Calculation update of strength of expanded clay fiber-reinforced concrete elements with mixed reinforcement

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Abstract. The paper deals with the methodology for calculating the strength of the expanded clay fiber-reinforced concrete elements with mixed reinforcement with the most complete view of the post-yield properties of prestressed and non-stressed rebars, as well as the influence of prestressing on the properties of the expanded clay fiber-reinforced concrete.

1. Introduction
The developed method of calculation is based on a complete account of the work of prestressed and non-stressed rebars beyond the conditional yield point, as well as the effect of prestressing on the properties of the expanded clay fiber-reinforced concrete.

For these purposes we need to use analytical dependencies that describe the complete actual diagrams of the tensile strength of high-strength reinforcement and the effect of prestress on them.

Currently, there are many proposals for an analytical description of steel deformation diagrams [1-5], from which there were selected the dependencies obtained in the Russian State Social University [4,5].

2. Experimental program and research results
These dependencies are flexible, because they are suitable for all types of steel, simple and extra-strong.

According to [5], the steel deformation diagram is divided into two sections: elastic (from 0 to $\sigma_{el}$) and non-elastic (from $\sigma_{el}$ to $\sigma_u$) work. Within the section from the elastic limit $\sigma_{el}$ to the temporary resistance of steel $\sigma_u$, the analytical description of the deformation diagram has the following form:

$$\sigma_s = S - K' u \frac{10(\varepsilon, 10^3 + L)}{\varepsilon, 10^3 + L},$$

where $S = \sigma_{0.2}$ (1.475 $\sigma_u / \sigma_{0.2}$ -0.475)-30; (1)

$$1$$

$$2$$

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\[ K = 3700 \left[ \frac{\sigma_{0.2} (\sigma_u / \sigma_{0.2} - 1) + 150}{1000} \right] ^2 - 96, \]  
(3)

\[ L = \sigma_{0.2} \left[ \frac{12.5(\sigma_u / \sigma_{0.2} - 1) - 1000}{E_s} \right] + 0.96, \]  
(4)

The relationship between deformations and stresses within the section of the elastic limit of steel before the development of deformations up to 1.5% in it can be represented as:

\[ \varepsilon_s \cdot 10^3 = \frac{K_0}{A - \sigma_s} - D + \frac{\sigma_s \left( 1.9 \cdot 10^5 - E_s \right)}{190 \cdot E_s}, \]  
(5)

where

\[ K_0 = 2.931 \cdot 10^{-2} (\sigma_u / \sigma_{0.2} - 1)^2 \sigma_{0.2}^2 + 84, \]  
(6)

\[ A = (1.186 \sigma_{0.2} - 231) \left( \sigma_u / \sigma_{0.2} - 0.5 \right)^2 + 0.675 \sigma_{0.2}^2 + 70, \]  
(7)

\[ D = 0.01 \cdot \sigma_{0.2} \left( 1.6 \cdot \sigma_u / \sigma_{0.2} - 2.126 \right) + 0.813. \]  
(8)

In formulas (1)-(8), the values of stress and modulus of elasticity are substituted in MPa.

Expressions (1)-(8) show that the parameters \( S, K, L, K_0, A, D \) depend on the known characteristics of the steel \( \sigma_{0.2}, \sigma_u, E_s \) so they can be easily determined for each type of steel[6,7]. After substituting the numerical values of steel characteristics in expressions (2)-(4) and (6)-(8), the parameters \( S, K, L, K_0, A, D \) for each type of steel will get certain numerical values, and the formula (1) and (5) will be significantly simplified.

When prestressing or reloading the reinforcement due to the choice of plastic deformations, the values of the conditional elastic and yield stresses increase. In an extremely stretched up to the stress \( \sigma_{sp} \) rebar the stresses are reduced over time due to the losses from stress relaxation \( \sigma_1 \) and other types of losses, the total value of which is here indicated as \( \sigma_{los} \). In the state of steady pre-stresses \( \sigma_{sp2} \) in rebar \( \sigma_{sp} \) the selected plastic deformations will be \( \varepsilon_{sp,pl} + \sigma_1 / E_s \), where \( \varepsilon_{sp,pl} \) - is the plastic deformation of steel when it is stressed to the level exceeding the elastic limit \( \sigma_{el} \). When an external tensile force is applied, the diagram can be assumed to be linear to the point of intersection with the original curve. This point corresponds to a new increased value of the elastic limit \( \sigma'_{el} \). The deformation corresponding to this stress is the following:

\[ \varepsilon'_el = \varepsilon_{sp} + \Delta \varepsilon_s, \]  
(9)

or according to [5]

\[ \varepsilon'_el = \varepsilon_{sp} + 7.7 \cdot 10^{-7} \sigma_{0.2}, \]  
(10)

The new value of the elastic limit can be determined by the formula (1) with \( \varepsilon_s = \varepsilon'_el \), and the new value of the conditional yield strength by the formula [5]:

\[ \sigma'_{0.2} / \sigma_{0.2} = 1 + \left( \sigma_u / \sigma_{0.2} - 1 \right) \left( \frac{0.127}{1.2214 - \sigma_{0.2}} - 0.157 \right). \]  
(11)
Let us determine the stress in a rebar when some external load effects on the element containing a pre-stretched \( A_{sp} \), and non-stressed \( A_s \) rebar. The initial condition is the equality of incremental strain of deformations from external load in the entire rebar located at the same distance from the most compressed face [8,9]. If the distance of the reinforcement from the specified face is different (not single-row reinforcement), the relationship between the incremental strain of deformations of the pre-stretched \( \varepsilon^N_{sp} \), and non-stressed \( \varepsilon^N_s \) rebar is expressed as follows:

\[
\frac{\varepsilon_h + \varepsilon^N_{sp}}{h_{sp}} = \frac{\varepsilon_h + \varepsilon^N_s}{h_{os}},
\]

where \( h_{sp} \) and \( h_{os} \) - is the distance from the most compressed face of the element to the resultant forces in the specified types of reinforcement.

In the reinforcement located within a zone stretched from an external load before applying the external stress load, the stresses will be equal: in a pre-stretched \( \sigma^2_{sp} \) (point \( A_1 \) in figure 1), and in non-stressed \( \sigma_{so} \) (point \( B_1 \)).

If we take the increment of deformations from the external load in the entire reinforcement to be the same and equal \( \varepsilon^N_s \), then the total deformations will be:

\[
\begin{align*}
\text{in rebar } A_{sp} \text{ (point } A \text{ in figure 1)} & : \varepsilon_s = \varepsilon^N_{sp} + \varepsilon^N_s, \\
\text{in rebar } A_s \text{ (point B)} & : \varepsilon_{sp} = \varepsilon^N_s - \varepsilon_{so}.
\end{align*}
\]

The stresses corresponding to these deformations are \( \sigma_s \) and \( \sigma_{sd} \). The increment of deformations in the reinforcement when the compressed zone of the element is destroyed is expressed by a known dependence:

\[
\varepsilon^N_s = \frac{\varepsilon_{bu}}{1 - \omega/1.1} \left( \frac{\omega}{\zeta} - 1 \right),
\]

and stress in elastic operation and with respect to prestress \( \sigma^2_{s2} \) - according to the formula of standards

\[
\begin{align*}
\sigma^N_s &= R \left( \omega / \zeta - 1 \right) + \sigma_{s2}, \\
R &= \sigma_{sc,ul} / (1 - \omega/1.1),
\end{align*}
\]

Deformation included in expression (13)

\[
\varepsilon^N_{sp2} = \varepsilon_{sp} - (\sigma_{los} - \sigma_i) / E_s,
\]

where \( \varepsilon_{sp} \) at \( \sigma_{sp} > \sigma_{el} \) is defined by the formula (5) taking \( \sigma_i = \sigma_{sp} \), and at \( \sigma_{sp} \leq \sigma_{el} \) deformation

\[
\varepsilon_{sp} = \sigma_{sp} / E_s.
\]
Figure 1. Stresses in a prestressed and non-stressed reinforcement when it is located in a cross-section zone that is stretched from the external load.

Deformations of non-stressed reinforcement $\varepsilon_{so} = \frac{\sigma_{so}}{E_s}$, where $\sigma_{so}$ - numerically equal to the prestress loss.

The stresses in the rebar $A_{sp}$ after applying an external load at $\sigma_s \leq \sigma_{cl}'$ is

$$\sigma_s = (\varepsilon_{sp2} - \varepsilon_{sp,pl} - \sigma_1/E_s + \varepsilon_s^N)E_s = \left[\varepsilon_{sp} - \left(\sigma_{los} - \sigma_1\right)/E_s - \varepsilon_{sp} + \sigma_{sp}/E_s - \sigma_1/E_s + \varepsilon_s^N\right]E_s,$$

or $\sigma_s = \bar{R}_s (\omega/\xi - 1) + \sigma_{sp2}$, \hspace{1cm} (19)

a well-known standard formula is obtained.

At $\sigma_s > \sigma_{cl}'$ the total deformation of a rebar $A_{sp}$ according to (13), (17) and (18) shall be

$$\varepsilon_s = \bar{R}_s (\omega/\xi - 1)/E_s + \varepsilon_{sp} - \left(\sigma_{los} - \sigma_1\right)/E_s$$

\hspace{1cm} (20)
where $\varepsilon_{sp}$ is determined by the formula (5) $\sigma_s = \sigma_{sp}$.

The stress corresponding to the strain $\varepsilon_s$ is determined by the formula (1).

In a non-stressed reinforcement $A_s$ the total deformation according to (14), (15), (17) is equal to

$$\varepsilon_{sd} = \overline{R}_s(\omega/\xi - 1)/E_s - \sigma_{so}/E_s,$$

(21)

Corresponding stresses at $\sigma_{sp} \leq \sigma_{el}$ are equal to

$$\sigma_{sd} = \overline{R}_s(\omega/\xi - 1) - \sigma_{so},$$

(22)

For non-elastic work ($\sigma_{sd} > \sigma_{el}$) they are determined by the formula (1), for deformations by (21).

In reinforcement located in a zone compressed from external load (figure 2), compressive stresses from the external load corresponding to the deformations $\varepsilon_{sd}^N$ are added to the steady-state stresses $\sigma_{sp}^2$ (point A) and $\sigma_{sd}^2$ (point B).

In a rebar $A_{sp}^s$ compressive stresses are added to the tensile stresses, so it works elastically and the total stresses (point A) are determined by the formula (19). Note that for a rebar located in the compressed zone, the first term of the right-hand side of this expression is negative and its highest value is equal to $\sigma_{sc} = -\sigma_{sc,u} + \sigma_{sp}^2$.

In a non-stressed rebar $A_{s}^i$ deformations $\varepsilon_{sd}^i$ (point B) are determined by the formula (21), and the stresses at $\sigma_{sp}^i \leq \sigma_{el}$ - by the formula (22). At $\sigma_{sp}^i \leq \sigma_{el}$ stresses $\sigma_{sp}^i$ are determined by the formula (1).

In a rebar of the compressed cross-section zone, the stress at the destruction of the element reaches the calculated resistance at $x/a' \geq 2$, where $a'$ - is the distance from the compressed face to the axis of the longitudinal reinforcement[9,10]. This condition is usually always met in compressed elements.

In case of its violation in the intervals $1 < x/a' < 2$ the deformations in a rebar change according to the linear law:

$$\varepsilon_{sc} = \varepsilon_{bu}(\tilde{h}_l/a' - 1) \leq \varepsilon_{bu},$$

(23)

The reinforcement stresses in a specified range will be:

in the rebar $A_{sp}^i$ $\sigma_{sc} = \sigma_{sp}^2 - \sigma_{sc,u}(\tilde{h}_l/a'_p - 1)$

(24)

in the rebar $A_{s}^i$ the stresses are equal to: $\sigma_{sd}^i = -\sigma_{su}^i - \sigma_{sc,u}(\tilde{h}_l/a'_s - 1)$

(25)

In formulas (24)-(25) $a'_p$ and $a'_s$ - is the distance from the longitudinal axes of the rebar $A_{sp}^i$ and $A_{s}^i$ to the most compressed face.
The given expressions for determining a reinforcement stress in rebars are used in the stress equilibrium equations. The first of them – the equation of moments relative to the line of action of the resultant external forces - is used to determine the height of the compressed cross-section zone and the stresses in the rebar of the stretched (or less compressed) cross-section zone[6,7]. The second - the equation of moments relative to the axis passing through the point of application of the resultant force in the rebar of the stretched zone - is used to determine the bearing capacity of the element.

When the compressed stress element is destroyed, the reinforcement stresses in the most compressed zone with a sufficiently large value of its height ($h \geq 2a'$) will obtain the maximum increment $\sigma_{sc,u}$ and will be:

in pre-stretched $A_{s(2)}$. 

Figure 2. Stresses in a prestressed and non-stressed rebar when it is located in a cross-section zone compressed from external load.
\[ \sigma_s(2) = \sigma_{sc,at} - \sigma_{s2(2)} \]  

(26)

in non-stressed \( A_{s(4)} \) -

\[ \sigma_{s(4)} = \sigma_{sc,at} - \sigma_{s2(4)} \]  

(27)

In expression (27) the stresses \( \sigma_{s2(4)} \) are compressive and are entered with the sign (-), so the total stresses \( \sigma_{s(4)} \) increase.

The equation of moments relative to the line of action of an external force will take the form:

\[ \sigma_{s(i)} S_{(1)} + \sigma_{s(3)} S_{(3)} = \gamma_{Rb} R_b bx(e + 0.5x - h_{oz}) + (\sigma_{sc,at} - \sigma_{s2(4)}) S_{(4)} + (\sigma_{sc,at} - \sigma_{s2(2)}) S_{(2)}, \]  

(28)

where:

\[ S_{(i)} = A_{s(i)}(e + a_z - a_{(i)}) \quad \text{at} \quad i = 1;3. \]

\[ S_{(i)} = A_{s(i)}(e_2 + a_{(i)}) \quad \text{at} \quad i = 2;4. \]

\( \gamma_{Rb} \) is a coefficient of concrete working conditions, taking into account the previous stress history.

For elastic operation of the reinforcement of the stretched zone, the expressions for the stresses according to (16) and (17) are represented as:

\[ \sigma_{s(1)} = \frac{b}{\gamma_{Rb}} b_{(1)} + x(\sigma_{s2(1)} - \gamma_{Rb}), \]  

(29)

\[ \sigma_{s(3)} = \frac{b}{\gamma_{Rb}} b_{(3)} + x(\sigma_{s2(3)} - \gamma_{Rb}), \]  

(30)

Substituting (29)-(30) in (28), after the transformations we get the cubic equation

\[ Ax^3 + Bx^2 + Cx + D = 0, \]  

(31)

where: \( A = 0.5\gamma_{Rb} b \)

\[ B = \gamma_{Rb} R_b b(e - h_{oz}), \]  

(33)

\[ C = SL - SK \]  

(34)

\[ SL = \sigma_{sc,at}(S_{(1)} + S_{(2)}) - \sigma_{s2(4)} S_{(4)} - \sigma_{s2(2)} S_{(2)}, \]  

(35)

\[ SK = \sigma_{s2(1)} S_{(1)} + \sigma_{s2(3)} S_{(3)} - \gamma_{Rb}(S_{(1)} + S_{(3)}), \]  

(36)

\[ D = -\gamma_{Rb}(S_{(1)} b_{(1)} + S_{(3)} b_{(3)}). \]  

(37)

Having determined from (28) the value of \( x \), and from (29)-(30) \( \sigma_{s(1)}, \sigma_{s(3)} \), the latter are compared with the new values of the elastic limits of steel \( \sigma_{e1} \). If at least one of the stresses exceeds \( \sigma_{e1} \) a new stress value is determined. To do this the deformation \( \varepsilon_{s(i)} \) is determined by formula (20), and according to it the stress \( \sigma_{s(i)} \) by formula (1). After determining the stresses \( \sigma_{s(1)} \) and \( \sigma_{s(3)} \) and substituting them into equation (28), the latter is reduced to the square equation:

\[ Ax^2 + Bx + C = 0, \]  

(38)

from which a new value of \( x \), after which the values of \( \sigma_{s(1)} \) and \( \sigma_{s(3)} \) are again determined.
The carrying capacity of an element is determined at $x \geq 2a'$ and $x \leq 0$ (with substitution $x=0$) by the formula:

$$N = \frac{1}{e} \left[ R_{y} \gamma_{Rc} x b (h_{bc} - 0.5x) + A_{s(2)} (\sigma_{sc,u} - \sigma_{s2(2)} h_{bc} - a_{(2)}) + A_{d(4)} (\sigma_{sc,u} - \sigma_{s4(4)} h_{bc} - a_{(4)}) \right], \quad (39)$$

and at $0 > x > 2a'$ by the formula:

$$N = \frac{1}{e} \left[ R_{y} \gamma_{Rc} x b (h_{bc} - 0.5x) + A_{s(2)} \left( \sigma_{sc,u} (x/a_{(2)} - 1) - \sigma_{s2(2)} h_{bc} - a_{(2)} \right) + A_{s(4)} \left( \sigma_{sc,u} (x/a_{(4)} - 1) - \sigma_{s4(4)} h_{bc} - a_{(4)} \right) \right]. \quad (40)$$

3. Conclusion

The above given method allows you to refuse determining the calculation case and the coefficient that takes into account the work of the reinforcement beyond the conditional yield point, moreover, this method for calculating the strength of compressed elements with mixed reinforcement is represented according to the calculation program on a computer.

4. References

[1] Baykov V N 1980 Stress-strain behaviour of reinforced concrete elements based on generalized experimental dependences of physical and mechanical characteristics of concrete and reinforcement Questions of strength, deformability and crack resistance of reinforced concrete 8 (Rostov-on-Don)

[2] Gushcha Y P 1983 About accounting for non-elastic deformations of concrete and rebar in calculation of reinforced concrete structures Improvement of structural forms, methods of calculation and design of reinforced concrete structures (Moscow: Scientific Research Institute of Concrete and Reinforced Concrete) pp 11-8

[3] Madatyan S A 1980 Technology of reinforcement tension and load-bearing capacity of reinforced concrete structures (Moscow: Stroizdat) p 196

[4] Mailyan R L and Mekerov B A 1983 Method of accounting for the effect of pre-stressing when calculating the strength of reinforced concrete elements Concrete and reinforced concrete 9 28-30

[5] Mekerov B A and Mailyan R L 1982 Analytical descriptions for the tensile diagram of high-strength reinforcing steel New types of reinforcement and its welding. (Collection of reports of the All-Union meeting in Volgograd: Moscow) pp 166-71

[6] Ryabukhin A and Matisy S 2013 In-situ measurement of the anchor-pile displacement in the geotechnical conditions of Sochi Proceedings of the 5th International Young Geotechnical Engineers' Conference (IYGEC 2013) (France. Paris :Ecole des PontsParisTech) pp 576–8

[7] Ryabukhin A K, Lesnoy V A and Kalashov D Y 2019 Study of the stability of soil slopes and downhill with the use of fiberglass dowels IOP Conf. Ser.: Mat. Sc. and Eng. 698(7) 077003

[8] Bezuglova E. and Matisy S 2013 Engineering and geological grounds of landslide protection reliability of structures Landslide Science and Practice: Risk Assessment, Management and Mitigation 6 709-14

[9] Bezuglova E, Matisy S and Podtelkov V. 2016 Landslide risk management at transport facilities Landslides and Engineered Slopes. Experience, Theory and Practice 2, 2016. 405-9

[10] Matisy S I and Sukhlyaeva L A 2019 Antimud flow protection with rigid thorough structures/ Geotechnics Fundamentals and Applications in Construction: New Materials, Structures, Technologies and Calculations (Proc. of the Int. Conf. on Geotechnics Fundamentals and Applications in Construction: New Materials, Structures, Technologies and Calculations, GFAC) pp 178-81