Sequence Prediction under Missing Data: An RNN Approach without Imputation

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ABSTRACT

Missing data scenarios are very common in ML applications in general and time-series/sequence applications are no exceptions. This paper pertains to a novel Recurrent Neural Network (RNN) based solution for sequence prediction under missing data. Our method is distinct from all existing approaches. It tries to encode the missingness patterns in the data directly without trying to impute data either before or during model building. Our encoding is lossless and achieves compression. It can be employed for both sequence classification and forecasting. We focus on forecasting here in a general context of multi-step prediction in presence of possible exogenous inputs. In particular, we propose novel variants of Encoder-Decoder (Seq2Seq) RNNs for this. The encoder here adopts the above mentioned pattern encoding, while at the decoder which has a different structure, multiple variants are feasible. We demonstrate the utility of our proposed architecture via multiple experiments on both single and multiple sequence (real) data-sets. We consider both scenarios where (i)data is naturally missing and (ii)data is synthetically masked.

CCS CONCEPTS

• Computing methodologies → Neural networks.

KEYWORDS

Recurrent Neural Networks, Encoder-Decoder, Seq2Seq, time-series, Missing Data.

1 INTRODUCTION

Sequence based prediction (classification [38] and forecasting [24]), though an old research area continues to be very relevant given recent improvements and constant emergence of newer applications. RNNs are arguably one of the most popular state-of-the-art sequence predictive modeling approaches. Deep RNNs have achieved astounding success over the last decade in domains like NLP, speech and audio processing [35], computer vision [36], time series classification, forecasting and so on. In particular, it has achieved state-of-the-art performance (and beyond) in tasks like handwriting recognition [10], speech recognition [9, 11], machine translation [4, 33] and image captioning [20, 39], to name a few.

Sequence Prediction under missing data also has a long literature. Missing data could arise due to a variety of reasons like sensor malfunction, maintenance OR even high noise (noise level could be so high that considering data as missing would be more meaningful). Researchers have provided a variety of techniques for data imputation over the years [8, 17, 21, 28, 31], which is typically followed by the predictive modelling step. RNNs in particular have also been explored for sequence modelling under missing data over the years. This paper addresses sequence prediction under missing data using RNNs in a novel way. While our proposed ideas can be employed on both classification and forecasting, we focus on forecasting in this paper. In particular, we consider a general multi-step forecasting scenario with possibly additional exogenous inputs to be handled.

Encoder-Decoder (ED) OR Seq2Seq architecture used to map variable length sequences to another variable length sequence were first successfully applied for machine translation [4, 33] tasks. From then on, the ED framework has been successfully applied in many other tasks like speech recognition[26], image captioning etc. Given its variable length Seq2Seq mapping ability, the ED framework has also been utilized for accurate multi-step (time-series) forecasting where the target vector length can be variable and independent of the input vector [37]. The missing data RNN architecture proposed here for forecasting is essentially built on the basic ED architecture.

1.1 Problem & Motivation

We consider real-valued time series data where time-stamps are assumed to be discrete (integers or natural numbers). Sequential data can come as a single long sequence OR in the form of multiple sequences (which can be short). We restrict ourselves to forecasting task alone here for both single and multi-sequence data case with data points potentially missing. The forecasting task considered here involves, predicting one or more endogenous variables over a
multi-step forecast horizon, in the presence of possible exogenous inputs, which influence the evolution of the endogenous variables. Such forecasting tasks arise in diverse applications. A retailer may be interested in forecasting one or more products sales (modelled as endogenous variables) in the presence of exogenous price variation. In electricity markets, forecasting power demand (endogenous) in different geographical regions by factoring temperature influences/fluctuations (exogenous) is another example. We consider multi-step forecasting in such scenarios under missing data.

1.2 Contributions

The traditional approach for predictive modelling under missing data has been to impute first and then perform model learning of one’s choice. Accordingly, there have been a wide variety of data imputation techniques proposed in literature. However, this sequential approach of impute and model building is very sensitive to the quality of imputation. Also, the imputation step itself can be computationally intensive. To circumvent the above limitations of this approach, researchers have proposed approaches where imputation and model learning are carried out jointly [22]. Among RNN based missing data works, many approaches follow the jointly impute and learn approach [1, 3].

Our proposed method in this paper adopts a fundamentally different (or distinct) approach compared to the above two broad class of approaches. We try to avoid imputation almost completely either before OR during model building. Our approach performs predictive model building by directly trying to learn the missingness patterns observed in the data. This novel general approach can be particularly useful in scenarios where the sequential data is missing in large consecutive chunks. In such scenarios, the existing impute based approaches will tend to suffer significant imputation errors.

Our contribution involves a novel ED architecture using two encoders, while allowing for intelligent choices/variants in the decoder which incorporates multi-step (target) learning. Overall contribution summary is as follows:

- We propose a novel encoding scheme of the input window without imputation, which is applicable for both sequence classification and forecasting. For multi-step forecasting in presence of exogenous inputs, we propose a novel ED based architecture which employs the above encoding scheme in the encoder.
- The missingness pattern in the input window of an example is encoded as it is (without imputation) using two encoders with variable length inputs. The encoding is lossless with significant compression feasible. The output window information can be translated into the decoder in multiple interesting ways.
- To utilize the proposed scheme for multiple sequence data, where per sequence data is less, we propose a heuristic procedure. We demonstrate effectiveness of our architecture variants on diverse real data sets.

2 RELATED WORK

Prediction under missing data is a classic problem with an old literature. One broad class of methods to address this is to first impute followed by predictive modeling. The simpler approaches towards data imputation include smoothing, interpolation, splines [17] which capture straightforward patterns in the data. The more sophisticated approaches include spectral analysis [28], matrix factorization [21], kernel methods [31], EM algorithm [8], GANs (Generative Adversarial Networks) [27] etc. These sophisticated approaches can be computationally expensive. Imputation methods typically assume restricted scenarios like low missing rates, missing at random and so on which may not hold in practice. The two step sequential process of imputation followed by prediction can suffer from imputation errors while the predictive model so built is blind to any missingness pattern structure.

Sequential Prediction under missing data has also been referred to as learning under irregularly sampled time-series data with applications ranging from Electronic health records, climate science, ecology and so on. Researchers have explored diverse techniques like Gaussian processes [23], GANs [14, 22], RNNs [2, 3, 6], Ordinary Differential Equations [19, 32] etc. towards this. Please refer to [22] for a recent overview on this interesting and wide range of techniques. Given that our proposed approach is RNN based, in this paper we focus our comparisons and bench-marking against RNN based approaches. Further while irregularly sampled time-series approaches typically allow the time-stamps to be continuous, our approach here assumes time-stamps to be discrete (or integer-valued).

Over the years, RNNs have been explored for time-series tasks (sequence classification in particular) with missing data. The early RNN approaches have predominantly adopted a strategy of jointly imputing and model building. One of the first approaches based on non-gated RNNs seems to be in [1]. The missing values here are initialized to an unconditional expected value and the RNN is unfolded in time for a few extra steps (allowed to relax) to allow the missing values to settle to something more reasonable while the other inputs are clamped to their observed values. The learning criterion is an output error minimization based on all time steps in the unfolded RNN. In [34], to mitigate errors while predicting in a free running OR teacher forcing mode, a linear correlated error model is assumed in addition to the NARX (nonlinear auto-regressive with exogenous inputs) model. The overall learning scheme here involves a mix of Real-time Recurrent Learning for the nonlinear map and an EM algorithm for the error model. An RNN approach for speech recognition under missing data (on account of noise) was proposed in [29] which uses an adaption of the architecture in [1]. In particular, [1] uses a Jordan network [18] with feedback or recurrent connections from output to hidden units. In contrast, [29] avoided feedback from the output but instead considered recurrent connections from the hidden layer to the input and itself (with a unit-delay) on the lines of an Elman network [7].

Among the recent RNN approaches, [25] considers diagnosis classification under (irregularly recorded) clinical time series. It learns missingness patterns without any explicit imputation during model building or prediction. In this sense, our approach perhaps comes closest to this in principle. [25] encodes the missingness pattern with a simple additional binary vector (indicating presence or absence of data) as input while training.

More recently, an approach GRU-D [3] which incorporates both missingness pattern (as in [25]) and the joint impute and learning strategy was proposed. Here, the missing data points are assumed
to be a convex combination of the last observed value and the unconditional mean. The key intuition is that weight on the last observed value drops (decays) as the time point under consideration moves away from the last observed value (OR time delay). This decay factor is parameterized via a simple perceptron like nonlinear function of this time-delay whose weights are jointly learnt with the RNN (GRU) weights.

Among the recent RNN based forecasting approaches, [6] proposes an attention-based encoder-decoder approach of imputing first either using padding (based on left end) OR interpolation (one can use any technique) and then learning by incorporating suitable re-weighting of the impact of these imputed values. This approach doesn’t handle exogenous inputs. The first missing data variant imputes using padding from left-end and considers learning another weight, which models the influence of these imputed values using an exponential decay based on the distance from the last available data point. The second variant uses interpolation (based on both left and right end values) to impute and then employs a re-weighting scheme using both left and right end. It divides a gap into 3 regions uniformly and learns a weight for each of these regions.

2.1 Proposed architecture in perspective

Our overall approach is to avoid the impute first to the extent possible. The proposed ED idea employs a unique approach of encoding the missingness pattern in the input window as it is without any imputation. Instead of using a single encoder with a binary encoding of the missingness pattern as in [25], we employ two encoders resulting in a compressed and lossless encoding. This feature of our approach is very different from all existing approaches to the best of our knowledge.

Our novel idea of encoding the input window cannot be employed on the output window. This is because the decoder is unfolded exactly to the extent of forecast horizon and each time-step of the decoder is used to separately forecast for a particular future time-step. However, as we discuss later, one can encode the missing exogenous inputs using the binary encoding of [25], which is one of our proposed variants on the decoder. A more powerful variant would be to employ the GRU-D approach on the decoder with exogenous future variables as inputs.

In summary, our approach incorporates useful aspects of [25] and [3] on the decoder, while it retains its unique distinguishing feature on the encoder side.

Compared to multi-step forecasting approach of [6], our approach can handle multi-step prediction in the presence of exogenous inputs also. Further, we avoid any imputation of any of the input-window missing values unlike [6], by encoding the missingness pattern directly. While [6] tries to mitigate the imputation errors by intelligently learning to weight the influence of the imputed values towards prediction, it can potentially still amplify the imputation errors.

3 PROPOSED METHODOLOGY

Amongst the three standard recurrent structure choices of plain RNN (without gating), LSTM [15] and GRU [5], we choose the GRU in this paper. Like the LSTM unit, the GRU also has a gating mechanism to mitigate vanishing gradients and have more persistent memory. But the lesser gate count in GRU keeps the number of weight parameters much smaller. GRU unit as the building block for RNNs is currently ubiquitous across sequence prediction applications [3, 12, 13, 30]. A single hidden layer plain RNN unit’s hidden state can be specified as

\[
h_t = \sigma(W^h h_{t-1} + W^u u_t + b)
\]

where \(W^h, W^u\) are weight matrices associated with the state at the previous time-instant \(h_{t-1}\) and the current input \((u(t))\) respectively, \(\sigma(.)\) denotes sigmoid function. GRU based cell computes its hidden state (for one layer as follows)

\[
z_t = \sigma(W^z u_t + U^z h_{t-1} + b_z)
\]

\[
r_t = \sigma(W^r u_t + U^r h_{t-1} + b_r)
\]

\[
\tilde{h}_t = \tanh(U(r_t \odot h_{t-1}) + W u_t + b)
\]

\[
h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t
\]

where \(z_t\) is update gate vector and \(r_t\) is reset gate vector. If the two gates were absent, we essentially have the plain RNN. \(h_t\) is the new memory (summary of all inputs so far) which is a function of \(u_t\) and \(h_{t-1}\), the previous hidden state. The reset signal controls influence of the previous state on the new memory. The final current hidden state is a convex combination (controlled by \(z_t\)) of the new memory and memory at the previous step, \(h_{t-1}\). All associated weight matrices \(W^z, W^r, W, U^z, U^r, U\) and vectors \(b_z, b_r\) and \(b\) are trained using back-propagation through time (BPTT).

3.1 ED Architecture for Missing Data

In this section, we propose a novel encoder-decoder architecture which can tackle missing data scenarios without imputation (to the extent possible). Most methods for time-series prediction under missing data either (a) impute the data first and then predict (using standard techniques) OR (b) jointly (or simultaneously) learn the best imputation function and final predictive model. In contrast, we propose a general approach which doesn’t try to learn the imputation function as in the above two approaches. It rather tries to learn the final predictive model without imputation with the missing data entries as they are. In other words, it bases its predictions on the available data and the missingness pattern in the input.

Please note while our approach is very general, it can be particularly attractive in certain situations. In some time series scenarios, nature of data missingness could be such that when data is missing, it mostly happens in a medium/large window of consecutive time points. For instance in health care applications, it is natural for certain patient health parameters, not to be monitored for certain long duration of time depending on the diagnosis. In such scenarios, data imputation can be pretty inaccurate especially at time points well into the window. Further if the underlying time series has a high total variation, these imputation errors can be more pronounced. Total variation of a \(T\) length sequence \(x\) is defined as

\[
TV = \sum_{i=2}^{T} |x(i + 1) - x(i)|
\]

Our approach can be particularly useful in such situations.
3.1.1 Proposed Encoding. We consider real-valued sequences or time series where the time of occurrence is an integer (or natural number). In particular, we denote a time series of length \( N \) as follows.

\[
< (x(1), y(1), (x(2), y(2)), \ldots (x(t), y(t)) \ldots (x(N), y(N)) >
\]

(7)

where \( t \in \mathbb{N} \) (set of natural numbers), \( y(t) \) is the endogenous variable (could be vector-valued) which needs to be forecast and \( x(t) \) (could be vector-valued) is the exogenous variable whose values need to be input into the forecast horizon to forecast the endogenous variable \( y(t) \).

**Assumptions:** The assumption of time-series with integer/natural number ticks is necessary for our proposed method. Another key assumption around the type of missingness is to assume that a random variable \( x(t) \) follows.

**Key Idea:** The main idea is that the data points in the input window are split into two buckets, missing and present, and the two buckets are...
fed to separate encoders intelligently. This is achieved in a compressed and lossless fashion. This compact input representation can potentially aid the RNN learn better and faster.

Figs. 1 and 2 illustrate our overall encoding idea with a general example. The available data points in the input time window are fed in the order of their occurrence into the first encoder, in spite of the points not being consecutive. Fig. 1, gives an example input-output window at time tick \((t - 1)\). The input window dimension is 30 while the output window is of dimension 10. The exogenous and endogenous variable/value associated with each time tick of the input window are indicated above and below the (appropriately colored) ellipses respectively. The number in the interior of each of the ellipses indicate the position of the time-tick relative to the start of the window (input or output). Time-ticks where data is missing is additionally quantified with a ‘*’

Note that based on our encoding idea, all 10 available data points in the input window each marked in red. Observe that all these 10 points are fed sequentially to the first encoder as illustrated in Fig. 2.

The second encoder identifies the blocks of missing data in the input window. Each block can be uniquely identified by two fields: (a) Start time of the block with reference to the start of the input window (b) width of the block. In Fig. 2, there are 4 such missing data blocks. Each block of consecutive points is marked by a unique color (also grouped separately and shaded in yellow). In particular, the first block’s relative start position is 4, while its width is 7. These two bits of information which identify block 1 are fed as inputs to the first time-step of the encoder 2. In general, the (Start time, window width) information of the \(i^{th}\) missing data block in the input-window of the data is fed at the \(i^{th}\) time-step of the encoder 2. As illustrated in Fig. 2, the start, width of the 4 missing data blocks in input window of Fig. 1 are sequentially fed as inputs at 4 time-steps of encoder 2. Note the color coding of the 4 missing data blocks being consistent with the respective GRU block colors, to clearly demonstrate our proposed architecture.

Sensitivity considerations: While the length of first encoder would be exactly equal to the number of available data points in the input window, second encoder length would be sensitive to position of the missing points. In particular, one extreme would be if all missing points fall in one block, then second encoder length is the shortest and will be 1. This is the case of maximum compression. The other extreme would be if all missing points fall in distinct blocks, which means each block is of width 1. In such a case, the second encoder length is the longest and equal to the number of missing points. If there are no missing points in the input window, we still use the second encoder with length 1, where both inputs are taken to be 0.

This rearranged way of feeding input-window information (without imputation) using two encoders is clearly lossless as one can recover the original data in the input window from the encoded information. Further if data unavailability happens in blocks of large width, our scheme achieves significant compression.

Computational issues: Note that based on our encoding idea, the input part of a training example gets transformed into two vectors (fed to the two encoders) of variable dimension. We exploit this variable length information while training so that the two encoders are sequentially unfolded only to the extent needed. In the presence of data missing in blocks, our encoding scheme can potentially work with larger input windows in comparison to what an impute and predict strategy could, due to the vanishing gradient issue. This is because the number of unfolded steps in the two encoders via our scheme can be significantly lower than the input-window size. The example in Fig. 1 is a case in point where a standard encoder would have up-to 30 time steps, while our intelligent encoding leads to at most 10 steps per encoder. From a training run-time perspective, for the same input window size, the gradient computation can be faster (compared to an impute and predict strategy) via our two-encoder approach given the reduced number of steps in the unfolded structure.

3.1.2 Decoder Variants. Our above idea of intelligently encoding the available and missing data into two separate layers cannot be employed on the decoder. This is because the decoder is performing a (possibly variable length) multi-step forecasting, where at each time-step the decoder output is used to make a prediction. Basically one needs to retain the identity of each step in the forecast horizon in the unfolded decoder.

Figure 3: Illustration of the Decoder side (Impute with binary encoding) with reference to Example Sequence of Fig. 1. go, output from Fig. 2 is fed as initial state here.
**Simple Imputation:** Perhaps the simplest way to handle missing entries in the output window is to impute the missing entries first using a computationally cheap technique. This could be achieved by a simple mean/median imputation for instance. Now the decoder could be unfolded with inputs and targets at each time-step. Fig. 3 (with \( m(t) \) input ignored) illustrates this for example sequence in Fig. 1. Note the imputed values of \( x \) and \( y \) are denoted as \( \hat{x} \) and \( \hat{y} \) respectively.

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**Figure 4:** Variable Length Unfolding in Decoder (based on Input-Output Window of Fig. 1).

**Variable length unfolding:** A natural way to tackle missing values in output window is to unfold the decoder based on the first set of consecutively available points from the output window (essentially up to the first missing point). For example, in Fig. 1, we unfold the decoder up to two time-steps only (Fig. 4).

**Binary Encoding:** The previous approach while plausible would ignore a training example completely if the data at the first time-step of the output window is missing. But this can lead to severe loss of examples. A more intelligent approach would be to additionally introduce a binary vector indicating the presence or absence of data at a time-tick as in [25]. This means for every input-output window, we obtain an example (we don’t miss any example) with decoder unfolded for all steps in the output window on each example. The missing values could be imputed based on simple imputation as described earlier. Fig. 3 indicates the decoder structure with the additional binary input vector \( (m(t)) \) and imputed missing values for the example in Fig. 1.

**GRU-D based Decoder:** As explained earlier in related work, GRU-D [3] in addition to using the masking binary vector also adopts a novel joint impute-learn strategy.

We define formally the binary masking variable, \( m_t \), as

\[
m_t = \begin{cases} 
1, & \text{if } x_t \text{ is observed} \\
0, & \text{otherwise}
\end{cases}
\]  

(8)

GRU-D also maintains a time-interval \( \delta_t \), denoting the distance from its last observation. Formally, for integer time-ticks,

\[
\delta_t = \begin{cases} 
1 + \delta_{t-1}, & \text{if } t > 1, m_{t-1} = 0 \\
1, & \text{if } t > 1, m_{t-1} = 1 \\
0, & t = 1
\end{cases}
\]  

(9)

which is a recursive way of defining \( \delta_t \).

GRU-D essentially models a decay mechanism based on the last observation’s distance. The decay factor is modelled for each input variable using a monotonically decreasing function of \( \delta_t \) ranging between 0 and 1.

\[
y_t = \exp\{-\max(0, W_y \delta_t + b_y)\}
\]  

(10)

Here \( W_y \) and \( b_y \) are weights that need to be learnt. The modified input that is input to the GRU unit is now

\[
\hat{x}(t) = m_t x(t) + (1 - m_t)(y_t x^t(t) + (1 - \gamma_t)\tilde{x})
\]  

(11)

where \( m_t \) is the masking variable (eqn. (8)), \( x^t(t) \) is the last observation to the left of \( t \) and \( \tilde{x} \) is empirical mean of \( x(t) \). Replacing \( u(t) \) by \( \hat{x}(t) \) in eqns. (2)-(5) is what a GRU-D recurrent unit is by incorporating input decay alone. Enhancing the architecture of Fig. 3 using modified exogenous inputs based on eqns. (8)-(11) gives us a GRU-D based decoder. This variant performed best in our experiments.

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### 3.2 Training on multi-sequence data

Building sequence-specific models for multi-sequence data, when per-sequence data is less, may be a poor strategy. It can be further compounded if the exogenous variable additionally shows very little variation per sequence. We propose a heuristic strategy to adapt our above architecture (primarily for single sequences) to such scenarios. The idea is to build one common background model which can tackle the per-sequence data sparsity. We perform a sequence-specific scaling of each of the input-output windows (for both the endogenous and exogenous variables) depending on the sequence from which a particular input-output window was constructed. These normalized examples are used to train a common background model. During prediction the model output is re-scaled in a sequence-specific fashion.

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### 4 RESULTS

We first describe the data sets used for testing, followed by error metrics and hyper-parameters for evaluation, baselines and performance results in comparison to them.

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### 4.1 Data Sets

Our first data set D1 validates our proposed approach at a single sequence level with about 4 sequences. Synthetic masking is employed here. The next two data sets are used to vindicate our approach on multi-sequence data. In particular, synthetic masking is employed in D3, while D2 has missing data points naturally. We provide more details of these data-sets next.

- **D1:** M5 data-set is a publicly available data-set from Walmart and contains the unit sales of different products on a daily basis spanning 5.4 years. This data is distributed across 12 different aggregation levels. We pick sequences from level 12 where sales and prices are available at a product and store level (lowest level with no sales aggregation). Price is used as exogenous input here. While this level contains little more than 30,000 sequences, we pick 4 sequences from here with high total variation (referred to as D1). All 4 sequences turn out to belong to FOODS category. We essentially pick the hardest sequences which exhibit sufficient variation. We test our architecture separately on each of these 4 sequences by synthetically masking with large width windows.

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We consider the following two error metrics. Given \( X_i \) is the predicted value, while \( X \) is the true value, \( \text{RE} = \frac{\text{abs}(X_i - X)}{X} \). In the multi-step setting, APE is computed for each step and is averaged over all steps to obtain the MAPE for one window of the prediction horizon. APE while has the advantage of being a scale independent error metric which is scale-free could be MASE.

The MASE is computed with reference to a baseline metric. The choice of baseline is typically the \textit{copy previous} predictor, which just replicates the previous observed value as the prediction for the next step. For a given window of one prediction horizon of \( K \) steps ahead, let us denote the \( i^{th} \) step error by \( |X_i - X_i| \). The \( i^{th} \) scaled error is defined as:

\[
e_i = \frac{|X_i - X_i|}{\frac{1}{n_K} \sum_{j=K+1}^n |X_j - X_{j-K}|}
\]

where \( n \) is no. of data points in the training set. The normalizing factor is the average \( i^{th} \) step-ahead error of the copy-previous baseline on the training set. Hence the MASE on a multi-step prediction window \( w \) of size \( K \) will be:

\[
\text{MASE}(w, K) = \frac{1}{K} \sum_{j=1}^K e^j
\]

### 4.3 Baselines and Proposed Approaches

We benchmark two of the discussed decoder variants (Sec. 3.1.2): (a) Simple (Mean) Imputation denoted as DEMI (b) GRU-D based decoder imputation denoted as DEGD. DE refers to double encoder or the two RNN layers used to encode input window (Fig. 2). Recall from Sec. 3.1.2 that ‘variable length unfolding’ variant can suffer from severe loss of examples while the ‘Binary encoding’ variant is a special case of GRU-D based variant. Hence, we benchmark DEMI and DEGD only. The baselines we benchmark our method against are as follows:

1. **BEDXM** - post mean imputation (in all missing points) run a Basic Encoder-Decoder (with one encoder capturing immediate lags) with exogenous inputs as in [37].
2. **BEDXL** - Consider any band of missing points. The center of the band is imputed with the mean, while from both the left and right end, the missing points are now linearly interpolated. Next use [37] as above. It uses a simple imputation method but utilizes both the right and left end information of any gap, unlike [3].
3. **GRU-D** - at both the encoder and decoder, we use the GRU-D unit to achieve multi-step forecasting (something unexplored in the original [3] paper).
4. **WIAED[6]** - which learns the position-based weighted influence of imputed values followed by attention layer on the encoder-decoder framework.

#### 4.3.1 Assessing significance of mean error differences statistically.

We have conducted a Welch t-test (unequal variance) based significance assessment (across all relevant experiments) under both the mean metric (MASE, MAPE) differences (Proposed vs Baseline) with a significance level of 0.05 for null hypothesis rejection. The best performing methods’ error is highlighted in bold if its MASE/MAPE improvement over every other method is statistically significant. We allow for highlighting the second/third best errors in situations when the mean error differences between the best and second best errors are statistically insignificant.

#### 4.3.2 Synthetic Masking

The masking process adopted is as follows. We scan the time-series sequentially from the start. At each step, we essentially decide if a masking has to be carried out starting from the current step. Towards this, we toss a coin with a variable heads probability \( q \). If tails, we decide against masking from the current step and move on. On heads, we decide to mask from current step. After seeing a head every time, a decision of how many consecutive data points to mask needs to be taken. Towards

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1 https://www.kaggle.com/c/walmart-recruiting-store-sales-forecasting/data
this, we consider $T_w$, which is set of window lengths from which we uniformly sample a specific length. Let $T_w$ denote the window length sampled from $T_w$. For the next masking window starting position, we choose $q = 1/T_w$. This strategy assures that about 50% of the data points are missing because for every $T_w$ points removed, we retain the next $T_w$ points (in expectation). Initial $q$ is set to 0.05.

Synthetic masking has the following advantage. Since the underlying true data before masking is available, one can assess the efficacy of training on a separate test set for every step of the prediction horizon.

### 4.4 Results on D1 (after synthetic masking)

For D1, the forecast horizon was set to be 28 days ($K = 28$). The choice of $K = 28$ would mean about 4 weeks ahead (which is a meaningful and not an overly long forecast horizon). This was also the forecast horizon of the M5 challenge. A test size of 127 days (time-points) out of 1941 time points was set aside for each sequence in D1. This means we tested for 100 output windows of width 28 per sequence. Given close to 2K time points in each sequence, we chose a proportionately long input-window of size 200 here.

To illustrate on D1 (where data is not missing), we artificially simulate masking using long masking windows. Tab. 2 provides errors of all relevant methods. Results indicate superior performance of both DEMI and DEGD with best case MAPE improvements per baseline of 14%, 8%, 4% and 28% with respect to BEDXM, BEDXL, GRU-D and WIAED respectively. Similarly, the best case MAPE improvements per baseline were 0.17, 0.16, 0.05 and 0.39 respectively. Also note all improvements of the proposed approaches are statistically significant.

![Figure 5: Percentage of sequences (out of 940 on which at least one method’s MASE < 1) where DEGD does better.](image)

**Table 2: D1 results: $T_w = \{30, 31, 32, \ldots , 39, 40\}$, Input-Window width=200, Category=FOODS.**

| Seq ID | DEMI | DEGD | BEDXM | BEDXL | GRU-D | WIAED |
|--------|------|------|-------|-------|-------|-------|
| 3,714  | (0.47,38) | (0.46,38) | (0.56,46) | (0.56,46) | (0.49,40) | (0.48,39) |
| 3,252  | (0.51,23) | (0.51,23) | (0.54,26) | (0.57,25) | (0.55,20) | (0.74,33) |
| 3,080  | (0.16,71) | (0.15,71) | (0.32,21) | (0.31,15) | (0.20,12) | (0.54,35) |
| 3,090  | (0.29,19) | (0.29,19) | (0.41,26) | (0.45,27) | (0.32,21) | (0.56,41) |

It demonstrates that DEGD does better on at least 66% of sequences and up to 85%, compared to all baselines.

Tab. 3 gives the average, max and min across sequences (of MASE) for all methods. It demonstrates that on an average DEGD does better than all baselines. MASE improvements are up to 0.94. Tab. 4 looks at the (conditional) average MASE under two conditions with respect to each baseline: (i)average over those sequences on which DEGD fares better, (ii)average over those sequences on which the baseline does better. At this level, we observe MASE improvements of at least 0.22 while up to 1.58.

![Table 3: Max, Avg and Min of MASE across all relevant 940 sequences (on which at least one method’s MASE < 1)](image)

| Method | MASE based |
|--------|------------|
|        | Max | Avg | Min |
| DEMI   | 21.44 | 1.79 | 0.004 |
| DEGD   | 19.61 | 0.43 | 0.001 |
| BEDXM  | 17.26 | 1.34 | 0.001 |
| BEDXL  | 16.98 | 1.37 | 0.002 |
| GRU-D  | 16.87 | 0.52 | 0.0002 |
| WIAED  | 24.11 | 0.93 | 0.005 |

**Table 4: Average MASE on 940 relevant sequences when (i)DEGD fares better (ii)Baseline fares better.**

| Method | DEGD better | Baseline better |
|--------|-------------|----------------|
|        | Diff | Diff | Diff | Diff |
| DEMI   | 0.26 | 2.18 | 1.92 | 0.97 |
| BEDXM  | 0.21 | 1.75 | 1.54 | 0.94 |
| BEDXL  | 0.21 | 1.79 | 1.58 | 0.93 |
| GRU-D  | 0.24 | 0.46 | 0.22 | 0.80 |
| WIAED  | 0.34 | 0.98 | 0.64 | 0.95 |

### 4.5 Results on D2

For D2, input window size is chosen to be 20, while the prediction horizon chosen was of length $K = 30$ (decoder length OR output window size), which is a 3-month ahead forecast horizon. This choice of input and output window length (per example) results in a reasonably long forecast horizon while also generating sufficient number of input-output examples per sequence given that we have about 104 weeks of data points per sequence. We tested for 6 output windows of width 12 on a 17 week (separately kept) test set per sequence.

For about 348 items, $MASE > 1$, for all baselines and the proposed methods, which meant these items could be tackled better by the simple copy previous baseline. We hence kept these items aside and concentrate on the remaining 940, on each of which at least one of the baselines OR proposed approaches have $MASE < 1$. Fig. 5 gives a detailed breakup of the percentage of these 940 sequences on which DEGD did better compared to DEMI and the 4 baselines.

### 4.6 Results on D3 (after synthetic masking)

As explained earlier, we perform a synthetic masking first. Note the per-sequence length here is about 150 points. Compared to D1, the input window length has to chosen to be comparatively lower (chosen to be 20 here as in D2) to obtain reasonable number of input-output examples from each sequence. This means the masking window length has to be proportionately less (compared to that of D1).
A test size of 15 weeks (time-points) was set aside for each sequence (out of about 150 weeks) in D3. We choose $K = 10$ time-steps in the decoder (output window length) for training which means forecast time-horizon would be about 2.5 months, a reasonably long forecast window which also allows for sufficiently many input-output examples from each sequence. We tested for 6 output windows (as in D2) of width 10 on the 15 week test set per sequence.

Fig. 6 gives a detailed breakup of percentage of sequences on which DEMI did better compared to the 4 baselines. It demonstrates that DEMI does better on at least 52% of the sequences and up to 59% compared to all considered baselines. A similar plot is obtained when DEGD is compared with the 4 baselines as shown in Fig. 7. This clearly indicates similar performance of DEGD and DEMI on D3.

![Figure 6: Percentage of sequences where DEMI does better. $q = 0.05, T_w = \{6, 7, 8, 9, 10\}$.]

![Figure 7: Percentage of sequences where DEGD does better. $q = 0.05, T_w = \{6, 7, 8, 9, 10\}$.

Tab. 5 gives the average, max and min across sequences (of MASE and MAPE) for all methods. It demonstrates that on average DEMI does better than all baselines based on both these complementary metrics. MASE improvements are up to 0.10 while the MAPE improvements are up to 5%. Tab. 6 looks at the (conditional) average MASE under two conditions with respect to each baseline: (i) average over those sequences on which DEMI fares better, (ii) average over those sequences on which the baseline does better. At this level, MASE improvements of at least 0.28 while up to 0.60 are observed. Tab. 7 considers a similar (conditional) average MAPE. At this level of MAPE, there are improvements of at least 11% to up to 18%. Overall our results on D3 indicate that our approach is viable even when the masking window length is low.

Table 5: Max, Avg, Min of MASE/MAPE across all sequences

| Method | MASE | MAPE |
|--------|------|------|
| DEMI   | 4.03 | 0.93 |
| DEGD   | 3.77 | 0.94 |
| BEDXM  | 3.62 | 0.99 |
| BEDXL  | 3.43 | 0.96 |
| GRU-D  | 3.53 | 0.96 |
| WIAED  | 4.58 | 1.03 |

Table 6: Average MASE on sequences where (i)DEMI does better (ii)Baseline fares better.

| Method | DEMI better | Baseline better |
|--------|-------------|-----------------|
| DEMI   | 0.84        | 1.22            |
| DEGD   | 0.84        | 1.24            |
| BEDXM  | 0.84        | 1.18            |
| BEDXL  | 0.84        | 1.14            |
| GRU-D  | 0.86        | 1.46            |
| WIAED  | 0.84        | 1.34            |

Table 7: Average MAPE on sequences where (i)DEMI does better (ii)Baseline fares better.

| Method | DEMI better | Baseline better |
|--------|-------------|-----------------|
| DEMI   | 28          | 39              |
| DEGD   | 29          | 47              |
| BEDXM  | 32          | 47              |
| BEDXL  | 28          | 40              |
| GRU-D  | 28          | 44              |
| WIAED  | 28          | 47              |

5 CONCLUSIONS AND FUTURE WORK

For time-series under missing data, we proposed a novel lossless, compressed encoding scheme of the input window sequence using RNNs. This is equally applicable for classification and forecasting. Based on this novel scheme, we proposed an encoder-decoder framework for a general multi-step forecasting with possible exogenous inputs. We showed that multiple interesting variations are possible on the associated decoder. We demonstrated the utility of this ED framework on multiple data-sets where our proposed approach was outperforming the current state-of-the-art RNN baselines. In particular, our approaches demonstrated superior or comparable performances in both scenarios where (i) data was synthetically masked with varied masking window length choices (data-sets D1 and D3) and (ii) when data was naturally missing (as in D2). Further, our experiments also indicate that our DEGD variant was the more robust variant in performance (compared to DEMI which performed poorly on D2). We also demonstrated the applicability of our proposed architectures on both single (D1) and multi sequence data-sets (D2 and D3). As future work, we would like to validate our approach on classification tasks. We would also like to explore approaches where target imputation can be avoided (before model building) in forecasting applications.
