Jet evolution in Yang-Mills-Wong simulations

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Abstract

We present results for collisional energy loss and momentum broadening of high momentum partons in a hot and dense non-Abelian plasma obtained by solving the coupled system of Yang-Mills-Wong equations on a lattice in real time. Including hard elastic collisions among the particles we obtain cutoff independent results for the collisional energy loss $dE/dx$ and the transport coefficient $\hat{q}$. The latter is found to receive a sizable contribution from a power-law tail in the transverse momentum distribution of high-momentum partons. We further argue that the effect of instabilities on jet broadening should be accessible by experiment when employing jet cones with elliptical bases or studying correlations within the cone in full jet reconstruction methods.

1. Introduction

High transverse momentum jets produced in heavy-ion collisions represent a valuable tool for studies of the properties of the hot parton plasma produced in the central rapidity region [1]. We study the collisional energy loss and momentum broadening of high momentum partons employing a numerical simulation of the Boltzmann-Vlasov equation, which is coupled to the Yang-Mills equation for the soft gluon degrees of freedom. Soft momentum exchanges between particles are mediated by the fields, while hard momentum exchanges are described by a collision term including binary elastic collisions. This way, we are able to provide an estimate of the coupling of jets to a hot plasma which is independent of infrared cutoffs.

We also simulate plasmas with local momentum-space anisotropies, which occur in heavy-ion collisions due to the longitudinal expansion during the very early stages of the plasma evolution. These plasmas develop Chromo-Weibel instabilities [2], which lead to the formation of long-wavelength chromo-fields with $E_z > B_z$ and $B_\perp > E_\perp$. The strong fields affect the propagation of hard partons, leading to an asymmetry of the jet shape in rapidity $\Delta \eta$ and azimuthal angle $\Delta \phi$, which should be accessible using new jet reconstruction measurements [3].

2. Boltzmann-Vlasov equation for non-Abelian gauge theories

We solve the classical transport equation for hard gluons with SU(2) color charge including hard binary collisions

$$p^\nu \left[ \partial_{\mu} + gg^a F^a_{\mu\nu} \partial_{\nu} + g f^{abc} A^b_{\mu}(x) q f^c_{\nu} \right] f = C, \quad (1)$$

where $f = f(x, p, q)$ denotes the single-particle phase space distribution. It is coupled self-consistently to the Yang-Mills equation for the soft gluon fields

$$D_{\mu} F^{\mu\nu} = f' = g \int \frac{d^3 p}{(2\pi)^3} dq q^\nu f(x, p, q), \quad (2)$$
with \( \nu^\mu = (1, \mathbf{p}/p) \). The collision term contains all binary collisions, described by the leading-order \( gg \to gg \) tree-level diagrams.

We replace the distribution \( f(x, p, q) \) by a large number of test particles, which leads to Wong’s equations \([4]\)

\[
\begin{align*}
    \mathbf{x}_i(t) &= \mathbf{v}_i(t), \\
    \mathbf{p}_i(t) &= g\mathbf{q}_i^\mu(t) (\mathbf{E}^\mu(t) + \mathbf{v}_i(t) \times \mathbf{B}^\mu(t)) , \\
    \mathbf{q}_i(t) &= -ig\mathbf{v}_i^\mu(t) [\mathbf{A}_\mu(t), \mathbf{q}_i(t)],
\end{align*}
\]

for the \( i \)-th test particle, whose coordinates are \( \mathbf{x}_i(t) \), \( \mathbf{p}_i(t) \), and \( \mathbf{q}_i(t) \), the color charge. The time evolution of the Yang-Mills field is determined by the standard Hamiltonian method \([5]\) in \( A^0 = 0 \) gauge. See \([6, 7, 8, 9, 10]\) for more details. The collision term is incorporated using the stochastic method \([11]\). The total cross section is given by \( \sigma_{2\to2} = \int_{k^*}^{k/e} \frac{dp}{dq^2} \), where we have introduced a lower cutoff \( k^* \). To avoid double-counting, this cutoff should be on the order of the hardest field mode that can be represented on the given lattice, \( k^* \sim \pi/a \), with the lattice spacing \( a \). This way soft momentum exchanges are mediated by the fields while hard momentum exchanges are described by the collision term. For a more detailed discussion see \([10]\).

3. Jet broadening in an isotropic plasma

We first consider heat-baths of particles with different densities \( n_\xi \) and temperatures \( T \). We simulate an undersaturated gluon plasma due to numerical restrictions (see \([10]\)) and later extrapolate to thermal systems with physical densities and temperatures. For a given lattice (or \( k^* \)) we take the initial energy density of the thermalized fields to be \( \int d^3k/(2\pi)^3 k f_{\text{boson}}(k) \Theta(k^* - k) \), where \( f_{\text{boson}}(k) = n_\xi/(2T^3\zeta(3))/(e^{k/T} - 1) \) is a Bose distribution normalized to the assumed particle density \( n_\xi \), and \( \zeta \) is the Riemann zeta function. The initial field spectrum is fixed to Coulomb gauge and \( A_\xi = 1/k \).

We study colorless bunches of test particles that allow us to restrict to collisional energy loss and momentum broadening due to elastic collisions only. Both

\[
\hat{q} = \frac{1}{\lambda s} \int d^2p_\perp p_\perp^2 d\sigma \left( \frac{p_\perp^2(t)}{T} \right),
\]

and the differential energy loss \( dE/dx \) are independent of the separation scale \( k^* \). \( \lambda \) is the mean free path and \( p_\perp \) denotes the momentum transverse to the initial jet momentum. Fig.\([1]\) depicts the contributions to \( \hat{q} \) and \( dE/dx \) due to soft and hard collisions separately, as well as the total.

Studying the dependencies of \( \hat{q} \) and \( dE/dx \) on the density, temperature, and hard parton energy reproduces results from perturbative QCD (see \([10]\) for a detailed analysis). Using these dependencies we can extrapolate \( \hat{q} \) and \( dE/dx \) to temperatures around 400 MeV. Adjusting the color factors as appropriate for SU(3) and extrapolating to the thermal density of gluons we find \( \hat{q} \approx 3.6 \pm 0.3 \text{ GeV}^2 \text{ fm}^{-1} \) and \( dE/dx \approx 1.6 \pm 0.4 \text{ GeV} \text{ fm}^{-1} (E = 19.2 \text{ GeV}) \).

Next, we present the full \( p_\perp^2 \)-distribution of the high-momentum partons traversing the hot medium in order to assess the relative contributions from various processes to its first moment \( \hat{q} \). We find that over time the initial \( \delta \)-function broadens to a Gaussian distribution with a power-law tail. Fig.\([2]\) shows the distribution of the high-momentum test particles after \( t \approx 5.2 \text{ fm} \) in a double-logarithmic plot versus \( p_\perp^2/t \). The low-\( p_\perp \) part follows a Gaussian distribution in \( p_\perp \). The power-law tail at large \( p_\perp \) behaves approximately as \( p_\perp^{-4} \). This is expected for particles
experiencing only few scatterings since in the high-energy limit the differential cross section $d\sigma/dp_{\perp}^2 \sim p_{\perp}^{-4}$. We also indicate the value of $\hat{q}$ to show that the power-law tail contributes significantly to this transport coefficient.

We also provide an estimate for the nuclear modification factor $R_{AA}$ due to elastic energy loss in a classical Yang-Mills field. This is of relevance for collisions of heavy nuclei at high energies: the large number of gluons produced in the central rapidity region can be described as a classical field for a short time (see e.g. [12]) until the field modes decohere and thermalize [8]. From the simulations presented above, we obtained the elastic energy loss $dE/dx$. For large $k^*$ (on the order of the “saturation momentum” $Q_s$) most of the energy density is due to the classical field. In a thermal system at $T = 400$ MeV the energy density is about 17 GeV/fm$^3$, which is an appropriate average over the first 1 fm/c of a central Au+Au collision at RHIC energy [12]. An estimate for the nuclear modification factor $R_{AA}$ at the parton level (neglecting hadronization) can be written as [13, 10]

$$R_{AA}(p_{\perp}) = \frac{dN_i/d^2p_{\perp}dy}{dN_c/d^2p_{\perp}dy} = (1 - \epsilon(p_{\perp}))^n.$$  \hspace{1cm} (5)
\( \epsilon \) denotes the fractional energy loss up to \( \tau = 1 \) fm and \( dN_i/d^2 p_\perp dy \sim 1/p_n^{\perp+2} \) is the initial \( p_\perp \) distribution of jets, where \( n \approx 4 \). Fig. 3 shows the result for \( R_{AA} \). The experimentally observed flat \( R_{AA} \approx 0.2 \) cannot be fully accounted for by early-stage elastic energy loss in the classical field background, but our result shows that this contribution is significant and cannot be neglected.

4. Jet broadening in an unstable plasma

As studied in detail in [9], instability growth in unstable plasmas leads to a direction dependent \( \hat{q} \). We describe the broadening of the hard (now colored) test particles in the transverse and longitudinal \( z \) directions via the variances \( \hat{q}_\perp := \frac{d}{dt} \langle (\Delta p_\perp)^2 \rangle \), \( \hat{q}_z := \frac{d}{dt} \langle (\Delta p_z)^2 \rangle \). The ratio \( \hat{q}_z/\hat{q}_\perp \) can be roughly associated with the ratio of jet correlation widths in azimuth and rapidity: \( \sqrt{\hat{q}_z/\hat{q}_\perp} \approx \langle \Delta \eta \rangle / \langle \Delta \phi \rangle \). The numerical simulations of unstable plasmas obtain [9]

\[
\langle \Delta \eta \rangle / \langle \Delta \phi \rangle \approx 1.5.
\]

We propose to look for this effect by using the greatly improved experimental methods for full jet reconstruction [3, 14, 15]. In particular, one should use cones with an elliptical base and determine its major axis in the \( \phi-\eta \) plane by maximizing the energy contained in the cone. If the system is isotropic, the angle should be uniformly distributed while the effect of instabilities should lead to a preferred orientation of the major axis in the \( \eta \) direction. Another, possibly more practical way of experimentally determining such an anisotropy of the jet is to keep the circular base and study correlations among the particles within the cone.

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