An alternative approach to holographic dark energy

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Abstract. Here we consider a scenario in which dark energy is associated with the apparent area of a surface in the early universe. In order to resemble the cosmological constant at late times, this hypothetical reference scale should maintain an approximately constant physical size during an asymptotically de Sitter expansion. This is found to arise when the particle horizon—anticipated to be significantly greater than the Hubble length—is approaching the antipode of a closed universe. Depending on the constant of proportionality, either the ensuing inflationary period prevents the particle horizon from vanishing, or it may lead to a sequence of ‘big rips’.

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1. **Introduction**

The acceleration of our universe is now a well established phenomenon, and numerous theoretical approaches have attempted to provide an explanation. The cosmological constant is arguably the strongest candidate, although it has a number of well-documented issues, particularly with regard to its size. One of the more attractive methods of attaining such a small number is by relating it to a cosmological length scale.

The holographic principle speculates that all information stored within some volume is represented on the boundary of that region. A cosmological variation of the holographic principle was originally proposed by Fischler and Susskind [1]. In this scenario the cosmological horizon acts as a surface which obeys the Bekenstein bound, limiting the entropy of the enclosed volume, such that

\[ S \leq \frac{A}{4G}. \]  

(1)

This concept has been extended by Bousso [2] into a covariant and more general conjecture.

Cohen *et al* [3] took a slightly different approach, whereby the dark energy density is proportional to the inverse square of some cosmological length. This arises when constraining the energy in some volume to be less than a black hole of the same size.

\[ L^3 \rho_\Lambda \lesssim LM_P^2. \]  

(2)

The most natural choices for $L$ are the particle horizon, and the Hubble length. However Hsu [4] has shown that under adiabatic conditions, neither exhibit the equation of state necessary to mimic dark energy. One could enforce $w = -1$, by permitting energy exchange between the dark matter and dark energy; however this approach has not proved particularly successful [5].
Li [6] produced a model of holographic dark energy based on Cohen’s approach, which has subsequently been explored in further detail [7]–[10] and is compatible with current observational constraints [11]. By identifying \( L \) as the future event horizon, the required equation of state could emerge without invoking energy exchange. For a saturated inequality, the energy density is given by

\[
\rho_\Lambda = 3d^2 M_p^2 L^{-2}.
\]

The value \( d = 1 \) is usually adopted for the reasons outlined in [6]—in particular, \( d \geq 1 \) ensures that the entropy does not decrease. However, within Li’s model there remain two areas of concern. First of all, for self-consistency, there is the requirement that all forms of energy must decay at some time in the future. And secondly, the initial condition of a preset future event horizon appears a little unusual, raising the issue of causality.

Here we take a rather different approach, finding that a more natural form of dark energy may arise when considering the particle horizon. We argue that its associated area may possess greater physical significance than the length scale itself. In section 2 we explore the consequences of attributing an energy density to the area of the particle horizon within a closed universe. Section 3 establishes the equation of state, and explores different values of the constant of proportionality \( d \). In some cases this may lead to a ‘big rip’ scenario. Section 4 summarizes the current and future observational constraints on the model. We speculate on the possible physical interpretations in section 5.

2. Particle horizon

Here we consider the consequences of ascribing an energy density to the apparent area of the particle horizon (which represents the intersection of our past light cone with a particular point in time prior to inflation). Our parametrization of the proportionality constant, \( 12\pi d^2 \), retains the close relationship with (3) and will simplify our analysis in the following section.

\[
\rho_\Lambda = \frac{12\pi d^2 M_p^2}{A}.
\]

The area \( A \) associated with our past light cone in a closed universe is given by

\[
A = 4\pi R_C^2 \sin^2 \theta,
\]

where \( \theta = R_P/R_C \), as illustrated in figure 1, and the particle horizon \( R_P \) is

\[
R_P = a(t) \int_0^t \frac{dt'}{a(t')},
\]

and \( R_C = R_0a = H^{-1}\Omega_k^{-1/2} \) is the radius of curvature.

The vanishing particle horizon within a closed universe has caused some concern [1, 12] but since we are attributing its (inverse) size to the dark energy density, we find that this may produce cosmic acceleration, so the recession is strongly suppressed, and the entropy bound is protected (provided \( d \geq 1 \)).
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Figure 1. The spread of the particle horizon $R_P$ across a closed universe, with the radius of curvature $R_C$ and corresponding angle $\theta$ also labelled. Whilst the comoving area shrinks once beyond the halfway point, the physical area of the particle horizon never decreases provided $d \geq 1$. The residual distance is effectively the future event horizon.

3. The equation of state

Taking the time derivative of (4), and utilizing the relations

$$\dot{\rho} = -3\frac{a}{a} \rho (1 + w)$$

and

$$\dot{R}_P = 1 + H R_P$$

which arose from differentiating (6), we find that the equation of state can be expressed as a function of $\Omega_\Lambda = \rho_\Lambda / \rho_c$:

$$w = -\frac{1}{3} + \frac{2}{3} \frac{\sqrt{\Omega_\Lambda}}{d} \cos \theta.$$  

(9)

This clarifies the behaviour of $w$ as the particle horizon traverses the closed universe. In the following subsections we explore the significance of $d$, assuming that it exhibits no significant time variation.

3.1. The case $d = 1$: de Sitter

For the scenario in which $d = 1$ there are three distinct regimes

$$w = +\frac{1}{3} \quad (\Omega_\Lambda = 1, R_P \ll R_C)$$

(10)

$$w = -\frac{1}{3} \quad (\Omega_\Lambda = 0)$$

(11)

$$w = -1 \quad (\Omega_\Lambda = 1, R_P \simeq \pi R_C).$$

(12)
Both (10) and (11) will ultimately converge to (12). For this limiting case, we have $|\Omega_k| \ll 1$, and the particle horizon is in close proximity to the observer’s antipode. Then our expression for the energy density (4) simplifies to

$$\rho_\Lambda = \frac{d^2}{(\pi R_C - R_P)^2}$$

(13)

for $R_P/R_C \simeq \pi$. This leads to

$$w = -\frac{1}{3} - \frac{2}{3} \sqrt{\Omega_\Lambda}.$$  

(14)

Note that having set $d = 1$, the evolution of the equation of state matches that of Li’s approach, since the particle horizon is identified with the future event horizon (see figure 1). While the future event horizon was a free parameter in Li’s model, here it is dictated by global geometry, which in turn was determined by the suppression of $\Omega_k$ during inflation. Once inflation has been established, the particle horizon is frozen in at some fraction of the radius of curvature. Further growth of $\theta$ occurs during radiation and matter domination, scaling as $a$ and $\sqrt{a}$ respectively.

Given an observational fit of the form $w = w_0 + w_a (1 - a)$, and taking $\Omega_\Lambda = 0.75$, one expects

$$w \simeq -0.9 + 0.2(1 - a).$$

(15)

3.2. The case $d > 1$: decay

For greater values of $d$, there is initially a higher energy density, and thus acceleration arises earlier. This ensures that $\theta < \pi$ is sustained, although we still find $\theta$ converging to $\pi$ for all values of $d$. The only significant difference from the $d = 1$ case is the gradual decay of the vacuum energy. Again, for small $\Omega_k$ we have

$$w = -\frac{1}{3} - \frac{2}{3} \sqrt{\Omega_\Lambda}.$$  

(16)

3.3. The case $d < 1$: big rips

We can see from (9) that $d < 1$ may result in a phantom cosmology, characterized by $w < -1$. This leads to a divergence in the energy density, destroying all galactic and atomic structure within a finite time [13]. In this context, the value of the Hubble parameter is initially too low, so the particle horizon is not prevented from vanishing. Let us take $d = 0.5$ as an illustration. This provides an equation of state asymptotically approaching $w = -5/3$, and so $\rho_\Lambda \propto a^2$. The unusual scenario here is that at some point—which we take to be the Planck density—we pass $\theta = \pi$, and there is a radical transition in behaviour. The area of the particle horizon begins to increase in size once more, returning to its point of origin, and we find $w = 1$. We can then establish a series of ‘big rips’, each taking much longer than its predecessor, and with the number of e-folds dictated by $d$. For instance, with $d = 0.5$ a present day value of $\rho_\Lambda \sim 10^{-120}$ implies a further $\sim 140$ e-folds until the Planck density is reached. If $d = 0.8$, then $\rho_\Lambda \propto \sqrt{a}$, so a further $\sim 550$ e-folds are required. Note that $d < 0.5$ appears to be forbidden on the grounds that this could lead to an unphysical value of $w > 1$. 

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3.4. Entropy

Could the particle horizon still obey the constraint given by (1)? As explored by Fishler and Susskind, the most natural way to count the entropy enclosed within a given area is that which has passed through the past light cone. The most important consequence of this is that we bypass the vast volume of the particle horizon associated with inflation, as we need only consider the light cone after reheating. Thus one could potentially reconcile a large universe with the constraint given by (1). This bound is eventually violated when considering the $d < 1$ models. However, the significance of this is questionable, since a simple model of a collapsing universe would also lead to a violation.

3.5. Theoretical issues

One of the problems associated with dark energy is the question of coincidence. Why are we around at the time of transition from matter to dark energy domination? There are few scenarios which can genuinely address this problem. One is where recent events—such as the formation of structure—are directly related to cosmological kinetics. Efforts to explain dark energy via the gravitational influence of non-linearities have thus far proved unsuccessful. The second is one in which the lifetime of the universe is restricted. If either recollapse or the big rip occurs within the next $\sim 10^{12}$ years then we are no longer unusually close to the time of transition. This makes studying cosmologies with $w < -1$ particularly attractive, despite the difficulty in interpreting the thermodynamics.

The standard cosmological constant also runs into difficulty when we try to establish why the energy density appears so small. Conversely, here we must wonder why the energy density is not even smaller. If there were a few extra e-folds of inflation this would generate a much larger volume within which we would be more likely to exist. This could be alleviated if other important factors, such as the magnitude of primordial density perturbations, were also correlated with the number of e-folds of inflation.

4. Observational constraints

Since the equation of state with small $\Omega_k$, given by (16), matches earlier studies using the future event horizon, we can utilize constraints on $d$ from Chang et al [14]. They find that an equation of state of the form (16) remains compatible with current observational data from supernovae, the CMB, and large scale structure. There is however a preference for $d < 1$, a value which as we have seen leads to some intriguing behaviour. They also highlight how constraints are particularly sensitive to the Hubble parameter, with a lower $h$ favouring higher $d$.

Recently, it has been suggested that the controversial Heidelberg-Moscow claim of neutrinoless double beta decay [15] provides an indication of a phantom-like dark energy, with $w < -1$ [16,17]. However, it should be noted that even if the neutrino result is verified, the constant $w$ parametrization can provide misleading results [18,19], with an evolving equation of state mimicking the phantom behaviour. This leaves us with the unusual situation whereby evidence for $w < -1$ could still suggest $w(z) > -1$. 
5. Physical motivation

A potential relationship between cosmological length scales and dark energy has received a great deal of attention, with limited physical justification. Here we assess the possible interpretations and motivations for associating the particle horizon with the vacuum energy. At first glance, the area of the particle horizon simply corresponds to the surface of infinite redshift surrounding us. However this may take on greater physical significance if there was a critical change in the underlying physics in the very early universe. This would correspond to a particular point in time prior to inflation. We would then sample this ‘horizon’ surface which intersects our past light cone, in much the same way as our view of the last scattering surface samples a preferential scale. Properties of the present day universe may then be influenced by this reference length scale. This could be related to the proposal by Padmanabhan [20] in which vacuum fluctuations within finite regions of space are responsible for the cosmological constant.

This work was also partly motivated by the following prospect. If the universe is found to be slightly closed, with $\Omega_k \sim 0.01$, we would be pushed deeper into the coincidence problem. There appears little reason to believe that the curvature should be comparable to the matter and dark energy density. However if there was some relationship between the global curvature and the dark energy, this would partly alleviate the problem. Unfortunately we cannot make quantitative predictions on $\Omega_k$ here, since it present value will depend on the nature of inflation.

6. Discussion

An energy density associated with the size of the particle horizon within a closed universe has been shown to demonstrate a negative equation of state. Precise behaviour depends on the constant of proportionality, $d$. Whilst $d = 1$ is the most natural choice, imitating the cosmological constant at late times, current observational constraints slightly prefer $d < 1$. This corresponds to a cosmology with a sequence of ‘rips’, as the particle horizon repeatedly traverses the closed universe. Such a model has the advantage of addressing the coincidence problem, and has a simple mechanism for lowering the equation of state at late times. We have also established a robust lower bound of $d > 0.5$, which must be satisfied in order to ensure $w < 1$ in the early universe.

For the case $d = 1$ we find cosmic evolution equivalent to the model put forward by Li, which was based on associating the dark energy density with the future event horizon. However by using the particle horizon, this approach is causal, does not require the decay of matter, and has a natural reference scale—the radius of curvature.

As with other holographic models, this approach is rather speculative, but does feature a number of attractive properties. Given the close relationship predicted between $\Omega_\Lambda$ and $w$, data from future dark energy surveys such as SNAP should be capable of distinguishing this model from the cosmological constant.

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