Thermal and mechanical characteristics of contact friction pair based on 3-D wheel/rail-foundation contact vertical system

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Abstract. The main goal of this article is to analyze the influence of wheel/rail friction heat on the wheel/rail contact friction pair. The nonlinear constitutive relation and heat conduction equation are established firstly. In addition, a direct coupling model of 3-D wheel/rail-foundation contact vertical system is established. The mechanical, thermal and geometrical characteristics of wheel/rail friction pair are researched. The results show that the temperature rise on the wheel/rail contact surface is significant, and the influence of friction heat on the contact patch, contact pressure and stress is obvious. However, the influence focuses on the surface and subsurface of the wheel/rail contact. Once the depth reaches a certain distance below the contact surface, the influence can be ignored. The temperature does not change when the distance to the rail surface surpasses 3.28 mm; also the influence of temperature rise on the stress becomes very weak when the distance exceeds 8.05 mm.

1. Introduction
Comparing with the motor and airplane, railway transport has incomparable advantages. Railway has great influence on the communication, economy and environment, etc. The railway transport has increasingly developed in recent years. A lot of researchers widely carry out the studies about material strength, heavy haul railway, high-speed railway and wheel/rail relationship, etc. Wheel and rail are the important components of railway, playing vital roles in ensuring safety and transferring load. The research of wheel/rail relationship includes the analysis of fatigue damage, wear, creep and friction, etc. The friction is one of the common phenomena when a train is braking or passing through a small radius curved track. The friction heat can cause the peeling of the wheel/rail surface [1], and it is tightly linked with the surface scratch of wheel and rail. Recently, the friction and stress of wheel/rail contact has been widely researched. Li et al [2] established a 2-D rail model to investigate the influence of creep on the temperature and residual stress. The research results show that friction heat has a significant effect on the residual stress when the creepage is large. Kuminek et al [3], Wu [4] and Pun et al [5] widely researched the temperature and stress of wheel/rail under condition of sliding or rolling contact. Esmaeili et al [6] predicted that the growing of thermo-mechanical crack on the wheel treads when the tread bears mechanical and thermal load because of the rolling contact and braking. A 3-D model is used to analyze the temperature and strain feature of wheel/rail when a vehicle wheel slides on the rail surface in [7]. The calculation results show that the data of 3-D model are closer to the actual situation and more reasonable than the results gotten from plane model. Liu et al [8], Athukorala et al [9] and Nikas et al [10] studied the damage of wheel/rail contact.
Since there are many difficulties in accurately measuring the temperature and stress of wheel/rail contact region through experiment, the numerical simulation becomes a good solution. Many researchers studied the wheel/rail contact and friction heat by means of finite element methods. A large number of vital research results have been obtained [1-3,11-13]. Those research results have positive effect on the understanding of wheel/rail contact. In the previous studies, some of the models are 2-D models, the calculation parameter and the friction coefficient are not temperature-dependent. Besides, the friction heat of model sometimes is replaced by a moving heat source. Meanwhile, the tread profiles of wheel/rail are always simplified. In this article, the nonlinear constitutive relation and heat conduction equation are firstly established. The mechanical, thermal and geometrical features of wheel/rail friction pair are researched. The node temperature and the influences of temperature on node stress are deeply analyzed. Of course, the thermal and mechanical parameters of wheel/rail material are seen as the function of temperature. The geometry of wheel/rail are real. Moreover, the coupling of wheel/rail is direct coupling.

2. Coupling equation

2.1. Equation of friction heat transfer

Wheel/rail contact links the vehicle with the track. The force which acts on the contact surface of wheel/rail is especially complex. Large amount of friction heat will generate on contact surface during the sliding of wheel on rail surface. The schematic diagram of heat conduction on rail surface can be illustrated in figure 1 [7].

![Figure 1. Micro element of heat conduction.](image)

where $\Phi_x$, $\Phi_y$ and $\Phi_z$ is respectively the heat flow in three coordinate axes. $Q_{in}$ is internal heat generation per unit time and volume. $q_{rail}$ and $q_{wheel}$ are the heat flux transferred into the rail and wheel contact surface. They can be written as follows [7],

$$
\begin{align*}
q_{rail} &= \xi q_i = \xi \gamma \mu(T)vp(x, z) \\
q_{wheel} &= \xi (1-\gamma)q_i = \xi (1-\gamma)\mu(T)vp(x, z)
\end{align*}
$$

(1)

where $q_i$ is the total braking mechanical energy between wheel and rail in unit time and area. $\xi$ is heat generation rate. $\gamma$ is the heat partition coefficient between the wheel and rail, and its value is 0.5. $\mu(T)$ is temperature-dependent friction coefficient. $v$ is the sliding speed of wheel. $p(x, z)$ is contact pressure, it is [14]:

$$
p(x, z) = \begin{cases} 
0 & \text{if, } u > 0, \text{separation} \\
K_n \cdot u_n + \eta_{i+1} & \text{if, } u \leq 0, \text{contact}
\end{cases}
$$

(2)

where $K_n$ is wheel/rail contact stiffness, $u_n$ is the distance between contact interfaces. $\eta_{i+1}$ is
\[ \eta_{int} = \begin{cases} \eta_i + K_n \cdot u_n, & \text{if, } |u_n| > \Delta \\ \eta_i, & \text{if, } |u_n| < \Delta \end{cases} \] (3)

where \( \Delta \) is calculation tolerance of wheel/rail, \( \eta_i \) is Lagrange multiplier.

Based on the Fourier law and the law of energy conservation [7,8], the equation of heat conduction in the wheel/rail contact region can be written in integral form as follows,

\[
\int \rho C(T) \frac{\partial T}{\partial t} dV + \int \nabla \cdot [-k(T) \nabla T] dV = \int \sum Q_{in} dV + \int \sum [\xi(T) p(x, z) v] dxdz dV
\] (4)

where \( \rho \) is steel density, its value is 7790 \( \text{kg/m}^3 \), and it does not change with temperature. \( C(T) \) is specific heat capacity. \( k(T) \) is thermal conductivity. \( C(T) \) and \( k(T) \) are function of temperature (table 1). \( Q_{in} \) is internal heat source of wheel/rail, its value is 0. \( \nabla \) is Hamiltonian Operator.

**Table 1. Mechanical and thermal parameter.**

| T (°C) | Elastic modulus (GPa) | Poisson’s ratio | Yield Stress (MPa) | Tensile Strength (MPa) | \( C(T) \) (J·kg\(^{-1}·{°C}^{-1}) | \( k(T) \) (W·m\(^{-1}·{°C}^{-1}) | Coefficient of thermal expansion (°C\(^{-1}) | \mu(T) |
|-------|----------------------|----------------|-------------------|-----------------------|------------------|--------------------------|--------------------------|----------|
| 25    | 209                  | 0.30           | 608               | 1000                  | 490.1            | 47.7                     | 11.0×10\(^{-6})          | 0.334   |
| 100   | 207                  | 0.30           | 608               | 998.9                 | 499.9            | 48.9                     | 11.6×10\(^{-6})          | 0.301   |
| 650   | 105                  | 0.36           | 502               | 985.7                 | 571.5            | 57.8                     | 14.8×10\(^{-6})          | 0.139   |
| 1000  | 50                   | 0.39           | 237.9             | 740.9                 | 617.1            | 63.4                     | 15.7×10\(^{-6})          | 0.085   |
| 1450  | 2.1                  | 0.40           | 7                 | 42                    | 671.8            | 76.4                     | 16.1×10\(^{-6})          | 0.045   |

2.2. Equation of thermal stress

The thermal and mechanical parameters are seen as function of temperature in equation (4). At the same time, the above temperature-dependent parameters should be considered in the constitutive relation [15]. Based on the free energy of isotropic material, the constitutive relation considering the temperature-dependent parameters can be obtained as follows,

\[
\sigma_{ij} = 2G(T)e_{ij} + \lambda(T)e_{kk} \delta_{ij} - \beta(T) \Delta T \delta_{ij}
\] (5)

where \( e_{kk} \) is body strain, \( \delta_{ij} \) is Kronecker symbol. \( \Delta T \) is temperature difference, and the temperature is controlled by the equation (4). \( G(T) \), \( \lambda(T) \) and \( \beta(T) \) is respectively shear modulus, Lame constant and thermal stress coefficient. They can be described as follows,

\[
G(T) = G_0 + G_1 \cdot T^s, \quad \lambda(T) = \lambda_0 + \lambda_1 \cdot T^s, \quad \beta(T) = \beta_0 + \beta_1 \cdot T^s
\] (6)

where \( G_0, \lambda_0 \) and \( \beta_0 \) is respectively shear modulus, Lame constant and thermal stress coefficient at initial time. \( G_1, \lambda_1 \) and \( \beta_1 \) is respectively coefficient of influence of the unit relative temperature difference on the shear modulus, Lame constant and thermal stress coefficient. \( T^s \) is relative temperature difference, it is equal to

\[
T^s = \Delta T / T_0
\] (7)

where \( T_0 \) is initial temperature.

2.3. Thermal boundary condition

The thermal boundary conditions of wheel/rail friction can be written as follows [16]:

\[
q_e = h(T - T_s), \quad q_s = \varepsilon \sigma (T^s - T_s^4)
\] (8)

where \( h \) is heat transfer coefficient, \( \varepsilon \) is emissivity, \( \sigma \) is Stefan-Boltzmann constant. \( T \) is temperature of wheel/rail. \( T_s \) is surrounding temperature.
3. Finite element model
The finite element model of wheel/rail-foundation contact vertical system is built by the ANSYS software [14]. Solid226 is selected to simulate the wheel/rail component. Conta174 and Targe170 are adopted to simulate the contact behavior of wheel/rail. Fasteners and foundation are simulated by Combin14. The Solid226, Conta174 and Targe170 have structural-thermal capability. In order to reduce the element and node numbers, the multiple point constraint (MPC) method is used. The model has 48672 nodes and 42101 elements. The smallest element size is 0.5 mm in this article [7,8]. At the same time, the coupling of both heat and force is strong coupling, and the full integration method is adopted. At last, the Solid186 is selected to establish temperature-independent model, in which the temperature is not considered. And calculation model is shown in figure 2.

Figure 2. 3-D finite element model. (a) Wheel/rail-foundation contact system, (b) Detail of contact and (c) Contact pair.

When the rail length exceeds 4.2 m, the rail length has little effect on the result. Therefore, the length of rail in FEM is 4.2 m [8]. The longitudinal and transverse displacement on the two sides of rail are constrained. The vertical displacement at the bottom of foundation springs is constrained. For the fastener springs, the vertical displacement at the top is constrained. The wheel can slide on the rail surface along longitudinal direction, but it can not move in the transverse direction.

The wheel load is 100 kN. The initial temperature is 20°C, the heat transfer coefficient is 25 W·m⁻²·K⁻¹. Stefan-Boltzmann constant is 5.68×10⁻⁸ W·m⁻²·K⁻⁴. The stiffness of single fastener is 2 MN·m⁻¹, and the foundation stiffness is 50 MN·m⁻¹ [17]. The wheel sliding speed is equal to 1.0 m·s⁻¹ [7]. The sliding distance is 0.1 m. The material properties used for the FEA are listed in table 1 [18].

4. Result

4.1. Stress of wheel/rail friction pair
In this section, the basic features of stress on wheel/rail friction pair are illustrated. The contact characteristics is very fundamental for wheel/rail friction pair. The maps of von Mises stress on the wheel/rail surface at the beginning of sliding are presented in figure 3. The von Mises stress fields obtained by sectioning the wheel/rail through the contact center in two directions are illustrated in figure 4. The contact patch of wheel/rail is shown in figure 6.

Figure 3 shows the von Mises stress of wheel/rail surface is very large in the contact center. On wheel and rail surface, the maximum von Mises stress is respectively 600.3 MPa and 585.6 MPa at the initial time (t=0 s, sliding distance (s) is equal to 0 m). The maximum stress is close to the yield stress of wheel/rail.

Figure 4 shows the phenomenon of stress concentration is very obvious near the wheel/rail contact region. The von Mises stress fields of wheel/rail contact region are similar to figure 4 in two directions during each iteration. The calculation result shows that the maximum von Mises stress occurs at the subsurface of wheel/rail. In the range of 3.93 mm below the wheel/rail surface, von Mises stress is close to the maximum value. The stress of the wheel/rail contact area is very large, which is one of the reasons that leads to the damage of wheel and rail (figure 5 [19,20]). The result calculated in this paper is in agreement with the cases of wheel/rail damage.
Figure 3. Von Mises stress (t=0 s, s=0 m, Pa). (a) Wheel surface and (b) Rail surface.

Figure 4. Von Mises stress fields (t=0 s, s=0 m, Pa). (a) Transverse direction and (b) Longitudinal direction.

Figure 5. Damage of wheel/rail.

Figure 6. Contact patch. (a) t=0 s, s=0 m ($P_{max}=1440$ MPa, $A=153.1$ mm$^2$), (b) t=0.05 s, s=0.05 m ($P_{max}=881$ MPa, $A=245.4$ mm$^2$) and (c) t=0.1 s, s=0.1 m ($P_{max}=827$ MPa, $A=272.6$ mm$^2$).

Figure 6 shows the shape of the wheel/rail contact patch on the rail surface is similar to ellipse. The shape does not change so much in the process of sliding. The minor axis and major axis of contact patch
at the initial contact is respectively 11.03 mm and 23.15 mm. However, because of temperature rise, the contact area increases by 78.1% and the maximum pressure decreases by 42.6% at the end of sliding time (figure 6).

4.2. Temperature of wheel/rail friction pair

The stripping and abrasion of wheel/rail surface are closely related to friction heat. Huge friction heat generates during the wheel sliding, and the heat will influence the stress of wheel/rail contact region (equations (4) and (5)). Figure 7(a) is temperature sectioning maps in longitudinal and transverse direction through the contact point. Figures 7(b) and 7(c) is temperature field of the wheel and rail surface respectively.

![Figure 7. Temperature field (t=0.1 s, s=0.1 m, ℃). (a) Slicing map, (b) Wheel surface and (c) Rail surface.](image)

The temperature field is similar to the figure 7(a) at every moment during wheel sliding. The contact location of wheel does not change, so the temperature on the wheel surface is much higher than that on the rail surface. The high temperature region locates on the wheel/rail surface. However, the influence area and depth of temperature are limited. For wheel and rail surfaces, figure 7 shows the highest temperature is respectively 1390℃ and 798.5℃. On the rail, the influence of temperature on the depth is no more than 3.28 mm. However, the temperature influence depth on wheel is two times as large as that on rail.

4.3. Node temperature and stress of rail

In order to research the thermo-mechanical coupling effect, seven nodes are selected from rail surface and subsurface (figure 8(a)). The nodes locate at the depth of 0~14.92 mm below the rail surface. The relationship between node temperature and wheel sliding distance is presented in figure 8(b). The von Mises stress curve obtained from temperature-dependent model is shown in figure 8(c). Figure 8(d) illustrates the von Mises stress curve obtained from the temperature-independent model. Moreover, the distribution of maximum von Mises stress of nodes along depth is plotted in figure 8(e).
Figure 8(b) shows the temperature curve of node 1 is kept at room temperature before wheel/rail contact. The temperature rises rapidly during wheel/rail contact and then decreases gradually. The temperature time history curve of node 1 corresponds to three processes: the wheel approaching the node 1 (stage I), sliding over the node 1 (stage II) and departing away the node 1 (stage III). However, the temperature of nodes 2 ascends slowly with the sliding distance increasing. And the temperature change of node 3 is not obvious.

Both figure 8(c) and (d) indicate that there are three stages in von Mises stress curves. However, the curves of node 1 and 2 in figure 8(c) are obviously different from the curves at the same locations in figure 8(d). Figure 8(c) shows that the curves of both node 1 and 2 decrease gradually after reaching the maximum values. It is mainly because the temperature rise of node 1 and 2 is obvious in stage II and III (figure 8(b)). The temperature has obvious influence on the von Mises stress of node 1 and 2. In the stage III, the von Mises stress of both node 1 and 2 in figure 8(c) is respectively 70.9 MPa and 170.2 MPa higher than that in figure 8(d).

Figure 8(e) presents the change law of two curves is same. The maximum von Mises stress of nodes increases slowly first and then decreases gradually with the increasing of depth. The maximum value occurs at the rail subsurface. Because the influence of temperature on the von Mises stress descends gradually with the growth of the depth, the maximum von Mises stress calculated through two models are almost the same when the depth exceeds 8.05 mm.

5. Conclusions
In this paper, the stress and temperature of wheel/rail sliding contact are studied. The calculation results indicate that the friction heat has a significant effect on the stress and the contact patch. Main conclusions are as follows:

- The maximum von Mises stress of wheel and rail surface is respectively 600.3 MPa and 561.7 MPa at initial time. In the range of 3.93 mm below the wheel/rail surface, the von Mises stress is close to the maximum value. At the initial contact moment, the minor axis and major axis of contact patch is 11.03 mm and 23.2 mm respectively.
- The temperature of wheel/rail surface increases with the sliding distance increasing. Comparing
with the initial contact, the contact area ascends by 78.1% and the maximum pressure decreases by 42.6% because of temperature rise at the end of sliding time. And the highest temperature of wheel and rail is respectively 1390°C and 798.5°C. However, the influence of temperature on depth is limited, and the value is no more than 3.28 mm on the rail.

- The stress curves of rail surface and subsurface (node 1 and 2) obtained from the temperature-dependent model are obviously different from the curves in temperature-independent model. Especially in the stage III of the curves, the von Mises stress obtained from the temperature-dependent model is significantly higher than the stress calculated by the temperature-independent model.
- The maximum von Mises stress of nodes ascends slowly first and then descends gradually with the growth of depth. The maximum value occurs at the rail subsurface. However, when the depth exceeds 8.05 mm, the difference of von Mises stress in two models can be ignored.
- Based on the actual geometric dimension and the thermal property of material, the mechanical, thermal and geometrical characteristics of wheel/rail friction pair are truly reflected.

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References
[1] Ahlstrom J and Karlsson B 2002 Modeling of heat conduction and phase transformations during wheel sliding-theoretical predictions and comparison with results of full-scale experiments Wear 253 291-300
[2] Li W, Wen Z F, Jin X S and Wu L 2014 Numerical analysis of rolling-sliding contact with the frictional heat in rail Chinese J. Mech. Eng-En. 27 41-9
[3] Tomasz K and Krzysztof A 2014 Methodology and verification of calculations for contact stresses in a wheel-rail system Vehicle Syst. Dyn. 52 111-24
[4] Wu B, Wen Z F, Wu T and Jin X S 2016 Analysis on thermal effect on high-speed wheel/rail adhesion under interfacial contamination using a three-dimensional model with surface roughness Wear 366-367 95-104
[5] Pun C L, Kan Q, Mutton P J, Kang G and Yan W 2014 A single parameter to evaluate stress state in rail head for rolling contact fatigue analysis Fatigue Fract. Eng. M. 37 909-19
[6] Esmaeili A, Walia M S, Handa K, Ikeuchi K, Ekh M, Vernersson T and Ahlström J 2017 A methodology to predict thermomechanical cracking of railway wheel treads: From experiments to numerical predictions Int. J. Fatigue 105 71-85
[7] Wu Y P, Wei Y P, Liu Y, Duan Z D and Wang L B 2017 3-D analysis of thermal-mechanical behavior of wheel/rail sliding contact considering temperature characteristics of materials Appl. Therm. Eng. 115 455-62
[8] Liu Y, Jiang S, Wu Y P, Duan Z D and Wang L B 2016 Effects of spalling on rail thermo-elasto-plasticity in wheel-rail sliding contacts J. Traffic Transp. Eng. 16 47-54 (in Chinese)
[9] Athukorala A C, De Pellegrin D V and Kourousis K I 2017 A unified material model to predict ratcheting response in head-hardened rail steel due to non-uniform hardness distributions Tribol. Int. 111 26-38
[10] Nikas D, Ahlström J and Malakizadi A 2016 Mechanical properties and fatigue behaviour of railway wheel steels as influenced by mechanical and thermal loadings Wear 366-367 407-15
[11] Ahlström J, Kabo E and Ekberg A 2016 Temperature-dependent evolution of the cyclic yield stress of railway wheel steels Wear 366-367 378-82
[12] Vo K D, Tieu A K, Zhu H T and Kosash P B 2015 The influence of high temperature due to high adhesion condition on rail damage Wear 330-331 571-80
[13] Meysam N, Li S G, Li Z L, Wu R 2017 Thermomechanical analysis of the wheel-rail contact
using a coupled modelling procedure *Tribol. Int.* **117** 250-60

[14] Alawadhi and Esam M 2015 *Finite Element Simulations Using Ansys* 2nd Ed. (Boca Raton: CRC Press)

[15] Huang H G 1989 *Introduction to Thermoelasticity* (Beijing: Tsinghua University Press) (in Chinese)

[16] Li Z L 2002 *Wheel-Rail Rolling Contact and Its Application to Wear Simulation* (Delft: Delft University Press)

[17] Wang K Y, Cai C B and Zhu S Y 2013 Vertical dynamic model and vibration characteristic of rail fastening system *Engineering Mechanics* **30** 146-9 (in Chinese)

[18] Chen Y C and Lee S Y 2009 Elastic-plastic wheel-rail thermal contact on corrugated rails during wheel braking *J. Tribol.* **131** 1-9

[19] Nan X 2011 Method of thermo-mecanical fatigue strength assessment on railway heavy-haul freight car wheel plate (Beijing: Beijing Jiaotong University) p 2 (in Chinese)

[20] Xing L X 2008 Research on defect characteristics and classification of higher speed rails (Beijing: China Academy of Railway Sciences) p 66 (in Chinese)