Abstract—Accurately modeling generator frequency response to power disturbances is essential for assessing frequency control performance in power grids. A widely used technique is to aggregate the frequency response of coherent generators into a single effective machine. Previous work has demonstrated that the best choice of inertial and damping coefficients for the effective generator is obtained by adding among all the corresponding generator parameters. However, in the presence of turbine dynamics, the proper choice of turbine time constants is challenging.

We introduce a novel framework to approximate the aggregate frequency dynamics of coherent synchronous generators. By leveraging recent results on dynamics concentration of tightly-connected networks, we show that the aggregate dynamics are high-order, therefore cannot be accurately represented by a single machine. Instead, we develop a hierarchy of reduced order models—based on balanced truncation—that accurately approximate the aggregate system response. Our results outperform existing aggregation techniques and can be shown to monotonically improve the approximation as the hierarchy order increases.

Techniques for deriving reduced order approximations of large-scale power networks based on coherence and aggregation have been investigated for decades [1]. Generally, a group of generators is considered coherent if they have similar frequency responses when subject to power imbalance. When modeling the frequency response of power grids, a widely used technique is to aggregate the frequency response of coherent generators into a single effective machine [2]–[5].

1) Aggregate Dynamics of Coherent Generators: Consider a power network with n coherent generators. For each generator \( i \), the transfer function from electric power imbalance to frequency is given by \( g_i(s) \). We show that

\[
\hat{g}(s) = \left( \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}
\]

is an accurate aggregate model for the network in the asymptotic senses [6]: As the connectivity of the underlying network increases, the network aggregate dynamics eventually converge to \( \hat{g}(s) \).

Our result illustrates, in particular, the difficulty on aggregating generators with heterogeneous turbine time constants. For example, for the standard swing equations with first order turbines, \( g_i(s) = \frac{r_i s + 1}{(r_i s + 1)(m_i s + d_i) + r_i^{-1} s} \), where \( d_i \), \( r_i \), and \( \tau_i \) are the damping, droop, and turbine time constants, the aggregate model (1) is given by

\[
\hat{g}(s) = \frac{1}{\hat{m} s + \hat{d} + \sum_{i=1}^{n} \tau_i^{-1} s + 1},
\]

where \( \hat{m}, \hat{d} \) are the sum of \( m_i, d_i \), respectively. When all generators have the same turbine time constant \( \tau_i = \hat{\tau} \), then \( \hat{g}(s) \) in (2) reduces to the typical effective machine model with parameters \( (\hat{m}, \hat{d}, \hat{\tau}, \hat{\tau}) \), where \( \hat{r}^{-1} = \sum_{i=1}^{n} \tau_i^{-1} \). However, if the \( \tau_i \) are heterogeneous, then \( \hat{g}(s) \) is a high-order transfer function and finding its low-order approximation is harder.

2) Reduced Order Model for Coherent Generators: To solve this problem, we use frequency weighted balanced truncation [7] to approximate \( \hat{g}(s) \). Frequency weighted balanced truncation identifies the most significant dynamics with respect to particular LTI frequency weight by computing the weighted Hankel singular values, the square root of eigenvalues of \( X_c Y_o \), where \( X_c \) and \( Y_o \) are the frequency weighted controllability and observability gramians of the system to be reduced. In many cases, the Hankel singular values decay fast, allowing us to accurately approximate high-order systems.

We propose two approaches to obtain \( k \)th order reduction model of \( \hat{g}(s) \): 1) Replace the high-order turbine dynamics \( \sum_{i=1}^{n} \tau_i^{-1} s + 1 \) by its \((k-1)\)-th order approximation by weighted balanced truncation; 2) \( k \)-th order balanced truncation on closed-loop system \( \hat{g}(s) \) directly.

We compare all proposed reduction models by balanced truncation: 2nd and 3rd order balanced truncation on turbine dynamics \( \hat{g}_2^{tb}(s) \) (BT2-b), \( \hat{g}_3^{tb}(s) \) (BT3-b) with frequency weight \( W_{tb}(s) = \frac{s + 3 \cdot 10^{-2}}{s + 10^{-4}} \); 2nd and 3rd order balanced truncation on closed-loop dynamics, \( \hat{g}(s) \), given by \( \hat{g}_2^{cl}(s) \) (BT2-cl), \( \hat{g}_3^{cl}(s) \) (BT3-cl) with frequency weight \( W_{cl}(s) = \frac{s + 8 \cdot 10^{-2}}{s + 10^{-4}} \). The step response along with step response error with respect to \( \hat{g}(s) \) are shown in Fig. 1.

Fig. 1. Comparison of all reduced order models by balanced truncation

While it is not surprising that approximation models with higher order outperform all second order models, we highlight that with only a third order model one can accurately approximate the aggregate response. Moreover, a 3rd order reduced model by balanced truncation can be interpreted as a swing model back-fed with 2nd order turbine dynamics.
By partial fraction expansion on the turbine dynamics, it can be regarded as two first order turbine in parallel.

Secondly, we find that reduction on the closed-loop dynamics improves the approximation compared to reduction on turbine dynamics. The main reason is that reduction on closed-loop dynamics has the flexibility to choose the equivalent inertia and damping of the reduced order model in order to better approximate the response.

Lastly, we compare reduced order models by balanced truncation on closed-loop dynamics $\tilde{g}_{cl}^{2}(s)$, $\tilde{g}_{cl}^{3}(s)$ to proposed models in [3], [4]. The step responses and the approximation errors are shown in Fig. 2.

In the comparison, $\tilde{g}_{cl}^{3}(s)$ outperforms all other reduced order models and it is the most accurate reduced order model of $\hat{g}(s)$. It is also worth noting that $\tilde{g}_{cl}^{3}(s)$ has the least approximation error among all 2nd order models. In general, such results suggest us that to improve the accuracy of reduced order model of coherent dynamics of generators $\hat{g}(s)$, we should consider: 1) increasing the complexity (order) of the reduction model; 2) reduction on closed-loop dynamics instead of on turbine dynamics.

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