A Common Origin for all CP Violations

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We put forward the conjecture that all CP violating phenomena may have a common origin. In order to illustrate our idea, we present a minimal model where CP is spontaneously broken at a high energy scale, through the phase in the vacuum expectation value of a complex scalar singlet. This single phase is the origin of both low energy CP violation in the quark and leptonic sectors, as well as leptogenesis. We also show that in this framework the strong CP problem may be solved in a simple way through the introduction of a $Z_4$ symmetry which allows for the implementation of the Nelson-Barr mechanism.

I. INTRODUCTION

The phenomenon of CP violation plays a central rôle in Particle Physics and it has profound implications for Cosmology, since it is one of the necessary ingredients for generating the observed baryon asymmetry of the Universe (BAU). From a phenomenological point of view, one may consider the following four aspects of CP violation:

(i) Quark Sector: CP violation was first discovered in the K-meson sector about four decades ago 1 and recently was also detected in the B-sector through CP asymmetries in neutral B-meson decays 2. 3.

(ii) Lepton Sector: In the Standard Model (SM), neutrinos are strictly massless and therefore there is neither leptonic mixing nor CP violation in the leptonic sector, in the context of the SM. However, any extension of the SM which accounts for the recently observed neutrino oscillations through nonzero neutrino masses implies, in general, CP violation in the leptonic sector which might be detected in future experiments performed at neutrino factories. Forthcoming experiments on neutrinoless double beta decay may also give indirect evidence for the presence of a non-vanishing phase in the leptonic mixing matrix.

(iii) Generation of BAU: One may also interpret the existence of a matter dominated Universe as another evidence for CP violation. It has been established that within the framework of the SM it is not possible to generate the observed size of BAU, due in part to the smallness of CP violation in the SM. This provides motivation for considering new sources of CP violation beyond the Kobayashi-Maskawa (KM) mechanism. A very interesting scenario for generating BAU is that provided by leptogenesis 4, where first the out-of-equilibrium decay of righthanded neutrinos creates a lepton asymmetry which is then converted into a baryon asymmetry through B-violating but (B-L) conserving sphaleron mediated processes 5. An interesting question is whether, within the framework of leptogenesis, it is possible to relate the CP violation necessary to generate BAU, with leptonic CP violation observable at low energies 6, 7, 8. It has been shown that this connection exists only in specific models 9, 10.

(iv) The Strong CP problem: Another aspect of CP violation has to do with the fact that in the context of the SM and taking into account nonperturbative instanton effects, strong interactions do violate CP. This leads to the so-called strong CP problem 11 for which various solutions have been proposed.

In this paper we address the question of whether it is possible to find a framework where all these manifestations of CP violation have a common origin. In particular, we describe a minimal model with spontaneous CP violation, where CP breaking both in the quark and leptonic sectors arises solely from a phase $\alpha$ in the vacuum expectation value of a complex scalar singlet $S$, with $\langle S \rangle = \frac{1}{2} \exp(i\alpha)$. Since $S$ is an SU(2) × U(1) × SU(3)_c singlet, $V$ can be much larger than the electroweak breaking scale. Therefore, in this framework CP violation is generated at a high energy scale. In order for the phase $\alpha$ to generate a non-trivial phase at low energies in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one is led to introduce at least one vector-like quark, whose left-handed and righthanded components are singlets under SU(2). In the leptonic sector, righthanded neutrinos play the rôle of the vector-like quarks, establishing the connection between CP breaking at high and low energies. With the introduction of a $Z_4$ symmetry in the Lagrangean it is possible to find a solution to the strong CP problem, of the type proposed by Nelson 11 and Barr 12. We show that in the leptonic sector, one can get CP violation required to have a viable leptogenesis while also generating CP violation at low energies, detectable for instance through neutrino oscillations.

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II. THE MODEL

We add to the SM the following fields: a singlet charge \( \frac{1}{3} \) vectorial quark \( D^0 \), three righthanded neutrino fields \( \nu_R^0 \) (one per generation) and a neutral scalar singlet field, \( S \). We impose a \( Z_4 \) symmetry, under which the fields transform in the following manner:

\[
D^0 \rightarrow -D^0, \quad S \rightarrow -S \\
\psi_l^0 \rightarrow i\psi_l^0, \quad e_R^0 \rightarrow ie_R^0, \quad \nu_R^0 \rightarrow i\nu_R^0
\]

where \( \psi_l^0 \) denotes the lefthanded lepton doublets, while \( e_R^0, \nu_R^0 \) stand for the righthanded charged lepton and neutrino singlets, respectively. All other fields remain invariant under the \( Z_4 \) symmetry. Furthermore, we impose CP invariance on the Lagrangian, thus constraining all Yukawa couplings to be real. In any weak basis (WB) the Yukawa terms can be written as:

\[
L_Y = L_q + L_l \\
L_q = \overline{\psi}_u^0 G_u \phi u_R^0 + \overline{\psi}_d^0 G_d \phi d_R^0 + + (f_1 S + f'_1 S^*) \overline{D}_L^0 \psi_l^0 + \overline{D}_R^0 M_D^0 + h.c. \\
L_l = \overline{\psi}_l^0 G_l \phi e_R^0 + \overline{\psi}_l^0 G_u \phi \nu_R^0 + \frac{1}{2} \nu_R^0 \lambda \overline{D}_L^0 S (f_l S + + f_{l'} S^*) \nu_R^0 + h.c.
\]

Here \( \psi_u^0, \psi_d^0 \), and \( \nu_R^0 \) are the SM quark fields, and \( \phi \) is the SM Higgs doublet. Notice that an additional bare mass term of the form \( \overline{D}_L^0 \overline{D}_R^0 \) was included in \( L_q \). This term is both gauge and \( Z_4 \) invariant and is present in the Lagrangian together with the mass terms arising from the Yukawa interactions upon \( SU(2) \times U(1) \times Z_4 \) symmetry breaking. The scalar potential will contain terms in \( \phi \) and \( S \) with no phase dependence, together with terms of the form \( (\mu^2 + \lambda_1 S S + \lambda_3 \phi^3)(S^2 + S'^2) + \lambda_3 (S^4 + S'^4) \) which, in general, lead to the spontaneous breaking of T and CP invariance \[13\] with \( \phi \) and \( S \) acquiring vacuum expectation values (vevs) of the form:

\[
\langle \phi \rangle = \frac{v}{\sqrt{2}}, \quad \langle S \rangle = \frac{V \exp(i\alpha)}{\sqrt{2}}
\]

III. THE HADRONIC SECTOR

A crucial aspect of this model is the fact that the phase \( \alpha \equiv \arg(S) \) at a high energy scale does generate at low energies a CP violating phase \( \delta_{KM} \) in the \( 3 \times 3 \) sector of the mixing matrix connecting standard quarks. In this respect, the presence of the vector-like quark \( D^0 \) plays a crucial rôle, since it is through the couplings \( (f_1 S + f'_1 S^*) \overline{D}_L^0 \psi_l^0 \) that the phase \( \alpha \) appears in the effective mass matrix for the down standard-like quarks. Without loss of generality, one may choose to work in a weak basis where the up quark mass matrix is diagonal. In this basis, it can be readily shown \[14\] that the \( 3 \times 3 \) \( V_{CKM} \) matrix, mixing the standard quarks in the charged weak current is obtained through the following relations:

\[
V_{CKM}^{-1} h V_{CKM} = d^2
\]

\[
h \equiv m_d^0 m_u^0 - (m_d^0 M_D^0 M_D^0 m_u^0) / M^2
\]

where \( d^2 = \text{diag}(m_d^2 m_d^2, m_u^2) \), \( m_d^0 = \sqrt{2} G_d \), \( M^2 = M_D M_D^0 + M^2 \) and \( M_D = \frac{1}{\sqrt{2}} M_{15} \cos(\alpha) + i f_{l}^0 \sin(\alpha) \), with \( f_\pm \equiv f_1 \pm f_{l}' \).

It is clear from Eq. (6) that the phase \( \delta_{KM} \), generated through spontaneous CP violation is not suppressed by factors of \( \frac{v}{\sqrt{2}} \). Note that we are assuming that the mass terms \( (M_D) \) are of the same order of magnitude as \( M \). This is a reasonable assumption since both terms are \( SU(2) \times U(1) \times SU(3) \), invariant. For very large V (e.g. \( V \sim M_{GUT} \sim 10^{15} \text{ Gev} \), \( \delta_{KM} \) is the only leftover effect at low energies, from spontaneous CP breaking at high energies. For not so large a value of \( V \) (e. g., \( V \) of the order of a few Tev) the appearance of significant flavour changing neutral currents (FCNC) in the down quark sector leads to new contributions to \( B_d \rightarrow \overline{\nu} \nu \) and \( B_s \rightarrow \overline{\nu} \nu \), mixing which can alter \[15\] some of the predictions of the SM for CP asymmetries in B meson decays. These FCNC are closely related to the non-unitarity of the \( 3 \times 3 \) CKM matrix, with both effects suppressed by powers of \( \frac{v^2}{V^2} \).

As a result of the \( Z_4 \) symmetry, this model satisfies the Nelson-Barr criteria \[11,12\] and therefore the \( \Theta \) parameter is zero in tree approximation. Recall that the parameter \( \Theta \) associated with strong CP violation can be written as \( \Theta = \Theta_{QCD} + \Theta_{QFD} \), where \( \Theta_{QCD} = g_s^2 FF / 32 \pi^2 \), and \( \Theta_{QFD} = \arg(det m) \) denoting the quark mass matrix. In this model CP is a good symmetry of the Lagrangian, only spontaneously broken by the vacuum, which implies \( \Theta_{QCD} = 0 \). Furthermore, \( \Theta_{QFD} \) vanishes at tree level \[14\] in a natural way so that higher order corrections to \( \Theta \) are finite and calculable. The symmetry \( Z_4 \) plays a crucial rôle in the vanishing of the argument of the determinant of the down type quark mass matrix \( M_D \). One-loop corrections are suppressed by small Yukawa couplings which is a general property of this class of models, as pointed out by Nelson \[11\]. A nice feature of this model is that one loop corrections are further suppressed by the ratio \( v^2 / V^2 \).

IV. THE LEPTONIC SECTOR

In the leptonic sector, after spontaneous symmetry breakdown, one obtains from Eq. (1) the following mass terms:

\[
\mathcal{L}_m = - \left[ \overline{\nu}_R^0 m_R^0 + \frac{1}{2} \nu_R^0 \nu_R^0 + \overline{\nu}_L^0 m_l^0 \right] + h.c. = \left[ \frac{1}{2} \overline{\nu}_L^0 C M^* \nu_L + \overline{\nu}_L^0 m_l^0 \right] + h.c.
\]
where $m$, $M$ and $m_l$ denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix and the charged lepton mass matrix, respectively, and $n_L = (\nu^0_L, (\nu^0_R)^\dagger)$. In this model we have:

$$\mathcal{M} = \begin{pmatrix} 0 & m \cr m^T & M \end{pmatrix}, \quad m_l = \frac{v}{\sqrt{2}} G_l, \quad m = \frac{v}{\sqrt{2}} G_p$$

$$M = \frac{V}{\sqrt{2}} (f^+_\nu \cos(\alpha) + i f^-_\nu \sin(\alpha))$$

with $f^+\nu \equiv f_\nu + f_\nu^\dagger$. It is clear that $m_l$ and $m$ are real and $M$ is complex and symmetric. In the leptonic sector the $Z_4$ symmetry prevents the existence of a mass term of the form $\frac{1}{2} \nu_R^0 \mathcal{C} \mathcal{M} \nu_R^0$. Yet, a term of this form will be generated through the couplings of $\nu_R^0$ to the scalar singlet $S$, after $Z_4$ breaking.

In the weak basis where $m_l$ is chosen to be diagonal and real the light neutrino masses and the low energy mixing are obtained from the diagonalization of the effective neutrino mass matrix $m_{\text{eff}} \equiv -\frac{1}{16\pi} m^T$:

$$-K^i m^1 \frac{1}{M} m^T K^* = d_\nu,$$  \hspace{1cm} (10)

In this weak basis, the $V_{\text{MNS}}$ matrix is given by $K$ after eliminating three of its factorizable phases. Although $m$ is a real matrix, since $M^{-1}$ is a generic complex symmetric matrix, $m_{\text{eff}}$ is also a generic complex symmetric matrix. Therefore $K$ has three CP violating phases, one Dirac-type and two Majorana-type. On the other hand, the heavy neutrino masses are, to an excellent approximation, the eigenvalues of the matrix $M$. In the WB where both $m_l$ and $M$ are diagonal and real, the lepton-number asymmetry, resulting from the decay of a heavy Majorana neutrino $N^3$ into charged leptons $l_i^\pm$ ($i = e, \mu, \tau$) is given by:

$$A^j = \frac{g^2}{M_{W*}} \sum_{k,j} \text{Im} \left( (m^1 m)_{jk} (m^1 m)_{jk} \right) \times \left[ \frac{1}{16\pi} \left( I(x_k) + \sqrt{\frac{1}{2}} \left( 1 - x_k \right) \frac{1}{(m^1 m)_{jj}} \right) \right],$$  \hspace{1cm} (11)

with the lepton-number asymmetry from the $j$ heavy Majorana particle, $A^j$, defined in terms of the family number asymmetry $\Delta A^j_i = N^j_i - \overline{N}^j_i$ by:

$$A^j = \frac{\sum_i \Delta A^j_i}{\sum_i (N^j_i + \overline{N}^j_i)}$$  \hspace{1cm} (12)

the sum in $i$ runs over the three flavours $i = e, \mu, \tau$, $M_k$ are the heavy neutrino masses, the variable $x_k$ is defined as $x_k = \frac{M_k^2}{M_W^2}$ and $I(x_k) = \sqrt{x_k} \left( 1 + (1 + x_k) \log \left( \frac{1}{1 + x_k} \right) \right)$. From Eq. (11) it can be seen that the lepton-number asymmetry is only sensitive to the CP-violating phases appearing in $m^1 m$ in this WB.

We show next that in the present model $m^1 m$, in the WB where $m_l$, $M$ are diagonal real and positive, will contain in general the CP violation required by leptogenesis.

CP violation in the general case where $m_l$, $m$ and $M$ are complex has been discussed in a previous work [8]. It was shown that in the special WB where $m_l$ and $M$ are diagonal real and positive, all CP violating phases appear in the matrix $m$, which is a general complex matrix, and therefore can be written as $m = W^\dagger dV = UH$, where in the first equality $W$ and $V$ are general unitary matrices with $d$ diagonal real and positive, and the second equality is the polar decomposition into the product of a unitary and a hermitian matrix. Three phases in $U$ can be eliminated and $m$ is left with six independent phases. Whilst low energy CP violation is only sensitive to the three phases appearing in $V_{\text{MNS}}$, leptogenesis only sees the three phases appearing in $m^1 m = H^1 H$.

In our special framework, in the WB where $m_l$ and $M$ are real diagonal and positive, the most general matrix $m$ can be written as $m = O^T d T$ with $O$ orthogonal real and $T$ unitary. Since three of the factorizable phases in $T$ do not commute with the matrix $O$, $m$ still has six independent phases. The important point is that in this model the product $m^1 m = T^1 d^2 T$ is entirely general and therefore one may have CP violation required by leptogenesis. It can be readily seen from the definition of $m_{\text{eff}}$ that in this framework the absence of CP violation at high energies (i.e., a real matrix $T$) immediately implies no CP violation at low energies (i.e., no CP violation in $K$). On the other hand, if $T$ is a complex matrix, in general it is not possible to have $m_{\text{eff}}$ real, thus implying CP violation also at low energies. A distinctive feature of this scenario is the fact that $(m^1 m^1)$ is a now a real matrix. Note that in supersymmetric seesaw models the predictions for $\text{Br}(l_i \rightarrow l_j \gamma)$ are directly related to the size of $(m^1 m^1)_{jj}$ and are potentially large [17]. Furthermore, it has been shown [8] that in the limit of exactly degenerate heavy neutrino masses, in the general case, CP violation in charged lepton flavour violating processes arises only from the phase contained in $(m^1 m^1)$. Since in the present model $(m^1 m^1)$ is real, CP violation in those processes vanishes in this limit. Similar arguments apply to the electric dipole moments of the charged leptons which are already strongly suppressed in the above limit [18]. It is important to emphasize that in the limit of degenerate heavy neutrinos non-trivial phases can still be generated in $V_{\text{MNS}}$.

V. CONCLUSIONS

We suggest that all physical manifestations of CP violation may have a common origin. In order to illustrate our conjecture, we have presented a specific minimal model where CP is spontaneously broken at a higher energy scale, through the vacuum expectation value of one complex scalar singlet. A vector-like quark is added to the spectrum of the SM and its couplings to standard quarks play a crucial role for the generation of a CP violating phase in the CKM matrix. Righthanded neutrinos acquire complex mass terms through their couplings.
to the complex scalar singlet. We have shown that the model has the remarkable feature that although it has only one fundamental CP violating phase, there is CP violation at low energies both in the quark and lepton sectors and furthermore there is sufficient CP violation in order to have viable leptogenesis.

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