Theoretical analysis of the resistively-coupled single-electron transistor

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Abstract

The operation of resistively-coupled single-electron transistor (R-SET) is studied quantitatively. Due to the Nyquist noise of the coupling resistance, degradation of the R-SET performance is considerable at temperatures $T$ as small as $10^{-3}e^2/C$ (where $C$ is the junction capacitance) while the voltage gain becomes impossible at $T \gtrsim 10^{-2}e^2/C$. 

Single-electron tunneling attracts considerable theoretical and experimental attention and can be potentially used in important applications including ultradense digital electronics. The simplest and most thoroughly studied single-electron device is the single-electron transistor (SET) which consists of two tunnel junctions in series. The current through this double-junction system depends on the background charge $Q_0$ of the central electrode ("island") which can be controlled with an additional external electrode thus providing the transistor effect. In the usual capacitively-coupled SET (C-SET) the charge $Q_0$ is controlled via the gate capacitance while the other possibility is to use the coupling resistance $R_g$ (R-SET) – see Fig. 1a.

C-SET can be relatively easy realized experimentally that also motivated numerous theoretical studies of different problems related to C-SET. In contrast, R-SET has almost not been studied theoretically after the initial proposal even in the simplest approximation (RC-SET with combined coupling has been considered in Ref.). The reason is the difficulty of experimental realization of R-SET. In order not to smear the discreteness of the island charge by quantum fluctuations, the gate resistance should be sufficiently large,

$$R_g \gg R_Q = \frac{\pi \hbar}{2e^2} \approx 6.5 \text{k}\Omega,$$

and simultaneously the geometrical size of the resistor should be relatively small so that its stray capacitance does not significantly increase the total capacitance of the island. The progress in fabrication of such resistors has been achieved only recently.

R-SET could be a very useful element for the integrated single-electron digital devices. At present the majority of the proposals for single-electron logic (see Ref.) are based on the capacitively-coupled devices which suffer from the principal problem of fluctuating background charges (the solution is known so far only for memory devices). The use of R-SET which is not influenced by background charges would allow to avoid this problem. Another anticipated advantage of R-SET is the possibility of much larger voltage gain than for C-SET. The potential importance for integrated devices and the possibility of the experimental demonstration of R-SET in the nearest future makes urgent the basic theoretical analysis of
R-SET operation. In this paper we consider the I-V curve and the dependence on the gate potential. We also discuss the smearing of the Coulomb blockade and the reduction of the voltage gain at finite temperatures.

Assuming sufficiently large gate resistance (Eq. (1)) and tunnel resistances, $R_{1,2} \gg R_Q$, and using the “orthodox” theory of single-electron tunneling, we describe the internal dynamics of the R-SET by the following master equation:

$$
\begin{align*}
\dot{\sigma}(Q) &= \Gamma^{-}(Q + e)\sigma(Q + e) + \Gamma^{+}(Q - e)\sigma(Q - e) \\
&- [\Gamma^{+}(Q) + \Gamma^{-}(Q)]\sigma(Q) \\
&+ \frac{1}{R_g C_\Sigma} \frac{\partial}{\partial Q}[(Q - \tilde{Q})\sigma(Q)] + \frac{T_r}{R_g C_\Sigma} \frac{\partial^2 \sigma(Q)}{\partial Q^2}.
\end{align*}
\tag{2}
$$

Here $\sigma(Q)$ is the probability density to find the total charge $Q$ on the island, $C_\Sigma = C_1 + C_2$ is the total island capacitance, and $\tilde{Q} = U C_\Sigma - V C_2$ corresponds to the equality between the gate potential $U$ and the island potential $\phi = Q/C_\Sigma + V C_2/C_\Sigma$. The last term in Eq. (2) describes the Nyquist noise of the gate resistance being at temperature $T_r$ which can in principle differ from the temperature $T$ of the electron gas in tunnel junctions (we assume $T_r = T$). $\Gamma^\pm(Q) = \Gamma^+_1(Q) + \Gamma^+_2(Q)$ where $\Gamma_i^\pm$ are the rates of tunneling through $i$th junction increasing (+) or decreasing (−) the island charge:

$$
\begin{align*}
\Gamma_i^\pm &= \frac{W_i^\pm}{e^2 R_i [1 - \exp(-W_i^\pm/T)]}, \\
W_i^\pm &= \frac{e}{C_\Sigma} \left[ \pm \left( Q + (-1)^i V \frac{C_1 C_2}{C_i} \right) - \frac{e}{2} \right].
\end{align*}
\tag{3}
$$

In this paper we analyze only dc characteristics of R-SET, so $\dot{\sigma}(Q) = 0$ is assumed in Eq. (4).

At $T = 0$ the Coulomb blockade state is realized when $\phi = U$ and the voltages across both tunnel junctions are less than the tunneling threshold,

$$
|U| < e/2C_\Sigma, \ |V - U| < e/2C_\Sigma.
\tag{4}
$$

Outside the blockade range the average currents through junctions,
\[ I_i = (-1)^{i+1}e \int [\Gamma_i^+(Q) - \Gamma_i^-(Q)]\sigma(Q)dQ , \] (5)

can be different because of finite gate current \( I_g = I_2 - I_1, I_g = [U - \int \phi(Q)\sigma(Q)dQ]/R_g. \)

The analysis can be considerably simplified in the limit \( R_g \gg R_{1,2}. \) Then it is useful to separate the total charge
\( Q = Q_0 + ne \) into the part \( Q_0 \) supplied via \( R_g \) and the integer charge \( ne \) due to tunneling (initial background charge is included in \( Q_0 \)). Because of \( R_g \gg R_{1,2}, \) the change of \( Q_0 \) is slow and the first averaging can be done over the fast tunneling events exactly like for C-SET, that gives \( e \)-periodic dependencies \( \bar{\phi}(Q_0) \) and \( \bar{I}(Q_0) \) (the currents through junctions are equal in this approximation).

If the Nyquist term in Eq. (2) can be neglected \( (T_r = 0) \), then \( \dot{Q}_0 = (U - \bar{\phi})/R_g. \) In the case when \( \min_{Q_0} \bar{\phi}(Q_0) < U < \max_{Q_0} \bar{\phi}(Q_0) \), the stationary state with \( I_g = 0 \) will be eventually reached. (This condition is satisfied by two values of \( Q_0 \) per period with the stable state determined by \( \partial \bar{\phi}/\partial Q_0 > 0. \) It is interesting that in this case the I-V curve of R-SET can have negative differential conductance (see also Ref. 4) which is realized when
\( (\partial \bar{I}/\partial V) < (\partial \bar{I}/\partial Q_0)(\partial \bar{\phi}/\partial V)/(\partial \bar{\phi}/\partial Q_0). \)

If the gate voltage \( U \) is outside the range \( (\min \bar{\phi}, \max \bar{\phi}) \), then the stationary state for \( Q_0 \) is impossible and the current through R-SET will perform single-electron oscillations with the period \( \tau = \int_0^e R_g/[U - \bar{\phi}(Q_0)]dQ_0 \) while the average gate current \( I_g = e/\tau. \) The average output current does not depend on \( R_g \) and can be easily calculated using the numerical solution for \( Q_0(t). \)

When the ratio \( R_g/R_{1,2} \) is finite, the stationary solution of full Eq. (2) can be found numerically (we will discuss the numerical methods elsewhere). Figure 1b shows the currents \( I_1 \) (solid line) and \( I_g \) (dashed line) for the symmetric R-SET \( (C_1 = C_2 = C, R_1 = R_2 = R) \) as functions of the bias voltage \( V \) for \( T = 0, R_g/R = 10, \) and different gate voltages \( U. \) Notice strong asymmetry of the I-V curve shape near two thresholds of the Coulomb blockade for \( U \neq 0. \) The slope of the step-like feature grows with the increase of \( R_g/R \) (the perfect step is realized for \( R_g/R = \infty \) as follows from the analysis above). In the large-bias limit \( (V \gg e/C, V - U \gg e/C) \) the currents can be found analytically using simple Kirchhoff
analysis and taking into account the effective voltage shift $e/2C_\Sigma$ (opposite to the current direction) in each tunnel junction: $I_1 = [V(R_2 + R_g) - UR_2 - (e/2C_\Sigma)(2R_g + R_2)]/A$ and $I_g = [U(R_1 + R_2) - VR_2 + (e/2C_\Sigma)(R_2 - R_1)]/A$ where $A = (R_1R_2 + R_1R_g + R_2R_g)$. The voltage offset between the positive and negative asymptotes of $I_1(V)$ is equal to $(e/C_\Sigma)(2R_g + R_2)/(R_g + R_2)$.

Figure 2 illustrates the effect of the temperature on the I-V curve of R-SET. One can see that in contrast to the C-SET, even small temperature significantly smears the Coulomb blockade threshold. The finite temperature changes the tunneling rates (Eq. (3)) and also causes the Nyquist noise of the gate resistance. The effect of the tunneling rates change is similar to that in C-SET and leads to the smearing of sharp features within a voltage range on the order of $T/e$; hence, it is quite small at $T \lesssim 0.01e^2/C_\Sigma$. The effect of the Nyquist noise is much more important at relatively low temperatures. In absence of the tunneling current within the Coulomb blockade, even for arbitrary large $R_g$ (that reduces the noise – see Eq. (2)) the fluctuations of $Q_0$ should satisfy the thermal distribution leading to r.m.s. values

$$\delta Q_0 = (TC_\Sigma)^{1/2}, \ \delta \phi = (T/C_\Sigma)^{1/2}.$$ \hspace{1cm} (6)

The scaling as $T^{1/2}$ makes the effect significant even for $T \sim 10^{-3}e^2/C_\Sigma$ and thus creates a serious problem for the practical use of R-SET. (Notice that Nyquist noise was similarly the main obstacle for the wide use of resistively-coupled SQUIDs.)

For $i$th junction biased below the blockade threshold, the noise-induced tunneling rate can be estimated as $\Gamma_i \simeq \int_0^\infty (x/eR_i)(C_\Sigma/2\pi T)^{1/2}\exp[-(x + \Delta_i)^2C_\Sigma/2 T]dx$, where $\Delta_1 = e/2C_\Sigma - (V - U)$ and $\Delta_2 = e/2C_\Sigma - U$ ($\Delta_i \gg T/e$). However, the numerical results show that the leakage current is typically few times larger (can be much larger) than this estimate. The reason is the positive feedback from the gate resistance. For example, when the positive charge tunnels to the island through the first junction, it causes some negative gate current. Hence, after the charge escapes through the second junction, the voltage across the first junction is increased in comparison with the situation before tunneling. This effect
enhances the “clustering” of tunneling events above the level determined by Nyquist random
walk and further increases the shot noise (which in this case is considerably higher than the
Schottky level). The leakage current typically grows with $R_g$ because at relatively small $R_g$
the train of tunneling events can be stopped by the single charge escape through the gate
resistance.

The strong smearing of the Coulomb blockade at finite temperatures significantly reduces
the R-SET voltage gain. Figure 3 shows the control curves at different temperatures of the
inverter made of symmetric R-SET ($R_g = 10R$) loaded with resistance $R_L = 10R$ and biased
by $V_B = 0.5e/C$. The voltage $V = V_B - I_1R_L$ is the output of the inverter while $U$ is the
input voltage. One can see that the voltage gain $K_V = |dV/du|$ becomes less than unity at
the negative slope of the $V-U$ dependence at temperatures as low as $\sim 10^{-2}e^2/C$ (while $K_V$
can be arbitrary large at $T = 0$). To check that the main reason for low $K_V$ is the Nyquist
noise of the gate resistance, we also performed calculations for $T_r = 0$ while $T$ is nonzero.
Dashed line in Fig. 3 shows such a result for $T = 0.005e^2/C$. For this curve the maximum
$K_V \simeq 7$, to be compared with $K_V \simeq 1.2$ for the corresponding curve with $T_r = T$.

The inset in Fig. 3 shows the control curves on the larger scale. The asymptotes of
$V-U$ dependence can be calculated similar to that for the I-V curve,

$$V = \frac{V_BA + \left(U \mp \frac{e}{2C_\Sigma} R_2 R_L\right)}{A + R_L(R_2 + R_g)}.$$  However, in the case $R_g \gg R_i$ the $V-U$ asymptotes
are reached only at very large $U$ because it requires sufficiently large junction currents,
$|I_i| \gtrsim 2e/R_iC_\Sigma$.

In Fig. 4 the inverter bias voltage $V_B = e/C_\Sigma$ is equal to the maximum Coulomb block-
ade threshold. The increase of $V_B$ destroys Coulomb blockade even for $T = 0$ leading to
additional smoothing of the negative slope range. The decrease of $V_B$ creates the plato on
the control curve when $V$ is limited by $V_B$.

Figure 4 illustrates the dependence of inverter control curves on the load and gate re-
sistances. At finite temperature the increase of $R_L$ shifts the negative slope range to lower
input voltages and also decreases the output voltage both before and after this range. In-
crease of $R_g$ for fixed $R_L$ produces similar effects. Notice that the maximum voltage gain
typically grows with the increase of \( R_g \) and \( R_L \).

The optimal loading and the voltage symmetry is provided by complementary R-SETs. In this case (similar to the case \( R_L \to \infty \)) the maximum temperature \( T_{\text{max}} \) at which \( K_V > 1 \) is still achievable, is close to \( 0.011e^2/C \) for \( R_g/R = 10 \) (\( 0.010e^2/C \) for \( R_g/R = 3 \) and \( 0.012e^2/C \) for \( R_g/R = 30 \)). This value is less than one half of \( T_{\text{max}} = 0.026e^2/C \) for the inverter based on the C-SETs (moreover, for C-SET it is achieved at twice larger total island capacitance).

In conclusion, while R-SET outperforms C-SET at \( T = 0 \) (in terms of the voltage gain), its characteristics degrade with temperature much faster than for C-SET due to the Nyquist noise of the gate resistance (because of \( T^{1/2} \) scaling). As a result, at \( T \gtrsim 10^{-2}e^2/C \), the R-SET performance becomes comparable or even worse than that of C-SET. Nevertheless, insensitivity to the background charge and the nonoscillatory dependence on the gate voltage can still be the principle advantages of the R-SET for some applications.

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REFERENCES

1. D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991), p. 173.

2. A. N. Korotkov, in *Molecular Electronics*, edited by J. Jortner and M. Ratner (Blackwell, Oxford, 1997), p. 157.

3. K. K. Likharev, IEEE Trans. on Magn. 23, 1142 (1987).

4. A. N. Korotkov, Phys. Rev. B 49, 16518 (1994).

5. L. S. Kuzmin, Yu. V. Nazarov, D. B. Haviland, P. Delsing, and T. Claeson, Phys. Rev. Lett. 67, 1161 (1991).

6. L. S. Kuzmin and Yu. A. Pashkin, Physica B 194-196, 1713 (1994).

7. T. Henning, D. B. Haviland, and P. Delsing, Supercond. Sci. Technol. 10, 727 (1997).

8. Sh. Farhangfar, J. J. Toppari, Yu. A. Pashkin, A. J. Manninen, E. B. Sonin, and J. P. Pekola, preprint (1998).

9. W. Zheng, J. R. Friedman, D. V. Averin, S. Han, and J. E. Lukens, to be published.

10. P. Joyez, D. Esteve, and M. H. Devoret, preprint (1998).

11. K. K. Likharev and A. N. Korotkov, in: *Proceedings of ISDRS'95* (Charlottesville, VA, 1995), p. 355.

12. I. O. Kulik and R. I. Shekhter, Sov. Phys. JETP 41, 308 (1975).

13. K. K. Likharev, *Dynamics of Josephson junctions and circuits* (Gordon and Brich, NY, 1986), Ch. 7.

14. A. N. Korotkov, R. H. Chen, and K. K. Likharev, J. Appl. Phys. 78, 2520 (1995).
FIGURES

FIG. 1. (a) Schematic of the R-SET. (b) The currents $I_1$ (solid line) and $I_g$ (dashed line) as functions of the bias voltage $V$ at $T = 0$. The gate voltages (from top to bottom): $U/(e/C) = -1/2$, -3/8, -1/4, -1/8, 0, 1/8, 1/4, 3/8, 1/2. The curves are shifted vertically (by $\Delta I = 0.4U/R$) for clarity.

FIG. 2. The I-V curves of the R-SET for different temperatures.

FIG. 3. The control curves of resistively loaded R-SET (inverter) at different temperatures. Dashed line shows the result for $T = 0.005e^2/C\Sigma$ neglecting Nyquist noise. Inset shows the same curves on the larger scale.

FIG. 4. The control curves of the inverter at $T = 0.005e^2/C$ for different (a) load resistances $R_L$ and (b) gate resistances $R_g$. 
\[ V_{Q} = Q_{0} + ne \]

\[ I_1 = C_1 R_1 \]
\[ I_2 = C_2 R_2 \]

\[ T = 0 \]
\[ R_g = 10 R \]
\[ C_1 = C_2 = C \]
\[ R_1 = R_2 = R \]

**Fig. 1**

**Fig. 2**
Fig. 3

Fig. 4