Contact stress analysis and optimization of spur gears

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Abstract. Gear drive is one of the most important components in mechanical power transmission systems. Gears in gearbox generally fail when the working contact stress exceeds the contact fatigue limit. In the scope of this paper, the researcher studies gear design methods based on ISO 6336 and draws a comparison with AGMA 9001. In addition, this paper also presents the optimal gear design results with 3D gear models designed in CAD software, then imported into CAE software for topology and shape optimization. More specifically, the amount of material that can be removed from the gear body without affecting the properties of the gear mesh is considered and calculated. Consequently, the aim of this research is to reduce the weight of the gears and choose the optimal parameters of gear pair by contact stress.

Keywords: CAD/CAE Software, Contact stress, Shape Optimization, Spur Gears, Topology

1. Introduction
The design and analysis of optimal structures play an increasingly important role in the mechanical field with the goals of saving costs, time, and resources while still ensuring the quality and costs. The development of CAD / CAE software is a great and urgent support tool to facilitate the optimal design calculation to be further completed in theory and applications. The optimization problems of gear drive include [3-8]:
- Optimization of geometry parameters for teeth according to contact stress.
- Topology, shape, and size optimization of gears.
- Optimization of gearboxes and reducer with the goals of achieving minimum total shaft distance, minimum gearbox mass, equality strength of contact and bending stress, maximum efficiency, and minimum angle error.
- Optimization of the gearbox and reducer housing.

This paper concentrates on topology, shape and size optimization of spur gears, and performs an analysis of those factors affecting the calculation of teeth design according to contact stress.

1.1. Topology Optimization Using CAD / CAE Software
The design process has been continuously improved over time to create the products that satisfy the increasing demands of people. Previously, through the utilization process, defects will appear on the products, so the manufacturer has to correct them to meet the needs of users. These improvements gradually create the optimal product design, shape, quality as well as price. Therefore, it takes a very
long time, maybe a few years or even decades, to get the best results for the products. The overall design development time above can be significantly reduced when CAD and CAE tools are employed in the design process.

Currently, the product design process is a cycle utilizing CAD and CAE tools (Figure 1): from modeling on CAD software, to topology, shape and size optimizing designs on CAE. The design input involves designing task, loads, boundary condition, manufacturing method, cost criteria, design space, design style, designs with the same function, and much more. At the beginning, a new idea is given based on the designer's knowledge and experience. Next comes the calculation of optimal design (topology, shape and size) of details based on criteria or by analysis. The design is then re-modeled using CAD software and re-calculated by CAE software.

![Figure 1. The design sequence of topology optimization using CAD/CAE software.](image)

CAD systems build 3D parametric models, and then export them through CAE software which will mesh, set boundary conditions, loads, material properties and related information.

The method of structural design optimization plays an increasingly important role in product design and development. The basis of the design topology optimization process is finding the distribution of the material in a given area called the design space, so that the structure can respond to load conditions and vibration frequency, and at the same time satisfy the constraints on weight, volume, fabrication ability. Many topology optimization methods have been developed, some of which are homogenous method, density method, and evolutionary method.

In general, design optimization includes optimization of continuous structure design and optimal discrete structure design. Optimizing continuous structure design refers to the optimization of cubic structures made up of consecutive elements, which can be either solid or hollow. Optimizing discrete structure designs involves solving optimal problems for the structures formed by discrete elements such as frames or trusses.

The design topology optimization process follows the six main steps as shown in Figure 2:
1. Create initial shape space for the part and set boundary condition for load parameters.
2. Define the objective function, the constraint function, and control parameters.
3. Conduct calculation based on an algorithm to gradually remove unnecessary materials (according to the algorithm's shell type criteria).
4. Determine whether the structure satisfies the binding conditions and specifications.
5. If satisfactory, go to step 6; otherwise, go back to step 3.
6. View the outcomes.

Currently, commercial CAE software such as MSC Nastran, Ansys, Altair, and Hyperworks [9] are calculated based on density method, using isotropic material model and only one design variable which is material density. The structure is separated into elements by the finite element method. The optimized density method is based on the standard of structural hardness, whereby the found structural hardness will reach the maximum value corresponding to the mass of material used. 3D-printed objects are created from a digital file of designed products using 3D printing technology.
2. Gear design calculation based on ISO 6336, AGMA 9001

Currently in the world, there are many standards for calculating and designing gears, such as ISO 6336 (1996, 2006, 2019), AGMA 9001, Merrit, Legacy ANSI, Bach ... Depending on the purpose of utilization, the standard and suitable ones for design will be particularly chosen. In the scope of this paper, the researcher will only study gear design methods based on ISO 6336 [1, 9, 10] and comparison will be made with AGMA 9001 [2] for the specific case.

According to ISO 6336 the contact stresses are calculated from the following formula [1]:

$$\sigma_H = Z_B \sigma_{H0} \sqrt{K_A K_V K_H K_H a} \leq \sigma_{HP}$$

(1)

Where:

$$\sigma_{H0} = Z_H Z_E Z_{\beta} \frac{F_i (u + 1)}{d_b u}$$

Calculated contact stress $\sigma_{HP}$ is determined as follows:

$$\sigma_{HP} = \frac{\sigma_{HG}}{S_{H lim}} = \frac{\sigma_{H lim} Z_{NT} Z_{L} Z_{R} Z_{W} Z_{X}}{S_{H lim}}$$

(2)

The bending stress is calculated from the formula:

$$\sigma_F = \sigma_{FG0} K_A K_V K_{F_{\beta}} K_{F_{\alpha}} \leq \sigma_{FP}$$

(3)

Where:

$$\sigma_{FG0} = \frac{F_i}{b_m^n} Y_{FS} Y_{Sa} Y_{S} \frac{Y_{L} Y_{R} Y_{X}}{Y_{\beta}} = \frac{F_i}{b_m^n} Y_{FS} Y_{\beta}$$

Calculated bending stress $\sigma_{FP}$ is determined as follows:

$$\sigma_{FP} = \frac{\sigma_{FG}}{S_{FP min}} = \frac{\sigma_{FG0} Y_{S} Y_{ST} Y_{S} Y_{R}}{Y_{Rot} Y_{S} \frac{S_{FP min}}{S_{min}} Y_{FS} Y_{\beta}}$$

(4)

2.1. Analysis and design of spur gear in CAD software

To draw a comparison, the researcher conducts calculations for specific values, calculating with input parameters for compressor equipment with the following specific data: Power P (5.66kW), number of revolutions n (298 rpm), precision level 7, and working 8000 hours. Calculation is made with the standard transmission ratio $u = 2.0; 2.5; 3.15; 4.0; 5.0; 6.3$, using materials with medium and high
contact fatigue strength: normalized 36Mn5 steel (with $\sigma_{\text{Hlim}} = 520$ MPa, $\sigma_{\text{Flim}} = 372$ MPa) and surface hardened 36Mn5 steel (with $\sigma_{\text{Hlim}} = 1140$ MPa, $\sigma_{\text{Flim}} = 352$ MPa).

With heat treatment the contact fatigue limit increases significantly, but the bending fatigue limit decreases. The remaining calculated parameters are selected according to the standard series. The results calculated according to ISO and AGMA standards are shown in the following tables and figures.

Table 1. Comparison of Calculation Results of Normalized 36Mn5 Steel Spur Gear.

| Gear Radio | ISO | AGMA |
|------------|-----|------|
| a          | Z1  | $S_H$ | $S_F$ | $d_1$ | V  | a | $S_H$ | $S_F$ | $d_1$ | V  |
| 2          | 250 | 37    | 1.237 | 8.74  | 166.5 | 1742 | 200 | 30    | 1.273 | 7.872 | 135 | 1145 |
| 2.5        | 280 | 35    | 1.228 | 8.205 | 157.5 | 1559 | 224 | 28    | 1.253 | 7.461 | 126 | 998  |
| 3.15       | 315 | 34    | 1.207 | 7.893 | 153   | 1471 | 250 | 27    | 1.213 | 7.066 | 121.5 | 928 |
| 4          | 400 | 35    | 1.283 | 8.12  | 157.5 | 1559 | 315 | 28    | 1.288 | 7.512 | 126 | 998  |
| 5          | 450 | 33    | 1.223 | 7.663 | 148.5 | 1386 | 355 | 26    | 1.229 | 6.953 | 117 | 860  |
| 6.3        | 560 | 34    | 1.268 | 7.766 | 153   | 1471 | 450 | 27    | 1.28  | 7.279 | 121.5 | 928 |

Table 2. Comparison of Calculation Results of Surface Hardened 36Mn5 Steel Spur Gear.

| Gear Radio | ISO | AGMA |
|------------|-----|------|
| a          | Z1  | $S_H$ | $S_F$ | $d_1$ | V  | a | $S_H$ | $S_F$ | $d_1$ | V  |
| 2          | 125 | 18    | 1.369 | 4.117 | 81 | 412 | 100 | 15    | 1.255 | 2.473 | 67.5 | 286 |
| 2.5        | 140 | 18    | 1.339 | 3.877 | 81 | 412 | 125 | 16    | 1.390 | 2.805 | 72  | 326 |
| 3.15       | 180 | 19    | 1.398 | 3.829 | 85.5 | 459 | 140 | 15    | 1.282 | 2.486 | 67.5 | 286 |
| 4          | 224 | 20    | 1.501 | 3.781 | 90 | 509 | 180 | 16    | 1.399 | 2.768 | 72  | 326 |
| 5          | 250 | 18    | 1.332 | 3.382 | 81 | 412 | 200 | 15    | 1.298 | 2.944 | 67.5 | 286 |
| 6.3        | 315 | 19    | 1.422 | 3.446 | 85.5 | 459 | 224 | 14    | 1.224 | 2.303 | 63  | 249 |

2.2. Compare Calculation Results
The charts below compare the results according to ISO 6336 and AGMA 2001, the chart on the left uses 36Mn5 normalized steel, and the right one uses 36Mn5 hardened steel (Figure 3-5).
Dimensions calculated according to ISO 6336 (when using both types of steel) are higher than those calculated according to AGMA 2001 - D04: 2005. However, when the 36Mn5 surface hardened steel is used, the safety factor of contact strength when calculated according to ISO 6336 is about 10% higher and the safety factor for bending strength when calculated according to ISO 6336 is higher up to about 40%. Similarly, as researchers calculated according to AGMA 2001 - D04: 2005, the calculating contact stress will be higher than ISO 6336 with the corresponding rate. The safety factor for the bending strength is significantly reduced to nearly reach the minimum limit when surface hardened steel is used.

![Figure 4](image1.png)  
**Figure 4.** Comparison of factor of safety $S_H$.  

![Figure 5](image2.png)  
**Figure 5.** Volume comparison $V$.  

![Figure 6](image3.png)  
**Figure 6.** Teeth mesh and the main generation of spur gears.
This assembly 3D model (Figure 6) can be transferred or edited back and forth in any CAD/CAE software such as: CATIA, Inventor, and Solidworks, with similar dimensions. Gear topology optimization is performed with the finite element method using ANSYS software.

3. Analysis and optimization of spur gear with Finite Element Method – ANSYS Workbench

3.1. Topology Optimization of Gears
After the driving gear has been designed in CAD software, the researcher will export to ANSYS software to analyse and optimize the design (Figure 7-10). We will proceed with keyhole cutting and disk creation in the Space Claim environment.

![Figure 7. Meshed 2D Model of Spur gear.](image1)

![Figure 8. Model with 60% material removal.](image2)

A design calculation is performed with the model with 60% material removal (Figure 8). This is obtained after the material particles are removed and holes of the corresponding hole or groove shape are cut, as long as the fabrication conditions are fulfilled and aesthetic factors are guaranteed. It is then possible to replace the holes with corresponding large and small round holes, which is very convenient in machining, and optimal shape (Figure 9) is ensured.

Based on the results of the standard material density distribution in the above gear models, we will opt for the appropriate gear model option which is close to the optimal result (Figure 19). However, the gears of the given shape are intended to guarantee optimal geometry only, and may have non-technologic shapes despite satisfying the geometrical optimization. Hence, in the given shape, the form which can be fabricated will be chosen for utilization.

Contact stress test: The permissible contact stress is determined (2) with a value of 391.3 N/mm² (MPa). To ensure durable conditions, the force is put at in the most dangerous position on the teeth (right at the top of the teeth). When two gears in mesh, only 1 or 2 pair teeth are touching at a time. The value of the force acting on the tooth is equal to the value of the force during optimal design process.

3.2. Shape Optimization of Gears
In fact, for large gears (diameter > 400mm), 4-6 round holes are usually cut. Besides, in addition to the common round whole profile, other profiles are also cut, such as alternating grooves, large and small round holes (Figure 9). However, the number of holes must be an even number so as to avoid the eccentricity when the gears are operating.
Based on the element removal results in Section 2.1 and practical experience, the researcher cuts the 3D gear model at the disk part with different shapes and rechecks the maximum stress (Figure 10).

**Figure 9.** Common shape of hole in gear structure.

**Figure 10.** Maximum stress results for different shapes.
To test the teeth strength, we use ISO 6336 [2]. Allowable contact stress is determined by formula (2) to obtain a value of 608.8 N/mm$^2$ (MPa). The volume percent reduction, compared to the original model according to the different shapes, is shown in Table 3.

| Shape | Holes | Material Reduction Percentage |
|-------|-------|------------------------------|
| 1 (Fig. 14.a) | 4 holes $\varnothing$70mm | $\%V_{decrease} = \frac{V_0 - V_1}{V_0} = \frac{\Delta V}{V_0} = \frac{4839427}{11097613} = 43.6\%$ |
| 2 (Fig. 14.b) | 8 holes $\varnothing$60mm, 4 holes $\varnothing$30mm | 46.1\% |
| 3 (Fig. 14.c) | 8 holes $\varnothing$50mm | 43.7\% |
| 4 (Fig. 14.d) | 4 grooves | 43.8\% |

As mentioned above, during the operation of the gear, there is only one bearing tooth at a time when the gear is engaged. The force acting on the tooth is equal to the value of the force used in optimal process. Compared with the allowable stress value, the stress result after optimal calculation is satisfied.

The four types of holes and grooves are optimized and re-tested according to the contact strength. The volume of the gear is significantly reduced according to the shape. For those gears with extremely large or very small diameters (used in watches) a material reduction is essential. The whole profiles and grooves are remarkably diverse and widely used, depending on the increasingly modern machining methods which contribute to help the optimization process become more convenient and efficient.

From the aforementioned theory and applications, once again the correctness and the necessary role of optimization in engineering are confirmed. The optimal design not only finds the most technically optimal model, but also the most economically optimal. The application of the CAD / CAE systems in structural optimization brings high efficiency in the design processes. Considerably, the possibilities of structural optimization can go even further in the future as more and more people are engaged in the research and employment of design support software.

4. Study on the Effects of Factors on Contact Stress

4.1. Distribution of Maximum Stress at Point of Contact

The workability of gear teeth is determined according to the contact stress; therefore, the study into the parameters affecting the contact strength to choose the reasonable range of values is remarkably crucial. With gear transmission, there are many studies about the effects of geometric parameters on contact strength [4, 6, 10, and 11]. Currently, with such modern tools as CAD/CAE software with modules design of machine elements, we can quickly find out these effects of gear module m and pressure angle $\alpha$ on contact stress.

![Figure 1](image1.png)  
(a)  
(b)  

**Figure 11.** Distribution of maximum stress at point of contact
First, we respectively compare the results calculated with the analytical formula according to ISO and with FEA (ANSYS software). When simulated on ANSYS, the maximum value of contact stress is equal to 371.84 N/m² (Figure 11). Then, another calculation is made with the analytical formula according to ISO 6336 using Equation (1) is 381,142 N/mm² (MPa). Compare the % difference between the two results:

\[
\text{% ERROR} = \frac{383,142 - 371,84}{383,142} \approx 2.9\%
\]  

(5)

There is no significant difference between the values obtained when calculations are done respectively by ISO 6336 standard and by FEA on ANSYS software. The stress determined when the researcher conducts calculation using ISO 6363: 1996 is slightly greater. However, according to [6], an error of less than 4% is acceptable; this shows that the simulation performed in ANSYS is compliant with ISO 6336.

4.2. Influence of Teeth Modulus and Angle of Pressure on Contact Strength

In this section, to study the influence of the gear module and the pressure angle on the contact stress of gear teeth, the gear transmission uses the parameters in gear transmission in item 2, with the pressure angle of 17.5°; 20°; 22.5° and the modules are respectively changed to 4, 6, 8 and 10. The results are illustrated in Figure 12.

![Figure 12. Diagram of contact stress with modules and pressure angle.](image)

When the module increases, the number of teeth decreases correspondingly, the calculated contact stress also decreases. Then the amount of cutting volume of the teeth will rise; when the pressure angle increases, the contact stress also decreases. When the pressure angle is large, the undercut of gears when cutting a gear with a small number of teeth can be overcome, but it is easy to happen the tip relief of the gear tooth surface. When the pressure angle is small, the undercut of gears becomes apparent.

5. Conclusions

Thus, this article has studied the design calculation and the optimization of gears with some following conclusions and directions for development:

- Some studies have been conducted into the world standards for gear calculations, especially those calculations made according to the contact strength when Autodesk Inventor Professional software is utilized to calculate the data and compare calculation results. When calculated according to ISO, the size is about 20 to 30% larger than when AGMA is used; this difference is due to the different calculation coefficients between the two standards. In addition, other sets of standards can be studied and in fact all of these standards design gears based on bending and contact strength.

- ANSYS application is utilized for topology optimal design. With its optimized shape, the gear saves about 43.6 to 46.1% of the material.
- The gears are optimized for shape by cutting holes on large diameter conventional discs. When the diameter is large (usually more than 500mm), it is possible to create grooves or holes by stamping, then, the work surfaces can be additionally improved while the teeth are machined by envelope or copying methods. In some cases, it is possible to use castings.

- Design software is used to create and test a design 3D model after the optimization of a shape. In addition, it is necessary to conduct experimental research to verify the theoretical and simulation results.

- The researcher studies on calculating and simulating the effects of geometric parameters on gear contact strength. In fact, when the coupling angle increases, the contact stress decreases. When the researcher calculates according to ISO standards and compares the results with mathematical simulation software, the difference is about 2.9%.

- If more time is allowed, the researcher can learn more about the factors that influence the contact strength and bending strength, such as helical angle, number of teeth, rim width, pitch diameter, etc. to form a basis for gear design research.

- More different shapes of the disc profile can be expanded. Besides, it is possible to optimize the coronary region. In addition to the trunk profile, some more other profiles can be studied.

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