Experimental and numerical study of the natural convection in dispersed systems in a heated rectangular cell

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Abstract. In this work, we study the appearance and development of convective flows in a dispersed medium by the experimental methods and mathematical modeling. An experimental study of the dynamics of thermoconvective flows is carried out in a specially designed cell, which is a rectangular vertical cavity with the possibility of heating the walls. A series of experiments were carried out at different temperature gradients. Various modes of convective flow have been obtained. To simulate the dynamics of convective flows in a rectangular cavity, a mathematical model based on the equations of thermal convection in the diffusion approximation has been created. Convective flow patterns for a dispersed medium have been obtained. Numerical calculations coincide with the experimental results. Numerical simulation of the problem allows us to estimate the distribution of velocities, temperature, and concentration of the dispersed phase inside the cell during the formation of convective flows and the established convection regime.

1. Introduction

Free convective flows are widespread both in nature and in many technological processes. The presence of convection in the industry can have both positive and negative impacts on various industrial processes [1]. An example of the positive effects of convection is to increase the dissolution rate of impurities during the production of mixtures. The negative impact of free convective flows is manifested, for example, in the growth of crystals. Convective flows lead to defects and inhomogeneities [2].

It is worth noting the influence of convective flows on technological processes in the oil and gas industry. Separation of oil-water emulsions during the preparation of market oil is one of the main tasks in the oil and gas business. Heat treatment of oil is a widely applicable method for the destruction of emulsion systems [3]. The appearance and development of thermoconvective flows, in this case, is inevitable. To apply the method effectively, it is necessary to know the threshold of heat exposure, since with intense heating the speed of convective flows can be so high that it can prevent the separation of the emulsion system [4].

Currently, there are many experimental and theoretical studies of free convective flows [5, 6]. Among the early experimental studies it is worth distinguishing the work by Benard [7], who investigated the occurrence of convection, created by surface tension in a horizontal layer of liquid when heated from below. Major theoretical works on the study of convective flows belong to such authors as Rayleigh, who determined the convection threshold for studying convective stability in a
horizontal fluid layer [8], and G.Z. Gershuni and E. M. Zhukhovitsky who studied the stability and nonlinear convection regimes in various systems [9, 10].

Despite a large number of works devoted to the study of convective flows, there are still many unsolved problems in this area. Therefore, the research and obtaining new knowledge about the dynamics of various media in the conditions of natural convection is relevant.

The purpose of this work is to study the dynamics of dispersed systems in a rectangular vertical cell under conditions of natural convection created by heating its walls using methods of physical (experimental) and mathematical (numerical) modeling. In the first approximation, solid particles suspended in a viscous fluid are considered as a disperse system.

2. Experimental setup and procedure

To study the dynamics of dispersed systems, an experimental cell was developed. It is a rectangular vertical cavity with internal dimensions of 500x50x10 mm (figure 1). Internal vertical walls of the cell are made of polycarbonate, horizontal of copper.

The movement of a coarse-dispersed system, which is based on solid polyethylene particles suspended in a viscous fluid (glycerol), is simulated in the cell. The viscosity of glycerol was measured on a viscometer Brookfield DV-I+ Pro. The average dynamic viscosity was \( \mu = 6.25 \text{ Pa}\cdot\text{s} \), which well agrees with the previous results [11]. The size of the polyethylene particles was determined using an optical microscope Olympus IX71, and the obtained images were processed using ImageJ software.

To prepare the mixture, particles were added to glycerol at a concentration of 1.5% vol. The entire volume was evacuated to get rid of air bubbles formed during the mixing process. The resulting mixture was placed inside the cell. The temperature gradient is maintained by circulating water in copper tubes, and its temperature is set using two thermostats LAUDA Alfa A6 and LOIP LT-117b operating in the temperature range from 20 to 100°C (figure 1).

3. Computational fluid dynamics study

To simulate the results of experimental studies and further study of the effect of natural convection on the dynamics of disperse systems, a mathematical model based on the system of heat convection equations in the Boussinesq approximation [12] with the addition of the diffusion equation, which takes into account the dispersion of the medium [4], was built:

\[
\frac{\partial \bar{u}}{\partial t} + (\bar{u}\nabla)\bar{u} = -\frac{1}{\rho} \nabla p + \nabla (\nabla \cdot \bar{u}) + \tilde{g} (1 - \beta \gamma T)
\]

\[
\rho C_p \left( \frac{\partial T}{\partial t} + (\bar{u}\nabla)T \right) = \lambda \Delta T
\]

\[
\nabla \bar{u} = 0
\]
where $\bar{u}$ is the convection velocity vector of the dispersed system, $p$ is the pressure, $t$ is the time, $g$ is the acceleration of gravity, $T$ is the temperature, and $D$ is the diffusion coefficient. The density of the dispersed medium $\rho$, specific heat capacity $C_p$, thermal diffusivity $\lambda$, and thermal expansion coefficient $\beta_T$ are calculated as additive values depending on the concentration $C$. The kinematic viscosity coefficient $\nu$ is determined from the Einstein’s empirical equation [13]. The use of this approximation is acceptable for most cases of small concentrations of non-deformable particles [14]. The sedimentation rate $\bar{u}_{sed}$ is calculated according to the Stokes law [15]:

$$\bar{u}_{sed} = \frac{2}{9} \rho_0 \frac{g}{\mu_1} \left( \rho_2 - \rho_1 \right)$$

where $\rho_1$, $\rho_2$, $\mu_1$, $\mu_2$ is the density and dynamic viscosity of the dispersion medium, and $\rho_2$, $r_0$ is the density and radius of the particles.

The simulation of the problem is aimed at numerical reproduction of experimental studies and detailed research of the appearance and development of convective flows in a dispersed medium inside the cell. The cell size for the numerical calculations corresponds to the size of the experimental cell containing the dispersed medium. The numerical solution of equations (1) – (4) describing the motion of a dispersed medium inside the cavity was carried out using the control volume method in an open-source CFD software OpenFOAM. Since the velocity of the appeared convective flows is rather small, the laminar flow regime is considered. The third kind of boundary condition is set for the concentration at the horizontal boundaries, which allows compensating for the sedimentation flow and ensures a constant average concentration in the cell, and the second kind of boundary condition is specified on the vertical walls. The fixed temperature at the horizontal boundaries is set for the heat equation. On the vertical walls, the heat exchange with the environment is set. The no-slip velocity conditions are applied at all boundaries.

4. Results

A series of experiments and numerical calculations were carried out to study the dynamics of dispersed systems in a rectangular vertical cell under conditions of natural convection. During the simulation, different vertical temperature gradients were set. The picture of the distribution of dispersed particles in the experimental cell when the temperature of the bottom is 40°C and the temperature of the top is 20°C is presented in figure 2. The given image is one frame from the video obtained with the stationary regime of convection. The arrows indicate the trajectory of the particles. With these experimental parameters, two almost symmetrical ring-type flows arise. It can be seen that the bulk of the particles concentrate in the core of the flow and rise by about 2/3 of the entire height of the cell. In this case, the emerging convective flows prevent the separation of a dispersed medium.

![Figure 2. The trajectory of particles in convective flows at temperatures in the bottom – 40°C and in the top – 20°C.](image_url)
Numerical simulation was carried out based on experimental data. In figure 3, you can see the stages of appearance and development of convective flow with a temperature difference between horizontal boundaries of 20°C. Initially, the environment was at rest. Then, under the influence of temperature, the medium is out of equilibrium (figure 3a). Then convective flows develop (figure 3b). Ultimately, a flow forms in the form of two rings directed to the vertical center of the cell (figure 3c), and does not change over time. The obtained flow patterns and the values of the maximum particle velocity in convective flows are in good agreement with the experimental results (figure 2). The Rayleigh number at the fully developed flow corresponds to the value $Ra = 1.6 \cdot 10^5$.

![Figure 3](image.png)

**Figure 3.** Stages of appearance and development of convective flows: (a) $t = 2 \cdot 10^2$ s, (b) $t = 1.1 \cdot 10^3$ s, (c) $t = 10^4$ s.

The distribution of concentration and temperature under the fully developed convective flows is shown in figure 4. Figure 4a shows that only a small part of the particles are concentrated in the lower corners of the cell, and the bulk of the dispersed phase is concentrated in the core of the convective flow. In the central vertical section of the cell, a rarefied region with a low concentration of particles is formed, which also agrees with the experimental results (figure 2). That is explained by the fact that under these conditions the maximum sedimentation rate of particles $\bar{u}_{\text{sed}} = 5.6 \cdot 10^{-6}$ m/s is two orders of magnitude lower than the value of the maximum velocity of convective flows, equal to $2.3 \cdot 10^{-4}$ m/s. In the region where this condition is broken, the stratification of the dispersed system is observed. Figure 4b shows a significant distortion of isotherms. That is due to the presence of thermoconvective flows, in which the liquid heated from below is carried up along the sides, and the liquid cooled near the top is carried down the center.

![Figure 4](image.png)

**Figure 4.** The distribution of the concentration of the dispersed phase (a) and the temperature distribution (b) in the dispersed medium.

Besides, experimental and numerical studies of the appearance and development of thermoconvective flows under Rayleigh numbers from $8 \cdot 10^4$ to $4 \cdot 10^5$ were carried out. It is worth noting that in this range, the formation of three- and four-vortex structures of convective flows has been detected.
Conclusion
A series of experiments and numerical calculations were carried out to study the dynamics of dispersed systems in a rectangular vertical cell under conditions of natural convection, which occurs when the bottom of the cell is heated, and the top of the cell is cooled. It was shown that at $Ra = 1.6 \times 10^5$, two almost symmetrical ring-type flows appear in the cell. The stages of the origin and development of convective flows, obtained by numerical simulation of the process, are given. It has been established that the rate of convective flows substantially exceeds the value of the sedimentation rate. In this case, the bulk of the particles are concentrated in the flow core. It is noticed that the emerging convective flows impede the separation of the dispersed medium. In the central vertical section of the cell, a rarefied area with a low concentration of particles is formed. The numerical results obtained for different Rayleigh numbers are in good agreement with the results of experimental studies. The mathematical model, considered in the paper, describes quite well the processes in a dispersed system with a low concentration of particles. However, in the conditions of dispersed systems stratification, the mathematical model requires clarification.

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