The finiteness of the four dimensional antisymmetric tensor field model in a curved background

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Abstract. A renormalizable rigid supersymmetry for the four dimensional antisymmetric tensor field model in a curved space–time background is constructed. A closed algebra between the BRS and the supersymmetry operators is only realizable if the vector parameter of the supersymmetry is a covariantly constant vector field. This also guarantees that the corresponding transformations lead to a genuine symmetry of the model. The proof of the ultraviolet finiteness to all orders of perturbation theory is performed in a pure algebraic manner by using the rigid supersymmetry.

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1 Introduction

One of the most exciting investigations of the last decade was the study of certain problems arising in gauge theory, which led to important developments and deep insights into the topology and geometry of low dimensional manifolds. A well-known example is the analysis of topological invariants \cite{1,2} of four dimensional manifolds by Donaldson \cite{3,4}. Another contribution was given by Witten \cite{5}, namely the construction of the so-called topological Yang–Mills model on a four dimensional manifold. After that many such topological models, like Chern–Simons theory, BF models and others have been discussed and many new features, as the description of invariants of knots in terms of the Chern–Simons theory \cite{6}, have been found.

The main property of topological field theories \cite{7} is that the observables only depend on the global structure of the space–time manifold on which the model is defined. Particularly this implies that these quantities are independent of any metric which may be used to construct the classical theory. There exist two different types of topological field theories, namely the Witten–type models and the Schwarz–type models. The first one is characterized by the fact, that the whole gauge fixed action can be written as a BRS variation, whereas for the second one only the gauge–fixing part of the action is given by a BRS variation. The most famous example of a Witten–type model is the topological Yang–Mills theory and representatives of the Schwarz–type models are the Chern–Simons theory and the BF models.

In particular, the topological BF models describe the coupling of an antisymmetric tensor field to the Yang–Mills field strength. Chronologically, such models have been first used in interacting string theories and nonlinear sigma models \cite{8,9,10,11,12,13,14}. Their topological nature has been analyzed much later \cite{1,2}. Furthermore, these models are also studied because of their connection with lower dimensional quantum gravity. Especially, the Einstein–Hilbert gravity in three space–time dimensions, with and without cosmological constant, can be naturally formulated in terms of the BF models \cite{15,16}.

In general, it is known that the BF models, due to the presence of zero modes, require a highly nontrivial quantization \cite{17,18,19,20,21}, which implies several ghost generations for the gauge–fixing procedure. This can be done in an elegant manner by using the Batalin–Vilkovisky quantization procedure \cite{22}.

The aim of this work is to analyze the perturbative finiteness of the four dimensional BF model. We generalize the discussion already carried out in the flat space–time limit \cite{20} and take into consideration the presence of a curved background. For this purpose, we will use the concept of the extended BRS symmetry \cite{23} and we will follow the way demonstrated for the Chern–Simons theory \cite{24}. A renormalizable rigid supersymmetry \cite{25,26,27} will play an important role which, in the flat space–time limit, is a common feature of many topological field theories \cite{27,28,29,30,31}. We will see that the algebra between the BRS operator and the generators of translation and rigid supersymmetry closes on–shell. Furthermore, the closure of the algebra also requires a constraint for the corresponding infinitesimal supersymmetry parameter. This fact limited our analysis to be only valid for a curved manifold admitting a gradient vector.
Our present work is organized as follows: in Section 2 we give an overview concerning the classical algebraic properties of the four dimensional BF model. Next, we construct the rigid supersymmetry and analyze the off–shell algebra. In Section 3 we will discuss the stability of the model by using cohomology techniques, and see that the symmetries do not allow any deformation of the classical action. The last section is devoted to the study of anomalies, which will complete our proof of the perturbative finiteness. Some details concerning the trivial counterterms can be found in the final appendix.

2 The classical BF model

The BF models can be defined on manifolds $\mathcal{M}$ in arbitrary dimensions $(n + 2)$, with a gauge group $G$, according to \[1, 2, 7, 31\]

\[S_{BF} = Tr \int_\mathcal{M} BF = \frac{1}{2n!} Tr \int d^{n+2}x \varepsilon^{\mu_1 \cdots \mu_{n+2}} B_{\mu_1 \cdots \mu_n} F_{\mu_{n+1} \mu_{n+2}} , \quad (2.1)\]

where the two–form

\[F = dA + \frac{1}{2}[A, A] = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu , \quad (2.2)\]

is the Yang–Mills field strength of the gauge connection one–form $A = A_\mu dx^\mu$ and the field

\[B = \frac{1}{n!} B_{\mu_1 \cdots \mu_n} dx^{\mu_1} \cdots dx^{\mu_n} \]

is a $n$–form. Of course, this action being metric independent has a topological character \[1, 2\].

2.1 The four dimensional BF model in flat space–time

In terms of differential forms we start with the topological invariant classical action on an arbitrary space–time four–manifold $\mathcal{M}$

\[S_{inv} = Tr \int_\mathcal{M} BF = \int_\mathcal{M} B^a F^a , \quad (2.3)\]

where the two–forms for the antisymmetric tensor field $B^a$ and the field curvature $F^a$ are given by

\[B^a = \frac{1}{2} B_{\mu\nu} dx^\mu dx^\nu , \]

\[F^a = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu = \frac{1}{2} (\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + f^{abc} A^{b}_\mu A^{c}_\nu) dx^\mu dx^\nu , \quad (2.4)\]

with the gauge field $A^a_\mu$. The fields belong to the adjoint representation of the gauge group $G$, assumed to be compact and semi–simple\footnote{Gauge group indices are denoted by Latin letters $(a, b, c, \ldots)$ and refer to the adjoint representation, $[T^a, T^b] = f^{abc} T^c$, $Tr(T^a T^b) = \delta^{ab}$}.

In the case of flat space–time, with a metric $\eta_{\mu\nu}$, the action (2.3) can be rewritten as

\[S_{inv} = \frac{1}{4} \int d^4x \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} B^a_{\rho\sigma} , \quad (2.5)\]
where $\varepsilon^{\mu\nu\rho\sigma}$ denotes the totally antisymmetric tensor of rank four.

The action (2.5) possesses two kinds of invariances, given by

\[
\begin{align*}
\delta^{(1)} A^a_\mu &= -(D_\mu \theta)^a = -\left( \partial_\mu \theta^a + f^{abc} A^b_\mu \theta^c \right), \\
\delta^{(1)} B^a_{\mu\nu} &= f^{abc} \theta^b B^c_{\mu\nu},
\end{align*}
\]  

(2.6)

and

\[
\begin{align*}
\delta^{(2)} A^a_\mu &= 0, \\
\delta^{(2)} B^a_{\mu\nu} &= -(D_\mu \varphi_\nu - D_\nu \varphi_\mu)^a,
\end{align*}
\]  

(2.7)

with $\theta^a$ and $\varphi^a_\mu$ as local parameters for the two symmetries. Remark, that the second symmetry contains zero modes [17, 18, 19], which we will take into account in the next subsection for the general case of a curved space–time.

### 2.2 The BF model in curved space–time

From now on, we are discussing the BF model on an arbitrary four–manifold, endowed with an Euclidean metric $g_{\mu\nu}$. Rewriting (2.3) in components one obtains for the invariant classical action in curved space–time

\[
S_{\text{inv}} = \frac{1}{4} \int d^4 x \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} B^a_{\rho\sigma},
\]  

(2.8)

where the symbol $\varepsilon^{\mu\nu\rho\sigma}$ now represents, contrary to that one in (2.3), a totally antisymmetric tensor density with weight 1. Furthermore, the determinant of the metric $g = \det(g_{\mu\nu})$ has weight 2 and the volume element density $d^4 x$ carries weight $-1$. The relation between the contravariant and covariant $\varepsilon$–tensor densities is given by

\[
\varepsilon_{\mu\nu\rho\sigma} = g_{\mu\alpha} g_{\nu\beta} g_{\rho\gamma} g_{\sigma\delta} \frac{1}{g} \varepsilon^{\alpha\beta\gamma\delta},
\]  

(2.9)

where the weight of $\varepsilon_{\mu\nu\rho\sigma}$ is $-1$. Therefore, the action (2.8) is, besides the symmetries (2.6) and (2.7), also invariant under diffeomorphisms with the corresponding infinitesimal parameter $\varepsilon^\mu$.

In the following we will use the BRS formalism [32], which requires the introduction of Faddeev–Popov ghosts $c$ and $\xi$ of ghost number one for the infinitesimal parameters $\theta$ and $\varphi$. Collecting both symmetries in (2.6) and (2.7) we get, in a first step, for the BRS transformations of the gauge field $A^a_\mu$ and the antisymmetric tensor field $B^a_{\mu\nu}$

\[
\begin{align*}
sA^a_\mu &= -(D_\mu c)^a = -\left( \partial_\mu c^a + f^{abc} A^b_\mu c^c \right), \\
sB^a_{\mu\nu} &= -(D_\mu \xi_\nu - D_\nu \xi_\mu)^a + f^{abc} c^b B^c_{\mu\nu},
\end{align*}
\]  

(2.10)

which leave the action (2.8) invariant. A special care has to be taken for the $\xi$–symmetry due to the presence of the so–called reducible symmetry in the BRS transformations given above. To show this fact, we rewrite the BRS transformation of the $B$–field in terms of
forms and analyze only the part containing \( \xi \), namely
\[
s_\xi B^a = (D\xi)^a = -(DD\phi)^a = -f^{abc} F^b \phi^c = -f^{abc} \delta S^\text{inv}_{B^b} \phi^c ,
\]
which vanishes on–shell. This symmetry is said to be on–shell reducible. Therefore, the whole set of BRS transformations is given by
\[
\begin{align*}
  s & A^a_\mu = -(D_\mu c)^a = -(\partial_\mu c^a + f^{abc} A^b_\mu c^c) , \\
  s & B^a_{\mu\nu} = -(D_\mu \xi^\nu - D_\nu \xi^\mu)^a + f^{abc} c^b B^c_{\mu\nu} , \\
  s & c^a = \frac{1}{2} f^{abc} c^b c^c , \\
  s & \xi^a_\mu = (D_\mu \phi)^a + f^{abc} c^b \xi^c _\mu , \\
  s & \phi^a = f^{abc} c^b \phi^c .
\end{align*}
\]
(2.12)
After some calculations one finds
\[
s^2 B^a_{\mu\nu} = -\frac{1}{2} \delta_{\mu\nu\rho\sigma} f^{abc} \delta S^\text{inv}_{B^b} \phi^c 
\text{ and } s^2 = 0 \text{ for the other fields .}
\] (2.13)

The quantization of the model is not straightforward due to the presence of zero modes [17, 18, 19] and can be performed by using the Batalin–Vilkovisky scheme [22]. In the present work we will not follow this way, but will use another equivalent procedure [31]. Following [20], the gauge–fixing action in the Landau–type gauge, adapted to the case of curved space–time, is given by
\[
S_{gf} = s \int d^4x \sqrt{g} \left[ c^a g^{\mu\nu} \nabla_\mu A^a_\nu + g^{\mu\alpha} g^{\nu\beta} \tilde{c}^a \nabla_\alpha B^a_{\mu\nu} + \tilde{\phi}^a g^{\mu\nu} \nabla_\mu \xi^a _\nu \
+ \tilde{\xi}^a_\mu g^{\mu\nu} \nabla_\nu c^a + \tilde{\phi}^a \chi^a \right] ,
\]
(2.14)
with the covariant space–time derivative \( \nabla_\mu \) defined by
\[
\nabla_\mu X_\nu = \partial_\mu X_\nu - \Gamma_\mu^\lambda \nabla_\nu X_\lambda ,
\]
where \( \Gamma_\mu^\lambda \) denotes the Christoffel symbol,
\[
\Gamma_\mu^\lambda = \frac{1}{2} g^{\lambda\rho} (\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}) ,
\]
(2.15)
which is symmetric in the lower indices assuming a torsion–free manifold. Notice that the gauge–fixing action in (2.14) contains inhomogeneous gauge conditions [33] for the fields \( B^a_{\mu\nu} \) and \( \xi^a_\mu \).

The corresponding antighosts and Lagrange multiplier fields are introduced in BRS–doublets
\[
\begin{align*}
  s & \bar{c}^a = b^a , \quad s b^a = 0 , \\
  s & \bar{\xi}^a_\mu = h^a_\mu , \quad s h^a_\mu = 0 , \\
  s & \bar{\phi}^a = \omega^a , \quad s \omega^a = 0 , \\
  s & c^a = \lambda^a , \quad s \lambda^a = 0 .
\end{align*}
\]
(2.16)

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5The BRS transformations of the gauge ghost \( c^a \), the vector ghost field \( \xi^a_\mu \) and the scalar ghost field \( \phi^a \) are defined by the requirement of the nilpotency of \( s \).

6Remark, that the metric \( g_{\mu\nu} \) is covariantly constant, i.e. \( \nabla_\rho g_{\mu\nu} = 0 \).
The gauge–fixing action \((2.14)\) depends on the metric explicitly and hence it has no more a topological character, but it is still invariant under diffeomorphisms. Furthermore, the metric plays the role of a gauge parameter which we also let transform as a BRS–doublet

\[
sg_{\mu\nu} = \hat{g}_{\mu\nu} \; , \; s\hat{g}_{\mu\nu} = 0 \; ,
\]

\hspace{1cm} (2.17)

in order to guarantee its non–physical meaning [24]. This is understood as the concept of extended BRS symmetry [23]. Note that the BRS transformation of the inverse of the metric is given by

\[
sg_{\mu\nu} = -g^{\mu\alpha}g^{\nu\beta}\hat{g}_{\alpha\beta} = -\hat{g}_{\mu\nu}.
\]

The canonical dimensions of the fields, the assigned ghost numbers and the corresponding weights are given in Table 1.

|  | \(A^a_\mu\) | \(B^a_{\mu\nu}\) | \(c^a\) | \(\bar{c}^a\) | \(h^a\) | \(\xi^a_\mu\) | \(\bar{\xi}^a_\mu\) | \(\phi^a\) | \(\bar{\phi}^a\) | \(\omega^a\) | \(e^a\) | \(\lambda^a\) | \(g_{\mu\nu}\) | \(\hat{g}_{\mu\nu}\) | \(\sqrt{g}\) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| dim | 1 | 2 | 0 | 2 | 2 | 1 | 1 | 1 | 0 | 2 | 2 | 2 | 2 | 0 | 0 | 0 |
| \(\phi\pi\) | 0 | 0 | 1 | -1 | 0 | 1 | -1 | 0 | 2 | -2 | -1 | 0 | 1 | 0 | 1 | 0 |
| weight | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Table 1: Dimensions, ghost numbers and weights.

Due to the gauge–fixing action term \((2.14)\) the BRS transformation of the \(B\)–field is no more nilpotent on–shell, since some of the terms in the equation of motion, stemming from the gauge–fixing part, are missing. In order to reestablish the nilpotency for the \(B\)–field one has to modify its BRS transformation according to

\[
sB^a_{\mu\nu} = -(D_\mu \xi_\nu - D_\nu \xi_\mu)^a + \varepsilon^{abc}B^c_{\mu\nu} + \varepsilon_{\mu\nu\rho\sigma}f^{abc}\sqrt{g}g^{\rho\alpha}g^{\sigma\beta}(\partial_\alpha \bar{\xi}^b_\beta)\phi^c \; .
\]

\hspace{1cm} (2.18)

This requires to add a further term in the gauge fixed action \((S_{inv} + S_{gf})\) to make it invariant under the modified BRS transformation\(^7\)

\[
S_{inv} + S_{gf} = \frac{1}{4} \int d^4x \varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma} - s \int d^4x \sqrt{g} \left[ g^{\mu\nu}(\partial_\mu \bar{c}^a_\nu)A^a_\nu + g^{\mu\alpha}g^{\nu\beta}(\partial_\alpha \bar{\xi}^a_\beta)B^a_{\mu\nu}
- g^{\mu\nu}(\partial_\mu \bar{\phi}^a_\nu)\xi^a_\nu - g^{\mu\nu}\bar{\xi}^a_\mu \partial_\nu e^a - \bar{\phi}^a \lambda^a \right]
- \frac{1}{2} \int d^4x \varepsilon^{\mu\nu\rho\sigma}(\partial_\mu \bar{c}^a_\nu)(\partial_\rho \bar{c}^a_\sigma)\phi^c \; ,
\]

\hspace{1cm} (2.19)

with

\[
s(S_{inv} + S_{gf}) = 0 \; .
\]

\hspace{1cm} (2.20)

Remark, that the last term in \((2.19)\) does not disturb the topological character of the theory.

The BRS transformations of the fields introduced so far read:

\[
sA^a_\mu = -(D_\mu c^a)^a = -(\partial_\mu c^a + f^{abc}A^b_\mu c^c)^a \; ,
\]

\hspace{1cm} (2.19)

\hspace{1cm} \(^7\)In \((2.19)\) we have performed an integration by parts and have used the fact that the Christoffel symbol is symmetric in the lower indices.
\[ s B^a_{\mu\nu} = -(D_\mu \xi_\nu - D_\nu \xi_\mu)^a + f^{abc} B^b_{\mu\nu} + \varepsilon_{\mu\nu\rho\sigma} f^{abc} \sqrt{g} g^{\alpha\beta} \partial_\alpha \xi_\beta \phi^c, \]
\[ s c^a = \frac{1}{2} f^{abc} c^b c^c, \]
\[ s c^a = b^a, \quad s b^a = 0, \]
\[ s g_{\mu\nu} = \tilde{g}_{\mu\nu}, \quad s \tilde{g}_{\mu\nu} = 0. \]

These transformations are nilpotent on-shell:
\[ s^2 B^a_{\mu\nu} = -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} f^{abc} \frac{\delta (S_{inv} + S_{gf})}{\delta B^b_{\rho\sigma}} \phi^c \quad \text{and} \quad s^2 = 0 \quad \text{for the other fields}. \]

### 2.3 Supersymmetry–like transformations

Besides the BRS symmetry and the invariance under diffeomorphisms, the action could possess a further supersymmetric–like invariance given by\(^8\)

\[ \delta^S_{(r)} A^a_\mu = -\varepsilon_{\mu\rho\sigma} \tau^\nu \sqrt{g} g^{\rho\alpha} \partial_\alpha \xi_\mu, \]
\[ \delta^S_{(r)} B^a_{\mu\nu} = -\varepsilon_{\mu\rho\sigma} \tau^\rho \sqrt{g} g^{\sigma\alpha} \partial_\alpha \xi_\mu, \]
\[ \delta^S_{(r)} c^a = -\tau^\mu A^a_\mu, \]
\[ \delta^S_{(r)} c^a = 0, \]
\[ \delta^S_{(r)} b^a = L_\tau \xi_\mu = \tau^\mu \partial_\mu \xi_\mu, \]
\[ \delta^S_{(r)} \xi_\mu = \tau^\mu B^a_{\mu\nu}, \]
\[ \delta^S_{(r)} \xi_\mu = \tau^\mu \xi_\mu, \]
\[ \delta^S_{(r)} \xi_\mu = 0, \]
\[ \delta^S_{(r)} \phi^a = L_\tau \phi^a = \tau^\mu \partial_\mu \phi^a, \]
\[ \delta^S_{(r)} \phi^a = \tau^\mu \phi^a, \]
\[ \delta^S_{(r)} \phi^a = 0, \]
\[ \delta^S_{(r)} \lambda^a = L_\tau \lambda^a = \tau^\mu \partial_\mu \lambda^a, \]
\[ \delta^S_{(r)} \lambda^a = \tau^\mu \lambda^a, \]
\[ \delta^S_{(r)} g_{\mu\nu} = L_\tau g_{\mu\nu} = \tau^\rho \partial_\rho g_{\mu\nu}, \]
\[ \delta^S_{(r)} g_{\mu\nu} = \tau^\rho \partial_\rho g_{\mu\nu} + (\partial_\mu \tau^\rho) g_{\rho\nu} + (\partial_\nu \tau^\rho) g_{\mu\rho}, \]

with the corresponding infinitesimal parameter \( \tau^\mu \) and the Lie derivative \( L_\tau \). The resultant algebra between the BRS operator \( s \), the generator of diffeomorphisms \( \delta^D_{(s)} \) and the

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\(^8\)In order to get fermionic generators we assign to the the parameter of the diffeomorphisms \( \varepsilon^\mu \) ghost number 1 and to the parameter of the supersymmetry–like transformations \( \tau^\mu \) ghost number 2.
generator of superdiffeomorphisms $\delta^S_{(\tau)}$ closes on–shell

$$\{s, s\} = 0 + \text{equations of motion},$$
$$\{s, \delta^S_{(\tau)}\} = \mathcal{L}_\tau + \text{equations of motion},$$
$$\{\delta^S_{(\tau)}, \delta^S_{(\tau')}\} = 0,$$  \hfill (2.24)

whereby the gauge fixed action $(S_{\text{inv}} + S_{gf})$ obeys the following symmetries:

$$s(S_{\text{inv}} + S_{gf}) = \delta^D_\varepsilon (S_{\text{inv}} + S_{gf}) = \delta^S_{(\tau)} (S_{\text{inv}} + S_{gf}) = 0.$$  \hfill (2.25)

At this stage we have to make some comments about the algebra concerning the parameter $\tau^\mu$ of the susy–like transformations. Contrary to the case of flat space–time, where one has instead of $\tau^\mu$ a constant parameter for the translations, the algebra in the present case does not close a priori. In particular, for the antisymmetric tensor field $B^a_{\mu\nu}$ one has

$$\{s, \delta^S_{(\tau)}\} B^a_{\mu\nu} = \mathcal{L}_\tau B^a_{\mu\nu} + \varepsilon_{\mu\rho\sigma} \tau^\rho \frac{\delta (S_{\text{inv}} + S_{gf})}{\delta A^a_\sigma} - \varepsilon_{\mu\rho\sigma} f^{\rho\delta\epsilon} \sqrt{g} g^{\sigma\alpha} (\nabla_\alpha \tau^\rho) \bar{\phi}^b \phi^c.$$  \hfill (2.26)

In order to implement a closed (at least on–shell) algebra, the last term in (2.26), which is quadratic in the quantum fields, has to vanish, since it cannot be absorbed in the equation of motion. Hence, we require

$$g^{\rho\alpha} (\nabla_\alpha \tau^\sigma) - g^{\sigma\alpha} (\nabla_\alpha \tau^\rho) = 0.$$  \hfill (2.27)

Therefore, the susy–like symmetry is only realizable on manifolds where (2.27) has a solution. This guarantees that the algebra closes on–shell on the Lie derivative. In particular, a solution of (2.27) is given by

$$\tau^\mu = g^{\mu\nu} \partial_\nu \Lambda.$$  \hfill (2.28)

For completeness we remark that also the closure of $\{\delta^S, \delta^S\}$ is disturbed by a term of this kind

$$\{\delta^S_{(\tau)}, \delta^S_{(\tau')}\} A^a_\mu = \varepsilon_{\mu\rho\sigma} \sqrt{g} g^{\rho\alpha} \bar{\phi}^a (\tau^\nu \nabla_\alpha \tau^\rho + \tau^\nu \nabla_\alpha \tau^\sigma).$$  \hfill (2.29)

Furthermore, when the susy–like operator $\delta^S_{(\tau)}$ acts on the gauge fixed action we get the breaking

$$\delta^S_{(\tau)} (S_{\text{inv}} + S_{gf}) = -s \int d^4 x \left( \sqrt{g} g^{\mu\alpha} (\nabla_\alpha \nabla_\nu) \bar{\phi}^a B^a_{\mu\nu} + \varepsilon^{\mu\nu\rho\sigma} g_{\mu\lambda} (\nabla_\nu \tau^\lambda) (\partial_\rho \bar{\xi}_a) \bar{c}^a \right),$$  \hfill (2.30)

which contains analogous terms. So one can see that with the help of (2.28) one gets (2.24) and the last equation in (2.25).

Finally, as we will explain in the next section, the constraint (2.27) requires that the parameter of diffeomorphisms $\varepsilon^\mu$ must be a killing vector such that $\mathcal{L}_\varepsilon g_{\mu\nu} = 0$. As a consequence, instead of diffeomorphism invariance we have translation invariance with vector parameter a killing vector.
2.4 The off–shell algebra

In order to translate the BRS invariance of the gauge fixed action into a Slavnov identity, one has to couple the nonlinear parts of the BRS transformations (2.21) to external sources, which lead to the following metric–independent action term

\[
S_{\text{ext}} = \int d^4x \left[ \frac{1}{2} \gamma^{\alpha\mu}(sB_{\mu}^\alpha) + \Omega^{\alpha\mu}(sA_{\mu}^\alpha) + L^a(sc^a) + D^a(s\phi^a) + \rho^{\alpha\mu}(s\xi_{\mu}^a) \right] + \frac{1}{8} \int d^4x f^{abc} \varepsilon_{\mu\nu\rho\sigma} \gamma^{\alpha\mu} \gamma_{b\rho\sigma} \phi^c ,
\]

(2.31)

whereby all external sources carry weight one and do not transform under the BRS operator. Let us remark, that the last additional term in the external action (2.31) has an analogous origin as the one in the gauge–fixing action (2.19). It ensures the Slavnov identity (2.33) in presence of the (on–shell nilpotent) BRS transformations (2.21) (see also [22]).

The dimensions, ghost numbers and weights of the sources are given in Table 2.

| \( \gamma^{\alpha\mu} \) | \( \Omega^{\alpha\mu} \) | \( L^a \) | \( D^a \) | \( \rho^{\alpha\mu} \) |
|----------------|----------------|------|------|------|
| dim            | 2              | 3    | 4    | 4    | 3    |
| \( \phi^\pi \) | -1             | -1   | -2   | -3   | -2   |
| weight         | 1              | 1    | 1    | 1    | 1    |

Table 2: Dimensions, ghost numbers and weights of the external sources.

The complete classical action

\[
\Sigma = S_{\text{inv}} + S_{gf} + S_{\text{ext}}
\]

(2.32)

obeys the Slavnov identity

\[
S(\Sigma) = 0 ,
\]

(2.33)

where

\[
S(\Sigma) = \int d^4x \left( \frac{1}{2} \frac{\delta \Sigma}{\delta \gamma^{\alpha\mu}} \frac{\delta \Sigma}{\delta \gamma^{\alpha\mu}} + \frac{\delta \Sigma}{\delta \Omega^{\alpha\mu}} \frac{\delta \Sigma}{\delta A_{\mu}^\alpha} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta \phi^a} + \frac{\delta \Sigma}{\delta D^a} \frac{\delta \Sigma}{\delta \phi^a} + \frac{\delta \Sigma}{\delta \rho^{\alpha\mu}} \frac{\delta \Sigma}{\delta \xi_{\mu}^a} \right) + b^a \frac{\delta \Sigma}{\delta \xi_{\mu}^a} + h_\mu^a \frac{\delta \Sigma}{\delta \phi^a} + \omega^a \frac{\delta \Sigma}{\delta \phi^a} + \lambda^a \frac{\delta \Sigma}{\delta \phi^a} + \frac{1}{2} \frac{\delta \phi^a}{\delta \phi^a} \frac{\delta \Sigma}{\delta \phi^a} .
\]

(2.34)

It is straightforward to verify that the corresponding linearized Slavnov operator

\[
S_\Sigma = \int d^4x \left( \frac{1}{2} \frac{\delta \Sigma}{\delta \gamma^{\alpha\mu}} \frac{\delta \Sigma}{\delta \gamma^{\alpha\mu}} + \frac{1}{2} \frac{\delta \Sigma}{\delta \Omega^{\alpha\mu}} \frac{\delta \Sigma}{\delta A_{\mu}^\alpha} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta \phi^a} + \frac{\delta \Sigma}{\delta D^a} \frac{\delta \Sigma}{\delta \phi^a} + \frac{\delta \Sigma}{\delta \rho^{\alpha\mu}} \frac{\delta \Sigma}{\delta \xi_{\mu}^a} \right) + b^a \frac{\delta \Sigma}{\delta \xi_{\mu}^a} + h_\mu^a \frac{\delta \Sigma}{\delta \phi^a} + \omega^a \frac{\delta \Sigma}{\delta \phi^a} + \lambda^a \frac{\delta \Sigma}{\delta \phi^a} + \frac{1}{2} \frac{\delta \phi^a}{\delta \phi^a} \frac{\delta \Sigma}{\delta \phi^a} ,
\]

(2.35)
is nilpotent, i.e.
\[ \{S_\Sigma, S_\Sigma\} = 0 \, . \] (2.36)

At the functional level, the invariance of the classical action under translations can be expressed by an unbroken Ward identity
\[ \mathcal{P}(\varepsilon) \Sigma = 0 \, , \] (2.37)
where \( \mathcal{P}(\varepsilon) \) denotes the corresponding Ward operator
\[ \mathcal{P}(\varepsilon) = \int d^4x \sum_\varphi (\mathcal{L}_\varphi \varepsilon) \frac{\delta}{\delta \varphi} \, , \] (2.38)
for all fields \( \varphi \). Of course \( \mathcal{L}_\varepsilon g_{\mu\nu} = 0 \), which is the Killing condition (see below, (2.53)).

Concerning the invariance under the rigid susy–like transformation, the related Ward operator \( \mathcal{V}^S_\tau \) is given by
\[ \mathcal{V}^S_\tau \Sigma = \Delta^c_\tau \, , \] (2.40)
where the breaking writes as
\[ \Delta^c_\tau = \int d^4x \left[ -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \tau^\rho (\sqrt{gg} g^{a\alpha} g^{b\beta} \partial_\alpha \xi^a_\mu + \frac{1}{2} \gamma^{ab\alpha} \right) \frac{\delta}{\delta A^a_\mu} - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \tau^\rho (\sqrt{gg} g^{a\alpha} g^{b\beta} \partial_\alpha \epsilon^a_\mu + \frac{1}{2} \gamma^{ab\alpha} \right) \frac{\delta}{\delta B^a_\mu} - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \tau^\rho (\sqrt{gg} g^{a\alpha} g^{b\beta} \partial_\alpha \phi^a_\mu + \frac{1}{2} \gamma^{ab\alpha} \right) \frac{\delta}{\delta \Omega^a_\mu} - \tau^\mu D^a_\mu - \tau^\mu L^a_\mu - \tau^\mu \rho^{a\mu} \right] \, . \] (2.41)

After tedious calculations the corresponding Ward identity takes the form
\[ \mathcal{V}^S_\tau \Sigma = \Delta^c_\tau \, , \] (2.40)
where the breaking writes as
\[ \Delta^c_\tau = \int d^4x \left[ -\frac{1}{2} \gamma^{a\mu\nu} \mathcal{L}_\tau B^a_\mu - \Omega^{a\mu} \mathcal{L}_\tau A^a_\mu + L^a_\tau \mathcal{L}_\tau e^a - D^a_\tau \mathcal{L}_\tau \phi^a + \rho^{a\mu} \mathcal{L}_\tau \xi^a_\mu \right. \]
\[ \left. - \varepsilon_{\mu\nu\rho\sigma} \Omega^{a\mu} \tau^\rho s(\sqrt{gg} g^{a\alpha} g^{b\beta} \partial_\alpha \xi^a_\mu) - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \gamma^{a\mu\nu} \tau^\rho s(\sqrt{gg} g^{a\alpha} g^{b\beta} \partial_\alpha \epsilon^a_\mu) \right] \, . \] (2.41)

The breaking is linear in the quantum fields and therefore harmless in the context of the renormalization procedure [31].

It is straightforward to verify that the classical action \( \Sigma \) fulfills, besides the gauge–fixing conditions,
\[ \frac{\delta \Sigma}{\delta b^a_\mu} = \partial_\mu (\sqrt{gg} g^{a\mu} A^a_\mu) \, , \]
\[ \frac{\delta \Sigma}{\delta h^a_\mu} = -\partial_\nu (\sqrt{gg} g^{a\mu} g^{b\beta} B^a_{\alpha\beta}) + \sqrt{gg} g^{a\mu} \partial_\nu e^a \, , \]
\[ \frac{\delta \Sigma}{\delta \omega^a_\mu} = \partial_\mu (\sqrt{gg} g^{a\mu} \xi^a_\mu) + \sqrt{gg} \lambda^a \, , \]
\[ \frac{\delta \Sigma}{\delta \lambda^a} = -\partial_\mu (\sqrt{gg} g^{a\mu} \xi^a_\mu) - s(\sqrt{gg} \phi^a) \, , \] (2.42)
also the corresponding antighost equations,
\[
\frac{\delta \Sigma}{\delta c^a} + \partial_\mu \left( \sqrt{g} g^{\mu \nu} \frac{\delta \Sigma}{\delta \Omega_{\mu \nu}} \right) = -\partial_\mu \left( s(\sqrt{g} g^{\mu \nu}) A_\nu^a \right),
\]
\[
\frac{\delta \Sigma}{\delta \xi_\mu^a} - \partial_\nu \left( \sqrt{g} g^{\mu \alpha} \gamma_\nu^{\beta} \frac{\delta \Sigma}{\delta \gamma^{\mu \alpha \beta}} \right) = \partial_\nu \left( s(\sqrt{g} g^{\mu \alpha} \gamma^{\rho \beta}) P_{\alpha \beta}^a \right) - s(\sqrt{g} g^{\mu \nu} \partial_\nu \xi^a),
\]
\[
\frac{\delta \Sigma}{\delta \phi^a} - \partial_\mu \left( \sqrt{g} g^{\mu \nu} \frac{\delta \Sigma}{\delta \rho^{\mu \nu}} \right) = \partial_\mu \left( s(\sqrt{g} g^{\mu \alpha}) \xi_\alpha^a \right) + s(\sqrt{g} \lambda^a),
\]
\[
\frac{\delta \Sigma}{\delta e^a} = -\partial_\mu \left( s(\sqrt{g} g^{\mu \alpha}) \xi_\alpha^a + \sqrt{g} g^{\mu \nu} h_\nu^a \right),
\]
which one usually obtains by (anti-)commuting the gauge conditions (2.42) with the Slavnov identity (2.33).

Finally, the action (2.32) obeys a further integrated constraint, namely the ghost equation
\[
\mathcal{G}^a \Sigma = \Delta^a,
\]
where the integrated ghost operator is given by
\[
\mathcal{G}^a = \int d^4x \left( \frac{\delta}{\delta \phi^a} - f_{abc} \phi^b \frac{\delta}{\delta \phi^c} \right),
\]
and
\[
\Delta^a = \int d^4x f_{abc} \left( \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} \gamma^{b \mu \nu} \sqrt{g} g^{a \rho \sigma} \partial_\alpha \xi_\beta^c + D^b \phi^c + \rho^{b \mu} A_{\mu}^c + \frac{1}{8} \epsilon_{\mu \nu \rho \sigma} \gamma^{b \mu \nu} \gamma^{c \rho \sigma} \right).
\]

As a conclusion of this section we display the complete nonlinear algebra generated by all the operators defined above, with \(\Gamma\) an arbitrary functional depending on the fields of the model,
\[
\begin{align*}
S_\Gamma S(\Gamma) &= 0, \\
S_\Gamma P_\epsilon(\Gamma) + P_\epsilon S(\Gamma) &= 0, \\
\{P_\epsilon, P_{\epsilon'}\} \Gamma &= -P_{\{\epsilon, \epsilon'\}} \Gamma = 0, \\
S_\Gamma (\mathcal{V}^S_{\tau}(\Gamma - \Delta^a_{\tau})) + \mathcal{V}^S_{\tau}(\Gamma) S(\Gamma) &= P_\tau(\Gamma), \\
\{\mathcal{V}^S_{\tau}, \mathcal{V}^S_{\tau'}\} \Gamma &= 0, \\
P_\epsilon(\mathcal{V}^S_{\tau}(\Gamma - \Delta^a_{\tau})) + \mathcal{V}^S_{\tau}(P_\epsilon \Gamma) &= -\mathcal{V}^S_{\{\epsilon, \tau\}} \Gamma = 0,
\end{align*}
\]
\[
\begin{align*}
\mathcal{G}^a S(\Gamma) - S_\Gamma (\mathcal{G}^a \Gamma - \Delta^a) &= \mathcal{F}^a \Gamma - \Theta^a, \\
\mathcal{G}^a P_\epsilon(\Gamma) - P_\epsilon (\mathcal{G}^a \Gamma - \Delta^a) &= 0, \\
\mathcal{G}^a (\mathcal{V}^S_{\tau}(\Gamma - \Delta^a_{\tau})) - \mathcal{V}^S_{\tau}(\mathcal{G}^a \Gamma - \Delta^a) &= 0, \\
\mathcal{G}^a (\mathcal{G}^b \Gamma - \Delta^b) - \mathcal{G}^b (\mathcal{G}^a \Gamma - \Delta^a) &= 0, \\
\mathcal{F}^a S(\Gamma) + S_\Gamma (\mathcal{F}^a \Gamma - \Theta^a) &= 0, \\
\mathcal{F}^a P_\epsilon(\Gamma) + P_\epsilon (\mathcal{F}^a \Gamma - \Theta^a) &= 0, \\
\mathcal{F}^a (\mathcal{V}^S_{\tau}(\Gamma - \Delta^a_{\tau})) + \mathcal{V}^S_{\tau}(\mathcal{F}^a \Gamma - \Theta^a) &= 0, \\
\mathcal{F}^a (\mathcal{G}^b \Gamma - \Delta^b) - \mathcal{G}^b (\mathcal{F}^a \Gamma - \Theta^a) &= 0, \\
\mathcal{F}^a (\mathcal{F}^b \Gamma - \Theta^b) + \mathcal{F}^b (\mathcal{F}^a \Gamma - \Theta^a) &= 0,
\end{align*}
\]
where

\[
\mathcal{F}^a = \int d^4x f^a_{
abla} \left( -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} (\sqrt{g} g^{\mu
u} g^{\beta\gamma} \partial_\alpha \bar{c}_\beta + \frac{1}{2} \gamma_{b\mu
u} + \frac{1}{2} \delta B_{\rho\sigma} + \rho^b_{\mu\nu} \delta \Omega_{\epsilon\mu\nu} \right) - D^b \frac{\delta}{\delta \epsilon^c} \left( \varepsilon^b_{\delta\epsilon\mu\nu} \right) - D^b \frac{\delta}{\delta \epsilon^c} - A^b_{\epsilon\mu} \frac{\delta}{\delta \epsilon^c} - \frac{\partial b_{\delta\epsilon\mu}}{\delta \epsilon^c} + \omega_{b\delta\epsilon\mu} \delta \Omega_{\epsilon\mu\nu} \right),
\]

(2.49)

with

\[
\Theta^a = \int d^4x f^a_{\nabla} \left( \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \gamma^{b\mu\nu} s(\sqrt{g} g^{\rho\sigma} g^{\alpha\beta} \partial_\alpha \bar{c}_\beta) \right).
\]

(2.50)

If the functional \( \Sigma \) is a solution of the Slavnov identity and of the Ward identities of translations and rigid supersymmetry the off–shell algebra (2.47) reduces to the linear algebra

\[
\{ \mathcal{S}_\Sigma, \mathcal{S}_\Sigma \} = 0,
\]

\[
\{ \mathcal{S}_\Sigma, \mathcal{P}(\varepsilon) \} = 0,
\]

\[
\{ \mathcal{P}(\varepsilon), \mathcal{P}(\varepsilon') \} = -\mathcal{P}(\{\varepsilon,\varepsilon'\}) = 0,
\]

\[
\{ \mathcal{S}_\Sigma, \mathcal{V}_\tau \} = \mathcal{P}(\tau),
\]

\[
\{ \mathcal{P}(\varepsilon), \mathcal{V}_\tau \} = -\mathcal{V}^S_{[\varepsilon,\tau]} = 0,
\]

\[
\{ \mathcal{V}^S_{(\tau)}, \mathcal{V}^S_{(\tau')} \} = 0,
\]

(2.51)

with the graded Lie brackets

\[
\{ \varepsilon, \varepsilon' \}_\mu = \mathcal{L}_\varepsilon \varepsilon'^\mu = \varepsilon'^\nu \partial_\nu \varepsilon^\mu + \varepsilon^\nu \partial_\nu \varepsilon'^\mu,
\]

\[
[\varepsilon, \tau]_\mu = \mathcal{L}_\varepsilon \tau^\mu = \varepsilon'^\nu \partial_\nu \tau^\mu - \tau^\nu \partial_\nu \varepsilon^\mu.
\]

(2.52)

Let us give some remarks concerning the above results. First, to get the Ward operator for the rigid susy–like transformations on the right hand side of the fifth identity in (2.51) the vector parameter \( [\tau, \varepsilon]_\mu \) must obey the constraint (2.27). This requirement leads exactly to a further constraint, indeed:

\[
\mathcal{L}_\varepsilon g_{\mu\nu} = 0,
\]

(2.53)

which means that \( \varepsilon^\mu \) is a Killing vector. On the other hand, from the fourth identity in (2.51) and the requirement above (2.53) we conclude that also the vector \( \tau^\mu \) has also to fulfill the Killing condition (2.53). To summarize, we have

\[
\mathcal{L}_\tau g_{\mu\nu} = \nabla_\mu \tau_\nu + \nabla_\nu \tau_\mu = 0,
\]

\[
\partial_\mu \tau_\nu - \partial_\nu \tau_\mu = \nabla_\mu \tau_\nu - \nabla_\nu \tau_\mu = 0,
\]

(2.54)

both equations imply that the vector \( \tau^\mu \) has to be covariantly constant,

\[
\nabla_\mu \tau_\nu = 0.
\]

(2.55)

As a consequence the graded Lie brackets of two covariantly constant vectors is zero, therefore the results in (2.51).

As expected from the case of flat space–time [20], the rigid supersymmetry anticommutated
with the BRS transformations yield translations. In our case, however, the curved manifold possesses a covariantly constant vector.

Finally, let us remark that the classical action obeys a further invariance, namely the rigid gauge invariance

\[ \mathcal{H}_{\text{rig.}}^a \Sigma = 0 , \quad \text{(2.56)} \]

with the corresponding Ward operator

\[ \mathcal{H}_{\text{rig.}}^a = \sum \varphi \int d^4x f^{abc} \varphi^b \frac{\delta}{\delta \varphi^c} , \quad \text{(2.57)} \]

where \( \varphi \) stands for all fields.

### 3 Stability

Till now we have been concentrated on the classical analysis of the model and its symmetries. In this section, we will discuss the problem of stability of the theory, which can be formulated as a cohomology problem. By stability we mean that the most general counterterm provides a redefinition of the fields and/or a renormalization of the parameters of the theory which are already present at the classical level. Let us explicitly consider the perturbed action

\[ \Sigma' = \Sigma + \Delta , \quad \text{(3.1)} \]

where \( \Sigma \) is the original action (2.32) and \( \Sigma' \) is an arbitrary functional which satisfies the Slavnov identity (2.33), the Ward identities for translations (2.37) and rigid susy–like transformations (2.40), as well as the gauge conditions (2.42), the antighost equations (2.43) and the ghost equation (2.44). The perturbation \( \Delta \) is an integrated local polynomial of dimension four and ghost number zero.

The consistency with the above constraints requires the quantity \( \Delta \) to fulfill the following set of equations:

\[ \frac{\delta \Delta}{\delta b^a} = 0 , \quad \text{(3.2)} \]

\[ \frac{\delta \Delta}{\delta h^a_\mu} = 0 , \quad \text{(3.3)} \]

\[ \frac{\delta \Delta}{\delta \omega^a} = 0 , \quad \text{(3.4)} \]

\[ \frac{\delta \Delta}{\delta \lambda^a} = 0 , \quad \text{(3.5)} \]

\[ \frac{\delta \Delta}{\delta \xi^a_\mu} + \partial_\nu \left( \sqrt{g} g^{\mu\nu} \frac{\delta \Delta}{\delta \Omega^{a\mu}} \right) = 0 , \quad \text{(3.6)} \]

\[ \frac{\delta \Delta}{\delta \xi^a_\mu} - \partial_\nu \left( \sqrt{g} g^{\mu\nu} g^{\nu\beta} \frac{\delta \Delta}{\delta \gamma^{a\alpha\beta}} \right) = 0 , \quad \text{(3.7)} \]
\[
\frac{\delta \Delta}{\delta \phi^a} - \partial_\mu \left( \sqrt{g} g^{\mu \nu} \frac{\delta \Delta}{\delta \rho^{\mu \nu}} \right) = 0 ,
\]
(3.8)

\[
\frac{\delta \Delta}{\delta e^a} = 0 ,
\]
(3.9)

\[
S_{\Sigma} \Delta = 0 ,
\]
(3.10)

\[
\mathcal{V}_{(r)} \Delta = 0 ,
\]
(3.11)

\[
\mathcal{P}_{(\varepsilon)} \Delta = 0 ,
\]
(3.12)

\[
\int d^4 x \frac{\delta \Delta}{\delta \phi^a} = 0 .
\]
(3.13)

The equations (3.2)–(3.5) and the equation (3.9) imply that the perturbation \( \Delta \) does not depend on the fields \( b^a, h_\alpha^a, \omega^a, \lambda^a \) and \( e^a \), whereas the equations (3.6)–(3.8) imply that the fields \( (\bar{c}^a, \Omega^{a \mu}), (\bar{\xi}^{a \mu}, \gamma^{a \mu \nu}), (\bar{\phi}^a, \rho^{a \mu}) \) can appear in \( \Delta \) only through the following combinations:

\[
\tilde{\Omega}^{a \mu} = \Omega^{a \mu} + \sqrt{g} g^{\mu \nu} \partial_\nu \bar{c}^a ,
\]
\[
\tilde{\gamma}^{a \mu \nu} = \gamma^{a \mu \nu} + \sqrt{g} g^{\mu \alpha} g^{\nu \beta} (\partial_\alpha \bar{\xi}^a_\beta - \partial_\beta \bar{\xi}^a_\alpha) ,
\]
\[
\tilde{\rho}^{a \mu} = \rho^{a \mu} - \sqrt{g} g^{\mu \nu} \partial_\nu \bar{\phi}^a .
\]
(3.14)

The redefinitions of the external sources (3.14) imply that \( \Delta \) is independent of the antighosts

\[
\frac{\delta \Delta}{\delta \bar{c}^a} = 0 , \quad \frac{\delta \Delta}{\delta \bar{\xi}^a_\mu} = 0 , \quad \frac{\delta \Delta}{\delta \bar{\phi}^a} = 0 .
\]
(3.15)

This means that our problem reduces to finding all possible \( \Delta \)'s such that

\[
\Delta = \Delta(A^a_\mu, \xi^a_\mu, \phi^a, \gamma^{a \mu \nu}, \bar{c}^a, \Omega^{a \mu}, \bar{\xi}^{a \mu}, \bar{\phi}^a, L^a, D^a, g_{\mu \nu}, \hat{g}_{\mu \nu}) .
\]
(3.16)

Equations (3.10)–(3.12) can be collected [14] to produce a single cohomology problem

\[
\delta \Delta = 0 .
\]
(3.17)

The operator \( \delta \) is given by

\[
\delta = S_{\Sigma} + \mathcal{P}_{(\varepsilon)} + \mathcal{V}_{(r)} + \int d^4 x (-\tau^\mu) \frac{\delta}{\delta \bar{c}^a} .
\]
(3.18)

At this point let us remark that due to the redefinitions of the external sources (3.14) the Slavnov operator and the Ward operator of susy–like transformations are now given by

\[
S_{\Sigma} = \int d^4 x \left( \frac{1}{2} \frac{\delta \Sigma}{\delta \bar{c}^a} \frac{\delta}{\delta B^a_{\mu \nu}} + \frac{1}{2} \frac{\delta \Sigma}{\delta \bar{\xi}^a_\mu} \frac{\delta}{\delta B^{\mu \nu}_{a}} + \frac{\delta \Sigma}{\delta \bar{\phi}^a} \frac{\delta}{\delta A^a_\mu} + \frac{\delta \Sigma}{\delta \bar{\xi}^a_\mu} \frac{\delta}{\delta \Omega^{a \mu}} + \frac{\delta \Sigma}{\delta \bar{\phi}^a} \frac{\delta}{\delta \bar{c}^a}
\]
\[
+ \frac{\delta \Sigma}{\delta \bar{c}^a} \frac{\delta}{\delta L^a} + \frac{\delta \Sigma}{\delta \bar{\phi}^a} \frac{\delta}{\delta D^a} + \delta^a_\mu \delta \bar{\xi}^a_\mu + \frac{\delta \Sigma}{\delta \bar{\phi}^a} \frac{\delta}{\delta \bar{c}^a} + \frac{1}{2} \hat{g}_{\mu \nu} \frac{\delta}{\delta \bar{c}^a} \right) ,
\]
(3.19)

\footnote{Furthermore, the Ward operators are already restricted to the actual field content.}
\[
\mathcal{V}_S^{(\tau)} = \int d^4x \left( -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \tau^\nu \hat{\gamma}_a \, \delta \frac{\delta A^a_\mu}{\delta A^a_\mu} - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \tau^\rho \hat{\Omega}^a_{\sigma} \frac{\delta}{\delta B^a_{\mu\nu}} - \tau^\mu A^a_\mu \frac{\delta}{\delta c^a} + \tau^\nu B^a_{\mu\nu} \frac{\delta}{\delta \xi^a_\mu} \right) + \tau^\mu \xi^a_\mu \frac{\delta}{\delta \phi^a} - \tau^\mu \hat{\rho}^a_{\mu} \frac{\delta}{\delta \hat{\rho}^a_{\mu}} - \tau^\nu \hat{\rho}^a_{\nu} \frac{\delta}{\delta \hat{\rho}^a_{\nu}} \right) .
\]

(3.20)

The main property of the above constructed operator $\delta$ is its nilpotency

\[
\delta^2 = 0 .
\]

(3.21)

Thus, one can easily check that the cohomology problem (3.17) possesses solutions of the form $\delta = \delta \hat{\Delta}$. These are called trivial solutions because the nilpotency of $\delta$ implies that any expression of the form $\delta \hat{\Delta}$ is automatically a solution of (3.17).

In the following we will call cohomology of $\delta$ the space of all solutions of (3.17) modulo trivial solutions. In other words, we are looking for $\Delta = \Delta_c + \delta \hat{\Delta}$, where $\Delta_c$ is $\delta$–closed ($\delta \Delta_c = 0$) but not trivial ($\Delta_c \neq \delta \hat{\Delta}$). We therefore introduce an operator $\mathcal{N}$, the filtering operator

\[
\mathcal{N} = \int d^4x \sum_f f \frac{\delta}{\delta f} ,
\]

(3.22)

where $f$ stands for all fields on which $\Delta$ depends. Here, we have assigned to each field homogeneity degree 1. The operator $\mathcal{N}$ induces a decomposition of $\delta$ according to

\[
\delta = \delta_0 + \delta_1 .
\]

(3.23)

The operator $\delta_0$ in (3.23) has the property that it does not increase the homogeneity degree when it acts on a field polynomial, whereas $\delta_1$ increases the homogeneity degree by 1. Due to the nilpotency of $\delta$ one has also

\[
\delta_0^2 = \{\delta_0, \delta_1\} = \delta_1^2 = 0 .
\]

(3.24)

An obvious identity which follows from (3.23) and (3.17) reads

\[
\delta_0 \Delta = 0 .
\]

(3.25)

Due to the nilpotency of $\delta_0$ (3.24), the above equation (3.25) defines a further cohomology problem.

At this stage we will use the important result [35, 36] given by the theorem, which states that the cohomology of the operator $\delta$ is isomorphic to a subspace of the cohomology of the operator $\delta_0$.

Next, we will solve the cohomology of $\delta_0$, which is easier to solve than the cohomology of $\delta$ and by using the theorem mentioned above we will determine the solution of $\delta \Delta = 0$. The action of the operator $\delta_0$ on the fields is explicitly given by

\[
\delta_0 A^a_\mu = -\partial_\mu c^a ,
\]

\[
\delta_0 c^a = 0 ,
\]

\[
\delta_0 B^a_{\mu\nu} = -\partial_\mu \xi^a_\nu + \partial_\nu \xi^a_\mu ,
\]

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\[ \begin{align*}
\delta_0 \xi^a_\mu &= \partial_\mu \phi^a, \\
\delta_0 \phi^a &= 0, \\
\delta_0 g_{\mu\nu} &= \hat{g}_{\mu\nu}, \\
\delta_0 \hat{g}_{\mu\nu} &= 0, \\
\delta_0 \hat{g}^{a\mu} &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu B^a_{\rho\sigma}, \\
\delta_0 \hat{\gamma}^{a\mu\nu} &= \epsilon^{\mu\nu\rho\sigma} \partial_\rho A^a_\sigma, \\
\delta_0 L^a &= -\partial_\mu \tilde{\Omega}^{a\mu}, \\
\delta_0 \tilde{\rho}^{a\mu} &= \partial_\nu \hat{\gamma}^{a\mu\nu}, \\
\delta_0 \epsilon^a_\mu &= -\tau^a_\mu, \\
\delta_0 \tau^a_\mu &= 0.
\end{align*} \]
(3.26)

The first remark we make is that \( g_{\mu\nu} \) and \( \hat{g}_{\mu\nu} \) and also \( \epsilon^a_\mu \) and \( \tau^a_\mu \) transform as \( \delta_0 \)–doublets, therefore both pairs of fields are out of the cohomology [36], implying that \( \Delta_c \) is independent of \( g_{\mu\nu}, \hat{g}_{\mu\nu}, \epsilon^a_\mu \) and \( \tau^a_\mu \). In order to have a more compact notation we switch to the language of forms where \( d \) represents the nilpotent (\( d^2 = 0 \)) exterior derivative, given explicitly by
\[ d = dx^\mu \partial_\mu. \]
The gauge field \( A^a_\mu \) and the ghost field \( \xi^a_\mu \) are represented by the one forms
\[ A^a_\mu = A^a_\mu dx^\mu \]
and
\[ \xi^a_\mu = \xi^a_\mu dx^\mu, \]
and the antisymmetric tensor field \( B^a_{\mu\nu} \) by the two form
\[ B^a_{\mu\nu} = \frac{1}{2} B^a_{\mu\nu} dx^\mu dx^\nu. \]
The ghosts \( c^a \) and \( \phi^a \) are scalar fields or equivalently zero forms.

From the quantities \( \tilde{\gamma}^{a\mu\nu}, \tilde{\rho}^{a\mu}, \tilde{\Omega}^{a\mu}, L^a \) and \( D^a \) we construct the following dual forms:
\[ \begin{align*}
\ast \tilde{\gamma}^a &= \frac{1}{2!} \epsilon_{\mu\nu\rho\sigma} \tilde{\gamma}^{a\mu\nu} dx^\rho dx^\sigma, \\
\ast \tilde{\rho}^a &= \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} \tilde{\rho}^{a\mu} dx^\nu dx^\rho dx^\sigma, \\
\ast \tilde{\Omega}^a &= \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} \tilde{\Omega}^{a\mu} dx^\nu dx^\rho dx^\sigma, \\
\ast L^a &= \frac{1}{4!} \epsilon_{\mu\nu\rho\sigma} L^a dx^\mu dx^\nu dx^\rho dx^\sigma, \\
\ast D^a &= \frac{1}{4!} \epsilon_{\mu\nu\rho\sigma} D^a dx^\mu dx^\nu dx^\rho dx^\sigma.
\end{align*} \]
(3.27)

In terms of forms the nilpotent operator \( \delta_0 \) reads\[12\]
\[ \begin{align*}
\delta_0 &= \int_M \left( d\frac{\delta}{\delta A^a_\mu} A^a_\mu + d\frac{\delta}{\delta B^a_{\mu\nu}} B^a_{\mu\nu} + d\frac{\delta}{\delta \xi^a_\mu} \xi^a_\mu + 2d\frac{\delta}{\delta \phi^a_\mu} \phi^a_\mu + dB^a_{\mu\nu} \frac{\delta}{\delta \Omega^a_\mu} + d\frac{\delta}{\delta \Omega^a_\mu} \Omega^a_\mu - \frac{1}{2} \frac{\delta}{\delta D^a} \frac{\delta}{\delta D^a} \right) - d\frac{\delta}{\delta D^a} \Omega^a_\mu - \frac{1}{2} d\frac{\delta}{\delta \rho^a_\mu} \left( \frac{\delta}{\delta \rho^a_\mu} \right) - \int d^4x \left( \frac{1}{2} \hat{g}_{\mu\nu} \frac{\delta}{\delta \hat{g}_{\mu\nu}} - \tau^a_\mu \frac{\delta}{\delta \tau^a_\mu} \right).
\end{align*} \]
(3.29)

\[12\]The functional derivatives with respect to differential forms has to be understood as follows:
\[ \frac{\delta S}{\delta f} = X \quad \text{if} \quad S = \int_M f X, \]
(3.28)

where \( f \) and \( X \) are general differential forms.
For $\Delta_c$ being an integrated polynomial of form degree four and ghost number zero we can write $\Delta_c = \int_M \omega^0_4$, where $\omega^p_q$ is a field polynomial of form degree $q$ and ghost number $p$. Due to Stocks theorem and to (3.25), we note that $\delta_0 \Delta_c = \int_M \delta \omega^0_4 = 0$, which implies the following result

$$\delta_0 \omega^0_4 + d \omega^1_3 = 0.$$  

(3.30)

Using the algebraic Poincaré lemma [36] and the fact that $\delta_0$ and $d$ anticommute, $\{\delta_0, d\}$ = 0, we derive the following tower of descent equations

$$\delta_0 \omega^0_4 + d \omega^1_3 = 0,$$

$$\delta_0 \omega^1_3 + d \omega^2_2 = 0,$$

$$\delta_0 \omega^2_2 + d \omega^3_1 = 0,$$

$$\delta_0 \omega^3_1 + d \omega^4_0 = 0,$$

$$\delta_0 \omega^4_0 = 0.$$  

(3.31)

The only possible expression for $\omega^4_0$, restricted by its form degree and ghost number, is given by

$$\omega^4_0 = u \phi^a_0 \phi^a + v f^{abc} c^b \phi^c,$$  

(3.32)

where $u$ and $v$ are constant coefficients. In order to solve the tower of descent equations (3.31) we decompose the exterior derivative [37] according to

$$[\bar{\delta}, \delta_0] = d, \quad [\bar{\delta}, d] = 0.$$  

(3.33)

With the help of the operator (3.34) one finds

$$\omega^0_4 = \bar{\delta} \bar{\delta} \bar{\delta} \omega^4_0$$

$$= u \left( * L^a \phi^a + * \tilde{\phi}^a \phi^a + \frac{1}{2} B^a B^a \right)$$

$$+ v f^{abc} \left( * D^a c^b \phi^c - * \tilde{\rho}^a A^b \phi^c - * \tilde{\phi}^a c^b \phi^c + \frac{1}{8} * \tilde{\gamma}^a \tilde{\gamma}^b \phi^c + \right.$$  

$$+ \frac{1}{2} * \tilde{\gamma}^a A^b \phi^c + \frac{1}{2} A^a A^b B^c + \frac{1}{2} * \tilde{\gamma}^a c^b B^c + A^a c^b * \tilde{\phi}^c + \frac{1}{2} c^a c^b * L^c \right).$$  

(3.35)

After some calculations one can show the invariance of $\int_M \omega^0_4$ under the whole operator $\delta$ of (3.18)

$$\delta \int_M \omega^0_4 = 0.$$  

(3.36)

The general solution of the cohomology problem (3.17) has the form

$$\Delta = \Delta_c + \delta \Delta = \int_M C^0_4 + \delta \Delta.$$  

(3.37)

The nontrivial solution of the $\delta$–cohomology, $\int_M C^0_4$, must not necessarily be equal to the nontrivial solution of the $\delta_0$–cohomology $\int_M \omega^0_4$. In order to analyze this situation completely, we construct the most general trivial solution, restricted by the dimension, the ghost number and the weight, according to
Therefore, the counterterms (3.37) reduce to
\[
\Delta = \int d^4x \left( \alpha_1 \tilde{\Omega}^{a\mu} A^a_\mu + \alpha_2 \tilde{\gamma}^{a\mu} B^a_\mu + \alpha_3 L^a c^a + \alpha_4 \bar{\rho}^{a\mu} \zeta^a_\mu + \alpha_5 D^a \phi^a \\
+ \alpha_6 f^{abc} \tilde{\gamma}^{a\mu} A^b_\mu A^c_\mu + \alpha_7 f^{abc} \bar{\rho}^{a\mu} \bar{A}^{b\mu} \bar{A}^{c\mu} + \alpha_8 \tilde{\gamma}^{a\mu} \partial_\mu A^a_\nu + \alpha_9 \bar{\rho}^{a\mu} \bar{A}^a_\mu \hat{g}^\nu \hat{g}_{\nu} \\
+ \alpha_10 \int f^{abc} D^a c^b \tilde{c}^c + \alpha_{11} f^{abc} \tilde{\varepsilon}_{\mu\nu\rho\sigma} \tilde{\gamma}^{a\mu} \tilde{\gamma}^{b\nu} \tilde{\gamma}^{c\rho} \tilde{c} + \alpha_{12} D^a c^a \tilde{g}_{\mu\nu} \tilde{g}^{\mu\nu} + \alpha_{13} \bar{\rho}^{a\mu} \bar{\partial}_\mu c^a \\
+ \alpha_{14} \int \frac{1}{\sqrt{g}} \tilde{\gamma}^{a\mu} \tilde{\gamma}^{a\nu} \tilde{g}_{\mu\nu} \tilde{g}^{\mu\nu} + \alpha_{15} \sqrt{g} \tilde{\varepsilon}_{\mu\nu\rho\sigma} \tilde{\gamma}^{a\mu} \tilde{g}^{\rho\sigma} \tilde{g}^{\nu} \tilde{A}^a_\beta \\
+ \alpha_{16} \int \frac{1}{\sqrt{g}} f^{abc} \tilde{g}_{\mu\nu} \tilde{g}_{\rho\sigma} \tilde{\varepsilon}_{\mu\nu\rho\sigma} \tilde{\gamma}^{a\mu} \tilde{g}^{\rho\sigma} \tilde{g}^{\nu} \tilde{g}^{a\rho} \tilde{\gamma}^{a\mu} \tilde{B}^{a\beta} \right), \quad (3.38)
\]
with \( \alpha_i, i = 1, \ldots, 19 \) as constant coefficients.

The only acceptable counterterms for the action (2.32) must be independent of the vector parameters \( \varepsilon^\mu \) and \( \tau^\mu \). A careful analysis of the \( \varepsilon^\mu \)– and \( \tau^\mu \)–dependent part\(^{13} \) of \( \delta \Delta \) leads to the vanishing of all \( \alpha_i \), except \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \) and \( \alpha_5 \) which fulfill
\[
- \alpha_5 = \alpha_4 = - \alpha_3 = - 2 \alpha_2 = \alpha_1 \equiv \alpha . \quad (3.39)
\]
Therefore, the counterterms (3.37) reduce to
\[
\Delta = \int \mathcal{M} C^0 + \alpha \mathcal{S}_\Sigma \int d^4x \left( \tilde{\Omega}^{a\mu} A^a_\mu - \frac{1}{2} \tilde{\gamma}^{a\mu} B^a_\mu - L^a c^a + \bar{\rho}^{a\mu} \zeta^a_\mu - D^a \phi^a \right). \quad (3.40)
\]

An important fact is that the \( \alpha \)–proportional term in (3.40), after performing the \( \mathcal{S}_\Sigma \)–operation, gives identically the \( \nu \)–proportional part of (3.35). This means that (3.35) contains a trivial solution of the \( \mathcal{S}_\Sigma \)–cohomology, which can be reabsorbed in the trivial counterterms. Therefore, the complete expression for the counterterms (3.37) is given by
\[
\Delta = u \int \mathcal{M} \left( *L^a \phi^a + *\tilde{\Omega}^a \zeta^a + \frac{1}{2} B^a B^a \right) \\
+ \alpha \mathcal{S}_\Sigma \int d^4x \left( \tilde{\Omega}^{a\mu} A^a_\mu - \frac{1}{2} \tilde{\gamma}^{a\mu} B^a_\mu - L^a c^a + \bar{\rho}^{a\mu} \zeta^a_\mu - D^a \phi^a \right). \quad (3.41)
\]
Now, by using the ghost equation, or more precisely the constraint (3.13) we deduce the following result: the two constant coefficients \( u \) and \( \alpha \) present in the counterterms (3.41) are both equal to zero. This means that there is no possible deformation of the action (2.32), which is the most general local functional, solution of the Ward identities, the gauge conditions, the antighost equations as well as the ghost equation.

On the other hand, the result of this section implies that at the quantum level (if there are no anomalies) the four dimensional antisymmetric tensor field model does not admit any renormalizations (renormalization of the coupling constant or of the fields). In this case the theory is said to be finite. To prove the finiteness to all orders of perturbation theory one has to overcome another problem: the absence of anomalies. This is the subject of the last section.

\(^{13}\)The technical details can be found in the appendix.
4 Anomaly analysis

In the context of renormalization theory one has to investigate whether the symmetries, collected in $\delta$, are not disturbed by quantum corrections. If there is an anomaly, then it corresponds to $\delta \Sigma = A$, where $A$ is an integrated local field polynomial of form degree four and ghost number one ($A = \int M \omega_4^1$), that fulfills
\[
\delta A = 0 . 
\] (4.1)

Using the same strategy as in the previous section, we derive the following tower of descent equations
\[
\begin{align*}
\delta_0 \omega_4^1 + d \omega_3^2 &= 0 , \\
\delta_0 \omega_3^2 + d \omega_2^3 &= 0 , \\
\delta_0 \omega_2^3 + d \omega_1^4 &= 0 , \\
\delta_0 \omega_1^4 + d \omega_0^5 &= 0 , \\
\delta_0 \omega_5^0 &= 0 . 
\end{align*}
\] (4.2)

The only possible expression for $\omega_5^0$ is given by
\[
\omega_5^0 = x Tr(e^5) + y Tr(c^3 \phi) + z Tr(c^2 \phi^2) ,
\] (4.3)

with $x$, $y$ and $z$ constant coefficients. The last two terms in (4.3) are $\delta_0$ invariant, but not $\delta$ invariant expressions. Since a possible anomaly has to be invariant under the $\delta$ operation, one has to set the coefficients $y$ and $z$ equal to zero.

Using the decomposition operator (3.34), the solution of the descent equations is given by
\[
\begin{align*}
\omega_4^1 &= \delta \delta \delta \delta \delta \omega_5^0 = \\
&= x Tr(\delta \delta \delta \delta \delta C^4 - \delta \delta \delta \delta \delta \rho Ac^3 - \delta \delta \delta \delta \delta \rho c A c^2 - \delta \delta \delta \delta \delta \rho c^2 A c - \delta \delta \delta \delta \delta \rho c^3 A + \frac{1}{4} \delta \delta \delta \delta \delta \gamma c^3 + \frac{1}{4} \delta \delta \delta \delta \delta \gamma c^2 \gamma c^2 + \frac{1}{2} \delta \delta \delta \delta \delta \gamma A^2 c^2 + \\
&+ \frac{1}{2} \delta \delta \delta \delta \delta \gamma A c A c + \frac{1}{2} \delta \delta \delta \delta \delta \gamma A c^2 A c + \frac{1}{2} \delta \delta \delta \delta \delta \gamma c A c A + \frac{1}{2} \delta \delta \delta \delta \delta \gamma c^2 A^2 + A^4 c) ,
\end{align*}
\] (4.4)

which belongs not only to the cohomology of $\delta_0$ modulo $d$ but also to the cohomolgy of $\delta$ modulo $d$, i.e. $\delta \omega_4^1 + d \omega_3^2 = 0$. This implies that the anomaly candidate, i.e. solution of (4.1), is nothing else but
\[
A = x Tr \int_M (\delta \delta \delta \delta \delta C^4 - \delta \delta \delta \delta \delta \rho Ac^3 - \delta \delta \delta \delta \delta \rho c A c^2 - \delta \delta \delta \delta \delta \rho c^2 A c - \delta \delta \delta \delta \delta \rho c^3 A + \frac{1}{4} \delta \delta \delta \delta \delta \gamma c^3 + \frac{1}{4} \delta \delta \delta \delta \delta \gamma c^2 \gamma c^2 + \\
+ \frac{1}{2} \delta \delta \delta \delta \delta \gamma A^2 c^2 + \frac{1}{2} \delta \delta \delta \delta \delta \gamma A c A c + \frac{1}{2} \delta \delta \delta \delta \delta \gamma A c^2 A c + \frac{1}{2} \delta \delta \delta \delta \delta \gamma c A c A + \\
+ \frac{1}{2} \delta \delta \delta \delta \delta \gamma c^2 A^2 + A^4 c) .
\] (4.5)

As argued in [20], the anomaly candidate $A$ disappears due to the fact that all the fields considered so far take values in the adjoint representation of the gauge group. In this case the totally symmetric tensor defined by the symmetrized trace of the generators of the gauge group, $d^{abc} = \frac{1}{2} Tr(T^a \{ T^b, T^c \})$, which is present in the trace of (4.3), vanishes.
Therefore, the most general solution of $\delta \mathcal{A} = 0$ is a $\delta$–exact quantity given by $\mathcal{A} = \delta \hat{\mathcal{A}}$. This particularly means that the Slavnov identity, the translations and the rigid susy–like transformations Ward identities are anomaly free, thus can be promoted to the quantum level. Using standard arguments \cite{31} one can easily show that the constraints (3.2), (3.3) are anomaly free, hence valid to all orders of perturbation theory. Concerning the ghost equation (3.24), it can also be proven to hold at the quantum level. The proof may be carried out by following the lines of \cite{38}.

In this section we have showed that the four dimensional antisymmetric tensor field model in a curved background, admitting a covariantly constant vector, is anomaly free. Therefore, due to the results of the previous section, it is finite to all orders of perturbation theory.

5 Conclusion

We have shown in great details that the four dimensional antisymmetric tensor field model in a curved space–time is finite to all orders of perturbation theory. The proof was performed by an extensive use of the algebraic renormalization procedure, which does not depend on a particular regularization scheme such as the dimensional regularization or the BPHZ regularization. But, unfortunately, the use of the algebraic renormalization scheme requires the existence of a possible regularization \textit{a priori}. This fact limits our quantum analysis to be only valid in the case of a curved, topologically trivial and asymptotically flat manifolds admitting covariantly constant vector.

On the other hand, we have seen that the role played by the symmetry under the susy–like transformations was decisive in reducing the counterterms (3.37) to take the simpler form (3.40), which was forbidden by the ghost equation. This symmetry only exists, as it was shown in Section 2, on manifolds where the equation (2.27) has a solution. Remember also, that we have considered manifolds where the torsion vanishes. These are all the restrictions on the manifolds where our quantum analysis holds.

Appendix: Analysis of the trivial counterterms

We devote this appendix to give all the superdiffeomorphism parameter dependent field polynomials appearing in the trivial counterterms constructed in (3.37). With

$$\mathcal{V}_S^{(\tau)} \hat{\mathcal{A}} = \sum_{i=1}^{19} X_i \eqno(A.1)$$

these polynomials read as

$$X_1 = \int d^4 x \alpha_1 \left[ - L^a {\tau}^\mu A^a_\mu + \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \tilde{\Omega}^{a\mu} {\tau}^\nu \tilde{\gamma}^{a\rho\sigma} \right], \eqno(A.2)$$

$$X_2 = \int d^4 x \alpha_2 \left[ 2 \tilde{\rho}^{a\mu} {\tau}^\nu B_{\mu\nu}^a + \varepsilon_{\mu\nu\rho\sigma} \gamma^{a\mu\nu} \tilde{\tau}^\rho \tilde{\Omega}^{a\sigma} \right], \eqno(A.3)$$
\( X_3 = \int d^4x \, \alpha_3 \left[ - L^a \gamma_\mu A^a_\mu \right], \)  
(A.4)

\( X_4 = \int d^4x \, \alpha_4 \left[ - D^a \gamma_\mu \xi^a_\mu + \bar{\rho}^{a\mu} \gamma_\nu B^a_{\mu\nu} \right], \)  
(A.5)

\( X_5 = \int d^4x \, \alpha_5 \left[ - D^a \gamma_\mu \xi^a_\mu \right], \)  
(A.6)

\( X_6 = \int d^4x \, \alpha_6 \, f^{abc} \left[ 2 \tau^\nu \bar{\rho}^{a\mu} A^b_\mu A^c_\nu + \epsilon_{\mu\nu\rho\sigma} \gamma_{a\mu\nu} \bar{\gamma}_{b\rho\sigma} A^c_\nu \right], \)  
(A.7)

\( X_7 = \int d^4x \, \alpha_7 \, f^{abc} \left[ - \tau^\mu D^a A^b_\mu c^c - \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \tau^\nu \bar{\rho}^{a\mu} \gamma_{b\rho\sigma} c^c - \tau^\nu \bar{\rho}^{a\mu} A^b_\mu A^c_\nu \right], \)  
(A.8)

\( X_8 = \int d^4x \, \alpha_8 \left[ - \left( \tau^\mu \bar{\rho}^{a\mu} - \tau^\nu \bar{\rho}^{a\mu} \right) \bar{\gamma}_x A^a_\nu - \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \tau^\rho \left( \bar{\partial}_x \bar{\gamma}_{a\mu\nu} \right) \bar{\gamma}_{b\rho\sigma} \right], \)  
(A.9)

\( X_9 = \int d^4x \, \alpha_9 \left[ - \tau^\mu D^a A^b_\mu g^{\alpha\nu} \bar{g}_{\mu\nu} - \frac{1}{2} \epsilon_{\nu\alpha\beta\delta} \tau^\alpha \bar{\gamma}_x \bar{\gamma}_{b\rho\sigma} \gamma^{\alpha\beta\delta} g^{\rho\sigma} \right], \)  
(A.10)

\( X_{10} = \int d^4x \, \alpha_{10} \left[ 2 f^{abc} D^a \tau^\mu A^b_\mu c^c \right], \)  
(A.11)

\( X_{11} = \int d^4x \, \alpha_{11} \, f^{abc} \left[ - 4 \epsilon_{\mu\nu\rho\sigma} \tau^\mu \bar{\rho}^{a\mu} \gamma_{b\rho\sigma} c^c - \epsilon_{\mu\nu\rho\sigma} \bar{\gamma}_{b\rho\sigma} \tau^\alpha A^c_\alpha \right], \)  
(A.12)

\( X_{12} = \int d^4x \, \alpha_{12} \left[ D^a \tau^\rho A^b_\rho g^{\mu\nu} \right], \)  
(A.13)

\( X_{13} = \int d^4x \, \alpha_{13} \left[ - \tau^\mu D^a \bar{\partial}_\mu c^a + \left( \bar{\partial}_x \bar{\rho}^{a\mu} \right) \tau^\nu A^a_\nu \right], \)  
(A.14)

\( X_{14} = \int d^4x \, \Omega_{a\beta} \left[ 2 \tau^\mu \bar{\rho}^{a\mu} \gamma_{b\rho\sigma} \bar{g}_{\mu\nu} g^{\alpha\nu} + 2 \bar{\gamma}_{b\rho\sigma} \tau^\rho \bar{\rho}^{a\rho} \bar{g}_{\mu\nu} g^{\alpha\nu} \right], \)  
(A.15)

\( X_{15} = \int d^4x \, \Omega_{a\beta} \sqrt{g} \left( 2 \epsilon_{\mu\nu\rho\sigma} \tau^\nu \bar{\rho}^{a\mu} \left( \partial_x A^a_\beta \right) g^{\rho\sigma} g^{\sigma\beta} - \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \left( \partial_x \bar{\gamma}_{a\mu\nu} \right) g^{\rho\sigma} g^{\sigma\beta} \right), \)  
(A.16)

\( X_{16} = \int d^4x \, \Omega_{a\beta} \sqrt{g} \left( 4 \tau^\nu \bar{\rho}^{a\mu} \bar{\gamma}_{b\rho\sigma} c^c \bar{g}_{\mu\nu} g^{\alpha\nu} - \bar{\gamma}_{b\rho\sigma} \tau^\alpha A^c_\alpha \bar{g}_{\mu\nu} g^{\alpha\nu} \right), \)  
(A.17)

\( X_{17} = \int d^4x \, \Omega_{a\beta} \left[ - D^a \tau^\mu A^a_\mu g^{\rho\sigma} \bar{g}_{\rho\sigma} - \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \tau^\nu \bar{\rho}^{a\mu} \gamma_{a\mu\nu} \bar{\gamma}_{b\rho\sigma} g^{\alpha\beta} g_{\alpha\beta} \right], \)  
(A.18)

\( X_{18} = \int d^4x \, \Omega_{a\beta} \sqrt{g} f^{abc} \left( 2 \epsilon_{\mu\nu\rho\sigma} \tau^\nu \bar{\rho}^{a\mu} A^b_\mu A^c_\nu g^{\rho\sigma} g^{\sigma\beta} + \epsilon_{\mu\nu\rho\sigma} \bar{\gamma}_{a\mu\nu} \gamma_{b\rho\sigma} \bar{\gamma}_{c\rho\sigma} \right), \)  
(A.19)

\( X_{19} = \int d^4x \, \Omega_{a\beta} \sqrt{g} \left( 2 \epsilon_{\mu\nu\rho\sigma} \tau^\nu \bar{\rho}^{a\mu} B^a_{\beta\rho} g^{\rho\sigma} g^{\sigma\beta} + \epsilon_{\mu\nu\rho\sigma} \bar{\gamma}_{a\mu\nu} \gamma_{b\rho\sigma} \bar{\gamma}_{c\rho\sigma} \right), \)  
(A.20)

The sum of the above constructed polynomials (\( \sum_i X_i \)) has to vanish, otherwise we will get the participation of the vector parameter \( \tau^\mu \) in the expression of the counterterms (3.37), a fact which is not desirable. By direct computation one can convince himself that the only possible solution of the constraint

\[ \sum_{i=1}^{19} X_i = 0 \]  
(A.21)

is that all the \( \alpha_i \) vanish for \( 6 \leq i \leq 19 \). The remaining \( \alpha_i \) have to obey the following equalities

\[ - \alpha_5 = \alpha_4 = - \alpha_3 = -2 \alpha_2 = \alpha_1 \equiv \alpha \]  
(A.22)

In this way we could reduce the trivial counterterms given in (3.37) to the more simpler expression (3.41).
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