Baryon Modes of $B$ Meson Decays

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Abstract

The baryon decay modes of $B, \bar{B} \to N_1 \bar{N}_2(\bar{f}), \bar{N}_1 N_2(f)$ provide a framework to test $CP$-invariance in baryon sector. It is shown that in the rest frame of $B$, $N_1$ and $\bar{N}_2$ come out with longitudinal polarization $\lambda_1 = \lambda_2 = \pm 1$ with decay width $\Gamma_f = \Gamma^{++}_f + \Gamma^{--}_f$ and the asymmetry parameter $\alpha_f = \Delta \Gamma_f = \Gamma^{++}_f - \Gamma^{--}_f$. It is shown that $CP$ invariance prediction $\alpha_f = -\bar{\alpha}_f$ can be tested in these decay modes; especially in the time dependent decays of $B_0^0 \to B_0^0$ complex. Apart from this, it is shown that decay modes $B(\bar{B}) \to N_1 \bar{N}_2(N_1 \bar{N}_2)$ and subsequent non-leptonic decays of $N_2, \bar{N}_2$ or $(N_1, \bar{N}_1)$ into hyperon (anti-hyperon) also provide a framework to study $CP$-odd observables in hyperon decays.

1 Introduction

The $CP$-violation in kaon and $B_q^0 - \bar{B}_q^0$ systems has been extensively studied \cite{1}. There is thus a need to study $CP$-violation outside these systems. In hyperon decays, the observables are the decay rate $\Gamma$, asymmetry parameter $\alpha$, the transverse polarization $\beta$ and longitudinal polarization $\gamma$ \cite{2}. $CP$ asymmetry predicts $\bar{\Gamma} = \Gamma, \bar{\alpha} = -\alpha, \bar{\beta} = -\beta$, where these observables correspond to non-leptonic hyperon decays $N \to N'\pi$ and $\bar{N} \to \bar{N}'\pi$. Thus
to leading order $CP$-odd observables are \[3\]

\[ \delta \Gamma = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}, \quad \delta \alpha = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}, \quad \delta \beta = \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} \]

(1)

The decays of $B(\bar{B})$ mesons to baryon-antibaryon pair $N_1 \bar{N}_2$ ($\bar{N}_1 N_2$) and subsequent decays of $N_2, \bar{N}_2$ or $(N_1, \bar{N}_1)$ to a lighter hyperon (antihyperon) plus a meson provide a means to study $CP$-odd observables as for example in the process

\[ e^- e^+ \rightarrow B, \bar{B} \rightarrow N_1\bar{N}_2 \rightarrow N_1\bar{N}_2'\pi, \quad \bar{N}_1 N_2 \rightarrow \bar{N}_1 N_2'\pi \]

Apart from the above motivation, the baryon decay modes of $B$-mesons are of intrinsic interest by themselves as we discuss below. The baryon decay modes of $B^0_d - \bar{B}^0_d$ have also been discussed in a different context in \[4\].

In the rest frame of $B$, $N_1$ and $\bar{N}_2$ come out longitudinally polarized with polarization

\[ \left( \lambda_1 \equiv \frac{E_1}{m_1} n \cdot s_1 \right) = \left( \lambda_2 \equiv \frac{E_2}{m_2} (-n \cdot s_2) \right) = \pm 1, \]

where

\[ p_1 = |p| n, p_2 = -|p| n, s_1 = \frac{m_1}{E_1} n \]

\[ s_2 = -\frac{m_2}{E_2} n \]

$s_1^\mu, s_2^\mu$ are polarization vectors of $N_1$ and $\bar{N}_2$ respectively ($p_1 \cdot s_1 = 0, p_2 \cdot s_2 = 0, s_1^2 = -1 = s_2^2$). The decay $B \rightarrow N_1\bar{N}_2(f)$ is described by the matrix element

\[ M_f = F_q e^{+i\phi} [\bar{u}(p_1)(A_f + \gamma_5 B_f)v(p_2)] \]

(2)

where $F_q$ is a constant containing CKM factor, $\phi$ is the weak phase. The amplitude $A_f$ and $B_f$ are in general complex in the sense that they incorporate the final state phases $\delta_p^f$ and $\delta_s^f$. Note that $A_f$ is the parity violating amplitude ($p$-wave) whereas $B_f$ is parity conserving amplitude ($s$-wave). The $CPT$ invariance gives the matrix elements for the decay $\bar{B} \rightarrow \bar{N}_1 N_2(\bar{f})$:

\[ \bar{M}_f = F_q e^{-i\phi} [\bar{u}(p_2)(-A_f + \gamma_5 B_f)v(p_1)] \]

(3)

If the decays are described by a single matrix element $M_f$, then $CPT$ and $CP$ invariance give the same prediction viz

\[ \bar{\Gamma}_f = \Gamma_f, \quad \bar{\alpha}_f = -\alpha_f, \quad \bar{\beta}_f = -\beta_f, \quad \bar{\gamma}_f = \gamma_f \]

(4)

2
2 Decay Rate and Asymmetry Parameters:

The decay width for the mode $B \rightarrow N_1\bar{N}_2(f)$ is given by

$$\Gamma_f = \frac{m_1 m_2}{2\pi m_B^2} |p| |M_f|^2$$

$$= \frac{F_q^2}{2\pi m_B^2} |p| [(p_1 \cdot p_2 - m_1 m_2) |A_f|^2 + (p_1 \cdot p_2 + m_1 m_2) |B_f|^2] \quad (5)$$

In order to take into account the polarization of $N_1$ and $\bar{N}_2$, we give the general expression for $|M_f|^2$

$$|M_f|^2 = \frac{F_q^2}{16m_1 m_2} \text{Tr} \left[ (\not{p}_1 + m_1)(1 + \gamma_5 \gamma \cdot s_1)(A_f + \gamma_5 B_f)(\not{p}_2 - m_2) \times (1 + \gamma_5 \gamma \cdot s_2)(A_f^* - \gamma_5 B_f^*) \right]$$

$$= \frac{4F_q^2}{16m_1 m_2} \left[ |A_f|^2 (p_1 \cdot p_2 - m_1 m_2) + |B_f|^2 (p_1 \cdot p_2 + m_1 m_2) - (A_f B_f^* + B_f A_f^*)(m_2 p_1 \cdot s_2 + m_1 p_2 \cdot s_1) - i(A_f B_f^* - B_f A_f^*)(e^{i\nu\phi_\alpha} p_{\mu\nu\rho\lambda} s_1^\alpha s_2^\beta s_3^\gamma s_4^\delta) \right. \right.$$

$$\left. \left. + m_1 m_2 (|A_f|^2 + |B_f|^2) s_1 \cdot s_2 \right) \right]$$

$$+ (|A_f|^2 - |B_f|^2) (-p_1 \cdot p_2 s_1 \cdot s_2 + (p_1 \cdot s_2)(p_2 \cdot s_1)) \quad (6)$$

It is clear that Eqs. (5) follows from Eqs. (2) and (6). In the rest frame of $B$, we get from Eqs. (5) and (6)

$$|M_f|^2 = F_q^2 \frac{2E_1 E_2}{4m_1 m_2} \left[ |a_s|^2 + |a_p|^2 \right] \left\{ 1 + \alpha_f \left( \frac{m_1}{E_1} n \cdot s_1 - \frac{m_2}{E_2} n \cdot s_2 \right) \right. \right.$$

$$\left. \left. + \beta f n \cdot (s_1 \times s_2) + \gamma_f \left[ (n \cdot s_1)(n \cdot s_2) - s_1 \cdot s_2 \right] \right) \right. \right.$$

$$\left. - \frac{m_1 m_2}{E_1 E_2} (n \cdot s_1)(n \cdot s_2) \right\} \quad (7)$$

where

$$a_s = \sqrt{\frac{p_1 \cdot p_2 + m_1 m_2}{2E_1 E_2}} B, \quad a_p = -\sqrt{\frac{p_1 \cdot p_2 - m_1 m_2}{2E_1 E_2}} A \quad (8)$$

$$\alpha_f = \frac{2S_f P_f \cos(\delta_f^l - \delta_f^p)}{S_f^2 + P_f^2}, \quad \beta_f = \frac{2S_f P_f \sin(\delta_f^l - \delta_f^p)}{S_f^2 + P_f^2}$$

$$\gamma_f = \frac{S_f^2 - P_f^2}{S_f^2 + P_f^2}, \quad a_s = S_f e^{i\delta_f^l}, a_p = P_f e^{i\delta_f^p} \quad (9)$$

However in the rest frame of $B$, due to spin conservation

$$\frac{E_1}{m_1} n \cdot s_1 = \frac{E_2}{m_2} (-n \cdot s_2) = \pm 1 \quad (10)$$
Thus invariants multiplying $\beta_f$ and $\gamma_f$ vanish. Hence we have

$$|M_f|^2 = \left(\frac{2E_1E_2}{m_1m_2}\right) F^2_q (S^2_f + P^2_f) [(1 + \lambda_1\lambda_2) + \alpha_f(\lambda_1 + \lambda_2)] \quad (11)$$

$$\Gamma_f = \Gamma_f^+ + \Gamma_f^- = \frac{2E_1E_2}{2\pi m^4_B} |\vec{p}| F^2_q [S^2_f + P^2_f] = \bar{\Gamma}_f \quad (12)$$

$$\Delta \Gamma_f = \frac{\Gamma_f^{++} - \Gamma_f^{--}}{\Gamma_f^+ + \Gamma_f^-} = \alpha_f \quad \Delta \bar{\Gamma}_f = -\alpha_f \quad (13)$$

Eqs. (12) and (13) follow from CP or CPT invariance. It will be of interest to test these equations.

In this paper, we confine ourselves to decays $B \to N_1 \bar{N}_2 (\bar{B} \to \bar{N}_1 N_2)$ described by a single matrix element $M_f$ ($\bar{M}_f$) i.e. to the effective Lagrangians

$$L = V_{cb} V^*_{ub} [\bar{q}u]_{V-A} [\bar{c}b]_{V-A} + h.c. \quad (14)$$

$$L = V_{ub} V^*_{cq} [\bar{q}c]_{V-A} [\bar{u}b]_{V-A} + h.c. \quad (15)$$

where $q = d$ or $s$. For the decay modes described by the above Lagrangians, there are no contributions from the penguin diagrams. The Lagrangian given in Eq. (14) is relevant for the decays

\begin{itemize}
  \item[i)] $B_q^0 \to N_1 \bar{N}_2 (f); \quad \bar{B}_q^0 \to \bar{N}_1 N_2 (\bar{f})$
  \item[ii)] $B^+ \to N_1 \bar{N}_2 : \quad n\bar{\Lambda}_-, \quad \frac{1}{\sqrt{6}} \Lambda \bar{\Xi}^- - \frac{1}{\sqrt{2}} \Sigma^0 \bar{\Xi}^- (q = d)$
\end{itemize}

For the decay modes (i), the weak phase $\phi = 0$ and the decay matrix elements $M_f$ and $\bar{M}_f$ are given by Eqs. (2) and (3). For the Lagrangian given in Eq. (15), the relevant decay modes are

\begin{itemize}
  \item[ii)] $\bar{B}_q^0 \to N_1 \bar{N}_2 (f); \quad B_q^0 \to \bar{N}_1 N_2 (\bar{f})$
  \item[ii)] $B^- \to N_1 \bar{N}_2 : \quad n\bar{\Lambda}_-, \quad \frac{1}{\sqrt{6}} \Lambda \bar{\Xi}^- - \frac{1}{\sqrt{2}} \Sigma^0 \bar{\Xi}^- (q = d)$
  \item[ii)] $B^- \to N_1 \bar{N}_2 : \quad n\bar{\Lambda}_-, \quad \frac{1}{\sqrt{6}} \Lambda \bar{\Xi}^- - \frac{1}{\sqrt{2}} \Sigma^0 \bar{\Xi}^- (q = d)$
\end{itemize}

For various decay channels (i) and (ii), we have explicitly shown the $SU(3)$ factors. For the decay modes (ii), the weak phase $\phi = \phi_3/\gamma$, which arises
from $V_{ub} = |V_{ub}| e^{-i\gamma}$. For the decay modes (ii), the matrix elements $\bar{M}'_f$ and $M'_f$ are given by

$$\bar{M}'_f = e^{-i\phi_3} F'_q[\bar{u}(p_1)(A'_f + \gamma_5 B'_f)v(p_2)]$$

$$(16)$$

$$M'_f = e^{i\phi_3} F'_q[\bar{u}(p_2)(-A'_f + \gamma_5 B'_f)v(p_1)]$$

$$(17)$$

Hence the decay widths and $CP$-asymmetry parameters are given by

$$\bar{\Gamma}'_f = \Gamma'_f = \frac{2E_1 E_2}{8\pi m^2} |p| F'^2_q(S'^2_f + P'^2_f)$$

$$(18)$$

$$\bar{\alpha}'_f = -\alpha'_f = \frac{2S'_f P'_f \cos(\delta'_s - \delta'_p)}{(S'^2_f + P'^2_f)}$$

$$(19)$$

Now

$$F_q = \frac{G_F}{\sqrt{2}} (a_2, a_1) V_{cb} V_{ub}$$

$$(20)$$

$$F'_q = \frac{G_F}{\sqrt{2}} (a_2, a_1) |V_{ub}| V_{cq}$$

$$(21)$$

Define

$$r = \frac{F'_q}{F_q} = \frac{|V_{ub}| V_{cq}}{V_{cb} V_{ub}} = -\lambda^2 \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \text{ for } q = d$$

$$= \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \text{ for } q = s$$

$$(22)$$

$a_2$ ($a_1$) are factors which account for color suppressed (without color supressed) matrix elements. From Eqs. (12), (20), we get

$$\frac{\Gamma(B^0_q \to p\bar{\Lambda}_c^{-})}{\Gamma(B^0_q \to p\bar{\Lambda}^{-}_c)} = \lambda^2 \left(\frac{m_{B_d}}{m_{B_s}}\right)^2 \left|\frac{E_1 E_2}{E_1 E_2} \bar{p}\right|_{B_d} \xi^2$$

$$\approx \lambda^2 \xi^2$$

$$(23)$$

where $\xi$ is a measure of $SU(3)$ violation.

Now $B^0_q, \bar{B}^0_q$ annihilate into baryon-antibaryon pair $N_1\bar{N}_2$ through $W$-exchange as depicted in Figs (1a) and (1b). $B^- \to N_1\bar{N}_2$ through annihilation diagram is shown in Fig (2). It is clear from Fig (1a) and (1b), that we have the same final state configuration for $B^0_q, \bar{B}^0_q \to N_1\bar{N}_2$. Thus one would expect

$$S'_f = S_f, P'_f = P_f$$

$$\delta'_s = \delta'_s, \delta'_p = \delta'_p$$

$$(24)$$
Hence we have

\[ \Gamma'_f = \bar{\Gamma}_f = r^2 \Gamma_f \]  

\[ \bar{\alpha}'_f = -\alpha'_f = \alpha_f = -\bar{\alpha}_f \]  

\[ \frac{\Gamma(B^0_s \rightarrow p\bar{\Lambda}^-)}{\Gamma(B^0_s \rightarrow p\Lambda^-)} = (\bar{\rho}^2 + \bar{\eta}^2) \]  

\[ \frac{\Gamma(B^- \rightarrow \Lambda\bar{\Lambda}^-)}{\Gamma(B^0_d \rightarrow p\Lambda^-)} \approx \frac{2}{3} \left( \frac{\lambda a_1}{a_2} \right)^2 (\bar{\rho}^2 + \bar{\eta}^2) \]  

Eq.(28) is valid in SU(3) limit, but SU(3) breaking effects can be taken into account by using physical masses for proton and Λ hyperon in the kinematical factors.

Above predictions can be tested in future experiments on baryon decay modes of B-mesons. In particular \( \bar{\alpha}'_f = \alpha_f \) would give direct confirmation of Eqs.(24).

Finally, we discuss \( B^0_d \rightarrow p\bar{\Lambda}^- \) decay. For this decay mode the experimental branching ratio is \( (2.2 \pm 0.8) \times 10^{-5} \) \(^5\). Using the experimental value for \( \tau_{B^0_d} \), we obtain

\[ \Gamma(B^0_d \rightarrow p\bar{\Lambda}^-) = (9.46 \pm 3.44) \times 10^{-15} \text{ MeV} \]  

The decay width in terms of \([S^2_f + P^2_f]\) is given by

\[ \Gamma_f = \frac{G^2_F}{2} |V_{cb}|^2 |V_{ud}|^2 a_2 \left( S^2_f + P^2_f \right) \left[ \frac{2E_1E_2}{2m_B^2} |\mathbf{p}| \right] \]  

Using \( |V_{cb}| = 41.6 \times 10^{-3} \), \( |V_{ud}| = 0.97378 \) \(^5\), \( a_2 = 0.226 \) and noting that \( \frac{2E_1E_2}{2m_B^2} |\mathbf{p}| \approx 1.01 \text{ GeV} \)

we get

\[ \Gamma_f = [9.09 \times 10^{-25} \text{MeV}^{-3}] [S^2_f + P^2_f] \]  

Using Eq.(29), we get

\[ (S^2_f + P^2_f) = (1.04 \pm 0.38) \times 10^{10} \text{MeV}^4 \]  

In order to express \((S^2_f + P^2_f)\) in terms of dimensionless form factors, we use \( B^- \rightarrow l^-\bar{\nu}_l \) decay as a guide, which also occurs through a diagram similar to Fig 2.
For the decay $B^- \rightarrow l^- \bar{\nu}_l$,

$$\Gamma(B^- \rightarrow l^- \bar{\nu}_l) = \frac{G_F^2}{2} |V_{ub}|^2 \left( \frac{2E_1E_2}{2\pi m_B^2} \right) |p| \left[ S^2 + P^2 \right]$$

$$= \frac{G_F^2}{2} |V_{ub}|^2 \left( \frac{2E_1E_2}{2\pi m_B^2} \right) |p| 2(m_l^2 + m_{\nu_l}) f_B^2$$

(33)

Noting that

$$\frac{2E_1E_2 |p|}{m_B^2} \approx \frac{1}{4} m_B$$

we get

$$\Gamma(B^- \rightarrow l^- \bar{\nu}_l) \approx \frac{G_F^2}{8\pi} |V_{ub}|^2 m_B m_l^2 f_B^2$$

(34)

Thus we see that for this decay

$$S^2 + P^2 = 2(m_l^2 + m_{\nu_l}) f_B^2$$

(35)

Hence we can parametrize $(S_l^2 + P_l^2)$ in terms of two form factors $F_V^{\Lambda_c - p}(s)$ and $F_A^{\Lambda_c - p}(s)$:

$$P_l^2 = f_B^2 (m_{\Lambda_c} + m_p)^2 \left[ \left( \frac{m_{\Lambda_c} - m_p}{m_{\Lambda_c} + m_p} \right) F_V^{\Lambda_c - p}(s) \right]_{s=m_B^2}^2$$

$$S_l^2 = f_B^2 (m_{\Lambda_c} + m_p)^2 \left[ F_A^{\Lambda_c - p}(s) \right]_{s=m_B^2}^2$$

(36)

It is easy to see that for $F_V = 1$ and $F_A = 1$, it reduces to form of Eq.(35). Using the experimental values for the masses and $f_B \approx 180$ MeV, we get from Eq.(33)

$$(0.175)[F_V^{\Lambda_c - p}(m_B^2)]^2 + [F_A^{\Lambda_c - p}(m_B^2)]^2 = (3.1 \pm 1.1) \times 10^{-2}$$

(37)

The dominant contribution comes from the axial vector form factor. The decay $B_c^- \rightarrow n\bar{p}$ would give information for nucleon form factors:

$$P_f^2 = f_{Bc}^2 (m_n + m_p)^2 \left[ \frac{m_n - m_p}{m_n + m_p} F_V(s) \right]_{s=m_{B_c}^2}^2 \approx 0$$

$$S_f^2 = f_{Bc}^2 (m_n + m_p)^2 \left[ F_A^2(s) \right]_{s=m_{B_c}^2}^2$$

(38)

The baryon decay modes of $B$-mesons also provide the means to explore the baryon form factors at high $s$. Finally, we note that Eq.(36) give the $SU(3)$ breaking factor $\xi = \frac{f_{Bc}}{f_B}$ in Eq.(23).
3 Time-Dependent Baryon Decay Modes of $B_q^0$

Define the amplitudes

$$A_{\lambda_1\lambda_2}(t) = \frac{\left[ \Gamma(B_q^0(t) \to f) - \Gamma(\bar{B}_q^0(t) \to \bar{f}) \right]_{\lambda_1\lambda_2} + \left[ \Gamma(B_q^0(t) \to \bar{f}) - \Gamma(\bar{B}_q^0(t) \to f) \right]_{\lambda_1\lambda_2}}{\sum_{\lambda_1\lambda_2} \left[ \Gamma(B_q^0(t) \to f, f) + \Gamma(\bar{B}_q^0(t) \to \bar{f}, \bar{f}) \right]_{\lambda_1\lambda_2}}$$

$$= \frac{-2 \sin \Delta m t \left[ \Im e^{2i\phi_M} (M_f^* \bar{M}_f + M_{f'}^* \bar{M}_{f'}) \right]}{\sum_{\lambda_1\lambda_2} \left[ |M_f^2| + |\bar{M}_f^2| + |M_{f'}^2| + |\bar{M}_{f'}^2| \right]}$$

$$F_{\lambda_1\lambda_2}(t) = \frac{\left[ \Gamma(B_q^0(t) \to f) - \Gamma(\bar{B}_q^0(t) \to \bar{f}) \right]_{\lambda_1\lambda_2} - \left[ \Gamma(B_q^0(t) \to \bar{f}) - \Gamma(\bar{B}_q^0(t) \to f) \right]_{\lambda_1\lambda_2}}{\sum_{\lambda_1\lambda_2} \left[ \Gamma(B_q^0(t) \to f, f) + \Gamma(\bar{B}_q^0(t) \to \bar{f}, \bar{f}) \right]_{\lambda_1\lambda_2}}$$

$$= \frac{\cos \Delta m t \left[ |M_f^2| + |\bar{M}_f^2| - |M_{f'}^2| - |\bar{M}_{f'}^2| \right] - 2 \sin \Delta m t \left[ \Im e^{2i\phi_M} (M_f^* \bar{M}_f - M_{f'}^* \bar{M}_{f'}) \right]}{\sum_{\lambda_1\lambda_2} \left[ |M_f^2| + |\bar{M}_f^2| + |M_{f'}^2| + |\bar{M}_{f'}^2| \right]}$$

Thus

$$8 \left[ (S_f^2 + P_f^2) + r^2(S_{f'}^2 + P_{f'}^2) \right] A_{\lambda_1\lambda_2}(t)$$

$$= 2 \sin \Delta m t \begin{cases} \sin(\phi_M - \gamma) \left[ 2r(1 + \lambda_1\lambda_2)(S_f S_{f'} \cos(\delta_f - \delta_{f'}) + P_f P_{f'} \cos(\delta_f - \delta_{f'})) \right] \\
- \cos(2\phi_M - \gamma) \left[ 2r(\lambda_1 + \lambda_2)(S_f P_{f'} \sin(\delta_f - \delta_{f'}) + S_{f'} P_f \sin(\delta_f - \delta_{f'})) \right] \end{cases}$$

(41)
These are general expressions for the time-dependent decay modes in the rest frame of $B_q^0$. From Eqs. (41) and (42), the even and odd time-dependent
decay amplitudes are given by

$$A(t) \equiv (A^{++}(t) + A^{--}(t))$$

$$= \frac{2r \sin \Delta mt \sin(2\phi_M - \gamma) \left[ S_f S_f' \cos(\delta_s^f - \delta_s^f) + P_f P_f' \cos(\delta_p^f - \delta_p^f) \right]}{(S_f^2 + P_f^2) + r^2(S_f^2 + P_f^2)}$$

$$\Delta A(t) \equiv A^{++}(t) - A^{--}(t)$$

$$= -2r \sin \Delta mt \cos(2\phi_M - \gamma) \frac{\left[ S_f P_f' \sin(\delta_s^f - \delta_s^f) + S_f' P_f \sin(\delta_p^f - \delta_p^f) \right]}{(S_f^2 + P_f^2) + r^2(S_f^2 + P_f^2)}$$

$$F(t) = F^{++}(t) + F^{--}(t)$$

$$= \cos \Delta mt \left[ (S_f^2 + P_f^2) - r^2(S_f^2 + P_f^2) \right] + 2r \sin \Delta mt \cos(2\phi_M - \gamma) \frac{\left[ S_f S_f' \sin(\delta_s^f - \delta_s^f) + P_f P_f' \sin(\delta_p^f - \delta_p^f) \right]}{2 \left[ (S_f^2 + P_f^2) + r^2(S_f^2 + P_f^2) \right]}$$

$$\Delta F(t) \equiv F^{++}(t) - F^{--}(t)$$

$$= \cos \Delta mt \left[ (S_f^2 + P_f^2)(\alpha_f + \bar{\alpha}_f) - r^2(S_f^2 + P_f^2)(\bar{\alpha}_f' + \alpha_f') \right]$$

$$- 2r \sin \Delta mt \cos(2\phi_M - \gamma) \left[ S_f P_f' \cos(\delta_s^f - \delta_s^f) + P_f S_f' \cos(\delta_p^f - \delta_p^f) \right]$$

$$\frac{2 \left[ (S_f^2 + P_f^2) + r^2(S_f^2 + P_f^2) \right]}{(S_f^2 + P_f^2) + r^2(S_f^2 + P_f^2)}$$

For $B_d^0$, $r = -\lambda^2 \sqrt{\rho^2 + \eta^2} \approx -(0.02 \pm 0.006)$ [4], $\phi_M = -\beta$; for $B_s^0$, $r = -\sqrt{\rho^2 + \eta^2} \approx -(0.40 \pm 0.13)$ [4], $\phi_M = 0$. First term of Eq. (46) has an important implication: This term is zero, if $\alpha_f = -\bar{\alpha}_f$; $\bar{\alpha}_f' = -\alpha_f'$ as implied by $CP$-conservation. The finite value of this term would imply $CP$ violation in baryon decay. The above equations are simplified if we assume the validity.
of Eq. (24). In that case we have

\[ A(t) = \frac{2r \sin \Delta mt \sin(2\phi_M - \gamma)}{1 + r^2} \] (47)

\[ \Delta A(t) = 0 \] (48)

\[ F(t) = \frac{1 - r^2}{1 + r^2} \cos \Delta mt \] (49)

\[ \Delta F(t) = \frac{1 - r^2}{2(1 + r^2)} (\alpha_f + \bar{\alpha}_f) \cos \Delta mt \]
\[ - \frac{4r \sin \Delta mt \sin(2\phi_M - \gamma) S_f P_f}{(1 + r^2)(S_f^2 + P_f^2)} \] (50)

Eq. (47) gives a means to determine the weak phase \(2\beta + \gamma\) or \(\gamma\) in the baryon decay modes of \(B^0_d\) and \(B^0_s\) respectively. Non-zero \(\cos \Delta mt\) term in \(\Delta F(t)\) would give clear indication of \(CP\) violation especially for baryon decay modes of \(B^0_d\), for which \(r^2 \leq 1\), so that \(\frac{1 - r^2}{1 + r^2} \approx 1\). Assuming \(CP\)-invariance, we get from Eqs. (47) and (50)

\[ - 2S_f P_f = (S_f^2 + P_f^2) \frac{\Delta F(t)}{A(t)} \]
\[ = \{(1.04 \pm 0.38) \times 10^{10} \text{MeV}^4\} \frac{\Delta F(t)}{A(t)} \] (51)

The \(S_f P_f\) can be determined from the experimental value of \(\frac{\Delta F(t)}{A(t)}\) in future experiments.

The baryon decay modes of \(B\)-mesons not only provide a means to test prediction of \(CP\) asymmetry viz \(\alpha_f + \bar{\alpha}_f = 0\) for charmed baryons (discussed above) but also to test the \(CP\)-asymmetry in hyperon (antihyperon) decays viz absence of \(CP\)-odd observables \(\Delta \Gamma, \Delta \alpha, \Delta \beta\) discussed in [3]. Consider for example the decays

\[ B^0_q \rightarrow p\Lambda_c^- \rightarrow p\bar{p}K^0(p\Lambda\pi^- \rightarrow p\bar{p}\pi^+\pi^-), \]
\[ \bar{B}^0_q \rightarrow \bar{p}\Lambda_c^+ \rightarrow \bar{p}p\bar{K}^0(\bar{p}\Lambda\pi^+ \rightarrow \bar{p}p\bar{\pi}^-\pi^+). \]

By analyzing the final state \(\bar{p}p\bar{K}^0, ppK^0\), one may test \(\alpha_f = -\bar{\alpha}_f\) for the charmed hyperon. We note that for \(\Lambda_c^+\), \(c\tau = 59.9 \mu m\), whereas \(c\tau = 7.8 cm\) for \(\Lambda^-\)hyperon [4], so that the decays of \(\Lambda_c^+\) and \(\Lambda\) would not interfere with each other. By analysing the final state \(\bar{p}p\pi^-\pi^+\) and \(pp\pi^+\pi^-\), one may
check $CP$-violation for hyperon decays. One may also note that for $(B^0_d, \bar{B}^0_d)$ complex, the competing channels viz $B^0_d \to \bar{p}\Lambda^+_c, \bar{B}^0_d \to p\bar{\Lambda}^-_c$ are doubly Cabibbo supressed by $r^2 = \lambda^4 (\bar{\rho}^2 + \bar{\eta}^2)$ unlike $(B^0_s - \bar{B}^0_s)$ complex where the competing channels are supressed by a factor of $(\bar{\rho}^2 + \bar{\eta}^2)$. Hence $B^0_d( ~\bar{B}^0_d)$ decays are more suitable for this type of analysis. Other decays of intrest are

\begin{align*}
B^- & \to \Lambda\bar{\Lambda}^- \to \Lambda\bar{\Lambda}\pi^- \to p\pi^- \bar{p}\pi^+ \\
B^+ & \to \bar{\Lambda}\Lambda^+ \to \bar{\Lambda}\Lambda\pi^+ \to \bar{p}\pi^+ p\pi^- \\
B^-_c & \to \bar{p}\Lambda \to \bar{p}p\pi^- \\
B^+_c & \to p\bar{\Lambda} \to p\bar{p}\pi^+
\end{align*}

The non-leptonic hyperon (antihyperon) decays $N \to N'\pi (\bar{N} \to \bar{N}'\bar{\pi})$ are related to each other by $CPT$

\begin{equation*}
a_t(I) = \langle f_{\text{out}} | H_W | N \rangle = \eta_f e^{2i\delta_l(I)} \langle \bar{f}_{\text{out}} | H_W | \bar{N} \rangle = \eta_f e^{2i\delta_l(I)} \bar{a}^*_t(I)
\end{equation*}

Hence

\begin{equation*}
\bar{a}_t(I) = \eta_f e^{2i\delta_l(I)} a^*_t(I) = (-1)^{l+1} e^{i\delta_l(I)} e^{-i\phi} |a_t|
\end{equation*}

where we selected the phase $\eta_f = (-1)^{l+1}$. Here $I$ is the isospin of the final state and $\phi$ is the weak phase. Thus necessary condition for non-zero $CP$ odd observables is that the weak phase for each partial wave amplitude should be different [see ref [3] for details; for a review see first ref in [1]].

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**Figure Captions**

Figure1a: $W$-exchange diagram for $B^0_q \rightarrow N_1 \bar{N}_2 (M_f)$

Figure1b: $W$-exchange diagram for $\bar{B}^0_q \rightarrow N_1 \bar{N}_2 (\bar{M}_f)$

Figure2: Annihilation diagram for $B^- \rightarrow N_1 \bar{N}_2$
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