Hydrodynamics and Nonlocal Conductivities in Vortex States of Type II Superconductors

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(Received )

ABSTRACT

A hydrodynamical description for vortex states in type II superconductors is presented based on the time-dependent Ginzburg-Landau equation (TDGL). In contrast to the familiar extension of a single vortex dynamics based on the force balance, our description is consistent with the known hydrodynamics of a rotating neutral superfluid and correctly includes informations on the Goldstone mode. Further it enables one to examine nonlocal conductivities perpendicular to the field in terms of Kubo formula. Typically, the nonlocal conductivities deviate from the usual vortex flow expressions, as the nonlocality parallel to the field becomes weaker than the perpendicular one measuring a degree of positional correlations, and, for instance, the superconducting contribution of dc Hall conductivity nonlocal only in directions perpendicular to the field becomes vanishingly small in the situations with large shear viscosity, leading to an experimentally measurable relation $\rho_{xy} \sim \rho_{xx}^2$ among the resistivity components. Other situations are also discussed on the basis of the resulting expressions.

KEYWORDS: type II superconductor, vortex states, vortex flow conductivities
§1. Introduction

At present, understanding theoretically the nonlocality\(^1,2\) of linear resistivity in a clean system (with no pinning) seems to be one of central problems on the vortex states of a type II superconductor. By analogy to the usual viscous fluid, Marchetti and Nelson\(^1\) have previously argued within a linear hydrodynamics that, deep in the liquid regime, a shear-viscous force proportional to \(-\partial^2_\perp \partial_t \mathbf{s}\) will take the place of the shear term in the elastic force \(\mathbf{f}_{\text{el}}\) in the following force-balance equation for the vortex lattice

\[
\partial_t (-\Gamma_1 \mathbf{s} + \Gamma_2 (\hat{z} \times \mathbf{s})) + \mathbf{J} \times \mathbf{B} = \mathbf{f}_{\text{el}},
\]

and that consequently, a positive nonlocal term proportional to \(q^2_\perp\) should exist in the conductivity perpendicular to \(\mathbf{B}\). In eq. (1) (and through this paper,) \(\mathbf{s}\) denotes the displacement field of vortex positions, \(\mathbf{J}\) the external current, \(\mathbf{B}\) the uniform flux density parallel to \(z\) axis, \(q_\perp\) the wavevector perpendicular to \(B\), and \(\Gamma_1\) and \(\Gamma_2\) will be given later. This equation is essentially an extension\(^3\) of the single vortex dynamics to interacting vortex states. Subsequently, the presence of other nonlocal terms was proposed\(^2\) neglecting any sample disorder, although experiments have been performed in heavily twinned (disordered) samples\(^4\), and, the presence deep in the liquid regime of a large conductivity nonlocal in the field direction was argued. It is important to theoretically clarify to what degree these proposals can be justified beyond the phenomenology\(^1,2,5\) and within a basic dynamical model for type II superconductors such as the time-dependent Ginzburg-Landau equation (TDGL) which correctly describes the uniform linear dissipation in terms of Kubo formula.\(^6\) Actually, the eq.(1) is not correct in the following senses. Firstly, it is not compatible with calculations of conductivities based on the use of Kubo formula. Secondly, it is not clear from eq. (1), in which \(\mathbf{f}_{\text{el}}\) is independent of \(\mathbf{J}\) and of the time derivative \(\partial_t\), how the possible viscous (nonlocal) terms in the dc conductivity are changed as the vortex lattice is formed. Further, the dispersion \(\sim q^4_\perp\) of the shear mode found as a phase fluctuation formally and generally\(^7,8\) cannot be seen in eq.(1). Improving this is important in order to avoid misunderstanding conclusions on the phase correlations in refs. 7 and 8.

In the present paper, a correct hydrodynamical approach, or equivalently, a dynamical harmonic analysis on the transport in vortex states, particularly in the vortex lattice, is presented on the basis of TDGL. Our approach has no difficulties mentioned above and makes it possible to examine nonlocal conductivities above and below the melting transition, and several consequences of the resulting conductivities are discussed. As has been understood through studies\(^9\) on nonlocal and static linear responses, the use of the harmonic analysis for 3D and layered systems often leads to misunderstandings on physics in the liquid regime, and hence, we will not consider situations with negligibly small positional correlations of vortices.

§2. TDGL hydrodynamics

Our starting point is TDGL describing dynamics of the order parameter \(\psi\) in an isotropic type II superconductor

\[
a(\gamma + i\gamma')\partial_t \psi + [b(|\psi|^2 - \rho_0) + a\xi^2_0 (-i\partial - \frac{2\pi}{\phi_0} A)^2] \psi = 0,
\]

(2)
where $\text{curl}A = B \hat{z}$, $\gamma$, $a$, and $b$ are positive, $\phi_0$ is the flux quantum, $\xi_0$ the coherence length, and the $\rho_0$ is the spatial average of the mean squared order parameter. The inclusion of sample anisotropy is straightforward and will be done later. Since the linearized version of eq. (2) is considered through the paper, the noise term introducing fluctuations in equilibrium does not have to be included, and growths of time scales will be phenomenologically included later as the only fluctuation effects. Within the harmonic analysis given below, spatially varying gauge fields other than the external disturbance (see below) have to be excluded in order, for the derivation of conductivities in vortex states, to become consistent with the corresponding one, for instance, of the ac conductivity in the Meissner state $\sigma_{ij}(\omega) = \pi \rho_{s0} \delta(\omega) \delta_{ij}$ valid irrespective of the details of dynamics, where $\rho_{s0} = 2(2\pi \xi_0 / \phi_0)^2 \rho_0 a$ is the mean field superfluid density.

Particularly upon calculations of linear responses, it is convenient to work rather within the harmonic approximation of the corresponding Euclidean action with gauge field disturbance $\delta A$ (and with $\hbar = 1$)

$$S_{\text{har}} = \sum_{\Omega} \left[ \beta \sum_{ij} a(\gamma|\Omega| + i \gamma' \Omega)|\psi_\Omega|^2 + \int_0^\beta d\tau \left[ a\xi_0^2 |(-i \partial - \frac{2\pi}{\phi_0}(A + \delta A(\tau)))\psi(\tau)|^2 \right. \right.$$

$$
+ b \left( 2|\psi_0|^2 |\psi(\tau)|^2 + \frac{1}{2} (\psi_0^* \psi^2(\tau) + \text{c.c.}) \right) \right].$$

(3)

Since we are interested only in frequencies and wavevectors accompanying $\delta A$ and summations with respect to them are not performed, calculations of conductivities based on the action (3) are equivalent to the linearized analysis on eq. (2) and hence, formally independent of temperature. Here $\tau$ is the imaginary time, $\psi_\Omega$ is the temporal Fourier transform of $\psi(\tau)$, $\Omega$ is Matsubara frequency, $\beta$ the inverse temperature, and $\psi_0$ the mean field solution of $\psi$. The superconducting (and real) part $\sigma_{ij}$ of dc linear conductivity tensor should be always calculated in terms of Kubo formula

$$\sigma_{ij}(q) = \left. \frac{\partial \rho_{s,ij}(q, i\Omega_0)}{\partial \Omega_0} \right|_{\Omega_0 \rightarrow +0} = \left. \frac{\partial}{\partial \Omega_0} \frac{\delta^2(-\beta^{-1} \ln \text{Tr}_\psi \exp(-S_{\text{har}}))}{\delta A_i(q, \Omega_0) \delta A_j(-q, -\Omega_0)} \right|_{\Omega_0 \rightarrow +0},$$

(4)

where $\Omega_0$ is the external (Matsubara) frequency, $i, j = x$ or $y$, and the absence of dc uniform ($q = 0$) superfluid density was used (see below). More generally, the ac nonlocal conductivities are defined as the imaginary parts of the response function $\rho_{s,ij}(q, \omega + i0^+)$ divided by the real frequency $\omega$. However, since general expressions of the ac nonlocal conductivities are complicated and they always become the vortex flow results in $q \rightarrow 0$ limit ( of the present formulation ), the dc nonlocal conductivities will be only considered below. Since the harmonic modes appearing in eq. (4) are spatial and temporal variations around the mean field solution linearly excited by the gauge field disturbance, the linear response quantities in vortex states resulting from such a harmonic analysis are not accompanied by the temperature $\beta^{-1}$. In this sense, this approach is not a fluctuation theory and is invalid for the linear dissipation parallel to $B$ which is a consequence of critical fluctuations. On the other hand, the vortex flow conductivities are essentially mean field results. Therefore, we focus below on the configuration perpendicular to $B$. 

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Before presenting general results, it is useful to first see results on the linear conductivities in the Landau level (LL) approach\(^7\) within the lowest and next-lowest LLs. In such a high field approximation the mean field solution and the shear elastic mode are found within the lowest LL, and the next-lowest LL, which does not participate in constructing the mean field solution,\(^7,9\) provides the uniform displacement of vortices (see eqs.(4.12) and (4.16) in ref.7). Since, in constructing basis functions of eigenmodes, Matsubara frequencies in the Euclidean action play similar roles to the wavevectors \(q\) and \((4.16)\) in ref.7). Since, in constructing basis functions of eigenmodes, Matsubara frequencies in the Euclidean action play similar roles to the wavevectors \(q\) and \((4.16)\) in ref.7). Since, in constructing basis functions of eigenmodes, Matsubara frequencies in the Euclidean action play similar roles to the wavevectors \(q\) and \((4.16)\) in ref.7).

For later convenience, an amplitude-dominated mode in the lowest LL, denoted by \(\delta \rho\), will also be included. Then, to the lowest order in \(q_\perp\), the action (3) becomes \(S_{LL} = S_0 + S_1\), where

\[
S_0 \simeq \int_{\mathbf{r}} \left[ \beta a \sum_\Omega (\gamma \rho_0 |\Omega||\chi_\Omega|^2 + \gamma' \Omega \delta \rho_0^* \chi_\Omega) + \int d\tau \frac{b}{2} \delta \rho^2 + a \rho_0 \xi_0^2 (\partial_z \chi - \frac{2\pi}{\phi_0} \delta A_z)^2 + \frac{C_{66}}{2} r_B^4 (\partial_{\perp}^2 \chi)^2 \right],
\]

\[
S_1 \simeq \frac{\rho_0 a}{2 r_B^2} \int_{\mathbf{r}} \left[ \sum_\Omega \left( \gamma |\Omega||s_{0\Omega}|^2 + \gamma' \Omega (s_{0\Omega}^* \times s_{0\Omega})_z \right) + \xi_0^2 \int d\tau \left( (\partial_z s_0)^2 + 2 r_B^{-2} (s_0 + B^{-1} (\delta A_{\perp,0} \times \xi))^2 \right) \right],
\]

where \(s^T\) (the transverse component of \(s\)) is given\(^7,8\) by \(r_B^2 (\partial_{\perp} \chi \times \xi)\) with vortex spacing \(r_B = \sqrt{\phi_0 / 2\pi B}\), \(s_0\) the uniform displacement with vanishing \(q_\perp\), \(\chi\) a longitudinal phase variable, and \(C_{66}\) the resulting shear modulus. Variational equations with replacement \(i\Omega \to \omega\) (real frequency) give eigenmodes of the vortex lattice. Note that the resulting dispersion \(\omega = -i (a \gamma \rho_0)^{-1} C_{66} (q_\perp r_B)^4\) for the shear mode\(^7,8\) at zero \(q_z\) is different from that following from eq. (1), and hence that the extrapolation\(^1,3\) of the uniform (or equivalently, the single-) vortex dynamics to the interacting case with the shear mode is not justified within TDGL. Through the London limit (see the action (7) given below), the action (5) with the constraint \(s^T = r_B^2 (\partial_{\perp} \chi \times \xi)\) is found to correctly give hydrodynamical results at longer distances than

\[
l = r_B \sqrt{C_{66} r_B^2 / 2 a \rho_0 \xi_0^2},
\]

which is typically of the order \(r_B\).

The absence of the dc uniform superfluid density (helicity modulus) is obvious from the expressions (4) and (5)', implying that a constant twist pitch \(\delta A_0\) is transmuted into an uniform displacement of vortices keeping the free energy invariant. That is, as also indicated by Baym,\(^8\) situation is different from an elastic matter\(^1\) with free energy invariant with respect to uniform displacements. Using (4) and (5)' and eliminating \(s_0\), we easily obtain the mean field expressions on diagonal \((i = j)\) and Hall \((i \neq j)\) vortex flow conductivities

\[
\sigma_{ij}(q_\perp = 0, q_z) = \rho s_0 \frac{r_B^2}{2 \xi_0^2} \frac{\gamma \delta_{ij} + \gamma' \varepsilon_{ij}}{1 + r_B^2 q_z^2 / 2}.
\]
Due to the constraint $s^T = r_B^2(\partial_\perp \chi \times \hat{z})$ imposed before introducing $\delta A$, the action (5), in which the degree of the positional order is directly reflected, does not give any nonlocal (and longitudinal) conductivity even when $q_\perp \neq 0$. It should be noticed that the nonlocality of the denominator in eq. (6), which results from the $|\partial_\perp s|^2$ term of eq. (5)' and is a typical example leading to a possible negative $q^2$-term in $\sigma_{ij}$, is accompanied by a small length scale ($\sim r_B$) insensitive to temperature and hence, is not relevant to the ordering of the system. Consistently with eq. (6), the nonlocal superfluid density, namely the real part of $\sqrt{\rho}$ described according to the Gross-Pitaevskii equation. Consistently with refs.12 and 13, ref.9, this $q_\perp^4$ superfluid longitudinal which is necessary in recovering the limit of incompressible fluid (i.e., zero field case at such short lengths, while, if $q_\perp$ is nonzero in the action (7), which is necessary in considering longitudinal conductivity at low $q_\perp$ behavior does not change through a 3D melting transition.11)

Next, let us here comment on eigen modes of the action (3) in 2D and nondissipative ($\gamma = 0$) case, formally corresponding to the rotating superfluid $^4$He at zero temperature described according to the Gross-Pitaevskii equation. Consistently with refs.12 and 13, the order parameter in eq. (3) will be divided into the amplitude $\rho$ and phase $\varphi$: $\psi = \sqrt{\rho} \exp(i\varphi)$ and, just as in the usual London approximation, the presence of the field-induced vortices will be taken into account through the topological condition14) on $\partial_\perp \varphi = (\partial_\perp \varphi, \partial_r \varphi)$ by neglecting any fluctuation-induced vortices. As a result, we obtain the following harmonic action

$$S_{ph} = a \int d\tau \left(i \gamma' \delta \rho \partial_\tau \chi + \rho_0 \xi_0^2 (\partial_\perp \chi - r_B^{-2}(\hat{z} \times s))^2 + \frac{b}{2a} \delta \rho^2 + \frac{C_{66}}{2a} (\partial_j s_j^T)^2 \right). \quad (7)$$

The only difference of this action from (5) (in 2D) is that the longitudinal current

$$v_s = 2a \xi_0^2 (\partial_\perp \chi - r_B^{-2}(\hat{z} \times s^T)) \quad (8)$$

is nonzero in the action (7), which is necessary in considering longitudinal conductivity nonlocal in directions perpendicular to the field. When $q_\perp l > 1$, the minimal coupling in eq. (8) between $\chi$ and $s^T$ is removed, and in principle the phase fluctuation behaves like in zero field case at such short lengths, while, if $q_\perp l < 1$ and a gauge disturbance $\delta A_\perp (q_\perp \neq 0)$ is not substituted, $v_s \simeq 0$ is established and thus, (7) reduces to (5). Variational equations resulting from the action (7) agree with those following from a dual representation used in ref.12, where a role of nonzero $v_s$ at nonzero frequency was stressed in a context of superfluid $^4$He. The action (7) does not include the term

$$S_{inc} = i \int d\tau \gamma'' \frac{\rho_0 a}{2r_B^2} (s \times \partial_\tau s)_z, \quad (9)$$

which is necessary in recovering the limit of incompressible fluid (i.e., $\delta \rho = 0$ and $v_s = 0$). This additional term with the coefficient $\gamma''$, which should become $\gamma'$, cannot be detected in this phase-only analysis for the action (3) (Hereafter, we will assume $\gamma'' = \gamma'$). The eigenvalues following from $S_{ph} + S_{inc}$ precisely coincide with those derived by Sonin13) and give dispersions $\omega_{sh}^2 \simeq (a \gamma')^{-2} b C_{66} (q_\perp r_B)^4 / (1 + (q_\perp l_{inc})^2)$ and $\omega_c^2 \simeq (2 \xi_0^2 / \gamma'' r_B^2)^2 (1 + (q_\perp l_{inc})^2)$ of two modes corresponding to the shear (massless) and compression (massive) elastic modes, respectively, where $l_{inc} = r_B^2 \sqrt{b \rho_0 / 2a / \xi_0}$. The quadratic dispersion of $\omega_{sh}$ at low $q_\perp$ is an origin of the destruction of the long-ranged phase coherence in the
vortex lattice. If $\Gamma_1 = 0$, $\Gamma_2 = 2\pi a_0 \gamma'' B / \phi_0$, and $v_s = 0$, the variational equation, with replacement $-i\tau \to t$ (real time), with respect to the shear displacement of $S_{ph} + S_{inc}$ becomes the transverse part of eq. (1) with no external current. In this case the constraint $v_s = 0$ is different from that in the action (5) and rather corresponds to the limit of incompressible fluid. Following ref.13, this limit is valid only at shorter lengths than $l_{inc}$, which is at most of the order of several vortex spacings in a type II superconductor near the melting line in a moderate field.

The above agreement with the well-known rotating superfluid hydrodynamics justifies the presence\(^7,8\) of the minimal-coupling in $v_s$ between the phase field $\chi$ and shear displacement $s^T$. Based on this, we assume below that the $v_s^2$ term appearing in the London limit (7) will be found at the static level by summing up many higher LLs in GL harmonic analysis around the mean field state. This should be expected as far as mean field results in London limit are recovered in GL theory. When combining this picture with eqs. (5) and (5)', we are naturally led to invoking the following action\(^15\) appropriate to examining nonlocal conductivities:

\[
S_{nl} = \beta \sum_q \int \rho_0 a|\Omega|(\gamma|\chi_0 \Omega|^2 + \tilde{\gamma}_q r_B^{-2} |s_0 \Omega|^2) + \frac{C_{66}(q_s |\Omega|)}{2} q^\perp_{\perp}^2 |s^T_\perp|^2
\]

\[
+ a\xi_0^2 \rho_0 \int_s \int_\Omega d\tau \left[ (\partial_\perp \chi - r_B^{-2} (\hat{z} \times s^T) - \frac{2\pi}{\phi_0} \delta A^L_\perp)^2 + r_B^{-4} (s^L + B^{-1} (\delta A^T_\perp \times \hat{z}))^2 + \frac{r_B^{-2}}{2} (\partial_s \chi)^2 + (\partial_\perp \chi - \frac{2\pi}{\phi_0} \delta A_\perp)^2 \right],
\]

where $\delta A^L_\perp (\delta A^T_\perp)$ denotes the longitudinal (transverse) part of the gauge disturbance defined within the x-y plane. For later convenience, the shear modulus is assumed to have possible frequency and wavevector dependences, and the time scale of $s$ was phenomenologically changed taking account of a possibility that it may have nonlocal corrections $\tilde{\gamma}_q - \gamma (> 0)$, due to an origin\(^1-2\) other than the freezing to the vortex lattice and irrelevant to statics of vortex states at long distances. The amplitude mode leading to a $\Omega^2 \chi^2$ term was neglected. The term (9) has to be included when considering a nonlocality of Hall conductivity. Again, the variational equation with respect to $s^T$ of the action (10) is different from that of eq. (1) with $\Gamma_1 = 2\pi a_0 \gamma'' B / \phi_0$ due to the presence of nonzero $v_s$. Further, by neglecting the gauge disturbance $\delta A$ and examining the dispersion of the shear mode in the vortex lattice, it is found that, due to the presence of the first term $\sim |\Omega||\chi|^2$ in (10), setting $v_s = 0$ from the outset, as in eq. (1), always leads to an erroneous result on the dispersion. Rather, at low enough $q$, the constraint $v_s \simeq 0$ is established as in eq. (5), and consistently, the second (dynamical) term $|\Omega||s^T|^2$ in (10) becomes unimportant compared to the first term. In the action (10), we assumed the coefficient of $(\partial_\perp s)^2$ term to be the same as that in (5)'. Although this is not correct in the usual London limit, the details of this coefficient do not change main results in the next section. The first term in (10) having the same form as in the lowest LL case (5) is required within TDGL formalism and actually is justified, because the $\chi$ variable to be related to the shear displacement at small but nonzero $q_\perp$ was shown\(^7\) to, up to the lowest order in $q_\perp$, become a phase
change around $\psi_0$ even if higher LLs are fully included. It is clear that this dynamical term associated with the Goldstone (shear) mode is found only by taking account of interactions among vortices. On the other hand, in the derivation in ref.3 of the eq.(1), the origin of contributions at nonzero frequency was attributed to variations of the order parameter in the vicinity of each vortex core which were assumed to be those of a single vortex motion. Consequently, the first (dynamical) term of the action (10) is overlooked in such an extension of the single vortex dynamics to the vortex lattice. As is seen in the next section, due to the presence of this dynamical term, the destruction\cite{7,8} of the true off-diagonal long ranged order is reflected even in dc conductivities under a current perpendicular to $B$.

§3. Nonlocal Conductivities

It is straightforward, using the expressions (4) and (10), to find the nonlocal superconducting contributions to dc conductivities. First, let us discuss the vortex lattice (solid) where $C_{66}(q = 0, \Omega = 0) \neq 0$. In this case the longitudinal ($\| \mathbf{q}_\perp$) part of the diagonal conductivity $\sigma_{xx}$ and the Hall conductivity are given by

\[
\sigma_{xx}^{(s)}(q) = \rho s_0 \frac{r_B^2}{2\xi_0^2} \frac{\gamma q_2^4 + \gamma q_2^2 r_B^2 [q_z^2 + 2 (lq_\perp r_B^2)^2] / 2}{[q_z^2 (1 + q^2 r_B^2 / 2) + l^2 q^2 q_\perp^2 / 2],}
\]

\[
\sigma_{xy}^{(s)}(q) = \rho s_0 \frac{r_B^2}{2\xi_0^2} \frac{\gamma^\prime q_2^2}{(1 + q^2 r_B^2 / 2)[q_z^2 (1 + q^2 r_B^2 / 2) + q^2 q_\perp^2 / 2]},
\]

using the length $l \propto \sqrt{C_{66}}$ defined in §2. In a layered material with mass anisotropy $M/m(>1)$ and under a field perpendicular to the layers, $q_z^2$ appearing in expressions (11) is replaced by $2(1 - \cos(q_z s)) m/M s^2$, where $s$ is the layer spacing. The transverse part $\sigma_{xx}^T$ of the diagonal conductivity merely becomes, even in the liquid regime, $\tilde{\gamma} q \sigma_{xx}(q_\perp = 0, q_z / \gamma$ (using eq.(6)), and hence, in contrast to (11), is not affected by the positional correlation\cite{16} and not relevant to the channel flow situation, proposed in ref.1, where rather $\sigma_{xx}^L$ is measured. We note that the combination $\sim q_z^2 + q_\perp^2 l^2$ seen in denominators of (11) is the dispersion of the 3D shear (Goldstone) mode destroying the true off-diagonal long ranged order.\cite{7,8} In general, the magnitudes of the conductivities (11) significantly depend on the relative size of $|q_z|$ and $|q_\perp|$. For instance, when $|q_z| > l|q_\perp| / r_B \sim |q_\perp|$, both $\sigma_{xx}^{(s)} L$ and $\sigma_{xy}^{(s)}$ are well approximated by $\sigma_{ij}$ (expressions (6)) with replacement $\gamma \rightarrow \tilde{\gamma} q$. On the contrary, when $|q_z| < lq_\perp^2$, $\sigma_{xx}^{(s)} L$ typically becomes the value at zero $q_z$ (or in 2D case); $\gamma \rho s_0 / q_\perp^2 \xi_0^2$, which is the same as the 2D result in zero field.\cite{17} Although this result has also been suggested in ref.5, it is unclear to us whether the authors in ref.5 have distinguished the case $|q_z| > |q_\perp|$ from the case $|q_z| < |q_\perp|$. This divergent behavior is a reflection of the fact that, in this case, the conductivity is primarily determined by the phase variation $\chi$ and not by the so-called vortex motions $\partial_t \mathbf{s} = B^{-1} (\mathbf{E} \times \hat{z})$ implicit in the last term of the action (5)', where $\mathbf{E}$ is the external electric field. Correspondingly, $\sigma_{xy}$ decreases like $\sim (q_z / q_\perp^2)^2$ and vanishes, for arbitrary (nonzero) $q_\perp$, at zero $q_z$ or in 2D case. In other words, as the nonlocality parallel to $B$ becomes negligible compared to that perpendicular to $B$, contributions of a finite $\tilde{\gamma} q$ are covered and overcome by the perfect positional correlation (i.e., the infinite shear viscosity) making the Goldstone mode well-defined. Particularly, these features in $|q_\perp| > |q_z|$ are quite remarkable, because, as mentioned in relation to
the actions (5) and (5)', their origin (i.e., a nonvanishing \( \mathbf{v}_s \)) is of higher order in the wavevector \( q_\perp \) and hence, is usually neglected. On the other hand, these features are lost, at fixed \( q_\perp \) and \( q_z \), as \( l \) becomes shorter, i.e., with increasing field. It is consistent with the absence of the nonlocal conductivity in the vortex lattice constructed within the lowest LL (see a sentence following eq. (6)). Unfortunately, we cannot predict precisely a threshold field below which these nonlocal effects are measurable. In addition, note that, as seen above in \( q_z \to 0 \) limit, the infinite shear viscosity of the vortex lattice does not mean a zero value of the nonlocal resistivity. We emphasize that the remarkable difference between the uniform result (6) and this large conductivity in the case with zero \( q_z \) and small \( q_\perp \neq 0 \) is a consequence of the fact that, at zero \( q_\perp \), the shear displacement \( s^T \) decouples with the zero mode \( \chi \) and changes into the uniform displacement. As is seen by comparing (10) with (5)', the ac conductivities in the vortex lattice, when \( q_\perp \to 0 \), reduces to the vortex flow results (6) except corrections irrelevant to positional correlations. This is a typical example of a crucial difference between ac and dc responses in a kind of ordered state.

The above results that \( \sigma^{(s)L}_{xx} \sim q_\perp^{-2} \) and \( \sigma^{(s)}_{xy} = 0 \) when \( q_z = 0 \) and \( q_\perp \neq 0 \) are valid even for a state with nonzero \( C_{66}(q_\perp \neq 0, |\Omega| = 0) \) such as the 2D hexatic liquid state which may be possible in low enough fields (The 3D hexatic phase invoked in refs.1 and 10 is inconsistent with the first order freezing to the vortex lattice and never realized\(^{18}\)) in real superconductors, and hence need not to be considered). They are also applicable to real systems with pinnings and hence with a finite correlation length \( \xi_h \) of the bond orientational order, if \( q_\perp \xi_h > 1 \). Therefore, it is valuable to examine these dc linear responses in real experiments in relation to determinations of the position of the freezing to the vortex lattice and of the existence or absence of the hexatic phase in 2D systems. In particular, it should be noted that the behavior \( \sigma^{(s)L}_{xx} \sim q_\perp^{-2} \) at nonzero \( q_\perp \) as well as the vanishing \( \sigma^{(s)}_{xy} \) implies \( \rho_{xy} \simeq \rho_{xx}^2 \sigma_{xy,N} \), where \( \rho_{xx}(\rho_{xy}) \) is the total diagonal (Hall) nonlocal resistivity, and \( \sigma_{xy,N} \) is the extrapolated normal Hall conductivity.

When trying to understand the liquid regime (disordered phase) in the present approach, some comments are necessary. As shown in ref.9, the harmonic analysis cannot be used in the situation where the contributions with nonzero reciprocal lattice vectors are negligible even if the amplitude fluctuation of \( \psi \) around \( \psi_0 \) is negligible. Even in such situations the nonvanishing transverse diamagnetic susceptibility results from the static superconducting fluctuation\(^6\) and, in layered systems, shows a dimensional crossover\(^9\) due to a competition between the layer spacing and a finite phase coherence length parallel to \( B \) which cannot be detected in the harmonic analysis. Consistently, this dimensional crossover will be seen also in the nonlocal conductivity. Unfortunately, it is practically difficult at present to find possible nonlocal corrections, associated with the freezing to the vortex lattice, to the vortex flow expression (6) according to the fluctuation theory,\(^6\) but the mean field (i.e., harmonic) analysis may become a guideline to an extension of the analysis in ref.6. In addition, it is possible that the present analysis based on the phase-only model (i.e., based on the mean field solution) will provide an essentially correct result at relatively short scales in directions perpendicular to \( B \) in the disordered state.\(^9\) For these reasons, we will focus on the region just above the freezing point to the vortex lattice and only consider the nonlocal conductivities with vanishing \( q_z \) (see, however, § 4), in which case the dimensional crossover does not appear, and hence, the use of the
harmonic approximation may be permitted.

In this case, the corresponding results to (11) are given by

$$\sigma_{xx}^{(l)}(q_\perp; q_z = 0) = \rho_s \sigma_0 \frac{r_B^2}{2\xi_0} \frac{\tilde{\gamma}_q + \eta q^2 r_B^2 / \rho_0 a}{1 + q^2 r_B^2 (\tilde{\gamma}_q + \eta q^2 r_B^2 / \rho_0 a) / 2\gamma},$$

$$\sigma_{xy}^{(l)}(q_\perp; q_z = 0) = \rho_s \sigma_0 \frac{r_B^2}{2\xi_0} \frac{\gamma^\prime}{1 + q^2 r_B^2 (\tilde{\gamma}_q + \eta q^2 r_B^2 / \rho_0 a) / 2\gamma},$$

where the Maxwell form\(^{19}\) \(C_{66}(q, |\Omega|) \approx \eta|\Omega|\) with shear viscosity \(\eta\) was assumed\(^{20}\) for the dynamical shear modulus. Interestingly, the expressions (12) smoothly lead to the results (11) for the vortex lattice at zero \(q_z\) by taking \(\eta\) to be infinity. The terms \(1 + \eta(q_\perp r_B)^4 / 2\gamma \rho_0 a\) in the denominators of expressions (12) again originates, through a nonzero \(v_s\) (i.e., a higher spatial gradient), from the spectrum of the shear mode \(\omega \sim -i C_{66}(q_\perp r_B)^4\), which cannot be detected in eq. (1), while the nonlocality in the numerator of \(\sigma_{xx}^{(l)}\), resulting from nonlocalities of the time scale for the displacement field, is the same as that expected from (1). As a result, at large \(q_\perp\) of the order \(r_B^{-1}\), both (11) and (12) are well approximated by the usual one (6), suggesting that, for such a rapid variation of the external current, not only collective effects but also the sample disorder (pinning) is irrelevant as far as the time scale to be affected by the sample disorder is \(\tilde{\gamma}_q\) and not \(\gamma\). In general, in the case with a pinning effect enhancing \(\tilde{\gamma}_q\), the viscous effect accompanying the freezing to the vortex lattice becomes negligible.\(^{1}\) Further, the expressions (12) suggest that the low temperature limits of the nonlocal conductivities with nonzero \(q_\perp\) in 2D systems with pinnings, where we have no transitions at nonzero temperatures, again become the corresponding, and above-mentioned, results (independent of \(\tilde{\gamma}_q\)) in the vortex lattice, again leading to the relation \(\rho_{xy} \simeq \rho_{xx} \sigma_{xy, N}\). This relation should also be observed in a clear 3D-like situation with vanishing \(q_z\) and a large \(\eta\). An apparently similar relation was found\(^{21}\) in the pinning-dominated region of BSCCO.

§4. Discussions and Conclusion

We will discuss consequences of possible viscous effects\(^{1,2}\) deep in the liquid regime of pinning free systems, other than \(\eta\), which should appear through nonlocalities of \(\tilde{\gamma}_q\). According to eqs. (11) and (12), when \(|q_z| < |q_\perp|\), it is difficult to practically divide, in \(\sigma_{xx}^{(l)}\), contributions of \(\tilde{\gamma}_q\) from the positional correlation effects, and hence, we will only consider the case with vanishing \(q_\perp\) but nonzero \(q_z\), where the diagonal conductivities can be always expressed by (6) with replacement \(\gamma \rightarrow \tilde{\gamma}_q\). Further we will focus on the 3D region below the dimensional crossover\(^9\) (We note that this dimensional crossover, formally equivalent to that resulting from the entanglement picture,\(^1\) has nothing to do with that argued through experiments in ref.4)). For this case, it is often argued\(^{22}\) that thermally activated cutting (and reconnection) processes among vortices induced by a nonuniform \((q_z \neq 0)\) current will lead to a very long length scale and significantly increase \(\tilde{\gamma}_q\) even if the (thermally-induced) entanglement is absent.\(^2\) This picture seems to suggest that the time scale grows unlimitedly even in the vortex lattice upon cooling, and hence that the vortices bent by the current cannot move at low enough temperatures. It should be noted that such an argument would also be applicable to a nondissipative case (with zero \(\gamma\) but
nonzero $\gamma'$) by imagining a nonlocal growth of $\gamma'$. However, it is even unclear to us if this is a correct argument in the context of linear hydrodynamics. In superfluid $^4$He, the reconnection process does not\textsuperscript{23} seem to need a remarkable slow dynamics. In addition, the algebraic system-size dependence of an onset temperature\textsuperscript{4,24} of apparently nonlocal vortex motions is inconsistent with the argument based on the thermally activated vortex cutting processes in which the temperature variation is dominated by the Arrenius factor, because the measured algebraic size dependence inevitably means an algebraic decrease of the cutting barrier itself with increasing the system-size, although intuitively such a decrease of the barrier should not be expected. Further, the resistivity data\textsuperscript{24} parallel to $B$ in a heavily twinned sample seem to intuitively contradict those in a more 3D-like situation.\textsuperscript{25} Clearly, experiments in twin-free samples are necessary.\textsuperscript{26} Theoretically, there seems to be no acceptable reasons why such a nonlocal growth of $\tilde{\gamma}_q$ has to be expected. Firstly, our analysis shows that, in contrast to the phenomenology in ref.2 and 5, viscous terms possible in $\tilde{\gamma}_q$ cannot reduce to elastic terms in the vortex lattice. Secondly, the (if any) entanglement\textsuperscript{4} making a growth of $\tilde{\gamma}_q$ possible must disappear at least deep in the vortex lattice.\textsuperscript{27} In ref.9, the presence of a large and positive nonlocal ($\sim q_z^2$) contribution in $\tilde{\gamma}_q$ was questioned on the basis of the observation that, in contrast to the pinning-induced activation form for the time scale, any nonlocal (i.e., $q_z$-dependent) growth of the time scale may be incompatible with the uniform vortex flow due to the nonlinearity (i.e., summations with respect to internal $q_z$'s) in Kubo formula in the nonGaussian fluctuation theory.\textsuperscript{5} At the present stage, however, this opinion is not still conclusive.

As already commented in the context, the present analysis cannot clarify a characteristic field below which the positional correlation-induced nonlocal responses studied in this paper and the resulting relation $\rho_{xy} \sim \rho_{xx}^2$ become measurable. In order to estimate this theoretically, it is necessary to know how a nonzero $v_s$, required through the agreement with the rotating superfluid hydrodynamics, is found consistently with the action (9). From the fluctuation theory,\textsuperscript{6} the nonlocality in the numerator of $\sigma_{xx}^{(L)}$ may be expected to be detected as a consequence of renormalizations of the time scale for vortex motions. On the other hand, it is extremely difficult to detect nonlocalities in the denominators of conductivities (12), because their origin consists in the details of physics at short scales which, as mentioned in ref.9, cannot easily be captured in the fluctuation theory from higher temperatures.

In conclusion, a hydrodynamics for vortex states in type II superconductors consistent with the rotating superfluid hydrodynamics has been presented in order to study the nonlocal conductivities perpendicular to the magnetic field. In particular, the conductivities with nonlocality only in perpendicular directions to the field and in a situation with large shear viscosity are essentially the same as those in zero field case, and the vanishing superconducting part of the Hall conductivity under such a situation will be useful in experimentally judging the existence or absence of the 2D hexatic phase in type II superconductors. It is interesting to experimentally examine the predicted relation $\rho_{xy} \simeq \rho_{xx}^2\sigma_{xy,N}$ not only in dirty samples but in a clean sample such as untwinned YBCO.
Acknowledgement

The author acknowledges related discussions with Alan Dorsey, Wai Kwok, and Ulrich Welp, and is grateful to Physics Department, Indiana University for hospitality where this manuscript was written. This research was financially supported by Grant-in-Aid for Scientific Research from the Ministry of Education, Science, and Culture in Japan.

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10) T.Chen and S.Teitel: Phys.Rev.Lett. 72 (1994) 2085. It is clear that, as done by these authors, imposing the London gauge to the gauge disturbance in calculations of linear responses is misleading and has no theoretical foundation, because it would consistently require the absence of any longitudinal component ($\sigma_{xx}^L$) of the dc conductivity which has to exist in order to become consistent with the uniform ($\mathbf{q} = 0$) case (see also a sentence following eqs.(11)). The absence of the longitudinal component of the superfluid density tensor at zero frequency is easily found without any restriction on the gauge disturbance.
11) This means that the elastic terms associated with the compression mode are not affected by the 3D melting transition (see ref.9). It also invalidates a corresponding result $\rho_{s,xx}(q_\perp=0,q_z) \sim |q_z|$ [M. V. Feigel’man and L. B. Ioffe: JETP Lett. 61 (1995) 75] in a putative liquid phase resulting from the boson analogy even if the fluctuating magnetic field is neglected, because the divergent $\chi_{\perp}^{(c)} \sim \rho_{s,xx}(q_\perp=0,q_z)/q_z^2 \sim |q_z|^{-1}$ resulting from it would mean that such an intermediate phase has a stronger ordering than the unpinned vortex lattice in which $\chi_{\perp}^{(c)}$ is finite (see refs.9 and 16). Namely, the intermediate liquid phase with no linear dissipation parallel to $B$ is compatible not with the unpinned vortex lattice but with a pinned vortex lattice with nonzero $\rho_s(\mathbf{q}=0)$ (i.e., $\chi_{\perp}^{(c)} \sim q_z^{-2}$). This is quite consistent with the simulation results of Teitel and coworkers[Phys. Rev. B 49 (1994) 4136], where the unpinned 3D vortex lattice was not found.
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16) Correspondingly, we have two kinds of the transverse parts of diamagnetic susceptibility, closely related to the tilt moduli as mentioned in Ref.9, due to the presence of two (elastic) modes, and they are given by 
\[ \chi^{(s)}_\perp = r_B^2 \rho_s q_z^2 + 2(q_B r_B)^2 + 2l^2 q_z^4 \] 
and 
\[ \chi^{(c)}_\perp \approx r_B^2 \rho_s / 2 \] 
for the shear and compression modes, respectively. Note that the divergent behavior of \( \chi^{(s)}_\perp \) \( q_z \to 0; q_\perp = 0 \) does not contradict the vanishing superfluid density perpendicular to \( B \). Since we have neglected in eqs. (5)’ and (10) an \( O(q_z^2 s^{-1}) \) term playing no significant roles in nonlocal linear dissipations, the longitudinal susceptibility is zero in the present approach.

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20) It may be possible that, as far as the wavevector \( q_z \) is nonzero, the static (zero frequency) shear modulus deep in the liquid regime is nonzero and has the form 
\[ C_{66} \approx q_z^2 r_B^2 f(q) \] 
where \( f(0) = 0 \). Note a similarity between \( \Omega \) and \( q_z^2 \) in the action (10). In this case the susceptibility \( \chi^{(s)}_\perp \) (see Ref.16) is slightly enhanced when \( q_\perp \neq 0 \):
\[ \chi^{(s)}_\perp(q_\perp \neq 0) \approx \rho_s r_B^2 (1 + 2q_z^2 r_B^2 f(q)) / (2 + 2q_z^2 r_B^2 f(q)) \]

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26) According to a preliminary measurement [F. de la Cruz: in Int. Workshop on Vortex Dynamics, Lake Forest, June 1995 (unpublished)], the nonlocal effect on the resistivity just above the melting transition in an untwinned YBCO sample is not seen. This is consistent with ref.11 and with the statement in ref.4 that the nonlocal effect has something to do with the ‘shoulder’ in a resistivity curve due to the twin-boundary pinning.

27) In addition, the picture in ref.1 that the thermally-induced entanglement will become more remarkable with increasing field is questionable, because a situation dominated by the lowest LL mode, in which the entanglement is absent (see ref.9 and M. J. W. Dodgson and M. A. Moore: preprint), must become valid with increasing field.