South Korean Elementary Teachers’ Mathematical Knowledge for Teaching Numbers and Operations

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Doi:10.5901/mjss.2014.v5n15p336

Abstract

We examined South Korean elementary teachers’ mathematical knowledge for teaching numbers and operations and the factors that contribute to this knowledge. We used a translated and adapted version of the Learning Mathematics for Teaching Project measures and the Quadric Survey to collect information on the teachers’ knowledge of teaching the subject and their background information. Based on the statistical analysis (ANOVA and multiple regression mode), we found that the teachers’ knowledge for teaching mathematics might differ based on teaching experience, academic degree related to elementary mathematics education, certification level, and educational districts in which they work.

Keywords: mathematical knowledge for teaching; elementary teacher; teacher certification; teaching experiences; teacher education program;

1. Introduction

Recent studies propose that teachers’ mathematical knowledge for teaching mathematics is one of the major factors related to students’ achievement in the subject (An, Kulm, & Wu, 2004; Ball, Hill, & Bass, 2005). Teachers’ mathematical knowledge for teaching goes beyond simply being able to solve mathematics problems (Kwon, Thames, & Pang, 2012). In this respect, Ball and Bass (2000) argued that teachers should know how to represent mathematics and use that knowledge to explain core concepts. Common Core State Standards [CCSS] for Mathematics Initiative also proposes that teachers’ knowledge for teaching mathematics include both procedural skills and conceptual understanding to make sure students are learning and absorbing the critical information they need to succeed at higher levels (CCSS, 2010).

Elementary teachers’ mathematical knowledge for teaching is significant in students’ mathematics learning because teachers at this level affect younger students’ mathematics achievement scores more than those of older students (Hill, 2008; Konstantopoulos, 2011). However, elementary teachers’ mathematical knowledge for teaching has been widely recognized as weak compared to mathematics teachers at the secondary level (Ng, 2011). The fact that elementary teachers are usually trained to be generalists in charge of teaching all subjects may make the work of teaching mathematics challenging, and teachers may have difficulties developing profound knowledge for teaching mathematics (Grover & Conner, 2000).

In particular, results of studies on elementary teachers’ mathematical knowledge for teaching numbers and operations illustrates that the teachers have weak knowledge regarding number concepts and have difficulties representing the concepts during the mathematics instruction. Hopkins and Cady (2007) demonstrated that elementary teachers may not fully comprehend the whole number concepts. McClain (2003) also reported that elementary teachers have a weak understanding of number concepts; the teachers in the study identified place values improperly and seemed to misunderstand the relationship between numbers and operations (McClain, 2003). Elementary teachers’ weak
knowledge in numbers and operations may affect students’ learning because diverse studies have shown that there is a positive correlation between students’ development of mathematical proficiency and teachers’ knowledge for teaching mathematics, as noted previously (Ball et al., 2010). Thus, it is significant to improve teachers’ knowledge for teaching numbers and operations in developing students’ mathematical understanding.

The main prerequisite to improving elementary teachers’ mathematical knowledge for teaching numbers and operations may be to scrutinize teachers’ current statuses. In particular, studies on the factors that may contribute to the development of teachers’ mathematical knowledge for teaching may provide meaningful implications to policy makers, teacher preparation programs, and professional development programs.

The focus of this study encompasses factors that may contribute to elementary teachers’ mathematical knowledge for teaching numbers and operations in South Korea. Specifically, the purpose of this study is to investigate the following research questions:

• How does South Korean elementary teachers’ mathematical knowledge for teaching numbers and operations correspond to the teaching experiences, academic backgrounds, certification levels, gender, and locations at which the teachers work?
• Which of the above factors contribute most to South Korean elementary teachers’ mathematical knowledge for teaching numbers and operations?

2. Mathematical Knowledge for Teaching

Over past two decades, diverse studies have investigated how teachers apply their knowledge when teaching mathematics (e.g., Fennema & Franke, 1992; Ma, 2000). Effective teachers obtain in-depth knowledge of how to represent the subject matter to students (Parker & Heywood, 2000). Shulman (1986, 1987) named this profound knowledge pedagogical content knowledge (PCK), which provides a special amalgam of content and pedagogy. PCK also contains knowledge of how to transform content into forms that are adaptive to the variations in ability and background presented by the students based on the components of PCK (An et al., 2004), such as the knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes, and values, and their philosophical and historical bases (Shulman, 1987). Building on this work, Hill, Ball and Schilling (2008) developed a theory about the mathematics that elementary school teachers need to know and named it mathematical knowledge for teaching (MKT) (Table 1).

Table 1. Mathematical Knowledge for Teaching

| MKT       | Components                                                                 |
|-----------|-----------------------------------------------------------------------------|
| PCK       | Knowledge of content and students                                           |
|           | Knowledge of content and teaching                                           |
|           | Knowledge of content and curriculum                                         |
| Subject matter knowledge (SMK) | Common content knowledge |
|           | Horizon content knowledge                                                   |
|           | Specialized content knowledge                                               |

Hill et al. (2008) distinguished PCK and SMK within MKT first and relocated the other domains of teachers’ knowledge. Hill et al. (2008) took Shulman’s (1986) definition of PCK and established three subdomains: knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. However, SMK is not intertwined with PCK, and teachers need SMK for specific teaching tasks, such as explaining the purpose of mathematics education to parents (Ball, Thames, & Phelps, 2008). Ball et al. also argued that teachers needed to know the mathematical definition of a given concept and its alternative meaning and that they should be able to give a solid mathematical explanation. SMK has three subdomains: common content knowledge, horizon content knowledge, and specialized content knowledge.

In follow-up research, Delaney et al. (2008) developed assessment items that require teachers to identify the most appropriate answer for multiple-choice questions based on classroom-related scenarios. Using these items, studies revealed the positive relationship between teachers’ MKT and students’ achievement (e.g., Hill, Rowan, & Ball, 2005), as well as the quality of teachers’ mathematics instruction (Hill et al., 2008). Recent international studies also used these items as grounds for adapting the instruments for use outside of the United States (e.g., Ng, 2011; Delaney et al., 2008).
In particular, Kwon et al. (2012) examined the possibility of adapting the instruments for South Korea and confirmed that the items are available for use and that they maintain validity.

3. **Elementary Teacher Recruitment System in South Korea**

According to the Ministry of Education in South Korea (2012), there are 5,895 elementary schools in South Korea. Among them, only 76 schools (1.3%) are private schools. To teach in both public and private elementary schools in South Korea, preservice teachers should have a second-level elementary teacher's license. Elementary teacher candidates obtain the license by attaining bachelor's degrees from 13 specialized four-year universities designated by the South Korean government. All preservice teachers complete the same required courses for 120 credits, including seven credits for elementary mathematics education as well as an 11-week teacher-practicum and 20 elective credits for advanced subject matter education (Seoul National University of Education, 2012). For the advanced subject matter education, preservice teachers may choose one subject among 12 in elementary education as their specialty. For example, if preservice teachers want to study more about mathematics education, they may take courses related to mathematics education as their advanced subject matter. Five credits among the required 120 credits are related to mathematics education: two credits for mathematics content and three credits for elementary mathematics curriculum. After earning 140 credits, preservice teachers obtain both bachelor's degrees in elementary education and second-level elementary teachers’ licenses.

To work in a private elementary school, preservice teachers must apply for each school with second-level elementary teachers’ licenses. The school board employs individual teachers based on the school's needs. The Ministry of Education in South Korea also takes charge of balancing the supply with the demand for elementary teachers in public schools. Until 1990, preservice teachers who had an elementary teacher certification could work in a public school without taking a national recruitment examination. However, the government revised the education law regarding the teacher recruitment system in 1990. To become an elementary teacher in a public school, preservice teachers who acquired their teacher certification from a specialized university must also pass national recruitment examination. The Ministry of Education in South Korea guarantees tenures until the age of 62 for those who pass the examination. After 3 to 5 years of teaching experiences in both private and public elementary schools, all in-service teachers are required to obtain first-level elementary teachers’ licenses with at least 90 hours of credits in a professional development program (Yoo, 2012). With first-level elementary teachers' licenses, teachers are qualified to become head teachers in elementary schools. The Ministry of Education in South Korea asks elementary teachers who have both the first-level and second-level elementary teachers' licenses to participate in professional development programs for at least 60 hours every year. The teachers are free to choose the programs based on their own needs. Although it is not compulsory, the record of participating in a professional development program is reflected in teachers’ performance assessments every year.

Since 1990, only preservice elementary teachers who pass the national recruitment examination can work in public elementary schools. However, the Ministry of Education in South Korea allowed special employment, lowering the retirement age for elementary teachers from 65 to 62 in 1999. The total number of preservice elementary teachers who graduate from the specialized universities was not enough to fill the demand at that time. Thus, the Ministry of Education in South Korea hired 5,558 preservice secondary teachers from 1999 to 2001 under the condition that they complete 91 academic credits related to elementary education from the specialized universities (Kim, 2011). The Ministry of Education in South Korea provided the second-level elementary teachers’ licenses to those who completed the refresher program. As of 2012, there were 181,435 elementary teachers in South Korea (Ministry of Education in South Korea, 2012). The proportion of elementary teachers employed from the special employment was up to 5% in 2011 (Kim, 2011).

The Ministry of Education in South Korea also determines the distribution of elementary teachers. The Ministry allocates elementary teachers to the school and relocates them every 5 years by law, taking the teacher's residence into consideration. The Ministry of Education in South Korea tries to designate a school as close as possible to a teacher's home. The Ministry also deliberates educational districts when it assigns teachers. There are 11 educational districts in Seoul: GangDong, GangSeo, GangNam, DongJak, NamBu, SeoBu, BukBu, JungBu, DongBu, SeoungDong, and SeoungBuk. The Ministry decided the range of educational districts based on geographical accessibilities and economic status. Some educational districts are located in suburbs while others are in urban areas. If a teacher has worked in a suburban area for 10 years, the will relocate the teacher to an urban area.

4. **Methods**

Conducting a survey helped to develop a broader perspective of elementary teachers' knowledge for teaching
mathematics. The survey method is useful when the purpose of the study is to describe quantitatively specific aspects of a given population (Kraemer, 1991). If the survey obtained data based on a representative sample, the data can be generalizable to a population (Kelley et al. 2003). Therefore, 287 randomly selected South Korean elementary teachers were surveyed to ensure validity and generalize findings.

4.1 Participants

The target population of this study is South Korean elementary school teachers. We chose participants who work in Seoul because of its geographical accessibility. Of the 181,435 elementary teachers in South Korea in 2012, 29,762 of them work in Seoul (Ministry of Education in South Korea, 2012). From this number, 500 elementary teachers were randomly selected for this research. Among them, 287 elementary teachers participated in the survey. The response rate was 57.4%. Although 287 South Korean elementary teachers participated in this survey, the participants had the right to skip questions they did not want to answer. Therefore, the total number of participants may differ among questions. The demographic information of the participants is shown in Table 2.

Table 2. Demographic Information of the Participants

| Teaching Experience | Gender | Total Number |
|---------------------|--------|--------------|
|                     | Male (n) | Female (n) |        |
| 0–5 years           | 18 | 106 | 124 |
| 6–10 years          | 8 | 41 | 49 |
| 11–15 years         | 10 | 52 | 63 |
| 16–20 years         | 6 | 17 | 23 |
| 21 years or more    | 6 | 21 | 27 |
| Total               | 48 | 237 | 285 |

| Teacher Certification | Gender | Total Number |
|-----------------------|--------|--------------|
|                      | Male (n) | Female (n) |        |
| 1st level             | 28 | 131 | 159 |
| 2nd level             | 19 | 105 | 124 |
| Total                 | 47 | 236 | 283 |

| Academic Degree relates to elementary education (including elementary mathematics education) | Gender | Total Number |
|-----------------------------------------------------------------------------------------------|--------|--------------|
| Program*                                                                                        | 3     | 8           | 11     |
| Bachelor                                                                                         | 34    | 195         | 230    |
| Master                                                                                           | 11    | 33          | 44     |
| Doctor                                                                                            | .     | .           | .      |
| Total                                                                                           | 48    | 236         | 284    |

| Academic Degree relates to elementary mathematics education*** | Gender | Total Number |
|----------------------------------------------------------------|--------|--------------|
| Bachelor**                                                      | 7      | 43          | 50     |
| Master                                                          | 1      | 11          | 12     |
| Doctor                                                           | .      | .           | .      |
| None                                                            | 40    | 176         | 216    |
| Total                                                           | 48    | 230         | 278    |

| Educational District | Gender | Total Number |
|----------------------|--------|--------------|
|                      | Male (n) | Female (n) |        |
| GangDong             | 3       | 13          | 16     |
| GangSeo              | 1       | 16          | 17     |
| GangNam              | 14      | 30          | 44     |
| DongJak              | 1       | 19          | 20     |
| NamBu                | 1       | 4           | 5      |
| SeoBu                | 1       | 16          | 17     |
| BukBu                | 6       | 29          | 35     |
| JungBu               | 7       | 41          | 48     |
| DongBu               | 5       | 12          | 16     |
| SeoungDong           | 2       | 23          | 25     |
| SeoungBuk            | 8       | 31          | 39     |
| Total                | 48      | 234         | 282    |

* Program indicates the specialized course for teachers who had the secondary teacher license. After completing 91 credits from the program, the teachers acquired the second-level elementary teacher’s license. 
** Bachelor relates to elementary mathematics education representing teachers who had to take mathematics education courses for their 20 credits of advanced subject matter as preservice teachers. 
*** Teachers were requested to identify their highest attainment in elementary mathematics education. For example, if a teacher had a master’s degree as well as bachelor’s degree in mathematics education, he or she only indicated the master’s degree for this item.
We assumed that the location of the elementary school in which the teachers worked might also be a factor that could affect teachers’ mathematical knowledge for teaching. This assumption is based on Hill and Lubienski’s (2007) findings that there is a relationship between characteristics of the population of the students in the school and the teachers’ MKT. Based on this assumption, we analyzed the teachers’ MKT scores with regard to the educational districts in which their schools are located. The demographic information of the participants across the educational districts is shown in Table 3.

Table 3. Demographic Information of the Participants According to the Educational Districts

| Educational District | Gang-Dong | Gang-Seo | Gang-Nam | Dong-Jak | Nam-Bu | Seo-Bu | Buk-Bu | Jung-Bu | Dong-Buk | Seoung-Dong | Seoung-Buk | Total |
|----------------------|-----------|---------|----------|----------|--------|--------|--------|---------|----------|------------|-----------|-------|
| Teaching Experience  |           |         |          |          |        |        |        |         |          |            |           |       |
| 0-5                  | 14        | 9       | 13       | 6        | 5      | 8      | 15     | 17      | 7        | 5          | 22        | 121   |
| 6-10                 | 1         | 1       | 6        | 5        | 0      | 7      | 4      | 10      | 10       | 5          | 11        | 5     |
| 11-15                | 1         | 7       | 9        | 4        | 0      | 1      | 10     | 10      | 5        | 11         | 5         | 63    |
| 16-20                | 0         | 0       | 9        | 2        | 0      | 0      | 3      | 4       | 1        | 2          | 2         | 23    |
| > 21                 | 0         | 0       | 7        | 3        | 0      | 1      | 3      | 7       | 0        | 2          | 4         | 27    |
| Total                | 16        | 17      | 44       | 20       | 5      | 17     | 35     | 48      | 16       | 26         | 39        | 283   |
| Certification       |           |         |          |          |        |        |        |         |          |            |           |       |
| 1st                  | 2         | 8       | 33       | 14       | 0      | 6      | 20     | 31      | 8        | 20         | 18        | 160   |
| 2nd                  | 14        | 9       | 11       | 6        | 5      | 11     | 15     | 16      | 8        | 6          | 21        | 122   |
| Total                | 16        | 17      | 44       | 20       | 5      | 17     | 35     | 47      | 16       | 26         | 39        | 282   |
| Gender               |           |         |          |          |        |        |        |         |          |            |           |       |
| Male                 | 3         | 1       | 14       | 1        | 1      | 1      | 6      | 7       | 4        | 2          | 8         | 48    |
| Female               | 13        | 16      | 30       | 19       | 4      | 16     | 29     | 41      | 12       | 23         | 31        | 234   |
| Total                | 16        | 17      | 44       | 20       | 5      | 17     | 35     | 48      | 16       | 25         | 39        | 282   |
| Degree in Elementary Education | | | | | | | | | | | | |
| Program              | 0         | 2       | 0        | 0       | 0      | 0      | 1      | 2       | 2        | 1          | 3         | 9     |
| Bachelor             | 15        | 11      | 27       | 18       | 5      | 16     | 27     | 39      | 13       | 23         | 33        | 227   |
| Master               | 1         | 4       | 16       | 2        | 0      | 1      | 7      | 7       | 1        | 2          | 3         | 44    |
| Total                | 16        | 17      | 43       | 20       | 5      | 17     | 35     | 48      | 16       | 26         | 39        | 282   |
| Degree in Elementary Mathematics Education | | | | | | | | | | | | |
| Master               | 1         | 0       | 3        | 3        | 0      | 2      | 1      | 2       | 0        | 0          | 0         | 12    |
| None                 | 9         | 11      | 35       | 16       | 4      | 7      | 29     | 38      | 15       | 21         | 31        | 214   |
| Total                | 16        | 17      | 43       | 20       | 5      | 16     | 35     | 48      | 16       | 24         | 38        | 278   |

4.2 Research Instruments

Two instruments were used for the survey: the Qualtrics Online Survey System and the Learning Mathematics for Teaching (LMT) measures (Learning Mathematics for Teaching Project, 2006). The Qualtrics Online Survey System collects information on the number of years of teaching experience, the highest educational degree obtained, and gender. The LMT measures mathematical knowledge for teaching. In particular, we applied the number and operation section that consists of 15 questions based on grades 3 through 8. The content of the questions is typical of those that teachers should answer when teaching numbers and operations to elementary school students. All questions are in multiple-choice format and have a range of three to five possible answers. The reliability of the LMT measures is 0.8 (Hill, 2004).

The authors of this study translated and adapted LMT measures into Korean and conducted psychometric analyses to ensure the validity and reliability of the measures. Two authors of this study are native speakers of Korean and are fluent in English. The two native Korean authors also have more than 10 years of teaching experience at elementary level in South Korea each. To maintain validity, we translated the instrument using a double translation process and then adjusted cultural differences based on Kwon et al.’s (2011) assumptions of changes for translating the LMT measures into Korean: the researchers should consider cultural context, school context, language differences, mathematical substance, curriculum, students’ work, teachers’ work, and other features when translating the LMT measures (Kwon et al., 2011). Among these categories, we altered people’s names to make them familiar to South Korean elementary teachers and altered the language to reflect the mathematical substance (e.g., the words “flat,” “long,” and “cube” were changed to “hundreds model," “tens model,” and “a piece model,” respectively). Three specialists validated the translated instruments: a professor of elementary mathematics education from Seoul National University of Education who is fluent in both English and Korean and two elementary teachers who have more than 15 years of teaching experiences each. These experts verified the accuracy of translation and the changes regarding cultural fit in South Korea.
4.3 Procedures

To ensure the reliability of the instrument, we conducted a pilot survey with 50 South Korean elementary teachers in November 2013. We randomly selected 50 elementary teachers who had a variety of teaching experiences and certification levels to participate in the pilot survey. We obtained the reliability of the instrument from the principal component analysis; the reliabilities of both PMTE and MTOE were .751.

In December 2013, we administered a survey on South Korean elementary teachers’ mathematical knowledge for teaching mathematics to a sample of 500 elementary teachers. Institutional Review Board approval was obtained to conduct research with human subjects from Boston College. Working with the alumni of Seoul National University of Education, the participants for the survey were recruited via e-mail. Participation was voluntary.

A total score for all participants was generated based on the number of correct responses to the LMT measures. The raw total score was then converted to a Z-score. A Z-Score is a statistical measurement of a score’s relationship to the mean in a group of scores (Cronk, 2008). A Z-score of 0 means the score is the same as the mean; a Z-score can be positive or negative, indicating whether it is above or below the mean and by how many standard deviations. The Z-score represents an individual teacher’s mathematical knowledge for teaching.

Participants’ data were analyzed to examine the characteristics of the teachers in this study. Analyses of variance (ANOVA) were conducted to investigate whether the teachers were different in terms of their efficacy in teaching mathematics based on other background information. We also generated a multiple regression model to explore which of these variables contributed most to the teachers’ MKT. The computer statistical tool SPSS (SPSS Statistics 20) was used to analyze the data.

5. Results

5.1 Descriptive Statistics

The majority of the teachers in this study were female (83%); the percentage of female participants is acceptable to represent the population in this study when it considers teachers’ gender proportion in South Korea. More than half the participants had a first-level elementary teacher certification (53%). Considering that elementary teachers are required to achieve the first-level elementary teacher certification after 3 to 5 years of teaching, the proportion of teachers who have the first-level certification is acceptable for the survey. Most of teachers in this survey had a bachelor’s degree in elementary education (80%). Among the respondents, most of the teachers did not have an academic degree related to elementary mathematics education (78%). Table 4 summarizes the results from the survey. All results were reported at \( \alpha = 0.05 \).

| Variable                                      | N    | Percentage | MKT mean (SD) | p value |
|-----------------------------------------------|------|------------|---------------|---------|
| Gender                                        |      |            |               |         |
| Male                                          | 48   | 17         | -1.63 (1.11)  | 0.600   |
| Female                                        | 237  | 83         | 0.34 (0.97)   |         |
| Teaching experience                           |      |            |               |         |
| 0–5 years                                     | 124  | 43         | 0.99 (0.87)   | 0.014*  |
| 6–10 years                                    | 49   | 17         | 0.19 (0.96)   |         |
| 11–15 years                                   | 63   | 22         | -0.04 (1.06)  |         |
| 16–20 years                                   | 23   | 8          | -0.38 (1.27)  |         |
| > 21 years                                    | 27   | 10         | -0.47 (1.03)  |         |
| Certification                                 |      |            |               |         |
| Level 2                                       | 124  | 47         | 0.12 (0.87)   | 0.002*  |
| Level 1                                       | 160  | 53         | -0.95 (1.08)  |         |
| Academic Degree relates to elementary education|      |            |               |         |
| Program                                       | 11   | 3          | -0.47(1.10)   | 0.026*  |
| Bachelor’s                                    | 230  | 80         | 0.03 (0.99)   |         |
| Master’s                                      | 44   | 17         | -0.01(0.98)   |         |
| Academic Degree relates to elementary mathematics education | | | | |
| None                                          | 216  | 78         | 0.76 (0.10)   | 0.008*  |
| Bachelor’s                                    | 50   | 18         | 0.95 (0.27)   |         |
| Master’s                                      | 12   | 4          | 1.03 (0.59)   |         |


We converted the raw scores into Z-scores. Therefore, the mean MKT number and operations score of these teachers was 0 and the standard deviation was 1.

5.2 Analysis of Variance

There were significant differences between groups of teachers’ MKT based on their teaching experiences, certification levels, and academic degrees related to mathematics elementary education (p < .05). We used a one-way ANOVA to test for differences in MKT among three groups of teachers who have degrees related to mathematics education at the bachelor’s and master’s level and those who do not have academic degrees in this field. Academic degree related to elementary mathematics education differed significantly across the three groups (F = 4.86, p = 0.008). Post hoc tests, using the Bonferroni correction, revealed that the group of teachers that do not have degrees related to elementary mathematics education had statistically significant lower Z-scores for teaching mathematics than did the group of bachelor’s degrees teachers (p = 0.09).

5.3 Multivariate Regression Model

To identify which of the variables were significant predictors of South Korean elementary teachers’ mathematical knowledge for teaching numbers and operations, we applied a multiple linear regression model. In particular, we used a backward elimination selection process to eliminate variables that did not significantly predict the MKT numbers and operations score. We found that the MKT score regressed on the teachers’ teaching experiences. This predictor accounted for approximately 2.8% of the variance in MKT scores (R² = 0.031, β = -0.29, p = 0.003), which was significant at p = 0.001 level.

6. Discussions

There was no statistically significant difference in the mean mathematical knowledge for teaching numbers and operations scores between male and female teachers, although the mean for female teachers was higher than for male teachers. There were significant differences among groups of teachers in this study based on their years of teaching experience (p = 0.014). As shown in Figure 1, the greatest difference was between those who had 6 to 10 years of teaching experience and those who had taught for more than 21 years (p = 0.053). South Korean elementary teachers’ mathematical knowledge for teaching numbers and operations slightly increased up to 10 years of teaching experience, although it is not statistically significant (p = 1.00). There is a negative relationship between teaching experience and knowledge after 10 years of teaching experience.
Figure 1. Graph of teaching experience and mathematical knowledge for teaching

There are contradictory arguments regarding the relationship between mathematical knowledge for teaching across the countries. While Hill (2010) and Margolis et al. (2005) found that there is a positive correlation between the U.S. teachers’ mathematical knowledge for teaching and teaching experiences, Ng (2011) proposed that Indonesian elementary teachers’ mathematical knowledge for teaching mathematics seemed to decrease as teaching experiences increased. Ng (2011) explained that limited opportunities for quality professional development programs in Indonesia and lack of demand for Indonesian teachers to continue learning mathematics content might cause a decrease in MKT throughout their teaching experiences.

Similar to Indonesia, there are no required professional development programs related to mathematics education for elementary teachers in South Korea after they participate in the professional development program for acquiring the first-level elementary teacher certification. Although the Ministry of Education in South Korea encourages elementary teachers to participate in professional development programs for at least 60 hours annually, it does not need to be mathematics education.

Another plausible explanation for the negative relationship between teaching experiences and MKT might be the improvement of the quality of preservice teacher education programs. As discussed previously, only 13 specialized national universities in South Korea provide preservice teacher education programs. According to Hue (2010), the specialized national university changed its curriculum based on the changes of the National Curriculum and the needs of the society. As a result, the quality of the program has improved. Although we did not examine the quality of the program, the fact that the novice teachers with 0 to 5 years of teaching experience have higher MKT scores than those who have more than 15 years may suggest there needs to be more investigation regarding how the preservice teacher education program has been developed and how changes to the curricula in the universities affect teachers’ MKT.

The statistically significant difference among groups of teachers based on academic degrees ($p = 0.030$) also provides evidence that the preservice elementary teacher education program may affect the teachers’ MKT. As shown in Figure 2, the MKT of teachers who attained bachelor’s degrees from specialized universities is statistically significantly higher than those who acquired the teacher certification from the refresher program. However, MKT mean for teachers who have master’s degrees is slightly lower than those who have bachelor’s degrees ($p = 0.100$), though it is statistically significantly higher than those who participated in the refresher program ($p = 0.049$). However, we acknowledge that more investigation on the relationship between the MKT and academic degree is needed because the total number of teachers who attained the teacher certification from the refreshment program ($N = 11$) was quite small compared to other groups. Ng (2011) argued that the small sample size may affect the results of statistical analysis.
The finding confirms Ball et al.’s (2010) argument that elementary teachers’ MKT can be increased through teacher education programs. The finding that there are statistically significant differences among groups of teachers in this study based on their academic degrees related to elementary mathematics education also verifies the positive relationship between teacher education programs and their MKT. As shown in Figure 3, the MKT for the teachers who have bachelor’s degrees related to elementary mathematics education is statistically significantly higher than those who did not have degrees ($p = 0.009$). This may show the significance of teacher education programs in increasing elementary teachers’ MKT. Ng (2011) also argued that elementary teachers who are in longer preparation programs might have more opportunities to develop deeper knowledge of subject matter.

It is an interesting finding that the MKT for the teachers who have master’s degrees related to elementary mathematics education is lower than the MKT for the teachers who have bachelor’s degrees, although it is not statistically significant ($p = 0.100$). Several plausible reasons for this finding include the effects of teaching experiences and the content of a master’s program verses a bachelor’s program. As discussed previously, the teachers’ MKT tended to decrease throughout their teaching experiences. The teachers who have master’s degrees have more teaching experience because the master’s programs for elementary teachers focus on reeducation of experienced teachers (Seoul National University of Education, 2012). In addition, the master’s programs related to elementary mathematics education offers philosophical and theoretical understandings of elementary mathematics education rather than practical teaching skills, such as how to present mathematics concepts to elementary students, while the bachelor’s programs do the opposite (Lee & Jeon, 2002). The finding indirectly suggests that there is a specific type of teacher education program that may increase elementary teachers’ MKT.
Regarding the relationship between the MKT and the certification level, the teachers who had second-level elementary teacher's license had higher MKTs than those who had the first-level license ($p = 0.002$). Because this study revealed that there is a negative relationship between teaching experiences and the MKT, a possible explanation for this finding is that the teachers who have the first-level elementary teacher's license have more teaching experience than those who have second-level licenses. However, this may not indicate that the elementary teacher education program providing the first-level license does not affect the teachers' MKT. As shown in Figure 1, the teachers' MKT increases up to 10 years of teaching and decreases thereafter. Considering that most of the elementary teachers participated in the certification program with 3 to 5 years of teaching experience, we may assume that the MKT might increase after participation in the certification program, although the effect of the program on the MKT seemed not to last more than 5 years.

There were significant differences among groups of teachers in this study based on the educational districts in which the teachers worked ($p = 0.023$). The greatest difference was between those who worked at SeoBu and those who taught at DongBu ($p = 0.272$), as illustrated in Figure 4.

As discussed previously, the teachers who have less than 10 years of teaching experience have higher MKT scores than those who have more than 10 years of experience. Thus, we first looked at the proportion of the participants in each district according to the teachers' teaching experiences: if an educational district had more participants with fewer than 10 years teaching experience than the other districts, the MKT scores were higher. Of the respondent from SeoBu, which has the highest MKT scores of its teachers, 88% have less than 10 years of teaching experience, while 62% of participants from DongBu, which has the lowest MKT scores of its teachers, have less than 10 years of teaching experience. Thus, the proportion of teachers based on their teaching experiences may affect the differences among educational districts in terms of the teachers' MKT scores. The Ministry of Education in South Korea may place elementary teachers in each educational district based upon the teachers' residences. If there is a specific area in which teachers of certain ages prefer to live, the demographic of teachers might differ from other educational districts. Indeed, future work would be useful regarding the relationship between the teachers' proportion of each educational district, as we randomly selected the participants for this study. In addition, it may difficult to conclude that the teachers in an educational district that has more teachers with less than 10 years of teaching experience tend to have higher MKT scores because there are diverse factors that may affect those teachers' MKT. For example, Ma's (2000) investigation on both the U.S. and China elementary teachers' knowledge for teaching mathematics found that a school environment that encourages teachers' learning regarding how to present mathematics concepts to students might be a significant factor that increases teachers' knowledge for teaching mathematics. Thus, we suggest that there needs to be further investigation on the relationship between educational districts and teachers' MKT.

7. Conclusion

We identified that teaching experience levels might be the most significant factor contributing to South Korean elementary teachers' mathematical knowledge for teaching numbers and operations. Teachers with more than 10 years of teaching experience had lower mathematical knowledge for teaching than those who did not. The finding suggests that the Ministry of Education in South Korea should consider providing required professional development programs related to
mathematics education for teachers who have more than 10 years of teaching experience. However, more studies should be conducted regarding what causes the contradictory results between the studies conducted across countries because the result of this study contradicts findings form previous studies conducted in the United States (e.g., Hill, 2007).

In this study, we found that both preservice and in-service teacher education programs seem to play a significant role in increasing teachers' mathematical knowledge for teaching numbers and operations. In particular, the teachers who earned their elementary teacher certifications from refresher trainings had lower mathematical knowledge for teaching than those who had bachelor's degrees in elementary education. Among teachers who had bachelor's degrees, the teachers who acquired 20 extra credits related elementary mathematics education had better mathematical knowledge for teaching than those who did not. However, there are remaining questions. Will the linearity of the relationship between teacher education programs and teachers' mathematical knowledge for teaching mathematics continue or will it level off? Should teachers participate in teacher education programs to maintain or improve their mathematical knowledge for teaching periodically or is participation in only one program enough? Further studies need to be conducted to determine whether the effects of professional development programs on mathematical knowledge for teaching continues throughout teaching experiences.

It was an interesting finding that there are differences among the teachers who participated in this study according to the location of the school in which teachers work. South Korean elementary teachers took similar courses related to elementary mathematics education from 13 specialized universities and passed the national recruitment examination to teach in public schools. After 3 to 5 years of teaching, the teachers are required to participate in a teacher education program to achieve the first-level elementary teacher's license. In addition, the Ministry of Education in South Korea relocates elementary teachers every 5 years depending upon the teachers' residences. For all the efforts to homogenize public elementary schools in terms of the teachers, the fact still remains that there are differences in terms of teachers' mathematical knowledge for teaching mathematics, and that suggests that further studies are warranted to explore what causes these differences.

In this study, we examined diverse factors that may affect South Korean elementary teachers' mathematical knowledge for teaching. This study cannot be generalized to all elementary teachers in South Korea because the samples included only one city. However, we also acknowledge that one city may represent the whole system of education in South Korea with a centralized education system in South Korea. Therefore, the results of the study do point to the importance of mathematical knowledge for teaching and to the essential factors that can promote further understanding of teachers' knowledge for teaching mathematics. Based on the findings, we expect that further studies should be developed that engage the challenging and important question of how elementary teachers improve or maintain their mathematical knowledge for teaching.

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