Unitarity-safe models of non-minimal inflation in supergravity

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Abstract We show that models of chaotic inflation based on the φ⁵ potential and a linear non-minimal coupling to gravity, f_R = 1 + c_R φ, can be done consistent with data in the context of Supergravity, retaining the perturbative unitarity up to the Planck scale, if we employ logarithmic Kähler potentials with prefactors −p(1+n) or −p(n+1)−1, where −0.035 ≤ n ≤ 0.007 for p = 2 or −0.0145 ≤ n ≤ 0.006 for p = 4. Focusing, moreover, on a model employing a gauge non-singlet inflaton, we show that a solution to the µ problem of MSSM and baryogenesis via non-thermal leptogenesis can be also accommodated.

1 Introduction

Inflation established in the presence of a non-minimal coupling between the inflaton φ and the Ricci scalar R is called collectively non-minimal inflation (nMI) [1–13]. Between the numerous models, which may be proposed in this context, universal attractor models (UAMs) [14] occupy a prominent position since they exhibit an attractor towards an inflationary phase excellently compatible with data [15] for c_R ≫ 1 and φ ≤ m_P where m_P is the reduced Planck mass. UAMs consider a monomial potential of the type

\[ V_{CI}(\phi) = \lambda^2 \phi^p / 2^{p/2} m_p^{p-4} \]  

in conjunction with a strong non-minimal coupling [5,14]

\[ f_R(\phi) = 1 + c_R \phi / m_P \]  

with p = q. The emergence of an inflationary plateau in these models can be transparently shown in the Einstein frame (EF) where the inflationary potential, \( \tilde{V}_{\text{attr}} \), takes the form

\[ \tilde{V}_{\text{attr}} = V_{CI} / f_R^{2} \simeq \lambda^2 m_P^4 / c_R^2, \]

with the exponent in the denominator being related to the conformal transformation employed [1–5] to move from the Jordan frame (JF) to EF.

However, due to the large c_R values needed for the establishment of nMI with φ ≤ m_P, the inflationary scale, \( \tilde{V}_{\text{attr}}^{1/4} \), turns out to be larger than the Ultraviolet (UV) cut-off scale [16–18]

\[ \Lambda_{\text{UV}}^{\text{attr}} = m_P / c_R^{1/(q/2-1)} \]  

with 2 < q ≤ 14/3 of the corresponding effective theory, which thereby breaks down above it. A criticism of these results may be found in Ref. [19], where background-dependent cut-off scales, well larger than \( \tilde{V}_{\text{attr}}^{1/4} \), are evaluated. This practice is rather questionable, though, preventing the possibility of making perturbative extrapolations of the low-energy theory. Indeed, the low-energy theory expanded around the true low-energy vacuum should break at a scale that is calculable within the low-energy field expansion [20]. Therefore, the presence of \( \Lambda_{\text{UV}}^{\text{attr}} \) in Eq. (4) cannot be avoided, and it signals the breakdown of the theory in that field range. Several ways have been proposed to surpass the inconsistency above. E.g., incorporating new degrees of freedom at \( \Lambda_{\text{UV}}^{\text{attr}} \), or assuming additional interactions [22], or invoking a large inflaton vacuum expectation value (v.e.v.) \( \langle \phi \rangle \) [23–27], or introducing a sizable kinetic mixing in the inflaton sector which dominates over \( f_R \) [28–34].

Here we propose a novel solution – applied only in the context of Supergravity (SUGRA) – to the aforementioned problem, by exclusively considering q = 2 in Eq. (2) – cf. Ref. [35]. In this case, the canonically normalized inflaton \( \hat{\phi} \) is related to the initial field φ as \( \hat{\phi} \sim c_R \phi \) at the vacuum of the theory, in sharp contrast to what we obtain for q > 2 where \( \hat{\phi} \sim \phi \). As a consequence, the small-field series of the various terms of the action expressed in terms of \( \hat{\phi} \), does not contain c_R in the numerators, preventing thereby the reduction of \( \Lambda_{\text{UV}} \) below \( m_P \) [7,18]. The same conclusion may be drawn within the JF since no dangerous inflaton-inflaton-graviton

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interaction appears because $\bar{f}_{R,\phi} = 0$ for $q = 2$ [7]. Note that the importance of a scalar field with a totally or partially linear non-minimal coupling to gravity in unitarizing Higgs inflation within non-SUSY settings is highlighted in Refs. [36,37].

A permanently linear $\bar{f}_{R}$ can be reconciled with an inflationary plateau, similar to that obtained in Eq. (3), in the context of SUGRA, by suitably selecting the employed Kähler potentials. Indeed, this kind of models is realized in SUGRA, by suitably selecting the employed Kähler potentials [8–12] with the prefactor $(-N)$ of the logarithms being related to the exponent of the denominator in Eq. (3). Therefore, by conveniently adjusting $N$ we can achieve, in principle, a flat enough EF potential for any $p$ in Eq. (1) but taking exclusively $q = 2$ in Eq. (2). As we show in the following, this idea works for $p \leq 4$ in Eq. (1) supporting nMI compatible with the present data [15]. For $p = 4$ we also show that the inflaton may be identified with a gauge singlet or non-singlet field. In the latter case, models of non-minimal Higgs inflation are introduced, which may be embedded in a more complete extension of MSSM offering a solution to the $\mu$ problem [38] and allowing for an explanation of baryon asymmetry of the universe (BAU) [39] via non-thermal leptogenesis (nTL) [40,41]. The resulting models employ one parameter less than those used in Refs. [33,34] whereas the gauge-symmetry-breaking scale is constrained to values well below the MSSM unification scale contrary to what happens in Refs. [27,33,34].

Below, we first – in Sect. 2 – describe the SUGRA set-up of our models and prove that these are unitary-conserving in Eq. (3). Therefore, by conveniently adjusting $N$ we can achieve, in principle, a flat enough EF potential for any $p$ in Eq. (1) but taking exclusively $q = 2$ in Eq. (2). As we show in the following, this idea works for $p \leq 4$ in Eq. (1) supporting nMI compatible with the present data [15]. For $p = 4$ we also show that the inflaton may be identified with a gauge singlet or non-singlet field. In the latter case, models of non-minimal Higgs inflation are introduced, which may be embedded in a more complete extension of MSSM offering a solution to the $\mu$ problem [38] and allowing for an explanation of baryon asymmetry of the universe (BAU) [39] via non-thermal leptogenesis (nTL) [40,41]. The resulting models employ one parameter less than those used in Refs. [33,34] whereas the gauge-symmetry-breaking scale is constrained to values well below the MSSM unification scale contrary to what happens in Refs. [27,33,34].

Below, we first – in Sect. 2 – describe the SUGRA set-up of our models and prove that these are unitary-conserving in Sect. 3. Then, in Sect. 4, we analyze the inflationary dynamics and predictions. In Sect. 5 we concentrate on the case of nMI driven by a Higgs field and propose a possible post-inflationary completion. We conclude in Sect. 6. Unless otherwise stated, we use units where the reduced Planck scale $m_p = 2.4 \times 10^{18}$ GeV is set to be unity.

## 2 Supergravity framework

In Sect. 2.1 we describe the generic formulation of our models within SUGRA, and then we apply it for a gauge singlet and non-singlet inflaton in Sects. 2.2 and 2.3 respectively.

### 2.1 General framework

We focus on the part of the EF action within SUGRA related to the complex scalars $z^\alpha$ – denoted by the same superfield symbol – which has the form [8–11]

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} \bar{R} + K_{\alpha \bar{\beta}} \bar{D}_\mu \bar{z}^\alpha \bar{D}_\nu \bar{z}^{\bar{\beta}} - \bar{V} \right),$$  \hspace{1cm} (5a)

where $\bar{R}$ is the EF Ricci scalar curvature, $D_{\mu}$ is the gauge covariant derivative, $K_{\alpha \bar{\beta}} = K_{\bar{\alpha} z^{\bar{\beta}}} z^\alpha$, and $K_{\alpha \bar{\beta}} K^{\bar{\gamma} \gamma} = \delta^\alpha_\gamma -$ throughout subscript of type $z$ denotes derivation with respect to (w.r.t) the field $z$. Also, $\bar{V}$ is the EF SUGRA potential which can be found once we select a superpotential $W$ in Eq. (24) and a Kähler potential $\bar{K}$ via the formula

$$\bar{V} = e^K \left( K_{\alpha \bar{\beta}} D_\alpha W D_\bar{\beta} \bar{W} - 3|W|^2 \right) + \frac{g^2}{2} \sum a_i D_a^2,$$  \hspace{1cm} (5b)

where $D_a W = W_{, a} + K_{,a} W$ is the Kähler covariant derivative and $D_a = z^\alpha (T_a)_{\alpha \bar{\beta}} K^{\bar{\beta} \bar{\gamma}}$ are the D term corresponding to a gauge group with generators $T_a$ and (unified) gauge coupling constant $g$. The remaining terms in the right-hand side (r.h.s) of the equation above describes contribution from the $F$ terms. The contribution from the $D$ terms vanishes for a gauge singlet inflaton and can be eliminated during nMI for a gauge non-singlet inflaton, by identifying it with the radial part of a conjugate pair of Higgs superfields – see Sect. 2.3. In both our scenarios, we employ a “stabilizer” field $S$ placed at the origin during nMI. Thanks to this arrangement, the term $3|W|^2$ in $\bar{V}$ vanishes, avoiding thereby a possible run-away problem, and the derivation of $\bar{V}$ is facilitated since the non-vanishing terms arise from those proportional to $W, S$ and $W_S$, – see Sects. 2.2.2 and 2.3.2 below.

Defining the frame function as

$$-\Omega / N = \exp (-K / N) \Rightarrow K = -N \ln (-\Omega / N),$$  \hspace{1cm} (6)

where $N > 0$ is a dimensionless parameter, we can obtain – after a conformal transformation a long the lines of Refs. [8–11,29] – the EF form of $S$ which is

$$S = -\int d^4x \sqrt{-g} \left( \frac{\Omega}{2N} \bar{R} + \omega_{\alpha \bar{\beta}} D_\mu \bar{z}^\alpha D_\nu \bar{z}^{\bar{\beta}} - V \right. \left. - \frac{27}{N^3} \Omega A_\mu A_\mu \right) \text{ with } \omega_{\alpha \bar{\beta}} = \Omega_{\alpha \bar{\beta}} + \frac{3 - N \Omega_{\alpha \bar{\beta}}}{N} \Omega,$$  \hspace{1cm} (7a)

Here we use the shorthand notation $\Omega_{\alpha \bar{\beta}} = \Omega_{z^\alpha \bar{z}^{\bar{\beta}}}$ and $\Omega_{\bar{z}^{\bar{\alpha}}} = \Omega_{\bar{z}^{\bar{\alpha}}}$, and we also set $V = \bar{V} \Omega^2 / N^2$ and

$$A_\mu = -i N \left( \Omega_{\alpha \bar{\beta}} D_\mu z^\alpha - \Omega_{\bar{\alpha} \bar{\beta}} D_\nu \bar{z}^{\bar{\beta}} \right) / 6 \Omega.$$  \hspace{1cm} (7b)

Although the choice $N = 3$ ensures canonical kinetic terms in Eq. (7a), $N$ may be considered in general as a free parameter with interesting consequences not only on the inflationary observables [7,25,26,29,32,42,43] but also on the consistency of the effective theory, as we show below.
2.2 Gauge-singlet inflaton

Below, in Sect. 2.2.1, we specify the necessary ingredients (super- and Kähler potentials) which allow us to implement our scenario with a gauge-singlet inflaton. Then, in Sect. 2.2.2, we outline the derivation of the inflationary potential.

2.2.1 Set-up

This class of models requires the utilization of two gauge singlet chiral superfields, i.e., \( z^a = \Phi, S \), with \( \Phi \) (\( \alpha = 1 \)) and \( S \) (\( \alpha = 2 \)) being the inflaton and a “stabilizer” field respectively. More specifically, we adopt the superpotential

\[
W_{\text{CI}} = \lambda S \Phi^p/2, \tag{8}
\]

which can be uniquely determined if we impose two symmetries: (i) an \( R \) symmetry under which \( S \) and \( \Phi \) have charges 1 and 0; (ii) a global \( U(1) \) symmetry with assigned charges \(-1\) and \( 2/p \) for \( S \) and \( \Phi \). To obtain a linear non-minimal coupling of \( \Phi \) to gravity, though, we have to violate the latter symmetry as regards \( \Phi \). Indeed, we propose the following set of Kähler potentials

\[
\begin{align*}
K_1 &= -N \ln \left( 1 + c_R (F_R + \frac{F_R^s}{F_R^s} - F_-/N + F_{1S}) \right), \tag{9a} \\
K_2 &= -N \ln \left( 1 + c_R (F_R + \frac{F_R^s}{F_R^s} + F_{1S}) + F_- \right), \tag{9b} \\
K_3 &= -N \ln \left( 1 + c_R (F_R + \frac{F_R^s}{F_R^s} - F_-/N + F_{2S}) \right), \tag{9c} \\
K_4 &= -N \ln \left( 1 + c_R (F_R + \frac{F_R^s}{F_R^s}) + F_- + F_{2S} \right), \tag{9d} \\
K_5 &= -N \ln \left( 1 + c_R (F_R + \frac{F_R^s}{F_R^s}) + F_{3S} \right). \tag{9e}
\end{align*}
\]

Recall that \( N > 0 \). From the involved functions

\[
F_R = \Phi/\sqrt{2} \quad \text{and} \quad F_- = -\frac{1}{2} (\Phi - \Phi^*)^2 \tag{10}
\]

the first one allows for the introduction of the linear non-minimal coupling of \( \Phi \) to gravity whereas the second one assures canonical normalization of \( \Phi \) without any contribution to the non-minimal coupling along the inflationary path – cf. Refs. [14,30,31]. On the other hand, the functions \( F_{1S} \) with \( l = 1, 2, 3 \) offer canonical normalization and safe stabilization of \( S \) during and after nM1. Their possible forms are given in Refs. [33,34]. Just for definiteness, we adopt here only their logarithmic form, i.e.,

\[
\begin{align*}
F_{1S} &= -\ln(1 + |S|^2/N), \tag{11a} \\
F_{2S} &= \frac{N}{3} \ln(1 + |S|^2/N_S), \tag{11b} \\
F_{3S} &= \frac{N}{3} \ln(1 + F_-/N_S + |S|^2/N_S), \tag{11c}
\end{align*}
\]

with \( 0 \leq N_S < 6 \). Recall [8–11,44–46] that the simplest term \( |S|^2 \) leads to instabilities for \( K = K_1 \) and \( K_2 \) and light excitations for \( K = K_3 - K_5 \). The heaviness of these modes is required so that the observed curvature perturbation is generated wholly by our inflaton in accordance with the lack of any observational hint [39] for large non-Gaussianity in the cosmic microwave background. Note that all the proposed \( K \’s \) contain up to quadratic terms of the various fields. Also \( F_R \) and \( F_{1S} \) is exclusively included in the logarithmic part of the \( K \’s \) whereas \( F_- \) may or may not accompany it in the argument of the logarithm. Note finally that, although quadratic nM1 is analyzed in Refs. [6,7,14] too, the present set of \( K \’s \) is examined for first time.

2.2.2 Inflationary potential

Along the inflationary track determined by the constraints

\[
S = \Phi - \Phi^* = 0, \quad \text{or} \quad s = \bar{s} = \theta = 0 \tag{12}
\]

if we express \( \Phi \) and \( S \) according to the parametrization

\[
\Phi = \phi e^{i\theta}/\sqrt{2} \quad \text{and} \quad S = (s + i\bar{s})/\sqrt{2}, \tag{13}
\]

the only surviving term in Eq. (5b) is

\[
\hat{V}_{\text{CI}} = \hat{V}(\theta = s = \bar{s} = 0) = e^K K^{SS^*} |W_{\text{CI},S}|^2. \tag{14}
\]

which, for the \( K \’s \) in Eqs. (9a)–(9e), reads

\[
\hat{V}_{\text{CI}} = \frac{\lambda^2 \phi^p}{2p/2 f_R^{2/2}} \begin{cases} f_R & \text{for } K = K_1, K_2 \\ 1 & \text{for } K = K_3 - K_5 \end{cases} \tag{15}
\]

where we define the (inflationary) frame function as

\[
f_R = -\frac{\Omega}{N}|_{\text{Eq. (12)}} = 1 + c_R \phi. \tag{16}
\]

As expected, \( f_R \) coincides with \( \tilde{f}_R \) in Eq. (2) for \( q = 2 \). This form of \( f_R \) assures the preservation of unitarity up to \( m_p = 1 \) as explained in Sect. 1 and verified in Sect. 3. The last factor in Eq. (15) originates from the expression of \( K^{SS^*} \) for the various \( K \’s \). Indeed, \( K_{\alpha\beta} \) along the configuration in Eq. (12) takes the form

\[
(K_{\alpha\beta}) = \text{diag} \left( \kappa + N c_R^2/2 f_R^2, K_{SS^*} \right), \tag{17}
\]

where

\[
K_{SS^*} = \begin{cases} 1/f_R & \text{for } K = K_1, K_2 \\ 1 & \text{for } K = K_3 - K_5 \end{cases} \tag{18}
\]

and

\[
\kappa = \begin{cases} 1/f_R & \text{for } K = K_1, K_3 \\ 1 & \text{for } K = K_2, K_4, K_5. \end{cases} \tag{19}
\]
If we set

$$N = \begin{cases} p(n + 1) + 1 & \text{for } K = K_1, K_2 \\ p(n + 1) & \text{for } K = K_3 - K_5, \end{cases}$$  \hfill (20)

we arrive at a universal expression for \( \hat{V}_{\text{CI}} \) which is

$$\hat{V}_{\text{CI}} = \frac{\lambda^2}{2p^2} f_{\text{R}}^p (1 + n p) .$$  \hfill (21)

For \( n = 0 \) and \( p = 2 \), \( \hat{V}_{\text{CI}} \) reduces to \( \hat{V}_{\text{att}} \) in Eq. (3) whereas for \( p > 2 \), \( \hat{V}_{\text{CI}} \) deviates from \( \hat{V}_{\text{att}} \) although it develops a similar inflationary plateau, for \( c_R \gg 1 \), since both numerator and denominator are dominated by a term proportional to \( \phi^p \) as in the case of UAMs. The choice \( n = 0 \) is special since, for integer \( p \), it yields integer \( N \) in Eqs. (9a)–(9e), i.e., \( N = p + 1 \) for \( K = K_1 \) and \( K_2 \) or \( N = p \) for \( K = K_3 - K_5 \). Although integer \( N \)’s are more friendly to string theory – and give observationally acceptable results as shown in Sect. 4.2 –, non-integer \( N \)’s are also acceptable [25,26,29,32–34,42,43] and assist us to cover the whole allowed domain of the observables. More specifically, for \( n < 0 \), \( \hat{V}_{\text{CI}} \) remains an increasing function of \( \phi \), whereas for \( n > 0 \), it develops a local maximum \( \hat{V}_{\text{CI}}(\phi_{\text{max}}) \) where

$$\phi_{\text{max}} = \frac{1}{nc_R} \left( 1 + \frac{1 + n}{1 + np} \right) .$$  \hfill (22)

In a such case we are forced to assume that hilltop [47] nMI occurs with \( \phi \) rolling from the region of the maximum down to smaller values.

Defining the EF canonically normalized fields, denoted by hat, via the relations

$$\frac{d\hat{\phi}}{d\phi} = \sqrt{K_{\Phi \Phi'}} = J, \quad \hat{\phi} = J\phi \quad \text{and} \quad (\hat{s}, \hat{\bar{s}}) = \sqrt{K_{SS'}} (s, \bar{s}),$$  \hfill (23)

we can verify that the configuration in Eq. (12) is stable w.r.t the excitations of the non-inflaton fields. Taking the limit \( c_R \gg 1 \) we find the expressions of the masses squared \( \hat{m}^2_{\phi_{\text{max}}} \) (with \( z^a = \theta \) and \( s \)) arranged in Table 1, which approach rather well the quite lengthy, exact expressions taken into account in our numerical computation. We infer that \( \hat{m}^2_{\phi_{\text{max}}} = \hat{H}^2_{\text{CI}} = \hat{V}_{\text{CI}}/3 \) for \( 1 < N < 6 \) and \( K = K_1, K_2 \) or for \( 0 < N < 6 \) and \( K = K_3 - K_5 \). Therefore \( \hat{m}^2_{\phi_{\text{max}}} \) are not only positive but also heavy enough during nMI. In Table 1 we display the masses \( \hat{m}^2_{\psi_{\pm}} \) of the spinors \( \tilde{\psi}_+ = (\tilde{\psi}_S \pm \tilde{\psi}_\Phi)/\sqrt{2} \) too. We define \( \tilde{\psi}_S = \sqrt{K_{SS'}} \tilde{\psi}_S \) and \( \tilde{\psi}_\Phi = \sqrt{K_{SS'}} \tilde{\psi}_\Phi \) where \( \tilde{\psi}_\Phi \) and \( \tilde{\psi}_S \) are the Weyl spinors associated with \( S \) and \( \Phi \) respectively.

### 2.3 Gauge non-singlet inflaton

Following the strategy of the previous section, we show below, in Sect. 2.3.1, how we can establish a model of unitarity-conserving nMI driven by a gauge non-singlet inflaton and then, in Sect. 2.3.2, we outline the derivation of the corresponding inflationary potential.

#### 2.3.1 Set-up

In this case, as explained below Eq. (5b), we employ a pair of left-handed chiral superfields, \( \tilde{\Phi} \) and \( \Phi \), oppositely charged under a gauge group, besides the stabilizer, \( S \), which is a gauge-singlet chiral superfield. Here, we take for simplicity the group \( U(1)_{B-L} \) where \( B \) and \( L \) denote the baryon and lepton number respectively. We base our construction on the superpotential [48]

$$W_{\text{HI}} = \lambda S (\bar{\Phi} \Phi - M^2/4) ,$$  \hfill (24)

where \( \lambda \) and \( M \) are parameters which can be made positive by field redefinitions. \( W_{\text{HI}} \) is the most general renormalizable superpotential consistent with a continuous R symmetry [48] under which \( S \) and \( W_{\text{HI}} \) are equally charged whereas \( \Phi \) is uncharged. To obtain nMI – which is actually promoted to Higgs inflation – with a linear non-minimal coupling to gravity we combine \( W_{\text{HI}} \) with one of the Kähler potentials in Eqs. (9a)–(9e) where the functions \( F_{\text{R}} \) and \( F_- \) are now defined as

$$F_{\text{R}} = (\Phi \Phi')^{1/2} \quad \text{and} \quad F_- = (\Phi - \Phi')^2 .$$  \hfill (25)

Note that the proposed \( K \)’s respect the symmetries of \( W_{\text{HI}} \). As in the case of Sect. 2.2.1, \( F_- \) ensures that the kinetic

### Table 1

| Fields | Eigenstates | Masses squared |
|-------|-------------|----------------|
|       |             | \( K = K_1 \) | \( K = K_2 \) | \( K = K_3 \) | \( K = K_4 \) | \( K = K_5 \) |
| 4 Real | \( \tilde{s}, \tilde{\bar{s}} \) | \( \hat{m}^2_{\tilde{s}} \) | \( \hat{m}^2_{\tilde{\bar{s}}} \) | \( \hat{m}^2_{\tilde{s}} \) | \( \hat{m}^2_{\tilde{\bar{s}}} \) |
| Scalars | \( \tilde{\psi}_\pm \) | \( \hat{m}^2_{\psi_\pm} \) | \( \hat{m}^2_{\psi_\pm} \) | \( \hat{m}^2_{\psi_\pm} \) | \( \hat{m}^2_{\psi_\pm} \) |

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terms of $\bar{\Phi}$ and $\Phi$ do not enter the expression of $f_\mathcal{R}$ along the inflationary trough – cf. Refs. [27,29–32].

Comparing the resulting $K$’s with the ones used in Refs. [33,34] we may notice that here $F_-$ is not accompanied by an independent variable $c_-$ and the real function $F_+$ is here replaced by the combination of the holomorphic function $F_\mathcal{R}$ and its anti-holomorphic. Therefore, the present models are more economical since they include one parameter less. On the other hand, the presence of unity in the argument of the logarithms distinguishes clearly the present models from those in Ref. [27] where the absence of unity enforces us to invoke a large inflaton v.e.v.  

2.3.2 Inflationary potential

Employing the parameterization of $S$ in Eq. (13) and expressing $\Phi$ and $\bar{\Phi}$ as follows

$$\Phi = \phi e^{i\theta} \cos \theta_\phi / \sqrt{2} \quad \text{and} \quad \bar{\Phi} = \phi e^{i\bar{\theta}} \sin \theta_\phi / \sqrt{2},$$

with $0 \leq \theta_\phi \leq \pi / 2$, we can determine a D-flat direction from the conditions

$$\bar{s} = s = \theta = \bar{\theta} = 0 \quad \text{and} \quad \theta_\phi = \pi / 4.$$  

(27)

Along this path the only surviving term is again given by Eq. (14) which now reads – cf. Eq. (15)

$$\tilde{V}_{HI} = \frac{\lambda^2 (\phi^2 - M^2)^2}{16 f_\mathcal{R}^2 N} \begin{cases} f_\mathcal{R}^2 & \text{for } K = K_1, K_2, \\ 1 & \text{for } K = K_3 - K_5, \end{cases}$$

(28)

and $f_\mathcal{R}$ is again given by Eq. (16). If we take into account the definition of $n$ in Eq. (20) we end up with

$$\tilde{V}_{HI} = \frac{\lambda^2 (\phi^2 - M^2)^2}{f_\mathcal{R}^2 (1 + n)}.$$  

(29)

which can be approached rather well by Eq. (21) for $p = 4$, when $M \ll 1$, and $\lambda^2$ replaced with $\lambda^2 / 4$.

To specify $\hat{\phi}$ in the present case we note that, for all $K$’s in Eqs. (9a)–(9e) with $f_\mathcal{R}$ and $F_-$ given in Eq. (25), $K_{\alpha\beta}$ along the configuration in Eq. (27) takes the form

$$K_{\alpha\beta} = \text{diag} (M_\pm, K_{SS'})$$,

(30)

where

$$M_\pm = \left[ \begin{array}{cc} \kappa + N c_\mathcal{R}^2 / 4 f_\mathcal{R}^2 & N c_\mathcal{R}^2 / 4 f_\mathcal{R}^2 \\ N c_\mathcal{R}^2 / 4 f_\mathcal{R}^2 & \kappa + N c_\mathcal{R}^2 / 4 f_\mathcal{R}^2 \end{array} \right]$$

(31)

with $\kappa$ given in Eq. (19). Upon diagonalization we obtain the following eigenvalues

$$\kappa_+ = \kappa + N c_\mathcal{R}^2 / 2 f_\mathcal{R}^2 \quad \text{and} \quad \kappa_- = \kappa.$$  

(32)

Inserting Eqs. (26) and (30) in the second term of the r.h.s of Eq. (5a) we can define the EF canonically normalized fields, as follows

$$d\hat{\phi} / d\phi = J = \sqrt{\kappa_+}, \quad \hat{\theta}_\phi = \theta_\phi \sqrt{\kappa_-} (\theta_\phi - \pi / 4),$$

(33a)

$$\hat{\theta}_\pm = J \theta_\pm / \sqrt{2} \quad \text{and} \quad \hat{\theta}_\mp = \sqrt{\kappa_-} \theta_\pm / \sqrt{2},$$  

(33b)

where $\theta_\pm = (\bar{\theta} \pm \theta) / \sqrt{2}$ and the normalization of $\hat{\phi}, \hat{s}$ and $\hat{\bar{s}}$ coincides with that found in Eq. (23). Note, in passing, that the spinors $\psi_{\Phi \pm}$ associated with the superfields $\Phi$ and $\bar{\Phi}$ are similarly normalized, i.e., $\tilde{\psi}_{\Phi \pm} = \sqrt{\kappa_\pm} \psi_{\Phi \pm}$ with $\psi_{\Phi \pm} = (\psi_\Phi \pm \bar{\psi}_\Phi) / \sqrt{2}$.

To check the stability of inflationary direction in Eq. (27) w.r.t the fluctuations of the non-inflaton fields, we derive the mass-squared spectrum of the various scalars defined in Eqs. (33a) and (33b). Taking the limit $c_\mathcal{R} \gg 1$, we find the approximate expressions listed in Table 2 which are rather accurate at the horizon crossing of the pivot scale. As in case of Table 1, we again deduce that $m_{\Phi \pm}^2 \gg \tilde{V}_{HI} / 3$ for $1 < N < 6$ and $K = K_1, K_2$ or for $0 < N_5 < 6$ and $K = K_3 - K_5$. In Table 2 we also display the masses $M_{BL}$ of the gauge boson $A_{BL}$ and the corresponding fermions. The non-vanishing of $M_{BL}$ signals the fact that $U(1)_{B-L}$ is broken during nMI and so no cosmic string are produced at

| Fields | Eigenstates | Masses squared |
|--------|-------------|----------------|
| 4 Real | $\hat{\theta}_+$ | $\tilde{m}_{\hat{\theta}_+}^2$ | $3(1 - 1/N) \tilde{H}_{HI}^2$ | $3 \tilde{H}_{HI}^2$ |
| Scalars | $\hat{\theta}_\phi$ | $\tilde{m}_{\hat{\theta}_\phi}^2$ | $M_{BL}^2 + 6 \tilde{H}_{HI}^2$ | $M_{BL}^2 + 6(1 + 1/N_5) \tilde{H}_{HI}^2$ |
| $\bar{s}, \bar{\bar{s}}$ | $\tilde{m}_{\bar{s}}^2$ | $6 \tilde{H}_{HI}^2 c_\mathcal{R}^2 / N$ | $6 \tilde{H}_{HI}^2 / N_5$ |
| 1 Gauge boson | $A_{BL}$ | $M_{BL}^2$ | $g^2 \phi^2 / f_\mathcal{R}$ | $g^2 \phi^2$ |
| 4 Weyl | $\tilde{\psi}_{\phi\pm}$ | $\tilde{m}_{\tilde{\psi}_{\phi\pm}}^2$ | $(c_\mathcal{R} (N - 5) \phi - 4) \tilde{H}_{HI}^2 / N c_\mathcal{R}^2 \phi^2$ | $(c_\mathcal{R} (N - 4) \phi - 4) \tilde{H}_{HI}^2 / N c_\mathcal{R}^2 \phi^2$ |
| Spinors | $\lambda_{BL}, \tilde{\psi}_{\Phi \pm}$ | $M_{BL}^2$ | $g^2 \phi^2 / f_\mathcal{R}$ | $g^2 \phi^2$ |
its end. Finally, the unspecified eigenstates \( \hat{\psi}_\pm \) are defined as \( \hat{\psi}_\pm = (\hat{\psi}_\phi \pm \hat{\psi}_\lambda) / \sqrt{2} \) – cf. Table 1.

### 3 Effective cut-off scale

The motivation of our proposal originates from the fact that \( f_R \) in Eq. (16) assures that the perturbative unitarity is retained up to \( m_p \) although the attainment of nMI for \( \phi \leq m_p \) requires large \( c_R \)'s – as expected from the UAMs [5–7] and verified in Sect. 4 below. To show that this achievement is valid, we extract below the UV cut-off scale, \( \Lambda_{UV} \), expanding the action in the JF – see Sect. 3.1 – and in the EF – see Sect. 3.2. Throughout this section we find it convenient to restore \( m_p \) in the formulas. We concentrate also on the versions of our model with gauge singlet inflaton. However, this analysis covers also the case of a gauge non-singlet inflaton for \( M \ll m_p \), \( p = 4 \) and \( \lambda^2 \) replaced by \( \lambda^2/4 \).

Although the expansions about \( \langle \phi \rangle = 0 \), presented below, are not valid [19] during nMI, we consider \( \Lambda_{UV} \) extracted this way as the overall cut-off scale of the theory for two reasons: (i) the reheating phase – realized via oscillations about \( \hat{\phi} \) – is an unavoidable stage of the inflationary dynamics; (ii) the result is within the range of validity of the low-energy theory and so this can be perturbatively extrapolated up to \( \Lambda_{UV} \).

#### 3.1 Jordan frame computation

Thanks to the special dependence of \( f_R \) on \( \phi \) in Eq. (16), there is no interaction between the excitation of \( \phi \) and the graviton, \( h^{\mu \nu} \) which can jeopardize the validity of perturbative unitarity. To show this, we first expand \( g_{\mu \nu} \) about the flat spacetime metric \( \eta_{\mu \nu} \) and the inflaton \( \phi \) about its v.e.v.,

\[
g_{\mu \nu} \simeq \eta_{\mu \nu} + h_{\mu \nu} / m_p \quad \text{and} \quad \phi = 0 + \delta \phi. \tag{34}\]

Retaining only the terms with up to two space-time derivatives of the excitations, the part of the lagrangian corresponding to the two first terms in the r.h.s of Eq. (7a) takes the form [19,25,26]

\[
\delta \mathcal{L} = - \frac{\langle f_R \rangle}{8} G_{\text{EH}} \left( h^{\mu \nu} \right) + \frac{1}{2} \langle f_K \rangle \partial_\mu \delta \phi \partial^\mu \delta \phi
+ \frac{m_p}{2} G_R \left( h^{\mu \nu} \right) \left( \frac{\langle f_{R,\phi} \rangle}{2} \delta \phi + \sum_{\ell = 2}^\infty \frac{1}{\ell!} \left( \frac{d^\ell f_R}{d\phi^\ell} \right) \delta \phi^\ell \right)
= - \frac{1}{8} G_{\text{EH}} \left( \delta h^{\mu \nu} \right) + \frac{1}{2} \partial_\mu \delta \phi \partial^\mu \delta \phi, \tag{35}\]

where the functions \( G_{\text{EH}} \) and \( G_R \) are identical to \( F_{\text{EH}} \) and \( F_R \) defined in Refs. [25,26] – \( F_R \) should not to be confused with that given in Eqs. (10) and (25). From Eq. (7a) we compute \( f_K = \omega_{\phi, \phi^*} = 1/\kappa + (N - 3)c_R^2 / 2 f_R \). For \( \langle \phi \rangle = 0 \) we get \( \langle f_K \rangle = 1 + (N - 3)c_R^2 / 2 \), \( \langle f_R \rangle = 1 \) and \( \langle f_{R, \phi} \rangle = c_R / m_p \). The JF canonically normalized fields \( \tilde{h}_{\mu \nu} \) and \( \delta \phi \), which diagonalise the quadratic part of the form above, are defined as

\[
\delta \phi \simeq \sqrt{\frac{N}{2}} c_R \delta \phi \quad \text{and} \quad \tilde{h}_{\mu \nu} = h_{\mu \nu} + c_R \eta_{\mu \nu} \delta \phi. \tag{36}\]

The UV behavior of the scattering amplitudes is determined by the operators with dimension higher than four which arise from the last term in the second line of Eq. (35). The resulting interactions are proportional to the quantity \( \delta \phi^2 \) with \( \tilde{h}/m_p^{-1} \) with \( \tilde{h} = \partial_\nu \partial^\nu \delta \phi - c_R - c. \) Refs. [19,24–26,35]. Given that \( d^\ell f_R / d\phi^\ell \) vanishes for \( \ell > 2 \), though, no interaction of that type appears and so the theory does not face any problem with the perturbative unitarity.

#### 3.2 Einstein frame computation

Alternatively, \( \Lambda_{UV} \) can be determined in EF, analyzing the small-field behavior of our models following Ref. [18]. We focus first on the second term in the r.h.s of (5a) for \( \mu = v = 0 \) and \( K_{\phi, \phi^*} = J^2 \) given in Eq. (23). Expanding it about \( \langle \phi \rangle = 0 \), we arrive at

\[
J^2 \delta \phi^2 = \left( 1 + \frac{1}{2} N c_R^2 - \frac{3}{2} N c_R^4 \frac{\delta \phi^2}{m_p^2} + \frac{3}{2} N c_R^4 \frac{\delta \phi^2}{m_p^2} + \cdots \right) \delta \phi^2, \tag{37}\]

where \( \delta \phi \) is defined in Eq. (34) and we set \( \phi = \delta \phi \). If the EF (canonically normalized) inflaton, \( \hat{\phi} \), was identical to \( \delta \phi \) at the vacuum of the theory – as happens for the UAMs [16–18] with \( q > 2 - \Lambda_{UV} \) would have been equal to \( m_p / c_R^3 \) since this is the lowest of the denominators above for \( c_R \gg 1 \). However, here we have \( \hat{\phi} = \langle J \rangle \phi \gg \delta \phi \) since

\[
\langle J \rangle = \sqrt{1 + N c_R^2 / 2 \simeq} c_R \sqrt{N/2}. \tag{38}\]

Therefore, expressing Eq. (37) in terms of \( \delta \phi \) and \( \hat{\phi} = \langle J \rangle \delta \phi \), we see that the \( c_R \)'s are cancelled out and we end up with the result

\[
J^2 \delta \phi^2 = \left( 1 - 2 \frac{\sqrt{2}}{\sqrt{N}} \frac{\delta \phi}{m_p} + \frac{6}{N} \frac{\delta \phi^2}{m_p^2} - 8 \frac{\sqrt{2}}{N^2} \frac{\delta \phi^3}{m_p^3} + \cdots \right) \delta \phi^2, \tag{39a}\]

where we neglect terms suppressed by powers of \( c_R \) in the denominator. This expression is valid for both cases in Eq. (17) and agrees with that in Ref. [7] for \( N = 3(1 + n) \). Expanding similarly \( \bar{V}_{\text{CI}} \), see Eq. (21), in terms of \( \delta \phi \) we
4 Inflation analysis

In Sects. 4.1 and 4.2 below we examine semi-analytically and numerically respectively, if \( \hat{V}_{CI} \) in Eq. (21) may be consistent with a number of observational constraints. The analysis can be easily adapted to the case of \( \hat{V}_{HI} \) in Eq. (29) performing the replacements mentioned in Sect. 3.

4.1 Semi-analytic results

The period of slow-roll nMI is determined in the EF by the condition – see, e.g., Refs. [49,50]:

\[
\max[\hat{c}(\phi), |\hat{\eta}(\phi)|] \leq 1, \tag{40a}
\]

where the slow-roll parameters \( \hat{c} \) and \( \hat{\eta} \) read

\[
\hat{c} = \left( \frac{\hat{V}_{CI,\phi}}{\sqrt{2\hat{V}_{CI}}} \right)^2 \quad \text{and} \quad \hat{\eta} = \frac{\hat{V}_{CI,\phi\phi}}{\hat{V}_{CI}}, \tag{40b}
\]

and can be derived employing \( J \simeq \sqrt{N/2\phi^2} \) in Eq. (23) without express explicitly \( \hat{V}_{CI} \) in terms of \( \hat{\phi} \). Since \( J \) for \( K = K_1, K_3 \) deviates slightly from that for \( K = K_2, K_4, K_5 \) – see Eq. (19) – we have a discrimination as regards the expressions of \( \hat{c} \) and \( \hat{\eta} \) in these two cases. Indeed, our results are

\[
\hat{c} \simeq \frac{p^2 f_n^2}{c_R^2 \phi^2} \left\{ \begin{array}{ll} 1/N & \text{for } K = K_1, K_3, \\ 1/(N + 2\phi^2) & \text{for } K = K_2, K_4, K_5, \end{array} \right. \tag{41}
\]

where \( f_n = 1 - n c_R \phi \). Similarly, we obtain

\[
\frac{\hat{\eta}}{2p} \simeq \left\{ \begin{array}{ll} (pf_n^2 - f_n)/Nc_R^2 \phi^2 & \text{for } K = K_1, K_3, \\ (Nc_R \phi f_n + (N + 2\phi^2)(c_R \phi(f_n - 1) + pf_n^2))/c_R^2 \phi^2(N + 2\phi^2) & \text{for } K = K_2, K_4, K_5. \end{array} \right. \tag{42}
\]

For any of the \( K \)'s above we can numerically verify that Eq. (40a) is saturated for \( \phi = \phi_I \), which is found from the condition

\[
\hat{c}(\phi_I) \simeq 1 \quad \Rightarrow \quad \phi_I \simeq \frac{p}{c_R N - n^2 p^2}. \tag{43}
\]

Apart from irrelevant constant prefactors, the formulas above for \( \phi^\star \) and \( K \)'s above we can numerically verify that Eq. (40a) is saturated for \( \phi = \phi_I \), which is found from the condition

\[
\hat{c}(\phi_I) \simeq 1 \quad \Rightarrow \quad \phi_I \simeq \frac{p}{c_R N - n^2 p^2}. \tag{43}
\]

The number of e-foldings \( \hat{N} \) that the pivot scale \( k_\star = 0.05/\text{Mpc} \) experiences during nMI and the amplitude \( A_s \) of the power spectrum of the curvature perturbations generated by \( \phi \) can be computed using the standard formulae

\[
\hat{N}_\star = \int_{\phi_I}^{\phi_\star} d\dot{\phi} \frac{\hat{V}_{CI}}{\hat{V}_{CI,\phi}}, \quad \text{and} \quad A_s = \frac{1}{12\pi^2} \frac{\hat{V}_{CI,\phi}^2(\phi_\star)}{\hat{V}_{CI,\phi}}. \tag{44}
\]

where \( \phi_\star [\hat{\phi}_\star] \) is the value of \( \phi [\hat{\phi}] \) when \( k_\star \) crosses the inflationary horizon. Taking into account \( \phi_\star \gg \phi_I \), we can derive \( \hat{N}_\star \). We single out the following cases:

- For \( n = 0 \) and any \( K \) in Eqs. (9a)–(9e), we obtain

\[
\hat{N}_\star = \frac{Nc_R}{2p} \phi_\star \Rightarrow \phi_\star \simeq \frac{2p\hat{N}_\star}{Nc_R}. \tag{45a}
\]

Note that for \( n \neq 0 \) the formulas below for \( \hat{N}_\star \) cannot be reduced to the previous one.

- For \( n \neq 0 \) and \( K = K_1, K_3 \), we obtain

\[
\hat{N}_\star = -\frac{N \ln f_{n\star}}{2n(1 + n)p} \Rightarrow \phi_\star \simeq \frac{1 - e_n}{nc_R}. \tag{45b}
\]

where \( e_n = e^{2pn(n+1)} \hat{N}_\star/N \) and \( f_{n\star} = f_n(\phi_\star) \).

- For \( n < 0 \) and \( K = K_2, K_4, K_5 \), we obtain

\[
\hat{N}_\star = -\frac{N \ln f_{n\star}}{2n(1 + n)p} \phi_\star^2 \Rightarrow \phi_\star \simeq \sqrt{\frac{N}{2(n + 1)}} W_0(y). \tag{45c}
\]

where \( W_k \) is the Lambert or product logarithmic function [51] with \( k = 0 \) and
$y = 2(1 + n) c_n^2 / N n^2 c_{\sR}^2.$  \hfill (45d)

- For $n > 0$ and $K = K_2, K_4, K_5$, we obtain

$$\hat{N}_s = -\frac{2 + n c_{\sR}^2 N}{2 n c_{\sR}^3 n^3 (1 + n) p} \ln f_{n*} - \frac{3 + f_{n*}}{n^2 p c_{\sR} \phi_*}. \quad (45e)$$

Here we are not able to solve the equation above w.r.t $\phi_*$. As a consequence, it is not doable to find an analytical expression for $\phi_*$ and the inflationary observables – see below. Therefore, in this portion of parameter space, our last resort is the numerical computation, whose the results are presented in Sect. 4.2.

In all cases above, there is a lower bound on $c_{\sR}$, above which $\phi_* \leq 1$, as in the original UAMs [14]. E.g., for $n = 0$

$$\phi_* \leq 1 \implies c_{\sR} \geq 2 p \hat{N}_s / N \gg 1, \quad (46)$$

with $\hat{N}_s \approx (52 - 59) / p$ for $p = 2 - 4$. As shown in Sect. 3, these large $c_{\sR}$’s do not disturb the validity of perturbative unitarity up to $m_p$. On the other hand, the EF field, $\hat{\phi}$ may be transplanckian, since integrating the first equation in Eq. (23) with $\hat{\phi}$ given below Eq. (40b), we find

$$\hat{\phi} \approx \hat{\phi}_e + \sqrt{N/2} \ln \phi, \quad (47)$$

with $\hat{\phi}_e$ being a constant of integration. If we set, e.g., $\hat{\phi}_e = 0$ and $\phi \approx (0.001 - 1)$, then $|\phi| > 1$. Despite this fact, our proposal is stabilized against corrections from higher order terms in $W_{\sCl}$ and/or $K$’s in Eqs. (8) and (9a)–(9e), since these terms are exclusively expressed as functions of the initial field $\Phi$ and remain harmless for $\phi = \sqrt{2} |\Phi| \leq 1$ – cf. Ref. [14].

From Eq. (44b) we can derive a relation between $\lambda$ and $c_{\sR}^{p/2}$ for fixed $n$, in sharp contrast to UAMs where the same condition implies a relation between $\lambda$ and $c_{\sR}$ [6, 12, 13]. Given that $c_{\sR}$ assumes large values, we expect that $\lambda$ increases with $p$ and rapidly (for $p \geq 5$ as we find numerically) violates the perturbative bound $\lambda \leq 2 \sqrt{\sR} \simeq 3.5$. In particular, our results can be cast as following:

- For $n = 0$ and any $K$ in Eqs. (9a)–(9e), we obtain

$$\lambda = 2^{3/4 + 2} \pi \sqrt{3 N A_s c_{\sR}^{p/2}} / \hat{N}_s. \quad (48a)$$

- For $n \neq 0$ and $K = K_1, K_3$, we obtain

$$\lambda = 2^{3/4 + 2} p \pi \sqrt{3 A_s (1 - n f_{R*}^2) f_{R*}^{mp-1} c_{\sR}^{p/2} / N^{1/2}}, \quad (48b)$$

where $f_{R*} = f_{\sR}(\phi_*)$. Taking into account that $n \ll 1$ and $f_{R*}$ is almost proportional to $c_{\sR}$ for large $c_{\sR}$, we can easily convince ourselves that the output above implies that $\lambda / c_{\sR}^{p/2}$ remains constant for fixed $n$.

- For $n < 0$ and $K = K_2, K_4, K_5$, we obtain

$$\lambda = 2^{3/4 + 2} p \pi \sqrt{3 A_s (1 - n f_{R*}) f_{R*}^{mp-1} c_{\sR}^{p/2}} \left(2 f_{R*}^2 + N c_{\sR}^2 \right)^{1/2}, \quad (48c)$$

from which we can again verify that the approximate proportionality of $\lambda$ on $c_{\sR}^{p/2}$ holds.

The remaining inflationary observables – i.e., the (scalar) spectral index $n_s$, its running $a_s$, and the scalar-to-tensor ratio $r$ – are found from the relations [49, 50]

$$n_s = 1 - 6 \hat{c}_s + 2 \hat{N}_s, \quad r = 16 \hat{c}_s, \quad (49a)$$

$$a_s = 2 \left(4 \hat{N}_s^2 - (n_s - 1)^2\right) / 3 - 2 \hat{c}_s, \quad (49b)$$

where the variables with subscript $\ast$ are evaluated at $\phi = \phi_*$ and $\hat{c}_s = \hat{V}_{\sCl} / V_{\sCl}$. Inserting $\phi_*$ from Eqs. (45a) (45b) and (45c) into Eq. (40b) and then into equations above we can obtain some analytical estimates. Namely:

- For $K = K_1$ and $K_3$ we end up with a unified result

$$n_s \approx 1 - 4 \frac{p}{N f_{R*}} - 2 \frac{p^2}{N} \left(1 - \frac{1}{f_{R*}}\right) \left(1 - \frac{2}{N f_{R*}}\right)/n_p; \quad (50a)$$

$$r \approx \frac{16 p^2}{N} \left(1 - \frac{1}{f_{R*}}\right)^2 \left(1 - \frac{2}{N f_{R*}}\right)/n_p; \quad (50b)$$

$$a_s \approx 8 p^2 (n f_{R*} - 1) \left(1 + (1 + n) p + f_{R*} (1 - n p)\right) / N^2 f_{R*}^3; \quad (50c)$$

For $n = 0$ the above results are also valid for $K = K_2, K_4$ and $K_5$ and yield observables identical with those obtained within UAMs [6, 8–12].

- For $n < 0$ and $K = K_2, K_4$ and $K_5$ we arrive at the following results:

$$n_s \approx 1 + \frac{2 p c_{\sR}^4}{f_{R*}^2 (2 f_{R*}^2 + N c_{\sR}^2)} \left(4 f_{R*}^3 (n f_{R*} - 1)^2 - 2 f_{R*}^2 (2 f_{R*}^2 + N c_{\sR}^2) p\right); \quad (51a)$$

$$r \approx \frac{16 p^2}{N} \frac{f_{R*}^2}{N^2 f_{R*}^2 / c_{\sR}^2} \left(1 - \frac{1}{f_{R*}}\right)^2 \quad (51b)$$

with negligibly small $a_s$, as we find out numerically. Contrary to our previous results, here a $c_{\sR}$ dependence arises which complicates somehow the investigation of these models – see Sect. 4.2.
4.2 Numerical results

The conclusions above can be verified and extended for any \( n \) numerically. In particular, we confront the quantities in Eq. (44) with the observational requirements [39]

\[
\hat{N}_s \simeq 61.3 + \frac{1 - 3w_{th}}{12(1 + w_{th})} \ln \frac{\pi^2 g_{th} T_{th}^4}{3V_C(t_*) f_R(t_*)^2} + \frac{1}{2} \ln \left( \frac{V_C(t_*) f_R(t_*)}{\sqrt[1/6]{g_{th}} V_C(t_*) 1/2} \right) \quad \text{and} \quad A_s \simeq 2.141 \cdot 10^{-9},
\]

where we assume that nMI is followed in turn by a oscillatory phase, with mean equation-of-state parameter \( w_{th} \simeq 0 \) or 1/3 for \( p = 2 \) or 4 respectively [39], radiation and matter domination. Also \( T_{th} \) is the reheat temperature after nMI, with energy-density effective number of degrees of freedom \( g_{th} = 228.75 \) which corresponds to the MSSM spectrum – see Sect. 5.3.1.

Enforcing Eq. (52a) we can restrict \( \lambda/c_{\lambda}^{p/2} \) and \( \phi_* \) and compute the model predictions via Eqs. (49a) and (49b), for any selected \( p \) and \( n \). The outputs, encoded as lines in the \( n_s - r_{0.002} \) plane, are compared against the observational data [39,52] in Fig. 1 for \( K = K_1 \) and \( K_3 \) (left plot) or \( K = K_2, K_4 \) and \( K_5 \) (right plot) – here \( r_{0.002} = 16\hat{g}(\phi_{0.002}) \) where \( \phi_{0.002} \) is the value of \( \phi \) when the scale \( k = 0.002/\text{Mpc} \), which undergoes \( \hat{N}_{0.002} = (\hat{N}_s + 3.22) \) e-foldings during nMI, crosses the horizon of nMI. We draw dashed [solid] lines for \( p = 2 \) [\( p = 4 \)] and show the variation of \( n \) along each line. We take into account the data from Planck and Baryon Acoustic Oscillations (BAO) and the BK14 data taken by the BICEP2/Keck Array CMB polarization experiments up to and including the 2014 observing season. Fitting the data above \([15,52]\) with \( \Lambda \text{CDM} + r \) model we obtain the marginalized joint 68\% [95\%] regions depicted by the dark [light] shaded contours in Fig. 1. Approximately we get

\[
(a) \quad n_s = 0.968 \pm 0.009 \quad \text{and} \quad (b) \quad r \leq 0.07,
\]

at 95\% confidence level (c.l.) with \( |a_3| \ll 0.01 \).

From the left plot of Fig. 1 we observe that the whole observationally favored range of \( n_s \) is covered varying \( n \) which, though, remains close to zero signaling an amount of tuning. In accordance with Eqs. (50a) and (50b), we find the allowed ranges

\[
0.42 \gtrsim n/0.01 \gtrsim -1.4 \quad \text{and} \quad 3 \lessgtr r/10^{-3} \lessgtr 9.8 \quad (54a)
\]

for \( p = 2 \), whereas for \( p = 4 \) we have

\[
5.2 \gtrsim n/0.001 \gtrsim -9 \quad \text{and} \quad 3 \lessgtr r/10^{-3} \lessgtr 11. \quad (54b)
\]

As \( n \) varies in its allowed ranges above, we obtain

\[
1.3 \lessgtr 10^5 \lambda/c_\lambda \lessgtr 2.3 \quad \text{or} \quad 2.1 \lessgtr 10^5 \lambda/c_\lambda \lesssim 3.7, \quad (55)
\]

for \( p = 2 \) or \( p = 4 \) respectively in agreement with Eqs. (48a) and (48b). If we take \( n = 0 \), we find \( n_s = 0.963, a_3 \simeq -6.7 \cdot 10^{-4} \) and \( r = 0.004 \) for \( p = 2 \) demanding \( \hat{N}_s \simeq 53 \), whereas for \( p = 4 \) we get \( n_s = 0.967, a_3 \simeq -5.6 \cdot 10^{-4} \) and \( r = 0.005 \) requiring \( \hat{N}_s \simeq 58.6 \). Therefore, for integer prefactors of the logarithms in Eqs. (9a) and (9c), \( n_s \) converges towards its central value in Eq. (53) and practically coincides with the prediction of the UAMs [5,12,14]. The results for \( p = 4 \) are rather close to those achieved in Ref. [27] for \( K = K_{1R} \) and \( K_{2R} \) and approach the ones found in Refs. [33,34] for the largest possible value of the parameter \( r_+ \).

![Fig. 1](image-url)
Fixing, in addition, $n = 0$ and $K = K_1$, $\phi_* = 1$ – i.e. confining the corresponding $c_R$ and $\lambda$ values to their lowest possible values enforcing Eq. (48a) – we illustrate in Fig. 2 the structure of $\bar{V}_c$ as a function of $\phi$ for $p = 2$ (light gray line) or $p = 4$ (gray line). More specifically, we find $\lambda = 1.173 \cdot 10^{-3}$ or $0.257$ and $c_R = 75$ or $99$ for $p = 2$ or $p = 4$ respectively. We see that in both cases $\bar{V}_c$ develops a plateau with magnitude $10^{-10}$ which is similar to that obtained in Starobinsky inflation [24,44–46] but one order of lower than that obtained from the models analyzed in Refs. [32–34] where $r$ is a little more enhanced. Contrary to Starobinsky inflation, though, $\bar{V}_c$ is well defined at the origin as happens within the original UAMs and those in Refs. [32–34]. Obviously, the requirement – mentioned in Sect. 3 – $\bar{V}_c^{1/4} \ll 1$ dictated from the validity of the effective theory is readily fulfilled.

Practically the same observables for $n = 0$ are shown in the right plot of Fig. 1. In that plot, though, we see that the models’ predictions are confined to $n_s \lesssim 0.974$, $n$ may deviate more appreciably from zero (mainly for $p = 2$) and the maximal possible $r$ is somewhat larger. Moreover, these predictions depend harder on $c_R$ for $|n| > 0.01$, as expected from Eqs. (51a) and (51b). Therefore, in that regime, we could say that these models are less predictive than those based on $K = K_1$ and $K_3$. Our results below are presented for $c_R$ such that $\phi_* \simeq 1$. Namely, for $p = 2$, we find

$$0.7 \lesssim n/0.01 \lesssim 3.5, \quad 9.57 \lesssim n_s/0.1 \lesssim 9.68, \quad (56a)$$

$$1.3 \lesssim r/10^{-3} \lesssim 20 \quad \text{and} \quad 0.98 \lesssim 10^5 \lambda/c_R \lesssim 2.8, \quad (56b)$$

whereas for $p = 4$ we have

$$0.6 \lesssim n/0.01 \lesssim 1.45, \quad 9.57 \lesssim n_s/0.1 \lesssim 9.72, \quad (57a)$$

$$2 \lesssim r/10^{-3} \lesssim 14 \quad \text{and} \quad 1.8 \lesssim 10^5 \lambda/c_R^2 \lesssim 3.6. \quad (57b)$$

The latter results deviate more drastically from those in Refs. [27,33,34]. From the data of both plots of Fig. 1, we remark that $r \gtrsim 0.0013$. These $r$ values are testable by the forthcoming experiments [53–56], which are expected to measure $r$ with an accuracy of $10^{-3}$. The tuning, finally, required for the attainment of hilltop nMI for $n > 0$ is very low, since $\phi_{\text{max}} \gg \phi_*$. However, $\lambda/c_R^{p/2}$ is constant for fixed $n$, the amplitudes of $\lambda$ and $c_R$ can be bounded. This fact is illustrated in Fig. 3 where we display the allowed values $c_R$ versus $\sqrt{\lambda}$ for $p = 4$ and $K = K_1$ (gray lines) or $K = K_5$ (light gray lines). We take $n = 0$ (solid lines) and $n = -0.004$ (dashed lines). As anticipated in Eq. (46) for any $n$ there is a lower bound on $c_R$, above which $\phi_* \leq 1$ stabilizing thereby the results against corrections from higher order terms – e.g., $(\Phi \Phi')^l$ with $l > 1$ in Eq. (24). The perturbative bound $\lambda = 3.5$ limits the various lines at the other end. We observe that the ranges of the allowed lines are much more limited compared to other models – cf. Refs. [12,30,31] – and displaced to higher $\lambda$ values as seen, also, by Eqs. (48a) and (48b). We find that for $p = 5$, $\lambda$ corresponding to lowest possible $c_R$ violates the perturbative bound and so, our proposal can not be applied for $p > 4$.

5 A post-inflationary completion

In a couple of recent papers [27,33,34] we attempt to connect the high-scale inflationary scenario based on $W = W_{\text{HI}}$ in Eq. (24) with the low energy physics, taking into account constraints from the observed BAU, neutrino data and MSSM phenomenology. It would be, therefore, interesting to check if this scheme can be applied also in the case of our present set-up where $W_{\text{HI}}$ in Eq. (24) cooperates with the K’s in Eqs. (9a) – (9e) where $F_R$ and $F_-$ given in Eq. (25). The necessary extra ingredients for such a scenario are described in Sect. 5.1. Next, we show how we can correlate nMI with...
the generation of the $\mu$ term of MSSM – see Sect. 5.2 – and
the generation of BAU via nTL – see Sect. 5.3. Hereafter, we
restore units, i.e., we take $m_p = 2.433 \cdot 10^{18}$ GeV.

5.1 Relevant set-up

Following the post-inflationary setting of Refs. [33,34] we
consider a $B - L$ extension of MSSM with the field content
charged under $B - L$ and $R$ as displayed in Table 1 therein.
The superpotential of the model contains $W_{\text{HI}}$ in Eq. (24),
the superpotential of MSSM with $\mu = 0$ and the following
two terms

$$W_\mu = \lambda_{\mu} S H_u H_d, \quad (58a)$$
$$W_{\text{RHN}} = \lambda_{N^c} \hat{N}^c_i h_{N^c} L_j H_u. \quad (58b)$$

From the terms above, the first one inspired by Ref. [38] helps
to justify the existence of the $\mu$ term within MSSM, whereas
the second one allows for the implementation of (type I) see-
saw mechanism (providing masses to light neutrinos) and
supports a robust baryogenesis scenario through nTL. Let
us note that $L_i$ denotes the $i$-th generation $SU(2)_L$ doublet
left-handed lepton superfields, and $H_u [H_d]$ is the $SU(2)_L$
doublet Higgs superfield which couples to the up [down]
quark superfields. Also, we assume that the superfields $N^c_i$
are rotated in the family space so that the coupling constants $\lambda_i$
are real and positive. This is the so-called [24,33,34] $N^c_i$
basis, where the $N^c_i$ masses, $M_{iN^c}$, are diagonal,
real and positive.

We assume that the extra scalar fields $X^\theta = H_u, H_d, \tilde{N}^c_i$
have identical kinetic terms as the stabilizer field $S$ expressed
by the functions $f_S$ with $l = 1, 2, 3$ in Eqs. (11a) – (11c) –
see Refs. [33,34]. Therefore, $N_S$ may be renamed $N_X$
hereafter. The inflationary trajectory in Eq. (12) has to be sup-
plemented by the conditions

$$H_u = H_d = \tilde{N}^c_i = 0, \quad (59)$$

and the stability of this path has to be checked, parameterizing
the complex fields above as we do for $S$ in Eq. (13). The
relevant masses squared are listed in Table 3 for $K = K_1 - K_5$, with hatted fields being defined as $\hat{s}$ and $\hat{s}$ in Eq. (23).

Also we set

$$\hat{\nu}_\pm = (\tilde{\nu}_u \pm \tilde{\nu}_d) / \sqrt{2} \quad \text{and} \quad \tilde{\hat{\nu}}_\pm = (\tilde{\nu}_u \pm \tilde{\nu}_d) / \sqrt{2}. \quad (60)$$

In Table 3 we see that $\tilde{m}_{\nu}^2 \gtrsim \tilde{m}_{\nu}^2$ and $\tilde{m}_{\nu}^2 \gtrsim \tilde{m}_{\nu}^2$ for every
$\phi$, whereas imposing $\tilde{m}_{\nu}^2 \gtrsim \tilde{m}_{\nu}^2$ dictates

$$\lambda_{\mu} \lesssim \frac{\lambda \phi^2}{4} \cdot \begin{cases} 
\frac{(2/3 + c_R \phi / N) / f_R}{(2/3 + 1/N_X)} 
\text{for} \ K = K_1, K_2, \\
\frac{1.5 \cdot 10^{-5}}{5.3 \cdot 10^{-5}} \text{for} \ K = K_3 - K_5. 
\end{cases} \quad (61)$$

Both bounds above depend on $\phi$ and assume their lowest
values for $\phi \simeq \phi_l$. Taking, e.g., $n = 0$ and $N_X = 2$ the equations above imply

$$\lambda_{\mu} \lesssim \frac{1.5 \cdot 10^{-5}}{5.3 \cdot 10^{-5}} \text{ for} \ K = K_1, K_2, \\
\frac{1.5 \cdot 10^{-5}}{5.3 \cdot 10^{-5}} \text{ for} \ K = K_3 - K_5. \quad (62)$$

Since $\phi_l$ is inverse proportional to $c_R$ and $\lambda$ is proportional
to $c_R^2$ according to Eqs. (43) and (48a) respectively, the
bounds above are practically independent of the variation of
$\lambda$ and $\phi_l$. Similar bounds are obtained in Refs. [24,27,33,34]
and should not be characterized as unnatural, given that the
Yukawa coupling constant which provides masses to the up-
type quarks, is of the same order of magnitude at a high scale
cf. Ref. [57].

5.2 Solution to the $\mu$ problem of MSSM

Supplementing, in Sect. 5.2.2, with soft SUSY breaking
terms the SUSY limit of the SUGRA potential – found
in Sect. 5.2.1 – we can show that our model assists us to under-
stand the origin of $\mu$ term of MSSM, consistently with the
low-energy phenomenology – see Sect. 5.2.3.

5.2.1 SUSY potential

The presence of unity in the arguments of the logarithms of
$K$’s in Eqs. (9a)–(9e) with $F_R$ and $F_-$ defined in Eq. (25)
allows us to expand them for $m_p \rightarrow \infty$ up to quadratic terms
obtaining $\tilde{K}$. Focusing on the $\mathcal{S} - \tilde{\Phi} - \Phi$ system we obtain

\begin{table}[h]
\centering
\caption{Mass-squared spectrum of the non-inflaton sector for various $K$’s along the path in Eqs. (27) and (59)}
\begin{tabular}{|c|c|c|c|}
\hline
Fields & Eigenstates & Masses squared & \multicolumn{2}{c|}{\text{Sect. 5.2.2.}} \\
\hline
10 Real & $\tilde{\nu}_+, \tilde{\nu}_-$ & $\tilde{m}_h^2$ & $3 \tilde{m}_{\nu}^2 (1 + c_R \phi / N \pm 4 \lambda_{\mu} f_R / \lambda \phi^2)$ & $3 \tilde{m}_{\nu}^2 (1 + 1/N_X \pm 4 \lambda_{\mu} / \lambda \phi^2)$ \\
\hline
Scalars & $\tilde{\nu}_j^c, \tilde{\nu}_j^c$ & $\tilde{m}_{\nu}^2 \tilde{\nu}_j^c$ & $3 \tilde{m}_{\nu}^2 (\phi / N + 16 \lambda_{\nu}^2 \tilde{\nu}_j^c / \lambda^2 \phi^2)$ & $3 \tilde{m}_{\nu}^2 (1 + 1/N_X + 16 \lambda_{\nu}^2 \tilde{\nu}_j^c / \lambda^2 \phi^2)$ \\
\hline
3 Weyl spinors & $\hat{N}^c_i$ & $\tilde{m}_{\nu}^2 \tilde{N}^c_i$ & $48 \tilde{m}_{\nu}^2 \lambda_{\nu}^2 / \lambda^2 \phi^2$ & \\
\hline
\end{tabular}
\end{table}
\[ \bar{K} = - N m p c R (F_R + F_R^*) + \frac{1}{2} \tilde{c}^2 N (F_R + F_R^*)^2 + F_+ + |S|^2 \]  

(63)

from which we can then compute

\[ (\bar{K}_{a\bar{b}}) = \text{diag}(\tilde{M}_\pm, 1) \]  

(64a)

where the matrix \( \tilde{M}_\pm \) reads

\[ \tilde{M}_\pm = \begin{bmatrix} 1 + N c_R^2 |\Phi|^2 / 4 |F_R|^2 & N c_R^2 \Phi^* / 4 |F_R|^2 \\ N c_R^2 \Phi^* / 4 |F_R|^2 & 1 + N c_R^2 |\Phi|^2 / 4 |F_R|^2 \end{bmatrix} \]  

(64b)

After calculating \( \bar{K}^{-1} \) and substituting it into the SUSY limit

\[ V_{\text{SUSY}} = \bar{K}_{a\bar{b}} W_{H_a} W_{H_a} + g_s^2 \sum_a D_a D_a, \]  

(65)

of \( \bar{V}_{\text{HI}} \) in Eq. (5b), we find

\[ V_{\text{SUSY}} = \lambda^2 |\Phi| - 1 - M^2 |^2 + \frac{\lambda^2 |S|^2}{\det \tilde{M}_\pm} \left( |\Phi|^2 + |\Phi|^2 \right) \]  

+ \frac{g_s^2}{2} \left( |\Phi|^2 - |\Phi|^2 \right)^2, \]  

(66)

where the non-diagonal contributions in the F terms proportional to \( W_{H_a} W_{H_a} \), and \( W_{H_1} \Phi W_{H_1} \), \( W_{H_1} \Phi W_{H_1} \), \( W_{H_1} \Phi W_{H_1} \), and \( W_{H_1} \Phi W_{H_1} \), are cancelled out and

\[ \det \tilde{M}_\pm = 1 + c_R^2 N \left( |\Phi|^2 + |\Phi|^2 \right) / 4 |F_R|^2. \]  

(67)

From Eq. (66), we find that the SUSY vacuum lies along the D-flat direction \( |\Phi| = |\Phi| \) with

\[ \langle S \rangle = 0 \quad \text{and} \quad \langle |\Phi| \rangle = \langle |\Phi| \rangle = M / 2. \]  

(68)

As a consequence, \( \langle \Phi \rangle \) and \( \langle \tilde{\Phi} \rangle \) break spontaneously \( U(1)_{R-L} \) down to \( Z_2^{R-L} \). Since \( U(1)_{R-L} \) is already broken during nMI, no cosmic string are formed. Given, finally, that \( X^\beta \) participates in \( \bar{K} \), see Eq. (63), with terms similar to the one we have for \( S \), we can easily verify that their v.e.v.s lie along the direction in Eq. (59).

5.2.2 Generation of the \( \mu \) term of MSSM

The contributions from the TeV scale soft SUSY breaking terms, although negligible during nMI may shift slightly \( \langle S \rangle \) from zero in Eq. (68). The relevant potential terms are

\[ V_{\text{soft}} = \left( \lambda A_\lambda \Phi \Phi - a_S S \lambda \lambda M^2 / 4 + \text{h.c.} \right) + m_\alpha^2 |X^\alpha|^2, \]  

(69)

where \( m_\alpha \), \( A_\lambda \), and \( a_S \) are soft SUSY breaking mass parameters. Considering \( V_{\text{soft}} \) together with \( V_{\text{SUSY}} \) from Eq. (66) we end up with the total low energy potential

\[ V_{\text{tot}} = V_{\text{SUSY}} + V_{\text{soft}}. \]  

(70)

which, replacing \( \Phi \) and \( \tilde{\Phi} \) by their SUSY v.e.v.s from Eq. (66), can be rephrased as

\[ \langle V_{\text{tot}}(S) \rangle = \lambda^2 M^2 S^2 / (N c_R^2 / 2 + \lambda a_S / 2 m_3 / 2 M S^2), \]  

(71a)

if we neglect \( m_\alpha \ll M \) and set

\[ |A_\lambda| + |a_S| = 2 a_S / 2 m_3 / 2. \]  

(71b)

Here \( m_3 / 2 \) is the gravitino (\( \tilde{G} \)) mass and \( a_S / 2 \geq 0 \) a parameter of order unity which parameterizes our ignorance for the dependence of \( |A_\lambda| \) and \( |a_S| \) on \( m_3 / 2 \) – note that the phases of \( A_\lambda \) and \( a_S \) have been chosen so that \( \langle \Phi \rangle \) is minimized and \( S \) has been rotated in the real axis by an appropriate \( R \)-transformation. The extremum condition for \( \langle V_{\text{tot}}(S) \rangle \) in Eq. (71a) w.r.t \( S \) leads to a non-vanishing \( \langle S \rangle \) as follows

\[ \frac{d}{dS} \langle V_{\text{tot}}(S) \rangle = 0 \quad \Rightarrow \quad \langle S \rangle \simeq a_S / 2 m_3 / 2 (N c_R^2 / 2 + 2 \lambda). \]  

(72a)

The generated \( \mu \) term from \( W_\mu \) in Eq. (58a) is

\[ \mu = \lambda_\mu \langle S \rangle \simeq \lambda_\mu a_S / 2 m_3 / 2 (N c_R^2 / 2 + 2 \lambda). \]  

(72b)

which, although similar, is clearly distinguishable from the results obtained in Refs. [24, 27, 33, 34]. The resulting \( \mu \) above depends only on \( n \) and \( \lambda_\mu \) since \( \lambda / c_R^2 \) is fixed for frozen \( n \) by virtue of Eqs. (48a)–(48c). As a consequence, we may verify that any \( |\mu| \) value is accessible for the \( \lambda_\mu \)’s allowed by Eq. (62) without any ugly hierarchy between \( m_3 / 2 \) and \( \mu \).

5.2.3 Link to the MSSM phenomenology

The subgroup, \( Z_2^R \) of \( U(1)_R \) – which remains unbroken after the consideration of the SUSY breaking effects in Eq. (69) – combined with the \( Z_2^L \) fermion parity yields the well-known \( R \)-parity. This symmetry guarantees the stability of the lightest SUSY particle (LSP), providing thereby a well-motivated cold dark matter (CDM) candidate.

The candidacy of LSP may be successful, if it generates the correct CDM abundance [39] within a concrete low-energy framework. In the case under consideration [33, 34] this could
be the Constrained MSSM (CMSSM), which employs the following free parameters

$$\text{sign}\mu, \tan \beta = (H_u) / (H_d), M_{1/2}, m_0 \text{ and } A_0,$$

where sign$\mu$ is the sign of $\mu$, and the three last mass parameters denote the common gaugino mass, scalar mass and trilinear coupling constant, respectively, defined (normally) at $M_{\text{GUT}}$. The parameter $|\mu|$ is not free, since it is computed at low scale by enforcing the conditions for the electroweak symmetry breaking. The values of the parameters in Eq. (73) can be tightly restricted imposing a number of cosmo-phenomenological constraints from which the consistency of LSP relic density with observations plays a central role. Some updated results are recently presented in Ref. [58], where we can also find the best-fit values of $|A_0|$, $m_0$ and $|\mu|$ listed in the second, third and fourth leftmost columns of Table 4. We see that there are four allowed regions characterized by the specific mechanism for suppressing the relic density of the LSP which is the lightest neutralino ($\chi$) – note that $\tilde{\tau}_1, \tilde{\tau}_1$ and $\tilde{\chi}^\pm_1$ stand for the lightest stau, stop and chargino eigenstate.

The inputs from Ref. [58] can be deployed within our setting, if we identify, e.g.,

$$m_0 = m_{3/2} \text{ and } |A_0| = |A_\text{L}| = |A_\text{S}|.$$  \hspace{1cm} (74)

Fixing also $n = 0$, we first derive $a_{3/2}$ from Eq. (71b) – see fifth column of Table 4 – and then the $\lambda_{\mu}$ values which yield the phenomenologically desired $|\mu| -$ ignoring renormalization group effects. The outputs w.r.t $\lambda_{\mu}$ of our computation are listed in the fourth rightmost columns of Table 4 for $K = K_1 - K_5$. From these we infer that the required $\lambda_{\mu}$ values vary slightly depending on $N$ and $c_R$ required by each $K$ and, besides the ones written in italics, are comfortably compatible with Eq. (62). Therefore, the whole inflationary scenario can be successfully combined with all the allowed regions CMSSM besides the $\tilde{\tau}_1 - \chi$ coannihilation region for $K = K_1$ and $K_2$. On the other hand, the $m_{3/2}$’s used in all the CMSSM regions can be consistent with the gravitino limit on reheat temperature $T_{\text{rh}}$, under the assumption of the unstable $\tilde{G}$, for the $T_{\text{rh}}$ values necessitated for satisfactory leptogenesis – see Sect. 5.3.2.

5.3 Non-thermal leptogenesis and neutrino masses

Besides the generation of $\mu$ term, our post-inflationary setting offers a graceful exit from the inflationary phase (see Sect. 5.3.1) and explains the observed BAU (see Sect. 5.3.2) consistently with the $\tilde{G}$ constraint and the low energy neutrino data. Our results are summarized in Sect. 5.3.3.

5.3.1 Inflaton mass and decay

When nMI is over, the inflaton continues to roll down towards the SUSY vacuum, Eq. (68). Soon after, it settles into a phase of damped oscillations around the minimum of $\tilde{V}_{\text{HI}}$. The (canonically normalized) inflaton,

$$\hat{\delta} \phi = \langle J \rangle \delta \phi \text{ with } \delta \phi = \phi - M,$$

where $\langle J \rangle$ is estimated from Eq. (38), acquires mass given by

$$\hat{m}_{\delta \phi} = \left( \tilde{V}_{\text{HI}} \delta \phi \right)^{1/2} = \frac{\lambda M}{\sqrt{2} \langle J \rangle} \simeq \frac{\lambda M}{\sqrt{N c_R}}.$$  \hspace{1cm} (76)

As we see, $\hat{m}_{\delta \phi}$ depends crucially on $M$ which is bounded from above by the requirement $\langle f_R \rangle = 1$ ensuring the establishment of the conventional Einstein gravity at the vacuum. This bound is translated to an upper bound on the mass $\langle M_{\text{BL}} \rangle$ that the $B-L$ gauge boson acquires for $\phi = \langle \phi \rangle$ – see Table 2. Namely we obtain $\langle M_{\text{BL}} \rangle \lesssim 10^{14}$ GeV, which is lower than the value $M_{\text{GUT}} \simeq 2 \cdot 10^{16}$ GeV dictated by the unification of the MSSM gauge coupling constants – cf.Refs. [27, 33, 34]. However, since $U(1)_{B-L}$ gauge symmetry does not disturb this unification, we can treat $\langle M_{\text{BL}} \rangle = g M$ as a free parameter with $g \simeq 0.5 - 0.7$ being the value of the GUT gauge coupling at the scale $\langle M_{\text{BL}} \rangle$.

During the phase of its oscillations at the SUSY vacuum, $\delta \phi$ decays perturbatively reheating the Universe at a reheat temperature given by [59]
where the unusual – cf. Refs. [27,33,34] – prefactor is consistent with $w_{\text{th}} \simeq 0.33$ [59] and we set $g_{\text{th}} = 228.75$ as in Eq. (52a). The total decay width of $\delta \phi$ is found to be

$$
\Gamma_{\delta \phi} = \Gamma_{\delta \phi \rightarrow N_i^c} + \Gamma_{\delta \phi \rightarrow H} + \Gamma_{\delta \phi \rightarrow XYZ},
$$

(77b)

where the individual decay widths are

$$
\Gamma_{\delta \phi \rightarrow N_i^c} = \frac{g_i^2 N_c}{16 \pi} \frac{M_i^2}{m_{\delta \phi}} \left( 1 - \frac{4 M_i^2}{m_{\delta \phi}^2} \right)^{3/2},
$$

(78a)

$$
\Gamma_{\delta \phi \rightarrow H} = \frac{2}{8 \pi} g_H^2 m_{\delta \phi},
$$

(78b)

$$
\Gamma_{\delta \phi \rightarrow XYZ} = \frac{g_y^2}{27 \pi} \frac{m_{\delta \phi}^3}{m_p^2},
$$

(78c)

and the relevant coupling constants are defined as follows

$$
g_{i \, N_c} = \frac{\lambda_{i \, N_c}}{(M)}, \quad g_H \simeq \lambda_{\mu} / \sqrt{2} \quad \text{and} \quad g_y = y_3 N_c (J) c_R / 2.
$$

(79a)

(79b)

The decay widths above arise from the lagrangian terms

$$
\mathcal{L}_{\delta \phi \rightarrow N_i^c N_i^c} = -\frac{1}{2} e^{K / 2 m_{\tilde{\phi}}^2} W_{RHN, N_i^c N_i^c} \left( N_i^c N_i^c + \text{h.c.} \right) + \cdots,
$$

(80a)

$$
\mathcal{L}_{\delta \phi \rightarrow H_a H_d} = -e^{K / m_{\tilde{\phi}}^2} K^{SS} \left| W_{\mu, S} \right|^2 \quad \text{and} \quad \mathcal{L}_{\delta \phi \rightarrow H_a H_d} = -e^{K / m_{\tilde{\phi}}^2} \left( H_a^* H_d + \text{h.c.} \right) + \cdots,
$$

(80b)

$$
\mathcal{L}_{\delta \phi \rightarrow X \psi_Y \psi_Z} = -\frac{1}{2} e^{K / m_{\tilde{\phi}}^2} \left( W_{Y, Y} \psi_Y \psi_Z \right) + \text{h.c.},
$$

(80c)

describing respectively $\delta \phi$ decay into a pair of $N_i^c$ with masses $M_{1, N_c} = \lambda_{i, N_c} M < m_{\delta \phi} / 2$, $H_a$ and $H_d$ and three MSSM (s)-particles $X$, $Y$, $Z$ involved in a typical trilinear superpotential term $W_{Y, Y} = y X Y Z$. Here, $\psi_X$, $\psi_Y$ and $\psi_Z$ are the chiral fermions associated with the superfields $X$, $Y$ and $Z$ whose scalar components are denoted with the superfield symbols and $y = y_3 \simeq (0.4 - 0.6)$ is a Yukawa coupling constant of the third generation.

5.3.2 Lepton-number and gravitino abundances

For $T_{\text{th}} < M_{1, N_c}$, the out-of-equilibrium decay of $N_i^c$ generates a lepton-number asymmetry (per $N_i^c$ decay), $\varepsilon_i$ estimated from Ref. [60]. The resulting lepton-number asymmetry is partially converted through sphaleron effects into a yield of the observed BAU

$$
Y_B = -0.35 \frac{3}{2} \frac{T_{\text{th}}}{m_{\delta \phi}} \sum_i \frac{\Gamma_{\delta \phi \rightarrow N_i^c}}{\Gamma_{\delta \phi}} \varepsilon_i,
$$

(81)

where $\langle H_a \rangle \simeq 174 \text{ GeV}$, for large $\beta$ and $m_{1D}$ is the Dirac mass matrix of neutrinos, $\nu_i$. The ratio $(3 / 2)$ is again [59] consistent with $w_{\text{th}} = 0.33$. The expression above has to reproduce the observational result [39]

$$
Y_B \simeq \left( 8.64 \pm 0.15 \right) \cdot 10^{-11}.
$$

(82)

The validity of Eq. (81) requires that the $\delta \phi$ decay into a pair of $N_i^c$’s is kinematically allowed for at least one species of the $N_i^c$’s and also that there is no erasure of the produced $Y_L$ due to $N_i^c$ mediated inverse decays and $\Delta L = 1$ scatterings. These prerequisites are ensured if we impose

(a) $m_{\tilde{\delta \phi}} \geq 2M_{1, N_c}$ and (b) $M_{1, N_c} \gtrsim 10T_{\text{th}}$.

(83)

The quantity $\varepsilon_i$ can be expressed in terms of the Dirac masses of $\nu_i$, $m_{1D}$, arising from the third term of Eq. (24). Employing the (type I) seesaw formula we can then obtain the light-neutrino mass matrix $m_\nu$ in terms of $m_{1D}$ and $M_{1, N_c}$. As a consequence, nTL can be nicely linked to low energy neutrino data. We take as inputs the recently updated best-fit values [61] – cf. Refs. [33, 34] – on the neutrino mass-squared differences, $\Delta m_{21}^2 = 7.56 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 2.55 \cdot 10^{-3} \text{ eV}^2$ [49].

The mixing angles, $\sin^2 \theta_{12} = 0.321$, $\sin^2 \theta_{13} = 0.02155$ [$\sin^2 \theta_{13} = 0.0214$] and $\sin^2 \theta_{23} = 0.43$ [$\sin^2 \theta_{23} = 0.596$] and the CP-violating Dirac phase $\delta = 1.44 \pi$ [$\delta = 1.44 \pi$] for normal [inverted] ordered (NO [IO]) neutrino masses, $m_\nu$’s. Furthermore, the sum of $m_{1D}$’s is bounded from above at $95\%$ c.l. by the data [39]

$$
\sum_i m_{1D} \leq 0.23 \text{ eV}.
$$

(84)

The required $T_{\text{th}}$ in Eq. (81) must be compatible with constraints on the $\tilde{G}$ abundance, $Y_{3/2}$, at the onset of nucleosynthesis (BBN), which is estimated to be [62–67]

$$
Y_{3/2} \simeq 1.9 \cdot 10^{-22} \frac{T_{\text{th}}}{\text{GeV}},
$$

(85)

where we take into account only thermal production of $\tilde{G}$, and assume that $\tilde{G}$ is much heavier than the MSSM gauginos. Non-thermal contributions to $Y_{3/2}$ [68] are also possible but strongly dependent on the mechanism of soft SUSY breaking. No precise computation of this contribution exists within nMI adopting the simplest Polonyi model of SUSY breaking [69, 70]. It is notable, though, that the non-thermal contribution to $Y_{3/2}$ in models with stabilizer field, as in our case, is significantly suppressed compared to the thermal one.
On the other hand, \( Y_{3/2} \) is bounded from above in order to avoid spoiling the success of the BBN. For the typical case where \( \hat{G} \) decays with a tiny hadronic branching ratio, we obtain \([64–67]\) an upper bound on \( T_{\text{th}} \), i.e.,

\[
T_{\text{th}} \lesssim 5.3 \begin{cases} 
10^7 \text{ GeV} \\
10^6 \text{ GeV} \\
10^9 \text{ GeV}
\end{cases} \text{ for } m_{3/2} \simeq \begin{cases} 
0.69 \text{ TeV} \\
10.6 \text{ TeV} \\
13.5 \text{ TeV}
\end{cases}
\]

The bounds above can be somehow relaxed in the case of a stable \( \hat{G} \).

5.3.3 Results

Confronting with observations \( Y_B \) and \( T_{\text{th}} \) which depend on \( \hat{m}_{\delta_\phi} \), \( M_{1N^c} \) and \( m_{1D}'s \) – see Eqs. (81) and (86) – we can further constrain the parameter space of the our models. In our investigation we follow the bottom-up approach detailed in Refs. [33, 34], according to which we find the \( M_{1N^c}'s \) by using as inputs the \( m_{1D}'s \), a reference mass of the \( v_i's \) – \( m_{1D} \) for NO \( m_{1V}'s \), or \( m_{3V} \) for IO \( m_{1V}'s \), the two Majorana phases \( \varphi_1 \) and \( \varphi_2 \) of the PMNS matrix, and the best-fit values for the low energy parameters of neutrino physics mentioned in Sect. 5.3.2. In our numerical code, we also estimate [33, 34] the renormalization-group evolved values of the latter parameters at the scale of nTL, \( \Lambda_L = \hat{m}_{\delta_\phi} \), by considering the MSSM with tan \( \beta \simeq 50 \) as an effective theory between \( \Lambda_L \) and the soft SUSY breaking scale, \( M_{\text{SUSY}} = 1.5 \text{ TeV} \). We evaluate the \( M_{1N^c}'s \) at \( \Lambda_L \), and we neglect any possible running of the \( m_{1D}'s \) and \( M_{1N^c}'s \). The so obtained \( M_{1N^c}'s \) clearly correspond to the scale \( \Lambda_L \).

Some representative values of the parameters which yield \( Y_B \) and \( T_{\text{th}} \) compatible with Eqs. (82) and (86), respectively, are arranged in Table 5. We take \( n = 0 \) – to avoid any tuning as regards the inflationary inputs –, \( \lambda_\mu = 10^{-6} \) in accordance with Eq. (62), and \( \langle M_{BL} \rangle = 10^{12} \text{ GeV} \). Note that we consider \( \langle M_{BL} \rangle \) as a free parameter since the unification value – imposed in Refs. [27, 33, 34] – is not reconciled with the reappearance of Einstein gravity at low energies, i.e., \( \langle f_{\text{R}} \rangle = 1 \). Setting \( g = 0.7 \) in the formula giving \( M_{BL} \) in Table 2, we obtain \( M = 1.43 \cdot 10^{12} \text{ GeV} \) resulting via Eq. (76) to \( 2.8 \leq \hat{m}_{\delta_\phi}/10^9 \text{ GeV} \leq 4.1 \) – the variation is due to the choice of \( K \). Although this amount of uncertainty does not cause any essential alteration of the final outputs, we mention just for definiteness that we take throughout \( K = K_1 \) corresponding to \( \hat{m}_{\delta_\phi} = 3.3 \cdot 10^{10} \text{ GeV} \). We consider NO (cases A and B), almost degenerate (cases C, D and E) and IO (cases F and G) \( m_{1V}'s \). In all cases, the current limit in Eq. (84) is safely met. This is more restrictive than the 90% c.l. upper bound arising from the effective electron neutrino mass \( m_\beta \) in \( \beta \)-decay [71–73] by various experiments. Indeed, the current upper bounds on \( m_\beta \) are comfortably satisfied by the values found in our set-up.

### Table 5 Parameters yielding the correct BAU for \( K = K_1 \) or \( K_2 \), \( \langle M_{BL} \rangle = 10^{12} \text{ GeV} \), \( n = 0 \), \( \lambda_\mu = 10^{-6} \), \( y_3 = 0.5 \) and various neutrino mass schemes

| Cases | A | B | C | D | E | F | G |
|-------|---|---|---|---|---|---|---|
| Parameters | Normal Hierarchy | Almost Degeneracy | Inverted Hierarchy |
| Low scale parameters (masses in eV) | | | | | | | |
| \( m_{1V}/0.1 \) | 0.01 | 0.1 | 0.5 | 0.6 | 0.7 | 0.51 | 0.5 |
| \( m_{2V}/0.1 \) | 0.09 | 0.13 | 0.51 | 0.61 | 0.71 | 0.52 | 0.51 |
| \( m_{3V}/0.1 \) | 0.5 | 0.51 | 0.71 | 0.78 | 0.5 | 0.1 | 0.05 |
| \( \sum m_{1V}/0.1 \) | 0.6 | 0.74 | 1.7 | 2 | 1.9 | 1.1 | 1 |
| \( \varphi_1 \) | \( -\pi/9 \) | \( -2\pi/3 \) | \( \pi \) | \( \pi/2 \) | \( \pi/2 \) | \( 3\pi/5 \) | \( 2\pi/3 \) |
| \( -\varphi_2 \) | \( \pi/2 \) | \( 0 \) | \( -\pi/2 \) | \( 2\pi/3 \) | \( 2\pi/3 \) | \( \pi/2 \) | \( \pi/2 \) |
| Leptogenesis-scale mass parameters in GeV | | | | | | | |
| \( m_{1D}/0.1 \) | 0.38 | 0.69 | 3.23 | 2.25 | 0.94 | 50 | 1.9 |
| \( m_{2D} \) | 1 | 5 | 0.2 | 0.15 | 0.3 | 0.097 | 0.2 |
| \( m_{3D} \) | 1 | 10 | 0.5 | 10 | 9 | 6 | 5 |
| \( M_{1N^c}/10^8 \) | 5.4 | 8.4 | 9.9 | 4.7 | 2.2 | 7.9 | 12.6 |
| \( M_{2N^c}/10^9 \) | 13 | 755 | 2.5 | 0.96 | 1.57 | 219 | 3.5 |
| \( M_{3N^c}/10^{11} \) | 3.2 | 18.8 | 1.6 | 6.1 | 5 | 12.7 | 12.1 |
| Decay channels of \( \delta_\phi \) | \( \delta_\phi \rightarrow N_i^c \) | \( N_i^c \) | \( N_i^c \) | \( N_{i,2}^c \) | \( N_{i,2}^c \) | \( N_i^c \) | \( N_1^c \) |
| Resulting B-yield | \( Y_B/10^{-11} \) | 8.7 | 8.6 | 8.6 | 8.6 | 8.5 | 8.7 | 8.6 |
| Resulting \( T_{\text{th}} \) (in GeV) | \( T_{\text{th}}/10^7 \) | 2.8 | 3 | 3 | 3.1 | 2.7 | 2.9 | 3.3 |
inferred from Table 5, we examine also how the central value of \( m_{B} \) in Eq. (5.8) consistently with the inflationary requirements, in the a \( \langle M_{BL} \rangle - m_{1D} \) and b \( \langle \tilde{m}_{\delta} \rangle - M_{1N^c} \) plane. We take \( K = K_1 \), \( n = 0 \), \( \lambda_{\mu} = 10^{-6} \) and the values of \( m_{1v} \), \( m_{2D} \), \( m_{3D} \), \( \varphi_1 \) and \( \varphi_2 \) which correspond to the cases A (solid line), C (dashed line) and E (dot-dashed line) of Table 5.

\[
0.002 \leq m_\beta/eV \leq 0.036, \tag{87}
\]

where the lower and upper bound corresponds to case A and C respectively.

The gauge symmetry considered here does not predict any particular Yukawa unification pattern and so, the \( m_{1D} \)'s are free parameters. This fact allows us to consider \( m_{1D} \)'s which are not hierarchical depending on the generation. Also, it facilitates the fulfillment of Eq. (83b) since \( m_{1D} \) affects heavily \( M_{1N^c} \). Care is also taken so that the perturbativity of \( \lambda_{1N^c} \) – defined below Eq. (80c) – holds, i.e., \( \lambda_{1N^c}^2/4\pi \leq 1 \). The inflaton \( \delta \phi \) decays mostly into \( N^c_1 \)'s – see cases A – E. In all cases \( \Gamma_{\delta \phi \rightarrow N^c_1} < \Gamma_{\delta \phi \rightarrow H} \) and so the ratios \( \Gamma_{\delta \phi \rightarrow N^c_1}/\Gamma_{\delta \phi} \) introduce a considerable reduction in the derivation of \( Y_B \).

\[
0.07 \lesssim \frac{\Gamma_{\delta \phi \rightarrow N^c_1}}{\Gamma_{\delta \phi}} \lesssim 0.35 \tag{88}
\]

where the lower [upper] bound comes out in case E [G]. In Table 5 we also display the values of \( T_{th} \), the majority of which are close to \( 3 \cdot 10^7 \) GeV, and consistent with Eq. (86) for \( m_{3/2} \gtrsim 1 \) TeV. These values are in nice agreement with the ones needed for the solution of the \( \mu \) problem of MSSM – see, e.g., Table 4. Thanks to our non-thermal set-up, successful leptogenesis can be accommodated with \( T_{th} \)'s lower than those necessitated in the thermal regime – cf. Ref. [74].

In order to investigate the robustness of the conclusions inferred from Table 5, we examine also how the central value of \( Y_B \) in Eq. (82) can be achieved by varying \( \langle M_{BL} \rangle \), or \( \tilde{m}_{\delta \phi} \), and adjusting conveniently \( m_{1D} \) or \( M_{1N^c} \) – see Fig. 4a and b respectively. We fix again \( n = 0 \) and \( \lambda_{\mu} = 10^{-6} \). Since the range of \( Y_B \) in Eq. (82) is very narrow, the 95% c.l. width of these contours is negligible. The convention adopted for the various lines is also depicted. In particular, we use solid, dashed and dot-dashed line when the remaining inputs – i.e. \( m_{1v}, m_{2D}, m_{3D}, \varphi_1, \) and \( \varphi_2 \) – correspond to the cases A, C and E of Table 5, respectively. At the lower limit of these lines \( n_{TL} \) becomes inefficient (due to low \( T_{th} \)) failing to reach the value in Eq. (82). At the other end, these lines terminate at the values of \( m_{1D} \) beyond which Eq. (83b) is violated and, therefore, washout effects start becoming significant. Along the depicted contours, the resulting \( M_{2N^c} \) and \( M_{3N^c} \) remain close to their values presented in the corresponding cases of Table 4. As regards the other quantities, in all we obtain

\[
0.04 \lesssim T_{th}/10^8 \text{ GeV} \lesssim 13, \tag{89a}
\]
\[
0.03 \lesssim \tilde{m}_{\delta \phi}/10^{10} \text{ GeV} \lesssim 4.64, \tag{89b}
\]

with the lower and upper bound obtained in the limits of the solid line which represent the most ample region of parameters satisfying the imposed requirements. These values are much lower than those obtained in Ref. [27] and a little lower than those found in Refs. [33,34], mainly due to lower \( \langle M_{BL} \rangle \)'s employed here.

As a bottom line, \( n_{TL} \) is a realistic possibility within our setting. It can be comfortably reconciled with the \( \tilde{G} \) constraint even for \( m_{3/2} \sim 1 \) TeV as deduced from Eqs. (89b) and (86) adopting a sufficiently low \( \langle M_{BL} \rangle \).

6 Conclusions

Motivated by the fact that a strong linear non-minimal coupling of the inflaton to gravity does not cause any problem with the validity of the effective theory up to the Planck scale, we explored the possibility to attain observationally viable...
nMI (i.e. non-minimal inflation) in the context of standard SUGRA by strictly employing this coupling. We showed that nMI is easily achieved, for $p \leq 4$ in the superpotential of Eq. (8), by conveniently adjusting the prefactor $(-N)$ of the logarithmic part of the relevant Kähler potentials $K$ given in Eqs. (9a)–(9e), where the relevant functions $F_R$ and $F_-$ are shown in Eq. (10) for a gauge-singlet inflaton. For appropriately selected integer $N$'s – i.e., setting $n = 0$ in Eq. (20) –, our models retain the predictive power of well-known universal attractor models – which employ a non-minimal coupling functionally related to the potential – and yield similar results. Allowing for non-integer $N$ values, this predictability is lost since the observables depend on the adopted $K$ and $n$ in Eq. (20) and may yield any $n_s$ in its allowed region and 0.0013 $\leq r \leq 0.02$.

This scheme works also for a gauge non-singlet inflaton employing the superpotential shown in Eq. (24) and the functions $F_R$ and $F_-$ in Eq. (25). Embedding these models within a $B - L$ extension of MSSM, we showed that a $\mu$ term is easily generated and the baryon asymmetry in the Universe is naturally explained via non-thermal leptogenesis. The $B - L$ breaking scale $\langle M_{BL} \rangle$, though, has to take values lower than the MSSM unification scale and so, the present scheme is similarly predictive with that of Refs. [33,34] which employs one more parameter in the Kähler potentials but allows for $\langle M_{BL} \rangle$'s fixed by the gauge coupling unification within MSSM. Our scenario can be comfortably tolerated with almost all the allowed regions of the CMSSM with gravitino as low as 1 TeV. Moreover, leptogenesis is realized through the out-of-equilibrium decay of the inflaton to the right-handed neutrinos $N^i_1$ and/or $N^i_2$ with masses lower than 2.32 $\cdot 10^9$ GeV, and a reheat temperature $T_{rh} \leq 10^9$ GeV taking $\langle M_{BL} \rangle \leq 10^{13}$ GeV.

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