Out-of-Bounds Hydrodynamics in Anisotropic Dirac Fluids

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Out-of-bounds hydrodynamics in anisotropic Dirac fluids

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We study hydrodynamic transport in two-dimensional, interacting electronic systems with merging Dirac points at charge neutrality. The dispersion along one crystallographic direction is Dirac-like, while it is Newtonian-like in the orthogonal direction. As a result, the electrical conductivity is metallic in one and insulating in the other direction. The shear viscosity tensor contains six independent components, which can be probed by measuring an anisotropic thermal flow. One of the viscosity components vanishes at zero temperature leading to a generalization of the previously conjectured lower bound for the shear viscosity to entropy density ratio.

Hydrodynamic flow in quantum many-body systems is essential in systems as diverse as superfluid Helium [1], (Al,Ga)As heterostructures [2], cold atomic “gases” [3–5], and the quark-gluon plasma [5, 6]. Recently, it has become possible to study in greater detail the hydrodynamic flow of electrons [7, 8] via transport measurements in graphene, yielding a breakdown of the Wiedemann-Franz law [9], super-ballistic transport [10, 11], negative local resistance [12, 13], and giant magnetodrag [14] (for a recent review see Ref. 15 and 16). Other key examples are ultra-pure PdCoO$_2$ [17] and Weyl semimetals [18]. The appeal of these experiments is that they allow for an investigation of the universal collision-dominated dynamics of the pure electron fluid, largely independent of its couplings to the lattice and impurities: the hydrodynamic flow is expected when electron-electron scattering dominates over impurity and electron-phonon scattering processes [15].

Hydrodynamics is also one of the most successful applications of the duality between strongly coupled gauge theories and gravity theory [19], leading to the lower bound [20] for the ratio of the shear viscosity and entropy density

$$\frac{\eta}{s} \geq \frac{\hbar}{(4\pi k_B)} . \quad (1)$$

While originally derived as an equality for a specific strongly-coupled field theory, the bound was conjectured to apply to all single-component non-relativistic fluids [20]. Thus, to identify a scenario where Eq. (1) is explicitly violated is of fundamental importance. It is also of practical relevance as a small viscosity implies a strong tendency towards turbulent flow [21]. Eq. (1) can already be rationalized using Boltzmann transport theory: Let $s \approx k_B \lambda^{-d}$ be the entropy density (in $d$ dimensions) in terms of the thermal de Broglie wave length $\lambda$ and $\eta \approx \epsilon \tau \lambda^{-d}$ the shear viscosity with energy density $\epsilon$ and scattering time $\tau$. A quasiparticle description of transport suggests that $\epsilon \tau \gg \hbar$, leading to the above bound (up to numerical coefficients of order unity). A more formal reasoning can be made using scaling arguments. We rescale distances according to $x \rightarrow x' = x/b$ with the scaling factor $b$. Momentum conservation and hyperscaling for critical systems imply that both the viscosity and entropy density change according to $\eta(T) = b^{-d}\eta(b^2T)$ and $s(T) = b^{-d}s(b^2T)$, with the dynamic scaling exponent $z$. Thus, the entropy density and shear viscosity have the same scaling dimension. If the system approaches a fixed point, the ratio $\eta/s$ should approach a universal value in units of $\hbar/k_B$. This is analogous to the electrical conductivity in $d = 2$ that approaches a universal value in units of $e^2/h$, a result that follows from $\sigma(T) = b^{2-d}\sigma(b^2T)$.

The bound (1) appears to be violated in gravity theories dual to an anisotropic version of a super-Yang-Mills plasma [22–24] with applications to cold gases [25]. It is of great interest to identify a solid-state system where such a violation might occur.

In this Letter, we analyze the hydrodynamic behavior in an anisotropic Dirac system, where two Dirac cones merge in momentum space [26]. Such a model is relevant to the organic conductor $\alpha$-(BEDT-TTF)$_2$I$_3$ under pressure [27] and the heterostructure of the 5/3 TiO$_2$/VO$_2$ supercell [28, 29]. Similar behavior is expected in the surface modes of topological crystalline insulators with unpinned surface Dirac cones [30] and quadratic double Weyl fermions [31]. In the collision-dominated regime at charge neutrality, we predict extremely anisotropic electrical transport exhibiting either insulating or metallic behavior depending on the orientation of the applied electric field relative to the crystallographic axes. Similarly, the electronic shear viscosity strongly depends on the flow direction, exhibiting fundamentally different temperature behavior. As a result, at low temperatures the viscosity to entropy density ratio may diverge, stay constant, or vanish, depending on the spatial direction. In the latter case, the Eq. (1) is violated, an effect with experimentally measurable consequences through viscous thermal Hagen–Poiseuille flow. We explain the anisotropic transport in terms of the emergence of multiple length scales. In addition we propose a generalization of the viscosity bound to two-dimensional anisotropic sys-
Recently investigated in Ref. [26] within a large-
electron-electron scattering dominates.

To reach a sufficiently wide temperature regime where
Ref. 32, it follows that the dimensionless strength of the
dimension [33, 34] (26)

\[
\frac{C}{s} \propto \frac{1}{4\pi k_B}, \quad \frac{\eta_{\alpha\beta\gamma\delta}}{s} \propto \frac{1}{4\pi k_B},
\]

which includes the electrical conductivity anisotropy as an
additional observable. The numerical coefficient 1/4\pi is consistent with Ref. 23.

The model. Anisotropic Dirac systems are described by the model Hamiltonian, \(H = H_0 + H_C\), where the single-
particle part is

\[
H_0 = \int d^2r \psi^\dagger \left(-\frac{1}{2m} \nabla^2 \sigma_x - i\nu \nabla_y \sigma_y \right) \psi,
\]

and \(H_C\) is the electron-electron Coulomb interaction with two-body potential \(V(r-r') = e^2/|r-r'|\). Here, \(\nu\) is the velocity along the \(y\)-direction and the Pauli matrices \(\sigma_{x,y}\) describe the pseudo-spin space. The dispersion in the \(x\)-direction is characterized by the mass \(m\) or, alternatively, by the momentum scale \(k_0 = \nu \sigma\). The anisotropy in Eq. (3) is enforced by the underlying lattice. Specifically, the direction of the parabolic dispersion (the \(x\)-direction) is along the axis of the two merging Dirac points, see

Fig. 1. In the organic conductor \(\alpha-(BEDT-TTF)_2I_3\) [27], the two Dirac cones merge together upon applying uniaxial pressure. According to Ref. 27, an anisotropic Dirac cone is expected to form at \(P = 40\) kbar, see Fig. 1. From Ref. 32, it follows that the dimensionless strength of the Coulomb interaction is of order unity, which is important to reach a sufficiently wide temperature regime where electron-electron scattering dominates.

The renormalization group behavior of this model was recently investigated in Ref. [26] within a large-\(N\) expansion \[33, 34\] \(N\) is the number of fermion flavors; \(N = 2\) for

\[
\frac{d\alpha}{d\log b} = -\frac{0.362}{N} \alpha, \quad \frac{dk_0}{d\log b} = \frac{0.2374}{N} k_0.
\]

This gives rise to two characteristic length scales

\[
\lambda_x \propto T^{-\phi/z}, \quad \lambda_y \propto T^{-1/z},
\]

with the dynamic scaling exponent, \(z = 1 - 0.362/N\). The additional crossover exponent \(\phi = (1 - 0.2374/N)/2\) is a measure of the anisotropy. \(z < 1\) reflects an increase of the velocity at low energies and \(\phi < 1/2\) implies that interactions make the anisotropy even stronger if compared to the bare spectrum of Eq.(3). The fact that \(\phi \neq 1\) is the most crucial ingredient of our subsequent discussion. The violation of the lower bound only requires \(\phi \neq 1\) as one viscosity component vanishes faster than \(s\) for \(T \to 0\) even if the large-\(N\) approach turns out to be quantitatively inaccurate.

Scaling. Now we consider the constitutive relations in anisotropic systems. The electrical conductivity is a rank-two tensor defined in the standard way, \(\sigma_{\alpha\beta} = \sigma_{\alpha\beta} E_{\beta}\) (with \(\alpha, \beta \in \{x, y\}\)). The viscosity is a rank-four tensor connecting the dissipative part of the stress tensor \(\tau_{\alpha\beta}\) and the flow velocity gradient, \(\tau_{\alpha\beta} = \sum_{\delta, \gamma} \eta_{\alpha\beta\gamma\delta} \partial_{\gamma} u_{\delta}\). The number of independent conductivity and viscosity coefficients can be found from symmetry arguments \[35\]. In a rotationally invariant system, both the conductivity and shear viscosity are each characterized by a single independent coefficient: \(\sigma_{\alpha\beta} = \sigma_{\alpha\beta} \sigma_{\alpha\beta}\) and \(\eta_{\alpha\beta\gamma\delta} = \eta (\delta_{\alpha\gamma} \delta_{\beta\delta} + \delta_{\alpha\delta} \delta_{\beta\gamma} - \delta_{\alpha\beta} \delta_{\gamma\delta})\) \(35\). In this Letter, we focus on incompressible fluids and hence do not consider the bulk viscosity. In contrast, the Hamiltonian (3) is not rotationally invariant and is characterized by two conductivity elements \(\sigma_{xx}\) and \(\sigma_{yy}\) and six independent viscosity coefficients with \(\eta_{\alpha\beta\gamma\delta} = \eta_{\alpha\beta\gamma\delta}\), such that

\[
\begin{pmatrix}
\tau_{xx} \\
\tau_{xy} \\
\tau_{yx} \\
\tau_{yy}
\end{pmatrix}
=
\begin{pmatrix}
\eta_{xxxx} & 0 & 0 & \eta_{xxyy} \\
0 & \eta_{yyxx} & \eta_{yxxy} & 0 \\
0 & \eta_{yxxy} & \eta_{xxxy} & 0 \\
\eta_{yyyy} & 0 & \eta_{yyxy} & \eta_{yyy}\n\end{pmatrix}
\begin{pmatrix}
\partial_{xx} u_x \\
\partial_{xy} u_y \\
\partial_{yx} u_x \\
\partial_{yy} u_y
\end{pmatrix}
\]

Below we show that the “off-diagonal” momentum relaxation along the \(y\)-direction (with linear dispersion) due to a flow with velocity along the \(x\)-direction (of parabolic dispersion) described by \(\eta_{xyxy}\) is clearly different from the “opposite” case, \(\eta_{yxxy}\).

The scaling behavior of the conductivity and the viscosity follows from the Kubo formalism \[36, 37\]. If one takes charge conservation into account, the scaling dimension of the conductivity is a purely “geometric” effect that involves the length scales \(\lambda_x\). If \(b\) is a scaling parameter for length scales along the \(y\)-direction, it follows
\[ \sigma_{xx}(T) = b^{-(3-\phi)} \sigma_{xx}(b^2 T), \]
\[ \sigma_{yy}(T) = b^{-(3-\phi)} \sigma_{yy}(b^2 T). \] (7)

Fixing the coefficient \( b \) via \( b^2 T = \text{const} \) reveals that \( b \) is given by the ratio of the two length scales \( b^{\phi-1} \rightarrow \lambda_x/\lambda_y \).

It immediately follows that \( \sigma_{yy} \propto T^{(\phi-1)/z} \) diverges as \( T \to 0 \), while \( \sigma_{xx} \propto T^{(1-\phi)/z} \) vanishes. The system is insulating along the direction with parabolic dispersion and metallic in the other direction. Below, we confirm this behavior within an explicit kinetic theory and see that this behavior is pronounced below the characteristic temperature \( k_b T_0 \equiv m^* \sigma \approx 1.5 \text{ eV} \) in the organic salts.

Similar behavior emerges for the entropy density and the components of the viscosity tensor. For the entropy density, it follows from the usual scaling behavior of anisotropic systems \([39]\), \( s(T) = b^{-(1-\phi)} s(b^2 T) \), yielding \( s(T) \propto k_B/\theta_x \). To determine the behavior of the viscosity tensor we use again the Kubo formalism (the details are summarized in the supplementary material \([40]\)). The result is that most tensor elements \( \eta_{\alpha \beta \gamma \delta} \) have the same scaling dimension as \( s(T) \). However, there are two crucial exceptions:

\[ \eta_{xyxy}(T) = b^{-(3-\phi)} \eta_{xyxy}(b^2 T), \]
\[ \eta_{xxyx}(T) = b^{-(3\phi-1)} \eta_{xxyx}(b^2 T). \] (8)

It immediately follows that \( \eta_{xyxy}/s \propto T^{2(1-\phi)/z} \) and \( \eta_{xxyx}/s \propto T^{2(1-\phi)/z} \). Unless the system is isotropic and \( \phi = 1 \), one component vanishes and the other diverges. Thus, regardless of the numerical coefficient of \( \eta_{xyxy} \), it will violate the bound Eq. (1) at sufficiently low temperatures since \( \phi < 1 \). Below we obtain this behavior from Boltzmann theory as well. The scaling analysis reveals that the charge transport and momentum transport are closely related to each other. This allows us to construct combinations of physical observables that have scaling dimension zero. These combinations are listed in Eq. (2) and give rise to the generalized lower bound for the viscosity tensor.

Hydrodynamics. The scaling behavior can be obtained from the kinetic equation

\[ \frac{\partial f_{\mu k}}{\partial t} + v_{\mu k} \cdot \frac{\partial f_{\mu k}}{\partial x} + eE \cdot \frac{\partial f_{\mu k}}{\partial k} = I_{\mu k}^c, \] (9)

where \( f_{\mu k} \) is the distribution function for a quasiparticle from the band \( \mu \) and with the quasi-momentum \( k \), and \( I_{\mu k}^c \) is the collision integral due to the Coulomb interaction. The latter we treat in perturbation theory in \( 1/N \) (for details see Refs. 40 and 41).

Following the derivation of the hydrodynamic theory in the limit of an incompressible fluid \([21]\), we integrate the kinetic equation (9) and obtain generalizations of the Navier-Stokes equation at the charge neutrality point. Flow along the direction of the parabolic dispersion is described by a Navier-Stokes equation similar to that of a Galilean invariant system,

\[ m_\mathrm{s} n (\partial_t u_x + u_i \partial_i u_x) + \partial_x P = \mathcal{F}_{s,x} + m_\mathrm{s} n \delta j^i_1, \] (10)

where \( m_\mathrm{s} \approx 1.37 m \), \( n \) is the total quasiparticle density, \( \delta j_1 \) is the dissipative correction to the total quasiparticle current, \( P \) is the hydrodynamic pressure, and \( \mathcal{F}_{s,\alpha} = \partial_\alpha \tau_{\alpha \beta} = \partial_\alpha \eta_{\alpha \beta \gamma \delta} \partial_\gamma u_\delta \) is the Stokes force. Flow in the \( y \)-direction obeys the equation similar to that in graphene \([42, 43]\)

\[ Ts (\partial_t u_y + u_i \partial_i u_y) + \partial_y P = \mathcal{F}_{s,y} = u_y E \delta j_1 + u_y \partial_x \delta j_1. \]

The dissipative terms include the corrections to the electric current, \( \delta j_1 \), and the energy current, \( \delta j_2 \).

The simplest nontrivial solution of these equations can be obtained in the linearized stationary regime in the absence of the electric field. Using Eq. 6, we find

\[ \partial_\alpha P = \tilde{\eta}_{\alpha \alpha \alpha \alpha} \partial^2_\alpha u_\alpha + 2 \tilde{\eta}_{\alpha \alpha \alpha \alpha} \partial_\alpha \partial_\alpha u_\alpha + \tilde{\eta}_{\alpha \alpha \alpha \alpha} \partial_\alpha u_\alpha \] (12)

with \( \tilde{\eta}_{\alpha \beta \gamma \delta} = 1/2 (\eta_{\alpha \beta \gamma \delta} + \eta_{\beta \alpha \gamma \delta}) \), and \( \pi = y \) and vice versa. Note that \( \tilde{\eta}_{xyxy} = \eta_{xyxy} \) and same for \( \tilde{\eta}_{xxyx} \). From the kinetic equation it also follows that the heat current, \( j_\sigma \approx (5/3) \epsilon u \), is solely determined by the flow velocity. This is similar to the particle current in Galilean-invariant systems and reflects the fact that the thermal conductivity of a Dirac system at neutrality is infinite in the limit of infinite size \([44, 45]\). Using \( \partial_\alpha P = -s \partial_\alpha T \) at neutrality, we can solve the above equations for a finite geometry, find the velocity profile \( u \), and determine the thermal current from \( j_\sigma \). Note that this is how the entropy density enters our theory.

Consider now a flow in a system of width \( w \). In this geometry, there is no net flow in the lateral \( x \)-direction. The solution of Eq. (12) with the no-slip boundary conditions, \( u_y (x = \pm w/2) = 0 \), yields the standard Hagen-Poiseuille profile

\[ u_y (x) = \frac{s}{2 \tilde{\eta}_{xyxy}} \left( \frac{w^2}{4} - x^2 \right) \partial_y T. \] (13)
Integration over the cross section yields the total thermal current \( I_e = w \kappa_{yy} \partial_T \) with the thermal conductivity \( \kappa_{yy} = (5e\nu^2/24)(s/\eta_{xxyy}) \). Similar analysis of the flow along the \( x \)-direction yields \( \kappa_{xx} = (5e\nu^2/24)(s/\eta_{xxyy}) \). In the case of no-stress boundary conditions it is better to analyze conical flow.

The above results demonstrate that the thermal Hagen–Poiseuille flow is determined precisely by those viscosity tensor elements (8) that violate the ordinary scaling behavior. Thus, the ratios \( \eta_{xxyy}/s \) and \( \eta_{xxyy}/s \) matter and are in fact the easiest to observe. The other tensor components emerge only when the flow direction is not aligned with one of the crystalline axes. In this case a transverse temperature gradient builds up, see Fig. 2.

**Kinetic theory.** Finally, we use the microscopic quantum kinetic equation approach to find the conductivities and viscosities. The former can be found in the standard way [21]: applying a weak electric field, \( E \), we drive the system out of equilibrium where the distribution function \( f_{\mu k} \) acquires a nonequilibrium correction proportional to the field: \( \delta f_{\mu k} = f_{\mu k}^{(0)}(1 - f_{\mu k}^{(0)})h_{\mu k}/T, \) \( h_{\mu k} = \mu_{\mu k} E k T E_{\mu k} \).

Solving the kinetic equation for the functions \( g_{E_{\mu k}} \), we find \( g_{E_{\mu k}}(T) \propto N^2(e^2/h)(T/T_0)^{2/(1-\phi)/z} \) in agreement with the scaling results.

To determine the viscosity, we have to find the stress tensor within linear response to the external shear force. In terms of the nonequilibrium distribution function, the stress tensor is given by \( \tau_{\alpha \beta} = \sum_{\mu k} f_{\mu k} v_{\mu k}^\alpha k^\beta \delta f_{\mu k} \). The correction \( \delta f_{\mu k} \) is proportional to the velocity gradients with \( h_{\mu k} = \sum_{\alpha \beta} v_{\mu k}^\alpha k^\beta - \delta_{\alpha \beta} \epsilon_{\mu k}/2 ) \partial_{\alpha} u_{\beta} g_{\alpha k} \). Now we expand the functions \( g_{\alpha k} \) in the basis of the eigenfunctions of the linearized kinetic equation \[43, 46\].

The dominant contribution comes from the two modes describing the energy and the energy band index. This allows us to represent the kinetic equation (9) in the matrix form, \( M_{\alpha \beta} \psi_{\alpha} = G_{\alpha \beta} \psi_{\alpha} \), where the matrix \( M_{\alpha \beta} \) corresponds to the collision integral due to Coulomb interaction. The exact form of \( M_{\alpha \beta} \) and \( G_{\alpha \beta} \) can be found in Ref. 40. Solving the matrix equation, we find \( g_{\alpha k} \) and hence the viscosity coefficients, \( \eta_{\alpha \beta} = \sum_{\mu k} \mu_{\mu k} k^\beta (\epsilon_{\mu k} / 2 ) g_{\alpha k} \partial_{\alpha} \mu_{\mu k} \left( 1 - f_{\mu k}^{(0)} \right) / \sqrt{T} \). At charge neutrality, the resulting viscosities are given by \( \eta_{\alpha \beta} = N^2 C_{\alpha \beta} (k^0 T) (T/T_0)^{\epsilon_{\alpha \beta} / \phi_{\alpha \beta}} \), where \( C_{\alpha \beta} \) are numerical coefficients of order unity. The exponents \( \phi_{\alpha \beta} \) coincide with the results of the above scaling analysis.

The linear-response solution of the quantum kinetic equation yields the entropy density in the scaling form, \( s = N C_k (k^0 h^2 (T/T_0)^{1-\phi} / T_i \). Due to the linear dispersion in the \( y \)-direction, the velocity component \( v_y \) at the scale \( T \) is larger than \( v_x \). The viscosity coefficient \( \eta_{xxyy} \) describes the flow of the momentum \( k_x \) with the larger velocity component \( v_y \), leading to the diverging ratio \( \eta_{xxyy}/s \). In contrast, the viscosity coefficient \( \eta_{yyxx} \) corresponds to the flow of the momentum \( k_y \) with the much slower velocity \( v_x \), leading to the violation of the bound. The ratios of the viscosity coefficients to the entropy density are shown in Fig. 3.

**Summary.** In this letter, we have shown that anisotropic Dirac systems are fascinating new materials with unparalleled transport properties in the hydrodynamic regime. The same material exhibits both insulating and metallic behavior depending on the direction of the applied electrical field. Furthermore, the shear viscosity is represented by a fourth rank tensor with six independent components with the ratio \( \eta_{\alpha \beta} / s \) that may either vanish or diverge, see Fig. 3. In the former case, the universal bound (1) appears to be violated. We demonstrated that these viscosity tensor elements can be measured via viscous thermal flow, where the more electrically conducting direction is also the direction of larger thermal conductivity. The thermal flow in the direction with the linear spectrum is expected to be highly susceptible to turbulence and should lead to large transverse temperature variations. (see right panel of Fig.2). Similar behavior will also occur in other anisotropic systems, such as critical bosonic systems at a Lifshitz point [39].

The ratio \( \eta/s \) (1) was introduced to have a measure for the strength of interaction in a quantum fluid. Our analysis shows that the violation of the bound for anisotropic systems is not necessarily an indicator for extreme interactions, but reflects the fact that \( \eta/s \) is no longer an appropriate measure of the interaction strength in anisotropic systems. We have suggested a generalization of the lower bound that takes into account the anisotropy. The generalized bound (2) offers a better quantification of fluid interactions. Nevertheless, the smallness of the viscosity to entropy density ratio \( \eta/s \ll h/(4\pi B) \) for anisotropic systems remains a strong indicator for a tendency towards turbulent flow.

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[1] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor & Francis Ltd, 1990).

[2] M. J. M. de Jong and L. W. Molenkamp, Phys. Rev. B 51, 13389 (1995).

[3] C. Chaín and T. Schäfer, Phys. Rev. A 87, 023629 (2013).

[4] T. Enss, R. Haussmann, and W. Zwerger, Ann. Phys. (Amsterdam) 326, 770 (2011).

[5] T. Schäfer, Annu. Rev. Nucl. Part. Sci. 64, 124 (2014).

[6] E. Shuryak, Prog. Part. Nucl. Phys. 53, 273 (2004).

[7] R. N. Gurzhi, Zh. Eksp. Teor. Fiz. 44, 771 (1963), [Sov. Phys. JETP 17, 521 (1963)].

[8] R. N. Gurzhi, Usp. Fiz. Nauk 94, 689 (1968), [Sov. Phys. Usp. 11, 255 (1968)].

[9] J. Crossno, J. K. Shi, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, Science 351, 1055 (2016).

[10] L. A. Ponomarenko, G. Falkovich, I. V. Grigorieva, L. S. Levitov, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, and A. K. Geim, Nature Physics 13, 1182 (2017).

[11] S. Cremonini, Mod. Phys. Lett. B 30, 1058 (2016).

[12] H. Guo, E. Ilseven, G. Falkovich, and L. S. Levitov, PNAS 114, 3068 (2017).

[13] R. Krishna Kumar, D. A. Bandurin, F. M. D. Pellegrino, Y. Cao, A. Principi, H. Guo, G. H. Auton, M. B. Shalom, L. A. Ponomarenko, G. Falkovich, I. V. Grigorieva, L. S. Levitov, M. Polini, and A. K. Geim, Nature Physics 13, 1182 (2017).

[14] D. A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Science 351, 1055 (2016).

[15] L. S. Levitov and G. Falkovich, Nature Phys. 12, 672 (2016).

[16] M. Titov, R. V. Gorbachev, B. N. Narozhny, T. Tudorovskiy, M. Schütt, P. M. Ostrovsky, I. V. Gornyi, A. D. Mirlin, M. I. Katsnelson, K. S. Novoselov, A. K. Geim, and L. A. Ponomarenko, Phys. Rev. Lett. 111, 166601 (2013).

[17] B. N. Narozhny, I. V. Gornyi, A. D. Mirlin, and J. Schmalian, Annu. Rev. Condens. Matter 30, 053001 (2018).

[18] B. N. Narozhny, I. V. Gornyi, A. D. Mirlin, and J. Schmalian, Annu. Rev. Condens. Matter 30, 053001 (2018).

[19] J. Crossno, J. K. Shi, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, Science 351, 1055 (2016).

[20] H. Guo, E. Ilseven, G. Falkovich, and L. S. Levitov, PNAS 114, 3068 (2017).

[21] R. Krishna Kumar, D. A. Bandurin, F. M. D. Pellegrino, Y. Cao, A. Principi, H. Guo, G. H. Auton, M. B. Shalom, L. A. Ponomarenko, G. Falkovich, I. V. Grigorieva, L. S. Levitov, M. Polini, and A. K. Geim, Nature Physics 13, 1182 (2017).

[22] D. A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Science 351, 1055 (2016).

[23] L. S. Levitov and G. Falkovich, Nature Phys. 12, 672 (2016).

[24] M. Titov, R. V. Gorbachev, B. N. Narozhny, T. Tudorovskiy, M. Schütt, P. M. Ostrovsky, I. V. Gornyi, A. D. Mirlin, M. I. Katsnelson, K. S. Novoselov, A. K. Geim, and L. A. Ponomarenko, Phys. Rev. Lett. 111, 166601 (2013).

[25] B. N. Narozhny, I. V. Gornyi, A. D. Mirlin, and J. Schmalian, Annu. Rev. Condens. Matter 30, 053001 (2018).

[26] B. N. Narozhny, I. V. Gornyi, A. D. Mirlin, and J. Schmalian, Annu. Rev. Condens. Matter 30, 053001 (2018).

[27] J. M. Kadow, G. Vignale, M. Carrega, and M. Polini, Phys. Rev. B 93, 125410 (2016).

[28] D. E. Sheehy and J. Schmalian, Phys. Rev. Lett. 99, 226803 (2007).

[29] R. Dornheim, M. Luban, and S. Smirikman, Phys. Rev. Lett. 35, 1675 (1975).

[30] E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. S. Levitov, M. Polini, and A. K. Geim, Nature Physics 13, 1182 (2017).

[31] D. A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Science 351, 1055 (2016).

[32] L. S. Levitov and G. Falkovich, Nature Phys. 12, 672 (2016).

[33] M. Titov, R. V. Gorbachev, B. N. Narozhny, T. Tudorovskiy, M. Schütt, P. M. Ostrovsky, I. V. Gornyi, A. D. Mirlin, M. I. Katsnelson, K. S. Novoselov, A. K. Geim, and L. A. Ponomarenko, Phys. Rev. Lett. 111, 166601 (2013).

[34] B. N. Narozhny, I. V. Gornyi, A. D. Mirlin, and J. Schmalian, Annalen der Physik 529, 1700043 (2017).

[35] A. Lucas and K. C. Fong, Journal of Physics: Condensed Matter 30, 053001 (2018).

[36] P. J. W. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie, Science 351, 1061 (2016).

[37] J. Gooth, F. Menges, C. Shekhar, V. Stiβ, N. Kumar, Y. Sun, U. Drechsler, R. Zierold, C. Felser, and B. Gotsmann, (2017), arXiv:1706.05925.