Research Article

Process Monitoring for Gamma Distributed Product under Neutrosophic Statistics Using Resampling Scheme

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In this article, a repetitive sampling control chart for the gamma distribution under the indeterminate environment has been presented. The control chart coefficients, probability of in-control, probability of out-of-control, and average run lengths have been determined under the assumption of the symmetrical property of the normal distribution using the neutrosophic interval method. The performance of the designed chart has been evaluated using the average run length measurements under different process settings for an indeterminate environment. In-control and out-of-control nature of the proposed chart have been described. The comparison of the proposed chart has been made with the existing chart. A real-world example from the healthcare department has been included for the practical application of the proposed chart. It has been observed from the simulation study and real example that the proposed control chart is efficient in quick monitoring of the out-of-control process. It can be concluded that the proposed control chart can be applied effectively in uncertainty.

1. Introduction

The control chart is considered as the most efficient, fabulous, and powerful tool of statistical process control. The control charts have been widely used in various fields. Suman and Prajapati [1] discussed the application in the healthcare department. Zaman et al. [2] applied a control chart in the wind turbine field. Hossain et al. [3] discussed the application of a control chart for monitoring the glass fiber process. The effectiveness and efficiency of the control chart are judged by its reaction behavior against changes in its designed parameters. There are two types of changes observed in the control chart literature, i.e., common changes and special changes. Common changes also known as common causes are natural and have no threatening effect on the interested quality characteristic as compared to the special changes or special causes [4]. The early and quick detection of the special cause of variation is the prime property of any control chart which not only detects the out-of-control process quickly but also timely stops the process from producing a bulk of defective items which ultimately cause a bad impression for the producer and results in heavy losses [5]. The idea of the control chart was floated by Shewhart during the 1920s [6], and researchers are endeavoring to propose a robust control chart since its inception but remained unsuccessful. The proposed chart is an efficient struggle for the quick monitoring of the manufacturing process. The variable control chart is used when the data obtained from the measurement process and attribute control charts are applied when the data is obtained from the counting process. Abbas et al. [7] proposed the control chart for monitoring healthcare. Aslam et al. [8] designed the control chart for the process capability index. Nazir et al. [9] proposed the improved control chart for the industrial processes. Saghir et al. [10] proposed the improved control chart for modified gamma data. Saghir et al. [11] incorporated auxiliary information and repetitive sampling for the monitoring of the process.

Repetitive sampling scheme (RSS) is an efficient sampling scheme for the statistical process control techniques that attracted the attention of many researchers during the last two decades. The RSS was basically introduced by
Several control chart schemes have been developed for the processes having clear, certain, determined, and crisp observations of the interested quality characteristic. There are many situations when the observations are unclear, uncertain, vague, indeterminate, incomplete, and fuzzy. Bradshaw [34] developed a control chart for monitoring the observations from the fuzzy set theory. Williams and Zigi [35] proposed charts for fuzzy logic for the service industry. Taleb and Limam [36] constructed procedures for monitoring of linguistic data based on probability and fuzzy theory. Gülbay et al. [37] developed a fuzzy control chart for linguistic data. Hsieh et al. [38] explained a Poisson-based control chart for monitoring wafer defects for fuzzy theory. Sorooshian [39] investigated the fuzzy theory for monitoring attribute quality characteristics.

The neutrosophic logic which is the extension of the fuzzy logic was proposed by Smarandache [40]. The neutrosophic provides information about the measure of indeterminacy which fuzzy logic is unable to provide. Smarandache [41] discussed the generalization of intuitionistic fuzzy logic. Smarandache [42] introduced neutrosophic theory using the generalization form of the fuzzy set theory. Abu Qamar and Hassan [43] and Abu Qamar and Hassan [44] discussed Q-neutrosophic with appellations in decision-making. More information on the applications of neutrosophic logic can be found in the study carried out by Alhabib et al. [45], Abdel-Baset et al. [46], and Jana and Pal [47].

Smarandache [48] introduced the generalized class of the traditional statistics under the neutrosophic logic and called it the neutrosophic statistics. The neutrosophic statistics tend to transform to the classical statistics if all the observations are clear, certain, complete, or determined. Chen et al. [49] analyzed the scale effect and anisotropy for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. Aslam [50] introduced a new sampling plan for the indeterminate environment under the process loss consideration. Aslam et al. [51] studied the indeterminate environment for testing of grouped product using the Weibull distribution. Aslam and Raza [8] developed a novel neutrosophic sampling plan for the multiple manufacturing lines using an exponentially weighted moving average and classical process capability index under the neutrosophic optimization solution method. Recently, Aslam et al. [52] designed the control chart for the gamma distribution using the indeterminate environment. More information regarding the control charts can be found in the study carried out by Intaramo and Pongpullponsak [53], Charongrattanasakul and Pongpullponsak [54], Panthong and Pongpullponsak [55], Aslam et al. [29], Aslam et al. [56], Fernández [57], Khan et al. [26], Aslam et al. [58], and Mashuri and Ahsan [59].

Average run length (ARL) is used very commonly in control chart literature as the evaluation tool of any proposed chart. ARL is defined as the average number of samples falling inside the control limits before the process shows an out-of-control condition Montgomery [4]. In a statistically controlled process, the values of neutrosophic ARL (NARL) must be larger, but for the shifted process, the
smaller NARL values are preferred under the indeterminate environment for quick indication of out-of-control process and thus resulting in a smaller amount of defective items. More information about ARL can be found in the study carried out by Woodall [60], Molnau et al. [61], Kim [62], Knoth [63], Li et al. [64], Chananet al. [65], and Phanyama et al. [66].

In this article, a control chart scheme has been developed for a repetitive sampling scheme using the gamma distribution for the indeterminate environment with the objective that it will be an efficient monitoring scheme. To the best of the author’s knowledge, no work has been done on a repetitive sampling control chart for gamma distribution using the indeterminate environment. The rest of the paper is organized as follows. The Neutrosophic gamma distribution is introduced in Section 2. The design of the proposed neutrosophic gamma distribution chart has been given in Section 3. In Section 3, the control chart for \( a_N \in [3, 5] \) and \( b_N \in [1.9, 2.1] \) and \( a_N \in [5, 10] \) and \( b_N \in [1.45, 1.55] \) has been discussed. In addition, tables of NARLs have been generated and the simulation study of the neutrosophic statistics has been explained. In Section 4, a comparison of the proposed chart with an existing chart has been given. In Section 5, a real example has been explained for the practical application of the proposed chart. Conclusion and the direction for future research have been given in the Section 6.

2. Neutrosophic Gamma Distribution

Let the neutrosophic failure time be \( T_N \in [T_L, T_U] \), where \( T_L \) and \( T_U \) represent the indeterminacy interval of lower and upper failures of an item that follows the neutrosophic gamma distribution with neutrosophic scale parameter \( b_N \in [b_L, b_U] \) and neutrosophic shape parameter \( a_N \in [a_L, a_U] \). Then, the neutrosophic probability density function (npdf) of the neutrosophic gamma distribution is given as

\[
f(t_N) = \frac{b_N^{a_N} t_N^{a_N-1} e^{-b_N t_N}}{\Gamma(a_N)}; \quad t_N \in [a_N b_N, b_N],
\]

where \( \Gamma(x) \) the neutrosophic gamma function; for more details, readers may refer to [20].

The resultant neutrosophic cumulative distribution (ncdf) of the neutrosophic gamma distribution (NGD) is

\[
P(T_N \leq t_N) = 1 - \sum_{j=1}^{a_N-1} \frac{e^{-(t_N/b_N)} (t_N/b_N)^j}{j!}; \quad T_N \in [T_L, T_U], \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U].
\]

(2)

It is to be noted that the NGD under the classic statistics is the generalization of the traditional gamma distribution. The mean and variance of the neurtosophic statistics can be written as

\[
\mu_N = \frac{a_N}{b_N}; \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U],
\]

\[
\sigma_N^2 = \frac{a_N}{b_N^2}; \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U].
\]

To construct control chart, we need the neutrosophic normal distribution which is developed using the approximation developed by [67] as \( T_N^* = T_N^{1/3} \) and \( T_N \in [T_L, T_U] \). More information regarding neutrosophic distribution can be found in the study carried out by Smarandache [48], Peng and Dai [68], Peng and Dai [69], Aslam et al. [51], Aslam et al. [51], Aslam and Raza [8], and Aslam [50]. Then, the mean and variance of the transformed neutrosophic distribution \( T_N \in [T_L^*, T_U^*] \) can be written as

\[
\mu_{T_N} = \frac{b_N^{1/3} \Gamma(a_N + 1/3)}{\Gamma(a_N)}; \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U],
\]

\[
\sigma_{T_N}^2 = \frac{b_N^{2/3} \Gamma(a_N + 2/3) - (b_N^{1/3} \Gamma(a_N + 1/3))^2}{\Gamma(a_N)}; \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U].
\]

(4)

3. Design of the Proposed Control Chart

In this section, we described the designing of the proposed neutrosophic control chart for the transformed variable \( T_N^* = T_N^{1/3}, T_N^* \in [T_L^*, T_U^*] \). According to Wilson and Hilferty [67], the random variable \( T_N^* = T_N^{1/3}, T_N^* \in [T_L^*, T_U^*] \), has the symmetry property of the normal probability distribution. We developed the neutrosophic control chart using the neutrosophic statistical interval method under the condition that the interested quality characteristic follows the NGD.

As mentioned by Wilson and Hilferty [67], the transformed variable \( T_N^* = T_N^{1/3}, T_N^* \in [T_L^*, T_U^*] \), has the symmetry property of the neutrosophic normal distribution. We propose the following control chart under the NISM when the quality of interest follows the NGD. The following two steps have been adopted to develop the neutrosophic control chart:

1. Determine \( T_N^* = T_N^{1/3} \), where \( T_N^* \) is the transformed random variable based on the randomly selected items from the manufacturing process.

2. Using control limits, plot \( T_N^* \); then, declare the process as out-of-control when \( T_N \geq UCL_{IN} \) or \( T_N \leq LCL_{IN} \), where \( LCL_{IN} \in [LCL_{IN}, LCL_{IN}] \) and \( UCL_{IN} \in [UCL_{IN}, UCL_{IN}] \) are neutrosophic lower and upper control limits, respectively. Note here that the decision about the process is out-of-control and is taken if \( T_N^* \) is beyond the outer of neutrosophic control limits.

The proposed neutrosophic control chart under the neutrosophic statistical interval method is the extension of the Sheu and Lin [70] control chart under the classical statistics. The proposed chart converts to Sheu and Lin [70]
control chart when developed under the crisp, complete, or certain observations. Let the process lie in-control state under the neutrosophic scale parameter $b_{0N} \in [b_{0L}, b_{0U}]$. Then, the control limits of the proposed neutrosophic control chart can be developed as

$$
\begin{align*}
\text{LCL}_{1N} &= \mu_{T^*_N} - k_{1N} \sigma_{T^*_N} = \frac{b_{1N}^{1/3} \Gamma(a_N + 1/3)}{\Gamma(a_N)} - k_{1N} \left( \frac{b_{0N}^{2/3} \Gamma(a_N + 2/3)}{\Gamma(a_N)} - \mu_{T^*_N} \right)^2, \\
\text{LCL}_{2N} &= \mu_{T^*_N} - k_{2N} \sigma_{T^*_N} = \frac{b_{1N}^{1/3} \Gamma(a_N + 1/3)}{\Gamma(a_N)} - k_{2N} \left( \frac{b_{0N}^{2/3} \Gamma(a_N + 2/3)}{\Gamma(a_N)} - \mu_{T^*_N} \right)^2, \\
\text{UCL}_{1N} &= \mu_{T^*_N} + k_{1N} \sigma_{T^*_N} = \frac{b_{1N}^{1/3} \Gamma(a_N + 1/3)}{\Gamma(a_N)} + k_{1N} \left( \frac{b_{0N}^{2/3} \Gamma(a_N + 2/3)}{\Gamma(a_N)} - \mu_{T^*_N} \right)^2, \\
\text{UCL}_{2N} &= \mu_{T^*_N} + k_{2N} \sigma_{T^*_N} = \frac{b_{1N}^{1/3} \Gamma(a_N + 1/3)}{\Gamma(a_N)} + k_{2N} \left( \frac{b_{0N}^{2/3} \Gamma(a_N + 2/3)}{\Gamma(a_N)} - \mu_{T^*_N} \right)^2.
\end{align*}
$$

(5)

where $k_{1N} \in [k_{1L}, k_{1U}]$ and $k_{2N} \in [k_{2L}, k_{2U}]$ are the neutrosophic control limit coefficients.

Furthermore, we define

$$
\begin{align*}
\text{LL}_{1N} &= \left[ \frac{\Gamma(a_N + 1/3)}{\Gamma(a_N)} - \frac{\Gamma(a_N + 2/3)}{\Gamma(a_N)} \right]^2, \\
\text{LL}_{2N} &= \left[ \frac{\Gamma(a_N + 1/3)}{\Gamma(a_N)} - \frac{\Gamma(a_N + 2/3)}{\Gamma(a_N)} \right]^2, \\
\text{UL}_{1N} &= \left[ \frac{\Gamma(a_N + 1/3)}{\Gamma(a_N)} + \frac{\Gamma(a_N + 2/3)}{\Gamma(a_N)} \right]^2, \\
\text{UL}_{2N} &= \left[ \frac{\Gamma(a_N + 1/3)}{\Gamma(a_N)} + \frac{\Gamma(a_N + 2/3)}{\Gamma(a_N)} \right]^2.
\end{align*}
$$

(6)

Therefore, the neutrosophic control limits can be written as follows:

$$
\begin{align*}
\text{LCL}_{1N} &= b_{1N}^{1/3} \mu_{T^*_N}, \\
\text{LCL}_{2N} &= b_{1N}^{1/3} \mu_{T^*_N}, \\
\text{UCL}_{1N} &= b_{1N}^{1/3} \mu_{T^*_N}, \\
\text{UCL}_{2N} &= b_{1N}^{1/3} \mu_{T^*_N}.
\end{align*}
$$

(7)

For a shifted process, note that a shift occurs in the neutrosophic statistical scale parameter, whereas the shape parameter remains constant. Then, the probability under the neutrosophic statistical interval method of the in-control process when the process shows the state of in-control can be calculated as

$$
\begin{align*}
P_{\text{out},N}^0 &= P(T^*_N < \text{LCL}_{1N}|b_N = b_{0N}) + P(T^*_N > \text{UCL}_{1N}|b_N = b_{0N}), \\
\text{or } P_{\text{out},N}^0 &= 1 - \sum_{j=1}^{a_{N-1}} \frac{e^{-\text{LL}_{1N}^3}}{j!} + \sum_{j=1}^{a_{N-1}} \frac{e^{-\text{UL}_{1N}^3}}{j!}, \\
P_{\text{rep},N}^0 &= P(\text{LCL}_{1N} < T^*_N < \text{LCL}_{2N}|b_N = b_{0N}) + P(\text{UCL}_{1N} < T^*_N < \text{UCL}_{2N}|b_N = b_{0N}), \\
P_{\text{rep},N}^0 &= \sum_{j=1}^{a_{N-1}} \frac{e^{-\text{UL}_{2N}^3}}{j!} - \sum_{j=1}^{a_{N-1}} \frac{e^{-\text{LL}_{1N}^3}}{j!} + \sum_{j=1}^{a_{N-1}} \frac{e^{-\text{UL}_{2N}^3}}{j!} - \sum_{j=1}^{a_{N-1}} \frac{e^{-\text{LL}_{2N}^3}}{j!}.
\end{align*}
$$

(8)

The probability of out-of-control under neutrosophic statistics is given by

$$
P_{\text{out}}^0 = \frac{P_{\text{out},N}^0}{1 - P_{\text{rep},N}^0}.
$$

(10)
As mentioned earlier the ARL is used to evaluate the developed scheme for its efficiency to declare the shifted process as out-of-control quickly. So, the neutrosophic ARL (NARL) for the in-control process \( \text{ARL}_{0N} \) can be defined as

\[
\text{ARL}_{0N} = \frac{1}{P_{\text{out}}}; \quad \text{ARL}_{0N} \in [\text{ARL}_{0L}, \text{ARL}_{0U}].
\]  

(11)

We will measure the efficiency of the proposed control chart under the neutrosophic average run length (NARL) which shows on the average when the process is out-of-control and is defined by

\[
P_{\text{out},N} = P(T_N < \text{LCL}_{1N}|b_N = cb_{0N}) + P(T_N^* > \text{UCL}_{1N}|b_N = cb_{0N}),
\]

(13)

or

\[
\begin{align*}
P_{\text{out},N} &= 1 - \sum_{j=1}^{a_N-1} \frac{e^{(-\text{LCL}_{1N}/c)}(\text{LCL}_{1N}/c)^j}{j!} + \sum_{j=1}^{a_N-1} \frac{e^{(-\text{UL}_{1N}/c)}(\text{UL}_{1N}/c)^j}{j!}, \\
\text{ARL}_{0N} &= \frac{1}{P_{\text{out},N}}\quad \text{ARL}_{0N} \in [\text{ARL}_{0L}, \text{ARL}_{0U}].
\end{align*}
\]

(16)

Using the abovementioned equations, the R-language code program was written to estimate the neutrosophic parameters of the proposed chart for different process settings. Tables 1 and 2 have been generated for \( a_N \in [3, 5] \) and \( b_N \in [1.9, 2.1] \) and \( a_N \in [5, 10] \) and \( b_N \in [1.45, 1.55] \) with NARL values for different shifts from 1.0 to 4.0.

Table 1 provides NARL values for the in-control \( \text{NARL}_{0N} \in [200, 300, 370] \) and \( k_{dN} = [4.594878, 5.233344], [5.282686, 5.430229], [5.000939, 5.409798] \) and \( k_rN = [1.527915, 2.881848], [0.3242994, 2.6622] \) and \([0.9223276, 4.060355]\). Figure 1 has been given for the plotting of \( a_N \in [3, 5] \) and \( b_N \in [1.9, 2.1] \).

From Tables 1 and 2, we made the following trends in NARL:

1. As the values of the shift \( c \) increase from 1.0 to 4.0, the indeterminacy intervals \( \text{ARL}_{1N} \in [\text{ARL}_{1L}, \text{ARL}_{1U}] \) decrease.

2. As the values of \( a_N \in [a_{1L}, a_{1U}] \) and \( b_N \in [b_{1L}, b_{1U}] \) increase from \( a_N \in [3, 5] \) and \( b_N \in [1.9, 2.1] \) to \( a_N \in [5, 10] \) and \( b_N \in [1.45, 1.55] \), the indeterminacy intervals decrease.

\[
\text{ARL}_{0N} = \frac{1}{P_{\text{out}}}; \quad \text{ARL}_{0N} \in [\text{ARL}_{0L}, \text{ARL}_{0U}].
\]

(12)

Let a shift occur in the process; then, the process is shifted from the targeted \( b_{0N} \in [b_{0L}, b_{0U}] \) to \( b_{1N} = cb_{0N}, b_{1N} \in [b_{1L}, b_{1U}] \), where the constant \( c \) shows the shift in the process. Then, the probability of the out-of-process under the neutrosophic statistical interval method can be developed as

\[
P_{\text{out},N} = P(T_N^* > \text{UCL}_{1N}|b_N = cb_{0N}),
\]

(14)

4. Comparison of the Proposed Chart with the Existing Chart

In this section, the comparative advantages and efficiency of the proposed chart over the existing chart of the traditional chart for gamma distribution under the indeterminacy environment have been discussed with the help of the simulated data. For the purpose of fair comparison, we fixed the same values of the process parameters. Table 3 shows the in-control \( \text{NARL}_0 \) and out-of-control \( \text{NARL}_1 \) values for different shifts from 1.0 to 4.0.

A simple comparison shows that the proposed chart has smaller \( \text{NARL}_1 \) values as compared to the existing chart [52]. For example, when \( c = 1.1 \), the indeterminacy intervals of NARL for the existing chart is \( \text{ARL}_{1N} \in [89.86, 101.98] \) and for the proposed chart is \( \text{ARL}_{1N} \in [80.02, 86.99] \). From this comparison, it can be concluded that the proposed control chart will indicate the shift in the process between 80\textsuperscript{th} to 86\textsuperscript{th} samples. On the contrary, the chart proposed by Aslam et al. [8] will indicate the shift in the process between 89\textsuperscript{th} and 101\textsuperscript{th} samples. Therefore, the proposed control chart has the ability to detect a shift in the process earlier than the existing control chart.

We will now discuss the efficiency of the proposed control chart over the existing control chart using the simulated data. According to the proposed chart, the process is said to out-of-control if \( T_N^* \geq \text{UCL}_{1N} \) or \( T_N^* \leq \text{LCL}_{1N} \). The first 20 observations are generated from the neutrosophic gamma distribution when the process is an in-control state.
The next 20 observations are from the out-of-control process when \( c = 1.4 \). The proposed control chart for simulated data is shown in Figure 1. The existing control chart for the simulated data is shown in Figure 2. From Table 1, it is expected that the shift should be detected between 16th sample to 22nd sample. From Figure 1, it can be seen that the proposed control chart detects a shift in the process according to expectation. The determinate part (lower value) of the statistic \( T_N^{*} \) is beyond UCL\( N \) between 16th samples to 22nd sample. We also note that several observations are within indeterminacy interval and resampling areas. On the contrary, the existing control chart does not show any shift in the process. From this simulation study, it is concluded that the proposed chart has the ability to detect a shift in the process as compared to the existing control chart.

### 5. Application of the Proposed Chart

In this section, we will discuss the application of the proposed control chart in the healthcare department. A large hospital management is interested to track the urinary tract infections (UTIs) patients. According to Santiago and Smith [71], "data were provided from a large hospital system concerned with a very high rate of hospital-acquired UTIs. Specifically, the hospital would like to track the frequency of patients being discharged who had acquired a UTI while in the hospital as a way to quickly identify an increase in infection rate or, conversely, monitor whether the forthcoming process or material changes result in fewer infections because the root cause often differs based on gender, male and female patients." The UTIs' data of male

| \( k_a_N \) | [4.594878, 5.233344] | [5.282686, 5.430229] | [5.000939, 5.409798] |
| \( k_r_N \) | [1.527915, 2.881848] | [0.3242994, 2.662222] | [0.9223276, 4.060355] |
| \( a_N \) | [3, 5] | [3, 5] | [3, 5] |
| \( b_N \) | [1.9, 2.1] | [1.9, 2.1] | [1.9, 2.1] |
| \( c \) | ARL\( N \) | 1.0 | 200, 200.01 |
| | | 1.1 | [80.02, 86.99] |
| | | 1.2 | [37.51, 43.40] |
| | | 1.3 | [19.92, 24.14] |
| | | 1.4 | [11.71, 14.68] |
| | | 1.5 | [7.50, 9.62] |
| | | 1.6 | [5.18, 6.72] |
| | | 1.7 | [3.81, 4.96] |
| | | 1.8 | [2.97, 3.85] |
| | | 1.9 | [2.42, 3.11] |
| | | 2.0 | [2.05, 2.66] |
| | | 2.3 | [1.55, 1.88] |
| | | 2.5 | [1.32, 1.54] |
| | | 2.8 | [1.18, 1.33] |
| | | 3.0 | [1.13, 1.25] |
| | | 4.0 | [1.03, 1.08] |

| \( k_a_N \) | [4.006202, 4.571112] | [3.939843, 4.788404] | [4.17499, 4.867394] |
| \( k_r_N \) | [1.086602, 2.200099] | [2.939107, 1.814869] | [1.414571, 1.799174] |
| \( a_N \) | [5, 10] | [5, 10] | [5, 10] |
| \( b_N \) | [1.45, 1.55] | [1.45, 1.55] | [1.45, 1.55] |
| \( c \) | ARL\( N \) | 1.0 | [200.01, 200] |
| | | 1.1 | [58.92, 72.91] |
| | | 1.2 | [21.72, 31.51] |
| | | 1.3 | [9.65, 15.64] |
| | | 1.4 | [5.06, 8.74] |
| | | 1.5 | [3.09, 5.42] |
| | | 1.6 | [2.15, 3.68] |
| | | 1.7 | [1.68, 2.72] |
| | | 1.8 | [1.42, 2.15] |
| | | 1.9 | [1.27, 1.79] |
| | | 2.0 | [1.18, 1.57] |
| | | 2.3 | [1.07, 1.27] |
| | | 2.5 | [1.03, 1.15] |
| | | 2.8 | [1.01, 1.08] |
| | | 3.0 | [1.01, 1.05] |
| | | 4.0 | [1, 1.01] |

Table 1: Neutrosophic average run length of the proposed chart for \( a_N \in [3, 5] \) and \( b_N \in [1.9, 2.1] \).

Table 2: Neutrosophic average run length of the proposed chart for \( a_N \in [5, 10] \) and \( b_N \in [1.45, 1.55] \).
patients are selected from [8] and shown in Table 4. From the UTIs’ data, it is clear that the data is presented in the interval. Therefore, the existing control chart proposed by [71] cannot apply for the monitoring of UTIs patients. The hospital management can apply the proposed control chart for tracking UTIs patients. Suppose that $ARL_{0N} \in [370, 370]$, $a_N \in [7.6666, 7.7777]$, $b_N \in [1.0959, 1.1559]$, and $n_N \in [50, 50]$. The control limit coefficients are $k_{1N} \in [3.3590, 3.7703]$ and $k_{2N} \in [0.1637, 2.0479]$. Figure 3 shows the proposed control chart for UTIs patients. From Figure 3, it can be seen that two points are outside the upper control limits. Aslam et al. [8] presented a control chart for UTIs data. The neutrosophic control chart proposed by Aslam et al. [8] shows that all points are within the control limits. In addition, it can be noted from the proposed chart that several points are within the indeterminacy interval and between repetitive areas. It means that the hospital management can be indeterminate about the several observations in the UTIs data and need to repeat the process from those observations in the repetitive areas. By comparing the proposed UTIs chart with the UTIs chart proposed by Aslam et al. [8], it can be concluded that the proposed control chart clearly indicates some issues in tracking the UTIs’ patient, and therefore, the hospital management should take action to bring back the process to in-control state. The proposed control chart can be applied to any other data in the same way.
**Figure 2:** The existing control chart for simulated data when $a_N \in [3, 5], b_N \in [1.9, 2.1], \eta_N \in [20, 20], k_{1N} \in [4.5948, 5.2333]$, and $k_{2N} \in [1.5279, 2.8818]$.

**Table 4:** The neutrosophic UTIs’ data.

| Sr# | $T_N$          | $T^*_N$       |
|-----|----------------|---------------|
| 1   | [13.13, 13.56] | [2.35, 2.38]  |
| 2   | [3.57, 15.55]  | [1.52, 2.49]  |
| 3   | [4.31, 16.50]  | [1.62, 2.54]  |
| 4   | [2.76, 25.53]  | [1.40, 2.94]  |
| 5   | [7.75, 15.38]  | [1.97, 2.48]  |
| 6   | [11.45, 13.18] | [2.25, 2.36]  |
| 7   | [9.20, 15.18]  | [2.09, 2.47]  |
| 8   | [5.51, 9.77]   | [1.76, 2.13]  |
| 9   | [8.18, 13.07]  | [2.01, 2.35]  |
| 10  | [7.07, 19.91]  | [1.91, 2.71]  |
| 11  | [7.35, 14.89]  | [1.94, 2.46]  |
| 12  | [5.62, 11.09]  | [1.77, 2.23]  |
| 13  | [8.38, 16.72]  | [2.03, 2.55]  |
| 14  | [9.49, 10.06]  | [2.11, 2.15]  |
| 15  | [4.90, 23.67]  | [1.69, 2.87]  |
| 16  | [4.45, 14.68]  | [1.64, 2.44]  |
| 17  | [7.11, 16.44]  | [1.92, 2.54]  |
| 18  | [9.37, 15.95]  | [2.10, 2.51]  |
| 19  | [12.00, 16.38] | [2.28, 2.53]  |
| 20  | [7.41, 16.62]  | [1.95, 2.55]  |
| 21  | [10.64, 15.15] | [2.19, 2.47]  |
| 22  | [6.63, 11.21]  | [1.87, 2.23]  |
| 23  | [2.87, 14.27]  | [1.42, 2.42]  |
| 24  | [6.87, 10.37]  | [1.90, 2.18]  |
| 25  | [6.16, 18.85]  | [1.83, 2.66]  |
| 26  | [6.53, 12.47]  | [1.87, 2.31]  |
| 27  | [6.85, 12.13]  | [1.89, 2.29]  |
| 28  | [8.08, 22.69]  | [2.00, 2.83]  |
| 29  | [11.61, 17.14] | [2.26, 2.57]  |
6. Concluding Remarks

In this article, we presented the control chart using repetitive sampling under neutrosophic statistics when the data follow the gamma distribution. We presented some necessary measures to evaluate the proposed control chart. A simulation study and real example from the healthcare were included to show the efficiency of the proposed control chart over the existing control chart. From the study, it is observed that the proposed chart is an efficient addition in the tool kit of the quality control personnel. The proposed scheme can be extended for the multivariate case as future research. The proposed control using some other transformation for nonnormal distribution and different datasets can be considered as future research. The proposed chart using the cost model can be studied as future research. The proposed

![Figure 3: The proposed control chart for UTIs’ patients.](image-url)
control chart for monitoring imbalanced data can be considered as future research.

**Data Availability**

The data used to support the findings of the study are included within the article.

**Conflicts of Interest**

The authors declare no conflicts of interest regarding this paper.

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**References**

[1] G. Suman and D. R. Prajapati, "Control chart applications in healthcare: a literature review," *International Journal of Metrology and Quality Engineering*, vol. 9, p. 5, 2018.

[2] B. Zaman, M. H. Lee, M. Riaz, and M. Ramat Abujiya, "An improved process monitoring by mixed multivariate memory control charts: an application in wind turbine field," *Computers & Industrial Engineering*, vol. 142, Article ID 106343, 2020.

[3] M. P. Hossain, M. H. Omar, M. Riaz, and S. Y. Arafat, "On designing a new control chart for Rayleigh distributed processes with an application to monitor glass fiber strength," *Communications in Statistics—Simulation and Computation*, 2020.

[4] D. C. Montgomery, *Introduction to Statistical Quality Control*, John Wiley & Sons, Inc., New York, NY, USA, 6th edition, 2009.

[5] L. Ahmad, M. Aslam, and C.-H. Jun, "The design of a new repetitive sampling control chart based on process capability index," *Transactions of the Institute of Measurement and Control*, vol. 38, no. 8, pp. 971–980, 2016.

[6] W. A. Shewhart, "Quality control charts," *Bell System Technical Journal*, vol. 5, no. 4, pp. 593–603, 1926.

[7] T. Abbas, M. Riaz, M. Tahir, H. Z. Nazir, and M. Abid, "A comparative analysis of robust dispersion control charts with application related to health care data," *Journal of Testing and Evaluation*, vol. 48, no. 1, Article ID 20180572, 2019.

[8] M. Aslam and M. A. Raza, "Design of new sampling plans for multiple manufacturing lines under uncertainty," *International Journal of Fuzzy Systems*, vol. 21, no. 3, pp. 978–992, 2019.

[9] H. Z. Nazir, T. Hussain, N. Akhtar, M. Abid, and M. Riaz, "Robust adaptive exponentially weighted moving average control charts with applications of manufacturing processes," *The International Journal of Advanced Manufacturing Technology*, vol. 105, no. 1–4, pp. 733–748, 2019.

[10] A. Saghir, L. Ahmad, and M. Aslam, "Modified EWMA control chart for transformed gamma data," *Communications in Statistics—Simulation and Computation*, 2019.

[11] A. Saghir, L. Ahmad, M. Aslam, and C.-H. Jun, "A EWMA control chart based on an auxiliary variable and repetitive sampling for monitoring process location," *Communications in Statistics—Simulation and Computation*, vol. 48, no. 7, pp. 2034–2045, 2019b.

[12] R. E. Sherman, "Design and evaluation of a repetitive group sampling plan," *Technometrics*, vol. 7, no. 1, pp. 11–21, 1965.

[13] S. Balamurali, H. Park, C.-H. Jun, K.-J. Kim, and J. Lee, "Designing of variables repetitive group sampling plan involving minimum average sample number," *Communications in Statistics—Simulation and Computation*, vol. 34, no. 3, pp. 799–809, 2005.

[14] S. Balamurali and C.-H. Jun, "Repetitive group sampling procedure for variables inspection," *Journal of Applied Statistics*, vol. 33, no. 3, pp. 327–338, 2006.

[15] L. Ahmad, M. Aslam, and C.-H. Jun, "Coal quality monitoring with improved control charts," *European Journal of Scientific Research*, vol. 125, no. 2, pp. 427–434, 2014.

[16] M. Azam, O. H. Arif, M. Aslam, and W. Ejaz, "Repetitive acceptance sampling plan based on exponentially weighted moving average regression estimator," *Journal of Computational and Theoretical Nanoscience*, vol. 13, no. 7, pp. 4413–4426, 2016.

[17] C.-H. Yen, C.-H. Chang, and M. Aslam, "Repetitive variable acceptance sampling plan for one-sided specification," *Journal of Statistical Computation and Simulation*, vol. 85, no. 6, pp. 1102–1116, 2015.

[18] O. A. Adeoti and J. O. Olaomi, "Capability index based control chart for monitoring process mean using repetitive sampling," *Communications in Statistics—Theory and Methods*, vol. 45, no. 3, 2017.

[19] M. Aslam, N. Khan, M. Azam, and C.-H. Jun, "Designing of a new monitoring t-chart using repetitive sampling," *Information Sciences*, vol. 269, pp. 210–216, 2014.

[20] M. Aslam, O.-H. Arif, and C.-H. Jun, "A control chart for gamma distribution using multiple dependent state sampling," *Industrial Engineering and Management Systems*, vol. 16, no. 1, pp. 109–117, 2017a.

[21] C.-H. Jun, H. Lee, S.-H. Lee, and S. Balamurali, "A variables repetitive group sampling plan under failure-censored reliability tests for Weibull distribution," *Journal of Applied Statistics*, vol. 37, no. 3, pp. 453–460, 2010.

[22] S.-W. Liu and C.-W. Wu, "Design and construction of a variables repetitive group sampling plan for unilateral specification limit," *Communications in Statistics—Simulation and Computation*, vol. 43, no. 8, pp. 1866–1878, 2014.

[23] R. Radhakrishnan and P. K. Sivakumaran, "Construction of six sigma repetitive group sampling plans," *International Journal of Mathematics & Computation*, vol. 1, no. 8, pp. 75–83, 2008.

[24] H. A. Al-Oraimi and M. A. Rahim, "Economic statistical design of X control charts for systems with Gamma (A, 2) in-control times," *Computers & Industrial Engineering*, vol. 43, no. 3, pp. 645–654, 2002.

[25] Z. G. B. Stoumbos and M. R. Reynolds, "Robustness to nonnormality and autocorrelation of individuals control charts," *Journal of Statistical Computation and Simulation*, vol. 66, no. 2, pp. 145–187, 2000.

[26] N. Khan, M. Aslam, L. Ahmad, and C.-H. Jun, "A control chart for gamma distributed variables using repetitive sampling scheme," *Pakistan Journal of Statistics and Operation Research*, vol. 13, no. 1, pp. 47–61, 2017.

[27] H. Aksoy, "Use of gamma distribution in hydrological analysis," *Turkish Journal of Engineering and Environmental Sciences*, vol. 24, no. 6, pp. 419–428, 2000.

[28] D. K. Bhaumik and R. D. Gibbons, "One-sided approximate prediction intervals for at least p of m observations from a
gamma population at each of r locations,” Technometrics, vol. 48, no. 1, pp. 112–119, 2006.

[29] M. Aslam, N. Khan, and C.-H. Jun, “A control chart using belief information for a gamma distribution,” Operations Research and Decisions, vol. 26, no. 4, pp. 5–19, 2016.

[30] D. Jeurkapporn, D. C. Montgomery, G. C. Runger et al., “Process monitoring for correlated gamma-distributed data using generalized-linear-model-based control charts,” Quality and Reliability Engineering International, vol. 19, no. 6, pp. 477–491, 2003.

[31] C. W. Zhang, M. Xie, J. Y. Liu, and T. N. Goh, “A control chart for the Gamma distribution as a model of time between events,” International Journal of Production Research, vol. 45, no. 23, pp. 5649–5666, 2007.

[32] F. L. Chen and C. H. Yeh, “Economic statistical design of non-uniform sampling scheme X bar control charts under non-normality and gamma shock using genetic algorithm,” Expert Systems with Applications, vol. 36, no. 5, pp. 9488–9497, 2009.

[33] I. M. Gonzalez and E. Viles, “Design of R control chart assuming a gamma distribution,” Economic Quality Control, vol. 16, no. 2, pp. 199–204, 2001.

[34] C. W. Bradshaw Jr., “A fuzzy set theoretic interpretation of economic control limits,” European Journal of Operational Research, vol. 13, no. 4, pp. 403–408, 1983.

[35] R. H. Williams and R. M. Zigli, “Ambiguity impedes quality in the service industries,” Quality Progress, vol. 20, no. 7, pp. 14–17, 1987.

[36] H. Taleb and M. Limam, “On fuzzy and probabilistic control charts,” International Journal of Production Research, vol. 40, no. 12, pp. 2849–2863, 2002.

[37] M. Gülbay, C. Kahraman, and D. Ruan, “α-cut fuzzy control charts for linguistics data,” International Journal of Intelligent Systems, vol. 19, no. 12, pp. 1173–1195, 2004.

[38] K.-L. Hsieh, L.-I. Tong, and M.-C. Wang, “The application of control chart for defects and defect clustering in IC manufacturing based on fuzzy theory,” Expert Systems with Applications, vol. 32, no. 3, pp. 765–776, 2007.

[39] S. Sorooshian, “Fuzzy approach to statistical control charts,” Journal of Applied Mathematics, vol. 2013, Article ID 745153, 6 pages, 2013.

[40] F. Smarandache, Neutrosophy: Neutrosophic Probability, Set, and Logic, vol. 105, pp. 118–123, ProQuest Information & Learning, Ann Arbor, MI, USA, 1998.

[41] F. Smarandache, “Neutrosophic set-a generalization of the intuitionistic fuzzy set,” International Journal of Pure and Applied Mathematics, vol. 24, no. 3, p. 287, 2005.

[42] F. Smarandache, “Neutrosophic logic-A generalization of the intuitionistic fuzzy logic,” Multispace & Multistructure, Neutrosophic Transdisciplinarity (100 Collected Papers of Science), North-European Scientific Publishers, vol. 4, p. 396, Hanko, Finland, 2010.

[43] M. Abu Qamar and N. Hassan, “Q-neutrosophic soft relaition and its application in decision making,” Entropy, vol. 20, no. 3, p. 172, 2018.

[44] M. Abu Qamar and N. Hassan, “An approach toward a Q-neutrosophic soft set and its application in decision making,” Symmetry, vol. 11, no. 2, p. 139, 2019.

[45] R. Alhabib, M. M. Ranna, H. Farah, and A. Salama, “Some neutrosophic probability distributions,” Neutrosophic Sets and Systems, vol. 22, 2018.

[46] M. Abdel-Baset, V. Chang, and A. Gamal, “Evaluation of the green supply chain management practices: a novel neutrosophic approach,” Computers in Industry, vol. 108, pp. 210–220, 2019.

[47] C. Jana and M. Pal, “A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making,” Symmetry, vol. 11, no. 1, p. 110, 2019.

[48] F. Smarandache, Introduction to Neutrosophic Statistics: Infinite Study, Romania-Educational Publisher, Columbus, OH, USA, 2014.

[49] J. Chen, J. Ye, and S. Du, “Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics,” Symmetry, vol. 9, no. 10, p. 208, 2017.

[50] M. Aslam, “A new sampling plan using neutrosophic process loss consideration,” Symmetry, vol. 10, no. 5, p. 132, 2018.

[51] M. Aslam, N. Khan, and M. Khan, “Monitoring the variability in the process using neutrosophic statistical interval method,” Symmetry, vol. 10, no. 11, p. 562. 2018.

[52] M. Aslam, G. Rao, A. Al-Marshadi, L. Ahmad, and C.-H. Jun, “Control charts for monitoring process capability index using median absolute deviation for some popular distributions,” Processes, vol. 7, no. 5, p. 287, 2019.

[53] R. Intaramo and A. Pongpulpornsak, “Development of fuzzy extreme value theory control charts using α-cuts for skewed populations,” Applied Mathematical Sciences, vol. 6, no. 117, pp. 5811–5834, 2012.

[54] P. Charongrattanakul and A. Pongpulpornsak, “Economic model for fuzzy Weibull distribution,” in Proceedings of the International Conference Applied Statistics 2014 (ICAS 2014), Khon Kaen, Thailand, May 2014.

[55] C. Panthong and A. Pongpulpornsak, “Non-normality and the fuzzy theory for variable parameters control charts,” Thai Journal of Mathematics, vol. 14, no. 1, pp. 203–213, 2016.

[56] M. Aslam, G. Srinivasa Rao, L. Ahmad, and C.-H. Jun, “A control chart for multivariate Poisson distribution using repetitive sampling,” Journal of Applied Statistics, vol. 44, no. 1, pp. 123–136, 2017.

[57] M. N. P. Fernández, “Fuzzy theory and quality control charts,” in Proceedings of the 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), London, UK, July 2017.

[58] M. Aslam, N. Khan, and M. Albassam, “Control chart for failure-censored reliability tests under uncertainty environment,” Symmetry, vol. 10, no. 12, p. 690, 2018.

[59] M. Mashuri and M. Ahsan, “Performance fuzzy multinomial control chart,” Journal of Physics: Conference Series, vol. 1028, no. 1, Article ID 012120, 2018.

[60] W. H. Woodall, “The distribution of the run length of one-sided CUSUM procedures for continuous random variables,” Technometrics, vol. 25, no. 3, pp. 295–301, 1983.

[61] W. E. Molnau, G. C. Runger, D. C. Montgomery, K. R. Skinner, E. N. Loredo, and S. S. Prabhu, “A program for ARL calculation for multivariate EWMA charts,” Journal of Quality Technology, vol. 33, no. 4, pp. 515–521, 2001.

[62] M.-J. Kim, “Number of replications required in control chart Monte Carlo simulation studies,” Communications in Statistics—Simulation and Computation, vol. 36, no. 5, pp. 1075–1087, 2007.

[63] S. Knoth, “Accurate ARL calculation for EWMA control charts monitoring normal mean and variance simultaneously,” Sequential Analysis, vol. 26, no. 3, pp. 251–263, 2007.

[64] Z. Li, C. Zou, Z. Gong, and Z. Wang, “The computation of average run length and average time to signal: an overview,” Journal of Statistical Computation and Simulation, vol. 84, no. 8, pp. 1779–1802, 2014.

[65] C. Chanenet, S. Sukparungsee, and Y. Areepong, “The ARL of EWMA chart for monitoring ZINB model using Markov
chain approach,” *International Journal of Applied Physics and Mathematics*, vol. 4, no. 4, pp. 236–239, 2014.

[66] S. Phanyaem, Y. Areepong, and S. Sukparungsee, “Numerical integration of average run length of CUSUM control chart for ARMA process,” *International Journal of Applied Physics and Mathematics*, vol. 4, no. 4, pp. 232–235, 2014.

[67] E. B. Wilson and M. M. Hilferty, “The distribution of chi-square,” *Proceedings of the National Academy of Sciences*, vol. 17, no. 12, pp. 684–688, 1931.

[68] X. Peng and J. Dai, “Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function,” *Neural Computing and Applications*, vol. 29, no. 10, pp. 939–954, 2018.

[69] X. Peng and J. Dai, “A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017,” *Artificial Intelligence Review*, vol. 53, pp. 199–255, 2018.

[70] S. H. Sheu and T. C. Lin, “The generally weighted moving average control chart for detecting small shifts in the process mean,” *Quality Engineering*, vol. 16, no. 2, pp. 209–231, 2003.

[71] E. Santiago and J. Smith, “Control charts based on the exponential distribution: adapting runs rules for the t chart,” *Quality Engineering*, vol. 25, no. 2, pp. 85–96, 2013.