Oscillations of $K^0$ Mesons in the Phase Volume Approach in the Standard Model and in the Model of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa Matrices

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Abstract

The elements of the theory of dynamical expansion of the theory of weak interaction working at the tree level, i.e. the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices, are given.

The equation for mass difference of $K_1^0$, $K_2^0$ mesons or the length of $K^0$, $\bar{K}^0$ meson oscillations in the phase volume approach in the framework of the standard model of the weak interactions and the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices is calculated (in this model the oscillations of $K^0$, $\bar{K}^0$ mesons arise at violation of strangeness by $B$ bosons). Comparison of the length of oscillations in the diagram and phase volume approaches was done. The length of $K^0 \leftrightarrow \bar{K}^0$ oscillations in the phase volume approach is much more than the length of these oscillations in the diagram approach.

PACS: 12.15 Ff Quark and lepton masses and mixing.
PACS: 12.15 Ji Aplication of electroweak model to specific processes.

1 Introduction

At the present time the theory of electroweak interactions has a status of theory which is confirmed with a high degree of precision. However, some experimental results (the existence of the quarks and the leptons families, etc.) have not got any explanation in the framework of the theory. One part of the electroweak theory is the existence of quark mixings introduced by the Cabibbo-Kobayashi-Maskawa matrices (i.e.,
these matrices are used for parametrization of the quark mixing).

In previous works [1] a dynamical mechanism of quark mixing by the use of four doublets of massive vector carriers of weak interaction $B^\pm, C^\pm, D^\pm, E^\pm$, i.e. expansion of the standard theory of weak interaction (the theory of dynamical analogy of the Cabibbo-Kobayashi-Maskawa matrices) working at the tree level, was proposed.

The vacuum oscillation of neutral $K$ mesons is well investigated at the present time [2]. This oscillation is the result of $d, s$ quark mixings and is described by Cabibbo-Kobayashi-Maskawa matrices [3, 4]. The angle mixing $\theta$ of neutral $K$ mesons is $\theta = 45^O$ since $K^o, \bar{K}^o$ masses are equal (see $CPT$ theorem). Besides, since their masses are equal, these oscillations are real, i.e. their transitions to each other are going without suppression.

This work is devoted to the study of $K^0, \bar{K}^0$ oscillations in the phase volume approach in the frame work of the standard model and the model of dynamical analogy of the Cabibbo-Kobayashi-Maskawa matrices.

At first, we will give the general elements of the model of dynamical analogy of the Cabibbo-Kobayashi-Maskawa matrices, the $K^0, \bar{K}^0$ oscillations will be considered in the frame work of these models and then the comparison of length of oscillations in the diagram and phase volume approaches is fulfilled.

2 The Theory of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa matrices

In the case of three families of quarks the current $J^\mu$ has the following form:

$$J^\mu = (\bar{u}c\bar{d})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{1}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

where $V$ is a Kobayashi-Maskawa matrix [4].
Mixings of the $d, s, b$ quarks are not related to the weak interaction (i.e., with $W^\pm, Z^0$ bosons exchanges). From equation (1) it is well seen that mixings of the $d, s, b$ quarks and exchange of $W^\pm, Z^0$-bosons take place in an independent manner (i.e., if matrix $V$ were diagonal, mixings of the $d, s, b$ quarks would not have taken place).

If the mechanism of this mixings is realized independently of the weak interaction ($W^\pm, Z^0$-boson exchange) with a probability determined by the mixing angles $\theta, \beta, \gamma, \delta$ (see below), then this violation could be found in the strong and electromagnetic interactions of the quarks as clear violations of the isospin, strangeness and beauty. But, the available experimental results show that there are no clear violations of the number conservations in strong and electromagnetic interactions of the quarks. Then we must relate the non-conservation of the isospins, strangeness and beauty (or mixings of the $d, s, b$ quarks) with some type of interaction mixings of the quarks. We can do it introducing (together with the $W^\pm, Z^0$-bosons) the heavier vector bosons $B^\pm, C^\pm, D^\pm, E^\pm$ which interact with the $d, s, b$ quarks with violation of isospin, strangeness and beauty.

We shall choose parametrization of matrix $V$ in the form offered by Maiani [5]

$$V = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{array} \right) \left( \begin{array}{ccc} c_\beta & 0 & s_\beta \exp(-i\delta) \\ 0 & 1 & 0 \\ -s_\beta \exp(i\delta) & 0 & c_\beta \end{array} \right) \left( \begin{array}{ccc} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{array} \right),$$

where $c_\theta = \cos \theta, s_\theta = \sin \theta, \exp(i\delta) = \cos \delta + i \sin \delta$. (2)

To the nondiagonal terms in (2), which are responsible for mixing of the $d, s, b$-quarks and $CP$-violation in the three matrices, we shall correspond four doublets of vector bosons $B^\pm, C^\pm, D^\pm, E^\pm$ whose contributions are parametrized by four angles $\theta, \beta, \gamma, \delta$. It is supposed that the real part of $Re(s_\beta \exp(i\delta)) = s_\beta \cos \delta$ corresponds to the vector boson $C^\pm$, and the imaginary part of $Im(s_\beta \exp(i\delta)) = s_\beta \sin \delta$ corresponds to the vector boson $E^\pm$ (the couple constant of $E$ is an imaginary value!). Then, when $q^2 << m_W^2$, we get:
\[ \tan \theta \simeq \frac{m_W^2 g_B^2}{m_B^2 g_W}, \quad \tan \beta \simeq \frac{m_W^2 g_C^2}{m_C^2 g_W}, \]
\[ \tan \gamma \simeq \frac{m_W^2 g_D^2}{m_D^2 g_W}, \quad \tan \delta \simeq \frac{m_W^2 g_E^2}{m_E^2 g_W}. \] (3)

If \( g_{B \pm} \simeq g_{C \pm} \simeq g_{D \pm} \simeq g_{E \pm} \simeq g_{W \pm}, \) then
\[ \tan \theta \simeq \frac{m_W^2}{m_B^2}, \quad \tan \beta \simeq \frac{m_W^2}{m_C^2}, \]
\[ \tan \gamma \simeq \frac{m_W^2}{m_D^2}, \quad \tan \delta \simeq \frac{m_W^2}{m_E^2}. \] (4)

Concerning the neutral vector bosons \( B^0, C^0, D^0, E^0, \) the neutral scalar bosons \( B', C', D', E' \) and the GIM mechanism [6], we can repeat the same arguments which were given in the previous work [1].

Using the data from [2] and equation (4), we have obtained the following masses for \( B^\pm, C^\pm, D^\pm, E^\pm \)-bosons:
\[ m_{B\pm} \simeq 169.5 \div 171.8 \text{ GeV}; \]
\[ m_{C\pm} \simeq 345.2 \div 448.4 \text{ GeV}; \] (5)
\[ m_{D\pm} \simeq 958.8 \div 1794 \text{ GeV}; \]
\[ m_{E\pm} \simeq 4170 \div 4230 \text{ GeV}. \]

3 Oscillations of Neutral \( K^0, \bar{K}^0 \) Mesons in the Phase Volume Approach in the Standard Model and in the Model of Dynamical Analogy of Kabibbo-Kobayashi-Maskawa Matrices

The oscillations of \( K^0, \bar{K}^0 \) mesons are characterized by two parameters: angle mixing–\( \theta \) and length–\( R \) or time oscillations–\( \tau \).
At first, we will consider mixings of $K^0$ mesons in the phase volume approach and obtain the expression for matrix elements and the time of $K^0, \bar{K}^0$ transitions arising for existence of the strangeness violation of the weak interaction through $\sin \theta W$ or $B$ exchange in the standard or in our model of dynamical analogy of Kabibbo-Kobayashi-Maskawa matrices. The general scheme of $K^0, \bar{K}^0$ meson oscillations is given in the end of the article.

In further considerations, for transition from the standard model to our model, the following values are used for $\sin \theta$, $\cos \theta$ and $G_F$:

$$\sin \theta \approx \frac{m^2_W g_B^2}{m^2_B g_W^2},$$

$$\cos^2 \theta = 1 - \sin^2 \theta,$$

$$G_F^2 = \frac{g_W^2}{32 m^4_W}.$$ (6)

**A) Mixings of $K^0, \bar{K}^0$ Mesons**

Let us consider mixings of $K^0, \bar{K}^0$ mesons. $K^0$ and $\bar{K}^0$ consist of $\bar{d}, s, d, \bar{s}$ quarks and have the same masses (it is a consequence of the CPT invariance) but their strangennesses are different $s_{K^0} = -1, s_{\bar{K}^0} = 1$. Since the weak interaction through $B$ boson exchanges changes the strangeness, then the $K^0, \bar{K}^0$ are mixed.

The mixings of $K^0$ mesons can be considered by using the following nondiagonal mass matrix of $K^0$ mesons:

$$\begin{pmatrix}
m^2_{K^0} & m^2_{K^0 \bar{K}^0} \\
m^2_{K^0 \bar{K}^0} & m^2_{\bar{K}^0}
\end{pmatrix}.$$ (7)

Since $m^2_{K^0} = m^2_{\bar{K}^0}$, the angle $\theta'$ of $K^0, \bar{K}^0$ mixing, or the angle rotation for diagonalization of this matrix

$$\begin{pmatrix}
m^2_1 & 0 \\
0 & m^2_2
\end{pmatrix},$$

is given by the expression:

$$\tan 2\theta' = \frac{m^2_{K^0 \bar{K}^0}}{m^2_{K^0} - m^2_{\bar{K}^0}}.$$
and is equal to \((\theta' = \frac{\pi}{4})\) and

\[
m_{1,2}^2 = \frac{1}{2}(m_{K^0}^2 - m_{\bar{K}^0}^2) \pm \sqrt{(m_{K^0}^2 - m_{\bar{K}^0}^2)^2 + 4(m_{K^0\bar{K}^0}^2)^2},
\]

\(m_1^2 - m_1^2 = m_{K^0\bar{K}^0}^2.
\)

Then the following new states \(K_1^0, K_2^0\) arise:

\[
K_1^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}}, \quad K_2^0 = \frac{K^0 - \bar{K}^0}{\sqrt{2}}
\]

In the quark model we can take a mass matrix with masses in the first degree \((m_{ab}^2 \rightarrow m_{ab})\).

Below the \(CP\) –invariance is supposed to be strong conserved (the consequences which arise of the \(CP\) violation were discussed in work \([7]\)), and then the following decays are possible: \((CP\) parity of \(K_1^0\) is \(P(K_1^0) = +1\), and \(\bar{K}_2^0\) is \(P(\bar{K}_2^0) = -1)\)

\[
K_1^0 \rightarrow 2\pi, \quad \bar{K}_2^0 \rightarrow 3\pi.
\]

**B). Estimation of Probability or Time of \(K^0 \leftrightarrow \bar{K}^0\) Meson Transitions in the Phase Volume Approach**

The calculation of probability or time of \(K^0 \leftrightarrow \bar{K}^0\) meson transitions will be executed in the framework of the weak quark interactions by using the following Feynman diagram:
where $\sin \theta W$ and $B$ are a bosons changing the strangeness; $u, c$ are quarks (for simplification the $t$ quark is not taken into account).

We do not give here the details of the calculation on this diagram, since they were widely discussed in literature [8, 9]. Using the standard Feynman rules for the weak interaction (after integrating on the inside lines, twice using the Fierz rules and without the outside momenta), we obtain the following expression for the amplitude of $K^0 \rightarrow \bar{K}^0$ transition:

$$M(K^0 \rightarrow \bar{K}^0) = -\frac{G^2 f_K^2 m^2 c \sin^2 \theta \cos^2 \theta}{8\pi^2} dQ_{\alpha} \bar{s} dQ_{\alpha} s =$$

$$= G dQ_{\alpha} \bar{s} dQ_{\alpha} s,$$

where

$$G = \frac{G^2 f_K^2 m^2 c \sin^2 \theta \cos^2 \theta}{8\pi^2}, \quad Q_{\alpha} = \gamma_{\alpha} (1 - \gamma_5).$$

Then we use the following phenomenological expression:

$$<0 | \bar{s} Q_{\alpha} d | K^0 > = \phi_K f_K p_{\alpha},$$

(13)

where $\phi_K$ is a $K$-meson wave function and $| \phi_K |_2 = 1$, $f_K$ is a $K$ constant decay and $f_K \cong 1.27 f_\pi$ ($f_\pi \cong 125$ MeV), $p_{\alpha}$ is a $\pi$ four-momentum.

Also, we suppose that the beginning state $\bar{s} Q_{\alpha} d$ corresponds to $K^0$ meson.

Introducing the values $F_{\alpha} = f_K \phi_K p_{\alpha}, Q_\alpha = \bar{d}_L \gamma_5 u_L$, we rewrite exp.(12) in the following form:

$$M = GF_{\alpha} Q_\alpha =,$$

(14)

If we use the expression $\hat{p}_\pi = \hat{p}_s + \hat{p}_{\bar{d}}$ and the Dirac equation

$$(\hat{p} - m) u = 0$$

(where $u$ is a quark wave function), then one can rewrite Eq.(14) in the following form:

$$M = G f_K \phi_K (m_s + m_{\bar{d}}) \bar{d}_L \gamma_5 s_L.$$  

(15)

Using the standard procedure for $| M |^2$, one obtains the following expression:

$$| M |^2 = G^2 f_K^2 (m_s + m_{\bar{d}})^2 4(p_{\bar{d}} p_s) \cong$$
\[ \cong 4G^2 f_K^2 (m_s + m_d)^2 m_K E_d, \]  
where \( E_d \) is the energy of \( d \) quark in \( K^0 \) meson \(((p_d, p_s) = (p_d, (p_K - p_d)) = m_K E_d - m_d^2 \approx m_d m_K, m_d^2 \approx 0). \]

\[ E_d = m_K (1 - \frac{m_s^2}{m_K^2}). \]

Then the probability \( W(...) \) of \( K^0 \leftrightarrow \bar{K}^0 \) meson transitions is

\[ W(K^0 \leftrightarrow \bar{K}^0) = \frac{2|\tilde{M}|^2}{2m_{K^0}^2} \int \frac{d^3p_d}{2E_d(2\pi)^3} \frac{d^3p_s}{2E_s(2\pi)^3} (2\pi)^4 \delta(p_d + p_s - p_{\pi}) = \]

\[ = \frac{2|\tilde{M}|^2}{4\pi m_{K^0}} \int \delta(E_{\bar{d}} + E_s - m_{K^0}) \frac{E_{\bar{d}}dE_{\bar{d}}}{E_{u}} \cong \]

\[ \cong \frac{2|\tilde{M}|^2}{4\pi} \frac{E_{\bar{d}}}{m_{K^0}^2}, \]

where \( E_d \) is given by expression (17).

Then using the expression (16) for \( |\tilde{M}|^2 \), one can rewrite equation (4) in the form:

\[ W(K^0 \leftrightarrow \bar{K}^0) \cong \frac{G^2 f_K^2 (m_s + m_d)^2 m_{K^0}}{8\pi}, \]  
and

\[ \tau_0 = \frac{1}{W_0(...).} \]

Then the time \( \tau(...) \) of \( K^0 \leftrightarrow \bar{K}^0 \) transition is

\[ \tau(K^0 \leftrightarrow \bar{K}^0) = \frac{1}{W(K^0 \leftrightarrow \bar{K}^0)}. \]

The mass difference of \( K_1^0 - K_2^0 \) is

\[ \Delta m = m_1 - m_2 \approx \frac{h}{\tau_{K^0}}. \]

The length \( L \) of \( K^0 \leftrightarrow \bar{K}^0 \) oscillations is

\[ L_{phas} = \tau_{K^0}(...) u_{K^0}, \]  

8
where $v_{K^0}$ is the velocity of $K^0$ meson.

and

$$L_{\text{phas}} = \tau V = \frac{8\pi p_{K^0}}{G^2 f_{K^0}^2 m_s^2 m_{K^0}^2}. \quad (23)$$

Transition to the length of $K^0$ oscillations for $L_{\text{phys}}$ in the model of dynamical analogy of Cabibbo-Kobayashi- Maskawa matrices is given by Exp. (22) by using Exp. (6).

**C) The scheme of $K^0$, $\bar{K^0}$ oscillations**

As an example of this oscillation we consider the oscillation of $K^0$ mesons produced in the reaction $\pi^- + P \rightarrow K^0 + \Lambda$. At $t = 0$ there is the state $K^0(0)$, then in time $t \neq 0$ for $K^0(t)$, if to take into account equation (10), we get $K^0$:

$$K^0(t) = \frac{1}{2}[(K^0 + \bar{K}^0)\exp(-im_1 t - \frac{\Gamma_1 t}{2}) + (K^0 - \bar{K}^0)\exp(-im_2 t - \frac{\Gamma_2 t}{2})] =$$

$$= \frac{1}{2}K^0\exp(-im_2 t)[\exp(-i\Delta mt - \frac{\Gamma_1 t}{2}) + \exp(-\frac{\Gamma_2 t}{2})] + \frac{1}{2}\bar{K}^0\exp(-im_1 t)[\exp(i\Delta mt - \frac{\Gamma_1 t}{2}) + \exp(-\frac{\Gamma_2 t}{2})]. \quad (24)$$

From (23) it is clear that on the background of $K^0$ meson decays, the oscillations of $K^0$ mesons take place [8, 10].

4 **Comparisons of $K^0 \leftrightarrow \bar{K}^0$ Oscillations in the Diagram and the Phase Volume Approaches**

The length of $K^0 \leftrightarrow \bar{K}^0$ in diagram approach is [11]

$$L_{\text{diag}} = 2\pi \frac{2p_{K^0}}{|m_1^2 - m_2^2|} = \pi \frac{2p_{K^0}}{m_{K^0} \Delta m}, \quad (25)$$

where $\Delta m$ is

$$\Delta m = \frac{8}{3} m_{K^0} f_{K^0}^2 G, \quad (26)$$

and $G$ is given by expression (12).
The relation of these two lengths is:

\[ \frac{L_{osc}}{L_{phas}} = \frac{3}{32} Gm_s^2, \]  

(27)

and

\[ \frac{L_{osc}}{L_{phas}} << 1, \]  

(28)
i.e., the length of \( K^0 \leftrightarrow \bar{K}^0 \) oscillations in the phase volume approach in the standard model and in the model of dynamical analogy of Cabibbo-Kobayashi- Maskawa matrices is much more than the length of these oscillations in the diagram approach and in the model of dynamical analogy of Cabibbo-Kobayashi- Maskawa matrices. It means that on the background of \( K^0 \) decays it is hard to register the phase volume \( K^0 \) oscillations in contrast to \( K^0 \) oscillations in the diagram approach.

5 Conclusion

The elements of the theory of dynamical expansion of the theory of weak interaction working at the tree level, i.e. the model of dynamical analogy of Cabibbo-Kobayashi- Maskawa matrices, were given.

The equation for mass difference of \( K_1^0, K_2^0 \) mesons or the length of \( K^0, \bar{K}^0 \) meson oscillations in the phase volume approach in the frame work of the standard model of the weak interactions and the model of dynamical analogy of Cabibbo-Kobayashi- Maskawa matrices have been calculated (in this model the oscillations of \( K^0, \bar{K}^0 \) mesons arise at violation of strangeness by \( B \) bosons). Comparison of the length of oscillations in the diagram and phase volume approaches was done. The length of \( K^0 \leftrightarrow \bar{K}^0 \) oscillations in the phase volume approach is much more than the length of these oscillations in the diagram approach.

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