Fuzzy Programming Approach to a Multi-Objective Fuzzy Stochastic Routing and Siting Hazardous Wastes

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ABSTRACT

The aim of the research article is not only to propose a solution procedure to solve multi-objective fuzzy stochastic programming problem by using genetic-algorithm-based fuzzy programming method, but also to apply the computational techniques for transportation of the hazardous waste materials. In this article, routing and siting problems for nuclear hazardous waste material are studied and solved. The amount of waste materials generated in the nuclear reactors follows normal distribution. The two considered objective functions are about route selection which includes minimum travel time and minimum number of houses along the way, taking the safety measures into consideration. A multi-objective fuzzy stochastic mathematical model is formulated with the above mentioned objective functions and the route selection as the constraints. The proposed solution procedure is illustrated by a numerical example and a case study.

Keywords: Fuzzy Stochastic Programming; Multi-Objective Programming; Fuzzy Programming; Genetic Algorithm

1. Introduction

Hazardous materials and wastes have created a major problem throughout the world. The planning and design of hazardous waste management system involves allocation of hazardous wastes, selection of treatment and disposal facilities, waste residues from generator to the treatment in disposal sites, and selection of the transportation routes. The shipment, dumping of hazardous materials has emerged as a critical problem for decision makers. Some of work done on it can be found in (Ardjmand et al.[1-2], Karadimas et al.[3], Samanioglu[4], Current and Ratick[5], and Giannikos[6]). Another issue associated with transporting the hazardous materials is siting problem which may increase the transportation cost if it wasn’t considered. The risk factors and the cost associated with transporting the waste materials from source to destination is of great concern for all the countries which can be seen in (List and Mirchandani[7,8], Nema and Gupta[9], and Gomez et al.[10]).

These problems are handled with different planning and objectives to meet the common goal of safety and minimization of cost and time. Different methods and approaches are being used and studied to solve the problem in order to make better and accurate decision (Warmerdam and Jacobs[11], Liu[12], Chang and Davila[13]). The problem of hazardous wastes treatment demands not only optimal solution but also precise and accurate solution. One of such approach is fuzzy programming approach which is used in various fields to obtain an optimal solution. Some of the applications where the fuzzy programming approaches are used can be found in (Bit et al.[14], Hulsurkar et al.[15], Kumar et al.[16], Liu and Kao[17]).

In order to have optimal precise solution, the im-
preciseness and uncertainty which exist in various parameters need to be handled. This can be done using both the fuzzy theory Zadeh\(^{(18)}\) and stochastic programming Charnes and Cooper\(^{(19)}\). The idea on the fuzz-ifying approach to multi-objective stochastic programming problem were developed by Mohan and Nguyen\(^{(20)}\). Recent developments in fuzzy stochastic problem can be found in (Acharya and Biswal\(^{(21)}\), Sakawa et al.\(^{(22)}\), Wang and Watada\(^{(23)}\), Mousavi et al.\(^{(24)}\), Sakawa and Matsui\(^{(25)}\), Aiche et al.\(^{(26)}\), Acharya et al.\(^{(27,28)}\), Li et al.\(^{(29)}\)).

Genetic Algorithm (GA) are based on the concept of the biological process of natural selection, and developed by Holland\(^{(30)}\). Liu\(^{(12)}\) showed that the existing chance constrained programming models for fuzzy decision systems are essentially maximin model. Also, he analysed that fuzzy simulation based genetic algorithm was suitable for the minimax model. Liu and Iwamura\(^{(11)}\) formulated fuzzy simulation based genetic algorithm for solving chance constrained programming models with fuzzy decision. A few researches have been done to handle fuzzy and stochastic using GA (Jana and Biswal\(^{(32,33)}\), Dutta et al.\(^{(34,35)}\)).

The article is organized as follows. Following the introduction, basic preliminaries are presented in Section 2. The mathematical model of multi-objective fuzzy stochastic problem is presented describing the alpha-cuts in Section 3. GA based fuzzy programming approach procedure is provided in Section 4. Solution procedure is presented in Section 5. A numerical example and a case study are provided in support of the proposed method in Section 6. Finally, conclusion is provided in Section 7.

2. Basic preliminaries

**Definition 2.1**

A fuzzy number \(\tilde{A}\) is a convex normalized fuzzy set of the real line \(\Re\), with membership function \(\mu_A(x) : \Re \rightarrow [0,1]\), satisfying the following conditions:

1. There exist unique interval J such that \(\mu_A(x) = 1; x \in J\)
2. The membership function \(\mu_A\) is piecewise continuous.

**Definition 2.2**

A fuzzy number \(\tilde{B} = (B^0, B^-\alpha, B^-\beta)\) is said to be triangular if its membership function is strictly increasing in the interval \((B^0, B^-\alpha)\) and strictly decreasing in \((B^-\alpha, B^-\beta)\) and \(\mu_B(B^-\alpha) = 1\), where \(B(m)\) is core, \((B^-\alpha, B^-\beta)\) is left spread and \((B^0, B^-\beta)\) is right spread of the fuzzy number \(\tilde{B}\).

**Definition 2.3**

\(\alpha\) cut of the fuzzy number \(\tilde{A}\) is the set \(\{x | \mu_A(x) \geq \alpha\} \cap [0,1]\) for \(0 \leq \alpha < 1\) and denoted by \(\tilde{A}(\alpha)\).

**Definition 2.4**

Let \(\bar{A} = (A^p, A^-o, A^-\delta)\) and \(\tilde{B} = (B^p, B^-\alpha, B^-\beta)\) be two fuzzy numbers with \(\alpha\) cuts \(\tilde{A}(\alpha) = [A^o, A^-\delta] \) and \(\tilde{B}(\alpha) = [B^o, B^-\beta] \) respectively, then \(\tilde{A} \leq \tilde{B}\) if and only if \(A^o \leq B^o\) and \(\bar{A} \leq \bar{B}\).

**Definition 2.5**

A fuzzy random variable is a random variable whose parameter is fuzzy number. Let \(\tilde{X}\) be continuous random variable with fuzzy parameter \(\tilde{\alpha}\) and \(\tilde{\beta}\) as fuzzy probability, then \(\tilde{X}\) is said to be continuous fuzzy random variable with density function \(f(x, \tilde{\alpha}, \tilde{\beta})\), \(\tilde{\beta}(X \leq x) = \beta\), where \(0 \leq \beta \leq 1\) : \(\beta = (\beta^p, \beta^-\alpha, \beta^-\beta)\), \(\beta^o \geq 0\) and \(\beta^p > 0\).

**Definition 2.6**

Let \(E = [c, d]\) be an event. Then the probability of the event \(E\) of continuous fuzzy random variable \(\tilde{X}\) is a fuzzy number whose \(\alpha\) cut is\(^{(39)}\):

\[
\bar{P}[c \leq \tilde{X} \leq d] = \left[ \min \left\{ \int_{-\infty}^{b} f(x, \tilde{\alpha}) dx | \tilde{\alpha} \right\} ; \int_{-\infty}^{\infty} f(x, \tilde{\alpha}) dx = 1 \right] \\
\max \left\{ \int_{-\infty}^{b} f(x, \tilde{\alpha}) dx | \tilde{\alpha} \right\} = [B^o, B^-\beta(\alpha)]
\]

**Bounded Random Number (BRN):** For any type of C compiler, the subroutine of generating pseudo random number has been given in the C library as: include `< stdlib.h >` and rand function rand () which produces an integer between 0 and RAND MAX, where RAND MAX is in # stdlib as (2\(^{15}\) - 1). Therefore, a random number on an interval [0, 1] can be generated as:

1. Step 1: \(m = \text{rand}()\)
2. Step 2: \(m \leftarrow (m \text{RAND MAX})\)

**Fuzzy Normal Distribution:** A random variables has a fuzzy normal distribution if its probability density function (pdf) is given by:

\[
f(s) = \frac{1}{\sigma \sqrt{2\pi}} e^{- \frac{(s - \mu)^2}{2\sigma^2}}, \quad -\infty < s < \infty
\]

Denote the pdf as \(\tilde{FNL}(\bar{\mu}, \sigma^2)\), where \(\bar{\mu}\) is the mean and \(\sigma^2\) is the variance

1. Step 1: Generate \(n_1\) and \(n_2\) from BRN (0, 1)
2. Step 2: \(m = [-2 \ln(n_1)]^{0.5} \sin(2\pi n_2)\)
3. Mathematical model of multi-objective fuzzy stochastic programming problem

The mathematical programming model for multi-objective fuzzy stochastic programming (MOFSP) problem is expressed as

\[
\min Z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, \ldots, K
\]  

(3.1)

Subject to

\[
\bar{P} \left( \sum_{j=1}^{m} a_{ij} x_j \leq \bar{b}_i \right) \geq (1 - \beta_i) \quad i = 1, 2, \ldots, m
\]  

(3.2)

\[
x_j \geq 0, \quad \forall j
\]  

(3.3)

where \( \beta_i \) is the fuzzy number and \( c_j^k \in \mathbb{R}, \forall j, k \).

Depending on the different distribution of the random variables, following cases are considered in this article.

Case 1: Let \( b_i, i = 1, 2, \ldots, m \) are independent FRVs distributed normally. Let the FRVs \( b_i \) be denoted as \( \bar{b}_i \).

The \( \alpha \)-cut of the probabilistic constrained defined above can be expressed as:

\[
\bar{P} \left( \sum_{j=1}^{m} a_{ij} x_j \leq \bar{b}_i \right)[\alpha], i = 1, 2, \ldots, m
\]  

(3.4)

\[
\mathcal{P}(A_i \leq b_i)[b_i \in \bar{b}_i[\alpha]], where A_i = \sum_{j=1}^{m} a_{ij} x_j
\]  

(3.5)

Using fuzzy inequality, the \( \alpha \)-cut of the fuzzy constraints (3.4 - 3.5) is expressed as:

\[
\bar{P} \left( \sum_{j=1}^{m} a_{ij} x_j \leq \bar{b}_i \right)[\alpha] \geq (1 - \beta_i[\alpha]) = \mathcal{P}(A_i \leq b_i) \geq (1 - \beta_i[\alpha])
\]  

(3.6)

where \( [b_i[\alpha], \beta_i[\alpha]] \in \bar{b}_i[\alpha] \) and \( [\beta_i[\alpha], \beta_i'[\alpha]] \in \bar{b}_i[\alpha] \)

Case 2: Let \( \bar{b}_i, i = 1, 2, \ldots, m \) are independent FRVs distributed Weibull. Let the FRVs \( b_i \) be denoted as \( \bar{b}_i \).

The \( \alpha \)-cut of the probabilistic constrained defined above can be expressed as:

\[
\bar{P} \left( \sum_{j=1}^{n} a_{ij} x_j \leq \bar{b}_i \right)[\alpha], i
\]  

(3.7)

\[
\mathcal{P}(A_i \leq b_i)[b_i \in \bar{b}_i[\alpha]], where A_i = \sum_{j=1}^{n} a_{ij} x_j
\]  

(3.8)

Using fuzzy inequality, the \( \alpha \)-cut of the fuzzy constraints (3.7 - 3.8) is expressed as:

\[
\bar{P} \left( \sum_{j=1}^{n} a_{ij} x_j \leq \bar{b}_i \right)[\alpha] \geq (1 - \beta_i[\alpha]) = \mathcal{P}(A_i \leq b_i) \geq (1 - \beta_i[\alpha])
\]  

(3.9)

where \( [b_i[\alpha], \beta_i[\alpha]] \in \bar{b}_i[\alpha] \) and \( [\beta_i[\alpha], \beta_i'[\alpha]] \in \bar{b}_i[\alpha] \)

4. Fuzzy programming approach simulation based GA

The fuzzy programming simulation-based GA is design to solve the fuzzy probabilistic programming problems. A fuzzy stochastic simulation-based GA algorithm is described in Dutta et al.\[35\]. We describe the steps of the algorithm as follows:
Algorithm: Fuzzy Programming GA

\[ P = (x_{i1}, x_{i2}, \ldots, x_{in}), \ n \in N \] - Initial Population
\[ D = (r_{11}, r_{12}, \ldots, r_{in}), \ n \in N \] - Distribution Parameter

\[
gen = \text{generation} \\
x_{in} = \text{decision variables} \ n \in N \\
i_e = \text{Lower bound} \ n \in N \\
u_i = \text{Upper bound} \ n \in N \\
C_i = \text{Constraints} \ n \in N \\
x_{in}, x_{jm} = \text{New Child} \ n,m \in N \\
x_{m} = \text{Mutated Child} \ n \in N \\
x_{m} = \text{Best Solution} \ n \in N \\
\text{max-gen} = \text{maximum generation}
\]

Begin

\[
\text{generate } D \\
// \text{Generating Distribution Parameter}
\text{init } P \\
// \text{Initializing Population for each objective function}
\text{gen} \leftarrow 0 \\
i_e \leq x_{in} \leq u_i \\
// \text{Applying bounds separately for each objective function}
\]

\[
\text{while } (\text{gen} \leq \text{max-gen}) \text{ do} \\
\left[ C_i x_{in} - r_{in} \right] \alpha \\
// \text{Applying } \_ \text{-cut separately for each objective function}
\text{x}_{in} \leftarrow \text{select best} \\
// \text{Applying Selection separately for each objective function}
\text{x}_{in}, x_{jm} \rightarrow \text{x}^1_{in}, x^1_{jm} \\
// \text{Crossover}
\text{x}_{in} \rightarrow \text{x}_{in}' \\
// \text{Mutation}
\text{Evaluate Fi} \\
// \text{Function value of each objective function separately}
\text{Pr}(C_i x_{in} - r_{in}) \\
// \text{Probability Criteria}
\text{if} (\text{Probability Criteria is satisfied}) \text{ then}
\text{Elitism}
\text{else}
\text{goto init}
\text{gen} \rightarrow \text{gen} + 1
\text{end if}
\]

\[
x^*_{in} \\
// \text{Ideal Solution for each objective function separately}
\]
End

The Flow Diagram of Fuzzy Stochastic GA is shown in Figure 1.

4.1 Representation and initialization

A population of potential solution \( x_1, x_2, \ldots, x_n \) is generated and initialized. Each solution is called chromosomes can be represented as \( X_p = (x_1, x_2, \ldots, x_n)_p \), where \( p = 1, 2, \ldots, p_{\text{size}} \) and \( p_{\text{size}} \) being the size of the population.

4.2 Checking constraints by the fuzzy simulation

The constraints of the model are represented as fuzzy probabilistic constraints. Consider the fuzzy probabilistic constraints

\[
\bar{P} \left( \sum_{j=1}^{n} a_{ij} x_j \leq b_i \right) \alpha \geq (1 - \beta_i \alpha), \ i = 1, 2, \ldots, m
\]  \hspace{1cm} \text{(4.1)}

We defuzzifying the constraints using the \( \alpha \)-cut and inequality conditions, so that the constraint reduces to

\[
\{ P(A_i \leq b_{i\alpha}) \geq (1 - \beta_i \alpha), j = 1, 2, \ldots, n \}
\]  \hspace{1cm} \text{(4.2)}

The above inequality can be represented by

\[
\{ P(A_i \leq b_{i\alpha}) \geq (1 - \beta_i \alpha), j = 1, 2, \ldots, n \}
\]  \hspace{1cm} \text{(4.3)}

\[
P(s_i(a_{ij} x_j s_i) \leq 0) \geq (1 - \beta_i \alpha), j = 1, 2, \ldots, n
\]  \hspace{1cm} \text{(4.4)}

where \( s_i = b_{i\alpha}, i = 1, 2, \ldots, m \); \( x_j = (x_1, x_2, \ldots, x_n) \) is the decision variables.

Let \( N_{ij} \leq N \), \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \) be the number of times the following relation are satisfied:

\[
s_i(a_{ij} x_j) \leq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n
\]

And \( s_i(a_{ij} x_j) \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \), where \( N \) is the initial population. Then, by the definition of probability, (3.2) hold, if \( \frac{N_{ij}}{N} \geq (1 - \beta_i) i = 1, 2, \ldots, m; j = 1, 2, \ldots, n 

4.3 Fitness

The value of objective function is called fitness value if the given constraints are satisfied.
4.4 Selection

Selection operator selects the best chromosomes depending on the fitness values. In this article, we choose Binary Tournament Selection (BTS) for selection process.

4.5 Crossover

Probability of crossover (pc) is assigned, and a random number is generated within [0, 1] for each pair of chromosomes. If the random number is less than (pc), then the chromosomes are selected for mating.

4.6 Mutation

Probability of mutation (pm) is assigned, and a random number is generated within (0, 1) for each chromosome. If the random number is less than (pm), the chromosome will undergo mutation.

4.7 Termination

When number of iterations becomes equal to the generation number as defined, the execution will be stopped.

5. Solution procedure

Using GA based fuzzy programming approach, we find out the ideal solution for each objective function in the following way:

Step 1: Initialize the population $P_1(t)$, $P_2(t)$, ..., $P_n(t)$, for each objective function ($Z_1(x)$, $Z_2(x)$, ..., $Z_n(x)$) separately, keeping the constraints same for all the objective functions.

Step 2: Apply fuzzy stochastic GA based approach to find the ideal solution for each objective function.

(See Figure 1)

Step 3: Construct a pay-off matrix containing the ideal solution and functional values.

Step 4: From the constructed pay-off matrix, determine the bounds for $m$-th objective function $Z_m(x)$; $m = 1, 2, ..., n$. As there are only two possible choices for the bound, either a lower bound or an upper bound. Find the best lower bound $L_m^*$ and worst upper bound $U_m^*$ in case an objective function being of minimization type and in case of maximization type finding the best upper bound $U_m^*$ and worst lower bound $L_m^*$; $m = 1, 2, ..., n$.

Step 5: Define fuzzy membership function $\mu_{zm}$ for the $m$-th objective function $Z_m(x)$ as:

$$
\begin{align*}
1, & \quad \text{if } Z_m(x) \geq U_m^*, \quad m = 1, 2, ..., n \\
& \quad \frac{Z_m(x) - L_m^*}{U_m^* - L_m^*}, \quad \text{if } L_m^* < Z_m(x) < U_m^* \\
& \quad 0, \quad \text{if } Z_m(x) \leq L_m^* 
\end{align*}
$$

(5.1)

Or

$$
\begin{align*}
1, & \quad \text{if } Z_m(x) \geq L_m^*, \quad m = 1, 2, ..., n \\
& \quad \frac{U_m^* - Z_m(x)}{U_m^* - L_m^*}, \quad \text{if } L_m^* < Z_m(x) < U_m^* \\
& \quad 0, \quad \text{if } Z_m(x) \geq U_m^*
\end{align*}
$$

(5.2)

| $X_1(x)$ | $Z_1(x)$ | $Z_2(x)$ | $Z_n(x)$ |
|----------|----------|----------|----------|
| $Z_{11}$ |
| $Z_{12}$ | $Z_{1n}$ |
| $Z_{21}$ |
| $Z_{22}$ | $Z_{2n}$ |
| $...$ |
| $...$ |
| $Z_{n1}$ |
| $Z_{n2}$ | $Z_{nn}$ |

Table 1. Pay-off matrix
Step 6: (i) Formulate a single objective mathematical programming problem using maxmin operator with augmented variable $\lambda$

\[
\begin{align*}
\text{max:} & \quad \lambda \\
\text{Subject to} & \quad \lambda \leq \mu z_p(x), p = 1, 2, 3, \ldots, R \\
& \quad x \in S
\end{align*}
\]

where $S$ is the feasible region of the MOFSP model.

(ii) Similarly, using min-max operator with augmented variable $\lambda$, and formulating a single objective mathematics programming problem as:

\[
\begin{align*}
\text{min:} & \quad \lambda \\
\text{Subject to} & \quad \lambda \geq \mu z_p(x), p = 1, 2, 3, \ldots, R \\
& \quad x \in S
\end{align*}
\]

Step 7: Finally, solve the single objective mathematical programming problem (5.3) - (5.5) or (5.6) - (5.8) using GA-based fuzzy stochastic approach to obtain the Pareto solutions.

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**Figure 1.** Flow diagram of fuzzy stochastic fuzzy programming approach GA.
6. Numerical example
Consider the following example:
\[
\begin{align*}
\max & = 5x_1 + 7x_2 \\
\max & = 10x_1 - 2x_2 \\
\max & = 9x_1 + 8x_2 \\
\text{Subject to} & \\
\bar{p}(2x_1 + x_2 \leq \bar{b}_1) & \geq 0.7 \\
\bar{p}(x_1 + 2x_2 \leq \bar{b}_2) & \geq 0.60 \\
2x_1 + 2x_2 & \leq 4 \\
x_1, x_2 & > 0
\end{align*}
\]
where \(\bar{b}_1\) and \(\bar{b}_2\) follows Weibull distribution with
\[
\bar{F}_W(\bar{b}_1, \bar{y}_1) = \bar{F}_W(\bar{4}, \bar{2}), \quad \bar{F}_W(\bar{b}_2, \bar{y}_2) = \bar{F}_W(\bar{5}, \bar{2}) \quad \text{where} \quad \bar{2} = (2/3/4); \quad \bar{4} = (4/5/6); \quad \bar{5} = (5/6/7) \text{are fuzzy triangular numbers respectively.}
\]
After applying the steps of fuzzy programming approach, we obtain three ideal solutions as \(X_1(x) = (0.90458; 1.08882); \quad X_2(x) = (1.09666; 0.06076); \quad X_3(x) = (1.0709; 0.9253); \quad Z_3(x) = 17.0405.

Use max-min operator with augmented \(\lambda\) the
\[
\bar{p}(x_1 + 2x_2 \leq \bar{b}_2) \geq 0.60
\]
MOFSP reduces to MOFSGP problem
\[
\min: \lambda \\
\text{Subject to} \\
5x_1 + 7x_2 + 6.23628 \geq 12.1446 \\
10x_1 - 2x_2 + \lambda 3.97694 \geq 10.8451 \\
9x_1 + 8x_2 + 6.68448 \geq 17.0405 \\
\bar{p}(2x_1 + x_2 \leq \bar{b}_1) \geq 0.70
\]

The proposed GA based fuzzy programming approach is coded in C++ in VB2010 professional. The population size is taken as 100. Optimum solution obtained for different values of probability of crossover, mutation and \(\alpha = 0.5\) over 100 generation is presented.

| pc | pm | Optimum Solution \(X = (x_1, x_2)\) | \(Z = (Z_1, Z_2, Z_3)\) | \(\lambda\) |
|----|----|---------------------------------|---------------------|-----|
| 0.6 | 0.01 | 1.0481, 0.4921 | 8.6852, 9.4968, 13.3697 | 0.9 |
| 0.6 | 0.03 | 1.0614, 0.5415 | 9.0975, 9.531, 13.8846 | 0.5 |
| 0.6 | 0.05 | 1.05855, 0.7448 | 10.5064, 9.0959, 15.4854 | 0.9 |
| 0.6 | 0.08 | 1.0481, 0.60515 | 9.47655, 9.2707, 14.2741 | 0.9 |
| 0.7 | 0.01 | 1.06045, 0.8702 | 11.3937, 8.8641, 16.5056 | 0.5 |
| 0.7 | 0.03 | 1.0519, 0.76 | 10.5795, 8.999, 15.5471 | 0.5 |
| 0.7 | 0.05 | 1.0405, 0.88255 | 11.3804, 8.6399, 16.4249 | 0.6 |
| 0.7 | 0.08 | 1.06995, 0.68685 | 10.1577, 9.3258, 15.1244 | 0.9 |
| 0.8 | 0.01 | 1.03385, 0.6099 | 9.43855, 9.1187, 14.1839 | 0.8 |
| 0.8 | 0.03 | 1.06805, 0.4522 | 8.5065, 9.7761, 13.23 | 0.8 |
| 0.8 | 0.05 | 1.05285, 0.84265 | 11.1628, 8.8432, 16.2169 | 0.6 |
| 0.8 | 0.08 | 1.0614, 0.89395 | 11.5647, 8.8261, 16.6 | 0.6 |
6.1 Case study

The need for the energy in the world is kept on increasing, and one of the major sources of energy is nuclear power. As a result of using nuclear as a fuel to generate energy, the waste products generated in the reactor needs to be handled and disposed very carefully, as they are hazardous to the environment and mankind. To dispose the nuclear waste in different sites away from the reactor requires shipment across a transportation network, which has become one of the major problems. To transport the nuclear waste material from the reactor to different sites, one has to look into several factors, such as environment issues, accident prone area, time taken to reach the sites, distance covered from reactor to different sites, population along the road or rail through which waste will be transported, and cost of transportation.

This problem was studied by ReVelle et al.\cite{40}. Here, in this case study, we have considered minimum time required to reach different sites, and minimum number of houses along the different routes as the objective functions. One has to consider which site is open for disposal among the different available sites, and which route could be most appropriate at the same time. The two objective functions are conflicting in nature as time taken is more where number of houses are less, and vice versa. If the two objectives are not conflicting in nature then it would have become a simple multi-objective problem. The amount of the waste products generated follows fuzzy normal distribution. The pictorial representation and the information for constructing the model are given below.

| Source | Route 1/Site 1 | Route 1/Site 2 | Route 2/Site 1 | Route 2/Site 2 |
|--------|---------------|---------------|---------------|---------------|
| 1      | Time(hours)   | 5             | 7             | 13            | 18            |
|        | No. of Houses | 900           | 400           | 3000          | 2000          |
| 2      | Time(hours)   | 8             | 9             | 12            | 15            |
The mathematical model of the case study can be expressed as follows:

Min: $Z_1 = 5x_{111} + 7x_{112} + 13x_{121} + 18x_{122} + 8x_{211} + 9x_{212} + 12x_{221} + 15x_{222} + 16x_{311} + 18x_{312} + 10x_{321} + 13x_{322}$

(6.16)

Min: $Z_2 = 900x_{111} + 400x_{112} + 3000x_{121} + 2000x_{122} + 1800x_{212} + 2500x_{221} + 1000x_{222} + 3000x_{311} + 1700x_{312} + 1500x_{321} + 800x_{322}$

(6.17)

Subject to

$\bar{P}(x_{111} + x_{112} + x_{121} + x_{122} \leq b_1) \geq 0.75$

(6.18)

$\bar{P}(x_{211} + x_{212} + x_{221} + x_{222} \leq b_2) \geq 0.85$

(6.19)

$\bar{P}(x_{311} + x_{312} + x_{321} + x_{322} \leq b_3) \geq 0.70$

(6.20)

Using min-max operator with augmented $\lambda$ the MOFSP reduces to MOFSGP

$m\text{in}: \lambda$

Subject to

$5x_{111} + 7x_{112} + 13x_{121} + 18x_{122} + 8x_{211} + 9x_{212} + 12x_{221} + 15x_{222} + 16x_{311} + 18x_{312} + 10x_{321} + 13x_{322} + 0.645\lambda \geq 35.065$

(6.24)

$900x_{111} + 400x_{112} + 3000x_{121} + 2000x_{122} + 1800x_{212} + 2500x_{221} + 1000x_{222} + 3000x_{311} + 1700x_{312} + 1500x_{321} + 800x_{322} + 2097\lambda \geq 6914.185$

(6.25)

$\bar{P}(x_{111} + x_{112} + x_{121} + x_{122} \leq b_1) \geq 0.75$

(6.26)

The proposed GA based fuzzy programming approach is coded in C++ in VB2010 professional. The population size is taken as 100. Optimum solution Table (6.14) obtained for different values of probability of crossover (pc), mutation (pm) and $\alpha$ over 100 generations are presented.

| No. of Houses | 1800 | 800 | 2500 | 1000 |
|--------------|------|-----|------|------|
| Time(hours)  | 6    | 18  | 10   | 13   |
| No. of Houses| 3000 | 1700| 1500 | 800  |

Table 4. Model information

| c            | 34.4201 | 6914.185 |
|--------------|---------|----------|
| $X_2$        | 35.065  | 4817.165 |

Table 5. The pay-off matrix
| pc | pm  | Optimum Solution X= | Z= Z_1 Z_2 | λ | Sites (1 or 2) |
|----|-----|----------------------|-------------|---|----------------|
| 0.6| 0.01| 1.1725, 0.265, 0.749, 0.2133, 0.5584, 0.1282, 0.2908, 0.3076, 0.1451, 0.5796, 0.5356, 0.6214. | 61.2071 8698.27 | 0.1 | 0 1 1 |
| 0.6| 0.03| 0.1803, 0.0363, 0.21, 0.1583, 0.1822, 0.208, 0.4912, 0.2266, 0.9943, 0.1825, 0.4113, 0.1649 | 56.9915 7126.55 | 0.3 | 1 0 1 |
| 0.6| 0.05| 0.3684, 0.0418, 0.5224, 0.7017, 0.3178, 0.6922, 0.2032, 0.106, 0.1836, 0.1429, 0.7556, 0.3057 | 51.3969 7230.37 | 0.2 | 0 1 0 |
| 0.6| 0.08| 1.1032, 0.4003, 0.2419, 0.1539, 0.6142, 0.1882, 0.5956, 0.2164, 0.3926, 0.155, 0.3123, 0.4091 | 48.7465 7385.05 | 0.5 | 1 0 0 |
| 0.7| 0.01| 1.1274, 0.0979, 0.1154, 0.65, 0.6466, 0.1942, 0.121, 0.2098, 0.4003, 0.2056, 0.1275, 0.2551 | 45.739 6477.31 | 0.3 | 1 0 1 |
| 0.7| 0.03| 0.375, 0.3376, 1.1087, 0.1616, 0.145, 0.2758, 0.4114, 0.2776, 0.2881, 0.3453, 0.3211, 0.2474 | 51.5553 8040.46 | 0.5 | 1 0 1 |
| 0.7| 0.05| 0.1847, 0.0583, 0.3717, 0.5928, 0.2872, 0.5542, 0.325, 0.3748, 0.4641, 0.3464, 0.3035, 0.2265 | 53.2818 7255.5 | 0.7 | 1 0 1 |
| 0.7| 0.08| 0.4421, 0.6061, 0.2419, 0.419, 0.3142, 0.3178, 0.2914, 0.1042, 0.6137, 0.4311, 0.2111, 0.1044 | 51.5756 6833.62 | 0.6 | 1 0 0 |
| 0.8| 0.01| 0.3112, 0.2298, 0.1891, 0.386, 0.1216, 0.349, 0.3106, 0.1234, 0.5444, 0.1528, 0.7413, 0.8348. | 51.9891 6782.03 | 0.1 | 1 0 0 |
| 0.8| 0.03| 0.606, 0.7216, 0.1198, 0.4443, 0.1756, 0.466, 0.1618, 0.1156, 0.6962, 0.3409, 0.2936, 0.4674 | 53.198 6773.47 | 0.2 | 0 1 0 |
| 0.8| 0.05| 0.2122, 0.8382, 0.3101, 0.3618, 0.5932, 0.3946, 0.5626, 0.1846, 0.3332, 0.4982, 0.1814 | 0.7 | 0 1 0 |

10
| pc  | pm  | Optimum Solution X=  |
|-----|-----|-----------------------|
|     |     | x₁, x₂, ..., x₁₂      |
|     |     | Z = Z₁, Z₂             |
|     |     | λ                      |
|     |     | Sites (1 or 2)          |
| 0.8 | 0.08| 0.2045, 0.4982, 0.1814 | 47.2436 5823.34 | 0.7 | 0 1 1 |
|     |     | 0.7677, 0.253, 0.1473, | 46.9333 6863.1 | 0.9 | 0 1 0 |
|     |     | 0.3156, 0.2698, 0.2914, |               |     | 1 0 1 |
|     |     | 0.5416, 0.1486, 0.2232, |               |     |     |
|     |     | 0.2771, 0.1649, 0.43    |               |     |     |
| 0.9 | 0.01| 0.8                     |
|     |     | 0.7677, 0.253, 0.1473,  |
|     |     | 0.3156, 0.2698, 0.2914,  |
|     |     | 0.5416, 0.1486, 0.2232,  |
|     |     | 0.2771, 0.1649, 0.43    |
| 0.9 | 0.03| 0.2859, 0.5246, 0.1165,  |
|     |     | 0.3739, 0.6892, 0.5686,  |
|     |     | 0.3502, 0.652, 0.3981,  |
|     |     | 0.1451, 0.1374, 0.6269   |
| 0.9 | 0.05| 0.5785, 0.2321, 0.3277,  |
|     |     | 0.2958, 0.2692, 0.3334,  |
|     |     | 0.2584, 0.436, 0.1847,   |
|     |     | 0.2628, 0.7677, 0.2243   |
| 0.9 | 0.08| 0.3959, 0.3883, 0.1616,  |
|     |     | 0.1836, 0.3688, 0.1174,  |
|     |     | 0.5776, 0.6406, 0.3387,  |
|     |     | 0.3431, 0.5323, 0.2386   |
| 0.6 | 0.01| 0.9558, 0.066, 0.1374,  |
|     |     | 0.2089, 0.349, 0.2542,  |
|     |     | 0.154, 0.61, 0.1814,     |
|     |     | 0.5466, 0.672, 0.3211    |
| 0.6 | 0.03| 0.573, 0.0099, 0.397,    |
|     |     | 0.1506, 0.5812, 0.5002,  |
|     |     | 0.5626, 0.1558, 0.1099,  |
|     |     | 0.1671, 0.9613, 0.3145   |
| 0.6 | 0.05| 0.1913, 0.0264, 0.21,    |
|     |     | 0.1154, 0.505, 0.2236,   |
|     |     | 0.6946, 0.2134, 0.3013,  |
|     |     | 0.2892, 0.1352, 0.8029   |
| 0.6 | 0.08| 0.4454, 0.4378, 0.1154,  |
|     |     | 0.3937, 0.2008, 0.2686,  |
|     |     | 0.1462, 0.1576, 0.1869,  |
|     |     | 1.1186, 0.4905, 0.3156   |
| 0.7 | 0.01| 0.2782, 1.1417, 0.1968,  |
|     |     | 0.2463, 0.1504, 0.3052,  |
|     |     | 0.1528, 0.2932, 0.5312,  |
|     |     | 0.4322, 0.3013, 1.1351   |

Table 6. α = 0.1
| α   | 0.03 | 0.848, 0.1683, 0.1077, 0.9503, 0.5326, 0.4894, 0.1852, 0.2566, 0.1363, 0.1066, 0.2749, 0.3222 | 0.496976 6384.25 | 0.4 | 1 0 1 0 1 0 |
|-----|------|-----------------------------------------------------------------------------------------------|------------------|-----|-----------|
| α   | 0.05 | 0.2837, 0.0055, 0.8359, 0.1066, 0.6358, 0.373, 0.2764, 0.1654, 0.2254, 0.1011, 0.6874, 0.3926 | 0.458877 7470.92 | 0.1 | 0 1 0 1 0 |
| α   | 0.08 | 0.7512, 0.4411, 0.2155, 0.144, 0.2356, 0.1822, 0.433, 0.172, 0.2188, 0.5521, 0.3222, 0.2254 | 0.564621 5883.28 | 0.5 | 1 0 1 0 0 |
| α   | 0.01 | 0.32, 0.3894, 0.4322, 0.1539, 0.3574, 0.3796, 0.2962, 0.3502, 0.7644, 0.2177, 0.2628, 0.1176 | 0.481034 7237.43 | 0.7 | 1 0 1 0 0 |
| α   | 0.03 | 0.8777, 0.0011, 0.1583, 0.1616, 0.5572, 0.3028, 0.1954, 0.6142, 0.2837, 0.1847, 0.4069, 0.2958 | 0.479282 5952.5 | 0.6 | 1 0 1 0 0 |
| α   | 0.05 | 0.1902, 0.5225, 0.1473, 0.2892, 0.1846, 0.2122, 0.5488, 0.6772, 0.2045, 0.3343, 1.1285, 0.1044 | 0.537908 6909.8 | 0.3 | 1 0 1 0 0 |
| α   | 0.08 | 0.5565, 0.3564, 0.3871, 0.1022, 0.2038, 0.2746, 0.4528, 0.1144, 0.3112, 0.4003, 1.0449, 0.1022 | 0.507133 7108.6 | 0.6 | 1 0 0 1 1 |
| α   | 0.01 | 0.3277, 0.1936, 0.1209, 0.2254, 0.7024, 0.5158, 0.5968, 0.2488, 0.3937, 0.595, 0.2177, 0.3849 | 0.560704 7432.8 | 0.5 | 1 0 1 0 0 |
| α   | 0.03 | 0.2837, 0.8745, 0.265, 0.177, 0.1564, 0.3082, 0.1, 0.2638, 0.5202, 0.7127, 0.1847, 0.5026 | 0.528856 6247.33 | 0.7 | 1 0 1 0 0 |
| α   | 0.05 | 0.9514, 0.3817, 0.1737, 0.3101, 0.313 0.4342, 0.2746, 0.517, 0.3794, 0.177, 0.8678, 0.1231 | 0.536695 7105.18 | 0.2 | 1 0 1 0 0 |
| α   | 0.08 | 0.3739, 0.4113, 0.2386, 0.1649, 0.3334, 0.3706, 0.3658, 0.1936, 0.133, 0.2199, 0.6137, 0.386 | 0.46082 5558.24 | 0.7 | 0 1 1 0 0 |

Table 7. $\alpha = 0.2$
Table 8. \( \alpha = 0.3 \)

| pc | pm | Optimum Solution \( X = x_1, x_2, \ldots, x_{12} \) | \( Z = Z_1, Z_2 \) | \( \lambda \) | Sites (1 or 2) |
|----|----|-----------------------------------------------|-----------------|-----|----------------|
| 0.6 | 0.01 | 0.1308, 0.4455, 0.1253, 0.1132, 0.208, 0.184, 0.2836, 0.1582, 0.9448, 0.3156, 0.5015, 0.1099 | 43.7765 6498.11 | 0.8 | 1 0 1 0 1 0 |
| 0.6 | 0.03 | 0.3464, 0.088, 0.6808, 0.2166, 0.3298, 0.2014, 0.1288, 0.3688, 0.2155, 0.6566, 0.32, 0.6291 | 53.2709 7014.12 | 0.0 | 0 1 0 1 0 1 |
| 0.6 | 0.05 | 1.0526, 0.0858, 0.2254, 0.1198, 0.1012, 0.5056, 0.3382, 0.1726, 0.2155, 1.1593, 0.2848, 0.2716 | 56.0141 6766.35 | 0.2 | 0 1 0 1 0 1 |
| 0.6 | 0.08 | 0.3321, 0.0968, 0.8942, 0.2078, 0.4282, 0.3742, 0.1642, 0.2644, 0.3651, 0.5092, 0.3508, 0.1319 | 50.4198 7751.89 | 0.2 | 0 1 0 1 0 1 |
| 0.7 | 0.01 | 0.177, 0.5357, 0.3046, 0.1363, 0.586, 0.1804, 0.2488, 0.2254, 0.2309, 0.7479, 0.5158, 0.5334 | 52.9751 6771.05 | 0.7 | 1 0 1 0 1 0 |
| 0.7 | 0.03 | 0.6599, 0.4378, 0.2012, 0.3739, 0.5782, 0.3754, 0.2986, 0.2128, 0.2606, 0.5114, 0.2694, 0.1077 | 47.9582 6562.25 | 0.2 | 0 1 0 1 0 1 |
| 0.7 | 0.05 | 0.3992, 0.1782, 0.188, 0.3376, 0.1768, 0.1882, 0.6106, 0.4648, 0.6027, 0.5092, 0.3508, 0.1319 | 55.4298 7297.05 | 0.3 | 0 1 0 1 0 1 |
| 0.7 | 0.08 | 0.2672, 0.5158, 0.4806, 0.155, 0.6814, 0.4438, 0.409, 0.1288, 0.3849, 0.2012, 0.1418, 0.6049 | 49.3315 7124.82 | 0.7 | 0 1 0 1 1 0 |
| 0.8 | 0.01 | 0.4245, 0.0561, 1.0834, 0.1253, 0.1954, 0.2038, 0.112, 0.538, 0.3178, 0.3607, 0.1594, 0.7336 | 54.3744 7630.62 | 0.2 | 0 1 0 1 0 1 |
| 0.8 | 0.03 | 0.6412, 0.5665, 0.6995, 0.177, 0.1102, 0.4582, 0.4102, 0.1522, 0.4454, 0.5301, 0.1088, 0.3057 | 53.3921 7643.93 | 0.1 | 0 1 0 1 0 1 |
| 0.8 | 0.05 | 0.947, 0.4301, 0.1176, 0.2056, 0.2404, 0.1582, 0.4258, 0.1276, 0.2804, 0.2045, 0.6225, 0.3541 | 45.1457 5948.4 | 0.8 | 1 0 0 1 0 1 |
| 0.8 | 0.08 | 0.8018, 0.8338, 0.5015, 0.6258, 0.3118, 0.2992, 0.637, 0.1378, 0.1517, 0.1297, 0.2452, 0.1011 | 51.0558 7466.41 | 0.8 | 0 1 0 1 0 1 |
| 0.9 | 0.01 | 0.661, 1.0043, 0.1891, 0.2518, 0.5632, 0.4132, 0.3334, 0.6826, 0.2441, 0.1319, 0.122, 0.859 | 58.4568 6754.67 | 0.3 | 0 1 0 1 0 1 |
| 0.9 | 0.03 | 0.9327, 0.9394, 0.1737, 0.419, 0.4468, 0.4942, 0.2392, 0.2662, 0.3035, 0.3002, 0.1154, 0.2188 | 50.183 6407.07 | 0.8 | 1 0 1 0 1 0 |
| 0.9 | 0.05 | 0.5598, 0.1903, 0.3959, 0.1407, 0.4108, 0.7006, 0.2332, 0.5506, 0.1022, 0.353, 0.2936, 0.771 | 53.4078 6446.46 | 0.6 | 1 0 1 0 1 |
| 0.9 | 0.08 | 0.6786, 0.6193, 0.331, 0.1792, 0.3754, 0.1924, 0.2362, 0.1144, 0.2837, 0.6214, 0.8293, 0.7226 | 57.9531 7473.91 | 0.1 | 0 1 0 1 0 1 |
| 0.6  | 0.05 | 0.4784, 0.1562, 0.7215, 0.2078, 0.1768, 0.4132, 0.349, 0.163, 0.4498, 0.1242, 0.1044, 0.2309 | 41.8496 6659.3 | 0.7 | 1 0 1 0 1 0 |
| 0.6  | 0.08 | 0.3618, 0.7029, 0.441, 0.1286, 0.544, 0.187, 0.397, 0.2692, 0.2232, 0.5785, 0.4465, 0.5257 | 54.8974 7320.84 | 0.3 | 1 0 1 0 1 0 |
| 0.7  | 0.01 | 0.3585, 0.0671, 0.3662, 0.111, 0.2284, 0.3496, 0.4732, 0.1414, 0.2012, 0.7215, 0.3167, 0.166 | 50.214 6130.18 | 0.9 | 0 1 0 1 1 0 |
| 0.7  | 0.03 | 0.452, 0.7689, 0.3332, 0.155, 0.1618, 0.5398, 0.5278, 0.4162, 0.43, 0.2122, 0.4498, 0.1957 | 51.2348 6964.74 | 0.3 | 0 1 0 1 0 1 |
| 0.7  | 0.05 | 1.1549, 0.5863, 0.1099, 0.7919, 0.6982, 0.2278, 0.2308, 0.6472, 0.1033, 0.188, 0.1924, 0.7446 | 62.3155 7364.41 | 0.5 | 1 0 1 0 1 0 |
| 0.7  | 0.08 | 0.1473, 0.0484, 0.5345, 0.4971, 0.3454, 0.289, 0.163, 0.2062, 0.1132, 0.6742, 0.1583, 0.8667 | 54.1817 6632.8 | 0.7 | 0 1 0 1 0 1 |
| 0.8  | 0.01 | 0.2232, 0.0385, 0.1264, 0.2727, 0.1168, 0.6154, 0.6868, 0.2548, 0.5345, 0.6368, 0.2628, 0.5785 | 58.785 7360.45 | 0.4 | 1 0 1 0 1 0 |
| 0.8  | 0.03 | 0.4487, 0.3332, 0.1055, 0.1231, 0.6286, 0.5614, 0.1378, 0.5536, 0.3871, 0.122, 0.1803, 1.0141 | 51.5781 6028.94 | 0.5 | 1 0 1 0 0 1 |
| 0.8  | 0.05 | 0.9943, 0.0814, 0.419, 0.2826, 0.1258, 0.3934, 0.148, 0.304, 0.3508, 0.1275, 0.3662, 0.4124 | 52.759 6122.03 | 0.9 | 1 0 1 0 0 1 |
| 0.8  | 0.08 | 0.2617, 1.0086, 0.3134, 0.3926, 0.13, 0.2158, 0.2686, 0.238, 0.3211, 0.5499, 0.6467, 0.1099 | 52.2166 6636.61 | 0.4 | 0 1 0 1 1 0 |
| 0.9  | 0.01 | 0.3827, 0.3938, 0.3893, 0.1902, 0.1576, 0.412, 0.1786, 0.2458, 0.3937, 0.9976, 0.9734, 0.4619 | 63.9483 8062.47 | 0.4 | 1 0 1 0 1 0 |
| 0.9  | 0.03 | 0.265, 0.4487, 0.5257, 53.1761 7035.98 | 0 1 0 1 |
| pc | pm | Optimum Solution | Z = Z₁, Z₂ | λ | Sites (1 or 2) |
|----|----|------------------|-------------|---|---------------|
| 0.6 | 0.01 | X=x₁, x₂, x₁₂ | 0.1319, 0.1564, 0.2287, 0.10053 | 0.7 | 1 0 1 0 1 |
| 0.6 | 0.03 | | 0.2584, 0.4246, 0.3233, 0.2331 | 0.7 | 1 0 1 0 1 |
| 0.6 | 0.05 | | 0.1286, 0.1825, 0.7303 | 0.8 | 1 0 1 0 1 |
| 0.6 | 0.08 | | 0.5048, 0.3304, 0.1286, 0.2936 | 0.1 | 1 0 1 0 1 |
| 0.7 | 0.01 | | 0.5675, 0.4798, 0.1011, 0.2936 | 0.4 | 1 0 1 0 1 |
| 0.7 | 0.03 | | 0.133, 0.2932, 0.1264, 0.3761 | 0.6 | 1 0 1 0 1 |
| 0.7 | 0.05 | | 0.3046, 0.4528, 0.1209, 0.4432 | 0.6 | 1 0 1 0 1 |
| 0.7 | 0.08 | | 0.1638, 0.4624, 0.7611 | 0.9 | 1 0 1 0 1 |
| pc  | pm  | X= x₁, x₂, ..., x₁₂ | Solution | Z= Z₁, Z₂ | λ  | Sites (1 or 2) |
|-----|-----|---------------------|----------|------------|----|----------------|
| 0.8 | 0.01| 0.6665, 0.4235, 0.1374, 0.5004, 0.2554, 0.2902, 0.1042, 0.226, 0.1495, 0.2133, 0.8084, 0.6698 | 58.3371 5929.11 | 0.9 | 0 0 1          |
| 0.8 | 0.03| 0.3695, 0.6358, 0.3827, 0.2155, 0.1996, 0.5146, 0.2794, 0.1816, 0.7897, 0.3134, 0.1176, 0.6621 | 55.6803 7425.15 | 0.1 | 0 1 0          |
| 0.8 | 0.05| 0.7853, 0.0154, 0.5136, 0.1341, 0.3328, 0.3484, 0.2248, 0.1372, 0.5994, 0.4168, 0.1517, 0.4608 | 60.2807 7213.84 | 0.1 | 1 0 0          |
| 0.8 | 0.08| 0.7237, 0.0506, 0.1044, 0.188, 0.3664, 0.3976, 0.2632, 0.3844, 0.188, 1.068, 0.8557, 0.1451 | 56.8232 7160 | 0.5 | 1 0 1          |
| 0.9  | 0.01| 0.2815, 0.2914, 0.1671, 0.4278, 0.244, 0.1354, 0.1816, 0.7036, 0.7523, 0.54, 0.4003, 0.2177 | 57.8137 7381.44 | 0.5 | 0 1 1          |
| 0.9  | 0.03| 0.7006, 1.0142, 0.4773, 0.3651, 0.1636, 0.5434, 0.6652, 0.3274, 0.1814, 0.1231, 0.1022, 0.2584 | 51.9713 7031.41 | 0.1 | 0 1 0          |
| 0.9  | 0.05| 0.518, 0.0462, 0.2375, 0.6621, 0.13, 0.2296, 0.178, 0.1366, 0.3981, 0.5917, 0.2166, 0.3871 | 49.4286 6355.43 | 0.7 | 0 1 0          |
| 0.9  | 0.08| 0.3585, 0.7084, 0.5961, 0.4036, 0.1684, 0.2668, 0.1336, 0.1366, 0.3596, 0.3552, 0.5235, 0.4377 | 56.3198 7010.8 | 0.9 | 0 1 1          |

Table 10. $\alpha = 0.5$
| 0.6  | 0.05 | 0.3607, 0.6611, 0.2606, 0.4674, 0.1666, 0.2878, 0.2248, 0.1012, 0.144, 0.7567, 0.7446, 0.8282 | 60.508 6996.84 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.6  | 0.08 | 0.8667, 0.011, 0.3233, 0.1, 0.4162, 0.4006, 0.3022, 0.4864, 0.3827, 0.1572, 0.4641, 0.9811 | 54.6189 7162.24 | 0.8 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.7  | 0.01 | 0.914, 0.297, 0.4025, 0.3871, 0.5044, 0.2836, 0.1384, 0.1696, 0.2793, 0.2056, 0.2606, 0.3277 | 44.6774 6413.98 | 0.6 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.7  | 0.03 | 0.3805, 0.8084, 0.1451, 0.32, 0.1756, 0.3976, 0.1288, 0.4372, 0.4179, 0.1242, 0.4212, 0.2881 | 45.1737 5461.59 | 0.9 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0.7  | 0.05 | 0.2166, 0.4147, 0.4201, 0.1066, 0.1606, 0.277, 0.1978, 0.3556, 0.199, 1.2132, 0.2551, 0.3849 | 55.4277 6545.11 | 0.2 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.7  | 0.08 | 0.2529, 0.5939, 0.2529, 0.287, 0.4558, 0.5848, 0.226, 0.3148, 0.5565, 0.1484, 0.3992, 0.4256 | 51.3191 6827.01 | 0.3 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0.8  | 0.01 | 0.1528, 0.6281, 0.5257, 0.5004, 0.5308, 0.5116, 0.2782, 0.643, 0.1913, 0.1814, 0.1891, 0.1055 | 52.4247 6920.21 | 0.3 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.8  | 0.03 | 0.2023, 0.5082, 0.2507, 0.2529, 0.271, 0.3982, 0.1552, 0.2548, 0.5917, 0.1836, 0.4168, 0.3189 | 51.4945 6066.54 | 0.9 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0.8  | 0.05 | 0.3486, 0.2199, 0.2584, 0.5477, 0.148, 0.4234, 0.3634, 0.6172, 0.1495, 0.3288, 0.4333, 0.6632 | 56.3785 6591.09 | 0.5 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0.8  | 0.08 | 0.4091, 0.2475, 0.3673, 0.1726, 0.2218, 0.478, 0.3784, 0.1702, 0.7105, 0.2001, 0.1517, 0.4036 | 48.5855 6836.25 | 0.1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.9  | 0.01 | 0.8898, 0.0781, 0.1957, 0.6148, 0.3442, 0.3304, 0.124, 0.3832, 0.7138, 0.1638, 0.2254, 0.4234 | 53.6968 7322.52 | 0.2 | 0 | 1 | 0 | 1 | 0 | 1 |
0.9 0.03 0.6863, 0.7314, 0.3607, 0.3431, 0.262, 0.331, 0.2716, 0.5176, 0.4047, 0.1814, 0.2562, 0.3783 52.7347 6820.95 0.7 0 1 0 1 0 1
0.9 0.05 1.0691, 0.1914, 0.1132, 0.2023, 0.1564, 0.1576, 0.4474, 0.2446, 0.6819, 0.2463, 0.2595, 0.2749 54.297 6636.51 0.4 1 0 0 1 0 1
0.9 0.08 0.4322, 0.386, 0.4014, 0.1176, 0.5782, 0.3412, 0.2896, 0.6586, 0.1396, 0.276, 0.9833, 0.122 51.8692 7139.65 0.4 0 1 0 1 1 0

| pc | pm | Optimum Solution X=x₁, x₂,..., x₁₂ | Z= Z₁, Z₂ | λ | Sites (1 or 2) |
|----|----|---------------------------------|----------|----|----------------|
| 0.6 | 0.01 | 0.2958, 0.3388, 0.8348, 0.2034, 0.4564, 0.2344, 0.1024, 0.1486, 0.2881, 0.2639, 0.43, 1.134 | 55.9846 7591.71 | 0.6 | 1 0 1 0 1 0 |
| 0.6 | 0.03 | 0.6324, 0.6358, 0.21, 0.1341, 0.1972, 0.2704, 0.1552, 0.2884, 0.7831, 0.232, 0.4179, 0.3992 | 49.0302 6659.27 | 0.5 | 1 0 1 0 1 0 |
| 0.6 | 0.05 | 0.1495, 0.3201, 0.1836, 0.6918, 0.2152, 0.283, 0.385, 0.13, 0.1528, 0.1935, 0.4432, 0.5994 | 46.818 5834.92 | 0.7 | 1 0 1 0 1 0 |
| 0.6 | 0.08 | 0.3552, 0.2651, 0.3915, 0.331, 0.1564, 0.1606, 0.4114, 0.1846, 0.1605, 0.2793, 0.2331, 0.9261 | 47.0473 5932.16 | 0.9 | 1 0 1 0 1 0 |
| 0.7 | 0.01 | 0.1231, 0.9141, 0.1363, 0.1011, 0.1438, 0.337, 0.3652, 0.3094, 0.4564, 0.375, 0.3068, 0.5059 | 47.5098 5709.99 | 0.9 | 0 1 0 1 0 1 |
| 0.7 | 0.03 | 0.188, 0.671, 0.1825, 0.6775, 0.7114, 0.283, 0.2146, 0.1186, 0.1297, 0.5598, 0.3706, 0.1737 | 50.9126 6537.74 | 0.6 | 0 1 0 1 0 1 |
| 0.7 | 0.05 | 0.7226, 0.5456, 0.1, 0.1913, 0.1618, 0.3622, 0.2416, 0.4246, 0.166, 0.683, 1.0581, 0.3035 | 55.4745 6649.83 | 0.2 | 1 0 1 0 1 0 |
| 0.7 | 0.08 | 1.0526, 0.506, 0.3816, 0.4157, 0.1648, 0.3088, | 54.1981 6434.47 | 0.3 | 0 1 1 0 1 0 |
| pc | pm | Optimum Solution | Z = Z₂ | λ | Sites (1 or 2) |
|----|----|------------------|---------|---|----------------|
| 0.6 | 0.01 | \(x = x_1, x_2, ..., x_{12}\) | 54.9261 6594.92 | 0.5 | 0 1 0 |
| 0.6 | 0.03 | \(x = x_1, x_2, ..., x_{12}\) | 51.5267 6130.15 | 0.8 | 0 1 0 |

Table 12. \(\alpha = 0.7\)
| 0.6 | 0.05 | 0.694, 0.2486, 0.2716, 1.1087, 0.1288, 0.2752, 0.1438, 0.1426, 0.1605, 0.2925, 0.3508, 0.3651 | 52.1567 6507.37 | 0.9 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.6 | 0.08 | 0.1638, 0.5192, 0.1143, 0.3574, 0.121, 0.4504, 0.433, 0.3898, 0.3145, 0.1924, 0.3662, 0.5554 | 47.8145 5727.42 | 0.9 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0.7 | 0.01 | 0.727, 0.3146, 0.2485, 0.1088, 0.4852, 0.2458, 0.1852, 0.5506, 0.2045, 0.5708, 0.7391, 0.4058 | 53.8141 6843.99 | 0.6 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.7 | 0.03 | 0.1308, 0.6699, 0.2991, 0.4003, 0.1504, 0.6106, 0.3286, 0.1204, 0.1781, 0.5983, 0.1616, 0.3783 | 56.1935 5888.29 | 0.5 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.7 | 0.05 | 0.7941, 0.1364, 0.4256, 0.3981, 0.5122, 0.4654, 0.2878, 0.229, 0.188, 0.2683, 0.1, 0.5708 | 53.4879 6716.21 | 0.8 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.7 | 0.08 | 0.4971, 0.5863, 0.1308, 0.5125, 0.3706, 0.1756, 0.1858, 0.1654, 0.5719, 0.3365, 0.7182, 0.3706 | 53.978 7198.3 | 0.2 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.8 | 0.01 | 0.2122, 0.8272, 0.1297, 0.5664, 0.2218, 0.214, 0.2284, 0.4786, 0.1484, 0.5356, 0.2859, 0.7545 | 57.0356 6051.97 | 0.5 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.8 | 0.03 | 0.2221, 0.0836, 0.199, 0.452, 0.3538, 0.661, 0.1204, 0.2464, 0.1077, 0.804, 0.1682, 0.7072 | 53.4097 5955.33 | 0.7 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0.8 | 0.05 | 0.4949, 0.8404, 0.1165, 0.4498, 0.3076, 0.1246, 0.3616, 0.568, 0.3398, 0.2199, 0.2419, 0.4806 | 52.4714 6296.59 | 0.5 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.8 | 0.08 | 0.5829, 0.5698, 0.2958, 0.1198, 0.1738, 0.4252, 0.3382, 0.4378, 0.1484, 0.3112, 0.9943, 0.6225 | 54.759 6779.52 | 0.1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0.9 | 0.01 | 1.1054, 0.3003, 0.2419, 0.1539, 0.6142, 0.187, 0.595, 0.2158, 0.1121, 0.1561, 0.3134, 0.5499 | 61.6598 6634.89 | 0.9 | 0 | 1 | 1 | 0 | 1 | 0 |
Table 13. $\alpha = 0.8$

| pc | pm | Optimum Solution $X=x_1, x_2, \ldots, x_{12}$ | $Z=Z_4, Z_2$ | $\lambda$ | Sites (1 or 2) |
|----|----|----------------------------------|----------------|--------|----------------|
| 0.6| 0.01| 0.5422, 0.1947, 0.5576, 0.5146, 0.2133, 0.2674, 0.1066, 0.8878, 0.3453, 0.5477 | 57.4254 7260.85 | 0.6 | 0 1 0 1 0 1 0 1 |
| 0.6| 0.03| 0.9745, 0.3102, 0.1616, 0.8435, 0.3233, 0.127, 0.4906, 0.6634, 0.388, 0.2837, 0.1693, 0.8381, 0.3552 | 54.7615 7480.33 | 0.1 | 1 0 1 0 1 0 1 0 |
| 0.6| 0.05| 0.287, 0.5258, 0.177, 0.1836, 0.3646, 0.5608, 0.175, 0.496, 0.4476, 0.2991, 0.1957, 0.9437 | 54.9959 6305.02 | 0.8 | 1 0 1 0 1 0 1 0 |
| 0.6| 0.08| 0.7336, 0.4323, 0.4454, 0.155, 0.3898, 0.1474, 0.526, 0.1996, 0.1451, 0.2287, 0.5125, 0.1517 | 42.5606 6527.72 | 0.9 | 1 0 0 1 0 1 0 |
| 0.7| 0.01| 0.5455, 0.9922, 0.32, 0.1418, 0.3544, 0.262, 0.1876, 0.1012, 0.3618, 0.3519, 0.0218, 0.4364 | 43.5657 5728.43 | 0.7 | 1 0 1 0 1 0 1 0 |
| 0.7| 0.03| 0.5312, 0.8657, 0.2584, 0.4234, 0.1888, 0.3526, 0.4066, 0.1696, 0.1121, 0.2342, 0.3453, 0.3046 | 48.7659 5753.99 | 0.8 | 1 0 1 0 1 0 1 0 |
| 0.7| 0.05| 1.0977, 0.4257, 0.1924, 0.2771, 0.2272, 0.424, 0.496, 0.1036, 0.3486, 0.2298, 0.87, 0.6577 | 56.1043 7657.99 | 0.1 | 1 0 1 0 1 0 1 0 |
| 0.7| 0.08| 0.3255, 0.1947, 0.1539, 0.4567, 0.2235, 0.2346, 0.3456, 0.3456, 0.2346, 0.1539, 0.1947, 0.3255 | 52.1845 6773.66 | 0.4 | 1 0 1 0 1 0 1 0 |

21
| Table 14. $\alpha = 0.9$ |
|-------------------------|
| 0.8 0.01 | 0.1968, 0.3696, 0.1935, 0.4861, 0.3634, 0.2704, 0.208, 0.646, 0.3321, 0.9008, 0.3442, 0.1825 | 59.7058 7104.06 | 0.7 | 0 1 0 |
| 0.8 0.03 | 0.342, 0.1518, 0.8557, 0.2276, 0.2716, 0.1576, 0.2428, 0.4108, 0.1517, 0.507, 0.2144, 0.2529 | 47.6452 6864.5 | 0.3 | 1 0 1 |
| 0.8 0.05 | 0.1407, 0.3839, 0.1814, 0.1825, 0.2002, 0.1978, 0.3244, 0.6772, 0.2023, 0.8898, 0.4718, 0.7864 | 60.661 6652.57 | 0.7 | 0 1 0 |
| 0.8 0.08 | 0.5576, 0.1375, 0.4344, 0.4553, 0.118, 0.4612, 0.2998, 0.1468, 0.3651, 0.4245, 0.4762, 0.1143 | 48.218 6870.99 | 0.5 | 1 0 1 |
| 0.9 0.01 | 0.5576, 0.165, 0.1132, 0.3585, 0.4168, 0.148, 0.1846, 0.4312, 0.5246, 0.7963, 0.7127, 0.2012 | 57.6868 7543.3 | 0.9 | 1 0 1 |
| 0.9 0.03 | 0.419, 0.5301, 0.188, 1.1989, 0.1282, 0.178, 0.3034, 0.145, 0.5026, 0.2859, 0.1264, 0.5455 | 59.8166 7447.43 | 0.8 | 1 0 1 |
| 0.9 0.05 | 0.738, 0.4356, 0.3156, 0.2298, 0.6556, 0.1078, 0.1948, 0.4108, 0.4509, 0.1814, 0.1792, 0.8304 | 52.7598 7003.16 | 0.7 | 1 0 1 |
| 0.9 0.08 | 0.1869, 0.7304, 0.2001, 0.5158, 0.6676, 0.436, 0.1036, 0.3574, 0.3596, 0.1913, 0.5884, 0.2507 | 57.8918 6752.07 | 0.7 | 1 0 1 |
7. Conclusion and results

In this article, the multi-objective fuzzy stochastic programming problem for hazardous wastes have been handled using GA based fuzzy programming approach. The bounds of the problem and case study are described as fuzzy Weibull distribution and fuzzy normal distribution, and the confidence levels are treated as fuzzy number. The Table 6-14 show the optimum Pareto’s solutions for different values. The Figure 2 and 3 present the graphical representation of the optimum solutions for $\alpha = 0.5$. One of the major advantages of the procedure is that the computational time is greatly reduced to solve the main problem. Another advantage is that it provides the decision maker to have alternative Pareto solutions which readily help them in making appropriate decision regarding the problem. The results of the numerical example and case study showed that the above procedure is very efficient. The number of variables in the problem increases as we moved to some large and complex problems which put the limitation to the proposed methods. The extension of the proposed research model can be made to problems of transportation, transhipment, solid transportation, inventory, etc.

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