Cyclotron radiation and emission in graphene — a possibility of Landau-level laser

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Abstract. The cyclotron radiation and emission in graphene in magnetic fields are theoretically examined in terms of the optical conductivity and relaxation rates. Their peculiarities are found to favor a realization of the Landau level laser, proposed decades ago [H. Aoki, Appl. Phys. Lett. 48, 559 (1986)].

Introduction
While the physics of “massless Dirac” particles in graphene has been kicked off by the observation of an anomalous quantum Hall effect[1, 2], interests begin to extend to optical properties, which include recent spectroscopic studies on unevenly spaced Landau levels $\propto \sqrt{N}$[1, 3–6].

In the present paper we point out that the graphene Landau levels should have an interesting implication on the Landau-level laser, which was proposed decades ago[7] and whose basic idea is simple enough: we can exploit the unusual coalescence of the energy spectrum into a series of line spectrum (Landau levels) to realize a laser from a spontaneous emission if we can make a population inversion. There, the photon energy will be the cyclotron energy, which is tunable and falls on the terahertz region for $B \sim 10T$. However, the difficult part is the population inversion, since if we optically pump the system, the excitation would go up the ladder of equidistant Landau levels indefinitely.

This has motivated us to put a question (Fig.1): will the graphene Landau levels, with uneven spacing among their peculiarities, favor in realizing such a population inversion?

Having this in mind we have calculated the optical conductivity for graphene in magnetic fields including the effect of disorder. We show the following: (i) The unevenly spaced Landau levels do give rise to an interesting situation, where the $N = 0$ Landau level stands alone for an appropriate level of disorder. This should help in realizing the population inversion (across $N = 0$ and $N \geq 1$ here) necessary for lasing. (ii) We have then examined the relaxation processes, with an extention of the treatment for ordinary quantum Hall systems,[8, 9] to show that

![Figure 1](image_url). Cyclotron absorption (green) and emission (red) processes schematically depicted for the ordinary(a) and graphene(b) quantum Hall systems. Black lines represent the band dispersion, while blue lines Landau levels.

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graphene’s peculiar cyclotron energy $\propto \sqrt{B}$ along with its 2D nature favor the cyclotron emission over other relaxation processes[10]. (iii) We have further explored the optimal condition for the effects of disorder on the optical conductivity.

**Optical conductivity in graphene**

Low-energy physics around the Fermi energy in graphene is described by the massless Dirac Hamiltonian,[4] and in magnetic fields the energy spectrum is quantized into Landau levels, $\varepsilon_n = \text{sgn}(n)\sqrt{n}\hbar\omega_c, \omega_c = \sqrt{2v_F/\ell} = v_F\sqrt{2eB/\hbar}$, for a clean system, where $n = 0, \pm 1, \ldots$ is the Landau index, $\ell = \sqrt{\hbar/eB}$ the magnetic length, and $v_F$ the Fermi velocity. Here we consider realistic systems having disorder with the self-consistent Born approximation (SCBA) [4, 11] to calculate Green’s function $G$, with the self-energy $\Sigma_n(\varepsilon) = \Gamma/4 \sum_{n'} [\varepsilon - \text{sgn}(n)\sqrt{|n|} - \Sigma_{n'}(\varepsilon)]^{-1}$, where the Landau level broadening is given by $\Gamma^2/4 = n_0V_0^2$ if we assume for simplicity short-ranged random potentials of density $n_0$ and intensity $V_0$.

We have numerically obtained the Green’s function and optical conductivity with Kubo formula. While in usual cases the broadened Landau levels are uniformly merged as $\Gamma$ is increased, there is a striking difference for graphene, where the Landau levels ($\propto \sqrt{n}$) are unevenly spaced, so that the broadened Landau levels overlap to lesser extent as we go to the central one ($n \rightarrow 0$), as typically depicted in Fig.2. Namely, for an intermediate value of $\Gamma/\omega_c$ only the $n = 0$ Landau level stands alone while the other levels form a continuous spectrum.

We now look at the optical conductivity in Fig.3 for the Fermi energy at $\varepsilon_F = 0$ (energy for the Dirac point), each resonance peak can be assigned to an allowed transition with the selection rule ($|n| \leftrightarrow |n| + 1$). The largest peak around $\omega/\omega_c = 1$ corresponds to the transition between $n = 0 \leftrightarrow \pm 1$, while the peaks at higher frequencies come from the transition across the Fermi energy, $-n \leftrightarrow n \pm 1$. If we turn to the temperature dependence in the figure, there is a peak, in the region $\omega/\omega_c < 1$, that grows for higher $T$. We can identify this as coming from the uneven Landau levels in graphene: As $T$ is raised, higher Landau levels begin to be occupied, which enables the transitions among higher Landau levels, $n \leftrightarrow n \pm 1$, to take place, while this would not cause new lines to appear for equidistant Landau levels.

Previously, the optical conductivity has been obtained by Gusynin et al.,[12, 13] and Sadowski et al.[14]. While they did not calculate the self-energy self-consistently, we have adopted the self-consistent Born approximation. The present result qualitatively agrees with the former studies, but the new findings here are, first, the full dependence on the $k_B T/\hbar\omega_c$, including the
growing of low-frequency peaks at low temperatures. Secondly, we point out that the situation as depicted in Fig. 2 should be interesting for the cyclotron resonance and emission in non-equilibrium situations. Namely, the electrons excited to higher energies will relax down to the $n = 1$ level across the continuum spectrum, so that the population inversion across $n = 0$ and $n \geq 1$ should be easier to be realized.

**Relaxation processes**

To quantify this idea, we have to consider the relaxation processes which should control the population inversion. For the ordinary quantum Hall systems the relaxation processes have been extensively discussed. Specifically, Chaubet et al. [8, 9] discussed dissipation mechanisms, where spontaneous photon radiation and coupling with phonons are examined on the basis of Fermi’s golden rule. When the wavelength of light is much larger than the cyclotron radius, as usually the case, we have for the photon emission efficiency

$$W_{i \rightarrow f} = \frac{2\pi V}{h} \frac{\epsilon}{\pi c^3} \int \omega^2 d\omega' \frac{e^2 h}{2\epsilon_0 V \omega'} |\langle i |v|f \rangle|^2 \delta(h\omega' + \epsilon_f - \epsilon_i) = 4\alpha \left( \frac{|\langle i |v|f \rangle|}{c} \right)^2 \omega,$$

where $|\langle i |v|f \rangle|$ is the wavefunction (energy) in the initial state while $f$ stands for the final states, $c$ is the velocity of light, $\alpha = e^2/(4\pi \epsilon_0 hc)$ the fine-structure constant, and we put $\hbar \omega = \epsilon_f - \epsilon_i$ to be the cyclotron energy $\hbar \omega_c$.

A peculiarity of graphene appears in the current matrix element, for which the rate of spontaneous emission reads

$$W_{\text{graphene}}^{n+1 \rightarrow n} = \left\{ \begin{array}{ll}
\frac{2\alpha (\frac{\epsilon}{c})^2}{\alpha (\frac{\epsilon}{c})^2} \omega & (n = 0), \\
\alpha (\frac{\epsilon}{c})^2 \omega & (n \neq 0). 
\end{array} \right. \tag{1}$$

This expression shows that the spontaneous emission rate depends linearly on the cyclotron energy and quadratically on the Fermi velocity. This is in sharp contrast with the ordinary QHE systems. In this case the velocity matrix element $|\langle n |v|n + 1 \rangle|^2 = (n + 1)\hbar \omega/2m^*$ should be plugged in eqn.(1), which yields $W_{\text{2DEG}}^{n+1 \rightarrow n} = 2(n + 1)\alpha \hbar \omega^2/m^*c^2$. This reveals a dramatic difference between graphene and usual 2DEG, where the emission rate in the latter is proportional to the square of the cyclotron energy.

We can quantitatively realize the difference: with $B = 1 \text{ T}$, the cyclotron energies are $\hbar \omega = \hbar eB/m^* /c \approx 1.7 \text{ meV}$ for GaAs, and $v_Fv\sqrt{2\hbar eB} \simeq 37 \text{ meV}$ for graphene. So with the value of graphene Fermi velocity $v_F = 1.06 \times 10^6 \text{ m/s}$[14], we obtain

$$W_{i \rightarrow f} \left\{ \begin{array}{ll}
\propto B^2 \simeq 6 \times 10^4 (s^{-1}) & (\text{GaAs}), \\
\propto \sqrt{B} \simeq 1 \times 10^7 (s^{-1}) & (\text{graphene}),
\end{array} \right.$$  

where we show the $B$-dependence along with numerical values for $B = 1 \text{ T}$. A conspicuous difference, $\propto B^2$ in the former and $\propto \sqrt{B}$, leads to the spontaneous photon emission rate enhanced orders of magnitude in graphene in moderate magnetic fields (as in the above numbers quoted for $B = 1 \text{ T}$.) This indicates that the present system is indeed favorable for a realization of the envisaged Landau level laser. We can further argue that competing processes such as phonon emission is negligible in [10].

**Dependence on the disorder**

We finally examine the dependence of the optical conductivity on the degree of disorder. As we vary impurity concentration $\Gamma$, the Landau-quantized structure in DOS broadens (Fig. 4(a)). As the disorder is increased, higher Landau levels begin to merge and $n = 0$ Landau level stands
alone due to uneven energy spacing, until the $n=0$ Landau level eventually merges with others. Accordingly, the peaks in the optical conductivity in Fig.4(b) decay due to $\sigma(\omega)$. To realize our scenario of population inversion, the disorder should be strong enough to make $n=0$ Landau level stand alone, but should be not too strong to make $n=0$ Landau level merged or the peak in the optical conductivity small for an efficient optical pumping. From the result we can see that there is such a region for an intermediate strength of $\Gamma$.

Summary
We conclude that the uneven Landau levels, unusual cyclotron energy, unusual transition selection rules in graphene all work favorably for a population inversion envisaged for the Landau level laser. An estimate of the photon emission rate shows that the emission rate is orders of magnitude more efficient than in the ordinary QHE system. Important future problems include the examination of the actual lasing processes including the cavity properties.

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