Swap Action in a Solid-State Controllable Anisotropic Heisenberg Model

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Abstract

Correct swap action can be realized via the control of the anisotropic Heisenberg interaction in solid-state quantum systems. The conditions of performing a swap are derived by the dynamics of arbitrary bipartite pure state. It is found that swap errors can be eliminated in the presence of symmetric anisotropy. In realistic quantum computers with unavoidable fluctuations, the gate fidelity of swap action is estimated. The scheme of quantum computation via the anisotropic Heisenberg interaction is implemented in a one dimensional quantum dots. The slanting and static magnetic field can be used to adjust the anisotropy.

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I. INTRODUCTION

The physical implementation of quantum computation is thought of as a fundamental step for quantum information processing [1, 2, 3]. In recent years, some proposals have been offered about the realization of quantum computation, using atoms or photons in cavity [4, 5], trapped ions [6, 7], and bulk NMR techniques [8, 9]. In the implementation of these quantum computation architectures, it seems very difficult to perform the actual large-scale quantum computation. However, the quantum solid-state computation now attracts a considerable interest because of the feasible manipulation of many qubits represented by superconducting Cooper pairs [10], electron spin in quantum dots [11, 12, 13], orbital energy levels in nanostructures [14], donor nuclear spins [15, 16] and newly defined pseudospins [17]. In principle, the amazing large-scale quantum computation can be realized in such quantum solid-state systems. The schemes based on quantum dots have unique advantages in the actual physical implementation. In quantum dots, the microscopic systems of two discrete levels can serve as a qubit carrying the elementary quantum information. The electron spin as a natural two-level quantum system can be considered as one good qubit with the long coherence time [18]. The pseudospin using the orbit degrees of freedom is another one which can be controlled easily [19]. More recently, the proposal by combining the spin and orbit degrees of freedom has been introduced [17]. Based on it, coherently controlling single electron spin is possible. It seems that both good qualities about the long coherence time and easy manipulation are shown by this newly defined quantum system. Regardless of the definition of qubit, most of effective interaction between two coupled qubits are modeled by the Heisenberg exchange interaction. It has been shown that the universal quantum gates [20] can be realized via the isotropic Heisenberg interaction and the uniform magnetic field [12]. In the realistic spin-based quantum computation, errors from inhomogeneous Zeeman field [21] and anisotropic interactions [22, 23] are regarded as a major obstacle in quantum dots. The errors from inhomogeneous field cannot be eliminated completely [21, 24]. Meanwhile, the anisotropy induced by the spin-orbit coupling can lead to the nontrivial error with the order of $10^{-4}$ [23]. Therefore, it is necessary to study the method of implementing the quantum computation in the anisotropic Heisenberg model. The construction of correct swap action is one crucial step for possible quantum computation.
In this paper, the correct swap action can be performed in the anisotropic Heisenberg $XXZ$ model. In section II, the conditions of performing correct swap action are analyzed in detail. The impacts of the certain fluctuations on swap action are estimated by the gate fidelity from the Heisenberg interaction, the anisotropy and the effective Zeeman field. In section III, the possible physical implementation of such swap action is presented. A discussion concludes the paper.

II. SWAP ACTION IN ANISOTROPIC HEISENBERG MODEL

In many protocols about the solid-state quantum computation, the isotropic Heisenberg model is always used because the universal quantum gates can be constructed in this ideal model. Nevertheless, in realistic quantum computers, there are always anisotropic exchange interactions. Therefore, the general case of anisotropic Heisenberg $XXZ$ model needs to be studied. The Hamiltonian of two coupled qubits $i$ and $j$ can be given as

$$H_{ij} = J(S^x_i S^x_j + S^y_i S^y_j + \Delta S^z_i S^z_j) + \Gamma(S^z_i + S^z_j)$$  \hspace{1cm} (1)

where $S^\alpha_i = \frac{1}{2} \sigma^\alpha_i (\alpha = x, y, z)$ are three components of qubit $i$ operator and $\sigma^\alpha_i$ is the Pauli operator. For the convenience of computation, $|0\rangle_i, |1\rangle_i$ are assumed to be the eigenstates of $\sigma^z_i$ with the corresponding eigenvalues $\pm 1$, $J$ is the effective strength of Heisenberg exchange interaction and $\Delta$ is the anisotropy parameter. If the external magnetic field $\vec{B}$ is along $z$ direction, $\Gamma$ is the effective Zeeman splitting energy with $\Gamma = g \mu_B B$, $g$ is the effective $g$ factor and $\mu_B$ is the Bohr magneton. For the possible realization of quantum computation, the parameters $J, \Delta$, and $\Gamma$ can be tunable in quantum computers.

To show the dynamics of the anisotropic Heisenberg $XXZ$ model, the eigenstates $|\psi\rangle$ and corresponding eigenvalues $E$ of $H_{ij}$ need to be derived. In this model, the total spin is conserved since $[H_{ij}, S^z_i + S^z_j] = 0$. In the product space of two qubits, $|\psi\rangle$ and $E$ can be easily obtained by $E_1 = \frac{3\Delta}{4} + \Gamma$, $E_2 = \frac{J(\Delta-2)}{4} - \Gamma$, $E_3 = -\frac{J(\Delta+2)}{4}$ and $|\psi_1\rangle = |00\rangle, |\psi_2\rangle = |11\rangle, |\psi_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), |\psi_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. The general unitary transformation on qubits $i$ and $j$ can be expressed by

$$U_{ij}(t) = T \exp\{-i \int_0^t H_{ij}(t') dt'\}$$  \hspace{1cm} (2)

where $T$ is the time ordering operator. The swap action is just one of the unitary operations $U_{sw}$ by which the states at qubit $i$ and $j$ can be exchanged. Without losing of the generality,
an initial arbitrary quantum product state of qubits $i$ and $j$ can be assumed to be

$$|\Psi_m\rangle = (\alpha_1|0\rangle_i + \alpha_2|1\rangle_i) \otimes (\beta_1|0\rangle_j + \beta_2|1\rangle_j)$$

(3)

where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are arbitrary complex coefficients which satisfy $|\alpha_1|^2 + |\alpha_2|^2 = 1$ and $|\beta_1|^2 + |\beta_2|^2 = 1$. In the process of time evolution, the reduced density matrix $\rho_i$ and $\rho_j$ are generally turned into the mixed ones, i.e., $\rho_{ij} \neq \rho_i \otimes \rho_j$, which is useless for the setup of the swap action. Only if the density matrix $\rho_{ij}$ can be expressed by the product of $\rho_i$ and $\rho_j$, it can be used to construct the swap action. The method is very crucial by which the pure state $\rho_i$ and $\rho_j$ can be determined at a certain time. To obtain the swap action, the theorem about an arbitrary $2 \times 2$ matrix $A$ is introduced by

$$A^2 - Tr(A)A = Det(A)I$$

(4)

where $Tr(A)$ is the trace norm of matrix $A$, $Det(A)$ is the determinant of $A$ and $I$ is the unity matrix. If $A$ is a pure state, the determinant satisfies $Det(A) \equiv 0$. Thus, by means of calculating the determinant of $\rho_i$ or $\rho_j$, the reduced density matrix possibly denotes a pure state at certain time when the determinant is zero. A swap action can be finally constructed.

The quantum state $|\Psi(t)\rangle$ at time $t$ is given by

$$|\Psi(t)\rangle = \alpha_1\beta_1e^{-i(\phi_H+\phi_Z/4)}|00\rangle + \frac{1}{2}[\gamma_1e^{i(\phi_Z/4-\phi_X/2)} + \gamma_2e^{i(\phi_Z/4+\phi_X/2)}]|01\rangle$$

$$+ \alpha_2\beta_2e^{i(\phi_H-\phi_Z/4)}|11\rangle + \frac{1}{2}[\gamma_1e^{i(\phi_Z/4-\phi_X/2)} - \gamma_2e^{i(\phi_Z/4+\phi_X/2)}]|10\rangle$$

(5)

where $\gamma_1 = \alpha_1\beta_2 + \alpha_2\beta_1$ and $\gamma_2 = \alpha_1\beta_2 - \alpha_2\beta_1$. The phase angles are given by $\phi_H = \int_0^t \Gamma dt'$, $\phi_Z = \int_0^t J\Delta dt'$ and $\phi_X = \int_0^t Jdt'$. The reduced density matrix $\rho_i$ can be easily obtained by

$$\rho_i(t) = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix}$$

(6)

Here the elements of the matrix are calculated as

$$a_{00} = |\alpha_1\beta_1|^2 + \frac{1}{4}(|\gamma_1|^2 + |\gamma_2|^2 + \gamma_1^*\gamma_2e^{-i\phi_X} + \gamma_1^*\gamma_2e^{i\phi_X})$$

$$a_{01} = a_{10}^* = \frac{1}{2}\alpha_1\beta_1e^{-i(\phi_Z/2+\phi_H)}(\gamma_1^*e^{i\phi_X/2} - \gamma_2^*e^{-i\phi_X/2})$$

$$+ \frac{1}{2}\alpha_1\beta_1e^{i(\phi_Z/2-\phi_H)}(\gamma_1e^{-i\phi_X/2} - \gamma_2e^{i\phi_X/2})$$

$$a_{11} = |\alpha_2\beta_2|^2 + \frac{1}{4}(|\gamma_1|^2 + |\gamma_2|^2 - \gamma_1\gamma_2^*e^{-i\phi_X} - \gamma_1^*\gamma_2e^{i\phi_X})$$

(7)
The determinant is given by \( \det(\rho_i) = a_{00}a_{11} - a_{01}a_{10} \) and can be simplified by

\[
\det(\rho_i) = \left| \alpha_1\alpha_2\beta_1\beta_2 - \frac{1}{4} [\gamma_1^2 e^{i(\phi_Z - \phi_X)} - \gamma_2^2 e^{i(\phi_Z + \phi_X)}] \right|^2 
\]  

(8)

If the density matrix \( \rho_i \) is a pure state, the determinant is zero. For arbitrary complex coefficients \( \alpha_1, \alpha_2, \beta_1, \beta_2 \), the condition of \( \det(\rho_i) \equiv 0 \) needs the phase angles which simultaneously satisfy \( \phi_Z - \phi_X = 2n\pi \) and \( \phi_Z + \phi_X = 2m\pi \) with \( m \neq n \) and \( m, n = 0, \pm 1, \pm 2, \ldots \). That is, \( \phi_Z = (m + n)\pi \) and \( \phi_X = (m - n)\pi \). Furthermore, when the value of \( |m - n| \) is even at certain time \( \tau_o \), the state of qubit \( i \) can be expressed by \( |\psi_i(\tau_o)\rangle = \alpha_1|0\rangle_i + \alpha_2\exp[i(\pi n + \phi_H(\tau_o))]|1\rangle_i \) with the additional phase to the original state \( |\psi_i(0)\rangle \). When the value of \( |m - n| \) is odd at another certain time \( \tau_s \), the state is \( |\psi_i(\tau_s)\rangle = \beta_1|0\rangle_i + \beta_2\exp[i(\pi n + \phi_H(\tau_s))]|1\rangle_i \). If the phase angle \( \phi_H(\tau_s) = n\pi \), \( |\psi_i(\tau_s)\rangle = |\psi_j(0)\rangle \), which is just the original state of qubit \( j \). The correct swap action can be performed at this moment. Similarly, \( |\psi_i(\tau_o)\rangle = |\psi_i(0)\rangle \) at \( \phi_H(\tau_o) = n\pi \) and the state of qubit \( i \) keeps invariant. In some solid-state quantum computation architecture, the anisotropy \( \Delta \) can be adjusted by the time independent parameter. Thus, a correct swap action \( U_{sw} \) will be set up if the conditions are satisfied by

\[
\int_0^{\tau_s} J\Delta dt' = (m + n)\pi, \quad \int_0^{\tau_s} Jdt' = (m - n)\pi, \quad \int_0^{\tau_s} \Gamma dt' = n\pi 
\]  

(9)

By combining the corresponding single qubit operations and the swap action, the other two-qubit gate like CNOT gate will be easily constructed [11].

In the previous work [25], Yin et. al. discussed the Heisenberg XXZ model for quantum swap action. By means of the time evolution of one single-qubit reduced density matrix, they found that the Heisenberg XXZ model of \( 0 \leq \Delta < 1 \) cannot be used to perform the exact swap action. However, the anisotropy \( \Delta > 1 \) needs to be included in the general Heisenberg XXZ model. Different from the result of [25], the general solution of a swap action is obtained by Eq. (4) when the anisotropy is \( \Delta > 1 \). From Eq. (9), it is clear that the anisotropy of \( 0 \leq \Delta < 1 \) cannot be used to perform a swap action. This is consistent with the previous work [25].

Apart from the case of Eq. (9), the swap errors cannot be neglected. In realistic quantum computation, certain fluctuations [26, 27] from internal and external impacts are unavoidable. For quantum gates, the fluctuations from \( \phi_X, \phi_Z \) and \( \phi_H \) will influence the performance of \( U_{sw} \). For the weak coupling, the Gaussian distributions of
\( \phi_X \sim N(\bar{\phi}_X, \lambda_X), \phi_Z \sim N(\bar{\phi}_Z, \lambda_Z) \) and \( \phi_H \sim N(\bar{\phi}_H, \lambda_H) \) can be reasonably assumed. The Gaussian distribution \( N(\bar{\phi}, \lambda) = \frac{1}{\sqrt{2\pi}\lambda} e^{-(\phi - \bar{\phi})^2/2\lambda^2} \) with the mean value \( \bar{\phi} \) and the standard deviation \( \lambda \). These fluctuations are possible attributable to those of exchange interaction \( J \), anisotropy \( \Delta \) and the effective Zeeman field \( \Gamma \). To evaluate the effects of such fluctuations on \( U_{sw} \), the gate fidelity \[ F = \langle \Psi_{in} | U_{sw}^\dagger \rho U_{sw} | \Psi_{in} \rangle \] where the over-line denotes the average for all the possible initial states. After the calculation, the general fidelity is expressed by

\[
F(\phi_X, \phi_Z, \phi_H) = \frac{1}{5} + \frac{8}{15} \sin^2 \frac{\phi_X}{2} + \frac{4}{15} \sin \frac{\phi_X}{2} \sin(\frac{\phi_Z}{2} + \phi_H) \tag{10}
\]

When the mean value of the distributions is chosen to be those given by Eq. (9), the average fidelity \( F_A \) with fluctuations is obtained by

\[
F_A = \frac{7}{15} + \frac{4}{15} (e^{-\lambda_X^2/2} + e^{-(\lambda_X^2 + \lambda_Z^2 + 4\lambda_H^2)/8}) \tag{11}
\]

It is seen that the fluctuation from \( \phi_X \) mainly determined by the Heisenberg interaction \( J \) is always dominate in contrast to others. If the deviations \( \lambda_X, \lambda_Z, \lambda_H \to \infty \), the limit of the average fidelity is \( F_A \to \frac{7}{15} \). In Fig. 1, the condition of \( \lambda_X = \lambda_Z \) is clearly shown. The gate fidelity is decreased more rapidly with \( \lambda_X \) than that with \( \lambda_H \).

### III. IMPLEMENTATION BASED ON ONE PSEUDOSPIN

From one kind of newly defined pseudospin \[ [17], the implementation of quantum computation can be possibly performed via the controllable anisotropic Heisenberg model in quantum dots. As an effective qubit, this pseudospin can be constructed in a \( z \)-directional quantum dot. The either end of the quantum dot is applied by a ferromagnetic gate electrodes that creates a magnetic field gradient \( \vec{b} \) along \( x \) axis. If another external magnetic field \( \vec{B}_0 \) is applied along \( z \) axis, the total slanting magnetic field is \( \vec{B} = B_0 \vec{e}_z + zb\vec{e}_x \). It is noticed that the slanting magnetic field is static and tunable. The Hamiltonian of single electron in the parabolic confinement potential like \( GaAs \) can be expressed by

\[
h_i = -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + \frac{m\omega_0^2 z^2}{2} + g\mu_B (B_0 S^z_i + zb S^x_i) \tag{12}
\]

where \( m \) is the effective mass, \( \omega_0 \) is the frequency of the potential. The last term at the right hand side of Eq. (12) is the Zeeman splitting energy from the slanting magnetic
field $\vec{B}$. The amplitude of $\vec{B}$ is quite large that the effect of the Zeeman splitting energy cannot be neglected. If $|g_{\mu B}B| < \hbar \omega_0$, the effective two-level quantum system can be formed at the ground state. In general, the amplitude of field gradient $\vec{b}$ is smaller than that of external field $\vec{B}_0$. Thus, it is reasonable to apply the perturbation method to the splitting of $H'_i = g_{\mu B} z_b S^z_i$. For the convenience, the length of the confinement potential is chosen as $L = \sqrt{\frac{2\hbar}{m \omega_0}}$. In the product space of spin and orbit degree of freedom $\{|n, s\}, n = 0, 1, 2, \cdots, s = \pm\}$, the ground-state energy $E_{0,s}$ can be given by the second order approximation,

$$E_{0,s} \approx E_{0,s}^{(0)} + \frac{|\langle 0, s | H'_i | 1, -s \rangle|^2}{E_{0,s}^{(0)} - E_{1,-s}^{(0)}}$$

(13)

where $E_{0,s}^{(0)} = (n + \frac{1}{2})\hbar \omega_0 + g_{\mu B} B_0 s$ is the zeroth order energy and $\langle 0, s | H'_i | 1, -s \rangle = -\sqrt{\frac{\pi}{4}} g_{\mu B} b L$. Simultaneously, the ground state is calculated by the first order approximation,

$$|\varphi(0, s)\rangle \approx |0, s\rangle + C_{is} |1, -s\rangle$$

(14)

where the coefficient for pseudospin $i$ is $C_{is} = \frac{\langle 0, s | H'_i | 1, -s \rangle}{E_{0,s}^{(0)} - E_{1,-s}^{(0)}}$. The two-level states of this pseudospin are described by $|\varphi(0, +)\rangle, |\varphi(0, -)\rangle$ with the corresponding splitting energy $E_{0,+}, E_{0,-}$. Therefore, the effective Hamiltonian for this pseudospin $i$ can be written as $h_{eff} = \omega S^z_i$ where the transition frequency $\omega = |E_{0,s} - E_{0,-s}|$.

Based on this pseudospin, two coupled quantum dots $i$ and $j$ can be constructed in series [19]. After the introduction of the tunneling and inter-dot interaction, the effective Hamiltonian $H_{eff}$ mapped into the qubits is obtained by [17]

$$H_{eff} = J_{eff} (S^z_i S^z_j + S^y_i S^y_j + \Delta S^z_i S^z_j) + \tilde{\omega} (S^z_i + S^z_j)$$

(15)

where the effective interaction $J_{eff} = \frac{U_{ij}}{U-V}$, the effective anisotropy $\tilde{\Delta} = \frac{t_{+}^2 + t_{-}^2}{2t_{+} t_{-}} - \frac{f^2}{t_{+} t_{-} [1 - \Delta^2 (U-V)^2]}$ and the effective Zeeman splitting $\tilde{\omega} = \omega [1 - \frac{2f^2}{(U-V)^2 - \omega^2}]$ with $f = \frac{1}{2} (f_{+} + \frac{g_{i} b_{i}}{g_{j} b_{j}} f_{-})$. It is noted that the inhomogeneity from $g_{i} \neq g_{j}$ and $b_{i} \neq b_{j}$ is considered. The parameter $U$ is the charge energy, $V$ is the strength of the inter-dot interaction, $t_{\pm}$ and $f_{\pm}$ are the tunneling terms. The expressions of $t_{\pm}$ and $f_{\pm}$ are given by $t_{\pm} = t_{00} + C_{i\pm} C_{j\pm} t_{11}$ and $= (C_{i\pm} + C_{j\pm}) t_{12}$ where $t_{mn}$ is the tunneling amplitude from level $m$ in dot $i$ to level $n$ in dot $j$. Although the parameters of the effective Hamiltonian are complicate, the Hamiltonian is the anisotropic Heisenberg XXZ model discussed in Sec. II. The effective interaction $J_{eff}$, the anisotropy $\tilde{\Delta}$ and the Zeeman splitting energy $\tilde{\omega}$ can be adjusted via the static

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slanting magnetic field. When the conditions given in Eq. (9) are satisfied, the correct swap action can be performed in this quantum computer. It is also shown that the study of how to perform a swap action in anisotropic Heisenberg model is very instructive.

IV. DISCUSSION

The correct swap action can be performed in the anisotropic Heisenberg $XXZ$ model via the control of anisotropic interactions and the effective Zeeman field. The conditions of performing a perfect swap action are derived by the dynamics of arbitrary bipartite pure initial state. Considering the fluctuations in realistic quantum computers, the gate fidelity is used to estimate the robust ability of swap action against noise. It is found that the impact of the phase fluctuations $\phi_X$ from the Heisenberg interaction is dominant in contrast to those of $\phi_Z$ and $\phi_H$. Based on the newly introduced pseudospin [17], the possible physical realization of swap action is illustrated. Swap errors can be eliminated in the model of tunable anisotropic interactions.

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[1] A. Ekert and R. Jozsa, Rev. Mod. Phys. 68, 733(1996).
[2] A. Steane, Rep. Prog. Phys. 61, 117(1998).
[3] C. H. Bennett and D. P. DiVincenzo, Nature(London) 404, 247(2000).
[4] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, Phys. Rev. Lett. 75, 4710(1995).
[5] T. Sleator and H. Weinfurter, Phys. Rev. Lett. 74, 4087(1995).
[6] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091(1995).
[7] C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. 75, 4714(1995).
[8] D. Cory, A. Fahmy, and T. Havel, Proc. Natl. Acad. Sci. USA 94, 1634(1997).

[9] N. A. Gershenfeld and I. L. Chuang, Science 275, 350(1997).

[10] A. Shnirman, G. Schon, and Z. Hermon, Phys. Rev. Lett. 79, 2371(1997).

[11] D. Loss and D. P. DiVincenzo, Phys. Rev. A 57, 120(1998).

[12] G. Burkard and D. P. DiVincenzo, Phys. Rev. B 59, 2070(1999).

[13] A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. 83, 4204(1999).

[14] N. H. Bonadeo, J. Erland, D. Gammon, D. Park, D. S. Katzer, and D. G. Steel, Science 282, 1473(1998).

[15] B. E. Kane, Nature(London) 393, 133(1998).

[16] V. Privman, I. D. Vagner, and G. Kventsel, Phys. Lett. A239, 141(1998).

[17] Y. Tokura, W. G. van der Wiel, T. Obata, and S. Tarucha, Phys. Rev. Lett. 96, 047202(2006).

[18] J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science 309, 2180(2005).

[19] W. G. van der Wiel, S. De Franceschi, J. L. Elzerman, T. Fujisawa, S. Tarucha, and L. P. Kouwenhoven, Rev. Mod. Phys. 75, 1(2003).

[20] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, UK, 2000).

[21] X. Hu, R. de Sousa, and S. Das Sarma, Phys. Rev. Lett. 86, 918(2001).

[22] G. Burkard and D. Loss, Phys. Rev. Lett. 88, 047903(2002).

[23] K. V. Kavokin, Phys. Rev. B64, 075305(2001).

[24] X. Hu and S. Das Sarma, Phys. Rev. A68, 052310(2003).

[25] W. Yin, J. Q. Liang, and Q. W. Yan, Phys. Lett. A339, 472(2005).

[26] G. Vidal and R. Tarrach, Phys. Rev. A59, 141(1999).

[27] S. Bandyopadhyay and D. Lidar, Phys. Rev. A70, 010301(R)(2004).

[28] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 78, 390(1997).
Fig. 1
The average gate fidelity is plotted when \( \lambda_X = \lambda_Z \).
