Cosmic Acceleration in a Model of Interacting Massive Gravitons Dark Energy

E. Koorambas

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Abstract We propose an alternative understanding of gravity, resulting from the extension of N. Wu’s gauge theory of gravity with massive gravitons, which are minimally coupled to massless gravitons. Based on this, we derive the equations of state for massive gravitons. We study the dynamics of these massive gravitons in a flat, homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe. We calculate the critical points of the massive graviton dark energy interacting with background perfect fluid. These calculations may have crucial implications for the massive gravitons and dark energy theories. They could, therefore, have important repercussions for current cosmological problems.

Keywords Field theory · Dark energy · Massive gravitons

1 Introduction

Observations of type Ia supernovae and of large-scale structure (LSS), in combination with measurements of the characteristic angular size of fluctuations in the cosmic microwave background (CMB) [1–10] show that the expansion of the Universe is accelerating. This acceleration is attributed to the “dark energy”, a hypothetical energy with negative pressure [1–6]. Evidence for the presence of a dark energy is also provided by an independent, albeit more tentative, investigation of the integrated Sachs-Wolfe (ISW) [11]. The dark energy may result from Einstein’s cosmological constant (which is of a phenomenally small value); from evolving scalar fields [12]; and from a weakening of gravity in our 3 + 1 dimensions by leaking into the higher dimensions, as required in string theories [13]. These explanations may have crucial implications for both massive gravitons and dark energy. This has stimulated further efforts to confirm the initial results on dark energy, test possible sources of error and extend our empirical knowledge of this newly discovered component of the Universe.

Although most discussions of magnetic fields in the universe use a scalar field (because of its simplicity), the vector field also has applications in modern cosmology. Some workers,
for instance, consider that universe inflation is driven by the vector field \([14, 15]\). Others study the Lorentz-violated vector field and its effects to the universe \([16, 17]\); the quantum fluctuations of vector fields produced at the first stage of reheating after inflation \([18]\); and the cosmology of massive vector fields with SO(3) global symmetry is investigated in \([19]\) (for overviews of the literature on magnetic fields in the universe see \([20–22]\) and references therein).

Building on this earlier work, N. Wu proposed a mechanism which introduces massive gravitons without violating the local gauge symmetry of the Lagrangian \([23–26]\). The third-order gravitational gauge field \(C^a_{3\mu}\) is massive if mass is very large. Such a field has no contribution to the long-range gravitational force. Long-range gravitational force results exclusively from the contribution of the fourth-order gravitational gauge field \(C^a_{4\mu}\) and obeys inverse square law.

If the mass term of gravitational gauge field is extremely small, however, the third-order gravitational gauge field \(C^a_{3\mu}\) will contribute to the middle range gravitational force within an approximate range \(L \approx \frac{h}{m}\) (where \(h\) is the Plank constant and \(c\) is the speed of light). For graviton mass \(2 \times 10^{-7}\) eV the gravitational force range will be about one meter.

In the traditional gauge treatment of gravity the Lorentz group is localized. The gravitational field is, thus, not represented by gauge potential, but by the metric field \(g_{\mu\nu}\).

Here, we propose an alternative understanding of gravity, resulting from the extension of N. Wu’s gauge theory of gravity with massive gravitons that are minimally coupled to massless gravitons. Based on this, we derive the equations of state for massive gravitons. We study the dynamics of these massive gravitons in a flat, homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe and calculate the critical points of the massive graviton dark energy interacting with background perfect fluid.

These calculations may have crucial implications for the theory of both massive gravitons and dark energy. They could, therefore, have important repercussions for current cosmological problems.

### 2 Gauge Theory of Gravity with Massive Graviton

Taking Wu’s gauge model as our starting point \([23–32]\), we introduce two gravitational gauge fields \((C^a_{\mu}, C^a_{2\mu})\) simultaneously. Since \((C^a_{\mu}, C^a_{2\mu})\) are vectors in Lie algebra \([8, 9, 25]\), they can be expanded as

\[
C_\mu = C^a_{\mu} \hat{P}_a, \quad C_{2\mu} = C^a_{2\mu} \hat{P}_a. \tag{1}
\]

These correspond with two gauge covariant derivatives

\[
D_\mu = \partial_\mu - igC_\mu(x), \quad D_{2\mu} = \partial_\mu + iagC_{2\mu}(x) \tag{2}
\]

and two different field strengths, given by

\[
F_{\mu\nu} = \frac{1}{-ig}[D_\mu, D_\nu], \quad F_{2\mu\nu} = \frac{1}{iag}[D_{2\mu}, D_{2\nu}]. \tag{3}
\]

The Lagrangian of the system is given by

\[
\mathcal{L}_0 = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\rho\gamma} F^\beta_{\mu\nu} F^\gamma_{\rho\sigma} - \frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\rho\gamma} F^\beta_{2\mu\nu} F^\gamma_{2\rho\sigma}
- \frac{m^2}{2(1 + a^2)} \eta^{\mu\nu} g_{\rho\gamma} (C^\beta_\mu + aC^\beta_{2\mu})(C^\gamma_\nu + aC^\gamma_{2\nu}), \tag{4}
\]

where \(m\) is the constant mass parameter.
Equation (4) gives the mass term in gravitational gauge fields. To obtain the eigenstates of mass matrix the following rotation is needed

\[ C_{3\mu} = \cos \theta C_\mu + \sin \theta C_{2\mu}, \]
\[ C_{4\mu} = -\sin \theta C_\mu + \cos \theta C_{2\mu}, \]

where the angle $\theta$ is given by

\[ \cos \theta = \frac{1}{\sqrt{1 + a^2}}, \quad \sin \theta = \frac{a}{\sqrt{1 + a^2}}. \]

After transformation (5), the Lagrangian of the system is given by

\[ \mathcal{S}_0 = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\beta\gamma} F_{30\mu\nu}^{\beta} F_{30\rho\sigma}^{\gamma} - \frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\beta\gamma} F_{40\mu\nu}^{\beta} F_{40\rho\sigma}^{\gamma} - \frac{m_c^2}{2} \eta^{\mu\nu} g_{\beta\gamma} C_{3\mu}^\beta C_{3\nu}^\gamma + \mathcal{S}_I, \]

where $F_{30\mu\nu}^{\alpha}$, $F_{40\mu\nu}^{\alpha}$ are given by

\[ F_{30\mu\nu}^{\alpha} = \partial_\mu C_\alpha^{3\nu} - \partial_\nu C_\alpha^{3\mu}, \]
\[ F_{40\mu\nu}^{\alpha} = \partial_\mu C_\alpha^{4\nu} - \partial_\nu C_\alpha^{4\mu}. \]

From the above follows that the gauge field, $C_{3\mu}^{\alpha}$ has mass $m_c$, while the gravitational gauge field, $C_{4\mu}^{\alpha}$ is massless.

Transformations (5) are pure algebraic operations which do not affect the gauge symmetry of the Lagrangian [23]. They can, therefore, be regarded as redefinitions of gauge fields. The local gauge symmetry of the Lagrangian is still strictly preserved after field transformations. In other words, the symmetry of the Lagrangian before transformations is absolutely the same with the symmetry of the Lagrangian after transformations; therefore, we do not introduce any kind of symmetry breaking.

### 3 Interacting Massive Gravitons Dark Energy

Our aim is to study the dynamics of massive graviton $C_{3\mu}^{\alpha}$ in the presence of the massless gravitational field $C_{4\mu}^{\alpha}$. Here, the Lagrangian $\mathcal{S}_0$ that describes interactions between massive gravitons and massless gravitons is

\[ \mathcal{S}_0 = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\beta\gamma} F_{30\mu\nu}^{\beta} F_{30\rho\sigma}^{\gamma} - \frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\beta\gamma} F_{40\mu\nu}^{\beta} F_{40\rho\sigma}^{\gamma} - \frac{m_c^2}{2} \eta^{\mu\nu} g_{\beta\gamma} C_{3\mu}^\beta C_{3\nu}^\gamma + V(C^2) + \mathcal{S}_m, \]

where $F_{30\mu\nu}^{\alpha}$, $F_{40\mu\nu}^{\alpha}$ are given by (8), $C^2 = \eta^{\mu\nu} g_{\beta\gamma} C_{3\mu}^\beta C_{3\nu}^\gamma$, $m_c$ is the graviton mass, and $\mathcal{S}_m$ is the Lagrangian density of the matter field $\psi$. The potential term $V(C^2)$ is introduced into the theory without violating the strict local gravitational gauge symmetry.

In order to resume the gravitational gauge symmetry of the action, we introduce an essential factor in the form of

\[ e^{I(C)} = e^{\eta_{\alpha\mu} C^{\alpha}_\mu}, \quad I(C) = g\eta_{\alpha\mu} C^{\alpha}_\mu. \]

The full Lagrangian $\mathcal{S}$ is given by

\[ \mathcal{S} = e^{I(C)} \mathcal{S}_0, \]
and the action $S$ is defined by
\[ S = \int d^4x \mathcal{S}. \]  
(12)

It can be proven that this action has local gravitational gauge symmetry [23]. According to the gauge principle, the global symmetry gives out a conserved current, which is
\[ T_{\mu}^{\alpha} = e^{l(C)} \left( - \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{3v})} \partial_\alpha C_\mu^{3v} - \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{4v})} \partial_\alpha C_\mu^{4v} + \eta_\mu^{\alpha} \mathcal{S}_0 \right). \]  
(13)

We call this quantity inertial energy-momentum tensor [23]. The Euler-Lagrange equations for $C_\alpha^{3\mu}$ and $C_\alpha^{4\mu}$ gauge fields are
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{3v})} = - \frac{\partial \mathcal{S}_0}{\partial C_\alpha^{3v}}, \]  
(14)
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{4v})} = - \frac{\partial \mathcal{S}_0}{\partial C_\alpha^{4v}}. \]  
(15)

These forms are identical with those that occur in quantum field theory [23]. By inserting (11) into (14) and (15), we get
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{3v})} = - \frac{\partial \mathcal{S}_0}{\partial C_\alpha^{3v}} = \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{4v})} = 0. \]  
(16)

Suppose that the massless gravitational gauge field $C_\mu^{\alpha}$ is very weak in vacuum $gC_\mu^{\alpha} \approx 0$. Then, in leading order approximation, by substituting (9) to (16) and (17) we obtain
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{3v})} = 0. \]  
(18)
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{4v})} = 0. \]  
(19)

The equations of motion for massive and massless gravitons, thus, become
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{3v})} = -m_c^2 C_\alpha^{3v} + 2 \frac{dV}{dC} C_\alpha^{3v}, \]  
(20)
\[ \frac{\partial}{\partial \mathcal{S}_0} \frac{\partial \mathcal{S}_0}{\partial (\partial_\mu C_\alpha^{4v})} = 0. \]  
(21)

We now study the dynamics of these massive gravitons in a flat, homogeneous and isotropic FRW universe with metric
\[ ds^2 = -dt^2 + a^2(t)dx^2. \]  
(22)

An ansatz for the massive gravitons that turns out to be compatible with the symmetries (homogeneity and isotropy) of this metric is
\[ C_\mu^{\alpha} = \delta_\mu^{\alpha} C(t) \beta. \]  
(23)

Substituting ansatz (23) and the metric (22) into the massive graviton equations of motion (20) we find
\[ \ddot{C} + 3H \dot{C} + \left( H^2 + \frac{\ddot{a}}{a} - m_c^2 \right) C + \frac{dV}{dC} = 0 \]  
(24)
where $a$ dot stands for a derivative with respect to cosmic time $t$. Note that the zero component of (20) forces $C_0^a$ to vanish, as in the ansatz (23).

There are two independent Friedmann equations for modeling a homogeneous, isotropic universe. They are:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{2g^2}{3}\rho_{\text{total}} + \frac{m^2}{2},$$

(25)

and

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{g^2}{3}(\rho_{\text{total}} + 3p_{\text{total}}) + \frac{m^2}{2}$$

(26)

where $H = \dot{a}/a$ is the Hubble parameter, $g^2 = 4\pi G$. Here, the term $m^2/2$ can be regarded as late-time positive effective cosmological constant [56].

Using (25), (26) can be rewritten as

$$\dot{\rho}_{\text{total}} = -3H(\rho_{\text{total}} + P_{\text{total}}).$$

(27)

Equation (27) eliminates $m^2$ and expresses the conservation of mass-energy. Equations (25), (26) are sometimes simplified by replacing

$$\rho_{\text{total}} \rightarrow \rho_{\text{total}} - \frac{m^2}{2g^2},$$

(28)

$$p_{\text{total}} \rightarrow p_{\text{total}} + \frac{m^2}{2g^2}$$

(29)

to give Friedmann equations and Raychaudhuri equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{2g^2}{3}\rho_{\text{total}} = \frac{2g^2}{3}(\rho_C + \rho_m),$$

(30)

$$\dot{H} + H^2 = \left(\frac{\ddot{a}}{a}\right) = -\frac{g^2}{3}(\rho_{\text{total}} + 3p_{\text{total}}) = -\frac{g^2}{3}[\rho_C + \rho_m + 3(p_C + p_m)],$$

(31)

$$\dot{H} = -\frac{g^2}{3}(\rho_{\text{total}} + P_{\text{total}}) = -\frac{g^2}{3}(\rho_C + \rho_m + p_C + p_m).$$

(32)

Note that the simplified form of (31) is invariant under this transformation.

The corresponding energy density $\rho_C$ and (isotropic) pressure $p_C$ of massive gravitons are given by

$$\rho_C = \frac{3}{2}(\dot{C} + HC)^2 + 3V(C^2),$$

(33)

$$p_C = \frac{1}{2}(\dot{C} + HC)^2 - 3V(C^2) + 2\frac{dV}{dC^2}C^2.$$  

(34)

Note that the equation of motion (24) can be also derived from the condition

$$\dot{\rho}_C + 3H(\rho_C + p_C) = 0.$$  

(35)

The most remarkable feature of the massive graviton dark energy is that its Equations of State (EoS) $w_C = p_C/\rho_C$ can be smaller than $-1$, while possessing a conventional positive kinetic term. This is due to the additional term in proportion to $dV/dC^2$ in $p_C$. While the energy density $\rho_C$ is positive, we find that the condition for $w_C < -1$ is

$$\frac{dV}{dC^2}C^2 < -(\dot{C} + HC)^2.$$  

(36)

Thus, it is necessary that $dV/dC^2 < 0.$
4 Dynamical System of Massive Graviton Dark Energy Interacting with Background Perfect Fluid

Let us now consider the interaction between massive gravitons dark energy and background matter. Here, the background matter is described by a perfect fluid with barotropic equation of state

\[ p_m = w_m \rho_m = (\gamma - 1) \rho_m \]  

(37)

where the barotropic index \( \gamma \) is a constant satisfying the condition \( 0 < \gamma < 2 \). In particular, \( \gamma = 1 \) and \( \gamma = 4/3 \) correspond to dust matter and radiation, respectively. We assume that massive graviton dark energy and background matter interact through an interaction term \( B \), as follows

\[
\begin{align*}
\dot{\rho}_C + 3H(\rho_C + p_C) &= -B, \\
\dot{\rho}_m + 3H(\rho_m + p_m) &= B,
\end{align*}
\]

(38)  (39)

which preserves the total energy conservation equation \( \dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0 \).

It should be noted that the equation of motion (24) should be changed when \( B \neq 0 \). In this case, a new term due to \( B \) will appear in its right hand side. Following [34–50], we introduce the following dimensionless variables

\[
x = \frac{k \dot{C}}{\sqrt[6]{6H}}, \quad y = \frac{k \sqrt{V}}{\sqrt[3]{3H}}, \quad z = \frac{k \sqrt{\rho_m}}{\sqrt[3]{3H}}, \quad u = \frac{k C}{\sqrt{6}}.
\]

(40)

With the application of (30)–(34), the evolution (38) and (39) can be rewritten as a dynamical system [33]:

\[
\begin{align*}
x' &= 6 \left[ (x + u)^2 + \frac{\gamma}{4} z^2 + \Theta \right] (x + u) - 2\Theta u^{-1} - 3x - 2u - B_1, \\
y' &= 6y \left[ (x + u)^2 + \frac{\gamma}{4} z^2 + \left(1 + \frac{1}{3} xy^{-2} u^{-1}\right) \Theta \right], \\
z' &= 6z \left[ (x + u)^2 + \frac{\gamma}{4} z^2 + \Theta - \frac{\gamma}{4} \right] + B_2, \\
u' &= x,
\end{align*}
\]

(41)  (42)  (43)  (44)

where

\[
\Theta = \frac{u^2}{H^2} \frac{dV}{dC^2}
\]

(45)

and

\[
B_1 = \frac{k^2 B}{18H^3} (x + u)^{-1}, \quad B_2 = \frac{z B}{2H\rho_m},
\]

(46)

a prime denoting derivative with respect to the so-called e-folding time \( N = \ln a \). Here we use

\[
-\frac{\dot{H}}{H^2} = 6 \left[ (x + u)^2 + \frac{\gamma}{4} z^2 + \Theta \right].
\]

(47)

Friedmann constraint equation (30) then becomes

\[
3 \left[ (x + u)^2 + y^2 \right] + z^2 = 1
\]

(48)

(see [51]).
The fractional energy densities of massive gravitons dark energy $\Omega_C$ and background matter $\Omega_m$ are given by
\[
\Omega_C = \frac{\kappa^2 \rho_C}{3H^2} = 3[(x + u)^2 + y^2], \quad \Omega_m = \frac{\kappa^2 \rho_m}{3H^2} = z^2,
\]
(49) respectively.

The EoS of massive graviton dark energy and the effective EoS of the whole system are
\[
w_C = \frac{p_C}{\rho_C} = \frac{(x + u)^2 - 3y^2 + 4\Theta}{3[(x + u)^2 + y^2]},
\]
(50)
and
\[
w_{\text{eff}} = \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \Omega_C w_C + \Omega_m w_m,
\]
(51) respectively.

From (50), it is seen that the condition for $w_C < -1$ is
\[(x + u)^2 + \Theta < 0,
\]
(52) which is equivalent to (49).

Finally, it is worth noting that $y \geq 0$ and $z \geq 0$ by definition and that, in what follows, we only consider the case of expanding universe with $H > 0$. With these conditions, equations (41)–(44) become an autonomous system when the potential $V(C^2)$ is chosen to be an inverse power-law or exponential potential and the interaction term $B$ is chosen to be of suitable form.

Let us consider the model with exponential potential. The interaction between massive graviton dark energy and background perfect fluid (the most familiar interaction term, extensively considered in the literature [35–55]), is expressed by $B = 3\sigma H \rho_C$, where $\sigma$ is a dimensionless constant.

Here, we consider the massive graviton dark energy model with an exponential potential
\[
V(C^2) = V_0 \exp(-\lambda g^2 C^2),
\]
(53)
where $\lambda$ is a positive dimensionless constant (required by the condition $(dV/dC^2 < 0)$). In this case,
\[
\Theta = -3\lambda u^2 y^2.
\]
(54)
The critical points $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$ of the dynamical system expressed by (41)–(44) can be obtained from (53) and (54) by imposing the conditions $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = 0$. Note that these critical points must satisfy the Friedmann constraint (48), $\bar{x} > 0$, $\bar{z} > 0$ and the requirement of $\bar{x}$, $\bar{y}$, $\bar{z}$, $\bar{u}$ all being real.

To study the stability of these critical points, we substitute linear perturbations $x \to \bar{x} + \delta x$, $y \to \bar{y} + \delta y$, $z \to \bar{z} + \delta z$, and $u \to \bar{u} + \delta u$ about the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$ into the dynamical system of (41)–(44) with (53) and (54) and linearize these equations. Due to the Friedmann constraint (48), there are only three independent evolution equations, namely
\[
\delta x' = 6 \left[ 3(1 + 3\lambda \bar{u}^2)(\bar{x} + \bar{u})^2 - 2\lambda \bar{u}(\bar{x} + \bar{u}) + \left( \lambda \bar{u} + \frac{\gamma}{4} \right) \bar{z}^2 - \lambda \bar{u}^2 - \frac{1}{2} \right] \delta x
\]
\[+ 4\bar{z} \left[ 3(\bar{x} + \bar{u}) \left( \lambda \bar{u}^2 + \frac{\gamma}{4} \right) - \lambda \bar{u} \right] \delta z + 2 \left[ 18\lambda \bar{u}(\bar{x} + \bar{u})^3 + 3(9\lambda \bar{u}^2 + 3 - \lambda)(\bar{x} + \bar{u})^2
\]
where $\delta B_1$ and $\delta B_2$ are the linear perturbations stemming from $B_1$ and $B_2$, respectively. The three eigenvalues of the coefficient matrix of the above equations determine the stability of the corresponding critical point.

Let us now consider the case when $B = 3\sigma H\rho_C$. In this case,

$$B_1 = \frac{3}{2} \sigma [(x + u)^2 + y^2](x + u)^{-1} = \frac{\sigma}{2} (1 - z^2)(x + u)^{-1},$$

$$B_2 = \frac{9}{2} \sigma [(x + u)^2 + y^2]z^{-1} = \frac{3}{2} \sigma (z^{-1} - z).$$

The physically reasonable critical points of the dynamical system equations (41)–(44) with (53)–(54) are shown in Table 1, where

$$r_1 = \frac{\sigma}{-4 + 3\gamma},$$

$$r_2 = \sqrt{-3\gamma \lambda \sigma (\gamma + \sigma) + (\gamma - \gamma \lambda + \sigma)^2}.$$

Next, we consider the stability of these critical points. Substituting

$$\delta B_1 = -\frac{\sigma}{2}(\bar{x} + \bar{u})^{-2}(1 - \bar{z}^2)\delta x - \sigma \bar{z}(\bar{x} + \bar{u})^{-1}\delta z - \frac{\sigma}{2}(\bar{x} + \bar{u})^{-2}(1 - \bar{z}^2)\delta u,$$

$$\delta B_2 = -\frac{3}{2}\sigma (\bar{z}^{-2} + 1)\delta z,$$

and the corresponding critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$ into (55)–(57), we find that (E.I.1) is unstable if it can exist at all, while points (E.I.2) and (E.I.3) exist and are stable in a proper parameter-space (see [51] for more details). The late time attractors (E.I.2) and (E.I.3) both have

$$\Omega_C = \frac{\gamma}{\gamma + \sigma}, \quad \Omega_m = \frac{\sigma}{\gamma + \sigma}, \quad w_C = -1 - \sigma, \quad w_{\text{eff}} = -1,$$

which are scaling solutions. Note that $w_C$ of attractors (E.I.2) and (E.I.3) are both smaller than $-1$, since $\sigma > 0$ is required by their corresponding $\Omega_m$.

In the scaling attractor, the effective densities of massive graviton dark energy and background matter decrease in the same manner with the expansion of our universe, and the ratio

| Label | Critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$ |
|-------|-------------------------------------------------|
| E.I.1 | $0, 0, (3r_1)^{1/2}, \pm(\frac{1}{3} - r_1)^{1/2}$ |
| E.I.2 | $0, \frac{\sqrt{2\gamma (1 + \gamma + \sigma) - r_1}}{6\lambda (\gamma + \sigma)}, \pm\frac{\sqrt{2\gamma (1 + \gamma + \sigma) + 2r_2}}{6\lambda (\gamma + \sigma)}$ |
| E.I.3 | $0, \frac{\sqrt{2\gamma (1 + \gamma + \sigma) - r_1}}{6\lambda (\gamma + \sigma)}, \pm\frac{\sqrt{2\gamma (1 + \gamma + \sigma) + 2r_2}}{6\lambda (\gamma + \sigma)}$ |

Table 1 Critical points for $B = 3\sigma H\rho_C$ in the model with massive gravitons exponential potential. $r_1$ and $r_2$ are given in (60) and (61) respectively [51].
of massive graviton dark energy and background matter becomes a constant. So, it is not strange that we are living in a $B = 3\sigma H\rho_C$ epoch when the densities of massive graviton dark energy and matter are comparable. In this sense, the (first) cosmological coincidence problem is alleviated (see [35–55] for examples).

As it is explicitly shown in this work, for suitable interaction forms (for instance $B = 3\sigma H\rho_C$), regardless of the model with exponential potential, there are some attractors with $w_C < -1$ while their corresponding $\Omega_C$ and $\Omega_m$ are comparable in the interacting massive graviton dark energy model. In the case $B = 3\sigma H\rho_C$, all stable attractors have these desirable properties. So, for a fairly wide range of initial conditions with $w_C > -1$, the universe will eventually evolve to the scaling attractor(s) with $w_C < -1$. This solution alleviate the (first) and the second cosmological coincidence problems [35–55] at the same time.

5 Conclusions

We propose an alternative understanding of gravity resulting from the extension of N.Wu’s gauge theory of gravity with massive gravitons, which are minimally coupled to massless gravitons. The mass term of gravitational gauge field is introduced into the theory without violating the strict local gravitational gauge symmetry. For these massive gravitons, we derive the equations of state that satisfy the dark energy condition.

We study the dynamics of these massive gravitons in a flat, homogeneous and isotropic FRW universe and calculate the critical points of the massive graviton dark energy interacting with background perfect fluid.

These calculations may have crucial wider implications for the physics of massive gravitons and dark energy. They could, therefore, have important repercussions for current cosmological problems.

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