DEMIXING STRUCTURED SUPERPOSITION SIGNALS FROM PERIODIC AND APERIODIC NONLINEAR OBSERVATIONS

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ABSTRACT

We consider the demixing problem of two (or more) structured high-dimensional vectors from a limited number of nonlinear observations where this nonlinearity is due to either a periodic or an aperiodic function. We study certain families of structured superposition models, and propose a method which provably recovers the components given (nearly) \( m = \mathcal{O}(s) \) samples where \( s \) denotes the sparsity level of the underlying components. This strictly improves upon previous nonlinear demixing techniques and asymptotically matches the best possible sample complexity. We also provide a range of simulations to illustrate the performance of the proposed algorithms.

1. INTRODUCTION

1.1. Motivation

The demixing problem involves disentangling two (or more) high-dimensional vectors from their linear superposition, and has several applications in signal and image processing, statistics, and data analysis \([3, 9]\). In applications involving signal recovery, such superpositions can be used to model situations where there is some ambiguity in the components (e.g., the true components can be treated as “ground truth” + “outliers”) or when there is some existing prior knowledge that the true underlying vector is a superposition of two components. Mathematically, suppose that the underlying signal is given by

\[
y = \Phi \theta_1 \Psi \theta_2 + e,
\]

where \( y \) denotes a nonlinear link function and \( e \) denotes observation noise. This is akin to the Generalized Linear Model (GLM) as well as the Single Index Model (SIM) from statistical learning \([9]\). Here, the problem is to estimate \( \theta_1 \) and \( \theta_2 \) from the observations \( y \) with as few measurements as possible. The estimation problem \( (1) \) is challenging in several different aspects:

(i) there is a basic identifiability issue of obtaining \( \theta_1 \) and \( \theta_2 \) even with perfect knowledge of \( \beta \);

(ii) there is a second identifiability issue arising from the nontrivial null-space of the design matrix (since \( m \ll n \)); and

(iii) the nonlinear nature of \( g \), as well as the presence of noise \( e \) can further confound recovery.

Standard techniques to overcome each of these challenges are well-known. By and large, these techniques all make some type of sparsity assumption on the components \( \theta_1 \) and \( \theta_2 \) \([8]\); some type of incoherence assumption on the bases \( \Phi \) and \( \Psi \) \([10, 11] \); and some regularity condition on \( g \) (which we elaborate later). However, to our knowledge, the confluence of the three above challenges have not been simultaneously addressed in the literature.

1.2. Summary of contributions

In this paper, we focus on the case where the components \( \theta_1, \theta_2 \) obey certain structured sparsity assumptions. Structured sparsity models are useful in applications where the support patterns (i.e., the coordinates of the nonzero entries) belong to model-specific restricted families (for example, the support is assumed to be group-sparse \([12]\)). It is known that such assumptions can significantly reduce the required number of samples for estimating the underlying signal, compared to generic sparsity assumptions \([13, 14]\). We consider two classes of link functions: aperiodic and periodic functions, and accordingly, two different demixing approaches. Our approach in this paper builds upon and extends our recent previous work on nonlinear demixing \([4, 6] \).

In the aperiodic case, we follow the setup of \([4] \) where \( g \) is assumed to be monotonic; satisfies some type of restricted strong convexity (RSC) \([7]\); and some type of restricted strong smoothness (RSS) assumptions \([16]\). For this case, we develop a non-convex iterative algorithm that stably estimates the components \( \theta_1 \) and \( \theta_2 \).

In the periodic case, we use the approach of \([17]\) both for designing the matrix \( X \) and the link function \( g \). Specifically, we let \( X \) be factorized as \( X = DB \), where \( D \in \mathbb{R}^{m \times q} \) and \( B \in \mathbb{R}^{q \times n} \) have some specific structures; please see Section 2 for details. Again, for this case, we demonstrate a novel two-stage algorithm that stably estimate the components \( \theta_1 \) and \( \theta_2 \).

For both cases considered above, we show that under certain sufficiency conditions, the performance of our methods strictly improves upon previous nonlinear demixing techniques, and asymptotically matches (close to) the best possible sample-complexity.

1.3. Prior work

The demixing problem has been a recent focus in several fields including signal and image processing, machine learning, and computational physics \([2]\). The majority of the literature on the demixing problem studies the case of linear superposition of two or more components where these components can be modeled in various ways such as sparse vectors \([3]\), low-rank and sparse matrices \([18]\), and...
manifold models [14][19]. Recently, a few papers have addressed the nonlinear setting where the observations are index-wise nonlinear functions of the superposition of the components [4][8]. This nonlinear demixing framework can also be considered as a special instance of nonlinear signal recovery which has recently received broad attention [20][23]. For instance, [4] considers the nonlinear demixing of a pair of sparse vectors with arbitrary supports where the nonlinearity is a monotonic function, obtains a sufficient condition on the number of samples for achieving a desired estimation accuracy. On the other hand, [17] studies the problem of nonlinear signal recovery and demixing, where the nonlinearity is a period function.

We note that demixing approaches in high dimensions with structured sparsity assumptions have appeared before in the literature [21][22]. However, our method differs from these earlier works in a few different aspects. The majority of these methods involve solving a convex relaxation problem; in contrast, our algorithm is manifestly non-convex. Despite this feature, for certain types of structured superposition models, our method provably recovers the components given (nearly) \( m = O(s) \) samples. Moreover, these earlier methods have not explicitly addressed the nonlinear observation model (with the exception of [25]). In this paper, we leverage the structured sparsity assumptions to our advantage, and show that this type of structured sparsity priors significantly decreases the sample complexity (both for periodic and aperiodic nonlinearities) for estimating the signal components.

2. PRELIMINARIES

Let \( \| \cdot \| \) denote the \( \ell_q \)-norm of a vector. Denote the spectral norm of the matrix \( X \) as \( \| X \| _{\text{spect}} \). The true parameter vector, \( \theta = [\theta_1; \theta_2] \in \mathbb{R}^{2n} \) as the vector obtained by stacking the true and unknown coefficient vectors, \( \theta_1, \theta_2 \). For simplicity of exposition, in this paper we suppose that the components \( \theta_1 \) and \( \theta_2 \) exhibit block sparsity with sparsity \( s \) and block size \( b \) [13]. (Analogous approaches apply for other structured sparsity models.)

The problem (1) is inherently ill-posed. To resolve this issue, we need to assume that the coefficient vectors \( \theta_1, \theta_2 \) are somehow distinguishable from each other. This is characterized by a notion of incoherence of the components \( \theta_1, \theta_2 \) [6].

Definition 1. The bases \( \Phi \) and \( \Psi \) are called \( \varepsilon \)-incoherent if \( \varepsilon = \sup_{\| \Phi u \| \leq 1, \| \Psi v \| \leq 1} \| \langle \Phi u, \Psi v \rangle \| / \min(\| \Phi u \| , \| \Psi v \| ) \).

For the analysis of aperiodic link functions, we need the following standard definition [7]:

Definition 2. A function \( f : \mathbb{R}^{2n} \rightarrow \mathbb{R} \) satisfies Structured Restricted Strong Convexity/Smoothness (SRSC/SRSS) if:

\[
m_{\text{SRSC}} \leq \| \nabla^2 f(t) \| \leq M_{\text{SRSS}}, \quad t \in \mathbb{R}^{2n},
\]

where \( \xi = \sup_{t_i} \{ t \in \mathbb{R}^{2n} \} \cup \{ t \in \mathbb{R}^{2n} \} \) such that \( t_i \) belongs to \( (2s,b) \) block-sparse vectors for \( i = 1, 2 \), and \( m_{\text{SRSC}} \) and \( M_{\text{SRSS}} \) are (respectively) the SRSC and SRSS constants. Also, \( \nabla^2 f(t) \) denotes a \( 4s \times 4s \) sub-matrix of the Hessian matrix \( \nabla^2 f(t) \) comprised of rows/columns indexed by \( \xi \subseteq [2n] \).

Furthermore, for aperiodic functions, we assume that the derivative of the link function is strictly bounded either within a positive interval, or within a negative interval. In addition, let \( \beta_d \) denotes the \( j \)-th entry of the signal \( \beta \in \mathbb{R}^n \). Also, for \( j \in \{1, 2, \ldots, q\} \), \( \beta(j : q : (k-1)q + j) \in \mathbb{R}^{2d} \) denotes the sub-vector of \( \beta \) starting at index \( j + qr \), where \( r = 0, 1, \ldots, k - 1 \). Finally, \( Y'(j : q : (k-1)q + j) \) represents the sub-vector made by picking the \( j \)-th column of any matrix \( Y \) and choosing the entries of this column as stated.

For the analysis of periodic link functions, by following the approach of [17], we let the design matrix \( X \) be factorized as \( X = DB \), where \( D \in \mathbb{R}^{m \times q} \) and \( B \in \mathbb{R}^{q \times n} \). We assume that \( m \) is a multiple of \( q \), and that \( D \) is a concatenation of \( k \) diagonal matrices of \( q \times q \) such that the diagonal entries in the blocks of \( D \) are i.i.d. random variables distributed uniformly within an interval \([-T, T]\) for some \( T > 0 \). The choice of \( B \) is flexible and can be chosen such that it supports stable demixing. In particular, as [4] has shown, \( B \) can be any random matrix with independent subgaussian rows.

Overall, our low-dimensional observation model can be written as:

\[
y = g(DB\beta) + e = g(DB(\Phi\theta_1 + \Psi\theta_2)) + e,
\]

where \( g \) is either sinusoidal function, or any periodic function such that in each period, it behaves monotonically. Furthermore, \( D = [D_1, \ldots, D_k]^T \) comprises \( k \) diagonal matrices \( D_i \)'s, and \( e \in \mathbb{R}^m \) denotes additive noise such that \( e \sim \mathcal{N}(0, \sigma^2 I) \). The goal is to stably recover \( \theta_1, \theta_2 \) from the embedding \( y \). The structural parameter of the matrix \( D \) reduces the the final recovery of underlying components to first obtaining a good enough estimation of \( B\beta \), and then using a linear demixing approach from a (possibly noisy) estimate of \( B\beta \) will lead to the estimation of \( \theta_1, \theta_2 \).

3. ALGORITHMS AND ANALYSIS

In this section, we describe our algorithm and theoretical result for both aperiodic and periodic link functions.

3.1. Aperiodic link functions

To solve the demixing problem in (1), we consider the minimization of a special loss function \( F(t) \), following [6]:

\[
\min_{t \in \mathbb{R}^{2n}} F(t) = \frac{1}{m} \sum_{i=1}^{m} \Theta(x_i^T \Gamma t) - y_i x_i^T \Gamma t \quad \text{s. t.} \quad t \in D,
\]

where \( \Theta'\cdot \phi(q\Gamma), \Gamma = [\Phi, \Psi], x_i \) is the \( i \)-th row of the design matrix \( X \) and \( D \) denotes the set of length-2p vectors formed by stacking a pair of \((s, b)\) block-sparse vectors. The objective function in (3) is motivated by the single index model in statistics; for details, see [6]. To approximately solve (3), we propose an algorithm which we call it Structured Demixing with Hard Thresholding (STRUCT-DHT). The pseudocode of this algorithm is given in Algorithm 1. At a high level, STRUCT-DHT tries to minimize loss function defined in (3) (tailed to \( g \)) between the observed samples \( y \) and the predicted responses \( X\tilde{\Gamma}t \), where \( \tilde{\Gamma} = [\tilde{\theta}_1; \tilde{\theta}_2] \) is the estimate of the parameter vector after \( N \) iterations. The algorithm proceeds by iteratively updating the current estimate of \( \tilde{\Gamma} \) based on a gradient update rule followed by (myopic) hard thresholding of the residual onto the set of \( s \)-sparse vectors in the span of \( \Phi \) and \( \Psi \). Here, we consider a version of DHT [6] which is applicable for the case that coefficient vectors \( \theta_1 \) and \( \theta_2 \) have block sparsity. For this setting, we use component-wise block-hard thresholding, \( P_{s,b}^{(B)} \). Specifically, \( P_{s,b}^{(B)} \) projects the vector \( \hat{f}^s \in \mathbb{R}^{2n} \) onto the set of concatenated \((s, b)\) block-sparse vectors by projecting the first and the second half of \( \hat{f}^s \) separately. Now, we provide the theorem supporting the convergence analysis and sample complexity (required number of observations for successful estimation of \( \theta_1, \theta_2 \)) of STRUCT-DHT.

Theorem 3. Consider the observation model (1) with all the assumption and definitions mentioned in the section 2. Suppose
Algorithm 1 Structured Demixing with Hard Thresholding (STRUCT-DHT)

**Inputs:** Bases $\Phi$ and $\Psi$, design matrix $X$, link function $g$, observation $y$, sparsity $s$, block size $b$, step size $\eta$.

**Outputs:** Estimates $\hat{\theta} = \Phi \hat{\theta}_1 + \Psi \hat{\theta}_2, \hat{\theta}_1, \hat{\theta}_2$

**Initialization:**
\[ (\beta^0, \theta_1^0, \theta_2^0) \leftarrow \text{RANDOM INITIALIZATION} \]

$k \leftarrow 0$

while $k \leq N$ do

\[ t^k \leftarrow [\theta_1^k; \theta_2^k] \]  
\[ t_1^k \leftarrow \frac{1}{\eta} X^T (g(X \beta^k) - y) \]

\[ t_2^k \leftarrow \frac{1}{\eta} \nabla F_k (t_1^k; t_2^k) \]  
\[ i^k \leftarrow t^k - \eta \nabla F_k \]  
\[ [\theta_1^k; \theta_2^k] \leftarrow \mathcal{P}_{s,b} \{ i^k \} \]  
\[ \hat{\beta}^k \leftarrow \Phi \theta_1^k + \Psi \theta_2^k \]  
\[ k \leftarrow k + 1 \]

end while

Return: $(\hat{\theta}_1, \hat{\theta}_2) \leftarrow (\theta_1^N, \theta_2^N)$

**Proof sketch.** The proof follows the technique used to prove Theorem 4.6 in [4]. The main steps are as follows. Let $b' \in \mathbb{R}^{2b} = [b_1; b_2] = t^k - \eta \nabla F(t^k), b = t^k - \eta \nabla F(t^k)$ where $J := J_k \supseteq \supp(\beta^k) \cup \supp(\beta^{k+1}) \cup \supp(\theta_1^k; \theta_2^k)$. Also define $t^{k+1} = \mathcal{P}_{s,b}(b') = [\mathcal{P}_s(b_1); \mathcal{P}_b(b_2)]$. By the triangle inequality, we have: $\|t^{k+1} - \theta\|_2 \leq \|t^{k+1} - b\|_2 + \|b - \theta\|_2$. The proof is completed by showing that $\|t^{k+1} - b\|_2 \leq 2\|b - \theta\|_2$. Finally, we use the Khintchine inequality [25] to bound the expectation of the $\ell_2$-norm of the restricted gradient function, $\nabla F(\theta)$ (evaluated at the true stacked signal $\theta$) with respect to the support set $J$.

The inequality [4] indicates the linear convergence behavior of our proposed algorithm. Specifically, in the noiseless scenario to achieve $\kappa$-accuracy in estimating the parameter vector $\hat{\theta} = [\hat{\theta}_1; \hat{\theta}_2]$, STRUCT-DHT only requires $\log \left( \frac{s}{\kappa} \right)$ iterations. We also have the following theorem regarding the sample complexity of Alg. [1]

**Theorem 4.** If the rows of $X$ are independent subgaussian random vectors [25], then the required number of samples for successful estimation of the components is given by $O(\frac{K}{\kappa} \log \frac{1}{\delta})$. Furthermore, if $b = \Omega(\log \frac{s}{\kappa})$, then the sample complexity of our proposed algorithm is given by $m = O(s)$, which is asymptotically optimal.

**3.2. Periodic link functions**

In this section, we focus on the periodic link functions which are either sinusoidal (complex-exponential), or any periodic function such that it is monotonic within each period. We start with the sinusoidal (complex-exponential) link function and follow the approach of [17]. In [17], the authors proposed an algorithm called MF-Sparse for recovering an underlying signal which is arbitrary sparse, or is the superposition of two arbitrary sparse components, but they only considered sinusoidal link functions. This algorithm has two steps: first step outputs a vector $z$ as the estimate of $z = B\beta$ from measurement $y$ in [3]. The idea is to use leverage the structure of the block diagonal matrix $D$ to decouple the estimation of each entry in $z$ through a tone estimation algorithm proposed in [27]. Then, $\hat{z}$ is used as the input for the second step where any sparse recovery technique can be used to estimate the underlying signal. The CoSaMP algorithm [23] has been used for the second step.

In our case, we use MF-Sparse algorithm as a core algorithm for estimating the underlying components albeit with two differences: first, we might have a preprocessing step before tone estimation depending on the periodic nonlinearity. More precisely, if we use a link function except sinusoidal, we first map the observation vector $y$ to $\tilde{y}$ through a sinusoidal function and use this new observation vector $\tilde{y}$ as the input to the second step, tone estimation. To give a explanation why this method works, we note that in each period, the link function is assumed to be monotonic; as a result, for each entry of $\tilde{y}$, there is one and only one entry from $y$. Thus, we can use the method of recovery under sinusoidal nonlinearity to estimate the underlying components $\hat{\theta}_1, \hat{\theta}_2$. Second, for the third stage, we invoke STRUCT-DHT with identity link function $g(x) = x$ instead of any regular sparse recovery method. We call the resulting algorithm MF-STRUCT-DHT and is given in Algorithm 2.

By combining Theorem 4 and Theorem 2.1 in [17], we obtain the sample complexity of the MF-STRUCT-DHT scheme to achieve $\kappa$-accuracy.

**Theorem 5** (Sample complexity of MF-STRUCT-DHT). Consider the measurement model in [4] without any additive noise. Assume that the nonzero entries of block diagonal matrix $D$ are i.i.d. random variables, distributing uniformly within the interval $[-T, T]$, and...
the rows of $B$ are independent subgaussian random vectors (normalized by $\frac{1}{\sqrt{n}}$). Moreover, assume $\|x\|_2 \leq R$ for some constant $R > 0$. If we set $m = kq$ where $k = c_1 \log \left( \frac{2\kappa}{\omega} \right)$ for some $\kappa > 0$, $q = \mathcal{O} \left( \frac{\kappa}{\omega} \right)$, $\omega = c_2 R$, and $\Omega = [ -\omega, \omega ]$, the MF-STRUCT-DHT scheme provides an estimate $\hat{\beta}$, such that $\| \hat{\beta} - \beta \|_2 = \mathcal{O}(\kappa)$, with probability at least $1 - \delta$. Here, $c_1, c_2$ are constants. Furthermore, if $b$ scales as $b = \mathcal{O}(\log \frac{2}{\omega})$, then the sample complexity of the MF-STRUCT-DHT scheme is given by $m = \mathcal{O}(s)$, which is asymptotically optimal.

Proof. The proof follows from a straightforward application of Theorem 2.1 in [17]. According to this result, one can estimate $\hat{\beta}$ by $B\hat{\beta}$ up to $\nu$-accuracy if $T$ scales as $|T| = \mathcal{O}(\frac{1}{\nu})$. Under this choice for $T$, the required number of block diagonal matrices in $D$ to achieve $\nu$ accuracy for estimating $z$ is given by $k = \mathcal{O}(\log(\frac{1}{\nu}))$ where $|T| = \mathcal{O}(R)$ (see Algorithm 2). Now by choosing the design matrix $B \in \mathbb{R}^{s \times n}$, final accuracy parameter $\kappa$ as $\nu = \mathcal{O}(\frac{\kappa}{\omega})$, and choosing $q = \mathcal{O}(\frac{\kappa}{\omega})$ according to Theorem 2, the result follows.

Note that big-Oh constant does not depend on the bounds on the derivative of the link function since it is a identity function. In addition, if the periodic link function $g$ is set to the sinusoidal (complex-exponential), then the additive noise can be added to $\hat{z}$. In this case, the sample complexity is increased by a multiplicative factor equals to $1 + \sigma^2$ where $\sigma^2$ denotes the variance of the Gaussian noise; see [17] for details.

4. NUMERICAL RESULTS

To show the efficacy of STRUCT-DHT for demixing components with structured sparsity for aperiodic link functions, we numerically compare STRUCT-DHT with ordinary DHT (which does not leverage structured sparsity), and also with an adaptation of a convex formulation described in [16] that we call Demixing with Soft Thresholding (DST). We first generate true components $\theta_1$ and $\theta_2$ with length $n = 2^{16}$ with nonzeros grouped in blocks with length $b = 16$ and total sparsity $s = 656$. The nonzero (active) blocks are randomly chosen from a uniform distribution over all possible blocks.

We construct a design (observation) matrix following the construction of [29]. Finally, we use a (shifted) sigmoid link function $g(x) = \frac{1}{1 + e^{-x}}$ to generate the observations $y$. Figure 1(a) shows the performance of the three algorithms with different number of samples averaged over 10 Monte Carlo trials. In Figure 1(b), we plot the probability of successful recovery, defined as the fraction of trials where the normalized error is less than 0.05. Figure 1(b) shows the normalized estimation error for these algorithms. As we can observe, STRUCT-DHT shows much better sample complexity (the required number of samples for obtaining small relative error) as compared to DHT and DST.

We conduct a similar experiment for two periodic link functions: sinusoidal and sawtooth (modulo) functions with period $2\pi$ and amplitude 1. The parameters are as before, except we set $n = 2^{14}$, $s = 160$, and $k = 4$. We numerically compare MF-STRUCT-DHT scheme with the case where we do not consider the structured sparsity, and with a convex relaxation formulation [16]. Figures 1(c) and (d) show the probability of success for the sinusoidal and sawtooth cases, respectively. Again, we get the same conclusion as in the aperiodic case: our proposed algorithm achieves far improved sample complexity over previous existing methods that solely rely on sparsity assumptions.

5. CONCLUSIONS

In this paper, we addressed the problem of demixing from a set of limited nonlinear measurements in high dimensions. Specifically, we considered two nonlinearities: aperiodic and periodic link functions and the structured sparsity in the underlying signal components. For each of these nonlinearities, we proposed an algorithm and support them with sample complexity analysis. As a result of our proposed schemes, we showed that having structured sparsity assumption in the underlying components can significantly reduce the sample complexity compared to the case where we just have regular sparsity prior in these components. Finally, we verified our theoretical claims with some experimental results.
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