Generation of uniform synthetic magnetic fields by split driving of an optical lattice

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We describe a method to generate a synthetic gauge potential for ultracold atoms held in an optical lattice. Our approach uses a time-periodic driving potential based on two quickly alternating signals to engineer the appropriate Aharonov-Bohm phases, and permits the simulation of a uniform tunable magnetic field. We explicitly demonstrate that our split driving scheme reproduces the behavior of a charged quantum particle in a magnetic field over the complete range of field strengths, and obtain the Hofstadter butterfly band-structure for the Floquet quasienergies at high fluxes.

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Introduction – Systems of ultracold atoms held in optical lattice potentials have an exceptionally high degree of controllability and excellent coherence properties. As such they have proven to be excellent systems for simulating Hamiltonians arising in diverse areas of physics, such as graphene [2, 3], Majorana fermions [4], Veselago optics [5], and models of high-temperature superconductivity [6]. A topic of intense current activity is how to reproduce the effects of gauge fields in these systems. This would not only extend the use of cold-atom simulators to new domains, but is of considerable interest to applications such as quantum computation. A particularly important example is the $U(1)$ gauge of electromagnetism. The ability to simulate magnetic fields would give exciting new ways to explore quantum Hall physics and related effects such as topological insulators and anyon physics, together with the realization of phenomena such as the Hofstadter butterfly [7], a fractal energy spectrum that occurs in lattice systems exposed to high magnetic fluxes.

Many efforts to simulate an applied magnetic field with cold atoms have used laser-driven transitions between internal atomic states to generate phases [8, 10] which mimic the Aharonov-Bohm phases that would be experienced by charged particles moving in a uniform magnetic field. An attractive alternative to these schemes is to use inertial forces, which do not require a specific internal state structure, and so are applicable to a wider range of atomic species. First efforts of this kind [11] used rotation to generate a Coriolis force, which has an analogous form to the Lorentz force of electromagnetism. Only weak fields, however, were accessible by this method.

An alternative inertial approach, which directly modifies the intersite tunneling, is to periodically accelerate (or “shake”) the lattice to produce the effect known as coherent destruction of tunneling, a quantum coherent effect in which the driving renormalizes the tunneling amplitude [12]. This effect has been directly observed in the expansion dynamics of atomic clouds [13, 14]. More recently, it has also been noted that careful control of the phase of the driving field at the moment it is switched on can render the tunneling complex [15, 17], which gives the prospect of generating synthetic magnetic fields. This possibility has been explored experimentally in one-dimensional lattices [18], and later in triangular lattices where this effect was used to create a staggered field [19].

Generating a uniform field on a square lattice, however, is not straightforward, and an interesting proposal by Kolovsky [20] was later found to contain a problem that limited it to inhomogeneous fields [21]. Very recently, experimental progress has been made in countering this problem [22, 23] by introducing additional lattice potentials and a strong magnetic field gradient, which indeed permitted a uniform field to be generated. In this work we address the problem in a different way, by borrowing the split-operator technique from quantum simulation. By dividing the time-dependent Hamiltonian into two parts and applying them sequentially, we show that it is possible to simulate a uniform magnetic field of arbitrary strength. Our scheme requires only small driving amplitudes, thus limiting heating effects that can hinder observation of Hofstadter physics [24], and depends only on the manipulation of the time-dependent driving potential, which can be controlled experimentally to a high degree of precision.

Method – To illustrate the main ideas we schematically show a possible arrangement in Fig.1. We consider a two-dimensional optical lattice, formed by the superposition of two orthogonal standing waves. When the optical lattice is sufficiently deep, a system of cold atoms can be described well by a tight-binding (hopping) Hamiltonian

$$H(t) = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + H.c.) + H_f(t)$$

(1)

where $J$ is the hopping between nearest-neighbors $\langle i,j \rangle$, and $a_i^\dagger / a_i$ are the standard particle annihilation/creation operators. An acceleration of the lattice in the $x$-direction can be viewed in the rest frame of the lattice as an inertial force, described as a scalar potential depending linearly on $x$, $H_f(t) = V(t) \sum_j x_j n_j$ where $x_j$ is the $x$-coordinate of lattice-site $j$, $n_j$ is the number operator, and

$$V(t, \phi) = V_0 + K \sin (\omega t + \phi).$$

(2)
of the phase with ping processes (marked with arrows); a linear variation of produce a spatially-dependent phase. To reproduce the Landau two-dimensional standing wave between incoming laser beams.

FIG. 1: (a) Schematic representation of a possible implementation of the method. The optical lattice is formed as a two-dimensional standing wave between incoming laser beams (not shown). Two additional running-wave beams, with a frequency difference $\omega$ and crossing at an angle of $2\theta$, produce a spatially-dependent phase. To reproduce the Landau gauge the induced phase appears only on the horizontal hopping processes (marked with arrows); a linear variation of the phase with $y$, $\chi(m) = m\Phi$, produces a uniform flux $\Phi$ threading each plaquette. (b) The split-operator scheme. In the first half-period, an oscillating potential is applied in the $x$-direction (upper figure) to induce the Aharonov-Bohm tunneling phases, while the tunneling in the $y$ direction is ($J_y$) suppressed (lower figure). In the second half-period the $x$-coordinate is uniformly accelerated back to its original position, suppressing $x$-tunneling ($J_x$) via the Wannier-Stark effect, while tunneling in the $y$ direction is restored. This pattern of driving is periodically repeated.

Here $V_0$ and $K$ parameterize the constant and time-dependent components of the acceleration respectively, $\omega$ is the frequency of the oscillating component, and $\phi$ is its phase. The driving starts at $t = 0$.

We begin by considering the case of a one-dimensional lattice (i.e. a single row of the system shown in Fig.1), and restrict ourselves to the case of resonant driving, $V_0 = n\omega$ where $n$ is an integer. In the limit of high frequencies, $\omega \gg J$, it can be shown using perturbational Floquet theory [17], that the system can be described by an effective static hopping Hamiltonian, with an effective hopping given by

$$J_{\text{eff}} = J e^{-i(K/\omega)\cos \phi} e^{in(\phi+\pi/2)} J_n (K/\omega),$$

for right-to-left hopping, and $J'_{\text{eff}}$ for left-to-right movement. Note that for $\phi = \pi/2$ this reduces to the well-known Bessel function renormalization of a driven lattice [26], $J_{\text{eff}} = (-1)^n V J_n (K/\omega)$. From Eq. [3] we can see that the effective tunneling has a phase $\chi = -(K/\omega)\cos \phi + n(\phi + \pi/2)$, which can be used to induce transport [16, 17], and has been experimentally observed in experiments on “super-Bloch oscillations” [26, 27].

The magnetic flux passing through a plaquette is evaluated by summing the complex phases acquired by each tunneling process as the plaquette is traversed anti-clockwise. The specific case of the Landau gauge $\chi(m) = m\Phi$, where $m$ indexes the $y$-coordinate, is shown in Fig.1a, and it can be readily seen that each plaquette is threaded by a net flux $\Phi$. A natural way to try to reproduce this configuration is to make $\phi$ similarly space-dependent $\phi \to m\phi$. From Eq. [3] this gives a flux per plaquette of

$$\Phi = n\phi - (K/\omega)[\cos(m+1)\phi - \cos m\phi].$$

The first term of this expression indeed corresponds to a uniform field, while the second corresponds to a flux that varies with $y$. Its value, however, is bounded $|\Phi_{\text{var}}| \leq 2|K/\omega \sin \phi/2|$, and so it can be controlled by setting $K/\omega$ sufficiently small. Making $K/\omega$ small is also advantageous to limit lattice heating effects, although as this reduces the amplitude of $J_{\text{eff}}$ this also has the effect of slowing the system's dynamics.

In experiment this spatial dependence of $\phi$ could be introduced in a number of ways. In the scheme shown in Fig.1 it is produced by using a pair of far-detuned running-wave beams, with a frequency difference of $\omega$ between them. This produces an optical potential $V_R \cos^2(ky \sin \theta - \omega t/2)$, where $2\theta$ is the angle between the beams, which effectively leads to $\Phi$ with $\phi$ replaced by $\phi(m) = m\phi$, where $\phi = 2k d_L \sin \theta$ and $d_L$ is the optical lattice spacing. The phase can thus be controlled by altering the angle between the running wave beams, or by varying their wavelength $2\pi/k$. Unfortunately, when $\phi$ varies in this way, the potential [2] has the undesired effect of also driving tunneling in the $y$-direction since the potential difference between a site and its neighbors in the $y$-direction, $\tilde{V}(t, m\phi) - \tilde{V}(t, m\phi \pm \phi)$, is in general a time-dependent quantity, oscillating with frequency $\omega$. In consequence, driving naively in this way also renormalizes the $y$-tunneling [21] in a space-dependent fashion, destroying the homogeneity of the simulated field.

To eliminate this “accidental renormalization” of the tunneling perpendicular to the driving, we apply the split-operator technique familiar from numerical studies of quantum systems, and divide the time-evolution operator over a short time-interval into two parts, as shown
in Fig. 1. In the first interval the lattice is driven by the potential \( V \) with \( \phi \) replaced by \( m \phi \), but the \( y \)-hopping is suppressed (for example, by increasing the depth of the optical lattice in the \( y \)-direction). In this interval \( H(t) \) can thus be replaced by an effective static Hamiltonian, \( H_x^{\text{eff}} \), which contains only \( x \)-hopping terms in which the tunneling has been renormalized according to Eq. \( \text{(3)} \) with \( \phi \to m \phi \). In the second interval we restore the \( y \)-hopping, and instead suppress the \( x \)-hopping, so that the system evolves under \( H_y \), a time-independent Hamiltonian containing only the \( y \)-hopping operators. A convenient way to do this is to flip the sign of the acceleration of the lattice (\( V_0 \to -V_0 \) while setting \( K = 0 \)) so that the intersite tunneling in the \( x \)-direction is destroyed by Wannier-Stark localization. This has the additional practical benefit of keeping the average displacement of the lattice zero \([28]\), otherwise the constant acceleration would quickly move the lattice out of the experimental area.

This division amounts to a Suzuki-Trotter decomposition of \( H(t) \)

\[
e^{-i(H_x^{\text{eff}} + H_y)\Delta t} \approx e^{-iH_x^{\text{eff}}\Delta t} e^{-iH_y\Delta t}. \tag{5}
\]

As \( H_x \) and \( H_y \) do not commute, the leading error in this decomposition is given by the Baker-Campbell-Hausdorff formula \( \Delta t \left( [H_x^{\text{eff}}, H_y] \right) \approx J_{xy} \Delta t^2 \). More complicated decompositions can be used in which the error term decays more rapidly \([29]\), but for simplicity we limit ourselves to the most primitive form. For good accuracy we must take \( \Delta t \) to be as small as possible, but to obtain the effective renormalization of the tunneling \([3]\) \( \Delta t \) must be larger than the driving period \( T = 2\pi/\omega \). Numerically we have found the minimum period to be \( \Delta t = 3T \); below this value the renormalization effect is abruptly lost. All the results we show below are obtained for a time-interval of \( \Delta t = 8T \).

Results – At low values of magnetic flux, we can understand the behavior of the system semiclassically. We consider a \( 15 \times 15 \) lattice with open boundary conditions, initialise the system as a narrow Gaussian wavepacket, and propagate it in time using the full time-dependent Hamiltonian \([11]\). In Fig. 2a we show the effect of giving the initial state a kick in the \( +y \) direction at \( t = 0 \) for various values of \( \phi \). We can clearly see that in each case the center of mass follows a circular trajectory, analogous to the Larmor orbit of a classical particle under the Lorentz force. As the magnetic flux is increased, the radius of the orbit decreases proportionately, as expected. Since the wavepacket contains a number of different quasimomenta, it spreads during the time-evolution, however, and eventually contacts the edge of the lattice, distorting the circular motion of the center of mass.

In Fig. 2b we show the time evolution of a Gaussian wavepacket in the presence of a parabolic trap potential, for a weak magnetic flux of \( \phi = 0.02 \). At \( t = 0 \) the trap potential is abruptly shifted 4 lattice spacings to the left, thereby exciting the wavepacket into motion. In the absence of the magnetic flux, the wavepacket would simply oscillate from one side of the trap to the other \([30]\). However, with the flux present the wavepacket experiences a Lorentz force perpendicular to its direction of motion, causing its center of mass to trace out the characteristic rosette pattern seen in Fig. 2b. This is precisely analogous to the path traced by a Foucault pendulum subjected to the Coriolis force. Physical parameters: driving frequency \( \omega = 50J \), driving amplitude \( K = 0.05\omega \), and \( n = 1 \).

FIG. 2: Weak field results. (a) The system was initialized as a Gaussian wavepacket of width \( \sigma^2 = 5 \), and kicked in the \( +y \) direction at \( t = 0 \) by applying a phase imprinting \( \exp(i\phi) \). The centre of mass of the wavepacket describes a circular orbit, the radius of which is inversely proportional to the applied field, in analogy to the cyclotron orbit of a charged particle in a uniform magnetic field. (b) Here the system is initialized in the ground state of a parabolic trap potential \( V = Kx^2/2 \), with curvature \( k = 0.1J \), which is then shifted 4 lattice spacings to the left. The centre of mass traces out a rosette pattern, precisely analogous to the path of a Foucault pendulum subjected to the Coriolis force. Physical parameters: driving frequency \( \omega = 50 J \), driving amplitude \( K = 0.05\omega \), and \( n = 1 \).
tistinguish two kinds of states; those localized in the bulk of the lattice, and those that are localized on the edge. The bulk states contain a self-similar series of gaps, which become fractal in the limit of large system size. The edge states, which would be absent in an infinite system or a system with periodic boundary conditions, lie within these gaps, and are chiral transporting states. It can be clearly seen that for a given value of flux the edge states occur in pairs with opposite slope, corresponding to propagation in either the clockwise or anti-clockwise sense around the boundary of the lattice.

In Fig. 3 we take the lower, and more experimentally realistic, value of \( \omega = 10J \) to demonstrate that our simulation procedure is robust. Reducing the value of \( \omega \) affects the results in two ways; firstly the perturbation theory yielding Eq. 3 becomes increasingly less accurate, and secondly the error term arising from the Suzuki-Trotter decomposition 4 becomes more significant. These two sources of inaccuracy are responsible for the differences which Fig. 3 shows with respect to the ideal behavior of Fig. 3b. The butterfly structure is still clearly evident in the results, although the difference between the edge and bulk states is less sharp than for the high-frequency case, and some differences with the Harper-Hofstadter spectrum now appear. In particular we can observe that a subset of the edge states show an almost flat dependence on the flux, indicating that they have zero group velocity, and so are no longer transporting. Although these differences become more significant as \( \omega \) is reduced further, we have checked that the broad structure of the Hofstadter butterfly is still reproduced even for driving frequencies as low as \( \omega = 2J \).

Conclusions – In summary, we have described a method of using a periodic driving potential to produce a synthetic gauge field of arbitrary strength. To achieve this we have made use of a split-driving approach, in which the desired time-evolution operator is constructed from a sequence of unitary operations that separate the \( x \) and \( y \) degrees of freedom, and so avoid the problems in Kolovsky’s original proposal 20. Our method is robust and simple, avoids the need for any additional fields and, unlike schemes based on hyperfine transitions, it does not require a specific internal structure for the atoms. Interestingly, the Hofstadter butterfly of Fig. 3 involves quasienergies and not energies, as has so far been conventional. In the limit \( \omega \rightarrow \infty \), the quasienergies become the energies of the idealized tight-binding Hamiltonian with a constant flux per plaquette.

This method opens the prospect of using the single-site addressability and fine experimental control of cold atom systems to study the quantum Hall effect and topological insulator systems in a novel way. A particularly appealing application is to ladder systems 31, which provide a convenient bridge between one-dimensional topological insulators and the full two-dimensional case. Exciting future developments would be the generalization of these results to non-Abelian gauge theories, and to the rapidly developing field of Floquet topological insulators, in which the Floquet quasienergy spectrum itself has a non-trivial topology 32.

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