

(1)

$\Psi$s are free massless fermion fields, $B_j$ are scalar fields with mass $m_j$, $C_j$ are massless ghosts and $m_j$ are related to the vector boson mass term, $[M^2_0]_{jk}$ in the Lagrangian by

$$[M^2_0]_{jk} = m^2_j \delta_{jk} - \sum_\alpha \frac{|e|_{j\alpha} |e|_{k\alpha}}{\pi}$$

(2)

The long-distance anomalous dimension of a gauge invariant operator comes from the free fermion propagators and the exponential of the ghost field propagators.

We consider gauge invariant operators that have at most one power of each type of fermion field (LH and RH count as different types but we do not allow $\psi$ and $\psi^*$ of the same type and

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1There is a literature on confinement and the slightly different effect of screening in 1+1 dimension. See \cite{1} and references therein.
handedness). Such operators appear as the first term in an operator product expansion (to all orders in perturbation theory) of the product of the component fields with no necessity for normal ordering and since the short distance physics is free, we can bring the fermion fields to the same point without divergences. But more importantly, these operators have the lowest dimensions at long distances, and we are particularly interested in low-dimension operators. They have the form

\[ O_N = \prod_{\alpha=1}^{n_F} \left( \psi_{\alpha 1}^* \overline{n}_1 \alpha \right) \left( \psi_{\alpha 2}^* \overline{n}_2 \alpha \right) \left( \psi_{\alpha 1} \right)^{n_1 \alpha} \left( \psi_{\alpha 2} \right)^{n_2 \alpha} \]  

(3)

where the numbers of LH and RH \( \psi^* \)s, \( n_1 \alpha \) and \( n_2 \alpha \), and \( \psi \)s, \( n_1 \alpha \) and \( n_2 \alpha \) satisfy \( n_j \alpha + n_j \alpha = 0 \) or 1 for \( j = 1 \) or 2 and for all \( \alpha \). Gauge invariance requires

\[ \sum_{\alpha=1}^{n_F} e_{j\alpha}(\overline{n}_1 \alpha + \overline{n}_2 \alpha - n_1 \alpha - n_2 \alpha) = 0 \ \forall j \]  

(4)

In the classic Sommerfield and Schwinger models, there is only one kind of gauge boson and fermion, so we can drop the \( j \) and \( \alpha \). Then the local “unparticle” operators look like

\[ \psi_1^*(x) \psi_2(x) \ \text{and} \ \psi_2^*(x) \psi_1(x) \]  

(5)

To compute the dimension of the operator in the long-distance theory, we calculate the 2-point function of the operator and its complex conjugate, which for this class of operators is

\[ \langle 0 | T O_N(x) O_N^*(0) | 0 \rangle = (-1)^s \times \left( S_1(x) \right)^{\sum_{\alpha} (\overline{n}_1 \alpha + n_1 \alpha)} \left( S_2(x) \right)^{\sum_{\alpha} (\overline{n}_2 \alpha + n_2 \alpha)} \] 

\[ \exp \left[ \sum_{j=1}^{n_A} \frac{e_{j\alpha}e_{j\alpha}}{2\pi m_j^2} \left[ K_0 \left( m_j \sqrt{-x^2 + i\epsilon} \right) + \ln \left( \xi m_j \sqrt{-x^2 + i\epsilon} \right) \right] \right] \]  

(6)

where

\[ e_{j\alpha} = \sum_{\alpha=1}^{n_F} e_{j\alpha}(\overline{n}_1 \alpha - \overline{n}_2 \alpha - n_1 \alpha + n_2 \alpha) \]  

(7)

and \((-1)^s\) is a calculable signature factor that keeps track of operator ordering.

For (4), \( j = 1 \) because there is only one gauge boson, \( \overline{n}_1 \alpha + n_1 \alpha = \overline{n}_2 \alpha + n_2 \alpha = 1 \) and \( e_{j\alpha} = 2e \) and (6) becomes

\[ \frac{(\xi m)^{2(e^2/\pi)/m^2}}{(2\pi)^2} \exp \left[ \frac{2e^2}{\pi m^2} \left[ K_0 \left( m \sqrt{-x^2 + i\epsilon} \right) \right] \right] \left( \frac{1}{-x^2 + i\epsilon} \right)^{1-(e^2/\pi)/m^2} \]  

(9)

At long distances, the exponential goes to one and we can read off the dimension of the operator as

\[ 1 - (e^2/\pi)/m^2 \]  

(10)
The dimension $10$ is always greater than $0$ in the Sommerfield model, but it goes to zero at the “Schwinger point” where the bare mass goes to zero and $m^2 \to e^2/\pi$ (see (2)). Thus the Schwinger model has a mass gap and there is no conformal sector. In the operator language, what is happening is that the effect of the ghost propagator in the exponential is to decrease the dimension from its free-field value. At the Schwinger point, this exactly cancels the free field behavior from the fermion propagators and the 2-point function goes to a constant at long distances. Then cluster decomposition implies that the operators must have non-zero vacuum expectation values, because it must be that

$$\langle 0| T O_N(x) O^*_N(0) |0 \rangle \xrightarrow{-x^2 \to \infty} \langle 0|O_N(0)|0 \rangle \langle 0|O^*_N(0)|0 \rangle$$

(11)

This means the vacuum at the Schwinger point must be degenerate with

$$\langle 0|O_N(0)|0 \rangle = \frac{\xi m}{2\pi} e^{i\theta} \quad \langle 0|O^*_N(0)|0 \rangle = \frac{\xi m}{2\pi} e^{-i\theta}$$

(12)

where $\theta$ is the parameter that labels the vacuum state [9, 10, 11].

The primary goal of our note is to search for similar effects in generalized Schwinger models. We will look for (and find) other examples of long-distance 0-dimension operators, which we call zdops (pronounced zee-dops). Because the smallest dimension in the long-distance conformal theory consistent with unitarity is 0 [15], zdops can only occur at a boundary of parameter space, like the Schwinger point. They do not occur in generalized Sommerfield models. For this reason, we study generalized Schwinger models, in which all the vector boson mass terms in the Lagrangian vanish, so we can write (2) as

$$m_j^2 \delta_{jk} = \sum_\alpha \left[ e \right]_{j\alpha} \left[ e \right]_{k\alpha}$$

(13)

We will see that not only do zdops exist in some generalized Schwinger models, but they produce new physical effects that have no analog in the original Schwinger model.

Before describing models with zdops, we should note that the existence of zdops is not generic. It requires special relationships among couplings. A simple example without zdops is a model with two massless fermions with couplings $e$ and $2e$ to one vector boson. At the Schwinger point there are non-derivative composite operators with long-distance dimensions $1/5$, $4/5$, and greater, but no zdops!

$n$-flavor Schwinger model

A simple generalization to consider is $n_F = 2$ with $[e]_1 = [e]_2 = e_0^2$ so that (13) gives

$$m^2 = \frac{2e_0^2}{\pi}$$

(14)

$^2$As in the standard model, we refer to “flavor” as an attribute of multiple fermion fields with the same gauge couplings. Below, we will discuss the analog of “color”.

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This model has a classical chiral $U(2) \times U(2)$ symmetry which is broken by the chiral anomaly down to $SU(2) \times SU(2) \times U(1)$. There are fermion-anti-fermion operators like the zdops in the Schwinger model, which in this model transform like the $(2, 2)$ representation of the chiral symmetry. They are shown below

$$O_{k12}^j = \psi_{j1}^* \psi_{k2} \quad \text{and} \quad O_{k21}^j = \psi_{j2}^* \psi_{k1}$$

At the Schwinger point, because $e_0^2/m^2$ is half what it is in the 1-flavor Schwinger model, the ghost contributions to the long-distance anomalous dimensions are just half what they are in the Schwinger model and so do not cancel the free fermion contributions to the 2-point functions (see [16]). Instead, these are unparticle operators with long-distance dimension $1/2$. But the cancellation does take place in the 2-point function of the chiral $SU(2) \times SU(2)$ singlet operators

$$O_{12}^{2-\text{flavor}} = \psi_{11}^* \psi_{21} \psi_{12} \psi_{22} \quad \text{and} \quad O_{21}^{2-\text{flavor}} = \psi_{12}^* \psi_{22} \psi_{11} \psi_{21}$$

for which

$$\langle 0|T O_{12}^{2-\text{flavor}}(x) O_{21}^{2-\text{flavor}}(0)|0 \rangle = \frac{(\xi m)^4}{16 \pi^4} \exp \left( 4K_0 \left( m\sqrt{-x^2 + i\epsilon} \right) \right)$$

These are the zdops. Cluster decomposition requires that these operators have VEVs

$$\langle 0|O_{12}^{2-\text{flavor}}(0)|0 \rangle = e^{2i\theta} \left( \frac{\xi m}{4\pi^2} \right)^2 \quad \text{and} \quad \langle 0|O_{21}^{2-\text{flavor}}(0)|0 \rangle = e^{-2i\theta} \left( \frac{\xi m}{4\pi^2} \right)^2$$

where $\theta$ is the parameter that labels the vacuum state. [9] [10] [11]

Similar behavior occurs in the massless $n$-flavor Schwinger model, where a completely analogous argument implies a non-zero $2n$-fermion condensate for the $SU(n) \times SU(n)$ singlet operators

$$O_{12}^{n-\text{flavor}} = \left( \prod_{\ell=1}^n \psi_{\ell1}^* \right) \left( \prod_{\ell=1}^n \psi_{\ell2} \right) \quad \text{and} \quad O_{21}^{n-\text{flavor}} = \left( \prod_{\ell=1}^n \psi_{\ell2}^* \right) \left( \prod_{\ell=1}^n \psi_{\ell1} \right)$$

for which

$$\langle 0|T O_{12}^{n-\text{flavor}}(x) O_{21}^{n-\text{flavor}}(0)|0 \rangle = \left( \frac{\xi m}{4\pi^2} \right)^{2n} \exp \left( 2n\pi K_0 \left( m\sqrt{-x^2 + i\epsilon} \right) \right)$$

so cluster decomposition requires that these operators have VEVs

$$\langle 0|O_{12}^{n-\text{flavor}}(0)|0 \rangle = e^{i\phi} \left( \frac{\xi m}{4\pi^2} \right)^n \quad \text{and} \quad \langle 0|O_{21}^{n-\text{flavor}}(0)|0 \rangle = e^{-i\phi} \left( \frac{\xi m}{4\pi^2} \right)^n$$

The phase of the $2n$-fermion chiral $SU(n) \times SU(n)$ singlet condensate can by changed by $2\pi/n$ by a chiral $SU(n)$ transformation — hence the $2\theta$ in (18) and the $n\theta$ in (21).
Conformal coalescence

The first example we will discuss of the nonperturbative effects of zdops is on the 2-point function of the operators \( \{ O_{jk} \} \) in the 2-flavor Schwinger model. The 2-point function is (note that \( O_{k12} \) and \( O_j^{21} \) are hermitian conjugates)

\[
\langle 0| T \, O_{k12}^{j1}(x) O_{j221}^{k2}(0) |0 \rangle = \delta_{j2}^{j1} \delta_{k1}^{k2} \frac{\xi m}{(2\pi)^2} \exp \left[ K_0 \left( m\sqrt{-x^2 + i\epsilon} \right) \right] \left( \frac{1}{-x^2 + i\epsilon} \right)^{1/2} \tag{22}
\]

This goes to the free-field result \( \propto 1/x^2 \) for small \( x \) and scales with anomalous dimension \( 1/2 \) at long distances. The zdops produce non-perturbative corrections to (22). The 3-point correlation function with an added zdop is

\[
\langle 0| T \, O_{12}^{2-\text{flavor}}(z) O_{k12}^{j1}(x) O_{k21}(0) |0 \rangle = \langle 0| T \, O_{21}^{2-\text{flavor}}(z) O_{k12}^{j1}(x) O_{k221}(0) |0 \rangle = -\epsilon^{j1j2} \epsilon_{k1k2} \times \frac{(\xi m)^3}{(2\pi)^4} \exp \left[ -2 \left( K_0 \left( m\sqrt{-z^2 + i\epsilon} \right) + K_0 \left( m\sqrt{(z-x)^2 + i\epsilon} \right) \right) \right] \left( \frac{1}{-x^2 + i\epsilon} \right)^{1/2} \tag{23}
\]

Now here is the crucial point. Cluster decomposition can be applied to (23) just as it can in (11) and (20). We can pull the zdop away to infinity and replace it by its VEV, (18), then the exponential in (23) goes to 1 and what remains is a nonperturbative contribution to the 2pt functions of the long-distance-dimension \( 1/2 \) operators. Thus

\[
\langle 0| T \, O_{k12}^{j1}(x) O_{k221}^{j2}(0) |0 \rangle = -\epsilon^{-2i\theta} \epsilon^{j1j2} \epsilon_{k1k2} \frac{(\xi m)}{(2\pi)^2} \left( \frac{1}{-x^2 + i\epsilon} \right)^{1/2} \tag{24}
\]

\[
\langle 0| T \, O_{k12}^{j1}(x) O_{k212}^{j2}(0) |0 \rangle = -\epsilon^{-2i\theta} \epsilon^{j1j2} \epsilon_{k1k2} \frac{(\xi m)}{(2\pi)^2} \left( \frac{1}{-x^2 + i\epsilon} \right)^{1/2} \tag{25}
\]

The zdop VEV has given us a nonperturbative contribution to the 2-point function that is fixed by the calculable 3-point function. It is amusing that we can calculate this exactly. But there are more surprises in store. Define (using summation convention)

\[
O_{k\pm}^j \equiv \left( O_{k12}^j \pm e^{2i\theta} e^{j\ell} \epsilon_{k2m} O_{m21}^\ell \right) \quad \text{and the h.c.} \quad \overline{O}_{j\pm}^k \equiv \left( O_{j21}^k \pm e^{-2i\theta} e^{km} \epsilon_{j\ell} O_{m12}^\ell \right) \tag{25}
\]

Then

\[
\langle 0| T \, O_{k12}^{j1}(x) \overline{O}_{j221}^{k2}(0) |0 \rangle = 0 \tag{26}
\]

and

\[
\langle 0| T \, O_{k1\pm}^{j1}(x) \overline{O}_{j2\mp}^{k2}(0) |0 \rangle = \delta_{j2}^{j1} \delta_{k1}^{k2} \frac{(\xi m)}{(2\pi)^2} \left( \exp \left( K_0 \left( m\sqrt{-x^2 + i\epsilon} \right) \right) + 1 \right) \left( \frac{1}{-x^2 + i\epsilon} \right)^{1/2} \tag{27}
\]

\[\text{Our convention is that } \epsilon^{12} = \epsilon_{12} = 1 \text{ and so that } \epsilon^{j1k1j2k2} \epsilon_{k1k2} = \epsilon^{j1j2}.
\]

\[\text{In general, we might have to include the contributions from } n \text{-point functions with more zdops, but in this example, these do not give any new contributions.}\]
The penultimate factor in (27) is the shocker. At short distances, the exponential dominates for both + and − and (along with the last factor) produces the expected free-fermion scaling. But at long distances, while $O_{k-}$ goes smoothly to a conformal operator, the $O_{k+}$ correlator goes to zero exponentially. Half of the operators disappear from the conformal theory as the pairs in $O_{k-}$ coalesce!

This analysis is also important because it illustrates the difference between a zdop and a tadpole (see [17]). Both involve VEVs, but tadpoles arise from VEVs of local operators with positive dimension. They are contributions to individual Feynman diagrams and to get their full contributions one must sum over all possible tadpole insertions. Zdops, in contrast, appear in complete correlators with other operators and are not summed. Cluster decomposition applied to correlators involving many zdops does not give independent contributions to correlators of positive long-distance-dimension operators. There are only a finite number (2 in this example) of independent effects.

**Diagonal color and related VEVs**

Because we wish to focus on models that are solvable in the same sense as the Schwinger model, we will consider only models with diagonal gauge couplings, and thus $SU(n_c)$ color is not available to us. However, a particularly interesting related class of models with multiple gauge bosons has been studied in the literature under the name “diagonal color” [18] and it will be useful to discuss the general issues of models with multiple gauge bosons in this example. In an $SU(n_c)$ diagonal color model, there are $n_c$ colors of the fermions and $n_c-1$ gauge bosons with couplings proportional to the diagonal traceless $n_c\times n_c$ matrices in the conventional basis, so that the $(n_c-1)\times n_c$ gauge coupling matrix has matrix elements

$$[e]_{ja} = \frac{e_j}{\sqrt{2j(j+1)}} \begin{cases} 1 & \text{for } \alpha \leq j \\ -j & \text{for } \alpha = j + 1 \\ 0 & \text{for } j + 1 < \alpha \leq n_c \end{cases} \quad \text{where } e_j = \sqrt{2\pi m_j} \quad (28)$$

We have chosen the normalizations so that $ee^T/\pi$ is the diagonal mass-squared matrix of the gauge bosons, as in (13) so we are at the Schwinger point. In general, the physics depends on the masses and (related) couplings of the different gauge bosons. In particular, dimensional factors in the correlators depend on the individual gauge boson masses, $m_j$. However, long-distance anomalous dimensions depend only on $e^T M^{-2} e/\pi$ which is independent of $m_j$.

The simplest interesting example is “diagonal color $SU(3)$”, in which there are three massless fermions (“colored quarks”) but we keep only the gluons $G_3$ and $G_8$, with diagonal couplings to the colored quarks [18]. Here $e$ is a matrix

$$e = \sqrt{2\pi} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & 0 \\ \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & 0 \end{pmatrix} \Rightarrow \frac{1}{\pi} e e^T = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \quad (29)$$

Using (29), (6), and (7) to search for gauge invariant operators that do not require normal ordering we find long-distance dimensions $0, \frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{3}{2},$ and $3$.

\[6\text{Diagonal color } SU(2) \text{ is nothing new because it can be related to the 2-flavor Schwinger model.}\]
The 6 zdops can be labeled by an ordered pair of color indices

\[ O_{jk} \equiv \psi_{j1}^* \psi_{k2}^* \psi_{k1} \psi_{j2} \quad \text{for} \quad j \neq k = 1 \text{ to } 3 \text{ satisfying } O_{jk} = O_{kj}^* \]  

(30)

with non-zero 2-point functions

\[
\langle 0 | T O_{12}(x) O_{21}(0) | 0 \rangle = \frac{(\xi m_1)^4}{16 \pi^4} \exp \left(4K_0 \left(m_1 \sqrt{-x^2 + i \epsilon}\right)\right)
\]  

(31)

\[
\langle 0 | T O_{j3}(x) O_{3k}(0) | 0 \rangle = \delta_{jk} \frac{\xi^4 m_1 m_2}{16 \pi^4} \exp \left(K_0 \left(m_1 \sqrt{-x^2 + i \epsilon}\right)\right) \times \exp \left(3K_0 \left(m_2 \sqrt{-x^2 + i \epsilon}\right)\right) \quad \text{for} \quad j, k = 1 \text{ or } 2
\]  

(32)

Cluster decomposition of the 2-point functions fixes the absolute values of the zdop VEVs, but does not determine the phases (complex conjugate pairs, \(O_{jk}\) and \(O_{kj}\) must have opposite phases). The 3-point function puts additional constraints on the VEVs. The 3-point function is completely symmetric and there are two combinations of zdops that give a non-zero result: those with indices in cyclic order; and those in anti-cyclic order. The explicit calculation and the usual cluster argument then implies

\[
\langle 0 | O_{12} | 0 \rangle \langle 0 | O_{23} | 0 \rangle \langle 0 | O_{31} | 0 \rangle = \langle 0 | O_{13} | 0 \rangle \langle 0 | O_{21} | 0 \rangle \langle 0 | O_{32} | 0 \rangle = \left(\frac{\xi^2 m_1 m_2}{4 \pi^2}\right)^3
\]  

(33)

Combined with complex conjugation, (33) implies that only two independent phases are allowed for the zdop VEVs, and the VEVs are

\[
\langle 0 | O_{12} | 0 \rangle = e^{i \theta_1} \frac{\xi^2 m_1^2}{4 \pi^2} \quad \text{and} \quad \langle 0 | O_{23} | 0 \rangle = e^{i \theta_2} \frac{\xi^2 \sqrt{m_1 m_2^3}}{4 \pi^2} \Rightarrow
\]

\[
\langle 0 | O_{31} | 0 \rangle = e^{-i(\theta_1 + \theta_2)} \frac{\xi^2 \sqrt{m_1 m_2^3}}{4 \pi^2}, \quad \langle 0 | O_{13} | 0 \rangle = e^{i(\theta_1 + \theta_2)} \frac{\xi^2 \sqrt{m_1 m_2^3}}{4 \pi^2},
\]  

(34)

\[
\langle 0 | O_{21} | 0 \rangle = e^{-i \theta_1} \frac{\xi^2 m_1^2}{4 \pi^2}, \quad \langle 0 | O_{32} | 0 \rangle = e^{-i \theta_2} \frac{\xi^2 \sqrt{m_1 m_3^3}}{4 \pi^2}
\]

Having one independent \(\theta\) angle for each gauge field is consistent with the connection between \(\theta\) angles and background electric fields [3, 19].

**Beyond coalescence**

So far, we have focused on the zdops themselves and their nonperturbative effects on 2-point functions. This is a sensible place to start because the 2-point functions determine the operator structure of the long-distance conformal theory. But zdop cluster decomposition can impose non-perturbative constraints on higher correlators as well. One might wonder whether these effects can all be explained by conformal coalescence. For example, it could be that the full nonperturbative \(n\)-point functions could be constructed from the perturbative \(n\)-point functions by some sort of projection onto the operators that survive at long
distances. But that turns out not to be the case. New effects emerge specifically for the higher correlators. The simplest example we have found is in a 4-fermion, 2-gauge-boson model with

\[ e = \left( \begin{array}{cccc} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & \frac{0}{4} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & -\frac{1}{4} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{array} \right) e_0 \]  

and two zdops,

\[ \psi_1^* \psi_2 \psi_2^* \psi_1 + \text{h.c.} \]  

where the 3-point function of two long-distance-dimension 5/12 operators and one long-distance-dimension 2/3 operator can be calculated nonperturbatively and cannot be related to a projection of the perturbative 3-point function.

Conclusions

We hope that we have convinced our readers that it is interesting to study generalized Schwinger models with long-distance 0-dimension operators. While many of the properties of these models are dependent on the special properties of massless fermions in 1+1 dimensions, some of the new phenomena we observe here such as conformal coalescence could occur in more physical theories. We look forward to further exploration of this class of models.

It is tempting to speculate about whether there could be analogs of our zdops in 3+1 dimensions. At first sight, this appears impossible because we could not have a long-distance conformal theory with very small anomalous dimensions. But the mechanism does not rely on the existence of a non-trivial conformal theory (as the example of the simple Schwinger model shows). The zdops are simply local operators in the full theory that just happen to have VEVs and couple only to massive states, so perhaps one could imagine that a chiral U(1)-breaking operator in massless QCD could be a zdop whose correlators are associated with the breaking of the anomalous symmetry. If so, this might provide a different way to study the breakdown of anomalous symmetry in which different chiral states are separated not in an extra dimension (as in [20]), but in real space.

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References

[1] J. S. Schwinger, “Gauge invariance and mass. II,” \textit{Phys. Rev.} \textbf{128} (1962) 2425.

[2] K. G. Wilson, “Confinement of Quarks,” \textit{Phys. Rev.} \textbf{D10} (1974) 2445–2459.

\[ \text{H. Georgi and B. Warner, manuscript in preparation.} \]
[3] S. R. Coleman, R. Jackiw, and L. Susskind, “Charge Shielding and Quark Confinement in the Massive Schwinger Model,” *Annals Phys.* **93** (1975) 267.

[4] D. J. Gross, I. R. Klebanov, A. V. Matytsin, and A. V. Smilga, “Screening versus confinement in (1+1)-dimensions,” *Nucl. Phys.* **B461** (1996) 109–130, arXiv:hep-th/9511104 [hep-th]

[5] N. K. Nielsen and B. Schroer, “Topological fluctuations and breaking of chiral symmetry in gauge theories involving massless fermions,” *Nucl. Phys.* **B120** (1977) 62.

[6] G. Maiella and F. Schaposnik, “The Role of Pseudoparticle Configurations in the Schwinger Model,” *Nucl. Phys.* **B132** (1978) 357–364.

[7] C. Adam, “Instantons and vacuum expectation values in the Schwinger model,” *Z. Phys. C63* (1994) 169–180.

[8] A. V. Smilga, “Instantons and fermion condensate in adjoint QCD in two-dimensions,” *Phys. Rev.* **D49** (1994) 6836–6848, arXiv:hep-th/9402066 [hep-th].

[9] A. V. Smilga, “On the fermion condensate in the Schwinger model,” *Phys. Lett.* **B278** (1992) 371.

[10] C. Jayewardena, “SCHWINGER MODEL ON S(2),” *Helv. Phys. Acta* **61** (1988) 636–711.

[11] J. E. Hetrick and Y. Hosotani, “QED ON A CIRCLE,” *Phys. Rev.* **D38** (1988) 2621.

[12] H. Georgi and B. Warner, “Generalizations of the Sommerfield and Schwinger models,” arXiv:1907.12705 [hep-th]

[13] C. M. Sommerfield, “On the definition of currents and the action principle in field theories of one spatial dimension,” *Ann. Phys.* **26** (1964) 1.

[14] T. Banks and A. Zaks, “On the phase structure of vector-like gauge theories with massless fermions,” *Nucl. Phys.* **B196** (1982) 189.

[15] G. Mack, “All unitary ray representations of the conformal group SU(2,2) with positive energy,” *Commun. Math. Phys.* **55** (1977) 1.

[16] H. Georgi, “The Schwinger Point,” arXiv:1905.09632 [hep-th].

[17] S. R. Coleman and S. L. Glashow, “Departures from the eightfold way: Theory of strong interaction symmetry breakdown,” *Phys. Rev.* **134** (1964) B671–B681.

[18] P. J. Steinhardt, “Two-dimensional Gauge Theories With Diagonal SU(N) Color,” *Annals Phys.* **132** (1981) 18.
[19] S. R. Coleman, “More about the massive Schwinger model,”
\textit{Ann. Phys.} \textbf{101} (1976) 239.

[20] D. B. Kaplan, “A Method for simulating chiral fermions on the lattice,”
\textit{Phys. Lett.} \textbf{B288} (1992) 342–347, \texttt{arXiv:hep-lat/9206013} [hep-lat].