Parametric injection for monoenergetic electron acceleration

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Abstract. Electrons are accelerated in the laser wakefield (LWFA). This mechanism has been studied by 2D or 3D Particle In Cell simulation. However, how the electrons are injected in the wakefield is not understood. In this paper, we consider about the process of self–injection and propose new scheme. When plasma electron density modulates, parametric resonance of electron momentum is induced. The parametric resonance depends on laser waist modulation. We carried out 2D PIC simulation with the initial condition decided from resonance condition. Moreover, we analyze experimental result that generated 200-250 MeV monoenergetic electron beam with 400TW intense laser in CAEP in China.

1. Introduction
To generate a high monoenergetic electron beam, various experiments and numerical analyses were reported [1][2]. Recently, several groups succeeded in generating accelerated electrons over 1GeV with tens of TW lasers. For applications, it is important to control the quality of electron beams like total charge number of electrons, kinetic energy, and energy spread. Those parameters depend on the way of plasma electron injection for further acceleration. It is well known that the condition of trapped electrons in acceleration field is out of the phase of plasma wave. However, wave-breaking theory is not enough to explain the generation of monoenergetic beam.

In this paper, we discuss an electron multiple self-injection provoked by the parametric resonance [3] in relativistic plasmas and present results of simulation for 200 TW focused laser pulses in uniform plasma.

2. Parametric resonance
The mechanism of electron injection can be described by the motion of plasma electrons. As one conceivable scenario, we propose multiple self-injection. Some trapped electrons by the pulse are blown out by their Coulomb force, and flow behind the pulse. Those energetic electrons can be injected in the first cavity. The electrons can be trapped in the second cavity even if not trapped in the first cavity. In addition, the electrons might be blown off again without being accelerated even if
trapped in the cavity. In this section, we propose a new injection scheme with hydrodynamics of energetic plasma electrons.

2.1. Resonance of electron momentum

In 1D geometry, the hydrodynamic equations in the laboratory frame have the following form:

\[
\frac{\partial \epsilon}{\partial \tau} = n_e \frac{p}{\gamma}, \quad \frac{\partial \epsilon}{\partial x} = n_i - n_e, \quad \frac{\partial n_e}{\partial \tau} + \frac{\partial}{\partial x} \left( \frac{p}{\gamma} n_e \right) = 0
\]

\[
\frac{\partial p}{\partial \tau} + \frac{p}{\gamma} \frac{\partial p}{\partial x} = -\epsilon + f_p (x - V_g \tau), \quad \gamma = \sqrt{1 + p^2} \tag{1}
\]

where \( n_e = N_e / N_0, n_i = N_i / N_0, \) and \( N_e, N_i \) are the electron and ion densities, \( N_0 \) is the normalizing density; \( \epsilon \) is the electric field in \( e/mc^2 \omega_{p0} \) units with \( \omega_{p0} \) the plasma frequency at \( N_e = N_0 \); \( p \) is electron momentum normalized on \( mc \); \( f_p \) is a drive force moving with a group velocity \( V_g \). The coordinates \( x \) is in \( k \varepsilon / \omega_{p0} \) units, and \( \tau = \omega_{p0} t \).

When there is a weak temporal modulation of electron density, one may seek a small correction for the stationary solution of Eq.(1) in the moving reference frame in the following form

\[ p'(x', \tau') = p'(x) + \delta p(x', \tau) \] and \( \epsilon'(x', \tau') = \epsilon(x') + \delta \epsilon(x', \tau') \). The corrections obey the following relations:

\[
\frac{\partial \delta \epsilon}{\partial \tau'} = n_e' \frac{\partial p}{\gamma'}, \quad \frac{\partial \delta p}{\partial \tau'} = -\delta \epsilon \text{ or } \frac{\partial^2 \delta p}{\partial \tau'^2} = -n_e \frac{\partial p}{\gamma'} \tag{2}
\]

Since the self-injection appear only when \( p' , \ll 1 \), we set \( \gamma' \approx \sqrt{1 + \delta p^2} \). Equation (2) describes that there should be a long-wave oscillation of a cold electron layer. The initial conditions are \( \delta p(0) = 0 \) and \( (\partial \delta p / \partial \tau')(0) = -u \), the derivative \( u \) depends on the initial electron displacement. For \( n_e' = 1 \), this equation has a periodic solution:

\[
\int_0^1 dz / \sqrt{1 + u^2 / 4 - \sqrt{1 + z^2}} = -2 \tau',
\]

with its minimal amplitude \( \delta p_{\text{min}} = -\sqrt{(1 + u^2 / 4) - 1} \). If \( \delta p_{\text{min}} > p'_{\text{min}} \), there is no self-injection.

The resonance is stabilized by an increase in the relativistic factor \( \gamma' \) and, therefore, appears repeatedly. When the electron plasma density modulates, the amplitude of electron momentum also oscillates.

2.2. Laser pulse oscillation of waist diameter of laser beam focused into plasma channel

Usually, the waist of a laser pulse self-guiding in underdense plasma oscillates. In the paraxial approximation, dynamics of pulse waist, \( w(x) \), in a plasma channel with \( N_e(r) = N_e(0) + N_i(R_0) r^2 / R_0^2 \) obeys the following equation [4],

\[
\frac{d^2 U}{dx^2} = \frac{2}{kL_R} \frac{1}{U^3} - \frac{N_e(R_0)}{N_{e,0}} \left( \frac{w_o}{R_0} \right)^2 \frac{U}{\gamma} - \frac{N_e(0)}{N_{e,0}} \frac{a_o^2}{4\gamma U^3} - \frac{4}{kL_R \gamma^2} \left[ \frac{1}{U^3} - \left( \frac{w_o}{R_0} \right)^2 \frac{N_e(R_0)}{N_{e,0}} \right] + 2 \frac{a_o^2}{kL_R \gamma^2 U^3}
\]
where $L_R = \frac{k w_0^2}{2}$ is the Rayleigh length, $a_0$ is the normalized laser field, $U = \frac{w(x)}{w_0}$, $x$ is normalized on $w_0$, $k = \frac{\omega}{c}$, $\gamma = \left(1 + \frac{a_0^2}{2}\right)^{1/2}$. Period and amplitude of this oscillation depend on the initial laser diameter, laser intensity, plasma density, and the transverse plasma density gradient. When laser pulse is propagating in plasma, its radius perpendicular to longitudinal direction oscillates. Pulse waist modulation changes plasma electron density around the buckets. It can be thought that this density change causes the parametric resonance. When the normalized momentum increases by the resonance, electron velocity exceeds the plasma phase velocity. Therefore, electron injection should be affected by the parameter of pulse waist modulation.

2.3. Simulation results

At first, we carry out the 2D simulation of laser pulse propagation in a 6 mm plasma channel. The plasma channel has the minimal density $N_{\text{min}} = 1.2 \times 10^{18}$ cm$^{-3}$ and the maximal $N_{\text{max}} = 4 \times 10^{18}$ cm$^{-3}$; diameter of a parabolic channel is 25 µm. The laser intensity is $I = 4 \times 10^{19}$ W/cm$^2$, the focus diameter is $D = 18$ µm; the duration of the laser pulse is 30 fs. At the laser vacuum interface the density linearly decreases to zero in 150 µm length. Such a density gradient in low-density plasma gives negligibly small injection owing to the density ramp.

Results of the calculation for LWFA in the plasma channel are given in Fig.1 and Fig.2. One can see the multiple self-injection process in the first bucket behind the laser pulse. In Fig.1 one can find up to 4 consequent injections those appear in 4 mm propagation distance, which are seen as small density bunches in the first bucket. This repetitive injection can be explained by parametric resonance.

Secondary, we changed above initial condition. The diameter of plasma channel is 60 µm, the other conditions were not changed. The simulation results are shown in Fig.3 and Fig.4, these figures indicate when the pulse propagated 3.2 mm. In Fig.4, accelerated electrons around 40 MeV and 200 MeV can be observed. These electrons are injected in the second and the following buckets. If the initial condition was slightly changed, the pulse waist modulation also changed. It seems to affect electron injection.

3. CAEP LWFA experiment [5]

Recently, the LWFA with highly relativistic laser pulses, $a_0 > 4$, have been performed; the monoenergetic bunch with its energy $E \sim 200$-250 MeV has been observed. We analyzed this experimental result with similar parameters by 2D PIC simulation. The plasma channel length is 4.5 mm with including the regions with 1.1 mm length where the plasma densities linearly decrease to
zero at both side of the plasma channel. The density of uniform plasma is $4.3 \times 10^{18}$ cm$^{-3}$. The intensity of the laser beam focused into the spot with diameter $D=16$ µm is $I=4.3 \times 10^{19}$ W/cm$^2$; the duration of the laser pulse is 30fs. We neglect the effects of the laser pre-pulse in the present calculation. Figure 4 indicates electron distribution function when the pulse propagated 2.5mm. Monoenergetic electron beam with the energy of 100-120MeV and 2% energy spread can be observed. In the present calculation the maximal energy of electrons is smaller than that measured in the experiment [7]. It may be a result of numerical error in the calculation of the group velocity of laser pulses in lower density plasmas.

4. Conclusion
In summary, we have carried out fully relativistic 2D PIC simulations to study the processes of electron self-injection of the monoenergetic electron laser wake field acceleration in underdense plasma. The pulse waist modulation causes parametric resonance of electron momentum. Then, some electrons exceed plasma group velocity and result in self-injection. Pulse waist modulation strongly depends on plasma channel gradient and laser intensity. Therefore, the qualities of electron beam also depend on those parameters. In case of parametric resonance, high quality monoenergetic electron beam can be generated efficiently.

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