Wolfenstein-like Parametrization of the Neutrino Mixing Matrix

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Abstract

We show that the $3 \times 3$ lepton flavor mixing matrix $V$ can be expanded in powers of a Wolfenstein-like parameter $\Lambda \equiv |V_{\mu 3}| \sim 1/\sqrt{2}$, which measures the strength of flavor conversion in atmospheric neutrino oscillations. The term of $O(\Lambda^2)$ is associated with the large mixing angle in solar neutrino oscillations. The Dirac phase of CP violation enters at or below $O(\Lambda^8)$, and the Majorana phases of CP violation are not subject to the $\Lambda$-expansion. Terrestrial matter effects on this new parametrization in realistic long-baseline neutrino oscillation experiments are briefly discussed. Some comments are also given on the possible relation between $\Lambda$ and a relatively weak hierarchy of neutrino masses.

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The KamLAND neutrino experiment [1] has recently confirmed the large-mixing-angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution [2] to the solar neutrino problem. Meanwhile, the K2K long-baseline neutrino experiment [3] has unambiguously observed a reduction of $\nu_\mu$ flux and a distortion of the energy spectrum. These new measurements, together with the robust SNO evidence [4] for the flavor conversion of solar $\nu_e$ neutrinos and the compelling Super-Kamiokande evidence [5] for the deficit of atmospheric $\nu_\mu$ neutrinos, convinces us that the hypothesis of neutrino oscillations is indeed correct! We are then led to the exciting conclusion that neutrinos do have masses and lepton flavor mixing does exist.

A global analysis of today’s solar neutrino data [6] indicates that the maximal mixing is strongly disfavored for the LMA solution. The mixing factor of atmospheric neutrino oscillations is found to be almost maximal [7], on the other hand. Taking account of the KamLAND, K2K, SNO, Super-Kamiokande and CHOOZ results [8], we expect that the $3 \times 3$ lepton flavor mixing matrix $V$ is typically of a constant pattern [9] in the leading-order approximation:

$$V = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\sqrt{\frac{2}{4}} & \sqrt{\frac{6}{4}} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{4} & -\sqrt{\frac{6}{4}} & \frac{\sqrt{2}}{2} \end{pmatrix}. \tag{1}$$

Namely,

$$\{\theta_{\text{sun}}, \theta_{\text{atm}}, \theta_{\text{chz}}\} = \{30^\circ, 45^\circ, 0^\circ\}, \tag{2}$$

or $\sin^2 2\theta_{\text{sun}} = 3/4$, $\sin^2 2\theta_{\text{atm}} = 1$ and $\sin^2 2\theta_{\text{chz}} = 0$. Note that $|V_{e3}| = |V_{\mu3}|^2$ holds in this simplified neutrino mixing pattern. It implies that an expansion of $V$ in terms of $V_{\mu3}$ is actually possible. Note also that one may introduce a small perturbation into $V$, such that:

1. $\theta_{\text{sun}}$ gets closer to its best fit value ($\theta_{\text{sun}} \sim 32^\circ$ [6]);
2. $\sin^2 2\theta_{\text{atm}}$ deviates slightly from unity ($\sin^2 2\theta_{\text{atm}} > 0.92$ [7]);
3. $|V_{e3}| = \sin \theta_{\text{chz}} \neq 0$ appears ($\sin^2 2\theta_{\text{chz}} < 0.1$ [8]);
4. a Dirac phase of CP violation can naturally be included into $V$ [10].

Motivated by these observations, we proceed to propose a new parametrization of the lepton flavor mixing matrix $V$, in which all matrix elements are expanded in powers of a parameter $\Lambda \equiv |V_{\mu3}| \sim 1/\sqrt{2}$. Clearly $\Lambda$ can be regarded as the leptonic analog of the well-known Wolfenstein parameter $\lambda \equiv |V_{us}| \approx 0.22$ for quark flavor mixing [11].

The first step is to parametrize $V$ in the limit of $V_{e3} = 0$. We obtain

$$V = \begin{pmatrix} \sqrt{1 - A^2 \Lambda^4} & A\Lambda^2 & 0 \\ -A\Lambda^2 \sqrt{1 - A^2} & \sqrt{(1 - \Lambda^2)(1 - A^2 \Lambda^4)} & \Lambda \\ A\Lambda^3 & -\Lambda \sqrt{1 - A^2 \Lambda^4} & \sqrt{1 - \Lambda^2} \end{pmatrix}, \tag{3}$$

where $A = |V_{e2}|/|V_{\mu2}|$.
where $A$ is a positive coefficient of $\mathcal{O}(1)$. If $A = 1$ and $\Lambda = 1/\sqrt{2}$ are taken, the constant pattern of $V$ in Eq. (1) can straightforwardly be reproduced from Eq. (3). It is obvious that $\Lambda$ measures the strength of flavor mixing in atmospheric neutrino oscillations, while $A\Lambda^2$ characterizes the magnitude of flavor mixing in solar neutrino oscillations.

The second step is to introduce small corrections to $V$ in Eq. (3), such that $|V_{e3}| \neq 0$ appears. Because $|V_{e3}| < 0.16$ is required \[\text{[8]},\] we may take $|V_{e3}| \sim \mathcal{O}(A^8) \sim 0.06$ as a typical possibility for $\Lambda \sim 1/\sqrt{2}$. Smaller values of $|V_{e3}|$ are certainly allowed. In a number of phenomenological models for lepton flavor mixing \[\text{[12]},\] however, $|V_{e3}| \sim \sqrt{m_e/m_\mu} \sim 0.07$ is naturally predicted. Hence $\mathcal{O}(A^8)$ could be the plausible order of $|V_{e3}|$. Let us fix the matrix elements $V_{\mu 2}$, $V_{e3}$ and $V_{\mu 3}$ by use of four independent parameters:

$$V_{\mu 3} = \Lambda, \quad V_{e2} = A\Lambda^2, \quad V_{e3} = B\Lambda^8 e^{-i\delta},$$

(4)

where $B$ is of $\mathcal{O}(1)$ or smaller, and $\delta$ denotes the Dirac phase of leptonic CP violation. Given Eq. (4), one may make use of the unitarity of $V$ to work out exact analytical expressions for the other six matrix elements. The relevant results are quite complicated and will be presented elsewhere. We find that it is more instructive to approximate $V$ as

$$V = \begin{pmatrix}
\sqrt{1 - A^2\Lambda^4} & A\Lambda^2 & B\Lambda^8 e^{-i\delta} \\
-A\Lambda^2\sqrt{1 - \Lambda^2} & \sqrt{(1 - \Lambda^2)(1 - A^2\Lambda^4)} & \Lambda \\
\Lambda^3 [A - B\Lambda^8 \sqrt{(1 - \Lambda^2)(1 - A^2\Lambda^4)} e^{i\delta}] & -A\sqrt{1 - A^2\Lambda^4} & \sqrt{1 - \Lambda^2}
\end{pmatrix}.$$  (5)

In this approximation, the unitary normalization relations of $V$ keep valid to $\mathcal{O}(\Lambda^11) \sim 2\%$. Hence Eq. (5) is sufficiently accurate to describe lepton flavor mixing, not only in solar and atmospheric neutrino oscillations, but also in some of the proposed long-baseline neutrino oscillation experiments where leptonic CP violation is not concerned \[\text{[13]},\] As the unitary orthogonality relations of $V$ in the above approximation are valid to $\mathcal{O}(\Lambda^8) \sim 6\%$, the leptonic unitarity triangles can also be described by Eq. (5) to a reasonably good degree of accuracy. A rephasing-invariant measure of leptonic CP violation is the well-known Jarlskog parameter $J$ \[\text{[13]},\] whose magnitude must be proportional to $\sin \delta$ in our parametrization. The explicit expression of $J$ reads

$$J = ABA^{11}\sqrt{(1 - \Lambda^2)(1 - A^2\Lambda^4)} \sin \delta.$$  (6)

Note again that $\Lambda^{11} \sim 0.02$ for $\Lambda \sim 1/\sqrt{2}$. Thus $|J|$ may be at the percent level, if $\delta \sim \pm 90^\circ$ holds. No matter whether neutrinos are Dirac or Majorana particles, the strength of CP and T violation in normal neutrino-neutrino and antineutrino-antineutrino oscillations is always governed by $J$ or by the Dirac phase $\delta$ \[\text{[13]},\]. Furthermore, the off-diagonal asymmetries of the lepton flavor mixing matrix $V$ \[\text{[16]}\] read as

$$\mathcal{A}_L \equiv |V_{e2}|^2 - |V_{\mu 1}|^2 = |V_{\mu 3}|^2 - |V_{\tau 2}|^2 = |V_{\tau 1}|^2 - |V_{e3}|^2$$

$$= A^2\Lambda^6,$$

$$\mathcal{A}_R \equiv |V_{e2}|^2 - |V_{\mu 3}|^2 = |V_{\mu 1}|^2 - |V_{\tau 2}|^2 = |V_{\tau 3}|^2 - |V_{e1}|^2$$

$$= \Lambda^2 (A^2\Lambda^2 - 1).$$  (7)
We see that $A_L > 0$ holds definitely. In comparison, the sign of $A_R$ cannot be fixed from the present experimental data. It is actually possible to obtain $A_R = 0$, when $A^2\Lambda^2 = 1$ is satisfied. In this interesting case, the lepton flavor mixing matrix $V$ is exactly symmetric about its $V_{e3}V_{\mu2}V_{\tau1}$ axis \[7\].

The final step is to incorporate $V$ with two Majorana phases of CP violation, provided neutrinos are Majorana particles. To do so, we simply multiply $V$ on its right-hand side with a pure phase matrix; i.e.,

$$V \rightarrow VP, \quad P = \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}, \quad (8)$$

in which $\rho$ and $\sigma$ are the Majorana-type CP-violating phases \[10\]. The phase convention of $P$ chosen in Eq. (8) is to make the CP-violating phase $\delta$ not to manifest itself in the effective mass term of the neutrinoless double beta decay:

$$\langle m \rangle_{ee} = \left| m_1 \left(1 - A^2\Lambda^4 \right)e^{2i\rho} + m_2A^2\Lambda^4e^{2i\sigma} + m_3\Lambda^8 \right|, \quad (9)$$

where $m_i$ (for $i = 1, 2, 3$) are physical neutrino masses. This result can somehow get simplified, if a specific pattern of the neutrino mass spectrum is assumed. The present experimental upper bound is $\langle m \rangle_{ee} < 0.35$ eV (at the 90% confidence level \[18\]), from which no constraint on $\rho$ and $\sigma$ can be got.

Now let us establish the direct relations between ($\Lambda$, $A$, $B$) and ($\theta_{\text{atm}}$, $\theta_{\text{sun}}$, $\theta_{\text{chz}}$). With the help of Eq. (4) and

$$|V_{e2}|^2 = \frac{\cos^2 \theta_{\text{chz}}}{2} - \frac{\sqrt{\cos^4 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}}}{2},$$

$$|V_{e3}|^2 = \sin^2 \theta_{\text{chz}},$$

$$|V_{\mu3}|^2 = \sin^2 \theta_{\text{atm}}, \quad (10)$$

which have been obtained in Ref. \[19\], we arrive at

$$\Lambda = \sin \theta_{\text{atm}},$$

$$A = \frac{\sqrt{\cos^2 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}}}{\sqrt{2} \sin^2 \theta_{\text{atm}}},$$

$$B = \frac{\sin \theta_{\text{chz}}}{\sin^8 \theta_{\text{atm}}}. \quad (11)$$

Once the mixing angles $\theta_{\text{atm}}$, $\theta_{\text{sun}}$ and $\theta_{\text{chz}}$ are precisely measured, we may use Eq. (11) to determine the magnitudes of $\Lambda$, $A$ and $B$. For the purpose of illustration, we typically take $0.25 \leq \sin^2 \theta_{\text{sun}} \leq 0.40$ \[3\], $\sin^2 2\theta_{\text{atm}} > 0.92$ \[7\] and $\sin^2 2\theta_{\text{chz}} < 0.1$ \[8\] to calculate the allowed regions of $\Lambda$, $A$ and $B$. Then we obtain $0.6 \leq \Lambda \leq 0.8$ straightforwardly. The numerical results for $A$ and $B$ are presented in Fig. 1, from which $0.8 \leq A \leq 1.94$ and $0 \leq B \leq 9.5$ can directly be read off.
It is also worthwhile to connect \((\Lambda, A, B)\) to \((\theta_{12}, \theta_{23}, \theta_{13})\), which are three mixing angles of the “standard parametrization” of \(V\) \(^{20}\). We find

\[
\sin \theta_{12} = \frac{A\Lambda^2}{\sqrt{1 - B^2\Lambda^4}} \approx A\Lambda^2, \\
\sin \theta_{23} = \frac{\Lambda}{\sqrt{1 - B^2\Lambda^4}} \approx \Lambda, \\
\sin \theta_{13} = BA^8. \quad (12)
\]

In addition, the Dirac phase of CP violation in the standard parametrization is exactly equal to \(\delta\) defined in the present Wolfenstein-like parametrization.

An interesting point is that the effective lepton flavor mixing matrix in matter, which is denoted as \(\tilde{V}\) \(^{21}\), can similarly be parametrized in terms of four matter-corrected parameters \(\tilde{\Lambda}, \tilde{A}, \tilde{B}\) and \(\tilde{\delta}\):

\[
\tilde{V} = \begin{pmatrix}
\sqrt{1 - \tilde{A}^2\tilde{\Lambda}^4} & \tilde{A}\tilde{\Lambda}^2 & \tilde{B}\tilde{\Lambda}^8 e^{-i\tilde{\delta}} \\
-\tilde{A}\tilde{\Lambda}^2\sqrt{1 - \tilde{\Lambda}^2} & \sqrt{(1 - \tilde{\Lambda}^2)(1 - \tilde{A}^2\tilde{\Lambda}^4)} & \tilde{\Lambda} \\
\tilde{\Lambda}^3 \left[ \tilde{A} - \tilde{B}\tilde{\Lambda}^5 \sqrt{(1 - \tilde{A}^2)(1 - \tilde{A}^2\tilde{\Lambda}^4)} e^{i\tilde{\delta}} \right] & -\tilde{\Lambda}\sqrt{1 - \tilde{A}^2\tilde{\Lambda}^4} & \sqrt{1 - \tilde{\Lambda}^2}
\end{pmatrix}. \quad (13)
\]

Clearly there exist the same relations as those given in Eq. (12) between the effective mixing angles of \(\tilde{V}\) (i.e., \(\tilde{\theta}_{12}, \tilde{\theta}_{23}\) and \(\tilde{\theta}_{13}\)) and the corresponding new parameters \((\tilde{\Lambda}, \tilde{\tilde{A}}\) and \(\tilde{\tilde{B}}\). It has been shown that \(\sin \tilde{\theta}_{23} \approx \sin \theta_{23}\) and \(\sin \tilde{\delta} \approx \sin \delta\) hold to leading order for a variety of terrestrial long-baseline neutrino oscillation experiments \(^{22}\). Therefore, we have

\[
\tilde{\Lambda} \approx \Lambda, \quad \tilde{\delta} \approx \delta. \quad (14)
\]

This result implies that \(\Lambda\) and \(\delta\) are essentially stable against terrestrial matter effects. Hence the expansion of \(\tilde{V}\) in powers of \(\tilde{\Lambda} \approx \Lambda\) makes sense. Only \(A\) and \(B\) in \(V\) are sensitive to the matter-induced corrections. Because of \(\tilde{\tilde{A}} \propto \sin \tilde{\theta}_{12}\) and \(\tilde{\tilde{B}} \propto \sin \tilde{\theta}_{13}\), two remarkable conclusions can be drawn from Ref. \(^{22}\) for our new parameters: (a) \(\tilde{\tilde{A}}/A\) is suppressed up to the order \(\Delta m^2_{\text{sun}}/\Delta m^2_{\text{atm}}\); and (b) \(\tilde{\tilde{B}}/B\) may have the resonant behavior similar to the two-neutrino MSW resonance \(^{2}\).

Finally we give some speculation on the physical meaning of \(\Lambda\). It is well known that the Wolfenstein parameter \(\lambda \approx 0.22\) can be related to the ratios of quark masses in the Fritzsch ansatz of quark mass matrices \(^{23}\) or its modified versions \(^{24}\):

\[
\lambda \approx \sqrt{\frac{m_u}{m_c} - e^{i\phi_\lambda} \sqrt{\frac{m_d}{m_s}}}, \quad (15)
\]

where \(\phi_\lambda\) denotes the phase difference between the (1,2) elements of up- and down-type quark mass matrices. Confronting Eq. (15) with current experimental data on the Cabibbo angle and quark masses leads to \(\phi_\lambda \sim \pm 90^\circ\) \(^{24}\). Such a result for \(\phi_\lambda\) is also consistent with the large CP-violating effect observed in \(B^0_d\) vs \(\bar{B}^0_d \rightarrow J/\psi K_S\) decays at KEK and SLAC \(B\)-meson factories \(^{25}\). Eq. (15) indicates that the smallness of \(\lambda\) is a natural consequence
of the strong quark mass hierarchy. Could the largeness of $\Lambda$ be attributed to a relatively weak hierarchy of three neutrino masses? The answer is indeed affirmative in the Fritzsch texture of lepton mass matrices, which coincides with current experimental data on neutrino oscillations if the masses of three neutrinos perform a normal but weak hierarchy (typically, $m_1 : m_2 : m_3 \approx 1 : 3 : 10$) \cite{26}. In this phenomenological model, we approximately obtain

$$\Lambda \approx \sqrt{\frac{m_2}{m_3}} - e^{i\phi_\Lambda} \sqrt{\frac{m_\mu}{m_\tau}},$$

(16)

where $\phi_\Lambda$ denotes the phase difference between the (2,3) elements of charged lepton and neutrino mass matrices. We find that $\phi_\Lambda \sim \pm 180^\circ$ is practically favored \cite{26}, in order to obtain a sufficiently large $\Lambda$. To illustrate, we typically take $m_2/m_3 \sim 0.3$ as well as $m_\mu/m_\tau \approx 0.06$ \cite{20}. Then we arrive at $\Lambda \sim 0.8$, a result compatible with our empirical expectation for the order of $\Lambda$. \footnote{Assuming a somehow stronger mass hierarchy for three neutrinos, Kaus and Meshkov \cite{27} have proposed a different expansion of the neutrino mixing matrix in terms of $\Lambda = \sqrt{m_2/m_3} = (\Delta m^2_{\text{sun}}/\Delta m^2_{\text{atm}})^{1/4} \sim 0.37$. This parameter is associated with $V_{e2}$ instead of $V_{\mu3}$, therefore it is sensitive to the matter effect. In contrast, our parametrization does not rely on the assumption of neutrino mass hierarchy, and its expansion parameter is insensitive to the matter-induced corrections.}

In summary, we have shown that the $3 \times 3$ lepton flavor mixing matrix can actually be expanded in terms of a Wolfenstein-like parameter $\Lambda \sim 1/\sqrt{2}$. This parameter measures the strength of flavor mixing in atmospheric neutrino oscillations, thus it is insensitive to the matter effect. In our new parametrization, the term of $\mathcal{O}(\Lambda^2)$ is associated with the flavor mixing angle of solar neutrino oscillations. The Dirac-type CP-violating phase enters at or below the level of $\mathcal{O}(\Lambda^8)$, while the Majorana-type CP-violating phases are not subject to the $\Lambda$-expansion. Direct relations between the parameters in this Wolfenstein-like representation and those in the standard representation have been established. We expect that such a new description of lepton flavor mixing can be very useful in phenomenology of neutrino physics.

Relating our new parametrization to the models of lepton mass matrices is not the subject of this short note. Nevertheless, we have taken the Fritzsch ansatz for example to give a simple interpretation of the Wolfenstein parameter $\lambda$ in the quark sector and its analog $\Lambda$ in the lepton sector. We find that their different magnitudes reflect different mass hierarchies of quarks and leptons. This observation is very suggestive, although it is quite preliminary. Further attempts are therefore desirable, towards deeper understanding of both similarities and differences between lepton and quark mass spectra and flavor mixing schemes.

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REFERENCES

[1] KamLAND Collaboration, K. Eguchi et al., [hep-ex/0212021].
[2] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); S.P. Mikheyev and A.Yu. Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985).
[3] K2K Collaboration, M.H. Ahn et al., [hep-ex/0212007].
[4] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002); Phys. Rev. Lett. 89, 011302 (2002).
[5] For a review, see: C.K. Jung, C. McGrew, T. Kajita, and T. Mann, Ann. Rev. Nucl. Part. Sci. 51, 451 (2001).
[6] See, e.g., V. Barger and D. Marfatia, [hep-ph/0212126]; G.L. Fogli et al., [hep-ph/0212127]; P. Creminelli, G. Signorelli, and A. Strumia, [hep-ph/0102234], version 4, 9 December (2002); M. Maltoni, T. Schwetz, and J.W.F. Valle, [hep-ph/0212129]; W.L. Guo and Z.Z. Xing, [hep-ph/0212147]; J.N. Bahcall, M.C. Gonzalez-Garcia, and C. Pena-Garay, [hep-ph/0212147]; P. Aliani, V. Antonelli, M. Picariello, and E. Torrente-Lujan, [hep-ph/0212212].
[7] M. Shiozawa (Super-Kamiokande Collaboration), talk given at Neutrino 2002, Munich, May 2002.
[8] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 420, 397 (1998); Palo Verde Collaboration, F. Boehm et al., Phys. Rev. Lett. 84, 3764 (2000).
[9] C. Giunti, [hep-ph/0209103].
[10] H. Fritzsch and Z.Z. Xing, Phys. Lett. B 517, 363 (2001).
[11] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[12] For recent reviews with extensive references, see: H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000); G. Altarelli and F. Feruglio, [hep-ph/0206077] to appear in Neutrino Mass - Springer Tracts in Modern Physics, edited by G. Altarelli and K. Winter (2002).
[13] See, e.g., B. Autin et al., CERN 99-02 (1999); D. Ayres et al., [physics/9911009]; C. Albright et al., [hep-ex/0008063]; and references therein.
[14] C. Jarlskog, Phys. Rev. Lett. B 55, 1039 (1985).
[15] V. Barger, K. Whisnant, and R.J.N. Phillips, Phys. Rev. Lett. 45, 2084 (1980).
[16] Z.Z. Xing, Nuovo Cim. A 109, 115 (1996).
[17] Z.Z. Xing, Phys. Rev. D 65, 113010 (2002).
[18] H.V. Klapdor-Kleingrothaus (Heidelberg-Moscow Collaboration), [hep-ph/0103074]; and references therein.
[19] Z.Z. Xing, Phys. Rev. D 65, 077302 (2002).
[20] Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002). A systematic classification of different angle–phase parametrizations for the $3 \times 3$ flavor mixing matrix can be found in: H. Fritzsch and Z.Z. Xing, Phys. Rev. D 57, 594 (1998).
[21] Z.Z. Xing, Phys. Lett. B 487, 327 (2000); Phys. Rev. D 63, 073012 (2001); Phys. Rev. D 64, 033005 (2001).
[22] W. Zaglauer and K.H. Schwarzer, Z. Phys. C 40, 273 (1988); M. Freund, Phys. Rev. D 64, 053003 (2002); Z.Z. Xing, Phys. Rev. D 64, 073014 (2001).
[23] H. Fritzsch, Phys. Lett. B 73, 317 (1978); Nucl. Phys. B 155, 189 (1979).
[24] See, e.g., H. Fritzsch and Z.Z. Xing, Phys. Lett. B 353, 114 (1995); Phys. Lett. B 413, 396 (1997); Nucl. Phys. B 556, 49 (1999); hep-ph/0212195; J.L. Chkareuli and C.D. Froggatt, Phys. Lett. B 450, 158 (1999); Nucl. Phys. B 626, 307 (2002); and references therein.
[25] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 89, 201802 (2002); Belle Collaboration, K. Abe et al., Phys. Rev. D 66, 071102 (2002).
[26] Z.Z. Xing, Phys. Lett. B 550, 178 (2002).
[27] P. Kaus and S. Meshkov, hep-ph/0211338.
FIG. 1. Allowed regions of $A$ and $B$ changing with $\sin^2 2\theta_{\text{chz}}$, where $0.25 \leq \sin^2 \theta_{\text{sun}} \leq 0.40$ and $\sin^2 2\theta_{\text{atm}} > 0.92$ have typically been input.