Entanglement-assisted Orientation in Space

ˇCaslav Brukner,1,2 Nikola Paunkovi´c,3 Terry Rudolph,4 and Vlatko Vedral5,1

1Institut f¨ur Experimentalphysik, Universit¨at Wien, Boltzmanngasse 5, A–1090 Wien, Austria
2Institut f¨ur Quantenoptik und Quanteninformation, ¨Osterreichische Akademie der Wissenschaften, Boltzmanngasse 3, A–1090 Wien, Austria
3Fondazione I.S.I., Villa Gualino, Viale Settmario Severo 65, 10133 Torino, Italy
4Department of Physics, Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2BW, UK
5The School of Physics and Astronomy, University of Leeds, Leeds, LS2 9JT, United Kingdom

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We demonstrate that quantum entanglement can help separate individuals in making decisions if their goal is to find each other in the absence of any communication between them. We derive a Bell-like inequality that the efficiency of every classical solution for our problem has to obey, and demonstrate its violation by the quantum efficiency. This proves that no classical strategy can be more efficient than the quantum one.

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Quantum entanglement is a phenomenon in which two or more quantum systems have to be described with reference to each other regardless of their spatial separation [1]. It leads to correlations that are inconsistent with local realism [2], as demonstrated by violation of Bell’s inequalities [3]. Although entanglement does not involve information transfer, it surprisingly can produce effects over arbitrary distances as if information had been transferred. It can substitute or even eliminate any need of communication that is classically necessary for achieving a goal of common interest of separated parties [4, 5, 6]. Entanglement can thus reduce the communication complexity of certain problems; the key ingredient for this is violation of local realism (quantum non-locality) [7].

Here we show that entanglement can help individuals in making decisions if their goal is to find each other, even if there is no communication between them. This gives the problem properties of “pseudo-telepathy” [8]. Ref. [9] suggests a similar mechanism to explain cooperation of insects and demonstrates an advantage of the use of entanglement over some classical strategies. To provably demonstrate that no classical solution of our problem exists that can achieve the efficiency of the entanglement-based quantum solution, we derive a Bell-type inequality that every classical efficiency has to obey and demonstrate its violation by the quantum efficiency.

In a broader context we show that quantum non-locality - one of the most peculiar features of quantum physics - can be used to solve everyday if somewhat unusual problems, as similarly suggested by [10, 11].

We now define the problem in detail. We let two partners be on the two poles of the Earth, Alice on the North pole and Bob on the South pole. From either of the poles there are three paths (red 1, yellow 2 and blue 3) and for each path there are two directions (+ and −) (right, view from the North pole). Which path and direction should the partners take to find each other at the equatorial line in the lack of any communication? (see text for details)

FIG. 1: Two partners are on the two poles of the Earth (left). From each pole there are three paths (red 1, yellow 2 and blue 3) and for each path there are two directions (+ and −) (right, view from the North pole). Which path and direction should the partners take to find each other at the equatorial line in the lack of any communication? (see text for details)

With the aim to find each other, each partner chooses a path and direction and follows it along the great circle of the globe until she/he reaches the equatorial line. Arriving there the partners certainly meet if they have taken the same path and the same direction (either {+, +} or {−, −}). Alternatively, they may miss each other if they have taken either different paths or different directions of the same path. In the latter case the partners will end at two opposite points of the equator. In the former case they will end at two points that are separated either under angle of 60°, in the case they have taken different directions (either {+, −} or {−, +}), or under angle 120°, if they have taken the same directions (either {+, +} or {−, −}). We will assume that both partners have a far-reaching view with an opening angle of 60° so that even if they took different paths but opposite directions they
can see and subsequently find each other on the equator.

We assume that the partners can not communicate nor that they have agreed in advance which path and direction to take in order to find each other. Furthermore, we assume that the choices of paths are completely independent, random and given with equal probabilities [13]. However, in choosing the directions Alice and Bob are allowed to use some previously shared classically correlated random strings or quantum entanglement. We therefore consider a situation in which Alice and Bob share no information whatsoever about which path each other’s partner intends to take [14].

In order to achieve their goal with the highest possible efficiency Alice and Bob should maximize both the probability to take the same directions if they choose the same path and the probability to take opposite directions if they choose different paths. The overall probability of success is given by

$$P = \frac{1}{9} \left( \sum_{i=1}^{3} P_{ii} \text{(same)} + \sum_{i \neq j=1}^{3} P_{ij} \text{(opp)} \right), \quad (1)$$

where, e.g., $P_{ij} \text{(opp)}$ is the probability that the partners take opposite directions if Alice chooses path $i$ and Bob $j$. The factor 1/9 is due to the assumption that every possible combination of the two paths taken by Alice and Bob is equally probable. We will now give the quantum solution of the problem. It is based on an example of quantum non-locality presented by Mermin [12].

Suppose that the partners share a maximally entangled pair of photons in the state $|\phi^\pm\rangle = 1/\sqrt{2}(|H\rangle|H\rangle + |V\rangle|V\rangle)$, where $|H\rangle$ denotes horizontal polarization and $|V\rangle$ vertical polarization of a photon. Every partner chooses a path at random from the set $\{1, 2, 3\}$, independently of each other. The choice of paths determines a choice of directions of polarization measurements, given by the angles $\{0^\circ, 120^\circ, -120^\circ\}$: if the taken path is 1, the measurement angle is $0^\circ$, while in case of paths 2 and 3, measurement angles are $120^\circ$ and $-120^\circ$, respectively. Polarization measurements are performed on each partner’s photons. If the outcome is $H$, the partner takes direction +, and if it is $V$ she/he takes direction −. In 3/9 fraction of cases the partners choose the same measurement directions and obtain the same outcomes with certainty. In the remaining 6/9 fraction of cases they choose different measurement directions, in which case the probability of them obtaining opposite results is 3/4. Thus, one has $P = 5/6 \approx 83\%$ for the success rate in the quantum protocol.

In a classical protocol, instead of an entangled pair of photons, Alice and Bob can use some previously distributed classically correlated random strings of variables which may improve the success of the protocol. On the basis of these strings they could make their decision which directions (+ or −) to choose, but the choice of the path (red, yellow, or blue) is, as already mentioned, assumed not to be dependent on the strings.

The best classical strategy is successful in $7/9 \approx 78\%$ of the cases. This can be achieved in a model where Alice and Bob share classically correlated variables and in which each variable carries an internal code, determining whether the outcome $H$ or $V$ will emerge, for each of the three possible choices of paths 1, 2 and 3. Thus, the set of all possible codes for both variables given to Alice and Bob is: $\{(HHH), (VVV), (HVH), (VHH), (HVV), (VHV), (VHV)\}$. Here the position in each code plays the role of the measurement angle, while the corresponding value ($H$ or $V$) at that position corresponds to a measurement outcome. Imagine that each pair of variables, one given to Alice and the other to Bob, carries the same code, so as to have the maximal possible probability of success in the case when both partners choose the same path: $\frac{1}{3} \sum_{i=1}^{3} P_{ii} \text{(same)} = 3/9$. For any of those pairs of codes if the choices of paths are different, then the probability of emerging opposite outcomes is at most 2/3, which can be easily checked by simple inspection (the value 7/9 is obtained if neither of codes $\{(HHH), (VVV)\}$ is used). Thus, the efficiency is $P = (3/9) + (6/9)(2/3) = 7/9$ in this protocol.

No classical protocol can be more efficient than this, in the lack of any communication between the two parties. This is based on the proof given below that the combination of probabilities as given in Eq. 11 satisfies a Bell inequality with 7/9 being a local realistic bound. Thus, the probability of success of any classical protocol is bounded, while those of quantum protocols can exceed the limit by utilizing quantum non-locality.

We now give the proof that

$$\sum_{i=1}^{3} P_{ii} \text{(same)} + \sum_{i \neq j=1}^{3} P_{ij} \text{(opp)} \leq 7, \quad (2)$$

for all local realistic models. This new Bell’s inequality with three possible measurement settings per observer is a byproduct of our analysis. Consider two observers, Alice and Bob, and allow each of them to choose between three dichotomic observables, determined by some local parameters denoted here $a_1$, $a_2$ and $a_3$ for Alice and $b_1$, $b_2$ and $b_3$ for Bob. The assumption of local realism implies the existence of three numbers $A_1$, $A_2$ and $A_3$ for Alice and $B_1$, $B_2$ and $B_3$ for Bob, where the numbers take values +1 or −1 and describe the predetermined results of corresponding measurements. In other words, for given specific numerical values, $(A_1 A_2 A_3)$ is a code from our previous example, while the values +1 and −1 play the role of $H$ and $V$, respectively. In a specific run of the experiment the correlations between two observations can be represented by the product of the type $A_i B_j$. The correlation function is then the average over many runs of the experiment $E(a_i, b_j) = \langle A_i B_j \rangle$.

The following combination of the predetermined results has a maximal value as given by:

$$\text{max}\{A_1(B_1 - B_2 - B_3) + A_2(B_2 - B_1 - B_3) + A_3(B_3 - B_1 - B_2)\} = 5$$
After averaging this expression over the ensemble of the runs of the experiment, one obtains the following Bell inequality:

\[ | \sum_{i=1}^{3} E(a_i, b_i) - \sum_{i \neq j}^{3} E(a_i, b_j) | \leq 5. \quad (4) \]

Using the connection between correlation functions and probabilities \[^{[14]}\], \( E(a_i, b_i) = 2P_{ij} \text{(same)} - 1 \) and \( E(a_i, b_j) = 1 - 2P_{ij} \text{(opp)} \) when \( i \neq j \), one finally obtains inequality \[^{[2]}\]. Its local realistic limit demonstrates that the efficiency of the particular classical solution discussed above is the optimal one. The quantum solution is based on violation of the inequality by a factor 7.5 with the quantum entangled state \( |\psi^+\rangle \).

Our problem can be seen as an instance studied in the field of communication complexity. Typically, in communication complexity problems two spatially separated parties receive local input data, e.g. one party receives number \( i \) and the other \( j \). Their goal is to compute a given function \( f(i, j) \). In one class of these problems only a restricted amount of communication between the parties is allowed, so that in general they can not arrive at the correct value of the function with certainty. While an error is allowed, the parties try to compute the function correctly with as high probability as possible.

In our example no communication between Alice and Bob is allowed. The input data \( i \) and \( j \) correspond to Alice’s and Bob’s choices of paths, respectively. Function \( f(i, j) \) is defined in the following way: \( f(i, i) = 1 \) and \( f(i, j) = -1 \) if \( i \neq j \). On the basis of \( i \) and \( j \) Alice and Bob produce new local data \( A \) and \( B \) (obtained as results +1 or −1 of local measurements of polarization along directions \( i \) and \( j \) in quantum protocol) that define the directions of their movements along paths \( i \) and \( j \), respectively. Only if \( f(x, y) = A \cdot B \) they will find each other (in which case they also compute the function correctly).

In conclusion, we have demonstrated that quantum entanglement can help separated individuals to orient themselves on a sphere if their aim is to find each other, even in the absence of any communication between them. In future it will be interesting to investigate whether there are further problems in life science, economics or every-day situations whose solutions favor quantum correlations over classical ones.

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[1] E. Schrödinger, Naturwissenschaften 23, 807-812; 823-828; 844-849 (1935). Translation published in Proc. Am. Phil. Soc. 124, p. 323-338 and in Quantum Theory and Measurement edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton), p. 152-167. A copy can be found at http://www.tu-harburg.de/rzt/rzt/it/QM/cat.html.

[2] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47 777 (1935).

[3] J. Bell, Physics (Long Island City, N.Y.), 1, 195 (1964).

[4] A.C.-C. Yao, in Proc. of the 11th Annual ACM Symposium on Theory of Computing, 209-213 (1979).

[5] R. Cleve and H. Buhrman, Phys. Rev. A 56, 1201 (1997).

[6] G. Brassard, e-print quant-ph/0101005 (2001).

[7] Č. Brukner, M. Zukowski, J.-W. Pan and A. Zeilinger, Phys. Rev. Lett. 92, 127901 (2004).

[8] G. Brassard, A. Broadbent, A. Tapp, e-print quant-ph/0306042 (2003).

[9] J. Summhammer, e-print quant-ph/0503136 (2005).

[10] A.M. Steane and W. van Dam, Physics Today 53, 35-39 (2000).

[11] K. Jacobs, and H. Wiseman, e-print quant-ph/0504192 and references therein.

[12] N.D. Mermin, Am. J. Phys. 49, 940 (1981).

[13] Later in the text we will see that this corresponds to the common assumption of experimenter’s freedom in choosing between different possible measurement settings in the context of Bell’s inequalities. See, for example, J.S. Bell, Free Variables and Local Causality, Dialectica 39, 103-106 (1985).

[14] Such a formulation of the problem invalidates a trivial possibility for partners to share a common set of instructions of the type ”take yellow path, direction +”.

[15] \( E(a_i, b_j) = \langle A_i B_j \rangle = \sum_{A_i, B_j \in \{-1, 1\}} P(A_i, B_j)A_i B_j = P_{ij}(\text{same}) - P_{ij}(\text{opp}) = 2P_{ij}(\text{same}) - 1 = 1 - 2P_{ij}(\text{opp}) \), as \( P_{ij}(\text{same}) + P_{ij}(\text{opp}) = 1 \). Here \( P(A_i, B_j) \) is probability that Alice obtains \( A_i \) and Bob \( B_j \), for the choices of paths \( i \) and \( j \), respectively.