Interaction of quantum Hall systems with waveguide elastic modes

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An interaction of non-uniform plane elastic modes of the waveguide type with monolayer and double-layer quantum Hall systems is considered. It is shown, that unlike the case of the surface acoustic wave propagation, the restriction on maximal values of the wave vectors for which the velocity shift can be observed experimentally does not take place for the waveguide modes. In case of study of incompressible fractional quantum Hall states, the effect can be used for measuring a dependence of the effective magnetic length on the filling factor and for observing phase transitions in double-layer systems under the interlayer distance variation

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I. INTRODUCTION

The interaction of surface acoustic waves (SAW) with quantum Hall systems was intensively studied both experimentally [1] and theoretically [2, 3] during the last few years. Measurements of both velocity shift and absorption of SAW’s provide an indirect method of studying the dynamical conductivity of a two dimensional electron(2DE) gas. The SAW experiments (together with a study of the magnetic focussing effect [10, 11] and of a temperature dependence of the d.c. conductivity of the quantum Hall system [4, 5]) are considered as an experimental proof for the composite fermion model [6, 7] of the fractional quantum Hall(FQH) effect.

In experiments, the SAW is localized on the surface of the sample, while the two dimensional electron layer is placed at some depth d (typical values of d are 10³ ÷ 5 · 10³Å). Therefore the study of a momentum (q) dependence of the conductivity is restricted by the inequality q < q₀, where q₀ is of order of d⁻¹.

The measurements beyond that region are especially important for an investigation of dynamical properties of incompressible FQH states. It was shown in Ref. [7], that at filling factors (ν) corresponding to such states an interaction of acoustic phonon with 2DEs results in an essential phase velocity shift (∆v) for phonons at finite wave vectors. According to Ref. [7], the function ∆v(q) oscillates at finite q. The period of oscillation is ∼ λeff⁻¹, where λeff is the effective magnetic length. Therefore, the measurements of ∆v(q) can be used as a way of an observing the λeff(ν) dependence. It was shown Ref. [8, 9], that the measurements of the velocity shifts of high frequency phonons interacting with a double layer FQH system can be also used as a method of a registration of phase transitions between FQH states described by different quantum numbers. (They are the generalized Laughlin states, which correspond to the Halperin wave function [10]. The composite fermion theory for such states was developed by Lopez and Fradkin [20].)

The values of the velocity shifts obtained in Refs. [7, 18] correspond to bulk acoustic waves interacting with a superlattice of 2DE layers. For a single monolayer (or a double layer) system, the elastic wave should be localized near the electron layer to provide the values of the effect, which can be observed experimentally. At large q it is not the case of SAW, which is localized near the surface. The experimental geometry required can be realized in layered elastic media, where waveguide elastic modes may exist. We consider the case of an infinite elastic medium which incorporates an elastic layer with the acoustic properties different from the ones for the bulk. The interaction of non-uniform transverse elastic waves propagating in such a medium with a quantum Hall system is studied. We consider the case, for which the waves vectors and the polarization vectors of the non-uniform modes are parallel to the interface boundaries. The velocity of the transverse sound for the layer is assumed to be smaller then for the bulk matrix (it is the condition of the existence of such modes). The piezoelectric interaction mechanism is implied.

Heterostructures AlₓGa₁₋ₓAs – GaAs used in experimental studies of double layer quantum Hall systems obey the elastic properties required. For instance, the structures A – B – A – B – A and A – B – A (where A - AlₓGa₁₋ₓAs, B - GaAs) were used in Refs. [21, 22]. Such structures can be approximately described by a model of a bulk sample of AlGaAs incorporating a layer of GaAs. (One can expect that the effect of the surface, the substrate and the thin central layer (for the A – B – A – B – A structure) can be neglected for the wave vectors considered). The transverse sound velocity for GaAs is smaller then for AlₓGa₁₋ₓAs. Therefore, these exist non-uniform elastic modes localized near the GaAs layer. As it follows from our results, the values of the velocity shifts of those modes are within the experimental resolution, while they are smaller than in the SAW experiments. We also consider the geometry for which the heterostructure is places in the matrix of another material. For the last case the values of the effect are of the same order or higher than for the SAW.
II. MATRIX ELEMENTS OF INTERACTION

Let us start from a simple problem of renormalization of the phase velocity of acoustic phonons interacting with 2DEs. The phonon part of Hamiltonian has the form

\[ H_{ph} = \sum_{q} \omega_q (b_q^+ b_q + \frac{1}{2}), \]

where \( \omega_q = v_q \) is the phonon spectrum, \( b_q^+ (b_q) \) are the phonon creation (annihilation) operators. The Hamiltonian of the electron-phonon interaction is chosen in the form

\[ H_{int} = \frac{1}{\sqrt{S}} \sum_{q,k} \int d^2r \ g_{qk} \Psi_{rk}^+ \Psi_{rk} e^{iqr} (b_q + b_{-q}^+), \]

where \( \Psi^+ (\Psi) \) are the electron creation (annihilation) operators, \( k \) is the electron layer index, \( S \) is the area of the layer, \( g \) are the matrix elements of the interaction.

The velocity shift under such a choice of \( H_{ph} \) \( H_{int} \) is described by the expression

\[ \frac{\Delta v}{v} = \frac{1}{v_q} \sum_{q,k,k'} g_{qk} g_{qk'} D_{kk'} (q,vq), \]

where \( D(q,\omega) \) is the electron density-density response function. In the composite fermion approach \([3,16]\) an additional statistical interaction between electrons is introduced, which results in a modification of the functions \( D \) in Eq. \((3)\).

We will use the approach similar to that of Ref. \([3]\) for the calculations of the matrix elements \( g_{qk} \). The piezoelectric interaction of the elastic wave with the electrons corresponds to the Hamiltonian

\[ H = \sum_{k} \int d^2r \ e \varphi_{rk} \Psi_{rk}^+ \Psi_{rk}, \]

where \( \varphi \) is the electric potential, generated by the elastic mode. According to the linear response theory \( \varphi \) in Eq. \((3)\) is the external potential. Therefore, to calculate its value one should neglect the influence of the electron system.

We will use the smallness of the piezoelectric interaction (the explicit expression for the small parameter is given below). Let us discuss the solution of the elastic problem with the piezoelectric constant set to zero. Let the interfaces be in the \( x-y \) planes with \( z = \pm a \) coordinates. We denote \( c_1 \), the transverse sound velocity in the layer \( (|z| < a) \), and \( c_0 \), the transverse sound velocity in the bulk matrix \( (|z| > a) \). The condition \( c_0 > c_1 \) is implied. For definiteness, let us assume that the crystal symmetry of the bulk matrix and of the layer is the cubic one, and the elastic moduli \( c_{44} \) are the same for the layer and for the matrix, while they have different densities. (We use such a special choice of parameters to simplify the expressions obtained. This choice corresponds approximately to the parameters for AlGaAs and GaAs.) We consider the case when the axis \( x \) is chosen along the [100] direction and the \( z \) axis - along the [001] direction.

Let the wave vector and the displacement vector of the elastic wave are directed along the \( x \) and \( y \) axes correspondingly. The wave equation for the Fourier component of the displacement \( u_q (q,z,\omega) = u \) reads as

\[ \frac{\partial^2 u}{\partial z^2} - (c_{44}^2) \eta^2 - \omega^2 u = 0. \]

One can find from the boundary conditions that the function \( u(q,z) \) and \( \partial u(q,z)/\partial z \) are continuous ones at the interfaces. The localized solutions of Eq. \((5)\) have the form

\[ u_e (q,z) = A_q^e \begin{cases} \cos(\eta qa) & |z| > a \\ \cos(\eta qz) & |z| \leq a \end{cases} \]

(the even mode),

\[ u_o (q,z) = A_q^o \begin{cases} \sin(\eta qa) & z \geq a \\ \sin(\eta qz) & z \leq a \end{cases} \]

(the odd mode). In Eqs. \((5)\) \( \lambda = \sqrt{1 - v^2/c_b^2}, \eta = \sqrt{v^2/c_t^2 - 1} \). The velocity \( v \) for the even mode is given by the solution of the equation

\[ \tan(\eta qa) = \lambda/\eta, \]

and for the odd mode - by the solution of the equation

\[ \cot(\eta qa) = -\lambda/\eta. \]

At \( q < \pi/(2a\sqrt{c_t^2/c_b^2} - 1) \) there exists only one (even) mode. At larger \( q \) other solutions of Eqs. \((5)\) emerge (waveguide modes).

The transverse component of the displacement vector can be rewritten in terms of \( b \) operators:

\[ u_{e,o} (r,z) = \sum_q u_{e,o} (q,z) e^{iqr} (b_q + b_{-q}^+). \]

Inserting \( u_{e,o} (r,z) \) into the expression for the energy of elastic vibrations

\[ E_u = \frac{1}{2} \int d^2r dz [\rho(z) (\partial u/\partial t)^2 + c_{44} \left( (\partial u/\partial x)^2 + (\partial u/\partial z)^2 \right) ] \]

and comparing the result with Eq. \((1)\), we determine the values of the normalization factors in Eqs. \((5)\):

\[ A_q^e = A_q^o = \frac{1}{\sqrt{2}} \frac{v}{\sqrt{2Sc_{44}}}, \]

where \( \alpha = (qa(1 + \eta^2) + 1/\lambda)^{1/2} \).

The electric potential can be found from the solution of the Poisson’s equation
\[ \Delta \varphi = \frac{4\pi}{\epsilon} \beta_{i,jm} \partial_i u_{jm}, \]  

(13)

where \( u_{jm} \) is the strain tensor, \( \epsilon \) is the dielectric constant, \( \beta_{i,jm} \) is the piezoelectric tensor. In cubic crystals all nonzero component of \( \beta_{i,jm} \) (with \( i \neq j \neq m \)) are equal to \( \beta \).

The solution of Eq. (13) under substitution of Eqs.(10-12) can be written in the form

\[ \varphi_{cz} = i\chi \sqrt{\frac{2\pi e}{S \epsilon}} \sum_q \gamma_q(z) e^{i\pi q} (b_q + b_q^+), \]  

(14)

where \( \chi = \beta \sqrt{16\pi / (\epsilon e^4)} \) is the dimensionless parameter of the piezoelectric interaction (\( \chi \) is the small parameter of our consideration), \( \gamma_q \) is the structural factor, which is determined by the boundary conditions on \( \varphi \) and by the type of the elastic mode. By using Eqs. (2,4,14), we arrive at the expression

\[ g_{eq} = i \sqrt{\frac{2\pi e}{\epsilon}} \phi \gamma_q(z_k), \]  

(15)

where \( z_k \) is the \( z \) coordinate of the electron layer. The explicit expressions for \( \gamma_q(z_k) \) will be presented in Sec.IV, when we specify the geometry of the model.

III. RESPONSE FUNCTIONS IN THE COMPOSITE FERMION MODEL

Let us discuss the calculation of the response functions \( D \) for the composite fermion model. At the beginning, consider the monolayer electron system. The random phase approximation gives the answer

\[ D = (1 - D_0 V)^{-1} D_0, \]  

(16)

where \( V = 2\pi e^2 / \epsilon q \) is the Fourier component of the Coulomb interaction, and \( D_0 \) can be expressed though the polarization operator \( (\Pi_{\mu \nu}) \) of the electromagnetic field

\[ D_0 = \frac{1}{\epsilon^2} \Pi_{00}. \]  

(17)

We use the approach which is the Lagrange formulation of the modified random phase approximation (Ref. [23]) for the calculation of \( \Pi_{00} \). In Ref. [23] the Landau Fermi liquid interaction was taken into account. The flav constant (see, for example, Ref. [24]) of this interaction is determined by the ratio \( m^*/m_b \), where \( m^* \) is the composite fermion effective mass, \( m_b \) - is the band mass of electrons. It was shown in Ref. [23], that taking into account that interaction one obtains the response function, which satisfy the \( f \)-sum rule.

The Lagrangian of composite fermions can be written in the form

\[ L = \Psi^* (i\partial_t + \mu_F - a_0 - eA_0 - \frac{1}{2m^*}(i\nabla_i + a_i + eA_i + b_i)^2) \Psi + \frac{1}{4\pi}\rho \phi \partial_i a_j + \frac{1}{2}\xi b_i^2, \]  

(18)

where \( \mu_F \) is the potential of the Chern-Simons gauge field, \( A_\mu \) is the electromagnetic field potential, \( e \) is fully asymmetric unit tensor, \( \mu_F \) is the chemical potential, \( \psi \) is the parameter of the model, which corresponds to the number of the gauge field flux quanta carrying by the composite quasiparticle \( (\psi \) is the even number), \( b_i \) is an auxiliary field, which is introduced for modelling the Fermi liquid interaction. In Eq. (18) the transverse gauge for the \( a \) field is implied.

The value of \( \xi \) can be determined from the condition that the Lagrangian (18) provides the Galilean invariant value for the electric current (see, for example, Ref. [24]):

\[ \frac{\delta L}{\delta A_i} = \delta j_i - \frac{\epsilon n_0}{m^*} b_i = \frac{m^*}{m_b} < j_i >, \]  

(19)

where \( j_i = (e/m^*) \Psi^* (i\nabla_i + a_i + eA_i) \Psi, n_0 \) is the uniform electron concentration. The mean value of the field \( b \) is determined by the equation

\[ \frac{\delta L}{\delta b_i} = 0 = \frac{1}{e} < j_i > - \frac{n_0}{m^*} < b_i > + \xi < b_i >. \]  

(20)

The condition of consistency of Eqs. (19-21) yields \( \xi = n_0 / (m^* - m_b) \).

The self-consistent effective field acting on the composite fermions is \( B_{eff}^f = B(1 - \nu_F) \) (where \( B \) is the external magnetic field). At filling factors \( \nu_f = N/(N_\psi \pm 1) \) (\( N \) is an integer value) \( B_{eff}^f = \pm n_0 \phi_0 / N, \) where \( \varphi_0 \) is the magnetic field flux quantum. The field \( B_{eff}^f \) corresponds to the integer value \( N \) of filled Landau levels. Therefore, \( \nu_f \)'s correspond to the incompressible states. The field \( B_{eff}^f \) defines the effective cyclotron energy \( \omega_{ce} = 2\pi n_0 / m^* N \) and the effective magnetic length \( \lambda_{eff} = (N/2\pi n_0)^{1/2} \).

To determine the value of \( \Pi_{00} \) one can calculate the quadratic part of the effective action for the electromagnetic field. To do that we calculate the functional integral over the fields \( \Psi \) in the expression for the partition function of the system, described by the Lagrangian (18). Then we calculate the functional integral over the fields \( a \) and \( b \) in the vicinity of the saddle point. Here we present the final result at \( T = 0 \)

\[ \Pi_{00} = \frac{e^2 q^2 S_0}{2\pi \omega_{ce} \Delta_1}, \]  

(21)

where

\[ S_0 = \Sigma_0 - p(\Sigma_0(\Sigma_2 + N) - \Sigma_1^2), \]  

(22)

\[ \Delta_1 = (1 - \psi \Sigma_1)^2 - \psi^2 \Sigma_0(\Sigma_2 + N) - pF, \]  

(23)

\[ F = \Sigma_2 + N + (\omega / \omega_{ce})^2 S_0 \]  

(24)

with \( p = (m^* - m_b) / (m^* N) \),
\[ \Sigma_j = (\text{sgn}(B_{eff}))i\epsilon e^{-2} \sum_{n=0}^{\infty} \sum_{m=N}^{\infty} \frac{n!}{m!} \times \frac{x^{m-n-1}(m-n)}{(\omega/\omega_c)^2 - (m-n)^2} \left(L_n^m(x)\right)^2 - j \times \left[(m-n-x)L_n^m(x) + 2x \frac{dL_n^m(x)}{dx}\right]^2. \]  

In Eq. (24), \( x = ((q \lambda_{eff})^2/2, L_n^m(x)\) is the generalized Laguerre polynomial. Substituting Eqs. (15-17, 21) into Eq. (3) we obtain \n
\[ \Delta v = \chi^2 \gamma_\nu^2 (z_1) \frac{f_q S_0}{\Delta_1 - f_q S_0}, \]  

where \( f_q = e^2 q/(\epsilon \omega_c) \). The case of the double layer Hall system is considered by a similar way. Two types of the Chern-Simons fields and the fermion fields are introduced (they correspond to the two electron layers). The free part of the Chern-Simons Lagrangian includes non-diagonal terms with respect to the layer index.

\[ L_{CS} = \frac{1}{4\pi} \epsilon_{ij} a_{0k} M_{kk'} \partial_l a_{jkl} \]  

with \( M = \frac{1}{\psi^2 - s^2} \left( \begin{array}{cc} \psi & -s \\ -s & \psi \end{array} \right) \),

where \( s \) (an integer number) is the number of the Chern-Simons flux quanta of the type “1”, carrying by the quasi-particle of the type “2” and vice versa (there exists the interlayer statistical interaction in the system). Here we specify the case of two equivalent electron layers. At \( \psi = s \) it is better to rewrite the Lagrangian in terms of the in-phase and out-of-phase combinations of the fields \( a_k \).

The incompressible states for the double layer system correspond to the filling factors \( \nu_f = \frac{N}{N(\psi + s)} \pm 1 \) (calculated for one layer). The expressions for \( B_{eff}, \omega_c, \lambda_{eff} \) remain unchanged. Eq. (24) obeys the matrix form

\[ V = \frac{2\pi \epsilon e^2}{eq} \left( \begin{array}{cc} 1 & e^{-qd_0} \\ e^{-qd_0} & 1 \end{array} \right), \]

where \( d_0 \) is the distance between the electron layers.

The expression for the velocity shift is modified to the following one

\[ \frac{\Delta v}{v} = \frac{1}{2} \gamma^2 S_0 f_q \left\{ \frac{(\gamma_q(z_1) + \gamma_q(z_2))^2}{\Delta_+ - f_q E + S_0} + \frac{(\gamma_q(z_1) - \gamma_q(z_2))^2}{\Delta_- - f_q E - S_0} \right\}, \]

where \( E_\pm = 1 \pm \exp(-qd_0) \).

\[ \Delta_\pm = (1 - (\psi \pm s) \Sigma_1)^2 - (\psi \pm s)^2 \Sigma_0 (\Sigma_2 + N) - pF. \]  

The ground state of the double layer system is described by the quantum numbers \( \psi \) and \( s \). One can see from Eq. (29), that the same \( \nu \) may correspond to different sets of \( \psi \) and \( s \) (different phases). If the correlation between the layers is a small one (at large \( d_0 \)), the phase with \( s = 0 \) is realized. Reduction of the interlayer distance may result in a phase transition into the state with \( s \neq 0 \) (see, for instance, Ref. [24]). The transition may be accompanied by the shift of the velocity of the acoustic wave, interacting with the electrons, as described by Eq. (31).

IV. VELOCITY SHIFT FOR THE WAVEGUIDE ELASTIC MODES

Eqs. (26, 31) define the value of \( \Delta v \) relative the velocity for the systems without the electron-photon interaction. In experiments the value of \( \Delta v \) is measured relative to the velocity at \( B = 0 \). To compare the theoretical value with the experimental data one should know the value of \( \sigma_{xx}(q, \omega) \) (longitudinal component of the conductivity tensor) at zero magnetic field. One can use the approximation \( \sigma_{xx} \rightarrow \infty \) at \( B = 0 \), but in that case the momentum dependence \( \sigma_{xx}(q, \omega) \) at \( B = 0 \) is neglected. We propose to measure the difference between the sound velocities at \( \nu = \nu_f \) and \( \nu = 1 \). The case of \( \nu = 1 \) is also described by Eqs. (26), (31), if one sets \( \psi = s = 0 \), \( N = 1 \) \( \nu = m_0 \).

Thus, the value we calculate is given by the expression

\[ \frac{\Delta v}{v} = \left. \frac{\Delta v}{v} \right|_{\nu = \nu_f} - \left. \frac{\Delta v}{v} \right|_{\nu = 1}. \]

Let us consider the structure \( Al_{0.3}Ga_{0.7}As - GaAs - Al_{0.3}Ga_{0.7}As \), which incorporates two electron layers at \( z_k = \pm a \) (\( d_0 = 2a \)). The parameters \( \beta \) and \( \epsilon \) are implied to be uniform for the whole system. Then, the boundary conditions for \( \varphi \) require the continuity of the values of \( \varphi(q, z) \) and \( \partial \varphi(q, z)/\partial z \) at the interfaces.

The solution of Eq. (24) for the even mode (Eq. (3)) gives an odd function of \( \varphi(z) \), while for the odd mode (Eq. (3)) - an even function of \( \varphi(z) \).

At \( \nu = 1/5 \) one can expect the phase transition between \( (\psi = 1, s = 0) \) and \( (\psi = 2, s = 2) \) states. We see from Eq. (31), that the transition is accompanied by the shift of the velocity of the even elastic mode, while the odd mode does not show such a behavior.

For the even mode the values \( \gamma_q(z_k) \) are found to be

\[ \gamma_q(a) = -\gamma_q(-a) = \alpha^{-1} \frac{\cos(qqa)}{1 + \eta^2}. \]
The function of $\Delta v/v$ on the wave vector are shown in Fig.1 at $\psi = 4$, $s = 0$ and $\psi = 2$, $s = 2$. The parameters $\beta = 4.5 \cdot 10^4$ dyn$^{1/2}$/cm, $\epsilon = 12.5$, $n_0 = 10^{11}$ cm$^{-2}$, $m = 0.07m_e$ ($m_e$ is the bare electron mass), $m^* = 4m_e$, $c_{44} = 6 \cdot 10^{11}$ dyn/cm$^2$, $\rho_1 = 5.3$ g/cm$^3$, $c_b/c_l = 1.05$, $d_0 = 300\lambda$ are used for the calculations. We see, that at finite $q$ the value of the velocity shift is within the experimental resolution, while it is rather small due to the smallness of $\Delta c = (c_b - c_l)/c_l$.

Now we analyze the geometry for which one could expect larger values of the effect. Let the heterostructure contains two electron layers at $z = \pm d_0/2$, then the phase transition between $\psi = 4$, $s = 0$ and $\psi = 2$, $s = 2$ states is accompanied by the shift of the velocity of the even mode only, as for the previous case. The maximal value of the effect takes place for one of higher harmonics (we specify the case of $d_0 \ll 2a$).

The dependencies $\Delta v/v$ on $q$ are shown in Fig.2 at $\nu = 2/5, 3/7, 4/9$ ($N = 2, 3, 4$) for the lowest odd elastic mode. The parameters $c_b/c_l = 1.5$, $2a = 1500\lambda$ are used (other parameters are the same ones). We see, that the function of $\Delta v(q)$ oscillates and the period of oscillation becomes smaller when $\nu$ approaches to $\nu = 1/2$ (it reflects the fact that the effective magnetic length is increased).

The absolute value of the velocity shift is much higher that for the previous case. The effect is lowered at larger $a$.

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The structural factor in Eq. (26) is equal to

$$\gamma_q(0) = \alpha^{-1} \frac{\eta}{1 + \eta^2} \left( 1 - \frac{\cos(\eta qa)}{\cosh(qa)} \right) \quad (35)$$

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The structural factor is equal to

$$\gamma_q(d_0/2) = -\gamma_q(-d_0/2) = \alpha^{-1} \frac{\eta}{1 + \eta^2} \left( \frac{\sin(\eta qa) \sinh(qd_0/2)}{\cosh(qa)} \right) \quad (36)$$

The dependencies $\Delta v/v$ on $q$ are shown in Fig.3 for the third even harmonics ($d_0 = 300\lambda$, $2a = 1500\lambda$).
Thus, the measurements of the velocity shifts for the non-uniform plane elastic modes of the waveguide type propagating in heterostructures AlGaAs – GaAs can be considered as a possible way of studying the dynamical properties of the monolayer and double layer quantum Hall systems. Unlike the case of the surface acoustic wave propagation, the restriction on maximal values of the wave vectors, for which the velocity shift can be observed experimentally, does not take place for the waveguide modes. In case of study of incompressible fractional Hall states, the effect can be used for measuring the dependence of the effective magnetic length on the filling factor and for observing the phase transitions in the double-layer systems under the interlayer distance variation.

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