Accounting for outliers and calendar effects in surrogate simulations of stock return sequences

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Abstract

Surrogate Data Analysis (SDA) is a statistical hypothesis testing framework for the determination of weak chaos in time series dynamics. Existing SDA procedures do not account properly for the rich structures observed in stock return sequences, attributed to the presence of heteroscedasticity, seasonal effects and outliers. In this paper we suggest a modification of the SDA framework, based on the robust estimation of location and scale parameters of mean-stationary time series and a probabilistic framework which deals with outliers. A demonstration on the NASDAQ Composite index daily returns shows that the proposed approach produces surrogates that faithfully reproduce the structure of the original series while being manifestations of linear-random dynamics.

Key words: Surrogate Data Analysis, Least Median of Squares, heteroscedasticity, Chaos, Financial Time Series Analysis.
1 Introduction

The search for nonlinear deterministic dynamics in stock market prices has been an intensive area\textsuperscript{1} for research, and especially active in the recent years with the advances in Econophysics (9; 10). The accurate determination of stock return dynamics and their distributional properties is of main concern here, as they can significantly improve portfolio formation and risk evaluation practices, as well as allow the fine tuning of asset valuation procedures.

There have been several indications that stock prices do not fluctuate as randomly as they should, according to the underlying theoretical equilibrium framework (see discussions in Ref. 11; 12; 13; 14), and exhibit rich and complex structures (15; 16; 17). However, earlier research (18; 19; 20; 21; 22; 23) has not provided a clear answer towards the presence or absence of nonlinear determinism and chaos. Hence, the candidacy of deterministic chaos as an alternative hypothesis to randomness, has not enjoyed popularity among the ranks of economists. Limitations posed by the quantity and quality of data, computational power and the absence of a widely acceptable and appropriate theoretical and statistical framework, have also been factors that contributed to the dispute against chaotic dynamics in finance and economics.

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\textsuperscript{1} For early influential work and discussions refer to (1; 2; 3; 4; 5; 6; 7; 8).
However, a Monte-Carlo simulation-based statistical hypothesis testing framework for detecting weak chaos, appears to have been ignored by and large till recently in finance. This framework is called Surrogate Data Analysis (SDA) and has preceded historically a significant amount of influential research of chaotic dynamics in finance and economics.

SDA (see section 3 for details on how this methodology works) has been primarily designed to ensure the validity of results of investigations for nonlinear determinism and the presence of weak chaos. Similar investigations have been mainly focusing on the examination of invariant measures, such as dimension based statistics for the characterization of strange attractors. However such measures have been shown to provide misleading, biased or inconclusive results, due to the presence of noise or the lack of sufficient observations in the data sets examined. Though SDA can provide the means to bypass some of the limitations posed by the quality and quantity of the sequences under examination, still the structure of their underlying dynamics and their noise content can pose serious considerations. The above discussion comes into context in the analysis of financial time series, where the nature of the data generating processes and the noise components are still largely unknown, while the “mechanics” and the equilibrium conditions of the market systems examined often appear empirically to be ill or loosely defined. Especially, the presence of heteroscedastic noise in stock returns and their nonstationary fluctuations among other stylized facts, can mask the presence of low dimensional nonlinear determinism. As mentioned earlier, the greatest disadvantage of the nonlinear statistics based on invariant measures is their lack of power, especially in financial applications. SDA enables us to
bypass this limitation. However, heteroscedasticity may render this exercise useless, as the existing surrogate methods are designed for homoscedastic time series. Their application on noisy and heteroscedastic sequences may lead to misleading results and biased or inaccurate conclusions (39; 40; 41). Since SDA is essentially a simulation of the linear characteristics of a time series, it should be able to deal with heteroscedasticity, outliers and calendar effects, which are major features of financial time series. In this paper we demonstrate how to modify one of the most advanced and popular surrogate methods, the Iterative Amplitude Adjusted Fourier Transformed surrogates (IAAFT) (42), in order to account for important stylized facts regarding heteroscedasticity, calendar effects and outliers in stock returns sequences.

2 Dealing with heteroscedasticity and outliers

A time series sequence is subject to heteroscedasticity when the variance is time-varying. Empirical research on stock returns has shown that from time to time the variance fluctuates, volatility appears to be clustering, while outliers appear in the time series, often attributed to exogenous factors and random events. The use of robust statistics is justified for the identification and characterization of the underlying dynamics. Robust statistics were developed principally during the 70’s with a few related but major methodologies appearing the following decade. In this paper we make use of the Least Median of Squares (LMS) concept introduced by (43; 44). The LMS estimator minimizes the median of the squared discrepancies rather than the mean, as in the Ordinary Least Squares (OLS) methodology. Hence, LMS estimators may produce results which are relatively immune to the presence of the outliers.
and the non-normality of the errors’ distribution. One disadvantage of LMS estimators is that they are considerably less efficient in the case of normally distributed errors. However, it is well established that the distributions of the first logarithmic differences of stock prices (i.e., logarithmic returns) fail normality tests, and exhibit strong leptokurtic features, and this justifies the applicability of the LMS concept.

Since outliers may pose considerations under the SDA framework, it is necessary to follow a policy for their classification and characterization. For the purposes of our approach here, in order to isolate the outliers of a given data set \( x \), we suggest the following steps in the spirit of the wider LMS literature:

1. Find the LMS location parameter of the data set:

   \[
   \text{loc} = \text{argmin}(\text{med}(x - \theta^2)),
   \tag{1}
   \]

   i.e., determine the value of a parameter \( \theta \) which minimizes the median of the squared deviations from the median. This can be easily achieved by sorting the data set and calculating the midpoint of the range of the 50% of the densest data.

2. Find the LMS scale parameter of the data set:

   \[
   \text{scale} = 1.4826 \times (1 + \frac{5}{N - 1}) \times \text{med}(r^2),
   \tag{2}
   \]

   where \( r \) is the residuals’ vector obtained from the previous step and the consistency constant 1.4826 comes from the square root of the median of the chi-square distribution with one degree of freedom (43). Hence, this scale parameter can be calculated once the LMS location parameter \( \theta \) is
(3) Calculate the $z_{LMS}$-score: $z_{LMS} = (x - \text{loc})/\text{scale}$ i.e., normalize the data according to the LMS concept.

Rousseeuw and Leroy (44) propose the following fuzzy model (see also Fig. 1) for determining the degree $\lambda$ of a residual not being an outlier:

- If $|z_{LMS}| \leq 2.0$ then $\lambda = 1.0$ and $x$ is not an outlier,
- if $2.0 < |z_{LMS}| \leq 3.0$ then $\lambda = 3.0 - |z_{LMS}|$, and $x$ is not an outlier with degree $\lambda$, and
- if $3.0 < |z_{LMS}|$ then $\lambda = 0.0$, and $x$ is an outlier.

Our approach converts the above fuzzy model to probabilistic. In other words, every time we run the surrogate data algorithm we consider a probability equal to the degree $\lambda$ that a data point $x$ is classified or not as an outlier. Thus, we classify as “outliers” values with a corresponding $|z_{LMS}|$ score more than 3.0, and as non-outliers the values with a corresponding $|z_{LMS}|$ score less than 2.0. A random number generator that produces uniformly distributed random values in [0,1] helps on the intermediate $|z_{LMS}|$ scores (i.e., scores between 2 and 3). For example if a data point $x$ has $|z_{LMS}|$ score of 2.8, a corresponding random number of 0.2 or greater will classify it as an outlier, while a corresponding random number less than 0.2 will not classify it as one.
3 The methodology of the Probabilistic IAAFT surrogates

The SDA methodology focuses on producing simulated sets from a sequence which capture only the linear properties of the original data. Then a discriminating pivotal statistic is chosen. Sufficient evidence for rejecting the null of linear stochastic dynamics is given when the value of the statistic calculated on the original data, differs significantly from its values obtained from the surrogate sets. The simulation procedures for generating surrogate data differ according to the null being considered. For example, a simple reshuffling of the original sequence can test for white noise, whereas more complicated reshuffling exercises may test for linearly filtered noise or monotonic nonlinear transformations of linearly filtered noise. Usually the last case is regarded as the most interesting, as the other procedures may produce spurious results in the presence of linearly correlated noise that has been transformed by a static, monotone nonlinearity. The SDA technique is different to the Bootstrap (45) as it refers to a constrained randomization simulation based hypothesis testing framework, found in permutation tests (46).

To test for the original sequence being a monotonic nonlinear transformation of linearly filtered noise, one has to simulate surrogates according to the following steps (47; 48):

1. Starting with the original sequence $x$, generate an individually and identically distributed (i.i.d.) Gaussian data set $y$ and reorder according to the ranking of $x_n$. In this way we can rescale the original sequence to a normal distribution.
2. Produce the Fourier transform of the rescaled sequence $y$ and assign a
random phase to each (positive) frequency.

(3) Take the inverse transform of above step’s sequence, say \(y^*\). This stage ensures that the surrogates will exhibit the same power spectrum as the originating sequence \(x\).

(4) Reorder the original data \(x\) to generate a surrogate \(x_s\) which will have the same rank distribution as \(y^*\). In this way we are certain that not only the spectrum but also the distribution of the original sequence \(x\) is preserved in \(x_s\).

The above surrogates are referred to as “Amplitude Adjusted Fourier Transformed” surrogates or AAFT for short. AAFT surrogates will have the same distributions and amplitudes with the original sequence but will not exhibit the same power spectra. To achieve the latter, an improved, iterative version of AAFT surrogates (termed IAAFT) has been proposed. To produce IAAFT surrogates one has to follow the steps below:

(1) Apply a Fourier transform to the original sequence \(x\) and save the amplitudes \(\alpha\). Produce a shuffled surrogate sequence \(x'_s\) from the original \(x\), apply a Fourier transform to \(x'_s\) and preserve the phases \(\phi\). Finally, construct a vector \(\vec{r}\) that contains the ranking of \(x\).

(2) Produce a phase randomized (AAFT) surrogate sequence \(x''_s\) combining \(\alpha\) and \(\phi\). Compare the rank orders of \(x''_s\) and \(\vec{r}\). If these are the same, proceed to the next step, otherwise the vector \(\vec{r}\) hosts the rankings of \(x''_s\), \(\phi\) hosts the phases of \(x''_s\), and the procedure of this step is repeated. This step can also be terminated if the maximum number of iterations defined by the user (e.g., 1000) is reached. Thus we avoid strong discrepancies between the surrogates and the original sequence’s spectrum.

(3) Force \(x''_s\) to follow the distribution of \(x\), by assigning on its indices the
corresponding values of $x$.

The IAAFT surrogates ensure that the main linear features of a time series will be faithfully preserved. However, the above procedure has been designed for stationary time series and therefore cannot cope with the presence of heteroscedasticity and outliers. In other words and with respect to the classification produced in section 2, the IAAFT surrogates have been designed for time series where all the observations are subject to $|z_{\text{LMS}}| \leq 2$. According to the proposed framework in this paper and in order to take into account the outliers that are observed in stock returns, we have to modify the surrogate generating algorithm according to the following steps:

1. Calculate the LMS location parameter of the time series.
2. Calculate the LMS scale parameter of the time series.
3. Calculate the $z_{\text{LMS}}$ for each observation.
4. Convert the $z_{\text{LMS}}$ to $\lambda$, according to section 2.
5. Create a new series of uniformly distributed random numbers in [0,1], say $u$, with length equal to the length of the original time series.
6. Create a new time series $x_s$, which contains all the values of $x$ that correspond to $\lambda_i \geq u_i$.
7. Apply the IAAFT surrogate algorithm to $x_s$.
8. The final surrogate sequence will preserve the values of $x$ that correspond to $\lambda_i < u_i$, in exactly the same positions as in the original sequence, and will receive the surrogate of $x_s$ for $\lambda_i \geq u_i$, to fill the remaining gaps.

Our experiments below show that according to the above procedure (termed Probabilistic IAAFT, or PIAAFT for short), the outliers, volatility clustering and hence heteroscedasticity can be faithfully reproduced with a “reason-
able” probability, according to their level of presence in the original sequence. Moreover, the rest of the desirable properties of the IAAFT surrogates are preserved.

4 Calendar Correction

So far we have described a surrogates generation procedure which is able to account for heteroscedasticity. In this section we also demonstrate how to account for the calendar effects. As a first step we have to define what we imply here by the term “calendar effects”. Since there is no universal definition, we presume eight kinds of calendar effects. The first five effects, and the least important ones, are the five weekdays. Next and of greater importance, the first and last trading days of a month (day-of-month) are being considered as calendar effects. Finally, we have the holiday effect, which is also assumed here to be the most important. For example, if a trading day can be characterized as both a pre-holiday and end-of-month day, the holiday effect applies. Following the same rationale, if a trading day is both a Thursday and the first day of a trading month, it is classified according to the latter effect.

In order to specialize the algorithm given in section 3, we have to reconsider its first 3 steps for the “calendar-wise” time series. To achieve it, we normalize (using the LMS parameters) every calendar-wise distribution. The rest of the steps are followed without any change, save for the 7th step which has to be adapted according to the calendar structure of the time series. This procedure is the Calendar Corrected version of the PIAAFT (hence CCPIAAF...
5 Empirical Results

This section compares the surrogates produced by the proposed CCPIAFFT algorithm to the surrogates of the IAAFT algorithm. Our time series is the NASDAQ Composite Index, daily closings, from 5-Feb-1971 to 31-Dec-2003. There are totally 8311 observations. Since all the surrogates generating algorithms need the original time series to be at least mean stationary, we work with the first logarithmic differences of the daily closing prices (i.e., the continuously compounded returns).

[ Insert Fig. 2 about here. ]

[ Insert Fig. 3 about here. ]

As the Fig. 2 and 3 show, there is no need for specific statistical tests to realize the difference between the compared surrogate algorithms. The CCPIAAFT surrogates “imitate” extremely well the heteroscedasticity caused by volatility clustering in the original time series and the trend changes that are implied. In Fig. 4 and 5 we utilize the correlation integral (37, CI:) to demonstrate that the CCPIAAFT surrogates result a CI much more closer to the one of the original time series.

[ Insert Fig. 4 about here. ]

[ Insert Fig. 5 about here. ]

Considering the IAAFT surrogates as our null hypothesis implies that we theorize that extreme events (such as the oil crisis of 1973, the Black Monday of
1987 and the recent bubble of 2000) can occur with equal probability, a premise that voluminous research in finance has challenged so far. Certain events that trigger unanticipated changes, occur due to exogenous political and economic (and not necessarily) market dynamics. Therefore, if these unsystematic fluctuations could be preserved, along with any other calendar effects, one could produce financial surrogates that faithfully reproduce certain market realities. The linear correlations and the randomization of the returns should only affect the systematic components. Hence, CCPIAAFT surrogates essentially isolate the systematic from the unsystematic changes. The degree to which this is achieved is highlighted in Fig. 2 and 3. Fig. 6 and 7 also refer to various realizations of CCPIAAFT surrogates for comparison purposes.

[ Insert Fig. 6 about here. ]

[ Insert Fig. 7 about here. ]

6 Conclusions

In this paper we suggest a method which embodies the outliers and calendar effects on the production of surrogate data. In financial time series where heteroscedasticity, in the sense of volatility clustering, is the most striking feature, the proposed method yields simulated sequences which are more similar to the original time series, when compared with other surrogate data generating methods. Moreover, the proposed approach has the advantage of behaving as the IAAFT algorithm when no heteroscedasticity or calendar effects are present. We do not assume (G)ARCH volatility structures, however our strat-
A strategy can be modified to accommodate such a case. We reserve this as an area for future research.

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Fig. 1. The model proposed by Rousseeuw and Leroy (1987) regarding the distinction between outliers and the bulk of the observations, according to the $|z_{LMS}|$ score. In this model $\lambda$ on the vertical scale represents the degree of a point not being an outlier. Observations with $|z_{LMS}| < 1$ are not considered outliers, and observations with $|z_{LMS}| > 3$ are surely considered outliers. In between these two extremes, the degree falls linearly.
Fig. 2. The original time series (bottom) and 5 surrogates (from top to bottom): the shuffled surrogates (top), the phase randomized surrogates, the AAFT surrogates, the IAAFT surrogates and the CCPIAAFT surrogates. It is evident that the CCPIAAFT series preserve the salient features of the original sequence, especially the volatility clustering and the outliers (shocks) which are linked to well known historical events such as the crash of 1987 and the uncertainty after the burst of the more recent financial bubble.
Fig. 3. The levels of the time series shown in Fig. (2). The CCPIAAFT surrogate series levels (2nd from bottom) preserve exactly the trends that the original time series exhibit, while the all the other sequences above follow a general trend with no time-specific characteristics.
Fig. 4. The correlation integral on the series of Fig. (2) with embedded dimensions (a) $m = 2$ and (b) $m = 3$. 
Fig. 5. The logarithm of the norm-2 difference between the correlation integral of the original time series and the surrogates, shown in Fig. (2). We observe that in both cases the CCPIAAFT surrogates show the smallest difference compared to their counterparts, implying that the CCPIAAFT surrogates provide improved simulations of the original time series.
Fig. 6. A comparison of the original time series and 4 CCPIAAFT surrogate series. Which one is the original? (Answer: the 4th from above).
Fig. 7. The levels of the series shown in Fig. (6). In this graph the differentiation from the original time series is obvious in very few specific time domains. More precisely, we can observe that the drop of the index related to the 1974 crisis and the increase related to the 2000 bubble, appear to be smoother in all surrogate series. This is attributed to the small daily changes in each case being considered as part of the normal fluctuations of the original time series by the CCPIAAFT procedure.