Performance Analysis of Relay-Assisted OWC Over Foggy Channel with Pointing Error

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Abstract—Signal fading due to atmospheric channel impairments and pointing error is a major bottleneck for the performance of optical wireless communication (OWC). In this paper, we consider an amplify-and-forward (AF) optical relaying to enhance the performance of the OWC system with a negligible line-of-sight (LOS) link under the combined effect of fog and pointing error. We analyze the end-to-end performance of the relay-assisted system, which consists of complicated probability distribution functions. We derive analytical expressions of the outage probability, average signal-to-noise ratio (SNR), and ergodic rate in terms of OWC system parameters. We also develop an exact integral-form expression of these performance metrics using the half-harmonic mean of individual SNRs to validate the tightness of the derived analytical expressions. The numerical and simulation analysis shows that the proposed dual-hop relaying has significant performance improvement when compared to the direct transmission over considered channel impairments. Compared to the direct transmission, the relay-assisted system requires almost 30 times less transmission power to achieve the same outage probability. The considered system also provides a significant gain in the average SNR and ergodic rate for practical scenarios of OWC deployment.

Index Terms—Exotic channels, fog, optical wireless communication, performance analysis, outage probability, pointing error, relaying, SNR.

I. INTRODUCTION

Optical wireless communication (OWC) is an emerging broadband technology that transmits data through an unguided atmospheric channel in the unlicensed optical spectrum [1], [2]. However, signal transmission at small wavelengths encounters different channel impairments such as atmospheric turbulence (due to the scintillation effect of light propagation), pointing error (caused by the misalignment between the transmitter and receiver), and fog. The pointing error caused by the dynamic wind loads, weak earthquakes and thermal expansion has a statistical effect on the signal quality and presents a major challenge in the OWC deployment [3], [4]. On the other hand, the impact of foggy conditions on OWC systems depends on the intensity of fog, which ranges between light, medium and dense. It is noted that turbulence and fog may not co-exist since both are inversely correlated with each other. This allows us to investigate the effect of turbulence and foggy conditions separately on the OWC performance.

The use of relaying to improve the performance of OWC systems has been extensively studied under the effect of turbulence and/or pointing errors [5]–[17]. Hybrid RF/OWC systems, where relays act as an interface between RF and optical links, have been studied in [5]–[10]. Considering a dual-hop transmission with a single-relay and ignoring the direct transmission, the authors in [5]–[7] have analyzed the OWC performance under the turbulence and pointing error. The use of multiple relays in parallel with active relay-selection is shown to improve the reliability of an OWC system in [8], [9]. A multi-user dual-hop system using a multi-aperture RF relay over mixed links is investigated in [10].

Recently, relaying for all-optical OWC systems has gained considerable research interests [11]–[17]. In the seminal work [11], both multi-hop and cooperative relaying coupled, with amplify-and-forward and decode-and-forward modes has been considered only for turbulence channel. The authors in [12], [13] provided asymptotic expressions for outage probability, average bit-error-rate (BER) and ergodic capacity under turbulence and pointing errors by considering a single amplify-and-forward (AF) relay with fixed and variable gain. Multi-hop relaying with AF and decode-and-forward (DF) protocol, under the Gamma-Gamma turbulence and pointing error, has been considered in [14], [15]. The authors in [16] have considered inter-relay cooperation to boost the reliability of an OWC system. In a line-of-sight communication, a single relay can be used to enhance the direct transmission both the received signals at the destination, as shown in [17].

In the aforementioned and related research, the statistical effect of foggy channels has not been considered. Traditionally, signal attenuation due to the fog was quantified using a visibility range, less in light fog and more in dense fog [18], [19]. However, recent studies confirm that signal attenuation due to fog is not deterministic but Gamma distributed [20]–[23]. Following this direction, the authors in [24], [25] have analyzed the impact of fog on OWC using various performance measures such as average signal-to-noise ratio (SNR), ergodic rate, outage probability, and BER. They have shown that OWC performance is significantly limited in dense fog, but can provide acceptable performance in light fog over short links. However, combining the effect of pointing error with fog shows high degradation in performance even in light foggy conditions [26], [27]. The authors in [27] have considered three techniques to mitigate this effect, those being multi-hop relay systems using DF, active laser selection and parallel RF/OWC links. Laser selection and hybrid transmission techniques require feedback from the receiver to the transmitter, thereby increasing the overhead. Multi-hop relaying on the
other hand requires channel state information (CSI) at each
relay, to decode the signal, which can be hard in practice. It
is noted that there are no analyses of average SNR, ergodic
rate, and diversity order for the performance improvement of
a relay-assisted OWC system under the effect of fog and
pointing errors.

In this paper, we analyze the end-to-end performance of a
relay-assisted OWC system under the combined effect of fog
and pointing error. We consider a single optical-relay with AF
relaying, assuming no direct transmission to the destination.
Using an upper bound approximation of the end-to-end SNR,
we derive closed-form expressions for the outage probability,
average SNR, and ergodic rate of the relay-assisted OWC
system. It should be noted that it requires novel approaches
to analyze the end-to-end system since distribution functions
of individual links consists of incomplete gamma functions
and exponential integrals. The analysis show that the proposed
dual-hop relaying improves the performance of the OWC sys-
tem significantly. We also develop an integral-form expression
for the probability density function (PDF) of the end-to-end
SNR to validate the tightness of the derived analytical expres-
sions. Numerical evaluation is presented to validate the results
of the analytical formulae, demonstrating the improvement in
performance compared to the direct transmission.

II. SYSTEM MODEL

We consider an OWC system using intensity modula-
tion/direct detection (IM/DD). It consists of a single-aperture
transceiver system, with a negligible line-of-sight (LOS) link,
under fog and pointing error. The signal received at the
receiver aperture, $y$, is given as

$$y = h_1 h_p R x + w,$$

where $x$ is the transmitted signal, $R$ represents the detector
responsivity (in amperes per watt), and $w$ represents an Add-
itive White Gaussian Noise (AWGN) with variance $\sigma_w^2$. The
terms $h_1$ and $h_p$ are the random states of the foggy channel
and pointing error, respectively.

We define $\gamma = \gamma_0 |h|^2$ as the SNR, where $h = h_1 h_p$, \(\gamma_0 = 2P_t R^2/\sigma_w^2\), and $P_t$ is the average optical transmitted power.

Denoting the Gamma function as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ and
the incomplete Gamma function as $\Gamma(a, t) = \int_0^t s^{a-1} e^{-s} ds$, the PDF of SNR for the OWC system can be easily derived using the distribution of $|h|^2$ [27]:

$$f_{\gamma}(\gamma) = \frac{z^\beta e^{\frac{z^2}{\beta}}}{\Gamma(\beta+1)} \left( \frac{z}{\gamma_0} \right)^{\beta-1}$$

$$\times \left[ \Gamma(\beta) - \Gamma(\beta, m \ln(\gamma_0/\sqrt{\gamma/\gamma_0})) \right], \quad \gamma \leq A_0^2 \gamma_0$$

where, $k > 0$ is the shape parameter, $\beta > 0$ is the scale
parameter, $z = 4.343/\beta d$, $d$ (in km) the transmission link
length between source and destination, and $m = z - \rho^2$. The
parameter $\rho = \omega_z d / 2\sigma_s$ is the ratio between the equivalent
beam radius and the standard deviation of the pointing error
displacement at the receiver [27]. The parameter $A_0$ is the
fraction of collected power given as $A_0 = (\text{erf}(v))^2$ where
$v = \sqrt{\pi/2} a/\omega_z$, and $a$ is the receiver aperture radius. Since
experimental data for pointing error parameters $\rho$ and $A_0$
is available only for few link ranges, we obtain a simple
expression to determine these parameters for various link
distances for numerical analysis in Section IV.

Denoting $E_n(a, r) = \int_1^{ar} \frac{e^{-t}}{t} dt$ as the two-argument ex-
ponential integral, we can obtain the cumulative distribution
function (CDF) of foggy channel with pointing error [27]:

$$F_{\gamma}(\gamma) = \frac{z^\beta}{\Gamma(\beta)} \left[ 1 - e^{-\rho^2 \ln(\gamma_0/\sqrt{\gamma/\gamma_0})} \right]$$

$$\Gamma(\beta, m \ln(\gamma_0/\sqrt{\gamma/\gamma_0}))$$

$$+ (\rho^2 + m \ln(\gamma_0/\sqrt{\gamma/\gamma_0}))^{-1}$$

$$\left[ m \ln(\gamma_0/\sqrt{\gamma/\gamma_0})^k (e^{-\rho^2 \ln(\gamma_0/\sqrt{\gamma/\gamma_0})} + (k-1)E_n(2k, \rho^2 \ln(\gamma_0/\sqrt{\gamma/\gamma_0})) \right]$$

(3)

For the case of relayed transmission, the expressions for
signals received at the relay and destination are:

$$y_r = h_{r1} h_{p1} Rx + w_1$$

$$y_d = h_{r2} h_{p2} Gy_r + w_2$$

where $h_{r1}$, $h_{p1}$ and $h_{r2}$, $h_{p2}$ are random fog and pointing
error states between source-relay and relay-destination, respec-
tively, each having $w_1$ and $w_2$ as AWGNs. $h_{r1} = h_{r2} h_{p1}$
is the combined channel between source and relay, and
$G = \sqrt{\frac{P_t}{2\pi R^2 (\sigma_w^2 + \sigma_e^2)}}$ is the antenna gain. Here, $x$ and $y_r$ are
transmitted from the source and relay respectively.

Taking $P_t = P_r$, we get the instantaneous SNRs of signals
received at the relay and receiver as $\gamma_1 = \gamma_0 h_1^2$ and $\gamma_2 = \gamma_0 h_2^2$, respectively. Using these, the expression for the end-to-end
SNR is given as [13]:

$$\gamma_{e2e} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}$$

(6)

To derive tractable analytical expressions using the well known
half harmonic mean expression, we ignore the 1 (since the PDF
value is small for $\gamma < 1$) in the numerator to get:

$$\gamma_{e2e} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \leq \min(\gamma_1, \gamma_2)$$

(7)

We assume similar foggy conditions (i.e., same $k$ and $\beta$) for
both links. This assumption is practical since foggy conditions
do not change over distances of a few kilometers, over which
a majority of OWC systems work. However, we get different
values for $\rho$ and $A_0$ depending on the individual link lengths.
We also consider that the relay is situated at the mid-way
between the source and destination since this results into
optimum performance as demonstrated in numerical section.
This causes the parameters $A_0, \omega_z, \rho, m$ and $z$ same for
both links (i.e., a symmetric condition) facilitating insightful
analytical expressions.

III. PERFORMANCE ANALYSIS

In this section, we present an integral-form expression to
determine the exact outage probability, average SNR and
ergodic rate. Considering the intractability of the integration,
we use the upper bound approximation of the harmonic mean and derive closed-form analytical bounds of these parameters. The derived expressions show the system behavior in a relay-assisted environment under the combined effect of pointing errors and fog.

A. Distribution Function of End-to-End SNR

We first present an exact expression (with an exception of ignoring 1 in the denominator of (9)) for the PDF of end-to-end SNR and then represent a tractable PDF for the performance analysis. Denoting the PDF of $\gamma_1$ and $\gamma_2$ as $f_1(\gamma)$ and $f_2(\gamma)$, respectively, we evaluate the PDFs of $1/\gamma_1$ and $1/\gamma_2$ using the inverse distribution, and employ the PDF of their sum [28] eq.5.55 to get resulting PDF of $\gamma_{e2e}$ as

$$f(\gamma) = \gamma \int_{t_{\min}}^{t_{\max}} f_1\left(\frac{\gamma}{t}\right) f_2\left(\frac{\gamma}{1-t}\right) \frac{dt}{t^2(1-t)^2} \tag{8}$$

Considering the logarithmic term in (2) positive and applying this constraint to $f_1\left(\frac{\gamma}{t}\right) f_2\left(\frac{\gamma}{1-t}\right)$, we get $\int f_{\delta_{j\gamma,\gamma}} > 0 \implies t > \frac{A^2_{\delta_{j\gamma,\gamma}}}{\gamma} = t_{\min}$, and $\int f_{\delta_{j\gamma,\gamma}} > 0 \implies t < 1 - \frac{A^2_{\delta_{j\gamma,\gamma}}}{\gamma} = t_{\max}$. Note that using these limits, we can get the exact PDF in (8). This representation is used to numerically evaluate the OWC performance as a benchmark for our derived analytical expressions.

Since the integral-form expression in (8) is intractable for the PDF in (8), we use an upper-bound approximation $\gamma_{e2e} = \min(\gamma_1, \gamma_2)$ to get the cumulative distribution function (CDF) and PDF [13]:

$$F(\gamma) = F_1(\gamma) + F_2(\gamma) - F_1(\gamma) F_2(\gamma) \tag{9}$$

$$f(\gamma) = f_1(\gamma) + f_2(\gamma) - f_1(\gamma) f_2(\gamma) - f_2(\gamma) f_1(\gamma) \tag{10}$$

where $f_1(\gamma)$, $f_2(\gamma)$, $F_1(\gamma)$ and $F_2(\gamma)$ can be obtained from (2) and (3) using $d_1 = d_2 = d/2 = d_r$, where $d_1$ is the distance between source to relay and $d_2$ is the distance from relay to the destination. It is noted that distribution functions in (9) and (10) involves incomplete gamma functions and exponential integrals which requires novel approaches to performance analysis.

B. Outage Probability

Outage probability is performance measure to demonstrate the effect of fading channel. It defined as the probability of failing to reach a specified quality of service (QoS), for example, an SNR threshold value $\gamma_{th}$. Using the PDF in (9), an exact integral representation of the outage probability is given as

$$P_{out} = P(\gamma < \gamma_{th}) = \int_0^{\gamma_{th}} f(\gamma) d\gamma \tag{11}$$

Further, we can use the CDF in (9) with $\gamma = \gamma_{th}$ to get a closed-form approximation of the outage probability.

**Proposition 1:** If $k$ and $\beta$ are the parameters of foggy channel, $A_0$ and $\rho$ are the parameters of pointing error, and $z_r = 4.343/\beta d_r$ with $d = 2d_r$ as the transmission link length for a relay-assisted OWC system, then an expression for the outage probability is given as

$$P_{out} = 2P'_{out} - (P''_{out})^2, \text{ where} \tag{12}$$

$$P'_{out} \approx \left(\frac{z_r}{\gamma_{th} - \rho^2}\right)^k \left(\frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}}\right)^{-\rho^2/2} - \frac{z_r^k}{\Gamma[k]} \left(\frac{\ln(\frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}})}{\gamma_{th}}\right)^{k-1} \left(\frac{A_0}{\sqrt{\gamma_{th}}}\right)^{-\rho^2/2} + \frac{z_r^k}{\ln(\frac{\pm A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}})} \Gamma[k] + \frac{(k-1)z_r^k}{\Gamma[k]} \ln(\frac{A_0}{\sqrt{\gamma_{th}}}) \left(\frac{\ln(\frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}})}{\gamma_{th}}\right)^{k-2} \left(\frac{A_0}{\sqrt{\gamma_{th}}}\right)^{-\rho^2/2} \tag{13}$$

**Proof:** Using $\gamma = \gamma_{th}$, $m = z_r - \rho^2$, $E_n(2 - k, z_r \ln u) = (z_r \ln u)^{1-k} \Gamma[k-1, z_r \ln u]$ and approximation of incomplete Gamma function $\Gamma[k, m \ln u] \approx u^{-m}(m \ln u)^{k-1}$ in (9), we get the terms of (13), and thus (12).

It can be seen from the outage probability expression that the exponent of the SNR $A_{\delta_{j\gamma,\gamma}}$ is $z_r/2$. Thus, the diversity order, $M \approx \frac{0.1715}{\beta d_r}$.

C. Average SNR and Ergodic Rate

Exact expressions of the average SNR $\bar{\gamma}^{exact}$ and ergodic rate $\bar{\eta}^{exact}$ are:

$$\bar{\gamma}^{exact} = \int_0^{\gamma_{max}} \gamma f(\gamma) d\gamma \text{ and } \bar{\eta}^{exact} = \int_0^{\gamma_{max}} \log(1 + \gamma) f_{\gamma}(\gamma) d\gamma \tag{14}$$

where $\gamma_{max} = \frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}}$ and $f(\gamma)$ as given in (8). Note that we have assumed that relay requires negligible time to relay the data while computing the ergodic rate. Considering the intractability of the above integral, we use the PDF in (10) to derive a closed-form approximations. Since the fog and pointing error parameters are the same for source to relay and relay to the destination, expressions for the average SNR and ergodic rate are

$$\bar{\gamma} = 2 \int_0^{\gamma_{max}} \gamma [f(\gamma) - f\gamma(\gamma) F(\gamma)] d\gamma \tag{14}$$

$$\bar{\eta} = 2 \int_0^{\gamma_{max}} \log(1 + \gamma) [f(\gamma) - f\gamma(\gamma) F(\gamma)] d\gamma \tag{15}$$

**Theorem 1:** If $k$ and $\beta$ are the parameters of foggy channel, $A_0$ and $\rho$ are the parameters of pointing error, and $z_r = 4.343/\beta d_r$ with $d = 2d_r$ as the transmission link length for a relay-assisted OWC system, then an approximation of average SNR and ergodic rate are given in (16) and (17).

**Proof:** Substituting $u = \frac{A_{\delta_{j\gamma,\gamma}}}{\sqrt{\gamma_{th}}}$, $m = (z_r - \rho^2)$ and $E_n(2 - k, z_r \ln u) = \Gamma[k-1, z_r \ln u]$ in (14):

$$\bar{\gamma} = 2 \int_0^{\gamma_{max}} \frac{A_{\delta_{j\gamma,\gamma}}^{2k + \rho^2}}{\Gamma[k]} u^{\rho^2/2} - \frac{\Gamma[k] m \ln u}{u^{\rho^2/2}} - \frac{z_r^{k+m-k}}{\Gamma[k]} u^{-\rho^2/2} + \frac{z_r^{k+m-k}}{\Gamma[k]} u^{-\rho^2/2} \int_0^{\gamma_{max}} \left(\frac{\ln(\frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}})}{\gamma_{th}}\right)^{k-1} \left(\frac{A_0}{\sqrt{\gamma_{th}}}\right)^{-\rho^2/2} + \frac{(k-1)z_r^{k+m-k}}{\Gamma[k]} \ln(\frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}}) \left(\frac{\ln(\frac{A_{\delta_{j\gamma,\gamma}}}{\gamma_{th}})}{\gamma_{th}}\right)^{k-2} \left(\frac{A_0}{\sqrt{\gamma_{th}}}\right)^{-\rho^2/2} \tag{18}$$
\[
\tilde{\gamma} \approx A_0^2 \rho^2 z_r \gamma_0 \left[ m^{-2k} \left( \frac{1-2(2m+2\rho^2)}{1+\rho^2} \right)^{-k} z_r^k + m^k \left( -2z_r^{-1-k}(2+\rho^2+z_r)^{-k} + 2-3\rho^2+2(1+\rho^2) \frac{2m+2\rho^2}{2+3\rho^2+\rho^4} z_r^{-k} + 2(1+\rho^2) \frac{2m+2\rho^2}{2+3\rho^2+\rho^4} z_r^{-k} \right) \right] + z_r^{-2+k}(1+z_r)^{-1-k} \frac{(m+2mz_r-z_r^2)\Gamma[-1/2+k]}{m^2\sqrt{\pi}k} \]
\]

\[
\tilde{\eta} \approx \frac{1}{\log 2} \left[ 2m^{-k} \rho^2 z_r^k \left( 2(-1+\rho^2 \log A_0) + 2z_r^{-1-k}(\rho^2+z_r)^{-1-k}(k-(\rho^2+z_r) \log A_0) \right) \right] \]

\[
\frac{2}{\rho^2} \left( -1 + \rho^2 \log A_0 + \frac{(m+2\rho^2)^{-k}}{m^k} (m+2(1+\rho^2)^{-k}(m+2\rho^2) \log A_0) \right) + \frac{2}{\rho^2} \left( 1 - \rho^2 \log A_0 + \frac{(2m+2\rho^2)^{-k}}{m^k} (2m+2\rho^2) \log A_0 \right) + \log_{\rho^2} + \left( \frac{z_r^{-k}}{\rho^2} \right) \log_{\rho^2} \left( \frac{1-2k}{k} \log_{\rho^2} \right) \]

\[
\left( -m^{-2k}(1+\rho^2)(2\log A_0+\log \gamma_0)) \right) - \rho^2 \Gamma[-\frac{1+k}{2}](\frac{1}{2}+\rho^2)(2\log A_0+\log \gamma_0) \right] \]

We use approximation of incomplete Gamma function \(\Gamma[k, m \ln u] \approx u^{-m}(m \ln u)^{k-1}\) in (15), and apply the following identities on the resultant integrals:

\[
\int_1^{\infty} (\ln u)^p u^{-n} du = \frac{\Gamma[p+1]}{[n-1]^{p+1}} \quad (19)
\]

\[
\int_1^{\infty} u^{-n} \Gamma[k, m \ln u] du = \frac{1-n^{(2n-1-k)^{\Gamma[k]}}}{n-1} \quad (20)
\]

To get the identity in (19), we substitute \(\ln u = t\) and apply the definition of Gamma function. Further, the identity in (20) can be found by substituting \(u \ln u = t\) and using the well known identity \(\int_0^{\infty} e^{-at} \Gamma(b, t) dt = a^{-1}\Gamma(b)(1-(a+1)^{-b})\) [29]. Using (19) and (20) in (18) with some algebraic simplifications, we get (16) of Theorem 1. Similarly to obtain (17), we use the inequality \(\log(1+\gamma) \geq \log(\gamma)\) in (15) and follow the same procedure used in deriving expression of the average SNR.

As noted in [27], the OWC performance is not sufficient for high-speed data transmission under moderate \((k > 2)\) foggy conditions with pointing error. We derive an exact expressions on the average SNR and ergodic rate for \(k = 2\) (i.e., under light foggy conditions).

**Lemma 1:** For light foggy condition \(k = 2\), expressions for average SNR and ergodic rate are given as:

\[
\tilde{\gamma} = 2A_0^2 \rho^2 z_r \gamma_0 \left( \frac{1}{(2+2\rho^2)(2+z_r^2)} - \frac{1+2\rho^2}{4(1+\rho^2)(1+z_r^2)(2+2\rho^2+z_r^2)} \right)
\]

\[
\tilde{\eta} : 2A_0^2 \rho^2 z_r \gamma_0 \left( \frac{1}{(2+2\rho^2)(2+z_r^2)} \right)
\]

Note that the first term in (21) and (22) with \(d_\tau = d\) corresponds to twice of the average SNR and ergodic rate without relaying. Thus, \(\tilde{\gamma}_{\text{direct}} = \frac{2A_0^2 \rho^2 z_r \gamma_0}{(2+2\rho^2)(2+z_r^2)}\) [26]. Since all the terms in (21) is positive, we expect a higher average SNR with relaying. This has been extensively studied through
Outage probability.

The source-relay distance from expansion and it is taken to be constant with distance. The parameter optimal performance for the OWC system. Note that the analysis considered in the paper is justified giving a near-occurs at $d = 1$ km.

We can use the linear approximation $w = \frac{2.5d}{\lambda}$, where $\lambda$ is the light wavelength. For a helium neon laser ($\lambda = 650$ nm) with $w_0 = 5$ mm, we get Rayleigh distance of 50 m. Thus, we can use the linear approximation $w_z = \theta d$, where $\theta$ is beam divergence. Using $w_z = 2.5m$ at $d = 1$ km $\left(\frac{2.5}{\lambda}\right)$, we get $w_z = \frac{2.5d}{\lambda}$ and thus $v = \sqrt{\frac{\pi}{2}} a/\omega_z = \sqrt{\frac{\pi}{2}} a/\omega_0$. Finally, using $w_{z_{eq}} = w_z \text{erf}(v)/(2v \exp(-v^2))$, we can get $A_0 = (\text{erf}(v))^2 \text{ and } \rho = \omega_{z_{eq}}/2\sigma_s$, where $\sigma_s = 0.28$ m $[3]$. The parameter $\sigma_s$ is caused by building swaying and thermal expansion and it is taken to be constant with distance.

First, we analyze the optimal location of relay by changing the source-relay distance from 250 m to 750 m for an OWC system with the source-to-destination distance of 1 km. It can be seen from Fig. 1a that the minimum outage performance occurs at $d_r = 500$ m. Thus, we see that the symmetric analysis considered in the paper is justified giving a near-optimal performance for the OWC system. Note that the similar conclusion holds when we analyze the optimal relay location using average SNR and ergodic rate.

In this section, we use numerical analysis and Monte Carlo simulation (averaged over $10^6$ channel realizations) to demonstrate the outage probability, average SNR, and ergodic rate performance of the relay-assisted OWC system under the combined effect of fog and pointing error. We provide a comparison between the performance of direct and relay-assisted transmissions at various link distances. We use the light foggy conditions and consider the experimental values of foggy channel parameters ($k = 2$ and $\beta = 13.12$) $[24]$. We consider the range for transmitted power as $P_t = 0$ dBm to 30 dBm, receiver with AWGN power $\sigma_w^2 = 10^{-14}$, and detector responsivity $R = 0.41$ A/W.

Since the experimental values of pointing error parameters $\rho$ and $A_0$ are available for 1 km $[22]$, we use Gaussian beam optics to determine these parameters for other link distances for a better estimate of performance. The radius of a Gaussian beam $w_z$ (i.e., the beam waist) increases non-linearly with distance near the focal point of the transmitter but after the Rayleigh distance $\frac{\pi w_0^2}{\lambda}$, the beam waist increases linearly with the distance. Here, $w_0$ is the waist size at the focal point, and $\lambda$ is the light wavelength. For a helium neon laser ($\lambda = 650$ nm) with $w_0 = 5$ mm, we get Rayleigh distance of 50 m. Thus, we can use the linear approximation $w_z = \theta d$, where $\theta$ is beam divergence. Using $w_z = 2.5m$ at $d = 1$ km $\left(\frac{2.5}{\lambda}\right)$, we get $w_z = \frac{2.5d}{\lambda}$ and thus $v = \sqrt{\frac{\pi}{2}} a/\omega_z = \sqrt{\frac{\pi}{2}} a/\omega_0$. Finally, using $w_{z_{eq}} = w_z \text{erf}(v)/(2v \exp(-v^2))$, we can get $A_0 = (\text{erf}(v))^2 \text{ and } \rho = \omega_{z_{eq}}/2\sigma_s$, where $\sigma_s = 0.28$ m $[3]$. The parameter $\sigma_s$ is caused by building swaying and thermal expansion and it is taken to be constant with distance.

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In this paper, we investigated the performance of an AF relaying based OWC system under the combined effect of fog
and pointing error. We provided a detailed analysis for outage probability, average SNR, and ergodic capacity in order to show the benefit of proposed dual-hop relaying for the OWC system with weak direct links. Numerical analysis and Monte Carlo simulations show that the derived analytical bounds are close to the complicated integral expressions, and thus can be implemented for real-time tuning of the system parameters for optimized performance. It was also demonstrated that the relay-assisted system shows better performance than the single-link transmissions. The relaying scheme requires almost 30 times less transmission power to achieve the same outage probability and achieves an SNR gain of approximately 4.7 dB compared to the direct transmission. The extended version of the work will involve a consideration of asymmetric situations with different channel statistics at the relay and destination.

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