We investigate the potential of the ILC for measuring anomalous quartic gauge couplings, both in production of three electroweak gauge bosons as well as in vector boson scattering. Any new physics that could possibly couple to the electroweak gauge bosons is classified according to its spin and isospin quantum numbers and parameterized in terms of resonance parameters like masses, widths, magnetic moment form factors etc. By a maximum log-likelihood fit, the discovery reach of a 1 TeV ILC for scalar, vector and tensor resonances is examined.

1 Parameterization of new physics in terms of resonances

The Standard Model (SM) with all yet discovered particles (fermions and gauge bosons) can be described by a non-linear sigma model for the electroweak (EW) interactions, dictated by the invariance under $SU(2)_L \times U(1)$ transformations (see e.g. [2, 3]). In this EW chiral Lagrangian the Higgs boson is absent, and the model has to be renormalized order by order, adding new higher-dimensional operators. Any new physics beyond the SM can then be parameterized in terms of these operators in a quite generic way. The building blocks of this (bottom-up) approach are the SM fermions, $\psi$, the $SU(2)_L$ gauge bosons, $W^a_{\mu}$, the hypercharge gauge boson, $B_\mu$, and the nonlinear representation of the Goldstone bosons: $\Sigma = \exp \left[-i v w^a \tau^a \right]$. The longitudinal vector bosons are built from the Goldstone bosons within the vector $V = \Sigma (D \Sigma)^\dagger$. To describe isospin-breaking effects, one singles out the neutral component: $T = \Sigma \tau^3 \Sigma^\dagger$. With these prerequisites we can write the minimal SM Lagrangian (without the yet unobserved Higgs boson) including all the EW gauge interactions as

$$ L_{\text{min}} = \sum_\psi \bar{\psi} (i \gamma^\mu D_\mu) \psi - \frac{1}{2 g^2} \text{tr} \{W_{\mu\nu} W^{\mu\nu}\} - \frac{1}{2 g'^2} \text{tr} \{B_{\mu\nu} B^{\mu\nu}\} + \frac{v^2}{4} \text{tr} \{(v D_\mu \Sigma)(v D^\mu \Sigma)\} $$

The complete Lagrangian, since non-renormalizable, contains infinitely many higher-dimensional operators and, hence, infinitely many parameters:

$$ L_{\text{eff}} = L_{\text{min}} - \sum_\psi \bar{\psi} L_0 \Sigma M \psi_R + \beta_1 L_0 + \sum_i \alpha_i L_i + \frac{1}{v^2} \sum_i \alpha_i^{(5)} L_i^{(5)} + \frac{1}{v^4} \sum_i \alpha_i^{(6)} L_i^{(6)} + \ldots $$

All of flavor physics is contained in the fermion mass matrix $M$, but is ignored for the rest of the paper, since we are interested mainly in the bosonic EW structure. Indirect information on new physics is encoded in the $\rho$ (or $T$) parameter $\beta_1$, the $\alpha$ parameters and higher-dimensional coefficients. The parameters above can be expressed in terms of the fundamental building blocks (for more details cf. [4]):
The $\alpha$ parameters can be measured at ILC with an expected accuracy at least an order of magnitude better than at LEP, which allows to access new physics scales that lie outside the kinematical range of LHC. One of the tasks of this paper is to study the sensitivity of ILC for new physics scales in the bosonic EW sector, parameterized by the $\alpha_i$. From the LEP experiments we already know that the $\alpha$ parameters must be quite small, $\alpha_i \ll 1$. If new physics coupled to the EW sector is present, we expect the parameters to be of the order of $\alpha_i \gtrsim 1/16\pi^2 \approx 0.006$, because the higher-dimensional operators renormalize divergences which appear with $O(1)$ coefficients, $16\pi^2 \alpha_i \gtrsim 1$.

**Table 1**: Classification of resonances that could possibly couple to the sector of EW bosons according to their spin and isospin quantum numbers, together with some simple examples for them.

| $I$ | $J = 0$ | $J = 1$ | $J = 2$ |
|-----|---------|---------|---------|
| 0   | $\sigma^\mu$ (Higgs ?) | $\sigma^\mu$ (Graviton ?) | $J^\mu$ (Graviton ?) |
| 1   | $\pi^\pm, \pi^0$ (2HDM ?) | $\rho^\pm, \rho^0$ (W' / Z' ?) | $a^\pm, a^0$ |
| 2   | $\phi^{\pm \pm}, \phi^\pm, \phi^0$ (Higgs triplet ?) | — | $\ell^{\pm \pm}, \ell^\pm, \ell^0$ |

**Figure 1**: Left: Combined fit for $WWZ/ZZZ$ production at $\sqrt{s} = 1$ TeV, 1 $ab^{-1}$, both beams polarized. Right: Expected sensitivity (combined fit for all processes) to quartic anomalous couplings for a 1 $ab^{-1}$ $e^+e^-$ sample in the conserved $SU(2)_c$ case. Solid lines represent 90% CL, dashed ones 68%.
A single new physics scale, $\Lambda$ or $\Lambda^*$, by which the higher-dimensional operators are suppressed in the form of $\alpha_i \sim v^2/\Lambda^2$, is in itself not a very meaningful quantity. Furthermore, it cannot be unambiguously extracted, since the operator normalization is arbitrary as long as the full theory is unknown. And, as we will demonstrate below, the power counting can be quite intricate, such that there is no simple one-to-one correspondence between new physics and chiral Lagrangian parameters.

To be specific: we consider resonances that couple to the EW symmetry breaking sector of the SM. The resonance masses will give detectable shifts in the $\alpha_i$ parameters. These resonances could either be quite narrow in which case we would call them “particles” or rather wide where they would be accounted for as a “continuum”. In that sense, the approach we are using here accounts for both weakly and strongly interacting models. In Tab. 1 we classified all possibilities of resonances that can couple to the EW sector according to their spin and isospin quantum numbers. A special case is the parameter $\beta_1$ (“$\rho$” parameter) being much smaller than the others as it expresses the $SU(2)_c$ custodial symmetry almost respected by the SM Lagrangian. The custodial symmetry is broken by the hypercharge gauge interactions $g' \neq 0$ and the fermion masses.

The most reliable way to take the effects of heavy resonances on the EW Lagrangian into account is to integrate them out in the path integral by completing the square in the Gaussian integration. Considering the leading order effects of resonances on the EW sector, integrating out a resonance $\Phi$ generates higher-dimensional current-current interactions:

$$\mathcal{L}_\Phi = z \left[ \Phi \left( M_\Phi^2 + DD \right) \Phi + 2\Phi J \right] \Rightarrow \mathcal{L}_{\Phi}^{\text{eff}} = -\frac{z}{M^2} JJ + \frac{z}{M^4} J(DD)J + O(M^{-6})$$

Table 2: Left: Generated processes and cross sections for signal and background for $\sqrt{s} = 1$ TeV, polarization 80% left for electron and 40% right for positron beam. For each process, those final-state flavor combinations are included that correspond to the indicated signal or background subprocess. Right: The expected sensitivity from 1 ab$^{-1}$ $e^+e^-$ sample at 1 TeV, asymmetric 1 sigma errors.

| Process          | Cross Section [fb] | $SU(2)_c$ conserved | $SU(2)_c$ broken |
|------------------|--------------------|--------------------|-----------------|
| $\nu\bar{\nu}, q\bar{q}q\bar{q}$ | $\nu\bar{\nu}, q\bar{q}q\bar{q}$ | $23.19$              | $-1.41$ $1.38$  |
| $\nu\bar{\nu}, g\bar{g}g\bar{g}$ | $\nu\bar{\nu}, g\bar{g}g\bar{g}$ | $7.624$              | $-1.16$ $1.09$  |
| $V \rightarrow VVV$ | $V \rightarrow VVV$ | $9.344$              |                 |
| $W W \rightarrow W W$ | $W W \rightarrow W W$ | $132.3$             | $-2.72$ $2.37$  |
| $Z Z \rightarrow ZZ$ | $Z Z \rightarrow ZZ$ | $2.09$               | $-2.46$ $2.35$  |
| $W W \rightarrow W Z$ | $W W \rightarrow W Z$ | $414.$               |                 |
| $e^{-}e^{-} \rightarrow t\bar{t}$ | $e^{-}e^{-} \rightarrow t\bar{t}$ | $331.768$           | $-3.22$ $3.31$  |
| $q\bar{q}q\bar{q}$ | $q\bar{q}q\bar{q}$ | $3560.108$          | $-3.93$ $5.53$  |
| $e^{-}e^{-} \rightarrow eeW$ | $e^{-}e^{-} \rightarrow eeW$ | $7.624$             | $-2.46$ $2.35$  |
| $e^{-}e^{-} \rightarrow eeZ$ | $e^{-}e^{-} \rightarrow eeZ$ | $134.935$           | $-3.93$ $5.53$  |
| $X$ | $X$ | $1637.405$ | $-5.55$ $4.55$ |

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Figure 2: Mass of the scalar singlet resonance in the isospin-conserving case as a function of $\alpha_5$, with the resonance’s width to mass ratio $f_\sigma$ equal to 1.0 as full, 0.8 as dashed, 0.6 as dot-dashed, and 0.3 as dotted line, respectively. The left vertical line in the plot is the 1 $\sigma$ limit on $\alpha_5$, the right one the 2 $\sigma$ limit.

which leads to an effective Lagrangian with the following anomalous quartic couplings $\alpha_5 = \frac{g^2_\sigma \cdot v^2}{(8M^2_\sigma)}$, $\alpha_7 = 2g_\sigma h_\sigma \cdot v^2/(8M^2_\sigma)$, $\alpha_{10} = 2h^2_\sigma \cdot v^2/(8M^2_\sigma)$. A special case of this would be the SM Higgs with $g_\sigma = 1$ and $h_\sigma = 0$. (Another example for such states would be the light pseudoscalars present in Little Higgs models [5]).

Assuming that this scalar resonance is much heavier than the EW gauge bosons ($M_\sigma \gg M_W, M_Z$) we can neglect mass effects and calculate its width:

$$\Gamma_\sigma = \frac{g^2_\sigma + \frac{1}{2}(g^2_\sigma + 2h^2_\sigma)^2}{16 \pi} \left(\frac{M^3_\sigma}{v^2}\right) + \Gamma(\text{non-}WW, ZZ)$$

For a broad continuum the largest allowed coupling would result in a width that equals the resonance’s mass, $\Gamma \sim M \gg \Gamma(\text{non-}WW, ZZ) \sim 0$. This limiting case translates into bounds for the effective Lagrangian (e.g. in the case of a scalar singlet with no isospin violation):

$$\alpha_5 \leq \frac{4\pi}{3} \left(\frac{v^4}{M^4_\sigma}\right) \approx \frac{0.015}{(M_\sigma \text{ in TeV})^4} \Rightarrow 16\pi^2\alpha_5 \leq \frac{2.42}{(M_\sigma \text{ in TeV})^4}$$

In performing the power counting in a similar manner for other resonances one would naively conclude the following dependence of the anomalous couplings on the resonance masses:

| Type     | $\Gamma \sim g^2M^3$, $\alpha \sim g^2/M^2$ | $\alpha_{\text{max}} \sim 1/M^4$ |
|----------|-----------------------------------------------|----------------------------------|
| Scalar   | $\Gamma \sim g^2M^3$, $\alpha \sim g^2/M^2$ | $\alpha_{\text{max}} \sim 1/M^4$ |
| Vector   | $\Gamma \sim g^2M$, $\alpha \sim g^2/M^2$    | $\alpha_{\text{max}} \sim 1/M^2$ |
| Tensor   | $\Gamma \sim g^2M^3$, $\alpha \sim g^2/M^2$ | $\alpha_{\text{max}} \sim 1/M^4$ |

This naive power counting fails in providing the correct answer (for the technical details see [4]). Here the $1/M^2$ term only renormalizes the kinetic energy (i.e. $v$), and hence is unobservable.

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So for vector resonances, all $\alpha_i \sim 1/M^4$, except for the $\rho$ parameter $\beta_1 \sim \Delta\rho \sim T \sim h^2/M^2$. Of course, if new physics resonances couple with non-negligible parameters to the SM fermions, there will be 4-fermion contact interactions that scale like $J_\mu J^{\mu} \sim 1/M^2$ and constitute effective $T$ and $U$ parameters. Since these are the most constrained cases (and those most investigated in the literature) we focus here on physics where these interactions can be neglected compared to those to the bosonic EW sector. As a remark of caution we mention that there is also the possibility of a coupling of the EW current due to new resonances to the longitudinal EW bosons which also leads to an effective $S$ parameter $J_\mu V^{\mu} \sim 1/M^2$. It induces a mismatch between the measured fermionic and bosonic couplings $g$ [6, 7]. The presence of heavy vector resonances leads to the following effects: for the triple gauge couplings at $O(1/M^2)$ to a renormalization of the $ZWW$ coupling, at $O(1/M^4)$ to shifts in $\Delta g_1^2$, $\Delta\kappa^\gamma$, $\Delta\kappa^Z$, $\lambda^\gamma$, $\lambda^Z$; for the quartic gauge couplings at order $O(1/M^4)$ to shifts in the $\alpha$ parameters that are orthogonal to the scalar case in the $\alpha_4$–$\alpha_5$ space.

## 2 Results and Interpretation

There are two ways to study quartic gauge couplings at the ILC, namely triple boson production and vector boson scattering. Concerning the first case, we consider the processes $e^+e^- \to WWZ/ZZZ$, which depend on the combinations $(\alpha_4 + \alpha_6)$, $(\alpha_5 + \alpha_7)$, $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$, respectively. Polarization populates the longitudinal modes and drastically suppresses the SM background. The simulations for the processes discussed here have been performed with the WHIZARD package [8, 9, 10], which is ideally suited for physics beyond the SM [11].

For the triple boson production we assumed a 1 TeV ILC with 1 ab$^{-1}$ integrated luminosity. The complete six-fermion final states generated with WHIZARD have been piped through the SIMDET fast simulation. As observables we used $M^2_{WW}$, $M^2_{WW}$, and the angle between the incoming electron and the $Z$. We considered the three cases A) unpolarized, B) 80% $e^-_R$, C) 80% $e^-_L$, 60% $e^+$. One has a branching ratio of 32% hadronic decays, for which we used the Durham jet algorithm. The most severe SM background is $t\bar{t} \to 6$ jets being vetoed against by a missing energy variable cut, $E^2_{\text{mis}} + p^2_{\perp,\text{mis}}$. So far, no angular correlations have been used in this analysis yet. The result is shown for the combined WWZ/ZZZ case in the left of Fig. [1]

Vector boson scattering – as the second process where quartic gauge couplings could be measured – has been studied for a 1 TeV ILC with 1 ab$^{-1}$, full six-fermion final states, 80% $e^-_R$ and 60% $e^+_L$ polarization. The contributing channels are mainly $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$, in more detail in the left of Tab. [2]. We performed a binned log-likelihood analysis for all different spin-/isospin combinations listed in Tab. [1]. To interpret the ILC reach as limits

| Spin | $I = 0$ | $I = 1$ | $I = 2$ |
|------|---------|---------|---------|
| 0    | 1.55    |        | 1.95    |
| 1    |         | 2.49    | -       |
| 2    | 3.29    | 4.30    |         |

Table 3: Accessible scale $\Lambda$ in TeV for all possible spin/isospin channels, derived from the analysis of vector-boson scattering at the ILC.

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on resonances, we consider the width to mass ratio, $f = \Gamma / M$, by which we can trade the unknown parameters (i.e. coupling constants) by experimentally accessible resonance parameters like the position and shape of the resonance.

As the simplest example, we show the $SU(2)$ conserving scalar singlet in Fig. 2. Here the relation between the resonance mass, the $\alpha$ parameters and the width-to-mass ratio, $M_\sigma = v ((4\pi f_\sigma) / (3\alpha_5))^{1/4}$, can easily be solved. Extracting limits for resonances with $SU(2)$ breaking or higher isospin gets more and more complicated. The most complex case is the $SU(2)$ broken vector triplet: since the effects from the presence of the vector resonance enter only at $O(1/M^4)$ one has to consider all operators at this order. This includes also magnetic moments of the vector resonances. Assuming also $SU(2)_c$ breaking the system contains too many unknown parameters. The missing information can be gained from the investigation of the triple gauge couplings: we used the covariance matrix from this measurement [12] to find the minimum in the multi-dimensional parameter space for these cases.

ILC has the ability to detect new physics in the EW sector even if it is kinematically out of reach. Our results are summarized in Tab. 3. For the case of a scalar singlet with conserved $SU(2)$ we combined triple boson production and boson scattering, shown on the right of Fig. 1 and Tab. 2. The limits are translated into resonance masses from the 1 $\sigma$ limits on the $\alpha$s. In general, the limit lies in the range from 1 – 6 TeV, getting better the more internal degrees of freedom are contributing (higher spin and isospin). It is important to note that these limits apply for narrow resonances as well as broad continua.

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