The evolution of a gas bubble in a liquid near a flat wall

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Abstract. The axisymmetric dynamics of a cavitation bubble in a liquid near a flat solid wall is studied. The liquid is considered to be ideal, incompressible, its flow is potential, the gas in the bubble is homobaric. The Euler method is used for determining the position of the bubble surface and the velocity potential on it at every time moment and the boundary elements method is applied for determining the normal velocity component. The possibility of transforming an initially one-connected bubble into a toroidal one is taken into account. The process of expansion and subsequent contraction of a bubble in water under the influence of the excessive internal pressure in it under room conditions is considered. The shape of the bubble surface, the velocity and pressure fields in the surrounding liquid are determined. The results of calculations are given for the case when the ratio of the initial distance from the center of the bubble to the wall to the bubble maximum radius is 0.6, and also the results of comparing the calculations with known experimental data are given.

1. Introduction

Investigation of bubble dynamics near a wall is important for estimating the effect of cavitation on the surfaces of solids. A large number of publications are devoted to this topic, among which there are many papers in which the effect of the initial distance \( h_0 \) from the bubble center to the wall on the dynamics of a bubble during its expansion and subsequent compression is studied [1-3]. In most papers, the results are given for \( \gamma = h_0 / R_{\text{max}} \geq 0.7 \) [2, 3], where \( R_{\text{max}} \) is the maximum bubble radius reaching by the bubble in the course of its expansion in an unbounded volume of liquid. At lower values of \( \gamma \) at the end of the expansion phase, a thin layer of liquid between the wall and the bubble surface is formed, which also persists on the toroidal phase of motion [4, 5]. The presence of such a layer complicates the implementation of numerical algorithms. Therefore, in a number of articles a thin layer of liquid is artificially eliminated by considering the bubble in contact with the wall [6, 7]. Under the assumption of the potentiality of the liquid flow, the most common technique used in calculations is to combine the time-step method with the boundary element method [1, 8-10]. The numerical instability of such an approach is eliminated by various smoothing procedures applied to the shape of the bubble surface and the potential values on it [11, 12]. In this paper, we present the results of calculations for the low-studied case \( \gamma = 0.6 \), obtained using the technique [8, 12].
2. Problem statement and computational technique

The process of expansion and subsequent contraction of a cavitation bubble in water near a flat solid wall under room conditions is considered (the liquid density is \( \rho = 1000 \text{ kg/m}^3 \), the liquid pressure at a large distance from the bubble is \( p_\infty = 1 \text{ bar} \)). At the moment of maximum expansion, the volume of the bubble \( V \) is equal to the volume of a sphere of radius \( R_{\text{max}} = 1.45 \text{ mm} \). It is assumed that at the initial time \( t = 0 \) the bubble has the form of a sphere of a small radius \( R_0 = 0.25 \text{ mm} \), the center of which is distant from the wall by a distance \( h_0 = 0.6R_{\text{max}} \), the pressure in the bubble is \( p_{b0} = 88 \text{ bar} \). The liquid is considered to be an ideal incompressible, its motion is potential, the surface tension is not taken into account. The pressure in the bubble varies according to the law

\[
p_b = p_{b0}(V_0/V)^\kappa,
\]

where \( V_0 \) is the initial volume of the bubble, \( \kappa \) is the adiabatic exponent (\( \kappa = 1.4 \)).

For the given data, two phases are distinguished in the evolution of the bubble at the wall. The first phase includes the expansion stage of the bubble and a part of the subsequent stage of its contraction up to the instant \( t = t_c \), when the cumulative jet arising on the bubble surface impacts the layer of liquid between the bubble and the wall. The second phase includes the motion of the toroidal bubble formed as a result of the impact of the jet onto the liquid layer.

In accordance with this, the numerical solution algorithm is also divided into two stages. A detailed description of the calculation technique used in this paper is presented in [12]. It should be noted, that in [12], in contrast to other authors, smoothing by a cubic spline is used to eliminate the numerical instability of the functions that determine the bubble contour and the velocity potential value on it.

The following dimensionless quantities are utilized when presenting the results: \( p^* = p/p_\infty \), \( v^* = v/(p_\infty/p)^{1/2} \), \( r^* = r/R_{\text{max}} \), \( z^* = z/R_{\text{max}} \), \( t^* = t/[R_{\text{max}}(p/p_\infty)^{1/2}] \), where \( p \) is the pressure, \( v \) is the velocity value, \( r, z \) are the radial and axial coordinates of the axisymmetric reference system with the origin on the wall and with the \( z \) axis passing orthogonally to the wall through the center of the bubble.

3. Results

To evaluate the reliability of the obtained results, a comparison is made with the experimental data [13] at four time moments both at the expansion stage and the stage of the subsequent contraction. The results of the comparison are shown in figure 1, where the upper row \((a)\) presents the experimental data, the lower row \((b)\) presents the numerical results. One can see their good qualitative agreement.

![Figure 1](image.png)

**Figure 1.** Comparison of the numerical results \((b)\) of the present work with the experimental data \([13] (a)\).
The results of calculations of the bubble motion in its first phase, i.e. prior to the moment $t^*$ corresponding to the impact of the cumulative jet onto the thin liquid layer, are shown in figure 2. Figure 2a illustrates the change in the shape of the bubble surface during its expansion and contraction. In this figure, the first moment corresponds to the beginning of the process, the second moment corresponds to the maximum expansion of the bubble, the third and fourth moments correspond to the collapse with the formation of the cumulative jet and the fifth moment corresponds to the instant of the jet impact onto the thin liquid layer between the bubble and the wall. It can be seen that the part of the bubble surface near the wall at the moment of the maximum expansion of the bubble and then during its subsequent contraction is nearly flat. At that, the thickness of the liquid layer between the bubble and the wall at the moment $t^*_c$ is approximately equal to 0.006$R_{max}$.

**Figure 2.** Bubble contours at five consecutive moments of time: 1 corresponds to $t^* = 0$, 2 to $t^* = 1.18$, 3 to $t^* = 1.89$, 4 to $t^* = 2.17$, 5 to $t^* = t^*_c = 2.24$ (a) and the pressure field in the liquid surrounding the bubble at the moment $t^*_c$ (b)
Figure 3. The contour of the bubble at the toroidal phase of motion (a), the pressure field in the liquid in the area of contact between the jet and the layer (b), the pressure profiles along the symmetry axis (c) and the radial pressure profiles on the wall (d) at three moments of time: 1 corresponds to \( t^* - t^*_c = 0.0034 \), 2 to \( t^* - t^*_c = 0.0136 \), 3 to \( t^* - t^*_c = 0.069 \).

Figure 2b shows the pressure field in the liquid surrounding the bubble at the end of the first phase. It is seen that the pressure in the vicinity of the jet end is approximately equal to the pressure in the bubble \( p^* = 0.8 \), and the largest pressure \( p^* = 3.3 \) is observed at the base of the jet. The jet end velocity in the final of the first phase is \( \dot{v} = 7.04 \).

The results of calculations on the toroidal phase of motion are shown in figure 3. It can be seen (figure 3a), that at the beginning of this phase the part of the bubble surface near the wall, as in the first phase, remains practically flat. As a result of the jet impact onto the liquid layer, a thin ring-like liquid splash directed into the bubble interior appears on the bubble surface at the impact region periphery [2]. With time it moves away from the axis of symmetry and its amplitude increases. Figure 3b illustrates the changes of the liquid pressure field in the vicinity of the torus hole. It is seen that with time the torus hole increases, the maximum pressure on the wall near the axis of symmetry decreases from 45.4 to 17.6. The liquid pressure profiles along the symmetry axis are shown in figure 3c. It can be seen that the maximum pressure due to the jet impact is realized on the wall and rapidly decreases with time and with increasing \( z^* \). The radial pressure profiles depicted in figure 3d show that the greatest pressure observed at the axis of symmetry, with time, as the liquid splash on the bubble surface develops, becomes comparable with the pressure in the region of the splash (\( t^* - t^*_c = 0.069 \)).

4. Conclusion
Using the time-stepping method and the boundary element method, the dynamics of a cavitation bubble in water near a solid wall has been studied in the case the pressure in the bubble at the initial instant of time considerably exceeds the pressure in liquid and the thickness of the liquid layer between the bubble and the wall is 0.6 of the radius of the bubble at the moment of its maximum expansion. The numerical results have been compared with the experimental data by other authors. Good agreement is obtained. It is shown that at the beginning of the impact of the cumulative jet, a pronounced pressure maximum at the wall arises in the center of the region of action. Over time, this maximum decreases, becoming comparable with the peripheral local maximum in the region of the liquid splash into the bubble.

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