Spin Transport in Cold Fermi gases: A Pseudogap Interpretation of Spin Diffusion Experiments at Unitarity

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(Dated: January 11, 2013)

We address recent spin transport experiments in ultracold unitary Fermi gases. We provide a theoretical understanding for how the measured temperature dependence of the spin diffusivity at low $T$ can disagree with the expected behavior of a Fermi liquid (FL) while the spin susceptibility (following the experimental protocols) is consistent with a Fermi liquid picture. We show that the experimental protocols for extracting $\chi_s$ are based on a FL presumption; relaxing this leads to consistency within (but not proof of) a pseudogap-based approach. Our transport calculations yield insight into the measured strong suppression of the spin diffusion constant at lower $T$.

PACS numbers: 03.75.Ss, 67.10.Jn, 67.85.De

Recent measurements associated with mass transport [1] and spin transport [2] in the ultracold Fermi gases and superfluids are of great interest principally because they provide detailed information about the excitation spectrum, thereby strongly constraining microscopic theories. Equally important are their widespread implications for a host of different strongly correlated systems ranging from quark-gluon plasmas to the high $T_c$ cuprates [3]. Common to these materials are the short mean free paths which lead to “near-perfect fluidity” [1], “bad metallicity” [1] and now bad spin conductivity [2]. These latter are Fermi-gas-based materials with small spin diffusivities $D_s$ that approach the quantum limited value $\hbar/\tau$ as the temperature is lowered. However, the nature of the excitations in these Fermi gases is currently under debate [3, 4] and a controversy has emerged as to whether the associated normal state of these superfluids is a Fermi liquid or contains an excitation (pseudo)gap. Moreover, there are mixed features in recent spin transport experiments [2] which are indicative of both pairing and Fermi liquid theory.

The goal of this paper is to address these spin transport experiments by Sommer et al. [2] which measure the spin diffusivity $D_s$, and the spin conductivity $\sigma_s$, and thereby deduce the spin susceptibility $\chi_s = \sigma_s/D_s$, as functions of temperature at unitarity. Our calculations, based on a pseudogap approach to BCS-BEC crossover theory [7], have the aim of resolving apparent discrepancies between interpretations of $\chi_s$, $D_s$ and $\sigma_s$. It should be stressed, however, that we focus on lower $T$ (below the pairing onset $T^*$) than in Ref. 2 where one can fully contrast Fermi liquid and pseudogap theories. Indeed, if one is to sort out whether the normal state of a unitary gas has a pseudogap or is a Fermi liquid, it is essential to address these spin (as well as mass) transport experiments using multiple theoretical frameworks.

Our key observation is that one must differentiate between the spin susceptibility as computed assuming a constant number of carriers in $\sigma_s$ and one which is calculated assuming the number of carriers in $\sigma_s$ increases with $T$ reflecting the presence of a pairing gap. For the latter case one recovers a consistent interpretation of both $\chi_s$ and $D_s$, in contrast to the former Fermi liquid approach, which leads to this mixed interpretation.

The physics of transport in the unitary gas is complicated because, if a pseudogap is present in the normal state, one should accomodate both fermions and fermionic pairs. In recent papers we have included both types of quasi-particles in addressing [8] the observed anomalously low shear viscosity [1] both above and below $T_c$, as well as magnetic and non-magnetic Bragg scattering [2]. While previous studies of spin transport in the cold Fermi gases have employed, for example, Quantum Monte Carlo, Boltzmann transport and variational approaches [10, 11], we choose to use the Kubo formalism which more readily addresses conservation laws and sum rules. The underlying theory is a BCS-BEC crossover theory in a t-matrix approximation [7]. At $T = 0$, the system is in the BCS-Leggett ground state where there are only superconducting pairs (characterized by the gap $\Delta_{sc}$). For temperatures $0 < T \leq T_c$, there is a comixture of superconducting pairs, noncondensed pairs (characterized by the gap $\Delta_{pg}$), and fermionic quasiparticle excitations.

Essential to satisfying conservation laws and sum rules is the incorporation of Ward identities and collective mode physics. (The latter applies only to the mass transport or “electromagnetic” response.) These issues are
addressed in Refs. 2, 13. The presence of both condensed and non-condensed pairs in transport leads to the usual Maki-Thompson (MT) and Aslamazov-Larkin (AL) diagrams, which are demonstrably consistent with gauge invariance. A different diagram subset necessar-
ily appears in the spin response, as compared with the
mass-transport (or electromagnetic response); in the for-
ging the AL diagrams are not present, nor do collective
masses (or phononic Goldstone bosons) enter [9, 14]. The
AL diagrams are demonstrated assuming a temperature independent effective number of carriers and normalized by the value at \( T_c \). At low \( T \), Figs. (a), (b) and (c) can be directly compared to Figs. 2, 3 and 4a respectively in Ref. 2.

\[
\Sigma(p, i\omega_n) = -i\gamma + \frac{\Delta_{pg}^2}{i\omega_n + \xi_p + i\gamma} + \frac{\Delta_{sc}^2}{i\omega_n + \xi_p}
\]

where \( \xi_p \) is the free quasiparticle dispersion and \( \gamma \) is the fermionic inverse-lifetime associated with the conversion from fermions to non-condensed pairs. This general form for \( \Sigma \) in Eq (1) has been applied in both experimental [17] and theoretical radio frequency (RF) studies of the cold Fermi gases. In the weak dissipation limit where \( \gamma \) is small, there is little distinction between condensed and non-condensed pairs, while in the strong dissipation limit, this distinction is enforced. In this paper we find that these two approaches tend to converge for \( s \)-wave pairing and the lifetime parameters obtained from RF experiments [12]. For reasons of transparency, for the most part the equations we present are in the weak dissipation limit.

We write the key equations first for the dc spin conductivity in the more general strong dissipation limit and second for the the spin-spin correlation function at general \((\mathbf{q}, \omega)\) (called \(Q_{00}(\mathbf{q}, \omega)\)) but in the limit in which the non-condensed pair lifetime is relatively long:

\[
\sigma_s = -\lim_{\omega \to 0} \lim_{q \to 0} \frac{1}{6m^2\omega} \text{Im} \sum_p p^2 \left[ G_p + G_p - F_{sc,p} + F_{sc,p} - F_{pg,p} + F_{pg,p} - F_{pg,p} - F_{pg,p} \right]_{\omega \to \omega^+}, \tag{2}
\]

\[
Q_{00}(\mathbf{q}, \omega) = \sum_p \left[ \frac{E_p^+ E_p^- E_p^+ E_p^- - \xi_p^+ \xi_p^- - \Delta_{sc}^2 - \Delta_{pg}^2}{\omega^2 - (E_p^+ + E_p^-)^2} (1 - f(E_p^+) - f(E_p^-)) - \frac{E_p^+ E_p^- \xi_p^+ \xi_p^- + \Delta_{sc}^2 + \Delta_{pg}^2}{\omega^2 - (E_p^+ - E_p^-)^2} (f(E_p^+) - f(E_p^-)) \right], \tag{3}
\]

where \( P^+ = (p + q, i\omega_n + i\Omega_l), P = (p, i\omega_n), \) and \( \omega^+ = \omega + i0^+ \). The quantity \( i\omega_n \) (\( i\Omega_l \)) is a fermionic (bosonic) Matsubara frequency. The excitation energy is \( E_p = \sqrt{\xi_p^2 + \Delta^2} \) where \( \Delta = \sqrt{\Delta_{sc}^2 + \Delta_{pg}^2} \). The superscripts \( \pm \) on \( \xi_p \) and \( E_p \) indicate the momentum argument \( p \pm \frac{\mathbf{q}}{2} \). Here \( f \) is the Fermi function and \( G = G(\Sigma) \) is the dressed Green’s function. Note that while one can interpret \( F_{sc} \) as the usual Gor’kov Greens function reflecting superconducting order, there must also be a counterpart \( F_{pg} \) (discussed in detail elsewhere [8] which reflects non-condensed pairs). Interestingly, in the weak dissipation
limit the spin transport correlation functions depend only on the total pairing gap. When pg effects are dropped, these equations reduce to their usual BCS counterparts.

Any consistent theory of the spin transport must be compared with the f-sum rule

$$\int_{-\infty}^{\infty} d\omega \chi''(q, \omega)/\pi = nq^2/m$$

(4)

where \(\chi''(q, \omega) = -\text{Im}Q''_{00}/\pi\) and we have verified from Eq. (3) that this can be proved analytically.

In the weak dissipation limit, it follows from Eq (2) and Ref 9 that the spin conductivity \(\sigma_s = -\lim_{\omega \to 0q \to 0} \text{Im} Q''_{00}(q, \omega)/\pi\). Importantly, it is common to interpret the spin conductivity in terms of an effective carrier number \([n/m(T)]_{\text{eff}}\)

$$\sigma_s = \left[\frac{n}{m(T)}\right]_{\text{eff}} \tau_s(T) = \frac{2}{3} \tau_s \sum_{p} v_{p}^2 \left(-\frac{\partial f_{p}}{\partial E_{p}}\right)$$

(5)

where \(\tau_s = 1/\gamma\) which can be directly associated with \(1/\tau_s\), the spin drag relaxation time. Here \(v_p = \partial E_p/\partial p\).

The microscopic Kubo calculation presented here leads to an identification between the spin-drag lifetime, \(\tau_s\) and the quasi-particle lifetime \(\gamma^{-1}\), since spin and “charge” or mass are carried by the same quasi-particles. This identification was also observed in previous Kubo calculations of spin diffusion in Helium-3 [19]. That the interconversion between fermions and bosons is the physical origin of the lifetime leads us to speculate that \(\tau_s\) is smallest for the unitary gas [3] where the number of bosonic pairs and fermions are roughly comparable. This appears consistent with the findings in Ref. 2. The spin susceptibility is \(\chi^{\text{pd}}_s = 2\sum_p \left(-\frac{\partial f_{p}}{\partial E_{p}}\right)\). Alternatively, from the small \(\omega\) and \(q\) hydrodynamics, it is seen that \(\chi_s = \sigma_s/D_s\). [20]. This latter approach is used in recent experiments [2].

We now turn to these experiments. In Fig 1(a), the values of \(\tau^s_s\) inferred from RF data [12] are plotted in units \(E_F\) [8]. The lifetime increases for lower temperatures since inter-conversion processes cease when noncondensed bosons disappear; in this way fermions become long lived. At the lowest \(T\) considered, this figure shows similar trends to Fig 2 in Ref. [2]. Here one associates \(\Gamma_{sd}\) with \(\tau^-_s\)

The microscopically computed spin diffusivity \(D_s\) is shown in Fig 1(b). Importantly we find the spin diffusivity is suppressed at low temperatures reflecting the suppression in the spin conductivity \(\sigma_s\) as a result of the reduced number of carriers. As in experiment, \(D_s\) is theoretically found to decrease and (at the lower \(T\)) this figure shows similar trends to Fig. 3 in Ref. [2]. We estimate \(D_s\) at \(T_c\) is \(0.61 h/m\), which when corrected by a factor of \(5.3\), estimated to account for trap effects [2], is in reasonable agreement. As \(T \to 0\) we cannot rule out an upturn in \(D_s\), depending on \(\tau_s(T)\).

The protocol used to deduce the spin susceptibility in Ref. 2 is based on

$$\chi^{FL}_s = \sigma_s/D_s \equiv \frac{n}{m} \tau_s/D_s \frac{n}{m} = \text{constant}$$

(6)

We use the superscript \(FL\) to emphasize that this analysis builds in a Fermi liquid interpretation by assuming that the carrier number is a constant in temperature and that no excitation gap is present. Qualitatively similar, low \(T\) experimental behavior is shown in Fig. 4a of Ref. 2. Nevertheless, this agreement between theory and experiment should not be assumed to support a Fermi-liquid based interpretation— but rather to establish internal consistency. [If the behavior for \(\sigma_s\) is assumed to be Fermi liquid like, the computed \(\chi_s\) will also be so]. As in experiment, \(\chi^{FL}_s\) increases for low temperature and appears incompatible with a pairing theory since the spin susceptibility does not vanish for low temperatures.

This will be contrasted with the microscopic calculation of the spin susceptibility, based on \(\sigma_s\) (Eq. (5)), within a pseudogap formulation, where \((n/m)_{\text{eff}}\) is strongly temperature dependent. Plotted in Fig 2(a) is this effective carrier number \((n/m)_{\text{eff}}\) as a function of temperature while the inset indicates the behavior for \(\sigma_s\), computed from Eq. (2). Both \(\sigma_s\) and \((n/m)_{\text{eff}}\) vanish with decreasing temperature due to the pairing gap.

Our calculation of the spin susceptibility \(\chi^{\text{pd}}_s = \sigma_s/D_s\) is shown in Fig 2(b). This has the expected temperature dependence associated with a pseudogap. Our key observation here is that one must differentiate between

Figure 2: (a) The effective carrier number \((n/m)_{\text{eff}}\) for a homogenous unitary Fermi gas in our microscopic theory normalized by the value at \(T_c\). It vanishes at low temperature because of the pairing gap \(\Delta\). (a) Inset: The spin conductivity \(\sigma_s\) for a homogenous unitary Fermi gas in the microscopic theory normalized by the value at \(T_c\). (b) The spin susceptibility \(\chi^{\text{pd}}_s\) for a homogenous unitary Fermi gas in our microscopic theory normalized by the \(T_c\) value.
the spin susceptibility as computed assuming a constant number of carriers in $\sigma_s$ and one which is calculated assuming the number of carriers in $\sigma_s$ increases with $T$ reflecting the presence of pairing. For the latter case one recovers a consistent interpretation of both $\chi_s$ and $D_s$, in contrast to the former Fermi liquid approach.

We have not, in this paper proved one way or the other whether there is strong evidence for a pseudogap associated with the experimental results presented in [2]. What we have established is that the indications from the spin susceptibility, which are used to support Fermi liquid theory are based on Eq. (6) which is associated with a Fermi liquid like behavior. This does not establish the absence of a pseudogap. Other claims for this absence, based on thermodynamics of a balanced unitary gas [3], can be countered by noting that the same thermodynamic power fits can be found for a non-Fermi liquid (for example in BCS theory below $T_c$). Interpretation of recent experiments [4] which address the low $T$ normal phase associated with a Fermi gas of arbitrary imbalance also fall in this category. The only theoretical phase diagram [21] (of which we aware)– addressing where pseudogap and Fermi liquid phases are stable– predicts this low $T$ phase should be Fermi liquid, as observed [6].

A suggestion for establishing how to rule in or rule out a pseudogap in the normal phase of the unitary gas is discussed in Ref. 22. Essential is that one first confirm the known characteristics of the superfluid phase, such as a suppressed spin susceptibility or entropy, and then establish that these features vary rather smoothly persisting somewhat above the transition. These observations are the counterpart of those first applied to the cuprates and recent cold gas experiments [24] have suggested new techniques for measuring these potential spin susceptibility suppressions.

Of great interest is the relation between spin and mass transport. Because spin and “charge” or mass are carried by the same quasi-particles, even in the presence of a pseudogap [9], spin and mass transport behave similarly. The analysis applied in this paper was used to anticipate that the anomalously low shear viscosity [8] of the normal state should persist down to $T \approx 0$, as now observed [1]. As in Ref. 24 we find the excitation gap is responsible for this behavior. It would similarly explain bad metallicity [4] in the pseudogapped high $T_c$ superconductors [3]. It is striking that experiments from the high $T_c$ community [23] have now tended to focus on the temperature dependence of the effective number of carriers (in the presence of a pseudogap) and note that it will affect transport “because $n_{\text{eff}}$ may be changing with $T$. These commonalities highlight the importance of the ultracold gases as a powerful simulation tool for a wide class of condensed matter systems.

In summary, in this paper we presented a theory of spin transport in a non Fermi liquid (FL) scenario and have shown that these experiments do not provide evidence against a pseudogap. Following the experimental protocols for extracting the spin susceptibility we find that $\chi_s$ appears FL like, but we emphasize this reflects a FL-starting point (presuming $T$-independent carrier number in $\sigma_s$) in the protocols. As in experiment, at lower $T$ we find that $D_s$ is suppressed. Moreover $\sigma_s$ is even more so, as both reflect the suppressed carrier number due to pseudogap effects.

This work is supported by NSF-MRSEC Grant 0820054. C.C.C. acknowledges the support of the U.S. Department of Energy through the LANL/LDRD Program.

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