Nonlinear Model Predictive Control for Robust Bipedal Locomotion
Exploring CoM Height and Angular Momentum Changes

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Abstract— Human beings can make use of various reactive strategies, e.g., foot location adjustment and upper-body inclination, to keep balance while walking under dynamic disturbances. In this work, we propose a novel Nonlinear Model Predictive Control (NMPC) framework for versatile bipedal gait pattern generation, with the capabilities of footstep adjustment, Center of Mass (CoM) height variation and angular momentum adaptation. These features are realized by constraining the Zero Moment Point motion with considering the variable CoM height and angular momentum change of the Inverted Pendulum plus Flywheel Model. In addition, the NMPC framework also takes into account the constraints of footstep location, CoM vertical motion, upper-body inclination and joint torques, and is finally formulated as a quadratically constrained quadratic program. Therefore, it can be solved efficiently by Sequential Quadratic Programming. Using this unified framework, versatile walking pattern with exploiting time-varying CoM height trajectory and angular momentum changes can be generated based only on the terrain information input. Furthermore, the improved capability for balance recovery under external pushes has been demonstrated through simulation studies.

I. INTRODUCTION

Humanoid robots have attracted a lot of attention for their potential capabilities in accomplishing challenging tasks in real-world environments. With several decades passed, state-of-the-art humanoids such as ASIMO [1], Atlas [2], WALK-MAN [3] and CogIMon [4] have been developed for this purpose. However, due to the complex nonlinear dynamics and intense environmental interactions during walking, enhancing balancing capabilities for bipedal locomotion, which is one of the fundamental abilities to make humanoids practical, still needs further improvements and studies. In this paper, inspired from human beings, who can use various strategies, such as ankle, hip and stepping strategies, to realize balance recovery [5]–[7], we are interested in developing a versatile and robust walking pattern generator which could integrate multiple modulation strategies in a unified way that is consistent with the environmental constraints.

To generate the walking pattern, simplified models have been proposed, among which the Linear Inverted Pendulum Model (LIPM) is widely used [8]. Based on the LIPM, Kajita et al. introduced preview control method for walking pattern generation [9]. Then, considering the feasibility constraints, Wieber et al. [10] formulated a Model Predictive Control (MPC) framework for pattern generation, which was later extended to deal with footstep adaption [11]. Since then, using stepping strategy (footstep adjustment), robust walking on uneven surface, unknown slope, and recovery from external push have been realized [12]–[15]. However, the lack of considering the time-varying vertical Center of Mass (CoM) motion and the effect of angular momentum about CoM limits the humanoid’s capabilities against large disturbances.

To deal with the vertical CoM motion, Nishiwaki et al. proposed a trajectory planning algorithm for generating variable height motions in the real time [16]. However this work did not consider the effect that variable CoM height has on the Zero Moment Point (ZMP) dynamics. The same problem also existed in [17]. Englsberger et al. [18] generalized the 3D divergent component of motion to solve the height motion. Yet, the method proposed an analytic solution and did not deal with feasibility constraints. Based on MPC framework, Brasseur et al. limited the nonlinear part of the dynamic feasibility constraint between properly chosen extreme values and obtained the 3D natural walking gait [19]. Then, using the floating-base inverted pendulum model, Caron et al. proposed a Nonlinear MPC (NMPC) strategy to realize stable walking on uneven terrains [20]. For unknown CoM height, Liu et al. proposed 1-step terrain adaptation strategy for humanoid walking based on the 3-D actuated Dual-SLIP model [21]. Van Heerden solved a NMPC via Sequential Quadratic Programming (SQP) after formulating the problem as a quadratically constrained quadratic program (QCQP) [22]. Nevertheless, above work did not take into account the effect of change of angular momentum.

On the other side, momentum optimization has gained more attention these years, such as the work on whole body motion [23], [24]. Focusing on bipedal gait, Aftab et al. integrated the ankle, hip and stepping strategies by using one single NMPC. But they didn’t take into account the height change of CoM [25]. Using the concept of capture point, Englsberger et al. proposed a measurement-based tracking controller with integration of vertical CoM motion and angular momentum [26]. However, this work focused on the tracking control and did not further reveal the effect on robustness of disturbance rejection. Recently, Zhao et al. proposed a hybrid phase-space method to realized dynamic walking on random but pre-known uneven terrain, based on centroidal momentum dynamics [27]. Yet, this method needed a set of keyframe states as input. Lack et al. [28] and Shafiee-Ashtiani et al. [29] have taken into consideration the effects of angular momentum change and vertical height variation using the MPC framework, but they took the height variations into account as keyframes.

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trajectory as input and just followed the pre-defined height trajectory strictly.

Inspired by above work, especially the work in [22], we propose a NMPC-based walking pattern generator with following contributions. Firstly, the adjustment of foot location, variation of CoM height trajectory and change of angular momentum are integrated into one single NMPC and thus forms a versatile framework for locomotion generation and control, which can dramatically enhance the robustness in compensation for severe external disturbance. Secondly, with taking into consideration the dynamics effects that caused by the CoM height variation and change of angular momentum, the proposed approach can generate stable walking patterns with merely using step parameters references, consisting of step locations reference and step timing reference. Based on state feedback, this approach can generate time-varying CoM height trajectory and body inclination in real time instead of strictly tracking the pre-defined ones. Even with different task authorities, the framework can work effectively.

The rest of this paper is organized as follows. In Section II we briefly review the CoM dynamics and the procedure of SQP for NMPC solution. In Section III, we introduce the problem formulation. In IV, the simulation results are discussed. Finally, in Section V, we draw the conclusions.

II. OVERVIEW OF COM DYNAMICS, MPC AND SQP

A. CoM Dynamics

The LIPM [8], which is widely used as a linear approximation of humanoid walking dynamics, is based on the following assumptions: 1) the robot has a lumped mass body; 2) legs are mass-less and telescopic; 3) CoM moves at a constant height. These assumptions, which over-constrain the robot’s motion capabilities, limit the robot’s performance undergoing external perturbations. Therefore, in order to maximize the bipedal mobility, we propose to use the Inverted Pendulum plus Flywheel Model (IPFM) to utilize the humanoid’s full-body ability, especially variable CoM height motion and upper-body inclination, for versatile locomotion on different terrains and under large disturbances.

The IPFM, as can be seen in Fig. 1, assuming that 1) the flywheel has rotational inertia; 2) legs are mass-less and telescopic; 3) the CoM is located at the hip joint; 4) the CoM moves arbitrarily as long as physical limits are satisfied. Thus, IPFM can be used to model angular momentum and vertical body motion. As the result, the Zero Moment Point (ZMP), which must be inside the robot’s support polygon, of the IPFM can be calculated by

\[
p_x = c_x - \frac{c_z - d_z}{g + c_z} \frac{\dot{L}_y}{m(g + c_z)}, \quad \dot{L}_y = I_y \ddot{\theta}_p, \tag{1}
\]

\[
p_y = c_y - \frac{c_z - d_z}{g + c_z} \frac{\dot{L}_x}{m(g + c_z)}, \quad \dot{L}_x = I_x \ddot{\theta}_i, \tag{2}
\]

where \([p_x, p_y]^T\), \([c_x, c_y, c_z]^T\) and \([d_x, d_y, d_z]^T\) denote the position of ZMP, CoM and supporting foot, respectively. \(L_x\) and \(L_y\), \(I_x\) and \(I_y\), \(\theta_i\) and \(\theta_p\) denote angular momentum, moment of inertia and flywheel rotation angle about \(x\) and \(y\)-axis (where, \(x\)- and \(y\)-axis point to the forward movement in the sagittal plane and the right movement in the coronal plane, respectively), respectively. \(m\) is the overall mass, and \(g\) is the gravitational acceleration.

B. General Framework of Model Predictive Control

Assuming constant jerks of CoM trajectory and body inclination over the time interval \(\Delta t\), we can compute the corresponding state at time \(t_k\),

\[
\hat{x}(k+1) = \hat{A} \hat{x}(k) + \hat{B} \hat{r}(k), \tag{3}
\]

where \(\hat{x}(k) = [x(k), \dot{x}(k), \ddot{x}(k)]^T\) is the current state, \(x \in \{c_x, c_y, c_z, \theta_i, \theta_p\}\),

\[
\hat{A} = \begin{bmatrix} 1 & \frac{\Delta t}{2} & \frac{\Delta t^2}{6} \\ 0 & 1 & \frac{\Delta t}{2} \\ 0 & 0 & 1 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} \frac{\Delta t^3}{6} \\ \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}.
\]

Using (3), we can derive relationships between the jerk, its position, velocity and acceleration over the prediction horizon, of length \(N_h\),

\[
\hat{X}(k) = \hat{P}_{ps} \hat{x}(k) + \hat{P}_{ps} \hat{X}(k), \quad \hat{X}(k) = \hat{P}_{vs} \hat{x}(k) + \hat{P}_{vs} \hat{X}(k), \tag{4}
\]

\[
\hat{X}(k) = \hat{P}_{as} \hat{x}(k) + \hat{P}_{as} \hat{X}(k),
\]

where \(\hat{X}(k) = [x(k+1), \ldots, x(k+N_h)]^T\), \(\hat{X}(k) = [\ddot{x}(k+1), \ldots, \ddot{x}(k+N_h)]^T\), \(\hat{X}(k) = [\dot{x}(k+1), \ldots, \dot{x}(k+N_h)]^T\), \(X \in \{C_x, C_y, C_z, \Theta_i, \Theta_p\}\) representing the future state of CoM along \(x\)-, \(y\)- and \(z\)- axis and the body inclination state about \(x\)- and \(y\)- axis during the prediction horizon, where, e.g. \(C_x(k) = [c_x(k+1), \ldots, c_x(k+N_h)]^T\). \(\hat{P}_{ps}, \hat{P}_{ps}, \hat{P}_{vs}, \hat{P}_{as}, \hat{P}_{as}\) can be obtained by calculating (3) recursively [11].

C. Sequential Quadratic Programming

A nonlinear quadratically constrained quadratic program is expressed as follows,

\[
\min_{\mathbf{X}} \quad f(\mathbf{X}) = \mathbf{X}^T \mathbf{G} \mathbf{X} + \mathbf{g}^T \mathbf{X}, \tag{5}
\]

s.t. \(h_j(\mathbf{X}) \leq 0, \quad h_j(\mathbf{X}) = \mathbf{X}^T \mathbf{V}_j \mathbf{X} + \mathbf{v}_j^T \mathbf{X} + \sigma_j, \quad j \in \{1, \ldots, N_c\}, \)

where \(\mathbf{X} \in \mathbb{R}^{N_x}\) is the state vector, \(N_c, N_t\) are the number of constraints and state variables, respectively. \(\mathbf{G}, \mathbf{V}_j \in \mathbb{R}^{N_t \times N_t}, \mathbf{v}_j \in \mathbb{R}^{N_t}, \) and \(\sigma_j \in \mathbb{R}\) are the parameters that specify the objective function and constraints, respectively.
The problem can be easily solved by the SQP algorithm,\[\text{min}_{\Delta X} \frac{1}{2} \Delta X \nabla^2 f(X) \Delta X + (\nabla X f(X))^T \Delta X,\]
s.t.\[\nabla X h_j(X) \Delta X + h_j(X) \leq 0,\]
where \( j \in \{1, \ldots, N_x\} \).
\[\nabla^2 f(X) = 2G, \quad \nabla X f(X) = 2GX + g, \quad \nabla^2 (h_j(X)) = 2V_j, \quad \nabla X (h_j(X)) = 2V_jX + v_j.\]

With (6), the NMPC is transformed as the Quadratic Problem (QP). The solution of the QP (\(\Delta X \in \mathbb{R}^{N_x}\)) is then used to update the state variable (\(X\)), via \(X = X + \Delta X\). Once completed, (6) is repeated with the new \(X\) value for \(N_s\) times until it satisfies the convergence condition, which will be discussed in detail in following parts.

III. PROBLEM FORMULATION

Considering the nonlinear CoM dynamics and physical limits, a NMPC is established and solved to generate walking patterns. This section discusses the objective function and the feasibility constraints in detail.

A. Objective Function

Given the reference footstep location and step time, we need to minimize the error between actual and desired CoM positions, the error between actual and desired upper-body inclination angles, and the error between actual and desired footstep locations. Also, we minimize the velocities and jerks of CoM movement and body inclination. Thus, at the \(k\)th sampling time, we have the objective function as follows:

\[f = \sum_{X} \left\{ \frac{\alpha_X}{2} \| \ddot{X}(k) \|^2 + \frac{\beta_X}{2} \| X(k) - X_{\text{ref}}(k) \|^2 \right. \]
\[\left. + \frac{\gamma_X}{2} \| \dddot{X}(k) \|^2 \right\} + \sum_{U} \frac{\delta_U}{2} \| U(k) - U_{\text{ref}}(k) \|^2.\]

(7)

Here \(\alpha_X, \beta_X, \gamma_X\) and \(\delta_U\) are the velocity, position tracking, jerk, support position tracking penalties, respectively, and these penalties should be greater than zero so that \(G\) is positive-definite. \(X_{\text{ref}}(k) = [x_{\text{ref}}(k+1), \ldots, x_{\text{ref}}(k+N_h)]^T\) and \(U_{\text{ref}}(k) = [u_{\text{ref}}(k+1), \ldots, u_{\text{ref}}(k+N_h)]^T\) are the reference state of CoM position and upper-body inclination angles over the prediction horizon. \(U(k) = [u(k+1), \ldots, u(k+N_h)]^T\) and \(U_{\text{ref}}(k) = [u_{\text{ref}}(k+1), \ldots, u_{\text{ref}}(k+N_h)]^T\) are the actual and reference future footstep locations over the prediction horizon, respectively (\(u \in \{d_x, d_y, d_z\}\)). \([D_{x}(k), D_{y}(k), D_{z}(k)]^T\) are the actual horizontal footstep locations over the prediction horizon, where, e.g. \(D_{x}(k) = [d_x(k+1), \ldots, d_x(k+N_h)]^T\). Particularly, in this paper, the actual footstep height \((D^e_{z}(k))\) is set to be the desired step height \((D_{z}(k))\), which is determined offline by the surface.

1In this paper, we use \([d_x(k+1), \ldots, d_x(k+N_h)]^T\) to denote the future footstep locations of different walking cycles falling on the prediction horizon, and \([d_x(k+1), \ldots, d_x(k+N_h)]^T\) to denote footstep positions at different sampling times over the prediction horizon.

Under this objective function, the parameters for (5) can be calculated by (4), with more details can be found in Appendix A. During each supporting period, the reference CoM positions along \(x\) and \(y\) axis are set to be the center of the reference supporting footstep locations. Besides, the reference roll angles and pitch angles are set to be zero during the whole walking process. Furthermore, the reference CoM height is the sum of a constant height difference between CoM and the supporting foot (denoted by \(h_{\text{ref}}\)) and the reference footstep height. That is, the reference body inclination and CoM height are determined by

\[\left\{ \begin{array}{l}
\theta^\text{ref}_{i(k+i)} = \theta^\text{ref}_{p(k+i)} \equiv 0, \quad i \in \{1, \ldots, N_h\}, \\
n^\text{ref}_{i(k+i)} = h^\text{ref}_{i} + d^\text{ref}_{i(k+i)}, \quad i \in \{1, \ldots, N_h\},
\end{array} \right.\]

where \([\theta^\text{ref}_{i(k+i)}, \ldots, \theta^\text{ref}_{i(k+N_h)}]\) and \([n^\text{ref}_{i(k+i)}, \ldots, n^\text{ref}_{i(k+N_h)}]\) denote the reference roll angles and pitch angles during the prediction horizon, respectively, \(h^\text{ref}_{i}\) is the constant height difference determined by physical structure, \([n^\text{ref}_{i(k+i)}, \ldots, n^\text{ref}_{i(k+N_h)}]\) denote the reference CoM height, \([\theta^\text{ref}_{x(k+i)}], \ldots, \theta^\text{ref}_{x(k+N_h)}]\) denote the reference footstep height which is determined by the surface height configuration.

Seen from (6), we do not design the specific body inclination angles and CoM height trajectories according to different walking scenarios in advance. Even though, by using the objective function (7), the optimal time-varying body inclination angle and the CoM height trajectory can be generated online when faced with external disturbances, which will be demonstrated in following sections.

B. Constraints

To guarantee the feasibility, the proposed framework takes into account the constraints of ZMP movement, footstep location variation, CoM vertical motion, upper-body inclination and hip joint torques output. Furthermore, these constraints are expressed in quadratic forms in this work.

1) Constraints on ZMP trajectory: The ZMP should be inside the support polygon to keep walking stability. Although the support polygon changes when switching from single supporting to double supporting, we only consider the single supporting during the walking process. Since those constraints of single supporting are the most restrictive and the sampling time can be large enough in MPC framework, this simplification is reasonable, as pointed out in [11]. At the \(k\)th sampling time, taking the movement in sagittal plane for instance, the following constraint needs to be satisfied:

\[P_{x_{\text{min}}} \leq P_{x(k+i)} - d_{x(k+i)} \leq P_{x_{\text{max}}}, \quad i \in \{1, \ldots, N_h\},\]

(9)

where \([P_{x(k+1)}, \ldots, P_{x(k+N_h)}]\) denote the actual ZMP trajectory over the prediction horizon along the \(x\)-axis, \([d_{x(k+1)}, \ldots, d_{x(k+N_h)}]\) denote the actual foot location over the prediction horizon along the \(x\)-axis, \(P_{x_{\text{min}}}\) and \(P_{x_{\text{max}}}\) are the lower and upper ZMP boundary along \(x\)-axis, which are determined by physical structure of supporting foot.

The ZMP movement in the coronal plane should also satisfy constraint conditions. Using CoM dynamics expressed
in [1] and [2], the ZMP constraints are nonlinear inequalities. Inspired by [22] and [30], they are formulated as quadratic forms, with more details can be found in Appendix B.

2) Constraints on footstep location: The objective function takes the footstep locations as variables that can change arbitrarily (except for footstep height). However, we need to make sure that these modifications can be realized physically with considering feasibility limitations, such as maximal leg length, maximal joint velocities, self-collision avoidance etc. In this paper, the following limitations are considered.

Firstly, due to structural limitation and actuation capability limitation, the range of step parameters including step length and step width should be constrained. At the present, these limitations are simplified to be following linear inequalities (taking the step length for instance):

\[
\begin{align*}
    d_{x}^{\text{min}} & \leq d_{x}(k,i) - \dot{d}_{x}(k) \leq d_{x}^{\text{max}}, & i = 1, \\
    d_{x}^{\text{min}} & \leq d_{x}(k,i) - d_{x}(k,i-1) \leq d_{x}^{\text{max}}, & i \in \{2, ..., N_f\},
\end{align*}
\]

where \(\dot{d}_{x}(k)\) denotes the current supporting foot position along the \(x\)-axis, \(d_{x}^{\text{min}}\) and \(d_{x}^{\text{max}}\) are lower and upper boundaries of step length.

Secondly, since the future foot location corresponding to the same walking cycle is re-computed in every loop, it may change in the real time. However, the position of swing foot, which is related to the future footstep, can not change rapidly due to the joint velocity limitation. In this paper, to be brief, rather than constraining the swing foot trajectory strictly, we limit the change of future footsteps corresponding to different control loops. Since this constraint is imposed at each time interval, limiting the next one future step is enough. Thus, we have:

\[
d_{x}^{\text{min}} \Delta t \leq d_{x}(k,1) - d_{x}(k-1,1) \leq d_{x}^{\text{max}} \Delta t, \tag{11}
\]

where \(d_{x}(k,1)\) and \(d_{x}(k-1,1)\) are the next one footstep position corresponding to the same period computed by current and last optimization loop, \(d_{x}^{\text{min}}\) and \(d_{x}^{\text{max}}\) are the lower and upper velocity boundaries along the \(x\)-axis.

Thirdly, as mentioned above, the actual footstep height is merely determined by the surface height configuration. Thus, we have the equality constraint on footstep height as follows:

\[
d_{z}(k,i) = d_{z(k,i)}^{\text{ref}}, i \in \{1, ..., N_h\}, \tag{12}
\]

3) Constraints on CoM motion: The time-varying CoM vertical height can change quickly to stabilize the ZMP, but this type of motion should be constrained to avoid unreliable trajectories when considering the kinematic constraints on a real robot. The limitation of the CoM vertical motion leads to following constraints:

\[
h_{c}^{\text{min}} \leq c_{z}(k,i) - d_{z}(k,i) - d_{z(k,i)}^{\text{ref}} \leq h_{c}^{\text{max}}, i \in \{1, ..., N_h\}, \tag{13}
\]

where \(h_{c}^{\text{min}}\) and \(h_{c}^{\text{max}}\) are the lower and upper boundaries of the vertical height variance, \([d_{z}(k+1), ..., d_{z(k+N_h)}]\) denote the actual vertical height of the supporting foot.

Additionally, since the ground surface only generates unilateral reactive forces, the CoM acceleration in the downward direction should be limited to the rate of free fall. That is:

\[
\ddot{c}_{z(k+i)} \geq -g, i \in \{1, ..., N_h\}. \tag{14}
\]

4) Constraints on body inclination: The trunk rotation is limited by articulation constraints. With such a simple model, it will be constrained by allowable bounds (taking the roll angle for instance):

\[
\theta_{r}^{\text{min}} \leq \theta_{r}(k+i) \leq \theta_{r}^{\text{max}}, i \in \{1, ..., N_h\}, \tag{15}
\]

where \(\theta_{r}^{\text{min}}, \theta_{r}^{\text{max}}\) are the lower and upper boundaries of roll angle.

5) Constraints on hip torque: Taking the roll direction for instance:

\[
\tau_{r}^{\text{min}} \leq I_{r} \ddot{\theta}_{r}(k+i) \leq \tau_{r}^{\text{max}}, i \in \{1, ..., N_h\}, \tag{16}
\]

where \(\tau_{r}^{\text{min}}\) and \(\tau_{r}^{\text{max}}\) are the lower and upper limits of roll torque.

**IV. SIMULATION RESULTS**

In this section, we demonstrate the effectiveness of the proposed framework by generating walking pattern on uneven terrain and under external disturbances. The IPFM simulations and the the whole-body humanoid simulations are conducted. For both parts, the fixed time interval \(\Delta t\) is 0.1 s and the predictive length \(N_{h}\) is 15. Besides, the walking cycle \((T)\) is a constant (0.8 s). Other basic parameters for simulations are listed in Appendix C.
A. IPFM Simulations on CogIMoN humanoid robot

This part was conducted on CogIMoN robot [4]. The reference step length and width were 0.3 m and 0.4 m.

1) 3D walking on uneven terrain: Firstly, the proposed framework was tested by a 3D walk scenario which requires walking upstairs and downstairs. All stairs had the same height (0.15 m) and the same length (0.3 m).

Seen from Fig. 2 with keeping the ZMP trajectory within the allowable region formed by the supporting foot, the proposed method is able to generate 3D gait pattern with exactly tracking the reference gait parameters while satisfying all the constraints. In addition, for the vertical motion, seamless transition between CoM trajectory in frontal plane was realized, as can be seen in Fig. 3. Within one gait cycle, the frontal CoM motion depicted the butterfly shape, which is similar to what has been observed in human walking [7]. Furthermore, as seen in Fig. 4 the robot also slightly rotated the upper-body to maintain stability when going upstairs and downstairs. Since the swing foot would swing from the back to the front relative to the supporting foot while the body remains on the same side relative to the supporting foot during one supporting period, the frequency of pitch inclination are double of that of the roll inclination. Due to the integration of the body vertical motion, the upper-body inclination is suppressed.

2) Balance recovery from external disturbances: In this part, we analyze the recovery capability from external push by using footstep adjustment (strategy 1), using footstep adjustment and body inclination adaptation (strategy 2) and integrating footstep adjustment, body inclination adaptation and vertical height motion (strategy 3). The strategy 1 and strategy 2 can be easily realized by imposing additional equality constraints on the generated CoM height trajectory, roll angle and pitch angle. The external forces along x- and y- axis were applied to the pelvis at 2 s, lasting 0.25 s.

The walking patterns generated by different strategies, under the same external push (forward 180 N, lateral 127 N in this case) are shown in Fig. 5. For the first test, only the step location adjustment (strategy 1) was activated, which has been used in many previous work such as [11]. As can be seen in the left part of Fig. 5 the robot timely adjusted the step locations after the external push occurred. And after six steps, the robot finally fully recovered from the push and tracked the reference footstep locations exactly again.

On the other hand, by combining the footstep adjustment with the online optimization of the change of the body angular momentum, less steps were needed to recover from the same push force, as indicated in the center part of Fig. 5. Moreover, compared with strategy 1, smaller step length and width variations were needed.

Using strategy 3, which can be seen in the right part of Fig. 5 least number of reactive steps with smallest variation of step length and step width were needed to overcome the same external disturbance. Furthermore, due to the vertical CoM variation shown in Fig. 6, less body inclination and smaller torque output than strategy 2 were also observed, as can be seen in Fig. 7 and Fig. 8. Different from [28] and [29], the time-varying CoM trajectory was generated online by the proposed approach.

Further analysis also revealed that, more reactive strategies improve the ability recovering from large external pushes. As listed in Table I, strategy 1 could only withstand 207 N forward force ($F_x$) and 127 N lateral force ($F_y$) while strategy 2 could reject 225 N forward force and 142 N lateral force. As expected, by integrating all the three balancing strategies, the robot could recover from much larger pushes (326 N forward force and 226 N lateral force), hence, improving its robustness for push recovery.

B. Whole-body simulation on COMAN humanoid robot

To further demonstrate the effectiveness of this method, the whole-body simulations were conducted using the physical characteristic of COMAN robot [31]. The swing foot trajectory was generated by the 5th polynomial interpolation. The animation can be seen in [32].

1) 3D walking on uneven terrain: Whole-body simulation demonstrated that, with using CoM height adaptation, the robot can walk stably on the stairs with 9 cm height (21% of the leg length) while only 3 cm height without CoM vertical motion. The snapshots of walking on 7 cm stairs are shown in Fig. 9. To be brief, only the CoM trajectory and foot trajectories are shown in Fig. 10. Seen from the Fig. 10 the framework generated feasible CoM trajectory and swing foot locations when walked on the uneven terrain.

2) Balance recovery from external pushes: Similar with Section IV-A.2 we analyze the recovery capability from external push when using three different strategies. The external forces were applied to the pelvis at 3.6 s and lasted 0.1 s. To be brief, only the maximal tolerant push forces are listed in Table II. Again, the robot achieves the strongest recovery capability from external pushes by integrating the
Fig. 5: Horizontal CoM trajectory, ZMP trajectory and footstep locations when faced with external push generated by using three strategies, the green blocks represent footstep locations.

Fig. 6: Vertical height trajectories generated by different strategies.

Fig. 7: Body inclination angles generated by different strategies.

Fig. 8: Hip torques generated by different strategies.

Fig. 9: COMAN robot walked on uneven terrain, the stair height is 7 cm (16% of the leg length).

Fig. 10: CoM trajectory and foot trajectories of COMAN robot when walking on 7 cm stairs.

### TABLE II: Maximal tolerant push forces the robot can reject under different strategies

| Force | Strategy | strategy 1 | strategy 2 | strategy 3 |
|-------|----------|------------|------------|------------|
| $F_x$/N | 169 | 187 | 275 |
| $F_y$/N | 156 | 170 | 251 |

C. Computation Efficiency

In this paper, we use the SQP approach (6) to solve the QCQP (5). Different from [22], in this paper, the SQP loop would terminate when satisfying following condition:

$$ \| \Delta x \| \leq \varepsilon \quad \text{or} \quad N_s > 3. \quad (17) $$

Under this termination condition, the C++ optimization library QuadProg++ (available under GNU General Public License) is used to solve the NMPC problem. It turns out that the time cost for each NMPC loop is less than 8 ms on a 3.0 GHz quad-core CPU. As a result, it can be implemented on a real robot in real time.

V. Conclusion

In this paper, we proposed a versatile and robust walking pattern generation framework based on NMPC for bipedal locomotion. The proposed framework is formulated as a QCQP and solved via the off-the-shelf SQP technique.

Using the IPFM, the ZMP constraints are formulated in a quadratic form with the consideration of variable CoM height and angular momentum change. Combining with footstep location adjustment, angular momentum optimization and vertical height adaptation in this framework.

In this paper, we proposed a versatile and robust walking pattern generation framework based on NMPC for bipedal locomotion. The proposed framework is formulated as a QCQP and solved via the off-the-shelf SQP technique.

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adjustment and other feasibility constraints, robust walking was realized with the capabilities of reactive stepping, CoM height variation and upper-body inclination. In contrast to the previous work, the proposed framework can generate versatile walking patterns without strictly tracking the reference CoM height motion and angular momentum changes. Simulation studies showed that, the robot was able to achieve higher adaptability under realistic scenarios.

We believe a promising future about this framework, since it can be used to generate the walking patterns and to keep balance as well for humanoids in the real environment. At the present, we set the penalties in objective function out of experience, which may be unnatural or energy-consuming for humanoids. Thus, studying the priority of different strategies under realistic scenarios can be a next focus. In addition, experiments on a real robot is current work.

VI. APPENDIX
A. Parameters for Objective Function

For the objective function defined in (7), using 4, the G, g and state X in (5) are expressed as follows:

\[
G = \text{diag}(\theta_{c_1}, \theta_{c_2}, \theta_{\Theta_1}, \theta_{\Theta_2}, \phi_{D_1}, \phi_{D_2}, \phi_{D_3}),
\]

\[
\partial_x = \frac{\gamma S}{2} I_{N_h \times N_h} + \frac{\alpha_x}{2} P_v^T P_v + \frac{\beta_x}{2} P_u^T P_u,
\]

\[
\phi_u = \frac{\delta_u}{2} I_{N_f \times N_f},
\]

\[
g = \begin{bmatrix}
\alpha_c P_v^T P_v + \beta_c P_u^T P_u, \\
\alpha_c P_v^T P_v + \beta_c P_u^T P_u, \\
\alpha_c P_v^T P_v + \beta_c P_u^T P_u, \\
(\alpha_{\Theta_1} P_v^T P_v + \beta_{\Theta_1} P_u^T P_u)\theta_{\Theta_1} - \beta_{\Theta_1} P_u^T P_u, \\
(\alpha_{\Theta_2} P_v^T P_v + \beta_{\Theta_2} P_u^T P_u)\theta_{\Theta_2} - \beta_{\Theta_2} P_u^T P_u, \\
-\delta_d D_x^T, \\
-\delta_d D_x^T, \\
-\delta_d D_x^T,
\end{bmatrix}
\]

\[
X^k = [\bar{C}_x(k); \bar{C}_y(k); \bar{C}_z(k); \bar{\Theta}_1(k); \bar{\Theta}_2(k); \bar{D}_x(k); \bar{D}_y(k); \bar{D}_z(k)],
\]

where, \(\text{diag}()\) is a function that produces a diagonal matrix with the given parameters in the diagonal positions, \(X \in \{C_1, C_2, C_3, \Theta_1, \Theta_2\}\), \(U \in \{D_x, D_y, D_z\}\).

B. Quadratic Form of Feasibility Constraints

All the constraints introduced in Section III-B can be formulated in quadratic form as expressed in (5). At the \(k\)-th sampling time, defining the selection matrices, the predictive CoM ject and foot locations can be expressed as follows:

\[
U_{(k)} = S_u X^k, \quad S_u \in \mathbb{R}^{N_y \times N_i},
\]

\[
\bar{X}_{(k)} = S_x X^k, \quad S_x \in \mathbb{R}^{N_h \times N_i},
\]

\[
\bar{x}_{(k+j)} = S_j \bar{X}_{(k)}, \quad S_j \in \mathbb{R}^{1 \times N_h}, j \in \{1, \ldots, N_h\}.
\]

TABLE III: Weight coefficients and other model parameters

| \(\alpha_{c_1}\) | \(10/10\) | \(\alpha_{c_2}\) | \(10/10\) |
| \(\alpha_{\Theta_1}\) | \(10/10\) | \(\alpha_{\Theta_2}\) | \(10/10\) |
| \(\alpha_{\Theta_3}\) | \(10/5 \times 10^4\) | \(\alpha_{\Theta_4}\) | \(10/5 \times 10^5\) |
| \(\beta_{c_1}\) | \(10/5 \times 10^5\) | \(\beta_{c_2}\) | \(5 \times 10^5\) |
| \(\beta_{\Theta_1}\) | \(10/5 \times 10^4\) | \(\beta_{\Theta_2}\) | \(10/5 \times 10^5\) |
| \(\beta_{\Theta_3}\) | \(10/5 \times 10^5\) | \(\beta_{\Theta_4}\) | \(10/5 \times 10^5\) |
| \(\gamma_{c_1}\) | \(1/1\) | \(\gamma_{c_2}\) | \(1/1\) |
| \(\gamma_{\Theta_1}\) | \(1/100\) | \(\gamma_{\Theta_2}\) | \(1/100\) |
| \(\gamma_{\Theta_3}\) | \(1/100\) | \(\gamma_{\Theta_4}\) | \(1/100\) |
| \(\delta_{D_1}\) | \(5 \times 10^5 / 5 \times 10^7\) | \(m_{[kg]}\) | \(100/31\) |
| \(b_{ref}[m]\) | \(1.02/0.466\) | \(g_{[m/s^2]}\) | \(9.8/9.8\) |
| \(I_x(J_f)[kg\cdot m^2]\) | \(16/1.24\) | \(N_f\) | \(2/2\) |

where, \(\bar{D}_x(k) = [d_{ref}^{(k+1)}; \ldots; d_{ref}(k+N_h)]^T\) consists of the foot locations over the prediction horizon, \(\bar{d}_x(k)\) denotes position of the current supporting foot, the \(e_1(k)\) and \(e_1(k)\) are mapping matrix, with more details can be found in [11].

And then, we have position of supporting foot as:

\[
d_x(k+j) = S_j \bar{D}_x(k), j \in \{1, \ldots, N_h\}. \tag{21}
\]

**quadratic form of ZMP constraints:** To be brief, we only discuss the upper boundary. Taking the motion along the \(x\)-axis for instance, substituting (18) into (9), we have:

\[
(c_x(k+j) - d_x(k+j) - p_x^{max})(g + \dot{c}_x(k+j)) - (c_x(k+j) - d_x(k+j))\dot{c}_x(k+j) - I_y \dot{y}(k+j)/m \leq 0 \tag{22}
\]

Then, by substituting (18), (19) and (21) into (22) and collecting terms, the quadratic form of ZMP constraints is:

\[
V_{p_{(j)}} = m(S_j^T P_{pu} P_{wu} S_{j} P_{wu} S_{j} - S_j^T E_{j}(k) S_j P_{wu} S_{j}), \tag{23}
\]

\[
V_{p_{(j)}} = m(c_{j}(k+1) P_{pu} S_{j} P_{wu} S_{j} S_j P_{wu} S_{j} - c_{j}(k) P_{pu} S_{j} P_{wu} S_{j} + c_{j}(k) P_{pu} S_{j} P_{wu} S_{j} S_j P_{wu} S_{j}) + d_{j}(k) S_j P_{wu} S_{j}
\]

\[
- (c_{j}(k) P_{pu} S_{j} J_{f} E_{j}(k) S_j P_{wu} S_{j} + \dot{d}_x(k) S_j P_{wu} S_{j}) - g S_j E_{j}(k) S_j P_{wu} S_{j} - \dot{p}_x^{max} S_j P_{wu} S_{j} - I_y S_j P_{wu} S_{j}), \tag{24}
\]

\[
\sigma_{p_{(j)}} = m(c_{j}(k) P_{pu} S_{j} P_{wu} S_{j} - c_{j}(k) P_{pu} S_{j} P_{wu} S_{j}) + d_{j}(k) S_j P_{wu} S_{j}
\]

\[
- (c_{j}(k) P_{pu} S_{j} J_{f} E_{j}(k) S_j P_{wu} S_{j} + \dot{d}_x(k) S_j P_{wu} S_{j}) - g S_j E_{j}(k) S_j P_{wu} S_{j} - \dot{p}_x^{max} S_j P_{wu} S_{j} - I_y S_j P_{wu} S_{j}), \tag{25}
\]

C. Parameters Setup for Simulations

The basic parameters for IPFM simulation and whole-body humanoid simulation can be seen in Table III and Table IV. For each item in Table III and Table IV, the right side is for IPFM simulation and the left side is for whole-body simulation.
TABLE IV: Parameters for feasibility constraints

| Constraint          | Lower Bound | Upper Bound | Description                  |
|---------------------|-------------|-------------|------------------------------|
| Footstep location   |             |             | Body inclination Constraints |
| $d_{min}^m$ [m]     | -0.5/-1     | 3/3         | $\dot{x}_{min} [-\dot{x}_{max}]/N \cdot m$ |
| $d_{max}^m$ [m]     |             |             | $\dot{x}_{min} [-\dot{x}_{max}]/N \cdot m$ |
| $\dot{\theta}_{min}$ [rad] | -0.15/-0.1  | 0.15/0.1    | CoM output constraints        |
| $\dot{\theta}_{max}$ [rad] |             |             | $\dot{\theta}_{min} [-\dot{\theta}_{max}]/N \cdot m$ |
| $\dot{h}_{min}$ [m] | -0.25       | 0.7         | $h_{min}$                      |
| $\dot{h}_{max}$ [m] |             |             | $h_{max}$                      |
| $\dot{s}_{t}$ [m/s] | -0.15       | 0.15        | $s_{t}$                        |
| $\dot{s}_{f}$ [m/s] | -0.15       | 0.15        | $s_{f}$                        |

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