A Low Complexity joint synchronization algorithm based on CAZAC sequence for OFDM system

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Abstract. To solve the problem of timing ambiguity and reduce the complexity of integer frequency offsets (IFO) estimation for OFDM system, this paper proposes a novel synchronization algorithm based on CAZAC sequence in time domain. The algorithm constructs a training sequence with conjugate and symmetry properties. Firstly, timing synchronization and IFO estimation are completed simultaneously by utilizing the cross-correlating of the training sequence and local signal based on the analysis of the impact of IFO on the timing of metric peak positions. Then the fractional frequency offsets (FFO) estimation is done using the repeatability of the symmetrical training sequence. Compared with the existing representative algorithms, the proposed algorithm achieves better synchronous performances with lower computational complexity and acceptable transmission efficiency.

1. Introduction
Orthogonal frequency division multiplexing (OFDM) has been extensively adopted in wireless communication due to its high spectrum efficiency and excellent performance in anti-multipath fading. Meanwhile, it’s very sensitive to synchronization errors[1-2]. Therefore, time and frequency synchronization should be paid much attention to.

In recent years, some kinds of synchronization algorithms in OFDM have been proposed, including auto-correlation based approaches, weighted based approaches and cross-correlation based approaches. S&C considered a training sequence consisting of two identical parts to reduce the multipath fading effect, but the presence of the CP causes a plateau of timing peaks[3]. Minn’s method reduced timing estimation variance by smoothing the timing metric values within the prefix length, but still had the problem of timing ambiguity in multipath channels[4]. Park designed a preamble structure with conjugate and symmetry properties, which effectively improved the sharpness of the timing peak, but there still existed two subpeaks[5]. Inspired by these typical algorithms,a kind of hybrid approaches combing delay and symmetric auto-correlation were proposed, they searched the first path of multi-path channel to achieve accurate timing by threshold detection[6-7], but large-scale detection causes too much calculation. Ren introduced a weighted correlation operation to eliminate the peak plateau caused by CP, but increased the difficulty of FFO estimation. Fan analyzed the relationship between timing sidelobe sharpness and weighted factor, and proposed an average weighted auto-correlation timing scheme to avoid the “plateau effect”[9]. In [11], the author proposed a novel algorithm based on CAZAC sequence weighted by a new factor, which had no subpeaks or timing platform but doubled calculation compared with pseudorandom noise (PN) weighted algorithms. Malik designed a preamble with two conjugate ZC sequence, although it further enhanced the synchronization performance, the transmission efficiency was lower due to using two OFDM symbols, and timing performance was largely determined by the accuracy of IFO estimation.
The complexity is also an important issue that synchronization algorithms should consider[1], therefore, the existing problem is how to balance the synchronization performance, transmission efficiency and computational complexity in a low signal-to-noise (SNR) multipath channel as existing algorithms can already achieve accurate timing in good channel environment. In order to solve above problems, we design the training preamble with only one symbol consisting of four different CAZAC sequences conjugate and symmetric to each other. Then we propose a time-domain timing and frequency offset estimation scheme that can accomplish accurate and stable timing and obtain IFOs simultaneously. It can also overcome the range limitation of FFO estimation.

2. System Model For OFDM Signal
In OFDM baseband transmission system, the time-domain sequence generated by N-points IFFT transform of frequency data symbols can be expressed as:

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp \left( j \frac{2\pi kn}{N} \right), -N \leq n \leq N - 1 \]  

(1)

Where \( X(k) \) denotes the data in frequency-domain transmitted on the k-th subcarrier, \( n \) refers to the sampling index, \( N_s \) is the length of CP attached in front of each OFDM symbol to protect the receive symbols from ISI and ICI caused by the overlap of preceding OFDM symbol.

In time-varying multipath channels, the n-th sample of the received sequence is given as:

\[ r(n) = e^{j \frac{2\pi}{N} n} y(n - \delta) + w(n), 0 \leq n \leq N - 1 \]  

(2)

Let \( \epsilon \) represent Normalized carrier frequency deviation caused by oscillator instability between receiver and transmitter, and \( \delta \) denote the normalized time delay in sampling cycles. \( w(n) \) is the n-th additive white gaussian noise (AWGN) sample. Here we focus signal transmission in multipath channel, so in addition to noise, there also exists multipath effects. Then \( y(n) \) can be written as:

\[ y(n) = \sum_{l=0}^{L} h(l)x(n-l), n = 0,1,2\ldots N - 1 \]  

(3)

Where \( h(l) \) is the normalized average power of the L-th path, \( l \) is the normalized non-sample-spaced delay of the l-th path. In this paper, we assume that \( L < N_g \), where \( L \) represents the maximum delay.

2.1. The Auto-Correlation Timing-Metric
The A1-Scheme illustrated in [2] utilized the preamble \( P_{\text{a1}} = [A_{N/2} A_{N/2}] \). It considered the sum of \( N/2 \) products of \( r_m \) and \( r_{m+N/2} \) as timing metric function that were suitable for the channel environments with large carrier frequency offset and large delay. At the point \( d \), it can be denoted as :

\[ P_{a1}(d) = \sum_{m=0}^{N/2-1} r_m^* (d+m) r(d+m+N/2) \]  

(4)

There is another typical timing method based on symmetric auto-correlation, such as the one proposed by Park. In order to eliminate the peak platform caused by the repeatability of the prefix and training sequence, he designed a novel preamble \( P_{\text{pno}} = [A_{N/4} B_{N/4} A_{N/4}^* B_{N/4}^*] \) and timing method as:

\[ P_{a2}(d) = \sum_{m=0}^{N/4-1} r_m^* (d-m) r(d+m) \]  

(5)

2.2. The Weighted-Sequence Timing-Metric
Compared with PN sequences, CAZAC sequences have better auto-correlation characteristics. Therefore, in order to further improve the performance of the synchronization algorithm, paper [8] introduced CAZAC sequence to construct the preamble, structurally consistent with the literature [3]. Moreover, it used PN sequence to take weighting operation to break the repeatability of the prefix and CAZAC sequences, as \( P_{\text{wz}} = [A_{N/2} \cdot S_{N/2} \quad A_{N/2} \cdot S_{N/2}^*] \), and \( S_{N/2} \) take the value +1 or -1. The proposed time-meric in [8] is given as:
Based on the above methods, [11] inherited Park’s preamble structure, the difference is that \( B_{N/4} \) and \( A_{N/4} \) also have a conjugate relationship. Moreover, another weighted factor \( \nu(n) = \exp(-j \frac{2\pi u_n^2}{N}) \) was introduced. So the timing Metric function is given by:

\[
P_{k}(d) = \sum_{m=0}^{N/4-1} r'(m)(d+m+N/4)r'(d+m+d+N/4)\]

As described in [11], CAZAC sequence is defined as:

\[
c(n) = \exp(j \frac{\pi u_n^2}{N}), 0 \leq n \leq N \]

Where \( u \) is an integer that is relative-prime to \( N \), usually takes 1 or \( N-1 \).

In the noiseless and no CFOs case, the formula (7) can be rewritten as:

\[
P_{k}(d) = \sum_{m=0}^{N/4-1} \exp(-j \frac{\pi u_m^2}{N}) \exp(-j \frac{\pi u_m^2}{N}) \exp(j \frac{\pi u_m^2}{N}) \exp(j \frac{\pi u_m^2}{N}) \exp(j \frac{\pi u_m^2}{N}) \exp(j \frac{\pi u_m^2}{N}) \]

Obviously, \( P_{k}(d) \) have only a peak value when \( d \) is zero, which indicates that this method can more effectively eliminate side lobes. Meanwhile, it also need double complex multiplication calculations.

2.3. The Cross-Correlation Timing-Metric

In [13], Malik analyzed the influence of ZC sequence’s root index on time synchronization in the presence of IFO, and designed a preamble with CP and CS, \( P_{\text{Meik}} = \{C_{j}, C_{j} \} \), \( C_{j} \) is Zadoff-Chu (ZC) sequence. The cross-correlation between local training ZC sequence and received data samples is the timing metric function as follow:

\[
P_{c}(d) = \sum_{n=0}^{N-1} y(n+d)x^*(n) \]

Substituting equation (3) into equation (10), then

\[
P_{c}(d) = \sum_{n=0}^{N-1} y(n+d)x^*(n) \]

According to the analysis of IFO’s effect in [13], equation (12) can be further simplified as:

\[
P_{c}(d) = \sum_{d=0}^{N/2-1} h(l)\delta(d-\delta - N_{l} - l+\hat{s}_{l}) \]

Where \( s = d - \delta - N_{l} \), according to that the former and the latter sequence’s peaks have different shifts, Malik applied the minimum difference between the front and back cross-correlation values to estimate the IFOs:

\[
\hat{s}_{l} = \frac{1}{2\pi} \arg \min_{\theta=-N_{l}} \left| P_{c}(\theta + \hat{d}) - P_{c}(\theta + \hat{d} + N + 2N_{l} - 2) \right| \]

Where \( \hat{d} \) is the coarse time point, which can be deduced as \( d = \hat{d} - s \hat{d} \), indicated that the accuracy of the frequency offset estimation determines whether the timing result is desirable.
3. The Synchronization Method

3.1. The Synchronization Preamble Design

In order to eliminate subpeaks and side lobes, and overcome the limitations of FFOs estimation, a new preamble based on the CAZAC sequence is proposed in this paper:

\[ P_{\text{pre}} = [A \ A^* \ B \ B^*] \]

\( A \) is a part of \( c(n) \), as \( A = \exp(- \pi n^2 / N), \ n = 0, 1, 2, \ldots, N/4 - 1 \), and \( B \) is a sequence that is conjugate and symmetry to \( A \), as \( B = \exp(- \pi n^2 / 4) / (1 - n^2 / N) \).

3.2. The Timing Metrics And IFO Estimation

Based on the conjugate property of the proposed preamble, we propose the following timing metrics:

\[ P_1(d) = \left| \sum_{n=0}^{N/4-1} [r(n+d)A^*(n) + r(n+d+N/2)B^*(n)] \right| \]
\[ P_2(d) = \left| \sum_{n=0}^{N/4-1} [r(n+d+N/4)A^*(n) + r(n+d+3N/4)B^*(n)] \right| \]

Where \( d_1 \) and \( d_2 \) respectively represent the point that \( P_1(d) \) and \( P_2(d) \) take the maximum. Assume that the period of is \( N_{i} \), \( N \), may take \( N/2 \), \( N/4 \) or \( N \), then

\[ P_1(d) = 2 \sum_{i=1}^{N/4-1} \sum_{l=1}^{N-1} h(l)e^{j \pi (d-N_{g} - l)} + w(n) \exp(-j \pi n^2 / N_{i}) \]

\[ = e^{j \pi N_{i} / 2} \sum_{i=1}^{N/4} h(l) \exp(-j \pi (d-N_{g} - l)) \times \left( \sum_{i=1}^{N-1} e^{j \pi (u-d-N_{g} - l)} \exp(-j \pi n^2 / N_{i}) \right) + 2 \sum_{n=1}^{N/4} w(n) \exp(-j \pi n^2 / N_{i}) \]

Ignore the noise, when \( u(d-N_{g} - l) + \epsilon / p = mN_{i} \), where \( p = N/N \), we assume that \( \epsilon \) is \( i \) is a integer), the correct symbol starting point appears at \( l = 0 \), then the condition can be described as:

\[ u(d_1 - N_{g}) + i = mN_{i} \]

When \( i = 1 \), \( u(d_1 - N_{g}) + 1 = mN_{i} \). Assuming that when \( m = m_1 \), the hypothesis \( d_1 = d_{i-1} + (d_1 - N_{g}) \) satisfies equation (18), when \( i = 2 \):

\[ u(d_2 - N_{g}) + 2 = u((d_1 + d_1 - N_{g}) - N_{g}) \]

\[ = 2u(d_1 - N_{g}) + 2 = 2m_{1}N_{i} = m_{2}N_{i} \]

Similarly,

\[ u(d_j - N_{g}) + i = u(d_{i-1} + d_1 - N_{g}) \]

\[ = i = m_1N_{i} = m_1N_{i} \]

From the above formulas, we can derive the relationship between the shift of the timing peaks \( \Delta d \), the training sequence length \( N \), root index \( u \) and integer frequency offset \( \epsilon_i \):

\[ \Delta d = \epsilon_i (d_1 - N_{g}) / p \]

Therefore, the cross-correlation peaks of \( A \) and \( B^* \) parts shift to the left, and the cross-correlation peaks of \( A^* \) and \( B \) parts shift to the right. Then:

\[ d_1 = d + s \epsilon_i / p \]

\[ d_2 = d - s \epsilon_i / p \]

Further, the symbol starting point and IFOs can be derived as:

\[ d = (d_1 + d_2) / 2 \]

\[ \epsilon_i = P(d_1 - d_2) / (2s) \]
Obviously, the timing point and the IFOs can be estimated simultaneously, which means that timing synchronization is not affected by IFOs estimation. The difference between \(d_1\) and \(d_2\) is \(2se_i/p\). The integer frequency offset that can be estimated must be even when \(P\) takes 1, so once \(\text{mod}(d_i - d_2)/2\neq 0\), we need to modify the above results. In order to ensure that the timing point can fall within the cyclic prefix, and the estimations are redefined as: \(\hat{e}_i = p(d_i - d_2 + 1)/2s\) and \(\hat{d} = (d_i + d_2 - 1)/2\).

3.3. FFO Estimation

The \(A\), \(B^\ast\) parts and \(A^\ast\), \(B\) parts of the proposed preamble are symmetrically repeated, so IFO can be estimated by:

\[
\hat{e}_i = \frac{1}{\pi} \angle \left[ \sum_{n=0}^{N/2+1} p_s(n) \right]
\]  

(26)

The Phase difference metric function is defined as:

\[
p_s(n) = r^*(\hat{d} + n)r(\hat{d} + N - n - 1)
\]  

(27)

The above analysis indicate that the method can realize the IFO estimation in the range of [-1,1].

4. Simulation Results And Analysis

In Simulation, we compare five algorithms [3,5,8,11,13] and the proposed algorithm’s the mean square error of symbol timing offsets (MSE of STO), IFO and FFO. The simulation parameters in OFDM system are set as: system bandwidth 10MHZ, \(N\) is 256, \(s\) is 32, \(e\) is 4.3, the root index \(u\) takes 1 and scale parameter \(P\) takes 1 or 2. The ITU-VA and COS207-RA channel model (hereinafter referred to as VA and RA) are adopted. The relative delays of the VA channel and RA channel are [0,0.31,0.71,1.09,1.73,2.51], [0,0.1,0.2,0.3,0.4,0.5] (unit: us), the average powers are [0, -1, -9, -10, -15, -20] and [0, -4, -8, -12, -16, -20] (unit: dB). We simulate 5000 times for each SNR.

| Time metric | IFO Estimation | FFO Estimation |
|-------------|---------------|---------------|
| Complex Multiplication | Complex Multiplication | Complex Multiplication |
| Addition | (exclude FFT) | Addition |
| Multiplication | 3N/2+2 | N-2 |
| S&C | N+3 | N-2 |
| Park | N+4 | N |
| Ren | 3N/2+3 | 3N/2-2 |
| Shao | 2N+1 | 3N/2-2 |
| Malik | 2N+2 | 2N |
| Proposed | N+2 | N-2 |

Table 1 shows the computational statistics of timing and frequency synchronization estimation algorithms (“no” in the table shows that algorithm did not analyze this part), the proposed algorithm requires the minimal calculation as N+2 times complex multiplication and N-2 times addition in time synchronization, and N/2+1 times complex multiplication and N/2-1 times addition in frequency synchronization.

The MSE curves in figure 1 reflects the timing performance of different algorithms. It can be seen that under different SNRs and channel conditions, the proposed algorithm’s MSE keeps the lowest, and especially at low SNRs, indicating that the timing performance is optimal. Under AWGN channels, except for the SC algorithm, other algorithms can achieve 100% accurate timing when SNR > 0. Compared with Ren’s algorithm, the Shao’s algorithm introduced a new weighting factor to effectively reduce the timing MSE at a low SNR. However, effected by multipath delay, there is still a timing error even if the channel conditions are good, and increased SNRs does not significantly improve the timing performance.
It can be seen in Figure 2 that the presented method has the minimal MSE when SNR > -10dB under AWGN channels, and is almost zero when SNR is greater than -5. However, the MSE of Shao algorithm always keeps constant with the increase of SNR. The only defect is that the performance of IFO estimation is worse than Malik's algorithm at -10dB, this is because Malik's algorithm obtains better performance at the cost of higher calculations and lower transmission efficiency. Considering synchronization performance, system transmission efficiency and computational complexity comprehensively, the proposed algorithm is optimal. In Figure 3, S&C utilized the complete repeatability to estimate FFO, so the performance is best. Ren’s algorithm destroyed the repeatability due to the weighted operation, resulting in performance degradation at low SNRs. The sampling points used for correlation calculation are reduced as Malik only used CS, leading to the Gaussian channel is inferior to that of S&C algorithm. The proposed algorithm is basically same to Shao’s algorithm, and is slightly superior under VA channels. The above analysis indicate that the algorithm using the repeatability of the training sequence to estimate the FFO performs better.

5. Conclusion
This paper presents an OFDM synchronization algorithm based on CAZAC sequence, we complete the entire synchronization process with only one training symbol, so it improves the system transmission efficiency. In addition, once symbol timing point is determined, the IFOs is also estimated, and effectively reduces the calculation complexity. At the same time, compared to traditional synchronization algorithms, the entire synchronization process is completed in the time domain without FFT operation that effectively reduces the calculation complexity. Simulation and analysis show that the proposed algorithm has lower mean square error of timing and integer frequency offset estimation under the influence of low SNRs and multipath, which is more suitable for multipath fading channels and practical applications.
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