Vehicular traffic flow at a intersection controlled by signal light with a new probability

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We introduced a probability for traffic light, $P_L$, at an intersection when approaching cars in two roads are in same conditions. As a application, we proposed a modified Nagel-Schreckenberg cellular automata model for describing a conflicting vehicular traffic flow at the intersection. The results show that the plateau region in the fundamental diagrams, caused by the effect of interaction, is dependent not only on the probability $P_L$, but also on the adaptive schemes.

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Efficient transportation systems are essential for the every day activities of modern industrialized societies. The urgency to reduce CO$_2$ emissions and fuel consumption, and the excessive, unpredictable travel times during traffic congestion call for more efficient control approaches, in particular, the optimization of traffic lights in urban road networks. During past years, various traffic models have been developed to investigate the problems of traffic jams [1, 2], car accidents [3, 4, 5] and energy dissipation [6, 7] within the framework of single lane traffic models. Recently, a research focus was put on urban road networks, which required extending one-dimensional traffic models in order to cope with situations [8, 9]. Evidently the optimization of traffic flow at a single intersection is preliminary but important step for not only global optimization in city networks but also local optimization. After the first model for simulation two crossing roads [10], controlling traffic flow at intersections have attracted notable attentions [11].

In principle, the traffic flow at the intersection of two roads can be controlled via two distinctive schemes, with or without signalized traffic light. It is evident that traffic light is unavoidable as the density of cars increases. And it is agreed that a further improvement of the traffic flow requires applying flexible strategies than fixed-time controls. In this letter, we developed a Nagel-Schreckenberg cellular automata model for describing a conflicting vehicular traffic flow at an intersection and present our simulation results for three intelligent controlling schemes. Our control schemes are inspired by the model proposed by Fouladvand [11]. In that work, approaching car to the intersection yield to traffic at the perpendicular direction by adjusting its velocity to a safe value to avoid collision. And the priority is given to the nearest car to the intersection according to driving rules. A nature and important question is that which one will get the priority when two approaching cars are in same conditions?

In this letter, we introduce a probability for traffic light, $P_L$, while that happens. In order to capture the basic feature of this problem, we have constructed a modified NS cellular automata model with three traffic responsive schemes applying $P_L$.

We first discuss the traffic adaptive controlling scheme in which the light signalization is adapted to the traffic. Nowadays advanced traffic control systems anticipate the traffic approaching intersections. The data obtained via traffic detectors installed at the intersection make it possible to measure the velocity and distance of an approaching car and estimate the time the car successively passing through the intersection [12]. That supports us to promote three possible traffic adaptive controlling schemes.

In each scheme, the distance to the intersection, the velocity and the time passing the crossing point of the approaching cars are denoted by $d_1, v_1, t_1$ and $d_2, v_2, t_2$ for the first and second road, respectively. Generally, both roads are green. By green (red) road we mean the road for which the traffic light is green (red). If the two approaching cars both can pass the crossing point at the next step, one road should change to red to avoid collision under the next three algorithms.

1. The movement priority is given to the nearest car ($d_{min}$) to the crossing point, and the car adjusts its velocity as usual with its leading car. In contrast, the other road is turned red and the car brakes irrespective of its direct gap. The simplest way to take into this cautionary braking is to adjust the gap with the crossing point itself. If both the approaching cars have the same distance, $d_1 = d_2$, the priority is given to one road with probability $P_L$, and the other with $1 - P_L$.

2. The priority is given to the car that use less time ($t_{min}$) for pass through. While $t_1 = t_2$, the priority is given to one road with probability $P_L$, and the other with $1 - P_L$.

3. No matter what the $d_1, v_1, t_1$ is, randomly chose one
road and give it the movement priority with probability $P_L$, and the other with $1 - P_L$.

We do not use the velocity alone to form a new scheme not only because of the traffic safety. If we give the priority to the fastest car ($v_{\text{max}}$), the approaching car in red road will lose its velocity. For high densities, at the intersection, the approaching car in the green road will always be faster than that in the red road, and the red road will keep in red. With the velocity priority scheme, the current of the system rises to its maximum value rapidly, and then exhibits linear decrease versus density the same manner as in the fundamental diagram of a single road (data not shown).

Referring to Fig. [1] we consider two perpendicular one-dimensional chains which represent urban roads accommodating unidirectional vehicular traffic flow. They cross each other at the sites $i_1 = i_2 = 0$ on the first and the second chains respectively. With the closed boundary conditions, the system is equivalent to cars moving in two circles cross at one point with a traffic light. The roads, in principle, can each carry two opposite flows of vehicles. With no loss of generality, we take the direction of traffic flow in the first chain from north to south and in the second chain from west to east. Space and time are discretized in such a way that each chain is divided in the second chain from west to east. Space and time are assumed to elapse in discrete steps of 1s. We take the number of cells to be $L_1 = L_2 = 500$ for both roads. Each cell can be either occupied by a car or be empty. Moreover, each car can take discrete-valued velocities $0, 1, 2, ..., v_{\text{max}}$ in which $v_{\text{max}}$ is the maximum velocity of cars. To be more specific, at each step of time, the system is characterized by the position and velocity configurations of cars and the traffic light state at each road.

The system evolves under the NS dynamics [13]. Let us briefly explain the NS updating rules which synchronously evolve the system state from time $t$ to $t + 1$. We denote position, velocity of the $i$th car at time step $t$ by $x(i, t)$ and $v(i, t)$, respectively. The same quantities for its leading car are correspondingly denoted by $x(i + 1, t)$ and $v(i + 1, t)$. The number of empty cells in front of the $i$th car is denoted by $d(i, t) = x(i + 1, t) - x(i, t) - 1$. Concerning the above considerations, the following updating steps evolve the position and the velocity of each car in parallel.

1. Acceleration:
   $v(i, t + 1/3) \rightarrow \min[v(i, t) + 1, v_{\text{max}}]$;
2. Slowing down:
   $v(i, t + 2/3) \rightarrow \min[v(i, t + 1/3), d(i, t)]$;
3. Stochastic braking:
   $v(i, t + 1) \rightarrow \max[v(i, t + 2/3) - 1, 0]$ with the probability $p$;
4. Movement:
   $x(i, t + 1) \rightarrow x(i, t) + v(i, t + 1)$.

The state of the system at time $t + 1$ is updated from that at time $t$ by applying the modified NS dynamical rules. After transients, two roads maintain steady-state currents, defined as the number of vehicles passing from a fixed location per a definite time interval, denoted by $J_1$, $J_2$ and the average $J_{\text{avg}} = (J_1 + J_2)/2$. They are functions of the global densities $\rho_1 = N_1/L_1$, $\rho_2 = N_2/L_2$ and $\rho = (\rho_1 + \rho_2)/2$, where $N_1$ and $N_2$ are the number of vehicles in the first and the second road, respectively. We kept the global density $\rho_1 = \rho_2$ and $v_{\text{max}} = 5$ during the simulations. Figure 2 exhibits the fundamental diagram of the isolate system, $J_{\text{avg}}$ versus $\rho$, and the difference in three schemes at the fixed $P_L = 0.5$, $p = 0$.

It is observed that, in scheme 1 and 3, for small den-
ties $\rho$, $J_{avg}$ rises to its maximum value rapidly, and then undergoes a short rapid decrease after which a lengthy plateau region is formed. The current in the plateau region is independent of $\rho$. In scheme 2, $J_{avg}$ rises to the plateau without any decrease. After the plateau, in all three schemes, $J_{avg}$ exhibits linear decrease versus $\rho$ in the same manner as in the fundamental diagram of a single road. It is known that local non-linear interactions can lead to system-wide patterns of motion and a local defect can affect the low-dimensional non-equilibrium automata models describing vehicular traffic flow. As discussed in [11], the intersection makes the cross point as a sitewise dynamical defect whose effect is to form the plateau region in our case. In fact, when $P_L = 0.5$, the model yield to a non-sigaled intersection and the scheme one is equal to that discussed by Foulaadvand. We observed that the $J_{avg}$ in scheme 2 is higher than others. The reason is that, in scheme 2, the car got the priority requires less time to pass through. This raises the efficiency of the intersection compared to the two other schemes hence the second scheme is more optimal than them.

Intersection of two chains makes the intersection point appear as a sitewise dynamical defective site, and the smaller the $P_L$ is, the stronger the dynamic defect is. For each value of $P_L$, consider the current $J_1(J_2)$ in each road, the larger the $P_L$ is, the plateau region is wider and the current value is more reduceddata not shown. In order to find deeper insight, it would be illustrative to look at the behavior of average current as a function of density. Fig. E show that in scheme 3, with fixed $p = 0$, $\rho_1 = \rho_2$ and $0 \leq P_L \leq 0.5$. The results show that the plateau region of $J_{avg}$ also becomes wider and the value becomes smaller as $P_L$ increases. Because of the equivalence of the two roads, the condition inverts while $0.5 \leq P_L \leq 1$. In particular, both with $P_L = 0$ and $P_L = 1$, the fundamental diagram exhibits the same manner as that of a single road. The results are same as that in scheme 2. But there is little different in scheme 1. As shown in Fig 4 with fixed $p = 0$, $\rho_1 = \rho_2$ and $0 \leq P_L \leq 0.5$, while the length of the plateau region grows with the $P_L$ increase, the height of the plateau do not show significant dependence on $P_L$. While for low density of roads, $P_L$ should be adapted according the situation for higher $J_{avg}$, for high density, $P_L$ of two roads should be set the same, $P_L = 0.5$, for optimal purpose. While for low density, $P_L$ should be adapted according the situation for higher $J_{avg}$

We then examined $J_{avg}$ versus $P_L$. Because the two roads are equivalent in our model, the symmetry centered at $P_L = 0.5$ is expected, as shown in Fig 6. In general, the dependence of $J_{avg}$ on $P_L$ depends on the value of $\rho$. For large values of $\rho$, $\rho_{c1} > 0.70$ in scheme 1 and $\rho_{c2} > 0.61$ in scheme 2 and $\rho_{c3} > 0.66$ in scheme 3, $J_{avg}$ increases with $P_L$, then starts its decrease after the maximum at $P_L = 0.5$. For $\rho < \rho_{c3}$, the results are different in three schemes. In scheme 2, $J_{avg}$ increases with $P_L$ to the maximum at $P_L = 2c$, then the value reduced until $P_L = 0.5$. After that point, $J_{avg}$ exhibits an increase up to $P_L'$ and decrease subsequently. We note that the region, $P_L \in [P_{L2c}, P_L']$, is interested. For each value of $\rho$, this region is on the same line, though $P_{L2c}$ and $P_L'$ are different. That is the same as shown in scheme 3. In scheme 1, there is little different that, for each $\rho$ except large values, $J_{avg}$ is almost independent of $P_L$ and remains constant in $P_L \in [P_{L2c}, P_L']$. In fact, $P_L \in [P_{L2c}, P_L']$ denotes the appearance of the plateau region as shown in Fig 2. The results show that, for each $\rho$, the height of the plateau region is independent on $\rho$. In particular, that is also independent of $P_L$ in scheme 1. As shown in Fig 5 at fixed $\rho = 0.3$ and brake possibil-
ity \( p = 0 \), while \( J_{avg} \) changes little in scheme 1, \( J_{avg} \) rises to its maximum value up to 0.363 and 0.364 in scheme 2 and 3 respectively. Then \( J_{avg} \) reduces rapidly until \( P_L = 0.5 \), and the condition inverts subsequently. In scheme 1, the most commonly used method in non-signal intersections, the value of \( J_{avg} \) is less than that in others and insensitive to \( P_L \). It is easy to see that the scheme 2 is the optimal one. While it is difficult for drivers to get the velocity and distance to the cross point of another approaching car, the traffic light with traffic detectors is unavoidable even at \( P_L = 0.5 \).

In summary, we introduced a probability of traffic light, \( P_L \), while cars upon reaching the intersection are in same conditions. We have investigated the flow characteristics in a signalized intersection via developing a NS model applying \( P_L \). We have considered three types of controlling schemes. In particular, we have obtained the fundamental diagrams and the dependence of average current on the probability \( P_L \) road densities. Our findings show the probability of priority at the intersection gives rise to formation of plateau regions in the fundamental diagrams and an optimal method for controlling the traffic is unavoidable. The findings may shed light on the way to further study the urban cross road.

\[ \text{Figure 5: } J_{avg} \text{ versus } P_L \text{ at } \rho = 0.3 \text{ and } p = 0. \]

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