Determination of $J/\psi$ chromoelectric polarizability from lattice data

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The chromoelectric polarizability of $J/\psi$ is extracted from lattice QCD data on the nucleon-$J/\psi$ potential in the heavy quark limit. The value of $\alpha(1S) = (1.5 \pm 0.6) \text{GeV}^{-3}$ is obtained. We also comment on the possibility of hadrocharmonia.

I. THE EFFECTIVE QUARKONIUM-BARYON INTERACTION

The interaction of a heavy quarkonium with a baryon is dominated in the heavy quark limit by the emission of two virtual color-singlet chromoelectric dipole gluons [1, 2] and described by an effective potential in terms (for $S$-wave quarkonia) of the quarkonium chromoelectric polarizability $\alpha$ and energy-momentum tensor (EMT) densities of the baryon as [3]

$$V_{\text{eff}}(r) = -\alpha \frac{4\pi^2}{b} \frac{g_2^2}{g_s^2} \left( \nu T_{00}(r) - 3 p(r) \right), \quad \nu = 1 + \xi_s \frac{b g_s^2}{8\pi^2}. \quad (1)$$

Here $T_{00}(r)$ and $p(r)$ are the energy density and pressure inside the baryon [4], which satisfy respectively

$$\int d^3r \, T_{00}(r) = M_B, \quad \int d^3r \, p(r) = 0, \quad (2)$$

and $b = (\frac{4}{3} N_c - \frac{4}{3} N_f)$ is the leading coefficient of the Gell-Mann-Low function, $g_s$ ($g_2$) is the strong coupling constant renormalized at the scale $\mu_s$ ($\mu_s$) associated with the heavy quarkonium (baryon) state. The parameter $\xi_s$ denotes the fraction of the baryon energy carried by gluons at the scale $\mu_s$ [5]. The derivation of Eq. (1) is justified in the limit that the ratio of the quarkonium size is small compared to the effective gluon wave-length [2], and a numerically small term proportional to the current masses of the light quarks is neglected.

Due to Eq. (2) the effective potential has the following normalization and mean square radius

$$\int d^3r \, V_{\text{eff}}(r) = -\alpha \frac{4\pi^2}{b} \frac{g_2^2}{g_s^2} \nu M_B, \quad \langle r^2_{\text{eff}} \rangle = \frac{\int d^3r \, r^2 V_{\text{eff}}(r)}{\int d^3r \, V_{\text{eff}}(r)} = \langle r^2 \rangle - \frac{12 d_1}{5 \nu M_B^2}, \quad (3)$$

with the mean square radius of the energy density $\langle r^2 \rangle = \int d^3r \, r^2 T_{00}(r)/M_B$ and the $D$-term $d_1 = \frac{5}{2} M_B \int d^3r \, r^2 p(r)$ [4, 6]. The large-distance behavior of $V_{\text{eff}}(r)$ in chiral limit can for our purposes be conveniently expressed as

$$V_{\text{eff}}(r) = \frac{27}{16 \pi^2} \frac{1 + \nu}{\nu} \frac{g_A^2}{M_B F_\pi^2 r^6} \int d^3r' V_{\text{eff}}(r') \quad \text{for } r \text{ large,} \quad (4)$$

where $F_\pi = 93 \text{MeV}$ is the pion decay constant, and $g_A$ is the axial coupling constant with $g_A = 1.26$ for the nucleon. Notice that this result refers to the leading order of the expansion in a large number of colors $N_c$ [14] with $N_c \to \infty$ taken first, and $m_\pi \to 0$ taken second (in general these limits do not commute). For finite $m_\pi$ the behavior is $V_{\text{eff}}(r) \propto \exp(-2m_\pi r/r^2)$ at $r \gg 1/m_\pi$ [14].

II. CHROMOELECTRIC POLARIZABILITIES

The chromoelectric polarizabilities $\alpha$ are important properties of quarkonia. Little is known about them especially for charmonia. The chromoelectric polarizabilities were calculated in the large-$N_c$ limit in the heavy quark approximation [7]. Applying the results to the charmonium case one finds [3]

$$\alpha(1S)_{\text{pert.}} \approx 0.2 \text{GeV}^{-3}, \quad (5a)$$

$$\alpha(2S)_{\text{pert.}} \approx 12 \text{GeV}^{-3}, \quad (5b)$$

$$\alpha(2S \to 1S)_{\text{pert.}} \approx -0.6 \text{GeV}^{-3}. \quad (5c)$$
Independent phenomenological information on the value of the $2S \to 1S$ transition polarizability is available from analyses of data on the decay $\psi' \to J/\psi \pi\pi$ \cite{2}

$$|\alpha(2S \to 1S)| \approx 2 \text{GeV}^{-3} \quad \text{(phenomenology)}.$$ \hspace{1cm} (6)

In the heavier bottomonium system $1/\Nc$ corrections to $\alpha(1S)$ are of $\mathcal{O}(5\%)$ \cite{3}. In the charmonium system presently no information is available on the chromoelectric polarizabilities besides the perturbative estimates \cite{2} and the phenomenological value for the $2S \to 1S$ polarizability \cite{2} which is only in rough agreement with the perturbative prediction, see Eq. (5c) vs (6).

In this situation, independent information on the chromoelectric polarizabilities of charmonia is of importance.

### III. Extraction of the chromoelectric polarizability of $J/\psi$

The recent lattice QCD data on the effective charmonium-nucleon interaction \cite{10} put us in the position to extract the chromoelectric polarizability $\alpha(1S)$ of $J/\psi$. The extraction assumes that the charm-quark mass is sufficiently large to neglect heavy quark mass corrections, but is otherwise model-independent. This assumption can be tested with future lattice QCD data. Below we will see that the current lattice data are compatible with this assumption. From Eq. (3) we obtain

$$\alpha = - \frac{b}{4\pi^2 \kappa M N} \frac{g_s^2}{g_c^2} \int d^3r \; V_{\text{eff}}(r). \quad \text{(7)}$$

Let us discuss the different factors which play a role in the extraction of $\alpha$ and their uncertainties.

The coefficient $\kappa$ introduced in Eq. (1) was estimated on the basis of the instanton liquid model of the QCD vacuum and the chiral quark soliton model, where the strong coupling constant freezes at scale set by the nucleon size at $g_s^2/(4\pi) \approx 0.5$. Assuming $\xi_s \approx 0.5$ as suggested by the fraction of nucleon momentum carried by gluons in DIS at scales comparable to $\mu_s$ one obtains the value $\kappa \approx 1.5$ \cite{3}. This is supported by the analysis of the nucleon mass decomposition \cite{3} with $\xi_s \approx \frac{1}{3}$ leading to $\kappa \approx 1.4$. Based on these results we will use

$$\kappa \approx 1.5 \pm 0.1 \quad \text{(8)}$$

in this work. Let us remark that a similar result $\kappa = (1.45 \ldots 1.6)$ was obtained for the pion in Ref. \cite{5}.

In order to estimate the factor $g_s^2/g_c^2$ we use two extreme approaches. One estimate is based on effective nonperturbative methods. For that we use the non-perturbative result $g_s^2/(4\pi) \approx 0.5$ from the instanton vacuum model mentioned above which refers to a low scale of the nucleon, see above. Interestingly, phenomenological calculations of charmonium properties require $g_s^2/(4\pi) = 0.5461$ at a scale associated with charmonia \cite{11}. This indicates that $g_s^2/g_c^2 \sim 1$ which is a reasonable assumption \cite{3}. Another “extreme” result is provided by the leading-order QCD running coupling constant. We follow Ref. \cite{12} where the description of the strong coupling constant was optimized to guarantee perturbative stability down to a low initial scale $\mu^n_Q = 0.26 \text{GeV}^2$ of the parametrizations for the unpolarized parton distribution functions. In this way we obtain $g_s^2/(4\pi) = 0.46$ at a scale set by the nucleon mass, while $g_s^2/(4\pi) = (0.27 \ldots 0.36)$ depending on whether one evaluates the running coupling constant at the scale $\mc$ or $2\mc$ (the leading-order derivation of Eq. (11) does not fix the scale, and both choices are equally acceptable). In this way we obtain the “leading-order perturbative estimate” $g_s^2/g_c^2 \sim (1.3 \ldots 1.7)$. This indicates that this quantity is associated with a substantial theoretical uncertainty. In order to cover both extreme cases, we will assume that

$$\frac{g_s^2}{g_c^2} \approx 1.37 \pm 0.37 \quad \text{(9)}$$

The information on $\int d^3r \; V_{\text{eff}}(r)$ is obtained from the lattice QCD calculation \cite{10} performed with unphysical light quark masses such that $m_\pi = 875 \text{MeV}$ and $M_N = 1816 \text{MeV}$ but with a physical value of $m_c$. In the heavy quark limit the effective potential factorizes in the chromoelectric polarizability $\alpha$ and nucleonic properties, and we can expect the extracted value of $\alpha$ to be unaffected by the unphysical light quark masses. (The heavy quark mass corrections are sensitive to light quark masses, but we assume them to be small which can be verified by future lattice calculations.) In the lattice calculation $V_{\text{eff}}(r)$ was computed in the region $0 \leq r \leq 1.7 \text{fm}$ in the angular momentum channels $J = \frac{1}{2}$ and $J = \frac{3}{2}$ as shown in Fig. \ref{fig:1}. The lattice data in both channels can be fitted with functions of the form

$$V_{\text{eff}}(r) = C_0 e^{-\frac{r}{\rho_0}} \frac{1}{1 + \frac{r}{\rho_1}} + C_2 \; e^{-\frac{r^2}{\rho_2^2}} \quad \text{(10)}$$

\begin{table}
\caption{The coefficients $C_0, C_1, C_2$ and the parameter $\rho$ are determined from the lattice data.}
\begin{tabular}{|c|c|c|}
\hline
Channel & $C_0$ & $C_1$ \\
\hline
\frac{1}{2} & 1.4 & 0.3 \\
\frac{3}{2} & 1.2 & 0.2 \\
\hline
\end{tabular}
\end{table}
where the first term is defined such that at large \( r \) it has the form dictated by chiral symmetry, while the second term constrains the parametrization in the small-\( r \) region. The best fit parameters in the channel \( J = \frac{1}{2} \) are as follows

\[
\begin{align*}
C_0^{(1/2)} &= (201.8 \pm 4.4) \text{ MeV}, & r_0^{(1/2)} &= (0.354 \pm 0.027) \text{ fm}, & r_1^{(1/2)} &= (0.421 \pm 0.085) \text{ fm}, \\
C_2^{(1/2)} &= (-133.8 \pm 5.0) \text{ MeV}, & r_2^{(1/2)} &= (0.090 \pm 0.004) \text{ fm}, & \chi^2_{\text{d.o.f.}} &= 0.21, \\
\end{align*}
\]

and the best fit parameters in the channel \( J = \frac{3}{2} \) are as follows

\[
\begin{align*}
C_0^{(3/2)} &= -(181.5 \pm 4.0) \text{ MeV}, & r_0^{(3/2)} &= (0.372 \pm 0.028) \text{ fm}, & r_1^{(3/2)} &= (0.416 \pm 0.081) \text{ fm}, \\
C_2^{(3/2)} &= -(114.9 \pm 4.6) \text{ MeV}, & r_2^{(3/2)} &= (0.087 \pm 0.005) \text{ fm}, & \chi^2_{\text{d.o.f.}} &= 0.20. \\
\end{align*}
\]

The fits are shown in Fig. 1. Several remarks are in order.

First, the potentials in both channels are very similar, and agree with each other within \( \pm 5\% \) relative accuracy. In fact, except for the point at \( r = 0 \) both lattice data sets are compatible with each other within error bars. Let us remark that, if heavy quark mass corrections play a role, one should expect them to have an impact especially in the region of small \( r \lesssim 1/m_c \approx 0.13 \text{ fm} \). The independence of \( V_{\text{eff}}(r) \) of \( J = \frac{1}{2} \) or \( \frac{3}{2} \) is an important consistency check of our approach. The effective potential is universal in our approach, and differences due to different \( J \) are expected to be suppressed in the heavy quark limit, as we observe. Thus, we have no indication that heavy quark mass corrections are significant for \( V_{\text{eff}}(r) \) in the charmonium system. As mentioned above, this point can be tested quantitatively with future lattice data.

Second, chiral symmetry dictates \( r_0 = (2m_{\pi})^{-1} = 0.11 \text{ fm} \). The fits are a factor of 3 off. Notice, however, that the lattice data clearly constrain \( V_{\text{eff}}(r) \) in both channels up to only about to \( r \lesssim 1 \text{ fm} \). It is likely that this limited \( r \)-region does not extend far enough to see the chiral asymptotics. Indeed, for \( 1 \text{ fm} < r < 1.7 \text{ fm} \) the lattice data on \( V_{\text{eff}}(r) \) are actually compatible with zero within error bars, see the inserts in Fig. 1. Notice, however, that a fit with the fixed parameter \( r_0 = (2m_{\pi})^{-1} \) (with \( m_{\pi} = 875 \text{ MeV} \) here) has still an excellent \( \chi^2 \) per degree of freedom of \( \chi^2_{\text{d.o.f.}} = 0.4 \) for both channels. This is remarkable and indicates that the lattice data are compatible with chiral symmetry.

Third, we explored also other shapes for the fit functions with practically no difference in the region \( r \lesssim 1 \text{ fm} \) where the lattice data have the strongest constraining power. We will comment below on the region \( r > 1 \text{ fm} \).

In order to evaluate \( \int d^3r V_{\text{eff}}(r) \) we consider separately the region \( r < 1 \text{ fm} \) where the lattice data are clearly non-zero, and \( r \geq 1 \text{ fm} \) where the lattice data are compatible with zero within error bars (including the region \( r > 1.7 \text{ fm} \) with no available lattice data), see the insert in Fig. 1. In the region \( r < 1 \text{ fm} \) the fit in Eqs. \((10)\) \((12)\) yield

\[
\int_{r<1\text{ fm}} d^3r V_{\text{eff}}(r) = \begin{cases} (-9.3 \pm 0.8) \text{ GeV}^{-2} & \text{for } J = \frac{1}{2}, \\
(-8.9 \pm 0.8) \text{ GeV}^{-2} & \text{for } J = \frac{3}{2}. \end{cases}
\]

The uncertainty of these results is due to the statistical uncertainty of the lattice data. We tried several other fit Ansätze which all had larger \( \chi^2_{\text{d.o.f.}} \), and gave results compatible with \((13)\) within statistical error bars. The systematic uncertainty due to the choice of fit Ansatz is therefore negligible compared to the statistical uncertainty of the fits.

In the region \( r > 1 \text{ fm} \) systematic uncertainties due to the choice of fit Ansatz are not negligible. The form \((10)\) of the best fit is well-motivated by chiral symmetry. But the lattice data \((10)\) have a modest constraining power.

\[\text{FIG. 1: Effective } J/\psi\text{-nucleon potential } V_{\text{eff}}(r) \text{ as function of } r \text{ from the lattice QCD calculation } (10) \text{ and the best fits } (10) (12) \text{ in the channels: (a) } J = \frac{1}{2}, \text{ and (b) } J = \frac{3}{2}. \text{ The shaded areas show the } 1\sigma \text{ regions of the fits. The inserts show the regions of } 1 \text{ fm} < r < 1.7 \text{ fm} \text{ where the available lattice data are compatible within error bars also with zero or with chiral predictions.}\]
for $1 \text{ fm} < r < 1.7 \text{ fm}$, and no lattice data are available beyond that. To proceed we assume that the fits \cite{10,12} give useful estimates for the central values of contributions from $r > 1 \text{ fm}$ to the integrals over $V_{\text{eff}}(r)$, and assign a systematic error by using two extreme estimates. For the first estimate we approximate $V_{\text{eff}}(r) = 0$ for $r \geq 1 \text{ fm}$, which fits the lattice data in the region $1 \text{ fm} < r < 1.7 \text{ fm}$ with a $\chi^2_{\text{d.o.f.}} = 0.7$, and certainly leads to overestimates of the contributions from the large-$r$ region to $\int d^3r V_{\text{eff}}(r)$ in both channels. For the second extreme estimate we assume $V_{\text{eff}}(r) \propto 1/r^6$ with the coefficient given by Eq. (4). Notice that the coefficient strictly speaking needs the full result for $\int d^3r V_{\text{eff}}(r)$ which we do not yet know. At this point one could design an iterative procedure, but for our purposes it is sufficient to assume that $\int d^3r V_{\text{eff}}(r) \approx -(10\ldots20) \text{ GeV}^{-2}$. This is also compatible with the lattice data (a fit assuming $\int d^3r V_{\text{eff}}(r) = -15 \text{ GeV}^{-2}$ has $\chi^2_{\text{d.o.f.}} = 0.20$ and is shown in Fig. 1) and certainly leads to an underestimate of the large-$r$ contribution to the integral. To summarize, in the large-$r$ region we obtain

$$\int_{r \geq 1 \text{ fm}} d^3r V_{\text{eff}}(r) = \begin{cases} 0 & J = \frac{1}{2}, \frac{3}{2} \text{ extreme estimate (i): } V_{\text{eff}}(r) = 0 \text{ for } r > 1 \text{ fm}, \\ -(3.8 \pm 2.3) \text{ GeV}^{-2} & J = \frac{1}{2} \text{ extrapolation based on the best fit in Eqs. (10, 11),} \\ -(4.1 \pm 2.5) \text{ GeV}^{-2} & J = \frac{3}{2} \text{ extrapolation based on the best fit in Eqs. (12, 13),} \\ -(3.3 \ldots 6.6) \text{ GeV}^{-2} & J = \frac{1}{2}, \frac{3}{2} \text{ extreme estimate (ii): } V_{\text{eff}}(r) \text{ with “chiral tail” for } r > 1 \text{ fm.} \end{cases}$$

(14)

We use the best fit results as central values and the extreme estimates to assign a systematic uncertainty as follows

$$\int_{r \geq 1 \text{ fm}} d^3r V_{\text{eff}}(r) = \begin{cases} -3.8^{+3.8}_{-2.8} \text{ GeV}^{-2} & J = \frac{1}{2}, \\ -4.1^{+4.1}_{-2.5} \text{ GeV}^{-2} & J = \frac{3}{2}. \end{cases}$$

(15)

Combining Eqs. (13, 15) the final result for the full integral of the effective potential is

$$\int d^3r V_{\text{eff}}(r) = \begin{cases} -13.1 \pm 0.8^{+3.8}_{-2.8} \text{ GeV}^{-2} & J = \frac{1}{2}, \\ -13.0 \pm 0.7^{+4.1}_{-2.5} \text{ GeV}^{-2} & J = \frac{3}{2}. \end{cases}$$

(16)

where the first error is due the statistical accuracy of the lattice data in the region $r < 1 \text{ fm}$ and the second error is due to the systematic uncertainty in the extrapolation for $r > 1 \text{ fm}$.

From Eqs. (8, 9, 16) we obtain the value for the chromoelectric polarizability

$$\alpha(1S) = \begin{cases} (1.48 \pm 0.09^{+0.43}_{-0.23} \pm 0.38 \pm 0.10) \text{ GeV}^{-3} & J = \frac{1}{2}, \\ (1.49 \pm 0.08^{+0.49}_{-0.37} \pm 0.40 \pm 0.10) \text{ GeV}^{-3} & J = \frac{3}{2}. \end{cases}$$

(17)

with the errors due to the following uncertainties (in this order): statistical accuracy of the lattice data in the region $r < 1 \text{ fm}$, systematic uncertainty of $\int d^3r V_{\text{eff}}(r)$ due to extrapolation in the region $r > 1 \text{ fm}$, uncertainty of the ratio $(g_c/g_s)^2$ and that of $\nu$. Combining the systematic uncertainties in quadrature we obtain

$$\alpha(1S) = \begin{cases} (1.48 \pm 0.09^{+0.50}_{-0.46} \pm 0.38 \pm 0.10) \text{ GeV}^{-3} & J = \frac{1}{2}, \\ (1.49 \pm 0.08^{+0.64}_{-0.55} \pm 0.40 \pm 0.10) \text{ GeV}^{-3} & J = \frac{3}{2}. \end{cases}$$

(18)

Rounding off and combing all sources (statistical and systematic) of uncertainties, we obtain for both channels

$$\alpha(1S) = (1.5 \pm 0.6) \text{ GeV}^{-3}.$$  

(19)

IV. POSSIBILITY FOR HADROCHARMONIA

The charmonium-nucleon potential is attractive and we can study the possibility of a bound state - hadrocharmonium \cite{13}. A candidate for such a state with a mass around 4450 MeV was recently observed by LHCb \cite{16}. To do this we rescale the lattice effective potential by the factor $M_N^{\text{phys}}/M_N^{\text{lattice}}$, where $M_N^{\text{phys}} = 940 \text{ MeV}$ is physical nucleon mass and $M_N^{\text{lattice}} = 1816 \text{ MeV}$ the nucleon mass obtained in lattice measurements of $M_N^{\text{lattice}}$ \cite{10}. We need this rescaling to ensure the physical normalisation condition \cite{3} for the effective potential.

Solving the Schrödinger equation for the rescaled potential we confirm the conclusion of Ref. \cite{10} that $J/\psi$ does not form the bound state with the nucleon. Now we can study the possibility of a nucleon bound state with $\psi(2S)$. To do this we note that according to Eq. (1) the shape of the nucleon-$\psi(2S)$ potential is the same as for the corresponding potential for $J/\psi$, the only difference is the overall normalisation factor due to chromoelectric polarizability.

Using the results for the shape of the effective potential extracted here from the lattice and treating $\alpha(2S)$ as a free
parameter, we obtain the following results:

- The nucleon-ψ(2S) bound states can form if \( \alpha(2S) \geq \alpha_{\text{crit}}(2S) = (7.5 \pm 3.0) \text{ GeV}^{-2} \), where error bars are due to statistical and systematic error of our fit, and due to uncertainty of \((g_s/g_c)^2\), see Eq. (9). Note that in the ratio \( \alpha(2S)/\alpha(1S) \) many systematic uncertainties are canceled. For this ratio we obtain \( \alpha_{\text{crit}}(2S)/\alpha(1S) = (5.0 \pm 0.5) \). The values of \( \alpha_{\text{crit}}(2S) \) from the \( J = 1/2 \) and \( J = 3/2 \) potentials are indistinguishable within error bars. The obtained value of \( \alpha_{\text{crit}}(2S) \) is compatible with those obtained in Refs. [3,13] in completely different frameworks.

- For \( \alpha(2S) = (22 \pm 9) \text{ GeV}^{-2} \) the bound state with mass 4450 MeV is formed. It may correspond to the narrow LHCb pentaquark \( P_c(4450) \). Again we have a good agreement with findings of Refs. [3,13]. In terms of the ratio \( \alpha(2S)/\alpha(1S) \) the hadrocharmonium \( P_c(4450) \) exists for \( \alpha(2S)/\alpha(1S) = (15 \pm 1) \).

- From the data [10] for \( J = 1/2 \) and \( J = 3/2 \) effective potentials we are able to estimate the hyperfine splitting between \( \frac{3}{2}^- \) and \( \frac{1}{2}^- \) components of \( P_c(4450) \). We find the hyperfine mass splitting \((30 \pm 30) \text{ MeV}\) with tendency for \( J = 3/2 \) to be heavier. This is compatible with both zero and with the estimate of 5-10 MeV obtained in [3].

We see that the lattice data of [10] confirm the conclusions about nucleon-ψ(2S) bound state made in Refs. [3,13]. It would be very interesting to make independent lattice measurement of the nucleon-ψ(2S) effective potential.

V. CONCLUSIONS

The chromoelectric polarizability \( \alpha(1S) \) of \( J/\psi \) was extracted on the basis of the formalism [1] from the lattice QCD data [10] on the effective nucleon-\( J/\psi \) potential \( V_{\text{eff}} \). Besides assuming the heavy quark limit the extraction is model-independent. The lattice data in the angular momentum channels \( J = \frac{1}{2} \) and \( J = \frac{3}{2} \) are compatible with each other indicating that heavy quark mass corrections for \( V_{\text{eff}} \) are not large in the charmonium system. The final result is \( \alpha(1S) = (1.5 \pm 0.6) \text{ GeV}^{-2} \) and significantly larger than the perturbative prediction \( \alpha(1S)_{\text{pert}} \approx 0.2 \text{ GeV}^{-2} \) [2,7] which was basically the only available information on the chromoelectric polarizability of \( J/\psi \).

We studied the possibility of the nucleon-ψ(2S) bound state. We came to conclusions which are similar to those in Refs. [3,13], and support the interpretation of \( P_c(4450) \) as a ψ(2S)-nucleon bound state with \( \alpha(2S)/\alpha(1S) \approx 15 \). Our result is compatible with the value of \( \alpha(2S) \approx 17 \text{ GeV}^{-2} \) obtained in Refs. [3,13] in completely different frameworks. This is remarkable, considering that in Refs. [3,13] chiral models were used with massless [3] and physical [13] pion masses, while here we used lattice data obtained at large unphysical \( m_\pi \). The results for the \( \psi(2S) \) chromoelectric polarizability obtained in Refs. [3,13] and here are based on the interpretation of \( P_c(4450) \) as a hadrocharmonium. Our analysis also provides independent support for this interpretation.

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