Raman - Nath approximation for diffraction of atoms in the laser field taking into account spontaneous emission of atoms for ground and high energy level

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Abstract. The problem of resonant Kapitza–Dirac diffraction is solved in Raman–Nath approximation out of familiar Bessel function approximation (applicable in zero and very large resonance detuning cases). It shows new and promising results for the atom optics and atom interferometry if the atomic momentum state has been prepared in a form of discrete Gaussian distribution. Namely, instead of monotonic broadening within the Bessel function approximation, our formula yields in splitting of initial distribution into two identical peaks, which preserving form, which symmetrically move away from the distribution center for the interaction time. A table-shaped form for the ultimate momentum distribution also is in frame of new distributions. As to relaxation processes, they have only quantitative influence on the pattern of diffraction.

Introduction
Atom scattering in the field of laser counterpropagating waves attracts a constant attention in atom optics and atom interferometry. The mechanism of scattering is emission of photons from one of the counterpropagating waves into the other counterpropagating wave, when due to each reemitted photon the atom acquires recoil momentum $2\hbar k$, where $k$ is the wave vector. Outside the Bragg regime of diffraction analytical results for the scattering amplitudes was possible to obtain also for short times of interaction when the kinetic energy operator of the atom may be neglected (Raman-Nath approximation). Under exact resonance and far off-resonance conditions, the probability amplitude of the $n$-th order diffraction (acquiring of an additional momentum $2n\hbar k$) has the following well-known form:

$$a_n(t) = i^n J_n(\Omega_{Rabi}t),$$

(1)

where the argument of the Bessel function $J_n(\Omega_{Rabi}t)$ where the argument of the Bessel function $J_n(\Omega_{Rabi}t)$ is the product of Rabi frequency $\Omega_{Rabi}$ and the interaction time $t$. The evolution of momentum state (1) is shown in Figure 1. The characteristic feature of it is the monotonic unlimited enlargement of the area covered by the momentum states. Therefore, high-order Kapitza-Dirac
diffraction appears not useful for atom interferometry. A careful analysis, proceeding from (1), shows that the superpositional nature of the initial state of atomic center of mass translational motion also does not give the desirable breakthrough in the problem. Here, we reexamine the problem deriving an expression for the atom scattering amplitude \( a_n(t) \) without putting any restrictions on the value of resonance detuning \( \Delta = \omega - \omega_0 \), where \( \omega \) is the laser field frequency and \( \omega_0 \) is the atomic transition frequency. The amplitude is expressed by means of a definite integral (see (9)) and can be viewed as some type of generalization of the Bessel function. Although it is true, that the difference between (9) and (1) has only a quantitative nature, one has totally diverse behavior if the atom initially is in the superposition of many equidistant momentum states. Evolution of diffraction picture acquires a qualitatively new content, which is very promising for the large-scale beam splitter atom interferometry, which we call here two bunch atom interferometry.

![Figure 1](image)

**Figure 1.** The picture of resonant Kapitza-Dirac diffraction in Raman-Nath approximation \( W_n = \left| a_n(t) \right|^2, p_n = 2 \hbar k \), in case of a definite value of atomic initial momentum \( a_n(t=0) = \delta_{n0} \).

### 2. Scattering amplitudes

The problem under consideration is evolution of the momentum distribution of a two-level atom due to interaction with the field of counterpropagating waves. The strength of electric field is

\[
E(z,t) = E_1(t) e^{i(kz-\omega t)} + E_2(t) e^{i(kz-\omega t)} + c.c
\]

(2)

where \( k = \omega / c \) and \( z \) is the atomic center of mass coordinate. Wave amplitudes \( E_1(t) \) and \( E_2(t) \) are assumed of simplest step-wise form, which turn on at the moment \( t = 0 \). The Hamiltonian in dipole approximation is

\[
\hat{H}_0 = \frac{\hbar^2}{2M} \frac{\partial^2}{\partial z^2} + \hat{H}_0 \cdot d \cdot E(z,t)
\]

(3)

where \( \hat{H}_0 \) is the Hamiltonian of free and immovable atom, \( M \) is atomic mass and \( d \) is the dipole moment operator. The atomic wavefunction can be written in the following form:
\[ \psi(z, r, t) = a(z, t) \psi_1(r) e^{i \frac{E_1}{\hbar} t} + b(z, t) \psi_2(r) e^{i \frac{E_2}{\hbar} t}, \]  
\tag{4}

where \( a(z, t) \) and \( b(z, t) \) are the target probability amplitudes of atom to reside in ground and excited energy levels respectively, \( E_1, E_2 \) and \( \psi_1(r), \psi_2(r) \) are eigenvalues and eigenfunctions of the free Hamiltonian, \( r \) is the radius-vector of atomic optical electron relative to the center of mass. The Raman-Nath approximation is an approximation of short interaction times when the corresponding atomic displacement is much smaller than the spatial period of the standing wave (constructed by the pair of counterpropagating waves). In some sense it means an immovable atom, giving a right to ignore the kinetic energy operator in the Hamiltonian (3). For illustrating the effect of relaxation of high energy level due to spontaneous emission on probability amplitudes we choose the simplest model: instead of \( \omega_0 \) we took \( \omega_0 + i \gamma \). As long as we have a resonance detuning greater than the line broadening, this procedure describes the damping phenomena very well. Then, for \( t \gg 0 \) we obtain the equations

\[ i \hbar \frac{\partial}{\partial t} a(z, t) = -d \left( E_1 e^{i k z} + E_2 e^{i k z} \right) b(z, t) \]  
\tag{5}

\[ \left( i \hbar \frac{\partial}{\partial t} - \hbar \Delta + i \hbar \gamma \right) = -d * \left( E_1 e^{i k z} + E_2 e^{i k z} \right) a(z, t). \]  
\tag{6}

The right-hand side coefficients do not have any time dependence and the equation for amplitude \( a(z, t) \) can be obtained by acting on (5) with operator \( i \hbar \frac{\partial}{\partial z} - \hbar \Delta \) and then using Eq. (6). This yields

\[ \left( \frac{\partial}{\partial z} + i \Delta + \gamma \right) i \hbar \frac{\partial}{\partial t} a(z, t) = -\frac{|d|^2}{\hbar^2} \left( E_1^2 + E_2^2 + 2 E_1 E_2 \cos(2 k z) \right) a(z, t) \]  
\tag{7}

Solution of this equation in momentum space should be the Fourier series

\[ a(z, t) = \sum_{n=-\infty}^{\infty} a_n(t) e^{i n 2 k z}, \]  
\tag{8}

Unknown amplitude \( a_n(t) \) will be sought in the form of a definite integral

\[ a_n(t) = \frac{1}{\pi} \int e^{i \hat{\lambda}(\phi) t} \cos(n \phi) d \phi \]  
\tag{9}

with an unknown function \( \hat{\lambda}(\phi) \). Substitution of (8) and (9) into (7) gives two possible expressions for \( \hat{\lambda}(\phi) \):

\[ \hat{\lambda}_{1,2}(\phi) = \frac{i \gamma - \Delta}{2} \pm \frac{|A|}{2} \sqrt{1 - \frac{\gamma^2}{\Delta^2} + \frac{2 i \gamma}{\Delta} + \xi(\phi)}, \]  
\tag{10}

where \( \xi(\phi) = \xi_1^* + \xi_2 + 2 \sqrt{\xi_1^* \xi_2^*} \cos(\phi) \) and \( \xi_{1,2} = 4 d^2 E_{1,2} / \hbar^2 \Delta^2 \).
The general solution is a superposition of two linearly independent solutions:

\[ a_n(t) = C_1 \int_0^\pi e^{i \lambda_1 \phi} \cos(n \phi) d\phi + C_2 \int_0^\pi e^{i \lambda_2 \phi} \cos(n \phi) d\phi, \quad (11) \]

where coefficients \( C_1 \) and \( C_2 \) have to be determined from initial conditions. If, for example, the atom initially was in rest and resided on the ground state, then

\[ C_{1,2} = \frac{1}{2} \pm \frac{\text{sign} \{ \Delta \}}{4 \sqrt{1 - (\frac{\lambda_1}{\lambda_2})^2} + 2 i \frac{\lambda_1}{\lambda_2} + \xi}, \quad (12) \]

where \( F(x) \) is the elliptic integral of the first kind. Equation (11) with notations (10) and (12) is a generalization of familiar (1) for the case of arbitrary resonance detuning.

3. Numerical calculation

Qualitative differences from the Bessel function regularities are obtained when the atomic motional state is prepared in a quantum superposition state. They are especially prominent, when the superposition is of discrete Gaussian form:

\[ a(z, t = 0) = \sum_{n=-\infty}^{\infty} s_n(t) e^{i n 2k z}, \quad (13) \]

\[ s_n = \frac{1}{\sqrt{\pi}} e^{i \alpha n} \frac{1}{\sqrt{\sigma}} e^{-i \alpha v \frac{(n \sigma)^2}{2\sigma}}, \quad (14) \]

where \( \sigma \) is the FWHM of distribution and \( \alpha \) is the initial phase (this case is depicted in Figure 2).

Then the diffraction amplitude (11) gets the following form:

\[ a_n(t) = \frac{1}{\sqrt{\pi}} e^{i \alpha n} \frac{1}{\sqrt{\sigma}} \left( C_1 \int \exp\left(\frac{i \phi \lambda_1}{2 \sigma} \sqrt{1 - \frac{\lambda_1^2}{\lambda_2^2} + 2i \frac{\lambda_1}{\lambda_2} + \xi(\phi)} \right) \sum_{n=-\infty}^{\infty} \exp(-i \alpha v \frac{(n \sigma)^2}{2\sigma}) \cos(v \phi) d\phi + C_2 \int \exp(-\frac{i \lambda_2 \phi}{2 \sigma} \sqrt{1 - \frac{\lambda_1^2}{\lambda_2^2} + 2i \frac{\lambda_1}{\lambda_2} + \xi(\phi)} \right) \sum_{n=-\infty}^{\infty} \exp(-i \alpha v \frac{(n \sigma)^2}{2\sigma}) \cos(v \phi) d\phi. \quad (15) \]

To illustrate the new regularities in the diffraction pattern, we will concentrate on more important cases \( \alpha = 0 \) and \( \alpha = \pi / 2 \). Bessel function approximation in case of \( \alpha = 0 \) results in a monotonic increment in the width of prepared Gaussian distribution. Numerical calculations on base of (15) give a qualitatively different result. As is seen in Figure 3, initial distribution splits into two identical parts, which preserving their form, symmetrically move away from each other (or the centre of initial distribution). It should be noted experimentally it has not been possible to increase the distance between the two peaks up to few hundreds of photon momentum yet, while in principle this is realizable under the interaction scheme suggested in the paper. Note, that the relaxation has no role under conditions of Figure 3. The occurrence of relaxation in general affects the picture of evolution through the product parameter \( \gamma t \), where \( \gamma \) is the rate of relaxation. Such a case with \( \gamma t = 0.3 \) is
presented in Figure 4. As is seen, the influence on the form of distribution is slightly visible. Main deviation from the reference picture in Figure 3 is the loss of full symmetry of the picture. As would be expected, the symmetry can't be restored totally by means of some new value of $\gamma t$ or remaining parameters. The optimum form of distribution in case of $\gamma t = 0.3$ is depicted in Figure 5.

![Figure 2](image2.png) Initial momentum distribution with phase parameter $\alpha = 0$ and half-width $\sigma = 10$.

![Figure 3](image3.png) Diffracting in the field of far off-resonance counterpropagating waves in Raman-Nath approximation, atom momentum distribution splits into two symmetrically posed paths. Parameters are the similar to the ones in Figure 2, $\gamma = 0, t = 18 \times 10^6$ (in units $\Delta^{-1}$).

Not less interesting is the case $\alpha = \pi$, when a table-shaped form of momentum distribution is obtained from the initial Gaussian distribution as is shown in Figure 6. This strongly recommended in high resolution spectroscopy$^5$ distribution is robust with respect to relaxation processes: affecting the

![Figure 4](image4.png) Parameters are the similar to the ones in Figure 3, $g = 0.3, t = 18 \times 10^6$ (in units $\Delta^{-1}$).

![Figure 5](image5.png) Parameters are the similar to the ones in Figure 2, $g = 0.3, t = 17.82 \times 10^6$ (in units $\Delta^{-1}$).

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absolute value of probability amplitudes, the relaxation preserves the table-shaped form of the momentum distribution.

\[ W_n = \frac{n(2 \pi k)}{n(2 \pi k)} \]

Figure 6. Generation of table-form momentum distribution for a moving atom in the Raman-Nath approximation: \( \alpha = \pi \), \( \sigma = 10 \), \( t = 17.8 \times 10^6 \).

4. Conclusion
The problem of atomic Kapitza-Dirac diffraction is solved in Raman-Nath approximation for arbitrary values of resonance detuning including the relaxation. The obtained formula for the scattering amplitudes generalizes the known expression given by means of the first kind Bessel function. Remarkable point here is the ability to obtain double-peak and table-like forms in momentum distribution, desirable in applied atomic optics and atom interferometry.

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5. References
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