One-Loop Calculation of the Oblique S Parameter
in Higgsless Electroweak Models

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1. Motivation

i) The Standard Model (SM) provides an extremely successful description of the electroweak and strong interactions.

ii) A key feature is the particular mechanism adopted to break the electroweak gauge symmetry to the electroweak subgroup, $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{QED}}$, so that the $W$ and $Z$ bosons become massive.

iii) The LHC has just discovered a new particle around 125 GeV*.

iv) What if this new particle is not a Higgs boson? Or a not standard one? Or a scalar resonance? We should look for alternative mechanisms of mass generation.

vi) Strongly-coupled models: usually they do contain resonances. Many possibilities in the market: Technicolour, Walking Technicolour, Conformal Technicolour, Extra Dimensions…

v) They should fulfilled the existing phenomenological tests.

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* Preliminary CMS and ATLAS Collaborations.

** Peskin and Takeuchi '92.
Similarities to **Chiral Simmetry Breaking in QCD**

i) In the limit where the $U(1)_Y$ coupling $g'$ is neglected, the Lagrangian is invariant under global $SU(2)_L \times SU(2)_R$ transformations. The **Electroweak Symmetry Breaking** (EWSB) turns out to be $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ (custodial symmetry).

ii) Absolutely similar to the **Chiral Symmetry Breaking** (ChSB) occuring in QCD. So the same pion Lagrangian describes the Goldstone boson dynamics associated with the EWSB, being replaced $f_\pi$ by $v=1/\sqrt{2G_F}=246$ GeV. Same procedure as in **Chiral Perturbation Theory** (ChPT)*.

$$\Delta L^{(2)}_{\text{ChPT}} = \frac{f_\pi^2}{4} \langle u_\mu u^\mu \rangle \rightarrow \Delta L^{(2)}_{\text{EW}} = \frac{v^2}{4} \langle u_\mu u^\mu \rangle$$

iii) We can introduce the **resonance fields** needed in strongly-coupled Higgsless modes in a similar way as in ChPT: **Resonance Chiral Theory** (RChT)**.

✓ Note the implications of a naïve rescaling from QCD to EW:

$$\begin{align*}
M_\rho &= 0.770 \text{ GeV} & \rightarrow & M_V &= 2.1 \text{ TeV} \\
M_{a_1} &= 1.260 \text{ GeV} & \rightarrow & M_A &= 3.4 \text{ TeV}
\end{align*}$$

iv) Actually, the estimation of the $S$ parameter in strongly-coupled EW models is equivalent to the determination of $L_{10}$ in ChPT***.

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* Weinberg ’79
* Gasser & Leutwyler ’84 ’85
* Bijnens et al. ’99 ’00
**Ecker et al. ’89
** Cirigliano et al. ’06
*** Pich, IR, Sanz-Cillero ’08.
2. Oblique Electroweak Observables

- Universal oblique corrections via the EW boson self-energies (transverse in the Landau gauge)
  \[ \mathcal{L}_{\text{v.p.}} = - \frac{1}{2} W_\mu^3 \Pi_{33}^{\mu\nu}(q^2) W_\nu^3 - \frac{1}{2} B_\mu \Pi_{00}^{\mu\nu}(q^2) B_\nu - W_\mu^3 \Pi_{30}^{\mu\nu}(q^2) B_\nu - W_\mu^+ \Pi_{WW}^{\mu\nu}(q^2) W_\nu^-. \]

- S parameter: new physics in the difference between the Z self-energies at \(Q^2=M_Z^2\) and \(Q^2=0\).
  \[ e_3 = \frac{g}{g'} \tilde{\Pi}_{30}(0), \quad \Pi_{30}(q^2) = q^2 \tilde{\Pi}_{30}(q^2) + \frac{g^2 \tan \theta_W}{4} v^2, \quad S = \frac{16\pi}{g^2} (e_3 - e_{3\text{SM}}). \]

- We follow the useful dispersive representation introduced by Peskin and Takeuchi.*
  \[ S = \frac{16}{g^2 \tan \theta_W} \int_0^\infty \frac{ds}{s} \left( \text{Im} \tilde{\Pi}_{30}(s) - \text{Im} \tilde{\Pi}^{\text{SM}}_{30}(s) \right) = \]
  \[ = \int_0^\infty \frac{ds}{s} \left( \frac{16}{g^2 \tan \theta_W} \text{Im} \tilde{\Pi}_{30}(s) - \frac{1}{12\pi} \left[ 1 - \left( 1 - \frac{M_H^2}{s} \right)^3 \theta (s - M_H^2) \right] \right) \]
  \[ \text{The convergence of the integral needs a vanishing } \text{Im} \tilde{\Pi}_{30}(s) \text{ at short distances.} \]
  \[ \text{S parameter is defined for a reference value for the SM Higgs mass.} \]

* Peskin and Takeuchi ’92.

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The Oblique S Parameter in Higgsless Electroweak Models, I. Rosell

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3. The Effective Lagrangian

Let us consider a low-energy effective theory containing the SM gauge bosons coupled to the electroweak Goldstones and the lightest vector and axial-vector resonances:

\[ \mathcal{L} = \mathcal{L}^{(2)}_{\text{EW}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{\text{kin}}^{VV} + \mathcal{L}_{\text{kin}}^{AA} + \mathcal{L}_{VA} \]

\[ \mathcal{L}^{(2)}_{\text{EW}} = -\frac{1}{2g^2} \langle \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \rangle - \frac{1}{2g'^2} \langle \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \rangle + \frac{v^2}{4} \langle u_\mu u^\mu \rangle, \quad \mathcal{L}_{\text{GF}} = -\frac{1}{2\xi} (\partial^\mu \hat{W}_\mu)^2, \]

\[ \mathcal{L}_V + \mathcal{L}_A = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f^\mu_{\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^{\nu}] \rangle + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f^\mu_{\nu} \rangle, \]

\[ \mathcal{L}_{\text{kin}}^{RR} = -\frac{1}{2} \langle \nabla^\lambda R_\lambda R_\nu R^\nu - \frac{M^2_R}{2} R_{\mu\nu} R^{\mu\nu} \rangle, \quad (R = V, A) \]

\[ \mathcal{L}_{VA} = i \lambda_2^{VA} \langle [V^\mu_{\nu}, A_{\nu\alpha}] h^\alpha_\mu \rangle + i \lambda_3^{VA} \langle [\nabla^\mu V_{\mu\nu}, A^{\nu\alpha}] u_\alpha \rangle + i \lambda_4^{VA} \langle [\nabla_\alpha V_{\mu\nu}, A^{\nu\mu}] u_\mu \rangle + i \lambda_5^{VA} \langle [\nabla_\alpha V_{\mu\nu}, A^{\mu\nu}] u^\alpha \rangle, \quad \kappa = -2\lambda_2^{VA} + \lambda_3^{VA}, \quad \sigma = 2\lambda_2^{VA} - 2\lambda_3^{VA} + \lambda_4^{VA} + 2\lambda_5^{VA}. \]

We have seven resonance parameters: \( F_V, G_V, F_A, \kappa, \sigma, M_V \) and \( M_A \).

The high-energy constraints are fundamental.
4. The Calculation of S

i) At leading-order (LO)*

\[ \Pi_{30}(s)|_{LO} = \frac{g^2 \tan \theta_W}{4} s \left( \frac{v^2}{s} + \frac{F_V^2}{M_V^2} - s - \frac{F_A^2}{M_A^2} - s \right) \]

\[ S_{LO} = 4\pi \left( \frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right) \]

* Peskin and Takeuchi '92.
ii) At next-to-leading order (NLO)**

\[
\Pi_{30}(s) = \Pi_{30}(0) + \frac{s}{\pi} \int_0^\infty \frac{dt}{t(t-s)} \text{Im}\Pi_{30}(t)
\]

- Once-subtracted dispersive relation
- Contributions from $\pi\pi$, $V\pi$ and $A\pi$ cuts, since higher cuts are supposed to be suppressed.
- $F_R^r$ and $M_R^r$ are renormalized couplings which define the resonance poles at the one-loop level.

\[
\Pi_{30}(s) |_{\text{NLO}} = \frac{g^2 \tan \theta_W}{4} s \left( \frac{v^2}{s} + \frac{F_V^r}{M_V^r} - s - \frac{F_A^r}{M_A^r} - s + \overline{\Pi}(s) \right)
\]

\[
S_{\text{NLO}} = 4\pi \left( \frac{F_V^r}{M_V^r} - \frac{F_A^r}{M_A^r} \right) + \overline{S}
\]

* Barbieri et al.’08
* Cata and Kamenik ’08
* Orgogozo and Rynchov ’08
5. High-energy Constraints

- We have seven resonance parameters: $F_V, G_V, F_A, \kappa, \sigma, M_V$ and $M_A$.
- The number of unknown couplings can be reduced by using short-distance information.
- In contrast with the QCD case, we ignore the underlying dynamical theory.

i) Weinberg Sum Rules (WSR)*

\[
\Pi_{30}(s) = \frac{g^2 \tan \theta_W}{4} s \left[ \Pi_{VV}(s) - \Pi_{AA}(s) \right]
\]

\[
\begin{align*}
\frac{1}{\pi} \int_0^\infty dt \left[ \text{Im} \Pi_{VV}(t) - \text{Im} \Pi_{AA}(t) \right] &= v^2 \\
\frac{1}{\pi} \int_0^\infty dt t \left[ \text{Im} \Pi_{VV}(t) - \text{Im} \Pi_{AA}(t) \right] &= 0
\end{align*}
\]

i.0) LO

\[
F_V^2 - F_A^2 = v^2 \\
F_V^2 M_V^2 - F_A^2 M_A^2 = 0
\]

i.ii) Imaginary NLO

\[
\text{Im} \Pi_{30}(s) \sim \mathcal{O} \left( \frac{1}{s} \right)
\]

i.iii) Real NLO: fixing of $F_{V,A}^r$ or lower bounds**

\[
F_V^{r,2} - F_A^{r,2} = v^2 (1 + \delta^{(1)}_{\text{NLO}}) \\
F_V^{r,2} M_V^{r,2} - F_A^{r,2} M_A^{r,2} = v^2 M_V^{r,2} \delta^{(2)}_{\text{NLO}}
\]

* Weinberg’67
* Bernard et al.’75.
** Pich et al.’08
ii) Additional short-distance constraints

ii.1) $W_L W_L \rightarrow W_L W_L$ scattering*  
\[ G_V = \frac{v}{\sqrt{3}} \]

ii.2) Vector Form Factor**  
\[ F_V G_V = v^2 \]

ii.3) Axial Form Factor***  
\[ F_V - 2G_V = F_A (2\kappa + \sigma) \]

✓ We have up to 9 (7) constraints with 2 (1) WSR and 7 resonance parameters: we cannot consider all the constraints at the same time, some approximately.

✓ As a check of consistency we consider different combination of constraints.

* Bagger et al.’94
** Barbieri et al.’08
*** Ecker et al.’89
**** Pich et al.’08
6. Phenomenology

\[ S = 0.04 \pm 0.10 \times (M_H=0.120 \text{ TeV}) \]

i) LO results

i.i) 1st and 2nd WSRs

\[ S_{LO} = \frac{4\pi v^2}{M_V^2} \left( 1 + \frac{M_V^2}{M_A^2} \right) \]

\[ \frac{4\pi v^2}{M_V^2} < S_{LO} < \frac{8\pi v^2}{M_V^2} \]

i.ii) Only 1st WSR

\[ S_{LO} = 4\pi \left\{ \frac{v^2}{M_V^2} + F_A^2 \left( \frac{1}{M_V^2} - \frac{1}{M_A^2} \right) \right\} \]

\[ S_{LO} > \frac{4\pi v^2}{M_V^2} \]

At LO \( M_V > 1.5 \text{ TeV} \) at 3\( \sigma \)

* Gfitter
* LEP EWWG
* Zfitter
ii) NLO results: 1st and 2nd WSRs

- 1st and 2nd WSRs at LO and at NLO:
  - 6 constraints
  - $M_V$ the only free parameter
- 8 solutions.
- Only 2 approximately compatible with VFF, AFF and scattering constraints (green).

- If, alternatively, we consider the 1st and the 2nd WSR only at NLO with the VFF and AFF constraints (6 constraints), a heavier result is found: $M_V > 2.4$ TeV at $3\sigma$.

At NLO with the 1st and 2nd WSRs $M_V > 1.8$ TeV at $3\sigma$.
iii) NLO results: only 1st WSR

- 1st WSR at NLO + VFF and AFF constraints:
  - 5 constraints
  - \( M_V \) and \( M_A \) the only free parameters are.

- Without the 2nd WSR we can only derive lower bounds on \( S \).

- Imposing that \( F_v^2 - F_A^2 > 0 \) we have found only 2 solutions.

- One of them (red) is clearly disfavoured:
  - Sharply violation of the 2nd WSR at LO and at NLO
  - Large NLO correction
  - Big splitting between \( M_V \) and \( M_A \).

- Without the 2nd WSR it is possible the analysis with only the \( \pi\pi \) cut. The same result is found: \( M_V > 1.8 \text{ TeV at 3} \sigma \).
7. Summary

1. What?

One-loop calculation of the oblique $S$ parameter within Higgsless models of EWSB

2. Why?

What if this new particle around 125 GeV is not a Higgs boson?

- We should look for alternative ways of mass generation: strongly-coupled higgsless models.
- They should fulfill the existing phenomenological tests.

3. Where?

Effective approach

- $\text{EWSB: SU}(2)_L \times \text{SU}(2)_R \Rightarrow \text{SU}(2)_{L+R}$: similar to ChSB in QCD: ChPT.
- Strongly-coupled Higgsless models: similar to resonances in QCD: RChT.
- General Lagrangian with at most two derivatives and short-distance information.

4. How?

Dispersive representation of Peskin and Takeuchi'92.
✓ Improvements over previous NLO calculation:
  ✓ Dispersive calculation: no unphysical cut-offs.
  ✓ A more general Lagrangian.
  ✓ Short-distance information as a crucial ingredient.

✓ We have considered different possibilities:
  ✓ LO
  ✓ NLO with the 1st and 2nd WSR
  ✓ NLO with only the 1st WSR

✓ Similar results:
  ✓ At LO $M_V > 1.5$ TeV at 3$\sigma$.
  ✓ At NLO $M_V > 1.8$ TeV at 3$\sigma$.

✓ In these reasonable strongly coupled models the S parameter requires a high resonance mass scale, beyond the 1 TeV.
Consideration of this new scalar with a mass around 125 GeV in our calculation:

- Higgs boson? Which one?
- A scalar resonance of strongly-coupled models?

Oblique T parameter

A new $S\pi$ or $H\pi$ cut!, but only at NLO.

Absence of a known dispersive representation.