THE EFFECT OF RADIATION REACTION FROM THE BURSTS OF MAGNETARS

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ABSTRACT

The two radio bursts from SGR 1935+2154 detected by Bochenek et al. (2020) and The CHIME/FRB Collaboration et al. (2020b) are similar to the features of cosmological fast radio bursts (FRBs). In this paper, we try to clarify that whether these two radio bursts are FRBs. First, we compare these two radio bursts with previous observations of normal pulsars and magnetars, and find that these two radio bursts should be emitted from positions different from that of radio pulses of normal pulsars/magnetars. Furthermore, we infer that if radio/X-ray/γ-ray bursts of a magnetar are emitted from these positions, an extra torque will exert on the magnetar so that an extra increment will contribute to the glitch of a magnetar (no matter the glitch originates mainly from a starquake or a catastrophic superfluid vortex unpinning), and a precession will be induced. We argue that these two radio bursts don’t have the same origin with cosmological FRBs by discussing the cause of formation of periodically repeating FRBs at last.

Keywords: fast radio bursts - stars: magnetars - stars: neutron stars

OBSERVATIONS

Bochenek et al. (2020) and The CHIME/FRB Collaboration et al. (2020b) report the detection of two bright radio pulses from SGR 1935+2154. These two radio pulses are different from previous radio pulses from pulsars. We list some aspects as follows.

For SGR 1935+2154 (in an X-ray active period now):
(i) Never have we seen such energetic radio bursts from Milk Way neutron stars (NSs).
(ii) There is frequency up-drift.
(iii) Time interval between these two radio bursts is $\sim 37$ ms.

For the rest radio pulsars and magnetars:
(a) No frequency drifting has been detected, even if the pulsar/magnetar shows glitches or outbursts.
(b) Magnetars show long-lasting continuous radio emission during the outburst (Olausen & Kaspi 2014).
(c) The rotation periods of magnetars are usually $1 - 10$ s, so the time interval of $\sim 37$ ms in SGR 1935+2154 seems not to be the rotation period.

Therefore, we conclude from these differences that these two radio bursts from SGR 1935+2154 are emitted from positions different from that of periodic radio pulses of pulsars/magnetars. Note that multiband observation confirms the radio burst/X-ray burst associations (Mereghetti et al. 2020; Li et al. 2020; Ridnaia et al. 2020; Tavani et al. 2020,?), similarly, the X-ray bursts and gamma-ray bursts induced by some instabilities may also occur at positions different from that of normal radio pulses. This is, of course, a mediocre conclusion.

There is another issue that is there any relationship between these two radio bursts and fast radio bursts (FRBs). If FRBs have the same generation mechanism with that of the two radio bursts (part of FRBs also show frequency drifting), the FRBs should also not emitted from the position where the normal radio pulses emitted. Especially, under the scenario of magnetic reconnection, the radiation locations of FRBs should be lower than that of the two bursts, since the closer to the magnetar surface, the larger the magnetic energy density.

But before using the galactic radio bursts to study the cosmic FRBs, we should consider the compatibility between the two X-ray burst-induced radio bursts and cosmological FRBs (especially the periodically repeating FRBs).
1. RADIATION REACTION

If the reaction of a burst on an NS is not centripetal, an extra torque may exert on the NS. The upper limit of the change in angular momentum can be estimated as

$$\Delta J_{\text{max}} = \frac{E_{\text{burst}}}{c} \cdot r = 3.3 \times 10^{40} \left( \frac{E_{\text{burst}}}{10^{45} \text{ erg}} \right) \left( \frac{r}{10^{6} \text{ cm}} \right) \text{ erg} \cdot \text{s},$$

(1)

where $c$ is the speed of light, $E_{\text{burst}}$ is the total energy release of the burst (see, e.g., SGR 1806-20, Hurley et al. 2005), and $r$ is the distance from the acting point of force to the center of mass. Correspondingly, the upper limit on change of rotation frequency is

$$|\Delta \nu|_{\text{max}} = \frac{\Delta J_{\text{max}}}{2\pi I} = 5.3 \times 10^{-6} \left( \frac{E_{\text{burst}}}{10^{45} \text{ erg}} \right) \left( \frac{r}{10^{6} \text{ cm}} \right) \left( \frac{I}{10^{45} \text{ g cm}^2} \right)^{-1} \text{ s}^{-1},$$

(2)

where $I$ is the rotational inertia of the magnetar. Therefore, this radiation reaction could be one of the sources of a glitch. And, the upper limit on induced angular velocity of precession is given by

$$\dot{\varphi}_{\text{max}} \sim 1.6 \times 10^{-5} \left( \frac{L_{\text{burst}}}{3 \times 10^{45} \text{ erg s}^{-1}} \right) \left( \frac{r}{10^{6} \text{ cm}} \right) \left( \frac{I}{10^{45} \text{ g cm}^2} \right)^{-1} \left( \frac{P}{1 \text{ s}} \right) \text{ rad} \text{ s}^{-1},$$

(3)

where $L_{\text{burst}}$ is the luminosity of the burst, and $P$ is the rotational period.

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**Figure 1.** The geometry between the frame of principal axis of inertia $Ox'y'z'$ and lab frame $Oxyz$. $\varphi$ (precession angle), $\psi$ (rotation angle), $\theta$ (nutation angle) are three Euler angles. $ON$ is the intersecting line between the area $Ox'y'$ and area $Oxy$.

Except for the precession, there should be also an induced nutation. Let’s consider this problem more analytically. We treat an NS as an Euler gyro that the rotational inertias along the $x'$-axis and $y'$-axis are the same (see Figure 1), i.e., $I'_{11} = I'_{22}$. The rotation of this NS is determined by Euler’s dynamics equations (in the frame $Ox'y'z'$) \(^1\)

$$I'_{11}\dot{\omega}'_x - (I'_{22} - I'_{33})\omega'_y\omega'_z = M'_x,$$

(4)

$$I'_{22}\dot{\omega}'_y - (I'_{33} - I'_{11})\omega'_z\omega'_x = M'_y,$$

(5)

$$I'_{33}\dot{\omega}'_z = M'_z,$$

(6)

\(^1\) The derivation can be found in many textbooks of theoretical mechanics.
where

\[ M'_i = \eta_i \frac{L_{\text{burst}}}{c} r, \quad (i = x, y, z) \]  

(7)

is the component of the torque induced by radiation reaction with \( \eta_i \in (0, 1) \) being the contribution of the burst luminosity to the torque (note that \( \eta_x, \eta_y \) and \( \eta_z \) compete with each other), and \( \omega'_i \) is the component of the spin vector.

Under the initial conditions that \( \omega'_x(t = 0) = \omega'_y(t = 0) = \omega'_z(t = 0) = 0 \), and the assumption that

\[ L_{\text{burst}} = \begin{cases} L_p \left( \frac{t}{t_p} \right)^\alpha & \text{if } 0 < t < t_p, \\ L_p \left( \frac{t}{t_p} \right)^\beta & \text{if } t_p < t < t_e, \end{cases} \]

(8)

with \( L_p \) being the peak luminosity, \( t_p \) being the peak time and \( t_e \) being the duration time of the burst, we may solve the above equations (4)-(6). From equation (6), there is

\[ \omega'_z(t) = \begin{cases} \frac{\eta_x L_{\text{burst}}}{I_{zz} c} \left[ \frac{t^{\alpha+1}}{t_p^\alpha} + t_p^\alpha \right] + \omega_0 if 0 < t < t_p, \\ \frac{\eta_x L_{\text{burst}}}{I_{zz} c} \left[ \frac{t^{\beta+1}}{(\beta+1)t_p^\beta} + t_p^\beta \right] + \omega_0 if t_p < t < t_e. \end{cases} \]

(9)

From equations (4) and (5), we have

\[ \dot{\omega'_x} = \frac{\omega'_y}{\omega'_x} + \varepsilon^2 \omega'_z \dot{\omega'_x} = \frac{M'_x}{I_{11}} \varepsilon \frac{M'_y}{I_{22}} \omega'_z - \frac{M'_z}{I_{11}} \omega'_x, \]

(10)

and

\[ \dot{\omega'_y} = \frac{\omega'_z}{\omega'_y} + \varepsilon^2 \omega'_z \dot{\omega'_y} = \frac{M'_y}{I_{22}} \varepsilon \frac{M'_z}{I_{11}} \omega'_x - \frac{M'_z}{I_{22}} \omega'_y, \]

(11)

where \( \varepsilon = (I_{zz} - I_{11})/I_{22} \). Therefore, equations (10) and (11) can be solved numerically for the given \( \eta_i \) and \( \varepsilon \). Since the value of \( \varepsilon \) should be very small, we simplify the discussion as

\[ I_{11} \dot{\omega'_x} \approx M'_x, \]

(12)

\[ I_{22} \dot{\omega'_y} \approx M'_y, \]

(13)

so that there are

\[ \omega'_x(t) = \begin{cases} \frac{\eta_x L_{\text{burst}}}{I_{11} c} \left[ \frac{t^{\alpha+1}}{t_p^\alpha} + t_p^\alpha \right] & \text{if } 0 < t < t_p, \\ \frac{\eta_x L_{\text{burst}}}{I_{11} c} \left[ \frac{t^{\beta+1}}{(\beta+1)t_p^\beta} + t_p^\beta \right] & \text{if } t_p < t < t_e, \end{cases} \]

(14)

and

\[ \omega'_y(t) = \begin{cases} \frac{\eta_y L_{\text{burst}}}{I_{22} c} \left[ \frac{t^{\alpha+1}}{t_p^\alpha} + t_p^\alpha \right] & \text{if } 0 < t < t_p, \\ \frac{\eta_y L_{\text{burst}}}{I_{22} c} \left[ \frac{t^{\beta+1}}{(\beta+1)t_p^\beta} + t_p^\beta \right] & \text{if } t_p < t < t_e. \end{cases} \]

(15)

Clearly, \( \omega = \sqrt{\sum_{i=x,y,z} \omega'_i^2} \) is not a constant, precession and nutation should exist simultaneously.

After the burst, we input the zeroth approximation

\[ \begin{align*}
\omega'_x(t = t_e) &= \frac{\eta_x L_{\text{burst}}}{I_{11} c} \left[ \frac{t^{\alpha+1}}{t_p^\alpha} + t_p^\alpha \right] + \omega_0 \\
\omega'_y(t = t_e) &= \frac{\eta_y L_{\text{burst}}}{I_{22} c} \left[ \frac{t^{\alpha+1}}{t_p^\alpha} + t_p^\alpha \right] + \omega_0 \\
\omega'_z(t = t_e) &= \frac{\eta_z L_{\text{burst}}}{I_{33} c} \left[ \frac{t^{\alpha+1}}{(\alpha+1)t_p^\alpha} + t_p^\alpha \right] + \omega_0
\end{align*} \]

(16)

into the source free Euler’s dynamics equations as the new initial conditions. The solution under \( t > t_e \) is

\[ \begin{align*}
\omega'_x(t) &= A \cos \left[ \varepsilon \omega'_x(t = t_e) t + \phi_0 \right] \\
\omega'_y(t) &= A \sin \left[ \varepsilon \omega'_y(t = t_e) t + \phi_0 \right] \\
\omega'_z(t) &= \omega'_z(t = t_e)
\end{align*} \]

(17)
where
\[ \phi_0 = \arctan \left( \frac{\omega'_x(t = t_e)}{\omega'_y(t = t_e)} \right) - \varepsilon \omega'_z(t = t_e) t_e, \]  
(18)
\[ A = \frac{\omega'_x(t = t_e)}{\cos[\varepsilon \omega'_z(t = t_e)t_e + \phi_0)}. \]  
(19)

After the burst, there is a precession left since \( \omega = \sqrt{\sum_{i=x,y,z} \omega'_i^2} \) is a constant. The evolution of the three Euler angles can be easily solved through the conservation of angular momentum \(^2\), i.e.,
\[ J \sin \theta \sin \psi = I'_{11} A \cos(\varepsilon \omega'_z(t = t_e)t + \phi_0), \]
(20)
\[ J \sin \theta \cos \psi = I'_{11} A \sin(\varepsilon \omega'_z(t = t_e)t + \phi_0), \]
(21)
\[ J \cos \theta = I'_{33} \omega'_z, \]
(22)
where
\[ J = \sqrt{2I'_{11}^2 A^2 + I'_{33}^2 \omega'_z^2(t = t_e)}. \]
(23)
is the total angular momentum.

Combining equations (20)-(22) gives
\[ \dot{\varphi} = \omega'_z(t = t_e)(1 + \varepsilon) \sec \theta_0, \]
(24)
where
\[ \theta_0 = \arccos \left( \frac{I'_{33} \omega'_z(t = t_e)}{J} \right). \]
(25)
So the precession period can be estimated as \( P_p = \frac{2\pi}{\dot{\varphi}}. \)

2. DISCUSSION

The periodically repeating FRBs suggest that these FRB sources may be binaries or precessing NSs (Chime/Frb Collaboration et al. 2020a). If one considers that FRBs have same origins as that of the two radio bursts, there is a problem that where the periods of there periodically repeating FRBs comes from? No hint shows SGR 1935+2154 belongs to a binary (all the detected magnetars are isolated). Therefore, the sources of periodically repeating FRBs seems quite unlikely binaries. If the sources of periodically repeating FRBs are just binaries, there is a problem that how the orbital motion induce the period of these radio bursts associated X-ray bursts. Let’s turn to the precession scenario. The precessing FRB models demand a fixed angle (position) between the radio beams of the bursts and \( \vec{J} \), this needs the associated X-ray bursts to also occur at a fixed position (this seems to be too optimistic). Besides, if the precession is induced by the radiation reaction, there is still a problem that the burst will easily change the precession (see equation (24)), as well as the periods of periodically repeating FRBs. But current observations have not yet shown distinct change in the FRB periods. So we prefer to believe that, at least, the two radio bursts of SGR 1935+2154 have different origins from that of periodically repeating FRBs Unless all the FRBs are produced by asteroids colliding with NSs (Geng & Huang 2015; Dai 2020), so that the origin of periods of periodically repeating FRBs can be attributed to “external factor ” (asteroid belts).

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