LEP, SLC AND THE STANDARD MODEL

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Abstract

A simple way of deriving and analyzing electroweak radiative corrections to the Z boson decays is presented in the framework of the Standard Model. This paper is an updated and revised version of a plenary talk given at the Second Sakharov Conference; Lebedev Physical Institute, Moscow, May 20-24, 1996.

1 LEP-I and SLC

LEP-I (CERN) and SLC (SLAC) electron-positron colliders had started their operation in the fall 1989. The sum of energies of $e^+ + e^-$ was chosen to be equal to the Z boson mass. LEP-I was terminated in the fall of 1995 in order to give place to LEP-II which operated at energy 135 GeV in spring 1996, 172 GeV in spring 1997 and will finally reach about 200 GeV in 1999–2000. Meanwhile SLC continues at energy close to 91 GeV. The reactions, which have been studied at LEP-I (detectors: ALEPH, DELPHI, L3, OPAL) and SLC (detector SLD) may be presented in the form:

$$e^+ e^- \rightarrow Z \rightarrow f \bar{f},$$

where

$$f \bar{f} = \nu \bar{\nu} (\nu_e \bar{\nu}_e, \nu_{\mu} \bar{\nu}_{\mu}, \nu_{\tau} \bar{\nu}_{\tau}) - \text{invisible},$$
$$l \bar{l} (e \bar{e}, \mu \bar{\mu}, \tau \bar{\tau}) - \text{charged leptons},$$
$$q \bar{q} (u \bar{u}, d \bar{d}, s \bar{s}, c \bar{c}, b \bar{b}) \rightarrow \text{hadrons}.$$
About 20,000,000 Z bosons has been detected at LEP and about 100,000 at SLC (but here electrons are polarized). Fantastic precision has been reached in the measurement of the Z boson properties \cite{1,2,3}:

\[ M_Z = 91,186.3 \pm 1.9\text{MeV} , \quad \Gamma_Z = 2,494.7 \pm 2.6\text{MeV} , \]

\[ \Gamma_h \equiv \Gamma_{\text{hadrons}} = 1,743.6 \pm 2.5\text{MeV} , \quad R_l = \frac{\Gamma_h}{\Gamma_l} = 20.783 \pm 0.029 , \]

\[ \Gamma_{\text{invisible}} = 499.5 \pm 2.0\text{MeV} , \]

where \( \Gamma_l \) refers to the width of a single charged leptonic channel. (As a rule we use the most recent data submitted to the XXIInd Rencontre de Moriond Conference, Les Arcs/Savoie, France, March 15-22, 1997 \cite{1}. These data have tiny deviations from the data reported at the 28th ICHEP, Warsaw, Poland, 25-31 July, 1996 \cite{2}, or from the data from the latest Review of Particle Physics \cite{3}.)

By comparing the \( \Gamma_{\text{invisible}} \) with theoretical predictions for neutrino decays it was established that the number of neutrinos which interact with Z boson is 3: \( N_\nu = 2.989 \pm 0.012 \). This is a result of fundamental importance.

More than 2,000 experimentalists and engineers and hundreds of theorists participated in this unique collective quest for truth.

2 Theoretical analysis: the fundamental parameters.

The theoretical study of electroweak radiative corrections started in 1970’s and was elaborated by a number of theoretical groups in many countries \cite{3} - \cite{21}. This study has been summarized in Yellow CERN Reports of working groups on precision calculations for the Z resonance in 1995 \cite{22}. The results of different groups are in good agreement. The deviations in theoretical calculations are by order of magnitude smaller than the experimental uncertainties. In what follows we mainly use our own calculations \cite{21} in which we tried to simplify the work of previous authors \cite{3} - \cite{20} by separating genuine electroweak radiative corrections from purely electromagnetic ones.

It is instructive to compare the electroweak theory with the quantum electrodynamics (QED). In the latter there are two fundamental parameters: mass of the electron, \( m_e \), and its charge, \( e \), or fine structure constant \( \alpha = \frac{e^2}{\pi m_e c} \).
\( e^2/4\pi \) (we use units \( \hbar, c = 1 \)). Both are not only fundamental, but also known with high precision. Every observable in QED can be expressed in terms of \( m_e \) and \( \alpha \) (and, of course, of energies and momenta of particles participating in a given process).

In the electroweak theory the situation is more complex for several reasons:

1. There are more fundamental charges and masses.

2. They are not independent of each other.

3. Not all of them are known with high accuracy.

4. On the other hand, as a basic parameter of the theory a quantity is used, which is known with highest accuracy, but which is not fundamental, the four-fermion coupling of the muon decay, \( G_\mu \).

The fundamental masses of the electroweak theory are masses of \( W \) and \( Z \) bosons, \( m_W \) and \( m_Z \). Among the masses of fermions, the most important for the \( Z \)-decay is the mass of the top-quark, \( m_t \).

The fundamental couplings of the electroweak theory are \( e, f, g \), or
\[
\alpha = e^2/4\pi, \quad \alpha_Z = f^2/4\pi, \quad \alpha_W = g^2/4\pi:
\]
e is the coupling of photons to electrically charged particles, \( f \) is the coupling of \( Z \) bosons to weak neutral current, e.g. \( \bar{\nu}\nu \), \( g \) is the coupling of \( W \) bosons to weak charged current, e.g. \( \bar{e}\nu \).

While the charged current is a purely V-A current of the form \( \gamma_\alpha(1 + \gamma_5) \), the ratio \( g_{Vf}/g_{Af} \) between the vector and axial vector neutral currents depends on the third projection of the isotopic spin of the fermion \( f \), \( T_3^f \), and on its electric charge \( Q_f \). The decay amplitude of the \( Z \) boson may be written in the form:

\[
M(Z \to f\bar{f}) = \frac{1}{2} f \bar{\psi}_f (g_{Vf} \gamma_\alpha + g_{Af} \gamma_\alpha \gamma_5) \psi_f Z^\alpha ,
\]

where \( \bar{\psi}_f \) is the wave function of emitted fermion, \( \psi_f \) corresponds to the emitted antifermion (or absorbed fermion), \( Z^\alpha \) is the wave function of the \( Z \) boson. At the tree level (see, e.g. text-books [23]):

\[
g_{Af} = T_3^f , \quad g_{Vf} = T_3^f - 2Q_f s_W^2 .
\]
Here

\[ T^f_3 = +1/2 \text{ for } f = \nu, u, c; \ T^f_3 = -1/2 \text{ for } f = l, d, s, b. \]

Thus

\[ g_{Vf}/g_{Af} = 1 - 4|Q|s_w^2. \]

In the above expressions \( s_W \equiv \sin \theta_W \), where \( \theta_W \) is the so called weak angle.

At the tree level (no loops): \( e/g = s_W \), \( g/f = c_W \), \( m_W/m_Z = c_W \), where \( c_W \equiv \cos \theta_W \).

The four-fermion coupling constant \( G_{\mu} \) is extracted from the life-time of the muon, \( \tau_\mu \), after taking into account the well-known kinematic and electromagnetic corrections:

\[ 1/\tau_\mu = \Gamma_\mu = \frac{G_{\mu}^2 m_\mu^5}{192\pi^3}(1 + \text{well known corrections } \sim (\frac{m_e}{m_\mu})^2, \alpha), \]

\[ G_{\mu} = (1.16639 \pm 0.00002) \cdot 10^{-5}\text{GeV}^{-2}. \]

In the tree approximation the four-fermion coupling constant \( G_{\mu} \) can be expressed in terms of \( W \) boson coupling constant \( g \) and its mass \( m_W \):

\[ G_{\mu} = \frac{g^2}{4\sqrt{2}m_W^2} = \frac{\pi\alpha}{\sqrt{2}m_W^2 s_w^2} = \frac{\pi\alpha}{\sqrt{2}m_Z^2 s_w^2 c_w^2}. \]

(The last two expressions are derived by using the relations \( e/g = s_W \), \( \alpha = e^2/4\pi \), \( m_W/m_Z = c_W \)).

3 Theoretical analysis: the running \( \alpha(q^2) \).

It has been well known since 1950’s that electric charge \( e \) and hence \( \alpha \) logarithmically depend on the square of the four-momentum of the photon, \( q^2 \) [24]. (For a real photon \( q^2 = 0 \), for a virtual one \( q^2 \neq 0 \)). This phenomenon is usually referred to as "the running of \( \alpha \)." It is caused by vacuum polarization, by loops of virtual charged particles: charged leptons, \( l\bar{l} \), and quarks, \( q\bar{q} \), inserted into the propagator of a photon. As a result \( \bar{\alpha} \equiv \alpha(q^2 = m_Z^2) \) is approximately by 6% larger than \( \alpha \equiv \alpha(0) \).
The relation between $\bar{\alpha}$ and $\alpha$ is obtained by summing up an infinite chain of loops:

$$\bar{\alpha} = \alpha/(1 - \delta \alpha);$$

$\delta \alpha = \delta \alpha_l + \delta \alpha_h$, where $\delta \alpha_l$ is the one-loop contribution of three charged leptons, while $\delta \alpha_h$ – is that of five quarks (hadrons). The leptonic contribution is predicted with very high accuracy: $\delta \alpha_l = \frac{\alpha}{4\pi} \sum_i \left( \ln \frac{m^2_i}{m^2_{e^+e^-}} - \frac{5}{3} \right) = 0.03141$.

The hadronic contribution is obtained on the basis of dispersion relations and low-energy experimental data on $e^+e^-$-annihilation into hadrons [25]: $\delta \alpha_h = 0.02799(66)$.

The value of $\alpha(0)$ is known with extremely high accuracy: $\alpha \equiv \alpha(0) = 1/137.035985(61)$; the accuracy of $\alpha$ is very important for QED, but irrelevant to electroweak physics.

The value of $\bar{\alpha}$ is less accurate: $\bar{\alpha} = 1/128.896(90)$ [25], but $\bar{\alpha}$ is pivotal for electroweak physics. Let us stress that the running of $\alpha(q^2)$ is a purely electromagnetic effect, caused by electromagnetic loops of light fermions. Contributions of $t\bar{t}$ and $W^+W^-$ are negligibly small and may be taken into account together with purely electroweak loops.

Unlike $\alpha(q^2)$, two other electroweak couplings $\alpha_Z(q^2)$ and $\alpha_W(q^2)$ are not running but ”crawling” in the interval $0 \leq q^2 \leq m^2_Z$ [26]:

$$\alpha_Z(m^2_Z) = 1/22.91, \quad \alpha_Z(0) = 1/23.10;$$

$$\alpha_W(m^2_Z) = 1/28.74, \quad \alpha_W(0) = 1/29.01.$$

The natural scale for $Z$-physics is $q^2 = m^2_Z$. Therefore it is evident that $\bar{\alpha} \equiv \alpha(m^2_Z)$, not $\alpha \equiv \alpha(0)$ is the relevant parameter. In fact, in all computer codes, dealing with $Z$-physics, $\bar{\alpha}$ enters at a certain stage and substitutes $\alpha$. But this occurs inside the ”black box” of the code, while $\alpha$ formally plays the role of an input parameter. In these codes the running of $\alpha$ is considered as (the largest) electroweak correction. We consider this running as purely electromagnetic one and define our Born approximation in terms of $\bar{\alpha}$, $G_\mu$ and $m_Z$.

Instead of angle $\theta_W$, we define angle $\theta$ [11, 21] ($s \equiv \sin \theta$, $c \equiv \cos \theta$) in the following way:

$$G_\mu = \frac{g^2(q^2 = 0)}{4\sqrt{2}m^2_W} \simeq \frac{g^2(q^2 = m^2_Z)}{4\sqrt{2}m^2_W},$$
where the second equality is based on the ”crawling” of \( g(q^2) \): \( g(0) \simeq g(m_Z^2) \). We use it to define the angle \( \theta \):

\[
G_\mu \equiv \frac{e^2(m_Z^2)}{4\sqrt{2}s^2 m_W^2} = \frac{\pi \alpha}{\sqrt{2}s^2 c^2 m_Z^2}.
\]

Thus,

\[
\sin^2 2\theta = \frac{4\pi \alpha}{\sqrt{2}G_\mu m_Z^2} = 0.71078(50),
\]

\( s^2 = 0.23110(23), \ c^2 = 0.76890(23), \ c = 0.87687(13) \).

Our Born approximation starts with the most accurately known observables: \( G_\mu, m_Z, \bar{\alpha} \) (or \( s^2 \)) [21].

Traditionally parametrization in terms of \( G_\mu, \alpha, \) and Sirlin’s \( \theta_W \) [6] \( s_W^2 \equiv 1 - m_W^2/m_Z^2 \) is widely used in the literature (see e.g. the review [3] and references therein). This parametrization is less convenient (\( s_W \) has poor accuracy: \( \Delta m_W = \pm 80 \text{ MeV} \); running of \( \alpha \) is not separated from electroweak corrections and overshadows them). (In references [15] the Born approximation is also defined by \( s^2 \), but purely electromagnetic ”radiators” are included in the definitions of electroweak observables.)

4 Theoretical analysis: one-loop electroweak corrections.

For the sake of brevity let us choose two observables:

\[
s_W^2 \equiv 1 - \frac{m_W^2}{m_Z^2}, \ s_l^2 \equiv \frac{1}{4}(1 - \frac{g_{Vl}}{g_{Al}}).
\]

In the Born approximation \( s_W^2 = s_l^2 = s^2 \). From UA2 and CDF experiments measuring the mass of \( W \) boson [2]:

\[
s_W^2 = 0.2231(16), \ 5\sigma \ \text{away} \ \text{from} \ s^2 = 0.23110(23).
\]

From the parity violating asymmetries (and, hence \( g_{Vl}/g_{Al} \)) measured at LEP and SLC:

\[
s_l^2 = 0.23141(28), \ 1\sigma \ \text{away} \ \text{from} \ s^2.
\]
(Note the high experimental accuracy of $s^2_l$ compared to that of $s^2_W$.)

In the one-loop approximation

$$s^2_l = s^2 - \frac{3}{16\pi c^2 - s^2} V_{R_l}(m_t, m_H),$$

where $c^2 - s^2 = 0.5378$; index $R_l$ stands for the ratio $g_{Vl}/g_{Al}$, and the radiative correction depends on the masses of the top quark and higgs. These masses enter via loops containing virtual top quark, or higgs. The coefficient in front of $V_{R_l}$ is chosen in such a way that $V_{R_l}(m_t, m_H) \approx t \equiv (m_t/m_Z)^2$ for $t \gg 1$. The same asymptotic normalization is used for radiative corrections to other electroweak observables \[21\]. The good agreement, within $1 \sigma$, between experimental value of $s^2_l$ and its Born value means that corresponding electroweak radiative correction is anomalously small \[21\]. The unexpected smallness of $V_{R_l}$ is the result of cancelation between large and positive contribution from the $t$-quark loops and large and negative contribution from loops of other virtual particles. This cancelation, which looks like a conspiracy, occurs when $m_{top}$ is around 160 GeV, if higgs is light ($m_H \leq 100$ GeV). If higgs is heavy ($m_H = 1000$ GeV) it occurs when $m_{top}$ is around 210 GeV.

Thus, vanishing electroweak radiative corrections told us that top was heavy before it was discovered.

5 LEPTOP and the general fit.

The analytical formulas for all electroweak observables have been incorporated in our computer code which we dubbed LEPTOP \[27\]. The fit of all electroweak data by LEPTOP gives:

$$m_t = 181 \pm 5^{+17}_{-21} \text{ GeV}.$$  

The central value ($181 \pm 5$) corresponds to $1/\bar{\alpha} = 128.896$ and $m_H = 300$ GeV; the shifts (+17, -21) – to $m_H = 1000$ and 60 GeV, respectively. This prediction is in perfect agreement with the recent (spring 1997) data on the direct measurements of the top mass by two collaborations at FNAL:

$$m_t = 175.6 \pm 5.5 \text{ (CDF/D0) \[4\].}$$

Unfortunately the electroweak radiative corrections depend on $m_H$ only weakly (are proportional to $\ln m_H/m_Z$). This weak dependence results in a
rather poor accuracy for $m_H$. The values of $m_t$ and $m_H$ from four parameter fit $(m_t, m_H, \hat{\alpha}_s, \bar{\alpha})$ with the constraints by world average values of $\hat{\alpha}_s^{\text{world}} = 0.118 \pm 0.003$ and $\bar{\alpha}^{\text{world}} = 1/128.896(90)$ are:

$$m_t = 172.6 \pm 5.3 \text{ GeV},$$

$$m_H = 134_{-72}^{+119} \text{ GeV},$$

$$\log m_H = 2.13^{+0.28}_{-0.33},$$

$$\chi^2/n.d.f. = 21/15,$$

where all LEP/SLC, $\nu N$, $M_{W}$ and the direct CDF/D0 $m_{top}$ measurements are taken into account ($\log m_H$ is digital logarithm and $m_H$ is taken in GeV). The error of the higgs mass does not take into the account the theoretical uncertainties, which were estimated in [1] to be $\delta \log m_H = 0.1$. We see that the upper bound on $m_H$ from the analysis of electroweak radiative corrections is not yet very strong, about 1000 GeV at $3\sigma$ level. One should not give too much credit to the central value of $m_H$ from the fit. One way to demonstrate this is to remove $\bar{\alpha}^{\text{world}}$ from the input data set and determine $\bar{\alpha}$ value only from four-parameter ($m_t, m_H, \hat{\alpha}_s, \bar{\alpha}$) fit to LEP, SLC, $p\bar{p}$, $\nu N$ results:

| Constraint | $\alpha_s$ is not constrained | $\alpha_s$ is constrained: |
|------------|-------------------------------|--------------------------|
| $m_t$      | $173.8 \pm 5.3$ GeV           | $173.2 \pm 5.3$ GeV      |
| $m_H$      | $336_{-328}^{+623}$ GeV       | $248_{-236}^{+423}$ GeV  |
| $\log m_H$| $2.53^{+0.46}_{-1.66}$        | $2.39^{+0.43}_{-1.35}$   |
| $\hat{\alpha}_s$ | 0.121 \pm 0.004 | 0.119 \pm 0.002 |
| $1/\bar{\alpha}$ | 129.107 \pm 0.254 | 129.050 \pm 0.262 |
| $\chi^2/n.d.f.$ | 20/13 | 21/14 |

We see that the fit of the $Z$ boson decay gives a value of $\bar{\alpha}$, compatible with that derived from the low-energy $e^+e^-$ data, although with larger uncertainties. We also see that a slight decrease of the central value of $\bar{\alpha}$ leads to a drastic increase of the central value of $m_H$ (see also [28]). This anticorrelation follows from the expression for $s_l^2$ given at the beginning of
chapter 4 and the decrease of the function $V_R$, with increase of the $m_H$ value (similar consideration holds for $s_W^2$).

Hadronic decays of $Z$ are sensitive to the value of the gluonic coupling $\alpha_s$:

$$\Gamma_q \equiv \Gamma(Z \to q\bar{q}) = 12\Gamma_0[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}],$$

where $q$ is a generic quark, and

$$\Gamma_0 = \frac{G_F m_Z^3}{24\sqrt{2}\pi} = 82.944(6) \text{ MeV}.$$

The "radiators" $R_{Aq}$ and $R_{Vq}$ contain QCD and QED corrections caused by the final state emission and exchange of gluons and photons. As is well known [3], in the first approximation

$$R_{Vq} = R_{Aq} = 1 + \frac{\hat{\alpha}_s}{\pi},$$

where $\hat{\alpha}_s \equiv \alpha_s(m_Z^2)$. The LEPTOP fit of all electroweak data gives:

$$\hat{\alpha}_s = 0.122(3)_{-1}^{+2}.$$

Here the central value (0.122 ± 0.003) corresponds to $m_H = 300$ GeV; the shifts +0.002 and −0.001 to $m_H = 1000$ GeV and 60 GeV, respectively.

Let us note, that low energy processes (deep inelastic scattering, Υ-spectroscopy) give much smaller values of $\hat{\alpha}_s$, around 0.110, when extrapolated to $q^2 = m_Z^2$. There are different opinions on the seriousness of this discrepancy.

Another problem was connected with the experimental value of the width of the decay $Z \to b\bar{b}$. Theoretically the ratio $R_b = \Gamma_b/\Gamma_h$ is not sensitive to $\hat{\alpha}_s$, $m_t$ and $m_H$; the theory predicts:

$$R_b = 0.2154(2)_{-17}^{+7},$$

where again the central value (0.2154 ± 0.0002) corresponds to $m_H = 300$ GeV, whilst shifted by -0.0007 at $m_H = 1000$ GeV and by +0.0007 at $m_H = 60$ GeV. Up to 1996 the experimental value was $R_b = 0.2219(17)$, which was 4σ larger than the theoretical prediction based on the Standard Model.

In May 1996 we ended the talk at the Sakharov Conference by saying: "Both problems (of $\hat{\alpha}_s$ and of $R_b$) if they exist may be solved by new physics. But maybe, experimentalists, can also change their numbers?" Since that time the experimental value of $R_b$ decreased by 0.0040 to 0.2179(11); now it is only 2σ away from the theoretical prediction.

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