G-Matrix Equation in the Resonating-Group Method

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abstract

The G-matrix equation is most straightforwardly formulated in the resonating-group method if the quark-exchange kernel is directly used as the driving term for the infinite sum of all the ladder diagrams. The inherent energy-dependence involved in the exchange term of the normalization kernel plays the essential role to define the off-shell T-matrix uniquely when the complete Pauli-forbidden state exists. We analyze this using a simple solvable model with no quark-quark interaction, and calculating the most general T-matrix in the formulation developed by Noyes and Kowalski. This formulation gives a certain condition for the existence of the solution in the Lippmann-Schwinger resonating-group method. A new procedure to deal with the corrections for the reduced masses and the internal-energy terms in the $\Lambda N$-$\Sigma N$ coupled-channel resonating-group equation is proposed.

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1 Introduction

The $G$-matrix formalism is one of the best known frameworks to study the effective baryon-baryon interaction in a nuclear medium. \cite{1, 2} When the free baryon-baryon scattering is described by the non-relativistic Schrödinger equation with a simple local potential, the derivation of the $G$-matrix equation is straightforward even for the most general off-shell $G$-matrix used in the many-body calculations. This is not the case if the basic baryon-baryon interaction is formulated as a composite-particle interaction in the framework of the resonating-group method (RGM). \cite{3, 4} It is well known that the relative wave-function between clusters has a different normalization property from the wave-function of the ordinary Schrödinger equation, owing to the antisymmetrization of constituent quarks ($q$). \cite{5} It is, therefore, claimed that the quark-model potential derived from the $(3q)-(3q)$ RGM \cite{6} should be defined by rewriting the original RGM equation to the Schrödinger-type equation. One of the merits of this method is that it is possible to eliminate the explicit energy dependence of the quark-exchange kernel, which inherently appears in the RGM equation. The essential point of this procedure is to renormalize the RGM relative wave-function $\chi(r)$ by the exchange normalization kernel $K$ as $\psi(r) = \sqrt{1 - K}\chi(r)$. This decent prescription, however, needs a special care when a complete Pauli-forbidden state exists.

In this paper, we discuss roles of the Pauli-forbidden state in the formulation of the $G$-matrix equation for the RGM. It is shown that the direct use of the quark-exchange kernel for the driving term of the $G$-matrix or $T$-matrix equation is the simplest and the most natural procedure in the sense that the orthogonality condition \cite{9} to the complete Pauli-forbidden state is automatically incorporated in the structure of the exchange kernel. The concept of the orthogonality need not be applied only to the relative wave-function, but could also be applied to any other physical quantities appearing in the $T$-matrix formulation of the scattering in a nuclear medium. The energy dependence involved in the normalization exchange kernel is not an unfavorable feature, but is essential to represent the effect of the compositeness of the nucleon clusters. To show these, we use a simplest version of the Saito’s orthogonality condition model (OCM) \cite{8, 10, 11} with no $q$-$q$ interaction. In this case, the exact solution of the complete off-shell $T$-matrix is analytically given. If we solve this problem in the Noyes \cite{7} and Kowalski \cite{8} method, it is easy to see what kind of condition is necessary to guarantee the
existence of the solution of the basic Lippmann-Schwinger-type equation. In the general RGM equation, this condition is automatically satisfied. When we need to modify the exchange kernel, the modification should be made in such a way that this condition is still satisfied. An example of this kind of modification is the correction of the reduced masses and a small readjustment of the threshold energies in the coupled-channel RGM (CCRGGM). One can preserve a realistic kinematics of the baryon-baryon scatterings even in a very rigorous framework of the CCRGM.

In the next section, we first briefly illustrate the RGM formalism to show the notation used in this paper. An orthogonality condition model is introduced as a simplified version of the RGM. The analytic solution of the \( T \)-matrix equation is given in a heuristic way. Next we consider the basic equations for the most general \( T \)-matrix in the Noyes and Kowalski method. The orthogonality relations to the Pauli-forbidden state are analyzed in \( \S \)3 with respect to the simplest version of the OCM. The modification of the exchange kernel of the RGM equation is discussed in \( \S \)4. The final section is devoted to discussion and a brief summary.

## 2 Formulation

### 2.1 RGM equation

The RGM equation for the relative-motion wave-function \( \chi(r) \) is usually formulated from the variational equation [4]

\[
\langle \phi_{\text{int}} | E - H | \mathcal{A} \{ \phi_{\text{int}} \chi \} \rangle = 0 ,
\]

where \( \phi_{\text{int}} \) is an appropriate internal cluster function and the total Hamiltonian consists of

\[
H = \sum_{i=1}^{6} t_i - T_G + \sum_{i<j}^{6} v_{ij} .
\]

Here we particularly consider (3q)-(3q) RGM [3] for the baryon-baryon interaction. The normalization kernel \( N \) stands for

\[
N = \langle \phi_{\text{int}} | \mathcal{A} | \phi_{\text{int}} \rangle = 1 - K ,
\]
where the exchange normalization kernel $K$ sometimes allows a complete Pauli-forbidden state $|u\rangle$ satisfying

\[ K|u\rangle = |u\rangle \quad \text{and} \quad \mathcal{A}\{\phi^\text{int}\} = 0. \quad (4) \]

We use a common notation, $\Lambda = 1 - |u\rangle\langle u|$, to denote the projection operator on the Pauli-allowed space. For the direct term, we express $H$ as

\[ H = H_\text{int} + H_0 + \sum_{i=1}^{3} \sum_{j=4}^{6} v_{ij}, \quad (5) \]

and define the internal-energy term $E_\text{int}$ and the direct potential $V_D$ by

\[ \langle \phi^\text{int} | H_\text{int} | \phi^\text{int} \rangle = E_\text{int} = E^K_\text{int} + E^V_\text{int}, \]

\[ \langle \phi^\text{int} | \sum_{i=1}^{3} \sum_{j=4}^{6} v_{ij} | \phi^\text{int} \rangle = V_D. \quad (6) \]

For the exchange term, we use the notation

\[ G = G^K + G^V, \]

\[ G^K = \langle \phi^\text{int} | (\sum_{i=1}^{6} t_i - T_G)(\mathcal{A} - 1) | \phi^\text{int} \rangle + E^K_\text{int} K, \]

\[ G^V = \langle \phi^\text{int} | (\sum_{i<j}^{6} v_{ij})(\mathcal{A} - 1) | \phi^\text{int} \rangle + E^V_\text{int} K. \quad (7) \]

The exchange kinetic-energy kernel $G^K$ is symmetric, since $E^K_\text{int}$ is common for all channels while $G^V + E^V_\text{int} K$ is symmetric for the exchange interaction kernel. Using these notations, Eq. (1) is converted into the usual RGM equation:

\[ (\varepsilon - H_0 - V_{\text{RGM}})\chi = 0, \quad (8) \]

with $\varepsilon = E - E_\text{int}$ and $V_{\text{RGM}} = V_D + G + \varepsilon K$. \footnote{Here we use the simplest center-of-mass coordinate $X = (x_1 + x_2 + x_3)/3$ for the $(3q)$ clusters and assume that the effect of the flavor symmetry breaking is respected only in the original Hamiltonian $H$.}
The essential feature of the RGM equation, Eq. (8), is the existence of the trivial solution $\chi = u$; namely, we can write Eq. (8) as

$$\Lambda(\varepsilon - H_0 - V_{\text{RGM}})\Lambda \chi = 0.$$  

(9)

This equation is rewritten as

$$(\varepsilon - H_0 - V)\chi = 0,$$  

(10)

with

$$V = V(\varepsilon) + v,$$

$$V(\varepsilon) = (\varepsilon - H_0) - \Lambda(\varepsilon - H_0)\Lambda,$$

$$v = \Lambda V_{\text{RGM}}\Lambda = \Lambda(V_D + G + \varepsilon K)\Lambda.$$  

(11)

We note simple relations, $\Lambda V(\varepsilon)\Lambda = 0$ and $v = \Lambda v\Lambda$. Actually, $v$ also has a weak $\varepsilon$ dependence. If the effect of the Pauli principle has a simple structure such as in the deuteron-deuteron system, the $\Lambda K\Lambda$ term exactly vanishes. In this particular case, we can also show that $G^K \sim \Lambda H_0\Lambda - H_0$ and $\Lambda G^K\Lambda \sim 0$. This implies that $V(\varepsilon)$ term in Eq. (11) stands for the dominant part of $G^K + \varepsilon K$ in the original RGM kernel $V_{\text{RGM}}$, and $v \sim \Lambda(V_D + G^V)\Lambda$.

From these observations, we can conclude that the most essential part of the Pauli principle of the RGM equation is already retained in a simple OCM-type equation

$$(\varepsilon - H_0)\psi = V(\varepsilon)\psi,$$  

(12)

and the general RGM equation, Eq. (8), can be obtained by adding to $V(\varepsilon)$ the potential term $v$ having the property $v = \Lambda v\Lambda$. We will consider the $T$-matrix of Eq. (12) in the next subsection.

\subsection*{2.2 $T$-matrix of the simple OCM equation}

In this subsection we derive a complete off-shell $T$-matrix for the simple OCM equation

$$\Lambda(\varepsilon - H_0)\Lambda \psi = 0,$$  

(13)
which is equivalent to Eq. (12). Since the solution of Eq. (13) has an ambiguity of $u$, we first consider a more general equation

$$(\omega - H_0)\psi = V(\varepsilon)\psi \quad \text{with} \quad \omega \neq \varepsilon ,$$

and take the limit $\omega \to \varepsilon$ in the final expression. One can formulate this in two ways. The first method is to consider the solution of

$$(\omega - H_0)\psi = V(\varepsilon)\psi \quad \text{with} \quad (\omega - H_0)|\psi\rangle = 0 ,$$

and the second method is to use

$$(\omega - H_0)(\psi - \phi) = V(\varepsilon)\psi \quad \text{with} \quad (\varepsilon - H_0)|\phi\rangle = 0 .$$

The latter equation is motivated by the correlation function technique for the $G$-matrix, for which $\chi = \phi - \psi$ corresponds to the so-called defect function. In this case, the starting energy $\omega$ is usually negative. For $\omega \neq \varepsilon$ one can easily prove $\langle u|\psi\rangle = 0$ for the solution of Eq. (15) and $\langle u|\chi\rangle = 0$ for the solution of Eq. (16).

Both equations, Eqs. (15) and (16), lead to the same definition for the $T$-matrix, given by

$$T(\omega, \varepsilon) = V(\varepsilon) + V(\varepsilon)G_0^{(+)}(\omega)T(\omega, \varepsilon)$$

with

$$G_0^{(+)}(\omega) = \frac{1}{\omega - H_0 + i0} .$$

The solution of Eq. (17) is derived as follows. We first define the free Green function in the allowed space by

$$G_\Lambda(\omega) = G_0^{(+)}(\omega) - G_0^{(+)}(\omega)|u\rangle \frac{1}{\langle u|G_0^{(+)}(\omega)|u\rangle} \langle u|G_0^{(+)}(\omega) ,$$

which satisfies

$$\Lambda(\omega - H_0)\Lambda G_\Lambda(\omega) = G_\Lambda(\omega)\Lambda(\omega - H_0)\Lambda = \Lambda .$$

\(^2\)The on-shell $T$-matrix for more general OCM equations than Eq. (13) with local potentials has been extensively studied by many authors. See, for example, Refs. [10], [11], [12] and [13].
The Green function for $\omega - H_0 - V(\varepsilon)$ can be easily found to be

$$G(\omega, \varepsilon) = \frac{1}{\omega - H_0 - V(\varepsilon)} = G_\Lambda(\omega) + |u\rangle \frac{1}{\omega - \varepsilon} \langle u| .$$  \hspace{1cm} (20)

Since this $G(\omega, \varepsilon)$ is the formal solution of

$$G(\omega, \varepsilon) = G_0^{(+)}(\omega) + G_0^{(+)}(\omega)V(\varepsilon)G(\omega, \varepsilon) ,$$  \hspace{1cm} (21)

the $T$-matrix solution of Eq. (17) is derived from

$$G(\omega, \varepsilon) = G_0^{(+)}(\omega) + G_0^{(+)}(\omega)T(\omega, \varepsilon)G_0^{(+)}(\omega) .$$  \hspace{1cm} (22)

We find

$$T(\omega, \varepsilon) = - \frac{|u\rangle \langle u|}{\langle u|G_0^{(+)}(\omega)|u\rangle} + (\omega - H_0)|u\rangle \frac{1}{\omega - \varepsilon} \langle u|(\omega - H_0) .$$  \hspace{1cm} (23)

This expression has seemingly a singularity at $\omega = \varepsilon$ in the second term. There is, however, no such singularity for the initial state $|\omega\rangle$ or $|\phi\rangle$:

$$T(\omega, \varepsilon)|\omega\rangle = - \frac{|u\rangle \langle u|\omega\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} ,$$

$$T(\omega, \varepsilon)|\phi\rangle = - \frac{|u\rangle \langle u|\phi\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} + (\omega - H_0)|u\rangle \langle u|\phi\rangle .$$  \hspace{1cm} (24)

In particular, the on-shell $T$-matrix given by

$$\langle \omega|T(\omega, \varepsilon)|\omega\rangle = - \frac{\langle \omega|u\rangle \langle u|\omega\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} ,$$  \hspace{1cm} (25)

has no $\varepsilon$-dependence. The $T$-matrix elements we need in a nuclear medium is

$$\langle \phi'|T(\omega, \varepsilon)|\phi\rangle = - \frac{\langle \phi'|u\rangle \langle u|\phi\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} + (\omega - \varepsilon') \langle \phi'|u\rangle \langle u|\phi\rangle ,$$  \hspace{1cm} (26)

where

$$\langle \varepsilon' - H_0|\phi'\rangle = 0 .$$  \hspace{1cm} (27)
Summarizing this subsection, we find that, to define the complete off-shell $T$-matrix, it is convenient to start with Eq. (16), which is similar to the Schrödinger-type equation for the defect function in the $G$-matrix formalism. The energy $\varepsilon$ in $V(\varepsilon)$ is the relative energy for the initial two-particle state, and should not be mixed up with the energy $\omega$ in the free Green function. The energy $\omega$ is usually negative and sometimes referred as the starting energy in $G$-matrix calculations for ground state properties. In the former equation Eq. (15) the orthogonality is imposed on $\psi$, while the defect function $\chi = \psi - \phi$ respects the orthogonality to the Pauli forbidden state $u$.

2.3 Noyes-Kowalski equation

The Lippmann-Schwinger-type equations in the momentum representation are nicely solved by the techniques developed by Noyes [7] and Kowalski [8]. For details of this method, the original papers should be referred to. Here we recapitulate only minimum equations necessary for the following discussion. Let us consider the Lippmann-Schwinger equation for the wave-function

$$|\psi^{(+)}\rangle = |\phi\rangle + G^{(+)}_0(E)V|\psi^{(+)}\rangle$$

with

$$(E - H_0)|\phi\rangle = 0 ,$$

(28)

or the $T$-matrix equation

$$T(E) = V + VG^{(+)}_0(E)T(E) .$$

(29)

We assume that the partial-wave decomposition is already made, and write the free Green function as

$$G^{(+)}_0(E) = \mathcal{P} \frac{1}{E - H_0} - i\pi \delta(E - H_0) = \mathcal{P}G_0(E) - i\pi |\phi\rangle \langle \phi| .$$

(30)

Then, it is convenient to deal with the $K$-matrix equation

$$R(E)|\phi\rangle = V|\phi\rangle + VPG^{(+)}_0(E)R(E)|\phi\rangle ,$$

(31)

instead of $T(E)|\phi\rangle$, which is obtained from

$$R(E)|\phi\rangle = T(E)|\phi\rangle [1 - i\pi \langle \phi|T(E)|\phi\rangle]^{-1} .$$

(32)
The essential point of the Noyes-Kowalski formalism is to use
\[ W = V - V|\phi\rangle \frac{1}{\langle \phi|V|\phi\rangle} \langle \phi|V , \] (33)
satisfying \( \langle \phi|W = 0 \) and \( W|\phi\rangle = 0 \). The solution of Eq. (31) is factorized as
\[ R(E)|\phi\rangle = f|\phi\rangle \langle \phi|R(E)|\phi\rangle . \] (34)
Then the square integrable function \( |f\rangle = f|\phi\rangle \langle \phi|V|\phi\rangle \) should satisfy the basic equation
\[ |f\rangle = V|\phi\rangle + WG_0(E)|f\rangle . \] (35)
Note that \( WG_0(E) \) with \( G_0(z) = 1/(z - H_0) \) is the Hilbert-Schmidt kernel, and the solution of Eq. (35) satisfies a trivial relationship \( \langle \phi|f|\phi\rangle = 1 \). The on-shell \( T \)-matrix is calculated from
\[ \langle \phi|R(E)|\phi\rangle = [1 - \langle \phi|V\mathcal{P}G_0(E)f|\phi\rangle]^{-1} \langle \phi|V|\phi\rangle , \] (36)
and
\[ \langle \phi|T(E)|\phi\rangle = [1 + i\pi \langle \phi|R(E)|\phi\rangle]^{-1} \langle \phi|R(E)|\phi\rangle . \] (37)
The half off-shell \( T \)-matrix is given by
\[ T(E)|\phi\rangle = f|\phi\rangle \langle \phi|T(E)|\phi\rangle . \] (38)
In order to obtain the complete off-shell \( T \)-matrix, we should generalize Eq. (35) as
\[ f = V\langle \phi|V|\phi\rangle^{-1} + WG_0(E)f , \] (39)
and its transpose
\[ \tilde{f} = \langle \phi|V|\phi\rangle^{-1}V + \tilde{f}G_0(E)W . \] (40)
The full \( T \)-matrix is given by
\[ T(E) = f|\phi\rangle \langle \phi|T(E)|\phi\rangle \langle \phi|\tilde{f} + \left\{ \begin{array}{c} f\langle \phi|V|\phi\rangle - f|\phi\rangle \langle \phi|V \\ \langle \phi|V|\phi\rangle \tilde{f} - V|\phi\rangle \langle \phi|\tilde{f} \end{array} \right\} . \] (41)
Let us discuss the condition for which the solution of Eq. (35) exists. From the Fredholm’s alternative theorem, it is essential to examine if there exist square integrable functions $\langle \psi \rangle$ for the conjugate homogeneous equation

$$\langle \psi \rangle (1 - W G_0(E)) = 0 .$$

If there exist such solutions, the necessary and sufficient condition for the unique solution of Eq. (35) is

$$\langle \psi \rangle V \langle \phi \rangle = 0 ,$$

for all $\psi$. Suppose there exists only one $\psi$ for Eq. (12), and Eq. (35) has a special solution $|f_0\rangle$. Then the general solution of Eq. (35) is given by

$$|f\rangle = |f_0\rangle + C(E - H_0)|\psi\rangle$$

for any arbitrary constant $C$. If $\psi$ does not satisfy Eq. (43), Eq. (35) is unsolvable. We can also rewrite the integral equation Eq. (12) to the form of the differential equation

$$\langle \psi \rangle (E - H_0 - W)|\psi\rangle = 0 ,$$

with the proper boundary condition. The plane wave $|\phi\rangle$ fulfills Eq. (13), but is not a solution of Eq. (12) since it is not square integrable. Unfortunately, we can not prove or disprove the existence of the positive-energy bound state for $W$ in Eq. (13), even for the standard potential $V$. In the following, we assume that there is no such solution which satisfied Eq. (12) for the standard potentials $V$.

When we apply the present formalism to the simple OCM equation Eq. (12) or to the RGM equation Eq. (8), we find an apparent solution $|\psi\rangle = |u\rangle$. In fact, for $(\varepsilon - H_0)|\phi\rangle = 0$, we find that $W = W(\varepsilon)$ for $V = V(\varepsilon)$ is given by

$$W(\varepsilon) = \frac{\langle \varepsilon - H_0\rangle |u\rangle \langle u| (\varepsilon - H_0)}{\varepsilon - \varepsilon_0} \quad \text{with} \quad \varepsilon_0 = \langle u|H_0|u\rangle .$$

This $W(\varepsilon)$ satisfies

$$W(\varepsilon)|u\rangle = (\varepsilon - H_0)|u\rangle \quad \text{and} \quad \langle u|W(\varepsilon) = \langle u| (\varepsilon - H_0) ,$$

which is similar to

$$V(\varepsilon)|u\rangle = (\varepsilon - H_0)|u\rangle \quad \text{and} \quad \langle u|V(\varepsilon) = \langle u| (\varepsilon - H_0) .$$
Thus we can easily see that $|\psi\rangle = |u\rangle$ is a solution of Eq. (43); i.e., $(\varepsilon - H_0 - W(\varepsilon))|u\rangle = 0$, and satisfies $\langle u | V(\varepsilon) | \phi \rangle = 0$. When we apply the present formulation to the RGM equation Eq. (10), $W$ for $V = V(\varepsilon) + v$ is no more simple like Eq. (46), but the basic relationship Eq. (47) for $W(\varepsilon)$ and $V(\varepsilon) \rightarrow V$ is still valid owing to the property $v = \Lambda v \Lambda$. We assume that there is no other solution $\psi$ for Eq. (12).

Let us rederive the $T$-matrix of §2.2 for the simple OCM. For the on-shell and the half off-shell $T$-matrix, the direct use of Eq. (46) in Eq. (35) allows the solution

$$|f_0\rangle = (\varepsilon_0 - H_0)|u\rangle \langle u| \phi \rangle, \quad C = \frac{1}{\varepsilon - \varepsilon_0} \langle u| f \rangle.$$ (49)

Here $f_0$ is a special solution which satisfies $\langle u| f_0 \rangle = 0$. Using this solution, we can easily derive

$$\langle \phi | R(\varepsilon) | \phi \rangle = - \frac{\langle \phi | u \rangle \langle u | \phi \rangle}{\langle u | PG_0(\varepsilon) | u \rangle}, \quad \langle \phi | T(\varepsilon) | \phi \rangle = - \frac{\langle \phi | u \rangle \langle u | \phi \rangle}{\langle u | G_0^+(\varepsilon) | u \rangle}.$$ (50)

For the complete off-shell $T$-matrix, we again need to assume $\omega \neq \varepsilon$, since otherwise Eq. (46) applied to Eq. (39) leads to the condition $\langle u | (\varepsilon - H_0) = 0$, which is apparently not satisfied for a general $\varepsilon$. The decomposition of $V(\varepsilon)$ with respect to $(\omega - H_0)|\omega\rangle = 0$ leads to the result

$$W(\omega, \varepsilon) = V(\varepsilon) - V(\varepsilon)|\omega\rangle \frac{1}{\langle \omega | V | \omega \rangle} \langle \omega | V(\varepsilon)$$

$$= \frac{(\omega - H_0)|u\rangle \langle u| (\omega - H_0)}{2\omega - \varepsilon - \varepsilon_0}.$$ (51)

After some calculations, the solution of Eq. (39) with $E$, $|\phi\rangle$, $V$ and $W$, being replaced with $\omega$, $|\omega\rangle$, $V(\varepsilon)$ and $W(\omega, \varepsilon)$, respectively, is found to be

$$f = \frac{1}{\langle \omega | u \rangle \langle u | \omega \rangle} \left\{ |u\rangle \langle u| - \frac{(2\omega - \varepsilon - H_0)|u\rangle \langle u| (\omega - H_0)}{(2\omega - \varepsilon - \varepsilon_0)(\omega - \varepsilon)} \right\}.$$ (52)

This expression leads to some simple relations

$$f|\omega\rangle = \frac{|u\rangle}{\langle \omega | u \rangle}, \quad \langle \omega | f | \omega \rangle = 1,$$

$$\langle u | f \rangle = \frac{1}{\langle \omega | u \rangle \langle u | \omega \rangle} \left\{ |u\rangle - \frac{1}{\omega - \varepsilon} \langle u| (\omega - H_0) \right\},$$

$$\langle u | f | \omega \rangle = \frac{1}{\langle \omega | u \rangle}.$$ (53)
Similarly, the solution of Eq. (40) is given by
\[ \tilde{f} = \frac{1}{\langle \omega | u \rangle} \left\{ \langle u | u \rangle - \frac{(\omega - H_0)|u\rangle\langle u|(2\omega - \varepsilon - H_0)}{(\omega - \varepsilon)(2\omega - \varepsilon - \varepsilon_0)} \right\}, \quad (54) \]
and
\[ \langle \omega | \tilde{f} \rangle = \frac{\langle u \rangle}{\langle u | \omega \rangle}, \quad \langle \omega | \tilde{f} | \omega \rangle = 1, \]
\[ \tilde{f}|u\rangle = \frac{1}{\langle \omega | u \rangle} \left\{ \langle u \rangle - \frac{1}{\omega - \varepsilon}(\omega - H_0)|u\rangle \right\}, \]
\[ \langle \omega | \tilde{f} | u \rangle = \frac{1}{\langle u | \omega \rangle}. \quad (55) \]

Finally, we use Eqs. (41) with \(|\phi\rangle \rightarrow |\omega\rangle\), (25), and (52) \(\sim (53)\), to reconstruct \(T(\omega, \varepsilon)\). The final result is, of course, equal to Eq. (23).

### 3 The orthogonality in the \(T\)-matrix for the simple OCM equation

We can use the full expression of the \(T\)-matrix derived in the preceding section, to investigate how the idea of the orthogonality to the Pauli-forbidden state is preserved in the simple OCM. Let us first consider the \(\omega \rightarrow \varepsilon\) limit in the two expressions in Eq. (24). These two expressions correspond to the wave-function of the ordinary scattering problem Eq. (15) (with \(\varepsilon\) being a simple parameter) and to the off-shell \(T\)-matrix in the \(G\)-matrix formalism in Eq. (26), respectively. We find
\[ \lim_{\omega \rightarrow \varepsilon} T(\omega, \varepsilon) |\omega\rangle \neq \lim_{\omega \rightarrow \varepsilon} T(\omega, \varepsilon) |\phi\rangle. \quad (56) \]

In the left-hand side of Eq. (56) the wave-function is orthogonal to the Pauli-forbidden state \(|u\rangle\), while in the right-hand side, the portion corresponding to the defect function of the \(G\)-matrix is orthogonal. When \(\omega = \varepsilon\), the solution of Eq. (13) has the ambiguity for any admixture of the \(|u\rangle\) component, which resolves the discrepancy of the two different half off-shell \(T\)-matrices.
in Eq. (56). In order to see this, we assume \( \omega = \varepsilon \) and derive the wave-function, leaving the ambiguity of \(|u\rangle\). The solution Eq. (44) with Eq. (49) immediately gives

\[
f|\phi\rangle = \frac{1}{(\varepsilon_0 - \varepsilon)}|u\rangle \langle u|\phi\rangle + (\varepsilon - H_0)|u\rangle C. \tag{57}
\]

The new parameter \( c \) is related to \( C \) through

\[
c = \frac{1}{(\varepsilon_0 - \varepsilon)}\langle u|\phi\rangle \langle u|\phi\rangle \left[ \langle u|\phi\rangle + C \right]. \tag{58}
\]

On the other hand, the wave-function is derived from

\[
|\psi(+)\rangle - |\phi\rangle = G_0^{(+)}(\varepsilon) T(\varepsilon)|\phi\rangle = G_0^{(+)}(\varepsilon) f|\phi\rangle \langle \phi| T(\varepsilon)|\phi\rangle,
\]

\[
|\psi_R\rangle - |\phi\rangle = \mathcal{P} G_0(\varepsilon) R(\varepsilon)|\phi\rangle = \mathcal{P} G_0(\varepsilon) f|\phi\rangle \langle \phi| R(\varepsilon)|\phi\rangle, \tag{59}
\]

where \( |\psi_R\rangle \) is the standing-wave solution for \( \mathcal{P} G_0(\varepsilon) \). For simplicity, we use the shorthand notation

\[
D(\varepsilon) = \langle u| \mathcal{P} G_0(\varepsilon)|u\rangle,
\]

\[
D^{(+)}(\varepsilon) = \langle u| G_0^{(+)}(\varepsilon)|u\rangle = D(\varepsilon) - i\pi \langle u|\phi\rangle \langle \phi| u\rangle, \tag{60}
\]

and express the on-shell \( T \)-matrix as

\[
\langle \phi| R(\varepsilon)|\phi\rangle = -\frac{\langle \phi| u\rangle \langle u|\phi\rangle}{D(\varepsilon)}, \quad \langle \phi| T(\varepsilon)|\phi\rangle = -\frac{\langle \phi| u\rangle \langle u|\phi\rangle}{D^{(+)}(\varepsilon)}. \tag{61}
\]

From Eq. (57), \( T(\varepsilon)|\phi\rangle \) etc. are obtained as

\[
T(\varepsilon)|\phi\rangle = f|\phi\rangle \langle \phi| T(\varepsilon)|\phi\rangle = \frac{|u\rangle \langle u|\phi\rangle}{D^{(+)}(\varepsilon)} + (\varepsilon - H_0)|u\rangle \langle u|\phi\rangle B, \tag{62}
\]

where the third parameterization of the \(|u\rangle\) component, \( B \), is given by

\[
B = -c \frac{\langle \phi| u\rangle}{D^{(+)}(\varepsilon)} = -\frac{1}{(\varepsilon_0 - \varepsilon)D^{(+)}(\varepsilon) \langle u|\phi\rangle} \left[ \langle u|\phi\rangle + C \right]. \tag{63}
\]
We have various orthogonalities, as shown in Table I, depending on what values we take for the arbitrary $C$, $c$ or $B$. These half off-shell $T$-matrices have all equal qualifications for the solution of Eq. (12).

Summarizing this section, we have found that the Lippmann-Schwinger equation Eq. (17) is a very general equation describing not only the free scattering but also the correlation for the scattering in a nuclear medium. For the simple OCM Eq. (13), the solution of the half off-shell $T$-matrix depends on how the model is formulated from the more general equation in which the Pauli-forbidden state does not exist. The second method in Eq. (16) seems to be more natural than Eq. (15), since it has a direct physical meaning of summing up all the ladder diagrams in the $G$-matrix formulation. In this case, the orthogonality to the Pauli-forbidden state is represented with respect to the correlation function, instead of the wave-function itself.

Table 1: The orthogonality properties for the simple OCM in Eq. (12) or Eq. (13).

| orthogonality       | $C$       | $c$       | $B$       |
|---------------------|-----------|-----------|-----------|
| $\langle u|\psi^{(+)}\rangle = 0$ | $-\langle u|\phi\rangle$ | 0         | 0         |
| $\langle u|\psi^{(+)} - \phi\rangle = 0$ | $-\left[1 + (\varepsilon_0 - \varepsilon)D^{(+)}(\varepsilon)\right]\langle u|\phi\rangle$ | $-\frac{D^{(+)}(\varepsilon)}{\langle \phi|\psi\rangle}$ | 1         |
| $\langle u|\psi_R - \phi\rangle = 0$ | $-\left[1 + (\varepsilon_0 - \varepsilon)D(\varepsilon)\right]\langle u|\phi\rangle$ | $-\frac{D(\varepsilon)}{\langle \phi|\psi\rangle}$ | $\frac{D(\varepsilon)}{D^{(+)}(\varepsilon)}$ |
| $\langle u|f\rangle = 0$ | 0         | $\frac{1}{(\varepsilon_0 - \varepsilon)\langle \phi|\psi\rangle}$ | $-\frac{1}{(\varepsilon_0 - \varepsilon)D^{(+)}(\varepsilon)}$ |
4 Modification of the RGM kernel

The uniqueness of the solution for the Noyes-Kowalski equation Eq. (33) is derived from the structure

\[ V(\varepsilon) = (\varepsilon - H_0) - \Lambda(\varepsilon - H_0)\Lambda \]

and the relationship \( v = \Lambda v\Lambda \). It does not depend on the explicit form \( v = \Lambda V_{\text{RGM}}\Lambda = \Lambda (V_D + G + \varepsilon K)\Lambda \). We can modify the internal-energy part and reduced masses of the CCRGM kernel, by using this property. Namely, we leave \( \varepsilon \) in \( v \) as it is, and set up the RGM equation as

\[
(\varepsilon_{\text{exp}} - H_{0\text{exp}}) \chi = (V_{\text{exp}}(\varepsilon) + v) \chi , \\
V_{\text{exp}}(\varepsilon) = (\varepsilon_{\text{exp}} - H_{0\text{exp}}) - \Lambda (\varepsilon_{\text{exp}} - H_{0\text{exp}})\Lambda ,
\]

(64)

where

\[
\varepsilon_{\text{exp}} = E - E_{\text{int}} , \quad H_{0\text{exp}} = -\frac{\hbar^2}{2\mu_{\text{exp}}} \left( \frac{\partial}{\partial r} \right)^2 = \frac{\mu}{\mu_{\text{exp}}} H_0
\]

(65)

are the empirical relative energies and free kinetic-energy operators, respectively. We denote the modification of these quantities by

\[
\Delta E_{\text{int}} = E_{\text{int}}^{\text{exp}} - E_{\text{int}} , \quad \Delta H_0 = H_{0\text{exp}} - H_0 = \left( \frac{\mu}{\mu_{\text{exp}}} - 1 \right) H_0 ,
\]

(66)

and use the notation

\[
\Delta G = \Lambda (\Delta E_{\text{int}} + \Delta H_0)\Lambda - (\Delta E_{\text{int}} + \Delta H_0) .
\]

(67)

The new RGM equation Eq. (64) is equivalent to

\[
\Lambda (\varepsilon_{\text{exp}} - H_{0\text{exp}} - V_{\text{RGM}})\Lambda \chi = 0 ,
\]

(68)

if we use \( v = \Lambda V_{\text{RGM}}\Lambda \). This implies that we have replaced \( \varepsilon \) and \( H_0 \) in the direct term as

\[
\varepsilon \rightarrow \varepsilon_{\text{exp}} , \quad H_0 \rightarrow H_{0\text{exp}} ,
\]

(69)

in the allowed model space, without changing \( \varepsilon \) in \( v = \Lambda V_{\text{RGM}}\Lambda = \Lambda (V_D + G + \varepsilon K)\Lambda \). In the original form of the RGM equation Eq. (8), it is easy to see that Eq. (8) should be modified to

\[
(\varepsilon_{\text{exp}} - H_{0\text{exp}} - V_{\text{RGM}} - \Delta G)\chi = 0 .
\]

(70)
This equation indicates that an extra modification of adding $\Delta G$ is required, in addition to the modification Eq. (67) of the direct term.

As an example, let us consider $\Lambda N$-$\Sigma N (I = 1/2)$ CCRGM. The scattering problem of this system is solved in Ref. [15], using Lippmann-Schwinger RGM (LS-RGM) formalism. Since this system involves a complete Pauli-forbidden state in the $(11)_{s} SU_{3}$ representation for the $^{1}S_{0}$ state, [16] we need a special care for the treatment of the Pauli principle. In our previous publications, the realistic treatment of the reduced mass in the direct term, using the empirical $\Lambda$ and $\Sigma$ masses, needed some modification of the exchange kinetic-energy kernel. The procedure adopted in Ref. [16], multiplying the exchange kinetic-energy kernel $G^{K}$ by the factor $\sqrt{\mu_{\alpha}/\mu_{\text{exp}}}$ from the bra and ket sides, as $\sqrt{\mu_{\alpha}/\mu_{\text{exp}}} G^{K}_{\alpha\alpha'} \sqrt{\mu_{\alpha'}/\mu_{\text{exp}}}$, is not actually accurate for the coupled-channel problems. As the result, the previous calculation has given a catastrophic resonance behavior for the $\Lambda N$ $^{1}S_{0}$ phase shift in the low-momentum region around $p_{\Lambda} \sim 100$ MeV/$c$, if we take too many mesh points for the momentum discretization. We, therefore, use the following two-step modification for the $\Lambda N$ and $\Sigma N$ reduced masses. We fist multiply all the channels by the common $\mu_{\alpha=1}/\mu_{\text{exp}}$ factor for the incident baryon channel, just as done for the single-channel problem (see Ref. [17]). This process is necessary to reduce too strong effect of the momentum-dependent Darwin term involved in the Fermi-Breit interaction. The Pauli principle is exactly preserved at this stage with respect to the kinetic-energy term. Next we introduce a small modification for the reduced mass of the second baryon channel with respect to the direct term. The modification of the exchange term is carried out by using $\Delta G$ in Eq. (67). Since the magnitude of $\Delta H_{0}$ term is at most a few MeV, the error caused by this approximate treatment of the exchange term should be more than one order smaller in comparison with the "exact" value of the exchange kernel (even if it is possible to evaluate). In practice, we augment the exchange kernel $G(q_{f}, q_{i})$ with the $S$-wave Born kernel

$$
\Delta G^{K}(q_{f}, q_{i}) = \left( \frac{\hbar^{2}}{2\mu} \right) f \frac{9}{100} \begin{pmatrix}
\frac{9}{40\pi} & -\frac{10}{7} q_{f}^{2} + \frac{27}{40\pi} \\
-\frac{10}{7} q_{f}^{2} + \frac{27}{40\pi} & -\frac{10}{7} q_{f}^{2} + \frac{27}{40\pi} + \frac{81}{40\pi}
\end{pmatrix},
$$

$$
\Delta G^{K}(q_{f}, q_{i}) = \left( \frac{\hbar^{2}}{2\mu} \right) f \frac{9}{100} \begin{pmatrix}
\frac{9}{40\pi} & \frac{10}{7} q_{i}^{2} + \frac{3}{40\pi} \\
-\frac{10}{7} q_{i}^{2} + \frac{3}{40\pi} & -\frac{10}{7} q_{i}^{2} + \frac{3}{40\pi}
\end{pmatrix}.
$$
for $\Lambda N$-incident and $\Sigma N$-incident scatterings, respectively. Here we have defined $\Delta \hbar^2/2\mu = \hbar^2/2\mu_{\Sigma N}^\text{exp} - \hbar^2/2\mu_{\Lambda N}^\text{exp}$, $b$ is the harmonic-oscillator width parameter, and the $S$-wave spatial function $f$ is given by

$$f = f(q_f, q_i) = \left(\frac{8\pi}{3}b\right)^3 \exp\left\{-\frac{1}{3}b^2(q_f^2 + q_i^2)\right\}.$$  

(72)

The condition $\langle u|V_RGM + \Delta G|\phi \rangle = 0$ for the Pauli-forbidden state $|u\rangle$ and $|\phi\rangle$ satisfying $(\varepsilon^\text{exp} - H_0^\text{exp})|\phi\rangle = 0$ guarantees the existence of the solution of Eq. (70). If we assume $\Delta G = 0$ in Eq. (70), we obtain the expression

$$\langle u|V_RGM|\phi \rangle = -\langle u|\Delta G|\phi \rangle = \langle u|(E_{\text{int}} + H_0)|\phi \rangle.$$  

(73)

This expression is actually 0 if the incident momentum of $\Lambda$ is below the $\Sigma N$ threshold. This is because the plane wave solution $|\phi\rangle$ has the non-zero component only for the $\Lambda N$ channel and the $2 \times 2$ matrix $\Delta(E_{\text{int}} + H_0)$ has a non-zero component only for the second diagonal channel. In this particular case, neglecting $\Delta G$ in the second step is harmless, even if we modify only the direct term.

The modification of the internal-energy term is not necessary in the $\Lambda N$-$\Sigma N$ coupled-channel problem in the isospin basis, since the model parameters are usually fixed to reproduce the mass difference of the $\Lambda$ and $\Sigma$ in our previous models, FSS and RGM-H. [18, 19, 20] This is, however, no more valid in the particle-basis calculation [21] to study the charge symmetry breaking, since a consistent description of the baryon-mass splitting in terms of the up-down quark mass difference and the Coulomb energies is not always successful. It may, therefore, be useful to present a general expression of the extra Born kernel Eq. (67) in the following:

$$\Delta G_{\alpha\alpha'}(q_f, q_i) = \Delta G_{\alpha\alpha'}^\text{int}(q_f, q_i) + \Delta G_{\alpha\alpha'}^K(q_f, q_i),$$

$$\Delta G_{\alpha\alpha'}^\text{int}(q_f, q_i) = c_\alpha c_{\alpha'} f \left[ -\left(\Delta E_{\alpha}^\text{int} + \Delta E_{\alpha'}^\text{int} \right) + \sum_{\alpha''} c_{\alpha''}^2 \Delta E_{\alpha''}^\text{int} \right],$$

$$\Delta G_{\alpha\alpha'}^K(q_f, q_i) = c_\alpha c_{\alpha'} f \left[ -\left(q_f^2 \Delta \hbar^2/2\mu_{\alpha} + q_i^2 \Delta \hbar^2/2\mu_{\alpha'} \right) + \frac{9}{4b^2} \sum_{\alpha''} c_{\alpha''}^2 \Delta \hbar^2/2\mu_{\alpha''} \right].$$  

(74)
where $f$ is given in Eq. (72) and

$$
\Delta E^\text{int}_\alpha = \left( E^\text{int}_\alpha \right)^\text{exp} - \left( E^\text{int}_\alpha \right)^\text{cal},
$$

$$
\Delta \frac{\hbar^2}{2\mu^2} = \frac{\hbar^2}{2\mu^2_\alpha} - \frac{\hbar^2}{2\mu^2_{\alpha=1}}. \quad (75)
$$

The normalized eigenvector $c_\alpha$ of the Pauli-forbidden state in the flavor space is obtained from the eigenvalue equation

$$
\sum_{\alpha'} \left( \lambda \delta_{\alpha\alpha'} + (X_N)_{\alpha\alpha'} \right) c_{\alpha'} = 0 \quad \text{with} \quad \lambda = 1, \quad (76)
$$

by using the spin-flavor-color factors of the exchange normalization kernel $X_N$. The result of Eq. (71) is obtained if one uses $c_1 = 1/\sqrt{10}$, $c_2 = 3/\sqrt{10}$ for the $\Lambda N$-incident channel and $c_1 = 3/\sqrt{10}$, $c_2 = 1/\sqrt{10}$ for the $\Lambda N$-incident channel. This formula can also be used for the $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ CCRGM with the strangeness $S = -2$.

## 5 Discussion and summary

In this paper we have discussed what kind of equation we should solve for the off-shell $T$-matrix derived from the $(3q)-(3q)$ resonating-group method (RGM) for the baryon-baryon interaction. This is an important issue since the present-day quark-model description for the nucleon-nucleon ($NN$) and the hyperon-nucleon ($YN$) interactions is very accurate, and the realistic calculation of the hypertriton and baryonic matter etc. using directly the quark-exchange kernel of these interactions is feasible to investigate the important off-shell effect and intricate behavior of the short-range correlations.\footnote{For the $G$-matrix calculation of the $NN$ and $YN$ systems, using the model FSS and RGM-H, see Refs. \[22\] and \[23\].}

The off-shell $T$-matrix in RGM requires a rather involved formulation when the complete Pauli-forbidden state exists. Since the relative wavefunction can contain an arbitrary admixture of the redundant components, the half off-shell $T$-matrix should be defined as a limit of some definite $T$-matrix equation without this ambiguity. As a possible choice of such equation, we have proposed a standard $G$-matrix equation which uses the quark-exchange kernel $V_{\text{RGM}} = V_D + G + \varepsilon K$ directly as the driving term for the
infinite sum of all the ladder diagrams. The relative energy $\varepsilon$ should be taken with respect to the initial two-particle state.\textsuperscript{3} By using a simplified version of Saito's orthogonality condition model (OCM),\textsuperscript{9} we have argued that this inherent energy-dependence of the Born kernel is essential to preserve the major role of the Pauli principle, which is sometimes represented in the form of the orthogonality condition to the Pauli-forbidden state. In the ordinary formulation of OCM this orthogonality condition is applied to the relative wave-function,\textsuperscript{3, 4} while in the present $G$-matrix formulation the defect-function part of the correlation function is orthogonal. We can also start from the OCM-type RGM equation like Eq. (10), and derive a different type of $G$-matrix equation for the quark-model interaction. But such a formulation needs a careful treatment of the Pauli forbidden state, which is represented by $V(\varepsilon)$ in Eq. (11) in this paper. The present method is the simplest and the most natural in the sense that such a term is automatically incorporated in the structure of the exchange kernel for the normalization and kinetic-energy terms.

We have also clarified the condition for which the integral equation derived by Noyes\textsuperscript{7} and Kowalski\textsuperscript{8} is solvable and has a unique solution for the most general off-shell $T$-matrix. For the on-shell and half off-shell $T$-matrices, the conjugate homogeneous equation has a trivial solution if the Pauli-forbidden state $\vert u \rangle$ exists. The condition $\langle u \vert V_{\text{RGM}} \phi \rangle = 0$ is automatically satisfied for the plane-wave solution $\vert \phi \rangle$ in the initial channel, as long as the energy $\varepsilon$ in $V_{\text{RGM}}$ is fixed to the energy of $\vert \phi \rangle$. We can use this relationship to test if the exchange kernel is correctly calculated. When one needs to modify the exchange kernel, the modification should be made such a way that this condition is still satisfied. An example of this kind of modification is the correction of the reduced masses and a small readjustment of the threshold energies in the coupled-channel RGM (CCRGM). In the rigorous framework of the RGM, it is sometimes difficult to reproduce correct reduced masses and internal energies starting from a unique Hamiltonian. In particular, the baryon masses for the inertia mass and the rest mass are sometimes inconsistent in the framework of the non-relativistic quark model, due to the residual $qq$ interaction. On the other hand, the correct kinematics is mandatory for the realistic description of the baryon-baryon interaction.

\textsuperscript{5}In this sense, the prescription to neglect the $\varepsilon K$ term in $V_{\text{RGM}}$, as in Ref.\textsuperscript{24}, is not correct.
We have shown that, if one uses the empirical reduced masses and threshold energies, an additional exchange term $\Delta G(q_f, q_i)$ should be added in general to the Born amplitude $V_{RGM}(q_f, q_i)$. We have given the explicit expression for $\Delta G(q_f, q_i)$, which can be used for $\Lambda N-\Sigma N$ and $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ CCRGM.

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