Parametric control of a superconducting flux qubit

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Quantum state engineering has become one of the most important arenas in quantum physics. In particular, the coherent control of quantum two-state systems (TSS), which are applicable to quantum bits (qubit), has attracted increasing interest in the context of quantum computing and quantum information processing. Various candidate physical systems are being studied for the future implementation of qubits. These include artificial quantum TSS like superconducting qubits and naturally existing quantum TSS like nuclear spins. Of the many candidates that may enable us to realize quantum computation, superconducting qubits based on Josephson junctions have gained increasing importance because of their potential controllability and scalability.

The coherent control of a single qubit has been demonstrated in many types of superconducting circuits, such as charge, charge-phase, phase, and flux qubits. Recently, two-qubit operation has been demonstrated in charge and phase qubits. In addition to qubit operation, the superconducting qubit offers a testing ground for exploring interactions between photons and artificial macroscopic objects, which we shall refer to as “atoms”. In the weak-driving limit, the interaction between a single “atom” and a single microwave photon has been demonstrated with a charge qubit, which is strongly coupled to a superconducting transmission line resonator. In the strong-driving regime, superconducting qubits have exhibited nonlinear optical responses: multi-photon Rabi oscillations have been observed in a charge qubit by using microwave pulses, and under continuous microwave irradiation, multi-photon absorption has been observed in a phase and a flux qubit.

In this Letter, we describe the parametric control of a superconducting flux qubit with two-frequency microwave pulses. We have succeeded in observing two-photon Rabi oscillations of the qubit caused by a parametric transition when the qubit Larmor frequency matches either the sum of the two microwave frequencies or the difference between them. We also show multi-photon Rabi oscillations corresponding to one- to four-photon resonances by applying single-frequency microwave pulses. The parametric control demonstrated in this work widens the frequency range of microwaves for controlling the qubit and offers a high quality testing ground for exploring nonlinear quantum phenomena.
the flux qubit measurement system. On-chip components are set in the sample holder. We put adequate copper powder filters CP and LC filters F and attenuators A for each line.

resistors. There are two well-controlled resonance modes for each line.

The flux qubit consists of a loop of on-chip components close to the qubit. One is the dc-SQUID’s plasma mode with a frequency of 1.0 GHz, which is formed in the two symmetrical loops, each being composed of L, C, 2C, and the SQUID’s Josephson inductance. The other is the harmonic LC resonance mode with a frequency of 4.311 GHz, which is produced in the larger loop consisting of the two L’s and the two C’s. In this way, we have achieved an artificial TSS in a well-controlled environment.

The three Josephson junctions of the qubit form a double-well potential in the space of the Josephson phase when about half a flux quantum threads the qubit loop. We use the two lowest levels in the potential as the qubit states, which are well separated from the higher levels. Thus, the qubit is described by the Hamiltonian $H_{qb} = (\hbar/2)(\varepsilon \sigma_z + \Delta \sigma_x)$, where $\sigma_x, \sigma_z$ are the Pauli spin matrices. The eigenstates of $\sigma_z$ describe clockwise and counter-clockwise persistent currents in the qubit. The qubit tunnel splitting is described by $\hbar \Delta$, and $\hbar \varepsilon = 2I_p \Phi_0 / (\Phi_0 - 1.5)$ is the energy imbalance between the two potential wells caused by the externally applied magnetic flux threading the qubit loop $\Phi_{qb}$, where $\Phi_0 = \hbar/2e$ is the flux quantum and $I_p = I_c \sqrt{1 - (1/2\alpha)^2}$ is the magnitude of the qubit persistent current when the qubit is in the $\sigma_z$ eigenstates and $I_c$ is the critical current of the larger junctions. The energy difference between the ground state $|g\rangle$ and the first excited state $|e\rangle$ of the qubit is $\hbar \omega_{qb} = h f_{gb} = \hbar \sqrt{\varepsilon^2 + \Delta^2}$. Assuming that the applied microwaves are in coherent states, we may describe the qubit under microwave irradiation by the Hamiltonian

$$\hat{H} = \frac{\hbar}{2} (\varepsilon \sigma_z + \Delta \sigma_x) + \sum_{k=1}^{l} \frac{2\hbar g_k \alpha_k \sigma_z \cos \omega_{MW_k} t}, \quad (1)$$

where $l$ is one (two) in the case of a single- (two-) frequency microwave and $g_k$ is the coupling between the qubit and the $k$-th microwave ($MW_k$), whose amplitude and frequency are $\alpha_k$ and $\omega_{MW_k} = \omega_{MW_k}/2\pi$, respectively. Solving the Schrödinger equation with the Hamiltonian (1) without the rotating wave approximation, we obtain the time evolution of the probability $P_e(t)$ with which we find the qubit in $|e\rangle$. The probability $P_e(t)$ oscillates periodically under resonant conditions $\omega_{qb} = n_1 \omega_{MW1}$ (for the single-frequency microwave) or $\omega_{qb} = n_1 \omega_{MW1} \pm n_2 \omega_{MW2}$ (for the two-frequency microwave), where $n_k$ is the MWk photon number. When we operate the qubit away from its degeneracy point $\varepsilon \neq 0$, the frequency of the oscillation is given for the single-frequency microwave irradiation by

$$\Omega_{Rabi} = \frac{\Delta}{A} J_{n_1} \left( A \frac{g_{10}}{\omega_{MW1}} \right) \quad (2)$$

and for the two-frequency microwave irradiation by

$$\Omega_{Rabi} = \frac{\Delta}{A} J_{n_1} \left( A \frac{g_{10}}{\omega_{MW1}} \right) J_{n_2} \left( A \frac{g_{20}}{\omega_{MW2}} \right). \quad (3)$$

Here $J_{n_k}$ is the $n_k$-th order Bessel function of the first kind and $A \equiv \varepsilon / \omega_{qb} \approx 1$. This approximation is valid when $\varepsilon >> \Delta$. The dressed atom approach gives results similar to Eq. (2) and (3).

The measurements were carried out in a dilution refrigerator. The sample was mounted in a gold plated copper box that was thermalized to the base temperature of 20 mK ($k_B T << \hbar \omega_{qb}$). To produce two-frequency microwave pulses, we added two microwaves MW1 and MW2 with frequencies of $f_{MW1}$ and $f_{MW2}$, respectively, by using a splitter SP (Fig. 1(b)). Then we shaped them into microwave pulses through two mixers. We measured the amplitude of MWk $V_{MWk}$ at the point between the attenuator and the mixer with an oscilloscope. We confirmed that unwanted higher-order frequency components in the pulses, for example $f_{MW1} \pm f_{MW2}$, $2f_{MW1}$, and $2f_{MW2}$ are negligibly small under our experimental conditions. First, we choose the operating point by setting $\Phi_{qb}$ around 1.5$\Phi_0$, which fixes the qubit Larmor frequency $f_{qb}$. The qubit is thermally initialized to be in $|g\rangle$ by waiting for 300 $\mu$s, which is much longer than the qubit energy relaxation time (for example 3.8 $\mu$s at $f_{qb} = 11.1$ GHz). Then a qubit operation is performed by applying a microwave pulse to the qubit. The pulse, with an appropriate length $t_p$, amplitudes $V_{MWk}$, and frequencies $f_{MWk}$, prepares a qubit in the superposition state of $|g\rangle$ and $|e\rangle$. After the operation, we immediately apply a dc readout pulse to the dc-SQUID. This dc pulse consists of a short (70 ns) initial pulse followed by a long (1.5 $\mu$s) trailing plateau that has 0.6 times the amplitude of the initial part. For $\Phi_{qb} < 1.5\Phi_0$, if the qubit is detected as being in $|e\rangle$, the SQUID switches to a voltage state and an output voltage pulse should be observed.
Four-photon Rabi oscillations when one to four-photon absorption processes. The solid curves are 10.25 GHz. The external flux is $\Phi$.

FIG. 2: Experimental results with single-frequency microwave dots represents the resonant frequencies $f_{\text{res}}$ caused by the one to four-photon absorption processes. The solid curves are numerical fits. The dashed line shows a microwave frequency $f_{\text{MW1}}$ of 10.25 GHz. (a) One-photon Rabi oscillations of $P_{\text{sw}}$ with exponentially damped oscillation fits. Both the qubit Larmor frequency $f_{\text{QB}}$ and the microwave frequency $f_{\text{MW1}}$ are 10.25 GHz. The external flux is $\Phi_{\text{QB}}/\Phi_0 = 1.4944$. (c) Four-photon Rabi oscillations when $f_{\text{QB}} = 4.0$ GHz, $f_{\text{MW1}} = 10.25$ GHz, and $\Phi_{\text{QB}}/\Phi_0 = 1.4769$. (d) The microwave amplitude dependence of the Rabi frequencies $\Omega_{\text{Rabi}}/2\pi$ up to four-photon Rabi oscillations. The solid curves represent theoretical fits.

otherwise there should be no output voltage pulse. By repeating the measurement 8000 times, we obtain the SQUID switching probability $P_{\text{sw}}$, which is directly related to $P_e(t_p)$ for the dc readout pulse with a proper amplitude. For $\Phi_{\text{QB}} > 1.5\Phi_0$, $P_{\text{sw}}$ is directly related to $1 - P_e(t_p)$.

We performed a spectroscopy measurement of the qubit with long (50 ns) single-frequency microwave pulses. We observed multi-photon resonant peaks ($\Phi_{\text{QB}} < 1.5\Phi_0$) and dips ($\Phi_{\text{QB}} > 1.5\Phi_0$) in the dependence of $P_{\text{sw}}$ on $f_{\text{MW1}}$ at a fixed magnetic flux $\Phi_{\text{QB}}$. We obtained the qubit energy diagram by plotting their positions as a function of $\Phi_{\text{qb}}/\Phi_0$ (Fig. 2a). We took the data around the degeneracy point $\Phi_{\text{qb}} \approx 1.5\Phi_0$ by applying an additional dc pulse to the microwave line to shift $\Phi_{\text{qb}}$ away from $1.5\Phi_0$ just before the readout, because the dc-SQUID could not distinguish the qubit states around the degeneracy point. The top solid curve in Fig. 2a represents a numerical fit to the resonant frequencies of one-photon absorption. From this fit, we obtain the qubit parameters $E_1/h = 213$ GHz, $\Delta/2\pi = 1.73$ GHz, and $\alpha = 0.8$. The other curves in Fig. 2a are drawn by using these parameters for $n_1 = 2, 3,$ and 4.

Next, we used short single-frequency microwave pulses with a frequency of 10.25 GHz to observe the coherent quantum dynamics of the qubit. Figures 2b and (c) show one- and four-photon Rabi oscillations observed at the operating points indicated by arrows in Fig. 2a) with various microwave amplitudes $V_{\text{MW1}}$. These data can be fitted by damped oscillations $\alpha \exp(-t_p/T_\text{d}) \cos(\Omega_{\text{Rabi}}/2\pi)$, except for the upper two curves in Fig. 2b). Here, $t_p$ and $T_\text{d}$ are the microwave pulse length and qubit decay time, respectively. To obtain $\Omega_{\text{Rabi}}$, we performed a fast Fourier transform (FFT) on the curves that we could not fit by damped oscillations. Although we controlled the qubit environment, there were some unexpected resonators coupled to the qubit, which could be excited by the strong microwave driving or by the Rabi oscillations of the qubit. We consider that these resonators degraded the Rabi oscillations in the higher $V_{\text{MW1}}$ range of Fig. 2 b). Figure 2(d) shows the $V_{\text{MW1}}$ dependences of $\Omega_{\text{Rabi}}/2\pi$ up to four-photon Rabi oscillations, which are well reproduced by Eq. 2. Here, we used only one scaling parameter $a(10.25$ GHz $) = 0.013$ defined as $a(f_{\text{MW1}}) \equiv 4g_1\alpha_1/\omega_{\text{MW1}}V_{\text{MW1}}$, because it is hard to measure the real amplitude of the microwave applied to the qubit at the sample position. The scaling parameter $a(f_{\text{MW1}})$ reflects the way in which the applied microwave is attenuated during its transmission to the qubit and the efficiency of the coupling between the qubit and the on-chip microwave line. In this way, we can estimate the real microwave amplitude and the interaction energy between the qubit and the microwave $2\hbar g_1\alpha_1$ by fitting the dependence of $\Omega_{\text{Rabi}}/2\pi$ on $V_{\text{MW1}}$. These results show that we can reach a driving regime that is so strong that the interaction energy $2\hbar g_1\alpha_1$ is larger than the qubit transition energy $\hbar\omega_{\text{qb}}$.

We have also performed experiments with two microwave frequencies $f_{\text{MW1}}$ and $f_{\text{MW2}}$. First, we carried out a spectroscopy measurement by using long (50 ns) two-frequency microwave pulses. In addition to resonances caused by the multi-photon absorption processes at multiples of each microwave frequency ($f_{\text{QB}} = n_1 f_{\text{MW1}}, n_2 f_{\text{MW2}}$), we also clearly observed those due to parametric processes ($f_{\text{QB}} = |f_{\text{MW1}} \pm f_{\text{MW2}}|$) (not shown).

We next investigated the coherent oscillations of the qubit through the parametric processes by using short two-frequency microwave pulses. Figure 2a) (b) shows
superconducting flux qubit. First, we observed multi-photon Rabi oscillations caused by up to four-photon transitions by using single-frequency microwave pulses. The microwave amplitude dependences of the Rabi frequencies are well reproduced by Bessel functions derived from a semi-classical model. Furthermore, we successfully demonstrated parametric control of the qubit by using two-frequency microwave pulses. We observed Rabi oscillations of the qubit caused by parametric transitions when \( f_{qb} = |f_{MW1} \pm f_{MW2}| \). The Rabi frequencies as a function of the microwave amplitudes are well described by the product of two Bessel functions. These results indicate that the flux qubit offers a good testing ground for exploring quantum nonlinear phenomena in a macroscopic quantum object. Furthermore, these multi-photon processes observed in our experiment widen the frequency range of microwaves for controlling flux qubit.

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