Magnetized ion-acoustic shock waves in degenerate quantum plasma

S. Jahan\textsuperscript{1,*}, T.S. Roy\textsuperscript{2,**}, B.E. Sharmin\textsuperscript{1,***}, N.A. Chowdhury\textsuperscript{3,†}, A. Mannan\textsuperscript{1,§}, and A.A. Mamun\textsuperscript{1,¶}

\textsuperscript{1} Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh
\textsuperscript{2}Department of Physics, Bangladesh University of Textiles, Tejgaon Industrial Area, Dhaka-1000, Bangladesh
\textsuperscript{3}Plasma Physics Division, Atomic Energy Centre, Dhaka-1000, Bangladesh

\textsuperscript{*}e-mail: jahan88phy@gmail.com, \textsuperscript{†}tanu.jabi@gmail.com, \textsuperscript{‡}sharmin14phy@gmail.com, nurealam1743phy@gmail.com, \textsuperscript{¶}abdulmannan@juniv.edu, \textsuperscript{§}mamunwphys@juniv.edu

Abstract

A theoretical investigation has been carried out to examine the ion-acoustic shock waves (IASHWs) in a magnetized degenerate quantum plasma system containing inertialess ultra-relativistically degenerate electrons, and inertial non-relativistic positively charged heavy and light ions. The Burgers’ equation is derived by employing reductive perturbation method. It can be seen that under consideration of non-relativistic positively charged heavy and light ions, the plasma model supports only positive electrostatic shock structure. It is also observed that the charge state and number density of the non-relativistic heavy and light ions enhance the amplitude of IASHWs, and the steepness of the shock profile is decreased with ion kinematic viscosity ($\eta$). The findings of our present investigation will be helpful in understanding the nonlinear propagation of IASHWs in white dwarfs and neutron stars.

Keywords: Shock waves; Burgers’ equation; Degenerate quantum plasma; Reductive perturbation method

1. Introduction

The research regarding the propagation of nonlinear electrostatic excitations in degenerate quantum plasma system (DQPS) has received a substantial attention to the plasma physicist due to its ubiquitous existence in white dwarfs \cite{1,2,3} and neutron stars \cite{4,5,6}. It is believed that the components of the DQPS are electrons, positively charged heavy ions (e.g., $^{56}$Fe \cite{4}), $^{85}$Rb \cite{5} and positively charged light ions (e.g., $^{3}$H \cite{2}, $^{12}$C \cite{6}). A number of authors investigated nonlinear waves in DQPS having positively charged heavy and light ions, and electrons \cite{11,12,13,14}.

The characteristics of DQPS are comprehensively governed by the number density of the plasma species of DQPS, and it has been observed that the electron number density in white dwarfs is in the order of $10^{26}$ cm$^{-3}$ to $10^{29}$ cm$^{-3}$, and even more in neutron stars \cite{4,10}. The dynamics of these high-dense plasma species in DQPS can be predicted by the Heisenberg uncertainty principle and Pauli exclusion principle, and under consideration of these two principles, the plasma species can create degenerate pressure which is readily outward directional, and is not similar to the thermal pressure in normal plasmas. In extremely high-dense plasma, the degenerate pressure usually exceeds the thermal pressure. Therefore, the degenerate pressure has to be taken into account to model the dynamics of the DQPS. The degenerate pressure associated with degenerate electrons, heavy ions, and light ions can be given by \cite{2}

$$P_s = \dot{\mathbf{K}}_s N_s^\gamma,$$

where $s$ represents electron or heavy ion or light ion species, i.e., $s = e$ for the electron species, $s = 1$ for the heavy ion species, and $s = 2$ for the light ion species;

$$\gamma = \frac{5}{3}; \quad \dot{K}_s = \frac{3}{5}(\pi)^\frac{3}{4} \frac{\pi \hbar^2}{m_s} \approx \frac{3}{5} \Lambda_s \hbar c,$$

for non-relativistic limit [with $\Lambda_s = \pi \hbar/m_s c$, $\hbar$ is the Planck constant ($h$) divided by 2$\pi$, $m_s$ is the mass of species $s$, and $c$ is the speed of light in vacuum], and

$$\gamma = \frac{4}{3}; \quad \dot{K}_s = \frac{3}{4}(\pi)^\frac{3}{4} \frac{h \pi}{9} \approx \frac{3}{4} \hbar c,$$

for ultra-relativistic limit \cite{13,14}. The degenerate pressure depends only on the number density of the plasma species but not on their temperature \cite{13,14}. For stable configuration of the DQPS, the outward directional degenerate pressure is counter-balanced by the inward gravitational pressure.

The electrostatic shock wave profile, which may arise due to the Landau damping and kinematic viscosity of the medium, is governed by the Burgers’ equation \cite{15,16,17,18}. Alteya et al. \cite{15} examined the ion-acoustic (IA) shock waves (IASHWs) in DQPS, and reported that the amplitude of the positive shock profile increases with the increase of electron number density. Abdelwahed et al. \cite{16} investigated IASHWs in non-thermal plasma, and found that the steepness of the shock profile decreases with ion kinematic viscosity.

The external magnetic field has been considered to investigate the electrostatic shock \cite{19,20,21} and solitary \cite{22,23} waves in plasmas. Hossen et al. \cite{21} examined the IASHWs in the presence of external magnetic field, and highlighted that the amplitude of IASHWs increases with increasing the angle between the wave propagation vector and the direction of external magnetic field (via $\delta$). Shaukat \cite{22} studied IA solitary waves.
in degenerate magneto-plasma. Ashraf et al. [23] observed that the amplitude of the electrostatic shock wave increases with oblique angle.

Recently, Islam et al. [44] investigated envelope solutions in a three-component DQPS containing relativistically degenerate electrons, positively charged heavy and light ions. To the best knowledge of the authors, no attempt has been made to study IASHWs in a magnetized DQPS having positively charged non-relativistic heavy and light ions, and ultra-relativistically degenerate electrons. Therefore, the aim of our present investigation is to derive the Burgers’ equation and by employing its shock solution, we will numerically analyze the IASHWs in a magnetized DQPS.

The manuscript is organized in the following way: The governing equations are described in section 2. The derivation of the Burgers’ equation and its shock solution are demonstrated in section 3. The results and discussion are presented in section 4. The conclusion is provided in section 5.

2. Governing Equations

We consider a magnetized DQPS consisting of inertially positively charged non-relativistic heavy ions (mass \(m_1\); charge \(q_1 = +eZ_1\); number density \(N_1\); pressure \(P_1\)), positively charged non-relativistic light ions (mass \(m_2\); charge \(q_2 = +eZ_2\); number density \(N_2\); pressure \(P_2\)), and inertialless ultra-relativistically degenerate electrons (mass \(m_e\); charge \(-e\); number density \(N_e\); pressure \(P_e\)); where \(Z_1 (Z_2)\) is the charge state of the heavy (light) ion. We also assume an uniform external magnetic field \(B\) in the direction of z-axis (\(B = B_0\)). The propagation of IASHWs is governed by the following equations:

\[
\begin{align*}
\frac{\partial N_1}{\partial t} + \nabla \cdot (N_1 U_1) &= 0, \\
\frac{\partial U_1}{\partial t} + (U_1 \cdot \nabla)U_1 &= -\frac{Z_1 e}{m_1} \Phi + \frac{Z_1 e B_0}{m_1} (U_1 \times \hat{z}) - \frac{1}{m_1 N_1} \nabla P_1 + \eta \nabla^2 U_1, \\
\frac{\partial N_2}{\partial t} + \nabla \cdot (N_2 U_2) &= 0, \\
\frac{\partial U_2}{\partial t} + (U_2 \cdot \nabla)U_2 &= -\frac{Z_2 e}{m_2} \Phi + \frac{Z_2 e B_0}{m_2} (U_2 \times \hat{z}) - \frac{1}{m_2 N_2} \nabla P_2 + \eta \nabla^2 U_2, \\
\nabla^2 \Phi &= 4\pi n_e (N_e - Z_2 N_2 - Z_1 N_1), \\
\nabla \Phi - \frac{1}{e N_e} \nabla P_e &= 0.
\end{align*}
\]

Now, we have introduced the normalizing parameters as follows: \(n_1 \to N_1 / n_{01}; n_2 \to N_2 / n_{02}; n_e \to N_e / n_{0e}; u_1 \to U_1 / C_1; u_2 \to U_2 / C_2; \phi \to e \Phi / m_e c^2; t \to T / \omega_{p1}^{-1}; \nabla \to \nabla / \lambda_{01}; \eta = \eta / (\omega_{p1}^{-1} \lambda_{01}^2)\) [where \(C_1 = (Z_1 m_e c^2 / n_{01})^{1/2}\); the plasma frequency \(\omega_{p1}^{-1} = (m_1 / 4\pi Z_1^2 e^2 n_{01})^{1/2}\); the Debye length \(\lambda_{01} = (m_e e^2 / 4\pi Z_1^2 e^2 n_{01})^{1/2}\)]. At equilibrium, the quasi-neutrality condition can be written as \(n_{0} = Z_1 n_{10} + Z_2 n_{20}\). By using these normalizing parameters, Eqs. (4)-(8) can be expressed as

\[
\begin{align*}
\frac{\partial n_1}{\partial t} + \nabla \cdot (n_1 u_1) &= 0, \\
\frac{\partial u_1}{\partial t} + (u_1 \cdot \nabla)u_1 &= -\phi + \Omega_1 (u_1 \times \hat{z}) - \frac{\mu_1 K_1}{n_1} \nabla \eta^2 + \eta \nabla^2 u_1, \\
\frac{\partial n_2}{\partial t} + \nabla \cdot (n_2 u_2) &= 0, \\
\frac{\partial u_2}{\partial t} + (u_2 \cdot \nabla)u_2 &= -\mu_2 \phi + \mu_2 \Omega_1 (u_2 \times \hat{z}) - \frac{\mu_2 K_2}{n_2} \nabla \eta^2 + \eta \nabla^2 u_2, \\
\nabla^2 \phi &= (1 + \mu_4) n_e - \mu_4 n_2 - n_1, \\
\end{align*}
\]

where the plasma parameters are: \(\Omega_1 = \omega_{p1} / \omega_{p1}\) [where \(\omega_{p1} = Z_1 e B_0 / (m_1)\); \(\mu_1 = m_1 / Z_1 m_e; \mu_2 = Z_2 m_1 / Z_1 m_2; \mu_3 = n_{0e} / Z_1 n_{10}; \mu_4 = Z_2 n_{20} / Z_1 n_{10}\); \(K_1 = n_{10}^{-1} K_e / m_e c^2; K_2 = n_{20}^{-1} K_e / m_2 c^2\) and \(\gamma = \alpha = 5 / 3\)] (for non-relativistic limit). Now, by normalizing and integrating Eq. (9), the number density of the inertialless electrons can be obtained in terms of electrostatic potential \(\phi\) as

\[n_e = \left[1 + \frac{\gamma_e - 1}{K_3 \gamma_e}\right]^{\frac{\mu_4}{\mu_2} - 1},\]

where \(K_3 = n_{0e}^{-1} K_e / m_e c^2\) and \(\gamma = \gamma_e = 4 / 3\) (for ultra-relativistic limit). Now, expanding the right hand side of Eq. (15) and substituting in Eq. (14), we can write

\[
\nabla^2 \phi + n_1 + \mu_4 n_2 = 1 + \mu_4 + \sigma_1 \phi + \sigma_2 \phi^2 + \sigma_3 \phi^3 + \cdots,
\]

where \(\sigma_1 = [(\mu_4 + 1) / (\alpha K_3)]; \sigma_2 = [(\mu_4 + 1)(2 - \gamma_e) / 2(\alpha K_3)^2];\) and \(\sigma_3 = [(\mu_4 + 1)(2 - \gamma_e)(3 - 2 \gamma_e) / 6(\alpha K_3)^3]\).

3. Derivation of the Burgers’ Equation

To study IASHWs, we derive the Burgers’ equation by introducing the stretched coordinates for independent variables as [21, 24]:

\[
\xi = (l_x x + l_y y + l_z z - v_p t),
\]

\[
\tau = e^2 t,
\]

where \(v_p\) is the phase speed and \(e\) is a smallness parameter measuring the weakness of the dissipation (0 < \(e < 1\)). The \(l_x, l_y, \) and \(l_z\) (i.e., \(l_x^2 + l_y^2 + l_z^2 = 1\)) are the directional cosines of the wave vector \(k\) along \(x, y, \) and \(z\)-axes, respectively. The depen-
dent variables can be expressed in power series of $\epsilon$ as \[21\]

\[
n_1 = 1 + \epsilon n_1^{(1)} + \epsilon^2 n_1^{(2)} + \cdots, \quad (19)
\]

\[
n_2 = 1 + \epsilon n_2^{(1)} + \epsilon^2 n_2^{(2)} + \cdots, \quad (20)
\]

\[
u_{\text{ly}} = \epsilon^2 \nu_{\text{ly}}^{(2)} + \epsilon^3 \nu_{\text{ly}}^{(3)} + \cdots, \quad (21)
\]

\[
u_{\text{zy}} = \epsilon^2 \nu_{\text{zy}}^{(2)} + \epsilon^3 \nu_{\text{zy}}^{(3)} + \cdots, \quad (22)
\]

\[
u_{\text{iz}} = \epsilon \nu_{\text{iz}}^{(2)} + \epsilon^2 \nu_{\text{iz}}^{(3)} + \cdots, \quad (23)
\]

\[
u_{\text{iz}} = \epsilon \nu_{\text{iz}}^{(2)} + \epsilon^2 \nu_{\text{iz}}^{(3)} + \cdots, \quad (24)
\]

\[
\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots. \quad (25)
\]

Now, by substituting Eqs. \((17)-(25)\) in Eqs. \((10)-(13)\) and \((16)\), and collecting the terms containing $\epsilon$, the first-order equations reduce to

\[
n_1^{(1)} = \frac{\epsilon^2}{(v_p^2 - \alpha \mu \epsilon K_1)} \phi^{(1)}, \quad (26)
\]

\[
u_{\text{iz}}^{(1)} = \frac{v_p^2}{(v_p^2 - \alpha \mu \epsilon K_1)} \phi^{(1)}, \quad (27)
\]

\[
n_2^{(1)} = \frac{\mu \epsilon^2}{(v_p^2 - \alpha \mu \epsilon K_2)} \phi^{(1)}, \quad (28)
\]

\[
u_{\text{iz}}^{(1)} = \frac{\mu \epsilon^2}{(v_p^2 - \alpha \mu \epsilon K_2)} \phi^{(1)}. \quad (29)
\]

Now, the phase speed of IASHWs can be written as

\[
v_p = v_p^+ = \sqrt{\frac{m_2 + \sqrt{m_2^2 - 4m_1m_3}}{2m_1}}, \quad (30)
\]

\[
v_p = v_p^- = \sqrt{\frac{m_2 - \sqrt{m_2^2 - 4m_1m_3}}{2m_1}}, \quad (31)
\]

where $m_1 = \sigma_1$, $m_2 = 1 + \mu_2 \mu_4 - \alpha \sigma_1 \mu_1 K_2 - \alpha \sigma_1 \mu_1 K_1$, and $m_3 = \alpha \mu_1 K_2 + \alpha \mu_1 \mu_2 K_1 + \sigma_1 \alpha^2 \mu_1^2 K_1 K_2$. The $x$ and $y$-components of the first-order momentum equations can be manifested as

\[
u_{\text{iz}}^{(1)} = -\frac{l_v^2}{\Omega_1(v_p^2 - \alpha \mu \epsilon K_1)} \frac{\partial \phi^{(1)}}{\partial \xi}, \quad (32)
\]

\[
u_{\text{iz}}^{(1)} = -\frac{l_v^2}{\Omega_1(v_p^2 - \alpha \mu \epsilon K_1)} \frac{\partial \phi^{(1)}}{\partial \xi}, \quad (33)
\]

\[
u_{\text{iz}}^{(1)} = -\frac{l_v^2}{\Omega_1(v_p^2 - \alpha \mu \epsilon K_2)} \frac{\partial \phi^{(1)}}{\partial \xi}, \quad (34)
\]

\[
u_{\text{iz}}^{(1)} = -\frac{l_v^2}{\Omega_1(v_p^2 - \alpha \mu \epsilon K_2)} \frac{\partial \phi^{(1)}}{\partial \xi}. \quad (35)
\]

Now, by taking the next higher-order terms, the equation of continuity, momentum equation, and Poisson’s equation can be written as

\[
\frac{\partial n_1^{(1)}}{\partial \tau} - v_p \frac{\partial n_1^{(2)}}{\partial \xi} + l_v \frac{\partial u_1^{(1)}}{\partial \xi} + l_v \frac{\partial u_1^{(2)}}{\partial \xi} + l_v \frac{\partial u_2^{(1)}}{\partial \xi} + l_v \frac{\partial u_2^{(2)}}{\partial \xi} = 0, \quad (36)
\]

\[
\frac{\partial u_1^{(1)}}{\partial \tau} - v_p \frac{\partial u_1^{(2)}}{\partial \xi} + l_v \frac{\partial u_1^{(1)}}{\partial \xi} + l_v \frac{\partial u_1^{(2)}}{\partial \xi} + l_v \frac{\partial u_2^{(1)}}{\partial \xi} + l_v \frac{\partial u_2^{(2)}}{\partial \xi} = 0, \quad (37)
\]

\[
\frac{\partial u_1^{(2)}}{\partial \tau} - v_p \frac{\partial u_1^{(3)}}{\partial \xi} + l_v \frac{\partial u_1^{(2)}}{\partial \xi} + l_v \frac{\partial u_1^{(3)}}{\partial \xi} + l_v \frac{\partial u_2^{(2)}}{\partial \xi} + l_v \frac{\partial u_2^{(3)}}{\partial \xi} = 0, \quad (38)
\]

\[
\frac{\partial u_2^{(2)}}{\partial \tau} - v_p \frac{\partial u_2^{(3)}}{\partial \xi} + l_v \frac{\partial u_2^{(2)}}{\partial \xi} + l_v \frac{\partial u_2^{(3)}}{\partial \xi} + l_v \frac{\partial u_2^{(4)}}{\partial \xi} + l_v \frac{\partial u_2^{(4)}}{\partial \xi} = 0, \quad (39)
\]

Finally, the next higher-order terms of Eqs. \((10)-(13)\) and \((16)\), with the help of Eqs. \((26)-(30)\), can provide the Burgers’ equation as

\[
\frac{\partial \Phi}{\partial \tau} + A \frac{\partial \Phi}{\partial \xi} = \frac{\partial^2 \Phi}{\partial \xi^2}, \quad (41)
\]

where $\Phi = \phi^{(1)}$ for simplicity. In Eq. \((41)\), the nonlinear coefficient $A$ and dissipative coefficient $B$ are, respectively, given by

\[
A = P(Q + R - 2\sigma_{2}), \quad (42)
\]

\[
B = Q, \quad (43)
\]

where

\[
P = \frac{(v_p^2 - \alpha \mu \epsilon K_1)^2(v_p^2 - \alpha \mu \epsilon K_2)^2}{2v_p^2[v_p^2(1 + \mu_2 \mu_4) + \alpha^2 \mu_1^2 K_1^2 + \mu_2 \mu_4 K_1^2 - M]}, \quad (44)
\]

\[
M = 2\alpha \mu_1^2 v_p (K_2 + K_1 \mu_2 \mu_4), \quad (45)
\]

\[
Q = \frac{\mu \epsilon^2 \nu_{\text{iz}}^{(3)} + \nu_{\text{iz}}^{(2)} (\alpha \sigma_1 - 2)}{(v_p^2 - \alpha \mu \epsilon K_2)^2}, \quad (46)
\]

\[
R = \frac{\mu \epsilon^2 \nu_{\text{iz}}^{(3)} + \nu_{\text{iz}}^{(2)} \epsilon \mu_2 \mu_4 (\alpha \sigma_1 - 2)}{(v_p^2 - \alpha \mu \epsilon K_2)^2}. \quad (47)
\]

Now, we look for stationary shock wave solution of this Burgers’ equation by considering $\zeta = \xi - U_0 \tau$ and $\tau = \tau'$ (where $U_0$ is the speed of the shock waves in the reference frame). These allow us to write the stationary shock wave solution as

\[
\Phi = \Phi_0 \left[1 - \tanh \left(\frac{\zeta}{\Delta}\right)\right], \quad (48)
\]

where the amplitude $\Phi_0$ and width $\Delta$ are given by

\[
\Phi_0 = \frac{U_0}{A}, \quad \text{and} \quad \Delta = \frac{2B}{U_0}. \quad (49)
\]
It is clear from Eqs. [42] and [45] that the IASHWs exist, which are formed due to the balance between nonlinearity and dissipation, because $B > 0$ and the IASHWs with $\Phi > 0$ ($\Phi < 0$) exist if $A > 0$ ($A < 0$) because $U_0 > 0$.

### 4. Results and Discussions

Our present investigation is valid for white dwarfs and neutron stars in which both non-relativistic positively charged heavy ions (e.g., $^{56}\text{Fe}$ [4], $^{83}\text{Rb}$ [5], $^{96}\text{Mo}$ [6]), and light ions (e.g., $^1\text{H}$ [8], $^3\text{He}$ [9], $^{12}\text{C}$ [10]), and ultra-relativistically degenerate electrons are exist. For numerical analysis, we have considered $Z_1 = 20 \sim 60$, $Z_2 = 1 \sim 12$, $n_{10} = 1 \times 10^{30} \text{cm}^{-3} \sim 9 \times 10^{30} \text{cm}^{-3}$, $n_{20} = 2 \times 10^{30} \text{cm}^{-3} \sim 8 \times 10^{30} \text{cm}^{-3}$, and $n_{30} = 10^{33} \text{cm}^{-3} \sim 3 \times 10^{33} \text{cm}^{-3}$. The IASHW is governed by the Burgers’ equation [21], and the positive (negative) shock potential can exist corresponding to the limit of $A > 0$ ($A < 0$). The variation of $A$ with $\mu_4$ can be seen from Fig. 1 and it is clear from this figure that our plasma model supports only positive shock potential under consideration of both non-relativistic positively charged heavy and light ions (i.e., $\alpha = 5/3$), and ultra-relativistically degenerate electrons (i.e., $\gamma_e = 4/3$).

The parameter $\delta$ reveals the angle between the direction of the wave propagation and the direction of the external magnetic field, and the effects of $\delta$ on the formation of IASHWs can be seen in Fig. 2. When the oblique angle ($\delta$) increases, the magnetic effect becomes more significant, and therefore the amplitude of the shock wave increases, and this result agrees with the result of Hossen et al. [21].

Figure 3 illustrates the effects of the non-relativistic heavy and light ion’s kinematic viscosity on the positive potential (i.e., $\Phi > 0$) under consideration of $A > 0$. It is really interesting that the steepness of the shock profile decreases with an increase in the value of the non-relativistic heavy and light ion’s kinematic viscosity but the amplitude of shock profile is not affected by the kinematic viscosity ions, and this result agrees with the previous work of Abdelwahed et al. [19].

The variation of IASHWs with electron number density ($n_{e0}$) under consideration of both non-relativistic positively charged heavy and light ions (i.e., $\alpha = 5/3$), and ultra-relativistically degenerate electrons (i.e., $\gamma_e = 4/3$) can be observed in Fig. 4. It is clear from this figure that as we increase the electron number density, the amplitude of the IASHWs associated with $\Phi > 0$ (i.e., $A > 0$) increases. So, the ultra-relativistic electrons...
enhance the amplitude of the IASHWs in a magnetized DQPS having non-relativistic positively charged heavy and light ions, and ultra-relativistically degenerate electrons.

The effects of the charge state of non-relativistic heavy and light ions species on the formation of IASHWs in a magnetized DQPS can be seen in Fig. 5 and 6, respectively. It is obvious from these figures that the charge state of both non-relativistic heavy and light ion species enhances the amplitude of IASHWs associated with $\Phi > 0$ (i.e., $A > 0$) under consideration of $\alpha = 5/3$ and $\gamma_e = 4/3$. Physically, both non-relativistic heavy and light ion species, due to both are positively charged, play same role in the dynamics of magnetized DQPS as well as the configuration of IASHWs. Similarly, the number density of the non-relativistic heavy and light ion species can play significant role in the formation of IASHWs. It is clear from Figs. 5 and 6 that the amplitude of the IASHWs associated with $\Phi > 0$ (i.e., $A > 0$) and under consideration of $\alpha = 5/3$ and $\gamma_e = 4/3$ increases with the number density of both non-relativistic heavy and light ion species.

5. Conclusion

We have investigated the fundamental characteristics of IASHWs in a magnetized DQPS having inertial non-relativistic positively charged heavy and light ions, inertialess ultra-relativistically degenerate electrons. The reductive perturbation method [27] has been employed to derive Burgers’ equation. The results which have been found from present study can be pinpointed as follows:

- The plasma model supports only positive shock potential under consideration of both non-relativistic positively charged heavy and light ions (i.e., $\alpha = 5/3$), and ultra-relativistically degenerate electrons (i.e., $\gamma_e = 4/3$).
- The greater number density of ultra-relativistic electrons enhances the amplitude of the IASHWs.
- The increasing charge state and number density of the non-relativistic heavy and light ion species enhance the amplitude of the IASHWs associated with $\Phi > 0$ (i.e., $A > 0$).
- The steepness of the shock profile is decreased with the kinematic viscosity ($\eta$) of ions.
The amplitude of the shock profile is found to increase as the oblique angle increases.

It may be noted here that the self-gravitational effects of the DQPS are really important to include in the governing equations but beyond the scope of our present work. However, We are optimistic that the outcomes from our present investigation will be useful to understand the propagation of IASHWs in white dwarfs and neutron stars in which the non-relativistic positively charged heavy and light ions, and ultra-relativistically degenerate electrons are exist.

Acknowledgments

Authors would like to acknowledge “UGC research project 2018-2019” for their financial supports to complete this work.

References

[1] S. Chandrasekhar, Philos. Mag. 11, 592 (1931).
[2] S. Chandrasekhar, Astrophys. J. 74, 81 (1931).
[3] H.M. Van Horn, Science 252, 384 (1991).
[4] A. Vanderburg, et al., Nature 526, 546 (2015).
[5] A. Witze, Nature 510, 196 (2014).
[6] R.S. Fletcher, et al., Phys. Rev. Lett. 96, 105003 (2006).
[7] T.C. Killian, Nature 441, 297 (2006).
[8] R.H. Flower, J. Astrophys. Astron 15, 105 (1994).
[9] D. Koester and G. Channugam, Rep. Prog. Phys. 53, 837 (1990).
[10] D. Koester, Astron Astrophys. Rev. 11, 33 (2002); N. Ahmed, et al., Chaos 28, 123107 (2018); N.A. Chowdhury, et al., Vacuum 147, 31 (2018); R.K. Shikha, et al., Eur. Phys. J. D 73, 177 (2019); S. Banik, et al., Eur. Phys. J. D 75, 43 (2021); N.A. Chowdhury, et al., Contrib. Plasma Phys. 58, 870 (2018); M. Hassan, et al., Commun. Theor. Phys. 71, 1017 (2019); S. Jahan, et al., Plasma Phys. Rep. 46, 90 (2020).
[11] A.A. Mamun, Phys. Plasmas 25, 022307 (2018).
[12] A.A. Mamun, Contrib. Plasma Phys. 25, e201900080 (2019).
[13] S. Islam, et al., Phys. Plasmas 24, 092308 (2017).
[14] S. Islam, et al., Phys. Plasmas 24, 092115 (2017).
[15] A. Atteya, et al., Eur. Phys. J. Plus 132, 109 (2017).
[16] H.G. Abdelwahed, et al., J. Exp. Theor. Phys 122, 1111 (2016).
[17] S.A. Tantawy, Astrophys. Space Sci. 361, 249 (2016).
[18] A.N. Dev and M. K. Deka, Phys. Plasmas 25, 072117 (2018).
[19] A.S. Bains and M. Tribeche, Astrophys. Space Sci. 351, 191 (2014).
[20] A.S. Bains, et al., Astrophys. Space Sci. 360, 17 (2015).
[21] M.M. Hossen, et al., High Energy Density Phys. 24, 9 (2017).
[22] M. I. Shaukat, Phys. Plasmas 24, 102301 (2017).
[23] S. Ashraf, et al., Astrophys. Space Sci. 348, 495 (2013).
[24] H. Washimi and T. Tanuti, Phys. Rev. Lett. 17, 996 (1966).
[25] V.L. Karpman, Nonlinear Waves in Dispersive Media, (Pergamon Press, Oxford, 1975).
[26] A. Hasegawa, Plasma Instabilities and Nonlinear Effects, (Springer-Verlag, Berlin, 1975).
[27] M.H. Rahman, et al., Phys. Plasmas 25, 102118 (2018); N.A. Chowdhury, et al., Phys. Plasmas 24, 113701 (2017); M.H. Rahman, et al., Chin. J. Phys. 56, 2061 (2018); N.A. Chowdhury, et al., Plasma Phys. Rep. 45, 459 (2019); S.K. Paul, et al., Pramana J. Phys 94, 58 (2020); T.I. Rajib, et al., Phys. plasmas 26, 123701 (2019); N.A. Chowdhury, et al., Chaos 27, 093105 (2017); S. Jahan, et al., Commun. Theor. Phys. 71, 327 (2019).