A view on Connectedness and Compactness in Fuzzy Soft Bitopological Spaces

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Abstract. In the present paper, we introduce the notions of \((1, 2)^*\)-fuzzy soft b-separated sets, \((1, 2)^*\)-fuzzy soft b-connectedness and \((1, 2)^*\)-fuzzy soft b-compactness in fuzzy soft bitopological spaces. Then, some basic topological properties of these notions are investigated. Also, some illustrative examples are given to show the importance of the obtained theorems.

2020 Mathematics Subject Classifications: 54A05, 54A40, 54F99.

Key Words and Phrases: \((1, 2)^*\)-fsb-separated, \((1, 2)^*\)-fsb-connected, \((1, 2)^*\)-fsb-compact

1. Introduction

In 1965, Zadeh [36], introduced the concept of fuzzy set theory and its applications can be found in many branches of mathematical and engineering sciences including management science, control engineering, computer science and artificial intelligence (see, [5, 7]).

In 1999, Russian researcher Molodtsov [16], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences (see, [20, 28]). In 2003, Maji, Biswas and Roy [22], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, D. Chen [6], presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory.

Recently, on soft sets, soft topological space has been studied increasingly Shabir and Naz [32] defined the theory of soft topological space over an initial universe with a fixed set of parameters. Çağman et al. [18] introduced a topology on a soft set called “soft topology” and presented the foundations of the theory of soft topological spaces. Moreover, many authors studied soft topology and its applications (e.g. [8, 9, 11]). Later Tanay and Kandemir [35] introduced fuzzy soft topological space and established the basic
definitions of fuzzy soft topological space by incorporating the fuzzy topology and soft set. Fuzzy soft topological space was applied in various ways say, game theory, analysis, etc. Fuzzy soft set in topological space further studied by Roy [27]. The authors [13, 19, 33] are successfully applied fuzzy soft topological space in real life.

In 1963, Kelly [14], first initiated the concept of bitopological spaces and other authors have contributed to development and construction of some properties of such spaces (see, [23, 24]) as generalizations of which are in general topology.

In 2014, Ittanagi [12], introduced and studied the concept of soft bitopological spaces and other authors have contributed to development and construction of some properties of such spaces (see, [2, 3, 25, 26]).

The notion of soft bitopological space was introduced using different soft topologies on an initial universe set. On the other hand, the mixed type of soft set theory was given using different soft topologies (see, [1, 4, 10, 21, 34]).

In 2015, Mukherjee and Park [17], first introduced the notion of fuzzy soft bitopological space and they introduced the notions of \(\tau_1\tau_2\)-fuzzy soft open(closed) sets, \(\tau_1\tau_2\)-fuzzy soft interior (resp. closure) and studied some of their basic properties. Also, Sayed ([29–31]) were extension and continuation of studying in this trend by characterizing new concepts in fuzzy soft bitopological spaces. In the present paper, we introduce the notions of 

\[(1,2)^*\]-fuzzy soft b-separated sets, 

\[(1,2)^*\]-fuzzy soft b-connectedness and 

\[(1,2)^*\]-fuzzy soft b-compactness in fuzzy soft bitopological spaces. Then, some basic topological properties of these notions are investigated. Also, some illustrative examples are given to show the importance of the obtained theorems.

2. Preliminaries

In this section we are going to present the basic definitions and results of fuzzy soft set and fuzzy soft bitopological space which will be a central role in our paper. Throughout our discussion, \(X\) refers to an initial universe, \(E\) the set of all parameters for \(X\) and \(P(X)\) denotes the power set of \(X\).

Definition 1. [36] A fuzzy set \(A\) in a non-empty set \(X\) is characterized by a membership function \(\mu_A : X \rightarrow [0,1] = I\) whose value \(\mu_A(x)\) represents the "degree of membership" of \(x\) in \(A\) for every \(x\) in \(X\). Let \(I^X\) denotes the family of all fuzzy sets on \(X\).

Definition 2. [36] The empty fuzzy set on \(X\) denoted by \(\tilde{0}\) is a function which maps each \(x \in X\) to 0. That is, \(\tilde{0}(x) = 0\) for all \(x \in X\).

A universal fuzzy set denoted by \(\tilde{1}\) is a function, which maps each \(x \in X\) to 1. That is, \(\tilde{1}(x) = 1\) for all \(x \in X\).

Definition 3. [16] Let \(A \subseteq E\). A pair \((F,A)\) is called a soft set over \(X\) if \(F\) is a mapping given by \(F : A \rightarrow P(X)\).

Definition 4. [15] Let \(A \subseteq E\). A pair \((f,A)\), denoted by \(f_A\), is called a fuzzy soft set over \(X\), where \(f\) is a mapping given by \(f : A \rightarrow I^X\) defined by \(f_A(e) = \mu_{f_A}^e\) where
Definition 5. [22] A fuzzy soft set \( f_A \in (X, E) \) is said to be:

(a) NULL fuzzy soft set, denoted by \( \tilde{\phi} \), if for all \( e \in A \), \( f_A(e) = \tilde{0} \).

(b) absolute fuzzy soft set, denoted by \( \tilde{1}_E \), if for all \( e \in E \), \( f_A(e) = \tilde{1} \).

Definition 6. [27] The complement of a fuzzy soft set \( f_A \in (X, E) \) is given by

\[
\mu_{f_A}^c = \begin{cases} \tilde{0}, & \text{if } e \notin A; \\ \text{otherwise, } & \text{if } e \in A. \\ \end{cases}
\]

\((X, E)\) denotes the family of all fuzzy soft sets over \((X, E)\).

Definition 7. [27] Let \( f_A, g_B \in (X, E) \). \( f_A \) is fuzzy soft subset of \( g_B \), denoted by \( f_A \subseteq g_B \), if \( A \subseteq B \) and \( \mu_{f_A}^e \leq \mu_{g_B}^e \) for all \( e \in A \), that is, \( \mu_{f_A}^e(x) \leq \mu_{g_B}^e(x) \) for all \( x \in X \) and for all \( e \in A \).

Definition 8. [27] Let \( f_A, g_B \in (X, E) \). The union of \( f_A \) and \( g_B \) is also a fuzzy soft set \( h_C \), where \( C = A \cup B \) and for all \( e \in C \), \( h_C(e) = \mu_{h_C}^e = \mu_{f_A}^e \lor \mu_{g_B}^e \). Here we write \( h_C = f_A \cup g_B \).

Definition 9. [27] Let \( f_A, g_B \in (X, E) \). The intersection of \( f_A \) and \( g_B \) is also a fuzzy soft set \( d_C \), where \( C = A \cap B \) and for all \( e \in C \), \( d_C(e) = \mu_{d_C}^e = \mu_{f_A}^e \land \mu_{g_B}^e \). Here we write \( d_C = f_A \cap g_B \).

Definition 10. [27] A fuzzy soft topology \( \tau \) over \((X, E)\) is a family of fuzzy soft sets over \((X, E)\) satisfying the following properties:

(i) \( \tilde{0}_E, \tilde{1}_E \in \tau \),

(ii) if \( f_A, g_B \in \tau \), then \( f_A \cap g_B \in \tau \),

(iii) if \( f_A \in \tau \) for all \( \alpha \in \Delta \) an index set, then \( \bigcup_{\alpha \in \Delta} f_A \in \tau \).

Definition 11. [17] If \( \tau \) is a fuzzy soft topology on \((X, E)\), then the triple \((X, E, \tau)\) is said to be a fuzzy soft topological space. Also each member of \( \tau \) is called a fuzzy soft open set in \((X, E, \tau)\).

The complement of a fuzzy soft open set is a fuzzy soft closed set.

Definition 12. [17] Let \((X, E, \tau_1)\) and \((X, E, \tau_2)\) be two different fuzzy soft topologies on \((X, E)\). Then \((X, E, \tau_1, \tau_2)\) is called a fuzzy soft bitopological space on which no separation axioms are assumed unless explicitly stated.

The members of \( \tau_i(i = 1, 2) \) are called \( \tau_i(i = 1, 2) \)-fuzzy soft open sets and the complement of \( \tau_i(i = 1, 2) \)-fuzzy soft open sets are called \( \tau_i(i = 1, 2) \)-fuzzy soft closed sets.
Definition 13. [17] A fuzzy soft set \( f_E \subseteq (\tilde{X}, \tilde{E}) \) is called \( \tau_1 \tau_2 \)-fuzzy soft open set if \( f_E = g_E \cup h_E \) such that \( g_E \subseteq \tau_1 \) and \( h_E \subseteq \tau_2 \).

The complement of \( \tau_1 \tau_2 \)-fuzzy soft open set is called \( \tau_1 \tau_2 \)-fuzzy soft closed set.

The family of all \( \tau_1 \tau_2 \)-fuzzy soft open (closed) sets in \((X, E, \tau_1, \tau_2)\) is denoted by \( \tau_1 \tau_2 \text{FSO}(X, \tau_1, \tau_2)_E \) (\( \tau_1 \tau_2 \text{FSC}(X, \tau_1, \tau_2)_E \)) respectively.

Definition 14. [17] Let \((X, E, \tau_1, \tau_2)\) be a fuzzy soft bitopological space and \( f_E \subseteq (\tilde{X}, \tilde{E}) \). Then the \( \tau_1 \tau_2 \)-fuzzy soft closure of \( f_E \), denoted by \( \tau_1 \tau_2 \text{cl}(f_E) \), is the intersection of all \( \tau_1 \tau_2 \)-fuzzy soft closed supersets of \( f_E \).

Clearly, \( \tau_1 \tau_2 \text{cl}(f_E) \) is the smallest \( \tau_1 \tau_2 \)-fuzzy soft closed set over \((X, E)\) which contains \( f_E \).

Definition 15. Let \((X, E, \tau_1, \tau_2)\) be a fuzzy soft bitopological space and \( f_E \subseteq (\tilde{X}, \tilde{E}) \). Then \( f_E \) is called \((1, 2)^*\)-fuzzy soft b-open set (briefly, \((1, 2)^*\)-fsb-open) if
\[
\forall \tau_1 \tau_2 \text{int}(f_E) \subseteq \tau_1 \tau_2 \text{cl}(f_E) \bigcup \tau_1 \tau_2 \text{cl}(\tau_1 \tau_2 \text{int}(f_E)).
\]

Definition 16. [30] Let \((X, E, \tau_1, \tau_2)\) be a fuzzy soft bitopological space and \( f_E \subseteq (\tilde{X}, \tilde{E}) \).

(i) \((1, 2)^*\)-fuzzy soft b-closure (briefly \((1, 2)^*\)-fsbcl\((f_E)\)) of a set \( f_E \) in \((X, E, \tau_1, \tau_2)\) defined by
\[
(1, 2)^*\text{fsbcl}(f_E) = \bigcap \{ g_E \subseteq f_E : g_E \text{ is a } (1, 2)^*\text{-fuzzy soft b-closed set in } (X, E, \tau_1, \tau_2) \}.
\]

(ii) \((1, 2)^*\)-fuzzy soft b-interior (briefly \((1, 2)^*\)-fsbint\((f_E)\)) of a set \( f_E \) in \((X, E, \tau_1, \tau_2)\) defined by
\[
(1, 2)^*\text{fsbint}(f_E) = \bigcup \{ g_E \subseteq f_E : g_E \text{ is a } (1, 2)^*\text{-fuzzy soft b-open set in } (X, E, \tau_1, \tau_2) \}.
\]

\((1, 2)^*\)-fsbcl\((f_E)\) is the smallest \((1, 2)^*\)-fuzzy soft b-closed set in \((X, E, \tau_1, \tau_2)\) which contains \( f_E \) and \((1, 2)^*\)-fsbcl\((f_E)\) is the largest \((1, 2)^*\)-fuzzy soft b-closed set in \((X, E, \tau_1, \tau_2)\) which is contained in \( f_E \).

Definition 17. [31] A fuzzy soft mapping \((\varphi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)\) is said to be \((1, 2)^*\)-fuzzy soft b-continuous (briefly \((1, 2)^*\)-fsb-continuous) the inverse image of every \( \sigma_1 \sigma_2 \)-fuzzy soft open set in \((Y, K, \sigma_1, \sigma_2)\) a \((1, 2)^*\)-fuzzy soft b-open set in \((X, E, \tau_1, \tau_2)\).

Definition 18. [31] A fuzzy soft mapping \((\varphi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)\) is said to be \((1, 2)^*\)-fuzzy soft b-irresolute mapping (briefly \((1, 2)^*\)-fsb-irresolute) \((\varphi, \psi)^{-1}(g_K)\) is \((1, 2)^*\)-fuzzy soft b-closed set in \((X, E, \tau_1, \tau_2)\) for every \((1, 2)^*\)-fuzzy soft b-closed set \( g_K \) in \((Y, K, \sigma_1, \sigma_2)\).

3. \((1, 2)^*\)-Fuzzy Soft b-Connectedness

In this section we introduce the concepts of \((1, 2)^*\)-fuzzy soft b-separated sets and \((1, 2)^*\)-fuzzy soft b-connectedness in fuzzy soft bitopological spaces. Also, some of the main results and properties are studied and discussed.

Definition 19. Two non-empty fuzzy soft subsets \( f_E, g_E \) of \((\tilde{X}, \tilde{E})\) are said to be fuzzy soft disjoint if \( f_E \cap g_E = \tilde{0}_E \).
Definition 20. Let \((X,E,\tau_1,\tau_2)\) be a fuzzy soft bitopological space. Two non-empty fuzzy soft disjoint fuzzy soft subsets \(f_E, g_E\) of \((X,E)\) are called
(i) \((1,2)^*\)-fuzzy soft separated sets over \(X\) if \(\tau_1\tau_2\operatorname{cl}(f_E)\setminus g_E = f_E\setminus \tau_1\tau_2\operatorname{cl}(g_E) = \mathring{0}_E\).
(ii) \((1,2)^*\)-fuzzy soft b-separated ((1,2)^*-fsb-separated) sets over \(X\) if
\((1,2)^*\)-fuzzy soft separated.

Remark 1. From the fact that \((1,2)^*\)-fsbcl \((f_E)\subseteq \tau_1\tau_2\operatorname{cl}(f_E)\), for every fuzzy soft subset \(f_E\) of \((X,E)\), every \((1,2)^*\)-fuzzy soft separated set is \((1,2)^*\)-fuzzy soft b-separated. But the converse may not be true.

Definition 21. A \((1,2)^*\)-fuzzy soft b-separation ((1,2)^*-fsb-separation) of a fuzzy soft bitopological space \((X,E,\tau_1,\tau_2)\) is a pair of \((1,2)^*\)-fuzzy soft b-separated sets \(f_E\) and \(g_E\) whose fuzzy soft union is absolute fuzzy soft set \(1_E\) (that is \(f_E\cap g_E = \mathring{1}_E\)).

Definition 22. Let \((X,E,\tau_1,\tau_2)\) be a fuzzy soft bitopological space. Then \((X,E,\tau_1,\tau_2)\) is called \((1,2)^*\)-fuzzy soft b-connected space if \(1_E\) can not be expressed as the fuzzy soft union of two \((1,2)^*\)-fuzzy soft b-separated sets.

Remark 2. In a fuzzy soft bitopological space \((X,E,\tau_1,\tau_2)\):
(i) A fuzzy soft empty set is trivially \((1,2)^*\)-fuzzy soft b-connected set.
(ii) Every fuzzy soft singleton set is \((1,2)^*\)-fuzzy soft b-connected, since it can not be expressed as a fuzzy soft union of two non-empty \((1,2)^*\)-fuzzy soft b-separated sets.

Theorem 1. Let \((X,E,\tau_1,\tau_2)\) be a fuzzy soft bitopological space. Then the following statements are equivalent:
(i) \((X,E,\tau_1,\tau_2)\) is a \((1,2)^*\)-fuzzy soft b-connected space.
(ii) \(1_E\) and \(\mathring{0}_E\) are the only \((1,2)^*\)-fuzzy soft b-clopen (that is, closed and open) sets in \((X,E,\tau_1,\tau_2)\).
(iii) \(1_E\) can not be expressed as the fuzzy soft union of two fuzzy soft disjoint non-empty \((1,2)^*\)-fuzzy soft b-clopen sets.
(iv) \(1_E\) can not be expressed as the fuzzy soft union of two fuzzy soft disjoint non-empty \((1,2)^*\)-fuzzy soft b-closed sets.

Proof. (i) \(\Rightarrow\) (ii): Let \((X,E,\tau_1,\tau_2)\) be a fuzzy soft bitopological space. Let \(f_E\) be non-empty proper fuzzy soft subset of \((X,E)\) that is \((1,2)^*\)-fuzzy soft b-clopen. Then \(1_E\setminus f_E\) is a non-empty \((1,2)^*\)-fuzzy soft b-clopen set and \(1_E = f_E \cup (1_E \setminus f_E)\). This is a contradiction to \((X,E,\tau_1,\tau_2)\) is a \((1,2)^*\)-fuzzy soft b-connected space. Therefore \(1_E\) and \(\mathring{0}_E\) are the only \((1,2)^*\)-fuzzy soft b-clopen sets in \((X,E,\tau_1,\tau_2)\).

(ii) \(\Rightarrow\) (iii): Assume that \(1_E\) and \(\mathring{0}_E\) are the only \((1,2)^*\)-fuzzy soft b-clopen sets in \((X,E,\tau_1,\tau_2)\). Suppose (iii) is false. Then \(1_E = f_E \cup g_E\) where \(f_E\) and \(g_E\) are fuzzy soft disjoint non-empty \((1,2)^*\)-fuzzy soft b-clopen sets. Then \(g_E = 1_E \setminus f_E\) is \((1,2)^*\)-fuzzy soft b-closed and non-empty. Thus \(g_E\) is a non-empty proper \((1,2)^*\)-fuzzy soft b-clopen set in \((X,E,\tau_1,\tau_2)\), which contradicts (ii).

(iii) \(\Rightarrow\) (iv): Assume \(1_E\) cannot be expressed as the fuzzy soft union of two fuzzy soft disjoint non-empty \((1,2)^*\)-fuzzy soft b-clopen sets. Suppose (iv) false. Then \((1,2)^*\)-fuzzy
soft $b$-closed sets. Then $f_E = \overline{I}_E \setminus g_E$ and $g_E = \overline{I}_E \setminus f_E$ are fuzzy soft disjoint non-empty $(1,2)^*\text{-fuzzy soft}$ b-open sets in $(X, E, \tau_1, \tau_2)$. Thus $\overline{I}_E$ is the fuzzy soft union of two fuzzy soft disjoint non-empty $(1,2)^*\text{-fuzzy soft}$ b-open sets. This contradicts $(iii)$.

$(iv) \Rightarrow (i)$: Suppose $(X, E, \tau_1, \tau_2)$ is not $(1,2)^*\text{-fuzzy soft}$ b-connected space. Then $\overline{I}_E = f_E \cup g_E$ where $f_E$ and $g_E$ are fuzzy soft disjoint non-empty $(1,2)^*\text{-fuzzy soft}$ b-open sets. Then $f_E = \overline{I}_E \setminus g_E$ and $g_E = \overline{I}_E \setminus f_E$ are fuzzy soft disjoint non-empty $(1,2)^*\text{-fuzzy soft}$ b-closed sets in $(X, E, \tau_1, \tau_2)$. This is a contradiction to $(iv)$.

**Proposition 1.** Every $(1,2)^*\text{-fuzzy soft}$ b-connected space is $(1,2)^*\text{-fuzzy soft}$ connected.

**Proof.** Let $f_E$ be a $(1,2)^*\text{-fuzzy soft}$ b-connected set in the fuzzy soft bitopological space $(X, E, \tau_1, \tau_2)$. Then there does not exist a $(1,2)^*\text{-fuzzy soft b-separation}$ of $f_E$. Since every $\tau_1\tau_2$-fuzzy soft open set is a $(1,2)^*\text{-fuzzy soft b-open set}$, there does not exist a $(1,2)^*\text{-fuzzy soft separation}$ of $f_E$. Hence $f_E$ is a $(1,2)^*\text{-fuzzy soft}$ connected set in the fuzzy soft bitopological space $(X, E, \tau_1, \tau_2)$.

The converse is not true as shown in the following example.

**Example 1.** $(1,2)^*\text{-fuzzy soft}$ connectedness does not imply $(1,2)^*\text{-fuzzy soft b-connectedness}$. Let $(X, E, \tau_1, \tau_2)$ be a fuzzy soft bitopological space, where $X = \{ x, y \}, E = \{ e_1, e_2 \}$ and let $\tau_1 = \{ \overline{0}_E, \overline{1}_E, f_1^E, f_2^E, f_3^E \}, \tau_2 = \{ \overline{0}_E, \overline{1}_E, g_1^E, g_2^E \}$, where $f_1^E = \{ f_1(e_1) = \{ x/0.2, y/0.0 \}, f_1(e_2) = \{ x/0.0, y/0.0 \} = \emptyset \}$, $f_2^E = \{ f_2(e_1) = \{ x/0.0, y/0.0 \} \}, f_3^E = \{ f_3(e_1) = \{ x/0.2, y/0.1 \}, f_3(e_2) = \{ x/0.7, y/0.0 \} \}$, $g_1^E = \{ g_1(e_1) = \{ x/0.0, y/0.0 \} = \emptyset, g_1(e_2) = \{ x/0.7, y/0.0 \} \}$ and $g_2^E = \{ g_2(e_1) = \{ x/0.0, y/0.0 \} \}$. Then $\tau_1\tau_2$-fuzzy soft open sets are $\{ \overline{0}_E, \overline{1}_E, f_1^E, f_2^E, f_3^E, g_1^E, g_2^E \}$ and $\tau_1\tau_2$-fuzzy soft closed sets are $\{ \overline{0}_E, \overline{1}_E, f_1^E, f_2^E, f_3^E, g_1^E, g_2^E \}$ where $f_1^E = \{ f_1(e_1) = \{ x/0.0, y/0.1 \}, f_1(e_2) = \{ x/0.7, y/0.4 \} \}$, $f_2^E = \{ f_2(e_1) = \{ x/0.0, y/0.1 \}, f_2(e_2) = \{ x/0.0, y/0.4 \} \}$, $f_3^E = \{ f_3(e_1) = \{ x/0.0, y/0.0 \} \}$. It is clear that $(X, E, \tau_1, \tau_2)$ is $(1,2)^*\text{-fuzzy soft}$ connected since the only $(1,2)^*\text{-fuzzy soft}$ clopen sets are $\overline{0}_E, \overline{1}_E$. Also $(1,2)^*\text{-fuzzy soft b-open sets}$ are $\{ \overline{0}_E, \overline{1}_E, f_1^E, f_2^E, f_3^E, g_1^E, g_2^E, g_3^E, g_4^E \}$, where $f_1^E, f_2^E, f_3^E, g_1^E$ and $g_2^E$ are defined as above and $f_4^E = \{ f_4(e_1) = \{ x/0.0, y/0.1 \}, f_4(e_2) = \{ x/0.7, y/0.4 \} \}$. And $(1,2)^*\text{-fuzzy soft b-closed sets}$ are $\{ \overline{0}_E, \overline{1}_E, f_1^E, f_2^E, f_3^E, f_4^E, f_5^E, g_1^E, g_2^E, g_3^E, g_4^E \}$, where $f_1^E, f_2^E, f_3^E, f_4^E$ and $f_5^E$ are obtained as above and $f_6^E = \{ f_6(e_1) = \{ x/0.2, y/0.0 \}, f_6(e_2) = \{ x/0.0, y/0.0 \} \}$. 


Let $f_E^c = \{f^c(1) = \{x/0.2, y/0.0\}, f^c(2) = \{x/0.7, y/0.0\}\},
g_E^c = \{g^c(1) = \{x/0.0, y/0.0\} \cup g^c(2) = \{x/0.7, y/0.4\}\},\text{ and }
g_E^g = \{g^g(1) = \{x/0.0, y/0.1\}, g^g(2) = \{x/0.7, y/0.4\}\}.

where $1_E = f_{1_E} \cup f_{2_E},$ then $(1, 2)^*-fsbcl(f_{1_E}) = f_E^{c*}$, $(1, 2)^*-fsbcl(f_{2_E}) = g_E^{c*}$, and $(1, 2)^*-fsbcl(f_{1_E}) \cap f_{2_E} = 0_E$. Hence $1_E$ can be expressed as a fuzzy soft union of two $(1, 2)^*-fuzzy soft b-separated sets $f_{1_E}, f_{2_E}$. There $(X, E, \tau_1, \tau_2)$ is not $(1, 2)^*-fuzzy soft b-connected.

**Example 2.** $(1, 2)^*-fuzzy soft b-connectivity is not hereditary property.

Consider the fuzzy soft bitopological space $(X, E, \tau_1, \tau_2)$, where $X = \{x, y\}, E = \{e_1, e_2\}$ and let $\tau_1 = \{0_E, 1_E, f_{1_E}, f_{2_E}\}, \tau_2 = \{0_E, 1_E, g_{1_E}\},$ where $f_{1_E} = \{f_1(e_1) = \{x/0.2, y/0.0\}, f_1(e_2) = \{x/0.0, y/0.0\} = \emptyset\},$ $f_{2_E} = \{f_2(e_1) = \{x/0.2, y/0.0\}, f_2(e_2) = \{x/0.7, y/0.0\}\}$ and $g_{1_E} = \{g_1(e_1) = \{x/0.2, y/0.1\}, g_1(e_2) = \{x/0.0, y/0.0\} = \emptyset\}$.

Then $\tau_{12}$-fuzzy soft open sets are $\{0_E, 1_E, f_{1_E}, f_{2_E}, g_{1_E}, h_{1_E}\},$ where $h_{1_E} = \{h_1(e_1) = \{x/0.2, y/0.0\}, h_1(e_2) = \{x/0.7, y/0.0\} = \emptyset\}$.

Also, $(1, 2)^*-fuzzy soft b-open sets are $\{0_E, 1_E, f_{1_E}, f_{2_E}, g_{1_E}, h_{1_E}, h_{2_E}, h_{3_E}, h_{4_E}\},$ where $h_{2_E} = \{h_2(e_1) = \{x/0.2, y/0.0\}, h_2(e_2) = \{x/0.0, y/0.4\}\},$ $h_{3_E} = \{h_3(e_1) = \{x/0.2, y/0.0\}, h_3(e_2) = \{x/0.7, y/0.0\}\}$ and $h_{3_E} = \{h_3(e_1) = \{x/0.0, y/0.0\}, h_3(e_2) = \{x/0.7, y/0.0\}\}$. Then $(1, 2)^*-fuzzy soft b-closed sets are $\{0_E, 1_E, f_{1_E}, f_{2_E}, g_{1_E}, h_{1_E}, h_{2_E}, h_{3_E}, h_{4_E}\},$ where $f_{1_E} = \{f_1(e_1) = \{x/0.0, y/0.1\}, f_1(e_2) = \{x/0.7, y/0.4\}\},$ $f_{2_E} = \{f_2(e_1) = \{x/0.0, y/0.1\}, f_2(e_2) = \{x/0.0, y/0.4\}\},$ $g_{1_E} = \{g_1(e_1) = \{x/0.0, y/0.0\} = \emptyset, g_1(e_2) = \{x/0.7, y/0.4\}\},$ $h_{1_E} = \{h_1(e_1) = \{x/0.0, y/0.0\} = \emptyset, h_1(e_2) = \{x/0.0, y/0.4\}\},$ $h_{2_E} = \{h_2(e_1) = \{x/0.0, y/0.0\}, h_2(e_2) = \{x/0.7, y/0.0\}\},$ $h_{3_E} = \{h_3(e_1) = \{x/0.0, y/0.1\}, h_3(e_2) = \{x/0.0, y/0.0\} = \emptyset\}$ and $h_{4_E} = \{h_4(e_1) = \{x/0.0, y/0.1\} = \emptyset, h_4(e_2) = \{x/0.0, y/0.4\}\}.

It is clear that $(X, E, \tau_1, \tau_2)$ is $(1, 2)^*-fuzzy soft b-connected, since the only $(1, 2)^*-fuzzy soft clopen sets are $0_E$ and $1_E$. Let $Y = \{x\} \subseteq X \text{ and } E = \{e_1, e_2\}$. Let $\sigma_1 = \{0_E, 1_E, f_{1_E}\}, \sigma_2 = \{0_E, 1_E, h_{1_E}\}$. Then $\sigma_1 \sigma_2$-fuzzy soft open sets are $\{0_E, 1_E, f_{1_E}, h_{1_E}\}$. Also $(1, 2)^*-fuzzy soft b-clopen sets are $\{0_E, 1_E, f_{1_E}, h_{1_E}\}$. Clearly $(Y, E, \sigma_1, \sigma_2)$ is not $(1, 2)^*-fuzzy soft b-connected; since $f_{1_E}$ and $h_{1_E}$ are two $(1, 2)^*-fuzzy soft b-clopen sets other than $0_E$ and $1_E$.

**Proposition 2.** Let $f_E$ be a $(1, 2)^*-fuzzy soft b-connected set, $g_E$ and $h_E$ are $(1, 2)^*-fuzzy soft b-separated sets. If $f_E \subseteq g_E$ or $f_E \subseteq h_E$ then either $f_E \subseteq g_E$ or $f_E \subseteq h_E$.

**Proof.** Let $f_E$ be a $(1, 2)^*-fuzzy soft b-connected set, $g_E$ and $h_E$ are $(1, 2)^*-fuzzy soft b-separated sets such that $f_E \subseteq g_E \cup h_E$. Let $f_E \subseteq g_E$ and $f_E \subseteq h_E$. Suppose $k_E = g_E \cap f_E \neq 0_E$ and $l_E = h_E \cap f_E \neq 0_E$ then $f_E = k_E \cup l_E$.

Since $k_E \subseteq g_E$, $(1, 2)^*-fsbcl(k_E) \subseteq (1, 2)^*-fsbcl(g_E)$. Also $(1, 2)^*-fsbcl(g_E) \cap h_E = 0_E$ then $(1, 2)^*-fsbcl(k_E) \cap h_E = 0_E$. Since $l_E \subseteq h_E$, $(1, 2)^*-fsbcl(l_E) \subseteq (1, 2)^*-fsbcl(h_E)$. Also $(1, 2)^*-fsbcl(h_E) \cap g_E = 0_E$ then $(1, 2)^*-fsbcl(l_E) \cap k_E = 0_E$. But $f_E = k_E \cup l_E$. 
therefore $f_E$ is not $(1,2)^*$-fuzzy soft $b$-connected set which is not a contradiction. Then either $f_E \subseteq g_E$ or $f_E \subseteq h_E$.

**Theorem 2.** If $f_E$ is a $(1,2)^*$-fuzzy soft $b$-connected set and $f_E \subseteq g_E \subseteq ((1,2)^*-fsbcl(f_E))$ then $g_E$ is a $(1,2)^*$-fuzzy soft $b$-connected.

**Proof.** Suppose $g_E$ is not $(1,2)^*$-fuzzy soft $b$-connected then there exists two non-empty fuzzy soft sets $f_{1E}$ and $f_{2E}$ such that $((1,2)^*-fsbcl(f_{1E})) \cap f_{2E} = f_{1E} \cap ((1,2)^*-fsbcl(f_{2E})) = \emptyset_E$ and $f_E = f_{1E} \cup f_{2E}$. Since $f_E \subseteq g_E$ then either $f_E \subseteq f_{1E}$ or $f_E \subseteq f_{2E}$.

Suppose $f_E \subseteq f_{1E}$, then $((1,2)^*-fsbcl(f_E)) \subseteq ((1,2)^*-fsbcl(f_{1E}))$, thus $((1,2)^*-fsbcl(f_E)) \cap f_{2E} = f_E \cap ((1,2)^*-fsbcl(f_{2E})) = \emptyset_E$. But $f_{2E} \subseteq g_E \subseteq ((1,2)^*-fsbcl(f_E))$, thus $((1,2)^*-fsbcl(f_E)) \cap f_{2E} = f_{2E}$. Therefore $f_{2E} = \emptyset_E$, which is a contradiction.

If $f_E \subseteq f_{2E}$, then by the same way we can prove that $f_{1E} = \emptyset_E$. This is a contradiction. Thus $g_E$ be a $(1,2)^*$-fuzzy soft $b$-connected.

**Theorem 3.** If $f_E$ is a $(1,2)^*$-fuzzy soft $b$-connected set, then $(1,2)^*-fsbcl(f_E)$ is $(1,2)^*$-fuzzy soft $b$-connected.

**Proof.** Suppose $f_E$ is $(1,2)^*$-fuzzy soft $b$-connected and $(1,2)^*-fsbcl(f_E)$ is not $(1,2)^*$-fuzzy soft $b$-connected. Then there exist two $(1,2)^*$-fuzzy soft $b$-separated sets $f_{1E}$ and $f_{2E}$ such that $(1,2)^*-fsbcl(f_1E) = f_{1E} \cup f_{2E}$. But $f_E \subseteq (1,2)^*-fsbcl(f_E)$ then $f_E = f_{1E} \cup f_{2E}$ and since $f_E$ is $(1,2)^*$-fuzzy soft $b$-connected set, then either $f_E \subseteq f_{1E}$ or $f_E \subseteq f_{2E}$.

If $f_E \subseteq f_{1E}$ then $(1,2)^*-fsbcl(f_E) \subseteq (1,2)^*-fsbcl(f_{1E})$. But $(1,2)^*-fsbcl(f_{1E}) \cap f_{2E} = \emptyset_E$, hence $(1,2)^*-fsbcl(f_E) \cap f_{2E} = \emptyset_E$. Since $f_{2E} \subseteq (1,2)^*-fsbcl(f_E)$, then $(1,2)^*-fsbcl(f_E) \cap f_{2E} = f_{2E}$, hence $f_{2E} = \emptyset_E$ which is a contradiction.

If $f_E \subseteq f_{2E}$, then by the same way we can prove that $f_{1E} = \emptyset_E$, which is a contradiction. Therefore $(1,2)^*-fsbcl(f_E)$ is $(1,2)^*$-fuzzy soft $b$-connected.

**Theorem 4.** The fuzzy soft union $f_E$ of any family $\{f_iE : i \in I\}$ of $(1,2)^*$-fuzzy soft $b$-connected sets having a non-empty fuzzy soft intersection is $(1,2)^*$-fuzzy soft $b$-connected.

**Proof.** Let $f_E$ be fuzzy soft union of any family of $(1,2)^*$-fuzzy soft $b$-connected sets having a non-empty fuzzy soft intersection. Suppose that $f_E = f_{1E} \cup f_{2E}$, where $f_{1E}$ and $f_{2E}$ form a $(1,2)^*$-fuzzy soft $b$-separation of $f_E$. By hypothesis, we may choose a fuzzy soft point $f_\in\cap_{i \in I} f_iE$. Then $f_\in \in f_{iE}$ for all $i \in I$. If $f_\in \in f_2E$, then either $f_\in \in f_{1E}$ or $f_\in \in f_{2E}$ but not both. Since $f_{1E}$ and $f_{2E}$ are fuzzy soft disjoint, we must have $f_{iE} \subseteq f_{1E}$, since $f_{iE}$ is $(1,2)^*$-fuzzy soft $b$-connected and it is true for all $i \in I$, and so $f_\in \subseteq f_{1E}$. From this we obtain that $f_{2E} = \emptyset_E$; which is a contradiction. Thus, there does not exist a $(1,2)^*$-fuzzy soft $b$-separation of $f_E$. Therefore, $f_E$ is a $(1,2)^*$-fuzzy soft $b$-connected set.

**Theorem 5.** (i) If $\psi : (X, E, \tau_1, \tau_2) \rightarrow (Y, E, \sigma_1, \sigma_2)$ is a $(1,2)^*$-fuzzy soft $b$-continuous surjection and $(X, E, \tau_1, \tau_2)$ is $(1,2)^*$-fuzzy soft $b$-connected then $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft connected.

(ii) If $\psi : (X, E, \tau_1, \tau_2) \rightarrow (Y, E, \sigma_1, \sigma_2)$ is a $(1,2)^*$-fuzzy soft $b$-irresolute surjection and $(X, E, \tau_1, \tau_2)$ is $(1,2)^*$-fuzzy soft $b$-connected then $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft $b$-connected.
Proof. (i) Suppose \((Y, E, \sigma_1, \sigma_2)\) is not \((1, 2)^*\)-fuzzy soft connected. Let \(Y = f_E \hat{\cup} g_E\), where \(f_E\) and \(g_E\) are fuzzy soft disjoint non-empty \(\sigma_1\sigma_2\)-fuzzy soft open sets in \((Y, E, \sigma_1, \sigma_2)\). Since \(\psi\) is a \((1, 2)^*\)-fuzzy soft b-continuous and onto; \(g_E = \psi^{-1}(f_E) \cup \psi^{-1}(g_E)\), where \(\psi^{-1}(f_E)\) and \(\psi^{-1}(g_E)\) are fuzzy soft disjoint non-empty \((1, 2)^*\)-fuzzy soft open sets in \((X, E, \tau_1, \tau_2)\). This contradicts the fact that \((X, E, \tau_1, \tau_2)\) is \((1, 2)^*\)-fuzzy soft b-connected. Hence \((Y, E, \sigma_1, \sigma_2)\) is \((1, 2)^*\)-fuzzy soft connected.

(ii) Suppose \((Y, E, \sigma_1, \sigma_2)\) is not \((1, 2)^*\)-fuzzy soft b-connected. Let \(Y = f_E \hat{\cup} g_E\), where \(f_E\) and \(g_E\) are fuzzy soft disjoint non-empty \((1, 2)^*\)-fuzzy soft b-open sets in \((Y, E, \sigma_1, \sigma_2)\). Since \(\psi\) is \((1, 2)^*\)-fuzzy soft b-irresolute and onto; then \(g_E = \psi^{-1}(f_E) \cup \psi^{-1}(g_E)\), where \(\psi^{-1}(f_E)\) and \(\psi^{-1}(g_E)\) are fuzzy soft disjoint non-empty \((1, 2)^*\)-fuzzy soft b-open sets in \((X, E, \tau_1, \tau_2)\). This contradicts the fact that \((X, E, \tau_1, \tau_2)\) is \((1, 2)^*\)-fuzzy soft b-connected. Hence \((Y, E, \sigma_1, \sigma_2)\) is \((1, 2)^*\)-fuzzy soft b-connected.

4. \((1, 2)^*\)-Fuzzy Soft b-Compactness

In this section \((1, 2)^*\)-fuzzy soft b-compactness is defined and some of the characterizations are proved.

Definition 23. A collection \(\{f_{iE} : i \in \Lambda\}\) of \((1, 2)^*\)-fuzzy soft b-open sets in fuzzy soft bitopological space \((X, E, \tau_1, \tau_2)\) is called a \((1, 2)^*\)-fuzzy soft b-cover of \(f_E\) if \(f_E \hat{\subseteq} \bigcup\{f_{iE} : i \in \Lambda\}\).

Definition 24. A fuzzy soft bitopological space \((X, E, \tau_1, \tau_2)\) is called a \((1, 2)^*\)-fuzzy soft b-compact if every \((1, 2)^*\)-fuzzy soft b-cover of \(1_E\) has a finite subcover.

Definition 25. A fuzzy soft subset \(f_E\) of fuzzy soft bitopological space \((X, E, \tau_1, \tau_2)\) is said to be \((1, 2)^*\)-fuzzy soft b-compact if for every collection \(\{f_{iE} : i \in \Lambda\}\) of \((1, 2)^*\)-fuzzy soft b-open subsets of \((X, E, \tau_1, \tau_2)\) such that \(f_E \subseteq \bigcup\{f_{iE} : i \in \Lambda\}\) there exists a finite subset \(\Lambda_0\) of \(\Lambda\) such that \(f_E \subseteq \bigcup\{f_{iE} : i \in \Lambda_0\}\).

Definition 26. A fuzzy soft subset \(f_E\) of fuzzy soft bitopological space \((X, E, \tau_1, \tau_2)\) is said to be \((1, 2)^*\)-fuzzy soft b-compact if \(f_E\) is \((1, 2)^*\)-fuzzy soft b-compact as a subspace of \((X, E, \tau_1, \tau_2)\).

Theorem 6. Every \((1, 2)^*\)-fuzzy soft closed subset of fuzzy \((1, 2)^*\)-fuzzy soft b-compact space \((X, E, \tau_1, \tau_2)\) is \((1, 2)^*\)-fuzzy soft b-compact relative to \(1_E\).

Proof. Let \(f_E\) be a \((1, 2)^*\)-fuzzy soft closed subset of \((X, E, \tau_1, \tau_2)\). Then \(f_E\) is a \((1, 2)^*\)-fuzzy soft open set in \((X, E, \tau_1, \tau_2)\). Let \(S = \{g_{iE} : i \in \Lambda\}\) be a cover of \(f_E\) by \((1, 2)^*\)-fuzzy soft open subsets in \((X, E, \tau_1, \tau_2)\). Then \(\bigcup f_{iE}\) is a \((1, 2)^*\)-fuzzy soft b-open cover for \(1_E\). Since \((X, E, \tau_1, \tau_2)\) is a \((1, 2)^*\)-fuzzy soft b-compact; it has a finite subcover say \(S = g_{1E} \cup g_{1E} \cup \ldots \cup g_{nE} \cup f_E, g_{iE} \in S, i = 1, 2, \ldots, n\). But \(f_E\) and \(f_E\) are fuzzy soft disjoint. Hence \(f_E \subseteq g_{1E} \cup g_{2E} \cup \ldots \cup g_{nE} \subseteq S\). Thus we have shown that any \((1, 2)^*\)-fuzzy soft b-open cover has a finite subcover. Therefore \(f_E\) is \((1, 2)^*\)-fuzzy soft b-compact relative to \(1_E\).
Theorem 7. A $(1,2)^*\text{-fuzzy soft}$ $b$-continuous image of a $(1,2)^*\text{-fuzzy soft}$ $b$-compact space is $(1,2)^*\text{-fuzzy soft}$ compact.

Proof. Consider $\psi : (X, E, \tau_1, \tau_2) \to (Y, E, \sigma_1, \sigma_2)$ be a $(1,2)^*\text{-fuzzy soft}$ $b$-continuous function. Let $\{f_{i_E} : i \in \Lambda\}$ be a $\sigma_1\sigma_2$-fuzzy soft open cover of $1_E$ in $(Y, E, \sigma_1, \sigma_2)$. Then $\{\psi^{-1}(f_{i_E}) : i \in \Lambda\}$ is a $(1,2)^*\text{-fuzzy soft}$ $b$-open cover of $1_E$ in $(X, E, \tau_1, \tau_2)$. Since $(X, E, \tau_1, \tau_2)$ is $(1,2)^*\text{-fuzzy soft}$ $b$-compact; it has a finite subcover say, $\{\psi^{-1}(f_{1_E}), \psi^{-1}(f_{1_E}), ..., \psi^{-1}(f_{n_E})\}$. Since $\psi$ is onto, $\{f_{1_E}, f_{1_E}, ..., f_{n_E}\}$ is a $\sigma_1\sigma_2$-fuzzy soft open cover of $1_E$ in $(Y, E, \sigma_1, \sigma_2)$ and hence $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*\text{-fuzzy soft}$ compact.

Theorem 8. If a map $\psi : (X, E, \tau_1, \tau_2) \to (Y, E, \sigma_1, \sigma_2)$ is a $(1,2)^*\text{-fuzzy soft}$ $b$-irresolute and a fuzzy soft subset $f_E$ of $(X, E, \tau_1, \tau_2)$ is $(1,2)^*\text{-fuzzy soft}$ compact relative to $1_E$ then the image $\psi(f_E)$ is $(1,2)^*\text{-fuzzy soft}$ compact relative to $1_E$ in $(Y, E, \sigma_1, \sigma_2)$.

Proof. Let $\{f_{i_E} : i \in \Lambda\}$ be a collection of $(1,2)^*\text{-fuzzy soft}$ $b$-open sets in $(Y, E, \sigma_1, \sigma_2)$ such that $\psi(f_E) \subseteq \bigcup\{f_{i_E} : i \in \Lambda\}$. Then $f_E \subseteq \bigcup\{\psi^{-1}(f_{i_E}) : i \in \Lambda\}$, where $\psi^{-1}(f_{i_E})$ is $(1,2)^*\text{-fuzzy soft}$ $b$-open in $(X, E, \tau_1, \tau_2)$ is $(1,2)^*\text{-fuzzy soft}$ compact relative to $1_E$ in $(X, E, \tau_1, \tau_2)$, there exists a finite sub collection $\{f_{1_E}, f_{2_E}, ..., f_{n_E}\}$ such that $f_E \subseteq \bigcup\{\psi^{-1}(f_{i_E}) : i = 1, 2, 3, ..., n\}$ that is, $\psi(f_E) \subseteq \bigcup\{f_{i_E} : i = 1, 2, 3, ..., n\}$. Hence $\psi(f_E)$ is $(1,2)^*\text{-fuzzy soft}$ compact relative to $1_E$ in $(Y, E, \sigma_1, \sigma_2)$.

5. Conclusion

In this paper, we introduced the notions of $(1,2)^*\text{-fuzzy soft}$ $b$-separated sets, $(1,2)^*\text{-fuzzy soft}$ $b$-connectedness and $(1,2)^*\text{-fuzzy soft}$ $b$-compactness in fuzzy soft bitopological spaces. Then, some basic topological properties of these notions were investigated. Also, some illustrative examples were given to show the importance of the obtained theorems. We hope that this paper will be important for researchers to studying many other concepts and also the generalization for some important results in topology.

Acknowledgements

The author is very grateful to the editor and the reviewers for their valuable suggestions.

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