Novel depiction of love wave dispersion and inversion for inversely dispersive medium by full SH-wavefield reflectivity method – part I: numerical example

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Abstract. Recently, Love wave applications on geological and geophysical problems have been frequently reported by numerous authors. Unlike Rayleigh wave, Love wave modelling has a simpler algorithm since depends only on three earth properties: SH-wave velocity, density, and thickness of layers. The fewer the parameters more stable the inversion result. There are several tasks, however, that have not been reported related to the modelling and inversion of Love wave data. The tasks are to give answer to several following questions: (a) can Love wave propagate in an inversely dispersive medium of the earth? (b) what is the method recommended to use for explaining appropriate Love wave dispersion curve for the inversely dispersive medium? (c) is that possible to have ‘effective’ Love wave dispersion curve for the inversely dispersive medium and to utilize it in the inversion process? Numerical simulations show that modal matrix method cannot prepare appropriate Love wave dispersion for such a medium. The resulting Love wave dispersion curve still shows high phase velocities at low frequencies, which is not as desired. Full SH-waveform method, in the other hand, successful simulates appropriate Love wave dispersion for such a medium. By this method, an effective Love wave dispersion curve is also introduced. Inversion of the effective dispersion curve can excellently estimate the shear-wave velocity.

1. Introduction

Applications of surface wave methods for mapping near shear-wave velocity variation with depth have become common practice in recent years. Most reports of surface wave analysis incorporated modal matrix methods for modeling the phase velocity dispersion (e.g. [1], [2], [3], and [4]). The methods involve compound matrix [5], which are restricted to plane modal dispersion curve and require proper mode identification. The proper modal dispersion curve, in fact, is usually difficult to be identified at many sites, e.g. when velocity reversal or inverse stiffness of layers exist [6].

Identification and incorporation of higher mode energy when the shallow stiffness exists had been cited by [7]. The presence of higher modes may lead to generation of ‘effective’ surface wave dispersion. Many authors have employed such ‘effective’ dispersion modelling, for soil [8] and asphalt [9] evaluation. These methods usually employ an impulse-response transfer-function, which can simulate higher modes, spreading wavefronts and body waves, but not usually recording aperture and processing effects, nor leaky waves, e.g. multiply-reflected or guided P-waves [10].

Full wavefield reflectivity modeling has been applied to account for all of Rayleigh and Love wave higher modes, near-field effects, leaky-mode and body-wave contributions. This method essentially simulates the complete acquisition-processing flow as described in [10], [11], [12], [13] and [14]. All
the authors have shown advantages of full P-SV wavefield reflectivity (and so Rayleigh wave) in simulating and estimating any Earth’s layered model, in particular for depiction and delineation of the velocity reversals and or low velocity layer (LVL). Application of Rayleigh wave method for modelling and delineation of an inversely dispersive medium has been addressed by several authors. Modelling Love wave data (e.g., waveform and phase velocity dispersion) for the inversely dispersive medium and its inversion, so far however, have not been reported yet. A research on application of Love wave method for the inversely dispersive medium is, therefore, important to be carried out in order to make clear: a) whether Love wave can propagate in such a medium or not, and b) whether the resulting data can be utilized to estimate the desired medium properties.

Most of the reports of Love wave applications used multimode Love wave dispersion in the inversion process. In the other hand, ‘effective’ phase velocity dispersion, which is a superposition of several modes of propagations, is commonly used in analysis of Rayleigh wave data. Here, a curiosity appears related to possibility to have an effective Love wave phase velocity dispersion. Is it possible to have such effective phase velocity dispersion for Love wave? And can it be applied to delineate near-surface shear wave velocity? These questions appear because, so far, there are no any discussions and reports related to the effective Love wave phase velocity dispersion and its inversion.

Based on the matters explained above, this research is carried out to give answer to several following questions:

1. Can Love wave propagate in an inversely dispersive medium of the Earth?
2. What is the method recommended to use for explaining appropriate Love wave dispersion curve for the inversely dispersive medium?
3. Is that possible to have effective Love wave dispersion curve for the inversely dispersive medium and to utilize it in the inversion process?

For giving answers to the above questions, we have to investigate firstly the most appropriate modelling method to prepare the effective Love phase velocity dispersion, whether modal matrix method or full SH wavefield method. Therefore, we will start our task by addressing numerical test using the full wavefiled reflectivity matrix of Muller [15], a method that has been used successfully for modelling and inversion of surface wave dispersion.

2. Experimental Methods

2.1. Model parameters

Three layers including half-space are considered (Table 1). The data represents an inversely dispersive property of the Earth layered medium. The first layer has 3 m thick with a shear-wave velocity of 240 m/s. This upper-most layer is embedded by a softer layer of 180 m/s with a thickness of 7 m over a half-space. The half-space is considered to have velocity of 80 m/s. This medium example mimics the presence of softer layers under a hard asphalt-road base.

| Layer   | Thickness (m) | Shear-wave Velocity (m/s) | Density (g/cc) |
|---------|---------------|----------------------------|----------------|
| 1st layer | 3             | 240                        | 1.8            |
| 2nd layer | 7             | 180                        | 1.8            |
| Half-space | ∞             | 80                         | 1.8            |

In this test, we incorporate full SH wavefield reflectivity method. The discussion involves depiction of waveform, phase velocity curve, and comparison to the normal mode dispersion curve.
2.2. SH body and full SH waveform

Generations of SH wavefields are carried out by incorporating three earth parameters only, which are thickness ($h$), density ($\rho$), and shear-wave velocity ($V_s$). This is because SH wave and Love wave are independent to $V_p$ property. While generation of the SH wavefield shot gathers is conducted, we deal with two types of waves, namely SH body wave and full SH (body and Love) wave. This is necessary in order to make clear whether the waves obtained representing SH body waves only or full (SH body and Love) waves.

The generated SH wavefield images shown in Figure 1 demonstrate the SH body waves (Figure 1a), and full (SH body and Love) waves (Figure 1b). When the Figure 1a is inspected carefully, it does not reveal a dispersive characteristic of waves. The waves tend to show a single ‘flat’ wave pulses. This is a presage that the generated waves are SH body wave only. It is different with the wave shown in Figure 1b. Here, the waves propagate in several wave pulse modes (dispersive). This characteristics obviously show that the SH surface (Love) waves are also generated, together with the SH body waves. Therefore, it is suitable to call such waves as full SH wavefields. Since Love waves have more dominant energy than SH body waves, the dispersive characteristics of the full SH wavefields are influenced more dominantly by the Love waves. The difference of SH body waves and Love waves may be inspected furthermore from their corresponding phase velocity dispersion curves.

2.3. Generating theoretical phase velocity dispersion curve

Generation of Love wave phase velocity is carried out by employing the full SH reflectivity method of [10]. The steps of process are started with calculation of waveforms, followed by extraction of phase-velocity dispersion curves using plane-wave transform. Figure 2 displays waveforms and curves of phase velocity versus frequency for the SH body waves (Figures 2a and b) and the full SH wavefields (Figures 2c and d), respectively. The SH body wave phase velocities (Figure 2b) are non-dispersive (e.g. relatively constant with frequencies) at high frequency range (20-80 Hz), with values of about 248 m/s that may represent the first layer of the profile. The decreasing phase-velocity trend at low frequencies is most likely due to near-field effect. A manner to reduce this near-field effect will be addressed hereafter.

Unlike the SH body waves, the nature of phase-velocity curve of the Love waves changes with frequencies (e.g. called as dispersive), showing several modes of propagation, separated at frequencies of 20 and 40Hz (Figure 2d). The curve seems representative for the inversely dispersive profile, showing decreasing phase-velocity dispersion trend at low frequencies.

As cited above that there is a decreasing phase-velocity trend at low frequencies when only the SH body waves generated. Reducing the near-field effect may be carried out by increasing the distance of nearest offset. While the nearest offset is increased from 1 m to 10 m (Figure 3a), the near-field effect is reduced significantly (Figure 3b). Here, the phase-velocities tend to a constant value of about 248 m/s at a longer frequency range, i.e. 4-80 Hz. Again, only the first layer of model may be represented from the wave, and other layers may not be figured out. This result made clearer information on the non-dispersive nature of the SH body wave phase velocities. Therefore, we may also say that the phase velocity curve of the SH body wave could not be employed to figure out the inversely dispersive medium.

2.4. Comparison to modal matrix method

Overlaying the phase velocity dispersion curve of Love waves resulted from Figure 2d on the modal dispersion function of [16] shows obvious mismatching, especially for the first higher mode and the fundamental (Figure 4). The fundamental mode resulted from the full SH wavefield reflectivity shows a decreasing trend at low frequencies, whereas the modal matrix displays a vice versa. The decrease of phase velocity dispersion curve at low frequency tends to represent a model response of an inversely dispersive medium. This result indeed gives a new depiction. In one hand, many authors had cited that the conventional modal matrix method does not recognize such a decreasing trend of the Love dispersion curve at low frequencies. In the other hand, the full-SH wavefield reflectivity modelling can provide such a curve. This is an important empirical result that has so-far not been reported. This result gives a
new perspective in imaging Love wave propagation in an inversely dispersive medium and shows the excellence of the full SH wavefield reflectivity modelling. The further inspection of the phase velocity dispersion curve generated by the full SH wavefield reflectivity shows a superposition of three higher modes plus fundamental over frequencies 2 – 80 Hz. This finding describes the presence of an effective Love wave dispersion curve for an inversely dispersive medium, and gives a new perspective on the use of Love wave dispersion curve in the inversion process hereafter. Ability of the effective Love wave dispersion to estimate shear-wave velocity versus depth through the inversion process will be addressed below.

2.5. Inversion of synthetic effective dispersion curve
Inversion of the effective Love wave dispersion curve of Figure 2d was carried out using a linearized inversion of Occam’s algorithm. Inversion procedure referred to that of [10]. In this inversion procedure, the model response of phase velocity dispersion was calculated using the full SH wavefield reflectivity. Density values were assumed known of 1.8 g/cc for each layer. This assumption was usually given due to insensitivity of this parameter to the phase velocity dispersion change. Here, we used the true thickness of layers, and initial shear-wave velocity model were given a priori. The results of the inversion processes are shown in Figure 5. The obtained results are excellent. This is supported by a small RMS error (i.e., 0.60%) between the theoretical and calculated phase velocity data (Figure 5a), and the accuracy of the shear-wave velocity value estimated for each layer (Figure 5b). The appropriate of the estimated shear-wave velocity was also denoted by small relative error of each inverted shear-wave velocity (Figure 5c). This numerical inversion test also gives a new depiction, that is, inversion of the effective Love wave dispersion curve can provide accurate shear-wave velocity variation with depth for an inversely dispersive medium.

Figure 1: Waveform images for inversely dispersive profile listed in Table 1: (a) for reflected SH body waves, and (b) for full (SH body and Love) waves.
Figure 2: (a) SH body waveform and (b) its corresponding phase-velocity versus frequency, (c) full SH (body and Love) waveform and (d) its corresponding Love wave dispersion curve. Nearest offset is 1 m for both waveforms.

Figure 3: (a) SH body waveform with nearest offset of 10 m and (b) corresponding curve of phase-velocity versus frequency. Near field effect in the Figure 2b is reduced by increasing nearest offset to 10 m.
Figure 4: Overlaying the Love wave dispersion curve of Figure 2d extracted from the full SH waveform on the corresponding modal dispersion function of [16].

Figure 5: Inversion results of Love wave phase velocity dispersion in Figure 2d (a) comparison of measured and calculated dispersion curve, (b) Inverted shear-wave velocity versus depth, and (c) relative error for each inverted shear-wave velocity.
3. Conclusion
Full wavefield reflectivity method is a most powerful method for modelling surface wave compared to other existing surface wave modelling. The method can simulate any Earth’s subsurface profile. Full SH wavefield reflectivity method can successfully simulate Love wave propagation and relating phase velocity dispersion for an inversely dispersive profile. Such a subsurface profile cannot be simulated properly by the commonly used modal surface wave method. The modal matrix method provides undesired dispersion curves, where the resulting phase velocities still increase at low frequency end. This is different with the expectation, where the Love wave dispersion curve should decrease at the low frequency end. Meanwhile, the full SH wavefield reflectivity method simulates successfully a proper Love wave dispersion for such a medium, where the smaller the frequency the smaller the phase velocity. By the full SH wavefield reflectivity method, an effective Love wave dispersion curve of the medium is also introduced. Inversion of the effective dispersion curve can excellently estimate the shear-wave velocity.

4. References
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