Vacuum force for a massive scalar field in a multiply warped braneworld

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Abstract. We determine the finite temperature Casimir force for a scalar massive field for a pair of parallel plates in a 6D non-factorizable geometry. In this contribution we use the Green’s function method to determine the Casimir force. It can be expressed in terms of two four-dimensional Casimir force contributions: one for the zero mode and the other for a tower of massive modes associated with the extra dimensions. An uncertainty band of 10% around the theoretical standard 4D Casimir force was set up, by limiting the effective 4D Casimir force within such band yields the bound \( k/r_z \gg 1.97 \times 10^9 \) TeV.

1. Introduction

The idea of considering our observable four-dimensional universe as a subspace of a higher dimensional spacetime is seriously considered in high energy physics. These models with extra dimensions were proposed to solve various fundamental problems such as the problem of hierarchy among others (see [1, 2, 3]). Several ideas in this direction have evolved to explore various implications in cosmology, astrophysics and particle physics.

A recent work [4] has considered extension of the Randall-Sundrum model to more than one extra dimension, where the warped compact dimensions get successive more
warping. The resulting geometry of the $D$ dimensional spacetime reduces to $M^{1,D-1} \rightarrow \left(\left[ M^{1,3} \times S^1 / Z_2 \right] \times S^1 / Z_2 \right) \times \ldots$ with $D-4$ warped spacelike dimensions. One of the interesting characteristics of such a model is that the resulting geometry is similar to a combination RS and ADD scheme of compactification. The Kaluza-Klein (KK) modes for a massive scalar field and gravitational field have been studied in [5] and [6].

The prevalence of spacetimes with extra dimensions has motivated research on possible corrections to Casimir effect acting between parallel plates due to the existence of extra dimensions. In general the Casimir effect may be defined as the stress on the boundary surface when a quantum field in its vacuum state is confined to a finite volume of space.

In this work we analyse the Casimir force in this model as a test for the phenomenological viability of such multiply warped spacetime. To obtain the Casimir force for the 6D massive scalar field we use the Green’s function method.

2. The model
We work in a 6D non-factorizable geometry, which is doubly warped and compactified with a $Z_2$ orbifold in each of the extra dimensions [4]. The metric $g_{MN}$ in this scenario is given by

$$ds^2_6 = b^2(z) [a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu - R_y^2 dy^2] - r_z^2 dz^2,$$

(1)
greek indexes denote 4D space-time coordinates, $\eta_{\mu\nu}$ is the Minkowski metric with signature $(+, -, -, -)$, while the coordinates $y$ and $z$ represent the extra spatial dimensions. $R_y$ and $r_z$ are the two moduli corresponding to the $y$ and $z$ directions and $a(y)$ and $b(z)$ are the corresponding warp factors. The warp factors are given by

$$a(y) = e^{-\rho |y|}, \quad \rho = \frac{R_y k}{r_z \cosh k \pi},$$

(2)

$$b(z) = \frac{\cosh (kz)}{\cosh (k \pi)}, \quad k = r_z \sqrt{-\Lambda / M^4},$$

(3)

where $M$ is the 6D Planck constant and $\Lambda$ the bulk cosmological constant.

It is worth mentioning that this model can be seen as a box-like picture of the bulk, where the walls of the box are 4-branes, and the intersection of two 4-branes may be identified with a 3-brane. The Standard Model brane (SM brane) depends on the values of the parameters, which are determined in terms of $\Lambda$, $r_z$, $R_y$. We can identify our SM brane by requiring the desired TeV scale, therefore we must identify the SM brane with the one at $y = \pi$, $z = 0$, while the 3-brane located at $y = 0$, $z = \pi$ must be identified with the Planck brane. The other two 3-branes located at $y = 0$, $z = 0$, and $y = \pi$, $z = \pi$ have intermediate energy scale.

For the resolution of the hierarchy problem in this model we need that

$$10^{-16} = \frac{e^{-\rho \pi}}{\cosh k \pi}.$$ 

(4)

This equation, along with the condition (2), implies large warping in one direction and small in the other.

Finally, we mention that in this model the KK modes of bulk scalar fields have been studied [5], however, when the higher dimensional scalar field is massive, there does not exist a zero mode solution, so in order to overcome this problem we would now address the nature of Kaluza Klein modes of scalar fields in the six dimensional braneworld.
3. Massive scalar field

In this section we start by computing the mass spectrum and eigenfunctions for the massive scalar field. Let us consider the 6D action for a massless scalar field $\Phi$ in the metric (1)

$$S = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz \sqrt{-g} \left[ g_{MN} \partial_M \Phi \partial_N \Phi - M_5^2 \Phi^2 \right], \tag{5}$$

where $g_{MN}$ is the six dimensional metric (1). As it is well known [7], when the higher dimensional scalar field is massive there does not exist a zero mode solution. In order to obtain a zero mode we need to modify the action and include boundary mass terms, as it was made in [8]. Thus we propose the following boundary mass terms

$$S' = -\int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz \sqrt{-g} \left\{ \frac{2\tilde{c}\rho}{R^2b^2(z)} [\delta(y) - \delta(y - \pi)] + \frac{2\tilde{d}k}{r_z^2} [\delta(z) - \delta(z - \pi)] \right\} \Phi^2, \tag{6}$$

where $\tilde{c}$ and $\tilde{d}$ are dimensionless constants. Assuming solutions of the form $\Phi(x^\mu, y, z) = \phi(x^\mu) \xi(y) \chi(z)$ and performing separation of variables in the field equations for the massive scalar field, it is straightforward to obtain the three differential equations

$$\left( \Box_4 + m_{nN}^2 \right) \phi(x^\mu) = 0, \tag{7}$$

$$\frac{1}{R_y^2} \partial_y \left( a^4 \partial_y \xi(y) \right) + m_n^2 \phi \xi(y) - m_{nN}^2 \phi a^2 \xi(y) - \frac{2\tilde{c}a^4}{R^2} \rho [\delta(y) - \delta(y - \pi)] \xi(y) = 0, \tag{8}$$

$$\frac{1}{r_z^2} \partial_z \left( b^5 \partial_z \chi(z) \right) + m_n^2 \phi \chi(z) - M^2 b^5 \chi(z) - \frac{2\tilde{d}k}{r_z^2} b^5 [\delta(z) - \delta(z - \pi)] \chi(z) = 0. \tag{9}$$

Notice that $m_n$ and $m_{nN}$ correspond to the spectra of the modes for the two compact coordinates $z$ and $y$ respectively. From the 4D point of view Eq. (7) corresponds to an effective massive scalar field $\phi(x^\mu)$ whose mass $m_{nN}$, picks up two independent contribution corresponding to the $y$ and $z$ coordinates. Our first task is to determine this mass spectrum and the associated eigenfunctions.

3.1. Mode descomposition

In this subsection we focus on the $y, z$ dependence, Eqs. (8) and (9). To obtain the KK mass spectrum we first determine the tower denoted by $m_n$ and then using it as the input to the Eq. (8) to determine the mass tower $m_{nN}$.

In order to implement the boundary conditions in the $z$ direction, we integrate the equation (9) over an interval across the two boundaries at $z = 0, \pi$, respectively, which allows us to match the modes across the brane along the coordinate $z$.

For reasonable values of the parameter $k$, we consider the following form of the warp factor $b(z) \sim e^{-k(\pi - z)} = e^{-kz'}$. Performing the change of variable $\tilde{z} = \frac{r_z m_n}{k} e^{kz'}$ and redefining the function as $\tilde{\chi} = e^ {\nu k z'} \chi$, $\nu = 5/2$, one gets the Bessel equation:

$$\tilde{z}^2 \frac{d^2 \tilde{\chi}_n}{d\tilde{z}^2} + \tilde{z} \frac{d\tilde{\chi}_n}{d\tilde{z}} + (\tilde{z}^2 - \alpha^2) \tilde{\chi}_n = 0, \tag{10}$$
where $\alpha = \sqrt{\nu^2 + \frac{M^2 r_z^2}{k^2}}$. When $m_{n=0} = 0$ the solution has the form

$$
\chi_0(z) = \begin{cases} 
\sqrt{\frac{2\nu}{\pi}} e^{\nu k z} Y_\nu(k z), & \nu > 0 \text{ localized in } z = \pi \\
\sqrt{\frac{2\nu}{\pi}} e^{-\nu k z} J_\nu(k z), & \nu < 0 \text{ localized in } z = 0 
\end{cases}
$$

In the case $m_n > 0$ the normalized modes become

$$
\chi_n(z) = \frac{e^{\nu k z}}{d_1} \left[ J_\alpha \left( \frac{r_z m_n}{k} e^{k z} \right) + d_2 Y_\alpha \left( \frac{r_z m_n}{k} e^{k z} \right) \right],
$$

where $d_{1,2}$ are arbitrary constants and $J_\alpha$ and $Y_\alpha$ are Bessel’s functions of order $\alpha$. The constants in (12) are chosen in such a way that the orthogonality relations are

$$
\int_{-\pi}^{\pi} dz b^3(z) \chi_n^*(z) \chi_m(z) = \delta_{mn},
$$

fulfilling the boundary condition $\left. \frac{d\chi_n(z)}{dz} - \tilde{d} k \chi_n(z) \right|_{z=0,\pi} = 0$. The KK masses $m_n$ are determined by imposing the boundary condition on the solution (12). Notice that the appropriate boundary condition depends on whether the 5D field is odd or even under the orbifold $Z_2$ symmetry. In this work we consider that the 5D field is even under the $Z_2$ symmetry. Thus we have

$$
\chi_n(z) \rightarrow \chi_n(-z).
$$

Imposing the boundary conditions in the low energy regime $m_n r_z \ll k$ and $k r_z \gg 1$, we have the approximated KK mode masses

$$
m_n \simeq \left( n + \frac{\alpha}{2} - \frac{3}{4} \right) \frac{\pi k e^{-k \pi}}{r_z}, \quad n = 1, 2, \ldots, \alpha > 0.
$$

After obtaining the $z$ dependent part of the KK modes and the corresponding spectrum, we now can solve the $y$ dependent part of the modes. Adopting a similar technique, we can rewrite the Eq. (8) as

$$
y^2 \frac{d^2 \tilde{\xi}_N}{dy^2} + y \frac{d \tilde{\xi}_N}{dy} + (y^2 - \beta^2) \tilde{\xi}_N = 0,
$$

where $\beta = \sqrt{4 + \left( \frac{m_n R}{\rho} \right)^2}$ and, once again, we have performed the change of variable $\tilde{y} = \frac{R m_n e^{\rho y}}{\rho} \tilde{y}$ and redefining the function as $\xi = \tilde{y}^2 \tilde{\xi}$. It is worth mentioning that the equation (8) is invariant under a change of sign in the coordinate $y$ and therefore it is enough to solve the equation in the region $y > 0$.

When $m_{n=0,N=0} = 0$, the solution of Eq. (16) yields a zero mode

$$
\tilde{\xi}_0(y) = \begin{cases} 
\sqrt{\frac{(-1)\rho}{R e^{(-1)\rho} e^{-\rho} - 1}}, & \tilde{c} - 1 > 0 \text{ localized in } y = \pi \\
\frac{1}{\sqrt{2\rho}}, & \tilde{c} - 1 = 0 \text{ no localization}, \\
\sqrt{\frac{(-1)\rho}{R e^{(-1)\rho} e^{-\rho} - 1}}, & \tilde{c} - 1 < 0 \text{ localized in } y = 0
\end{cases}
$$
As a first step in the calculation of the Casimir force one has to find the Green’s function for the unbounded space coordinates along the plates. For the geometry of parallel plates the 4D Green’s function we need to solve the equation (7). To obtain the explicit form of the 4D Green’s function we need to solve the equation (7). To obtain the approximation KK tower

\[ m_{0N} \approx N - \frac{1}{2} \frac{\rho e^{-\rho \pi}}{R}, \quad N = 1, 2, \ldots, m_n = 0, \]

\[ m_{nN} \approx N + \sqrt{\frac{1}{2} + \left[ \frac{N + \frac{25}{16} + \left( \frac{2\rho^2}{k^2} - \frac{3}{4} \frac{kR e^{-\frac{\pi}{2}}}{2\rho} \right)^2}{4} \right] \frac{\rho e^{-\rho \pi}}{R},} \]

\[ N = 1, 2, \ldots, m_n \neq 0. \]

We have obtained the mass spectrum of the scalar field modes by looking at the \( y, z \) dependence, our next task is to solve Eq. (7) for the \( x \) dependence and incorporate them all to get the Green’s function.

### 3.2. Green’s function

As a first step in the calculation of the Casimir force one has to find the Green’s function for the 6D field equations. The corresponding Green’s function satisfies

\[
\begin{align*}
\left[ \frac{1}{\mu^2 a^2} \Box + \frac{1}{\sqrt{-g}} \partial_y \left( \sqrt{-g} \partial_y \right) - \frac{1}{\sqrt{-g}} \partial_z \left( \sqrt{-g} \partial_z \right) + M^2 + \frac{2\rho}{R^2 a^2} \left[ \delta(y) - \delta(y - \pi R) \right] + \frac{2d\kappa}{r_z^2} \left[ \delta(z) - \delta(z - \pi r_z) \right] \right] G_{6D}(x, y, z, x', y', z') &= \frac{\delta(x - x')\delta(y - y')\delta(z - z')}{\sqrt{-g}}.
\end{align*}
\]

We solve this equation using the eigenfunction expansion for the Green’s function

\[
G_6(x, y, z, x', y', z') = \sum_n \sum_N \chi_n^*(z') \chi_n(z) \xi_N(y') \xi_N(y) G_4(x, x'),
\]

with \( * \) denoting complex conjugation and \( G_{4D} \) is the 4D Green’s function. To determine the explicit form of the 4D Green’s function we need to solve the equation (7). To obtain the solutions for \( \phi \) it is convenient to split up the 3D position vector as follows, \( \vec{x} = (\vec{x}_\perp, x_3) \), where \( x_3 \) is the coordinate orthogonal to the plates and \( \vec{x}_\perp \) denotes a 2D vector in the direction of the unbounded space coordinates along the plates. For the geometry of parallel plates the 4D Green’s function is given by (see for example [9])

\[
G_{4D}(x, x') = \int \frac{d\omega d^2 k_{\perp}}{2\pi (2\pi)^2} e^{-i\omega(t - t')} e^{i k_{\perp}(\vec{x}_\perp - \vec{x}'_\perp)} g(x_3, x_3'),
\]
where
\[
g(x_3, x'_3) = \begin{cases} \frac{1}{\sin(\lambda \ell)} \sin(\lambda x_3) \sin(\lambda x'_3 - \ell), & 0 \leq x_3, x'_3 \leq \ell, \\ \frac{1}{\sin(\lambda (x_3 - \ell))} \exp(i \lambda (x'_3 - \ell)), & \ell \leq x_3, x'_3. \end{cases}
\] (26)

here \(\ell\) is the separation between plates, \(\lambda^2 = \omega^2 - \vec{k}_\perp^2 - m_{nN}^2\), \(x_3>(x_3<)\) represents the greater (lesser) of \(x_3\) and \(x'_3\).

4. Casimir Force

Knowing the Green’s function we can calculate the force on the bounding surfaces from the stress tensor. This can be done evaluating the discontinuity in the flux of the stress tensor across the plate, i.e., the corresponding normal-normal component of the stress tensor at \(x_3 = \ell\)

\[
F = \int_0^A \int_{-\pi}^{\pi} \frac{d\omega d\vec{k}_\perp}{(2\pi)^3} \sqrt{|g_{\text{plate}}|} \left[ \langle T_{33}\rangle_{\text{in}}|_{x_3=\ell} - \langle T_{33}\rangle_{\text{out}}|_{x_3=\ell} \right],
\] (27)

where \(A\) is the area of the plate and \(g_{\text{plate}}\) is the induced metric on the physical plate located a \(x_3 = \ell\), \(\sqrt{|g_{\text{plate}}|}\) contributes with the term \(R\ell a^2\).

As it is well known [9], the normal-normal component of the stress tensor is related to the Green’s function through

\[
\langle T_{33}\rangle_{\text{in}}|_{x_3=\ell} \sim \frac{1}{2i} \partial_{x_3} G_{\text{in}}(x_3, x'_3)|_{x_3=x'_3=\ell} = \frac{i}{2} \lambda \cot(\lambda \ell),
\] (28)

\[
\langle T_{33}\rangle_{\text{out}}|_{x_3=\ell} \sim \frac{1}{2i} \partial_{x_3} G_{\text{out}}(x_3, x'_3)|_{x_3=x'_3=\ell} = \frac{\lambda}{2}.\] (29)

So, now we can find the force per unit area on the conducting surface

\[
f_{\text{eff}} \equiv \frac{F}{A} = \sum_n \sum_{N} \int \frac{d\omega d\vec{k}_\perp}{(2\pi)^3} \left( \frac{i}{2} \lambda \cot(\lambda \ell) - \frac{\lambda}{2} \right),
\] (30)

where the dependence on the \(y\) and \(z\) coordinates vanishes completely by relations (13) and (19). Using dimensional regularization (see [10] for more details) we obtain that

\[
f_{\text{eff}} = -\sum_{n,N} 2 \left( \frac{m_{nN}}{4\pi} \right)^{\frac{d+2}{2}} K_{\frac{d+2}{2}} \left( \frac{2\kappa \ell m_{nN}}{\kappa^2} \right) - \frac{2m_{nN}}{\ell} \sum_{\kappa=1}^{\infty} K_{\frac{d+2}{2}} \left( \frac{2\kappa \ell m_{nN}}{\kappa^2} \right),
\] (31)

and we can rewrite the full Casimir force as

\[
f_{\text{eff}} = -\sum_{n=0}^{\infty} \sum_{N=0}^{\infty} \frac{m_{nN}^2}{8\pi^2} \left[ \frac{3}{\ell} \sum_{\kappa=1}^{\infty} K_{\frac{2}{2}} \left( \frac{2\kappa \ell m_{nN}}{\kappa} \right) - \frac{2m_{nN}}{\ell} \sum_{\kappa=1}^{\infty} K_{\frac{2}{2}} \left( \frac{2\kappa \ell m_{nN}}{\kappa} \right) \right],
\] (32)

where we have taken the limit \(d \to 2\) in (31). Sometimes this expression is presented in a slightly different way, which can be obtained by using the identity \(K_{\nu}(z) = K_{\nu-2}(z) + \frac{2(\nu-1)}{z} K_{\nu-1}(z)\). In this case

\[
f_{\text{eff}} = -\sum_{n=0}^{\infty} \sum_{N=0}^{\infty} \frac{m_{nN}^2}{8\pi^2} \left[ \frac{3}{\ell} \sum_{\kappa=1}^{\infty} K_{\frac{2}{2}} \left( \frac{2\kappa \ell m_{nN}}{\kappa} \right) + \frac{2m_{nN}}{\ell} \sum_{\kappa=1}^{\infty} K_{\frac{1}{2}} \left( \frac{2\kappa \ell m_{nN}}{\kappa} \right) \right].
\] (33)

The above result (33) represents the Casimir force due to a massive scalar field \(\Phi\). It is worth mentioning that for \(n = 0\), \(N\) can have both zero and non-zero values. But for \(n \neq 0\), \(N\) can
have only nonzero values, as is evident from equations (21)-(22). Thus, we can write the Casimir force as

\[
\begin{align*}
    f_{\text{eff}} &= -\frac{\pi^2}{480 \ell^4} \sum_{N=1}^{\infty} \frac{m_{0N}^2}{8\pi^2} \left[ \frac{3}{\ell^2} \sum_{\kappa=1}^{\infty} \frac{K_2(2\kappa \ell m_{0N})}{\kappa^2} + \frac{2m_{0N}}{\ell} \sum_{\kappa=1}^{\infty} \frac{K_1(2\kappa \ell m_{0N})}{\kappa} \right] \\
    &\quad - \sum_{n=1}^{\infty} \sum_{N=1}^{\infty} \frac{m_{nN}^2}{8\pi^2} \left[ \frac{3}{\ell^2} \sum_{\kappa=1}^{\infty} \frac{K_2(2\kappa \ell m_{nN})}{\kappa^2} + \frac{2m_{nN}}{\ell} \sum_{\kappa=1}^{\infty} \frac{K_1(2\kappa \ell m_{nN})}{\kappa} \right].
\end{align*}
\]

To complete the analytical study, we proceed now to evaluate numerically \( f_{\text{eff}} \). Using the fact that \( e^{\rho \pi} = 10^{16}/\cosh(k\pi) \) (as demanded by the hierarchy problem) and the definition of \( \rho = Rk/[r_z \cosh(k\pi)] \), the equation (34) depends on two extra free parameters in addition to the plates separation \( \ell \): the mass of the 6-dimensional scalar field \( M_\Phi \) and the size of one extra space dimension \( r_z \) (more precisely \( k/r_z \)).

In Fig. 1, the Casimir force as a function of the plate separation \( \ell \) is shown for two \( k/r_z \) values. Due to the warp factor along of the \( y \) direction, the Casimir force is very sensitive to the \( k/r_z \) value. It should be noted that the measurements of Casimir force between parallel plates are in the 0.5 \( \mu m \) to 2 \( \mu m \) range [11].

Let us set an uncertainty 10% for the theoretical standard 4D Casimir force. This gives rise to an uncertainty band for the Casimir force for a massless scalar field. Thus, limiting \( f_{\text{eff}} \) within such band for different values of \( k/r_z \) \((k/r_z = M_\Phi)\), we obtain the following bound for \( k/r_z \gg 1.97 \times 10^6 \) TeV (see the figure 1).

\[ \begin{align*}
    \text{Figure 1.} & \quad \text{The Casimir force for a massive scalar field is shown as a function of the separation between plates for two values of } k/r_z. \text{ In order to get a rough bound on } k/r_z \text{ we also include a 10\% error in the Casimir force. Such a region is represented by the two solid lines.} \\
    \text{5. Discussion} & \quad \text{In this work we have studied the Casimir force between two parallel plates in a doubly warped compactified six-dimensional spacetime with a } Z_2 \text{ orbifolding in each of the extra dimensions.}
\end{align*} \]
We adopt the Green’s function approach to do so, therefore we first analyzed the modes for the 6D massive scalar field.

The KK modes can be obtained by imposing the boundary condition on the solutions for the scalar field, however, the appropriate boundary conditions depend on the parity of the scalar field under the \( Z_2 \) symmetry, thus without loss of generality we have chosen the fact that the 6D massive scalar field is even under the orbifold symmetry in each of the extra dimensions.

To satisfy the boundary condition at \( z = 0, \ z = \pi, \ y = 0 \) and \( y = \pi \) the argument of the eigenfunctions (Bessel functions) has to satisfy a general equation which yields a discrete KK spectrum (22) in the limit \( m_n r_z \ll k, kr_z \gg 1 \) and (21) in the limit \( m_n N R \ll k, \rho R \gg 1 \).

Having summarized the KK tower for a massive scalar field, we proceeded with the calculation of the Casimir force. The mass of the KK modes yield a Casimir force naturally splitting into a leading order term given by a massless mode of the KK tower plus a correction term coming from the massive KK modes. The massive KK modes contain mixed information of the two extra dimensions (22). Usually in other works of the Casimir force in braneworlds the effective mass in 4D for a massless scalar field is the sum of the tower KK for each extra dimension [12, 13]. Such leading order term coincide with the standard 4D Casimir force for a massless scalar field.

We have also studied numerically the effective Casimir force (34) due to the extra dimensions. To do this, an uncertainty band around the theoretical force for a massless scalar field \( f_{s,f}^{4D} = -\hbar c \pi^2 / (480 \ell_4^4) \) was set, corresponding to an uncertainty of the 10%. Limiting \( f_{eff} \) within such band yields the bound for the scenario parameter.

In summary, it should be noted that although the Green’s function technique involves further details of the mode decomposition of the corresponding fields, it is due to their orthogonality relations which involve the correct hypervolume factor of the plates, that one can eliminate the mode eigenfunctions from the Casimir force, one can conclude that localization of the field modes does not play a role as far as the Casimir force is concerned.

6. Acknowledgments
We thank Hugo A. Morales-Técotl for discussions and comments on this paper. HH was supported by Conacyt grant CB-2008-01/101774.

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