Symmetric Rydberg $C_Z$ gates with adiabatic pulses

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We analyze neutral atom Rydberg $C_Z$ gates based on adiabatic pulses applied symmetrically to both atoms. Analysis with smooth pulse shapes and Cs atom parameters predicts the gate can create Bell states with fidelity $F > 0.999$ using adiabatic rapid passage (ARP) pulses in a room temperature environment. With globally optimized pulse shapes for stimulated Raman adiabatic passage (STIRAP) we generate Bell states with fidelity $F = 0.998$. The analysis fully accounts for spontaneous emission from intermediate and Rydberg states. The gate protocols do not require individual addressing and are expected to be robust against Doppler shifts due to atomic motion.

I. INTRODUCTION

Qubits encoded in hyperfine states of neutral atoms can be entangled using $C_Z$ or CNOT gates mediated by Rydberg state interactions. Recent experimental progress has demonstrated entanglement fidelity of $F \simeq 0.97$ in a one-dimensional geometry and $F \simeq 0.89$ in a two-dimensional qubit array. Numerous pulse protocols for Rydberg entanglement have been proposed. Detailed analysis of gates using constant amplitude Rydberg excitation pulses has predicted a fidelity limit of $F < 0.999$ for Rb or Cs atoms in a room temperature background environment. Using Cs atoms the theoretical limit has been extended to $F > 0.9999$ with smooth, analytic pulses in, and with a dark state mechanism in.

Although there now exist protocols for entangling gates that offer $F > 0.9999$, a performance which is expected to be sufficient for scalable quantum computation, there is a fidelity gap between predicted performance and the best experimental results. Some of the missing fidelity can be ascribed to known technical imperfections including laser noise, Doppler broadening from finite atomic temperature, and the possible influence of background electric fields. Many Rydberg entanglement experiments have suffered from unexpectedly large loss of atoms that are left in Rydberg states during intermediate steps of the entanglement protocol.

In order to reduce any excess Rydberg state loss it is desirable to develop protocols that do not lose population in Rydberg states where they are subject to relatively fast decoherence during intermediate stages of the gate. The standard Rydberg blockade $C_Z$ pulse sequence consists of a $\pi$ pulse on the control qubit, a $2\pi$ pulse on the target qubit, and a $\pi$ pulse on the control qubit, with each pulse resonant between a ground hyperfine qubit state and a Rydberg level. If the control qubit enters the gate in state $|1\rangle$ it is Rydberg excited and will sit in the Rydberg level during the $2\pi$ pulse on the target qubit. It has been observed that this leads to larger loss, as compared to the case of a continuous $2\pi$ pulse on the ground - Rydberg transition. For this reason it appears advantageous to develop protocols that continuously drive the ground - Rydberg - ground transition. We note that since the logical action of a $C_Z$ gate in a quantum circuit is symmetric with respect to the control and target inputs, it is natural to seek a gate protocol that is also symmetric with respect to interchange of qubits as regards the applied pulses.

There has been previous work using symmetric driving of both atoms for Rydberg gate protocols. Entanglement was demonstrated using continuous driving of a ground-Rydberg-ground transition on both atoms with constant amplitude pulses, and with symmetric but not continuous pulses in. A $C_Z$ gate protocol with continuous driving of both atoms is possible using an adiabatic pulse with time varying Rabi frequency $\Omega(t)$ and Rydberg level detuning $\Delta(t)$ in the limit of $|\Omega| > B$, where $B$ is the Rydberg-Rydberg interaction strength. Unfortunately, the requirement of adiabaticity renders the gate slow and susceptible to spontaneous emission from the Rydberg level. A careful optimization of the pulse parameters resulted in prediction of a fidelity of not more than $F \sim 0.98$. A version of the adiabatic gate using adiabatic rapid passage (ARP) pulses together with electric field switching of Rydberg Förster resonances yielded a simulated Bell state fidelity of $F = 0.996$. A related proposal for an adiabatic gate in the intermediate regime $|\Omega| \sim B$ yielded $F \sim 0.95$. Analysis of entanglement creation with $F < 0.999$ based on stimulated rapid adiabatic passage (STIRAP) pulses and evolution of a two-atom dark state in the blockade regime of $|\Omega| \ll B$ was presented. It was also shown how to implement an adiabatic phase gate, but a complete fidelity analysis was...
not performed. Additional variations of STIRAP pulses for entangling gates were analyzed in [28], although the effect of a finite Rydberg lifetime was not included.

It is also possible to achieve a fast phase gate using non-adiabatic, constant amplitude pulses that continuously drive both atoms. The challenge in making this work is that when only one atom is in the ground state that is Rydberg coupled (input states |01⟩ or |10⟩) the coupling rate is given by the one-atom Rabi rate Ω. However, if the input state is |11⟩ the effective Rabi rate due to the Rydberg blockade mechanism is \( \Omega_2 = \sqrt{2} \Omega \). Therefore a 2π pulse for states |01⟩ or |10⟩ will be a 25/2π pulse for |11⟩ leading to large gate errors. This problem was solved in [8, 9] using a combination of a larger pulse area and finite detuning Δ from the Rydberg level. Unfortunately, due to the larger pulse area, there is increased spontaneous emission from the Rydberg state and predicted gate fidelities are less than \( \mathcal{F} \sim 0.999 \).

In this paper we revisit adiabatic \( C_Z \) protocols in the blockade regime of \( \Omega \ll B \), and show that using rapid adiabatic methods [27], together with optimized pulse shapes, we can achieve \( C_Z \) gates with high fidelities. With ARP pulses that drive the ground-Rydberg transition of both atoms simultaneously, and are almost continuous, we reach \( \mathcal{F} > 0.999 \). With STIRAP pulses the fidelity is \( \mathcal{F} > 0.98 \) for analytical pulse shapes, which we further improve to \( \mathcal{F} = 0.998 \) with an optimized control version of the STIRAP pulses. The design methodology for either ARP or STIRAP versions of the gate are closely related to adiabatic protocols that have been previously studied for gates acting on multi-atom ensemble qubits [28, 29].

The rest of the paper is organized as follows. In Sec. III we present ARP pulses for implementing a \( C_Z \) gate and calculate the resulting Bell state fidelity. In Sec. IV we analyze a \( C_Z \) gate based on symmetric driving with STIRAP pulses. Two versions of analytical pulse shapes are presented in Sec. IV A and in Sec. IV B we consider globally optimized STIRAP pulses that provide higher fidelity entanglement. The results are summarized in Sec. V.

II. \( C_Z \) GATE WITH ARP PULSES

Consider excitation of a ground state |1⟩ to Rydberg state |r⟩ with a one-photon transition as shown in Fig. 1. States |1⟩, |r⟩ are coupled by a laser giving Rabi frequency \( \Omega(t) \) and detuning \( \Delta(t) \). The \( C_Z \) protocol relies on driving a 2π rotation on both atoms. The symmetric states evolve as |01⟩ → |0r⟩ → \( e^{i\phi_1}|01⟩ \), |10⟩ → |r0⟩ → \( e^{i\phi_2}|10⟩ \), whereas the symmetric state |00⟩ is dark to the gate pulses and in the limit of strong blockade the state |11⟩ evolves as |11⟩ → \( \frac{|r1⟩ + |1r⟩}{\sqrt{2}} \) → \( e^{i\phi_2}|11⟩ \). The logical transformation in the basis \{00⟩, |01⟩, |10⟩, |11⟩\} is therefore diag[1, \( e^{i\phi_1}, e^{i\phi_1}, e^{i\phi_2} \)]. To achieve a maximally entangling \( C_Z \) gate we may set \( \phi_1 = \phi_2 = \pi \). This can be achieved by using a double ARP pulse with the detuning reversed in the second pulse [28] as shown in Fig. 2b.

The gate was analyzed by numerical integration of the two-atom master equation in Lindblad form

\[
\frac{d\rho}{dt} = i[H, \rho] + \mathcal{L}[\rho]
\]

with initial conditions \( \rho(0) = \rho_c(0) \otimes \rho_t(0) \) where c, t label control and target qubits. For the Hamiltonian we use \( \mathcal{H} = \mathcal{H}_c \otimes I + I \otimes \mathcal{H}_t + B|rr⟩⟨rr| \) with

\[
\mathcal{H}_{c/t} = \frac{\Omega(t)}{2}|r⟩⟨c/t| |1⟩ + \text{H.c.} + \Delta(t)|r⟩⟨r| .
\]

We neglect optical excitation of the |0⟩ state due to the large detuning, \( \gamma_q \gg \Omega \). We verify below that the additional gate errors from off-resonant excitation of this state, as well as off-resonant excitation of neighboring Rydberg states, is negligible for the chosen parameters. The decay term is

\[
\mathcal{L}[\rho] = \sum_{\ell=c,t,j=0,1,d} L_{\ell}^j \rho L_{\ell}^j\dag - \frac{1}{2} L_{\ell}^j\dag L_{\ell}^j \rho - \frac{1}{2} \rho L_{\ell}^j\dag L_{\ell}^j
\]

with \( L_{\ell}^j = \sqrt{b_{\ell r}} |\gamma_r⟩⟨\ell| \) where \( \gamma_r = 1/\gamma_r \) is the population decay rate of the Rydberg state and the \( b_{\ell r} \) are branching ratios to lower level \( j \). The levels |0⟩, |1⟩, |d⟩ are taken to be stable. The uncoupled state |d⟩ represents all the ground hyperfine states outside the qubit basis. Although some of these states are at the same energy as |1⟩, and can be resonantly excited, we neglect such dynamics, thereby making the worst case assumption that all population leakage into |d⟩ is an uncorrectable error.

Figure 2 shows the population evolution for states |10⟩ and |r0⟩ for parameters corresponding to excitation of the Cs107p3/2 state. For both one and two Rydberg coupled atoms the populations faithfully execute a 2π rotation between ground and singly excited Rydberg states.

![FIG. 1. (color online) Energy level structure of neutral atom qubits with ground states |0⟩, |1⟩. Rydberg states |r⟩ interact with strength B. One-photon excitation with Rabi frequency \( \Omega(t) \) at detuning \( \Delta(t) \). Level |d⟩ is an uncoupled state that accumulates spontaneous emission from |r⟩ which has lifetime \( \tau_r = 1/\gamma_r \) and decays to states |0⟩, |1⟩, |d⟩ with branching ratios \( b_{0r}/16, b_{1r}/16, b_{dr}/7/8 \).](image-url)
had a duration of start and stop points. For the data in the figure each pulse a pulse with ARP pulses. a) Time dependence of $\Omega(t)$ and $\Delta(t)$. b) Populations of the $|10\rangle$ and $|d\rangle$ states for the initial state $|10\rangle$. The population in $|d\rangle$ is defined as $1 - \text{Tr}_{0,1,2}[\rho]$. c) Populations of the $|11\rangle$, $|1r\rangle$ and $|0\rangle$, $|r\rangle$, $|d\rangle$ states for the initial state $|11\rangle$. Parameters were $\Omega_{\text{max}}/2\pi = 17$ MHz, $\Delta_{\text{max}}/2\pi = 23$ MHz, $B/2\pi = 100$ MHz, total gate time $T = 0.54$ $\mu$s, $\gamma = 1/(540$ $\mu$s) $b_{0r} = 7/8$, $b_{0r} = b_{1r} = 1/16$. The atomic parameters correspond to the Cs $107p_{3/2}$ state with spontaneous decay randomly distributed among the 16 ground hyperfine states. The temporal shape of the detuning was a quarter period of a sin function for each pulse and the Rabi drive was of the form $\Omega(t) = \Omega_{\text{max}} \left[ e^{-(t-t_0)^2/r^2} - a \right] / (1 - a)$ with $t_0$ the center of the pulse, and the offset $a$ set to give zero amplitude at the start and stop points. For the data in the figure each pulse had a duration of $T/2$ and $\tau = 0.173T$.

Although we are analyzing the ARP protocol as a one-photon excitation process, it could also be implemented as a two-photon transition for each pulse. This can be done for example by keeping the frequency of the first photon constant, with nonzero detuning from an intermediate level, and sweeping the frequency of the second photon. The one-photon analysis gives an upper limit to the gate fidelity. A two-photon implementation will suffer additional errors due to scattering from the intermediate state. The additional error can be made negligible provided there is sufficient laser power available to allow for large intermediate state detuning. It is also the case that for both one- and two-photon implementations there will be additional contributions to $\Delta(t)$ from the dynamic Stark shifts of the ground and Rydberg states. We do not explicitly include these shifts in the analysis. They can be corrected for either by modifying $\Delta(t)$ to compensate the Stark shifts, or by adding frequency sidebands to the excitation lasers to cancel the shifts.

To generate entanglement we start with the state $|ct\rangle = |11\rangle$, apply a Hadamard gate to each qubit, the Rydberg $C_2$ operation, and a final Hadamard to the target qubit which ideally prepares the Bell state $|B\rangle = |100\rangle + |111\rangle$. The Bell fidelity can then be defined as $30$

$$F = \frac{\rho_{100} + \rho_{111}}{2} + |\rho_{1010}|.$$  

Assuming perfect Hadamard operations, and the same parameters as in Fig. 2 we find a Bell fidelity of $F = 0.9994$ at $B/2\pi = 3$ GHz which is close to the maximum possible for the Cs $107p_{3/2}$ state using the pulse shapes from Ref. 10. The fidelity exceeds 0.99 for $B/2\pi = 300$ MHz with the dependence of fidelity on $B$ or, equivalently, interatomic spacing $R$ shown in Fig. 3. The fidelity is accurately described by a small offset due to spontaneous emission, plus the scaling $(\Omega_{\text{max}}/B)^2$ which reflects blockade leakage allowing for finite excitation of the $|rr\rangle$ state.

The highest fidelity result uses a maximum Rabi frequency of only $17$ MHz which implies leakage errors due to excitation of $|0\rangle$, or due to excitation of $|1\rangle$ to a different Rydberg level that are bounded by $\epsilon \sim (\Omega_{\text{max}}/\Delta_{\text{min}})^2$ where $\Delta_{\text{min}}$ is the smallest of $\omega_i$ or any of the detunings.
from $|0\rangle$ or $|1\rangle$ to nearby Rydberg levels. As detailed in [10] for the Cs $107\rho_{3/2}$ state $\Delta_{\text{min}}/2\pi \simeq 3$ GHz giving $\epsilon = 3 \times 10^{-5}$ which is negligible relative to the calculated fidelity.

Beyond the convenience of symmetric driving of the qubits the use of adiabatic pulses makes the gate less sensitive to Doppler detuning and less sensitive to small variations in laser amplitude than a gate using constant amplitude pulses.

III. $C_Z$ GATE WITH STIRAP PULSES

A STIRAP gate version using two-photon excitation is also possible as originally suggested in [25]. Consider excitation of a ground state $|1\rangle$ to Rydberg state $|r\rangle$ with two-photons that are near resonant with intermediate state $|p\rangle$ as shown in Fig. 4. States $|1\rangle, |p\rangle$ are coupled by a laser giving Rabi frequency $\Omega_1(t)$ and detuning $\Delta_1$, while states $|0\rangle, |r\rangle$ are coupled by a second laser with Rabi frequency $\Omega_2(t)$ and detuning $\Delta_2$. The two-photon detuning is $\Delta = \Delta_1 + \Delta_2$ which will be set to zero for resonant excitation. The gate is again modeled by Eq. (1) with the replacements

$$H_{c/t} = \left[ \frac{\Omega_1(t)}{2} |p\rangle_c \langle 1| + \frac{\Omega_2(t)}{2} |r\rangle_c \langle p| + \text{H.c.} \right]$$

$$+ \Delta_1(t) |p\rangle_c \langle p| + \Delta_2(t) |r\rangle_c \langle r|$$

and

$$\mathcal{L}[p] = \sum_{\ell=c,t} \sum_{j,k=0,1,d,p,r} I_{jk}^{(\ell)} \rho_{jk} I_{jk}^{(\ell)\dagger} - \frac{1}{2} L_{jk}^{(\ell)\dagger} L_{jk}^{(\ell)} - \frac{1}{2} \mu_{jk} \rho_{jk}$$

where $L_{jk}^{(\ell)} = \sqrt{b_j \gamma_k} |j\rangle \langle k|$ for $j < k$ and 0 otherwise. As with the analysis of the one-photon excitation ARP protocol we neglect any re-excitation of atoms that decay to $|d\rangle$.

A. Analytical STIRAP pulses

The requirement for a phase gate is that the pulse shapes $\Omega_1(t), \Omega_2(t)$, and detunings $\Delta_1, \Delta_2$ are chosen such that all three states $|01\rangle, |10\rangle, |11\rangle$ return to the ground state after the applied pulse with phases $\phi_1, \phi_1, \phi_2$. This is possible using the counterintuitive STIRAP sequence with $\Omega_2(t)$ preceding $\Omega_1(t)$, as shown in Fig. 5a). The population dynamics for one and two atoms in the Rydberg coupled state show high accuracy transfer to the Rydberg state and back to the ground state, as is shown in panels b) and c). Using the STIRAP pulse sequence, states $|01\rangle$ and $|10\rangle$ follow an adiabatic dark state with zero eigenvalue so there is no dynamical phase accumulation and $\phi_1 = 0$. An entangling $C_Z$ gate can then be obtained if $\phi_2 = \pi$. If the intermediate STIRAP detuning is set to $\Delta_1 = 0$ the state $|11\rangle$ will follow a two-atom dark state in the presence of strong blockade and no dynamical phase is accumulated[22]. However, such an approach is not useful when $|p\rangle$ is subject to radiative decay[22]. Instead, we use $\Delta_1 \neq 0$ so there is minimal excitation of state $|p\rangle$. The dynamics do not follow a dark state, and the $2\pi$ rotation $|11\rangle \rightarrow \frac{|11\rangle + |10\rangle}{\sqrt{2}} \rightarrow |11\rangle$ gives a dynamical phase of $\phi_2 = \pi$.

Using the parameters of Fig. 4 and the same steps as for the ARP protocol we prepare the Bell state $|B'\rangle = \frac{|11\rangle + |10\rangle}{\sqrt{2}}$. This is different than the state prepared with ARP pulses due to the different choices of $\phi_1, \phi_2$. For the parameters of Fig. 5 we find that the state $|B'\rangle$ is created with fidelity $F = 0.976, 0.978, 0.979$ at $B/2\pi = 500, 1500, 3000$ MHz.

The fidelity of the STIRAP gate is substantially lower than that achieved with ARP pulses. The reason is that the parameters used are not sufficiently adiabatic giving imperfect following of the dark state. Calculations show that $\phi_1 = -18$ deg. and $\phi_2 = -171.5$ deg., So there is a phase error of 18 deg. for one atom excited and 9.5 deg. for two atoms. Although tests with slower, more adiabatic pulses, with spontaneous emission turned off result in Bell states with arbitrarily high fidelity the challenge is to design pulse shapes that are both adiabatic and sufficiently fast to prevent spontaneous emission errors. One approach may be to compensate the imperfect dynamical phase with a geometrical phase by adjusting the relative phase of $\Omega_1$ and $\Omega_2$[33].

Alternatively we may use optimized analytical pulse shapes[34] to improve the entanglement fidelity as shown in Fig. 6. This design uses a double STIRAP sequence with switching of the sign of the detuning and the phase of the $\Omega_2$ pulse halfway through the gate.
The curve labeled $\Omega_b$ of the pulse, and the offset $\Omega_a$ for the initial state $|\psi\rangle$. For $\Omega_a = \frac{T}{4}$ in [11, 28]. To achieve high fidelity we use the optimized pulse shapes $^{34}$

$$\Omega_1(t) = \Omega_0 F(t - t_1) \sin \left[ \frac{\pi}{2} f(t - t_1) \right] + \Omega_0 F(t - t_2) \cos \left[ \frac{\pi}{2} f(t - t_2) \right], \quad (2a)$$

$$\Omega_2(t) = \Omega_0 F(t - t_1) \cos \left[ \frac{\pi}{2} f(t - t_1) \right] - \Omega_0 F(t - t_2) \sin \left[ \frac{\pi}{2} f(t - t_2) \right], \quad (2b)$$

with $F(t) = e^{-((t/2)^2)}$ and $f(t) = \left(1 + e^{-4t/\tau}\right)^{-1}$. The times $t_1, t_2$ correspond to the centers of the $\Omega_1$ pulses shown in Fig. 4a). Note that in addition to the sign of the

This is a modification of the scheme considered earlier in [11, 28]. To achieve high fidelity we use the optimized pulse shapes $^{34}$

$$\Omega_1(t) = \Omega_0 F(t - t_1) \sin \left[ \frac{\pi}{2} f(t - t_1) \right] + \Omega_0 F(t - t_2) \cos \left[ \frac{\pi}{2} f(t - t_2) \right], \quad (2a)$$

$$\Omega_2(t) = \Omega_0 F(t - t_1) \cos \left[ \frac{\pi}{2} f(t - t_1) \right] - \Omega_0 F(t - t_2) \sin \left[ \frac{\pi}{2} f(t - t_2) \right], \quad (2b)$$

with $F(t) = e^{-((t/2)^2)}$ and $f(t) = \left(1 + e^{-4t/\tau}\right)^{-1}$. The times $t_1, t_2$ correspond to the centers of the $\Omega_1$ pulses shown in Fig. 4a). Note that in addition to the sign of the
intermediate state detuning changing in the middle of the gate, the phase of \( \Omega_2 \) changes by \( \pi \) in the second half of the gate. In the ideal case of no spontaneous emission and infinite blockade these pulses result in the state evolution \(|00\rangle \rightarrow |00\rangle, |01\rangle \rightarrow -|01\rangle, |10\rangle \rightarrow -|10\rangle, |11\rangle \rightarrow -|11\rangle \) which implements a controlled phase gate. Numerical simulations of the dynamical evolution with finite blockade and Rydberg lifetime are shown in Fig. 7. The predicted Bell state fidelity is \( F = 0.990, 0.991 \) for \( B/2\pi = 500, 1500 \) MHz. The infidelity is reduced by about a factor of 2 relative to the pulse scheme of Fig. 4 although the dominant error source is still scattering from the intermediate \( |p\rangle \) state leading to growth of population in the uncoupled ground state \( |d\rangle \). This error can be reduced using larger \( \Delta_1 \) and the Bell state fidelity increased to 0.996, but only with unrealistically high Rabi frequencies.

B. Global optimization of STIRAP pulses

We employ global optimization algorithms to search for pulse profiles that deliver feasible pulse sequences that improve the gate fidelity. In this approach we divide \( f(t) \) (\( \Omega_1(t), \Omega_2(t), \Delta_1(t) \)) into \( 2N \) equal length segments that are piecewise continuous and parameterized by the magnitude of each segment \( f_i(t) \), with the segments connected by error functions according to

\[
f(t) = \frac{f_t + f_{t+1}}{2} + f_{t+1} - f_t \text{erf} \left[ \frac{5}{\Delta t} \left( t - \frac{t_i + t_{i+1}}{2} \right) \right],
\]

where \( t_i \leq t \leq t_{i+1} \) for the \( i^{th} \) segment, each of which has length \( \Delta t \). The use of error functions as building blocks facilitates constructing smooth pulse shapes that respect constraints imposed by available devices for optical modulation. Using pulses that are symmetric about the central time of the gate the optimization problem is reduced to finding \( 3N \) variables (for \( \Omega_1(t), \Omega_2(t), \Delta_1(t) \)) that optimize the entanglement fidelity \( F \) while satisfying a constraint on maximum modulation rate.

As the above optimization problem is highly constrained with a non-convex objective function \( F \), we employ an off-the-shelf global optimization algorithm to solve the problem. We use a parallelized version of differential evolution \( \text{DE} \), motivated by previous work on quantum control problems \( \text{DE} \). Optimized pulse shapes are shown in Fig. 7 and the coefficients for each segment are listed in Table I. These pulses give \( F = 0.997 \) for a gate time of 1 \( \mu s \) and \( F = 0.998 \) for a gate time of 0.5 \( \mu s \). We see that the entanglement fidelity is significantly improved compared to STIRAP with analytic pulse shapes, and is slightly better than the fidelity of the ARP gate at the same blockade strength.

IV. DISCUSSION

In summary we have presented adiabatic pulses that lead to Rydberg gate fidelities of \( F = 0.9994 \) at a blockade strength of \( B/2\pi = 3 \) GHz and gate duration of 0.54 \( \mu s \) for ARP pulses and \( F = 0.998 \) at a blockade strength of \( B/2\pi = 0.5 \) GHz and gate duration of 1.0 \( \mu s \) for feasible STIRAP pulses obtained by employing a differential evolution algorithm. The pulses are applied simultaneously to both atoms which removes the need for high speed switching of lasers between different spatial locations. The results account fully for spontaneous emission from all participating excited atomic states in a room temperature environment. The reported gate fidelities are defined as the fidelity of a Bell state prepared with the gate assuming there are no other control errors, zero excess laser noise, and no errors due to atomic motion. Compared to Rydberg gates using constant amplitude,

| Functions | Segment number |
|-----------|----------------|
| \( \Omega_1(t)/2\pi \) | \( \Omega_2(t)/2\pi \) | \( \Delta_1(t)/2\pi \) |
| 1.381 | 10.299 | 25.538 |
| 165.091 | 199.992 | 198.145 |
| 198.872 | 198.872 | 200.000 |
| 173.485 | 200.000 | 173.485 |
| 392.573 | 363.476 | 364.356 |
| 360.993 | 416.450 | 420.394 |

| Functions | Segment number |
|-----------|----------------|
| \( \Omega_1(t)/2\pi \) | \( \Omega_2(t)/2\pi \) | \( \Delta_1(t)/2\pi \) |
| 24.502 | 71.652 | 178.489 |
| 179.604 | 180.728 | 59.400 |
| 91.861 | 0.000 | 11.486 |
| 950.029 | 907.110 | 946.848 |
| 961.995 | 969.995 | 980.487 |
or other non-adiabatic pulses, we expect that ARP and STIRAP will provide improved robustness in the presence of Doppler shifts at finite atomic temperature.

These results, while promising, should not be considered as an ultimate limit on the Rydberg gate fidelity with simultaneous addressing of both atoms. The space of possible pulse designs is large, and further exploration allowing for a wider range of pulse shapes with more degrees of freedom to optimize over is likely to lead to higher fidelity limits.

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