Scalable quantum computing requires gate operations with fidelities above a certain threshold $1^2$. Reaching this threshold remains challenging, primarily due to two issues: (i) the control fields driving the gate operations have often experimental uncertainties, such as amplitude errors, that are bigger than the allowed deviations and (ii) the quantum systems cannot be completely shielded from the effects of a noisy environment. Different techniques have been developed to overcome these obstacles, such as dynamical decoupling (DD) for suppressing the effect of environmental noise and thus extend the coherence time of the system. An ideal dynamical decoupling sequence completely eliminates the interactions of the system with its environment and thereby “freezes” the system, apart from the evolution under the internal system Hamiltonian. This approach can therefore implement high-fidelity quantum memories. If, however, the specific application requires that the system evolves under a suitable control Hamiltonian, such as in a quantum information processor, the protection scheme and the control operations must be applied simultaneously. It is then necessary to design gates that combine processing with decoupling in such a way that the effect of the control fields survives, while all unwanted interactions are eliminated $3^4$.

Measuring the fidelity of gate operations in the region of the threshold for scalable quantum computation cannot be done on individual gates, since the measurement errors can be comparable to or greater than the gate errors. For this purpose, the “randomized benchmarking” procedure was developed $4^5$. It measures an average gate fidelity of a random selection of gates. This allows a much more precise determination of the average gate fidelity. It does not provide information about the fidelity of individual gates, but it provides estimates of the fidelity of sequences of gates, which is the relevant quantity for the analysis of large-scale computation.

In this paper, we present several families of gate operations that combine robustness against unwanted variations in the amplitudes of the control fields (flip angle errors) with protection against environmental noise. For this purpose, we use Clifford gates, which allow universal quantum computation when combined with magic states $5^8$. We determine their fidelities experimentally, using a nuclear spins as qubit and a different nuclear spin system as the noisy environment.

We consider a system qubit $S$, which is coupled to a noisy environment via a dephasing interaction

$$\mathcal{H}_{SE} = b(t)S_z,$$

where the (semi-)classical field $b(t)$ has a finite correlation time $\tau_c$. Dynamical decoupling by sequences of inversion pulses can suppress the dephasing effect of this interaction and prevent the decay of the coherence in the system $8^9$. This is well explored in the context of quantum memories, where the goal is to keep a specific quantum state unchanged. Here the effect of DD can be roughly described as a prolongation of the dephasing time $T_2$.

In the case of quantum computing, the goal is not the preservation of a quantum state, but the precise control of a quantum system such that it evolves along a well-defined path in Hilbert space. Dephasing then creates errors roughly $\propto (1 - e^{-\tau/T_2}) \approx \tau/T_2$, where $\tau$ is the duration of the gate operation and $T_2$ the dephasing time. Extending the dephasing time by DD therefore helps also for quantum computing. However, in this case, an additional complication arises: the dynamical decoupling sequences that are used for protecting quantum memories would equally decouple the system from the external fields that are applied to control the system’s path. It is therefore essential to combine dynamical decoupling and gate operations in such a way that they do not interfere destructively. Two general approaches for this have been proposed for solving this problem: to apply gate operations and DD operations successively $10^9$, or to interleave gate operation and DD by splitting the gate operation into as many elements as there are delays between the DD pulses and modify them in such a way that the effect of the DD pulses is to revert these modifications and the

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High-fidelity gate operations for quantum computing beyond dephasing time limits.

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The implementation of quantum gates with fidelities that exceed the threshold for reliable quantum computing requires robust gates whose performance is not limited by the precision of the available control fields. The performance of these gates also should not be affected by the noisy environment of the quantum register. Here we use randomized benchmarking of quantum gate operations to compare the performance of different families of gates that compensate errors in the control field amplitudes and decouple the system from the environmental noise. We obtain average fidelities of up to 99.8%, which exceeds the threshold value for some quantum error correction schemes as well as the expected limit from the dephasing induced by the environment.

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overall effect becomes that of the targeted gate operation \[3, 10, 16\].

A third option is to design gate operations that are robust against experimental imperfections and have ‘built-in’ the effect of a dynamical decoupling sequence. Such a decoupling effect is, e.g., built into \(\pi\)-pulses around an axis in the \(xy\)-plane: they invert the system operator \(S_z\) and therefore the system environment interaction given in eq. 1. The sequences that we consider for randomized benchmarking consist to 50% of \(\pi\)-pulses. Accordingly, they automatically reduce the effect of the system-environment interaction, although their efficiency may be lower than that of specialized DD-sequences.

Apart from reducing the coherence time, high fidelity operations also need to be robust against errors in the control fields driving the gate operations. The BB1 composite pulse (see figure 1g) was introduced by Wimperis [17] as a possible scheme for generating composite rotation pulses that are well compensated against amplitude errors. The sequence for generating a compensated rotation by an angle \(\theta\) around an axis in the \(xy\)-plane is

\[
R_{\varphi}(\theta)R_{\beta+\varphi}(\pi)R_{\beta+\varphi}(2\pi)R_{\beta+\varphi}(\pi).
\]

Here, \(\varphi\) defines the orientation of the rotation axis and \(\beta = \cos^{-1}(-\theta/4\pi)\). For the present purpose, we consider the cases \(\theta = \pi/2\) and \(\pi\), where \(\beta_{90} \approx 1.7\) and \(\beta_{180} \approx 1.8\). If the four \(\pi\)-pulses of this sequence are separated in time by a delay that is twice the duration of the initial \(\theta\)-pulse, this yields a sequence that is not only robust to amplitude errors, but also to environmental noise: it conforms to the “compute, then decouple” approach and the four \(\pi\)-pulse generate the DD cycle.

A robust \(\pi\)-pulse can also be generated by concatenating 5 \(\pi\)-pulses with the phase \(\pi/6, 0, \pi/2, 0, \pi/6\) [18] (see figure 1f). This generates an inversion of the \(z\)-component, but in addition also a \(-\pi/3\) rotation around the \(z\)-axis. We will refer to this pulse as KDD-5 = \(R_{\varphi}(-\pi/3)R_0 = R_0R_{\varphi}(\pi/3)\), where \(R_0\) is the targeted \(\pi\)-rotation around the \(x\)-axis. If it is used in a sequence like randomized benchmarking, the additional \(z\)-rotation can be taken into account by adjusting the coordinate system.

To determine the average fidelity of the gate operations, we use the scheme proposed by [19, 20]. It requires the application of \(\pi/2\) and \(\pi\) rotations around the axes of the coordinate system. The individual gates are 1-qubit Clifford gates \(C = PG\), where \(P\) indicates a unit operation or a \(\pi\)-rotation around one of the coordinate axes (8 different operations) and \(G\) a \(\pi/2\) rotation around a coordinate axis (6 different operations, 48 total). Rotations around \(\pm z\) axes are implemented through a change to the rotating frame definition [19, 20]. At the end of the sequence a recovery operation is applied that corresponds to the inverse of the sequence up to this point, so that the full sequence becomes a unit operation in the absence of imperfections. For optimal randomization, the recovery operation itself consists of two random \(P\) operations sandwiching the complement \(R\), which is another Clifford gate. For a single qubit, the effect of a non-ideal identity operation can be written as

\[
\rho_{\text{out}} = (1 - d)\rho_{\text{in}} + \frac{d}{2}\mathbf{1}.
\]

Here, \(\rho_{\text{in}}\) is the initial state, \(\mathbf{1}\) the unit operator and \(d\) is the depolarizing parameter. Here, we use the fact that for a sequence of Clifford gates, the result can be represented as a depolarization channel [21, 22]. The trace fidelity is then

\[
F = \text{tr}\{\rho_{\text{out}}\rho_{\text{in}}\} = 1 - \frac{d}{2}.
\]

and the error per gate is the difference

\[
E_{PG} = 1 - F = \frac{d}{2}.
\]

If the error per gate is purely random, we expect that the average fidelity for sequences with \(m\) gate operations decays as

\[
F = \frac{1}{2}(1 + (1 - d)^m) \approx \frac{1}{2}(1 + e^{-md}).
\]

For the experiment, we used the \(^{13}\)C nuclear spins of adamantane, a molecular crystal. The protons in the same crystal provide the noisy environment, which leads to a dephasing time of 360 µs. A single refocusing pulse (Hahn echo) can extend the dephasing time to 740 µs. The ultimate limit on the information storage in this system is given by the energy relaxation time \(T_1 = 1.52\) s.

The system was initially in thermal equilibrium, with \(\rho_{\text{in}} \propto S_z\). For each measurement, we averaged over a set of 32 random sequences composed of 1 to 80 gate operations. After the restore operation, the signal was measured by applying one additional readout pulse that converted the remaining \(S_z\) component of the density operator into transverse magnetization. The free precession signal was acquired, Fourier-transformed, and the signal corresponding to the \(CH_2\) carbon was integrated and used as a measure for the survival probability \(e^{-md}\).

Figure 11 gives an overview over the different sequences tested in this work. First (not shown in the figure) we tested gates without DD protection, using two different implementation for P and G rotations: i) simple rectangular pulses and ii) BB1 pulses, which compensate errors in the amplitudes of the control fields. In the second approach (11a) we generated a protected G rotation by separating the four \(\pi\)-pulses of the BB1 composite rotation by a delay that is twice the duration of the initial \(\pi/2\)-pulse. The P rotation in this scheme was implemented by one KDD-5 pulse. The third set of sequences (11b-e) was designed to reduce experimental imperfections and decouple the system from its environment by interleaving the P and G rotations with dynamical decoupling pulses in different ways, using BB1 pulses for all PG rotations.
FIG. 1: Different sequences of gate operations compared in this work. The double arrows indicate the gate duration \( \tau \), defined as the time period between the beginning of one P gate to the beginning of the next P gate. a) A protected G gate to the beginning of the next P gate. a) A protected G pulse, while in d) and e) the DD pulses were implemented as rectangular pulses. In b) and c) the DD pulses were also implemented as BB1 pulses and interleaved in different ways with the DD sequence XY-16 (see text). f) and g) show the KDD-5 and BB1 pulses.

In b) and c) the DD pulses were also implemented as BB1 pulses, while in d) and e) the DD pulses were implemented by rectangular pulses.

FIG. 2: Experimental survival probabilities for sequences of BB1-gates as a function of the number of steps. The circles represent the survival probability averaged over 32 gate sequences. The solid lines are represent an exponential fit and theoretical predictions (see text).

Figure 2 shows the experimental results obtained when composite BB1 pulses were used to implement the Clifford gates. Here, the gate time was 76 \( \mu s \). The blue and green lines show the theoretical prediction for pure dephasing with times of 750 \( \mu s \) and 340 \( \mu s \), respectively, which would result in EPGs of 3.2\% and 6.7\%. The straight lines through the experimental data points are fits to exponential decays, the corresponding EPG is 0.32 \( \pm \) 0.03\%, which is significantly better than the EPG extrapolated from the \( T_2 \) values. This is a clearly indication that the environment is refocused during the benchmarking experiment, even without applying DD pulses. In Figure 3 we compare the BB1 pulses with the three other cases:

1) The approach defined in figure 1a, ii) interleaving BB1 pulses with the DD sequence XY-16 (figure 1c) and iii) rectangular pulses. The gate time of the first scheme is 88 \( \mu s \) and its performance is comparable to that of BB1, the observed EPG is 0.34 \( \pm \) 0.03\%. In the case of interleaving BB1 pulses with DD, the gate time is 152 \( \mu s \), which is approximately twice the gate time for BB1 gates without DD protection. However the observed EPG is 0.22 \( \pm \) 0.03\%, showing that dynamical decoupling sequences can further increase the fidelity of quantum gates. The rectangular pulses do not exhibit an exponential decay, since those pulses are not built to compensate r.f. field inhomogeneities. In this case we cannot assign a single value to the EPG. For a quantitative evaluation, we fitted the experimental data to the function \( a^{n_k} \). The parameters obtained from this fit are \( a \sim 0.88 \) and \( k \sim 0.44 \).

Figure 3 compares the decay of the survival probability vs. time for the case of a benchmarking sequence whose gates are protected against environmental noise by an XY-4 DD sequence with the decay of coherence if only DD is applied, with no interleaved gate operations. For the benchmarking experiment the initial state was thermal equilibrium, with \( \rho_{in} \propto S_z \), while in the pure DD experiment the system was initialized in a superposition state. Clearly, the benchmarking results in much slower decays than the XY-4 sequence. This result can be traced to the fact that the pulse imperfections in an XY-4 cycle result in a propagator \( U_{4} \approx 1 + Q \), where \( Q \) is the error term. In the case of DD, the overall propagator is \( U_{4} = U_{4}^n \approx 1 + nQ \), for small number of cycles \( n \), the error grows linearly. In the case of benchmarking, the gates that are interleaved with the DD sequence modulate the error in a random manner, resulting in a
random walk in Liouville space. As a result, an increase $\alpha n$ may be expected for a random sequence. This is an expression of the general principle that in most cases, a decoupling scheme exists that is better than a simple repetition of the same basic cycle.

The experimental data clearly show that a robust design of the gate operations can make their performance largely independent of experimental imperfections. This does not mean that the error per gate can be made arbitrarily small, since not all interactions that contribute to the errors are actually suppressed. In particular, two types of environmental noise cannot be refocused by explicit or implicit DD: (i) interactions that are not linear in the qubit operators $S$ and (ii) interactions whose correlation time is shorter than the experimentally accessible time scales. In the present system, $^{13}$C-$^{13}$C couplings are an example of (i): they are bilinear in the spin interactions and are not eliminated by the DD sequences used here, although other sequences exist for suppressing them [23, 24]. Processes of category (ii) are those that contribute to the energy relaxation ($T_1$-processes): their correlation times must be comparable to or shorter than the Larmor precession period. Their contributions can be estimated from available experimental data. The longest dephasing times that have been reached in this system for isotropic decoupling sequences indicate that $\tau_1 \approx 50$ ms [25]. This value was achieved by using robust pulses and interpulse delays close to zero and it is also in qualitative agreement with numerical estimates for the broadening by $^{13}$C-$^{13}$C dipolar coupling in natural abundance adamantane. The contribution from fast processes can be estimated from the energy relaxation time as $\tau_2 = T_1 \approx 1.52$ s. We can therefore neglect processes of type (ii). This indicates that further improvements would require pulses that compensate also for homonuclear (bilinear) interactions. If the remaining dephasing rate is $T_2^{-1}$, this should contribute an error per gate of $\approx \tau/T_2$.

Table I summarizes the results. For each type of gate it shows the observed EPG$_{\exp}$. All gate operations include decoupling elements and thus result in an error per gate that is lower than the error rate EPG$_M$ estimated from the dephasing time measured by Hahn Echo. This is a clear indication that gate operations alone provide some dynamical decoupling. Each sequence is also compared to the minimum EPG$_m$ values we could reach in the present system, EPG$_m \approx \tau/T_2$, which is expected from pure dephasing due to the remaining environmental noise. These values were obtained by DD with robust pulses and very short delays between the pulses [25]. The best performance was achieved by the gate type (c), which corresponds to BB1 gates protected by the DD sequence XY-16.

The implementation of robust high-fidelity gate operations is an essential step towards reliable and scalable quantum computing. In this work we use randomized benchmarking of single qubit quantum gates to compare the performance of different families of gates that compensate errors in the control field amplitudes and decouple the system from the environmental noise. In some cases the total duration of the experiments (from $\sim 1$ ms to $\sim 30$ ms) exceeds the dephasing time, measured by Hahn echoes (760 $\mu$s), by almost two orders of magnitude. We could obtain average fidelities exceeding the expected limit from the dephasing induced by the environment. This is a clear indication that the effect of the noisy environment is reduced by the sequence of applied gate operations. The best average fidelity observed, 99.8%, was achieved by interleaving gate operations implemented by composite pulses with the dynamical decoupling sequence XY-16.

The improvements in gate accuracy by decoupling methods, as observed in this work, implies a reduction of the overhead cost of QEC since more noise can be tolerated by QEC codes combined with DD than by QEC alone [10]. Therefore, future work will be devoted to investigate the performance of QEC codes combined with dynamical de-
coupling sequences and to test the performance of DD protected gates in systems with multiple qubits. This work is supported by CNPq, FAPERJ, the Brazilian National Institute of Science and Technology for Quantum Information (INCT-IQ) and CAPES Ciência sem Fronteiras program (Grant 084/2012).

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