Photovoltaic performance of one axis multiple-position sun-tracked PV panels

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Abstract. In this article, the photovoltaic performance of one-axis multiple-positions sun-tracked photovoltaic panels (MP-PV) is investigated based on solar geometry and dependence of photovoltaic conversion efficiency on the incident angle (IA) of solar rays on PV panels. For such PV system, the azimuth angle (AZA) of PV panels is daily adjusted several times (M) from eastward in the morning to westward in the afternoon by rotating PV panels about inclined north-south axis (INSA) to ensure the projected incident angle (PIA) of solar rays on the plane perpendicular to INSA is always less than the specified angle \( \theta_a \). Results show that, the annual electricity generation (AEG) of MP-PV increases with the increase of M, but such increase is not noticeable when M>5. For MP-PV with the tilt-angle (\( \beta \)) of INSA being yearly fixed (1T-MP-PV), the optimal \( \theta_t \) of 3P-, 5P- and 7P-PV for maximizing AEG are respectively 24°, 15° and 11.5°, and their AEGs are respectively about 92%, 94% and 95% of that from similar 2-axis tracked PV panels (2A-PV). Whereas for MP-PV with the \( \beta \) being yearly adjusted four times at three tilts (3T-MP-PV), the optimal \( \theta_t \) of 3P-, 5P- and 7P-PV are respectively about 22.5°, 14.5° and 11°, and the Pₜ are respectively about 96%, 98% and 99% of that of similar 2A-PV systems.

1. Introduction

In recent years, solar energy is gaining much attention over the world due to increasing energy demand and depletion of available fossil fuels. The conversion of solar energy into electricity is usually performed by solar photovoltaic (PV) and concentrating solar thermal power systems. The output power from PV panels highly depends on the collectible radiation. Therefore, solar panels are usually yearly oriented in such a way that it maximizes the annual collectible radiation [1-4]. However, to further increase the solar gain, it is required to track the sun. There are basically two kinds of sun tracking (ST) techniques: single- and dual-axis.

Theoretical analysis based on the extraterrestrial radiation indicated that the annual collectible radiation (ACR) on a polar mount sun-tracked collector was about 96% of that a 2-axis sun-tracked
collector annually captured [5]. Theoretical calculations by Chang [6] based on the extraterrestrial radiation indicated that the gain in ACR on an inclined north-south axis (INSA) sun-tracked solar panel over a fixed panel was about 50% in areas with the site latitude below 65°. Chang and Li found that the ACR on east-west axis tracked panels was much less than that on INSA tracked panels [7, 8]. The study conducted by Ghosh et Al. [9] showed that solar panels tracking the sun about polar-axis annually collected about 98.22% of that a 2-axis tracked solar panel annually collected. Studies by Li and co-authors [10, 11] showed that the ACR on vertical axis and INSA tracked solar panels are respectively about 96% and 97% of that on a full 2-axis tracked panel. It is also found that the ACR on INSA tracked panels can be greatly improved by yearly adjusting the tilt-angle of INSA four times at three tilts, and the optimal tilt angle of vertical axis tracked panels for maximizing ACR is almost linearly proportional to the site latitude. The study by Morcos [12] indicated that the ACR on a solar collector was increased by 29.2% by setting the azimuth and tilt angles to their optimal values daily [12]. Measurements conducted by Al-Mohamad showed that compared to a fixed PV module, the daily electricity generation from a vertical axis tracked PV module was increased by more than 20% [13]. An experimental study by Abdallah [14] showed that compared with the fixed PV module in Jordan, there were electrical increases up to 43.87%, 37.5%, 34.43% and 15.69% for 2-axis, INSA, vertical axis and horizontal north-south axis sun-tracked PV panels, respectively. Measurements by Kacira [15] showed that compared to fixed PV modules in Turkey, the solar and power gains of 2-axis sun-tracked PV modules were 29.3% and 34.6%, respectively, on a particular day on July. Gómez-Gil [16] compared the real annual electricity generation (AEG) from fixed, 1-axis and 2-axis tracked PV systems installed in Spain in 2009. It was found that compared with the fixed systems, 1- and 2-axis tracking systems had 22.3% and 25.2% increase in the AEG, respectively, but less than 32.1% and 38.7% predicted based on the data from Photovoltaic Geographical Information System (PVGIS). Chin [17] presented the design, modeling and testing of an active single axis solar tracker, and a 20% increase in efficiency over a fixed panel design was found. In recent years, solar panels are being widely used for water pumping [18,19], but the utilization of collectible radiation on PV panels for pumping water is low as pumps operate only when the solar intensity on PV modules is higher than the level required to start pumps. Experimental tests by Vilela showed that compared to fixed photovoltaic water pumping system, the daily solar gain on north-south axis tracked PV panels were about 19 to 24%, but the gain in the pumped water volume were about 37 to 41% [20].

A number of studies indicated that the ST was an effective way to increase the solar radiation collection and electricity generation of PV panels. Generally, benefits due to ST were about 20-40% [6]. The benefits are high in the areas with abundant solar radiation resources and low in the sites poor in solar radiation resources as ST increases the collection of beam radiation. Among various ST strategies, 2-axis ST is the most efficient, and followed by INSA ST and the horizontal east-west ST is the worst [8]. As compared to 1-axis ST strategies, 2-axis ST enabled PV panels to increase energy gains about 3-5% [21]. It was also found that the power gains of tracked PV panels were commonly higher than the solar gains thanks to the increased photovoltaic conversion efficiency resulting from the reduced incident angle (IA) of solar rays on solar panels [22]. Some researchers suggested that 2-axis trackers were more efficient and could be cost-effective if bigger systems were implemented [23, 24]. But some argued that ST could eventually disappear due to their complexity and running
costs [25]. Recent publication of Nicola Pearsall indicated that, at present level of PV prices for large-scale fixed ground mounted systems, the 25% annual gain in the electricity generation due to ST would be covered by the cost of 200 kg metal needed per kWp to build the 2-axis tracker, but the optimized 1-axis systems only required about 100 kg metal per kWp [26]. This means that, at present level of PV prices, the 2-axis sun-tracking PV systems are not economically attractive, but 1-axis tracking systems are economically attractive compared to fixed PV systems.

However, for one-axis ST systems, a complicated and expensive ST system is commonly required, hence, tracked PV systems often suffers from mechanical and control failures in practical applications. To simplify ST techniques, Huang and Sun proposed a one-axis three-position ST system (1A-3P) [27]. The tests by Huang over 13 months in Taipei showed that as compared to a fixed PV, the AEG from 1A-3P tracking PV was increased by 23.6% [28]. Calculations showed that if 1A-3P tracking PV was used in the region with abundant solar resources, there can be an increase of AEG higher than 37.5% in comparison with fixed PV system, close to the AEG from a 2-axis continuous tracking PV. A theoretical study by Zhong indicated that the ACR of the 1A-3P sun-tracked solar panels was above 96% of that on 2-axis tracked PV panels [29]. The study on the performance of PV modules with daily two-position ST was presented by Tomson [30]. Results indicated that as compared to a fixed collector, the seasonal energy gain was 10-20%.

For non-concentrating PV modules, the aim of using ST is to reduce the IA of solar ray on PV modules, hence ST increases the collection of beam radiation meanwhile improves the photovoltaic conversion efficiency. However, the reduction in the radiation collection and photovoltaic conversion efficiency of PV modules due to even 20° deviation of incident rays from the normal incidence is insignificant as the collectible beam radiation is proportional to the cosine of IA and the photovoltaic conversion efficiency is almost kept unchanged when the IA is less than 40° [22]. This means that, to improve the power output from non-concentrating PV modules, a simple tracking technique which makes the IA always within a specified angle, such as 20°, may be sufficient. In this work, a new design, INSA multiple positions sun-tracking PV system (MP-PV) was proposed. As shown in figures 1-2, for such PV system, the PV panel is oriented towards the east in the early morning, and the azimuth angle (AZA) is daily adjusted several times (M) from eastward in the morning to westward in the afternoon by rotating the PV panel about INSA to ensure the projected incident angle (PIA) of solar rays on the plane perpendicular to INSA always less than the specified angle $\theta_s$. To investigate the performance of the proposed system, a mathematical procedure was suggested based on solar geometry and dependence of photovoltaic conversion efficiency on IA of solar rays on PV panels with the aim to find the optimal design for maximizing AEG and compare its AEG with those from fixed, 1-axis and 2-axis tracked PV systems.
2. Mathematical procedures to calculate solar gain and electricity generation of PV panels

2.1. Fixed south-faced PV panels

It is assumed that the directional intensity of sky diffuse radiation from all directions of the sky dome is identical and the radiation reflecting from the ground is not considered. Hence, at any time of a day, the collectible radiation on unit area of fixed south-faced PV panels inclined from the horizon is given by:

\[
I_0 = I_b \cos \theta_{in,0} g(\theta_{in,0}) + 0.5(1+\cos \beta) I_d
\]  

(1)

where \(I_b\) is intensity of beam radiation on the surface normal to incident rays, \(I_d\) is sky diffuse radiation on the horizon, \(\theta_{in,0}\) is the IA of solar rays on fixed solar panels. To make the analysis easy and convenient, a coordinate system fixed on south-faced PV panels tilted at \(\beta\) from the horizon was employed. As shown in figure 1 and figure 2, the x-axis is normal to solar panels, y-axis is parallel to the horizon and pointing to the east, and z-axis is pointing to the northern sky dome. Hence, the unit vector from the earth to the sun in the coordinate system is expressed by [5, 10]:

\[
n_s = (n_x, n_y, n_z)
\]  

(2)

in which
\[
\begin{align*}
\mathbf{n}_x &= c \cos \phi - \cos \theta \, \cos \phi \sin \theta \sin \phi \\
\mathbf{n}_y &= -c \cos \phi - \cos \theta \, \cos \phi \sin \theta \cos \phi \\
\mathbf{n}_z &= -c \sin \phi - \cos \theta \, \sin \phi
\end{align*}
\]  
\tag{3}
\]

where \( \lambda \) is the latitude, \( \omega \) is the hour angle, and \( \delta \), the declination of the sun, is subjected to expression as [5]:

\[
\sin \delta = -\sin 23.45 \cos [360(n+10)/365.25]
\]  
\tag{4}
\]

where \( n \) is the day number counting from the first day of a year. The \( \theta_{in,0} \) in Eq.1 can be calculated by:

\[
\cos \theta_{in,0} = n \cdot (1,0,0)
\]  
\tag{5}
\]

as unit vector of the normal to fixed PV panels is (1,0,0). The \( g(\theta_{in,0}) \) in Eq.1 is a control function, it is 1 when \( \cos \theta_{in,0} > 0 \) and 0 when \( \cos \theta_{in,0} \leq 0 \). The power output from fixed PV panels is calculated by:

\[
P = I_d \eta_{pv}(\theta_{in,0}) + P_d = I_d \eta_{pv}(\theta_{in,0}) + P_d
\]  
\tag{6}
\]

where \( \eta_{pv}(\theta_{in,0}) \) is the photovoltaic conversion efficiency of solar cells for radiation incident at \( \theta_{in,0} \).

The electricity output from a PV panel is usually affected by many factors. To make the comparison of AEG from different sun-tracked PV systems meaningful, it is assumed that except IA, effects of all factors on the photovoltaic conversion efficiency of solar panels are identical, and the angular dependence of photovoltaic conversion efficiency of solar panels is subjected to [22]:

\[
\eta_{pv} = \begin{cases} 
15.5494 + 0.02325 \theta_{in} - 0.00301 \theta_{in}^2 + 9.4685 \times 10^{-5} \theta_{in}^3 - 1.134 \times 10^{-6} \theta_{in}^4 & (0 < \theta_{in} < 65^\circ) \\
41.52 - 0.4784 \theta_{in} & (65^\circ \leq \theta_{in} < 90^\circ)
\end{cases}
\]  
\tag{7}
\]

\( P_d \) in Eq.6 is the electricity produced by sky diffuse radiation. For isotropic sky diffuse radiation, it can be estimated by [31]:

\[
P_d = C_{d, pv} I_d
\]  
\tag{8}
\]

\[
C_{d, pv} = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta \eta_{pv}(\theta) d\theta
\]  
\tag{9}
\]

where \( \phi_b = 0 \) when \( \theta \leq 0.5 \pi - \beta \), and it is given by \( \cos \phi_b = \tan \beta \tan \phi \) for \( 0.5 \pi - \beta < \theta \leq 0.5 \pi \). Eq.9 shows that \( C_{d, pv} \) is a function of \( \beta \) as it depends on the sky dome viewed from the tilted PV surface. Hence, given \( \beta \), \( C_{d, pv} \) is a constant and can be numerically calculated. The daily radiation on fixed PV panels can be obtained by integrating Eq.1 over the daytime as:

\[
H_{day, \theta} = \int_{t_0}^{t_1} I \eta \, g(\theta_{in,0}) \, dt + 0.5(1 + \cos \beta) H_d
\]  
\tag{10}
\]

And the daily electricity from fixed PV panel is calculated by:

\[
P_{day, \theta} = \int_{t_0}^{t_1} I \eta \, g(\theta_{in,0}) \eta_{pv}(\theta_{in,0}) \, dt + C_{d, pv} H_d
\]  
\tag{11}
\]

t_0 in Eqs.10-11 is the sunset time in the day and can be determined based on \( \lambda \) and \( \delta \).
2.2. Two-axis sun-tracked PV panels

For 2-axis sun-tracked solar panels (2A-PV), the normal of PV panels is required to point towards the sun, hence, the tilt-angle ($\beta_{2A}$) of 2A-PV is equal to the zenith angle of the sun ($\theta_z$). Therefore, the daily radiation collection is given by:

$$H_{day,2A} = \int_{t_b}^{t_e} [I_b + 0.5(1 + \cos \theta_z)I_d]dt$$  \hspace{1cm} (12)

The daily power output is calculated by:

$$P_{day,2A} = \int_{t_b}^{t_e} [I_b\eta_{pv}(0) + C_{d, pv}I_d]dt$$  \hspace{1cm} (14)

2.3. Inclined north-south single axis sun-tracked PV panels

For PV panels tracking the sun about INSA (1A-PV), to minimize the IA, it is required that the normal of PV panels always overlays the projection of incident solar rays on the plane perpendicular to INSA, thus, the vector of the normal to PV panels is given by:

$$n_{m,1A} = (n_x, n_y, 0) / (\sqrt{n_x^2 + n_y^2})$$  \hspace{1cm} (15)

The IA of solar rays on 1A-PV can be calculated by:

$$\cos \theta_{m,1A} = n_x \cdot n_{m,1A} = \sqrt{n_x^2 + n_y^2}$$  \hspace{1cm} (16)

The tilt angle of 1A-PV relative to the horizon is given by:

$$\cos \beta_{1A} = n_x \cdot n_{m,1A} = (\cos \beta, 0, \sin \beta) \cdot (n_x, n_y, 0) / (\sqrt{n_x^2 + n_y^2}) = \cos \beta / \sqrt{n_x^2 + n_y^2}$$  \hspace{1cm} (17)

as the vector of the normal to the horizon is $n = (\cos \beta, 0, \sin \beta)$ in the suggested coordinate system. Therefore, the daily radiation on 1A-PV is given by:

$$H_{day,1A} = \int_{t_b}^{t_e} [I_b \cos \theta_{m,1A} + 0.5(1 + \cos \beta_{1A})I_d]dt$$  \hspace{1cm} (18)

The daily electricity generation is calculated by:

$$P_{day,1A} = \int_{t_b}^{t_e} [I_b \cos \theta_{m,1A}\eta_{pv}(\theta_{m,1A}) + C_{d, pv}I_d]dt$$  \hspace{1cm} (19)

The $C_{d, pv}$ in the above equation is a function of $\beta_{1A}$, and calculated by Eq.9.

As shown in figure 2, to ensure the projection of solar rays on the plane perpendicular to INSA within the specified angle $\theta_a$, the AZA of solar panels should be adjusted based on projected angle $\theta_{p,0}$ and number of daily AZA adjustment. As an example, for 7P-PV, it is given by:

$$\gamma' = \begin{cases} 0 & (\theta_{p,0} \leq \theta_a) \\ 2\theta_a & (\theta_a < \theta_{p,0} \leq 3\theta_a) \\ 4\theta_a & (3\theta_a < \theta_{p,0} \leq 5\theta_a) \\ 6\theta_a & (\theta_{p,0} > 5\theta_a) \end{cases}$$  \hspace{1cm} (20)

As shown in figure 2, the $\theta_{p,0}$, measuring from x-axis, is calculated by:

$$\theta_{p,0} = \begin{cases} \text{Atn}[n_y / n_x] & (n_x > 0) \\ 0.5\pi & (n_x = 0) \\ 0.5\pi + \text{Atn}[n_y / n_x] & (n_x < 0) \end{cases}$$  \hspace{1cm} (21)
It is noted that \( \gamma \), measuring from x-axis to west, is positive in the afternoon. Hence, the vector of the normal of MP-PV at \( i^{th} \) ST position \((i=0, \pm 1, \pm 2\ldots)\) is given by:

\[
n_{n,MP} = (\cos \gamma, -\sin \gamma, 0)
\]  

(22)

The IA of solar rays on MP-PV is calculated by:

\[
\cos \theta_{in,PM} = n_x \cdot n_{n,MP} = n_x \cos \gamma_y - n_y \sin \gamma_i
\]  

(23)

The tilt angle of MP-PV is given by:

\[
\cos \beta_{MP} = n_h \cdot n_{n,MP} = (\cos \beta_x, 0, \sin \beta) \cdot n_{n,MP} = \cos \beta \cos \gamma_i
\]  

(24)

It is known from Eq.24 that the \( \beta_{MP} \) differs at different ST position, hence, the daily collectible radiation on MP-PV should be calculated by:

\[
H_{day,MP} = \int_{t_0}^{t_f} [I_s \cos \theta_{in,PM} + 0.5(1 + \cos \beta_{MP})I_d]dt
\]  

(25)

Also, the daily electricity generation from MP-PV is calculated by:

\[
P_{day,MP} = \int_{t_0}^{t_f} [I_s \cos \theta_{in,MP} \eta_{pv}(\theta_{in,MP}) + C_{d,pv}I_d]dt
\]  

(26)

The \( C_{d,pv} \) in the above equation is a function of \( \beta_{MP} \), thus differs at different sun-tracking position, and can be calculated by Eq.9.

Analysis in the above shows that at any moment of a day, the position of the sun in the sky dome can be determined in terms of \( n_x \), then the IA of solar rays on fixed and sun-tracked PV panels can be obtained, meanwhile, the tilt-angle of tracked PV panels also can be determined. Therefore, the daily collectible radiation and electricity generation from fixed and tracked PV panels could be numerically obtained on the time variations of \( I_s \) and \( I_d \) in the day, then ACR \((S_n)\) and AEG \((P_d)\) can be obtained.

In the consequent calculations, the monthly horizontal radiation averaged over many years in five sites of China with typical climatic conditions (Beijing and Lhasa are abundant but Chongqing poor in solar radiation resources; Shanghai, rainy in winters and hot in summers; Kunming, rainy in summers and sunny in winters) was employed for the analysis [32]. The monthly average daily sky diffuse radiation on the horizon \((H_d)\) and time variations of \( I_s \) and \( I_d \) in a day are estimated based on the correlations proposed by Collares-Pereira and Rabl [33]. The interval of time for the calculation of daily radiation collection and electricity generation is taken to be 1 minute, the interval of \( \theta \) for calculating \( C_{d,pv} \) is set to be 0.2°. For fixed south-faced PV panels, the tilt angle is set to be latitude \( (\lambda)\). For INSA sun-tracked PV panels \((1A-PV)\) and MP-PV, two cases with the tilt-angle \( (\beta)\) of INSA being yearly fixed and yearly adjusted four times are considered. For the case with the \( \beta \) yearly fixed, \( \beta \) is set to \( \lambda \), whereas for the case with \( \beta \) yearly adjusted four times, \( \beta \) is set to \( \lambda \) during the periods of 23 days before and after both equinoxes, and taken to be \( \lambda +23^\circ \) and \( \lambda -23^\circ \) in winters and summers, respectively[10]. For the MP-PV addressed here, the AZA is symmetrically adjusted about the solar-noon \((\gamma_0=0 \text{ at solar-noon})\), hence, the number \((M)\) of daily AZA adjustments is an odd, such as 3, 5 and 7. Theoretically, with the increase of \( M \), the AEG of MP-PV \( (P_{a,MP})\) increases, but more frequent AZA adjustment in a day is not practical, thus, the analysis in this work is limited to \( M=3, 5 \text{ and } 7 \).
3. Results and discussions

3.1. Performance of 1A-PV and 2A-PV

Table 1 presents the ACR (S_<sub>a</sub>,0, S_2A) and AEG (P_<sub>a</sub>,0, P_2A) of fixed and 2-axis sun-tracked PV systems. It is seen that as compared to fixed PV system, the increases of ACR (C_<sub>a</sub>,2A,0=S_2A/S_<sub>a</sub>,0) and AEG (C_<sub>p</sub>,2A,0=P_2A/P_<sub>a</sub>,0) of 2A-PV are sensitive to solar radiation resources in the site. For a given site, the AEG increase factors, C_<sub>p</sub>,2A,0, are commonly higher than the ACR increase factors, C_<sub>a</sub>,2A,0, a result of the fact that ST results in the improvement of photovoltaic conversion efficiency of PV panels. It also shows that, in the sites with abundant solar radiation resources, such as Beijing and Lhasa, the AEG from a 2A-PV is increased by above 37.5% in comparison with similar fixed PV, highly agreed with the results expected from PVGIS by Gomez-Gil and Huang [16, 28]. This indicates that the method presented in this work can reasonably expect the performance of tracked PV system in terms of the AEG increase factor (C_<sub>p</sub>,2A,0 or C_<sub>p</sub>,1A,0).

| site       | λ   | K_i | S_<sub>a</sub>,0 | P_<sub>a</sub>,0 | S_<sub>2A</sub> | P_<sub>2A</sub> | C_<sub>a</sub>,2A,0 | C_<sub>p</sub>,2A,0 |
|------------|-----|-----|------------------|----------------|--------------|---------------|----------------|----------------|
| Beijing    | 39.95 | 0.522 | 6096 | 856 | 7836 | 1178 | 1.285 | 1.375 |
| Shanghai   | 31.20 | 0.416 | 4942 | 687 | 5948 | 881 | 1.203 | 1.281 |
| Lhasa      | 29.71 | 0.732 | 9323 | 1306 | 12357 | 1872 | 1.325 | 1.434 |
| Chongqing  | 29.50 | 0.324 | 3735 | 516 | 4287 | 626 | 1.148 | 1.212 |
| Kunming    | 25.03 | 0.49 | 6216 | 864 | 7635 | 1139 | 1.228 | 1.319 |

Table 2. Performance of INSA sun-tracked PV panels

| site       | S_<sub>1T</sub> | P_<sub>1T</sub> | S_<sub>3T</sub> | P_<sub>3T</sub> | C_<sub>s</sub>,1T,0 | C_<sub>s</sub>,3T,0 | C_<sub>p</sub>,1T,2A | C_<sub>p</sub>,3T,2A |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Beijing    | 7536 | 1131 | 7776 | 1170 | 1.236 | 1.276 | 0.96 | 0.994 |
| Shanghai   | 5736 | 852 | 5915 | 877 | 1.161 | 1.197 | 0.967 | 0.995 |
| Lhasa      | 11940 | 1802 | 12277 | 1862 | 1.281 | 1.317 | 0.963 | 0.995 |
| Chongqing  | 4161 | 607 | 4266 | 623 | 1.114 | 1.142 | 0.971 | 0.996 |
| Kunming    | 7424 | 1104 | 7599 | 1135 | 1.194 | 1.222 | 0.969 | 0.996 |

The performance of INSA tracked PV systems (1A-PV) is presented in table 2. It is seen that for 1A-PV with the β of INSA yearly fixed (1T-1A-PV), the AEG is about 96-97% (see the column of C_<sub>p</sub>,1T,2A) of that from similar 2A-PV; whereas for 1A-PV with the β yearly adjusted four times at three-tilts (3T-1A-PV), the AEG is almost identical to that of similar 2A-PV systems (see the column of C_<sub>p</sub>,3T,2A). This means that INSA sun-tracking technique is very efficient to improve the performance of non-concentrating PV systems.

3.2. Performance and optimal design of MP-PV

3.2.1 Performance and optimal design of 1T-MP-PV. Analysis in the previous sections shows that in a specific site, the ACR and AEG of MP-PV with the β of INSA yearly fixed (1T-MP-PV) are sensitive to daily AZA adjustment number (M) and θ_a. Hence, given M, ACR (S_<sub>a</sub>) and AEG (P_<sub>a</sub>) depend on θ_a, hence the optimal θ_a for maximizing ACR and AEG can be respectively found by iterative calculations. As shown in figures 3-4, compared to fixed PV systems, the AEG increase factors
(C_{p,MP-0}) of 1T-MP-PV are commonly higher than the ACR increase factors (C_{s,MP-0}), and the optimal \( \theta_a \) for maximizing ACR are about 2\(^\circ\) higher than that for maximizing AEG. It is also found that the reduction in \( P_s \) due to 2\(^\circ\) deviation of \( \theta_a \) from the optimal value for maximizing AEG is less than 0.1%.

![Figure 3](image-url)  
**Figure 3.** Effects of \( \theta_a \) on the ACR and AEG of 1T-3P-PV

![Figure 4](image-url)  
**Figure 4.** Effects of \( \theta_a \) on the ACR and AEG of 1T-5P-PV

| site    | \( \theta_a,3P \) | \( \theta_a,5P \) | \( \theta_a,7P \) | \( C_{p,3P-0} \) | \( C_{p,5P-0} \) | \( C_{p,7P-0} \) | \( C_{p,3P-2A} \) | \( C_{p,5P-2A} \) | \( C_{p,7P-2A} \) |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Beijing | 24.8           | 15.8           | 11.8           | 1.262          | 1.295          | 1.305          | 0.917          | 0.942          | 0.949          |
| Shanghai| 23.0           | 14.6           | 11.0           | 1.198          | 1.222          | 1.229          | 0.935          | 0.953          | 0.959          |
| Lhasa   | 25.8           | 16.6           | 12.2           | 1.318          | 1.355          | 1.366          | 0.919          | 0.945          | 0.953          |
| Chongqing| 23.0          | 14.0           | 11.0           | 1.145          | 1.164          | 1.168          | 0.945          | 0.96           | 0.964          |
| Kunming | 23.0           | 14.8           | 11.0           | 1.233          | 1.26           | 1.268          | 0.935          | 0.955          | 0.961          |

Table 3 presents the optimal \( \theta_a \) of 1T-MP-PV (M=3,5,7) for maximizing AEG and performance of optimized 1T-MP-PV in terms of \( C_{p,MP-0} \) and \( C_{p,MP-2A} \), the ratios of AEG from MP-PV to that from
similar fixed and 2A-PV systems, respectively. It is seen that, the $\theta_{a,3p}$, the optimal $\theta_a$ of 3P-PV, ranges from 23° in the sites with poor solar resources to about 25° in the sites with abundant solar resources, hence $\theta_{a,3p}$ =24° is advisable as the optimal design of 1T-3P-PV because the reduction in AEG due to 2° deviation of $\theta_a$ from the optimal value is insignificant. Similarly, $\theta_{a,5p}$ =15° and $\theta_{a,7p}$ =11.5° are respectively recommended as the optimal design of 5P- and 7P-PV. Table 3 shows that the AEG in terms of $C_{p,MP,0}$ increases with the increase of M, but such increase is not noticeable when M>5. Because the collectible radiation on PV panels is proportional to cos $\theta_a$, the $\theta_a$ is always kept within 16° when M>5 and the average value of cos $\theta_a$ from 0 to 16° is 0.987, hence a further increase of collectible radiation by reducing IA is not noticeable. This means that the 1T-MP-PV with M>5 are not advisable in practical application to simplify ST. It is also seen that, the AEG of 3P-, 5P- and 7P-PV are about 92%, 94% and 95% of that from the similar 2A-PV system, respectively.

### 3.2.2 Performance and optimal design of 3T-MP-PV

The optimal design of MP-PV with $\beta$ of INSA yearly adjusted four times at three tilts (3T-MP-PV) and performance of optimized 3T-MP-PV are presented in Table 4. It is seen that, in a given site, the optimal $\theta_{a,3p}$ of 3T-3P-PV is about 1.5° lower than that of similar 1T-3P-PV, and the optimal $\theta_{a,5p}$ and $\theta_{a,6p}$ are about 0.5 lower than those of similar 1T-5P-, and 1T-7P-PV, respectively. Hence, $\theta_{a,3p}$ =22.5°, $\theta_{a,5p}$ =14.5° and $\theta_{a,7p}$ =11° are respectively recommended as the optimal design of 3T-3P-, 3T-5P- and 3T-7P-PV. It also shows that the AEG of 3T-3P-, 3T-5P- and 3T-7P-PV are respectively about 96%, 98% and 99% of that from similar 2A-PV.

| site       | $\theta_{a,3p}$ | $\theta_{a,5p}$ | $\theta_{a,7p}$ | $C_{p,3P,0}$ | $C_{p,5P,0}$ | $C_{p,7P,0}$ | $C_{p,3P,2A}$ | $C_{p,5P,2A}$ | $C_{p,7P,2A}$ |
|------------|------------------|------------------|-----------------|--------------|--------------|--------------|---------------|---------------|---------------|
| Beijing    | 23.4             | 15.2             | 11.2            | 1.316        | 1.346        | 1.355        | 0.957         | 0.979         | 0.986         |
| Shanghai   | 21.0             | 14.0             | 10.0            | 1.239        | 1.261        | 1.268        | 0.967         | 0.984         | 0.989         |
| Lhasa      | 24.2             | 15.6             | 11.6            | 1.37         | 1.404        | 1.414        | 0.955         | 0.979         | 0.987         |
| Chongqing  | 21.0             | 14.0             | 10.0            | 1.182        | 1.197        | 1.202        | 0.975         | 0.987         | 0.992         |
| Kunming    | 21.8             | 14.2             | 10.6            | 1.272        | 1.297        | 1.305        | 0.965         | 0.984         | 0.99          |

### 3.2.3 Optimal design of 3T-MP-PV for maximizing power output in each of four seasons

Analysis in the previous sections shows that the performance of MP-PV can be greatly improved by yearly adjusting $\beta$ of INSA four times at three tilts. In this section, an attempt is made to find the optimal design of 3T-MP-PV for maximizing electricity generation in each of four seasons. In this case, the $\beta$ of INSA is yearly adjusted four times at three tilts, and the $\theta_a$ in each of four seasons is set to such value that maximizes the electricity generation in the season. As shown in Table 5, optimal values of $\theta_a$ in springs ($\theta_{a,sp}$), summers ($\theta_{a,au}$), autumns ($\theta_{a,sea}$) and winters ($\theta_{a,sea}$) are slightly different as the solar radiation resource and distribution over different seasons commonly differ, but the AEG ($P_{a,sea}$) from seasonally optimized 3T-3P-PV, is almost identical to $P_s$ from yearly optimized 3T-3P-PV. This implies that a further design optimization of 3T-3P-PV by maximizing power output in each of four seasons is meaningless.

| site       | $\theta_{a,sp}$ | $\theta_{a,au}$ | $\theta_{a,sea}$ | $P_{a,sea}$ | $P_s$ |
|------------|------------------|------------------|-----------------|-------------|-------|
| Beijing    | 24.4             | 24.8             | 20.4            | 1127.9      | 1126.8 |
| Shanghai   | 21.5             | 21.8             | 20.0            | 852         | 852   |
| Lhasa      | 25.6             | 25.8             | 22.0            | 1790        | 1788.6 |
| Chongqing  | 20.0             | 20.0             | 20.0            | 610.5       | 610.2 |
| Kunming    | 22.5             | 22.8             | 21.4            | 1099.1      | 1098.8 |
4. Conclusions

Results show that as compared to fixed PV system, the AEG increase factors of sun-tracked PV systems are commonly higher than the ACR increase factors as a result of the fact that tracking the sun not only increases the collection of beam radiation but also improves the photovoltaic conversion efficiency due to reduced IA. Calculations show that the AEG from INSA sun-tracked PV system (1A-PV) with the $\beta$ of INSA yearly adjusted four times is highly close to that from similar 2-axis sun-tracked PV system (2A-PV). Compared to fixed PV systems, the AEG from similar 2A-PV is increased about 38% in sites with abundant solar radiation resources, and this result is in agreement with those reported in the literatures where PVGIS and MATLAB/Simulink model were employed. This implies that the mathematical method presented in this work can reasonably predict the photovoltaic performance of sun-tracked PV systems in terms of AEG increase factor, although it cannot reasonably estimate the real AEG from sun-tracked PV systems.

Results show that the AEG of MP-PV increases with the increase of daily AZA adjustment number (M), but such increase was not noticeable when M>5. For MP-PV with the $\beta$ of INSA yearly fixed (1T-MP-PV), the optimal $\theta_c$ of 3P-, 5P- and 7P-PV for maximizing the AEG are respectively 24°, 15° and 11.5°, and the their AEGs are respectively about 92%, 94% and 95% of that from similar 2A-PV systems. Whereas for MP-PV with the $\beta$ of INSA yearly adjusted four times at three tilts (3T-MP-PV), the optimal $\theta_c$ of 3P-, 5P- and 7P-PV are respectively about 22.5°, 14.5° and 11°, and the AEGs are respectively about 96%, 98% and 99% of that of similar 2A-PV systems. It is also found that a further optimization of 3T-3P-PV by maximizing the power output in each of four seasons is meaningless as the AEG from seasonally optimized 3T-MP-PV is almost identical to that from the one yearly optimized.

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Glossary

$C_{d,pv}$: ratio of electricity generation of solar PV contributed by sky diffuse radiation to diffuse radiation on the horizon (dimensionless)

$C_{P_{A,2A}}$: annual power output increase of 2A-PV compared to similar fixed PV system (dimensionless)

$C_{P_{A,1A}}$: ratio of $P_a$ from 1A-1A-PV to that of similar 2A-PV system (dimensionless)

$C_{P_{A,3T}}$: ratio of $P_a$ from 3T-1A-PV to that of similar 2A-PV system (dimensionless)

$C_{P_{A,MP}}$: annual power output increase of MP-PV compared to similar fixed PV system (dimensionless)

$C_{P_{A,MP}}$: ratio of $P_a$ from MP-PV to that of similar 2A-PV system (dimensionless)

$C_{S_{A,2A}}$: annual solar gain increase of 2A-PV over similar fixed PV system (dimensionless)

$C_{S_{A,1A}}$: annual solar gain increase of 1A-1A-PV over similar fixed PV system (dimensionless)

$C_{S_{A,3T}}$: annual solar gain increase of 3T-1A-PV over similar fixed PV system (dimensionless)

$C_{S_{A,MP}}$: annual solar gain increase of MP-PV over similar fixed PV system (dimensionless)

$C_{S_{A,MP}}$: ratio of $S_a$ from MP-PV to that of similar 2A-PV system (dimensionless)

$H$: daily solar gain (MJ/m$^2$);

$I$: instantaneous radiation intensity (W/m$^2$)

$K_t$: annual average atmosphere clearness index (dimensionless)

$M$: number of daily azimuth angle adjustment

$N$: days counting from equinoxes

$n$: unit vector of normal to a surface

$n_s$: unit vector of incident solar rays

$P_a$: annual power output of PV panels (MJ/m$^2$)

$S_a$: annual collectible radiation on PV panels (MJ/m$^2$)

$t$: solar time (s)

Greek letters

$\beta$: tilt-angle of INSA or PV panel relative to the horizon, radian

$\gamma$: azimuth angle of MP-PVs relative to $x_0$-axis, radian

$\delta$: declination of the sun, radian

$\eta_{pv}(\theta_m)$: photovoltaic efficiency of solar cells as a function of $\theta_m$, dimensionless

$\lambda$: site latitude, radian in expressions and degree in text
$\theta_a$: allowable maximum projected incident angle of solar rays on MP-PV panel
$\theta_{in}$: incidence angle of solar rays on PV panels, radian
$\omega$: hour angle, radian

**Subscripts**
- 0: sunset; fixed solar panel
- 1A: 1-axis sun-tracked PV panel
- 2A: 2-axis sun-tracked PV panel
- a: annual
- b: beam radiation
- d: sky diffuse radiation
- day: daily
- h: horizon
- $i$: at $i^{th}$ sun-tracking position
- MP: multiple-positions sun-tracked PV panel
- p: power
- s: solar gain

**Abbreviations**
- 1A-PV: INSA tracked PV panel
- 2A-PV: 2-axis tracked PV panel
- 1T-MP-PV: MP-PV with the tilt angle of INSA yearly fixed
- 3T-MP-PV: MP-PV with the tilt angle of INSA yearly adjusted four times at three tilts
- AEG: annual electricity generation
- ACR: annual collectible radiation
- AZA: azimuth angle of MP-PV
- IA: incident angle
- INSA: inclined north-south axis
- MP-PV: INSA multiple-position tracked PV panels
- PIA: projected incident angle
- ST: sun-tracking