An Optimal Two-level Supply Chain Model for Small and Medium-sized Enterprises with a rework for New Products and Price-Dependent Demand

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Abstract

Today, the advent of new technologies, globalization of markets, customers' varied needs, and increasingly fierce competition, are making SMEs seek to improve engagement with their suppliers, and cost management practices in order to survive. SMEs clearly need to focus on the interests of the entire supply chain by enacting win-win policies. In this study, we investigate a two-level inventory model featuring a manufacturer and a buyer in the competitive market with the policy of producing new products. Imperfect quality products and the capacity to rework are also considered in the model. In other words, because of the competitive nature of the market, any increase in price leads to a decrease in demand. The mathematical model is proposed over two scenarios: a scenario where shortage can occur, and one with no possibility of shortage. The objective function of the mathematical model revolves around the central goal of maximizing the total profit of the supply chain considering both independent and joint optimization by the supply chain members. A new algorithm is proposed to solve the mathematical model whose applicability is evaluated by giving a numerical example to the analysis software MATLAB. The results are analyzed and discussed using a sensitivity analysis approach.

Keywords: supply chain management (SCM), imperfect quality products, rework, inventory management, and new products, small and medium-sized enterprises (SMEs)
1  **Introduction**

Cutting global competition forces manufacturing organizations to develop their internal manners and improve collaborative networks with their suppliers and clients to produce mutual advantages. They have recognized that solely optimizing the present manufacturing foundation would fail to manage and conserve a competitive advantage. Therefore, they promote human capital to convert the shop floor into an adaptable system capable of promptly reacting to customers’ demands. Flourishing manufacturing corporations have succeeded by performing versatile principles to integrate supply chain collaborators with their manufacturing approaches. For these causes, organizations place a high priority on the improvement of long-term connections with internal and external supply chain partners [1]. Due to these reasons, researchers believe entirely that the formulation and implementation of supply chain policies need substantial administration practices and the highest administration support [2]. Directors in the thriving entrepreneurial-oriented firms, who have ambidextrous administration behaviors and placed great emphasis on maximizing the integration with suppliers and clients, succeeded in enhancing business responsiveness [3, 4]. Moreover, the study found that both strategic management and supply chain integration are critical components for companies to promote agility in humanitarian operations [5].

In the real world, each business industry wants more profit with a smaller production cost. Client satisfaction is another significant issue for any business, and for that, the quality of the commodity should also be improved. A tremendous amount of demand for any commodity results is more profit for the corporations, and more shortages and defective items may also occur due to the high demand. As a result, shortages and faulty items are an excellent issue for
consumer satisfaction, for which protection stocks may provide an answer. An integration in SMEs’ supply chain can increase the quality of a product such as rework process which is beneficial to any industry such procedures can also enhance the total profit. [6].

Some of the commodities produced in the company are faulty due to many reasons, including the failure of equipments, the mistake of operators, and the low quality of primary substances. So, deciding on faulty items is one of the significant concerns faced by corporations and production units. There are two kinds of defective items: scrap and repairable. Scrapped items can be sent to manufacturer at a single shipment. But, repairable items are sold after rework at a lower price. Repairable items need reworking, which is usually kept in a warehouse until the rework process is called for. Thus, a general method must be determined with regard to the cost of an intended system to do the rework process so as to obtain it at the least cost. This method should cover all types of states that can happen in SMEs’ supply chain. It must provide a proper method and schedules to the managers based on the best result. This study helps administrators to recognize the best strategy for the rework process based on SMEs' supply chain conditions. Also, it attempts to provide programmers and managers with guidelines to choose the optimal production quantity based on maximizing total profit. Includes production cost for the rework and normal production process, holding cost for imperfect and perfect items, and manufacturing cost and rework cost.

This paper, has been emphasized on the importance of paying attention to rework of defective items. In this supply chain inventory system, items are inspected by buyer after production and sending to buyer. Good quality items are stocked and sold to customer immediately. Defective items scheduled for rework. We assume buyer and supplier make decision integerally and provide some types of equipments and facilities to rework imperfect items in the buyer place.
Therefore, the buyer will pay lower price for imperfect goods, which reworked by itself. After inception the vendor send all of the defective commodities to manufacturer as a shipment.

This research work develops an optimal inventory model in two level supply chain for small and medium-sized enterprises with a rework for new products and price-dependent demand. One of the novelty of this study is that the defective items are assumed to be scrape or reworkable and the defective items will be reworked at the buyer location and inception will carry outed by distributor. Rework can be drawn as the conversion of production rejects, failed, or non-conforming objects into re-usable commodities of the equivalent or lower quality during or after review. Rework is significant and useful, mainly if materials are restricted in availability and also pricey. Furthermore, rework can be an excellent contribution to a ‘green image environment.’ Sometimes produced faulty items can be improved and reworked. Throughout the last decade, investment in rework on optimal replenishment decisions has been grown-up extensively. An other novelty of this study is that we have employed two strategy base on the real world’ supply chain for SMEs: first, the manufacturer and distributor make decision independently, second, the manufacturer and distributor share many types of their useful information and facilities to make decision integrately. The third novelty is considering new products and price-dependent demand for supply chain in SMEs. The aim of this study is to determine an optimal supply chain for SMEs with a rework for new products and price-dependent demand so that the total inventory profit is maximized.

In the next section, we will describe the detail about the objective of this investigation and the research gaps that we are trying to cover. The remainder of this paper is as follows. The next section reviews the literature on the models with integration in the two-level supply chain, price-
dependent demand, imperfect quality items, new product, rework, rework in the supply chain, SMEs, and the two-level supply chain in SMEs. This section is followed by Section 3, which discusses the research gap and the objectives of this study. Section 4, initially, develops the Hypotheses and Symbols of this model. Sections 5 discusses the material and methods. Sections 6 the paper then provides some numerical studies and sensitivity. The paper finally concludes in Section 7.

2 Literature review

2.1 Integration in Two-level supply chain

Supply chain integration, as a way to align and coordinate the production within a supply chain, has become a well-recognized industrial practice. The combination of different grades of the supply chain, from raw substance generation to sub-assembly/part production, commodity assembly, delivery, and selling, together with the corresponding warehousing, and shipping are receiving more and more notice from both manufacturing and academia. To meet customers’ requirements, commodity quality reliability and punctuality of distribution should be ensured. In the meanwhile, the cost of the total supply chain should be optimized [7]. [8] provided a production-inventory integration model for supplier, producer and retailer in supply chain with imperfect quality products. [9] presented the integration and generalization of the supply chain model under a trade policy. The trade credit is expressed in two levels, in which the supplier sells the retailer with a permissible delay of M and the retailer sells products to the
customer with a permissible delay of \( n \). This model is provided to find the optimal retail price, EOQ (Economic Order Quantity) and the number of shipments from the supplier to the single-time retailer and also to integrate the inventory system based on both the payment and price policies related to the demand rate.

[10] focuses on the coordination of the supplier network in an integrated inventory model. The inventory model is presented for a heterogeneous buyer and supplier and a single product, based on the reduction of system overall cost. [11] revised the traditional economical orders size with the assumption of perfect products. Their model and analysis are described under two conditions, first, There is no relationship between buyer's purchase price, buyer's sales price and customer demand, and second, there is a relationship between the buyer's purchase price, the buyer's sales price, and customer demand. [12] provided a new model by revising the lot size model that affects inventory level and decisions between buyer and seller as well as supply chain performance. This model (JELS) determines the system cost, the order quantities, the optimum size of the products, as well as the optimal number of shipments. [13] presented a vendor-buyers integrated inventory model including quality enhancement investment in a supply chain. [14] explained an integrated just-in-time inventory model where the demand rate is linearly diminishing with time, the generation rate is limited, and transfer time is constant and deterministic. [15] presented the integration of inventory and sales decisions model for a two-level supply chain, taking into account imperfect quality products and backorders. [16] investigated a two-level supply chain, including a wholesaler and a retailer, in which the retailer works as a vendor and sells the commodities to the final clients. This chain only creates a single deteriorating good with a continuous deteriorating rate. In this chain, demand is deterministic, and lead time for replenishment and replacement of the vendor's product is considered to be zero.
The purpose of this study is to maximize the total chain's profit by defining the optimal values of the vendor's selling price \((p)\) and order cycle length \((T)\). [17] proposed an optimal integrated vendor-buyer inventory model with imperfect items. Most researches for faulty items considered that an inspection process is carried out by the buyer. They assumed that the vendor conveys the inspection process and disposes of faulty items in multiple shipments.

[18] assumed a buyer and multi-supplier, that the production process in the supplier is affected by training, which reduces production costs and increases the supplier's production capacity. Assuming that suppliers are able to reduce their sales price that result in reducing the purchaser's acquisition costs. [19] provided a multi-level supply chain in which equal and unequal batches can be transmitted to different levels, and if the delivery time is not on time, a fine will be imposed on the system. Unlike previous works, this research does not limit the number of steps and analyzes a general case. First, an analytical model of the supply chain is developed and then the model specifications are expressed. Then a simulation study is performed on the model analysis. [20] investigated a model with a single buyer, a single product, and multiple vendors. The seller is selected based on reducing the shipping and handling costs. This research suggests that fixed shipments may result in a reduction in the total cost. [21] provided a supply chain model with a vendor and several retailers that uses reversible transportation, such as containers and boxes. [22] introduced the supply chain construction consists of a single producer with a multi-buyer where the producer requests a determined number of raw substances from outdoor suppliers, processes the documents, and delivers the finished commodities in unequal purchases to each client. [23] showed a model which deals with the difficulty of the joint measurement of the selling price, replenishment lot size, and the number of purchases for an economic production quantity (EPQ) model with the rework of faulty items where the multi-shipment method is used.
Proposed a new multi-objective model for dynamic and joined the network plan of a sustainable closed-loop supply chain network, which points to optimize financial, environmental, and social anxieties, concurrently. In order to have a dynamic perspective, multiple police periods are considered throughout the outlining horizon. Moreover, various short-term decisions are integrated with long-term decisions related to the network layout problem. Two of these short-term decisions are discovering the selling price of commodities in transmitting logistics and purchasing the price of used stocks from client zones in reverse logistics.

2.2 Price-dependent Demand

Since market demand intensively prices sensitive, pricing has taken a significant and vital role for any corporation. Consequently, the businesses hire the pricing strategy as one of the decision policies to optimize their costs in order to absorb clients and obtain more profit. In supply chain, when the demand is not fixed and is price-sensitive, it is possible to influence the costs to the producer and the buyers, most significantly their holding cost. In this situation, the unit procurement cost for the buyers may also vary if the producer fixes its sales price depending against its cost incurred in producing the items [25].

[26] provided a model by assuming that a product is supplied in a market that the demand is sensitive to the price and quality. [17] suggested a two-layer game method on pricing, ordering, and allocation policies in a supply chain, including a supplier and two retailers. [27] Studied a two-level supply chain model consisting of the producer and retailer where the manufacturer sells the outcome in both direct and retail channels. Additionally, they assume that the customers’ demand is sensitive to the selling price of each member, delivery time for direct channel, and retail service for the business where the demand in the direct business has a
negative effect on higher selling price, lengthy delivery time for direct market and more retail servicing from the retail market.

2.3 Imperfect quality items

Producing defective items is an unavoidable issue to which companies are faced. Additionally, the imperfect products have a significant value to the company. [28] summarizes the researches regarding the EOQ model for imperfect quality products. In the classical EOQ models that are also widely in use, it is assumed that all products are perfect and this is different from actual production systems and imperfect products are common in production process [29]. [30] extended an integrated lot-sizing and production model for a faulty and unpredictable production method. [31] considered an EPQ model for imperfect objects with a screening and shortage constraint. Later [32] proposed an EPQ model for an imperfect production process allowing backordered shortages.

2.4 New product

The nature of competitive businesses continuously promotes manufacturers to develop new products to meet the frequently diversified client demands. One advantage of globalization is the expanding opportunities for products the market can afford. On the demand side, consumers are enjoying the developing choices they have when choosing commodities that satisfy their needs. On the supply side, this globalization trend pushes manufacturers to promote new products that can fulfill the upcoming demands for customized commodities at competitive prices. To meet client demands and sustain their competitiveness, producers keep improving new products.
[33] analyzed the relationship between the competitiveness of informal (unregistered) companies and the new products development (NPD) and official (registered) companies. They analyzed 9,000 official companies in Eastern Europe and Central Asia and concluded that NDP is an effective factor to differentiate formal companies from informal companies, in this study the direct effect of NDP was taken to account and this effect was also moderated based on the characteristics of competition and organizational institutions. Given the significance of product design and its profound impact on the total supply chain performance, more and more people recognize the necessity to integrate commodity improvement decisions, and supply chain decisions and trying to bring the integration of suppliers one step earlier – to the design phase [34]. Joint product and supply chain configuration/design is a cross-domain, cross-space, cross-level, and cross-echelon difficulty that explores to optimize many decisions all at once [35]. [7] investigated a study based on the interests of the decision-maker, this dilemma may demand the incorporation of other components, such as consumer favorite, demand prediction, product and production process design, supplier preference, stock locating, logistic planning, pricing, production planning, and so on. This joint optimization obstacle can become significantly involved when commodity variants are considered. The supply chain domain is at the intermediate level, and the goods domain is at the lower level. The necessities of clients will affect the product design and supplier choice. Once the product and supply chain design is fixed, creation will be arranged and regulated into the supply chain, and corresponding substances will flow from suppliers to end clients. [36] focus on the mathematical modeling of commodity, process, and supply chain design (i.e. three-dimension concurrent engineering, 3D-CE) a classifies extant research based on the dimensions considered.
2.5 Rework

Rework process is likewise one crucial issue backward coordination where utilized items are modified to reduce total inventory cost, waste, and environmental pollution. [26] studied the production line for the imperfect quality products at each level. The imperfect quality products enter a reworked process, which is done incompletely and the products that are still imperfect for the second time are considered to be scrap and no rework can be done on them. This model is optimized for a multi-level production-inventory system and the system performance is optimized based on the production time, the production process performance, and shipment frequency. [37] studied the integration model of production problems, and in this model, the preventive maintenance (PM) and inventory screening are done in the production process, and after rework and repairs, the imperfect quality products are returned to the inventory. Preventive maintenance is done during the control phase. The result of proper preventive maintenance is the reduction in the rate of imperfect quality products, and also performing a faulty preventive maintenance in the system result in making it out of control with a specific probability. When the production process is out of control, the system produces imperfect quality products with a specific probability. Rework is typical in Paper Industries, Semiconductor, Chemical, Food industries, Textile industries, Pharmaceutical, Glass industries, Metal processing industries, and Plastic industries [38-40]. The rework process has attracted significant consideration because of the decline of natural sources and the rise in the cost of raw material. Rework processes perform an essential role in eradicating waste and efficiently controlling the cost of manufacturing in a production system. Therefore, determining the optimal lot size in a system that allows reworking is a valuable objective to maximize the total inventory profit. Rework process diminishes energy usage and preserves more natural resources for future generations. Therefore, corporations are
offering sustainable improvement. Also [41] explained an integrated inventory product model with a single-stage production process and reworking faulty objects. [42] presented rework with multi-production fixings. [43] investigated an economic generation quantity model for three kinds of faulty goods with rework the economic production quantity (EPQ) model problem for a single object below the hypotheses of the defective commodity, and complete rework has been performed by [44]. [27, 45] developed the models under multiple conditions with regard to the planned method, one of which examines an imperfect generation system that produces remarkable low-quality commodities in lot-sizing restrictions. [17] optimized a stochastic lot-sizing difficulty for a single faulty stage generation system with paramount rework, faulty preventive preservation, and minimal repair. [46] studied replenishment and pricing strategies for a defective process with paramount rework and various deliveries. [47] solved the multi-delivery lot-sizing dilemma with a partial rework. [6] proposed a lot-sizing model for a production system with back-ordering and outsourcing rework. They concluded that repairable products were not reworked at the firm and sent to a repair store. [6] and [48] dealt with defective economic production quantity models with back-ordered demand and reworked process, respectively.

2.6 Rework in Supply chain

[44] investigated the combined results of defective quality goods, the faulty inspection process, the rework process, and sales return under two-level trade credit. [49] suggested a single-vendor single-buyer inventory model for an imperfect production system with various production set-up and rework.
2.7 Small and Medium-sized Enterprises (SMEs)

SMEs perform an essential role in every economy, both in prosperous improved countries and developing countries. They perform some critical roles like employment creation [50], wealth creation, which enhances the economy capacity. Furthermore, SMEs utilize their restricted sources efficiently and promote entrepreneurial abilities. Investigations carried out by multiple authors, recommends that SMEs contribute significantly to the social-economy and political foundation of promoting and developed countries. [51] have done an empirical study on inventory management of retailers in SMEs in India. In this research, the benefits, barriers and impact of inventory management implementation of SMEs are evaluated. [52] provided a supplier inventory integration model as a multi-stage dynamic program, and in their study, an inventory optimization policy for the supplier is provided in two cases, first, there is no access to external finance. And the second case is that access to external finance through inverse or traditional factors is possible. The importance of having a strategy in a competitive supply chain should not be underestimated, for SMEs the implementation of supply chain management has been difficult. In spite of these challenges, growth resulting from globalization for automotive SMEs, outsourced constructing activities by developed countries because of own uncompetitive high prices, etc., are the evidences that further confirm the need for greater investigation into the problems faced by this section. It is also interesting to note that a diversity of perspectives have been reported on explaining SME organizations.

SCM has achieved a reputation around the globe due to its importance in relation to enhancing marketing performance. Although large enterprises have been capable of performing SCM successfully, it is varied in SMEs as they are faced with various restrictions that prevent the practical implementation of SCM. Variables such as technological barriers, poor coordination
with the supplier’s absence of government support, sources, funds, access to markets and top
management support, issues in information sharing, inventory administration, poor SCM
knowledge, and uneducated workforce all influence the SCM implementation in SMEs [53].

2.8 Two Level Supply Chain in SMEs

The supply chain is vital in business activities, including for small and medium enterprises. The
importance of all parties ranging from suppliers, producers, distributors, retailers, and clients
working together in the manner of commodity production to the distribution of goods to end
customers produces the appearance of the concept of supply chain management [54]. [55] try to
find the supply chain for SMEs. This research is based on investigating the implementation of
SCM by SMEs and the factors affecting the success these enterprises in achieving the balance
between supply chain capacities and customer needs. The research approach was based on a case
study that has been done on five SMEs. [56] did a comparative analysis of the effect of SCM and
information systems (IS) on operating functionality in small and medium enterprises in two
markets of Turkey and Bulgari. Then, he investigated the moderating and inhibiting effects of
SCM-IS related to the inhibitors of the SCM, IS and OPER SME approaches. SMEs tend to aim
in the brief term, merely acting to arising positions and focusing on their incredible survival.
Furthermore, they share less formalized constructions and classified methods while they are most
usually owner-managed, resulting in a management and administration management culture.
SMEs can obtain sales by reaching out to clients immediately and promptly within e-business
when starting new products and by penetrating into markets that might be impassable due to
distribution or infrastructural limitations [57]. [58] studied the inventory model for s SMEs in a
two-level supply chain of several buyers and several vendors for seasonal products. In this
model, it is assumed that every buyer has a warehouse. The location of the buyers is specified and the capacities of the warehouses are limited, and the buyers order different products to the seller with a discount policy. The optimal location of potential vendors is determined based on minimizing the maintenance and ordering costs and also the size of order. [53] analyzed how the influence of SCM on enterprise performance. The SCM strategy is performed with four independent variables, namely sharing information, trust, long-term connection, and collaboration. This investigation proposes to analyze the influence of SCM on corporate performance. The sampling technique utilizes a random sampling technique with a total example of 75 people consisting of small and medium-sized leather shoes in Medan. [54] were measured SCM restrictions in Tanzanian SMEs. A mixed-method method was used in this study. Definitive statistics were then used to maintain the connection between SCM impediments and effective SCM implementation in Tanzanian SMEs. Frequency and percentages were computed to achieve the study results. According to Cscmp.org (2019), “SCM includes the planning and administration of all activities from sourcing raw substances into manufacturing these and delivering them to the consumers. It also involves coordination among the supply chain co-workers such as suppliers, clients, intermediaries, and third-party consumers. Additionally, SCM integrates supply and demand across companies.” [59] discovered that SCM undoubtedly and significantly promotes SMEs to achieve liquidity, working capital and to improve member connections. Notwithstanding the advantages of SCM in institutions, numerous investigations have documented the difficulties that attain in the way of successful SCM implementation. Newly, [60] discovered that firms still encounter problems in information sharing in their supply chain while utilizing blockchains. [61] research on the mining area in Zimbabwe revealed an absence of fundamental and organizational change necessitated to promote sustainable SCM.
implementation. Furthermore, [62] showed that firms in Zimbabwe face SCM restrictions such as an unfriendly market condition, restrictive tax management. A poor regulatory method, a lack of stable government policy, financial limitations, poor procurement, and inventory policies and poor logistics and communication. [63] determined that a shortage of purposes, short-term objectives, a lack of collaboration, poor willingness between the supply chain partners and economic, managerial, and technological limitations to be issued preventing SCM in SMEs in the Gulf countries. [64] discovered that supplier matters influence better SCM in UK aerospace SMEs. [65] published financing limitations to be the dominant problem affecting the supply chain in SMEs.

3 Research gaps and objectives

The review of the literature above illustrates that several topics have frequently been studied in the past, and indeed the number of publications in several individual areas is on the rise. It is, however, clear that there has been less attention from researchers to integrate several topics into a single frame to investigate the rework in the imperfect quality goods in the two-level supply chain for SMEs’ points of view. For instance, the available literature does not support an optimal decision for an optimal inventory model in two-level for SMEs that base various strategies the shipments have been screened and make the decision about imperfect items with considering shortage, new products. To contribute to close the identified research gap and to build a joint two-level supply chain for SMEs with imperfect quality items, rework, new products, shortages, screening, this paper revisits the model of [15]. As underlined before, the study of [15] is one of the perfect models in this research stream that has received favorable attention from researchers. Our study also considers the significant extension of the basic model of [15]. Their study has
regarded as the research [29]; [66]; [67]; [11]; [68] and extends them to take into account an
optimal decision making in the two-level supply chain for large companies. This helps us to
study an optimal inventory model in the two-level supply chain for SMEs under different
assumptions and to investigate the effect of the rework process on the optimal policies of this
work. Specifically speaking, we investigate some scenarios that may occur in real-world supply
chain decision making for SMEs, including kinds of imperfect items, the inspection process,
rework process, decision making in the two-level supply chain in SMEs. We mainly attempt to
address the research question of how policy-makers in SMEs should modify their inventory
decision metrics when intend to rework the imperfect goods in the supply chain into account.

This study develops an optimal inventory model in two level supply chain SMEs with a rework
for new products and price-dependent demand. The main contributions are listed as follows:

1. This study provides an inventory model in a two-level supply chain for SMEs with a single
manufacturer and a single buyer.

2. This paper considered imperfect quality items and reworked, which have performed by the
buyer in SMEs’ supply chain, which is quite near the real world.

3. In our models, two different practical situations are considered: (1) the manufacturer and the
buyer make decision independently about their issues such as inventory level, number of
shipments, end price, order quantity, and so on. Also, they attempt to maximize their profit. (2)
the manufacturer and the buyer make the integrated decision about their issues. Therefore, they
can take a joint maximize profit.

4. One of the novel contributions of the model is price-dependent demand, which is common in a
two-level supply chain for SMEs.
5. Another important novelty in our model’s contribution, which is a significant production’s difficulties in real-world is that the imperfect productions classified into two segments: scrape, reworkable.

6. Finally, we conduct a sensitivity analysis to provide some management insights.

4 Hypotheses and Symbols

In this section, the model's assumptions and symbols are discussed.

4.1 Symbols

The symbols used in the model are described in two parts of the parameters and the decision variables:

Demand Function \( D(p) \)

Scaling Factor Of The Demand Function \( \alpha \)

Index Of Price Elasticity Of The Demand Function \( \beta \)

Vendor’s Set Up Cost \( S \)

Buyer’s Ordering Cost \( A \)

Sales Price Per Unit For The Buyer (Decision Variable) \( p \)

Order Quantity (Decision Variable) \( Q \)

Number Of Shipments (Decision Variable) \( n \)
Backorders (Decision Variable) \( b \)

Vendor’s Inventory Holding Cost \( h_v \)

Buyer’s Inventory Holding Cost \( h_b \)

Rework’s Inventory Holding Cost \( h_{rw} \)

Backordering Cost \( \pi \)

Unit Production Cost \( C \)

Unit Rework Cost \( C' \)

Buyer’s Screening Cost \( C_{ib} \)

Vendor’s Warranty Cost For Defective Units \( C_w \)

Fixed Transportation Cost Per Shipment \( F \)

Unit Variable Cost For Handling Or Receiving an item \( V \)

Rate of defectives, which is uniformly distributed on \( y \)

Mean Rate Of Defectives \( Y \)

Variance Of Defective Rate \( \text{var} \)

Unit Purchase Price \( W(Y) \)

Maximum Unit Purchase Price, which is charged for defect-free shipments \( m \)

Reduction Slope of the Purchase Price \( K \)

Screening Rate of the Buyer \( x_b \)

Production Rate \( R \)

Ratio Of The Market Demand Rate To The Production Rate \( q = D/R \)

Order Cycle \( \rho \)

Buyer’s Expected Profit [$/year] Under Independent Optimization \( ETP_B \)

Vendor’s Expected Profit [$/year] Under Independent Optimization \( ETP_V \)

Expected Total System Profit [$/year] Under Independent Optimization \( ETP_I \)

Expected Joint Total Profit \( EJTP \)

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4.2 Assumptions

The assumptions of the proposed model are as follows:

1. A two-level supply chain for SMEs with a vendor (manufacturer) and a distributor and a new product is presented.

2. The demand rate is a decreasing function of the selling price, \( D(p) = \alpha p^{-\beta}, \alpha > 0, \beta > 1 \) and \( \beta \) is the price flexibility index [69] [70].

3. if there are imperfect quality products in the shipment that are not reworkable, the distributor faces shortages.

4. Each shipment delivered to the distributor includes a rate of imperfect quality products, \( y \) is the rate of imperfect quality products, which included two rates of \( y_1 \) (scrap products) and \( y_2 \) (reworkable imperfect products), \( y = y_1 + y_2 \).

5. The purchase price to be paid by the distributor to the manufacturer (\( W \)) is a decreasing function of the mean rate of the imperfect quality products: \( w(Y) = m - KY, (m > K > 0) \), \( Y = y_1 + y_2 \) assuming that \( W(Y) \) is greater than the cost of production per unit.

6. Each shipment delivered to the distributor is inspected 100% by the distributor at the screening rate of \( x_b \), perfect products are separated from imperfect quality products. In order to avoid shortages during screening, the screening rate is considered to be higher than the demand rate, and the lowest number of perfect products per shipment is equal to the demand at that time and this number is constant and definite.

7. The expected perfect products in each shipment are equal to, which is equal \( Q(1 - y) \) to the demand during the order period of \( T \) [71].
8. The screening process is considered without any error, and the first type error and the second type error has been removed.

9. At the end of the screening, the distributor pays the purchase price of perfect and the reworked products to the producer in one payment. And if any imperfect products without the capability of being reworked are found, these products are returned to the producer by one single shipment.

10. In order to avoid the complexity of the model, it is assumed that the shortages that are caused by the distributor are compensated in the next shipment before the start of the screening, and all these products are considered to be perfect.

5 Material and methods

The proposed model is an inventory model for SMEs with imperfect quality products and backorders. The existence of imperfect quality products in the production system incurs costs and damages; thus, manufacturers and distributors adopt various policies to eliminate these products from the supply chain, one of which is to rework imperfect quality products. In addition to reducing scrap and raw material waste, reworking imperfect quality products can partly compensate for the manufacturer and distributor's losses caused by these items. Imperfect quality products can, at the same time, result in shortage for the distributor, which also imposes the additional cost of lost orders to the distributor. Therefore, reworking the returned products can play a significant role in reducing supply chain costs and help increase profit.

All members of the supply chain in SMEs, including the manufacturer-distributor, adopt preventive policies to reduce the production rate of imperfect quality products because of the
costs these products impose on the production system. The most important of these policies is reworking. In real-world production systems, manufacturing imperfect quality products during the production process of producing new products is considered inevitable. In order to create a real inventory model, the proposed model is expected to increase the total joint profit of all supply chain members and to obtain the optimal amount of order, number of shipments, and the selling price, by taking into account the imperfect quality products as well as the rework procedure required to recycle these items. The goal of this study is to provide an integrated inventory model for a two-level supply chain in SMEs with a manufacturer, a distributor, and manufacturing a new product, assuming that the screening and rework procedures are done by the distributor. A numerical test is carried out to evaluate the efficiency of the model and the profit rate improvement. Finally, a sensitivity analysis of the results is performed to decide whether it is economically worthwhile to rework the imperfect quality products or not.

5.1 Inventory model with allowable shortages

In this section, the inventory model is presented fewer than two conditions of allowable shortages and without shortages for independent and joint optimizations of the supply chain for SMEs. In the unintegrated model of the supply chain SMEs, there is no integration or coordination between members of the supply chain, and all the members of the supply chain make their own pricing decisions independently, thus the manufacturer (seller) and distributor profits will be maximized independently. In this case, the distributor will make sales and order decisions independently in order to maximize his or her own profits. Therefore, the expected profit for the distributor in SMEs in the independent optimization model can be calculated as follows:
\[
ETP_B(p, Q, b) = \left[ p - w(Y) - CI_B - C'(Y_1 + Y_2) \right] \alpha p^{-\beta} - \frac{(A + F)\alpha p^{-\beta}}{Q(1 - Y_1)} \\
\quad - \frac{h_b Q(1 - Y_1)}{2} - \frac{h_{rw} Q(Y_1 + Y_2)}{2} - \frac{(\pi + h_b) b^2}{2Q(1 - Y_1)} + b h_b
\]  
(1)

The manufacturer in the independent optimization model makes decisions in order to maximize his or her profits, as well as reduces costs of production, transportation and operating and acts independently from other members of the supply chain members, so in this case, the profit for the manufacturer can be expressed as follows:

\[
ETP_V(n) = \left[ w(Y) - C - C_w(Y_1 + Y_2) \right] \alpha p^{-\beta} - \frac{S\alpha p^{-\beta}}{nQ(1 - Y_1)} - \frac{h_v Q}{2} [(2 - n)\rho] + n - 1
\]  
(2)

When members of the supply chain (distributor-manufacturer) decide independently, their purpose is to maximize their own profits separately, and the expected benefits of the entire supply chain system for SMEs are calculated by the following equation:

\[
ETP_I = ETP_B(p, Q, b) + ETP_V(n)
\]  
(3)

Here, the inventory models with and without shortages are investigated in the case that the manufacturer and distributor decisions are dependent together. In this case, the joint expected benefit in each unit can be expressed as follows:
\[ EJTP(p, Q, n, b) \]
\[ = \left[ p - CI_B - V - C - (Y_1 - Y_2)(C_w + C') \right] \alpha p^{-\beta} \]
\[ - \frac{\alpha p^{-\beta}}{Q} \left[ A + F + \frac{S}{n} \right] \]
\[ - \frac{Q}{2} \left\{ h_b (1 - Y_1) + h_{rw} (Y_1 + Y_2) - h_v [(2 - n) \rho + n - 1] \right\} \]
\[ - \left( \frac{\pi + h_b}{2} \right) b^2 + 2Q(1 - Y_1) + b h_b \]

The goals are to maximize the Equation (4) and find the optimal amounts for \( p, b, n \) and \( Q \) variables. The optimum amount of \( b \) can be calculated as follows:

\[ b^* = \frac{h_b Q (1 - Y_1)}{\pi + h_b} \] (5)

Equation (5) is given in study [15] and the \( b^* \) value is calculated. By replacing Equation (5) into Equation (4):

\[ EJTP(p, Q, n) = \left[ p - CI_B - V - C - (Y_1 - Y_2)(C_w + C') \right] \alpha p^{-\beta} \]
\[ - \frac{\alpha p^{-\beta}}{Q} \left[ A + F + \frac{S}{n} \right] - \frac{Q}{2} h_b \left\{ (1 - Y_1) + \frac{h_b (1 - Y_1)}{\pi + h_b} \right\} \]
\[ + h_{rw} (Y_1 + Y_2) + h_v [(2 - n) \rho + n - 1] \] (6)

The second-order partial derivative of Eq. (6) with respect to \( Q \) is:

\[ \frac{\partial^2 EJTP(p, Q, n)}{\partial Q^2} = \frac{2 \alpha p^{-\beta} (A + F + \frac{S}{n})}{Q^3} < 0 \] (7)
So, the optimum amount of $Q$ for fixed values of $n, p$ is calculated as follows:

$$\frac{\partial EJTP(p, Q, n)}{\partial Q} = 0$$

$$Q^* = \sqrt{\frac{2 \alpha p^{-\beta} \left[ A + F + \frac{S}{n} \right]}{H(n)}}$$  \hspace{1cm} (8)$$

And, for more simplicity, is can be expressed as:

$$H(n) = h_b \left[ (1 - Y_1) + \frac{h_b (1 - Y_1)}{\pi + h_b} \right] + h_{rw} (Y_1 + Y_2) + h_v [(2 - n) \rho + n - 1]$$  \hspace{1cm} (9)$$

The expected joint total profit as a function of $p$ and $n$ is calculated by replacing the Equation 9 into the Equation 6:

$$EJTP(p, n) = [p - C I_B - V - C - (Y_1 + Y_2)(C_w + C')] \alpha p^{-\beta}$$

$$- \sqrt{2 \alpha p^{-\beta} \left[ A + F + \frac{S}{n} \right] H(n)}$$  \hspace{1cm} (10)$$

For a specific amount of $p$, $EJTP(p, n)$ can be maximized by minimizing the following expression:

$$EJTP' = 2 \alpha p^{-\beta} \left[ A + F + \frac{S}{n} \right] H(n)$$  \hspace{1cm} (11)$$

First, it is assumed that $n$ is a continuous variable [15] and $n$ value can be calculated by solving Equation (2):
Due to the complexity of the equations, the MATLAB software is used to solve them. The optimum amount of p is calculated using equation (10).

5.2 Inventory model in joint optimization without shortages

In this section, the integration of inventory model without shortages is discussed. The expected joint total profit without shortages can be expressed as follows:

\[
EJTP_{WS}(p, Q, n) = [p - CI_B - V - C - (Y_1 - Y_2)(C_w + C') ]ap^{-\beta}
\]

\[
- \frac{\alpha p^{-\beta}}{Q} \left[ A + F + \frac{S}{n} \right] - \frac{Q}{2} \left\{ h_b(1 - Y_1) + h_{rw}(Y_1 + Y_2) + h_v[(2 - n)\rho + n - 1] \right\}
\]

And the optimal amount of Q can be calculated as:

\[
\frac{\partial EJTP}{\partial Q} = 0
\]

\[
Q^* = \sqrt{\frac{2\alpha p^{-\beta} \left[ A + F + \frac{S}{n} \right]}{H(n)}}
\]

By placing the Equation (14) in Equation (13):
\[ EJT_PW_S(p, n) = [p - CI_B - V - C - (Y_1 + Y_2)(C_w + C')] \alpha p^{-\beta} \]

\[- \sqrt{2\alpha p^{-\beta} \left[ A + F + \frac{S}{n} \right] H(n)} \]

462  \( H(n) \) is defined as follows to simplify the equations:

\[ H(n) = h_b(1 - Y_1) + h_{rw}(Y_1 + Y_2) + h_v[(2 - n)\rho + n - 1] \]

463  For a specific amount of \( p \), \( EJT_PW_S(p, n) \) is maximized after the following expression gets its own optimal value:

\[ EJT'_P = 2\alpha p^{-\beta} \left[ A + F + \frac{S}{n} \right] H(n) \]

465  \( n \) value can be calculated as follows:

\[ \frac{\partial EJT_PW_S'}{\partial n} = 0 \]

\[ n^* = \frac{S[h_b(1 - Y_1) + h_{rw}(Y_1 + Y_2) + 2\rho h_v + h_v]}{\sqrt{\rho h_v(A + F)(\rho - 1)}} \]

466  The value of \( p \) can be calculated using the Equation (15):

\[ \frac{\partial EJT_P(p, n)}{\partial p} = 0 \]
\[
\frac{\partial EJTP_{WS}(p, n)}{\partial p} = \alpha p^{-\beta} - \alpha \beta p^{-\beta-1} [p - CI_B - V - C - (Y_1 + Y_2)(C_w + C')] \\
- \frac{\beta p^{-\beta/2 - 1}}{2} \sqrt{2\alpha \left[ A + F + \frac{S}{n} \right] H(n)}
\]

(19)

MATLAB software can be used to do the work and solve the equations.

6 Solution algorithm

In this section, an algorithm is introduced that can be used to find the optimum values for the inventory model without shortages. This algorithm is based on the algorithms provided in the previous studies Rad and khoshalhan [15, 68, 72]. Chen and Kang [68] utilize a similar solution procedure for carrying out the price-negotiation system based on the difference of total profits of three models. Rad and khoshalhan [15] attempt to use a solution algorithm to calculate a optimal solution for one of their models. So to speak, a solution procedure can be summarized as follows:

1. Computing \( n \) from Equation (11) using MATLAB software
2. Computing \( p \) based on Equation (10)
3. Computing \( Q \) using Equation (7)
4. Computing \( b \) based on Equation (4)
5. Calculate the amount of \( EJTP(p^n, Q^n, b^n, n) \) using Equation (4)
6. \( n = n + 1 \) for steps (2) to (5) is calculated too, So \( EJTP(p^n, Q^n, b^n, n) \) (Goyal, 1977)
7. If \( EJTP(p^n, Q^n, b^n, n) \geq EJTP(p^{(n-1)}, Q^{(n-1)}, b^{(n-1)}, n) \) the step (6) is repeated again, and the optimal values are equal to:

\[
EJTP(p^*, Q^*, b^*, n^*) \geq EJTP(p^{(n-1)}, Q^{(n-1)}, b^{(n-1)}, n-1)
\]

7 Results

The numerical example presented in this section is similar to the studies conducted by Rad and khoshalhan and Chen and Kang [15, 68].

\[
\begin{align*}
A &= 100 \\
F &= 100 \\
S &= 1200 \\
C &= 2.5 \\
V &= 1 \\
CW &= 11 \\
CIb &= 0.1 \\
\pi &= 1.5 \\
P &= 0.8 \\
hv &= 0.25 \\
M &= 9 \\
K &= 20 \\
\alpha &= 30000 \\
B &= 125 \\
hrw &= 0.96 \\
D(p) &= 30000p - 1.25 \\
W(Y) &= 9 - 20Y
\end{align*}
\]

\[PI = \frac{EJTP - ETP_i}{ETP_i} \times 100 \] (20)

\[\Delta\% = \frac{EJTP_{WS} - EJTP}{EJTP} \times 100 \] (21)

7.1.1 PI is the percentage of profit improvements in the supply chain with integration for SMEs Rad and khoshalhan [15]. Where \( EJTP \) is the expected joint total profit, while the \( ETPi \) (expected total profit improvement) rate without shortage is expressed by \( \Delta\% \). Based on the results, it can be concluded that the optimal \( EJTP \) improvement rate is
9.91%, which is higher than the (5.5%) which is achieved when the supply chain members act independently.

### 7.2 Theoretical analysis

Based on the equations and results, the following findings may be pointed out:

1. Reduction in the selling prices leads to reduction in the optimal size of the orders decreases, (Equation (8));

2. Reduction in the optimal size of the shortage leads to reduction of the selling prices (Equation (8));

3. When the distributor increases the cost of shortage, the value of $H(n)$, selling prices, and total cost increase. This, in turn, reduce the optimal order size and shortage (Equation (5) and (8)). These changes are more severe when demand is sensitive to price changes than when demand is fixed.

4. The average impact of imperfect quality products $Y$ on the optimal value of shortage $b^*$ can be expressed as:

#### 7.2.1 Independent optimization for supply chain members in SMEs

The optimal value of shortage is considered as the denominator of the fraction and the value of $b^*$ increases when the value of $Y$ increases. When demand is fixed, the optimal amount of shortage decreases when the value of $Y$ increases. Contrastingly, when demand is sensitive to...
price changes, the selling prices and the optimal amount of shortage may either increase or
decrease depending on the conditions.

7.2.2 Joint optimization for supply chain members in SMEs

The optimal value of the shortage is calculated through Equation (5) and can be analyzed as
described in the previous section. It can be concluded that the optimal value of the shortage may
increase or decrease when the value of \( Y \) increasing.

7.3 Sensitivity analysis for flexibility index (\( \beta \))

As can be seen in the results listed in Table 1, in cases where no shortage are allowed and for
independent and joint optimizations, the values of \( PI \) (percentage of improvement in the \( EJTP \))
and \( i \) (percentage of improvement in the \( ETPI \) without shortage) increase when the value of
\( \beta \) increases. It can be concluded that unallowable shortage and the presence of coordination in
the supply chain in SMEs with the possibility of doing rework on new products play an
important role in solidifying the firms' market position and boost overall customer satisfaction,
especially when market demand in SMEs is highly sensitive to the price. This case can be more
common in competitive markets in SMEs where new products have little discrepancy and
customers focus on the price. Under this condition, the integration of supply chains in SMEs and
the rework of new products that help to eliminate shortage can be used to improve the position of
organizations and enterprises in the market. As can be seen in Table 1, and confirmed by prior
research, the more sensitive the demand is to the price, the lower the optimal selling prices will
be.
### 7.4 Sensitivity analysis of the effects of $\beta$ changes on $\Delta \%$

Figure (1) indicates that increasing the value of $\beta$ when all imperfect quality products are reworkable and no shortage occurs in the system, the $EJTP$ for SMEs will be much greater than when the system suffers from shortage because of non-reworkable products. This means that, under the condition that all imperfect quality products are reworkable, doing so will lead to a higher total profit. It is also evident that in markets where demand is sensitive to price changes, when the distributor manages to rework the returned products and eliminate all shortages, then the total profit and optimal value of orders increase in proportion with the value $\beta$. Based on these results, it can be concluded that the equations proposed by previous studies are not suitable in cases where imperfect quality products can be reworked, especially when the market demand is highly sensitive to the price.

#### 7.4.1 Figure (1)

Figure (1) shows that the impact of changes in the values $\beta$ on $\Delta\%$. The improvement rate of $ETPI$ (expected total system profit) without shortage is expressed by $\Delta\%$, while $\beta$ represents the index of price elasticity of the demand function.

### 7.5 Sensitivity analysis of the effects of $\beta$ changes on $PI$

As can be seen in Figure (2), when the supply chain members optimize their operations independently, allowable rework on imperfect quality products will result in increased total profit. The results show that the possibility of rework in independent and joint profit models in SMEs leads to an increase in total profits. The increase of $PI$ in the model proposed in this study
is greater than same value in the similar model proposed by [15]. Product rework is allowable in our model, as well. It can be seen that doing rework on imperfect products in both cases of independent and joint optimization of inventory models leads to an increase in the total supply chain profit of SMEs. The $EJTP$ of the supply chain reaches its highest value under the conditions that all imperfect quality products in the system can be reworked and the system experiences no shortage. In this case, in markets where demand is sensitive to price changes, increasing the value of $\beta$ can lead to an increase in the optimal order size.

7.5.1 Figure (1)

It can be concluded from Figure (2) that the impact of changes in the value of $\beta$ on $PI$, where $PI$ is the percentage of profit improvements in the integrated supply chain of the SMEs and $\beta$ represents the index of price elasticity of the demand function.

7.6 Sensitivity analysis for expected rates of imperfect quality products $Y = E(Y)$

In this section, the imperfect quality products rates are the same as the numerical example used in the research done by Rad and khoshalhan [15]. As described in the previous sections, the rate of imperfect quality products in the equations of this study is $Y = Y_1 + Y_2$ and the values of $Y_1$ and $Y_2$ are shown in table 2. The two variables $Y_1$ and $Y_2$ can be used to determine the $EJTP$ and the values of decision variables. As can be seen, an increase in the value of $Y_1$ (average rate of scrap products) and a decrease in the value of $Y_2$ (average rate of reworkable imperfect products) may lead to an increase in the expected joint total profit of supply chain system, as it is assumed that shortage can occur in the model if scrap products be produced in the system, so an
increase in $Y_1$ results in an increase in $b$. Also, it is clear that, due to the model assumptions in addition to the fact that scrap quality products are costly for the system, increasing the amount of scrap per shipment leads to the reduction of the purchase price. Based on the results of Table 2, the increase in the average rate of re-work able imperfect quality products, the expected joint total profit in the case of unallowable shortage, $\Delta\%$ increases. Additionally, the impact of the average rate of re-workable imperfect quality products on decision variables can be different.

Integration in the supply chain always leads to a decrease in the average price of imperfect quality products which, in turn, leads to an increase in total profit. When the supply chain members i.e. the manufacturer and buyer optimize their inventory models independently, or when they receive shipments which contain a high percentage of imperfect quality products, if the manufacturer quotes a low selling price, it can be very profitable business for the distributor. Prior research have also arrived at this conclusion [66, 73].

8 Discussion and conclusions

This paper aspired to present a supply chain inventory model for SMEs consisting of one distributor and one manufacturer. Product shortage, backorders, rework and the demand sensitivity to price are considered in the proposed model which is based on such variables as the expected total system profit ($ETPI$), optimal selling prices, number of orders, amount of shortage, calculation of the number of shipments, and both joint and independent inventory model optimization by the distributor and manufacturer. Using numerical tests and sensitivity analysis, the impact of various factors on the $ETPI$ was also investigated and the results indicate that the optimal scenario is the one in which no shortage occurs, doing rework on faulty new products is permitted by the distributor, especially under the condition that demand sensitivity to
price, backorder costs, and average rate of imperfect quality products are high. The results also suggest that the integration of the manufacturer and distributor's decisions, with the possibility of rework by the distributor in the SMEs, may significantly improve the total profit.

As evident by the results, supply chain integration in SMEs, especially when demand is sensitive to price changes, seems able to greatly increase the total profit recorded by supply chain members.

When the supply chain members in SMEs act independently as regards optimization, distributors often tend to purchase from manufacturers that generate a high rate of imperfect products; whereas manufacturers tend to offer a lower purchase price based on the average rate of imperfect quality products per total production.

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10 List of Captions:

8. Figure 1: Δ% changes
9. Figure 2: PI changes
10. Table 1: Sensitive analysis of average of imperfect products rate
11. Table 2: Joint optimization & Independent optimization (non-integrated)

11 figures and tables

Figure 1 Δ% changes

Figure 2 PI changes
| β   | Joint optimization | PI | Δ% |
|-----|-------------------|----|----|
|     | Model with shortage |     |     |
| p   | Q     | b  | n  | EJTP | p  | Q  | n  | EJTP ws |
| 1.05 | 89.2  | 1081 | 390.09 | 13 | 227520 | 87.1 | 662 | 13 | 227700 | 2.43 | 0.079 |
| 1.10 | 46.05 | 1390 | 501.53 | 13 | 185010 | 44.89 | 1002 | 13 | 185370 | 4.27 | 0.19 |
| 1.25 | 20.73 | 1718 | 619.71 | 13 | 111310 | 20.18 | 1651 | 13 | 111920 | 9.91 | 0.54 |
| 1.50 | 12.98 | 1671 | 602.65 | 13 | 51260  | 12.7  | 2205 | 13 | 51891  | 22.31 | 1.23 |
| 2.00 | 8.38  | 1363 | 491.75 | 13 | 16972  | 8.16  | 2906 | 13 | 17321  | 64.87 | 2.05 |
|     | No shortage model |     |     |

| β   | Independent optimization (non-integrated) | PI | Δ% |
|-----|------------------------------------------|----|----|
| p   | Q     | b  | n  | ETPI | p  | Q  | n  | ETPI |
| 1.05 | 205.17 | 526.24 | 193.704 | 11 | 222120 | 1.10 | 106.97 | 790.62 | 291.01 | 11 | 177420 |
| 1.25 | 47.7  | 1309.9 | 482.15 | 11 | 101270 | 1.50 | 29.15 | 1781.7 | 655.82 | 11 | 45381 |
| 2.00 | 19.63 | 2281.3 | 839.71 | 11 | 10294 |

**Table 1** (Joint optimization & Independent optimization (non-integrated))

| β   |    | p  | Q    | b  | n  | EJTP |
|-----|----|----|------|----|----|------|
|     |    |    |      |    |    |      |
| Y₁  | 0  | 29.72 | 1338 | 0  | 12 | 101930 |
| Y₂  | 0.17 | 29.73 | 1318 | 42.425 | 14 | 101820 |
| Y₃  | 0.17 × 0.25 | 29.74 | 1299 | 39.439 | 13 | 101710 |
| Y₄  | 0.17 × 0.5 | 29.86 | 1211 | 15.453 | 12 | 101590 |

**Table 2** (Sensitive analysis of average of imperfect products rate)
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