NUMERICAL SIMULATIONS OF 
ELECTROWEAK BARYOGENESIS AT PREHEATING*

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It has recently been suggested that the baryon washout problem of standard electroweak baryogenesis could be avoided if inflation ends at a low enough energy density and a parametric resonance transfers its energy rapidly into the standard model fields. We present preliminary results of numerical simulations in a SU(2)×U(1) gauge-Higgs model in which this process was studied.

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1 Introduction

In order to explain the observed baryon asymmetry of the universe, a theory must satisfy three conditions:

1. Baryon number violation,
2. C and CP violation and
3. Deviation from thermal equilibrium.

In principle, they all seem to be fulfilled in the electroweak theory and the standard cosmological Big Bang scenario. At high temperatures, baryon number is violated by non-perturbative sphaleron processes, which change the Chern-Simons number and consequently, due to a quantum anomaly, also the baryon number. The electroweak phase transition makes the system fall out of equilibrium in a natural way. The strength of CP violation in the electroweak theory is too small, but even that is not a severe problem because the constraints for CP violation arising from beyond the standard model are fairly weak.

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However, the baryon asymmetry generated in the electroweak phase transition gets easily washed out. Although sphaleron processes become less frequent after the phase transition, they don’t stop completely. Instead, their rate is proportional to \( \exp(-M_{sph}/T) \), where \( M_{sph} \) is proportional to the expectation value \( \phi \) of the Higgs field, and unless \( \phi \) is large enough, the baryon asymmetry is washed out. This can only be avoided if the transition is strongly enough first order so that the discontinuity of the Higgs field is \( \Delta \phi \gtrsim T \).

In the minimal standard model, the Higgs mass \( m_H \) is the only unknown parameter, and lattice simulations have revealed that, whatever its value, the transition is not strong enough. In more complicated models, such as MSSM, there are more unknown parameters and this freedom makes it possible to satisfy the constraint, but only barely.

In an alternative scenario proposed recently by two groups, the baryon asymmetry is generated by sphaleron processes during a period of preheating after inflation. This requires that inflation ends at an energy scale that is below the electroweak scale and that a large fraction of the energy of the inflaton is transferred rapidly to the standard model fields by a parametric resonance. In the resulting non-equilibrium power spectrum, all the fermionic fields and the short-wavelength modes of the bosons are practically in vacuum, but the long-wavelength bosonic modes have a high energy density. The sphaleron rate depends strongly on the temperature of these long-wavelength modes and is therefore very high, and the out-of-equilibrium processes can generate a large baryon asymmetry very quickly. Eventually, the system equilibrates and the effective temperature decreases by a rate that is much faster than the decay rate of baryons. The final temperature \( T_{\text{reheat}} \) is determined by the initial energy density and provided that it is low enough, \( T_{\text{reheat}} \lesssim 0.6T_c \), the sphaleron rate becomes negligible and the baryon washout is avoided.

Simulations of the dynamics of preheating in an Abelian gauge-Higgs system has confirmed this qualitative picture, but they cannot address the issue of baryogenesis directly. In this talk, we discuss the electroweak theory with the full gauge group \( SU(2) \times U(1) \), and argue that reliable simulations are possible using reasonable approximations. We present some preliminary results and discuss the prospect of determining the generated baryon asymmetry using these simulations.

2 Simulations

Let us consider a simple model of inflation, in which the expansion of the universe is driven by the potential energy of a scalar field, the inflaton, rolling slowly down its potential towards its minimum at the origin. Inflation dilutes
away all inhomogeneities and thus all the standard model fields are in vacuum when inflation ends, and the inflaton has a large homogeneous expectation value. It goes on rolling down its potential and starts to oscillate about the minimum of its potential. We assume that it is coupled to the Higgs field and the two fields start to resonate, whereby a large amount of energy is rapidly transferred from the inflaton to the long-wavelength modes of the Higgs. The details of this process of preheating depend on the properties of the inflaton, which are unfortunately unknown. Therefore we simply assume that the energy transfer is extremely efficient and results in a state in which the energy is concentrated in the Higgs modes with very long wavelengths. From the point of view of the microscopic physics that we want to describe, this is practically equivalent to the Higgs having a very large value $\phi_0$. Furthermore, we assume that the effect of the inflaton to the later dynamics of the system is negligible, and therefore we don’t include it in our simulations as a dynamical field. Thus we can simply consider the time evolution of the standard model fields with the special initial conditions in which the Higgs has initial value $\phi_0$ and all the other fields are in vacuum.

As previous simulations have shown, the system will quickly reach a quasi-equilibrium state in which the long-wavelength modes of the Higgs and gauge fields are approximately in equilibrium at a high effective temperature. The decays of the bosons transfer energy slowly into the fermions and gluons, and the effective temperature of the long-wavelength bosonic modes decreases. As the fermions and gluons are initially in vacuum, this process can be described perturbatively by a damping term, whose magnitude $\Gamma \approx 2 \text{ GeV}$ is obtained from the observed lifetime of $W$ and $Z$ bosons. This approximation is valid until $t \sim \Gamma^{-1}$.

Thus, the only fields that we have to consider are the gauge bosons and the Higgs field. For baryogenesis, the relevant degrees of freedom are the long-wavelength modes, and they have large occupation numbers. Therefore they behave classically, and thus we can study the dynamics of the system simply by solving the classical equations of motion

$$
\begin{align*}
\partial_0^2 \phi &= D_i D_i \phi + 2\lambda \left(\frac{1}{2} v^2 - \phi \phi^\dagger\right) - \Gamma \partial_0 \phi,
\partial_0^2 B_i &= -\partial_j B_{ij} + g' \text{Im} \phi \partial_i \phi - \Gamma \partial_0 B_i,
\partial_0^2 W_i &= -[D_j, W_{ij}] + ig \left(\phi (D_i \phi)^\dagger - \frac{1}{2} (D_i \phi)^\dagger \phi - \text{h.c.}\right) - \Gamma \partial_0 W_i,
\end{align*}
$$

where $\phi$ is the Higgs field, $B_i$ is the U(1) gauge field and $W_i$ is the SU(2)
gauge field, and the covariant derivative is

\[ D_i = \partial_i - \frac{i}{2} g W_i - \frac{i}{2} g' B_i. \]  

(2)

In addition, both gauge fields must satisfy the corresponding Gauss laws

\[ \partial_i E_i = g' \text{Im} \pi \phi, \]

\[ [D_i, F_i] = ig \left( \pi \phi^\dagger - \frac{1}{2} \phi \pi - h.c. \right), \]  

(3)

where

\[ \pi = \partial_0 \phi, \quad E_i = -\partial_0 B_i, \quad F_i = -\partial_0 W_i. \]  

(4)

Although the classical equations of motion describe the dynamics of the long-wavelength modes, they fail to describe the early stages of the thermalization, when the quantum fluctuations play an important role. Therefore we approximate them by adding to the initial field configuration Gaussian fluctuations with the same two-point correlation function as in the quantum theory at tree level. For each real field component \( Q \) of mass \( m \) and its canonical momentum \( P \), this means

\[ \langle Q^\star(t, \vec{k}) Q(t, \vec{k}') \rangle = \frac{1}{2 \sqrt{\vec{k}^2 + m^2}} (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}'), \]

(5)

\[ \langle P^\star(t, \vec{k}) P(t, \vec{k}') \rangle = \frac{\sqrt{\vec{k}^2 + m^2}}{2} (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}'). \]

(6)

In a sense, this means that the quantum effects are approximated to leading order in perturbation theory. Just like real quantum fluctuations, these fluctuations generate radiative corrections to the couplings, and they must be taken into account, i.e., the parameters must be renormalized. The situation is made more complicated by the damping term \( \Gamma \), which dampens also the fluctuations and therefore the mass divergence and consequently the mass counterterm also decrease with time. The parameter with the largest radiative corrections in the Higgs mass, and we have calculated the necessary renormalization counterterm at one-loop level in perturbation theory

\[ m_{\text{latt}}^2 \approx m_H^2 - \left( 6\lambda + \frac{9}{4} g^2 + \frac{3}{4} g'^2 \right) \frac{0.226}{h x^2} e^{-\Gamma t}. \]  

(7)

In a similar way, when we plot \( \langle \phi^\dagger \phi \rangle \), the quantity is actually

\[ \langle \phi^\dagger \phi \rangle \approx \langle \phi^\dagger \phi \rangle_{\text{latt}} - \frac{0.452}{h x^2} e^{-\Gamma t}, \]  

(8)

where we have subtracted the dominant ultraviolet divergence.
With these approximations, the dynamics of the system depends on only two unknown parameters, the Higgs mass $m_H$, for which we used the value $m_H = 100$ GeV, and the initial value $\phi_0$, which is constrained by the requirement that when the system equilibrates, it is already deep enough in the broken phase to prevent the washout of the baryon asymmetry by sphalerons. It has been estimated that this requires $\phi \gtrsim T_{\text{reheat}}$, where $T_{\text{reheat}}$ is the final temperature. On the other hand, conservation of energy implies
\begin{equation}
T_{\text{reheat}} \approx \left( \frac{30 \lambda}{g_* \pi^2} \right)^{1/4} \phi_0 \approx 0.2 \phi_0, \tag{9}
\end{equation}
which leads to the constraint
\begin{equation}
\phi_0 \lesssim 600 \text{ GeV}. \tag{10}
\end{equation}
Since we were only interested in the qualitative behaviour and not in precise numbers, we used in the simulations the value $\phi_0 = 700$ GeV, which is slightly larger than the constraint (10).

3 Results

Starting from the initial configuration described above, we solved numerically the equations of motion (1) on a $60^3$ lattice with lattice spacing $\delta x = 3$ TeV$^{-1}$ and time step $\delta t = 0.6$ TeV$^{-1}$. The time evolution of $|\phi|^2$ is shown in Fig. 1. Its qualitative behaviour is similar to that in the Abelian theory. Note that in the presence of fluctuations, $|\phi|^2$ is never zero, but from its qualitative behaviour we can deduce that the electroweak symmetry is effectively restored until $t \sim 0.8$ GeV$^{-1}$. In the absence of a Higgs condensate, the damping term is namely expected to cause $|\phi|^2$ to decrease as $\exp(-\Gamma t)$, which is exactly what we observe. At $t \sim 0.7$ GeV$^{-1}$, $|\phi|^2$ starts to grow towards its vacuum expectation value, just as it is expected to do in the broken phase. During this period of non-thermal symmetry restoration, baryon number is not conserved, and the out-of-equilibrium processes can generate a non-zero baryon asymmetry.

4 Baryon asymmetry

So far, we have concentrated on understanding the qualitative dynamics of the electroweak theory during preheating. However, if we really want to test the scenario of electroweak baryogenesis at preheating, we have to be able to measure the baryon asymmetry generated during the transition, and that involves many technical problems.
Figure 1. The time evolution of $|\phi|^2$ with the initial value $\phi_0 = 700 \text{ GeV}$. The dashed line shows the vacuum expectation value. Until $t \sim 0.8 \text{ GeV}^{-1}$, the curve decreases exponentially, indicating that the Higgs condensate is absent and the symmetry is restored. Eventually, $|\phi|^2$ starts to grow towards its vacuum value, which means that the condensate develops and the symmetry is broken.

In principle, we can measure the change of the baryon number even though we don’t have fermions in our system, because a quantum anomaly links it to the changes of the Chern-Simons number of the SU(2) gauge field

$$\Delta B = 3\Delta N_{CS} = \frac{1}{16\pi^2} \int_0^t dt \int d^3 x \epsilon_{ijk} E_a^i F_{jk}^a. \quad (11)$$

By measuring $E_a^i$ and $F_{jk}^a$, we could then find the change of the baryon number.

However, as $N_{CS}$ is a topological quantity, it does not have a natural definition on a lattice, and attempts to measure $\epsilon_{ijk} E_a^i F_{jk}^a$ in lattice theories have shown that it is dominated by ultraviolet fluctuations. However, at least in thermal equilibrium, it is possible to remove these fluctuations by cooling the system, which leads to a more reliable result.

Another, more serious problem is that in order to generate the observed baryon asymmetry, the theory must violate CP. If this effect arises from heavy degrees of freedom, it can be approximated by an effective term in the La-
$$\Delta L = \frac{\delta_{\text{CP}}}{M_{\text{new}}^2} \phi^\dagger \phi \frac{3g^2}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu},$$  \hspace{1cm} (12)$$

where $M$ is the mass of the heavy fields and $\delta_{\text{CP}}$ parameterizes the strength of the CP violation. Unfortunately, it is very difficult to add this term in the equations of motion, because it is extremely sensitive to ultraviolet fluctuations.

Thus, it seems that the only way to measure the generated baryon asymmetry is to treat $\delta_{\text{CP}}$ as a linear perturbation. One way to do that is to use a Boltzmann-type equation:\textsuperscript{12}

$$\frac{dn_B}{dt} = \frac{\Gamma_{\text{sph}}}{T_{\text{eff}}} \frac{\delta_{\text{cp}}}{M_{\text{new}}^2} \langle \phi^2 \rangle,$$  \hspace{1cm} (13)$$

and measure $\Gamma_{\text{sph}}$, $T_{\text{eff}}$ and $\langle \phi^2 \rangle$ in the simulation. However, this approximation may not always take all the relevant effects into account.\textsuperscript{13}

5 Conclusions

We have studied numerically some aspects of the behaviour of the electroweak theory during preheating and found that it is possible to restore the symmetry non-thermally for a short time, which allows the baryon asymmetry to be generated. When the Higgs and gauge bosons decay into fermions, the temperature decreases so rapidly that this baryon asymmetry does not have time to be washed out.

While the results presented in this talk support the scenario of electroweak baryogenesis at preheating, they are qualitative in nature and do not let us deduce the generated amount of baryon asymmetry. However, more precise simulations are under way.\textsuperscript{14}

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