A fractional-order model of coronavirus disease 2019 (COVID-19) with governmental action and individual reaction

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The deadly coronavirus disease 2019 (COVID-19) has recently affected each corner of the world. Many governments of different countries have imposed strict measures in order to reduce the severity of the infection. In this present paper, we will study a mathematical model describing COVID-19 dynamics taking into account the government action and the individuals reaction. To this end, we will suggest a system of seven fractional deferential equations (FDEs) that describe the interaction between the classical susceptible, exposed, infectious, and removed (SEIR) individuals along with the government action and individual reaction involvement. Both human-to-human and zoonotic transmissions are considered in the model. The well-posedness of the FDEs model is established in terms of existence, positivity, and boundedness. The basic reproduction number (BRN) is found via the new generation matrix method. Different numerical simulations were carried out by taking into account real reported data from Wuhan, China. It was shown that the governmental action and the individuals’ risk awareness reduce effectively the infection spread. Moreover, it was established that with the fractional derivative, the infection converges more quickly to its steady state.

KEYWORDS
basic infection reproduction number, Caputo fractional-order derivative, COVID-19, numerical simulation, sensitivity analysis

MSC CLASSIFICATION
34A08; 37N25; 78A70; 26A33

1 INTRODUCTION

Coronavirus disease 2019 (COVID-19) is still targeting a great number of individuals worldwide. With more than one million confirmed cases and eighty thousand deaths in only the last few months, COVID-19 is considered a public health
priority. It is worthy to notice that the first human coronavirus was first identified in 1965.\textsuperscript{1} At the end of the year 2019, a new type of coronavirus caused by SARS-CoV-2 have been determined, COVID-19. The name is derived from the virus crown-like appearance and the year of identification.\textsuperscript{2,3} So far, the ways of COVID-19 transmission are via respiratory coughs or sneezes within nearly two meters and also by indirect contact of anterior contaminated surfaces\textsuperscript{4,5}

Knowing that, until now, there is no cure against COVID-19; several mathematical models have been deployed in order to understand, to predict, and to act for reducing or even stop the spread of the disease. Recently, in this year, 2020, several works have investigated the dynamics of COVID-19 by modeling the infection by a system of ordinary differential equations (ODEs). Indeed, the effect of intervention, prediction, and risk estimation of COVID-19 has been studied via an ODE model.\textsuperscript{6} Likewise, the meteorological effect and different policy measures on COVID-19 were performed via an ODE model.\textsuperscript{7} Moreover, an ODE system of equations reporting the interaction between susceptible, infected, and recovered have been tackled in Volpert et al\textsuperscript{8} along with the involvement of the quarantine effect.

Since the disease still causes a huge number of infections worldwide, many governments have taken several drastic measures in order to reduce the severity of this deadly disease. Good reaction of individuals toward those measures leads to significant control of COVID-19 epidemic spread. A recent work\textsuperscript{9} has included these two major factors of government policy as well as individual reaction into the suggested ODE model describing the COVID-19 outbreak. We continue in this paper the investigation on COVID-19 by extending the latter work to a fractional deferential equation (FDE) model. Indeed, modeling with fractional order derivatives becomes an important tool to study various phenomena; for instance, Kumar and Erturk\textsuperscript{10} give an effective study on the application of generalized Caputo type fractional operator in ecology. In the same context, other works have used FDEs to study various epidemic dynamics.\textsuperscript{11,12} Also, recent papers studied the dynamics of COVID-19 by means of FDEs.\textsuperscript{13–19} Hence, the mathematical model for COVID-19 that we consider is formulated as follows:

$$\begin{align*}
D^aS &= -\frac{\beta_0 SF}{N} - \frac{\beta(t) SI}{N} - \mu S, \\
D^aE &= \frac{\beta_0 SF}{N} + \frac{\beta(t) SI}{N} - \sigma E - \mu E, \\
D^aI &= \sigma E - \gamma I - \mu I, \\
D^aR &= \gamma I - \mu R, \\
D^aN &= -\mu N, \\
D^aD &= d\gamma I - \lambda D, \\
D^aC &= \sigma E,
\end{align*}$$

(1)

where the classes of our problem are as follows: $S$ indicates the number of susceptible individuals, $E$ represents the number of exposed individuals, $I$ denotes the number of infectious individuals, $R$ stands for the number of recovered individuals, $N$ indicates the size total population, $D$ represents the public risk awareness, and $C$ stands for the number of cumulative cases. The parameters of our problem are as follows: $F$ is the number of zoonotic cases, $\beta_0$ represents the transmission rate, $\sigma^{-1}$ stands for the mean latent period, $\gamma^{-1}$ denotes the mean infectious period, $d$ depicts the proportion of severe cases, and $\lambda^{-1}$ represents the mean duration of public reaction. In this model, we will assume two mode of transmissions, the first is zoonotic, denoted by $\beta_0$ and human-to-human transmission denoted by $\beta(t) = \beta_0 (1 - \rho)(1 - \frac{D}{N})^\kappa$, where $\rho$ is governmental action strength, $\kappa$ is intensity of response, and $\mu$ is the emigration rate. In this last mode of transmission, the governmental action is taken into consideration. In this paper, we will study mathematically the problem (1), and we will perform some numerical simulations by taking into account real reported data from Wuhan, China.

The schematic representation of the viral dynamics of the problem under study is demonstrated via Figure 1.

The structure of the article is presented into many sections, which start with some basic definitions related to fractional derivatives and integrals given. The next part of the article presents the well-posedness of the model in terms of existence, positivity, and boundedness. The BRN and sensitivity analysis are investigated in Section 4. The numerical technique for solving the model, the simulations, and concluding remarks are presented in the last portion of the article.

2 | PRELIMINARY TOOLS

In the present section, we give some necessary tools on fractional operators and Mittag-Leffler (ML) function.
Definition 2.1. The fractional integral of order $\alpha > 0$ of a function $\Psi : \mathbb{R}_+ \to \mathbb{R}$ is defined by

$$I^\alpha \Psi(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \Psi(s)ds,$$

(2)

where $\Gamma(.)$ represents the well-known Gamma function.

Definition 2.2. The fractional operator of Caputo kind having order $\alpha > 0$ of a function $\Psi : \mathbb{R}_+ \to \mathbb{R}$ is presented in the subsequent manner

$$D^\alpha \Psi(t) = I^{n-\alpha}D^n \Psi(t),$$

(3)

where $D = d/dt$ and $n - 1 \leq \alpha \leq n$, $n \in \mathbb{N}$.

Also, if $0 < \alpha \leq 1$, we have

$$D^\alpha \Psi(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\Psi'(s)}{(t-s)^\alpha}ds.$$

(4)

Definition 2.3. Let $\alpha > 0$. The function $E_\alpha$ written as

$$E_\alpha(z) = \sum_{j=0}^{+\infty} \frac{z^j}{\Gamma(\alpha j + 1)}$$

(5)

is called the ML function.

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ where $n \geq 1$. Let us assume a fractional-order system

$$D^\alpha X(t) = f(X),$$

(6)

$$X(0) = X_0,$$

(7)

with $0 < \alpha \leq 1$ and $X_0 \in \mathbb{R}^n$. To study the global existence of solution of the problem (6), we have the subsequent lemma as follows.
Lemma 2.1. Assume that \( f \) holds the following subsequent conditions:

(i) \( f(X) \) and \( \frac{\partial f}{\partial X}(X) \) are continuous on \( \mathbb{R}^n \).
(ii) \( \|f(X)\| \leq c_1 + c_2\|X\| \) for all \( X \in \mathbb{R}^n \), where \( c_1 \) and \( c_2 \) are positive constants.

Then, the system (6) has a unique solution defined on \( [0, +\infty) \).

3 | WELL-POSEDNESS RESULT

3.1 | Existence, positivity, and boundedness

Since our problem is associated with population dynamics, the variables should be nonnegative and bounded. Hence, in this subsection, we study the existence, positivity, and boundedness result of solutions to the problem (1). First, for biological purposes, the initial data \( S_0, E_0, I_0, R_0, N_0, D_0, \) and \( C_0 \) should be larger than or equal to zero. For the existence, positivity, and boundedness of the problem solution, we have the following subsequent result:

**Proposition 3.1.** For any nonnegative initial conditions \( (S_0, E_0, I_0, R_0, N_0, D_0, C_0) \), the system (1) has a unique solution. Moreover, this solution is nonnegative and bounded for all \( t \geq 0 \).

**Proof.** First, the model (1) can be rewritten as follows:

\[
D^X = A_1X + \frac{1}{N}A_2X + I\left(1 - \frac{D}{N}\right)X,
\]

where

\[
X = \begin{pmatrix} S \\ E \\ I \\ R \\ N \\ D \\ C \end{pmatrix},
\]

\[
A_1 = \begin{pmatrix} -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\mu + \sigma) & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma & -(\gamma + \mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & d\gamma & 0 & 0 & -\lambda & 0 \\ 0 & 0 & \sigma & 0 & 0 & 0 & 0 \end{pmatrix},
\]

\[
A_2 = \begin{pmatrix} -\beta_0 F & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 F & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\]

and

\[
A_3 = \begin{pmatrix} -\beta_0(1 - \rho) & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0(1 - \rho) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\]

From (8), we have

\[
\|D^X\| \leq (\|A_1\| + \|A_2\| + \|A_3\|)\|X\|.
\]
Therefore, by using Lemma 2.1, the system (1) has a unique solution on \([0, +\infty)\).
Now, we will establish the nonnegativity of the solution. We have

\[
\begin{align*}
D^\alpha S|_{S=0} &= 0 \geq 0, \\
D^\alpha E|_{E=0} &= \frac{\beta_0 S F}{N} + \frac{\beta S I}{N} \geq 0, \\
D^\alpha I|_{I=0} &= \sigma E \geq 0, \\
D^\alpha R|_{R=0} &= \gamma I \geq 0, \\
D^\alpha N|_{N=0} &= 0 \geq 0, \\
D^\alpha D|_{D=0} &= d\gamma I \geq 0,
\end{align*}
\]

and

\[
D^\alpha C|_{C=0} = \sigma E \geq 0.
\]

This shows that the solution of system (1) is nonnegative.

About the boundedness of the solutions, we assume that

\[
T = S + E + I + \frac{1}{2}R + N + \frac{1}{2d}D,
\]

and then, we have

\[
D^\alpha T(t) \leq -\mu T(t),
\]

and therefore,

\[
T(t) \leq T(0)E_a(-\mu t^\alpha),
\]

where \(E_a(u) = \sum_{j=0}^{+\infty} \frac{u^j}{\Gamma(\alpha j+1)}\) is the ML function.

Since \(0 \leq E_a(-\mu t^\alpha) \leq 1\), then

\[
T(t) \leq T(0).
\]

This fact implies that \(S, E, I, R, N,\) and \(D\) are bounded.

About the boundedness of \(C\), we use the last equation of (1); we have

\[
D^\alpha C = \sigma E.
\]

Then,

\[
C(t) \leq C(0) + \sigma\|E\|_{\infty}.
\]

Then the variable \(C\) is also bounded.

\[\square\]

4 \ THE BRN AND SENSITIVITY ANALYSIS

As it is well known, the BRN is described as the average number of novel possible cases of an infection due to one infected person, in a certain population that consists of susceptibles only. We will utilize the next generation matrix \(FV^{-1}\) to evaluate the BRN \(R_0\). The BRN is presented by the formula as follows: \(R_0 = \varphi(FV^{-1})\), where \(\varphi\) indicates the spectral radius, \(F\) represents the nonnegative matrix of newly infection cases, while \(V\) stands for the matrix of the transition infections pertaining to the model (1).

\[
F = \begin{pmatrix} 0 & \beta_0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \sigma + \mu & 0 \\ -\sigma & \gamma + \mu \end{pmatrix}.
\]
Therefore,

\[
V^{-1} = \frac{1}{(\sigma + \mu)(\gamma + \mu)} \begin{pmatrix} \gamma + \mu & 0 \\ \sigma & \sigma + \mu \end{pmatrix}.
\]

Then,

\[
FV^{-1} = \frac{1}{(\sigma + \mu)(\gamma + \mu)} \begin{pmatrix} \beta_0 \sigma & \beta_0 (\sigma + \mu) \\ 0 & 0 \end{pmatrix}.
\]

This fact implies that the BRN is formulated as follows:

\[
R_0 = \frac{\beta_0 \sigma}{(\gamma + \mu)(\sigma + \mu)}.
\]

The sensitivity analysis is utilized mainly to find out which parameter of the model can change remarkably dynamics of infection. This permits to figure out the parameters that have a higher influence on \( R_0 \). To study such analysis, we have the requirement of the subsequent normalized sensitivity index of \( R_0 \) w.r.t. any given parameter \( \theta \):

\[
\varphi_\theta = \frac{\partial R_0}{\partial \theta} \frac{\theta}{R_0}.
\]

So we obtain

\[
\varphi_{\beta_0} = 1,
\]
\[
\varphi_\mu = -\frac{\mu (\sigma + \gamma + 2\mu)}{(\gamma + \mu)(\sigma + \mu)},
\]
\[
\varphi_\sigma = \frac{\mu}{\sigma + \mu}.
\]

**TABLE 1**  The sensitivity index of \( R_0 \)

| Parameters | Sensitivity index |
|------------|------------------|
| \( \beta_0 \) | 1 |
| \( \mu \) | -0.1514 |
| \( \sigma \) | 0.0585 |
| \( \gamma \) | -0.907 |

**FIGURE 2**  Surface plot of \( R_0 \) versus \( \mu \) and \( \beta_0 \) [Color figure can be viewed at wileyonlinelibrary.com]
and

\[ \varphi_r = \frac{-\gamma}{\gamma + \mu}. \]

It can be noticed from Table 1 that the parameters \( \beta_0 \) and \( \sigma \) are positive sensitivity indices and the other rest of parameters \( \mu \) and \( \gamma \) are negative sensitivity indices. It is noticed that the parameters \( \beta_0 \) and \( \gamma \) have larger magnitude, in their absolute values, which implies that they are the most sensitive parameters of the considered model. It demonstrates that any enhancement in the value of the parameters \( \beta_0 \) leads an enhancement of \( R_0 \), which would result to a rise in the infection. On the other hand, an enhancement in the value of the parameters \( \gamma \) will decrease \( R_0 \) which leads to a decline of the infection.

From Figure 2, one can notice that for \( \beta_0 = 1 \) and \( \mu = 0 \), the value of \( R_0 \) extends to its maximum value of 4.97. By enhancing \( \mu \) from 0 to 1 and decreasing \( \beta_0 \) from 1 to 0, we see that the value of \( R_0 \) gradually reduces and moves toward \( 1.03 \times 10^{-4} \) (when \( \beta_0 = 0; \mu = 1 \)). It noticeably reflects the influence of the efficiency in terms of controlling the infection. Similarly, we demonstrate with the aid of a surface plot of \( R_0 \) in Figure 3 that for \( \beta_0 = 1 \) and \( \sigma = 1 \), the value of \( R_0 \) comes to its maximum value of 4.44. By decreasing \( \beta_0 \) and \( \mu \) from 1 to 0, we see that the value of \( R_0 \) gradually falls of and inclines toward \( 5.39 \times 10^{-4} \) (for \( \beta_0 = 0; \sigma = 0 \)). It obviously reflects the influence of the effectiveness in terms of controlling the infection. Figure 4 shows that for \( \beta_0 = 1 \) and \( \gamma = 0 \), the value of \( R_0 \) becomes its maximum value of 44.83. By enhancing the value of \( \gamma \) from 0 to 1 and decreasing \( \beta_0 \) from 1 to 0, we see that the value of \( R_0 \) gradually drops and moves toward \( 4.61 \times 10^{-4} \) (at \( \beta_0 = 0; \gamma = 1 \)). The influence of the efficiency in terms of controlling the infection is clearly evident.
Finally, from Figure 5, we can conclude that at \( \sigma = 1 \) and \( \gamma = 0 \), the value of \( R_0 \) reaches its maximum value of 30.57. By enhancing \( \gamma \) from 0 to 1 and decreasing \( \sigma \) from 1 to 0, we see that the value of \( R_0 \) gradually reduces and approaches \( 1.52 \times 10^{-2} \) (for \( \sigma = 0; \gamma = 1 \)). This undoubtedly reflects the influence of the effectiveness in terms of controlling the infection.

5 | NUMERICAL SIMULATIONS

In the present portion, several numerical simulations will be performed by taking real reported data from Wuhan, China, and discussing in detail.

First, let us describe the numerical method we will use to perform simulations for the studied problem. Following the idea of Atangana and Owolabi,²⁰ we take a general fractional differential equation given as

\[
\quad{}_0D_t^\alpha x(t) = F(t,x(t)) \quad \text{with} \quad x(0) = x_0, \tag{9}
\]

with fundamental theorem of fractional calculus, the above differential equation can be converted into

\[
x(t) - x(0) = \frac{1}{\Gamma(\alpha)} \int_0^t F(r,x(r))(t - r)^{\alpha - 1} dr. \tag{10}
\]

Now, we choose the following uniform grid:

\[
h = \frac{T}{N}, \quad t_n = nh, \quad \text{for} \quad n = 0, 1, 2, \ldots, N, \quad t_0 = 0 \quad \text{and} \quad T_n = T.
\]

For a given \( t = t_{n+1} \), \( n = 0, 1, 2 \ldots, N \), it yields

\[
x(t_{n+1}) = x_0 + \frac{1}{\Gamma(\alpha)} \int_0^{t_{n+1}} F(s,x(s))(t_{n+1} - s)^{\alpha - 1} ds
\]

\[
= x_0 + \frac{1}{\Gamma(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} F(s,x(s))(t_{n+1} - s)^{\alpha - 1} ds. \tag{11}
\]

It is well known that composite Lagrange interpolation consists in splitting the interval in many subintervals and use a lower order Lagrange interpolation in each subinterval, in order to have a good approximation of a function. Therefore,
TABLE 2 The problem parameters, their meaning and values

| Parameters | Meaning                     | Value or range                     | Reference               |
|------------|-----------------------------|------------------------------------|-------------------------|
| $\beta_0$  | Transmission rate           | $[0.56, 0.6553]$ (day$^{-1}$)      | -                       |
| $F$        | Number of zoonotic cases    | $[0, 10]$                          | Wu et al.$^{21}$        |
| $\rho$     | Governmental action strength| $[0, 0.4239, 0.8478]$              | He et al.$^{22}$        |
| $\kappa$   | Intensity of responds       | 1117.3                             | He et al.$^{22}$        |
| $\mu$      | Emigration rate             | $[0, 0.0205]$ (day$^{-1}$)         | South China Morning Post$^{23}$ |
| $\sigma^{-1}$ | Mean latent period       | 3 (days)                           | Worldometers$^{24}$     |
| $\gamma^{-1}$ | Mean infectious period   | 5 (days)                           | Worldometers$^{24}$     |
| $d$        | Proportion of severe cases  | 0.2                                | Atangana$^{25}$         |
| $N_0$      | Initial population size     | 14 million                         | South China Morning Post$^{23}$ |
| $\alpha$  | Fractional derivative order | -                                  | -                       |

on each subinterval $[t_k, t_{k+1}]$, we approximate $F(s, x(s))$ with a Lagrange interpolant polynomial:

$$P_k(x) = \frac{s - t_{k-1}}{t_k - t_{k-1}} F(t_k, x(t_k)) - \frac{s - t_k}{t_k - t_{k-1}} F(t_{k-1}, x(t_{k-1}))$$

$$= \frac{F(t_k, x(t_k))}{h} (s - t_{k-1}) - \frac{F(t_{k-1}, x(t_{k-1}))}{h} (s - t_k)$$

$$\approx \frac{F(t_k, x_k)}{h} (s - t_{k-1}) - \frac{F(t_{k-1}, x_{k-1})}{h} (s - t_k).$$

Coming back to (11), we get the following:

$$x_{n+1} = x_0 + \frac{1}{\Gamma(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \frac{F(t_k, x(s))}{h} (s - t_{k-1}) (t_{n+1} - s)^{\alpha-1} ds$$

$$- \frac{1}{\Gamma(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \frac{F(t_{k-1}, x_{k-1})}{h} (s - x_k) (t_{n+1} - s)^{\alpha-1} ds.$$

(12)

Next, we compute the following coefficients:

$$A_{a,k,1} = \int_{t_k}^{t_{k+1}} (s - t_{k-1}) (t_{n+1} - s)^{\alpha-1} ds,$$

and

$$B_{a,k,2} = \int_{t_k}^{t_{k+1}} (s - t_k) (t_{n+1} - s)^{\alpha-1} ds.$$

A simple integration leads to

$$A_{a,k,1} = \frac{(n + 1 - k)^\alpha (n - k + 2 + \alpha) - (n - k)^\alpha (n - k + 2 + 2\alpha)}{\alpha^2 + \alpha},$$

(13)

and

$$B_{a,k,2} = \frac{(n + 1 - k)^{\alpha+1} - (n - k)^{\alpha} (n - k + 1 + \alpha)}{\alpha^2 + \alpha}.$$
Inserting (13) and (14) in Equation (12) gives the following approximation:

\[ x_{n+1} = x_0 + \sum_{k=0}^{n} Q_k ((n + 1 - k)^\alpha (n - k + 2 + \alpha) - (n - k)^\alpha (n - k + 2 + 2\alpha)) - \sum_{k=0}^{n} Q_{k-1} ((n + 1 - k)^\alpha - (n - k)^\alpha (n - k + 1 + \alpha)). \]

\[ Q_k = \frac{h^\alpha F(t_k, x_k)}{\Gamma(\alpha + 2)} \quad \text{and} \quad Q_{k-1} = \frac{h^\alpha F(t_{k-1}, x_{k-1})}{\Gamma(\alpha + 2)}. \]

With the help of the above numerical technique, the studied model has been solved and the numerical have been obtained with the help of the Matlab software.

The parameters for the numerical simulations are given in Table 2.

Remark 1. The value of \( \beta_0 \) is derived from Wu et al.\textsuperscript{21} and He et al.\textsuperscript{22} Since \( R_0 = 2.8 \), we have \( \beta_0 = 0.56 \) or \( \beta_0 = 0.6553 \).

From Figures 6 and 7, concerning the COVID-19 dynamics, we observe that the recovered population grows significantly. This means that the majority of the population will recover. On the other hand, the population of those infected and exposed decrease significantly. This means that the majority of the population will be recovered. which results in a decrease in the deaths caused by COVID-19.

In Figures 8 and 9, we give the behavior of the susceptibles and the recovered population, we observe that if \( \alpha \) is decreased, the amount susceptibles and the recovered population will be reduced. Hence, a small memory of the infection effect (higher value of \( \alpha \)) maximizes the number of COVID-19 healthy individuals.
Figures 10 and 11 illustrate the behavior of the susceptibles and the infected population for distinct values of the parameter $\rho$ representing the governmental action strength. We observe that if the government takes strong actions, the number of susceptibles is maximized and oppositely a decrease of infected population is remarked. Consequently, the governmental type of actions is very important to avoid any undesirable infection progression.

Finally, in Figures 12 and 13, we observe the evolution of the susceptibles and the infected population during the period of observation. We clearly see that when we increase $\kappa$, the amount susceptibles is reduced and the number of infected population is increased. This shows the effect of the population response intensity to the governmental action in reducing the infection severity.
FIGURE 11 The infected population as a function of time for different values of $\rho$ [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 12 The susceptibles population as a function of time for different values of $\kappa$ [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 13 The infected population as a function of time for different values of $\kappa$ [Color figure can be viewed at wileyonlinelibrary.com]

6 | CONCLUSION

In this article, we have studied the behavior of the late coronavirus infection (COVID-19). To this end, we have taken into account a fractional derivative SEIR model by considering the effect of governmental action characterized by the term $(1 - \rho)(1 - \frac{D}{N})^{\kappa}$. The effect of the memory on the dynamics of COVID-19 is represented by the fractional derivative order incorporated in the different components of our problem. First, we have established the well-posedness of our problem in terms of proving that our model admits a unique positive solution. Next, we have performed a sensitivity analysis of
our problem in order to show the effect of some parameters on the value of the BRN. Finally, some numerical results are illustrated in order to observe the behavior of the infection under the effect of different key parameters. It was observed that the governmental action and quality of population responds may play an essential role in reducing the infection severity.

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