Formation of two-dimensional photopolymer diffractive optical elements for converting light beams with two-beam interactions taken into account

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Abstract. Bessel light beams are used in many fields, such as nonlinear optics, optical metrology, design of atmospheric wireless telecommunication lines, nanoscale objects manipulating, etc. Bessel beams do not diffract, they have restoration properties, they retain high intensity in the axial region at large distances. Thus, the search for an affordable method of forming such beams seems very relevant. In this paper, we study the process of two-dimensional diffractive optical elements (DOEs) holographic formation in photopolymer materials (PPM) with allowance for two-beam interactions. DOEs is intended for converting light beams with a Gaussian distribution into a Bessel-like ones.

1. Introduction
One of the most effective method of fundamental mode transformation into an arbitrary complex distribution is diffraction optics. As it is known, one of the most effective methods of diffractive elements formation is holographic one (using photosensitive media) [1–5]. The effectiveness of this method is due to the fact that the refractive index or absorption coefficient distributions are entirely determined by the scheme of holographic recording, as well as by the physicochemical processes occurring in medium. Thus, by changing the conditions of holographic formation, arbitrarily complex diffraction (photon) structures can be created.

2. Theoretical model
Let two monochromatic light beams with amplitude distributions \( E_0(r) \) (Gaussian distribution) and \( E_1(r) \) (Bessel-like distribution), wave vectors \( k_0 \) and \( k_1 \) fall on the boundary of the sample with thickness \( d \) containing a photopolymer material at angles \( \theta_0 \) and \( \theta_1 \) (Figure 1a). The optical field at the input boundary is described as [6–9]:

\[
E(t, r) = \sum_{j=0,1} e_j \cdot E_j \cdot \exp \left[ i \left( \omega t - k'_j \cdot r \right) \right] + \text{c.c.},
\]

where \( e_j \) – polarization vectors; \( r \) – radius-vector; \( k'_j = k \cdot N_j \), \( k = n \cdot \omega / c \) – wave number and \( N_j \) – normal to the wave front, \( n \) – refraction index.
When two light beams with distributions \( E_0(r) \) and \( E_1(r) \) fall onto the PPM, the intensity distribution of recording field \( I(t,r) \) changes under the influence of diffraction. In areas of low contrast (<0.1, Figure 1b), an additional diffraction grating (DG) is formed under the influence of two-beam interaction. The formation of an additional grating causes inhomogeneity of the amplitude-phase profile of the entire grating, which leads to an exchange of energy between the beams and a shift of the Bragg angle when reading the DG.

The intensity distribution of recording light field taking into account the two-beam interaction is described by the equation [9]:

\[
I(t,r) = I^0(x,z) \left[ 1 + m(x,z) \cos(Kr) + \frac{E_0(x,z)\delta E^*_1(t,x,y,z)}{I^0(x,z)} e^{-ikr} + \text{c.c.} \right],
\]

where \( I^0(x,z) = [I_0(x,z) + I_1(x,z)] \), \( J_1(x,z) = |E_j(x,z)|^2 \), \( j = 0,1 \), \( m(x,z) = 2\sqrt{I_0(x,z)I_1(x,z)}(e_1 \cdot e_2)/[I_0(x,z) + I_1(x,z)] \) – local contrast of interference pattern, \( \delta E_1(t,x,y,z) = -iGE_0(x) \int_0^1 n_1(t,x,z,y')dy' \) – light wave \( E_1 \) change in the field of interaction, \( G = \pi I \left[ \lambda \cos(\theta_0) \right] \), \( \lambda,\theta_0 \) – wavelength and recording angle in the material of \( E_0 \) wave, \( n_1 \) – first harmonic of the refractive index grating.

Equation for the amplitude of refractive index first harmonic taking into account two-beam interaction:

\[
n_1(t,x,y,z) = n_{p}(t,x,y,z) + n_{H}(t,x,y,z),
\]

where \( n_{p}(t,x,y,z) \) – contribution of photopolymerization-diffusion processes to the amplitude of the first harmonic, \( n_{H}(t,x,y,z) \) – contribution of self-diffraction of recording light beams [9]:

\[
n_{p}(t,x,y,z) = \delta n_p F_2(x,z) \sqrt{m_0(x,z)} \int_0^1 R(\tau,x,z)d\tau,
\]

\[
n_{H}(t,x,y,z) = \delta n_H F_2(x,z) \sqrt{m_0(x,z)} \int_0^1 R(\tau,x,z)H(\tau,x,y,z)d\tau,
\]

where \( \delta n_p \) and \( \delta n_H \) – model parameter characterizing the change of \( n \) due to polymerization and diffusion of material components; \( \tau = t/T_m \) – relative time; \( T_m = 1/(K^2_1D_m) \) – diffusion time; \( K_1 = |K_1| \) – first harmonic wave number; \( D_m \) – initial diffusion coefficient; \( F_2(x,z) = \frac{2k^2}{b_x} \frac{2k}{1 + m_0(x,z)} \); \( m_0(x,z) = I_0(x,z)/I_1(x,z) \) – ratio of intensities of recording light beams; \( b_x = b(x,z) = T_p(x,z)/T_m \); \( T_p(x,z) = \frac{h^{-1}}{[I^0(x,z)]^2} \) – local polymerization time.
$$R(\tau, x, y, z) = \frac{M_0(\tau, x, y, z)}{M_n} - \left(\frac{2^k}{b_k} - C_n\right) \cdot \int_0^\tau M_0(\tau', x, y, z) \cdot e^{-\int_0^\tau F(\tau', x, y, z) d\tau'} \cdot M_0(\tau, x, y, z) \cdot \text{monomer concentration for zero harmonic};$$

$$M_n = \text{initial monomer concentration}; \quad C_n = \delta n_i / \delta n_p;$$

$$F(\tau, x, y, z) = \frac{2^k}{b_k} - b_n(\tau, x, y, z); \quad b_n(\tau, x, y, z) = \exp\left[-s \left(1 - M_0(\tau, x, y, z) / M_n\right)\right];$$

$$H(\tau, x, y, z) = \frac{i \cdot F_2(x, z) \cdot \Gamma}{y / d} \cdot \int_0^\tau R(\tau', x, y, z) d\tau' \cdot J_1 \left[2 \sqrt{i \cdot F_2(x, z) \cdot \Gamma \cdot y / d} \cdot \int_0^\tau R(\tau', x, y, z) d\tau' \right];$$

$$J_1[x, z] = \text{Bessel function}; \quad \Gamma = \delta n_p \cdot G \cdot d = \omega \cdot d \cdot \delta n_p \cdot 2 \cdot c \cdot \cos(\phi) \quad \text{normalized coupling coefficient, characterizing the efficiency of interaction of light waves with the grating.}$$

The resulting equation (3) describes the temporal dynamics of DG spatial distribution, taking into account the two-beam interaction.

3. Experimental setup

Figures 2, 3 show experimental setups for the holographic formation and reading of DOE [10–13]. A helium-neon (He-Ne) laser with a radiation wavelength of 633 nm forms a reference light beam with a Gaussian light distribution of 1 mm in diameter and a power of 2 mW. After reflection from the mirror (M), the beam is divided into two using a beam-splitting cube (BSC). Further, the signal beam with a Gaussian intensity distribution through an AT is transformed into a Bessel-like one [14]. AT has a concentric slit width of 0.05 mm and a ring diameter of 0.4 mm. The distance from the AT to the lens (L) and from the lens to the sample corresponded to the focal length (20 cm). The angle of incidence of the reference and signal beam is 5 degrees. The reference beam after the mirror (M) was broadened using a collimator (C) to an aperture value of 4 mm. In the bulk of the PPM sample, the reference and signal beams interfere. Further, following to the holographic principle, a phase transmission hologram is formed in the sample.

Photopolymer films “GFPM633.5” produced by LLC Polymer Holograms-Novosibirsk with a layer thickness of 45 ± 5 μm on a glass substrate with a thickness of 1 ± 0.1 mm were used as PPM. The laser beam analyzer (A) captures the intensity distribution of the transmitted signal and reference beam. To read the obtained hologram, the signal light beam was blocked by a shutter (S). At the output of a formed structure, the analyzer recorded the intensity distribution of the diffracted light field.

![Figure 2. Experimental setups: (a) for DOE formation and (b) for DOE reading](image-url)
The electric field in front focal plane of lens corresponds to the Fourier transform of the incident electric field. Due to the circular symmetry of the system, this Fourier transform can be written as the Fourier-Bessel transform [15]:

\[ E_{\text{front}}(r) = A_i \frac{2\pi r_i}{i\lambda f} J_0 \left( \frac{2\pi r}{\lambda f} \right), \]

where \( A_i \) – wave amplitude, \( r_i \) – gap diameter, \( f \) – focal length. Equation (6) has the same form as the equation of zero order Bessel function.

Equation 6 is further used in the numerical simulation of the signal light beam. Geometric shadow length \( z_0 \) can be written as [15]:

\[ z_0 = \frac{Df}{2R}, \]

where \( D \) – imaging lens diameter, \( R \) – ring diameter.

Based on equation 7, the length of the geometric shadow at which a Bessel-like light beam retains diffraction-free properties was calculated. It is worth noting that these properties have not deteriorated in a diffracted light beam with side maxima amplified in terms of level (Figure 6).

4. Experimental and numerical simulation results

Figure 4a shows normalized profiles of light intensity distributions for diffracted beam, calculated by equation \( k_m[m(x, z)] = \eta_{d,i} \frac{\eta_{d,x}}{\eta_{d,i}} [m(x, z)] \) using theoretical model without two-beam interaction taken into account (\( \eta_{d,i} \)) and with it (\( \eta_{d,x} \)). Figure 4a shows that diffracted beam’s lateral maxima are amplified by two-beam interaction effect. This effect is more pronounced at the low contrast areas (\( m<0.1 \)).

Figures 4c and 4d contain experimental distributions of light intensity for signal and diffracted beams. Figure 4b shows normalized profiles of diffracted beam, calculated numerically and obtained experimentally.

standard deviation of the theoretical results from the experiment is 7.4%. A theoretical and experimental study confirms the need to take into account the influence of two-beam interaction in regions of low contrast on the diffraction characteristics of a holographically formed DOE.
Figure 4. (a) Normalized profiles of a diffracted beam’s theoretical intensity distribution along the $x$ coordinate with two-beam interaction taken into account and without; (b) Normalized profiles of diffracted beam, calculated numerically and obtained experimentally; Intensity distributions of signal (c) and diffracted (d) light beams.

Using the experimental setups shown on Figure 2, measurements of light beams diameters were made by displacing the laser beam analyzer along the direction of light beams propagation from 10 to 28 cm relative to the lens. Figure 5 shows two-dimensional intensity profiles of the Gaussian signal and diffracted beam versus the distance between the lens and the beam analyzer. From a qualitative comparison of changes in the intensity profiles from 10 to 28 cm, it can be seen that the width of signal beam remains almost constant, while the width of the Gaussian beam varies many times over a small distance. The diffracted beam repeats the nature of the signal beam.

Figure 5. Two-dimensional intensity profiles of a Gaussian, signal and diffracted beam versus the distance between the lens and the laser beam analyzer.

To quantify the change in the width of Gaussian beam and the width of central maximum of signal and diffracted beams according to the intensity level 0.5, a normalized (relative to the beam width in the waist) plot is made, illustrated in Figure 6. It can be seen from Figure 6 that at a distance of 18 to
28 cm the width of diffracted and signal beams coincides, while the Gaussian beam’s width increases by a factor of 6. This effect confirms the ability of signal and diffracted beams to compensate the diffraction by the side supply of radiation energy.

To observe the unique properties of Bessel-like beams, an obstacle was placed at a distance of 2 cm from the DOE along the diffracted beam propagation. The obstacle was a thin wire which diameter was comparable to the width of diffracted beam’s central maximum. Figure 7 shows two-dimensional profiles of the diffracted beam at a distance of 2 cm from the DOE (Figure 7a), with the introduction of an obstacle (Figure 7b) and the reconstructed beam at a distance of 6 and 16 cm (Figures 7c and 7d), respectively.

Figure 6. The measurement result with a longitudinal displacement

Figure 7. The two-dimensional intensity profiles of (a) diffracted beam, (b) diffracted beam with an obstacle, and (c, d) the restored beam

Figure 7 shows that a qualitative restoration of the central region of a Bessel-like light field was observed at a distance of 6 cm from the obstacle.

5. Conclusion
Thus, in this work we developed a theoretical model of two-dimensional photopolymer DOE holographic formation with allowance for two-beam interaction, which allows us to convert light fields into Bessel-like ones.

An experimental study was also carried out. On the base of numerical simulations and experimental results it has been shown that the level of lateral maxima of a diffracted light beam is amplified with respect to the recording field due to the influence of two-beam interaction in regions of low contrast. Therefore, in order to determine the spatial distribution of PPM refractive index during DOEs holographic formation, it is necessary to take into account the two-beam interaction of recording light beams. It will make it possible to more accurately determine the diffraction characteristics of formed optical elements.

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