Constraining Bianchi type V universe with recent $H(z)$ and BAO observations in Brans–Dicke theory of gravitation

R. Prasad\textsuperscript{1,a}, Avinash Kr. Yadav\textsuperscript{2,b}, Anil Kumar Yadav\textsuperscript{3,c}

\textsuperscript{1} Department of Physics, Galgotias College of Engineering and Technology, Greater Noida 201310, India
\textsuperscript{2} Department of Mathematics, United College of Engineering and Research, Greater Noida 201310, India
\textsuperscript{3} Department of Physics, United College of Engineering and Research, Greater Noida 201310, India

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Abstract In this paper, we investigate a transitioning model of Bianchi type V universe in Brans–Dicke theory of gravitation. The derived model not only validates Mach’s principle but also describes the present acceleration of the universe. In this paper, our aim is to constrain an exact Bianchi type V universe in Brans–Dicke gravity. For this sake, firstly we obtain an exact solution of field equations in modified gravity and secondly constrain the model parameters by bounding the model with recent $H(z)$ and Baryon acoustic oscillations observational data. The current phase of accelerated expansion of the universe is also described by the contribution coming from cosmological constant screened scalar field with deceleration parameter showing a transition redshift of about $z_t = 0.79$. Some physical properties of the universe are also discussed.

1 Introduction

The supernovae Ia observations \cite{1,2} have exhibited a strong evidence that our universe is dominated by two types of dark components at present epoch. These two components of present universe are named as dark matter and dark energy. Today, it is one of the major issues in modern cosmology to describe the nature of dark matter and dark energy. The dark matter has not been directly observed but there are many evidences such as galaxy rotation curves, gravitational effects, gravitational lensing, etc., which support the existence of dark matter. The dark energy is an unknown form of energy that pervades the whole universe. It is believed to have negative pressure, and the dark energy causes acceleration in the present universe. According to WMAP observations \cite{3–5}, the universe energy density appears to consist of approximately 4\% of that of visible matter, 21\% of that of dark matter and 75\% of that of dark energy. In the literature, the acceleration in the present universe is described by two ways: (1) inclusion of dark energy in right side of Einstein’s equation,
i.e., by modifying energy-momentum tensor, and (2) modification in left side of Einstein’s equation, i.e., geometric modification. The authors of Refs. [6,7] have described late-time acceleration of the universe by considering dark energy and modified gravity, respectively. Later on, numerous cosmological models have been investigated in General Relativity (GR) with inclusion of dark energy [8–15] and in modified theories of gravity without inclusion of dark energy [16–23]. Even after all these attempts, the reliable nature of dark energy has not been convincingly explained yet.

The Brans–Dicke (BD) theory [24], which is a natural generalization of GR, provides a worthy framework for dynamical dark energy models. In this theory, the scalar field \( \phi \) is being time-dependent and it is equivalent to \((8\pi G)^{-1}\). Therefore, in BD scalar–tensor theory, the scalar field \( \phi \) couples to the gravity with a dimensionless coupling parameter \( \omega \). It is worth to note that BD theory of gravitation commits expanding solutions for scalar field and average scale factor which are compatible with the solar system observations. In Refs. [25–27], the authors have investigated that BD theory explains the late-time accelerated expansion of the universe and also conciliates the observation data. It is also to be noticed that BD theory of gravitation reduces to GR if scalar field is constant and \( \omega \to \infty \) [28,29]. Some new agegraphic dark energy models in Brans–Dicke gravity have been investigated [30–33]. These models explain the late-time accelerated expansion of the universe with evolution of scalar field as power law of scale factor. In the literature, BD theory is invoked to fulfill the requirement of Mach’s principle [24,34–37]. In Sen and Sen [38], authors have investigated that a perfect fluid cannot support acceleration but a fluid with dissipative pressure can drive late-time acceleration of current universe. The present cosmic acceleration without resorting to a cosmological constant or quintessence matter has been investigated in BD theory but then Brans–Dicke coupling constant asymptotically acquires a small negative value for an accelerating universe at late time [39], while in Ref. [25], authors have obtained solution for accelerating universe with \( \phi^2 \) potential for large BD coupling constant without considering positive energy condition for matter and scalar field both. Recently, Akarsu et al. [40] have investigated some particular negative range of \( \omega \) and positive large value of \( \omega \) that lead to acceleration in massive Brans–Dicke gravity. Some large-angle anomalies viewed in cosmic microwave background (CMB) radiations [5] are favoring the presence of anisotropies in the early stage of the universe, which violate the isotropic nature of the observable universe and hence to clearly describe the early universe, spatially homogeneous but anisotropic Bianchi models play a significant role. In the literature, several Bianchi-type models have been investigated with different matter distribution in Brans–Dicke theory of gravitation. In particular, Kiran et al. [41] have investigated an interacting Bianchi V cosmological model within the framework of Brans–Dicke cosmology. In the recent past, some Brans–Dicke anisotropic models have been studied to discuss the late-time accelerated expansion of the universe [42–45]. Some useful applications of Bianchi-type models compatible with astrophysical observations are given in Refs. [15,46–49].

In this paper, we have investigated a Bianchi type V model of the universe filled with pressureless matter and cosmological constant at present in Brans–Dicke gravity. Firstly, we have obtained an exact Brans–Dicke universe and then found constraints on model parameters by using recent H(z) and BAO observational data. The rest of the paper is organized as follows: In Sect. 2, we present the model and its basic equations. In Sect. 3, we describe the method and likelihoods. In Sect. 4, we discuss the physical and kinematic properties of the model under consideration. The summary of our findings is presented in Sect. 5.
2 The model and basic equations

The Einstein’s field equations in Brans–Dicke theory are given by

\[ R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = \frac{8 \pi}{c^2} T_{ij} - \frac{\omega}{\phi^2} (\phi_i \phi_j - \frac{1}{2} g_{ij} \phi_k \phi^k) - \frac{1}{\phi} (\phi_{ij} - g_{ij} \Box \phi) \]  

(1)

and

\[ (2 \omega + 3) \Box \phi = \frac{8 \pi T}{c^4} + 2 \Lambda \phi \]  

(2)

where \( \omega \) is the Brans–Dicke coupling constant; \( \phi \) is Brans–Dicke scalar field and \( \Lambda \) is the cosmological constant.

The Bianchi type V space-time is read as

\[ ds^2 = dt^2 - A(t)^2 dx^2 - e^{2 \alpha x} [B(t)^2 dy^2 + C(t)^2 dz^2] \]  

(3)

where \( A(t), B(t) & C(t) \) are scale factors along \( x, y \) and \( z \) directions, respectively, and average scale factor is defined as \( a = (ABC)^{\frac{1}{3}} \). The exponent \( \alpha \neq 0 \) in (3) is an arbitrary constant.

The energy momentum tensor of perfect fluid is given by

\[ T_{ij} = (p + \rho) u_i u_j - p g_{ij} \]  

(4)

Here, \( p \) and \( \rho \) are the isotropic pressure and energy density of the matter under consideration. Also \( u^i u^j = -1 \) and \( u^i \) is the four velocity vector.

The field equations (1) for space-time (3) are read as

\[ \ddot{B} + \ddot{C} + \frac{\dot{B} \dot{C}}{BC} = \frac{\alpha^2}{A^2} + \frac{\omega \dot{\phi}^2}{2 \phi^2} + \frac{\dot{\phi}}{\phi} \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{\dot{\phi}}{\phi} = -\frac{8 \pi p}{\phi} + \Lambda \]  

(5)

\[ \ddot{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = \frac{\alpha^2}{A^2} + \frac{\omega \dot{\phi}^2}{2 \phi^2} + \frac{\dot{\phi}}{\phi} \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{\dot{\phi}}{\phi} = -\frac{8 \pi p}{\phi} + \Lambda \]  

(6)

\[ \ddot{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = \frac{3 \alpha^2}{A^2} - \frac{\omega \dot{\phi}^2}{2 \phi^2} + \frac{\dot{\phi}}{\phi} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{8 \pi \rho}{\phi} + \Lambda \]  

(7)

\[ 2 \frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \implies A^2 = BC \]  

(9)

\[ \frac{\ddot{\phi}}{\phi} + 3 \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \frac{\dot{\phi}}{\phi} = \frac{8 \pi (\rho - 3 p)}{(2 \omega + 3) \phi} + \frac{2 \Lambda}{2 \omega + 3} \]  

(10)

where overdot denotes derivatives with respect to time \( t \).

The equation of continuity is read as

\[ \dot{\rho} + (1 + \gamma) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \rho = 0 \]  

(11)
where γ is the equation of state parameter of perfect barotropic fluid and it is defined as $\gamma = \frac{p}{\rho} = \text{constant}$. The pressure of dark matter is zero, which can be recovered from barotropic equation of state by choosing $\gamma = 0$.

2.1 Solution of Einstein’s field equations

Equations (5)–(7) lead to the following system of equations

$$\frac{\dddot{A}}{A} - \frac{\dddot{B}}{B} + \frac{\dddot{C}}{C} - \frac{\dddot{A}}{A} \frac{\dddot{B}}{B} + \frac{\dddot{C}}{C} \frac{\dddot{A}}{A} + \left( \frac{\dddot{A}}{A} - \frac{\dddot{B}}{B} \right) \frac{\dot{\phi}}{\phi} = 0 \tag{12}$$

$$\frac{\dddot{B}}{B} - \frac{\dddot{C}}{C} + \frac{\dddot{A}}{A} \frac{\dddot{B}}{B} - \frac{\dddot{C}}{C} \frac{\dddot{A}}{A} + \left( \frac{\dddot{B}}{B} - \frac{\dddot{C}}{C} \right) \frac{\dot{\phi}}{\phi} = 0 \tag{13}$$

$$\frac{\dddot{C}}{C} - \frac{\dddot{A}}{A} + \frac{\dddot{C}}{C} \frac{\dddot{A}}{A} - \frac{\dddot{B}}{B} \frac{\dddot{C}}{C} + \left( \frac{\dddot{C}}{C} - \frac{\dddot{A}}{A} \right) \frac{\dot{\phi}}{\phi} = 0 \tag{14}$$

Equations (12)–(14) are the system of three equations with four unknown variables $A$, $B$, $C$ and $\phi$. So, one cannot solve these equations in general. In connection with Eq. (9), one may propose the following relation among the metric functions

$$B = AD \quad \& \quad C = \frac{A}{D} \tag{15}$$

where $D = D(t)$ measures the anisotropy in universe. For $D = 1$ and $\alpha = 0$, Bianchi V universe recovers the case of FRW universe.

Equations (13) and (15) lead to

$$\frac{\ddot{D}}{D} - \frac{\dot{D}^2}{D^2} + \frac{\dot{D}}{D} \left( 3 \frac{\dddot{A}}{A} + \frac{\dot{\phi}}{\phi} \right) = 0 \tag{16}$$

After integration of Eq. (18), we obtain

$$D = \exp \left[ \int \frac{k}{A^3} \, dt \right] \tag{17}$$

Now, the average scale factor is computed as

$$a^3 = ABC = A^3 \Rightarrow a = A \tag{18}$$

Therefore, Friedmann equations (5) and (8), respectively, recast as follows:

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{\dot{\phi}^2}{a^2} + \frac{k^2}{a^6 \phi^2} + \frac{\omega \dot{\phi}^2}{2 \phi^2} + 2 \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} = -\frac{8\pi \rho}{\phi} + \Lambda \tag{19}$$

$$3\frac{\dot{a}^2}{a^2} - \frac{3\alpha^2}{a^2} - \frac{k^2}{a^6 \phi^2} - \frac{\omega \dot{\phi}^2}{2 \phi^2} + 3 \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} = \frac{8\pi \rho}{\phi} + \Lambda \tag{20}$$

2.2 The model: Brans–Dicke anisotropic universe

The density parameters are read as

$$\Omega_m = \frac{8\pi \rho_m}{3H^2 \phi}, \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}, \quad \Omega_\sigma = \frac{k^2}{3H^2 a^6 \phi^2}, \quad \Omega_\alpha = \frac{\alpha^2}{a^2 H^2} \tag{21}$$

where $\rho_m = (\rho_m)_0 a^{-3}$ is the energy density of pressureless matter and $\Omega_m$, $\Omega_\Lambda$ and $\Omega_\sigma$ represent the dimensionless density parameters for dark matter, $\Lambda$ energy, shear anisotropy and $\alpha$ parameter, respectively. $H$ is Hubble’s parameter and it is defined as $H = \frac{\dot{a}}{a}$. 

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The deceleration parameter $q$ and scalar field deceleration parameter $q_\phi$ are read as

$$q = -\frac{\ddot{a}}{aH^2}, \quad q_\phi = -\frac{\ddot{\phi}}{\phi H^2} \quad (22)$$

Dividing Eq. (13) by $H^2$ and then using Eq. (21), we have

$$\Omega_m + \Omega_A + \Omega_\sigma + \Omega_\alpha = 1 + \psi - \frac{\omega}{6} \psi^2 \quad (23)$$

where $\psi = \frac{\dot{\phi}}{\phi H}$. After some manipulations, finally we obtained

$$\phi = \phi_0 \left( \frac{a}{a_0} \right)^\psi \quad & \quad \psi = \frac{1}{\omega + 1} \quad (24)$$

where $a_0$ is the present value of scale factor. Thus, Eq. (23) reduces to

$$\Omega_m + \Omega_A + \Omega_\sigma + \Omega_\alpha = 1 + \frac{5\omega + 6}{6(\omega + 1)^2} \quad (25)$$

If we define the density of scalar field $\phi$ as

$$\Omega_\phi = -\frac{5\omega + 6}{6(\omega + 1)^2} \quad (26)$$

then Eq. (25) is recast as

$$\Omega_m + \Omega_A + \Omega_\sigma + \Omega_\alpha + \Omega_\phi = 1 \quad (27)$$

The scale factor $a$ and $\phi$ in connection with $z$ are read as

$$a = \frac{a_0}{1 + z}, \quad \phi = \frac{1}{(1 + z)^{1+\omega}} \quad (28)$$

Equations (21), (27) and (28) lead to

$$H_{\sigma BD} = \frac{H_0}{(1 - \Omega_\phi)^\frac{1}{2}} \left[ \Omega_{m0}(1 + z)^{2.5(\sqrt{1 - \Omega_\phi} - 1)^3 + \Omega_{\sigma 0}(1 + z)^{5(\sqrt{1 - \Omega_\phi} - 1)^6 + \Omega_\alpha(1 + z)^2} + \Omega_{A0} \right]^\frac{1}{2} \quad (29)$$

where $H_0$, $\Omega_{m0}$, $\Omega_{\sigma 0}$ and $\Omega_{A0}$ denote present values of Hubble constant and density parameters due to dark matter, anisotropy and cosmological constant, respectively.

3 Method and likelihoods

In this section, we briefly describe the observational data and the statistical methodology to constrain the Bianchi V universe as discussed in the previous section.

– Observational Hubble data (OHD) We adopt 46 $H(z)$ data points over the redshift range of $0 \leq z \leq 2.36$ obtained from cosmic chronometric (CC) technique. We have compiled all 46 $H(z)$ data points in Table 1.
| S. No. | $z$ | $H(z)$ [Gyr$^{-1}$] | $\sigma_i$ [Gyr$^{-1}$] | References |
|-------|----|-----------------|----------------|-------------|
| 1     | 0  | 0.069           | 0.0013         | [55]        |
| 2     | 0.07| 0.069           | 0.020          | [56]        |
| 3     | 0.09| 0.071           | 0.012          | [57]        |
| 4     | 0.01| 0.071           | 0.012          | [58]        |
| 5     | 0.12| 0.071           | 0.027          | [56]        |
| 6     | 0.17| 0.07            | 0.0081         | [58]        |
| 7     | 0.179| 0.085          | 0.0041         | [59]        |
| 8     | 0.1993| 0.077        | 0.0051         | [59]        |
| 9     | 0.2 | 0.077           | 0.030          | [56]        |
| 10    | 0.24| 0.075           | 0.0026         | [60]        |
| 11    | 0.27| 0.081           | 0.014          | [58]        |
| 12    | 0.28| 0.079           | 0.035          | [56]        |
| 13    | 0.35| 0.091           | 0.0085         | [61]        |
| 14    | 0.352| 0.085       | 0.0143         | [59]        |
| 15    | 0.38| 0.085           | 0.0019         | [62]        |
| 16    | 0.3802| 0.083      | 0.0137         | [63]        |
| 17    | 0.4 | 0.097           | 0.0173         | [57]        |
| 18    | 0.4004| 0.079       | 0.0104         | [63]        |
| 19    | 0.4247| 0.089      | 0.0114         | [63]        |
| 20    | 0.43| 0.088           | 0.0038         | [60]        |
| 21    | 0.44| 0.084           | 0.008          | [64]        |
| 22    | 0.4449| 0.095       | 0.013          | [63]        |
| 23    | 0.47| 0.091           | 0.051          | [65]        |
| 24    | 0.4783| 0.083       | 0.009          | [63]        |
| 25    | 0.48| 0.099           | 0.061          | [58]        |
| 26    | 0.51| 0.092           | 0.0019         | [62]        |
| 27    | 0.57| 0.106           | 0.0035         | [53]        |
| 28    | 0.593| 0.106         | 0.0132         | [59]        |
| 29    | 0.6 | 0.089           | 0.0062         | [64]        |
| 30    | 0.61| 0.099           | 0.0021         | [62]        |
| 31    | 0.68| 0.094           | 0.0082         | [59]        |
| 32    | 0.73| 0.099           | 0.0072         | [64]        |
| 33    | 0.781| 0.107       | 0.012          | [59]        |
| 34    | 0.875| 0.128         | 0.0173         | [59]        |
| 35    | 0.88| 0.092           | 0.041          | [58]        |
| 36    | 0.9 | 0.120           | 0.0234         | [58]        |
| 37    | 1.037| 0.157        | 0.020          | [59]        |
| 38    | 1.3 | 0.172           | 0.0173         | [58]        |
| 39    | 1.363| 0.164        | 0.0343         | [66]        |
| 40    | 1.43| 0.181           | 0.0183         | [58]        |
| 41    | 1.53| 0.143           | 0.0143         | [58]        |
| 42    | 1.75| 0.207           | 0.041          | [58]        |
Table 1 continued

| S. No. | z      | $H(z)$ [Gyr$^{-1}$] | $\sigma_i$ [Gyr$^{-1}$] | References |
|--------|--------|----------------------|--------------------------|------------|
| 43     | 1.965  | 0.191                | 0.0514                   | [66]       |
| 44     | 2.3    | 0.229                | 0.0082                   | [67]       |
| 45     | 2.34   | 0.227                | 0.0072                   | [68]       |
| 46     | 2.36   | 0.231                | 0.0082                   | [69]       |

-- Baryon acoustic oscillations (BAO) We use ten baryon acoustic oscillations data extracted from the 6dFGS [50], SDSS-MGS [51], BOSS [52], BOSS CMASS [53] and WiggleZ [54] surveys.

Note that in the above references, $H(z)$ and error $\sigma_i$ are in the unit of km s$^{-1}$ Mpc$^{-1}$. In this paper, we have converted these quantities in the unit of Gyr$^{-1}$.

For all analysis, we have defined a $\chi^2$ for parameters with the likelihood given by $\zeta \propto e^{-\chi^2/2}$. Therefore, the $\chi^2$ function for $H(z)$ data is written as

$$\chi^2_H = \sum_{i=1}^{46} \left( \frac{H(z_i, s) - H_{\text{obs}}(z_i)}{\sigma_i} \right)^2$$  \tag{30}$$

where $s$ and $\sigma_i$ denote the parameter vector and standard error in experimental values of Hubble’s function $H$, respectively.

Similarly, the joint $\chi^2$ is read as

$$\chi^2_{\text{joint}} = \chi^2_H + \chi^2_{\text{BAO}}$$  \tag{31}$$

Figures 1 and 2 exhibit the one-dimensional marginalized distribution and two-dimensional contours with 68% CL and 95% CL for parameter space $\Theta_{\sigma\beta\Delta}$ using $H(z)$ and combined $H(z) + \text{BAO}$ data, respectively. The numerical result of statistical analysis is listed in Table 2.

We have summarized the numerical result of statistical analysis in Table 2. From Table 2, it has been observed that the estimated constraints on $H_0$ as 0.0719 Gyr$^{-1} \sim 70.4$ km s$^{-1}$ Mpc$^{-1}$ and 0.0717 Gyr$^{-1} \sim 70.2$ km s$^{-1}$ Mpc$^{-1}$ are closer to other investigations [70–73]. The best fit curve of Hubble rate versus redshift of derived model is shown in Fig. 3. In this paper, our aim is also to constrain the density of scalar field $\Omega_\phi$ and the estimated constraints on $\Omega_\phi$ with $H(z)$ and $H(z) + \text{BAO}$ data are as $\Omega_\phi = 0.0098$ and $\Omega_\phi = 0.014$, respectively. In Amirhashchi and Yadav [74], we also find constraint on scalar field density as $\Omega_\phi = 0.010$ by using different observational datasets. In Table 2, $\chi^2_i$ is read as $\chi^2_v = \chi^2_{\text{min}}/\text{dof}$ where dof is abbreviation of degree of freedom and it is defined as the difference between all observational data points and the number of free parameters. It should be noted that for $\chi^2_v \leq 1$, the fitting of model with observed data is considered as the best fitting model.
Fig. 1  One-dimensional marginalized distribution and two-dimensional contours with 68% CL and 95% CL for parameter space $\Theta_{\sigma BD}$ using $H(z)$ data

4 Properties of the model

4.1 The deceleration parameter

The deceleration parameter in terms of redshift is read as

$$q(z) = -1 + \left(1 + z\right) \frac{H'_{\sigma BD}}{H_{\sigma BD}}$$  \hspace{1cm} (32)

Here, $H'_{\sigma BD}$ denotes first derivative of $H_{\sigma BD}$ with respect to $z$.

Using Eq. (29), Eq. (32) is recast as

$$q(z) = -1 + \frac{(z+1) \left(2.5\Omega_{m0} (1-\sqrt{1-\Omega_{\phi}} + 0.2)(z+1)^{2.5\sqrt{1-\Omega_{\phi}}-0.5} + 2(z+1)\Omega_{\alpha 0} + q_1\right)}{2 \left(\Omega_{\Lambda 0} + \Omega_{m0}(z+1)^{2.5\sqrt{1-\Omega_{\phi}}+0.5} + (z+1)^2\Omega_{\alpha 0} + \Omega_{\sigma 0}(z+1)^{5\sqrt{1-\Omega_{\phi}}+1}\right)}$$  \hspace{1cm} (33)
Fig. 2  One-dimensional marginalized distribution and two-dimensional contours with 68% CL and 95% CL for parameter space $\Theta_{\sigma_{BD}}$ using $H(z) + BAO$ data

Table 2  Summary of statistical analysis

| Model parameters | $H(z)$ | $H(z) + BAO$ |
|------------------|--------|--------------|
| $H_0$            | 0.0719 (Gyr$^{-1}$) | 0.0717 (Gyr$^{-1}$) |
| $\Omega_{m0}$    | 0.258  | 0.261        |
| $\Omega_{\Lambda 0}$ | 0.742  | 0.733        |
| $\Omega_{\phi}$  | 0.0098 | 0.014        |
| $\chi^2_{\text{min}}$ | 24.343 | 38.779       |
| $\chi^2_{\nu}$   | 0.579  | 0.745        |
where \( q_1 = \Omega_{\sigma 0} \left( 5 \sqrt{1 - \Omega_\phi} + 1 \right) (z + 1)^5 \sqrt{1 - \Omega_\phi} \).

The present value of deceleration parameter is obtained as

\[
q_0 = -1 + \frac{2\Omega_\alpha 0 + \Omega_m 0 \left( 2.5 \left( \sqrt{1 - \Omega_\phi} - 1 \right) + 3 \right) + \Omega_\sigma 0 \left( 5 \left( \sqrt{1 - \Omega_\phi} - 1 \right) + 6 \right)}{2 (\Omega_\alpha 0 + \Omega_\Lambda 0 + \Omega_m 0 + \Omega_\sigma 0)}
\]

(34)

The dynamics of deceleration parameter with the age of the universe ID is depicted in Fig. 4. The derived model represents a transitioning universe with a transition redshift of about \( z_t = 0.79 \). We observe that the current universe is in accelerating phase, while it was in decelerating phase of expansion in the past. The present value of deceleration parameter \( q_0 \) is about \(-0.61 \). This value of \( q_0 \) is in excellent agreement with recent observations.

4.2 The age of the universe

The age of the universe is obtained as

\[
\int t = -\frac{dz}{(1 + z) H_{\sigma BD}} \Rightarrow \int_t^0 dt = -\int_z^0 \frac{1}{(1 + z) H_{\sigma BD}} dz
\]

(35)

Equations (29) and (35) lead to

\[
t_0 - t = \int_0^z \frac{(1 - \Omega_\phi)^\frac{1}{2} dz}{H_0(1 + z) \left[ \Omega_m 0 (1 + z)^{2.5 (\sqrt{1 - \Omega_\phi} - 1)} + 3 + \Omega_\sigma 0 (1 + z)^{5 (\sqrt{1 - \Omega_\phi} - 1)} + 6 + \Omega_\alpha (1 + z)^2 + \Omega_\Lambda 0 \right]^\frac{1}{2}}
\]

(36)
Fig. 4  Plot of deceleration parameter versus the redshift $z$. The transition redshift is $z_t = 0.79$.

Fig. 5  Plot of $H_0(t_0 - t)$ versus the redshift $z$ for $\Omega_{m0} = 0.261$, $\Omega_\Lambda = 0.733$ and $\Omega_\phi = 0.014$.

Here, $t_0$ is the present age of the universe. Hence,

$$t_0 = \lim_{s \to \infty} \int_0^\infty \frac{(1 - \Omega_\phi)^{\frac{1}{2}} dz}{H_0(1 + z) \left[ \Omega_{m0}(1 + z)^2(\sqrt{1 - \Omega_\phi}) + 3 + \Omega_\sigma0(1 + z)^3(\sqrt{1 - \Omega_\phi}) + 6 + \Omega_\sigma(1 + z)^2 + \Omega_\Lambda0 \right]^{\frac{1}{2}}}$$

(37)

Integrating Eq. (37), we get

$$H_0 t_0 = 0.977621$$

(38)

From Eq. (38), the present age of the universe is read as $t_0 = 0.977621 H_0^{-1} \sim 13.65$ Gyrs. The plot of $H_0(t_0 - t)$ versus redshift $z$ is graphed in Fig. 5. From Fig. 5, we observe that at the present time, i.e., for $z = 0$, $H_0(t_0 - t)$ is null which turns to imply $t = t_0$. 

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4.3 The particle horizon

The particle horizon is the furthest distance from which one can retrieve information from the past and hence defines the observable universe [75]. Thus, the particle horizon is represented by proper distance measured by light signal coming from $t = 0$ to $t = t_0$.

Here, we assume light signal is emitted from a source along x-axis. The proper distance of the source will be $a_0 x$, and we are receiving that signal at the present time $t_0$. Thus, the proper distance of the source from us is calculated as $a_0 \int_{t_0}^{t_p} \frac{dt}{a(t)}$ where $t_p$ is the time in the past at which the light signal was transmitted from the source.

Therefore, the particle horizon is computed as

$$R_p = \lim_{t_p \to 0} a_0 \int_{t_0}^{t_p} \frac{dt}{a(t)} = \lim_{z \to \infty} \int_0^z \frac{dz}{H_\sigma BD}$$

Using Eq. (29), Eq. (39) becomes

$$R_p = \lim_{z \to \infty} \int_0^z \frac{(1 - \Omega_\phi)^{\frac{1}{2}} dz}{H_0 \left[ \Omega_m (1 + z)^2 (1 - \Omega_\phi) + \Omega_\sigma (1 + z)^2 (1 - \Omega_\phi) + \Omega_\Lambda \right]^\frac{1}{2}}$$

Integrating Eq. (40) for $\Omega_m = 0.261$, $\Omega_\Lambda = 0.733$ and $\Omega_\phi = 0.014$, we obtain

$$R_p = \frac{2.668}{H_0}$$

Figure 6 shows variation in proper distance versus redshift. From Fig. 6, we observe that at present, i.e. for $z = 0$, $a_0 H_0 x$ is null which turns to imply that $x \to \infty$. Thus, we are at infinite distance from the first event occurred in the past.

Fig. 6  Plot of proper distance $a_0 H_0 x$ versus the redshift $z$ for $\Omega_m = 0.261$, $\Omega_\Lambda = 0.733$ and $\Omega_\phi = 0.014$
4.4 The jerk parameter

The jerk parameter \( j \) [76] in terms of redshift is given by

\[
j = 1 - (1 + z) \frac{H'_{\sigma BD}}{H_{\sigma BD}} + \frac{1}{2} (1 + z)^2 \frac{[H''_{\sigma BD}]^2}{[H_{\sigma BD}]^3}
\]  
(42)

Equations (29) and (42) lead to

\[
j = 1 - (1 + z) \xi_1 + (1 + z)^2 \xi_2
\]  
(43)

where

\[
\xi_1 = \frac{2.5 \Omega_{m0} \left( \sqrt{1 - \Omega_{\Lambda}^5} + 0.2 \right) (z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 2(z + 1) \Omega_{m0} + \Omega_{\sigma0} \left( 5 \sqrt{1 - \Omega_{\Lambda}^5} + 1 \right) (z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1}{2 \sqrt{1 - \Omega_{\Lambda}^5} \sqrt{\Omega_{\alpha0} + \Omega_{m0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1}}
\]

\[
\xi_2 = \frac{(w_3 - w_1)^2}{w_4}
\]

\[
w_1 = \frac{6.25 \left( \Omega_{m0} (1 - \sqrt{1 - \Omega_{\Lambda}^5} + 0.2) (z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + (z + 1)^2 \Omega_{m0} + \Omega_{\sigma0} (z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1 \right)^2}{(z + 1)}
\]

\[
w_2 = \frac{4 \left( \Omega_{\alpha0} + \Omega_{m0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + (z + 1)^2 \Omega_{m0} + \Omega_{\sigma0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1 \right)}{(z + 1)^3 (z + 1)}
\]

\[
w_3 = \frac{\Omega_{m0} w_5 (z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + (z + 1)^2 \Omega_{m0} + \Omega_{\sigma0} (z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1)}{(-12.5 \Omega_{\Lambda}^5 + 2.5 \sqrt{1 - \Omega_{\Lambda}^5} + 12.5) (z + 1)^3 \sqrt{1 - \Omega_{\Lambda}^5}}
\]

\[
w_4 = \frac{32 w_6 \left( \Omega_{\alpha0} + \Omega_{m0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + (z + 1)^2 \Omega_{m0} + \Omega_{\sigma0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1 \right)}{3 z + 3}
\]

\[
w_5 = \frac{6 \left( \Omega_{\alpha0} + \Omega_{m0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + (z + 1)^2 \Omega_{m0} + \Omega_{\sigma0}(z + 1)^2 \sqrt{1 - \Omega_{\Lambda}^5} + 1 \right)}{3 z + 3}
\]

In 2004, Blandford et al. [77] have described the features of the jerk parameterization which gives an alternative approach to describe cosmological models close to ΛCDM model. A powerful feature of the jerk parameter is that for the CDM model \( j = 1 \). In Refs. [78, 79], the authors have investigated the important features of \( j \) for discriminating different dark energy models. The value \( j \neq 1 \) would favor a non-ΛCDM model. In the considered model, the explicit behavior of \( j \) is shown in Fig. 7. We observe that the jerk parameter of the considered model does not have \( j = 1 \).

4.5 Shear scalar and relative anisotropy

The shear scalar is read as

\[
\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}
\]  
(44)

where \( \sigma_{ij} = u_{i; j} - \theta (g_{ij} - u_i u_j) \)

In derived model, the shear scalar is given by

\[
\sigma^2 = \frac{\dot{D}^2}{D^2} = k^2 (1 + z)^{6+5 \sqrt{1 - \Omega_{\Lambda}^5} - 1}
\]  
(45)

Thus, the relative anisotropy is obtained as

\[
A_m = \frac{\sigma^2}{\rho_m}
\]  
(46)
From Eq. (46), it is clear that relative anisotropy depends on redshift $z$. For high value of redshift, the relative anisotropy is large and it decreases with low value of $z$ and finally becomes null at $z \rightarrow 0$. This behavior of relative anisotropy $A_m$ is depicted in Fig. 8.

5 Concluding remarks

In this paper, we have investigated a transitioning model of an isotropic universe in Brains–Dicke theory of gravitation. We describe that the current phase of accelerated expansion of the universe is due to contribution coming from A screened scalar field and the transition redshift is $z_t = 0.79$. For redshift $z > z_t$, the universe was in decelerating phase of expansion. Some important features of derived model are as follows:

1. The derived model obeys Mach’s principle.
2. We find constraints on $H_0$, $\Omega_m$ and $\Omega_\Lambda$ by bounding the model under consideration with recent OHD and BAO data. The best fit values of $H_0$ are closer to other investigations.
Thus, we conclude the present OHD and BAO data provide well-constrained values of $H_0$ and our model has good consistency with recent observations.

3. We have estimated the present age of the universe as $t_0 = 13.65$ Gyrs. This age of the universe nicely matches with those obtained by Plank collaboration.

4. The dynamics of deceleration parameter shows a signature flipping from early decelerating phase to current accelerating phase at $z_t = 0.79$. The present value of deceleration parameter is computed as $q_0 = -0.61$.

5. In the derived model, particle horizon exists and its value is different from $Λ$CDM model of universe.

6. In the derived model, $j \neq 1$. Therefore, the derived solution describes the model of universe other than $Λ$CDM and the deviation from $j = 1$ investigates the dynamics of different kinds of dark energy models other than $Λ$CDM. Some important applications of non-$Λ$CDM model of the universe are given in Refs. [80,81].

References

1. A.G. Riess et al., Astron. J. 116, 1009 (1998)
2. S. Perlmutter et al., Astrophys. J. 517, 565 (1999)
3. C.L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003)
4. G. Hinshaw et al., Astrophys. J. Suppl. 148, 135 (2003)
5. D.N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003)
6. E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)
7. K. Bamba et al., Astrophys. Space Sci. 342, 155 (2012)
8. O. Akarsu, C.B. Killinc, Gen. Relativ. Gravit. 42, 119 (2010)
9. S. Kumar, C.P. Singh, Gen. Relativ. Gravit. 43, 1427 (2011)
10. A.K. Yadav, Astrophys. Space Sci. 335, 565 (2011)
11. A.K. Yadav, L. Yadav, Int. J. Theor. Phys. 50, 218 (2011)
12. A.K. Yadav, F. Rahaman, S. Ray, Int. J. Theor. Phys. 50, 871 (2011)
13. A.K. Yadav, Astrophys. Space Sci. 361, 276 (2016)
14. H. Amirhashchi, Phys. Rev. D 96, 123507 (2017)
15. H. Amirhashchi, S. Amirhashchi, Phys. Rev D 99, 02316 (2018)
16. P.H.R.S. Moraes, P.K. Sahoo, Eur. Phys. J. C 77, 480 (2017)
17. A.K. Yadav, Euro Phys. J. Plus 129, 194 (2014)
18. A.K. Yadav, A.T. Ali, Int. J. Geom. Methods Mod. Phys. 15, 1850026 (2017)
19. V. Singh, C.P. Singh, Int. J. Theor. Phys. 55, 1257 (2015)
20. R. Myrzakulov, Eur. Phys. J. C 72, 2203 (2012)
21. M.J.S. Houndjo, Int. J. Mod. Phys. D 21, 1250003 (2012)
22. F. Kiani, K. Nozari, Phys. Lett. B 728, 554 (2014)
23. A.K. Yadav, Braz. J. Phys. 49, 262 (2019)
24. C. Brans, R.H. Dicke, Phys. Rev. 124, 925 (1961)
25. O. Bertolami, P.J. Martins, Phys. Rev. D 61, 064007 (2000)
26. H. Kim, Mon. Not. R. Astron. Soc. Lett. 364, 813 (2005)
27. T. Clifton, J.D. Barrow, Phys. Rev. D 73, 104022 (2006)
28. S.K. Rama, S. Gosh, Phys. Lett. B 383, 32 (1996)
29. S.K. Rama, Phys. Lett. B 373, 282 (1996)
30. A. Sheykhi, Phys. Rev. D 81, 023525 (2010)
31. A. Sheykhi, M. Jamil, Phys. Lett. B 694, 284 (2011)
32. A. Pasqua, S. Chattopadhyay, Astrophys. Space Sci. 348, 284 (2013)
33. V. Fayaz, Astrophys. Space Sci. 361, 86 (2016)
34. Y. Fujii, K.-I. Maeda, The Scalar–Tensor Theory of Gravitation (Cambridge University Press, Cambridge, 2003)
35. V. Faraoni, Cosmology in Scalar–Tensor Gravity (Kluwer Academic Publishers, Dordrecht, 2004)
36. K. Uehara, C.W. Kim, Phys. Rev. D 26, 2575 (1982)
37. D. Lorenz-Petzold, Phys. Rev. D 29, 2399 (1984)
38. S. Sen, A.A. Sen, Phys. Rev. D 63, 124006 (2001)
39. N. Banerjee, D. Pavon, Phys. Rev. D 63, 043504 (2001)
40. O. Akarsu, N. Katirci, N. Ozdemir, J.A. Vazque, Euro. Phys. J. C 80, 32 (2020)
41. M. Kiran et al., Astrophys. Space Sci. 356, 407 (2015)
42. K.S. Adhav et al., Astrophys. Space Sci. 353, 249 (2014)
43. G. Ramesh, S. Umadevi, Astrophys. Space Sci. 361, 50 (2016)
44. D.R.K. Reddy et al., Astrophys. Space Sci. 361, 349 (2016)
45. K.D. Naidu, D.R.K. Reddy, Y. Aditya, Euro. Phys. J. Plus 133, 303 (2018)
46. H. Amirhashchi, Phys. Rev. D 97, 063515 (2018)
47. O. Akarsu, S. Kumar, S. Sharma, L. Tedesco, Phys. Rev. D 100, 023532 (2019)
48. H. Amirhashchi, S. Amirhashchi. arXiv:1802.04251v4 [astro-ph.CO] (2019)
49. G.K. Goswami, M. Mishra, A.K. Yadav, A. Pradhan, Mod. Phys. Lett. A (2020). https://doi.org/10.1142/S0217732320500868
50. F. Beutler et al., Mon. Not. R. Astron. Soc. 423, 3430 (2012)
51. A.J. Ross et al., Mon. Not. R. Astron. Soc. 449, 835 (2015)
52. S. Alam et al. [BOSS Collaboration]. arXiv:1607.03155 [astro-ph.CO] (2017)
53. L. Anderson et al., BOSS Collaboration. Mon. Not. R. Astron. Soc. 441, 24 (2014)
54. E.A. Kazin et al., Mon. Not. R. Astron. Soc. 441, 3524 (2014)
55. E. Macaulay et al.. arXiv:1811.02376 (2018)
56. C. Zhang et al., Res. Astron. Astrophys. 14, 1221 (2014)
57. J. Simon, L. Verde, R. Jimenez, Phys. Rev. D 71, 123001 (2005)
58. D. Stern et al., JCAP 1002, 008 (2010)
59. M. Moresco et al., JCAP 08, 006 (2012)
60. E. Gazta Naga et al., MNRAS 399, 1663 (2009)
61. C.H. Chuang, Y. Wang, MNRAS 435, 255 (2013)
62. S. Alam et al. arXiv:1607.03155 (2016)
63. M. Moresco et al., JCAP 05, 014 (2016)
64. C. Blake et al., MNRAS 425, 405 (2012)
65. A.L. Ratsimbazafy et al., MNRAS 467, 3239 (2017)
66. M. Moresco, MNRAS 450, L16 (2015)
67. N.G. Busca et al., Astron. Astrophys. 552, 18 (2013)
68. T. Delubac et al., Astron. Astrophys. 574, A59 (2015)
69. A. Font-Ribera et al., BOSS Collaboration. JCAP 1405, 027 (2014)
70. G. Chen, B. Ratra, B, PASP 123, 1127 (2011)
71. E. Aubourg et al., Phys. Rev. D 92, 123516 (2015)
72. G. Chen, S. Kumar, B. Ratra, Astrophys. J. 835, 86 (2017)
73. G. Hinshaw et al., Astrophys. J. Suppl. Ser. 208, 25 (2013)
74. H. Amirhashchi, A. K. Yadav. arXiv:1908.04735 [gr-qc] (2019)
75. B.M. Bentabol, J.M. Bentabol, J. Cepa, J. Cosmol. Astropart. Phys. 02, 015 (2013)
76. P. Mukherjee, S. Chakrabarti. arXiv:1908.01564 [gr-qc] (2019)
77. R.D. Blandford et al., ASP Conf. Ser. 339, 27 (2004). [arXiv:astro-ph/0408279]
78. V. Sahni, T.D. Saini, A.A. Starobinsky, U. Alam, JETP Lett. 77, 201 (2003)
79. U. Alam, V. Sahni, T.D. Saini, A.A. Starobinsky, MNRAS 344, 1057 (2003)
80. O. Akarsu, T. Dereli, Int. J. Theor. Phys. 51, 2995 (2012)
81. J.K. Singh, R. Nagpal. arXiv:1910.09289 [physics.gen-ph] (2019)
82. O. Akarsu et al. arXiv:1903.06679v1 [gr-qc] (2019)
83. S. Kumar, A.K. Yadav, Mod. Phys. Lett. A 26, 647 (2011)