The Role of Downflows in Establishing Solar Near-surface Shear

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1. Introduction

Helioseismology has revealed the presence of two boundary layers of shear at the top and bottom of the solar convection zone (CZ). In the tachocline at the bottom, differential rotation in the CZ transitions sharply to solid-body rotation in the radiative zone below (see Figure 1). At the top of the CZ, there is a 5% reduction in rotation rate with increasing radius over a depth of ~35 Mm, which is largely a uniform feature at all latitudes. This latter feature is known as the near-surface shear layer (NSSL). Both boundary layers may play a significant role in the solar dynamo, since rotational shear creates a toroidal field from a poloidal field through the \( \Omega \)-effect. However, the dynamical origins of these boundary layers are still not well understood.

The solar differential rotation (from Figure 1, the equator rotates about 30% faster than the poles) is believed to be the result of convectively driven Rossby waves, which manifest in the solar CZ as Busse columns, also known as banana cells in the literature (e.g., Gilman 1972; Brun & Toomre 2002; Busse 2002; Nelson et al. 2018). Busse columns are convective rolls of fluid aligned with the rotation axis. Each roll has a cross-sectional tilt in the equatorial plane such that upflows move prograde and downflows move retrograde, the net result being the transport of angular momentum away from the rotation axis (see Figure 6 of Busse 2002). The Busse columns thus tend to spin up the equator (which is far from the rotation axis) compared to the poles.

By contrast, in the NSSL (see Figure 1), the surface layers rotate 5% slower than the layers just below. Foukal & Jokipii (1975) hypothesized the following explanatory mechanism for the formation of the NSSL: fluid particles conserve their specific angular momentum \( \mathcal{L} \) in the outermost layers of the Sun as they move in the radial direction. Thus, for a steady-state system, the angular momentum profile \( \mathcal{L}(r) \) should be constant with radius. Since specific angular momentum is related to the local fluid rotation rate \( \Omega \) through \( \mathcal{L} = \Omega r^2 \sin^2 \theta \), this would imply \( \Omega \propto 1/r^2 \) along a radial line in the outermost fluid layers—a decrease in rotation rate with radius.

In order to successfully homogenize angular momentum, the flows need to be free from rotational constraint so that they do not get captured by Busse column rolls. The degree of rotational constraint is parameterized by the Rossby number (Ro), which is the ratio of rotational period to convective overturning time. Thus, according to Foukal & Jokipii (1975), the Sun must possess a region near the outer surface in which

\[
\text{Ro} \equiv \frac{v'}{2\Omega_0 L} \gtrsim 1, \quad (1)
\]

where \( v' \) is a typical velocity of the convective flow structures (that may vary with radius), \( L \) is their typical length scale, and \( \Omega_0 \) is the frame rotation rate. Thus, a rotationally unconstrained fluid structure is one that is both fast and small-scale. In the models explored here, we find that only the downflows (in particular, structures we call “downflow plumes”) are sufficiently fast and small-scale to be rotationally unconstrained.

Previous numerical simulations of rotating, spherical-shell convection that capture aspects of the solar NSSL have all possessed a layer of rotationally unconstrained fluid near the outer surface. Guerrero et al. (2013) saw an NSSL arise near the equator of their models due to the mixing of angular momentum by fast, high-Rossby-number convective motions near the outer surface. In modeling the band structure observed in zonal flows on the gas and ice giants, Gastine et al. (2013) found that high density contrast across the spherical shell (≈150) enables low-Rossby-number Busse columns to exist in the deep fluid layers and high-Rossby-number, small-scale
convection to occur near the outer surface. Hotta et al. (2015) came closest to a self-consistent reproduction of an NSSL in a model with a density contrast across the shell of ≈613. They successfully achieved a Rossby-number transition with a thin rotationally unconstrained outer layer and reported prominent near-surface shear at high latitudes.

Features of near-surface shear thus seem to appear in models with high density contrast. This is because increasing the density contrast across the spherical shell—while keeping the density at the inner surface fixed—decreases the near-surface density scale height, which is a good representative length scale of the convection. It also accelerates the downflows to higher speeds through the buoyancy force. Both effects serve to increase the Rossby number near the outer surface. Furthermore, these high-contrast models have two types of flow structures: Busse columns at low latitudes in the deep fluid layers and small-scale, fast flows near the outer surface.

In this work, we systematically investigate the nature of rotationally unconstrained near-surface layers across a range of density contrasts in rotating spherical shells. We explicitly analyze the simulations at the two ends of this range, with contrasts of ≈20 and ≈150. We find that the low-contrast simulation is almost entirely dominated by Busse columns at low latitudes, as expected. The high-contrast simulation, on the other hand, exhibits rotationally unconstrained fast flows near the outer surface, as in previous work. However, our analysis reveals that only the downflows are rotationally unconstrained and transport angular momentum inward, creating shear. The upflows still exist in Busse columns and transport angular momentum outward, even near the outer surface.

In Section 2, we present the mathematical equations that are solved in our models and describe the parameter space we explore. In Section 3, we discuss the global character of the flows achieved, both instantaneously and averaged over time. Sections 4 and 5 deal with the structure and evolution of Busse columns and downflow plumes, respectively. In Section 6, we discuss the dynamical balance of torque in our models and its relation to the simulated features of near-surface shear. In Section 7, we examine in detail the Reynolds stress from Busse columns and downflow plumes, which manifest in the separate upflow and downflow contributions to the angular momentum flux. In Section 8, we discuss our results in the general context of meridional force balance.

2. Numerical Model

We numerically evolve a rotating, stratified shell of fluid representative of the CZ of a star. We use the MHD code Rayleigh 0.9.1 (Featherstone & Hindman 2016a; Matsui et al. 2016; Featherstone 2018), published under the GPL3 license. The computational domain of the layer consists of a spherical shell with inner radius \( r_i \) and outer radius \( r_o \). We generally use the standard spherical coordinates \((r, \theta, \phi)\) and corresponding unit vectors \((\hat{r}, \hat{\theta}, \hat{\phi})\), although for some topics, we also use the cylindrical coordinates \((\lambda, \phi, z) = (r \sin \theta, \phi, r \cos \theta)\) and unit vectors \((\hat{\lambda}, \hat{\phi}, \hat{z})\).

Rayleigh makes use of the anelastic approximation (e.g., Gough 1969; Gilman & Glatzmaier 1981) to increase the maximum allowable time step by removing sound waves. The thermodynamic reference state is chosen to be temporally steady and spherically symmetric with adiabatic stratification (see Jones et al. 2011 for a complete description). We denote the pressure, density, temperature, and entropy by \( P, \rho, T, \) and \( S \), respectively. We use overbars on the thermodynamic variables to denote the fixed reference state and a lack of overbars to denote deviations from the reference state. The equations representing conservation of mass, momentum, and energy are then given by (e.g., Featherstone & Hindman 2016a)

\[
\nabla \cdot (\rho \mathbf{v}) = 0, \quad (2)
\]

\[
\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -2\rho \mathbf{\Omega}_0 \times \mathbf{v} - \rho \nabla \left( \frac{P}{\rho} \right) - \frac{\rho S}{c_p} \mathbf{g} + \nabla \cdot \mathbf{D}, \quad (3)
\]

and

\[
\rho T \left[ \frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S \right] = \nabla \cdot \left[ \kappa \rho \nabla T \nabla S \right] + Q + 2\nu \left( \epsilon_{ij} \epsilon_{ij} - \frac{1}{3} (\nabla \cdot \mathbf{v})^2 \right), \quad (4)
\]

respectively. Here \( \mathbf{v} = (v_r, v_\theta, v_\phi) \) is the local fluid velocity (in spherical coordinates) in the rotating frame, \( c_p \) is the specific heat at constant pressure, \( \mathbf{g} \) is the local gravitational acceleration due to a solar mass \( M_\odot \) at the origin, and \( \kappa \) is the thermometric conductivity. The expression for momentum conservation—Equation (3)—employs the Lantz–Braginsky–Roberts approximation, which is exact for adiabatic reference states (Lantz 1992; Braginsky & Roberts 1995). Both the diffusivities \( \nu \) and \( \kappa \) are fixed constants in space for our models. The tensors \( \mathbf{D} \), \( \epsilon_{ij} \), and \( \delta_{ij} \) refer to the standard Newtonian viscous stress, rate of strain, and Kronecker delta, respectively. In Equation (4), the standard summation convention for repeated indices is used. The internal heating function, given by the negative divergence of the radiative energy flux \( Q = -\nabla \cdot \mathbf{Q}_{\text{rad}} \), is chosen to have the fixed radial profile \( Q(r) = \alpha (P(r) - P(r_o)) \), with the normalization constant \( \alpha \) chosen such that a solar luminosity \( L_\odot \) is forced through the domain (see Featherstone & Hindman 2016a). This prescription for the internal heating function coincides well with the
radiative heating calculated by more sophisticated solar models (see model S described in Christensen-Dalsgaard et al. 1996, for example).

The equation set is closed with an equation of state for the thermodynamic variables, which consists of a perfect gas subject to small thermodynamic perturbations about the adiabatic reference state:

$$\frac{\rho}{\rho_0} = \frac{P}{P_0} = \frac{T}{T_0} = \frac{P}{\gamma \rho T} = c_p.$$  

Here $\gamma$ is the ratio of specific heats.

We adopt boundary conditions on the velocity field to conserve angular momentum and mass. These are the stress-free and impenetrability boundary conditions:

$$v_r = \left( \frac{\partial}{\partial r} \right) \left( \frac{v_\theta}{r} \right) = \left( \frac{\partial}{\partial r} \right) \left( \frac{v_\phi}{r} \right) = 0 \text{ at } r = r_i, r_o.$$  

The conditions on the entropy are such that heat does not enter through the inner boundary and there is constant entropy at the outer boundary:

$$\frac{\partial S}{\partial r} = 0 \text{ at } r = r_i \text{ and } S = 0 \text{ at } r = r_o.$$  

The outer boundary condition on the entropy ensures that as the system equilibrates, a sharp gradient in the spherical mean of $S$—i.e., a thermal boundary layer—develops near the outer surface, such that a solar luminosity is ultimately carried out of the layer via thermal conduction. This stands in contrast to the solar CZ, where the energy is ultimately carried out by radiative cooling.

In this work, we compare two models with different density contrasts. Each rotates at roughly 3 times the solar Carrington rate to ensure that a solar-like differential rotation (fast equator and slow poles) is attained. The relevant model parameters are shown in Table 1. Here $N_r$ refers to the number of density scale heights across the domain. The density contrast from the inner to the outer surface (also shown in Table 1) is related to the number of scale heights by $\eta_i/\eta_o = \exp(N_r)$, where $\eta_i$ and $\eta_o$ refer to the values of $\eta$ at the inner and outer boundaries, respectively. The thermal diffusion time $T_{\text{diff}}$ estimates how long it takes for heat to diffuse across the full spherical shell. The averaging time refers to the time interval used in the temporal averages of fluid quantities—e.g., differential rotation, meridional circulation, and Reynolds stress. This interval coincides with the time from equilibration (total energy flux constant with radius) to the end of the simulation. All input parameters are fixed in the two models, except for $N_r$, which takes on the values 3 and 5 (and the grid resolution, which is higher for the case with greater density contrast). We refer to the resulting models as cases N3 and N5, respectively.

### 3. Global Flow Properties

![Image](https://example.com/image.jpg)

Figure 2 shows the radial velocity of the flow on spherical surfaces near the outer boundary and at mid-depth for cases N3 and N5. The pairs of alternating lanes of upflow and downflow parallel to the rotation axis indicate the locations of pairs of Busse columns. The columns at mid-depth have connectivity to their counterparts in the near-surface layers, but they are significantly slower, and the downflow lanes are thicker. At high latitudes, there is less noticeable alignment of the fluid structures with the rotation axis, although this may in part be due to angular distortion effects of the spherical projection. We note that in the near-surface layers of case N5, there are downflow lanes oriented north–south (parallel to the rotation axis) and east–west (parallel to lines of latitude). We call the places where the two types of lanes cross “interstices.” We shall see that the interstices are sources of prominent downflow plumes, which evolve independently from the Busse columns and transport angular momentum in the opposite direction.

We turn next to the mean flow properties of our models. The average radial profile of rotation rate at various latitudes for cases N3 and N5 is shown in Figure 3, and the full rotation rate in the meridional plane is shown in Figure 4. Case N3 exhibits the stronger differential rotation, with a variation of $\Delta \Omega = 155$ nHz from equator to pole. This corresponds to a differential rotation fraction of $\Delta \Omega / \Omega_0 = 0.125$. As the density contrast across the layer increases, the overall differential rotation $\Delta \Omega$ decreases, along with the differential rotation rate. Case N5 has a differential rotation from equator to pole of only $\Delta \Omega = 81$ nHz and a differential rotation fraction of $\Delta \Omega / \Omega_0 = 0.065$. We note that both of these values of fractional differential rotation are significantly smaller than the value $\Delta \Omega / \Omega_0 \approx 0.3$ observed in the Sun.

While the rotation rate in case N3 increases monotonically in both radius and latitude, case N5 exhibits features reminiscent of near-surface shear. The most prominent of these is a “dip” in the rotation rate at low latitudes near the outer surface: here the rotation rate decreases with radius by about 2.7%. The effect is comparable in magnitude to the Sun’s NSSL, which is
Figure 2. Mollweide projection of radial velocity $v_r$ on spherical surfaces for cases N3 and N5. Red tones indicate upflows ($v_r > 0$), while blue tones indicate downflows ($v_r < 0$). The spherical surfaces are at two radii, one at a near-surface layer 5% of the way from top to bottom ($r/r_o = 0.988$) and one at a layer midway through the domain ($r/r_o = 0.881$). In panel (c), the color bar is binormalized to highlight the asymmetry in the amplitudes of upflows vs. downflows near the outer surface for case N5. The dashed $40^\circ \times 40^\circ$ box at the center of each of panels (a), (c), and (d) shows the regions that are magnified in later figures.
characterized by a roughly 5% decrease in rotation rate near the top of the CZ. This low-latitude dip has been a robust feature of other work (e.g., Brun & Toomre 2002; Brandenburg 2007; Gastine et al. 2013; Guerrero et al. 2013; Hotta et al. 2015) and is also referred to as a dimple in the literature. At high latitudes in case N5, there are some signs of shear as well, although the overall effect is much weaker, corresponding to a reduction in angular velocity of only $\sim 0.5\%$. Furthermore, the shear has both a negative and positive radial gradient. Hotta et al. (2015) saw this phenomenon as well.

The magnitude of the zonally averaged mass flux for cases N3 and N5 is shown in Figure 5. At low latitudes in the northern hemisphere (for both cases), there are three cylindrically stacked circulation cells, the inner and outer cells being counterclockwise and the sandwiched inner cell being clockwise. The central clockwise cell is large in case N3 but small in case N5, while the opposite is true for the counterclockwise cell near the outer surface. Furthermore, case N5 has two additional clockwise cells at high latitudes, one near the outer boundary and one near the inner boundary. For both cases, the large counterclockwise cell at high latitudes is concentrated in a thin band near the outer surface, where there is strong poleward flow. In the southern hemisphere for each case, the circulation patterns are the same but with the clockwise/counterclockwise sense of each cell reversed.

Figure 6 shows the breakdown of radial energy fluxes for case N5. These fluxes are defined (e.g., Featherstone & Hindman 2016a) to be

\[ F_{\text{rad}}(r) \equiv \frac{1}{r^2} \int_r^\infty Q(x)x^2dx. \]  
\[ F_{\text{cond}}(r) \equiv -\kappa \frac{\partial S}{\partial r}, \]  
\[ F_{\text{KE}}(r) \equiv \frac{1}{2} \mathcal{P} \left( \frac{\partial S}{\partial r} \right), \]  
\[ F_{\text{visc}}(r) \equiv -\left( v \cdot D \right), \]

Here the angular brackets denote spherical averages. For the most part, four main fluxes contribute: the radiative flux $F_{\text{rad}}$, the conductive flux $F_{\text{cond}}$, the enthalpy flux $F_{\text{enth}}$ (which represents the convective transport of heat), and the kinetic energy flux $F_{\text{KE}}$. In the bottom layers ($r/r_0 \lesssim 0.85$), energy is transported primarily by the radiative flux (with about 25% of the energy being transported by the conductive flux). As the radiative flux decreases, the enthalpy flux begins to take over. Around $r/r_0 \approx 0.97$, the convective heat flux is dominant. Finally, near the outer surface, the boundary conditions on the velocity and entropy force all fluxes to vanish except for the conductive flux, which carries a solar luminosity out of the domain in a narrow thermal boundary layer. The extreme flatness of the total energy flux in case N5 indicates a mature state of statistical equilibrium for the energy transport.
4. Busse Columns

We recall that Busse columns are convective rolls of fluid aligned with the rotation axis. Adjacent rolls have opposite senses of spin, so that each columnar downflow lane traces the region in between two Busse column rolls. In Figure 8, we show the equatorial cross section of radial velocity for case N3. The prograde tilt of the downflow lanes is obvious; the portions of the lane close to the outer surface are at a higher longitude than the portions of the column close to the inner surface. As a general rule, the columns extend in depth all the way through the layer; however, several structures (especially the downflows in the upper half of the layer) only extend through half the layer or less.

The character of the Busse columns depends on the level of rotational constraint in the simulations. We show the radial profile of the Rossby number for cases N3 and N5 in Figure 7. The near-surface outer layers of case N5 are rotationally unconstrained (Ro > 1), in contrast to those of case N3 (see also Figures 2(a) and (c)). In the deep layers, on the other hand, both cases are rotationally constrained, with low Rossby numbers and definitive Busse column structure (Figures 2(b) and (d)).

In the deep layers of the shell, the Busse column structure is clearest (e.g., Figures 2 and 8). We estimate the zonal wavenumber of the Busse columns in each case by finding the peak of the power spectrum of the sectoral spherical harmonics (first averaging the spectrum in time and radius over the inner half of the shell). We find a clear peak in each power spectrum, with m_{peak} \approx 28 for case N3 and m_{peak} \approx 24 for case N5. For both cases, a typical Busse column thus has a zonal extent comparable to the depth of the shell.

Featherstone & Hindman (2016b) showed that the wavenumber of the Busse columns scales with the Rossby number like m_{peak} \sim Ro^{-1/2}. Near the inner surface, the ratio of the Rossby numbers between cases N3 and N5 is \approx 0.76 (see Figure 7). This corresponds to a ratio of Busse column wavenumbers of (0.76)^{-1/2} \approx 1.15, in agreement to first order with our estimates of m_{peak} from the power spectra (28/24 \approx 1.17). The comparable values of the Busse column wavenumbers in cases N3 and N5 illustrate the fact that although the near-surface layers of the two cases have very different flow structures, the deep layers of the two simulations (where the Rossby numbers are only slightly different) have similar flow structures.

Figure 9 shows the temporal evolution of the near-surface flow field in case N3. To better see the evolution, we magnify a 40° \times 40° patch centered at the equator, indicated by the dashed box in Figure 2(a). Some downflow lanes (which trace the regions in between adjacent Busse column rolls) have been labeled with capital letters to indicate how they are advected and distorted by the flow. Although the downflow lanes are rather long-lived (lane A, for instance, maintains its structure for case N5).
for several rotation periods before getting absorbed by another lane), they are not simply advected passively by the flow. They frequently merge, disappear, and reappear, indicating that no one Busse column lasts for a protracted interval. Furthermore, the lanes (and, hence, the columns) only extend coherently to about $\pm 15^\circ$; beyond this latitude range, the lanes move more slowly, eventually breaking off and joining the swirling small-scale flow at high latitudes. We note that the differential rotation of case N3 (seen in Figures 3(a) and 4(a)) correspondingly occupies mainly the narrow latitude band between $\pm 15^\circ$. Each lane is advected by several degrees over the whole rotation period, corresponding to a pattern speed of the Busse columns that is $\sim 40 \text{ m s}^{-1}$ faster than the background rotation rate. In other words, the Busse columns superrotate with respect to the background flow.

5. Downflow Plumes

Figure 10 shows the evolution of the near-surface flow field for case N5, with several interstices (regions where the north–south and east–west downflow lanes cross) labeled with capital letters. We see that each upflow is surrounded by a more or less polygonal network of downflow lanes and thus may be regarded as a cell. The cells are stacked in the axial direction such that there are 10 or so downflow lanes that connect throughout the whole patch north–south, analogous to the downflow lanes between pairs of Busse column rolls in case N3 (Figure 9). On average, the interstices move in the prograde direction, indicating that they are superrotating like the Busse columns. The interstices have much shorter lifetimes than the Busse columns, splitting up and merging several times over the course of a rotation period. In Figure 10(b), for example, interstice A has split into two interstices, A1 and A2, while in Figure 10(c), the two interstices have merged again.

At any given instant of time, the interstices shown in Figure 10 are the sources of downflow plumes. The plumes can be seen by following the interstices down in depth. In Figure 11, we consider the patch from Figure 10(a) and examine the connectivity of the downflows from the near-surface layers to mid-depth. As the patches get successively deeper, the downflow associated with each interstice intensifies in amplitude and becomes more localized. We refer to the entire structure (interstice to localized downflow near mid-depth) as a downflow plume. It is important to note that the plumes do not coincide with the trajectory of a fluid parcel launched downward from the interstice. Since the interstices move prograde in time, each radial point on the plume corresponds to a fluid parcel that was launched when the parent interstice (top of the plume) was positioned retrograde from its present location.

The coherence of the north–south downflow lanes in Figure 11 increases with depth, and the plumes slowly fade. No plume extends in depth more than $\sim 0.1 r_o$ (or about 2/5 the depth of the layer) from the near-surface layer shown in Figure 11(a). This is consistent with the ephemeral nature of the interstices; over the radial extent of a plume, the fastest speeds are on average $\sim 400 \text{ m s}^{-1}$ in the plume core, while the lifetime of an individual interstice is $\sim 2$ days. Thus, the total extent in depth of the plumes should be $\sim (400 \text{ m s}^{-1}) \times (2 \text{ days}) \approx 0.1 r_o$. 

Figure 8. Instantaneous profile of the radial velocity $v_r$ on an equatorial cut of the computational domain for case N3. The view is from the north pole, so the longitude $\phi$ increases in a counterclockwise sense. The radial velocity has been divided by its rms value at each radius, and the color bar is binormalized to show the asymmetry in the upflow and downflow speeds with respect to the rms.

Figure 9. Temporal evolution of the near-surface radial velocity $v_r$ for case N3. Spherical surfaces are taken at 5% depth ($r/r_o = 0.988$). Each panel shows a $40^\circ \times 40^\circ$ patch of the spherical surface centered at the equator, with successive patches equally spaced in time by roughly a quarter of a rotation period. The frame of the patch is rotating at the local equatorial rotation rate in order to see the superrotation of the columns.
6. Torque Balance

The steady-state distribution of angular momentum (and, by extension, the differential rotation) can be understood in terms of the angular momentum transport, or torque, due to various aspects of the flow. In equilibrium, the torque balance (e.g., Elliot et al. 2000; Brun & Toomre 2002; Miesch & Hindman 2011) is expressed as

$$
\tau_r + \tau_{\text{mc}} + \tau_v \equiv 0,
$$

(6)

where

$$
\tau_r \equiv - \nabla \cdot [\rho r \sin \theta \langle v_z' v_{r}' \rangle],
$$

(7a)

$$
\tau_{\text{mc}} \equiv - \langle \rho v_m' \rangle \cdot \nabla \mathcal{L},
$$

(7b)

and

$$
\tau_v \equiv \nabla \cdot [\rho_0 r^2 \sin^2 \theta \nabla \Omega].
$$

(7c)

Here $v_m \equiv v_r \hat{e}_r + v_\theta \hat{e}_\theta$ is the meridional part of the fluid velocity and $\mathcal{L} \equiv r \sin \theta (\Omega \sin \theta + \langle v_\theta \rangle) = \Omega r^2 \sin^2 \theta$ is the fluid’s specific angular momentum in the nonrotating lab frame. Angular brackets indicate a combined temporal and zonal average, and the primes indicate deviations from the average.

Figure 12 shows the balance of torque for cases N3 and N5. At each point in the meridional plane, the magnitude of the sum of the torques is a factor of $\sim 100$ smaller than the magnitude of any of the three torques individually. Furthermore, following equilibration in each simulation, the kinetic energy in the differential rotation varies only by a few percent from its temporally averaged value, as shown in Figure 13. We have also confirmed that there is no secular drift in the kinetic energy with time over several thousand rotation periods, indicating a mature state of equilibrium in the torque balance.

For each case, the torque is roughly constant on cylinders near the equator, whereas it is roughly constant on spheres at high latitudes. The main feature that sets case N5 apart from case N3 is the strong band of negative Reynolds stress torque near the outer surface of case N5, extending across all latitudes; in case N3, there is only negative Reynolds stress torque at high latitudes, and it is significantly weaker than in case N5.

A decomposition of $\langle v'_r v'_m \rangle$ into its two component correlations $\langle v'_r v'_v \rangle$ and $\langle v'_r v'_\theta \rangle$ reveals that the Reynolds stress torque is almost entirely dominated by the radial turbulent angular momentum flux,

$$
\mathcal{F}_r \equiv \rho r \sin \theta \langle v'_r v'_v \rangle,
$$

(8)

for both cases N3 and N5. We now argue that this radial flux is produced by the two types of flow structures mentioned previously, Busse columns and downflow plumes.

The cross-sectional tilt of the Busse columns (Figure 8) tends to give upflows ($v'_r > 0$) a positive $v'_r$ and downflows ($v'_r < 0$) a negative $v'_r$. Within a Busse column, both upflows and downflows thus yield a positive (radially outward) angular momentum flux. The net result is that angular momentum is taken away from the inner layers (negative torque) and deposited in the outer layers (positive torque). Figure 14(a) shows the low-latitude torque balance for case N3. The Reynolds stress torque is positive in roughly the upper quarter of the domain and negative in the lower three-quarters, indicating that Busse columns dominate the Reynolds stress at all depths. In case N5, however, the Reynolds stress torque is
negative near the outer surface (Figure 14(c)), indicating the influence of an additional transport mechanism.

The downflow plumes do not follow the Busse column tilt. There are only a few other physical mechanisms to create correlations in \( v_r \) and \( v_f \), and the most obvious of these is deflection by the Coriolis force. For downflow plumes (\( v_r < 0 \)), the deflection is prograde (\( v_f > 0 \)), corresponding to a negative (radially inward) transport of angular momentum. This picture is consistent with the Reynolds stress torque at high latitudes for both cases (Figures 14(b) and (d)): the Reynolds stress torque is negative in the outer layers and positive in the inner layers, corresponding to the inward transport of angular momentum from the outer surface to the bottom.

At low latitudes, the presence of downflow plumes is most prominent in case N5 near the outer surface, where the Reynolds stress torque is negative. Correspondingly, there is a dip in rotation rate at low latitudes for case N5, but not for case N3. This lends substantial support to the argument of Foukal & Jokipii (1975), at least at low latitudes: downflow plumes are Coriolis-deflected (or, equivalently, they conserve their angular momentum), transporting angular momentum radially inward and creating near-surface shear.

The torque from the meridional circulation is determined by the alternating pattern of clockwise and counterclockwise cells in Figure 5, coupled with the fact that angular momentum in our models is roughly constant on cylinders (\( \lambda = \text{constant} \)). From Equation (7b), this means that the torque has a sign opposite to that of \( \langle p v_r \rangle \). Near the equatorial boundaries of the circulation cells, the meridional flow is dominated by the radial component \( \langle p v_r \rangle = \langle p v_\lambda \rangle \). Consequently, \( \tau_{mc} \) changes sign in Figures 14(a) and (c) at the radial locations of the north–south cell boundaries.

At high latitudes, a decomposition of the meridional circulation into its radial and latitudinal components reveals that only the latitudinal term \( -\langle p Y \rangle (\partial Y / \partial \theta) \) contributes significantly to the torque. Geometrically, this is due to the shape of the near-surface counterclockwise cell of meridional circulation in both cases, which is most intense in a region highly elongated in the latitudinal direction. The resultant poleward flow dredges up high angular momentum fluid from the equator and brings it to high latitudes, thereby creating a positive near-surface torque.

The viscous flux of angular momentum \( -p \nu r^2 \sin^2 \theta \nabla \Omega \) is proportional to the negative gradient of the rotation rate. This is
similar to a Fickian diffusivity, meaning that viscosity simply tends to bring the rotation rate to a constant value (i.e., it eliminates shear). In this sense, the viscous torque is “passive,” responding only to counteract the shear produced by the Reynolds stress and meridional circulation. Thus, in Figure 14, the viscous torque is simply negative or positive according to the sign of the combined Reynolds stress and meridional circulation torques.

7. Angular Momentum Transport by Upflows and Downflows

Figure 15 shows the Reynolds stress angular momentum flux $\mathcal{F}_r$ broken up into upflow and downflow components for cases N3 and N5 in the high- and low-latitude regimes. For case N3 at low latitudes, both components of the flux have the same sign: positive in most of the domain except for a narrow region of very weak negative flux near the outer boundary. The strong positive fluxes are consistent with the tilts of the Busse columns in the equatorial plane, which makes the correlation $\langle v'_r v'_\phi \rangle$ positive for both upflows and downflows. In the middle layers, the magnitude of the upflow flux is about twice as large as that of the downflow flux. This asymmetry may be attributed to the fact that although most downflows are entrained in Busse columns, there are also downflows whose speeds are large enough that they are really plumes that lack a negative $v'_\phi$.

The overall positive correlation $\langle v'_r v'_\phi \rangle$ is thus weaker for the downflows.

For case N5, the low-latitude upflow flux looks rather similar to the upflow flux in case N3, except that it peaks in the middle of the layer, as opposed to peaking in the upper half. The downflow flux in the bottom half of the layer ($r/r_o \lesssim 0.87$) is positive, consistent with the organization of both upflows and downflows into Busse columns in the deeper layers. The top half of the shell, by contrast, has negative downflow flux, whose maximum magnitude is over twice as great as the maximum magnitude of the positive downflow flux. Thus, at low latitudes, the inward transport of angular momentum comes only from the downflow plumes; the upflows are dominated by Busse columns.

When the low-latitude upflow and downflow fluxes are added, there is a positive slope in the radial profile of the total angular momentum flux $\mathcal{F}_r$. The effect on the Reynolds stress torque, which scales like the radial derivative of $r^2 \mathcal{F}_r$, is to produce the narrow region of negative torque near the outer boundary as seen in Figure 14(c). This negative torque is responsible for maintaining the low-latitude near-surface shear against viscosity.

At high latitudes in both cases, the upflow and downflow fluxes have the same sign: negative in most of the fluid layer, except near the shell boundaries. Near the outer surface, there is significant asymmetry in the flux magnitudes between cases N3 and N5 and separately between the upflow and downflow flux of case N5. We argue that this arises mostly from the asymmetry in upflow and downflow speeds. Figure 16 shows the rms radial speeds of upflows and downflows in cases N3 and N5 as functions of radius. In case N5, the downflows are about twice as fast as the upflows near the outer surface. The Reynolds stress, which scales like $v'_r^2$, is correspondingly larger for the downflows than the upflows. Similarly, since both the upflows and downflows are about twice as fast in case N5 than
in case N3 near the outer surface, both the upflow and downflow fluxes are greater in case N5.

We note here that the Rossby number associated with the rms radial downflow speed at the base of the thermal boundary layer is \( \sim 0.2 \) for case N3 and \( \sim 1.2 \) for case N5. Here we define the downflow Rossby number as \( \psi_{\text{rms}}' (r) / 2H f_i (r) \), where \( \psi_{\text{rms}}' (r) \) is the rms convective speed of the downflows averaged over spherical surfaces of radius \( r \). Hence, the downflows in case N5 are, on average, rotationally unconstrained according to Equation (1). This agrees with the fact that the downflow flux is negative in the outer layers of case N5; the downflow plumes conserve their angular momentum and thus transport it radially inward.

We now verify that it is only the downflow plumes that carry angular momentum inward at low latitudes, not the slower downflow lanes associated with Busse columns. Figure 17 shows a similar “dive” through the fluid layer as Figure 11, this time illustrating the instantaneous angular momentum flux in the patch. We see that at \( r/r_o = 0.957 \) (which is close to \( r/r_o = 0.942 \), the extremum of the downflow flux in Figure 15(b)), the central region of nearly every plume is associated with a positive \( \psi_o \) and thus a negative angular momentum flux. The notable exception is plume D, which has the deepest extent of any of the plumes. However, plume D has negative flux in the deepest layers, as seen in Figure 17(d).

In contrast to the plumes, the columnar downflow lanes in the deep layers (Figures 17(b)–(d)) mostly have positive angular momentum flux, consistent with the tilts of the Busse columns. It is also interesting to note that in between the axially aligned downflow lanes, the upflows are split into two regions with an alternating sign of the flux—positive on the right and negative on the left. This is simply due to the manner in which upflows diverge and recirculate (in the center of the upwell) to accommodate the impenetrable outer boundary and maintain mass conservation. Note, however, that in the deeper layers, the positive flux in the upflows dominates the negative flux. This explains the weakness of the positive upflow flux between \( r/r_o \sim 0.96 \) and \( r/r_o \sim 1 \) in Figure 15(c). Near the outer surface, the upflow rolls diverge symmetrically in both the positive and negative zonal directions, and their net angular momentum transport cancels out almost completely.

Finally, we note that near the outer surface, the east–west downflow lanes have largely the opposite sign of flux compared to the upflows in which they are embedded: negative on the right and positive on the left for the upflow column straddling the central meridian. This is due to the tendency of the fluid in the east–west lanes to flow sideways toward the interstices, which have an extremely low pressure compared to their surroundings.

8. Discussion

In this work, we have explored the development of near-surface shear in 3D spherical-shell models of solar-like convection as a natural consequence of increasing the density contrast across the shell. We find that increased stratification does indeed foster more rapid flow structures with Reynolds stresses that enhance near-surface shear. However, this proves insufficient to create a solar-like NSSL.

Our highest-contrast model (case N5) contains two types of flow structures that influence differential rotation: rotationally constrained Busse columns and rotationally unconstrained downflow plumes. The Busse columns transport angular momentum outward at low latitudes and thus maintain a fast equator and slow poles. The plumes transport angular momentum inward in the outer half of the layer at all latitudes. The influences of Busse columns and downflow plumes on the Reynolds stress are cleanly summarized in Figure 18, which shows the correlation coefficient

\[
C \equiv \langle \psi_o' \psi_o' \rangle / [\langle \psi_o^2 \rangle \langle \psi_o' \rangle]^{1/2}
\]

plotted in the meridional plane for the upflows and downflows.
in case N5. This correlation contains the same information as the angular momentum fluxes depicted in Figure 15, but the signal has been normalized and the geometric factor $r \sin \theta$ has been removed.

For the upflows, the correlation is everywhere positive at low latitudes, implying outward angular momentum transport—and thus dominance by Busse columns—at all depths. It is negative (except near the inner boundary) at high latitudes, consistent with inward angular momentum transport through Coriolis deflection. The correlation for the downflows is mostly negative everywhere, except in the inner half of the shell at low latitudes (where the Busse columns dominate) and near the inner boundary at high latitudes. The strong positive correlation near the inner boundary for both upflows and downflows is due to the impenetrability condition maintained by the pressure force, which we do not investigate in detail here.

Figure 18 shows that the dip in angular velocity at low latitudes in case N5, while similar in amplitude and radial extent to the solar NSSL, arises for the wrong reasons. The Busse columns transport angular momentum outward, while the plumes transport angular momentum inward. The radial location where the upward flux from the Busse columns cancels the downward flux from the plumes—i.e., the spherical surface where the total correlation is zero in Figure 18(c)—corresponds roughly to the peak in rotation rate at low latitudes. The low-latitude NSSL in case N5 thus does not represent a layer that the downflow plumes slow down, but rather a layer that the Busse columns fail to speed up. In the Sun, by contrast, the rotation curves have only a small positive slope with radius (see Figure 1), implying that Busse columns, if they are present in the Sun, do not transport angular momentum in the same way as they do in simulations, or that the plumes reach more deeply than in our models.

The essential role played by Busse columns in maintaining numerical models’ near-surface shear is even more apparent at high latitudes in case N5. Here the Busse columns have little effect on angular momentum transport (the correlations in Figure 18 are mostly negative for both upflows and downflows), while fast, small-scale downflow plumes conserve angular momentum in radial motion, transporting it inward. Although this Coriolis deflection effect is, in fact, stronger at high latitudes than at low latitudes, there is basically no high-latitude near-surface shear in case N5. Thus, the mechanism described by Foukal & Jokipii (1975)—namely, that there is a rotationally unconstrained fluid layer near the outer surface of the solar CZ—cannot, by itself, explain the NSSL, at least not using current numerical models.

Miesch & Hindman (2011) made a similar point in their investigation of how meridional circulation and differential rotation respond to a negative axial torque. In our case N5, there is a negative torque due to the Reynolds stress. At high latitudes, the torque balance is primarily between the Reynolds stress and meridional circulation, reducing the torque balance in Equation (6) to $-\tau_{\text{rc}} = \tau_{\text{cy}}$. The definition of $\tau_{\text{rc}}$ in Equation (7) then leads to the following equilibrium relationship between meridional circulation and differential rotation:

$$\mathbf{\bar{\rho}} \bar{v}_m \cdot \nabla (\Omega r^2 \sin^2 \theta) = \tau_{\text{cy}}.$$  

(9)

The preceding equation may be satisfied in multiple ways, since both the rotation rate $\Omega$ and the meridional circulation $\mathbf{\bar{\rho}} \bar{v}_m$ appear on the left-hand side. All local models that seek to explain the solar NSSL by inward angular momentum transport—for example, the Coriolis deflection of downflow plumes in our case N5—prescribe a negative right-hand side, e.g., a negative $\tau_{\text{cy}}$. This does not, however, constrain the rotation rate until another equation—namely, that of meridional force balance—is specified to determine the meridional circulation. Since the meridional circulation is fundamentally a global phenomenon, it is unlikely that any local model of angular momentum transport will explain the NSSL.

An examination of the high-latitude meridional circulation profile in the context of Equation (9) reveals why near-surface shear is nearly absent at high latitudes in our case N5 and the simulation of Hotta et al. (2015). In both simulations, there is a narrow band of poleward meridional circulation near the outer surface (see Figure 5 of this work and Figure 10 of Hotta et al. 2015), which brings high angular momentum fluid from equator to pole. If, on long timescales, this process pumps angular momentum to high latitudes more efficiently than the Reynolds stress can remove it, there will be no shear at high latitudes in equilibrium.

Figure 19 shows the radial profiles of Reynolds stress torque, latitudinal mass flux, and rotation rate averaged over high latitudes for case N5. Clearly, case N5 satisfies Equation (9) in its outer layers by having the meridional circulation near the outer surface inherit the radial profile of negative Reynolds stress torque, while the near-surface rotation rate is mostly flat. In other words, although we might expect the strongly negative Reynolds stress torque created by downflow plumes in the outer layers of case N5 to drive near-surface shear, the particular profile of the near-surface meridional circulation opposes this driving mechanism almost completely.

Because the molecular viscosity in the solar interior is so low, the balance in Equation (9) likely holds in the Sun, possibly with the added complication of a Maxwell torque from the magnetic field. This work has shown explicitly that the only way to understand the solar NSSL in the context of local angular momentum transport—i.e., the right-hand side of
Figure 19. Radial profiles of $\tau_m$, $\langle \rho \nu_\theta \rangle$, and $\Omega$ averaged over high latitudes (between $\pm 45^\circ$ and $\pm 60^\circ$) for case N5. We scale $\langle \rho \nu_\theta \rangle$ by its maximum absolute value to make its radial profile lie in the range $[-1, 1]$. We scale $\Omega$ and $\tau_m$ to match the shape of their profiles in Figures 3 and 14, respectively. The shaded region indicates the near-surface layers where the radial profile of the meridional circulation follows that of the Reynolds stress torque.

Equation (9)—is through a detailed understanding of how the meridional circulation is established in the Sun. Once this is understood, the origins of the NSSL can be determined by Equation (9).

Featherstone & Miesch (2015) and Hotta et al. (2015) made substantial progress in understanding meridional force balance, with the latter being concerned specifically with the balance required to maintain an NSSL. Hotta et al. (2015) argued that the high-latitude feature of near-surface shear achieved in their simulation represents a significant deviation from the Taylor–Proudman state, which is achieved by turbulent transport of latitudinal momentum in the radial direction—i.e., the correlation $\langle \nu' \nu' \rangle$. This correlation is, in turn, produced by the sign of the radial derivative of the meridional flow ($\partial (\nu_\theta) / \partial r$) in their NSSL. Although this indeed appears to be a promising mechanism to break the Taylor–Proudman constraint, we argue that it cannot provide the full picture for meridional force balance in the solar NSSL. The Sun has a rotation rate constant roughly on radial lines at high latitudes and thus is not in a Taylor–Proudman state either within the shear layer or without. Thus, breaking the Taylor–Proudman state is a necessary but insufficient condition to create near-surface shear.

Although the detailed dynamical maintenance of the solar meridional circulation is unclear, helioseismic observations provide fairly rigorous constraints on the circulation profile itself, at least in the NSSL. In particular, there is little near-surface variation of latitudinal flow $\langle \rho \nu_\theta \rangle$ with radius in the Sun as compared to case N5 (e.g., Giles et al. 1997; Zhao & Kosovichev 2004; Hathaway 2012; Chen & Zhao 2017; Mandal et al. 2018). Given that both the meridional circulation $\langle \rho \nu_\theta \rangle$ and rotation rate $\Omega$ are observationally constrained in the NSSL, it would be worthwhile to measure the Reynolds stress torque in the NSSL as well, using high-resolution ring analysis (e.g., Greer et al. 2014, 2015). Once this is done, the relationship between meridional circulation and differential rotation in the NSSL will be elucidated. This will be an important guide for future simulations attempting to capture near-surface shear.

In summary, we have determined that the solar NSSL is still an unsolved problem. The argument that a rotationally unconstrained layer near a convecting spherical shell’s outer surface efficiently mixes angular momentum (and thus creates an NSSL) is oversimplified, since only the downflows are sufficiently rotationally unconstrained to efficiently pump angular momentum inward. At low latitudes, the downflows must continually fight the upflows, which still exist in Busse columns and transport angular momentum outward. In our models, this battle between upflows and downflows seems essential to create substantial shear. By contrast, at high latitudes, the inward transport of angular momentum by both upflows and downflows only produces very weak shear due to the global character of the meridional circulation.

In future work, it would be useful to analyze the nature of the feedback between the meridional circulation and differential rotation achieved in spherical-shell models of convection, in particular its behavior in highly stratified regimes and in the presence of magnetic fields. This feedback is likely relevant in a broader context than simply NSSL dynamics. Since the meridional circulation is dynamically linked to the solar dynamo cycle both observationally and theoretically (e.g., Wang et al. 1989; Chou & Dai 2001; Ghizaru et al. 2010; Charbonneau 2014; Komk et al. 2015), dissecting the nature of feedback between the solar NSSL and meridional circulation will provide important theoretical constraints on the processes by which the Sun forms its magnetic field.

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**References**

Braginsky, S. I., & Roberts, P. H. 1995, *GAFD*, 79, 1
Brandenburg, A. 2007, in IAU Symp. 239, Convection in Astrophysics, ed. F. Kupka, I. Roxbury, & K. L. Chan (Cambridge: Cambridge Univ. Press), 457
Brun, A. S., & Toomre, J. 2002, *ApJ*, 570, 865
Busse, F. H. 2002, *PhFl*, 14, 1301
Charbonneau, P. 2014, *ARA&A*, 52, 251
Chen, R., & Zhao. J. 2017, *ApJ*, 849, 144
Chou, D. Y., & Dai, D. C. 2001, ApJL, 559, L175
Christensen-Dalsgaard, J., Dappen, W., AjuKov, S. V., et al. 1996, Sci, 272, 1286
Elliot, J. R., Miesch, M. S., & Toomre, J. 2000, ApJ, 533, 546
Featherstone, N. 2018, Rayleigh 0.9.1, Zenodo, doi:10.5281/zenodo.1236565
Featherstone, N. A., & Hindman, B. W. 2016a, ApJ, 818, 38
Featherstone, N. A., & Hindman, B. W. 2016b, ApJL, 830, L15
Featherstone, N. A., & Miesch, M. S. 2015, ApJ, 804, 67
Foukal, P. A., & Jokipii, J. R. 1975, ApJL, 199, L71
Gastine, T., Wicht, J., & Aurnou, J. M. 2013, Icar, 225, 156
Ghizaru, M., Charbonneau, P., & Smolarkiewicz, P. K. 2010, ApJL, 715, L133
Giles, P. M., Duvall, T. L., Scherrer, P. H., & Bogart, R. S. 1997, Natur, 390, 52
Gilman, P. A. 1972, SoPh, 27, 3
Gilman, P. A., & Glatzmaier, G. A. 1981, ApJS, 45, 335
Gough, D. O. 1969, JAtS, 26, 448
Greer, B. J., Hindman, B. W., Featherstone, N. A., & Toomre, J. 2015, ApJL, 803, L17
Greer, B. J., Hindman, B. W., & Toomre, J. 2014, SoPh, 289, 2823
Guerrero, G., Smolarkiewicz, P. K., Kosovichev, A. G., & Mansour, N. N. 2013, ApJ, 779, 176
Hathaway, D. H. 2012, ApJ, 760, 84
Hotta, H., Rempel, M., & Yokoyama, T. 2015, ApJ, 798, 51
Howe, R., Christensen-Dalsgaard, J., Hill, F., et al. 2000, Sci, 287, 2456
Jones, C. A., Boronski, P., Brun, A. S., et al. 2011, Icar, 216, 120
Komm, R., Gonzalez Hernandez, I., Howe, R., & Hill, F. 2015, SoPh, 290, 3113
Lantz, S. R. 1992, PhD thesis, Cornell Univ., http://adsabs.harvard.edu/abs/1992PhDT........78L
Mandal, K., Hanasoge, S. M., Rajaguru, S. P., & Antia, H. M. 2018, ApJ, 863, 39
Matsui, H., Heien, E., Aubert, J., et al. 2016, GGG, 17, 1586
Miesch, M. S., & Hindman, B. W. 2011, ApJ, 743, 79
Nelson, N. J., Featherstone, N. A., Miesch, M. S., & Toomre, J. 2018, ApJ, 859, 117
Wang, Y.-M., Nash, A. G., & Sheeley, N. R., Jr. 1989, Sci, 245, 712
Zhao, J., & Kosovichev, A. G. 2004, ApJ, 603, 776