AN INSTRUMENT FOR INVESTIGATION OF THE COSMIC MICROWAVE BACKGROUND RADIATION AT INTERMEDIATE ANGULAR SCALES

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ABSTRACT

We describe an off-axis microwave telescope for observations of the anisotropy in the cosmic microwave background (CMB) radiation on angular scales between 0°.5 and 3°. The receiver utilizes cryogenic high electron mobility transistor (HEMT) amplifiers and detects the total power in multiple 3 GHz wide channels. Both frequency and polarization information are recorded allowing discrimination between CMB radiation and potential foreground sources and allowing checks for systematic effects. The instrumental radiometric offset is small (~1 mK). Data are taken by rapidly sampling while sweeping the beam many beamwidths across the sky. After detection, a spatio-temporal filter is formed in software that optimizes the sensitivity in a multipole band in the presence of atmospheric fluctuations. Observations were made from Saskatoon, Saskatchewan (SK), Canada, during the winter of 1993 with six channels between 27.6 and 34.0 GHz, in 1994 with 12 channels between 27.6 and 44.1 GHz, and in 1995 with six channels between 38.2 and 44.1 GHz. The performance of the instrument and assessment of the atmospheric noise at this site are discussed.

Subject headings: cosmic microwave background — cosmology: observations — instrumentation: detectors — space vehicles

1. INTRODUCTION

In most cosmological models, the anisotropy in the cosmic microwave background (CMB) is a direct probe of the conditions in the early universe. A complete characterization of this ~30 µK signal can potentially tell us about the large-scale geometry of the universe, the Hubble constant, the source of primordial density fluctuations, the fraction of the universe made of baryons, and the ionization history (Bond et al. 1994; Jungman et al. 1996). Measurements of the CMB anisotropy over a wide range of frequencies and angular scales are currently being conducted. Reviews of the results and theoretical issues may be found in Bond (1997), Readhead & Lawrence (1992), and White, Scott, & Silk (1994). In this paper, we describe a ground-based telescope that operates between 28 and 45 GHz and measures angular scales between 0°.5 and 3° (roughly between multipoles l = 60 and 360). Data have been taken with both Ka-band (28–34 GHz) and Q-band (38–45 GHz) receivers. Observations were made in the winters of 1993, 1994, and 1995 at an observing site in Saskatoon, Saskatchewan (SK), Canada. We refer to the results from the three years as SK93, SK94, and SK95. Previous results and limited information about the instrument are given in Wollack et al. (1993) (SK93), Wollack (1994) (SK93), Wollack et al. (1994), Page et al. (1994), Netterfield (1995) (SK94), and Netterfield et al. (1995) (SK93 to SK94 comparison). An analysis of the data from all three years is in Netterfield et al. (1997).

Figure 1 shows the configuration of the SK experiment. A mechanically cooled HEMT-based (high electron mobility transistor) receiver senses the sky in three frequency bands and two polarizations. The beam is formed by a cryogenic feed and an ambient temperature off-axis parabolic reflector. The beam is steered on the sky by a large computer-controlled chopping plate that oscillates about the vertical axis at ~4 Hz. As the plate moves, the receiver outputs are rapidly sampled. The elevation of the beam is fixed at 52°, the latitude of Saskatoon, SK. To point the telescope, the receiver, parabolic reflector, near ground-screen, electronics, and chopping plate are moved in azimuth as a unit. All of the beam-forming optics are inside a large fixed aluminum ground screen.

For all measurements of the anisotropy of the CMB, the signal is between 10⁻⁶ and 10⁻⁸ of the ambient temperature. Because it takes anywhere from a few hours to hundreds of hours of integration to distinguish the celestial signal from the atmospheric emission and instrument noise, great care must be taken to ensure that the signal is fixed to the sky and not due to a systematic effect buried in the instrument noise. Generally, systematic effects modulate the baseline or “offset” of the radiometer. The offset is the instrumental contribution to a differential measurement of a uniform sky. In principle, this is zero. In practice, offsets are typically between tens of µK to a few mK. It may be caused by atmospheric gradients, thermal or emissivity gradients in the optics, polarized emission from the optics, misalignment of the optics, asymmetric sidelobe contamination, microphonics, or electronic pickup. In most all experiments (one exception is the OVRO RING experiment; see Myers, Readhead, & Lawrence 1993), this term is subtracted from the data before analysis. The larger the offset, the more vulnerable the data are to the environment and quirks of the instrument. Just as important as the magnitude is the offset’s stability.

Since the discovery of the CMB, researchers have realized that the radiometric offset can be minimized by moving only the optical element furthest from the receiver (Partridge & Wilkinson 1967; Fabbri et al. 1980; Lubin, Epstein, & Smoot 1983; Davies et al. 1987; Dall’Oglio & de Bernardis 1988; Dragovan et al. 1994). If the beam is scanned only in azimuth, the atmospheric signal is mini-
mized. The remaining dominant sources are modulated emission from and spill past the edges of the chopping plate. For the Saskatoon experiments, the offset depends on the year and the observing strategy. In 1993 and 1994, it was \( \sim 400 \) \( \mu \)K; in the worst case in 1995, it was 2.2 mK. The change in average offset was less than 7 \( \mu \)K day\(^{-1}\) for all observations.

For clarity, this paper focuses on the \( K_s \)-band system, but frequent references are made to the two different Q-band configurations that share the same observing platform. In §2 the receiver is discussed. In §3 we outline the telescope design and performance. Calibration, radiometric offsets, and atmospheric seeing are reviewed in §§4, 5, and 6, respectively.

2. THE RECEIVER

The receiver is comprised of a mechanically cooled Dewar at 15 K, which houses the HEMT amplifiers and a single feed horn, and a thermally regulated 300 K enclosure, which houses the warm amplifiers and band-defining microwave components. Figure 2 shows a schematic of the \( K_s \) receiver. It operates in total-power base-band mode; that is, there are no Dicke switches, mixers, or local oscillators. Similar receivers are discussed in Gaier et al. (1992) and Gundersen et al. (1994). The receiver is positioned so that the feed horn's phase center is within 0.5 cm of the rotation axis of the chopper, which is in turn near the center of a large fixed ground screen. The geometry is shown in Figure 1.

2.1. Receiver RF Design

Radiation from the sky enters the receiver through a 9 cm diameter vacuum window made of 0.38 mm polypropylene. Two overlapping aluminum baffles define the entrance aperture. One is anchored to the \( \sim 70 \) K stage, the other to ambient temperature. Strips of aluminized Mylar electrically connect the top of the feed to the warm baffle and suppress RF interference. To inhibit the formation of frost on the vacuum window, warm air is forced into a volume in front of the window defined by a low-loss polystyrene foam ring (Eccofoam PS, Emerson & Cuming Inc., Canton, MA) covered with a 0.09 mm polyethylene sheet.

The beam is formed by a conical corrugated scalar feed (see, e.g., Clarricoats & Olver 1984, and references therein). At the base of the feed, a wide band \( TE_{11} \)-to-\( HE_{11} \) mode converter (James & Thomas 1982; Wollack 1994) matches the circular waveguide to the hybrid mode. For the \( K_s \) system, an electroformed adiabatic round-to-square transition matches the feed to an orthomode transducer (OMT), which splits the incident radiation into vertical and horizontal linear polarizations. The low end of the feed's bandpass is defined by the input waveguide cutoff, and the upper end is set by the excitation of \( TM_{11}^\circ \) in the horn-to-circular-guide transition. Over the 26-to-36 GHz bandpass of the \( K_s \) receiver, the reflection coefficient is less than \(-26 \) dB and between 23 and 48 GHz it is less than \(-20 \) dB. The Q-band feed for SK94 is scaled from the \( K_s \)-band system and has similar performance. The design parameters for all of the feeds are given in Table 1.

The outputs of the OMT are fed directly into two National Radio Astronomy Observatory (NRAO) amplifiers (Pospieszalski et al. 1988; Pospieszalski 1992). These devices have approximately 10 GHz of available bandwidth and +30 dB of RF gain. When cooled to \( \sim 15 \) K, the average noise temperatures of the \( K_s \) and Q-band amplifiers are \( \approx 50 \) and \( \approx 20 \) K, respectively.

The HEMT amplifiers are connected to ambient temperature isolators by a 10 cm length of 0.25 mm wall stainless steel WR28 waveguide (WR22 for Q-band), gold plated to minimize attenuation. The waveguide vacuum windows between the refrigerator and the room temperature receiver box are made of 0.013 mm thick kapton sheet. Following the isolators are commercial wideband amplifiers with \( \sim +50 \) dB of gain. These amplifiers have a noise figure of 4–7 dB (440–1200 K). Split-block attenuators are used to flatten the bandpass and set the signal level presented to the
The highest usable passbands in order to maximize spectral discrimination and passband flatness, and minimize noise. The highest usable frequency is set by the OMT. In the E-plane port (called A channels), the upper band edge is set at 35.6 GHz to avoid spurious spikes due to the mode in the OMT. The highest frequency band circulators form a filter bank as shown in Figure 2.

In the receiver, half-wave waveguide filters and full-band circulators form a filter bank as shown in Figure 2. The 10 GHz HEMT bandwidth is divided into three passbands in order to maximize spectral discrimination and passband flatness, and minimize noise. The highest usable frequency is set by the OMT. In the E-plane port (called A channels), the upper band edge is set at 35.6 GHz to avoid spurious spikes due to the mode in the OMT. The passband of the H-plane port (called B channels) smoothly degrades above 36 GHz. The highest frequency filters are tailored for maximum bandwidth in response to this difference. Some care must be taken with the lowest channel filter. Waveguide dispersion moves the "second harmonic" of the filter response to lower frequencies than is expected in a TEM structure (Matthaei, Young, & Jones 1980). A "picket fence" filter with a 26-to-32 GHz bandpass in series with the filter is necessary to prevent leakage near 38 GHz into the high-frequency channels. The large number of waveguide joints in the filter bank make them especially prone to microphonics. This problem was overcome by embedding them in a mechanically stable and electrically lossy contoured support structure. In the Q-band system, all the filtering is done with a custom built, monolithic, channel-dropping multiplexor. A listing of the major components used in the two receivers is given in Table 2.

A negative polarity Schottky barrier diode is mounted on the output of each waveguide filter. A typical sensitivity is 1500 mV mW~1~, and the response is linear in power for input levels small compared to −10 dBm (with a 1 MΩ diode detectors in the K_a receiver. In the Q-band receiver, a relatively large HEMT gain slope and the frequency multiplexor design necessitated located the level set attenuator before the room temperature amplifiers to avoid gain compression. In both receivers, the contribution to the total system temperature from the components after the HEMT amplifiers is less than 2 K.

In the K_a receiver, half-wave waveguide filters and full-band circulators form a filter bank as shown in Figure 2. The ≈10 GHz HEMT bandwidth is divided into three passbands in order to maximize spectral discrimination and passband flatness, and minimize noise. The highest usable frequency is set by the OMT. In the E-plane port (called A channels), the upper band edge is set at 35.6 GHz to avoid spurious spikes due to the TM_{10} mode in the OMT. The passband of the H-plane port (called B channels) smoothly degrades above 36 GHz. The highest frequency filters are tailored for maximum bandwidth in response to

### Table 1

**Telescope Specifications**

| Property                                      | SK93K_a | SK94K_a/Q | SK95Q |
|-----------------------------------------------|---------|-----------|-------|
| Feed semiangle, $\theta_0$ (deg)             | 6.0     | 6.0       | 4.4   |
| Feed aperture diameter, $d_y$ (cm)           | 4.2     | 4.7/3.0   | 2.1   |
| Feed normalized phase error, $\Delta$        | $<0.1$  | $<0.1$    | $<0.06$ |
| Feed phase center, $(r_y)$ (cm)              | 1.5     | 1.5/1.0   | 0.24  |
| Feed hybrid frequency, $v_h$ (GHz)           | 29.9    | 29.9/39.7 | 39.7  |
| Feed beamwidth, $\theta_{beam}$ (deg)        | 18      | 18/20     | 27    |
| Primary edge illumination (dB)               | $<-24$  | $<-24$    | $<-27$ |
| Primary offset angle, $\theta_{off}$ (deg)   | $+55.0$ | $+55.0$   | $+55.0$ |
| Primary minor axis, $2a$ (cm)                | 58.4    | 58.4      | 122.0 |
| Primary major axis, $2a$ (cm)                | 65.8    | 65.8      | 137.5 |
| Primary focal length, $f$ (cm)               | 50.0    | 50.0      | 75.0  |
| Primary surface tolerance, $\Delta_{mm}$ (μm) | 8       | 8         | 10−20 |
| Primary aperture efficiency, $\eta_{ap}$     | 0.52    | 0.54      | 0.56  |
| Primary beamwidth on sky, $\theta_{beam}$ (deg) | 1.44   | 1.42/1.04 | 0.5   |
| Chopper edge illumination (dB)               | $<-45$  | $<-45$    | $<-30$ |
| Chopper shield edge illumination (dB)        | $<-65$  | $<-65$    | $<-50$ |
| Far ground screen edge illumination (dB)     | $<-65$  | $<-65$    | $<-65$ |
| Chopper width, $w_c$ (cm)                    | 91.4    | 91.4      | 146.1 |
| Chopper height, $h_c$ (cm)                   | 121.9   | 121.9     | 207.6 |
| Chopper surface tolerance, $\Delta_{mm}$ (μm) | $<-40$  | $<-40$    | $<-40$ |
| Chopper aperture efficiency, $\eta_{ap}$     | 0.16    | 0.17      | 0.27  |
| Chopper step response transition time (ms)   | 29      | 32        | ...   |
| Chopper frequency, $f_v$ (max) (Hz)          | 10      | 6         | 3     |
| Chopper amplitude in AZ, $\phi_v$ (max) (deg) | ±2.5    | ±3.5      | ±3.5  |

### Table 2

**Radiometer Components**

| Component                  | Part Number | Source                                      |
|----------------------------|-------------|---------------------------------------------|
| SK-K_a                    |             | Atlantic Microwave Corp., Bolton, MA       |
| Orthomode transducer      | P/N OM3800  | NRAO, Charlottesville, VA                   |
| Cryogenic amplifiers      | S/N 6, 14   | Microwave Resources Inc., Chino, CA        |
| Warm amplifiers           | P/N SMW92-1953 | Avantek, Inc., Santa Clara, CA             |
| Bandpass filters "A"      | P/N BFF-28-27.5, 30.5, 33.5 | Dorado Int. Corp., Seattle, WA             |
| Bandpass filters "B"      | P/N F27.5-6, 30.5-6, 34.0-6 | Space Labs, Santa Barbara, CA             |
| Diodes                    | P/N DXP-28  | Millitech Corp., South Deerfield, MA       |
| SK-Q                      |             | Gamma-f Corp., Torrance, CA                |
| Orthomode transducer      | P/N 110265-1 | Microwave Tech, South Deerfield, MA       |
| Cryogenic amplifiers      | S/N 6, 14   | Hughes, Microwave Prod. Div., Torrance, CA |
| Isolators                 | P/N 45162H-1000 | DBS Microwave, El Dorado Hills, CA        |
| Warm amplifiers           | P/N DB93-0474 | Pacific Millimeter Products, Golden, CO    |
| Triplexers                | P/N 3647    | Millitech Corp., South Deerfield, MA       |
| Diodes                    | P/N DXP-22  | Millitech Corp., South Deerfield, MA       |
audio impedance. Optimal performance is achieved with 
−18 dBm of RF power at the diode. By tuning each chain's
split-block attenuator, all channels are within 3 dB of this
value. The typical DC level is ≈ 20 mV. A low-noise preamp
with a gain of 100 and a 2 kΩ input impedance is mounted
on the SMA output of each diode. This impedance is a
reasonable compromise between the reduction in sensitivity
(compared to 1 MΩ) and increased signal linearity and
thermal stability. The radiometer noise is approximately 10
times the preamp noise floor.

2.2. Receiver Thermal and Mechanical Considerations

The cryogenic components are cooled by a CTI 350
refrigerator, which allows uninterrupted operation of the
receiver for extended periods of time. All mechanical and
electrical connections from the cold stage to 300 K are
thermally anchored to the refrigerator ≈ 70 K stage. The
bias wiring, waveguide connections to the room tem-
perature receiver, and radiation loading of the refrigerator
result in a total cold stage thermal load of 950 mW. With a
CTI 8500 compressor, the cold stage runs at ≈ 12 K,
roughly 2 K warmer than without any thermal load. A rms
thermal variation of ≈ 40 mK synchronous with the
refrigerator 1.2 Hz drive is measured at the cold station.
This signature is not detected in the radiometer outputs.

By design, the cold stage vibrational amplitude is less
than 0.013 mm (CTI 1971). The vibration power spectrum is
dominated by a 1.2 Hz line from the cold head cycle and by
the broadband feature at ≈ 200 Hz resulting from gas flow
through the refrigerator. Slight changes in RF impedance
causred by vibrations can be synchronously modulated by
the chopper. This microphonic sensitivity was eliminated
through the use of monolithic subassemblies, adequate
strain relief throughout the receiver chain, and a waveguide
flange alignment of less than 0.05 mm. In particular, micro-
phonics were traced to loose attenuator vanes, isolator
tuning networks, and waveguide joints.

The room temperature receiver box is rigidly mounted on
the Dewar. It has two levels of RF shielding and is filled
with microwave absorber to limit air convection as well as
damp microwave energy. A G-10 fiberglass spacer mechanically
supports the inner box and provides the thermal path
between the boxes with a several hour time constant.

The diode sensitivity and amplifier gain are a function of
temperature. Double regulation of the temperature of the
receiver enclosure is required to allow field operation
between −45° C and 0° C. The outside of the enclosure is
regulated to be 10° ± 2° C, while the inside is regulated at
28° C to less than ±0.035° C. In addition, the RF com-
ponents are fitted with insulation to prevent convective
cooling and are thermally anchored to a common aluminum
mounting plate. With the inner regulator on, the gain
coefficient is less than −0.03 dB K−1 for temperature
changes on the outside of the receiver box.

2.3. Radiometer Noise and Passbands

We use two different means to measure the system noise
in the laboratory. In the first method, a variation on the
Y-factor technique (see, e.g., Pozar 1990), the power from a
cold load is varied and the radiometer response is recorded
for a series of points. Over the region of linear response, the
sensitivity and noise are related to the slope and the inter-
cept of the resulting data. This technique measures the
mean noise power. In the second method, the audio fre-
quency power spectrum of the calibrated receiver is mea-
sured directly with a spectrum analyzer. It is important to
recall that an estimate of the spectral density determined
from the system temperature (e.g., via the Y-factor
 technique) and the RF effective bandwidth underestimates
the actual noise because of the presence of low-frequency
fluctuations. Only at frequencies large compared to the
receiver 1/f knee will these two techniques yield the same
result.

In the power spectrum of the noise at the diodes, as
shown in Figure 3, the 1/f character is readily apparent.
This was shown to be due to gain fluctuations in the HEMT
amplifiers (Jarosik et al. 1993). The total power relative
variance is well modeled by a modified radiometer equation
(e.g., Rohlfs 1990),

\[
\frac{\Delta T}{T_{sys}} = \frac{1}{\Delta v_{rf} \tau} + \left[ \frac{\Delta G(f)}{G} \right]^2
\]

where \((\Delta G/G)^2 \propto 1/f^2\) is the square of the fractional receiver
gain variation. The dominant contribution to the gain fluc-
tuations in the system is the cryogenic HEMT amplifier.
The measured variations in the Kα band HEMT gain are
\((\Delta G/G) \sim 2 \times 10^{-2}\) at 8 Hz with \(\alpha \approx 0.9\). Both the magni-
tude of the fluctuations and \(\alpha\) are weakly dependent on the
transistor bias (Wollack 1995). A summary of the measured
sensitivities is given in Table 3.

The gain fluctuations correlate data from different chan-
nels of a single amplifier chain. With the independent A and

![Fig. 3.—Typical power spectra, \(S_{\nu}\), of the receiver output. The data
were taken while observing the sky during good weather. For SK95Q, the
two-point and three-point offsets (see §§ 4.1 and 5) are manifest as the
spectral features at 3 and 6 Hz (\(\zeta < 1.5 \text{ mK s}^{1/2} \text{ deg}^{-1}\)). For SK94Kα
only the two-point offset at 4 Hz is evident (\(\zeta < 0.9 \text{ mK s}^{1/2} \text{ deg}^{-1}\)). Data taken
under laboratory conditions have an indistinguishable 1/f component that
originates from the cooled HEMTs. Given the approximately equal band-
widths in the \(K_\alpha\) and Q systems, the higher Q-band 1/f knee reflects a lower
gain stability. At \(f \gg 100\) Hz the power spectra for both radiometers agree
with the sensitivity derived from the measured RF bandwidth and system
temperature. At 8 Hz, these results may be compared to \(S_{\nu}\) in Table 3.

The slight differences between the plot and the table result from the atmo-
spheric noise contribution.
channel correlation, instrumental or atmospheric in origin, value as measured in the lab with a cold load. Any inter-
S
Noor, the cross-polarization frequency-frequency corre-
atmospheric fluctuation approaches the receiver noise
those intrinsic to each amplifier. As the magnitude of the
common source, for instance a fluctuating atmosphere, and
those intrinsic to each amplifier. As the magnitude of the
atmospheric fluctuations approaches the receiver noise
those intrinsic to each amplifier. As the magnitude of the

tions, one can distinguish between correlations due to a
channel correlation, instrumental or atmospheric in origin, value as measured in the lab with a cold load. Any inter-
S
Noor, the cross-polarization frequency-frequency corre-
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those intrinsic to each amplifier. As the magnitude of the

\[ \Delta v_{cf} = \int_{0}^{G(v)} (G(v) dv) \]

where \( G \) is the power response profile of the microwave
bandpass (Dicke 1946). In a typical channel, the bandpass ripple results in a \( \sim 5\% \) reduction in sensitivity relative to an ideal filter response. Similarly, the center frequency is
computed from

\[ v_c = \frac{\int_{0}^{\infty} v T_b(v) G(v) dv}{\int_{0}^{\infty} T_b(v) G(v) dv} \]

where \( T_b \) is the brightness temperature of the calibration source. The center frequency for a blackbody and a syn-
chrotron calibration source (e.g., Cas A) differ by \( \sim 200 \)
MHz for a typical channel. The effective bandwidths and center frequencies for each channel are given in Table 3.

### 2.4. The Data Collection System

An 80386-based computer located outside the fixed
ground screen controls the telescope and data acquisition.
The six receiver channels are read with V/F (voltage-to-
frequency) converters on the telescope. Counter cards on
the PC backplane synchronously “integrate” the V/F
output for each sample (see Table 4). To have sufficient
dynamic range, the DC level of each channel is subtracted
by a low-noise bucking circuit and the signal is then amplified
by a factor of 5. The signals are filtered by eight-pole
low-pass Bessel filters with a time constant determined by
the sample rate. The system can respond to temperatures
within \( \pm 5 \) K from the nominal level. In practice, we find a
4 K window is sufficient for all the usable sky data (see § 6).
The chopper and base position are recorded at the same
rate as the primary data. Housekeeping data are read by a
12 bit A/D in the PC at a slower rate. One master clock
controls the V/F converters, the counter gates, and the
chopper/base pointing update. In 1995, roughly 1 Gbyte

### Table 3

| Channel | \( v_c \) (GHz) | \( \Delta v_{cf} \) (GHz) | \( T_b \) (K) | \( S_m(b) \) (mK \( s^{1/2} \)) | \( S_e(a) \) (mK \( s^{1/2} \)) |
|---------|----------------|----------------------|-------------|-----------------|-----------------|
| K_a-A1  | 27.6 ± 0.15    | 2.8 ± 0.2            | 44          | 1.1             | 1.6             |
| K_a-A2  | 30.5 ± 0.15    | 2.9 ± 0.2            | 38          | 1.5             | 1.9             |
| K_a-A3  | 33.4 ± 0.15    | 2.8 ± 0.2            | 53          | 1.3             | 1.6             |
| K_a-B1  | 27.5 ± 0.15    | 2.9 ± 0.2            | 29          | 1.1             | 1.5             |
| K_a-B2  | 30.5 ± 0.15    | 2.8 ± 0.2            | 47          | 1.3             | 1.7             |
| K_a-B3  | 34.0 ± 0.15    | 3.7 ± 0.2            | 51          | 1.4             | 1.7             |

* The reported receiver noise temperatures, \( T_b \), are measured at the
  OMT circular waveguide input flange. The uncertainty is \( \pm 3 \) K.

b Receiver noise (total power) measured in the lab at 8 Hz using a 15 K
cold load for \( K_a \), and 20 K for the Q-band system. The increase over the
ideal is due to imperfect passbands and "1/f" noise in the HEMT ampli-

| Channel | \( v_c \) (GHz) | \( \Delta v_{cf} \) (GHz) | \( T_b \) (K) | \( S_m(b) \) (mK \( s^{1/2} \)) | \( S_e(a) \) (mK \( s^{1/2} \)) |
|---------|----------------|----------------------|-------------|-----------------|-----------------|
| Q-B1    | 38.2 ± 0.12    | 2.5 ± 0.1            | <10         | 1.8             | 2.3             |
| Q-B2    | 40.7 ± 0.12    | 4.1 ± 0.1            | 10          | 2.5             | 3.0             |
| Q-B3    | 44.1 ± 0.12    | 3.3 ± 0.1            | 15          | 4.0             | 5.1             |

### Table 4

| Parameter                        | SK93K_a | SK94K_a/Q | SK95Q |
|----------------------------------|---------|-----------|-------|
| Elevation angle, \( \theta_{ele} \) (deg) | +52.25 ± 0.06 | +52.16 ± 0.06 | +52.24 ± 0.01 |
| Primary beamwidth, \( \theta_{bwm} \) (deg) | 1.44 ± 0.02 | 1.42 ± 0.02 | 1.04 ± 0.02 |
| Point source sensitivity, \( T(v) \) (\( \mu K \) Jy \( ^{-1} \)) | 49      | 50/52     | 230    |
| Chopper frequency, \( f_c \) (Hz) | 3.906   | 3.906     | 2.970  |
| Sample frequency, \( f_s \) (Hz) | 62.5    | 250       | 500    |
| Sky detection frequency (Hz)     | 12      | 8–32      | 6–70   |
| Samples per chop, \( N_s \)       | 16      | 0.6 ± 0.1 | 0.0 ± 0.1 | 0.0 ± 0.002 |
| Chopper phase offset, \( \delta N \) | 64      | 168       | 12     |
| Beam per throw                   | 4       | 5/7       | 17     |
| Chopper amplitude on sky, \( \theta_{chp} \) (deg) | ±2.45   | ±3.51/±3.68 | ±5.97  |
| Base wobble amplitude on sky, \( \theta_{bw} \) (deg) | ±4.91   | ±4.42     | ±4.50  |
| Observing polarization, \( A \)   | ↑→      | ↑→        | ↑→     |

* Note — The vertical polarization for SK94Q is indicated by " ↑" in the table. A waveguide joint in the Q-band
radiometer was damaged during shipping, and the data was not used in analysis (see Netterfield et al. 1995).

The bandpass is determined by measuring the power response in each band with a leveled Hewlett Packard
8690B RF swept source at the orthomode transducer flange. The effective bandwidth is defined as

\[ \Delta v_{cf} = \frac{\int_{0}^{G(v)} (G(v) dv)^2}{\int_{0}^{G(v)} G(v)^2 dv} \]

where \( G \) is the power response profile of the microwave

B polarizations, which are not correlated by gain fluctuations,
one can distinguish between correlations due to a
common source, for instance a fluctuating atmosphere, and
those intrinsic to each amplifier. As the magnitude of the
atmospheric fluctuations approaches the receiver noise
floor, the cross-polarization frequency-frequency corre-
lations \( \langle AA \rangle, \langle BB \rangle \) approach the same value as measured in the lab with a cold load. Any inter-
channel correlation, instrumental or atmospheric in origin,
must be accounted for in assessing the statistical signifi-
cance of the data (Wollack et al. 1993; Dodelson,
Kosowsky, & Myers 1994).
day$^{-1}$ of data were recorded. The telescope-control/data-acquisition PC is connected via Ethernet to a workstation where the data are stored, reduced, and analyzed.

### 2.5. Electrical Offsets and Interference

Synchronous electrical pickup and crosstalk in the system are small. A monitor diode and preamp in the receiver with a factor of $1 \times 10^3$ higher audio gain than the other channels indicates that synchronous electrical signals are less than 0.1 $\mu$K. The receiver was tested in the lab with a regulated cold load as the radiation source while the chopper underwent a three position 4 Hz chop (more gentle motions were used in the field). This maximizes the chopper drive current and vibration. After a $\sim 1$ day coherent average of the data, an absolute upper limit of less than 10 $\mu$K is set on all audio frequencies of interest on any potential electrical-magnetic or microphonic pickup. In the field, the signals from the quadrature phase$^3$ of the weighted data are stable and are typically $\sim 20 \pm 10$ $\mu$K; in worst case for SK95Q, less than 70 $\mu$K.

The Saskatoon airport radar operates in L-band ($\sim 1$ GHz) and sweeps our site every 8 s. This signal is not detected in the data or in the monitor channel at the 20 $\mu$K level. In the laboratory the receiver and data collection system were illuminated with a $+12$ dBm L-band source, again no signal was seen at the 10 $\mu$K level. There is no indication of RF interference in any of the data.$^4$

### 3. Antenna Design and Performance

An offset-parabolic reflector fed by a corrugated feed has minimal blockage, approximately equal E/H-plane beamwidths, and relatively low sidelobe response (Rudge et al. 1982, chap. 3). These properties are ideal for CMB observations. The radiometer beam is formed by a cooled corrugated feed horn that underilluminates an ambient temperature paraboloid, which in turn illuminates the chopping flat. There are two levels of ground screen shielding: the near ground screen moves with the beam-forming optics, shielding the telescope base and deckling, and the stationary far ground screen shields the Earth and Sun. The telescope is optimized for observations at the elevation of the north celestial pole (NCP) from the site. See Figure 1 and Table 1 for a summary of the telescope geometry and system parameters.

#### 3.1. Primary Illumination and The Main Beam

When an underilluminated parabolic section is fed with a diffraction-limited feed, the resulting beam size is approximately frequency independent. This occurs because the competing effects of the diffraction-limited feed and the

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$^3$ The quadrature phase, which is not sensitive to a signal on the sky, is shifted $90^\circ$ relative to the chopper phase, which is sensitive to the celestial signal. See §4.

$^4$ In the last decade the population of satellites using $K_a$-band communication channels has increased dramatically. There are now approximately 70 satellites in the Space Network Listing (RCB 1993) with on-board equipment operating between 20 and 40 GHz. The bulk of these transmit in our lowest frequency channel from geosynchronous orbits. Given typical satellite beam coverage, orbital parameters, and our scan strategy, the likelihood of RF contamination from a communications satellite is small.

---

underfilled primary cancel. In this limit, the effective diameter of the primary illumination is inversely proportional to frequency.

For a corrugated feed with a small aperture phase error, the full width at half-maximum (FWHM) beamwidth is

$$
\theta_f \approx \sin^{-1} \left( \kappa_b \frac{\lambda}{d_h} \right),
$$

where $d_h$ is the effective aperture diameter and $\kappa_b \approx 78^\circ$ is the beam constant for HE$_{11}$ aperture illumination ($\Delta \approx 0.1$). To the same order of accuracy, the main beamwidth is given by

$$
\theta_b \approx \kappa_p \frac{\lambda}{d_p},
$$

where $\kappa_p \approx 60^\circ$ is the beam constant for the primary illumination profile.

The aperture phase error is the difference in wavelengths between the path from the feed apex to the edge of the aperture and from the apex to the center of the aperture (see, e.g., Thomas 1978). It is defined as

$$
\Delta \equiv \frac{l_h}{\lambda} (1 - \cos \theta_0) = \frac{d_h}{2} \tan \left( \frac{\theta_0}{2} \right),
$$

where $l_h$ is the horn slant length, $d_h$ is the aperture diameter, and $\theta_0$ is the horn semiangle angle. Small $\Delta$ horns are diffraction-limited and thus have a frequency-dependent beam size. Large $\Delta$ feeds have a frequency-independent beam size (the phase error averaged over the feed aperture effectively washes out coherence).

We define $d_p \equiv 2f_{\text{eff}} \tan (\theta_f)$ as the effective aperture illumination diameter and $f_{\text{eff}} \equiv 2f_0/\left[1 + \cos (\theta_{\text{par}})\right]$ as the distance from the focal point to the center of the off-axis parabolic section. Solving for the main beamwidth in the limit $\theta_0 \ll 1$, we find

$$
\theta_b(v) \approx \theta_b(v) \left[ 1 - \epsilon + e \left( \frac{v_c}{v} \right)^2 + \cdots \right],
$$

where

$$
\theta_b(v) = \frac{\kappa_p}{\kappa_b} \frac{d_h}{2f_{\text{eff}}} (1 + \epsilon)
$$

and

$$
\epsilon = \frac{1}{2} \left( \frac{\kappa_h \lambda}{d_h} \right)^2 \ll 1.
$$

The center frequency, $v_c$, is $\sim 30.5$ GHz for $K_a$ and $\sim 41.0$ GHz for Q-band. The approximate magnitudes of $\epsilon$ are 0.051, 0.055, and 0.11 for SK93K, SK94Q, and SK95Q, respectively.$^5$

The telescope is focused by placing the feed phase center at the primary's focal point. For a corrugated horn the

---

$^5$ Eq. (7) suggests that in order to reduce the frequency dependence of the main beamwidth, the overall size of the optical system should be as large as possible for a fixed $\theta_f$. The residual contributions to the beam's frequency dependence that arise from nonideal feed performance and finite edge illumination should also be considered in this limit.
location of the phase center behind the plane of feed aperture, \(t\), for the fundamental Gaussian mode can be expressed as (Martin 1990)

\[
\frac{t}{\lambda} = \frac{(\gamma \Delta)^2}{1 + (\gamma \Delta)^2},
\]

where \(\gamma \equiv 2\pi c^2 \approx 2.603\), \(\lambda_p\) is the slant length of the horn, and \(\Delta\) is the aperture phase error. Thus, for a small \(\Delta\) horn, the phase center is near the aperture of the horn, while for large \(\Delta\) horns the phase center is near the apex. The change in the position of the phase center with wavelength is given by

\[
\frac{\partial t}{\partial \lambda} = -\frac{2\lambda_p}{\lambda} \frac{(\gamma \Delta)^2}{1 + (\gamma \Delta)^2} \left[ 1 - \frac{(\gamma \Delta)^2}{1 + (\gamma \Delta)^2} \right].
\]

Designs for frequency-insensitive waist positions exist for \(\Delta > 1.8\) and \(\Delta < 0.2\). We require that the change in waist position between 26 and 36 GHz is less than \(\sim 1\lambda\), the allowed defocusing error. The feed horn position was adjusted to maximize the forward gain in the center channel. As a result, the upper and lower frequency channels are not optimally focused but the average instrument allowed defocusing error. The feed horn position was insensitive to less than 1.5\% changes, consistent with the model prediction.

The telescope angular response was computed by aperture integration (Sletten 1988). We model the near field feed amplitude and phase with the first 20 modes in a Gaussian-Laguerre expansion (Jaakkola, & Tuovinen (Friberg, 1992; Sletten 1988). We model the near-field feed (Tuovinen 1992) by integrating the resulting fields in the chopper plane.

We ignore the effects of the feed choke rings and vacuum screen edge illumination, phase, and fabrication tolerances. For the SK95Q beam results from a 0.4 cm spacer (inadvertently omitted during alignment), which vertically shifted the feed in the focal plane from the design geometry. The observed increase in the beamwidth along \(\theta_x\) and decrease along \(\theta_y\) is consistent with the phase center offset.

Figure 4 shows a comparison between the computed and measured beam profiles. A summary of the main beamwidth for each channel and polarization is given in Table 5.

The A and B polarizations were characterized in both the E and H planes by illuminating the telescope with a coherent RF source. The far ground screen, which has negligible effect on the main beam, was not present during these measurements. The measured beam efficiency, \(\eta_b \approx 0.99 \pm 0.01\), is in agreement with the model.6 After correction for the 80 m source-to-observation distance (Silver 1949), the measured and computed beamwidths agree to within \(\sim 2\%\). The magnitude of this correction is \(\leq 1\%, 0.4\%,\) and 12\%, respectively, for the SK93K\(_a\), SK94Q, and SK95Q main beamwidths. The computed magnitudes are used to predict the optical performance. However, for the data analysis we rely on celestial calibrators to characterize the telescope angular acceptance.

The primary used for SK93/94 is machined out of a solid QC7 aluminum plate. The final surface has a rms of 8 \(\mu\)m < 0.001\(\lambda\) (Crone 1993). The SK95 mirror (Fixsen 1995) was made out of 6061-T6 aluminum plate in nine pieces, a

TABLE 5

| SK Measured and Modeled Beamwidths |
|-----------------------------------|
| Band     | FWHM | A1/B1 (deg) | A2/B2 (deg) | A3/B3 (deg) | \(\sigma_\theta\) (deg) |
|----------|------|-------------|-------------|-------------|------------------|
| SK93K    | \(\theta_x \approx \theta_y\) | 1.46 | 1.44 | 1.41 | +0.02 |
| SK94K    | \(\theta_x \approx \theta_y\) | 1.44 | 1.42 | 1.41 | +0.02 |
| Modeled  | \(\langle \theta_{v1} \rangle\) | 1.47 | 1.44 | 1.41 | +0.03 |
| SK94Q    | \(\theta_x\) | 1.09 | 1.08 | 1.07 | +0.02 |
|          | \(\theta_y\) | 1.012 | 1.004 | 0.993 | +0.006 |
| Modeled  | \(\langle \theta_{v2} \rangle\) | 1.06 | 1.04 | 1.00 | +0.03 |
| SK95Q    | \(\theta_x\) | 0.471/0.486 | 0.443/0.461 | 0.453/0.496 | +0.01 |
|          | \(\theta_y\) | 0.567/0.538 | 0.525/0.513 | 0.570/0.591 | +0.01 |
| Modeled  | \(\langle \theta_{v3} \rangle\) | 0.513 | 0.505 | 0.499 | +0.02 |

Note.—The best fit to the measured beamwidths in the vertical and horizontal planes is denoted by \(\theta_x\) and \(\theta_y\). The E plane for the “A” polarization is along \(\theta_x\) and for “B” it is along \(\theta_y\). The modeled full width at half-maximum response, \(\theta_b\), is given for the center frequency for the design geometry. The uncertainty reflects the variation in beamwidth with polarization and cut plane (E, H, and diagonal), and the uncertainty in the aperture illumination and telescope alignment. The discrepancy between the modeled and measured response for the SK95Q beam results from a 0.4 cm spacer (inadvertently omitted during alignment), which vertically shifted the feed in the focal plane from the design geometry. The observed increase in the beamwidth along \(\theta_x\) and decrease along \(\theta_y\) is consistent with the phase center offset.

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6 For the SK telescope the aperture diameter is defined by the edge of the paraboloid. The illumination function for the reflector encompasses the first null of the feed pattern over the entire RF bandwidth. In this limit, the beam efficiency is equal to the feed beam efficiency. In estimating the total main-beam solid angle we integrate out to 2.5\(\theta_b\)RF. This is approximately the position of the first null in the response in the limit of a vanishing aperture phase error. The modeled magnitude includes the estimated ohmic, diffractive, and depolarization losses intrinsic to the telescope. The atmospheric transmission efficiency at the site (which can be derived from Fig. 12) is not included in \(\eta_b\).
The rotation axis of the assembly is adjusted to be at the center of mass, thus the front surface moves a small amount laterally as the chopper rotates. The coils are positioned so that an impulse produces minimal force on the pivot. Permanent magnets are mounted to a reaction bar that is also pivoted at its center of mass. A second set of “coupling” coils couples the reaction bar to the frame using a PD control loop. Without them, the reaction bar hits the chopper mount when the telescope base is rotated.

When the coupling coils are turned off, the reaction bar and chopping plate are well modeled as a driven linear oscillator (Radford et al. 1990). The driving force is \( F = 2\pi \rho N_{\text{eff}} i B \), where \( \rho \) is the radius of the coil, \( N_{\text{eff}} \approx 120 \) turns is the number of turns in the magnetic field \( B \approx 2000 \) G, and \( i \) is the peak current through the coils. For the SK94 chopper, the force is approximately 30 N. Air resistance is negligible. A good estimate of the chopper throw is \( \phi_{\max} \approx 2F/\omega^2 \), where \( I \) is the moment of inertia, \( r \) is the distance from the center of the plate to the coils, and \( \omega \) is the drive frequency. Using the values in Table 6, \( \phi_{\max} \approx 4.3 \),

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The design is an extension of smaller choppers built at Princeton by M. Dragovan, J. Peterson, and G. Wright. We benefited from their insights and previous work.

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58 cm center section surrounded by eight petals with a \( \sim 10-20 \mu \text{m} \) rms surface.

3.2. The Chopping Flat

A large flat aluminum honeycomb plate driven by custom voice coils sweeps the beam across the sky in azimuth. A computer sends the requested position signal to a proportional-integral-derivative (PID) control loop similar to that used by Payne (1976) and Radford, Boynton, & Melchiorri (1990). A high-power op-amp drives current through the coils in response to an error signal (e.g., Kuo 1991).

A schematic of the chopper is shown in Figure 5, and the components and dimensions are given in Tables 1 and 6. Coils are wound on and epoxied to a Kevlar substrate, which is in turn epoxied to the back of the chopping flat.

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The figure shows the main beam and feed horn response. The measured and theoretical response for the corrugated feed horns used in the K_a and Q-band systems are presented in the left panels. The solid lines are the E plane, the dashed lines are the H plane, and the symbols are the measured response. The right panels display the measured and computed telescope main-beam maps. The theory curves were computed by aperture integration of the modeled feed illumination and phase. For clarity, only the measured main-beam E plane data for the horizontal polarization (channel B) is plotted. All measurements are at the feed hybrid frequency \( \nu_{\text{f}} \) (see Table 1).
whereas the actual value was 3.5. The measured 10% to 90% response time is ~30 ms. Though the chopper can be square-wave chopped, sinusoidal and rounded triangle motions are used for observing because they result in lower frame vibration and require less electrical power.

The azimuthal angle of the flat is measured with respect to the frame with a rotary-variable-differential transformer (RVDT) calibrated to 0.006 with the absolute encoder that measures the base motion. The angle is sampled at the data rate. For SK93/94 the chopper angle varied by less than 0.01 from the requested position. For SK95, the chopper size was increased from 91 \times 123 \text{ cm} to 146 \times 208 \text{ cm}. Springs were attached to the flat and it was driven on resonance at 3 Hz. This led to a large reduction in the required power (SK94/SK93: 200 W, SK95: 25 W, despite the increase in size) but degraded positioning. At the worst times, the rms deviation from the requested position was 0.045. This has no measurable effect on the data.

The chopper’s magnetic fields have been mapped and do not interfere with the receiver (see §2.5). The DC field within each magnet assembly is \approx 2000 \text{ G} over a 200 cm\textsuperscript{3} volume and drops to less than 5 \text{ G} at 20 cm. The permanent fields are oriented so that the AC components produced by each coil are in opposite direction near the receiver. At the receiver, the field at 3.9 Hz is \approx 0.02 \text{ G}.

The heat produced by the chopper coils is removed by DC fans. Seven sensors monitor the temperature distribution of the front of the plate. The temperature difference between the center and the coil position is about 7 K over 26 cm and is constant.

The surface of the chopping plate is not perfectly flat. At 2.5 cm scales, the deviations are random with a root-mean-squared amplitude of less than 40 \text{ k m}. This results in a transfer of less than 0.005 of the relative forward gain into diffuse sidelobes. (See Ruze 1966; also, Dragone & Hogg 1963 for effects on the wide-angle antenna response.) Print-through of the aluminum honeycomb hexagon pattern produces a periodic array with a \lambda_{\text{hex}} = 0.6 \text{ cm} wavelength and amplitude of 4 \text{ \mu m}. An absolute upper bound on relative gain loss of less than 0.03 is computed by increasing the Ruze estimate by the total number of scatters, \(N \approx (D_{\text{mirror}}/\lambda_{\text{hex}})^2\). From the measured telescope response, we conclude that these surface irregularities have an insignificant effect on the forward gain and sidelobe performance.

TABLE 6
SK94 CHOPPER COMPONENTS

| Assembly               | Subassembly     | Physical Characteristics/Source |
|------------------------|-----------------|---------------------------------|
| Chopping plate        | Chopper flat    | Thickness = 2.59 cm (front/back face = 0.305 mm/0.406 mm) |
|                        |                 | Mass = 5.8 kg, I = 0.36 kg m\textsuperscript{2} |
|                        |                 | Shape: 3' by 5', approximately ellipsoidal |
|                        |                 | Al Honeycomb, M. C. Gill Co., El Monte, CA |
| Flex pivots           |                 | Diameter = 1.27 cm |
|                        |                 | P/N 5016-800, Lucas Aerospace, Utica, NY |
| Angle encoder         | Rotary-Variable-Differential Transformer (RVDT) | |
|                        |                 | P/N RSYN-8/30, Lucas-Schaevitz, Pennsauken, NJ |
| Servo drive           | Nominal 4 Hz Triangle: \pm 70 V DC at 3.4 A (rms) | |
|                        |                 | PA-04, Apex Microtechnology Corp., Tucson, AZ |
| Reaction bar........... | Flex pivots     | Diameter = 1.91 cm |
|                        |                 | P/N 5024-400, Lucas Aerospace, Utica, NY |
| Position encoder      | Linear Variable Differential Transformer (LVDT) | |
|                        |                 | Model 503XS-A, Lucas-Schaevitz, Pennsauken, NJ |
| Servo drive           | Nominal 4 Hz triangle: \pm 15 V DC at 1.4 A (rms) | |
|                        |                 | PA-12, Apex Microtechnology Corp., Tucson, AZ |
| Coils/magnets .......... | Coil            | Resistance = 13.5 \Omega, inductance = 20 \text{ mH} |
|                        |                 | 24 Ga. Cu wire, diameter = 7.9 cm, axis distance = 29 cm |
|                        | Permanent magnets | Diameter = 5.08 cm, length = 4.76 cm |
|                        | NdFeB grade 35, Magnetic sales and Mfg., Culver City, CA |
|                        | Yokes           | Diameter = 12.7 cm, length = 12.7 cm |
|                        |                 | CMI-C low carbon iron, Connecticut Metals Incorp., Waterbury, CT |
3.3. The Ground Screens

The near and far ground screens both block the relatively bright signals from the Earth and Sun and reflect the antenna sidelobes to cold sky. The far ground screen size and angle are designed to shadow the top edge of the chopper baffle for incident rays greater than the angle at which undiffracted rays are reflected normal to the panels (see Fig. 1). The far ground screen is more than 17° from the main beam, a clearance of more than 3.5 times the width of the parabolic section. The ground screen is fabricated out of aluminum angle and 1" × 4" × 8" sheets of metallized housing insulation (Energy Shield, Atlas Roofing Company, Meridian, MS). The metal foil covered side of this housing insulation has an antenna temperature measured at 31.4 GHz of ~320 mK for a 45° incidence angle. This is a factor of ~1.2 times the emissivity computed from the DC conductivity of the aluminum surface. Deviations in the surface of this material are on the order of 0.2 cm over a span of a few centimeters and large-scale camber errors are not uncommon. In effect the sky, which has an order of magnitude lower antenna temperature than the ground, is being used as a termination for the antenna’s sidelobes.

The sidelobe response of the telescope, including the near and far ground screens, was measured. The structure was illuminated by a θFWHM = 7° coherent RF source placed at eight locations around the perimeter of the ground screen and on the roof of a building behind the ground screen. The source was pointed ~9° upward from horizontal, aimed at the edge of the closest panel, from 12 m away. It was modulated by a “hand-chopped” AN 73 Eccosorb (Emerson Cuming, Canton, MA) sheet in front of the transmitting horn. The measured sidelobe level is less than ~115 dB for rays impinging upon the ground screen from behind.8 For rays illuminating the front ground screen edges, the response was ~−95 dB. When the mirror is chopped, the demodulated signal was measured to decrease ~10 dB in the E and H planes relative to the direct signal from the source. Similar results were obtained for the Q-band system at 39.5 GHz. The attenuation provided by the ground screen, found by measuring the response to a source directed at the telescope with and without the ground screen erected, is greater than 36 dB. Leakage through the lower seams and panel glue joints limit the net attenuation. The maximum “anisotropy” emission from the ground screen is estimated to be less than 1 μK (see Table 7).

With the exception of the Sun, most foreground sources are more homogeneous than the test source. To the extent signals are not blocked by the ground screens and are inhomogeneous after spatial averaging by the telescope sidelobes, a difference signal is produced. The Kα beam solid angle is ~10−4 sr. The Sun is a 8200 K source at ~1 cm with a solid angle of 6 × 10−5 sr (Allen 1976). Thus, a sidelobe level of ~−100 dB is required to give a response of ~1 μK from incident solar radiation. For the Earth, a 300 K source subtending 2π sr, a sidelobe of ~−130 dB is required if a complete modulation of the incident radiation is assumed. Conservative analytical estimates indicate that differential contributions to the antenna temperature from the sidelobes are less than 10 μK. A host of effects enter at this level: signals can result from diffracted Earth/Sun-shine, changes in sidelobe illumination with base position, and gradients in the ground screen panel temperature and emissivity.

In 1993, the ground screen did not geometrically block the Sun when it was directly behind the rear corners (see Fig. 1). Once the problem was noticed, the ground screen was modified. To be conservative, data taken with the Sun illuminating the top of the chopper were blanked (Wollack et al. 1993). Independent of this precaution, for a given position on the sky, data recorded during the day and night are in agreement and the telescope offsets are independent of the time of day. To accommodate the size of the SK95 optics, the front edges of the far ground screen were extended 46 cm and rolled with a radius of curvature of r ø 23 cm, to reduce the level of diffracted radiation (Keller 1959). With these modifications, the net shielding from the front section of the far ground screen is similar to the level used in SK93/94.

4. Flux Scale Calibration and Pointing

The primary calibrator for both the Kα and Q radiometers is Cassiopeia A.9 Telescope efficiency and atmospheric attenuation are intrinsically included. The calibration signal is on the order of ~10 mK; thus, receiver nonlinearity is not significant. To determine the beamwidth and pointing, the beam is swept in azimuth with the chopper while the Earth’s rotation moves the source through the beam. The data are reduced with an optimal filter for a point source to make a two dimensional map of Cas A. See Figure 6. A two dimensional Gaussian is fit to the resultant map to determine the beamwidth and pointing. A summary of measured beamwidths is given in Table 5. The pointing is stable for the duration of an observing trip, attesting to the rigidity and stability of the telescope platform and footings.

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8 The sidelobe level is measured relative to power received by the main beam, 10 log (P_d/P_0).

9 3C 461, 2321 + 583 IAU(1950), l = 111°7, b = -2°1.
The flux density scale for Cas A was obtained by fitting a power law to data compiled by Baars et al. (1977) from 8.2 to 31.4 GHz and to a measurement by Mezger et al. (1986) at 250 GHz. We find $S_\nu(Cas\ A) = (2070 \pm 162)\nu^{-0.695 \pm 0.029}$ Jy, where $\nu$ is the frequency in GHz (epoch 1994). A secular decrease of the form, $\eta = 0.9903 \pm 0.003 \log(\nu)$ (Baars et al. 1977), where $N_s$ is number of years since the measurement, is assumed in correcting the Cas A flux scale to the observing epoch. The measured point source sensitivity of the telescope, $\Gamma = \lambda^2/2k \delta \Omega$, where $\delta \Omega$ is the total beam solid angle, is used in assigning the flux scale. See Table 4 for the magnitude of $\Gamma(\nu)$ at the synchrotron-spectrum weighted centroid, $\nu_c$. Once the flux scale is assigned, the data are converted to the CMB thermodynamic temperature scale by dividing by the derivative of the Plank function with respect to temperature $< T_{an}/T_{CMB} > = x^2 e^x/(e^x - 1)^2$, where $x \equiv h \nu/kT_{CMB}$. Using $T_{CMB} \approx 2.726$ K (Mather et al. 1994; Gush et al. 1990), the conversion factors to the CMB temperature scale are 1.02 and 1.05 for 27.5 and 44.1 GHz, respectively.

The error in the temperature scale is dominated by the uncertainty in the knowledge of the flux of Cas A and its environment. Cas A passes through an elevation of $52^\circ$:24 twice a day. For Q-band there was $\sim 7\%$ discrepancy and for K, a $\sim 12\%$ discrepancy between the results of measurements taken in the morning and those taken in the evening. This effect is due to $2311 + 611, a \sim 25$ Jy H I region (Kallas & Reich 1980; Fich 1986; Becker, White, & Edwards 1991), which is intercepted by the beam during the evening runs, but not during the morning runs. The uncertainty in our measurement of the relative temperature scale of Cas A is $\sim 5\%$, affecting all channels equally, and the uncertainty in the spectral index is $\pm 0.1$. The temperature scale derived from measurements of Cas A is within 20% of the laboratory sensitivity measurements. The calibration is also affected by small errors introduced by inaccuracies in the beam $\theta_{FWHM}$ (2%), the phase of the recorded signals with respect to the optical axis (<1%), and the finite size of Cas A (<1%). The combination of all of these errors leads to a $\pm 14\%$ uncertainty (1 $\sigma$) in the CMB temperature scale. Reduction of this error will require a more accurate determination of the flux of Cas A.

4.1. Observing Strategy and Beam Synthesis

To measure the sky, the beam is swept in azimuth with a computer commanded pattern, $x_\theta$, with frequency $f_c$ and amplitude $\theta_i$ on the sky. As a second level of modulation, the base is pointed at $x_\theta$; $\theta_i$ is the height of the north celestial pole (NCP) for $\sim 20$ s and $\theta_i$ is east of the NCP for $\sim 20$ s in a manner similar to Timbie & Wilkinson (1990). The instantaneous beam position is given by

$$x' = x_\theta + x_\theta(\theta_i, t) ,$$

where $x_\theta$ and $x_\theta$ are parallel to the horizon. Data taken with $x_\theta$ in the east and west base positions are analyzed independently.

The repositioning of the telescope base in azimuth, or wobbling, allows observation of the same patch of sky approximately every 12 hours. Thus, a real sky signal can be differentiated from a residual 24 hr diurnal effect in the data set. This symmetry on the sky is not quite perfect because circles of constant elevation are not great circles. Nevertheless, the symmetry is good enough to provide convincing systematic checks. Due to the geometry of the ground screens, a signal produced by the Sun is expected to follow a 9 hr--15 hr--9 hr cycle. If such a signal were present, it would be stronger in either the east or west and would interchange in base position from morning to evening. No such signals were observed.

In order to probe a range of angular scales the beam is scanned many beamwidths while rapidly sampling the receiver output. By specifying the relative weight of each sample in software, an effective antenna pattern can be synthesized. For example, if the samples with the chopper positioned in the east are weighted with “-1” and the east samples are “+1,” the beam pattern resembles a classic single difference. In considering the general case, it is useful to define a weighting vector of the form

$$w_i = \sum_{m=1}^{m_{(max)}} a_m(n) \cos(m \omega_c t_i + \phi_m) + b_m(n) \sin(m \omega_c t_i + \phi_m) ,$$

where $\omega_c = 2\pi f_c$ and “$n$” is a convenient label for the vector. The time samples

$$t_i = [i(\tilde{x}') - \tilde{x}']f_c N_s ,$$

are evaluated at the midpoint of each integration bin, where $i$ is the time sample index and $N_s$ is the total number of samples in a chop. A phase difference between the chopper

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11. The chopper azimuth, $\phi_i$, is related to sky throw by, $\theta_i \approx 2\phi_c \cos(\theta_{az})$, where $\theta_{az}$ is the beam elevation.

12. We label a synthesized beam by the observing run year (SK93/94/95), the receiver waveguide band designation (K/Q), and the number of lobes in the response ($n$). For example, for SK93K, a sinusoidal modulation of the beam was used and $a_n$ was the only term in the weighting vector. The resulting pattern has two negative and one positive lobe ($n=3$). This pattern qualitatively resembles a “double-difference” or “three-beam-chop” and is referred to as the three-point response. For the general case, the dominant component of $w_i$ is used to uniquely specify $n$. 

13. The beam solid angle, $4\pi$ divided by the directivity of the antenna, is related to the main lobe solid angle by $\delta \Omega = \eta \delta \Omega$, where $\eta$ is the beam efficiency.
motion and the recorded data stream, \( \delta \phi_x = 2\pi \delta N_x / N_s \), results from the mechanical and electrical responses (see Table 4).

With our chopper phase convention, the \( a_m \) (the signal phase) are sensitive to the sky signal and the \( b_m \) (the quadrature phase) are sensitive to any instrumental effects. The desired number and position of the synthesized lobes on the sky are formed from an appropriate sum of \( a_m \). Physically, the quadrature phase is the difference between the data taken during the clockwise chopper scan minus the data taken during the counterclockwise scan.

As the beam sweeps across the sky, spatial frequencies along the direction of motion are detected over a range of audio frequencies. The anti-aliasing filters and integrators (see Table 4) filter our measurement of the angular power spectrum of the CMB. In 1993, the effect was the largest (\( \sim 5\% \)) and was corrected in the frequency domain by a direct multiplication of the data. Because the response of the combination of the filters is well approximated by a Gaussian, the effect may also be corrected by expressing the component of the main beamwidth parallel to the direction of the chopper throw as a function of the position in the sweep. In other words, the correction is made as a convolution in the spatial domain. This was done in 1994 and 1995 to correct a 12% effect at the highest frequency.

The data are normalized so a small change in temperature of \( \delta T \) from a 2.7 K blackbody filling the positive lobes of the synthesized beam gives a detected signal of \( \delta T \). Thus, the synthesized beams are normalized such that the integral of the area under the positive lobes of the effective antenna pattern is equal to 1:

\[
\frac{1}{2} \int |H(\hat{x}, n)| d\hat{x} = 1,
\]

(14)

where \( H \) is the oriented antenna pattern for the \( n \)-point response. The oriented antenna pattern is

\[
H(\hat{x}, n) = \left\langle \sum_{i=1}^{N_t} w(n) G[\hat{x} - \hat{x}(t_i)] \right\rangle,
\]

(15)

where \( G \) is the antenna angular response, \( \hat{x} \) denotes the arrival direction of photons incident on the telescope, \( \hat{x}' \) is the beam position corresponding to sample \( t_i \) as defined in equation (11), and the angle brackets indicate an average over the field of view of the observation.\(^{13}\) We approximate the telescope angular response by a Gaussian,

\[
G(\hat{x} - \hat{x}') \approx \frac{1}{2\pi \sigma_x \sigma_y} \exp \left[ -\frac{\left| x_\perp - \hat{x}'(t_i) \right|^2}{2\sigma_x^2} - \frac{\left| x_\parallel - \hat{x}'(t_i) \right|^2}{2\sigma_y^2} \right],
\]

(16)

where \( x = x_\perp + x_\parallel, \sigma = \theta_{200}/[8 \ln(2)]^{1/2} \) is the beamwidth, and the subscripts "\( \perp \)" and "\( \parallel \)" indicate the components perpendicular and parallel to the direction of the chopper throw. The gain is normalized with an effective beam solid angle equal to the best fit to the measured antenna response, \( \delta \Omega_b = 2\pi \sigma_x \sigma_y \) (the main-beam response has a greater than 0.98 overlap with a Gaussian spatial distribution). With these conventions, the signal for the \( j \)-th pixel on the sky is

\[
\Delta T^{(j)} = \int H(\hat{x}, n) T_b(\hat{x}) d\hat{x},
\]

(17)

where \( T_b \) is the celestial brightness distribution. Typical examples of the resulting beam sensitivities are given in Figures 7 and 8.

The rms of the weighted data depends on the weighting vector used. A measure of this is \( \kappa_n \) defined as the ratio of the rms of the weighted data to the rms of the raw data,

\[
\kappa_n = \left[ \frac{N_s}{\sum_{i=1}^{N_s} \sigma_i(n)^2} \right]^{1/2}.
\]

(18)

Table 8 gives \( \kappa_n \) for all the weighting vectors. The overall sensitivity decreases (\( \kappa_n \) increases) as the synthesized beam spacing approaches the intrinsic telescope beamwidth. In practice, \( \sim 2.5 \) time samples per beamwidth are required in order to avoid a significant loss of resolution.

A figure of merit for the noise level in a particular weighting scheme is \( \kappa_n S_n(f_{obs}) \), where \( f_{obs} \) is the effective sampling frequency associated with the weighting vector (see Table 8) and \( S_n \) is the power spectrum of the receiver measured while staring at the sky (see Fig. 3). Notice that the reduction in instrumental \( 1/f \) noise with increasing frequency tends to compensate the increase in the system’s effective noise level due to \( \kappa_n \).

The angular window function is used to characterize the instrument's spatial filtering properties. From equation (15), one sees that the choice of weighting vector, \( w(n) \), determines the synthesized beam pattern. Thus, this choice is intimately linked to the observing strategy’s angular

\[\text{Fig. 7.—Contours of the SK94Q synthesized beam pattern. Dashed and solid lines, respectively, indicate negative and positive beam lobe responses during a chop cycle. The telescope is in the east base position. Note, lines of constant beam elevation curve upward with respect to lines of constant right ascension. The north celestial pole (NCP) is at (0, 0); \theta_s and \theta_a \text{ are given in Table 5.}}\]
window function,

\[ W_l^i(n) = \int d\hat{x}_1 \int d\hat{x}_2 \ H_i(\hat{x}_1, n) \ H_j(\hat{x}_2, n) \ P_l(\hat{x}_1 \cdot \hat{x}_2), \quad (19) \]

where \( P_l \) is a Legendre polynomial of order \( l \). One generally desires a window function with uniform value over a well-defined range of spherical harmonics, \( l \). In practice, if the beam is scanned with \( x_n \) following a triangular pattern, then the best \( n \)-point weighting is \( a_m = a_{n-1} = 1 \) (before normalization) and all other \( a_m, b_m \) equal to zero. Achieving overlap between observations with differing spatial coverage and limiting the atmospheric noise contribution also provide practical constraints to be addressed during the selection of the weights. Examples of the window functions used for the analysis of the SK data set can be found in Netterfield et al. (1996). The question of optimization of the weighting vectors \( w_i \), has been addressed in detail by Tegmark (1996) and Knox (1996).

5. RADIOMETRIC OFFSETS

We use the following model to identify contributions to the offset:

\[
T_{\text{ant}} \approx (1 - \varepsilon_0) \left[ \eta_s (T_{\text{sky}} + T_{\text{atm}}) + (1 - \eta_s) T_{\text{spill}} \right] + \varepsilon_0 \eta_b \ T_{\text{plate}} \quad (20)
\]

where \( \eta_s \approx 1 \) is the beam efficiency, \( \varepsilon_0 \ll 1 \) is the emissivity of the aluminum chopping plate, \( T_{\text{sky}} \) is the brightness temperature of the sky measured from the surface of the Earth, \( T_{\text{atm}} \) is the atmospheric brightness temperature, \( T_{\text{plate}} \) is the physical temperature of the chopping plate, and \( T_{\text{spill}} \) is the termination temperature of the sidelobes. When the beam is moved on the sky, the change in antenna temperature is

\[
\delta T_{\text{ant}} \approx \delta \eta_s (T_{\text{sky}} + T_{\text{atm}} - T_{\text{spill}}) + \delta \varepsilon_0 \eta_b \ T_{\text{plate}} + \eta_b (\delta T_{\text{atm}} + \varepsilon_0 \delta T_{\text{plate}}) + (1 - \eta_s) \delta T_{\text{spill}}, \quad (21)
\]

where contributions are grouped by changes in coupling efficiency, emission, and termination temperature.\(^{14}\) For simplicity, the sky is assumed to be uniform and only the dominant contributions to the offset are retained. Offsets due to modulated spillover radiation are controlled by baffle geometry and illumination level. Underillumination of the optical elements results in \( \delta \eta_s \approx 0 \). The offsets from atmospheric emission are minimized through telescope alignment and by rapidly differentiating regions with the same temperature. Under stable atmospheric conditions, the dominant contributions are due to modulation of the antenna spill and plate emissivity.

5.1. The Chopping Plate Emission Offset

The process of scanning with the chopper produces a synchronous modulation of the plate emissivity when viewed from the feed. As the angle of incidence is varied, the magnitude of the chopper's surface brightness temperature changes. For a good conductor with skin depth \( \delta = (1/\mu_0 \pi v_0 \sigma)^{1/2} \), where \( \sigma \) is the conductivity of the metal surface and \( v_0 \) is the observing frequency, the parallel and perpendicular emissivities are

\[
\varepsilon_\parallel \approx \varepsilon_0 \cos (\theta), \quad (22)
\]

with \( \delta/\lambda_0 \ll \cos (\theta) \), and

\[
\varepsilon_\perp \approx \varepsilon_0 \cos (\theta), \quad (23)
\]

with \( \delta/\lambda_0 \ll 1 \). The incident angle of the radiation is given by

\[
\theta(\phi_c) = \cos^{-1} \left[ -\hat{k}_i \cdot \hat{n}(\phi_c) \right], \quad (24)
\]

where \( \hat{k}_i \) is the propagation vector and \( \hat{n} \) is the plate normal vector in the detector frame (see, e.g., Landau & Lifshitz 1960; Rytov, Kravtsov, & Tatarskii 1978). The emissivity of

\(^{14}\) Modulation of the receiver input match can also produce synchronous offsets. The beam switch in the SK telescope is produced by both the chopper and the base motions. Due to the off-axis telescope geometry and relatively wide channel bandwidth, the receiver noise temperature and amplitude response are essentially unmodulated by beam switching.
a 6061-T6 aluminum sheet at 31.4 GHz is \( \varepsilon_0 = 4\pi \delta /\lambda_0 \approx 9 \times 10^{-4} \). We normalized equations (22) and (23) to the DC conductivity of the surface. The brightness temperature of the plate is

\[
T_{\text{plate}} \approx T_{\text{phys}}[\varepsilon_{\parallel}(\theta)|\vec{E}_{\parallel}(\theta)|^2 + \varepsilon_{\perp}(\theta)|\vec{E}_{\perp}(\theta)|^2],
\]

where \( T_{\text{phys}} \) is the physical temperature of the plate, and \( \vec{E}_{\parallel} \) and \( \vec{E}_{\perp} \) are the perpendicular and parallel electric field projections onto the chopping flat normal. The resulting emission can be expressed as

\[
\Delta T_{\text{plate}} \approx \int H(\vec{x}, \eta) T_{\text{plate}}(\vec{x}) d\vec{x},
\]

where \( H \) is the antenna pattern of the synthesized beam (see § 4). Since the chopping plate essentially fills the entire field of view, the emission offset is only a function of the chopper position angle and the weighting vector. The antenna temperature as a function of the physical chopping plate angle with \( w_i = 1 \), is given in Figure 9. For polarization \( \phi_h = 0^\circ \) (E vector \( \perp \) to the chopper axis) and \( 90^\circ \), the antisymmetric component is eliminated. With \( \phi_h = \pm 45^\circ \) the symmetric term is minimized. For an unpolarized detector, the antisymmetric component is eliminated by the average over feed polarization angle. The measured and computed three-point offsets for each radiometer are summarized in Table 9.

With \( \phi_h = \pm 45^\circ \), the two-point offset was measured to be \( \delta \Delta T_{\text{plate}} / \delta \phi_c \approx (\pm 3.3 \pm 0.3) \text{ mK deg}^{-1} \). The predicted magnitude is \( \approx 3.5 \text{ mK deg}^{-1} \) for the aluminum chopping plate surface. In the vertical/horizontal feed polarization configuration \( (\phi_h = 0^\circ \) and \( 90^\circ) \) a stainless steel sheet was rigidly attached to the chopper surface. The measured offset increased by a factor of \( \approx 7 \) consistent with the increase in emissivity.

Changes in the plate temperature influence the offset magnitude. The chopping flat temperature is monitored at seven locations. While taking astrophysical data, the average chopping plate temperature varied by as much as 30 K. This results in a maximal \( \pm \sim 5\% \) variation in the offset magnitude as a function of the ambient temperature.

Due to the differences in the DC conductivity of aluminum, the relative emissivity varies by a factor up to \( \sim 1.5 \) depending upon alloy type (e.g., Weast 1982). Surface treatment and finish can increase the RF emissivity by an additional factor between 1.1 and 1.5 in typical microwave components.

![Fig. 9.—Computed antenna temperature due to plate emission as a function of chopper azimuthal position. In the field, feed polarization "A" (long dashed line) is vertical and "B" (short dashed line) is horizontal. For the calculation, the observation angle is \( \vartheta = 37:85 \) and the radial extent of the plate is taken as infinite. A physical temperature of \( T_{\text{phys}} = 300 \text{ K} \) and an emissivity of \( \varepsilon_0 = 9 \times 10^{-4} \) are assumed. The solid line is 1/10 of the computed magnitude for the feed polarization with \( \phi_h = 45^\circ \). The response of an ideal unpolarized detector is indicated by the solid bold line. The average emission \( \langle T_{\text{plate}} \rangle \approx 400 \text{ mK} \), is a function of the feed polarization orientation and observation angle. To facilitate comparison, this constant term has been subtracted from the computed response. Calculated and measured magnitudes are given in Table 9.

There is a small correlation between the chopping temperature and the offset. However, changes in ambient temperature can affect the offset through other mechanisms (see § 5.4), so one cannot conclude that the correlation is causal. Removal of the correlated signal has negligible effect on the reported results.

5.2. The Chopping Plate Spill Offset

As the plate moves, the chopping edge diffraction changes. This effect is minimized by underilluminating the flat. By evaluating the directivity in the Gaussian optics approximation we obtain an estimate of the chopping edge illumination (Martin 1990; see also Murphy, Egan, & Withington

| Parameter                  | SK93K_A/B (\( \mu \text{K} \)) | SK94K_A/B (\( \mu \text{K} \)) | SK94Q_A/B (\( \mu \text{K} \)) | SK95Q_A/B (\( \mu \text{K} \)) |
|----------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \( \Delta T_{\text{plate}} \) | +50/−190                       | +120/−430                       | +130/−500                       | +150/−570                       |
| \( \Delta T_{\text{em}} \)   | −150                            | −5                              | −10                             | −10                             |
| \( \Delta T_{\text{pill}} \) | −10                             | −10                             | −10                             | −500                            |
| \( \Delta T_{\text{theo}} \) | −110/−350                       | +105/−445                       | +110/−520                       | −460/−1080                      |
| \( \Delta T_{\text{theo}} \) | −60/−360                        | +117/−350                       | −695                            | −616/−2182                      |

**Note.**—The three-point offsets are computed from the measured plate emissivity, telescope alignment, and spill. \( \Delta T_{\text{pill}} \) is an order-of-magnitude estimate based on the telescope configuration. The theoretical offset is estimated from \( \Delta T_{\text{theo}} = \Delta T_{\text{plate}} + \Delta T_{\text{em}} + \Delta T_{\text{pill}} \) using the measured telescope parameters. The data from the SK94Q vertical polarization (channel "A") was not used and the corresponding radiometric offset was not investigated. The offset for the SK95Q three-point is larger than the simple three term model would predict. We believe the excess offset is due to a 0.04 cm wide seam in one side of the chopper, which moves laterally with respect to the center of the beam during a chop cycle.
for the SK93 optics it is $-46$ dB relative to the near field main beam, consistent with measurement. In addition, a stationary baffle that extends 15 cm past the chopper edge shields the chopper from behind. The computed illumination level at this edge is more than 65 dB lower than the main beam in the chopper plane. For the SK93 and SK94 optical designs, the offset due to modulated spill is immeasurably small. For SK95, the edge illumination was increased to produce a narrower beam. This contributed to the larger three-point offset that year.

5.3. The Atmospheric Offset

The chopping plate axis is aligned with the vertical in order to minimize the atmospheric offset, $\Delta T_{\text{atm}}$. If the plate axis is rotated in the plane of the plate, $\delta \theta_{\parallel}$, a two-point response results. A base or chopper throw leveling error would induce this offset. A vertical axis error due to a rotation perpendicular to the plane of the plate, $\delta \theta_{\perp}$, generates a three-point response. A typical example of this is an elevation pointing error. We model the atmospheric brightness temperature as

$$T_{\text{atm}} = \langle T_{\text{atm}} \rangle + \frac{\partial T_{\text{atm}}}{\partial \psi} [\delta \psi_{||} + \delta \psi_{\perp}] ,$$

where $\delta \psi_{||} \ll 1$ is the variation in angle due to chopper misalignment from a fixed zenith angle, $\theta_z$. We assume a gradient in the atmospheric temperature of the form

$$\frac{\partial T_{\text{atm}}}{\partial \psi} \approx T_z \tan (\theta_z) \sec (\theta_z) ,$$

where $T_z$ is the zenith temperature. The variation in the sky signal due to a small rotation about the axis parallel to the chopper normal is approximated by

$$\delta \psi_{||} \approx 2 \delta \theta_{\parallel} \sin (\theta_z) \sin (\phi),$$

where $\delta \theta_{\parallel} \ll 1$. The leading factor of 2 results from the reflection of the beam off the plate. Similarly, for a small rotation of the chopper vertical axis about the perpendicular to the plate normal

$$\delta \psi_{\perp} \approx -2 \delta \theta_{\perp} \cos (\theta_z) \cos (\phi),$$

for $\delta \theta_{\perp} \ll 1$. From equation (27), the resulting atmospheric offset is

$$\Delta T_{\text{atm}}^{(n)} \approx \int H(\hat{\epsilon}, n) T_{\text{atm}}(\hat{\epsilon}) d\hat{\epsilon} .$$

In Figure 10, the atmospheric brightness temperature as a function of chopper position is plotted for typical chopper alignment errors. In practice, temporal variations in the horizontal atmospheric temperature profile can mask the effects of the parallel axis misalignment on short timescales.

5.4. Limits to Offset Stability

The stability of the offset is as important as its magnitude. For example, COBE/DMR had offsets on the order of hundreds of mK (Kogut et al. 1992) but the extreme stability of the space environment allowed it to produce the highest quality data set to date. Stability is especially important for the SK experiment because the sky is reobserved only every 12 hours. Possible causes for a drift include changes in the chopping plate temperature profile, thermal contraction of the mounts, icing of the optics, and changes in the system gain from thermal variations. The largest drift is $7 \mu K \ day^{-1}$ in the SK95 three-point data, more typically the drift is less than $4 \mu K \ day^{-1}$.

There may be drifts in the data on faster timescales but the receivers are not sensitive enough to detect them. In 12 hours, the three-point SK94Q data (Fig. 11) can only measure $20 \mu K$ with a signal-to-noise of 1. Even when data from multiple channels are combined, the noise is not
During January through March, Saskatoon is relatively dry, clear, and cold. Precipitable water vapor has a mean value of 5 mm and drops to ≤2 mm on the coldest winter days (Jones 1993). From ground-based measurements of the wind speed/direction, temperature, pressure and relative humidity at the site, the following general trends were noted: "good data" are typically obtained during periods with little or no wind (<5 knots), temperatures less than −5°C, atmospheric pressure greater than 710 torr, stable weather systems, and low relative humidity. However, at times the effects of atmospheric turbulence were seen in the data despite clear skies and favorable weather station readings, suggesting that the offending fluctuations were far removed from the telescope. In general, the presence of high-altitude clouds or ice fog did not increase the atmospheric noise.

The atmospheric zenith temperature is measured by tilting the chopping plate ±5° and measuring the total power at the detector. We compute the theoretical atmospheric zenith temperature assuming a dependence for the column depth using a model based on Liebe (1985) and Danese & Partridge (1989). The results are presented in Figure 12. Data suitable for CMB analysis lay within a 4 K atmospheric temperature window above the minimum recorded magnitude at the site.

In addition to the flat spectral component from atmospheric thermal emission, there is also a "1/f" component originating from atmospheric turbulence (Tatarski 1961; Andreani et al. 1990; Church 1995). There also appear to be other mechanisms that enter on longer timescales. The spatial power spectrum of the emission is dominated by

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**Fig. 12.**—Atmospheric zenith temperature in Saskatoon, SK on a typical calm winter day (5 mm precipitable water vapor). The units are antenna temperature and the error bars reflect the uncertainty in the calibration scale. The solid curve is the modeled radiometric brightness temperature & Partridge A Van Vleck-Weisskopf (Liebe 1985; Danese 1989). A Van Vleck-Weisskopf line profile is assumed for each atmospheric component in computing the integrated emission.

**Fig. 13.**—Distribution of fluctuations for 20 minute averages of the two-point data as a function of cut level: SK93K, with the 27.5 GHz channel, ξ < 2.2, 3.0, 4.5 mK s^1/2 deg^-1, and all data; SK94K, ξ < 1.7, 2.5 mK s^1/2 deg^-1, and all data; and SK94Q with the 38.5 GHz channel, ξ < 1.7, 2.5 mK s^1/2 deg^-1, and all data. The receiver noise floors for 1993 and 1994 K were the same; however, with the increased chopper throw in 1994, the sensitivity to atmospheric fluctuations increased. In 1993, there were relatively few days that were detector-noise dominated. In 1994, the observing season was longer and the data were of better overall quality. (See Fig. 3 of Netterfield et al. 1997.) To convert to the equivalent NET, multiply by: θ_{out}/κ_1 = 1.68; for SK93K, θ_{out}/κ_2 = 2.25; for SK94K, and for SK94Q, θ_{out}/κ_2 = 2.37. Note that the receiver noise contribution is included in ξ.

**Fig. 14.**—Atmospheric noise structure function. The boxes are a 20 minute average of the structure function (SK94Q; ξ < 2.5 mK s^1/2 deg^-1). The solid line (log ⟨(T(θ) − T(0))⟩ = 1.66 log (θ) − 0.41) is a fit to the data.
thermal gradients spanning more than 4° as determined from the two-point data. These gradients have been observed to persist for up to 1 hour. The three-point data, which are sensitive to the curvature rather than the gradient, typically have a factor of 8 less noise.

The atmospheric cut for selecting astrophysical data (three-point and higher) is based on the stability of the statistically independent two-point data. This is done by evaluating the two-point mean absolute deviation, \( \eta \), of the 8 s averages for a 20 minute segment of data. The mean deviation is less sensitive than the standard deviation to outliers. To compare with other experiments, the cut levels are converted to units of mK s\(^{1/2}\) deg\(^{-1}\) by multiplying \( \eta \) by the sensitivity of the synthesized beam pattern to a 1 mK deg\(^{-1}\) horizontal atmospheric gradient

\[
\zeta = \frac{K}{\theta_{\text{eff}}} \times \text{NET} \approx \left( \frac{\pi}{2} \right)^{1/2} \frac{\eta}{\theta_{\text{eff}}}, \tag{32}
\]

where \( \text{NET} \) is the equivalent noise temperature of the cut level, \( \tau \) is the integration time, and \( \theta_{\text{eff}} = \frac{1}{f} \theta(\hat{x})H(\hat{x})d\hat{x} \) is the two-point effective beam separation angle. This conversion assumes Gaussian fluctuations, which is not necessarily valid. Figure 13 shows the distribution of the spatial gradient fluctuations, \( \zeta \), at several cut levels. The distributions from SK94, K, and Q have some qualitative differences. The lower cutoff on both distributions is due to the system noise, which is higher in Q than in K. The second hump located in the vicinity of \( \sim 200 \) mK s\(^{1/2}\) deg\(^{-1}\) is due to data acquired during a 2 week period of very poor weather in 1994 January. Similarly, the bump in SK93 results from data taken during the spring thaw in March of 1993. During these periods the zenith temperature is unstable and the atmospheric water content relatively high.

For roughly 25% of any campaign, the receivers are detector noise dominated (Netterfield et al. 1996). As an example of the atmospheric stability, the power spectrum for an 18 hr contiguous stretch of good data (\( \zeta < 2.5 \) mK s\(^{1/2}\) deg\(^{-1}\)) is shown in Figure 11. The two-point response, SK94Q(2), has a 1/f knee of \( \sim 1 \) mHz and the three-point response, SK94Q(3), is featureless. For \( n > 2 \) the responses are qualitatively similar to SK94Q(3). In Figure 14, the structure function (see Tatarski 1961; Church 1995),

\[
D(\theta) = \langle [T(\theta) - T(0)]^2 \rangle, \tag{33}
\]

is computed for a 20 minute average of the data. For beam separations less than 7°, the average of the east and west data sets is used, while for angles greater than 7°, the east and west data are combined into a long scan across the sky. The absence of evidence of saturation in \( D(\theta) \) suggests this component of the atmospheric noise results from angular scales greater than the base wobble angle.

7. SUMMARY

The instrument uses a HEMT-based total-power receiver at the focus of an off-axis parabolic primary. The beam is swept many beamwidths on the sky with a large under-illuminated movable flat. The clear optical path, polarization orientation, and mechanical stability result in small and stable radiometric offsets. To provide spectral discrimination, each receiver observes in three frequency bands. Due to the fast radiometer response, the outputs can be sampled rapidly compared to the beam scan rate. By weighting each sample in software, a set of synthesized beams is constructed. This allows one to simultaneously probe a range of angular scales, optimize spatial frequency coverage while minimizing atmospheric contamination, and synthesize beam patterns for a direct comparison with other experiments.

The data from this instrument are of generally high quality, and we are not aware of any instrument-based systematic effects that could compromise them. The data selection criteria are not based on the data used for the astrophysical analysis but rather on an independent weighting scheme. Also, the symmetries in the observing strategy and the three years of observations provide many internal consistency checks. The dominant contribution to the 14% calibration uncertainty is from the inaccuracy in our knowledge of our calibration source, Cas A. Future observations using the techniques described here will enhance our confidence in the data and lead to a better understanding of the anisotropy in the CMB.

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\(^{16}\) The mean absolute deviation of a signal is defined as \( \eta = \langle (1/N) \sum |i| \rangle - \langle \zeta \rangle \), where \( i \) is a data vector of length \( N \) with an average value of \( \langle \zeta \rangle \).

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