Direct observation of the particle exchange phase of photons

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Quantum theory stipulates that if two particles are identical in all physical aspects, the allowed states describing the system are either symmetric or antisymmetric with respect to permutations of single-particle states6,7. Experimentally, the symmetry of the states can be inferred indirectly from the fact that neglecting the correct exchange symmetry in the theoretical analysis leads to dramatic discrepancies with the observations20,21. The only way to directly unveil the symmetry of the states is by measuring the phase associated with the permutation process, the so-called particle exchange phase6. Following this idea, we have observed the exchange phase of indistinguishable photons, providing direct evidence of their bosonic character.

In three spatial dimensions, quantum physics distinguishes between two fundamental types of particle, bosons and fermions7. Such particle classification arises from the fact that indistinguishable bosons obey the Bose–Einstein statistics and indistinguishable fermions obey the Fermi–Dirac statistics. Simply put, this means that fermions cannot occupy the same quantum state, as dictated by the Pauli exclusion principle2, while bosons are allowed to 'condensate' into the same state3. Remarkably, these distinctive particle statistics are key ingredients that give form to all the existing elements and fields as we know them in the Universe.

From a theoretical perspective, it has been postulated that the correct statistics for bosons and fermions can only be observed provided the associated states are symmetric and antisymmetric, respectively4. This means that, under permutations of any pair of single-particle states, the physically allowed states of a system comprising $N$ identical bosons must remain unchanged, $\hat{P}_{ij} |N\text{ bosons}\rangle = |N\text{ bosons}\rangle$, while the states of a system of $N$ identical fermions must undergo a sign change, $\hat{P}_{ij} |N\text{ fermions}\rangle = - |N\text{ fermions}\rangle$. Here, $\hat{P}_{ij}$ represents the permutation operator that interchanges the arbitrary single-particle states $|i\rangle$ and $|j\rangle$.

Certainly, the (anti-)symmetric nature of identical particle states has profound implications for quantum science and technology25. In this respect, perhaps the most prominent example is the so-called Hong–Ou–Mandel effect16, which gives rise to maximally mode-entangled two-photon states by interfering two indistinguishable photons25. Importantly, the observation of mode entanglement only depends on the symmetry of the two-photon wave function15.

Recently, it has been recognized that (anti-)symmetric multiparticle states belong to a special set of quantum states referred to as decoherence-free subspaces17, which are quantum states that are immune to the impact of environmental noise. Even more importantly, it has been shown that quantum superpositions of indistinguishable states provide a natural control for the generation of noise-free entanglement14, and as such those superpositions represent an appealing source of quantum coherence that functions even when the associated particles are prepared independently4. Further, it is worth noting that, contrary to what was thought in the past, the entanglement of identical particles is truly physical20–22, as demonstrated experimentally in refs. 23,24, and it is known to provide metrological advantages for estimating phase shifts in systems of identical bosons3.

Beyond explorations of the technological advantages offered by quantum indistinguishable particles, many researchers have investigated the validity of the symmetrization postulate in a variety of experiments ranging from spectroscopy6,26, via quantum chemistry27 and ultracold atoms13. Yet in all these experiments the postulate has been demonstrated indirectly, for example by examining the absence of particular states that are forbidden by the postulate20–22.

In physical terms, the permutation of single-particle states in a (anti-)symmetrized state of identical (fermions) bosons implies the existence of a definite relative phase between the unswapped and swapped states2. This concept is more intuitively illustrated for the particular case of two indistinguishable particles occupying two different spatial modes $a$ and $b$, $|\Psi\rangle = |1_a, 1_b\rangle = \hat{a}^\dagger \hat{b}^\dagger |0\rangle$, where $\hat{a}^\dagger$ and $\hat{b}^\dagger$ are the corresponding single-particle creation operators. The exchange symmetry of $|\Psi\rangle$ is readily unveiled by applying a physical swap operation, which is encoded in the transformations $\hat{a} \rightarrow e^{i\phi} \hat{b}$ and $\hat{b} \rightarrow e^{-i\phi} \hat{a}$, yielding the state

$$|\Psi\rangle_{\text{swap}} \rightarrow e^{i\phi} |\Psi\rangle,$$

with $\phi = 0$ for bosons and $\phi = \pi$ for fermions. The argument $\phi$ is termed the particle exchange phase (EP) and, as we show here, it is amenable to direct measurement26. Clearly, a measurement of $\phi$ would directly reveal the fundamental exchange symmetry of $|\Psi\rangle$ (ref. 27). For a physical description of the symmetrization postulate and its technological applications we refer the reader to ref. 27.

In optics, the only way to measure phase differences is via interferometry. Hence, to observe the EP we have to superpose a reference two-particle state $|\Psi\rangle$, with its physically permuted version $e^{i\phi} |\Psi\rangle$, yielding a quantum interference term $(1 + e^{i\phi}) |\Psi\rangle$. However, care has to be exercised: exchanging the position, or other degrees of freedom, of two identical particles yields a final state

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They are split 50:50 on the second PBS. The beams acquire a phase difference $\phi_6$ impinging the state $|\rightarrow, \downarrow_2\rangle$ on paths 1 and 2, respectively. Crucially, this state is permuted an interferometer that superposed a reference state of two indistinguishable photons occupying two spatial modes with a total wave function lacking any meaningful symmetry property. This observable is found to be the combined photon coincidence rate of the four detectors at the outputs

$$\langle \tilde{I} \rangle \equiv \langle \hat{n}_1\hat{n}_4 + \hat{n}_2\hat{n}_3 - \hat{n}_1\hat{n}_3 - \hat{n}_2\hat{n}_4 \rangle = \frac{1}{2} \cos(\phi_1 + \phi_2 + \pi - \phi_3),$$

where $\phi_1$ and $\phi_2$ are two known reference phases, which are adjusted deterministically using the two mirrors in arms 1 and 2 attached to two piezo-elements. To be precise, the reference phases $\phi_1$ and $\phi_2$ are obtained a priori by launching a diagonally polarized attenuated laser beam into the input port 2, and taking the difference of the single-photon click rates between the detectors $(\hat{n}_1, \hat{n}_2)$ and $(\hat{n}_3, \hat{n}_4)$ to give $\langle \hat{n}_2 - \hat{n}_1 \rangle = \frac{1}{2} \cos(\phi_1)$ and $\langle \hat{n}_3 - \hat{n}_4 \rangle = \frac{1}{2} \cos(\phi_2)$ (see Fig. 3a and Supplementary Notes 1 and 2 for more details). At this stage, the accidental coincidence clicks within a window of 400 ns (for example from dark counts) are disregarded.

In equation (2) we have included an additional phase $\pi$ to account for the geometric phase contributed by the physical swap-operation on the two-photon states. Indeed, as alluded to above, the relative

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**Fig. 1 | Conceptual sketch of the interferometer setup.**

**a.** In the first measurement step, a diagonally polarized attenuated laser beam is incident on input port 2 (blue beam) and split 50:50 on the first PBS. The half-wave plate (HWP) brings the separate beams 1 and 2 back into diagonal polarization and they are split 50:50 on the second PBS. The beams acquire a phase difference $\phi_6$ between the paths 1 and 2 (3 and 4) and are recombined on two non-polarizing beam splitters (BS). In the second stage, two indistinguishable photons in orthogonal polarization are injected simultaneously into input port 1 (red beam).

**b.** At the second PBS, we consider two possible paths of the two-photon state: one path where the photon in beam 1 is transmitted and the photon in beam 2 is reflected (top) and the other way around (bottom). The interferometric superposition of these two paths reveals the EP of photons.
phase between the states is not only determined by the particles’ fundamental statistics \( \phi_1 \), but also by the dynamic phase \( \phi_g \) and the geometric phase \( \phi_0 \), as defined by Aharonov and Anandan\(^{29} \). In general, the physical swapping of the single-particle states of two indistinguishable particles yields \( \phi_g = \pi \), while the dynamic phase vanishes \( \phi_d = 0 \) (Supplementary Note 3).

To measure the observable \( \langle \hat{I} \rangle \) as a function of the total reference phase \( \phi_{12} \), we first determine the actual phases \( \phi_1 \) and \( \phi_2 \) using the calibration laser beam (for further details see Supplementary Note 4). Next, we turn off the calibration beam and launch photon pairs from the SPDC source (post-selected detection rate of approximately 4,200 pairs per min) and measure photon coincidences to obtain \( \langle \hat{I} \rangle \) (see Supplementary Note 5 for additional information on the two-photon source). By repeating this process and changing the voltages applied to the two piezo elements after each measurement, we collect the coincidences for several values of \( (\phi_1 + \phi_2) \), as shown in Fig. 3b. In order to exclude indeterministic thermal fluctuations of \( \phi_1 \) and \( \phi_2 \), we set the accumulation time for one measurement point to 1 s, which is much smaller than the time scale (several hours to days) over which thermal phase drifts have been observed in our setup.

Notably, for the observable \( \langle \hat{I} \rangle \), the losses, dark counts, and imperfect indistinguishability (86 ± 5% in our case, where 100% corresponds to perfect indistinguishability) only contribute as a visibility reduction, and as an offset in the \( \langle \hat{I} \rangle \) signal, which is independent of \( \phi_1 \) and \( \phi_2 \). This implies that our results are robust against systematic errors due to experimental imperfections because the particle EP \( \phi_0 \) is given as a horizontal displacement of \( \langle \hat{I} \rangle \) along the \( (\phi_1 + \phi_2) \) axis (see Supplementary Notes 6 and 7 for further details).

In Fig. 3c we show the measured coincidence rate \( \langle \hat{I} \rangle \) for different values of \( (\phi_1 + \phi_2) \) (blue dots). The data points are sorted into bins of width 0.1 rad, with respect to the sum \( (\phi_1 + \phi_2) \), and the mean value of the bins is calculated with the standard error as an uncertainty (Fig. 3d). To model the measured data and to determine the phase experimentally we use a slight variation of equation (2), \( \langle \hat{I} \rangle = A \cos(\phi_1 + \phi_2 - \phi_0 + \pi) + C \), where the amplitude \( A \) has to be positive and \( C \) describes a constant offset. Both constants characterize the combined effects of the brightness of the two-photon source, detection efficiencies, detector noise, distinguishability of the photon pairs and the integration times. As described above, these effects do not contribute to a horizontal displacement of the signal (along the \( (\phi_1 + \phi_2) \) axis) and, therefore, have no systematic impact on the obtained value for \( \phi_0 \). The fit (red solid line) in Fig. 3d reveals an EP of \( \phi_0 = (-0.04 \pm 0.07) \) rad (95% confidence interval). This result includes an EP of zero revealing the symmetric nature of the two-photon state and agrees with the expectation that photons are indeed bosons. Furthermore, this result demonstrates that it is crucial to consider the geometric phase associated with the swapping process, otherwise our measurements would lead to the erroneous conclusion that two-photon states are antisymmetric.

At this point, it is interesting to note that many textbooks introduce the symmetrization postulate stating that quantum-mechanical systems comprising \( N \) identical particles are either totally symmetric or antisymmetric under the exchange of any pair of particles, for example ref. \(^{5} \). Such statement seems to imply that the physical situation must remain unaffected if the particles undergo a physical swap-transformation. However, as demonstrated here, this is not the case. Instead, an observable geometric phase arises when the positions (or any degree of freedom) of a pair of identical particles are physically exchanged. In this respect, our work will serve as a reference to clarify the definition of the symmetrization postulate from a physical point of view. Furthermore, our results provide a first bound for a possibly non-vanishing EP of photons, and are a starting point for precision measurements on tests of the symmetry of multiparticle wave functions.
Looking forward, our optical setup may be further improved and optimized toward enhanced accuracy. For example, a brighter and more stable photon-pair source would allow for the acquisition of more data points and thus improve the statistical significance of the result. Also the setup may be implemented using integrated and passively stable optical elements, in order to minimize the influence of thermal phase fluctuations. Thus, in the coming years a steady increase in accuracy and reduction of the bound for a non-vanishing EP can be achieved. Experimental tests of the EP with fermions would be of high interest. In this regard, recent advances in the coherent control of the rotational degree of freedom of a two-ion Coulomb crystal have paved the way towards the implementation of the state-dependent transport protocol with (fermionic) $^{40}$Ca$^+$ ions\textsuperscript{31}. Also, the achievement of trapping electrons in a microwave Paul trap at room temperature is promising, which may allow a similar state-dependent transport protocol involving electrons in the future\textsuperscript{31}.

To summarize and conclude, we have developed an interferometric technique to directly measure the particle EP of indistinguishable photons. Within the margin of error our results confirm the symmetric nature of states that consist of two indistinguishable photons. We have demonstrated that it is crucial to consider the additional geometric and dynamic phases accumulated during the state-dependent transport protocol. Furthermore, our work constitutes an experimental technique to generate and certify spatially symmetrized two-photon states, which can find applications in transferring quantum information through random media\textsuperscript{17}, or to test entanglement of identical particles\textsuperscript{22}. Finally, our experiment is another example of how non-classical photon-pair sources have now entered the field of precision measurements to investigate the validity of fundamental laws of quantum mechanics\textsuperscript{34}.

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References
1. Leinaas, J. M. & Myrheim, J. On the theory of identical particles. Il Nuovo Cimento B (1971-1996) 37, 1–23 (1977)
2. Pauli, W. Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren. Zeitschrift für Physik 31, 765–783 (1925)
3. Davis, K. B. et al. Bose-Einstein condensation in a gas of sodium atoms. Phys. Rev. Lett. 75, 3969–3973 (1995)
4. Messiah, A. M. L. & Greenberg, O. W. Symmetrization postulate and its experimental foundation. Phys. Rev. 136, B248–B267 (1964).
5. Sakurai, J. J. & Napolitano, J. Modern Quantum Mechanics 2nd edn (Cambridge Univ. Press, 2017).
6. Hilborn, R. C. & Yuca, C. L. Spectroscopic test of the symmetrization postulate for spin-0 nuclei. Phys. Rev. Lett. 76, 2844–2847 (1996).
7. Modugno, G., Inguscio, M. & Tino, G. M. Search for small violations of the symmetrization postulate for spin-0 particles. Phys. Rev. Lett. 81, 4790–4793 (1998).
8. English, D., Yashchuk, V. V. & Budker, D. Spectroscopic test of Bose-Einstein statistics for photons. Phys. Rev. Lett. 104, 253604 (2010).
9. Ramberg, E. & Snow, G. A. Experimental limit on a small violation of the Pauli principle. Phys. Lett. B 238, 438–441 (1990).
10. de Angelis, M., Gagliardi, G., Gianfrani, L. & Tino, G. M. Test of the symmetrization postulate for spin-0 particles. Phys. Rev. Lett. 76, 2840–2843 (1996).
11. DeMille, D., Budker, D., Derr, N. & Devaney, E. Search for exchange-antisymmetric two-photon states. Phys. Rev. Lett. 83, 3978–3981 (1999).
12. Ospelkaus, S. et al. Quantum-state controlled chemical reactions of ultracold potassium-rubidium molecules. Science 327, 853–857 (2010).
13. Levin, K., Fetter, A. L. & Stamper-Kurn, D. M. Ultracold Bosonic and Fermionic Gases 1st edn (Cambridge Univ. Press, 2012).
14. Roos, C. F., Alberti, A., Meschede, D., Hauke, P. & Häffner, H. Revealing quantum statistics with a pair of distant atoms. Phys. Rev. Lett. 119, 160401 (2017).
15. Walmsley, I. Quantum interference beyond the fringe. Science 358, 1001–1002 (2017).
16. Hong, C. K., Ou, Z. Y. & Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. Phys. Rev. Lett. 59, 2044–2046 (1987).
17. Perez-Leija, A. et al. Endurance of quantum coherence due to particle indistinguishability in noisy quantum networks. npj Quant. Info. 4, 45 (2018).
18. Nosrati, F., Castellini, A., Compagno, G. & Lo Franco, R. Robust entanglement preparation against noise by controlling spatial indistinguishability. npj Quant. Info. 6, 39 (2020).
19. Castellini, A. et al. Indistinguishability-enabled coherence for quantum metrology. Phys. Rev. A 100, 012308 (2019).
20. Sperling, J., Perez-Leija, A., Busch, K. & Walmsley, I. A. Quantum coherences of indistinguishable particles. Phys. Rev. A 96, 032334 (2017).
21. Lo Franco, R. & Compagno, G. Indistinguishability of elementary systems as a resource for quantum information processing. Phys. Rev. Lett. 120, 240403 (2018).
22. Morris, B. et al. Entanglement between identical particles is a useful and consistent resource. Phys. Rev. X 10, 041012 (2020).
23. Sun, K. et al. Experimental quantum entanglement and teleportation by tuning remote spatial indistinguishability of independent photons. Opt. Lett. 45, 6410–6413 (2020).
24. Barros, M. R. et al. Entangling bosons through particle indistinguishability and spatial overlap. Opt. Express 28, 38083–38092 (2020).
25. Mirman, R. Experimental meaning of the concept of identical particles. Il Nuovo Cimento B (1971-1996) 18, 110–122 (1973).
26. Landshoff, P. & Stapp, H. Parastatistics and a unified theory of identical particles. Ann. Phys. 45, 72–92 (1967).
27. van Enk, S. J. Exchanging identical particles and topological quantum computing. Preprint at https://arxiv.org/abs/1810.05208 (2018).
28. Peres, A. Quantum Theory: Concepts and Methods 1st edn (Springer, 2002).
29. Aharonov, Y. & Anandan, J. Phase change during a cyclic quantum evolution. Phys. Rev. Lett. 58, 1593–1596 (1987).
30. Wang, K., Weimann, S., Nolte, S., Perez-Leija, A. & Szameit, A. Measuring the Aharonov-Anandan phase in multiport photonic systems. Opt. Lett. 41, 1889–1892 (2016).
31. Altschul, B. Testing photons’ Bose-Einstein statistics with Compton scattering. Phys. Rev. D 82, 101703 (2010).
32. Urban, E. et al. Coherent control of the rotational degree of freedom of a two-ion Coulomb crystal. Phys. Rev. Lett. 123, 133202 (2019).
33. Matthiesen, C., Yu, Q., Guo, J., Alonso, A. M. & Häffner, H. Trapping electrons in a room-temperature microwave Paul trap. Phys. Rev. X 11, 011019 (2021).
34. Sinha, U., Couteau, C., Jennewein, T., Laflamme, R. & Weihs, G. Ruling out the Pauli principle. Nuovo Cimento B (1971-1996) 238, 438–441 (1990).
35. Messiah, A. M. L. & Greenberg, O. W. Symmetrization postulate and its experimental foundation. Phys. Rev. A 1st edn (Springer, 2002).
36. Mirman, R. Experimental meaning of the concept of identical particles. Il Nuovo Cimento B (1971-1996) 18, 110–122 (1973).
37. van Enk, S. J. Exchanging identical particles and topological quantum computing. Preprint at https://arxiv.org/abs/1810.05208 (2018).
38. Peres, A. Quantum Theory: Concepts and Methods 1st edn (Springer, 2002).
39. Aharonov, Y. & Anandan, J. Phase change during a cyclic quantum evolution. Phys. Rev. Lett. 58, 1593–1596 (1987).
40. Wang, K., Weimann, S., Nolte, S., Perez-Leija, A. & Szameit, A. Measuring the Aharonov-Anandan phase in multiport photonic systems. Opt. Lett. 41, 1889–1892 (2016).
41. Altschul, B. Testing photons’ Bose-Einstein statistics with Compton scattering. Phys. Rev. D 82, 101703 (2010).
42. Urban, E. et al. Coherent control of the rotational degree of freedom of a two-ion Coulomb crystal. Phys. Rev. Lett. 123, 133202 (2019).
43. Matthiesen, C., Yu, Q., Guo, J., Alonso, A. M. & Häffner, H. Trapping electrons in a room-temperature microwave Paul trap. Phys. Rev. X 11, 011019 (2021).
44. Sinha, U., Couteau, C., Jennewein, T., Laflamme, R. & Weihs, G. Ruling out the Pauli principle. Nuovo Cimento B (1971-1996) 238, 438–441 (1990).
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Data availability
The authors declare that the main data supporting the findings of this study are available within the article and its Supplementary Information. Extra data are available from the corresponding authors upon reasonable request.

Code availability
The code that was used to analyse the experimental data is available from the corresponding authors upon reasonable request.

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Author contributions
A.P.-L., K.T., O.B. and K.B. initiated the study and guided the work. K.T., C.M., T.K., M.S. and J.W. designed the interferometer. M.S., C.M. and T.K. set up the interferometer. C.M. and M.S. performed the optical measurements. C.M. and K.T. analysed and interpreted the experimental data. K.T. and A.P.-L. developed the theory. K.T., C.M. and A.P.-L. wrote the manuscript with input from all co-authors.

Competing interests
The authors declare no competing interests.

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