The ANC of $^{16}$O subthreshold states from $^{12}$C($^6$Li,d) reaction at energies near the barrier

Sucheta Adhikari *, Chinmay Basu

Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata-64, India

Abstract

Sub-Coulomb transfer reactions are being used in the ANC method [1] to determine the astrophysical S-factor of capture reactions taking part in the nucleosynthesis process. This method is more important for reactions where direct measurement of capture cross-section is difficult due to very low cross-section. The ANC method relies on the peripheral nature of the reaction process that makes the calculations free from the geometrical parameters $(R,a)$ of the binding potential of the nucleus of interest. Moreover the reaction if performed at sub-Coulomb energies is expected to remove the dependence on the entrance and exit channel potentials. The $^{12}$C($\alpha,\gamma$) reaction is an important astrophysical reaction that determines the ratio of $^{16}$O to $^{12}$C at the end of helium burning in stars. Alpha transfer reactions $^{12}$C($^7$Li,t) and $^{12}$C($^6$Li,d) at sub-Coulomb energies have been carried out by Brune et al. [2] to determine the astrophysical S-factor for the $^{12}$C($\alpha,\gamma$) reaction at 300 keV. This alpha capture reaction essentially proceeds through two subthreshold states of $^{16}$O at 6.92 ($2^+$) and 7.12 ($1^-$) MeV. ANC of these two states of $^{16}$O were determined in [2] from a measurement of the total transfer cross-section. However, a more complete picture of the extent of agreement of the theoretical calculation with respect to the measured cross-section is obtained from the angular distributions. Though the ANC at sub-Coulomb energies for $^{13}$C($\alpha,n$) [3] and $^{14}$C($\alpha,\gamma$) [4] reaction has been extracted from transfer angular distribution, there is no such work for the $^{12}$C($\alpha,\gamma$) case.

In this work, we explore for the first time the angular distribution method for this reaction. Unfortunately, there is no sub-Coulomb measurements of the deuteron and triton angular distributions for either $^{12}$C($^6$Li,d) or $^{12}$C($^7$Li,t) reaction. The only existing data is the $^{12}$C($^6$Li,d) reaction at $E(^6\text{Li}) = 9$ MeV ($E_{cm} = 6$ MeV) [5] near the Coulomb barrier. The motivation of the present work is to determine the ANC of the $1^-$ and $2^+$ states from this near barrier data and to investigate their dependence on the nuclear potentials involved in the calculations. The ANCs were extracted by comparison of this near barrier data with finite range DWBA theory. The reaction at this energy is found to be highly peripheral in comparison to that at above barrier energies [6]. Effect of the entrance and exit channel potentials on the angular distribution is found to be consistently small at the grazing angle where the ANCs are extracted.

The alpha spectroscopic factor ($S_\alpha$) of a nuclear state can be extracted from alpha transfer reaction by a normalization of the experimental data with the theoretical cross-section. Thus for the $^{12}$C($^6$Li,d) reaction $S_\alpha$ is given by [7]

$$\frac{d\sigma}{d\Omega}\text{ exp} = S_1 S_\alpha \frac{d\sigma}{d\Omega}\text{ theo}$$

(1)

where $S_1$ is the spectroscopic factor for the $\alpha + d$ configuration of the $^6\text{Li}$ ground state and $S_\alpha$ is the $\alpha + ^{12}\text{C}$ spectroscopic factor for a state of $^{16}$O. The square of the ANC ($C^2$) of a particular state is related to the alpha spectroscopic factor ($S_\alpha$) via the single particle ANC $b$ as

$$C^2 = S_\alpha b^2$$

(2)

The single particle ANC $b$ is the normalization of the bound state wave function of $^{16}$O at large radii with respect to the Whittaker function. In this work, we calculate the $\alpha$-transfer angular distributions using the code FRESCO (version fres2.4) [8] in the framework.
of the FRDWBA theory. The code requires (i) the entrance channel 
\((^6\text{Li} + ^{12}\text{C})\) potential, (ii) the exit channel \((d + ^{16}\text{O})\) potential, (iii) the core–core \((d + ^{12}\text{C})\) potential and the (iv) \(\alpha + d\) and (v) \(\alpha + ^{12}\text{C}\) binding potentials respectively for \(^6\text{Li}\) and \(^{16}\text{O}\). The binding potentials for \(^6\text{Li}\) and \(^{16}\text{O}\) are obtained from \([9]\) and \([10]\) respectively.

The entrance channel potential are chosen from \([5]\) as they were obtained in the same experiment in which transfer angular distributions were measured. The exit channel potentials are those mentioned in \([5]\). In Fig. 1(a) we show the Finite Range DWBA calculations using the three sets of entrance channel potentials as given in \([5]\) with the core–core potential a global deuteron potential \([11]\). The calculations explain the observed data at the forward angles with an underprediction at backward angles due to contribution from compound nuclear process. The compound nuclear (CN) contribution is calculated using the Hauser–Feshbach code CINDY \([12]\) and is shown by dashed line in Fig. 1. The effect of the CN process on the direct calculation is insignificant at forward angles. The alpha spectroscopic factors of the \(1^−\) and \(2^+\) states are extracted by a normalization of the experimental angular distributions (within 40 degrees in Fig. 1(b)) in terms of the calculated values. We used the spectroscopic factor of \(^6\text{Li}\) to be 0.8 ± 0.1 as given in \([6]\). This value of \(S_1\) alongwith the \((\alpha + d)\) binding potential of \(^6\text{Li}\) \([9]\) gives the square of the ANC to be 5.14 ± 0.64 fm\(^−1\) [6]. This value is also close to the value used in \([2]\). Since the experimental data is a sum of the deuteron angular distributions populating the \(1^−\) and \(2^+\) states of \(^{16}\text{O}\) we re-write Eq. (1) for the present case as

\[
\frac{d\sigma}{d\Omega}^{6.92+7.12}_{\text{expt}} = S_1 \left( S^{6.92}_\alpha \frac{d\sigma}{d\Omega}^{6.92}_{\text{theo}} + S^{7.12}_\alpha \frac{d\sigma}{d\Omega}^{7.12}_{\text{theo}} \right)
\]

where \(\frac{d\sigma}{d\Omega}^{6.92+7.12}_{\text{expt}}\) represents the summed angular distribution data, \(S^{i}_\alpha\) and \(d\sigma^{i}_{\text{theo}}\) denotes respectively the alpha spectroscopic factors and the theoretical (FRDWBA) calculations for the state \(i\) (\(i = 6.92\) or 7.12 MeV state of \(^{16}\text{O}\)). The two \(S_\alpha\) values are obtained by fitting the experimental data in terms of the calculated values and adopting a \(\chi^2\) minimization process. The separate \(S_\alpha\) values depend upon the shapes of the theoretical curves that introduce uncertainties from different sources. The angular distribution of the 7.12 MeV state has a falling trend after 15° whereas the 6.92 MeV state shows a rising trend up to 24° (Fig. 2). Due to this difference in shape the \(\chi^2\) minimization yields the set of \(S_\alpha\) values for the two states. The shape of the theoretical angular distribution depends sensitively on the entrance and exit channel optical potentials and also contribution from compound nuclear process. Besides uncertainties arise from the errors in the measured cross-sections. The ANCs (C) are obtained using the relation in Eq. (2). The single particle ANC (b) are evaluated by fitting the bound state wavefunctions of the two states by a Whittaker function at large radii. In Fig. 2 we show the calculated angular distributions for each state with two choices of radial nodes for the 7.12 MeV state. In all other calculations we choose \((2,1)\) configuration for the 7.12 MeV state because in this configuration the relative behaviour of the three sets are almost similar as for the 6.92 MeV state and also the extracted ANC do not depend on the choice of radial node. In order to apply the ANC method, the foremost criterion is the peripheral nature of the reaction and to test this we varied the single particle ANC (by a variation of the geometrical parameters of the binding potential). The extracted alpha spectroscopic factors, \(S_\alpha\) and its variation with respect to \(b\) is shown in Fig. 3(a) and (b). The corresponding ANC for the two states are shown in Fig. 4(a) and (b). The ANC shows a very small variation with respect to a variation of the single particle ANC b, a feature exhibited by a peripheral reaction [7]. However, the dependence of the ANC on the entrance and exit channel potentials remain.

Brune et al. \([2]\) suggested a measurement of total transfer cross-section at deep sub-Coulomb energies in order to avoid the po-
tential dependence in the entrance and exit channels. At these energies it was suggested that the observed cross-sections can be explained with a pure Coulomb (nuclear potential switched off) potential. At energies not sufficiently below the Coulomb barrier Johnson et al. [3] noted the effects of the uncertain nuclear potential. Their calculations with only Coulomb interaction differ from the observed angular distribution by 40%. To understand this aspect we as a representative case calculated the total transfer cross-section using the code FRESCO for the 7.12 MeV state of $^{16}$O using the $^{12}$C($^6$Li,d) reaction at $E(^6$Li) = 2.7–7.0 MeV as in [2]. The calculations are shown in Fig. 5(a) for two cases one with a Coulomb plus nuclear interaction and second with the nuclear interaction switched off. The two calculations agree except at energies above 5 MeV. The deuteron angular distributions were calculated with and without the nuclear potential and are found to be similar at deep sub Coulomb energies. However the angular distributions with and without the nuclear potential at 4 MeV (Fig. 5(b)) shows a difference (10% at backward angles and 24% at forward angles) though the total transfer cross-sections are similar at this energy. Therefore in addition to the total transfer cross-section the angular distribution data should also be analyzed for a more complete extraction of the ANCs. In the present case since the reaction energy is near but above the barrier it is expected that there will be nuclear effects particularly at backward angles.

The errors involved in the evaluated ANCs in this work using near barrier angular distribution data have been also analyzed. It is observed that the uncertainty from the entrance channel potential in C is reduced to 8% for 6.92 MeV state and 13% for the 7.12 MeV state if the normalization is performed at the grazing angle (22 degrees). The sensitivity of our calculation on the exit channel ($d + ^{16}$O) optical potential was also examined using three sets of potential from [13] at 5 MeV (typical energy of deuteron for the two excited states of $^{16}$O in the transfer reaction) and shown in Fig. 6. The uncertainty in C from the variation in the exit channel potentials [5,13] at the grazing angle for the 7.12 and 6.92 MeV states are respectively 6% and 10%. The dependence on the core-core ($d + ^{12}$C) potential variation is almost negligible at forward angles as shown in Fig. 7 where the potentials are adopted from [11] and [14]. Besides the contribution to the error in C from the compound nuclear process is 10.8% for 6.92 and 19.5% for 7.12 MeV state. The uncertainty due to the experimental errors is 10–20%. The error in the ANC (C) of $^6$Li is estimated to be about 6.25%. Adding these errors in quadrature the error in C for the two states are about 20% for the 2$^+$ state and 27% for the 1$^-$ state assuming...
Table 1
Comparison of ANC from present work with earlier works.

| State (MeV) | ANC (fm^{-1/2}) |
|------------|-----------------|
|            | Brune [2]       | Belhout [6] |
| 6.92 (3, 2) | 1.136 ± 0.1 x 10^5 | 3.445 ± 0.5 x 10^5 |
| 7.12 (2, 1) | 2.08 ± 0.2 x 10^14 | 5.068 ± 0.6 x 10^14 |
| 7.12 (4, 1) | 1.702 ± 0.46 x 10^15 | 1.996 ± 0.54 x 10^15 |

Fig. 6. The sensitivity of the exit channel potential on the angular distribution of 7.12 MeV state and 6.92 MeV states. In these calculations the entrance channel potential is set B of [5] and EP1, EP2 and EP3 denotes the three sets of d + ^{16}O potential at 5 MeV [13].

10% experimental error. The above error analysis assume that the normalization can be performed at the grazing angle, which is not possible with the presently available data. There is an additional systematic error resulting from the separation of the 1^- and 2^+ contributions which is likely to be significant, particularly for the 1^- contribution.

In Table 1 we show the extracted ANC in this work and those extracted by Brune et al. [2] from sub-Coulomb total cross-section measurement and Belhout et al. [6] from above barrier angular distribution measurement. The ANC for the 2^+ state is in agreement with the other works whereas the ANC of the 1^- state deviates by an order of magnitude (most likely due to the problem of separating 1^-/2^+ contribution in the present analysis). Since the data analyzed in the present work is of limited quality it is desirable to have additional deuteron angular distribution data for the 2^+ and 1^- states separately for ^{12}C(6Li,d) reaction at lower energies in order to extract the ANC more accurately.

We have calculated using a near barrier ^{12}C(6Li,d) angular distribution the ANC of two subthreshold states of ^{16}O. These two states play crucial role in the ^{12}C(\alpha, \gamma) capture reaction. The present calculation shows that if the normalization is done at the grazing angle the uncertainty from the optical potentials can be reduced. The analysis of the angular distribution in the present work instead of the total cross-section provides additional information. Since the data analyzed in the present work are not separated for the two states of ^{16}O, improved measurement of the ^{12}C(6Li,d) angular distribution near the barrier is required for a more accurate extraction of the ANCs.

References
[1] R.E. Tribble, Nucl. Instr. Meths. Phys. Research B 241 (2005) 204.
[2] C. Brune, et al., Phys. Rev. Lett. 83 (1999) 4025.
[3] E.J. Johnson, et al., Phys. Rev. Lett. 97 (2006) 192701.
[4] E.J. Johnson, et al., Phys. Rev. C 80 (2009) 045805.
[5] D.J. Johnson, M.A. Waggoner, Phys. Rev. C 1 (1970) 41.
[6] A. Belhout, et al., Nucl. Phys. A 793 (2007) 178.
[7] L.J. Thompson, F.M. Nunes, Nuclear Reactions for Astrophysics, Cambridge University Press, 2009.
[8] J.L. Tribble, Comput. Phys. Rep. 7 (1988) 167.
[9] K.J. Kubo, M. Hirata, Nucl. Phys. A 187 (1972) 186.
[10] C.A. Bertulani, Phys. Rev. C 49 (1994) 2686.
[11] H. An, C. Cai, Phys. Rev. C 73 (2006) 054605.
[12] E. Sheldon, V.C. Rogers, Comput. Phys. Commun. 6 (1973) 99.
[13] N.E. Davisson, et al., Can. J. Phys. 48 (1970) 2235.
[14] H. Wilsch, G. Clausnitzer, Nucl. Phys. A 160 (1971) 609.