SINGLE-SPIN ASYMMETRIES WITH TWO-HADRON FRAGMENTATION FUNCTIONS

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Using the formalism of two-hadron fragmentation functions, we discuss single-spin asymmetries occurring in the production of two hadrons in the current region of deep inelastic scattering, with a particular emphasis on transversity measurements.

Single-spin asymmetries in single-hadron production have been a subject of intense activity on the theoretical and experimental sides in the last years, the reason being that they provide access to the yet unmeasured quark transversity distribution, and that they involve interesting effects related to spin, intrinsic transverse momentum, orbital angular momentum and T-odd distribution and fragmentation functions.

Some drawbacks unfortunately affect these observables and hinder the extraction of clean information on the distribution and fragmentation functions:

- there is no proof of factorization for transverse-momentum dependent observables up to subleading twist (only very recently a preprint on the leading-twist proof appeared [1]);
- the distribution and fragmentation functions appear in convolutions;
- the evolution equations for transverse momentum dependent functions is unknown (only very recently a preprint on this subject appeared [2]);
- the expressions describing the asymmetries have several competing contributions.

Two-hadron production asymmetries are free from the first three problems, as they can be integrated over intrinsic transverse momenta, and less affected by the last one. Asymmetries are in this case proportional to the product of a parton distribution function times a two-hadron fragmentation function. Single-spin asymmetries contain in particular the so-called interference fragmentation functions, which are T-odd, i.e. they are odd under naive time reversal.

Interference fragmentation functions were studied in Refs. [3,4,5]. The complete analysis has been carried out up to leading-twist in Ref. [6] and up to subleading-twist in Ref. [7]. Positivity bounds and the expansion in the partial waves of the two hadrons were presented in Ref. [8]. While we refer to this list of references
for further details, here we describe some of the most interesting observables to be measured in semi-inclusive DIS.

The process we are considering is $l p \to l' h_1 h_2 X$, where both hadrons are produced in the current fragmentation region. The outgoing hadrons have momenta $P_1$ and $P_2$, masses $M_1$ and $M_2$, and invariant mass $M_h$ (which must be much smaller than the virtuality of the photon, $Q$). We introduce the vectors $P_h = P_1 + P_2$ and $R = (P_1 - P_2)/2$, i.e. the total and relative momenta of the pair, respectively. The angle $\theta$ is the polar angle in the pair’s center of mass between the direction of emission (which happens to be back-to-back in this frame) and the direction of $P_h$ in any other frame [8]. We introduce also the invariant

$$|R| = \frac{1}{2M_h} \sqrt{M_h^2 - 2(M_1^2 + M_2^2) + (M_1^2 - M_2^2)^2}. \quad (1)$$

Cross-sections are assumed to be differential in $d\cos \theta dM_h^2 d\varphi_R dz dx dy d\varphi_S$, where $z, x, y$ are the usual scaling variables employed in semi-inclusive DIS and the azimuthal angles are defined so that (see Fig.1)

$$\cos \varphi_S = \frac{(\hat{q} \times l)}{|\hat{q} \times l|} \cdot \frac{(\hat{q} \times S)}{|\hat{q} \times S|}, \quad \sin \varphi_S = \frac{(1 \times S) \cdot \hat{q}}{|1 \times l| \cdot |\hat{q} \times S|}, \quad (2)$$

$$\cos \varphi_R = \frac{(\hat{q} \times l)}{|\hat{q} \times l|} \cdot \frac{(\hat{q} \times R_T)}{|\hat{q} \times R_T|}, \quad \sin \varphi_R = \frac{(1 \times R_T) \cdot \hat{q}}{|1 \times l| \cdot |\hat{q} \times R_T|}, \quad (3)$$

where $\hat{q} = q/|q|$ and $R_T$ is the component of $R$ perpendicular to $P_h$.

In writing the following cross sections, it is understood that distribution functions have a flavor index $a$ and depend on $x$, fragmentation functions have a flavor index $a$ and depend on $z$, $\cos \theta$ and $M_h^2$. We introduce the functions

$$A(y) = 1 - y + y^2 / 2, \quad B(y) = 1 - y,$$

*Note that there is a difference of sign between the angles used here and those used in Ref. [7], to conform to the so-called Trento conventions.*
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\[ V(y) = 2 (2 - y) \sqrt{1 - y}, \quad W(y) = 2 y \sqrt{1 - y}. \]

The unpolarized cross section up to subleading twist is

\[ d^7\sigma_{UU} = \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y} \left\{ A(y) f_1 D_1 - V(y) \cos \varphi_R \sin \theta \frac{|R_i|}{Q} \left[ \frac{f_1}{z} \tilde{D} + \frac{M_x}{M_h} \frac{1}{z} G_{1}^{\perp} \right] \right\} \tag{4} \]

When the target is polarized opposite to the beam direction the polarized part of the cross section is

\[ d^7\sigma_{UL'} = - \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y} \sin \varphi_R \left\{ |S_L| V(y) \sin \theta \frac{|R_i|}{Q} \left[ \frac{M_x}{M_h} \frac{h_L H_1^\perp + \frac{1}{z} g_1 G_{1}^{\perp}}{h_1} \right] \right\} \tag{5} \]

where \(|S_L| = 2 |S_L| M x Q y / Q\). When the target is polarized perpendicular to the beam direction we have

\[ d^7\sigma_{UT} = - \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y} |S_L| \left\{ B(y) \sin(\varphi_R + \varphi_S) \sin \theta \frac{|R_i|}{M_h} h_1 H_1^\perp \right. \]

\[ + V(y) \sin \varphi_S \frac{M_h}{Q} \left\{ h_1 \left( \frac{1}{z} H + \sin^2 \theta \frac{|R_i|^2}{M_h^2} H_{1}^{3o(1)} \right) - \frac{M_x}{M_h} x f_T D_1 \right\} \tag{6} \]

When the beam is longitudinally polarized we have

\[ d^7\sigma_{LU} = - \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y} \lambda_e W(y) \sin \varphi_R \sin \theta \frac{|R_i|}{Q} \left[ \frac{M_x}{M_h} e H_1^\perp + \frac{1}{z} f_1 G_{1}^{\perp} \right], \tag{7} \]

where \(\lambda_e\) denotes the helicity of the lepton.

In Wandzura-Wilzcek approximation, all fragmentation functions with a tilde vanish. Of particular interest is the partial-wave expansion of some of the fragmentation functions involved, truncated at the \(p\)-wave level (only \(s\) and \(p\) waves contribute at low invariant mass):

\[ D_1(z, \cos \theta, M_h^2) \approx D_{1, uu}(z, M_h^2) + D_{1, uf}(z, M_h^2) \cos \theta + D_{1, ll}(z, M_h^2)^{3 \cos^2 \theta - 1} \tag{8} \]

\[ H_{1}^{\perp}(z, \cos \theta, M_h^2) \approx H_{1, uu}(z, M_h^2) + H_{1, ul}(z, M_h^2) + H_{1, ll}(z, M_h^2) \cos \theta. \tag{9} \]

The partial-wave expansion shows that, for instance, Eq. \(8\) can be integrated over \(\cos \theta\) without washing out completely the term proportional to the transversity distribution. Unfortunately, the dependence on the invariant mass of \(H_{1, uu}\) is not known, requiring a study in separate invariant-mass bins. Vice-versa, we expect the term \(H_{1, ll}^{\perp}\) to show the Breit-Wigner invariant-mass shape typical of the \(\rho\) resonance (for two-pion production): in this case Eq. \(8\) could be integrated over \(M_h^2\) in the neighborhood of the \(\rho\) mass, but should be studied in separate \(\cos \theta\) bins in order to disentangle the \(H_{1, ll}^{\perp}\) contribution.

In conclusion, the measurement of single-spin asymmetries in two-hadron production in DIS can provide a good way to extract information on the transversity distribution function \(h_1(x)\) and on the distribution function \(e(x)\) in a cleaner way.
compared to single-hadron production. This kind of measurements is currently under way at HERMES.

Two-hadron fragmentation functions can be studied also in $e^+e^-$ and $pp$ collisions. The first process has been studied in detail in Ref. [9] and it is under experimental study by the BELLE Collaboration [10]. The second process can be measured by the PHENIX and STAR collaborations [12,13] and allows the measurement of a convolution of the transversity distribution and the function $H_1^<$ [11] when employing one polarized proton, and of a convolution of two $H_1^<$ functions when employing two unpolarized protons and detecting two hadron pairs [14].

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