Torsional vibration analysis in turbo-generator shaft due to mal-synchronization fault

Abhishek Bangunde*, Tarun Kumar, Rajeev Kumar and S C Jain
School of Engineering, Indian Institute of Technology Mandi, H.P., 175005, India
*Corresponding author E-mail: abhishekbanagunde@gmail.com

Abstract: A rotor of turbo-generator shafting is many times subjected to torsional vibrations during its lifespan. The reasons behind these vibrations are three-Phase fault, two-phase fault, line to ground fault, faulty-mal synchronization etc. Sometimes these vibrations can cause complete failure of turbo-generator shafting system. To calculate moment variation during these faults on the shafting system vibration analysis is done using Finite Elements Methods to calculate mass and stiffness matrix. The electrical disturbance caused during Mal-synchronization is put on generator section, and corresponding second order equations are solved by using “Duhamel Integral”. From the moment variation plots at four sections critically loaded sections are identified.

1. Introduction:
In a power plant generator subjected to fault causes change in torque requirement in very short duration of time. This change in torque requirement generates very high impulsive forces in generator shafting which produces torsional vibration in throughout turbo-generator shafting system. The causes of fault are a) Short circuit at generator terminals b) Dynamic instability in turbogenerator shafting system c) Mal-synchronization d) Non-uniform electric field [1]. The amplitude of torsional vibrations is 3 to 6 times the nominal torque [1]. Same phenomena is observed in fighter aircraft when it drops a heavy bomb or missile there is a sudden decrement in load required in an engine which causes torsional vibrations in the shaft of the engine [2]. These vibrations cause the partial or complete failure of shafting system, which means complete shutdown of power plant. Therefore, vibration analysis of turbo-generator shafting is necessary. Synchronization is a process of connecting to a generator when its field winding is excited [3]. An AC generator is unable to deliver power to electrical grid system until both system frequencies, magnitude and phase are same. There are two types of generator synchronization available in the power plant 1) Manual synchronization 2) Automatic synchronization. Both have their advantages and disadvantages.

1.1 System Description:

![Figure 1: Turbo-generator shafting system](image-url)
In the given system as shown in Fig. 1 turbine and generator is connected by intermediate shaft. This intermediate shaft is transferring power from turbine to generator. Another intermediate shaft is connecting and transferring power between generator and Slip ring. At the end of the shaft, SSS clutch is placed which is connected directly to slip ring.

2. Mathematical Modelling:

2.1 Finite Element Modelling of shaft:

![Figure 2: Two nodded Torsional Element](image)

The turbo-generator shafting system consists of three types of elements, these are solid cylindrical section, hollow cylindrical section and solid tapered section. For all three types of section mass/moment of inertia and stiffness matrix equations are derived for two nodded one degree of freedom i.e. rotational element Fig. 2 using Finite Element Method. Therefore, mass and stiffness matrix can be written as:

$$[I^{(e)}] = \int \rho J_s \begin{bmatrix} N_1^2 & N_1 N_2 & N_2^2 \end{bmatrix} dx$$ and $$[K^{(e)}] = \int GJ_s \begin{bmatrix} (\frac{dN_1}{dx})^2 & (\frac{dN_1}{dx})(\frac{dN_2}{dx}) & (\frac{dN_2}{dx})^2 \end{bmatrix} dx$$

(1)

Where $N_1$ and $N_2$ are shape functions $N_1 = 1 - \frac{x}{L}$ and $N_2 = \frac{x}{L}$

Cylindrical Element: After integrating equation (1) for two nodded cylindrical element Mass/moment of inertia and stiffness matrix will become

$$I^{(e)} = \begin{bmatrix} I_{11} & I_{12} \\ I_{13} & I_{14} \end{bmatrix}$$ and $$K^{(e)} = \begin{bmatrix} K_{11} & K_{12} \\ K_{13} & K_{14} \end{bmatrix}$$

(2)

Tapered Element:

![Figure 3: Pictorial representation of Tapered Element](image)
After integrating equation (2) two nodded tapered element as shown in Fig. 3 whose diameter is increasing from left to right, the mass/moment of inertia and stiffness matrix will become

$$
I^{(e)} = \begin{bmatrix}
I_{21} & I_{22} \\
I_{23} & I_{24}
\end{bmatrix}
$$

and

$$
K^{(e)} = \begin{bmatrix}
K_{21} & K_{22} \\
K_{23} & K_{24}
\end{bmatrix}
$$

(3)

2.2 Torque variation during Mal-synchronization fault:

The expression for torque variation during Mal-synchronization fault at is given as [3].

$$
T(t) = \frac{3E_f^2}{\omega x_d} \sin \delta \left( 1 - \cos(\omega t) + \tan \frac{\delta}{2} \sin(\omega t) \right)
$$

(4)

Where, \( \omega \) = electrical angular velocity

\( E_f \) = Excitation emf, \( \delta \) = synchronizing angle

\( x_d \) = sub-transient reactance and \( t \) = time in seconds

2.3 Vibration Analysis:

The second order forced damped vibration equation is employed here in the present study as given below,

$$
[I]\{\dot{\theta}\} + [C]\{\dot{\theta}\} + [K]\{\theta\} = \{T(t)\}
$$

(5)

Where damping constant \( \zeta = 0.1 \) value is used in equation (5). \( \{T(t)\} \) is external electromagnetic disturbance occurred during Mal-synchronization fault in the generator which is varying from 0 to 94.2478 radian (0.3 seconds) as shown in Fig. 4 Now model analysis is performed to decouple these equations and solved by using “Duhamel Integral” [4].

3. Result and discussion

3.1 Validation:

In the present study, the verification of the present solution has been carried out by comparing the results obtained from the present method with those given by Ref. [3], where they have calculated the electromagnetic torque variation during mal-synchronization with angular displacement of rotor given by equation (4) at synchronizing angle \( \delta = 2\pi/3 \). From Fig. 4 it has seen that MATLAB (2013a) code to calculate electromagnetic torque given by equation (4) is efficiently matching with Ref. [3]. The verification angular displacement in the shaft is carried out by comparing with two degrees of freedom system given in Ref. [4]. Damped forced vibration analysis of two degrees of freedom system is carried out. In this system time, varying force is applied on the system and displacement vs time plot is obtained using “Duhamel Integral” in MATLAB (2013a). Fig. 5 clearly validates the exact matching results of system with given solution.
3.2 Parametric study:

In figure 6, the Electromagnetic torque variation is plotted against different rotor angle (θ) at various synchronizing angles (δ) (i.e. π/6, π/3 and 2π/3). It has been observed that with change in synchronizing angle, there is significant variation of amplitude of torque. Torque amplitude at a synchronizing angle (δ) π/6 and 2π/3, found lowest and highest respectively, whereas, at δ = π/3, intermediate vibration amplitude is observed. Fig. 7 shows variation of maximum value of torque with respect to synchronization angle. From figure, it is observed that electromagnetic torque amplitude increases very rapidly from δ = 0 to 2π/3, it becomes maximum at synchronizing angle of δ = 2π/3. Then there is rapid decrement till the value of δ becomes 5 radians and finally, there is gradual decay in amplitude from δ = 5 to 2π. It is also found that at δ = 0, the amplitude of vibration is zero. This is on expected line because at δ = 0, the generator is synchronized, therefore the variation in electromagnetic torque will not occur.

![Figure 4: Electromagnetic torque variation during Mal-synchronization vs rotor angular displacement](image)

Figure 4: Electromagnetic torque variation during Mal-synchronization vs rotor angular displacement

Moment variation for four sections of turbo-generator shafting system is calculated and plotted as shown in Fig. 8. It is seen that the maximum moment variation limits between section turbine and intermediate shaft is $1.2717 \times 10^7$ Nm to $-1.5662 \times 10^7$ Nm. For section between the intermediate shaft and generator, maximum moment variation limits are $1.1279 \times 10^7$ Nm to $-4.4972 \times 10^6$ Nm. For section between generator and slip ring, maximum moment variation limits are $4.6635 \times 10^6$ Nm to $-1.3615 \times 10^7$ Nm. For section slip ring and SSS clutch maximum moment variation limits is $5.6082 \times 10^5$ Nm to $-1.9587 \times 10^5$ Nm.
Figure 5: Displacement vs time plot a) First displacement b) Second displacement

Figure 6: Electromagnetic torque variation at different Synchronizing angles
In Figs 8a, the variation of amplitude of torsional vibration (torque) developed in between the
turbine and intermediate shaft with time is depicted. It is found that the amplitude of vibration reflects
the fluctuation response throughout the time span. It is noteworthy that the amplitude of vibration
initially increases, then decreases with time. On the other hand, in Fig 8b, the amplitude of torsional
vibration developed in between intermediate shaft and generator is plotted with time. Although in
both the figures 8 (a) and (b), the amplitude of vibration shows the fluctuation response but response
observed in fig.8b is opposite then the response found in fig 8a. While in Fig 8c-d the amplitude of
torsional vibration is constant throughout the time in their corresponding sections.

Figure 7: Variation of Max. Torque at different synchronizing angle

Figure 8: Torsional vibration during mal-synchronization at delta = 2π/3 radian between a) Turbine and int. shaft b) Int. shaft and generator c) Generator and slip ring d) Slip ring and SSS clutch
4. Conclusion

For a given generator, the electromagnetic torque variation is maximum at synchronization angle \( \delta = \frac{2\pi}{3} \) radian. The amplitude of variation of electromagnetic torque is much higher than nominal electromagnetic torque. To know the torsional behavior of the turbo-generator shaft during transient time period vibration analysis is performed. For this mass/Moment of inertia and stiffness matrix is calculated by Finite Element Method. Governing differential equations are solved by using Duhamel Integral and results at desired sections are plotted in graph. The amplitude of Torsional vibration is highest in section turbine and intermediate shaft then the section between generator and slip ring, in the section between intermediate shaft and generator in decreasing order. Torsional vibration is lowest in section between section slip ring and SSS clutch maximum moment variation limits. But the number of damage cycles is more in section generator and slip ring, therefore this section is critical. Also, it should be noted that the section between the turbine and intermediate shaft is subjected to elevated temperature, if we consider temperature factor then this section is also critical compared to remaining two sections. So there are more chances of failure of turbo-generator shafting system either at section between turbine and intermediate shaft or section between generator and slip ring.

References:

[1] Bovsunovskii AP, Chernousenko O Yu, Shtefan E V and Bashta D A2010 Fatigue damage and failure of steam turbine rotors by torsional vibrations *Strength of Materials* **42**, No. 1
[2] Huang HZ,Gong J,Ming J Z,Zhu S P and Liao Q 2012 Fatigue life estimation of an aircraft engine under different load spectrums *International Journal of Turbo Jet-Engines* **29** 259–267
[3] Machowski J, Bialek J W and Bumby J R2008 *Power system dynamics and stability control vol2* (Wiltshire: John Willey & sons, Ltd.) p165 and 166
[4] S. S. Rao *Mechanical Vibrations* 2012 vol 4(New Delhi: Pearson publications) p 450.
[5] Shul’zhenko N G, Gontarovskii P P, Garmash NG and Grishin N N2015 Torsional vibrations and damageability of turboset shaftings under extraordinary generator loading *Strength of Materials* **47**, No. 2
[6] Xiang L, Yang S and Chunbiao G2012 Torsional vibration of a shafting system under electrical disturbances *Shock and Vibration* **19** 1223–1233
[7] Booyse,n C, Heyns P S, Hindley M P and Scheepers R2015 Fatigue life assessment of a low pressure steam turbine blade during transient resonant conditions using a probabilistic approach *International Journal of Fatigue* **73** 17–26