Real Differences between OT and CRDT in Correctness and Complexity for Consistency Maintenance in Co-Editors

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OT (Operational Transformation) was invented for supporting real-time co-editors in the late 1980s and has evolved to become a core technique used in today’s working co-editors and adopted in major industrial products. CRDT (Commutative Replicated Data Type) for co-editors was first proposed around 2006, under the name of WOOT (WithOut Operational Transformation). Follow-up CRDT variations are commonly labeled as "post-OT" techniques and have made broad claims of superiority over OT solutions, in terms of correctness, time and space complexity, simplicity, etc. Over one decade later, however, OT remains the choice for building the vast majority of co-editors, whereas CRDT is rarely found in working co-editors. Why?

In prior work, we have revealed that CRDT is like OT in following the same general transformation approach and CRDT is not natively commutative for concurrent operations in co-editors, which helps to clarify what CRDT really is and is not for co-editors. In this article, we reveal OT and CRDT differences in correctness and complexity by dissecting and examining representative OT and CRDT solutions. We explore how different basic approaches — the concurrency-centric approach taken by OT and the content-centric approach taken by CRDT — had resulted in different technical challenges, correctness and complexity issues, and solutions. Moreover, we reveal hidden algorithmic flaws with representative CRDT solutions, and discuss common myths and facts related to correctness, time and space complexity, and simplicity of OT and CRDT.

We present facts and evidences that refute CRDT claimed advantages over OT. We hope the discoveries from this work help clear up common myths and confusions surrounding OT and CRDT, and accelerate progress in co-editing technology for real world applications.

CCS Concepts: • Information Systems → Group and Organization Interfaces; Synchronous Interaction, Theory and Model.

KEYWORDS
Operational Transformation (OT), Commutative Replicated Data Type (CRDT), concurrency control, consistency maintenance, real-time collaborative editing, distributed/Internet/cloud computing technologies and systems, Computer Supported Cooperative Work (CSCW) and social computing.

1 INTRODUCTION
Real-time co-editors allow multiple geographically dispersed people to edit shared documents at the same time and see each other's updates instantly [1,6,13,14,15,16,38,43,54,55,60,72,78]. One major challenge in building such systems is consistency maintenance of documents in the face of concurrent editing, under high communication latency environments like the Internet, and without imposing interaction restrictions on human users [13,54,55].

Operational Transformation (OT) was invented to address this challenge [13,54,61,72] in the late 1980s. OT introduced a framework of transformation algorithms and functions to ensure consistency in the presence of concurrent user activities. The OT framework is grounded in established distributed computing theories and concepts, principally in concurrency and context theories [24,54,66,67,83,84]. Since its inception, the scope of OT research has evolved from the initial focus on consistency maintenance to include a range of key collaboration-enabling capabilities, including group undo [38,44,57,58,66,67], and workspace awareness [1,19,60]. In the past decade, a main impetus to OT research has been to move beyond plain-text co-editing [6,13,
20,38,43,54,55,58,61,70,71,77], and support real-time collaboration in rich-text co-editing in word processors [60,65,68,82], HTML/XML Web document co-editing [10], spreadsheet co-editing [69], 3D model co-editing in digital media design tools [1,2], and file synchronization in cloud storage systems [3]. Recent years have seen OT being widely adopted in industry products as the core technique for consistency maintenance, ranging from battle-tested online collaborative rich-text editors like Google Docs[11], to emerging start-up products, such as Codox Apps\(^3\).

A variety of alternative techniques for consistency maintenance in co-editors had also been explored in the past decades [14,16,18,41,42,72]. One notable class of techniques is CRDT\(^3\) (Commutative Replicated Data Type) for co-editors [4,5,8,25,32,37,39,40,41,45,47,48,79,80,81]. The first CRDT solution for plain-text co-editing appeared around 2006 [40,41], under the name of WOOT (WithOut Operational Transformation). One motivation behind WOOT was to solve the FT (False Tie) puzzle in OT [53,55] (also discussed in detail in Section 3.3 and Section 5 in this article), using a radically different approach from OT. Since then, numerous WOOT revisions (e.g. WOOTO [80], WOOTH [4]) and alternative CRDT solutions (e.g. RGA [45], Logoot [79,81], LogootSplit [5]) have appeared in literature. CRDT has often been labeled as a "post-OT" technique that makes concurrent operations natively commutative, and does the job "without operational transformation" [40,41], and even "without concurrency control" [25]. CRDT solutions have made broad claims of superiority over OT solutions, in terms of correctness, time-space complexity, simplicity, etc. After over one decade, however, CRDT solutions are rarely found in working co-editors or industry co-editing products, and OT solutions remain the choice for building the vast majority of co-editors.

The contradictions between realities and CRDT’s purported advantages have been the source of much confusion and debate in co-editing research and developer communities\(^4\). What is CRDT really to co-editing? What are the real differences between OT and CRDT for co-editors? What are the key factors that may have affected the adoption of and choice between OT and CRDT for co-editors in the real world? We believe that a thorough examination of these questions is relevant not only to researchers exploring the frontiers of collaboration-enabling technologies and systems, but also to practitioners who are seeking viable techniques to build real world collaboration tools and applications.

To seek answers to these questions and beyond, we set out to conduct a comprehensive review and comparative study on representative OT and CRDT solutions and working co-editors based on them, which are available in publications or from publicly accessible open-source project repositories. In this work, we explored what, how, and why OT and CRDT solutions are different and the consequences of their differences from both an algorithmic angle and a system perspective. From this exploration, we made a number of discoveries, some of which are rather surprising. One such discovery is that CRDT is actually the same as OT in following a general transformation approach to achieving consistency in co-editors. Moreover, we have examined major CRDT claims over OT, and provided evidences that refute those claims.

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1 https://www.google.com/docs/about/
2 https://www.codox.io
3 In literature, CRDT can refer to a number of different data types [48]. In this paper, we focus exclusively on CRDT solutions for text co-editors, which we abbreviate as “CRDT” in the rest of the paper, though occasionally we use “CRDT for co-editors” for emphasizing this point and avoiding misinterpretation.
4 We posted an early version of our report on this work at https://arxiv.org/abs/1810.02137, in Octo. 2018, which attracted wide interests and discussions in public blogs (among both academics and practitioners) and private communications (between readers and authors). This link, at https://news.ycombinator.com/item?id=18191867, hosts some representative comments and opinions on various issues addressed in our article. For example, a well-known CRDT researcher commented: "The argument of Sun's paper seems to be that CRDTs have hidden performance costs. Perhaps this is true. This completely misses the main point. OT is complex, the theory is weak, and most OT algorithms have been proven incorrect (…). AFAIK, the only OT algorithm proved correct is TTF, which is actually a CRDT in disguise. In contrast, the logic of CRDTs is simple and obvious. We know exactly why CRDTs converge. ... Disclaimer: I did not read the paper in detail, just skimmed over it." The above comments basically reiterated and further amplified some typical CRDT claims and criticisms against OT, which reconfirms the liveness of these issues, and warrants thorough examination of them. This series of articles reveal facts and evidences that refute all these CRDT claims. Readers may check whether any of the above mentioned points missed and make independent judgement on these issues.
In this work, we focus on OT and CRDT solutions to consistency maintenance in real-time co-editing, as it is the foundation for other co-editing capabilities, like group undo and issues related to non-real-time co-editing, which we plan to cover in future work. We know of no existing work that has made similar attempts.

The topics and bulk of outcomes from this study are comprehensive, complex and diverse, which are of interest and accessible to readers with different needs and backgrounds. To cope with the complexity and diversity of topics and readership and take into account of feedback to a prior version of our report on this work (see footnote Error! Bookmark not defined.), we have organized the materials into three parts and presented them in a series of three independent and complementary articles.

1. **Real differences between OT and CRDT under a general transformation framework for consistency maintenance in co-editors** [73]. In this article, we review the basic ideas of OT and CRDT and present a general transformation framework for consistency maintenance in co-editors. Furthermore, we reveal that CRDT is like OT in following the general transformation approach, and is not natively commutative for concurrent operations in co-editors. Uncovering the hidden transformation nature and demystifying the commutativity property of CRDT provides much-needed clarity about what CRDT really is and is not to co-editing, which in turn brings out the real differences between OT and CRDT for co-editors. Materials in this article are presented at high levels and require no in-depth co-editing technical background from readers.

2. **Real differences between OT and CRDT in correctness and complexity for consistency maintenance in co-editors** (this article). We dissect and examine representative OT and CRDT solutions, and explore how different basic approaches – the concurrency-centric approach taken by OT versus the content-centric approach taken by CRDT – had resulted in different technical challenges and consequential correctness and complexity issues. Moreover, we reveal hidden algorithmic flaws with some representative CRDT solutions, and discuss common myths and facts related to correctness, time and space complexity, and simplicity of OT and CRDT solutions. Materials in Part II are technical in nature, but a large part of them are described at high levels and should be understandable by people with general knowledge of OT and CRDT literature. However, in-depth understanding of the technical contents in this article require advanced co-editing technical background from readers.

3. **Real differences between OT and CRDT in building co-editing systems and real world applications** [74]. In this article, we examine the role of building co-editing systems in OT and CRDT research, and the consequential differences in the adoption and choice between OT and CRDT in real world co-editors. In particular, we review the evolution of co-editors from research vehicles to real world applications, and discuss representative OT-based co-editors and alternative approaches in industry products and open source projects. Moreover, we evaluate CRDT-based co-editors in relation to published CRDT solutions, and clarify myths surrounding system implementation and “peer-to-peer” co-editing. Materials in this article should be understandable by people with general knowledge in co-editing, and of particular interest to practitioners seeking viable techniques for building real world applications.

For improving readability and self-containment, we include the same introduction and references in three articles. At the end of each article, the conclusion section summarizes main results and contributions covered in that individual article. In the third article [74], the conclusion gives a summary of major results and contributions presented in the series of three articles.

### 2 CONCURRENCY-CENTRIC VS CONTENT-CENTRIC APPROACHES

In [73], we have revealed that CRDT is like OT in following the same general transformation approach (albeit indirectly), and is not natively commutative for concurrent operations in co-editors. Uncovering the hidden transformation nature and demystifying the commutativity property of CRDT provides much-needed clarity about what CRDT really is and is not to co-editing, which in turn brings out what CRDT is really different from OT.
From dissecting and examining representative OT and CRDT solutions, we have drawn some key insights on their fundamental differences in realizing the general transformation. One insight is the direct vs indirect transformation approaches (which is covered in [73]), and another insight is the concurrency-centric vs content-centric realization approaches, taken by OT and CRDT, respectively. The OT approach is concurrency-centric in the sense that an OT solution treats generic concurrency issues among operations with the first priority at its core by means of control algorithms (Section 3.1), and separately handles application-specific data and operation modelling issues within transformation functions (Section 3.2). In contrast, the CRDT approach is content-centric in the sense that a CRDT solution is designed around manipulating special contents, including an object sequence, identifiers, and schemes for searching and applying identifier-based operations in the object sequence, which are directly related to application data and operation models (Section 4). In effect, CRDT solutions treat application-specific data and operation issues with the first priority, but mix concurrency issues within object search and manipulation schemes.

In this article, we elaborate the concurrency-centric versus concurrency-centric approaches, taken by OT and CRDT respectively, and explore how the different basic approaches had resulted in different technical challenges and had major impacts on the correctness, complexity and efficiency of OT and CRDT solutions.

3 KEY TECHNICAL ISSUES AND SOLUTIONS WITH OT

3.1 Control Algorithms and the dOPT Puzzle

Control algorithms are at the core of the OT concurrency-centric approach. Designing correct and efficient OT control algorithms used to be a major challenge [54]. Under the first control algorithm – dOPT (distributed OPerational Transformation) [13], two operations could be transformed with each other as long as they had a concurrency relationship, which turned out to be inadequate. This algorithmic flaw, named the dOPT puzzle later, was subtle and had taken a few years for several researchers to independently discover and resolve it [54].

The key to resolving this puzzle is to ensure the two input operations to a transformation function are not only concurrent, but also defined on the same document state, or equivalent contexts [55]. With the guarantee of the context-equivalence condition, a transformation function can compare parameters of input operations to derive their concurrency-impact on each other. Detecting and resolving the dOPT puzzle has led to the invention of multiple OT control algorithms capable of ensuring the context-equivalence condition, and the establishment of the theory of operation context [54,66,67], which become a cornerstone of OT correctness. For a comprehensive review of independent solutions to the dOPT puzzle, the reader is referred to [54].

Some OT control algorithms, including adOPTed [43], GOT [53,55], GOTO [54], and SOCT2 [51], had the quadratic time complexity in transforming a remote operation – O(c^2), where c is the number of concurrent operations involved in transforming an operation. Though this theoretic complexity was not a concern in real-time co-editors because typically c \( \leq 10 \) [61], more efficient control algorithms were proposed (e.g. Jupiter [36], NICE [49], TIBOT [26,84], SOCT4 [77], Google Wave and Docs OT [11,34,78], COT [66,67], and POT [84]), with a time complexity of \( O(c) \) for transforming a remote operation. For processing local operations, OT solutions generally have the constant time complexity \( O(1) \).^6

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5 In this paper, we focus exclusively on OT solutions that separate generic control algorithms from application-specific transformation functions [1-3,6,10,11,13,15,20,26,31,34,36,38,43,44,49-58,60,65-72,76-78,82-84], as they represent the majority and mainstream OT solutions, on which existing OT-based co-editors are built. In literature, however, there are other OT solutions (e.g. [27-30,46]), in which control procedures are not generic but dependent on specific types of operation and data, and transformation procedures may examine concurrency relationships among other operations as well. In those OT solutions, "control procedure and transformation functions are not separated as in previous works – instead, they work synergistically in ensuring correctness" [30], and different correctness criteria were used as well [27-30,46,61].

6 The local operation processing time covers the period during which the local operation is timestamped and saved in the buffer (ready for propagation), as sketched in OT LOH in Table 1 [73]. For most OT solutions, a local operation in the buffer is propagated as-is without further processing. For a few OT solutions, notably Google Wave and Docs OT, TIBOT, and SOCT4, local operations may wait in the buffer until certain conditions are met, e.g., a local operation is not propagated
For recording concurrency-impact, an OT solution keeps a buffer of operations. The space complexity of this buffer is characterized by two variables \( c \) and \( m \), where \( c \) is the same as above, and \( m \) is the number of users in a session, and depends on whether using scalars or vectors for timestamping operations, and whether maintaining single or multiple transformation paths in the buffer, as summarized in Table 1.

Due to its concurrency nature, the variable \( c \) has two properties: (1) \( c \) is often bounded by a small value, e.g. \( c \leq 10 \), in real-time co-editing sessions, in which the number \( (m) \) of users is small (e.g. \( m \leq 5 \)), and the number of operations each user may generate per second is small (e.g. \( \leq 2 \)) due to the relative slow pace of human interactions and operation composing schemes commonly used in real-time co-editors [61,78]; (2) \( c \) can be reduced to zero by applying garbage collection schemes that remove buffered operations whenever it is no longer possible for them to be concurrent with future operations [55,61,67,84]. Garbage collecting non-concurrent operations has been well-established not only in theory, but also commonly adopted in OT-based co-editors, including Google Wave and Docs [11,78], CoWord [60], Codox Apps, etc.

|                 | Single-T-Path | Multi-T-Path |
|-----------------|---------------|--------------|
| Scalar Timestamps | \( O(c)[11,26,36,49,77,78] \) | \( O(c-m)[84] \) |
| Vector Timestamps | \( O(c+m)[13,51,53,54,55] \) | \( O(c-m^2)[43,67] \) |

### 3.2 Transformation Functions and the False-Tie Puzzle

For an OT solution to work, transformation functions need to preserve some transformation properties under certain conditions (e.g. what control algorithms are used in the OT solution).

**Convergence Property 1 (CP1):** Given \( O_a \) and \( O_b \) defined on the document state \( DS \), and a transformation function \( T \), if \( O_a' = T(O_a, O_b) \), and \( O_b' = T(O_b, O_a) \), the following holds:

\[
DS \circ O_a \circ O_b' = DS \circ O_b \circ O_a',
\]

which means applying \( O_a \) and \( O_b' \) in sequence on \( DS \) produces the same state as applying \( O_b \) and \( O_a' \) in sequence on \( DS \).

Preserving CP1 is the key for OT to make concurrent editing operations commutative. Numerous transformation functions capable of preserving CP1 (under text and other document models) have been designed [1,2,3,43,69,70,71]. Further research found that CP1 alone may not be sufficient to ensure convergence (for OT solutions supporting more than 2 users); an additional property CP2 may be required under certain conditions (more elaboration on those conditions in Section 3.3 and Section 5) [43,67,84].

**Convergence Property 2 (CP2):** Given \( O_a, O_b \), and \( O_c \), defined on the same state, and a transformation function \( T \), if \( O_a' = T(O_a, O_b) \), and \( O_b' = T(O_b, O_c) \), the following holds:

\[
T(T(O_a, O_b), O_c) = T(T(O_a, O_c), O_b'),
\]

which means transforming \( O_a \) against \( O_b \) and then \( O_b' \) produces the same operation as transforming \( O_a \) against \( O_c \) and then \( O_b' \).

It is worth highlighting that the transformation function \( T \), document state \( DS \), and operation \( O \) are all unspecified in CP1 and CP2 specifications, meaning CP1 and CP2 are applicable to transformation functions defined for any document states and operation models. *Establishing general transformation conditions and properties for correctness, including CP1 and CP2, is one of major achievements in past OT research* [13,38,43,54,55,61,66,67,70,71,84].

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until the prior propagated local operation has been acknowledged (for Google Wave and Docs OT); a local operation is not propagated until all remote operations with time-interval-based timestamps (for TIBOT) or global sequence numbers (for SOCT4) that are earlier than that of the waiting local operation have been received and processed. While waiting in the buffer, a local operation may be transformed with incoming remote operations, and such processing time is part of handling a remote operation by ROH (with the complexity \( O(c) \)). Handling a local operation is completed at the moment when an OT solution is able to handle another local operation (by LOH) or remote operation (by ROH).
Unlike CP1, which is relatively easy to preserve, CP2 is non-trivial to preserve and could be violated under certain circumstances. The first CP2-violation case in text editing was reported in [53], later named the False-Tie (FT) puzzle as it always involves an insert-insert-tie that does not exist in original user-generated operations but occurs only between transformed operations [61]. It turned out that detecting and resolving the FT puzzle and CP2-violation in general had enormous impact on follow-up development of OT (with respect to CP2-correctness), the invention of WOOT, and other OT alternatives as well (see more discussions in Sections 4 and 5) [21,40,41,42].

3.3 Solving the CP2 Issue and the FT Puzzle under the OT Approach

One fundamental point we should highlight at the start is that CP2 is not unconditionally required, but required for transformation functions only if the control algorithms in the OT solution transform concurrent operations in arbitrary orders, or more precisely, transform the same pair of concurrent operations in different contexts [43,61,66,67,84]. This CP2 precondition has played a key role in the invention and verification of OT solutions to the CP2 issue along two alternative paths.

First, the CP2 issue can be addressed by designing OT control algorithms that avoid transforming operations in arbitrary orders (the CP2-avoidance approach, which is generic and independent of operation types and data models). When control algorithms with the CP2-avoidance capability are used in an OT solution, CP2 is no longer a correctness requirement for corresponding transformation functions. Control algorithms capable of avoiding CP2 are numerous [11,26,34,36, 49,50,66,67,77,78,84]. It should be stressed that CP2-avoidance solutions impose no restriction on users’ ability to edit the shared document freely and concurrently, and were designed with a variety of communication structures and protocols, with or without using a server for operation transformation and propagation, and using either vectors or scalars for timestamping operations. It is mistaken to label CP2-avoidance solutions as being unsuitable for peer-to-peer co-editing (e.g. [4]). A comprehensive study of CP2-avoidance control algorithms is available in [83,84].

Second, the CP2 (and FT) issue can also be addressed by designing transformation functions capable of preserving CP2, without changing underlying operation and data models. This CP2-preserving approach is suitable to OT solutions with control algorithms that transform operations in arbitrary orders (e.g. adOPTed [43], GOTO [54], and SOCT2 [51]). Obviously, transformation functions capable of preserving CP2 (and CP1) can also be used in combination of control algorithms with the CP2-avoidance capability. Verified transformation functions capable of preserving CP2 (and CP1) for string-wise data and operation models can be found in [71].

The CP2-avoidance approach is often favored as it is generally applicable to data and operation models for a wide range of different applications beyond plain-text editing. OT solutions for advanced co-editing applications, such as 2D spreadsheets [69], 3D digital media design systems [1,2], and shared workspaces in cloud storage systems [3], have all used a combination of transformation functions for preserving CP1 (plus application-specific combined effects for concurrent operations [1,3,61,70]) and control algorithms for avoiding CP2[67,83,84].

There were other attempts to solve the FT puzzle and the CP2-violation issue [21,22,27,28,29, 30,40,41,42,52], which did not follow above CP2-avoidance or preserving strategies. We will discuss them, together with misconceptions surrounding FT and CP2, in Section 5.

3.4 Summary of Correctness and Complexity of OT Solutions

In over two-decades, numerous OT solutions have been invented to address OT-special technical challenges, including the dOPT puzzle and the FT puzzle, and to support building real world co-editors from text editing to more complex editing applications. The correctness of key OT components, including generic control algorithms and transformation functions for a range of commonly used operation and data models (e.g. string-wise plain-text editing and beyond) [1,3,69, 70,71], has been established under well-defined conditions and properties [13,31,38,43,54,55,58, 61,67,71,84]. State-of-the-art OT solutions have achieved the space complexity $O(c)$ or $O(c^m)$, and the time complexity $O(1)$ and $O(c)$ for processing local and remote operations, respectively.

4 KEY TECHNICAL ISSUES AND SOLUTIONS WITH CRDT

In over one decade, CRDT has evolved from the initial WOOT to a myriad of CRDT solutions for co-editing [4,5,8,25,32,37,39,40,41,45,47,48,79,80,81]. CRDT solutions have been commonly
designed around internal object sequences, object identifiers, identifier-based operations, and schemes for searching and manipulating internal object sequences, which we characterized as the content-centric approach. However, concurrency issues are inherent and unavoidable in unconstrained co-editing (and they are the same to both OT and CRDT), so CRDT solutions have to face and handle concurrency issues in content-specific ways, meaning to mix concurrency solutions within specific object sequences, identifiers, and associated search and manipulation schemes, leading to CRDT-special correctness and complexity problems.

4.1 Big C/C Complexities in CRDT Solutions

A direct consequence of the CRDT content-centric approach is that time and space complexities of CRDT solutions are inherently determined by a big C (for Contents) or C_i (for Contents with tombstones) variable – the number of objects in the internal object sequence. In contrast to c (for concurrency) in OT, C often takes a big value, e.g. $10^3 \leq C \leq 10^6$, for common text document sizes ranging from 1KB to 1MB; C_i is much bigger than C due to tombstones (more elaboration below).

At first glance, CRDT object sequences may appear simple and understandable by anyone with basic knowledge of data structures and algorithms in computer science, without necessarily knowing advanced concurrent and distributed computing techniques. Unfortunately, this is an illusion. While linear object sequences and search algorithms in sequential processes are well-understood in computer science, understanding such data structures and algorithms under real-time concurrent co-editing scenarios is a different matter altogether and turned out to be a big challenge.

4.1.1 WOOT with Time Complexity $O(C^3)$

In WOOT (the first CRDT), for example, to achieve object sequence consistency in the presence of concurrent inserts, designers had to devise a recursive procedure IntegrateIns to search the object sequence in multiple and nested loops in order to determine the correct target location. The resulting IntegrateIns procedure not only had the high time complexity of $O(C^3)$ [40,41], but also was quite intricate to devise (and to understand, and to reason about its correctness) and had taken the designers multiple iterations to get it to work, as reported in [40].

4.1.2 WOOTO and WOOTH with Time Complexity $O(C^2)$

The $O(C^3)$ complexity was obviously too costly for executing a single insert operation (at both the local and remote object sequences) in WOOT, so various improvements were proposed, including WOOTO[80], which used a degree scheme to capture the relative ordering of concurrent object creations and save one round of object sequence search; and WOOTH [4], which used a hash scheme to speed up the search of neighboring objects. Both WOOTO and WOOTH achieved the time complexity $O(C^2)$, which is better than $O(C^3)$, but still not cheap given the big value of C.

4.1.3 RGA with Time Complexity $O(C_i)$ or $O(C)$

RGA (Replicated Growable Array) was another CRDT solution for co-editing [45], which also adopted the tombstone-based approach to representing the internal object sequence, but used a hash scheme, and vector-based total ordering which respects causal-ordering to speed up the search of the target object or location in the object sequence, and achieved the time complexity of $O(C_i)$ (or $O(C)$ with tombstone garbage collection, discussed below) for executing an insert operation, but with additional costs in maintaining the hash table and issues related to vector clocks [40,41,79]. RGA is often quoted as the fastest tombstone-based CRDT solution (e.g. [4]).

4.1.4 Object Sequence Search with Time Complexity $O(C)$ or $O(\log(C))$

The cost of applying an identifier-based operation in the object sequence (e.g. the cost of IntegrateIns) is only one part of handling a local or remote operation in WOOT (and its variations). Another often-occurring cost is searching the object sequence, which has the time complexity $O(C_i)$, and may occur at a number of places, e.g. converting a position-based operation into an identifier-based operation at the local site (using the position as the search key), checking the existence of an object in the sequence for determining the condition for accepting each remote identifier-based operation (using the identifier as the key), and converting an identifier-based operation to a position-based operation for replay at a remote site (counting the number of visible objects in the sequence), etc.
Table 2. Key features of representative CRDT solutions. C, and C represent the number of objects in the internal object sequence with or without tombstones, respectively. Time complexity is for handling one insert operation internally, which incurs at both local and remote sites. G.C. stands for Garbage Collection.

| Object sequence | WOOT [40,41] | WOOTO [80] | WOOTH [4] | RGA [45] | Logoot [79] | Logoot-Undo [81] |
|-----------------|---------------|------------|-----------|----------|-------------|-----------------|
| Identifier-based ops | \(I(id_s, c)\) and \(D(id_t)\); \(c\) is a character | | | | | |
| Object          | \(<id_s, idp, idn, v, c>\) | \(<id_s, deg, v, c>\) | \(<id_s, deg, nl, klink, v, c>\) | \(<id_s, idp, nl, link, klink, v, c>\) | \(<id_s>\) | \(<id_s>\) |
| Identifier (id) | \(<sid, seq>\) | \(<sn, sum, sid, seq>\) | \(<id_t>s<sid>, seq>\) | \(<id_t>s<seq>\) | \(<id_t>s<seq>\) | \(<id_t>s<seq>\) |
| Identifiers (ids) | \(<idp, idc, idn, deg>\) | \(<idp, idc, idn, deg>\) | \(<idc, idp>\) | \(NA\) | \(NA\) | \(NA\) |
| Identifier ordering | without additional constraint | + causal order | + positional order | | | |
| Timestamp | Scalar-based | Vector-based | NA (but require causally ordered broadcast) | | | |
| Time complexity | \(O(C^3)\) | \(O(C^2)\) | \(O(C)\) with G.C. | \(O(C)\) for local operations; | \(O(C)\) for remote operations; | \(O(C)\) for local operations; |
| Space complexity | \(O(C)\) | \(O(C)\) with G.C. | \(O(C)\) with G.C. | | | |

A more recent paper [8] tried to address the CRDT performance problem in the upstream processing (i.e., processing local operations). This paper attributed the problem to the time complexity \(O(C_i)\) (or \(O(C)\)) for the search process in converting a position-based operation into an identifier-based operation. The proposed solution was to add one extra binary tree to speed up the local search process. With this extra tree, together with its additional complication and costs, the time complexity of this search process could be reduced to \(O(\log(C_i))\), which improves the upstream execution time by “several orders of magnitude” as claimed in [8]. For comparison, OT solutions commonly have the complexity \(O(1)\) for the whole process of handling a local operation.

4.1.5 Space Complexity \(O(C)\) or between \(O(C)\) and \(O(C^2)\)

Apart from time complexities, space complexities of CRDT solutions are also characterized by the big \(C\) or \(C\) variable. The internal object sequence has the space complexity \(O(C)\) for CRDT solutions with tombstones (e.g. WOOT variations), or between \(O(C)\) and \(O(C^2)\) for those without maintaining tombstones (e.g. Logoot [79]). In Table 2, we give a summary of objects, identifiers, time and space complexity, and other characteristic features of representative CRDT solutions.

The actual size of each object in the sequence deserves attention too: for each character of one byte in a text document, the corresponding internal object may have a size of 14, 22, or 26 bytes for WOOT variations, 42 bytes for RGA, or a variable size (bounded by \(C\)) for Logoot variations. This means that the space over head of a CRDT object sequence is at least 14 to 42 times larger than the original size of the external document, without counting tombstones.

4.2 Tombstone Overhead Issues in WOOT Variations and RGA

While WOOT variations and RGA are different from each other in concrete representations of internal object sequences and operations, they have one thing in common: the use of tombstones to

\[ ^{7} \text{In [8], the authors acknowledged "upstream execution – and thus responsiveness of CRDT algorithms often performs poorly," but still claimed "downstream execution of CRDT algorithms is more efficient by a factor between 25 to 1000 compared to representative OT algorithms", quoting the evaluation results in [4], where the referred OT algorithm is the TTF solution [42], which is examined in Section 5.2.} \]
represent deleted objects in the object sequence. The very need for tombstones was to support identifier-based search of the object sequence in the face of concurrent operations. The major problem with tombstones is the time and space overhead. Here is a telling example of the unacceptable costs caused by tombstones [79], reported directly by WOOT researchers:

"For the most edited pages of Wikipedia, the tombstone storage overhead can represent several hundred times the document size. Tombstones are also responsible of performance degradation. Indeed, in all published approaches, the execution time of modification integration depends on the whole document size — including tombstones. ... While the 'George W. Bush' page contains only about 553 lines, the number of deletions is about 1.6 million. As a consequence, tombstones-based systems are not well-suited for such documents since we obtain 1.6 million tombstones for only 553 lines."

While the prohibitive tombstone costs ($C_t = C \times 2894$ in this example) quoted above are not necessarily generalizable, the tombstone overhead was indeed a major and inherent issue with WOOT variations and RGA.

Various attempts were made trying to remove tombstones as garbage. In RGA [45], a garbage collection scheme was proposed, which was based on the following conditions\(^8\): a tombstone object can be removed only if: (1) no future operation will be concurrent with the delete operation that converted the object into a tombstone (similar to the garbage collection condition and the vector-clock-based scheme in GOT [55]); and (2) no future operation will have a total order that is earlier than the total order of the object immediately after the tombstone in the object sequence (an additional condition related to RGA’s special way of maintaining and using tombstones). However, WOOT variations have no garbage collection schemes of their own, and also cannot adopt the garbage collection scheme from RGA due to the absence of vector-clocks and different ways of maintaining and using tombstones and neighboring objects in WOOT variations under complex concurrent co-editing scenarios [4,41,80].

4.3 Identifier Length and Space Explosion Issues with Logoot

As the tombstone issue could be a road blocker for WOOT variations to be applicable to real world co-editors, an alternative CRDT solution, named Logoot [79], was proposed to avoid using tombstones in the object sequence, but this CRDT solution imposes a special positional ordering constraint on identifiers: object identifiers must have a total ordering that is consistent with the positional ordering of the corresponding objects in the object sequence.

As shown in Table 2, all CRDT solutions, with or without tombstones in object sequences, have imposed a total ordering on identifiers for various purposes, e.g. to determine the location of an insert operation when multiple concurrent operations are inserting at the same location in the object sequence (for WOOT variations and RGA). The positional ordering constraint for Logoot (and its variations) identifiers is quite special, which is to determine the location of all operations (being insert or delete, concurrent or sequential) in the object sequence (at remote sites) — the key to Logoot’s correctness and complexity, and hence worth further elaboration.

For any two objects A and B in the object sequence, with identifiers ida and idb, respectively, if A is positioned before B in the object sequence, ida must be ordered before idb in the identifier ordering as well. This property is essential for searching target objects or locations in the object sequence, without the help of tombstones, in the face of concurrent editing. For Logoot to work, the key is to devise identifiers that really possess uniqueness, immutability and positional ordering properties, which turned out to be (and remains) a major challenge.

The basic idea in the Logoot identifier scheme is to assign each object with an integer in a number system of a chosen base (e.g. $2^{64}$) according to the object’s position in the object sequence. For any two objects A and B, if A precedes B in the object sequence, the integer assigned to A must be smaller than that to B. To insert a new object X between two existing objects A and B with integers $p$ and $q$ in their identifiers, respectively, the identifier scheme will assign object X with an integer $i$, which is randomly taken between $p$ and $q$, such that $p < i < q$.

---

\(^8\) It is noteworthy that the key for RGA to improve WOOT and its variations (in time complexity and garbage collection) was due to the use of vector clocks and causal-ordering, which were also adopted in some OT solutions, but dismissed by WOOT (and other CRDT proposals) on the ground of being non-scalable and unsuitable for peer-to-peer co-editing [74].
One issue with this basic scheme stems from the nature of concurrent editing: what if two users are inserting concurrently between the same pair of objects and the identifier schemes at the two sites generate the same random integer $i$? To break the concurrent-insert-tie, Logoot coupled the random integer with a site identifier (sid) to make a tuple $<i, sid>$. This extension, however, was still not enough. Another issue with the identifier scheme is related to the limited space between any two integers: what if a user inserts a large number of objects between two existing objects, and the required integers for identifying those objects exceed the available integers between the identifiers of neighboring objects? This led to further extensions of the Logoot identifier from a single tuple to a variable number of tuples, which is bounded by the big $C$, and an additional local operation sequence number ($seq$) was added to maintain uniqueness (see Table 2).

The variable and potentially lengthy identifiers could lead to high space and time costs in the object sequence representation and searching [79]. Average lengths of identifiers depend on editing patterns, and the worst case complexity of Logoot identifiers is $O(C)$ [79,81] – to put this in plain terms: the length of a Logoot identifier to represent a single character is proportional to the length of the document, which led to the space complexity $O(C^2)$ for the object sequence. The time complexity for handling a local insert is $O(C)$, but $O(C \cdot \log(C))$ for a remote insert [4,79,81].

The space and time costs in Logoot identifiers turned out to undermine what Logoot was originally proposed to achieve (i.e. to avoid tombstone-related costs in WOOT variations). To get the identifier length and object sequence space explosion issues under control, various patches were added to the Logoot identifier scheme. Those patches, unfortunately, led to increasingly complicated schemes, which can generate identifiers violating required properties, and manifest as inconsistencies in document states, as revealed in the following section.

### 4.4 Multiple Unsolved Correctness Issues in Logoot

In this section, we present multiple correctness issues in Logoot that we discovered as part of this investigation. In Fig. 1, we use one scenario with three operations to illustrate these issues. For convenience, we introduce the following notations: ES for External State (visible to users), EO for (position-based) External Operation (generated from ES by users), and IS for Internal State (the object sequence manipulated by Logoot).

In the following examples, we use the identifier generation scheme described in [79,81] (both versions are basically the same). For simplicity, we use 10 as the integer base for identifiers, MAX as a symbolic value larger than the biggest integer 9 in the base, $<0, NA, NA>$ and $<MAX, NA, NA>$, which were also used in [81], as two special identifiers marking the start and end of the object sequence. Initially (the portion above the first horizontal dashed line in Fig. 1), the external state contains two characters "XY", which correspond to the two internal objects with identifiers $<1,1,1>$ and $<3,1,2>$, respectively. In each tuple of an identifier, the first number is an integer, the second number is the site identifier, and the third number is the sequence number.

#### 4.4.1 Concurrent-Insert-Interleaving Puzzle

Among all issues with the Logoot identifier scheme, the first critical one is the random interleaving anomaly, which could occur whenever two users concurrently insert continuous characters between the same pair of existing characters. As shown in Fig. 1, User A generates $EO_1 = I(1, "ab")$ to insert two characters "ab" between "X" and "Y"; and concurrently User B generates $EO_2 = I(1, "AB")$ to insert two characters "AB" between "X" and "Y". Internally, Logoot at the User A site first uses the position 1 in $EO_1$ to find the insert location and obtain two neighboring identifiers $<1,1,1>$ and $<3,1,2>$ in IS$_A$. Since there exists only one available integer between 1 and 3 (in the first tuples of the two neighboring identifiers), a new tuple has to be added to generate two new identifiers for the two inserted characters. According to the identifier generation scheme in [79,81], a range of legitimate identifiers (shown in Box A in Fig. 1) are available for random choices. Without losing generality, we choose two identifiers $<1,1,1><9,1,3>$ and $<2,1,4><8,1,4>$ from this range to identify the two inserted characters. Then, Logoot inserts these two identifiers at the location 1 in the object sequence. Afterwards, two identifier-based operations are propagated to User B. At the User B site, a similar process occurs, two identifiers $<1,1,1><9,2,1>$ and $<2,2,2><0,2,2>$ are generated.
(from Box B in Fig. 1) to identify the two inserted characters, and two identifier-based operations are propagated to User A.

After receiving identifier-based operations from a remote site, Logoot uses the identifiers in remote operations to determine the insert locations in the object sequence. After applying the identifier-based concurrent operations at each other sites, the combined final (both internal and external) states are consistent, but the four characters and their corresponding internal identifiers are *interleaved*: the external document state becomes "XaAbBY", rather than "XabABY" (or "XABabY"), which would normally be expected by users.

It should be highlighted that the four identifiers are *legitimate* identifiers (according to [79, 81]), and meet the *uniqueness, immutability* and *position-ordering* properties required by Logoot, but their orders are *randomly interleaved*. This anomaly is rooted in Logoot's fundamental random positional identifier generation scheme.

![Diagram of multiple correctness issues in Logoot](image)

**Fig. 1.** Illustration of multiple correctness issues, including concurrent-insert-interleaving, inconsistent-position-integer-ordering (and infinite loop flaw), and position-order-violation, in Logoot.

### 4.4.2 Inconsistent-Position-Integer-Ordering Puzzle

Before the second horizontal dashed line in Fig. 1, the concurrent-insert-interleaving abnormality has manifested itself, but there is another hidden problem inside the internal states (both IS$_{12}$ and IS$_{22}$): the two adjacent identifiers $id_1 = <2,1.4><8,1.4>$ and $id_2 = <2,2.2><0,2.2>$ have an inconsistent-position-integer-ordering problem, meaning their positional order ($id_1 < id_2$) is inconsistent with their integer order ($28 > 20$). This hidden problem would manifest as an infinite loop in the Logoot identifier generation scheme when any user inserts a character between these two adjacent objects. For example, when User A generates $EO_3 = I(4, "cd")$ to insert two characters
"cd" at position 4, i.e. between "b" and "B" in the external state ES₁₂, as shown in Fig. 1, Logoot would run into an infinite loop and fail to generate identifiers.

In general, the Logoot identifier generation scheme will run into an infinite loop and fail to generate any new identifier between two neighboring identifiers with position integers p (for left identifier) and q (for right identifier) if p ≥ q (in other words, their position and integer orders are inconsistent), which could occur when two users are inserting at the same location concurrently.

The infinite loop flaw was in the first Logoot paper [79], remained the same in a late version [81], and never corrected in late CRDT publications. However, we found several patches introduced to avoid the infinite loop in open source codes⁹,¹⁰, which implemented the Logoot identifier scheme. Nevertheless, none of those patches could really solve the problem without running into new problems, which are illustrated in the next subsection.

4.4.3 Position-Order-Violation Puzzle
We use the patch in the Logoot library (referred to in footnote 9) implemented by Logoot authors [8], to illustrate the position-order-violation puzzle, which is caused by the patches introduced to resolve the infinite loop flaw, as shown under the second horizontal dashed line in Fig. 1.

When User A generates EO₃ = I(4, "cd") to insert two characters "cd" between "b" and "B", the Logoot library would avoid the infinite loop by changing the right neighbor identifier from <2,2,2><0,2,2> into <3,2,2><0,2,2> (only inside the identifier generation algorithm, not in the internal state), to allow the use of a range of illegitimate identifiers from <2,1,4><8,1,4><1,1,9> to <2,1,4><0,2,2><9,1,9>, shown in Box C in Fig. 1. However, this patch could cause numerous abnormal cases that cannot be handled by the original identifier generation scheme (e.g. the ConstructId function in [81]), which in turn requires additional patches to deal with. For example, the tuple <0,2,2> in identifiers <2,1,4><0,2,2><*₁,*₉> is inherited from the corresponding tuple in the right neighboring identifier <3,2,2><0,2,2>, and this inheritance is forced by one patch in the Logoot library¹¹. Unfortunately, the position-order-violation problem manifests itself among these illegitimate identifiers, e.g. between the two identifiers <2,1,4><8,1,4><9,1,5> and <2,1,4><0,2,2><4,1,6> (and among other pairs as well) in this range. Trouble would occur when these two identifiers are assigned to the two new objects for the two characters "cd".

This position-order-violation does not immediately cause a trouble at the local site as the local insertion position is determined by the position number 4, rather than by these identifiers; the trouble occurs when Logoot at the remote User B site uses these identifiers to determine their positions in IS₂₃, and inserts "c" and "d" at corresponding positions, which results in inconsistent internal states, i.e., IS₁₃ ≠ IS₂₃, which in turn leads to incorrect external (transformed) operation EO₃, which finally results in inconsistent external states, i.e. ES₁₃ ≠ ES₂₃, as shown in Fig. 1.

It should be pointed out that the inconsistent-position-integer-ordering problem (and the associated infinite loop flaw), and the position-order-violation puzzle could occur under numerous circumstances, e.g. the reader can use different identifier combinations in Boxes A, B and C in Fig. 1 to create varieties of similar puzzles.

It remains a critical open issue for Logoot to find a correct identifier scheme. Logoot variations, such as LogootSplit [5] and LogootUndo [81], tried to extend Logoot from supporting character-wise to string-wise operations and from supporting do to undo. Unfortunately, new identification schemes for string operations and undo were even more complicated than that for character-wise operations and do-only Logoot solutions (see Table 2), and their correctness was not verified either.

4.5 General Correctness Issues with CRDT for Co-Editors
In general, the correctness of key components in CRDT solutions, e.g. object identifiers, object sequences, object sequence searching and manipulation schemes, remains to be verified, using

⁹ https://github.com/coast-team/replication-benchmarker.
¹⁰ https://github.com/rudi-c/alchemy-book. It is worth noting that the author of this work also detected various issues in Logoot and pointed out Logoot "missing what I think are key details on how to handle certain edge cases", and devised his own patches to deal with those missing key details. Those patches also came with problems which may result in state inconsistencies or system crashes. Detailed analysis of those issues is beyond the scope of this article.
¹¹ This patch is implemented in functions generateLineIdentifiers and constructIdentifier in https://github.com/coast-team/replication-benchmarker/blob/master/src/main/java/jbenchmarker/logoot/.
well-defined criteria, which are yet to be established as well. In contrast, representative OT solutions have their key components, i.e. control algorithms and transformation functions, theoretically verified under well-established algorithmic correctness criteria (see "the multi-facets of OT correctness" in [61]) – context-based conditions and transformation properties (discussed in Section 3), and experimentally validated in working co-editors based on these solutions (see [74]).

Consistency (e.g. convergence) claims of a technical solution cannot be derived from any theory or assumption that all components of this solution will work according to theoretic requirements. For example, Logoot identifiers are required to possess the positional ordering property – one necessary condition for Logoot to achieve convergence, but the designed Logoot identifier schemes actually violated this required property and led to inconsistency, and even run into infinite loops and failed to generate any identifier, as illustrated in Fig. 1. Claiming "we know exactly why CRDTs converge" (see foot node 4) can neither eliminate those algorithmic flaws, nor guarantee the correctness of any CRDT solution for co-editors.

4.5 Summary of Correctness and Complexity of CRDT Solutions
One motivation of the first CRDT solution (WOOT) was to address the FT puzzle and the CP2-violation issue in OT, which have been solved under the OT approach (see Section 3.3). As an OT-alternative in co-editing, CRDT solutions did its job without using OT algorithms, but with CRDT-special object sequences, identifier-based operations, and schemes for manipulating such sequences and operations, which came with CRDT-special correctness and complexity issues (see time and space complexities of representative CRDT solutions in Table 2). Some CRDT issues (e.g. tombstone overhead for WOOT variations, inconsistent-position-integer-ordering and position-order-violation for Logoot variations, etc.) are open for resolution, but others (e.g. big C/C complexities in time and space for all CRDT solutions, and concurrent-insert-interleaving for Logoot variations, etc.) are inherent to the basic approaches taken by those CRDT solutions, and it is unclear whether they can be solved without fundamental changes to the basic approaches.

5 DISCUSSION ON CORRECTNESS AND COMPLEXITY OF OT AND CRDT
CRDT was proposed to address basically the same consistency maintenance problem in co-editors and to meet the same consistency requirements as OT [73], but OT predated CRDT for over one decade. To justify later proposals, CRDT articles often start by dismissing OT, with messages like: "most OT algorithms have been proved incorrect", "the only OT algorithm proved correct is TTF", and "CRDT algorithms is more efficient by a factor up to 1000 compared to representative OT algorithms" (e.g. see footnotes 4 and 7), by quoting some results from some early publications (e.g. [4,21,42]) without independent validation; then proceed to self-justify yet another CRDT by comparing with prior CRDT solutions only, as if OT had been ruled out (for being incorrect and inefficient) in the "post-OT" era. These fallacious messages had caused major confusions and misled quite some people in the co-editing community, hindering progress in co-editing research and application. To clear up those confusions, we examine major myths and misconceptions surrounding correctness and complexity of OT and CRDT, and explain why the above messages are groundless and false.

5.1 Misconceptions about OT Correctness: the CP2 Syndrome
A large body of CRDT literature has built on an assertion: OT is incorrect. This assertion has served as a kind of justification for any non-OT proposal, with allowance to have high costs (e.g. "…CRDTs have hidden performance costs. Perhaps this is true. This completely misses the main point... " in footnote 4) and even algorithmic flaws (e.g. see Fig. 1). Unfortunately, this assertion is false as it is based on misconceptions about OT correctness, which we examine in this section.

Achieving convergence is one key requirement for OT and any consistency maintenance solution to co-editing. Two transformation properties CP1 and CP2 are directly relevant to achieving convergence in OT solutions [13,54,55]. In [43], one theorem established that CP1 and
CP2\textsuperscript{12} are two \textit{necessary} and \textit{sufficient} conditions to achieve \textit{convergence} under the adOPTed algorithm proposed as a solution to the dOPT puzzle\textsuperscript{13}. This theorem is useful and applicable to \textit{transformation functions} in OT solutions that allow concurrent operations to be transformed in \textit{arbitrary orders} or under \textit{different contexts} \cite{67}. Unfortunately, this theorem has often been misinterpreted—a source of common misconceptions surrounding OT correctness, particularly CP2 correctness, which is collectively called the \textit{CP2 syndrome} below.

\subsection*{5.1.1 Are CP1 and CP2 Necessary and Sufficient Conditions for OT Correctness?}

The top symptom of the CP2 syndrome is to misinterpret CP1 and CP2 as \textit{necessary} and \textit{sufficient} conditions for the \textit{correctness} of transformation functions or even an OT solution as a whole. This had distorted the real meaning and importance of CP1 and CP2, and misled many to treat CP1 and CP2 as two golden rules for evaluating OT correctness, resulting in numerous flawed claims in co-editing literature.

In fact, CP1 and CP2 are \textit{neither} necessary \textit{nor} sufficient for the correctness of transformation functions, let alone for the correctness of a whole OT solution. CP1 and CP2 are \textit{unnecessary} for transformation functions because they can be avoided by using OT control algorithms (e.g. CP2-avoidance algorithms \cite{11,26,34,36,49,50,67,77,78,84}). CP1 and CP2 are \textit{insufficient} because they govern only \textit{convergence}, but not \textit{intention preservation} (e.g. the \textit{combined effects} of concurrent operations in text editing \cite{70,71}) in co-editors \cite{55}.

Without intention preservation, transformation functions can preserve CP1 and CP2 \textit{trivially}. For example, the function \textit{trivial-TF}(O, O_x) = O' transforms O against O_x to produce O', where O' is an operation that always replaces existing contents of the document with a number X. It can be shown that this \textit{trivial-TF} preserves CP1 and CP2. By assigning X to an arbitrary number, one can get an infinite number of transformation functions capable of preserving CP1 and CP2, but none of them is \textit{meaningful}, let alone be \textit{correct} for co-editing.

\subsection*{5.1.2 Is CP2-Violation the Root of All Inconsistency Problems?}

Another symptom of the CP2 syndrome is to attribute the root of every puzzle to CP2-violation as long as the puzzle may result in divergence. In fact, divergence is only a \textit{symptom} of a puzzle and could be \textit{caused} by other factors unrelated to CP2 \cite{61}.

For example, the famous dOPT puzzle had resulted in divergent states, and it was \textit{not} caused by CP2-violation, but by the violation of the \textit{context-equivalence} condition, which was a crucial understanding in the process of resolving the dOPT puzzle \cite{54}. After the dOPT puzzle had been resolved for over a decade, however, we still see publications (e.g. \cite{42,45}) trying to attribute this puzzle to \textit{CP2-violation} and to relate proposed CP2 solutions (e.g. TTF \cite{42}) to the dOPT puzzle.

Accurate attribution of an OT puzzle to the root transformation condition is crucial not only to resolving the puzzle, but also to evaluating the correctness of OT solutions in general. The incorrect attribution of the dOPT puzzle to CP2-violation not only reflects misunderstanding fundamental OT correctness conditions (e.g. context-based conditions \cite{54,67,84}), but also exemplifies a common phenomenon in literature, i.e. to inflate the importance of CP2-related work.

\subsection*{5.1.3 Are Most OT Solutions Incorrect with Respect to CP2?}

Yet another and most common symptom is to dismiss most OT solutions for \textit{not} preserving CP2 \cite{4,5,8,21,40,42,45,81}. Arguments along this line could be traced back to the early history of exploring alternative solutions to the FT puzzle (a case of CP2-violation).

After the discovery of the FT puzzle reported in \cite{53,55}, numerous attempts were made to resolve this puzzle, resulting in a large number of proposals \cite{21,22,27,28,29,30,40,41,42,52}. Though different from each other, those works share some common characteristics. One was to improperly \textit{amplify} the importance of CP2 and CP2-related work (see Sections 5.1.1 and 5.1.2). Another one, seemingly justifiable under the inflated CP2 importance, was to make radical changes to core OT components, e.g. data and operation models (e.g. tombstone-based TTF \cite{42}), or the

\textsuperscript{12} In \cite{43}, CP1 and CP2 are named as TP1 (Transformation Property 1) and TP2 (Transformation Property 1), respectively.

\textsuperscript{13} The adOPTed algorithm is able to resolve the dOPT puzzle due to its capability of ensuring the \textit{context-equivalence} condition \cite{54}, rather than requiring functions to preserve CP1 and CP2. In \cite{43}, however, the \textit{context-equivalence} condition was not explicitly stated but implied in the description of the adOPTed algorithm, whereas CP1 and CP2 were explicitly stated in a theorem, which was often misinterpreted as the reason for the adOPTed algorithm to resolve the dOPT puzzle.
basic structure of separating generic control algorithms and application-specific transformation functions (see footnote 5), which are fundamental to OT, but have little to do with the FT puzzle. Consequently, such changes often brought in new issues, in both correctness and efficiency, which were far more complicated than the original FT problem they were proposed to address.

There were quite some publications claiming to have disproved all prior (by then) OT solutions in terms of CP2-correctness (e.g. using theorem provers\textsuperscript{14} \cite{21,22}), and proposed new solutions that were proven to be CP2-correct (using the same theorem provers or math proofs); but those proposals or verifications were repeatedly found to be flawed later (e.g. see counter-examples reported in \cite{21,27,29,30,52}). Erroneous results and misconceptions generated from those attempts had the effect of creating the illusion that OT was full of puzzles spiraling out of control, which had caused major confusions among practitioners and later researchers entering the field.

5.1.4 Basic Facts about the FT Puzzle and OT Correctness with Respect to CP2

To help clear up these misconceptions, we highlight the following basic facts.

1. Despite a variety of CP2-violation puzzles or counter-examples reported in those \textit{FT-solution-hunting} attempts (discussed in Section 5.1.3), all those reported puzzles were just \textit{variations} of the same FT puzzle, meaning none of them could be counted as a new discovery, or \textit{derivatives} of erroneous solutions proposed to solve the original FT puzzle \cite{61}.

2. All those attempts had been confined to a primitive model for text editing with \textit{character-wise} \textit{insert} and \textit{delete} operations. Based on exhaustive examination of all possible transformation cases under this primitive model, it has been proven that the FT puzzle is the only possible CP2-violation case in OT solutions that support commonly adopted combined-effects for pair-wise concurrent operations in text editing \cite{70}.

3. Last and the most important fact is: all possible CP2-violation cases under a more general \textit{string-wise} operation model\textsuperscript{15} for text editing have been detected and solved by (verified) solutions based on both \textit{CP2-preservation} (applicable to text editing) and \textit{CP2-avoidance} (applicable beyond text editing) strategies (see detailed elaborations in Section 3).

In a nutshell, the notion that "most OT algorithms have been proved incorrect" (with respect to CP2 or as a whole) is plainly groundless and false.

5.2 Twin Solutions to CP2-Violation: WOOT and TTF

Among numerous alternative proposals (other than the ones summarized in Section 3.3) to address the CP2-violation issue, a pair of solutions are particularly noteworthy: one is the WOOT solution, and the other is the TTF (Tombstone Transformation Functions) solution, both of which were based on the same idea of \textit{tombstone\textsuperscript{16}-based} object sequences, and proposed at nearly the same time by the same authors \cite{40,41,42}. Basically, WOOT was proposed as an OT alternative \textit{(without} using OT, and also \textit{without} the label of CRDT) capable of avoiding CP2-violation; and the TTF solution was proposed as an OT solution capable of preserving CP2 at the transformation function level.

What made WOOT and TTF special was not so much due to their alternative ways of dealing with the CP2 issue, but their \textit{unjustified} claims and \textit{special} roles in follow-up development: WOOT was late named as the \textit{first} CRDT and further mystified to possess, among other superiorities over OT, a capability of making concurrent operations \textit{natively} commutative \cite{4,5,8,25,32,39,40,41,45,47}.

\textsuperscript{14} Another myth about OT correctness is that verification of CP1 and CP2 is an \textit{exponential} explosion problem, which is too complex and "even impossible" for manual checking, so theorem-provers and model checkers had to be used for this purpose \cite{7,21,22}. As reported in \cite{7}, hundreds of thousands of states had to be checked in order to verify a co-editing scenario with a few operations (e.g. 331,776 states for merely 4 operations). However, the state explosion problem was not inherent to OT verification itself, but caused by the specific verification method in \cite{7}, in which concurrency relationships (taken care of by OT control algorithms) and positional relationships (responsible by transformation functions) among operations were mixed, rather than separated from each other ─ a common pitfall in some OT work. In \cite{70}, different verification methods were used to \textit{exhaustively} cover all possible transformation cases (less than 100 cases) under the same data and operations models as \cite{7}, and those transformation cases can be easily checked even manually.

\textsuperscript{15} In \cite{71}, an additional False-Border (FB) puzzle under a pair of string-wise insert and delete operations was detected and resolved, and it was shown that the FT and FB puzzles were the only two possible CP2-violation cases in OT solutions supporting string-wise operations (with commonly adopted combined effects of concurrent operations) in text co-editors.

\textsuperscript{16} To our knowledge, the AST (Address Space Transformation) solution in \cite{18} was the first to use \textit{marker} (tombstone-like) objects to record deleted characters in co-editors.
5.2.1 TTF as the Sole Correct OT for Comparison with CRDT

Being claimed as the sole correct OT solution, TTF was often used as the OT representative in comparison with CRDT solutions. Quite some claims about CRDT superiority over OT were based on the comparison between CRDT solutions (e.g., Logoot, RGA, etc.) and TTF (typically integrated with the SOCT2 control algorithm [51]). For example, the TTF solution was reported to be outperformed by Logoot and RGA for a factor up to 1000 in [4]. This 1000-times-gain claim was highly remarkable and widely cited as an experimental evidence for CRDT’s performance superiority over OT [4,5,8,45,81] (also reflected in footnote 7).

Validating whether and how Logoot and RGA had actually achieved 1000-time-gain over TTF (+SOCT2) would be interesting, but outside the scope of this paper. What we want to point out here is that those CRDT and TTF claims are groundless and false, because: (1) they are contradicted by the facts that numerous OT solutions have been proven to be correct with respect to well-established conditions and properties (including CP1 and CP2, among others) before and after TTF and WOOT solutions appeared (see Section 3); and (2) they are also mistaken about what TTF really is: TTF is a hybrid of CRDT and OT, as elaborated below.

5.2.1 What is TTF? Really?

In the TTF solution, an internal tombstone-based object sequence is maintained, which is a characteristic CRDT component (like WOOT). In addition, an existing OT control algorithm (e.g., SOCT2 [51]) and specially designed CP2-preserving transformation functions (i.e., TTF [42], defined for a pair of character-wise insert and delete operations on a sequence of objects with tombstones) were used to transform operations, which is similar to OT. One subtle but crucial detail deserves attention: operations being transformed by TTF are not user-generated operations as in typical OT solutions, but internal operations which are defined on and only applicable to the internal object sequence. Consequently, additional conversions between internal and external operations are required, which is typical to CRDT solutions (like WOOT).

Due to its hybrid nature, the TTF solution bears the costs of both CRDT and OT (SOCT2 has the time complexity of $O(c^2)$, with the main costs dominated by its CRDT components, which include the maintenance of the tombstone-based object sequence and associated schemes (each with the time complexity $O(C)$), where $C$ is multiple orders of magnitude larger than $c$ and even $c^2$ for converting between internal and external operations. We refrain from detailed comparison of TTF with OT or CRDT in this paper, but will present comprehensive comparisons of OT, CRDT, TTF, and other alternatives (including those in [18,27,28,29,30]), that are based on the same general transformation approach in a future paper.

5.3 Comparison of OT and CRDT in Time and Space Complexity

The notion that OT is complex is also often used to justify CRDT. In addition to the 1000-times-gain claim over OT (actually TTF), CRDT has been claimed to have general superiority over OT in time and space complexity. For comparison, we have summarized complexities of representative OT and CRDT solutions in Table 3, which clearly disprove CRDT claims in complexity.

We highlight that the real complexity differences between OT and CRDT solutions should be examined not only by theoretic expressions using the big-O notation, but also by practical evaluation of the input variables in those theoretic expressions: $c$ is often bounded by a small value, e.g. $0 \leq c \leq 10$, for real-time sessions with a few users; $C$ is typically orders of magnitude larger

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17 It is worth pointing out that the performance results reported in [4] did not cover WOOT for the reason “it is obviously outperformed by its optimized versions WOOT and WOOTII algorithms”, and did not have a comparison of WOOT with TTF+SOCT2 (used as the OT representative) either. Given the significant differences between $O(C^2)$ (for WOOT) and $O(c^2)$ (for SOCT2) and between $C$ and $c$, it is reasonable to derive that WOOT is obviously outperformed by TTF+SOCT2, and, by deduction, outperformed by Logoot and RAG for a factor of more than 1000 as well, according to [4].

18 It is noteworthy that, after the post of the first version of this article (at arxiv.org, in 2018), a well-known CRDT researcher commented at https://news.vcombinator.com/item?id=18191867: “the only OT algorithm proved correct is TTF, which is actually a CRDT in disguise”, which further amplifies a typical CRDT criticism against OT, i.e. no single OT algorithm proved correct, by reclassifying TTF as a CRDT solution.
than $c$, e.g. $10^3 \leq C \leq 10^6$, for common text document sizes ranging from 1K to 1M characters, while $C_i$ is much larger than $C$ with the inclusion of tombstones. In practice, the following *inequality* commonly holds: $C_i \gg C \gg c$. These practical differences are often more significant than the theoretic differences in real-world real-time co-editing applications.

To reveal the roots of CRDT false superiority claims in complexity, we point out two major *flaws* in CRDT articles in characterizing OT and CRDT time and space complexities below.

| OT | CRDT |
|----------------|----------------|
| Tombstone-based WOOT variations [4,41,80] + RGA [45] | Non-tombstone-based Logoot variations [5,79,81] |
| m (usually $1 < m \leq 5$) is the number of users in a real-time co-editing session. |

Table 3 Space and time complexities of representative (not exhaustive) OT and CRDT solutions.

1. One flaw was to replace the variable $c$ (for OT) – the number of *concurrent* operations in the history buffer, with another variable of very different nature $H$ – the total number of operations executed so far in a co-editing session, which are accumulated in the history buffer and may grow large for long-lasting sessions.

2. Another closely related flaw was to use the same $H$ to replace $C/C_i$ (for CRDT) – the number of objects in the internal object sequence, which must include the objects for representing characters in the *initial* document, but $H$ does not capture the size of initial document contents.

In fact, nearly all CRDT articles ignored the existence and impact of initial document contents in calculating the size of the internal object sequence and analyzing CRDT complexity.

With such replacements, the time complexity $O(c^2)$ or $O(c)$ for OT solutions (in transforming a remote operation) were *distorted* into $O(H^2)$ or $O(H)$; and the time complexity $O(C^2)$, $O(C^3)$, or $O(C)$ for CRDT solutions were *disguised* as $O(H^2)$, $O(H^3)$, or $O(H)$, which, plus some additional twists, were then used to make CRDT superiority claims in complexity [4,5,8, 40, 41, 42, 45, 79, 80, 81].

5.4 Myths and Misconceptions in Simplicity Arguments of CRDT over OT

In conjunction with the notion that OT is *complex*, CRDT is often claimed to be *simple*. This section tackles various versions and arguments on CRDT simplicity, which we have found scattered in published literature [4,8,39,47,48,81] and discussions among developers.

One version of the CRDT simplicity argument can be sketched as follows: CRDT works without OT, thus avoids complex issues with OT, hence CRDT is simple. The fallacies of this argument are: the simplicity of any approach *cannot* be logically argued on the basis of being different from or working without another approach. The relevant questions that should be asked are: *what* special issues one approach brings in, *whether* those particular issues are easy to solve and have been solved, and *whether* solutions (if exist) to such issues are actually simple. In previous sections, we have provided ample evidences that show: CRDT has its own challenging issues and many of them remain unsolved (see Section 4); and CRDT solutions are not simple but more complex than OT solutions, as shown in Table 3.

Another version of the CRDT simplicity is argued along the line how OT and CRDT achieve *commutativity* of concurrent operations in co-editors. CRDT has been broadly defined as a data type or solution that makes “concurrent operations commute with one another” [39,47], but this *commutativity* concept was not new as OT had been known for its very capability of making concurrent editing operations commutative on replicated documents long before CRDT came into being (see [73]). To differentiate CRDT from OT, CRDT inventors formed the following argument:
CRDT makes concurrent operations \textit{natively commutative} (or \textit{by design}), whereas OT makes concurrent operations commutative \textit{after the fact}; hence, CRDT is \textit{simpler} and more \textit{elegant} than OT \cite{47,48} (also see footnote 4). The above CRDT argument and reasoning is \textit{deceptive} because the CRDT \textit{identifier}-based operations are \textit{not} native to editors \cite{73}; CRDT is \textit{not} different from OT in making concurrent \textit{position}-based operations commutative on replicated documents \textit{after the fact} \cite{73}; and the CRDT approach to achieving the same OT commutativity has much higher complexity than OT (see Table 3 and analysis in this article). The notion that CRDT is \textit{elegant} is also contradicted by the fact that representative CRDT solutions are known to be \textit{intricate} (e.g., the core \textit{IntegrateHns} algorithm, with $O(C^3)$ time complexity, in WOOT), and \textit{error-prone} (e.g., multiple algorithmic flaws in the core identifier scheme in Logoot), as we revealed in Section 4.

Still another notion of simplicity is related to implementation: \textit{OT was reported to be hard to implement correctly, so CRDT must be simpler to implement as CRDT is non-OT}. We refrain from discussing fallacies in this notion here as this topic is best examined in the context of implementing co-editors based on OT and CRDT in another article dedicated to this topic \cite{74}.

6 CONCLUSIONS

Based on a comprehensive review and comparison of OT and CRDT for consistency maintenance in real-time co-editing and in building real world co-editors, we have made a number of discoveries, which contribute to the advancement of the state-of-the-art knowledge on collaboration-enabling technology in general, and on OT and CRDT in particular.

In \cite{73}, we have presented a general transformation framework for describing, examining and comparing a variety of concurrency control solutions in co-editing (e.g. OT and CRDT solutions, among others), and revealed previously hidden but critical facts about CRDT: CRDT is like OT in following the same general transformation approach to consistency maintenance in real-time co-editors; CRDT is the \textit{same} as OT in making user-generated operations commutative \textit{after the fact}; and CRDT operations are \textit{not} natively commutative to text editors, but require additional conversions between CRDT internal operations and external editing operations. Revealing these facts helps demystify what CRDT really \textit{is} and \textit{is not} to co-editing, and in turn bring out the \textit{real differences} between OT and CRDT for co-editors — their radically different ways of realizing the same general transformation approach, which is covered in detail in this paper.

One key insight from probing what really differentiates OT and CRDT is: OT is \textit{concurrency-centric} in the sense it treats \textit{generic} concurrency issues among operations as its \textit{first priority} at the core control algorithms, and \textit{isolates} the handling of \textit{application-specific} data and operation modelling issues in transformation functions; whereas CRDT is \textit{content-centric} in the sense that it takes the \textit{first priority} to manipulate internal application-related \textit{contents}, including object sequences and schemes for searching and applying identifier-based operations in the object sequence, but \textit{mixes} the handling of concurrency issues within object search and manipulation schemes. This \textit{concurrency-centric vs content-centric} difference is fundamental and has profound implications to OT and CRDT solutions.

The \textit{first} significant implication is found in the different \textit{design and correctness issues} in OT and CRDT solutions. Key OT design issues include designing control algorithms to deal with \textit{generic} concurrency issues, and designing \textit{separate} transformation functions to handle \textit{application-specific} issues; OT-special challenges and puzzles (all solved), such as ensuring context-based conditions (e.g. the \textit{dOPT} puzzle was a case of violating the \textit{context-equivalence} condition), and avoiding or preserving CP2 (e.g. the \textit{FT} puzzle was a case of violating the \textit{CP2 property}), were derived from and solved under the concurrency-centric approach. The \textit{correctness} of key OT components, including generic control algorithms and transformation functions for a range of commonly used operation and data models (e.g. string-wise plain-text editing and beyond), has been established under well-defined conditions and properties.

In contrast, key CRDT design issues include designing CRDT-special data structures and schemes for representing and manipulating object sequences, searching and executing identifier-based operations in the object sequence, and conversions between internal \textit{identifier-based} operations and external \textit{position-based} operations, which \textit{collectively} deal with both application-
specific and concurrency issues in co-editing. This approach has induced a myriad of CRDT-specific challenges and puzzles, such as tombstone overhead, variable and lengthy identifiers, and the correctness of CRDT key data structures and functional components. In this work, we have detected multiple correctness problems with Logoot: inconsistent-position-integer-ordering and infinite loop flaws, position-order-violation puzzles, and concurrent-insert-interleaving puzzles. It remains an open challenge to resolve these issues under the CRDT approach to co-editing. The claims that CRDT solutions are simple and obvious in correctness are contradicted by the facts that the correctness of key components in various CRDT solutions, e.g. object identifiers and sequences, and object sequence searching and manipulation schemes, remains to be verified, using well-defined criteria, which are yet to be established as well.

The second significant implication is found in the different time and space complexities among OT and CRDT solutions. OT complexity is determined by a variable \( c \) (for concurrency) – the number of concurrent operations involved in transforming an operation; CRDT complexity is dominated by a variable \( C \) (for Contents) or \( C_t \) (for Content with tombstones) – the number of objects maintained in the internal object sequence. In terms of theoretic complexity (see details in Table 3), representative OT solutions have achieved the time complexity \( O(1) \) for processing local operations, and \( O(c) \) or \( O(c^2) \) for processing remote operations; and the space complexity \( O(c) \), \( O(c.m) \), or \( O(c.m^2) \), where \( m \) is the number of real-time co-editing users in a session (usually \( m < 5 \)), under various OT system architectures and protocols. In contrast, representative CRDT solutions have the time complexity ranging from \( O(C_t^3) \), \( O(C_t^2) \), to \( O(C) \) (for tombstone-based solutions), \( O(C \cdot \log(C)) \) (for non-tombstone-based solutions); and the space complexity ranging from \( O(C) \) (for tombstone-based solutions, without tombstone garbage collection) or \( O(C) \) (for tombstone-based solutions, with tombstone garbage collection), to between \( O(C) \) and \( O(C^2) \) (for non-tombstone-based solutions).

In addition to examining the theoretic complexity differences, we highlight the practical differences of the input variables in those complexity expressions: \( c \) is often bounded by a small value, e.g. \( 0 \leq c \leq 10 \), for a real-time session with a few (e.g. less than 5) users; \( C \) is orders of magnitude larger than \( c \), e.g. \( 10^3 \leq C \leq 10^6 \), for common plain text document sizes ranging from 1K to 1M characters, while \( C_t \) could be much larger than \( C \) due to the inclusion of tombstones. In real-time text co-editing, the following inequality commonly holds: \( C \gg C_t \gg c \). It remains an open challenge to devise \( C_t/C \)-based CRDT solutions that are superior over \( c \)-based OT solutions in time and space complexity and in practical performance.

The third implication is in the generality and extendibility of OT and CRDT solutions for co-editors. OT solutions separate generic concurrency issues from application-specific data and operation issues, with the core control algorithms being generally applicable to different application domains beyond text editing. Extensions of existing OT solutions can be and have been achieved by designing new transformation functions for new applications, without reinventing its core control algorithms. In contrast, CRDT solutions mix concurrency issues with application-specific data and operation issues, with key CRDT components being intricately related to each other and coupled with application-specific object sequences and operations. So far, most CRDT solutions for co-editing has been confined to plain-text editing.

In [74], we examine the role of building working co-editors in co-editing research, and its impact in shaping OT and CRDT research and solutions. Moreover, we discuss some myths and facts related to OT and CRDT implementation and “peer-to-peer” co-editing.

We hope discoveries from this work will help clear up common myths and misconceptions surrounding OT and CRDT, inspire new and fruitful explorations of novel collaboration techniques, and accelerate progress in co-editing and collaboration-enabling technology innovation and real world applications.

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REFERENCES

1. Agustina, Liu, F., Xia, S., Shen, H.F. and Sun, C. CoMaya: Incorporating advanced collaboration capabilities into 3D digital media design tools. ACM CSCW (2008), 5-8.

2. Agustina and Sun, C. Dependency-conflict detection in real-time collaborative 3D design systems. ACM CSCW (2013), 715-728.

3. Agustina and Sun, C. Operational transformation for real-time synchronization of shared workspace in cloud storage. ACM GROUP (2016), 61-70.

4. Ahmed-Nacer, Ignat, M. C.-L., Oster, G., Roh, H.-G. and Urso, P. Evaluating CRDTs for real-time document editing. ACM DocEng (2011), 103–112.

5. André, L., Martin, S., Oster, G. and Ignat, C.-L. Supporting adaptable granularity of changes for massive-scale collaborative editing. IEEE CollaborateCom (2013), 50–59.

6. Begole, J., Rosson, M.B. and Shaffer, C.A. Flexible collaboration transparency: supporting worker independence in replicated application-sharing systems. ACM TOCHI 6, 2 (1999), 95 – 132.

7. Boucheneb, B. and Imine, A. On model-checking optimistic replication algorithms. FORTE on Formal Techniques for Distributed Systems (2009), 73 – 89.

8. Briot, L., Urso, P. and Shapiro, M. High responsiveness for group editing CRDTs. ACM GROUP (2016), 51–60.

9. Crowley, C. Data structures for text sequences. Computer Science Department, University of New Mexico, 1996. http://www.cs.unm.edu/~crowley/papers/ads/ads.html

10. Davis, A., Sun, C. and Lu, J. Generalizing operational transformation to the standard general markup language. ACM CSCW (2002), 58-67.

11. Day-Richter, J. What’s different about the new Google Docs: Making collaboration fast. https://drive.googleblog.com /2010/09/whats-different-about-new-google-docs.html

12. Drucker, Peter F. A brief glance at how various text editors manage their textual data. https://ecc-comp.blogspot.com/2015/05/a-brief-glance-at-how-5-text-editors.html

13. Ellis, C. A. and Gibbs, S. J. Concurrency control in groupware systems. ACM SIGMOD (1989), 399–407.

14. Fraser, N. Differential Synchronization. ACM DocEng (2009), 13–20.

15. Gentle, J. ShareJS: Collaborative editing in any app. https://github.com/josephg/ShareJS

16. Greenberg, S. and Marwood, D. Real time groupware as distributed system: concurrency control and its effect on the interface. ACM CSCW (1994), 207 – 217.

17. Grudin, J. Why CSCW applications fail: problems in the design and evaluation of organizational interfaces. ACM CSCW (1988), 85-93.

18. Gu, N., Yang, J. and Zhang, Q. Consistency maintenance based on the mark & retrace technique in groupware systems. ACM GROUP (2005), 264 – 273.

19. Gutwin, C. and Greenberg, S. The effects of workspace awareness support on the usability of real-time distributed groupware. ACM TOCHI, 6(3), 1993, 243-281.

20. Ignat, C. and Norrie, M.C. Customizable collaborative editor relying on treeOPT algorithm. ECSCW (2003), pp. 315 – 334.

21. Imine, A., Molli, P., Oster, G. and Rusinowitch, M. Proving correctness of transformation functions in real-time groupware. ECSCW (2003), 277 – 293.

22. Imine, A., Rusinowitch, M., Oster, G. and Molli, P. Formal design and verification of operational transformation algorithms for copies convergence. Theoretical Computer Science (2006), 351(2):167–183.

23. Laird, Avery. Text Editor:Data Structures. www.averylaird.com/programming/the%20text%20editor/2017/09/30/the-piece-table.

24. Lamport, L. Time, clocks, and the ordering of events in a distributed system. CACM 21, 7 (1978), 558-565.

25. Mihai Letia, M., Preguica, N., Shapiro, M. CRDTs: Consistency without concurrency control. RR-6956, INRIA. 2009.

26. Li, R., Li, D. and Sun, C. A time interval based consistency control algorithm for interactive groupware applications. IEEE IPCADS (2004), 429-436.

27. Li, D. and Li, R. Ensuring content and intention consistency in real-time group editors. IEEE ICDCS (2004), 748–755.

28. Li, R. and Li, D. A landmark-based transformation approach to concurrency control in group editors. ACM GROUP (2005), 284–293.

29. Li, D. and Li, R. An approach to ensuring consistency in Peer-to-Peer real-time group editors. JCSCW 17, 5-6 (2008), 553 – 611.

30. Li, D. and Li, R. An admissibility-based operational transformation framework for collaborative editing systems. JCSCW 19, 1 (2010): 1 – 43.

31. Liu, Y., Xu, Y., Zhang, S. and Sun, C. Formal verification of operational transformation. Proc. of 19th International Symposium on Formal Methods, 2014. LNCS Vol. 8442, 432-448.

32. Lv, X., He, F., Cai, W., and Cheng, Y. A string-wise CRDT algorithm for smart and large-scale collaborative editing systems. Advanced Engineering Informatics (2017), 33: 397 - 409.

33. Koch, M. and Schwabe, G. Interview with Jonathan Grudin on Computer-Supported Cooperative Work and Social Computing. Bus Inf Syst Eng, DOI 10.1007/s12599-015-0377-1. Published online: 03 March 2015.

34. MacFadden, M. The client stop and wait operational transformation control algorithm. Solute Consulting, San Diego, CA, 2013.

35. MacFadden, M., Agustina, Ignat, C., Gu, N. and Sun, C. The fifteenth international workshop on collaborative editing systems. Companion of ACM CSCW (2017) workshop program, 351-354. http://cooffice.ntu.edu.sg/sigce/iwces15/.
36. Nichols, D., Curtis, P., Dixon, M. and Lamping, J. High-latency, low-bandwidth windowing in the Jupiter collaboration system. *ACM UIST* (1995), 111-120.
37. Nicolaescu, P., Jahns, K., Derntl, M. and Klamma, R. Near real-time peer-to-peer shared editing on extensible data types. *ACM GROUP* (2016), 39–49.
38. Prakash, A. and Knister, M. A framework for undoing actions in collaborative systems. *ACM TOCHI* 1, 4 (1994), 295–330.
39. Preguiça, N., Marquès, J. M., Shapiro, M. and Letia, M. A commutative replicated data type for cooperative editing. *IEEE ICDCS* (2009), 395–403.
40. Oster, G., Urso, P., Molli, P. and Imine, A. Real time group editors without operational transformation. Research Report RR-5580, INRIA, May 2005.
41. Oster, G., Urso, P., Molli, P. and Imine, A. Data consistency for p2p collaborative editing. *ACM CSCW* (2006), 259–268.
42. Oster, G., Molli, P., Urso, P. and Imine, A. Tombstone transformation functions for ensuring consistency in collaborative editing systems. *IEEE CollaborateCom* (2006), 1-10.
43. Ressel, M., Ruhlnd, N. and Gunzenhauser, R. An integrating, transformation-oriented approach to concurrency control and undo in group editors. *ACM CSCW* (1996), 288 – 297.
44. Ressel, M. and Gunzenhauser, R. Reducing the problems of group undo. *ACM GROUP* (1999), 131–139.
45. Roh, H.-G., Jeon, M., Kim, J.-S. and Lee, J. Replicated abstract data types: Building blocks for collaborative applications. *JPDC*, 71, 3. (2011), 354–368.
46. Shao, B., Li, D., Lu, T. and Gu, N. An operational transformation based synchronization protocol for web 2.0 applications. *ACM CSCW* (2011), 563 – 572.
47. Shapiro, M. and Preguiça, N. Designing a commutative replicated data type. arXiv:0710.1784v1 [cs.DC] 9 Oct 2007.
48. Shapiro, M., Preguiça, N., Baquero, C. and Zawirsy, M. Conflict-free replicated data types. *SSDS* (2011), 386–400.
49. Shen, H.F. and Sun, C. Flexible notification for collaborative systems. *ACM CSCW* (2002), 77 – 86.
50. Spiewak, D. Understanding and applying operational transformation. www.codecommit.com/blog/java/java/
51. Suleiman, M., Cart, M. and Ferrié, J. Serializaton of concurrent operations in a distributed collaborative environment. *ACM GROUP* (1997), 435 – 445.
52. Suleiman, M., Cart, M. and Ferrié, J. Concurrent operations in a distributed and mobile collaborative environment. *IEEE ICDE* (1998), 36–45.
53. Sun, C., Jia, X., Zhang, Y., Yang, Y., and Chen, D. A generic operation transformation scheme for consistency maintenance in real-time cooperative editing systems. *ACM GROUP* (1997), 425 – 434.
54. Sun, C. and Ellis, C. Operational transformation in real-time group editors: issues, algorithms, and achievements. *ACM CSCW* (1998), 59 – 68.
55. Sun, C., Jia, X., Zhang, Y., Yang, Y., and Chen, D. Achieving convergence, causality-preservation, and intention-preservation in real-time cooperative editing systems. *ACM TOCHI* 5, 1 (1998), 63 – 108.
56. Sun, C. Optional and responsive fine-grain locking in Internet-based collaborative systems.” *IEEE TPDS*, 13, 9 (2002), 994-1008.
57. Sun, C. Undo any operation at any time in group editors. *ACM CSCW* (2000), 191-200.
58. Sun, C. Undo as concurrent inverse in group editors. *ACM TOCHI* 9, 4 (2002), 309 – 361.
59. Sun, C. Consistency maintenance in real-time collaborative editing systems. Talk and demo at Microsoft Research (Redmond, USA) in Feb 2003. Video: http://ciscooffice.ntu.edu.sg/cos/wordsoftes/lEcute.htm.
60. Sun, C., Xia, S., Sun, D., Chen, D., Shen, H. and Cai, W. Transparent adaptation of single-user applications for multi-user real-time collaboration. *ACM TOCHI* 13, 4 (2006), 531 – 582.
61. Sun, C. OTFAQ: operational transformation frequently asked questions. http://ciscooffice.ntu.edu.sg/otfaq.
62. Sun, C. Issues and experiences in designing real-time collaborative editing systems. Tech talk and demo at Google (Mountain View, USA), 17 Nov, 2008. Video: https://www.youtube.com/watch?v=sRz2pXUQHHC.
63. Sun, C. Operational transformation theory and practice: empowering real world collaborative applications. *ACM CSCW* (2011) tutorial. http://cscw2011.org/program/6.html
64. Sun, C., Agustina, and Xu, Y. Exploring operational transformation: from core algorithms to real-world applications. *ACM CSCW* (2011) demo. http://cscw2011.org/program/demos.html
65. Sun, D., Xia, S, Sun, C. and Chen, D. Operational transformation for collaborative word processing. *ACM CSCW* (2004), 437 – 446.
66. Sun, D. and Sun, C. Operation context and context-based operational transformation,” *ACM CSCW* (2006), 279 – 288.
67. Sun, D. and Sun, C. Context-based operational transformation in distributed collaborative editing systems. *IEEE TPDS* 20, 10 (2009), 1457 – 1470.
68. Sun, D., Sun, C., Xia, S. and Shen, HF. Creative conflict resolution in collaborative editing systems. *ACM CSCW* (2012), 1411-1420.
69. Sun, C. Wen, H. and Fan, H. Operational transformation for orthogonal conflict resolution in collaborative two-dimensional document editing systems. *ACM CSCW* (2012), 1391 – 1400.
70. Sun, C., Xu, Y. and Agustina. Exhaustive search of puzzles in operational transformation. *ACM CSCW* (2014), 519-529.
71. Sun, C., Xu, Y. and Agustina. Exhaustive search and resolution of puzzles in OT systems supporting string-wise operations. *ACM CSCW* (2017), 2504 – 2517.
72. Sun, C. Some reflections on collaborative editing research: from academic curiosity to real-world application. *IEEE CSCWD* (2017), New Zealand, 10-17.
73. Sun, C., Sun, D., Agustina, Cai, W. Real differences between OT and CRDT under a general transformation framework for consistency maintenance in co-editors. Submitted to arxiv.org on May 2, 2019.
74. Sun, D., Sun, C., Agustina, Cai, W. Real differences between OT and CRDT in building co-editing systems and real world applications. Submitted to arxiv.org on May 2, 2019.
75. Valdes, R. Text editors: algorithms and architectures, not much theory but a lot of practice. *Dr. Dobb's J.* (1993), 38-43.
76. Valdes, R. The secret sauce behind Google Wave. May 31, 2009. https://blogs.gartner.com/ray_valdes/2009/05/31/the-secret-sauce-behind-google-wave/.
77. Vidot, N., Cart, M., Ferrie, J. and Suleiman, M. Copies convergence in a distributed real-time collaborative environment. *ACM CSCW* (2000), 171 – 180.
78. Wang, D., Mah, A. and Lassen, S. Google wave operational transformation. http://www.waveprotocol.org/whitpapers/operational-transform.
79. Weiss, S., Urso, P. and Molli, P. Logoot: A scalable optimistic replication algorithm for collaborative editing on p2p networks. *IEEE ICDCS* (2009), 404–412.
80. Weiss, S., Urso, P. and Molli, P. Wooki: a p2p wiki-based collaborative writing tool. *WISE* (2007), 503–512.
81. Weiss, S., Urso, P. and Molli, P. Logoot-undo: Distributed collaborative editing system on p2p networks. *IEEE TPDC* 21, 8 (2010), 1162–1174.
82. Xia, S., Sun, D., Sun, C., Shen, H.F. and Chen, D.: Leveraging single-user applications for multi-user collaboration: the CoWord approach, *ACM CSCW* (2004). 162–171.
83. Xu, Y., Sun, C. and Li, M. Achieving convergence in operational transformation: conditions, mechanisms, and systems. *ACM CSCW* (2014), 505-518.
84. Xu, Y. and Sun, C. Conditions and patterns for achieving convergence in OT-based co-editors. *IEEE TPDC* 27, 3 (2016), 695-709.