Investigation of the interference pattern from a spherical particle at different viewing angles

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Abstract. The work aim is to investigate the influence of the parameters of the laser interference method experimental setup on the obtained images. Using computer modeling based on the diffraction theory and physical experiment, the change in the parameters of the interference pattern for different viewing angles had been shown. The results can be used to develop a single algorithm for processing images of the laser interference method.

1. Introduction
The laser interference method (LIM), also called IPI, was proposed by Rof in 1993. This method is based on recording the image of particles in scattered light using a CCD or CMOS camera, which allows to simultaneously determine the particles velocities and sizes. Scattering by large (relative to the radiation wavelength) particles is described by two mechanisms: reflection, rays refraction by a particle within the geometric optics laws, as well as wave diffraction on a particle.

To obtain an image interference pattern of flying liquid droplets or air bubbles, data recording must be performed with a defocused image using a digital camera. The resulting interference patterns are a circle with a set of alternating dark and light fringes. By the number of interference fringes, it is possible to determine the bubble diameter and using the cross-correlation method, its velocity.

The interference fringes number depends on the camera viewing angle $\theta$ relative to the optical axis. The larger the angle $\theta$, the smaller the number of interference fringes. The angle $\theta$ varies in the range from 0 to 90 degrees.

The two-beam interferometric particle imaging (DIPI) method has been developed to measure the size of a nontransparent metal droplet. It is adapted from traditional interferometric particle imaging (IPI) for transparent particles. Measuring the size of a micron-sized metal drop is very popular in industrial applications, for example, in 3D printing [1].

In recent years, interest in the laser interference method has increased due to the requirement to measure the characteristics of droplets and micron-sized particles [2–4]. This method allows the analysis of a larger flow area compared to point methods. However, the analysis of domestic and foreign literature shows that despite the theoretical validity and analytical logic of interferometric imaging methods, there is still no single approach and algorithm for calculating the particle diameter distribution and the vector field of particle velocities in the flow, especially at the droplets high density. In almost all calculation methods, the initial conditions come to the fore: the environment in
which the research is carried out, the equipment used during the experiment, even the human factor plays an important role (the accuracy of positioning the laser plane, focusing the video camera, competence in this science field). To build the single algorithm for processing images of the laser interference method, it is necessary to determine the general characteristics of the measuring setup parameters influence on the resulting images.

2. Investigation of the influence of the observation angle on the interference pattern using mathematical modeling

In most sources, the resulting interference pattern (IP) is considered from the point of geometric optics view. The phase difference between the transmitted and reflected beam is defined as

$$\delta_{\text{r}} - \delta_{\text{t}} = 2 \frac{\pi \cdot d}{\lambda} \cdot (n \cdot \cos(\alpha') - \cos(\alpha') - n \cdot \cos(\alpha')$$

$$= 2 \frac{\pi \cdot d}{\lambda} \left[ \sqrt{n^2 - 2n \cdot \cos(\theta/2) + 1} - n \cdot \sin(\theta/2) \right],$$

(1)

where \(d\) – bubble diameter, \(\lambda\) – radiation wavelength, \(n\) – relative refractive index.

The bubble diameter is calculated using the following equation

$$d = \frac{2 \gamma N}{n \alpha} \frac{1}{\cos(\theta/2) - \frac{\sin(\theta/2)}{\sqrt{n^2 - 2n \cos(\theta/2) + 1}}},$$

(2)

where \(N\) – interference fringes number, \(\lambda\) – radiation wavelength, \(n\) – relative refractive index, \(\theta\) - scattering angle, \(\alpha\) – aperture angle.

Let’s consider the diameter and velocity diagnostics of a single optically transparent spherical particle from the point of diffraction theory view. So let’s consider the interference phenomenon in the scattering of a Gaussian beam on a moving spherical particle. In this case, the wave is decomposed into two components and the scattered radiation is the interference of diffracted and transmitted radiation.

$$Ed = \int \frac{\exp\left(-i \cdot k \cdot \frac{x - vt}{R} + \frac{x^2 + y^2}{2R(z)} + \varphi(z)\right)}{w(z)} \, dx \, dy \, dz,$$

$$Eg = \frac{w_0}{w(z)} \exp\left[i \cdot \left(\frac{k \cdot z + k \cdot (x^2 + y^2)}{2R(z)} + \varphi(z)\right) - \frac{x^2 + y^2}{w^2(z)}\right],$$

(3)

$$R_0 = \frac{k \cdot w_0^2}{2}, \quad w(z) = w_0 \left[1 + \left(\frac{z}{R_0}\right)^2\right], \quad R(z) = z \cdot \left[1 + \left(\frac{R_0}{z}\right)^2\right], \quad \varphi(z) = \arctg\left(\frac{z}{R_0}\right),$$

$$I = Ed \cdot Ed^* + Eg \cdot Eg^* + 2 \, \text{Re}(Eg \cdot Ed^*),$$

where \(Ed\) – diffracted wave tension, \(Eg\) – Gaussian beam tension, \(I\) – diffraction pattern intensity, \(r\) - particle radius, \(v\) – particle velocity, \(t\) – time, \(k\) – wave number, \(R\) – distance from the particle to the registration plane, \(x\) and \(y\) – coordinates in the registration plane, \(\eta\) and \(\xi\) – coordinates in the diffraction plane, \(w_0\) – beam waist, \(w\) – beam radius, \(\varphi\) – phase shift along the beam axis.
The simulation was performed under the following conditions: \( x, y = \text{const}, \ t = \text{var} \) and the dependencies are orthonormalized in such way that not only the main maximum, but also the rest of the distribution is visible. The following parameters were used for the construction: the radiation wavelength is 0.6328 \( \mu \text{m} \), the particle radius is 50 \( \mu \text{m} \), the laser beam radius is 1.85 mm, the particle velocity is \( 1153 \times 10^{-4} \text{mm/s} \), the distance from the particle to the registration plane is 480 mm.

The particle begins its movement from the center of the laser beam. The obtained results of modeling the dependence of the scattered radiation intensity on time are shown in figure 1.

By modeling interference patterns in this way, it becomes clear that the particle in the laser plane, as well as the viewing angle (its analogue in modeling is the variable \( x \)), has a great influence on the IP parameters.

It is possible to consider the intensity dependences of the radiation scattered on the particle from the observation point at object different positions. In this case, the coincidence of the particle center and the Gaussian beam corresponds to the time \( t = 0 \) s. The simulation results are presented in figure 2. From the dependencies in figure 2, it can be seen that if the observation angle is chosen incorrectly, it can not get the interference pattern at all.
3. Experimental investigation of the observation angle influence on the interference pattern

Before assembling a laboratory sample of the setup, laser modules were checked. Results are demonstrated in figure 2. A signal from each module is present to a greater extent only in one of three channels. Thus selection of RGB laser sources has been carried out correctly.

Figure 3 shows a scheme of an optical electronic setup for simultaneous size and velocity determination of air bubbles in water by the laser interference method. The radiation of He-Ne laser (1) with the wavelength of \( \lambda = 0.6328 \) µm and a power of 10 mW passes through an optical system (2) for forming a laser plane consisting of spherical and cylindrical lenses. Next, the radiation is directed through a glass plate (3) made of K8 glass with a refractive index \( n = 1.52 \), with the gas bubble (5) under research. The radiation scattered on the bubble is recorded using a digital video camera (6) and transmitted to a computer (7). The bubbles diameters were measured in advance using a comparator with an error of \( 10^{-3} \) mm.

The video is recorded when the image is defocused, which is the interference pattern. The image is transmitted to the computer using special software with the Gigabit Ethernet interface. Next, filtering and subsequent processing of the obtained images is performed using specially developed programs.
A rail with the video camera is installed on the rotary device. Due to this, the video camera has the ability to move around the glass plate relative to the bubble center. Figure 4 shows the IP when the camera is rotated by different values of the angle $\theta$ relative to the optical axis for a bubble with a diameter of 50 $\mu$m.

As can be seen from the obtained IP, the fringes number decreases with increasing angle $\theta$. Figure 5 shows a plot of the interference fringes number dependence on the camera rotation angle for the simulated and experimentally obtained values.

![Figure 3. Scheme of the experimental setup.](image1)

**Figure 3.** Scheme of the experimental setup.

![Figure 4. IP when the camera is rotated at different angles $\theta$.](image2)

**Figure 4.** IP when the camera is rotated at different angles $\theta$. 

\[ \theta = 20^\circ \quad \theta = 30^\circ \quad \theta = 40^\circ \quad \theta = 50^\circ \quad \theta = 60^\circ \]
4. Conclusion
The laser interference method makes it possible to determine the bubbles size from 30 µm to 2 mm. The maximum measured velocity of gas bubbles in water at camera shooting speed of 26 frames per second is 21 cm/s. When processing images using the laser interference method, it is necessary to take into account the viewing angle influence, since its inaccurate measurement can make a strong error in the results.

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