Analysis of a transformer converter of the number of phases with symmetric elements on the low side of the transformer

N V Vasilev, F D Kosoukhov, V A Ruzhev, N Yu Krishtopa and A L Boroshnin
Saint-Petersburg State Agrarian University, 2, Peterburgskoe ave., Saint-Petersburg, 196601, Russia
E-mail: profkom_gau@mail.ru

Abstract. The analysis of a transformer converter of the number of phases TCNP-2 with balancing elements on the low side of a step-down transformer is given. The purpose and objectives of the analysis of PFHF-2 are formulated. In the analysis of TCNP-2, 15 equations were compiled in a complex form, as a result of which three main equations were obtained for complex currents at the input of TCNP-2. Having decomposed the obtained system of equations into symmetrical components of the currents of the forward, reverse, and zero sequences, we determined the resistance and capacitances of the balancing capacitors. Thus, the dependences of the capacitor parameters on the current and power factor of a single-phase load are established. Having solved the system of equations of three currents TCNP-2, we determined the dependences of the currents and voltages of the primary and secondary windings of the transformer and the balancing elements on the load current. The dependences of the capacities of the transformer and phase-converting elements on the power of a three-phase load are also determined. The authors developed and built on a complex plane a vector diagram of voltages and currents TCNP-2, with the help of which a method for phase converting a single-phase current to a three-phase one for a two-wire power transmission system is disclosed.

1. Introduction
The electric power industry is one of the most relevant and advanced branches of agricultural production [1]. Issues related to the economical and high-quality transmission of electrical energy in extended areas and in sparsely populated areas occur in the scientific works of scientists and engineers [2].

At the Department of Electric Power Engineering and Electrical Equipment of Saint-Petersburg State Agrarian University (SPbGAU) we developed a two-wire power transmission system with transformer phase number converters for which a patent was obtained [3]. This patent describes two devices:

- a two-wire power transmission system with the transformer converter of the number of phases and capacitor banks on the high side of the transformers TCNP-1 (Figure 1);
- a two-wire power transmission system with the transformer converter of the number of phases and capacitor banks on the low side of the transformers TCNP-2 (Figure 2).

The patented device contains two transformer converters of the number of phases. The first transformer converter of the number of phases (TCNP-1) (Figure 1) converts the symmetric three-phase current system of the power source 2 into a single-phase current, while simultaneously increasing the voltage of the two-wire line 4, through which electric energy is transmitted for a considerable distance to the second TCNP-2. In it, a voltage decrease occurs and at the same time a single-phase current is converted into a symmetric three-phase current system of the receiver 9. Thus, the main energy devices of this power transmission system are transformers for converting the number of phases, which transform the voltage and convert the number of phases.
The transformer phase number converter (TCNP-1) contains a three-phase three-core transformer (Figure 1.) with primary windings with the same number of turns $W_1$. Secondary windings have $W_1$ number of turns, also the same. The design of the transformer TCNP-1 does not differ from the design of an ordinary three-phase power transformer. The primary windings are connected to a star. A symmetric three-phase voltage system is supplied to them. Secondary windings are connected according to a special scheme: phase a and phase b windings are connected in series and counterclockwise; the phase c winding is connected to the phase a winding in series and according to $i.e.$ all three secondary windings are connected in series.

Two capacitor banks $C_{11}$ and $C_{21}$ are used as balancing elements in TCNP-1, battery $C_{21}$ is connected in series with the secondary winding of phase c of the transformer, and battery $C_{11}$ is connected to the output terminals a;b of the transformer.

The purpose of the TCNP-2 analysis is to establish in principle the possibility of converting a single-phase current to a three-phase current using a three-phase transformer and two capacitor banks on the low side of the transformer.

The tasks of the analysis of TCNP-2 include:
1) Establishment of the dependences of the change in capacitance of capacitor banks TCNP-2 on the current value and power factor of a single-phase load.
2) Determination of the dependence of currents and voltages of the primary and secondary windings of the transformer and balancing capacitors on the load current.
3) Determination of the dependence of the capacities of the transformer and balancing capacitors on the power of a single-phase load. Balancing capacity.

2. Materials and methods

The method of analysis of the transformer converter of the number of phases was developed by professor Kosoukhov F.D. and was first published by him in a scientific article [4].

A method for analyzing the transformer converter of the number of phases with balancing elements on the low side of a transformer was developed jointly with F.D. Kosoukhov and published in the article [4].
3. **The study of the structure of the modified lead-tin-base bronze**

Equations for TCNP-2.

Scheme TCNP-2 is presented in Figure 3.

![Figure 3. Scheme TCNP-2](image)

**Assumptions are made in the analysis of PFHF-2:**

1. The TCNP-2 transformer (Figure 3) is taken perfect [8], therefore, the complex voltage transformation coefficient for an ideal transformer does not depend on the load:

   \[
   n = \frac{U_A}{U_a} = \frac{U_B}{U_b} = \frac{U_C}{U_c} = ne^{j180°} = -n, \tag{1}
   \]

   where \( n = \frac{w_1}{w_2} \); \( U_A, U_B, U_C \) – phase voltage complexes of the transformer primary winding; \( U_a, U_b, U_c \) – phase voltage complexes of the secondary winding of the transformer.

2. Losses in capacitor banks are neglected, i.e. complex resistance of capacitor banks:

   \[ Z_1 = -jx_1 = x_1e^{-j90°}, \quad Z_2 = -jx_2 = x_2e^{-j90°}. \tag{2} \]

3. The three-phase load is symmetrical, has an inductive character.

   The complex resistance of one phase:

   \[ Z = R + jx = Ze^{j90°}. \tag{3} \]

4. With a symmetric three-phase load, the node voltage \( U_{MN_1} = 0 \), then the voltage on the secondary windings of the transformer:

   \[
   U_A' = U_A, \quad U_B' = U_B, \quad U_C' = U_C. \tag{4}
   \]

   \[
   U_A' = U_B', \quad U_B' = U_C', \quad U_C' = U_A'. \tag{5}
   \]

   Given equalities (4), we have:

   \[
   U_A = \frac{U}{n}, \quad U_B = \frac{U}{n}, \quad U_C = \frac{U}{n}. \tag{6}
   \]

   \[
   U_B = \frac{U}{n}, \quad U_C = \frac{n}{n}Z_2 \tag{7}
   \]

   In the analysis of TCNP-2, we direct the single-phase voltage vector \( U = U \) along the axis of the real ones.

   We compose the following equations for the circuit (Figure 3):

   \[
   U_B - U - U_B + U_A = 0, \quad \text{where} \quad U = U_A - U_B + U_C; \tag{6}
   \]

   \[
   U_B - U_B - U_B = 0, \quad \text{or} \quad U_1 = U_A - U_B = U_{AB}; \tag{7}
   \]

   \[
   U_B - U_B + U_2 = 0, \quad \text{or} \quad U_2 = U_C - U_B = U_{BC}; \tag{8}
   \]

   Based on the equilibrium law of magnetomotive force along a closed transformer circuit, we compose two equations [5]:

   \[ -I_Aw_1 + I_Bw_2 - I_A'w_2 + I_Bw_1 = 0; \tag{9} \]
\[-I_b w_1 + I'_a w_2 - I'_c w_2 + I_c w_1 = 0. \quad (10)\]

The remaining equations are compiled according to the laws of Kirchhoff:

Node No. 1: \[I_a + I_b + I_c = 0; \quad (11)\]

Node a: \[-I_a - I_a + I_a + I'_a = 0; \quad (12)\]

Node in: \[I_1 + I'_a - I_n = 0; \quad (13)\]

Node with: \[I'_c - I_c - I_c = 0 \quad (14)\]

\[I_A = I_c = -I_B = \frac{1}{I}. \quad (15)\]

As a result of solving the system of equations (1) – (15), we obtain:

\[I_a = \frac{U}{2nZ} \frac{Z_2 Z_2 - 2Z_2 - 2Z_2}{A}; \quad (16)\]

\[I_n = \frac{U}{2nZ} \frac{Z_2 Z_2 + 2Z_2 + 2Z_2}{A}; \quad (17)\]

\[I_c = -\frac{U}{2nZ} \frac{Z_1 Z_2 + 2Z_2 + 2Z_1}{A} \quad (18)\]

where \(A = 2Z_1 Z_2 + 4Z_1 Z_4 + Z_2 \).

Determination of parameters of balancing elements TCNP-2

We decompose the system of equations (16) - (18) into symmetrical components and determine the complex coefficient of the reverse current sequence \(K_2 = \frac{I_c}{I_1} \).

The current vector in the system of symmetric coordinates [6]:

\[[S]^{-1} = \frac{1}{Z} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix}. \quad (21)\]

 Resistances of balancing elements:

\[x_1 = \sqrt{3} \frac{Z^2 \cos \varphi + Z \sin \varphi}{Z \cos \varphi} = \frac{Z^2 \cos \varphi}{Z \cos \varphi}. \quad (22)\]

\[\begin{align*}
\frac{2\sqrt{3} Z}{\sqrt{3} \sin \varphi - Z \cos \varphi} &= \frac{2\sqrt{3} Z^2}{Z(\sqrt{3} \sin \varphi - \cos \varphi)} = \frac{2\sqrt{3} Z}{\sqrt{3} \sin \varphi - \cos \varphi} \\
&= \frac{2\sqrt{3} Z}{2(\frac{\sqrt{3}}{Z} \sin \varphi - \frac{1}{Z} \cos \varphi)} = \frac{\sqrt{3} Z}{\cos 30^\circ \sin \varphi - \sin 30^\circ \cos \varphi} = \frac{\sqrt{3} Z}{\sin(\varphi - 30^\circ)}. \quad (23)\end{align*}\]

 Capacities of balancing elements. Capacities of the first capacitor bank:

\[C_1 = \frac{2 \cos \varphi n_{ld}}{\omega \sqrt{2} U} = \frac{4 n_{ld} \cos \varphi}{\omega \sqrt{2} U}. \quad (24)\]
Determination of currents of a transformer converter of the number of phases TCNP-2.

In mathematical expressions (16) - (18), the complex currents of a three-phase symmetrical load are expressed in terms of the complex load resistances $Z$ and of the balancing elements $Z_1$ and $Z_2$. We define these currents through the load resistance modules $R$, $x$ and the balancing elements $x_1$ and $x_2$:

$$I_a = \frac{U}{2nZ} e^{j(120^\circ - \varphi)},$$  
$$I_b = \frac{U}{2nZ} e^{-j\varphi},$$  
$$I_c = \frac{U}{2nZ} e^{-j(120^\circ + \varphi)}.$$  

Currents of balancing elements TCNP-2:

$$I_1 = \frac{U \cos \varphi}{nZ} e^{-j120^\circ}.$$  
$$I_2 = \frac{U \sin(\varphi - 30^\circ)}{2nZ}.$$  

Complex currents in the secondary windings of the TCNP-2 $I'_a$, $I'_b$, $I'_c$:

$$I'_a = I'_c = -\frac{U}{2nZ} \left[ \cos \varphi + j \sin(60^\circ - \varphi) \right] = \frac{U}{4nZ} \left[ 2 \cos \varphi e^{-j120^\circ} - e^{-j\varphi} \right];$$  
$$I'_b = \frac{U}{nZ} \left[ \cos \varphi + j \sin(60^\circ - \varphi) \right] = \frac{U}{2nZ} \left[ e^{-j\varphi} - 2 \cos \varphi e^{-j120^\circ} \right].$$  

Complex currents in the primary windings of the TCNP-2 transformer $I_A$, $I_B$, $I_C$:

$$I_A = I_c = \frac{3U}{4n^2Z} \left[ \cos \varphi + j \sin(60^\circ - \varphi) \right];$$  
$$I_B = -\frac{U}{4n^2Z} \left[ \cos \varphi + j \sin(60^\circ - \varphi) \right].$$  

Table 1 shows the results of the calculation of currents and capacities of TCNP-2 depending on the angle $\varphi$ of the load. According to this table, Figures 4 and 5 show the corresponding dependencies.

### Table 1. The results of the calculation of currents and capacities of TCNP-2 depending on the angle $\varphi$ of the load at $n = 1,0, I_n = I_{n(nom)}$

| Physical quantity | Measurements | Result of calculation |
|-------------------|--------------|-----------------------|
| $\varphi$ (degrees) | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
| $I_a = I_b = I_c$ | p.u. | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| $I'_a = I'_c$ | p.u. | 1.32 | 1.25 | 1.14 | 1.0 | 0.84 | 0.67 | 0.5 |
| $I'_b$ | p.u. | 2.64 | 2.5 | 2.28 | 2.0 | 1.68 | 1.34 | 1.0 |
| $I_A = I_B = I_C$ | p.u. | 1.98 | 1.88 | 1.71 | 1.5 | 1.26 | 1.0 | 0.75 |
| $I_1$ | p.u. | 2.0 | 1.97 | 1.88 | 1.73 | 1.53 | 1.29 | 1.0 |
| $I_2$ | p.u. | -0.5 | -0.34 | -0.174 | 0 | 0.174 | 0.34 | 0.5 |
| $C_1$ | p.u. | 1.15 | 1.14 | 1.08 | 1.0 | 0.88 | 0.74 | 0.58 |
| $C_2$ | p.u. | -0.29 | -0.2 | -0.1 | 0 | 0.1 | 0.2 | 0.29 |
Figure 4. Dependences of the capacities of TCNP-2 on the angle $\varphi$ of the load at $n = 1.0$, $I_n = I_{\text{nom}}$

Figure 5. Dependences of the parameters of the balancing elements TCNP-2 on the angle $\varphi$ of the load at $n = 1.0$, $I_n = I_{\text{nom}}$

Voltage transformer converter of the number of phases TCNP-2.

Phase voltages of a three-phase symmetric receiver in accordance with equalities:

$$
U_a = I_a Z; \quad U_b = I_b Z; \quad U_c = I_c Z.
$$

$$
\begin{align*}
U_a &= \frac{U}{2n} e^{j120^\circ} = U' a; \\
U_b &= \frac{U}{2n} = U' b; \\
U_c &= \frac{U}{2n} e^{-j120^\circ} = U' c
\end{align*}
$$

(35)

Voltages on the primary windings of the transformer $U_A$, $U_B$, $U_C$ in accordance with equalities:
\[
\begin{aligned}
U_A &= ne^{j180^\circ}, \quad U_{120^\circ} = \frac{U}{2} e^{-j60^\circ}, \\
U_B &= ne^{j180^\circ}, \quad U_{120^\circ} = \frac{U}{2} e^{j180^\circ} = -\frac{U}{2}, \\
U_C &= ne^{j180^\circ}, \quad U_{120^\circ} = \frac{U}{2} e^{j120^\circ} = \frac{U}{2} e^{j60^\circ}.
\end{aligned}
\] (36)

Input voltage TCNP-2 according to equation:

\[
U = U_A - U_B + U_C = \frac{U}{2} \left(e^{-j60^\circ} + 1 + e^{j60^\circ}\right) = U. \tag{37}
\]

From equation (60) \(U = 2U_B\).

Voltage of balancing elements:

\[
U_1 = \frac{\sqrt{3}U}{2n} e^{j150^\circ}. \tag{38}
\]

\[
U_2 = \frac{\sqrt{3}U}{2n} e^{-j90^\circ}. \tag{39}
\]

In the vector diagram (Figure 6), the equations for the voltages and for the currents are satisfied, which confirms the correctness of the solution of the system of equations for TCNP-2.

Figure 6. Vector diagram for TCNP-2 with \(\phi = 0, n = 1.0, I_n = I_{n(nom)}\)

Power TCNP-2.

Three-phase load power

\[
\begin{aligned}
S_n &= \frac{3U^2}{4n^2Z}, \\
P_n &= \frac{3U^2 \cos \phi}{4n^2Z}, \\
Q_n &= \frac{3U^2 \sin \phi}{4n^2Z}.
\end{aligned}
\] (40)

Power TCNP-2

\[
S = U \cdot I^* = \frac{3U^2}{4n^2Z} \left[\cos \phi - j \sin(60^\circ - \varphi)\right] = \frac{3U^2 \cos \phi}{4n^2Z} - j \frac{3U^2}{4n^2Z} \sin(60^\circ - \varphi); 
\]
\[ P = \frac{30^2 \cos \varphi}{4n^2Z}, \quad Q = -\frac{30^2 \sin(60^\circ - \varphi)}{4n^2Z}. \]  

(41)

Power of the first balancing element

\[ S_1 = U_1 I_1^* = \frac{\sqrt{2}}{2n} e^{j150^\circ}, \quad U_1 \cos \varphi e^{j120^\circ} = \frac{\sqrt{3}U_1^2 \cos \varphi e^{-j30^\circ}}{2n^2Z} ; \quad Q_1 = \frac{\sqrt{3}U_1^2 \cos \varphi}{2n^2Z}. \]  

(42)

Power of the second balancing element

\[ S_2 = U_2 I_2^* = \frac{\sqrt{2}}{2n} e^{-j30^\circ}, \quad U_2 \sin(\varphi - 30^\circ) e^{-j30^\circ} = \frac{\sqrt{3}U_2^2 \sin(\varphi - 30^\circ)}{4n^2Z} e^{-j30^\circ} ; \quad Q_2 = \frac{\sqrt{3}U_2^2 \sin(\varphi - 30^\circ)}{4n^2Z}. \]  

(43)

4. Conclusion

1) A method for the analysis of transformer converters of the number of phases with balancing elements on the low side of the transformer has been developed.

2) The named method is applied to the analysis of a transformer of a phase number converter of a two-wire power transmission system for which patent No. 2532534 was obtained.

3) As a result of the analysis of the transformer converter of the number of phases with the balancing elements on the low side of the transformer, the parameters of the balancing elements are determined at which a symmetric system of currents is created at the converter output.

4) The dependences of the voltages and currents of a three-phase power source, primary and secondary windings of a transformer, balancing elements on the current and power factor of a three-phase load are determined.

5) A vector diagram of voltages and currents TCNP-2 was constructed in the coordinates of the complex plane, with the aid of which the phase conversion of a single-phase current to a three-phase current for a two-wire transmission system is disclosed.

5. Acknowledgments

The analysis of TCNP-1 with capacitor banks on the high side of the transformer (Figure 1) was performed and published in the journal «Electricity» [4]. The feasibility study of the new power transmission system is published in publications [7].

An analysis of a two-wire power transmission system with the transformer converter of the number of phases and capacitor banks on the low side of the transformer was performed by us and published in the journal [4]. Experimental studies of two-wire and three-wire power transmission systems were performed on a physical model in the research laboratory of SPbGAU.

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