Harvesting entanglement by non-identical detectors with different energy gaps

Hui Hu, Jialin Zhang and Hongwei Yu

Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, 36 Lushan Rd., Changsha, Hunan 410081, China

E-mail: 2316172529@qq.com, jialinzhang@hunnu.edu.cn, hwyu@hunnu.edu.cn

ABSTRACT: It has been shown that the vacuum state of a free quantum field is entangled and such vacuum entanglement can be harvested by a pair of initially uncorrelated detectors interacting locally with the vacuum field for a finite time. In this paper, we examine the entanglement harvesting phenomenon of two non-identical inertial detectors with different energy gaps locally interacting with massless scalar fields via a Gaussian switching function. We focus on how entanglement harvesting depends on the energy gap difference from two perspectives: the amount of entanglement harvested and the harvesting-achievable separation between the two detectors. In the sense of the amount of entanglement, we find that as long as the inter-detector separation is not too small with respect to the interaction duration parameter, two non-identical detectors could extract more entanglement from the vacuum state than the identical detectors. There exists an optimal value of the energy gap difference when the inter-detector separation is sufficiently large that renders the harvested entanglement to peak. Regarding the harvesting-achievable separation, we further find that the presence of an energy gap difference generally enlarges the harvesting-achievable separation range. Our results suggest that the non-identical detectors may be advantageous to extracting entanglement from vacuum in certain circumstances as compared to identical detectors.

KEYWORDS: Effective Field Theories, Quantum Dissipative Systems

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1 Introduction

Quantum entanglement is an intriguing phenomenon in quantum physics and a crucial resource in quantum information technologies, e.g., quantum communication [1–3], quantum teleportation [4], dense coding [2] and quantum key distribution [5]. In recent years, a lot of progress has been made in understanding entanglement in quantum field theory. It has been demonstrated that the vacuum state of a free quantum field is entangled in the framework of the formal algebraic quantum field theory [6, 7], in the sense that the vacuum state can maximally violate Bell’s inequalities. Interestingly, such vacuum entanglement was found to be able to even be extracted by a pair of initially uncorrelated particle detectors/atoms interacting locally with vacuum fields for a finite time [8–10]. More recently, this extraction process has been further operationalized in a protocol with the Unruh-DeWitt (UDW) detector model of two levels [11, 12], and the phenomenon has been known as entanglement harvesting [12, 13].

The entanglement harvesting phenomenon has attracted a lot of attention and been examined in a wide variety of circumstances, e.g., with detectors in accelerated motion, in the presence of completely reflecting boundaries and in spacetimes with nontrivial topology. It was demonstrated that the acceleration of detectors, which leads to the well-known Unruh effect, can surprisingly assist entanglement harvesting in certain cases [12, 14] and the reflecting boundary plays a double edged role in entanglement harvesting, i.e., degrading in general the harvested entanglement while enlarging the entanglement harvesting-achievable parameter space [15]. More intriguingly, one also finds that entanglement harvesting is quite sensitive to the intrinsic properties of spacetime including its dimension [13], topology [16], curvature [17–22], and causal structure [23–25], and thus the phenomenon of entanglement harvesting may provide a tangible way to probe the properties of spacetime [16, 18, 21, 22] and even distinguish a thermal bath from an expanding universe at the same temperature [17]. Experimentally, some feasible setups for entanglement harvesting, such as atomic systems, superconducting circuits and a microcavity, have been proposed in several exploratory studies [26–30].
So far, the study of entanglement harvesting is focused on the situation of two detectors with an equal energy gap, i.e., two identical detectors, whereas little is known for the case of non-identical detectors. Actually, according to the entanglement harvesting protocol, an energy gap difference between two detectors manifests in both detectors’ transition probabilities and the nonlocal correlations of the quantum field in vacuum which in turn jointly determine the entanglement harvesting. Hence, it is quite interesting and sensible to explore entanglement harvesting for non-identical detectors and see whether novel properties emerge. In this paper, we present such a study on entanglement harvesting for two non-identical detectors and address the question whether non-identical detectors could harvest more entanglement than identical ones with the entanglement harvesting protocol and how an energy gap difference between the two detectors affects the harvesting phenomenon.

The rest of this paper is organized as follows. We begin in section 2 by reviewing the basic formalism for the UDW detectors locally interacting with vacuum scalar fields in the framework of the entanglement harvesting protocol. In section 3, we analyze the properties of entanglement harvesting for two inertial detectors with different energy gaps, and make a cross-comparison with that for two identical ones with an equal energy gap. Finally, we finish with conclusions in section 4. Throughout the paper the natural units $\hbar = c = 1$ are adopted.

## 2 The basic formulas

We consider a pair of point-like two-level static detectors (labeled by $A$ and $B$) with $|g\rangle$ and $|e\rangle$ denoting the ground and excited states respectively, which locally interact with a massless scalar field $\phi$ in vacuum, and assume that the interaction Hamiltonian takes the following form in the interaction picture

$$H_D(\tau) = \lambda \chi(\tau)[e^{i\Omega_D^+} \sigma^+ + e^{-i\Omega_D^-} \sigma^-]\phi[x_D(\tau)], \quad D \in \{A, B\}$$

(2.1)

where $\lambda$ is the coupling strength and subscript $D$ specifies which detector we are considering, $\chi(\tau) := \exp[-\tau^2/(2\sigma^2)]$ is a switching function with parameter $\sigma$ controlling the duration of interaction, $\Omega_D$ is the energy gap of the detector, $\sigma^+ = |e\rangle_D \langle g|_D, \sigma^- = |g\rangle_D \langle e|_D$ are the SU(2) ladder operators acting on the Hilbert space of the detector, and $x_D(\tau)$ is the spacetime trajectory of the detector parameterized by its proper time $\tau$.

Initially, we suppose that the two UDW detectors are prepared in their ground state and the field is in the vacuum state $|0\rangle_M$. With the interaction Hamiltonian eq. (2.1), the final reduced density matrix of the system (two detectors) can be obtained in the basis \{\{g\}_A|g\rangle_B, |g\>_A|e\>_B, |e\>_A|g\>_B, |e\>_A|e\>_B\}, after some algebraic manipulations based on the perturbation theory [13, 16, 22]

$$\rho_{AB} := \text{tr}_\phi (U |\Psi\rangle_i \langle \Psi|_i U^\dagger) = \begin{pmatrix} 1 - P_A - P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4), \quad (2.2)$$
where
\begin{equation}
P_D := \lambda^2 \int d\tau d\tau' \chi(\tau) \chi(\tau') e^{-i\Omega_D (\tau - \tau')} W(x_D(\tau), x_D(\tau')) \quad D \in \{A, B\},
\end{equation}
\begin{equation}
C := \lambda^2 \int d\tau d\tau' \chi(\tau) \chi(\tau') e^{-i(\Omega_A \tau - \Omega_B \tau')} W(x_A(\tau), x_B(\tau')) ,
\end{equation}
\begin{equation}
X := -\lambda^2 \int d\tau d\tau' \chi(\tau) \chi(\tau') e^{-i(\Omega_A \tau + \Omega_B \tau')} \left[ \theta(\tau' - \tau) W(x_A(\tau), x_B(\tau')) + \theta(\tau - \tau') W(x_B(\tau'), x_A(\tau)) \right]
\end{equation}
with $W(x, x') := \langle 0|_M \phi(x)\phi(x') |0\rangle_M$ denoting the Wightman function of the field and $\theta(\tau)$ the Heaviside step function. Here, we have used the fact that $t = \tau$ for detectors at rest. Let us note that $P_D$ is the transition probability of detector $D$, and the quantities $C$ and $X$ characterize the field correlations $[22]$. We utilize concurrence as a measure of entanglement harvested by the UDW detectors, which can be evaluated straightforwardly from the $X$-type density matrix eq. (2.2) to give $[16, 19, 22]$
\begin{equation}
C(\rho_{AB}) = 2 \max \left[ 0, |X| - \sqrt{P_A P_B} \right] + O(\lambda^3).
\end{equation}
So, the concurrence quantifying the harvested entanglement is a result of the competition between off-diagonal matrix element $X$ and the geometric mean of transition probabilities $P_A$ and $P_B$. For two identical detectors at rest, $P_A = P_B$, and the concurrence is simply determined by $|X| - P_D$.

3 Entanglement harvesting for non-identical detectors

We now examine entanglement harvesting for two static non-identical detectors with different energy gaps. Without loss of generality, we assume the detector labelled by $A$ has a comparatively smaller energy gap, i.e., $\Delta \Omega := \Omega_B - \Omega_A \geq 0$ throughout the paper. In the four dimensional Minkowski spacetime, the Wightman function for the massless scalar field is given by $[31]$
\begin{equation}
W(x, x') = -\frac{1}{4\pi^2} \left( \frac{1}{(\tau - \tau' - i\epsilon)^2} - |x - x'|^2 \right).
\end{equation}
Substituting eq. (3.1) into eq. (2.3), one can find the transition probability $[15, 16]$
\begin{equation}
P_D = \frac{\lambda^2}{4\pi} \left[ e^{-\sigma^2 \Omega_D^2} - \sqrt{\pi} \Omega_D \sigma \text{Erf}(\sigma \Omega_D) \right], \quad D \in \{A, B\},
\end{equation}
where Erfc($x$) := $1 - \text{Erf}(x)$. We suppose that two detectors are separated by a distance $L$, then the correlation term $X$ can be straightforwardly worked out from eq. (2.5)(see appendix. A)
\begin{equation}
X = -\frac{\lambda^2 \sigma}{8\sqrt{\pi} L} e^{-s^2(2\Omega_A + \Delta \Omega)^2 + L^2} \left[ e^{i\Delta \Omega L/2} \text{Erfi} \left( \frac{L - i\sigma^2 \Delta \Omega}{2\sigma} \right) \right. \left. + e^{-i\Delta \Omega L/2} \text{Erfi} \left( \frac{L + i\sigma^2 \Delta \Omega}{2\sigma} \right) + 2i \cos \left( \frac{\Delta \Omega L}{2} \right) \right],
\end{equation}
where the imaginary error function is defined as $\text{Erfi}(x) := -i\text{Erf}(ix)$. Let us note here that the correlation term $X$ diverges in the limit of $L \to 0$. This divergence actually arises from the ill-defined point-like approximation of the UDW detector model in the entanglement harvesting protocol when $L/\sigma \ll \lambda$, and a finite-size detector model with a spatial smearing function should be called for to resolve the issue [13].

Although further analytical behaviors of the transition probabilities $P_D$ and the correlation term $X$ are not obtainable, approximate expressions can however be found in some special cases. For energy gaps extremely small as compared to the duration of interaction parameter $\sigma$ ($\Omega_A\sigma \leq \Omega_B\sigma \ll 1$), we have

$$\sqrt{P_A P_B} \approx \frac{\lambda^2}{4\pi} e^{-\sigma^2(\Omega_A^2 + (\Omega_A + \Delta\Omega)^2)/2}, \quad (3.4)$$

whereas for large energy gaps ($1 \ll \Omega_A\sigma \leq \Omega_B\sigma$),

$$\sqrt{P_A P_B} \approx \frac{\lambda^2}{8\pi} \frac{e^{-\sigma^2[\Omega_A^2 + (\Omega_A + \Delta\Omega)^2]/2}}{\Omega_A(\Omega_A + \Delta\Omega)\sigma^2} \left[1 - \frac{3}{4\sigma^2\Omega_A^2} - \frac{3}{4\sigma^2(\Omega_A + \Delta\Omega)^2}\right]. \quad (3.5)$$

Regarding the correlation term (3.3), we find that it can be approximated, when $L/\sigma \ll 1$, as

$$X \approx -\frac{\lambda^2}{4\sqrt{\pi}} \frac{e^{-\sigma^2(2\Omega_A + \Delta\Omega)^2/4}}{4\sqrt{\pi} L} \left[\frac{i\sigma}{L} + \frac{e^{-\Delta\Omega^2\sigma^2/4}}{\sqrt{\pi}} + \frac{\Delta\Omega\sigma}{2} \text{ Erf} \left(\frac{\Delta\Omega\sigma}{2}\right)\right], \quad (3.6)$$

while $L/\sigma \gg 1$, the approximation is

$$X \approx -\frac{\lambda^2}{4\sqrt{\pi} L} \left[\frac{2\sigma^2}{L}\int_0^\Omega e^{-\Delta\Omega^2\sigma^2/4} \frac{\Delta\Omega\sigma}{2} \text{ Erf} \left(\frac{\Delta\Omega\sigma}{2}\right)\right]. \quad (3.7)$$

Therefore, one can estimate the concurrence for small inter-detector separations ($L/\sigma \ll 1$) to be

$$C(\rho_{AB}) \approx \frac{\lambda^2\sigma}{2L\sqrt{\pi}} e^{-\sigma^2(2\Omega_A + \Delta\Omega)^2/4}, \quad (3.8)$$

while that for large inter-detector separations ($L/\sigma \gg 1$)

$$C(\rho_{AB}) \approx \begin{cases} \max \left\{ \frac{\lambda^2}{2\pi} e^{-\sigma^2[\Omega_A^2 + (\Omega_A + \Delta\Omega)^2]/2} \left(\frac{2\sigma^2}{L^2} - 1\right) \right\}, & \Omega_A\sigma \leq \Omega_B\sigma \ll 1; \\
\max \left\{ \frac{\lambda^2}{2\pi} e^{-\sigma^2[\Omega_A^2 + (\Omega_A + \Delta\Omega)^2]/2} \left[\frac{2\sigma^2}{L^2} - \frac{1}{2\Omega_A(\Omega_A + \Delta\Omega)\sigma^2}\right] \right\}, & \Omega_B\sigma \geq \Omega_A\sigma \gg 1. \end{cases} \quad (3.9)$$

As can be seen from eq. (3.8), the entanglement harvested by time-like separated detectors with very small inter-detector separation ($L/\sigma \ll 1$) is an obviously decreasing function of the energy gap difference, while it follows from eq. (3.9) that for space-like separated detectors\(^1\) entanglement harvesting is only possible for detectors with an energy gap larger than the Heisenberg energy (i.e, $\Omega_B \geq \Omega_A \gg 1/\sigma$). This can be understood as follows.

\(^1\)Strictly speaking, detectors with a Gaussian switching function are never spacelike separated. However, after a time of several $\sigma$, the interaction will be exponentially suppressed by several e-folds and can thus be considered as effectively turned off. Here, we regard an inter-detector’s separation as space-like if it is larger than about $3\sigma$ and time-like if less.
For a very small inter-detector separation \( L/\sigma \ll 1 \), one can see from eq. (2.6) and eq. (3.6) that the correlation term \( X \) will be a decisive player in the concurrence and the term related to the transition probabilities can be neglected, leading to that the concurrence behaves as a decreasing function of the energy gap difference. However, for a large inter-detector separation \( L/\sigma \gg 1 \), the term related to the transition probabilities can no longer be neglected because now the correlation term \( X \) for two space-like separated detectors also becomes small. For detectors with an energy gap much smaller than the Heisenberg energy (i.e, \( \Omega_A \ll 1/\sigma \) ), the probabilities term exceeds the correlation term, thus making the concurrence vanish, whereas for detectors with an energy gap much larger than the Heisenberg energy (\( \Omega_A \gg 1/\sigma \) ), the probabilities term can be smaller than the correlation term \( X \) as long as the inter-detector separation is not too large, since the detector’s transition probability is now exponentially suppressed, yielding a non-zero concurrence.

Since the concurrence is determined by the competition between the correlation \( X \) and the transition probabilities, one may infer that for any detectors with given energy gaps the entanglement will not be able to be harvested for a very large but finite inter-detector separation. In other words, there exists a separation range for entanglement harvesting to be possible. For convenience, we introduce parameter \( L_{\text{max}} \) to characterize the maximum harvesting-achievable separation beyond which entanglement harvesting ceases to occur. It easy to deduce from eq. (3.9) that \( L_{\text{max}} \), for very large energy gaps (\( \Omega_A \gg 1 \)), takes the form

\[
L_{\text{max}} \approx 2\sigma^2 \sqrt{\Omega_A(\Omega_A + \Delta \Omega)} ,
\]

and this reveals that an energy gap difference can enlarge the harvesting-achievable range.

In order to show the behavior of the entanglement harvested by non-identical detectors in more general cases, we will now perform numerical evaluations on the concurrence.
In figure 1, the concurrence is plotted as a function of $L/\sigma$ for various $\Delta \Omega/\Omega_A$. Remarkably, regardless of the value of $\Delta \Omega/\Omega_A$, the harvested entanglement decreases as the detector separation increases, which is in accordance with the existing results of entanglement harvesting by identical detectors ($\Delta \Omega = 0$) in the literature [14–16, 19, 22]. So, the existence of an energy gap difference does not change the property of concurrence as a monotonically decreasing function of the detector separation. However, it is noteworthy that the gap difference does impact the amount of the entanglement harvested. As shown in figure 1, only for small detector separations ($L/\sigma < 1$) could the identical detectors with an equal energy gap harvest more entanglement, and an evident crossover when non-identical detectors harvest more entanglement would emerge as the inter-detector separation increases to sufficiently large as compared to the interaction duration parameter $\sigma$. The smaller the gap difference, the sooner the crossover occurs as the separation increases. This implies that non-identical detectors with an energy gap difference at a sufficiently large separation are bound to harvest more entanglement via locally interacting with vacuum fields than identical detectors. In this sense, the presence of an energy gap difference is conducive to entanglement harvesting. It should be emphasized that the appearance of the crossover phenomenon is irrelevant to the value of $\Omega_A$ in the sense that a larger $\Omega_A\sigma$ just pushes the crossover point to a larger inter-detector separation $L/\sigma$, making the quantitative details of the crossover point a little different for different values of $\Omega_A\sigma$ (compare figures 1a and 1b).

We could qualitatively understand the appearance of the crossover phenomenon as follows. As can be seen from eq. (3.8), for a very small inter-detector separation, the two detectors with an equal energy gap ($\Delta \Omega = 0$) harvest more entanglement as factor $e^{-\sigma^2(2\Omega_A+\Delta \Omega)^2/4}$ attains the maximum value for $\Delta \Omega = 0$. However, for not too small $L/\sigma$, the transition probabilities, $P_A$ and $P_B$, cannot be neglected in the concurrence (2.6) any more. We could roughly estimate the derivative of the concurrence with respect to $\Delta \Omega$ as follows

$$
\frac{\partial C(\rho_{AB})}{\partial (\Delta \Omega)} \approx \frac{\lambda^2 \sigma}{4\sqrt{\pi}} \sqrt{\frac{P_A}{P_B}} \operatorname{Erfc}[(\Omega_A + \Delta \Omega)\sigma] - \frac{(\Omega_A + \Delta \Omega)\sigma^4}{L^2} e^{-\sigma^2(\Omega_A^2 + (\Omega_A + \Delta \Omega)^2)/2} e^{-(\Omega_A + \Delta \Omega)\sigma^3/4L^2} \operatorname{Erfc}[(\Omega_A + \Delta \Omega)\sigma] - \frac{(\Omega_A + \Delta \Omega)\sigma^3}{L^2} \right)
$$

In the last line of eq. (3.11), the approximation $P_D \propto e^{-\sigma^2\Omega_D^2}$, i.e., eq. (3.2), has been utilized. If the inter-detector separation is large enough ($L \gg \sigma$), we have $\partial C(\rho_{AB})/\partial (\Delta \Omega) > 0$. This means that the concurrence is now an increasing function of $\Delta \Omega$, which also explains why the non-identical detectors could harvest more entanglement than the identical detectors in the regime of large inter-detector separations. Moreover, figure 1 also demonstrates that the harvesting-achievable separation range is singularly affected by the energy gap difference. In general, the identical detectors would have a relatively small harvesting-achievable separation range in comparison with non-identical detectors. More details of this property will be discussed later.

To gain a better understanding of the influence of an energy gap difference on entanglement harvesting, we further plot the concurrence versus the energy gap difference in figure 2. As we can see, the entanglement in general degrades with the increasing energy...
The concurrence versus $\Delta \Omega / \Omega_A$ for various $L/\sigma$ with $\Omega_A \sigma = 0.50$ in (a) and $\Omega_A \sigma = 1.20$ in (b). The vertical dashed lines in the plots indicate where the peaks are located.

Figure 2. The concurrence versus $\Delta \Omega / \Omega_A$ for various $L/\sigma$ with $\Omega_A \sigma = 0.50$ in (a) and $\Omega_A \sigma = 1.20$ in (b). The vertical dashed lines in the plots indicate where the peaks are located.

gap difference $\Delta \Omega$ (in the region of large $\Delta \Omega / \Omega_A$) no matter whether $\Omega_A \sigma$ is small or large, but it remarkably displays a peaking behavior when the inter-detector separation is sufficiently larger than the duration parameter $\sigma$. This reveals that there is an optimal value of the energy gap difference (denoted by $\Delta \Omega_p$) that renders the concurrence to peak. Moreover, we also find that the larger the separation, the larger the optimal value of the energy gap difference, and the larger $\Omega_A \sigma$, the larger the inter-detector separation with which the peaking behavior occurs. This property can actually be deduced from eq. (3.11). Since $e^{x^2} \text{Erfc}(x)$ is an exponential-like monotonically decreasing function of its argument [32], a larger inter-detector separation requires a larger $\Omega_A \sigma$ or $\Delta \Omega \sigma$ to make $\partial C(\rho_{AB}) / \partial (\Delta \Omega) = 0$ which governs where the peak locates.

In order to analyze the concurrence peaking phenomenon more clearly, we plot the behaviors of both $|X|$ and $\sqrt{P_A P_B}$ versus $\Delta \Omega / \Omega_A$ in figure 3. In fact, according to eq. (3.2) and eq. (3.3), it is easy to see that both the geometric mean of the transition probabilities $\sqrt{P_A P_B}$ and the correlation term $|X|$ decrease as the energy gap difference $\Delta \Omega$ increases. But the rate of decrease is different. As shown in figure 3, the difference between $|X|$ and $\sqrt{P_A P_B}$ at the beginning grows with increasing $\Delta \Omega$. However, when the energy gap difference grows across $\Delta \Omega_p$ (indicated by the vertical dashed line in figure 3), the correlation term $|X|$ would decrease more rapidly than $\sqrt{P_A P_B}$, resulting in a peak of $|X| - \sqrt{P_A P_B}$ therein. As a consequence, the concurrence would remarkably peak at the optimal value of the energy gap difference $\Delta \Omega_p$. Here, we have plotted $\Delta \Omega_p / \Omega_A$ versus $L/\sigma$ in figure 4 as a supplement to analyze how the inter-detector separation influences the optimal value of the energy gap difference. It is easy to discern that the optimal energy gap difference is in general an increasing function of $L/\sigma$ over the region of not too small $L/\sigma$, and a large $\Omega_A \sigma$ renders the curve of function $\Delta \Omega_p / \Omega_A$ to move rightward along the axis of $L/\sigma$. 

Figure 3. The correlation term $|X|$ and the geometric mean of the detectors’ transition probabilities $\sqrt{P_A P_B}$ are plotted as a function of $\Delta\Omega/\Omega_A$ with $\Omega_A \sigma = 0.50$ and $L/\sigma = 2.00$. Here, the dashed black line represents the difference between $|X|$ and $\sqrt{P_A P_B}$, and the vertical dashed line indicates where the peak concurrence occurs, i.e., $|X| - \sqrt{P_A P_B}$ is maximum.

Figure 4. The plot of $\Delta\Omega_p/\Omega_A$ versus $L/\sigma$ for various $\Omega_A \sigma = \{0.20, 0.50, 1.00, 1.20\}$. Here, $\Delta\Omega_p$ denotes the optimal value of the energy gap difference between the two detectors that peaks the concurrence. It is easy to get that $\Delta\Omega_p/\Omega_A$ is a general increasing function in the region of large enough $L/\sigma$. 
The maximum harvesting-achievable separation, $L_{\text{max}}/\sigma$, is plotted as a function of $\Delta \Omega/\Omega_A$ for $\Omega_A \sigma = \{0.20, 0.50, 1.00, 1.20\}$. The dashed horizon lines indicate the corresponding values of $L_{\text{max}}/\sigma$ for identical detectors (i.e., $\Delta \Omega = 0$). It is easy to see the oscillatory behavior of $L_{\text{max}}/\sigma$ would emerge for large $\Omega_A \sigma$.

Now, we turn to investigate the influence of an energy gap difference on the harvesting-achievable separation range. As mentioned before, for extremely large energy gaps ($\Omega_A \sigma \gg 1$), the energy gap difference does enlarge the harvesting-achievable range. For not too large energy gaps, the detailed behavior of the harvesting-achievable separation range is further depicted in figure 5. Obviously, the parameter $L_{\text{max}}$ generally is an increasing function of $\Delta \Omega$ for small $\Omega_A \sigma$. This means that the non-identical detectors possess more spacious “room” for entanglement extraction, and an energy gap difference between two detectors assists entanglement harvesting in the sense of harvesting-achievable range. As is shown in figure 5, as the energy gap $\Omega_A$ grows close to and a little beyond $1/\sigma$, the harvesting-achievable range ($L_{\text{max}}/\sigma$) overall is still an increasing function, but with obvious oscillations as a result of the oscillatory cosine and exponential functions of $\Delta \Omega$ in eq. (3.3). Such obvious oscillations can even make the value of $L_{\text{max}}/\sigma$ have one opportunity to be smaller than that for identical detectors (see $\Omega_A \sigma = 1.20$ in figure 5). Hence, the energy gap difference between the two detectors does have significant influence on entanglement harvesting phenomenon, i.e., overall it enlarges the harvesting-achievable separation range and results in a peak of the entanglement harvested for a sufficiently large inter-detector separation.

4 Conclusion

We have performed a detailed investigation on the entanglement harvesting phenomenon of two non-identical detectors with different energy gaps locally interacting with massless scalar fields in (3+1)-dimensional Minkowski spacetime. We analyzed the influence of an energy gap difference between the two detectors on entanglement harvesting from two perspectives: the amount of entanglement harvested and the harvesting-achievable separation, mainly focusing on the question whether the non-identical detectors are more easily to get
entangled than the identical detectors and when the non-identical detectors could harvest more entanglement from the fields in vacuum during the entanglement harvesting process.

For the perspective of the amount, the harvested entanglement in general degrades with the increasing separation between the two detectors regardless of the energy gap. However, the non-identical detectors could extract more entanglement from the vacuum state than the identical detectors if the inter-detector separation is not too small with respect to the interaction duration parameter. There seems to be an optimal value of the energy gap difference between the non-identical detectors that renders the harvested entanglement to peak if the inter-detector separation is sufficiently large. In addition, we also demonstrate how the optimal value of energy gap difference depends on the inter-detector separation with the energy gap of one detector ($\Omega_A$) fixed. We find that the larger the inter-detector separation, the greater the optimal value of the energy gap difference.

Regarding the harvesting-achievable range of the inter-detector separation, we find that the harvesting-achievable range is an increasing function of the energy gap difference when the energy gap of the detector with a smaller gap, $\Omega_A$, is very large or small as compared to the Heisenberg energy $1/\sigma$. However, when $\Omega_A$ becomes comparable to the Heisenberg energy $1/\sigma$, the harvesting-achievable range overall increases as the gap difference increases, although with obvious oscillations in the regime of large $\Delta\Omega/\Omega_A$. Therefore we conclude that the presence of an energy gap difference generally has a positive influence on the harvesting-achievable range. In other words, the presence of an energy gap difference is conducive to entanglement harvesting of two detectors in the sense of harvesting-achievable separation range.

In summary, non-identical detectors with different energy gaps could harvest more entanglement from the vacuum state of quantum fields than identical detectors as long as the inter-detector separation is not too small. Furthermore, the presence of an energy gap difference can in general enlarge the harvesting-achievable range of the separation between detectors. Our results illustrate that the energy gap difference is an important physical quantity like those characterizing spacetime topology, curvature and detector’s motion to manipulate entanglement harvesting.

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A The derivation of $X$

To verify eq. (3.3), let us begin from eq. (2.5), which satisfies

$$X = -\lambda^2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' \chi(\tau) \chi(\tau') \left[ e^{-i(\Omega_A + \Omega_B)\tau'} W(x_A(\tau'), x_B(\tau)) + e^{-i(\Omega_A + \Omega_B)\tau} W(x_B(\tau'), x_A(\tau)) \right]$$

$$= -\lambda^2 \int_{-\infty}^{\infty} du \chi(u) \chi(u-s) e^{-i(\Omega_A + \Omega_B)u} \int_0^{\infty} ds \left[ e^{i\Omega_A s} W(s) + e^{i\Omega_B s} W(s) \right], \quad (A.1)$$
where we have introduced \( u := \tau \), \( s := \tau - \tau' \) and considered \( W(s) = W(x_A(\tau), x_B(\tau)) = W(x_B(\tau'), x_A(\tau)) \) due to the fact that two detectors are at rest with respect to one another. Carrying out the integration with respect to \( u \), we have

\[
X = -\lambda^2 \sqrt{\pi} \sigma \sigma^2 (\Omega_A + \Omega_B)^{3/4} \int_0^\infty ds e^{-\frac{s^2}{4\sigma^2}} \left[ e^{i(\Omega_A - \Omega_B)s/2} W(s) + e^{-i(\Omega_A - \Omega_B)s/2} W(s) \right].
\]  

(A.2)

The Wightman function eq. (3.1) for two static detectors then can be written as

\[
W(s) = -\frac{1}{4\pi^2} \frac{1}{(-s - i\epsilon)^2 - L^2},
\]  

(A.3)

Substituting eq. (A.3) into eq. (A.2) yields

\[
X = \frac{\lambda^2 \sigma}{4\pi^{3/2}} e^{-\sigma^2(2\Omega_A + \Delta \Omega)^{2/4}} \int_{-\infty}^\infty ds e^{-s^2/(4\sigma^2)} e^{-i\Delta \Omega s/2} ds - \frac{i \lambda^2 \sigma}{4\sqrt{\pi} L} e^{-s^2/(4\sigma^2)} \left[ e^{i\Delta \Omega L/2} \text{Erfi}\left(\frac{L - i\sigma^2 \Delta \Omega}{2}\right) + \text{Erfi}\left(\frac{L + i\sigma^2 \Delta \Omega}{2}\right) \right],
\]  

(A.4)

where we have defined \( \Delta \Omega := \Omega_B - \Omega_A \) and utilized the Sokhotski formula

\[
\frac{1}{x \pm i\epsilon} = P \frac{1}{x} \mp i\pi \delta(x).
\]  

(A.5)

Here, the corresponding integration in eq. (A.4) can be carried out by using the Fourier transforms, that is

\[
P \int_{-\infty}^\infty \frac{e^{-s^2/(4\sigma^2)} e^{-i\Delta \Omega s/2} ds}{s^2 - L^2} = P \int_{-\infty}^\infty \frac{2e^{-t^2/\sigma^2}}{4t^2 - L^2} e^{-i\Delta \Omega t} dt = \mathcal{F}\left[2e^{-t^2/\sigma^2} \cdot \frac{1}{4t^2 - L^2}\right] = \frac{1}{2\pi} \mathcal{F}\left[2e^{-t^2/\sigma^2}\right] * \mathcal{F}\left[\frac{1}{4t^2 - L^2}\right] = -\frac{\pi}{2L} e^{-L^2/(4\sigma^2)} e^{-i\Delta \Omega L/2} \left[ e^{i\Delta \Omega L/2} \text{Erfi}\left(\frac{L - i\sigma^2 \Delta \Omega}{2}\right) + \text{Erfi}\left(\frac{L + i\sigma^2 \Delta \Omega}{2}\right) \right],
\]

(A.6)

where the symbols “\( \mathcal{F} \)” and “\( * \)” stand for the Fourier transform and the convolution of two functions, respectively.

Substituting eq. (A.6) into eq. (A.4), one can straightforwardly obtain \( X \) in the terms of error functions

\[
X = -\frac{\lambda^2 \sigma}{8\sqrt{\pi} L} e^{-s^2/(4\sigma^2) + \Delta \Omega^2} \left[ e^{i\Delta \Omega L/2} \text{Erfi}\left(\frac{L - i\sigma^2 \Delta \Omega}{2\sigma}\right) + e^{-i\Delta \Omega L/2} \text{Erfi}\left(\frac{L + i\sigma^2 \Delta \Omega}{2\sigma}\right) + 2i \cos\left(\frac{\Delta \Omega L}{2}\right) \right].
\]  

(A.7)
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