A Critique of Pure String Theory: Heterodox Opinions of Diverse Dimensions

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ABSTRACT

I present a point of view about what M Theory is and how it is related to the real world that departs in certain crucial respects from conventional wisdom. I argue against the possibility of a background independent formulation of the theory, or of a Poincare invariant, Supersymmetry violating vacuum state. A fundamental assumption is black hole dominance of high energy physics. Much of this paper is a compilation of things I have said elsewhere. I review a crude argument for the critical exponent connecting the gravitino mass and the cosmological constant, and propose a framework for the finding a quantum theory of de Sitter space.
1. Introduction: The Conventional Wisdom

String theory, although it is a theory of gravity, is a creation of particle physicists. Traditional string phenomenology shows its pedigree by asking for an exact solution of a purported theory of everything, which exhibits exact Poincare symmetry (a symmetry which is clearly only approximate in the real world). This theory is supposed to describe the scattering of particles in the real world, which is thus postulated to be insensitive to the cosmological nature of the universe.

The basis for this assumption is locality, a property that is evidently only approximately true of string theory at low energy. Super Planckian scattering is dominated by black hole production\(^1\), and the spectrum and properties of black holes of sufficiently high energy are definitely affected by the global structure of the universe. By continuity, there are effects on low energy physics as well. The only question is how large they are.

At any rate, a principal defect of this approach is that it already postulates two mathematically consistent solutions of the theory of everything, namely the real, cosmological, world, and the exact Poincare invariant solution. In fact, as is well known, the situation is much worse than that. There are many disconnected continuous families of Poincare invariant solutions of string theory. They have various dimensions, low energy fields, and topologies, but they all share the property of exact SUSY. The program of string phenomenology is to find a SUSY violating, Poincare invariant solution of the theory, which describes low energy scattering in the real world. In \(^2\) I expressed the opinion that no such solution exists. Be that as it may, the string phenomenologist, having found the holy grail of a Poincare invariant, SUSY violating, ”realistic” theory, will still be faced with the question of why it is preferred over all of the vacuum states with exact supersymmetry.

By contrast, if one adopts the hypothesis of cosmological SUSY breaking (CSB) proposed in \(^2\), this problem is solved at a stroke. The theory of the real world has a finite number of states\(^1\) and can be neither Poincare invariant, nor supersymmetric. Since the number of states in the real world is \(e^{10^{120}}\), it would not be surprising to find that some of the properties of the real world are well approximated by those of a Poincare invariant solution.

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\(^1\) The suggestion that the dS entropy represented a bound on the number of states in dS space first arose in conversations initiated by W. Fischler. I asked Fischler to be a coauthor on \(^3\), but he declined on the grounds that he had not contributed to the ideas about SUSY breaking. Fischler talked about the finite number of states at the Festschrift for G. West in the Spring of 2000\(^3\).
theory, which I will call the \textit{limiting vacuum}. By the arguments (reviewed below) of [2], this limiting theory must be SUSic and have no moduli. The combination of these two properties and the general structure of SUSic theories imply that it must be four dimensional, with only $N = 1$ SUSY, and have an exact complex R symmetry$^3$. This puts strong constraints on the low energy effective theory in the limiting vacuum. I have described approaches to the low energy phenomenology of CSB in a recent paper$^4$.

In this paper I want to summarize a collection of ideas that I have been playing with since 1999. They form a context in which the hypothesis of CSB is seen as a natural extension of the facts we already know about M-theory. These ideas are only loosely connected and have not yet jelled into a consistent alternative to the conventional wisdom about the way in which string theory is connected to the real world. I am setting them down here in the hope that others can make more progress thinking about them than I have. If these ideas are even partially correct, then they imply that some of our fundamental assumptions have been wrong, and I think it is important that we revise them.

The key concepts revolve around the search for the fundamental degrees of freedom of a quantum theory of gravity, and the conviction that these are intimately connected with the high energy behavior of the theory. All of our experience with quantum theories suggests this connection. This is nowhere more evident than in Feynman’s path integral formulation of quantum mechanics. The key step in the derivation of the path integral is the exact evaluation of the short time propagation kernel, and the key assumption is that the short time behavior is dominated by a free theory. In this view, all of the formalism of classical mechanics and canonical quantization is a consequence of the assumption of what we have learned to call a Gaussian fixed point. For this reason, I will call theories whose short time behavior is dominated by a Gaussian fixed point, Lagrangian theories.

Wilson’s view of general quantum field theory as constructed from relevant perturbations of general fixed point theories, may be thought of as an extension of Feynman’s principle. Again, the high energy behavior defines the theory. It has been our fortune/misfortune to, for the most part, be able to access non-Gaussian fixed point theories as infrared (IR) limits of Lagrangian theories. Another avenue to non-Gaussian fixed point theories has been through cutoff models, mostly arising from statistical mechanics. The resemblance of the statistical sums in these models to discretized path integrals has helped to obscure the non-Lagrangian nature of the fixed points. It is only with the discovery of fixed points

\textsuperscript{2} One must also use the fact that the theory has a dS deformation to prove this.
like the $(2,0)$ theory in six dimensions, which have not yet been realized as infrared limits of Gaussian models (this is impossible) or discretized statistical sums (this is possible but unknown at the moment) that we have been forced to face the truly radical departure from Lagrangian dynamics that a non-Gaussian fixed point implies.

It is my opinion, that in attempting to construct a theory of quantum gravity we should again look to the high energy behavior of the theory. When we do so, we are faced with several shocks. Firstly, the traditional connection between high energy and short distance disappears. Even in perturbative string theory, high energy physics is dominated by long strings. More generally, in any theory containing gravity there will be black holes. The Bekenstein-Hawking formula for the entropy of black holes suggests that they dominate the high energy spectrum, and semiclassical black hole dynamics suggests that they are metastable. Recent verifications that the Bekenstein-Hawking formula indeed refers to a microscopic count of all of the states of a black hole, lend credence to this point of view. Arguments to be reviewed below suggest that high energy scattering processes are dominated by black hole production. The result of these considerations is a radical new principle, which I consider to be the ultimate form of the UV/IR correspondence: High Energy Dynamics is dominated by large black holes, some of whose properties can be calculated using the semiclassical Lagrangian formulation of general relativity. At the Davidfest in Santa Barbara, I called this principle Asymptotic Darkness.

The fact that certain features of black holes are describable in the IR limiting theory is a direct consequence of the UV/IR connection. The GR description of black holes is however incomplete. It can give partial evidence for a huge set of states associated with the black hole but cannot give a microscopic quantum description of their properties.

The assumption that black holes dominate the high energy physics of quantum gravity, and thus should be taken as a clue to the whereabouts of the fundamental degrees of freedom, has several dramatic consequences. First of all, it immediately suggests the Holographic Principle: degrees of freedom should be associated with $(d - 2)$ dimensional areas in spacetime, rather than with points. At very high energy densities, space is filled with black holes and the area scaling of entropy becomes manifest. Fischler, Susskind and Bousso have shown how to formulate this principle for a general spacetime. In spacetimes with appropriate asymptotic boundaries one can see that this suggests a formulation

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3 I will explain below why I think this idea does not generalize to the spacelike boundaries of asymptotically de Sitter (dS) spaces.
in terms of degrees of freedom on the boundary. This fits in with arguments from string theory and quantum gravity, that the only observables in a theory of quantum gravity in asymptotically flat or AdS spacetimes are boundary correlators like the S-matrix.

The AdS/CFT \[9\] correspondence is the most complete and successful realization of this idea. I want to emphasize that one can view the above line of reasoning as a way of guessing or deriving the AdS/CFT correspondence. Namely, the spectrum of black holes in asymptotically AdS spacetimes is that of a conformal field theory living on the boundary. Asymptotic Darkness, and the associated Holographic Principle then suggest that the dynamics of the spacetime is completely captured by such a conformal field theory (or a relevant perturbation of it). We now know that this is true in many cases.

For asymptotically flat spacetimes the consequences of the UV/IR connection are more dramatic. Black hole dominance implies that quantum correlation functions of operators that do not distinguish the degenerate microstates of black holes are not tempered distributions in time, nor even the more singular distributions of quasilocal field theories \[8\] \[10\]. This means there is no way to localize the theory in time. Curiously, the black hole spectrum is consistent with locality (quasilocality for four asymptotically flat dimensions) in light cone time, which might suggest a reason for the ubiquitous presence of the light cone frame in Hamiltonian formulations of quantum gravity in asymptotically flat spacetime.

A holographic formulation of nonperturbative quantum gravity in certain asymptotically flat spacetimes is provided by Matrix Theory \[11\]. At present it is formulated only in the approximation of discrete Light Cone Quantization (DLCQ).

It is tempting to try to formulate a theory of asymptotically flat quantum gravity in more covariant terms, as a theory on null infinity. Existing descriptions of massless particles at null infinity suggest that one should not think of this as a dynamical theory. All the coordinates of null infinity are spatial (in the sense that longitudinal and transverse coordinates are treated as spatial coordinates in light cone frame). Instead, dynamics is encoded in the fact that null infinity is not a manifold, and splits naturally into two disjoint cones with a common boundary at spatial infinity. Nontrivial correlation functions in the theory are those which contain points on both components, and are nothing but the matrix elements of the S-matrix. The question that arises (a question to which there is as yet no answer) is what the dynamical principle is that determines the S-matrix. I will not discuss null infinity much in this paper.
The above discussion, and [12] make it clear (to me at least) that the old dream of background independence in string theory is a chimera. We already know that the various asymptotically AdS spacetimes for which we have discovered the full quantum theory, are not vacua of the same theory. They are unitary quantum theories without degenerate vacua\(^4\). Some of them are related by deformation by relevant or marginal parameters or by compactifying one field theory and taking a limit, but this is not what we usually mean by a theory having multiple vacua. It is also significant that the cosmological constant in these theories is a discretely tunable, fundamental parameter which encodes properties of the fundamental UV theory, rather than a low energy effective parameter, characteristic of a given IR representation of the algebra of quantum operators (what we usually mean by a choice of vacuum in QFT).

It is even more obvious that these are not vacua of a theory that also includes asymptotically flat spacetimes, since the latter have a radically different spectrum of high energy states, and their state spaces carry representations of different maximal spacetime symmetry groups. Rather the two classes of theories are related by the fact that the AdS theories are decoupling limits of certain configurations in asymptotically flat space. It is also possible, that one can recover certain flat space theories by taking large radius limits of AdS theories with free parameters\(^1\). It is virtually certain that not all asymptotically flat vacua can be retrieved in this fashion.

In [12] I argued that asymptotically flat theories also broke up into disjoint families that are not states in the same theory. I will review these arguments below.

The key features unifying all of these bizarre properties of quantum gravity are the fact that geometry responds to dynamics, and the UV/IR connection, which intimately entwines the large scale geometry (which in the traditional view is the vacuum dependent part of the dynamics, to which the high energy behavior is insensitive) with the high energy spectrum (which I have argued should be thought of as the domain where the fundamental degrees of freedom are defined). This viewpoint suggests very strongly that our traditional view of the cosmological constant problem is in error. The traditional view is that there is one theory, which can have various infrared behaviors, characterized by positive, negative or vanishing cosmological constant. The calculation of the cosmological constant

\(^4\) Even the famous moduli spaces are not there if we think of these as theories in AdS space, so that the CFT is compactified on a sphere, rather than as theories of branes embedded in flat space.
constant in any given vacuum is a dynamical problem: it is the calculation of the effective potential for some low energy effective field. Instead, the UV/IR connection suggests that the cosmological constant is an input, since it controls the very different behaviors of the high energy density of states in the different theories. This is indeed true in AdS/CFT. In known, supersymmetric, versions of this correspondence, the cosmological constant is determined by $N$, a parameter that characterizes the number of degrees of freedom in the CFT. More generically, it will be completely determined by the fixed point theory, even in those cases where the full theory is a relevant deformation of the CFT. The cosmological constant is a property of the large scale, asymptotically AdS geometry, which, by the UV/IR correspondence\,[\text{14}], is dual to the UV fixed point. The concept of an off-shell effective potential, which combines information about theories with different values of the cosmological constant, cannot be meaningful if this point of view is correct. At best it corresponds to an approximate concept, valid only in extreme regions of moduli space.

This line of thought leads inevitably to the conclusion of \,[\text{2}][\text{3}], that the cosmological constant of an asymptotically de Sitter (AsdS) space is a fundamental parameter counting the number of states in the quantum theory. I will review the arguments for this below, and contrast this point of view with that of \,[\text{15}] and \,[\text{16}] which try to make a parallel with AdS/CFT involving the asymptotic spacelike boundaries of AsdS spaces. I will argue that the latter approach neglects important back reaction corrections to linearized classical gravity, and grossly overestimates the number of observables in an AsdS space. The key point is that most of the measurements one can make on the (past) boundary actually destroy the large scale geometry of dS space at a finite time in the future (the CPT conjugate of this statement is also true).

The final topic that I will discuss here is a similar sensitivity of asymptotically flat geometry to the dynamics assumed for matter inside it. As of this date, we know of no example of a controllable approximation to a theory of quantum gravity that leads to a nonsupersymmetric theory in asymptotically flat, Poincare invariant spacetime. Within the regimes in which we are able to calculate, we always generate a potential for moduli. In some cases we can argue that the moduli can be stabilized with positive or negative cosmological constant. In other cases, we find Big Bang cosmological solutions with Newton’s constant going to zero in the asymptotic future.

This failure, and a perturbative string theory argument due to Kutasov and Seiberg \,[\text{17}] lead me to conjecture that there are no SUSY violating, Poincare invariant theories
of quantum gravity. Here again, I believe the fundamental issue is the divergent spectrum of black hole states in asymptotically flat geometries. SUSY cancellations are, I believe, necessary for the existence of a sensible Poincare invariant S-matrix with such a spectrum. This argument leads directly to the conjecture\cite{2} that all SUSY breaking in the real world should be associated with the existence of a positive cosmological constant. The latter conjecture has undergone a fundamental change since I first suggested it. In \cite{2} I conjectured that it was virtual contributions of the largest black hole states in dS space, which led to large renormalizations of the classical formula for the gravitino mass. Psychologically, this had to do with my argument that it was the black hole spectrum in asymptotically flat space that required SUSy cancellations. I have since realized that most of the states in dS space are not seen as black holes by any given static observer. Rather, they are states on his horizon, representing localized excitations seen by other observers. They have static energy at most of order the dS temperature. In the limit in which dS space becomes Minkowski space, these low energy states decouple from the Hilbert space of scattering states. I will argue below that the gravitino mass is due to its interaction with these ultra-low energy states on the horizon, which decouple from the limiting Poincare invariant theory.

The rest of this paper is an elaboration of the message of the introduction. Some of the arguments will be repeated in various ways. Feynman once said that if you have lots of arguments to prove a given point, then that is a sign that you have no good arguments. I stand vulnerable to such an accusation. I’m writing this in the hope that someone smarter than I will pay enough attention to these issues to come up with a good argument.

2. Supersymmetric Quantum Theories of Gravity

I believe that the most cogent summary of what was achieved by the String Duality Revolution is the statement that what we used to call String Theory is really just the collection of Supersymmetric Quantum Theories of Gravitation. Recall that using BPS arguments, we can derive perturbative Type II string theories compactified on tori as limits of the moduli space of a quantum theory, which has another limit where its low energy dynamics is that of eleven dimensional supergravity (SUGRA) (what some call M-theory but I would prefer to call the 11D SUGRA limit of M-theory).

Similarly there is a collection of moduli spaces of theories with 16 supercharges, the most well known of which is 11D SUGRA on $K3 \times T^d$, with $d \leq 3$, also known as the heterotic string compactified on tori. Again, SUSY enables us to derive the existence of this
moduli space, including its weak coupling stringy limits. Perturbative string theory calculations give an independent check that the SUSY arguments are valid. In the perturbative regime, we can of course do much more detailed calculations of amplitudes.

The different moduli spaces discussed so far are disconnected, and the arguments of [12], reviewed below, show that they cannot be viewed as part of the same theory. If we descend to 8 supercharges, it may be that the situation improves. There are many different descriptions of moduli spaces with 8 supercharges, the simplest of which is Type II string theory compactified on CY$_3$ manifolds. The phenomenon of extremal transitions [18] lends credence to the conjecture that the moduli space of compactifications to four asymptotically flat dimensions with 8 supercharges is a connected space (though not a manifold). Furthermore, there exists the possibility that we can recover all of the theories with more supercharges by taking limits on this moduli space.

We also know how to obtain moduli spaces of SUSY theories with AdS or linear dilaton asymptotics, as limits of brane configurations in asymptotically flat theories. These limiting theories are quantum field theories and Little String Theories, respectively. In these theories, the meaning of moduli space is somewhat different [19]. Fluctuating modes in these theories satisfy a normalizability criterion at infinity, while changes in the moduli correspond to non-normalizable perturbations.

In the case where the quantum theory is a QFT it is clear that even changes in the moduli correspond to changes in the theory and do not resemble the traditional notion of change of vacuum state at all. That is, they are simply changes in parameters specifying a quantum field theory. Presumably, the same is true for the theories with linear dilaton asymptotics.

In AdS/CFT we can find consistent relevant perturbations of the CFT, which violate SUSY. These have to be interpreted as quantum theories of spacetimes which are asymptotically AdS but have local distortions. Note that these theories are not even Poincare invariant, since in order to interpret them in terms of AdS spacetime, we have to study them on the sphere. But the full superconformal algebra is restored asymptotically in spacetime.

There is a subtle issue here. We can certainly recover decompactified theories with more SUSY by taking limits on this moduli space. The question is whether we can actually take limits on the 8 SUSY moduli space which give us the entire moduli spaces with more SUSY. If we can only access these by decompactification and recompactification then they are not really continuously connected.
Asymptotically AdS theories that break SUSY everywhere in spacetime are harder to come by. We can, if we wish interpret any conformal field theory as a theory of quantum gravity in asymptotically AdS spacetime with radius of curvature of order the Planck or string scale. Since there is no regime in which ordinary low energy theories of gravity are valid in such a spacetime, there is no calculation to compare the CFT results to, which might contradict such an assertion. The hypothesis that there exist SUSY violating conformal field theories that can be interpreted as AdS spacetimes with large radius leads to some puzzles, which we will discuss below.

Apart from this possibility, we know of no examples of consistent theories of quantum gravity in a large smooth spacetime, which violate supersymmetry. If we try to break SUSY in perturbative string theory, we generate potentials for moduli that lead either to runaways or AdS vacua. The runaway solutions always drive the system into regions where the perturbation series breaks down. There is one class of models (the $O(16) \times O(16)$ heterotic string in 10 dimensions is a typical example) where there are solutions of the loop corrected equations of motion in which the dilaton always stays in the weak coupling region although the geometry has a Big Bang or Big Crunch singularity. It is possible that nonperturbative analysis can lead to consistent theories of quantum gravity based on these solutions.

In my view, all of these facts are evidence that SUSY is a much more crucial ingredient in theories of quantum gravity, than semiclassical analysis has led us to suspect. This property of quantum gravity is not at all evident in the field theory approximation. One of string theory’s most important contributions to our understanding is to bring it to the fore. I suggest that we have not paid enough attention to what the theory is trying to tell us. Followed to its logical conclusion, this clue resolves many paradoxes and helps us to understand how string theory is related to the real world.

I have left for last the discussion of supersymmetric theories of gravity with Poincare invariance and only four supercharges. These live in four or fewer spacetime dimensions. It is likely that there are infrared problems with the definition of asymptotically flat theories of quantum gravity below four dimensions, so I will restrict my attention to $\mathcal{N} = 1$ SUSY in four dimensions. Here, the equations for SUSic, Poincare invariant vacua, $W = D_i W = 0$ are over determined and we expect few solutions, typically isolated. It seems likely that such isolated points have an enhanced discrete $R$ symmetry that accounts for the vanishing of the superpotential. Indeed, in the presence of such an $R$ symmetry it is sometimes
natural to have moduli spaces of $\mathcal{N} = 1$ vacua\footnote{Witten has pointed out another way to find moduli spaces of $\mathcal{N} = 1$ vacua by using holomorphy and showing that the superpotential vanishes order by order in the expansion about a weak coupling or large radius region of moduli space. One must have control over multi-instanton calculations to use this method.}. According to the results of \cite{12} we should think of each of these disconnected components of the four supercharge moduli space as an alternative theory of gravitation in four asymptotically flat dimensions. We will see that the isolated points in this moduli space are of particular interest. None is yet known, because they are not amenable to perturbative analysis. Indeed, we would hope that they are few in number. Below, I will argue that the real world is described by a quantum theory of gravity with a finite number of states, which approaches an isolated minimal 4D Super Poincare invariant theory in the limit that the number of states goes to infinity. The limiting theory has to obey certain additional constraints. If there is more than one such limit, we would have to add experimental input beyond the existence of gravitation and quantum mechanics to decide which of them is related to the real world. The more isolated $\mathcal{N} = 1$ vacua there are, the less predictive will be our theoretical framework.

In the following sections, I will try to attain a deeper understanding of why SUSY is so important, and precisely what SUSY is.

3. Classical Considerations or: Why General Relativity is Not a Field Theory

One way to approach the quantum theory of gravity is to try to quantize the classical Einstein equations. Traditionally this has been done by treating general relativity as a field theory. Canonical quantization of a classical system proceeds by finding a polarization of the symplectic structure on its phase space; the space of solutions of the classical equations of motion. The phase space of field theories is generally described by invoking the Cauchy-Kowalewska theorem. For hyperbolic theories in $d$ dimensions, solutions are determined by fixing the field and its normal derivative on some spacelike surface of dimension $d - 1$. The phase space is then said to consist of one pair of canonical variables per space point.

While there does exist a Cauchy formulation of Einstein’s equations, two facts mitigate its utility. The first is that the geometry of spatial surfaces is time dependent and this (at least if we imagine imposing a fixed spatial cutoff of short distances, like a simplicial decomposition of spatial hypersurfaces) appears to present problems with unitarity. The number of canonical degrees of freedom can change with time.
More problematic is the generic occurrence of singularities. The Cauchy-Kowalewska theorem only guarantees local existence of solutions. We have little rigorous data on the global structure of the phase space of General Relativity. What is known is that if we parametrize solutions in terms of scattering data in asymptotically flat space-time, or appropriate boundary data in asymptotically AdS space-time (I will refer to both sorts of data as scattering data from now on), then many singularities are cloaked in black hole horizons. Classically, this means that they do not have any effect on the deterministic evolution of the system outside the horizon. A conservative formulation of the Cosmic Censorship Conjecture is that this is true of all solutions determined in terms of scattering data. I will assume this form of Cosmic Censorship.

When combined with the Bekenstein-Hawking entropy formula for black holes, this suggests that the field theoretic “per unit volume” counting of degrees of freedom is wrong in General Relativity. Instead, as I will explain in more detail in the next section, we have a holographic counting of degrees of freedom. Note this is also suggested by the idea that a complete set of solutions should be parametrized by scattering data.

Classical black holes are completely stable, and a specification of scattering data would also have to list the number and positions of incoming and outgoing black holes. Quantum mechanically, because of Hawking evaporation, particle scattering data are sufficient.

We see then that the attempt to quantize Einstein’s equations semiclassically leads us immediately into deep waters, and suggests a crucial role for Black Holes in the fundamental formulation of the theory. In the next section I will argue that a more general approach to the quantum theory leads to the same conclusion.

4. Short times at energy high

The key step in the Feynman path integral formulation of quantum mechanics is the approximation of the short time evolution operator by a perturbation of a free system. This leads to an expression for amplitudes as a path integral over the exponential of the classical Lagrangian. From these formulae one can derive the standard canonical quantization procedure. K. Wilson realized that this prescription did not always work in quantum field theory, and needed to be generalized. The Wilsonian definition of a

7 Black Hole Complementarity is the assertion that the same is true in the quantum theory, with deterministic replaced by unitary.
quantum system begins with a conformal fixed point theory and realizes a general theory as a relevant perturbation of the fixed point. The richness of ordinary quantum mechanics and the relative scarcity of consistent quantum field theories is a consequence of the fact that $1+0$ dimensional Gaussian fixed points have a lot of relevant perturbations.

The above description of ordinary quantum mechanics and quantum field theory highlights the fact that the fundamental description of the system is obtained by looking at the high energy limit of its spectrum. The “degrees of freedom” (DOF) of the system are a parametrization of the high energy spectrum. In models where the high energy behavior is described by a Gaussian fixed point this coincides with the classical definition of degrees of freedom. For more general fixed points the degrees of freedom have to do with the structure of the operator algebra. In integrable CFT’s there is usually a small set of generating operators which can be thought of as the fundamental set of degrees of freedom. At more general fixed points we do not yet know whether there is such a simplification of the operator algebra.

*Asymptotic Darkness* is the conjecture that in all quantum theories of gravity, the asymptotic spectrum of states is dominated by black holes. There are several reasons for making this conjecture. The simplest is that the black hole spectrum grows so rapidly at high energies and we have never discovered another class of states with such rapid increase. More convincing are arguments I will review below, which show that high energy scattering processes produce black holes over a range of impact parameters, which grows with the energy. If these arguments are correct, then all attempts to probe the theory at high energies, probe black hole physics. Finally, the AdS/CFT correspondence, our most rigorously formulated quantum theory of gravity, exhibits Asymptotic Darkness quite explicitly. The form of the high energy entropy, up to a multiplicative constant, follows from symmetry arguments in the CFT and agrees with the Bekenstein-Hawking entropy of AdS-Schwarzchild or Kerr black holes. In the one case where we can calculate the constant reliably from the field theory, the answers also agree.

Asymptotic Darkness also sheds light on both the UV/IR connection and the holographic principle. If generic high energy states are black holes of larger and larger mass, then we know that their gravitational (and other) fields outside the horizon are smooth and have low curvature. Thus, there must be a description of some of their properties in terms of long wavelength effective field theory. Since the general laws of thermodynamics have to be obeyed in the effective field theory approximation, it is not surprising that semiclassical methods can be used to calculate the entropy of black holes, even though
the quantum states involved cannot be treated correctly by effective field theory. This, I believe, is the ultimate form of the UV/IR connection. Note that it also implies that the form of the high energy spectrum is dependent on the shape of spacetime at arbitrarily large distances. Ultimately, it is the latter fact which is responsible for the failure of the field theory paradigm of effective potentials and superselection sectors.

The holographic principle follows from Asymptotic Darkness, because the latter principle shows that the counting of high energy states in the theory scales with the area, rather than the volume, of the spacetime region they occupy. In trying to probe the theory to find the volume’s worth of DOF we might have expected, we would be forced to do scattering experiments at high energies and small impact parameters. Classical GR suggests strongly that such experiments will always produce larger and larger black holes, rather than probing short distances.

The intuitive argument for black hole creation in high energy collisions goes as follows: Imagine that a finite fraction, $M$, of the energy of the collision remains for some time in a region bounded by something of order the impact parameter. Then we have, at large distances from this region, a Schwarzschild field with mass $M$. For large $M$ and fixed impact parameter, the Schwarzschild radius is larger than the region in which the energy is concentrated and so the system must be a black hole. An obvious loophole is the possibility that all but a finite amount of the energy is radiated away.

The argument has been made more rigorous by a number of developments\textsuperscript{1}. In $2+1$ dimensions, with negative cosmological constant, the problem of colliding Aichelberg-Sexl waves has been completely solved and black holes are indeed formed under the stated conditions. In $3+1$ dimensions, Penrose showed that a trapped surface forms for collisions of zero impact parameter. If one invokes Cosmic Censorship, this shows that a black hole forms and the size of the trapped surface puts a lower bound on the mass of the black hole. d’Eath and Payne studied this problem in more detail. Eardley and Giddings have recently generalized Penrose’s argument to nonzero impact parameter. Thus, at the level of classical general relativity the question of black hole formation in such collisions has been reduced to the proof of the Cosmic Censorship conjecture within the class of solutions of Einstein’s equations with scattering boundary conditions.

It is important to realize that once this classical argument is completed, we should believe that it is telling us something correct about the quantum theory. For large enough energy, the fields outside the trapped surface are low curvature and therefore well described by low energy classical field theory. Again we see a manifestation of the UV/IR connection.
These classical arguments are not sufficient to tell us about the detailed quantum mechanics of the final states of high energy collisions. They do however tell us that it will be very complicated. High energy collisions at a range of impact parameters from zero to an upper bound that grows like $E^{\frac{1}{D-3}}$ will be thermodynamic in nature, and the relevant thermodynamics is that of black holes.

A final argument for black hole dominance in high energy collisions comes from the AdS/CFT correspondence. In this context, black hole dominance just means thermalization. There is a lot of evidence that the high energy spectrum of states in the relevant conformal field theories can indeed be identified with AdS-Kerr black holes. Thus if we inject energy into the system by making boundary perturbations, it follows from the assumption that the dynamics of the CFT is not exactly integrable, and the standard derivation (such as it is) of statistical mechanics from quantum mechanics, that at sufficiently high energy the system will thermalize. The identification of black holes with generic thermal states of the system then shows us that high energy scattering leads to black hole production.

To summarize, quantum theories are defined by their high energy behavior. General arguments suggest that the slogan which captures the essence of the high energy behavior of quantum theories of gravity is Asymptotic Darkness. We will see that this has important implications for understanding how the various incarnations of M-theory are related to each other and to the real world.

5. Against Independence

I will use the term M-theory to refer to a collection of models of the quantum theory of gravity, which we have been studying since 1984 (well, some of us (J. Schwarz) have been studying them since 1974). They are for the most part SUSic, though there are some SUSY violating systems that can be studied fairly reliably. An important question is the extent to which these models are “all part of the same theory”, often called the question of background independence. It is important to understand the precise meaning of this, since any two separable Hilbert spaces are of the same dimension are unitarily isomorphic to each other. So it is trivial to map one theory onto another. This is surely not what we mean by background independence.

Our paradigm for what we do mean is classical field theory. There we have a Lagrangian density and different vacuum states of the same theory mean different solutions of the same equations of motion that preserve a maximal spacetime symmetry group. In
this definition, we lump together Minkowski (M) space, Anti de Sitter (AdS) space and de Sitter (dS) space, even though the meaning of symmetry generators in the latter case is quite different. I will argue that the conflation of these different kinds of spacetime is incorrect in the quantum theory.

If we ignore the quantum mechanics of spacetime, and consider quantum fluctuations in a fixed Minkowski space then there is a nice quantum analog of this classical paradigm. Unitary Quantum field theories are defined by relevant and marginal perturbations of conformally invariant fixed point theories. Unitary Conformal field theories have a unique conformally invariant vacuum state. They are defined by a set of primary fields and their descendants under the conformal group. This set of fields is in one to one correspondence with the finite norm states of the theory. The algebra of fields closes, in the sense that for every pair of fields we have an operator product expansion (OPE):

\[ A(x)B(0) = \sum C_i^n x^{d_A - d_A - d_B} O_i^n(0), \]  

which converges when applied to the vacuum state.

Some conformal field theories (usually SUSic ones) have a moduli space of non conformally invariant vacua. These are unitarily inequivalent representations of the same local operator algebra. The maximal symmetry of a state in these representations is the Poincare subgroup of the conformal group. These are examples of what we mean quantum mechanically by different states of the same theory. The local operator algebra does not mix up these different representations. The complete Hilbert space of one representation is obtained by taking limits of polynomials in smeared local fields (with test functions of compact support) acting on the vacuum. Often, the theories on the moduli space have a particle interpretation. That is, there are other bases for the Hilbert space which consist of incoming and outgoing multiparticle states. The relation between the two descriptions of the Hilbert space is given by the LSZ formula. In this case the theory on the moduli space always contains Goldstone bosons of spontaneously broken scale symmetry.

Another way to get nonconformally invariant theories is to perturb conformal field theories by a relevant operator. This is the Wilsonian definition of general quantum field theory. Field theories are parametrized by relevant perturbations of all possible fixed points. We do not normally think of field theories with different values of their parameters as different states of the same theory. And indeed, there is a difference between the breaking of conformal invariance along a moduli space, and explicit breaking of conformal
invariance by relevant operators. In the former case there is always a massless dilaton in the theory, whose low energy couplings to other states is characterized by conformal Ward identities. So given a set of Green functions which violate conformal invariance but approach a conformal field theory at short distances we can tell whether they represent a perturbed CFT or a moduli space of vacua by looking for dilaton singularities.

Of course, nonconformally invariant field theories can also have degenerate vacua. Again these are inequivalent representations of the underlying operator algebra. In generic CFT’s the algebra is harder to characterize. Although it is in principle determined by the first few terms in the short distance expansion, this expansion is no longer convergent and the precise mathematical characterization of what is going on is more difficult. In QCD this has led to endless questions about whether the perturbation expansion determines the theory. Nonetheless, the general picture is clear. Different states of a quantum field theory are inequivalent representations of an underlying operator algebra that can be extracted from a universal short distance behavior of Green’s functions.

There is a conceptually quite different way to discuss inequivalent vacua in QFT. Namely, given a single vacuum state, one can, by injecting enough energy, construct regions of arbitrary size that resemble another. For vacua that are continuously connected this is fairly trivial to do. For isolated vacua it is a consequence of the existence of static domain walls interpolating between two vacua. These are limits of large, long lived bubbles of one vacuum, which can form inside another. I now want to discuss whether either of these two methods of connecting vacua, works in quantum theories of gravity.

5.1. Disconnected asymptotically flat vacua

In the last subsection, I reviewed two field theory methods for judging when we have two vacuum states of the same theory. The message of [12] was that neither of these two methods of verifying the existence of multiple states of the same theory work in theories of quantum gravity unless there is a moduli space of vacua. Both fail because of the existence of black holes in asymptotically flat space. Quantum gravity appears to be a holographic theory, which means that the only gauge invariant observable in asymptotically flat space is the S-matrix. The closest analog of short distance behavior is the study of scattering matrix elements in the limit that all kinematic invariants are large. In this regime the considerations of the an earlier section lead us to expect scattering to be dominated by

\[8\] though in theories of gravity it turns out to be surprisingly intricate [12].
the creation of supermassive black holes. The Hawking temperature of such holes is very low and thus the final amplitudes are sensitive to the infrared structure of the theory in its particular vacuum state. There is no analog of the universal short distance behavior of different states of a QFT.

Similarly, a simple scaling argument shows that the attempt to construct metastable bubbles of another vacuum generically leads to the creation of a black hole. Again the decay of the black hole is sensitive to the nature of the external vacuum and contains no trace of the putative information about the vacuum state inside the black hole.

These arguments lead me to expect that isolated vacua of AF quantum gravity are not “states of the same theory”. Something similar can be said about disconnected pieces of moduli space. For example, M-theory with 16 SUSYs has a disconnected moduli space [21], one branch of which is the conventional heterotic string on a torus. The above arguments apply to the question of whether these different branches are “different states of the same theory”.

How then are the various moduli spaces of vacua connected to each other? For vacua with at least 8 supercharges there is a plausible conjecture. We have learned from Matrix Theory that, in theories of quantum gravity, compactified spacetimes have more degrees of freedom than their noncompact limits. We also know that many compactifications to four dimensions with eight supercharges, lie on moduli spaces, and that many different moduli spaces can be connected by extremal transitions. It would not contradict anything we know, to conjecture that there is a single stratified manifold of four dimensional compactifications with eight supercharges. The quantum scattering matrix would vary smoothly through the extremal transitions, as long as we kept all stable states in the theory at all points in the moduli space. We could further conjecture that all theories with at least 8 supercharges and/or larger numbers of AF dimensions could be achieved as limits along asymptotic directions in this moduli space. Many asymptotically AdS compactifications

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9 This remark is correct within the context of moduli spaces with 16 SUSYs. If the maximal speculations about the connectivity of the moduli space of 4D, 8 supercharge theories are correct, and if we can obtain finite points in the 16 SUSY moduli space by taking limits of the 8 SUSY moduli space, then we could view all moduli spaces with 8 or more supercharges as one connected theory.

10 Strictly speaking, there is no scattering matrix in four dimensions, because of IR divergences. However, there is a generalized S-matrix with finite amounts of classical gravitational radiation in initial and final states.
could also arise as low energy limits of brane configurations in these vacua. This is, I believe, the maximal amount of “background independence” that we can hope for in a quantum theory of gravity.

The reader may wonder why I do not include theories with four supercharges or fewer AF dimensions. The latter are ruled out because they do not really have S-matrices. Indeed, although there are perfectly good free string theories with 1-3 AF dimensions, the vertex operator correlation functions that define the perturbative S-matrix, do not exist. There is a good reason for this. Massive states, including most multiparticle states of massless particles, distort the asymptotic geometry of spacetime, so we can no longer talk about scattering theory in AF spacetime. A hint of this problem already occurs in 4 dimensions, as a consequence of IR divergences. Although a classical proposal for a kind of scattering operator in $2+1$ dimensions exists\cite{22}, it is far from clear that there is a good quantum theory. Furthermore, SUSic theories always contain massless scalars as well as gravitons, so the mass of massive states often appears infinite because of the contribution of their long range scalar fields. Thus Super-Poincare invariant theories of quantum gravity in $2+1$ dimensions may not even have a collection of asymptotically locally flat scattering spacetimes as in the proposal of \cite{22}.

Four dimensional gravitational theories with $N=1$ SUSY do not have a moduli space, at least generically. The equations $D_i W = W = 0$ are overdetermined. They are likely to have solutions in only two circumstances. The first is at points of enhanced discrete R symmetry. If some of the fields at such an enhanced symmetry point have R charge zero, then we get a continuous moduli space. Otherwise, we expect only isolated vacua. There are also apparently\cite{23} examples where instanton calculations and holomorphy guarantee that the superpotential vanishes identically along some subspace of a classical moduli space.

The disconnected moduli spaces of e.g. ten dimensional theories with 16 SUSYs may be connected through the moduli space of theories with 8 SUSYs in four dimensions. But there is no apparent way to connect disconnected pieces of the moduli space of four dimensional theories with minimal SUSY. We had previously hoped to connect them by “going over the potential barrier”, but the considerations of \cite{12} show that this does not happen. Indeed, these considerations suggest that the notion of an off-shell effective potential has no exact meaning in quantum gravity, and is a useful tool only in extreme regions of moduli space, if it is useful at all .

The latter remark is the most disturbing aspect of this critique of the conventional wisdom. So much of our effort has gone into thinking about the calculation of the off-shell
potential in string theory that it is difficult to give up this tool. Much interesting recent work has gone into finding potentials which can stabilize the moduli in regions where approximate methods of calculation are valid. There do seem to be examples where the quantum gravity effects I have discussed should not disturb the existing calculations. However, the interpretation of the potentials is far from straightforward, and deep issues of quantum gravity in non-perturbative regimes are encountered in this interpretation. Another disturbing aspect of these calculations is that the potential exhibits a plethora of AdS vacua. Conventional wisdom would lead us to expect them to be connected. However, as the next section will show, this is certainly untrue.

5.2. The teachings of don juan: an AdS/CFT way of knowledge

Our most comprehensive nonperturbative formulation of a quantum theory of gravity is the AdS/CFT correspondence. In this subsection I want to explain how the ideas of asymptotic darkness, inequivalence of vacuum states, and the nature of the cosmological constant, appear in this formalism.

The Bekenstein-Hawking formula for black hole entropy in AdS spacetimes is identical with that of a conformal field theory. This can in some ways be seen as a derivation of the AdS/CFT correspondence, if we take asymptotic darkness as a fundamental rule: black holes always dominate the high energy density of states. That is, accepting this rule, we derive from it the fact that quantum gravity in asymptotically AdS spacetimes is a quantum theory which approaches a conformal field theory in the UV (at least with regard to its density of states). But this is nothing more nor less than the Wilsonian definition of a generic quantum field theory. The difference between general QFTs and those which are actually conformally invariant has to do with the rate at which AdS asymptotics are approached. Thus, AdS/CFT is a confirmation of asymptotic darkness, which is a more general principle.

The AdS/CFT correspondence also throws light on the connectedness of vacuum states. In field theory, given two disconnected SUSic vacuum states of the same theory, we expect a BPS domain wall interpolating between them. In fact, there are BPS domain wall solutions of supergravity in situations where the AdS/CFT correspondence applies. However, their meaning is radically different from what it was in field theory. The kind of BPS domain wall that has been discovered in AdS/CFT is identified with a renormalization group flow between two different quantum field theories. Only one of these field theories has the full asymptotic density of states of the system. The other is
a limiting infrared subspace of the space of states. Indeed, there is a c-function for this flow, which decreases along it. In the SUGRA approximation, decrease of the c-function is a consequence of the same dominant energy condition that leads to the black hole area theorem.

Strictly speaking, the BPS domain wall is not a part of quantum gravity in an asymptotically AdS space at all. The theory with AdS boundary conditions is really the CFT on the sphere, and there is no global BPS solution, because the domain wall has a translational invariant Poincare energy density on a Poincare slice (and so is singular when mapped to the sphere). In the CFT this corresponds to the fact that, on the sphere, RG flow is cut off in the IR by the finite volume. Indeed, AdS/CFT describes two related but distinct gravitational systems: globally asymptotically AdS space, and the near horizon geometry of a BPS brane in asymptotically flat space. The BPS domain wall, like the moduli spaces of vacua of the CFT, really exist only in the latter interpretation of the theory. However, the two forms of the theory each contain complete information about how to construct the other, so perhaps this is merely a technical quibble.

The important point is that, although BPS domain walls exist in quantum gravity, they no longer have the significance of connections between two vacua of the same theory. Rather they represent a theory with a dimensionful parameter, whose renormalization group flow interpolates between two conformal (and therefore asymptotically AdS in the gravitational interpretation) theories. However, it is only the UV fixed point, which has the full set of degrees of freedom of the system, and represents its true asymptotic behavior in spacetime.

It is important to note that the holographic RG flows are a special kind of BPS domain wall, between a Breitenlohner-Freedman allowed AdS maximum of the supergravity potential, and an AdS minimum. Other kinds of domain walls, including that between two minima, do not have a C function which would enable us to interpret them as RG flows (the C function flows backward for one class of such walls, and is not monotonic for the other). There is no known dual description of such walls in CFT. It seems entirely plausible that they are a feature of low energy supergravities which does not have a quantum mechanical realization in a complete theory.

There is an extension of these arguments which deals with the question of whether AdS and Minkowski vacua are part of the same theory. There are no known examples of

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11 I would like to thank M. Cvetic for teaching me about the different kinds of BPS domain walls in AdS SUGRA.
a relevant perturbation of a superconformal theory that could be identified with a domain wall between an AdS and a Minkowski vacuum. Let us recall the geometric C-function that was discovered \cite{25} in the parallel between the structure of SUGRA domain walls and renormalization group flows \cite{26}. The dominant energy condition gives a directionality to the domain wall structure that mirrors the loss of entropy as one flows from the UV to the IR in a field theory. In domain walls interpolating between Minkowski and AdS vacua, the C-function decreases from the Minkowski side. It is clear that the Minkowski side of the domain wall is always in the UV and the AdS side in the IR. Thus it is not surprising that one cannot find domain walls that interpolate between Minkowski and AdS space as relevant perturbations of a conformal field theory (for any such theory is AdS invariant in the UV). AdS spaces should thus be thought of as infrared limits of Minkowski space where a large number of degrees of freedom decouple\textsuperscript{12}.

The conclusion one is forced to accept, is that two different AdS vacua are different quantum systems, rather than two states of the same system. The clearest way of explaining the disconnectedness of AdS vacua, is to note that the cosmological constant in Planck units is, \textit{in every AdS/CFT correspondence of which I am aware, a discrete tunable variable, which characterizes different field theories (e.g. a power of the }N\text{ of an }SU(N)\text{ gauge group)}. Nothing could be further from the idea of the value of an effective potential of a given theory at its different minima. The effective potential is a strictly IR concept, and different ground states of a field theory have the same UV behavior. By contrast, in AdS/CFT, different values of the cosmological constant correspond to different high energy behaviors, since }N\text{ controls the density of states at high energy. These results are in accord with (in fact were the origin of) the conjecture of Asymptotic Darkness. The generic high energy state in AdS space is a black hole, and the nature of the black hole spectrum is crucially dependent on the cosmological constant, because the entropy of a black hole is determined by the area of its horizon.

I conclude from this that the field theoretic notions of off shell effective potential and the possibility of connecting different vacuum states “through the off shell configuration

\textsuperscript{12} On the other hand, if one has a sequence of AdS spaces with radii increasing to infinity, it would not be surprising to recover an asymptotically flat theory in the limit. This is the approach taken in \cite{13}. It should be noted that, although the arguments of these authors are plausible, their construction is in no way a complete argument for the existence of the unitary asymptotically flat S-matrix that is their goal. More importantly, it does not seem likely that most asymptotically flat vacua of M-theory can be recovered in this way.

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space”, are not valid concepts in theories of quantum gravity in asymptotically AdS space. The infrared properties of spacetime, which we generally associate with the nature of the vacuum state, are determined by the UV properties of the quantum field theory which defines quantum gravity in asymptotically AdS spaces.

Since the quantum theory of gravity in asymptotically AdS spaces is defined by a quantum field theory, it is interesting to ask what the interpretation of QFT superselection sectors, multiple vacuum states, and effective potentials are in the spacetime interpretation. The key to this question is the proper formulation of asymptotically $AdS_d$ gravity as QFT quantized on $S^{d-2} \times R$. We have already seen that (if $d > 3$) this eliminates the moduli spaces of CFT’s. However, it might appear that we could still get multiple vacua, by considering relevant perturbations of the CFT that lead to a theory with a field theoretic (FT) effective potential with multiple minima. Relevant perturbations of a CFT lead to a quantum theory of spacetimes which asymptotically approach AdS but not quite fast enough that the perturbation can be considered a normalizable fluctuation. They seem to be perfectly legitimate quantum theories of gravity, which describe inhomogeneous spacetimes with only spherical symmetry and a single global timelike Killing vector. All other generators of the conformal group have been broken by the combination of the relevant perturbations and the asymptotically AdS boundary conditions. Perhaps general renormalizable QFTs should be thought of as theories of defects with long range gravitational fields, embedded in an AdS spacetime.

When renormalizable QFTs are quantized in Minkowski space, they can have multiple vacua. Once again, however, the fact that we are quantizing on a compact space defeats our attempt to find multiple vacuum states. Discrete degeneracies of the classical FT potential on a compact space, lead to a unique ground state, which in the the semiclassical approximation is a superposition of semiclassical ground states in each of the degenerate wells. At large AdS radius, the quantization sphere is large, so the tunneling amplitudes between semiclassical ground states are small, of order $e^{-(mR_{AdS})^{d-2}}$, where $m$ is the mass scale induced by the relevant perturbation. However, the large distance behavior in space time is UV behavior in the field theory and is insensitive to the choice of ground state. Thus, although we could set up long lived states which live close to one minimum of the FT effective potential, the asymptotic observer will not view them as different vacua. Similarly, if we construct a FT effective potential with a metastable minimum, we will indeed find a metastable state in the quantum theory, but its decay will look nothing like a Coleman
DeLuccia\cite{27} bubble to the asymptotic observer. Rather, it will look like the decay of a localized object, which does not affect the asymptotic structure of spacetime.

I find these translations of the known field theoretic structures that we have been trying to mimic in thinking about vacuum states of quantum gravity, to be the most convincing evidence that we have been deluding ourselves. The notions of vacuum, effective potential, and vacuum decay from field theory, are not correct ones in quantum gravity\cite{13}

Similar remarks could have been made about asymptotically flat vacua in the context of Matrix Theory, but the problem of seeing phenomena associated with the vacuum in light cone quantum field theory, made one suspicious of the conclusions.

I want to end this subsection with a few remarks about SUSY breaking in AdS spaces. One way to do this is to add relevant SUSY violating operators to a known large radius AdS/CFT correspondence. This is not terribly interesting. As we take the AdS radius to infinity, we find that most of the spacetime is SUSic. We are really looking at some sort of SUSY violating defect in a SUSic vacuum state. It is more interesting to study SUSY violating fixed point theories. Here the problem is to find large radius examples. An interesting class of examples is given by the large $N$ version of the fixed points studied in \cite{29}. These are large $N$ gauge theories with massless fermions in $N_f$ copies of the fundamental. $N$ is taken to $\infty$ with $N/N_f$ fixed in such a way that the $\beta$ function for the gauge coupling has a zero in the perturbative regime $g^2 N \sim \epsilon \ll 1$. One expects, on grounds of continuity, and because of the known examples with $\mathcal{N} = 1$ SUSY, that there is a collection of such fixed points with $\epsilon$ ranging from a very small number to something of order 1. These theories have a $1/N$ expansion, whose leading term consists of planar diagrams with an arbitrary number of fermion loops (holes). The planar limit is not a free string theory, because there is no restriction of $SU(N_f) \times SU(N_f) \times U(1)$ gauge invariance. The only obvious conjecture to make about what the large $N$ expansion means in a dual spacetime picture is that it is the low energy SUGRA expansion. This would be remarkable if true because it would mean that we could solve a rather nontrivial four dimensional CFT by solving Einstein’s equations. However, because the theory has a huge number of operators whose dimension is a finite multiple (as $N \to \infty$) of that of the stress

\footnote{As discussed in\cite{28}, this might not be true in de Sitter quantum gravity. As that theory has not yet been constructed, it is harder to assess the validity of these concepts in a rigorous manner. There seems to be a sensible semiclassical theory of dS decay into other dS spacetimes or into negatively curved FRW universes with vanishing cosmological constant.}
tensor, it seems that, as in perturbative string theory with AdS radius of order the string scale, there is no regime in which SUGRA is a good approximation.

A putative AdS dual of these field theories would have some peculiar properties. The global $SU(N_f) \times SU(N_f) \times U(1)$ symmetry of the boundary CFT implies a huge spacetime gauge group in the limit of large AdS radius. The simplest context for investigating this phenomenon is probably the $\mathcal{N} = 2$ superconformal analogs of these non-SUSic fixed points. A possible interpretation of the huge spacetime gauge group is the following: $A_n$ singularities in string theory can produce $SU(n+1)$ gauge groups. D-branes at the weakly coupled string orbifold singularity of $A_n$ type give rise to a $\mathcal{N} = 2$ quiver gauge theory\cite{20}. The string theory orbifold differs from the configuration with the true $A_n$ singularity by having NS B fields on the shrunken cycles, but the B field can be continuously dialed to zero. One then conjectures that the superconformal theory with $N_f = 2N_C$ can be achieved as a marginal perturbation of the quiver theory, and that its spacetime interpretation is that of $N$ branes at an $A_{2N-1}$ singularity, without B flux\cite{21}.

One can obtain non-SUSic fixed points without flavor groups by studying large $N$ theories with a variety of fermion representations, tuned so that the leading coefficient of the Callan-Symanzik function is small. Little is known about such theories, but the AdS/CFT correspondence suggests that they have an AdS dual with radius large compared to the Planck scale.

There is however a general problem with all of these constructions of SUSY violating, large radius AdS space. By making $N$ large, we guarantee that the AdS radius is large compared to the Planck scale (comparing the Bekenstein-Hawking entropy formula with the CFT entropy formula). But there is no guarantee that it is large compared to the string scale. All of these constructions are reminiscent of large $N$ gauge theories with fixed ’t Hooft coupling. Such theories have a large number of operators (going to infinity with $N$) whose anomalous dimension is a finite multiple of that of the stress tensor. Thus, as in string theory with weak string coupling, there is a spectrum of particles whose mass is of order the inverse AdS radius, in the limit that the Planck mass goes to infinity. Furthermore, E.Gorbatov\cite{22} has argued using the Horowitz-Polchinski correspondence principle, that in this range of couplings there will be no black hole states with size less than the AdS radius. The large $N$ limit of these theories is a free theory of an infinite number of

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14 This conjecture was developed with B. Acharya and H. Liu.

15 E. Gorbatov, private communication
free massive particles in AdS space, rather than an interacting theory of quantum gravity in flat space. This is consistent with our conjecture in a later section, that SUSY violating theories of gravity in asymptotically flat space do not exist.

Special mention should be made here of the SUSY violating orbifolds of $AdS_5 \times S^5$ gravity. These are described by non-supersymmetric quantum field theories, which nevertheless have a line of fixed points in the planar limit. The gap in dimensions between the stress tensor and (most) other operators, which we expect in a theory which describes a space with curvature low compared to the string scale, is guaranteed by the analogous phenomenon in the SUSic parent theory.

However, like all tree level SUSY breaking in string theory, this example is highly unstable. Non-planar corrections give infinite mass to the scalars and the theory nominally flows to an asymptotically free theory of gauge bosons and fermions. If we try to tune the scalar masses to zero we encounter the Halperin-Lubensky-Ma-Coleman-Weinberg first order fluctuation induced phase transition, rather than a conformally invariant fixed point. Even if we assume that some conformal point can be found non-perturbatively, the nonplanar corrections to the 't Hooft coupling’s Callan-Symanzik function show that any such conformal theory will be an isolated fixed point with $g^2N$ of order 1. In other words, these orbifold theories behave in a manner similar to Scherk-Schwarz orbifolds of toroidal string theories. At tree level they give a SUSY violating string spectrum in a large smooth spacetime with a maximally symmetric subspace. Once loop corrections are taken into account this picture is not even approximately valid. As a field theory the theory likely becomes asymptotically free in the UV and has no large smooth spacetime interpretation. It is possible that there is a conformal point that can be defined as a limit of the cutoff version of this field theory, but it can at best be interpreted as an AdS spacetime with radius of order string scale.

Finally, I would like to mention SUSY violating flux compactifications of string theory, which can describe large radius AdS space-times for appropriate values of flux. There is no contradiction between these models and the statements I have made here, because the SUSY violation goes away as the radius is taken to infinity. Nonetheless, they are peculiar from the point of view of AdS/CFT. Since many of them are effectively perturbative, we might expect them to be related to large $N$ gauge theories in three dimensions. Thus, for large values of the flux, these perturbative vacua predict infinite sequences of fixed points at large $g^2N$ in some kind of large $N$ gauge theory. It would be of great interest to get a handle on these peculiar conformal field theories. Alternatively, evidence that they did not exist would throw doubt on the effective potential calculations that went into the effective field theory constructions of these systems.
5.3. Flat contradictions

The ordering of Minkowski with respect to AdS in our discussion of domain walls, fits with a striking difference between the black hole entropy formulae for the two spacetimes. Indeed, using the logic of the previous subsection, it is clear that quantum gravity in asymptotically flat spacetimes is a different kind of beast from quantum field theory, with a high energy density of states unlike any quantum theory we have dealt with before. It grows more rapidly than an exponential of the energy. This means that Green functions of generic Heisenberg operators:

\[
< 0|O(t_1) \ldots O(t_n)|0 >
\] (5.2)

are not tempered distributions\textsuperscript{[5][10]} . That is, if we smear the operators with smooth functions of compact support, they are not finite. Consequently, we should not expect to be able to localize operators in time\textsuperscript{16}. Much of the conventional framework of quantum mechanics is lost.

This new feature of asymptotically flat space is also suggested by the structure of the boundary of Minkowski space, and the description of physics on it. Ashtekar\textsuperscript{[33]} has described the quantum theory of massless particles on null infinity. In \(d\) dimensional AF spacetimes, future (or past) null infinity is a \(d - 1\) manifold with a degenerate conformal structure. There exists a coordinate system \((u, \Omega)\) where \(u\) is null and \(\Omega\) coordinatizes a manifold with metric conformal to the round \(d - 2\) sphere. Ashtekar uses Fock space methods to define multiple asymptotic massless particle states in terms of fields defined on null infinity. The commutation relations of the fields have the form

\[
[F(u, \Omega), F(u', \Omega')] = 3D\Delta(u - u', \Omega, \Omega')
\] (5.3)

. Thus, all of the coordinates of null infinity are spatial. The fields do not solve any dynamical equations on null infinity, but are independent variables at each point. Thus, we should not expect a parallel to the AdS paradigm of ”correlation functions on the boundary”, for asymptotically flat spaces. Instead, the dynamics will be incorporated in a

\textsuperscript{16} The exceptions would be operators that resolve the degeneracy of black hole states and have matrix elements to only a few states of arbitrarily high energy. Given the thermal nature of black hole decay, it is likely that the description of such operators in terms of the asymptotic particle basis is hopelessly complicated.
mapping (the S-matrix) between formulations of the theory on past and future null infinity. That is, the boundary of asymptotically flat space splits into two pieces, neither of which is compact. They are joined along an asymptotic region (spacelike infinity) that they both share. The Hilbert spaces associated with the two boundaries carry unitarily equivalent representations of the Poincare group, but there is a nontrivial Poincare invariant unitary operator, the S-matrix, which connects them. What we do not yet have is an exact prescription for calculating that S-matrix.

Perturbative string theory describes it as an asymptotic series in a small parameter. Matrix theory \[1\] describes it as the (conjectural, Poincare invariant) large \(N\) limit of the S-matrix of an \(N \times N\) matrix quantum mechanics. Polchinski and Susskind \[13\] have suggested a way to obtain it as a limit of CFT correlation functions. Aside from questions of convergence, it is clear that none of these prescriptions applies to all situations in which we expect to have Poincare invariant vacua of M-theory.

Another approach to a holographic description of asymptotically flat spacetime is to use the light-front gauge, as in perturbative string theory and Matrix theory. The black hole spectrum throws light on the ubiquity of light-front gauge in Hamiltonian descriptions of asymptotically flat M-theory (AFM). Indeed, for 5 or more asymptotically flat directions, the light cone energy spectrum (equivalently, the \(M^2\) spectrum for fixed longitudinal and transverse momentum) grows more slowly than an exponential so that conventional quantum mechanical formulae make sense in light cone time. It is quite interesting that this argument (marginally) fails in four dimensions, where asymptotic darkness predicts a Hagedorn-like exponential spectrum. Perhaps this means that the Hamiltonian theory of four dimensional asymptotically flat spacetime is a quasi-local field theory, or little string theory, in light cone time.

The special role of four dimensions appears in a number of other contexts. It is the lowest dimension in which asymptotically flat spacetime has black hole excitations, indeed the lowest dimension in which it has any massive excitations at all (and remember that generic multiparticle states of massless particles are massive). It is also the dimension where the S-matrix ceases to exist in any quantum theory of gravity. Infrared divergences cause the vanishing of all amplitudes, which do not have an infinite number of gravitons in the final state. As a consequence, one must invent a generalized S-matrix between states with coherent classical gravitational radiation. The asymptotic symmetry group
of this class of spacetimes is the Bondi- Metzner-Sachs (BMS) group and the theory is undoubtedly more complicated than it is in higher dimensions.

At the present time, we do not have a general prescription for writing down the quantum theory for asymptotically flat spacetimes. For spacetimes with six or more asymptotically flat dimensions and 16 or 32 supercharges, Matrix Theory provides a plausible answer, though of course one has yet to prove Poincare invariance of the large $N$ limit. Matrix Theory is supposed to be the Discrete Light Cone Quantization (DLCQ) of M-theory. The spectrum of the DLCQ theory diverges more rapidly at large energy than that of the limiting, decompactified theory, for 9 or fewer asymptotically flat dimensions. At $D = 5$ the DLCQ spectrum blows up faster than an exponential of light cone energy and we don’t know how to define it. It is of the greatest interest to work out the form of the decompactified quantum theory.

Another approach to flat spacetime is to take the large radius limit of AdS/CFT. This applies to even fewer examples, and the construction is on a much less firm footing. In Matrix Theory, one has a well defined, unitary scattering matrix and one must show that its limit exists and is Poincare invariant. In the AdS/CFT approach one has a complicated definition of scattering matrix elements, and one must prove that they form a unitary matrix as well as proving that the proper flat space symmetry group (which is larger than the contraction of the AdS/CFT symmetry group) is restored.

6. The Peculiar Position of Perturbative String Theory

In my description of M-theory in the second section, strings were exiled to certain extreme regions of moduli space. Perturbative string theory was useful for confirming

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17 In fact, the relevant Lie algebra may be even more interesting than that of BMS. Let $(u, \Omega)$ be the coordinates of (say) future null infinity, $u$ a null coordinate and $\Omega$ a coordinate on the $d-2$ sphere. The BMS algebra is the semidirect product of vector fields of the form $f(\Omega)\partial_u$, with $f$ an arbitrary function on the sphere, and the conformal algebra of the sphere (with conformal factor also rescaling $u$). For $d = 4$ the conformal algebra is the infinite dimensional Virasoro algebra. The correct symmetry algebra of the formalism is probably this large extension of the BMS algebra, or some interesting subalgebra of it e.g. $f$ might be restricted to be the sum of a (locally) holomorphic and anti-holomorphic function on the sphere.

18 at least for toroidal compactification. The authors of [34] claim that for Calabi-Yau compactification the DLCQ is some kind of $3 + 1$ dimensional field theory.
dualities, and for doing certain exact calculations, which are protected by SUSY and have a nonperturbative significance.

It is clear however that there is more to perturbative string theory than that. It contains baby versions of the holographic principle, the UV/IR connection, the growth of the high energy spectrum of states, all of which we have seen to be central features of the quantum theory of gravity. String theory also teaches us about the importance of SUSY in the theory. There are no known asymptotically flat string vacua with broken SUSY. Kutasov and Seiberg[17] have given a very general argument for why this is so.

Historically, perturbative heterotic string theory gave us the first indication that a theory of gravity could also explain the standard model of particle physics. The more recent and more general understanding of how non-abelian gauge groups and chiral fermions arise from singular limits of Kaluza-Klein SUGRA does not diminish the historical importance of the perturbative string results. Indeed, the geometric picture was derived [35] by trying to understand how to reproduce the heterotic string results in the dual Type II picture.

It is even more remarkable that, in a variety of situations, perturbative string theory and duality have allowed us to guess/derive the exact non-perturbative formulation of the theory. We would have neither Matrix Theory nor the AdS/CFT correspondence if it were not for the tools of perturbative string theory.

These facts suggest a deeper role for perturbative string theory than I have allowed it in these ruminations. What it might be is beyond my comprehension. It seems unlikely to me that the answer is String Field Theory. Witten’s formulation of classical open bosonic string field theory is undoubtedly elegant[36]. But this classical theory misses a lot of gravitational physics that seems essential. In particular, it makes no distinctions between phenomena on D-branes of different co-dimension. It is clear that once gravitational interactions come into play, the corrections to classical open string field theory for D-branes of low codimension are large (generally infinite). More generally, the loop effects which give rise to gravitons in open string perturbation theory are singular. If the graphs are regularized in any known way, one finds that one must add a divergent series of corrections to the Lagrangian to reproduce perturbation theory to all orders (generally one has to add an explicit closed string field as well). So String Field theory does not give us a non-perturbative definition of a quantum theory.

It seems more likely to me that the elegant connection between classical string theory and the world sheet renormalization group is the place to search for a deeper connection
between the perturbative string formalism and a well defined non-perturbative theory of quantum gravity.

One avenue of research which, as a consequence of the considerations of this paper, seems very unlikely to lead to successful results, is the search for a perturbative string theory resolution of space-like cosmological or black hole singularities. This search was based on the notion that string theory was the correct high energy description of theories of quantum gravity. The breakdown of perturbative string theory in a wide range of the kinematic regime that we can call high energy (including both the tradition Regge and fixed angle regimes) suggests very strongly that it is not.

7. de Sitter Space: the importance of being finite

Asymptotically dS spaces, according to the conjecture of \cite{2,3}, are described quantum mechanically by finite dimensional Hilbert spaces. This fits in well with what we have learned from the AdS/CFT correspondence. We have learned that negative values of the cosmological constant are discrete parameters (partially) determining different theories of quantum gravity, and controlling the high energy density of states in the theory. Similarly, the finite Hilbert space conjecture identifies positive values of the cosmological constant as a discrete parameter, wholly or partially determining different theories of quantum gravity. In this case the cosmological constant determines an upper cutoff on the energy spectrum of the theory (in static coordinates), equal to the mass of the Nariai black hole.

A question that arises immediately is where representations of the dS group fit into such a story. There are several answers to this question, depending on what one is trying to accomplish. The simplest\cite{15} is to claim that since the global dS spacetime has no spatial boundary, all observables, and physical states are dS invariant. However, this does not take into account the fact that observation in physics always consists in separating the world into an experimental apparatus, and a system. A realistic measuring apparatus will follow a timelike trajectory in dS space and determine a static coordinate system. The cosmological horizon volume of this static system is, in $dS_d$, invariant under an $R \times SO(d-1)$ subgroup of $SO(1,d)$. $R^+$ is generated by the static Hamiltonian. This subgroup will act as a group of global symmetries on the quantum mechanics of this observer. Thus, by a choice of gauge, we introduce a boundary into the system, on which to define those generators of the dS group which preserve the gauge as global symmetry operators. Gomberoff and Teitelboim\cite{37} have given a rather explicit description of how this works for all Kerr-de
Sitter spacetimes. In an idealized eternal dS space, all other generators should be viewed, as advocated by Witten, as gauge transformations, which map one horizon volume into another. Different static observers are different gauge equivalent descriptions of the same physics.

In a universe which is only asymptotically dS in the future, we might be interested in these different gauge copies because the past dynamics of the system might set up different initial conditions in them. We might be interested in the fate of galaxies which we used to be able to see, but which have passed out of our horizon. Even in an eternal dS space, a nostalgic observer might want to learn something about the fate of a friend who had been foolish enough to let go of her hand, and found himself swept away by the Hubble flow.

It is extremely important to understand how the Poincare group emerges from dS space in the limit that the cosmological constant goes to zero. The discussion above indicates that it should not be thought of as the limit of the dS group, most of which consists of gauge transformations. Global symmetry generators arise in General Relativity, by imposing boundary conditions on hypersurfaces in spacetime. This is demonstrated quite elegantly in \[37\]. The natural hypersurface in dS space is the cosmological horizon of a given timelike observer. The generators which preserve this hypersurface form the $R \times SO(d-1)$ subgroup of the dS group that we have referred to above.

Near the future cosmological horizon, the metric of dS space takes the form

$$ds^2 = R^2 dudv + R^2 d\Omega_{d-2}^2.$$  \hspace{1cm} (7.1)

$v \to 0$ is the locus of the horizon. The $SO(d-1)$ invariance is manifest, while the static dS Hamiltonian is the infinitesimal boost of the coordinates $u$ and $v$. This should be contrasted with the metric of asymptotically flat spacetime near future null infinity.

$$ds^2 = \frac{dudv + d\Omega_{d-2}^2}{v^2}.$$ \hspace{1cm} (7.2)

Future null infinity is the conformal compactification of the $v \to 0$ limiting manifold, described by a conformal structure equal to that of the round sphere plus a single null coordinate, $u$ \[33\]. The full asymptotic symmetry group is the BMS group, consisting of vector fields of the form $f(\Omega)\partial_u$, semi-direct product with the conformal group of the sphere (also acting by conformal rescaling of $u$). This large group arises because relativists want to classify spaces with classical gravitational radiation in the initial and final states as asymptotically flat. In spacetime dimension higher than four, there is no infrared problem
for gravity, and a quantum S-matrix with finite numbers of particles and no classical radiation, exists. In these dimensions one can restrict attention to the Poincare subgroup of the BMS group (this why we have never seen the BMS group in perturbative string theory). In a conformal gauge in which the metric on the sphere at infinity is round, the Poincare subgroup is obtained by restricting $f$ to either $f^0 = 1$ or $f^i = n^i$, the unit vector on the sphere. These transformations obviously commute, and when account is taken of the conformal rescaling of $u$, it is easy to see that they transform as a $d$ vector under the Lorentz group (the conformal group of the sphere).

It is clear that the only remnant of the dS group, which carries over to the Poincare group, is the group of $SO(d - 2)$ rotations. The Minkowski translations and boosts arise only in the limit $R \to \infty$. One may be puzzled by the fact that the dS Hamiltonian is a symmetry generator which is there for all finite $R$, but seems to disappear in the limit. We will see below that in the limit, this Hamiltonian has a degenerate subspace which becomes infinite dimensional and of infinitely low energy. The space of states on which the Poincare generators act is orthogonal to all of these states and matrix elements of all reasonable measurements (for the asymptotic observer in asymptotically flat space) between these states and scattering states, go to zero. The upshot of this is that the static dS Hamiltonian does not act on the limiting Hilbert space of scattering states, which is the space on which the Poincare generators act. We will describe the physical basis for this mathematical behavior below, when we discuss measurement theory in dS space.

In asymptotically flat or AdS spaces, one can talk about idealized measurements on the boundary of spacetime, which have no effect on the system in the interior, or rather effects that can be very precisely encoded in the statement that there are a certain number of incoming and outgoing particles (using language appropriate to the flat case) of certain types. The acts of measuring these particles do not effect what has happened to them in the interior. In dS space, no such precise separation is possible.

The latter statement may seem peculiar to someone who is used to thinking about the global coordinates for dS space that are emphasized in [15] and [16]. Indeed, dS space has a boundary, past and future null infinity $\mathcal{I}_\pm$ which is conformal to two spheres. It is tempting to view data depending on a finite number of points on each of these spheres as an analog of the scattering matrix of asymptotically flat, or the boundary correlation functions of asymptotically AdS, space times. This would seem to imply an infinite number of states since, in a semiclassical approximation one can think of an infinite number of well separated “particles” propagating to the past or the future, with low energies. Witten
has suggested that this apparent contradiction with the finite number of states could be resolved if this infinite dimensional S-matrix was not a unitary operator, but a degenerate matrix of finite rank. He also emphasized that these quantities were “meta-observables” which could not be measured by any given observer.

I think that there is a much more subtle problem with this analysis, which has to do with our lack of knowledge of the phase space of classical gravity. The phase space of a general Lagrangian system is the space of solutions of its classical equations of motion, perhaps restricted by appropriate asymptotic boundary conditions in the case of field theory. As noted in previous sections, it is conventional to describe this in terms of the field variables and their first time derivatives at fixed time, invoking the Cauchy-Kowalevska theorem. Note that such a description seems to contradict any possible holographic interpretation of a field theory since we are presented with “one degree of freedom per space point”. This is in fact correct for non-gravitational theories, but I claim the analysis fails for theories including gravity. A first indication of this has been encountered by numerous people who have thought about the cosmology of compact universes. Invoking a Planck scale spatial cutoff, one would apparently be faced with a change of the number of degrees of freedom with time, since the dimensions of the universe expand or contract.

In fact, the Cauchy-Kowalevska analysis is only valid for some finite time interval and does not discuss the question of global, nonsingular solutions. For ordinary field theories the existence of singularities does not qualitatively change the number of solutions, but in General Relativity (assuming cosmic censorship) singularities correspond to the formation of black holes, and thus to drastic distortions in the geometry of spacetime itself. Given data on a spacelike slice, it is not easy to specify which solutions will evolve into black holes. This problem is ameliorated if we pose our boundary value problem on the boundary of asymptotically flat spacetime, and insist that it corresponds to finite numbers of particles coming in from (going out to) infinity. We have a rough idea, in terms of the kinematics of incoming and outgoing particles, of which configurations lead to black hole formation. Quantum mechanically, because of Hawking radiation, (and ignoring the infrared problem in four dimensions) even processes that classically form black holes are really scattering processes involving finite numbers of particles. So, in asymptotically flat spacetime, scattering data give us a good estimate of the number of classical solutions and therefore of the number of quantum states of the theory. Note that although this number is infinite, it is a surface infinity - a holographic counting of degrees of freedom.
In dS space on the other hand, I claim that the analysis at $I_\pm$ is misleading. If I send in some number of particles from $I_-$ and assume that they do not materially alter the dS geometry, then I can estimate their energy density in global coordinates, at the time the dS sphere shrinks to its minimal size. Obviously, the density becomes larger than Planck density for some finite number of particles, as long as I do not make the formal classical approximation of saying that each particle carries negligible energy, because its classical field is infinitesimal. In fact, long before this occurs the geometry will be distorted. It is likely that a typical asymptotic condition on $I_-$ leads to a solution with a Big Crunch singularity. Similarly, typical data on $I_+$ came from a Big Bang. If I try to put scattering data on both past and future, then generically there will be no sensible solution at all. More precisely, I believe that the phase space of gravity coupled to a generic set of physically sensible fields, with the boundary conditions that the solutions be smooth except for isolated singularities hidden behind black hole horizons, and the same asymptotic dS space in the past and future, is compact\(^{19}\).

I believe that in this manner, the nonlinear Einstein equations are trying to hint to us about the finiteness of the number of states in AsdS spacetimes\(^{20}\). A rigorous proof that the phase space of asymptotically dS solutions of Einstein’s equations is compact is, along with Cosmic Censorship for scattering solutions, an important problem in classical GR whose solution would lend more credence to the speculations in this paper.

There is another presentation of the semiclassical physics of dS space, the Euclidean functional integral, that makes the point in an even more striking manner. I presented this analysis in \cite{2} but it seems to have been completely ignored. Euclidean dS space is a sphere. In the formal\(^{21}\) Euclidean quantization one expands around the sphere. Rotations

\(^{19}\) Preliminary results to this effect were obtained in unpublished work of G.Horowitz and N.Itzhaki.

\(^{20}\) It is often said that the Bekenstein bound cannot be seen in a classical analysis because the Planck length goes to zero in the classical limit. In the above paragraph we have evaded this argument by talking about classical solutions that carry a finite amount of energy in the classical limit (what we called particles). In the standard classical limit, a particle’s Compton wavelength is kept fixed as $\hbar \to 0$, so its mass is taken to zero and we could have an infinite number of particles with finite energy. A purely classical statement with the same content, would refer to the compactness of the phase space with AsdS boundary conditions in both past and future.

\(^{21}\) In two space time dimensions Euclidean functional integral quantization of dS space is completely rigorous and gives the tree approximation to string theory. On the other hand, it has no interpretation in terms of local physics on the string world sheet. Polchinski\cite{38} has pointed out potential problems with the Euclidean formalism in dimensions $> 2$. term in the action.

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of the sphere are diffeomorphisms and one is instructed to mod out by them since the sphere is compact. The first correction to the classical saddle point defines free quantum field theory on the sphere. Analytically continuing this to Minkowski signature one obtains free quantum field theory on the static patch of dS space, with metric

$$ds^2 = -dt^2(1 - r^2/R^2) + \frac{dr^2}{(1 - r^2/R^2)} + r^2d\Omega^2,$$

(7.3)

in the thermal state of the static Hamiltonian. The dS group, which is the analytic continuation of the rotation group of the sphere, maps one static patch into another (the static Hamiltonian and rotation group of the \((d - 2)\) sphere leave a given patch invariant). The thermal correlation functions can then be viewed as obtained from a particular dS invariant Gaussian state of field theory on global dS space, by tracing over degrees of freedom outside the static patch.

However, the Euclidean path integral prescription instructs us to think of dS transformations as gauge transformations\(^{22}\). Thus the formulation of the theory in global coordinates contains an infinite number of gauge copies of the system in a static patch. The gauge fixed theory in a single static patch contains all of the physics of dS space. Note that, although, quantum field theory in the static patch has an infinite number of states, here the infinite entropy is associated with large energies, or infinitesimal regions near the horizon. For energies above the Planck scale, typical localized states in the static patch are black holes and are not well described by field theory. Thus, here it is easy to see that the infinity might be illusory. However, one is led to ask how it can be that the static patch contains all the physics of dS space, if the static coordinate system does not cover the entire manifold?

This question is of course reminiscent of the Black Hole Information Paradox. There, a static coordinate system, appropriate to an observer at infinity also gives rise to a thermal state. The region behind the horizon of a black hole is not gauge equivalent to the rest of the space. But the resolution of the Paradox proposed by 't Hooft \[^{39}\] and by Susskind and collaborators \[^{40}\] has a very similar flavor to our description of dS space. It goes under the name of Black Hole Complementarity, and consists of the claim that the descriptions

\(^{22}\) Naively, one is also instructed to treat the static Hamiltonian and rotation group as gauge transformations. However Gomberoff and Teitelboim have shown that if we consider dS space to be the zero mass limit of a dS black hole, then the Euclidean spacetime has a boundary and these generators are global symmetries on the boundary.
of physics by external and infalling observers utilize the same set of states but measure quantum mechanically complementary observables.

Fischler and I [41] provided a new rationale for this principle and connected it to the Problem of Time. We also generalized it to the case of cosmological horizons. The essential point is that the vector fields corresponding to time evolution as viewed by these two observers do not commute with each other. Thus, even semiclassically, the quantum theories describing the experience of observers related by a general coordinate transformation, use non-commuting time evolution operators. It is not surprising then that physics as viewed by one observer is not quantum mechanically compatible with physics as viewed by the other. Indeed, in discussions of the Problem of Time in canonical approaches to quantizing General Relativity, the idea that there may be many different Hamiltonians that describe the same physics is often discussed. The necessity for this point of view is evident even in the quantization of relativistic particles and strings, viewed as generally covariant systems in one and two dimensions. What is new in low curvature spacetimes with a horizon is the possibility of having two different \textit{semiclassical} descriptions of the same physical system, which are quantum mechanically complementary to each other. Following ’t Hooft and Susskind, this would be the correct quantum mechanical way to describe regions of spacetime which, in the classical approximation, lose causal contact with each other. Rather than being described by independent sets of commuting variables, they are non-commuting descriptions of the same Hilbert space, each of which separately has a semiclassical interpretation.

I would claim that the rules of Euclidean Path Integral quantization of dS space, as adumbrated above, give a semiclassical derivation of this complementarity (dubbed Cosmological Complementarity in [41]) principle for the case of a spacetime that is AsdS in both its past and future. That is, if we imagine a large radius dS space and different semiclassical observers in it, who are outside each other’s cosmological horizon, the rule tells us that each of them has a complete description of the physics of all the others. The different descriptions are gauge equivalent to each other via the global dS group.

This is not so interesting in an exact dS space where there is a symmetry relating local observations of two different static observers. Consider however a universe (like our own?) which began with a Big Bang and asymptotes to dS in the future. There will again be a multitude of static observers but now they are all different. For example, observers in our own galaxy and in some other galaxy, (say the Sombrero galaxy, ) not gravitationally bound to us, will, if there is a nonzero cosmological constant, eventually be outside each
other’s horizon. Observers in our galaxy might well ask where, in their description of the world, information about the evolution of the Sombrero galaxy is encoded. There is an obvious answer to this question, suggested by the earliest studies of black hole physics.

From the point of view of a static observer, nothing ever really goes through the horizon. That is, the entire region of spacetime covered by his coordinates ends at the horizon. Instead, as something approaches the horizon it gets squashed into a smaller and smaller spacelike region, but never quite disappears. Thus, in such a coordinate system, it is natural to associate things that go through the horizon with states localized on the horizon.

It is now important to understand how the number of states localized on the cosmological horizon of a particular observer compares to the number that this observer views as localized in the bulk of spacetime that he can explore. The key to understanding this is the Bekenstein-Hawking bound on the entropy of localized systems. The largest entropy one can fit into a region of spacetime is the entropy of a black hole whose horizon is the boundary surrounding that region. More properly stated: when localized energy density is concentrated in a region, if its entropy is large enough it must form a black hole with the requisite radius.

It is well known that the Nariai solution is the maximal black hole that fits into dS space, and that its entropy is only two thirds of the dS entropy. Thus, for a large radius dS space, the overwhelming majority of quantum states must be viewed by any given observer as being localized on her cosmological horizon, rather than on systems localized within the dS bulk. The Nariai solution represents a very special class of excitations, in which most of the degrees of freedom of dS space are frozen and the system explores only a small number of its available states. By contrast, the dS vacuum (the thermal Gibbons-Hawking state of the static observer) and small excitations of it, have a much larger entropy. This counting of states seems peculiar to the local static observer, who in trying to construct a Nariai black hole, follows her local rule for maximizing entropy. The resolution of this tension is the realization that local degrees of freedom are “stolen” from the horizon. If we make a small local excitation we do not substantially alter the bulk of the degrees of freedom, but the Nariai excitation can only be achieved in a very special class of states.

For example, if the interpretation of cosmological data in terms of a cosmological constant is correct then the dS entropy of the universe is larger by a factor of $10^{25}$ than the entropy of everything we see as localized excitations (including hypothetical supermassive black holes in the centers of all large galaxies). Thus, the classical picture that dS space
has an infinitely larger set of degrees of freedom than what can be seen in a given horizon, becomes correct in the limit of infinite dS radius, if we restrict attention to those states that the static observer views as being localized in the bulk.

The answer to the question: “What goes on in the Sombrero galaxy after it has passed through our horizon?” is encoded in the microscopic quantum state of the cosmological horizon. This viewpoint also relieves a certain amount of unease that might be caused by applying the ideas of Black Hole Complementarity to the universe. Proponents of complementarity often claim that observation of the Hawking radiation from a black hole puts the system in a quantum state which has no classical interpretation for an infalling observer. In the black hole context this is palatable because the infalling observer eventually gets crushed in the singularity, and there is no compatible notion of simultaneity for the two observers. However, we would be disturbed to find that our measurements could destroy the semiclassical coherence of observers in the Sombrero galaxy. This paradox is resolved because we do not have the possibility of constructing an apparatus that can measure such a huge number of states. If we tried to do so we would collapse into a black hole. Even if we imagine being able to measure things by using the microstates of black holes to construct the measuring apparatus, the maximal dS black hole has only one third of the entropy of dS space, so for large dS radius it cannot measure the state of most of the degrees of freedom we would like to assign to localized systems outside the horizon.

If we restrict attention to well understood measuring devices, there is a sense in which the global coordinate picture of many commuting sets of degrees of freedom is valid in the proper quantum theory of dS space. Let us use the phrase “field theoretic” to refer to states inside each horizon volume which are well described by quantum field theory. In particular we do not allow black holes whose size scales to infinity with the dS radius. To

23 In “nice slice” coordinates, which try to describe both the interior and exterior on a surface of simultaneity, the internal observer has to make measurements with super Planckian time resolution on the slices where the external observer has absorbed a significant fraction of the Hawking radiation. Because of the UV/IR connection she can no longer be considered a local observer. This means that “nice slice” coordinates do not really exist over time scales comparable to the time it takes for the external observer to extract information from the black hole. The internal part of a nice slice at these late times, is never well described by local physics.

24 Indeed, in dS space, a perfectly nonsingular “Nice Slice ”, the global coordinate system, exists. Physics described in this system must be gauge equivalent to the static patch physics. The global observer sees nothing happening to the Sombrero galaxy as a result of our measurements.
describe a three dimensional region of size $R$ in terms of field theory, we must insist on a UV cutoff such that the typical state (which in field theory means states near the UV cutoff) has Schwarzschild radius less than $R$. Thus, in Planck units, $M^4 R^3 < R$. The field theory entropy is of order $M^3 R^3 \sim R^{3/2}$. In four spacetime dimensions, the total entropy of dS space indicates the possibility of constructing of order $R^{1/2}$ mutually commuting sets of “field theoretic” degrees of freedom, each of which could describe the field theoretic states in a given horizon volume. Thus, in the field theory approximation, i.e. restricting attention to only states well described by local field theory (but allowing black holes with radii much smaller than the dS radius), the picture of a global dS space is approximately valid unless I try to study correlations between more that $R^{1/2}$ disjoint horizon volumes (which I can only do in the far past or future).

On the other hand, if I construct states with horizon scale black holes in a single horizon volume then it is no longer possible to speak of many other independent commuting degrees of freedom. The horizon size black hole carries a finite fraction of the total number of degrees of freedom in the system. The field theory description of an asymptotically infinite dS space is not far wrong for large $R$, as long as we restrict attention to low energy processes. It is only when a given observer begins to construct black holes of order his horizon size that he begins to have a significant effect on the local physics in other horizon volumes. If we are not too ambitious, we won’t have to worry about our erstwhile friends in the Sombrero galaxy.

It would be very interesting to put some more mathematical detail on these arguments by studying multiple black hole solutions in dS space. Indeed, even a global coordinate description of single black hole solutions would illuminate these points. As far as I know, no such formulae have appeared in the literature.

In the last few paragraphs, we have touched on the issue of measurement theory in dS space, which was discussed in [42]. Measuring devices in dS space must be large classical devices, approximately describable by local field theory, and thus much smaller than the dS horizon size. In the large $R$ limit, there are two classes of interesting measuring devices in a given horizon volume: free falling devices, and devices bound to the measured system at the origin. The dS Hamiltonian is the appropriate description of physics as measured by the latter class of devices. In the large $R$ limit, the collection of measurements made by all freely falling devices, far away from the origin but long before they fall through the horizon, becomes the Scattering Matrix of the limiting Minkowski space (we are talking here of an eternal dS space with both a past and future cosmological horizon, not an AsdS.
space which arose from a Big Bang.). The approximate Poincare generators, whose algebra converges to the Poincare algebra as $R \to \infty$ are symmetries that act on the measurements made by the free falling observers.

7.1. Entropy and the number of states

In a quantum system, the entropy of a given density matrix is $-\text{Tr} \rho \ln \rho$. Up to this point, we have been conflating the idea of entropy with the logarithm of the number of states. This is only valid for the completely uncertain density matrix on a finite dimensional Hilbert space. One can have finite entropy in an infinite system. The additional semiclassical input that we need to prove finiteness of the number of states in dS space, is the fact that the density matrix is thermal, and that there is an upper bound on the energy spectrum, given by the mass of the Nariai black hole. These two facts, combined with finite entropy, tell us that the number of states is finite, but not that it is equal to the exponential of the dS entropy.

There are two clues which help us to understand the origin of the dS temperature. The entropy of localized excitations of finite energy is bounded by that of black holes and is, for large $R$, much smaller than the total dS entropy. The classical energy of empty dS space is zero (see e.g. [37]). This indicates that the empty dS entropy should be thought of as coming from states of zero classical energy, which are the static observer’s view of the world behind her horizon.

I propose that in the quantum theory, the spectrum of the Hamiltonian at low energies is that of a random $N \times N$ Hermitian matrix $H_V$ with an energy cutoff $\Delta$ of order the dS temperature. A possible way of implementing the cutoff might be to choose $H_V$ from the Gaussian ensemble with covariance $\Delta$. The thermal density matrix $e^{-\beta_{dS} H_V}$ is very close to the normalized projection operator on this $N$ dimensional subspace of low energy states, so that its entropy is nearly the same as $\ln N$. It is in this sense that the dS entropy is actually a count of the number of states in the quantum theory. States with energies higher than the dS temperature have considerably less entropy, both because they are fewer in number, and because they are Boltzmann suppressed in the thermal ensemble. The full Hamiltonian for quantum dS space will have the form $H = H_V + H_I + H_{loc}$, where $H_{loc}$ gives approximate eigenstates for localized objects in a single horizon volume and $H_I$ represents interactions between the vacuum ensemble of states and the localized states. One would like to show that these interactions result in a thermalization of the localized
degrees of freedom, at the dS temperature. Thus the origin of the dS temperature will be
the dense set of eigenstates whose dynamics is approximately governed by $H_V$.

To summarize: dS spacetime should be described in quantum theory by a system
with a finite number of quantum states. The positive cosmological constant is a discrete,
tunable parameter, closely related to the logarithm of the number of states. Only of order $R^{3/2}$ of the degrees of freedom can be viewed as local excitations in a given horizon volume.
This indicates that the global coordinate picture of many commuting degrees of freedom is
approximately correct for large $R$. That is, we can rigorously talk of order $R^{1/2}$ commuting
sets of degrees of freedom, each of which describes field theory in a single horizon volume
(including small black holes). If the observer in a single horizon volume tries to make a
horizon scale black hole, this picture is no longer valid and most of the degrees of freedom
in other horizon volumes are also frozen into large black hole configurations. For a given
observer, the Hamiltonian of dS space contains a dense and chaotic spectrum of energy
levels below the dS temperature. These represent the observer’s view of things outside the
horizon. In addition there are localized excitations, the simplest of which are large black
holes. These have finite energy, given by the mass parameter in the black hole solution (for
zero angular momentum). The entropy of dS space is the thermal entropy of this system,
but for the states with no black holes, it is very closely approximated by the logarithm of
the number of states below the dS temperature.

In fact, the black hole states are not really eigenstates of the system, except in some
approximate sense. Kerr-dS black holes evaporate, and most of their decay products fall
through the horizon. The end product of the decay might be a stable massive remnant
or nothing. In either case the classical approximation to the Hamiltonian \[37\] in which
there are very high energy stable eigenstates, must be modified by interactions between the
degrees of freedom associated with the black hole, and those associated with the horizon,
in such a way that the only exact eigenstates of the Hamiltonian correspond to the dS
vacuum and a small number of stable “particle” excitations of it.

A more detailed attempt to construct a toy model of dS quantum mechanics was
described in my talks at the Davis Inflation Conference\[43\]

7.2. Physics, metaphysics, and mathematics of a quantum dS universe

The concept of measurement is essential to all discussions of physics. Until very re-
cently physicists thought of their endeavours as the description of isolated systems. Exper-
imenters performed measurements on these systems from the outside and theorists wrote
mathematical formulae which were supposed to predict the results of those measurements.
With the advent of quantum mechanics we have had to pay much more attention to the concept of measurement, in order to account for the robust and apparently deterministic nature of measurements in a world where we believed that all physical systems were subject to the intrinsically probabilistic laws of the quantum theory. Starting from the work of Von Neumann it has been argued that the nature of a measurement in quantum mechanics is correlation of some complete orthonormal set of basis states $|s\rangle$ of the system with “pointer states” $|P_s\rangle$ of the apparatus. That is, unitary evolution is supposed to take an initially uncorrelated stated of the combined system, $\sum a_s |s\rangle |N\rangle$, into $\sum a_s |s\rangle |P_s\rangle$. One then tries to argue that further measurements of system observables in this correlated state will reproduce their expectation values in the system density matrix $\rho = \sum |a_s|^2 |s\rangle \langle s|$. That is, after the measurement, as long as it remains in interaction with the measuring apparatus, the system will obey the laws of classical probability theory with sample space given by the particular orthonormal basis which has been measured by the apparatus, and probability distribution given by the square of its initial wave function.

There has been much discussion of the necessity of including interactions with a large, random, unmeasured environment to explain why particular pointer states of the apparatus lead to decoherence in this manner. Without quarreling with those discussions and their applicability to realistic measurements, I would like to suggest that environmental decoherence is not a logically necessary component of quantum measurement theory. Large systems with local interactions (i.e. quantum field theories with infrared and (perhaps) ultraviolet cutoffs) provide examples of systems in which decoherence can occur without a stochastic environment. Consider a spin one half particle, and an apparatus enclosed in a volume $V$. The apparatus has two pointer states which are to be correlated with the $\sigma_3$ eigenstates of the particle when the two systems come into contact. We can make a mathematical model of such an apparatus as a cutoff $\phi^4$ field theory in the volume $V$, with a potential with two degenerate minima. We postulate a non-local coupling $\sigma_3 \int \phi(x) \chi_V(x_P)$ between the two systems. Here $x_P$ is the particle position and $\chi_V$ is the characteristic function of the volume $V$. This non-local interaction is a cartoon of the amplification system that is required to correlate the state of a quantum spin with a macroscopic pointer.

Once this correlation has been established, it is very robust. Further operations that can be modeled by the action of more or less local operators in the field theory will not detect interference between the two pieces of the wave function. In the limit $V \to \infty$ we have an exact decomposition of the Hilbert space into superselection sectors, which never communicate. Thus, when $V$ is large in microscopic units, we can say that an
almost classical measurement has been made. Only tunneling effects, of order $e^{-V}$, are sensitive to the coherent phases in the correlated wave function. Thus, a system is a good measuring device, if the quantum fluctuations of its pointer observables are analogous to those of a vacuum order parameter in quantum field theory. The tunneling time for fluctuations between different pointer positions is of order the inverse of the number of states associated with the pointer.

In de Sitter space, there is a bound on the size of a system that can be described by local field theory. The tunneling time for the largest possible field theoretic machine is of order $e^{R^{3/2}}$, much shorter than the recurrence time in the dS space\[45\]. Even if we tried to go beyond the well understood realm of field theoretic machines, and imagined that we could use localized black hole eigenstates to construct classical measuring devices we would still find that the tunneling time between pointer states of such a machine was much shorter than the recurrence time, because the maximal black hole entropy is much smaller than the entropy of empty dS space. Thus predictions about phenomena on time scales as long as the recurrence time, have no operational meaning.

More importantly, we learn that many details of the mathematical quantum theory of de Sitter space, are in principle unobservable. This means that there will be many mathematical theories that have the same consequences for all observations, within the bounds of precision that are allowed by the above arguments. One should thus view the quantum theory of dS space as a universality class of theories, describing the critical limit $\Lambda \rightarrow 0$. Our considerations of measurement theory suggest that all members of the universality class should give results for experiments which do not produce black holes of order the horizon scale, which are in agreement to all orders in powers of $\Lambda$. For such low energy processes, the restrictions on measurements lead to exponentially small inaccuracies as the cosmological constant vanishes.

Having established the nature of realistic measurements in a putative finite dimensional quantum theory of dS space, and the consequent ambiguity in the mathematical description of this theory, let us turn to a vexing metaphysical problem raised by this proposal. \emph{If the number of states is finite, what determines it?}

There are I think, two possible responses to this question, which I would call the Anthropic answer, and the Pythagorean answer. Actually, the Pythagorean answer will be seen to require a very weak form of the anthropic principle as well. The Anthropic answer invokes the results of Weinberg \[16\] to claim that a value of the (positive) cosmological constant larger than what is observed would lead to a universe devoid of galaxies and
thus (presumably) of living organisms of any kind. Actually the galaxy bound exceeds the observed value by a factor of order 100 and one must resort to arguments about what a “typical” universe obeying the bound would look like. Intrinsic to any such discussion is an \textit{a priori} notion of what the ensemble of possible values of $\Lambda$ is and what the probability density on this ensemble looks like. The arguments of Weinberg assume a fairly flat density in the vicinity of $\Lambda = 0$. If on the other hand, we associate $\Lambda$ with the number of states, then the small $\Lambda$ region is the region with a large number of states and a flat probability density near vanishing cosmological constant is assuming a cutoff on the number of states. This does not appear reasonable, and puts in a scale by hand.

Cosmological SUSY breaking can help to solve this problem. It implies that systems with a large number of states become more and more SUSic. In the limit of small gravitino mass, atoms and nuclei can decay to a bose condensed ground state by gravitino and photino emission. Life is impossible in a very SUSic universe. Note that, although this argument uses life of our type as its basis, it may still be a weak anthropic argument. It is possible that the constraints\textsuperscript{I7} on the limiting SUSic vacuum of dS space are so strong that there is only one solution and it predicts the low energy supersymmetric standard model with all of its parameters. Thus, purely mathematical arguments might lead to the unique conclusion that nuclear physics and chemistry are as they are in the real world, whenever the cosmological constant is small. This would determine the possible types of life. The precise value of the cosmological constant would then be predictable only by anthropic arguments.

Given the latter assumption we can get even stronger lower bounds on $\Lambda$ by combining CSB, the anthropic principle, and the assumption that the weak scale is determined by the scale of SUSY breaking. If $\Lambda$ is too small then the weak scale will be so small that the dominant contribution to the proton neutron mass difference will be electromagnetic.\textsuperscript{25} Protons will decay rapidly due to the relatively strong weak interactions, and there will be no atoms or heavy nuclei. Dimopoulos and Thomas \textsuperscript{48} estimate that the weak scale can be no lower than a factor of three smaller than its actual value, to prevent this disaster. According to CSB, the weak scale vanishes like $\Lambda^{1/8}$, so we have an anthropic lower bound on $\Lambda$ which is about a factor $10^{-5}$ smaller than its “real” value. Because of the weak power law dependence of $M_W$ on $\Lambda$ it seems unlikely that refinements of these arguments could produce a really tight lower bound on $\Lambda$.

\textsuperscript{25} This argument about the effect of lowering the weak scale is due to Dimopoulos and Thomas.
A possible way to do better would be to take up the suggestion I made in [47] that some of the small parameters in the quark mass matrix could be functions of Λ as well. The motivation for this is that the discrete $R$ symmetry which guarantees Poincare invariance of the limiting SUSic theory, and which is broken by interactions with the horizon states in dS space, might be related to the discrete flavor symmetries which constrain the quark masses. If this were the case for the up and down quark masses, then the proton neutron mass difference would be a more rapidly varying function of Λ. If we assume the entire ratio of the up to top quark masses is due to a power of Λ, then the proton neutron mass difference scales like $Λ^{1/6}$ and the anthropic lower bound on Λ is about $10^{-3}$ smaller than its real value.

As an aside, I should mention that although the anthropic determination of Λ is often considered a great success for the anthropic principle, there are a large number of hidden assumptions in such a statement. We have already seen that the apparently innocuous assumption of a flat probability distribution near $Λ = 0$ does not make much sense if we think of Λ as a parameter controlling the number of states in the quantum theory. We had to use the additional assumption that any $Λ = 0$ theory was exactly SUSic to make any use of the anthropic principle with the more plausible assumption of a uniform probability distribution on the number of states. Even then we had to use the hypothesis of CSB and even more speculative assumptions about dependence of low energy parameters on Λ to get a reasonably tight lower bound. Furthermore, Weinberg’s upper bound on Λ assumes that the dark matter density at the beginning of inflation, and the amplitude of primordial density fluctuations are fixed to there values in the real world. In most attempts to motivate anthropic arguments by assuming an elaborate potential energy landscape and the ability to jump between local minima (a picture that is on very shaky grounds if one believes the arguments of this paper), both of these parameters would be expected to fluctuate randomly. The anthropic prediction for the central values in this multidimensional parameter space is not very impressive. Thus, one really needs a theory like CSB where only the cosmological constant is allowed to vary, to claim that the anthropic determination of Λ is successful.

Perhaps the most attractive feature of the anthropic argument is that it does not require us to know much about the Meta-theory, which determines the probability distribution of the cosmological constant. One requires only that such a theory exists and that the probability distribution in the vicinity of the anthropic bound is nonzero, and reasonably smooth. The lack of dependence on details of the Meta-theory is important,
because it is unlikely that any of those details could be checked by experiment. If we needed to understand an elaborate mathematical theory, most of whose structure could never be tested, in order to believe in the anthropic bound, then that bound would appear much less plausible.

The Pythagorean answer to the question of the number of states, is an attempt to build a Meta-theory using number theoretic concepts. One imagines a “universe machine”, which (in a time which has nothing to do with any time coordinate in our universe) spits out some number \( n \) of commuting Pauli spin operators (or some other elementary quantum system like the spinor variables of the holographic cosmology described below) and allows them to interact according to the rules of quantum asymptotically de Sitter space-time. \( n \) is to be chosen by some number theoretic criterion, built into the (hypothetically elegant) structure of the universe machine. An ideal Pythagorean solution would find \( n \) to be uniquely defined by some simple criterion. For example, if Fermat’s theorem were false, and had a unique but huge counterexample, it might have fit the bill. What is required is a number theoretic problem that has a unique solution, which happened to be the value of \( n \) that fits the cosmological constant. If such a problem could be found, we might believe in the theory even if no other experimental checks of the mechanism behind the universe machine could be done.

A more plausible construction might rely on a number theoretic problem that had sparse solutions. As an example, we might consider the requirement that \( n \) be a Mersenne prime. These are primes of the form \( 2^k - 1 \), and there are only 39 of them known. The largest has \( k \) of order \( 13.5 \times 10^6 \). The resulting value of \( n \) is much too large to fit the observed value of the cosmological constant. Indeed, there are no Mersenne primes which fit the right value, all giving answers which are much too large or much too small.\(^{26}\) However, one can imagine a similar problem in number theory, which, for some value of \( k \) hit the cosmological constant on the nose, and missed by a huge margin for all other values of \( k \). We could then used this elegant number theoretic machinery to construct a much more satisfying version of the anthropic argument. The \textit{a priori} probability distribution for the cosmological constant would have point support and all but one of the points would violate the anthropic upper and lower bounds that we have described, by large amounts. Thus, a Pythagorean choice of \( N \) would ultimately depend on anthropic arguments, but would be much more compelling than the argument of Weinberg. There would be a very

\(^{26}\) The two closest Mersenne primes predict a cosmological constant of order \( 10^{-157} \) or \( 10^{-38} \).
sparse set of choices for this fundamental integer and only one of them would be compatible with a very weak version of the anthropic principle.

I must admit to a great deal of unease in talking about these arguments. Consider the following model of a Meta-theory: A supreme being plays dice with himself, and on the basis of each throw, decides to construct a universe with a finite number of quantum states obeying the famous, yet to be constructed, rules for quantum cosmology in such a universe. Only the number of spins $n$ is decided by the throw of the dice. We then apply the anthropic argument. As theoretical physicists, we would certainly find an elegant mathematical model of a Meta-theory more satisfying than the supreme being model, but our inability to perform experiments for the values of $n$ that are ruled out by the anthropic argument, leaves us with no experimental proof that the supreme being model is any less right than the mathematical one. We must ask ourselves whether we are really doing science. So must anyone who indulges in anthropic speculation.

8. Supersymmetry

8.1. Breaking SUSY on the horizon

It is clear that dS space violates SUSY. There is a dS analytic continuation of the AdS SUSY algebra, but it has no unitary representations and is not compatible with quantum mechanics. The basic problem is that the dS group has no highest weight generators (it is isomorphic to the Lorentz group in an appropriate number of dimensions) and so no bosonic generator can be written as a positive product of supercharges. Furthermore, if we believe the arguments above, the dS group itself should be viewed as a group of gauge transformations, with the coset of the static subgroup not acting on the Hilbert space of states of a given static observer. The question is, by how much does dS space with a given cosmological constant violate SUSY?

This question touches on a nastier one, namely what are the precisely defined mathematical observables in dS space, or are there any at all? I have discussed this briefly in a previous section, but it is beside the main point. Whatever the precise definition of observables in AsdS spaces, it must be true that there is some approximate notion of low energy physics described by an effective Lagrangian. In this context, breaking of SUSY can always be described as spontaneous, as long as the gravitino mass is much smaller than the Planck scale. The hypothesis of CSB guarantees that this is so, for it links the SUSY breaking scale to a positive power of the cosmological constant, which, according
to the hypothesis, is a tunable parameter. Furthermore, since the limiting theory, with vanishing cosmological constant, is supersymmetric, the goldstino must be part of a linear supermultiplet, and SUSY breaking must be described by some standard (or novel) low energy mechanism. The novelty of the current approach is that one is led to accept the existence of what appear to be fine tuned relevant parameters in the low energy effective Lagrangian. For someone like myself, who has spent a good part of his career in physics searching for dynamical explanations of mass hierarchies, this seems like a revolting and reactionary approach to the problem.

The crucial point however is that the dynamics which explains these finely tuned numbers is, according to CSB, a new critical dynamics of the large set of degrees of freedom that become available as the cosmological constant goes to zero. In thinking about quantum gravity, we have grown used to invoking various kinds of infrared critical behavior (asymptotic freedom and nontrivial fixed points). We have imagined that since the theory contains an apparent UV cutoff scale, the Planck mass, that there was no problem of an ultraviolet infinity of degrees of freedom. I have stated above, that this is wrong. In fact, the UV/IR connection, as manifested in the black hole spectrum in asymptotically flat space suggests that the theory with vanishing cosmological constant indeed has a UV critical problem, of a type never encountered before. This is a critical problem to which the standard paradigm of Lorentz invariant conformal field theory (in space time) simply does not apply. In [2] I invoked this new critical phenomenon as an explanation for the change of the exponent, $\alpha$, in the relation, $m_{3/2} \sim \Lambda^\alpha$ (Planck units), between the gravitino mass and cosmological constant. This led me to suggest virtual black holes as the mechanism which renormalizes the critical exponent.

I no longer believer this argument. Although it is easy to argue for unsuppressed virtual black hole production in the high energy parts of Feynman graphs, the probability of tying all the black hole decay products back into a single particle (in order to renormalize the gravitino mass) seems very small. More importantly, I realized that, as discussed above, black holes seen by a static observer account for only a tiny fraction of the states of dS space. Indeed, there is another, peculiar, IR critical phenomenon that occurs as the cosmological constant goes to zero. In the holographic picture of dS space, the states on the cosmological horizon must be not only degenerate, but have very small eigenvalues of the static Hamiltonian. This is required, in order for them to participate in thermodynamics at the (very low) Hawking temperature. In my current view, it is these states that are responsible for the relatively large renormalization of the gravitino mass.
The classical low energy Lagrangian for SUGRA coupled to chiral and abelian vector superfields has a potential of the form

$$V = e^K [F_i \bar{F}_j K^{ij} - 3|W|^2] + D_a^2,$$

where $K$ is the Kahler potential and the $F$ and $D$ terms have their usual expressions in terms of the chiral fields. If the cosmological constant is tuned to zero, the gravitino mass (interpreted as the mass of a scattering state in asymptotically flat space) is given by $\sqrt{e^K |F|^2}$, where we have introduced a shorthand for the norm squared of the one form $F_i dz^i$ in the Kahler metric. If we turn on a positive cosmological constant this term no longer has the same meaning, but it still serves as the coefficient of the nonderivative quadratic term in the gravitino Lagrangian. We will take it as our estimate for the size of SUSY breaking. It is clear that this Lagrangian has no a priori connection between the size of SUSY breaking and the cosmological constant. However, a cosmological constant, $\Lambda$ much smaller than the SUSY breaking scale is achieved only by subtracting two terms, each of which is many orders of magnitude larger than $\Lambda$. Quantum corrections in field theory seem to restore the problem solved by the classical fine tuning.

A number of authors[49] have suggested that these calculations are faulty because many of the intermediate states used in these calculations suffer large gravitational distortions. In particular, Cohen, Kaplan, and Nelson propose a calculational procedure in which the radiative corrections to the cosmological constant are consistent with observational bounds, and normal particle physics calculations are affected at a level below, but in some cases close to, current experimental precision. Thomas has proposed a different way of modifying the results of QFT calculations. In contrast to CKN he uses only the holographic bound on states, and does not predict such large corrections to other low energy calculations.

These calculations show how plausible holographic constraints on the field theory formalism can remove the technical problem of loop corrections to the fine tuning, but the problem is deeper than that. The effective Lagrangian formalism is more background independent than we have argued the fundamental quantum theory of gravity has any right to be. It treats asymptotically flat, dS and AdS spaces as part of the same theory and the cosmological constant as a calculable parameter. Quantum mechanically, these systems have very different structure: dS space has a finite number of states, while the infinite AF and AdS Hilbert spaces have a radically different behavior of the high energy density of states.
The cosmological constant is always a regulator of the growth of the number of states at high energy. It is a strict UV cutoff in dS space and a crossover scale between superexponential and subexponential growth of the density of states in AdS space (with the AF case viewed as the limit where this scale goes to infinity). As such it seems like a fundamental parameter of the theory rather than a parameter which can suffer renormalization. Indeed, as I have emphasized, in AdS/CFT the cosmological constant in Planck units is identified with a power of \( N \), a fundamental integer characterizing the quantum theory, rather than an effective parameter.

From this point of view, the fine tuning of the cosmological constant in the effective theory seems to be merely a way of putting this fundamental piece of high energy information into the theory. It is not that different than imposing a symmetry whose justification comes from the high energy theory. What about the relation between the cosmological constant and the SUSY breaking scale?

Our discussion of the structure of Hilbert space in a dS spacetime leads us to consider contributions to the renormalization of the gravitino mass coming from diagrams like those of Fig. 1, in which the internal gravitino line propagates out to the cosmological horizon, and back. The external gravitino lines are localized in a small, approximately flat region of spacetime whose scale is somewhat bigger than the gravitino Compton wavelength.

In propagating out to the horizon, the gravitino line enters what, for the static Hamiltonian, is a region of infinitely high temperature. It is able to interact with the mysterious horizon states. While amplitudes like these are suppressed by \( e^{-cm^2/2R} \) due to virtual propagation over a large spacelike distance, there is a potential contribution from interaction with of order \( e^{R^2M_P^2} \) near horizon states. Until we understand how to compute these interactions, we cannot claim that these renormalization effects are small, or estimate how they depend on \( R \).

In [47], I argued that the only sort of conventional low energy SUSY breaking mechanism consistent with CSB and some rudimentary standard model phenomenology was one in which a combination of F and D term constraints for fields charged under a new \( U(1) \) gauge theory with Fayet-Iliopoulos (FI) term, leads to a vacuum with spontaneously broken SUSY. It remains to be seen whether a phenomenologically viable model of this type can be found. In the meantime I have discovered another class of models, based on dynamical SUSY breaking, that also seems to be consistent with CSB.

Since my understanding of the phenomenological implications of CSB is much less complete than I had originally thought, I will not review this work here. However, there
are a few points that are essential to the following argument. Despite the fact that dS space breaks SUSY, at the level of the low energy effective Lagrangian this breaking must appear spontaneous, unless the scale of the gravitino mass is above the Planck scale. This follows from the necessity of SUSY Ward identities to any low energy theory of gravitinos interacting with gravity. Since SUSY is local, we can always make its breaking look spontaneous, by introducing a nonlinear Goldstino field. Moreover, by assumption we have a one parameter set of theories with a SUSic limit. Near the limit, the scale of SUSY breaking goes to zero, and the Goldstino must actually fit into a linear SUSY multiplet. Finally, to naturally assure a zero cosmological constant limit, we invoke a discrete complex R symmetry of the limiting theory.

The discrete R symmetry is explicitly broken by terms that vanish with $\Lambda$. Thus, we are led to postulate an R symmetric low energy Lagrangian with a Poincare invariant, SUSic, vacuum state. When explicit R breaking terms (including a constant in the superpotential that we can use to fine tune $\Lambda$) are added, the model must spontaneously break SUSY.

Our goal is to estimate the size of the R breaking terms that are induced by interactions with the cosmological horizon. These are given by Feynman diagrams with vertices localized near a given static observer, and lines carrying R-charge out to the horizon. The gravitino always carries R-charge, and in most models will be the lightest R charged particle. We will see that graphs with gravitino lines going to the horizon are the important ones. Consider a graph with one such line (Fig. 1). As noted above, it will contain a factor $e^{-m_{3/2}R}$, where $R$ is the spacelike distance to the horizon.

There is a set of arguments that recovers the relation $m_{3/2} = M_P(\Lambda/M_P^4)^{1/4}$ from such configurations. First we must assume, based on the area scaling of horizon entropy, that we can view the states on the horizon as distributed uniformly over it. Then the interaction of some localized particle with the horizon, would be only with those states concentrated in the area of the horizon explored by the particle. A particle of mass $m$ can move a proper distance of order $1/m$ along a null surface like a horizon. For longer proper distances it must be considered to follow a timelike trajectory and cannot stay in contact with the horizon. However, from the point of view of a particle which must return to the observer at the origin (as must be the case for the virtual line in Fig. 1) the horizon is a very hot place, and the particle undergoes strong interactions as it moves around near the horizon. In particular, it’s position is subjected to a random kick every time it moves a Planck distance. So its motion should be viewed as a random walk, and in a proper distance $1/m$ it moves only a distance $\sqrt{1/m}$ from its starting point. Thus, it explores an
area \sim \frac{1}{m^{27}}. Note that these random kicks only move the particle within the horizon. In the static frame there is a huge inertial potential energy which pins it to the horizon. And if it needs any repetition: in the holographic description there is no place outside the horizon.

To understand the order of magnitude of the correction we have to say something about a model of physics at the horizon. The area scaling of entropy might suggest a cutoff field theory model, a sort of quantization of the fields of the membrane paradigm. This I believe to be wrong. Consider for example a cutoff field theory model of a black hole horizon. Along with an area’s worth of entropy, it would predict an area’s worth of energy density. But a black hole’s area scales with a power of the energy greater than one, so this is inconsistent. In a field theory, the splittings between horizon states would scale like an inverse power of the area. Instead, we expect the splittings between levels of a black hole to be exponentially small in the area (the inverse of the spectral density) rather than the power laws that would be predicted by a field theory.

A better model can be constructed in 4 spacetime dimensions (which may be the only place we need it for dS space). The horizon is a two sphere. Consider free nonrelativistic fermions propagating on this two sphere, in the presence of the background magnetic field of a monopole at the center of the sphere. The fermions are doublets of an $SU(2)$ isospin symmetry and their isospin operators do not appear in the Hamiltonian. Take the monopole charge to be large, and consider a completely filled lowest Landau level. The isospin degeneracy gives a number of degenerate states exponential in the area of the sphere, in units of the quantized Larmor area, which should evidently be of order the Planck area. This will be our model of the degenerate horizon states.

It is well known that there are linear combinations of single particle states in the first Landau level which are approximately localized within a quantized Larmor area. Particles in the bulk of dS space will be assumed to interact with these horizon states via a localized function of the difference between the particle coordinate on the sphere and the fermion guiding center coordinates. The localization length of this function is of order the Planck length. The interaction is again independent of the fermion isospin operators.

With this model of horizon states and our description of the random walk of bulk gravitinos on the sphere, it is clear that the gravitino interacts with of order $e^{1/m}$ states. In this simple model, the interaction is insensitive to the degeneracy and so we sum coherently

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$^{27}$ These estimates are done in four dimensions. The general exponent is $A \sim m^{1-d/2}$. 

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over the degenerate states. More generally, if the contributions of a finite fraction\footnote{Finite means finite as $R \to \infty$. Actually, we want the fraction to vanish as a power of $R$, but this is a subleading correction to the exponential terms we are studying.} of these states to the renormalization of the effective Lagrangian adds coherently, then we will have

$$\delta \mathcal{L} \sim e^{-mR} e^{1/m}$$  \hspace{1cm} (8.2)

Only if $m \sim R^{-1/2} \sim \Lambda^{1/4}$ will this contribution be neither exponentially growing or vanishing as $R \to \infty$.

I would like to view this as part of a self consistent calculation of the gravitino mass, in the spirit of the Nambu-Jona-Lasinio model. That is, we add R violating terms to the low energy Lagrangian which induce a gravitino mass because they lead to low energy spontaneous SUSY breaking. Then we ask for what value of this mass the R violating terms will be induced by diagrams like that of Fig. 1. If our crude estimates of the exponential enhancement are correct, then only the scaling exponent $1/4$ is self consistent. If the gravitino mass were smaller than this, then our estimate would give an exponentially larger contribution, so a smaller mass is not consistent. Similarly, a larger mass, would predict an exponentially small contribution from the horizon. However, we know that there are no other sources for large $\Lambda$-dependent renormalizations of $m_{3/2}$. So a larger mass is also inconsistent.

Why should we imagine that the above estimate applies to the gravitino mass, rather than that of some other state? At the moment, my only argument is based on the low energy effective Lagrangian. In \cite{47} I argued that this had to be a system which was an R violating relevant perturbation of a supersymmetric, R symmetric system. In that context, there are no SUSY splittings that are smaller than the gravitino mass\footnote{In fact in the explicit models constructed in the above reference, all other SUSY splittings are much larger than the gravitino mass.}. Furthermore, it is clear that the contribution from the horizon is dominated by interactions with the lightest particle available. The particle in Fig. 1 must carry R charge, in order to generate R violating interactions. It is very plausible phenomenologically, and certainly true in the models of \cite{47} that the gravitino is the lightest R charged particle.

Together, these arguments suggest a radical change in our thinking about the cosmological constant problem. That problem itself is insoluble (see however the section on
Metaphysics above) - the cosmological constant in Planck units is simply the inverse entropy of the universe, and is built into the structure of the Hilbert space. The crucial relation between the scale of SUSY breaking and $\Lambda$ depends on physics that we do not have good control over, but I have argued that the eventual quantum theory of dS space will give a value for the SUSY breaking scale which is compatible with observations.

8.2. Uniqueness of the limiting SUSic vacuum

A crucial question for the program described in this paper is the uniqueness of the limiting SUSic vacuum state. Our claim is that there is a quantum theory of dS spacetime with any finite number of states and that the zero c.c. limit of these theories must be a Super Poincare invariant, R invariant system (with an invariant vacuum state) satisfying several other properties. Nothing in our argument tells us that there cannot be many such systems, perhaps even an infinite number.

This would be something of a disaster for our program. It is likely that the low energy dynamics of any theory satisfying our criteria would be sufficiently complicated that we would have little chance of deciding whether complex, intelligent organisms could evolve in these alternative universes. There would be many theories of quantum gravity in asymptotically dS space with a fixed value of the cosmological constant, and so our theoretical framework would not be terribly predictive. Many key features of the world we know, like the choice of the low energy gauge group, would be random accidents, correlated to the structure of our particular form of life, but having no explanation at a more fundamental level. The best we could hope to do would be to find a theory in this class with the right low energy gauge group, and calculate the parameters in the standard model from first principles. The task of theoretical physics would end with the establishment of the existence of a theory which contained the right gauge groups and the right parameters.

It is to be hoped then that the limiting SUSY theory is unique or is member of a small finite family. Infinite sets of theories with low energy dynamics trivial enough to rule out the possibility of complex organisms would also be acceptable.

The most important question is to find an algorithm, which will generate all possible theories with isolated SUSic vacua. Here I want to suggest something which seems to run contrary to (but is in fact completely compatible with) my claim that off shell effective potentials are not sensible objects in string theory. If we look at compactifications to four dimensions with four supercharges, there are a variety of regions in which we seem to
obtain moduli spaces of limiting theories with exact super-Poincare invariance. Examples are the heterotic string, or large radius compactification of 11D SUGRA on $G_2$ manifolds or $CY_3 \times S^1/Z_2$. For finite values of the string coupling or radial moduli we expect a superpotential to be generated. Furthermore, the superpotential has an expansion in $e^{-aS}$ for some $a$, which can, in principle, be computed by instanton analysis. Moreover, there is every indication that these expansions have a finite radius of convergence. It would seem that these expansions unambiguously define a section of a locally holomorphic line bundle on some kind of complex moduli space.

We cannot say the same about the Kahler potential, which is given by a divergent series. However, the question of the existence of Super-Poincare invariant vacuum states is independent of the Kahler potential, and indeed has the same answer for any choice of the connection on the line bundle. One is led to conjecture the existence of a topological version of string/M theory with 4 SUSYs, which would define this complex moduli space in a non-perturbative fashion, and compute the number of solutions to the equations $W = dW = 0$. Perhaps it will also give answers to questions about the spectrum of massless states at each solution. Techniques for answering the analogous question about the spectrum of BPS branes and their world volumes superpotentials in compactifications to four dimensions with eight supercharges have been developed by Douglas and others [50].

One would hope to be able to do more, that is to find a constructive prescription for computing the full scattering matrix (here I abuse language and neglect the infrared problem) at each of these solutions. Thus, the goal that I propose is to find an algorithm for constructing all Minimally Super Poincare invariant $S$-matrices for quantum gravity in 4 dimensions \[^30\]. They are rare jewels and constructing a machine that produces them would seem to be a worthy goal for much future research in string theory. I believe that it is of great interest even if one rejects the wilder conjectures in this paper.

8.3. SUSY and the Holographic Screens: An Idea for An Idea

The current section is even more speculative than the rest of this paper. Given the importance of supersymmetry in string theory, one is led to ask whether there is a more geometrical way of understanding the necessity for incorporating supersymmetry into a

\[^30\] Again, I am using $S$-matrix as a shorthand for a more sophisticated object, which includes soft graviton bremsstrahlung.
quantum theory of gravity. In my lecture at the Strings at the Millenium Conference in January of 2000[51], I suggested that the geometrical origin of local SUSY was the holographic principle. The formulation of the holographic principle for asymptotically flat spacetimes[52] makes it apparent that the choice of a holographic screen (holoscreen) is a gauge choice. In extant formulations in asymptotically flat space-times, the holoscreen is chosen to be a light plane, corresponding to a particular choice of light front gauge.

Bousso’s[53] general formulation of the holographic principle makes it abundantly clear that there are many ways to project the data in a given spacetime into collections of holoscreens. This suggests that any quantum theory of gravity based on the holographic principle must have a new gauge invariance, going beyond (but intertwined with) general coordinate invariance. It was obvious that this must be related in some way to local SUSY.

The mathematical connection comes from the Cartan-Penrose (CP) equation, relating a null direction to a pure spinor. In fact, a pure spinor not only defines a null direction, but also a holoscreen transverse to that direction. One can think of the choice of a pure spinor at each point in space-time as a choice of a holographic screen on which the data at that point is projected. A local change of the choice of spinor corresponds to the holoscreen gauge invariance noted above, and smells like it has something to do with local SUSY.

The spinors in the classical CP equation are bosonic, but projective. Nothing in the correspondence depends on the overall complex scale of the spinor. The current, $\tilde{\psi}\gamma^\mu\psi$ constructed from the pure spinor, is a null direction, again a projective object. The equation knows only about the conformal, and not the metrical structure of space-time. In particular, although the classical CP equation makes a natural connection between the choice of a pure spinor at a point in spacetime, a null direction, and the orientation of a holographic screen, it says nothing about how far the screen is from the point, nor the geometrical extent of the screen. The geometrical interpretation of pure spinors originated in the work of Cartan[54]. In four dimensions, Penrose described the geometry with the picturesque phrase “A spinor is a flagpole (null direction) plus a flag (holoscreen).”[55].

Below we will turn the CP spinors into quantum operators. The quantization procedure breaks the classical projective invariance of the CP equation, leaving over a discrete phase invariance. This should be viewed as an additional gauge symmetry of the quantum theory. We will show that, after performing a Klein transformation using a $\mathbb{Z}_2$ subgroup of this gauge symmetry, the CP spinors become Fermions and $\mathbb{Z}_2$ is just $(-1)^F$. 57
The motivation for this particular method of quantizing the classical variables, combines the classical relation between spinors and holoscreens with the BHFSB relation between area and entropy. In this way the breaking of projective invariance is precisely equivalent to the introduction of metrical structure on space-time. The detailed answer to the question of the size and location of the holoscreen is thus quantum mechanical. As we will see, it also depends on a priori choices of boundary conditions for the space-time and on a choice of gauge. Below, we will concentrate on non-compact, 11 dimensional cosmological space-times which expand eternally.

8.4. Quantizing the CP equation

As explained above, the connection between local SUSY and holography comes via the Cartan-Penrose relation between spinors and null directions. Given a null direction $p^\mu$ in $d$ dimensions, there are always $2^{[d/2]-1}$ solutions of the Cartan-Penrose equation

$$p_\mu \gamma^\mu \psi = 0.$$ 

Furthermore, solutions of this equation for general $p^\mu$ can be characterized as the submanifold of all (projective) spinors satisfying $\bar{\psi} \gamma^\mu \psi \gamma_\mu \psi = 0$. The null direction, of $p^\mu$ is determined by this equation, but since the spinor is projective, not its overall scale.

A choice of pure spinor completely determines a null direction and a $(d-2)$ dimensional hyperplane transverse to it. The orientation of the hyperplane, but not its extent, or location in space-time is determined by the nonzero components of $\bar{\psi} \gamma^{[\mu_1...\mu_k]} \psi$ for all $k$. Equivalently, given a point in spacetime, the choice of a pure spinor at that point can be thought of as determining the direction and orientation of the holoscreen on which the information at that point is encoded. A hypothetical holoscreen gauge invariance can thus be thought of as a local transformation that changes a pure spinor at each point. In 3, 4, 6 and 10 dimensions, pure spinors can be obtained as the projective space defined by a linear representation of the Lorentz group (Dirac, Weyl, Symplectic Majorana, and Majorana-Weyl). In other dimensions there is no canonical way to associate a general spinor with unique pure spinors. We can always find a basis for the spinor representation consisting entirely of pure spinors. Indeed, consider two null directions, one with positive and the other with negative time component. The pure spinor conditions for these two null vectors define two subspaces of the space of all spinors, with half the dimension of the total space.
Furthermore, since a pure spinor uniquely determines its null direction, the two subspaces are linearly independent, and thus form a basis for the entire Dirac spinor space. However, in general this splitting is not canonical - the choices of null vectors are arbitrary. In 3, 4, 6, and 10 dimensions, there is a canonical way to decompose general spinors into irreducible representations of the Lorentz group, such that each spinor in the irrep is pure.

In any spacetime which is a compactification of a ten dimensional theory, we can decompose a general spinor into pure spinors in a unique way, so a general spinor at a point can be thought of as defining the direction and orientation of a pair of holographic screens on which to project the information from that point. One of the screens can be thought of as being in the past of the point and the other in its future, according to the sign of $p^0$. As a consequence, a gauge principle that refers to the ambiguity of choosing a spinor at each point in spacetime (which is to say, local SUSY), can be thought of as the holographic gauge invariance implicit in the work of Bousso.

One of the lacunae in my understanding of this subject is the lack of an analogous statement about eleven dimensions. I know of no way to canonically split a general spinor into pure spinors in eleven dimensions. In the discussion below, this will be dealt with in an unsatisfactory manner. We will discuss eleven dimensional Big Bang cosmology, where we will see that the fundamental variables are past directed pure spinors. This will enable us to sidestep the necessity to decompose a general spinor into pure spinors.

Aficionados of the superembedding formalism [56] will undoubtedly appreciate the deep connection between what I am trying to do here and that formalism. Superembedding is particularly powerful in eleven dimensions. I regret that my own understanding of this formalism is so rudimentary that I cannot exploit its elegance in the present work.

The projective invariance in the definition of pure spinors is a classical gauge invariance. It is real or complex, depending on the nature of the Lorentz irreps in the given space-time dimension. When we quantize the pure spinors, we will break most of this invariance, but there will always be at least a $Z_2$ gauge invariance left over in the quantum theory. We will see that this $Z_2$ can be identified with Fermi statistics. Indeed, the classical spinors of the CP equation are bosonic. We will quantize them as compact bosons - generalized spin operators. Each such operator will correspond to a new bit of holographic screen which is added to the Hilbert space describing the interior of a backward light-cone in a Big Bang space-time, as one progresses along a timelike trajectory. Operators corresponding to independent areas on the screen commute with each other. However, the $Z_2$ gauge invariance of the formalism will enable us to perform a Klein transformation.
which makes all the operators into Fermions. In this new basis for the operator algebra, the $Z_2$ is just $(-1)^F$. In some dimensions there will be a larger group of discrete gauge transformations, which survives quantization of the CP equation. I would like to interpret this as a discrete R symmetry.

In the last section, I will use quantized CP spinors to construct a holographic formulation of quantum cosmology.

8.5. SUSY and Poincare: a marriage of convenience or necessity?

The arguments of the preceding subsections suggest strongly that the asymptotically flat limit of a dS spacetime will be SUSic. It is tempting to speculate that this is a special case of a more general principle: quantum gravity in AF spacetimes must be SUSic. The temptation comes mostly from our abject failure to find asymptotically flat SUSY violating vacua of perturbative string theory. Until recently, all attempts to violate SUSY lead either to tachyonic instabilities, or the generation of a potential for moduli. In some of the latter situations (those where the potential is positive and draws the system into the semiclassical region of moduli space), it appears reasonable to conjecture that there is a cosmological FRW spacetime with flat or negatively curved spatial sections and only a Big Bang singularity. In cases where the potential is negative and draws the system into a strongly coupled, string scale region, it is likely that the AF perturbative vacuum has no resemblance to the real physics. These models either do not really exist, or are related to SUSY violating small radius AdS vacua. More generally, the failure to find an asymptotically flat SUSY violating quantum state for gravity, is what is conventionally called the cosmological constant problem. There are plausible constructions of SUSY violating AdS vacua, but they all have the property that SUSY breaking vanishes as the AdS radius is taken to infinity. There is no known example of a SUSY violating AdS vacuum in which SUSY violation appears to survive the large radius limit. Furthermore, all known SUSY violating AdS vacua that can be constructed as concrete conformal field theories have “radius of order the string scale” in the sense that they contain a plethora of operators which correspond to particles of Compton wavelength of order the AdS radius.

In a beautiful paper, Kutasov and Seiberg exhibited a clear connection between SUSY violation and tachyonic instability in perturbative string theory. A stringy Hagedorn spectrum generally implies, via modular invariance, tachyonic instability, which can only be removed by asymptotically exact SUSY cancellations. One is tempted to speculate
that the much more rapidly diverging spectrum of black hole states in AF spacetime might lead to instability if SUSY were not exact.

To attack this question, one must find a replacement for modular invariance, which is a perturbative string theory concept. I would like to suggest that the replacement is simply crossing symmetry, analyticity, and unitarity of the S-matrix. The two body scattering amplitude for spinless massless particles is an analytic function of the kinematic invariants: $A(s,t)$. Here $s > 0$ is the square of the center of mass energy and $t < 0$ the invariant momentum transfer. Crossing symmetry is the claim that when analytically continued to $s < 0$, $t > 0$ this is the same amplitude (and a similar statement when the Mandelstam $u = -s - t$ variable is the center of mass energy).

In the analysis of hadronic scattering amplitudes, it is well known that the large $s$ fixed $t$ behavior of amplitudes is dominated by singularities that probe the spectrum of the theory. In particular, for large impact parameter we are probing the lightest states in the theory, and would also encounter tachyons if there were any.

In quantum gravity, asymptotic darkness implies that scattering amplitudes at large $s$ and values of impact parameter that grow like a power of $s$ are dominated by the production of black holes. Elastic cross sections fall off exponentially in this range of impact parameters because the probability that a black hole formed in a two body collision will decay into two bodies is Boltzmann suppressed. The two body final state has very little entropy. The optical theorem must be satisfied by the contribution of states at very large impact parameter. The Froissart bound violating growth of black hole cross sections is enough to prove the existence of massless states in the theory, and, at least at the level of naive Regge pole analysis, the fact that there has to be a massless spin two particle. Of course, we already know that large impact parameter collisions are likely to be dominated by (eikonal) graviton exchange.

I believe that a more careful analysis of the analyticity, unitarity and angular momentum structure of high energy amplitudes in the light of black hole dominance of high energy collisions, might lead to an analog of the Kutasov-Seiberg result in perturbative string theory. That is, analyticity, crossing and unitarity of the S-matrix, plus black hole dominance of high energy cross sections might generically imply a tachyon in the crossed channel. Delicate supersymmetric cancellations might be the only way to eliminate the tachyon. In an exactly SUSic theory, any amplitude for production and decay of a black hole will be exactly related to another amplitude in which an additional soft gravitino
is emitted, by a SUSY Ward identity. The two amplitudes will behave differently under crossing symmetry.

The idea that there are no Poincare invariant vacua that are not Super-Poincare invariant is of course consistent with (though not strictly implied by) the idea of cosmological SUSY breaking. It remains to be seen whether the almost forgotten lore of analytic S-matrix theory will prove useful in addressing the problem of the nature of quantum gravity and proving a deep connection between SUSY and stability in quantum gravity.

8.6. Discussion

This section has been concerned with the role of SUSY in theories of quantum gravity, and consists of three distinct claims. The first (in a different order than I have presented them) is that local SUSY is connected with the Holographic principle, and is the local field theory limit of a gauge invariance corresponding to the freedom of changing holographic screens. The conventional wisdom holds that non-supersymmetric theories of quantum gravity might be sensible, while our claim is that every consistent theory of quantum gravity is holographic and therefore a theory of supergravity.

The second claim is that Poincare invariance occurs in theories of quantum gravity only as a subgroup of super Poincare invariance. The evidence for this contention is our abject failure to find a counterexample (which is, in some sense, what we usually call the cosmological constant problem). I suggested a more fundamental approach to this claim, based on examining the compatibility of a black hole spectrum of states with the existence of a unitary, Poincare invariant, analytic S-matrix. Another approach to this question might be via the AdS/CFT correspondence. Unfortunately, we know too little about how the full S-matrix is constructed in taking the large AdS radius limit, to make much progress here.

I then presented a heuristic calculation of the scaling law $m_{3/2} \sim \Lambda^{1/4}$ relating the gravitino mass to the radius of de Sitter space. This calculation makes TeV scale superpartners a natural consequence of the (apparently) observed value of the cosmological constant. The calculation is only consistent if the effective field theory explanation of SUSY breaking is non-gravitational. It also puts additional constraints on the low energy theory, and implies in particular that moduli must be frozen by SUSic dynamics at a high scale. There is no cosmological moduli problem in a theory with cosmological SUSY breaking. There are other consequences for low energy phenomenology. Models where dark matter is a neutralino are incompatible with this mechanism for SUSY breaking. Finally, a discrete
R-symmetry is crucial to the mechanism. It might be related to a family symmetry for quarks and leptons.

There are two directions for further work on this mechanism. The first is to develop a more detailed mathematical model of the quantum mechanics of a dS universe, which could lead to a more rigorous derivation of the scaling law. The second is to explore the constraints on low energy dynamics more fully and come up with an attractive, phenomenologically viable and predictive model of TeV scale physics, based on these ideas. Both directions are being pursued.

9. Holographic cosmology

In this section I will recall some ideas about holographic cosmology that were presented in [58], and extend them by choosing the fundamental variables of the theory to be quantized versions of the pure spinors of Cartan and Penrose. The formulation of quantum cosmology that I will present is Hamiltonian, and therefore necessarily gauge fixed. Therefore, the connection between CP spinors and a gauge principle for holographic screens will not be immediately obvious. However, we will see that the fundamental variable of the theory is a path dependent spinor. I view it as the gauge fixed, holographic version of the quantized gravitino field.

9.1. Prolegomenon to a Holographic Theory of Space Time

The treatment of cosmology in string theory has, for the most part, been an exercise in effective field theory. Many cosmological solutions of the equations of low energy perturbative string theory can be found, but like most time dependent solutions of Einstein’s equations, they contain Big Bang or Big Crunch singularities. This indicates the necessity for a more profound approach to the problem. Fischler and Susskind provided a fundamental new insight into cosmology, and Big Bang singularities, by trying to impose the holographic principle in a cosmological context[59]. This work was followed by Bousso’s construction of a completely covariant holographic entropy bound[53].

I believe that the work of F(ischler) S(usskind) and B(ousso) provided us with the foundations of a quantum theory of cosmology. There are three important principles that are implicit in the work of FSB:
1. The holographic principle is consistent with the idea of a particle horizon, a notion which we generally derive from local field theory. More generally, it is consistent with the
idea that a *causal diamond* in spacetime contains an operator algebra that describes all measurements, which can be performed within this diamond. In many cases, the holographic principle implies that the dimension of this operator algebra is finite. In this case there is a unique Hilbert space representation of the algebra.

2. In particular, the interiors of backward light cones in a Big Bang spacetime must have finite operator algebras. Furthermore, the FSB entropy bound implies that the dimension decreases (apparently to zero) as we go back to the Big Bang singularity. These results lead one to conjecture that instead, the (reverse) evolution stops when the Hilbert space has some minimal dimension. They also lead one to some guesses about the fundamental formulation of quantum cosmology, which I will sketch below.

3. Within the semiclassical approximation the holographic principle is compatible with F(riedman)-R(obertson)-W(alker) cosmology at early times if and only if the stress tensor satisfies the equation of state $p = \rho$, with the entropy density related to the energy density by $\sigma \propto \rho^{1/2}$. This is a peculiar new form of matter. Fischler and I [60] gave a heuristic picture of such a system as a “dense fluid of black holes”, but a more precise quantum description still eludes us.

I would like to sketch the outlines of a quantum cosmology based on these principles. This sketch is an update of ideas presented in [58] [61]. It is still less than a full dynamical quantum theory of spacetime. In presenting it, I have used the following strategy. I utilize spacetime concepts to motivate quantum mechanical constructions. Eventually, one would like to turn everything around, and present a set of purely quantum axioms from which we derive a classical spacetime. The reader should keep in mind the dual purpose of this discussion and, as it were, try to read every argument both backwards and forwards.

Consider a timelike trajectory (perhaps a geodesic) in a Big Bang spacetime, and a sequence of backward light cones whose tips end on this trajectory. The FSB bound implies that the Hilbert space describing all measurements in the interior of each of these light cones is finite dimensional. Let us define the entropy to be the logarithm of this dimension (it is the entropy of the maximally uncertain density matrix on this Hilbert space). Let us for the moment restrict attention to a period in which the universe is expanding. Then the entropy decreases as we follow the trajectory back to the Big Bang.

The concept of particle horizon means that each of these Hilbert spaces should have a self contained description of all of the physics that goes on inside it. That is, there

31 The latter condition means that the homogeneous modes of minimally coupled scalars *do not* satisfy the requirements of holography.
should be a sequence of unitary transformations describing time evolution inside each backward light cone, without reference to any of the larger light cones. On the other hand, the dynamics in a large light cone should be restricted by consistency with earlier light cones in the sequence. The reason that I insist on a sequence of unitaries, rather than a continuous one parameter family (a groupoid\(^{32}\)) is that the system in any one light cone is finite dimensional. A finite system can have a continuous time evolution if it is in contact with an external classical measuring apparatus, but, because of the time-energy uncertainty relations (whose precise form depends on the spectrum of the finite system) it does not make sense to talk about infinitely precise time resolution as a measurement performed by the system on itself. A more fundamental reason for discreteness will be discussed below.

This suggests, that as time goes on and the particle horizon expands, more and more precise time resolution becomes available. Thus, the time intervals between unitary transformations in the sequence should not be thought of as defining the Planck time. Instead, I insist that they define time slices in which the FSB area increases by some minimal amount (to be quantified below). Call the sequence of Hilbert spaces \( \mathcal{H}_n \). \( \mathcal{H}_n \) has dimension \( K^n \), and we think of it as defining the Hilbert space inside a backward light cone whose holographic screen has FSB area \( 4n \ln K \) in Planck units. In each \( \mathcal{H}_n \) there is a sequence of unitary transformations \( U_n(k) \) for \( n \geq k \geq 0 \). One further assumes that \( \mathcal{H}_n = H \otimes \mathcal{H}_{n-1} \), where \( H \) is a \( K \) dimensional space. The maps \( U_n(k) \) are required to factorize in a manner compatible with this concatenated tensor factorization of the Hilbert space. For example, for every \( n \) and \( k \), \( U_n(k) \) for \( k < n \) is a tensor product of \( U_{n-1}(k) \) and a \( K \) dimensional unitary transformation on \( H \). Below, we will choose the number \( K \) in a natural way that depends on the dimension of spacetime.

This definition gives us some idea of how much time is represented by each unitary evolution in the sequence. An area \( 4n \ln K \) in \( d \) spacetime dimensions, allows the creation of black holes of energy of order \( (4n \ln K)^{(d-3)/(d-2)} \). The inverse of this energy is the maximum time resolution that such a system can have. On the other hand, if we make some other assumption about the state of the system, we may have less time resolution than this. Thus, we can begin to see a correlation between the spacetime geometry and the matter content of the system.

\(^{32}\) I would like to thank G. Moore for explaining the mathematical name for the composition property for time evolution operators in time dependent quantum mechanics.
We also see the fundamental reason for discreteness in these equations. The FSB areas of backward light cones in a Big Bang space-time are quantized because they refer to the logarithms of Hilbert space dimensions.

So far of course we have defined much less than a full spacetime. To go on, we need to consider neighboring timelike trajectories, and we must introduce the dimension of spacetime. To do this, introduce a $d$ dimensional cubic lattice, and assign Hilbert spaces and unitary operators to each vertex of the lattice.

There are several disturbing things about this (as far as I can see) unavoidable introduction of dimensions. The is that we believe that we can define cosmologies in string theory, that interpolate between spaces of different dimension. For example, the Kasner cosmologies studied in [62] can interpolate between heterotic strings on tori and 11D SUGRA on K3 manifolds. It is not clear to me that this is a difficulty. We are not describing local field theories here, and our description might be valid in all regions of moduli space, even though defined with respect to one. What is certain, is that all of these dualities involve the nontrivial topology of the compactification manifold. We can for the moment restrict our attention to describing the noncompact part of space, with the compact parts described by the structure of the spectrum of states in Hilbert space. However, there is obviously much to be understood about this question. In the present paper I will restrict attention to non-compact 11 dimensional cosmologies.

The second disturbing aspect of our construction will be an asymmetry between space and time. It is intrinsic to our formulation of the problem in terms of time evolution in Hilbert space (rather than some sort of path integral formalism). We have chosen a rather particular gauge, in which every point on a time slice has a backward lightcone with equal FSB area. One could make different choices, but none would be gauge independent. No physical Hamiltonian of a general covariant theory can be gauge independent, since the choice of time evolution is a choice of gauge. Only in spacetimes with a fixed classical asymptotic boundary can we imagine a gauge independent choice of Hamiltonian. We will introduce the asymmetry between space and time into our notation by labeling points in the lattice by a $d$ vector of integers ($t, x$).

Now we have to address the question of how the Hilbert spaces and time evolution operators corresponding to different points on the lattice, are related to each other. It is here that the formalism parts company with a lattice field theory like system, where each point should have independent degrees of freedom. In fact, since we are associating the observables with experiments done in the backward light cone of the point, there should
be a large degree of overlap between nearest neighbors. Indeed, we defined the smallest
time difference by insisting that the Hilbert space at time \( n \) have only \( K \) times as many
states as that at time \( n - 1 \). If \( K \) were 2, this would be the minimum increase compatible
with the notion that the new particle horizon has some independent degrees of freedom in
it that were not measurable in the old one. Similarly we will require a maximal overlap for
nearest neighbor points on the lattice. That is, the Hilbert spaces \( \mathcal{H}_n(x) \) and \( \mathcal{H}_n(x + \mu) \)
should each factorize as

\[
\mathcal{H}_n(x) = H(x) \otimes O(x, x + \mu) \quad \mathcal{H}_n(x + \mu) = H(x + \mu) \otimes O(x, x + \mu),
\]

where for each \( y \), \( H(y) \) is a \( K \) dimensional space.

We will choose \( K \) in a manner motivated by our remarks about the connection between
supersymmetry and holography. Let \( S_\alpha \) transform in the irreducible spinor representation
of the Lorentz group \( Spin(1, d - 1) \). The details of the construction will depend somewhat
on the properties of spinors in various dimensions, so I will restrict attention to \( d=11 \).
We will insist that the spinor be pure, that is, that \( \bar{S}\gamma^{\mu}S\gamma_\mu S = 0 \). Such spinors have 16
independent real components. In the quantum theory, they will be quantum operators, \( S_a \),
\( a = 1 \ldots 16 \). We also restrict attention to past directed pure spinors - the associated null
vector is past directed. In choosing to describe the pure spinor in terms of only sixteen
variables, we have chosen a gauge for local Lorentz gauge symmetry. In principle, one could
keep 32 components and a local symmetry which allowed us to reduce to 16. However,
the Lorentz connection would have to be a constrained variable, in order not to introduce
new degrees of freedom into the system. We are aiming toward a completely gauge fixed
Hamiltonian description of our cosmology. Below, we will introduce a mapping \( \Psi \) between
the operator algebras in Hilbert spaces at different points on the lattice. In particular,
that mapping will relate the spinor basis at one point to that at another. \( \Psi \) implicitly
contains the gauge fixed Lorentz connection.

We have seen that, classically, a past directed pure spinor determines a past directed
null direction. We think of the physical interpretation of this null direction in terms of two
holographic screens for an observer traveling along the timelike trajectory between \((t, x)\)
and \((t + 1, x)\). The physics inside the backward light cone of the observer at these two
points, can be projected onto a pair of holographic screens, both in the past of the tips
of the light cones. In a geometrical picture, the information that is not contained in the
smaller screen can be communicated to the observer at some point, \( P \), on his trajectory.
between the tips of the two light cones. The new pure spinor that we add to the system may be thought of as the instruction for building the new piece of the holographic screen, on which the information at P is to be projected. Of course, this classical language can have only a poetic meaning at the time scales on which we are making our construction.

It is important to note that the paragraph above contains the answer to the question of where and how large the holographic screen is. If we assume that the quantum formalism will, in the limit of large Hilbert spaces, indeed determine a classical geometry consistent with the words we have been using, then the bit of holographic screen that is added by the operator $S_a(t,x)$ is located on the FSB surface of the backward light-cone from $(t+1,x)$, and has area $4 \ln 256$ in Planck units. The null vector which would specify precisely where on that screen this particular variable is, is the bilinear current constructed from this pure spinor. It is a quantum operator, and so only describes probability amplitudes for the bit of screen to be at specific points on the FSB surface. The FSB surface itself is constructed out of all the spinors in the Hilbert space $\mathcal{H}_{n+1}$, so its quantum fluctuations are small in the limit of large area.

As anticipated above, we will build the Hilbert space $\mathcal{H}(t+1,x)$ by adding operators $\hat{S}_a(t+1,x)$ to the Hilbert space $\mathcal{H}(t,x)$. These will commute with all of the operators in the latter space. The defining relation for a pure eleven dimensional spinor is invariant under real projective transformations of the spinor. We will break this invariance in the quantum theory.

Thus, we postulate that

$$[\hat{S}_a, \hat{S}_b]_+ = 2\delta_{ab}. \quad (9.2)$$

Up to normalization, this is the unique ansatz that gives a finite dimensional Hilbert space, and is invariant under the $SO(9)$ group of rotations that leave the null vector invariant. These postulates break the projective invariance except for a factor of $(-1)$. We will treat the latter factor as a $Z_2$ gauge transformation, which will eventually be seen as Fermi statistics. The fact that the classical projective gauge symmetry of the CP equation is broken down to $Z_2$ has to do with the fact that our spinor carries information about the conformal factor of the spacetime geometry, as well as its causal structure. Indeed, the commutation relations determine the dimension of the new Hilbert space, and thus the area of the new holographic screen. The logarithm of the dimension of the new Hilbert space increases by $8 \ln 2$, which corresponds to an increase in area (Planck units) of $32 \ln 2$.

We can now turn the $\hat{S}_a$ into Fermions, by defining $S_a = (-1)^F \hat{S}_a$, where, $(-1)^F$ is the product of all of the previous $S_a$ operators (note that the number of these operators
is always even). In other words, we start with the irreducible representation of the Clifford algebra. This defines the smallest possible Hilbert space at the moment of the Big Bang. Then we build successive Hilbert spaces along a given timelike trajectory, by tensoring in one more commuting copy of the minimal Clifford representation. We then do a Klein transformation to present the full algebra as a larger Clifford algebra. The Klein transformation is a $Z_2$ gauge transformation, which is the quantum remnant of the projective invariance of the Cartan-Penrose equation. It is Fermi statistics of the Klein transformed operators. Note that all operators transforming in integer spin representations of the Lorentz group, will be even functions of the $S_a$, so the connection between spin and statistics is built into the formalism.

To recapitulate, the quantum description of the causal pasts of a sequence of points along a given timelike trajectory in a Big Bang cosmology is described by a sequence of Hilbert spaces $\mathcal{H}_n$. The operator algebra of the $k$th Hilbert space is the Clifford algebra generated by operators $S_a(n)$ with $1 \leq n \leq k$:

$$[S_a(n), S_b(m)]_+ = \delta_{ab} \delta_{mn} \quad (9.3)$$

The operator $S_a(n)$ in each Hilbert space may be identified with the operator with the same labels in any other Hilbert space. We will see later that this identification may be viewed as a gauge choice for the discrete analog of local SUSY.

Dynamics is defined by a sequence of unitary transformations, $\{U(n)\}$, in each Hilbert space, satisfying a simple compatibility condition, which will be discussed below. In principle, we could introduce a continuous unitary groupoid $U(t, t_0)$ such that the unitary transformations in the sequence could be viewed as the values of $U(t_n, 0)$ at a sequence of times. In this way the formalism becomes that of ordinary quantum mechanics, with a time dependent Hamiltonian, but changes in the groupoid, which do not change the values at the special times $t_n$, should be viewed as physically equivalent.

The sequence in $\mathcal{H}_k$ has $k$ steps, and should be thought of as the evolution operators over the time steps determined by the sequence of points on our timelike trajectory. The fundamental consistency condition is that the operator $U_k(n)$ in $\mathcal{H}_k$ with $n < k$ should be a tensor product of the ($k$th copy of) $U_n(n)$ with an operator that depends only on the $\hat{S}_a(m)$ with $m > n$. Thus

$$U_k(n) = U_n(n)V_k(n), \quad (9.4)$$
where $V_k(n)$ is a function only of $S_a(m)$ with $m > n$. We will impose the $Z_2$ gauge invariance on all of these unitaries, so that they are even functions of the fundamental variables, and we can ignore the distinction between the hatted and bare headed variables. Note also, that in writing the last equation we have used the same notation for the operator $U_n(n)$ and the copy of this operator in every $\mathcal{H}_k$ with $k > n$.

These rules define a quantum system, which is compatible with the notion of particle horizon in a Big Bang cosmology. The Hilbert space $\mathcal{H}_k$ describes all measurements that can be done inside the particle horizon at time $t_k$, in a manner compatible with the fact that measurements inside earlier particle horizons commute with measurements that can only be made at later times. Each particle horizon has its own time evolution operator, but the evolution operators at early times, agree with those in previous particle horizons, in their action on those variables that are shared between the two systems.

The system is also compatible with the holographic principle in that we will identify the dimension of the Hilbert space with the area of the FSB surface on the past light cone. This statement does not have much content until we enrich our system and show that it does have a spacetime interpretation.

Indeed, the conditions we have stated so far are very easy to satisfy, and most solutions do not resemble spacetime in any obvious way. What is missing is the notion that the new

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33 From here on I will stop insisting that the shared operators are really copies of the operators at earlier times. The reader will have to supply this pedantry by himself. Its importance will be apparent when we discuss copies of operators associated with other timelike trajectories.

34 This identification is only appropriate for the past light cones on trajectories in an eternally expanding universe. In contracting universes, the FSB area can sometimes decrease as one goes into the future. It must then be interpreted as the entropy of a density matrix more pure than the uniform probability density. The interpretation of this is that the assumed spacetime geometry and matter content is a very special class of states of the system. More generic initial conditions at times before the FSB area began to decrease would have led to a different spacetime in these regions. Of course, the latter statement could also be made about, e.g., the future evolution of an expanding matter dominated FRW universe. However, in this case we can still imagine exciting a more general configuration in the future by creating lots of black holes. In contracting regions, certain possible excitations of the system at early times are ruled out by the assumption that the geometry behaves in a particular classical manner. In this connection note that if we examine contracting FRW universes with matter with equation of state $p = \rho$, corresponding to a maximal entropy black hole fluid, then the FSB area of backward light cones always increases. It is only the assumption that low entropy systems with soft equations of state persist into contracting regions that leads to the phenomenon of decreasing area.
degrees of freedom that come into a particle horizon “come from other points in space”. To implement this, we return to our hypercubic eleven dimensional lattice with points labeled \((t, x)\) and Hilbert spaces \(\mathcal{H}(t, x)\). The sequence of Hilbert spaces at fixed \(x\) has all the properties we have described above.

To understand the geometric interpretation of, e.g., the Hilbert space \(\mathcal{H}(t, x + e_1)\), where \(e_1\) is some unit lattice vector, introduce a time slicing of our Big Bang spacetime by the rule that the past light cone of every point on a time slice has equal FSB area (for FRW these are just slices of cosmic time, but a single unit in \(t\) does not correspond to a fixed unit of cosmic time, but rather a fixed unit of FSB area). Now, starting at a point labeled \(x\) on a fixed time slice, choose a spacelike direction on the slice and move along it to a new point, labeled \(x + e_1\). The intersection of the past light cones of these two points has an almost everywhere null boundary but is not a full light cone. Choose the point (for very small distance between the two points on the time slice, it will be unique) inside the intersection whose past light cone has the largest FSB area, and call this the FSB area of the intersection. The point \(x + e_1\) is chosen such that the FSB area of the intersection is smaller than the FSB areas of the causal pasts of \(x\) and \(x + e_1\), by precisely the fundamental unit. Now proceed to do the same in the negative \(e_1\) direction and in 9 other locally independent directions. Then repeat the same procedure for each of these new points and so on ad infinitum (for this paper, we restrict attention to spatial topologies, which are trivial and extend to infinity in all dimensions). Repeat the same for each time slice. This picture motivates our lattice of Hilbert spaces.

The crucial step now is to introduce maps between a tensor factor of the operator algebra (equivalently, the Hilbert space, since everything is finite dimensional) in \(\mathcal{H}(t, x)\) and that in \(\mathcal{H}(t, x + e_1)\) for every (positive and negative) direction. The common factor Hilbert space has dimension smaller by a factor of \(\frac{1}{256}\). Equivalently we can think of this as a relation, which defines a copy of the generators of the algebra at \(x\) in the Hilbert space at \(x + e_1\).

\[
S_a(t_i; t; x; x + e_1) = \Psi_{ab}(t_i, t_j; x, x + e_1)S_b(t_j, bfx + e_1)
\]

(9.5)

The labels \(0 \leq t_i \leq t, 0 \leq t_j \leq t\) on the operators, remind us that the Hilbert space \(\mathcal{H}(t, x)\) contains operators that have been copied from the Hilbert spaces at all previous times. The map \(\Psi\) is part of the definition of the dynamics of this quantum spacetime. It is subject to a large number of constraints. Viewed as a matrix on the \(256t\) dimensional
space of $S$ components, it should have rank $256(t - 1)$. Precisely 256 generators of the algebra at $x$ should have vanishing representative in the nearest neighbor Hilbert space. Furthermore, the different $\Psi$ maps at different points of the spacetime lattice must all be compatible with each other.

A much stronger set of constraints comes from requiring that the unitary transformations $U(t_k, 0)$ in each Hilbert space be compatible with each other after application of the map $\Psi$. This is a system of mutual compatibility constraints between the $\Psi$ maps and the unitary transformations. Indeed, one is tempted to conjecture that any lattice of Hilbert spaces, $\Psi$ maps and unitary transformations satisfying all of these axioms should be viewed as a consistent quantum mechanical description of a Big Bang cosmology. I am not prepared to make such a bold conjecture at this time. Many examples will have to be discovered and worked out before we can hope to understand this formalism, and whether it needs to be supplemented with additional axioms.

The bilateral relations between nearest neighbor Hilbert spaces on the lattice, enable us construct copies of subalgebras of the operators in any Hilbert space, inside the operator algebra of any other. For a pair of points on the lattice, this correspondence will be path dependent. Thus, in some sense, the fundamental dynamical variables in the theory are the path dependent objects

$$S^\Gamma_a(t, x; t', x').$$  \hspace{1cm} (9.6)

In words, this is the copy of $S_a(t, x)$, in $\mathcal{H}(t', x')$ obtained by concatenating the $\Psi$ maps along the path $\Gamma$ between the two points. The $\Psi$ map gives us a special case of these variables for the minimal path between nearest neighbor points. Thus, $S_a(t, x; t, x + e_1)$ can be thought of as a discrete analog of the gravitino field $\psi_{\alpha \mu} d e_1^\mu$ integrated along the link between two nearest neighbor lattice points.

We now see that the simple mapping between the operator algebras at different times, at the same spatial point, can be viewed as a gauge choice for the time component of the gravitino field. We do not yet have any evidence that this formalism reduces to some kind of classical field theory in limiting situations, but it seems like that if it does, that field theory will be locally SUSic.
9.2. Discussion

The system that we have been discussing bears some resemblance to a lattice quantum field theory. This is both misleading, and suggestive. It is misleading because the fields at different points of the lattice at the same time do not (anti)-commute with each other. Their commutation relations are complicated and depend on the choice of Ψ mappings. This choice is part of the specification of the dynamics of the system. Note further that if the true connection between geometry and quantum mechanics is to be extracted from the entropy/area relation, the space-time geometry will not be that of the lattice. The lattice does serve to specify the topology of the spacetime.

The relation to lattice field theory is however suggestive of the possibility that in some dynamical circumstances, suitable subsets of the variables of this system might behave like quantum fields.

9.3. What is To Be Done?

The answer to this question is of course: “Almost everything”. More specifically, the most urgent problem is to find one example of a solution to the constraints postulated above, and show that in the large time limit it has an approximate description in terms of quantum fields in curved spacetime. The obvious case to start with is that of a homogeneous isotropic universe. That is, every sequence of Hilbert spaces $\mathcal{H}(x, t)$ has the same set of unitary maps $U(s, s - 1)$ for $s \leq t$. There is essentially a single Ψ mapping, which must be consistent with the unitary dynamics. This problem is still complicated enough that no solutions have been found as yet.

With W. Fischler, I have conjectured a possible solution, corresponding to a homogeneous spatially flat universe with equation of state $p = \rho$. Write $U(s, s - 1) = e^{H(s)}$ and expand $H(s)$ in powers of the Fermion operators. In particular, there will be a quadratic term

$$H_2(s) = \sum_{p, r < s} S_a(p) h(s|p, r) S_a(r)$$  \hspace{1cm} (9.7)

Now, for each $s$, let $h(s|p, r)$ be a random antisymmetric matrix, chosen from the Gaussian ensemble. It is well known\textsuperscript{13} that large Gaussian random antisymmetric matrices have a spectral density that behaves linearly in a larger and larger region around zero eigenvalues. Thus, for large $s$, the spectrum of the time dependent Hamiltonian $H_2(s)$ has
a universal behavior that looks like that of a system of free massless Fermions in $1+1$ dimensions. Furthermore, with the exception of a single marginally relevant four Fermion operator (the analog of the Bardeen-Cooper-Schrieffer operator), this low energy spectrum will not be disturbed by higher order polynomials in Fermions. The universal behavior of the spectral density is shared by a large class of random Hamiltonians for the Fermion system.

Thus, for large $s$, although we are dealing with a problem with time dependent Hamiltonian, we approach a system whose spectral density becomes time independent and satisfies the energy/entropy relation $\sigma \sim \sqrt{\rho}$ of a $p = \rho$ fluid. Note that, in the hypothetical translation of this physics into a spacetime picture, the energy of this system at time $t$ would be interpreted as the energy density at the tip of the backward light cone, $(t, x)$.

On the other hand, because for each $s$ we make an independent choice of random Hamiltonian, there is no sense in which the quantum state of the system settles down to the ground state of any given Hamiltonian, even after time averaging. All of the degrees of freedom of the system remain permanently excited. The density matrix of the system is completely random, maximizing the entropy, but certain energetic properties become smooth and universal for large $s$. To prove that this system satisfies our axioms, one would have to exhibit a $\Psi$ mapping compatible with this prescription for time evolution. This has not yet been done.

9.4. Discussion

Our discussion has been restricted to Big Bang cosmological spacetimes. I have emphasized above and elsewhere [64] that one should expect the fundamental formulation of quantum gravity to depend on the asymptotic geometry of spacetime. Gravity is not a local theory, once one goes beyond the realm of classical geometry (where the degree of non-locality can be controlled by the choice of initial conditions of the classical solution), and its fundamental formulation has every right to depend on the boundary conditions. Nonetheless, the considerations of the present paper suggest the possibility of a more local, but perhaps gauge dependent formulation, as has been advocated by Susskind. Again, the idea is that the choice of holographic screen is a gauge artifact. In asymptotically flat or AdS spacetimes, it may be convenient and elegant to place the screen at infinity, but there may also be other gauges where the same information is mapped onto a collection of local screens. Our formulation of quantum cosmology has fixed a particular gauge defined by equal area time slices.
In asymptotically flat spacetime, the causal diamond formed by the intersection of the causal past of a point with the causal future of point in that causal past, has finite FSB area. Thus, one can imagine assigning finite dimensional operator algebras to causal diamonds and trying to imitate the formalism of this paper. Any given finite dimensional algebra would be embedded in a sequence of algebras corresponding to larger and larger causal diamonds. The limiting Hilbert space would be infinite and the limiting time evolution operator in this space would approach the scattering matrix. Again, the formalism would be constrained by the requirement of consistency with many partially overlapping sequences corresponding to nested causal diamonds centered around different points in space.

In asymptotically flat spacetime we expect to have an exact rotational symmetry. Thus, it makes sense to choose holographic screens which are spherically symmetric. One would want to represent the pure spinor operators for a finite causal diamond as something like elements of a spinor bundle over a fuzzy sphere in order to have a formalism which preserves rotational invariance at every step. Furthermore, the explicit breaking of TCP invariance which was evident in our treatment of Big Bang cosmologies, should be abandoned. Each causal diamond of FSB area $K^N$ should have a sequence of unitary operators $U(t_k, -t_k)$ for $1 \leq t_k \leq N$, which commute with an anti-unitary TCP operator. In the limit as $N \to \infty$, $U(t_N, -t_N)$ would become the S-matrix. An important aspect of this limit is that the finite $N$ fuzzy sphere should become a conformal sphere as $N \to \infty$, in order to obtain a Lorentz invariant S-matrix (the Lorentz group is realized as the conformal group of null-infinity).

In asymptotically AdS spacetime, things are more complicated. The causal past of a point includes all of AdS space prior to some spacelike slice. Thus if we try to construct the causal diamond corresponding to a pair of timelike separated points, it becomes infinite when the timelike separation is of order the AdS radius, and the backward and forward light-cones of the two points intersect the boundary of AdS space before intersecting each other. This suggests that there should be a sequence of finite dimensional operator algebras which cuts off at some finite dimension of order $e^{R^{(d-2)}_{AdS}}$. Since we already have a “complete” formulation of the quantum theory of AdS spacetimes, it would seem to be a good strategy to search for such a sequence of nested operator algebras within the Hilbert space of conformal field theory. This would be a new approach to the puzzle of how local data is encoded in the CFT.
10. Conclusions

It is unfortunate but perhaps inevitable that the negative conclusions of a paper like this are on a firmer footing than the attempts to make progress in new and positive directions. Unless one rejects the AdS/CFT prescription for quantum gravity in Anti de Sitter space, it is difficult to defend the idea that there is a unique theory of quantum gravity, with different realizations of it corresponding to minima of an effective potential. This field theory inspired picture is based on a separation between UV and IR physics which is simply not there in theories of quantum gravity. I have tried to investigate both real \[12\] and virtual \[28\] transitions between vacua with different values of the cosmological constant, or isolated vacua with the same values of the cosmological constant and found that they do not occur - black holes get in the way. There remains one question in this general category whose answer remains unclear: is there a meaning to meta-stable dS minima which can decay into negatively curved FRW spacetimes with vanishing cosmological constant. It would seem that the primary challenge here would be to establish the existence of a quantum theory corresponding to the Big Bang FRW universe, into which the metastable dS states are supposed to decay. Only with a well controlled quantum theory of this Big Bang spacetime in hand, could we hope to make a rigorous verification of the existence of dS “resonances”. Calculations of approximate effective potentials without an underlying high energy theory, cannot resolve this question.

These results, to my mind, establish the existence of a variety of consistent mathematical models of quantum gravity and lead to the question of what distinguishes our world from among them. I believe that the key question to ask here is “What is Supersymmetry and Why Don’t We See It?”.

I have given a variety of partial answers to this question, of varying degrees of plausibility. At the deepest level, I suggested that local SUSY was connected to the holographic principle via the Cartan-Penrose equation. I have a strong feeling that there is something right about this idea, and an even stronger one that I have as yet expressed it only clumsily.

The question of global SUSY depends, as does everything in theories of quantum gravity, on asymptotic boundary conditions in space-time. In AdS space-time, global SUSY seems to be necessary in order to obtain an AdS radius large compared to the string scale. SUSY violating AdS theories with radius larger than the string scale have been exhibited using low energy effective Lagrangians, and flux compactification. There is as yet no proposed CFT dual for these models. At any rate, the amount of SUSY violation
vanishes as the AdS radius goes to infinity. Consistent with this, we have no example of a SUSY violating theory of quantum gravity in asymptotically flat space-times. I have conjectured that none exists. The two avenues by which one might try to establish or falsify this conjecture are to search for SUSY violating sequences of CFT's with AdS radius going to infinity in string units, or to use the combination of unitarity, crossing, analyticity and black hole dominance of high energy processes to prove the necessity of SUSY.

The Poincare → Super-Poincare conjecture leads to, but is stronger than, the CSB conjecture that SUSY breaking in the world we see is connected to the positive value of the cosmological constant. I have given an argument for the anomalous scaling $m_{3/2} \sim \Lambda^{1/4}$, but to make it rigorous one really needs a complete mathematical quantum theory of dS space. Absent such a theory, there is still an interesting calculation which can be done to make this speculation more plausible. I have argued that the global coordinate picture of an infinitely expanding sphere may actually be sensible in the field theory approximation, if we make an infrared cutoff when the volume of the sphere exceeds $R_{\text{dS}}^{\frac{3}{2}}$ in Planck units. Perturbative quantum gravity calculations in dS space are fraught with IR divergences. In particular, one might imagine that the gravitino mass term in the low energy effective Lagrangian might have (probably logarithmic) IR divergent loop corrections. Since we have argued that the IR cut-off is the dS radius, this would suggest an anomalous dependence of the gravitino mass on the cosmological constant. Perhaps one could even get the right critical exponent by adroit resummation of the perturbation series.

I will end by mentioning two other directions of research that are suggested by CSB. The first is the program begun in [4] to find a low energy description of the SUSY breaking mechanism. One must find a SUSic, R-symmetric theory, which, when perturbed by R violating terms which are a function of the cosmological constant, breaks SUSY and gives a gravitino mass of order $\Lambda^{1/4}$. The second is the search for an algorithm which describes all $\mathcal{N} = 1, d = 4$ SUSic compactifications of M-theory, including isolated ones. Arguments of holomorphy suggest that the otherwise oxymoronic program of “computing the superpotential on moduli space”, and finding stationary points where it vanishes is a good first attempt at such an algorithm. It would be of great interest to find a non-perturbative formulation of this problem, perhaps as a sort of topological version of M-Theory.
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