Anatomy of Gossamer Superconductivity

Stephan Haas,1 Kazumi Maki,1 Thomas Dahm,2 and Peter Thalmeier3

1Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484
2Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany
3Max-Planck Institute for Chemical Physics of Solids, Nöthnitzer Str. 40, D-01187 Dresden, Germany

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There are many systems in which two order parameters compete with each other. Of particular interest are systems in which these order parameters are both unconventional. In this contribution, we examine the representative example of a d-wave superconductor in the presence of a d-wave density wave, which has been suggested as a model for the pseudogap phase in the high-Tc superconductors. The physical properties of unconventional superconductivity in the presence of an anisotropic charge density wave are investigated within mean field theory. This model describes many features that were anticipated by an earlier phenomenological treatment of Tallon and Loram. In addition, the quasiparticle density of states in the presence of these two order parameters is calculated, which should be accessible by scanning tunneling microscopy.

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I. INTRODUCTION

The discovery of unconventional high-temperature superconductivity by Bednorz and Müller in 1986 took many in the community by surprise. Subsequent developments and some of the initial confusions are documented in the textbook by Enz. Among many others, Anderson pointed out in 1987 that high-Tc superconductivity should be understood in terms of a competition between a Mott insulating state in the limit of zero hole doping and a superconducting state at optimal doping. Meanwhile, experimental evidence for the d-wave nature of high-Tc superconductivity was found and general features of the phase diagram were established. However, the nature of the pseudogap phase is still being debated. However, now there is increasing consensus that this regime can be described by in terms of a d-density-wave.

Recently, Laughlin proposed a very intuitive formulation regarding how to deal with superconductors close to a Mott insulating state. This “gossamer superconductivity” is characterized by a very small superfluid density and spectral weight of the superconducting quasiparticles. In this contribution, we will apply Laughlin’s idea to a more general situation: unconventional superconductivity in the presence of unconventional density wave (UDW) order. More precisely, for high-Tc cuprates we study d-wave superconductivity in the presence of a d-density wave. This could either be a charge density wave or a spin density wave order parameter, although the distinction between these two possibilities is not of importance for the following discussion. Here, we wish to examine whether some features of the pseudogap phase can follow from the proposed model, as it was suggested by Tallon and Loram based on a phenomenological approach. Furthermore, the recently observed weakening of superconductivity in the κ-(ET)2 salts when the sample is rapidly cooled may be interpreted within the model we are presenting in this paper.

When both the superconductivity and the density wave are conventional, this competition is not of great interest as long as the system can be reduced to a single band or a single Fermi surface. In this case, there is typically little room for coexistence, and one of the order parameters, usually the superconductivity, dominates over the other one. A similar statement holds if only one of the two competing order parameters is conventional. However, the situation changes completely when two unconventional order parameters are considered. If we look at heavy fermion systems and organic superconductors, we see signs for gossamer superconductivity almost everywhere, for example in CeCu2Si2, URu2Si2, UPd2Al3, CeRhIn5, CeCoIn5, and the κ-(ET)2 salts. Therefore gossamer superconductivity as defined above may be a pervading feature in high-Tc cuprates, heavy fermion superconductors and organic superconductors.

Here, we report calculations for nodal superconductivity in the presence of nodal density wave order. For simplicity, we limit ourselves to quasi-two-dimensional systems with cylindrical Fermi surfaces. We denote the superconducting and density wave order parameters by \(\Delta_1(\mathbf{k})\) and \(\Delta_2(\mathbf{k})\), respectively, with angular dependence \(\Delta_1(\mathbf{k}) = \Delta_1 f(\mathbf{k})\) and \(\Delta_2(\mathbf{k}) = \Delta_2 g(\mathbf{k})\), where \(\Delta_1\) and \(\Delta_2\) are the maximum values of the corresponding order parameters. Further, we limit ourselves to \(f = \cos(2\phi)\) (i.e. \(d_{x^2-y^2}\)-wave) while for \(g\) we consider the two cases (a) \(g = \cos(2\phi)\) and (b) \(g = \sin(2\phi)\). The first case describes many features of the superconductivity in the pseudogap phase in high-Tc cuprates, while we conjecture that the second case applies for the glassy phase of κ-(ET)2X with X=Cu[N(CN)2]Br.
II. QUASIPARTICLE GREEN FUNCTION

Let us consider a one-band quasi-two-dimensional Fermi surface where a nodal superconductor and a nodal density wave coexist. As a model Hamiltonian we can think of a generalized Hubbard model\textsuperscript{21}. The Green function is given in $4 \times 4$ spinor space as

$$G^{-1}(\mathbf{k}, E) = E - \xi \rho_3 \sigma_3 - \eta \rho_3 - \Delta_1 f \sigma_1 \rho_1 - \Delta_2 g \rho_1 \sigma_3$$

(1)

where the Pauli matrices $\rho_i$ and $\sigma_i$ are operating on the particle-hole space and the $\mathbf{k}$ and $\mathbf{k} + \mathbf{Q}$ space, respectively, where $\mathbf{Q}$ is the nesting vector for the nodal density wave. We have assumed here that the UDW is a charge density wave. The Green function possesses poles at the quasiparticle energies

$$E = \pm \sqrt{\left(\sqrt{\xi^2 + \Delta_1^2} \pm \eta \right)^2 + \Delta_2^2 f^2}$$

(2)

which is well known in the literature\textsuperscript{9,15}. Here, $\eta$ is the imperfect nesting term. Less well-known, perhaps, is the quasiparticle density of states (DOS) given by

$$\frac{N(E)}{N_0} \equiv G(E) = \frac{1}{2} \sum_{\pm} \mathrm{Re} \left\{ \frac{|E|}{\sqrt{\left(E^2 - \Delta_1^2 f^2 \right)^2 - \Delta_2^2 f^2}} \right\}$$

(3)

where the sum has to be taken over the two terms with $\pm \eta$. Here, $\langle \ldots \rangle$ denotes the angular average over $\phi$. In the limit $\eta \to 0$ this expression reduces to a very familiar form

$$G(E) = \mathrm{Re} \left\{ \frac{|E|}{\sqrt{E^2 - \Delta_1^2 f^2 - \Delta_2^2 g^2}} \right\}$$

(4)

In particular, for case (a) when $g = f = \cos(2\phi)$ we have

$$G(E) = \left\{ \begin{array}{ll}
\frac{2}{\pi} x K(x) & \text{for } x < 1 \\
\frac{2}{\pi} K(x^{-1}) & \text{for } x > 1
\end{array} \right.$$  

(5)

where $x = E/\sqrt{\Delta_1^2 + \Delta_2^2}$ and $K$ is the complete elliptic integral of the first kind. This has the same form as in $d$-wave superconductors\textsuperscript{22} with the only exception that the $d$-wave gap has to be replaced by the total gap $\sqrt{\Delta_1^2 + \Delta_2^2}$. On the other hand, for case (b) when $f = \cos(2\phi)$ and $g = \sin(2\phi)$ the DOS develops a complete gap below $E = \min(\Delta_1, \Delta_2)$ and we find

$$G(E) = \left\{ \begin{array}{ll}
0 & \text{for } E < \Delta_1 \\
\frac{2}{\pi} \sqrt{E^2 - \Delta_1^2} y K(y) & \text{for } y < 1 \\
\frac{2}{\pi} \sqrt{E^2 - \Delta_2^2} K(y^{-1}) & \text{for } y > 1
\end{array} \right.$$  

(6)

where $y = \sqrt{(E^2 - \Delta_1^2)/(\Delta_2^2 - \Delta_1^2)}$. Here we have assumed $\Delta_1 < \Delta_2$. The corresponding DOS with $\Delta_1/\Delta_2 = 0.1$, 0.3, and 0.5 is shown in Fig. 1(b). Except for the opening of the energy gap with increasing $\Delta_1$, the DOS looks very similar to the case of $\Delta_1 = 0$, with a characteristic logarithmic singularity at $|E| = \Delta_2$.

As we shall see in the following, for case (a) a non-vanishing $\eta$ is crucial for the presence of two order parameters. In Fig. 1(a) we show the corresponding DOS for the choices $\Delta_1 = 0$ ($\eta = 0$), $\Delta_1 = 0.1\Delta_2$ ($\eta = 0.15\Delta_2$), and $\Delta_1 = 0.3\Delta_2$ ($\eta = 0.4\Delta_2$). Due to the presence of two order parameters, the logarithmic singularity at $|E| = \Delta_2$ splits into two singularities, which are now located at $|E| = \sqrt{(\Delta_2 \pm \eta)^2 + \Delta_1^2}$. We expect this double-peak structure to be accessible to scanning tunneling microscope measurements in the pseudogap phase of the high-T$_c$ cuprates.

Very recently, new measurements of the DOS in underdoped Bi2212 were reported\textsuperscript{23}. Indeed, they show a clear double-peak structure. Unfortunately, it was pointed out later that the secondary peak is a likely artifact of the heating process\textsuperscript{24,25}. However, based on the present model calculation we conclude that a second peak should indeed be visible in a careful experiment.
III. GAP EQUATIONS

For case (a) with \( f = g \), the gap equations are given by

\[
\lambda_1^{-1} = 2\pi T \sum_n \text{Re} \left\{ \frac{2f^2}{\sqrt{\left(\sqrt{\omega_n^2 + \Delta_1^2 f^2} + i\eta\right)^2 + \Delta_2^2 f^2}} \right\} \quad (7)
\]

\[
\lambda_2^{-1} = 2\pi T \sum_n \text{Re} \left\{ \frac{2f^2}{\sqrt{\left(\sqrt{\omega_n^2 + \Delta_2^2 f^2} + i\eta\right)^2 + \Delta_1^2 f^2}} \right\} \quad (8)
\]

for \( \Delta_1 \) and \( \Delta_2 \) respectively. Here, \( \lambda_1 \) and \( \lambda_2 \) are the dimensionless coupling constants for superconductivity and density wave. Also, it is assumed here that \( \lambda_1 < \lambda_2 \), because otherwise there is no chance for the UDW to occur. Furthermore, for the survival of a finite superconducting component we also need \( \eta \neq 0 \). Then, for small \( \eta \) one finds \( \Delta_1(0)/\Delta_2(0) \sim (\eta/\Delta_2(0))^{3/2} \), where \( \Delta_1(0) \) and \( \Delta_2(0) \) are the corresponding order parameters in the zero-temperature limit. Moreover, for the energy gap that can be observed by scanning tunneling microscopy we find at low temperatures \( 0.5(\sqrt{\Delta_2 + \eta} + \sqrt{\Delta_2 - \eta}) \approx \Delta_2(1 + \Delta_2^2/(2(\Delta_2^2 - \eta))) \). Thus, in the limit \( \Delta_1/\Delta_2 \ll 1 \) and \( |\eta|/\Delta_2 \ll 1 \), scanning tunneling microscopy will still identify \( \Delta_2(0) \). Therefore, the puzzling result that the detected gap \( \Delta(0) \approx 2.14T_c \), where \( T_c \) is the transition temperature of the pseudogap phase, can be naturally interpreted in terms of a \( d \)-density wave. Finally, since the corresponding superfluid density at zero-temperature is given by

\[
\rho_s(0,\eta) = \frac{\Delta_2^2(0)}{\Delta_1^2(0) + \Delta_2^2(0)}, \quad (9)
\]

there should be a substantial reduction of this quantity in the coexistence regime.

For the case (b) where \( f \neq g \) the gap equations are now given by

\[
\lambda_1^{-1} = 2\pi T \sum_n \text{Re} \left\{ \frac{2f^2}{\sqrt{\left(\sqrt{\omega_n^2 + \Delta_1^2 f^2} + i\eta\right)^2 + \Delta_2^2 g^2}} \right\} \quad (10)
\]

\[
\lambda_2^{-1} = 2\pi T \sum_n \text{Re} \left\{ \frac{2f^2}{\sqrt{\left(\sqrt{\omega_n^2 + \Delta_2^2 f^2} + i\eta\right)^2 + \Delta_1^2 g^2}} \right\} \quad (11)
\]

In contrast to case (a), here we always encounter coexistence of two order parameters, even when \( \eta = 0 \). In particular,
for the case \( \eta = 0 \), Eqs. 10 and 11 can be solved at zero temperature.

\[
\Delta_2(0) + \Delta_1(0) = 2\sqrt{E_1E_2}\exp\left[-(\lambda_1^{-1} + \lambda_2^{-1})/2\right]
\]

(12)

\[
\frac{\Delta_2(0) - \Delta_1(0)}{\Delta_2(0) + \Delta_1(0)} = \lambda_1^{-1} - \lambda_2^{-1},
\]

(13)

where \( E_1 \) and \( E_2 \) are cut-off energies. Furthermore, the zero-temperature superfluid density is given by

\[
\rho_s(0,0) = \frac{\Delta_1(0)}{\Delta_2(0) + \Delta_1(0)}.
\]

(14)

Although the superfluid density is reduced in the present case, the reduction is not as severe as it is for case (a).

IV. CONCLUDING REMARKS

In this paper, we have analyzed the properties of a nodal superconductor in the presence of a nodal density wave. Our preliminary results look very consistent with experimental observations that have been reported for the pseudogap region of the high-\( T_c \) cuprates. For example, in contrast to the large quasiparticle energy gap observed by scanning tunneling microscopy, the corresponding superfluid density and the specific heat associated with superconductivity are much reduced. The present model calculation appears to describe these features consistently, although more measurements on this issue are clearly necessary. The application of this model to other potential “gossamer superconductors” is promising.

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