What’s Live? Understanding Distributed Consensus

Saksham Chand  Yanhong A. Liu

Computer Science Department, Stony Brook University, Stony Brook, NY 11794, USA
{schand,liu}@cs.stonybrook.edu

Abstract

Distributed consensus algorithms such as Paxos have been studied extensively. They all use the same definition of safety. Liveness is especially important in practice despite well-known theoretical impossibility results. However, many different liveness properties and assumptions have been stated, and there are no systematic comparisons for better understanding of these properties.

This paper systematically studies and compares different liveness properties stated for over 30 prominent consensus algorithms and variants. We introduce a precise high-level language and formally specify these properties in the language. We then create a hierarchy of liveness properties combining two hierarchies of the assumptions used and a hierarchy of the assertions made, and compare the strengths and weaknesses of algorithms that ensure these properties. Our formal specifications and systematic comparisons led to the discovery of a range of problems in various stated liveness properties, from too weak assumptions for which no liveness assertions can hold, to too strong assumptions making it trivial to achieve the assertions. We also developed TLA+ specifications of these liveness properties, and we use model checking of execution steps to illustrate liveness patterns for Paxos.

1 Introduction

Distributed systems are pervasive, where processes work with each other via message passing. For example, this article is available to the current reader owing to some distributed system. At the same time, distributed systems are highly complex, due to their unpredictable asynchronous nature in the presence of failures—processes can fail and later recover, and messages can be lost, delayed, reordered, and duplicated. This paper considers the critical problem of distributed consensus—a set of processes trying to agree on a value or a continuing sequence of values—under these possible failures.

Distributed consensus under these possible failures is essential in services that must maintain a state, including many services provided by companies like Google with, e.g., the

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Chubby distributed lock service [9] and the BigTable distributed storage system [14]. This is because such services must use replication to tolerate failures caused by machine crashes, network outages, etc. Replicated servers must agree on the state of the service or the sequence of operations that have been performed—for example, a customer order has been placed and paid but not yet shipped—so that when some servers become unavailable, the remaining servers enable the service to function correctly.

Many algorithms and variations have been created for this consensus problem, including the most prominent ones listed in Table 1. These algorithms share similar ideas, with Paxos [39] becoming the most well-known name and the focus of many studies. These algorithms are required to be safe in that only a single value or a single sequence of values have been agreed on at any time, and the value or values agreed on are among the values proposed by the processes. Proving safety of these algorithms has been of utmost interest since time immemorial—with varying degrees of rigor, from informal proof sketches to fully formal machine-checked proofs.

However, liveness of these algorithms—certain desired progress has been made in reaching agreement—is not always given the same kind of attention. For example, out of the 31 algorithms listed in Table 1, only 18 (58%) of them mention liveness (Table 3) and only 11 (35%) of them give some kind of liveness proof (Column “Proofs” of Table 3). This is intriguing because real-world implementations of consensus algorithms are required to be live, a.k.a. “responsive”, despite the FLP impossibility results [22]. Furthermore, prima facie, it is sometimes unclear or ambiguous what exactly liveness means for these algorithms and under what exact conditions they are claimed to satisfy those properties.

This paper. This paper considers consensus algorithms and variants listed in Table 1 and examines and assembles their liveness properties as stated in the literature, including the assumptions under which the properties are satisfied. We formally specify and precisely compare these liveness properties and present our analysis for all of them (summarized in Table 3). We categorize the assumptions into two classes: (1) link assumptions, stating what is assumed about the communication links, and (2) server assumptions, stating what is assumed about the servers in the system.

We identify three different kinds of link assumptions, seven different kinds of server assumptions, and six different kinds of liveness assertions capturing different kinds of progress. We relate different assumptions and assertions by creating three hierarchies: a 3-element total order for link assumptions, a 7-element partial order for server assumptions that includes two diamonds, and a 6-element partial order for liveness assertions that includes a diamond in the middle. Together they form an overall hierarchy of liveness properties, which includes all the liveness properties stated for the algorithms and variants considered.

We introduce a precise high-level language and formally specify all these assumptions and assertions in the language. In fact, it was the precise language and formal specifications that allowed us to establish the precise relationships among all the assumptions and assertions. The overall hierarchy not only helps in understanding liveness better, but also helps in finding tighter results. It led us to the discovery of a range of problems in various stated liveness properties, from lacking assumptions or too weak assumptions under which
no liveness assertions can hold, to too strong assumptions making it trivial or uninteresting to satisfy the liveness assertions. For example,

- EPaxos [53] only assumes that eventually, there is always some quorum of servers that is non-faulty (what we call Alw-Q), but this is too weak for reaching consensus, because each such quorum might not stay non-faulty long enough to make progress.

- Zab [30], Paxso-EPR [59], and some others assume that eventually, there is a server P and a quorum Q of servers, such that always, P is non-faulty and is the primary and Q is non-faulty (what we call PQ-Alw). Chubby-Live [12] even assumes that eventually, all servers are always non-faulty. Such strong assumptions make it trivial to reach consensus.

This also led us to the question of what the right or best liveness properties are among the many possible combinations of assumptions and assertions. In fact, many protocols assert stronger properties than necessary.

We also developed TLA+ specifications of these liveness properties. We show, in Appendix A that model checking execution steps using TLC can illustrate liveness patterns for single-value Paxos on up to 4 proposers and 4 acceptors in a few hours, but becomes too expensive for multi-value Paxos or more processes. Appendices E, C, D, and F show the liveness properties and assumptions statements that we extracted and assembled from all the algorithms and variants considered.

2 Preliminaries

This section introduces the terminology used in this paper, the algorithms and variants considered, and the high-level structure of these algorithm and variants.

Distributed systems and failures considered. A distributed system is a set of processes each operating on its local data and communicating with other processes by sending and receiving messages. A process may crash and may later recover, and a message may be lost, delayed, put out of order, and/or duplicated. A process is said to be non-faulty if “it eventually performs the actions that it should, such as responding to messages” [41].

Distributed consensus. The basic consensus problem, called single-value consensus, is to ensure that a single value is chosen by non-faulty processes from among the set of values proposed by the processes. The more general consensus problem, called multi-value consensus, is to choose a single sequence of values, instead of a single value.

Terminology. We identify two types of processes: servers, which execute some consensus algorithm and together provide consensus as a service, and clients, which use this service. In case of multi-value consensus, clients send values to servers for them to order, and each server maintains a log, which is a sequence of chosen values. The indices of entries in the log are called slots.
A server is said to be primary if it is explicitly elected by the algorithm to be the single primary or leader among all servers in the system. Some other names for primary that appear in the literature are president [39], leader [11, 19, 37, 30, 50], and coordinator [51, 20]. In general, the defining property of the primary is that it is the only server that can propose values to other servers. Note that some consensus algorithms do not use a primary.

A quorum system $Q_s$ is a set of subsets, called quorums, of servers such that any two quorums overlap, that is, $\forall Q_1, Q_2 \in Q_s : Q_1 \cap Q_2 \neq \emptyset$. The most commonly used quorum system $Q_s$ takes any majority of servers as an element in $Q_s$. A quorum is said to be non-faulty if each of its servers is non-faulty, and faulty otherwise.

**Algorithms and variants.** Table 1 lists 31 prominent algorithms and variants for distributed consensus in the literature, together with the languages in which they are expressed.

- The first group (1-5) comprises systems and algorithms based on what is known as Virtual Synchrony [6], the first algorithm for reliable group communication using broadcast primitives.
- The second group (6-7) is for what is called Viewstamped Replication [55], the earliest algorithm that has a primary-backup architecture.
- The third group (8-11) is for Paxos [39] and core variants, the earliest algorithm based on symmetric leader election and state-machine replication. Basic Paxos solves the problem of single-value consensus. Multi-Paxos, hereafter simply called Paxos, solves the problem of multi-value consensus.
- The fourth group (12-13) is for two core classes of single-value consensus algorithms with failure detection, one where servers follow crash failure model (12) and the other where servers follow crash-recover failure model (13).
- The fifth group (14-31) is for other algorithm variations. The earliest two (14-15) and latest seven (25-31) in this group are variants specified formally, in a language with machine-supported syntax and semantics checking, and with proof supports or compilation and execution.

Note that the similarity and differences among the names do not necessarily reflect those in the algorithms or variants. For example, Paxos-VS differs from Paxos much more than Raft differs from Paxos, whereas Chubby and Raft are very similar despite having completely different names.

**High-level algorithm structure.** At a (sufficiently) high-level, all of the algorithms in Table 1 are quorum-based consensus algorithms that execute in rounds. A round, also known as a ballot, view, epoch, and term, is identified by a unique number, and is initiated by at most one server. Multiple rounds may run simultaneously.

- A server initiates a new round once it thinks that the current round has died, i.e., stopped making progress. Different algorithms use different conditions to decide that a round is considered dead.
| Name            | Description                                                                 | Language used          |
|-----------------|-----------------------------------------------------------------------------|------------------------|
| VS-ISIS         | Reliable group communication, Birman-Joseph 1987 [7]                        | English (items)        |
| VS-ISIS2        | Virtual synchrony, Birman-Joseph 1987 [6]                                  | pseudocode             |
| EVS             | Extended virtual synchrony for network partition, Amir et al 1995 [2]       | pseudocode             |
| Paxos-VS        | Virtually synchronous Paxos, Birman-Malkhi-van Renesse 2012 [8]             | pseudocode             |
| Derecho         | Virtually synchronous state machine replication, Jha et al 2019 [29]        | pseudocode             |
| VR              | Viewstamped replication, Oki-Liskov 1988 [55]                               | pseudocode (coarse)    |
| VR-Revisit      | VR revisited, Liskov 2012 [46]                                              | English (items)        |
| Paxos-Synod     | Paxos in part-time parliament, Lamport 1998 [39]                            | TLA [38]               |
| Paxos-Basic     | Single-value Paxos, Lamport 2001 [10]                                       | (single-value)         |
| Paxos-Fast      | Single-value Paxos with replicas proposing, Lamport 2006 [41]               | English (items), TLA+  |
| Paxos-Vertical  | Single-value Paxos with external starting of leader election, Lamport-Malkhi-Zhou 2009 [44] | pseudocode, PlusCal [42] |
| CT              | Single-value consensus with crash failures, Chandra-Toueg 1996 [13]         | pseudocode             |
| ACT             | Single-value consensus in crash-recovery model, Aguilera-Chen-Toueg 2000 [1] | pseudocode             |
| Paxos-Time      | Paxos with time analysis, De Prisco-Lampson-Lynch 2001 [19]                 | IOA [49]               |
| Paxos-PVS       | Single-value Paxos for proof, Kellomäki 2004 [35]                           | PVS [57]               |
| Chubby          | Paxos in Google’s Chubby lock service, Burrows 2006 [9]                     | English (partial items) |
| Chubby-Live     | Chubby in Paxos made live, Chandra-Griesemer-Redstone 2007 [12]             | English                |
| Paxos-SB        | Paxos for system builders, Kirsch-Amir 2008 [37]                            | pseudocode             |
| Mencius         | Paxos with leaders proposing in turn, Mao et al 2008 [51]                   | English (items)        |
| Zab             | Yahoo/Apache’s Zookeeper atomic broadcast, Junqueira-Reed-Serfani 2011 [30] | pseudocode             |
| Zab-FLE         | Zab with fast leader election, Medeiros 2012 [52]                            | pseudocode             |
| EPaxos          | Egalitarian Paxos, Moraru-Andersen-Kaminsky 2013 [53]                       | pseudocode             |
| Raft            | Consensus in RAMCloud, Ongaro-Ousterhout 2014 [54]                           | pseudocode             |
| Paxos-Complex   | Paxos made moderately complex, van Renesse-Altinbuken 2015 [62]             | pseudocode, Python     |
| Raft-Verdi      | Raft for proof using Coq, Wilcox et al 2015 [64]                            | Verdi [64]             |
| IronSL          | Paxos in Microsoft’s IronFleet for proof, Hawblitzel et al 2015 [20]        | Dafny [45]             |
| Paxos-TLA       | Paxos for proof using TLAPS, Chand-Liu-Stoller 2016 [11]                    | TLA+                   |
| LastVoting-PSync| Single-value Paxos in Heard-Of model for proof, Drágoi-Henzinger-Zufferey 2016 [20] | PSync [20] |
| Paxos-EPR       | Paxos in effectively propositional logic for proof, Padon et al 2017 [59]   | Ivy [60]               |
| Paxos-Decon     | Paxos deconstructed, Garcia et al 2018 [24, 25]                             | Scala/Akka [28]        |
| Paxos-High      | Paxos in high-level executable specification, Liu-Chand-Stoller 2019 [47]    | DistAlgo [48]          |

Table 1: Distributed consensus algorithms and variants, and languages used to express them.
A server learns a value upon either receiving votes from a quorum of servers for that value in some round or a message from some other server informing it about the value learned by that server.

If the server is maintaining a state machine, it then executes the learned value on the state machine.

The result of executing a value is sent to the client as a response to the value it initially requested.

3 Language

To describe properties, especially liveness properties, of distributed systems, we introduce a precise high-level language. It supports direct use of sets and predicate logic to capture system state and state properties, and extends them with time and temporal expressions to capture properties related to time.

System state. The following set and predicate model processes and process failures in the system:

- servers — the set of all servers in the system. This set may be dynamically updated, because servers can be added or removed.
- clients — the set of all clients in the system. Clients send values to servers for the servers to form consensus on.
- \( p.\text{nf} \) — true iff process \( p \) is non-faulty.

To model messages, there are two built-in history variables for each process \( p \):

- \( p.\text{sent} \) — the sequence of all messages sent by \( p \), optionally including the receiver process with each message.
- \( p.\text{received} \) — the sequence of all messages received by \( p \), optionally including the sender process with each message.

For ease of reading, we use \( p.\text{sent} \ m \ \text{to} \ p_2 \) to mean \((m,p_2)\) in \( p.\text{sent} \), and use \( p.\text{received} \ m \ \text{from} \ p_2 \) to mean \((m,p_2)\) in \( p.\text{received} \). The clauses to \( p_2 \) and from \( p_2 \) are optional.

For consensus algorithms, the following variables and predicates are used:

- quorums — the set of quorums of servers used by the algorithm.
- values — the set of proposed values from which a value or a sequence of values can be agreed on.
- \( p.\text{is\_primary} \) — whether \( p \) is currently the primary server.
• **rounds** — the total-ordered set of round identifiers. Each uniquely determines a server and its current attempt at being the unique primary among all servers.

**Set expressions and quantifications.** We use queries for expressing computations at a high level. A query can be a comprehension, aggregation, or quantification over sets or sequences.

- A comprehension, also called a set former, \( \{ e : v_1 \text{ in } s_1, \ldots, v_k \text{ in } s_k, \text{cond} \} \), returns the set of values of \( e \) for all combinations of values of variables that satisfy all \( v_i \text{ in } s_i \) clauses and condition \( \text{cond} \).

- An aggregation, \( \text{agg } s \), where \( \text{agg} \) is an aggregation operator such as \( \text{count} \) or \( \text{max} \), returns the value of applying \( \text{agg} \) to the set value of \( s \).

- A universal quantification, \( \text{each } v_1 \text{ in } s_1, \ldots, v_k \text{ in } s_k \text{ has } \text{cond} \), returns true iff for each combination of values of variables that satisfy all \( v_i \text{ in } s_i \) clauses, \( \text{cond} \) holds.

- An existential quantification, \( \text{some } v_1 \text{ in } s_1, \ldots, v_k \text{ in } s_k \text{ has } \text{cond} \), returns true iff for some combination of values of variables that satisfy all \( v_i \text{ in } s_i \) clauses, \( \text{cond} \) holds. When the query returns true, variables \( v_1, \ldots, v_k \) are bound to a combination of satisfying values, called a witness.

**Time and time scope.** There is a built-in notion of time, a number that ranges from the start of time, 0, to unbounded time later, \( \text{inf} \). We omit the unit for time because the discussion is general and any unit can be used.

A time interval denotes all times between two times. Time intervals are represented using common notation for intervals. For example, \( (t_1, t_2] \) is the time interval from \( t_1 \) and \( t_2 \) excluding \( t_1 \) but including \( t_2 \), and \( [0, \text{inf}) \) is the entire range of time. A duration is the difference between two times.

To specify temporal properties, each expression has a time parameter specifying when the value of the expression is taken.

- \( e \text{ at } t \) — the value of expression \( e \) at time \( t \). This includes the case that \( e \) is a Boolean-valued condition.

- \( e \text{ at } . \) — for when the time parameter is implicit, where “.” (dot) denotes the implicit time in the scope of \( e \). For an outermost expression, the implicit time is 0.

In any scope, the time parameter distributes to subexpressions, that is, \( \text{at} \) is distributive. For example, \( (x + y) \text{ at } t = (x \text{ at } t) + (y \text{ at } t) \).

**Temporal expressions.** There are two most important temporal operations: \( \text{alw} \), read as “always”, and \( \text{evt} \), read as “eventually”, defined as follows, where \( t \) is a fresh variable not used in the scope of the expression being defined.
• \texttt{alw \ cond} = \texttt{each } t \texttt{ in } [., \inf) \texttt{ has } (\texttt{cond at } t). \texttt{ That is, } \texttt{cond} \texttt{ holds at each time starting from the time in scope.}

• \texttt{evt \ cond} = \texttt{some } t \texttt{ in } [., \inf) \texttt{ has } (\texttt{cond at } t). \texttt{ That is, } \texttt{cond} \texttt{ holds at some time starting from the time in scope.}

The implicit time can be made explicit by following the definitions. For example, for an expression at the outermost scope, we have the following:

\[
\texttt{alw \ evt \ cond} = (\texttt{alw } \texttt{(evt cond)}) \texttt{ at } 0 \\
= \texttt{each } t \texttt{ in } [0, \inf) \texttt{ has } ((\texttt{evt cond}) \texttt{ at } t) \\
= \texttt{each } t \texttt{ in } [0, \inf) \texttt{ has } ((\texttt{some } t2 \texttt{ in } [t, \inf) \texttt{ has } (\texttt{cond at } t2)) \texttt{ at } t)
\]

For convenience, we define operator \texttt{during}, where again \( t \) is a fresh variable:

• \texttt{cond during intvl} = \texttt{each } t \texttt{ in intvl: cond at } t

We also define operators \texttt{lasts} and \texttt{after}:

• \texttt{cond lasts dur} = \texttt{cond during }[., .+\texttt{dur}]

• \texttt{cond after dur} = \texttt{cond during }(.+\texttt{dur}, \inf)

**Shorthand for non-faulty processes.** Finally, we define a shorthand for stating whether a set \( ps \) of processes are all non-faulty, where \( p \) is a fresh variable:

\[
\texttt{ps nf} = \texttt{each } p \texttt{ in } ps \texttt{ has } p.f\texttt{n}
\]

### 4 Liveness specification and liveness hierarchy

We formally specify the liveness properties for all the algorithms and variants listed in Table 1 as stated by their authors, including assumptions about the communication links and about the servers, which we call link assumptions and server assumptions, respectively. We create a total order of link assumptions, a partial order of server assumptions, and a partial order of liveness properties. Elements in these three hierarchies can be straightforwardly combined to form a single liveness hierarchy.

Table 2 shows the total order of link assumptions, in increasing strength from top to bottom. Figure 9 shows the hierarchies for server assumptions (on the left) and liveness assertions (on the right). A solid arrow from node \( A \) to node \( B \) denotes that \( A \) implies \( B \), that is, if \( A \) is true, then \( B \) must be true; we say that \( A \) is stronger than \( B \). A dashed arrow (only from \texttt{Resp} to \texttt{Each-Exec} on the right) denotes that if \( A \) is true, then \( B \) may be true. Note that weaker assumptions are better, and stronger assertions are better. To relate to Lamport’s notion of weak and strong fairness [38], note that the assumptions made by weak fairness are stronger than the assumptions made by strong fairness.

The total order for link assumptions is proved precisely in Theorem 1. The partial orders for server assumptions and liveness assertions are argued informally but can be proved precisely in a similar fashion as for link assumptions.
4.1 Liveness assumptions

Link assumptions. Each algorithm makes some assumptions about the communication links used, forming three different kinds of link assumptions, which are used to separate the algorithms.

1. **Raw** — messages sent may or may not be received. This includes algorithms and variants that do not discuss liveness propositions, or do not precisely describe the assumptions made about links in their liveness discussion. For example, VR-Revisit only states that some processes “are able to communicate” [46, Section 8].

2. **Fair** — all links between servers are fair; that is “if a correct process repeatedly sends a message to another correct process, at least one copy is eventually delivered” [62]. We also include algorithms and variants that assume no communication failures in this category under the assumption that copies of a same message are identical, meaning that retransmitting a message does not change the set of sent messages. With this view, fair links assumptions states that every message in the set of sent messages is eventually delivered—which coincides with no communication failure.

3. **Sure** — the time between a message being sent and the message being received has a known upper bound. Note that this implies that every sent message is received. This differs from fair links by having a known upper bound.

Figure 1 shows the three kinds of link assumptions specified formally, where the known bound for **Sure** is captured by the explicit parameter \( D \). It is easy to see precisely that the difference between **Raw** and **Fair** is the outermost existential vs. universal quantification, and the difference between **Fair** and **Sure** is the eventual condition vs. the known duration.

Table 2 shows the algorithms and variants in Table 1 grouped into three categories based on the link assumptions they stated. We see that almost half of the 31 algorithms and variants fall under **Raw**.

Theorem 1 shows that the three kinds of link assumptions form a total order. It is equally easy to show that, for any duration \( D_1 \) shorter than or equal to \( D_2 \), **Sure** \((D_1)\) implies **Sure** \((D_2)\).

**Theorem 1.** **Fair** implies **Raw**, and **Sure** implies **Fair**.

**Proof.** By definitions of **Raw**, **Fair**, and **Sure**, and definitions of \( \text{evt} \) and \( \text{after} \), we have the following:
Table 2: Assumptions about links used in algorithms and variants in Table 1.

| Assumption | Algorithms and Variants | Count |
|------------|-------------------------|-------|
| Raw        | VS-ISIS, VS-ISIS2, Derecho, VR, VR-Revisit, Paxos-Basic, Paxos-Vertical, Paxos-PVS, Chubby, Zab-FLE, Raft, Paxos-TLA, Raft-Verdi, Paxos-Decon, Paxos-High | 15    |
| Fair       | EVS, Paxos-VS, Paxos-Fast, CT, ACT, Paxos-SB, Mencius, EPaxos, Paxos-Complex, Paxos-EPR | 10    |
| Sure       | Paxos-Synod, Paxos-Time, Chubby-Live, Zab, IronRSL, LastVoting-PSync | 6     |

When any message is sent, top-level universal quantification (each) in \textit{Fair} implies the existential quantification (some) in \textit{Raw}, thus \textit{Fair} implies \textit{Raw}. Some d in [0,\infty) in \textit{Sure} implies some d in [0,\infty) in \textit{Fair}, thus \textit{Sure} implies \textit{Fair}.

Server assumptions. The server assumptions form a 7-element partial order, as described below. Figure 2 shows formal specifications of the server assumptions. Figure 6-left shows the hierarchy they form.

1. **Alw-Q** — eventually, there is always some quorum of servers that is non-faulty. Note that no fixed quorum is non-faulty. This is the most basic server assumption because in quorum-based consensus algorithms, the system is said to be down if no quorum is non-faulty. In other words, this assumption states that eventually, the system is always up. This is called Strong-L1 in [37]. We include EPaxos in this category.

2. **Q-Alw** — eventually, there is some quorum of servers that is always non-faulty. This is called Weak-L1 in [37] and is used to argue liveness of Paxos-SB. This assumption is stronger than **Alw-Q** because it assumes a fixed non-faulty quorum. We also include EVS, CT, ACT, and Paxos-SB in this category.

3. **P-Alw-Q** — eventually, there is a server P such that, always, P is non-faulty and is the primary and there is some quorum Q of servers that is non-faulty. **P-Alw-Q** is stronger than **Alw-Q** because it assumes a fixed primary that is eventually always non-faulty. If we assume that primary is always part of the non-faulty quorum, then **P-Alw-Q** can be interpreted as that after some time, it always holds that some quorum containing the primary is non-faulty. We include Paxos-VS, Derecho and VR-Revisit in this category.

4. **PQ-Alw** — eventually, there is a server P and a quorum Q of servers, such that always, P is non-faulty and is the primary and Q is non-faulty. This assumption is
Figure 2: Server assumptions expressed precisely.

stronger than $Q$-$\text{Alw}$ because it assumes a fixed primary and is stronger than $P$-$\text{Alw}$-$Q$ because it assumes a fixed non-faulty quorum. Zab’s liveness is proved using this. Paxos-Fast, IronRSL, and Paxos-EPR use a slight generalization of this assumption which substitutes a single non-faulty quorum with a set of non-faulty quorums.

5. $\text{Alw}$ — eventually, all servers are always non-faulty, i.e., non-faulty forever from that time on. This is obviously the strongest assumption. This assumption is used while checking liveness of Chubby-live. The authors begin by testing for only safety while their failure injector injects random failures into the system. After some time, the failure injector is stopped and the system is given time to fully recover. Once recovered, they do not inject any more failures and check for liveness violations. The purpose of their liveness test is to check that the system does not deadlock after a sequence of failures. ACT gives liveness performance guarantees assuming this.

The next two assumptions differ from the ones described above in the sense that these two assume that eventually some set of processes remains non-faulty for some duration of time, not eventually always.

6. $\text{PQ-Dur}$ — eventually, there is a server $P$ and a quorum $Q$ such that $P$ and $Q$ are non-faulty and $P$ is the primary, for some (sufficiently long) duration of time. $\text{PQ-Dur}$ is the equivalent of $\text{PQ-Alw}$ but with time analysis. Paxos-Time and LastVoting-PSync use this assumption.

7. $\text{PQ-Extra-Dur}$. Lamport states the following about liveness of Paxos-Synod in [39]:

\[\text{PQ-Extra-Dur}(D_1,D_2) = \exists t \in [0,\infty) \text{ such that} (\forall t_2 \in [t,t+D_1+D_2) \text{ server at } t_2 = \text{server at } t) \text{ and} (\exists p \in \text{servers} \text{ such that} ((p.nf \text{ and } p.is\_primary) \text{ during } [t+D_1,t+D_1+D_2])) \text{ and} (\exists q \in \text{quorums} \text{ such that} (q.nf \text{ during } [t,t+D_1+D_2])) \text{)}\]
Presidential selection requirement: If no one entered or left the Chamber, then after $T$ minutes exactly one priest in the Chamber would consider himself to be the president.

Liveness property: If the presidential selection requirement were met, then the complete protocol would have the property that if a majority set of priests were in the chamber and no one entered or left the Chamber for $T + 99$ minutes, then at the end of that period every priest in the Chamber would have a decree written in his ledger.

Using $D_1$ and $D_2$ instead of $T$ and 99, respectively, we can summarize the assumptions as having a duration of time lasting $D_1 + D_2$ time units such that (1) the set of non-faulty servers does not change in the duration, (2) after $D_1$ time units, primary is selected, and (3) at least a fixed quorum of servers is non-faulty during the entire duration - this is because “no one enters or leaves the chamber” and “a majority of priests were in the chamber ... for $D_1 + D_2$ minutes”.

PQ-Extra-Dur is weaker than Alw because it allows some servers to be faulty in the pertinent duration. PQ-Extra-Dur is stronger than PQ-Dur, but is not comparable to PQ-Alw—it is weaker in using limited durations instead of alw, and it is stronger in requiring that no server fails during $D_1 + D_2$ whereas PQ-Alw allows for servers to fail as long as P and Q are non-faulty.

4.2 Liveness assertions

The liveness assertions form a 6-element partial order, as described below; for brevity, we omit slots from the description below. Figure 3 shows formal specifications of the liveness assertions; Figure 5 shows the corresponding assertions with slots for multi-value consensus. Figure 6-right shows the hierarchy these liveness assertions form.

1. Each-Vote — eventually in some round, each server of some quorum sends a vote for the same value in that round. Each-Vote is crucial for Paxos and its variants because Paxos’ safety relies on the invariant that if a quorum votes on the same value in some round, then only that value can be proposed in subsequent rounds. Each-Vote property is formally verified for Paxos-EPR, as well as single-value Paxos and Stoppable Paxos [50] in the proof system Ivy [60].

2. Some-Learn — eventually, some non-faulty server learns a value. This is stronger than Each-Vote, because the sent votes might not be received by any server. In Paxos-Fast, it is informally proven that Paxos-Basic and Paxos-Fast satisfy this property. We consider this, not Each-Vote, as the most basic property that any live implementation of any consensus algorithm is expected to guarantee. Figure 4 shows that Raft admits behaviors of infinite length in which Each-Vote is eventually always satisfied but Some-Learn is never satisfied.
Each-Vote = $\text{evt some } r \text{ in rounds, } q \text{ in quorums, } v \text{ in values has}$

$\text{each } p \text{ in } q \text{ has } p.\text{voted } (r,v)$

Some-Learn = $\text{evt some } p \text{ in servers, } v \text{ in values has } p.\text{learned } (v)$

Each-Learn = $\text{evt some } q \text{ in quorums, } v \text{ in values has}$

$\text{each } p \text{ in } q \text{ has } p.\text{learned } (v)$

Some-Exec = $\text{evt some } p \text{ in servers, } v \text{ in values has } p.\text{executed } (v)$

Each-Exec = $\text{evt some } q \text{ in quorums, } v \text{ in values has}$

$\text{each } p \text{ in } q \text{ has } p.\text{executed } (v)$

Resp = $\text{evt each } c \text{ in clients has some } v \text{ in values has}$

$c.\text{received ('resp',v)}$

Figure 3: Liveness assertions expressed precisely for single-value consensus. For a server $p$, round $r$, and value $v$, $p.\text{voted } (r,v)$ is true iff $p$ voted for value $v$ in round $r$, $p.\text{learned } (v)$ is true iff $p$ learned that value $v$ was chosen, $p.\text{executed } (v)$ is true iff $p$ has executed $v$ in its local copy of the replicated state machine. For a client $c$ and value $v$, $c.\text{received ('resp',v)}$ is true iff $c$ received value $v$ in response.

|   | S1 | S2 | S3 |
|---|----|----|----|
| (a) | 1 | 1 | 3 |
| (b) | 1 | 3 | 3 |
| (c) | 3 | 1 | 3 |
| (d) | 1 | 1 | 3 |

Figure 4: Demonstration of Raft satisfying Each-Vote but not Some-Learn. The system involves three servers, S1, S2, and S3. The primary is marked with a thick border. No value has been committed yet. A faded server is currently failed. (a) The initial state of the server logs. S1 has proposed value 1 which S2 has received and written in its log thus satisfying Each-Vote for value 1. Assume that S1 fails before realizing the majority, thus Some-Learn does not satisfy. (b) S3 becomes the new primary. (c) S3 enforces its log onto S2 replacing S1’s proposed value. Now Each-Vote is satisfied for value 3. (d) Before Some-Learn could be satisfied, S3 fails and S1 becomes the new leader. It enforces its log onto S2. Thus, Each-Vote is satisfied for value 1 coming back to configuration (a).
3. **Each-Learn** — eventually, each server of some quorum learns the same value. Obvi-
ously, **Each-Learn** is stronger than **Some-Learn**. This property is informally proven 
for Paxos-Synod and formally proven for LastVoting-Psync.

4. **Some-Exec** — eventually, some non-faulty server executes a value. This is stronger 
than **Some-Learn** because a value has to be learned before it can be executed. **Some-
Exec** and **Each-Learn** do not compare because execution details differ between al-
gorithms. While some would allow execution immediately after learning, others might 
require replication to occur before execution. It is claimed that Paxos-SB and EPaxos 
satisfy this property, and formally verified in Dafny that IronRSL satisfies this property.

5. **Each-Exec** — eventually, each server of some quorum executes the same value. Obvi-
ously, this is stronger than **Some-Exec** and **Each-Learn**. It is informally proven that 
EVS satisfies this property. In Paxos-Time, IOA specifications of single-value Paxos 
and Paxos are informally proven to satisfy this property. A proof sketch is given for 
Zab satisfying this property.

6. **Resp** — eventually, each client request is responded to. This property is stronger than 
**Some-Exec** because client expects the result of executing the requested value. **Resp** 
may or may not be stronger than **Each-Exec**. For example, normal case execution 
of Paxos-VS satisfies **Each-Exec** before **Resp**, that is, **Resp** implies **Each-Exec** for 
Paxos-VS. Meanwhile, normal case execution of VR-Revisit satisfies **Some-Exec** and 
then **Resp** but not **Each-Exec**, that is, **Resp** implies **Some-Exec** but not **Each-
Exec** for VR-Revisit. Hence, the dashed arc from **Resp** to **Each-Exec**.

For multi-value consensus, the liveness assertions are specified precisely in Figure 5. 
Compared with assertions for single-value consensus in Figure 3, the changes are:

- for each assertion except for **Resp**, add (1) parameter \( n \) for the assertion, (2) clause 
  \( \text{each } s \text{ in } 1..n \) on the first line, and (3) component \( s \) before \( v \) to the arguments of 
  **sent** and **executed** on the last line.

- for assertion **Resp**, add (1) clause \( v \text{ in } \text{values} \) on the first line, (2) the second line giving
  the condition for implication, and (3) the last component \( \text{res}(v) \) to the argument of 
  **received** on the last line. \( \text{res}(v) \) is the result of executing value \( v \).

5 Liveness properties and analysis results

This section describes liveness properties stated for the algorithms and variants considered, 
and their proofs and discussions given by their respective authors. We then present our 
analysis of these liveness properties and of the best possible liveness properties.


```
Each-Vote(n) = evt each s in 1..n has
  some r in rounds, q in quorums, v in values has
  each p in q has p.voted (r,s,v)

Some-Learn(n) = evt each s in 1..n has
  some p in servers, v in values has p.learned (s,v)

Each-Learn(n) = evt each s in 1..n has
  some q in quorums, v in values has
  each p in q has p.learned (s,v)

Some-Exec(n) = evt each s in 1..n has
  some p in servers, v in values has p.executed (s,v)

Each-Exec(n) = evt each s in 1..n has
  some q in quorums, v in values has
  each p in q has p.executed (s,v)

Resp = evt each c in client, v in values has
  c.sent ('req', v) implies c.received ('resp', v, res(v))
```

Figure 5: Liveness assertions expressed precisely for multi-value consensus. For a server p, round r, slot s, and value v, p.voted (r,s,v) is true iff p voted for value v in slot s in round r, p.learned (s,v) is true iff p learned that value v was chosen in slot s, p.executed (s,v) is true iff p has executed v in its local copy of the replicated state machine in slot s. For a client c and value v, c.sent ('req', v) is true iff c requested v to be executed, c.received ('resp', v, res(v)) is true iff c received a response that value v was executed with result res(v).
Figure 6: Hierarchy of assumptions about servers (left) and hierarchy of liveness assertions (right) of 18 algorithms, as stated by the authors of the algorithms, comprising of (i) the 16 algorithms and variants in the last two rows in Table 2, and (ii) 2 algorithms from the first row—Derecho and VR-Revisit. Others in Table 1 do not state liveness assumptions or assertions. There are 17 entries on the left, because Mencius and Paxos-Complex do not provide server assumptions, and Paxos-SB describes two variants of assumptions. There are 18 entries on the right, but Derecho and Mencius only mentions "liveness" without explanations, IronRSL proves two properties, and Paxos-SB states two properties.
5.1 Liveness properties stated and their proofs given

We organize liveness properties by the assumptions and assertions stated by the authors, for all the algorithms and variants shown in Figure 6, 18 total. Table 3 displays these in columns “Assumptions” and “Assertions”. Column “Proofs” states the kinds of proofs of liveness properties given by the authors of the algorithms and variants. We categorize the proofs into the following four kinds:

1. **Sketch** — a proof sketch is given explaining the idea but many details are omitted.

2. **Prose** — a proof in ordinary prose is given. The proof is usually provided in paragraphs and may omit some details.

3. **Systematic** — a proof written in the step-wise hierarchical structure [43]. The proof is more systematic than ordinary prose proofs.

4. **Formal \((T)\)** — a proof that is machine-checked, using the proof system \(T\).

Note that 7 of the 18 algorithms and variants do not have proofs of any kind given with the stated properties.

5.2 Analysis of liveness properties

We present our analysis results showing the range of issues from lacking assumptions or too weak assumptions for which no liveness assertions can hold, to too strong assumptions making it uninteresting to achieve the assertions. Column “Analysis” of Table 3 summarizes these results. For example,

- **EPaxos** assumes **Alw-Q**, that is, eventually, there is always some quorum of servers that is non-faulty. By Proposition 7, this is too weak for reaching consensus, because each such quorum might not stay non-faulty long enough to make progress.

- **Zab** assume **PQ-Alw**, that is, eventually, there is a server \(P\) and a quorum \(Q\) of servers, such that always, \(P\) is non-faulty and is the primary and \(Q\) is non-faulty. Chubby-Live even assumes **Alw**, that is, eventually, all servers are always non-faulty. By Corollary 5.1 this is too strong, making it trivial to reach consensus.

**Proposition 1** (Each-Vote insufficient). **Each-Vote** is not strong enough as a liveness assertion.

*Proof.* **Each-Vote** is insufficient to be considered as an important assertion for liveness—a sentiment even echoed in the liveness properties proved by Lamport in both Paxos-Synod and Paxos-Fast. As an example, consider Raft, which may satisfy **Each-Vote** with a quorum voting for a value \(v\), but not reaching consensus on \(v\). This happens with view changes, where a different value may be chosen as shown in Figure 4.

\[\square\]
Table 3: Assumptions, assertions, and proofs, as provided by the respective authors, together with our analysis, about the liveness properties for 18 algorithms, comprising of (i) the 16 algorithms and variants in the last two rows in Table 2, and (ii) 2 algorithms from the first row—Derecho and VR-Revisit. Others in Table 1 do not provide assumptions, assertions, or proofs about liveness.

“−” in Assumptions and Proofs indicates that the information is not provided; for proofs, it means that the property is only claimed to follow from the assumptions, but no proof was given.

“+” in Assertions indicates that the calculation of the bound after which the assertion would be satisfied is also given.
Proposition 2 (None from Raw). No algorithm can satisfy any liveness assertion in Figures 3 and 5 under Raw.

Proof. Under Raw all messages can be lost and therefore none of the liveness assertions in Figures 3 and 5 can be satisfied. □

Proposition 3 (All from Q-Alw single). There is an algorithm that solves single-value consensus and satisfies all liveness assertions in Figure 3 under Fair and Q-Alw.

Proof sketch. This is due to ACT. Aguilera et al. [1, Theorem 5] proves that ACT satisfies Each-Learn. By the hierarchy in Figure 6 it also satisfies Each-Vote and Some-Learn.

ACT does not specify processes executing values, and nor does it specify clients. However, it is straightforward to extend ACT to have them. If a process can both learn and execute values, then the process executes upon learning them. Otherwise, upon learning a value, processes send a ‘learned’ message with the value to all the processes that execute values. Fair link assumption states that these messages will be eventually received. Therefore, Some-Exec and Each-Exec follow from Some-Learn and Each-Learn, respectively.

ACT can be further extended by adding clients. Upon executing a value, processes send a ‘resp’ message with the executed value to all the clients. Fair and Some-Exec then imply Resp. □

Proposition 4 (All from Q-Alw). There is an algorithm that claims to solve consensus and satisfies all liveness properties shown in Figure 7 under Fair and Q-Alw.

Proof sketch. This is due to Paxos-SB. The authors state that it satisfies Some-Exec and Each-Exec. By the hierarchy structure, it also satisfies Each-Vote, Some-Learn, and Each-Learn.

However, if clients do not re-send request messages, Paxos-SB does not satisfy Resp, because only the server that received a client’s request can respond to the client [30]. Figure 10, Lines D3-D4. Thus, we can construct an adversarial run in which the server that received the client’s request executes the request but fails before replying to the client and all other servers are always non-faulty, thereby maintaining Q-Alw. If the client does not re-send request message, it will never receive any response. As a remedy, we must have clients re-send their requests to different servers upon not receiving a response within some timeout. Then, Resp can be satisfied. □

Proposition 5 (Resp from P-Alw-Q). There is an algorithm that claims to solve consensus and satisfy Resp under Fair and P-Alw-Q.

Proof sketch. This is due to Paxos-VS. □

Corollary 5.1 (Weak from PQ-Alw). A liveness property that assumes PQ-Alw is a weak property, because the assumption is stronger than necessary.

Proof. This is because PQ-Alw is a stronger assumption than P-Alw-Q, yet Paxos-VS solves consensus under P-Alw-Q. Note that, as a result, Alw is even more so—it is too strong making it trivial to achieve liveness. □
Proposition 6 (Not Resp). Paxos-Complex cannot satisfy Resp, even with Alw.

Proof sketch. In Paxos-Complex, servers may concurrently propose values sent by clients on the same slot. However, for each slot, only one value can be chosen. If the value proposed by a server is not chosen on the slot proposed by the server, the server re-proposes the value on a different slot. This can repeat forever. Thus, Paxos-Complex does not satisfy Resp. \( \square \)

The following impossibility result has also been discussed by Kirsch and Amir \[36\,\text{Strong L1}\]. It is proved by Keidar and Shraer \[33, 34\] in a general round-based algorithm framework called GIRAF.

Proposition 7 (None from Alw-Q). No quorum-based consensus algorithm that executes in rounds can satisfy Some-Learn under Fair and Alw-Q.

Proof sketch. If the algorithm does not use a primary, take the server that has proposed the highest round seen by some quorum as the primary. An adversarial run can be constructed where the messages sent by the primary are delayed long enough for a new primary to be elected before the messages are received. Because every sent message is received, Fair is satisfied, and because at most one server is faulty at any point, Alw-Q is satisfied. \( \square \)

5.3 What is the right or best liveness property, or properties?

The link assumptions in Figure 1 form a 3-element linear order, but as Figure 6 shows, the hierarchy of server assumptions includes two diamonds, and the hierarchy of liveness assertions includes a diamond in the middle and a branching at the bottom. That is, different server assumptions and liveness assertions are not merely a degree of strength but include a combination of different choices.

The main question remaining is, what is a right or best liveness property that can be satisfied? This property must give the best liveness assertion and make the weakest assumptions among all choices.

Responding to clients from reaching consensus. It is clear, from the perspective of truly solving the consensus problem, that Resp is the desired and best liveness assertion in Figure 6-right. Anything weaker does not guarantee the responsiveness of the servers to the requests of the clients.

Note that the partial order of liveness assertions could be made into a lattice, by adding a bottom element, Each-Resp, that is stronger than both Each-Exec and Resp, to close the branching at the bottom. Each-Resp would capture that clients have received not only the desired ‘resp’ messages, but received them from each server in some quorum. However, this has no benefit but only incurs extra cost. In fact, it is clear from Figure 6-right that neither Each-Exec nor Each-Learn is needed to ensure Resp.

As discussed in proofs above, Each-Vote is insufficient for liveness, but once Some-Learn is satisfied, Some-Exec and Resp are easy to satisfy with Fair. Indeed, it is known that consensus is reached when Some-Learn is satisfied.
Weakest assumptions for consensus. So what are the weakest link and server assumptions that can ensure Some-Learn? As discussed in proofs above, Raw and Alw-Q are both too weak, but ACT and Paxos-SB achieve consensus and more with Fair and Q-Alw with systematic proofs, and Paxos-VS and Derecho claim to achieve with Fair and P-Alw-Q without proofs.

Additionally, Paxos-Time achieves consensus and more with Sure and PQ-Dur, noted as “assuming primary” in Table 3. Is it possible to achieve consensus without assuming primary, that is, with a new element, Q-Dur, in Figure 6 that is weaker than both PQ-Dur and Q-Alw? We tend to think so, likely by strengthening analysis for ACT and Paxos-SB, but there are no existing proofs or even claims for this. Finally, with use of duration in Q-Dur, and with Sure, liveness assertions should also be strengthened to give upper bounds after which the assertions would be satisfied, similar to what Paxos-Time achieves. The ultimate goal is, of course, the most efficient algorithm with the most precise bound, and possibly with probabilistic guarantees. Much is open for future studies.

6 Related work and conclusion

There is a large literature on distributed consensus algorithms and variants. This work systematically studies, formalizes, and compares liveness assumptions and assertions in over 30 prominent consensus algorithms and variants under the asynchrony caused by process and link failures. Many of these algorithms do not discuss liveness properties, as described in Section 4 and summarized in Table 3. Among those that do, except Amir [2], none of them specify liveness properties formally in their papers, as shown in Appendices B, C, D, E, and F. In [2], Amir formalizes a liveness property similar to Each-Exec and provides a systematic proof of it for EVS, a group communication based consensus algorithm. Other such algorithms, including Congruity [4, 5] and COReL [31, 32], also study a similar liveness property.

Kirsch and Amir [37, 36] discuss two different liveness properties, called Weak L1 and Strong L1, and state that Paxos-SB satisfies Weak L1. They further state that the assumptions of the other liveness property, Strong L1, are too weak and could not guarantee liveness. Server assumptions for these properties are shown in Figure 6.

Keidar and Shraer [33, 34] develop GIRAF, a generalized framework that models consensus algorithms as round-based. The framework is a generalization of Gafni’s Round-by-Round Failure Detector (RRFD) [23]. The framework is formally specified in IOA [33, Algorithm 1]. Computation in this model proceeds in rounds and, in each round, processes may query an oracle. They give algorithms that satisfy Each-Learn under different liveness assumptions in their model and also give an impossibility result for another liveness assumption.

Van Renesse et al. [63] explore the similarities and differences between VR, Paxos-Synod, and Zab, and discuss liveness of these algorithms with particular emphasis on recovery after

1Ironfleet and EPR contain formal liveness properties in web links but do not present them in the papers.
process failures both with and without stable storage. Van Renesse and Altinbuken [62] describe \texttt{Resp} as the “ideal liveness condition” for Paxos and, like [39], detail certain practical considerations that would affect liveness including bounded clock drift, timeouts, and stable storage.

The language we introduced is powerful, easy to use, and highly effective. It is more general than temporal logic \cite{61,27} by supporting also time intervals, similar to many temporal logic extensions, e.g., \cite{16,15}. It captures commonly-used predicate logic over time intervals in simple forms that are easy to use for understanding and reasoning. It allowed us to easily express the wide range of liveness properties clearly and precisely as needed. The resulting specifications then allowed us to directly and precisely relate all different combinations of assumptions and assertions and to find weaknesses of many of them, as shown in Section 5.

In conclusion, this paper is a first step in precise specification of the wide variety of liveness assumptions and assertions used for distributed consensus algorithms and variants under asynchrony. Such precise specification is essential for comparing the algorithms and variants, including new algorithms as they are developed, and helps better understand all of them. Much future work is needed to formally verify liveness properties of consensus algorithms, especially the ones that give stronger liveness guarantees with weaker assumptions. Much remains open for precise time complexity analysis of these algorithms, as discussed in Section 5.3 and of Byzantine consensus algorithms, including precise probabilistic modeling and reasoning for understanding the performance of these algorithms analytically.

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A Liveness model checking

We developed TLA+ specifications of the liveness properties we formalized, and used TLC, the model checker for TLA+, to model check execution steps to illustrate liveness patterns for Paxos. We use the TLA+ specifications of Paxos from [10]\(^2\); note that nondeterminism in the models already allows all failures considered to happen. We add a tick variable in the models. This variable is initialized to 0 and is incremented by 1 with each action execution. Therefore, its value corresponds to the number of actions executed by the system.

We show trends and patterns about the liveness properties using two values about the number of execution steps.

- Each run of a model takes a stable duration start—the minimum number of actions after which liveness assumptions hold. For Paxos, the liveness assumptions used in the literature, as shown in Table 3, are PQ-Extra-Dur, PQ-Dur, and PQ-Alw, for Paxos-Synod, Paxos-Time, and Paxos-Fast, respectively. We make these assumptions hold by (1) ensuring that a leader election starts, if not already started, when the value of tick equals the stable duration start, and (2) checking that there are no new leader election from this point on.

- In each model, we use TLC to find the value of stable duration length—the maximum number of actions after which consensus is reached. For Paxos, the liveness assertions used in the literature, as shown in Table 3, are Each-Learn, Each-Exec, and Some-Learn, for Paxos-Synod, Paxos-Time, and Paxos-Fast, respectively. We use the value of tick after which Each-Vote holds; Each-Vote ensures that consensus is reached in Paxos [40], and Paxos does not have the infinite behavior of Raft shown in Figure 4.

We use TLC to find the value of stable duration length given the value of stable duration start, and run TLC on models with different number of servers. Paxos uses two kinds of

\(^2\)Available online at https://github.com/DistAlgo/proofs
servers: proposers and acceptors. A proposer may become a leader and propose values, and an acceptor may vote for leaders and values. Any majority set of acceptors is a quorum. To tolerate failures of 1 proposer and 1 acceptor, there must be at least 2 proposers and 3 acceptors.

We present experiments and results for single-value Paxos, which we call Paxos-Basic, because liveness properties using *tick* for multi-value Paxos are too expensive to check using TLC. We run 6 models of Paxos-Basic; for 2, 3, or 4 proposers, in combination with 3 or 4 acceptors. Each run finished in a few hours, but larger numbers are also too expensive to check using TLC. We name the model with *i* proposers and *j* acceptors *iPjA*.

**Results.** Figure 7 shows our model checking results using line graphs, where each line represents a different combination of numbers of proposers and acceptors:

- green for 2 proposers — lower line (circle marks) for 3 acceptors, and higher line (square marks) for 4.
- blue for 3 proposers — lower line (plus marks) for 3 acceptors, and higher line (cross marks) for 4.
- red for 4 proposers — lower line (triangle marks) for 3 acceptors, and higher line (diamond marks) for 4.

In the graphs for numbers of states and distinct states and model checking time, the line for 4P4A is cut in order to make the differences among the other lines visible.

1. Figure 7a shows the relationship between stable duration start and stable duration length. We see that, for each model, stable duration length grows linearly with stable duration start up to an inflection point, and remains unchanged after this point.

   - Runs with a later stable duration start allow more proposers in Paxos to compete to be the leader, and thus take a longer time to reach consensus, i.e., have a larger stable duration length. This is up to an inflection point, because the number of proposers in a run is fixed.
   - Models with fewer proposers reach the inflection point earlier, because fewer proposers have less competition and reach consensus faster.
   - Additionally, models with fewer acceptors reaches consensus faster, i.e., have a shorter stable duration length, because with majority quorums for reaching consensus, 2 votes are needed for 3 acceptors, and 3 votes are needed for 4 acceptors.

The rate at which stable duration length grows, when it does, equals the number of acceptors. The precise formula is,

\[
y = \begin{cases} 
  jx + (j + 2 + \left\lceil \frac{j + 1}{2} \right\rceil), & \text{if } x \leq i \\
  ji + (j + 2 + \left\lceil \frac{j + 1}{2} \right\rceil), & \text{otherwise}
\end{cases}
\]
Figure 7: Model checking execution steps for Paxos-Basic. (a) stable duration start and length as measured by TLC. (b) number of states generated by TLC. (c) model checking time (in seconds) taken by TLC. (d) number of distinct states generated by TLC.
where \( y \) is stable duration length and \( x \) is stable duration start. The term \((j + 2 + \left\lceil \frac{j+1}{2} \right\rceil)\) is the stable duration length when stable duration starts at tick 0.

2. Figures 7b and 7d show the number of states and the number of distinct states, respectively, generated by TLC for each model and stable duration start value. For models with 4 proposers, we see that the number of states (and distinct states) generated by TLC grows exponentially as stable duration start increases. For models with 3 proposers, we see a subexponential but superlinear growth. For models with 2 proposers, the growth is approximately linear for number of states, and is exactly linear for the number of distinct states.

3. Figure 7c shows the running time (in seconds) that TLC took to finish checking each model. For models with 4 proposers, we again see that the growth of model checking time is exponential. However, for models with 3 proposers, unlike the trends for number of states and number of distinct states, we see that the growth of model checking time is superlinear and larger. For models with 2 proposers, the growth appears to be linear.

In summary, after the liveness assumptions hold, consensus is reached within a fixed duration determined by the number of proposers and number of acceptors. Furthermore, consensus is reached faster when liveness assumptions hold earlier.

B Virtual Synchrony and variants

B.1 VS-ISIS [7]

This work [7] describes the fundamental communication and maintenance primitives used in the ISIS system. These primitives help build fault-tolerant process groups. The system ensures that the processes belonging to a fault-tolerant process group will observe consistent orderings of events affecting the group as a whole, including process failures, recoveries, migration, and dynamic changes to group properties like member rankings.

The system supports three modes of communication with varying ordering constraints:

- **GBCAST.** The order in which GBCASTs are delivered relative to the delivery of all other sorts of broadcasts (including other GBCASTs) is the same at all overlapping destinations. Additionally, if a process is observed to fail, a failure GBCAST message notifying its failure must be delivered after any other messages.

- **ABCAST.** These are labeled broadcasts used for maintaining a replicated queue. The order in which ABCASTs are delivered is the same for every intended recipient per label but is independent of other labels and broadcasts.

- **CBCAST.** These are similar to ABCAST with the added property that these broadcasts are not independent of other labels. For example, if a replicated variable \( x \) is
being maintained, and commands setting $x$ to 0 and incrementing $x$ are to be broadcast in that order, not only is it important that all processes receive the same order of commands but there must be a way to ensure the order.

The authors then detail their implementation that guarantees these primitives and give informal correctness (safety) proof for each implementation. For each primitive, atomicity is proven, i.e., every message is either delivered to all or none of its recipients. Then, the particular ordering property for each broadcast is proven. The proofs are manual pen-and-paper proofs. However, the exact properties being proven are not clearly stated all the times. Liveness is neither proven nor clearly stated. Following is the excerpt from the paper on liveness:

| On Liveness |
|-------------|
| In the interest of brevity, we omit a formal proof that the protocols given above are free of deadlock and livelock. |

### B.2 VS-ISIS2 [6]

This work describes ISIS$_2$. Built upon ISIS, this system also supports the same broadcast primitives: GBCAST, ABCAST, and CBCAST. This paper essentially details some parts of the systems and explains some optimizations. While performance is discussed, safety and liveness are not. Following are the assumptions about the system:

| Assumptions |
|-------------|
| In this work, we assume that a distributed system consists of processes with disjoint address spaces communicating over a conventional LAN using message passing. Processes are assumed to execute on computing sites. Individual processes and entire sites can crash; the former type of crash is assumed detectable by some monitoring mechanism at the site of the process, while the latter can only be detected by another site by means of a timeout. It is assumed that failing processes send no incorrect messages. Our system tolerates message loss, but not partitioning failures (wherein links that interconnect groups of sites fail). Partitioning could cause parts of our system to hang until communication is restored. |

### B.3 EVS [2, 54]

Extended Virtual Synchrony (EVS) extends Virtual Synchrony by adding more delivery primitives and includes mechanisms to handle system partition.
Notation

- $S$ is the servers group.
- $a_{s,i}$, is the $i^{th}$ action performed by server $s$.
- $D_{s,i}$, is the state of the database at server $s$ after actions $1..i$ have been performed by server $s$.
- $\text{stable\_system}(s,r)$ is a predicate that denotes the existence of a set of servers containing $s$ and $r$, and a time, from which on, that set does not face any communication or server failure. Note that this predicate is only defined to reason about the liveness of certain protocols. It does not imply any limitation on our practical protocol.

Liveness property

If server $s$ performs an action and there exists a set of servers containing $s$ and $r$, and a time, from which on, that set does not face any communication or processes failures, then server $r$ eventually performs the action.

Formalization

$\Diamond (\exists a_{s,i} \land \Box \text{stable\_system}(s,r)) \Rightarrow \exists a_{r,i}$

B.4 Paxos-VS

This work focuses on a dynamic reconfiguration model similar to the view model in VR (Appendix C) and unifies two widely popular approaches to the problem of consensus—Virtual Synchrony (Appendix B.1) and state machine replication, particularly Paxos (Appendix D). Many algorithms and variants are proposed exhibiting availability conditions. We focus on one of these algorithms—Virtually Synchronous Paxos. The algorithm bases on the concept of memberships similar to views in VR. A primary is elected in each membership. During steady-state operation, client requests received by non-primary servers are forwarded to the primary. Upon receiving a request, the primary assigns it to a slot and broadcasts the request and its slot number to the servers in current membership. If the primary fails, reconfiguration is triggered (instead of electing a new primary for the membership even if a majority of servers are non-faulty). Following is mentioned about liveness:
Assumptions

Our aim is to provide services in asynchronous systems whose set of servers is changed through explicit reconfigurations. We assume an additional set, potentially overlapping, of clients. We do not assume any bounds on message latencies or message processing times (i.e., the execution environment is asynchronous), and messages may get lost, re-ordered, or duplicated on the network. However, we assume that a message that is delivered was previously sent by some live member and that correct members can eventually communicate any message.

Liveness Assumption

Throughout the lifetime of a membership $M$, a majority of servers are correct.

Liveness Property

Every client request is eventually replied to.

B.5 Derecho [29]

Derecho is a fast state machine replication algorithm. Like VS-ISIS and Paxos-VS, this algorithm also executes on the principle of process groups and memberships. Unlike Paxos-VS though, where a non-primary server routes client requests to the primary, in Derecho each server appends incoming client requests in a local queue and informs other servers in the current group of this. If the group contains $N$ servers and server $i \in [0, N - 1]$ inserts a request in index $k$ of its local queue, then the request is virtually assigned the slot number $N \ast k + i$. For example, if the group has 3 servers, then server 0 inserts requests in slots $\{0, 3, 6, \ldots\}$, server 1 inserts requests in slots $\{1, 4, 7, \ldots\}$, and server 2 inserts requests in slots $\{2, 5, 8, \ldots\}$, that is, servers simultaneously propose requests but always in disjoint slots. Liveness is discussed in [29, Appendix C]. Following is the liveness assumption:

Liveness Assumption

Derecho will make progress if (1) no more than a minority of the current view fails and (2) the failure detector only reports a failure if the process in question has actually failed or has become unreachable.

C Viewstamped Replication and variants

C.1 VR [55]

This work presents the Viewstamped Replication algorithm based on the primary-backup approach. One server is designated as the primary; it executes procedure calls, and partici-
pates in two-phase commit. The remaining servers are backups, which are essentially passive and merely receive state information from the primary. Servers run in views. Multiple events can cause a server to trigger view change, for example, not receiving timely heartbeat messages from some other server in the view, receiving heartbeat messages from a new server, recovering after a crash, etc. This paper does not mention liveness.

### C.2 VR-Revisit [46]

This work gives an updated description of Viewstamped Replication. The description of the core algorithm is simplified and nuances of the application using it are separated out. It explains various optimizations and reconfiguration in detail. Both safety and liveness of the View change, Recovery and Reconfiguration algorithms are discussed and informally proven in this paper.

| Liveness         |
|------------------|
| View change. The protocol executes client requests provided at least $f + 1$ non-failed replicas, including the current primary, are able to communicate. |
| Recovery. The protocol is live, assuming no more that $f$ replicas fail simultaneously. |

The exact liveness assumptions for Reconfiguration are not explicitly mentioned but it can be seen that the algorithm requires at least $f + 1$ non-failed replicas of the old configuration to be non-faulty for the new configuration to be installed.

### D Original Paxos and variants

#### D.1 Paxos-Synod [39]

The seminal Part-time Parliament paper explains Paxos in full and also gives a liveness condition.

| Presidential selection requirement |
|-------------------------------------|
| If no one entered or left the Chamber, then after $T$ minutes exactly one priest in the Chamber would consider himself to be the president. |

| Liveness Property |
|-------------------|
| If the presidential selection requirement were met, then the complete protocol would have the property that if a majority set of priests were in the chamber and no one entered or left the Chamber for $T + 99$ minutes, then at the end of that period every priest in the Chamber would have a decree written in his ledger. |
Still assuming that $p$ was the only priest initiating ballots, suppose that he were required to initiate a new ballot iff (i) he had not executed step 3 or step 5 within the previous 22 minutes, or (ii) he learned that another priest had initiated a higher-numbered ballot. If the Chamber doors were locked with $p$ and a majority set of priests inside, then a decree would be passed and recorded in the ledgers of all priests in the Chamber within 99 minutes. (It could take 22 minutes for $p$ to start the next ballot, 22 more minutes to learn that another priest had initiated a larger-numbered ballot, then 55 minutes to complete steps 1-6 for a successful ballot.) Thus, the progress condition would be met if only a single priest, who did not leave the chamber, were initiating ballots.

This work assumes bounded message-delay (4 minutes) and bounded action execution time (7 minutes):

They determined that a messenger who did not leave the Chamber would always deliver a message within 4 minutes, and a priest who remained in the Chamber would always perform an action within 7 minutes of the event that caused the action.

The discussions above mention a president. However, the paper does not specify how a president can be elected:

The Paxons chose as president the priest whose name was last in alphabetical order among the names of all priests in the Chamber, though we don’t know exactly how this was done.

Given the sophistication of Paxon mathematicians, it is widely believed that they must have found an optimal algorithm to satisfy the presidential selection requirement. We can only hope that this algorithm will be discovered in future excavations on Paxos.

### D.2 Paxos-Basic [40]

The primary focus of this paper is to give a simpler explanation of Paxos and consistency proof, focusing on single-value consensus. Liveness however is not considered formally in this work. Lamport explicitly states in Section 2 of the paper: “We won’t try to specify precise liveness requirements”. Following is the informal discussion from the paper:
Liveness Discussion

To guarantee progress, a distinguished proposer must be selected as the only one to try issuing proposals. If the distinguished proposer can communicate successfully with a majority of acceptors, and if it uses a proposal with number greater than any already used, then it will succeed in issuing a proposal that is accepted. By abandoning a proposal and trying again if it learns about some request with a higher proposal number, the distinguished proposer will eventually choose a high enough proposal number.

If enough of the system (proposer, acceptors, and communication network) is working properly, liveness can therefore be achieved by electing a single distinguished proposer.

D.3 Paxos-Fast [41]

This paper introduces Fast Paxos, which uses the concept of fast quorums to achieve consensus in one less round than Paxos-Basic [40]. Incidentally, this paper discusses liveness of Paxos-Basic [40].

Liveness Assumptions

An agent is defined to be nonfaulty iff it eventually performs the actions that it should, such as responding to messages. Define a set G of agents to be good iff all the agents in G are nonfaulty and, if any one agent in G repeatedly sends a message to any other agent in G, then that message is eventually received—more precisely, the message is eventually delivered or G is eventually not considered to be good. Being nonfaulty and being good are temporal properties that depend on future behavior.

For any proposer p, learner l, coordinator c, and set Q of acceptors, define \( LA(p, l, c, Q) \) to be the condition that asserts:

LA1. \( \{p, l, c\} \cup Q \) is a good set.

LA2. p has proposed a value.

LA3. c is the one and only coordinator that believes itself to be the leader.

Liveness Property

For any learner l, if there ever exists proposer p, coordinator c, and majority set Q such that \( LA(p, l, c, Q) \) holds from that time on, then eventually l learns a value.
D.4 Paxos-Vertical [44]

This paper describes Vertical Paxos, which adds reconfiguration to Paxos. Liveness is considered beyond the scope of the paper:

The liveness property satisfied by a Vertical Paxos algorithm is similar to that of ordinary Paxos algorithms, except with the added complication caused by reconfiguration—a complication that arises in any algorithm employing reconfiguration. A discussion of liveness is beyond the scope of this paper.

E Consensus with failure detection

E.1 CT [13]

This work introduces the concept of unreliable failure detectors and studies how they can be used to solve consensus in asynchronous systems with crash failures (no recovery). Two algorithms are presented which solve consensus using (1) any Strong failure detector and, (2) any Eventual strong failure detector. These failure detectors satisfy strong completeness and (eventual) weak accuracy. Lemma 6.2.2 of the paper defines the liveness property as:

| Liveness Property |
|--------------------|
| Every correct process eventually decides some value. |

A sketch of the proof (\(\frac{1}{3}\) page) is given for the algorithm that uses Strong failure detector and a systematic proof (1 page) is given for the algorithm that uses Eventual strong failure detector. Likewise, the liveness assumptions for the two algorithms are different. Algorithm that uses Strong failure detector tolerates at most \(n-1\) process crashes where \(n\) is the total number of processes. This is not a quorum-based algorithm and therefore not considered in this work. The second algorithm that uses Eventual strong failure detector is quorum-based and its liveness assumption is:

| Liveness Assumptions |
|----------------------|
| • If we assume that the maximum number of faulty processes is less than half then we can solve Consensus using \(\Diamond J\), a class of failure detectors that satisfy only eventual weak accuracy. |
| • Every nonfaulty process proposes some value. |
E.2 ACT [1]

This work studies the problems of failure detection and consensus in asynchronous systems in which processes may crash and recover (unlike the predecessor CT where crashed processes do not recover), and links may lose messages. New types of failure detectors are developed to suit the crash-recovery model as opposed to prior work that requires unstable processes to be eventually suspected forever. Using one of these failure detectors, the authors develop a consensus algorithm without requiring stable storage and two algorithms with stable storage, but using two different failure detectors. Liveness assumptions and property are as follows:

| Liveness Assumption |
|---------------------|
| • A majority of processes are good. |
| • Every good process proposes a value. |

| Liveness Property |
|-------------------|
| Every good process eventually decides some value. |

A process is called good if it is either always up or eventually always up. Detailed systematic proofs are given for each of the algorithms.

F Other consensus algorithms and variants

F.1 Paxos-Time [19]

This paper performs a time analysis of a specification of Paxos-Basic ($S_{PAX}$) and Paxos in IOA. This work gives upper bounds on the running time and number of messages assuming upper bounds on message delay and action execution time.

| Liveness |
|----------|
| Let $\alpha$ be a nice execution fragment of $S_{PAX}$ starting in a reachable state and lasting for more than $24l + 10nl + 13d$. Then the leader $i$ executes $Decide(v')_i$ by time $21l + 8nl + 11d$ from the beginning of $\alpha$ and at most $8n$ messages are sent. Moreover by time $24l+10nl+13d$ from the beginning of $\alpha$ any alive process $j$ executes $Decide(v')_j$ and at most $2n$ additional messages are sent. |

Informally, a nice execution fragment is one in which (1) at least a majority of processes are non-faulty, (2) all messages are delivered in a bounded time. The exact definitions, including original markings from the paper, are:
Definitions

**Definition 2.1.** A step \((s_{k-1}, v(t), s_k)\) of a Clock GTA is called *regular* if \(s_k.Clock - s_{k-1}.Clock = t\). [GTA stands for General Timed Automaton. \(v(t)\) is the time-passage action that, when executed by the automaton, increments the \(Clock\) by an amount of time \(t' \geq 0\), independent of the amount \(t\). \(s_i\) is a state of the automaton.]

**Definition 2.2.** A timed execution fragment \(\alpha\) of a Clock GTA is called *regular* if all the time-passage steps of \(\alpha\) are regular.

**Definition 3.1.** Given a process automaton \(\text{PROCESS}_i\), we say that an execution fragment \(\alpha\) of \(\text{PROCESS}_i\) is *stable* if process \(i\) is either stopped or alive in \(\alpha\) and \(\alpha\) is regular.

**Definition 3.2.** Given a channel \(\text{CHANNEL}_{i,j}\), we say that an execution fragment \(\alpha\) of \(\text{CHANNEL}_{i,j}\) is *stable* if no \(\text{Lose}_{i,j}\) and \(\text{Duplicate}_{i,j}\) actions occur in \(\alpha\) and \(\alpha\) is regular.

**Definition 3.5.** Given a distributed system \(S\), we say that an execution fragment \(\alpha\) of \(S\) is *stable* if: (i) for all automata \(\text{PROCESS}_i\) modelling process \(i\), \(i \in S\) it holds that \(\alpha|\text{PROCESS}_i\) is a stable execution fragment for process \(i\); (ii) for all channels \(\text{CHANNEL}_{i,j}\) with \(i, j \in S\) it holds that \(\alpha|\text{CHANNEL}_{i,j}\) is a stable execution fragment for \(\text{CHANNEL}_{i,j}\).

**Definition 3.6.** Given a distributed system \(S\), we say that an execution fragment \(\alpha\) of \(S\) is *nice* if \(\alpha\) is a stable execution fragment and a majority of the processes are alive in \(\alpha\).

F.2 Paxos-PVS [35]

This paper presents a machine-checked safety proof of Paxos-Basic specified in the logic of the PVS theorem prover. Liveness is neither specified or proved.

F.3 Chubby [9]

This paper describes the authors’ experiences with the Chubby lock service, which is intended to provide coarse-grained locking as well as reliable (though low-volume) storage for a loosely-coupled distributed system. It describes the initial design and expected use of the service, and then the actual use and the modifications made to accommodate actual use. Safety and liveness are not discussed.

F.4 Chubby-Live [12]

This paper details some of authors’ experiences with Chubby lock service and its deployment. The authors describe their testing apparatus—a tool that tests the system’s safety and liveness:
Liveness Property

1. **Safety mode.** In this mode, the test verifies that the system is consistent. However, the system is not required to make any progress. For example, it is acceptable for an operation to fail to complete or to report that the system is unavailable.

2. **Liveness mode.** In this mode, the test verifies that the system is consistent and is making progress. All operations are expected to complete and the system is required to be consistent.

Our tests start in safety mode and inject random failures into the system. After running for a predetermined period of time, we stop injecting failures and give the system time to fully recover. Then we switch the test to liveness mode. The purpose for the liveness test is to verify that the system does not deadlock after a sequence of failures.

Formalization

None

Proof

None

F.5 Paxos-SB [37, 36]

This work gives low-level pseudocode for Paxos that was implemented in C++ by the authors.

| Stable Set |
|------------|
| A stable set is a set of processes that are eventually alive and connected to each other, and which can eventually communicate with each other with some (unknown) bounded message delay. |

| Paxos-L1 (Progress) |
|---------------------|
| If there exists a stable set consisting of a majority of servers, then if a server in the set initiates an update, some member of the set eventually executes the update. |

The following property states that after the system becomes "nice", if some nice server initiates an update, then some nice server eventually executes it, i.e., consensus was reached and at least one server recognized that.
Eventual Replication

If server $s$ executes an update and there exists a set of servers containing $s$ and $r$, and a time after which the set does not experience any communication or process failures, then $r$ eventually executes the update.

The authors claim that the progress property Paxos-L1 can be realized in the following two ways—Strong L1 and Weak L1. Weak L1 assumes that at least a single majority remains alive during the execution, whereas Strong L1 allows majorities to change during the execution such that no majority remains alive during the whole execution. The authors further claim that ensuring progress under Strong L1 is difficult in real world and doubt that any real Paxos-like implementation guarantees that.

**Strong L1**

If there exists a time after which there is always a set of running servers $S$, where $|S|$ is at least the majority, then if a server in the set initiates an update, some member of the set eventually executes the update.

**Weak L1**

If there exists a stable component consisting of a majority of servers, and a time after which the set does not experience any communication or process failures, then if a server in the set initiates an update, some member of the set eventually executes the update.

**Formalization**

None

**Proof**

None

Note that Paxos-L1 implies Weak L1. And the authors claim that the leader election protocol in Section 5.3 meets Paxos-L1. The following excerpt from the paper sheds more light as to why they define Weak and Strong L1. It gives their conjecture that any Paxos-like algorithm cannot achieve Strong L1 whereas group communication based protocols can achieve Weak L1:
Liveness discussion

Strong L1 requires that progress be made even in the face of a (rapidly) shifting majority. We believe that no Paxos-like algorithm will be able to meet this requirement. If the majority shifts too quickly, then it may never be stable long enough to complete the leader election protocol. Weak L1, on the other hand, reflects the stability required by many group communication based protocols, as described above. It requires a stable majority component, which does not receive messages from servers outside the component. Since the leader election protocol specified in Section 5.3 meets Paxos-L1, it also meets Weak L1. We note that group communication based protocols can most likely be made to achieve Paxos-L1 by passing information from the application level to the group communication level, indicating when new membership should be permitted (i.e., after some progress has been made).

F.6 Mencius [51]

Mencius is a protocol for general state machine replication that has high performance in a wide-area network. Mencius has high throughput under high client load and low latency under low client load even under changing wide-area network environment and client load. Mencius was developed as a derivation from Paxos. Unlike Paxos, it is a multi-leader protocol where each leader proposes values on mutually exclusive slots. For example in a two server system, one server proposes for odd-numbered slots and the other for even-numbered slots. Even though liveness is not explicitly proven, the authors do mention that Mencius assumes eventual delivery for liveness:

Liveness Assumption

Since \( \mathcal{P} \) (Mencius is \( \mathcal{P} \) combined with certain optimizations) only relies on the eventual delivery of messages for liveness, adding Optimization 2 and Accelerator 1 to protocol \( \mathcal{P} \) still implements replicated state machines correctly.

F.7 Zab [30]

Zab is an atomic broadcast protocol for primary-backup systems. Zab is shown to guarantee the following liveness property:
Suppose that:

- a quorum $Q$ of followers is up;
- the followers in $Q$ elect the same process $l$ and $l$ is up;
- messages between a follower in $Q$ and $l$ are received in a timely fashion.

If $l$ proposes a transaction $\langle v, z \rangle$, then $\langle v, z \rangle$ is eventually committed.

They provide an informal proof of liveness that is about 2 paragraphs long.

F.8 Zab-FLE [52]

This work provides pseudocode of the three phases of Zab—Discovery, Synchronization, and Broadcast phases. It then explains a Fast Leader Election (FLE) algorithm that was adopted in Zab, and replaces the Discovery and Synchronization phases with a single Recovery phase. This optimization was later shown to produce a liveness issue where the system may get caught in an infinite loop [52 Section 4.2]. This paper presents the fix adopted by Zab—re-introduce certain state variables that were removed in the (manual) optimization process. It is then informally argued that the re-introduction of previously maintained state variables solves the issue but no proof is given.

F.9 EPaxos [53]

EPaxos, short for Egalitarian Paxos, can be understood as a variant of Fast Paxos that (1) decides a sequence of commands instead of just one command, and (2) achieves consensus in a leaderless setting where commands may interfere with each other. Liveness is mentioned but not proved:

| Liveness |
|----------|
| Finally, liveness is guaranteed with high probability as long as a majority of replicas are non-faulty: clients and replicas use time-outs to resend messages, and a client keeps retrying a command until a replica succeeds in committing that command. |

F.10 Raft [56]

Raft is another consensus algorithm based on Paxos, inspired by the primary-backup approach of VR. This paper gives a liveness argument for the algorithm stating that the system will continue to make progress as long as the following timing condition is satisfied:
Timing condition

\[ \text{broadcastTime} \ll \text{electionTimeout} \ll \text{MTBF} \]

In this inequality \text{broadcastTime} is the average time it takes a server to send RPCs in parallel to every server in the cluster and receive their responses; \text{electionTimeout} is the election timeout described in Section 5.2; and \text{MTBF} is the average time between failures for a single server.

However, it is not clear what they exactly mean by progress.

F.11 Paxos-Complex [62]

The paper discusses liveness in Section 3. But does not make clear what liveness means for them. They write the following as an "example of a liveness property", but do not explicitly say that they are invested in achieving this.

Liveness Property

If a client broadcasts a new command to all replicas, it eventually receives at least one response.

The paper however discusses liveness in the presence of the following assumptions and how to achieve these assumptions in practice but does not prove liveness:

Assumptions

1. the clock drift of a process, that is, the rate of its clock is within some \[\text{bounded}\] factor of the rate of real time

2. the time between when a nonfaulty process initiates sending a message and the message has been received and handled by a nonfaulty destination process \[\text{is bounded}\].

The authors also claim that their liveness assumption on messages (Assumption 2 above), can be weakened to fair links assumption. They informally argue that liveness would hold in either case.

However, there is the issue of starvation in the context of the liveness property discussed here. To counter, the authors suggest that \textit{in the absence of new requests}, liveness could be satisfied:

Assumptions

\[\ldots\text{Should a competing command get decided, the replica will reassign the request to a new slot and retry. Although this may lead to starvation, in the absence of new requests, any outstanding request will eventually get decided in at least one slot.}\]
F.12 Raft-Verdi [64]

This paper presents Verdi—a verification framework for distributed algorithms—and a machine- checked proof of linearizability of Raft as a case study. Liveness is left as future work:

Liveness

This paper focuses on safety properties for distributed systems; we leave proofs of liveness properties for future work.

F.13 IronRSL [26]

IronRSL is part of the IronFleet project of Microsoft. It gives machine-checked proofs of one of Microsoft’s State machine replication engines based on Paxos. The proofs are in the Dafny system. They prove many liveness properties, most notably:

Liveness Property

If the network is eventually synchronous for a live quorum of replicas, then a client repeatedly submitting a request eventually receives a reply.

Following are their liveness assumptions (from the github repository linked from their project page given in the paper):

Liveness Assumptions

**NoZenoBehavior.** Time goes forward without Zeno behaviors

**ClockAmbiguityLimitedForHosts.** Live replicas have fairly accurate time sync [bound by asp.max_clock_ambiguity]

**LiveQuorum.** The live quorum is a set of replica indices and is big enough to constitute a quorum

**HostExecutesPeriodically.** Each host in the live quorum executes periodically, with period asp.host_period

**PersistentClientSendsRequestPeriodically.** The persistent client sends its request periodically, with period asp.persistent_period

**NetworkSynchronousForHosts.** The network delivers packets among the client and live quorum within time asp.latency_bound

**NetworkDeliveryRateBoundedForHosts.** The network doesn’t deliver packets to any host in the live quorum faster than it can process them
F.14 Paxos-TLA [11]

This work gives a machine-checked proof of safety of a specification of Paxos written in TLA+. This work does not discuss liveness formally.

F.15 LastVoting-PSync [20]

LastVoting is Paxos in Heard-Of (HO-) model [17]. This protocol proceeds in synchronous rounds where messages from old rounds (except the last one) are discarded. Leader selection in this protocol is intrinsic, based on the current round number \( r \), which every alive process has access to. PSync is an automatic verification tool based on HO-model and distributed algorithms written in a certain specification logic called CL:

| Specification logic |
|---------------------|
| For fault-tolerant distributed algorithms, the specification, the liveness assumptions, and the inductive invariants, require reasoning about set comprehensions, cardinality constraints, and process quantification. To express these properties and check their validity we use a fragment of first-order logic, called CL, and its semidecision procedure [21]. |

The authors discuss liveness of LastVoting as:

| Liveness |
|----------|
| In order to ensure termination LastVoting assumes that eventually there exists a sequence of four rounds, starting with Collect, where the coordinator is in the HO-set of every process, and during the Collect and Quorum rounds of this sequence the HO-set of each process contains at least \( n/2 \) processes. |

F.16 Paxos-EPR [59, 58]

This work aims at specifying Paxos in a constrained logic, EPR (Effectively Propositional Logic), similar to PSync [20, 21], and then proving its safety [59] and liveness [58] properties automatically. The authors prove liveness of Stoppable Paxos [50] under the following assumptions:
Liveness

Paxos eventually reaches a decision provided that there is a node \(l\) and a majority of nodes \(Q\) such that

- no action of \(l\) or of any node in \(Q\) can become forever enabled and never executed,
- every message sent between \(l\) and the nodes in \(Q\) is eventually delivered, and
- eventually, no node different from \(l\) tries to propose values

F.17 Paxos-Decon [24]

This work aims at specifying and verifying complex Paxos variants by (1) devising composable specifications for implementations of Paxos-Basic, and (2) engineering disciplines to reason about protocol-aware, semantics-preserving optimizations to Paxos-Basic. This work provides safety proof for Paxos-Basic and explains a method to derive Paxos from Paxos-Basic while preserving the safety property. Liveness is not discussed.

F.18 Paxos-High [47]

This work aims at giving precise high-level executable specifications of Paxos and its variants, based on lower level pseudo-code in Paxos-Complex [62]. These specifications are written in DistAlgo [48]. The authors demonstrate how writing high-level specifications led them to find liveness violations, if messages can be lost, in Paxos-Complex—an observation confirmed by the authors of Paxos-Complex. They also demonstrate how they could easily fix the violation using smart retransmission because of the high-level nature of their specifications. Despite this, liveness is neither formally nor informally described or proven.