Configuration mixing effects in neutron-rich carbon isotopes

C.X. Yuan\textsuperscript{1}, F.R. Xu\textsuperscript{2}*, C. Qi\textsuperscript{3}

\textsuperscript{1} The Franco-Chinese Institute of Nuclear Energy, Sun Yat-sen University, Tangjiawan, Zhuhai, Guangdong, 519082, China
\textsuperscript{2} State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
\textsuperscript{3} KTH (Royal Institute of Technology), Alba Nova University Center, SE-10691 Stockholm, Sweden
E-mail: frxu@pku.edu.cn

Abstract.
Shell model calculations are done to study the structure of neutron-rich carbon isotopes. For both even-A and odd-A neutron-rich carbon isotopes, the energy levels are strongly affected by the configuration mixing of valence neutrons. The calculated energy levels in the nucleus \textsuperscript{17}C are significantly improved compared with experimental values when the model space of the three valence neutrons is enlarged from pure \(\nu(0d_{5/2})^3\) configuration to full \(sd\) space. We also investigate the configuration mixing effect on the \(B(E2)\) values in even-even nuclei \textsuperscript{16−20}C.

1. Introduction
The shell structure evolves when we go from the beta-stability line to drip lines \cite{1}. New magic numbers emerge in neutron or proton-rich nuclei, such as the new \(N = 14\) and \(N = 16\) magic numbers in oxygen isotopes \cite{2, 3}. The proton-neutron interaction, especially its tensor component, plays an important role in the shell evolution \cite{1, 4, 5, 6, 7}. In case of the \(N = 14\) shell evolution, from oxygen to carbon isotopes, two protons are removed from the \(0p_{1/2}\) orbit. \(0p_{1/2}\) proton attracts \(0d_{5/2}\) neutron more than \(1s_{1/2}\) neutron \cite{8}. The \(N = 14\) gap thus becomes smaller in carbon isotopes than in oxygen isotopes. The attractive neutron-neutron interaction also contributes to the \(N = 14\) shell evolution \cite{8}. When the \(N = 14\) shell gap becomes smaller, neutrons move from \(0d_{5/2}\) to \(0s_{1/2}\) orbit. The attractive neutron-neutron interaction \(V_{\nu\nu}^{nm}\) and \(V_{0d_{5/2}0d_{5/2}}^{nm}\) thus make the \(N = 14\) shell gap even smaller \cite{8}. The \(N = 14\) shell existing in oxygen isotopes reduces in nitrogen isotopes \cite{9} and disappears in carbon isotopes \cite{10} which is caused by both proton-neutron and neutron-neutron interaction \cite{8}. The configuration mixing between neutron \(0d_{5/2}\) and \(1s_{1/2}\) orbits becomes important in carbon isotopes. In this paper, we will analyze effects caused by configuration mixing in neutron-rich carbon isotopes.

2. Shell-model framework
The model space we used in this paper is the \(psd\) model space, which includes \(0p_{3/2}, 0p_{1/2}, 0d_{5/2}, 1s_{1/2}\), and \(0d_{3/2}\) orbits \cite{11}. The well-established WBP \cite{12}, WBT \cite{12}, and MK \cite{13} effective Hamiltonians are used. We restrict that maximum zero or two nucleons can be excited from \(p\)
Figure 1. Probabilities of each configuration in the ground states of neutron-rich carbon, nitrogen, and oxygen isotopes calculated with WBP interaction. \( n \) indicates the number of valence neutrons in \( sd \) shell. \( n = 2, 4, 6 \) correspond to \(^{16}\text{C},^{18}\text{C}, \) and \(^{20}\text{C} \), respectively.

Figure 2. Energy levels of \( N = 11 \) isotones. Experimental values \([3, 9, 10]\) are compared with WBP results with two model space for valence neutron. One is full \( sd \) model space, the other is \( \nu(0d_{5/2})^3 \) which means the three valence neutrons are restricted on \( 0d_{5/2} \) orbit.

to \( sd \) shell, which are denoted as \( 0\hbar \omega \) and \( 2\hbar \omega \), respectively. The calculations are carried out with a newly-established parallel shell-model code described in Ref. [14]

In the case of \( 2\hbar \omega \) calculations, the spurious states caused by the center-of-mass motion need to be removed. The shell model Hamiltonian \( H \) can be modified to be \( H' = H + \beta H_{c.m.} \), where \( H_{c.m.} \) is the center-of-mass Hamiltonian [15]. The center-of-mass excitations can be moved to high excitation energies by setting a large positive \( \beta \) value. Thus the spurious states do not appear at low excitation energy. We use \( \beta = 100 \) MeV in the present work, the same as in previous works [16, 17, 18].

3. Calculations and discussions

The \( E(2^+_1) \) in \( N = 14 \) isotones decreases quickly from oxygen to carbon isotopes which indicates that the \( N = 14 \) shell gap existing in oxygen isotopes disappears in carbon isotopes [3, 10]. The excitation energies in carbon isotopes are strongly affected by the configuration mixing between neutron \( 0d_{5/2} \) and \( 1s_{1/2} \) orbit. In \(^{18}\text{O},^{20}\text{O}, \) and \(^{22}\text{O} \), the valence neutrons which occupy \( 0d_{5/2} \)
orbit couple to $J = 0$ in ground states. When excited, the $2^+_1$ states gain energy by the valence neutrons coupling to $J = 2$ in $^{18}\text{O}$ and $^{20}\text{O}$. In $^{22}\text{O}$ of which valence neutrons have fully occupied the $0d_{5/2}$ orbit, the $E(2^+_1)$ is gained by exciting a neutron from $0d_{5/2}$ to $1s_{1/2}$ orbit. In case of C isotopes, the $N = 14$ shell is disappeared. So the valence neutrons in C isotopes are not only occupy $\nu0d_{5/2}$ orbit but also $\nu1s_{1/2}$ orbit [8]. The $2^+_1$ states of $^{16}\text{C}$, $^{18}\text{C}$ and $^{20}\text{C}$ are excited by both neutrons coupling to $J = 2$ in $0d_{5/2}$ and a neutron moving to $1s_{1/2}$ orbit. This simple consideration is supported by shell-model calculations, as shown in Fig. 1. The neutron-rich oxygen isotopes contain very large (around 80%) pure $\nu0d_{5/2}$ configuration. On the other hand, the strong configuration mixing, especially the mixing between $\nu0d_{5/2}$ and $\nu1s_{1/2}$ orbits, exists in neutron-rich carbon isotopes.

The energy levels of odd-$A$ neutron-rich carbon isotopes also show the effects of configuration mixing between $\nu0d_{5/2}$ and $\nu1s_{1/2}$ orbits. The energy levels of $N = 11$ isotones are hard to be exactly described by shell model because of the strong configuration mixing [19]. A recent suggested Hamiltonian can well reproduce the energy levels of $^{17}\text{C}$, $^{18}\text{N}$, and $^{19}\text{O}$ [20]. Figure 2 shows how the configuration mixing drive the energy levels of $^{17}\text{C}$, $^{18}\text{N}$, and $^{19}\text{O}$. Here we present the $0\hbar\omega$ results, as the $2\hbar\omega$ results do not show much difference. In the case of $0\hbar\omega$ calculations, the three valence neutrons in $^{17}\text{C}$, $^{18}\text{N}$, and $^{19}\text{O}$ can be active in three $sd$ orbits. When we enlarge the model space of valence neutrons from pure $\nu(0d_{5/2})^3$ to full $sd$ configuration, the energy levels of these nuclei can be much improved compared with experimental values, especially for $^{17}\text{C}$.

Compared with valence neutrons in neutron-rich carbon isotopes, the valence protons in such nuclei are less active as the $Z = 6$ gap is much larger than the $N = 14$ gap. However, the configuration mixing between valence protons strongly affects the $E2$ transition in neutron-rich carbon isotopes [8].

The shell-model $B(E2)$ results agree well with the observed value in neutron rich C isotopes [21], also as show in Fig. 3. In our calculations, an approximate $1/A$ dependence effective charge is used [25]. The very small $B(E2)$ value is from an inelastic scattering measurement [22]. To clarify the contribution from valence neutrons and protons, we present the $e^{eff}_{p(n)}A_{p(n)}$ in Fig. 4. Both proton and neutron contribute a lot in $B(E2)$ values. There is a sudden increase of $e^{eff}_{p}A_{p}$

**Figure 3.** Calculated $B(E2)$ values for even-even $^{16}$–$^{20}\text{C}$. Experimental values are taken from Refs. [21, 22, 23, 24].
at $^{20}\text{C}$ when the $e_n^{eff} A_n$ changes little from $^{16}\text{C}$ to $^{20}\text{C}$. As discussed in Ref. [21], the reduce proton $\pi 0p_{1/2} - \pi 0p_{3/2}$ gap from $^{16}\text{C}$ to $^{20}\text{C}$ is a reason that the proton excitation enlarges from $^{16}\text{C}$ to $^{20}\text{C}$. When the neutron number increasing, the $\nu 0d_{5/2}$ attracts $\pi 0p_{1/2}$ orbit more than $\pi 0p_{3/2}$ orbit both in WBT and MK interaction. The energy gap between $\pi 0p_{1/2}$ and $\pi 0p_{3/2}$ decreases from $^{16}\text{C}$ to $^{20}\text{C}$.

We present in Fig. 5 the proton configuration of $^{16}\text{C}$, $^{18}\text{C}$ and $^{20}\text{C}$ calculated with WBT interaction. From $^{16}\text{C}$ to $^{20}\text{C}$, the $(\pi 0p_{1/2})^0$ configuration reduces and $(\pi 0p_{1/2})^1$ configuration increases. The $Ap$ is most contributed by the transition from $(\pi 0p_{1/2})^0$ in ground state to $(\pi 0p_{1/2})^1$ in $2^+_1$ state and from $(\pi 0p_{1/2})^1$ in ground state to $(\pi 0p_{1/2})^0$ in $2^+_1$ state. These two proton transitions increase from $^{16}\text{C}$ to $^{20}\text{C}$. This simple analysis does not consider the neutron configurations. Both $(\pi 0p_{1/2})^0$ and $(\pi 0p_{1/2})^1$ relate to many neutron configurations. Considering all possible configurations, $e_p^{eff} A_p$ of $^{20}\text{C}$ is 1.5 times more than that of $^{18}\text{C}$ as shown in Fig. 4. In calculation of $B(E2)$ values, this $e_p^{eff} A_p$ is squared and lead to the large $B(E2)$ value in $^{20}\text{C}$.

**Figure 4.** Calculated E2 transition matrix elements for even-even $^{16-20}\text{C}$.

**Figure 5.** Calculated WBT result of proton configurations for even-even $^{16-20}\text{C}$.
4. Summary
The effects of configuration mixing on neutron-rich carbon isotopes are analyzed within the frame work of shell model. The energy levels of neutron-rich carbon isotopes are strongly affected by the configuration mixing of valence neutrons. The configuration mixing of valence protons influence little on the energy levels. However it contributes significantly on the large $B(E2)$ values in $^{20}$C.

5. Acknowledgement
This work has been supported by the National Natural Science Foundation of China under Grant Nos. 10975006 and 11235001, and the Swedish Research Council (VR) under grant No. 621-2010-4723.

References
[1] Sorlin O and Porquet M G 2008 Prog. Part. Nucl. Phys. 61 602
[2] Janssens R V F 2009 Nature (London) 459 1069
[3] Stanoiu M et al. 2004 Phys. Rev. C 69 034312
[4] Otsuka T, Fujimoto R, Utsuno Y, Brown B A, Honma M, and Mizusaki T 2001 Phys. Rev. Lett. 87 082502
[5] Otsuka T, Suzuki T, Fujimoto R, Grawe H, and Akaishi Y 2005 Phys. Rev. Lett. 95 232502
[6] Otsuka T, Suzuki T, Honma M, Utsuno Y, Tsumoda N, Tsukiyama K, and Hjorth-Jensen M 2010 Phys. Rev. Lett. 104 012501
[7] Suzuki T and Otsuka T 2009 Int. J. Mod. Phys. E 18 18
[8] Yuan C X, Qi C, and Xu F R 2012 Nucl. Phys. A 883 25
[9] Sohler D et al. 2008 Phys Rev. C 77 044303
[10] Stanoiu M et al. 2008 Phys. Rev. C 78 034315
[11] Brown B A 2001 Prog. Part. Nucl. Phys. 47 517
[12] Warburton E K and Brown B A 1992 Phys Rev. C 46 923
[13] Millener D J and Kurath D 1975 Nucl. Phys. A 255 315
[14] Qi C and Xu F R 2008 Chin. Phys. C 32 Suppl. 112
[15] Gloeckner D H and Lawson R D 1974 Phys. Lett. B 53 313
[16] Umeya A, Kaneko G, Haneda T, and Muto K 2008 Phys. Rev. C 77 044301
[17] Ma H L, Dong B G, and Yan Y L 2010 Phys. Lett. B 688 150
[18] Utsuno Y and Chiba S 2011 Phys. Rev. C 83 021301
[19] Wiedeking M et al. 2008 Phys Rev. C 77 054305
[20] Yuan C X, Suzuki T, Otsuka T, Xu F R, and Tsumoda N 2012 Phys Rev. C 85 064324
[21] Petri M et al. 2011 Phys. Rev. Lett. 107 102501
[22] Elekes Z et al. 2009 Phys. Rev. C 79 011302(R)
[23] Wiedeking M et al. 2008 Phys. Rev. Lett. 100 152501
[24] Ong H J et al. 2008 Phys. Rev. C 78 014308
[25] Sagawa H, Zhou X R, Zhang X Z, and Suzuki T 2004 Phys. Rev. C 70 054316