Perspectives on Lorentz and CPT Symmetry
Violation

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Abstract.
Lorentz and CPT symmetries are fundamental to our understanding of the standard model and general relativity, but there has recently been a great deal of interest in the possibility that they may not be exact in nature. There is a systematically developed effective field theory, the standard model extension, that describes such symmetry violations. Many types of experiments have been used to place constraints on the parameters characterizing this theory. Some of the best constraints come from high-energy astrophysics. Observations of the radiation from ultrarelativistic electrons make it possible to map out the electron energy-momentum relation with great precision. Using observations of synchrotron and inverse Compton emissions from a variety of sources, the coefficients for spin-independent Lorentz violation in the electron sector can be constrained at the $10^{-15}$–$10^{-20}$ levels.

1. Introduction
In the past fifteen years, there has been an explosion of interest in the topic of Lorentz symmetry. So far as we know, all physical processes are invariant under rotations and Lorentz boosts. Yet an important question is whether this seemingly sacrosanct symmetry might actually be broken in nature. If a new experiment showed that Lorentz invariance does not hold exactly, that would be evidence of profoundly new physics.

Lorentz invariance is also closely related to discrete symmetries. Parity (P) and time reversal (T) are elements of the full $O(3,1)$ transformation group. Moreover, Lorentz invariance and CPT invariance are especially closely connected.

Of course, experimenters have been performing tests of Lorentz invariance since before relativity was even recognized. The famed Michelson-Morley experiment was a test of both the rotation and boost invariances that make up Lorentz invariance. The experiment’s null result was a surprise at the time, but we now understand it in the framework of special relativity, as indicating that there is no preferred rest frame for the universe.

Since Einstein’s introduction of special relativity in 1905, it has been subjected to a consistent barrage of experimental tests. However, until recently, such tests generally involved searches for purely ad hoc modifications of Einstein’s framework; there was no attempt to study possible forms of Lorentz symmetry breaking systematically. More recently though, theorists and experimenters have begun looking at the problem in a much more thorough and organized fashion, as it was realized that most possible forms of Lorentz violation had been completely overlooked in earlier analyses. The awareness that there are many more ways that Lorentz
symmetry could be broken than was previously appreciated has lead to a tremendous surge of interest in the subject.

There are two major reasons why the possibility of Lorentz invariance violation is interesting, and the two reflect differing “top-down” and “bottom-up” approaches to fundamental physics. The “top-down” reason for considering violations of Lorentz invariance is that most of the theories that are considered candidates to explain quantum gravity suggest the possibility of Lorentz symmetry violation in some regimes. These includes string theory [1], loop quantum gravity [2], noncommutative geometry theories [3], and others. However, none of these theories is yet a viable theory of four-dimensional quantum gravitation, and none of them are sufficiently well understood to be able to tell whether Lorentz violations (such as preferred frame effects) are real features of the theories—and if they are real, whether they are generic features of the theories.

The “bottom-up” approach suggests that Lorentz and CPT invariances should be tested simply because they appear to be so fundamental. Any violation of such symmetries would be a revolutionary discovery, since both the standard model of particle physics and the general theory of relativity are based on local Lorentz invariance; the entire edifice of fundamental physics is supported by this symmetry. A breaking of this symmetry would signal something profoundly new in physics and would be a critically important clue about the underlying nature of the universe. Much of the story of modern fundamental physics concerns how extremely useful and important symmetries—such as isospin, parity, and time reversal—were found not to be exact symmetries of nature. Knowing why these symmetries are violated is crucial to our understanding of the standard model.

Both Lorentz and CPT symmetries are fundamental to our understanding of the standard model, and the two are closely intertwined. It is impossible to break CPT symmetry in a local quantum theory without also breaking Lorentz invariance [4]. This means that many traditional tests of CPT can also be interpreted as tests of Lorentz invariance, and vice versa. On the other hand, many traditional phenomenalistic models of CPT violation (such as having particles and antiparticles with different masses) cannot be accommodated in a quantum field theory as we understand it.

2. The Standard Model Extension

Any realistic theory of Lorentz and CPT violation must also be capable of describing what we already know. All the fundamental physics that we current understand is described by the standard model (an effective quantum field theory) and general relativity. If we want to introduce small Lorentz-violating corrections to the standard model, we can continue to use the formalism of effective field theory, and the usual effective field theory that describes Lorentz and CPT violation is known as the standard model extension (SME) [5, 6].

The construction of the SME is straightforward. It involves only those fields (scalar, spinor, and gauge) that are part of the standard model, although the generalization to additional fields is very simple. However, in addition to the usual standard model terms, the SME Lagrange density includes all possible operators that can be constructed from these fields, subject to some appropriate restrictions. The standard model itself contains operators that are local, gauge invariant, Lorentz invariant, and renormalizable, but the SME relaxes these requirements. In particular, operators appearing in the Lagrangian do not need to be Lorentz scalars.

For example, while the usual mass term for a Dirac fermion is $L_m = -m\bar{\psi}\psi$, the SME also includes mass-like terms such as $L_\alpha = -a_0 \bar{\psi}\gamma^0 \psi$. This operator transforms like the time component of a four-vector, and so it is not invariant under boosts. The coefficient $a_0$ parameterizes the size of the Lorentz violation. $a_0$ also represents a preferred spacetime direction. If Lorentz violation is broken spontaneously (which may be required for consistency with general relativity [7]), $a_0$ would be the vacuum expectation value (vev) of a vector-valued dynamical
field. In the case of \( a_0 \), that vev is purely timelike, but models with both spacelike and timelike expectations may be viable [8]. There will also be SME coefficients with multiple indices, and each of these would correspond to the vev of an appropriate tensor-valued field [9].

Of course, the number of operators than can be constructed just from standard model fields is infinite, so some other restrictions are needed in order to have a predictive theory. The most commonly considered subset of possible operators are those that constitute the minimal SME, which contains only operators that are local, gauge invariant, and power counting renormalizable (i.e., of dimension 3 or 4). The minimal SME has become the standard framework from parameterizing the results of experimental Lorentz and CPT tests. However, there are certainly situations in which more general operators (which may be nonlocal, violate charge conservation, or have higher dimension) are of significant interest.

The minimal SME Lagrange density for a single species of fermion is

\[
\mathcal{L}_\psi = \bar{\psi}(i\Gamma^\mu \partial_\mu - M)\psi,
\]

where

\[
M = m + \not{\beta} - \gamma_5 + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu} + im_5 \gamma_5,
\]

and

\[
\Gamma^\mu = \gamma^\mu + e^{\rho\mu} \gamma_\rho - d^{\rho\mu} \gamma_\rho + \epsilon^{\mu} + if^{\mu} \gamma_5 + \frac{1}{2} \gamma^{\lambda\mu} \sigma_{\lambda\nu}.
\]

\( M \) and \( \Gamma \) contain coefficients with all possible Lorentz structures allowed by their respective dimensions. These represent all the superficially renormalizable couplings that are possible in a purely fermionic theory. While each fermion species will have a full set of coefficients associated just with the behavior of that one species, there is no requirement that the Lorentz violation coefficients be diagonal in flavor space. Off-diagonal coefficients would generate effects such as neutrino oscillations.

However, some of the coefficients are more interesting than others. In particular, \( m_5 \) does not violate Lorentz symmetry, and it may be absorbed into the other coefficients by means of a field redefinition [10]. Moreover, \( e, f, \) and \( g \) are inconsistent with the coupling of the fermion field to a chiral gauge field.

As noted, CPT violation in quantum field theory implies a breaking of Lorentz symmetry as well. However, the converse does not hold; not all forms of Lorentz violation necessarily violate CPT. In fact, only those coefficients with odd numbers of indices (\( a, b, e, f, \) and \( g \)) are odd under CPT. The various operators also have different transformations under \( C, P, \) and \( T \) separately. The discrete transformation properties of the various fermion bilinears are well known, and they have important experimental consequences. Most experiments are sensitive only to a small subset of the minimal SME coefficients—those coefficients that have a certain set of symmetry properties. (For example, tests involving parity are insensitive to \( a_0 \), which is odd only under \( C \), while tests of time reversal are insensitive to \( b_0 \), which is odd only under \( P \).) Moreover, at nonrelativistic energies, some of the coefficients have identical effects; these means only certain special linear combinations are observable in nonrelativistic experiments.

The photon sector of the SME involves additional coefficients. The photon Lagrange density is

\[
\mathcal{L}_A = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} k_{AF}^\mu \epsilon_{\mu\rho\sigma\tau} F^{\rho\sigma} A^\tau - \frac{1}{4} k_{F}^{\mu\rho\sigma} F_{\mu\rho} F_{\sigma\tau}.
\]

As in the fermion sector, the coefficient tensor \( k_{AF} \) with one index is CPT-odd, while \( k_F \) with four indices is CPT-even. The operator parameterized by \( k_{AF} \) has a Chern-Simons form, and its Lagrange density is not actually gauge invariant. It changes by a total derivative under a gauge transformation, but this means that the integrated action \( S = \int d^4 x \mathcal{L} \) is fully gauge invariant. However, the unusual gauge transformation properties of \( k_{AF} \) made it a subject of
significant controversy; it was unclear whether the $k_{AF}$ term could be generated by radiative corrections [11, 12, 13, 14]. Ultimately, it was found the the radiative corrections involved were actually undetermined.

Nonvanishing values of the various SME coefficients could have many different kinds of effects. One obvious consequence of rotation invariance would be angular momentum nonconservation. A sphere made from material with nonvanishing $c_{jk}$ coefficients would behave as if its tensor of inertia were not $I_{jk} = I_0 \delta_{jk}$ but $I_{jk} = I_0 \left[ \delta_{jk} + \frac{1}{2} c_{jk} \right]$, where $c_{(\mu\nu)} = c_{\mu\nu} + c_{\nu\mu}$. This would cause the sphere to wobble as it rotates. The absence of such wobbling in pulsar timing data makes it possible to place bounds on the $c$ coefficients for the neutrons that compose such pulsars [15].

While the pulsar measurements are not a particularly sensitive way to constrain Lorentz violation, there are many systems and reaction processes that can be used to set strong bounds on Lorentz and CPT violation. One obvious option would be experiments with neutral mesons. CP violation has only been observed in meson oscillation experiments, which also offer a convenient way to search for CPT violation. Tests of Lorentz and CPT violation are now a standard part of meson oscillation experiments using $K$, $D$, and $B$ mesons [16, 17, 18].

The rate of CPT violation in meson experiments is controlled by the parameter $\delta$. In terms of the SME coefficients, the $\delta$ for kaon oscillations is

$$\delta_K = \gamma \frac{v_{\mu} \left( a_{d}^{\mu} - a_{s}^{\mu} \right)}{m_{KL} - m_{KS}}.$$  

This depends on the $a$ coefficients for the two quark species $d$ and $s$ that make up the mesons, the $K_L$-$K_S$ mass difference of $3.5 \times 10^{-12}$ MeV, and the velocity and Lorentz factor of the kaons. The dependence on velocity has important effects. Prior to the development of the SME, sensitivity to CPT violation in kaon experiments was generally parameterized as a sensitivity to a $K^0$-$\bar{K}^0$ mass difference. However, such a particle-antiparticle mass difference is forbidden in a local quantum field theory. Two experiments with the same naive mass difference sensitivity may have very different sensitivities to the SME $a$ parameters; experiments done with more highly boosted mesons are more sensitive to local CPT violation. The best current experimental constraints on the SME coefficients coming from meson data are at the $10^{-21}$ GeV level for kaons and the $10^{-16}$ for other meson species.

While these meson experiments focus primarily on searches for CPT violation, many more coefficients can be studied by looking at violations of rotation and boost symmetries. For testing such symmetries, precision laboratory experiments are obvious options. Atomic clock and resonant cavity measurements have produced highly sensitive bounds on proton, neutron, electron and photon coefficients. Such experiments use rotating apparatuses to test whether observed resonant frequencies depend on the absolute orientation of electron, nuclear, and photons spins. The rotating setup may be implemented in two ways. The simpler way is just to utilize the Earth’s rotation, which changes the orientation of a fixed device in the lab frame over the course of the planet’s sidereal rotation period. More elaborate experiments also use an active rotation in the laboratory, placing the entire experiment on a turntable. The two forms of rotation complement each other; for example, by combining them, it is possible to study all the spacelike components of the various vectors $b$.

These tests of isotropy may also be extended to tests of boost invariance, although this requires a moving laboratory. As the Earth revolves around the sun, our velocity changes slightly, and so by making precision clock measurements at different times of year, it is possible to constrain $b_0$ and other boost invariance violation coefficients. However, the constraints on boost invariance violation are generally weaker; they are naturally suppressed by a factor of $v_0 \approx 10^{-4}$, since they involve comparisons of measurements made in rest frames that differ by that small velocity.
Atomic clock experiments and other precision laboratory experiments (such as measurements of muon magnetic moments, Penning trap frequencies, and short-distance spin-dependent forces) are extremely useful for constraining Lorentz and CPT violation. However, many of the strongest constraints actually come from astrophysical experiments that have relative poor intrinsic sensitivities. These low sensitivities can be overcome by using either of two phenomena that can be found astrophysically but not in the lab: very long distances and very high energies.

Observations of cosmologically distant objects, via photons that have propagated for $10^8$–$10^9$ years, can be very sensitive to small alterations in the electromagnetic sector. For example, most of the $k_F$ coefficients and all of the $k_{AF}$ coefficients give rise to birefringence in vacuum. Photons with different polarizations propagate at different speeds, leading to gradual a rotation in the polarization direction for linearly polarized radiation. Yet no evidence of such birefringence is seen, even at extreme distances [19, 20, 21, 22, 23]. In quasar jets, for example, the polarized radiation is mostly produced by the synchrotron process, with electrons orbiting around magnetic field lines that point approximately parallel to the jet itself. The radiation from the electrons is preferentially polarized perpendicular to the field direction, and this polarization structure is exactly what is observed in the light from most quasars, independent of how far they are away.

The synchrotron processes that produce this radiation operate at extremely high energies, sometimes above 1 PeV. Just as a long line of sight can magnify a very small photon propagation effect, these immense electron energies can magnify other kinds of Lorentz-violating phenomena. In particular, observations of the synchrotron and inverse Compton radiation emitted by these incredibly energetic electrons provide the best direct tests of boost invariance violation for electrons.

3. Bounds From Synchrotron Radiation

Radiation by energetic electrons can be a sensitive probe of the electron $c$ coefficients. It has been possible to place very strong bounds on $c$ by looking at the relationships between energy, momentum, and velocity in the $c$-modified theory. At ultrarelativistic energies, the bounds on $c$ that can be derived from observing the emissions of an electron with Lorentz factor $\gamma$ scale as $\gamma^{-2}$. This strong dependence on $\gamma$ is a consequence of the rapid growth in the importance of $c$ with energy. If $c$ is $O(m/M_P)$, its effects will become dominant at scale $E \sim \sqrt{mM_P}$, and the Lorentz factor at this scale is $\gamma \sim \sqrt{M_P/m}$. So if no effects of Lorentz violation are observed up to some Lorentz factor $\gamma$, this constrains $c$ to be smaller than $O(\gamma^{-2})$.

If rotation and boost symmetries are broken, the conventional relations between energy, momentum, and velocity $-E = \gamma m$ and $\vec{p} = \gamma m \vec{v}$ no longer need to hold. In the presence of $c$, the velocity of an electron with momentum $\vec{p}$ and energy $E$ is [24]

$$v_k = \frac{1}{E + c_{(ij)} \pi_j} (\pi_k - c_{kj} \pi_j - c_{jk} \pi_j + c_{jk} c_{jl} \pi_l) - c_{0k}. \quad (6)$$

This formula can be found either as a group velocity or using the Heisenberg equation of motion for $\pi$.

From eq. (6), we can see why the effects of $c$ become large at the scale $\sqrt{mM_P}$. According to eq. (6), the velocity might become superluminal when $|\vec{\pi}|/E \approx 1 - |c|$, where $|c|$ is a characteristic size for the Lorentz-violating coefficients. This gives us an estimate of the maximum value of $\gamma$ that can be achieved before new physics must come into play if some form of causality is to be preserved. For ultrarelativistic particles, $\gamma \approx \left[2(1 - |\vec{\pi}|/E)\right]^{-1/2}$, and this diverges at an energy scale $E_{\text{max}} \sim m/|c| \sim \sqrt{mM_P}$. Above this scale, the description of the Lorentz violation through an effective field theory containing only $c^\mu \alpha$ terms will generally break down, and higher dimension operators should become important.

We can exploit the Lorentz violation in the relationship between momentum and velocity to place bounds on $c$. There are two crucial and complementary effects. The first effect is that the
maximum electron speed in a given direction \( \hat{v} \) is generally different from 1. The value of this new maximum velocity can be easily calculated to first order in \( c \),

\[
(v_j \hat{e}_j)_{\text{max}} = 1 - c(\mathbf{k}) \hat{e}_k - c(\mathbf{0}) \hat{e}_j,
\]

(7)

If this is less than one, it can have readily observable consequences for the synchrotron spectrum.

The observable effects derive from the fact that the radiation by a charged particle depends on its velocity—not its momentum or its energy. This continues to be the case even in the presence of Lorentz violation, since the velocity dependence is a consequence of gauge invariance alone. The power emitted by a synchrotron electron is spread over frequencies less than the critical frequency

\[
\nu_c = \frac{3}{4\pi} \gamma^3 \omega_B \sin \theta = \frac{3}{4\pi m_e^2} \gamma^2 \left| eB \right| \sin \theta,
\]

(8)

where \( \theta \) is the angle between the magnetic field and the electron velocity \( \vec{v} \). Most of the power is emitted close to this frequency, and above \( \nu_c \), the radiated power falls off very rapidly.

The \( \nu_c \) in (8) represents the critical frequency for the emissions of a single electron. If a source contains significant numbers of electrons with velocities up to some maximum Lorentz factor \( \gamma_{\text{max}} \), then the observed cutoff in the spectrum will be at the cutoff frequency for the most energetic electrons—\( \nu_c = \frac{3}{4\pi m_e^2} \gamma_{\text{max}}^2 \left| eB \right| \sin \theta \).

This result for \( \nu_c \) allows one to infer \( \gamma_{\text{max}} \) from observations of synchrotron spectra. However, the analysis required to find \( \gamma_{\text{max}} \) is fairly involved. From (8), we can infer that \( \gamma_{\text{max}} \propto \sqrt{\frac{\nu_c}{|B|}} \).

The cutoff at \( \nu_c \) is an obvious feature of synchrotron spectra, and a lower bound on this cutoff frequency can easily be obtained from any spectrum that clearly has a synchrotron origin. However, the tricky part is calculating the strength of the magnetic field. The magnetic field is generally taken to be the minimum energy field which can generate the low-frequency part of the synchrotron spectrum. This turns out to depend on most of the parameters that must be fitted from the observed spectrum. Moreover, by considering only the low-frequency part of the spectrum, the value of \( B \) can be inferred independently of the measurement of \( \nu_c \) that characterizes the high-energy part of the spectrum. Ultimately, \( \gamma_{\text{max}} \) depends on at most the seventh root of any fit parameter (except \( \nu_c \)) that might be in error, and so its value can be quite robust.

If the existence of electrons with Lorentz factors up to some value \( \gamma_{\text{max}} = (1 - v_{\text{max}}^2)^{-1/2} \) moving in the \( \hat{e} \)-direction has been observed, this implies

\[
c_{jk} \hat{e}_j \hat{e}_k + c(\mathbf{0}) \hat{e}_j < \frac{1}{2 \gamma^2_{\text{max}}}.
\]

(9)

Radiation from relativistic sources is tightly beamed, into a narrow pencil of angles around the velocity \( \vec{v} \) of the emitter. This means the relevant direction \( \hat{v} \), in the bounds based on observations of a given source, is the direction from the source to the Earth. The most energetic observed synchrotron spectrum is that of the nearby Crab nebula, which contains electrons with Lorentz factors up to \( \gamma_{\text{max}} \approx 3 \times 10^5 \) [25, 26, 27]. For more distant, less well studied high-energy sources, the largest inferred Lorentz factors are typically in the \( 10^7-10^8 \) range. (While many of these sources probably do have electron populations extending up into the PeV range, the small number of electrons present at the high-energy end of the spectrum makes observations of their distinctive emissions difficult.)

However, these synchrotron observations are insufficient to place two-sided bounds on the electron \( c \) coefficients. The reason is simple. If Lorentz violation decreases electron velocities, there must be a maximum electron velocity and hence Lorentz factor; this maximum \( \gamma \) obviously cannot be smaller than any \( \gamma \) that is actually observed. If the Lorentz violation increases electron velocities instead, there will be no such maximum \( \gamma \); there will, however, be a complementary effect, whose effects would also be observable.
4. Bounds From Inverse Compton Scattering

The complementary effect is that there may be a maximum energy available to electrons with subluminal velocities. This will also generally depend on the direction of a particle’s motion. The maximum energy for a particle moving with a speed less than 1 in the direction \(\hat{e} \) is

\[
\frac{E}{m} = \frac{1}{\sqrt{-2c_{ik}\hat{e}_k\hat{e}_i - 2c_{(ij)\hat{e}_j}}}, \tag{10}
\]

which depends on the same combination of Lorentz violation coefficients as did the maximum value of \( \gamma \). However, the maximum subluminal energy \( E \) is proportional to the inverse square root of \( c \), which is not surprising, since for vanishing \( c \), \( E \) must be infinite. As is obvious from (10), such a maximum value for \( E \) need not always exist. In fact, according to (7), this maximal subluminal energy does not exist precisely when the maximum speed in the relevant direction is less than or equal to one.

An obvious question is what happens to particles moving with speeds \( v > 1 \). The answer is that there is vacuum Cerenkov radiation, a phenomenon that is unique to Lorentz-violating theories. This process is analogous to ordinary Cerenkov radiation in matter; when charged particles move faster than the phase velocity of light, they will emit radiation. This can occur as a result of Lorentz violation in either the electromagnetic or matter sectors. The process has been studied in the context of the minimal SME \([28, 29, 30, 31, 32, 33]\), but understanding it in detail is difficult, since new physics may appear at the same scale as the Cerenkov threshold given in eq. (10). What is clear about vacuum Cerenkov radiation is that it is an extremely efficient energy loss mechanism. (A naive estimate of the radiation rate is \( 10^{14} \text{ GeV s}^{-1} \).) Electrons moving superluminally would radiate almost all their energy away in times that are minuscule compared with their synchrotron lifetimes; they would have effectively no time to interact in other ways before losing almost all their energies.

So if it is possible to observe conventional radiation patterns coming from electrons with energies up to some value \( E_{\text{max}} \), such electrons must be moving with speeds \( v < 1 \). This immediately implies

\[
-\frac{1}{2(E_{\text{max}}/m)^2} < c_{ik}\hat{e}_k\hat{e}_i + c_{0j}\hat{e}_j, \tag{11}
\]

which is the other side of the one-sided bound found in eq. (9). While synchrotron radiation provides a relatively direct probe of a charge’s velocity, inverse Compton (IC) radiation provides a lower bound on \( E_{\text{max}} \). The highest-energy photons that are emitted by astrophysical sources arise in IC processes. Low-energy photons (which often come from synchrotron emission) scatter off ultrarelativistic electrons. An electron may transfer a substantial fraction of its own energy to a photon during such a collision, resulting in the emission of photons whose energies may range almost up to the scale of the highest energy electrons.

All collisions of photons with ultrarelativistic electrons are practically all head-on when viewed in the electron’s frame. Overtaking collisions are extremely rare, occurring only in a minuscule range of observer frame solid angles. In the observer frame, this means that the emitted photon is propagating in essentially the same direction as the initial electron. In the observer’s rest frame, the relationship between the initial and final photon energies \( \epsilon_i \) and \( \epsilon_f \) in the observer frame is

\[
\epsilon_f = \epsilon_i \gamma^2 \frac{(1 - \cos \psi_i)(1 + \cos \theta \cos \psi'_f)}{1 + \gamma^2 \frac{m}{\epsilon_i}(1 - \cos \theta)}, \tag{12}
\]

(where \( \psi_i, \psi'_f \), and \( \theta \) are the scattering angles). The Lorentz factor \( \gamma \gg 1 \) is that of the electron, and depending on the value of \( \gamma \epsilon_i/m \), the final photon energy may be \( \mathcal{O}(\gamma) \) to \( \mathcal{O}(\gamma^2) \) larger than its initial energy. If \( \gamma \epsilon_i/m \approx 0.1 \), for example, then the maximum energy that can be carried off
by the photon is \((\epsilon_f)_{\text{max}} \approx 0.4\gamma m\). So the IC process allows some electrons to transfer sizable fractions of their energy to photons, and observed IC photon energies range up to \(\sim 100\) TeV.

Models of a source’s structure can yield information about the energies of the electron responsible for the inverse Compton emission. In general, the models typically require maximum electron energies that are several times larger than the highest observed photon energies, because IC scattering events do not transfer all of the high-energy electrons’ energies to the photons. However, to get a more robust bound, one may take \(E_{\text{max}}\) to be the highest actually observed photon energy; this conservative estimate usually differs from a model-derived bound by less than an order of magnitude. The only input required from a model is that a source’s TeV \(\gamma\)-ray emission is well described by the IC process. This makes it easier to get reliable lower bounds on \(E_{\text{max}}\) than on \(\gamma_{\text{max}}\), and there has been a remarkable explosion in the number of TeV \(\gamma\)-ray sources observed in the last ten years, thanks especially to the H.E.S.S atmospheric Cerenkov telescope in Namibia.

There are nine components of \(c\) that can be bounded with the astrophysical data—the three \(c_{(ij)}\) and the six \(c_{(jk)}\). Each of the inequalities derived from (9) and (11) generally couples all nine of the coefficients in a nontrivial way. These bounds may be fairly awkward. However, the coupled bounds may be translated into bounds on the separate coefficients by means of linear programming, with the ultimate result that all nine of the coefficients are bounded on both sides, at the \(10^{-15}–10^{-17}\) levels.

5. Conclusion
Other high-energy astrophysical processes can also be used to produce competitive bounds on the \(c\) coefficients. For example, if \(c\) is nonzero, the photon decay process, \(\gamma \rightarrow e^+ + e^-\) may be allowed for sufficiently energetic electrons. The fact that IC photons do not decay in this fashion generates another bound, although a slightly weaker one than that discussed above. However, the bounds related to the absence of photon decay can be extended to other species of daughter particles; any \(\gamma \rightarrow X^+ + X^-\) will occur rapidly if it is allowed, above a threshold \(\sim m_X/\sqrt{|c_X|}\). The bounds are weaker than the electron bounds by a factor of \(m_X^2/m_e^2\) [34, 35].

There are also generalizations of all these bounds to higher-dimension operators. However, the number of anisotropic operators increases rapidly with the operator dimension, and there has so far been no systematic study of all the possible bounds to be derived from the astrophysical observations. However, there have been analyses of isotropic Lorentz violating operators (that is, operators that violate boost invariance only) at higher dimensions.

Lorentz and CPT violation are active areas of study in physics. Understanding the possible ways that these symmetries could be violated has become a major focus of precision in recent years. There is an active experimental program testing these fundamental symmetries, and if violations are discovered, that would be of momentous importance and would provide important guidance in the future searches for new physics. And there are many interesting forms of Lorentz and CPT violation that still remain to be constrained.

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