Analytical determination of Kondo and Fano resonances of
electron Green's function in a single-level quantum dot

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Abstract. The Kondo and Fano resonances in the two-point Green's function of the single-level quantum dot were found and investigated in many previous works by means of different numerical calculation methods. In this work we present the derivation of the analytical expressions of resonance terms in the expression of the two-point Green's function. For that purpose the system of Dyson equations for the two-point nonequilibrium Green's functions in the complex-time Keldysh formalism was established in the second order with respect to the tunneling coupling constants and the mean field approximation. This system of Dyson equations was solved exactly and the analytical expressions of the resonance terms are derived. The conditions for the existence of Kondo or Fano resonances are found.

Keywords: quantum dot, Green's function, nonequilibrium, resonance.

1. Introduction

The electrons transport through a single-level quantum dot (QD) connected with two conducting leads has been the subject for theoretical and experimental studies in many works since the early days of nanophysics [1–22]. Two observable physical quantities, which can be measured in experiments on electron transport, are the electron current through the QD and the time-averaged value of the electron number in the QD. Both can be expressed in terms of the single-electron Green's functions. In the pioneering theoretical works [1, 3] on the electron transport through a single-level QD, the differential equations for the real-time Green's functions were derived with the use of the Heisenberg equations of motion for the electron destruction and creation operators. Due to the presence of the strong Coulomb interaction between electrons in the QD the differential equations for the single-electron Green's functions contain multi-electron Green's functions, and all the coupled equations for these Green's functions form an infinite system of differential equations. In order to have a finite closed system of equations, one can assume some approximation to decouple the infinite system of equations. Moreover, since the electron transport is a nonequilibrium process, one should work with the Keldysh formalism of nonequilibrium complex-time Green's functions [23, 24].

In the study of the nonequilibrium complex-time Green's functions by means of the perturbation theory with respect to the Coulomb interaction, one usually retains some chain of ladder diagrams and assumes noncrossing approximation (NCA). The systems of equations for the Green's functions were solved by means of different numerical methods, for example the Quantum Monte Carlo technique [13] and the numerical renormalization group method [10, 12, 14, 17]. Electron two-point Green's functions were shown to have the resonance related to the Kondo effect. Beside of this Kondo
resonance, the Fano quasi-bound state in the energy spectrum of the electron system of the QD and the leads might also give some resonant contribution.

In this work, the exact analytical expressions of the Kondo and the Fano resonance terms are derived by explicitly solving the equations. From these analytical expressions we obtain the whole set of resonances and the conditions for their existence. In particular, we shall demonstrate the distinction between the Kondo and the Fano resonances, if they do exist.

2. System of Dyson Equations

Consider the nanosystem consisting of a single level QD connected to two leads through the potential barriers with the total Hamiltonian

\[
H = E \sum_{\sigma} c_\sigma^+ c_\sigma + U N_\uparrow N_\downarrow + \sum_k \left\{ E_a(k) a_\sigma^+(k) a_\sigma(k) + E_b(k) b_\sigma^+(k) b_\sigma(k) \right\} + \sum_k \left\{ V_a(k) a_\sigma^+(k) c_\sigma + V_b(k) b_\sigma^+(k) c_\sigma + V_a(k) c_\sigma^+ b_\sigma(k) \right\}
\]  

(1)

where \( a_\sigma(k), b_\sigma(k) \) and \( a_\sigma^+(k), b_\sigma^+(k) \) are the annihilation and creation operators of the electrons with momenta \( k \) in the two leads, and denote \( G_{\sigma\sigma}(t-t')_C \) the complex time single-electron Green's function

\[
G_{\sigma\sigma}(t-t')_C = \delta_{\sigma\sigma} G(t-t') = -i \left[ T_C [c_\sigma(t)c_\sigma^+(t')] \right]
\]  

(2)

which is a set of four functions

\[
G_{\sigma\sigma}(t-t') = \delta_{\sigma\sigma} G(t-t') , \quad \alpha, \beta = 1, 2
\]  

(3)

with the Fourier transforms

\[
\tilde{G}_{\sigma\sigma}(\omega)_{\alpha\beta} = \delta_{\sigma\sigma} \tilde{G}(\omega)_{\alpha\beta}.
\]  

(4)

Due to the presence of the Coulomb interaction term in the Hamiltonian (1) the differential equation for the Green's function (2) contains a new Green's function \( H(t-t')_C \) defined as

\[
H_{\sigma\sigma}(t-t')_C = \delta_{\sigma\sigma} H(t-t')_C = -i \left[ T_C [N_{\sigma\sigma} c_\sigma(t)c_\sigma^+(t')] \right].
\]

From the Heisenberg equations of motion for the operators \( c_\sigma(t), a_\sigma(k,t) \) and \( b_\sigma(k,t) \)

\[
i \frac{dc_\sigma(t)}{dt} = Ec_\sigma(t) + UN_{\sigma\sigma}(t)c_\sigma(t) + \sum_k \left\{ V_a(k)^* a_\sigma(k,t) + V_b(k)^* b_\sigma(k,t) \right\},
\]  

(5)

\[
i \frac{da_\sigma(k,t)}{dt} = E_a(k) a_\sigma(k,t) + V_a(k)c_\sigma(t),
\]  

(6)

\[
i \frac{db_\sigma(k,t)}{dt} = E_b(k) b_\sigma(k,t) + V_b(k)c_\sigma(t),
\]  

(7)

we can derive the differential equations for the Green's functions. Applying the mean field approximation to the products of four operators and neglecting the terms of the fourth and higher orders with respect to the tunnelling coupling constants \( V_a(k) \) and \( V_b(k) \), we obtain the system of Dyson equations:

\[
\left\{ \frac{i}{dz} - E \right\} G(t-t')_C = \delta(t-t')_C + UH(t-t')_C + \left[ dt'' \Sigma^{(1)}(t-t'')_C G(t''-t')_C \right],
\]  

(8)

\[
\left\{ \frac{i}{dt} - E - U \right\} H(t-t')_C = n\delta(t-t')_C + \rho(t-t')_C + \left[ dt'' \Sigma^{(2)}(t-t'')_C H(t''-t')_C - \left[ dt'' \Sigma^{(3)}(t-t'')_C G(t''-t')_C \right] \right],
\]  

(9)
where

\[ \Sigma^{(1)}(t-t')_C = \sum_k \left| \Psi_a(k) \right|^2 S^{E_a(k)}(t-t')_C + (a \to b), \]

\[ \Sigma^{(2)}(t-t')_C = \sum_k \left| \Psi_a(k) \right|^2 [2S^{E_a(k)}(t-t')_C + S^{2E+U-E_a(k)}(t-t')_C] + (a \to b), \]

\[ \Sigma^{(3)}(t-t')_C = \sum_k \left| n_a(k) \right|^2 \left[ S^{E_a(k)}(t-t')_C + S^{2E+U-E_a(k)}(t-t')_C \right] + (a \to b), \]

\[ \rho(t-t')_C = \sum_k \left| l_a(k) \Psi_a(k) \right|^2 \left[ S^{E_a(k)}(t-t')_C - S^{2E+U-E_a(k)}(t-t')_C \right] + (a \to b), \]

\[ n_a(k) = \left\{ a_a^+(k) a_a(k) \right\} = \frac{e^{-\beta E_a(k)}}{1 + e^{-\beta E_a(k)}}, \]

\[ n = \left\{ c_{a}^+ c_{a} \right\} = -i G(-0)_{11}, \]

\[ l_a(k) = \frac{1}{Z} \left[ e^{-\beta E} - [1 + e^{-\beta E}] n_a(k) \right] + e^{-\beta E} \frac{e^{-\beta (E+U)} - [1 + e^{-\beta (E+U)}] n_a(k)}{E + U - E_a(k)}, \]

\[ Z = 1 + 2 e^{-\beta E} + e^{-2(E+U)} , \]

and \( S^E(t-t')_C \) is the complex time Green's function of the free electrons at the single energy level \( E \). It satisfies the differential equation

\[ i \frac{d}{dt} S^E(t-t')_C = \delta(t-t')_C. \]

The terms containing \( S^{2E+U-E_a(k)}(t-t')_C \) in the r.h.s. of equations (11)–(13) are the crossing terms. In the NCA they are omitted.

The system of Dyson equations (8) and (9) is more rigorous than those derived previously by many other authors. In particular, in the derivation of equations (8) and (9), the contributions of all crossing terms have been taken into account: there was no necessity to use NCA. Moreover, in all previous works on the Green functions of quantum dots in the Keldysh formalism, the solution of the system of equations was solved by numerical methods. In the present work we derive the analytical expressions for the exact solution of the system of Dyson equations.

In terms of the 2×2 matrices \( \hat{G}(\omega) \), \( \hat{H}(\omega) \), \( \hat{\Sigma}^{(i)}(\omega) \) and \( \hat{\rho}(\omega) \) with the matrix elements being the Fourier transforms of the functions \( G(t-t')_{a'b'} \), \( H(t-t')_{a'b'} \), \( \Sigma^{(i)}(\omega)_{a'b'} \) and \( \rho(\omega)_{a'b'} \), the system of equations (8) and (9) becomes that of the matrix equations

\[ \hat{G}(\omega) = \hat{S}^E(\omega) + U \hat{S}^E(\omega) \hat{H}(\omega) + \hat{S}^E(\omega) \hat{\Sigma}^{(1)}(\omega) \hat{\eta} \hat{G}(\omega), \]

\[ \hat{H}(\omega) = n \hat{S}^{2E+U}(\omega) + \hat{S}^{2E+U}(\omega) \hat{\rho}(\omega) + \hat{S}^{2E+U}(\omega) \hat{\Sigma}^{(1)}(\omega) \hat{\eta} \hat{H}(\omega) - \hat{S}^{2E+U}(\omega) \hat{\Sigma}^{(1)}(\omega) \hat{\eta} \hat{G}(\omega), \]

where

\[ \hat{\eta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]

From these two matrix equations we derive two similar systems of algebraic ones, each of which consists of four equations of \( G(\omega)_{\alpha 1} \) and \( H(\omega)_{\alpha 1} \) or \( G(\omega)_{\alpha 2} \) and \( H(\omega)_{\alpha 2} \), \( \alpha = 1, 2 \). For investigating the electron transport through a QD it is necessary to study the function \( G(\omega)_{11} \). By
solving the system of equations for the functions $G(\omega)_{el}$ and $H(\omega)_{el}$, we have derived the expression of this function in the form

$$G(\omega)_{11} = \frac{Z(\omega)}{Y(\omega)},$$  \hspace{1cm} (22)$$

where $Z(\omega)$ and $Y(\omega)$ are the polynomials of $S^F(\omega)_{\alpha\beta}$, $S^E(\omega)_{\alpha\beta}$, $\Sigma^{(i)}(\omega)_{\alpha\beta}$ and $\rho(\omega)_{\alpha\beta}$. The explicit forms of the rather cumbersome numerator $Z(\omega)$ and denominator $Y(\omega)$ in equation (22) were found by exactly solving equations (19) and (20) and presented in the Appendix.

3. Kondo and Fano Resonances

Now we consider the resonances of the Green's function (22). The expression of the denominator $Y(\omega)$ consists of the terms of two types: finite terms which do not contain self-energies $\Sigma^{(i)}(\omega)_{\alpha\beta}$ and those proportional to $\Sigma^{(i)}(\omega)_{\alpha\beta}$ or their products. Because $\Sigma^{(i)}(\omega)_{\alpha\beta}$ are the linear combinations of the squared moduli of small tunnelling coupling constants $V_a(k)$ and $V_b(k)$ they give small contributions to $Y(\omega)$ at their regular points. However some of them may be divergent at some points, and near these divergence points $Y(\omega)$ may vanish: there appear the resonances. The study of the resonances of Green's functions requires the determination of the divergence points of functions $\Sigma^{(i)}(\omega)_{\alpha\beta}$. From equations (10)–(12) it follows that $\Sigma^{(i)}(\omega)_{12}$, $\Sigma^{(i)}(\omega)_{21}$ and the imaginary parts of $\Sigma^{(i)}(\omega)_{11}$ and $\Sigma^{(i)}(\omega)_{22}$, $i = 1, 2, 3$, are always finite and only the real parts of $\Sigma^{(i)}(\omega)_{11}$ and $\Sigma^{(i)}(\omega)_{22}$ may be divergent at some points. Introducing the spectral functions

$$\Gamma^{(p)}_{\alpha\beta}(\omega) = \pi \sum_k \left[ \frac{1}{1 + e^{-\beta E_{\alpha\beta}(k)}} \right] \left| \psi_{\alpha\beta}(k) \right|^2 \delta(\omega - E_{\alpha\beta}(k)),$$  \hspace{1cm} (23)$$

$p = 0, 1, 2$, we can express the functions

$$\Sigma^{(i)}(\omega) = \text{Re} \Sigma^{(i)}(\omega)_{11} = -\text{Re} \Sigma^{(i)}(\omega)_{22}$$  \hspace{1cm} (24)$$
in the form convenient for the investigation of their divergence

$$\Sigma^{(0)}(\omega) = \frac{1}{\pi} \int d\omega' \frac{1}{\omega - \omega'} \left[ \Gamma^{(0)}_{\alpha\beta}(\omega') + \Gamma^{(0)}_{\beta\alpha}(\omega') \right],$$  \hspace{1cm} (25)$$

$$\Sigma^{(1)}(\omega) = \frac{1}{\pi} \int d\omega' \frac{1}{2 \omega - \omega'} + \frac{1}{\omega - 2E - U + \omega'} \left[ \Gamma^{(0)}_{\alpha\beta}(\omega') + \Gamma^{(0)}_{\beta\alpha}(\omega') \right],$$  \hspace{1cm} (26)$$

$$\Sigma^{(2)}(\omega) = \frac{1}{\pi} \int d\omega' \frac{1}{\omega - \omega'} + \frac{1}{\omega - 2E - U + \omega'} \left[ \Gamma^{(0)}_{\alpha\beta}(\omega') + \Gamma^{(0)}_{\beta\alpha}(\omega') \right].$$  \hspace{1cm} (27)$$

Near the divergence points of the functions $\Sigma^{(i)}(\omega)$ we can neglect $\Sigma^{(i)}(\omega)_{12}$ and $\Sigma^{(i)}(\omega)_{21}$ as well as the imaginary parts of $\Sigma^{(i)}(\omega)_{11}$ and $\Sigma^{(i)}(\omega)_{22}$, $i = 1, 2, 3$, and have following asymptotic expression of function (22):

$$G_{11}(\omega) = \frac{A(\omega)}{B(\omega)},$$  \hspace{1cm} (28)$$

with

$$A(\omega) = \omega - E - U[1 - n - \rho(\omega)] - \Sigma^{(1)}(\omega),$$  \hspace{1cm} (29)$$

$$B(\omega) = (\omega - E - U)(\omega - E) - (\omega - E - U)\Sigma^{(1)}(\omega) - (\omega - E)\Sigma^{(2)}(\omega) + U\Sigma^{(3)}(\omega) + \Sigma^{(1)}(\omega)\Sigma^{(2)}(\omega),$$  \hspace{1cm} (30)$$
\[ \rho(\omega) = \frac{1}{\pi} \mathcal{P} \int \frac{d\omega'}{\omega'} \left[ \frac{1}{\omega - \omega'} - \frac{1}{\omega' - 2E - U + \omega'} \right] \lambda_{a,b}(\omega) + \lambda_{b,a}(\omega), \]

(31)

\[ \lambda_{a,b}(\omega) = \pi \sum_k |l_{a}(k)|^2 |V_{a,b}(k)|^2 \delta(\omega - E_{a,b}(k)). \]

(32)

The functions \( \Sigma^{(i)}(\omega) \) and \( \rho(\omega) \) contain the dispersion integrals with the spectral functions \( \Gamma_{a,b}^{(p)}(\omega) \) and \( \lambda_{a,b}(\omega) \). Denote \( \mu_a, \mu_b \) and \( \Omega_a, \Omega_b \) the chemical potentials and the tops of the energy bands of the systems of conducting electrons in the leads “a” and “b”, respectively. Because

\[ E_{a,b}(k) = E_{a,b}^{(0)}(k) - \mu_{a,b}, \]

where \( E_{a,b}^{(0)}(k) \) are the kinetic energies of the conducting electrons in the leads, \( 0 \leq E_{a,b}^{(0)}(k) \leq \Omega_{a,b} \), the spectral functions \( \Gamma_{a,b}^{(p)}(\omega) \) and \( \lambda_{a,b}(\omega) \) vanishes for \( \omega < -\mu_{a,b} \) and \( \omega > \Omega_{a,b} - \mu_{a,b} \). For the study of the divergence of \( \Sigma^{(i)}(\omega) \) and \( \rho(\omega) \) we set \( \mu_a = \mu_b = \Omega_a = \Omega_b = \Gamma_{a,b}^{(0)}(\omega), \lambda_{a,b}(\omega) = \lambda_{b,a}(\omega) \) and approximately replace the values of these functions in the interval \( -\mu < \omega < \Omega - \mu \) by some constants \( \Gamma \) and \( \lambda \), respectively. Then we have following divergent asymptotic expressions of functions \( \Sigma^{(i)}(\omega) \) and \( \rho(\omega) \):

\[ \Sigma^{(1)}(\omega) = \frac{1}{2} \Sigma^{(2)}(\omega) = \Sigma^{(3)}(\omega) = \frac{\Gamma}{\lambda} \rho(\omega) = \frac{2\Gamma}{\pi} \ln \left| \frac{\omega + \mu}{\Omega} \right| \quad \text{at} \ T = 0 \ \text{and} \ \omega = -\mu, \]

(33)

\[ \Sigma^{(3)}(\omega) = -\frac{2\Gamma}{\pi} \ln \left| \frac{\omega}{\mu} \right| \quad \text{at} \ T = 0 \ \text{and} \ \omega = 0, \]

(34)

\[ \Sigma^{(2)}(\omega) = \Sigma^{(3)}(\omega) = -\frac{\Gamma}{\lambda} \rho(\omega) = -\frac{2\Gamma}{\pi} \ln \left| \frac{\omega - 2E + U - \mu}{\Omega} \right| \quad \text{at} \ T = 0 \ \text{and} \ \omega = 2E + U + \mu, \]

(35)

\[ \Sigma^{(3)}(\omega) = \frac{2\Gamma}{\pi} \ln \left| \frac{\omega - 2E - U}{\mu} \right| \quad \text{at} \ T = 0 \ \text{and} \ \omega = 2E + U. \]

(36)

The formulae (33) and (34) contain some unknown finite constant \( \Omega \) of the order of the width of the energy band of the conducting electrons in the leads. However we shall formulate the conclusion which does not depend on the concrete value of this constant.

Consider now the behaviour of function (28) in the neighbourhoods of the divergence points of functions \( \Sigma^{(i)}(\omega) \), neglecting the finite very small terms of the second order with respect to the tunnelling coupling constants.

a) At \( \omega \to -\mu \) and low temperature \( T = 0 \), all functions \( \Sigma^{(i)}(\omega) \) and also \( \rho(\omega) \) tend to \(-\infty\). Function (28) has following asymptotic expression in the neighbourhoods of this point:

\[ G(\omega)_{11} = \frac{\lambda}{\Gamma} U(E + 2U + \mu)^{-1} \left[ \frac{1}{2}(E + \mu) + \frac{2\Gamma}{\pi} \ln \left| \frac{\omega + \mu}{\Omega} \right| + 2i\Gamma \right]^{-1} - E + \mu + \frac{\lambda}{\Gamma} U(E + 2U + \mu)^{-1} \left[ \frac{1}{2}(E + U + \mu) + \frac{2\Gamma}{\pi} \ln \left| \frac{\omega + \mu}{\Omega} \right| + 2i\Gamma \right]^{-1}. \]

(37)

If \( E + \mu > 0 \), then \( G(\omega)_{11} \) has two resonances at two points

\[ \omega^1_{a} = -\mu + \Omega \sqrt{\frac{\pi}{4\Gamma}(E + \mu)} \]

and two resonances at two points
\[ \omega_2^{(\pm)} = -\mu \pm \Omega e^{-\frac{\pi}{2\Gamma} (E+U+\mu)} . \]  

Between these four resonances there are the dips. If \( E + \mu < 0 \) but \( E + U + \mu > 0 \), then \( G(\omega)_{11} \) has only two resonances at the points \( \omega_2^{(\pm)} \). If \( E + U + \mu < 0 \), then function \( G(\omega)_{11} \) has no resonance in the neighbourhood of the point \( \omega = -\mu \). All four points \( \omega_2^{(\pm)} \) are very close to the point \( \omega = -\mu \) and the resonances at \( \omega_2^{(+)} \) and \( \omega_2^{-} \) look like a resonance at \( \omega = -\mu \). The origin of these resonances is the presence of the Fano quasi-bound state at the lower edge of the energy band of the conducting electrons. If they exist, they would be called the Fano resonances.

b) At \( \omega \to 0 \) and \( T = 0 \), functions \( \Sigma^{(1)}(\omega) \), \( \Sigma^{(2)}(\omega) \) and \( \rho(\omega) \) are bounded but the functions \( \Sigma^{(3)}(\omega) \) tends to \( +\infty \). Function \( G(\omega)_{11} \) has following asymptotic expression:

\[ G(\omega)_{11} = \left[ E + (1-n)U \right] \left[ E(E+U) + \frac{2U}{\pi} \ln \left| \frac{\mu}{\omega} \right| + 2i(3E+2U)\Gamma \right]^{-1} . \]  

If \( E(E+U) < 0 \), then \( G(\omega)_{11} \) has two resonances at the points

\[ \omega_2^{(\pm)} = \pm \omega_0 \pm \frac{\pi}{2\Gamma} \frac{E(E+U)}{U} , \]  

which are very close to the point \( \omega = 0 \). At \( \omega = 0 \) and \( 0 < T < T_K \),

\[ T_K = \frac{\pi}{kU} \exp \left\{ \frac{-\pi}{2} \frac{E(E+U)}{\Gamma U} \right\} , \]  

where \( k \) is the Boltzmann constant. Instead of formula (40) we have

\[ G(0)_{11} = \frac{\pi}{2} \frac{E+(1-n)U}{\Gamma U} \frac{1}{\ln[T/T_K] - i\pi(3E+2U)\Gamma} . \]  

The resonances in the neighbourhood of the point \( \omega = 0 \) have the same physical origin as the Kondo effect in the scattering of electrons by a magnetic impurity. They are the Kondo resonances.

c) At \( \omega \to 2E+U \) and \( T = 0 \) the functions \( \Sigma^{(1)}(\omega) \), \( \Sigma^{(2)}(\omega) \) and \( \rho(\omega) \) are bounded, but the function \( \Sigma^{(3)}(\omega) \) tends to \( +\infty \). Function \( G(\omega)_{11} \) has following asymptotic expression:

\[ G(\omega)_{11} = \left[ E + nU \right] \left[ E(E+U) + \frac{2U}{\pi} \ln \left| \frac{\mu}{\omega - 2E - U} \right| + 2i\Gamma \right]^{-1} . \]  

Therefore if \( E(E+U) > 0 \), then \( G(\omega)_{11} \) has also two resonances at the points

\[ \omega_4^{(\pm)} = 2E+U \pm \mu e^{-\frac{\pi}{2\Gamma} \frac{E(E+U)}{U}} , \]  

which are very close to the point \( \omega = 2E+U \). At \( \omega = 2E+U \) and \( 0 < T < T_K \),

\[ T_K = \frac{1}{kU} \exp \left\{ \frac{-\pi}{2} \frac{E(E+U)}{\Gamma U} \right\} , \]  

instead of equations (42) we have

\[ G(\omega)_{11} = \frac{\pi}{2} \frac{E+nU}{\Gamma U} \frac{1}{\ln[T/T_K]-i\pi\Gamma} . \]  

The resonances in the neighbourhood of the point \( \omega = 2E+U \) are the Kondo resonances of the crossing terms.
d) At $\omega \to 2E + U + \mu$ and low temperature $T = 0$, the functions $\Sigma^{(1)}(\omega)$, $\Sigma^{(2)}(\omega)$ tend to $-\infty$ and $\rho(\omega)$ tends to $+\infty$. Function $G(\omega)_{11}$ has following asymptotic expression:

$$ G(\omega)_{11} = -U(E + \mu)^{-1}\left[1 - n + (E + U + \mu)\frac{\lambda}{\Gamma}\right]\left[1 + E + U + \mu + \frac{2\Gamma}{\pi}\ln\left|\frac{\omega - 2E - U - \mu}{\Omega} - 2\Gamma\right|\right]^{-1}. \quad (48) $$

If $E + U + \mu > 0$, then $G(\omega)_{11}$ has two resonances at the points

$$ \omega_{\pm}^{(2)} = 2E + U + \mu \pm \Omega \frac{\pi}{\Gamma}, \quad (49) $$

which are very close to the point $\omega = 2E + U + \mu$. They are the Fano resonances of the crossing terms.

### 4. Conclusion

By means of the equation of motion method the system of Dyson equations for the complex-time non-equilibrium electron Green's functions of the system consisting of a single-level QD connected with two conducting leads was derived in the mean field approximation with respect to the products of four creation and destruction operators of the electron and in the second order with respect to the effective tunnelling coupling constants. All the crossing terms are included into the equations. The exact solution of the system of Dyson equations may have the resonances of four types in the dependence on the physical parameters of the system: the Kondo resonances at the Fermi surface, whose origin is similar to that of the Kondo effect in the scattering of electrons on magnetic impurities, the Fano resonances due to the presence of the electron quasi-bound state at the lower edge of the energy band of the conducting electrons, the Kondo resonances in the crossing terms and the Fano resonances in the crossing terms. The analytical asymptotic expressions of the single-electron Green function at these resonances were derived.

The results of the present study are the complement of and agree with the numerical calculations in previous works [6–19] on the electron Green's functions in QD.

### 5. Appendix

$$ Z(\omega) = \left\{ B(\omega)[1 + \Omega^{(1)}(\omega)_{22}] - UD(\omega)_{22} \right\}\left\{ B(\omega)S^{E}(\omega)_{11} + UC_{1}(\omega) \right\} - \left\{ B(\omega)\Omega^{(1)}(\omega)_{12} - UD(\omega)_{12} \right\}\left\{ B(\omega)S^{E}(\omega)_{21} + UC_{2}(\omega) \right\}, \quad (A.1) $$

$$ Y(\omega) = \left\{ B(\omega)[1 - \Omega^{(1)}(\omega)_{11}] + UD(\omega)_{11} \right\}\left\{ B(\omega)[1 + \Omega^{(1)}(\omega)_{22}] - UD(\omega)_{22} \right\} + \left\{ B(\omega)\Omega^{(1)}(\omega)_{12} - UD(\omega)_{12} \right\}\left\{ B(\omega)\Omega^{(1)}(\omega)_{21} - UD(\omega)_{21} \right\}, \quad (A.2) $$

$$ B(\omega) = [1 + \Omega^{(2)}(\omega)_{22}][1 - \Omega^{(2)}(\omega)_{11}] + \Omega^{(2)}(\omega)_{12}\Omega^{(2)}(\omega)_{21}, \quad (A.3) $$

$$ C_{i}(\omega) = \left\{ S^{E}(\omega)_{11}[1 + \Omega^{(2)}(\omega)_{22}] - S^{E}(\omega)_{12}\Omega^{(2)}(\omega)_{21} \right\}\lambda(\omega)_{11} - \left\{ S^{E}(\omega)_{12}\Omega^{(2)}(\omega)_{12} + S^{E}(\omega)_{12}[1 - \Omega^{(2)}(\omega)_{11}] \right\}\lambda(\omega)_{21}, \quad (A.4) $$

$$ D(\omega)_{ij} = \left\{ S^{E}(\omega)_{11}[1 + \Omega^{(2)}(\omega)_{22}] - S^{E}(\omega)_{12}\Omega^{(2)}(\omega)_{21} \right\}\Omega^{(3)}(\omega)_{1j} - \left\{ S^{E}(\omega)_{12}\Omega^{(2)}(\omega)_{12} + S^{E}(\omega)_{12}[1 - \Omega^{(2)}(\omega)_{11}] \right\}\Omega^{(3)}(\omega)_{2j}, \quad (A.5) $$

$$ \Omega^{(1)}(\omega)_{ij} = S^{E}(\omega)_{11}\Sigma^{(1)}(\omega)_{1j} - S^{E}(\omega)_{12}\Sigma^{(1)}(\omega)_{2j}, \quad (A.6) $$
\[ \Omega^{(2)}(\omega)_{ij} = S^{E+U}(\omega)_{1l} \Sigma^{(2)}(\omega)_{1j} - S^{E+U}(\omega)_{12} \Sigma^{(2)}(\omega)_{2j}, \]  
\[ \Omega^{(3)}(\omega)_{ij} = S^{E+U}(\omega)_{1l} \Sigma^{(3)}(\omega)_{1j} - S^{E+U}(\omega)_{12} \Sigma^{(3)}(\omega)_{2j}, \]
\[ \lambda(\omega)_{ij} = nS^{E+U}(\omega)_{ij} + S^{E+U}(\omega)_{1l} \rho(\omega)_{1j} - S^{E+U}(\omega)_{12} \rho(\omega)_{2j}, \]  
i = 1, 2, \quad j = 1, 2.

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