Strength reduction factor of square reinforced concrete column

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Abstract: This paper investigates the strength reduction factor (φ) of reinforced concrete (RC) columns using Monte-Carlo simulation (MCS). The main objective of this paper is to evaluate the strength reduction factor of the RC using the authors’ developed code. This code is important for further research to check other important effects when high-strength materials are used. The investigated RC column concrete compressive strengths (f_c) are 40 and 60 MPa while the rebar strengths (f_y) are set to 320, 400, and 500 MPa. Fiber-based cross-sectional analysis is used to compute the axial-moment interaction capacity of the RC column. The concrete compressive block is used to model the concrete contribution and the bilinear stress-strain model is adopted for the rebar. These simplifications can reduce the difficulties when solving the equilibrium of the forces in the sectional analysis. The parameters used in the sensitivity analysis of the strength reduction factor (φ) are the concrete compressive strength (f_c), the rebar yield strength (f_y), the longitudinal rebar ratio (ρ), and the column size (b,h). The effect of the coefficient of variations for each material on the resistance variation coefficient of the RC is also investigated. From the analysis, it can be concluded that when the RC column falls in the tension-controlled region, the obtained strength reduction factor is 0.93 which is slightly higher than the value of φ in ACI 318-19. On the other hand, when the RC column fails in the compression-controlled region, the obtained strength reduction factor is 0.6 which is lower than the value of φ in ACI 318-19 which is 0.65.

Keywords: Reinforced concrete, strength reduction factor, reliability index, Monte Carlo simulation.

INTRODUCTION

The strength reduction factor of the reinforced concrete (RC) plays an important role to ensure its safety. Failure of the RC column can be devastating as it supports the RC beam and carries the applied load on the structure. These loads can generate both axial and bending moment forces in the RC columns. Although this issue has been addressed in many building codes, it is important to note that studies on the reliability of RC columns considering the usage of high-strength materials are found to be rare. To investigate this issue further, the authors developed a computer code that was based on two-dimensional meshed elements sectional analysis with fiber-based method and can be assigned with any constitutive model of the materials [1]. The carried-out analysis in this paper is limited to the use of stress-block parameters for concrete and bilinear elastic-perfectly plastic model for the steel reinforcing bar. Another implementation of two-dimensional meshed elements for nonlinear analysis can be found in [2, 3].

Before looking at the advanced material properties, the developed code should be firstly evaluated with the well-known strength of materials and compared with the available building code. The strengths of concrete material considered in the analysis are 40 and 60 MPa which represent the normal- to medium-strength concrete. On the other hand, the steel yield strength considered is 320, 400, and 500 MPa which are available widely for construction in Indonesia. For verification of the strength reduction factor, the strength reduction factor from ACI 318-19 [4] is used and is compared with the strength reduction factor obtained from the analysis.

To evaluate the strength reduction factor of the RC column, the first-order reliability method (FORM) can be used. The reliability analysis (FORM) is generally performed by evaluating the probability of failure of the limit state functions (G) by using the certain value safety index (β)[5]. The limit state function is also known as the objective function and can be obtained by subtracting the applied load (S) from the resistance (R).

The resistance of a reinforced concrete column generally is very complex due to the complexity of the materials model. Therefore, an alternative technique is needed to evaluate the column resistance thoroughly. Several techniques can be used to determine the level of relationship between several variables, and one of them is Monte-Carlo Simulation (MSC). MSC is a really popular, powerful method, easy to implement, and can solve probabilistic problems with a fairly wide scope ranging from simple to more complex [6]. MSC is defined as a statistical sampling technique that can be used as a solution to quantitative problems. MSC combines the deterministic relationship between the performance of a system and each variable that affects that performance as well as the statistical properties of the distribution of all known variables [7]. Hence, in this paper, MCS along with the FORM to investigate the strength-reduction factor of the RC column is used.

RESEARCH SIGNIFICANCE

A more rational probabilistic theory approach using the combination of MCS and the first-order reliability method (FOSM) is used in this paper to evaluate the effect of various variables on the strength reduction factor of the square reinforced concrete column. The developed code is based on the two-dimensional meshed elements sectional analysis with the fiber-based method which can be extended further to include complex material constitutive models, irregular sections, and nonlinear sectional analysis of RC columns.

METHODOLOGY

The methodology in this paper consisted of three stages. The first stage consisted of merging the two-dimensional meshed elements sectional analysis with fiber-based method [2, 3] which replaces the standard sectional analysis of RC column [1] inside the MCS and FORM.
To evaluate the strength reduction factor of 
\( f_{cr} \)

\[ \sigma_{cr} = \sqrt{2 \log (1 / x_c)} \cos (2 \pi x_c) \] (6)\n
where \( f_{cr} \) and \( \sigma_{cr} \) are the mean concrete compressive strength and the standard deviation of \( f_{cr} \), respectively. In Eqn.(5), the margin 1.34 is related to the probability of failure allowed to nine percent.

To generate the rebar yield input, the following expressions are used:

\[ f_{sy} = f_{sr} + 1.125 \sigma_{sy} \]

\[ \sigma_{sy} = \sqrt{2 \log (1 / x_y)} \cos (2 \pi x_y) \] (8)\n
where \( f_{sy} \) and \( \sigma_{sy} \) are the mean rebar yield strength and the standard deviation of \( f_{sy} \), respectively. In Eqn.(7), the margin 1.125 is related to the probability of failure allowed to thirteen percent.

C. MONTE-CARLO SIMULATION (MCS) AND FIRST ORDER RELIABILITY METHOD (FORM)

Monte-Carlo simulation (MCS) is often used to generate the probabilistic distribution of a deterministic system. MCS works by repeating calculations with random variables as the inputs. The random variables are prepared using Box and Muller method as previously discussed. The objective function \( G(R, S) \) to be satisfied can be evaluated with:

\[ G(R, S) = R - S \] (9)

\[ R = \left[ P^2 + \left( \frac{M}{h} \right)^2 \right]^{\alpha_3} \] (10)

\[ S = \left[ (D + L) + \left( \frac{(D + L) e}{h} \right) \right]^{\alpha_3} \] (11)

where \( R \) and \( S \) are the resistance and the applied loads, respectively. In Eqn.(10), \( P \) is the axial load, \( M \) is the bending moment, and \( h \) is the column height or width in the loading direction. In Eqn.(11), \( D \) is the dead load, \( L \) is the live load, \( e \) is the load eccentricity.

By knowing the ratio of live to dead loads (\( R_L / R_D \)) and the safety index of the system, it is possible to compute the mean resistance \( \bar{R} \) as:

\[ \bar{R} = \frac{-2 \left[ R_{LD} + 1 \right] \sqrt{x}}{2 \left( \frac{\left[ \Omega_{KL} \right]}{1} \right)^2} \] (12)

\[ x = \left[ \frac{1}{2} \left[ \frac{1}{2} \left( \frac{\Omega_{KL}}{1} \right)^2 - 4 \left( \frac{\beta \Omega_{KL}}{1} \right)^2 \right] - \frac{R_{LD} + 1}{1} \right] \] (13)

where \( \beta \) is the safety index, \( \Omega_K \) and \( \Omega_D \) are the coefficient of variation of the resistance and the dead load, respectively.

Finally, the strength reduction factor (\( \phi \)) can be computed as:

\[ \phi = 1 - \left( \frac{\alpha_2 \beta \Omega_{KL}}{\sqrt{\alpha_2 \beta \Omega_{KL}}} \right) \] (14)
Table 1 Input data for parametric study

| No | Evaluated Variables                          | Column Dimension | ρ (%) | f'c (MPa) | fy (MPa) | Ωconcrete | Ωrebar |
|----|---------------------------------------------|------------------|-------|-----------|-------|-----------|--------|
| 1  | Global variation of the RC column           | 500 x 500        | 3%    | 40        | 400   | 20%       | 8%     |
| 2  | Effect of the concrete compressive strength | 500 x 500        | 3%    | 40        | 400   | 20%       | 8%     |
| 3  | Effect of the steel rebar yield strength    | 500 x 500        | 3%    | 40        | 320   | 20%       | 8%     |
| 4  | Variation in the steel rebar yield strength (Ωx = 0) | 500 x 500        | 3%    | 40        | 400   | 0%        | 8%     |
| 5  | Effect of variation in the steel reinforcing bar quality (Ωx = 0) | 500 x 500        | 3%    | 40        | 400   | 0%        | 8%     |
| 6  | Effect of variation in the concrete material quality (Ωx = 0) | 500 x 500        | 3%    | 40        | 400   | 30%       | 0%     |
| 7  | Effect of variation in the longitudinal bar reinforcement ratio (ρ) | 500 x 500        | 3%    | 40        | 400   | 20%       | 8%     |
| 8  | Effect of the RC column cross-sectional area | 500 x 500, 600 x 600, 700 x 700 | 3%    | 40        | 400   | 20%       | 8%     |

Note:
Ωc: Coefficient of variation for the concrete material
Ωs: Coefficient of variation for the steel reinforcing material

\[ \alpha_s = \frac{\sigma_s}{\sqrt{\sigma_{\varepsilon_s}^2 + \sigma_{\sigma_0}^2 + \sigma_{\sigma_t}^2}} \] (15)

In the above, \( \sigma_{\varepsilon_s} \) is the ratio between the nominal to mean value of the strength reduction factor, \( \sigma_0 \) is the standard deviation of the resistance, \( \sigma_0 \) and \( \sigma_t \) is the standard deviation for the dead and live loads. The sequence from Eqns.(12) to (15) is also known as the first-order reliability method or FORM which evaluates the direction cosines of the failure surface to compute the reliability of the system.

Table 1 shows the input data used in the parametric study of the strength reduction factor for the square RC column. As shown in Table 1, the input variables consisted of variation in the steel rebar yield strength, variation in the concrete compressive strength, variation in the steel reinforcing bar quality, variation in the concrete material quality, effect of variation in the longitudinal bar reinforcement ratio, and the effect of variation in the RC column cross-sectional area.

**ANALYSIS AND DISCUSSION**

A. PARAMETRIC STUDY OF VARIATION COEFFICIENT

This section shows the result of the simulation of the concrete square column by using Monte Carlo Simulation (MCS) combine with the first-order reliability method (FORM). Figure 1 shows the effect of eccentricity on the resistance global coefficient of variation resistance. In Figure 2, the value of \( \Omega_{concrete} = 20\% \) and \( \Omega_{steel} = 8\% \) were used. Figure 2 also shows the global coefficient of variation by setting the \( \Omega_{concrete} = 20\% \) and \( \Omega_{steel} = 0\% \), and \( \Omega_{concrete} = 0\% \) and \( \Omega_{steel} = 8\% \). The purpose was to gain insight into how the concrete or the steel rebar materials affects the resistance global coefficient of variation. From Figure 2, it can be inferred that at a small value of \( e/h \), the contribution of concrete material to the RC column resistance is much higher than the steel rebar. On the other hand, as the ratio of \( e/h \) is greater than two (see Figure 2), the steel rebar material dominated the portion of the RC column resistance. Another thing that can be investigated from Figure 2 is that when the eccentricity ratio \( (e/h) \) is greater than equal to two, the resistance global coefficient of variation is asymptotic to a value of 6.89 % which was lower than the expected value of eight percent. The same thing goes for the resistance global coefficient \( (\Omega_R) \) of variation at a small value of \( e/h \). The \( \Omega_R \) at a small value of \( e/h \) is around 17.01 % which was lower than twenty percent.

Figure 2. Coefficient of variation of the resistance \( (\Omega_R) \) \( (\Omega_{concrete} = 20\%, \Omega_{steel} = 8\%, f'_c = 40 \text{ MPa}, f_y = 400 \text{ MPa}) \)

Figure 3 shows the effect of concrete compressive strength on the \( \Omega_R \). As shown in Figure 3, for a small eccentricity ratio \( (e/h = 0.1) \), the value of \( \Omega_R \) drop from 17.01 % to 16.04 % when the concrete compressive strength increases from 40 to 60 MPa. On the other hand, for a large eccentricity ratio \( (e/h = 5) \), the value of \( \Omega_R \)
increases from 6.89% to 7.60% for 40 and 60 MPa concretes, respectively. A slight increase or decrease in the $\Omega_R$ can be well understood because the height of the stress block is a function of the concrete compressive strength. For a small eccentricity ratio ($e/h = 0.1$), the lower height of the stress block reduces the concrete contribution to carry the axial load and thus renders the steel rebar to contribute more. This explains why the value of $\Omega_R$ drops about -5.7% for large eccentricity ratio ($e/h = 5$), with the same reason as the small eccentricity ratio ($e/h = 0.1$) lead into increases value of $\Omega_R$ about 10.3%.

![Figure 3. Effect of the concrete compressive strength on the $\Omega_R$](image)

Figure 3. Effect of the concrete compressive strength on the $\Omega_R$

Figure 4 shows the effect of the steel yield strength on the $\Omega_R$. As shown in Figure 4, the reduced value of $\Omega_R$ when the eccentricity ratio is small ($e/h = 0.1$) was found to be quite significant. The value of $\Omega_{rebar}$ at this state reduced from 18.11% to 14.85% (18% drops in $\Omega_R$) when the yield strength increases from 320 to 500 MPa. The significant drop in $\Omega_R$ was owned by the increased contribution of steel rebar to carry compression load. On the other hand, for the large eccentricity ratio ($e/h = 5$), the difference was found to be small ($\Delta \Omega_R$ value decreases from 7.34% to 6.64%). This small decrease can be understood as the mean yield strength of the 500 MPa rebar is higher than the 320 MPa rebar which then reduces the variation in $\Omega_R$.

![Figure 4. Effect of the yield strength of the steel rebar to the $\Omega_R$](image)

Figure 4. Effect of the yield strength of the steel rebar to the $\Omega_R$

To further isolate the effect of steel rebar yield strength to the $\Omega_R$, the coefficient of variation in the concrete material ($\Omega_{concrete}$) is set to zero. Figure 5 shows the effect of $\Omega_{concrete}$ equal to zero. As shown in Figure 5, for a small eccentricity ratio ($e/h = 0.1$), it was found out that there was no significant difference between $f_y$ equal to 320 and 400 MPa. When $f_y$ changes to 500 MPa from 400 MPa, the value of $\Omega_R$ drops from 1.57% to 1.03%. Furthermore, for 500 MPa steel rebar yield strength, when the eccentricity ratio equal to 0.6, the value of $\Omega_R$ drops further to 0.32%. At this point, it was unclear the reason for this further drop. For the large eccentricity ratio ($e/h = 5$), the value of $\Omega_R$ for $f_y$ equal to 500 MPa was between the $f_y$ equal to 320 and 400 MPa which the authors also found to be an anomaly.

![Figure 5. Effect of the yield strength of steel rebar to the $\Omega_R$ when $\Omega_{concrete} = 0$](image)

Figure 5. Effect of the yield strength of steel rebar to the $\Omega_R$ when $\Omega_{concrete} = 0$

For the large eccentricity, the coefficient of variation of the steel rebar material quality to $\Omega_R$ with $\Omega_{concrete} = 0$ resulted in a higher value of $\Omega_R$. For small the small eccentricity ratio ($e/h = 0.1$), the difference of $\Omega_R$ was found to be insignificant. The increase of $\Omega_R$ when changing $\Omega_{rebar}$ from 6% to 10% was increased from 1.26% to 2.05%. However, for large eccentricity ratio ($e/h = 5$), the difference of $\Omega_R$ was significant with the increase of $\Omega_R$ from 4.62% to 7.62% for $\Omega_{rebar}$ equal to 6 and 10%, respectively. By looking more detail on the percentage of changes in $\Omega_R$, for small eccentricity ratio ($e/h = 0.1$) the percentage increase is 62.7% while for large eccentricity ratio ($e/h = 5$) the percentage increase is 64.9%. Hence, it can be concluded that the effect of steel reinforcement material quality affects the $\Omega_R$ for any eccentricity ratio.

![Figure 6. Effect of the steel reinforcement material quality to $\Omega_R$ with $\Omega_{concrete} = 0$](image)

Figure 6. Effect of the steel reinforcement material quality to $\Omega_R$ with $\Omega_{concrete} = 0$

Figure 7 shows the effect of concrete material quality value on the value of $\Omega_R$. To isolate the discussed effect, the coefficient of variation of the steel rebar ($\Omega_{rebar}$) is set to zero. As shown in Figure 7, the difference in the $\Omega_R$ magnitude was found to be significant when the eccentricity ratio is small ($e/h = 0.1$). The $\Omega_R$ values when $\Omega_{concrete}$ increased from 10 to 30 percent are 8.58% and
24.49 %, respectively. The percentage increase of \( \Omega_R \) for small eccentricity ratio (\( e/h = 0.1 \)) is 185 %. For large eccentricity ratio (\( e/h = 5 \)), the \( \Omega_R \) value are 1.16 % and 3.16 % when the \( \Omega_{\text{concrete}} \) are 10 % and 30 %, respectively. The increase of \( \Omega_R \) for large eccentricity ratio (\( e/h = 5 \)) is 172 %. By looking at the percentage increase of \( \Omega_R \) it can be concluded that the effect of concrete material quality also affects the \( \Omega_R \) for any eccentricity ratio.

Figure 7. Effect of the concrete material quality on the \( \Omega_R \) With \( \Omega_{\text{rebar}} = 0 \)

Figure 8. Effect of longitudinal rebar ratio to the \( \Omega_R \) (\( \Omega_{\text{concrete}} = 20\% \), \( \Omega_{\text{steel}} = 8\% \), \( f_c = 40 \) MPa, \( f_y = 400 \) MPa)

Figure 9. Effect of Variation in Column Dimension to \( \Omega_R \) (\( \Omega_{\text{concrete}} = 20\% \), \( \Omega_{\text{steel}} = 8\% \), \( f_c = 40 \) MPa, \( f_y = 400 \) MPa)

Figure 9 shows the effect of RC column dimension or cross-sectional area on the value of \( \Omega_R \). As shown in Figure 9, the effect of RC column dimension was barely noticeable and thus it can be neglected. It should be noted that the coefficient of variation for the column dimension is still not included in the MCS. It is possible that if the coefficient of variation for the column dimension is included. The global resistance coefficient of variation may be affected.

B. STRENGTH REDUCTION FACTOR OF SQUARE REINFORCED CONCRETE COLUMN

This section detailed discuss the strength reduction factor of the square RC column with varying geometry and material properties. The investigated column had a dimension of 400 x 400 mm and a concrete cover thickness of 30 mm. The hoops diameter is set to 10 mm dan the longitudinal rebar diameter is 22.70 mm. The investigated longitudinal rebar ratios (\( \rho \)) are 3, 5, and 8 % which are consisted of 12, 20, and 32 longitudinal bars (\( n_{\text{bar}} \)). To study the effect of material strengths, 40, 50, and 60 MPa concrete strengths and 320, 400, and 500 MPa rebar yield strengths are used in the simulation. Finally, three safety index (\( \beta_{\text{index}} \)) values are investigated which are 3, 3.5, and 4. For all cases, the standard input parameters besides the adjusted parameters previously are: \( \Omega_{\text{concrete}} = 20\% \), \( \Omega_{\text{steel}} = 8\% \), \( f_c = 40 \) MPa, \( f_y = 400 \) MPa, \( \beta_{\text{index}} = 3 \), ratio of live load to dead load is equal to 2.5, and \( n_{\text{bar}} = 12 \) or \( \rho = 3\% \).
was found to be lower than the simulation ($\phi_{\text{ACT,318-19}} = 0.65$ and $\phi_{\text{simulation}} = 0.6$). The higher-strength reduction factor of ACI 318-19 could be caused by the lower coefficient of variation of the concrete material.

As shown in Figure 12, the effect $\rho$ was found to be significant for any region. As the value $\rho$ increases, the initiation of $\phi$ higher than 0.9 was faster. This means that the point of $\varepsilon_t$ for the tension-controlled region can be shifted earlier and the expression for $\varepsilon_t$ can be formulated as a function of $\rho$. Figure 13 shows the effect of the reliability safety index ($\beta_{\text{index}}$) value to the function of $\varepsilon_t$. As shown in Figure 13, increasing the value of $\beta_{\text{index}}$ shifted the whole curve of $\phi$ downwards. From Figure 13, it can be inferred that the $\beta_{\text{index}}$ for the tension-controlled region is 3.5 and for the compression-controlled region is 3.0.

**CONCLUSIONS**

This paper has presented a complete evaluation of the governing parameters that affect the resistance coefficient of variation ($\Omega_R$) of the RC column. Among all the investigated parameters, only column dimension showed the negligible effect to the $\Omega_R$. All parameters related to concrete effects $\Omega_R$ more for a small ratio of eccentricity. On the other hand, all parameters affecting the steel rebar effect $\Omega_R$ more for a large ratio of eccentricity. Typically, a ratio of eccentricity higher than unity can be sufficiently large to fully utilize the reinforcing bar in carrying load. Some anomaly was found when only rebar yield strength was isolated and a 500 MPa rebar yield strength is used. The value for $\Omega_R$ drops when $e/h$ is equal to 0.6. The longitudinal rebar ratio was also found to affects the $\Omega_R$ for small ratio of eccentricity. This can be well understood as the rebar also carry loads in compression.

From the study of strength reduction factor for RC columns with varying $e/h$, it can be concluded that material strengths affect the strength reduction factors. It is important to note that the ACI 318-19 strength reduction factor was somewhat conservative for the tension-controlled region with a value of 0.9 and is lower than the simulated strength reduction factor which is 0.93. For the compression-controlled region, the ACI 318-19 strength reduction factor was found to be less conservative with a value of 0.65 and is higher than the simulated strength reduction factor which is 0.6. The longitudinal rebar ratio affects the whole strength reduction factor curve and shifting the tension-controlled strain limits to be higher than -0.005. Changing the value of the reliability safety index shifted the whole strength reduction factor curve downward which means an increase in the safety level of the RC column.

A possible avenue of future work may consist of investigating prestressed concrete spun piles with varying load levels and eccentricities. Extending the random input data to consider all the material and geometric properties can also be investigated in the future. It should be noted that in this paper, all the random data is prepared using normal distribution data. Some of the input data, naturally may not be normally distributed, and therefore it should be included in the future evaluation of RC members. A further extension to include other load combinations should also be investigated and the safety envelope of the strength reduction factor should be based on all of the possible configurations used in the design of RC structures.
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