Correlation functions of one-dimensional polar-molecules on optical lattices

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We combine a slave-spin approach with a mean-field theory to develop an approximate theoretical scheme to study the charge, spin, and, pairing correlation functions of fermionic polar molecules. We model the polar molecules subjected to a one-dimensional periodic optical lattice potential using a generalized \( t-J \) model, where the long-range part of the interaction is included through the exchange interaction parameter. For this model, we derive a set of self-consistent equations for the correlation functions, and evaluate them numerically for specific cases. We find that the pairing correlations are related to spin correlations through the doping level and the slave-spin correlations. Further, our calculations indicate that the long-range character of the interaction can be probed through these correlation functions. In the absence of exact solutions for the one-dimensional \( t-J \) model, our approximate theoretical treatment can be treated as a useful tool to study one dimensional long-range correlated fermions.

I. INTRODUCTION

Recent developments in laser technology and current extraordinary experimental advances in trapping and manipulating of ultra-cold atomic gases provide a wonderful path for the quantum simulation of many-body physics. Due to the unprecedented control over various experimental parameters, such as spatial dimensionality, lattice structures and geometries, interaction parameters, and atomic species, ultra-cold atomic systems are considered as promising playgrounds for studying fundamental condensed matter phenomena [1–2]. In addition to the experimental systems of ultra-cold bosons and fermions with short-range interactions on optical lattices [1–2], a great deal of excitement offered towards the polar-molecules as their long-range dipolar interactions give rise to more exotic many-body phenomena [10–12]. Unlike condensed matter counterpart part with Coulomb interactions, the dipolar-type long-range interactions are not affected by screening. As a result, the dynamics of polar molecules are described by tunable extended Hubbard-type Hamiltonians [13–15]. The tunability of all Hamiltonian parameters can be achieved by manipulating the anisotropic dipole-dipole interactions between the homonuclear and heteronuclear molecules via external dc and ac fields in the microwave regime [16–21]. Some of the predicted exciting novel many-body phenomena due to the effect of long-range character and the anisotropy of the dipolar interactions [22–29], have already been investigated in experiments [30–32].

In this work, we study density (charge), spin, and pairing correlation functions of an one-dimensional chain of polar molecules in a lattice. The dynamics of the polar molecules on the lattice can be represented by a generalized \( t-J \) model with long range interactions between molecules in the form \( 1/r^3 \), where \( r \) is the spatial distance between molecules. The usual nearest neighbor \( t-J \) model without this long range interaction part is the strong coupling limit of the well-known Hubbard model away from half filling. The strong coupling limit of the Hubbard model is defined as the limit \( U \gg t \), where \( U \) is the on-site interaction and \( t \) is the hopping or tunneling amplitude. There is no exact solutions even for this nearest neighbor only one dimensional \( t-J \) model, except for special cases [33–35]. The strong coupling limit of the Hubbard model \( U \gg t \) is equivalent to the \( t-J \) model when \( J \ll t \), where \( J \) is the exchange spin coupling of the \( t-J \) model. The exact Bethe-ansatz solutions for the nearest neighbor only one dimensional model is available only for \( J \ll t \) limit [35] and \( J = 2t \) limit [34–35]. Even for these limits, the Bethe-ansatz wave functions provide limitations for the calculation of physical quantities due to the complexity of Bethe-ansatz solutions. Therefore, the study of the system of long-range interacting one dimensional lattice polar molecules requires heavily numerical or advanced novel theoretical techniques. It is the purpose of this paper to develop such a technique to tackle polar interacting molecules on optical lattices.

We use a constraint free slave-spin approach to represent the quasi-particle operators and transform the interacting Hamiltonian into a combined slave spin and spinless fermion model. In this slave-spin representation, the original quasi-particle operators are decomposed into a bosonic pseudo-spin field and a fermionic field. However unlike other slave-boson representations, the charge degrees of freedom and the spin degrees of freedom are combinations of these two fields. We then evaluate the charge, spin, and pairing correlation functions after decoupling the slave-spin and the spinless fermion sectors by using a mean-field theory. These correlation functions need to be derived self-consistently due to the coupling of pseudo-spin field and fermion field through their mean-field values. Our decoupled spinless fermion part of the Hamiltonian has the form of topological Hamiltonians presented in Refs. [36–40]. However, the effective interaction parameters in our transformed fermion part need to be calculated self consistently in combination with slave-spin sector. We find that the zero tempera-
ture correlation functions show characteristic oscillatory decay, where oscillatory part originates from the pseudo-spin correlation and the algebraic decay originates from both pseudo-spin and spinless fermion correlations.

The paper is organized as follows. In section II, we introduce an effective model for the polar molecular system. The model is a generalization of the well known $t-J$ model, modified to include the long-range dipolar interactions. In section III, we introduce a slave-spin approach and convert our model Hamiltonian into a coupled spin-particle Hamiltonian. In section IV, we use a mean-field decoupling scheme to decouple the pseudo spin and spinless particle sectors of the Hamiltonian. In section V and VI, we solve the spin and spinless fermions parts independently in momentum space. In section VII, we combined the solutions of two sectors and derive self-consistent equations for the unknown mean field parameters. We dedicate section VIII to discuss the correlation functions and to provide our results. Finally, we present our conclusions with a short discussion in section IX.

II. THE MODEL

Our model describing the polar molecules in optical lattices is given by \[14\],

\[ H = -t \sum_{\langle ij \rangle} (c_{i,s}^\dagger c_{j,s} + h.c) \tag{1} \]

\[ + \sum_{i,j>0} \frac{1}{|i-j|^3} \left[ J_\perp (S_{i}^+ S_{j}^- + S_{i}^- S_{j}^+) + J_z S_{i}^z S_{j}^z \right]. \]

As usual, $c_{i,s}^\dagger$($c_{i,s}$) are fermionic creation (annihilation) operator for a fermionic polar molecule with spin $s = \uparrow, \downarrow$ on lattice site $i$. The components of spin operators are defined as $S_{i}^+ = c_{i,\uparrow}^\dagger c_{i,\downarrow}$, $S_{i}^- = c_{i,\downarrow}^\dagger c_{i,\uparrow}$, and $S_{i}^z = (c_{i,\uparrow}^\dagger c_{i,\uparrow} - c_{i,\downarrow}^\dagger c_{i,\downarrow})/2$. When deriving this model, it has been assumed that double occupations at a single site are not allowed due to the strong on-site interactions, thus the fermionic Hilbert space is projected onto the space with no doublons. The model is a straightforward generalization of the well-known $t-J$ model proposed for condensed matter systems \[41,44\]. All model parameters in our Hamiltonian and filling factors can be controlled independently. Here we restrict ourselves to a simple experimental realization of the model by setting $J_\perp = 2J_\parallel = J$ and assume hopping is restricted only to the nearest neighbors.

III. SLAVE SPIN REPRESENTATION OF THE MODEL

Slave-particle approaches are very common in studying strongly correlated systems due to their simplic-

ity in applying computational techniques and capability of accounting particle correlations beyond standard mean-field theories. While most variational approaches are valid only at zero temperature, the mean-field theories are unable to capture quantum fluctuations. However, slave particle theories are valid at both zero and finite temperatures and capable of capturing quantum fluctuations \[45,48\]. Further, it has been shown that slave-particle approaches are equivalent to a statistically-consistent Gutzwiller approximation \[49\]. In slave particle formalisms, the original local Fock space of the problem is usually mapped onto a larger local Fock space that contains more states due to the introduction of auxiliary particles. In general these nonphysical states are removed in enlarge Hilbert space by imposing constraints. These constraints introduce additional self-consistent equations for the calculation. However in this work, we apply constraint-free, invertible canonical transformation proposed in Ref. \[50\] to the correlated polar molecules on optical lattices. The transformation is more effective than other slave-particle transformations as the basis states of the Hilbert spaces of a molecule on a single site has one-to-one mapping. This one-to-one mapping excludes the additional constraint equations in this slave-spin scheme. In this representation, the quasi-particle is described as a composite of spin-ness and Fermi-ness. While the spin-ness is described by a Pauli operator, the Fermi-ness is described by a spinless fermion \[50\]. The physical spin and the physical particle number are related to both Pauli operators and spinless fermion number.

In this slave-spin approach, the particle operator is decoupled into a spinless fermion and a Pauli operator that carry the charge and spin degrees of freedom, respectively. First, the quasi-particle operator $c_{i,\uparrow/\downarrow}$ that creates an atom with spin $\uparrow/\downarrow$ at site $i$ is expressed as

\[ c_{i,\uparrow} = (a_{i,\uparrow} + a_{i,\downarrow})\sigma_{i}^\dagger/2, \tag{2} \]

and

\[ c_{i,\downarrow} = [(a_{i,\uparrow} (1 - \sigma_{i}^z) - a_{i,\downarrow} (1 + \sigma_{i}^z))/2. \tag{3} \]

Notice that a typo of missing factor 1/2 in Ref. \[50\] is corrected in Eq. (2). Under this transformation, the original number operators transform as $\hat{n}_{i,\uparrow} = (1 + \sigma_{i}^z)/2$ and $\hat{n}_{i,\downarrow} = 1/2 + (1/2 - \hat{n})\sigma_{i}^z$, where $\hat{n} = a_{i,\uparrow}^\dagger a_{i,\downarrow}$ is the spinless fermion number operator. The physical spin $S_{i}$ takes the form $S_{i} = \hat{n}_s \sigma_{i}/2$. Furthermore, the Hubbard-type interaction $(\hat{n}_{i,\uparrow} - 1/2)(\hat{n}_{i,\downarrow} - 1/2) = (1/2 - \hat{n})/2$. Therefore, the new Pauli operator $\sigma$ represents the spin of the particles in the presence of spinless fermions and the strongly correlated nature of the original quasi-particles are captured by that of the spinless fermions. Further, the physical pairing operator whose components are given by $P_{i}^z = (\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} - 1)/2$ and $P_{i}^+ = c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger$ has the form
in new representation $\vec{F}_i = (\hat{n}_i - 1)\hat{\sigma}_i/2$. In terms of new variables, our Hamiltonian becomes,

$$H = -\frac{t}{2} \sum_{(ij)} (a_i^\dagger a_j + h.c)(1 + \hat{\sigma}_i \cdot \hat{\sigma}_j)$$

$$-t \sum_{(ij)} (a_i^\dagger a_j + h.c)(1 - \hat{\sigma}_i \cdot \hat{\sigma}_j)$$

$$+ \frac{J}{4} \sum_{ij, j > 0} \frac{1}{|i - j|^2} \hat{n}_i \hat{n}_j \hat{\sigma}_i \cdot \hat{\sigma}_j.$$ 

The last term is simply the $S_i^z S_j^z \rightarrow \hat{n}_i \hat{n}_j \hat{\sigma}_i \cdot \hat{\sigma}_j/4$ and the first two terms originates from the transformed hopping part of the Hamiltonian.

IV. DECOUPLING SPIN AND FERMIONS

We decouple the transformed Hamiltonian by using a mean-field description. By introducing four local mean-field parameters, $\langle a_i^\dagger a_j + h.c \rangle_a = \chi_{ij}$, $\langle a_i^\dagger a_j^\dagger + h.c \rangle_a = \Delta_{ij}$, $\langle \hat{n}_i \hat{n}_j \rangle_a = D_{ij}$, and $\langle \hat{\sigma}_i \cdot \hat{\sigma}_j \rangle_s = m_{ij}$, our Hamiltonian $H$ becomes the sum of independent spin and fermion parts: $H = H_s + H_a$. This will lead to the $H_s$ part being an interacting pure spin model and the $H_a$ part being an interacting spinless fermion part. The subscript $a$ or $s$ means that the quantum and thermal expectation values must be taken with respect to the spin and fermion sectors, respectively. After performing the decoupling scheme, the fermion and spin parts of the Hamiltonian become,

$$H_a = -\frac{t}{2} \sum_{(ij)} (1 + m_{ij})(a_i^\dagger a_j + h.c)$$

$$-t \sum_{(ij)} (1 - m_{ij})(a_i^\dagger a_j^\dagger + h.c)$$

$$+ \frac{J}{4} \sum_{ij, j > 0} \frac{m_{ij}}{|i - j|^2} \hat{n}_i \hat{n}_j,$$

$$H_s = -\frac{t}{2} \sum_{(ij)} (\chi_{ij} - \Delta_{ij})\hat{\sigma}_i \cdot \hat{\sigma}_j$$

$$+ \frac{J}{4} \sum_{ij, j > 0} \frac{D_{ij}}{|i - j|^2} \hat{\sigma}_i \cdot \hat{\sigma}_j$$

$$-\frac{t}{2} \sum_{(ij)} (\chi_{ij} + \Delta_{ij}).$$

V. SOLUTION OF THE SPIN PART

By defining $|i - j| = ld_i$ with lattice constant $d_i$ and integer $l$, and rearranging the dummy variables in the sum, the spin part of the Hamiltonian can be casted as,

$$H_s = -\sum_{l=1}^{L-1} \sum_{j=1}^{r} J^z_l \hat{\sigma}_j \cdot \hat{\sigma}_{j+l}.$$ 

Here, we define $J^z_l = K_l \delta_{il} + J_l$, where $K_l = t(\chi_l - \Delta_l)/2$ and $J_l = JD_l/(4l \alpha^2)$, where we used a compact notation for local mean-field parameters, $\chi_{ij} = \chi_l$ for $|i - j| = ld_i$ etc. The parameter $L$ is the number of lattice sites, $r$ in general represents the range of interaction and $\delta_{il}$ is the usual discrete Kronecker delta function. Notice that the decoupled pseudo spin part of the Hamiltonian is still an interacting long-range spin Hamiltonian. The classical ground state of this one-dimensional effective pseudo spin Hamiltonian on a Bravais lattice is a single-$Q$ spiral state. Therefore, first we make a coordinate transformation by rotating the local axis by an angle $\theta_l = \hat{Q} \cdot \hat{r}_i$, such that a new axis-$\xi$ coincides with the classical solution of the pseudo-spin orientation,

$$\sigma_i^x = \sigma_i^0$$

$$\sigma_i^y = \sigma_i^0 \cos \theta_l + \sigma_i^0 \sin \theta_l$$

$$\sigma_i^z = -\sigma_i^0 \sin \theta_l + \sigma_i^0 \cos \theta_l.$$ 

The resulting Hamiltonian then becomes,

$$H_s = -\sum_{l=1}^{r} \sum_{j=1}^{L-1} J^z_l \{\sigma_j^0 \sigma_{j+l} + \sin(lQd_i)[\sigma_j^0 \sigma_{j+l} + \sigma_{j+l}^0 \sigma_{j+l}]$$

$$+ \cos(lQd_i)[\sigma_j^0 \sigma_{j+l} + \sigma_{j+l}^0 \sigma_{j+l}^0]\}.$$ 

Representing pseudo spin operators by bosonic Holstein-Primakoff local operators, $\sigma_i^0 = B_i^\dagger + B_i$, $\sigma_i^+ = B_i^\dagger - B_i$, and $\sigma_i^z = 1 - 2B_i^\dagger B_i$, and restricting ourselves to the quadratic order, the effective pseudo spin Hamiltonian in Fourier space can be written as,

$$H_s = -\sum_k \{2J^z(k)[B_k B_{-k} + B_k^\dagger B_{-k}^\dagger] - 8J^z(k)B_k^\dagger B_k$$

$$+ [2J^z(k) + J^z(k + Q) + J^z(k - Q)][B_k B_{-k}^\dagger + B_k^\dagger B_{-k}]\}.$$ 

where $J^z(k) = \sum_{l} J^z_l e^{ikld_i}$ is the Fourier transform of the effective coupling constant. The Hamiltonian can be brought to a diagonal form by usual Bogoliubov transformation, $A_k^\dagger = \cos \alpha_k B_k^\dagger + \sin \alpha_k B_k$,

$$H_s = \sum_k \omega_k \left( A_k^\dagger A_k + \frac{1}{2} \right),$$

where the effective spin wave dispersion $\omega_k = \sqrt{M_{11}(k) - M_{21}(k)}$. Here the matrix elements of the Hamiltonian matrix $M_{11}(k)$ and $M_{12}(k)$ are given by,
Hamiltonian for the spinless fermion sector has the form

\[ H = J^r(Q) - J^s(Q) - (1/2)[J^c(k + Q) + J^c(k - Q)] \]

and \( M_1(k) = 2J^r(Q) - J^r(Q) - (1/2)[J^c(k + Q) + J^c(k - Q)] \)

The diagonal form of the Hamiltonian allows us to calculate the local mean field parameter, \( m_{ij} = \langle \bar{\sigma}_i \cdot \bar{\sigma}_j \rangle_s \) through the Bogoliubov quasi particles. Assuming \( j = i + 1 \) and we find \( m_{ij} \) through the Fourier transform \( m_k = X^-(k) + [X^+(k - Q) + X^+(k + Q)]/2 \),

\[ H_\psi(k) = g_k \mathbb{1} + h_k \sigma_z + f_k \sigma_y + e_k \sigma_z \]

where \( \sigma_z \)’s are components of usual Pauli matrices and \( \mathbb{1} \) is the identity matrix. This is one of the most general Hamiltonians responsible for non-trivial topological quantum states [51]-[53]. The Hamiltonian can be diagonalized by using usual Bogoliubov transformation through a new fermionic quasiparticles, represented by the operator \( \gamma_k = e^{i\phi_k} \cos(\theta_k/2) a_k + e^{i\phi_k} \sin(\theta_k/2) a_k^\dagger \). The resulting diagonalized Hamiltonian has the form,

\[ H_s = \sum_{k,\lambda} E_{k\lambda} \gamma_\lambda^\dagger \gamma_{k\lambda}, \]

where the eigenvalues \( E_{k\pm} = g_k \pm \sqrt{f_k^2 + h_k^2} \). This allows us to calculate the previously defined local mean-field parameters, \( \Delta_l = \langle a_l^\dagger a_0 + h.c \rangle_a = \Delta_l + \Delta_l^* \)

\[ \Delta_l = \sum_k F_k e^{i(kd_l + \phi_k)}, \]

where,

\[ F_k = \frac{1}{2} \sum_\lambda \left( n_{k\lambda} - 1 \right) \sin(\theta_k), \]

and \( \chi_l = \langle a_l^\dagger a_0 + h.c \rangle_a = \tilde{\chi}_l + \tilde{\chi}_l^* \)

\[ \tilde{\chi}_l = \sum_k G_k e^{ikd_l}, \]

where

\[ G_k = \cos^2(\theta_k/2) n_{k+} - \sin^2(\theta_k/2) n_{k-} + \sin^2(\theta_k/2). \]

Here we defined, \( \phi_k \equiv \phi_1 - \phi_2 = \arctan[f_k/h_k] \)

and \( n_{k\lambda} = (e^{E_{k\lambda}/k_BT} - 1)^{-1} \). The parameter \( \theta_k \) is determined by \( \tan(\theta_k) = \sqrt{2} f_k \sin(2\phi_k) + h_k \cos(2\phi_k) / e_k \) with the constraint \( \phi_1 + \phi_2 = \pi/4 \).

VI. SOLUTION OF THE FERMION PART

First we tackle the spinless fermions density-density interaction term by a mean-field approximation, \( n_i n_j \equiv n_a(a_i^\dagger a_i)^2 + (\hat{\sigma}_j a_i^\dagger a_j + h.c) \), where \( n_a = \langle a_i^\dagger a_i \rangle_s \), \( \hat{\sigma}_j = (a_i^\dagger a_j) \), and \( \chi_i = \langle a_i^\dagger a_j \rangle_s \). Here h.c stands for complex conjugate. Notice that previously defined mean-field parameters are related to these as \( \Delta_i = \Delta_{ij} + \Delta_{ij}^* \) and \( \chi_i = \chi_{ij} + \chi_{ij}^* \). This leads to the fermion part of the Hamiltonian in real space,

\[ H_s = - \sum_l \sum_{j=1}^{L-1} \left( -e^{i\phi_j} a_j^\dagger a_j + \Delta_i e^{i\theta_j} a_j^\dagger a_j + h.c \right) \]

\[ + \mu \sum_{j=1}^{L-1} a_j^\dagger a_j \]

where we defined \( \omega_j e^{i\phi_j} = \frac{1}{2} \sum k a_k e^{ikL} \) with \( \Delta_l = \langle a_l^\dagger a_0 + h.c \rangle_a \)

\[ \text{VII. SELF-CONSISTENT EQUATIONS} \]

The spinless particle correlations \( \Delta_l \) and \( \chi_l \) must be calculated through the self-consistent equations (12), (17), and (19). Using these three equations, first we derive system parameters, \( e_k \), \( f_k \), \( g_k \), \( h_k \), and \( J^s(k) \) in terms of the functions \( G_k \) and \( F_k \), and \( m_k \).

\[ e_k = \frac{J}{8d_l^2} \sum_{k',k''} G_{k'} m_{k''} \left\{ 8n_{k''} - n_{q+k} - n_{q-k} \right\} \]

\[ - \frac{t}{2} \left[ 1 + \sum_k m_{k'} \cos(k'd_l) \right] \cos(kd_l). \]
\[ f_k = \frac{J}{8d_1^2} \sum_{k',k''} F_{k'} \sin(\phi_{k'}) m_{k''} [\Re \Gamma_{q-k} - \Re \Gamma_{k-q}] + \frac{J}{8d_1^2} \sum_{k',k''} F_{k'} \cos(\phi_{k'}) m_{k''} [\Im \Gamma_{q-k} + \Im \Gamma_{k-q}] \]
\[ = \frac{t}{2} \left[ 1 - \sum_{k'} m_{k'} \cos(k'd_i) \right] \sin(kd_i), \quad (21) \]
\[ g_k = \frac{J}{8d_1^2} \sum_{k',k''} G_{k'} m_{k''} [\Re \Gamma_{q-k} - \Re \Gamma_{q-k}] + \frac{t}{2} \left[ 1 + \sum_{k'} m_{k'} \sin(k'd_i) \right] \sin(kd_i), \quad (22) \]
\[ h_k = \frac{J}{8d_1^2} \sum_{k',k''} F_{k'} \cos(\phi_{k'}) m_{k''} [\Re \Gamma_{q-k} - \Re \Gamma_{k-q}] + \frac{J}{8d_1^2} \sum_{k',k''} F_{k'} \sin(\phi_{k'}) m_{k''} [\Im \Gamma_{q-k} - \Im \Gamma_{k-q}] \]
\[ = -\frac{t}{2} \left[ 1 - \sum_{k'} m_{k'} \sin(k'd_i) \right] \sin(kd_i), \quad (23) \]

and
\[ J^e(k) = 2t \sum_{k'} \left[ 2G_{k'} \cos(k'd_i) + F_{k'} \cos(k'd_i + \phi_{k'}) \right] \]
\[ \times \cos(kd_i) + \frac{J}{d_1^2} \left( \sum_{k'} G_{k'} \right)^2 \Re \Gamma_k, \quad (24) \]

where we have defined \( q = k' + k'' \), and \( \Re \Gamma_k \) and \( \Im \Gamma_k \) are real and imaginary parts of the third order polya- 

Within the slave-spin approach, the density (charge) 

correlation function \( \langle \hat{n}_c \hat{n}_c \rangle \), where \( \hat{n}_c = \sum_i \hat{n}_{i,c} \) is 
the total particle number operator at site \( i \) with \( \hat{n}_{i,c} = c_{i,c}^\dagger c_{i,c} \), the spin correlation function \( \langle \hat{S}_i \cdot \hat{S}_j \rangle \), and the 
pairing correlation function \( \langle \hat{P}_i \cdot \hat{P}_j \rangle \), all can be written in 
terms of spinless fermion density correlation \( D_l \), pseudo 
spin correlation \( m_l \), and the particle (or hole) 
doping parameter \( \delta \). These correlation functions have the forms, 
\[ \langle \hat{n}_c \hat{n}_c \rangle = (1 - 2n_a + Di) n_i, \quad \langle \hat{S}_i \cdot \hat{S}_j \rangle = Di m_j / 2 + m_l / 4, \quad \langle \hat{P}_i \cdot \hat{P}_j \rangle = \langle \hat{S}_i \cdot \hat{S}_j \rangle - n_i m_j / 2 + m_l / 4, \]

where the doping parameter enters through the on-site spinless fermion 
occupation number \( n_a = \langle a_i^\dagger a_i \rangle \). Here \( D_l \) is the 
spinless fermion density correlation function defined through 
\( D_l = \langle \hat{n}_c \hat{n}_c \rangle \). Using bi-linear decoupling for the four 
spinless fermion operators, this spinless density correlations 
can be written as \( D_l = n_i^2 + |\Delta_t|^2 - |\chi_l|^2 \). Finding these 
correlation functions are still a challenging task due to 
the self-consistency and the momentum dependence of 
the equations. However without loss of generality, we 
consider a special case where the spiral wave-vector 
for pseudo spin sector \( \hat{Q} = 0 \) at zero temperature. For 
this case, the mean-field parameter \( m_l \) is momentum 

\[ G_k = \sum_l \left( A_g (-1)^l e^{-\xi_k l} + B_g l^4 \right) \cos(kl) \]
\[ F_k \cos(\phi_k) = \sum_l \left( A_f (-1)^l e^{-\xi_k l} + B_f l^3 \right) \cos(kl) \]
\[ F_k \sin(\phi_k) = \sum_l \left( A_f (-1)^l e^{-\xi_k l} + B_f l^3 \right) \sin(kl) \quad (25) \]

where \( A_{g,f}, B_{g,f}, \) and \( \xi_{g,f} \) are interaction dependent 
constants. Assuming these solutions, we solve our self consis-
tent equations variationally for six variational parameters 
\( A_{g,f}, B_{g,f}, \) and \( \xi_{g,f} \). First, we insert these variational 
\( G_k \) and \( F_k \) in our self consistent equations, and then 
derive new expression for these functions using Eqs. (17) 
and (19). Second, by expanding the variational functions 
and newly constructed functions as powers of momenta 
\( k \) and equating the first three non-zero orders for both 
functions, we construct six equations for variational 
parameters \( A_{g,f}, B_{g,f}, \) and \( \xi_{g,f} \). As we are equating the 
non-zero lowest order three coefficients of power series, 
our result may valid only for the low energy sector. We 
numerically solve the six variational equations and find 
that the solutions exist only in the \( 1/\xi_{g,f} \to 0 \) limit. This 
indicates the absence of exponential behavior in correla-
tion functions. This result is not surprising as exponen-
tial behaviour originates from the massless edge modes.
and algebraic tails originates from the bulk of the system [30]. As we have used periodic boundary conditions, our system does not produce edge modes. Without, periodic boundary conditions, the correlation functions show both power law and exponential behavior. The algebraic behavior of the correlation functions for our closed spinless fermions sector is consistent with the findings of [36, 40]. However, the effective interaction parameters in our model need to be calculated self consistently. As a result, we have a set of self-consistent equations to be solved with combination of pseudo spin part of the Hamiltonian. As a result, the behavior of correlation functions of the $t$ -- $J$ model has different parameter dependence. Further, we justify the absence of exponential behaviour by using purely exponential and algebraic decay variational functions and solving the variational equations for $A_{g,f}$ and $ξ_{g,f}$.

Notice that the density correlations, the spin correlations, and the pairing correlations depend on the spinless particle density correlation $D_l$, and pseudo spin correlation $m_l$ and the doping level $δ$ that can be defined as $n_a = 1 ± δ$, where the upper sign is for particle doping and the lower sign is for hole doping. As all physical correlation functions depend on $D_l$, without specifying the doping amount, first we show $D_l - n_a^2$ as function of lattice coordinates in FIG. 1 for series of $J/t$ values at $Q = 0$ special case. Notice that $D_l$ as function of lattice coordinate $l$ shares the same qualitative features for all $J < 2t$. This behaviour has been seen for nearest neighbor only $t$ -- $J$ model also. Using Monte Carlo simulations [54], variational approaches [55], and finite-size scaling [56], it has been shown that there are no qualitative changes in the static properties of the regular nearest neighbor only $t$ -- $J$ model for $J ≤ 2t$. As seen from FIG. 1 we find that $D_l$ starts to demonstrate qualitatively different behavior for larger values of $J > 2t$. This different behavior may be attributed to two reasons. First, it may be due to the phase separation of particle and holes as suggested by the studies of nearest neighbor only $t$ -- $J$ model [54, 55, 57]. Second, as the slave-boson theories are accurate for stronger coupling limits ($U ≫ t$) of the Hubbard model, our calculation may not be valid for $J ≫ t$ limit of our $t$ -- $J$ model. As the $J ≪ t$ limit is equivalent to the strong coupling limit of the Hubbard model, our results may not be accurate for larger $J$ values. Further, earlier studies show that phase-separation transition occurs closer to $J = 3t$, thus our results may not valid for the phase separated state [58, 59].

As the spin correlation function and the pairing correlation function are related through the doping $δ$ and pseudo spin correlation function $m_l$, we provide only the spin correlation function for a demonstrative value of hole doping $δ = 0.15$ in FIG. 2. In contrast to the $D_l$ values in FIG. 1 the spin correlations do not decay as fast as spinless density correlations. This is due to the $l$ dependence on $m_l$ and its oscillatory behaviour. Due to the fact that pairing correlation has $m_l$ dependence and it is connected to the spin correlations, the pairing correlations also shows qualitatively similar features as a function of lattice coordinate $l$. There is no much quantitative difference for other values of doping parameters.

Notice that when we create FIG. 1 and FIG. 2, we treated lattice site separation $l$ as a continuous variable, however the lattice sites are represented by integer values of $l$. The main reason for this is to show the characteristic behavior of these correlation functions. For example, the period and the amplitude of the spin correlation function depends on the spiral wave-vector for pseudo spin operator $Q$. In our model calculation, we have set $Q$, which is proportional to the spin ordering wave-vector $q$ to be zero for simplicity. As a result, the spin correlation function has zeros at integer values of $l$. The oscillatory behavior of the spin correlation function at $Q = 0$ indicates spin long-range order exists in a topological sense. This can be justified by an analytical spin correlation func-

![FIG. 1](image1.png)  (color online) The spinless particle density correlation as a function of lattice coordinate $l$ for series of interaction parameters.

![FIG. 2](image2.png)  (color online) The spin correlation as a function of lattice coordinate $l$ for a series of interaction parameters. The hole doping concentration is set to be $δ = 0.15$. 

tion \((1 - \cos q d_t) / 8\) for a limiting case when spin-flip term and long-range part are absent at \(J = 4t\) \[58\]. Spinless fermion density correlation at \(J = 0\) limit has the same features as those of smaller \(J\) values. The limit \(J \to 0\) is the stronger coupling limit where \(U \to \infty\) where charge dynamics can be considered as non-interacting spinless fermions. The charge correlation has an analytic form at this limit \(\langle n_{tc} n_{jc}\rangle \propto \cos(\pi l) - 1 / (2\pi^2)\) \[58\]. As \(\langle n_{tc} n_{jc}\rangle = (1 - 2n_a + \Delta t)(m_f^2)\), this includes contributions from both spinless density correlation and spin pseudo spin correlation.

In a recent density matrix renormalization (DMRG) study on one-dimensional \(t - J\) model without the long-range part, different phases have been characterized by density and spin correlation functions \[60\]. Here we find that the long-range interactions have significant influence on the ground state properties. Indeed, DMRG studies on long-range dipolar interactions, a novel metallic phase with a spin-gap has been discovered \[61\]. Extending the DMRG scheme to study correlation functions for long-range interactions, it has been shown that transverse spin correlation function and the charge correlation function has algebraic decays with exponent 8.7 \[58\]. We are unable to make a quantitative comparison of the decay exponent as our model calculation provides the spinless fermions density correlation for a specific value of pseudo spin spiral wave-vector \(\hat{Q}\).

IX. DISCUSSION AND CONCLUSIONS

The model studied in this paper relevant for dipolar fermions is a generalized version the original \(t - J\) model defined on a lattice whose sites can be either occupied by one particle or not. This is the strongly correlated limit of the single band Hubbard model away from half filling. The spin exchange, particle repulsion, and hopping of atoms between lattice points are all taken into account through the model. Even in the one-dimensional case of our interests, the \(t - J\) model is not exactly solvable, except for extreme limiting cases with only nearest neighbor interactions. Therefore, our approximate theoretical solutions of the long-range \(t - J\) model relevant for dipolar fermions serve as a valuable tool for studying the strongly correlated atoms. In addition to calculating the correlation functions, our method would allow one to investigate thermodynamic properties of the dipolar fermions.

On the experimental side, the dipolar fermionic molecules and atoms, such as \(^{40}\)K\(^{85}\)Rb, \(^{27}\)Na\(^{8}\)Li, \(^{167}\)Er, \(^{53}\)Cr and \(^{161}\) Dy, can be promising candidates for realizing a truly long-range \(t - J\) model \[10, 16, 62, 67\]. A degenerate Fermi gas of ultracold polar molecules of potassium-rubidium. \(^{40}\)K\(^{87}\)Rb has already been brought to Fermi degeneracy \[68\]. The long range interactions further can be controlled via time-dependent dipole orientation or state-dressing \[63, 70\]. The correlation functions discussed in this paper can be measured with currently available experimental techniques in cold gas experiments. For example, spin and particle correlations can be detected by employing coherent microwave spectroscopy \[31\], using spin blockade effects \[71\], coupling atoms to light \[72\], using quantum noise analysis techniques \[72\], measuring the fraction of atoms residing in a lattice site due to the loss dynamics \[74\], using cold atom microscopy \[73\], and applying other spectroscopic techniques \[76, 81\] or periodic force techniques \[82\].

In conclusion, we have developed an approximate theoretical scheme to calculate the charge, spin, and pairing correlation functions for long range one-dimensional fermions subjected to a periodic optical potential. By combining a constraint free slave-spin approach with a mean field theory, we derived a set of self-consistent equations for the correlation functions. The calculated charge, spin, and pairing correlation functions are related through the spinless fermion density correlation and pseudo-spin correlation originated from slave-spin sector.

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