Nucleon-nucleon correlations and the single-particle strength in atomic nuclei

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We propose a phenomenological approach to examine the role of short- and long-range nucleon-nucleon correlations in the quenching of single-particle strength in atomic nuclei and their evolution in asymmetric nuclei and neutron matter. These correlations are thought to be the reason for the quenching of spectroscopic factors observed in (e, e′p), (p, 2p) and transfer reactions. We show that the recently observed increase of the high-momentum component of the protons in neutron-rich nuclei is consistent with the reduced proton spectroscopic factors. Our approach connects for the first time results on short-range correlations from high-energy electron scattering experiments with the quenching of spectroscopic factors and addresses quantitatively this intriguing question in nuclear physics. We also speculate about the nature of a quasi-proton (nuclear polaron) in neutron matter and its kinetic energy, an important quantity for the properties of neutron stars.

Many-body quantum mechanical systems consisting of interacting particles are encountered in many fields of modern physics, including condensed matter, atomic and nuclear physics. In general, it is not possible to obtain analytical solutions of the equations governing the dynamics of particles within such quantum systems, starting explicitly from the individual particle-particle interactions. Quantum Monte-Carlo and other numerical techniques can be used to obtain numerical solutions, but these methods are computationally intense and are therefore limited to few interacting particles [1–3]. To overcome these limitations, many-body systems are often described in terms of independent particles moving in an effective mean-field potential that reflects the average influence of all individual particle-particle interactions.

In fermionic systems, neglecting any residual interactions between the particles (beyond those captured by the effective mean-field potential), one can define a Fermi level below which all quantum states are occupied. In the presence of residual interactions between fermions, important correlations arise that deplete the occupancy of states below the Fermi level and populate states above it, thus making the Fermi surface diffused.

The atomic nucleus consists of strongly interacting nucleons forming a dense quantum system. It is noteworthy that for such strongly interacting quantum system the independent-particle model is proven to be a valid approximation and has provided the framework to explain many nuclear properties. Nevertheless, correlations between nucleons modify the mean-field approximation and dilute the pure independent-particle picture. These nucleon-nucleon (NN) correlations are often distinguished into long-range correlations (LRC) and short-range correlations (SRC), referring to their spatial separation and the part of the NN potential they are most sensitive to [4–6]. Studies show that for stable nuclei at any given moment, only 60% – 70% of the states below the Fermi momentum are occupied, with 30% – 40% of the nucleons participating in LRC and SRC configurations [4, 7–14]. Therefore, both LRC and SRC deplete the occupancy of single-particle states, with LRC primarily mixing states near the nuclear Fermi-momentum and SRC populating states well above it.

There are two questions regarding this depletion that require further study, and have attracted the attention of the Nuclear Physics community:

- What are the individual contributions of LRC and SRC to the observed single-particle depletion?
- What is the isospin (neutron-proton asymmetry) dependence of LRC and SRC, and how do they compete in very asymmetric nuclei?

Inspired by recent results from Jefferson Lab [15], where the ratio of the fraction of high- to low-momentum protons (where high and low are relative to the Fermi momentum) was measured, we propose a phenomenological model that directly connects these new results with the reduction of single-particle strength in atomic nuclei. Our approach captures both LRC and SRC ingredients and allows one to extract their individual contributions as well as their evolution with mass number and isospin.

Experimentally, the depletion of single-particle states is quantified as quenching of spectroscopic factors (SFs) with respect to the independent particle model (IPM) limit, observed in (e, e′p) [9, 10], (p, 2p) [13] and transfer reactions [11, 12]. This is reflected in the probability to end up at a given final state after a nucleon is removed from the parent nucleus compared to theoretically calculated cross sections for the same reaction. At this point it is important to note that the quenching extracted from (e, e′p) measurements may depend on the momentum transfer, Q 2 [16, 17]. While it is not clear whether this is an artifact of the reaction theory or a real dependence of the SFs with Q 2, here we analyze the (well established) low-Q 2 data, where the scale resolution
should be sensitive to probe the quenching due to both SRC and LRC [16].

Recently, single-nucleon removal [18, 19] and hadron-induced quasi-free scattering (QFS) reactions in inverse kinematics [20–22] have been employed using radioactive ion beams and probed the quenching of SFs across a wider region of isospin asymmetry, exploring its isospin dependence. These experiments agree on the depletion of the single-particle strength for nuclei near stability but report significantly different isospin dependency. The reported discrepancy has triggered an active debate on the validity of the reaction models used in the analysis and the extent to which this can lead to an over-estimation of isospin effects [11, 12, 18–20].

In parallel, electron scattering experiments indicate a high-momentum tail extending far beyond the Fermi momentum [23, 24] attributed to SRC between a pair of strongly interacting nucleons [5, 25]. A value of about 20% SRC contribution was indirectly inferred from scaling inclusive measurements of the fraction of high-momentum nucleons in nuclei relative to deuterium [5, 25–29]. Proton and electron scattering studies of $^{12}$C showed that SRC are predominantly neutron-proton (np) pairs, as opposed to proton-proton (pp) or neutron-neutron (nn) pairs that are favored at lower momenta [30–32]. This was interpreted as manifestation of the tensor part of the NN interaction, which at short distances ($q \approx 2 \text{ fm}^{-1}$) favors the $S = 1$ (T = 0) channel [5, 25, 33–37]. Follow-up works extended these findings to both lighter and heavier nuclei, see e.g. Refs. [38–40].

Finally, the Jefferson Lab results from Ref. [15] revealed that in neutron-rich nuclei the ratio of the fraction of high- to low-momentum protons increases as a linear function of the ratio of neutron to proton number (N/Z), while the equivalent fraction for neutrons is rather constant or possibly decreasing slightly. This indicates that the percentage of protons participating in SRC pairs increases for neutron-rich systems and consequently depletes the proton strength from the region below the Fermi momentum, which is probed in measurements of SFs. Hence, the SRC dependence with isospin asymmetry should be reflected in the quenching of the proton SFs.

To study the consistency between SRC experimental results and SFs, we introduce a phenomenological model to estimate the total “missing strength” in terms of contributions from LRC (defined here as pairing [41] and particle-vibration coupling) and SRC components. While generally in low-energy nuclear structure one refers to pairing correlations as the short-range part of the force, compared to the quadrupole force which is of longer range, within the context of this paper pairing is not part of the SRC associated with high-momentum components. We approximate the wave function of a “dressed” nucleon (quasi-particle) in the nuclear medium in the following form:

$$|\text{qp}| = K_{\text{sp}}|\text{sp}| + K_{\text{PVC}}|\text{PVC}| + K_{\text{PC}}|\text{PC}| + K_{\text{SRC}}|\text{SRC}|,$$

where the terms on the right-hand side are assumed to be orthogonal. The first term represents the pure single-particle configuration, and the following three terms the particle-vibration coupling (PVC), pairing correlations (PC) and SRC induced configurations, respectively. The probability to find a nucleon in the pure single-particle configuration is $R = K_{\text{sp}}^2$. For non-interacting nucleons $R = 1$, while in the presence of correlations $R < 1$ (quenched). The missing part of the single-particle strength is distributed to the correlation terms with probabilities given by the square of the corresponding amplitudes in the wavefunction of eq. (1), i.e. $R_{\text{PVC}} = K_{\text{PVC}}^2$, $R_{\text{PC}} = K_{\text{PC}}^2$ and $R_{\text{SRC}} = K_{\text{SRC}}^2$. The quenched single-particle strength, $R$, can then be expressed in terms of these three independent components:

$$R = 1 - (R_{\text{PVC}} + R_{\text{PC}} + R_{\text{SRC}}).$$

In this approach, we associate $R$ to the overall quenching of SFs reported in $(e,e'p)$ and $(p,2p)$ measurements and extract the weighting of each of the three components entering eq. (2) as fitting parameters.

The trend of the SRC component as a function of isospin is derived from Ref. [15]. The measured relative fractions of high to low momentum nucleons in nuclei relative to $^{12}$C are reproduced in Fig. 1 (after transforming their N/Z axis to (N−Z)/A). Here we make the assumption that the neutron momentum fraction measured for neutron-rich systems (neutrons being the majority nucleons) can be used as the proton momentum fraction in a proton-rich system (protons being the majority nucleons). Using the fitted slopes $SL_n^{\text{SRC}} = 2.8 \pm 0.7$ and $SL_p^{\text{SRC}} = 0.3 \pm 0.2$, see Fig. 1, we write the following expressions:

$$N > Z : \text{R}_{\text{SRC}} = \gamma \left(1 + SL_n^{\text{SRC}} \frac{N - Z}{A}\right),$$

$$N < Z : \text{R}_{\text{SRC}} = \gamma \left(1 + SL_p^{\text{SRC}} \frac{N - Z}{A}\right).$$

The proton SF data used in this analysis are taken from $A(e,e'p)$ experiments of Ref. [10] and are summarized in Table I. The SFs include only reactions that have populated the ground state of the daughter nucleus. Also included in Table I is the quenching of SFs (R) with respect to large-scale shell-model (SM) calculations of Ref. [42]. In SM calculations the reported SFs for doubly-magic nuclei is almost the same to that expected from an IPM picture (indicated in Table I with an asterisk). In other words, there is no quenching predicted by SM for these closed-shell systems. This is inconsistent with the experimentally reported values for the quenching of SFs for
double-magic nuclei and reflects the fact that SM calculations cannot reproduce the full strength lost in LRC due to the yet limited model space used [8, 43], and the lack of the SRC component. Similarly, the reduced SFs obtained by the SM for non-doubly magic nuclei can also be regarded as additional LRC not completely captured by the SM even in a large model-space.

Realizing that the available data are somewhat limited we make first the simplifying assumption that \( R_{\text{SRC}} \) and \( R_{\text{LRC}} \) add linearly to the SRC contribution. Following Refs. [45, 46], this is proportional to the collectivity of the phonon (as measured by the dynamic deformation parameter \( \varepsilon_x \)) and the radial form factor, proportional to \( \partial V / \partial r \). The potential depth (V) for a proton is often parametrized, including a term that depends on the neutron excess, as:

\[
V = V_0 \left( 1 + \kappa \frac{N - Z}{A} \right). \tag{5}
\]

We thus expect,

\[
R_{\text{PVC}} \propto \left( \frac{\varepsilon_x}{\hbar \omega_0} \right)^2 \left( \frac{\partial V}{\partial r} \right)^2. \tag{6}
\]

Using the potential given in Ref. [46], we propose a
parametrization of the form
\[ R_{\text{PVC}} = \alpha \left(1 + \frac{33 \cdot N - Z}{51 \cdot A}\right)^2. \]  

(7)

For finite nuclei, \( \alpha \) can be taken as a constant, given the average dependence of \( \varepsilon_\alpha \) and \( \hbar \omega_0 \) with mass number. However, for infinite systems it should scale as \( 1/\Lambda^{1/3} \) reflecting the surface nature of the coupling.

In a similar way, we can estimate the effect of fragmentation due to pairing (vibration) correlations. The mixing amplitude should be proportional in lowest order to the ratio of the pairing gap to a typical shell gap, \( \Delta/\hbar \omega_0 \). With a typical parametrization \([47]\) of \( \Delta \) we obtain
\[ R_{\text{PC}} = \beta \left(1 - 6.07 \left(\frac{N - Z}{A}\right)^2\right)^2, \]  

(8)

here, \( \beta \) is also a constant. Note that, specifically for \( N = Z \), we have \( \delta = \alpha + \beta \). Turning again our attention to the doubly magic vibrations, for which to lowest order pairing vibrations will introduce 2p2h admixtures in the unperturbed 0p0h ground state configuration, one can make a simple estimate of \( \beta \) as \( ((7.55/\Lambda^{1/3})/(41/\Lambda^{1/3}))^2 \approx 0.03 \), using the values given in Ref. [47]. For the SRC contribution, we use the result of the fit of Fig. 2 where \( \gamma = 22\% \).

With the expressions in eqs. (2) to (4), (7) and (8) we attempt a fit of the experimental data on doubly magic nuclei. The result of the fit gives a PVC contribution of \( \alpha = 10\% \pm 2\% \). The SRC and PC contribution have been fixed to \( \gamma = 22\% \) and \( \beta = 3\% \), respectively, based on the above argumentation. The total fit (and the individual components) shown in Fig. 3 is in good agreement with the full \((e,e'p)\) data set. As discussed earlier, the agreement seen also for open-shell nuclei indicates a level of missing strength in the full shell-model results, due to LRC, similar to that in the IPM for doubly-magic systems. QFS \(^A\text{O}(p,2p)\) data [20] are also shown in the same plot. The reported QFS data are inclusive measurements to all bound final states and not only ground-state to ground-state transitions like the \((e,e'p)\) data. This means that part of the LRC correlations that distributes the single-particle strength to low-lying excitations in the final states is integrated in the experimental cross section. Indeed, repeating the fit for the \(^A\text{O}(p,2p)\) data, we obtain a PVC contribution of \( \alpha = 4\% \pm 2\% \). Actually, the \(^A\text{O}(p,2p)\) measurement has the only data point at negative asymmetry, which is nicely reproduced with the different slope of the SRC contribution for knocking-out a proton from a proton-rich system (see eq. (4)). In this case, the protons are the majority nucleons.

Another topic of current debate is the quenching observed in one-proton (and one-neutron) removal reactions carried out at intermediate energies \((\sim 100 \text{ MeV/nucleon})\). The study of Ref. [19] showed an unexpectedly strong dependence of the quenching, expressed as a function of the difference \(\Delta S\) in proton and neutron separation energies, \(S_p - S_n\) (\(S_n - S_p\)). The origin of this strong dependence (i.e. whether it is indeed due to NN correlations or due to the reaction model) is still an open question. To add to the discussion, it is interesting to compare our predictions (from Fig. 2) with the results of Ref. [19]. For this purpose, we use the equations given in Ref. [48] to convert \(A, Z,\) and \(N\) into \(S_p - S_n\). The two trends are shown as shaded areas in Fig. 4. Our results give a less pronounced dependence on \(\Delta S\).

As a final note, we can also speculate about the nature of a quasi-proton (nuclear polaron [49]) in neutron matter. In the limit of \(A \to \infty\) and \((N - Z)/A \to 1\), and neglecting both surface and pairing coupling terms, expected to be small for infinite matter at saturation density, we predict a proton quenching factor of \(R_{\text{PVC}} = 1 - \gamma - \gamma S_{\text{SRC}}^2 \approx 0.2\). The high relative kinetic energy components in the wavefunction, present in this limit, will give the proton an average kinetic energy:
\[ \langle T_p \rangle_{\text{nM}} = \left(R_{nM} + \left(1 - R_{nM}\right)\frac{5 \cdot p_{\text{Max}}}{3 \cdot p_F}\right)\langle E_F \rangle. \]  

(9)

With \(p_{\text{Max}} \sim 2p_F\), estimated from the uncertainty principle, we obtain \(\langle T_p \rangle_{\text{nM}}\) approximately 2.5 times that of a proton in a Fermi Gas, an important quantity for the properties of neutron stars [50]. Similarly, the average kinetic energy for the neutrons \(\langle T_n \rangle_{\text{nM}}\) (for which \(R_{\text{PC}}^2 = 1 - \gamma - \gamma S_{\text{SRC}}\approx 0.85\)) is estimated to be approximately 1.4 times that of a neutron in a Fermi Gas.

In summary, we presented a phenomenological model that connects the quenching of spectroscopic factors with the recent SRC Jefferson Lab study. We derived simple
phenomenological parametrizations for the combined effects of SRC, PVC, and PC that were used in an analysis of data from low-Q$^2$ electron scattering and proton induced QFS experiments. Our analysis shows that approximately 20% of the missing strength observed in the region of N = Z can be attributed to SRC, in agreement with reported expectations. Furthermore, we show how the missing strength, including contributions from LRC, is expected to evolve with (N−Z)/A and speculate an extrapolation to a quasi-proton in neutron matter and its kinetic energy, with implications to neutron stars. While perhaps rather speculative at this stage, given the available data, we trust our conjecture will stimulate further theoretical and experimental work. In particular for the latter, we highlight the need to: 1) Measure the quenching of spectroscopic factors in stable nuclei with higher precision, including transfer reactions, 2) Study of quenching of spectroscopic factors in stable nuclei with measurements in nucleon-removal reactions \cite{19} as a function of the difference in separation energies $S_p - S_n$. Our prediction (within 2σ) follows the red-shaded area.

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