Cosmological Constant Problem

J. W. Moffat

Perimeter Institute for Theoretical Physics, Waterloo, Canada

and

Department of Physics, University of Toronto, Ontario, Canada

Abstract

The cosmological constant problem is reviewed and a possible quantum gravity resolution is proposed. A space satellite Eötvös experiment for zero-point vacuum energy is proposed to see whether Casimir vacuum energy falls in a gravitational field at the same rate as ordinary matter.

1 Cosmological Constant Problem

• It is generally agreed that the cosmological constant problem (CCP) is one of the most severe problems facing modern particle and gravitational physics. It is believed that its solution could significantly alter our understanding of particle physics and cosmology [1].
  • The accelerating universe and dark energy. Is the cosmological constant the explanation for the accelerating universe? What is dark energy?
  • There have been many attempts to solve the CCP. Adjustment models do not avoid fine-tuning.
  • Higher-dimensional models of the brane-bulk type do not avoid fine-tuning.
  • Superstring theory has not yet provided a solution to the CCP.

In the following, I will describe a possible resolution of the CCP, based on a model of a nonlocal field theory and quantum gravity theory that suppresses the coupling of gravity to vacuum energy density. The violation of the equivalence principle in the theory can be tested by performing Eötvös experiments on Casimir vacuum energy in satellites.
2 Gravitational Coupling to Vacuum Energy and Quantum Gravity Theory

We can define an effective cosmological constant

$$\lambda_{\text{eff}} = \lambda_0 + \lambda_{\text{vac}},$$

(1)

where $\lambda_0$ is the “bare” cosmological constant in Einstein’s classical field equations, and $\lambda_{\text{vac}}$ is the contribution that arises from the vacuum density $\lambda_{\text{vac}} = 8\pi G \rho_{\text{vac}}$.

Already at the standard model (SM) electroweak scale $\sim 10^2$ GeV, a calculation of the vacuum density $\rho_{\text{vac}}$, based on local quantum field theory results in a discrepancy of order $10^{55}$ with the observational bound $\rho_{\text{vac}} \leq 10^{-47}$ (GeV)$^4 \sim (10^{-3} \text{eV})^4$. The WMAP and supernovae SNIa data require dark energy [2]. If the vacuum energy is the dark energy, then $\rho_{\text{vac}} \sim (10^{-3} \text{eV})^4$. There is an egregious discrepancy between the particle physics estimate of $\rho_{\text{vac}}$ and the cosmological observation.

There is a severe fine-tuning problem of order $10^{55}$, since the virtual quantum fluctuations giving rise to $\lambda_{\text{vac}}$ must cancel $\lambda_0$ to an unbelievable degree of accuracy. This is the “particle physics” source of the cosmological constant problem.

3 Nonlocal Quantum Gravity Model

Let us consider a model of nonlocal gravity with the action [3, 4]:

$$S = S_G + S_M,$$

(2)

where $(\kappa^2 = 8\pi G)$:

$$S_G = -\frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R[g, G^{-1}] + G^{-1}2\lambda_0)$$

(3)

and

$$S_M = \frac{1}{2} \int d^4x \sqrt{-g} \left( g^{\mu\nu} \nabla_\mu \phi F^{-1} \nabla_\nu \phi - m^2 \phi F^{-1} \phi \right).$$

(4)

$G$ and $F$ are nonlocal regularizing, entire functions. As an example, we can choose the covariant functions

$$G(x) = \exp \left[ -\mathcal{D}(x)/2\Lambda_G^2 \right].$$
\[ F(x) = \exp \left[ -(\mathcal{D}(x) + m^2)/2\Lambda_M^2 \right], \] (5)

where \( \mathcal{D} \equiv \nabla_\mu \nabla^\mu \) and \( \Lambda_G \) and \( \Lambda_M \) are (length)\(^{-1}\) (energy) scales.

We expand \( g_{\mu\nu} \) about flat Minkowski spacetime: \( g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \). The propagators for the graviton and the \( \phi \) field in a fixed gauge are given by

\[ D^\phi(p) = \frac{G(p)F(p)}{p^2 - m^2 + i\epsilon}, \] (6)

\[ D^{G\mu\nu\rho\sigma}(p) = \frac{(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma})G(p)}{p^2 + i\epsilon}. \] (7)

Because \( G \) and \( F \) are entire functions of \( p^2 \), preserving the Cutkosky rules, they do not violate unitarity. Gauge invariance can be maintained by satisfying certain constraint equations for \( G \) and \( F \) in every order of perturbation theory. This guarantees that \( \nabla_\nu T^{\mu\nu} = 0 \).

### 4 Resolution of the Cosmological Constant Problem

In flat Minkowski spacetime, the sum of all disconnected vacuum diagrams \( C = \sum_n M_n^{(0)} \) is a constant factor in the scattering S-matrix \( S' = SC \). Since the S-matrix is unitary \( |S'|^2 = 1 \), then we must conclude that \( |C|^2 = 1 \), and all the disconnected vacuum graphs can be ignored. This result is also known to follow from the Wick ordering of the field operators.

Due to the equivalence principle gravity couples to all forms of energy, including the vacuum energy density \( \rho_{\text{vac}} \), so we can no longer ignore these virtual quantum fluctuations in the presence of a non-zero gravitational field.

Quantum corrections to \( \lambda_0 \) come from loops formed from massive SM states, coupled to external graviton lines at essentially zero momentum.

Consider the dominant contributions to the vacuum density arising from the graviton-SM loop corrections. We shall adopt a simple model consisting of a massive scalar meson \( \phi \), which has the SM mass \( m \sim 10^2 \) GeV.

The lowest order correction to the graviton-scalar vacuum loop will have the form (in Euclidean momentum space):

\[ \Pi_{\mu\nu\rho\sigma}^{G\text{vac}}(p) = -\kappa^2 \int \frac{d^4q}{(q^2 + m^2)[(q - p)^2 + m^2]} \]
\[ \times K_{\mu\nu\rho\sigma}(p,q) \exp \left\{ \frac{-(q^2 + m^2)}{2\Lambda_M^2} \right\} \]
\[ - \left[ (q-p)^2 + m^2 \right]/2\Lambda_M^2 - q^2/2\Lambda_{G\text{vac}}^2 \right\}. \quad (8) \]

For \( \Lambda_{G\text{vac}} \ll \Lambda_M \), we observe that from power counting of the momenta in the loop integral, we get
\[ \Pi_{\mu\nu\rho\sigma}^{G\text{vac}}(p) \sim \kappa^2 \Lambda_{G\text{vac}}^4 N_{\mu\nu\rho\sigma}(p^2) \]
\[ \sim \Lambda_{G\text{vac}}^4 \frac{N_{\mu\nu\rho\sigma}(p^2)}{M_{PL}^2}, \quad (9) \]
where \( N(p^2) \) is a finite remaining part of \( \Pi^{G\text{vac}}(p) \).

We now have
\[ \rho_{\text{vac}} \sim M_{PL}^2 \Pi^{G\text{vac}}(p^2) \sim \Lambda_{G\text{vac}}^4. \quad (10) \]

If we choose \( \Lambda_{G\text{vac}} \leq 10^{-3} \text{ eV} \), then the quantum correction to the bare cosmological constant \( \lambda_0 \) is suppressed sufficiently to satisfy the observational bound on \( \lambda \), and it is protected from large unstable radiative corrections.

This provides a solution to the cosmological constant problem at the energy level of the SM and possible higher energy extensions of the SM. The universal fixed gravitational scale \( \Lambda_{G\text{vac}} \) corresponds to the fundamental length \( \ell_{G\text{vac}} \leq 1 \text{ mm} \) at which virtual gravitational radiative corrections to the vacuum energy are cut off.

The gravitational form factor \( \mathcal{G} \), when coupled to non-vacuum SM gauge boson or matter loops, will have the form in Euclidean momentum space
\[ \mathcal{G}^{GM}(q^2) = \exp \left[ -q^2/2\Lambda_{GM}^2 \right]. \quad (11) \]

If we choose \( \Lambda_{GM} = \Lambda_M > 1 - 10 \text{ TeV} \), then we will reproduce the SM experimental results, including the running of the SM coupling constants, and \( \mathcal{G}^{GM}(q^2) = F^M(q^2) \) becomes \( \mathcal{G}^{GM}(0) = F^M(q^2 = m^2) = 1 \) on the mass shell.

This solution to the CCP leads to a violation of the weak equivalence principle (WEP) for coupling of gravitons to vacuum energy and matter. This could be checked experimentally in a satellite Eötvös experiment on the Casimir vacuum energy.

We observe that the required suppression of the vacuum diagram loop contribution to the cosmological constant, associated with the vacuum energy
momentum tensor at lowest order, demands a low gravitational energy scale \( \Lambda_{G\text{vac}} \lesssim 10^{-3} \text{ eV} \), which controls the coupling of gravitons to pure vacuum graviton and matter fluctuation loops. This is essentially because the external graviton momenta are close to the mass shell, requiring a low energy scale \( \Lambda_{G\text{vac}} \).

In our finite, perturbative quantum gravity model nonlocal gravity produces a long-distance infrared cut-off of the vacuum energy density through the low energy scale \( \Lambda_{G\text{vac}} < 10^{-3} \text{ eV} \) [5]. Gravitons coupled to non-vacuum matter tree graphs and matter loops are controlled by the energy scale: \( \Lambda_{GM} = \Lambda_{SM} > 1 - 20 \text{ TeV} \).

The rule is: When external graviton lines are removed from a matter loop, leaving behind pure matter fluctuation vacuum loops, then those initial graviton-vacuum loops are suppressed by the form factor \( G_{G\text{vac}}(q^2) \) where \( q \) is the internal matter loop momentum and \( G_{G\text{vac}}(q^2) \) is controlled by \( \Lambda_{G\text{vac}} \leq 10^{-3} \text{ eV} \). On the other hand, e.g. the proton first-order self-energy graph, coupled to a graviton, is controlled by \( \Lambda_{GM} = \Lambda_M > 1 - 20 \text{ TeV} \) and does not lead to a detectable violation of the equivalence principle.

There are problematic issues associated with our nonlocal quantum gravity model. Since the nonlocal form factors \( G_{G\text{vac}} \) and \( F_M \) contain significantly different nonlocal energy scales \( \Lambda_G \) and \( \Lambda_M \), unitarity at every order of perturbation theory could pose problems. This requires further study.

Complete S-matrix scattering amplitudes can through crossing symmetry lead to large violations of causality for energies \( \gg \Lambda \). The nonlocal quantum gravity model should only be considered an effective theory that regularizes the quantum gravity perturbation calculations [6].

Gluon condensates \( \langle G_{\mu\nu}G^{\mu\nu} \rangle_0 \) formed in the QCD vacuum in phase transitions, due to a broken phase of chiral symmetry, produce a vacuum density that is far too large: \( \sim \Lambda_{\text{QCD}}^4 / 16\pi^2 \sim 10^{-4} \text{ GeV}^4 \), which is more than 40 orders of magnitude larger than \( \rho_{\text{crit}} \). The SM Higgs particle produces a Higgs condensate \( V(\phi = v) = -m^4 / 2\Lambda_c + V_0 \) which is catastrophically large. These are both non-perturbative phenomena. How do we explain a suppression of these condensates in a perturbative quantum gravity scheme?

- The scales \( \Lambda_M \) and \( \Lambda_{G\text{vac}} \) are determined in loop diagrams by the quantum non-localizable nature of the gravitons and SM particles.
- The gravitons coupled to matter and matter loops have a nonlocal scale at \( \Lambda_{GM} = \Lambda_M > 1 - 20 \text{ TeV} \) or a length scale \( \ell_M < 10^{-16} \text{ cm} \), whereas the gravitons coupled to pure vacuum energy are localizable up to an energy
The fundamental energy scales $\Lambda_{G\text{vac}}$ and $\Lambda_{G M} = \Lambda_M$ are determined by the underlying physical nature of the particles and fields and do not correspond to arbitrary cut-offs, which destroy the gauge invariance, Lorentz invariance and unitarity of the quantum gravity theory for energies $> \Lambda_{G\text{vac}} \sim 10^{-3}$ eV. The underlying explanation of these physical scales must be sought in a more fundamental theory.

5 Satellite Eötvös Experiment for Zero-point Vacuum Energy

We consider that the cosmological constant arises from zero-point vacuum energy, so that a violation of the WEP could be observed in an Eötvös experiment [7]. We propose [4], that a satellite experiment be performed in which the acceleration of a spherical thin shell of aluminum be compared to a test mass made of copper or silver. Aluminum has a sharp transition from reflectance to absorption of EM waves at photon energies of 15.5 eV. In a simple cutoff calculation, the magnitude of the missing zero-point energy density inside the aluminum sphere is

$$\mathcal{E} = \frac{4\pi}{(\hbar c)^3} \int_0^{E_{\text{max}}} dE E^3,$$

where $E_{\text{max}}$ is the energy at which aluminum becomes transparent.

For $E_{\text{max}} = 15.5$ eV, one obtains $\mathcal{E} = 2.37 \times 10^{19}$ eV/cm$^3$. The rest-mass energy density is $1.52 \times 10^{33}$ eV/cm$^3$. Thus, the ratio is

$$R = 1.6 \times 10^{-14}.$$ (13)

In drag-free satellite missions such as SEE, STEP, Galileo Galilei or MICROSCOPE, the Eötvös parameter $\eta = 2(a_1 - a_2)/(a_1 + a_2)$ can reach an accuracy of $\eta \sim 10^{-15} - 10^{-17}$, so that our Casimir vacuum energy Eötvös test could reach a 1% level.

The calculation of the Casimir vacuum energy for a thin spherical shell of matter is controversial, due to the non-trivial self-energy problem. For a thin hollow sphere a calculation of the Casimir energy depends on the radius of the sphere in a non-trivial manner and the sharp Dirichlet boundary
conditions on the surface of the sphere cause the calculation to be dependent on the material of the sphere [8]. The calculation of the zero-point vacuum energy for a thin hollow sphere is cutoff dependent, due to the emergence of divergences that cannot be removed by a renormalization scheme.

- If the Eötvös experiment shows that the vacuum energy falls at a significantly slower rate than ordinary matter in a gravitational field, then this is a strong indication that the coupling of gravity to the vacuum energy is suppressed compared to its coupling to ordinary matter. This would provide a significant clue as to the basic mechanism that results in a small cosmological constant.

6 Conclusions

- We have described a possible solution to the cosmological constant problem. The particle physics resolution requires that we construct a consistent quantum gravity theory, which has vertex form factors that are different for gravitons coupled to quantum vacuum fluctuations and matter.
  - This predicts a violation of the WEP for coupling to vacuum energy, but not to matter-graviton couplings or to non-vacuum matter loops. This leads to a suppression of all SM vacuum loop contributions and, thereby, avoids a fine-tuning cancellation between the “bare” cosmological constant $\lambda_0$ and the vacuum contribution $\lambda_{\text{vac}}$. It retains the experimental agreement of the SM predictions.
  - A satellite Eötvös experiment for Casimir vacuum energy could experimentally decide whether nature does allow a vacuum energy WEP violation and a significant suppression of vacuum energy density.
  - As a model of a future fundamental, nonlocal quantum gravity theory, it does provide clues as to the resolution of the “infamous” cosmological constant problem.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada.
References

[1] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989); N. Straumann, astro-ph/020333.

[2] D. N. Spergel et al., astro-ph/0302209; S. Perlmutter et al., Astrophys. J. 517, 565 (1999); A. G. Riess et al., Astron. J. 116, 1009 (1998).

[3] J. W. Moffat, hep-ph/0102088; J. W. Moffat, AIP Conf. Proc. 646, 130 (2003), hep-th/0207198.

[4] J. W. Moffat and G. T. Gillies, New J. Phys. 4 (2002) 92.1, gr-qc/0208005.

[5] R. Sundrum, JHEP 9907 (1999) 001, hep-ph/9708329; hep-th/0306106. In these papers, Sundrum proposes that the graviton is a composite object with an associated form factor and the cutoff $\Lambda_c \leq 10^{-3}$ eV plays a fundamental role in suppressing the vacuum energy.

[6] A. Jain and S. D. Joglekar, hep-th/0307208.

[7] D. K. Ross, Nuovo. Cim. B114, 1073 (1999).

[8] R. Jaffe, Talk given at the QFEXT03 Workshop, University of Oklahoma, Oklahoma, USA, September 15-19, 2003. To be published in the workshop proceedings.