Analysis of EMCCD and sCMOS readout noise models for Shack-Hartmann wavefront sensor accuracy

A. G. Basden,a
aDepartment of Physics, South Road, Durham, DH1 3LE, UK

Abstract. In recent years, detectors with sub-electron readout noise have been used very effectively in astronomical adaptive optics systems. Here, we compare readout noise models for the two key faint flux level detector technologies that are commonly used: EMCCD and scientific CMOS (sCMOS) detectors. We find that in almost all situations, EMCCD technology is advantageous, and that the commonly used simplified model for EMCCD readout is appropriate. We also find that the commonly used simple models for sCMOS readout noise are optimistic, and recommend that a proper treatment of the sCMOS rms readout noise probability distribution should be considered during instrument performance modelling and development.

Keywords: adaptive optics, EMCCD, scientific CMOS, sCMOS, detectors, numerical, Monte-Carlo.

Address all correspondence to: Alastair Basden, Durham University, Department of Physics, South Road, Durham, DH1 3LE, UK; Tel: +44 191 3342229; E-mail: a.g.basden@durham.ac.uk

1 Introduction

Within the last decade, the use of optical detector arrays with sub-electron readout noise has become common for wavefront sensors on astronomical adaptive optics (AO) systems. The majority of these detectors have used electron multiplying CCD (EMCCD) technology, for example as used by the CANARY wide-field AO demonstrator on the William Herschel Telescope (WHT) and the SPHERE eXtreme AO (XAO) system on the Very Large Telescope (VLT). However, scientific CMOS (sCMOS) technology is now also offering sub-electron readout noise, and is a potential alternative to EMCCDs, particularly when larger detector arrays are required, for example for laser guide star (LGS) wavefront sensors (WFSs) for Extremely Large Telescope (ELT)-scale instruments. An sCMOS camera has been used on-sky by CANARY during LGS commissioning.

EMCCD and sCMOS detectors have different readout noise characteristics. The relative effect of different readout noise models on Shack-Hartmann sensor (SHS) WFS images, and the corresponding wavefront slope estimation accuracy, has not previously been studied in depth.

1.1 EMCCD readout noise

EMCCDs work on the principle of impact ionisation, where as the signal in a given pixel (electrons) are transferred along a many-stage multiplication register, there is a small probability (typically of order 1%, \( p = 0.01 \)) that each photo-electron will generate an additional electron. These registers are many hundreds of elements long, and so a large mean multiplication (or gain) can be achieved, equal to \((1 + p)^n\) where \(n\) is the number of stages. Unfortunately, this multiplication process is stochastic, and for a given number of input photons in a given pixel, there is a wide range in the possible measured EMCCD output value in addition to photon shot noise which is always present. Typically, a gain of order 500-1000 is used for astronomical AO systems. After the signal has been multiplied in this way, it is then read out of the detector and digitised, introducing readout noise to the signal. This readout noise is dependent on readout speed, and typically has a rms of about 50 electrons for an EMCCD operated at high frame rates. We ignore thermal noise, since
EMCCDs typically operate at high frame rates, and are usually cooled to temperatures of around 220 K in commercial camera models.

When modelling the impact of detector performance on instrument designs, the combination of these sources of uncertainty leads to increased complexity. Therefore, simplified models are often used (for example in Ref. [6]: typically, when modelling an EMCCD, the detector quantum efficiency (QE) is halved (i.e. the input flux is halved) as an approximation of the effect of the stochastic gain mechanism, and a readout of around 0.1 electrons is assumed (the true readout noise divided by the gain). Here, we investigate the effect of these assumptions.

1.2 sCMOS readout noise

An sCMOS detector is an active pixel sensor, with each pixel having its own individual readout, rather than a single, or small number of, readout ports in the case of a CCD. Each sCMOS pixel will therefore have an associated readout noise level, which will differ from other readouts due to manufacturing imperfections, etc. Additionally, the readout noise introduced at each pixel will also vary with each frame readout, i.e. the readout noise of a given pixel has some rms value, with all pixels forming a rms readout noise probability distribution. Therefore, manufacturers of sCMOS cameras usually quote the median rms readout of the device, which is at the level of 0.8 electrons for the best current cameras. As with an EMCCD, this level will be dependent on readout speed, which is generally not user-selectable for current commercial sCMOS cameras. Fig. 1 shows the a histogram of variation of individual pixel rms readout for a typical sCMOS device.

Instrument modelling of sCMOS detectors has to date typically used a single rms readout value for all pixels (see for example Ref. [8]), and readout noise is often described using a single (unspecified) parameter, for example Ref. [9] However, this can lead to an overestimation of instrument performance, since the occasional pixels with far greater readout noise are not modelled.

Fig 1 The probability distribution for rms readout noise of individual sCMOS detector pixels, scaled to the frequency of the modal average. Inset is shown the distribution on a logarithmic scale.
1.3 Accurate readout noise modelling for Shack Hartmann wavefront sensors

This paper seeks to investigate the effect of accurate readout noise models on the performance of Shack Hartmann wavefront sensors commonly used for astronomical AO systems. In §2, we describe the models used, our performance verification and the implemented tests. In §3, we discuss our findings and summarise the results. We conclude in §4.

2 Modelling readout noise in Shack Hartmann wavefront sensors

To investigate the effect of sensor readout noise characteristics on Shack-Hartmann wavefront sensor performance, we perform Monte-Carlo simulations of a single Shack-Hartmann sub-aperture, investigating different spot sizes and different sub-aperture sizes (i.e. number of pixels) for a range of input flux signal levels. Our procedure, following Ref. [6], is as follows:

1. A noiseless sub-aperture spot is generated at a random position, and the centre of gravity calculated ($S_{x:\text{true}}$, $S_{y:\text{true}}$ for the x and y position respectively).

2. Random photon shot noise is introduced across the sub-aperture.

3. Detector readout is modelled (see §2.1 and §2.2).

4. The spot position is estimated using a centre of gravity algorithm ($S_{x:\text{estimated}}$, $S_{y:\text{estimated}}$ for the x and y position respectively).

5. Steps 1–4 are repeated many ($N$) times.

6. The performance metric, $R$, is calculated.

The performance metric is given by

$$R = \frac{\sum_{m=1}^{N} \sqrt{(S_{x:\text{true}}(m) - S_{x:\text{estimated}}(m))^2 + (S_{y:\text{true}}(m) - S_{y:\text{estimated}}(m))^2}}{N}$$

(1)

where $S(m)$ is the $m^{th}$ individual slope measurement (x or y, true or estimated) of $N$ Monte-Carlo measurements (typically ten thousand). Essentially, this is the mean distance of the estimated position from the true position. We refer to this interchangeably as the slope error (on figure axes), and as the slope estimation accuracy.

We use an Airy disk for the noiseless sub-aperture spot, the width of which is a parameter we investigate (to allow for performance estimates with different pixel scales and seeing conditions), which we define here as the diameter of the first Airy minimum in pixels. When processing the noisy images to compute the spot position, different background levels are subtracted to enable investigation of optimum background subtraction. The background level resulting in lowest slope error is then used.

Signal levels from 20 photons per sub-aperture (below what would be used effectively on-sky) to 1000 photons per sub-aperture (approaching a high light level condition) are used. We assume 100% QE for the detectors to simplify data analysis, except for the simplified EMCCD model where the excess noise factor means that effective QE is 50%. In practise, the QE of a back-illuminated EMCCD can approach 95%, while second generation sCMOS detectors have a QE...
greater than 70%. By scaling flux levels by the relevant QE (as we do in Fig. 14 to provide an example), a reader can evaluate detector performance for their particular image sensor.

Unless stated otherwise, we assume here a spot size of diameter 2 pixels (Airy ring minima), and a signal level of 50 photons per sub-aperture. We investigate these parameters, and the number of pixels within a sub-aperture.

2.1 EMCCD models

We introduce three models for EMCCD technology readout noise:

1. EMCCD Simple: The simple model, involving halving the effective detector quantum efficiency, and using a readout noise of 0.1 electrons rms, normally distributed.

2. EMCCD Stochastic: A full stochastic Monte-Carlo electron multiplication process is modelled, with the photo-electrons from each pixel being propagated through the multiplication register, with a small, random probability of being multiplied at each stage. A readout noise of 50 electrons rms is then applied.

3. EMCCD Distribution: The EMCCD output is obtained from the probability distribution given by Eq. 2, and a readout noise of 50 electrons rms is then applied.

For the stochastic and probability distribution models, we use a mean gain of 500 (unless otherwise stated), with 520 multiplication stages, and therefore a probability of about 1.2% of a new electron being generated at each stage, for each input electron. We also do not investigate other readout noises which could be introduced at different detector readout speeds. To achieve the same performance at other readout noise levels, the EMCCD gain could be altered.

The probability distribution for EMCCD output is given by Eq. 2, taken from Ref. 5. Additionally, here we also introduce an approximation for this distribution at higher light levels (e.g. for greater than 50 input photo-electrons):

\[
p(x) = \frac{x^{n-1} \exp\left(-x/g\right)}{g^n(n-1)!}\tag{2}
\]

And our high light level approximation

\[
p(x) = \frac{\exp\left(-\left(x - g(n-1)\right)^2 / (2g^2n)\right)}{\sqrt{2\pi g^2n}}
\]

where \( n \) is the number of input photo-electrons, \( g \) is the mean gain and \( x \) is the output of the probability distribution. We use the high light level approximation for input signal levels greater than 50 photo-electrons.
2.1.1 Thresholding schemes

We also investigate a thresholding scheme for EMCCD output data, as introduced by Ref. [5]. In particular, we use the Poisson Probability (PP) scheme. This concept involves taking the EMCCD output, dividing by the mean gain, and placing it into non-uniformly spaced bins, with the \( n \)th bin being interpreted as \( n \) detected photo-electrons. The positions (thresholds) of bin boundaries that we use are given by Ref. [5], placed where the probability of obtaining a given output signal for a light level of \( n \) photons and \( n + 1 \) photons is equal.

This thresholding scheme is non-linear, and as a result does not provide a calibrated flux measurement. Application of a photometric correction is therefore also investigated here, as given in Ref. [5]. We note that this scheme is far from perfect, as identified in Ref. [11], and so seek only to investigate whether performance improvements are possible when using it.

2.2 sCMOS models

We also introduce a number of models for sCMOS readout noise:

1. sCMOS Median: All pixels have the same rms readout noise, normally distributed, equal to the manufacturer quoted median readout noise.

2. sCMOS Mean: All pixels have the same rms readout noise, normally distributed, equal to the manufacturer quoted rms readout noise.

3. A different rms readout noise for each pixel following the probability distribution in Fig. [1] (Eq. [3]):
   
   (a) sCMOS Fixed: We investigate 10 different sub-apertures, each following this probability distribution for readout noise, with an individual pixel’s rms readout noise held constant over the entire Monte-Carlo simulation.

   (b) sCMOS Random: We also investigate performance when the readout noise of pixels within a sub-aperture are changed each iteration (obeying the probability distribution), to get a feel for what “overall” performance would be like (i.e. many sub-apertures on the detector). Effectively, we are sampling many different sub-apertures and obtaining a mean expected performance metric.

It should be noted that in the cases where rms readout noise follows the probability distribution Eq. [3] some pixels will have a much larger rms readout noise than others, and thus will have a negative effect on centroid estimates. We use a probability distribution that closely matches manufacturer data [4] given by:

\[
f = \frac{1}{N (\tanh \left( 10x - 6.5 \right) + 1)} \times \left( \frac{1}{x^{10} + 0.1} + \frac{1}{2 \left( x^4 + 0.1 \right)} + \frac{1}{2 \left( x^2 + 0.1 \right)} \right) \tag{3}
\]

where \( f \) is the probability of a given pixel having readout noise \( x \), and \( N \) is the normalisation factor. Fig. [1] shows this distribution. We assume a slow-scan readout scheme to generate this probability distribution: a fast-scan readout would introduce more noise, shifting the distribution.

The random nature of pixel rms readout noise obtained from this distribution means that some sub-apertures will behave very well (with low noise throughout), while others will contain one
Fig 2 The per-pixel rms readout noise for the 10 sample noise patterns used here for $16 \times 16$ pixel sub-apertures. Each value represents the rms readout noise of that particular pixel, which is then randomly sampled from a Gaussian distribution every Monte-Carlo iteration.

or more noisy pixels, particularly for larger sub-apertures. To get some feel for this effect, we randomly generate noise patterns for 10 different sub-apertures, which are then used throughout the simulations (and for interest are shown in Fig. 2 for the $16 \times 16$ pixel case). Additionally, to get a better estimate for the mean performance of the sCMOS detector, we also include results where a new rms readout noise pattern was obtained every Monte-Carlo iteration (using the probability distribution, Fig. 1), the “sCMOS Random” model. It is important to note that these readout noise patterns are not a static offset added to the image. Rather, they represent the rms readout noise of the individual pixels; for each Monte-Carlo iteration, this rms value is used to generate the particular number of noise electrons introduced, randomly distributed in a Gaussian distribution with a standard deviation equal to the rms.

3 Implications for instrumental modelling of low-noise detectors

The first question that we seek to answer is the appropriateness of using the simplified EMCCD model for instrument design decisions. Fig. 3 and Fig. 4 show slope error as a function of signal level for different sub-aperture sizes, when the different EMCCD readout models are used. It can be seen that the probability distribution model agrees very closely with full stochastic model.

At intermediate flux levels, the simple model underestimates slope error slightly for small spot sizes (Fig. 3), while there is a slight overestimation of slope error for larger spot sizes (Fig. 4). However, the difference between the simple model and stochastic model are small, and unlikely to be a dominant source of error for AO instrument models. We therefore recommend that it is appropriate to use the simple EMCCD model during AO system analysis and design.

For astrometry, the case is not so simple. Here, the difference in spot position determination accuracy between the models may be more significant. Therefore we recommend that design studies for astrometric instruments should at least investigate a full EMCCD stochastic model (or
**Fig 3** A figure showing slope error as a function of signal level for the different EMCCD readout models, as given in the legend. The groups of lines (differentiated by colour or shade) represent different sub-aperture sizes, as given by the annotations on the figure. This figure is for a narrow spot diameter (Airy minimum separated by 2 pixels).

**Fig 4** A figure showing slope error as a function of signal level for the different EMCCD readout models, as given in the legend. The groups of lines (differentiated by colour or shade) represent different sub-aperture sizes, as given by the annotations on the figure. This figure is for a wide spot diameter (Airy minimum separated by 10 pixels).
probability distribution model), rather than assuming that the simple model is accurate enough. We discuss this further in §3.4.

3.1 EMCCD gain

Throughout our modelling, we have used a mean EMCCD gain of 500, which is close to the value that we frequently use on-sky with CANARY. However, Fig. 5 also shows slope estimation error at different levels of mean gain, for different light levels. It is clear here, that at the lowest light levels performance predicted by the “EMCCD Simple” model is worse than that of other models. We note that at 100 photons per sub-aperture (and at higher light levels), the “EMCCD simple” model is optimistic. We also note that the “EMCCD Stochastic” and “EMCCD Distribution” models give almost identical performance.

3.2 Impact of thresholding schemes

Fig. 6 shows the improvement in slope error brought about by thresholding of the EMCCD output for different signal levels, as a function of EMCCD gain, when compared with an unprocessed stochastic multiplication model. It can be seen that by using the thresholding scheme and applying the photometric correction, a reduction in slope estimation error is achievable, reducing the error by up to 5% under certain signal level conditions. We note that the photometric correction is necessary: applying thresholding without this correction results in poorer performance. The reduction in slope error is at best about 5%, and at the lowest light levels performance is worse, and therefore we recommend that further investigation is required for a given situation (sub-aperture size, spot size, etc) before this strategy should be considered.
Fig 6 The improvement in slope estimation accuracy resulting from application of a thresholding scheme as a function of EMCCD gain at different signal levels (as given within the plots), compared to a stochastic multiplication gain model. An improvement is signified by a value greater than unity (the dotted line at unity refers to performance without thresholding). Results with, and without, photometric correction are given, and the sub-aperture size is $8 \times 8$ pixels, for a spot of diameter 4 pixels (first Airy minimum).

3.3 sCMOS model implications

The parameter most commonly given for sCMOS readout noise by camera manufacturers is the median value, which is as low as 0.8 photoelectrons for second generation devices. The root-mean-square (RMS) readout noise is also sometimes given, with typical values around 1.1 photoelectrons. For instrument design studies, it can be tempting to use either of these values, or something in between, when modelling sCMOS detectors, for example Ref. 8 use a value of 1 photoelectron as representative of sCMOS readout noise.

Here, we compare slope estimation accuracy using both the typical median and mean values, and also using models with inter-pixel variation in readout noise, following the distribution given in Fig. 1. This probability distribution gives a median readout noise of 0.8 photoelectrons, and a mean of 1.08.

Fig. 7 shows slope estimation accuracy comparing these different models when large sub-apertures ($16 \times 16$ pixels) are used. The EMCCD stochastic model performance is also shown for comparison. It is interesting to note that using the median and mean sCMOS models provides significantly better performance than the EMCCD model. At first sight, if one of these simple sCMOS models is used during instrument development it will appear that sCMOS technology is more appropriate for Shack-Hartmann wavefront sensing than EMCCD technology. However, once the probability distribution for readout noise is taken into account, this is clearly no longer the case. As Fig. 7 shows, true sCMOS performance is significantly worse than that predicted using the simple models.
Fig 7 A figure showing slope estimation error as a function of signal level for different detector readout models, for a sub-aperture with $16 \times 16$ pixels.

3.3.1 The spread of sub-aperture performance

The ten curves for sub-apertures with different fixed readout noise patterns in Fig. 7 show a significant spread in slope error. This is because some of these sub-apertures are “unlucky” (Fig. 2), in that they contain one or more pixels with readout noise in the tail of the probability distribution (Fig. 1). Even the “lucky” sub-apertures, which yield lowest slope error, still have performance significantly worse than simple readout models predict, and still significantly worse than EMCCD performance. This is because the pixels within these sub-apertures still have a range of readout noise levels (the highest noise pixel in the best sub-aperture having a readout noise of 5.95 electrons, and the highest noise pixel in the worst sub-aperture having a readout noise of 9.34 electrons).

To get an idea of “average” expected performance using a sCMOS detector, the “sCMOS Random” model was used: every Monte-Carlo iteration, each pixel is assigned a new rms readout noise from the probability distribution. This rms readout noise is then used to obtain the number of readout electrons introduced that iteration, using a Gaussian distribution with standard deviation equal to the rms. In effect, this allows us to sample average performance over a large number of sub-apertures, and results are given by the “sCMOS random noise pattern” curve in Fig. 7. It can be seen here that this offers significantly worse slope estimation accuracy than either the EMCCD or simple sCMOS models.

Currently available sCMOS detectors all have large pixel counts. Therefore, for applications requiring low order wavefront sensing, where fewer pixels are required, it may be possible to select an area of the sCMOS detector where rms readout noise is generally low. However, this will be device dependent, and we do not consider it further here.
3.3.2 Performance dependence on sub-aperture size

Fig. 8 shows slope estimation accuracy for different detector readout models on a $4 \times 4$ pixel sub-aperture. For all but the lowest light levels, the “average” expected performance using the sCMOS detector (the “sCMOS Random” model) is better than that of the EMCCD. It is interesting to note that some sub-apertures are “lucky”, with performance at the level of that predicted by simple sCMOS models (i.e. constant readout noise equal to median or mean). This is because, with far fewer pixels, there is a higher probability that all pixels within a sub-aperture can avoid the tail of the probability distribution. Of the 10 sub-aperture readout noise patterns used, the maximum rms readout noise varied between 0.94 (for the best sub-aperture) and 7 electrons (for the worst). The mean rms values ranged from 0.72 to 1.3 electrons.

Fig. 9 shows slope estimation accuracy as a function of sub-aperture size. It can be seen that EMCCD performance is better than sCMOS performance for sub-aperture sizes equal to and greater than $6 \times 6$ pixels. For comparison, the simple sCMOS model results are also provided, and show that performance will be greatly overestimated if these models are used.

Therefore, we recommend that proper models of sCMOS readout noise should always be used when modelling instrument performance.

3.3.3 Performance dependence on spot size

Fig. 10 shows slope estimation accuracy as a function of Shack-Hartmann spot size. For smaller sub-apertures, sCMOS performance is better than EMCCD performance. However, for larger sub-apertures, EMCCD performance is generally better, particularly as spot size increases (with the available flux being spread over more pixels). We note that performance of sCMOS technology predicted using the full noise distribution model is always significantly worse than performance predicted using a simple (constant rms readout noise) model for larger sub-apertures.
**Fig 9** A figure showing slope estimation error as a function of sub-aperture size for different detector readout models as given in the legend. The sub-aperture size refers to the linear dimension, i.e. the square root of the total number of pixels within a sub-aperture.

**Fig 10** A figure showing slope estimation error as a function of spot diameter (Airy ring first minimum diameter) for different detector readout models.
Fig 11 A figure showing slope estimation error as a function of signal level for different sCMOS readout noise models, for a $4 \times 4$ (left) and $16 \times 16$ (right) pixel sub-aperture. The dashed lines show slope error predicted using a constant rms readout noise, the value of which is given adjacent to the lines. The solid line shows the average expected slope area from a sCMOS detector (the “sCMOS Random” model). The dotted lines represent the “sCMOS Fixed” model, and are shown to give an idea of spread in performance.

3.3.4 A simple model for sCMOS readout noise

We have established that using the mean or median sCMOS rms readout noise when estimating instrumental performance is optimistic. Unfortunately, using a full probability distribution will introduce additional complexity to instrumental modelling, and increase the parameter space that requires exploration, in part due to the need to randomly sample different parts of the probability distribution (to sample different areas of a detector) to obtain an average expected performance. Therefore, if a single-parameter model sCMOS readout noise can be obtained, this will greatly simplify instrumental modelling.

Fig. 11 compares slope estimation error ($R$) for different readout noise models. These include the ten “sCMOS Fixed” models identified earlier (e.g. Fig. 2), the “sCMOS Random” model, and also models with a range of constant rms readout noise values. By comparing the “sCMOS Random” model with the closest constant rms model for a given signal level, we can get a feel for the effective readout noise of the detector for that particular case.

To make sense of this information, and to provide a useful reference for future instrument modelling, Fig. 12 shows the single-value rms readout noise that will provide the same performance as predicted by the “sCMOS Random” model, for different sub-aperture and spot sizes. To use this figure when modelling a specific AO instrument, the known sub-aperture and spot size can be used to read off an effective rms Gaussian readout noise for a sCMOS detector on the figure, i.e. a single readout value for the detector. This effective rms Gaussian readout noise can then be used to predict AO system performance, giving a similar result as that expected if the full randomly sampled rms readout noise probability distribution had been used, but with reduced complexity.

It is important to note that different sCMOS detector generations and chip sizes will have a
Fig 12 A figure showing the effective Gaussian sCMOS rms readout noise that predicts the same slope estimation performance as that predicted by random sampling of the rms readout noise probability distribution. Values are given as a function of sub-aperture size and spot diameter (Airy ring first minimum diameter). Six different signal levels are shown, with the number of photons per sub-aperture given inset in the figures.

different rms readout noise probability distribution. We therefore recommend that an equivalent to Fig. 12 should be generated for the specific detector family under consideration in an instrument design. Using this information will then allow a more accurate prediction of instrumental performance to be made.

3.3.5 Elongated spots for laser guide stars

So far, we have only considered Shack-Hartmann point spread functions (PSFs) with circular symmetry. However, it is also important to consider the case when extended LGS sources are used, producing elongated PSFs. Fig. 13 shows slope estimation error as a function of elongation for a $16 \times 16$ sub-aperture. A two-dimensional Gaussian model has been used for the LGS spot PSF, with the Gaussian standard deviation in one dimension investigated. We note that using a standard deviation of unity gives a spot of size broadly equivalent to an Airy disk with the diameter of the first minimum being six pixels. We apply the different readout noise models to these elongated spots as described previously.

It can be seen that EMCCD technology provides lowest error. All three EMCCD readout noise models predict very similar performance, and so only the “EMCCD Stochastic” model is shown for clarity. It is interesting to note that as the spot becomes more elongated, sCMOS performance predicted using the “sCMOS Mean” model becomes closer to that predicted by the “sCMOS Random” model, thus suggesting that a simple model for sCMOS readout noise is applicable for elongated LGS spots. We note that in a real AO system, the degree of LGS elongation will depend on sub-aperture position within the telescope pupil, and some sub-apertures will remain almost un-elongated. In this case, the simple sCMOS readout noise model is optimistic, and so we recommend that a full sCMOS readout noise model based on the rms readout noise probability distribution should be used whenever readout noise is a key instrument design consideration.
3.3.6 Considerations of quantum efficiency

We have so far ignored detector QE, and assumed identical QE for all detector models (though we halve the effective QE for the simple EMCCD model). The QE of EMCCD devices can reach 95% (e.g. the Andor iXon3), while for sCMOS detectors, it is closer to 70% (e.g. the Andor Zyla 4.2). Fig. [14] shows slope estimation error once QE is taken into account, and can be compared directly with Fig. [8] (which assumes identical QE). It can be seen here that EMCCD performance is now at least as good as that predicted by the “sCMOS Random” model, i.e. that in practice, an EMCCD detector is likely to perform as well as a sCMOS detector for $4 \times 4$ pixel sub-apertures (and, as we have seen previously, better for larger sub-apertures).

3.4 Astrometric accuracy

We have investigated the effect of detector readout noise models on image centroiding accuracy. In addition to importance for AO systems, accurate position determination is critical for astrometric techniques.

We have shown that there are only small differences in estimation accuracy between the commonly used simple EMCCD model, and a full stochastic gain mechanism model. However, for some astrometric observations, this difference may be critical, and therefore we recommend that the full stochastic gain mechanism (or the EMCCD output probability distribution model, which is almost identical) should be used, until it can be demonstrated that the simple model is sufficient for each particular instrument study. We note here that the stochastic model is computationally more expensive than other models.

When using sCMOS technology for astrometric applications, greater care is required. We have shown that a simple model of sCMOS readout noise based on a single rms value for all pixels
Fig 14 A figure showing slope estimation error as a function of incident signal level for a 4 × 4 pixel sub-aperture. A 70% QE is assumed for sCMOS models, and 95% QE for EMCCD models.

(whether the median or mean) is optimistic. Therefore, a model for sCMOS readout noise that uses the per-pixel probability distribution for rms noise, is essential. Further model improvements can be made if the precise rms readout noise pattern for a physical detector under consideration can be used (i.e. once the detector has been acquired), though we do not consider this further here.

4 Conclusions

We have investigated detector readout models for sCMOS and EMCCD technologies, and the effect that these models have on slope estimation accuracy for Shack-Hartmann wavefront sensors used in AO systems. Our findings are also relevant to any problem involving image centre of mass location, including astrometry. We find that in general EMCCD technology offers better performance than sCMOS technology for Shack-Hartmann wavefront sensors and other applications requiring centre of mass calculations.

We find that the commonly used simple model for EMCCD readout (halving the effective QE and assuming a sub-electron readout noise) is sufficient for AO applications with predicted slope estimation accuracy differing only slightly from when using a full Monte-Carlo stochastic gain mechanism model. A model based on EMCCD probability output distribution also performs almost identically to the stochastic gain model.

For sCMOS technology, we find that the commonly used model that uses a single rms readout value for all pixels (whether the median or mean) produces optimistic results, which can predict better performance than that obtained by EMCCD detectors. However, more reliable performance estimates during instrument development and design studies can be made by taking a typical sCMOS rms readout noise probability distribution into account, and we find that this model generally predicts worse performance than that obtained by EMCCD detectors. Ideally, many random samples of this distribution should be taken, so that an average (and worst case) performance estimate for sCMOS technologies can be obtained. A key finding is that using the median or mean sCMOS
rms readout noise value is not sufficient to accurately predict instrumental performance: the full probability distribution for sCMOS readout noise should be used.

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References

1. P. Jerram, P. J. Pool, R. Bell, D. J. Burt, S. Bowring, S. Spencer, M. Hazelwood, I. Moody, N. Catlett, and P. S. Heyes, “The llccd: low-light imaging without the need for an intensifier,” (2001).
2. R. M. Myers, Z. Hubert, T. J. Morris, E. Gendron, N. A. Dipper, A. Kellerer, S. J. Goodsell, G. Rousset, E. Younger, M. Marteaud, and A. G. Basden, “CANARY: the on-sky NGS/LGS MOAO demonstrator for EAGLE,” in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Presented at the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference 7015 (2008).
3. T. Fusco, G. Rousset, J. F. Sauvage, C. Petit, K. Beuzit, J. L. Dohlen, D. Mouillet, J. Charton, M. Nicolle, M. Kasper, P. Baudou, and P. Puget, “High-order adaptive optics requirements for direct detection of extrasolar planets: Application to the SPHERE instrument,” Opt. Express 14, 7515–7534 (2006).
4. C. Coates, B. Fowler, and G. Holst, “Scientific cmos technology: A high-performance imaging breakthrough,” http://www.scmos.com/files/low/scmos_white_paper_2mb.pdf (2009).
5. A. G. Basden, C. A. Haniff, and C. D. Mackay, “Photon counting strategies with low-light-level CCDs,” MNRAS 345, 985–991 (2003).
6. A. G. Basden, “Sensitivity improvements for shack-hartmann wavefront sensors using total variation minimisation,” MNRAS (2015).
7. PCO, “pco.edge 4.2 scientific cmos camera.” Company website. Product datasheet for the PCO.Edge 4.2 sCMOS camera.
8. A. G. Basden, “Visible near-diffraction limited lucky imaging with full-sky laser assisted adaptive optics,” MNRAS 442, 1142–1150 (2014).
9. P. Qiu, Y.-N. Mao, X.-M. Lu, E. Xiang, and X.-J. Jiang, “Evaluation of a scientific CMOS camera for astronomical observations,” Research in Astronomy and Astrophysics 13, 615–628 (2013).
10. M. S. Robbins and B. J. Hadwen, “The noise performance of electron multiplying charge-coupled devices,” IEEE Transactions on Electron Devices 50, 1227–1232 (2003).
11. O. Daigle, C. Carignan, and S. Blais-Ouellette, “Faint flux performance of an EMCCD,” in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series 6276, 1 (2006).

Alastair Basden has extensive expertise in low noise detectors and adaptive optics, including real-time control and simulation. He is an eternal postdoc at Durham University.