Symmetries in collinear effective theory

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Abstract

We consider various symmetries present in the collinear effective theory and their implications. There are collinear, soft and ultrasoft gauge symmetries and we discuss transformation properties of a collinear quark and gauge fields under these gauge transformations. If we require the gauge invariance order by order, the gauge fields have definite transformation properties. We also probe reparameterization invariance and residual energy invariance in the collinear effective theory and discuss their physical consequences.

When we describe strong interaction processes which include massless particles with large energy $E$, we can use the collinear effective theory [1–4]. In the collinear effective theory, the effective Lagrangian can be systematically expanded in powers of $\lambda \sim p_\perp / E$ where $p_\perp$ is the transverse momentum of a collinear quark. Even though there is one large parameter $E$, there are three scales involved in the effective theory. The momentum of an energetic particle can be written as $p^\mu = \frac{1}{2}(\vec{n} \cdot p)n^\mu + p_\perp^\mu + \frac{1}{2}(n \cdot p)\vec{p}^\mu$, where $\vec{n} \cdot p \sim \lambda^0$, $p_\perp \sim \lambda$, and $n \cdot p \sim \lambda^2$. If we include small fluctuations due to the strong interaction, the momentum $P^\mu$ of an energetic particle is decomposed as the sum of a large momentum $p^\mu$ with $\vec{n} \cdot p \sim \lambda^0$, $p_\perp^\mu \sim \lambda$ and $n \cdot p = 0$ and a small residual momentum $k^\mu \sim \lambda^2$:

$$P^\mu = p^\mu + k^\mu = \frac{n^\mu}{2} \vec{n} \cdot p + p_\perp^\mu + k^\mu.$$  

(1)

Here $p^\mu$ acts as a label momentum on collinear fields, and the residual momentum $k^\mu$ is a small fluctuation.

Since there are three distinct scales $E, E\lambda, E\lambda^2$, we have to construct effective theories at each scale. Here we consider an effective theory in the region $E\lambda <

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\( \mu < E \), in which collinear degrees of freedom above \( \mu \) are integrated out. This effective theory can be used in exclusive decays of a heavy meson into energetic light mesons. It has been applied to the resummation of Sudakov logarithms in \( B \) decays [1] and to the proof of factorization in \( \bar{B} \rightarrow D \pi \) [5]. And the calculation of the form factors in heavy-to-light transitions to order \( \lambda \) was performed [6].

The symmetry structure in the collinear effective theory is rich. There are interesting physical implications by combining the symmetries of the effective theory. In this Letter, we consider gauge invariance, reparameterization invariance and residual energy invariance.

In order to discuss gauge symmetries of the collinear effective theory, let us consider the full QCD first. The Lagrangian for a massless quark and gluons is given by

\[
\mathcal{L}_{\text{QCD}} = \bar{q}i\gamma^{\mu}D_{\mu}q - \frac{1}{4}G_{\mu
u}^{a}G^{\mu
u a},
\]

where the covariant derivative \( D_{\mu} \) is defined as \( D_{\mu} = \partial_{\mu} + igA_{\mu}^{a}T^{a} \) and \( G_{\mu
u}^{a} \) is the gluon field strength tensor. The quark field and the gauge fields transform under an arbitrary \( SU(3) \) gauge transformation of the form \( U = e^{i\omega^{a}(x)T^{a}} \) as

\[
q(x) \rightarrow U(x)q(x), \quad A_{\mu}^{a}(x) \rightarrow U(x)A_{\mu}^{a}(x)U^{\dagger}(x) - \frac{i}{g}U(x)\partial_{\mu}U^{\dagger}(x),
\]

and the Lagrangian is invariant under the gauge transformation.

In the collinear effective theory, we decompose each field into a collinear field, a soft field and an ultrasoft (usoft) field. For a collinear quark, we redefine the field by extracting large momenta as

\[
q_{n,p}(x) = \sum_{p} e^{-ip \cdot x}q(x),
\]

where \( p_{\perp} = \frac{1}{2}(\vec{n} \cdot p)n_{\mu} + p_{\perp \mu} \) is a large label momentum to order \( \lambda \). And using a projection operator \( \frac{\vec{n}}{4} \) to extract a large component in the \( n_{\mu} \) direction, we can express the effective Lagrangian for collinear quarks in terms of \( \xi_{n} \), which is defined as

\[
\xi_{n} = \frac{\vec{p}}{4}q_{n,p} \text{ with } \not{n}\xi_{n} = 0.
\]
of order $\lambda^2$. In order to facilitate the power counting in $\lambda$, it is convenient to define label operators $P$ and $P_{\perp}$, which pick out the label momenta of collinear fields of order $\lambda^0$ and $\lambda$ respectively. For example, $P \xi_n = \pi \cdot p \xi_n$. Then the operator which picks out label momenta can be written as $P = \frac{1}{2} P n + P_{\perp}$.

The gauge field $A_\mu$ is also decomposed as $A_\mu = A^c_\mu + A_s^\mu + A_u^\mu$, where $A^c_\mu$ is a collinear gauge field, $A^s_\mu$ is a soft gauge field and $A^u_\mu$ is an usoft gauge field.

The effective Lagrangian of the collinear quark sector in the collinear effective theory is given by

$$L = \bar{\xi}_n \{ n \cdot (i D - g A_n) \\
+ (P_{\perp} - g A_{\perp} + iP_{\perp}) \frac{1}{\pi \cdot (P - g A_n + iD)} (P_{\perp} - g A_{n} + iP_{\perp}) \} \frac{\pi}{2} \xi_n, \quad (6)$$

where we write the collinear gauge field $A^c_n = \sum_q e^{-iq \cdot x} A^c_{n}^\mu$ explicitly, and the covariant derivative contains only usoft gluons. The summation on the label momenta and the phases are omitted. Note that there appear no soft gluons in the collinear quark sector because the interaction of a soft gluon with a collinear quark makes the collinear quark off the mass shell.

From all the possible gauge transformation in the full QCD, the relevant ones in constructing the effective theory are those which have support over collinear, soft, and ultrasoft momenta. A collinear gauge transformation $U_c(x)$ is defined as the subset of gauge transformations where $\partial_\mu U_c(x) \sim (\lambda^2, 1, \lambda)$. A soft gauge transformation $U_s$ is the subset of gauge transformations with $\partial_\mu U_s(x) \sim \lambda$, and an usoft gauge transformation $U_u$ is the subset with $\partial_\mu U_u \sim \lambda^2$. And the collinear, soft, and usoft gluons are the gauge fields associated with these transformations.

Now we specify the transformation properties of all the fields in the effective theory under these gauge transformations. We require that the effective Lagrangian is invariant under these gauge transformations. Here the point is that we can assign arbitrary transformation properties to all the fields as long as the sum of the gauge fields $A^c_\mu + A^s_\mu + A^u_\mu$ transforms as

$$A^c_\mu + A^s_\mu + A^u_\mu \rightarrow U_i (A^c_\mu + A^s_\mu + A^u_\mu) U_i^\dagger - \frac{i}{g} U_i \partial^\mu U_i^\dagger, \quad (i = c, s, u). \quad (7)$$

This is because the gauge transformations $U_i$ are subsets of the gauge transformations of the full QCD and $A^c_\mu + A^s_\mu + A^u_\mu$ is the gauge field in the full QCD. Therefore there is arbitrariness in specifying the gauge transformation properties of each gauge field. However, if we require that the effective Lagrangian be invariant at each order in $\lambda$, the gauge transformation for each field becomes more specific.
Let us consider one possible choice in which the gauge fields transform under collinear gauge transformations as

$$A^\mu_c \rightarrow U_c A^\mu_c U_c^\dagger - \frac{i}{g} U_c \partial^\mu U_c^\dagger, \quad A^\mu_s \rightarrow U_c A^\mu_s U_c^\dagger, \quad A^\mu_u \rightarrow U_c A^\mu_u U_c^\dagger.$$ (8)

Here we put the inhomogeneous term coming from the derivative of the gauge transformation in the transformation of a collinear gauge field. The full effective Lagrangian in Eq. (6) is invariant under the gauge transformation in Eq. (8), but it does not make the Lagrangian gauge invariant at each order in \(\lambda\). In order to show this fact in a transparent way, let us write the collinear gauge transformation \(U_c\) as

$$U_c = \sum_Q e^{-iQ \cdot x} U,$$ (9)

where \(Q^\mu\) is the label momentum and \(\partial^\mu U \sim \lambda^2\). Then the gauge transformation in Eq. (8) becomes

$$A^\mu_n \rightarrow UA^\mu_n U^\dagger - \frac{1}{g} U\left[(P^\mu + i\partial^\mu)U^\dagger\right], \quad A^\mu_s \rightarrow UA^\mu_s U^\dagger, \quad A^\mu_u \rightarrow UA^\mu_u U^\dagger,$$ (10)

where the square bracket means that the operator acts only inside the bracket. And the collinear spinor \(\xi_n\) transforms as \(\xi_n \rightarrow U\xi_n\). Under this gauge transformation, \(P^\mu - gA^\mu_n + iD^\mu\) transforms as

$$P^\mu - gA^\mu_n + iD^\mu \rightarrow U\left(P^\mu - gA^\mu_n + iD^\mu\right)U^\dagger.$$ (11)

Note that the full Lagrangian contains only the combination \(P^\mu - gA^\mu_n + iD^\mu\) contracted with \(n^\mu\), \(\overline{n}^\mu\) or \(\gamma_\perp^\mu\). Therefore the whole Lagrangian is invariant under the gauge transformation of Eq. (10). However this gauge transformation does not make the Lagrangian invariant order by order in \(\lambda\).

We can expand the effective Lagrangian in Eq. (6) in powers of \(\lambda\) as \(\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \cdots\), where

$$\mathcal{L}_0 = \overline{\xi_n} \left\{ n \cdot (iD + P - gA_n) + \left(P_\perp - gA_n^\perp\right) \overline{\xi_n} \right\} \frac{1}{\overline{n} \cdot (P - gA_n)} \left(P_\perp - gA_n^\perp\right) \xi_n,$$

$$\mathcal{L}_1 = \overline{\xi_n} \left\{ \overline{iD_\perp} \frac{1}{\overline{n} \cdot (P - gA_n)} \left(P_\perp - gA_n^\perp\right) + \left(P_\perp - gA_n^\perp\right) \frac{1}{\overline{n} \cdot (P - gA_n)} iD_\perp \right\} \frac{\overline{\xi_n}}{2},$$

$$\mathcal{L}_2 = \overline{\xi_n} \left\{ \overline{iD_\perp} \frac{1}{\overline{n} \cdot (P - gA_n)} iD_\perp \right\} \frac{\overline{\xi_n}}{2}.$$
From this expansion, it is clear why the gauge transformation of Eq. (10) does not preserve the gauge invariance order by order. At each order, the operators appear in the combinations \( n \cdot (P - gA) n \), \( \overline{P} \perp - g/ A \perp n \), \( n \cdot iD \), contracted with \( n \cdot i/ D \perp n \). Therefore the basic combinations of the operators which appear in the Lagrangian are \( P^\mu - gA^\mu n \) and \( iD^\mu \), and they transform as

\[
P^\mu - gA^\mu_n \rightarrow U\left(P^\mu - gA^\mu_n\right)U^\dagger + U\left[i\partial^\mu U\right],
\]

\[
iD^\mu \rightarrow U\left[i\partial^\mu U\right] - U\left[i\partial^\mu U\right].
\]

(13)

The combination \( P^\mu - gA^\mu_n + iD^\mu \) transforms homogeneously, but the decompositions \( P^\mu - gA^\mu_n \) and \( iD^\mu \) have additional terms, which are opposite in sign. Therefore though the whole Lagrangian is invariant under this gauge transformation, the Lagrangian at each order is not invariant under the transformation of Eq. (10).

As a special example, we can choose \( U \) such that \( \partial^\mu U \) has only the component in the \( n^\mu \) direction. That is, gluons transform under a collinear transformation as

\[
A^\mu_n \rightarrow U A^\mu_n U^\dagger - \frac{1}{g} U \left[ (P^\mu + \frac{n^\mu}{2} i n \cdot \partial)U \right],
\]

\[
A^\mu_s \rightarrow U A^\mu_s U^\dagger, \quad A^\mu_n \rightarrow U A^\mu_n U^\dagger.
\]

(14)

In this case, since \( \partial^\mu U \) has only the component in the \( n^\mu \) direction, the effective Lagrangian is collinear gauge invariant order by order [6]. Though the collinear gauge transformation in Eq. (14) is a legitimate gauge transformation, it is a very restricted subset of the most general transformations with \( \partial^\mu U \sim \lambda^2 \). For the most general transformations of \( U \), we can write the derivative of \( U \) as

\[
\partial^\mu U = \frac{n^\mu}{2} n \cdot \partial U + \partial^\perp U + \frac{n^\mu}{2} n \cdot \partial U,
\]

(15)

where each term is of order \( \lambda^2 \).

If we use this most general collinear gauge transformation in Eq. (10), the collinear gauge transformation is troublesome if we describe gauge-invariant operators such as heavy-to-light current operators at each order in \( \lambda \). In general, we cannot make gauge-invariant operators at fixed order in \( \lambda \). In order to make an operator gauge invariant, we have to invoke operators with different powers of \( \lambda \).
In Ref. [4], they have considered the gauge symmetry of the leading-order Lagrangian. They also argue for physical reasons that the usoft field acts as a background field and they extend the transformation of gluons in the presence of a background usoft gluon. Following the background field method of Abbott [7], the correct transformation property in the background usoft field, with the assumption that $\partial^{\mu} U$ has only the $\vec{\pi}^{\mu}$ component, is given by

\[ A^{\mu}_{n} \rightarrow U A^{\mu}_{n} U^{\dagger} - \frac{1}{g} U \left[ (P^{\mu} + \frac{\vec{\pi}^{\mu}}{2} n \cdot iD) U^{\dagger} \right] - \frac{\vec{\pi}^{\mu}}{2} n \cdot A_{u}, \]
\[ A^{\mu}_{s} \rightarrow A^{\mu}_{s}, \quad A^{\mu}_{u} \rightarrow A^{\mu}_{u}, \quad (16) \]

replacing the derivative with a covariant derivative including a usoft background gluon. Under the collinear gauge transformation in Eq. (16), only the leading Lagrangian $L_0$ is invariant, but the Lagrangians $L_1, L_2$ are not invariant. Therefore the full Lagrangian is not gauge invariant. This is because the combination $P^{\mu} - g A^{\mu}_{n} + iD^{\mu}$ does not transform homogeneously under the transformation in Eq. (16), but only the combinations $n \cdot (P - g A^{\mu}_{n} + iD)$ and $P_{\perp} - g A_{\perp}$, which appear in $L_0$, transform homogeneously. However, those terms such as $i\not\! P_{\perp}$ and $\vec{\pi} \cdot iD$, which appear in the Lagrangian at higher order, do not transform homogeneously.

If we want to keep the Lagrangian gauge invariant at each order in $\lambda$, we require the gauge fields transform under a collinear gauge transformation as

\[ A^{\mu}_{n} \rightarrow U A^{\mu}_{n} U^{\dagger} - \frac{1}{g} U \left[ P^{\mu} U^{\dagger} \right], \quad A^{\mu}_{s} \rightarrow U A^{\mu}_{s} U^{\dagger}, \]
\[ A^{\mu}_{u} \rightarrow U A^{\mu}_{u} U^{\dagger} - \frac{i}{g} U \left[ \partial^{\mu} U^{\dagger} \right]. \quad (17) \]

One may wonder why usoft fields transform inhomogeneously under collinear gauge transformations since they fluctuate over wavelengths which cannot resolve the fast local change induced by $U_{c}(x)$. But this is true only when $\partial^{\mu} U = 0$, that is, when the collinear gauge transformation does not include any fluctuation of order $\lambda^2$ ($\partial^{\mu} U \sim \lambda^2$). If we include small fluctuations of order $\lambda^2$ which is in superposition with the fast local change, usoft gluons can also change under this small remnant fluctuation of the collinear gauge transformation. Note that the gauge transformation shown in Eq. (17) is not unique. We can devise other gauge transformations which make the effective Lagrangian invariant order by order. Eq. (17) is one possible choice which satisfies this requirement.

Under the gauge transformations given by Eq. (17), the decompositions $P^{\mu} - g A^{\mu}_{n}$ and $iD^{\mu}$ transform homogeneously as

\[ P^{\mu} - g A^{\mu}_{n} \rightarrow U (P^{\mu} - g A^{\mu}_{n}) U^{\dagger}, \quad iD^{\mu} \rightarrow U iD^{\mu} U^{\dagger}. \quad (18) \]
Since the effective Lagrangian contains either $P^\mu - g A^\mu_n$ or $i D^\mu$ at each order, the effective Lagrangian is invariant order by order under the collinear gauge transformation given by Eq. (17). Then it is possible to consider gauge-invariant operators at any fixed order in $\lambda$.

We can choose a different gauge such as the background gauge. It is convenient in calculating radiative corrections for the operators with external gluons. If we are interested in the operators with external usoft gluons, we can write the usoft field as $A^\mu_u = B^\mu_u + Q^\mu_u$, where $B^\mu_u$ is the background usoft field and $Q^\mu_u$ is the quantum usoft field. If we also require the collinear gauge invariance order by order in $\lambda$, each gauge field transforms as

$$A^\mu_n \rightarrow U A^\mu_n U^\dagger - \frac{1}{g} U [P^\mu U^\dagger], \quad A^\mu_s \rightarrow U A^\mu_s U^\dagger,$$

$$Q^\mu_u \rightarrow U Q^\mu_u U^\dagger - \frac{i}{g} U [\tilde{D}^\mu U^\dagger] - B^\mu_u,$$  

where $\tilde{D}^\mu = \partial^\mu + ig B^\mu_u$ [7]. Note that we cannot neglect the quantum degrees of freedom for usoft fields as long as we include small fluctuations of order $\lambda^2$ in the collinear gauge transformation.

Since the soft gauge transformation is not relevant in the collinear quark sector, we will consider the remaining usoft gauge transformation. Under a usoft gauge transformation, all the fields transform as in QCD. Gauge fields transform under usoft gauge transformations as

$$A^\mu_n \rightarrow U_n A^\mu_n U^\dagger_n, \quad A^\mu_s \rightarrow U_s A^\mu_s U^\dagger_s, \quad A^\mu_u \rightarrow U_u A^\mu_u U^\dagger_u - \frac{i}{g} U_u \partial^\mu U^\dagger_u,$$  

and the spinor $\xi_n$ transforms as $\xi_n \rightarrow U_n \xi_n$. The effective Lagrangian is invariant under the usoft gauge transformation in Eq. (20) order by order.

Next let us consider reparameterization invariance of the collinear effective theory. Reparameterization invariance appears whenever we decompose momenta into large and small components. For instance, in the heavy quark effective theory (HQET), we can decompose the momentum of a heavy quark as $p^\mu_Q = m_Q v^\mu + k^\mu$, where $k^\mu$ is the residual momentum of order $\Lambda_{QCD}$. We can shift $v^\mu$ by an amount of order $\Lambda_{QCD}/m_Q$ as

$$p^\mu_Q = m_Q v^\mu + k^\mu \rightarrow m_Q (v^\mu + \epsilon^\mu/m_Q) + k^\mu - \epsilon^\mu \equiv m_Q v'^\mu + k'^\mu.$$  

And the physics should be invariant under different decompositions of the heavy quark momentum [8].
In the collinear effective theory, the momentum of a collinear quark is written as \( P^\mu = \frac{1}{2}(\vec{\pi} \cdot p)n^\mu + p^\mu_\perp + k^\mu \). If we shift the largest quantity \( \frac{1}{2}(\vec{\pi} \cdot p)n^\mu \) by a small amount of order \( \lambda \) or \( \lambda^2 \), and if this change is compensated by the change in \( p^\mu_\perp + k^\mu \), the total momentum remains unchanged and the physics should be invariant under different decompositions of \( P^\mu \). There is another ambiguity because a different choice of the basis vectors \( n^\mu \) and \( \vec{\pi}^\mu \) cannot change the physics as long as they satisfy \( n^2 = 0, \pi^2 = 0 \) and \( n \cdot \pi = 2 \). However, a small shift in momentum can be achieved by a small change in \( n^\mu \) and \( \pi^\mu \).

Manohar et al. [9] have generalized the reparameterization invariance, which was first considered in Ref. [6], to three classes of reparameterization transformations under which the basis vectors \( n^\mu \) and \( \pi^\mu \) change while \( n^2 = 0, \pi^2 = 0 \) and \( n \cdot \pi = 2 \) are fixed. The effective Lagrangian in the collinear effective theory is invariant under these transformations. The three classes of transformations are given by

\[
\begin{align*}
(I) \quad & \quad n^\mu \rightarrow n^\mu + \Delta^\mu_\perp, \\
& \quad \pi^\mu \rightarrow \pi^\mu, \\
(II) \quad & \quad n^\mu \rightarrow n^\mu, \\
& \quad \pi^\mu \rightarrow \pi^\mu + \epsilon^\mu_\perp, \\
(III) \quad & \quad n^\mu \rightarrow (1 + \alpha)n^\mu, \\
& \quad \pi^\mu \rightarrow (1 - \alpha)\pi^\mu.
\end{align*}
\]

Here \( n \cdot \Delta_\perp = \pi \cdot \Delta_\perp = n \cdot \epsilon_\perp = \pi \cdot \epsilon_\perp = 0 \) to order \( \lambda^2 \). And the corresponding changes for arbitrary vectors and quantum fields are summarized in Ref. [9].

If we shift the largest component in the \( n^\mu \) direction by a magnitude of order \( \lambda \) or \( \lambda^2 \), we can compensate this shift by changing \( p^\mu_\perp + k^\mu \) such that the total momentum \( P^\mu \) remains unchanged. The reparameterization transformation is given by

\[
P^\mu = \frac{\pi \cdot p}{2} (n^\mu + \frac{2\epsilon^\mu}{\pi \cdot p}) + p^\mu_\perp + k^\mu - \epsilon^\mu \rightarrow \frac{\pi \cdot p}{2} n^\mu + p^\mu_\perp + k^\mu,
\]

with \( \pi^\mu \) fixed. The infinitesimal shift \( \epsilon^\mu \) includes momenta of order \( \lambda \) or \( \lambda^2 \) with \( n \cdot \epsilon = \pi \cdot \epsilon = 0 \) to order \( \lambda^2 \). Here the last expression shows that the shift in \( n^\mu \) can be achieved using the type-I transformation and \( p^\mu_\perp + k^\mu = p^\mu_\perp + k^\mu - \frac{1}{2}(\pi \cdot p)\epsilon^\mu - \frac{1}{2}(\epsilon_\perp \cdot p_\perp)\pi^\mu \). The collinear quark field also changes as \( \xi_n \rightarrow \xi_n + \delta \xi_n \) to satisfy \( \not{n} \xi_n = 0 \). Under the transformation

\[
n^\mu \rightarrow n^\mu + \frac{2\epsilon^\mu}{\pi \cdot p}, \quad \xi_n \rightarrow e^{i\epsilon \cdot x} \left( 1 + \frac{i}{\pi \cdot p 2} \right) \xi_n,
\]

the effective Lagrangian in Eq. (6) is invariant. Therefore the collinear effective theory has a reparameterization invariance. As a result, the kinetic energy term is not renormalized to all orders in \( \alpha_s \). For instance, the kinetic energy term
to order $\lambda^2$, which is given by

$$K = \overline{\xi}_{n} (n \cdot i\partial + \frac{p_{\perp}^2}{\overline{n} \cdot p} + \frac{2p_{\perp} \cdot i\partial_{\perp}}{\overline{n} \cdot p} + \frac{(i\partial_{\perp})^2}{(\overline{n} \cdot p)^2} - \frac{\overline{n} \cdot p}{\overline{n} \cdot p} \frac{p_{\perp}^2}{2} \xi_{n}) \overline{\xi}, \quad (25)$$

is not renormalized to all orders in $\alpha_s$. Here the first two terms are of order $\lambda^0$, the third term is of order $\lambda$, and the last two terms are of order $\lambda^2$.

The changes in $n^\mu$ of order $\lambda$ and $\lambda^2$ have different implications on the structure of the effective theory because the momentum has three scales in it. When we change $n^\mu$ by an amount of order $\lambda$, it cannot be compensated by the residual momentum $k^\mu$ which is of order $\lambda^2$, but can be compensated by the change in the label momentum $p'^\mu$. Therefore the change of order $\lambda$ induces a reparameterization invariance between label momenta. This reparameterization invariance is independent of the usoft fluctuation of the fields. The reparameterization invariance at order $\lambda$ was utilized in deriving heavy-to-light current operators to order $\lambda$ in Ref. [6].

If we change $n^\mu$ by an amount of order $\lambda^2$, then it can be compensated by the residual momentum $k^\mu$. In terms of the operator $P^\mu = n^\mu \overline{n}/2 + P_{\perp}^\mu$, and $i\partial^\mu$, the transformation in Eq. (24) causes

$$P^\mu \rightarrow P^\mu + \epsilon^\mu, \quad i\partial^\mu \rightarrow i\partial^\mu - \epsilon^\mu. \quad (26)$$

This implies that reparameterization invariant operators must be built out of the linear combination $P^\mu + i\partial^\mu$ and the usoft gauge invariance requires that $i\partial^\mu$ be replaced by the covariant derivative $iD^\mu = i\partial^\mu - gA^\mu_u$.

There is another way to shift the large component $1/2(\overline{n} \cdot p)n^\mu$ by shifting $\overline{n} \cdot p$ instead of $n^\mu$. This also has an analogue in HQET, namely, the residual mass effect [10]. We can shift the large momentum by shifting the heavy quark mass $m_Q$ as

$$p'^\mu_Q = m_Q v^\mu + k^\mu \rightarrow (m_Q + \delta m)v^\mu + k^\mu - \delta m v^\mu \equiv m'_Q v^\mu + k'^\mu, \quad (27)$$

which is another legitimate decomposition into a large and a small momenta as long as $\delta m$ is of order $\Lambda_{\text{QCD}}$. The effective theory using $m'_Q$ should lead to the same results as the effective theory using $m_Q$, and the combinations $m^*_Q \equiv m'_Q + \delta m$, $iD^\mu \equiv iD^\mu - \delta m v^\mu$ are invariant under redefinitions of $\delta m$.

As a consequence hadronic form factors in the HQET must be defined in terms of matrix elements containing the operator $iD^\mu = iD^\mu - \delta m v^\mu$. The physics should be unambiguous in defining a heavy quark mass and the corresponding change in the HQET is called the residual mass effect [10].
In the collinear effective theory, we can change $\mathbf{n} \cdot \mathbf{p} \rightarrow \mathbf{n} \cdot \mathbf{p} + 2\delta E$, which shifts $\mathbf{n} \cdot \mathbf{p}$ by an amount of $\lambda^2$. This can be achieved by shifting $\mathbf{n} \mu$ with fixed $\mathbf{n} \mu$, which corresponds to the type-II transformation. We can decompose the momentum of a collinear quark as

$$P^\mu = \frac{\mathbf{n} \cdot \mathbf{p}}{2} n^\mu + p_\perp^\mu + k^\mu$$

$$= \left( \frac{\mathbf{n} \cdot \mathbf{p}}{2} - \delta E \right) n^\mu + p_\perp^\mu + k^\mu + \delta E n^\mu \rightarrow \frac{\mathbf{n} \cdot \mathbf{p}}{2} n^\mu + p_\perp^\mu + k^\mu,$$  \hspace{1cm} (28)

where the last expression shows that this change can be achieved by the type-II transformation in Eq. (22), along with the corresponding change in $\xi_n$. When we shift $\mathbf{n} \mu \rightarrow \mathbf{n} \mu + \epsilon_\perp^\mu, \delta E$ is given by $\delta E = -\epsilon_\perp \cdot \mathbf{p}_\perp / 2$, and $p_\perp^\mu + k^\mu = p_\perp^\mu + k^\mu - \frac{1}{2}(\epsilon_\perp \cdot \mathbf{p}_\perp)n^\mu$. If $\epsilon_\perp$ is of order $\lambda$, $\delta E$ is of order $\lambda^2$, which can be compensated by the residual momentum $k^\mu$. The final decomposition in Eq. (28) is also a legitimate decomposition of the momentum $P^\mu$. And the effective theory is invariant under this “residual energy transformation”. We call the invariance under this transformation as “residual energy invariance” borrowing the terminology from the HQET.

Following the reasoning in HQET, the combinations

$$\frac{\mathbf{n} \cdot \mathbf{p}}{2} \equiv \frac{\mathbf{n} \cdot \mathbf{p}^*}{2} + \delta E, \quad \frac{iD^\mu}{2} \equiv iD^\mu - \delta E n^\mu$$  \hspace{1cm} (29)

are invariant under redefinitions of $\delta E$. But here comes a big difference between the HQET and the collinear effective theory. In HQET, we have to use the combinations $m_Q^* = m_Q^\prime + \delta m, \quad iD^\mu = iD^\mu - \delta m v^\mu$, which are invariant under redefinitions of $\delta m$, and these combinations should appear in the HQET Lagrangian. However, the Lagrangian of the collinear effective theory is already invariant under the residual energy transformation, which works even when the variation $\epsilon_\perp^\mu$ can be of order $\lambda^0$ [9]. In other words, if we rewrite the Lagrangian in terms of $\mathbf{n} \cdot \mathbf{p}^*$ and $iD^\mu$, there appears no term depending on $\delta E$. Therefore it is irrelevant to use either $\mathbf{n} \cdot \mathbf{p}, iD^\mu$ or $\mathbf{n} \cdot \mathbf{p}^*, iD^\mu$, and the physics is the same. This means that there is no ambiguity in the choice of the large momentum component of order $\lambda^0$. And there is no need to change the matrix elements containing the operator $iD^\mu$ and the nonperturbative parameters in the calculation of form factors. This need not be true for heavy-to-light currents.

We have considered various symmetries of the effective Lagrangian in the collinear effective theory and their implications. Collinear and usoft gluons are associated with collinear and usoft gauge transformations. If we require the gauge invariance order by order in $\lambda$, the gauge transformation of collinear gluons and usoft gluons takes a definite form. The requirement of having
the gauge invariance at each order in $\lambda$ is important in constructing gauge-invariant operators order by order.

Reparameterization invariance imposes a constraint on the form of the effective Lagrangian. The derivative term should appear in the combination $P^\mu + iD^\mu$, which relates quantities with different powers of $\lambda$. Also the kinetic energy term is not normalized to all orders in $\alpha_s$. Residual energy invariance means that there is no ambiguity in defining the large component. Though we change the largest momentum component by order $\lambda$ or $\lambda^2$, the physical effect under this small change does not appear.

We have considered heavy-to-light currents and their renormalization behavior to order $\lambda$ in Ref. [6]. If we consider heavy-to-light currents to order $\lambda^2$, reparameterization invariance both in the collinear effective theory and the HQET will be useful in constructing operators at order $\lambda^2$. If we use the reparameterization transformation and the residual energy transformation of order $\lambda$, they will change the heavy quark velocity $v^\mu = (n^\mu + \pi^\mu)/2$ by an order $\lambda$. This will confuse the power counting in $\lambda$ since the energy of the light degrees of freedom in a heavy quark is of order $\lambda^2$. However it is possible to make $v^\mu$ intact by combining the reparameterization transformation and the residual energy transformation. For example, we consider both transformations at order $\lambda^2$, or we can combine both transformations of order $\lambda$ such that the changes caused by these transformations cancel in such a way that the combination $n^\mu + \pi^\mu$ does not change. In this case, we can combine the reparameterization invariance in the HQET and the collinear effective theory to study the structure of higher-dimensional heavy-to-light current operators. Studies in this direction are in progress.

**Note added:**

While this paper was being written, Ref. [9] appeared. In Ref. [9], the authors extended the idea of reparameterization invariance to three possible cases of changing the basis vectors $n^\mu$ and $\pi^\mu$. We note that the reparameterization transformation and the residual energy transformation can be achieved by using the type-I and the type-II transformations respectively. And the type-III transformation is new.

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