Passive Decentralized Fuzzy Control for Takagi-Sugeno Fuzzy Model Based Large-Scale Descriptor Systems

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ABSTRACT This paper presents a decentralized control problem for stabilizing the nonlinear large-scale descriptor (LSD) systems that use the proportional-plus-derivative state (PD) feedback scheme. The descriptor systems have a great focus on the research because they can contain more physical characteristics than standard state-space systems. At first, each nonlinear subsystem in the LSD system can be represented by Takagi-Sugeno (T-S) LSD systems with interconnections. In order to be more realistic, we additionally consider external disturbances and perturbations that affect the system. Our goal is to design a decentralized (DPD) fuzzy controller, such that the T-S LSD system is stable while the external disturbances and perturbations that affect the system. For the controller design process, sufficient conditions were developed based on the quadratic Lyapunov approach, PD feedback scheme, robust constraint, passivity constraint, and linear matrix inequality (LMI) sufficient criteria. At last, two numerical examples are given to show the application of the main results.

INDEX TERMS Takagi-Sugeno model, decentralized control, proportional-plus-derivative state feedback, large-scale descriptor systems, robust control, passivity constraint.

I. INTRODUCTION

Due to the growing complexity and increasing size of dynamic systems, so the large-scale (LS) system has been proposed to reduce the complexity of the mathematical model [1]–[3]. The research on the LS systems has received great attention due to its can be applied to many systems, such as transportation systems, aerospace systems, and power systems, etc... The LS system is composed of several subsystems with strong interconnections and high dimensionality, which makes stability analysis and controller design more difficult. In order to solve the above problems, many useful control methods have been developed for the LS systems in recent years [4]–[6]. According to [7], the decentralized control technique is more effective than the centralized control technique. The basic concept of decentralized control is the entire system can be decomposed into several interconnected subsystems, then one can design a cluster controller to control the entire system instead of a single controller.

The decentralized control technology proposed for LS systems has been reported in many previous studies [8]–[10].

These days, the T-S fuzzy model (TSFM) [11], [12] have shown their significance since it can approximate nonlinear systems by a set of linear subsystems and nonlinear fuzzy membership functions. Another importance of the TSFM is that can use many recognized linear control theories to analyze the stability conditions of nonlinear systems [13]. Consequently, many stability analyses and controller syntheses of practical systems have been made through the TSFM, such as unmanned marine vehicles, ship fin stabilizing systems, maglev vehicle suspension systems [14]–[16]. On the other hand, the TSFM has been applied to LS systems. To mention a few, the decentralized for T-S LS system has been studied in [17]. In [18], the author gives the delay-dependent decentralized control for T-S LS system. In addition, in [19], the observer event triggered for nonlinear LS system is carried out.

In the past few decades, the descriptor systems have been efficiently applied to many practical systems, for example, network system, DC motor system, and bio-economic...
The contributions in this paper are presented below: (1) Based on system gain matrices, the T-S LSD system is approximated by employing the premise variables and fuzzy set. The PD feedback is employed to design the fuzzy controller for T-S LSD systems. (2) Some definitions are given to derive the stability conditions for the singular systems (Define the descriptor matrix as singular matrix), then the closed-loop system is regular, impulse free, and stable. (3) Free-weighting matrices method and Schur complement are employed to derive the stability conditions for enhancing controller design flexibility, and the passivity constraint performance is guaranteed. (4) The perturbations and external disturbances are all considered, which makes the considered model more general and the results can be applied to a wilder practical system. (5) The proposed method is applied in the numerical singular system and double-inverted pendulums system with DPD control. In addition, a comparison has been given to show the effectiveness and applicability.

The paper is organized as follows. In Section II, the nonlinear LSD system is represented by T-S LSD system with external disturbances and perturbations and some important definitions and lemmas are also given. In Section III, the PD feedback method is employed to design the robust fuzzy controller for the T-S LSD system. In Section IV, two examples are given to show the application of the proposed method. Finally, the conclusions are given in the last section.

Notations: \( \Re^n \) denotes the n-dimensional Euclidean space. \( P \) is the n × n dimensional identity matrix. \( X^T \) and \( X^{-1} \) denote the transpose and inverse of the matrix \( X \), \( \det (X) \) is the determinant of a matrix \( X \). \( \text{deg} \) (X) is the number of edges attached to each vertex of a matrix \( X \). The matrix denotes \( P \) is positive definite matrix. \( \text{diag} \{ \cdot \cdot \cdot \} \) is a block-diagonal matrix. We use * as the ellipsis for the term introduced by symmetry in the symmetric block matrix. \( \text{He} \{ X \} \) denotes shorthand \( P > 0 \) notation for \( X + X^T \).

II. PROBLEM FORMULATIONS AND PRELIMINARIES

A. THE DESCRIPTION OF T-S LSD SYSTEM

Based on the Fig. 1, let consider the LSD systems containing N nonlinear subsystems with interconnections. Assume that the i-th nonlinear subsystem is represented by the following T-S LSD model with disturbance and perturbations:

**Plant Rule** \( \forall^i \): If \( z_{il} (t) \) is \( X_{il}^t \) and \( \ldots, z_{ig} (t) \) is \( X_{ig}^t \), THEN

\[
\tilde{E}_{il} \dot{x}_{il} (t) = \tilde{A}_{il} x_{il} (t) + \tilde{B}_{il} u_{il} (t) + G_{il} v_{il} (t)
\]

\[+ \sum_{k=1, k \neq i} N \tilde{A}_{ilk} x_{kl} (t) \quad (1a)\]

\[y_{il} (t) = C_{il} x_{il} (t) + D_{il} v_{il} (t) \quad (1b)\]

where \( i = \{ 1, 2, \ldots, N \} \) and \( l \in \{ l_{i,j} = \{ 1, 2, \ldots, r_{i,j} \} \). For the i-th nonlinear subsystem, \( \forall^i \) denotes the i-th fuzzy membership rules and \( r_{i,j} \) signifies the number of plant rules. \( z_{il} (t), r_{il} (t), \ldots, z_{ig} (t) \) are the premise variables, \( X_{iq}^t (q = 1, 2, \ldots, g) \) are fuzzy sets. \( x_{il} (t) \in \forall_{il}^t \) is the system state, \( u_{il} (t) \in \forall_{il}^u \) is the control input, \( v_{il} (t) \in \forall_{il}^v \) is the disturbance input. The matrices \( \tilde{E}_{il} \) and \( \Delta E_{il} \), \( \tilde{A}_{il} = A_{il} + \Delta A_{il} \), \( \tilde{B}_{il} = B_{il} + \Delta B_{il} \) are the matrices \( A_{il}, B_{il}, C_{il}, D_{il} \) and \( G_{il} \) indicate the constants matrices of the i-th fuzzy subsystem. \( \tilde{A}_{ilk} \) is the interconnection matrix between the
i-th and the k-th nonlinear subsystems. \( E_{il} \) are descriptor matrix, if necessary singular.

The matrices \( \Delta E_{il}, \Delta A_{il} \) and \( \Delta B_{il} \) are the perturbations of the \( l \)-th model that satisfying the following conditions.

\[
[ \Delta E_{il} \ \Delta A_{il} \ \Delta B_{il} ] = H_{il} \Delta \alpha_{il} (t) [ R_{cil} \ R_{ail} \ R_{bil} ], \quad l \in L_i
\]  

(2)

where \( H_{il}, R_{cil}, R_{ail} \) and \( R_{bil} \) donate the known matrices.

Referring to [37], the unknown time-varying functions \( \Delta \alpha_{il} (t) \in \mathbb{R}^{n_{il} \times 2} \) is satisfying following

\[
\Delta \alpha_{il}^T (t) \Delta \alpha_{il} (t) \leq I_{s_{il}}, \quad l \in L_i
\]  

(3)

Defining the fuzzy set \( X_i^l := \prod_{\phi=1}^{l} X_{i_{\phi}} \) and membership function \( \mu_{il} (z_i (t)) \) in the following form:

\[
\mu_{il} (z_i (t)) = \frac{\prod_{\phi=1}^{l} \mu_{i_{\phi}} (z_{i_{\phi}} (t))}{\sum_{l=1}^{l} \prod_{\phi=1}^{l} \mu_{i_{\phi}} (z_{i_{\phi}} (t))} \geq 0
\]  

(4)

where \( \mu_{i_{\phi}} (z_{i_{\phi}} (t)) \) indicates the grade of membership of \( z_{i_{\phi}} (t) \) in \( X_{i_{\phi}} \). We will denote \( \mu_{il} = \mu_{il} (z_i (t)) \) in the following.

By using center-average defuzzifier method, the system (1) can be defuzzified as follows:

\[
\sum_{l=1}^{l} \mu_{il} \tilde{E}_{il} \tilde{x}_i (t)
\]

\[
= \sum_{l=1}^{l} \mu_{il} \left\{ \tilde{A}_{il} x_i (t) + \tilde{B}_{il} u_i (t) + \tilde{G}_{il} y_i (t) \right\} + \sum_{k=1, k \neq i}^{N} \tilde{A}_{ik} (\mu_i) x_k (t)
\]  

(5a)

\[
y_i (t) = \sum_{l=1}^{l} \mu_{il} \left\{ C_{il} x_i (t) + D_{il} v_i (t) \right\}
\]  

(5b)

To ensure the stabilization of the overall T-S LSD system, a DPD fuzzy controller is proposed. The DPD fuzzy controller shares the same fuzzy sets with the i-th subsystem and the controller is given by:

**Controller Rule** \( N_i^l \): If \( z_i^l \) is \( X_i^l \) and, \( \ldots \), \( z_i^l \) is \( X_i^l \), then

\[
u_i (t) = -F_{il} \dot{x}_i (t) + F_{il} x_i (t)
\]  

(6)

where \( F_{il} \) and \( F_{il} \) are control gain matrices.

Analogous to (5), the fuzzy controller for the i-th subsystem, is

\[
u_i (t) = -\sum_{l=1}^{l} \mu_{il} F_{il} \dot{x}_i (t) + \sum_{l=1}^{l} \mu_{il} F_{il} x_i (t)
\]  

(7)

Thus, combining the T-S LSD system in equation (5a) with the DPD fuzzy controller (7), the closed-loop subsystem becomes

\[
E_i (\mu_i) \dot{x}_i = A_i (\mu_i) x_i (t) + G_i (\mu_i) v_i (t)
\]  

(8a)

\[y_i (t) = C_i (\mu_i) x_i (t) + D_i (\mu_i) v_i (t)
\]  

(8b)

where

\[
\begin{align*}
E_i (\mu_i) &= \sum_{l=1}^{l} \sum_{j=1}^{r_i} \mu_{il} \mu_{ij} \left[ \tilde{E}_{il} + \tilde{B}_{il} F_{ij} \right], \\
C_i (\mu_i) &= \sum_{l=1}^{l} \mu_{il} C_{il} \\
A_i (\mu_i) &= \sum_{l=1}^{l} \sum_{j=1}^{r_i} \mu_{il} \mu_{ij} \left[ \tilde{A}_{il} + \tilde{B}_{il} F_{ij} \right], \\
D_i (\mu_i) &= \sum_{l=1}^{l} \mu_{il} D_{il} \\
\tilde{A}_{ik} (\mu_i) &= \sum_{l=1}^{l} \mu_{il} \tilde{A}_{ikl}, \\
G_i (\mu_i) &= \sum_{l=1}^{l} \mu_{il} G_{il}
\end{align*}
\]

**FIGURE 1.** Large-scale descriptor systems.

Before presenting the main results, the following definitions and lemmas should be given for our results. According to [13], the TSFM can extend linear control theories to design controllers for nonlinear systems. Therefore, the following definitions and lemmas can be extended to each nonlinear subsystem.

**B. DEFINITIONS AND LEMMNAS**

Consider the following linear descriptor system:

\[
E \dot{x} (t) = Ax (t) + Bu (t)
\]  

(10)

**Definition 1 ([32]):**

1. The system (10) is regular if \( \text{det} (sE - A) \neq 0 \).
2. The system (10) is impulse-free if the system is regular, and \( \text{deg} [\text{det} (sE - A)] = \text{rank} (E) \).
3. The system (10) is stable if \( \sigma (E, A) \) lies in the open left half-plane and the system is regular, impulse-free, where \( \sigma (E, A) \) denotes all the roots of \( \text{det} (sE - A) = 0 \).
(4) The system (10) is admissible if the system is regular, impulse-free, and stable.

The descriptor system (10) can be rewritten as follow if there exists a PD controller $u(t) = F_d x(t) - F_d \dot{x}(t)$.

$$(E + BF_d) \dot{x}(t) = (A + BF_d)x(t)$$

(11)

**Definition 2 ([32]):** The system (10) is normal and stable, if the closed-loop system (11) meet the following requirements:

(R1) The matrix $(E + BF_d)$ is non-singular matrix.

(R2) The closed-loop system (11) is stable.

**Remark 1 ([32]):** The system (11) can be rewritten as follows if $\det((E - A)) \neq 0$ holds.

$$\dot{x}(t) = (E + BF_d)^{-1} (A + BF_d)x(t)$$

(12)

It should be noted that when $\det((E - A)) \neq 0$ holds, then the system (12) is unique and exists. In addition, the system (12) is impulse-free because the system (12) has no infinite poles. Therefore, if the above definitions are held, then the system (10) can become normal and stable through the PD control law.

**Definition 4 ([24]):** For all terminal time $t_p > 0$, the T-S LSD system is called passive with $v_i(t), y_i(t)$ if there exist the matrices $U_{i1}, U_{i2}, U_{i3}$ and satisfying the following inequality

$$2 \int_0^{t_p} y_i^T(t) U_{i1} v_i(t) dt > \int_0^{t_p} v_i^T(t) U_{i2} y_i(t) dt + \int_0^{t_p} v_i^T(t) U_{i3} v_i(t) dt$$

for all trajectories of system (8) and $\theta$ is a positive scalar.

Before using the convex optimization algorithm, it must be ensured that the stability condition needs to be in LMI form. The following lemmas will be used in the following to run to LMI conditions.

**Lemma 1 ([38]):** For every real vector $\xi$ and $\rho$:

$$2\xi^T \rho \leq \xi^T Z \xi + \rho^T Z^{-1} \rho$$

(15)

for any definite positive matrix $Z > 0$.

**Lemma 2 ([37]):** For two positive integers $r, r_0$ and any positive semidefinite symmetric matrix $G \in \mathbb{R}^{r \times r}$ satisfy $G^T = G \geq 0$ and $r \geq r_0 \geq 1$, the following inequality is established

$$\left(\sum_{k=r_0}^{r} x(k)\right)^T G \left(\sum_{k=r_0}^{r} x(k)\right) \leq \tilde{r} \sum_{k=r_0}^{r} x^T(k) G x(k)$$

(16)

where $\tilde{r} = r - r_0 + 1$.

**Lemma 3 ([15]):** Let $\mathbf{H}$ and $\mathbf{R}$ be matrices with compatible, for any matrix $\xi > 0$ with the conditions $\Delta^T(t) \Delta(t) \leq \mathbf{I}$, one can find following result.

$$\mathbf{H} \Delta(t) \mathbf{R} + \mathbf{R}^T \Delta^T(t) \mathbf{H}^T \leq \xi \mathbf{H} \mathbf{H}^T + \xi^{-1} \mathbf{R}^T \mathbf{R}$$

(17)

**III. DPD FUZZY CONTROL OF T-S LSD SYSTEM**

In this section, a DPD fuzzy controller has been designed to stabilize the T-S LSD system (1). The main result is summarized in the following theorems.

**Theorem 1:** The DPD fuzzy controller (7) exists and guarantee the T-S LSD system (8) is stable if there exist the matrices $Q_i, F_{sid}, F_{sij}, F_{sid}, F_{sid}, L_{i1}, L_{i2}, L_{i3}, U_{i1}, U_{i2} \geq 0$ and $U_{i3}$ such that for all the following conditions hold:

$$\Theta_{ijl} < 0, \text{ for } l = 1 \cdots r_i$$

(18)

$$\Theta_{ijl} + \Theta_{jil} < 0, \text{ for } l < j = 1 \cdots r_i$$

(19)

where

$$\Theta_{ijl} = \begin{bmatrix}
L_{i3} + C_{dl}^T U_{i2} C_{dl} & * & * \\
Z_{l1} & I + H e & \left(\tilde{E}_{id} + \tilde{B}_{id} F_{sid}\right) L_{i2} & * \\
X_{l31} & -G_{dl}^T & X_{l33}
\end{bmatrix}$$

$$\Theta_{jil} = \begin{bmatrix}
L_{i3} + C_{dl}^T U_{i2} C_{dl} & * & * \\
Z_{l1} & I + H e & \left(\tilde{E}_{id} + \tilde{B}_{id} F_{sid}\right) L_{i2} & * \\
X_{l31} & -G_{dl}^T & X_{l33}
\end{bmatrix}$$

$$\Phi = (N - 1) \sum_{k=1,k \neq i}^{N} x_i^T(t) \tilde{A}_{ik} \tilde{A}_{ki} x_i(t), \quad Z_{l21} = L_{i4} - \left(\tilde{A}_{il} + \tilde{B}_{il} F_{sid}\right) Q_i + \left(\tilde{E}_{il} + \tilde{B}_{il} F_{sid}\right) L_{i1},$$

$$Z_{l22} = L_{i4} - \left(\tilde{A}_{il} + \tilde{B}_{il} F_{sid}\right) Q_i + \left(\tilde{E}_{il} + \tilde{B}_{il} F_{sid}\right) L_{i1},$$

$$X_{l31} = -U_{l1}^T C_{dl} + D_{il}^T U_{i2} C_{dl}, \quad X_{l33} = U_{i3} - D_{il}^T U_{i1} - U_{i2} D_{il} + D_{il}^T U_{i2} D_{il}.$$
where the free-weighting matrices $S_{i1}$ and $S_{i2}$ are the variables.

Taking (21) into the derivative of $V(x(t))$, then

$$
\dot{V}(x(t)) = \sum_{i=1}^{N} \left( 2x_i^T(t) P_{ji} x_i(t) + O_i \right) 
$$

$$
= \sum_{i=1}^{N} \left\{ 2x_i^T(t) P_{ji} x_i(t) + 2 \left[ x_i^T(t) S_{i1} + x_i^T(t) S_{i2} \right] \times \left[ -E_i(\mu_i) \dot{x}_i + A_i(\mu_i) x_i(t) + G_i(\mu_i) v(t) \right] + \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right\} 
$$

Based on Lemma 1, the following inequalities can be obtained from (22).

$$
\sum_{i=1}^{N} \left( 2x_i^T(t) S_{i1} \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right) \leq \sum_{i=1}^{N} \left( x_i^T(t) S_{i1} S_{i1}^T x_i(t) \right)
+ \sum_{i=1}^{N} \left\{ \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \times \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \right\} \tag{23}
$$

and

$$
\sum_{i=1}^{N} \left( 2x_i^T(t) S_{i2} \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right) \leq \sum_{i=1}^{N} \left( \dot{x}_i^T(t) S_{i2} S_{i2}^T \dot{x}_i(t) \right)
+ \sum_{i=1}^{N} \left\{ \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \times \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \right\} \tag{24}
$$

The inequality $\tilde{A}_{ik}(\mu_i) \geq \tilde{A}_{ik}(\mu_i)$ can be obtained due to $\sum_{i=1}^{N} \mu_i(\xi_i(t)) = 1$ and $0 \leq \mu_i(\xi_i(t)) \leq 1$, and we have the following inequality from above.

$$
\sum_{i=1}^{N} \left( \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right) \times \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \leq \sum_{i=1}^{N} \left( \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right) \times \left( \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right) \tag{25}
$$

According to Lemma B.2 on page 328 of the [39], one can know that

$$
\sum_{i=1}^{N} \left\{ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right\} \times \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] = \sum_{i=1}^{N} \left\{ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right\} \times \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \tag{26}
$$

From [40], the following inequality can be obtained by Lemma 2 when $k \neq i$.

$$
\sum_{i=1}^{N} \left\{ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right\} \times \left[ \sum_{k=1, k\neq i}^{N} \tilde{A}_{ik}(\mu_i) x_k(t) \right] \leq \sum_{i=1}^{N} \left( N - 1 \right) \sum_{k=1, k\neq i}^{N} x_i^T(t) \tilde{A}_{ik}^T \tilde{A}_{ik} x_i(t) \right\} \tag{27}
$$

Based on the inequalities (23-27), then one can rewrite the Lyapunov function (22) as follows:

$$
\dot{V}(x(t)) \leq \sum_{i=1}^{N} \chi_i Z_i \chi_i \tag{28}
$$

where (29), as shown at the bottom of the page.

\[\begin{align*}
\dot{\chi}^T &= \left[ x_i(t) \quad \dot{x}_i(t) \quad v_i(t) \right]^T, \\
\Phi &= (N - 1) \sum_{k=1, k\neq i}^{N} \chi_i^T(t) \tilde{A}_{ki}^T \tilde{A}_{ki} x_i(t), \\
Z_i &= \begin{bmatrix}
H e(S_{i1} A_i(\mu_i)) + S_{i1} S_{i1}^T + 2\Phi \\
P_i - E_i^T(\mu_i) S_{i1}^T + S_{i2} A_i(\mu_i) \\
G_i^T(\mu_i) S_{i1}^T - H e(S_{i2} E_i(\mu_i)) + S_{i2} S_{i2}^T \\
0
\end{bmatrix}
\end{align*}\]
can write:

\[
\dot{V}(x(t)) \leq \sum_{l=1}^{r} \mu_{il}^{2} x^T \tilde{\Xi}_{ill} \tilde{x} + \sum_{l<j}^{r} \mu_{il} \mu_{lj} x^T \left[ \tilde{\Xi}_{ilj} + \tilde{\Xi}_{ljl} \right] \tilde{x}
\]

(31)

where

\[
\tilde{\Xi}_{ill} = \begin{bmatrix}
\tilde{L}_{i3} & * & * \\
Z_{i21} & I + He \left( \left( \tilde{E}_{il} + \tilde{B}_{il} \tilde{F}_{il} \right) L_{i2} \right) & * \\
0 & -G_{il}^T & 0
\end{bmatrix}
\]

(32)

\[
\tilde{\Xi}_{ilj} = \begin{bmatrix}
\tilde{L}_{i3} & * & * \\
Z_{j21} & I + He \left( \left( \tilde{E}_{il} + \tilde{B}_{il} \tilde{F}_{il} \right) L_{i2} \right) & * \\
0 & -G_{il}^T & 0
\end{bmatrix}
\]

(33)

Based on the passivity theory, one can define the cost function as follows with zero initial condition:

\[
\Phi(x, v, t) = \int_{0}^{t} \dot{V}(x(t)) \, dt = \int_{0}^{t} \left[ \dot{V}(x(t)) \right]^T \dot{V}(x(t)) \, dt
\]

(34)

where

\[
\dot{V}(x(t)) = \sum_{l=1}^{r} \mu_{il}^{2} x^T \tilde{\Xi}_{ill} \tilde{x} + \sum_{l<j}^{r} \mu_{il} \mu_{lj} x^T \left[ \tilde{\Xi}_{ilj} + \tilde{\Xi}_{ljl} \right] \tilde{x}
\]

(35)

(36)

Substituting (2b) and (31) into (36), one has

\[
\Psi(x, v, t) = \sum_{l=1}^{r} \mu_{il}^{2} x^T \tilde{\Xi}_{ill} \tilde{x} + \sum_{l<j}^{r} \mu_{il} \mu_{lj} x^T \left[ \tilde{\Xi}_{ilj} + \tilde{\Xi}_{ljl} \right] \tilde{x}
\]

(37)

where (38)–(41), as shown at the bottom of the page.

Noticed that, if \( \tilde{\Xi}_{ill} < 0 \) and \( \tilde{\Xi}_{ilj} < 0 \) that implies \( \Psi(x, v, t) < 0 \) from (37). According to (34), the inequality \( \Phi(x, v, t) < 0 \) implies

\[
\Phi(x, v, t) < 0
\]

(42)

or

\[
2 \int_{0}^{t} y_{i}^T(t) U_{i1} v_{i}(t) \, dt
\]

(43)

Thus, the system is achieved passivity constraint if the conditions (38) and (39) are satisfied. Since (43) is equivalent to (13), the closed-loop systems (8) is passive. Next, we will prove that the system is asymptotically stable. By assuming \( v(t) = 0 \), one can find the following equation from (36) with \( \Psi(x, v, t) < 0 \).

\[
\dot{V}(x(t)) < \sum_{l=1}^{r} \mu_{il}^{2} x^T \tilde{\Psi}_{ill} \tilde{x} + \sum_{l<j}^{r} \mu_{il} \mu_{lj} x^T \left[ \tilde{\Psi}_{ilj} + \tilde{\Psi}_{ljl} \right] \tilde{x}
\]

(44)
where
\[ \bar{\Psi}_{ill} = \begin{bmatrix} \dot{L}_{il3} + C_{il}^T U_{il2} C_{il} \\ Z_{il21} \end{bmatrix} \begin{bmatrix} I + H e \left( (\dot{\bar{E}}_{il} + \dot{B}_il F_{il2}) L_{il2} \right) \end{bmatrix} \]  
(45)
and
\[ \bar{\Psi}_{ijl} = \begin{bmatrix} \dot{L}_{ij3} + C_{ijl}^T U_{ij2} C_{ijl} \\ Z_{ij21} \end{bmatrix} \begin{bmatrix} I + H e \left( (\dot{\bar{E}}_{ijl} + \dot{B}_ijl F_{ij2}) L_{ij2} \right) \end{bmatrix} \]  
(46)

If \( U_{il2} \geq 0 \) is held then \( \dot{V} (t) < 0 \) is easily found from (44). Since \( \dot{V} (t) \leq 0 \), the closed-loop system (5) is asymptotically stable. The proof of Theorem 1 is complete. However, the stability conditions of Theorem 1 are BMI forms that cannot be calculated by LMI-Toolbox. In order to enable the synthesis of BMI forms that cannot be calculated by LMI-Toolbox. In order to enable the synthesis of DPD fuzzy controller in LMI framework to be possible, the conversion of stability conditions is given below.

**Theorem 2:** The DPD fuzzy controller (7) exists and guarantees the T-S LQD system (8) is stable if there exist the scalar \( \xi > 0 \) and matrices \( Q_i, K_{il3}, K_{il2}, K_{ij2}, L_{il2}, L_{il3}, U_{il1}, U_{il2} \geq 0 \) and \( U_{il3} \) such that for all the following conditions hold:
\[ \bar{\Theta}_{ill} < 0, \quad \text{for} \ l = 1 \cdots r_i \]  
(47)
\[ \bar{\Theta}_{ijl} + \bar{\Theta}_{jil} < 0, \quad \text{for} \ l < j = 1 \cdots r_i \]  
(48)

where
\[ \bar{\Theta}_{ill} = \begin{bmatrix} X_{i11} & * & * & * & * \\ X_{i21} & X_{i22} & * & * & * \\ X_{i31} & -G_{il}^T X_{i33} & * & * \\ X_{i41} & X_{i42} & 0 & -\xi & * \\ L_{i3}^T & 0 & 0 & 0 & -I/2 \\ \bar{A}_{il} Q_i & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix}, \]  
\[ \bar{\Theta}_{ijl} = \begin{bmatrix} X_{j11} & * & * & * & * \\ X_{j21} & X_{j22} & * & * & * \\ X_{j31} & -G_{jl}^T X_{j33} & * & * \\ X_{j41} & X_{j42} & 0 & -\xi & * \\ L_{jl}^T & 0 & 0 & 0 & -I/2 \\ \bar{A}_{jl} Q_j & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix}, \]  
\[ \bar{\Theta}_{jil} = \begin{bmatrix} X_{j11} & * & * & * & * \\ X_{j21} & X_{j22} & * & * & * \\ X_{j31} & -G_{jl}^T X_{j33} & * & * \\ X_{j41} & X_{j42} & 0 & -\xi & * \\ L_{jl}^T & 0 & 0 & 0 & -I/2 \\ \bar{A}_{jl} Q_j & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix}, \]  
\[ \mathbf{X}_{66} = \begin{bmatrix} X_{j11} & * & * & * & * \\ X_{j21} & X_{j22} & * & * & * \\ X_{j31} & -G_{jl}^T X_{j33} & * & * \\ X_{j41} & X_{j42} & 0 & -\xi & * \\ L_{jl}^T & 0 & 0 & 0 & -I/2 \\ \bar{A}_{jl} Q_j & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix}, \]  
(49)

If \( U_{il2} \geq 0 \) is held then \( \dot{V} (t) < 0 \) is easily found from (44). Since \( \dot{V} (t) \leq 0 \), the closed-loop system (5) is asymptotically stable. The proof of Theorem 1 is complete. However, the stability conditions of Theorem 1 are BMI forms that cannot be calculated by LMI-Toolbox. In order to enable the synthesis of BMI forms that cannot be calculated by LMI-Toolbox. In order to enable the synthesis of DPD fuzzy controller in LMI framework to be possible, the conversion of stability conditions is given below.

**Proof:** Consider the perturbations fluence the system, substituting the matrices \( \dot{\bar{E}}_{il}, \bar{A}_{il}, \bar{B}_il \) into equation (18), then the matrix can be rewritten as
\[ \Theta_{ill} = \mathbf{A}_{ill} + \bar{\Theta}_{ill} \]  
(49)
where (50)–(53), as shown at the bottom of the page.
Now, replacing the perturbations (2) and we can rewrite (49) as:
\[ \Theta_{ill} = \mathbf{A}_{ill} + \Pi_{il} \Delta (t) \Gamma_{il} + ( \Pi_{il} \Delta (t) \Gamma_{il} )^T \]  
(54)
where
\[
\Pi_{il} = \begin{bmatrix} 0 & H_{il}^T \\ H_{il} & 0 \end{bmatrix}^T
\]
(55)
\[
\Gamma_{il} = \begin{bmatrix} -R_{ii}Q_{il} + R_{ei}L_{i1} - R_{bi}K_{1il} & R_{ei}L_{i2} + R_{bi}K_{2il} \\ 0 \end{bmatrix}
\]
(56)

Based on Lemma 3, the equation (54) can be written as following if there exist scalar \( \xi > 0 \).
\[
\Theta_{ilj} \leq \mathbf{A}_{il} + \xi \Pi_{il} \Pi_{il}^T + \xi^{-1} \Gamma_{il} \Gamma_{il}
\]
(57)

Using Schur complements and substituting \( \tilde{L}_{i3} \) into the inequality (57), then
\[
\Theta_{ilj} \leq \begin{bmatrix} \mathbf{A}_{il} + \xi \Pi_{il} \Pi_{il}^T & * \\ \mathbf{X}_{i11} & * & * & * & * \\ \mathbf{X}_{i21} & \mathbf{X}_{i22} & * & * & * \\ \mathbf{X}_{i31} & -\mathbf{G}_{T_{il}} & \mathbf{X}_{i33} & * & * \\ \mathbf{X}_{i41} & \mathbf{X}_{i42} & 0 & -\xi & * \\ \mathbf{L}_{i3}^T & 0 & 0 & 0 & -I/2 * \\ \tilde{\mathbf{A}}_{i3} \mathbf{Q} \tilde{\mathbf{A}}_{i3}^T & 0 & 0 & 0 & 0 \mathbf{X}_{66} \end{bmatrix}
\]
(58)

where
\[
\mathbf{X}_{i11} = \text{He}(\mathbf{L}_{i1}) + \mathbf{C}_{il}^T \mathbf{U}_{il} \mathbf{C}_{il},
\]
\[
\mathbf{X}_{i22} = \mathbf{I} + \text{He}(\mathbf{E}_{i2} \mathbf{L}_{i2} + \mathbf{B}_{i2} \mathbf{K}_{2il}) + \xi \mathbf{H}_{il} \mathbf{H}_{il}^T,
\]
\[
\mathbf{X}_{i41} = -R_{il} \mathbf{Q}_{il} + R_{ei} \mathbf{L}_{i1} - R_{bi} \mathbf{K}_{1il},
\]
\[
\mathbf{X}_{i42} = R_{il} \mathbf{L}_{i2} + R_{bi} \mathbf{K}_{2il},
\]
\[
\mathbf{X}_{66} = -2 (N - 1)^{-1} \varepsilon,
\]
\[
\tilde{\mathbf{A}}_{ili} = \left[ \tilde{\mathbf{A}}_{ili}^{(1)}, \tilde{\mathbf{A}}_{ili}^{(2)}, \ldots, \tilde{\mathbf{A}}_{ili}^{(N)} \right]^T,
\]
\[
\varepsilon = \text{diag} \left[ \mathbf{I}_{|i|-1} \right].
\]

Similarly, the matrix \( \Theta_{ijl} \) can be rearranged as
\[
\Theta_{ijl} \leq \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} & * & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{T_{il}} & \mathbf{X}_{l33} & * & * \\ \mathbf{X}_{l41} & \mathbf{X}_{l42} & 0 & -\xi & * \\ \mathbf{L}_{l3}^T & 0 & 0 & 0 & -I/2 * \\ \tilde{\mathbf{A}}_{ijl} \mathbf{Q} \tilde{\mathbf{A}}_{ijl}^T & 0 & 0 & 0 & 0 \mathbf{X}_{66} \end{bmatrix}
\]
(59)

where
\[
\mathbf{X}_{l21} = \mathbf{L}_{il} - \mathbf{A}_{il} \mathbf{Q}_{il} + \mathbf{E}_{il} \mathbf{L}_{il} - \mathbf{B}_{il} \mathbf{K}_{1ij},
\]
\[
\mathbf{X}_{l22} = \mathbf{I} + \text{He}(\mathbf{E}_{il} \mathbf{L}_{il} + \mathbf{B}_{il} \mathbf{K}_{2il}) + \xi \mathbf{H}_{il} \mathbf{H}_{il}^T,
\]
\[
\mathbf{X}_{l41} = -R_{il} \mathbf{Q}_{il} + R_{ei} \mathbf{L}_{i1} - R_{bi} \mathbf{K}_{1il},
\]
\[
\mathbf{X}_{l42} = R_{il} \mathbf{L}_{i2} + R_{bi} \mathbf{K}_{2il},
\]
\[
\mathbf{K}_{ijl} = \mathbf{F}_{ijl} \mathbf{Q}_{il} - \mathbf{F}_{ijl} \mathbf{L}_{i1},
\]
\[
\mathbf{K}_{2ijl} = \mathbf{F}_{ijl} \mathbf{L}_{i2},
\]

It is obviously found that the conditions in Theorem 2 belong to the LMI problem that can be directly solved by MATLAB LMI Toolbox for seeking feasible solutions. To clearly find the feedback gains, the following design procedure is proposed by using Theorem 2 to find the feedback gains.

Design Procedure:
Step 1: Check the satisfactions of Definitions and Remarks for the system.
Step 2: Set up the performance matrices \( \mathbf{U}_{i1}, \mathbf{U}_{i2} \geq 0 \) and \( \mathbf{U}_{i3} \).
Step 3: Applying Theorem 2 to obtain the variables \( \xi > 0, \mathbf{Q}_{il}, \mathbf{K}_{1il}, \mathbf{K}_{2il}, \mathbf{K}_{1ijl}, \mathbf{K}_{2ijl}, \mathbf{L}_{i1}, \mathbf{L}_{i2}, \mathbf{L}_{i3} \) by using MATLAB LMI Toolbox.
Step 4: According to \( \mathbf{K}_{ijl} = \mathbf{F}_{ijl} \mathbf{Q}_{il} - \mathbf{F}_{ijl} \mathbf{L}_{i1} \) and \( \mathbf{K}_{2ijl} = \mathbf{F}_{ijl} \mathbf{L}_{i2} \), one can find the feedback gains \( \mathbf{F}_{ijl}, \mathbf{F}_{ijl}, \mathbf{F}_{ijl} \) and \( \mathbf{F}_{ijl} \). In addition, the Free-weighting matrices \( \mathbf{S}_{il} \) and \( \mathbf{S}_{il} \) also can be obtained according to \( \mathbf{L}_{i1}^T = -\mathbf{Q}_{il} \mathbf{S}_{il} \mathbf{S}_{il}^{-1} \) and \( \mathbf{L}_{i2}^T = -\mathbf{S}_{il}^{-1} \).
Step 5: Based on the gains obtained by Step 4, the corresponding controller (7) can be designed to establish the PDSF controller.

Based on the above design procedure, the DPD fuzzy controller (7) can be designed for guaranteeing the stability of the T-S LSD system (5a) subject to passivity constraint. In the next section, the stability conditions of Theorem 2 are selected to test the effectiveness and performance of the proposed control method. Two examples were considered to demonstrate the feasibility of the developed method. The first example focuses on the application of the singular system and the second example provides the control result of the descriptor system.

IV. SIMULATION
A. EXAMPLE 1
In this example, let us define the descriptor matrix as a singular matrix, then the T-S LSD system can be described with two membership functions and two interconnected subsystems as follows:

Plant Rule \( \mathbf{N}_{l}^i \): IF \( x_{i1} (t) \) is \( \mathbf{X}_{l1}^i \), THEN
\[
\tilde{\mathbf{E}}_{il} \dot{\mathbf{x}}_k (t) = \tilde{\mathbf{A}}_{il} \mathbf{x}_k (t) + \tilde{\mathbf{B}}_{il} \mathbf{u}_i (t) + \mathbf{G}_{il} \mathbf{v}_i (t) + \sum_{k=1,k\neq i}^{2} \tilde{\mathbf{A}}_{il} \mathbf{x}_k (t)
\]
(60a)
\[
\mathbf{y}_i (t) = \mathbf{C}_{il} \mathbf{x} (t) + \mathbf{D}_{il} \mathbf{v}_i (t)
\]
(60b)
where \( i \in \{1, 2\} \) and \( l \in \{1, 2\} \).

The first subsystem parameters are given as follows:
\[
\mathbf{E}_{il} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{A}_{i1} = \begin{bmatrix} 0 & 1 \\ 6.81 & 0 \end{bmatrix},
\]
\[
\mathbf{A}_{i2} = \begin{bmatrix} 0 & 1 \\ -2.8 & 0 \end{bmatrix}, \quad \mathbf{B}_{i1} = \mathbf{B}_{i2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]
\[
\tilde{\mathbf{A}}_{i2} = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}, \quad \mathbf{C}_{i1} = \mathbf{C}_{i2} = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\]
The second subsystem parameters are given as follows:

$$
\begin{align*}
D_{21} &= D_{22} = 1, \\
G_{11} &= G_{12} = 0.1.
\end{align*}
$$

The second subsystem parameters are given as follows:

$$
\begin{align*}
E_{21} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\
A_{21} &= \begin{bmatrix} 0 & -1 \\ 6.81 & 0 \end{bmatrix}, \\
A_{22} &= \begin{bmatrix} 2.3 & 1 \\ 0 & 1 \end{bmatrix}, \\
B_{21} &= \begin{bmatrix} 1 \\ 0.25 \end{bmatrix}, \\
B_{22} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
C_{21} &= C_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
D_{21} &= D_{22} = 1, \\
G_{21} &= G_{22} = 0.1.
\end{align*}
$$

The perturbations have the form of (2) can be given as follows:

$$
\begin{align*}
\Delta A_{il} &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \sin(t) \begin{bmatrix} 0.05 & 0 \end{bmatrix}, \\
\Delta B_{il} &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \sin(t) \begin{bmatrix} 0.01 & 0 \end{bmatrix}, \\
\Delta B_{il} &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \sin(t) \begin{bmatrix} 0.01 & 0 \end{bmatrix},
\end{align*}
$$

Let assumed the disturbance $v(t)$ has the following form:

$$
v_i(t) = \begin{cases} 
0.1 \sin(2t), & 0 \leq t \leq 5 \\
0, & \text{else}
\end{cases}
$$

(61)

In this example, we consider the passivity performance (14) with $\theta = 1$ and the membership functions are given in Fig. 2. The MATLAB software is used to obtain the following controller gains.

For the subsystem 1:

$$
\begin{align*}
F_{d11} &= \begin{bmatrix} -64.1815 & -29.9614 \\ 0.5848 & 0.0198 \end{bmatrix}, \\
F_{d11} &= \begin{bmatrix} -54.5619 & -29.9557 \\ 0.5845 & 0.0198 \end{bmatrix},
\end{align*}
$$

(62a)

and

For the subsystem 2:

$$
\begin{align*}
F_{d21} &= \begin{bmatrix} -1.2982 & 1.8156 \\ 0.0792 & -0.1491 \end{bmatrix}, \\
F_{d21} &= \begin{bmatrix} 1.7068 & 2.4178 \\ 0.1410 & -0.2289 \end{bmatrix},
\end{align*}
$$

(62b)

The following matrix can be obtained by using LMI-Toolbox.

For the subsystem 1:

$$
\begin{align*}
Q_1 &= \begin{bmatrix} 0.7745 & -1.4815 \\ -1.4815 & 6.5056 \end{bmatrix}, \\
L_{11} &= \begin{bmatrix} -1.3163 & -73.7101 \\ 74.1572 & -1.6586 \end{bmatrix}, \\
L_{12} &= \begin{bmatrix} -1.3584 & -0.7908 \\ 80.2561 & -66.7790 \end{bmatrix}
\end{align*}
$$

(63a)

and

For the subsystem 2:

$$
\begin{align*}
Q_2 &= \begin{bmatrix} 1.2790 & -1.4328 \\ -1.4328 & 5.9195 \end{bmatrix}, \\
L_{21} &= \begin{bmatrix} -1.4845 & 4.8164 \\ -6.4350 & -1.3437 \end{bmatrix}, \\
L_{22} &= \begin{bmatrix} -1.4877 & 1.1105 \\ 1.1373 & 8.3378 \end{bmatrix}
\end{align*}
$$

(63b)

According to $Q_i = P_i^{-1}L_i^T = -Q_i S_i S_i^{-1}$ and $L_i^T = -S_i^{-1}$, one can obtained the matrix $P_i$ and free-weighting matrices $S_i$ and $S_i$ from above matrices.

For the subsystem 1:

$$
\begin{align*}
P_i &= \begin{bmatrix} 2.2878 & 0.5210 \\ 0.5210 & 2.2878 \end{bmatrix}, \\
S_{i1} &= \begin{bmatrix} 18.8034 & 20.0705 \\ 9.1881 & 10.7406 \end{bmatrix}, \\
S_{i2} &= \begin{bmatrix} 0.4331 & 0.5205 \\ -0.0051 & 0.0088 \end{bmatrix}
\end{align*}
$$

(64a)
For the subsystem 2:

\[
\begin{align*}
\mathbf{P}_2 &= \begin{bmatrix} 1.0727 & 0.2597 \\ 0.2597 & 0.2318 \end{bmatrix} \\
\mathbf{S}_{21} &= \begin{bmatrix} -0.2238 & -0.6077 \\ -0.5690 & -0.1041 \end{bmatrix} \\
\mathbf{S}_{22} &= \begin{bmatrix} 0.6101 & -0.0832 \\ -0.0813 & -0.1089 \end{bmatrix}
\end{align*}
\]  

(64b)

The initial conditions are given as \(x_1(0) = [0.5 \ 0.2]^T\) and \(x_2(0) = [0.08 \ 0.0]^T\) for the example 1. Bringing the controller gain obtained by Theorem 2 into the system, the state responses of the LSD system are shown in Fig. 3 and Fig. 4.

The following specific values can be calculated to verify the passivity constraints (14).

\[
\begin{align*}
\text{Subsystem 1} : 2 \int_0^T \dot{y}_1^T \dot{y}_1 (t) v_1 (t) dt &= 3.1154 \quad (65a) \\
\text{Subsystem 2} : 2 \int_0^T \dot{y}_1^T \dot{y}_1 (t) v_1 (t) dt &= 1.8838 \quad (65b)
\end{align*}
\]

\[\text{Time(s)}\]

\[\text{Subsystem 2} \]

\[\text{Subsystem 1} \]

\[\text{State responses for Example 1 based on Theorem 2.} \]

\[\text{FIGURE 3. State responses for Example 1 based on Theorem 2.} \]

\[\text{FIGURE 4. State responses for Example 1 based on Theorem 2.} \]

\[\text{FIGURE 5. A block diagram for the DPD fuzzy control.} \]

\[\text{FIGURE 6. The interconnected pendulum system can be linearizing as follows:} \]

\[\text{B. EXAMPLE 2} \]

Let consider the double-inverted pendulums system shown in Fig. 5. In this simulation, the purpose here is to design a DPD fuzzy controller such that the double-inverted pendulums system is stable. According to [37], we can get the following equations for the interconnected pendulum:

\[\dot{x}_1 = x_2 \]

\[\dot{x}_2 = -\frac{kr^2}{4J_i} x_1 + \frac{kr^2}{4J_i} \sin(x_1) x_2 + \frac{2}{J_i} x_2 + \frac{1}{J_i} u_i \]

(66a)

According to the membership functions of \(x_1(t)\) shown in Fig. 6, the interconnected pendulum system can be linearizing as follows:

\[\text{Plant Rule } \mathcal{R}_1^I : \text{IF } x_1(t) \text{ is } X_1^I, \text{ THEN} \]

\[\ddot{E}_{il} x_i(t) = \tilde{A}_{il} x_i(t) + \tilde{B}_{il} u_i(t) + G_{il} v_i(t) \]

\[+ \sum_{k=1, k\neq i}^{2} \tilde{A}_{iik} x_k(t) \]

(67a)

\[v_i(t) = C_{il} x(t) + D_{il} v_i(t) \]

(67b)

where

\[\ddot{E}_{il} := E_{il} + \Delta E_{il}, \tilde{A}_{il} := A_{il} + \Delta A_{il}, \tilde{B}_{il} := B_{il} + \Delta B_{il}, \]

\(i = \{1, 2\} \) and \(l = \{1, 2\} \).
The first subsystem parameters are given as follows:

\[
E_{1l} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 0 & 1 \\ 8.81 & 0 \end{bmatrix},
\]

\[
A_{12} = \begin{bmatrix} 0 & 1 \\ 5.38 & 0 \end{bmatrix}, \quad B_{1l} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},
\]

\[
\bar{A}_{12} = \begin{bmatrix} 0 & 0 \\ 0.25 & 0 \end{bmatrix}, \quad C_{11} = C_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix},
\]

\[
D_{11} = D_{12} = 1, \quad G_{11} = G_{12} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}.
\]

The second subsystem parameters are given as follows:

\[
E_{2l} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 1 \\ 9.01 & 0 \end{bmatrix},
\]

\[
A_{22} = \begin{bmatrix} 0 & 1 \\ 5.58 & 0 \end{bmatrix}, \quad B_{2l} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},
\]

\[
\bar{A}_{21} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}, \quad C_{21} = C_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix},
\]

\[
D_{21} = D_{22} = 1, \quad G_{21} = G_{22} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}.
\]

The perturbations considered in this example can be given as follows:

\[
\Delta A_{il} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \sin(t) \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},
\]

\[
\Delta E_{il} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \sin(t) \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}
\]

and

\[
\Delta B_{il} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \sin(t) \begin{bmatrix} 0.5 \end{bmatrix}.
\]

Choose the same passivity constraints performance and disturbance form (61) from example 1, the stability conditions (47) and (48) of Theorem 2 can be solved by the LMI-Toolbox with:

For the subsystem 1:

\[
F_{d11} = \begin{bmatrix} -76.1596 & -26.6270 \\ 0.6384 & -0.1431 \end{bmatrix}
\]

\[
F_{d12} = \begin{bmatrix} -44.9131 & -15.7261 \\ 2.1381 & 0.0291 \end{bmatrix}
\]

and

For the subsystem 2:

\[
F_{s21} = \begin{bmatrix} -55.4507 & -18.3755 \\ -0.1412 & -0.4758 \end{bmatrix}
\]

\[
F_{s22} = \begin{bmatrix} -35.7546 & -13.2318 \\ 1.0977 & -0.7490 \end{bmatrix}
\]

The following matrix can be obtained by using LMI-Toolbox.

For the subsystem 1:

\[
Q_1 = \begin{bmatrix} 2.6095 & -6.0801 \\ -6.0801 & 16.7679 \end{bmatrix}
\]

\[
L_{l1} = \begin{bmatrix} -8.6273 & 3.4224 \\ 2.4728 & -16.9850 \end{bmatrix}
\]

\[
L_{l2} = \begin{bmatrix} -8.1120 & 0.2956 \\ 8.3932 & -16.1581 \end{bmatrix}
\]

and

For the subsystem 2:

\[
Q_2 = \begin{bmatrix} 1.5272 & -3.1458 \\ -3.1458 & 8.2783 \end{bmatrix}
\]

\[
L_{l21} = \begin{bmatrix} -4.4023 & -3.2705 \\ 5.5905 & -7.8629 \end{bmatrix}
\]

\[
L_{l22} = \begin{bmatrix} -4.5911 & -3.5190 \\ 9.1307 & -9.4884 \end{bmatrix}
\]

According to \(Q_i = P_i^{-1}L_i^T = -Q_iS_iS_i^{-1}\) and \(L_i^T = -S_i^{-1}\), one can obtained the matrix \(P_i\) and free-weighting
matrices $S_{11}$ and $S_{12}$ from above matrices.

For the subsystem 1:

$$
P_1 = \begin{bmatrix} 2.4703 & 0.8957 \\ 0.8957 & 0.3844 \end{bmatrix}$$

$$
S_{11} = \begin{bmatrix} 2.3136 & 1.7653 \\ 0.8156 & 0.6907 \end{bmatrix}$$

$$
S_{12} = \begin{bmatrix} 0.1257 & 0.0653 \\ 0.0023 & 0.0631 \end{bmatrix}
$$

and

For the subsystem 2:

$$
P_2 = \begin{bmatrix} 3.0140 & 1.1453 \\ 1.1453 & 0.5560 \end{bmatrix}$$

$$
S_{21} = \begin{bmatrix} 2.4975 & 1.5766 \\ 0.9544 & 0.7044 \end{bmatrix}$$

$$
S_{22} = \begin{bmatrix} 0.1254 & 0.1206 \\ -0.0465 & 0.0607 \end{bmatrix}
$$

In order to show the advantage and effectiveness of the proposed controller design method, the proposed method has been compared with the previous controller designed method developed in [37]. The following controller can be designed from Theorem 3 of [37].

Rule 1:

IF $x_{i1} (t)$ is about 0 THEN

$$
u(t) = K_{il} \tilde{y}_i (t)$$

(71a)

Rule 2:

IF $x_{i1} (t)$ is about $\pm 88^\circ$ THEN

$$
u(t) = K_{il} \tilde{y}_i (t)$$

(71b)

where $\tilde{y}_i (t) = \tilde{C}_{il} x_i (t)$ and $\tilde{C}_{il} = [1 \ 1].$

The feedback gain solutions can be obtained by solving the conditions of Theorem 3 in [37] as follows:

$$
\begin{align*}
K_{11} &= -92.7592 \\
K_{12} &= -92.7251 \\
K_{21} &= -84.4707 \\
K_{22} &= -82.6375
\end{align*}
$$

(72a)

The initial conditions are defined as $x_1 (0) = [1.2 \ 0]^T$ and $x_2 (0) = [0.8 \ 0]^T$. The comparison of state responses of Example 2 are shown in Fig. 7 to Fig. 10.

To ensure that the closed-loop system satisfies the passive constraints, some results are given as follows:

Subsystem 1:

$$
\frac{2 \int_0^{t_p} y_1^T (t) v_1 (t) \, dt}{\int_0^{t_p} v_1^T (t) v_1 (t) \, dt} = 4.6039
$$

(73a)
According to the simulation results, we can note that all states converge to zero, which means the proposed control method can effectively control the descriptor system and singular system. Compared with the fuzzy control method proposed in [37], the DPD fuzzy controller proposed in this paper can make the LSD system have a shorter settling time. The passivity constraint can be used to successfully inhibit the external noise by using the proposed DPD fuzzy controller. Via the proposed DPD fuzzy controller design method, the T-S LSD systems can be controlled to satisfy stability, robust and passivity constraints, simultaneously.

V. CONCLUSION

In this paper, a DPD fuzzy controller has been designed for the stabilization of nonlinear LSD systems. In order to discuss the descriptor system more completely, we additionally consider the transformation of the descriptor system into a singular system and give relevant definitions to solve the problems of the singular system. Considering that the system is affected by external disturbance, the passive performance has been chosen to design the controller, which can effectively suppress disturbance interference. By using the Lyapunov functional, sufficient conditions were derived with the PD feedback method, robust constraint, and passive performance. Then the LMI method can be used to solve these sufficient conditions and obtain the controller gains for the LSD systems. Finally, two examples are given to illustrate the importance and applicability of PD feedback to the descriptor system. Considering that time delay is very common in the system, the PD fuzzy control of the T-S LSD system with time delay will be investigated in future work.

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