Core Collapse Supernovae —
Theory between Achievements and New Challenges

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Abstract. Multi-dimensional hydrodynamic simulations of the post-bounce evolution of collapsed stellar iron cores have demonstrated that convective overturn between the stalled shock and the neutrinosphere can have an important effect on the neutrino-driven explosion mechanism. Whether a model yields a successful explosion or not, however, still depends on the power of neutrino energy deposition behind the stalled shock. The neutrino interaction with the stellar gas in the “hot bubble” also determines the duration of the shock stagnation phase, the explosion energy, and the composition of the neutrino-heated supernova ejecta. More accurate models require a more precise calculation of the neutrino luminosities and spectra and of the angular distributions of the neutrinos in the heating region. Therefore it is necessary to improve the numerical treatment of the neutrino transport, to develop a better understanding of the neutrino opacities of the dense nuclear medium, and to take into account convective processes inside the newly formed neutron star.

1 Convective instabilities — crucial for the explosion?

1.1 First hints from observations

Supernova 1987A in the Large Magellanic Cloud, which was the nearest visible type II supernova for more than 380 years, brought a wealth of observational data. The neutrino measurements by the Kamiokande [24], IMB [6] and Baksan [1] laboratories confirmed expectations based on theoretical models that neutrinos play a crucial role during the collapse of the stellar core. The photon emission from the supernova revealed that large-scale deviations from spherical symmetry develop during the explosion. This was suggested by the fact that nickel clumps were seen moving at velocities much faster than predicted by spherically symmetric models for the layers where explosive nucleosynthesis of iron-group elements takes place. Very strong mixing of hydrogen deep into the stellar interior and of helium, metals and radioactive nuclei far out into the hydrogen envelope had to be invoked in order to reproduce the shape and the smoothness of the observed light curve [2, 17, 52] and to understand the early appearance of X-rays [17, 52] and γ-rays [36, 35, 15, 47] from SN 1987A. Efforts to explain the large extent of the mixing and the very high Ni velocities by hydrodynamic instabilities at the composition interfaces of the progenitor star failed [1, 10, 9, 31]. Therefore one was tempted to conclude that the anisotropies might originate from hydrodynamic instabilities during the very early moments of the explosion.

1.2 New dimensions in modeling

This inspired multi-dimensional hydrodynamic simulations of neutrino-driven supernova explosions [14, 22, 23, 27, 28, 34, 35, 41, 71] which indeed confirmed conjectures that the negative entropy gradient that is built up by neutrino heating might lead to convective instabilities between the layers of strongest energy deposition and the supernova shock [6].
The main processes of neutrino energy transfer to the stellar gas are the charged-current reactions $\nu_e + n \rightarrow p + e^-$ and $\bar{\nu}_e + p \rightarrow n + e^+$. The heating rate per nucleon ($N$) is approximately given by

$$Q^+_{\nu_e} \approx 110 \cdot \left( \frac{L_{\nu_e,52} \left\langle \epsilon_{\nu_e,15}^2 \right\rangle}{r_7^2 \left\langle \mu \right\rangle_{\nu_e}} \right) Y_n + \left( \frac{L_{\bar{\nu}_e,52} \left\langle \epsilon_{\bar{\nu}_e,15}^2 \right\rangle}{r_7^2 \left\langle \mu \right\rangle_{\bar{\nu}_e}} \right) Y_p \left[ \frac{\text{MeV}}{8 \cdot N} \right],$$

where $Y_n$ and $Y_p$ are the number fractions of free neutrons and protons, respectively, $L_{\nu_e,52}$ denotes the luminosity of $\nu_e$ or $\bar{\nu}_e$ in $10^{52}$ erg/s, $r_7$ the radial position in $10^7$ cm, and $\left\langle \epsilon_{\nu_e,15}^2 \right\rangle$ the mean value of the squared neutrino energy measured in units of 15 MeV. $\left\langle \mu \right\rangle_{\nu_e} = \left\langle \cos \theta_{\nu_e} \right\rangle$ is the cosine of the angle $\theta_{\nu_e}$ of the direction of neutrino propagation relative to the radial direction, averaged over the neutrino phase space distribution. This quantity is very small in the opaque regime where the neutrinos are isotropic, adopts a value of about 0.25 around the neutrinosphere, and approaches unity for radially streaming neutrinos very far out.

There is general agreement about the existence and the growth of hydrodynamic instabilities in a layer between the shock position and the radius of maximum neutrino heating (which is just outside the “gain radius”, i.e. the radius where neutrino cooling turns into net heating). However, the results are less definite concerning the question whether the convective overturn is strong enough to ensure the success of the neutrino-heating mechanism in driving the explosion of a core-collapse supernova.

1.3 Theory between success and failure

Two-dimensional simulations by Herant et al. [23] and Burrows et al. [14] yielded successful explosions in cases where spherically symmetric models fail. According to these simulations, the convective overturn in the neutrino-heated region has the following effects on the shock propagation. On the one hand, heated matter from the region close to the gain radius rises outward and at the same time is replaced by cool gas flowing down from the postshock region. Since the production reactions of neutrinos ($e^+$ capture on nucleons and thermal processes) are very temperature sensitive, the expansion and cooling of rising plasma reduces the energy loss by reemission of neutrinos. Moreover, the net energy deposition by neutrinos is enhanced as more cool material is exposed to the large neutrino fluxes just outside the gain radius where the neutrino heating rate peaks (the radial dilution of the fluxes roughly goes as $1/r^2$). On the other hand, hot matter floats into the postshock region and increases the pressure there. Thus the shock is pushed further out which leads to a growth of the gain region and therefore also of the net energy transfer from neutrinos to the stellar gas.

In contrast, Mezzacappa et al. [40] and Lichtenstadt et al. [34], using multi-energy-group instead of a simpler grey treatment of neutrino diffusion, found that the convective overturn and its associated effects are not strong enough to revive the stalled prompt supernova shock, although the outward motion of the shock is enhanced.

These results can be understood in view of parametric studies carried out by Janka & Müller [27, 28]. Varying the neutrino luminosities from the neutrinosphere Janka & Müller found that successful, strong explosions as well as weak ones, and even failures, can be obtained with and without convection. Their simulations showed that there is a threshold luminosity above which the models explode and the explosion energy varies sensitively with the value of the $\nu_e$ and $\bar{\nu}_e$ luminosities.
More generally, the outcome of the simulations is determined by the strength of the neutrino heating (minus neutrino cooling), which according to Eq. (1), depends on $L_\nu$ and $L_{\bar{\nu}}$, on the neutrino spectra via the mean squared neutrino energies $\langle \epsilon_\nu^2 \rangle$, and on the angular distributions of the neutrinos through the inverse of the “flux factors” $\langle \mu \rangle_\nu$, because the neutrino number densities at radius $r$ are given by $n_\nu(r) = L_\nu / (4\pi r^2 \bar{c} \langle \epsilon_\nu \rangle \langle \mu \rangle_\nu)$. From the results of Janka & Müller [27, 28] one can conclude that convective energy transport into the postshock region is crucial for getting an explosion only in a certain window of values of the products $L_\nu \langle \epsilon_\nu^2 \rangle$ for $\nu_e$ and $\bar{\nu}_e$ (Fig. 1). For higher values, the heating timescale is so short and the energy deposition so strong that explosions develop before convective instabilities can grow. For lower values, neither the neutrino heating nor the convection is efficient enough to prevent that most of the energy transferred to the stellar gas is advected down through the gain radius and therefore lost again due to neutrino cooling.

2 Routes to progress in modeling neutrino-driven explosions

The discrepant results of different simulations and the unpleasant sensitivity of the explosion to the strength of the heating by $\nu_e$ and $\bar{\nu}_e$ absorption demands calculation of the factors in Eq. (1) to the highest possible accuracy. For this purpose, the numerical
treatment of the neutrino transport in the models needs further improvement, and the processes must be understood in more detail which govern the emission of neutrinos from the nascent neutron star and which thus determine the luminosities and spectra. Current interest of supernova modelers is focused on the following aspects.

2.1 Neutrino transport

The flux factor $\langle \mu \rangle_\nu$ cannot be reliably computed by flux-limited diffusion methods which fail to yield a good approximation of neutrino transport in the semi-transparent layers where neutrinos begin to decouple from the stellar medium and where the neutrino heating takes place. The flux factors calculated with such approximate methods of neutrino transport are systematically too large [38, 58], implying that the energy deposition by $\nu_e$ and $\bar{\nu}_e$ absorption according to Eq. (1) is underestimated.

Therefore efforts are made to solve the Boltzmann transport equation in hydrodynamic simulations of the shock-stagnation and neutrino-heating phases. Progress is led by the Mezzacappa et al. group [39, 38] whose discrete ordinate ($S_N$) method provides an excellent description of the neutrino spectra and a significant improvement of the angular moments of the neutrino distribution, although the latter still deviate from very accurate Monte Carlo results at a level of about 10% when the computations are performed with an affordable number (6–8) of angular grid points [58]. Since the Boltzmann solver yields values of the flux factor which are somewhat lower than the Monte Carlo results, one expects a slight overestimation of the neutrino heating in this case, as can be verified from Eq. (1). Mezzacappa et al. [38], however, claim that despite of the remaining differences, convergence can be achieved for the net heating.

2.2 Convection in the nascent neutron star

The luminosities and spectra of neutrinos emitted from the neutrinosphere can be significantly affected by convective processes inside the nascent neutron star [11, 37, 55, 56, 33, 31, 26]. Since neutrinos can be carried to the neutrinosphere by mass motions much faster than by the slow diffusion in the opaque interior of the neutron star, the two-dimensional simulations of Keil et al. [33, 31, 26] showed an amplification of the neutrino luminosity by nearly a factor of two at about half a second after core bounce relative to spherically symmetric models without convection. Of course, this aids delayed explosions by neutrino heating.

Convection inside the newly formed neutron star can be driven by gradients of the entropy and/or proton (electron lepton number) fraction in the nuclear medium [18]. The type of instability which grows most rapidly, e.g., doubly diffusive neutron-finger convection [37, 55, 56] or Ledoux convection [1] or quasi-Ledoux convection [31, 26], is a matter of the properties of the nuclear equation of state which determines the magnitudes and signs of the thermodynamic derivatives [10]. It is also sensitive to the gradients that develop, and thus may depend on the details of the treatment of neutrino transport in the dense interior of the star.

Convection below the neutrinosphere seems to be disfavored during the very early post-bounce evolution by the currently most elaborate supernova models [8, 8, 31], but can develop deeper inside the nascent neutron star on a longer timescale ($\gtrsim 100$ ms after bounce) and can encompass the whole star within seconds [11, 26, 31, 26]. Recent calculations by Pons et al. [42] for the neutrino cooling of hot neutron stars were done with improved neutrino opacities of the nuclear medium which were described
consistently with the employed equation of state. Their models confirm principal aspects of previous simulations, in particular the existence of convectively unstable layers in the neutron star.

2.3 Neutrino opacities of dense matter

Another important issue of interest are the neutrino opacities in the dense and hot nuclear medium of the nascent neutron star. In current supernova models, the description of neutrino-nucleon interactions is incomplete because the standard approximations assume isolated and infinitely massive nucleons [54]. Therefore effects like the fermion phase space blocking of the nucleons, the reduction of the effective nucleon mass by momentum-dependent nuclear interactions in the dense plasma, and nucleon thermal motions and recoil are either neglected completely or approximated in a more or less controled manner [5, 12]. These effects have been recognized to be important [19, 20, 14, 15] for reliable calculations of the neutrino luminosities and spectra, but still await careful inclusion in supernova codes.

Many-body (spatial) correlations due to strong interactions [48, 25, 13, 46] and multiple-scattering effects by spin-dependent forces between nucleons (temporal spin-density correlations [14, 20]) are of particular interest, because they lead to a reduction of the neutrino opacities in the newly formed neutron star and are associated with additional modes of energy transfer between neutrinos and the nuclear medium.

A reduction of the neutrino opacities implies larger neutrino mean free paths and thus increases the neutrino luminosities [32, 42, 13]. Despite of accelerated neutrino diffusion, however, convection in the nascent neutron star turns out not to be suppressed, but to be still the fastest mode of energy transport [30]. Therefore Janka et al. [30] found that the combined effects of reduced opacities and convective energy transport do not appreciably change the convectively enhanced neutrino emission.

Energy transfer in neutrino-nucleon collisions due to nucleon recoil and inelastic $\nu NN \leftrightarrow NN\nu$ scatterings are ignored in current supernova simulations. In addition to the nucleon bremsstrahlung process $\nu\bar{\nu} NN \leftrightarrow NN$ [53] these effects accomplish a stronger energetic coupling between the stellar plasma and the neutrinos and thus could lead to a significant reduction of the mean spectral energies of emitted neutrinos [20, 21]. In particular, it must be suspected that the average energies of $\bar{\nu}_e$ and heavy-lepton neutrinos $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$ and $\bar{\nu}_\tau$ on the one hand, and $\nu_e$ on the other, could be much more similar than predicted by current supernova and neutron star cooling models.

It is not clear yet whether these discussed aspects of neutrino-nucleon interactions in dense nuclear matter have any implications for the explosion mechanism. But due to their consequences for the neutrino luminosities and spectra they are certainly important for the detection of neutrinos from Galactic supernovae with future experiments like OMNIS and LAND.

3 Summary

Hydrodynamic simulations have shown that the shock revival phase is a very turbulent epoch in the supernova evolution where violent convective overturn takes place in the region between the neutrinosphere and the stalled shock front. Energy transfer to the gas in this region by absorptions of $\nu_e$ and $\bar{\nu}_e$ emitted from the settling, newly formed
neutron star can deposit the energy for a powerful supernova explosion, provided
the neutrino luminosities are high enough and/or the neutrinos have sufficiently hard
spectra.

Self-consistent multi-dimensional simulations of stellar core collapse and of the
post-bounce evolution carried out by different groups still yield discrepant results.
While models with a simple grey (energy-integrated) approximation of neutrino dif-
fusion show successful explosions, more elaborate multi-energy group treatments of
neutrino diffusion have so far only produced duds.

At present it is not clear whether failures can be converted into successes if defi-
ciencies of the diffusion approximation of neutrino transport in the semi-transparent
layers are removed, which lead to an underestimation of the neutrino energy de-
position in the heating region. Efforts are currently made to solve the Boltzmann
transport equation in combination with the equations of hydrodynamics. It is hoped
that the more accurate treatment will yield larger neutrino luminosities as well as
higher efficiencies of neutrino energy transfer in the post-shock layer.

However, there are more factors of uncertainty still present in current supernova
simulations. On the one hand, neutrino-nucleon interactions are described by simpli-
ifying or even by ignoring a number of potentially very important complications, e.g.,
nucleon recoil and thermal motions and nucleon phase space blocking. Other aspects
like nucleon-nucleon correlations in the dense medium and multiple-scattering effects
due to spin-dependent nucleon interactions are incompletely understood. These ef-
fects could enhance the neutrino luminosities significantly and might also change the
emitted neutrino spectra considerably.

On the other hand, recent multi-dimensional hydrodynamic simulations have con-
firmed previous conclusions from one-dimensional models that convectively unstable
layers can develop in the nascent neutron star. The simulations show that convec-
tion can encompass the whole star within seconds and is more efficient than neutrino
diffusion in transporting energy to the surface. This leads to a sizable increase of
the neutrino luminosities and to a hardening of the emitted neutrino spectra. If
neutrino-driven explosions need several 100 ms after core bounce to develop, convec-
tive enhancement of the neutrino luminosities could play a helpful role.

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