Role of exact pairing in the description of nuclear level density and radiative strength function

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Abstract. An approach utilizing the microscopic pairing Hamiltonian is proposed to simultaneously describe the nuclear level density and radiative strength function, taking into account the thermal effects of the exact pairing as well as the giant resonances within the phonon-damping model. The good agreement between the results of calculations and experimental data extracted by the Oslo group for $^{170,171,172}$Yb isotopes demonstrates the importance of exact thermal pairing in the description of nuclear level densities at low and intermediate excitation energies and invalidates the assumption based on the Brink-Axel hypothesis in the description of the radiative strength functions.

1. Introduction
The level spacing in medium and heavy nuclei decreases rapidly with increasing the excitation energy so that the consideration of average properties of nuclear excitations becomes convenient and meaningful. The nuclear level density (NLD) and radiative $\gamma$-ray strength function (RSF) are two main quantities, which are often employed to describe these properties. The former is the number of excited levels per unit of excitation energy $E^*$, whereas the latter is the average transition probability per $\gamma$-ray energy $E_\gamma$. The NLD offers the information on the pairing correlations and nuclear thermodynamic properties such as temperature, entropy, heat capacity, etc. [1]. The RSF is a tool to study the average nuclear electromagnetic properties [2]. The study of NLD and RSF has been one of the most important topics in nuclear structure physics owing to their important contributions in the study of low-energy nuclear reactions.
and nuclear astrophysics [3]. The recent developments of the experimental technique proposed by Oslo’s group, which simultaneously extracts both NLD and RSF from the primary $\gamma$-decay spectra of highly-excited nuclei created in compound nuclear reactions [4, 5, 6], have made the study of NLD and RSF particularly attractive. Despite the concepts of NLD and RSF have been proposed long time ago [2, 7], no unified theory, which can describe simultaneously and microscopically both the NLD and RSF, has been developed so far. The finite-temperature shell model quantum Monte-Carlo method [8] describes quite well the NLD, but it is time consuming in its application to heavy nuclei. Moreover, it cannot handle the $\gamma$-strength functions, which involve giant resonances, and the related RSF. The Hartree-Fock BCS [9] and Hartree-Fock-Bogolyubov plus combinatorial method (HFBC) [10] offer a global description of NLD but they violate the particle number. As a result, to match the experimental data, the NLD has to be renormalized by using two parameters, whose values are extracted from the experimental analysis of the cumulative number of discrete levels and $s$-wave neutron resonance spacing at the neutron binding energy [11]. The predictive power of these theories is therefore limited. In the present talk I will report on a unified theoretical approach, proposed for the first time in Ref. [12], which can simultaneously describe both the NLD and RSF.

2. Formalism

The present approach includes both the effects of exact thermal pairing and temperature-dependent resonance width. Thermal pairing is treated by using the eigenvalues $E_n$, obtained by diagonalizing the pairing Hamiltonian $H = \sum_k \epsilon_k(a_{+k}^\dagger a_{+k} + a_{-k}^\dagger a_{-k}) - G \sum_{kk'} a_{+k}^\dagger a_{-k}^\dagger a_{-k'} a_{+k'}$ at zero temperature and different numbers of unpaired particles (seniorities) $S$. The notations $a_{\pm k}^\dagger(a_{\pm k})$ denote the creation (annihilation) operators of a nucleon with angular momentum $k$, projection $m_{\pm k}$, and energy $\epsilon_k$. The total seniorities $S$ are equal to 0, 2, ..., $\Omega$ (number of single-particle levels) for a system with an even number of particles and 1, 3, ..., $\Omega$ for a system with an odd number of particles. These exact eigenvalues are used to construct the partition function of the canonical ensemble (CE) [14], from which all the thermodynamic quantities such as free energy $F$, total energy $E$, entropy $S$, heat capacity $C$, and thermal pairing gap $\Delta$ [15, 16] are calculated. Because of the limitation by the size of the matrix to be diagonalized, the exact solutions of the pairing Hamiltonian are limited to the levels around the Fermi surface (truncated levels). The total partition function of the whole system is obtained by combining the exact CE partition function of the truncated levels with those obtained within the independent-particle model (IPM) [17] for the levels beyond the truncated space, where the independent motion of nucleons is assumed.

The density of state $\omega(E^*)$ at excitation energy $E^*$ as $\omega(E^*) = e^\beta/(T \sqrt{2\pi C})$ [18] is obtained from the inverse Laplace transformation of the partition function [1]. The total NLD $\rho(E^*)$ is calculated from the state density $\omega(E^*)$ as $\rho(E^*) = \omega(E^*)/(\sigma \sqrt{2\pi})$ [19], where $\sigma$ is the spin cutoff parameter. The latter, in axially deformed nuclei, has two components, the perpendicular $\sigma_\perp = I_\perp T/h^2$ and parallel $\sigma_\parallel = I_\parallel T/h^2$ ones, where $I_\perp$ and $I_\parallel$ are the perpendicular and parallel moment of inertia of the nucleus, respectively. Based on the limit of rigid body with the same density distribution as of the nucleus, $\sigma_\perp$ is empirically described as $\sigma_\perp^2 \approx 0.015A^{5/3}T$ [20], whereas $\sigma_\parallel$ is given as $\sigma_\parallel = \sigma_\perp \sqrt{(3 - 2\beta_2)/(3 + \beta_2)}$ [21], where $\beta_2$ and $A$ are the quadrupole deformation parameter and mass number, respectively. The collective vibrational and rotational excitations, not included in the pairing Hamiltonian, are expressed in terms of the vibrational $k_{vib}$ and rotational $k_{rot}$ enhancement factors [21, 22, 23], whose explicit forms are empirically given as $k_{vib} = \exp[0.0555A^{1/3}(T_4/3)]$ [23] and $k_{rot} = (\sigma_\perp^2 - 1)/(1 + (E^* - U_C)/D_C) + 1$, where $E^*$ is the excitation energy obtained within the exact CE of the pairing Hamiltonian plus the IPM (EP+IPM), whereas $D_C$ and $U_C$ are given as $D_C = 1400\beta_2^2 A^{-2/3}$, $U_C = 120\beta_2^2 A^{1/3}$ [21].

The final total NLD, including the effects of vibrational and rotational enhancements, is given as
we calculate the strength function varies (See Eqs. (14) – (17) in Ref. [5] or Eqs. (9) – (11) in Ref. [28]). In the present work, deformed Woods-Saxon potential [31]. The quadrupole deformation parameters function of is beyond the scope of this work.

The resonance width in the PDM is the sum of the quantal width Γ_{ph} for E1 excitations as a function of T_{ph}, whereas the widths for M1 and E2 excitations take their values at T = 0 as T varies (See Eqs. (14) – (17) in Ref. [5] or Eqs. (9) – (11) in Ref. [28]). In the present work, we calculate the strength function S_{X\lambda}(E_{\gamma}) within the Phonon Damping Model (PDM), where the temperature-dependent resonance width Γ_{X\lambda}(T) is obtained microscopically, including the effect of non-vanishing thermal pairing [29] by using the exact CE pairing mentioned above [30]. The resonance width in the PDM is the sum of the quantal width Γ_{Q} caused by coupling the collective giant excitations to the non-collective pb configurations at zero and finite T, and the thermal width Γ_{T} caused by coupling of giant resonances to pp and hh configurations at T \neq 0. The model has two parameters F_{1}^{(A)} and F_{2}^{(A)} for the couplings to ph, and pp (hh) configurations, respectively. The value of F_{1}^{(A)} is chosen to reproduce the resonance width Γ_{X\lambda}(T = 0), whereas F_{2}^{(A)} is selected at T = 0 so that the resonance energy E_{X\lambda} does not changes significantly as T varies. In numerical calculations within the present work, the small fluctuation of the resonance peak is neglected by setting the resonance energies E_{X\lambda} for E1, M1, and E2 excitations at their corresponding experimental values extracted at T = 0. The numerical calculations are carried out for 170,171,172 Yb isotopes, whose single-particle spectra are taken from the axially deformed Woods-Saxon potential [31]. The quadrupole deformation parameters β_{2} are 0.295 for 170,171 Yb and 0.296 for 172 Yb. The other parameters of the Woods-Saxon potential are the same as those in Refs. [15, 32]. The values of the pairing interaction parameter G for neutrons and protons are chosen so that the exact neutron and proton pairing gaps obtained at T = 0 reproduce the corresponding experimental values extracted from the odd-even mass formulas. The diagonalization of the pairing Hamiltonian is carried out for 12 doubly degenerate single-particle levels with 6 levels above and 6 levels below the Fermi surface. The exact CE chemical potential and pairing gaps are calculated, from which one obtains the quantities that mimic the “exact” quasiparticle energy E_{k}, the coefficients u_{k} and v_{k} of the Bogolyubov transformation between particles and quasiparticles, as well as the quasiparticle occupation numbers n_{k} based on their conventional definitions. These quantities are used as inputs in the RSF calculations within the PDM for the levels with pairing around the Fermi surface, whereas for the remaining spectrum, where u_{k} = 1 (0) and v_{k} = 0 (1) for k = p (h) according to the IPM, one has E_{k} = |e_{k} - e_{F}|, n_{p} = f_{p} and n_{h} = 1 - f_{h} with e_{F} and f_{k} being the Fermi energy and the single-particle occupation number described by the Fermi-Dirac distribution at finite T, respectively.

3. Analysis of numerical results

The results of the exact neutron (solid lines) and proton (dotted lines) gaps as functions of T are plotted in Figs. 1(a) – 1(c). They decrease with increasing T and remain finite up to T \sim 3 MeV, that is much larger than the critical temperature T_{c} \sim 0.57\Delta(T = 0), where the BCS gap collapses. A slight increase in the exact neutron gap at low T < 0.5 MeV takes place in 171 Yb due to the blocking effect from the odd neutron [33]. Because of this non-vanishing pairing gaps, the NLDs obtained within the EP+IPM (solid lines) agree well with the experimental data for all nuclei considered in the present paper as seen in Figs. 1(d) – 1(f). The NLDs obtained within the EP+IPM almost coincide with results of the global microscopic calculations within the HFBC for both negative (dashed lines) and positive (dotted lines) parities.
Figure 1. (Color online) Neutron and proton pairing gaps $\Delta$ [(a) – (c)] as functions of $T$ and total level densities $\rho$ [(d) – (f)] as functions of $E^*$ obtained within the EP+IPM in comparison with predictions of HFBC calculations for the positive and negative parities and the experimental data for $^{170,171,172}$Yb nuclei.

However, to have a good description of the experimental data the NLDs obtained within the HFBC have to be renormalized based on two phenomenological parameters. Moreover, since the HFBC was derived based on the partition function of the incoherent $ph$ states built on top of the HFB single-particle spectra, it fails to predict the NLD in the region of high excitation energy, where the contributions of the $pp$, $hh$, as well as of higher states like $2p2h$, $3p3h$, etc. become significant. Within the EP+IPM, the exact CE partition function is obtained from the direct diagonalization of the matrix elements of the Hamiltonian, which consist of all possible couplings between the $ph$, $pp$ and $hh$ states. Therefore, this exact CE partition in combination with that of the IPM is capable to describe the NLD up to high $E^*$ region. The insets of Figs. 1(d) – 1(f), where the NLDs obtained within the EP+IPM are compared with those obtained within the HFBC in the region $10 \leq E^* \leq 30$ MeV, clearly show that the former are significantly higher than the latter. Beyond the Woods-Saxon mean field, the EP+IPM uses only two temperature-independent parameters, namely the monopole pairing strengths $G$ for protons and neutrons. Shown in Fig. 2 are the RSF [(a) – (c)] and the sum $S_{PD,M}(E_\gamma)$ of the strength functions $S_{X\lambda}(E_\gamma)$ calculated within the PDM for $E_1$, $M_1$, and $E_2$ resonances at several values of $T \leq 0.7$ MeV [(d) – (f)]. They have been multiplied by the corresponding cross sections $\sigma(X\lambda)$ at their maxima and normalized by $(2\lambda + 1)$, namely $S_{PD,M}(E_\gamma) = \sigma(E_1(I))S_{E_1(I)}(E_\gamma)/3 + \sigma(E_1(II))S_{E_1(II)}(E_\gamma)/3 + \sigma(M_1)S_{M_1}(E_\gamma)/3 + \sigma(E_2)S_{E_2}(E_\gamma)/5$, where $E_1(I)$ and $E_1(II)$ correspond to the two components of the GDR determined from the photoabsorption experiments [5]. The values of resonance energies $E_{X\lambda}$, their FWHM $\Gamma_{X\lambda}$, and cross sections $\sigma(X\lambda)$ at $T = 0$ for $^{170,171,172}$Yb are taken from Table I of Ref. [5]. The GDR with the largest values of $\sigma(X\lambda)$ ($X\lambda = E_1(I), E_1(II)$) gives the largest contribution the total
strength function [Figs. 2(d) – 2(f)]. The widths of its two components remain nearly constant at $T \leq 0.4$ MeV and increase with $T$ at $T > 0.4$ MeV, resulting in a significant increase in the total RSF at low $E_\gamma < 4$ MeV as seen in Figs. 2(a) – 2(c). The RSFs obtained within the PDM at $T = 0.7$ MeV agree well with the experimental data for all nuclei under consideration. This value of $T$ is higher than that obtained from the fitting by using the KMF model in Ref. [5], which is below 0.4 MeV. This result invalidates the assumption of the Brink-Axel hypothesis [35], which states that the GDR built on an excited state should be the same as that built on the ground state, and based on which the experimental RSFs were extracted. Based on the fitting by using the KMF model, Ref. [5] has also suggested that there should appear a two-component pygmy dipole resonance (PDR) in the region $2.1 < E_\gamma < 3.5$ MeV in $^{171}$Yb and $^{172}$Yb. This two-component PDR was added on top of the GDR in fitting the experimental RSF in Ref. [5]. Within the PDM, it has been shown in Ref. [30] that exact pairing enhances the $E_1$ strength function in the region $E_\gamma < 5$ MeV. By including this exact pairing, the RSFs, predicted by the PDM, agree well with the experimental data [thick solid lines in Figs. 2(e) and 2(f)]. Hence, the enhancement of the experimental RSF at low $E_\gamma$, which was suggested to be caused by the PDR, is explained microscopically by the effect of exact thermal pairing within the PDM.

4. Conclusions
An approach utilizing the same microscopic pairing Hamiltonian to describe simultaneously both the nuclear level density (NLD) and radiative $\gamma$-ray strength function (RSF) is proposed for the very first time. It employs the exact solutions of the pairing problem to construct the partition function, based on which the NLD and thermal pairing gap are calculated. The latter is included
in the phonon-damping model to calculate the RSF. The good agreement between the results obtained within this approach and the experimental data for NLD and RSF in $^{170,171,172}$Yb demonstrates the crucial role of exact thermal pairing in the description of both NLD and RSF in the low and intermediate region of excitation and $\gamma$-ray energies. The results obtained also show that, to have a good description of the RSF, the microscopic strength function with the temperature-dependent width for the giant resonances should be used instead of the Brink-Axel hypothesis. The merits of this approach are its microscopic nature and the use of only the parameters taken over from previous calculations as well as the very short computing time, which takes less than five minutes even for a heavy nucleus.

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