To the calculation of reinforced concrete flexible elements with combined pre-stress, taking into account the complete diagrams of material deformation

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Abstract. The formulas for determining losses in pre-compressed high-strength reinforcement are given, as well as the condition for closing the initial technological cracks in bent elements with pre-compressed reinforcement necessary for calculating the reinforced concrete beams with pre-compressed and stretched high-strength reinforcement.

Introduction
To develop a method for calculating the reinforced concrete elements with combined pre-stress, taking into account the complete deformation diagrams of reinforcing steel and concrete, it is necessary to have an analytical description of them.

Main part
Among many expressions for the analytical description of concrete deformation diagrams, we have adopted the equation proposed by M. Sargin [1] and recommended by ECB-FIP [2]

\[ \sigma_\varepsilon = R_\varepsilon \cdot \frac{K \cdot \varepsilon_\varepsilon}{1 + (K - 2) \frac{\varepsilon_\varepsilon}{\varepsilon_{\varepsilon R}}} \]

(1)

where

\[ K = E_\varepsilon \frac{\varepsilon_{\varepsilon R}}{R_\varepsilon} \]

(2)

\[ R_\varepsilon \] – is the prismatic concrete strength;
\[ \varepsilon_{\varepsilon R} \] – is the corresponding deformation.

The acceptance of this equation is justified not only by its comparative simplicity and accuracy, but also by the possibility of its application to the analytical description of the high-strength reinforcing steel deformation diagram. This is confirmed by a number of previous studies [3].
To describe the curved part of the reinforcement steel deformation diagram that does not have a physical yield point, we use the following expression [4]

\[
\sigma_s = \sigma_{el} + \left(\sigma_u - \sigma_{el}\right) \frac{K_s \left(\varepsilon_s - \varepsilon_{el}\right) - \left(\varepsilon_u - \varepsilon_{el}\right)}{1 + \left(K_s - 2\right)\frac{\varepsilon_u - \varepsilon_{el}}{\varepsilon_u - \varepsilon_{el}}},
\]

(3)

Where

\[
K_s = \frac{E_s (\varepsilon_s - \varepsilon_{el})}{\sigma_u - \sigma_{el}},
\]

(4)

\(\sigma_{el}\) and \(\varepsilon_{el}\) – denote the apparent yield stress and the corresponding deformation;
\(\sigma_u\) and \(\varepsilon_u\) – denote the tensile strength and the corresponding deformation;
\(E_s\) – is the steel elasticity modulus.

By \(\varepsilon_s \leq \varepsilon_{el}\) or \(\sigma_s \leq \sigma_{el}\) chart \(\sigma_s \leq \sigma_u\) linear (straight 1 in Figure 1, a), therefore, instead of the expression (3), the Hooke’s law is accepted as: \(\sigma_s = E_s \varepsilon_s\).

When the armature is prestressed to the stresses above the conditional yield point \(\sigma_{sp} > \sigma_{el}\), prestress losses occur from the stress relaxation \(\sigma_1\) without changes in deformations (the vertical segment, Figure 1, a) and the remaining losses occur with changes in deformations. The sum of all losses is equal to \(\sigma_{pre}\), and the set pre-voltages are \(\sigma_{sp,pl}\).

As a result of prestressing and removing part of the plastic deformations, the conditional limits of plasticity from \(\sigma_{el}\) to \(\sigma_{el}'\) and the yield from \(\sigma_{el}'\) to \(\sigma_{el,y}'\) are determined (see Figure 1, a).

In contrast to the equation (3) describing the high-strength steel initial deformation diagram, after prestressing it will take the form:

\[
\sigma_s = \sigma_{el}' + \left(\sigma_u - \sigma_{el}'\right)K_s' \frac{\left(\varepsilon_s - \varepsilon_{el}'\right) - \left(\varepsilon_{el}' - \varepsilon_{el}\right)}{1 + \left(K_s' - 2\right)\frac{\varepsilon_u - \varepsilon_{el}'}{\varepsilon_u - \varepsilon_{el}'},}
\]

(5)

where

\[
K_s' = \frac{E_s (\varepsilon_s - \varepsilon_{el}')}{{\sigma_u - \sigma_{el}'}};
\]

(6)

\(\sigma_{el}'\) and \(\varepsilon_{el}'\) - are the new values of the elasticity and corresponding deformation conditional limit;
\(\varepsilon_{el}' = \sigma_{el}' / E_s\) - are measured from the new origin.

When \(\varepsilon_s \leq \varepsilon_{el}'\) or \(\sigma_s \leq \sigma_{el}'\) the dependence \(\sigma_s - \varepsilon_s\) is linear (line 2, Figure 1, a), it obeys the Hooke’s law.

The pressed strain at the pre-stress \(\sigma_{sp}\) will be \(\varepsilon_{sp,pl} + \sigma_1 / E_s\), where \(\varepsilon_{sp,pl}\) – is the plastic strain at the stress \(\sigma_{sp} > \sigma_{el}\) and \(\sigma_1 / E_s\) - is the same, with the loss of pre-stress from stress relaxation \(\sigma_1\).
The experimental data show that it is possible to accept the following relationship between $\varepsilon_{sp,pl}$ and $\sigma_{sp}$:

$$
\varepsilon_{sp,pl} = 0.25 \left( \frac{\sigma_{sp}}{R_{sp}} - 0.8 \right)^3.
$$

(7)

Change chart $\sigma_s - \varepsilon_s$ of the high-strength steel, in particular the increase in yield point caused by the prestressing reinforcement in norms, is taken into account only if pre-stress exceeding the apparent yield stress of steel is in the delivery state [5].

It should be noted, however, that this phenomenon is also observed at $\sigma_{sp} < \sigma_{el}$, i.e. at a relatively low level of pre-stresses, which is a consequence of the steel creep deformations’ development with a constant total deformation, which leads to the pre-stresses loss from stress relaxation equal to $\sigma_1$ (Fig. 1,b).

When the armature is prestressed within the limits of its elastic operation, the conditional elastic limit changes slightly – only due to residual deformations caused by the prestress relaxation.

In this regard the conditional elastic limit is assumed to be the same as for the non-stressed reinforcement, i.e. $\sigma_{el} = 0.8\sigma_{0,2}$.

At a higher level of prestressing norms recommend that the conditional elastic limit of high strength rod reinforcement is determined by the formula

$$
\sigma'_{el} = \beta\sigma_{0,2},
$$

(8)

where

$$
\beta = 0.5 \frac{\sigma_t}{\sigma_{0,2}} + 0.4 \geq 0.8.
$$

(9)

Pre-compression of the beam compressed zone rebar above the conditional elastic limit is impractical, since the stress increment in the rebar $S'_p$ under external loading it will lead to excess of the calculated resistance by the total tension $\eta R_s$. So, for a class of valve A1000 $\sigma_{el} = 0.8 \cdot 815 = 652$ MPa, and the stress increment in the long-term of the current external load is: $R_{scp} = 500$ MPa. The total tension $R_{scp} + \sigma_{el} = 1152$ MPa, that is higher - $\eta R_s = 1.1 \cdot 815 = 896.5$ MPa.

Therefore, the pre-compression of the $S'_p$ rebar should not exceed the value $\sigma'_{sp} = \eta R_s - R_{scp}$, which is lower than the conditional elastic limit.

This condition does not apply to a preliminary compression reinforcement $S_p$, as stress from the external loads have opposite sign, so the total prestressing is equal to the difference of these tensions.

For class A1000 rebar, which is used in reinforced concrete structures proposed by the author in Chapter 2, the deformation diagrams before and after prestressing have the form shown in Fig. 1, a. The diagram is based on the design characteristics of the specified valve, equal to $R_s = 815$ MPa; $\eta = 1.1$; $\alpha_0 = R\eta = 815 \cdot 1.1 = 896.5$ MPa; $\sigma_{el} = 652$ MPa; $E_s = 19 \cdot 10^3$ MPa; $\varepsilon_0 = 652/19 \cdot 10^3 = 0.34 \cdot 10^{-2}$; $\varepsilon_u = 5 \cdot 10^{-2}$. 
Figure 1. Deformation diagrams for rebar steel of class A1000:
1-initial, 2 - at a pre-stress higher than the elastic limit (a); the same, at a pre-stress not higher than the elastic limit (b).
After substituting the specified values in the formulas (3) and (4), we get the following expression describing the deformation diagram of A1000 class reinforcing steel in the delivery state (curve 1 in Figure 1):

\[ \sigma_s = 652 + \frac{1.907 \cdot 10^3 \Delta \varepsilon \sigma - 1.127 \cdot 10^5 \Delta \varepsilon^2}{133.9 \Delta \varepsilon - 1.495}, \quad (10) \]

where \( \Delta \varepsilon = \varepsilon_a - \varepsilon_{el} \).

In the developed method of calculation, instead of a rectangular stress plot in the compressed zone of the bent reinforced concrete element, a real curvilinear one is adopted, which in the case when the relative height of the section \( \zeta \) compressed zone is close to the boundary \( \zeta_R \), significantly clarifies the calculation. For this purpose, the dependence curve \( \sigma_a - \varepsilon_a \) obtained by the concrete prisms’ central compression is used. It should be noted that when we transfer the curve to the compressed area of the flexible element some inaccuracy associated with the fact that in contrast to the central grip, when all “fibers” of the cross section given the same longitudinal strain in the compressed area of the flexible element in the “fibers” located at different distances from the neutral axis, is allowed, and the deformation will be different. Less loaded “fibers” will exert a restraining influence on the neighboring more loaded “fibers”. This certain inadequacy significantly less affects the calculation accuracy than replacing the actual curved plot of compressive stresses in the concrete of the bent element with a rectangular one.

**Summary**

When constructing a concrete deformation diagram, we divide it into two sections. On the site \( 0 - 0.3R_a \) concrete practically works elastically dependence \( \sigma_a - \varepsilon_a \) can be assumed as linear (Fig. 2). In this case, the beginning of the curve described by the formulas (1) and (2) should be moved to a point with the coordinates \( 0.3R_a - \varepsilon_{el} \). The analytical description of the complete dependence \( \sigma_a - \varepsilon_a \) will take the form

\[ \sigma_a = 0.3R_a + \frac{K_1 \frac{\varepsilon_a - \varepsilon_{a,el}}{0.7R_a} - \left( \frac{\varepsilon_a - \varepsilon_{a,el}}{\varepsilon_{aR} - \varepsilon_{a,el}} \right)^2}{1 + (K_1 - 2)\left( \frac{\varepsilon_a - \varepsilon_{a,el}}{\varepsilon_{aR} - \varepsilon_{a,el}} \right)}, \quad (11) \]

where

\[ K_1 = \frac{E_a}{R_a} - \frac{E_{a,el}}{0.7R_a}. \quad (12) \]

Characteristics of ordinary heavy concrete of class B30 equal \( R_a = 17 \text{ MPa}; \varepsilon_{aR} = 2 \cdot 10^{-3}; \ E_a = 32.5 \cdot 10^3 \text{ MPa}; \varepsilon_{a,el} = 0.2 \cdot 10^{-3}; \ R_{aR} = 1.2 \text{ MPa}; \sigma_{a,el} = 0.3R_a = 5.1 \text{ MPa}; \varepsilon_{a,el} = 0.16 \cdot 10^{-3} \), dependency (1), will take the form (Figure 2.):

\[ \sigma_a = 5.1 + \frac{3.37 \cdot 10^4 \Delta \varepsilon_a - 3.5 \cdot 10^5 \Delta \varepsilon^2_a}{1.65 \cdot 10^3 \Delta \varepsilon_a + 0.737}, \quad (13) \]

where \( \Delta \varepsilon_a = \varepsilon_a - \varepsilon_{el} \).
Figure 2. Deformation diagram of class B30 heavy concrete under axial compression and its analytical description.

References

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