Diquark properties and their role in hadrons

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Abstract. Diquark correlations are important in baryons, which can be modeled as quark-diquark bound states. In addition, diquarks could play a role in non-standard hadrons such as tetraquarks and pentaquarks. Here, we obtain properties of these diquarks from the corresponding bound state equation, using a model for the effective quark-quark interaction that has proved successful in the light meson sector. Subsequently, we use the same model to estimate the masses of the lightest diquark-diquark and diquark-antidiquark states.

The set of Dyson–Schwinger equations [DSEs] form a useful tool to obtain a microscopic description of hadronic properties [1]. The simplest hadrons are mesons: color-singlet bound states of a quark and an antiquark. They are described by solutions of the homogeneous Bethe–Salpeter equation [BSE] for $q\bar{q}$ states. The Bethe–Salpeter amplitudes [BSAs] of different types of mesons, such as pseudo-scalar, vector, etc. are characterized by different Dirac structures.

In addition to $q\bar{q}$ bound states, one could also ask the question whether or not there are $qq$ states, by studying the corresponding BSE. Two quarks can be coupled in either a color sextet or a color antitriplet. Single gluon exchange leads to an interaction that is attractive for diquarks in a color antitriplet configuration. Furthermore, it is the diquark in a color antitriplet that can couple with a quark to form a color-singlet baryon. Thus we only consider $qq$ states in a color antitriplet configuration. As in the case of mesons, the different types of diquarks are characterized by different Dirac structures. Since the intrinsic parity of a $qq$ pair is opposite to that of a $q\bar{q}$ pair, a scalar diquark BSA has exactly the same form as a pseudoscalar meson BSA.

For practical calculations, we have to make a truncation of the set of DSEs [1]. Here we adopt the rainbow truncation of the quark DSE, in combination with the ladder truncation for the BSE

$$\Gamma(p_+, i p) = f_C \frac{Z}{q} G(q^2) D_{\mu\nu}^{\text{free}}(k)\gamma_\mu S(q_+)\Gamma_M(q_+, iq)S(q)\gamma_\nu i.$$  

where $p = p$, $P=2$ and $q = q$, $P=2$ are the relative quark momenta, $P$ the total meson momentum, $S(q)$ the nonperturbatively dressed quark propagator, $D_{\mu\nu}^{\text{free}}(k = p - q)$ the free gluon propagator in Landau gauge and $G(k^2)$ a phenomenological effective interaction [2, 3]. The color factor $f_C$ in the BSE is different for mesons and diquarks: $f_C = \frac{4}{3}$ for mesons, whereas $f_C = \frac{2}{3}$ for diquarks.

For the effective interaction, we use the model of Ref. [3]. Our results for the light meson and diquark masses are given in Table 1. The pseudoscalar and vector meson properties seem to be rather independent of the details of the effective interaction, as
long as the interaction generates the observed amount of chiral symmetry breaking \[3\]. However, the diquark masses are more sensitive to details of the effective interaction.

| TABLE 1. Masses (in GeV) of the light mesons \[3\] and diquarks \[4, 5\] for two different sets of parameters in the effective interaction. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| input parameters | meson           | 0\(^{+}\) diquark | 1\(^{+}\) diquark |
| $\omega$ (GeV)  | $D$ (GeV\(^2\)) | $m_\pi$  | $m_K$  | $m_{\rho^0}$ | $m_K$  | $m_{ud}$  | $m_{qs}$  | $m_{ud}$  |
| 0.40           | 0.93            | 0.138   | 0.495  | 0.742       | 0.936  | 0.821    | 1.10     | 1.02     |
| 0.50           | 0.79            | 0.138   | 0.495  | 0.74        | 0.94   | 0.688    | 0.96     | 0.89     |

For electromagnetic interactions, we need the quark-photon vertex, in addition to the dressed quark propagator and the BSAs. In impulse approximation (which we use here), current conservation is guaranteed as long as this vertex satisfies the vector Ward–Takahashi identity \[6\]. The solution of the inhomogeneous ladder BSE for the quark-photon vertex satisfies this constraint. Furthermore, this approach unambiguously includes effects from intermediate vector mesons, since they appear as poles in the dressed quark-photon vertex \[7\].

With the quark-photon vertex, we can now calculate the meson and diquark electromagnetic form factors by coupling the photon to each of the two (anti-)quarks in the bound state. The resulting pion form factor is quite well approximated by a monopole \[7\], at least up to about 2 or 3 GeV. However, the ud scalar diquark form factor falls off significantly faster than a monopole, at least at moderately small space-like values of $Q^2$, see Fig. \[1\]. We can obtain a good fit to our calculation with a form $[M^2=(Q^2+M^2)]^{1.4}$. Asymptotically however, both the pseudoscalar meson and scalar diquark form factors vanish like $1/Q^2$.

The ud diquark charge radius, $r_{ud} = 0.71$ fm, is about 8% larger than $r_\pi = 0.66$ fm. This suggests that these diquarks are somewhat larger in size than pions. These results appear to be insensitive to the details of the effective quark-quark interaction kernel, even though the actual diquark masses are sensitive to details of the interaction \[5\].

In addition to ordinary mesons and baryons (see e.g. Ref \[1\] and references therein), there could be non-standard hadrons such as tetraquarks and pentaquarks. Two color-antitriplet diquarks could form a color-triplet, which can be combined with an antiquark
into a color-singlet pentaquark \[8\]. Binding between two diquarks can come from two-quark exchange and meson exchange, but it is more likely dominated by gluon exchange. Gluon exchange also contributes to binding in tetraquarks, i.e. color-singlet \((qq)-\overline{qq}\) states. To get an idea about the amount of binding provided by gluon exchange, we calculate the scalar and vector bound states of two scalar colored particles in ladder approximation, ignoring the compositeness of the diquarks. For the interaction between the two scalar colored particles, we use the same model as that for the effective quark-quark interaction. The color factors are \(f_C = \frac{4}{3}\) for the color-singlet \((qq)-\overline{qq}\) states and \(f_C = \frac{2}{3}\) for the color-triplet \((qq)-\overline{qq}\) states, cf. the mesons and diquarks.

Our results for the masses and binding energies are given in Table 2. These results suggest that simple gluon-exchange does not provide enough binding for pentaquarks dominated by a \((qq)-\overline{qq}\)-\(q\) configuration. Furthermore, if diquarks are important for pentaquarks, we would also expect additional states in the meson sector, in particular a scalar nonet. Identification of these additional meson-like states however will be difficult, since they mix with meson molecules and ordinary mesons.

### TABLE 2

Bound state masses and binding energies in GeV of two scalar constituents in ladder approximation. The X’s indicate that these states are not allowed by the Bose statistics of the diquarks (the color-triplet states are anti-symmetric in color indices).

| \(m_{qq}\) | \(m_{qq}\) | color singlet SU(3) \(f\) nonet \(m_{qq}\) | isospin | scalar, \(0^+\) | vector, \(1\) | SU(3) \(f\) triplet SU(3) \(f\) anti-sexet \(m_{qq}\) | scalar, \(0^+\) | vector, \(1\) | \(M\) | \(E_B\) | \(M\) | \(E_B\) | \(M\) | \(E_B\) |
|----------|----------|-----------------|--------|------------|-----------|-----------------|--------|------------|-----|-----|-----|-----|-----|-----|
| 0.821    | 0.821    | 0               | 0      | 1.068      | 0.574     | 1.331           | 0.311  | X          | X   | 1.601 | 0.041 |
| 0.821    | 1.10     | \(\frac{1}{2}, \frac{1}{2}\) | 1.342  | 0.579      | 1.592     | 0.329           | 1.624  | 0.297     | 1.853 | 0.068 |
| 1.10     | 1.10     | 0, 1            | 1.597  | 0.603      | 1.841     | 0.359           | 1.888  | 0.323     | 2.099 | 0.101 |
| 0.688    | 0.688    | 0               | 1      | 1.034      | 0.342     | 1.377           | 0.0    | X          | X   | —     | —    |
| 0.688    | 0.96     | \(\frac{1}{2}, \frac{1}{2}\) | 1.290  | 0.358      | 1.615     | 0.033           | 1.528  | 0.120     | —    | —    |
| 0.96     | 0.96     | 0, 1            | 1.530  | 0.390      | 1.844     | 0.076           | 1.770  | 0.150     | —    | —    |

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