Spectroscopy of the modes of a non-linear superconducting microwave resonator via inter-mode coupling and bifurcation amplification

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Abstract. The Josephson non-linearity provides a route to high sensitivity measurements through the bifurcation of a resonant mode into two metastable dynamical states. We probe the bifurcation regime of a superconducting microwave co-planar resonator incorporating a controlled non-linearity made of an array of SQUIDs. The switching probability between metastable states may be determined by repeated measurements under nominally identical conditions. We show that measurements of this probability and the inter-mode coupling introduced by the SQUIDs may be used for spectroscopy of other resonant modes. We also find the width of the switching-curve is within a factor two of quantitative agreement with Dykman’s model, but that the switching probability is affected by a low frequency noise that has its source on-chip.

1. Introduction
The phenomena associated with the Josephson effect may be regarded as a method of introducing non-linearity into an electrical circuit. The concept is particularly important as non-linearity is a fundamental necessity in the field of superconducting quantum circuits and the Josephson junction is the only known method of introducing non-linearity without dissipation. In the present discussion the non-linearity is based on the fact that the inductance of a Josephson junction is a function of the current flowing through the junction. Including Josephson elements in an electrical resonator provides a method for controlling by design the degree of non-linearity introduced and considerably enhancing it so that it is the dominant non-linearity. In our case the resonator is constructed from a superconducting co-planar waveguide and the non-linearity is introduced by placing an array of SQUIDs at the centre of the resonator. See Figure 1.

The consequences of adding non-linearity are multiple [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. In the first instance, the resonant modes are no longer harmonic multiples of each other, this follows because the non-linear inductance (which controls the frequency of the resonant modes) depends on the current flowing in the circuit and this is different for the different resonant modes. Secondly each mode becomes non-linear such that at high oscillator energies the mode is first 'pulled' in frequency and then 'bifurcates'. In the present electrical circuit this is a consequence of the inductance changing with the power level applied or equivalently the magnitude of the electrical current through the Josephson junction. The bifurcation phenomenon occurs when (descriptively speaking) the peak of the resonance curve is pulled further than the resonant
Figure 1. The experimental arrangement for pulsed radiofrequency measurements of a non-linear superconducting resonator. Radiofrequency pulses with the envelope shown in insert d are fed to the sample (insert a, located at $T \approx 20 \text{ mK}$) through attenuators and lossy coaxial lines. The power and the frequency of the pulses are tuned so that the bifurcation regime of the non-linear resonator is probed. The transmitted signal is fed via circulators, superconducting coax, a cold amplifier and Cu coax to room temperature and then homodyne detected. Insert e shows the signal obtained when the sample does or does not exhibit bifurcation. Inserts b and c show the input and output capacitance and the SQUID array respectively.

linewidth, resulting in three possible amplitudes at the same drive frequency, one of which is unstable. Under these conditions at a given drive frequency the system must choose between a high amplitude state and a low amplitude state. The transition between the two states is narrower than the natural linewidth of the resonant mode and hysteretic. By analogy with the Kerr effect in the field of quantum optics, the bifurcation phenomenon may be viewed as being a consequence of the ‘self-Kerr effect’. Thirdly, the resonant modes become coupled. This follows because the current flowing in the Josephson junction and attributed to one mode alters the value of the inductance that must be considered for the evaluation of a different mode. This coupling is called the ‘cross-Kerr effect’. The dispersive coupling described by the cross-Kerr effect has not been widely explored but has, for example, been discussed in the context of quantum non-demolition measurements [11, 12, 13].

It is usual to regard resonant phenomena as ‘amplifiers’ that enhance measurements by a factor of order the Quality-factor of the resonator, at the expense of being required to wait long enough that the oscillator may follow the parameter being measured. The bifurcation phenomena may also be used to measure experimental parameters but it is, in contrast, a threshold measurement. The ‘bifurcation amplifier’ may be biased close to the transition between
the two states and the external parameter to be measured may be used to influence whether or not the bifurcation threshold is actually crossed. The output is therefore binary but the threshold is sharper than the natural linewidth of the resonant mode, hence there is a gain in sensitivity at the expense of dynamic range. Through repetitive measurements of whether the system crosses the threshold a switching probability may be determined. Measurements of the switching probability as a function of drive frequency (or power) result in a cumulative probability curve or 'S-curve', whose width is a measure of the sensitivity of the technique. In the experiments described below the natural linewidth of the \( \approx 2 \) GHz resonant mode is \( \approx 212 \) kHz, while the width of the S-curve is \( \approx 4.5 \) kHz.

Our device [14] consists of a 200 nm thick sputter-deposited Nb film fabricated into a meandering half-wavelength coplanar waveguide resonator through optical lithography, with end capacitors of design value \( C = 7 \) pF. The fundamental frequency is \( \approx 1.77 \) GHz. An array of \( N = 7 \) Al/AlO\(_x\)/Al SQUIDs was formed with standard e-beam lithography and double-angle shadow evaporation. For a fixed inductance, using an array rather than a single junction allows the critical current (and hence the magnitude of the non-linearities) to be varied at the design stage. In this design only the odd numbered modes couple to the SQUID array, the even numbered modes possess nodes of current at the location of the array.

In the present experiments we drive the third mode to the bifurcation regime and tune it to a switching probability of 10%. We then inject a low-power continuous-wave signal in the vicinity of the coupled mode to be detected and sweep its frequency across the mode to measure the response. As shown in Fig. 2, we were able to detect the change in photon occupation of the 1\(^{st}\), 5\(^{th}\), 7\(^{th}\) and 9\(^{th}\) modes via a variation in the switching probability of the 3\(^{rd}\) mode, thus demonstrating simultaneously both the self-Kerr bifurcation of the 3\(^{rd}\) mode and the cross-Kerr coupling between the modes. With this technique, the cross-Kerr effect enables spectroscopy of modes that lie outside our measurement bandwidth (because of the limited bandwidth of the circulators) and thus are undetectable via the usual transmission detection. An important figure of merit is obtained by comparing the S-curve width to the magnitude of the relevant inter-mode coupling strength, allowing us to conclude that our device is sensitive to \( \approx 10 \) photons in the 1\(^{st}\) mode. The resonant frequencies of the coupled modes are obtained via a Lorentzian fit providing a value for the participation ratio at zero magnetic flux of \( \beta = (2.55 \pm 0.1)\% \), which is compatible with the value obtained from flux modulated vector network analyser measurements. The 9\(^{th}\)
Figure 3. Measurements of the switching probability of the 3\textsuperscript{rd} mode of a superconducting non-linear resonator as a function of the driving frequency at 8mK and $\Phi/\Phi_0 = 0$. Each point represents the probability of switching of the mode as determined from 1000 pulses. 50 separate consecutively measured curves are shown. The white line is the average of the 50 curves. The expected statistical error of the black lines is approximately the thickness of the white line.

mode is split for reasons that we do not understand; possibly the mode is bifurcating.

Figure 3 shows 50 switching probability curves acquired consecutively under nominally identical conditions, shown as a function of the driving frequency. The 10\%-90\% width of the S-shaped averaged probability curve (white line) is $4.5 \pm 0.5$ kHz or 0.9 ppm of the frequency of the third mode. This value can be compared with a theoretical value calculated from Dykman’s model [15, 16] which predicts an S-curve width due to quantum fluctuations of $\approx 0.5$ ppm. Hence our experimental value is about twice the theoretical value. In attempting to reconcile this, we note that the bifurcation curves of Fig. 3 show that the switching probability is affected by low frequency fluctuations that significantly exceed the expected statistical fluctuations. A noise source, whose low frequency components appear in these measurements, may also be present at higher frequencies where it may increase the S-curve width and may be the source of the discrepancy between our experimental values and the theory of Dykman. Such a noise source would be parametrically coupled to our CJBA, either through the frequency of the cavity or through the amplitude and frequency of the biasing pulses. Estimates and measurements of the extrinsic sources of noises, such as temperature fluctuations and power (frequency) fluctuations of the biasing pulses, suggest that they cannot account for the observed noise fluctuations and we are led to conclude that they originate on-chip due to the parametric coupling of our oscillator to local fluctuators embedded in the chip.

In summary [17], we have created a device to assess the usefulness of on-chip superconducting microwave optical components. By inserting a controllable, non-linear element into a superconducting microwave resonator, we can simultaneously exploit the self- and cross-Kerr effects. We have shown that high-sensitivity spectroscopy of nearby modes may be performed through measurements of the probability of switching of another mode via non-linear coupling. Analysis of the device sensitivity suggests that there is a source of noise that manifests as fluctuations in the cavity resonant frequency, it has a low frequency component and it arises on-chip. The noise may be related to the ubiquitous noise seen in similar SQUID and qubit systems [18, 19]; we have excluded microscopic flux noise and the remaining sources include two-level fluctuators in the environment and critical current noise in the tunnel barriers. However, we note that the intrinsic sensitivity of the device is close to the limits given by the theory of Dykman. Further analysis of repetitively acquired switching data from Josephson bifurcation phenomena
can be further explored ([20] and poster No. 785, this conference).

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