Delamination assessment of composite curved angles using simplified FEA models build-up by 2-D layered shell elements

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Abstract. As widely known, an accurate analysis of the stress state across the thickness of a composite laminate requires the calculation of the interlaminar through-the-thickness stresses as well as the shear stress components at the layers’ interfaces. These so-called out-of-plane stresses are the key parameters when defining the delamination strength of curved composite laminates, particularly when their radius of curvature and the thickness of the composite are of the same size order. Since the use of layered solid elements is far from a reasonable approach in practical design applications, based on the finite element analysis results obtained through the use of conventional layered shell elements, the purpose of this paper is to propose a simplified post processing approach that enable to predict the interlaminar stress components originated across the thickness of symmetrically balanced curved composite laminates under the combined action of axial forces, shear forces and bending moments applied in curvature plane. A benchmark analysis to provide a critical comparison between a reference finite element model constructed based on 3-D layered solid elements and a simplified model build-up by 2-D layered shell elements is also considered.

1. Introduction

The use of fiber-reinforced composite laminates across a variety of industrial fields is becoming increasingly common due to their weight saving potential with respect to conventional metallic materials. On the other hand, the fibres lying in the plane of a laminate do not provide reinforcement through-the-thickness direction and consequently this leads to interlaminar failures which significantly reduce the overall strength and integrity of composite structures. As well-known, one of the primary failure modes experienced by the fiber-reinforced composite laminates is the delamination induced due to interlaminar stresses caused by inherent factors such as the free-edge effects, structural discontinuities, impact damage by foreign objects, stiffness mismatch between adjacent layers of different orientation and in particular, curved-shape geometries when folding-unfolding under the applied external loads.

Besides the stress concentration effects near the laminate free edges as well as the impact damage failure mechanisms which have been extensively studied so far by analytical, experimental or numerical methods, few works were focused on the laminate curvature effects especially for highly curved composite laminates relative to their thickness.

When curved composite laminates are subjected to outward bending by unfolding, due to the combined effects of bending, axial and shear forces acting in plane of curvature the inner layers experience tension generating circumferential tensile stresses, whereas the outer layers undergo circumferential compression stresses. Otherwise, folding by inward bending generate compression of inner layers while tensile theouters. In addition, the radial stresses developed due to the internal bending
moment tend to separate the interfaces while the shear stresses caused by the out-of-plane force components tend to shear apart the layers.

For design purposes, an efficient procedure to accurately predict the stress state across the thickness of a specific composite laminate requires the calculation of interlaminar through-the-thickness stresses as well as the shear stress components at the interfaces. These so-called out-of-plane stresses are the key parameters in failure analysis of curved composite laminates, particularly when their radius of curvature approaches the stacking thickness.

As concern the various strategies available at the moment to conduct the finite element analyses of highly curved composite laminates loaded in the curvature plane, there are actually two major complications that arise when trying to predict the interlaminar stresses. The first is that even the three-dimensional models built by layered solid elements allow a complete extraction of the interlaminar stress components; in general their practical use is far from a reasonable approach as it requires long computational time as well as the necessity to perform careful and well-controlled discretization in order to avoid the mesh dependencies of results. In fact, these are the main reasons why the finite element models used for structural stress analysis of lightweight composite structures are basically build based on layered shell elements in different depth of discretization. Nevertheless, the second issue is related to the limitations provided through the theoretical definition of layered shell elements that make them impracticable for the analysis of delamination failure of fiber-reinforced composite laminates. These are provided by the main assumptions of the classical laminate theory as it is built into the layered shell elements. Therefore, essentially, the structural response of the entire composite laminate is based on the definition of each individual lamina assuming the out-of-plane normal stress and the out-of-plane shear strains are vanishing so that, as mentioned above, the interlaminar stresses and the associated failures cannot be predicted by means of classical laminate theory. Moreover, as the curvature effects are not considered at all, when the ratio between the radius of curvature and the composite thickness tends to unity, the layered shell elements are becoming inadequate for predicting in-plane curvature stresses about circumferential direction.

Several computational models are available to calculate the closed-form analytical solutions for the out-of-plane stress components developing over the cross sections of curved layered composite plates or beams under plane strain or plane stress assumptions. However, their use as given in the original analytical form is quite difficult and often limited due to various geometries of parts involved in as well as some complex loading conditions. Lekhnitskii [1] has developed an analytical solution based on the theory of elasticity, for orthotropic curved beams loaded by concentrated shear forces and moments at the free ends. The results were extended by Ko and Jackson [2] to a multi-layered theory proposed for semi-circular laminted composite beams subjected to concentrated forces and moments applied at the free ends. O’Peck [3] derived closed-form two-dimensional solutions applicable both to sandwich composites as well as to fibre-reinforced laminates, to determine the displacements, strains and stress fields developed across the section of curved laminated orthotropic beams subjected to shear loads, moments and temperature changes. Martin [4] has investigated the delamination failure in unidirectional curved laminates. Closed form elasticity solutions and 2-D finite element analyses were conducted to locate the critical values of radial stress over the curvature region. The variation of the strain energy release rate with the delamination growth was then determined using the finite element analysis. Gonzalez-Cantero et al. [5], [6], developed a complete analytical model to determine the radial, circumferential and shear stress components developed over the cross section of curved laminate composites subjected to simple bending in the curvature plane as well as surface pressures, assuming that each ply is composed by a number of fictitious laminas. In such a way the problem was simplified supposing a beam model in each lamina and thus the equilibrium and compatibility equations were solved taking the limit when the number of fictitious laminae is tending to infinity. The model does not consider the shear deformations as the displacements of every lamina are approximated by the displacements of a homogeneous equivalent material. Some studies in the literature have focused on the development of special layered finite elements [7], [8], or other numerical calculation techniques [9], [10], to predict the interlaminar failure of highly curved composite laminate structural parts. However, such methods are less efficient in the preliminary design phases when certain reliable, simplified and
conservative analytical approaches in combination with some basic finite element analysis results are more effective.

Based on the finite element analysis results obtained through the use of conventional layered shell elements, the purpose of this paper is to propose a simplified post processing approach that enable to predict the interlaminar stress components originated across the thickness of symmetrically balanced curved composite laminates under the combined action of axial forces, shear forces and bending moments applied in curvature plane. A benchmark analysis to provide a critical comparison between a reference finite element model constructed based on 3-D layered solid elements and a simplified model build-up by 2-D layered shell elements is also considered.

2. Analysis

The numerical analyses were conducted on laminated curved angles made of a polyphenylene sulfide (PPS) thermoplastic resin reinforced with T300 JB continuous carbon fibers. The layout is virtually obtained by stacking a number of 13 plies of 0/90° bidirectional 5-harness satin fabric semi-impregnated with PPS polymer, following the sequence [(0,90)/(±45)6/(0,90)], which results in a 4 mm-thick laminate. The elastic properties of PPS lamina that were considered in the finite element analyses are reported in Table 1.

|   | $E_1$  | $E_2$  | $E_3$  | $v_{12}$  | $v_{13}$  | $v_{23}$  | $G_{12}$  | $G_{13}$ | $G_{23}$ |
|---|-------|-------|-------|-----------|-----------|-----------|-----------|---------|---------|
| MPa | 55400 | 55400 | 8500  | 0.05      | 0.35      | 0.35      | 3510      | 3510    | 3510    |

The laminate angle under analysis is provided with a ratio between the radius of curvature and the composite thickness, $R/h$ equal to 2. It has to be noted that this radius of curvature, denoted by $R$, refers to the centroidal axis that defines the middle plane of the laminate cross section.

The use of consistent element coordinate systems to define the material orientation in case of layered solid elements models enables a coherent definition of the laminate layout. This is particularly important when curved laminate angles are involved in, since the accuracy of the results can be significantly
affected due to the anisotropic properties, in particular between in-plane as well as through-thickness direction. However, such a task requires an accurate control of meshing procedure.

A side view containing an isometric detail of curvature area for the layered shell finite element model is presented in Figure 2. In this regard it has to be mention that both for the simplified model as well as for the reference model, the loading is virtually represented by two unit forces \( F_1 = 1kN; F_2 = 1kN \) and a unit couple \( M_3 = 1kNm \), applied on the free side of the angle in the curvature plane, with respect to the global reference coordinate system. Furthermore, as shown in Figure 1 and Figure 2, for both finite element models, the clamped boundary conditions at the bolt holes as well as the unit loads are applied by means of RBE2 elements.

As already explained in the introductory section, since the layered shell elements are inadequate for predicting the folding-unfolding interlaminar stresses, a simplified post-processing model based on shell forces and shell moments extracted at the elements located on the baseline of the curvature region is proposed. Such a model can be used to estimate the strength of highly curved quasi-isotropic laminates provided their symmetrically balanced layout configuration, as is the present case study of 0/90° bidirectional polyphenylene sulphide reinforced with continuous carbon fibers. Therefore, the overall equivalent elastic properties of the laminate may be considered in the analysis, as given in Table 2.

**Table 2.** The equivalent elastic properties of laminate.

| \( E_x \)  | \( E_y \)  | \( E_z \) | \( v_{xy} \) | \( v_{yz} \) | \( G_{xz} \) | \( G_{xy} \) | \( G_{yz} \) |
|---------|---------|---------|-----------|-----------|----------|----------|----------|
| [MPa]   | [MPa]   | [MPa]   | [-]       | [-]       | [MPa]    | [MPa]    | [MPa]    |
| 41031   | 41031   | 8500    | 0.30      | 0.26      | 0.35     | 14066    | 3510     | 3510     |

Assuming a planar stress state, the radial stress component \( \sigma_r \) and the shear stress component \( \tau_{r\theta} \) may be derived based on the general curved beam expressions:

\[
\sigma_r (\theta) = \frac{s(\theta)}{h};
\]

\[
\tau_{r\theta}(\theta) = \frac{3}{2h} \frac{m(\theta)}{r_i r_o};
\]

where \( s(\theta) \) is the flux of shear force (force per unit length, units [N/mm]) while \( m(\theta) \) is the moment flux (bending moment per unit length, units [Nmm/mm]). In the above expressions the thickness and the width of the laminate are denoted by \( h \) and \( b \) while \( r_i \) and \( r_o \) indicate the inner radius and the outer radius respectively.
As Figure 3 shows, the curvature fluxes \( s(\theta) \) and \( m(\theta) \) can be easily derived through the translation of the element fluxes \( s_i, n_i \) and \( m_i \), extracted at the baseline of the shell elements located on the ends of curvature. Therefore, the following transformations may apply:

\[
\begin{align*}
  s(\theta) &= s_i \cos(\theta) - n_i \sin(\theta) \\
  m(\theta) &= m_i + s_i R \sin(\theta) - n_i [R - R \cos(\theta)]
\end{align*}
\]

(2)

where \( R \) is the radius of centroidal axis of laminate:

\[
R = r_i + \frac{h}{2}.
\]

(3)

Within the above context it is important to note that the definition of the coordinate system is taken so that the positive \( s_i \) and \( m_i \) determine the angle opening (unfolding) while positive \( n_i \) causes angle closing (folding).

Figure 3. Data processing scheme based on the extracted fluxes.

Neglecting the contribution of in-plane circumferential stresses, according to Ref. [11] the quadratic failure criterion proposed by Brewer and Lagace may be used to predict the delamination onset:

\[
\left( \frac{\tau_{13}}{f_{13}} \right)^2 + \left( \frac{\tau_{23}}{f_{23}} \right)^2 + \left( \frac{\sigma_1}{f_1} \right)^2 = 1,
\]

(4)

where \( r_{13} \) and \( r_{23} \) are the (average) interlaminar shear stresses, \( \sigma_1 \) is the (average) radial stress and \( f_{13}, f_{23}, f_1 \) are the corresponding allowable (strength) values.

Based on the aforementioned criterion, for plane stress problems when positive radial stresses occur, the Factor of Safety may be determined as:

\[
FoS = \frac{1}{\sqrt{\left( \frac{\sigma_1}{f_1} \right)^2 + \left( \frac{r_{13}}{f_{13}} \right)^2}}, \text{ if } \sigma_3 > 0
\]

(5)

while for regions with compressive radial stresses, the expression of Safety Factor reduces to:

\[
FoS = \frac{f_{13}}{r_{13}}, \text{ if } \sigma_3 < 0
\]

(6)
3. Results

The variations of the interlaminar radial and shear stresses with the curvature angle between 0 and 90 deg., obtained by means of the reference layered solid model are presented in Figure 4 and Figure 5. The stress components are evaluated with respect to a local cylindrical coordinate system whose radial direction is denoted by 1 while the tangential direction is denoted by 2.

![Variation of the interlaminar radial stresses with the curvature angle between [0-90] deg.](image1)

![Variation of the interlaminar shear stress with the curvature angle between [0 - 90] deg.](image2)

The force and moment fluxes were extracted at both ends of the curved region. The local coordinate systems are defined for both angle sides so that their positive directions determine opening the angle. Figure 6 shows the selection of elements and the orientations of the local coordinate systems used for extraction of fluxes at the beginning and the end of curvature region.

![The elements selection and the local coordinate systems.](image3)

Figure 7 to Figure 12 present the fringe plots of component fluxes represented by the shell forces and shell moments at the baseline of curvature region, extracted both for horizontal and vertical sides of the angle. The corresponding local coordinate systems are considered as shown in Figure 6. It is to be noted that the sign of the shell moment is defined by the element in the sense that a positive moment produces compression on the positive element side. Thus, when post-processing data it is important to consider the element normal and change the shell moment signs accordingly.
Figure 7. Fringe plots of shell forces $n_i$ at the curvature base (vert. side).

Figure 8. Fringe plots of shell forces $s_i$ at the curvature base (vert. side).

Figure 9. Fringe plots of shell forces $m_i$ at the curvature base (vert. side).

Figure 10. Fringe plots of shell forces $n_i$ at the curvature base (horiz. side).

Figure 11. Fringe plots of shell forces $s_i$ at the curvature base (horiz. side).

Figure 12. Fringe plots of shell forces $m_i$ at the curvature base (horiz. side).

Figure 13 to Figure 16 show comparative plots of Safety Factor calculated for the reference solid model versus the simplified shell model, at the different sections of the laminate angle as denoted by S1, S2, S3 and S4 in Figure 1.

Figure 13. FoS comparison results of solid and shell model at section S4.

Figure 14. FoS comparison results of solid and shell model at section S3.
4. Conclusions

A simplified post-processing approach based on shell forces and shell moments extracted at the elements located on the baseline of the curvature region is proposed. The reliability of those input loads and thus the reliability of the method depends mainly on the level of discretisation of the shell-based finite element model. Hence, the approach described within the present paper might be successfully used in practical applications either as a preselection method that allows to identify the critical locations or as a pre-sizing method to calculate the reserve factors of highly curved angles made of quasi-isotropic laminates provided their symmetrically balanced layout configuration.

The base assumption of this approach is that the solution of the elastic problem is assumed to be bi-directional. It follows that the main components of stresses that are responsible for unfolding are through-the-thickness normal and shear stresses which may be easily calculated based on shear force fluxes and bending moment fluxes. Another conservative assumption is that the maximum stresses occur at the same location in the thickness direction.

The use of consistent element coordinate systems to define the material orientation in case of layered solid elements models enables a coherent definition of the laminate layout. This is particularly important when such curved laminate angles are involved in, since the accuracy of the results can be significantly affected due to the anisotropic properties, in particular between in-plane as well as through-thickness direction. However, an accurate control of meshing procedure is required. It is also to be noted that the sign of the shell moment is defined by the element in the sense that a positive moment produces compression on the positive element side. Thus, when post-processing data it is important to consider the element normal and change the shell moment signs accordingly.

The comparative critical analysis of the finite element results obtained by means of using layered solid elements with respect to the results computed based on processing the shell forces and shell moments extracted at the baseline of curvature region, show that the proposed simplified approach provides conservative results. Anyhow, further extension of the method is required to account for in-plane circumferential tensile/compression stresses as well as the use of an appropriate failure criterion.

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