Formal Semantic Geometry over Transformer-based Variational AutoEncoder

Yingji Zhang\textsuperscript{1}, Danilo S. Carvalho\textsuperscript{1,3}, Ian Pratt-Hartmann\textsuperscript{1}, André Freitas\textsuperscript{1,2,3}

\textsuperscript{1} Department of Computer Science, University of Manchester, United Kingdom
\textsuperscript{2} Idiap Research Institute, Switzerland
\textsuperscript{3} National Biomarker Centre, CRUK-MI, Univ. of Manchester, United Kingdom

\{firstname.lastname\}@\textsuperscript{[postgrad.]}\textsuperscript{1}\textsuperscript{manchester.ac.uk}

Abstract

Formal/symbolic semantics can provide canonical, rigid controllability and interpretability to sentence representations due to their localisation or composition property. How can we deliver such property to the current distributional sentence representations to control and interpret the generation of language models (LMs)? In this work, we theoretically frame the sentence semantics as the composition of semantic role - word content features and propose the formal semantic geometry. To inject such geometry into Transformer-based LMs (i.e. GPT2), we deploy Transformer-based Variational AutoEncoder with a supervision approach, where the sentence generation can be manipulated and explained over low-dimensional latent Gaussian space. In addition, we propose a new probing algorithm to guide the movement of sentence vectors over such geometry. Experimental results reveal that the formal semantic geometry can potentially deliver better control and interpretation to sentence generation.

1 Introduction

Language Models (LMs) have provided a flexible scaling-up foundation for addressing a diverse spectrum of tasks (Touvron et al., 2023). Nonetheless, the question remains: can we develop language representations/models that offer more granular levels of control and interpretation from the perspective of “formal/structural” semantics? Addressing this question will enable us to enhance the controllability, interpretability, and safety of LMs.

Formal semantics, which provides a canonical, granular, and rigid representation, have been investigated for thousands of years, such as Montague Semantics (Dowty et al., 2012), Davidsonian Semantics (Davidson, 1967), Abstract Meaning Representation Banarescu et al. (2013), Semantic Role Labelling Palmer et al. (2010), and Argument Structure Theory (AST, Jackendoff (1992)). One typical characteristic of such formal semantics is the localisation or composition property. For example, in sentence: \textit{animals require oxygen for survival}, the words are functionally combined into sentence semantics: $\lambda x (\text{animals}(x) \rightarrow \text{require}(x, \text{oxygen}))$ where $x$ is the variable of any entity within a logical structure. In this case, we can localise the sentence semantics by replacing $x$ with \textit{birds}, etc. This localised process indicates the interpretation in Cognitive Science (Smolensky, 2006; Lees, 1957). However, such localisation is precisely what current distributional semantics lack, thereby limiting their controllability and interpretability.

Disentanglement (Bengio, 2013), which refers to the feature-dimension alignment (i.e., privileged basis Elhage et al. (2022)), can potentially provide such localisation, which has been widely investigated to localise image features, such as \textit{nose} in facial images (Esser et al., 2020; Jeon et al., 2019; Liu et al., 2021). In Transformers (Vaswani et al., 2017), however, token embeddings, residual stream, and attention are non-privileged, meaning that multiple dimensions contribute to a feature. Although some prior studies explored the possibility of language disentanglement, most are focused on coarse-grained/task-specific semantic features, such as sentiment, within the context of

Figure 1: Overview: latent sentence semantics can be decomposed into semantic role- word content features.
In this work, we focus on the localisation of general semantic features of sentences over distributional space to shorten the gap between deep latent semantics and formal linguistic representations (Gildea and Jurafsky, 2000; Banerescu et al., 2013; Mitchell, 2023), integrating the flexibility of distributional-neural models with the properties of linguistically grounded representations, facilitating both interpretability and generative control from the perspective of formal semantics. We specifically choose the conceptual dense explanatory sentences from WorldTree (Jansen et al., 2018) due to their clear formal semantic representation designed in the Explanatory Reasoning task.

In the NLP domain, Variational AutoEncoders (VAEs, Kingma and Welling (2013)) have been recognized as a prominent foundation for investigating generation control and interpretation through the observable low-dimensional smooth and regular latent spaces (e.g., std Gaussian space) (John et al., 2019; Li et al., 2022b; Bao et al., 2019; Mercatali and Freitas, 2021; Felhi et al., 2022; Vasilakes et al., 2022). Therefore, we probe the localisation property of formal semantics over latent sentence spaces under VAE architecture. Specifically:

(1) We first propose a geometrical framework to present the formal semantic features of sentences as semantic role - word content pairs (denoted as role-content) from the perspective of AST (Jackendoff, 1992) within the compositional distributional model (Clark et al., 2008). Subsequently, (2) we introduce a supervised approach for learning the role-content features of explanatory sentences in latent spaces. (3) Additionally, we propose a method to control sentence generation by navigating the sentence vectors across different role-content features within our geometric framework. (4) Our findings reveal that the role-content features are encoded as a convex cone in the latent sentence space (Figure 1). This semantic geometry facilitates the localisation of sentence generation by enabling the manipulation of sentence vectors through traversal and arithmetic operations within the latent space.

2 Related work

**Formal-distributional semantics.** Integrating distributional semantics with formal / symbolic semantics is challenging in the field of artificial intelligence. In the Reasoning domain, for example, existing approaches usually perform symbolic behaviour via explicitly symbolic representation injection, including graph (Khashabi et al., 2018; Khot et al., 2017; Jansen et al., 2017; Thayaparan et al., 2021), linear programming (Valentino et al., 2022b; Thayaparan et al., 2024), adopting iterative methods, using sparse or dense encoding mechanisms (Valentino et al., 2020; Lin et al., 2020; Valentino et al., 2022a; Bostrom et al., 2021), or synthetic natural language expression (Clark et al., 2020; Yanaka et al., 2021; Fu and Frank, 2024), among others. Comparatively, we explore the formal semantic property over distributional semantics via latent sentence geometry, which can potentially deliver better interpretation to current LMs.

**Language geometry.** There is a line of work that studies the geometry of word and sentence representations (Arora et al., 2016; Mimno and Thompson, 2017; Ethayarajh, 2019; Reif et al., 2019; Li et al., 2020a; Chang et al., 2022; Jiang et al., 2024a). E.g., king – man + woman = queen, which the word vectors can be manipulated with geometric algebra. This phenomenon indicates the linear subspaces in language representations, similar features are encoded as a close direction in latent space, which has been widely explored ranging from word (Mikolov et al., 2013a) to sentences (Ushio et al., 2021), Transformer-based LMs (Merullo et al., 2023; Hernandez et al., 2023), and multi-modal models (Trager et al., 2023; Huh et al., 2024). Under the linear subspace hypotheses, a significant work explored the interpretability (Li et al., 2022a; Geva et al., 2022; Nanda et al., 2023) and controllability (Trager et al., 2023; Merullo et al., 2023; Turner et al., 2023) of neural networks. In this work, we emphasise the formal semantic geometry for bridging the distributional and formal semantics, which is currently under-explored.

**Language disentanglement.** Disentanglement, refers to separating features along dimensions (Bengio, 2013), leading to clear geometric and linear representations. In the NLP domain, many studies explored the disentanglement between specific linguistic perspectives, such as sentiment-content (John et al., 2019), semantic-syntact (Bao et al., 2019), and negation-uncertainty (Vasilakes et al., 2022), or syntactic-level disentanglement (Mercatali and Freitas, 2021; Felhi et al., 2022). However, a fundamental issue has been overlooked: the
definition of disentanglement in the image domain (Esser et al., 2020) cannot be directly applied to the context of computational linguistics due to the variability and complexity of language expression and high entanglement after current Transformer-based encoders. Therefore, we contribute to a new lens on the disentanglement (separation) of sentence features from the perspective of formal semantics.

3 Formal Semantic Geometry

In this section, we first define the sentence semantic features as semantic role - word content from the perspective of formal semantics. Then, we link the semantic features with distributional vector spaces. That is, each semantic role - word content is encoded as a convex cone in latent spaces.

**Formal semantic features.** For formal / structural semantics, Argument Structure Theory (AST) (Jackendoff, 1992; Levin, 1993; Rappaport Hovav and Levin, 2008) provides a model for representing sentence structure and meaning of sentences in terms of the interface between the their syntactic structure and the associated semantic roles of the arguments within those sentences. It delineates how verbs define the organisation of their associated arguments and the reflection of this organisation in a sentence’s syntactic realisation. AST abstracts sentences as predicate-argument structures, where the predicate $p$ (associated with the verb) has a set of associated arguments $arg_i$, where each argument has an associated positional component $i$ and a thematic/semantic roles $r_i$, the latter categorising the semantic functions of arguments in relation to the verb (e.g. agent, patient, theme, instrument). In the context of this work, the AST predicate-argument representation is associated with a lexical-semantic representation of the content $c_i$ of the term $t_i$.

In this work, we simplify and particularise the relationship between the argument structure and the distributional lexical semantic representation as a role-content relation, where the structural syntactic/semantic relationship is defined by its shallow semantics, i.e. as the composition of the content of the terms, their position in the predicate-argument (PArg) structure ($arg_i$) and their semantic roles (SRs) ($r_i$; pred, arg), as described below:

\[ \text{animals} \quad \text{require} \quad \text{oxygen} \quad \text{for survival} \]

Therefore, we define the semantics of sentences, $sem(s)$, as the compositions between role-content, which can be described as follows:

\[ sem(s) = t_1(c_1, r_1) + \cdots + t_i(c_i, r_i) \]

\[ \text{i.e., ARG0–animals PREP–survival} \]

Where $t_i(c_i, r_i) = c_i \otimes r_i$ represents the semantics of term $t_i$ with content $c_i$ (i.e., animals) and SRL $r_i$ (i.e., ARG0) in context $s$. $\otimes$: connects the meanings of words with their roles, using the compositional-distributional semantic notation of (Smolensky and Legendre, 2006; Clark and Pulman, 2007; Clark et al., 2008). $\oplus$: connects the lexical semantics (word content + structural role) to form the sentence semantics. To deliver the localisation or composition property, the sentence semantics should be able to present separation or disentanglement under connector $\oplus$. E.g., replacing ARG0-animals with ARG0-fishes.

**Formal semantic features in vector space.** After defining the semantic features of sentences, we propose the concept of a convex cone of semantic feature. In linear algebra, a cone refers to a subset of a vector space that is convex if any $\alpha v_i + \beta v_j$ if any $v_i$ and $v_j$ belong to it. $\alpha$ and $\beta$ are positive scalars. Formally, the definition of convex cone, $C$, is described as a set of vectors:

\[ C = \{ x \in V | x = \sum_{i=1}^{n} \alpha_i v_i, \alpha_i \geq 0, v_i \in R \} \]

where $x$ is an element vector in vector space $\mathbb{R}$, $v_i$ are the basis vectors, $\alpha_i$ are non-negative scalars. In this context, we consider each role-content feature as a convex cone, $C$, corresponding to a hyperplane in high-dimensional vector space:

\[ C_{c_i,r_i} = \{ t(c_i, r_i) | t(c_i, r_i) \in sem(s), s \in \text{corpus} \} \]

where $t(c_i, r_i)$ represents the basis vector in $C_{c_i,r_i}$ (Figure 2). According to set theory, we can define the formal semantic space as follows:

**Assumption1:** The sentence semantic space is the union of all unique $C_{c_i,r_i}$ convex cones:

\[ C_{c_1,r_1} \cup C_{c_2,r_2} \cup \cdots \cup C_{c_{|V|},r_{|V|}} \]

$V$ is the vocabulary of a corpus. Based on Assumption1, we can establish:

**Proposition1:** The geometrical location of sentence semantic vectors, $sem(s)$, can be determined by the intersection of different $C_{c_i,r_i}$:

\[ sem(s) = t_1(c_1, r_1) + \cdots + t_i(c_i, r_i) = \{ t_1(c_1, r_1) \} + \cdots + \{ t_i(c_i, r_i) \} = C_{c_1,r_1} \cap C_{c_2,r_2} \cap \cdots \cap C_{c_{|V|},r_{|V|}} \]

4 Geometrical Formal Semantic Control

In this section, we first show that our formal semantic geometry can interpret sentence generation,
such as arithmetic (Shen et al., 2020), and extend the “Linear Representation Hypothesis”. Then, we propose a new semantic control approach, which recursively traverses the latent dimensions to probe the semantic geometry over latent spaces.

**Geometrical algebra interpretability.** Arithmetic has been considered a common way to control word or sentence semantics over latent spaces (Mikolov et al., 2013b). E.g., the addition operation can steer the sentence semantics (Shen et al., 2020; Mercatali and Freitas, 2021; Liu et al., 2023, 2024), or linear interpolation can generate smooth intermediate sentences (Hu et al., 2022). However, they lack an explanation for these phenomena. In this section, we show how our geometrical framework can provide an intuitive explanation for these phenomena.

For linear interpolation, for example, it takes two sentences \( x_1 \) and \( x_2 \) and obtains latent vectors \( z_1 \) and \( z_2 \), respectively. It interpolates a path \( z_t = z_1 \cdot (1 - t) + z_2 \cdot t \) with \( t \) increased from 0 to 1 by a step size of 0.1. Given two sentences with one role-content set overlap, \( C_{c_i,r_j} \). We can describe:

\[
sem(s_1) \cap sem(s_2) = \{ C_{c_1,r_1} \cap \ldots \cap C_{c_i,r_i} \} \cap \{ C_{c_2, r_1} \cap \ldots \cap C_{c_i, r_i} \} = \{ C_{c_1, r_1} \cap \ldots \cap C_{c_i, r_i} \} \cap C_{c_2, r_1}^{s_1 (2)}
\]

According to the definition of convex cone, if \( z_1 \) and \( z_2 \) are left in \( C_{c_2, r_1}^{s_1 (2)} \), the weighted sum vector, \( z_t \), is also in \( C_{c_2, r_1}^{s_1 (2)} \). Therefore, the intermediate sentence semantics can be described as:

\[
sem(s_{1 \rightarrow 2}) = (1 - t) \times sem(s_1) + t \times sem(s_2) = \{ z_1 \cdot (1 - t) + z_2 \cdot t, \ldots \} \cap C_{c_i, r_j}^{s_1 (2)}
\]

That is, the intermediate sentences will hold the \( \{ c_j, r_j \} \) information during interpolation.

**Linear representation hypothesis.** “Linear representation hypothesis” refers to high-level concepts being represented linearly as directions in representation space, which has been widely evaluated to interpret Large LMs’ mechanism (Marks and Tegmark, 2023; Xie et al., 2021; Wang et al., 2024; Jiang et al., 2024b; Park et al., 2023, 2024). However, a main challenge for this hypothesis is that it’s not clear what constitutes a “high-level concept”.

Our geometrical framework can further support and extend this hypothesis by answering what and how they are “linearly” encoded? For example, given a set of \( N \) atomic sentences: \( s_i: bird \ is \ a \ kind \ of \ living \ thing \) varying the content of \( arg1 \). Their semantics can be described below:

\[
sem(s) = \{ C_{c_i, arg_1}^{s_1}, \ldots \} \cap \cdots \cap C_{living\, thing, arg_2}^{s_1}, \ldots \}
\]

In this case, the concept \( living \ thing \) is encoded as a convex cone where all different \( C_{c_i, arg_1}^{s_1} \) contribute to its boundary, leading to a direction. The hierarchical relations between \( living \ thing \) and \( bird, etc. \) are determined by the convex cones is a kind of.

**Guided traversal.** Since we describe different sentence semantic features, \( \{ c_i, r_i \} \), as distinct convex cones, \( C_{c_i, r_i} \), within a \( N \)-dimensional vector space, \( V \in \mathbb{R}^N \), we can linearly divide each basis dimension, \( i \in \{ 1, \ldots, N \} \), into different value regions, \( [a, b]^{(i)} \), based on minimal information entropy. Consequently, there is a sequence of dimensional subspaces for each semantic feature. Thus, movement between different \( C_{c_i, r_i} \) regions can be achieved by moving out the dimensional regions within this sequence. This process can be implemented via a decision tree. In figure 3, for example, we can move the sentence from \( C_{pred, causes} \) to \( C_{pred, means} \) by modifying the values started from \( dim 20 \leq -0.035 \), ..., ending at \( dim 10 \leq -1.11 \). By traversing the tree path, we can control the sentence generation by moving between convex cones, detailed in Algorithm 1.

Based on our algorithm, we can use classification metrics as proxy metrics to evaluate latent space geometry. E.g., accuracy and recall for measuring feature separability and density.
**Algorithm 1 Guided latent space traversal**

1: Datasets: $D = \{s_1, \ldots, s_n\}$
2: Labels: $Y = \{y_1, \ldots, y_n\}$, $y_i \in \{0, 1\}$
3: $0$: pred-causes, $1$: pred-means
4: Seed: $s = \text{fire causes chemical change}$
5: for $s_i \in D$ do
6: $z_i \leftarrow \text{Encoder}(s_i)$
7: end for
8: $X \leftarrow \{z_1, \ldots, z_n\}$
9: tree $\leftarrow \text{DecisionTreeClassifier}(X, Y)$
10: path $\leftarrow \text{filter}(\text{tree})$ # choose the shortest path between $C_0$ and $C_1$
11: $z \leftarrow \text{Encoder}(s)$
12: for node $\in$ path do
13: $(\text{dim}, \text{range}, \text{yes/no}) \leftarrow \text{node}$
14: if in current branch do
15: $z[\text{dim}] \leftarrow v \notin \text{range}$ if yes $v \in \text{range}$ else do
16: $z[\text{dim}] \leftarrow v \in \text{range}$ if yes $v \notin \text{range}$
17: end for
19: $s \leftarrow \text{Decoder}(z)$ # fire means chemical change

**Figure 3:** Traversal between different role-content sets by moving along the tree path.

## 5 SRL-Conditional VAE

In this section, we investigate the architecture of VAE to integrate the latent sentence space with LMs and propose a supervision approach to learn defined semantic features (i.e., role-content).

**Model architecture.** We consider Optimus (Li et al., 2020b) as the foundation which used BERT and GPT2 as Encoder and Decoder, respectively. In detail, the sentence representation, Embed($x$), encoded from BERT[cls] will first transform into a Gaussian space by learning the parameters $\mu$ and $\sigma$ through multilayer perceptions $W_\mu, W_\sigma$. The final latent sentence representations can be obtained via: $z = W_\mu \times \text{Embed}(x) + W_\sigma$, which, as an additional Key and Value, is concatenated into the original Key and Value weights of GPT2, which can be described as: Attention($Q, K, V$) = $\text{softmax}(\frac{Qz^{T}K}{\sqrt{d}})$ where $Q$ has the shape $R^{\text{seq} \times 64}$, $K, V$ has the shape $R^{(\text{seq} + 1)\times 64}$ ($64$ is dimension of GPT2 attention, seq is sequence length). Since $Q$ represents the target, $K$ and $V$ represent the latent representations. By intervening the $KV$ with $z$, we can learn the transformation between latent space and observation distribution.

**Optimisation.** It can be trained via the evidence lower bound (ELBO) on the log-likelihood of the data $x$ (Kingma and Welling, 2014). To bind the word content and semantic role information in latent space, we conditionally inject the semantic role sequence into latent spaces where the latent space $z$ and semantic role $r$ are dependent. The joint distribution can be described as:

$$P(x, y, z) = P(x|z, r) \times P(z|r) \times P(r)$$

Specifically, we use encoder (i.e., Bert) to learn the approximate posterior based on both semantic roles and tokens, and additionally, we separately inject the semantic roles into encoder to learn the prior distribution. Both semantic roles and latent variables are injected into the decoder to auto-encode the tokens. The CVAE is trained to maximize the conditional log-likelihood of $x$ given $r$, which involves an intractable marginalization over the latent variable $z$. Moreover, to avoid the KL vanishing problem, which refers to the Kullback-Leibler (KL) divergence term in the ELBO becomes very small or approaches zero, we select the cyclical schedule to increase weights of KL $\beta$ from 0 to 1 (Fu et al., 2019) and a KL thresholding scheme (Li et al., 2019) that chooses the maximum between KL and threshold $\lambda$. The final objective function can be described as follows:

$$L_{\text{CVAE}} = -\mathbb{E}_{q_{\phi}(z|x, r)} \log p_{\theta}(x|z, r) + \beta \sum_{i} \max[\lambda, \text{KL}(q_{\phi}(z_{i}|x, r)\|p(z_{i}|r))]$$

where $q_{\phi}$ represents the approximated posterior (i.e., encoder). $i$ is the $i$-th latent dimension.
6 Empirical analysis

In the experiment, we quantitatively and qualitatively evaluate the latent space geometry via 1.traversal, 2.arithmetic, and 3.guided traversal. All experimental details are provided in Appendix A.

6.1 Latent Traversal

Qualitative evaluation. Traversal refers to the random walk over latent space. It can be done by decoding the latent vector in which each dimension is resampled and other dimensions are fixed (Higgins et al., 2017; Kim and Mnih, 2018; Carvalho et al., 2023). Given a latent vector from a “seed” sentence, we can traverse its neighbours to evaluate the geometry. As illustrated in Table 1, those traversed sentences can hold the same content under different semantic roles as the input, such as automobile in ARG1, indicating role-content feature separation in latent spaces.

| an automobile is a kind of vehicle |
| an automobile is a kind of moving object |
| an automobile is a kind of object |
| an airplane is a kind of vehicle |
| a car is a kind of vehicle |

Table 1: Traversal showing held semantic factors in explanations corpus.

Quantitative evaluation. Next, we employ t-SNE (Van der Maaten and Hinton, 2008) to statistically examine role-content features cluster and separation over latent space (i.e., natural clustering property (Bengio, 2013)). In the corpus, however, due to the small number of data points within each role-content cluster, t-SNE cannot capture the differences between clusters well, resulting in the visualized latent space not displaying good role-content separability (top in figure 5). Therefore, we increase the number of data points in different role-content clusters by traversing each and keeping those resulting data points with the same role-content. Then, we visualise the role-content cluster at the bottom of figure 5. We can find that the features are clustered and separated over the latent space. If this was not the case, after traversing the resulting vectors from the same role-content cluster, the visualization should show the same entanglement as the original datapoints distribution.

6.2 Latent Arithmetic

Qualitative evaluation. In addition, we demonstrate the geometric properties via interpolation in Table 2. For the top-most one, we can observe

| a beach ball is a kind of container |
| 1. a pool table is a kind of object |
| 2. a balloon is a kind of object |
| 3. a magnet is a kind of object |
| 4. a neutron is a kind of particle |
| 5. a proton is a kind of particle |
| an atom is a kind of particle |
| protons are found in the nucleus of an atom |
| 1. protons are found in the nucleus of an atom |
| 2. 1 atom is positive 1 in electric charge |
| 3. 1 in 6000 is equal to 27 in 10 years |
| 4. if protons and neutrons have the same number of neutrons then those two particles are physically closer than one another |
| 5. if a neutron has a negative -10 electric charge then the atom will not be able to move |
| 6. if a neutron has a negative -10 electric charge then the neutron will not have a positive electric charge |
| if a neutral atom loses an electron then an atom with a positive charge will be formed |

Table 2: Interpolation examples (top: interpolation between sentences with similar semantic information, bottom: interpolation between sentences with different semantic information). Only unique sentences shown.

that sentences are smoothly moved from source to target (e.g., from beach ball to atom connected by ballon, magnet, neutron, and proton) where the same role-content (i.e., pred-is) unchanged. In contrast, the second case doesn’t display the smooth interpolation path. E.g., the third sentence con-
necting different semantic structures is unrelated to both source and target due to a discontinuous space gap between different clusters. Both indicate that the explanatory sentences might be clustered according to different semantic role structures.

| $s_1$: animals require food for survival |
| $s_2$: animals require warmth for survival |
| animals eat plants |
| animals produce milk |
| animals usually eat plants |
| animals eat berries; plants |
| animals require food to survive |
| animals require shelter to survive |

$\begin{array}{ll}
s_1: & \text{water vapor is invisible} \\
S_2: & \text{the water is warm} \\
\end{array}$

| igneous rocks are found under the soil |
| quartz is usually very small in size |
| quartz is formed by magma cooling |
| quartz is made of iron and zinc |
| silica is made of argon and argon |
| sedimentary is formed by lithosphere collapsing |

Table 3: $s_1 \pm s_2$ (top: addition, bottom: subtraction).

Following the definition of convex cone, we next traverse the resulting sentence after adding or subtracting two sentences with the same role-content feature. As illustrated in Table 3, the adding operation tends to hold the same role-content (e.g., $\text{ARG0-Animals}$) as inputs. In contrast, the subtraction loses such control, e.g., from $\text{ARG1-water}$ to $\text{ARG1-quartz}$. More similar observations are in Table 11. These results corroborate our geometry.

### Quantitative evaluation

Next, we quantitatively assess our geometry framework by calculating the ratio of the same role-content results from the vector addition and subtraction for all sentence pairs with a matching role. As illustrated in Figure 6, the ADDed results (dark blue) can greatly hold the same token-level semantics (role-content) as inputs, indicating our geometrical framework. In contrast, the SUBed results (shallow blue) suffer from semantic shift. Similar observations for VERB and ARG1 can be found in Figure 11 and 12. Besides, we can quantify each role-content cluster’s geometrical area by calculating the cosine similarity between randomly selected sentence pairs in this cluster. We report the maximal and minimal distance in Figure 7. Similar observations for VERB and ARG1 can be found in Figure 13 and 14.

### 6.3 Guided Latent Traversal

Finally, we examine the latent space geometry with our algorithm 1. The categories mentioned next are chosen based on their frequencies to ensure the balance during the training of the classifier.

### Qualitative evaluation

Firstly, we evaluate the traversal between different semantic role structures, e.g., conditional and atomic sentences. Table 4 shows that the cluster of the generated sentence changes as the values of different dimensions change sequentially (e.g., the first three sentences hold the same characteristic $\text{if ... then ...}$ as the input. The remaining sentences gradually move closer to the target characteristics, such as $\text{is}$). Meanwhile, the sentences can hold the subject, $\text{something}$, during the movement, corroborating our geometry framework. Next, we evaluate the traversal between predicates. Table 5 shows the movement between verbs ($\text{cause}$ and $\text{mean}$).
if something receives sunlight it will absorb the sunlight
Dim27: if a thing absorbs sunlight then that thing is warmer
Dim12: if something is eaten then that something produces heat
Dim08: if something gets too hot in sunlight then that something is less able to survive
Dim03: something contains physical and chemical energy
Dim21: something contains sunlight
Dim10: some things are made of matter
Dim00: something is made of atoms
Dim17: a forest contains life
Dim00: something that is cold has a lower temperature
Dim21: something rises in temperature
Dim00: something is formed from things dissolved in water
Dim30: something that is cold has fewer nutrients
Dim21: something that is not moved is dead

Table 4: Movement from conditional to atomic sentences.

Table 5: Movement between cause and mean.

Table 6: Movement from water to something.

Quantitative evaluation. Finally, we use classification metrics, including accuracy (separability) and recall (density), as proxy metrics to assess latent space geometry. As shown in Table 7, both predicate and argument1 show higher separation.

7 Conclusion and Future Work

In this study, we investigate the localisation of general semantic features to enhance the controllability and explainability of distributional space from the perspective of formal semantics, which is currently under-explored in the NLP domain. We first propose the formal semantic features as role-content...
and define the corresponding geometrical framework. Then, we propose a supervision approach to bind the semantic role and word content. In addition, we propose a novel traversal probing approach to assess the latent space geometry based on information set and entropy. We extensively evaluate the latent space geometry through the geometrical operations, such as traversal, arithmetic, and our guided traversal. Experimental results indicate the existence of formal semantic geometry. In the future, we will explore the In-context-learning of explanatory reasoning of LLMs based on our formal semantic geometry framework.

8 Limitations

1. Limitation of data source: this work only focused on explanatory sentences, such as atomic sentences. Whether the semantic separability of other corpora emerges over latent space is not explored.

2. Role-content clusters overlapping: the geometric analysis indicates that the role-content regions still have significant overlapping, so we can propose a new task, naming “sentence semantic disentanglement”, which is how we can better separate/disentangle the semantic features to provide better localisation or composition behaviour over distributional semantic spaces in Computational Linguistics.

References

Sanjeev Arora, Yuanzhi Li, Yingyu Liang, Tengyu Ma, and Andrej Risteski. 2016. A latent variable model approach to PMI-based word embeddings. Transactions of the Association for Computational Linguistics, 4:385–399.

Laura Banerescu, Claire Bonial, Shu Cai, Madalina Georgescu, Kira Griffitt, Ulf Hermjakob, Kevin Knight, Philipp Koehn, Martha Palmer, and Nathan Schneider. 2013. Abstract meaning representation for sembanking. In Proceedings of the 7th linguistic annotation workshop and interoperability with discourse, pages 178–186.

Yu Bao, Hao Zhou, Shujian Huang, Lei Li, Lili Mou, Olga Vechtomova, Xinyu Dai, and Jiajun Chen. 2019. Generating sentences from disentangled syntactic and semantic spaces. In Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics, pages 6008–6019.

Joshua Bengio. 2013. Deep learning of representations: Looking forward. In International conference on statistical language and speech processing, pages 1–37. Springer.

Kaj Bostrom, Xinyu Zhao, Swarat Chaudhuri, and Greg Durrett. 2021. Flexible generation of natural language deductions. In Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing, pages 6266–6278. Online and Punta Cana, Dominican Republic. Association for Computational Linguistics.

Danilo S. Carvalho, Yingji Zhang, Giangiacomo Mercatali, and Andre Freitas. 2023. Learning disentangled representations for natural language definitions. Findings of the European chapter of Association for Computational Linguistics (Findings of EACL).

Tyler A Chang, Zhuowen Tu, and Benjamin K Bergen. 2022. The geometry of multilingual language model representations. arXiv preprint arXiv:2205.10964.

Peter Clark, Oyvind Tafjord, and Kyle Richardson. 2020. Transformers as soft reasoners over language. arXiv preprint arXiv:2002.05867.

Stephen Clark, Bob Coecke, and Mehrnoosh Sadrzadeh. 2008. A compositional distributional model of meaning. In Proceedings of the Second Quantum Interaction Symposium (QI-2008), pages 133–140. Oxford.

Stephen Clark and Stephen G. Pulman. 2007. Combining symbolic and distributional models of meaning. In Quantum Interaction.

Bhavana Dalvi, Peter Jansen, Oyvind Tafjord, Zhengnan Xie, Hannah Smith, Leigehanna Pipatanangkura, and Peter Clark. 2021. Explaining answers with entailment trees.

Donald Davidson. 1967. The logical form of action sentences.

David R Dowty, Robert Wall, and Stanley Peters. 2012. Introduction to Montague semantics, volume 11. Springer Science & Business Media.

Nelson Elhage, Tristan Hume, Catherine Olsson, Nicholas Schieber, Tom Henighan, Shauna Kravec, Zac Hatfield-Dodds, Robert Lasenby, Dawn Drain, Carol Chen, Roger Grosse, Sam McCandlish, Jared Kaplan, Dario Amodei, Martin Wattenberg, and Christopher Olah. 2022. Toy models of superposition. Transformer Circuits Thread.

Patrick Esser, Robin Rombach, and Bjorn Ommer. 2020. A disentangling invertible interpretation network for explaining latent representations. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 9223–9232.
Kawin Ethayarajh. 2019. How contextual are contextu-
alized word representations? Comparing the geom-
etry of BERT, ELMo, and GPT-2 embeddings. In Pro-
ceedings of the 2019 Conference on Empirical
Methods in Natural Language Processing and the
9th International Joint Conference on Natural Lan-
guage Processing (EMNLP-IJCNLP), pages 55–65,
Hong Kong, China. Association for Computational
Linguistics.

Ghazi Felhi, Joseph Le Roux, and Djamé Seddah. 2022.
Towards unsupervised content disentanglement in
sentence representations via syntactic roles. arXiv
preprint arXiv:2206.11184.

Hao Fu, Chunyuan Li, Xiaodong Liu, Jianfeng Gao,
Asli Celikyilmaz, and Lawrence Carin. 2019. Cyclic-
al annealing schedule: A simple approach to mit-
igating KL vanishing. In Proceedings of the 2019
Conference of the North American Chapter of the
Association for Computational Linguistics: Human
Language Technologies, Volume 1 (Long and Short
Papers), pages 240–250, Minneapolis, Minnesota.
Association for Computational Linguistics.

Xiyuan Fu and Anette Frank. 2024. Exploring continual
learning of compositional generalization in nli. arXiv
preprint arXiv:2403.04400.

Matt Gardner, Joel Grus, Mark Neumann, Oyvind
Tafjord, Pradeep Dasigi, Nelson H S Liu, Matthew E.
Peters, Michael Schmitz, and Luke Zettlemoyer. 2017.
A deep semantic natural language processing
platform.

Mor Geva, Avi Caciularu, Kevin Ro Wang, and Yoav
Goldberg. 2022. Transformer feed-forward layers
build predictions by promoting concepts in the vo-
vocabulary space. arXiv preprint arXiv:2203.14680.

Daniel Gildea and Daniel Jurafsky. 2000. Automatic
labeling of semantic roles. In Proceedings of the 38th
Annual Meeting on Association for Computa-
tional Linguistics, ACL ’00, page 512–520, USA.
Association for Computational Linguistics.

Yuxuan Gu, Xiaocheng Feng, Sicheng Ma, Lingyuan
Zhang, Heng Gong, and Bing Qin. 2022. A distribu-
tional lens for multi-aspect controllable text gen-
eration. In Proceedings of the 2022 Conference on
Empirical Methods in Natural Language Processing,
pages 1023–1043, Abu Dhabi, United Arab Emirates.
Association for Computational Linguistics.

Yuxuan Gu, Xiaocheng Feng, Sicheng Ma, Lingyuan
Zhang, Heng Gong, Weihong Zhong, and Bing Qin.
2023. Controllable text generation via probability
density estimation in the latent space. In Proceed-
ings of the 61st Annual Meeting of the Association
for Computational Linguistics (Volume 1: Long Papers),
pages 12590–12616, Toronto, Canada. Association
for Computational Linguistics.

Evan Hernandez, Arnab Sen Sharma, Tal Haklay, Kevin
Meng, Martin Wattenberg, Jacob Andreas, Yonatan
Belinkov, and David Bau. 2023. Linearity of relation
decoding in transformer language models. arXiv
preprint arXiv:2308.09124.

Irina Higgins, Loic Matthey, Arka Pal, Christopher P.
Burgess, Xavier Glorot, Matthew M. Botvinick,
Shakir Mohamed, and Alexander Lerchner. 2017.
beta-vae: Learning basic visual concepts with a con-
strained variational framework. In ICLR.

Jinyi Hu, Xiaoyuan Yi, Wenhao Li, Maosong Sun, and
Xing Xie. 2022. Fuse it more deeply! a variational
transformer with layer-wise latent variable inference
for text generation. In Proceedings of the 2022 Con-
ference of the North American Chapter of the Asso-
ciation for Computational Linguistics: Human
Language Technologies, pages 697–716, Seattle, United
States. Association for Computational Linguistics.

Zhiqing Hu and Li Erran Li. 2021. A causal lens for
controllable text generation. Advances in Neural
Information Processing Systems, 34:24941–24955.

Minyoung Huh, Brian Cheung, Tongzhou Wang, and
Philip Isola. 2024. The platonic representation hy-
pothesis. arXiv preprint arXiv:2403.07987.

Ray S Jackendoff. 1992. Semantic structures, vol-
ume 18. MIT press.

Peter Jansen, Rebecca Sharp, Mihai Surdeanu, and Peter
Clark. 2017. Framing qa as building and ranking
intersentence answer justifications. Computational
Linguistics, 43(2):407–449.

Peter A Jansen, Elizabeth Wainwright, Steven Mar-
morstein, and Clayton T Morrison. 2018. Worldtree:
A corpus of explanation graphs for elementary sci-
ence questions supporting multi-hop inference. arXiv
preprint arXiv:1802.03052.

Giyoon Jeon, Haedong Jeong, and Jaesik Choi. 2019.
An efficient explorative sampling considering the
generative boundaries of deep generative neural
networks.

Yibo Jiang, Bryon Aragam, and Victor Veitch. 2024a.
Uncovering meanings of embeddings via partial or-
thogonality. Advances in Neural Information Pro-
cessing Systems, 36.

Yibo Jiang, Goutham Rajendran, Pradeep Ravikumar,
Bryon Aragam, and Victor Veitch. 2024b. On the
origins of linear representations in large language
models. arXiv preprint arXiv:2403.03867.

Vineet John, Lili Mou, Hareesh Bahuleyan, and Olga
Vechtomova. 2019. Disentangled representation
learning for non-parallel text style transfer. In Pro-
cedings of the 57th Annual Meeting of the Associa-
tion for Computational Linguistics, pages 424–434.

Daniel Khashabi, Tushar Khot, Ashish Sabharwal, and
Olga Vechtomova. 2019. How contextual are contextu-
alized word representations? Comparing the geom-
etry of BERT, ELMo, and GPT-2 embeddings. In Pro-
ceedings of the 2019 Conference on Empirical
Methods in Natural Language Processing and the
9th International Joint Conference on Natural Lan-
guage Processing (EMNLP-IJCNLP), pages 55–65,
Hong Kong, China. Association for Computational
Linguistics.

Dan Roth. 2018. Question answering as global rea-
sion for Computational Linguistics.

Giyoung Jeon, Haedong Jeong, and Jaesik Choi. 2019.
A corpus of explanation graphs for elementary sci-
ence questions supporting multi-hop inference. arXiv
preprint arXiv:1802.03052.

Ray S Jackendoff. 1992. Semantic structures, vol-
ume 18. MIT press.

Peter Jansen, Rebecca Sharp, Mihai Surdeanu, and Peter
Clark. 2017. Framing qa as building and ranking
intersentence answer justifications. Computational
Linguistics, 43(2):407–449.

Peter A Jansen, Elizabeth Wainwright, Steven Mar-
morstein, and Clayton T Morrison. 2018. Worldtree:
A corpus of explanation graphs for elementary sci-
ence questions supporting multi-hop inference. arXiv
preprint arXiv:1802.03052.

Giyoon Jeon, Haedong Jeong, and Jaesik Choi. 2019.
An efficient explorative sampling considering the
generative boundaries of deep generative neural
networks.

Yibo Jiang, Bryon Aragam, and Victor Veitch. 2024a.
Uncovering meanings of embeddings via partial or-
thogonality. Advances in Neural Information Pro-
cessing Systems, 36.

Yibo Jiang, Goutham Rajendran, Pradeep Ravikumar,
Bryon Aragam, and Victor Veitch. 2024b. On the
origins of linear representations in large language
models. arXiv preprint arXiv:2403.03867.

Vineet John, Lili Mou, Hareesh Bahuleyan, and Olga
Vechtomova. 2019. Disentangled representation
learning for non-parallel text style transfer. In Pro-
cedings of the 57th Annual Meeting of the Associa-
tion for Computational Linguistics, pages 424–434.

Daniel Khashabi, Tushar Khot, Ashish Sabharwal, and
Dan Roth. 2018. Question answering as global rea-
sion over semantic abstractions. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 32.
Tushar Khot, Ashish Sabharwal, and Peter Clark. 2017. Answering complex questions using open information extraction. *arXiv preprint arXiv:1704.05572*.

Hyunjik Kim and Andriy Mnih. 2018. *Disentangling by factorising*. In *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pages 2649–2658. PMLR.

Diederik P. Kingma and Max Welling. 2013. *Auto-encoding variational bayes*.

Diederik P. Kingma and Max Welling. 2014. *Auto-encoding variational bayes*.

Robert B Lees. 1957. *Syntactic structures*.

Beth Levin. 1993. *English verb classes and alternations: A preliminary investigation*. University of Chicago press.

Graham Neubig, Taylor Berg-Beth Levin. 1993. *Methods in Natural Language Processing and the Variational autoencoder with disentanglement priors for low-resource task-specific natural language generation*. In *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing*, pages 10335–10356, Abu Dhabi, United Arab Emirates. Association for Computational Linguistics.

Guangyi Liu, Zeyu Feng, Yuan Gao, Zichao Yang, Xiaodan Liang, Junwei Bao, Xiaodong He, Shuguang Cui, Zhen Li, and Zhiting Hu. 2023a. *Composable text controls in latent space with ODEs*. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pages 16543–16570, Singapore. Association for Computational Linguistics.

Sheng Liu, Lei Xing, and James Zou. 2023b. In-context vectors: Making in context learning more effective and controllable through latent space steering. *arXiv preprint arXiv:2311.06668*.

Yahui Liu, Enver Sangineto, Yajing Chen, Linchao Bao, Haoxian Zhang, Nicu Sebe, Bruno Lepri, Wei Wang, and Marco De Nadai. 2021. *Smoothing the disentangled latent style space for unsupervised image-to-image translation*.

Samuel Marks and Max Tegmark. 2023. The geometry of truth: Emergent linear structure in large language model representations of true/false datasets. *arXiv preprint arXiv:2310.06824*.

Giagiacomo Mercatali and André Freitas. 2021. Disentangling generative factors in natural language with discrete variational autoencoders. In *Findings of the Association for Computational Linguistics: EMNLP 2021*, pages 3547–3556.

Jack Merullo, Carsten Eickhoff, and Ellie Pavlick. 2023. Language models implement simple word2vec-style vector arithmetic. *arXiv preprint arXiv:2305.16130*.

Tomas Mikolov, Ilya Sutskever, Kai Chen, Greg S Corrado, and Jeff Dean. 2013a. Distributed representations of words and phrases and their compositionality. *Advances in neural information processing systems*, 26.

Tomas Mikolov, Wen-tau Yih, and Geoffrey Zweig. 2013b. Linguistic regularities in continuous space word representations. In *Proceedings of the 2013 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pages 746–751, Atlanta, Georgia. Association for Computational Linguistics.

David Mimno and Laure Thompson. 2017. *The strange geometry of skip-gram with negative sampling*. In *Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing*, pages 2873–2878, Copenhagen, Denmark. Association for Computational Linguistics.

Melanie Mitchell. 2023. How do we know how smart ai systems are?

Neel Nanda, Andrew Lee, and Martin Wattenberg. 2023. Emergent linear representations in world models of self-supervised sequence models. *arXiv preprint arXiv:2309.00941*.

Martha Stone Palmer, Daniel Gildea, and Nianwen Xue. 2010. *Semantic role labeling*, volume 6. Morgan & Claypool Publishers.
Kiho Park, Yo Joong Choe, Yibo Jiang, and Victor Veitch. 2024. The geometry of categorical and hierarchical concepts in large language models.

Kiho Park, Yo Joong Choe, and Victor Veitch. 2023. The linear representation hypothesis and the geometry of large language models. arXiv preprint arXiv:2311.03658.

Malka Rappaport Hovav and Beth Levin. 2008. The English dative alternation: The case for verb sensitivity. Journal of linguistics, 44(1):129–167.

Emily Reif, Ann Yuan, Martin Wattenberg, Fernanda B Viegas, Andy Coenen, Adam Pearce, and Been Kim. 2019. Visualizing and measuring the geometry of bert. Advances in Neural Information Processing Systems, 32.

Tianxiao Shen, Jonas Mueller, Regina Barzilay, and Tommi Jaakkola. 2020. Educating text autoencoders: Latent representation guidance via denoising. In International conference on machine learning, pages 8719–8729. PMLR.

Paul Smolensky. 2006. Harmony in linguistic cognition. Cognitive science, 30(5):779–801.

Paul Smolensky and Géraldine Legendre. 2006. The harmonic mind: From neural computation to optimality-theoretic grammar. Vol. 1, Cognitive architecture. MIT.

Mokanarangan Thayaparan, Marco Valentino, and André Freitas. 2021. Explainable inference over grounding-abstract chains for science questions. In Findings of the Association for Computational Linguistics: ACL-IJCNLP 2021, pages 1–12.

Mokanarangan Thayaparan, Marco Valentino, and André Freitas. 2024. A differentiable integer linear programming solver for explanation-based natural language inference. arXiv preprint arXiv:2404.02625.

Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaie, Nikolay Bashlykov, Soumya Batra, Prajwal Bhargava, Shrutiben Bhosale, et al. 2023. Llama 2: Open foundation and fine-tuned chat models. arXiv preprint arXiv:2307.09288.

Matthew Trager, Pramuditha Perera, Luca Zancato, Alessandro Achille, Parminder Bhatia, and Stefano Soatto. 2023. Linear spaces of meanings: compositional structures in vision-language models. In Proceedings of the IEEE/CVF International Conference on Computer Vision, pages 15395–15404.

Alex Turner, Lisa Thiergart, David Udell, Gavin Leech, Ulisse Mini, and Monte MacDiarmid. 2023. Activation addition: Steering language models without optimization. arXiv preprint arXiv:2308.10248.

Asahi Ushio, Luis Espinosa-Anke, Steven Schockaert, and Jose Camacho-Collados. 2021. Bert is to nlp what alexnet is to cv: Can pre-trained language models identify analogies? arXiv preprint arXiv:2105.04949.

Marco Valentino, Mokanarangan Thayaparan, Deborah Ferreira, and André Freitas. 2022a. Hybrid autoregressive inference for scalable multi-hop explanation regeneration. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 36, pages 11403–11411.

Marco Valentino, Mokanarangan Thayaparan, and André Freitas. 2020. Explainable natural language reasoning via conceptual unification. arXiv preprint arXiv:2009.14539.

Marco Valentino, Mokanarangan Thayaparan, and André Freitas. 2022b. Case-based abductive natural language inference. In Proceedings of the 29th International Conference on Computational Linguistics, pages 1556–1568.

Laurens Van der Maaten and Geoffrey Hinton. 2008. Visualizing data using t-sne. Journal of machine learning research, 9(11).

Jake Vasilakes, Chrysoula Zerva, Makoto Miwa, and Sophia Ananiadou. 2022. Learning disentangled representations of negation and uncertainty. In Proceedings of the 60th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 8380–8397, Dublin, Ireland. Association for Computational Linguistics.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Lukasz Kaiser, and Illia Polosukhin. 2017. Attention is all you need. Advances in neural information processing systems, 30.

Xinyi Wang, Wanrong Zhu, Michael Saxon, Mark Steyvers, and William Yang Wang. 2024. Large language models are latent variable models: Explaining and finding good demonstrations for in-context learning. Advances in Neural Information Processing Systems, 36.

Sang Michael Xie, Aditi Raghunathan, Percy Liang, and Tengyu Ma. 2021. An explanation of in-context learning as implicit bayesian inference. arXiv preprint arXiv:2111.02080.

Hitomi Yanaka, Koji Mineshima, and Kentaro Inui. 2021. SyGNS: A systematic generalization testbed based on natural language semantics. In Findings of the Association for Computational Linguistics: ACL-IJCNLP 2021, pages 103–119, Online. Association for Computational Linguistics.
A Experiment Setting

Dataset. Table 8 displays the statistical information of the datasets used in the experiment. The data of the two datasets partially overlap, so only the unique explanations are selected as the experimental data. The rationale for choosing explanatory sentences is that they are designed for formal/localised/symbolic semantic inference task in natural language form, which provides a semantically complex and yet controlled experimental setting, containing a both well-scoped and diverse set of target concepts and sentence structures, providing a semantically challenging yet sufficiently well-scoped scenario to evaluate the syntactic and semantic organisation of the space.

| Corpus                        | Num data | Avg. length |
|-------------------------------|----------|-------------|
| WorldTree (Jansen et al., 2018) | 11430    | 8.65        |
| EntailmentBank (Dalvi et al., 2021) | 5134     | 10.35       |

Table 8: Statistics from explanations datasets.

Table 9 illustrates the semantic, structure, and topic information of explanatory sentences over the latent space. The explanatory sentences are automatically annotated using the semantic role labelling (SRL) tool, which can be implemented via AllenNLP library (Gardner et al., 2017). We report in Table 10 the semantic roles from the explanations corpus.

Architecture. Figure 8 provides a visual representation of the connection between BERT and GPT2 within the AutoEncoder architecture.

![Figure 8: Latent sentence injection.](image)

To train the CVAE, we use a new embedding layer for semantic roles and separate MLP layers \( W^\text{sr}_\mu \) and \( W^\text{sr}_\sigma \) to learn prior distribution.

Hyperparameters. The training process of the decision tree binary classifier can be implemented via scikit-learn packages with default hyperparameters. As for Optimus, the latent space size is 32 in the experiment. The training details are following the original experiment from Optimus (Li et al., 2020b).

B Further Experimental Results

Traversal visualisation. PCA plots for ARG0, ARG1, and PRED are provided in Figure 9.

![Figure 9: PCA visualisation.](image)

In addition, we also provide the visualisation of word content animal with different semantic roles: ARG0, ARG1, ARG2, in Figure 10. From it, we can observe that the same content with different semantic roles can also be clustered and separated in latent space.

![Figure 10: Visualisation for animal-ARG0,1,2.](image)

Qualitative evaluation for arithmetic. Table 11 lists the traversed explanations after addition (blue) and subtraction (red) on different semantic role information. We can observe that the resulting sentences after addition can hold the same role-content as inputs, revealing latent space geometry.

Quantitative evaluation for arithmetic. Quantitative evaluation for our hypotheses via latent arithmetic. Both VERB and Object can perform high ratio after addition, indicating role-content separability.
Cluster | Theme and Pattern
--- | ---
0 | Theme: physics and chemistry. Pattern: *if then and as*. E.g., if a substance is mixed with another substance then those substances will undergo physical change.
1 | Theme: country, astronomy, and weather. E.g., new york state is on earth
2 | Theme: physics and chemistry. Pattern: *is a kind of*. E.g., light is a kind of wave.
3 | Theme: biology. E.g., a mother births offspring.
4 | Theme: synonym for verb. Pattern: *means and is similar to*. E.g., to report means to show.
5 | Theme: astronomy. E.g., the solar system contains asteroids.
6 | Theme: animal/plant. Pattern: *is a kind of*. E.g., a seed is a part of a plant.
7 | Theme: item. E.g., a telephone is a kind of electrical device for communication.
8 | Theme: synonym for life. Pattern: *means and is similar to*. E.g., shape is a kind of characteristic.
9 | Theme: geography. Pattern: *is a kind of*. E.g., a mountain is a kind of environment.
10 | Theme: animal and plant. Pattern: *if then and as*. E.g., if a habitat is removed then that habitat is destroyed.
11 | Theme: scientific knowledge. Pattern: (*; number and *). E.g., freezing point is a property of a (substance ; material).
12 | Theme: item. Pattern: *is a kind of object*. E.g., a paper is a kind of object.
13 | Theme: chemistry and astronomy. E.g., oxygen gas is made of only oxygen element.
14 | Theme: general about science. Pattern: (*). E.g., seed dispersal has a positive impact on (a plant ; a plant’s reproduction).
15 | Theme: item. Pattern: *is a kind of*. E.g., fertilizer is a kind of substance.
16 | Theme: physics and chemistry. Pattern: (*). E.g., the melting point of oxygen is -3618f ; -2188c ; 544k.
17 | Theme: animal. E.g., squirrels live in forests.
18 | Theme: nature. E.g., warm ocean currents move to cooler ocean regions by convection.
19 | Theme: life. E.g., pond water contains microscopic living organisms.

Table 9: Cluster Information.

| Semantic Tags | Prop. % | Description and Example |
|---------------|---------|-------------------------|
| ARGM-DIR      | 0.80    | Directionals. E.g. all waves transmit energy from one place to another |
| ARGM-PNC      | 0.08    | Purpose. E.g. many animals blend in with their environment to not be seen by predators |
| ARGM-CAU      | 0.05    | Cause. E.g. cold environments sometimes are white in color from being covered in snow |
| ARGM-PRP      | 1.30    | Purpose. E.g. a pot is made of metal for cooking |
| ARGM-EXT      | 0.04    | Extent. E.g. as the amount of oxygen exposed to a fire increases the fire will burn longer |
| ARGM-LOC      | 4.50    | Location. E.g. a solute can be dissolved in a solvent when they are combined |
| ARGM-MNR      | 2.00    | Manner. E.g. fast means quickly |
| ARGM-MOD      | 9.80    | Modal verbs. E.g. atom can not be divided into smaller substances |
| ARGM-DIS      | 0.07    | Discourse. E.g. if something required by an organism is depleted then that organism must replenish that something |
| ARGM-GOL      | 0.20    | Goal. E.g. We flew to Chicago |
| ARGM-NEG      | 1.20    | Negation. E.g. cactus wrens building nests in cholla cacti does not harm the cholla cacti |
| ARGM-ADV      | 6.70    | Adverbials |
| ARGM-PRD      | 0.20    | Markers of secondary predication. E.g. |
| ARGM-TMP      | 7.00    | Temporals. E.g. a predator usually kills its prey to eat it |
| O             | -       | Empty tag. |
| V             | 100     | Verb. |
| ARG0          | 32.0    | Agent or Causer. E.g. rabbits eat plants |
| ARG1          | 98.5    | Patient or Theme. E.g. rabbits eat plants |
| ARG2          | 60.9    | indirect object / beneficiary / instrument / attribute / end state. E.g. animals are organisms |
| ARG3          | 0.60    | start point / beneficiary / instrument / attribute. E.g. sleeping bags are designed to keep people warm |
| ARG4          | 0.10    | end point. E.g. when water falls from the sky that water usually returns to the soil |

Table 10: Semantic Role Labels that appears in explanations corpus.
### ADD and SUB arithmetic

| ARGUMENT 1:                  |
|-----------------------------|
| a needle is a kind of object|
| a tire is a kind of object  |
| a wire is a kind of object  |
| a stick is a kind of object |
| a ball is a kind of object  |
| a serotype is similar to intersex egg |
| a zygote contains many cell types |
| an xylem is made of two clumps |

| VERB:                       |
|-----------------------------|
| chromosomes are located in the cells |
| Australia is located in the southern hemisphere |
| stars are located in the solar system |
| Jupiter is located in the Milky way galaxy |
| aurora is located in the constellation of Leo |
| a crystal is made of metal |
| an alloy is made of iron and zinc |
| an aluminum plug is nonmagnetic |

| LOCATION:                   |
|-----------------------------|
| volcanoes are often found under oceans |
| mosquitoes can sense carbon dioxide in the air |
| polar ice sheets are located along rivers |
| hurricanes occur frequently along the coast in Africa |
| tide waves cause flooding in coastal waters |
| valley is a kind of location |
| shape is a property of rocks |
| desert is a kind of place |

| TEMPORAL:                   |
|-----------------------------|
| as the population of prey decreases competition between predators will increase |
| as competition for resources decreases the ability to compete for resources will increase |
| as the population of an environment decreases ecosystem function will decrease |
| as the spread of available air mass increases the population will increase |
| as the number of heavy traffic required increases the traffic cycle will decrease |
| some types of lizards live in water |
| a rose is rich in potassium |
| a fern grass roots foot trait means a fern grass |

| NEGATION:                   |
|-----------------------------|
| pluto has not cleared its orbit |
| sound can not travel through a vacuum |
| radio waves don’t have electric charge |
| electromagnetic radiation does not have a neutral electric charge |
| electromagnetic radiation contains no electric charge |
| Mars is a kind of moon / planet |
| Anothermic rock is a kind of metamorphic rock |
| Anal Cetus’s skeleton is a kind of fossil |

| Table 11: Latent sapce arithmetic for five semantic tags (blue: addition, red: subtraction). |
Figure 11: Predicate (VERB). The content shows the high ratio after subtraction, indicating that the V-is is widely distributed over the latent space.

Figure 12: Object (ARG1).
Figure 13: Cosine distance of sentence pairs in VERB-content clusters.

Figure 14: Cosine distance of sentence pairs in ARG1-content clusters.