A New Radar-Embedded Communication Waveform with Low Computational Complexity

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Abstract. Radar-Embedded Communication (REC) is a covert communication technology that utilizes a radio frequency (RF) tag or transponder to embed the communication signal into the radar backscatter. The RF tag must be triggered by the radar signal and quickly generate the required communication waveforms so as to ensure that these communication symbols can be timely and effectively embedded in the radar backscatter pulse. Such an operation poses a great challenge to the response speed of the tag transponder. The traditional method is based on multiple eigenvalue decompositions to generate quasi-orthogonal communication waveforms, which requires a large amount of calculation. This paper proposes a strategy that requires only one eigenvalue decomposition operation. However, such a strategy makes it possible to generate completely orthogonal communication waveforms, and at the same time, it will contribute to a great reduction in the computational pressure of the RF tag. In addition, by simulation analyses, the communication waveform design strategy proposed in this paper does not take the reliability and the low probability of interception (LPI) of the waveform as the cost. On the contrary, under some conditions, communication reliability and LPI characteristics can be improved.

1. Introduction

The utilization of wireless communications in the military may face two challenges in the future [1,2]. First, due to the openness and broadcast nature of traditional wireless communications technologies, electromagnetic signals are extremely easy to be captured, identified, and interfered [3]. In a war, a large number of key information such as instructions and intelligence are transmitted through wireless communication, which can easily cause information leakage. Second, in future information warfare, there will be massive amounts of data for command transmission and intelligence sharing. How to improve the usage efficiency of spectrum resources is also a problem that military communications must face in the future [4,5].

In 2013, the US Defense Advanced Research Projects Agency (DARPA) launched a research project named Shared Spectrum Access for Radar and Communications (SSPARC) [6], which aims to develop spectrum sharing technology between military radars and military communication systems. As one of the strong bidders for the SSPARC project, REC can meet the concealment requirements of military communications while realizing the shared spectrum of radar and communications [7]. The concept of REC was first proposed by Professor Shannon of the University of Kansas in 2007 [8,9]. [8] first established the system modeling of REC and proposed three algorithms for REC waveform design: Eigenvectors-as-Waveforms (EAW), Weighted-Combining (WC), and Dominant
Projection (DP). In addition, [8] also proposed two REC receiving methods: matched filter (MF) and decorrelation filter (DF) for REC cooperative receiver receivers. In 2011, [10,11] proposed a two-stage REC receiving method based on the Neyman-Pearson (NP) criterion, which can realize the constant false alarm rate (CAFR) detection of REC communication signals. In 2015, [12] proposed two new REC waveform design methods: Shaped Dominant Projection (SDP) and Shaped Water Filling (SWF). The reliability and anti-interception performance of these two waveform design methods are analyzed in detail, which provides a reference for REC waveform design and selection. In 2020, [13] proposed an Inverse Shaped Dominant Projection (ISDP) waveform design method, which reduces the interference of communication to the radar system.

Figure 1 shows the working principle of REC [7,8]. First, the REC working area is illuminated by cooperative or non-cooperative radar, and both friendly targets and cooperative receivers can receive radar signals. Secondly, the friendly target carries a radio frequency (RF) tag that can perceive radar signals. It can process the radar signals and generate concealed communication signals, which are sent synchronously with radar scattered echoes. The local scattered echo around the tag will serve as a hidden carrier for the communication signal. Finally, the cooperative receiver extracts and restores the communication signal to complete the covert communication. In addition, there may be an intercepted receiver to detect the communication signal.

The advantage of REC is that it does not need to modify the existing radar system, but only needs to add a simple RF tag and a covert communication system can be built. RF tag is required to be small and easy to move, and may even be passive, which is destined to greatly restrict the computing power of RF tag. But the REC system also requires RF tag to be able to embed communication signals quickly and accurately to ensure the effectiveness of covert communication, which puts forward requirements on the computing power of RF tags [14]. In the previous methods of communication waveform design, reliability and anti-interception performance were mainly considered as key optimization goals, while algorithm complexity was not an important indicator to focus on. This paper takes the complexity of the communication waveform generation algorithm as the main optimization goal. Based on the traditional DP algorithm, a REC waveform design algorithm based on Extraction Dominant Projection (EDP) is proposed, which can greatly reduce the complexity of communication waveform generation, thereby alleviating the computational pressure of the RF tag. In addition, this paper also analyses the reliability and anti-interception performance of the EDP waveform and compares it with the traditional DP waveform.

![Figure 1 REC working principle diagram](image)

**2. System model**

Figure 2 shows the channel model of REC, and its signal transmission path can be divided into forward link and backward link. The radar illuminates the REC working area through the forward link,
and the environment scatters the radar signal to produce backscattered echoes, which will become the hidden carrier of the communication waveform. The RF tag carried by the friendly target will collect and process the radar signal to generate a communication waveform with LPI performance, which is then mixed in the radar backscattered echo and sent to the cooperative receiver. The signal received by the cooperative receiver is a mixed signal of radar echo, REC communication signal and noise:

\[ r(t) = s(t) * p(t) + \alpha_k c_k(t) * h(t) + n(t), \]  

(1)

Figure 2 REC channel model

where \( r(t) \) represents the mixed signal received by the cooperative receiver, \( s(t) \) represents the radar signal, \( p(t) \) represents the environmental scattering sample or clutter, and \( h(t) \) represents the channel multipath response. \( c_k(t) \) represents the \( k \) embedded communication waveform, \( \alpha_k \) is the power constraint factor of the waveform \( c_k(t) \), and \( n(t) \) is the environmental noise. Note that the radar backscattered echo signal is modelled as the convolution of the radar signal \( s(t) \) and clutter \( p(t) \).

Define \( N \) as the number of sampling points of the radar signal satisfying the Nyquist sampling criterion, and \( M_c \) as the oversampling factor. The radar signal is sampled and discretely represented as

\[ s = [s_1, s_2, s_3, \ldots, s_{NM_c}], \]  

(2)

where \( s_1, s_2, s_3, \ldots, s_{NM_c} \) is the over-sampling data of the radar signal. The Toeplitz matrix is constructed by cyclic shifting \( s \) as

\[
S_b = \begin{bmatrix}
    s_{NM_c} & s_{NM_c-1} & \cdots & s_1 & 0 & \cdots & 0 \\
    0 & s_{NM_c} & \cdots & s_2 & s_1 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & s_{NM_c} & s_{NM_c-1} & \cdots & s_1
\end{bmatrix},
\]  

(3)

where \( S_b \in \mathbb{C}^{NM_c \times (2NM_c - 1)} \). Then the convolution process \( s(t) * p(t) \) can be discretely expressed as
\[
S_b \cdot p = \begin{bmatrix}
    s_{NM_1} & s_{NM_2} & \cdots & s_1 & 0 & \cdots & 0 \\
    0 & s_{NM_1} & \cdots & s_2 & s_1 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & s_{NM_1} & s_{NM_2} & \cdots & s_1
\end{bmatrix} \cdot p, \quad (4)
\]

where the vector \( p \) is the discretization of the backscattered sample \( p(t) \). Therefore, the formula (1) can be discretized as

\[
r = S_s \cdot p + \alpha_s c_s + n, \quad (5)
\]

where \( c_s, n \in \mathbb{C}^{NM_c} \), which respectively represent the sampling values of communication signals and environmental noise.

The RF tag needs to generate a set of communication waveforms related to the radar backscattered echo in order to obtain the LPI performance, so it needs to extract the features of the radar backscattered echo firstly. The method is to perform eigenvalue decomposition on \( S_b \)

\[
S_b S_b^H = Q \Lambda Q^H, \quad (6)
\]

where \( Q \in \mathbb{C}^{NM_c \times NM_c} \) is a unitary matrix, \( \Lambda = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_{NM_c}) \) is a diagonal matrix, \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{NM_c} \geq 0 \) and \( Q^H \) represents the conjugate transpose of \( Q \).

According to the size of eigenvalues, \( Q \) is further divided into dominant space and non-dominant space as

\[
S_b S_b^H = [Q_D \quad Q_{ND}] \begin{bmatrix}
    A_D & O \\
    O & A_{ND}
\end{bmatrix} [Q_D^H \quad Q_{ND}^H], \quad (7)
\]

where \( A_D \in \mathbb{C}^{M \times M} \) is a diagonal matrix composed of the first \( M \) larger eigenvalues, and \( A_{ND} \in \mathbb{C}^{(NM_c-M) \times (NM_c-M)} \) is a diagonal matrix composed of the last \( NM_c - M \) smaller eigenvalues. \( Q_D \in \mathbb{C}^{NM_c \times M} \) is the dominant space composed of the first \( M \) eigenvectors, and \( Q_{ND} \in \mathbb{C}^{NM_c \times (NM_c-M)} \) is the non-dominant space composed of the last \( NM_c - M \) eigenvectors.

3. Communication waveform design

The purpose of waveform design is to construct a set of \( K \) waveforms, each of which can represent \( \log_2 K \) bit binary information. In this section, we will first introduce the traditional DP waveform generation algorithm, and then design an EDP waveform generation algorithm with low computational complexity based on the DP algorithm.

3.1. DP algorithm (Dominant projection)

The DP method considers the projection method to generate communication waveforms. The algorithm process is as follows:

**Step1:** Generate the projection matrix as follows:

\[
P_l = Q_{ND} Q_{ND}^H, \quad (8)
\]

where \( P_l \in \mathbb{C}^{NM_c \times NM_c} \) represents the first projection matrix, and \( I_{NM_c} \in \mathbb{C}^{NM_c \times NM_c} \) is the identity matrix.

Multiply the projection matrix \( P_l \) and the column vector \( d_l \) known by the RF tag and receiver to get the first communication waveform as
\[ c_{DP,1} = \beta_{DP}^{1/2} P d_i \]
\[ = \beta_{DP}^{1/2} Q_{ND}^\dagger q_i, \]  \( (9) \)
where \( c_{DP,1} \in \mathbb{C}^{NM_e \times 1}, \quad d_i \in \mathbb{C}^{NM_e \times 1}, \quad q_i \in \mathbb{C}^{(NM_e-m) \times 1} \) and
\[ |d_i| \approx |q_i| \approx \frac{1}{NM_e}. \]  \( (10) \)

Thus \( \beta_{DP} \) is set to \( \frac{NM_e}{NM_e - m} \) to ensure that \( c_{DP,1} \) has unit energy.

**Step2:** Add \( c_{DP,1} \) to matrix \( S_h \) to form a new matrix
\[ S_{p1} = [S_h \ c_{DP,1}], \]  \( (11) \)
where \( S_{p1} \in \mathbb{C}^{NM_e \times 2NM_e} \), and the eigenvalue decomposition is also performed as
\[ S_{p1}^\dagger S_{p1} = Q_{p1} A_{p1} Q_{p1}^\dagger, \]  \( (12) \)
where \( A_{p1} \in \mathbb{C}^{NM_e \times NM_e}, \quad Q_{p1} \in \mathbb{C}^{NM_e \times NM_e} \). Similar to equation (7), we can divide \( Q_{p1} \) into dominant space and non-dominant space, then the new projection matrix is
\[ P_2 = Q_{p1,ND} Q_{p1,ND}^\dagger, \]  \( (13) \)
where \( Q_{p1,ND} \in \mathbb{C}^{NM_e \times (NM_e-m-1)}, \quad P_2 \in \mathbb{C}^{NM_e \times NM_e} \). The second REC communication waveform is constructed as
\[ c_{DP,2} = \beta_{DP}^{1/2} P_2 d_2 \]
\[ = \beta_{DP}^{1/2} Q_{p1,ND}^\dagger q_2, \]  \( (14) \)
where \( c_{DP,2} \in \mathbb{C}^{NM_e \times 1}, \quad d_2 \in \mathbb{C}^{NM_e \times 1}, \quad q_2 \in \mathbb{C}^{(NM_e-m) \times 1} \) and
\[ |d_2| \approx |q_2| \approx \frac{1}{NM_e}. \]

**Step3:** Generate \( K \) REC communications waveforms sequentially according step 1 and step 2.

The above is the generation process of the DP waveform. It can be seen that the DP waveform requires multiple eigenvalue decomposition to ensure the orthogonality between the communication waveforms (in fact, it is not completely orthogonal), which has a high computational complexity.

### 3.2. EDP algorithm (Extraction Dominant Projection)

Based on the DP algorithm, we construct the REC communication waveform by extracting the Eigenvectors of the dominant space. The algorithm steps are as follows:

**Step1:** Assuming that the \( NM_e - m \) eigenvectors of the non-dominant space are \( Q_{ND} = [q_1, q_2 \ldots q_{NM_e-m}] \). The number of communication symbols that need to be constructed is \( K \), then the sub-matrix of \( Q_{ND} \) is constructed by uniformly extracting the eigenvectors according to the following rules:
\[ Q_{\text{ND}}^{[1]} = [q_1, q_{1+K}, \cdots, q_{1+(L-1)K}] \]

\[ Q_{\text{ND}}^{[2]} = [q_2, q_{2+K}, \cdots, q_{2+(L-1)K}] \]

\[ \cdots \]

\[ Q_{\text{ND}}^{[K]} = [q_K, q_{K+K}, \cdots, q_{K+(L-1)K}] \]

(15)

where \( L \) represents the number of feature vectors in \( Q_{\text{ND}}^{[k]} \), which can be calculated by the following formula

\[ L = \left\lfloor \frac{NM_c - m}{K} \right\rfloor, \]

(16)

where \( \lfloor \cdot \rfloor \) represents the round-down operation.

Further, the formula (15) can be rewritten as

\[ Q_{\text{ND}}^{[k]} = [q_k, q_{k+K}, \cdots, q_{k+(L-1)K}], \]

(17)

where \( Q_{\text{ND}}^{[k]} \in \mathbb{C}^{NM_c \times L} \) and \( k = 1, \cdots, K \).

**Step2:** According to formulas (8) and (17), projection matrices can be constructed as follows

\[ P_k = Q_{\text{ND}}^{[k]} Q_{\text{ND}}^{[k]T}, k = 1, 2, \cdots, K. \]

(18)

**Step3:** \( K \) REC communication waveforms can be constructed as follows

\[ c_{\text{EDP},k} = \beta_{\text{DEP}}^{1/2} P_k d \]

\[ = \beta_{\text{DEP}}^{1/2} Q_{\text{ND}}^{[k]} Q_{\text{ND}}^{[k]T} d \]

\[ = \beta_{\text{DEP}}^{1/2} Q_{\text{ND}}^{[k]} q_{\text{ND},k}, k = 1, 2, \cdots, K \]

(19)

where \( c_{\text{EDP},k} \in \mathbb{C}^{NM_c \times 1} \), \( d \in \mathbb{C}^{NM_c \times 1} \), \( q_k \in \mathbb{C}^{L \times 1} \). And

\[ |d_{k,j}|^2 \approx |q_{k,j}|^2 \approx \frac{1}{NM_c}, k = 1, 2, \cdots, K, \]

(20)

Compared with the DP algorithm, the EDP algorithm only needs the tag and receiver to save a random vector \( d \), while the DP algorithm needs to save \( K \) random vectors. The EDP method is simpler and more practical than the DP algorithm. And since the eigenvectors contained in each projection matrix are not repeated, the EDP waveforms are completely orthogonal.

The above are the steps of the EDP waveform generation algorithm. After experiments, the number of eigenvectors contained in the sub-matrix \( Q_{\text{ND}}^{[k]} \) must be greater than 5, that is, \( L > 5 \), so as to ensure that the EDP waveform has no distortion in the spectrum. Otherwise, the EDP waveform will have obvious spikes on the frequency spectrum and cause it to lose LPI performance.

We choose the common linear frequency modulation (LFM) radar signal to construct the EDP waveform, the parameter is set to \( N = 100, M_c = 2, m = 100, L = 5 \) and the noise is Gaussian white noise, as shown in Figure 3 is the power spectrum of the EDP waveform. In addition, we have drawn the power spectrum of DP waveform, radar signal, and direct sequence spread spectrum (DSSS) symbol for comparison.

It can be seen that the energy distribution of the EDP waveform and the DP waveform in the frequency domain is almost the same, and is mainly distributed on the bleeding spectrum of the radar,
while the DSSS symbol power is evenly distributed in the frequency domain. The similarity of the energy distribution of the EDP and DP waveforms can be expected that the reliability and LPI performance of the EDP waveform will not be severely degraded (but the computational complexity is reduced).

![Figure 3](image_url)  
Figure 3  the power spectrum of the Radar, DP, EDP and DSSS waveform

4. Performance analysis

4.1. Computational complexity

For the DP waveform generation algorithm, it needs to perform $K$ eigenvalue decompositions, generate $K$ projection matrices $P_k$ from equation (8), and multiply the $N M_c \times N M_c$ dimensional matrix $P_k$ and the $N M_c \times 1$ dimensional random vector $d$ $K$ times. Therefore, the computational complexity of DP algorithm is

$$T_{DP} = O\left(K\left(NM_c\right)^3 + K\left(NM_c\right)^2 + K\left(NM_c\right)^3(NM_c - m)\right),$$  
$$= O\left(2K\left(NM_c\right)^3 - Km\left(NM_c\right)^3\right),$$  
(21)

For the EDP waveform generation algorithm, it also needs to generate $K$ projection matrices $P_k$ from equation (18), and multiply $P_k$ and the random vector $d$ $K$ times. The difference is that it only needs to perform eigenvalue decomposition once. Therefore, the computational complexity of the EDP algorithm is

$$T_{EDP} = O\left(\left(NM_c\right)^3 + K\left(NM_c\right)^2 + K\left(NM_c\right)^3 L\right),$$  
$$= O\left(\left(NM_c\right)^3 + K\left(L + 1\right)\left(NM_c\right)^2\right),$$  
(22)

From the formula (16), we can see that $K\left(L + 1\right) \approx NM - m$. Therefore, (22) can be further simplified as

$$T_{EDP} \approx O\left(\left(NM_c\right)^3 + \left(NM - m\right)\left(NM_c\right)^2\right),$$  
$$= O\left(2\left(NM_c\right)^3 - m\left(NM_c\right)^2\right),$$  
(23)

From formula (21) and (23), we can get

$$T_{DP} = KT_{EDP},$$  
(24)
Therefore, the complexity of the EDP algorithm is reduced by $K$ times compared with the DP algorithm, which can greatly reduce the calculation of the RF Tag and is very effective in the actual REC system. Because it can greatly reduce the response time of the RF tag to improve the embedding accuracy of the communication waveform, thereby improving the LPI performance.

### 4.2. Detection and LPI performance

The cooperative receiver uses the NP receiver in [10], which is a two-stage detection CFAR receiver, and its structure is shown in Figure 4.

![Figure 4 The structure of NP detector](image.png)

The cooperative receiver performs detection asynchronously within the sampling length of $r(n) = \begin{bmatrix} r(n) & r(n-1) & r(n-NM_c+1) \end{bmatrix}$. $w_k$ is the receiving filter and here we use a decorrelation filter (DF):

$$w_{DF,k} = (S_b S_b^H)^{-1} c_k, k = 1, 2, \cdots, K.$$  

(25)

The output of the filter bank is used to calculate the decision threshold to achieve CFAR detection. On the other hand, the maximum value of the filter output will be used for comparison. If the maximum value exceeds the detection threshold, it means that a waveform is embedded and the judgment and decoding will be executed. Otherwise, it will be regarded as no communication symbols exist, and decoding will not be performed.

For the interception receiver, we use the energy detection method to intercept the communication symbols [11] and use the probability of successful detection by the interception receiver to measure the LPI performance. Assuming that the intercepting receiver already knows the oversampling factor, the time-bandwidth product, and the size of the dominant space, this is a relatively bad situation. The interception receiver first projects the intercepted signal to the non-dominant space, and then performs energy detection, namely:

$$\varepsilon = r^H P_m r,$$  

(26)

where $P_m = Q_{ND,m} Q_{ND,m}^H$ and $\varepsilon$ is the energy output of the intercepted receiver. Similarly, the intercepting receiver will detect within the sampling length of $r(n) = \begin{bmatrix} r(n) & r(n-1) & r(n-NM_c+1) \end{bmatrix}$. When the output of the energy detector exceeds the detection threshold, it is considered that the receiving receiver has successfully detected the existence of the communication symbol.

We also use the chirp signal as the radar signal, the parameter is set to $N = 100, M_c = 2, L = 5$, the false alarm probability of the NP receiver and the intercepted receiver is set to $P_f = 10^{-5}$ and the CNR (clutter-to-noise ratio) is set to CNR = 30dB. The noise is Gaussian white noise and the number of communication symbols is $K = 4$. Figure 5-7 compares the detection probability and interception probability of the DP symbol and the EDP symbol under the condition of the dominant space size of $m = 50$, $m = 100$ and $m = 150$, and the abscissa is the SNR (signal-to-noise ratio).
Figure 5 is the curve of detection probability and interception probability when the size of the dominant space is $m = 50$. It can be seen that when the SNR is lower than 2dB, the detection probability of the EDP symbol is better than that of the DP waveform. But when the SNR is higher than 2dB, the detection probability of the EDP symbol is lower than that of the DP waveform, and the reliability performance is degraded. As for the LPI performance, it can be seen that the probability of interception of EDP symbols is always better than that of DP symbols, and the higher the signal-to-noise ratio, the greater the advantage of LPI.

Similarly, for the size of dominant space is $m = 100$, when SNR < 1dB, the detection performance of EDP symbols is better than that of DP symbols, and when SNR > 1dB, the detection probability of EDP symbols is lower than that of DP symbols. For LPI performance, the intercept probability of EDP symbols is always lower than that of DP symbols, and as the SNR increases, the advantage of LPI is greater.

For $m = 150$, as shown in Figure 7, it can be seen that the detection probability of EDP symbols is always better than that of DP symbols, which means that EDP symbols have better reliability performance. For LPI performance, it can be seen that the intercept probability of EDP symbols is slightly lower than that of DP symbols as a whole. This situation is a relatively satisfactory result because the communication reliability and LPI performance are relatively stable, and both are better than the original DP waveform.

![Figure 5](image1.png) **Figure 5** Probability of detection and intercept for DP and EDP waveform. $m = 50$

![Figure 6](image2.png) **Figure 6** Probability of detection and intercept for DP and EDP waveform. $m = 100$

![Figure 7](image3.png) **Figure 7** Probability of detection and intercept for DP and EDP waveform. $m = 150$

![Figure 8](image4.png) **Figure 8** SER performance of DP and EDP waveforms. $m = 150$
4.3. SER Performance

In Section 4.2, we simulated and analyzed the detection performance of EDP waveform under NP detector. We found that the detection probability of EDP waveform is greater than that of DP waveform under the condition that the size of the dominant space is \( m = 150 \). As shown in figure 8, we further simulate the relationship between the SER and the SNR of the EDP waveform. The parameter settings are the same as in section 4.2. We can see that under the same SER conditions, the SER of the EDP symbol is always lower than that of the DP symbol and has a gain 0–1 dB, which is consistent with the conclusion in Section 4.2.

5. Conclusion

Based on the DP algorithm, this paper proposes a new low-complexity REC communication waveform generation algorithm—EDP algorithm. First of all, compared with the classic DP waveform generation algorithm, the EDP algorithm can reduce the computational complexity by \( K \) times. In practice, the reduction in complexity can relieve the computational pressure of the RF tag, shorten the response delay of the system, and improve the time accuracy of symbol embedding, thereby providing a guarantee for the LPI performance of REC. Second, the simulation results show that when the main space size is selected as a relatively large value (\( m = 150 \)), the overall communication reliability of EDP symbols is better than that of DP symbols, the SER gain is 0–1 dB. Finally, when the main space size is \( m = 150 \), the LPI performance of the EDP symbols is also slightly enhanced.

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**Acknowledgments**

This work was supported by the Program for Innovative Research Groups of Hunan Provincial Natural Science Foundation of China under Grant 2019JJ10004.