Phonon-assisted luminescence of magnetoexcitons in semiconductor quantum wells

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We consider a line-shape of magnetoexciton photoluminescence from quantum wells when the disorder is sufficiently small. In this case the phonon-assisted optical transitions become important for the line formation. We study both inter-band and intra-band excitons. For inter-band excitons the width of a single peak emission line is calculated as a function of temperature and quantum well width. For intra-band excitons the double peak of the emission line is predicted when the electron filling factor is odd and greater or equal to three. In the latter case the lowest magnetoexciton dispersion curve has a minimum at non-zero momentum. Then the higher-energy peak results from the direct optical emission of zero-momentum excitons. The origin of the lower-energy peak is the phonon-assisted transitions from the non-zero momentum exciton states. With increasing temperature, the higher-energy peak becomes more pronounced and the lower-energy peak vanishes.

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A two-dimensional (2D) exciton in a strong magnetic field is optically active only when the electron and the hole of the exciton belong either to the same Landau level of the different subbands or to the adjacent Landau levels of the same subband. In both cases the optical exciton recombination is allowed only at zero 2D momentum of the exciton. Then the exciton emission consists of a single infinitely narrow line. To acquire the finite width of the line the exciton should transfer its momentum to a third body, which can be either a phonon or an impurity. In this paper we study the phonon-assistant exciton recombination, which can be important in pure enough samples. The material parameters used in this paper correspond to a GaAs quantum well.

To find the line-shape of the exciton photoluminescence we restrict ourselves to the second order phonon-photon processes. The electron-phonon Hamiltonian can be rewritten as a sum of the electron-phonon and hole-phonon contributions:

\[ H_{e,h-\text{pn}} = - \sum_{j,Q} \frac{M_{e,j}(\vec{Q})}{\sqrt{V}} Z_e(q_z) \left[ \hat{\rho}_e^+(\vec{q}) \hat{d}_j^+(\vec{Q}) + \hat{\rho}_e(\vec{q}) \hat{d}_j(\vec{Q}) \right] - \sum_{j,Q} \frac{M_{h,j}(\vec{Q})}{\sqrt{V}} Z_h(q_z) \left[ \hat{\rho}_h^+(\vec{q}) \hat{d}_j^+(\vec{Q}) + \hat{\rho}_h(\vec{q}) \hat{d}_j(\vec{Q}) \right], \]

where the capital letter (\(\vec{Q}\)) denotes the three dimensional (3D) vector, its projections are denoted by the corresponding small letters, \(\vec{Q} = (\vec{q}, q_z)\); \(j\) is labeling the phonon modes, \(j = 1\) for the longitudinal mode and \(j = 2, 3\) for two transverse modes, \(\hat{d}_j, \hat{d}_j^+\) are the creation and annihilation operators of a phonon in the \(j\)th mode, \(V\) is a normalization volume; \(M_{e,j}(\vec{Q})\) and \(M_{h,j}(\vec{Q})\) are the matrix elements of electron-phonon and hole-phonon interactions, which are determined by the deformation potential and piezoelectric coupling. In GaAs they have the form [1]:

\[
\begin{pmatrix}
M_{e,j}(\vec{Q}) \\
M_{h,j}(\vec{Q})
\end{pmatrix} = \sqrt{\frac{\hbar}{2\rho_0 sQ}} \left[ \begin{pmatrix} -eh_{14} Q_{x} Q_{y} \xi_{j,z} + Q_{y} Q_{z} \xi_{j,x} + Q_{z} Q_{x} \xi_{j,y} \over Q^2 + \frac{1}{\kappa e_{0}} - i \left\{ \frac{\Xi_e}{\Xi_h} \right\} (\vec{\xi}_j, \vec{Q}) \right],
\]

where \(\rho_0\) is the mass density of GaAs, \(h_{14}\), \(\Xi_e\), and \(\Xi_h\) are the parameters of piezoelectric and deformation potential couplings, \(\xi_j\) is the polarization vector of the \(j\)th phonon mode. In the above expressions we have used the isotropic Debye approximation with a linear dependence of the phonon frequency on the wave vector: \(\omega_j(Q) = s_j Q\), where \(s_j\) is the speed of sound of the \(j\)th mode.

The form factors \(Z_e(q_z)\) and \(Z_h(q_z)\) in equation (1) are determined by the electron and hole spreading in \(z\) direction and are given by the expression:

\[
\begin{pmatrix}
Z_e(q_z) \\
Z_h(q_z)
\end{pmatrix} = \int dq e^{i q_z z} \left\{ \left| \chi_e(z) \right|^2 \left| \chi_h(z) \right|^2 \right\},
\]

where \(e^{i q_z z}\) is the normalizing factor.

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where \( \chi_e(z) \) and \( \chi_h(z) \) are the wave functions associated with the electron and hole subbands, respectively. Below we use the infinite well potential with the same width \( a \) for both the electron and the hole.

In the second order photon-phonon transitions there are two processes which contribute into the exciton emission distribution. In the first process the exciton with the momentum \( \vec{k} \) makes the transition into a zero-momentum state with emission of a phonon with momentum \( \vec{Q} \). Then the zero-momentum exciton emits the photon with the energy: \( h\omega = E(k) - h\omega(Q) + \text{const} \), where \( E(k) \) is the binding energy of an exciton with the momentum \( \vec{k} \); the \( \text{const} \) includes the interband gap energy, for convenience we put this constant to zero. In the second process the exciton with the momentum \( \vec{k} \) absorbs the phonon with the momentum \( \vec{Q} \) and then emits the photon with the energy \( h\omega = E(k) + h\omega(Q) \). Taking into account both these processes we find the probability of emitting a photon with frequency \( \omega \)

\[
I(\omega) = \frac{2\pi}{\hbar} \sum_{\vec{q}} \sum_{\vec{q}'} f(q) \sum_{\vec{Q}} \left[ \frac{<\vec{q}',N(-\vec{Q}) + 1|H_{e,h-pn}|N(-\vec{Q}),0>^2}{h^2(\Delta \omega)^2} \delta(h\Delta \omega - \Delta E(q) - h\omega(Q)) + \frac{<\vec{q},N(\vec{Q})|H_{e,h-pn}|N(\vec{Q}),1,0>^2}{h^2(\Delta \omega)^2} \delta(h\Delta \omega - \Delta E(q) + h\omega(Q)) \right] |H_{pt}|^2 ,
\]

where \( <H_{pt}> \) is the matrix element of the exciton recombination, which is proportional to \( \delta_{n_e,n_h} \) and \( \delta_{n_e,n_h-1} \sqrt{n_h} \) for the inter-band and intra-band excitons, respectively, \( n_e \) and \( n_h \) denote the electron and the hole Landau level numbers; \( \Delta E(q) = E(q) - E(0) \), \( \Delta \omega = \omega - \omega_0 \), \( \omega_0 \) is the photon frequency of a direct optical emission; \( N(q) \) and \( f(q) \) are the phonon and exciton distribution functions, respectively. In expression (4) we took into account the conservation of momentum for the transitions into the intermediate state.

Below we consider two types of excitons: the inter-band exciton with the electron and the hole being in the conduction and valence bands, respectively, and the intra-band intra-subband exciton with the electron and the hole being in the same size-quantization subband but in the different Landau levels. The hole in the later case is the absence of the electron in the completely occupied Landau level.

For the inter-band exciton the optical transitions are allowed only if the electron and the hole have the same Landau level number. We study the case when they are both in the lowest Landau level. The exciton dispersion is given by the expression (3):

\[
E(q) = \frac{\hbar^2}{2m} \int_0^\infty dkF(k)e^{-k^2l^2/2}J_0(kql^2) ,
\]

where \( l \) is the magnetic length, \( J_m \) is the Bessel function of the order \( m \). The factor \( F(k) \) is due to the finite width of the electron and hole wave functions in \( z \)-direction and has the form (3):

\[
F(k) = \int dq e^{-k|z_1-z_2|} |\chi_e(z_1)|^2 |\chi_h(z_2)|^2 .
\]

The matrix element of the electron (hole) density operator is given by the expression: \( \rho(\vec{q}) = \exp(-q^2l^2/4) \). We introduce the temperature \( T \) of the system and assume that the exciton distribution function \( f(q) \) is proportional to \( \exp(-E(q)/k_BT) \). Substituting equations (1)-(3),(5) into equation (4) we find the emission intensity, \( I(\omega) \). The width of the emission line is given by the expression: \( \delta \omega = \sqrt{<\omega^2> - <\omega>^2} \), where \( <\omega> \) and \( <\omega^2> \) are the first and the second normalized moments of the spectra, respectively. In Fig.1(a) the typical emission line is shown for the width of the quantum well \( a \) equal to the magnetic length \( l \) and for the temperature \( T = 2.5K \). The line has a single peak. In Fig.1(b) the width of the peak is shown as a function of temperature for different well widths. One can see that the width of the line is saturated with increasing temperature. This results from the suppression of the phonon absorption (emission) for the values of \( q \) greater than the inverse magnetic length \( 1/l \). This suppression is due to the factor \( \rho^2(\vec{q}) = \exp(-q^2l^2/2) \) in the electron-phonon matrix elements. The magnetoeexciton mass increases with increasing the well’s width \( a \). Therefore, the width of the emission line decreases with increasing \( a \).

For the intra-band exciton the shape of the emission line can be more complex. We consider the case when the electron filling factor is equal to three. The occupied levels are the first Landau level (\( n = 0 \)) with both spin directions, \( S_z = +1/2 \) and \( S_z = -1/2 \), and the second Landau level (\( n = 1 \)) with spin \( S_z = +1/2 \). In this case the optically active excitons are formed by the two types of excitations: 1) the hole is in the second Landau level (\( n = 1, S_z = +1/2 \)) and the electron is in the third Landau level (\( n = 2, S_z = +1/2 \)) and 2) the hole is in the first Landau level (\( n = 0, S_z = -1/2 \)) and the electron is in the second Landau level (\( n = 1, S_z = +1/2 \)).
$S_z = -1/2$ and the electron is in the second Landau level \((n = 1, S_z = -1/2)\). These two types of excitations interact with each other. As a result of this interaction (level repulsion) the exciton dispersion curve has a minimum at non-zero momentum. The exciton dispersion for the lower branch is given by the expression 4:

$$E(q) = \frac{E_1(q) + E_2(q)}{2} - \sqrt{\left(\frac{E_1(q) - E_2(q)}{2}\right)^2 + \frac{4|V(q)|^2}{2}} ,$$

where the energies \(E_1(q)\), \(E_2(q)\), and \(V(q)\), written down in units of \(e^2/\kappa l\), are

$$E_1(q) = \frac{q}{2} F(q) e^{-q^2/2} - \int_0^\infty dk F(k) \left(1 - \frac{k^2}{2}\right) e^{-k^2/2} J_0(kq) ,$$

$$E_2(q) = q \left(1 - \frac{q^2}{4}\right)^2 F(q) e^{-q^2/2} - \int_0^\infty dk F(k) \left(1 - \frac{k^2}{2}\right) \left(1 - k^2 + \frac{k^4}{8}\right) e^{-k^2/2} J_0(kq) ,$$

and

$$V(q) = \frac{q}{\sqrt{2}} \left(1 - \frac{q^2}{4}\right)^2 F(q) e^{-q^2/2} ,$$

where the momentum is in units of \(1/l\).

The exciton dispersion is shown in Fig.2(a) for the two values of the well width. The dispersion has a minimum at a non-zero value of momentum. With increasing the well width the depth of the minimum decreases, which results from the decreasing of the effective electron-hole interaction. The exciton emission line is shown in Fig.2(b). At low temperature excitons occupy the non-zero momentum minimum of the dispersion curve. Therefore there are only phonon-assisted optical transitions from this state. The optical line is red-shifted from the direct line, which is at \(\Delta \omega = 0\). With increasing temperature the population of the zero-momentum exciton state increases and an additional line, corresponding to the optical transition from the zero-momentum exciton, appears. The position of this line is at frequency \(\omega_0\), which (according to the Kohn theorem) is equal to a cyclotron frequency. At this intermediate temperature the exciton emission line has a double-peak structure. At higher temperature there is again a single line which now is at \(\Delta \omega = 0\). This line corresponds to the transition from the zero-momentum exciton state without phonon emission or absorption. With increasing the well width the separation between the lines becomes smaller and the temperature, at which the lower energy peak can be observed, is decreasing.

The additional peaks in the exciton absorption line were discussed in the literature for the case of impurity scattering being the dominant mechanism of the momentum transfer.

In conclusion, we have studied the phonon-assisted exciton emission. For the inter-band exciton the width of the emission line was calculated as a function of well width and temperature. The line-width is smaller for wider wells and is saturated with increasing temperature. For the intra-band exciton the double-peak structure of the emission line is predicted if the electron filling factor is equal to \(2m + 1\), where \(m\) is an integer number. With increasing temperature the emission line acquires a blue shift.

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FIG. 1. (a) Inter-band exciton emission line for $a = l$ and $T = 2.5$K. The intensity is in arbitrary units, $\Delta \omega$ is in units of $e^2/\kappa l$. (b) The width of the exciton emission line as a function of temperature. The numbers near the lines show the quantum well width in units of $l$. The temperature is in Kelvin. The line width is in units of $e^2/\kappa l$.

FIG. 2. (a) Intra-band exciton dispersion as a function of the momentum $k$. The energy $E(k)$ and the momentum $k$ are in units of $e^2/\kappa l$ and $1/l$, respectively. The numbers near the lines show the width of the quantum well in units of magnetic length $l$. (b) Intra-band exciton emission line at $a = 0.25l$. The numbers near the lines show the temperature in Kelvin. The intensity is in arbitrary units and $\Delta \omega$ is in units of $e^2/\kappa l$. 

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