A Numerical Simulation of Adiabatic Shear Bands Formation in Hollow Cylinder

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Abstract. The process of torsion deformation of a hollow cylinder is considered. The mathematical model describing this process is formulated. Numerical algorithm which allows one to simulate the fully localized plastic flow is proposed. The influence of the geometrical parameters of the problem of plastic flow localization is studied. The limiting case when the problem of torsion deformation of a hollow cylinder transforms to the plane problem of deformation of infinite slab is considered. An influence of cylinder size to the formation of adiabatic shear bands is considered. It is shown that declining of inner size of cylinder results to a significant change of solution and displacement of localization area to the inner surface of a hollow cylinder.

1. Introduction
A formation of adiabatic shear bands (ASB) or localization of plastic deformations is one of important phenomena of materials science. Adiabatic shear bands is a narrow region where high temperature and deformations are exceeded in a short period of time. The localizations of deformation are located usually in places with geometry defects or thermal heterogeneity [1] – [5]. The reason of the ASB formation is a loss of stability of plastic flow occurs due to the thermal softening effect. In conditions close to adiabatic the work of plastic deformation converted into heat which causes thermal softening of material and leads to adiabatic shear bands formation. This phenomenon is often investigated in the physical experiments with torsion deformation of a hollow cylinder. But for a numerical simulation many authors consider a limiting case: a shear deformation of an infinite slab. In this work we investigate the original problem and compare it with the limiting case.

2. Mathematical formulation of problem
We consider the process of torsion deformation of hollow cylinder which inner radius is equal to $r_0$ and outer radius equal to $r_0 + H$. The inner surface is fixed and the outer surface of the cylinder is rotated at a constant angular velocity.

Mathematical formulation of cylindrical (a) and plane (b) problems includes four governing equations: the Cauchy momentum equation (1), the Hooke’s law (2), the equation of state or plasticity flow law (3) and the law of conservation of energy (4). So we obtain

\[ a) \quad \rho r \omega = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau \right), \quad b) \quad \rho v_t = \tau_y. \]  

(1)
\[\dot{\gamma} = \Phi^{-1}(\gamma, \tau, T), \quad \gamma = \int_0^t \dot{\gamma}(s)\,ds,\]  
\[CpT_t = k\Delta T + \tau\dot{\gamma}.\]  
where \(y, r\) are coordinates, \(t\) is time, \(\tau, \gamma, v, \omega, T\) are stress, strain, velocity, angular velocity and temperature respectively. The cylindrical problem approaches to the plane problem at \(r_0 \to \infty\) and change of variables: \(r = r_0 + y, \quad v = r\omega\).

Boundary conditions are similar for both tasks and have the form:

\[a)\quad \omega|_{r=r_0} = 0, \quad \omega|_{r=r_0+H} = \omega_0, \quad T|_{r=r_0} = T|_{r=r_0+H} = 0,\]
\[b)\quad v|_{y=0} = 0, \quad v|_{y=H} = v_0, \quad T_y|_{y=0} = T_y|_{y=H} = 0.\]

And initial conditions:

\[a)\quad \omega(r, 0) = \dot{\gamma}_0 \ln \frac{r}{r_0}, \quad T(r, 0) = T_0(r),\]
\[b)\quad v(y, 0) = \dot{\gamma}_0 y, \quad T(y, 0) = T_0(y),\]
\[\gamma_p|_{t=0} = 0, \quad \tau|_{t=0} = 0, \quad \dot{\gamma}_p|_{t=0} = \dot{\gamma}_0.\]

The boundary value problem is solved numerically. The algorithm consist of two stages. The first stage devoted to solution of mechanical part of the problem. For this purpose we use the Courant-Isaacson-Rees scheme [6] with the Newton iterations to system (1), (2), (3). On the second stage, we solve thermal part of the problem (4).

3. Results of numerical simulation
For numerical simulations, we use high-tensile steel HY-100. The Thermophysical parameters of the HY-100 steel can be found in table 1.

| \(\mu, \text{GPa}\) | \(\rho, \text{kg/m}^3\) | \(k, \text{W/m} \cdot \text{K}\) | \(C, \text{J/kg} \cdot \text{K}\) | \(\dot{\varepsilon}_y, \text{s}^{-1}\) | \(a, \text{K}^{-1}\) | \(\tau_0, \text{MPa}\) | \(m\) |
|---|---|---|---|---|---|---|---|
| 80 | 7860 | 49.2 | 473 | \(10^{-4}\) | \(6.43 \cdot 10^{-4}\) | 600 | 0.0251 |

Table 1. Thermophysical parameters of HY-100 steel.

Our experiment is based on the plane problem which was investigated in the works [2], [3]. For steel we use the Litonski plasticity flow law, so \(\Phi(\gamma, \dot{\gamma}, T)\) from equation (3) has the form:

\[\tau = \Phi(\gamma, \dot{\gamma}, T) = \tau_0 e^{-aT} \left(1 + \frac{\dot{\gamma}^n}{\dot{\varepsilon}_y}\right)^m.\]

We test block which has thickness \(H = 6\) mm, strain rate \(\dot{\gamma}_0 = 750 \text{ s}^{-1}\). At an initial moment temperature has gaussian-shaped distribution with a peak \(T_{\max} = 16.2 ^\circ\text{C}\) at center of a layer. Below we use dimensionless time \(\gamma_{nom} = \gamma_0 \cdot t\).

The first experiment is conducted at \(r_0 = 100\) m, the ratio \(y/r_0 \sim 10^{-6}\), so this is almost plane problem, the parameters \(H, \gamma_0, T_{\max, 0}\) correspond to the parameters in the works [2], [3]. In this case the initial perturbation results in the formation of single ASB in the center of the sample. Stages of ASB formation are shown at figures 1 and 2. Time dependence of the
maximum temperature is presented at figure 8. We can see rapid increase of the temperature at the center of slab beginning about $\gamma_{nom} = 0.27 - 0.38$. These results are completely consistent with the results of other authors. For plane problem graphs of temperature, stress and strain are symmetric with respect to $y = 0.5H$, but symmetry is breaking with decreasing of inner radius of the cylinder. The second experiment is conducted at $r_0 = 1$ m. Results are shown at figures 3 and 4. On the figure 4 we can see that distribution of temperature has two local maximum and new maximum is located at the inner surface. Thus we get the temperature at the inner surface is more than at the outer.

This effect is enhanced for less radius of the hole. Let’s decrease inner radius $r_0$ to 0.5 m. In this case the temperature maximum almost immediately arise at the boundary. And at infinite time we come to stationary distribution where the whole cylinder without one point (inner surface) is moving at constant angular velocity, which equals the velocity of the outer surface, and the one point is fixed. It is demonstrated at figures 5 and 6.

However, at $r_0$ we also can get ASB at the center of the layer. It happens if we set a larger peak of temperature at initial moment (160 °C, for example). Let’s consider a problem with stationary initial distribution of temperature. In this case we get a solution with one fixed point regardless of the inner radius. Results of experiment for $r_0 = 0.5$ m are shown at figures 9 and
Figure 5. Angular velocity at $\gamma_{nom} = 0.2, 0.25, 0.3$ (solid, dashed, dotted lines relatively) and $r_0 = 0.5$ m.

Figure 6. Temperature at $\gamma_{nom} = 0.2, 0.25, 0.3$ (solid, dashed, dotted lines relatively) and $r_0 = 0.5$ m.

Figure 7. Time dependence of the minimum stress for $r_0 = 100, 1, 0.5$ m (solid, dashed, dotted lines relatively).

Figure 8. Time dependence of the maximum temperature for $r_0 = 100, 1, 0.5$ m (solid, dashed, dotted lines relatively).

Figure 9. Angular velocity at $\gamma_{nom} = 0.2, 0.25, 0.3$ (solid, dashed, dotted lines relatively) and $r_0 = 0.5$ m.

Figure 10. Temperature at $\gamma_{nom} = 0.2, 0.25, 0.3$ (solid, dashed, dotted lines relatively) and $r_0 = 0.5$ m.

10. The plane task has completely different, simple solution: the slab uniformly heats up due to internal friction. But in cylindrical case, stress at inner surface is higher, so internal friction is also higher. It leads to rapid increasing of temperature at the boundary.
It is obvious that we cannot get such results in a physical experiment. Because any material has micro defects which could become centers for localization of plastic strain, it was demonstrated in work [4]. If we has impossible continuous material, then the inner surface of sample most likely will melt, and we will not be able to fix it anymore. So in this case cylinder will become to rotate at constant angular velocity. However, our model does not take into account phase transactions.

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