Non-universal gauge boson $Z'$ and the spin correlation of top quark pair production at $e^-e^+$ colliders

Chong-Xing Yue and Li-Na Wang

Department of Physics, Liaoning Normal University, Dalian 116029, China*

March 26, 2022

Abstract

In the off-diagonal basis, we discuss the contributions of the non-universal gauge boson $Z'$ predicted by the topcolor-assisted technicolor ($TC^2$) model to the spin configurations and the spin correlation observable of the top quark pair production via the process $e^-e^+ \rightarrow t\bar{t}$. Our numerical results show that the production cross sections for the like-spin states, which vanish in the standard model, can be significantly large as $M_{Z'} \approx \sqrt{S}$. With reasonable values of the $Z'$ mass $M_{Z'}$ and the coupling parameter $k_1$, $Z'$ exchange can generate large corrections to the spin correlation observable.

PACS number: 12.10.Dm, 14.65.Ha, 12.15.Lk, 12.60.Cn.

*E-mail:cxyue@lnnu.edu.cn
I. Introduction

Although the standard model (SM) has been precisely tested by experiments, it is widely believed to be only an effective theory valid below some high energy scales. New physics should exist at energy scales around $TeV$. Most of the new physics models predict the existence of the new neutral gauge boson $Z'$ associated with the extra $U(1)$ subgroup of the underlying group. The phenomenology of this kind of new particles has been extensively studied in either model-dependent or model-independent approaches[1], whose presence will be of great interest in the next generation collider experiments [2].

Topcolor scenario is attractive because it explains the large top quark mass and provides possible dynamics mechanism of electroweak symmetry breaking (EWSB)[3]. A common feature of this kind of models is that the SM gauge groups are extended at energy well above the weak scale. Breaking of the extended gauge groups to their diagonal subgroups produces new massive gauge bosons. To generate enough large mass for top quark, the extended gauge groups are generally flavor non-universal, which predict the existence of the non-universal gauge bosons. For instance, the topcolor-assisted technicolor (TC2) model [4] and the flavor universal TC2 model [5] predict the non-universal neutral gauge boson $Z'$, which treats the third generation fermions differently from those in the first and second generations. Possible signals of the gauge boson $Z'$ have been studied in the literature [6].

The top quark with a mass of the order of the electronweak scale, $m_t = 171.4 \pm 1.2(stat.) \pm 1.8(syst.) GeV$ [7], is the heaviest particle yet discovered, which is singled out to play a key role in probing the new physics beyond the SM. Very large mass of top quark prompts itself to rapidly decay before hadronization and the spin information of the top quark is preserved from its decay. Thus, the top spin configuration can provide additional observables for testing the SM and new physics beyond the SM. Spin correlation in top production and decay is a very interesting issue in top quark physics.

It is well known that, at the high energy linear $e^-e^+$ collider (ILC) experiments, the top quark pair ($t\bar{t}$) is produced in a unique spin configuration and the decay products of polarized top quark are strongly correlated to the spin axis. Thus, studying the $t\bar{t}$
spin correlations provides an interesting possibility to test the SM and search for new physics. This fact has made that the spin correlation of the $t\bar{t}$ production at the ILC experiments has been extensively studied in the SM and new physics beyond the SM [8,9,10]. This paper is aimed at investigating the effects of the non-universal neutral gauge boson $Z'$ predicted by the topcolor scenario on the top quark pair production via the process $e^-e^+ \rightarrow t\bar{t}$, and seeing whether the possible signatures of $Z'$ can be detected via measuring the $t\bar{t}$ spin correlation observable in the future ILC experiments.

The TC2 model is one of the phenomenologically viable models, which has almost all essential features of the topcolor scenario. So, in the rest of this paper, we will give our numerical results in detail in the context of the TC2 model. In the next section, we will give the calculation formula including the contributions of the non-universal gauge boson $Z'$ in the general spin basis. The corrections of $Z'$ to the $t\bar{t}$ production cross sections for different spin configurations in the off-diagonal basis are calculated in section III. The contributions of $Z'$ to the $t\bar{t}$ spin correlation coefficient $C$ is discussed in section IV. Our conclusions are given in section V.

II. The relevant formula of the $t\bar{t}$ production in the general spin basis

As it is well known, a large part of the top quark mass is dynamically generated by topcolor interaction at a scale of order $1\text{TeV}$ in the TC2 model. To ensure that the top quark condenses and receives a large mass while the bottom quark does not, a non-universal extended hypercharge group $U_y(1)$ is introduced, so the TC2 model predicts the existence of the extra $U_y(1)$ gauge boson $Z'$, which couples preferentially to the third generation fermions. Thus, the new particle $Z'$ should be easily detected via the processes involving the third generation fermions. The couplings of $Z'$ to ordinary fermions which are related our calculation can be written as [3,11]:

\[ \frac{1}{2} g_1 \frac{1}{3} \cot \theta' Z'_\mu (\bar{t}_L \gamma^\mu t_L + 2 \bar{t}_R \gamma^\mu t_R) + \tan \theta' Z'_\mu (\bar{e}_L \gamma^\mu e_L + 2 \bar{e}_R \gamma^\mu e_R), \]

where $g_1$ is the ordinary hypercharge gauge coupling constant, $\theta'$ is mixing angle with $\tan \theta' = \frac{g_1}{\sqrt{4\pi k_1}}$. To obtain the top quark condensation without forming a $b\bar{b}$ condensation, there must be $k_1 \leq 1$ [5]. In our numerical estimation, we will assume that the value of
$k_1$ is in the range of $0.1 \sim 0.9$.

Top quark pairs will be abundantly produced via the s-channel photon exchange and $Z$ exchange at the ILC experiments. The gauge boson $Z'$ predicted by topcolor scenario can give additional contributions to top quark pair production. In this paper, we will discuss $Z'$ effects on the spin configurations of the top quark pair production in the general spin basis. In this basis, the spin states of the top quark and top anti-quark are defined in their own rest-frame by decomposing their spins along reference axes $\bar{A}$ and $\bar{A}$. The reference axis $\bar{A}$ for top quark is expressed by an angle $\xi$ between the axis and the top anti-quark momentum in the rest frame of the top quark. The usual helicity basis can be obtained by taking $\xi = \pi$.

In the general spin basis, the differential cross sections of the process $e^- e^+ \rightarrow t\bar{t}$ for different spin configurations of the top quark pair ($\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow, \downarrow\uparrow$) can be written as:

$$
\frac{d\sigma_L}{d\cos \theta}(e^-_L e^+_R \rightarrow t \uparrow \bar{t} \uparrow \text{or } t \downarrow \bar{t} \downarrow) = \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_L - \sin\xi_L (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_L - \sin\xi_L (\cos\theta - \beta)}| g_{LL}} + \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_L + \cos\xi_L (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_L + \cos\xi_L (\cos\theta - \beta)}| g_{LR}}, \quad (2)
$$

$$
\frac{d\sigma_L}{d\cos \theta}(e^-_L e^+_R \rightarrow t \uparrow \bar{t} \downarrow \text{or } t \downarrow \bar{t} \uparrow) = \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_R - \sin\xi_R (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_R - \sin\xi_R (\cos\theta - \beta)}| g_{RR}} + \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_R + \cos\xi_R (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_R + \cos\xi_R (\cos\theta - \beta)}| g_{RL}}, \quad (3)
$$

$$
\frac{d\sigma_R}{d\cos \theta}(e^-_R e^+_L \rightarrow t \uparrow \bar{t} \uparrow \text{or } t \downarrow \bar{t} \downarrow) = \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_L - \sin\xi_L (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_L - \sin\xi_L (\cos\theta - \beta)}| g_{RR}} + \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_L + \cos\xi_L (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_L + \cos\xi_L (\cos\theta - \beta)}| g_{RL}}, \quad (4)
$$

$$
\frac{d\sigma_R}{d\cos \theta}(e^-_R e^+_L \rightarrow t \uparrow \bar{t} \downarrow \text{or } t \downarrow \bar{t} \uparrow) = \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_R - \sin\xi_R (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_R - \sin\xi_R (\cos\theta - \beta)}| g_{RR}} + \frac{3\pi \alpha_e^2 \beta}{8 S} \frac{\sqrt{1 - \beta^2 \sin\theta \cos\xi_R + \cos\xi_R (\cos\theta + \beta)}}{|\sqrt{1 - \beta^2 \sin\theta \cos\xi_R + \cos\xi_R (\cos\theta - \beta)}| g_{RL}}, \quad (5)
$$

with

$$
g_{IJ} = g_{IJ}^\gamma + g_{IJ}^Z (e^\gamma(t) g^Z(t) \frac{S}{S - M_Z^2 + i M_Z \Gamma_Z})
$$

$$
= g_{\gamma}(t) g_{\gamma}(t) + g_{\gamma}^Z(t) g^Z(t) \frac{S}{S - M_Z^2 + i M_Z \Gamma_Z}
$$
\[ S - M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}. \]

(6)

Where \( \theta \) is the scattering angle of the top quark with respect to the electron beam, \( t \uparrow (t \downarrow) \) denotes the top quark spin along (against) the reference axis \( \bar{A} \), and \( \beta = \sqrt{1 - \frac{4m_t^2}{S}} \). The angles \( \xi_L \) and \( \xi_R \) correspond to the left- and right- handed electron beam, respectively. \( \sqrt{S} \) is the center-of-mass (c.m.) of the ILC experiments. \( \Gamma_Z \) and \( \Gamma_{Z'} \) are the decay widths of the gauge bosons \( Z \) and \( Z' \), respectively. \( g^Z_{I}(e) \) and \( g^Z_{J}(t) \) (\( g^{Z'}_{I}(e) \) and \( g^{Z'}_{J}(t) \)) with \( I, J = L \) or \( R \) represent the couplings of the SM gauge boson \( Z \) (the new gauge boson \( Z' \)) to the electron and top quark, respectively. They can be written as:

\[
\begin{align*}
g_{\gamma}(e) &= -1, & g_{\gamma}(t) &= \frac{2}{3}; \\
g^Z_{L}(e) &= \frac{S_W}{C_W} - \frac{1}{2S_WC_W}, & g^Z_{R}(e) &= \frac{S_W}{C_W}; \\
g^{Z'}_{L}(e) &= \frac{1}{2S_WC_W} - \frac{2S_W}{3C_W}, & g^{Z'}_{R}(e) &= -\frac{2S_W}{3C_W}; \\
g^Z_{L}(t) &= -\frac{1}{4C^2_W\sqrt{\pi k_1}}, & g^{Z'}_{R}(t) &= \frac{1}{2C^2_W\sqrt{\pi k_1}}; \\
g^{Z'}_{L}(t) &= \frac{\sqrt{\pi k_1}}{3}, & g^{Z'}_{R}(t) &= \frac{2\sqrt{\pi k_1}}{3}. 
\end{align*}
\]

(7) - (11)

Where \( S_W = \sin \theta_W \), \( \theta_W \) is the Weinberg angle.

The lower limits on the mass \( M_{Z'} \) of the new gauge boson \( Z' \) predicted by topcolor scenario can be obtained via studying its effects on various observable, which has been extensively studied [3]. For example, Ref.[12] has shown that, to fit the electroweak precision measurement data, the \( Z' \) mass \( M_{Z'} \) must be larger than 1 TeV. The lower bounds on \( M_{Z'} \) can also be obtained from dijet and dilepton production at the Tevatron experiments [13], or from \( B \bar{B} \) mixing [14]. However, these bounds are significantly weaker than those from precisely electroweak data. Furthermore, Refs.[5,15] have shown that, for the coupling parameter \( k_1 < 1 \), the \( Z' \) mass \( M_{Z'} \) can be explored up to several TeV at the ILC experiment with \( \sqrt{S} = 500 \text{GeV} \) and the integrated luminosity \( \mathcal{L}_{int} = 100 \text{fb}^{-1} \). As numerical estimation, we will take \( M_{Z'} \) as a free parameter and assume that \( M_{Z'} \) is in the range of \( 1\text{TeV} \sim 2\text{TeV} \) throughout this paper. In this case, the total decay width
Γ_{Z'} is dominated by t\bar{t} and \bar{b}b modes, which can be approximately written as [16]:

\[ \Gamma_{Z'} \simeq \frac{g_1 c \cot^2 \theta' M_{Z'}}{12\pi} = \frac{1}{3} k_1 M_{Z'} \]  

(12)

In the following sections, we will use above equations to discuss the contributions of Z' to different spin configurations and the spin correlation observable for the t\bar{t} production at the ILC experiments.

III. Contributions of Z' to the cross sections of t\bar{t} production for different spin configurations

![Figure 1: The relative correction parameter R(t ↑ \bar{t} ↓) as a function of cosθ for √S = 500GeV, k_1 = 0.6, and three values of M_{Z'}.](image)

It has been shown that, in the SM, the special values of the angles ξ_L and ξ_R can make the production cross sections of the like-spin states t ↑ \bar{t} ↑ and t ↓ \bar{t} ↓ vanish for the left- and right- handed electron beam, respectively [17]. Since the production cross sections for off-diagonal spin states (t ↑ \bar{t} ↓ and t ↓ \bar{t} ↑) are non-zero, thus it is called the off-diagonal basis. From Eq.(2) and Eq.(4), we can see that, in the SM, the expressions
of the angles $\xi_L$ and $\xi_R$ corresponding the off-diagonal basis are taken to be:

$$\cos \xi_I = \frac{-A_I}{\sqrt{A_I^2 + B_I^2}}, \quad \sin \xi_I = \frac{-B_I}{\sqrt{A_I^2 + B_I^2}} \quad (13)$$

with

$$A_I = g_{IJ}^{SM}(\cos \theta + \beta) + g_{II}^{SM}(\cos \theta - \beta), \quad (14)$$

$$B_I = (g_{II}^{SM} + g_{IJ}^{SM}) \sin \theta \sqrt{1 - \beta^2}. \quad (15)$$

Obviously, there are two off-diagonal bases for $t\bar{t}$ production, one is for $e^-e^+_L$ scattering and the other is for $e^-e^+_R$. However, for the process $e^-e^+ \to t\bar{t}$, the two spin bases are almost identical. Thus, we will use the off-diagonal basis for $e^-e^+_L$ scattering even when discussing $e^-e^+_R$ scattering in our numerical calculation.

![Figure 2](image-url)

Figure 2: Same as Fig. 1 but for the relative correction parameter $R(t \downarrow \bar{t} \uparrow)$.

One interesting feature of the off-diagonal basis is that, in the $SM$, the production cross sections $\sigma_L(t \uparrow \bar{t} \downarrow)$ and $\sigma_R(t \downarrow \bar{t} \uparrow)$ are dominant for the production cross sections $\sigma_L$
of the process $e_L^+ e_R^- \rightarrow t\bar{t}$ and $\sigma_R$ of the process $e_R^+ e_L^- \rightarrow t\bar{t}$, respectively. The differential cross section is an observable containing more detailed information about the underlying dynamics of the relevant process. To see the effects of the non-universal gauge boson $Z'$ on the dominant production modes, we define the relative correction parameters as:

$$R(t \uparrow \bar{t} \downarrow) = \frac{d\sigma_{t \uparrow \bar{t} \downarrow}^{SM + Z'}}{d\cos\theta} - \frac{d\sigma_{t \uparrow \bar{t} \downarrow}^{SM}}{d\cos\theta}$$

and

$$R(t \downarrow \bar{t} \uparrow) = \frac{d\sigma_{t \downarrow \bar{t} \uparrow}^{SM + Z'}}{d\cos\theta} - \frac{d\sigma_{t \downarrow \bar{t} \uparrow}^{SM}}{d\cos\theta}.$$
Thus, their values are insensitive to the free parameter $k_1$. We have taken $k_1 = 0.6$ in Fig.1 and Fig.2. From these figures, one can see that the values of the relative correction parameters decrease as $M_{Z'}$ increasing and $\cos\theta$ increasing. For $1\,\text{TeV} \leq M_{Z'} \leq 2\,\text{TeV}$ and $\cos\theta \leq 0$, the values of $R(t \uparrow \bar{t} \downarrow)$ and $R(t \downarrow \bar{t} \uparrow)$ are in the ranges of $27\% \sim 2.3\%$ and $64\% \sim 7.7\%$, respectively. The contributions of $Z'$ to the process $e^-e^+ \to t \downarrow \bar{t} \uparrow$ are larger than those for the process $e^-e^+ \to t \uparrow \bar{t} \downarrow$, which is because the right-handed couplings of $Z'$ to fermions are larger than the left-handed couplings of $Z'$ to fermions.

Figure 4: The production cross section $\sigma$ for the like-spin states as a function of the c.m. energy $\sqrt{S}$ for different values of $M_{Z'}$ and $k_1$. 
In the off-diagonal basis, the cross section of $t\bar{t}$ production predicted by the SM vanish for the like-spin states ($\uparrow\uparrow, \downarrow\downarrow$). However, $Z'$ exchange can contribute to the relevant process $e_L^−e_R^+ + e_R^-e_L^+ \rightarrow t\bar{t}\uparrow\uparrow + t\bar{t}\downarrow\downarrow$. In Fig.3, we plot the production cross section $\sigma$ of this process as a function of $M_{Z'}$ for $\sqrt{S} = 500GeV$ and $k_1 = 0.6$. One can see from Fig.3 that, for $1TeV \leq M_{Z'} \leq 2TeV$, the value of $\sigma$ is in the range of $1.07fb \sim 4.2 \times 10^{-2}fb$. If we assume that the yearly integrated luminosity of the ILC experiment with $\sqrt{S} = 500GeV$ is $\mathcal{L}_{int} = 340fb^{-1}[18]$, then there will be several tens of the raw events of the like-spin top quark pairs to be generated per year. Certainly, the degrees of electron and positron polarization and the detecting efficiency for the final state particles are generally smaller than 100%, the number of observable events is smaller than the number of the raw events.

To see the effects of the c.m. energy $\sqrt{S}$ on the production cross section $\sigma$ for the like-spin states, $\sigma$ is shown in Fig.4 as a function of $\sqrt{S}$ for different values of $M_{Z'}$ and $k_1$. One can see from Fig.4 that the cross section resonance emerges when $M_{Z'}$ approaches the c.m. energy $\sqrt{S}$. The resonance values of the cross section $\sigma$ are strongly dependent on the $Z'$ mass $M_{Z'}$ and the coupling parameter $k_1$, which decrease as $M_{Z'}$ increasing and $k_1$ increasing. For $M_{Z'} = 1.5TeV, k_1 = 0.4$ and $0.8$, the maximum values of the production cross section $\sigma$ can reach $228fb$ and $57fb$, respectively. Thus, the resonance effects of the gauge boson $Z'$ on the process $e^-e^+ \rightarrow t\bar{t}\uparrow\uparrow + t\bar{t}\downarrow\downarrow$ might be observed in the future ILC experiments.

Of course, in practice, there are experimental issues to consider, that will dilute the signal. We must take into account the fact that the colliding electron and positron will not have 100% polarization. Moreover, there will also be a considerable dilution of the signal associated with the efficiency of measuring the polarization of the top quarks.

IV. The non-universal gauge boson $Z'$ and the $t\bar{t}$ spin correlation observable

From above discussions we can see that, at the ILC, the top quark pair $t\bar{t}$ can be produced in an unique spin configuration. In the familiar helicity basis, all spin configurations have contributions to the production cross section of the process $e^-e^+ \rightarrow t\bar{t}$. In the off-diagonal basis, the contribution from the like-spin states ($t\uparrow\bar{t}\uparrow$ and $t\downarrow\bar{t}\downarrow$) to
the $SM$ process $e^- e^+ \to t\bar{t}$ at the tree-level is zero, while the contributions from the spin states $t \uparrow \bar{t} \downarrow$ and $t \downarrow \bar{t} \uparrow$ are dominant for the cross sections of the processes $e_L^- e_R^+ \to t\bar{t}$ and $e_R^- e_L^+ \to t\bar{t}$, respectively. This means that, in the off-diagonal basis, all $t\bar{t}$ pairs predicted by the $SM$ at tree-level are of opposite spins, the decay products of polarized top quark are strongly correlated to the spin axis, the top quark events at the $ILC$ have a very distinctive topology. However, the non-universal gauge boson $Z'$ predicted by topcolor scenario has contributions to the like-spin states $t \uparrow \bar{t} \uparrow$ and $t \downarrow \bar{t} \downarrow$, which can change
this topology. Thus, the gauge boson $Z'$ can product corrections to the $t\bar{t}$ spin correlation coefficient $C$.

The $t\bar{t}$ spin correlation can be measured by analyzing the angular distributions of the $t$ and $\bar{t}$ decay products. In principle, the process $e^-e^+ \rightarrow t\bar{t} \rightarrow 6\text{ jets}$ is very complex. However, since $\Gamma_t \ll m_t$, the narrow width approximation approach is valid for $t$ and $\bar{t}$ quarks. Using this method, it has been shown that the best way to analyze the $t\bar{t}$ correlation is through angular correlation among the two charged leptons $l$ and $l'$ in the di-lepton final state \cite{19}. To obtain the differential cross section of the process $e^-e^+ \rightarrow t\bar{t} \rightarrow b\bar{b}l\nu l'\bar{\nu}$, the production and decay spin density matrixes are written as the usual approach. After integrating over the azimuthal angles of the charged leptons, one can obtain the following double differential distribution \cite{9}

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_d d\cos\theta_{l'}} = \frac{1}{4} (1 + B_1 \cos\theta_d + B_2 \cos\theta_{l'} - C \cos\theta_d \cos\theta_{l'}), \quad (18)$$

where $\theta_d (\theta_{l'})$ denotes the angle of the charged lepton $\bar{l}$ ($l'$) with respect to the chosen spin axis in the top quark (top antiquark) rest frame. The coefficients $B_1 (B_2)$ and $C$ are related to the mean $t(\bar{t})$ polarization and spin correlation projected into the direction of the spin axis. The spin correlation coefficient $C$ can be written as:

$$C = \alpha_1 \alpha_{l'} \langle s_t s_{\bar{t}} \rangle, \quad (19)$$

where $\alpha_i$ is the spin analyzing power of particle $i$, which have opposite signs for the charged leptons $\bar{l}$ and $l$. At the leading order, there are $\alpha_\bar{l} = -\alpha_l = -1$.

The contributions of the non-universal gauge boson $Z'$ to the spin correlation coefficient $C$ can be obtained by using the formula given in Sec.II. Our numerical results are given in Fig.5, in which we plot the relative correction parameter $R_c = \frac{\delta C}{C_{\text{SM}}}$ as a function of the c.m. energy $\sqrt{S}$ for different values of the $Z'$ mass $M_{Z'}$ and the coupling parameter $k_1$. From these figures one can see that, when the value of the $Z'$ mass $M_{Z'}$ approaches the c.m. energy $\sqrt{S}$, the gauge boson $Z'$ can generate significantly contributions to the observable $C$. For example, for $k_1 = 0.6$ and $M_{Z'} \simeq \sqrt{S} = 1.5\text{TeV}$ the value of the relative correction parameter $R_c$ can reach 12.5%. Obviously, when the $Z'$ mass $M_{Z'}$ is
much smaller or larger than the c.m. energy $\sqrt{S}$, the effects of the non-universal gauge boson $Z'$ on the spin correlation coefficient $C$ are very small, which can not be detected at the ILC experiments.

V. Conclusions

Top quark decays before it hadronizes and the spin information of the top quark is not spoiled and directly reflect to the distributions of its decay products. Hence, we can utilize the top quark spin for disentangling different top quark interactions efficiently. The spin correlation in top quark production and decays is an interesting issue in top quark physics.

The top quark pairs can be produced at large rates via the process $e^-e^+ \rightarrow t\bar{t}$ in a clear environment at the ILC, which can be seen as an ideal tool to make precision measurement of the top quark properties. In the off-diagonal basis, the process $e^-e^+ \rightarrow t\bar{t}$ induced by the SM has two characteristic features. One is that the production cross sections for the like-spin states vanish and the other is that the production cross section of the spin state $t \uparrow \bar{t} \downarrow (t \downarrow \bar{t} \uparrow)$ for the left-handed (right-handed) electron beam is dominant. The effects of the QCD corrections do not change these features. Thus, observation of sizable cross section for the like-spin states and large corrections to the cross sections for the $t \uparrow \bar{t} \downarrow$ and $t \downarrow \bar{t} \uparrow$ states can be seen as signals of new physics beyond the SM.

In this paper, we calculate the contributions of the non-universal gauge boson $Z'$ predicted by the TC'2 model to the spin configurations of the top quark pair production via the process $e^-e^+ \rightarrow t\bar{t}$ in the off-diagonal basis. Our numerical results show that $Z'$ exchange can generate significantly corrections to the differential cross sections for the $t \uparrow \bar{t} \downarrow$ and $t \downarrow \bar{t} \uparrow$ states. For $1TeV \leq M_{Z'} \leq 2TeV$ and $\cos \theta \leq 0$, the values of the relative correction parameters $R(t \uparrow \bar{t} \downarrow)$ and $R(t \downarrow \bar{t} \uparrow)$ are in the ranges of $26\% \sim 2.3\%$ and $64\% \sim 7.7\%$, respectively. The production cross section for the like-spin state $t \uparrow \bar{t} \uparrow + t \downarrow \bar{t} \downarrow$ is non zero induced by $Z'$ exchange, which is in the range of $1.07fb \sim 4.2 \times 10^{-2}fb$ for $\sqrt{S} = 500GeV$, $0.2 \leq k_1 \leq 0.8$, and $1TeV \leq M_{Z'} \leq 2TeV$. Furthermore, when the $Z'$ mass $M_{Z'}$ approaches the c.m. energy $\sqrt{S}$, the cross section resonance emerges. For $k_1 = 0.2$, $M_{Z'} \simeq \sqrt{S} = 1TeV$ and $1.5TeV$, the resonance values
of the cross section $\sigma(t \uparrow \bar{t} \uparrow + t \downarrow \bar{t} \downarrow)$ can reach 2586 fb and 912.5 fb, respectively.

To see whether the effects of $Z'$ on the $t\bar{t}$ production can be measured in the future $ILC$ experiments, we further calculate the contributions of $Z'$ to the $t\bar{t}$ spin correlation coefficient $C$. We find that, with reasonable values of the free parameters $M_{Z'}$ and $k_1$, the value of the relative correction parameter $R_c$ can be significantly large. Thus, we expect that the effects of the new gauge boson $Z'$ on the spin configurations of the $t\bar{t}$ production and the $t\bar{t}$ spin correlation coefficient $C$ might be observed at the future $ILC$ experiments.

Acknowledgments

This work was supported in part by Program for New Century Excellent Talents in University (NCET-04-0290), the National Natural Science Foundation of China under the Grants No.10475037 and 10675057.
References

[1] For a review, see A. Leike, Phys. Rep. 317(1999)143.

[2] G. Weiglein et al.[LHC/LC Study Group], hep-ph/0410364

[3] C. T. Hill and E. H. Simmons, Phys. Rept. 381(2003)235, [Erratum -ibid, 390(2004)553].

[4] C. T. Hill, Phys. Lett. B 345(1995)483; K. D. Lane and E. Eichten, Phys. Lett. B 352(1995)382; K. D. Lane, Phys. Lett. B 433(1998)96; G. Cvetic, Rev. Mod. Phys. 71(1999)513.

[5] M. B. Popcovic and E.H. Simmons, Phys. Rev. D 58(1998)095007; G. Burdman and N. J. Evans, Phys. Rev. D 59(1999)115005.

[6] A. A. Andrianov et al., Phys. Rev. D 58(1998)075001; E. Malkawi and C. P. Yuan, Phys. Rev. D 61(2000)015007; P. Langacker and M. Plumacher, Phys. Rev. D 62(2000)013006; K. R. Lynch et al., Phys. Rev. D 63(2001)035006; B. Murakami, Phys. Rev. D 65(2002)055003; Chong-Xing Yue, Yan-Ming Zhang, and Lan-Jun Liu, Phys. Lett. B 547(2002)252.

[7] The Tevatran Electroweak Working Group for CDF and D0 Collaborations, Combination of CDF and D0 Results on the Mass of the Top Quark, hep-ex/0608032.

[8] J. H. Kuhn, A. Reiter, P. M. Zerwas, Nucl. Phys. B 272(1986)560; J. G. Korner, A. Pilaftsis, and M. M. Tung, Z. Phys. C 63(1994)575; M. M. Tung, Phys. Rev. D 52(1995)1353; C. Schmidt, Phys. Rev. D 54(1996)3250; S. Groote and J. G. Korner, Z. Phys. C 72(1996)255; A. Brandenburg, M. Flesch, and P. Uwer, Phys. Rev. D 59(1999)014001; V. Ravindran, W. L. van Neerven, Nucl. Phys. B 589(2000)507; S. Groote, J. G. Korner, and J. H. Leyva, Phys. Lett. B 418(1998)192; H. X. Liu, C. S. Li, Z. J. Xiao, Phys. Lett. B 458(1999)393; G. J. Zhang, C. S. Li, J. J. Liu, and L. G. Jin, Commun. Theor. Phys. 40(2003)687; J. Kodaira, T. Naruno, S. J. Parke, Phys. Rev. D 59(1999)014023.
[9] M. Hori, Y. Kiyo, and T. Nasuno, *Phys. Rev. D* **58**(1998)014005; J. Kodaira, T. Nasuno, S. Parke, *Phys. Rev. D* **59**(1999)014023; Y. Kigo et al., *Nucl. Phys. Proc. Suppl.* **89**(2000)37; Z. H. Lin et al., *Phys. Rev. D* **65**(2002)014008; A. Brandenburg and M. Maniatis, *Phys. Lett. B* **558**(2003)79.

[10] Kang Young Lee et al., *Phys. Rev. D* **61**(2000)074005; Kang Young Lee et al., *Phys. Rev. D* **63**(2001)094010.

[11] G. Buchalla et al., *Phys. Rev. D* **53**(1996)5185.

[12] R. S. Chivukula and E. H. Simmon, *Phys. Rev. D* **66**(2002)015006.

[13] A. A. Andrianov et al., *Phys. Rev. D* **58**(1998)075001; K. R. Lynch et al., *Phys. Rev. D* **63**(2001)035006;

[14] E. H. Simmon, *Phys. Lett. B* **526**(2002)365.

[15] Chong-Xing Yue and Dong-Qi Yu, *J. Phys. G* **30**(2004)963.

[16] C. T. Hill and S. T. Parke, *Phys. Rev. D* **49**(1994)4454.

[17] S. J. Parke and Y. Shadmi, *Phys. Lett. B* **387**(1996)199.

[18] T. Abe et al. [American Linear Collider Group], *hep-ex/0106057*; J. A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group], *hep-ph/0106315*; K. Abe et al. [ACFA Linear Collider Working Group], *hep-ph/0109166*; G. Laow et al., ILC Technical Review Committee, second report, 2003, SLAC-R-606; E. Accomando et al. [CLIC Physics Working Group], *hep-ph/0412251*.

[19] M. Beneke et al., *Top quark physics, hep-ph/0003033*; D. Chakraborty, J. Konigsberg and D. Rainwater, *Ann. Rev. Nucl. Part. Sci.* **53**(2003)301; W. Wagner, *Rept. Prog. Phys.* **68**(2005)2409.