In a recent paper, Harada and Sachs suggested that the Fitzgerald-Lorentz contraction does not refer to a physical contraction, but can perhaps be better understood by comparing it to the conversion of currency between different countries; e.g. the conversion between the Japanese yen in the US dollar and vice versa. In the present paper, this interpretation of the Fitzgerald-Lorentz contraction will be inspected further.

**Key words:** Special Relativity, Einstein, Fitzgerald-Lorentz contraction, Multiple Observers.
I. Introduction.

The special theory of relativity (STR) is a well-established theory which nevertheless predicts some counter-intuitive effects. One of those counter-intuitive effects is the change of size of objects moving with a certain velocity relative to each other. Mathematically, this change in size is related to the Lorentz transformation of coordinates of an observer in space-time. The difficulty with this contraction is its physical interpretation. The main question is whether an object actually changes in size when it moves relative to a relatively stationary observer. Because in STR space and time are treated on equal footing, this question also is relevant to the twin paradox and its physical interpretation (A.D. Allan).

In their paper, Harada and Sachs\(^2\) propose to treat the Fitzgerald-Lorentz contraction similar to the change in currency between the money system of two different countries. This point is inspected by making use of two observers. In the present paper, a number of observers are employed and the interpretation is inspected on consistency. The conclusions reached are to the opinion of the author independent of a type of interpretation.

II. Multiple observers.

In the interpretation of Harada and Sachs, a difference is made between a unit length at relative rest and a unit length in relative motion. Suppose we inspect the situation for an arbitrary observer, \(O_k\). In the view of \(O_k\) we have

\[
L_k = L_{kj} \sqrt{1 - \beta_{kj}^2}.
\]

Here, \(L_k\) is the unit of length at rest in \(O_k\)’s frame of reference, while, \(\beta_{kj} = v_{kj}/c\), is the (normed) velocity of observer \(O_j\)’s frame relative \(O_k\)’s frame and \(L_{kj}\) is the unit of length in motion, relative to observer \(O_k\) and carried by observer \(O_j\). Harada and Sachs state that the factor, \((1 - \beta_{kj}^2)^{1/2}\) can be compared to a ‘currency conversion’. To be more specific, when \(O_k\) wants to measure length in \(O_j\)’s units, who moves with \(\beta_{kj}\) relative to \(O_k\), he has to make use of the (currency) conversion, \((1 - \beta_{kj}^2)^{1/2}\). The observer, \(O_j\), measures with \(L_i\), but \(O_k\) has to use, \(L_{kj}\), as ‘currency’.
Introducing more than two observers, O_k and O_j, we may introduce observer O_i and note that, \( L_k = L_{ki}(1 - \beta_{ki}^2)^{1/2} \). If, furthermore, \( 1 \geq \beta_{ki}^2 > \beta_{kj}^2 \geq 0 \), we may write for \( L_{ki} \) and \( L_{kj} \)

\[
L_{kj} = L_{ki} \sqrt{1 - \frac{\beta_{ki}^2}{\beta_{kj}^2}}
\]

with \( \beta_{ij}^2 \) denoting the 'velocity'

\[
\beta_{kj}^2 = \frac{\beta_{kj}^2 - \beta_{ki}^2}{1 - \beta_{ki}^2}.
\]

Note that, \( 1 - \beta_{ki}^2 \geq \beta_{kj}^2 \geq 0 \), hence, \( \beta_{ij}^2 \) in \([0,1]\).

In addition to the O_k 'point of view' we may also employ the O_j 'point of view'. In this case we write, \( L_j = L_{jk}(1 - \beta_{jk}^2)^{1/2} \), where \( L_j \) is the unit of length at rest in O_j’s frame, \( \beta_{jk} \) the velocity of O_j relative O_k and, \( L_{jk} \) the 'currency' of O_j when he wants to measure in O_k’s unit.

Because, \( \beta_{ki} = \beta_{kj} \), we only may have, \( L_{ik} = L_{kj} \), when, \( L_i = L_k \).

Furthermore, let us state that \( L_i \) is unequal to \( L_k \) (\( \|L_i - L_k\| > 0 \)) and introduce additional observers, O_p, O_q and O_r. In this case we may write, \( L_p = L_{pq}(1 - \beta_{pq}^2)^{1/2} \) together with \( L_q = L_{qp}(1 - \beta_{qp}^2)^{1/2} \). Hence, when the inequality \( 1 - \beta_{pq}^2 \geq \beta_{pq}^2 - \beta_{pr}^2 \), is valid then it follows, \( \beta_{pq}^2 \) in \([0,1]\).

Starting from the supposition that \( L_i \) is unequal to \( L_k \), we are still allowed to suppose that \( \beta_{pq} = \beta_{ki} \). Hence, when, \( \beta_{pq} \) unequal to \( \beta_{ki} \) and \( \beta_{pr} \) unequal to \( \beta_{kj} \), we still may have

\[
\frac{\beta_{pq}^2 - \beta_{pr}^2}{1 - \beta_{pr}^2} = \frac{\beta_{ki}^2 - \beta_{kj}^2}{1 - \beta_{kj}^2}
\]

is possible. This implies that

\[
\frac{L_{pr}}{L_{pq}} = \frac{L_{kj}}{L_{ki}}
\]

which ends in a contradiction when, \( L_p = L_{ik} \) and, \( L_q = L_{kj} \), because, it cannot be avoided that, given, \( \|L_i - L_k\| > 0 \), \( L_q = L_{kb} \).
III. Conclusion and discussion.

In this paper, we arrived at a contradiction when we started with a particular interpretation of the Fitzgerald-Lorentz contraction. The contradiction appears to be too straightforward to be dependent on a single specific interpretation. The possibility of the unearthed contradiction is demonstrated with a numerical study. E.g. given $\beta_{kj}=0.1438$, $\beta_{ki}=0.8833$, together with, $\beta_{pr}=0.0363$, $\beta_{pq}=0.8808$, we have within good approximation $\beta_{kj}=\beta_{pq}$. Starting from $L_{k}=1$, we see that with the previous parameters, $L_{kj}=1.0105$, $L_{ki}=2.1329$, together with, $L_{pq}=1.0098$, $L_{pr}=2.1329$, such that within good approximation, $(L_{pq}/L_{pr})=(L_{kj}/L_{ki})$. Given this, we find that, $L_{k}=0.9993$.

Moreover, an alternative route is to note that $L_{p}=L_{p}(1-\beta_{pq}^{2})^{1/2}$, leads to

$$\beta_{pq}^{2} = 1 - (L_{p}/L_{kj})^{2}$$

Moreover, $L_{p}=L_{k}(1-\beta_{pq}^{2})^{1/2}$, leading to

$$\beta_{pq}^{2} = 1 - (L_{p}/L_{ki})^{2}$$

From the previous two expressions we find that,

$$\beta_{kj}^{2} = 1 - (L_{kj}/L_{ki})^{2}$$

together with

$$\beta_{pq}^{2} = 1 - (L_{pq}/L_{ki})^{2}$$

such that when, $\beta_{pq}=\beta_{kj}$, it easily follows that, $L_{k}=L_{p}$, despite $L_{k}$ unequal to $L_{p}$.

Physically, the contradiction can, for instance, occur in the field of relativistic gas dynamics or, more generally, in relativistic hydrodynamics. Here, the ‘unit of length at rest’ can be related to ‘the radius of the particles at rest’ that make up the gas. We may have, $L_{p}$, for the radius of $O_{p}$ particles, $L_{k}$, for $O_{k}$ particles and $L_{q}$, for $O_{k}$ particles.

If, on the other hand, the contradiction does not occur in the physical reality of, for instance, a relativistic gas, there has to be an implicit physical reason why $\beta_{pq}$ cannot be equal to $\beta_{kj}$ for unequal mutual velocities; $\beta_{kj} \neq \beta_{pr}$, $\beta_{pq} \neq \beta_{pq}$. The question then arises which kind of physical interaction we are dealing with, because the contradiction is valid, even for an ideal gas.
1. A. D. Allan to be published in Physics Essays.

2. M. Harada and M. Sachs, *Physics Essays* 11 1998, 532.

3. L. J. Synge, *The Relativistic Gas*. North-Holland Amsterdam, John Wiley, 1957.

4. A. H. Taub, *Relativistic Hydrodynamics*, p 150, Studies in Appl. Math. Vol 7, 1971.
