Resolution analysis for geostationary spaceborne-airborne bistatic forward-looking SAR

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Abstract: Owing to the spatial separation of transmitter and receiver, bistatic SAR is able to image the area in front of the moving direction of transmitter or receiver, but the geometry model is more complex than that of monostatic SAR, which brings great difficulties to system design and imaging process. Besides, the special geometry configuration of geostationary spaceborne-airborne bistatic forward-looking SAR results in non-orthogonal and non-uniform ground resolution, which is not orthogonal in the traditional range and azimuth resolution. To solve this problem, the resolution ellipse based on the generalised ambiguity function (GAF) is used to maintain the best and worst ground resolution in the resolution cell. The receiver’s flight direction is designed to optimise the uniform ground resolution distribution, which has great value for system design and the receiver’s path planning. Finally, the accuracy of the resolution ellipse and correctness of the derived formulas are verified by simulation.

1 Introduction
Geostationary spaceborne-airborne bistatic forward-looking SAR is a kind of bistatic SAR that places the transmitter on the geostationary satellite and the receiver on the airborne platform [1]. The transmitter located at the geostationary satellite is stationary relative to the ground and has the advantages of large coverage and strong security [2–4]. The receiver placed on the airborne platform has the advantage of flexibility. At the same time, bistatic SAR has the capability of forward-looking imaging because the transmitter and receiver are located at different platforms [5].

Although the spatial separation of transmitter and receiver brings a lot of advantages, it also makes it difficult to calculate the resolution of bistatic SAR. There are two main methods for bistatic SAR resolution calculation: the gradient method and the generalised ambiguity function. The gradient method proposed in [6] can be used to determine the range resolution, Doppler resolution, and the cross resolution. The generalised ambiguity function derived in [7] can be used to calculate the spatial resolution expression of geostationary spaceborne-airborne bistatic forward-looking SAR. However, GEO SAR is a monostatic SAR, the geometry model and ambiguity function are quite different from that of bistatic SAR.

Thus, this article will derive the resolution ellipse for geostationary spaceborne-airborne bistatic forward-looking SAR and calculate the receiver’s flight direction for optimal uniformity of ground resolution distribution. First, the generalised ambiguity function is used to determine the spatial resolution expression of geostationary spaceborne-airborne bistatic forward-looking SAR. Then, the resolution ellipse for geostationary spaceborne-airborne bistatic forward-looking SAR is obtained, and the uniformity of ground resolution distribution is characterised by the ratio of major and minor axes of the ellipse. The flight direction is found through maximising the ratio. Finally, the calculation formulas for spatial resolution, the flight direction, and the resolution ellipse are verified by simulation.

2 Ground resolution calculation
2.1 Geometry model
To calculate the ground resolution of geostationary spaceborne-airborne bistatic forward-looking SAR, the geometry model needs to be established first. O is the origin of the scene coordinate system in Fig. 1, P is the target location, R is the location of the receiver, and T is the location of the transmitter. $\phi_t$ and $\phi_r$ are the incident angles of the receiver and transmitter, and $\phi_t$ is assumed to be $>\phi_r$ to comply with the actual situation. $V_r$ is the speed of the receiver and is supposed to have no component along Z-axis. $\beta$ is the bistatic angle, and $\theta$ is the projection of $\beta$ on the XOY plane. The angle $\theta$ is introduced to describe the flight direction of receiver and the relative position of transmitter and receiver on XOY plane. While the receiver platform is moving towards, $\beta$ is always changing, but $\theta$ is constant.

According to the geometry model, the slant range of receiver and transmitter can be expressed as:

$$
\begin{align*}
TP & = R_T[\sin\psi_t\sin\theta \quad \sin\psi_t\cos\theta \quad -\cos\psi_t]^T \\
RP & = R_R[0 \quad \sin\psi_r \quad -\cos\psi_r]^T
\end{align*}
$$

(1)
where $R_t$ is the length of vector TP, $R_p$ is the length of vector RP.

The velocity vector of the receiver can be expressed as

$$V_t = V_t[0 \ 1 \ 0]^T$$

(2)

2.2 Ground resolution

The generalised ambiguity function is given by

$$\chi = \exp\left(\frac{j2\pi}{\lambda} \frac{[\Phi_{TP} + \Phi_{RP}]^T(P - Q)}{c}ight) \cdot \rho \left(\frac{[\Phi_{TP} + \Phi_{RP}]^T(P - Q)}{c}ight) \cdot m_A\left(\frac{[V_{\perp} / R_t]^T(P - Q)}{\lambda}\right)$$

(3)

where $\Phi_{TP}$ and $\Phi_{RP}$ are the unit vector of TP and RP, respectively. $Q$ is another point in a vicinity of $P$. $\rho(\cdot)$ is the IFT of the signal power spectrum, and $m_A(\cdot)$ is the IFT of the normalised received signal magnitude pattern and responsible for the Doppler resolution $V_{\perp}$ is the component of $V_t$ that is perpendicular to RP and can be expressed as

$$V_{\perp} = V_t - (\Phi_{RP} V_t) \Phi_{RP}$$

(4)

and the vector $P - Q$ can be written as

$$P - Q = \Delta x \cdot X + \Delta y \cdot Y$$

(5)

Thus, combining (1)–(5), we can obtain

$$\chi = \exp\left(\frac{j2\pi}{\lambda} k_1 \Delta x + k_3 \Delta y \right) \sin\left(\frac{k_1 \Delta x + k_3 \Delta y}{\rho_r}\right) \sin\left(\frac{k_3 \Delta y}{\rho_a}\right)$$

(6)

where the coefficient $k_1 - k_3$ can be written as

$$k_1 = \cos^2 \varphi_r$$

$$k_2 = \sin \varphi_r \sin \theta$$

$$k_3 = \sin \varphi_r \cos \theta + \sin \varphi_t$$

(7)

$\rho_r$ and $\rho_a$ are the −4 dB resolution on the slant range plane

$$\begin{cases}
\rho_r = \frac{c}{B} \\
\rho_a = \frac{\lambda R_t}{V_t T_s}
\end{cases}$$

(8)

where $B$ is the signal bandwidth and $T_s$ is the integration time. The results shown in (8) are two times that of monostatic SAR, because the range resolution here refers to the resolution of total slant range, which equals to the sum of $R_t$ and $R_p$, and the Doppler bandwidth is contributed by receiver only.

From (6), the ground resolution in range direction and azimuth direction can be acquired

$$\begin{cases}
\rho_{\theta r} = \frac{\rho_r}{\sqrt{k_1^2 + k_3^2}} = \frac{c}{\sqrt{\sin^2 \varphi_r + \sin^2 \varphi_t + 2 \sin \varphi_r \sin \varphi_t \cos \theta}} \\
\rho_{\theta a} = \frac{\rho_a}{k_3} = \frac{\lambda R_t}{V_t T_s \cos \varphi_r}
\end{cases}$$

(9)

The range resolution on ground is along the direction bisector of bistatic angle, the azimuth resolution on ground is along the direction of $Y$-axis. The angle between the range resolution and azimuth resolution on ground is given by

$$\eta = \tan^{-1}\left(\frac{k_1}{k_3}\right) = \tan^{-1}\left(\frac{\sin \varphi_r \sin \theta}{\sin \varphi_r \cos \theta + \sin \varphi_t}\right)$$

(10)

It is obvious that the range of $\eta$ depends on the relative size of $\varphi_r$ and $\varphi_t$. However, in the case of geostationary spaceborne-airborne bistatic forward-looking SAR, the incident angle of receiver is usually greater than that of transmitter. That means $\tan \eta$ is finite and $\eta$ cannot reach $\pi/2$, which is expected in the current resolution design.

Fig. 2 shows the relationship between $\eta$ and $\theta$, while $\varphi_r = 45^\circ$ and $\varphi_t = 60^\circ$, the maximum value of $\eta$ is obtained while $\theta = 145^\circ$.

From (9) and Fig. 3, we can know that the greater $\theta$ is, the worse the range resolution on ground is. So how to choose between $\eta$ and $\rho_{\theta r}$ is a difficult problem.
Table 1 Simulation parameter

| Parameters               | value | unit |
|-------------------------|-------|------|
| slant range of transmitter \((R_t)\) | 37,418 | km   |
| slant range of receiver \((R_r)\)            | 1     | km   |
| incident angle of transmitter \((\phi_t)\) | 45    | deg  |
| incident angle of receiver \((\phi_r)\)      | 60    | deg  |
| flight direction \((\theta)\)                 | 132.5 | deg  |
| integration time          | 1     | s    |
| velocity of receiver      | 100   | m/s  |
| Bandwidth                 | 100   | MHz  |
| carrier frequency         | S-band|      |

Fig. 4 Imaging simulation results at \(\theta = 132.5^\circ\)

3 Receiver’s moving direction design

3.1 Resolution ellipse

To describe the non-uniformity of ground resolution area rather than range resolution of an azimuth resolution, the resolution ellipse is introduced. According to (6), the resolution area can be expressed as

\[
\frac{(k_1 \Delta x + k_2 \Delta y)^2}{\rho_1^2} + \frac{(k_2 \Delta x + k_1 \Delta y)^2}{\rho_2^2} = 1
\]  
(11)

(11) is the equation of oblique elliptic equation, the semi-major axis and semi-minor axis of the ellipse are solved as

\[
\begin{align*}
A &= 2\sqrt{\left(\frac{k_1^2 + k_2^2}{\rho_1^2}\right)} - \sqrt{\left(\frac{k_1^2 + k_2^2}{\rho_1^2}\right) - 4 \frac{k_1^2 k_2^2}{\rho_1^2 \rho_2^2}}  \\
B &= 2\sqrt{\left(\frac{k_1^2 + k_2^2}{\rho_2^2}\right)} - \sqrt{\left(\frac{k_1^2 + k_2^2}{\rho_2^2}\right) - 4 \frac{k_1^2 k_2^2}{\rho_1^2 \rho_2^2}}
\end{align*}
\]  
(12)

The semi-major axis and semi-minor axis of the ellipse are the directions that the resolution area obtains the worst and best resolution. Thus, the ratio of \(B\) and \(A\) is used to describe the uniformity of resolution area, and the ratio is defined as

\[
K_d = \frac{B}{A} = \frac{1 - \sqrt{1 - q}}{1 + \sqrt{1 - q}}
\]  
(13)

where \(q\) is expressed as

\[
0 < q = \frac{k_1 k_2 / \rho_1 \rho_2}{(k_1^2 + k_2^2) / \rho_1^2 + (k_2^2 + k_1^2) / \rho_2^2} \leq 1
\]  
(14)

\(K_d\) describes the difference between different direction. The bigger \(K_d\) is, the smaller the difference is, and when \(K_d = 1\), that means the ground resolution is same in any direction. However, it is possible for \(K_d\) to get 1 only if \(k_1 = 0\), but this is not satisfied under the assumption that \(\phi_t > \phi_r\). When \(q\) is maximum, \(K_d\) gets the maximum value. Combining (7) and (8) (14) can be written as

\[
q = \frac{\rho_1 \rho_2 \sin \phi_t \cos \phi_r \sin \theta}{\rho_1^2 \sin \phi_t^2 + \sin \phi_r^2 + 2 \sin \phi_t \phi_r \sin \phi_r \cos \theta + \rho_2^2 \cos \phi_r^2}
\]  
(15)

and (15) maintain the maximum value when \(\theta\) satisfies

\[
\theta = \cos^{-1}\left(\frac{2 \rho_1^2 \sin \phi_t \sin \phi_r}{\rho_1^2 (\sin \phi_t^2 + \sin \phi_r^2) + \rho_2^2 \cos \phi_r^2}\right)
\]  
(16)

At this time, the difference between different direction of the resolution area is smallest, and the ratio of resolution of any two directions is not smaller than \(K_d\). Equation (16) is the direction that obtains the best uniformity.

3.2 Simulation results

In order to verify the correctness of (16), the computer simulations were performed, simulation parameters are shown in Table 1. According to (16), the maximum value of \(K_d\) is taken when \(\theta\) is equal to 132.5° and the maximum value of \(K_d\) is 0.4937. Fig. 4 shows the simulation results of BP imaging algorithm using the parameter in Table 1, and the spatial resolution in different directions are listed in Table 2. In Fig. 4, the angle between range resolution and azimuth resolution on ground is 53.46°, which is quite close to the 53.32° calculated by (10). The results in Table 2 show that the resolution calculated by the formulas above are close to the measurement of image resolution. The results verified the correctness of the above formulas.

Fig. 5 shows the value of \(K_d\) at different \(\theta\) calculated by (13) and the solution of ambiguity function obtained by the numerical method. The maximum value of \(K_d\) calculated by numerical method is 0.4929, when \(\theta\) is 133 degrees. The results calculated by (13) are quite close to the theoretical value calculated by numerical method in most cases. When \(\theta\) is close to zero, the resolution ellipse is no longer suitable for describing resolution distribution.

Table 2 Spatial resolution when \(\theta = 132.5^\circ\)

| Spatial resolution | Calculation results | Simulation results |
|--------------------|---------------------|--------------------|
| range resolution on ground \((\rho_1)\) | 4.09 m | 3.92 m |
| azimuth resolution on ground \((\rho_2)\) | 3.54 m | 3.6 m |
| angle between \(\rho_1\) and \(\rho_2\)  | 53.32° | 53.46° |
| semi-major axis \((A)\) | 6.83 m | 7.00 m |
| semi-minor axis \((B)\) | 3.37 m | 3.43 m |
| uniformity \((K_d)\) | 0.4937 | 0.4900 |

Fig. 5 Relationship between \(K_d\) and \(\theta\)
4 Conclusion

Due to the characteristics of non-orthogonal and non-uniform ground resolution, traditional range resolution and azimuth resolution are not suitable to describe the resolution distribution of the geostationary spaceborne-airborne bistatic forward-looking SAR, but the resolution ellipse can represent the non-uniformity of ground resolution in different direction. The optimal flight direction calculated based on optimal uniformity can always find, and the results are verified by computer simulation. This work has great value for bistatic forward-looking SAR system design and the receiver's path planning.

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