Joint Hybrid Precoder and Combiner Design for mmWave Spatial Multiplexing Transmission

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Abstract—Millimeter-wave (mmWave) communications have been considered as a key technology for future 5G wireless networks because of the orders-of-magnitude wider bandwidth than current cellular bands. In this paper, we consider the problem of codebook-based joint analog-digital hybrid precoder and combiner design for spatial multiplexing transmission in a mmWave multiple-input multiple-output (MIMO) system. We propose to jointly select analog precoder and combiner pair for each data stream successively aiming at maximizing the channel gain while suppressing the interference between different data streams. After all analog precoder/combiner pairs have been determined, we can obtain the effective baseband channel. Then, the digital precoder and combiner are computed based on the obtained effective baseband channel to further mitigate the interference and maximize the sum-rate. Simulation results demonstrate that our proposed algorithm exhibits prominent advantages in combating interference between different data streams and offer satisfactory performance improvement compared to the existing codebook-based hybrid beamforming schemes.

Index Terms—Millimeter-wave communication, hybrid precoder, multiple-input multiple-output (MIMO), antenna arrays, beamforming.

I. INTRODUCTION

The rapid proliferation of wireless devices has raised high demand of increasingly high transmission data rate. Millimeter-wave (mmWave) wireless communications, operating in the frequency bands from 30-300 GHz, have been demonstrated to be an excellent candidate to solve the spectrum congestion problem in recent experiments and are defining a new era of wireless communications because of the significantly large and unexploited mmWave frequency bands [1]-[3]. Economic and energy-efficient analog/digital hybrid precoding and combining transceiver architecture has been widely used in mmWave massive multiple-input multiple-output (MIMO) systems.

The hybrid precoding approach applies a large number of analog phase shifters to implement high-dimesional radio frequency (RF) precoders to compensate the large path-loss at mmWave bands, and a small number of RF chains for low-dimensional digital precoders to provide the necessary flexibility to perform advanced multiplexing/multuser techniques. The major challenge in designing hybrid precoder is the practical constraints of the RF precoders, such as constant modulus, which is usually imposed by phase shifters. A popular solution to maximize the spectral efficiency of mmWave communications is to minimize the Euclidean distance between the hybrid precoder and the full-digital precoder. Thus, the hybrid precoder design is deemed as solving various matrix factorization problems with constant modulus constraints of the analog precoder [4]-[7].

The existing hybrid precoding designs typically assume that the analog beamformers are implemented with infinite resolution phase shifters. However, the components for realizing accurate phase shifters could be very complicated and expensive. Therefore, low-resolution phase shifters with discrete/quantized tunable phases are cost-effective and typically adopted in realistic systems. Particularly, according to the special characteristic of a mmWave channel, more practical codebook-based hybrid precoder design has been widely used [8]-[15], in which the columns of the analog precoder are selected from certain candidate vectors, such as array response vectors of the channel and discrete Fourier transform (DFT) beamformers which have constant modulus and discrete phases.

In the existing codebook-free and codebook-based algorithms mentioned above, the optimal hybrid precoder and combiner are individually designed to approximate, in the best Frobenius norm, the right and left singular vectors of the channel matrix, receptively. While the separate design for hybrid precoder and combiner can provide satisfactory performance in terms of spectral efficiency, the orthogonality of resulting spatial multiplexing channel cannot be guaranteed. Therefore, the conventional hybrid precoder and combiner designs may cause significant performance loss in realistic mmWave multiplexing systems. This motivates us to reconsider the hybrid precoder and combiner design and find a better way for spatial multiplexing in mmWave MIMO communications.

In this paper, we consider the problem of codebook-based joint hybrid precoder and combiner design for spatial multiplexing transmission in mmWave MIMO systems. We propose to jointly select the analog precoder and combiner pair for each data stream successively, which can maximize the channel...
gain as well as suppress the interference between different data streams. Then, the digital precoder and combiner are computed based on the obtained effective baseband channel to further mitigate the interference and maximize the sum-rate. Simulation results demonstrate that our proposed algorithm exhibits prominent advantages in combating the interference between different data streams and offers satisfactory performance improvement compared to the existing codebook-based hybrid beamforming schemes.

The following notation is used throughout this paper. Boldface lower-case letters indicate column vectors and boldface upper-case letters indicate matrices; $\mathbb{C}$ denotes the set of all complex numbers; $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and transpose-conjugate operation, respectively; $\mathbf{A}(\cdot, l)$ denotes the $l$-th column of matrix $\mathbf{A}$; $\mathbf{I}_L$ is the $L \times L$ identity matrix; $\mathbb{E}\{\cdot\}$ represents statistical expectation. Finally, $|\cdot|$, $\parallel\cdot\parallel_F$, and $\parallel\cdot\parallel_F$ are the scalar magnitude, vector norm, and Frobenius norm, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a single-user mmWave MIMO multiplexing system with hybrid precoder and combiner, as illustrated in Fig. 1. The transmitter employs $N_t$ antennas and $N_t^{RF}$ RF chains to simultaneously transmit $N_s$ data streams to the receiver which is equipped with $N_r$ antennas and $N_r^{RF}$ RF chains. To ensure the efficiency of the communication with limited number of RF chains, the number of data streams is constrained as $N_s = N_t^{RF} = N_r^{RF}$, while the results can be applied to the general cases.

The transmitted symbols are first processed by a baseband precoder $\mathbf{F}_{BB} \in \mathbb{C}^{N_t^{RF} \times N_s}$, then up-converted to the RF domain via $N_r^{RF}$ RF chains before being precoded with an analog precoder $\mathbf{F}_{RF}$ of dimension $N_t \times N_s^{RF}$. While the baseband precoder $\mathbf{F}_{BB}$ enables both amplitude and phase modifications, the analog precoder $\mathbf{F}_{RF}$ has a constant amplitude $1/\sqrt{N_s}$ for each element since it is implemented using analog phase shifters.

The discrete-time transmitted signal can be written as

$$\mathbf{x} = \sqrt{P}\mathbf{F}_{RF} \mathbf{F}_{BB} \mathbf{s} \quad (1)$$

where $\mathbf{s}$ is the $N_s \times 1$ symbol vector such that $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \frac{1}{N_s}\mathbf{I}_{N_s}$. $P$ represents transmit power, and the total transmit power constraint is enforced by normalizing $\mathbf{F}_{BB}$ such that $\parallel\mathbf{F}_{RF} \mathbf{F}_{BB}\parallel_F^2 = N_s$.

For simplicity, we consider a narrowband block-fading propagation channel, which yields the received signal as

$$\mathbf{y} = \sqrt{P}\mathbf{H}\mathbf{F}_{BB} \mathbf{s} + \mathbf{n} \quad (2)$$

where $\mathbf{y}$ is the $N_r \times 1$ received vector, $\mathbf{H}$ is the $N_r \times N_t$ channel matrix, and $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{N_r})$ is the complex Gaussian noise vector corrupting the received signal.

The receiver uses its $N_r^{RF}$ RF chains and phase shifters to obtain the processed receive signal which has a form of

$$\hat{\mathbf{s}} = \sqrt{P}\mathbf{W}_{BB}^H \mathbf{W}_{RF}^H \mathbf{H}\mathbf{F}_{BB} \mathbf{s} + \mathbf{W}_{BB}^H \mathbf{W}_{RF}^H \mathbf{n} \quad (3)$$

where $\mathbf{W}_{BB}$ is the $N_r^{RF} \times N_s$ digital baseband combiner, $\mathbf{W}_{RF}$ is the $N_r \times N_r^{RF}$ analog RF combiner.

B. Millimeter-Wave MIMO Channel Model

Due to high free-space pathloss and large tightly-packed antenna arrays, the mmWave propagation in a massive MIMO system is well characterized by a limited spatial selectivity or scattering model, e.g. the Saleh-Valenzuela model, which allows us to accurately capture the mathematical structure of mmWave channels [8]. The matrix channel $\mathbf{H}$ is assumed to be a sum contribution of $N_{cl}$ scattering clusters, each of which provides $N_{ray}$ propagation paths to the channel matrix $\mathbf{H}$. Therefore, the discrete-time narrow-band mmWave channel $\mathbf{H}$ can be formulated as

$$\mathbf{H} = \sqrt{N_r N_s} \frac{N_{cl}}{N_r N_r^{ray}} \sum_{i=1}^{N_{cl}} \sum_{l=1}^{N_{ray}} \alpha_{il} a_r(\theta_{il}^t) a_i(\theta_{il}^r)^H \quad (4)$$

where $\alpha_{il} \sim \mathcal{CN}(0, \sigma_{\alpha_{il}}^2)$ is the complex gain of the $l$-th propagation path (ray) in the $i$-th scattering cluster and it yields independent identically distribution (i.i.d.). Let $\sigma_{\alpha_{il}}^2$ represent the average power of the $i$-th cluster, and the total power satisfies $\sum_{i=1}^{N_{cl}} \sigma_{\alpha_{il}}^2 = N_{cl}$. $\theta_{il}^t$ and $\theta_{il}^r$ are the angles of departure (AoD) and the angle of arrival (AoA), respectively, which are assumed to be Laplacian-distributed with a mean cluster angle $\theta_{il}^t$ and $\theta_{il}^r$ and an angle spread of $\sigma_{\theta_{il}}^t$ and $\sigma_{\theta_{il}}^r$, respectively. Finally, the array response vectors $\mathbf{a}_t(\theta)$ and $\mathbf{a}_i(\theta)$ are the antenna array response vectors, which only depend on the antenna array structures. When the commonly used uniform linear arrays (ULAs) are considered, the receive antenna array response vector can be written as

$$\mathbf{a}_r(\theta) = \frac{1}{\sqrt{N_r}}[1, e^{j\frac{2\pi}{\lambda}d\sin(\theta)}, \ldots, e^{j(N_r-1)\frac{2\pi}{\lambda}d\sin(\theta)}]^T \quad (5)$$

where $\lambda$ is the signal wavelength, and $d$ is the distance between antenna elements. The transmit array response vector $\mathbf{a}_t(\theta)$ can be written in a similar fashion.

C. Problem Formulation

We consider the problem of codebook-based hybrid precoder and combiner design in a mmWave multiplexing system. Specifically, let $\mathcal{F}$ and $\mathcal{W}$ denote the beamsteering codebooks for the analog precoder and combiner, respectively. If $B_{RF}^t$ and $B_{RF}^r$ bits are used to quantize the AoD (AoA), $\mathcal{F}$ and $\mathcal{W}$
will consist of all possible analog precoding and combining vectors, which can be presented as

\[ F = \{ a_i(2\pi i/2^{B_{RF}}) : i = 1, \ldots, 2^{B_{RF}} \}, \]

\[ W = \{ a_i(2\pi i/2^{B_{RF}}) : i = 1, \ldots, 2^{B_{RF}} \}. \]

The columns of analog precoding (combining) matrix \( F_{RF} \) \((W_{RF})\) are picked from candidate vectors in \( F \) \((W)\), i.e. \( F_{RF}(;l) \in F, \forall l = 1, \ldots, N_t^{RF}, W_{RF}(;l) \in W, \forall l = 1, \ldots, N_t^{RF}. \)

When Gaussian symbols are transmitted over the mmWave MIMO channel, the achieved spectral efficiency is given by

\[
R = \log_2 \left( |I_{N_s} + \frac{P}{N_s} R_n^{-1} W_H BB H F_{RF} F_{BB}| \times \right.
\]
\[
F_{BB}^H F_{RF}^H H F_{RF} W_{RF} W_{BB} \Big| \right), \quad (8)
\]

where \( R_n \triangleq \sigma_n^2 W_{BB}^H W_{RF}^H W_{RF} W_{BB} \) is the noise covariance matrix after combining.

While most existing hybrid precoder and combiner design algorithms aim to maximize spectral efficiency in (8), we should point out that it is actually a performance upper bound for a general MIMO system. Even though precoder and combiner designs by approximating the right and left singular vectors of the channel matrix can provide satisfactory spectral efficiency performance, they cannot guarantee the orthogonality of the resulting effective spatial multiplexing channel for multiple data streams transmission. Therefore, a practical spatial multiplexing system needs a more reasonable performance metrics, such as the sum-rate of each data stream which is described as follows.

Given the received signal in (3), the signal-to-interference-plus-noise ratio (SINR) of the \( k \)-th data stream is formulated by

\[
\gamma_k = \frac{\frac{P}{N_s} | W(:, k)^H HF(:, k) |^2}{\frac{P}{N_s} \sum_{i=1, i \neq k} | W(:, k)^H HF(:, i) |^2 + \sigma_n^2 \| W(:, k) \|^2}, \quad (9)
\]

where \( F \triangleq F_{RF} F_{BB} \) and \( W \triangleq W_{RF} W_{BB}. \) The achievable sum-rate of the spatial multiplexing system is

\[
R_{sum} = \sum_{k=1}^{K} \log(1 + \gamma_k). \quad (10)
\]

In this paper, we aim to jointly design the precoders \( F_{RF}, F_{BB} \) and combiners \( W_{RF}, W_{BB} \) to maximize the sum-rate of the mmWave multiplexing system, which can be formulated as

\[
\{ F_{RF}^*, F_{BB}^*, W_{RF}^*, W_{BB}^* \} = \arg \max \sum_{k=1}^{N_s} \log(1 + \gamma_k)
\]

s. t. \( F_{RF}(;l) \in F, \forall l = 1, \ldots, N_t^{RF}, \)
\[
W_{RF}(;l) \in W, \forall l = 1, \ldots, N_t^{RF}, \]
\[
\| F_{RF} F_{BB} \|^2_F = N_s.
\]

The optimization problem of (11) is obviously a non-convex NP-hard problem. In the next Section, we turn to seek a suboptimal joint hybrid precoder and combiner design to reduce the complexity while achieving a satisfactory performance.

III. PROPOSED JOINT HYBRID PRECODER AND COMBINER DESIGN

To implement an efficient multiplexing system and maximize the sum-rate in (10), we need to design the precoder and combiner which can enhance the channel gain of each data stream as well as suppress the interference from each other. To this end, we propose to decompose the difficult optimization problem in (11) into a series of suboptimal problems which are much easier to be solved. In particular, by considering transmit/receive RF chain pairs one by one, we successively select analog precoder and combiner to maximize the corresponding channel gain while suppressing the cochannel interference. Then, the baseband digital precoder and combiner are computed to further mitigate the interference and maximize the sum-rate.

For the first data stream channel (i.e. \( k = 1 \)), we attempt to find the optimal analog precoder and combiner pair \((f_{RF,1}^*, w_{RF,1}^*)\) from codebooks in order to obtain the largest beamforming gain:

\[
\{ f_{RF,1}^*, w_{RF,1}^* \} = \arg \max_{w_{RF} \in W} |w_{RF}^H H f_{RF}^*| \quad (12)
\]

which can be easily solved by searching all candidate vectors in \( F \) and \( W \) with computational complexity \( O(|F||W|) \). Then, we assign \( f_{RF,1}^* \) and \( w_{RF,1}^* \) to the precoding and combining matrices

\[
F_{RF}^*(:,1) = f_{RF,1}^*, \quad (13)
\]
\[
W_{RF}^*(:,1) = w_{RF,1}^*. \quad (14)
\]

For the rest \( K-1 \) data streams, we attempt to successively select precoders and combiners to actively avoid the interference of the data streams whose precoders and combiners have been already determined. In particular, the component of previous determined precoders and combiners should be removed from the other data streams’ channels such that similar analog precoders and combiners will not be selected by the other data streams. To achieve this goal, we first initialize \( \tilde{H} = H \), which will be updated successively in each step of selecting the analog beamformer pair. Let \( p_1 \triangleq f_{RF,1}^* \) and \( q_1 \triangleq w_{RF,1}^* \), be the components of the determined analog precoder and combinder for the first data stream, respectively. Then, before choosing the next analog precoder and combiner pair, the channel should be updated by eliminating such a series of orthogonal components of previous determined analog beamformer pairs. Particularly, before finding the second (i.e. \( k = 2 \)) analog precoder and combiner pair, we need to update channel as

\[
\tilde{H} = (I_{N_s} - q_1 q_1^H) H (I_{N_s} - p_1 p_1^H) \quad (15)
\]
and then execute searching process as
\[
\{f^*_{RF_2}, w^*_{RF_2}\} = \arg \max_{\substack{w_{RF_2} \in \mathcal{W} \cap \mathcal{F}^*}} |w^H_{RF_2} \tilde{H} f_{RF_2}|.
\]
(16)

The analog precoders and combiners for the rest data streams can be successively selected using the above procedure. Note that when \( k > 1 \), the orthonormal component \( w_k \) of the selected precoder and combiner \( f_{RF_k} \) and \( w_{RF_k} \) should be obtained by a Gram-Schmidt based procedure:
\[
p_k = f^*_{RF_k} - \sum_{i=1}^{k-1} p_i^H f^*_{RF_i} p_i, \quad p_k = p_k/\|p_k\|, \quad k = 2, \ldots, K;
\]
(17)
\[
q_k = w^*_{RF_k} - \sum_{i=1}^{k-1} q_i^H w^*_{RF_i} q_i, \quad q_k = q_k/\|q_k\|, \quad k = 2, \ldots, K.
\]
(18)

After all analog beamformer pairs have been determined, we can obtain the effective baseband channel as
\[
H_{eff} = (W^*_{RF})^H H F^*_{RF}
\]
(19)

where \( F^*_{RF} \triangleq \{f^*_{RF_1}, \ldots, f^*_{RF_Ns}\} \) and \( W^*_{RF} \triangleq \{w^*_{RF_1}, \ldots, w^*_{RF_Ns}\} \). For the baseband precoder and combiner design, we perform singular value decomposition (SVD)
\[
H_{eff} = U \Sigma V^H
\]
(20)

where \( U \) is an \( N_r \times N_r \) unitary matrix, \( \Sigma \) is an \( N_r \times N_r \) diagonal matrix of singular values arranged in decreasing order, and \( V \) is an \( N_t \times N_t \) unitary matrix. Then, an SVD-based baseband digital precoder is employed to further suppress the interference and maximize the sum-rate:
\[
F^*_{BB} = V(:, 1 : N_s),
\]
(21)
\[
W^*_{BB} = U(:, 1 : N_s).
\]
(22)

Finally, we normalize the baseband precoder \( F^*_{BB} \) by
\[
F^*_{BB} = \frac{\sqrt{\mathbf{F}_{RF}^* F_{BB}^*}}{\|F^*_{RF} F_{BB}^*\|_F}.
\]
(23)

This joint hybrid precoder and combiner design algorithm is summarized in Table I.

**IV. SIMULATION RESULTS**

In this section, we illustrate the simulation results of the proposed joint hybrid precoder and combiner design. Both transmitter and receiver are equipped with a 128-antenna ULA and the antenna spacing is \( d = \frac{\lambda}{2} \). The number of RF chains at the transmitter and receiver are \( N_{RF} = 4 \), so is the number of data streams, \( N_s = 4 \). The channel parameters are set as \( N_{cl} = 10 \) clusters, \( N_{ray} = 10 \) rays per cluster, and the average power of the \( i \)-th cluster is \( \sigma^2_{\alpha,i} = c \frac{\pi}{\lambda} \) where \( c = \left( \sum_{i=1}^{N_{cl}} (\frac{\pi}{\lambda})^2 \right)^{-1} N_{cl} \). The azimuths of the AoAs/AoDs within a cluster are assumed to be Laplacian-distributed with an average spread of \( \sigma_{\theta} = 2.5^\circ \). The mean cluster AoDs are assumed to be uniformly distributed over \([0, 2\pi]\), while the mean cluster AoAs are uniformly distributed over an arbitrary \( \overline{\Omega} \) sector. Finally, we employ a codebook consisting of array response vectors with 64 uniformly quantized angle resolutions.

Fig. 2 shows the spectral efficiency versus signal-to-noise-ratio (SNR) over \( 10^6 \) channel realizations. For the comparison purpose, we also include two state-of-the-art algorithms: i) Spatially Sparse Precoding (SPP) [8], which is a classic codebook-based hybrid precoding design; ii) Alternating Minimization using Phase Extraction (PE AltMin) algorithm [6], which is a codebook-free hybrid precoding design. The optimal (OPT) full-digital beamforming scheme with the unconstrained SVD algorithm is also plotted as the performance benchmark. It can be observed that our proposed algorithm outperforms the codebook-based SPP algorithm. Note that the PE AltMin algorithm has a continuous phase on RF beamformer and can achieve extremely close performance to the optimal full-digital approach. Therefore, we just consider it as a reference for the codebook-free algorithms will not compare it with our proposed algorithm. Fig. 3 presents the sum-rate versus SNR with the same system settings as of Fig. 2. We can notice that the proposed joint hybrid precoder and combiner design has a significant performance advantage over the other two hybrid beamforming designs. This is because our joint analog precoder and combiner selection approach aims to mitigate the interference between different data streams. We also consider a different setting that the number of RF chains and data streams are both set as 8. In Figs. 4 and 5 the spectral efficiency and sum-rate are presented, respectively. From these two figures, we can draw similar conclusions that the

**TABLE I**

| Input | Output | Initialization |
|-------|--------|---------------|
| \( F, \mathcal{W}, H \) | \( F_{RF}, F^*_{BB}, W_{RF}, W^*_{BB} \) | \( H = H \) |

*for \( k = 1 : N_s \)*

\( \{f^*_{RF_k}, w^*_{RF_k}\} = \arg \max_{\substack{w_{RF_k} \in \mathcal{W} \cap \mathcal{F}^*}} |w^H_{RF_k} \tilde{H} f_{RF_k}|; \)

\( F_{RF}(; k) = f^*_{RF_k} \); \( W_{RF}(; k) = w^*_{RF_k} \).

if \( k = 1 \)

\( p_k = f^*_{RF_k}, q_k = w^*_{RF_k} \).

draw similar conclusions that the
proposed algorithm can significantly suppress the inter-stream interference and take advantages of spatial multiplexing.

In Figs. 6 and 7 we turn to illustrate how the number of transmit and receive antennas affects the spectral efficiency and sum-rate performance. We assume $N_t = N_r = N$ which is varying from 16 to 256. The SNR is set at 20dB and $N_{RF_t} = N_{RF_r} = N_s = 4$. It can be observed from these two figures that the proposed algorithm has significant superiority compared with the SSP approach in the spectral efficiency as well as sum-rate performance. Interestingly, we also notice that the SSP scheme may exhibit severe performance degradation when the resolution of codebook is less than the number of antennas. This phenomenon is clearly shown in Fig. 6 with a turning point around $N = 64$.

Figs. 8 and 9 provide the performance of spectral efficiency and sum-rate versus the number of data streams $N_s$, respectively. The number of transmit and receive RF chains varies along with $N_s$. We can see that three approaches can achieve comparable performance in terms of spectral efficiency. However, only our proposed algorithm can maintain a satisfactory sum-rate achievement with the increase number of data streams. In addition, the strong interference between different data streams even causes a performance loss of SSP algorithm.

V. CONCLUSIONS

This paper investigated the problem of codebook-based joint hybrid precoder and combiner design for spatial multiplexing transmission in mmWave MIMO systems. We proposed to jointly select analog precoder and combiner pair for each data stream successively aiming at maximizing the channel gain as well as suppressing the interference between different data streams. Then, the digital precoder and combiner were computed based on the obtained effective baseband channel to further mitigate the interference and maximize the sum-rate. Simulation results demonstrated the performance improvement of our proposed algorithm compared to the existing codebook-based hybrid beamforming schemes.
Fig. 7. Sum-rate versus the number of antennas (SNR=20dB, $N_t^{RF} = N_s^{RF} = 4$, $N_s = 4$).

Fig. 8. Spectral efficiency versus $N_s$ ($N_t = N_r = 128$, SNR=20dB).

Fig. 9. Sum-rate versus $N_s$ ($N_t = N_r = 128$, SNR=20dB).

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