Giant thermoemf in multiterminal superconductor/normal metal mesoscopic structures.

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(November 18, 2018)

We considered a mesoscopic superconductor/normal metal (S/N) structure in which the N reservoirs are maintained at different temperatures. It is shown that in the absence of current between the N reservoirs a voltage difference $V_T$ arises between the superconducting and normal conductors. The voltage $V_T$ oscillates with increasing phase difference $\varphi$ between the superconductors, and its magnitude does not depend on the small parameter $(T/\epsilon_F)$.

It is well known that if the terminals of a normal conductor are maintained at different temperatures, then in the absence of a current a thermoelectric voltage $(V_{\text{em,}f})$ appears between the terminals. The magnitude of $V_{\text{em,}f}$ is equal to $c_1(T/\epsilon_F)\delta T/e$, where $c_1$ is a factor of the order 1, $\epsilon_F$ the Fermi energy and $\delta T$ is the temperature difference (see for example Ref. [1]).

In this paper we analyse the thermoelectric effect in the mesoscopic structure shown in Fig.1. We show that when a temperature difference exists between the normal (N) reservoirs, a voltage between normal and superconducting circuits $V_T$ appears. Unlike in the normal case, the magnitude of this voltage does not depend on a small parameter $(T/\epsilon_F)$, and also this voltage oscillates as the phase difference $\varphi$ between the superconductors varies. We assume that the superconductors are connected via a superconducting loop and the phase difference between them $\varphi$ is controlled by an applied magnetic field. There is no current between the N reservoirs and the temperatures of the reservoirs are different: $T(\pm L) = T_o \pm \delta T$. We will calculate the electric potential in the N film and, in particular, the potential $V_T$ in the N reservoirs. Since we set the potential in the superconductors equal to zero, the potential $V_T$ is the voltage difference between the N reservoirs and superconductors which arises in the presence of the temperature difference $\delta T$.

In order to find the potential $V_T$, we need to determine the distribution functions $f_{\pm}$ and the condensate wave functions $\widehat{G}^{R(A)}$ induced in the N film. The distribution functions $f_{\pm}$ are related to the ordinary distribution functions for electrons $n_+\,\text{ and holes} \, n_-$: $f_{\uparrow} = f_{\downarrow} = 1 - (n_{\uparrow} + p_{\downarrow})$ and $f_{\uparrow} = f_{\downarrow} = -(n_{\downarrow} + p_{\uparrow})$ (we assume that there is no spin-dependent interaction in the system). The function $f_+$ determines the condensate current and the function $f_-$ determines the quasiparticle current and electrical potential (see for example Ref. [2], where $f_+$ and $f_-$ are denoted by $f$ and $f_1$ respectively, and Ref. [3] where the application of the Green’s function technique to the study of transport in S/N mesoscopic structures is discussed). These functions satisfy the kinetic equation (see Ref. [4])

$$L\partial_x[M_+\partial_x f_{\pm}(x) + J_S f_{\pm}(x)] \pm J_{\text{em,}f} = r[A_+ \delta(x - L_1) + \tilde{A}_+ \delta(x + L_1)],$$

where all the coefficients are expressed in terms of the retarded (advanced) Green’s functions: $\widehat{G}^{R(A)} = G^{R(A)}(x) + \widehat{R}^{R(A)}; M_+ = (1 - G^{R(A)} + \widehat{G}^{R(A)})/2; J_{\text{em,}f} = (\partial_x \widehat{G}^{R(A)} + \partial_x \widehat{R}^{R(A)})/2; J_S = (1/2)(\partial_x \widehat{G}^{R(A)} + \partial_x \widehat{R}^{R(A)})/2; A_+ = (\nu\nu_S + g_0)(f_{\uparrow} - f_{\downarrow}) - (g_{\uparrow\downarrow} f_{\uparrow} + g_{\downarrow\uparrow} f_{\downarrow}); g_{\uparrow\downarrow} = (1/4)\nu\nu_S \widehat{G}^{R(A)}(\widehat{R}^{R(A)}/\widehat{S}^{R(S)})/2; g_{\downarrow\uparrow} = (1/4)\nu\nu_S \widehat{G}^{R(A)}(\widehat{R}^{R(A)}/\widehat{S}^{R(S)})/2$. The parameter $r = R/R_o$ is the ratio of the resistance of the N wire $R$ and S/N interface resistance $R_o$; the functions $A_-$ and $\tilde{A}_+$ coincide with $A_-$ and $A_+$ if we make a substitution $\varphi \to -\varphi$. We introduced above the following notations $\widehat{G}^{R(A)} = G^{R(A)}(x) + \widehat{G}^{R(A)}; J_{\text{em,}f} = J_{\text{em,}f}(x) + J_{\text{em,}f}(x)$ etc.; $\nu, \nu_S$ are the density-of states in the N film at $x = L_1$ and in the superconductors. The functions $f_{\pm}$ obey the boundary conditions: $f_{\pm}(\pm L) = F_{\mp}(\pm L)$, where $F_{\mp}(L) = (1/2)[\tanh(\beta_o(\epsilon + eV_T)) \pm \tanh(\beta_o(\epsilon - eV_T))]$, $\beta_o = (2T_o)^{-1}$.

In order to clarify the physical meaning of different terms in Eq.(1), consider the equation for $f_-$. The first term in this equation is the partial quasiparticle current (the quasiparticle current at a given energy) in the N film. The second term is the condensate current and the third term is also the condensate current which appears under nonequilibrium conditions. The factors $A_-$ and $\tilde{A}_-$ are the partial currents through the S/N interfaces. The first term in $A_-$ is the partial quasiparticle current above (the term $\nu\nu_S f_{\mp}$ and below (the term $g_{\uparrow\downarrow} f_{\mp}$) the gap $\Delta$, and the last term is the condensate current. The factors $A_+$ and $\tilde{A}_+$ are equal to zero below the gap (complete Andreev reflection).
In this paper we consider the case when the S/N interface resistance is greater than or approximately equal to the resistance of the N wire ($r < 1$). However $r$ should not be too small because in Eq.(1) we neglected the inelastic collision integral. This implies that the condition

\[(\epsilon \tau_e)^{-1} << r \lesssim 1 \]  \hspace{1cm} (2)

must be fulfilled; here $\tau_e$ is the energy relaxation time, $\epsilon \approx \min\{T, \epsilon_L\}$, and $\epsilon_L = D/L^2$ is the Thouless energy. We consider the most interesting case of low temperatures ($T << \Delta$) when for characteristic energies $\epsilon << \Delta$ the functions $A_+$ and $A_-^*$ equal zero as in this case $\nu_s \approx 0$ and $\tilde{F}^R \equiv \tilde{F}^A$. The first integration of Eq.(1) yields

\[M_+ \partial_x f_+(x)+J_S f_-(x)+J_{an} \partial_x f_-(x)=J_+ \]  \hspace{1cm} (3)

where $J_+$ is a constant of integration. In the limit of the small parameter $r$, all the corrections related to the proximity effect are small (the functions $\tilde{F}^{R(A)}$ are proportional to $r$). Therefore in the main approximation in the parameter $r$, we have for the distribution function $f_+(x)$

\[f_+(x) \approx \delta \beta \epsilon (x/L) \cosh^{-2}(\beta_0 \epsilon) + f_{eq} \]  \hspace{1cm} (4)

In obtaining Eq.(4) we have taken into account that in the N reservoirs the function $f_+(\pm L)$ has an equilibrium form with different temperatures: $f_+(\pm L) = f_{eq} \pm \delta \beta \epsilon \cosh^{-2}(\beta_0 \epsilon)$, where $\delta \beta / \beta = -\delta T / T$, $\beta_0 = (2T_\nu)^{-1}$. Eq.(4) implies that the temperature gradient leads to a flow of nonequilibrium electrons and holes in the N wire. In order to find the distribution function $f_-(x)$, we integrate Eq.(1)

\[M_- \partial_x f_-(x)+J_S f_+(x)-J_{an} \partial_x f_+(x) = J_1 \Theta(L_1 - |x|) + J_{2+,2-} \Theta(|x| - L_1) \]  \hspace{1cm} (5)

\[J_{2+,2-} = \pm J_1 = \pm r[g_{1f} + f_+ - g_{zf} + \delta f_+ (\pm L)] \]  \hspace{1cm} (6)

where $\delta J_1 = J_1 - J_S$ and $\delta f_+ = f_+ - f_{eq}$. Integrating Eq.(5) outside the interval $(-L_1, L_1)$ and using the boundary condition for $f_-(x)$, we obtain $J_{2+,2-} = \pm (f_+ (\pm L) - f_-(\pm L_1)) / L_2$. Using this expression and Eq.(6), one can show that in the main approximation in $r$ the function $f_-(x)$ is almost constant along the N wire and equal to $f_-(x) \approx f_- (0) \approx f_+ (\pm L) = eV_T \beta \cosh^{-2}(\beta_0 \epsilon)$. Integrating Eq.(6) over energies we readily find $eV_T$

\[eV_T = \frac{L_1}{L} \frac{\delta T}{T_0} \int_{0}^{\infty} d\epsilon \ g_{z+} \epsilon \cosh^{-2}(\beta_0 \epsilon) / \int_{0}^{\infty} d\epsilon \ g_{1+} \cosh^{-2}(\beta_0 \epsilon) \]  \hspace{1cm} (7)

The potential $V_T$ equals approximately the voltage difference between the N reservoirs and superconducting loop. It is worth noting that $V_T$, determined by Eq.(7), does not depend on the small parameter $r$ because both functions $g_{z+}$ and $g_{1+}$ are proportional to $r$ (however one should have in mind that according to the condition (2) this parameter must not be too small). The integrand in Eq.(7) can be calculated if the function $\tilde{F}^R$ is known from an approximate or numerical solution of the Usadel equation. In the limit considered, of small $r$, the retarded (advanced) Green’s functions are readily found from the linearized Usadel equation. In this case we find

\[g_{z+} = r \text{Re}(F^R_S \frac{\sin \theta_2 \sin \theta_2}{\theta \sin (2\theta)}) \sin \varphi; \quad g_{1+} = r \text{Im}(F^R_S [\sin(\theta_2 + 2\theta_1) + \cos \varphi \sin \theta_2 \frac{\sin \theta_2}{\theta \sin (2\theta)}]) \]  \hspace{1cm} (8)

where $\theta = \theta_1 + i \theta_2, \theta_1(2) = k_x L_1(2), k_x = \sqrt{-2\epsilon / \epsilon_L}, L_2 = L - L_1, F^R_S = \Delta / \sqrt{\epsilon + i \Gamma}^2 - \Delta^2$ is the retarded Green’s function in the superconductor. One can see that the ratio in Eq.(7) indeed does not depend on $r$. Numerical analysis of the Usadel equation shows that at a characteristic energy $\epsilon \cong \epsilon_L$ the difference between the linearized and exact numerical solutions of the Usadel equation is less than 10% even for $r \approx 1$.

It follows from Eqs.(7-8) that the voltage $V_T$ caused by the temperature gradient is zero when the phase difference between the superconductors is zero and oscillates with increasing $\varphi$. One can easily estimates the order of magnitude of $V_T$. We find
\[ eV_T = \delta T (L_1/L) \sin \varphi \begin{cases} 
\frac{(T/\epsilon_L)C_1(\varphi), T << \epsilon_L}{(\epsilon_L/T)^{3/2}C_2(\varphi), T >> \epsilon_L} 
\end{cases} \] (9)

Here \( C_{1,2}(\varphi) \) are periodic functions of the phase difference \( \varphi \) of order 1 and are not zero when \( \varphi = 0 \).

If we define the thermoemf \( V_T \) as the voltage between the N reservoirs and the superconducting circuit, we can state that this thermoemf is much larger than the thermoemf between the N reservoirs in the absence of superconductors because \( V_T \) does not contain the small parameter \( (T/\epsilon_F) \) as is the case in a normal system (see for example Ref. [1]). In addition, \( V_T \) oscillates with an applied magnetic field \( H \) (\( \varphi \) is proportional to \( H \)) allowing one to detect small temperature gradients. We stress once again that the thermoemf analysed in this work arises not between the normal reservoirs, as takes place in case of the ordinary thermoelectric effect, but between the superconducting and normal circuits. In Fig.2 we plot the dependence \( V_N \) on \( \varphi \) for various \( \beta^{-1} = (2T) \). In Fig.3 the temperature dependence \( V_T \) at \( \varphi = \pi/2 \) is presented. We see that this dependence is non-monotonic with a maximum at \( T \approx \epsilon_L \) (reentrant behaviour). We note that the influence of the proximity effect on the ordinary thermoelectric effect was studied theoretically in [5]. We ignore this effect regarding the thermo electric current \( \alpha \nabla T \) as negligible, compared to the effect under consideration. In Ref. [6] the thermoelectric voltage was measured in complicated S/N structures which differ from the simple structure considered by us. The reentrant behaviour of \( V_T \) and its oscillations on \( \varphi \) were observed in this work. It is possible that the observed effects are related to those considered in this paper. It is worthwhile noting that the influence of the ordinary thermoelectric current \( \alpha \nabla T \) on the Josephson effect was studied long ago [7].

The physical explanation of the effect is the following. The temperature gradient creates a deviation of the distribution function \( \delta f_+ = -(\delta n + \delta p) \) from the equilibrium form. On the other hand the superconductors do not affect this function because complete Andreev reflection conserves the total number of excess electrons and holes. The function \( \delta f_+ \) changes the condensate current flowing across the S/N interface. If \( \delta f_+ \) is created by an electrical current flowing between the N reservoirs, it has the same sign at \( x = \pm L \) and leads to a change in the Josephson current. In the case considered here the function \( \delta f_+ \) has different signs at these points and leads to a variation of the condensate current of the same sign at different S/N interfaces. Therefore, the potential \( V_T \) arises in the N wire producing a subgap current \( r g_{1+} f_-(\pm L_1) \) which compensates the current \( \delta J_S \).

In summary, we have calculated the voltage \( V_T \) between the superconducting and normal circuits in a S/N mesoscopic structure where the normal reservoirs are maintained at different temperatures. This voltage arises due to a branch imbalance in the N film and oscillates with varying phase difference. Its magnitude does not contain the small parameter \( (T/\epsilon_F) \) which is present in normal systems and is of the order \( \delta T (L_1/L)/e \).

We are grateful to the Royal Society and to the EPSRC for their financial support. We are grateful to Yu.Gal’perin for his useful comments and to V.V.Pavlovskii for his assistance.

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FIG. 1 Schematic view of the 4-terminal S/N/S structure under consideration. The electric potential of the superconductors is zero. The N reservoirs are disconnected from the external circuit.

FIG. 2 The dependence of the normalised thermoelectrical voltage $\tilde{V}_T = eV_T/(\delta T L_1/L)$ on the phase difference $\varphi$ for various $\beta = T/\epsilon_L$ (the parameters are $\Delta/\epsilon_L = 10, L_1/L = 0.5, r = 0.3$).

FIG. 3 The temperature dependence of the normalised voltage $\tilde{V}_T$ at $\varphi = \pi/2$, for the same parameters as in Fig. 2.