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RVP-FLMS: A Robust Variable Power Fractional LMS Algorithm

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Abstract—In this paper, we propose an adaptive framework for the variable power of the fractional least mean square (FLMS) algorithm. The proposed algorithm named as robust variable power FLMS (RVP-FLMS) dynamically adapts the fractional power of the FLMS to achieve high convergence rate with low steady state error. For the evaluation purpose, the problems of system identification and channel equalization are considered. The experiments clearly show that the proposed approach achieves better convergence rate and lower steady-state error compared to the FLMS. The MATLAB code for the related simulation is available online at https://goo.gl/dGTGmP.

Index Terms—Least mean square (LMS), fractional calculus, plant identification, channel equalization, fractional LMS (FLMS), robust variable power FLMS (RVP-FLMS), robust variable step size (RVSS), high convergence, low steady state error, adaptive filter.

I. INTRODUCTION

The conventional integer value integration and differentiation is undoubtedly a fundamental tool of calculus being utilized by professionals to determine the solution of numerous problems in various fields. On the contrary fractional order calculus (FOC) deals with the derivative and integral calculated with the fractional power. Derivative with the fractional power is certainly a paradox which introduced a new aspect in mathematics. Though the concepts of FOC were developed earlier in the 16th century by Liouville, Reimann and Leibniz [1], tendency of its utilization in generic areas became notable in the past decade. With the maturation of theoretical and practical operators of FOC, researchers have shown an inclination towards the implementation of FOC to observe the solution of the problems that were solved using the traditional calculus [2].

For the problem of path tracking in autonomous vehicles FOC was used in [3]. FOC was used in [4] to give an experimental proof of a theoretical model proposed for the pulse propagation through porous medium. FOC has turned out to be a convenient tool in the applications of viscoelasticity [5], image processing [6], edge detection [7], control systems and many others [8]. Implementation of FOC in the field of signal processing has been suggested and opted by a number of researchers. In this context, the utilization of FOC in least mean square (LMS) adaptive filtering is being particularly focused.

FOC based least mean square algorithm was proposed in [9] as FLMS. The algorithm was implemented on system identification problem and the results were compared with LMS. Several variants of FLMS algorithm have been proposed showing better performance [10], [11], [12]. In this paper a contemporary modification of the FLMS algorithm is proposed. The proposal is to use a variable fractional power rather than a fixed one, the concept of which is inspired by variable learning rate least mean square algorithm (RVSS-LMS) [13]. The objective is to use variable power of the fractional derivative. The proposed algorithm is evaluated on the problem of equalization and plant identification, the results
of which are compared with the conventional LMS. The rest
of the paper is structured as follows: A brief literature review
of modified least mean algorithms is discussed in section
II. The details of proposed algorithm is discussed in section
III followed by the experimental setup in section IV. The
conclusion of the paper is drawn in section V.

II. LEAST MEAN SQUARE

Least mean square algorithm is an adaptive filter that
belongs to the group of stochastic gradient methods. LMS has
been extensively adopted to counter a number of problems in
the area of signal processing. A considerable amount of work
is done towards the mitigation of the drawbacks in LMS.

Researchers have used several techniques to optimize the
performance of LMS [10], [14], [15], [16], [17]. The theory of
FOC was applied to the LMS algorithm and a fractional least
mean square (FLMS) was proposed in [17]. The algorithm was
tested for the estimation of input nonlinear control autoregres-
sive (INCAR) models and the output resulted in better con-
vergence rates when compared to volterra least mean square
(VLMS) and kernel least mean square (KLMS). The modified
fractional least mean square (MFLMS) was proposed in [11].
The algorithm was designed to improve the computational
complexity involved in FLMS due to the complex gamma
function. MFLMS has therefore shown improved results com-
pared to the LMS and FLMS. In another work [12] the
adaptive weight gain parameters are incorporated. Gradient-
based approach is implemented on variable learning scheme
that changed the nature of the order of fractional derivative in
MFLMS from fixed to adaptive. They proposed adaptive step-
size modified fractional least mean square (AMFLMS) and
presented improvement by comparing their algorithm with the
LMS, FLMS and MFLMS.

In this research we propose to utilize the concept of robust
variable step size (RVSS) [13] for the variable fractional power
of FLMS algorithm. The proposed scheme is robust and com-
putationally less expensive. For the performance evaluation we
consider the problem of system identification and equalization.
The results are compared with the FLMS algorithm.

III. PROPOSED RVP FLMS

In [9] the weight update equation for FLMS is given as :

\[ w_k(n+1) = w_k(n) - \mu \frac{\partial J(n)}{\partial w_k} - \mu_f \left( \frac{\partial}{\partial w_k} \right)^v J(n) \quad (1) \]

where \( w_k(n) \) is the weight of the \( k^{th} \) tap at \( n^{th} \) iteration,
\( J(n) \) is the cost function defined in equation (2), \( v \) is the
fractional power of derivative, \( \mu \) and \( \mu_f \) are the step sizes.

\[ J(n) = \frac{1}{2} e(n)^2 = \frac{1}{2} (d(n) - y(n))^2 \quad (2) \]

where \( e(n) \) is the instantaneous error between the desired
output \( d(n) \) and the estimated output \( y(n) \) at \( n^{th} \) iteration.

According to chain rule \( \frac{\partial J(n)}{\partial w_k} \) and \( \left( \frac{\partial}{\partial w_k} \right)^v J(n) \) are
defined as :

\[ \frac{\partial J(n)}{\partial w_k} = \frac{\partial J(n)}{\partial e(n)} \frac{\partial e(n)}{\partial y(n)} \frac{\partial y(n)}{\partial w_k} \quad (3) \]

\[ \left( \frac{\partial}{\partial w_k} \right)^v J(n) = \frac{\partial J(n)}{\partial e(n)} \frac{\partial e(n)}{\partial y(n)} \left( \frac{\partial}{\partial w_k} \right)^v y(n) \quad (4) \]

solving equation (3) result in equation (5)

\[ \frac{\partial J(n)}{\partial w_k} = -e(n)x(n) \quad (5) \]

Using the Rieman-Lioville fractional derivative method

\[ (D^v f)(t) = \frac{1}{\Gamma(1+\alpha)} \int_0^t (t - \tau)^{\alpha - v} f(\tau) d\tau \quad (6) \]

\[ D^v (t - a)^\alpha = \frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha + v)} (t - a)^{\alpha - v} - v \quad (7) \]

equation (4) result in equation (8)

\[ \left( \frac{\partial}{\partial w_k} \right)^v J(n) = -e(n)x(n) \frac{w_k^{1-v}(n)}{\Gamma(2-v)} \quad (8) \]

Using equation (5) and (8) the weight update equation (1)
becomes :

\[ w_k(n+1) = w_k(n) + \mu e(n)x(n) + \mu_f e(n)x(n) \frac{w_k^{1-v}(n)}{\Gamma(2-v)} \quad (9) \]

For time varying fractional power the \( v \) can be replaced by

\[ w_k(n+1) = w_k(n) + \mu e(n)x(n) + \mu_f e(n)x(n) \frac{w_k^{1-v(n)}(n)}{\Gamma(2-v(n))} \quad (10) \]

The update rule for the time varying fractional power \( v \) using
the RVSS-LMS based method is defined as [13] :

\[ v(n+1) = \beta v(n) + \gamma p^2(n) \quad (11) \]

where \( (0 < \beta < 1), (\gamma > 0) \), \( p(n) \) is the average error
energy correlation and \( v(n+1) \) is set to \( v_{\text{min}} \) or \( v_{\text{max}} \)

\[ p(n) = \alpha p(n-1) + (1 - \alpha)e(n)e(n-1) \quad (12) \]

The positive constant \( \alpha \) \( (0 < \alpha < 1) \) is a weighting
parameter that governs the averaging time constant. It is in
actual the forgetting factor. The limits on \( v(n+1) \) is given by:

\[ v(n+1) = \begin{cases} v_{\text{max}} & \text{if } v(n+1) > v_{\text{max}} \\ v_{\text{min}} & \text{if } v(n+1) < v_{\text{min}} \\ v(n+1) & \text{otherwise} \end{cases} \quad (13) \]

where \( v_{\text{max}} > v_{\text{min}} > 0. \)
IV. SIMULATION SETUP AND RESULTS

For the evaluation of the proposed method the problem of system identification and equalization are considered.

A. System Identification

To estimate the mathematical model of a system, the system identification method is used. Literature including [9], [14], [17], [18], [19] show that the adaptive techniques produce good performance in the application of system identification. To assess the performance of the proposed RVP-FLMS, we consider a linear system, shown in Figure 1:

\[ y(t) = a_1 x(t) + a_2 x(t-1) + a_3 x(t-2) + n(t) \quad (14) \]

Equation (14) shows the mathematical model of the system, where \( x(t) \) is the input and \( y(t) \) is the output of the system, the disturbance model \( n(t) \) is assumed to be \( \mathcal{N}(0, \sigma_n^2) \), \( a_i \)'s depicts the polynomial coefficients representing number of zeros in the system. For the experiment, \( x(t) \) is taken to be a binary phase shift keying (BPSK) modulated 500 randomly generated samples. In Figure 1 the impulse response of the system is \( p(t) \) while \( \hat{y}(t) \) is approximate output, \( \hat{b}(t) \) is the approximate impulse response and \( b(t) \) is the error of estimation. The simulation parameters selected for the experiments are: \( a_1 = 0.9, \ a_2 = 0.3, \ a_3 = -0.1 \). The experiments are performed on two noise levels with the SNR values of 10 dB and 20 dB.

There are 3 number of taps for the model structure. The weights values were initialised to zero. For the FLMS and RVP-FLMS value of the \( \mu \) and \( \mu_f \) was set to \( 1 \times 10^{-4} \) and the initial value of the fractional power \( \nu \) is taken to be 0.5. For the proposed RVP-FLMS the values of \( \alpha, \beta \) and \( \gamma \) are all set to 0.9, 0.99, and 0.9 respectively. The value of \( \nu_{\max} \) and \( \nu_{\min} \) are chosen to be 0.5 and 1 respectively.

The experiment are performed on two noise levels the mean square error (MSE) curves are shown in Figure 2. For the SNR values of 10 dB and 20 dB the RVP-FLMS shows the best performance. The MSE of \(-10.44 \text{ dB}\) and \(-20.20 \text{ dB}\) is achieved in around 45 and 50 iterations respectively whereas the FLMS converges to the MSE value of \(-10.43 \text{ dB}\) and \(-20.17 \text{ dB}\) in around 80 and 90 iterations respectively.

To compare the time complexity of the suggested method with FLMS, we investigated the training time for 100 iterations. The proposed method utilizes 1.85 seconds whereas the FLMS takes 1.57 seconds. The experiment clearly shows that the proposed robust approach dynamically adapts the learning rate to achieve the minimum steady state error in lesser number of iteration.

B. Equalization

The equalizer is used to nullify the effect of noise and distortion caused by the channel. Adaptive learning techniques show good results in this context [20], [21], [22], [23]. To evaluate the effectiveness of the proposed RVP-FLMS, we consider a linear filter, shown in Figure 3:

\[ q(t) = a_1 y(t) + a_2 y(t-1) + a_3 y(t-2) \quad (15) \]

Equation (15) shows the mathematical model of the linear filter, where \( y(t) \) is the input and \( q(t) \) is the output of the filter. \( a_i \)'s depicts the polynomial coefficients which are actually the number of zeros in the system. For the purpose of this experiment the same plant defined in section IV-A is used.

In Figure 3 the impulse response of the filter is \( p(t) \) while the disturbance model \( n(t) \) is supposed to be \( \mathcal{N}(0, \sigma_n^2) \). \( \hat{y}(t) \) is the approximate input, \( x(t) \) is the approximate noisy output and \( e(t) \) is the error of estimation. The experiments are performed on two noise levels with the SNR values of 10 dB and 20 dB.

In the equalizer structure, 3 number of taps were chosen. The weights values were initialised to zero. For the FLMS and
The suggested method requires 1.92 seconds whereas the FLMS takes 1.63 seconds. The experiment clearly shows that the proposed robust approach dynamically adapts the learning rate to achieve the minimum steady state error in lesser number of iteration.

V. CONCLUSION

In this research an adaptive least mean square algorithm based on fractional derivative is proposed. In particular, the fractional power of the FLMS is made variable using the concept of robust variable step size. The performance of the proposed approach is evaluated on system identification and equalization problems. The results of the proposed algorithm are compared to the FLMS. The proposed algorithm attains better convergence rate and steady-state error and is therefore found to be superior to the FLMS algorithm.

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