Fractal Features of Urban Morphology and Simulation of Urban Boundary

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Abstract  Using fractal dimension to reflect and simulate urban morphology are two applications of fractal theory in city geography. As the only consistent description of a fractal, fractal dimension plays an important role in describing the basic features of fractals. Just like other fractals, our cities have similar characteristics. Fractal dimension to some extent is regarded as an indicator of urban expansion, and it may change with urban morphology in different time and space. Based on the Geographic Information System (GIS), taking Wuhan city as a test area, the fractal dimensions of different land use were calculated, and a linear regression equation was established to analyze the relationship between fractal dimension and residential areas. Then the author used fractal dimension to simulate the urban boundary which is an important part of urban morphology. A mid-point subdivision fractal generator is needed in the simulation process, and the shape of the generator is determined by fractal dimension. According to the fractal theory, fractal boundaries in different scales have self-similarity and they have the same fractal dimensions. Based on this fact, the simulation method the author used could quantitatively keep the similarity of configuration of the urban boundaries.

Keywords  fractal dimension; urban morphology; fractal simulation; fractal generator

CLC number  P237.3

Introduction

Ever since the establishment of fractal theory in the 1970s[1], it has become more and more widely used in city geography. Fractal theory provides both good mathematic tools of describing the urban geographic phenomena and the mathematical foundation for fractal simulation of urban spatial systems. Thereby, many scholars have done lots of researches on urban morphologic structure and its expansion.

As a key quantity of fractals, fractal dimensions are used as indicators of the complexity of curves and surfaces, hence, researching on the methods of calculating the fractal dimensions is an important part of the fractal city theory. There are some common fractal dimensions such as Hausdorff dimension, Box-Counting dimension, etc. Scholars have proposed some useful ways for deriving the fractal dimensions, for example, Structured Walk method, Equipaced Polygon method, Hybrid Walk method, Box-Counting method, etc. In this paper, Box-Counting method is adopted, and the geographical significance of fractal dimensions calculated with this method indicates the degree of similarity. Fractal dimensions of different geographic objects are obtained from the digital land use map, and changed with the urban expansion and evolution during different time periods. Then we establish a linear regression equation to analyze the relationship between fractal dimension and areas of residence.
Fractal dimension is employed to simulate the urban boundary which is an important part of urban morphology. In the experiment, the urban boundary is defined as a closed line with a fractal dimension of more than one but less than two. It’s hard to store data of every scale of urban boundaries as a linear part of the map. More details of the urban boundaries are expected to be exhibited as the map scale is enlarged. Some conventional methods pay too much attention to the smoothness of linear element in this process. However, they isolate the isotropy which is an obvious geographic feature of the spatial change\cite{2}. Simulation based on fractal dimension can avoid this disadvantage.

1 Exploring fractal features

1.1 Mathematic model of fractal dimension calculation

According to Mandelbrot, supposing the fractal dimension is $D$, then

$$N(r) = cr^{-D}$$

Where $N(r)$ is the number of measurements; $r$ is the yardstick; $c$ is a constant to be determined; $D$ is the fractal dimension.

The equivalent equation is:

$$\ln N(r) = -D \cdot \ln r + \ln c$$

In the research of city morphology, $r$ stands for the yardstick of certain type of land use. $N(r)$ stands for the measurement of this yardstick, if they have the law which do not change with the yardstick:

$$N(r) \propto r^D$$

It is considered that the configuration of city land use has fractal quality: the spatial configuration features do not change with the measurement of the yardstick.

Suppose $\ln N(r)$ equals $Y$, $\ln r$ equals $X$, $\ln c$ equals $C$, changing the yardstick $r$, several groups of $X$, $Y$ is measured, and the method of least squares is used to fit the line which is described by Eq.(2), fitting equation

$$N \cdot C + D \cdot \sum_{i=1}^{N} X_i = \sum_{i=1}^{N} Y_i$$

$$C \cdot \sum_{i=1}^{N} X_i + D \cdot \sum_{i=1}^{N} X_i^2 = \sum_{i=1}^{N} X_i Y_i$$

The fractal dimension $D$ is calculated by Eq.(4).

1.2 Fractal dimensions and result analysis

The data used in this study cover approximately the same spatial extent at the same scale, and the different land parcels are extracted in the polygon, and the urban boundary is extracted in line according to the digital urban planning maps of different time periods with the aid of GIS.

![Fig.1 The digital vegetation and urban boundary](image)

Thereby, five layers of each map, namely, residential area, industrial area, vegetation, water system and boundaries are available for calculating the fractal dimensions (Fig.1). Box-Counting method is carried out in our experiment which is widely used in geography. Firstly, we transformed the vector layers of different land use into raster images, then fractal dimensions of land parcels distributed in the images are evaluated with the help of GIS. The urban planning maps in our experiment represent a main process of evolution of urban spatial pattern of Wuhan which includes three time periods: at the beginning of the establishment of new China, in the early part of Reform and Opening, and the 1990s, a time at which urban expansion tide occurred.

| Type    | Industry | Residence | Vegetation | Water system | Boundary |
|---------|----------|-----------|------------|--------------|-----------|
| Year    | 1950     | 1.325 3   | 1.350 2    | 1.462 6      | 1.542 3   | 1.201 2  |
|         | 1959     | 1.396 5   | 1.361 4    | 1.572 1      | 1.538 9   | 1.189 9  |
|         | 1980     | 1.413 5   | 1.372 2    | 1.525 4      | 1.550 1   | 1.210 0  |
|         | 1989     | 1.437 7   | 1.441 4    | 1.520 8      | 1.547 8   | 1.191 2  |
|         | 1997     | 1.449 6   | 1.487 2    | 1.521 0      | 1.548 5   | 1.232 5  |

As shown in Table 1, we list some cross-sectional years. The first stage (1949~1960), the entire urban spatial configuration extended little by little. Apart from the water system, fractal dimensions of different land use increased. The fractal dimension of industrial area increased 0.7% every year. The second stage
(1960~1980), spatial configuration became more and more complex. Increments of fractal dimensions were smaller than the previous stage, fractal dimension of industrial and residential area increased 0.1% and 0.11% every year, respectively. Vegetation area was reduced due to the expansion of industry and habitation, thus fractal dimension also decreased. The fractal dimension of water system fluctuated within a very small range. At the third stage (1980~2000), the industry of Wuhan rushed into a high speed of development. Both the industrial and residential fractal dimensions apparently increased. With an increase of 11.5% every year of fractal dimension, the pattern of the residential area held a more complex spatial configuration.

There is an inherent constraint or effect on the relationship between fractal indicator of urban morphology and human activities. Here we discuss the changes of fractal dimensions over a period of time to explore the relationship between fractal dimensions and residential area, for which a linear regression equation was established[3].

\[
\ln Areas = 9.896 - 3D - 1.7274 \quad (5)
\]

| type year | Areas | lnAreas | Dimensions |
|-----------|-------|---------|------------|
| 1950      | 93338 | 11.4440 | 1.3502     |
| 1959      | 116442| 11.6651 | 1.3614     |
| 1980      | 190456| 12.1572 | 1.3722     |
| 1989      | 285423| 12.5617 | 1.4414     |
| 1997      | 413110| 12.9315 | 1.4872     |

Fig.2 The relationship of fractal dimensions and areas

From the result of the linear regression equation shown in Fig.2, the correlation coefficient calculated is 0.9527. It shows that the fractal dimensions of urban morphology are great in relation to residential areas.

2 Simulation with fractal features

Fractal dimension of spatial morphology has inherent evoulutional rule in different temporal sizes. Its change is greatly affected by human activity[4]. In the previous part of this paper, fractal dimensions of different land use in different time periods are derived using Box-Counting method. The quantitative research on urban spatial characteristics has significant practical application for urban management and planning[3], but just exploring the relationship between fractal dimension and urban morphology is not enough. Fractal theory should play a more important role in the research of geography, and fractal dimension will be more widely used in the area of spatial analysis. In this part of our paper we pay more attention to the urban boundary, which is an important part of city configuration. As well as the coastline, cloud and mist, lightning and some other fractals, urban boundary has self-similarity. It’s hard for the conventional Euclidean geometry to describe these phenomena. Since the establishment of fractal theory, some other subjects were promptly attracted by this theory[5] and some fractal generators are promoted to simulate these abnormal curves – for instance, mid-point subdivision fractal generator which is used in our research.

2.1 Fractal generator and its dimension

According to Eq.(1), fractal dimension \( D \) can be derived by

\[
D = \frac{\ln(N_{n+1}/N_n)}{\ln(r/n/r_{n+1})} \quad (6)
\]

Fig.3 shows the mid-point subdivision fractal generator.
According to Eq.(6), the fractal dimension of this generator is:

\[ D = \frac{\ln(2/1)}{\ln(r_n / r_{n+1})} \]  

(7)

It can be changed into

\[ D = \frac{\ln 2}{\ln 2 + \ln(\cos \alpha)} \]  

(8)

So the relationship between fractal dimension and the shape of generator be related by Eq.(9)

\[ \alpha = \arccos e^{\frac{-\ln 2}{2\ln 2}} \]  

(9)

If fractal dimension \( D \) can be calculated by some methods, then the value of \( \alpha \) will be derived by Eq.(9), thereby fractal generators ground on different dimensions will be defined. Based on this notion, abnormal curves with different dimensions could be simulated by these generators. As we can’t store data of every scale of urban boundary as a linear part of the map, how to generate the data from one scale to another without changing their configuration is an issue often mentioned in the research of automatic map generation[6]. Some conventional methods focus on maintaining the smoothness of linear elements. However, they isolate the isotropy which is an obvious geographic feature of spatial change. Simulation based on fractal dimension will take this factor into account.

2.2 Implementation of simulation

Simulation is used to construct the enlarged boundary by applying the generator to these line segments, then it will yield a new boundary with more details. This process is continued towards the limit which is the minimum length of the line segments at a certain scale.

2.2.1 Simulate a simple curve

Firstly, the experiment was implemented from a simple curve, whose fractal dimension has been calculated. According to some law, the shape of the curve alters with the iterative process, then every line segment of the curve is instituted by the fractal generator. If the minimum threshold is the minimum length of a chord, this process will be terminated.

As show in Fig.4, simulation results with different length limits were obtained.

The black curve is a source curve which is simulated using mid-point subdivision fractal generator; the pink curve is its analog. It is clear that simulation results are mainly affected by the limitation length which is counted by pixels. That means if a line segment is longer than the threshold, it will not be replaced by the generator. The maximum threshold increases as the details decrease, as shown in Fig.4. The threshold increased from 2 pixels to 40 pixels, and in the end, the source curve is overlapped by the simulation result. Actually, in a map, if a line segment of a boundary is a long one, it may be an edge of a regular land parcel which has no more fine features to display as the map scale is enlarged. So length is taken into consideration in the urban boundary simulation.

2.2.2 Define the generator and obtain the data of coordinates

Cities are usually depicted in the plane as two-dimensional phenomena thus their boundaries immediately imply some measure of area which compose the entire urban morphology[7]. In our experiment, the urban boundaries are defined as closed lines; their fractal dimensions during different time periods are derived and listed in Table 1. According to Eq.(9), the angle \( \alpha \) can be calculated as the fractal dimension \( D \) is available. Every line segment of the boundary has its own generator which is determined by the angle and the length of the line segment. Coordinates of every node are stored when digitizing the urban boundary so the length of every line segment can be calculated, as shown in Fig.5.

There is a curve with certain dimension \( D \), and \((X_0, Y_0), (X_1, Y_1)\cdots (X_4, Y_4)\) are the coordinates of these
nodes. Line segment $BC$ is taken as an example, as the fractal dimension, coordinates of node $B, C$ are available, so angle $\alpha$ can be acquired from equation (9), the length and the azimuth angle of line segment $BC$ can be easily obtained, then the fractal generator will be defined by these factors. Because the uncertainty of natural fractal objects and the mechanism of the spatial phenomena is not understood deeply and roundly, some random factors should be added into the simulation process that will make the results closer to reality, so the position of $N$ has two possibilities: above or below $BC$.

$$X_N = \frac{X_1 + X_2}{2} - J \times L \times \sin \beta + R$$
$$Y_N = \frac{Y_1 + Y_2}{2} + J \times L \times \cos \beta + R$$  \hspace{2cm} (10)$$

Where $J = 1$ or $-1$, it will decide whether node $N$ will be above or below $BC$; $L$ is the distance perpendicular to the midpoint of $BC$; $\beta$ is the azimuth angle of $BC$; $R$ is a random number which will make the location of $N$ or $N'$ randomly scattered in the dashed rectangle as show in Fig.5.

These disposals are the embodiment of isotropy which is an obvious geographic feature of spatial change, and may well reflect the complex and fine features of fractals.

2.2.3 Simulation of urban boundaries

At the beginning of simulation, several reference points are needed which approximately determine the configuration of urban boundaries, so when digitizing the urban boundaries we may as well choose points in the inflexion that will help to exhibit more details of these linear elements. The iterative process is confined by two thresholds—the maximum and minimum length which will be used to terminate the iterative operation. If a line segment is shorter than the minimum threshold there is no need for it to be established by the generator, and if a line segment is longer than the maximum threshold, it maybe the edge of a regular land parcel which form the entire urban boundary. For example, a wall of a steel factory is not suited for the iterative process. Therefore, another point should be recognized when digitizing the urban boundaries. That is, if there are no regular land parcels at the edge of the city, we intend to use small line segments to digitize the urban boundaries, and the maximum threshold is decided by the average chord length $d_{avg}$, and the length of every chord is computed using the equation:

$$d_{i,i+1} = \sqrt{(x_i + x_{i+1})^2 + (y_i - y_{i+1})^2}$$

$$i = 1, \ldots, n - 1,$$  \hspace{2cm} (11)$$

Where $n$ is the number of total nodes, then the length of the boundary can be summed as:

$$s = \sum_{i=1}^{n-1} d_{i,i+1}.$$  \hspace{2cm} (12)$$

The average chord length is therefore:

$$d_{avg} = \frac{s}{n-1}.$$  \hspace{2cm} (13)$$

The minimum threshold is decided by the change of map scale. The algorithm is summarized as:

1) enter the original digital geographic curve, which stores as an array of node coordinates;
2) calculate the fractal dimension $D$;
3) compute the average chord length of the curve and determine the minimum and maximum threshold;
4) identify whether each chord is between the minimum and maximum threshold, deriving the fractal generator using Eq.(10). This process will be continued towards the minimum threshold;
5) export the new curve.

Fig.6 shows a part of the simulation results of urban boundaries. From the result, two chords are not taken into the iterative process, as they are out of the
maximum threshold.

3 Conclusions

In this paper, urban spatial characteristics of Wuhan city are analyzed quantitatively using GIS. The spatial configuration characteristics and expansion of urban land use are discussed using the fractals, and the change of fractal dimension of the city fractal system is compared with the actual situation. The relationship between residential area and dimension are analyzed, which implies that fractal dimension can be used to show the rule of urban spatial morphology change, then an experiment of simulating the urban boundary with fractal dimension is performed in this paper.

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Notes to Contributors

Contributions are welcomed on one of the following subjects or in related areas:

- GIS
- Geodynamic
- Physical geo-surveying
- GPS
- Geo-surveying
- Engineering surveying
- RS
- Photogrammetry
- Mapping apparatus
- Cartology
- Graphics

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