DERIVING SOME PROPERTIES OF STANLEY-REISNER RINGS FROM THEIR SQUAREFREE ZERO-DIVISOR GRAPHS

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Abstract. Let Δ be a simplicial complex, \( I_\Delta \) its Stanley-Reisner ideal and \( R = K[\Delta] \) its Stanley-Reisner ring over a field \( K \). In 2018, the author introduced the squarefree zero-divisor graph of \( R \), denoted by \( \Gamma_{sf}(R) \), and proved that if \( \Delta \) and \( \Delta' \) are two simplicial complexes, then the graphs \( \Gamma_{sf}(K[\Delta]) \) and \( \Gamma_{sf}(K[\Delta']) \) are isomorphic if and only if the rings \( K[\Delta] \) and \( K[\Delta'] \) are isomorphic. Here we derive some algebraic properties of \( R \) using combinatorial properties of \( \Gamma_{sf}(R) \). In particular, we state combinatorial conditions on \( \Gamma_{sf}(R) \) which are necessary or sufficient for \( R \) to be Cohen-Macaulay. Moreover, we investigate when \( \Gamma_{sf}(R) \) is in some well-known classes of graphs and show that in these cases, \( I_\Delta \) has a linear resolution or is componentwise linear. Also we study the diameter and girth of \( \Gamma_{sf}(R) \) and their algebraic interpretations.

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