Heavy quarkonium correlators at finite temperature: QCD sum rule approach

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We investigate the properties of heavy quarkonia at finite temperature in detail using QCD sum rules. Extending previous analyses, we take into account a temperature dependent effective continuum threshold and derive constraints on the mass, the width, and the varying effective continuum threshold. We find that at least one of these quantities of a charmonium changes abruptly in the vicinity of the phase transition. We also calculate the ratio of the imaginary time correlator to its reconstructed one, $G/G_{\text{rec}}$, by constructing a model spectral function and compare it to the corresponding lattice QCD results. We demonstrate that the almost constant unity of $G/G_{\text{rec}}$ can be obtained from the destructive interplay of the changes in each part of the spectral modification which are extracted from QCD sum rules.

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I. INTRODUCTION

In-medium property of heavy quarkonia provides information on the confinement-deconfinement transition in QCD. In relativistic heavy ion collisions, final yields measured through dilepton channels depend on whether they can exist as bound states or not. If the deconfined plasma is produced, color Debye screening will melt the quarkonia and suppress the resultant yields \textsuperscript{1}. Although $J/\psi$ has been measured in heavy ion collisions at various energies, no quantitative understanding has been reached yet, because of the intrinsic complexities of processes in the heavy ion collisions. See Refs. \textsuperscript{2,3} for recent reviews. Therefore, it is important to investigate the properties of the quarkonia in an ideal environment to give a solid foundation, not only on the existence of a bound state but also on their detailed spectral modification such as the mass shift and broadening at finite temperature or density. In this respect, it was pointed out that a downward mass shift of $J/\psi$ in hadronic matter was caused by the decrease of string tension, which had been predicted by lattice QCD, and as such can be a precursor phenomena of the confinement-deconfinement transition \textsuperscript{2}. Therefore, the detailed determination of the spectral properties can play a key role in the study of QCD phase transition.

Properties of the bound states have been traditionally investigated with quantum mechanical potential models. It is known that the mass spectrum of heavy quarkonium can be described well by the so-called Cornell potential, which implements the Coulomb potential at short distance and linearly rising one at long distance \textsuperscript{2,6}. This approach can be extended to finite temperature by assuming that all the effects of temperature can be accounted for by the temperature dependent potential \textsuperscript{8}. In this approach, however, how to construct the potential relevant for Schrödinger equation is a non-trivial problem and various types have been examined by incorporating properties known from lattice QCD \textsuperscript{9,13}. Although the potential model approach is related to QCD only through the temperature-dependent potential computed with lattice QCD, recent development in weak coupling methods such as pNRQCD \textsuperscript{14,15} which is an effective field theory of QCD and a resummed perturbative approach \textsuperscript{17,18} shed light on more rigorous foundation of the heavy quark potential at finite temperature.

Direct evaluation of the quarkonium properties with lattice QCD has been carried out through the maximum entropy method (MEM) \textsuperscript{19,21}. One can reconstruct the spectral function of a given channel by inverting the dispersion relation of the current correlation function calculated in the imaginary time. Indeed, Ref. \textsuperscript{19} indicated the existence of $J/\psi$ bound state even in the deconfined phase up to $T \sim 1.6T_c$. At high temperature, however, lattice QCD suffers from the limited size of the temporal direction, which also affects the accuracy of the reconstruction of the spectral function in MEM \textsuperscript{21,22}. In this approach, the only reliable information seems to be the presence or disappearance of the first peak in the spectral density. A potential model calculation \textsuperscript{13,22} indicates that the first peak observed in MEM at high temperature can be attributed to the threshold enhancement and that $J/\psi$ has already melted at $T = 1.2T_c$.

Recently AdS/QCD approach has also been applied to the heavy quarkonium in medium \textsuperscript{24,25}. Although a direct relation to real QCD is still missing, the approach seems to give another insight to the problem from the viewpoint of the strongly coupled gauge field theory. Both Refs. \textsuperscript{24,25} show notable spectral change in $J/\psi$ around and above $T_c$.

In previous works \textsuperscript{26,29}, we have proposed another approach to study the properties of heavy quarkonia at finite temperature based on QCD sum rules; the approach extended the previous studies at nuclear medium \textsuperscript{31,31}. The QCD sum rule \textsuperscript{32,33} provides a systematic frame-
work which connects the current correlation function at deep Euclidean region to the spectral function integrated with respect to the energy variable with a weight that makes the integral dominated by the lowest pole. It has been applied to various aspects of hadrons quite successfully \cite{31,32}. Due to the asymptotic freedom, one can reliably compute the correlation function at the deep Euclidean region using perturbation theory via the operator product expansion (OPE) which provides non-perturbative correction through QCD condensates. For a heavy quarkonium, to a good approximation, one can truncate the expansion at the lowest dimensional local operator, which is the dimension four gluon condensate.

The aim of this work is to extend our previous works to a more systematic analysis by incorporating the continuum part of the model spectral function, applying a more sophisticated optimization procedure in determining the spectral parameters, and then making a comparison to the lattice QCD results. In this paper, we describe the detailed procedure based on the Borel transformation which is widely used in QCD sum rule applications. Then we discuss the spectral change of charmonia and bottomonia at finite temperature near and above $T_c$. Using the spectral parameters obtained in the QCD sum rules, we construct model spectral functions and compute the imaginary time correlators, which we will compare with lattice QCD.

This paper is organized as follows. In the next section, we briefly review the QCD sum rules for heavy quarkonium at finite temperature, then explain the procedure based on the Borel transformation. We will show the results of the spectral parameters in Sec. III. We will discuss the imaginary time correlators reconstructed from the spectral parameters in Sec. IV. Section V is devoted to the summary.

II. QCD SUM RULES FOR HEAVY QUARKONIUM IN MEDIUM

A. OPE for correlation function

We start with the current correlation function

$$\Pi^I(q) = i \int d^4x e^{iqx} \langle [J^I(x), J^J(0)] \rangle.$$  \hspace{1cm} (1)

We choose the currents for pseudoscalar ($P$), vector ($V$), scalar ($S$), and axial-vector ($A$) as

$$j^P = i \bar{h} \gamma_5 h$$  \hspace{1cm} (2)

$$j^V = \bar{h} \gamma_\mu h$$  \hspace{1cm} (3)

$$j^S = \bar{h} h$$  \hspace{1cm} (4)

$$j^A = \left( q_\mu q_\nu / q^2 - g_\mu\nu \right) \bar{h} \gamma_5 \gamma^\nu h$$  \hspace{1cm} (5)

with $h$ being the heavy quark operator, $c$ or $b$. For the axial-vector, we pick up the conserved components for $\chi_{c(1)}$ states. We will take the expectation value at finite temperature. Therefore, in general, there are two independent components in both the $V$ and the $A$ channels. We assume the momentum of the current to be $q^\mu = (\omega, 0)$, i.e., a pair of quark and antiquark at rest with respect to the medium such that only one component becomes independent. Then we define the dimensionless correlation function as

$$\tilde{\Pi}^{P,S}(q^2) = \frac{\Pi^{P,S}(q)}{q^2}$$  \hspace{1cm} (6)

$$\tilde{\Pi}^{V,A}(q^2) = \frac{\Pi^{V,A}(q)}{-3q^2}$$  \hspace{1cm} (7)

which can be expanded up to dimension four operators via OPE

$$\tilde{\Pi}^I(q^2) \simeq C_1^I + C_2^I G_0 + C_2^I G_2.$$  \hspace{1cm} (8)

with $G_0 = \langle \phi G_{\mu\nu}^a G^{a\mu\nu} \rangle$ and $G_2$ being the scalar and twist-2 gluon condensates, respectively. $G_2$ is defined as the traceless and symmetric part of the gluon operator as

$$\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a \rangle = \left( u_{\mu} u_{\nu} - \frac{1}{4} g_{\mu\nu} \right) G_2$$  \hspace{1cm} (9)

which vanishes at $T = 0$ according to the Lorentz invariance. The medium four velocity $u^\mu$ is set to $(1, 0, 0, 0)$. Hereafter we assume that all medium effects are imposed on the change of the local operators \cite{30}. This assumption is justified when the typical scale of the condensates is smaller than the separation scale \cite{29}, namely,

$$4m_h^2 - q^2 \gg \langle G \rangle \sim (\Lambda_{QCD} + aT + b\mu^2).$$  \hspace{1cm} (10)

with $m_h = m_c$ or $m_b$ being the heavy quark mass. The large heavy quark mass $m_h$ allows us to work even at the physical energy scale $q^2 = m_{J/\psi}^2$, $m_{\Upsilon}^2$, and so on, although somewhat marginal in reality \cite{37}. In this case, the OPE gives a formula for the bound state mass that is proportional to the change of the color electric field squared. This formula is the QCD second order Stark effect \cite{37,38} that can also be obtained from the leading order correction of the static potential in pNRQCD \cite{10}. Combining the formula with the temperature dependence of the condensate, one finds a sudden mass shift at $T_c$ \cite{39}. In the QCD sum rule, we go to the deep Euclidean region $q^2 = \omega^2 = -Q^2 \ll 0$, in which the condition (10) is well fulfilled. Therefore the Wilson coefficients $C_i$ are the same as the vacuum case and have been calculated in Refs. \cite{28,30,54,40}. See Ref. \cite{29} for a list.
B. Borel transformation and dispersion relation

The Borel transformation of the correlation function is defined as

$$\mathcal{M}^J(M^2) = \lim_{Q^2/n \to M^2} \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2}\right)^n \Pi^J(Q^2).$$  \hspace{1cm} (11)

If one does not take the limit, the derivative of the correlation function corresponds to the moment of the OPE which was used in the previous sum rule works for the heavy quarkonia \textit{[26–28, 30, 31, 33, 40].} Taking this limit corresponds to going to deeper Euclidean region for better perturbative expansion while retaining the connection to the resonance through large \( n \) \textit{[32].} Indeed, Eq. \textit{(10)} is expected to be better satisfied as the typical OPE term \( \left(\frac{1}{4m_n^2-Q^2}\right) (G^d) \) reduces to \( \frac{1}{d(M^2)} e^{-4m_n^2/M^2} (G^d) \) after the transformation, and hence the condensate contribution is further suppressed by \( \frac{1}{T} \).

In the heavy quarkonia, the momentum sum rule works well enough to extract the mass due to the large separation scale coming from the heavy quark mass. Nevertheless, the Borel transformation approach has several advantages for more systematic analysis as revealed below.

For the expanded heavy quarkonium correlation function \textit{(8)}, the Borel transformation can be analytically carried out as

$$\mathcal{M}^J(M^2) = e^{-\nu \pi A^J(\nu)} [1 + \alpha_s(M^2) a^J(\nu) + b^J(\nu) \phi_b(T) + c^J(\nu) \phi_c(T)] \hspace{1cm} (12)$$

with a dimensionless scale parameter \( \nu = 4m_n^2/M^2 \). The first line of Eq. \textit{(12)} is the same as that derived in Ref. \textit{[4]} except for the temperature dependency of the scalar gluon condensate term \( \phi_b \). The second line shows the twist-2 term which appears in the case of medium. \( \phi_b \) and \( \phi_c \) are defined as

$$\phi_b = \frac{4\pi^2}{9(4m_n^2)^2} G_0(T),$$  \hspace{1cm} (13)

$$\phi_c = \frac{4\pi^2}{3(4m_n^2)^2} G_2(T),$$  \hspace{1cm} (14)

as given in Ref. \textit{[26, 27]}. While \( A^J(\nu), a^J(\nu) \) and \( b^J(\nu) \) are given in Ref. \textit{[4]}, the transformed twist-2 coefficient \( c^J(\nu) \) is derived for the first time in this paper. For completeness, we list all the Borel transformed Wilson coefficients used in Eq. \textit{(12)} in Appendix \textit{A}. While Bertlmann worked on the on-shell renormalization of heavy quark mass in Ref. \textit{[4]}, we maintain the off-shell renormalization as a straightforward extension from Ref. \textit{[40]}. Hence, the correction term \( -\frac{\partial A^J(M^2)}{\partial (1/M^2)} \) is included throughout this calculation. Note that this term is also necessary to keep the perturbative correction term small enough in \( \mathcal{M}(M^2) \) and in \( -\frac{\partial A^J(M^2)}{\partial (1/M^2)} \) that is used later. Since this part is temperature independent, the difference does not affect our aim but enables us to proceed in a more transparent way by retaining the relation with the previous moment sum rule analyses. In Eq. \textit{(12)}, external inputs are heavy quark mass \( m_h \) (contained in \( A(\nu), \phi_b \) and \( \phi_c \)), strong coupling constant \( \alpha_s(M^2) \) and gluon condensates \( G_0(T) \) and \( G_2(T) \). In this paper, we put \( m_c(p^2 = -m_c^2) = 1.262 \) GeV inferred from the \( m_c(p^2 = -2m_c^2) = 1.24 \) GeV used in the previous works \textit{[26, 30]} and \( m_b(p^2 = -m_b^2) = 4.12 \) GeV. \( \alpha_s(M^2) \) is calculated from the running coupling formula from \( \alpha_s(8m_c^2) = 0.21 \) also used in the previous works. \( \alpha_s(8m_b^2) = 0.158 \) is used for bottomonia. The gluon condensates have been extracted from the results of pure SU(3) lattice gauge theory \textit{[42]} and shown in Fig. \texttt{1}. Here effective temperature dependent coupling constant \( \alpha_s(T) \) is used for the determination of \( G_2(T) \) which is the symmetric and traceless part of the gluon operator. This is done with the identification \( \alpha_s(T) = \langle \pi \pi \rangle (G_{\alpha \mu} G_{\beta}^{\alpha \mu}) \) based on the separation scale in the OPE. We adopted \( \alpha_s(T) \) shown Ref. \textit{[43]} and took the value at \( r = r_{\text{screen}} \). It is also shown in the bottom panel of Fig. \texttt{1}.

The current correlation function is related to the spectral function through the dispersion relation. At finite temperature, the spectral function is given by the imag-
inary part of the retarded correlation function $\Pi_R(\omega)$ which is in general different from Eq. (1). However, we can relate it to the spectral function by virtue of the fact that $\Pi_R(\omega) = \Pi(\omega^2)$ since $\omega^2 = -Q^2 < 0$ \cite{36}. Putting $\tanh(\sqrt{s}/2T) = 1$, which is safely satisfied for $\sqrt{s} \sim m_{hh}$, we have the dispersion relation for the Borel sum rule in the same form as the vacuum case

$$M^I(M^2) = \int_0^\infty ds \ e^{-s/M^2} \text{Im}\tilde{\Pi}^I(s). \quad (15)$$

Note that the weight factor in the dispersion integration is now exponential $e^{-s/M^2}$ while it was the inverse power $(s + Q^2)^{-n}$ in the moment sum rule.

C. Analysis procedure

Assuming the quark-hadron duality, we take a model spectral function of a given current as a simple ansatz for the imaginary part of the correlation function and call it the spectral function of a given current as a simple ansatz for $\Pi$.

We adopt the perturbative part of the spectral function including $\alpha_s$ correction but with the sharp threshold factor $\theta(s-s_0)$ as a model for the continuum; such form reproduces the corresponding part of the OPE side when putting $s_0 = 4m_h^2$. These functional forms are explicitly given in Ref. \cite{34} and listed in Appendix C for the completeness. Since there are known excited states such as $\psi'$ between the lowest lying state and the physical continuum threshold, one may think this model is an oversimplification of the real spectrum. However, due to the suppression coming from the Borel transformation, this simplification does not affect the property of the lowest pole. Instead, this form results in a little smaller continuum threshold value than that from the analysis incorporating the excited states explicitly as we will see later. Moving the continuum part to the OPE side in Eq. (15), one can isolate the pole term. There is an additional contribution to the spectral function from the absorption of the current by the thermally excited particle, i.e., Landau damping which shows up as a peak at $s = 0$ when $q = 0$. This was recognized in Ref. \cite{44} in a QCD sum rule framework and has been called the “scattering term”. Recently it has been emphasized that this gives constant contribution to the imaginary time correlator which is the basis of the spectral function study in the lattice QCD \cite{42}. In the QCD sum rule application in the deconfined phase, we can neglect this contribution, as explained in Ref. \cite{28,29}. The scattering term appears in the OPE through the bare heavy quark condensates $\langle h\Gamma\bar{h}D_D\bar{h}\rangle$ which is converted into the gluon condensates via heavy quark expansion at $T = 0$ \cite{46}. To a first approximation that assumes free heavy quarks in a medium, we can put the same quantity on the phenomenological side so that it cancels the corresponding contribution in the OPE.

Differentiating both side of Eq. (15) with respect to $1/M^2$ and taking its ratio to the original equation, one has

$$\frac{\partial}{\partial(1/M^2)} \left[ \mathcal{M}(M^2) - \mathcal{M}_{\text{cont}}(M^2) \right] = \frac{\int_0^\infty ds \ e^{-s/M^2} \text{Im}\tilde{\Pi}^\text{pole}(s)}{\int_0^\infty ds \ e^{-s/M^2} \text{Im}\tilde{\Pi}^\text{pole}(s)} \quad (19)$$

where $\mathcal{M}_{\text{cont}}(M^2)$ is the Borel-transformed continuum spectral function according to Eq. (15). One immediately finds that the right-hand side of Eq. (19) gives the squared pole mass $m^2$ for $\Gamma = 0$. There are three spectral parameters to be determined in Eq. (19): pole mass $m$, width $\Gamma$ and effective continuum threshold $s_0$. The strength parameter $f_0$ or $f$ contained in the pole term cancels by taking the ratio. We solve Eq. (19) for the mass $m$ as a function of the Borel mass $M^2$ (which we call Borel curve) with given sets of $\Gamma$ and $s_0$. While the extracted mass $m$ depends on $M^2$ through the left-hand side of Eq. (19) by construction, it should not do so since $M^2$ is an external parameter. The apparent dependence of $m$ on $M^2$ is due to the truncation of the OPE and the insufficient subtraction of excited states and the continuum part in the spectral density. The truncation of the OPE shows up as strong $M^2$ dependency at small $M^2$ while the insufficient subtraction does at large $M^2$. In practice, however, we can expect $M^2$ independent $m$ at intermediate $M^2$ region after tuning the effective threshold parameter $s_0$ in the continuum part \cite{46,47}. Introducing the finite width also affects the $m(M^2)$ in the small $M^2$ region \cite{48}. We thus introduce the following quantity to determine the best set of the parameters that
gives the flattest curve in the intermediate $M^2$ region;
\[ \chi^2 \equiv \frac{1}{M^2_{\text{max}} - M^2_{\text{min}}} \int_{M^2_{\text{min}}}^{M^2_{\text{max}}} dM^2 [m(M^2) - m(M^2)]^2 \]

(20)

where $M^2_0$ is defined by $dm(M^2)/dM^2|_{M^2=M^2_0} = 0$. The range of the intermediate $M^2 \in [M^2_{\text{min}}, M^2_{\text{max}}]$ is called Borel window, in which the convergence of the OPE and the pole dominance of the dispersion integral are satisfied. We fix $M^2_{\text{min}}$ by requiring the dimension four correction terms to be smaller than 30% of the total since it is expected that the contribution from the next higher dimensional operator is kept less than 10% of the total within this condition [32]. For bottomonium systems, while this condition is always fulfilled in typical $M^2$ values due to the larger quark mass in Eqs. (13) and (14), the perturbative radiative correction term $\alpha_s(M^2)\alpha^2(\nu)$ can be large enough to spoil the perturbative expansion. We therefore impose this term to be less 0.3 at $M^2 > M^2_{\text{min}}$. The pole dominance should be also imposed on the criteria to preserve the reliability on the extracted property of the pole part of the spectral density. We determine $M^2_{\text{max}}$ by requiring the continuum contribution to be less than 30% of the total perturbative term. As we shall see, reducing the value of the continuum threshold makes the Borel curve flatter. Therefore, this criterion does not affect the pole mass at the best fit while it becomes important when the Borel window is narrow. The $\chi^2$ measures the average deviation of $m(M^2)$ from its value at the $M^2$ “plateau” characterized by $dm(M^2)/dM^2|_{M^2=M^2_0} = 0$. While it vanishes in the case of completely $M^2$ independent mass, $\sqrt{\chi^2}$ can be regarded as a systematic error on the extracted mass $m(M^2_0)$ such that $m(M^2_0) + \sqrt{\chi^2}$ gives the upper limit. Note, however, that it gives much larger deviation than actual uncertainty when Borel window contains the strongly $M^2$ dependent part of the Borel curve at small $M^2$ which is due to the truncation of OPE. Hence, a refinement on the determination of the Borel window will give more quantitative insights on the uncertainty in the extraction of the mass parameter.

To illustrate the minimization procedure, we show below how the Borel curve obtained by solving Eq. (19) changes with respect to the external parameters. We depict some examples in Fig. 2. Let us start with the dotted curve corresponding to $T = 0, \sqrt{s_0} = 3.5\text{ GeV}$, and $\Gamma = 0$ for $J/\psi$. From the definition of the Borel mass $M^2$, one sees that the Borel curve looks like the dependence of the moment ratio shown in Refs. [20, 27] except for the reversed direction of the horizontal axis. Here we draw the lines only within the Borel window. Therefore the dotted line is truncated at $M^2 = 1.12\text{ GeV}^2$ and $3.69\text{ GeV}^2$. The arrow on each line indicates the location of $M^2_0$ and the upper limit evaluated from $\sqrt{\chi^2}$. As temperature increases, the gluon condensates decrease as shown in Fig. 11. This is reflected by the lowered curves obtained for $T = 1.12T_c$, as in line with the moment sum rule case [20, 27]. Since the condensate contribution becomes dominant in the OPE side at small $M^2$, one sees great reduction of mass in this region. Then the solid curve, for $T = 1.04T_c$, shows a minimum at smaller Borel mass. This corresponds to the shift of the minimum of the moment ratio to large $n$ in the moment sum rule. Regarding the solid curve as the base line, one sees that decreasing the continuum threshold flattens the curve at large $M^2$. In this case, however, reduction of the continuum threshold makes the $M^2_{\text{max}}$ smaller and thus the resultant Borel window becomes narrower. This is why the dash line ends at $M^2_{\text{max}} = 2.08\text{ GeV}^2$. One also sees that the curve is almost flat above $M^2 = 1.5\text{ GeV}^2$. This means that further reduction of the continuum threshold breaks the stability of the Borel curve, i.e., the mass decreases monotonically and no minimum.
would exist. $M^2_{\text{min}}$ does not change against the reduction of the continuum threshold since it depends only on temperature through the power correction terms in the OPE. On the other hand, if one increases the width, it raises the mass, especially at low $M^2$ as clearly seen in the short-dashed curve. One sees that, at $M^2$ far from the $M^2_{\text{min}}$, the two lines, one obtained by reducing the continuum threshold and the other by introducing the width, show almost similar flatness. In the present case, the rapid rise in the $\Gamma = 100$ MeV gives $\chi^2 = 8.55 \times 10^{-4}$ GeV$^2$ which is much bigger than $\chi^2 = 1.89 \times 10^{-4}$ GeV$^2$ of the $\sqrt{s_0} = 3.2$ GeV curve. However, this depends on the choice of the criteria in the determination of the Borel window. If one tightens the criterion, to 10% power correction for instance, $M^2_{\text{min}}$ becomes larger and then $\chi^2$ of the $\Gamma = 100$ MeV will be smaller. This indicates the difficulty in accurately determining the spectral parameters when one takes into account the change of both the width and the continuum threshold. Nevertheless, one can make the curve flatter by decreasing the continuum threshold without introducing broadening up to a certain temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature. Note that the arrows become shorter as the temperature.

In the following, we use the $\chi^2$ evaluation using Eq. (20) only for determining the best curve among those cases with the same continuum threshold but with different $\Gamma$, in order to avoid the biases imposed by the choice of the Borel window. In this way one fixes one edge of the curve $M^2_{\text{max}}$ so that $\chi^2$ measures only the effect of introducing the width. For some cases where $\Gamma = 0$ always gives the flattest curve, we may use $\chi^2$ to determine the best $s_0$ value, since the other edge of the Borel curve is fixed. For example, we can safely determine the best $s_0$ by evaluating $\chi^2$ at $T = 0$. Furthermore, though we maintain the criterion for the Borel window as explained, we can easily estimate how the best value changes with respect to the change of the criterion. If one relaxes the pole dominance condition, it extends the Borel window to larger $M^2$ therefore continuum threshold giving the best $\chi^2$ will become smaller. On the other hand, if one requires the smaller power correction, $M^2_{\text{min}}$ becomes larger and thus the $\chi^2$ will be more sensitive to the continuum. As long as we preserve reasonable values of these criterion, typically $10 - 30\%$ for power correction and less than $50\%$ for the continuum contribution, we find the uncertainty of the obtained mass to be about a few tens MeV. Since we maintain the same criterion even at different temperatures, the relative in-medium change of the spectral parameters is not affected by the particular choice of the criterion for the Borel window.

Before closing the section, we would like to address possible uncertainty on the extracted parameters. The source of uncertainty is roughly classified into two parts; one is temperature independent and the other is dependent quantities. The former consists of the gluon condensate at $T = 0$, strong coupling constant, and the heavy quark masses. Whereas these quantities certainly affect the value of the spectral parameters, the relative changes at finite temperatures from the vacuum values do not differ by changing them within the constraints from the experiment. Hence we focus on the effect of the temperature dependent part here. We estimate the $1\sigma$ and $2\sigma$ uncertainty of the OPE side through the temperature dependent part of the gluon condensates by reading
off that of the energy density and pressure in the lattice results shown in Ref. [42]. Then, we extract the spectral parameters with the OPE side shifted by $\pm 1\sigma$ and $\pm 2\sigma$. The resultant constraints which show up the typical correlation between the mass and the width together with the uncertainty deduced from the deviation of the OPE side are displayed in Figs. 4 and 5. The solid line in Fig. 4 shows the case corresponding to the long-dashed line in Fig. 2; the best Borel curve is obtained by reducing the threshold parameter without introducing width. This fact is reflected to the kink of the $\chi^2_{\text{min}}$ contour (thin solid line). If the OPE side increases, the minimum shifts to finite $T$ (see next section). The obtained constraint shows a clear correlation between the mass and the width as pointed out by us in Ref. [26]. Quantitatively the relation can vary within $\pm (10 - 20)$ MeV due to the change of the temperature dependency. Nevertheless the overall behavior of the correlation is preserved. Figure 5 shows a similar case but the finite width $\simeq 100$ MeV was chosen as the best fit. The correlation between the mass shift and the width does not differ from the other example but the shape of the $\chi^2$ contour slightly does, reflecting the straight contour of $\chi^2_{\text{min}}$. In these figures, the area surrounded by the $1\sigma$ or $2\sigma$ correlation lines and one of the $\chi^2$ contour can be regarded as a possible region of the mass shift and the width including uncertainty for a given $\sqrt{s_0}$.

### III. Temperature Dependence of Spectral Parameters

#### A. Implication from OPE side

Let us begin with examining how temperature dependence of the gluon condensates is related to the spectral parameters by looking at the dispersion relation (15). The temperature dependence of the OPE side [Eq. (12)] comes from the gluon condensate terms $b_\phi \phi(T)$ and $c_\phi \phi(T)$. In Fig. 6 we plot the OPE coefficients in Eq. (12). One sees that both the scalar gluon condensate contribution and the twist-2 one increase as temperature increases. These

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**FIG. 4:** (color online). Constraint on the mass shift and the width of $J/\psi$ at $T = 1.04 T_c$ and $\sqrt{s_0} = 3.28$ GeV. In addition to the thick lines denoting the constraint, contours of the equal $\chi^2$ values (unit of deviation from the minimum) are plotted as thin lines.

**FIG. 5:** (color online). Same as Fig. 4 but for $\sqrt{s_0} = 3.5$ GeV.

**FIG. 6:** (color online). OPE coefficients of $V$ (upper panel) and $S$ (lower panel) channels in charmonium systems. Solid lines denote the temperature independent perturbative correction term $\alpha_s(M^2) a(\nu)$. Long and short dashed lines stand for the scalar gluon condensate term $b(\nu) \phi(T)$ for $T = 0$ and $T = T_c$, respectively. Dotted line shows the twist-2 contribution $c(\nu) \phi(T)$ at $T = T_c$. 

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**TABLE 1:** OPE coefficients $\alpha_s, b(\nu), c(\nu)$ at $T = T_c$ and $T = 0$.
dependencies come from the fact that \( G_0(T) \) and \( G_2(T) \) are monotonically decreasing functions of the temperature while the coefficients \( b(\nu) \) and \( c(\nu) \) have always negative sign.\(^1\) Therefore, up to dimension four operators, \( \mathcal{M}(M^2) \) in the OPE side is always increasing function of the temperature.

One also sees that the expansion coefficients of the \( P \)-wave state are larger than \( S \)-wave’s at the same Borel mass \( M^2 \), as also seen in the moment sum rule case \([28, 29]\). These properties result in the correct mass splitting between the \( J/\psi \) and the \( \chi_c \) states, and induce larger mass shift for the \( P \)-wave states through the derivatives of these coefficient with respect to \( 1/M^2 \). From the behavior of the condensate contributions at low \( M^2 \), one realizes how the location of the Borel window in the case of charmonia changes with respect to the temperature. At temperatures close to \( T_c \), \( M_{\text{min}}^2 \) is determined by the scalar condensate and thus becomes smaller as temperature increases. It eventually starts to increase when the twist-2 contribution dominates over the scalar one, \(|c(\nu)\phi_0| > |b(\nu)\phi_0|\).

From the dispersion relation \([13]\), where the weight of the integral over the spectral function is positive definite, one obtains the constraint equation for the changes of the spectral parameters against the change of the OPE side discussed above. The phenomenological side \([16–18]\) has four parameters: effective continuum threshold \( s_0 \), pole mass \( m \), width \( \Gamma \), and overlap \( f \). If only one of these four quantities is allowed to vary as temperature increases, the respective changes of the parameters needed to match the OPE side are,

- \( s_0 \): decrease,
- \( m \): decrease,
- \( \Gamma \): increase,
- \( f \): increase.

In practice, all of these quantities can change not only to the expected direction but also to the opposite, as long as the total combined change of the spectral function matches the OPE side. The Borel transformation procedure explained in the previous section provides an optimization way to find out the best set of the changes.

### B. Results for charmonia

We carried out the analyses for the charmonia for various temperatures around \( T_c \). For a reference, we summarize the results of \( T = 0 \) in Table I. One sees that the masses are well reproduced by the common parameter set indicated before. Finer tuning on the quark mass, the coupling constant, and the gluon condensate may improve the small discrepancies with the experimental data but we are not intending to do so here since our aim is to investigate the relative change from the vacuum value induced by the medium.

![FIG. 7: (color online). Constraint on the spectral parameters obtained from the QCD sum rule for \( \eta_c \). Upper and lower panels show the extracted masses and widths as functions of the continuum threshold, respectively. Symbols guided by lines represent the different temperatures. The vertical and horizontal lines indicate the continuum threshold and the mass at \( T = 0 \), respectively.](image)

We display the charmonium masses and widths extracted using the method described in the subsection [11C] as functions of the effective continuum threshold \( \sqrt{s_0} \) in Figs [7–10]. One sees the constraints among \( \sqrt{s_0}, m \) and \( \Gamma \) for various temperatures around \( T_c \). Qualitatively all the four cases are quite similar; the mass increases almost lin-

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\(^1\) In \( P \) channel \( b(\nu) \) can be positive, as understood from Eq. [221], but we found it is mostly negative for values of \( \nu \) corresponding to \( M_0^2 \) and \( \mathcal{M}(M^2) \) retains the property of increasing function of the temperature.
determine the best \( s_0 \) at all the temperatures. While it shows linear increase, the slope of the mass changes depends on whether the width is zero or not. One can see kinks in the mass curves at the same values of the \( s_0 \), we notice that it takes the minimum near the kink among various \( s_0 \) values at a fixed temperature. At \( T = T_{\text{onset}} \), beyond which width must be nonzero to maintain the Borel stability, the widths shown in the lower panels show linearly increasing behavior with increasing continuum threshold. The onset temperatures are summarized in Table II. One sees \( \eta_c \) starts to broaden earlier than \( J/\psi \) while \( T_{\text{onset}} \) of the \( P \)-wave states are the same. This may indicate a different temperature effect on the spin-spin interaction responsible for the mass splitting in the \( S \)-wave states. In comparison with what we learned in Sec. III A one sees the result is more complicated than the simple analysis by the dispersion relation. One sees the reduction of the mass always couples with that of the effective threshold (see dashed curve). Thus the downward mass shift always occurs with the reduction of the effective continuum threshold if there is no broadening.

Behavior of the width at small \( \sqrt{s_0} \) and at \( T > T_{\text{onset}} \) seems different from the genuine linear behavior. It first decreases as \( \sqrt{s_0} \) increases, then turns to increase. Formally we could obtain stable Borel curves at higher temperatures than those shown in the figure. At such high temperature, the width is always at an order of hundreds MeV and the Borel curve is similar to those displayed in Fig. 8. We would like to stress, however, that this result may not be a physical one; in this region, the mass is also small while the width becomes 100 – 200 MeV or more. Clearly the Breit-Wigner ansatz in the phenomenological side [17] which cut off the lower energy part than \( 4m_c^2 \) does not match with the dispersion integral [15]. If we do not impose the cutoff, the strong suppression factor combined with the Breit-Wigner form in the Borel-transformed dispersion relation [15] leads to numerical artifacts such that the contribution coming from spectral function much below \( 4m_c^2 \) comprises a subdominant fraction of the total dispersion integral. We depict an example taken from \( J/\psi \) at \( T = 1.07T_c \) in Fig. 11. The same consideration also holds for \( \eta_c \), \( \chi_{c0} \), and \( \chi_{c1} \) at \( T \geq 1.07T_c \). One sees that the integral receives large contribution from the energy region much smaller than \( 4m_c^2 \) when the width becomes larger, despite the increase of the mass. For example, let us consider the change of the width when lowering the \( h\bar{h} \)
threshold by 1 GeV$^2$ in the data shown in Fig. 11. With this change, $\Gamma = 64$ MeV at $\sqrt{s_0} = 3.2$ GeV and $\Gamma = 110$ MeV at $\sqrt{s_0} = 3.4$ GeV become 30 MeV and 62 MeV, respectively. This is so because the contribution to the dispersion integral from $s = 4m_c^2 - 1$ to $s = 4m_c^2$ is large enough to compensate the smaller Breit-Wigner width. We notice, however, that the solution of the sum rule, Eq. (19), does not exist at near $M_{\text{min}}^2$ for even smaller threshold as in the case shown in Ref. [18].

We also notice that this artifact is absent in the moment sum rule up to $n = 20$ beyond which it breaks down. Therefore, changing lower limit of the integration range from $4m_c^2$ to 0 will not affect the previous results. At present, use of the vacuum dispersion relation, which cuts off the contribution below $4m_c^2$, seems effective to estimate the width when its magnitude is less than 100 MeV. To give more quantitative results, we may need to take into account more detailed structure beyond the Breit-Wigner ansatz. Recent resummed perturbative calculation [74] might provide useful information for a better modeling. Furthermore, Borel curves at low $M^2$ will be more influenced by higher dimensional operators we have neglected. Since the width is sensitive to the low $M^2$ behavior of the Borel curves, as shown in subsection II C, it may receive sizable correction from those operators. At present, temperature dependence of the higher dimensional operators is poorly known. More quantitative analysis of the width in the non-perturbative manner thus needs further efforts.

Since Figs. 7–10 give only constraints, one needs to specify one of those spectral parameters to discuss specific temperature dependencies of each parameter. Previous analyses [26–29] correspond to $s_0 \to \infty$ limit. For instance, if $\sqrt{s_0}$ retains the vacuum value, the mass decreases rapidly until $T = T_c$ in $\eta_c$ and $T = 1.02T_c$ in $J/\psi$. We do not show the results of constant $\sqrt{s_0}$ for the $P$-wave states, as Eq. (19) has no solution at certain region inside the Borel window and thus $\chi^2$ [Eq. (20)] cannot be evaluated before it reaches the minimum as a function of the width. This absence of the solution actually occurs in $S$-wave cases also, especially at larger $\sqrt{s_0}$ and comes from the non-monotonic behavior of Borel transformation of the Breit-Wigner function [18]. Note that this does not mean the corresponding parameter sets are completely excluded, since one may choose another (narrower in most cases) Borel window such that the solution exists in any $M^2 \in [M^2_{\text{min}}, M^2_{\text{max}}]$. Nevertheless, from the almost linear dependence of the mass and width on $\sqrt{s_0}$, one can extrapolate the lines up to the desired value to have a rough estimate. Then, one finds in all the channels that the mass first decreases then the width starts to increase, as the temperature increases when $\sqrt{s_0}$ is held fixed. Note that this transition of the temperature dependence of the mass is caused by the start of the broadening of its width. We would like to point out that the analysis at $s_0 = \text{constant}$ is not the same as that at $s_0 \to \infty$ in which the determination of the flattest mass curve by $\chi^2$ does not make sense. When the mass is held

| $T_{\text{onset}}$ | $\eta_c$ | $J/\psi$ | $\chi_{c0}$ | $\chi_{c1}$ |
|------------------|----------|----------|-------------|-------------|
| 1.04             | 1.07     | 1.05     | 1.05        |             |

FIG. 10: (color online). Same as Fig. 7 but for $\chi_{c1}$.
to its vacuum value, the constraint is satisfied by the increase in both $\sqrt{s_0}$ and the width. For example, in the $J/\psi$ case, $\sqrt{s_0}$ becomes 3.6 GeV and $\Gamma = 128$ MeV at $T = 1.04T_c$ while $\Gamma = 0$ is still possible if the mass and $\sqrt{s_0}$ decrease to 2.92 GeV and 3.16 GeV, respectively.

At present, the temperature dependence of the continuum threshold is not clearly known yet. In fact, our threshold parameter should be regarded as an effective one since we do not take radial excited states such as $\psi'(2S)$ and $\eta_c(2S)$ into account near temperature dependent behaviors near the threshold [12, 18, 23]. Nevertheless, if those states dissociate at low temperatures as $T_c$, one may regard $\sqrt{s_0}$ as a physical threshold within temperatures between $T_c$ and a certain temperature at which the pole position becomes so close to the threshold that the pole and the continuum part of the model spectral function starts to overlap. For the $P$-wave states, the model will be better due to the absence of excited states below the threshold. One might be able to interpret the asymptotic value of the quark-antiquark potential as the continuum threshold [11], which decreases as temperature increases irrespective of the choice of the potential [3, 51]. This fact might be related to the decrease of the mass and subsequent dissolution of the $D$ mesons [52]. In this case, the obtained constraints give the downward mass shifts in the all channels. If the reduction is strong, only the mass shift occurs without broadening up to $T = T_{\text{onset}}$. If not, the widths will start to broaden gradually together with the moderate downward shift of the mass.

Another external constraint can be obtained from the second order Stark effect in QCD [37, 53]. Although the applicability to the charmonium systems is marginal, it gives a genuine downward mass shift due to the rapid increase of the color electric condensate [39]. For illustration in the case of downward mass shift, we combined the result of Ref. [39] with those of Figs. 7 and 8 by finding the masses in Figs. 7 and 8 that matches with the results of the second order Stark effect and then looking at the corresponding continuum threshold and width. The results of the masses, continuum thresholds, binding energies defined by $E_{\text{bin}} = \sqrt{s_0} - m_c$ and the widths of $J/\psi$ and $\eta_c$ are displayed in Fig. 12. As explained, the results for $T > T_{\text{onset}}$ are marginal. Moreover, the second order Stark effect has also limitation of applicability at this temperature region as the change of the electric condensate value becomes too large. Indeed the mass obtained from the Stark effect at $T > 1.09T_c$ becomes smaller than the smallest mass in Figs. 7 and 8 indicating the breakdown of the OPE in the Stark effect. Hence, we emphasize that any extrapolation of Fig. 12 to higher temperature is not appropriate. Apart from the marginal region, one sees that the downward mass shift smaller than the maximum given by QCD sum rules, as already found in Ref. [39], leads to broadening just above $T_c$. These temperatures, $1.02T_c$ for $\eta_c$ and $1.04T_c$ for $J/\psi$, are lower than the corresponding onset temperatures. One also sees that the continuum thresholds suddenly decrease around $T_c$ as in the case for the masses. It is quite intriguing to see that similar results are obtained in the potential model approaches, which utilizes the confinement force that can not be derived within the OPE formalism. Since the continuum thresholds change more rapidly, the resultant binding energy also drastically decreases across $T_c$. At the marginal temperatures, $E_{\text{bin}}$ is still around 100–200 MeV but the widths also become sizable due to thermal activation by gluons. We cannot draw conclusion on the dissociation of the charmonia from these results, since $\Gamma > 100$ MeV will have to be examined more carefully by incorporating higher dimensional operators and more realistic spectral function. Below $T_{\text{onset}}$, one finds $E_{\text{bin}}$ is still larger than $\Gamma/2$, indicating binding just above $T_c$. Furthermore, one does not see any broadening below $T_c$. That is in line with our previous finding in Ref. [39], where the effect of the continuum was not taken into account. Finally we would like to stress that all the spectral parameters show sudden change across $T_c$, as shown in Fig. 12 reflecting the abrupt change of the gluon condensates at this temperature and thus the QCD phase transition. Moreover, as discussed before, even if one of these parameters remains constant and retains its vacuum value, the QCD sum rule constraints force other quantities to exhibit such critical behaviors.
TABLE III: Spectral parameters of $b\bar{b}$ systems at $T = 0$. Experimental masses are taken from Particle data book [59].

| System | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $m_{\text{exp}}$ [GeV] | $M^2_0$ [GeV$^2$] | $f_0$ [GeV$^2$] |
|--------|-------------------|-----------|---------------------|-----------------|--------------|
| $\eta_b$ | 10.28 | 9.392 | 9.389 | 12.31 | 2.199 |
| $\Upsilon$ | 10.34 | 9.447 | 9.460 | 13.68 | 2.034 |
| $\chi_{b0}$ | 10.73 | 9.949 | 9.859 | 13.08 | 0.8 |
| $\chi_{b1}$ | 10.34 | 10.09 | 9.893 | 14.13 | 0.492 |

C. Results for bottomonia

We also calculate the in-medium changes of the spectral property of the bottomonia using the same framework. Results for $T = 0$ are summarized in Table III. As in the charmonium case, the sum rule works well for the bottomonium masses in the vacuum. In the case of bottomonium, the relative contribution of the dimension four operator to the OPE is much smaller than that of the charmonium because of the $m_{\bar{c}c}^{-4}$ dependence in the

TABLE IV: Onset temperatures of the width $T_{\text{onset}}$ for bottomonia

| System | $T_{\text{onset}}$ [GeV] |
|--------|------------------|
| $\eta_b$ | 2.40 |
| $\Upsilon$ | 2.56 |
| $\chi_{b0}$ | 1.87 |
| $\chi_{b1}$ | 1.50 |
Wilson coefficient as seen in Eqs. (13) and (14). Hence, its spectral property is much less affected by the change of the gluon condensates coming from the temperature effects. This fact allows us to go to much higher temperatures than in the charmonium cases until the dimension four contributions become so large as to break the Borel stabilities. We display some of the OPE coefficients at $T = 0$ and $T = 1.5 T_c$ in Fig. [11]. One sees that the power correction terms are much smaller than the leading perturbative correction term and might in fact be similar in magnitude to that of the next higher order radiative correction $[54]$. Therefore, while the separation scale in the present case is large enough for the OPE to provide a qualitatively reliable guide, further efforts are needed to obtain a quantitatively accurate estimate of the spectral property. From Fig. [13] one also sees that the twist-2 term dominates the temperature effect at $1.5 T_c$ and also at temperatures where the spectral modification becomes sizable as will be seen below. Since the perturbative effects are more dominant in the present case than in the charmonium cases, a detailed comparison with the resummed perturbative approach $[50]$ might be useful to understand the interplay between the perturbative and the non-perturbative effects at these temperatures.

We show the constraints among the effective continuum threshold, mass and width for each of the bottomonium states in Figs. [14,17]. The basic features are the same as in the charmonium cases, except now the sudden change across $T_c$ has disappeared. As the position of the mass is far from $2 m_b = 8.24$ GeV, the unphysical behavior of the width seen at $T > T_{\text{onset}}$ in the charmonium cases is absent in the bottomonium cases. The onset temperatures are summarized in Table [14]. One sees that the S-wave states have the narrow pole solution up
to $T \sim 2.4T_c$ and $P$-waves do so up to $1.5T_c$, suggesting survival of these states up to somewhat higher temperature than those suggested by a potential model analysis [13]. The maximum mass shifts are obtained near the onset temperatures and are found to be around 200 MeV for all the channels except for the axial vector. The mass shift of the $P$-wave states is found to be twice as large as that of the $S$-wave states at a fixed temperature as in the charmonium states. Unlike the charmonium, one sees a significant difference in $T_{\text{onset}}$ between the $S$ and $A$ channel. This is a manifestation of the dependence of $T_{\text{onset}}$ on the Borel window that was discussed in Sec. [13]. In this case, the additional condition, the perturbative correction less than 0.3 leads to a significant difference of $M_{\text{min}}^2$ between $S$ and $A$ channel such that the Borel window of $A$ channel becomes much narrower. If one set a common $M_{\text{min}}^2$ for instance, $T_{\text{onset}}$ does not differ so much. Indeed the Borel stability in the $\chi_b$ states is more marginal than other cases, as indicated in Ref. [40] as the difficulty of establishing the “plateau” in the moment sum rule. For the $A$ channel in the present case, even at $T = 0$, $M_{\text{min}}^2$ is larger than $M_0^2$ at $\sqrt{s} = \infty$. Although one can obtain the stability by relaxing the criterion for the $M_{\text{min}}^2$, the result would be less reliable since the $M_{\text{min}}^2$ is so chosen as to validate the perturbative expansion. After all, while we could obtain reasonable description of $\chi_b$ states at $T = 0$ and plausible in-medium changes of them, it has an intrinsic ambiguity in the quantitative results.

For the $S$-wave states, we also extract the results from combining the constraints with the second order Stark effect which are expected to be more reliable in the bottomonium systems. Figure [18] shows the results for the mass, the effective continuum threshold, the binding energy, and width of $\Upsilon$ and $\eta_b$. One sees that the changes as a function of the temperature are rather moderate; this reflects the smaller effects from the gluon condensates. Especially there is no significant broadening in both channels up to $T = 2.2T_c$. One should note, however, that the second order Stark effect gives the larger mass shifts than that of maximum given by the QCD sum rule at $T = 2.4T_c$ in $V$ channel. This is similar to what happened at $T > 1.09T_c$ in the case of $J/\psi$ (See Sec. [III B]). The heavier quark mass enables us to extend the OPE to higher temperature, but it seems to break down at $T = 2.4T_c$. Since the $P$ channel exhibits larger spectral change than the $V$ channel as in the charmonium cases, the maximum mass shift of $\eta_b$ given by the QCD sum rule is always smaller than that from the second order Stark effect. One sees small broadening at $T \geq 2.2T_c$.

IV. IMAGINARY TIME CORRELATORS

The spectral parameters obtained from QCD sum rules at finite temperature have shown sizable modifications from the vacuum values. To confirm the findings, it is desirable to compare the results with the first principle lattice calculation. Unfortunately the direct evaluation of the spectral function of the heavy quarkonia through MEM has insufficient resolution to identify the spectral changes of order of 100 MeV. In this section, we will construct model spectral functions at finite temperatures as well as in the vacuum using the previously obtained QCD sum rule results. Then, we reconstruct the imaginary time correlators via the dispersion relation, discuss how the spectral modification affects the correlator, and then compare them with the lattice results which are more accurately calculated.

A. Relation of spectral function with the imaginary time correlator

The imaginary time correlator $G(\tau, T)$ is related to the spectral function via the dispersion relation

$$G(\tau, T) = \int_0^\infty d\omega K(\omega, \tau; T)\rho(\omega, T)$$

(21)

where the integration kernel $K(\omega, \tau; T)$ is

$$K(\omega, \tau; T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)},$$

(22)

of which the zero temperature limit is $e^{-\omega\tau}$. To see the temperature effect on the spectral function, one usually computes the ratio of this correlator to the reconstructed one $G(\tau, T)/G_{\text{rec}}(\tau, T)$ with $G_{\text{rec}}$ defined as

$$G_{\text{rec}}(\tau, T) = \int_0^\infty d\omega \rho(\omega, T = 0)K(\omega, \tau; T)$$

(23)

which has temperature dependence coming only from the kernel.

We construct a model spectral function to be put into Eq. (21) from the phenomenological side [16]–[17]

$$\rho_{\text{pc}}(\omega) = \frac{C_J\omega^2}{\pi} \left[\text{Im}\tilde{\Pi}^{\text{pole}}(\omega^2) + \text{Im}\tilde{\Pi}^{\text{cont}}(\omega^2)\right]$$

(24)

with $C_J = 1$ for $P$ and $S$ channels and 3 for $V$ and $A$ channels. The subscript “pc” denotes the “pole+continuum”. We relate them to the spatial components of the spectral function for $V$ and $A$ channel, in order to compare them with lattice calculation. In $A$ channel, although lattice calculation uses an axial vector current of $j_\mu = \bar{u}\gamma_\mu\gamma_5u$ while we use the conserved part $J^\mu = \eta_{\mu\nu}J_\nu$ by multiplying $\eta_{\alpha\beta} = (q_\alpha q_\beta/q^2 - g_{\alpha\beta})$, the above expression still holds.

Putting each part of the model spectral function into
Eq. (21), one obtains the following formulæ:

$$G_{\text{pole}}(\tau, T) = \begin{cases} \frac{C_{J}m_{f_{0}} \cosh[m(\tau - 1/2T)]}{2\pi \sinh(m/2T)} & \text{if } \Gamma = 0 \\
\frac{C_{J}f_{T}}{\pi} \int_{0}^{\infty} \frac{\omega^{3}d\omega}{\cosh[\omega(\tau - 1/2T)]} \sinh(\omega/2T) & \text{if } \Gamma \neq 0 \end{cases}$$

(25)

$$G_{\text{cont}}(\tau, T) = \frac{C_{J}}{\pi} \int_{\sqrt{s_{0}}}^{\infty} d\omega \omega^{2} \Im \Pi_{\text{pert}}(\omega^{2}) \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}.$$  

(26)

As for the peak strength parameter $f$ and $f_{0}$, one can obtain it by using the dispersion relation (15) after determining the other three parameters as

$$f_{0} = e^{m^{2}/M_{0}^{2}} [\mathcal{M}(M_{0}^{2}) - \mathcal{M}_{\text{cont}}(M_{0}^{2})],$$

(27)

$$f = \frac{\mathcal{M}(M_{0}^{2}) - \mathcal{M}_{\text{cont}}(M_{0}^{2})}{\Gamma \int_{4m_{b}^{2}}^{\infty} ds e^{-s/M_{0}^{2}} \sqrt{s(s - m^{2})^{2} + 4\Gamma^{2}}}.$$  

(28)

We adopt the value of the Borel mass at $M^{2} = M_{0}^{2}$ where the property of the pole part was determined. This choice, however, can result in an unphysical behavior of the reconstructed correlator ratio $G/G_{\text{rec}}$ near $\tau \approx 0$ due to the sensitivity to the high energy part of the spectral function. The continuum part in the $P$ and the $S$ channel has a singular behavior in the high energy limit such that a slight deviation in $\alpha_{s}$ can lead to a sizable difference in the imaginary time correlator near $\tau = 0$. While this does not matter at $\tau \geq 0.1$ fm and in the QCD sum rule analysis due to the large suppression of the high energy part by the Borel transformation, we use the same the value of $\alpha_{s}$ at finite temperature as that of $T = 0$ by fixing $M_{0}^{2}$ so that $G/G_{\text{rec}} \rightarrow 1$ as $\tau \rightarrow 0$ at any temperature.

Explicit temperature, not divided by $T_{c}$, need to be specified in the kernel. While our gluon condensates have been taken from the lattice calculation with $T_{c} = 264$ MeV [42], we normalize the temperature dependence in the imaginary time correlator calculation to $T_{c} = 295$ MeV, which corresponds to the normalization used in the lattice calculation that we will be comparing our results to [22].

It has been emphasized that a peak of the spectral function at $\omega = 0$ gives a constant contribution to the imaginary time correlator [14]. Although we have ignored this contribution in the QCD sum rules, as explained above, this term is necessary for proper comparison of $G(\tau, T)$. Here, we adopt the expression calculated for free heavy quarks which is proportional to $\omega \delta(\omega)$. In this case, the spectral functions have been calculated and given in [55].

The zero mode (scattering) parts for $V, P, S$ and $A$

| $P$ | $V(\rho^{0})$ | $V(\rho^{0})$ | $S$ | $A(\rho^{0})$ | $A(\rho^{0})$ |
|-----|---------------|---------------|-----|---------------|---------------|
| $c_{1}$ | 0 | 0 | $-2$ | 2 | 6 | 6 |
| $c_{2}$ | 0 | 2 | 2 | $-2$ | $-4$ | $-6$ |

The numerical constants $c_{1}$ and $c_{2}$ are summarized in Table V and

$$\rho_{\text{scat}}(\omega) = N_{c} \omega \delta(\omega) (c_{1}I_{1} - c_{2}I_{2}).$$

(29)

The numerical constants $c_{1}$ and $c_{2}$ are summarized in Table V and

$$I_{1} = -2 \int \frac{d^{4}k}{(2\pi)^{4}} \frac{dn_{k}}{d\omega_{k}} \frac{k^{2}}{(s - m^{2})^{2} + 4\Gamma^{2}},$$

(30)

$$I_{2} = -2 \int \frac{d^{4}k}{(2\pi)^{4}} \frac{dn_{k}}{d\omega_{k}} \frac{k^{2}}{(s - m^{2})^{2} + 4\Gamma^{2}}$$

where $n_{k} = (e^{\omega_{k}/T} + 1)^{-1}$ and $\omega_{k} = \sqrt{k^{2} + m^{2}_{b}}$. Putting these expressions into Eq. (21), finally one obtains the constant contribution to the imaginary time correlator

$$G_{\text{scat}}(\tau, T) = N_{c}T(c_{1}I_{1} - c_{2}I_{2}).$$

(31)

*Hereafter, we adopt the three component model $G(\tau, T) = G_{\text{pole}} + G_{\text{cont}} + G_{\text{scat}}$ with spectral parameters taken from the results of the QCD sum rule as our model imaginary time correlator which we compare with the lattice QCD result shown in Ref. [22].

Before proceeding, it is useful to see how the typical spectral changes seen in the QCD sum rule affect the ratio of the imaginary time correlator $G/G_{\text{rec}}$. In Fig. 19 we plot several examples in which only one of the four spectral parameters is changed. Similar to what was shown in subsection 1.1, one can understand the qualitative behavior from the dispersion relation for the imaginary time correlator (21) due to the positivity of the kernel. When the modification of the spectral parameters increases the spectral sum, $G/G_{\text{rec}}$ also increases. One sees that a rather small modifications of the continuum threshold and the mass, of order of 100 MeV, lead to more than 10% change in $G/G_{\text{rec}}$, whereas less than 3% changes have been observed in the lattice QCD calculation of the $P$ channel [22]. On the other hand, the change of width does not affect $G/G_{\text{rec}}$, as seen in the lower-right panel. This is so because the increase of width implies reduction of the peak height $\rho(\omega^{2} = m^{2}) = fm/\Gamma$. If one artificially tries to preserve the height by increasing $f$ as well as $\Gamma$, even small increase of the width makes $G/G_{\text{rec}}$ increase very quickly, as shown in the dotted line in the figure.
FIG. 19: (color online). $G/G_{\text{rec}}$ in several cases of modification of single spectral property. Upper left : continuum threshold, upper right : mass, lower left : width, and lower right : peak height. The zero temperature spectral function in $G_{\text{rec}}$ is taken from $\eta_c$ case thus constant contribution is absent. For the lower panel, we utilized $T = 0$ spectral parameters obtained by putting $\Gamma = 2$ MeV for comparison.

B. Comparison with lattice QCD results

1. Charmonium

We start with the charmonium in the $P$ channel ($\eta_c$) to which no zero mode contributes. Unfortunately, the available lattice data are for $T = 0.87T_c$, $1.07T_c$, $1.09T_c$ and so on while our sum rule results of interest are just around $T_c$. At $0.87T_c$, spectral change is almost negligible because of the tiny change of the gluon condensates. At the next lowest temperature, $1.07T_c$, it is already above the onset temperature therefore our result for the charmonium is quantitatively obscure. Between the lowest and the next lowest temperature, however, $G/G_{\text{rec}}$ on the lattice seems unchanged because it is almost unity at both temperatures. Sizable deviation from unity has been seen above $1.5T_c$ at which the spectral functions extracted by MEM also show notable modifications. Hence, for references, we calculate $G/G_{\text{rec}}$ at not only temperatures where the lattice QCD results are available but also at $T = T_c$ and at $T = 1.04T_c$ where our QCD sum rule works well and results of the lattice QCD are unambiguously anticipated.

Figure 20 displays $G/G_{\text{rec}}$ for the $P$ channel charmonium current. We plot the cases for several sets of the spectral parameters, summarized in Appendix C. At $T = T_c$, $G/G_{\text{rec}}$ of $\eta_c$ can be both below and above unity depending on the parameter set. The parameter set of $\sqrt{s_0} = 3.4$ GeV which seems most plausible according to Fig. 12 gives the closest result to unity. We anticipate the lattice QCD gives almost unity as in the higher temperature cases. Therefore the most plausible case gives the most consistent result with the lattice result. Qualitatively, our results indicate that the combination of the small decreases of both effective threshold and mass can give a consistent result. One sees, however, that all of the parameter sets lead to $G/G_{\text{rec}} > 1$ above $T > 1.04T_c$ in spite of the quite different spectral parameters. We would like to stress that $T = 1.04T_c$ is the onset temperature of $\eta_c$; for the parameter set with the smallest $\sqrt{s_0}$ shown in the figure, we need to introduce $\Gamma = 18$ MeV to stabilize the Borel curve. No Borel window is open for smaller $\sqrt{s_0}$. In the two higher temperature
cases, $T = 1.07T_c$ and $1.09T_c$, lattice data are shown together. Note that the lattice setup is different between these temperatures. The latter is obtained with finer lattice spacing. As shown in the figure, the lattice correlators do not exhibit sizable changes from unity, that would be interpreted as no spectral modification at these temperatures.

These disagreements with the lattice QCD results at $T \geq 1.04T_c$ do not immediately mean serious flaws in our approach. First, the applicability of our present approach to these temperatures are marginal and thus our results will be quantitatively improved by including correction mentioned above. Second, our model spectral function might be too simple to make such a comparison. This simplification does not matter in the QCD sum rule approach due to the strong suppression of the high energy part in the Borel transformed dispersion relation, but might cause this defect in the correlator because of its sensitivity to the continuum. Figure 21 displays the kernel in the dispersion relation (22) with typical values of parameters together with the kernel in the Borel transformation (15) where we rewrite the formula so that integration variable is $\omega = \sqrt{s}$ and integrand has a form of $K(\omega, M^2)\rho(\omega)$. One sees that the Borel transformation suppresses the high energy part of the spectral function much more strongly than the heat kernel (22). Note that temperature effects on the heat kernel are small around $T_c$; it almost behaves like $e^{-\omega \tau}$. This fact means that while the detailed structure of the continuum and the excited states do not affect the property of the pole part in the QCD sum rule approach, they do contribute non-trivially to the imaginary correlator. For example, compared to the realistic situation, we have neglected the contributions from the excited states ($\eta_c(2S)$ in $P$ channel) in the model spectral function. While such approximation does not affect the lowest pole in the QCD sum rule analysis, it will result in the smaller continuum threshold than a realistic value to compensate for the missing state. This discrepancy between effectively small continuum threshold and existence of excited states will result in different imaginary time correlators. If the $2S$ state melts just above (or below) $T_c$, our parametrization is in fact better above $T_c$. Hence, if one takes into account the excited state contribution explicitly, $G_{\text{rec}}(\tau, T)$ will increase while $G(\tau, T)$ remain unchanged, thus the ratio will now become close to unity. Indeed the effect of the excited state has been examined in the context of a temperature dependent potential model in Ref. [11]. The authors found that the $\eta_c(2S)$ state reduces $G/G_{\text{rec}}$ 10–20% because the 2S state melts above $T_c$ thus increases $G_{\text{rec}}$. One sees that the observed reduction is also relevant for our case. Although the result of Ref. [11] shows the $2S$ states melts at $T_c$ thus our agreement at this temperature might be changed to other set of parameters, the spectral property of the excited states at finite temperature is not clear yet. If the $2S$ state survives at $T_c$ and our analysis catches essential points for the correlator, the difference between $T = T_c$ and $T > T_c$ indicates that the $2S$ state dissolves at $T_c < T < 1.04T_c$.

In Fig. 22 we display the results of $G/G_{\text{rec}}$ for the vector channel ($J/\psi$). One sees a deviation from unity starting at small $\tau$ region. This behavior is caused by the zero mode contribution $[13, 15, 58]$. At $T = 1.07T_c$, our $G/G_{\text{rec}}$ with $\sqrt{s_0} \leq 3.2$ GeV agrees the lattice results at $\tau < 0.15$ fm in which the zero mode contribution is negligible. This means that $G/G_{\text{rec}} \simeq 1$ is achieved by a combination of the various spectral changes as seen in the case of $\eta_c$ at $T = T_c$. The zero mode contribution, however, overwhelms other changes at $T = 1.07T_c$ and $T = 1.09T_c$, as clearly seen from the figure. There are following possibilities:

1. High energy part of the model spectral function.

As in the case of $\eta_c$, we have neglected $\psi'(2S)$ contribution to the $T = 0$ spectral function. Including this leads to larger $G_{\text{rec}}$ thus reducing $G/G_{\text{rec}}$. There may be also the possibilities that we underestimated the pole modification by truncation of the OPE and other approximations. As for $d = 6$ contribution in the OPE, however, it is expected to reduce the spectral modification [57].

2. Free charm quark approximation in the zero mode contribution.

Indeed this might be a flaw because in more realistic situation quarks are interacting such that zero mode spectral function is smeared [58]. Of course, since we are looking at the integrated value of the spectral function, this smearing itself might not change the value so much. Nevertheless, there are further ambiguities in the zero mode calculation such as the value of the charm quark mass; within the quasiparticle picture, the thermal effect

![FIG. 22: (color online). $G/G_{\text{rec}}$ for the $V$ channel charmonium current.](image_url)
will effectively increase the quark mass such that the zero mode contribution is reduced according to the thermal distribution. As we shall see below, there is clearly something that cannot be understood with the free charm description in the zero mode. To avoid these ambiguities, subtraction of the zero mode contribution by taking derivative of the imaginary time correlator and then looking at the ratio \( \frac{G_{\text{rec}}(\tau,T=0)}{G(\tau,T=0)} \) will provide useful information.

In the vector channel, the first possibility can be explicitly checked by including \( \psi' \) contribution to the continuum part of the model spectral function at \( T = 0 \). Namely,

\[
\text{Im} \Pi^\psi(s) = f' \delta(s - m_{\psi'}^2),
\]

with \( f' = 0.276 \) GeV\(^2\) being obtained from the leptonic decay width given by the Particle Data Group, is added to the phenomenological side. With this implementation, we found that the resultant \( J/\psi \) mass changes only 0.3% (3.05 GeV) while the continuum threshold increases from \( \sqrt{s_0} = 3.54 \) GeV to 3.93 GeV. On the other hand, we also found that incorporating the \( \psi' \) to the spectral function in the dispersion relation of the imaginary time correlator \( G(\tau,T=0) \) leads to a sizable change. This fact exactly demonstrates our expectation discussed above: while the spectral property deduced from the QCD sum rule is independent of detailed structure of the higher energy part of the model spectral function as long as the pole dominance is well satisfied, the imaginary time correlator receives sizable change. Note that the physical meaning of the threshold parameter becomes different if one takes the excited state into account. When one includes it, now \( \sqrt{s_0} \) can be regarded as physically more relevant threshold while it represents an effective one controlling the contribution from the high energy part other than the lowest pole in the dispersion relations. For example, \( \sqrt{s_0} \) in Fig. 12 means a merely effective threshold parameter since we have neglected the excited state contribution throughout the calculation. If \( \psi' \) melts above \( T_c \), \( \sqrt{s_0} \) can be now regarded as more physical one. In this case the true threshold might vary more rapidly from 3.93 GeV to a lower value in the vicinity of \( T_c \).

Figure 23 shows the effect of \( \psi' \) on the imaginary time correlator. In the upper panel, we compare two \( G(\tau,T=0) \) with and without \( \psi' \) contribution. One sees 3% reduction of the ratio, which means that inclusion of the \( \psi' \) gives an enhancement large enough to affect \( G_{\text{rec}}/G \) comparison. The resultant \( G_{\text{rec}}/G \) are shown in the thick lines in the lower panel, where one sees the apparent reduction of \( G_{\text{rec}}/G \) when including \( \psi' \). Nevertheless, \( \tau \) dependence is still governed by the zero mode contribution, on which we will give further consideration.

As for the second possibility, one way to check the consistency is to look at other channels. We display the imaginary time correlators of the scalar and the axial-vector channels in Figs. 21 and 24 respectively. Similarly the spectral parameters are summarized in Table VIII and IX. These two channels show quite similar behavior so that the following discussion can be applied for both cases. First, no sizable difference among various parameter sets is seen as indicated by the complete overlaps of the lines. One sees the clear effect of the zero mode contribution and its agreement with the lattice results at \( T = 1.07T_c \) contrary to the \( V \) channel case. This might be partly attributed to the absence of the 2S state contribution below the continuum threshold in the \( S \) and the \( A \) channels; i.e., the single pole plus continuum ansatz at \( T = 0 \) is a better approximation in these channels than in the \( V \) and the \( P \) channels. At \( T = 1.09T_c \), however, this agreement is lost though qualitatively the lattice results indicate the dominance of the zero mode. One sees the value of \( G_{\text{rec}}/G \) is smaller at \( T = 1.09T_c \) than at \( T = 1.07T_c \) in the lattice results. If the spectral modification of the pole and the continuum part does not differ so much between these temperatures, this result seems to indicate the smaller zero mode contribution at higher temperature. This cannot be understood within the free charm quark approximation in which zero mode contribution increases as temperature increases if the charm quark mass is constant. Therefore, although

\[\text{FIG. 23}: \text{color online}. \text{Effect of } 2S \text{ state in the imaginary time correlator. Upper panel: ratio of the correlator with } \psi' \text{ to that with } \psi' \text{ at } T = 0. \text{ Lower panel: } G/G_{\text{rec}} \text{ at } T = 1.07T_c \text{ with } \psi' \text{ contribution being included in } G_{\text{rec}}. \text{ Thin lines are the same ones shown in Fig. 22 while thick lines denote the cases in which } \psi' \text{ contribution is included in } G_{\text{rec}}.\]
our results show agreement with the lattice results in the \( \tau \) range where zero mode contribution is relatively small, we cannot draw definite conclusion on the quantitative correctness of the zero mode from the results. One way to avoid the difficulty of the zero mode is to evaluate the derivative of the imaginary time correlator with respect to \( \tau \) [43]. Figures 26–28 show the ratio of the derivative of the imaginary time correlator \( G'/G'_{\text{rec}} \) such that \( G'/G'_{\text{rec}} \approx 1 \) and model calculations such as potential models. In \( \eta_c \) and \( J/\psi \), it explains successfully the
lattice data at $T = 1.2T_c$. Note that this is beyond our regime in the present work; we cannot apply our method at this temperature since we could not have the reliable Borel stability beyond $T_{\text{onset}}$. We can still see some indication of such an effect in the present data. One notes that the Borel curve becomes flatter and flatter, according to $\sqrt{\chi^2}$, as $\sqrt{s_0}$ decreases above $T = T_{\text{onset}}$. (See Tables VI and VII). It has minimum at the smallest $\sqrt{s_0}$ at which the Borel window is about to close. This fact means that if one relaxes the criterion for the pole dominance, one still obtains the stability for smaller $\sqrt{s_0}$. Though any extrapolation to higher temperature cannot be reliable due to the missing effects as discussed, one might expect the threshold to be close to the mass thus the spectral function exhibits the threshold enhancement. Whereas direct confirmation will not be possible with the present method by construction since pole dominance will be badly violated, this behavior of $\sqrt{\chi^2}$ may indicate a connection to the higher temperature regime which has been investigated only through the potential models so far.

2. Bottomonium

We compute $G/G_{\text{rec}}$ for the bottomonium currents in the same manner. Figure 30 displays the results at $T = 1.15T_c$ and $T = 1.54T_c$ of which lattice results are available with the finest lattice spacing in Ref. [22]. As in the charmonium cases, the lattice results show little deviation from unity. In the S-waves, while our results show variations among the parameter sets there is a certain range of the effective threshold parameter of which parameter set gives $G/G_{\text{rec}} \simeq 1$. The best agreement seems to lie between the data set with vacuum $\sqrt{s_0}$ (green, long-dashed) and that with Stark effect results (blue, short-dashed). The difference of these two cases is tiny at $T = 1.15T_c$ but sizable at $1.54T_c$, indicating the possibility of discriminating the spectral change from the imaginary time correlator. As before, one has to consider excited states to give a definite conclusion. The interpretation of the behavior of $G/G_{\text{rec}}$ depends on whether such states below threshold survive or not at these temperatures. From these agreements, a possible interpretation is that excited state are still surviving even at $T = 1.54T_c$ since a potential model calculation shows larger reduc-
We have demonstrated that the results obtained in the lattice analyses and potential model calculations. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) as a simple model of the spectral function.

V. SUMMARY AND OUTLOOK

We have analyzed the spectral changes of heavy quarkonia in the hot environment in a systematic way based on the QCD sum rule with Borel transformation technique. We have taken into account possible changes of the continuum spectrum by a temperature dependent effective continuum threshold. Although the temperature dependence of the OPE side allows various combination of the changes of the spectral parameters, we have given the constraints among them by an optimization procedure which has been widely used in QCD sum rule applications. We found that instability of the Borel curve in the \( \Gamma = 0 \) limit caused by the change of the gluon condensates can be cured by introducing a width although neglected contributions from higher dimensional operators might be non-negligible at the small Borel mass. The results, Figs. 7, 10 and 14, show the behaviors of the spectral parameters with respect to the change of temperature through the gluon condensates. As already argued in previous literatures, charmonia exhibit the critical behavior in at least one of the spectral parameters. Note that although the effective continuum threshold shares the effect from the change of the gluon condensates and thus reduces the change of other parameters, it is basically linked with the mass as seen in the Figs. 7, 10 and 14 as a result of stabilizing the Borel curve. While we do not taken into account the excited state explicitly, property of the lowest pole is not affected by this simplification since it is imposed on the effective threshold parameter. When one of the spectral parameters remains unchanged, rapid change of other parameters is inevitable. We found there is an onset temperature for each channel at which broadening must occur. Although the value of this temperature is affected by the assumption on the Borel window and does not necessarily mean there is no significant broadening below, the combined analysis with the QCD second order Stark effect shows that \( \chi_{b0} \) and \( J/\psi \) do not likely have significant width below \( T_c \) and all the spectral parameters of \( J/\psi \) and \( \eta_c \) change abruptly in the vicinity of \( T_c \). The same analysis procedure for the bottomonia shows the little change around \( T_c \) because of the much heavier quark mass, but eventually shows sizable changes with increasing temperature.

Such changes should be obtained from the lattice QCD also, while spectral analyses based on MEM do not have sufficient resolution. Therefore, we have computed the imaginary time correlator which is the basis of the MEM analyses and its derivative with respect to the Euclidean time \( \tau \) by putting the phenomenological side of the QCD sum rule analyses as a simple model of the spectral function. Then, we take the ratio \( G(G/G_{rec}) \) in order to see the temperature effect on the spectral function, as done in the lattice analyses and potential model calculations. We have demonstrated that the results obtained in the lattice calculation, namely \( G(G/G_{rec}) \approx 1 \), do not always mean absence of the spectral changes, but can mean a mixture of some sets of the change of the spectral properties. Similar observation has been done in some potential model analyses but they are at higher temperatures and mainly focused on the threshold enhancement. We showed that the rather small modification compared to the bound state masses gives the sizable change in \( G(G/G_{rec}) \) and the constrained parameter sets by the QCD sum rule can lead to \( G(G/G_{rec}) \approx 1 \). We have also pointed out that while similarity in the dispersion relation exists between the Borel-transformed current correlation func-
tion (15) and the imaginary time correlator (21), the former is much more dominated by the pole contribution. As a result, the QCD sum rule analysis is not much affected by taking into account the known excited states explicitly, while the imaginary time correlator shows a small but significant change over the uncertainties of the lattice QCD results. Due to this property and the poorly known spectral function near $2m_c$ threshold region in the case of finite $\Gamma$, more work is needed to give a precise quantitative determination of the spectral parameters.

Before closing, we comment on implication for the full QCD case by repeating our argument in Refs. [27, 28, 39] since the present work is based on quenched QCD. If one takes into account light dynamical quarks in the OPE, they appear as light quark condensate contributions and change of the temperature dependence of the gluon condensates in the OPE. The former can be safely neglected since it is at order $\alpha_s^2(q^2)$. The latter has been shown to lead smoother temperature dependence of $G_0$ near $T_c$ [27] reflecting the crossover nature of the transition. One also expects the similar change in $G_2$. Since the OPE side depends on temperature through these condensates, this will result in smoother spectral changes than those shown in the present work. While the spectral changes become smoother, the actual magnitude of changes at $T_c$ might not differ so much since the reduction of the scalar condensate at $T = T_c$ is almost the same as that for the quenched case [27]. This also implies that the broadening below $T_c$ could be negligible. On the other hand, the moderate decrease of the condensates above $T_c$ may lead to higher onset temperatures in the full QCD case.

The results shown above strongly indicate that our main results, the mass shifts and width broadening induced by the QCD phase transition, might be also realized in lattice QCD simulation. The agreement between the sum rule constraints and the lattice correlator ratio is obtained on the basis of the effect of the known excited state on the lattice correlator and the zero mode contribution of the free heavy quarks. The best set of the parameters depends on such external assumptions. Further assessments of these quantities as well as continuum spectrum at finite temperature will be required to check the consistency more quantitatively.

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Appendix A: Borel transformed Wilson coefficients

In this appendix, we list the Wilson coefficients seen in Eq. (12) originally obtained by Bertlmann in Ref. [41]. Hereafter

$$c_2 = \frac{\pi}{2} - \frac{3}{4\pi}$$  \hspace{1cm} (A1)

$$c_1 = \frac{\pi}{3} + \frac{1}{2} c_2$$  \hspace{1cm} (A2)

$$c_3 = \frac{\pi}{2} - \frac{3}{\pi}$$  \hspace{1cm} (A3)

and $G(a, b, \nu)$ is the Whittaker function defined as

$$G(a, b, \nu) \equiv \frac{1}{\Gamma(b)} \int_0^\infty ds e^{-s} s^{b-1} (\nu + s)^{-a},$$  \hspace{1cm} (A4)

with $\Gamma(b)$ being the Gamma function.
1. **V channel**

\[
A(\nu) = \frac{3}{16\pi^{3/2}} \frac{4m_h^2}{\nu} G\left(\frac{1}{2}, \frac{5}{2}, \nu\right),
\]

\[
a(\nu) = \frac{4}{3\sqrt{\pi}G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)} \left[\pi - c_1 G(1, 2, \nu) + \frac{1}{3} c_2 G(2, 3, \nu)\right] - c_2 - \frac{4 \ln 2}{\pi} h(\nu),
\]

\[
h(\nu) = \nu \frac{G\left(\frac{1}{2}, \frac{3}{2}, \nu\right)}{G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)},
\]

\[
b(\nu) = -\frac{\nu^2 G\left(-\frac{1}{2}, \frac{3}{2}, \nu\right)}{2 G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)},
\]

\[
c(\nu) = b(\nu) - \frac{2}{3} \nu^2 \frac{G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)}{G\left(\frac{1}{2}, \frac{3}{2}, \nu\right)}
\]

Derivatives with respect to \(\nu\) are used in Eq. \[19\].

\[
A'(\nu) = -\frac{3m_h^2}{4\pi^{3/2}} \left[\frac{G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)}{2} + \frac{1}{\nu} G\left(\frac{1}{2}, \frac{3}{2}, \nu\right)\right],
\]

\[
a'(\nu) = -\frac{4}{3\sqrt{\pi}G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)} \left\{-c_1 G(2, 2, \nu) + \frac{2}{3} c_2 G(3, 3, \nu) - \frac{G\left(\frac{3}{2}, \frac{5}{2}, \nu\right)}{2G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)} \left[\pi - c_1 G(1, 2, \nu) + \frac{1}{3} c_2 G(2, 3, \nu)\right]\right\}
- \frac{4 \ln 2}{\pi} h'(\nu),
\]

\[
h'(\nu) = \frac{G\left(\frac{1}{2}, \frac{3}{2}, \nu\right)}{G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)} - \frac{\nu}{2G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)^2} \left[G\left(\frac{1}{2}, \frac{3}{2}, \nu\right) G\left(\frac{1}{2}, \frac{5}{2}, \nu\right) - G\left(\frac{1}{2}, \frac{3}{2}, \nu\right) G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)\right],
\]

\[
b'(\nu) = \frac{-\nu}{G\left(\frac{1}{2}, \frac{3}{2}, \nu\right)} \left[G\left(-\frac{1}{2}, \frac{3}{2}, \nu\right) + \frac{\nu G\left(\frac{1}{2}, \frac{3}{2}, \nu\right)}{4} + \frac{\nu G\left(-\frac{1}{2}, \frac{3}{2}, \nu\right) G\left(\frac{3}{2}, \frac{5}{2}, \nu\right)}{4G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)}\right],
\]

\[
c'(\nu) = b'(\nu) - \frac{4}{3} \nu \frac{G\left(\frac{3}{2}, \frac{5}{2}, \nu\right)}{G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)} - \frac{2\nu^2}{3G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)^2} \left[\frac{1}{2} G\left(\frac{1}{2}, \frac{3}{2}, \nu\right) G\left(\frac{1}{2}, \frac{5}{2}, \nu\right) - \frac{3}{2} G\left(\frac{3}{2}, \frac{3}{2}, \nu\right) G\left(\frac{1}{2}, \frac{5}{2}, \nu\right)\right],
\]

where we used

\[
\frac{\partial}{\partial \nu} G(b, c, \nu) = -b G(b + 1, c, \nu).
\]
2. P channel

\[ A(\nu) = \frac{3}{16\pi^{3/2}} \frac{4m_{h}^{2}}{\nu} G \left( \frac{1}{2}, \frac{1}{2}, \nu \right), \]  

\[ a(\nu) = \frac{4}{3\sqrt{\pi}G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \left[ \pi - \frac{1}{2} c_{1} G \left( 1, 2, \nu \right) \right] - c_{2} + \frac{1}{\pi} \left[ \frac{8}{3} + S(\nu) \right] - \frac{4\ln 2}{\pi} h(\nu), \]  

\[ h(\nu) = \nu \frac{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)}, \]  

\[ S(\nu) = -4G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) + \frac{5}{\pi} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right), \]  

\[ b(\nu) = -\frac{1}{2} G \left( -\frac{3}{2}, \frac{3}{2}, \nu \right) - 6G \left( -\frac{1}{2}, \frac{3}{2}, \nu \right) \]  

\[ c(\nu) = b(\nu) - 4\nu \frac{G \left( -\frac{1}{2}, \frac{1}{2}, \nu \right)}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)}, \]  

\[ A'(\nu) = \frac{3m_{h}^{2}}{4\pi^{3/2}} \left[ \frac{1}{\nu} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) + \frac{1}{\nu} G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) \right], \]  

\[ a'(\nu) = \frac{2}{3\sqrt{\pi}G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \left[ c_{2} G \left( 2, 2, \nu \right) + \frac{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \left[ \pi - \frac{1}{2} c_{2} G \left( 1, 2, \nu \right) \right] \right] + \frac{1}{\pi} S'(\nu) - \frac{4\ln 2}{\pi} h'(\nu), \]  

\[ h'(\nu) = \frac{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} - \frac{\nu}{2[G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)]^{2}} \left[ G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) - G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) \right], \]  

\[ S'(\nu) = \frac{1}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \left[ 6G \left( \frac{1}{2}, \frac{3}{2}, \nu \right) + \frac{15}{\nu} G \left( \frac{1}{2}, \frac{3}{2}, \nu \right) \right] \left[ \frac{1}{2} G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) \right] + \frac{4}{6} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right), \]  

\[ b'(\nu) = \frac{-1}{\nu} G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) \left[ 3\nu G \left( -\frac{1}{2}, \frac{1}{2}, \nu \right) - 3\nu G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) + \left[ 1 + \frac{\nu G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)}{2 G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \right] G \left( -\frac{1}{2}, \frac{1}{2}, \nu \right) - 6G \left( -\frac{1}{2}, \frac{1}{2}, \nu \right) \right], \]  

\[ c'(\nu) = b'(\nu) - \frac{4G \left( -\frac{1}{2}, \frac{1}{2}, \nu \right)}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \frac{2\nu}{G \left( \frac{1}{2}, \frac{1}{2}, \nu \right)} \left[ G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) + G \left( -\frac{1}{2}, \frac{1}{2}, \nu \right) G \left( \frac{1}{2}, \frac{1}{2}, \nu \right) \right]. \]
3. S channel

\[ A(\nu) = \frac{9}{32\pi^{3/2}} \frac{4m^2}{\nu} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right), \quad (A29) \]

\[ a(\nu) = \tilde{a}(\nu) + \frac{1}{\pi} \left[ \frac{16}{3} + S_s(\nu) \right], \quad (A30) \]

\[ \tilde{a}(\nu) = \frac{8}{9\sqrt{\pi}} \frac{G(1, 2, \nu)}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \left[ \pi - 2c_3 \frac{G(2, 3, \nu)}{G(1, 2, \nu)} \right] - \frac{2}{3}c_3 - \frac{4\ln 2}{\pi} h(\nu), \quad (A31) \]

\[ h(\nu) = \frac{G \left( \frac{2}{2}, \frac{2}{2}, \nu \right)}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)}, \quad (A32) \]

\[ S_s(\nu) = -\frac{4}{3\nu G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \left[ 5G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) + \frac{3}{2} G \left( \frac{5}{2}, \frac{5}{2}, \nu \right) + \left( 5 - \frac{1}{\nu} \right) G \left( \frac{5}{2}, \frac{7}{2}, \nu \right) \right], \quad (A33) \]

\[ b(\nu) = -\frac{3\nu}{2G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \left[ G \left( -\frac{1}{2}, \frac{3}{2}, \nu \right) - \frac{2}{3} G \left( \frac{1}{2}, \frac{5}{2}, \nu \right) \right], \quad (A34) \]

\[ c(\nu) = b(\nu) + \frac{4\nu}{3} \frac{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)}, \quad (A35) \]

\[ A'(\nu) = -\frac{9m^2}{8\pi^{3/2}} \left[ \frac{3}{2} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) + \frac{1}{\nu} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) \right], \quad (A36) \]

\[ a'(\nu) = \tilde{a}'(\nu) + \frac{1}{\pi} S_s'(\nu), \quad (A37) \]

\[ \tilde{a}'(\nu) = -\frac{8}{9\sqrt{\pi}G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \left\{ -2c_3 \left[ 2G(3, 3, \nu) - \frac{G(2, 2, \nu) G(2, 3, \nu)}{G(1, 2, \nu)} \right] + \left[ \pi - 2c_3 \frac{G(2, 3, \nu)}{G(1, 2, \nu)} \right] \right\} \times \left[ \frac{G(2, 2, \nu)}{2} - \frac{3\nu}{2} \frac{G \left( \frac{1}{2}, \frac{5}{2}, \nu \right)}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \right] \left( \frac{4\ln 2}{\pi} h'(\nu), \quad (A38) \right. \]

\[ h'(\nu) = \frac{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} - \frac{\nu}{2G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)^2} \left[ G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) - G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) \right], \quad (A39) \]

\[ S_s'(\nu) = \frac{4}{3\pi G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \left\{ \frac{25}{2} G \left( \frac{1}{2}, \frac{5}{2}, \nu \right) - \frac{1}{\nu^2} G \left( \frac{5}{2}, \frac{7}{2}, \nu \right) + \frac{5}{2} \left( \frac{5}{2} - \frac{1}{\nu} \right) G \left( \frac{7}{2}, \frac{7}{2}, \nu \right) \right\}, \quad (A40) \]

\[ b'(\nu) = -\frac{3}{2G \left( \frac{1}{2}, \frac{5}{2}, \nu \right)} \left\{ \frac{\nu}{2} G \left( \frac{1}{2}, \frac{5}{2}, \nu \right) + \frac{1}{3} G \left( \frac{5}{2}, \frac{5}{2}, \nu \right) + \left[ 1 + \frac{3\nu}{2} \frac{G \left( \frac{5}{2}, \frac{7}{2}, \nu \right)}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} \right] G \left( -\frac{1}{2}, \frac{5}{2}, \nu \right) - \frac{2}{3} G \left( \frac{3}{2}, \frac{5}{2}, \nu \right) \right\}, \quad (A41) \]

\[ c'(\nu) = b'(\nu) + \frac{4G \left( \frac{1}{2}, \frac{5}{2}, \nu \right)}{3G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)} + \frac{4}{3} \frac{\nu^2}{G \left( \frac{3}{2}, \frac{3}{2}, \nu \right)^2} \left[ \frac{3}{2} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) G \left( \frac{5}{2}, \frac{5}{2}, \nu \right) - \frac{1}{2} G \left( \frac{3}{2}, \frac{3}{2}, \nu \right) G \left( \frac{5}{2}, \frac{5}{2}, \nu \right) \right]. \quad (A42) \]
4. A channel ($^3P_1$)

\[
A(\nu) = \frac{2}{3} A(\nu)|_{\text{Scalar}},
\]
\[
a(\nu) = \tilde{a}(\nu) + \frac{1}{\pi} \left[ \frac{4}{3} + S_A(\nu) \right],
\]
\[
S_A(\nu) = -\frac{4}{3G(\frac{3}{2}, \frac{5}{2}, \nu)} \left\{ G\left(\frac{5}{2}, \frac{5}{2}, \nu\right) - 2G\left(\frac{3}{2}, \frac{5}{2}, \nu\right) + \frac{5}{3} \left(1 - \frac{3}{2\nu}\right) G\left(\frac{7}{2}, \frac{7}{2}, \nu\right) \right\},
\]
\[
b(\nu) = -\frac{3}{2} \frac{\nu}{G(\frac{7}{2}, \frac{7}{2}, \nu)},
\]
\[
c(\nu) = b(\nu) + \frac{4\nu}{3} \frac{G\left(\frac{5}{2}, \frac{5}{2}, \nu\right)}{G\left(\frac{5}{2}, \frac{5}{2}, \nu\right)},
\]
\[
A'(\nu) = \frac{2}{3} A'(\nu)|_{\text{Scalar}},
\]
\[
a'(\nu) = \tilde{a}'(\nu) + \frac{1}{\pi} S'_A(\nu),
\]
\[
S'_A(\nu) = \left\{ \frac{15}{4} G\left(\frac{5}{2}, \frac{5}{2}, \nu\right) - \frac{3}{2\nu^2} G\left(\frac{3}{2}, \frac{5}{2}, \nu\right) + \frac{5}{3} \left(1 - \frac{3}{2\nu}\right) G\left(\frac{7}{2}, \frac{7}{2}, \nu\right) + 7G\left(\frac{9}{2}, \frac{9}{2}, \nu\right) \right\},
\]
\[
b'(\nu) = \frac{3}{2} \frac{\nu}{G\left(\frac{7}{2}, \frac{7}{2}, \nu\right)} \left\{ G\left(\frac{5}{2}, \frac{5}{2}, \nu\right) + \left(1 - \frac{3}{2\nu}\right) G\left(\frac{7}{2}, \frac{7}{2}, \nu\right) + 2G\left(\frac{9}{2}, \frac{9}{2}, \nu\right) \right\},
\]
\[
c'(\nu) = b'(\nu) + \frac{4}{3} \frac{G\left(\frac{5}{2}, \frac{5}{2}, \nu\right)}{G\left(\frac{5}{2}, \frac{5}{2}, \nu\right)} + \frac{4\nu^2}{3 \left[G\left(\frac{3}{2}, \frac{5}{2}, \nu\right)\right]^2} \left\{ \frac{3}{2} G\left(\frac{1}{2}, \frac{3}{2}, \nu\right) G\left(\frac{5}{2}, \frac{5}{2}, \nu\right) - \frac{1}{2} G\left(\frac{3}{2}, \frac{3}{2}, \nu\right) G\left(\frac{3}{2}, \frac{5}{2}, \nu\right) \right\}. 
\]

Appendix B: Spectral function for the continuum part

We summarize the continuum part of the phenomenological spectral function $\text{Im} \Pi_{J, \text{pert}}(s)$ which are taken from the perturbative QCD calculation up to $\mathcal{O}(\alpha_s)$ shown in Ref. [34]. Here,

\[
u = \sqrt{1 - \frac{4m_h^2}{s}},
\]

and

\[
\Delta = \frac{2\alpha_s}{\pi} \ln 2.
\]
Then,

\[ \text{Im} \tilde{\Pi}^{V, \text{pert}}(s) = \frac{u(3 - u^2)}{8\pi} \left[ 1 + \frac{4\alpha_s}{3\pi u} \left( \pi \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right) \right] - \frac{3(1-u^2)^2}{8\pi u} \Delta \]

\[ \text{Im} \tilde{\Pi}^{P, \text{pert}}(s) = \frac{3u}{8\pi} \left[ 1 + \frac{4\alpha_s}{3\pi u} \left( \pi u \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right) + u \right] - \frac{3(1-u^2)^2}{8\pi u} \Delta \frac{1-u^2}{u} \]

\[ \text{Im} \tilde{\Pi}^{S, \text{pert}}(s) = \frac{3u^3}{8\pi} \left[ 1 + \frac{4\alpha_s}{3\pi u^3} \left( \pi u^3 \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right) + \frac{15}{16} \frac{3u^2 - 33u^4}{8\pi} \ln \frac{1+u}{1-u} \right] \]

\[ \text{Im} \tilde{\Pi}^{A, \text{pert}}(u) = \frac{u^3}{4\pi} \left[ 1 + \frac{4\alpha_s}{3\pi u^3} \left( \pi u^3 \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right) + u^3 + \frac{3(15 - 7u^2 - 7u^4 - u^6)}{32} \ln \frac{1+u}{1-u} \right] \]

Appendix C: List of obtained spectral parameters

We summarize the results of the spectral parameters of the model spectral function at various temperatures for which we compared them with the lattice QCD results of the imaginary time correlators in Sec. IV. In the case of the smallest \( \Delta \) will provide a good estimation of the best Borel curve since \( M_{\text{min}}^2 \) is fixed. When \( \Delta > 0 \), however, this will no longer hold to choose the best one among curves with different \( \sqrt{s_0} \). Therefore, we used this criterion only to choose the best \( \Delta \) value among a fixed \( \sqrt{s_0} \) case for each temperature. In Tables [VI, IX] we list sets of resultant parameters obtained in this way (not all cases: we computed more cases of \( \sqrt{s_0} \) but we include the case of the smallest \( \sqrt{s_0} \) below which no Borel window is available). We list \( \sqrt{\Delta} \) instead of \( \Delta \) since it is related to a crude estimate of a systematic error in the mass as discussed in Sec. [1C].

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### TABLE VI: Spectral parameters for $J/\psi$ at finite temperature obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|---------|---------------------|-----------|----------------|----------------------|------------------|---------------------|
| 1.00    | 3.26                | 2.975     | 0              | 0.264                | 2.350            | 51.3                |
|         | 3.4                 | 3.010     | 0              | 0.326                | 1.841            | 31.9                |
|         | 3.5                 | 3.026     | 0              | 0.360                | 1.673            | 32.1                |
|         | 3.6                 | 3.055     | 32             | 0.130                | 1.666            | 43.8                |
| 1.04    | 3.16                | 2.920     | 0              | 0.211                | 1.751            | 18.3                |
|         | 3.28                | 2.947     | 0              | 0.258                | 1.375            | 11.6                |
|         | 3.4                 | 2.997     | 58             | 0.105                | 1.452            | 19.7                |
|         | 3.5                 | 3.035     | 100            | 0.123                | 1.487            | 29.2                |
| 1.07    | 3.05                | 2.868     | 1              | 0.0494               | 1.009            | 0.04                |
|         | 3.2                 | 2.935     | 64             | 0.0770               | 1.322            | 2.4                 |
|         | 3.4                 | 3.024     | 158            | 0.118                | 1.509            | 9.4                 |
|         | 3.6                 | 3.100     | 240            | 0.158                | 1.554            | 24.6                |
| 1.09    | 3.04                | 2.901     | 126            | 0.0586               | 1.024            | 0.48                |
|         | 3.2                 | 2.950     | 134            | 0.0838               | 1.254            | 1.2                 |
|         | 3.4                 | 3.037     | 216            | 0.125                | 1.560            | 2.9                 |
|         | 3.6                 | 3.121     | 306            | 0.169                | 1.626            | 13.1                |

### TABLE VII: Spectral parameters for $\eta_c$ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|---------|---------------------|-----------|----------------|----------------------|------------------|---------------------|
| 1.00    | 3.2                 | 2.915     | 0              | 0.262                | 1.732            | 43.3                |
|         | 3.3                 | 2.936     | 0              | 0.307                | 1.413            | 31.1                |
|         | 3.4                 | 2.950     | 0              | 0.340                | 1.287            | 33.6                |
|         | 3.5                 | 2.975     | 26             | 0.122                | 1.281            | 48.2                |
|         | 3.6                 | 2.992     | 42             | 0.133                | 1.266            | 67.5                |
| 1.04    | 3.02                | 2.834     | 18             | 0.0527               | 1.008            | 0.15                |
|         | 3.2                 | 2.920     | 104            | 0.0918               | 1.243            | 2.1                 |
|         | 3.4                 | 3.011     | 202            | 0.140                | 1.376            | 7.6                 |
|         | 3.6                 | 3.089     | 288            | 0.187                | 1.427            | 21.0                |
| 1.07    | 3.02                | 2.880     | 230            | 0.0713               | 1.022            | 0.78                |
|         | 3.1                 | 2.900     | 198            | 0.0827               | 1.186            | 2.6                 |
|         | 3.2                 | 2.942     | 216            | 0.104                | 1.367            | 3.8                 |
|         | 3.3                 | 2.987     | 252            | 0.127                | 1.548            | 4.5                 |
|         | 3.4                 | 3.033     | 296            | 0.152                | 1.794            | 4.9                 |
|         | 3.5                 | 3.079     | 342            | 0.179                | 2.133            | 5.4                 |
| 1.09    | 3.02                | 2.917     | 428            | 0.0902               | 1.029            | 1.5                 |
|         | 3.1                 | 2.915     | 296            | 0.0920               | 1.207            | 4.7                 |
|         | 3.2                 | 2.952     | 284            | 0.111                | 1.374            | 7.1                 |
|         | 3.3                 | 2.995     | 306            | 0.134                | 1.540            | 8.8                 |
|         | 3.4                 | 3.041     | 344            | 0.159                | 1.710            | 10.2                |
|         | 3.5                 | 3.088     | 390            | 0.186                | 1.899            | 11.2                |
TABLE VIII: Summary of spectral parameters for $\chi_{c0}$ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|---------|---------------------|-----------|----------------|--------------------------|---------------------|----------------------|
| 1.00    | 3.6                 | 3.288     | 0              | 0.205                    | 2.230               | 16.5                 |
| 3.7     | 3.314               | 0         | 0.240          | 2.094                    | 15.1                |
| 3.8     | 3.368               | 40        | 0.0924         | 2.170                    | 22.6                |
| 1.04    | 3.28                | 3.083     | 0              | 0.0901                   | 1.367               | 0.31                 |
| 3.4     | 3.156               | 46        | 0.0420         | 1.537                    | 2.8                 |
| 3.5     | 3.215               | 80        | 0.0548         | 1.651                    | 6.6                 |
| 3.6     | 3.270               | 110       | 0.0687         | 1.735                    | 12.0                |
| 1.07    | 3.34                | 3.166     | 194            | 0.0428                   | 1.428               | 0.15                 |
| 3.4     | 3.199               | 200       | 0.0496         | 1.599                    | 0.67                |
| 3.5     | 3.254               | 214       | 0.0630         | 1.691                    | 2.5                 |
| 3.6     | 3.297               | 218       | 0.0768         | 1.592                    | 8.1                 |
| 1.09    | 3.26                | 3.167     | 340            | 0.0404                   | 1.331               | 0.05                 |
| 3.4     | 3.226               | 296       | 0.0549         | 1.548                    | 0.76                |
| 3.5     | 3.279               | 296       | 0.0672         | 1.891                    | 0.47                |
| 3.6     | 3.329               | 300       | 0.0833         | 1.724                    | 2.7                 |

TABLE IX: Summary of spectral parameters for $\chi_{c1}$ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|---------|---------------------|-----------|----------------|--------------------------|---------------------|----------------------|
| 1.00    | 3.5                 | 3.349     | 0              | 0.128                    | 2.382               | 22.2                 |
| 3.6     | 3.364               | 0         | 0.144          | 2.355                    | 17.3                |
| 3.7     | 3.386               | 6         | 0.0520         | 2.331                    | 20.7                |
| 3.8     | 3.434               | 46        | 0.0615         | 2.428                    | 28.9                |
| 1.04    | 3.2                 | 3.170     | 4              | 0.0215                   | 1.486               | 0.14                 |
| 3.3     | 3.214               | 44        | 0.0267         | 1.621                    | 2.6                 |
| 3.4     | 3.257               | 76        | 0.0327         | 1.747                    | 7.0                 |
| 3.5     | 3.301               | 104       | 0.0397         | 1.866                    | 12.6                |
| 3.6     | 3.344               | 130       | 0.0472         | 1.953                    | 19.6                |
| 1.07    | 3.2                 | 3.261     | 250            | 0.0295                   | 1.489               | 0.03                 |
| 3.3     | 3.285               | 252       | 0.0340         | 1.647                    | 1.4                 |
| 3.5     | 3.352               | 264       | 0.0466         | 1.904                    | 8.4                 |
| 3.7     | 3.432               | 286       | 0.0632         | 2.083                    | 20.0                |
| 1.09    | 3.2                 | 3.328     | 422            | 0.0361                   | 1.495               | 0.004                |
| 3.3     | 3.331               | 388       | 0.0392         | 1.664                    | 0.66                |
| 3.5     | 3.384               | 362       | 0.0512         | 1.946                    | 5.7                 |
| 3.7     | 3.461               | 366       | 0.0681         | 2.140                    | 15.1                |
### TABLE X: Summary of spectral parameters for Υ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|-------|-------------------|-----------|---------------|---------------------|-----------------|-----------------|
| 1.15  | 10.04             | 9.369     | 0             | 1.670               | 16.52           | 44.3            |
| 1.16  | 9.401             | 0         | 1.845         | 14.86               | 25.0            |
| 1.28  | 9.428             | 0         | 1.959         | 13.71               | 24.9            |
| 1.34  | 9.440             | 0         | 2.023         | 13.26               | 29.3            |
| 1.44  | 9.480             | 40        | 0.694         | 13.31               | 29.3            |
| 1.54  | 10.0              | 9.350     | 0             | 1.613               | 16.20           | 38.3            |
| 1.01  | 9.377             | 0         | 1.739         | 14.74               | 27.9            |
| 1.02  | 9.401             | 0         | 1.857         | 13.68               | 21.6            |
| 1.034 | 9.441             | 22        | 0.647         | 13.03               | 23.0            |
| 1.05  | 9.516             | 108       | 0.738         | 13.46               | 30.3            |

### TABLE XI: Summary of spectral parameters for $\eta_b$ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|-------|-------------------|-----------|---------------|---------------------|-----------------|-----------------|
| 1.15  | 9.98              | 9.315     | 0             | 1.769               | 15.71           | 48.6            |
| 1.01  | 9.347             | 0         | 1.941         | 13.48               | 35.1            |
| 1.22  | 9.373             | 0         | 2.094         | 12.33               | 28.2            |
| 1.28  | 9.385             | 0         | 2.163         | 11.91               | 28.3            |
| 1.4   | 9.430             | 44        | 0.753         | 11.92               | 34.7            |
| 1.54  | 9.94              | 9.295     | 0             | 1.705               | 15.18           | 39.5            |
| 1.04  | 9.322             | 0         | 1.848         | 13.23               | 28.9            |
| 1.14  | 9.345             | 0         | 1.977         | 12.17               | 22.9            |
| 1.28  | 9.387             | 28        | 0.694         | 11.69               | 25.7            |
| 1.4   | 9.441             | 86        | 0.768         | 11.95               | 31.8            |

### TABLE XII: Summary of spectral parameters for $\chi_{b0}$ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|-------|-------------------|-----------|---------------|---------------------|-----------------|-----------------|
| 1.15  | 10.43             | 9.838     | 0             | 0.609               | 13.49           | 22.5            |
| 1.58  | 9.882             | 0         | 0.696         | 12.81               | 13.4            |
| 1.73  | 9.943             | 22        | 0.254         | 12.62               | 16.5            |
| 1.8   | 9.984             | 46        | 0.274         | 12.83               | 19.9            |
| 1.54  | 10.34             | 9.773     | 0             | 0.542               | 12.36           | 9.13            |
| 1.52  | 9.839             | 16        | 0.207         | 11.89               | 7.17            |
| 1.72  | 9.960             | 84        | 0.260         | 12.58               | 14.5            |
| 1.8   | 10.007            | 110       | 0.283         | 12.83               | 18.1            |

### TABLE XIII: Summary of spectral parameters for $\chi_{b1}$ obtained from QCD sum rules.

| $T/T_c$ | $\sqrt{s_0}$ [GeV] | $m$ [GeV] | $\Gamma$ [MeV] | $f$ or $f_0$ [GeV$^2$] | $M_0^2$ [GeV$^2$] | $\sqrt{\chi^2}$ [MeV] |
|-------|-------------------|-----------|---------------|---------------------|-----------------|-----------------|
| 1.15  | 10.18             | 10.043    | 0             | 0.445               | 13.53           | 1.9             |
| 1.26  | 10.049            | 0         | 0.460         | 13.52               | 2.0             |
| 1.34  | 10.087            | 34        | 0.157         | 13.88               | 3.9             |
| 1.4   | 10.114            | 56        | 0.165         | 14.13               | 5.6             |
| 1.54  | 10.1              | 10.005    | 20            | 0.133               | 12.76           | 0.02            |
| 1.22  | 10.063            | 72        | 0.148         | 13.36               | 1.5             |
| 1.34  | 10.114            | 114       | 0.163         | 13.87               | 3.7             |
| 1.4   | 10.142            | 136       | 0.172         | 14.13               | 5.4             |
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