Next-to-Leading Order QCD corrections to Three-Jet Rates with Massive Quarks

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The reaction $e^+e^-$ annihilation into three jets was recently computed for massive quarks at next-to-leading order in perturbative QCD. We discuss some results of our calculation for $b$ jets produced at the $Z$ resonance.

1. INTRODUCTION

Huge samples of $Z$ decays into jets of hadrons have been collected both at LEP and at SLC. These data allow for numerous precision tests of the theories of electroweak and strong interactions. In particular, large numbers of jet events involving $b$ quarks can be isolated using vertex detectors. For detailed investigations of $b$ jets, aiming at a precision at the few percent level, one must take into account in theoretical predictions that the $b$ quark mass is non-zero. In fact $b$ quark mass effects in jets produced at the $Z$ resonance have already been seen indirectly in the so-called “flavour-independence tests” of the strong coupling. Such tests have been performed both at LEP [1] and at SLC [2].

Three groups have recently reported [3–5] the calculation of the next-to-leading order (NLO) QCD corrections for $e^+e^- \rightarrow 3$ jets for massive quarks. This extends the well-known NLO 3-jet results for massless quarks obtained in the early eighties [6–8]. (For the NLO 2-jet cross section and leading order (LO) results for three, four, and five jet rates involving massive quarks see [9] and [10,11], respectively.) In this talk we discuss some results of our NLO computation of $\sigma_{3\text{jet}}$ and apply it to $b$ jets produced at the $Z$ resonance.

2. THE NLO CROSS SECTION

The calculation of the $e^+e^-$ annihilation cross section, $\sigma_{NLO}$, into three jets involving a massive quark antiquark pair to order $\alpha_s^2$ consists of two parts: First, the computation of the amplitude of the partonic reaction $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow Q\bar{Q}g$ at leading and next-to-leading order in the QCD coupling. Here $Q$ denotes a massive quark and $g$ a gluon. We have calculated the complete decay amplitude and decay distribution structure for this reaction. This allows for predictions including oriented three-jet events. The differential cross section involves the so-called hadronic tensor which contains five parity-conserving and four parity-violating Lorentz structures. Second, the leading order matrix elements of the four-parton production processes $e^+e^- \rightarrow Z^*, \gamma^* \rightarrow ggQ\bar{Q}, Q\bar{Q}q\bar{q}, Q\bar{Q}Q\bar{Q}$ are needed. Here $q$ denote light quarks which are taken to be massless.

The infrared (IR) and ultraviolet (UV) singularities, which are encountered in the computation of the one-loop integrals, are treated within the framework of dimensional regularization in $D = 4 - 2\varepsilon$ space-time dimensions. We remove the UV singularities by the standard $\overline{\text{MS}}$ renormalization. We have converted from the outset the on-shell mass of the heavy quark $Q$ into the corresponding running $\overline{\text{MS}}$ mass. It is known that far from threshold one thereby absorbs some large logarithms into the running mass.

An essential aspect of any NLO computation of jet rates is to show that the IR singularities of the virtual corrections are cancelled by the singularities resulting from phase space integration of
the squared tree amplitudes for the production of four partons. Different methods to perform this cancellation have been developed (see [12–14] and references therein). We use the so-called phase space slicing method elaborated in [12]. The basic idea is to “slice” the phase space of the four parton final state by introducing an unphysical parton resolution parameter $s_{\text{min}} \ll s_{\text{cut}}$, where $y_{\text{cut}}$ is the jet resolution parameter. The parameter $s_{\text{min}}$ splits the phase space into a region where all four partons are “resolved” and a region where at least one parton remains unresolved. For massless partons, the resolved region may be conveniently defined by the requirement that all invariants $s_{ij} = (k_i + k_j)^2$ constructed from the parton momenta $k_i$ are larger than the parameter $s_{\text{min}}$. We have modified this definition slightly to account for masses.

In the unresolved region soft and collinear divergences reside, which have to be isolated explicitly to cancel the singularities of the virtual corrections. This is considerably simplified due to collinear and soft factorizations of the matrix elements which hold in the limit $s_{\text{min}} \to 0$. (In the presence of massive quarks, the structure of collinear and soft poles is completely different as compared to the massless case.) After having cancelled these IR poles against the IR poles of the one-loop integrals entering the virtual corrections, one is left with a completely regular differential three-parton cross section which depends on $s_{\text{min}}$.

The contribution to $\sigma_{\text{NLO}}^3$ of the “resolved” part of the four-parton cross section is finite and may be evaluated in $D = 4$ dimensions, which is of great practical importance. It also depends on $s_{\text{min}}$ and is most conveniently obtained by a numerical integration. Since the parameter $s_{\text{min}}$ is completely arbitrary, the sum of all contributions to $\sigma_{\text{NLO}}^3(y_{\text{cut}})$ must not depend on $s_{\text{min}}$. In the soft and collinear approximations one neglects terms which vanish as $s_{\text{min}} \to 0$. This limit can be carried out numerically. Since the individual contributions depend logarithmically on $s_{\text{min}}$, it is a nontrivial test of the calculation to demonstrate that $\sigma_{\text{NLO}}^3$ becomes independent of $s_{\text{min}}$ for small values of this parameter [6].

The three jet cross section depends on the experimental jet definition. We consider here the JADE [13] and Durham [16] clustering algorithms, although other schemes [17] can also be easily implemented. We have checked that we recover the result of [13] in the massless limit.

Let us now discuss some results for the cross section $\sigma_{\text{NLO}}^{3,b}$ for $b$ quarks. An important point to notice is that we are concerned here with the computation of tagged cross sections – whereas the massless NLO results [8] apply to summing over all quark flavours in the final state. In the computation of tagged cross sections one encounters mass singularities in the real corrections – that are regulated by keeping the quark mass non-zero – which find no counterpart in the virtual corrections against which they can cancel. In the case at hand, to order $\alpha_s^2$, there is a contribution to $\sigma_{\text{NLO}}^{3,b}$ from the diagrams $e^+e^- \to q\bar{q}g^* \to q\bar{q}b\bar{b}$. This contribution contains large logarithms involving $m_b$. (They result from the region where the invariant mass of the virtual gluon becomes small.) One may simply keep these logarithms, but that is not satisfactory, in particular if $\sqrt{s} \gg m_b$. Eventually, one has to factorize these logarithms into a fragmentation function for a gluon into a $b$ hadron. A detailed discussion of the single $b$-tag cross section $\sigma_{\text{NLO}}^{3,b}$ will be given elsewhere [19].

Here we shall consider instead the following “double $b$-tag” three-jet cross section: we require that at least two of the jets that remain after the clustering procedure contain a $b$ or $\bar{b}$ quark. The cross section $\sigma_{\text{NLO}}^{3,b}$, where the contributions from the above diagrams are included, remains infrared-safe in the limit $m_b \to 0$.

As mentioned above we have expressed $\sigma_{\text{NLO}}^{3,b}$ in terms of the $b$ quark mass parameter $m_b(\mu)$ defined in the $\overline{\text{MS}}$ scheme at a scale $\mu$. The asymptotic freedom property of QCD predicts that this mass parameter decreases when being evaluated at a higher scale. (A number of low energy determinations of the $b$ quark mass have been made; see for instance [24] and references therein.) With $m_b(m_b) = 4.36$ GeV [21] and $\alpha_s(m_Z) = 0.118$ [23] as an input and employing the standard renormalization group evolution of the coupling and the quark masses, we use the value $m_b(\mu = m_Z) = 3$ GeV. These values for $\alpha_s$ and $m_b$ are used in
Figs. 1a,b where we plot $\sigma_{NLO}^{3,b}$ as a function of $y_{cut}$ together with the LO result at the $Z$ peak, both for the JADE and the Durham algorithm.

![Graph showing cross section vs. y_cut for JADE and Durham algorithms](image)

**Figure 1:** The cross section $\sigma_{NLO}^{3,b}$ as a function of $y_{cut}$ for the JADE and Durham algorithms, respectively. The dashed line is the LO result. The NLO results are for renormalization scale $\mu = m_Z$ (solid line), $\mu = m_Z/2$ (dotted line), and $\mu = 2m_Z$ (dash-dotted line).

(Initial state photon radiation and electroweak corrections are not included.) The QCD corrections to the LO result are quite sizable, as is known also in the massless case. At small values of $y_{cut}$, for instance below $y_{cut} \sim 0.01$ for the JADE algorithm, perturbation theory is not applicable. The NLO corrections reduce the dependence on the renormalization scale significantly, as compared to the leading order result for the three-jet cross section. This renormalization scale dependence, which is also shown in Figs. 1a,b, is modest in the whole $y_{cut}$ range exhibited for the Durham and above $y_{cut} \sim 0.01$ for the JADE algorithm.

The effects of the non-zero $b$ quark mass on the three-jet rate at the $Z$ peak is of the order of a few percent, depending on the jet algorithm and on $y_{cut}$. An interesting application is the determination of the $\overline{MS}$ mass parameter $m_b$ at a high scale by measuring a suitable double ratio of three-jet fractions, as proposed in [24], computed to NLO in [3], and experimentally pursued by the DELPHI collaboration [25]. In [3] we have computed a double ratio that is somewhat differently defined than the one used in [24]. Effects due to the non-zero $b$ quark mass may also be studied by means of differential two-jet distributions, for instance in the Durham scheme.

3. CONCLUSIONS

We have computed the complete differential distributions for $e^+e^-$ annihilation into three and four partons, including the full quark mass dependence, both on and off the $Z$ resonance. Apart from $\sigma_{NLO}^{3,b}$ these results allow the computation of a number of other observables at order $\alpha_s^2$. Our results may be applied to $b$ and $c$ quark jet production at various c.m. energies, and to theoretical investigations of top quark production at very high-energetic electron positron collisions.

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