FSM: FBS Set Management, An energy efficient multi-drone 3D trajectory approach in cellular networks

Mehdi Sookhak and Amir Hossein Mohajerzadeh

Abstract—Nowadays, Unmanned Aerial Vehicles (UAVs) have been significantly improved, and one of their most important applications is to provide temporary coverage for cellular users. Terrestrial Base Station cannot service all users due to disasters or events such as ground BS breakdowns, bad weather conditions, natural disasters, transmission errors, etc. The UAV can be sent to the target location and establishes the necessary communication links without requiring any predetermined infrastructure and covers that area. Finding the optimal location and the appropriate number (DBS) of drone-BS in this area is a challenge. In this paper, the optimal location and optimal number of DBSs are distributed in the current state of the users and the subsequent user states determined by the prediction. Finally, the DBS transition is optimized from the current state to the predicted future locations. The simulation results show that the proposed method can provide acceptable coverage on the network.

Index Terms—UAV, Trajectory, Optimization, Cellular Networks, DCO

I. INTRODUCTION

Due to characteristics such as high mobility, maneuverability, adaptive altitude, and low cost, Unmanned Aerial Vehicles (UAVs) have key applications in wireless networks. One of the main advantages of using UAVs as Base Stations (BSs) is that they do not need any predetermined infrastructure and can deploy anywhere. Also, they can change their positions on-demand, to increase the Quality of Service (QoS) and users coverage. However, the use of UAVs also has challenges such as determining the proper positions, trajectory design, the optimal number of drones, and so on [1][2].

One important application in the UAV mounted wireless network is the UAV trajectory. The UAV’s trajectory involves passing through several points obtained from the deployment problem. The literature of 2D and 3D positioning was discussed in [3][4]. Solving the optimal trajectory problem is challenging because of finding unlimited optimization variables (e.g. UAVs positions) [5]. In [6], a UAV path planning is proposed base on But Algorithm. The main goal of the paper is that UAVs can reach a safer and shorter path without crash through the start and endpoint in a war operation environment. Authors of [7] proposed a deep learning algorithm trained by Genetic Algorithm (GA). The GA collects states and paths from different scenarios and then uses them to train deep neural network so that when faced with familiar scenarios, could quickly provide an optimized path.

In [8] a multi-UAV trajectory planning is proposed. The proposed method for creating a trajectory optimization model based on a single time interval uses time segmentation instead of traditional station segmentation, which simplifies the calculation of cost functions. In [9] a secure cognitive UAV communication network has been studied using the trajectory and high flexibility of the UAV and the possibility of creating direct vision links. Authors of [10] investigated the problem of safe transmission in a cache enabled drone relay network with device-to-device communications, assuming the presence of a listener. In [11], authors are aimed to maximize the minimum power of users by joint optimizing the UAV trajectory and transmit power. The cache, trajectory and transmit power optimization variables are optimizing in three different blocks alternatively.

The goal of [12] is to maximizing the system power using joint 3D UAV trajectory, communication scheduling, and transmit power optimization. The authors first considered a specific case where the path between UAV-BS and UAV-AP is predetermined. Subsequently, for the general case, considering the optimization of the 3D UAV trajectory, an efficient iterative algorithm is proposed that alternately optimizes the divided sub-problems based on the technique of sequential convex approximation. In [13] the optimization problem is formulated as a non-convex problem considering the altitude of the UAV and the performance of the wireless coverage. To address this problem, the authors proposed an iterative algorithm with low complexity and break down the main problem into four sub-problems and optimize the variables in order. First, the convex minimization algorithm is used to find the optimal global 2D position of the UAV. After that, the optimal altitude of the UAVs is obtained. Then a multi-objective evolutionary algorithm based on the decomposition algorithm is proposed to control the phase of the antenna elements to achieve the desired performance. Finally, with the above-solved variables, the main problem is reformulated as a single-variable optimization problem, which charging time is then the optimization variable, and can be solved using standard convex optimization techniques.

In [14] the UAV is used in a wireless sensor network to collect data from multiple sensor nodes (SNs). The goal is maximizing collected data from SNs with the UAV 3D trajectory according to the scheduling. In the proposed system,
the effect of disconnection is also considered and the speed of data collection is modeled according to the communication channel. To solve the space problem, the 3D motion of the path is optimized repeatedly, once in horizontal mode and again in vertical mode. Authors of [15] have considered a UAV that sends a shared file to a set of ground terminals. Their goal is to optimize the path to minimize UAV mission time. The altitude assumed a constant value and equal to the lowest possible altitude for safe flight. The resulting path consists of a number of way points and is an extension of the travelling salesman problem, except that there is no need to return to the starting point. The problem is reconstructed in such a way that the paths are designed to meet the minimum connection time limit during which the horizontal distance between the drone and the terminal is less than a certain value. For the given path points, the optimal speed of the drone is obtained by solving a linear programming problem. In [16] a path for the UAV is designed to collect data from the most possible sensors. The goal is to maximize the data collected, using the travelling salesman problem and convex optimization.

Authors in [17] jointly optimized user scheduling and UAV trajectory to maximize average data rates among ground users. They envisioned a wireless communication system that the UAV serves to a number of ground users. The UAVs operate periodically, and each UAV must return to the starting point at the end of each T interval. The trajectory of the UAVs is also planned to take into account speed limits and collision avoidance. The goal of [18] is maximizing the average power through the joint design of transmission power and travel path for an activated network of UAVs. The authors proposed a new method of alternative optimization by combining power and path in an intermediate variable and then updating the power and the newly introduced variable. This new variable simplifies the analysis of the main problem by turning it into two convex sub-problems, namely one operational power maximization sub-problem and a feasibility sub-problem. As a result, both of these sub-problems can be solved globally.

As the best of our knowledge existing studies have considered single UAV trajectory and have solved the optimization problem using heuristics or evolutionary algorithms which are time consuming and can not reach the exact solution.

In this paper, we consider a cellular network demand in an urban area. We aim to cover users and serve their required data rate in a period of time using a 5G cellular network. The type of considered UAV in this scenario is The Scout B-330 [UAV] helicopter [11] which can fly up to 3 km height. In these scenarios, to find the most proper trajectory of UAVs, we first must find the best positions of UAVs in different snapshots. We consider orthogonal frequency reuse to avoid interference between UAVs in the network. We also consider the number of communication channels constraint in intra cellular network. To find the optimum position of UAVs in each snapshot, we use the mathematical model proposed in [3]. We consider Non-Line of Sight (NLoS) path loss in these scenarios and aim to cover all users in each snapshot. To find the optimum trajectory of UAVs, we propose a mathematical model based on transportation problem to minimize the total distance tracked by UAVs. In each step we solve the proposed mathematical model for transiting UAVs between two snapshots. We also consider that users can be placed in different altitudes on their positions follows the Poisson Point Process distribution and their mobility follows the random way point. The UAVs battery and flight limitations are also considered. To tackle the energy problem we introduce the [DCO] approach to avoid losing energy in idle hover mode.

The main contributions of this work are as follows:

- We propose a mathematical model to solve the 3D multi-UAV trajectory problem with an exact solution.
- The positioning and trajectory problem is extended for considering users altitudes (for example in urban and high-raised urban areas).
- Proposing DCO to avoid losing energy in idle modes.
- Considering channel interference in the mathematical model.
- Considering both hovering and trajectory UAV power consumption in the mathematical models.

The rest of paper is as follows in Section II the system model of this paper including considered situations and channel model is discussed. Section III includes the problem formulation and proposed positioning and trajectory mathematical models. At the end of this section the proposed algorithm for solving whole trajectory problem is placed. In Section IV, the test system, parameters and the numerical results are discussed. At last the conclusion is in Section V.

II. System Model

In this paper, we have considered a wireless cellular network in the urban environment. Users are present and gather in different parts of the area at different times, so it is necessary to dynamically change the location of the base stations to provide the best possible services to users. Since deploying a terrestrial base station in short-term scenarios is not economically viable, our goal is to cover and serve all cellular network users using Aerial Base Stations (ABSs). It is assumed that users in this system demand different data rates, which we have assumed it is random based on uniform distribution. Also, communication links between users and ABSs are assumed to be Line of Sight (LoS) and Non-Line of Sight (NLoS).

Our main goal in this paper is to solve the problem of the 3D trajectory of UAVs to cover and serve users with the least possible number of UAVs. To solve this problem, we have divided the problem into several snapshots. In each snapshot, we first solve the problem of finding the most suitable 3D positions and the minimum possible number of UAVs. Then, we solve the trajectory problem with the aim of minimizing the energy consumption of UAVs.

We solve the 3D positioning problem of UAVs in each snapshot based on the proposed model in [4] by extension of considering the NLoS link and assuming 100 percent user coverage. We also find the minimum number of required UAVs based on the bisection algorithm. The altitude of the UAVs can be assumed between the two values of \( H_{\text{min}} \) and \( H_{\text{max}} \) based on the characteristics of the UAV used. In addition, different altitudes have been assumed for users in this problem, which means the presence of users in high-rise buildings. In
order to prevent inter-cell interference, we have considered the limitation of the number of internal channels in the cell.

In the trajectory problem solving phase, we propose a linear mathematical model based on the transportation problem between two snapshots, which obtains the optimal path of each drone from origin to destination between two snapshots with the aim of minimizing the energy consumption of the UAVs. We also assume that users move based on a random way-point between each two snapshots.

### A. Air to Ground channel model

The deployment and service of ABSs are directly related to the Air-to-Ground (A2G) communications links. In the literature, various models for A2G channel have been introduced. In this paper, we use the presented A2G model in [19]. The channel model generally consists of two parts, LoS and NLoS. The possibility of having a LoS link between the ABS and the ground user depends on several factors, including the density of the buildings, the position of the UAV and the elevation angle between the UAV and the ground user. The equation for having the probability of LoS link in A2G channel is as follows:

\[ P_{\text{LoS}} = \frac{1}{1 + a \exp(-b(\theta_u - \alpha))} \] (1)

where \( a \) and \( b \) are environmental constants and \( \theta_u \) is the elevation angle between user \( u \) and the UAV which depends on their altitudes. It is calculated as

\[ \theta_u = \frac{\text{atan}^{-1}(h_{\text{UAV}} - h_u)}{d_u} \]

where \( h_{\text{UAV}} \) and \( h_u \) are the altitudes of UAV and user \( u \), respectively, and \( d_u \) is the distance between the UAV and user \( u \). It is calculated as

\[ d_u = \sqrt{(x_u - x_{\text{UAV}})^2 + (y_u - y_{\text{UAV}})^2 + (h_u - h_{\text{UAV}})^2} \]

It can be seen from (1) that the probability of LoS is increased with enlarging the elevation angle between the UAV and the user. The probability of having NLoS communication link is also calculated as follows:

\[ P_{\text{NLoS}} = 1 - P_{\text{LoS}} \] (2)

Hence, the mean path loss (in dB) for LoS and NLoS communication links are calculated as follows [20]:

\[ L_{\text{LoS}} = 20 \log\left(\frac{\pi f_c d_u}{C}\right) + \delta_{\text{LoS}} \] (3)

\[ L_{\text{NLoS}} = 20 \log\left(\frac{\pi f_c d_u}{C}\right) + \delta_{\text{NLoS}} \] (4)

where \( \delta_{\text{LoS}} \) and \( \delta_{\text{NLoS}} \) are mean losses in LoS and NLoS communication links, respectively. Also, \( C = 3 \times 10^8 \) is the light speed, and \( f_c \) is the carrier frequency. Therefore, the probabilistic long-term mean path loss is obtained as:

\[ L(d_u) = L_{\text{LoS}} \times P_{\text{LoS}} + L_{\text{NLoS}} \times P_{\text{NLoS}} \] (5)

### III. Problem Formulation

Here we aim to solve the UAV 3D trajectory problem in a wireless cellular network. Actually, to solve the trajectory problem, we must find an unlimited continuous points for UAVs positions which is an NP-hard problem. To tackle this, we divide the problem into several snapshots and find the optimal path of UAVs between each two snapshots considering UAVs energy consumption. We divide the problem into two phases. First we find the minimum number of required UAVs and their positions to cover and serve users. Then, we find the optimal path of each UAV from the origin to the destination.

#### A. Positioning phase

In the first phase, the objective function of the positioning problem is both minimizing the number of required UAVs and finding the most proper positions of UAVs to cover users. In this phase, we reformulate the [4] proposed method for UAVs positioning. As mentioned before, we consider NLoS links in this problem, and we aim to cover all users. We also consider ABS capacity to avoid inter-cell interference between covered users.

In this formulation, the considered decision variables are presented in Table I. \( x_{ij} \) is the variable to decide the user \( j \) is served by ABS in candidate point \( i \) position. \( m_i \) shows that if candidate point \( i \) is selected or not. \( h_i \) is the variable of the UAV’s altitude. The path loss between user \( j \) and the ABS in candidate point \( i \) position is decided by \( k_{ij} \) variable. The \( t_{ij} \) is an auxiliary decision variable. Also, parameters used in this formulation are represented in Table II.

| Decision variable | Description |
|------------------|-------------|
| \( x_{ij} \)    | \( 1 \), if user \( j \) is served by candidate point \( i \), and \( 0 \), otherwise. |
| \( m_i \)       | \( 1 \), if candidate point \( i \) is selected for \( \text{UAV} \) deploying, and \( 0 \), otherwise. |
| \( h_i \)       | The altitude of \( \text{UAV} \) is deployed at the candidate point \( i \). |
| \( k_{ij} \)    | The path loss between user \( j \) and candidate point \( i \), if user \( j \) is served by candidate point \( i \), and \( 0 \), otherwise. |
| \( t_{ij} \)    | Auxiliary decision variable. |

In the proposed model, the objective function (10a) is defined to minimize the sum of path loss. Constraint (10b) represent that each user must serve by only one UAV. Constraint (10c) states that each UAV can serve a limited number of users based on its number of channels. Constraint (10d) shows that user \( j \) can only serve by the UAV deployed on candidate point \( i \). The (10e) constraint states that all users must covered by \( \text{ABS} \). Constraint (10f) allows each \( \text{ABS} \) to serve the maximum data rate it can based on its backhaul. Constraint (10g) states that the model must select only \( P \) points from the candidate points given. The (10h) and (10i) constraints state that if candidate point \( i \) is selected as the position of an \( \text{ABS} \) the UAV must fly within the permissible range. The UAV altitude will be set to zero, if and only if the candidate point \( i \) is not selected by the model. Constraint (10j) prevents the assignment of users who are not in the \( \text{ABS} \)’s coverage range. Constraints (10k) and (10l) are the first order Taylor expansion of equation 5 which is proven in Lemma II. In constraints (10m)-(10o) the decision variable \( t_{ij} = x_{ij}h_i \) is used to reduce the nonlinear part to the multiplication of \( x_{ij} \) and \( h_i \), \( t_{ij} \) must be zero if \( x_{ij} \) or \( h_i \) are equal to zero. Constraints (10m) and (10n) state this requirement. Also, \( t_{ij} \)
must be equal to \(h_i\) when \(x_{ij}\) becomes 1. Constraint (10m) and (10e) satisfy this.

**Lemma 1.** Consider \(L(d_{ij}) = L_{\text{LoS}} + P_{\text{LoS}} + L_{\text{NLoS}} + P_{\text{NLoS}}\) is the path-loss function. As if \(L(d_{ij}) \geq \mathcal{P}_{\text{max}}\), then \(x_{ij}\) must be equal to 0, the statement can be rewritten as follows:

\[
x_{ij} = \begin{cases} 
0, & \text{if } L(d_{ij}) \geq \mathcal{P}_{\text{max}}, \\
0 \text{ or } 1, & \text{otherwise.}
\end{cases} \tag{6}
\]

**Proof.** We obtain a linear conditional statement in terms of \(h_i\) by replacing \(L(d_{ij})\) in (6) with a linear approximation achieved from Taylor expansion around some \(h_0\).

\[
L(d_{ij}) = L(h_i - h_0 + h_0) \approx L(h_0) + L'(h_0)(h_i - h_0) = [P_{\text{LoS}} \times (4\pi\frac{f_c}{C})^2d_{ij}^2 + h_0^2]x_{ij}
\]

\[
+ (4\pi\frac{f_c}{C})^2 \times 2 \times h_0 \times (h_i - h_0) + x_{ij} \times 10^{\delta_{\text{LoS}}}
\]

\[
+ [P_{\text{NLoS}} \times (4\pi\frac{f_c}{C})^2d_{ij}^2 + h_0^2]x_{ij}
\]

\[
+ (4\pi\frac{f_c}{C})^2 \times 2 \times h_0 \times (h_i - h_0) + x_{ij} \times 10^{\delta_{\text{NLoS}}}. \tag{7}
\]

Now we have

\[
x_{ij} = \begin{cases} 
0, & \text{if } [P_{\text{LoS}} \times (4\pi\frac{f_c}{C})^2d_{ij}^2 + h_0^2]x_{ij}
\]

\[
+ (4\pi\frac{f_c}{C})^2 \times 2 \times h_0 \times (h_i - h_0) + x_{ij} \times 10^{\delta_{\text{LoS}}}
\]

\[
+ [P_{\text{NLoS}} \times (4\pi\frac{f_c}{C})^2d_{ij}^2 + h_0^2]x_{ij}
\]

\[
+ (4\pi\frac{f_c}{C})^2 \times 2 \times h_0 \times (h_i - h_0) + x_{ij} \times 10^{\delta_{\text{NLoS}}},
\end{cases}
\]

\[
\geq \mathcal{P}_{\text{max}}. \tag{8}
\]

Assume that \(A = (4\pi\frac{f_c}{C})^2\). By simplifying the conditional expression we have

\[
x_{ij} = \begin{cases} 
0, & \text{if } x_{ij} \geq \frac{\mathcal{P}_{\text{max}} - (A \times (d_{ij}^2 + h_0^2)) \times (P_{\text{LoS}} + P_{\text{NLoS}})}{2Ah_0},
\end{cases}
\]

\[
0 \text{ or } 1, \text{ otherwise.} \tag{9}
\]

By defining \(a_{ij} = \frac{\mathcal{P}_{\text{max}} - (A \times (d_{ij}^2 + h_0^2)) \times (P_{\text{LoS}} + P_{\text{NLoS}})}{2Ah_0}\), the conditional expression will be simplified as follows

\[
x_{ij} = \begin{cases} 
0, & \text{if } x_{ij} \geq a_{ij},
\end{cases}
\]

\[
0 \text{ or } 1, \text{ otherwise.} \tag{10}
\]

To form (8) as a valid constraint in mathematical programming, we represent the following expression:

\[
x_{ij} \leq \frac{M - h_i}{M - a_{ij} + \frac{1}{2}}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, \tag{9}
\]

where \(M\) is a big number.

**B. Trajectory phase**

In the trajectory phase, we aim to find the best path for each \(\text{UAV}\) between two snapshots. In the problem formulation, as mentioned in Table III, \(d_{ij}\) is the decision variable which

| Parameters        | Description                     |
|-------------------|---------------------------------|
| \(f_c\)           | Carrier frequency               |
| \(C\)             | Speed of light                  |
| \(\mathcal{I}\)   | Set of candidate points         |
| \(\mathcal{J}\)   | Set of users                    |
| \(\beta\)         | \(\text{UAV}\) backhaul data rate |
| \(\theta\)        | \(\text{UAV}\) elevation angle  |
| \(P\)             | Number of \(\text{UAVs}\) to be deployed |
| \(U\)             | Number of users                 |
| \(H_{\text{min}}\)| Minimum allowed altitude        |
| \(H_{\text{max}}\)| Maximum allowed altitude        |
| \(\mathcal{P}_{\text{max}}\)| Maximum allowed path loss in the network |
| \(C_{\text{UAV}}\)| Data rate required by user \(j\) |
| \(D_j\)           | Number of inter-cell channels   |
| \(d_{ij}\)        | Distance between user \(j\) and candidate point \(i\) |
must be equal to 1 if $UAV_i$ moves to position $j$, otherwise it must be equal to 0. The objective function is minimizing total energy consumed by $UAVs$ due to their flight (11a). Constraint (11b) states that the model decides the path of each $UAV$ placed in the origin snapshot. The constraint (11c) is to assure that each position in the next snapshot is chosen for one $UAV$. $\mathcal{M}$ and $\mathcal{N}$ are sets of $UAVs$ positions in origin and destination snapshots, respectively. The size of each set is twice of the maximum required $UAVs$ in all snapshots to ensure that the mathematical model has feasible solution in every iteration. (11d) is the Drone Cell Off (DCO) constraint. When the number of required $UAVs$ in one snapshot is less than the other one, extra $UAVs$ are move to/from the base to save their energy and also recharge. The DCO concept is like the Cell Switch-Off (CSO). Constraint (11e) ensures that each $UAV$ has enough power to fly to the next position. The $Energy_{th}$ parameter is the minimum energy required for each $UAV$ to fly to the base. If a $UAV$ has not enough power to continue serving users, the $d_{ij}$ variable will choose the path to the base. The $UAV$ will land in a base and another $UAV$ will be used to fly from the base to the destination point. We call this process DCO.

$$\text{min} \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} d_{ij} \times E_{ij} \quad (11a)$$

s.t

$$\sum_{j \in \mathcal{N}} d_{ij} = 1, \quad \forall i \in \mathcal{M}, \quad (11b)$$

$$\sum_{i \in \mathcal{M}} d_{ij} = 1, \quad \forall j \in \mathcal{N}, \quad (11c)$$

$$\sum_{i \in \mathcal{M}, j \in \mathcal{N}} d_{ij} = \max(\mathcal{M}, \mathcal{N}), \quad (11d)$$

$$d_{ij} \times E_{ij} \leq Energy_{th}, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}, \quad (11e)$$

$$d_{ij} \times \text{dist}_{ij} \leq \text{Distance}_{th}, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}. \quad (11f)$$

The constraint (11f) states that each $UAV$ can not fly further than its threshold distance based on its constant velocity. The $\text{Distance}_{th}$ parameter derives as follows:

$$\text{Distance}_{th} = \text{Energy}_{th} \times t_s \quad (12)$$

where $t_s$ is the time interval between two snapshots. Trajectory parameters are described in Table IV.

Overall, to solve the whole problem following steps are presented. First, we divide the problem into several snapshots. In each snapshot, users may have different positions and different data rate demand. In each snapshot we solve the 3D positioning problem to find the minimum number of required $UAVs$ and their proper positions. Then, we solve the trajectory problem between two snapshots in a row using proposed mathematical model and updating $UAVs$ energy parameter considering trajectory and hovering power consuations. The proposed trajectory model decides the best path for each $UAV$ to minimize the total energy consumption. It also decides that if a $UAV$ must fly to the base (DCO concept) because of less $UAV$ requirement or battery recharging. Both proposed mathematical models are in linear form. The proposed positioning model is Mixed-Integer Linear Problem (MILP) and the proposed trajectory model is Binary Linear Problem (BLP). Therefore, the proposed method reaches the exact solution. The process continues until there is no remaining snapshot. The whole process is shown in Algorithm 1.

### IV. NUMERICAL RESULTS

We first introduce the test system and simulation parameters in this Section, then we discuss and compare the results of scenarios for 3D $UAV$ trajectory problem.

#### A. Test system

In the simulations we considered a centralized decision making for $UAVs$ positioning and trajectory. Referring to [20] and [21], as mentioned in Table V we consider a 5000 × 5000 urban area with scenarios including 80, 200, and 450 users with Poisson Point Process (PPP) distribution. The PPP parameter $\lambda = 20000$, and the environment parameters are as follows: $f_c = 2$ GHz, $PL_{\text{max}} = 110$ dB, and $(a, b, \delta_{\text{LoS}}, \delta_{\text{NLoS}}) = (9.61, 0.43, 0.1, 20)$ corresponding to urban environments. Also, We consider the data rate requested by users with a uniform distribution with a maximum value of 5 Mbps The $UAVs$ backhaul data rate is considered equal to 100 Mbps for each. $UAVs$ flying altitude range is between 110 and 2500 meters, and users can be placed in up to 100 meter height. We also consider 45 degrees for $UAVs$ elevation angle. For the positioning model, we use Merge cell method proposed in [4] to find candidate points. We used Cplex solver

### TABLE IV: Using parameters in trajectory phase.

| Parameters | Description |
|------------|-------------|
| $E_{ij}$   | The energy that the $UAV$ consumes to fly between position $i$ and position $j$ |
| $\mathcal{M}$ | Set of origin $UAVs$ positions |
| $\mathcal{N}$ | Set of destination $UAVs$ positions |
| $Energy_{th}$ | Minimum required energy for a $UAV$ to fly to the base |
| $\text{dist}_{ij}$ | The distance between position $i$ and position $j$ |
| $\text{Distance}_{th}$ | Maximum allowed distance that the $UAV$ can fly between two snapshots |
to solve the positioning and trajectory mathematical models. We ran the simulation and solve the models many times and the presented results are consequences of them.

B. Results

In the following we discuss about the results of proposed positioning and trajectory model and compare the results of three different number of users.

The number of required UAVs for each scenario is illustrated in Figure 1. As it shows, the number of required UAVs increases with the number of users. For 80 users 3 or less UAVs are needed to cover all users in each snapshot. For 200 and 450 users, on average about 6 and 13 UAVs are needed, respectively.

Each snapshot is assumed 15 minutes away from the next one. Then between every two positioning snapshots, a trajectory problem solves. Figure 2 illustrates the UAVs trajectories in two different situations. In Figure 2a it is shown that two UAVs fly from red points to blue points. One of them increases its altitude but the other one decreases its altitude. This shows that users moved to different positions, and a group of them became very dense. Figure 2b illustrates that in this situation we need one more UAV. Moreover, the power of the last two UAVs is not enough to do the next operation. Therefore, two UAVs which were operated in the last phase, landed in the base, and three full power UAVs flew to the blue points to operate. Also, the two UAVs which landed in the base will recharge for further missions.

Figure 3 illustrates the path of all UAVs in the whole scenario with 200 users in 15 snapshots. It shows that UAVs change their positions and altitudes and land to the base many times during the whole operation. They move to different positions and altitudes due to users’ movement and density and if their power was not enough, or the UAV was not needed, they land in the base.

Table V: Test parameters for evaluating the problem model.

| Parameters | Description |
|------------|-------------|
| Region     | 5000 × 5000 m |
| U          | 80, 200, 450 |
| β          | 100 Mbps |
| D_{avg}    | 2.5 Mbps |
| H_{min}    | 110 m |
| H_{max}    | 2500 m |
| f_{c}      | 2 GHz |
| P_{L_{max}}| 110 dB |
| a          | 9.61 |
| b          | 0.43 |
| δ_{LoS}    | 0.1 |
| δ_{NLoS}   | 20 |

Figure 4 shows the comparison of sum of UAVs flight distance during their operation. As the number of users increases,
the total flight distance of UAVs is increased. It is because of needing more UAVs and more flights to cover users.

In Figure 5, the average of UAVs flight energy consumption is compared in different scenarios. The energy includes hovering and trajectory power consumed. Although we might expect that with increase of number of UAVs, the average power consumption of each UAV must decrease, because of shorter paths and therefore less number of recharging, the average flight energy consumption in 200 users scenario is more than 80 users scenario.

Figure 6 shows the comparison of average solving time of trajectory problem. This figure represents that beside the proposed trajectory mathematical model have exact solution, the solving time of it is very low comparing the time interval between two snapshots. It means we have enough time to solve the problem in real world. By the way, it is expected to have more solving time with more number of UAVs required but some times to have the optimum solution of the problem and minimize the UAVs energy consumption considering the problem constraints, the solving time may increase. As the figure shows the average solving time of 80 users scenario is more than other scenarios but the standard deviation of it shows that in some cases the solving time of this scenario is less than the minimum solving time of other scenarios.

V. CONCLUSION

In this paper we proposed an approach with exact solution for multi UAV 3D trajectory problem considering UAVs energy consumption, operation time, flight distance limitation and inter-cell interference constraints. The approach contains two phases, UAVs positioning phase and UAVs trajectory phase. First we find the minimum number of required UAVs and their proper 3D positions in each snapshot. Then, the trajectory phase runs between each two snapshots. In this phase the optimum path between origin and destination is found using proposed binary linear problem (BPL) model considering UAVs energy consumption and flight distance limitation. As the proposed method is BPL, the solution is the optimum one. The results show that the proposed method is operational in real problems and results are as expected.

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