Taking the Final Step to a Full Dichotomy of the Possible Winner Problem in Pure Scoring Rules

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November 29, 2011

Abstract

The POSSIBLE WINNER problem asks, given an election where the voters’ preferences over the candidates are specified only partially, whether a designated candidate can become a winner by suitably extending all the votes. Betzler and Dorn [1] proved a result that is only one step away from a full dichotomy of this problem for the important class of pure scoring rules in the case of unweighted votes and an unbounded number of candidates: POSSIBLE WINNER is NP-complete for all pure scoring rules except plurality, veto, and the scoring rule with vector (2, 1, . . . , 1, 0), but is solvable in polynomial time for plurality and veto. We take the final step to a full dichotomy by showing that POSSIBLE WINNER is NP-complete also for the scoring rule with vector (2, 1, . . . , 1, 0).

1 Introduction

The computational complexity of problems related to voting systems is a field of intense study (see, e.g., the surveys by Faliszewski et al. [3,4] and Conitzer [5] and the bookchapters by Faliszewski et al. [6] and Baumeister et al. [7]). For many of the computational problems investigated, the voters are commonly assumed to provide their preferences over the candidates via complete linear orderings of all candidates. However, this is not the case in many real-life settings: Some voters may have preferences over some candidates only, or it may happen that new candidates are introduced to an election after some voters have already cast their votes. As mentioned by Chevaleyre et al. [8] and Xia et al. [9], such a situation may occur, for example, when a committee whose members are to schedule their next meeting date by voting over a set of proposed dates. After some committee members have cast their votes (and then have

\*This work was supported in part by DFG grants RO-1202/11-1, RO-1202/12-1, and RO-1202/15-1, the European Science Foundation’s EUROCORES program LogICCC, and the SFF grant “Cooperative Norm-setting” of Heinrich-Heine-Universität Düsseldorf. A preliminary version appeared as a short paper [2] in the proceedings of the 19th European Conference on Artificial Intelligence (ECAI-2010).
gone on vacation where they are unavailable via email or phone), it turns out that some additional dates are possible, so the remaining committee members have a larger set of alternatives to choose from. Since the meeting date has to be fixed before the traveling committee members return from vacation, it makes sense to ask whether the winning date can be determined via extending their partial votes into complete linear ones by inserting the additional alternatives. Similar situations may also occur in large-scale elections, where the computational aspects of the related problems have more impact than for small-scale elections.

In light of such examples, it seems reasonable to assume only partial preferences from the voters when defining computational problems related to voting. Konczak and Lang [10] were the first to study voting with partial preferences, and they proposed the Possible Winner problem that (for any given election system) asks, given an election with only partial preferences and a designated candidate \( c \), whether \( c \) is a winner in some extension of the partial votes to linear ones. This problem was studied later on by Xia and Conitzer [11], Betzler and Dorn [1], and Baumeister et al. [12], and closely related problems have been introduced and investigated by Chevaleyre et al. [8], Xia et al. [9], and Baumeister et al. [12]. In particular, Betzler and Dorn [1] established a result that is only one step away from a full dichotomy result of the Possible Winner problem for the important class of pure scoring rules.

Dichotomy results are particularly important, as they completely settle the complexity of a whole class of related problems by providing an easy-to-check condition that tells the hard cases apart from the easily solvable cases. The first dichotomy result in computer science is due to Schaefer [13] who provided a simple criterion to distinguish the hard instances of the satisfiability problem from the easily solvable ones. Hemaspaandra and Hemaspaandra [14] established the first dichotomy result related to voting. Their dichotomy result, which distinguishes the hard instances from the easy instances by the simple criterion of “diversity of dislike,” concerns the manipulation problem for the class of scoring-rule elections with weighted votes.

In contrast, Betzler and Dorn’s above-mentioned result that is just one step away from a full dichotomy is concerned with the Possible Winner problem for pure scoring rules with unweighted votes and any number of candidates [1]. In particular, they showed NP-completeness for all but three pure scoring rules, namely plurality, veto, and the scoring rule with scoring vector \((2,1,\ldots,1,0)\). For plurality and veto, they showed that this problem is polynomial-time solvable, but the complexity of Possible Winner for the scoring rule with vector \((2,1,\ldots,1,0)\) was left open. Taking the final step to a full dichotomy result, we show that Possible Winner is NP-complete also for the scoring rule with vector \((2,1,\ldots,1,0)\).

2 Definitions and Notation

An election \((C, V)\) is specified by a set \( C = \{c_1, c_2, \ldots, c_m\} \) of candidates and a list \( V = (v_1, v_2, \ldots, v_n) \) of votes over \( C \). In the most common model of representing preferences, each such vote is a linear order\(^1\) of the form \( c_{i_1} > c_{i_2} > \cdots > c_{i_m} \) where \( \{i_1, i_2, \ldots, i_m\} = \)

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\(^1\)Formally, a linear order \( L \) on \( C \) is a binary relation on \( C \) that is (i) total (i.e., for any two distinct \( c, d \in C \), either \( c L d \) or \( d L c \)); (ii) transitive (i.e., for all \( c, d, e \in C \), if \( c L d \) and \( d L e \) then \( c L e \)); and (iii) asymmetric.
\{1,2,\ldots,m\}$, and $c_i > c_k$ means that candidate $c_k$ is (strictly) preferred to candidate $c_i$. A voting system is a rule to determine the winners of an election. Scoring rules (a.k.a. scoring protocols) are an important class of voting systems. Every scoring rule for $m$ candidates is specified by a scoring vector $\vec{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$, where each $\alpha_i$ is a nonnegative integer. For an election $(C,V)$, each voter $v \in V$ gives $\alpha_j$ points to the candidate ranked at the $j$th position in his or her vote. Summing up all points a candidate $c \in C$ receives from all votes in $V$, we obtain $\text{score}_{(C,V)}(c)$, $c$'s score in $(C,V)$. Whoever has the highest score wins the election. If there is only one such candidate, he or she is the unique winner. Betzler and Dorn [1] focus on so-called pure scoring rules. A scoring rule is pure if for each $m \geq 2$, the scoring vector for $m$ candidates can be obtained from the scoring vector for $m-1$ candidates by inserting one additional score value at any position subject to satisfying $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$. We will study only the pure scoring rule that for $m \geq 2$ candidates is defined by the scoring vector $(2,1,\ldots,1,0)$: In each vote the first candidate gets two points, the last candidate gets zero points, and the $m-2$ other candidates get one point each. We thus distinguish between the first, a middle, and the last position in any vote.

The Possible Winner problem is defined for partial rather than linear votes. For a set $C$ of candidates, a partial vote over $C$ is a transitive, asymmetric (though not necessarily total) binary relation on $C$. For any two candidates $c$ and $d$ in a partial vote, we write $c \succ d$ if $c$ is (strictly) preferred to $d$. For any two sets $A, B \subseteq C$ of candidates, we write $A \succ B$ to mean that each candidate $a \in A$ is preferred to each candidate $b \in B$, i.e., $a \succ b$ for all $a \in A$ and $b \in B$. As a shorthand, we write $a \succ b$ for $\{a\} \succ \{b\}$.

A linear vote $v'$ over $C$ extends a partial vote $v$ over $C$ if $v \subseteq v'$, i.e., for all $c,d \in C$, if $c \succ d$ in $v$ then $c \succ d$ in $v'$. A list $V' = (v'_1,v'_2,\ldots,v'_n)$ of linear votes over $C$ is an extension of a list $V = (v_1,v_2,\ldots,v_n)$ of partial votes over $C$ if for each $i$, $1 \leq i \leq n$, $v'_i \in V'$ extends $v_i \in V$.

Given a voting system $\mathcal{E}$, Konczak and Lang [10] define the following problem:

| \text{\mathcal{E}-POSSIBLE WINNER} |
|---|
| **Given:** | A set $C$ of candidates, a list $V$ of partial votes over $C$, and a designated candidate $c \in C$. |
| **Question:** | Is there an extension $V'$ of $V$ to linear votes over $C$ such that $c$ is a winner of election $(C,V')$ under voting system $\mathcal{E}$? |

This defines the problem in the nonunique-winner case; for its unique-winner variant, simply replace “a winner” by “the unique winner.” We focus on the nonunique-winner case here, but mention that the unique-winner case can be handled analogously as described by Betzler and Dorn [1]. We may drop the prefix “$\mathcal{E}$-” and simply write POSSIBLE WINNER when the specific voting system used is either clear from the context or not relevant in the corresponding context.

(i.e., for all $c,d \in C$, if $cLd$ then $dLe$ does not hold). Note that asymmetry of $L$ implies irreflexivity of $L$. (i.e., for no $c \in C$ does $cLe$ hold).
3 The Final Step to a Full Dichotomy Result

Theorem 3.2 below shows that Possible Winner for the scoring rule with vector $(2, 1, \ldots, 1, 0)$ is NP-hard. Our proof of this theorem uses the notion of maximum partial score defined by Betzler and Dorn [1]. Fix any scoring rule. Let $C$ be a set of candidates, $c \in C$ a candidate we want to make win the election, and let $V = V^l \cup V^p$ be a list of votes over $C$, where $V^l$ contains only linear votes and $V^p$ contains partial (i.e., incomplete) votes such that $c$’s score is fixed, i.e., the exact number of points $c$ receives from any $v \in V^p$ is known, no matter to which linear vote $v$ is extended. For each $d \in C - \{c\}$, define the maximum partial score of $d$ with respect to $c$ (denoted by $s^\text{max}(d, c)$) to be the maximum number of points that $d$ may get from (extending to linear votes) the partial votes in $V^p$ without defeating $c$ in $(C, V')$ for any extension $V'$ of $V$ to linear votes. Since the score of $c$ is the same in any extension $V'$ of $V$ to linear votes, it holds that

$$s^\text{max}_p(d, c) = \text{score}_{(C, V')}(c) - \text{score}_{(C, V')}(d).$$

The following lemma will be useful for our proof of Theorem 3.2.

Lemma 3.1 (Betzler and Dorn [1]) Let $\vec{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ be any scoring rule, let $C$ be a set of $m \geq 2$ candidates with designated candidate $c \in C$, let $V^p$ be a list of partial votes in which the score of $c$ is fixed, and let $s^\text{max}_p(c', c)$ be the maximum partial score with respect to $c$ for all $c' \in C - \{c\}$. Suppose that the following two properties hold:

1. There is a candidate $d \in C - \{c\}$ such that $s^\text{max}_p(d, c) \geq \alpha_1 |V^p|$.

2. For each $c' \in C - \{c\}$, the maximum partial score of $c'$ with respect to $c$ can be written as a linear combination of the score values, $s^\text{max}_p(c', c) = \sum_{j=1}^{m} n_j \alpha_j$, with $m = |C|$, $n_j \in \mathbb{N}$, and $\sum_{j=1}^{m} n_j \leq |V^p|$.

Then a list $V^l$ of linear votes can be constructed in polynomial time such that for all $c' \in C - \{c\}$, $\text{score}_{(C, V')}(c') = \text{score}_{(C, V')}(c) - s^\text{max}_p(c', c)$, where $V'$ is an extension of $V^p$ to linear votes.

Theorem 3.2 Possible Winner (both in the nonunique-winner case and in the unique-winner case) is NP-complete for the pure scoring rule with scoring vector $(2, 1, \ldots, 1, 0)$.

Proof. Membership in NP is obvious. Our NP-hardness proof uses a reduction from the NP-complete Hitting Set problem (see, e.g., [15]), which is defined as follows:

| **Hitting Set** |
|-----------------|
| **Given:** A finite set $X$, a collection $\mathcal{S} = \{S_1, \ldots, S_n\}$ of nonempty subsets of $X$ (i.e., $\emptyset \neq S_i \subseteq X$ for each $i$, $1 \leq i \leq n$), and a positive integer $k$. |
| **Question:** Is there a subset $X' \subseteq X$ with $|X'| \leq k$ such that $X'$ contains at least one element from each subset in $\mathcal{S}$? |
Let \((X, \mathcal{S}, k)\) be a given HITTING SET instance with \(X = \{e_1, e_2, \ldots, e_m\}\) and \(\mathcal{S} = \{S_1, S_2, \ldots, S_n\}\). From \((X, \mathcal{S}, k)\) we construct a POSSIBLE WINNER instance with candidate set

\[
C = \{c, h\} \cup \{x_i, x_1^2, \ldots, x_i^n, y_i^1, \ldots, y_i^n, z_i^1, \ldots, z_i^n \mid 1 \leq i \leq m\}
\]

and designated candidate \(c\). The list of votes \(V = V^l \cup V^p\) consists of a list \(V^l\) of linear votes and a list \(V^p\) of partial votes. \(V^p = V_1^p \cup V_2^p \cup V_3^p\) consists of three sublists:

1. \(V_1^p\) contains \(k\) votes of the form \(h \succ C - \{h, x_1, x_2, \ldots, x_m\} \succ \{x_1, x_2, \ldots, x_m\}\).
2. \(V_2^p\) contains the following \(2n + 1\) votes for each \(i, 1 \leq i \leq m\):
   
   \[
   v_i : \ h \succ C - \{h, x_i, y_i^1\} \succ \{x_i, y_i^1\},
   \]

   \[
   v_i' : \ y_i^j \succ C - \{y_i^j, z_i^j, h\} \succ h \quad \text{for } 1 \leq j \leq n,
   \]

   \[
   w_i' : \ x_i^j \succ C - \{x_i^j, y_i^{j+1}, z_i^j\} \succ y_i^{j+1} \quad \text{for } 1 \leq j \leq n - 1,
   \]

   \[
   w_i^n : \ x_i^n \succ C - \{x_i^n, z_i^n, h\} \succ h.
   \]

3. \(V_3^p\) contains the vote \(T_j \succ C - \{T_j, h\} \succ h\) for each \(j, 1 \leq j \leq n\), where
   \[
   T_j = \{x_i^j \mid e_i \in S_j\}.
   \]

For each \(i, 1 \leq i \leq m\), and \(j, 1 \leq j \leq n\), the maximum partial scores with respect to \(c\) are set as follows:

\[
\begin{align*}
\upsilon^{\text{max}}_p(x_i, c) &= |V^p| - 1 \\
\upsilon^{\text{max}}_p(x_i^j, c) &= |V^p| + 1 \\
\upsilon^{\text{max}}_p(y_i^j, c) &= \upsilon^{\text{max}}_p(z_i^j) = |V^p| \\
\upsilon^{\text{max}}_p(h, c) &\geq 2|V^p|.
\end{align*}
\]

This means that each \(x_i\) must take at least one last position, which is possible in the votes from \(V_1^p\) and the votes \(v_i, 1 \leq i \leq m\), from \(V_2^p\). Since the candidates \(x_i^j\) can never take a last position, they may take at most one first position. For \(y_i^j\) and \(z_i^j\), the maximum partial scores with respect to \(c\) are set such that for each first position they take, they must also take at least one last position. Finally, \(h\) can never beat \(c\). By Lemma 3.1 we can construct a list of votes \(V^l\) such that all candidates other than \(c\) can get only their maximum partial scores with respect to \(c\) in the partial votes.

We claim that \((X, \mathcal{S}, k)\) is a yes-instance of HITTING SET if and only if \(c\) is a possible winner in \((C, V)\), using the scoring rule with vector \((2, 1, \ldots, 1, 0)\).

From left to right, suppose there exists a hitting set \(X' \subseteq X\) with \(|X'| \leq k\) for \(\mathcal{S}\). The partial votes in \(V^p\) can then be extended to linear votes such that \(c\) wins the election as follows:
Hence all candidates get more points in their maximum partial scores with respect to position in the votes of all remaining votes. For each candidate, there is at most one first position and therefore does not exceed his or her maximum partial score with respect to position. Due to this, the only vote in which the score of $x_i$ makes him or her a winner in this extension of the list $V_p$ of partial votes is placed at the first position in $V_p$. This is possible only if the candidate $x^i$, where $x_i$ takes the last position in a vote from $V_p$, must take the first position in any vote of $V_p$. This means that all first positions in the votes of $V_p$ must be taken by those $x_i^j$ for which $x_i$ takes the last position in a vote from $V_p$. This is possible only if the $x_i^j$ are not at the first position in $w_i^j$. Thus, $x_i^j$ cannot take a first position in any vote of $V_p$. Since no candidate exceeds his or her maximum partial score with respect to position, candidate $c$ is a winner in this extension of the list $V_p$ of partial votes. Conversely, assume that $c$ is a possible winner for $(C,V)$. Then no candidate may get more points in $V_p$ than his or her maximum partial score with respect to position. Since at most $k$ different $x_i$ may take a last position in $V_p$, at least $n-k$ different $x_i$ must take a first position in $V_p$. Fix any $j$ such that $x_i$ is ranked last in $V_p$. We now show that it is not possible that a candidate $x_i^j$ then takes a first position in any vote of $V_p$. Since $x_i$ takes the last position in $V_p$, $y_i^j$ takes a middle position in this vote and gets one point. The only vote in which the score of $y_i^j$ is not fixed is $v_i^j$. Without the points from this vote, $y_i^j$ already gets $|V_p| - 1$ points, so $y_i^j$ cannot get two points in $v_i^j$, and $z_i^j$ takes the first position in $v_i^j$. Without the points from $w_i^j$, $z_i^j$ gets $|V_p|$ points and must take the last position in $w_i^j$. The first position in $w_i^j$ is then taken by $x_i^j$, so $x_i^j$ cannot take a first position in any vote from $V_p$. Candidate $y_i^j$ gets one point in $w_i^j$, and by a similar argument as above, $x^j$ is placed at the first position in $w_i^j$. Repeating this argument, we have that for each $j$, $1 \leq j \leq n$, $x_i^j$ is placed at the first position in $w_i^j$ and thus cannot take a first position in a vote from $V_p$. This means that all first positions in the votes of $V_p$ must be taken by those $x_i^j$ for which $x_i$ takes the last position in a vote from $V_p$. This is possible only if the $x_i^j$ are not at the first position in $w_i^j$. Thus, $x_i^j$ must take this position. Due to $z_i^j$'s maximum partial score with respect to position, this is possible only if $z_i^j$ takes the last position in $v_i^j$. Then $y_i^j$ takes the first position in this vote. This is possible, since $y_i^j$ can take a middle position in $v_i$ for $j = 1$, and in $v_i$ for $2 \leq j \leq n$. Hence all $x_i^j$, where $x_i$ takes the last position in the votes of $V_p$, may take the first position in the votes of $V_p$. Thus, by the definition of $V_p$ (which, recall, contains the vote $T_j \succ C - \{T_j, h\} \succ h$ for each $j$, $1 \leq j \leq n$, where $T_j = \{x_i^j \mid e_i \in S_j\}$), the elements $e_i$ corresponding to those $x_i$ must form a hitting set of size at most $k$ for $\mathcal{F}$. 

| $V_p$ | $e_i \in X'$ | $e_i \notin X'$ |
|-------|----------------|-----------------|
| $V_1^p$ | $h > \cdots > x_i$ | $h > \cdots > x_i$ |
| $V_2^p$ | $v_i^j : h > \cdots > y_i^j$ | $h > \cdots > y_i^j$ |
|       | $w_i^j : z_i^j > x_i^j > \cdots > y_i^j$ | $z_i^j > y_i^j > \cdots > h$ |

Every $x_i$ takes one last position and get his or her maximum partial score with respect to position. For $e_i \in X'$, all $y_i^j$ take exactly one first, one last, and a middle position in all remaining votes. For $e_i \notin X'$, all $y_i^j$ take middle positions only. So they always get their maximum partial scores with respect to position. The candidates $z_i^j$ also get their maximum partial scores with respect to position. Since they always get one first position, one last position, and a middle position in all remaining votes. Every candidate $x_i^j$ gets at most one first position and therefore does not exceed his or her maximum partial score with respect to position. Since no candidate exceeds his or her maximum partial score with respect to position, candidate $c$ is a winner in this extension of the list $V_p$ of partial votes.
4 Conclusions and Future Research

In this paper, we have taken the final step to a full dichotomy theorem for the POSSIBLE WINNER problem with unweighted votes and an unbounded number of candidates in pure scoring rules. Our result complements the results of Betzler and Dorn [1] by showing that POSSIBLE WINNER is NP-complete for the pure scoring rule with vector \((2, 1, \ldots, 1, 0)\), the one missing case in [1].

Besides establishing this dichotomy theorem, our result has also other consequences. Since POSSIBLE WINNER is a special case of the SWAP BRIBERY problem introduced by Elkind et al. [16], Theorem 3.2 implies that this problem is NP-hard for the pure scoring rule with vector \((2, 1, \ldots, 1, 0)\) as well. Informally put, in a SWAP BRIBERY instance an external agent seeks to make a distinguished candidate \(c\) win the election by bribing some voters so as to swap adjacent candidates in their preference orders (see [16] for formal details).

On the other hand, the POSSIBLE WINNER problem generalizes the COALITIONAL UNWEIGHTED MANIPULATION problem where a group of strategic voters, knowing the preferences of the nonstrategic voters, seeks to make their favorite candidate win by reporting insincere preferences. An instance of this manipulation problem can be seen as a POSSIBLE WINNER instance in which all nonstrategic voters report (sincere) complete linear orderings of all candidates, whereas all strategic voters initially have empty preference lists, and the question is whether they can extend them to complete linear orderings of all candidates such that their favorite candidate wins.

The NP-hardness result of Theorem 3.2 has no direct consequence for the complexity of this more special problem, and neither so for other more special variants of POSSIBLE WINNER, such as POSSIBLE WINNER WITH RESPECT TO THE ADDITION OF NEW CANDIDATES (see Chevaleyre et al. [8], Xia et al. [2], and Baumeister et al. [12]). Note that the complexity of the COALITIONAL WEIGHTED MANIPULATION problem, where all votes are weighted and the weights of all manipulators are known initially in addition to the weights and preferences of the nonmanipulators, is well understood (see the work of Conitzer et al. [17]), and even a dichotomy theorem for scoring rules due to Hemaspaandra and Hemaspaandra [14] is known for weighted votes. However, the complexity of COALITIONAL UNWEIGHTED MANIPULATION is still unknown for many voting systems, including many scoring rules. Only recently Betzler et al. [18] and Davies et al. [19] independently showed that COALITIONAL UNWEIGHTED MANIPULATION, even for only two manipulators, is NP-complete for Borda elections, where Borda with \(m\) candidates is the scoring rule with vector \((m - 1, m - 2, \ldots, 0)\). Further complexity results regarding the COALITIONAL UNWEIGHTED MANIPULATION problem for various voting systems are due to Faliszewski et al. [20, 21], Narodytska et al. [22], Xia et al. [23, 24], and Zuckerman et al. [25, 26]. None of these papers establishes a dichotomy theorem for manipulation in the unweighted case, although dichotomy results for scoring rules are now known for two of its generalizations, the COALITIONAL WEIGHTED MANIPULATION problem (see [14]) and the (unweighted) POSSIBLE WINNER problem (see [1] and this paper). For future research, we propose to tackle the open problem of finding a dichotomy result for COALITIONAL UNWEIGHTED MANIPULATION in scoring rules.
Acknowledgments

We thank the anonymous ECAI-2010 and IPL reviewers for their expert comments on this paper that helped improving its presentation.

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