Experimental evidence of Phase Control method in chaotic Semiconductor Laser

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Abstract
we study how to control the dynamics of excitable systems by using the phase control technique. We study how to control nonlinear semiconductor laser dynamics with optoelectronic feedback using the phase control method. The phase control method uses the phase difference between a small added frequency and the main driving frequency to suppress chaos, which leads to various periodic orbits. The experimental studying for the evaluation of chaos modulation behavior are considered in two conditions, the first condition, when one frequency of the external perturbation is varied, secondly, when two of these perturbations are changed. The chaotic system becomes regular under one frequency or two frequencies. But in two frequencies, phase control showed an excellent ability to maintain regular behavior in chaotic window and reexcite chaotic behavior when destroyed. This dynamics of the laser output are analyzed by time series and bifurcation diagram.

Keywords: Chaos, Multi-frequency, Phase control, Amplitude.
Introduction

Chaos is a term used to describe the irregular behavior of dynamic systems resulting from a strictly deterministic evolution of time without any noise. [1] Chaotic behavior takes place in dynamic systems broadly and in a large number of applications. Chaotic behavior divided to either natural systems or man-made devices like in technological application, and usually represent an undesirable character or artifact. Chaos control methods had to be developed in the last years as a control system for chaotic behavior. [2]

By using a semiconductor laser, we can have chaos by a setting of the control parameters. Semiconductor lasers (class B) are defined by two variables: the photons intensity and population inversion. And so, an external modulation or feedback is needed to have chaos in class B, like optoelectronic or optical as a common feature in the semiconductor laser [3, 4]. One of the access control the parameter is the use of an external sinusoidal modulation of the injection current and that can perturb a semiconductor laser. Chaos control is considered most interesting and stimulating nonlinear dynamics methods, control methods are divided into two major categories depending on their interaction with the chaotic system, i.e. feedback and non-feedback methods [5, 6]. In the feedback method a small state dependent perturbation is used [7, 8]. While a tiny perturbation is used in non-feedback methods [9, 10].

In non-feedback techniques, the phase control in a chaotic system have a decisive role. The phase difference between the main drivers responsible for the appearance of chaos linked to external perturbation is considered the key parameter in the control method [11]. The most important factor of control Method is chaotic system associates an external frequency of modulation with the role of an intrinsic reference clock for a control frequency with an adjustable phase shift. The frequency ratio between (modulation and scanning) will be varied, which make an interesting phenomenon that gives importance to the method of chaotic encryption. Attention should be paid that control method can not be used to stabilize fixed-point or stable solutions [12, 13]. Yang et al. and Qu et al. showed that a precise alternative to the phase allows the amplitude of the perturbation added to suppress chaos to be attenuated or leads to the spread of periodic orbits [14, 15]. The dynamical state of a nonlinear system (semiconductor laser) has been changed by Sora et al. and this was done by adding noise. These changes dependencies on the noise amplitude. [16] The duffing oscillator had been used by Arecchi et al. as a paradigm for the phase control through numerical exploration and that made the important role played by the phase more obvious. [17] Also, a method of multi-frequency phase control used by K. Al Naimee et al. which applied to a discrete economic mode “cobweb-model” with adaptive price expectations using parameter values where regular and chaos windows appears. [18]

In this work, we described the experimental details where the phase control method is applied on a chaotic semiconductor laser. The behavior of the dynamical system and the effects of various external modulation frequencies are shown in “Controlled bifurcation scenarios”.

Experimental work and discussions

The phase control method is applied to the Experiment for controlling the chaotic behavior. To this purpose, we configure the experimental set up as shown in Figure-1, it is a system, includes a semiconductor laser (hp / Agilent model 8150A optical signal source) with Operating current 70µA and ac-coupled optoelectronic feedback. The laser provides an emission with wavelength center at 850 nm, transmit by single mode optical fiber(10-125 )µm core cladding diameter and maximum output power of 2mW at room temperature. The Photodetector that used in this setup is THORL ABC model D400FC. The model of function generator is UTG9002C with Range 2MHz.
Figure 1 - Experimental setup with OEFB for investigating phase control method in chaotic dynamics.

SL, 850 nm semiconductor laser; OF, optical fiber; DC 50:50, single-mode 2 × 2 directional coupler; PD, photodetector; FG, function generator; VGA, variable gain amplifier; DSO, digital storage oscilloscope.

The output laser beam is sent through an optical fiber to a photodetector, where the optical signal is converted to an electrical signal. The generated electrical current is proportional to the optical intensity. Then the electrical signal passes through a variable gain amplifier. After that, the electrical signal is feedback to the mixer then to the semiconductor laser. Feedback current mixed with bias current and coming two signals from the function generator mixed inserted in modulation pin by differential amplifier. The chaotic semiconductor laser output is modulated by external signal using a function generator then added other perturbing frequencies for scanning the main modulation frequency.

The dynamics of the field density S and the carrier density N are proposed by single-mode semiconductor laser with optoelectronic feedback model [3]:

\[ S' = \left[ \frac{g(N-N_t)}{eV} \right] S \]

\[ N' = \frac{I_0}{eV} + \frac{f_e(1)}{eV} \frac{\gamma_c}{V_e-N}g(N-N_t)S \]

\[ l' = -\gamma_f + kS' \]

where \( I \) = high-pass filtered feedback before the nonlinear amplifier, \( V \) = volume of active layer, \( N_t \) = carrier density at transparency, \( I_0 \) = bias current, \( e \) = electron charge, \( f_e \) = feedback amplifier function, \( \gamma_c \) = population relaxation rate, \( k \) = a proportional coefficient to the responsively of the photo detector, \( g \) = differential gain, \( \gamma_0 \) = photon damping, \( \gamma_f \) = the high-pass filter cut off frequency.

The field density and carrier density are two linked variables used to describe the complete dynamics in our system. These variables have two very different characteristic time-scales. The application of optoelectronic delay feedback shows two benefits: firstly, adds a third degree of freedom in our system, secondly, adds a third much slower time-scale.

There is a need to rewrite equations 1, 2 and 3 in dimensionless form for analytical and numerical purposes; therefore, new variables were inserted:
Then, the rate equations become:

\[
\begin{align*}
x' &= x(y - 1) \\
y' &= y(\delta_0 - y + f(w + x) - xy) \\
w' &= -\varepsilon(w + x) 
\end{align*}
\]

where \( S = \frac{V_c}{k} \), \( \delta_0 = \left( \frac{I_0}{I_{th}} \right) \) is the bias current, \( f(w + x) \equiv \frac{w + x}{1 + S(w + x)} \), \( \alpha = \frac{\gamma_0}{(\varepsilon V_c)(\frac{\gamma_0}{x} + N_t)} \) is the current of laser, \( \gamma = \frac{t_c}{V_0} \) For more simplifications of dimensionless equations, let \( z = w + \chi \).

The nonlinear dynamics of SLs with OEF is represented in equations 4 (a, b, and c), where the first equation represents the photon density or output laser ray intensity and the second equation represents the population inversion. The feedback needed to generate chaos is represented by the third equation. This feedback is composed of the bias current and the intensity of laser output.

In phase control, the method relies on the addition of small harmonic perturbation parameters with adjustable phase. The key parameter in this control remains the phase difference between the required frequency to generate chaos (considering a modulation dynamical system) and the external perturbation frequency. In the phase control method, a new term \((1 + K)\) is introduced to equation 4(c), where \( K \) is an external perturbation. The equations then become[19]:

\[
\begin{align*}
x' &= x(y - 1) \\
y' &= y \left( \delta_0 - y + \alpha \frac{z}{1 + S} - xy \right) \\
w' &= -\varepsilon(w + x)(1 + K)
\end{align*}
\]

where \( K = (1 + \text{Asin}(2\pi mf_1 t) + \phi)\text{sin}(2\pi f_2 t) \), \( f_1 \), \( f_2 \) are external perturbations frequencies ; \( m \) represents the frequency ratio where our investigations will assume integer and fractional values. After fixing \( m \), the perturbation strength \( \varepsilon \) and phase difference \( \phi \) becomes the two control parameters.

Phase control of a chaotic system consisting of two frequencies as disturbance to a regularly operated chaotic system and using the phase difference to change the system’s dynamic state. The most necessary properties of this method, Only an accurate selection of frequencies leads the system to different orbits once, so we tend to fix the amplitude and frequency modulation, and then increase the external frequency of the disturbance, minimizing the amplitude necessary to suppress chaos.

The phase then represents another degree of freedom that is crucial to achieving control. Two function generators are used as a multi-frequency provider to phase control the system, one of them fixed for about 200Hz and 0.6 volt for amplitude, another frequency scanned from (160-700) Hz which fixed at 0.06volt as amplitude. The behavior of the system after adding two frequencies as shown in Figure-2.
We’ve got that by increasing the perturbation frequency and fixed the another frequency which leads the system from chaos state (Figure-2(a)) to a hyper-chaos (Figure-2(b)) so back to chaos state(Figure-2(c)). The phase different between these frequencies (\( \phi \)) made these transformation in this nonlinear system. This state compared with the chaotic system but adding only one frequency as shown in Figure-3.
Figure 3: The time series and Fast Fourier Transformation (FFT) of chaos modulated with (a) 170Hz, (b) 320Hz, (c) 600Hz.

In Figure-3, The chaotic system for one frequency of the control parameters has the same behavior (the system is led from chaos to a hyper-chaos) but with shift from the chaotic system that has two frequencies.

The results indicated that the one frequency could be considered as a parameter which control the system’s collective dynamics, but, very interesting results have been obtained regarding by adding two frequencies on the chaotic systems, the phase different between these frequencies $\phi$ have been controlled the chaotic system from chaos to hyper chaos and then to chaos.

The bifurcation diagram for different frequencies is plotted together, the bifurcation diagram is used to check chaotic routes and changes in output for variations in control parameters in nonlinear systems. The variation of the two frequencies lead to shifting in the bifurcation diagram, this shift is indicated in Figure-4.
Figure 4-Bifurcation diagram with different modulation frequencies.

Figure four shows these frequencies lead the system to change from chaos to a hyper-chaos state so back to chaos. A second frequency appears with a further change in the control parameter compared with the one frequency. Then we begin with another dynamics, amplitude and Frequency are fixed at 0.8 volt, 200Hz, another frequency scanned from (160-700) Hz which amplitude is fixed at 0.08 volt. In this condition, we have found by increasing the perturbation frequency leads the chaotic system to a hyper-chaotic and then to steady state, this case compared with the chaotic modulation for only one frequency which has the same behavior but with shift, these states with different frequencies are plotted together as shown in Figure-5.

Figure 5-Bifurcation diagram with different modulation frequencies

The relationship between the frequency of modulation and the frequency of external oscillations cause the shift in the diagrams of bifurcation as shown in Figure-5. the main features of phase control method: that an adequate choice of the phase leads the system from chaos to different periodic states, and how an accurate choice of can minimize the amplitude of the applied perturbation. However a thorough exploration of the parameter space allows us to find also new patterns on how phase control acts in this system, that were not previously observed. We underline the strong dependence of this
control scheme on the symmetries of the system, and we can clearly visualize how the election of is strongly affected by the amplitude and the frequency of the harmonic perturbation. When the frequency of modulation is close to the frequency of external oscillations, the position of critical points are changed and leading to this shift.

Conclusions

Phase control of chaos, consisting of applying a small parametric harmonic perturbation to a periodically driven chaotic system and use the phase difference $\phi$ to vary the dynamical state of the system. The most important properties of this method: that only a correct choice of $\phi$ can lead the system to a periodic orbit (once we fix the amplitude) of the perturbation and that, by adequately selecting the phase, the necessary amplitude to suppress chaos can be minimized. The phase control method has crucial effects on the Chaotic, Semiconductor, Laser. If the quantitative relationship of frequency or amplitude is varied, this method improves behavior in regularandchaotic regime. We have shown that the system led from chaos to a hyper-chaos state by increasing the frequency of perturbation and then back to chaos. When the perturbation amplitude increasing we can lead the system from chaos to hyper chaos and then back steady state. These features can be used in telecommunications when a chaotic carrier is used to transfer information.

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References

1. Ohtsubo, J. 2012. Semiconductor lasers: stability, instability and chaos (Vol. 111). Springer.
2. Shinbrot, T., Grebogi, C., Yorke, J.A. and Ott, E. 1993. Using small perturbations to control chaos. *Nature*, 363(6428): 411.
3. Al-Naimee, K., Marino, F., Ciszak, M., Meucci, R. and Arecchi, F.T. 2009. Chaotic spiking and incomplete homoclinic scenarios in semiconductor lasers with optoelectronic feedback. *New Journal of Physics*, 11(7): 073022.
4. Uchida, A. 2012. *Optical communication with chaotic lasers: applications of nonlinear dynamics and synchronization*. John Wiley & Sons.
5. Schöll, E. and Schuster, H.G. eds. 2008. *Handbook of chaos control* (Vol. 2). Weinheim: Wiley-Vch.
6. Boccaletti, S., Grebogi, C., Lai, Y.C., Mancini, H. and Maza, D. 2000. The control of chaos: theory and applications. *Physics reports*, 329(3): 103-197.
7. Ott, E., Grebogi, C. and Yorke , J.A. 1990 Controlling chaos. *Phys Rev Lett*, 64: 1196.
8. Pyragas, K., 1992. Continuous control of chaos by self-controlling feedback. *Physics letters A*, 170(6): 421-428.
9. Meucci, R., Gadomski, W., Ciofini, M. and Arecchi, F.T. 1994. Experimental control of chaos by means of weak parametric perturbations. *Physical Review E*, 49(4): R2528.
10. Meucci, R., Euzzor, S., Pugliese, E., Zambrano, S., Gallas, M.R. and Gallas, J.A.C. 2016. Optimal phase-control strategy for damped-driven Duffing oscillators. *Physical review letters*, 116(4): 044101.
11. Chacón, R., García-Hoz, A.M., Miralles, J.J. and Martínez, P.J. 2014. Amplitude modulation control of escape from a potential well. *Physics Letters A*, 378(16-17): 1104-1112.
12. Bielawski, S., Bouazaoui, M., Derozier, D. and Glorieux, P. 1993. Stabilization and characterization of unstable steady states in a laser. *Physical Review A*, 47(4): 3276.
13. Ciofini, M., Labate, A., Meucci, R. and Galanti, M. 1999. Stabilization of unstable fixed points in the dynamics of a laser with feedback. *Physical Review A*, 60(1): 398.
14. Qu, Z., Hu, G., Yang, G. and Qin, G. 1995. Phase effect in taming nonautonomous chaos by weak harmonic perturbations. *Physical review letters*, 74(10): 1736.
15. Yang, J., Qu, Z. and Hu, G. 1996. Duffing equation with two periodic forcings: The phase effect. *Physical Review E*, 53(5): 4402.
16. Abdalah, S.F., Ciszak, M., Marino, F., Al-Naimee, K.A., Meucci, R. and Arecchi, F.T. 2012. Noise Effects on Excitable Chaotic Attractors in Coupled Light-Emitting Diodes. *IEEE Systems Journal*, 6(3): 558-563.
17. Zambrano, S., Allaria, E., Brugioni, S., Leyva, I., Meucci, R., Sanjuán, M.A. and Arecchi, F.T. 2006. Numerical and experimental exploration of phase control of chaos. Chaos: An Interdisciplinary Journal of Nonlinear Science, 16(1): 1736.

18. Arecchi, F.T., Meucci, R., Salvadori, F., Acampora, D. and Al Naimee, K. 2010. A Physicist’s Approach to Phase Controlling Chaotic Economic Models. In Decision Theory and Choices: a Complexity Approach (pp. 205-210). Springer, Milano.

19. Al Husseini, H., Abdalah, S.F., Al Naimee, K.A.M., Meucci, R. and Arecchi, F.T. 2018. Exploring phase control in a quantum dot light-emitting diode. Nanomaterials and Nanotechnology, 8: 184.