Cyclotron motion without magnetic field

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Abstract

Non-trivial Bloch band overlaps endow rich phenomena to a wide variety of quantum materials. The most prominent example is a transverse current in the absence of a magnetic field (i.e. the anomalous Hall effect). Here we show that, in addition to a dc Hall effect, anomalous Hall materials possess circulating currents and cyclotron motion without magnetic field. These are generated from the intricate wavefunction dynamics within the unit cell. Curiously, anomalous cyclotron motion exhibits an intrinsic decay in time (even in pristine materials) displaying a characteristic power law decay. This reveals an intrinsic dephasing similar to that of inhomogeneous broadening of spins. Circulating currents can manifest as the emission of circularly polarized light pulses in response to an incident linearly polarized (pulsed) electric field, and provide a direct means of interrogating a type of Zitterbewegung of quantum materials with broken time reversal symmetry.

In the presence of a perpendicular magnetic field, electrons in a two dimensional electron gas experience a fast cyclotron motion owing to the Lorentz force. Additionally, when an external electric field is applied, these electrons exhibit a slow guiding center Hall drift perpendicular to both electric field and magnetic field directions that leads to the conventional Hall effect. These fast (cyclotron) and slow (guiding-center) dynamics are intimately linked and form dual faces of magneto-transport behavior. In the absence of a magnetic field, broken time reversal symmetry (TRS) materials can also exhibit an anomalous Hall effect (AHE)\textsuperscript{[1–6, 36, 37].} This originates from the internal structure of the electron wavefunction in such materials\textsuperscript{[6].} For example, electron wavepackets possessing a finite Bloch band Berry curvature exhibit an anomalous velocity (AV) transverse to an applied electric field that contributes to the AHE. This transverse (to an applied electric field) wavepacket motion in anomalous Hall materials bears striking resemblance to the more familiar slow guiding center Hall drift found in conventional magnetotransport. Extending this analogy further, a natural question then arises: in such anomalous Hall materials, can cyclotron motion without external magnetic field also arise?

Here, we show that in addition to a slow transverse drift arising from AV, anomalous Hall materials exhibit fast dynamics with a characteristic anomalous cyclotron-like motion without magnetic field. For example, focussing on a simple model of an anomalous Hall material—a broken TRS gapped Dirac system with two sites per unit cell—we find that the probability to find an electron on either A or B sites (wavefunction square amplitude) can naturally oscillate between the sites when the system is pulsed (figure\textsuperscript{1} (a)). As we discuss below, even when the valence band is initially fully occupied, signatures of the pulse-activated pseudo-spin precession persist and manifest as a real-space cyclotron motion (of the entire electron gas). The motion can be thought of as a special kind of Zitterbewegung that requires broken TRS and are collectively excited without the need for isolating electron wave-packets\textsuperscript{[7–12].} The anomalous cyclotron motion (ACM) can be activated by either applying a short electric pulse (figure\textsuperscript{2}) or rapidly turning on an electric field (figure\textsuperscript{3}). Both approaches induce cyclotron currents that can be probed via emission of circularly polarized light when linearly polarized fields are incident on a sample (figure\textsuperscript{4}).
Similar to how slow guiding center motion (Hall drift) and fast cyclotron motion in the presence of a magnetic field are dual to each other, the slow transverse drift from semiclassical AV in anomalous Hall materials possesses an analogous duality to the fast ACM we describe here. This is because both ACM and AV arise from the same origin—a non-trivial Bloch band overlap encoded in the internal structure of the wavefunction (intra-unit cell structure) [1, 13, 36, 37]. This non-trivial Bloch band overlaps are commonly associated with quantum geometry [14, 15]. To see this, we recall that AV arises from the adiabatic motion of carriers projected onto a single band (see supplementary information is available online at stacks.iop.org/NJP/21/083026/mmedia). In contrast, ACM arises from the non-adiabatic dynamics of carriers and can be excited by a protocols that activates inter-band transitions between the bands. Much as AV displays the static dc response, ACM is the physical manifestation of the dynamical response of anomalous Hall materials [16–20].

However, in contrast to conventional cyclotron motion in a magnetic field, the ACM we discuss here exhibits intrinsic power-law decay, even in a pristine and intrinsic system. This decay arises from dephasing due to inhomogeneous broadening of the pseudo-spins (in-momentum space). When electric field is applied, pseudo-spins in different states exhibit multi-frequency Larmor precession (each pseudo-spin precesses with a different frequency and direction) thus creating dephasing. Nevertheless, the response is peaked for transitions close to the band edges, and displays a characteristic frequency that is tunable by the band gap in a massive Dirac system.

We begin by considering a system with two sites in a unit cell, thus having two bands. These two sites can correspond to two types of atoms or orbitals within the unit cell; this two-band model is a simple system exhibiting non-trivial intra-unit-cell structure [1, 4, 13, 36, 37]. We write the Hamiltonian as: $H = \tau \cdot d(p)$, where $\tau_{x,y,z}$ are Pauli matrices that act on the sublattice $A$ and $B$ sites, and $d(p)$ describes the band structure of the specific material, and $p$ is the quasi-momentum. The pseudo-spin dynamics compactly describes amplitudes and phases of the electron wavefunction on the $A$ and $B$ sites, $m_p = \langle \psi(p)|\tau|\psi(p)\rangle$, with $\psi(p) = (\psi_A(p), \psi_B(p))$. The dynamics of $m_p$ thus represent motion of an electron within the unit cell that can be tracked in the Bloch sphere as illustrated in figure 1(b). When $m_p$ is aligned to the north (south) pole of the Bloch sphere, the wave function has weight solely on $A$($B$) sublattice.

Importantly, the pseudo-spins obey the Bloch equation of motion

$$\frac{d}{dt} m_p(t) = \langle \psi(p)|i\hbar^{-1}\left[\tau, H(t)\right]|\psi(p)\rangle = \frac{2}{\hbar} d(p) \times m_p. \quad (1)$$

Without applied electric field $m_p = m_p^{(0)} = \frac{d(p)}{|d(p)|}$ corresponding to the pseudo-spins of conduction and valence band states respectively. At equilibrium, conduction (valence) band pseudo-spins $m_p^{(0)}$ are parallel (anti-parallel) to $d(p)$ yielding $dm_p/dt$ that vanishes. Here we assume that initially all electrons fully occupy the valence band and set $T = 0 \text{ K}$. When an external electric field is applied, $d$ is shifted by $\delta d = A\delta\phi (p = eA/c)$ to linear order. Here $A$ is the vector potential, related to electric field by $E = -\partial A/c$. As a result, $m_p$ and $d' = d(p - eA/c)$ are no longer aligned, turning on the dynamics of $m_p$.

The rate of change of the pseudo-spin, $dm_p/dt$, is perpendicular to $m_p$ (see equation (1)) which makes the pseudo-spin $m_p$ Larmor precesses around $d'$ similar to spin dynamics (see figure 1(b)); this occurs naturally whenever $m_p$ is cantled away from $d(p)$. This dynamics can be visualized as an oscillation of probability at each site (see figure 1(a)). As we will see, it is this pseudo-spin oscillation that leads to the unusual ACM we describe below. In the following we will capture the dynamics by writing $m_p(t) = m_p^{(0)} + \delta m_p(t)$. We note that...
equation (1) can be nonlinear since δm and δd are both proportional to E. In this work, however, we focus on linear response while leaving nonlinear effects as a subject of further study.

To understand the pseudo-spin dynamics in anomalous Hall materials, we specialize to TRS-broken gapped Dirac systems with d(p) = (χνpL, νpS, Δ) where χ = ±1 is the chirality, v is the Dirac velocity of electrons, and Δ is the energy difference between A and B sublattice sites. This system can be found in graphene or other honeycomb monolayers on top of magnetic substrates such as yttrium iron garnet [21, 22] or EuO [23]. Since TRS is broken, we will focus on just a single cone. Although we focus on the simplest model, equivalent qualitative results can be drawn for different systems such as anomalous Hall ferromagnets, topological insulators, and Weyl and Dirac semimetals (see also supplemental information for a discussion of the half-Bernevig–Hughes–Zhang (half-BHZ) model).

As we now argue, the Larmor-like pseudo-spin precession (figure 1(b)) persists in the macroscopic dynamics of the entire electron gas and manifests as ACM. To see this, we excite the system with linearly polarized light, E = E0e−iωt̂x̂. Collecting the linear terms of equation (1), we obtain

$$\frac{d}{dt} m_p(t) = \frac{2}{\hbar} (d(p) \times \delta m_p(t) + \delta d \times m_p(0)), \tag{2}$$

where $δd = i\chi vE_0/ω̂x̂$. Here we have focussed on linear response, dropping the nonlinear term $δm_p \times δd(p)$. We note that the dynamics of $δm_p$ is controlled by the internal structure (the first term in the right hand side (rhs) of equation (2)), as well as the external driving (the second term in rhs of equation (2)). The former gives rise to a precessional motion with frequency $2|d|/\hbar$ that corresponds to the oscillation frequency of interband transitions. We note, parenthetically, that this frequency is related to the Zitterbewegung of a particular p state [7–12]. We will show later that collective dynamics (of the entire electron gas) arising from this pesudospin precession will generate ACM in the TRS broken anomalous Hall material. As discussed below, this allows ACM to be observed via a simple pulsed protocol; this contrasts with the finite momentum electron wave packet protocols discussed in connection with the observation of Zitterbewegung more generally [8, 11, 12].

For a fully gapped system, with Fermi energy in the gap, the electrons are in the valence band. Solving for Fourier components $δm \sim e^{-iωt}$ in equation (2), we obtain:

$$δm_x(ω) = \frac{i\chi vE_0 (d^2_x + d^2_y)}{ω|d|((hω/2)^2 - |d|^2)} \tag{3}$$

$$δm_y(ω) = -\frac{\chi vE_0 ((hω/2) d_x + i d_y)}{ω|d|((hω/2)^2 - |d|^2)},$$

$$δm_z(ω) = \frac{\chi vE_0 ((hω/2) d_y - i d_x)}{ω|d|((hω/2)^2 - |d|^2)},$$

where we have used $m_p(0) = -d(p)/|d(p)|$ corresponding to the valence band.

The current response of the entire electron gas can be determined as $j = e\sum_p (\partial_p \mathcal{H}) = ev(\chi \sum_p δm_x, \sum_p δm_y, 0)$, see supplemental information (SI) for a full account. Here the sum is taken over the entire valence band. We note that terms in equation (3) that are odd in $d_x$ and $d_y$ cancel during p integration. In the static limit $ω → 0$, the non-vanishing integrand of $δm_p$ reproduces the Berry curvature of gapped Dirac systems, $\chi v^2 h^2 d_x/(2|d|^2)$; taking the integral of $δm_p$ over p, one can obtain the (half) quantized Hall conductivity in the static limit expected from a single gapped Dirac cone per spin.

To demonstrate the full response of pseudo-spin dynamics, access to a broad range of frequency is needed. This can be achieved, for example, by an ultra short light pulse. For simplicity, we first consider the form $E = E_0^x δ(τ) ̂x̂$ (see SI for a Gaussian profile for a comparison). Using equation (3), the current that develops at $t > 0$ in response to the delta function pulse is (see supplementary information):

$$j^x_0(t) = \frac{e^2}{2\hbar} \frac{2\Delta E_0^x}{2|d|^2} \left[ τ \left( \frac{\sin(τ)}{τ} - \frac{1}{2} \sin(τ) + \cos(τ) + \pi b(τ) \right) \right],$$

$$j^y_0(t) = \frac{\chi e^2}{2\hbar} \frac{2\Delta E_0^y}{2|d|^2} \left[ \frac{π}{2} - \sin(τ) \right], \tag{4}$$

where $τ = 2Δt/\hbar$, and $Si(τ) = \int_0^τ dx' \sin(x')/x'$. Here $j^x_0$ captures the dynamical anomalous Hall motion. The oscillatory motion of both $j^x_0$ (blue) and $j^y_0$ (red) with $χ = 1$ are shown explicitly in figure 2(a). This oscillation (finite frequency response) indicates the contribution from interband transitions.

Crucially, the oscillatory response of $j^x_0$ and $j^y_0$ are displaced in phase by $π/2$ and characterizes the ACM in gapped Dirac systems (figure 2(a) inset). The $π/2$ phase lag of $j^x_0$ versus $j^y_0$ is inherited from the pseudo-spin motion that leads to the Larmor precession of the individual spins in figure 1(b). Indeed, the plot $j^x_0$ versus $j^y_0$ in...
the inset of figure 2(a) shows a circulating current (ACM). The circulating current is centered at (0, 0) because the delta-pulse electric field is (instantaneously) applied only at \( t = 0 \).

To emphasize the real-space nature of ACM, we plot the change of polarization, \( \Delta P = \int_0^t j(t') \, dt' \) shown in figure 2(b). This demonstrates how the average displacement for the carriers circulates in a cyclotron fashion reminiscent of that found in the presence of a Lorentz force. This features a characteristic spiral pattern, picking out a handedness (determined by \( \chi \)) that describes the sense of rotation; broken TRS is integral to ACM.

We note, parenthetically, that before performing cyclotron motion (ACM), the carriers exhibit a (global) shift in position. Because of \( \delta(\tau) \) in \( f_\rho \) (see equation (4)), the electron gas is initially displaced by \( \pi \lambda_0^2 / 4 \) and returns to equilibrium in the long time limit. Meanwhile in the \( y \) direction, the electron gas is polarized by an (electric-field-dependent) value \( \chi \lambda_0^2 \), where \( \lambda_0^2 = \sigma_H E_0^2 \) and \( \sigma_H = e^2 / 2h \) is the dc Hall conductivity for a single gapped Dirac cone per spin (see supplementary information). This can be understood by noting that, in the long time limit, polarization \( \Delta P \) from the delta function pulse is determined solely by the dc conductivity arising from intra-band contributions; we note that the inter-band contribution oscillates and its contribution to the polarization vanishes in the long time limit (see supplementary information). As a result, only a transverse polarization (arising from non-vanishing dc Hall conductivity) manifests at long times; longitudinal polarization diminishes to zero since dc longitudinal conductivity vanishes for a fully gapped system. This change of transverse polarization corresponds to the canting of \( \mathbf{d} \rightarrow d' \) in figure 1(b) and appears due to the instantaneous drift induced by the delta function pulse field.

In contrast to Larmor precession for a single \( p \) state in figure 1(b), or to cyclotron motion in a magnetic field, the macroscopic ACM found in \( f_\rho \) deteriorates, and follows a power-law \( 1/t \) decay (see the dashed line in figure 2(a)). This decay can be understood as a dephasing arising due to inhomogeneous broadening. Whenever the electric field is applied, each pseudo-spin at state \( p \) precesses with different frequency and direction. At very short times, all the spins oscillate at the same time. However, after sometime they begin to oscillate out of phase and dephase. As a result, ACM (the sum total dynamics of all the spins) decays. We note that this intrinsic decay can be slower than extrinsic scattering processes which may cut this slow-relaxation; nevertheless, the slow relaxation—that occurs even in the absence of extrinsic scattering—is a hallmark of the non-trivial structure of the pseudo-spin dynamics. We observe an intrinsic power-law decay \( 1/t^{\alpha} \) that applies across different systems with \( \alpha \) depending on the specific Hamiltonian of the system. See below for slower decay in the case of flatband and in SI for the BHZ model (see supplementary information).

Although the delta function pulse excites the entire frequency range \( (\omega \in [0, \infty]) \), the temporal profile of \( f_\rho \) predominantly follows an oscillation frequency \( 2\Delta / h \). This corresponds to the bandgap (i.e. direct interband transition between band-edges) and is a physical manifestation of the sharp logarithmic resonance in \( \text{Re} \left[ \sigma_{xy}(\omega) \right] \) and \( \text{Im} \left[ \sigma_{xy}(\omega) \right] \) when \( \omega \) matches with energy gap \( 2\Delta \) (see supplementary information) [16]. This frequency further underscores the inter-band transition nature of ACM; the cyclotron-like pseudo-spin motion in these pulsed systems originate from inter-band wavefunction coherences induced by the pulse.

ACM (from inter-band coherence) can co-exist side-by-side with AV (from intra-band coherence). To see this, we apply a homogeneous electric field in space with a step function in time, \( E_0^2 \Theta(t) \hat{x} \), so as to get a dc response (corresponding to AV) as well as to incorporate a broad frequency range (to enable ACM discussed above). Using equation (3), we obtain the full current response at \( t > 0 \) (see supplementary information):

![Figure 2. Anomalous cyclotron motion. (a) In the presence of a linearly polarized pulse, \( E_0^2 \Theta(t) \hat{x} \), longitudinal current \( j^x_\rho \) (blue line) and Hall current \( j^y_\rho \) (red line) oscillate as a function of time. For \( j^y_\rho \) plotted here, we have used \( \chi = 1 \). Dashed line indicates \( 1/t \) decay. Here \( j^y_\rho = E_0^2 \sigma_H 2\Delta / h \), \( \sigma_H = e^2 / 2h \), and \( T = 0 \) K. (Inset) \( j^x_\rho \) versus \( j^y_\rho \) are \( \pi/2 \) out-of-phase leading to a macroscopic circulating current. (b) Polarization exhibits an initial shift in \( x \) due to the pulse. It subsequently spirals inward displaying a macroscopic (real-space) cyclotron motion of charge carriers in anomalous Hall materials. This is obtained from integrating \( f_\rho \) over time (see text) where \( \chi \lambda_0^2 = \sigma_H E_0^2 \).](image-url)
and are circularly polarized with a \( \pi/2 \) phase shift. For \( j^0_y \), we have selected \( \chi = 1 \). Dashed lines indicate \( 1/t \) decay. Here \( j^0_x = \sigma_l E^0_x \) with \( \sigma_l = e^2/2h \). (Inset) current density spirals about \((j^0_x, j^0_y) = (0, 1)\) displaying ACM (circulating current) coexisting with AV \((j^0_y = 1)\). (b) Polarization of carriers obtained from integrating \( j^0_t \) over time (see text). Here \( \lambda_0 = (\sigma_l/\nu)_\lambda E^0_y \) where \( \lambda_0 = h\nu/2\Delta \) is the Compton wavelength.

\[
 j^0_x(t) = \frac{e^2 E^0_x}{8h} \left( \frac{\tau^2}{\chi} - 2 \right) \left( \text{Si}(\tau) - \frac{\pi}{2} \right) + \tau \cos(\tau) + \sin(\tau),
\]
\[ j^0_y(t) = \frac{e^2 E^0_y}{2h} \left( 1 - \cos(\tau) + \tau \left( \frac{\pi}{2} - \text{Si}(\tau) \right) \right), \tag{5} \]

The step function (in time) and homogeneous (in space) field induces an oscillating response with frequency \( 2\Delta/\hbar \) in both \( j^0_x \) and \( j^0_y \) (blue and red lines, respectively in figure 3(a)).

The anomalous Hall current \( j^0_y \) for \( \chi = 1 \) (red line of figure 3(a)) oscillates around \( j^0_y = \sigma_l E^0_y \). While \( \sigma_l = e^2/2h \) arises from the intra-band part of the AHE, the full response we evaluate here modulates the Hall current and includes an inter-band contribution (that arises from the sharp turn-on).

Similar to ACM induced by a delta-function pulse (in equation (4) above), \( j^0_x \) and \( j^0_y \) decay as \( 1/t \) (dashed lines of figure 3(a)) and exhibits a phase lag of \( \pi/2 \). Plotting \( j^0_x \) versus \( j^0_y \) in the inset of figure 3(a), we display how ACM spirals inwards. We note that at long times this oscillation decays and approaches the value expected for adiabatic dc transport (i.e. a drift from AV); this is shown by the spiral that is centered around \( j^0_y = j^0_x \) indicating its long-time behavior. This shifted spiral indicates how ACM (most pronounced at short times) can co-exist with AV.

For the step-function \( E^0_y \Theta(t) \), the predominant (real-space) displacement (polarization) is in the transverse direction (y direction); it is accompanied by an oscillatory motion in x (figure 3(b)). The oscillatory motion comes from the fact that \( j^0_x \) oscillates around zero (figure 3(a)). We note, parenthetically, there also exists a parallel translation in x due to a non-zero integral of \( j^0_x \) over time. This translation is proportional to the applied electric field and \( \Delta^{-1} \) as determined by a quantity \( \lambda_0 = (\sigma_l/\nu)_\lambda E^0_y \) where \( \lambda_0 = h\nu/2\Delta \) is a Compton wavelength. For small gap \( 2\Delta = 10 \) meV, the Compton wavelength can be sizable, \( \lambda_0 = 65 \) nm. It corresponds to the build up of polarization along x.

ACM can be probed by radiation of electromagnetic (EM) waves from the anomalous cyclotron current. Interestingly, these EM waves are circularly polarized even though the input pulse is linearly polarized, as illustrated in figures 4(a) and (c). To illustrate this, we note that EM radiation from a current source is given by:

\[
 \mathbf{E}(r, t) = -\frac{1}{c^2} \int \frac{d^3r'}{|r' - r|} \mathbf{d}(r', t - \frac{|r' - r|}{c}), \tag{6} \]

where \( c \) is the speed of light. For the light pulse \( E^0_y \delta(t) \), we use a convenient choice for the laser spot size assuming it to be a circle with area 1 \( \mu m^2 \) for illustration. For the step function electric field \( E^0_y \Theta(t) \), we assume homogeneous current density \( j^0(t) \) over an area of 1 \( \mu m^2 \); other spot sizes can be chosen with no qualitative changes to the results we discuss below. The radiated electromagnetic waves \( \mathbf{E}^\text{E} \) and \( \mathbf{E}^\text{E} \) are evaluated at a position \( r = 1 \) \( \mu m \) above the source as shown in figures 4(b) and (d), respectively. With these parameters, \( \mathbf{E}^\text{E} \) is typically 100 times smaller than the effective incident fields \( E^0_y \). Both \( \mathbf{E}^\text{E} \) and \( \mathbf{E}^\text{E} \) follow a power law \( 1/t \) and are circularly polarized (see inset of figures 4(b) and (d)). The jumps of \( \mathbf{E} \) at \( t = |r|/c \) are due to the effect of retardation.

Figure 3. Coexistence of anomalous velocity and anomalous cyclotron motion. (a) In the presence of a step function and homogeneous (in space) electric field \( E^0_y \delta(t) \) at \( t > 0 \), longitudinal current \( j^0_x \) (blue line) and Hall current \( j^0_y \) (red line) oscillate in time with a \( \pi/2 \) phase shift. For \( j^0_y \), we have selected \( \chi = 1 \). Dashed lines indicate \( 1/t \) decay. Here \( j^0_x = \sigma_l E^0_x \) with \( \sigma_l = e^2/2h \). (Inset) current density spirals about \((j^0_x, j^0_y) = (0, 1)\) displaying ACM (circulating current) coexisting with AV \((j^0_y = 1)\). (b) Polarization of carriers obtained from integrating \( j^0_t \) over time (see text). Here \( \lambda_0 = (\sigma_l/\nu)_\lambda E^0_y \) where \( \lambda_0 = h\nu/2\Delta \) is the Compton wavelength.
Figure 4. Circularly polarized light emission without magnetic field. (a) A linearly polarized pulse \( E_p^x \delta(t) \) yields a circulating current \( j \) and circularly polarized light emission. (b) Radiated electric field from cyclotron current in \( x \) (blue line) and \( y \) (red line) detected from a position \( r = (0, 0, 1) \mu m \). Here \( \mathcal{E}^x = \mathcal{E}^y / (E_0^x 2\Delta / b) \). We use \( \chi = 1 \) and \( 2\Delta = 1 \) eV. (Inset) evolution of radiated electric field polarization. (c) Probing the pseudo-spin dynamics via emission of circular polarized light arising from a step function electric field \( E_{\Phi}^0 \delta(t) \). (d) Radiated electric field in \( x \) (blue line) and \( y \) (red line) from \( j^y \) with the same parameters as (b). Here \( \mathcal{E}^{xy} = \mathcal{E}^{x0} / E_0^{x0} \). (Inset) evolution of radiated electric field polarization. Jumps of \( \mathcal{E}^{y0} \) at \( t = |\tau|/c \) are due to retardation. Note that all \( \mathcal{E}^{xy} \) values in (b) and (d) have been multiplied by 100.

ACM that we discuss here morphs to resemble conventional magneto-cyclotron motion if the band structure is flat and possesses non-zero Chern number. In such a case, the topological flat bands mimic Landau levels in the presence of magnetic field. To illustrate this, we consider a broken TRS lattice model with \( d(p) = \hbar v / a \left( \cos \left( \frac{pa}{2T} \right) \cos \left( \frac{pa}{2T} \right), \sin \left( \frac{pa}{2T} \right) \sin \left( \frac{pa}{2T} \right), b \left( \cos \left( \frac{pa}{2T} \right) - \cos \left( \frac{2a}{2T} \right) \right) \right) \) with \( a \) being the lattice constant and half of the band gap is \( \Delta = \hbar v / 2b \). Here we set \( b = (2\sqrt{2})^{-1} \) to obtain the band width to band gap ratio \( \approx 0.2 \) and Chern number \( = -1 \) [24]. See SI for detailed band structure and density of states.

Applying a delta function pulse to the flat bands, we obtain longitudinal and Hall currents that persist over long times (see figure 5(a)). The oscillating currents \( j_{x,y} \) initially decays with a slower rate than that in gapped Dirac systems and then recovers and exhibits beating. Taking the Fourier transform (FT) of \( j_{x,y} \), we show that the ACM predominantly oscillates with frequency defined by the bandgap \( (\hbar \omega = 2\Delta) \) (inset of figure 5(a)). Small contributions above the band gap causes the beating in \( j_{x,y} \). Overall FT profiles of \( j_{x,y} \) mimic the joint density of states of the bands (see supplementary information). Indeed the \( j_x, j_y \) plot in figure 5(a) shows cyclotron current that goes around for very long times with a nearly circular orbit centered at \( (0, 0) \).

In summary, we demonstrate how fast pseudo-spin dynamics can be tracked through a pulsed non-adiabatic excitation, manifesting in ACM without magnetic field. ACM can be found in a variety of anomalous Hall materials for example intrinsic anomalous Hall systems in magnetic materials [25], ferromagnetic insulators: Cr–Ge–Te alloy films [26], CoFe2O4 [27], EuO [28]; dilute magnetic semiconductors: (Ga, Mn)As [29]; magnetically doped topological insulators: Cr doped (Bi, Sb)2Te3 [30], gapped 2D Dirac materials (graphene, silicene, germanene, and transition metal dichalcogenides) on top of magnetic substrates [21–23], and broken TRS 3D Dirac and Weyl semimetals. ACM displays a number of unusual characteristics including an intrinsic power-law decay, and real-space charge displacements that follow a spiral-like trajectory. This can be detected via circularly polarized emission induced by a linearly polarized pulsed excitation in anomalous Hall materials in the absence of an applied magnetic field. This pulsed scheme for detecting ACM contrasts with other protocols used to observe fast oscillating Zitterbewegung [8, 11, 12, 31–35]. Just as slow AV of carriers characterizes the intra-band Bloch band overlaps (e.g. Berry curvature) of anomalous Hall materials, ACM captures its inter-band coherences. ACM indeed resembles conventional magneto-cyclotron orbit when the bands are nearly flat and topological analogous to the Landau levels. Both ACM and AHE are rooted in the non-trivial Bloch band
overlaps and constitute different aspects of Bloch band geometry; ACM provides a dynamical window into the inner (often hidden) dynamics within the unit cell.

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References

[1] Adams E N II and Blount E I 1959 J. Phys. Chem. Solids 10 286–303
[2] Karplus R and Luttinger J M 1954 Phys. Rev. 95 1154–60
[3] Sundaram G and Niu Q 1999 Phys. Rev. B 59 14915–25
[4] Xiao D, Chang M-C and Niu Q 2010 Rev. Mod. Phys. 82 1959–2007
[5] Vanderbilt D 2018 Berry Phases in Electronic Structure Theory (Cambridge: Cambridge University Press)
[6] Nagaosa N, Sinova J, Onoda S, MacDonald A H and Ong N P 2010 Rev. Mod. Phys. 82 1539
[7] Winkler R, Zülicke U and Bolte J 2007 Phys. Rev. B 75 205314
[8] Zawadzki W and Rusin T M 2011 J. Phys.: Condens. Matter 23 143201
[9] Cserti J and Dávid G 2010 Phys. Rev. B 82 201405(R)
[10] Katsnelson M I 2006 Eur. Phys. J. B 51 157–60
[11] Schliemann J, Loss D and Westervelt R M 2005 Phys. Rev. Lett. 94 206801
[12] Rusin T M and Zawadzki W 2014 J. Phys.: Condens. Matter 26 215301
[13] Nagaosa N and Morimoto T 2017 Adv. Mater. 29 1603345
[14] Haldane D 2011 Phys. Rev. Lett. 107 116801
[15] Sodemann I and Fu L 2015 Phys. Rev. Lett. 115 216806
[16] Tse W-K and Macdonald A H 2010 Phys. Rev. Lett. 105 057401
[17] Nandkishore R and Levitov L 2011 Phys. Rev. Lett. 107 097402
[18] Lasia M and Brey I 2014 Phys. Rev. B 90 075417
[19] Rostami H and Asgari R 2014 Phys. Rev. B 89 115413
[20] Mukherjee S P and Carbotte J P 2017 Phys. Rev. B 96 085114
[21] Yang C, Cheng B, Aldosary M, Wang Z, Iang Z, Watanabe K, Taniguchi T, Bockrath M and Shi J 2018 APL Mater. 6 026401
[22] Wang Z, Yang C, Sachs R, Barlas Y and Shi J 2015 Phys. Rev. Lett. 114 016803
[23] Averyanov D V, Sokolov I S, Tokmachev A M, Parfenov O E, Karateev I A, Taldenkov A N and Storchak V G 2018 ACS Appl. Mater. Interfaces 10 20767–74
[24] Sun K, Gu Z, Katsumo H and Sarma D S 2011 Phys. Rev. Lett. 106 236803
[25] Nagaosa N, Sinova J, Onoda S I, MacDonald A H and Ong N P 2010 Rev. Mod. Phys. 82 1539
[26] Mogi M et al 2018 APL Mater. 6 091104
[27] Amamou W et al 2018 Phys. Rev. Mater. 2 011401(R)
[28] Swartz A G, Odenthal P M, Rodney Y H, Ruoff S and Kawakami R K 2012 ACS Nano 6 10063–9
[29] Dietl T and Ohno H 2014 Rev. Mod. Phys. 86 187
[30] Chang C-Z et al 2013 Science 340 167–70
  He K, Wang Y and Xue Q-K 2018 Ann. Rev. Condens. Matter Phys. 9 329–44
[31] Stepanov I, Ersfeld M, Poshakinskiy A V, Lepsa M, Ivchenko E L, Tarasenko S A and Beschoten B 2016 arXiv:1612.06190
[32] Le Blanc L J, Beeler M C, Jimenez-Garcia K, Perry A R, Sugawa S, Williams R A and Spielman I B 2013 New J. Phys. 15 073011
[33] Qu C, Hamner C, Gong M, Zhang C and Engels P 2013 Phys. Rev. A 88 021604(R)
[34] Lamata L, Leon J, Schatz T and Solano E 2007 Phys. Rev. Lett. 98 253005
[35] Gerritsma R, Kirchmair G, Zahringuer F, Solano E, Blatt R and Roos C F 2010 Nature 463 69
[36] Adams E N II 1953 J. Chem. Phys. 21 2013
[37] Blount E I 1962 Solid State Phys. 13 505–73