Research Article

Robust Stabilization of Nonlinear Fractional Order Interconnected Systems Based on T-S Fuzzy Model

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This paper concerns robust stabilization of nonlinear fractional order interconnected systems. Based on uncertain fractional order Takagi–Sugeno fuzzy model and the fractional order extension of lyapunov direct method, a parallel distributed compensate controller is designed to asymptotically stabilize the fractional order interconnected systems. Then, a sufficient condition is given in the format of linear matrix inequalities. Simulation example is given to validate the effectiveness of the approach.

1. Introduction

Fractional order systems have attracted more and more attention due to its demonstrated application in many fields ([1–4]). Recently, a considerable literature has grown up around the theme of fractional order systems. For example, the problem of state estimation and synchronization for fractional order neural networks was discussed in [5–7]. Control problem for fractional order multiagent systems were introduced in [8–10]. Dynamic properties, control, and synchronization of fractional order chaotic systems were discussed in [11–14].

Large-scale interconnected system consists of a number of independent subsystems connected by some interconnections. Because interconnected systems are efficiently applied to practical systems such as economic systems, computer communication networks, and transportation systems, a considerable amount of literature has been published ([15–17]). Much of the previous research has focused on integer order. In 2013, a class of fractional order linear interconnected systems’ stabilization problem was considered [18]. Then, the problem of robust resilient controllers synthesis for uncertain fractional order linear interconnected system was studied, and the state feedback nonfragile controller was designed under the additive and multiplicative gain perturbations [19]. Positive reduced-order functional observers for positive fractional order interconnected time-delay systems were designed by [20]. The problem of robust stabilization for positive fractional order interconnected systems with heterogeneous time-varying delays was proposed by [21]. The robust decentralized fault-tolerant resilient control for fractional order large-scale interconnected uncertain system was investigated in [22]. There are relatively few historical studies in the area of fractional order nonlinear interconnected systems.

Takagi–Sugeno (T-S) fuzzy model is one of the most common effective methods for approximating complex nonlinear systems. Over the past several years, there has been rapid development of interconnected systems by using T-S fuzzy model. For example, fuzzy large-scale interconnected systems were discussed in [23–25] and [26]. T-S fuzzy controllers for nonlinear multiple time-delay interconnected systems were studied in [27–29]. The finite-time stabilization problem for type-2 T-S interconnected nonlinear systems was investigated in [30, 31]. $H_{\infty}$ control design for fuzzy discrete-time interconnected systems based on T-S fuzzy model was studied in [32–34]. Stability and stabilization problem for T-S Interconnected Fuzzy Systems by using different Lyapunov functions with slack variables are considered in [23]. The LMI stability conditions of fractional order uncertain T-S system were introduced in [35, 36]. In this paper, we study the stability problem of
nonlinear fractional order interconnected systems based on T-S fuzzy model and the fractional order extension of Lyapunov direct method.

This paper is organized as follows. Some preliminaries and the problem formulation are introduced first. The main results on the sufficient conditions of stabilization of nonlinear fractional order interconnected system are derived in Section 3. Section 4 interconnected fractional order chaotic systems to illustrate the effectiveness of the proposed results. Finally, the conclusions are drawn in Section 5.

Notations. The transpose of a matrix A is denoted by $A^T$. Sym{$A$} is used to denote the expression $A^T + A$ and $*$ will be used in some expression to indicate a symmetric, i.e.,

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}.$$  

2. Preliminaries and Problem Formulation

Consider a fractional order nonlinear interconnected system composed of $J$ fractional order subsystems $N_j, j = 1, \ldots, J$. The $j$th fractional order subsystem $N_j$ is described as follows:

$$D^\alpha x_j(t) = f_j(x_j(t), u_j(t)) + \sum_{n=1, n\neq j}^J b_{nj}(x_n(t)), \quad \alpha < 1,$$

where $0 < \alpha \leq 1$ is the fractional commensurate order, $f_j(\cdot)$ is the nonlinear vector-valued function, $\Delta f_j(\cdot)$ is the system uncertainties, $b_{nj}$ is the nonlinear interconnection between the $n$th and $j$th subsystems, $x_j(t)$ is the state vector, and $u_j(t)$ is the input vector of the $j$th fractional order subsystem, respectively. The operator $D^\alpha$ denotes $D^\alpha V$.

A set of fractional order T-S fuzzy model is employed here to deal with the control design problem of the fractional order nonlinear interconnected systems $N$. The $i$th rule of the fuzzy model for the fractional order nonlinear interconnected subsystem $N_j$ is proposed as follows:

**Plant Rule i:**

If $z_{i1}(t)$ is $M_{i1j}$ and \ldots and $z_{ip}(t)$ is $M_{ipj}$, then

$$D^\alpha x_j(t) = (A_{ij} + \Delta A_{ij})x_j(t) + \sum_{n=1, n\neq j}^J \tilde{A}_{nj}x_n(t) + (B_{ij} + \Delta B_{ij})u_j(t),$$

where $i = 1, \ldots, r_j$, $r_j$ is the number of IF-THEN rules, $M_{ihk}(h=1,2,\ldots,p)$ are the fuzzy sets, and $z_{ij}(t), \ldots, z_{ij}(t)$ are the premise variables. $A_{ij}, \tilde{A}_{nj}$, and $B_{ij}$ are constant matrices with appropriate dimension, while $\Delta A_{ij}$ and $\Delta B_{ij}$ are real-valued function matrices representing the time-varying parameter uncertainties that have the following form:

$$[\Delta A_{ij}, \Delta B_{ij}] = [D_{Aij}F_{Aij}(t)E_{Aij} D_{Bij}F_{Bij}(t)E_{Bij}],$$

where $D_{Aij}, D_{Bij}, E_{Aij}, E_{Bij}$ are known constant matrices, and $F_{Aij}(t), F_{Bij}(t)$ are unknown matrices with Lebesgue measurable elements satisfying $F_{Aij}(t)F_{Aij}(t) \leq I, F_{Bij}(t)F_{Bij}(t) \leq I$. The final state of the fractional order fuzzy model is inferred as follows:

$$\begin{align*}
D^\alpha x_j(t) &= \sum_{i=1}^{r_j} h_{ij}(t)(A_{ij} + \Delta A_{ij})x_j(t) + \sum_{n=1, n\neq j}^J \tilde{A}_{nj}x_n(t) \\
&+ (B_{ij} + \Delta B_{ij})u_j(t), \\
&= \sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t), \quad i = 1, 2, \ldots, r_j.
\end{align*}$$

where $h_{ij}(t) = w_{ij}(t)/\sum_{i=1}^{r_j} w_{ij}(t), \quad w_{ij}(t) = \prod_{q=1}^{p} M_{iqj}(z_{qj}(t)), \quad M_{iqj}(z_{qj}(t))$ is the grade of the membership of $z_{qj}(t)$ in $M_{iqj}$. Notice the facts $w_{ij}(t) \geq 0$ for $i = 1, 2, \ldots, r_j$ and $\sum_{i=1}^{r_j} w_{ij}(t) > 0$ for all $t$. Therefore, $h_{ij}(t) \geq 0$ for $i = 1, 2, \ldots, r_j$ and $\sum_{i=1}^{r_j} h_{ij}(t) = 1$.

According to the decentralized fuzzy control scheme, a set of fuzzy controllers is synthesized via the parallel distributed compensation (PDC) to deal with the stabilization control for the fractional order nonlinear interconnected systems $N$. The $j$th model-based fuzzy controller is

**Control rule i:**

IF $z_{ij}(t)$ is $M_{ij}$ and \ldots and $z_{pj}(t)$ is $M_{pj}$, THEN $u_j(t) = K_{ij}x_j(t)$.

where $i = 1, 2, \ldots, r_j$.

Hence, the final output of the fuzzy controller has the form

$$u_j(t) = \sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t), \quad i = 1, 2, \ldots, r_j.$$  

Substituting (4) into equation (3) yields the $j$th closed-loop subsystem as follows:

$$D^\alpha x_j(t) = \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{il}(t)h_{ij}(t)
\cdot \left\{(A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij}\right\}x_j(t) + \sum_{n=1, n\neq j}^J \tilde{A}_{nj}x_n(t).$$

Lemma 1 (see [37]). Let $x = 0$ be an equilibrium point for the nonautonomous fractional order system $D^\alpha x(t) = f(t, x)$. Let us assume that there exist a continuous Lyapunov function $V(x(t), t)$ and a scalar class-K function $\gamma_1(\cdot)$ such that $\forall x \neq 0$

$$\gamma_1(\|x(t)\|) \leq V(x(t), t), \quad D^\alpha V(x(t), t) \leq 0, \quad \text{with} \ \alpha \in (0, 1],$$

then the origin of the system is Lyapunov stable.

Lemma 2 (see [37]). Let $x(t) \in R^n$ be a vector of differentiable functions. Then, for any time instant $t \geq t_0$, the following relationship holds:

$$\frac{1}{2}D^\alpha x^T(t)Px(t) \leq x^T(t)PD^\alpha x(t), \quad \forall \alpha \in (0, 1], \forall t \geq t_0.$$
where $P \in \mathbb{R}^{n \times n}$ is a constant, square, symmetric, and positive matrix.

**Lemma 3** (see [38, 39]). For any matrices $X$ and $Y$ with appropriate dimensions, we have $X^TY + Y^TX \leq \sigma X^TX + \sigma^{-1}Y^TY$ for any $\sigma > 0$.

**Lemma 4** (see [40]). Given matrices $T$, $\Pi$, $N(t)$, and $M$ of appropriate dimensions and with $M$ symmetrical, then $M + T N(t) \Pi + \Pi^T N^T(t) T^T < 0$ holds for any $N(t)$ satisfying $N^T(t) N(t) \leq I$ if and only if there exists $\varepsilon > 0$, such that $M + \varepsilon TT^T + \varepsilon^{-1} \Pi \Pi^T < 0$.

**Lemma 5** (Schur Complement), see [41]). For a given matrix $S = S^T$, the following assertions are equivalent:

\[
\begin{align*}
(1) & \quad S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} < 0; \\
(2) & \quad S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0; \\
(3) & \quad S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.
\end{align*}
\]

### 3. Main Results

In this section, the stability of the fractional order nonlinear interconnected system $N$ is studied. A sufficient condition is established for system (5). Then the following theorem presents the main result.

**Theorem 1.** The closed-loop fractional order nonlinear interconnected system (5) is asymptotically stable if there are symmetric positive definite matrices $Q_j (j = 1, \ldots, J)$, matrices $W_{ij} (i = 1, 2, \ldots, r_j)$, and real scalar constants $\varepsilon_{ij}$, $\eta_{ij}$, $\delta_{ij}$, $\mu_{ij}$, $\rho_{ij}$ and $\overline{\mu}$ such that

\[
\begin{align*}
\Lambda_{ij} &= \text{Sym}\left[ A_{ij} Q_j + B_{ij} W_{ij} \right] + \varepsilon_{ij} D_{Aij} D_{Aij}^T + \eta_{ij} D_{Bij} D_{Bij}^T \\
&\quad + \sum_{n=1}^{J} \overline{\mu} \tilde{A}_{mj} \tilde{A}_{mj}^T,
\end{align*}
\]

\[
\begin{bmatrix}
\Lambda_{ij} & Q_j E_{Aij}^T & W_{ij} E_{Bij}^T & Q_j \\
* & -\varepsilon_{ij} I & 0 & 0 \\
* & * & -\eta_{ij} I & 0 \\
* & * & * & -\overline{\mu} (J-1)^{-1} I
\end{bmatrix} < 0,
\]

\[
j = 1, \ldots, J, i = 1, \ldots, r_j.
\]

where

\[
\begin{bmatrix}
\Lambda_{ij} & Q_j E_{Aij}^T & W_{ij} E_{Bij}^T & Q_j \\
& -\varepsilon_{ij} I & 0 & 0 \\
& * & -\eta_{ij} I & 0 \\
& * & * & -\overline{\mu} (J-1)^{-1} I
\end{bmatrix} < 0,
\]

\[
\begin{bmatrix}
\Lambda_{ij} & Q_j E_{Aij}^T & W_{ij} E_{Bij}^T & Q_j \\
& -\varepsilon_{ij} I & 0 & 0 \\
& * & -\eta_{ij} I & 0 \\
& * & * & -\overline{\mu} (J-1)^{-1} I
\end{bmatrix} < 0,
\]

\[
\begin{bmatrix}
\Lambda_{ij} & Q_j E_{Aij}^T & W_{ij} E_{Bij}^T & Q_j \\
& -\varepsilon_{ij} I & 0 & 0 \\
& * & -\eta_{ij} I & 0 \\
& * & * & -\overline{\mu} (J-1)^{-1} I
\end{bmatrix} < 0,
\]

$\Lambda_{ij} = \text{Sym}\left[ A_{ij} Q_j + B_{ij} W_{ij} \right] + \varepsilon_{ij} D_{Aij} D_{Aij}^T + \eta_{ij} D_{Bij} D_{Bij}^T + \sum_{n=1}^{J} \overline{\mu} \tilde{A}_{mj} \tilde{A}_{mj}^T$

\[
\begin{bmatrix}
\Lambda_{ij} & Q_j E_{Aij}^T & W_{ij} E_{Bij}^T & Q_j \\
& -\varepsilon_{ij} I & 0 & 0 \\
& * & -\eta_{ij} I & 0 \\
& * & * & -\overline{\mu} (J-1)^{-1} I
\end{bmatrix} < 0,
\]

$1 \leq i \leq r_j.$

**Proof.** Let the Lyapunov function for the fractional order interconnected system $N$ be defined as

\[
V(t) = \sum_{j=1}^{J} v_j(t) = \sum_{j=1}^{J} x_j^T(t) P_j x_j(t).
\]

$P_j$ is real symmetric positive definite matrix. It follows from Lemma 1, the closed-loop fractional order nonlinear interconnected system (5) is asymptotically stable if $D^n V(x(t), t) \leq 0$. Note that

\[
D^n V(t) = \sum_{j=1}^{J} D^n v_j(t).
\]

Applying Lemma 2 to $D^n v_j(t)$, it can be obtained that

\[
D^n V(t) \leq \sum_{j=1}^{J} \left[ \left(D^n x_j(t) \right)^T P_j x_j(t) + x_j^T(t) P_j D^n x_j(t) \right].
\]
Right side of inequality (15) can be represented by

\[
\begin{align*}
\sum_{j=1}^{l} & \left\{ x_j^T(t) P_j \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{ij}(t) h_{lj}(t) \left[ (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \right] x_j(t) \\
+ \sum_{n=1, m \neq j}^{l} \tilde{A}_{mj} x_n(t) \right\} + \left[ \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{ij}(t) h_{lj}(t) \right] \left[ (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \right] x_j(t) \\
+ \sum_{n=1, m \neq j}^{l} \tilde{A}_{mj} x_n(t) \right\}^T P_j x_j \\
= & \sum_{j=1}^{l} \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{ij}(t) h_{lj}(t) \left\{ x_j^T(t) \left[ (P_j A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \right] x_j(t) \\
+ \left[ (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \right] x_j(t) \right\} \\
+ \sum_{j=1}^{l} \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} \sum_{n=1, m \neq j}^{l} h_{ij}(t) h_{lj}(t) \left\{ x_j^T(t) P_j \tilde{A}_{mj} x_n + x_n^T \tilde{A}_{mj}^T P_j x_j \right\}.
\end{align*}
\]

By applying Lemma 3, it can be obtained

\[
D^s V(t) \leq \sum_{j=1}^{l} \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{ij}(t) h_{lj}(t) \left\{ x_j^T(t) \left[ \text{Sym} \{ P_j (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \} \right] x_j(t) \\
+ \mu \left( \frac{l-1}{l} \right) x_j^T(t) x_j(t) + \mu^{-1} \left( x_j^T(t) P_j \tilde{A}_{mj} \tilde{A}_{mj}^T P_j x_j - \left( \frac{1}{l} \right) x_j^T(t) P_j \tilde{A}_{ij} \tilde{A}_{ij}^T P_j x_j \right) \right\}.
\]

In view of the matrix \( \tilde{A}_{ij} \) is equal to zero and \( \sum_{l=1}^{r_j} h_{lj} = 1 \), we have

\[
\begin{align*}
\sum_{j=1}^{l} \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{ij}(t) h_{lj}(t) x_j^T(t) & \left[ \text{Sym} \{ P_j (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \} \right] + \\
\sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{mj} \tilde{A}_{mj}^T P_j + \mu (J - 1) I & x_j(t) \\
= & \sum_{j=1}^{l} \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{lj}(t) x_j^T(t) \left[ \text{Sym} \{ P_j (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \} \right] + \\
\sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{mj} \tilde{A}_{mj}^T P_j + \mu (J - 1) I & x_j(t) + \sum_{j=1}^{l} \sum_{i=1}^{r_j} \sum_{l=1}^{r_i} h_{lj}(t) h_{lj}(t) x_j^T(t) \left( \frac{1}{2} \right) \cdot \text{Sym} \{ P_j (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} + (A_{ij} + \Delta A_{ij}) + (B_{ij} + \Delta B_{ij})K_{ij} \} \\
& + \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{mj} \tilde{A}_{mj}^T P_j + \mu (J - 1) I x_j(t) < 0.
\end{align*}
\]
If it is possible to assume each sum of (18) to be negative definite, respectively, then the fractional order nonlinear interconnected system is asymptotically stable.

First, assume that the first sum of the last equation in (18) is negative definite:

\[
\text{Sym}\left[ P \left( A_{ij} + \Delta A_{ij} \right) + \left( B_{ij} + \Delta B_{ij} \right) K_{ij} \right]
+ \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{m_{ij}} \tilde{A}_{m_{ij}}^T P_j + \mu (J-1)I < 0.
\] (19)

Equation (19) can be represented by

\[
\text{Sym}\left[ P_j (A_{ij} + B_{ij} K_{ij}) \right] + \text{Sym}\left[ P_j D_{Aij} F_{Aij} (t) E_{Aij} \right]
+ \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{m_{ij}} \tilde{A}_{m_{ij}}^T P_j + \mu (J-1)I < 0.
\] (20)

By applying Lemma 3 to (20), one obtains (20) holds if and only if there exist \( \epsilon_{ij} \) and \( \eta_{ij} \) such that

\[
\text{Sym}\left[ P_j (A_{ij} + B_{ij} K_{ij}) \right] + \epsilon_{ij} P_j D_{Aij} D_{Aij}^T P_j + \epsilon_{ij} E_{Aij} E_{Aij}
+ \eta_{ij} P_j D_{Bij} D_{Bij}^T P_j
+ \eta_{ij} \left( E_{Bij} K_{ij} \right)^T E_{Bij} K_{ij}
+ \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{m_{ij}} \tilde{A}_{m_{ij}}^T P_j + \mu (J-1)I < 0.
\] (21)

By the Schur complement, we can get

\[
\begin{bmatrix}
\Omega_{ij} & \left( E_{Bij} K_{ij} \right)^T & I \\
* & -\epsilon_{ij} I & 0 & 0 \\
* & * & -\eta_{ij} I & 0 \\
* & * & * & -\mu^{-1} (J-1)^{-1} I
\end{bmatrix}
< 0,
\] (22)

where

\[
\Omega_{ij} = \text{Sym}\left[ P_j (A_{ij} + B_{ij} K_{ij}) \right] + \epsilon_{ij} P_j D_{Aij} D_{Aij}^T P_j
+ \eta_{ij} P_j D_{Bij} D_{Bij}^T P_j
+ \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{m_{ij}} \tilde{A}_{m_{ij}}^T P_j.
\] (23)

Define transformation matrix as diag\( P_j^{-1} I \) and take a congruence transformation to (22); this yields

\[
\begin{bmatrix}
P_j^{-1} \Omega_{ij} P_j^{-1} & P_j^{-1} \left( E_{Bij} K_{ij} \right)^T & P_j^{-1} I \\
* & -\epsilon_{ij} I & 0 & 0 \\
* & * & -\eta_{ij} I & 0 \\
* & * & * & -\mu^{-1} (J-1)^{-1} I
\end{bmatrix}
< 0.
\] (24)

Denoting \( Q_j = P_j^{-1}, W_{ij} = K_j P_j^{-1}, \) and \( \mu = \mu^{-1}, \) we have

\[
\begin{bmatrix}
\Lambda_{ij} & Q_j & W_{ij}^T & Q_j \\
* & -\epsilon_{ij} I & 0 & 0 \\
* & * & -\eta_{ij} I & 0 \\
* & * & * & -\mu (J-1)^{-1} I
\end{bmatrix}
< 0,
\] (25)

where

\[
\Lambda_{ij} = \text{Sym}\left[ A_{ij} Q_j + B_{ij} W_{ij} \right] + \epsilon_{ij} D_{Aij} D_{Aij}^T + \eta_{ij} D_{Bij} D_{Bij}^T
+ \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{m_{ij}} \tilde{A}_{m_{ij}}^T.
\] (26)

The second LMI (11) can be established through a similar procedure. Assume that the second sum of the last equation in (18) is negative definite:

\[
\frac{1}{2} \text{Sym}\left[ P \left( A_{ij} + \Delta A_{ij} \right) + \left( B_{ij} + \Delta B_{ij} \right) K_{ij} + \left( A_{ij} + \Delta A_{ij} \right)
+ \left( B_{ij} + \Delta B_{ij} \right) K_{ij} \right]
+ \mu (J-1)I < 0.
\] (27)

Equation (27) can be represented by

\[
\frac{1}{2} \text{Sym}\left[ P_j \left( A_{ij} + B_{ij} K_{ij} \right) \right] + \epsilon_{ij} P_j D_{Aij} D_{Aij}^T + \eta_{ij} P_j D_{Bij} D_{Bij}^T
+ \sum_{n=1}^{l} \mu^{-1} P_j \tilde{A}_{m_{ij}} \tilde{A}_{m_{ij}}^T P_j + \mu (J-1)I < 0.
\] (28)

By applying Lemma 3 to (28), one obtains (28) holds if and only if there exist \( \gamma_{ij}, \delta_{ij}, \rho_{ij} \) and \( \mu_{ij}, \) such that
\[
\frac{1}{2} \text{Sym}\{P \left( A_{ij} + B_{ij}K_{ij} + A_{ij} + B_{ij}K_{ij} \right) \} + \gamma_{ij}P_{j}D_{Alj}D_{Alj}^{T}P_{j} + \gamma_{ij}^{I}E_{Alj}^{T}E_{Alj}
\]
\[
+ \delta_{ij}P_{j}D_{Blj}D_{Blj}^{T}P_{j} + \delta_{ij}^{I}(E_{Blj}K_{ij}^{T})^{T}E_{Blj}K_{ij} + \mu_{ij}P_{j}D_{Alj}D_{Alj}^{T}P_{j}
\]
\[
+ \mu_{ij}^{I}E_{Alj}^{T}E_{Alj} + \rho_{ij}P_{j}D_{Blj}D_{Blj}^{T}P_{j} + \rho_{ij}^{I}(E_{Blj}K_{ij}^{T})^{T}E_{Blj}K_{ij}
\]
\[
+ \sum_{n=1}^{J} \mu_{j}^{-1}P_{j}\tilde{A}_{nj}\tilde{A}_{nj}^{T}P_{j} + \mu(J-1)I < 0.
\] (29)

By the Schur complement, we can get

\[
\begin{bmatrix}
\Omega_{ij} & E_{Alj}^{T} \left( E_{Blj}K_{ij}^{T} \right) & E_{Alj}^{T} \left( E_{Blj}K_{ij}^{T} \right) & I \\
* & -\gamma_{ij}I & 0 & 0 & 0 & 0 \\
* & * & -\delta_{ij}I & 0 & 0 & 0 \\
* & * & * & -\mu_{ij}I & 0 & 0 \\
* & * & * & * & -\rho_{ij}I & 0 \\
* & * & * & * & * & -\mu^{-1}(J-1)^{-1}I
\end{bmatrix} < 0,
\] (30)

where

\[
\Omega_{ij} = \frac{1}{2} \text{Sym}\{P \left( A_{ij} + B_{ij}K_{ij} + A_{ij} + B_{ij}K_{ij} \right) \} + \gamma_{ij}P_{j}D_{Alj}D_{Alj}^{T}P_{j}
\]
\[
+ \delta_{ij}P_{j}D_{Blj}D_{Blj}^{T}P_{j} + \mu_{ij}P_{j}D_{Alj}D_{Alj}^{T}P_{j} + \rho_{ij}P_{j}D_{Blj}D_{Blj}^{T}P_{j}
\]
\[
+ \sum_{n=1}^{J} \mu_{j}^{-1}P_{j}\tilde{A}_{nj}\tilde{A}_{nj}^{T}P_{j}.
\] (31)

Define transformation matrix as diag\[P_{j}^{-1}I\] and take a congruence transformation to (30); this yields

\[
\begin{bmatrix}
P_{j}^{-1}\Omega_{ij}P_{j} & P_{j}^{-1}E_{Alj}^{T} \left( E_{Blj}K_{ij}^{T} \right) & P_{j}^{-1}E_{Alj}^{T} \left( E_{Blj}K_{ij}^{T} \right) & P_{j}^{-1}I \\
* & -\gamma_{ij}I & 0 & 0 & 0 \\
* & * & -\delta_{ij}I & 0 & 0 & 0 \\
* & * & * & -\mu_{ij}I & 0 & 0 \\
* & * & * & * & -\rho_{ij}I & 0 \\
* & * & * & * & * & -\mu^{-1}(J-1)^{-1}I
\end{bmatrix} < 0,
\] (32)
we have
\[
\begin{bmatrix}
\Lambda_{ij} & Q_j & E_{i,j}^T \\
* & -y_{ij} & 0 \\
* & * & -\delta_{ij} & 0 \\
* & * & * & -\mu_{ij} & 0 \\
* & * & * & * & -\rho_{ij} & 0 \\
\end{bmatrix} < 0,
\]
\[1 \leq i < l \leq r_j,
\]
(33)

where
\[
\Lambda_{ij} = \frac{1}{2} \text{Sym}\{A_{ij}Q_j + B_{ij}W_{ij} + A_{ij}Q_j + B_{ij}W_{ij}\}
\]
\[+ \gamma_{ij}D_{ij}D_{ij}^T + \delta_{ij}D_{ij}D_{ij}^T + \mu_{ij}D_{ij}D_{ij}^T + \rho_{ij}D_{ij}D_{ij}^T + \sum_{n=1}^{l} \bar{\mu}_{nj}A_{nj}A_{nj}^T.
\]
(34)

This completes the proof. □

When \( \Delta A_{ij} \) and \( \Delta B_{ij} \) are 0, it is easy to get the following Corollary.

Corollary 1. The closed-loop fractional order nonlinear interconnected system (5) is asymptotically stable if there are symmetric positive definite matrices \( Q_j (j = 1, \ldots, J) \), matrices \( W_{ij} (i = 1, 2, \ldots, r_j) \), and a real scalar constant \( \bar{\mu} \) such that
\[
\begin{bmatrix}
\Lambda_{ij} & Q_j \\
* & -\bar{\mu}(J - 1)^{-1}I
\end{bmatrix} < 0,
\]
\[1 \leq i < l \leq r_j,
\]
(35)

with
\[
\Lambda_{ij} = \text{Sym}\{A_{ij}Q_j + B_{ij}W_{ij}\} + \sum_{n=1}^{l} \bar{\mu}_{nj}A_{nj}A_{nj}^T,
\]
(36)

with
\[
\Lambda_{ij} = \frac{1}{2} \text{Sym}\{A_{ij}Q_j + B_{ij}W_{ij} + A_{ij}Q_j + B_{ij}W_{ij}\}
\]
\[+ \sum_{n=1}^{l} \bar{\mu}_{nj}A_{nj}A_{nj}^T.
\]
(37)

The asymptotically stabilizing state feedback gain matrix is \( K_{ij} = W_{ij}Q_j^{-1} \).

Remark 1. Since T-S fuzzy system can effectively approximate complex systems with nonlinearity, our model can be applied to a broad class of nonlinear fractional order interconnected systems. Most of all, stabilization of system can be developed by solving a set of LMIs. Moreover, when the number of the rules \( r_1, \ldots, r_j \) is one, our model can be applied to solving fractional order linear interconnected system with uncertainties.

4. Numerical Examples

In this part, in order to show the effectiveness of the proposed method, a numerical example on interconnected fractional order chaotic systems will be provided.

Consider the asymptotical stability of nonlinear fractional order interconnected systems, and each subsystem is a fractional order uncertain Lorenz chaotic system:
\[
\begin{align*}
D^a x_1(t) &= -(a + \Delta a)x_1(t) + (a + \Delta a)x_2(t), \\
D^a x_2(t) &= (c + \Delta c)x_1(t) - x_2(t) - x_1(t)x_3(t), \\
D^a x_3(t) &= x_1(t)x_2(t) - bx_3(t).
\end{align*}
\]
(38)

And when \( a = 10, \Delta a = \sin t, b = 8/3, c = 28, \Delta c = 0.14 \), and \( a = 0.993 \), chaotic behaviors of the fractional order uncertain Lorenz chaotic system are shown in Figure 1.

Let us consider two interconnected fractional order uncertain Lorenz chaotic system as follows:
Subsystem 1:

\[
\begin{align*}
D^\alpha x_{11}(t) &= -(a + \Delta a)x_{11}(t) + (a + \Delta a)x_{21}(t) + x_{12}(t) + x_{22}(t) + u_1(t), \\
D^\alpha x_{21}(t) &= (c + \Delta c)x_{31}(t) - x_{21}(t) - x_{11}(t)x_{31}(t) + x_{14}(t) + x_{32}(t), \\
D^\alpha x_{31}(t) &= x_{11}(t)x_{21}(t) - bx_{31}(t).
\end{align*}
\] (39)

Figure 2: The state of subsystem 1 in nonlinear fractional order interconnected systems.

Figure 3: The state of subsystem 2 in nonlinear fractional order interconnected systems.
Subsystem 2:

\[
\begin{align*}
D^\alpha x_{12}(t) &= -(a + \triangle a)x_{12}(t) + (a + \triangle a)x_{22}(t) + x_{11}(t) + x_{21}(t) + u_2(t), \\
D^\alpha x_{22}(t) &= (c + \triangle c)x_{12}(t) - x_{22}(t) - x_{12}(t)x_{32}(t) + x_{11}(t) + x_{31}(t), \\
D^\alpha x_{32}(t) &= x_{12}(t)x_{22}(t) - bx_{32}(t).
\end{align*}
\] 

(40)

Figure 4: Control results of the fractional order nonlinear interconnected system.

Figure 5: Control results of the fractional order nonlinear interconnected system.
The state curves Nonlinear Fractional Order Interconnected Systems that without control, i.e., \( u_1 \equiv 0, u_2 \equiv 0 \) are shown in Figures 2 and 3.

**Step 1.** To stabilize the above fractional order interconnected system, we firstly establish fractional order T-S fuzzy model for each nonlinear fractional order subsystem. Assume that \( x_{ij}(t) \in [-d, d] \) and \( d > 0, d = 30 \), then we have

The fuzzy model of Subsystem \( j \):

Rule 1: IF \( x_{ij}(t) \) is \( M_{1j} \), THEN
\[
D^\alpha x_{ij}(t) = (A_{1j} + \Delta A_{1j})x_{ij}(t) + \tilde{A}_{1ij}x_i(t) + B_{1j}u_j(t),
\]
Rule 2: IF \( x_{ij}(t) \) is \( M_{2j} \), THEN
\[
D^\alpha x_{ij}(t) = (A_{2j} + \Delta A_{2j})x_{ij}(t) + \tilde{A}_{2ij}x_i(t) + B_{2j}u_j(t).
\]

where \( x_i^T(t) = [x_{ij}(t)x_{ij}(t)x_{ij}(t)] \), \( i \neq j \) and \( j = 1, 2 \).

\[
A_{1j} = \begin{bmatrix}
-a & a & 0 \\
-b & c & -1 \\
0 & d & -b
\end{bmatrix},
\]
\[A_{2j} = \begin{bmatrix}
-a & a & 0 \\
-b & c & -1 \\
0 & d & -b
\end{bmatrix},
\]
\[
\tilde{A}_{1ij} = \tilde{A}_{2ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
\Delta A_{1j} = \Delta A_{2j} = \begin{bmatrix}
-\Delta a & \Delta a & 0 \\
\Delta c & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]
\[B_{1j} = B_{2j} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.
\]

Here \( \Delta A_{1j} \) and \( \Delta A_{2j} \) can be represented by
\[
\begin{bmatrix}
-0.1 & 0 & 0 \\
0 & 0.005 & 0 \\
0 & 0 & 0
\end{bmatrix},
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 10 & -10 \\ 0 & 0 & 0 \end{bmatrix}
\]

then we have
\[
D_{A_{1j}} = D_{A_{2j}} = \begin{bmatrix}
-0.1 & 0 & 0 \\
0 & 0.005 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]
\[
E_{A_{1j}} = E_{A_{2j}} = \begin{bmatrix} 10 & -10 & 0 \\ 28 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
D_{B_{1j}} = D_{B_{2j}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
E_{B_{1j}} = E_{B_{2j}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

and the membership functions for Rule 1 are:
\[
M_{1j}(x_{ij}(t)) = (1/2)(1 + x_{ij}(t)/10),
\]
\[
M_{2j}(x_{ij}(t)) = (1/2)(1 - x_{ij}(t)/10).
\]

Fuzzy controller of Subsystem \( j \):

Rule 1: IF \( x_{ij}(t) \) is \( M_{1j} \), THEN \( u_j(t) = K_{1j}x_j(t) \),

\[
K_{1j} = [-7.4134, -17.3762, 3.7393],
\]
\[
K_{21} = [-9.6733, -20.5994, 0.8517],
\]
\[
K_{12} = [-10.7060, -21.3431, 5.6562],
\]
\[
K_{22} = [-13.8411, -25.5120, 1.4921].
\]

Therefore, the fractional order nonlinear fractional order interconnected system under fuzzy control law is determined to be asymptotically stable. The simulation results of each subsystem states under control are illustrated in Figure 6: Control curve \( u_1(t) \) of the fractional order interconnected system.

Figure 7: Control curve \( u_2(t) \) of the fractional order interconnected system.

**Step 2.** By applying Theorem 1 and using packages YALMIP in Matlab, we find the LMI (9) and (11) in Theorem 1 is feasible, a feasible solution is as follows:

\[
K_{11} = [-7.4134, -17.3762, 3.7393],
\]
\[
K_{21} = [-9.6733, -20.5994, 0.8517],
\]
\[
K_{12} = [-10.7060, -21.3431, 5.6562],
\]
\[
K_{22} = [-13.8411, -25.5120, 1.4921].
\]
Figures 4 and 5 shows that it is asymptotically stable, and the control curve of system is shown in Figures 6 and 7.

5. Conclusion
This paper focuses on the stability of the nonlinear fractional order interconnected systems. A useful stabilization approach has been given. The basis of this approach is to apply fractional order uncertain T-S fuzzy model to nonlinear fractional order interconnected systems. The PDC control design is carried out based on the fractional order T-S fuzzy model and the fractional order extension of Lyapunov direct method, a sufficient condition was given in terms of LMI. Finally, nonlinear fractional order interconnected systems was given to illustrate the effectiveness of the proposed theoretical results.

Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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