A Novel Algorithm for Cooperative Distributed Sequential Spectrum Sensing in Cognitive Radio

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Abstract—This paper considers cooperative spectrum sensing in Cognitive Radios. In our previous work we have developed DualSPRT, a distributed algorithm for cooperative spectrum sensing using Sequential Probability Ratio Test (SPRT) at the Cognitive Radios as well as at the fusion center. This algorithm works well, but is not optimal. In this paper we propose an improved algorithm- SPRT-CSPRT, which is motivated from Cumulative Sum Procedures (CUSUM). We analyse it theoretically. We also modify this algorithm to handle uncertainties in SNR’s and fading.

Keywords- Cognitive Radio, Spectrum Sensing, Cooperative Distributed Algorithm, SPRT.

I. INTRODUCTION

Presently there is a scarcity of wireless spectrum worldwide due to an increase in wireless services. Cognitive Radios are proposed as a solution to this problem. They access the spectrum licensed to existing communication services (primary users) opportunistically and dynamically without causing much interference to the primary users. This is made possible via spectrum sensing by the Cognitive Radios (secondary users), to gain knowledge about the spectrum usage by the primary devices. However due to the strict spectrum sensing requirements and the various wireless channel impairments spectrum sensing has become the main challenge faced by the Cognitive Radios.

Cooperative spectrum sensing (24, 27) in which different Cognitive Radios communicate each other exploits spatial diversity among them effectively. This can largely solve the problems caused by shadowing, multipath fading and hidden node problem in spectrum sensing. Moreover it improves the probability of miss detection and the probability of false alarm. Cooperative spectrum sensing (17, 27) is called centralized, when a central unit gathers sensing data from the Cognitive Radios and identifies the spectrum usage. It is distributed if each local user uses the observations to make a local decision and sends this to the fusion center to make the final decision. Secondary users can either transmit a soft decision (summary statistic) or a hard decision (17). Soft decisions provide better performance but at the cost of higher bandwidth consumption by the control channels between the Cognitive Radio and the fusion center. However as the number of cooperative users increases, hard decisions can perform as well (4).

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An extensive survey of spectrum sensing methods is provided in (27). Recent spectrum sensing schemes are based on higher order statistics (14), wavelet transforms (22) and compressed sensing (23). One can use a fixed sample size (one shot) detectors or sequential detectors (5, 10, 20, 27). In fixed sample size detectors, the matched filter is optimal when there is complete knowledge of the primary signal. When the only a prior knowledge is about the noise power, then an energy detector is optimal in Neyman-Pearson criterion (8). However sequential detectors perform better. A recent survey is (12). The sequential detectors can detect change or test a hypothesis. Sequential hypothesis testing finds out whether the primary is ON or OFF, while the sequential change point detection detects the point when the primary turns ON (or OFF). Sequential change detection is well studied (see 2, 10, 12, 15 and the references therein). However the optimal solution in the distributed setup is still not available. Sequential hypothesis testing (5, 9, 20, 29) is useful when the status of the primary channel is known to change very slowly, e.g., detecting occupancy of a TV channel. Usage of idle TV bands by the Cognitive network is being targeted as the first application for cognitive radio. In this setup Walds’ Sequential Probability Ratio Test (SPRT) (25) provides the optimal performance for a single node (16, 29). But the cooperative setup is not well explored.

We consider cooperative spectrum sensing using sequential hypothesis testing. SPRT is used at both the secondary nodes and the fusion center. This has been motivated by our previous algorithm DualCUSUM for change detection (6). This algorithm is called DualSPRT and has been studied in (7, 9 and 20). As against (9) and (20), in (7) it has been analysed theoretically also and has been extended to cover channel and noise power uncertainties. Cooperative spectrum sensing via sequential detection is also considered in (28). But fusion center noise is not modelled in (28). Similarly (13) provides the optimal decentralized sequential hypothesis testing algorithms without considering fusion center noise. Neither does it consider SNR uncertainty and fading.

Although DualSPRT works well, it is not optimal. In this paper we improve over DualSPRT. Furthermore we introduce a new way of quantising the local nodes SPRT decisions. We call this algorithm SPRT-CSRT. We extend this algorithm to cover SNR uncertainties and fading channels. We also provide
its theoretical analysis. This paper is organised as follows. Section II describes the model. Section III starts with the DualSPRT algorithm. Then we provide SPRT-CSPRT and DualCSPRT algorithms developed in this paper. We compare their performance. Next we consider the receiver SNR uncertainty and slow fading channels. Section IV provides the theoretical analysis and compares to simulations. Section V concludes the paper.

II. MODEL

Consider a Cognitive Radio system with one primary transmitter and $L$ secondary users. The $L$ local nodes sense the channel to detect the spectral holes. The decisions made by the secondary users are transmitted to a fusion node via a Multiple Access Channel (MAC) for it to make the final decision.

Let $X_{k,l}$ be the observation made at secondary user $l$ at time $k$. We assume that $\{X_{k,l}, k \geq 1\}$ are independent and identically distributed (i.i.d.) and that the observations are independent across Cognitive Radios. Using the detection algorithm based on $\{X_{n,l}, n \leq k\}$ the secondary user $l$ transmits $Y_{k,l}$ to the fusion node. We also assume that the secondary nodes are synchronised so that the fusion node receives $Y_k = \sum_{l=1}^L Y_{k,l} + Z_k$, where $\{Z_k\}$ is i.i.d. receiver noise, it will be assumed to be zero mean Gaussian with variance $\sigma^2$. The fusion center observes $\{Y_k\}$ and decides upon the hypothesis.

The observations $\{X_{k,l}\}$ depend on whether the primary is transmitting (Hypothesis $H_1$) or not (Hypothesis $H_0$):

$$X_{k,l} = \begin{cases} Z_{k,l}, & k = 1, 2, \ldots, \text{under } H_0, \\ h_lS_k + Z_{k,l}, & k = 1, 2, \ldots, \text{under } H_1, \end{cases} (1)$$

where $h_l$ is the channel gain of the $l^{th}$ user, $S_k$ is the primary signal and $Z_{k,l}$ is the noise at the $l^{th}$ user at time $k$. We assume $\{Z_{k,l}, k \geq 1\}$ are i.i.d. Let the fusion center makes a decision at time $N$. We assume that $N$ is much less than the coherence time of the channel so that the slow fading assumption is valid. This means that $h_l$ is random but remains constant during the spectrum sensing duration.

The general problem is to develop a distributed algorithm in the above setup which solves the problem:

$$\min E_{DD} \triangleq E[N|H_i], \quad (2)$$

subject to $P_{FA} \leq \alpha$

where $H_i$ is the true hypothesis, $i = 0, 1$ and $P_{FA}$ is the probability of false alarm, i.e., probability of making a wrong decision. We will separately consider $E[N|H_1]$ and $E[N|H_0]$. It is well known that for a single node case ($L = 1$) Wald’s SPRT performs optimally in terms of reducing $E[N|H_1]$ and $E[N|H_0]$ for a given $P_{FA}$. Motivated by the good performance of DualCUSUM in ([11], [6]) and the optimality of SPRT for a single node, we proposed DualSPRT in [7] and studied its performance. Now we modify DualSPRT to SPRT-CSPRT and DualCSPRT and we present the theoretical analysis of this algorithms.

III. SEQUENTIAL SPECTRUM SENSING ALGORITHMS

We first present DualSPRT which was introduced in our previous work [7].

A. DualSPRT Algorithm

1) Secondary node, $l$, runs SPRT algorithm,

$$W_{0,l} = 0, \quad W_{k,l} = W_{k-1,l} + \log \left[ f_{1,l} \left( X_{k,l} \right) / f_{0,l} \left( X_{k,l} \right) \right], \quad k \geq 1 (3)$$

where $f_{1,l}$ is the density of $X_{k,l}$ under $H_1$ and $f_{0,l}$ is the density of $X_{k,l}$ under $H_0$.

2) Secondary node $l$ transmits a constant $b_1$ at time $k$ if $W_{k,l} \geq \gamma_1$ or transmits $b_0$ when $W_{k,l} \leq \gamma_0$, i.e.,

$$Y_{k,l} = b_11_{\{W_{k,l} \geq \gamma_1\}} + b_01_{\{W_{k,l} \leq \gamma_0\}}$$

where $\gamma_0 < 0 < \gamma_1$ and $1_A$ denotes the indicator function of set $A$. Parameters $b_1, b_0, \gamma_1, \gamma_0$ are chosen appropriately.

3) Physical layer fusion is used at the Fusion Centre, i.e.,

$$Y_k = \sum_{l=1}^L Y_{k,l} + Z_k,$$

where $Z_k$ is the i.i.d. noise at the fusion node.

4) Finally, Fusion center runs SPRT:

$$F_k = F_{k-1} + \log \left[ g_1 \left( Y_k \right) / g_0 \left( Y_k \right) \right], \quad F_0 = 0, \quad (4)$$

where $g_0$ is the density of $Z_k + \mu_0$ and $g_1$ is the density of $Z_k + \mu_1$, $\mu_0$ and $\mu_1$ being design parameters.

5) The fusion center decides about the hypothesis at time $N$ where

$$N = \inf \{ k : F_k \geq \beta_1 \text{ or } F_k \leq \beta_0 \}$$

and $\beta_0 < 0 < \beta_1$. The decision at time $N$ is $H_1$ if $F_N \geq \beta_1$; otherwise $H_0$.

B. SPRT-CSPRT Algorithm

In DualSPRT given above, observations to the fusion center are not always identically distributed. Till the first transmission from secondary nodes, these observations are i.i.d. $\sim \mathcal{N}(0, \sigma^2)$ where $\mathcal{N}(a, b)$ is the Gaussian pdf with mean $a$ and variance $b$. But after the transmission from the first local node and till the transmission from the second node, they are i.i.d. Gaussian with another mean and same variance $\sigma^2$. Thus the observations at the fusion center are no longer i.i.d. Since the optimality of SPRT is known for i.i.d. observations ([26], [16]), DualSPRT is not optimal.

The following heuristic arguments provide the motivation of the proposed modifications to DualSPRT. A sample path of the fusion center SPRT under the hypothesis $H_1$ is given in Figure 1. If the SPRT sum defined in (4) goes below zero it delays in crossing the positive threshold $\beta_1$. Hence if we keep SPRT sum at zero whenever it goes below zero, it reduces $E_{DD}$. This happens in CUSUM ([15], [16]). Similarly one can use a CUSUM type algorithm under $H_0$. Thus we obtain the following algorithm.

Steps (1)-(3) are same as in DualSPRT. The steps (4) and (5) are replaced by
4) Fusion center runs two algorithms:

\[ F^1_k = (F^1_{k-1} + \log [g_1 (Y_k) / g_0 (Y_k)]) + D_1^+ \]  
(5)

\[ F^0_k = (F^0_{k-1} + \log [g_1 (Y_k) / g_0 (Y_k)]) + D_0^- \]  
(6)

\[ F^1_0 = 0, F^0_0 = 0, \text{ where } (x)^+ = \max(0, x) \text{ and } (x)^- = \min(0, x). \]

D_1 \text{ and } D_0 \text{ are appropriately chosen constants to introduce bias to the drift.}

5) The fusion center decides about the hypothesis at time \( N \) where

\[ N = \inf\{k : F^1_k \geq \beta_1 \text{ or } F^0_k \leq \beta_0\} \]

and \( \beta_0 < 0 < \beta_1 \). The decision at time \( N \) is \( H_1 \) if \( F^1_N \geq \beta_1 \), otherwise \( H_0 \).

Under \( H_1 \), (5) has a positive drift and hence it approaches the threshold \( \beta_1 \) quickly, but under \( H_0 \), (5) will most probably be hovering around zero. Similarly under \( H_0 \), (6) moves towards \( \beta_0 \), but under \( H_1 \) will be mostly around zero. This means that \( P_{FA} \) for this algorithm is expected to be less compared to DualSPRT.

![Sample Path of F_k under SPRT Sum and CSPRT Sum for γ₁ = 8, β₁ = 20, µ₁ = 1 and µ₀ = −1](image)

We consider one more improvement. When a local Cognitive Radio SPRT sum crosses its threshold, it transmits \( b_1/b_0 \). This node transmits till the fusion center SPRT crosses the threshold. If it is not a false alarm, then its SPRT sum keeps on increasing (decreasing). But if it is a false alarm, then the sum will eventually move towards the other threshold. Hence instead of transmitting \( b_1/b_0 \) the Cognitive Radio can transmit a higher/lower value in an intelligent fashion. This should improve the performance. Thus we modify the step (3) in DualSPRT as follows. Secondary node \( l \) transmits a constant from \( \{b^1_1, b^1_2, b^1_3, b^1_4\} \) at time \( k \) if \( W_{k,l} \geq \gamma_1 \) or transmits from \( \{b^0_1, b^0_2, b^0_3, b^0_4\} \) when \( W_{k,l} \leq \gamma_0 \), as follows:

\[
Y_{k,l} = \begin{cases} 
  b^1_1 & \text{if } W_{k,l} \in \gamma_1, \gamma_1 + 2\Delta_1), \\
  b^1_2 & \text{if } W_{k,l} \in \gamma_1 + 2\Delta_1, \gamma_1 + 4\Delta_1), \\
  b^1_3 & \text{if } W_{k,l} \in \gamma_1 + 4\Delta_1, \gamma_1 + 6\Delta_1, \\
  b^1_4 & \text{if } W_{k,l} \in \gamma_1 + 6\Delta_1, \infty), \\
  b^0_1 & \text{if } W_{k,l} \in \gamma_0 - 2\Delta_0, \\
  b^0_2 & \text{if } W_{k,l} \in \gamma_0 - 2\Delta_0, \gamma_0 - 4\Delta_0), \\
  b^0_3 & \text{if } W_{k,l} \in \gamma_0 - 4\Delta_0, \gamma_0 - 6\Delta_0), \\
  b^0_4 & \text{if } W_{k,l} \in \gamma_0 - 6\Delta_0, -\infty). 
\end{cases}
\]  
(7)

where \( \Delta_1 \) and \( \Delta_0 \) are the parameters to be tuned at the Cognitive Radio. The expected drift under \( H_1 (H_0) \) is a good choice for \( \Delta_1 (\Delta_0) \).

We call the algorithm with the above two modifications as SPRT-CSPRT (with ‘C’ as an indication about the motivation from CUSUM).

If we use CSPRT at both the secondary nodes and the fusion center with the proposed quantisation methodology (we call it DualCSPRT) it works better as we will show via simulations in Section III C. In the Section IV we will theoretically analyse SPRT-CSPRT. As the performance of DualCSPRT (Table I) is nearer to that of SPRT-CSPRT, we analyse only SPRT-CSPRT.

C. Performance Comparison

Throughout the paper we use \( \gamma = -\gamma = \gamma, \beta_1 = -\beta_0 \) and \( \mu_1 = -\mu_0 = \mu \) for the simplicity of the simulation and analysis.

We apply DualSPRT, SPRT-CSPRT and DualCSPRT on the following example and compare their \( E_{DD} \) for various values of \( P_{FA} \). We assume that the pre-change distribution \( f_0 \) and the post change distribution \( f_1 \) are Gaussian with different means. This type of modelling is relevant when noise and interference are log-normally distributed [24]. This is also a useful model when \( X_{k,l} \) is the sum of energy of a large number of observations at the secondary node at low SNR.

Parameters used for simulation are as follows: There are 5 nodes \( (L = 5) \), \( f_{0,l} \sim N(0, 1) \), for \( 1 \leq l \leq L \). Primary to secondary channel gains are \( -0.5, -2.5, -4 \) and \( -6 \) dB respectively (the corresponding post change means of Gaussian distribution with variance 1 are 1, 0.84, 0.75, 0.63 and 0.5). We assume \( Z_k \sim N(0.5) \) and drift of DualSPRT and SPRT-CSPRT at the fusion center is taken as \( 2\mu Y_k \), with \( \mu \) being 1. We also take \( D_0 = D_1 = 0 \). \{b^1_1, b^1_2, b^1_3, b^1_4\} = \{1, 2, 3, 4\}, \{b^0_1, b^0_2, b^0_3, b^0_4\} = \{-1, -2, -3, -4\} \text{ and } b_1 = -b_3 = 1 \text{ (for Dual-SPRT). Parameters } \gamma \text{ and } \beta \text{ are chosen from a range of values to achieve a particular } P_{FA}. \text{ Table I provides the } E_{DD} \text{ and } P_{FA} \text{ via simulations. We see a significant improvement in } E_{DD} \text{ compared to DualSPRT. The difference increases as } P_{FA} \text{ decreases. The performance under } H_0 \text{ is similar.}

| \( E_{DD} \) | \( P_{FA} = 0.1 \) | \( P_{FA} = 0.001 \) | \( P_{FA} = 5 \times 10^{-5} \) |
| --- | --- | --- | --- |
| DualSPRT | 19.74 | 31.37 | 34.177 |
| SPRT-CSPRT | 15.52 | 22.95 | 25.073 |
| DualCSPRT | 14.90 | 21.92 | 21.88 |

\text{TABLE I} 
\text{COMPARISON AMONG DUALSPRT, SPRT-CSPRT AND DUALCSPRT FOR DIFFERENT SNR'S BETWEEN THE PRIMARY AND THE SECONDARY USERS, UNDER H₁}
D. Unknown Received SNR and Fading

In this section, now we consider the following setup. We use energy detector at the Cognitive Radios, i.e, the observations $X_{k,l}$ are a summation of energy of past $N_1$ observations received by the $l^{th}$ Cognitive Radio node. Then if $N_1$ is reasonably large, $X_{k,l}$ are approximately Gaussian. If the received SNR at the Cognitive Radio is not known then the hypothesis testing problem can be approximated as a change in mean of Gaussian distributions problem, where the mean $\theta_1$ under $H_1$ is unknown. For this case in [7] we used composite sequential hypothesis testing proposed in [11] at the secondary nodes and used SPRT at the fusion node. This was called GLR-SPRT [7]. Here, to take the advantage of CSPRT at the fusion node and the new quantisation technique we modify GLR-SPRT [7] to GLR-SPRT with appropriate local quantisation. Thus the secondary node’s hypothesis testing problem, SPRT, stopping criteria and decision are modified as follows.

$$H_0 : \theta = \theta_0 ; \quad H_1 : \theta \geq \theta_1 .$$

where $\theta_0 = 0$ and $\theta_1$ is appropriately chosen,

$$W_{n,l} = \max \left\{ \sum_{k=1}^{n} \log \frac{f_{\theta_0}(X_k)}{f_{\theta_1}(X_k)} : \sum_{k=1}^{n} \log \frac{f_{\theta_1}(X_k)}{f_{\theta_0}(X_k)} \right\} ,$$

$$N = \inf \{ n : W_{n,l} \geq g(cn) \} ,$$

where $g()$ is a time varying threshold and $c$ is the cost assigned for each observation. Its approximate expression is given in [11]. Also for Gaussian $f_{\theta_0}$ and $f_{\theta_1}$, $\theta \in [a_1,a_2]$ and $S_n$ as the summation of observations $X_{k,l}$ up to time $n$, $\hat{\theta}_n = \max\{a_1,\min\{S_n/n,a_2\}\}$. At time $N$ decide upon $H_0$ or $H_1$ according as $\hat{\theta}_N \leq \theta^*$ or $\hat{\theta}_N \geq \theta^*$, where $\theta^*$ is obtained by solving $I(\theta^*,\theta_0) = I(\theta^*,\theta_1)$, and $I(\theta,\lambda)$ is the Kullback-Leibler information number. Here, as the threshold is a time varying and decreasing function, the quantisation [7] is changed in the following way: if $\hat{\theta}_N \geq \theta^*$

$$Y_{k,l} = \begin{cases} b_1 & \text{if } W_{k,l} \in \left[ g(kc), g(kc3) \right) , \\ b_2 & \text{if } W_{k,l} \in \left[ g(kc3), g(kc2) \right) , \\ b_3 & \text{if } W_{k,l} \in \left[ g(kc2), g(kc) \right) , \\ b_4 & \text{if } W_{k,l} \in \left[ g(kc), \infty \right) , \end{cases}$$

If $\hat{\theta}_N \leq \theta^*$ we will transmit from $\{b_1^1, b_2^1, b_3^1, b_4^1\}$ under the same conditions. Here $\Delta$ is a tuning parameter and $0 \leq 3\Delta \leq 1$. The choice of $\theta_1$ in [8] affects the performance of $E[N|H_0]$ and $E[N|H_1]$ for the algorithm [9]-[10]. As $\theta_1$ increases $E[N|H_0]$ decreases and $E[N|H_1]$ increases.

The performance comparison of GLR-SPRT and GLR-CSPRT for the example in Section III C (with $Z_k \sim \mathcal{N}(0,1)$) is given in Table[11]. Here $\Delta = 0.25$. As the performance under $H_1$ and $H_0$ are different, we give the values under both. We can see that GLR-SPRT is always inferior to GLR-CSPRT. For $E_{DD}$ under $H_1$, interestingly GLR-CSPRT have lesser values than that of SPRT-CSPRT for $P_{FA} > 0.02$ (note that SPRT-CSPRT has complete knowledge of the SNRs), while under $H_0$ it has higher value than SPRT-CSPRT.

| $H_{yp}$ | $E_{DD}$ | $P_{FA} = 0.1$ | $P_{FA} = 0.05$ | $P_{FA} = 0.01$ |
|----------|----------|---------------|---------------|---------------|
| $H_1$    | SPRT-CSPRT | 1.615         | 2.450         | 4.28          |
| $H_1$    | GLR-SPRT  | 1.597         | 2.783         | 5.286         |
| $H_1$    | GLR-CSPRT | 1.148         | 2.224         | 4.533         |
| $H_0$    | SPRT-CSPRT | 1.633         | 2.334         | 4.226         |
| $H_0$    | GLR-SPRT  | 2.985         | 4.257         | 7.047         |
| $H_0$    | GLR-CSPRT | 2.424         | 3.744         | 5.72          |

IV. PERFORMANCE ANALYSIS OF SPRT-CSPRT

$E_{DD}$ and $P_{FA}$ analysis is same under $H_1$ and $H_0$. Hence we provide analysis under $H_1$ only.

A. $P_{FA}$ Analysis

Let $P_0$ and $P_1$ denote the probability measure under $H_0$ and $H_1$ respectively. Between each change of drift (which occurs due to the change in number of Cognitive Radios transmitting to the fusion node and due to the change in the value transmitted according to the quantisation rule (7)) at the fusion center, under $H_1$, [5] has a positive drift and behaves approximately like a normal random walk, [6] also has a positive drift, but due to the min in its expression it will stay around zero and as the event of crossing negative threshold is rare [6] becomes a reflected random walk between each drift change. The false alarm occurs when the reflected random walk crosses its threshold. Under $H_1$, let

$$\tau_{\beta} \triangleq \inf \{ k \geq 1 : F_k^0 \geq \beta \} .$$

We call $\tau_{\beta}$ the first passage time at the fusion center. Let $\tau_{\gamma,l}$ be the first passage time to threshold $\gamma$ by the $l^{th}$ node. Let $t_k$ be the $k^{th}$ order statistics of $L$ i.i.d. random variables. Then

$$T_{\gamma,l} = \inf \{ k \geq 1 : F_k^0 \geq \gamma \} .$$

We call $T_{\gamma,l}$ the first passage time at the fusion center. Let $\tau_{\gamma,l}$ be the first passage time to threshold $\gamma$ by the $l^{th}$ node. Let $t_k$ be the $k^{th}$ order statistics of $L$ i.i.d. random variables. Then
$P_{FA}$ at the fusion node, when $H_1$ is the true hypothesis is given by,

$$P_{H_1} (\text{False alarm}) = P_{H_1} (\text{False alarm before } t_1)$$

$$+ P_{H_1} (\text{False alarm between } t_1 \text{and } t_2)$$

$$+ P_{H_1} (\text{False alarm between } t_2 \text{and } t_3) + \ldots$$

The main contribution to $P_{FA}$ comes from the first term.

$$P_{H_1} (FA \text{ before } t_1)$$

$$= \sum_{k=1}^{\infty} P(\tau_\beta \leq k, t_1 < t_1)$$

$$= \sum_{k=1}^{\infty} P(\tau_\beta \leq k | t_1 < t_1) P(t_1 > k)$$

(14)

In the following we compute $P_0 \{ \tau_\beta > x | \tau_\beta < t_1 \}$ and $P[t_1 > k]$. It is shown in [13] that,

$$\lim_{\beta \to \infty} P_0 \{ \tau_\beta > x | \tau_\beta < t_1 \} = \exp(-\lambda_\beta x), x > 0.$$

(15)

By finding solution to the integral equation obtained via renewal arguments [19], we can obtain the mean $1/\lambda_\beta$ of first passage time, $\tau_\beta$ as (done in [11], [21]). Let $L(s)$ be the mean of $\tau_\beta$ with $F_0^a = s$ and $S_k = \log [g_1(Y_k) / g_0(Y_k)] + D_0$. From the renewal arguments, by conditioning on $S_0 = z$:

$$L(s) = F_S(-s)L(0) +$$

$$\int_{-s}^{\beta} L(s+z)dF_S(z)dz + P[S > \beta - s],$$

(16)

where $F_S$ is the distribution of $S_k$ before the first transmission from the local nodes. By solving these equations numerically, we get $\lambda_\beta = 1/L(0)$.

Next we consider the distribution of $t_1$. SPRT $\{W_{k,l}, k \geq 0\}$ is a random walk at each secondary node $l$. We assume $f_{0,l} \sim N(0, \sigma^2_0)$ and $f_{1,l} \sim N(\theta_l, \sigma^2_1)$, where $\theta_l$ is the post change mean and $\sigma^2_1$ is the variance for $l$th Cognitive Radio. Let mean and variance of the drift of $t_1$ be $\delta_l = E[H_l | \log (f_1(X_{k,l})/f_0(X_{k,l}))], \Sigma^2_l = Var[H_l | \log (f_1(X_{k,l})/f_0(X_{k,l}))]$ respectively. We know $\delta_l > 0$. The time $\tau_{\gamma,l}$ for $W_{k,l}$ at each local node $l$ to cross the threshold $\gamma$ satisfies $E[\tau_{\gamma,l}] \sim \gamma/\delta_l$ for large values of $\gamma$ (needed for small $P_{FA}$). Then by central limit theorem we can show that at each node $l$

$$\tau_{\gamma,l} \sim N\left(\frac{2\sigma^2_\gamma}{\theta^2_l}, \frac{8\sigma^4_\gamma}{\theta^4_l}\right).$$

(17)

Thus now [14] equals

$$\approx \sum_{k=1}^{\infty} (1 - e^{-\lambda_\beta k}) \prod_{l=1}^{L} (1 - \Phi_{\tau_{\gamma,l}}(k))$$

where $\Phi_{\tau_{\gamma,l}}$ is the Cumulative Distribution Function of $\tau_{\gamma,l}$, obtained from the Gaussian approximation [17].

Table IV provides comparison of $P_{FA}$ via simulation and analysis.

### B. $E_{DD}$ Analysis

In this section we compute $E_{DD}$ theoretically. $t_i$, $i$th order statistics of $L$ random variables $\tau_{\gamma,l}, 1 \leq l \leq L$, is the first time at which $i$ local nodes are transmitting. Mean of $t_i$ can be found out from the method explained in [3], for the finding $k$th central moment of non i.i.d. $L^{th}$ order statistics.

Between $t_i$ and $t_{i+1}$ the drift at the fusion center is not necessarily constant because there are four thresholds (each corresponds to different quantizations) at the secondary node. The transmitted value changes after crossing each threshold, $b_1 \to b_2 \ldots \to b_4$. Let $t_{j+1}^a \leq j \leq 3$ be the time points at which a node changes the transmitting values from $b_j$ to $b_{j+1}$ between $t_i$ and $t_{i+1}$. We assume that the probability of false alarm at the local nodes, $P_{fa}$ is very small. Also with a high probability the secondary node with the lowest mean in [17] will transmit first, the node with the second lowest mean will transmit second and so on. In the following we will make computations under this approximations. The time difference between $t_i^{th}$ and $t_{i+1}^{th}$ transmission can be calculated if we take the second assumption $= \Delta_1/\delta_l$. We know $E[t_i]$ for every $i$ from an argument given earlier. Suppose $l$th node transmits at $t_i^{th}$ instant and if $E[t_i] + \Delta_1/\delta_l < E[t_{i+1}]$ then $E[t_{i}^1 ] = E[t_i] + \Delta_1/\delta_l$. Similarly if $E[t_{i}^1 ] + \Delta_1/\delta_l < E[t_{i+1}]$ then $E[t_{i}^2 ] = E[t_{i}^1 ] + \Delta_1/\delta_l$ and so on. Let us represent the sequence $t = \{t_1, t_1^1, t_1^2, t_2, t_2^1, t_2^2, \ldots, t_5^2\}$ (entry only for existing ones by the above criteria) by $T = \{T_1, T_2, T_3, \ldots\}$.

Let $\mu_k$ be the mean drift at the fusion center between $T_k$ and $T_{k+1}$. Thus $T_k$’s are the transition epochs at which the fusion center drift changes from $\mu_{k-1}$ to $\mu_k$. Also let $F_{k} = E[F_{T_k-1}^{-}]$ be the mean value of $F_k$ just before the transition epoch $T_k$. With the assumption of the very low $P_{fa}$ at the local nodes and from the knowledge of the sequence $t$ we can easily calculate $\mu_k$ for each $T_k$. Similarly $F_{k+1} = F_k + \mu_k(E[T_{k+1}] - E[T_k])$. Then,

$$E_{DD} = E[T_j] + \frac{\beta - \bar{F}}{\mu_j}$$

(18)

where

$$j = \min\{i : \mu_i > 0 \text{ and } \frac{\beta - \bar{F}}{\mu_i} < E[T_{i+1}] - E[T_i]\}.$$

Table IV provides the simulation and corresponding analysis values. We used the same set-up as in Section III C (with $Z_k \sim N(0, 1)$)

### V. Conclusion

We consider the problem of cooperative spectrum sensing in this paper. We provide improved algorithms SPRT-CSPRT and DualCSPRT over a recent algorithm DualSPRT. We show that these algorithms can provide significant improvements.
We provide theoretical analysis of SPRT-CSPRT and compare to simulations. We further extend these algorithms to cover the case of unknown SNR and channel fading.

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