Supergravity with broken Lorentz invariance: theory and phenomenological consequences

Arthur Marakulin1,* and Sergey Sibiryakov1,2,3,⩾⩾

1Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia
2CERN Theory division, Geneva, Switzerland;
3Institut de Physique, EPFL, Lausanne, Switzerland

Abstract. We construct and study supersymmetric extension of the Einstein-aether gravitational theory. Using superfield formalism we carry out unique linearized Lorentz violating supergravity action invariant under super-gauge transformations and obtain its bosonic part expanding in the Grassman coordinates. The model is considered also at the component level. We study supercurrent supermultiplet in the theory as an alternative way to prove its uniqueness and to construct fermionic interaction in the model. The phenomenological consequences of the theory and experimental constraints on the coupling constant are also discussed.

1 Introduction

Recently theory and phenomenology of possible Lorentz violation at high energy scales became popular subjects of research in modern theoretical physics [1]. Several modified theories of gravity involving violation of the local Lorentz invariance based on the model suggested by Hořava [2] are widely discussed in the context of quantization of gravity [3, 4]. In this paper we consider Einstein-aether theory of gravity [1] which is closely related to the latter cases. In this model Lorentz violation is based on the dynamical timelike vector field called “aether”. Aether is constrained by the condition

\[ u^m u_m = -1 \]  

The most general action of the aether minimally interacting with Einstein gravity

\[ S = S_{GR} - \frac{1}{2} \int d^4 x \sqrt{-g} \left( c_1 (\nabla_m u_m)^2 + c_2 (\nabla_m u_m)^2 + c_3 \nabla_m u_m \nabla^n u^n - c_4 u^r u^s \nabla_r u_m \nabla_s u_m \right) \]  

contains four free parameters. These coupling constants of the Einstein-aether theory are related to PPN-parameters [1]

\[ \alpha_1 = -\frac{c_2^2 + c_1 c_4}{2c_1 - c_1^2 + c_3^2} \]  

* e-mail: marakulin@physics.msu.ru
⩾⩾ e-mail: sergey.sibiryakov@cern.ch
\[ \alpha_2 = \frac{\alpha_1}{2} - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_3 + c_3 + c_4)}{(c_1 + c_2 + c_3)(2 - c_1 - c_4)}, \]  

which are strictly constrained by the experimental data [7]:

\[ |\alpha_1| \lesssim 10^{-4}; \quad |\alpha_2| \lesssim 4 \times 10^{-7} \]  

Dimensionless constants \( c_i (i = 1, 2, 3, 4) \) are also under the same constraints in general case:

\[ c_i \lesssim 10^{-7}. \]  

The natural question is whether the Einstein-aether gravity is compatible with supersymmetry which can be considering as one of the mechanisms leading to emergent Lorentz invariance even if the theory includes Lorentz violation at large energy scales [8, 9]. Constructing of the supersymmetric extension of the aether was the first step in answering it. The authors of the paper [8] have suggested the supersymmetric version of the aether as the vector extension of well-known chiral superfield

\[ U^m = u^m(x_L) + \sqrt{2}\theta h^m(x_L) + \theta^2 G^m(x_L) \]  

where \( x_L^m = x^m + i\theta\sigma^m \bar{\theta}, \theta^\alpha \). The unit norm constraint on the super-aether is similar to the ordinary case:

\[ U m U^m = -1 \]  

The most general action of the model under consideration in terms of superfields

\[ S = \int d^8z f (U m U^m) + \int d^6z \Lambda (U m U^m + 1) \]  

leads to the theoretical constraints on the aether Lagrangian parameters:

\[ c_2 = -c_3, \]  

\[ c_4 = 0. \]  

In the present paper we construct the theory of supergravity with broken Lorentz invariance. First of all we discuss so-called \( N = 1 \) non-minimal supergravity model which allows to define chiral vector superfield in curved superspace (\( \bar{\nabla}_h U^m = 0 \)). Using superfield formalism we obtain the unique Lagrangian invariant under supergauge transformations and carry out new theoretical constraints on the general theory parameters expanding it in the Grassmann variables. Then we consider an alternative way to the same result using supercurrent formalism in terms of component fields. Phenomenological consequences from the constructed model are also discussed.

### 2 Supergravity with Lorentz violation

The action of the linearized non-minimal supergravity is given by [10]

\[ S_{SG} = \frac{1}{k^2} \int d^8z f \left( \frac{1}{4} \left( \partial_h H_m \right)^2 - (\Delta_h H_m)^2 \right) + \frac{n + 1}{2n} (\partial_m H^m)^2 + \frac{n + 1}{2} (\Delta_m H^m)^2 - i \frac{3n + 1}{2n} \partial_m H^m \left( \Gamma - \bar{\Gamma} \right) + \frac{3n + 1}{2} - \Delta_m H^m \left( \Gamma + \bar{\Gamma} \right) + \frac{9n^2 - 1}{8n} (\Gamma^2 + \bar{\Gamma}^2) + \frac{(3n + 1)^2}{4n} \Gamma \bar{\Gamma}. \]  


where $H^m$ is real gravitational superfield, $\Gamma$ is linear compensator ($\bar{D}^2\Gamma = 0$), $\Delta_k H_m = \frac{1}{2} \bar{\theta}^m \gamma^k [D_m, D_n] H_n$ and $n$ is Siegel parameter. It is invariant under transformations

$$
\delta H_m = \bar{D}_o L_o - D_n \bar{L}_n
$$

and

$$
\delta \Gamma = -\frac{n+n}{4(3n+1)} \bar{D}_m D^2 L_n + \frac{1}{4} \bar{D}_o D^2 L_o
$$

Let’s consider the most general coupling of the chiral vector superfield $U^a$ with the non-minimal supergravity superfields. We construct linearized. Super-aether is expanded as consisting of the real VEV and perturbation:

$$
U^a = w^a + V^a
$$

and its gauge transformation is given by

$$
\delta V^a = u^b M_{ab}^a
$$

where

$$
M_{ab} = \frac{1}{4} (\sigma_{ab})_\beta^\alpha D_\alpha \bar{D}^2 L_\beta + \frac{1}{4} (\bar{\sigma}_{ab})^\alpha_\beta \bar{D}^2 L_\alpha
$$

yields the chirality condition

$$
\bar{D}_\alpha V^c = -u^b \Phi_{abc}^c,
$$

where the superconnection is given by the expression

$$
\Phi_{abc} = -\frac{1}{4} (\sigma_{bc})^\beta_\alpha \bar{D}^2 D^\alpha H_\beta - (\bar{\sigma}_{bc})^\beta_\alpha \bar{D} \bar{D} \Gamma.
$$

The most general action for the Lorentz violating supergravity model in terms of aether and gravitational superfields which is invariant under its supergauge transformations reads:

$$
S_{ae} = C \frac{1}{2k^2} \int d^8 z \left[ V_a \bar{V}^a + iw^a u^b \partial_a H_b (\Gamma - \bar{\Gamma}) + w^a u^b \Delta_a H_b (\Gamma + \bar{\Gamma}) + \frac{1}{4} (\Delta_k H_m \Delta^k H^m - \partial_k H_m \bar{\partial}^k H^m - (\Delta_m H^m)^2 + (\partial_m H^m)^2) + \frac{i}{4} \partial_m H^m (\Gamma - \bar{\Gamma}) + \frac{1}{4} \Delta_m H^m (\Gamma + \bar{\Gamma}) + \frac{3}{8} (\Gamma^2 + \bar{\Gamma}^2) \right].
$$

The action can be obtained as the result of solving the system of linear equations for the coefficients in front of all the possible quadratic terms constructed from the relevant superfields (see [11]). Note that it contains only one dimensionless coupling constant $C$.

### 3 From superfields to component fields

Now we write down the bosonic part of the Einstein-aether supergravity Lagrangian terms of component fields expanding superfields in the Grassmann coordinates. The real gravitational superfield expansion terms with bosonic components fields in the WZ-gauge are given by

$$
H_m(x, \theta, \bar{\theta})_{box} = \frac{1}{2} \partial r^a \bar{\partial} e_m (x) + \theta^2 \bar{\theta}^n d_m (x);
$$

where we made the tetrad symmetric ($e_{mn} = e_{nm} = \frac{1}{2} h_{mn}$) by the local Lorentz transformations.
The same expressions for the linear compensator \( x'^m = x^m + i\theta\sigma^m \tilde{\theta}, \theta^4 \)

\[
\Gamma \left( x_L, \theta, \tilde{\theta} \right)_{\text{boson}} = \frac{n + 1}{2(3n + 1)} h(x_L) + \theta^2 B(x_L) + \theta \sigma^m \tilde{\theta} (v_a(x_L) + i\mu_a(x_L)) ;
\] (22)

and the super-aether

\[
V_b \left( x_L, \theta, \tilde{\theta} \right)_{\text{boson}} = v_b(x_L) + \theta^2 G_b(x_L) + \theta \sigma^k \tilde{\theta} f_{bk}(x_L),
\] (23)

where \( v_a = v^b_a + i v^b_a \) is the first order aether perturbation and

\[
f_{bk} = w_c \partial_\mu \partial_\nu \left( \sigma_{bc} \sigma_{pk} \right) + \frac{i}{2} w_c \partial_\mu \partial_{\nu,\mu} \left( \sigma_{bc} \sigma_{pk} \sigma_{pk} \right) + w_c (v_c + i \mu_c) \partial_\mu \partial_{\nu,\mu} \tilde{\theta} \sigma_{kp} .
\] (24)

are written using "left" coordinates \( x'^m = x^m + i\theta\sigma^m \tilde{\theta}, \theta^4 \).

As a result, we obtain the bosonic part of the Lagrangian in components after integrating out the auxiliary fields under the assumption \( C \ll 1 \) through order \( O(\sqrt{C}) \) and canonically normalizing the aether perturbations to make their leading kinetic term become of order 1: \( \theta^a \theta^I \rightarrow \theta^a \theta^I = \sqrt{C} \theta^a \theta^I \) [11]:

\[
L = \frac{1}{2\gamma^2} \left\{ \frac{1}{3} h_{mn} h^{mn} + \frac{1}{2} \partial^b h_{mn} \partial_b h^{mn} - \frac{1}{2} \partial_a h^{mn} \partial_a h_{mn} \right. + \\
+ \frac{1}{4} \partial_a \partial_b h^{mn} \partial_c \partial_d h_{mn} - \partial_a \partial_b h_{mn} \partial_c \partial_d h^{mn} - \partial_a \partial_b h^{mn} \partial_c \partial_d h_{mn} + \\
- \frac{C}{4} (w^b \theta^a \theta^I) \partial_\alpha \partial_\beta \epsilon_{\alpha\beta \gamma \lambda} \partial_\gamma \theta^I \theta^j m + O(C^{3/2}) \right\} ;
\] (25)

4. Supercurrent multiplet in the aether theory

An alternative way for constructing the Lagrangian and showing its uniqueness is Noether procedure and the supercurrent constructing as the first step of it. Let's write down on-shell supersymmetric aether Lagrangian

\[
L = c_1 \left( - \partial_m a_0 \partial_m a_0 + i \eta \partial_m \tilde{\eta} \tilde{\partial}_m \tilde{a} + \text{h. c.} \right)
\] (26)

It is invariant under super-gauge transformations

\[
\delta_\xi u_b = \xi \eta_b
\] (27)

\[
\delta_\xi \eta_{0b} = 2i (\sigma_a \xi)_{0b} \partial_m a_b
\] (28)

with respect to the equations of motion: \( \tilde{\partial}^2 u_{0b} = 0, \sigma^a \partial_m \eta_0 = 0 \).

Let us also consider the aether field as consisting of the real vacuum and small perturbation:

\[
u_b = u_b + v_b
\] (29)
Every supersymmetric theory contains conserving supercurrent. Superscurrent is connected with the Einstein energy-momentum tensor by the supersymmetric transformations in the supercurrent supermultiplet[12, 13]:

$$\delta \xi \Sigma_{\alpha} = 2 \left( \sigma^{\mu} \xi \right)_{\alpha} T_{mn} + \text{o. s. t.}$$

$$\delta \xi T_{mn} = i \left( \xi \sigma_{mp} \partial^{p} \Sigma_{n} + \xi \sigma_{np} \partial^{p} \Sigma_{m} \right) + \text{h. c.},$$

and the first order coupling condition takes the form

$$L_{\text{int}}^{(1)} = h_{mn} T_{mn} + \Psi^{n} \Sigma_{n} + \text{h. c.}$$

Einstein energy-momentum tensor in the super-aether theory takes the form

$$T_{ab} = c_{1} + c_{3} w_{k} \partial_{k} v_{a} - c_{1} + c_{3} w_{b} \partial_{k} v_{b} + c_{1} - c_{3} w_{a} \partial_{k} v_{b} - c_{3} g_{ab} w_{k} \partial_{k} v_{b} + (a \leftrightarrow b) + \text{h. c.}$$

Let’s carry out the conserving supercurrent in the supersymmetric extension of the Einstein-aether theory. The conserving condition

$$\partial_{b} \Sigma_{b} = 0$$

leads to the most general form of the supercurrent :

$$\Sigma_{b} = C_{1} \left( w_{k} \partial_{k} \eta_{b} - w_{b} \partial_{k} \eta_{k} \right) + C_{2} \left( w_{k} \sigma_{kp} \partial_{k} \eta_{p} + w_{p} \sigma_{kp} \partial_{k} \eta_{b} \right) +
+ C_{3} \left( w_{k} \sigma_{kp} \partial_{p} \eta_{b} - w_{b} \sigma_{kp} \partial_{p} \eta_{k} \right) + C_{4} \left( w_{k} \epsilon_{kpmn} \sigma_{pb} \partial_{m} \eta_{n} - w_{k} \epsilon_{kpmn} \sigma_{pm} \partial_{b} \eta_{n} \right) +
+ C_{5} w_{k} \epsilon_{kpmn} \sigma_{kp} \partial_{m} \eta_{n} + C_{6} w_{b} \epsilon_{kpmn} \partial_{k} \eta_{n} + C_{7} w_{b} \epsilon_{mkn} \sigma_{mp} \partial_{n} \eta_{b} + \text{h. c.}$$

The next step is quite similar to the superfield Lagrangian constructing procedure. To carry out the supercurrent of the theory one have to write down supersymmetric transformations of all possible independent parts of it and also supersymmetric transformations of the energy-momentum tensor. Coefficients in these transformations define the system of linear equation to be solved. As a result, this system can be solved only with the condition

$$c_{3} = 0,$$

leading to the unique solution

$$C_{1} = \frac{i}{2} c_{1}, C_{2} = ... = C_{7} = 0,$$

This leads to the fermionic first order coupling in the theory [13]

$$L_{\text{ferm}}^{(1)} \sim w_{b} \eta_{b} \Psi^{b} + \text{h. c.}$$
5 Discussion

As it has been already mentioned, the theory is described by a single coupling constant $C$. In the real world supersymmetry must be broken. As it was shown [8] is the supersymmetry in the aether theory is broken softly it leads to the mass of the imaginary part of the perturbations, but the real part stays massless. In the low energy limit the theory becomes Einstein-aether gravity model with certain values of parameters constrained by supersymmetry. Thus, the Lagrangian (25) coincides with the quadratic part of the $2$ with $c_1 = C$, $c_2 = c_3 = c_4 = 0$.

This leads to the particular expressions for the PPN-parameters

$$
\alpha_1 = 0, \quad \alpha_2 = -\frac{2C}{2-C},
$$

(39)

and (5) translates into constraint on $C$:

$$
C \lesssim 10^{-7}.
$$

(40)

Another phenomenological consequence concerns the spectrum of modes described by the bosonic part of Einstein-aether supergravity model. It is shown that the velocities of small perturbations in the relevant theory is only luminal or superluminal[11]. For the graviton modes

$$
s^2 = 1 + C,
$$

(41)

where $s$ describes the linear dispersion relation in the theory:

$$
E = s \cdot p.
$$

(42)

Recent observations of the gravitational waves let us find out the most strict experimental constraints of the theory [14]. From the comparison of graviton velocity with the speed of light[15] one can obtain that

$$
C \lesssim 10^{-15}.
$$

(43)

The next steps of studying the supersymmetric Lorentz violating gravity will be its generalization to the non-linear case using notifications of[16, 17], possible applications to cosmology. We leave it to the future papers.

The work has been supported by the Russian Science Foundation grant 14-22-00161.

References

[1] T. Jacobson, PoS QG-PH, 020 (2007).
[2] P. Horava, Physical Review D, 79, 8, 084008 (2009).
[3] D. Blas, O. Pujolas, O., and S. Sibiryakov, (2009). JHEP, 0910, 029 (2009)
[4] D. Blas, O. Pujolas, O., and S. Sibiryakov, Phys. Rev. Lett. 104, 181302, (2010)
[5] T. Jacobson, Phys. Rev. D 81, 101502 (2010) Erratum: [Phys. Rev. D 82, 129901 (2010)],
Phys. Rev. D 89, 081501 (2014)
[6] D. Blas, O. Pujolas and S. Sibiryakov, JHEP 1104, 018 (2011)
[7] B. Z. Foster and T. Jacobson. Physical Review D, 73, 6, 064015, 1-9 (2006).
[8] O. Pujolas and S. Sibiryakov, JHEP, 1201, 062 (2012).
[9] S. Groot Nibbelink and M. Pospelov, Phys. Rev. Lett. 94, 081601 (2005).
[10] I. L. Buchbinder and S. M. Kuzenko, *Ideas and methods of supersymmetry and supergravity, or A walk through superspace*. (Institute of Physics Pub., 1998).
[11] A. Marakulin and S. Sibiryakov, arXiv:1610.07805.
[12] I. Ogievetsky and L. Mesinchesku, Usp. Fiz. Nauk. 117. No. 4 (1975)
[13] Z. Komargodski and N. Seiberg, JHEP, **1007** 017 (2010)
[14] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Mod. Phys. Lett. A **31**, 1650155 (2016)
[15] B. P. Abbott et al., Astrophys. J. **848**, No. 2, L13 (2017)
[16] G. Girardi, R. Grimm, M. Muller and J. Wess, Z. Phys. C **26**, 123 (1984).
[17] G. Girardi, M. Muller and J. Wess, Z. Phys. C **26**, 427 (1984).