Anomalous transition temperature oscillations in LOFF state

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We consider Aharonov-Bohm effect at normal metal-inhomogeneous LOFF superconducting state transition. It is shown that magnetic flux can increase the transition temperature and AB oscillations can have the double-peak structure at one period. Expressions for fluctuational heat capacity and persistent current are calculated for a thin ring and a cylinder. We also discuss the effect of fluctuations interaction in the nonuniform states in the vicinity of the superconducting transition.

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INTRODUCTION

Spin polarization of the Cooper pairs in magnetic field destroys the superconducting state at Chandrasekhar-Clogstone limit when paramagnetic energy coincides with the superconducting condensation energy. Paramagnetic limit is attained at critical field \( H_p = \sqrt{2\Delta/2\mu_B} \), where \( \Delta \) is the superconducting gap, \( \mu_B \) is the Bohr magneton. Orbital pair breaking effect usually dominates over the paramagnetic limit. However, orbital effect could be suppressed in low dimensional systems (thin wires) or by applying magnetic field parallel to the conductive planes of quasi-two-dimensional systems.

Spin polarization in external magnetic field or intrinsic exchange fields could lead to the formation of inhomogeneous superconductivity. Larkin and Ovchinnikov [2], Fulde and Ferrell [3] predicted the existence of the nonuniform superconducting state in a ferromagnetic superconductors at low temperatures in magnetic field larger than critical \( H_p \) (see for a review [4, 5]).

LOFF state is formed by Cooper pairs with nonzero momentum \( \sim 2\mu_B H/v_F \), where \( v_F \) is the Fermi velocity, and at fields higher than the paramagnetic limit has lower energy compared to the uniform superconducting state. This finite momentum of Cooper pairs results in a spatial modulation of the superconducting order parameter.

Mathematically, LOFF state appears due to the change in sign of coefficient \( \beta \) at the gradient term of the Ginzburg-Landau (GL) free energy functional \( \beta |\nabla \Psi|^2 \), where \( \Psi \) is the order parameter. Coefficient \( \beta \) is a function of temperature and Zeeman energy \( \mu_B H \). In BCS model it becomes negative at high magnetic fields \( H > 1.07T_c(0)/\mu_B \) and at low temperatures \( T < 0.56T_c(0) \), where \( T_c(0) \) is the temperature of transition to superconducting state at zero magnetic field, signaling of the formation of nonuniform LOFF state. As a result one has to take into account higher terms in the GL functional expansion \( |\nabla^2 \Psi|^2 \).

The theoretical research of nonuniform LOFF state includes, for example, the study of impurities effect [6] that suppresses the region of nonuniform superconductivity, the LOFF-like proximity effect at the ferromagnetic-superconductor boundary [7], interplay of orbital and paramagnetic effects [8-10], the study of phase transitions in the vicinity of the tricritical point [11], intrinsic pinning of vortices in layered superconductors [12].

Heavy-fermion compound CeCoIn\(_5\) was found to show the signatures of LOFF phase. The existence of the LOFF state in heavy-fermion superconductor was experimentally investigated by specific heat measurements [13, 14, 15] and nuclear magnetic resonance [16, 17].

The phase transition between possibly the LOFF state and the homogenous superconducting state was reported for organic superconductors such as \( \lambda - (\text{BETS})_2\text{FeCl}_4 \) [18, 19, 20, 21] with quasi-2D electronic structures and organic (TMTSF)\(_2\)ClO\(_4\) [22] with quasi-1D electronic structure.

These experiments were focused on the identification of the phase transition inferred from a kink of thermal conductivity [18], observation of peculiar properties-dip structures in the resistance [19] and changes in the rigidity of the vortex system [20]. The thermodynamic evidence of the existence of narrow intermediate state attributed to LOFF state, which separates the uniform superconducting state and normal state based on specific heat measurements was presented in paper [21]. Finally, the transition temperature dependence on the strength and direction of the magnetic field was studied in paper [22].

Nonuniform state of condensate was uncovered in systems of ultra-cold atoms in optical lattices [23]. Experimentally optical lattices could be formed by standing wave laser which provides a periodical potential for ultra-cold atomic gas. Recently, the observation of phase transition between normal and nonuniform superfluid state was reported for systems of strongly interacting Fermi gas with imbalanced spin population [24].

Crossovers between different fluctuational regimes of paraconductivity and specific heat in the vicinity of the LOFF transition were discussed theoretically in paper [25]. Authors showed that these fluctuational contributions have specific temperature dependencies compared to the case of uniform superconductivity and could serve as an additional indicator of the LOFF state.

In present paper we consider hollow superconducting...
cylinder/ring threaded by magnetic flux (fig. 1). We calculate the expressions for the persistent current in thin superconducting ring and present numerical results for specific heat and persistent current for the cylinder. Magnetic flux dependence of the current demonstrates the double-peak structure in Aharonov-Bohm (AB) oscillations. We also study the effect of fluctuations interaction on the nonuniform states in low-dimensional inhomogeneous superconductors. This research is motivated by the fact that the fluctuation region in nonuniform systems is much larger than in the case of uniform states and requires separate theoretical study.

**GL FREE ENERGY**

We consider AB oscillations in quasi one dimensional ring and thin-walled cylinder which transverse size $d$ is much smaller than the radius $R$, see fig. 1. In case of second order normal metal-LOFF transition the Ginzburg-Landau free energy functional above transition temperature could be written as

$$ F = \int d\mathbf{r} \left( a(T - T_c)|\Psi|^2 + \beta |\mathbf{D}\Psi|^2 + \delta |\nabla^2 \Psi|^2 \right) $$

where $\beta = -|\beta|$ and $\mathbf{D} = -i\nabla - (2e/c)\mathbf{A}$, while tangent component of the vector potential is given as $A_r = \Phi/2\pi R$. Representing the order parameter as

$$ \Psi(r) = \sum_{n,k} \Psi_n(k) e^{i\alpha_n} e^{ikz} $$

where $z$ is coordinate along the cylinder. We can write the free energy functional as

$$ F = V \sum_{n,k} E_n(k)|\Psi_n(k)|^2 $$

where $V$ is volume of the sample and

$$ E_n(k) = a(T - T_c) + \frac{|\beta|}{2Q^2R^4} \left( |\Phi|^2 + (kR)^2 - (QR)^2 \right)^2 $$

Here $\Phi_0 = e/c\pi c$ is the flux quantum,

$$ Q = \sqrt{\frac{|\beta|}{2g}} $$

is the modulus of superconducting modulation wavevector in inhomogeneous LOFF state, and

$$ T_c = \tilde{T}_c + \frac{\beta^2}{4a\tilde{a}} $$

is the transition temperature at $R \to \infty$. Recently [20], we have examined the Aharonov-Bohm oscillations in thin ring near LOFF-metal transition. In contrast to the uniform superconductivity the applied magnetic flux can increase the transition temperature of LOFF state and AB oscillations could have double peak structure. The nature of these effects is the fluctuation energy spectrum of the inhomogeneous state. To see it in more detail let us consider the ring threaded by magnetic flux $\Phi$. In this case spectrum $E_n(k)$ is given by eq. (4) at $k = 0$

$$ E_n = a(T - T_c) + \frac{|\beta|}{2Q^2R^4} \left( |\Phi|^2 + (n - \frac{\Phi}{\Phi_0})^2 - \phi^2 \right)^2 $$

where we introduce $\phi = QR$.

The independence of spectrum on $k$ allows one to introduce flux dependent transition temperature, corresponding to $E_n = 0$ as

$$ T_\phi(\Phi) = T_c - \frac{|\beta|}{2aQ^2R^4} \min \left( |\Phi|^2 - (n - \frac{\Phi}{\Phi_0})^2 - \phi^2 \right)^2 $$

The transition temperature into the LOFF state of the superconducting ring is defined by $n_\pm$ which are nearest integers to the corresponding values $\Phi/\Phi_0 \pm \phi$.

Generally, integers $n_\pm$ do not correspond to the minimum of the energy [7] which provides the highest transition temperature [10] of the system. Thus, one can both increase or decrease $E_n$ and correspondingly the transition temperature $T_\phi(\Phi)$ by changing the radius of the ring or by applying magnetic flux $\Phi$. This is in contrast to the case of normal-uniform superconductor transition where transition temperature $T_\phi(\Phi)$ always decreases with applying magnetic flux. Moreover, the degeneracy of the energy spectrum allows the system to jump between $E_{n_+}$ and $E_{n_-}$ states leading to peculiar properties of AB effect such as double-peak structure per period of oscillations.

Let us discuss the possible temperature dependencies shown in fig. 2 for the case of large $\phi \gg 1$. In this regime the transition temperature behavior is given by

$$ (T_c(\Phi) - T_c)/T_c = -\frac{|\beta|}{2aT_c^2} \max(f_+(\Phi)^2, f_-(\Phi)^2) $$

Where $f_\pm(\Phi)$ is the distance between $\Phi/\Phi_0 \pm \phi$ and corresponding integer $n_\pm$.

If $0 < \phi < 0.5$ then for applied magnetic flux in the range $0 < \Phi < \Phi_0/2$ the transition temperature behavior is governed by $f_-$. When $\Phi$ reaches $\Phi_0/2$ the crossover from $f_-$ to $f_+$ takes place and the further magnetic flux dependence is described by $f_+$ leading to the double-peak
structure of oscillations. Note that if $0.5 < \phi < 1$ one has the opposite crossover from $f_+$ to $f_-$. Suppose the value of phase $\phi$ equals to the half an integer number (dotted line in fig. [2]). Transition temperature first decreases as $(T_c(\Phi) - T_c) / T_c \propto - (\Phi / \Phi_0)^2$ with increasing applied magnetic flux. The maximal depression of $T_c(\Phi)$ occurs when $\Phi / \Phi_0 = 1/2$ and has a value of $(T_c(\Phi) - T_c) / T_c = - |\beta| / 2\pi aT_c R^2$. Further increase of $\Phi$ leads to the increase of transition temperature as $(T_c(\Phi) - T_c) / T_c \propto (\Phi / \Phi_0)^2 - 1$. Here both $f_+$ and $f_-$ give equal dependence on $\Phi$.

Finally, if the value of $\phi$ equals to the integer number then one has the opposite case (dashed line in fig[2]). Transition temperature increases with applied magnetic flux starting from the value $(T_c(\Phi) - T_c) / T_c = - |\beta| / 2\pi aT_c R^2$. When $\Phi$ reaches $\Phi_0/2$, the crossover between $f_+$ and $f_-$ leads to further decrease of $(T_c(\Phi) - T_c) / T_c$.

The variations of $T_c(\Phi)$ quantitatively explain flux dependence of physical quantities of quasi one dimensional ring. Interestingly, summation over momentum $k$ in case of cylinder does not wash out the peculiarities of flux dependence. In this case $kR/2\pi$ plays a role of random phase and the cylinder could be considered as a set of rings with different phases $\phi$. Superposition of different types of oscillations as we will show below leads to fact that the oscillation peculiarities appear at lower temperatures and/or smaller radiiuses of the ring.

### SPECIFIC HEAT

Carrying out the integral over the real and imaginary parts of order parameter $\Psi_n(k)$ one obtains the expression for fluctuational part of thermodynamical potential

$$\Omega = T \sum_{n,k} \ln \left( \frac{E_n(k)}{\pi T} \right)$$

(10)

We accept units where $k_B = 1$. Fluctuational correction to specific heat is given as

$$C = -\frac{T}{V} \frac{\partial^2 \Omega}{\partial T^2}$$

where $V$ is the volume of the sample. Taking derivatives over the temperature dependence of $E_n(k)$ one obtains

$$C = \frac{(aT_c)^2}{V} \sum_{n,k} E_n^{-2}(k)$$

(11)

Performing the Poisson transformation one obtains the fluctuation specific heat of the superconducting cylinder at temperatures $T > T_c$

$$C = \frac{q}{VR(T/T_c - 1)^2} \sum_{m,k} \int \frac{e^{2\pi i m (\Phi / \Phi_0 + \epsilon)}}{[1 + (t^2 - z + (kR/q)^2)^2]} dt$$

(12)

where we introduce parameters

$$q = R \sqrt{2Q}$$

$$z = \frac{Q \zeta}{\sqrt{2}}$$

(13)

Parameter $\zeta$ characterizes the LOFF inhomogeneity of the fluctuations and is proportional to the number of modulations of superconducting fluctuations of correlation length $\zeta$.

Correlation length $\zeta$ measures the scale of superconducting fluctuations and is defined as

$$\zeta = \sqrt{\frac{|\beta|}{a(T - T_c)}}$$

(14)

Now one can integrate over $t$ and make summation over $m$. The expression for the specific heat is then

$$C = \frac{\pi q}{4VR(T/T_c - 1)^2} \sum_k \left[ f(\Phi) + f(-\Phi) \right]$$

(15)

Here

$$f(\Phi) = \left[ \frac{1}{g} + \frac{i/2}{g^2} \right] \frac{1 + e^{2\pi i \varphi}}{1 - e^{2\pi i \varphi}} + \left[ \frac{2\pi g}{g^2} \right] \frac{e^{2\pi i \varphi}}{1 - e^{2\pi i \varphi}}$$

(16)

while $\varphi = \Phi / \Phi_0 + g$ and $g = (z - (kR/q)^2 + i)^{1/2}$.

Specific heat of the ring is determined by $k = 0$ term in expression (15). The detailed analysis of the specific heat magnetic flux dependence for the case of thin superconducting ring was given in the paper [26]. There it was shown that magnetic flux can increase the critical temperature of LOFF state and AB oscillations could have double peak structure.

Here in fig[3] we present the typical magnetic flux dependencies of the specific heat of the ring. It is seen
Numerical results for thin superconducting cylinder are shown on fig.3. It is seen from the fig.3 that magnetic flux can also either increase or decrease specific heat and leads to the double-peak structure in oscillations. However, the transition to the double-peak structure regime appears at smaller radiuses of the cylinder compared to the radius of ring. This is the consequence of the averaging procedure over momentum $k$.

**PERSISTENT CURRENT**

In this section we will discuss the persistent current in AB effect. Expression for the persistent current is given as

$$I = -\frac{\partial \Omega}{\partial \Phi} = -T \sum_{n,k} \frac{\partial E_n(k)}{\partial \Phi}$$

(17)

Using equation (10) and performing the Poisson transformation we obtain at $T > T_c$

$$I = \frac{2T}{\Phi_0} \sum_{m,k} \int dt \frac{2t(t^2 + y^2 - z)}{1 + (t^2 + y^2 - z)^2} e^{2\pi im(\Phi/\Phi_0 + tq)}$$

(18)

Performing summation over $m$ and integration over $t$ we conclude with

$$I = -\frac{4\pi T}{\Phi_0} \sum_{k} \int dk \sin \left(2\pi \frac{\Phi}{\Phi_0} \right) \cos \left(2\pi \sqrt{\phi^2 - (Rk)^2 + iq^2} \right) e^{-\sqrt{2\pi R}/\zeta}$$

(19)

Persistent current of the ring is determined by term $k = 0$ in expression (19).

The regime of strong inhomogeneity $Q\zeta >> 1$

Let us first consider the temperature regime in the vicinity of the LOFF-metal transition which corresponds to the large number of modulations of superconducting fluctuations $Q\zeta >> 1$. We first suggest the radius of the superconducting ring/cylinder being larger than the correlation length $R \gg \zeta$.

To calculate the persistent current for the superconducting ring in this regime one has to take into account mode $m = 1$ in eq.(18) since higher modes will be exponentially suppressed. As a result, the persistent current in the ring can be estimated as

$$I \simeq -\frac{8\pi T}{\Phi_0} \cos(2\pi \phi) \sin(2\pi \Phi/\Phi_0) e^{-\sqrt{2\pi R}/\zeta}$$

(20)

Depending on the sign of the random phase factor $\cos(2\pi \phi)$ persistent current could produce either diamagnetic or paramagnetic response at small flux: fig.4. That
is in contrast to the case of homogenous superconductor-metal transition. The numerical result for the cylinder geometry is presented in the fig. One sees that the magnetic flux dependence of the persistent current is also sensitive to the random phase.

Now let us discuss the regime of nonuniform superconductivity in ring/cylinder with small radius $\zeta > R$.

Again we concentrate on the temperatures in the vicinity of the LOFF-metal transition. One obtains for the current in thin superconducting ring

$$I = -\frac{2\pi T}{\Phi_0} [f(\Phi) - f(-\Phi)] \quad (21)$$

where

$$f(\Phi) \simeq \frac{2 \sin (2\pi (\phi + \Phi/\Phi_0))}{1 + (\pi R/\zeta)^2 - \cos (2\pi (\phi + \Phi/\Phi_0))} \quad (22)$$

In this case one observes the pronounced double-peak structure of the persistent current oscillations in thin superconducting ring: fig. (5). The same result also holds for the superconducting cylinder and the numerical calculations of the current dependence on the magnetic flux are presented in the fig. (6). Again, the double-peak oscillations structure exhibits at smaller radiuses of the cylinder compared to the ring due to summation over momentum $k$.

It is of interest to compare the result obtained above with the case of homogenous superconductor-normal metal transition. In this regime the persistent current in the thin ring is given by the expression

$$I = -\frac{2\pi T}{\Phi_0} \frac{\sin (2\pi \Phi/\Phi_0)}{\cosh (2\pi R/\zeta) - \cos (2\pi \Phi/\Phi_0)} \quad (23)$$

Thus if the radius of the ring is larger than the coherence length then

$$I \simeq -\frac{4\pi T}{\Phi_0} e^{-2\pi R/\zeta} \sin (2\pi \Phi/\Phi_0) \quad (24)$$

Comparing expressions (20) and (21) with expressions (23) and (24) we see that the persistent current in nonuniform case is the result of the summation of two usual currents with phases shifted by $\pm \phi$.

**The regime of weak inhomogeneity $Q\zeta < 1$**

Finally, we consider the oscillation regime in the vicinity of metal - LOFF transition where $Q\zeta < 1$. This regime corresponds to weakly inhomogeneous superconducting fluctuations when $\beta \rightarrow 0$. The number of LOFF
modulations per correlation length \( \xi \) is small. In equation (19) for persistent current we will also consider the following condition

\[
QR > \frac{\zeta}{R}
\]

This condition implies that if the radius of the ring is larger than the correlation length the number of LOFF modulations per circumference of the ring \( QR \) should be large. This condition can be rewritten as

\[
R > \xi
\]

where now effective coherence length is given as

\[
\xi = \left( \frac{\zeta}{\sqrt{2Q}} \right)^{1/2}
\]

With these assumptions one concludes with the following expression for the persistent current of the thin ring

\[
I = -\frac{16\pi T}{\Phi_0} \sin \frac{2\pi \Phi}{\Phi_0} \cos \left( \sqrt{2Q/R} \right) e^{-\sqrt{\pi R/\xi}}
\]

This result is illustrated in the fig. (8), where the persistent current dependence on the magnetic flux is presented. One sees that the amplitude of the oscillations decreases compared to the temperatures regime in strongly inhomogeneous fluctuations \( Q\zeta > 1 \).

**APPLICABILITY OF GAUSSIAN APPROXIMATION**

Here we will examine applicability of gaussian approximation in the vicinity of LOFF - metal transition in general. A first step of estimating the fluctuation interaction correction above \( T_c \) is to take into account the contribution of neglected \( |\Psi|^4 \) and \( |\Psi|^6 \) terms. Last term is important in case of first order LOFF-normal metal transition. GL functional is then

\[
\mathcal{F} = F + \int \! d\mathbf{r} [\gamma |\Psi|^4 + \nu |\Psi|^6]
\]

where \( F \) is given by eq. (11). In the absence of the orbital magnetic field effect \( \mathbf{D} = i\nabla \). Here coefficient \( \gamma \) being a function of Zeeman energy and temperature could also change sign and become negative at high magnetic field and low temperatures. In clean superconductors with simple Fermi surface both coefficients \( \beta \) and \( \gamma \) change sign at the same point on the transition line - the so called tricritical point. Brazovskii [32] showed that coupling of fluctuations in inhomogeneous LOFF-like systems are important and could lead to the first-order type transition. This is why one has to keep term \( \sim |\Psi|^6 \) in GL functional.

The first order correction in \( \gamma \) is given by the bubble containing fluctuation propagator is shown on fig. (9). Fluctuations in LOFF-like systems are more singular near transition. Let us consider how they depend on dimensionality of the system. First fluctuation correction, which determines transition temperature shift \( a(T - T_c) \rightarrow a(T - T_c + \Delta T(D)) \) is given by expression

\[
a\Delta T(D) = \frac{V_D}{V} \int \! \frac{d^D p}{(2\pi)^D} a(T - T_c) + \frac{T}{2Q} (|\beta|^2 - Q^2)^2
\]

where \( V/V_3 = 1 \) for three dimensional system, \( V/V_2 = d \) for thin film with thickness \( d < \zeta \) and \( V/V_1 = S \) for thin wire with cross section area \( S \) and thickness less than \( \zeta \).

Calculating (31), we obtain at \( Q\zeta > 1 \)

\[
a\Delta T(D) \sim \frac{\gamma T_c V_D}{|\beta|} \zeta Q^{D-1}
\]

and in the regime of small \( |\beta| \) when \( Q\zeta = \sqrt{|\beta|} < 1 \) we estimate

\[
a\Delta T(D) \sim \frac{\gamma T_c V_D}{\delta} \left( \frac{\zeta}{Q} \right)^{D-1}
\]

Comparing temperature shift (31) (32) with \( T - T_c \), we obtain Levanuk - Ginzburg parameter \( \tau_{LG} \), which de-

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**FIG. 8:** Persistent current of the cylinder as a function of magnetic flux measured in units \( l_0 = 2T_c L/R\Phi_0 \). For the case of \( Q\zeta/\sqrt{2} = 0.1 \) and \( q = R\sqrt{2Q/\zeta} = [0.35, 0.4, 0.45, 0.5] \)

**FIG. 9:** First order fluctuation correction diagram
terminated the width of critical fluctuations region where gaussian approximation fails.

In the case $Q\zeta > 1$ the Levanuk - Ginzburg parameter can be estimated in different dimensions as

$$
\tau_{LG} \sim \left( \frac{T_c}{E_F} \right)^{4/3} G_3, D = 3
$$

$$
\tau_{LG} \sim \left( \frac{1}{\tilde{p}_F E_F} \right)^{2/3} G_2, D = 2
$$

$$
\tau_{LG} \sim (S\tilde{p}_F^2)^{-2/3} G_1, D = 1
$$

In estimation for quasi one dimensional wire $p_F$ is fermi momentum.

In expression \(G_D = \left[ \frac{|\beta|}{|\eta_0|} \right]^{2/3} \left[ \frac{|\partial|}{|\eta_0|} \right]^{(D-2)/3}, \beta_0 \sim 1/m \) and $\gamma_0 \sim T_c^2/nE_F$ are the values of the coefficients $\beta$ and $\gamma$ far from the LOFF - metal transition. $m$ and $n$ are the electron mass and the density, correspondingly. Thus one always has $|\beta_0| > 1$ and $|\gamma_0| > 1$ and $G_D$ is small parameter. In obtaining \(33\) we use estimation for $a \sim T_c/E_F$.

In the opposite case when $Q\zeta < 1$ the Levanuk - Ginzburg parameter can be estimated as

$$
\tau_{LG} \sim \left( \frac{T_c}{E_F} \right)^{4/5} \tilde{G}_3, D = 3
$$

$$
\tau_{LG} \sim \left( \frac{1}{\tilde{p}_F E_F} \right)^{2/3} \tilde{G}_2, D = 2
$$

$$
\tau_{LG} \sim (S\tilde{p}_F^2)^{-4/7} \tilde{G}_1, D = 1
$$

where now $\tilde{G}_D = \left[ \frac{|\beta|}{|\eta_0|} \right]^{4/(8-D)}$.

Indeed in three and two dimensional systems correction is much more singular and corresponding critical region is much larger than in case of uniform order parameter. In quasi 2D organic superconductors and in heavy - fermion CeCoIn$_5$ compound the critical fluctuations region is still very small provided $T_c/E_F \sim 10^{-2} - 10^{-3}$ \(21\) and $T_c/E_F \sim 0.15$ \(3\) correspondingly.

However in one dimensional case correction coincides with that for the case of uniform order parameter. In this sense the regimes of quasi zero dimensional ring ($R \sim \zeta$) and quasi one dimensional cylinder ($L \gg R, \zeta$) considered here are the same as in case of standard superconductors and do not deserve special discussion. Parameter, which determines smallness of the Levanuk - Ginzburg parameter in quasi one dimensional wire, is $S\tilde{p}_F^2 \gg 1$, i.e. large number of transverse modes.

If coefficient at term $|\Psi|^4$ changes sign and becomes negative then one should check if the first order type transition destroys the Gaussian approximation. Let us estimate the temperature width of first order type transition in case of supercooling.

Consider sample with size less or order of $\zeta$. Varying \(20\) with respect to the amplitude of the order parameter written as $\Psi(\mathbf{r}) = \Psi(\mathbf{Q}\mathbf{r})$ we obtain three solutions: $\Psi_0 = 0$ and

$$
\Psi_\pm = \frac{|\gamma|}{3\nu} \pm \sqrt{\frac{(\gamma^2 - a(T - T_c))}{3\nu}}
$$

Temperature of first order transition from $\Psi_0 = 0$ state to $\Psi_+$ state is determined by the condition

$$
\mathcal{F}(\Psi_+) = \mathcal{F}(\Psi_0) = 0
$$

and is larger than $T_c$.

Solution $\Psi_-$ corresponds to the maximum of functional \(29\)

$$
\mathcal{F}(\Psi_-) \sim V \frac{(a(T - T_c))^2}{4|\gamma|}
$$

The probability of thermal activation transition of the order parameter from the steady state $\Psi_0$ over the barrier of height $\mathcal{F}(\Psi_-)$ is proportional to the value

$$
\sim \exp \left( - \frac{\mathcal{F}(\Psi_-)}{T} \right)
$$

In case of $\mathcal{F}(\Psi_+) \gg T_c$ system will stay in supercooled state. Corresponding temperature region might be estimated as

$$
(T - T_c)/T_c > \frac{1}{aT_c} \left( \frac{T_c|\gamma|}{V} \right)^{1/2} \sim (N_0VT_c)^{-1/2}
$$

Here $N_0$ is electron density of states at Fermi level. In case of not too small sample, when $N_0VT_c >> 1$, supercooled $\Psi_0 = 0$ state might be extended at $(T - T_c)/T_c < 1$.

**SUMMARY**

To summarize, we showed that Aharonov - Bohm effect is very sensitive tool for studying the intrinsic properties of superconductors in the regime of inhomogeneous LOFF state.

Depending on the ratio of modulation period of superconducting order parameter and the radius of the ring/cylinder transition temperature in magnetic flux can be either increased or decreased, or even can have double-peak structure at one flux quantum.

These effects arise due to the degeneracy of fluctuation energy spectrum of nonhomogeneous LOFF state.

We calculated the fluctuation contribution for the persistent current in thin superconducting ring and presented numerical results for fluctuation specific heat and persistent current for the cylinder. Flux dependencies of persistent current and specific heat qualitatively correspond to that of transition temperature.
We showed that despite the enhancing of singularity due to fluctuation's interaction in higher dimensions, in quasi one dimensional system Livanuk-Ginzburg parameter coincides with that for homogeneous superconductor. Most studied reason for inhomogeneity is zeeman splitting due to external magnetic field or exchange splitting in ferromagnetic superconductor. In this case coefficients $\beta$, $\delta$ and temperature $\tilde{T}_c$ itself depend on the magnetic field. Thus AB oscillations are superimposed by monotonous dependence on magnetic field. In ring geometry these two could be separated by measuring of AB oscillations in tilted magnetic field.

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