Ion acoustic shock waves in weakly relativistic multicomponent quantum plasma

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Abstract. Ion acoustic Shock waves (IASWs) are studied in an collisionless unmagnetized relativistic quantum electron-positron-ion (e-p-i) plasma employing the quantum hydro dynamic (QHD) model. Korteweg-deVries-Burger equation (KdVB) is derived using small amplitude perturbation expansion method to study the nonlinear propagation of the quantum IASWs. It is found that the coefficients of the KdVB equation are significantly modified by the positron density, relativistic factor, temperatures, kinematic viscosity and quantum factor.

1. Introduction
The importance of quantum effects have received a great deal of attention in the recent years due to their relevance to microelectronic devices, in dense astrophysical plasma system, laser plasmas, quantum plasma instabilities, the self-consistent dynamics of Fermi gas, dusty plasmas, in nonlinear optics etc. The characteristics of quantum plasmas are their high densities and low temperatures in contrast to their counterpart low density high temperature classical plasmas.

Electron-positron plasmas have been observed early universe, center of our galaxy, in active galactic nuclei, in pulsar magnetosphere, in the polar region of neutron stars as well as in intense laser field and they behave quite differently from electron-ion (e-i) plasmas.

In many astrophysical situations, there exist a small number of ions along with electrons and positrons. and therefore, it is important to study nonlinear behavior of e-p-i plasma. Classical relativistic e-p-i plasma with Boltzmann electron positron distributions, relevant to many astrophysical structures, has recently been studied.

Several well known approaches used for quantum plasmas are the Schrodinger- Poisson, the Wigner-Poisson, and the Dirac-Maxwell, which describe the statistical and hydrodynamic behavior of the plasma particles at quantum scales. These models are the quantum analogues of fluid and kinetic models of the classical plasma physics. Manfredi[19] wrote a review article on
the Schrodinger-Poisson and the Wigner-Poisson models in a collisionless quantum plasma used to study these systems. The quantum hydrodynamic model (QHD) is an extension of classical fluid model in a plasma. It consists of a set of equations that describes the transport of momentum and energy of charged species. An additional term, the so-called 'Bohm potential' is introduced in the equation of motion to account for the quantum effects.

Earlier investigations are confined to study KdV or KP equation and their special soliton solutions. However, when the medium has both dispersive and dissipative properties, the propagation of small amplitude perturbations are governed by Korteweg-de-Vries Burger (KdVB) equation. The dissipation may be due to one or several mechanisms as wave-particle interaction, anomalous viscosity etc. Several theoretical investigations have been reported to study of collective processes in quantum plasmas [20,21,22,23,24,25,26,27]. Recently quantum e-p-i plasma [28,29,30] have been also been studied because of their relevance to astrophysical and cosmological environment.

In the present investigation, we have used reductive perturbation method (RPM) to derive KdVB equation in e-p-i plasma where cold ions are treated as relativistic. In section 2, equations governing the dynamic of KdVB equation are given. Last section deals briefly with the results and discussion of numerical computation.

2. Governing Equations

We consider a three species unmagnetized, collisionless relativistic quantum plasma system composed of electrons, positrons and ions to study the nonlinear propagation of the ion acoustic shock waves. Inertia of the electrons and positrons is ignored by assuming that the phase velocity of the wave is much less than the Fermi velocities of the electrons and positrons respectively. We assume that ions are cold and the electrons and positrons obey the following pressure law:

\[ p_a = \frac{m_a v_{Fa}^2 n_a^3}{3n_{a0}^2} \]  (1)

Where \( v_{Fa} = \sqrt{\frac{2k_B T_{Fa}}{m}} \) with \( a = e, p \) is the Fermi thermal speed, \( T_{Fa} \) is the particle Fermi temperature, \( k_B \) is Boltzmann’s constant and \( n_{a0} \) is the equilibrium particle number density.

The basic set of normalized QHD equations for one-dimensional QIASWs in relativistic e-p-i plasma is given as:

\[ 0 = \frac{\partial \rho}{\partial x} - 2n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2\mu} \frac{\partial}{\partial x} \left( \frac{\partial^2}{\partial x^2} \sqrt{n_e} \right) \]  (2)

\[ 0 = -\frac{\partial \rho}{\partial x} - 2\sigma n_p \frac{\partial n_p}{\partial x} + \frac{H^2}{2\mu} \frac{\partial}{\partial x} \left( \frac{\partial^2}{\partial x^2} \sqrt{n_p} \right) \]  (3)

\[ \frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0 \]  (4)
\[
\frac{\partial (\gamma u)}{\partial x} + u \frac{\partial (\gamma u)}{\partial x} = n \frac{\partial^2 (\gamma u)}{\partial x^2} 
\]

(5)

\[
\frac{\partial^2 \varphi}{\partial x^2} = \mu n_e - (\mu - 1)n_p - n
\]

(6)

where \( \sigma = \left( \frac{T_{fp}}{T_{Fe}} \right) = \left( 1 - \frac{1}{\mu} \right)^{\frac{3}{2}}, \eta_{\text{io}} = \mu \omega_i / c_s^2, H = \hbar \omega / (k_B T_{Fe}) \), \( \mu = 1 / (1 - p) \), \( p = \frac{n_e}{n_{po}} \) and \( \gamma = \left( 1 - \frac{u^2}{c^2} \right)^{-1/2} \).

The electrostatic potential \( \varphi \) is normalized by \( e \varphi / (k_B T_{Fe}) \), the ion fluid velocity \( u \), by quantum ion acoustic speed \( c_s = \sqrt{k_B T_{Fe} / m_i} \) and the number density \( n_a \) by their unperturbed densities \( n_{a0} \).

The space coordinate \( x \) is normalized by the ratio of ion plasma frequency \( \omega_{pi} = \sqrt{4 \pi n_e e^2 / m_i} \) and the quantum ion acoustic speed \( c_s \), and the time coordinate \( t \) is normalized by the ion plasma period \( \omega_{pi}^{-1} \).

Now integrating equations (2) and (3) with boundary conditions viz \( n_e \rightarrow 1, n_p \rightarrow 1 \) and \( \varphi \rightarrow 1 \) at \( x \pm \infty \), we have

\[
\varphi_e = -1 + n_e^2 - \frac{H^2}{2\mu} \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2}
\]

(7)

\[
\varphi_p = \sigma - \sigma n_p^2 + \frac{H^2}{2\mu} \frac{1}{\sqrt{n_p}} \frac{\partial^2 \sqrt{n_p}}{\partial x^2}
\]

(8)

In order to investigate the propagation of QIASWs and to derive the KdVB equation in relativistic \( e-p-i \) plasma, the independent variables are stretched as \( \xi = \varepsilon^{12} (x - \lambda t), \tau = \varepsilon^{3/2} t \) and \( \eta_i = \varepsilon^{1/2} \eta_0 \), while \( \eta_0 \) is a finite quantity of the order of unity, and the dependent variables are expended as
where $\varepsilon$ is the small nonzero parameter proportional to the amplitude of the perturbation. Now, substituting the expressions from equation (9) along with the stretching coordinates into equations (2)-(6) and collecting the terms in different power of $\varepsilon$, the lowest order of $\varepsilon$ yields the following dispersion relation,

$$
(\lambda - u_0)^2 = \frac{1}{\gamma_1} \left( \frac{2\sigma}{\mu(\sigma+1) - 1} \right)
$$

where $\gamma_1 = 1 + 3u_0^2 / 2c^2$. Now if $\gamma_1 \rightarrow 1$, then this dispersion relation has the same form as that derived by the Roy et al [30].

Now, in the next higher order of $\varepsilon$, we eliminate the second order perturbed quantities from the set of equations using standard procedure, to obtain the required KdVB equation for QIAWs:

$$
\frac{\partial \phi}{\partial t} + A \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} - C \frac{\partial^2 \phi}{\partial \xi^2} = 0
$$

where

$$
A = \frac{2^{1/2} \gamma_1^{1/2}}{(\mu(\sigma+1) - 1)^{1/2}} \left[ \frac{3\mu^2(\sigma+1)^2 + \mu(\sigma-7)(\sigma+1) + 4 - 8\sigma^2 \gamma_2}{2\sigma} \left( \frac{\mu(\sigma+1) - 1}{2\sigma} \right)^{3/2} \gamma_1^{-3/2} \right]
$$

$$
B = \frac{2^{1/2} \gamma_1^{1/2}}{(\mu(\sigma+1) - 1)^{1/2}} \left[ 1 - \left( \frac{1 + \sigma^{3/2} H^2}{16} \right) \right]
$$

$$
C = \frac{\eta_0}{2}
$$

and

$$
\gamma_2 = 3u_0^2 / 2c^2
$$

Here $A$, $B$ and $C$ are the coefficients of nonlinear, dispersion and dissipation, respectively. Our results reduce to that of Roy et al [30] in nonrelativistic limit.

### 3. Results and Discussion

It is observed that coefficients $A$ and $B$ are modified by the inclusion of positron density ($p$), relativistic factor ($U_r = u_0/c$) as well as quantum parameter ($H$). The positron density enters the expressions for $A$ and $B$ through density ratio $p$. The last viz. fourth term has its origin in the ion kinematic viscosity. In the absence of viscosity and non relativistic limit, the equation (11) reduces to KdV equation for QIAW. On the other hand, when $B$, $H$ and relativistic factor are missing, we recover well known Burger equation.
The analytical solution of KdVB equation is given by the following expression[31].

\[ \varphi = a_0 + a_1 \tanh(\zeta) + a_2 \tanh^2(\zeta) \]  

(15)

where

\[ a_0 = \frac{1}{A(V + 12B)}, a_1 = \frac{-12C}{5B}, a_2 = \frac{-12B}{A} \]

The above equation is special solution of shock wave solution of equation (11). The constants appearing are the function of several parameters. We have numerically evaluated expression (15) for the following set of parameters.

\[ H=6.89, \eta = 0.81, \sigma = 0.01 \text{ and } U_r=0.01 \]

We have taken three value of positron density \( p=0.5, p= 0.6 \text{ and } p= 0.7 \) for curves 1, 2 and 3 respectively plotted in figure 1. It is observed that strength of the shock is maximum for lower value of \( p \). Further, the steepness of the shock front also follows the same trend as that of the strength of the shock. Moreover, it is worth mentioning that increase in degree of steepness, which being the measure of change of parameters downstream, is drastic one.

In figure 2 we have chosen three values of quantum effect parameters \( H=2, H=5 \text{ and } H=8 \) for curves 1, 2 and 3 respectively at constant values of \( p = 0.3, \sigma =0.01, \eta = 0.81 \text{ and } U_r= 0.01. \)

The parameter \( H \), which describes the density correlation due to quantum fluctuations, plays here a crucial role on the characteristics of shock waves. For lower values of \( H \), the behavior is quite similar to those obtained in the figure 1. However increase of the \( H \), leads to interplay of dispersive and dissipative terms resulting in reversing of shock fronts. Further increase in \( H \) leads to increase reversal effect for shock front which dies down in small transit \( \zeta \).

Lastly we observe the effect of \( \eta \), the kinematic viscosity in figure 3, where \( \varphi \) as a function of \( \zeta \) is plotted for three values of \( \eta =0.2, \eta = 0.5 \text{ and } \eta = 0.8 \) for curves 1,2 and 3 respectively for fixed values of \( p= 0.3, H=4, \sigma =0.01 \text{ and } U_r= 0.01. \)

We here observe that increase of \( \eta \) results in increase of shock strength as well as steepness. Further in upstream, decrease in shock strength with increase in \( \eta \) is also observed.

4. Conclusion

QHD model is used to study ion acoustic shock waves in the multi-component e-p-i plasma. KdVB equation, which governs the dynamics of shock wave with some dissipation mechanisms, has been derived using RPM in small amplitude approximation. We have considered the dissipative mechanism as the kinematic viscosity in the present investigation. We have analytically solved KdVB equation for parameters relevant to astrophysical environment. When the ratio of electron to positron density decreases, the shock strength increases which implies increase in degree of steepening. Further, the increase of quantum parameter, which describe the density correlation due to quantum fluctuation, leads to interplay of dispersive and dissipative terms resulting in the reversing of shock fronts. Lastly, the increase in kinematic viscosity leads to increase of shock strength as well as steepening. Also in the upstream region, the reverse effect is observed i.e. increase in kinematic viscosity leads to decrease in shock strength.

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References

[1] Markowich A, Ringhofer C and Schmeuser C. *Semiconductor Equations (Springer, Vienna, 1990)*

[2] Jung Y D 2001. *Phys. Plasmas* 8 3842

[3] Marklund M and Shukla PK. 2006 *Rev. Mod. Phys* 78 591

[4] Suh N, Feix M R and Bertrand P. 1992 *J. Comput. Phys.* 94 403

[5] Bonitz M, 1994 *Phys. Plasmas*. 1 832

[6] Haas F, Manfredi G and Goedert J. 2001 *Phys. Rev. E* 64 026413

[7] Manfredi G and Haas F 2001 *Phys. Rev. B* 64 075316

[8] Shukla PK and Ali S 2005 *Phys. Plasmas* 12 114502

[9] Masood W, Mushrat A and Khan R 2007 *Phys. Plasmas* 14 123702

[10] Leontovich M 1994 *Izv. Akad. Nauk SSR Ser. Fiz. mat. Nauk* 8 16

[11] Aggarwal G, *Nonlinear fibre Optics* (Acedemic, San Diego 1995)

[12] Gibbons G W, Hawking S W and Siklos S. *The Very Early Universe Cambridge University Press, Cambridge 1983.*

[13] Mille H R and Witta P. *Active Galactic Nuclei* (Springer, Berlin 1987) p. 202

[14] Michel FC 1982 *Rev Mod. Phys.* 54 1

[15] Michel F C *Theory of Neutron Star Magnetosphere*(Chicago University Press, Chicago 1991)

[16] Berezhiani V, Tskhakaya D D and Shukla P K. 1992 *Phys. Rev. A* 46 6608

[17] Gill T S, Singh A, Kaur H, Saini N S and Bala P. 2007 *Phys. Letts. A.* 361 364

[18] Mushtaq A and Shah H A. 2005 *Phys. Plasmas* 12 0723306

[19] Manfredi G 2005 *Fields Ints. Commun* 46 263

[20] Mishra A P, Shukla P K and Bhowmik C 2007 *Phys Plasmas* 14 082309

[21] Mushtaq A and Khan S A. 2007 *Phys. Plasmas* 14 052307

[22] Haas F, Garcia L G, Goedert J and Manfredi G. 2003 *Phys Plasmas* 10 3858

[23] Shukla P K and Stenflo L 2006 *New J. Phys* 8 111

[24] Shukla P K and Stenflo L 2006 *Phys. Lett. A* 355 378

[25] Haas F, Manfredi G and Feix M R 2000 *Phys Rev. E* 62 2763

[26] Khan S A and Mushtaq A 2007 *Phys. Plasmas* 14 083703

[27] Moslem W M, Ali S, Shukla P K, Tang X Y and Rowlands G. 2007 *Phys. Plasmas* 14 082308

[28] Ali S, Moslem W M, Shukla P K and Schlickeiser R 2007 *Phys. plasmas* 14 82307

[29] Masood W, Mirza A M and Hanif M 2008 *Phys. Plasmas* 15 072106

[30] Roy K, Mishra A P and Chatterjee P 2008 *Phys. Plasmas* 15 032310

[31] Sahu B and Roychoudhury R 2003 *Czek. J. of Phys.* 53 6 517
Figure 1. Variation of $\phi$ vs. $\xi$ at different values of $p = 0.5, 0.6, 0.7$ for curves 1, 2 and 3 respectively at fixed values of $H = 6.89, \eta = 0.81, \sigma = 0.01$ and $U_r = 0.01$. 
Figure 2. Variation of $\phi$ vs. $\zeta$ at different values of $H = 2, 5$ and $8$ for curves 1, 2 and 3 respectively at fixed values of $p = 0.3, \eta = 0.81, \sigma = 0.01$ and $U_r = 0.01$. 
Figure 3. Variation of $\phi$ vs. $\xi$ at different values of $\eta = 0.2, 0.5$ and $0.8$ for curves 1, 2 and 3 respectively at fixed values of $p = 0.3$, $H=4$, $\sigma = 0.01$ and $U_r = 0.01$. 