J/ψ c̅c̅ production in e⁺e⁻ and hadronic interactions

A. B. Kaida lov
Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25,
Moscow 117259, RUSSIA
E-mail: kaidalov@heron.itep.ru

Predictions of the nonperturbative Quark Gluon Strings model, based on the 1/N-expansion in QCD and string picture of interactions for production of states containing heavy quarks are considered. Relations between fragmentation functions for different states are used to predict the fragmentation function of c̅-quark to J/ψ-mesons. The resulting cross section for J/ψ-production in e⁺e⁻-annihilation is in a good agreement with recent Belle result. It is argued that associated production of c̅c̅ states with open charm should give a substantial contribution to production of these states in hadronic interactions at very high energies.

Investigation of heavy quarkonia production at high energies provides an important information on QCD dynamics in an interesting region of intermediate distances from 1/m_Q to r_QQ, where m_Q is the heavy quark mass and r_QQ is the radius of a heavy quarkonia state. For c and b-quarks this is the region 0.05 fm < r < 1 fm. In this region both perturbative and nonperturbative effects can be important. Production of J/ψ-mesons is studied experimentally in e⁺e⁻-annihilation, γp, hp, hA and AA-collisions. Analysis of hadronic interactions show that the simplest perturbative approach (color singlet model) ¹ does not reproduce experimental data ². This observation lead to an introduction of the color octet mechanism ³ of heavy quarkonia production. In this approach a set of nonperturbative matrix elements is introduced, which is determined from a fit to data. A characteristic prediction of this approach is a large transverse polarisation of J/ψ and ψ' at large transverse momenta ⁴ is not supported by the Tevatron data ⁵.

A new mystery to the problem of heavy quarkonia production has added recent result of Belle Collaboration ⁶ on a large production of J/ψ-mesons with charmed hadrons. The observed cross section at √s = 10.6 GeV is an order of magnitude larger than theoretical predictions ⁷, based on perturbative QCD. It is interesting that at this energy an associated production of J/ψ with c̅c̅-pair is the dominant mechanism of J/ψ production ⁶.

In this paper a nonperturbative approach, based on 1/N-expansion in QCD and string picture of particle production is used for a description of heavy quarkonia production at high energies. The model based on this approach (the Quark Gluon Strings model (QGSM) ⁹) has been successfully applied to production of different hadrons at high energies. It has been also used for
description of inclusive spectra of hadrons containing heavy (c,b) and light quarks\textsuperscript{11,12,13}. In QGSM the fragmentation functions, which describe transitions of strings to hadrons in many cases can be predicted theoretically\textsuperscript{9,10} and are expressed in terms of intercepts of corresponding Regge trajectories. We will show that the model naturally leads to the cross section of $J/\psi$ production in $e^+e^-$ annihilation consistent with the Belle result. Estimate of the contribution of the same mechanism in hadronic interactions indicates that it can be important at energies $\sqrt{s} \geq 10^2$ GeV.

Let us first discuss heavy quarkonia production in $e^+e^-$ collisions. In these reactions $c\bar{c}$-pair is produced directly by a virtual photon. However a probability of transition of such a state at high energies (far above threshold of charm production) to $J/\psi$ is very small. A simplest diagram of QCD perturbation theory (Fig.1a) corresponds to a transition to a white $c\bar{c}$ state with relative momentum characteristic to $J/\psi$ by emission of two hard gluons. This cross section is suppressed at high energies by a factor $4m^2_c/s$ and at $\sqrt{s} = 10$ GeV constitutes $10^{-3}$ of the total $c\bar{c}$ cross section\textsuperscript{7}.

$J/\psi$ production in association with extra charmed pair (Fig.1b) does not have this suppression, but contains a smallness due to production of this pair and a high threshold of the processes. At high energies this mechanism can be considered as a fragmentation of $c(\bar{c})$ to $J/\psi$. Calculation in the lowest order of QCD perturbation theory\textsuperscript{7} shows that this mechanism is important at energies $\sqrt{s} \geq 50$ GeV, but at $\sqrt{s} = 10$ GeV is smaller than the mechanism of Fig.1a by an order of magnitude and is about 0.07 pb. This is in a sharp contradiction with Belle result: $\sigma(J/\psi c\bar{c}) = 0.87^{+0.21}_{-0.19} \pm 0.17$ pb.

Note that for states of comparatively large radius like $J/\psi$ and especially $\psi'$ or $\chi_c$ a nonperturbative fragmentation can be important. Thus I shall estimate a fragmentation of $c(\bar{c})$ into $J/\psi$ using the nonperturbative model mentioned above. In this model particle production is described in terms of production and fragmentation of quark-gluon strings. A behaviour of the fragmentation
functions is determined in the limit $z \to 1$ from the corresponding Regge limit
and is expressed in terms of Regge intercepts $\alpha_q(0)$ \cite{9,10}. The fragmentation function of $c$-quark to $J/\psi$ in this model is written in the form \cite{10}

$$D_c^\psi = a_\psi z^{-\alpha_\psi(0)}(1 - z)^{-\alpha_\psi(0)+\lambda}$$ \hspace{1cm} (1)

where $\alpha_\psi(0)$ is an intercept of the $J/\psi$ Regge trajectory, which is known from analysis of data on spectrum of $c\bar{c}$ states and analysis of inclusive spectra of charmed particles (see below), $\lambda = 2\alpha_{D^*}\bar{z}\pi_D \approx 1$. Thus this fragmentation function is characterized by one constant $a_\psi$. In order to determine this constant we will use a relation between fragmentation function of $c$-quark to $J/\psi$ and fragmentation function of a light quark to $D(D^*)$-meson in the limit $z \to 1$. According to rules formulated in refs.\cite{10,15} both functions have the same behavior on $z$: $(1 - z)^{(-\alpha_\psi(0)+\lambda)}$ as $z \to 1$ and differ only by a kinematic factor related to mass difference between $J/\psi$ and $D(D^*)$-meson.

$$R_{D^*/\psi} \equiv \frac{D_{u}^{D^*}}{D_{c}^\psi} = \left(\frac{s_{uc}^{0c}}{s_{0\psi}^{0\psi}}\right)^{2(1 - \alpha_{D^*}(0))} \left(\frac{s_{uc}^{0c}}{m_{D_L}^{2}}\right)^{2(1 - \alpha_{\rho}(0))} \left(\frac{m_{\pi_L}^{2}}{s_{0u}^{0u}}\right)^{2(1 - \alpha_{\rho}(0))} (1 - z)^{2(\alpha_\rho(0) - \alpha_{D^*}(0))}$$ \hspace{1cm} (2)

The quantities $s_{0i}$ will be determined below.

Now we shall find the fragmentation function $D_{u}^{D^*}$ in the limit $z \to 1$. In this limit it is related to the fragmentation function of a light quark to $\pi$ meson \cite{10}

$$R_{u^+/\pi^+} \equiv \frac{D_{u}^{D^*}}{D_{u}^{\pi^+}} = \frac{\Gamma^2(1 - \alpha_{D^*}(0))}{\Gamma^2(1 - \alpha_\rho(0))} \left(\frac{s_{uc}^{0c}}{m_{D_L}^{2}}\right)^{2(1 - \alpha_{D^*}(0))} \left(\frac{m_{\pi_L}^{2}}{s_{0u}^{0u}}\right)^{2(1 - \alpha_\rho(0))} (1 - z)^{2(\alpha_\rho(0) - \alpha_{D^*}(0))}$$ \hspace{1cm} (3)

where $\alpha_\rho, \alpha_{D^*}(0)$ are intercepts of $\rho$ and $D^*$ Regge trajectories. They are related to $\alpha_\psi(0)$ by the following equation \cite{14}

$$\alpha_\rho(0) + \alpha_\psi(0) = 2\alpha_{D^*}(0)$$ \hspace{1cm} (4)

I shall use the following values for these intercepts: $\alpha_\rho(0) = 0.5, \alpha_\psi(0) = -2$ and $\alpha_{D^*}(0) = -0.75$ in accord with eq.(4). An uncertainty in the value of $\alpha_\psi(0)$ discussed in ref. \cite{11} is eliminated at present by experimental data on inclusive spectra of charmed hadrons in hadronic collisions.

The gamma functions in eq.(3) appear from Regge residues of the corresponding trajectories, which were chosen in accord with dual models are in
a good agreement with data on widths of hadronic resonances. The coupling is assumed to be universal (with an account of SU(4) and heavy quark symmetry).

The quantities \( s_0 \) entering in eq.(3) can be easily calculated using formulas and parameters of ref. \(^\text{15} \)

\[
(s_0^{uc})^2 \alpha_\psi(0) (s_0^{uD}) \alpha_\psi(0); \quad (5)
\]

\[
s_0^{uu} = 4m_{u\perp}^2 + 1 \ \text{GeV}^2; \quad s_0^{uD} = (m_c + m_{u\perp})^2 \quad (6)
\]

With \( m_{u\perp} = 0.5 \ \text{GeV} \) and \( m_{c\perp} = 1.6 \ \text{GeV} \), we obtain \( (s_0^{uc}) = 3.57 \ \text{GeV} \). Using these values for \( s_0 \) in eq.(3) and \( m_{u\perp}^2 = 0.18 \ \text{GeV}^2; \quad m_{c\perp}^2 = 5 \ \text{GeV}^2 \), the fragmentation function \( D_\psi^\pi = 0.44 \) we obtain the function \( D_\psi^0 \) at \( z \to 1 \) in the form \( 0.01(1-z)(-\alpha_\psi(0) + \lambda) \). This value is in a reasonable agreement with phenomenological studies of charmed particle production in hadronic interactions in the framework of QGSM \(^\text{12,13} \).

The value of \( s_0^{uc} \) in eq.(2) can be calculated in the same way with the substitution \( s_0^{uD} \to s_0^{\psi D} = 6.72 \ \text{GeV}^2 \). Finally we obtain from eq.(2)

\[
D_\psi^0 = 0.05 (1-z)(-\alpha_\psi(0) + \lambda); \quad z \to 1 \quad (7)
\]

Thus \( a_\psi = 0.05 \).

At asymptotic energies \( s \to \infty \) cross section for \( J/\psi \) production in \( e^+e^- \) annihilation is equal to

\[
\sigma_\psi = 2 \sigma_{c\bar{c}} \int_0^1 D_\psi^0 (z) \, dz \quad (8)
\]

factor 2 in eq.(8) takes into account \( J/\psi \) production by both \( c \) and \( \bar{c} \) quarks. At energy \( \sqrt{s} \approx 10 \ \text{GeV} \) there is an extra suppression due to phase space corrections for production of a heavy state. We estimate it by introducing an extra factor \( \gamma = \sqrt{1 - 4M_{c\bar{c}}^2/\sqrt{s}} \) to eq.(8). Distribution in \( M_{c\bar{c}}^2 \) is related to the \( z \) distribution. It has a maximum at \( M^2 \approx 0.27s \). For energy of Belle experiment the correction factor \( \gamma = 0.7 \). Thus we obtain the following cross section for \( J/\psi \) \( c\bar{c} \) production at \( \sqrt{s} = 10.6 \ \text{GeV} \) \( \sigma_\psi = 1.2 \ \text{pb} \). This value is in a good agreement with Belle result \(^\text{6} \) and is much larger than perturbative QCD prediction \(^\text{7} \). An estimated uncertainty in the value of cross section due to possible variation of quantities \( s_0, m_{u\perp} \) and \( \alpha_i(0) \) is about 50%.

Let us consider now \( J/\psi \) production in hadronic interactions. In the approach based on \( 1/N \)-expansion \(^\text{8} \) the main diagrams for particle production correspond to two-chain configurations, shown for pp-interactions in Fig.2a \(^\text{9} \). They can be considered as production and fragmentation of two \( q - q \)}
strings. It is important to emphasize that production of one \( c\bar{c} \)-pair together with light quark pairs in this approach always leads to an open charm production (Fig.2b) and \( J/\psi \) in this case is produced by OZI forbidden mechanism\(^{17}\). This leads to a strong suppression (\( \sim 10^{-2} \)) for heavy quarkonia production in hadronic collisions compared to open charm (beauty) production. To produce \( J/\psi \) in the chains by OZI allowed mechanism it is necessary to produce 2 \( c\bar{c} \) pairs close in rapidity (Fig2.c). Though this mechanism is suppressed due to production of extra heavy quark pair it can compete at very high energy with the mechanism of single \( c\bar{c} \) pair production. Its contribution can be estimated from charm quark fragmentation into heavy quarkonia in \( e^{+}e^{-} \)-annihilation. Consider production of a \( c\bar{c} \)-pair in \( q - \bar{q}q \) string of Fig.2. In each of \( q - \bar{c} \) and \( c - q\bar{q} \) substrings an extra \( c\bar{c} \) pair can be produced and fragment to a given quarkonium state. So it is possible to use an estimate of the fragmentation function of \( c(\bar{c}) \) quarks given above or directly experimental data from \( e^{+}e^{-} \) to determine a contribution of the corresponding diagrams to quarkonia production. This calculation is rather straightforward except of a threshold suppression factor. It is clear that at energies of fixed target experiments \( \sqrt{s} = 10 \div 40 \text{ GeV} \) there is a strong suppression for production of \( J/\psi \) and extra \( DD \) pair. I shall estimate this suppression factor for an energy of HERA-B experiment\(^{18} \) \( E_{Lab} = 920 \text{ GeV} \). Let us denote an extra suppression factor compared to suppression of a single \( c\bar{c} \) pair by \( \gamma_{pp} \). For its estimation it is possible to introduce the same kinematical factors as in \( e^{+}e^{-} \) collisions for each subchains \( q - \bar{c} \) and \( c - q\bar{q} \). For \( J/\psi \) production at rapidity \( y=0 \) \( \gamma_{pp} \approx 0.5 \). Another estimate can be done by assuming that \( J/\psi \)- \( DD \) system is produced by a gluon fusion. This gives \( \gamma_{pp} \approx 0.4 \). Using these estimates and taking into account that \( \sigma_{pp}^{J/\psi} / \sigma_{pp}^{c\bar{c}} \approx 10^{-2} \) we obtain that associated production of \( J/\psi \) with

![Figure 2: Diagrams for \( J/\psi \) production in \( pp \)-interactions.](image)
charmed hadrons constitute at this energy $\sim 10\%$. At Tevatron energies the role of this mechanism is more important and it can (at least partly) explain an excess of $J/\psi$ production at Tevatron compared to color singlet model.

For $\psi'$ associated production with $c\bar{c}$ in $e^+e^-$ annihilation is not known experimentally yet. However its total inclusive yield is close to the one for $J/\psi$. If a probability of $\psi'$ production by $c$-quark fragmentation is the same as for $J/\psi$ it will have even stronger impact on $\psi'$ production in hadronic collision because experimentally for $\psi'$ cross section is smaller than for $J/\psi$: $\sigma_{pp}^{\psi'}/\sigma_{pp}^{J/\psi} \approx 1.6 \times 10^{-3}$ and associated production can constitute a large fraction of the $\psi'$ production.

In conclusion it was demonstrated that the nonperturbative QGSM model predicts a sizable $J/\psi$ $c\bar{c}$- production in $e^+e^-$ annihilation at high energies consistent with recent experimental result. In the approach based on $1/N$-expansion in QCD it was shown that a large fraction of $c\bar{c}$-quarkonia production in hadronic collisions at very high energies can be due to associated production with charmed hadrons.

Acknowledgments

I would like to thank K. Boreskov, O.V. Kancheli for useful discussions. I am especially grateful to M.V. Danilov for drawing my attention to this problem and discussion of results of Belle Collaboration.

This work is supported in part by the grants: INTAS 00-00366, NATO PSTCLG-977275, RFBR 00-15-96786, 01-02-17383.

References

1. J.H. Kuhn, J.Kaplan and E.G.O.Safiani, Nucl. Phys. B157 (1979) 125. C.H.Chang, Nucl. Phys. B172 (1980) 425; E.L. Berger and D.Jones, Phys. Rev. D23 (1981) 1521.
2. M. Cacciari, in Proc. of the XXXth Rencontres de Moriond, ed. by J.Tran Thanh Van, Editions Frontiers 1995, p.327.
3. E. Braaten and T.C.Yuan, Phys. Rev. Lett. 71 (1993) 1673; M. Cacciari, M. Greco, Phys. Rev. Lett. 75 (1994) 1584; D.P. Roy, K. Stridhar, Phys. Lett. B339 (1994) 141.
4. P. Cho and M.B. Wise, Phys. Lett. B346 (1995) 129.
5. CDF Collaboration, T.Affolder et al., hep-ex/0004027
6. K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 89 (2002) 142001.
7. P. Cho and A.K. Leibovich, Phys. Rev. D54 (1996) 6690.
8. G. ’t Hooft, Nucl Phys. B72 (1974) 461.  
   G. Veneziano, Phys. Lett. 52B (1974) 220 ; G. Veneziano,  
   Nucl. Phys. B117 (1976) 519.
9. A.B. Kaidalov, Sov. J. Nucl. Phys. 33 (1981) 733; A.B. Kaidalov,  
   Phys. Lett. 116B (1982) 459; A.B. Kaidalov and K.A. Ter-Martirosyan,  
   Phys. Lett. 117B (1982) 247; A.B. Kaidalov and K.A. Ter-Martirosyan,  
   Sov. J. Nucl. Phys. 39 (1984) 979; 40 (1984) 135.  
   A.B. Kaidalov, in ”QCD at 200 TeV”, ed. L.Cifarelli and Yu. Dokshitzer,  
   Plenum Press (1992), p.1.
10. A.B. Kaidalov, Sov. J. Nucl. Phys. 45 (1987) 902.
11. A.B. Kaidalov, O.I. Piskunova, Sov. J. Nucl. Phys. 43 (1986) 994.
12. O.I. Piskounova, Phys. At. Nucl. 56 (1993) 1094.
13. G.G. Arakelyan, Yad. Fiz. 161 (1998) 1682.
14. A.B. Kaidalov, Z. Phys. C12 (1982) 63.
15. K.G. Boreskov, A.B. Kaidalov, Yad. Phys. 37(1) (1983) 174
16. A.B. Kaidalov and P.E. Volkovitsky, Sov. J. Nucl. Phys. 35 (1982) 909.
17. G.G. Arakelyan, K.G. Boreskov and A.V. Turbiner, Sov. J. Nucl. Phys.  
   41(4) (1985) 651.
18. HERA-B: Report on Status and Prospects, DESY-PRC 00/04.
19. K.Abe et al. Phys. Rev. Lett. 88 (2002) 052001