Abstract. Taylor-Wiles type lifting theorems allow one to deduce that if \( \rho \) is a "sufficiently nice" \( l \)-adic representation of the absolute Galois group of a number field whose semi-simplified reduction modulo \( l \), denoted \( \overline{\rho} \), comes from an automorphic representation then so does \( \rho \). The recent lifting theorems of Barnet-Lamb-Gee-Geraghty-Taylor impose a technical condition, called \( m \)-big, upon the residual representation \( \overline{\rho} \). Snowden-Wiles proved that for a sufficiently irreducible compatible system of Galois representations, the residual images are big at a set of places of Dirichlet density 1. We demonstrate the analogous result in the \( m \)-big setting using a mild generalization of their argument.

Résumé

\( m \)-bigness dans les systèmes compatibles. Les Théorèmes de type Taylor-Wiles indiquent qu’une représentation \( l \)-adique du groupe Galois d’un corps de nombre est automorphe si sa réduction modulo \( l \) est automorphe et si cette représentation satisfait de bonnes propriétés. Une condition technique mais cruciale qui apparaît dans le travail récent de Barnet-Lamb-Gee-Geraghty-Taylor est que la représentation résiduelle soit \( m \)-big. Snowden-Wiles ont démontré que pour un système compatible de représentations suffisamment irréductibles, les images résiduelles sont alors big pour un ensemble de Dirichlet densité 1. Nous démontrons ici un résultat analogue dans le cadre de \( m \)-big par une généralisation de la démonstration de Snowden-Wiles.

1. Introduction

We begin by recalling the condition \( m \)-big (cf. [2, Definition 7.2]). Let \( m \) be a positive integer, let \( l \) be a rational prime, let \( k \) be a finite field of characteristic \( l \), let \( V \) be a finite dimensional \( k \)-vector space and let \( G \subset GL(V) \) be a subgroup. For \( g \in GL(V) \) and \( \alpha \in k \), we shall write \( h_g \) for the characteristic polynomial of \( g \) and \( V_{g,\alpha} \) for the \( \alpha \)-generalized eigenspace of \( g \).

Definition 1.1. The subgroup \( G \) is said to be \( m \)-big if it satisfies the following properties.

1. \( H^1(G, \text{ad}^r V) = 0 \)
2. For all irreducible \( G \)-submodules \( W \) of \( \text{ad}^r V \), there exists \( g \in G, \alpha \in k \) and \( f \in W \) such that:
   - The composite \( V_{g,\alpha} \hookrightarrow V \xrightarrow{f} V \rightarrow V_{g,\alpha} \) is non-zero.
   - \( \alpha \) is a simple root of \( h_g \).
   - \( \beta \in \mathbb{F} \) is another root of \( h_g \) then \( \alpha^m \neq \beta^m \).

Remark 1.2. The condition big appearing in [3] corresponds here to the condition 1-big.
Our main result is the following:

**Theorem 1.3.** Let $F$ be a number field, let $E$ be a Galois extension of $Q$, let $L$ be a full set of places of $E$ and for each $w \in L$, let $\rho_w : \text{Gal}(Q/F) \to \text{GL}_n(E_w)$ be a continuous representation and let $\Delta_w \subset \text{Gal}(Q/F)$ be a normal open subgroup. Assume that the following properties are satisfied.

- The $\rho_w$ form a compatible system of representations.
- $\rho_w$ is absolutely irreducible when restricted to any open subgroup of $\text{Gal}(Q/F)$ for all $w \in L$.
- $\text{Gal}(Q/F)/\Delta_w$ is cyclic of order prime to $l$ where $l$ denotes the residual characteristic of $w$, for all $w \in L$.
- $[\text{Gal}(Q/F) : \Delta_w] \to \infty$ as $w \to \infty$.

Then there exists a set of places $P$ of $Q$ of Dirichlet density $1/[E : Q]$, all of which split completely in $E$, such that, for all $w \in L$ lying above a place $l \in P$:

i) $\overline{\rho}_w(\Delta_w)$ is an $m$-big subgroup of $\text{GL}_n(F_l)$.

ii) $[\text{ker ad } \overline{\rho}_w : \Delta_w \cap \text{ker ad } \overline{\rho}_w] > m$.

Here, as usual, $\overline{\rho}_w$ denotes the semi-simplified reduction modulo $l$ of $\rho_w$. For the definition of a compatible system and a full set of places, see Section 6.

**Remark 1.4.** The first part of the theorem is a mild generalization, from the setting of bigness to $m$-bigness, of the main result of Snowden-Wiles [3]. The result shall be proved by considering their arguments in the $m$-bigness setting combined with a slight strengthening of [3, Proposition 4.1] by Proposition 3.1.

The second part of the theorem proves another technical result required for the application of automorphy lifting theorems (see [1]). The proof of this result uses an argument of Barnet-Lamb-Gee-Geraghty-Taylor that originally appeared in [1].

The format of this article mirrors that of Snowden-Wiles [3]. We content ourselves here to remark upon the minor changes to [3] that are needed to obtain the above result.

1.1. **Notation.** Our notation is as in Snowden-Wiles [3]. More specifically, unless explicitly mentioned otherwise, we adhere to the following conventions. Reductive algebraic groups are assumed connected. A semi-simple algebraic group $G$ defined over a field $k$ is simply connected if the root datum of $G_k$ is simply connected. If $S$ is a scheme, then a group scheme $G/S$ is semi-simple if it is smooth, affine and its geometric fibers are semi-simple connected algebraic groups.

**Acknowledgements**

I wish to thank Michael Harris for his continual support and direction. I would also like to thank the referee for their helpful comments.

2. **Elementary properties of $m$-bigness**

In [3, §2], a series of results concerning elementary properties of bigness are demonstrated. We remark that the arguments appearing there trivially generalize to give the following results on $m$-bigness.

**Proposition 2.1.** Let $H$ be a normal subgroup of $G$. If $H$ satisfies the properties $(B2)$, $(B3)$ and $(B4)$ then $G$ does as well. In particular, if $H$ is $m$-big and the index $[G : H]$ is prime to $l$ then $G$ is $m$-big.

**Proposition 2.2.** The group $G$ is $m$-big if and only if $k^xG$ is $m$-big where $k^x$ denotes the group of scalar matrices in $\text{GL}(V)$.

**Proposition 2.3.** Let $k'/k$ be a finite extension, let $V' = V \otimes_k k'$ and let $G$ be an $m$-big subgroup of $\text{GL}(V)$. Then $G$ is also an $m$-big subgroup of $\text{GL}(V')$. 
3. Highly regular elements of semi-simple groups

We recall the norm utilized by Snowden-Wiles [3 §3.2]. Let $k$ be a field, let $G/k$ be a reductive algebraic group and let $T_\mathcal{F}$ be a maximal torus of $G \times_k \overline{k}$. For $\lambda \in X^*(T_\mathcal{F})$ a weight, one defines $||\lambda|| \in \overline{k}$ to be the maximal value of $|\langle \lambda, \alpha \rangle|$ as $\alpha$ runs through the roots of $G \times_k \overline{k}$ with respect to $T_\mathcal{F}$. For $V$ a representation of $G$, one defines $||V||$ to be the maximal value of $||\lambda||$ where $\lambda$ runs through the weights appearing in $V \otimes_k \overline{k}$.

The following result is a slight strengthening of [3, Proposition 4.1].

**Proposition 3.1.** Let $k$ be a finite field of cardinality $q$, let $G/k$ be a semi-simple algebraic group, let $T$ be a maximal torus of $G$ defined over $k$, let $m$ and $n$ be positive integers and assume that $q$ is large compared to $\dim G$, $n$ and $m$. Then, there exists an element $g \in T(k)$ for which the map

$$\{\lambda \in X^*(T_\mathcal{F}) : ||\lambda|| < n\} \to \overline{k}^\times, \quad \lambda \mapsto \lambda(g)^m$$

is injective.

**Proof.** The proof shall follow that of [3, Proposition 4.1] with the difference that we are considering here characters of the form $\lambda^m$ instead of $\lambda$.

To begin let $S := \{\lambda \in X^*(T_\mathcal{F}) : \lambda \neq 1, \quad ||\lambda|| < 2n\}$. We claim that

$$T(k) \not\subset \bigcup_{\lambda \in S} \ker \lambda^m$$

This is equivalent to the statement

$$T(k) \not\subset \bigcup_{\lambda \in S} \ker \lambda^m \cap T(k)$$

The later statement shall be proved by considering the cardinality of the two sides. Firstly, by [3, Lemma 4.2], $|T(k)| \geq (q - 1)^r$ where $r$ denotes the rank of $T$.

Consider now the right hand side. We remark that for $\lambda \in X^*(T_\mathcal{F})$,

$$|\ker \lambda^m \cap T(k)| \leq R_{m,q}|\ker \lambda \cap T(k)|$$

where $R_{m,q}$ denotes the cardinality of the kernel of the map

$$k^\times \to k^\times, \quad k \mapsto k^m$$

Furthermore, we can ensure that $R_{m,q}/q$ is as small as desired simply by considering $q$ sufficiently large with respect to $m$. We can now bound the cardinality of the right hand side by

$$NR_{m,q}M$$

where the terms are defined as follows.

- $N$ is equal to the cardinality of $S$, which by [3 Lemma 4.3] is bounded in terms of $\dim G$ and $n$.
- $M$ is equal to the maximum cardinality of $\ker \lambda \cap T(k)$ for $\lambda \in S$, which by [3 Lemma 4.4] is bounded by $C(q+1)^{r-1}$ for some constant $C$ depending only upon $\dim G$ and $n$.

Thus for $q$ sufficiently large with respect to $\dim G$, $n$ and $m$, the cardinality of the right hand side is strictly less than that of the cardinality of the left hand side and the claim follows.

As such we can choose a $g \in T(k)$ such that $g \not\in \ker \lambda^m$ for all $\lambda \in S$. Then, for all $\lambda, \lambda' \in X^*(T_\mathcal{F})$ such that $\lambda \neq \lambda', ||\lambda|| < n$ and $||\lambda'|| < n$, we have that $\lambda - \lambda' \in S$ and it follows that $\lambda(g)^m \neq \lambda'(g)^m$. $\square$
4. \textit{m}-bigness for algebraic representations

We show here that [3, Proposition 5.1] naturally generalizes to the setting of \textit{m}-bigness.

\textbf{Proposition 4.1.} Let \( m \) be a positive integer, let \( k \) be a finite field, let \( G/k \) be a reductive algebraic group and let \( \rho \) be an absolutely irreducible representation of \( G \) on a \( k \)-vector space \( V \). Assume that the characteristic of \( k \) is large compared to \( m \), \( \dim V \) and \( ||V|| \). Then \( \rho(G(k)) \) is an \( m \)-big subgroup of \( GL(V) \).

\textit{Proof.} Firstly, we note that by [3, Proposition 5.1] the conditions (B1), (B2) and (B3) are satisfied. Thus, it only remains to check the condition (B4) (in the \( m \)-bigness setting). The proof is almost identical to the 1-bigness case (cf. [3, Proposition 4.1]); the sole difference comes from appealing to Proposition 3.1 in lieu of [3, Proposition 4.1].

More specifically, as in [3, Proposition 5.1], one begins by reducing to the case where \( G \) is semi-simple, simply connected and the kernel of \( \rho \) is finite. Choose a Borel \( B \) of \( G \) defined over \( k \); this is possible as every reductive group scheme defined over a finite field is quasi-split. Let \( T \) be a maximal torus of \( B \) and let \( U \) be the unipotent radical of \( B \). The representation \( V_k = V \otimes_k k \) decomposes via its weights:

\[ V_k = \bigoplus_{\lambda \in S} V_{\lambda} \]

where \( S \) denotes the set of weights of \((G_k, T_k)\). By Proposition 3.1, we can find a \( g \in T(k) \) such that

\[ \lambda(g)^m \neq \lambda'(g)^m \]

for all distinct \( \lambda, \lambda' \in S \).

We remark that (ignoring multiplicity) the eigenvalues of \( g \) in \( V_k \) are equal to \( \{ \lambda(g) : \lambda \in S \} \). It follows that the generalized \( g \)-eigenspaces are equal to the weight spaces:

\[ V_{\lambda, g} = V_{\lambda} \]

for all \( \lambda \in S \).

Let \( \lambda_0 \) be the highest weight space (with respect to \( B \)) and let \( V_{\lambda_0} := V_{\lambda_0} \) be the corresponding highest weight space. In fact \( V_{\lambda_0} = V \otimes_k k \) and as such \( \lambda_0(g) \in k \). By [3, Proposition 3.7], \( V_{\lambda_0} \) is 1-dimensional. That is, \( \lambda_0(g) \) is a simple root of \( h_g \), the characteristic polynomial of \( g \). Furthermore, by the properties of \( g \), the \( m \)-th powers of the roots of \( h_g \) are distinct.

Finally it is shown in the proof of [3, Proposition 5.1] that for each irreducible \( G \)-submodule \( W \) of \( \text{ad} V \), there exists a \( f \in W \) such that the composite

\[ V_{g, \lambda_0} \hookrightarrow V \xrightarrow{f} V \xrightarrow{g, \lambda_0} \]

is non-zero. \hfill \Box

5. \textit{m}-bigness for nearly hyperspecial groups

Let \( l \) be a rational prime, let \( K/\mathbb{Q}_l \) be a finite field extension, let \( O_K \) be the ring of integers and let \( k \) be the residue field. For \( G \) an algebraic group over \( K \), we define the following \( K \)-algebraic groups.

- \( G^0 \) : the connected identity component.
- \( G^\text{ad} \) : the adjoint algebraic group, which is the quotient of \( G^0 \) by its radical.
- \( G^\text{sc} \) : the simply connected algebraic group cover of \( G^\text{ad} \).

We have the natural maps:

\[ G \xrightarrow{\sigma} G^\text{ad} \xrightarrow{\tau} G^\text{sc} \]

Following Snowden-Wiles [3], we shall call a subgroup \( \Gamma \subset G(K) \) nearly hyperspecial if \( \tau^{-1}(\sigma(\Gamma)) \) is a hyperspecial subgroup of \( G^\text{sc}(K) \).
Proposition 5.1. Let $m$ be a positive integer, let $\Gamma$ be a profinite group and let $\Delta \subset \Gamma$ be an open normal subgroup. Let $\rho: \Gamma \to GL_n(K)$ be a continuous representation and let $G$ be the Zariski closure of its image. Assume that the following properties are satisfied.

- The characteristic $l$ of $k$ is large compared to $n$ and $m$.
- The restriction of $\rho$ to any open subgroup of $\Gamma$ is absolutely irreducible.
- The index of $G^\circ$ in $G$ is small compared to $l$.
- The subgroup $\rho(\Gamma) \cap G^\circ(K)$ of $G^\circ$ is nearly hyperspecial.
- $\Gamma/\Delta$ is cyclic of order prime to $l$.

Then the following holds.

- $\overline{\rho}(\Delta)$ is an $m$-big subgroup of $GL_n(k)$.
- There exists a constant $C$ depending only upon $n$ such that
  $$\frac{\ker \text{ad} \overline{\rho}: \Delta \cap \ker \text{ad} \overline{\rho}}{[\Gamma : \Delta]} > \frac{1}{C}$$

Proof. Let us remark that the first statement is proved in almost the same way as the proof of [3 Proposition 6.1]. There are two minor differences, firstly we appeal here to Proposition 2.1 instead of [3 Proposition 5.1] and secondly we use an argument of Barnet-Lamb-Gee-Geraghty-Taylor to deduce the $m$-bigness of $\overline{\rho}(\Delta)$ instead of $\overline{\rho}(\Gamma)$. The proof of the second statement is due to Barnet-Lamb-Gee-Geraghty-Taylor and originally appeared in [1].

Let $\Gamma^\circ = \rho^{-1}(G^\circ)$ and let $\Delta^\circ = \Delta \cap \Gamma^\circ$. Then $\overline{\rho}(\Delta^\circ)$ is a normal subgroup of $\overline{\rho}(\Delta)$ and its index divides $[G : G^\circ][\Gamma : \Delta]$, which, by assumption, is prime to $l$ (recall $l$ is sufficiently large with respect to $[G : G^\circ]$). Thus, by Proposition 2.1, to prove that $\overline{\rho}(\Delta)$ is $m$-big it suffices to prove that $\overline{\rho}(\Delta^\circ)$ is $m$-big. Similarly, to prove the second part of the theorem it clearly suffices to prove the analogous statement for $\Gamma^\circ$ and $\Delta^\circ$. As such, we can now assume that $G = G^\circ$.

Let $V = K^n$ be the representation space of $\rho$. By [3 Lemma 6.3], we can find the following.

- A $\Gamma$-stable lattice $\Lambda$ in $V$.
- A semi-simple group scheme $\tilde{G}/\mathcal{O}_K$ whose generic fiber is equal to $G^{sc}$.
- A representation $r: \tilde{G} \to GL(\Lambda)$ which induces the natural map $G^{sc} \to G$ on the generic fiber.

These objects can be chosen such that

- $\mathcal{O}_K^{\times} \cdot r(\tilde{G}(\mathcal{O}_K))$ is an open normal subgroup of $\mathcal{O}_K^{\times} \cdot \rho(\Gamma)$, whose index can be bounded by a constant $C$ defined in terms of $n$.

Furthermore, the generic fiber of $r$ is necessarily an absolutely irreducible representation of $\tilde{G}_K$ on $V$.

By [3 Proposition 3.5], $\Lambda \otimes_{\mathcal{O}_K} k$ is an absolutely irreducible representation of $\tilde{G} \times_{\mathcal{O}_K} k$ and its norm is bounded in terms of $n$. Now $\tilde{G} \times_{\mathcal{O}_K} k$ is a semi-simple, simply connected, algebraic group and hence a finite product of simple, simply connected, $k$-algebraic groups. As $l > 4$, we have that $\tilde{G}(k)$ is perfect (cf. [4]). It follows, as $\Delta$ is a normal subgroup of $\Gamma$ whose quotient is abelian, that we have the following chain of normal subgroups.

$$k^{x}r(\tilde{G}(k)) \leq k^{x}\overline{\rho}(\Delta) \leq k^{x}\overline{\rho}(\Gamma)$$

Furthermore, $[k^{x}\overline{\rho}(\Gamma) : k^{x}r(\tilde{G}(k))] < C$. The second part of the theorem is now immediate.

The first part of the theorem is proved as follows. Proposition 2.1 implies that $r(\tilde{G}(k))$ is $m$-big. Then, Proposition 2.2 and Proposition 2.1 allow one to deduce that $\overline{\rho}(\Delta)$ is $m$-big. 

□
6. m-Bigness for Compatible Systems

**Definition 6.1.** A group with Frobenii is a triple \( (\Gamma, \mathcal{P}, \{\mathcal{F}_\alpha\}) \) where \( \Gamma \) is a profinite group, \( \mathcal{P} \) is an index set and \( \{\mathcal{F}_\alpha\} \) is a dense set of elements of \( \Gamma \). The \( \mathcal{F}_\alpha \) are called the Frobenii of the group.

**Remark 6.2.** If \( F \) is a number field then the corresponding global Galois group \( \text{Gal}(\overline{\mathbb{Q}}/F) \) is naturally a group with Frobenii.

**Definition 6.3.** A compatible system of representations (with coefficients in a number field \( E \)) is a triple \( (L, \mathcal{X}, \{\rho_\lambda\}) \) where \( L \) is a set of places of \( E \), \( \mathcal{X} \subset \mathcal{P} \times L \) is a subset and each \( \rho_\lambda : \Gamma \to GL_n(E_\lambda) \) is a continuous representation, such that the following conditions are satisfied.

- For all \( \alpha \in \mathcal{P} \), the set \( \{\lambda \in L : (\alpha, \lambda) \not\in \mathcal{X}\} \) is finite.
- For all finite sets of places \( \lambda_1, \ldots, \lambda_k \in L \), the set \( \cap_{i=1}^k \{\mathcal{F}_\alpha : (\alpha, \lambda_i) \in \mathcal{X}\} \) is dense in \( \Gamma \).
- For all \( \alpha \in \mathcal{P} \), \( \lambda \in \mathcal{X} \), the characteristic polynomial of \( \rho_\lambda(\mathcal{F}_\alpha) \) has coefficients in \( E \) and depends only upon \( \alpha \).

The set of places \( L \) is said to be full if there exists a set \( L' \) of rational primes of Dirichlet density 1 such that for all places \( \lambda \) of \( E \) lying above an \( l \in L' \), we have that \( \lambda \in L \).

The main theorem can now be stated. It is a mild generalization of [3, Theorem 8.1] and is proved in the same way by simply appealing to Proposition [5, 1] instead of [3, Proposition 6.1]

**Theorem 6.4.** Let \( m \) be a positive integer, let \( (\Gamma, \mathcal{P}, \{\mathcal{F}_\alpha\}) \) be a group with Frobenii, let \( E \) be a Galois extension of \( \mathbb{Q} \), let \( L \) be a full set of places of \( E \) and for each \( w \in L \), let \( \rho_w : \Gamma \to GL_n(E_w) \) be a continuous representation and let \( \Delta_w \subset \Gamma \) be a normal open subgroup. Assume the following properties are satisfied.

- The \( \rho_w \) form a compatible system of representations.
- \( \rho_w \) is absolutely irreducible when restricted to any open subgroup of \( \Gamma \) for all \( w \in L \).
- \( \Gamma/\Delta_w \) is cyclic of order prime to \( l \) where \( l \) denotes the residual characteristic of \( w \).
- \( |\Gamma : \Delta_w| \to \infty \) as \( w \to \infty \).

Then there exists a set of places \( P \) of \( \mathbb{Q} \) of Dirichlet density \( 1/[E : \mathbb{Q}] \), all of which split completely in \( E \), such that, for all \( w \in L \) lying above a place \( l \in P \):

i) \( \overline{\rho}_w(\Delta_w) \) is an \( m \)-big subgroup of \( GL_n(F_l) \).

ii) \( |\ker \overline{\rho}_w : \Delta_w \cap \ker \overline{\rho}_w| > m \).

Theorem [3] is then the special case of the above theorem where \( \Gamma \) is the absolute Galois group of a number field.

7. Version Française Abrégée

Le but de cet article est de faire les modifications nécessaires au travail de Snowden-Wiles [3] afin de généraliser leurs résultats sur 1-big à \( m \)-big. La définition de \( m \)-big est rappelée dans Définition [1]. Elle est une condition technique qui apparaît dans les généralisations récentes de la méthode de Taylor-Wiles aux groupes unitaires (cf. [1]). Le résultat principal de cet article est le théorème suivant (cf. Théorème [1,3]).

**Théorème 7.1.** Soient \( m \in \mathbb{N} \), \( F \) un corps de nombres, \( E \) une extension galoisienne de \( \mathbb{Q} \) et \( L \) un ensemble plein de places de \( E \). Pour tous \( w \in L \), soient \( \rho_w : \text{Gal}(\overline{\mathbb{Q}}/F) \to GL_n(E_w) \) une représentation continue semi-simple et \( \Delta_w \subset \text{Gal}(\overline{\mathbb{Q}}/F) \) un sous-groupe normal ouvert. Supposons que les propriétés suivantes sont satisfaites :
- Les \( \rho_w \) forment un système compatible de représentations.
- Pour tout \( w \in L \), la restriction de \( \rho_w \) à n’importe quel sous-groupe ouvert de \( \text{Gal}(\mathbb{Q}/F) \) est absolument irréductible.
- Pour tout \( w \in L \), \( \text{Gal}(\mathbb{Q}/F)/\Delta_w \) est cyclique d’ordre premier à la caractéristique résiduelle de \( w \).
- \( \text{[Gal}(\mathbb{Q}/F) : \Delta_w] \to \infty \) lorsque \( w \to \infty \).

Alors il existe un ensemble de places \( P \) de \( \mathbb{Q} \) de densité \( 1/[E : \mathbb{Q}] \), qui sont toutes totalement déployées dans \( E \), et telles que, pour tout \( w \in L \) au-dessus une place \( l \in P \):

i) \( \mathcal{F}_w(\Delta_w) \) est un sous-groupe \( m \)-big de \( GL_n(F) \).

ii) \( \text{[ker ad } \rho_w : \Delta_w \cap \text{ker ad } \rho_w] > m \).

Remarque 7.1. La première partie de ce théorème est une généralisation du résultat principal de Snowden-Wiles [3]. La démonstration suit leurs arguments, en appliquant Proposition 3.1 au lieu de [3, Proposition 4.1]. La deuxième partie est un résultat de Barnet-Lamb-Gee-Geraghty-Taylor qui est apparu à l’origine dans [1].

Le plan de cet article est pareil à celui de [3]. Section 2 démontre quelques propriétés de \( m \)-big qui étaient démontrées pour 1-big dans [3, §2]. Section 3 démontre Proposition 3.1 qui améliore légèrement [3, Proposition 4.1]. Ce résultat est appliqué dans Section 4 pour démontrer que l’image de certaines représentations algébriques est \( m \)-big (cf. Proposition 4.1 qui améliore [3, Proposition 5.1]). Finalement, Section 5 et Section 6 appliquent ce résultat pour démontrer le théorème principal.

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