ADDITIVE LOCAL MULTIPLICATIONS
AND ZERO–PRESERVING MAPS ON \( C(X) \)

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Abstract. Suppose \( X \) is a compact Hausdorff space. In terms of topological properties of \( X \), we find topological conditions on \( X \) that are equivalent to each of the following: 1. Every additive local multiplication on \( C(X) \) is a multiplication, 2. Every additive local multiplication on \( C_K(X) \) is a multiplication, 3. Every additive map on \( C(X) \) that is zero-preserving (i.e., \( f(x) = 0 \) implies \( (Tf)(x) = 0 \)) has the form \( T(f) = T(1) \Re f + T(i) \Im f \).

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REFERENCES

[1] E. CHRISTENSEN, Derivations of nest algebras, Math. Ann. 229 (1977) 155–161.
[2] R. CRIST, Local derivations on operator algebras, J. Funct. Anal. 135 (1996) 76–92.
[3] L. GILLMAN, M. HENRIKSEN, Concerning Rings of Continuous Functions, Trans. Amer. Math. Soc. (1954) 340–362.
[4] L. GILLMAN, M. JERISON, Rings of continuous functions, Graduate Texts in Mathematics, No. 43, Springer-Verlag, New York-Heidelberg, 1976.
[5] HADWIN, Don Algebraically reflexive linear transformations, Linear and Multilinear Algebra 14 (1983) 225–233.
[6] THE HADWIN LUNCH BUNCH, Local multiplications on algebras spanned by idempotents, Linear and Multilinear Algebra 37 (1994) 259–263.
[7] D. HADWIN, J. W. KERR, Local multiplications on algebras, J. Pure Appl. Algebra 115 (1997) 231–239.
[8] D. HADWIN, J. LI, Local derivations and local automorphisms, J. Math. Anal. Appl. 290 (2004) 702–714.
[9] D. HADWIN, J. LI, Local derivations and local automorphisms on some algebras, J. Operator Theory 60 (2008) 29–44.
[10] D. HAN, S.-Y. WEI, Local derivations of nest algebras, Proc. Amer. Math. Soc. 123 (1995) 3095–3100.
[11] R. V. KADISON, Local derivations, J. Algebra 130 (1990) 494–509.
[12] A. K. MISRA, A topological view of P-spaces, General Topology and its Applications 2 (1972) 349–362.
[13] W. RUDIN, Homogeneity problems in the theory of Čech compactifications, Duke Math. J. 23 (1956) 409–419.
[14] P. ŠEMRL, Local automorphisms and derivations on \( B(H) \), Proc. Amer. Math. Soc. 125 (1997) 2677–2680.
[15] S. WILLARD, General topology, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont. 1970.
[16] E. WIMMERS, The Shelah P-point independence theorem, Israel J. Math. 43 (1982) 28–48.