Research Article

Mechanical Properties of Functionally Graded Concrete Lining for Deep Underground Structures

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With the mining depth of coal resources increasing, the thickness of traditional lining for deep mines becomes large. The bearing capacity of the outer lining cannot be fully utilized, so a new type of functionally graded lining structure (FGL) with radial Young’s modulus varying in gradient is proposed. In this study, through theoretical analysis and numerical simulation, the mechanical properties of the functionally graded lining were studied, including the characteristic of the elastic working state and the ultimate bearing state. The influences of the structural parameters of the functionally graded lining, including the inner radius, the thickness, Young’s modulus, the compressive strength of the concrete, Poisson’s ratio, and the number of layers on the mechanical properties were analyzed. The calculation formula of the ultimate bearing capacity of the lining and the calculation formula of the maximum tangential strain at the time of lining failure were put forward. The accuracy of the formula was verified by comparing with the numerical simulation results. The research results provide the basis for the construction of the design theory of a functionally graded lining structure and have great engineering significance for the construction of the kilometer-deep mines in the future.

1. Introduction

With the depletion of shallow coal resources in China, the coal mining is developing towards the direction of deep and large scale. The annual production capacity of the new and expanded large vertical mines has reached 10 million tons, and the maximum mining depth has reached 1500 m. At present, the total proved coal resources are about 5.57 trillion tons, and 2.95 trillion tons are under 1 km buried depth, accounting for 53% of the total. So far, 47 mines with mining depth greater than 1000 m have been built in China. In the next 5–10 years, more than 30 coal mines with a depth of more than 1 km will be built.

The in situ stresses increase with depth. For example, the vertical stress at 1910 m depth is about 43.5 MPa [1]. Therefore, the continuous deepening of the mining depth means that lining structures with higher bearing capacity must be adopted. In order to ensure that the lining works in a safe state, there are usually two ways to improve the bearing capacity of the lining: increasing the material strength and increasing the thickness of the lining. For the first way, the bearing capacity of the lining can be significantly improved by increasing the concrete strength. For example, if the concrete strength is increased by 10 MPa, the bearing capacity of the lining will be increased by about 13.8% [2]. The high-grade cement was used to improve the strength of concrete, resulting in the increasing cost and the brittle failure of lining. For the second way, the improvement effect of increasing lining thickness is limited. Statistics show that rock excavation accounts for 40%–60% of the total cost in shaft construction. For shafts extending to 1000 m depth, an increase of 10 mm in the lining thickness will increase the
total cost by 1% for reinforced concrete and 0.25% for plain concrete [3]. Both theoretical research and practical experience show that using a traditional lining structure in the complex stratum of over kilometer-deep mine will face the problem of increasing material and construction cost. The thickness of the traditional lining makes the bearing capacity of the outer half material ineffective, which leads to serious waste of economy and resources.

To solve this problem, the functionally graded material (FGM) is used to design the lining structure in this study. In 1987, Japanese scientists first proposed a new concept and idea of using ceramic functionally graded materials. The physical properties of materials show gradient change in space, so there is no sudden change in material properties, which can effectively avoid and reduce stress concentration, so as to meet the different needs of different parts of the structure, and finally achieve the purpose of optimal overall performance of the structure [4]. The first developed FGM uses ceramic materials at high temperature to improve its high temperature resistance, and metal materials at the cold end in the presence of liquid nitrogen and liquid oxygen to provide good thermal conductivity and mechanical strength. From the micro point of view, the continuous change in the interlayer material from the hot end to the cold end shows nonuniform material properties [5]. Later on, its applications have been expanded to also the components of chemical plants, solar energy generators, heat exchangers, nuclear reactors, and high-efficiency combustion systems [6]. At present, the application fields of FGMs involve commerce, electronics, automobile, defense, aerospace, medical treatment, thermal barrier coating, optoelectronics, industry, and other fields [7]. In the field of construction engineering, the concept of functionally graded materials has been widely applied in the topical composites technology to develop more efficient materials [8–11]. Under the specific application requirements, materials with different properties are selected, and advanced composite technology is adopted to make the composition and structure of materials continuously change in gradient, and the properties and functions of materials also change continuously along the thickness direction. According to the distribution of component phases, functionally graded materials can be divided into two types, that is, continuous or discontinuous (stepwise or layered) gradation of materials. In addition, based on manufacturing techniques, these can be further classified as thin and bulk functionally graded materials [12].

This study proposes the functionally graded lining (FGL) as a new type of rock support for kilometer-deep underground engineering construction in order to make full use of the bearing capacity of lining materials (see Figure 1). It is assumed that the lining is homogeneous and isotropic in the tangential direction, and the material parameters change in gradient in the radial direction. Scholars at home and abroad have studied the theory of stress solution and displacement solution of functionally graded thick-walled hollow cylinder under different stress conditions. Tutuncu [13] gave the closed solutions of stress and displacement of functionally graded cylindrical and spherical vessels under internal pressure by using the microelement theory of elasticity. Abdelhakim [14] obtained the analytical solution of radial displacement and stress distribution of hollow cylinder under uniform internal and external pressure, assuming that the elastic modulus changes nonlinearly along the radial direction of the material. Theotokoglou and Loannis [15, 16] gave the exact analytical solution of the radial nonuniform spherical shell with equal thickness and Young’s modulus distributed as power function and exponential function in plane coordinates and spherical coordinates, and derived the displacement field and stress field. Under the action of uniform magnetic field and internal pressure, Li [17] has derived the analytical solutions of radial displacement, strain and stress components of a thick-walled hollow cylinder, and the vector of disturbed magnetic field by using the Voigt method. Ahmad [18] used the meshless local Petrov–Galerkin method to study the dynamic response of a functionally graded viscoelastic hollow cylinder subjected to thermo-mechanical loads. Based on the 3-delicacy theory, Ye [19] analyzed the three-dimensional hygrothermal vibration of multilayer cylindrical shell under general boundary conditions. Yavar [20] obtained the field equations and general solutions of axisymmetric thick shells made of functionally graded incompressible hyperelastic materials. Wang [21] deduced the two-dimensional elastic solution under the generalized plane strain assumption and gave the solution process of the separable problem of multilayer cylinder. Nie [22] used the Airy stress function to derive the exact solution of the plane strain of a functionally graded hollow cylinder with isotropic incompressible linear elastic materials on the inner and outer surfaces with different boundary conditions. Chen [23] proposed the method of transmission matrix to solve the stress and displacement theoretical solutions of thick-walled cylinders made of multilayer functionally graded materials with arbitrary Young’s modulus. Shi and Xiang [24, 25] obtained the hypergeometric equation of the contact pressure between layers by the displacement method and obtained the exact solution of the n-layer elastic hollow cylinder under the...
compression of the inner and outer surfaces. Zhang [26] proposed a method to solve the stress and displacement of the plane strain on the inner and outer surface of the radial inhomogeneous cylinder.

In this study, based on the previous theoretical research results, through theoretical analysis and the numerical calculation method, the mechanical properties of functionally graded lining were systematically and comprehensively studied. By changing a single material parameter, the effects of lining parameters on its mechanical properties such as stress, strain, and displacement were analyzed. Finally, the calculation formula of the ultimate bearing capacity of lining and the calculation formula of the maximum tangential strain of lining inner edge are put forward. The accuracy of the formula was verified by comparing the calculation results with the numerical simulation results.

2. Theoretical Basis and Experimental Design of FGL

2.1. Exact Solution of the N-Layer FGL. The stress problem of shaft lining equals to the stress problem of the thick-walled hollow cylinder. The calculation model of N-layer functional graded lining is shown in Figure 2. It is specified that $E_i$ represents the elastic modulus of layer $i$ and $R_i$ represents the outer radius of layer $i$. The thickness of each layer is determined according to the number of layers, with average layers. Assume that each layer has the same value of Poisson’s ratio $\nu$. The lining is only subjected to the uniform load $Q$ on the outer surface. For convenience, the symbols $N_{i+1} =$

$$
\begin{align*}
\sigma_r(i) &= \frac{A_i}{r^2} + 2C_i, \\
\sigma_\theta(i) &= -\frac{A_i}{r^2} + 2C_i, \\
\upsilon_r(i) &= 1 - \frac{v^2}{E_i} \left[ (1 + \frac{v}{1-v}) \frac{A_i}{r} + 2 \left(1 - \frac{v}{1-v}\right) C_i r + I_i \cos \theta + K_i \sin \theta \right],
\end{align*}
$$

where $A_i$, $C_i$, $I_i$, and $K_i$ are constants to be determined. For the sake of symmetry of lining, we have $I_i = K_i = 0$. The radial stress and displacement at the interfaces each layer should be continuous, so

$$
\begin{align*}
\sigma_r(i)|_{r=R_i} &= \sigma_r(i+1)|_{r=R_i}, \\
\upsilon_r(i)|_{r=R_i} &= \upsilon_r(i+1)|_{r=R_i}. 
\end{align*}
$$

The relationship of extrusion stress between adjacent layers is as follows:

$$
\begin{align*}
\frac{A_i}{R_{i-1}^2} + 2C_i &= -q_{i-1}, \\
\frac{A_{i+1}}{R_{i+1}^2} + 2C_{i+1} &= -q_{i-1}.
\end{align*}
$$

Here, $q_{i-1}$ ($i=2, \ldots, N$) is the extrusion stress between the layer $i-1$ and the layer $i$. $q_0$ and $q_N$ are the inner and outer pressure of lining, respectively. Considering $q_0 = 0$ and $q_N = Q$, we have
\[
\begin{align*}
\frac{A_i}{R_i^2} + 2C_i &= -q_i = 0, \\
\frac{A_N}{R_N^2} + 2C_N &= -q_N = -Q.
\end{align*}
\]

Combining equations (1)–(3), we obtained the relationship between \((A_i, C_i)\) and \((q_{i-1}, q_{i+1})\) as follows:

\[
\begin{pmatrix}
\frac{A_i}{R_i^2} + 2C_i = -q_i = 0, \\
\frac{A_N}{R_N^2} + 2C_N = -q_N = -Q
\end{pmatrix}
\]

Here,
\[
\begin{align*}
\delta_i &= 1 + \frac{1}{2} \left(1 + \frac{\nu}{1 - \nu}\right) (1 - N_{i+1}) \left(\gamma_{i+1}^2 - 1\right) \\
-\frac{1}{2} \left(1 - \frac{\nu}{1 - \nu}\right) (1 - N_{i+1}) \gamma_i^2 \\
-\frac{1}{2} \left(1 - \nu\right) N_{i+1} \gamma_i^2 \gamma_{i+1}^2.
\end{align*}
\]

In any layer, the radial stress satisfies
\[
(\sigma_r)_i = \frac{A_i}{R_i^2} + 2C_i = -q_i.
\]

Substituting (5) into (7), we obtain
\[
q_{i+1} = \frac{\delta_i q_i - (\gamma_{i+1}^2 - 1) N_{i+1} q_{i-1}}{(\gamma_i^2 - 1) \gamma_{i+1}^2}.
\]

Therefore, (5) can be written as
\[
\begin{align*}
\frac{A_i}{R_i^2} &= \frac{1}{1 - \gamma_i^2} (q_{i-1} - q_i), \\
2C_i &= -\frac{1}{1 - \gamma_i^2} (q_{i-1} - \gamma_i^2 q_i).
\end{align*}
\]

In order to express \(q_i\), we define \(q_0\) and \(q_1\) first. According to (8), we get
\[
\Delta_i = -\frac{\delta_i \Delta_{i+1} - (\gamma_{i+1}^2 - 1) N_{i+1} \Delta_{i-2}}{(\gamma_i^2 - 1) \gamma_{i+1}^2},
\]

\((i = 2, \ldots, N),\)

where \(\Delta_0 = -1; \Delta_1 = \delta_1 / (\gamma_1^2 - 1) \gamma_2^2.\)

(8) can be written as
\[
q_i = -\Delta_{i-1} q_i, \\
(i = 2, \ldots, N).
\]

Considering \(q_N = Q,\) we have

\[
q_i = -\frac{Q}{\Delta_{N-1}}.
\]

Therefore, the extrusion stress of each layer can be obtained as
\[
q_i = -\Delta_{i+1} \frac{Q}{\Delta_{N-1}},
\]

\((i = 2, \ldots, N - 1).\)

Equation (13) gives the exact solution of the extrusion stress of two adjacent layers of the \(N\)-layer thick-walled cylinder. Therefore, the stress solution and displacement solution at any position of the thick-walled cylinder can be obtained by lamine’s solution [24].

2.2. Failure Criterion of Concrete. Many concrete strength criteria have been proposed, and many scholars would select appropriate concrete failure criteria according to their own requirements, including von Mises strength criterion [27, 28], Mohr–Coulomb strength criterion [29, 30], Hsieh–Ting–Chen strength criterion [31, 32], Bresler–Pester criterion [33], William–Warnke failure criterion [34], Drucker–Prager criterion [35, 36], Kupfer strength criterion [37, 38], Guo–Wang multi-axial strength criterion [39, 40], multiparameter unified strength criterion [41], and multi-directional stress state failure criterion [42, 43]. The lining is mainly subjected to external confining pressure, and the lining concrete is under multi-axial stress. The multi-axial strength of concrete should be fully exploited [44].

The power function failure criterion [45], which is in accordance with the experimental results and expressed dimensionless by octahedral stress, is adopted. The general equation is as follows:

\[
\frac{\tau_{oct}}{f_c^*} = a \left( \frac{\sigma_{oct}}{c - \sigma_{oct}} \right)^d.
\]

Here, \(c = c_s (\cos 3/20)^{1/5} + c_s (\sin 3/20)^{3/5}\); \(\tau_{oct}\) and \(\sigma_{oct}\) denote the stress components; \(f_c^*\) is the uniaxial compressive strength of concrete, which shall be taken as a standard value, design value, average value, or test value according to
the structural analysis method and limit state checking calculation needs; θ is similar angles. a, b, c1, c2, and d are constants be determined, which can be calibrated according to uniaxial compressive strength, uniaxial tensile strength, biaxial isobaric strength, triaxial isobaric strength, and triaxial isobaric strength.

According to the test results at home and abroad, the constant values, that is, a = 6.9638, b = 0.09, c1 = 12.2445, c2 = 7.3319, d = 0.9297, which can be applied to all kinds of test conditions and all multi-axial stress ranges, were obtained. The calculation accuracy of this failure criterion was relatively high [46].

Based on the abovementioned multi-axial failure criterion, the envelope equation of lining failure is established as follows:

\[ f = a \left( \frac{b - \sigma_{ocf}/f_c^*}{c - \sigma_{ocf}/f_c^*} \right)^{\frac{d}{c}} - \frac{\tau_{ocf}}{f_c} \]  \hspace{1cm} (15)

When \( f = 0 \), the lining concrete reaches the critical state of failure; when \( f > 0 \), the lining works normally; when \( f < 0 \), the lining is damaged.

2.3. Function of Young’s Modulus of FGL. The material parameters of functionally graded thick-walled hollow cylinder are assumed to be graded in the radial direction. Young’s modulus of the material changes in a radial direction, and Poisson’s ratio is assumed constant [13–16]. The compressive strength of the lining concrete remains the same. Common function forms include linear function [24, 25], exponential function [15, 22], power function [20, 22], and other function forms [23, 26]. For the purpose of plastic yielding of the whole lining at the same time, through the method of back analysis [47], Young’s modulus function is obtained by using the required stress distribution. Based on the unified strength theory, the function form is as follows [3]:

\[ E(r) = C \left[ \frac{1 - 2\nu}{1 - \nu} \ln \left( \frac{r}{R_0} \right) + 1 \right]^{2(1-\nu)/(1-2\nu)} \]  \hspace{1cm} (16)

where \( C \) is the integral constant.

By specifying Young’s modulus \( E^* \) at position \( R_0 + R_0/2 \), we have \( E^* = E \left( R_0 + R_0/2 \right) \). When \( \nu \) is known, \( E(r) \) can be determined directly. Finally, Young’s modulus of each layer of FGL is obtained. We have \( E_t = E \left( R_0 \right) \).

2.4. Test Design. The parameter equation affecting the ultimate bearing capacity of FGL is as follows:

\[ P = f \left( R_0, t, E^*, f_{ocf}^*, n, \nu \right) \]  \hspace{1cm} (17)

Here, \( t \) and \( n \) are the lining thickness and number of layers, respectively.

In order to study the influence of various structural parameters on the mechanical properties of FGL, a set of fixed parameters is taken as the typical parameters first and then as the control group. The values of FGL calculation parameters are listed in Table 1. When studying the mechanical properties of FGL under the elastic state, the liner is assumed to be linear elastic material, and its failure is not considered temporarily. Let load \( Q = 15 \text{ MPa} \).

3. Analysis of Mechanical Properties of FGL in Elastic Working State

The radial stress distribution trend of FGL is similar to that of single-layer homogeneous lining [24], and the tangential stress is more important in lining design, so only the tangential stress is analyzed. In the later analysis, the reference path is defined as the path from the inner side of the lining to the outer side of the lining on the radial section of the lining. By normalizing the reference path length and tangential stress, we get \( \Delta t/\ell \) and \( \sigma_\theta/Q \), respectively.

3.1. Influence of Inner Radius \( R_0 \) of FGL. Figure 3 illustrates the variations of tangential stress, radial displacement, and tangential strain of FGL with internal radius. In the section of lining, the distribution of tangential stress, radial displacement, and tangential strain of 3-layers functionally graded lining obtained by the change of inner radius are similar. From the point of view of the whole functional graded lining, the tangential stress of the lining increases gradually from the inside to the outside of the lining. From the point of view of each layer of lining, the tangential stress of the lining decreases gradually from the inside to the outside of the lining, which is the same as that of the homogeneous lining (Figure 3(a)). The radial displacement of FGL is gradually reduced from the inner side to the outer side of lining, and the variation range of a specific value is very small (Figure 3(b)). The tangential strain of FGL is obviously reduced from the inner side to the outer side of lining, and the value of the tangential strain of the inner and outer edge of lining is quite different (Figure 3(c)).

As seen in Figure 3, at the inner and outer edges of the functionally gradient lining, as the inner radius of the lining gradually increases, both the tangential stress and the tangential strain of the lining increase linearly, and the radial displacement of the lining increases nonlinearly. When the inner radius of the lining is 4 and 8 m, the tangential stress of the inner edge of the lining is 53.84 and 93.22 MPa, the tangential strain of the inner edge of the lining is 1580 and 2620 \( \mu e \), and the radial displacement of the inner edge of the lining is 6.31 and 20.98 mm. When the inner radius of lining is doubled, the tangential stress, tangential strain, and radial displacement of the inner edge of lining are increased by 73.14%, 65.8%, and 232.5%, respectively.

3.2. Influence of Thickness \( t \) of FGL. Figure 4 illustrates the variations in tangential stress, radial displacement, and tangential strain of FGL with thickness. In the section of lining, the distribution of tangential stress, radial displacement, and tangential strain of 3-layers functionally graded lining obtained by the change in the thickness are similar. With the increase in the lining thickness, the tangential stress, radial displacement, and tangential strain all decrease. The variations of the lining tangential stress tends to be constant with the increase of the lining thickness. Therefore, the reduction effect of stress concentration due to the
increase in the thickness of 3-layer functionally graded lining is limited, which is similar to that of single-layer lining (Figure 4(a)). When the lining thickness is small, the radial displacement of each point gradually decreases from inside to outside on the reference path of lining. With the increase in the lining thickness, the radial displacement increases first

Table 1: Calculation parameters of FGL.

| Parameter | $R_0$ (m) | $t$ (m) | $E^*$ (GPa) | $f^*_c$ (MPa) | $n$ | $\gamma$ |
|-----------|-----------|---------|-------------|---------------|-----|---------|
| Typical value | 4 | 1.5 | 36 | 27.5 | 3 | 0.2 |
| Range | 4–10 | 1–3 | 36–38 | 27.5–35.5 | 2–6 | 0.2–0.35 |
| Step size | 0.5 | 0.5 | 0.5 | 2 | 1 | 0.5 |

Figure 3: Variations in tangential stress, radial displacement, and tangential strain of FGL with internal radius $R_0$. (a) Tangential stress. (b) Radial displacement. (c) Tangential strain.
and then decreases in the reference path, and the maximum radial displacement of the lining occurs in the middle of the lining (Figure 4(b)).

3.3. Influence of Young’s Modulus $E^*$ of FGL. Figure 5 illustrates the variations in tangential stress, radial displacement, and tangential strain of FGL with Young’s modulus $E^*$. The change in Young’s modulus $E^*$ of functionally graded lining has no effect on the distribution and value of tangential stress. However, the tangential stress redistribution of FGL is greatly changed compared with that of homogeneous lining (Figure 5(a)). Young’s modulus $E^*$ affects the deformation behavior of functionally graded lining. The radial displacement and tangential strain of the inner and outer edge of FGL decrease linearly with the increase in Young’s modulus $E^*$ (Figure 5(b) and 5(c)). In the design of functionally graded lining, the appropriate Young’s modulus $E^*$ can effectively control the displacement within a safe and reasonable range.

3.4. Influence of Concrete Compressive Strength $f'_c$ of FGL. Figure 6 shows that the concrete compressive strength $f'_c$ has no direct influence on the stress distribution, radial displacement, and tangential strain of functionally graded lining under elastic state.

3.5. Influence of Poisson’s Ratio $\nu$ of Concrete. Figure 7 illustrates the variations in tangential stress, radial displacement, and tangential strain of FGL with Poisson’s ratio $\nu$. The change in Poisson’s ratio $\nu$ will have a slight
influence on the distribution of tangential stress. With the increase in Poisson’s ratio \( \nu \), the tangential stress increases in the inner edge of the lining and decreases in the outer edge of the lining, but the variation is slight overall (Figure 7(a)). When Poisson’s ratio increases, the tangential strain and radial displacement decrease (Figures 7(b) and 7(c)). When Poisson’s ratio \( \nu \) changes, the radial displacement distribution of lining section is different. When \( \nu = 0.2 \) and 0.35, radial displacement difference between inner and outer edges of lining is 0.23 and 0.74 mm, respectively. The change in Poisson’s ratio has a great influence on radial displacement, but has little influence on stress distribution, which is consistent with [48].

3.6. Influence of FGL Stratification Number \( n \). As shown in Figure 8(a), the distribution form of tangential stress on the lining section is similar by changing the number of layers, and the number of layers of FGL determines the number of segments of the tangential stress. The tangential stress of the inside and outside of the functionally gradient lining has the opposite trend. The tangential stress at the inner edge of the lining decreases, and the tangential stress at the outer edge of the lining increases with the increase in the number of layers.

As shown in Figures 8(b) and 8(c), the radial displacement and the tangential strain of the lining section have the same change trend with the change in the number of layers, and both decrease from the inside to the outside of the lining. Both the radial displacement and tangential strain...
increase with the increase in number \( n \) of layers. The difference of radial displacement between the inner edge and the outer edge of lining is between 0.21 and 0.25 mm. The difference of tangential strain between inner edge and outer edge of lining is between 450 and 496 \( \mu \varepsilon \).

3.7. Failure Mode of 3-Layer FGL. Figure 9 shows the failure mode of 3-layer FGL under horizontal confining pressure load. The \( f \) determined by (13) is used to judge whether the lining is damaged. The gray area indicates that the lining has reached stress failure, the white area indicates that the lining is still in elastic working state, and the blue dotted line is the critical line of stress failure of the lining in Figure 9. The failure mode of 3-layer FGL under horizontal confining pressure load can be divided into the following stages:

1. Normal working stage: the whole lining is in the elastic working state, that is, when \( Q = 10 \text{ MPa} \).
2. The inner edge of the first lining starts to break: the inner edge of the innermost lining starts to break, and the rest of the lining is in the elastic working state, that is, when \( Q = 12 \text{ MPa} \).
3. The second layer of lining starts to damage: the damage degree of the innermost layer of lining continues to increase, but not all of the lining has been damaged, while the second layer of lining starts to damage from the inner edge of the lining, and the...
rest of the lining is in the elastic working state, that is, when $Q = 24$ MPa.

(4) The first layer of lining is completely damaged: the inner layer of lining is completely damaged, the second layer of lining is continuously damaged, and the rest of lining is in the elastic working state, that is, when $Q = 30$ MPa.

(5) The outer layer of the lining starts to damage: the inner layer of the lining is completely damaged, the damage degree of the second layer of lining continues to increase, and the third layer of lining starts to damage from the inner edge of the lining, that is, when $Q = 34$ MPa.

(6) The first two layers of lining are destroyed completely; the second layer is destroyed completely, and the third layer is destroyed continuously.

(7) Complete failure stage of lining: complete failure of 3-layer lining.

When the first layer of the lining reaches failure, the lining can no longer be used in the actual project. Therefore, the failure of FGL depends on whether the lining inner edge is damaged.

4. Analysis of Mechanical Properties of FGL in Ultimate Bearing State

When the inner edge of the lining reaches stress failure, it is considered that the lining can no longer be used in a safe way. Therefore, the ultimate bearing capacity, the maximum radial displacement, and tangential strain of the inner side of FGL will be obtained. In engineering, the working status of the lining can be judged by monitoring the radial displacement or tangential strain of the inner edge of the lining [49].
4.1. Influence of Inner Radius $R_0$ of FGL. Figure 10 shows the relationship between the inner radius of FGL and its ultimate bearing capacity, maximum radial displacement, and maximum tangential strain. As seen in Figure 10(a), the change in the inner radius of lining will have a great influence on the ultimate bearing capacity of FGL. The ultimate bearing capacity of lining decreases with the increase in the inner radius. When the inner radius is 8 m, the ultimate bearing capacity of homogeneous lining and 3-layer FGL is 5.31 and 5.87 MPa, respectively, and the latter increases by 10.5%. When the inner radius is 4 m, the ultimate bearing capacity of homogeneous lining and 3-layer FGL is 8.59 and 10.16 MPa, respectively, and the latter increases by 18.3%. When the inner radius of the lining is small, the bearing capacity increases greatly.

The maximum radial displacement is approximately linear with the inner radius of the lining. The maximum radial displacement inside the lining increases with the increase in the inner radius (Figure 10(b)). The maximum tangential strain of lining inner edge is less affected by the change in the lining inner radius. Compared with the homogeneous lining, the maximum tangential strain of FGL changes more obviously with the inner radius (Figure 10(c)).
Figure 9: The failure mode of 3-layer FGL under horizontal confining pressure.

Figure 10: Continued.
4.2. Influence of NT_hickness $t$ of FGL. Figure 11 shows the relationship between the thickness of FGL and its ultimate bearing capacity, maximum radial displacement, and maximum tangential strain. As seen in Figure 11(a), both the ultimate bearing capacity of homogeneous lining and 3-layer FGL will be increased with the increase in the lining thickness. When the thickness of lining is 1 m, the ultimate bearing capacity of 3-layer FGM lining and homogeneous lining is 7.45 and 6.57 MPa, respectively. When the thickness of lining is 3 m, the ultimate bearing capacity of 3-layer FGL and homogeneous lining is 15.7 and 12.28 MPa, respectively, and the ultimate bearing capacity of both is increased by 110.7% and 86.9%, respectively. As seen in Figure 11(b) and 11(c), the maximum radial displacement and the maximum tangential strain of the lining have the same trend with the change in the thickness, and both increase with the increase in the lining thickness. The influence of lining thickness on the maximum radial displacement of homogeneous lining is less but on the 3-layer FGL is greater.

4.3. Influence of Young’s Modulus $E^*$ of FGL. Figure 12 shows the relationship between Young’s modulus $E^*$ of FGL and its ultimate bearing capacity, maximum radial displacement, and maximum tangential strain. The ultimate bearing capacity of lining is independent of Young’s modulus $E^*$ (Figure 12(a)). The maximum radial displacement and the maximum tangential strain of the lining increase linearly with the increase in Young’s modulus $E^*$ (Figure 12(a) and 12(b)).

4.4. Influence of Concrete Compressive Strength $f^*_{c}$ of FGL. Figure 13 shows the relationship between the concrete compressive strength $f^*_{c}$ of FGL and its ultimate bearing capacity, maximum radial displacement, and maximum tangential strain. As seen in Figure 13(a), the ultimate bearing capacity of lining depends on the compressive strength of concrete. The ultimate bearing capacity of lining increases linearly with the increase in compressive strength of lining concrete. Compared with the homogeneous lining, the ultimate bearing capacity of the 3-layer FGL increases slightly with the same compressive strength. When the compressive strength of lining concrete is 27.5 and 35.5 MPa, the ultimate bearing capacity of homogeneous lining is 8.59 and 11.09 MPa, and the ultimate bearing capacity of 3-layer FGL is 10.16 and 13.12 MPa. When the compressive strength of lining concrete increases from 27.5 MPa to 35.5 MPa, the ultimate bearing capacity of homogeneous lining and 3-layer FGL increases by 2.5 (29.10%) and 2.96 MPa (29.13%), respectively. As seen in Figures 13(b) and 13(c), when the compressive strength of lining concrete is only changed, the maximum radial displacement of lining is the same as the maximum tangential strain. The maximum radial displacement and the maximum tangential strain of the lining increase linearly with the concrete compressive strength of the lining.

4.5. Influence of Poisson’s Ratio $\nu$ of Concrete. Figure 14 shows the relationship between Poisson’s ratio $\nu$ of FGL and its ultimate bearing capacity, maximum radial displacement, and maximum tangential strain. As seen in Figure 14(a), the ultimate bearing capacity of lining will increase slightly with the increase in Poisson’s ratio of lining. When Poisson’s ratio increases from 0.2 to 0.35, the ultimate bearing capacity of homogeneous lining and 3-layer FGL increases by 0.4 (4.6%) and 0.45 MPa (4.4%), respectively. In practical operation, on the one hand, it is difficult to control Poisson’s ratio of concrete accurately when preparing concrete; on the other hand, Poisson’s ratio has relatively small impact on the mechanical properties of lining, so Poisson’s ratio of concrete is generally taken as 0.2 for calculation in design, which is relatively conservative and reasonable. As seen in
Figures 14(b) and 14(c), the maximum radial displacement and the maximum tangential strain of the lining decrease with the increase in Poisson’s ratio.

4.6. Influence of FGL Stratification Number $n$. Let $\lambda = t/(t + R_0)$, which is determined by the thickness and inner radial of the lining. Figure 15 shows the relationship between the number $n$ of layers of FGL and its ultimate bearing capacity and maximum tangential strain. Both the ultimate bearing capacity and maximum tangential strain of lining increase with the increase in lining layers number $n$ (Figure 15(a) and 15(b)).

4.7. Influence of $\lambda$ of FGL. Figure 16 shows the relationship between $\lambda$ and ultimate bearing capacity, maximum radial displacement, and maximum tangential strain. Both the ultimate bearing capacity and maximum tangential strain of lining increase approximately linearly with the increase in $\lambda$ (Figure 16(a) and 16(c)). There is no obvious rule between the maximum radial displacement of lining and $\lambda$ (Figure 16(b)).

4.8. Calculation Formula of Ultimate Bearing Capacity of FGL. According to the test results, the ultimate bearing capacity $P_u$ of FGL is related to the concrete compressive strength $f_{c}^{*}$, $\lambda$, Poisson’s ratio $\nu$, and stratification number $n$, among

\[
P_u = f_{c}^{*} \cdot \frac{t}{t + R_0} \cdot n \cdot (1 - \nu^2)
\]
which $f_c^*$ and $\lambda$ are the most important factors. By fitting the test results, the formula of ultimate bearing capacity of FGL can be deduced as follows:

$$P_u = 1.328\lambda^{0.987}f_c^*k_p m_p,$$

$$k_p = 0.9409 - 0.6867\lambda + 0.0601n + 0.2841\lambda n - 0.01739n^2 - 0.01926\lambda n^2 + 0.001387n^3,$$

$$m_p = -0.003891\nu^{-1.695} + 1.059.$$

Here, $k_p$ is a coefficient of stratification number $n$ ($n = 2, 3, 4, \ldots$) and $m_p$ is a coefficient of Poisson’s ratio.

In order to verify the accuracy of the fitted ultimate bearing capacity calculation formula, six group parameters of FGL are randomly selected, and the parameters are not listed in accordance with the typical parameters. For comparison, the result of the fitting formula and ABAQUS are listed in Table 2. The difference between the calculation result of fitting formula and that of ABAQUS is between 0.05 MPa, so the fitting formula can be considered accurate.

4.9. Calculation Formula of Maximum Tangential Strain of FGL. According to the test results, the maximum tangential strain $e_{\theta\nu}$ of lining inner edge is related to Young’s modulus $E^*$ of lining, the compressive strength $f_c^*$ of the concrete, $\lambda$, Poisson’s ratio $\nu$, and stratification number $n$, among which $E^*$, $f_c^*$, and $\lambda$ are the most important factors. By fitting the test results, the formula of maximum tangential strain of lining inner edge can be deduced as follows:
Here, $k_e$ is a coefficient of stratification number $n$ ($n = 2, 3, 4,...$) and $m_e$ is a coefficient of Poisson’s ratio.

In order to verify the accuracy of the fitted maximum tangential strain calculation formula, six group parameters of FGL are randomly selected, and the parameters are not listed in accordance with the typical parameters. For comparison, the results of the fitting formula and ABAQUS are listed in Table 3.

The difference between the calculation result of fitting formula

\[ \varepsilon_{th} = -1.57 \frac{f'_c}{E} \lambda^{0.08561} k_e m_e, \]

\[ k_e = 0.8692 - 1.184\lambda + 0.1208\lambda + 0.492\lambda \]

\[ - 0.03316n^2 - 0.03388\lambda n^2 + 0.002612n^3, \]

\[ m_e = -1.914\nu^2 + 0.6838\nu + 0.941. \]
Figure 14: Variations in ultimate bearing capacity, maximum radial displacement, and maximum tangential strain with Poisson’s ratio of concrete $\nu$. (a) Ultimate bearing capacity. (b) Maximum radial displacement. (c) Maximum tangential strain.
Figure 15: Variations in ultimate bearing capacity and maximum tangential strain with the number of layers $n$. (a) Ultimate bearing capacity. (b) Maximum tangential strain.

Figure 16: Continued.
and that of ABAQUS is between 6 με, so the fitting formula can be considered accurate.

5. Discussion and Conclusions

In order to improve the bearing capacity of lining and give full play to the performance of lining materials, the study propose a new circular concrete lining structure for underground structure and shaft. According to the concept of functionally graded concrete lining, multilayered lining is designed to be rock support in mine. The mechanical properties of multilayered FGL are studied completely in this article. Based on the exact solution of the N-layer thick wall cylinder and multi-axial failure criterion of concrete, the stress, deformation, and ultimate bearing capacity of FGL can be determined. Through the one-factor test, the influence of each structure parameter on tangential stress, radial displacement, and tangential strain of FGL is analyzed. The mechanical and deformation characteristics of lining under elastic working condition, as well as the ultimate bearing capacity, deformation characteristics, and failure characteristics of lining when it reaches failure are studied. According to the experimental results, the calculation formula of the ultimate bearing capacity of FGL and the

| Parameters | $E^*$ (GPa) | $t$ (m) | $R_0$ (m) | $f^*_c$ (MPa) | $n$ | $\nu$ | Equation (18) (MPa) | ABAQUS (MPa) | D-value (MPa) |
|------------|-------------|---------|------------|----------------|----|-----|-------------------|--------------|--------------|
| Example 1  | 36          | 2       | 8          | 35             | 4  | 0.3 | 9.96              | 9.97         | 0.01         |
| Example 2  | 36          | 1       | 6          | 30             | 3  | 0.2 | 5.84              | 5.79         | -0.05        |
| Example 3  | 36          | 3       | 6          | 28             | 6  | 0.25| 13.87             | 13.91        | 0.04         |
| Example 4  | 36          | 3       | 6          | 27.5           | 3  | 0.25| 12.57             | 12.6         | 0.03         |
| Example 5  | 36          | 2       | 4          | 29.5           | 3  | 0.2 | 13.23             | 13.28        | 0.05         |

| Parameters | $E^*$ (GPa) | $t$ (m) | $R_0$ (m) | $f^*_c$ (MPa) | $n$ | $\nu$ | Equation (19) (με) | ABAQUS (με) | D-value (με) |
|------------|-------------|---------|------------|----------------|----|-----|-------------------|--------------|--------------|
| Example 1  | 36          | 2       | 8          | 35             | 4  | 0.3 | -1343.6           | -1343.1      | 0.515        |
| Example 2  | 36          | 1       | 6          | 30             | 3  | 0.2 | -1110.9           | -1114.5      | -3.567       |
| Example 3  | 36          | 3       | 6          | 28             | 6  | 0.25| -1265.3           | -1271.2      | -5.870       |
| Example 4  | 38          | 3       | 6          | 27.5           | 3  | 0.25| -1025.6           | -1029.4      | -3.782       |
| Example 5  | 38          | 2       | 4          | 29.5           | 3  | 0.2 | -1110.0           | -1112.1      | -2.051       |
| Example 6  | 38          | 3       | 8          | 27.5           | 6  | 0.2 | -1133.8           | -1128.5      | 5.302        |

Figure 16: Variations in ultimate bearing capacity and maximum tangential strain with $\lambda$. (a) Ultimate bearing capacity. (b) Maximum radial displacement. (c) Maximum radial displacement.
maximum tangential strain of FGL inner edge is obtained. The accuracy of the formula is verified by comparing with the calculation results of ABAQUS. The conclusions are as follows:

1. The stress distribution of FGL is mainly related to the inner radius, thickness, and number of layers. The tangential stress increases with the increase in the inner radius and the decrease in the thickness of the lining, and the stress concentration decreases with the increase in the number of layers.

2. The deformation behavior of FGL is mainly related to the elastic modulus, radius, thickness, and Poisson’s ratio. The radial displacement and circumferential strain increase with the increase in the lining radius, thickness, Young’s modulus, Poisson’s ratio, and number of layers.

3. The ultimate bearing capacity of FGL is related to the concrete compressive strength, $\lambda$, Poisson’s ratio, and number of layers. The ultimate bearing capacity of lining increases with the decrease in the inner radius, the increase in the thickness, the increase in the compressive strength and Poisson’s ratio, and the increase in $\lambda$ and the number of layers.

4. The maximum tangential strain of lining inner edge is related to Young’s modulus, the compressive strength of the concrete, $\lambda$, Poisson’s ratio, and number of layers. The maximum tangential strain increases with the increase of Young’s modulus, compressive strength, Poisson’s ratio, number of layers, and $\lambda$.

5. As the same as the homogeneous single-layer lining, the damage of FGL starts from the inner edge of lining. The tangential strain of lining inner edge can be controlled within the maximum value to ensure that the lining works in elastic state in engineering.

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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