Tunable Kondo screening in a quantum dot device

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We consider electron transport along a single-mode channel which is in contact, via tunnel junctions, with two quantum dots. Electron tunneling to and from the dots contributes to the electron backscattering, and thus modifies the channel conductance. If the dots carry spin, the channel conductance becomes temperature dependent due to the Kondo effect. The two-dot device geometry allows for a formation of $S = 1$ localized spin due to the indirect exchange interaction, called Ruderman-Kittel-Kasuya-Yosida interaction. This device offers a possibility to study the crossover between fully screened and under-screened Kondo impurity. We investigate the manifestation of such crossover in the channel conductance.

Exchange interaction of the localized spins with the conduction electrons affects the electron transport. At low temperatures, the localized magnetic moment tends to be screened by the spins of the itinerant electrons. The screening manifests itself in thermodynamic characteristics and, more importantly, in conduction of a metal with magnetic impurities, giving rise to the Kondo effect. Depending on the value of impurity spin and on the hybridization of the localized and extended electron states, the localized spin may be screened partially or completely, or even over-screened by itinerant electrons. It is hardly possible to control the spin screening for an impurity in a metallic host matrix. However, quantum dot (QD) devices have allowed for a progress in this direction.

The high-spin or multi-channel Kondo effect in quantum dots has been studied both theoretically and experimentally. It has been shown that the high spin induced by the intra-dot exchange interaction plays an important role in the Kondo physics. Recent ideas of a very asymmetric double-dot device propose to study the Kondo effect around the quantum critical point corresponding to an over-screened localized spin. In this paper, we consider a double-dot device suitable for the investigation of an under-screened $S = 1$ state.

The double-dot system is especially interesting because it allows for a formation of $S = 1$ localized spin due to the Ruderman-Kittel-Kasuya-Yosida (RKKY) indirect exchange interaction. The RKKY interaction between spins in QDs has been discussed. Recently, an experimental evidence of the RKKY interaction in coupled dot system has been reported. In that experiment, two local spins in small peripheral dots were interacting with each other by the RKKY interaction mediated by conduction electrons in the large central dot. The sign of the RKKY interaction depends on the specific structure of the electron wave functions in the central dot, and is hard to predict in advance.

Here we consider a two-dot device geometry where two QDs are connected to different sides of a single-channel quantum wire. In this device, the exchange couplings can be continuously modulated by applying magnetic fields. We consider the evolution of the localized spin system between two different screened states. In the course of the evolution, the system passes through a special point where the local spin is under-screened. The advantage of the proposed geometry is a more robust under-screened state than in the previously considered single-dot devices and an easier control over the crossover to a fully screened state. Unlike the conventional quantum dot configuration, for the side-coupling channel geometry the Kondo effect appears as an anomalously strong backscattering (rather than transmission). The Kondo interaction between a single-mode quantum wire and two QDs ($n = 1, 2$) each having spin $1/2$, schematically shown in Fig. 1, is described by Hamiltonian

$$H_{ex} = \sum_{nkk'\sigma\sigma'} J_n(k, k') c_{n\sigma}^\dagger \sigma' c_{k\sigma'} \cdot S_n,$$

where $J_n(k, k') = -V_{nk'}^* V_{nk} U_n/(\{U_n + \epsilon_n\})$ and $S_n$ is the local spin at dot $n$ which has charging energy $U_n$ and single-particle energy $\epsilon_n$. The coupling strength is given by $V_{nk} = v_{nk} \exp(ikx_n)$ where the lateral coupling $v_{nk}$ depends on the overlap of wavefunctions between the dots.

FIG. 1: Quantum wire coupled with two quantum dots. Two quantum dots (QD-1 and -2) coupled to a quantum wire. $V_{nk}$ is the coupling constants between dot $n=1, 2$ and the left- or right-moving waves $k = L, R$. 
QD and the wire in the lateral direction. Two local spins in two dots interact with each other by the RKKY interaction, $H_{\text{RKKY}} = -J_{\text{RKKY}}(S_1 \cdot S_2)$, where $J_{\text{RKKY}}(R) = -\pi E_F \rho_j J_{\text{RKKY}}(2k_F R)$ and $J_n = |J_n(k_F, -k_F)|$; function $\sin(x) = -\int_x^\infty dt (\sin(\tau/\tau))$ is the sine integral function $[21]$. 

When two dots are on the opposite sides with zero longitudinal distance $R = x_1 - x_2 = 0$ between them, the RKKY interaction is ferromagnetic and the exchange coupling constant is maximal, $J_{\text{RKKY}} \sim E_F(\rho_j)^2$. In the case of ferromagnetic interaction, the total spin $S = S_1 + S_2$ of the two dots is $S = 1$, and within the triplet subspace of states $|t\rangle$ the spin difference $(t_1(\mathbf{S}_1 - \mathbf{S}_2)|t_2\rangle = 0$. The effective Hamiltonian within that subspace is

$$H_{\text{ex}} = S \cdot \frac{1}{T} \sum_{kk' \sigma' \sigma} \left( I^{RR} I^{RL}_k + \frac{1}{2} \right) e^{i \mathbf{F}_{kk'}^{\sigma' \sigma} \sum_{\sigma' \sigma} e_{\mathbf{F}_{kk'}^{\sigma' \sigma}} \right)$$

$$+ I^{LL} e^{i \mathbf{F}_{kk'}^{\sigma' \sigma}} \sum_{\sigma' \sigma} e_{\mathbf{F}_{kk'}^{\sigma' \sigma}}$$

$$+ I^{RL} e^{i \mathbf{F}_{kk'}^{\sigma' \sigma}} \sum_{\sigma' \sigma} e_{\mathbf{F}_{kk'}^{\sigma' \sigma}}$$

$$+ I^{LL} e^{i \mathbf{F}_{kk'}^{\sigma' \sigma}} \sum_{\sigma' \sigma} e_{\mathbf{F}_{kk'}^{\sigma' \sigma}}$$

$$= I(k_F, k_F), I^{LL} = I(-k_F, -k_F),$$

$$I^{RL} = I(-k_F, k_F) = (I^{LR})^*,$$

where

$$2I(k, k') = J_1(k, k') e^{i(k-k')/2} + 2I_2(k, k') e^{i(k-k')/2}.$$  

(5)

The $2 \times 2$ exchange term in Eq. 4 can be easily diagonalized by the unitary transformation of the basis $(c_{Rk\sigma}, c_{Lk\sigma})$; the result is

$$H_{\text{ex}} = S \cdot \frac{1}{T} \sum_{kk' \sigma' \sigma} \left( J_0 a^\dagger_{k\sigma} (a_{k\sigma} a_{k'\sigma}) + J_0 b^\dagger_{k\sigma} (a_{k\sigma} a_{k'\sigma}) \right),$$

$$J_a(b) = \frac{1}{2} \left( I^{RR} + I^{LL} \right)$$

$$\pm \left( I^{RR} - I^{LL} \right)^2 + 4(I^{RL})^2 \cos 2k_F R$$

$$+ 4(I^{RL})^2 \sin 2k_F R \left( 1/2 \right),$$

(6)

where

$$2I^{RR}_1 = J_1(k_F, k_F) \pm J_2(k_F, -k_F),$$

$$2I^{LL}_1 = J_1(-k_F, -k_F) \pm J_2(-k_F, k_F),$$

(7)

For symmetrically positioned dots ($R = 0$) and at $B = 0$, the exchange constants are $J_a = J_1 + J_2$ and $J_a = 0$. It means that one of the channels is decoupled from the spin and becomes free propagating. Note that the decoupling for the symmetric case at $B = 0$ always occurs, i.e., regardless of dot parameters $(U_i, \epsilon_0, V_\nu, \epsilon_{0\nu}, \ldots)$. The basis $(a_{k\sigma}, b_{k\sigma})$ is related to the original one, $(c_{Rk\sigma}, c_{Lk\sigma})$, by the unitary transformation:

$$\left( \begin{array}{c} c_{Rk\sigma} \\ c_{Lk\sigma} \end{array} \right) = \left( \begin{array}{cc} \cos \frac{\theta}{2} e^{-\gamma/2} & -\sin \frac{\theta}{2} e^{-\gamma/2} \\ \sin \frac{\theta}{2} e^{\gamma/2} & \cos \frac{\theta}{2} e^{\gamma/2} \end{array} \right) \left( \begin{array}{c} a_{k\sigma} \\ b_{k\sigma} \end{array} \right),$$

$$\cot \theta = \frac{I^{RR} - I^{LL}}{2 \sqrt{(I^{RL} \cos k_F R)^2 + (I^{RL} \sin k_F R)^2}}.$$
a on the site n and \( t_0 > 0 \) is the transfer integral between neighboring sites. When \( J > 0 \) two spins \( s_{a0} \) and \( S \) tend to form a doublet. To describe the low-temperature limit behavior of the system, the parameters of the effective Hamiltonian must be tuned to satisfy the condition \( |J| \gg t_0 \), see Ref. \[32\]. In this case the doublet states are \( \psi_{\pm} = (\sqrt{2/3}, -\frac{1}{2}, 1) - \sqrt{1/3}(\frac{1}{2}, 0, 0) \), and \( \psi_{\pm} = -\sqrt{2/3}(\frac{1}{2}, -1) + \sqrt{1/3}(\frac{1}{2}, 0, 0) \), where we have denoted the spin state as \( s_z = \pm 1/2 \) and \( S_z = 1, 0, -1 \) (see Fig. 2). This doublet is the effective spin 1/2 formed as the result of screening at \( T \ll T_a \). By doing the second-order perturbation theory in the transfer amplitude \( t_0 / J \) between the sites \( n = 0 \) and 1, we obtain Kondo Hamiltonian for the effective spin \[32, 34\] coupled ferromagnetically to the rest of the system, \( \mathcal{H}_{eff} = - \sum_n t_0 (a_{n+1}^\dagger a_n + c.c.) + 2J_{a1} s_{a1} \cdot S' \), where \( J_{a1} \sim t_0^2/JS < 0 \) and \( S' = S - 1/2 \).

We may perform the poor man’s scaling procedure then, to obtain the effective scale-dependent interaction constant \( \rho J = 1 / \ln(D_0/T) \). Taking into account that the initial stage of screening occurred at \( T \sim T_a \), we have to set \( D_0 \simeq T_a \). In order to evaluate conductance in the low-temperature limit, first we consider the effect of the doubleton formation. Let us start with \( S \) matrix for half-wire \( n > 0 \). The scattering state is expressed by \( \psi_{n2} = e^{-i\pi n} + e^{i\pi n} \). At \( T = 0 \), the scattering phase shift is \( \delta = \pi / 2 \), so \( \psi \propto \sin(\pi n / 2) \). If we define now the scattering matrix for the half-chain \( n \geq 1 \), thus excluding the site involved in the singlet formation, the corresponding phase shift (at the Fermi level) would be \( \delta = \delta + \pi / 2 \). This rule for the phase shifts tells us that small deviations from the unitary limit for the backward conductance lead to its reduction.

\[
G_{\text{back}} = G_0 \left[ 1 - \frac{3\pi^2}{16} \frac{1}{\ln^2(T_a/T)} \right].
\]  
(14)

One finds the full form of the low-temperature limit for conductance by substituting Eq. 14 in Eq. 11.

Tuning of \( J_b \) through the \( J_b = 0 \) state may be achieved by applying magnetic field to the device, which would cause the variation of the orbital parts of electron wave functions. Now we turn to the case of non-symmetric functions, in which case \( J_b \neq 0 \). There are two characteris-
$T_a/T_b = \exp(1/2pJ_a - 1/2pJ_b)$, we assume the lateral confinement of the wire at $x = 0$ is a square-well potential with width of $W = 50$ nm, and $g_0 = \pm 0.3W$. The Fermi energy is 10 meV corresponding to the Fermi wave length of 54 nm so that the wire has only one conducting channel. We choose $pJ = 0.2$, which is deduced in the typical measurement of the Kondo effect giving the upper Kondo temperature $T_a \sim 1$ K. A numerical calculation of the single-particle wavefunction for the square-well confinement gives $T_b \approx 0.25T_a$ and $J_{KKY} \approx 0.2$ meV at $B = 2.5$ T. This evaluation confirms that the two Kondo temperatures are in an accessible range in experiment while the RKKY interaction is still large enough to maintain the spin triple.

So far, we have assumed the wire has only one conducting channel where the under-screened Kondo effect can be experimentally realized without employing a complicated gate tuning. It is interesting to discuss how the Kondo screening changes when the channel number exceeds $2S = 1$. In the case of perfect alignment of the dots and at $B = 0$, it is still possible to single out one propagating mode interacting with the local spin, so the problem still maps on a single-channel Kondo problem ($T_a \neq 0, T_b = 0$). At $B \neq 0$, however, a number of modes couple to the local spin, so the intermediate-temperature behavior of the conductance is more complicated than the one given by Eqs. (15) and (16). However, at the lowest temperatures two channels coupled to the spin strongest still will lead to a full screening, see Eq. (17).

In summary, we discussed electron transport along a single-mode channel which is in contact with two side-coupled quantum dots. If each dot has a spin $1/2$, the two-dot device geometry allows for a formation of $S = 1$ localized spin due to the indirect RKKY exchange interaction. We investigate the temperature and magnetic-field dependence of the conductance for such a device which shows the crossover between fully screened and under-screened Kondo impurity.

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