A Dynamic Model of Otolith for Determinants of a BPPV Reposition Maneuver with Its Simulation

Yunjian Peng and Jiaxin Qi
School of Automation Science and Engineering, South China University of Technology, Guangzhou 510640, P. R. China
E-mail: pengyj@scut.edu.cn; auqijx@mail.scut.edu.cn

Abstract. Benign paroxysmal positional vertigo (BPPV) is a common cause of vertigo disease, and its treatment consists of several body movements to reposition the otolith. Due to the small scale of the otolith, it is impossible to observe it clinically and therefore the treatment process is empirical. The paper proposes an improved dynamic model of the movement of a single otolith in a single semicircular canal (SCC), which aims at simulating the movement behavior of the otolith. With simulation results of the otolith’s movements, we take two parameterized indices of the BPPV treatment maneuvers to assess their determinants, i.e., the rotational angular velocity and the pause time between operations. The former formulates the behavior of the otolith’s moving in the SCC, the latter is confirmed by simulation as an important index for a successful maneuver, which could provide guidance for clinical treatment and improve treatment efficiency.

1. Introduction
Benign paroxysmal positional vertigo (BPPV) is a common cause of vertigo disease. The vestibular system is an important structure for the human body to sense balance and spatial orientation, in which the semicircular canal (SCC) system with three mutually perpendicular SCC is responsible for sensing the rotational motion. Each canal has an ampulla that contains a cupula. BPPV is considered to be related to the otolith that wrongly appears in the SCC. There are two mechanisms commonly used to interpret BPPV, one owes the disease to the calcite granule that flow freely in a SCC (Canalithiasis), the other owe the disease to an otolith attached to a cupula (Cupulolithiasis). For the treatment of BPPV, manual repositional maneuver (RM) is used clinically like Epley RM [1] and Semont Maneuver (SM) [2].

Although there are many related qualitative studies on the fluid mechanism of normal vestibular structures, quantitative research on BPPV mechanism is still limited. Suzuki simulated BPPV with a bullfrog’s post SCC [3]. Obrist proposed an in vitro model and simulated BPPV in it where physical and geometric parameters are amplified [4] and idealized the radius of the cross section of a single SCC to be fixed, proposing a particle-fluid model [5].

Quantitative studies of BPPV can help us understand the BPPV mechanism in details to guide diagnosis and treatment. Therefore, we improve a BPPV model based on the results of Berselli et al.’s research [6]. The model describes the dynamics of a single otolith in a single SCC, which is greatly affected by the initial angular velocity of the otolith. Besides, we describe an ideal treatment operation, with two main parameterized indices (pause time between operations and angular velocity during operation). We try to find the relationship between these parameters and the movement of the otolith,
which is not clinically visible, in order to provide some basis for explaining the feasibility and improving the effect of the treatment operation.

2. Formulation of Otolith’s Behavior

Most doctors use “canalith repositioning procedure” like particle repositioning manoeuvre (PRM) to treat patients, which consists of a series of head maneuvers [7]. It is supposed to move the symptom-causing otoliths from canals into utricle where they won’t cause vertigo. The procedure is quite effective [8], which verifies that although the scale of the particles is small and the movement of them is unobservable, they do not do Brownian motion and can be moved by external forces, which is the premise of our study.

In this section, we formulate a model of the movement of a single otolith in the case of canalithiasis. The aim of the model is to analyse the unobservable behavior of the otolith in the SCC.

Through the anatomy of the inner ear and the three-dimensional imaging based on SCC medical images [9-11], the SCC proved to be irregular (as figure 1 shows). Besides, the shape of the rigid otolith is quite small and irregular (shown as figure 2) [12]. To simplify the issue, some assumptions are proposed below [6]:

• The otolith is considered as a sphere whose mass is concentrated at center and whose constant radius is negligible when compared to the SCC cross section.
• The scale of the SCC cross section, whose radius is considered as constant, is relatively small when compared to the curvature of the SCC.

Consider the case where the SCC plane is perpendicular to the ground and the ampulla is at the lowest point. Since the SCC plane is approximately circular, we use polar coordinates to represent the location of the otolith. The coordinate pole is at the center of the SCC, and the polar axis points to the direction of gravity. The position of the otolith is represented by $q_o = [\beta, r]$. The parameters used in the following are listed in table 1.

According to the characteristics of the force situation, SCC can be divided into five regions, as figures 3-4 shows. In region 5, the otolith is subjected to gravity ($f_g$), inertial force ($f_i$) arising during motion, and Stokes force ($f_s$, with small Reynolds number) due to the presence of SCC liquid. When the otolith is close to the SCC wall (in region 3, 4), the coefficient of Stokes force changes. Once the otolith is in contact with the wall (in region 1, 2), it will be subjected to additional forces related to the damping and stiffness of the wall.
Table 1. Parameters of the model.

| Description                     | Symbol | Value     | Unit    |
|---------------------------------|--------|-----------|---------|
| SCC mean radius                 | \( r_{\text{mean}} \) | \( 3.2 \times 10^{-3} \) | m       |
| SCC inner wall radius           | \( r_{\min} \)      | \( 3.12 \times 10^{-3} \) | m       |
| SCC outer wall radius           | \( r_{\max} \)      | \( 3.28 \times 10^{-3} \) | m       |
| SCC wall stiffness              | \( k_c \)           | \( 3 \times 10^{-5} \)   | N \cdot m\(^{-1}\) |
| SCC wall damping                | \( b_c \)           | \( 1 \times 10^{-7} \)   | N \cdot s \cdot m\(^{-1}\) |
| Particle mass                   | \( m_o \)           | \( 3.88 \times 10^{-12} \) | kg      |
| Particle radius                 | \( r_c \)           | \( 7 \times 10^{-6} \)   | m       |
| Fluid(endolymph) viscosity      | \( \eta \)          | \( 10^{-6} \)            | m\(^2\) \cdot s\(^{-1}\) |
| SCC-particle gap                | \( r_{\text{gap}} \) | \( 10^{-6} \)            | m       |
| Gravity acceleration            | \( g \)             | 9.81                   | m \cdot s\(^{-2}\) |

Figure 3. The five regions of the SCC.

region 1: \( x_3 \geq r_{\max} \),
region 2: \( x_3 \leq r_{\min} \),
region 3: \( r_{Mg} < x_3 < r_{\max} \),
region 4: \( r_{\min} < x_3 < r_{Mg} \),
region 5: \( r_{mg} < x_3 < r_{Mg} \)

Figure 4. Forces acting on the otolith.

Due to the different force conditions in the five regions, the motion laws of the otolith are different. Equation 1 is the matrix form of the dynamic equations.

\[
M_o (q_o) \ddot{q}_o + C(q_o, \dot{q}_o) \dot{q}_o + D(q_o) \dot{q}_o + g(q_o) + b(q_o, \dot{q}_o) = 0
\]

where
\[ M(o) = \begin{bmatrix} m_o r^2 & 0 \\ 0 & m_o \end{bmatrix} \]

\[ C(q_o, \dot{q}_o) = \begin{bmatrix} m_o r \dot{r} & m_o r \dot{\beta} \\ -m_o r \dot{\beta} & 0 \end{bmatrix} \]

\[ D(q_o) = \begin{bmatrix} d(r) r^2 & 0 \\ 0 & d(r) \end{bmatrix} \]

\[ g(q_o) = \begin{bmatrix} m_o g r \sin \beta \\ -m_o g \cos \beta \end{bmatrix} \]

\[ b(q_o, \dot{q}_o) = \begin{cases} b_i \dot{r} r^2 & \text{if } r \geq r_{max} \\ k_i (r - r_{max}) + b_i \dot{r} & \text{if } r \leq r_{min} \\ 0 & \text{otherwise} \end{cases} \]

\( M(q_o) \) is the inertia matrix of the otolith. Note that it is diagonal, because we assume the otolith to be a sphere whose mass is concentrated in the center.

The first two part of equation (1) \((M(q_o) \dot{q}_o) \) and \((C(q_o, \dot{q}_o)) \dot{q}_o \) reflects the inertial force in the form of polar coordinates, which acts on the otolith.

\( g(q_o) \) represents gravity in polar coordinate.

\( b(q_o, \dot{q}_o) \) is a simple model describing the effect of the SCC wall, which includes the stiffness (\( k_i \)) and damping (\( b_i \)) of it.

\( D(q_o) \) describes the coefficients of the Stokes drag.

We define the elements in \( D(q_o) \) as:

\[ d_s = 6 \pi \eta r_o \]

\[ r_{mg} = r_{min} + r_{gap} \]

\[ r_{Mg} = r_{max} - r_{gap} \]

\[ d(r) = \begin{cases} d_s & \text{if } r_{mg} < r < r_{Mg} \\ \frac{d_s r_{gap}}{r - r_{min}} & \text{if } r_{min} < r < r_{mg} \\ \frac{d_s r_{gap}}{r_{max} - r} & \text{if } r_{Mg} < r < r_{max} \\ 0 & \text{otherwise} \end{cases} \]

where \( r_{gap} = 10^{-6}[m] \). In this way, the adherence effects between the otolith and SCC wall is emulated by the varying radial damping coefficient.

### 3. Determinants for a Successful Maneuver

When treating BPPV due to canalithiasis of a posterior SCC, SM is a commonly used method. It consists of two steps that change the position of patient. First, the patient changes from sitting to lying with the head at a 45-degree angle to the opposite direction of movement. After remaining for a rest time, the patient is brought 180-degree to the opposite position. Finally, the patient is moved back to
the sitting position after a few minutes in this position. After parameterizing the SM, we find there are two main parameters: the rest time between two steps \( T_p \) and the angular velocity during two steps \( \dot{\alpha} \). With regard to \( T_p \) and \( \dot{\alpha} \), different suggested parameters have been proposed in clinical practice. In this section, we evaluate the parameters that have an effect on the efficacy of the maneuver.

For the present study, we consider the movement of a single otolith with different particle sizes and a range of different parameters. The tested maneuver velocities vary between 90 deg/s and 180 deg/s in steps of 45 deg/s. After each step, we consider that the maneuver is successful if the otolith reaches the lowest point in the SCC. When the simulation is completed, the maneuver is considered successful if \( \beta \) equals 0.

### 4. Results

#### 4.1. Particle Trajectories

Figures 5-6 shows the trajectory of the otolith particles when the operation is simulated. Figure 5 shows a failed repositioning operation. The rest time between steps is too short, and the particle fails to reach the lowest point before the next step is performed. Eventually the particle returns to the initial point. Figure 6 shows a successful operation. The rest time is long enough that the particle has reached the lowest point before the start of next step, and enters the utricle of SCC after all operations are completed.

#### 4.2. Variation of Rotation Velocity

We performed a series of simulation experiments with different rotation velocity, in which the diameter of the otolith is 7 mm. For rotation velocity of 90 deg/s, 135 deg/s, and 180 deg/s, the simulation can be successful, if the rest time is not less than 7.8 s, 12.12 s, and 18.12 s, respectively.

#### 4.3. Variation of Particle Size

To evaluate the effect of particle weight, the simulation experiments are repeated with a particle radius of 15 mm (table 2). The rest time \( T_p \) is a little longer than that for the smaller particle.

| Particle radius \( r \) | Initial angular velocity \( \beta \) |
|-------------------------|-------------------------------|
|                         | 90 deg/s | 135 deg/s | 180 deg/s |
| 7 mm                    | 12.78 s  | 12.40 s   | 12.18 s   |
| 15 mm                   | 12.91 s  | 12.55 s   | 12.35 s   |

### 5. Discussion

The main findings of these simulation experiments on operating parameters are as follows:

First of all, the premise of a successful operation is that there is a long enough rest time between operations. Secondly, in the test range, the effect of rotation velocity on the operation is small. However, we need to be aware of the limitations of current experiments. The effect of multiple otolith particles, different otolith shapes and sizes are not considered in the model. And in reality, the inner wall radius and outer wall radius of the SCC are not constant. Secondly, in actual SM, due to individual differences, the SCC plane cannot be guaranteed to be perpendicular to the horizontal plane.
6. Conclusion
In this paper, we have the dual results of improving a dynamic model of a single otolith in a single SCC in the context of BPPV treatment is difficult to standardize and otoliths are not clinically visible, based on the force of otolith, and applying the model to carry out some simulation experiments. Based on the current results, we explain the effect of interval time between operations on otolith movements and confirm that it is an important basis for successful SMs. As the time between operations is easy to measure clinically, this could provide theoretical support for clinical controlled trial.
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