Chiral-Scale Perturbation Theory
About an Infrared Fixed Point

Lewis C. Tunstall
B. Sc. (Hons)

Special Research Centre for the Subatomic Structure of Matter
and Department of Physics
University of Adelaide

A thesis submitted for the degree of

Doctor of Philosophy

June 2013
"An idea that is not dangerous is unworthy of being called an idea at all."

— Oscar Wilde: *The Critic as Artist*
Abstract

This work explores the infrared behaviour of the strong running coupling $\alpha_s$ in Quantum Chromodynamics (QCD). We propose that $\alpha_s$ runs non-perturbatively to an infrared fixed point $\alpha_{\text{IR}}$ for three light quark flavours $u, d, s$. At the fixed point, we show that the quark condensate spontaneously breaks scale and chiral $SU(3)_L \times SU(3)_R$ symmetry. Consequently, the low-lying spectrum contains nine pseudo-Nambu-Goldstone bosons: $\pi, K, \eta$ and a scalar-isoscalar QCD dilaton $\sigma$. We argue that $\sigma$ may be identified with the $f_0(500)$ resonance, a pole at a complex mass with real part $\lesssim m_K$. For low-energy expansions in $\alpha_s$ about $\alpha_{\text{IR}}$, we replace chiral $SU(3)_L \times SU(3)_R$ perturbation theory with a new model-independent theory $\chi\text{PT}_\sigma$ based on approximate scale and chiral $SU(3)_L \times SU(3)_R$ symmetry.

We examine the phenomenological consequences which arise from this framework by constructing effective Lagrangians which simulate strong, weak, and electromagnetic interactions. We also study the convergence properties of the effective theory, wherein we find that $\chi\text{PT}_\sigma$ converges much better than $\chi\text{PT}_3$ in the presence of both scalar-isoscalar channels and $O(m_K)$ extrapolations in momentum. We achieve this without spoiling the successful leading order predictions of $\chi\text{PT}_3$ elsewhere.

In our phenomenological investigations, we show that the $\Delta I = 1/2$ rule for non-leptonic $K$-decays emerges as a consequence of $\chi\text{PT}_\sigma$, with a $K_S\sigma$ coupling fixed by data for $\gamma\gamma \to \pi\pi$ and $K_S \to \gamma\gamma$. This constitutes our most important result.

We also apply the electromagnetic trace anomaly to QCD at the infrared fixed point and obtain the estimate $R_{\text{IR}} \approx 5$ for the non-perturbative Drell-Yan ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ at $\alpha_{\text{IR}}$. 
Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. I also give permission for the digital version of my thesis to be made available on the web, via the University’s digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

List of publications, workshop proceedings, and presentations based on this thesis.

- R. J. Crewther and L. C. Tunstall, "Origin of the $\Delta I = 1/2$ Rule for Kaon Decays: QCD Infrared Fixed Point", arXiv:1203.1321 [hep-ph] (submitted to Physical Review D).
- R. J. Crewther and L. C. Tunstall,"Infrared Fixed Point in the Strong Running Coupling: Unraveling the $\Delta I = 1/2$ Puzzle in K-Decays", arXiv:1306.4445 [hep-ph] (Contribution to the proceedings of the workshop “Determination of the Fundamental Parameters of QCD",”
Nanyang Technological University, Singapore, March 18-22, 2013, to be published in Mod. Phys. Lett. A).

- L. C. Tunstall, “QCD Dilatons and the Origin of the $\Delta I = 1/2$ Rule”, talk and poster presented at CoEPP Summer School and Workshop, Lorne, VIC, Australia, February 20-24, 2012.

- L. C. Tunstall, “Origin of the $\Delta I = 1/2$ Rule for Kaon Decays: QCD Infrared Fixed Point”, talks given at Universidad de los Andes and Universidad Nacional de Colombia, Bogotá, Colombia, August 1-2, 2012.
Acknowledgements

The completion of this thesis — and the work presented in it — would not have been possible without the input from many people, all of whom I owe a great debt of gratitude.

To my supervisors for their support, open door policy, sharing of ideas, and for giving me the freedom to pursue my own interests in physics. Their help proved invaluable in the final stages of writing this thesis and I thank them for critically reading the manuscript under rather trying circumstances. In particular, to Dr. Rod Crewther for teaching me countless things about theoretical physics, and for encouraging me to think about the ‘hard problems’. I thank Dr. Ross Young for selflessly giving up his time and raising questions which have sharpened my understanding of non-perturbative phenomena.

To those who have given an honest appraisal of the work in this thesis. In particular, Profs. Matthias Jamin, Heiri Leutwyler, Anthony Thomas, and Anthony Williams, whose comments have either forced me to re-examine an underlying assumption or have directed me to new avenues of research. Furthermore, I am grateful to Profs. Thomas and Williams for providing me with financial support and the opportunity to broaden my theoretical horizons in the final stages of my PhD.

To Dr. James Zanotti for pointing me to crucial lattice QCD papers on the \(f_0(500)\) resonance.

To my peers at Adelaide who have influenced my way of thinking over the last few years. In particular, I thank Andrew Casey, Cael Hasse, and Dale Roberts for sharing the highs and lows of research life.

To my friends outside of physics for all the good times. In particular
to James Parry for passing on the book ‘Hyperspace’ which started this whole venture.

To the CSSM ladies, Bronwyn Gibson, Silvana Santucci, and Sharon John-son for administering a most efficient research centre and most importantly, for ensuring that the coffee never ran out!

To the astro boys who convinced me that more fun can be had at the pub than the office.

To my soccer team, the aptly named ‘Colour Singlets’, and my band-mates in ‘Gage’ for providing me with an important outlet to relax.

To the universities of South Australia for supporting the Australian - American Fulbright Commission and in particular, my Fulbright scholar-ship. My experiences at UC Berkeley and the U.S. at large left a strong, positive impression with me. It is a period of my life which I will treasure long into the future.

To Prof. Mary K. Gaillard for both supporting my Fulbright application and ensuring that my time at Berkeley was fruitful. To the graduate stu-dents who shared their time and knowledge with me: Kevin Schaeffer for patiently explaining aspects of string theory to me; Patrick Zulkowski for being a great office mate; David Pinner and Hannes Roberts for the beers and all matters related to high energy particle physics.

Reaching this point in my education would have been impossible without the guidance from my undergraduate mentors. To Mr. Phil Richard-son, whose enthusiasm for physics was so infectious that I ultimately switched careers to become a physicist. Thank you for cultivating my early interest in the subject. To Dr. Melanie Johnston-Hollitt who rec-ognized early on that I was not cut out to be an astrophysicist and en-couraged me to pursue theoretical physics at Adelaide — it is one of the best decisions I have made. To Prof. Derek Leinweber whose encourage-ment during my time at Adelaide inspired me to become a fully fledged theorist.

To my parents Andrew and Rebeca who have unconditionally supported
my dreams, no matter how abstract. For the many sacrifices they have made so that I could have the best possible education. Thank you for instilling the determination required to overcome the obstacles in a project such as this.

To my wife (and newly minted Dr.) Sofia for sharing all the adventures of the last few years. Without her love, advice and encouragement — especially during the dark and disheartening times of the past year — I doubt this project would have been completed in time. As a (then) PhD student herself, she uniquely understood both the joys which come from finding things out for oneself and the ensuing frustration which comes when new ideas do not work. I thank you for the sacrifices you have made so that I can pursue a career in physics and most importantly, for brightening my life in innumerable ways.
# Contents

Contents     viii

1 **Prelude**     1

2 **Chiral Perturbation Theory**     4
  2.1 Chiral Symmetry .................................................. 5
  2.2 Perturbations about a Goldstone Symmetry ..................... 7
  2.3 Effective Lagrangians for Strong Interactions .................. 9
  2.4 Functional Methods and Gauge Interactions .................... 11
    2.4.1 Next-to-Leading Order Effects .............................. 13
  2.5 Effective Lagrangians for Weak Interactions ................... 16
    2.5.1 The $\Delta I = 1/2$ Puzzle ................................ 19
  2.6 The Lowest QCD Resonance: Problems with Chiral $SU(3)_L \times SU(3)_R$
    Expansions? ................................................... 20

3 **Asymptotia**     25
  3.1 Effective Charges .............................................. 25
    3.1.1 Effective Charges for QCD .......................... 29
  3.2 Non-perturbative Determinations of the Strong Running Coupling ... 32
    3.2.1 Schrödinger Functional Scheme .................. 32
    3.2.2 Dyson-Schwinger Equations .................. 35
  3.3 Varieties of Asymptotic Behaviour ................................ 36

4 **Chiral-Scale Perturbation Theory about an Infrared Fixed Point**     42
  4.1 Historical Overview and Modern Developments .................... 43
  4.2 Broken Scale Invariance ........................................ 48
4.3 Chiral-Scale Lagrangian .................................. 51
  4.3.1 Local Scale Invariance ................................ 53
  4.3.2 Equations of Motion .................................. 55
  4.3.3 Trace Anomaly in the Effective Theory ................... 56
  4.3.4 The Next-to-Leading Order Lagrangian .................. 57
  4.3.5 The One-Loop Effective Action ........................ 59
4.4 Strong Interactions ...................................... 62
  4.4.1 Sigma Terms ..................................... 63
  4.4.2 Determining \( F_\sigma \) ................................ 66
  4.4.3 The Scale of the Chiral-Scale Expansion ................. 67
4.5 Electromagnetic Interactions .............................. 69
  4.5.1 The Electromagnetic Trace Anomaly .................... 70
  4.5.2 The Drell-Yan Ratio in the Infrared Limit ................. 73
4.6 Weak Interactions ....................................... 75

5 Discussion and Directions for Further Research 79
  5.1 Summary of Results ..................................... 79
  5.2 Implications of this Work ................................. 81
  5.3 Limitations of the Theory ................................. 82
  5.4 Directions for Future Research ............................. 82

Appendix A: Feynman Rules 84
Appendix B: Equations of Motion 86
Appendix C: Tadpole Cancellation in the Background Field Method 89
Appendix D: Cancellation of UV Divergences in the \( \gamma \gamma \) Channel 92

References 97