ON DYNAMICS OF FRACTALITY IN CENTRAL C-CU COLLISIONS AT 4.5A GEV/c

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ABSTRACT

Fractal structure of charged particle distributions in pseudorapidity and transverse momentum in central C-Cu collisions at 4.5 A GeV/c is studied by means of intermittency approach and multifractal analysis. The modifications to take into account statistical bias are applied. Intermittency study in the pseudorapidity phase space indicates a non-thermal phase transition and different regimes of multiparticle production. Multifractality is observed within the both methods applied. The interrelation of the methods using the effective average multiplicity approach is studied. The findings support the idea of statistical significance of influence of finite multiplicities. In the transverse momentum spectrum no dynamical fluctuations are found.

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1 Introduction

Last decade investigations of the intermittency effect excited an enhanced interest. It is shown that this phenomenon can be a direct characteristic of a possible phase transition expected to be occurred in high energy collisions.

In this talk we present the results of a study of dynamics of multiparticle fluctuations (correlations) in relativistic nuclear collisions using the intermittency/fractality approach. Such investigations have a specific interest due to the expectation of quark-gluon plasma formation in high energy nuclear collisions and in a sense of a possible “soft origin” of intermittency and weakness of the effect with reaction “complexity” and energy increase. Note that the study presented continues our preceding investigations.

Dynamical fluctuations are indicated by a power-law,

\[ F_q \propto f_q M^{\varphi_q}, \quad 0 < \varphi_q \leq q - 1 \quad (q \geq 2), \quad (1) \]

of the qth order normalized scaled factorial moments (NSFM),

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aOn leave from Institute of Physics, Tbilisi 380077, Georgia.
\[ \mathcal{F}_q = M^{q-1} \sum_{m=1}^{M} \frac{\langle n_m^{[q]} \rangle}{(n_m)^q}, \]  

(2) 

over \( M \) number of the bins into which the phase space of produced particles is divided. Here \( n_m \) is the number of particles in the \( m \)-th bin for each event, \( \langle \cdots \rangle \) denote averaging over the events, and \( n^{[q]} = n(n-1) \cdots (n-q+1) \).

The intermittency indices \( \varphi_q \) characterize (multi)fractal structure of the distribution. Fractality is pointed out to be a reflection of the phase transition via the codimensions defined as

\[ d_q = \varphi_q/(q-1). \]  

(3) 

Monofractal patterns \([d_q = \text{const.}(q)]\) are associated with second-order phase transition (e.g. from quark-gluon plasma), while self-similar cascading is characterized by multifractals \([d_q > d_p \text{ at } q > p]\) with a possible “non-thermal” phase transition.

As a signal of the transition an existence of a minimum of the function

\[ \lambda_q = \langle \varphi_q + 1 \rangle/q \]  

at a certain “critical” value of \( q = q_c \) is argued. The minimum of Eq. 4 may be also a manifestation of coexistence of (many small) liquid-type fluctuations and dust-type (high density) ones.

To continue the study to noninteger \( q \)'s and to make a direct search for multifractality, the method of frequency moments was proposed. The moments are defined as

\[ G_q = \sum_{m=1}^{M} \left( \frac{n_m}{n} \right)^q \theta(n_m - q). \]  

(5) 

Here \( n \) is the number of particles per event and \( \theta(x) \) is a step function which is 1 if \( x \geq 0 \) and 0 otherwise.

The dynamics of the fluctuations observed is of self-similar nature, i.e.

\[ \langle G_q \rangle \propto g_q M^{-\tau_q}, \]  

(6) 

and in analogous to Eq. 3 indicates dynamical fluctuations:

\[ \tau_{q\text{dyn}} \approx q - 1 - \varphi_q. \]  

(7) 

Let us note that in our previous study\footnote{Recently the procedure of continuing the NSFM to noninteger \( q \)'s has been suggested.} of the \( G \)-moments multifractality of the \( \eta \)-spectra has been found with indication of the dynamical fluctuations.
2 Data Sample and Analysis

The data analyzed here come from interactions of the JINR Synchrophasotron (Dubna) 4.5 \(A\) GeV/c \(^{12}\)C beam with a copper target inside the 2m Streamer Chamber SKM-200. The central collision trigger was used: absence of charged particles with momenta \(p > 3\) GeV/c in a forward cone of 2.4° was required.

The scanning and handling of the film data were carried out on special scanning tables of the Lebedev Physical Institute (Moscow). The average measurement error in the momentum \(\langle \varepsilon_p/p \rangle\) was about 12%, and that in the polar angle measurements was \(\langle \varepsilon_\theta \rangle \approx 2^\circ\). A total of 305 events with charged particles in the pseudorapidity window \(\Delta \eta = 0.2 - 3.0\) are considered (\(\eta = -\ln \tan(\theta/2)\)). The accuracy \(\langle \varepsilon_\eta \rangle\) does not exceed 0.1. In addition particles with \(p_t > 1\) GeV/c are excluded from the investigation to eliminate the contribution of protons as far as no negative charged particles were observed with such transverse momentum in an assumption of equal numbers of positive and negative pions. The average multiplicity is of 21.2 ± 0.6.

The fluctuations are considered in the \(\eta\) and \(p_t\) phase spaces. To avoid the problem of the non-flat shape of the distributions \(\rho(x)\), \(x = \eta\) or \(\ln p_t\) we use a “cumulative” variable:

\[
\tilde{x} = \frac{\int_{x_{min}}^{x} \rho(x') dx'}{\int_{x_{min}}^{x_{max}} \rho(x') dx'},
\]

(8)

with the uniform spectrum \(\rho(\tilde{x})\) within the \([0,1]\) interval.

Further, we apply the modified SFM (MSFM) proposed to remove the biased estimator of the normalization of the NSFM (2) (especially in small bins):

\[
F_q = \frac{N_q}{M} \sum_{m=1}^{M} \frac{\langle n_{m}^{[q]} \rangle}{N_{m}^{[q]}}.
\]

(9)

Here \(N_m\) is the number of particles in the \(m\)th bin in all \(N\) events.

The modified scaled frequency moments (MSFrM) were suggested to be defined as

\[
G_q = \frac{N}{M} \sum_{m=1}^{M} \frac{\langle n_{m}^{[q]} \theta(n_m - q) \rangle}{N_{m}^{[q]}}.
\]

(10)

Later on the modified moments (9) and (10) instead of the biased ones (2) and (5) are studied. The scaling laws (6) are considered with respect to these unbiased quantities. Note that use of the “transformed” variable (8) along with the modifications allow to study higher-order moments.
3 Results

3.1 Intermittency in the Pseudorapidity Phase Space

Fig. 1 illustrates dependence of the $F_q$ on $M$, depicted for $q = 3, 4$ for pseudorapidity fluctuations. The different increase of the MSFM with $M$ for the different $M$-regions manifesting on the plots continues up to $q = 8$ (not shown) confirming our earlier results. Such a behavior of the moments lends support to the existence of distinguished regimes of particle creation at various bin averaging scales.

Figure 1: Log-log plots of the MSFM ($F_q$) vs. $M$ in the pseudorapidity phase space.

Figure 2: $d_q$ vs. $q$.

Figure 3: $\lambda_q$ vs. $q$. 
Fig. 2 presents the function $d_q$ for different $M$-intervals. A few intervals with sensitively distinguished increase of the $d_q$ are seen: $2 \leq M \leq 22$, $4 \leq M \leq 15$, $7 \leq M \leq 17$, and $10 \leq M \leq 17$. Multifractality observed, as mentioned above, points out a cascading scenario of particle production.

From Fig. 3 of displaying of $\lambda_q$ function one can conclude that at least two regimes of particle production exist: one with the phase transition at $q_c = 4$, and another one for which no critical behavior is reached. Taking into account multifractality, critical $q_c$ indicates a “non-thermal” phase transition. Although the interpretation may be a matter of debate the minimum was found earlier in hadronic interactions at small $p_t$ and very recently by means of the $M$-intervals study in ultra-high heavy ion collisions.

In Fig. 2 and 3 we show also the $\lambda_q$ predicted by the Gaussian approximation (GA) and Ochs-Wosiek approach (OWA). As seen the approximations meet difficulties, especially when $q_c$ exists; the same is found for the negative binomial distribution input. Since all these random cascading models are based on the second order MSFM, the difference indicates multiparticle character of a possible phase transition. This observation confirms our earlier results despite the restriction of transverse momenta.

3.2 Intermittency in the Transverse Momentum Phase Space

Fig. 4 shows the MSFM behavior with $M$ in the phase space of $p_t$. No intermittency effect is observed in this projection. The fluctuations in the transverse momentum distributions seem to be of statistical origin.

Figure 4: Log-log plots of the MSFM $\lambda$ vs. $M$ in the $p_t$ phase space.

3.3 Multifractality Analysis

The MSFrM at $q = 2, 3$ calculated in the pseudorapidity phase space as well as in the $p_t$-projection are shown in Fig. 5 in comparison with the statistical $G^\text{stat}_q$. A clear difference of the pseudorapidity fluctuations from the
statistical ones is seen, while the fluctuations coming from the distributions in the transverse momentum confirm their statistical nature.

The effect is much stronger for very small bins (large \( M \)’s). Taking into account that scaling laws (1) and (6) are satisfied (strictly) for large \( M \) (mathematically \( M \to \infty \)) this finding seems to be very important and shows real dynamical origin of the fluctuations in \( \eta \)-projection. Two different behaviors of the MSFrM – weak rising at \( M \leq 10 \) and its enhancement for \( M > 10 \) – indicate two possible regimes of particle production mentioned in Sec. 3.1.

Contrary to the observed decrease and fast saturation of the \( G \)-moments for \( q > 0 \) (as it was also shown by us) the MSFrM are increasing functions of \( M \) up to very small bins. Despite this, connected with the modified form (10), the strong increase indicates very multifractal structure of the \( \eta \)-spectrum.

Note that the method of frequency moments allows to build fractal spectral function which in a case of the phase transition should have zeros at \( q_c \). Let us mention that in our study of C-Ne collisions such a zero was observed.

The scaling indexes \( \varphi_q \) and \( \tau_q \) are connected via Eq. 7, approximate character of which is shown to be caused by influence of finite multiplicities. Very recently to solve the problem the procedure based on the method of the effective average multiplicity has been developed. It is proposed to use the generalized moments defined by multiplying (in our case) by the \( N_m^L \), \( L = 0, 1, 2, \ldots \) under the sum in the definitions (9) and (10), so that for \( L = 0 \) the standard forms are restored.

The generalized scaling laws,

\[
F_q \propto (\mathcal{N}/M)^{\theta - 1} \left[ N_0^{(F)} \right]^{-L} f_q M^{\varphi_q},
\]
\[G_q \propto [N_0^{(G)}]^{-L} f_q M^{\varphi_q},\]

are pointed out to characterize dynamical fluctuations. Here \(N_0^{(F),(G)}\) is the effective average multiplicity predicted to be independent of the type of the moments \((N_0^{(F)} = N_0^{(G)})\) as well as of the parameter \(L\).

Fig. 6 confirms the mentioned independences and shows a weak dependence of \(N_0^{(F),(G)}\) on the \(M\)-interval in both analysis performed. These findings gives evidence for real accounting of statistical contribution (shown to be sensitive) to find an exact relation between the slopes. Note in this sense the method of noninteger order factorial moments where multifractal analysis is continued to the intermittency approach.

### 4 Conclusions

Study of dynamics of fluctuations and fractality in the pseudorapidity and transverse momentum spectra of charged particles produced in central C-Cu collisions at 4.5 Gev/c is performed. The scaled factorial and frequency moments are calculated in the transformed variables and corrected to take into account the bias of infinite statistics. The fluctuations in the pseudorapidity phase space are found to have very multifractal structure and to be of dynamical nature, indicating possible non-thermal phase transition. Two different regimes of particle production during the hadronization cascade are indicated. The generalized scaling laws of the moments analyzed are studied via the
method of the effective average multiplicity. Correct relation between the scaling exponents are shown to be strongly influenced by finite multiplicities. The fluctuations in the transverse momentum projection are found to be statistical ones.

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