Stable multiquark states with heavy quarks in a diquark model

Su Houng Lee\textsuperscript{1a} and Shigehiro Yasui\textsuperscript{2b}

\textsuperscript{1} Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea
\textsuperscript{2} Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1, Oho, Ibaraki 305-0801, Japan

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Abstract. Based on the color-spin interaction in diquarks, we argue why some multiquark configurations could be stable against strong decay when heavy quarks are included. After showing the stability of previously discussed states we identify new possible stable states. These are the $T_0^0(udcb)$ tetraquark, the $\Theta_{bs}(udus\bar{b})$ pentaquark and the $H_c(udusuc)$ dibaryon, and so forth.

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1 Introduction

Multiquark configuration for the scalar nonet was first suggested by Jaffe in the framework of the MIT bag model \cite{1}. Recently, the discussion on possible multiquark mesons has gained more ground as the recently discovered $Z(4430)$ \cite{2}, $Z(4051)$ and $Z(4248)$ \cite{3} seem to be composed of $c\bar{c}ud$ type of four quarks, as they decay into $\psi'\pi$ and $\chi_{c1}\pi$ respectively. The discussion on possibly multiquark configurations that are stable against strong decay is an old one starting from the $H$ dibaryon $(udsuds)$ \cite{4}, multiply charmed $H$ dibaryon $(udscsc)$ or $(udsc\bar{c}\bar{s})$ \cite{5}, charmed pentaquarks $\Theta_{us}(udus\bar{c})$ \cite{6} to tetraquarks $T_{cc}(ud\bar{c}c)$ \cite{7} to tetraquarks $T_{cc}(ud\bar{c}c)$ \cite{8,9,10,11,12}. While there are different models predicting the stability of these states, in the constituent quark model, the stability of possible stable multiquark configurations originate from the strong scalar diquark attraction \cite{11,16}. The strong attraction in the color antitriplet channel is an important ingredient in hadron spectroscopy and is also responsible for color superconductivity at high density \cite{17}. In that sense, finding a stable multiquark configuration is also an important step toward understanding QCD at high density \cite{18}.

In this work, we introduce a simple model that can be used to analyze the stability of multiquark configuration against strong decay and show why some of the previously discussed states could indeed be stable. Then we discuss several new stable multiquark states, namely tetraquark, pentaquark and dibaryon containing charm and bottom quarks. The paper is organized as follows. In section II, we introduce our simple constituent quark model. Based on this model, we discuss possible stable multiquark states and their measurable decay modes in section III. The summary is given in section IV.

2 A schematic model for hadron mass differences

It is a well known fact that the color-spin interaction \cite{19} plays an important role in hadron mass differences. To illustrate the mechanism, we introduce the following simplified form for the color-spin interaction:

$$C_H = \sum_{i>j} s_i \cdot s_j \frac{1}{m_i m_j},$$

(1)

Here $m_i$ and $s_i$ are the mass and spin of the constituent quark $i$. The overall strength of the color-spin interaction is given as \cite{20}

$$C_H = v_0 \lambda_i \cdot \lambda_j (\delta(r_{ij})),$$

(2)

where $v_0$ is related to the coupling, the second part is the color factor and the third part is the strength of the wave function at zero separation. The color factor $\lambda_i \cdot \lambda_j$ would be $-8/3$ for diquarks in the color antitriplet channel and $-16/3$ for quark and anti-quark pair in the color singlet channel. It is a well known fact that the wave function effect can almost be treated as a constant \cite{18,22}. To illustrate this point, we assume the following constituent quark masses: $m_{u,d} = 300$ MeV, $m_s = 500$ MeV, $m_c = 1500$ MeV, and $m_b = 4700$ MeV.

Table 1 shows the mass differences between baryons that are sensitive to the color-spin interaction only. By fitting $C_B$ to $\Delta M$, we obtain $C_B/m_u^2 = 193$ MeV and find that the mass differences $M_{\Sigma} - M_{\Lambda}$, $M_{\Sigma_c} - M_{\Lambda_c}$ and

\textsuperscript{a} e-mail: suhoungh@phy.a.yonsei.ac.kr
\textsuperscript{b} e-mail: yasuis@post.kek.jp
Table 1. Baryon mass relations. The first column is fit to experiments.

| Mass Difference | MeV |
|-----------------|-----|
| \(M_2 - M_1\)   | 290 |
| \(M_3 - M_2\)   | 397 |
| \(M_4 - M_3\)   | 75  |
| \(M_5 - M_4\)   | 192 |

Table 2. Meson mass relations. The first column is fit to experiments.

| Mass Difference | MeV |
|-----------------|-----|
| \(M_\rho - M_\pi\) | 381 |
| \(M_\phi - M_\pi\) | 127 |
| \(M_\Delta^+ - M_\Lambda\) | 41  |
| \(M_\Delta^0 - M_\Lambda\) | 46  |

When both quarks are heavy, the value of \(C_H\) is expected to become larger as the strength of the relative wave function at the origin is substantially increased. Fitting instead its value to the mass difference between \(J/\psi\) and \(\eta_c\), we find \(C_{cc}/m_c^2 = 117\) MeV. Assuming that the corresponding attraction between charged diquark is three times smaller than that between the charm quark-antiquark pair as in the case of light quarks, we have \(C_{cc}/m_c^2 = 39\) MeV. In the mass difference between \(Y\) and \(\eta_b\), in which the latter has been recently observed by BaBar \[21\], we also find \(C_{bb}/m_b^2 = 71.4\) MeV, and in turn \(C_{bb}/m_b^2 = 23.8\) MeV. Therefore, we find \(C_{bb}\) is much larger than \(C_{cc}\). Concerning the value of \(C_{bc}\), there is no available experimental data for the mass difference between \(B_c^+\) and \(B_c\). However, we also expect that \(C_{bc}\) has flavor dependence and given by an average interpolating between that for the cc and the bb diquarks. In any case, we find the flavor dependence induces the larger value of \(C_H\) for heavier flavors. Therefore, we could introduce additional mass dependence in \(C_B\) and in \(C_M\) by fitting the mass differences in the strange, charm and bottom hadrons from Table I and II, respectively. However, these introduce only minor changes in the analysis to follow, and therefore we will just use the mass independent \(C_H\)'s obtained above.

3 Stable multiquark configuration

Using above parameters, one notes that the scalar diquark has a large attractive contribution to the mass. Any multiquark configuration that are stable should be the ones that are composed of such configurations.

Table 3. Type of attractive scalar diquarks \((q\bar{q}_2)\) composed of quark \(q_1\) and \(q_2\). The second line is their binding in MeV due to the color-spin force \(-\frac{1}{2}C_B\frac{m_{q_1}m_{q_2}}{m_q^2}\).

| Type of diquarks | Binding (MeV) |
|------------------|---------------|
| (ud)             | -144.75       |
| (us)             | -86.85        |
| (uc)             | -28.95        |
| (ub)             | -9.23         |

Table 3 lists the possible diquarks and their attractions due to Eq. (1) with parameters obtained above. Candidates for stable multiquark configurations can be obtained by combining diquarks and possible color source to make them color singlet.

3.1 Heavy tetraquark

Tetraquarks are obtained by combining a diquark and an antidiquark in Table 3. Let us start by looking at the tetraquark with total spin zero containing the \((ud)\) diquark and an anti-diquark of \((\bar{q}_1q_2)\); the configuration will be called \(T^0_{q_1\bar{q}_2}\). In the following discussions, we will only consider quarks in the s-state, and assume that the color-spin effect is as given in Eq. (1) with constant coefficients determined above. To investigate the stability of the \(T^0_{q_1\bar{q}_2}\) against strong decay, one has to consider its mass difference with the two pseudo scalar mesons \(M_1(\bar{u}q_1)\) and \(M_2(d\bar{q}_2)\), which we call the binding energy \(B^0_{T_{q_1\bar{q}_2}}\).

\[B^0_{T_{q_1\bar{q}_2}} = m_{T^0_{q_1\bar{q}_2}} - m_{M_1} - m_{M_2} - \frac{1}{2}C_B\left(\frac{1}{m_u} + \frac{1}{m_{q_1}}\right) - \frac{1}{2}C_M\left(\frac{1}{m_d} + \frac{1}{m_{q_2}}\right)\]

Since phenomenologically \(C_M\) is about 3 times larger than \(C_B\), the combinations of diquarks which make Eq. (1) negative are very limited. Only when \(q_1 = c\) and \(q_2 = b\), the binding energy becomes \(B^0_{T_{cb}} = -21.25\) MeV, and the configuration \((ud\bar{c}\bar{b})\) which we call \(T^0_{cb}\) will be stable against...
strong decay. We note that more realistically the binding energy of a diquark with two heavy quarks could be larger as noted previously. Therefore the binding energy we have obtained will be the lower bound.

This result can be understood simply in the heavy quark limit. The binding energy of \( T_{cc}^0 \) is supplied mainly from the binding energy of (ud) diquark, because the binding energy of (cb) diquark is suppressed by \( 1/m_c m_b \). On the other hand, the binding energies of the two meson states, \( u\bar{c} \) and \( d\bar{b} \), are also suppressed by \( 1/m_c \) and \( 1/m_b \), respectively. Therefore, the (ud) diquark is regarded as an attractive force which stabilizes \( T_{cc}^0 \). Then, we may ask if \( T_{cb}^0 \) including (us) or (ds) diquarks can be stable, because \( s \) quark may be considered to belong to the light quarks. We find that the configuration us\bar{c} could be stable against \( B^+ + D^- \) (\( B_{T_{cb}^0(uc)} = -1.1 \) MeV), however it is unstable against \( \bar{D}^0 + B_{T_{cb}^0(uds)} = +24.8 \) MeV because of the relatively weaker binding in the (us) diquark. Therefore, \( T_{cc}^0 \) will not form \( 3_f \) multiplet in light flavor SU(3) \( f \) symmetry.

Another tetraquark that could be stable is \( T_{bb}^1 \), that was discussed before within the present model in Ref. Therefore, \( T_{cc}^1 \) is a state with scalar diquark (ud) and axial vector diquark (cc), hence the total quantum number is \( J^P = 1^- \). This state can not decay strongly to two pseudo scalar mesons, but can to a pseudo scalar meson and a vector meson. Therefore, although the charmed anti-diquark is repulsive, the attractive (ud) diquark gives more binding than the pseudo scalar and vector mesons. That is,

\[
B_{T_{cc}^1} = m_{T_{cc}^1} - m_B - m_D^* = -3 C_B \frac{4 m_u m_c}{4 m_u^2} + 1 \frac{C_B}{4 m_b^2} + \frac{3 C_M}{4} \left( \frac{1}{m_u m_c} - \frac{1}{3 m_u m_c} \right) \sim -79.3 \text{ MeV}.
\]

In the same way, \( T_{bb}^1 \) can also be a stable tetraquark, whose binding energy is given by

\[
B_{T_{bb}^1} = m_{T_{bb}^1} - m_B - m_D^* = -3 C_B \frac{4 m_u}{4 m_u^2} + 1 \frac{C_B}{4 m_b^2} + \frac{3 C_M}{4} \left( \frac{1}{m_u m_b} - \frac{1}{3 m_u m_b} \right) \sim -124.3 \text{ MeV}.
\]

We note that \( T_{cc(ub)}^1 \) containing (us) or (ds) diquark instead of (ud) diquark can also be stable as shown in Table 4. For example, \( T_{cc(ub)}^1 \) has binding energy -8.7 MeV against \( \bar{D}^0 + D^{-} \), and \( T_{bb(ub)}^1 \) has -62.3 MeV against \( B^+ + B^0 \). Therefore, \( T_{cc(ub)}^1 \) will form \( 3_f \) multiplet.

In between \( T_{bb}^1 \) and \( T_{bb(ub)}^1 \) with spin triplet (cb) diquark is also interesting. The binding energy is obtained similarly as shown in Table 4. In this case, \( T_{bb}^1 \) with (ud) diquark is stable, while \( T_{bb(ub)}^1 \) with (us) and (ds) diquarks are not. Therefore, \( T_{bb}^1 \) would not form \( 3_f \) multiplet, as only \( T_{bb}^1 \) is bound. \( T_{bb}^1 \) is regarded as an excited state of \( T_{bb}^0 \), whose mass splitting is given by \( C_B / m_u m_b \). Therefore, if both \( T_{bb}^0 \) and \( T_{bb}^1 \) are observed and their mass splitting fits well to \( C_B / m_u m_b \), then it confirms that \( T_{bb} \) is really a tetraquark, and not a bound molecular state, which could otherwise be another possibility.

The presence of \( T_{cc}^0 \) and \( T_{bb}^1 \) can be observed experimentally through their hadronic weak decay. \( T_{cc}^0 \) contains the components of \( \bar{D}^0 + B^0 \) or \( B^- + \bar{B}^0 \), whose available decay modes are listed in Ref. [26]. For example, through the weak decay \( \bar{D}^0 \rightarrow K^+ \pi^- \) and \( B^0 \rightarrow K^+ \pi^- \), we could observe \( T_{cc}^0 \rightarrow K^+ \pi^- + K^+ \pi^- \), as far as \( T_{cc}^0 \) is really stable against the strong decay. While the expected branching ratio for the particular hadronic decay mode is very small because of the small branching ratio of \( B^0 \rightarrow K^+ \pi^- \), we need to search for such modes to reconstruct the mass of \( T_{cc}^0 \). Similarly, \( T_{cc}^1 \) contains \( \bar{D}^0 + D^- \) or \( D^0 + \bar{D}^- \). Then, using \( D^* \rightarrow D \pi \), an interesting decay mode is given as \( T_{cc}^1 \rightarrow K^+ \pi^- + K^+ \pi^- + \pi^- \). In the same way, \( T_{bb}^1 \) and \( T_{bb(ub)}^1 \) contain \( B + B^* \) and \( D + B^* \), respectively. However, the prediction of decay modes are not easy at present because the available decay modes of \( B^* \) are very limited.

### 3.2 Heavy pentaquark

Pentaquarks are obtained by taking any two diquarks as given in Table 4 together with an antiquark. Stability in this case is guaranteed when the binding energy of the pentaquark configuration is larger than the largest sum of baryon and meson binding energies. The diquarks should be of different types as they have to be in color antisymmetric configuration so that it combines with the anti-quark into a color singlet baryon. For the pentaquark to be stable, it would be the best to start with the diquarks with the largest bindings; that is (ud) and (us). Combining them with an antiquark \( q \), the binding energy of the pentaquark \( \Theta_{qs} \) (udus\bar{q}) becomes

\[
B_{\Theta_{qs}} = m_{\Theta_{qs}} - m_A - m_{u\bar{q}} = \frac{3 C_B}{4 m_u^2} - \frac{3 C_B}{4 m_u m_s} + \frac{3 C_B}{4 m_u} + \frac{3 C_M}{4 m_u m_q}.
\]
The first and third term cancels. Since $C_M$ is about larger than $C_B$ by a factor 3, it is clear that we will have binding only if $m_q > C_B m_s > 3 m_s$. Therefore, the binding is slightly positive for $q = c$ with 8.4 MeV as shown in Table 3. For $q = b$, one can certainly expect a bound $\Theta_{bs}$ whose binding is $-56.4$ MeV [18]. The large attraction gained from recombining it into a meson and a baryon is the reason why the light pentaquark can not be stable, as was shown in a more detailed constituent quark model [23].

Next, we consider the configuration of $(us)$ and $(ds)$ diquarks with anti-quark $\bar{q}$, which we denote as $\Theta_{qs}(usds \bar{q})$. In this case we have two possible final states; $\Xi + u \bar{q}$ and $\Lambda + s \bar{q}$. The binding energy for the former state is given by

$$B_{\Theta_{us}} = m_{\Theta_{us}} - m_{\Xi} - m_{u \bar{q}} = -2 \times \frac{3}{4} C_B m_s + \frac{3}{4} C_B - \frac{3}{4} M - m_q,$$  

and that for the latter state by

$$B_{\Theta_{us}} = m_{\Theta_{us}} - m_\Lambda - m_{s \bar{q}} = -2 \times \frac{3}{4} C_B m_s + \frac{3}{4} C_B + \frac{3}{4} M - m_q,$$  

The numerical result is shown in Table 3. In these configurations, we again find that the pentaquark becomes more stable as the included antiquark becomes heavier. Therefore, $\Theta_{cs}$ and $\Theta_{cs} \bar{q}$ are slightly unstable, while $\Theta_{bs}$ and $\Theta_{bs} \bar{q}$ are stable. Nonetheless, it is worth searching for all these states in the experiments.

The flavor structure of the stable pentaquark is obtained by following light flavor $SU(3)_f$ symmetry. In the present discussion, the stable pentaquark contains $(ud)$, $(us)$ and $(ds)$ diquarks together with a heavy quark, say $q = b$. Because these diquarks belong to $3_f^1$, the wave function is given as $3_f^1 \times 3_f^1 = 3_f^1 + 6_f^1$.

In our diquark model, $6_f^1$ vanishes because it is symmetric under the exchange of flavors and hence can be canceled by anti-symmetry in color. Therefore, the pentaquarks obtained above belong to the $3_f^1$ representation.

### Table 5.

| $\epsilon_{abc}$ | $M + B$ | $u$ | $s$ | $c$ | $b$ |
| --------------- | ---------- | --- | --- | --- | --- |
| $udus\bar{q}$   | 389.4      | 198.9 | 8.4 | -56.4 |    |
| $usds\bar{q}$   | 389.4      | 198.9 | 8.4 | -56.4 |    |

The pentaquarks have easily identifiable decay modes. For example, $\Theta_{cs}(uddsc) \rightarrow \Lambda + K^+ \pi^-$, and $\Theta_{bs}(udds\bar{b}) \rightarrow A + K^+ \pi^-$, as modes with the least number of particles in the final states. Similarly, $\Theta_{cs} \bar{q} \rightarrow \Lambda + \pi^+ \pi^- \pi^-$, and $\Theta_{bs} \bar{q} \rightarrow A + \pi^+ \pi^- \pi^+ + \pi^-$.

### 3.3 Heavy dibaryon

Dibaryons are obtained by taking three diquarks and combining them into a color singlet combination. Considering the color anti-triplet scalar diquarks $(q_1 q_2)$ in Table 3, the wave function is given as,

$$\epsilon_{abc}(q_1 q_2)^a (q_3 q_4)^b (q_5 q_6)^c,$$

where $a, b, c$ are the color indexes. The total number of states which can be obtained from these wave functions are 120. However, only a few of them are stable against strong decay to two baryons. To understand which configurations are stable, let us consider the decay of the dibaryon into two baryons in the following way. First of all, let us assume that the diquark $(q_1 q_2)$ is the most stable one among the three diquarks in Eq. [18]. If there are two more stable diquarks, one of them is assigned to $(q_1 q_2)$. When the diquark decays to two baryons, there should be a process where some diquarks in the dibaryon are broken into quarks. In the present discussion, because $(q_1 q_2)$ is the most stable diquark, the remaining two diquarks, $(q_3 q_4)$ and $(q_5 q_6)$, can be broken. First possible process occurs when one diquark is broken. When $(q_3 q_4)$ is more loosely bound than $(q_3 q_4)$, then the former diquark is broken to a pair of single quarks $q_5$ and $q_6$. Then they are combined with $(q_1 q_2)$ and $(q_3 q_4)$ to form two baryons that can be represented as either $(q_1 q_2) q_5$ and $(q_3 q_4) q_6$, or $(q_1 q_2) q_6$ and $(q_3 q_4) q_5$. This process is shown by the following equation,

$$\text{conf}_1 = (q_1 q_2) q_5 + (q_3 q_4) q_6,$$

or $(q_1 q_2) q_6 + (q_3 q_4) q_5$.  

(10)

Second possible process occurs when two diquarks are broken. In this case, the diquarks $(q_3 q_4)$ and $(q_5 q_6)$ are first broken into $q_3$, $q_4$, $q_5$ and $q_6$. Second, when the most stable diquark among possible combinations of these four quarks is $(q_5 q_6)$, then they are combined to two possible final baryon states that are either $(q_1 q_2) q_4$ and $(q_3 q_5) q_6$, or $(q_1 q_2) q_6$ and $(q_3 q_5) q_4$. This process is shown by,

$$\text{conf}_2 = (q_1 q_2) q_4 + (q_3 q_5) q_6,$$

or $(q_1 q_2) q_6 + (q_3 q_5) q_4$.  

(11)

The stability of dibaryons is determined by the mass difference between the dibaryon and two baryon states. Namely, when the mass of dibaryon is smaller than those of conf1 and conf2, then such dibaryon can be stable against the strong decay. Now we investigate the stability of the configurations of conf1 or conf2.

Let us first consider the stability of dibaryon against the configuration conf1. In the present diquark model, the diquark $(q_5 q_6)$ is a bound state for any flavor combinations with finite binding energy as given in Table 1. Therefore, the mass of dibaryon is always smaller than that of conf1 by the binding energy of $(q_5 q_6)$. Consequently, we obtain the following rule.
i) Any dibaryon constructed from Table 3 is stable against the decay to conf$_1$. However, the dibaryons are not necessarily stable against the decay to conf$_2$. To see this, let us analyze the stability against the decay to conf$_2$ in Eq. (11). For this purpose, we consider the difference between the binding energies of the dibaryon and conf$_2$. Omitting the common binding energy supplied from $(q_1q_2)$, the binding energy of the dibaryon is given by
\[ \text{B.E.} \left[(q_3q_4) + (q_5q_6)\right] = -\frac{3}{4} \frac{C_B}{m_3m_5} \left(\frac{m_5}{m_4} + \frac{m_3}{m_6}\right). \] (12)
while that of conf$_2$ is given by
\[ \text{B.E.} \left[q_3q_5\right] = -\frac{3}{4} \frac{C_B}{m_3m_5}. \] (13)
As can be seen above, the dibaryon is stable if the following condition is satisfied,
\[ \frac{m_5}{m_4} + \frac{m_3}{m_6} > 1. \] (14)
From this equation, we obtain two rules to realize a stable dibaryon.

ii-a) First, we consider the case where the two diquarks are in the same row in Table 3. Then condition (14) reduces to
\[ m_3 + m_5 > m_4. \] (15)
Because two diquarks are taken from a single row in Table 3, we have the following possibilities: \{(q_3q_4), (q_5q_6)\} = \{(us), (ds)\} for $q_4 = s$, \{(uc), (dc)\}, \{(uc), (sc)\} and \{(dc), (sc)\} for $q_4 = c$, \{(ub), (db)\}, \{(ub), (sb)\}, \{(ub), (cb)\}, \{(db), (bc)\} and \{(sb), (cb)\} for $q_4 = b$. Note $q_3 \neq q_5$ from Eq. (11). Among them, condition (15) is satisfied only for \{(q_3q_4), (q_5q_6)\} = \{(us), (ds)\}.

In other words, diquarks which contain at least two diquarks from either the third or the fourth row of Table 3 such as \{uc, dc\}, can not be stable against conf$_2$.  

ii-b) Second, we consider the case where the two diquarks are in different row of Table 3. In this case however, one finds that no configuration satisfies Eq. (14) when $q_3$ and $q_5$ are different light quarks.

Combining these two rules, one can obtain stable dibaryon configurations by starting from either the first or second row and picking up the remaining two diquarks by moving down in rows either straight along a single column or to different columns with Eq. (14) satisfied. Following these rules about decays to conf$_1$ and conf$_2$, it is easy to find that there are 16 stable dibaryons containing \{(ud)\} diquark and 9 more containing \{(us)\} or \{(ds)\} diquarks. The only exception to this rule is the \{(ud)(sc)\} as it can decay into \{(us)c\} + \{(ds)b\} type baryons. Table 6 lists all the possible configurations and the stable dibaryons whose binding energy are negative.

In Table 6, the stable dibaryons are categorized according to their contents of diquarks as represented in the first column: (qq)(qq)(qq), (qq)(qq)(qQ), (qq)(qQ)(qQ'), (qq)(qq)(Q'Q') and (qq)(qQ)(Q'Q) with $q = u, d, s$ and $Q, Q' = c, b$ which may be called $H, H_Q, H_{Q'Q}, H'_{Q'Q}$ and $H_{Q'Q}$, respectively. The second column indicates the flavor content through five integers $N_uN_dN_sN_cN_b$, representing the number of quark with $u, d, s, c$ and $b$ flavors respectively. The third column is the configurations of diquarks in dibaryons. Here, for short description, we use $qq$ instead of (qq). The fourth, fifth and sixth columns are isospin, its $z$ component and strangeness, respectively. The seventh column indicates the configurations of the two baryon states with lowest threshold. The binding energy of the dibaryon against the two baryon threshold is shown as their mass difference in the last column.

The $(qq)(qq)(qq)$ type becomes stable from the rule ii-a) for conf$_2$, while the other configurations are stable from the rule i) for conf$_1$. We mention that several dibaryons which are not accounted from the rules i), ii-a) and ii-b) are also stable as components in isospin multiplets as shown in Table 6. They are the flavor $N_uN_dN_sN_cN_b = 22110$ with $I = 1$ and $I_z = 0$ in $(qq)(qq)(qq) (Q = c)$, $22101$ with $I = 1$ and $I_z = 0$ in $(qq)(qq)(qq) (Q = b)$, $21111$ and $12111$ with $I = 3/2$ and $I_z = \pm 1/2$ in $(qq)(qQ)(qQ)$, and $11121$ with $I = 1, I_z = 0$ in $(qq)(qQ)(Q'Q')$.

Among these dibaryons, the following two dibaryon configurations are the most deeply bound states and are worth mentioning in details. The first one is the $H$ dibaryon with diquark configurations of \{(ud)(us)\}(ds). Again, the binding energy of the $H$ dibaryon can be obtained by comparing its color-spin interaction to that of two $\Lambda$'s,
\[ B_H = m_H - m_A - m_A = -\frac{3}{4} \frac{C_B}{m_u} - \frac{3}{4} \frac{C_B}{m_u} + \frac{3}{4} \frac{C_B}{m_u} + \frac{3}{4} \frac{C_B}{m_u}. \] (16)
The first term cancels the third term. Therefore, the stability of $H$ dibaryon depends on whether it is energetically favorable to have \{(us) + (ds)\} diquarks for a strong \{(ud)\} diquark, which in fact is the rule ii-a) above. Substituting the constants in our model, we find a binding of $B_H = -29$ MeV, which is consistent with most of the model calculations 21.

The second one is the dibaryon of the $(qq)(qq)(qQ)$ type with $Q = c$ containing a single charm, which will call the $H_c$ dibaryon. The most attractive dibaryon in such configuration is \{(ud)(us)(uc)\}, \{(ud)(dc)+(ds)(ub)\} and \{(ud)(ds)(dc)\} in the isospin $I = 1$ multiplet. Concerning \{(ud)(us)(uc)\}, the binding energy of $H_c$ can be investigated by comparing its color-spin interaction to that of the proton \{(ud)\} and $\Xi_c$ \{(uc)\},
\[ B_{H_c} = m_{H_c} - m_p - m_{\Xi_c} = -\frac{3}{4} \frac{C_B}{m_u} - \frac{3}{4} \frac{C_B}{m_u} - \frac{3}{4} \frac{C_B}{m_u} + \frac{3}{4} \frac{C_B}{m_u} + \frac{3}{4} \frac{C_B}{m_u} - \frac{3}{4} \frac{C_B}{m_u}. \] (17)
The same binding energy is also obtained for the other isospin partners. In another view, it can be regarded that the binding energy of these $H_c$ dibaryons is supplied from the binding energy of $(uc)$ or $(dc)$ diquarks. As a result, one obtains $B_{H_c} = -29$ MeV and a stable $H_c$ dibaryon. This is a new prediction and could be searched in future heavy ion collisions, where large number of charm quarks are expected to be produced and weakly decaying particle identifiable through vertex detectors [18]. As further possible bound states, $(ud)(us)(sc)$ and $(ud)(ds)(sc)$ in $I = 1/2$ and $(us)(ds)(sc)$ in $I = 0$ all have binding energies of $-17$ MeV. This binding energy is supplied from the binding of the $(sc)$ diquark, and hence is smaller than that of $H_c$ dibaryon. Nevertheless, these configurations are additional candidates for stable dibaryons containing a single charm.

The dibaryons stated above have relatively large binding energies as the $H$ dibaryon. The diquark model also predicts other possible dibaryon states which have smaller binding energies. In the $(qq)(qq)(qQ)$ type, we obtain stable configurations for $Q = b$ ($H_b$) with binding energies equal to $-9$ MeV for $I = 1$, and $-5$ MeV for $I = 1/2$ and for $I = 0$, as shown in Table 6. The binding energy is reduced in comparison with $Q = c$, simply because the $(ub)$, $(db)$ and $(sb)$ diquarks are more loosely bound. As for the dibaryons containing two heavy quarks, the $(qq)(qQ)(qQ')$ and $(qq)(qq')(QQ')$ types with $Q = c$ and $Q' = b$ ($H_{bc}$) and $H_{cc'}$ can also be stable. For the former, the binding energies are obtained as $-9$ MeV for $I = 1$ and $I = 3/2$, and $-5$ MeV for $I = 1/2$ which bindings are supplied from each of $(ub)$, $(db)$ and $(sb)$ diquarks. For the latter, the binding energies are $-1.8$ MeV for $I = 1/2$ and $I = 0$ which are supplied from the $(cb)$ diquark. The binding energy of $(cb)$ diquark is very small in comparison with other diquarks containing light flavors. Therefore, it supplies only small binding energy for the dibaryons. Lastly, we can have dibaryons containing three heavy quarks as $(qq)(qQ)(QQ')$ with $Q = c$ and $Q' = b$ ($H_{bcc}$). However, their binding energies are also small because they are supplied from the $(cb)$ diquark.

According to the $SU(3)_f$ light flavor symmetry, the dibaryons in Table 6 belong to the following multiplets. Noting $(qq)$ diquark belongs to $3_f$, the light flavor multiplets of the $(qq)(qq')(qq)$ type are obtained by $3_f \times 3_f \times \overline{3}_f = 1_f + 8_f + 10_f + 10_f$. Then the $H$ dibaryon $(ud)(us)(ds)$ belongs to $1_f$.

For $(qq)(qq)(qQ)$ with $Q = c$ or $b$ ($H_c$ or $H_b$), noting $(qQ)$ diquark belongs to $3_f$, the light flavor is decomposed as

$3_f \times 3_f \times \overline{3}_f = 3_f^3 + 6_f^2 + 3_f^2 + \overline{15}_f$.

Here $a$ ($s$) means anti-symmetry (symmetry) for a pair of two diquarks $(qq)$’s in $(qq)(qq)(qQ)$. Because of the anti-symmetry of color in the wave function in Eq. 4, $3_f^2$ and $\overline{15}_f$ which contain identical pair of diquarks in symmetric flavor $6_f^2$ vanish, and only $3_f^3$ and $6_f^2$ which contain anti-symmetric flavor $3_f^2$ survive. Concerning mixing of $3_f^3$ and $6_f^2$, some comments are in order. First, in $S = -1$ sector, the wave function in the $I = 0$ configuration in $3_f^3$ is given as

$\langle ud \rangle \langle (us)(dQ) - (ds)(uQ) \rangle$,

while that of $I_z = 0$ in $I = 1$ in $6_f$, which weight vector is on the same place, is given as

$\langle ud \rangle \langle (us)(dQ) + (ds)(uQ) \rangle$,

with appropriate normalization constants. As far as the isospin is conserved exactly, these two states are independent. The latter dibaryon with $I = 1$ is stable against the decay to $\Lambda + \Lambda Q$ because isospin symmetry forbids this decay. However, the former dibaryon with $I = 0$ is not stable, but decays to $\Lambda + \Lambda Q$. Therefore, in $S = -1$ sector, we have stable dibaryons in $I = 1$. Second, in $S = -2$ sector, the $I = 1/2$ configuration in $3_f^2$ are generally mixed with those in $6_f$. For example, the $I_z = 1/2$ components are given by

$\langle ud \rangle \langle (us)(sQ) \mp (us)(ds)(uQ) \rangle$.

for $3_f^2$ and $6_f$, respectively. However, from the rule to obtain the stable dibaryon, the dibaryon $(us)(ds)(uQ)$ is not stable against $\Lambda + \Xi_Q$ and $\Lambda_Q + \Xi$. Therefore, by considering a superposition of $3_f^2$ and $6_f$ and eliminating $(us)(ds)(uQ)$, we obtain the stable dibaryon $(ud)(us)(sQ)$. Third, in $S = -3$ sector, the $(us)$ and $(ds)$ pair in $(us)(ds)(sQ)$ is allowed to be $I = 0$ from anti-symmetry of color configuration. The resulting weight diagrams are shown in Fig. 1 and 2 for $Q = c$ and $b$, respectively.

Similarly, other multiplets for the configurations with two or three heavy quarks are obtained. As for $(qq)(qQ)(qQ')$ with $Q = c$ and $Q' = b$ ($H_{bcb}$), the light flavor is decomposed by

$3_f \times 3_f \times 3_f \times 3_f \times (3_f^3 + 6_f^2) = 3_f^3 + 6_f^2 + 3_f^2 + 15_f$.

We note that $3_f^2$ and $15_f$ containing symmetric flavor $6_f^2$ can survive, because $(qQ)$ cannot be identical to $(qQ')$ due to $Q \neq Q'$. From the configurations in Table we find that the stable dibaryons contain pairs of $(qQ)$ and $(qQ')$ with identical $q$, which are assigned to $6_f^2$. Hence, these dibaryons belong to $3_f^2$ or $15_f$. Here, we show that only $15_f$ is relevant to the stable dibaryons. First, in $S = -1$ sector, the $I = 1/2$ configuration in $3_f^2$ is at the same position in the weight diagram with the $I_z = \pm 1/2$ components in the $I = 3/2$ configuration in $15_f$. However, the $I = 1/2$ configuration is not stable, because, for example, the $I_z = 1/2$ component in $I = 1/2$ can decay to $\Lambda_b + \Xi_Q^i$ (or $\Lambda_c + \Xi_Q^i$). Hence the $S = -1$ sector in $3_f^2$ is irrelevant. Second, in $S = -2$ sector, any components in $I = 0$ in $3_f^2$ and in $I = 1$ in $15_f$ are not stable. In fact we have already discussed that $(ud)(sc)(sb)$ configuration is not stable. Therefore, $3_f^2$ can be discarded, and so can the $I = 1$ configuration in $S = -2$ in $15_f$. Concerning the $S = 0$ sector, we note that the $I_z = 0$ component of $I = 1$ cannot decay to $\Lambda_c + \Lambda_b$ because of isospin conservation, and hence can be stable. The resulting weight diagram is shown in Fig. 3. Here, the black blobs indicate the stable dibaryons, while the white ones indicate the unstable ones.
As for the $(qq)(qq)(QQ')$ type with $Q = c$ and $Q' = b$ $(H_{bc})$, the light flavor is decomposed as

$$\mathbf{3}_f \times \mathbf{3}_f = \mathbf{3}_f^a + \mathbf{6}_f^a.$$  

Because a pair of two $(qq)$'s in $\mathbf{6}_f^a$ vanishes due to antisymmetry in color, only $\mathbf{3}_f^a$ can survive because of isospin conservation. The weight diagram of $(qq)(qq)(QQ')$ is shown in Fig. 3.

Lastly, for dibaryons with three heavy quarks such as $(qq)(qQ)(QQ')$ with $Q = c$ and $Q' = b$ $(H_{bc})$, the light flavor is decomposed as

$$\mathbf{3}_f \times \mathbf{3}_f = \mathbf{1}_f + \mathbf{8}_f.$$  

In $S = -1$ sector, the $I = 0$ configurations in $\mathbf{1}_f$ are superposition of ($(us)(d\bar{s})-(d\bar{s})(u\bar{s}))((QQ')$ and $(ud)(sQ)(QQ')$, which are constructed from $(us), (ds)$ and $(ud)$ dibaryons.

Among them, however, the dibaryons containing $(us)$ or $(ds)$ are not stable against $A_h + \Omega_{bc}^+$ (or $A_c + \Gamma_{bc}^+$), and hence $(ud)(sQ)(QQ')$ containing only $(ud)$ dibaryon is stable. On the other hand, the $I = 1$ configuration of $((us)(d\bar{s})-(d\bar{s})(u\bar{s}))((QQ')$ in $\mathbf{8}_f$ can survive. In $S = -2$, there are no stable dibaryons. As a result, the weight diagram is obtained in Fig. 3.

Now, let us discuss possible decay modes from these diabaryons for experiments. Concerning dibaryon, the observation has been studied by weak decay, for example, by $H \to \Lambda + \Lambda \to p\pi^- + p\pi^-$. Similarly, we investigate possible decay modes of the stable dibaryon including heavy quarks.

The $(qq)(qq)(QQ)$ with $Q = c$, which would be most stable state, can have the decay modes as summarized in Table I. For example, the wave function of the $(ud)(uc)(uc)$ with $I_2 = 1$ in $I = 1$ would have an overlap with $A_c^0 + \Sigma^+$ and $p + \Xi_c^+$ with invariant mass 3475 MeV and 3406 MeV, respectively. Then, each component will decay by weak interaction; $A_c^0 + \Sigma^+ \to pK^-\pi^+ + p\pi^0$, and $p + \Xi_c^+ \to p + AK^-\pi^+\pi^+$, $\Sigma^+K^+\pi^+$, $\Xi^+\pi^+\pi^+\pi^+$, $\Xi^0\pi^+\pi^+\pi^+$ and $\Xi^-\pi^+\pi^+\pi^+\pi^+\pi^+$.

We note that, concerning $A_c$, we indicate only $A_c \to pK^-\pi^+$ as a simplest decay pattern. The $I_2 = -1$ component in $I = 1$ has also an overlap with $A_c^+ + \Sigma^-$ and $n + \Xi_c^0$ with invariant mass 3483 MeV and 3410 MeV, respectively, and the decay modes of each component are $A_c^+ + \Sigma^- \to pK^-\pi^+ + n\pi^-$, and $n + \Xi_c^0 \to n + pK^-K^-\pi^+$, $AK^-\pi^+\pi^+\pi^-$, $\Xi^+\pi^+\pi^+\pi^-$, $\Xi^0\pi^+\pi^+\pi^+$, $\Xi^-\pi^+\pi^+\pi^+\pi^-\pi^-$. The $I_2 = 0$ component in $I = 1$ would contain $A + \Sigma_c^0$ or $A_c^+ + \Sigma^-$ because of isospin symmetry, and hence its decay modes are $A + \Sigma_c^0 \to \Lambda \pi^- + \Lambda_c^0 \to p\pi^- + pK^-K^-\pi^+$, and $A_c^+ + \Sigma^- \to \Lambda \gamma + pK^-\pi^+$. However, in reality, explicit isospin breaking effect would induce strong decay much larger than weak decay. Hence, these states would first decay to $H_{bc} \to \Lambda + A_c^0$ by explicit isospin breaking, and second they would decay to $p\pi^- + pK^-\pi^+\pi^0$ by weak interaction. As for $I = 1/2$ and $I = 0$, which would be second most stable states, we obtain the possible decay modes as shown in Table I. Note that the isospin $I = 0$ indicates the wave function $\Xi^0\Xi_c^0 = \Xi^-\Xi_c^+$ and each component is shown in the table.

Similar decay patterns hold for $(qq)(qq)(QQ)$ with $Q = b$ as shown in Table II. However, because of the lack of experimental data about the decays of several bottom baryons, available decay patterns are limited at present. Nevertheless at least the $I = 1$ and $I = 1/2$ dibaryons can be detectable, while the $I = 0$ dibaryon needs experimental information about the decays of $\Xi_b^0$ or $\Xi_b^-$.  

As for dibaryons with two or three heavy quarks, only a few candidates are available. In $(qq)(qQ)(QQ')$ with $Q = c$ and $Q' = b$, the $I_2 = \pm 1$ component in $I = 1$ and $I = 3/2$ could be investigated by using experimentally known decay patterns as shown in Table II. In $(qq)(qQ)(QQ')$ with $Q = c$ and $Q' = b$, the $I_2 = \pm 1/2$ components in $I = 1/2$ could be also investigated as shown in Table II.  

Lastly, any components in $(qq)(qQ)(QQ')$ with $Q = c$ and $Q' = b$ would be difficult because of the lack of experimental information as shown in Table II.

Thus, the diquark model gives a list of possible $H$ dibaryons, $H_c$, $H_b$, $H_{bc}$, and $H_{bcc}$. Among them, the first promising candidate is $H_c$ with $I = 0$, and the second one is $H_b$ with $I = 1/2$ and 1. The other $H$ dibaryons, $H_b$, $H_{bc}$, and $H_{bcc}$, are less probable because of their small binding energies. Nevertheless, search of these particles is a challenging subject, not only for theory with more sophisticated analysis by lattice simulation, QCD sum rule, and so on, but also for experiments in high energy facilities.
Fig. 3. The weight diagram of $SU(3)_f$ for stable diquark of $qq qQ qQ'$ ($Q = c$ and $Q' = b$) type ($H_{bc}$) with strangeness $S$, isospin $I$ and binding energy $B_H$. The black blobs indicate the stable dibaryons, while the white blobs do the unstable dibaryons.

Fig. 4. The weight diagram of $SU(3)_f$ for stable diquark of $qq qq' Q Q'$ ($Q = c$ and $Q' = b$) type ($H_{cc}$) with strangeness $S$, isospin $I$ and binding energy $B_H$. The black blobs indicate the stable dibaryons.

Fig. 5. The weight diagram of $SU(3)_f$ for stable diquark of $qq qQ qQ'$ ($Q = c$ and $Q' = b$) type ($H_{bc}$) with strangeness $S$, isospin $I$ and binding energy $B_H$. The black blobs indicate the stable dibaryons, while the white blobs do the unstable dibaryons.

4 Summary

Based on the consideration of the color-spin interaction between diquarks, which describes well the mass splittings between many hadrons and their spin flipped partners, we have systematically investigated possible stable multiquark configurations. We have shown that the configurations becomes stable only when heavy quarks are included. We have identified several new possibly stable multiquark configurations. These are the $T_3^0(udc\overline{b})$ tetraquark, the $Q_{bs}(uds\overline{d})$ pentaquark and the $Q_{bs}(udsu\overline{c})$ dibaryon, and so forth. The representative decay modes for these states are $T_3^0 \rightarrow K^+\pi^- + K^+\pi^-$, $Q_{bs} \rightarrow AK^+\pi^-$ and $H_{bc} \rightarrow A + A^+_c \rightarrow p\pi^- + pK^-\pi^+\pi^0$. If one observes these states, these will be the first explicit flavor multiquark states, and will be a valuable stepping stone to understanding QCD at larger quark number.

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Table 6. The mass difference of $H$-dibaryons and two baryon state $B$ and $B'$. $B_H = m_H - m_B - m_{B'}$. The $H$-dibaryons can be stable for $B_H < 0$. $Q \neq Q'$. The unit is in MeV. Contrary to the text, we omit brackets denoting scalar and isoscalar diquark for simplicity.

| Flavor | config. of $H$ | $I$ | $I_s$ | $S$ | config. of $B + B'$ | $B_H$ [MeV] |
|--------|----------------|-----|-------|-----|----------------------|----------------|
| $H(qq\bar{q}q\bar{q})$ | $ud\, us\, ds$ | 0 | 0 | -2 | $ud\, s + ud\, s$ | -28.95 |
| $H_c(qq\bar{q}q\bar{c})$ | $ud\, us\, uc$ | 1 | 1 | -1 | $ud\, c + us\, u$, $ud\, u + us\, c$ | -28.95 |
| $H(qq\bar{q}q\bar{q})$ | $ud\, ds\, dc$ | 1 | 0 | -1 | $ud\, s + ud\, c$, $ud\, s + ud\, c$ | -9.23 |
| $H(qq\bar{q}q\bar{q})$ | $ud\, us\, sc$ | 1/2 | 1/2 | -2 | $ud\, c + us\, s$, $ud\, s + us\, c$ | -17.37 |
| $H(qq\bar{q}q\bar{q})$ | $ud\, ds\, sc$ | 1/2 | -1/2 | -2 | $ud\, c + ds\, s$, $ud\, s + ds\, c$ | -17.37 |
| $H_b(qq\bar{q}c\bar{q})$ | $ud\, uc\, ub$ | 1 | 1 | -1 | $ud\, b + uc\, u$, $ud\, u + uc\, b$ | -9.23 |
| $H_b(qq\bar{q}c\bar{q})$ | $ud\, dc\, db$ | 1 | -1 | -1 | $ud\, b + dc\, d$, $ud\, d + dc\, b$ | -9.23 |
| $H_b(qq\bar{q}c\bar{q})$ | $us\, uc\, ub$ | 3/2 | 3/2 | -1 | $us\, u + uc\, b$, $us\, b + uc\, u$ | -9.23 |
| $H_b(qq\bar{q}c\bar{q})$ | $us\, sc\, sb$ | 1/2 | 1/2 | -3 | $us\, b + sc\, s$, $us\, s + sc\, b$ | -5.54 |
| $H_b(qq\bar{q}c\bar{q})$ | $ds\, dc\, db$ | 3/2 | -3/2 | -1 | $ds\, b + dc\, d$, $ds\, d + dc\, b$ | -5.54 |
| $H_b(qq\bar{q}c\bar{q})$ | $ds\, sc\, sb$ | 1/2 | 1/2 | -3 | $ds\, b + sc\, s$, $ds\, s + sc\, b$ | -5.54 |
| $H_{bc}(qq\bar{q}c\bar{q})$ | $ud\, us\, eb$ | 1/2 | 1/2 | -1 | $ud\, b + us\, c$, $ud\, s + us\, b$ | -1.84 |
| $H_{bc}(qq\bar{q}c\bar{q})$ | $ud\, ds\, cb$ | 1/2 | -1/2 | -1 | $ud\, b + ds\, c$, $ud\, c + ds\, b$ | -1.84 |
| $H_{bc}(qq\bar{q}c\bar{q})$ | $us\, uc\, cb$ | 1 | 1 | -1 | $us\, b + uc\, c$, $us\, c + uc\, b$ | -1.84 |
| $H_{bc}(qq\bar{q}c\bar{q})$ | $ds\, dc\, cb$ | 1 | -1 | -1 | $ds\, b + dc\, c$, $ds\, c + dc\, b$ | -1.84 |
Table 7. \([qq qq qQ; Q = c \ (H_c)]\) The possible decay modes of \(H_c\)-dibaryons with configuration \(qq qq qQ\) \((Q = c)\) against two baryon state \(B\) and \(B'\). The numbers in the brackets are invariant mass of the two baryons at rest frame.

| flavor config. of \(H_c\) | \(I\) | \(I_s\) | \(S\) | config. of \(B + B'\) (MeV) | final states |
|--------------------------|-----|-----|------|-----------------|-------------|
| 31110 \(ud us uc\)     | 1   | 1   | -1   | \(A_{c}^{+} + \Sigma^{0}_{c}\) \((3475.83)\) | \(pK^{-} \pi^{+} + p\pi^{0}\) |
|                          |     |     |      | \(p + \Xi_{c}^{-}\) \((3406.17)\) | \(p + \Lambda K^{-} \pi^{+} \pi^{+}\) |
|                          |     |     |      |                            | \(p + \Sigma^{+} K^{-} \pi^{0}\) |
|                          |     |     |      |                            | \(p + \Xi^{0}_{c} \pi^{+}\) |
|                          |     |     |      |                            | \(p + \Xi^{-} \pi^{+} \pi^{0}\) |
|                          |     |     |      |                            | \(p + \Xi^{0}_{c} \pi^{+} \pi^{-}\) |
| 22110 \(ud \frac{1}{\sqrt{2}}(us dc + ds uc)\) | 1   | 0   | -1   | \(A + \Sigma^{0}_{c}\) \((3568.58)\) | \(p\pi^{-} + A_{c}^{0} \pi^{+}\) |
|                          |     |     |      | \(\Sigma^{0} + A_{c}^{0}\) \((3478.92)\) | \(A_{c}^{-} + pK^{+} \pi^{+}\) |
| 13110 \(ud ds dc\)     | 1   | -1  | -1   | \(A_{c}^{+} + \Sigma^{-}_{c}\) \((3483.91)\) | \(n + pK^{-} K^{-} \pi^{+}\) |
|                          |     |     |      | \(n + \Xi^{0}_{0}\) \((3410.56)\) | \(n + \Lambda K^{-} \pi^{-} \pi^{+}\) |
|                          |     |     |      |                            | \(n + \Xi^{-} \pi^{+}\) |
|                          |     |     |      |                            | \(n + \Xi^{-} \pi^{+} \pi^{-}\) |
|                          |     |     |      |                            | \(n + \Omega^{-} K^{+}\) |
| 21210 \(ud us sc\)     | 1/2 | 1/2 | -2   | \(A_{c}^{+} + \Xi^{0}_{c}\) \((3601.32)\) | \(p\pi^{-} + A_{c}^{-} \pi^{+}\) |
|                          |     |     |      | \(A + \Xi^{0}_{c}\) \((3583.58)\) | \(p\pi^{-} + \Lambda K^{-} \pi^{+} \pi^{+}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Sigma^{+} K^{-} \pi^{0}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Xi^{0}_{c} \pi^{+}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Xi^{-} \pi^{+} \pi^{0}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Xi^{0}_{c} \pi^{+} \pi^{-}\) |
| 12210 \(ud ds sc\)     | 1/2 | -1/2| -2   | \(A_{c}^{+} + \Xi^{0}_{c}\) \((3608.17)\) | \(p\pi^{-} + pK^{-} K^{-} \pi^{+}\) |
|                          |     |     |      | \(A + \Xi^{0}_{c}\) \((3586.68)\) | \(p\pi^{-} + \Lambda K^{-} \pi^{-} \pi^{+}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Xi^{-} \pi^{+}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Xi^{-} \pi^{+} \pi^{-}\) |
|                          |     |     |      |                            | \(p\pi^{-} + \Omega^{-} K^{+}\) |
| 11310 \(us ds sc\)     | 0   | 0   | -3   | \(\Xi^{0} + \Xi^{0}_{c}\) \((3785.86)\) | \(A_{c}^{0} + \Lambda K^{-} \pi^{+}\) |
|                          |     |     |      | \(A_{c}^{0} + \Xi^{-}\) \((3789.6)\) | \(A_{c}^{0} + \Lambda K^{-} \pi^{+} \pi^{-}\) |
|                          |     |     |      |                            | \(A_{c}^{0} + \Xi^{-} \pi^{+}\) |
|                          |     |     |      |                            | \(A_{c}^{0} + \Xi^{-} \pi^{+} \pi^{-}\) |
|                          |     |     |      |                            | \(A_{c}^{0} + \Omega^{-} K^{+}\) |
Table 8. \([qq qq qQ; \ b \ (H_b)]\) The possible decay modes of \(H_b\)-dibaryons with configuration \(qq qq qQ\) \((Q = b)\) against two baryon state \(B\) and \(B'\). The numbers in the brackets are invariant mass of the two baryons at rest frame.

| flavor | config. of \(H_b\) | \(I\) | \(I_z\) | \(S\) | config. of \(B + B'\) (MeV) | final states |
|--------|------------------|-----|-----|-----|-----------------------------|-------------|
| 31101  | \(ud us ub\)    | 1   | 1   | -1  | \(p + \Xi^0\) (6730.67)    | \(p + \Xi^0\) |
| 22101  | \(ud \frac{1}{\sqrt{2}}(us db + ds ub)\) | 0   | 0   | -1  | \(A + \Xi^0\) (6926.18)    | \(p + \pi^+ + p\mu^+\) |
| 13101  | \(ud ds db\)    | 1   | -1  | -1  | \(A_0\) (6812.84)          | \(A_0 + A_1\pi^+\) |
| 21202  | \(ud us sb\)    | 1/2 | 1/2 | -2  | \(A_0\) (6935.06)          | \(A_0 + A_1\pi^+\) |
| 12201  | \(ud ds sb\)    | 1/2 | 1/2 | -2  | \(A_0\) (6908.08)          | \(p + \pi^-\) |
| 11301  | \(us ds sb\)    | 0   | 0   | -3  | \(n + \Xi^0\) (6731.97)    | \(A_1\pi^-\) |

Table 9. \([qq qQ qQ'; Q = c, Q' = b (H_c)]\) The possible decay modes of \(H_c\)-dibaryons with configuration \(qq qQ qQ'\) \((Q = c, Q' = b)\) against two baryon state \(B\) and \(B'\). The numbers in the brackets are invariant mass of the two baryons at rest frame.

| flavor | config. of \(H_c\) | \(I\) | \(I_z\) | \(S\) | config. of \(B + B'\) (MeV) | final states |
|--------|------------------|-----|-----|-----|-----------------------------|-------------|
| 31011  | \(ud uc ub\)    | 1   | 1   | 0   | \(A_0 + \Sigma^+\) (8074.2) | \(A_0 + A_1\pi^+\) |
| 22011  | \(ud \frac{1}{\sqrt{2}}(uc db + dc ub)\) | 1   | 0   | 0   | \(\Xi^0 \pm p (+)\)         | \(n + \Xi^0\) |
| 13011  | \(ud dc db\)    | 1   | -1  | 0   | \(A_0 + \Sigma^+\) (8073.96) | \(n + \Xi^0\) |
| 30111  | \(us uc ub\)    | 3/2 | 3/2 | -1  | \(\Sigma^+ + \Xi^0\) (8246.4) | \(\Sigma^+ + \Xi^0\) |
| 21111  | \(\frac{1}{\sqrt{3}}(us uc db + us dc ub + ds uc ub)\) | 3/2 | 1/2 | -1  | \(\Sigma^+ + \Xi^0\) (8278.4) | \(\Sigma^+ + \Xi^0\) |
| 12111  | \(\frac{1}{\sqrt{3}}(us dc db + ds dc ub + ds uc db)\) | 3/2 | -1/2| -1  | \(\Sigma^+ + \Xi^0\) (8282.5) | \(\Sigma^+ + \Xi^0\) |
| 03111  | \(ds dc db\)    | 3/2 | -3/2| -1  | \(\Sigma^+ + \Xi^0\) (8246.16) | \(\Sigma^+ + \Xi^0\) |
| 10311  | \(us sc sb\)    | 1/2 | 1/2 | -3  | \(\Xi^0 + \Omega^0\) (8489.9) | \(\Xi^0 + \Omega^0\) |
| 01311  | \(ds sc sb\)    | 1/2 | -1/2| -3  | \(\Xi^0 + \Omega^0\) (8489.9) | \(\Xi^0 + \Omega^0\) |
Table 10. \([qq \bar{q} Q Q']\): \(Q = c, Q' = b (H_{bc}^+\) The possible decay modes of \(H_{bc}^+\)-dibaryons with configuration \(qq \bar{q} Q Q'\) \((Q = c, Q' = b)\) against two baryon state \(B\) and \(B'\). The numbers in the brackets are invariant mass of the two baryons at rest frame.

| flavor | config. of \(H_{bc}\) | \(I\) | \(I_s\) | \(S\) | config. of \(B + B'\) (MeV) | final states |
|--------|------------------------|------|-------|------|-----------------------------|-------------|
| 21111  | ud us cb               | 1/2  | 1/2   | -1   | \(A_0^+ + \Xi_{cc}^0\) (8088.1) | \(A_0^+ \pi^+ + \Xi_{cc}^0\) (8078.86) |
|        |                        |      |       |      | \(\Lambda_2^+ \pi^+ + \Xi_{cc}^0\) (8091.2) | \(\Lambda_2^+ \pi^+ + \Xi_{cc}^0\) (8260.3) |
| 12111  | ud ds cb               | 1/2  | -1/2  | -1   | \(A_0^+ + \Xi_{cc}^0\) (8263.4) | \(\Xi_{cc}^0\) (8088.1) |
|        |                        |      |       |      | \(\Xi_{cc}^0\) (8078.86) | \(\Xi_{cc}^0\) (8260.3) |

Table 11. \([qq qQ Q'Q']\): \(Q = c, Q' = b (H_{bc}^+\) The possible decay modes of \(H_{bc}^+\)-dibaryons with configuration \(qq qQ Q'Q'\) \((Q = c, Q' = b)\) against two baryon state \(B\) and \(B'\). The numbers in the brackets are invariant mass of the two baryons at rest frame.

| flavor | config. of \(H_{bc}\) | \(I\) | \(I_s\) | \(S\) | config. of \(B + B'\) (MeV) | final states |
|--------|------------------------|------|-------|------|-----------------------------|-------------|
| 21021  | ud uc cb               | 1/2  | 1/2   | 0    | \(A_0^+ + \Omega_{cc}^+\) (?) | \(A_0^+ \pi^+ + ?\) |
|        |                        |      |       |      | \(A_0^+ + \Xi_{cc}^0\) (?) | \(pK \pi^+ + ?\) |
| 12021  | ud dc cb               | 1/2  | -1/2  | 0    | \(A_0^+ + \Omega_{cc}^+\) (?) | \(A_0^+ \pi^+ + ?\) |
|        |                        |      |       |      | \(A_0^+ + \Xi_{cc}^0\) (?) | \(pK \pi^+ + ?\) |
| 20121  | us uc cb               | 1    | 1     | -1   | \(A_0^+ + \Omega_{cc}^+\) (?) | \(A_0^+ \pi^+ + ?\) |
|        |                        |      |       |      | \(A_0^+ + \Xi_{cc}^0\) (?) | \(\Xi_{cc}^0\) (8078.86) |
| 11121  | \(\sqrt{2}(us dc + ds uc)\) cb | 1    | 0     | -1   | \(A_0^+ + \Omega_{cc}^+\) (?) | \(A_0^+ \pi^+ + ?\) |
|        |                        |      |       |      | \(A_0^+ + \Xi_{cc}^0\) (?) | \(pK \pi^+ + ?\) |
| 01121  | ds dc cb               | 1    | -1    | -1   | \(A_0^+ + \Omega_{cc}^+\) (?) | \(A_0^+ \pi^+ + ?\) |
|        |                        |      |       |      | \(A_0^+ + \Xi_{cc}^0\) (?) | \(pK \pi^+ + ?\) |
| 01121  | ud sc cb               | 0    | 0     | -1   | \(A_0^+ + \Omega_{cc}^+\) (?) | \(A_0^+ \pi^+ + ?\) |
|        |                        |      |       |      | \(A_0^+ + \Xi_{cc}^0\) (?) | \(pK \pi^+ + ?\) |