Trends for ‘wiskunde’ or ‘wiskunst’? the case of students’ problem solving on elementary math problem (a little practical review from ‘revisiting mathematics education’)

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Abstract. This paper is a little practical reflection of the idea proposed by Freudenthal in his book titled ‘Revisiting Mathematics Education’. Theoretically, a fundamental study of more philosophical mathematical meanings was discussed by Freudenthal through the terms ‘wiskunde’ and ‘wiskunst’. In essence, wiskunde means mathematics as an exact science, while wiskunst means an art in mathematics and science. Although there has been a lot of research that shows the success of RME in learning, the results of research that can contrast explicitly how the wiskunde and wiskunst phenomena work in the process of solving mathematical problems are still rarely found. This study aims to see the tendency of students towards the phenomenon of wiskunde or wiskunst in working on elementary math problems then describe the psychological factors and impacts that might be influential. This study used literature review and qualitative methods through the interview process for students who were given two elementary school math problems. Four out of forty students from fourth semester were chosen to be interview respondents to represent variations of the unique answers. Data analysis was carried out through a hermeneutic approach to student answers and interview transcripts. Research shows that the wiskunde phenomenon looks more dominant than wiskunst. This is thought to be related to the symptoms of psychological blockages of student’s college who are predominantly working on elementary math problems still using the very formal formulas, they should still be able to show a way of thinking like an elementary school kid. The impact of this wiskunde tendency indicates the ‘matherate’ of students who have become blunt. The contrast of wiskunde and wiskunst's character and its reflection to the philosophy of education are discussed boldly in this paper.

1. Introduction
In the book ‘Reviewing Mathematics Education’, Freudenthal began the conversation by questioning the resolution of mathematics. This seems intended as a critical effort in reviewing mathematics education at that time. ‘Mathematics’ in French "Les Mathématiques" looks like a plural term. While according to people, free liberals, mathematics is free of quadrivium which contains four elements of arithmetic, geometry, astronomy, and music. In terms of language, mathematics comes from the Dutch language, namely Wiskunde, the science of something definite. Wis en zeker mean sure and certain, is something that produces no doubt; and Kunde meaning of knowledge, science, or theory [1].
It is clear that Freudenthal wants to explore a strong mindset about the point of view of mathematics itself by looking for a view that is more philosophical in interpreting mathematics than just a common sense. For this reason, Freudenthal refers to Stevin (1548-1620) who preferred to use the term Wiskunst rather than Wiskunde. Wiskunst means art in science, which more precisely refers to a mental art context where people feel closer to what they are doing as a process rather than prioritizing mathematics itself [1,2]. In other words, Freudenthal said that mathematics working in a context would be more meaningful for someone in building knowledge.

His book illustrates how Freudenthal is a philosophical figure. His expertise background is in mathematics science, not education. But with a strong philosophical basis, he questioned how the development of mathematics education at that time. Even in the ‘mathematization’ sub-chapter, he describes the current mathematical conditions that are trapped in modern mathematics where people prefer to work with something very abstract as a result of reasoning-game. As a result, mathematics looks very formal. Most people are taught mathematics as a set of rules or algorithms only, the someone's satisfaction seems measurable if they mastered it and they will be disappointed when they failed when doing math. In fact, not every problem is successfully resolved in a method/algorithm with one success immediately.

The above philosophical foundation is reinforced by Freudenthal through fundamental questions about differences in concepts and mental objects, such as, numbers and concepts of numbers, heat and concepts of heat, triangles and concepts of triangles, etc. Even mentioned in this chapter about teaching and teaching concepts, where the concept of teaching tends to create an illusion to strengthen understanding, for example by providing applicative examples in everyday life. This concept also stated by some ethnomathematics researcher [3,4,13–15,5–12] as a semiotics mediation, even though on the other hand this was also critically commented by other researchers [16–18]. Freudenthal viewed cognition does not begin with concepts, but on the contrary, concepts are the results of cognition. It is this rationale that initiated Freudenthal’s idea to divide the mathematical process into two approaches namely horizontal and vertical mathematical.

According to Freudenthal, mathematization is a key process in mathematics education. First, mathematics is not only an activity in a mathematician, but it can also familiarize students with a mathematical approach to everyday rules. Second, mathematics connects the idea of rediscovery, a process in which students formalize their informal understanding and intuition. The process of rediscovery involves two aspects namely horizontal mathematical and vertical mathematical. Horizontal mathematization concerned on the process of transforming real/everyday problems into symbols. While vertical mathematical is a process that occurs within the scope of the mathematical symbol itself. Horizontal mathematization examples are identifying, formulating and visualizing problems in different ways by students. While the example of vertical mathematization is the presentation of relationships in formulas, refining and adjusting mathematical models, the use of different models, formulation of mathematical models and generalization.

A realistic approach is an approach that contains two types of mathematization. Where, learning begins with the informal stage, which is then students are invited to do mathematical in the real world which is represented in the world of symbols. After that, students can do vertical mathematical, namely the process of using models to reach more general conclusions. In other words, mathematics arises from the mathematical process. The perspective of a person in interpreting mathematics influences his perspective on mathematics education. So even people who interpret mathematics as deductive science or formula catalogs will interpret mathematics with the same spirit, and vice versa.

The purpose of this study is to see the extent of students college who have taken the realistic mathematics class in 6 weeks are able to work on basic math problems according to the way of thinking that should be psychologically (12-14 years old). The questions given are intended for students of that age based on standardization of mathematics based on PISA. Naturally, the questions that will be used by students will be set so that psychologically suitable to their age. Through the analysis of the answer sheet we can see which subjects work with sufficient reasoning and which students work in a standard / very inductive manner.
2. Method
This study uses two approaches, literature review and qualitative studies. The literature study was conducted by reviewing a book written by Freudenthal entitled Revisiting Mathematics Education. The literature review technique is conducted through an academic discussion forum divided into several scholar groups and then an open discussion is held to accommodate various criticisms, reinforcement, and responses. This study also employed a qualitative approach to data collection and interpretation. Qualitative designs are naturalistic to the extent that the research takes place in a real-world setting [19–21]. Qualitative methods are conducted by case study and interviewing four of the forty students who were previously given some basic level math questions and have done a realistic mathematics class. Student answer papers are examined and analyzed openly with the research team. There are 7 basic math questions with realistic types of questions that represent the levels of primary and secondary education. But in this article only the (two) case for basic math problems is discussed. The questions given can be solved in many ways both deductively and inductively. The question is made to see the tendency of students to work with psychology in accordance with the level of the question (in the question explained that students must assume themselves as students in grades 5-6). The tendency of students to work with the style of *wiskunde* or *wiskunst* will be seen from the problem solving chosen.

3. Result and Discussion
There are two basic level math questions given to IV semester students, see figure 1 and figure 6. The first problem relates to the system of linear equations of two simple variables that are displayed visually. The second problem is related to the comparison. In elementary school mathematics competency standards, these two questions are not listed in the elementary mathematics syllabus but are expressed as junior secondary level material. The students who are accustomed to math competitions in their schools usually deal with questions on this material. Based on psychological considerations, age, experience and knowledge this is considered very reasonable to be tested on fourth semester students.

3.1 The Case of Linear Equation

![Figure 1. Problem 1 (Elementary Math Problem: Two Variables Linear Equations System)](image)

### 3.1.1 Case of Andra (Pseudonym)
In the figure 2, andra has worked by making mathematical modeling in the form of 3 systems of two variables linear equations. At first, Andra had suspected that this problem was a system of two variable equations so that it was enough to write just two equations, so the solution could be found. But he was curious about the condition of the question which included 3 parallels, even though there were two variables. Finally, he tried to write it in full. In the end, Andra was able to work on the questions.
procedurally by elimination. This method was chosen because he mastered it well based on previous knowledge he had learned since high school.

**Figure 2. Andra’s Work**

3.1.2 Case of Tilda (Pseudonym)
Figure 3 is Tilda's work that shows a different way of thinking with Andra and is not algorithmic at all. She works with reasoning by capturing patterns from mathematical modeling that she made. Psychologically, Tilda shows the ability to be an actual elementary school child. This is indicated by the form of the equation she wrote. She did not use the variable letters to represent pencils and erasers at all, but she preferred boxes or symbol symbols to represent them. The results of the interview showed that there were other factors in the academic experience she had gone through. Before she joined mathematics class at her university, she had experience taking courses at another campus with an elementary school major with one year. This makes her more mastering psychology to think of as an elementary school child.

**Figure 3. Tilda’s Work**

3.1.3 Case of Rima (Pseudonym)
In figure 4, Rima's work clearly shows that she is almost trapped in a system of three-variable equations system. She considers that because there are three equations, so the question is a system of three-variable equations. But finally, after she did the calculation in the first stage and she got the value of one of the variables, she immediately realized that what she was doing was a system of two-variable equations.
The way she worked was the same as Andra, solving the problem as algorithmic as the elimination method. Psychologically, Rima's work does not show the proper way of thinking based on age criteria.

3.1.4 Case of Adlan (Pseudonym)

Figure 5 shows that Adlan works in a very formal way. He wrote the system of equations in the form of matrices and worked on them with the method of Gauss's Elimination. He forgot that he was asked to position himself as an elementary school child who was doing mathematics. Even worse, he made a mistake in the calculation. Technically, he understands the procedure for working with the method used, but operationally it is less thorough in the calculation as a result he is stuck with complicated decimal calculations. This situation illustrates a very complex but not precise way of thinking.
and worthy of thought, they say "as long as this method is correct, it doesn't matter". Really this is not the expected reason. We were very forced to ask deeper, "is this related to what is taught in school?". Their answers tend to accuse this of the habit of thinking that they have often been trained to work their way. Especially if we look at the case of Adlan, he worked on the Gauss-Jordan method. Really this is a very algorithmic mathematical process. It seems that Adlan worked on the method because he had just studied the method in his course.

3.2 The Case of Comparation

3.2.1 Case of Andra (Pseudonym)

Figure 6. Problem 2 (Comparation)

Figure 7. Andras’ Work

Figure 7. shows that Andra works with heuristic strategies. He described that tap A in one minute is able to fill 1/48 volume and tap B in one minute can fill 1/80 volume, so that in 1 minute with both taps it can fill 8/240 volume. So, in two minutes it can fill 16/240 volumes and so on. Thus in 30 minutes he fulfills 1 volume of a full tub. There was one subject who did not answer and he was truly unable to work in this matter.

3.2.2 Case of Tilda (Pseudonym)

This question measures how the understanding of the concept of reverse comparation. The case on the question is able to measure how originality is thought of someone who relies heavily on memorizing concepts or does heuristic thinking. In figure 8, it appears that Tilda works with very natural reasoning. She made an unusual model, not in the form of algebra but with a heuristic strategy that she could understand. She tried to turn on both taps together, then the two met up to fill the full number of times in 240 minutes. She concluded that in the 240th minute the tub would have been filled with 8 full volumes so that each good for one volume was 30 minutes.
Figure 9 shows that Rima really works with the formula she memorizes. When interviewed how she obtained the formula, she said that this formula had been mastered by everyone well and was widely used in enriching students in Olympiad classes. She was unable to explain the process of how the formula was made, she justified it by giving examples through other questions. This case occurred in most of the forty students in the RME class.

3.2.4 Case of Adlan (Pseudonym)
Figure 10 shows that Adlan experienced a mathematical misconception. He forgets that the speed of the second tap is not the same. But in other conditions, he wrote the correct formula about the ratio of reversing values with the wrong calculation. The author does not pay much attention to miscalculation, but far from that the most important thing is to pay attention to the failure to understand the concept. This case is enough to show that someone who memorizes a concept and formula can still fail to understand the problem. This is in accordance with Freudenthal's previous opinion that algorithmic proficiency might deny its authenticity [1].
From the discussed cases, we can see the tendency of students to solve some very open-ended problems. Actually, the above questions can be solved in a more artful way, but that requires good reasoning and communication in making mathematical modeling. Most students prefer to work algorithmically and formally. We are very aware that this is not to be contradicted between right and wrong, because substantially all legitimate methods are used if the answer is correct. But far from that, the author sees another view of how one's instincts work in mathematics. From this case, it is clear that someone who knows a formula for solving problems does not need to work in another way, even though this is not so expected. This condition reflects a psychological obstacle as explained by Ambrosio [22].

Ambrosio stated: Before and outside school almost all children in the world become 'matherate'-that is to say, they develop their mathematics ability. In school 'the 'learned' mathercy eliminates the so-called 'spontaneous' mathercy. An individual who manages perfectly well numbers, operations, geometric forms and notions, when facing a completely new and formal approach to the same facts and needs creates a psychological blockage which grows as a barrier between different modes of numerical and geometrical thought'. As a consequence, 'the early stages of mathematics education offer a very efficient way of instilling in the children a sense of failure and dependency' [23].

To strengthen this argument, we look at the student's note (on their answer sheet) in mathematical work. Here are a few student strokes when counting

Figure 11 are some examples of how students can do the multiplication process. The author sees a phenomenon of how they are very dependent on an algorithm even though the questions are simple and can be calculated without a pencil. The author thinks that this is a proof of what D'Ambrosio meant [22]. When they calculate $12 \times 21$, they do not need to do calculations with standard procedures. They can use the property of distribution $(12 \times 20) + (12 \times 1)$ or $(21 \times 10) + (21 \times 2)$ in their imagination. This habit of thinking is always used by indigenous people in Ciamis whose academic history is only up to elementary school level [7]. We consider psychological blockages as a result of ways of thinking that are too dependent on the formulas that are owned as identified by researchers in the interview stage. In formal school they are taught multiplication with a very algorithm and they are used to it so even for simple cases they tend to use the algorithm. In this research issue, this condition shows that the wiskuntrend is not as strong as wiskunde.
This situation can be reflected in the conditions of the Indonesian Mathematics Education curriculum. Mathematics education oriented to mastery learning is prioritized in the process of ‘active learning’ to build knowledge through meaningful learning. On the other hand, the Indonesian curriculum provides a very large list of topics that must be covered in semester programs, with limited time. In fact, an effective learning process cannot be done in a hurry. Whereas to apply mathematics as wiskunst is not as easy as applying mathematics as wiskunde. In other words, it is as if the mathematics teacher is faced with the reality of mastery learning versus coverage. The author sees the concept of intertwining proposed by Freudenthal can be an alternative to combining several mathematical topics in thematic learning. This is exemplified by Mathematics in Context [24]. Nevertheless, critical and in-depth research needs to be continuously developed to achieve a more comprehensive and tested mathematical learning model.

4. Conclusion
Three of the four students explored showed a tendency to work mathematics with wiskunde characters, only one student described wiskunst characters. The four students have joined the RME class and they have understood what Wiskunde and Wiskunst are. They have also been taught how to apply psychology in learning (mathematical thinking in accordance with the characteristics of the problem), but they failed. Interviews showed that they considered procedural work with a formula to be considered sufficient for a mathematician to solve a problem. Art in mathematics is quite difficult for students to have. Psychological barriers occur due to the overly formal approach they usually receive in everyday learning. The case is that they are rarely asked to work on a problem in a new way but are more often faced with using a formula in a new problem. This hypothetical allegation requires a lot of validation; therefore, we sincerely hope that many future researchers will discuss this issue.

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References
[1] Freudenthal H 2002 Revisiting Mathematics Education (Translated by China Lectures) (Dordrecht, Boston, London: Kluwer Academic Publisher.)
[2] Stevin S 1995 De begrippen axioma en axiomatiek Wis-en Natuurkd. 39 156–75
[3] Bakker A and Hoffmann M H G 2005 Diagrammatic Reasoning as The Basis for Developing Concepts: A Semiotic Analysis of Students’ Learning about Statistical Distribution Educ. Stud. Math. 60 333–58
[4] Bussi M G B and Mariotti M A 2008 28 Semiotic mediation in the mathematics classroom Artifacts and signs after a Vygotskian perspective 746–83
[5] Yulianto E 2019 Merancang dan Mengenal Keindahan Fractal dalam Pembelajaran Matematika (Universitas Siliwangi) pp 123–38
[6] Yulianto E, Prabawanto S and Sabandar J 2019 Pola matematis dan sejarah batik sukupura : Sebuah kajian semiotika J. Penelit. Pendidik. dan Pengajaran Mat. 4 15–30
[7] Muzdalipah I and Yulianto E 2018 ETHNOMATHEMATICS STUDY : THE TECHNIQUE OF COUNTING FISH SEEDS (OSPHRONEMUS GOURAMY) J. Medives 2 25–40
[8] Font V, Godino J D and Amore B D 2007 An Onto-Semiotic Approach to Representations in Mathematics Education Learn. Math.
[9] Radford L 2002 The Seen, The Spoken and The Written: A Semiotic Approach to The Problem of Objectification of Mathematical Knowledge [ 1 ] Learn. Math. 22 14–23
[10] Radford L 2007 Iconicity and Contraction: A Semiotic Investigation of Forms of Algebraic Generalizations of Patterns in Different Contexts ZDM Math. Educ. 40 83–96
[11] Radford L 2000 Signs and Meanings in Students’ Emergent Algebraic Thinking: A Semiotic Analysis Educ. Stud. Math. 42 237–68
[12] Radford L 2010 Algebraic Thinking from A Cultural Semiotic Perspective Res. Math. Educ. 12 1–19
[13] Ernest P 2008 Towards A Semiotics of Mathematical Text (Part3*) Learn. Math. 28 42–9
[14] Ernest P 2006 A semiotic perspective of mathematical activity: the case of number Educ. Stud. Math. 61 67–101
[15] Winsløw C 2004 Semiotics as an Analytic Tool for The Didactics of Mathematics Nord. Stud. Math. Educ. 9 1–15
[16] Pais A 2016 At the intersection between the subject and the political: a contribution to an ongoing discussion Educ. Stud. Math. 92 347–59
[17] Pais A 2011 Criticisms and contradictions of ethnomathematics Educ. Stud. Math. 76 209–30
[18] Pais A 2013 An ideology critique of the use-value of mathematics Educ. Stud. Math. 84 15–34
[19] Creswell J W 2007 Qualitative Inquiry and Research Design: Choosing Among Five Approaches vol 2nd ed (Thousand Oaks: Sage Publication)
[20] Creswell J W 2015 Penelitian Kualitatif dan Desain Riset (Yogyakarta: Pustaka Belajar)
[21] Creswell J W 2007 Chapter 3: Designing a Qualitative Study Qual. Inq. Res. Des. Choos. among five approaches 35–41
[22] Gerdes P 1996 Ethnomathematics and Mathematics Education International Handbook of Mathematics Education ed A J Bishop (Netherlands: Springer Netherlands) pp 909–43
[23] d’Ambrosio U 1985 Ethnomathematics and its place in the history and pedagogy of mathematics Learn. Math. 5 44–8