Equivalence-Invariant Algebraic Provenance for Hyperplane Update Queries

Pierre Bourhis  
NRS, UMR 9189  
CRISTAL

Daniel Deutch  
Tel Aviv University

Yuval Moskovitch  
Tel Aviv University

Abstract

The algebraic approach for provenance tracking, originating in the semiring model of Green et. al, has proven useful as an abstract way of handling metadata. Commutative Semirings were shown to be the “correct” algebraic structure for Union of Conjunctive Queries, in the sense that its use allows provenance to be invariant under certain expected query equivalence axioms.

In this paper we present the first (to our knowledge) algebraic provenance model, for a fragment of update queries, that is invariant under set equivalence. The fragment that we focus on is that of hyperplane queries, previously studied in multiple lines of work. Our algebraic provenance structure and corresponding provenance-aware semantics are based on the sound and complete axiomatization of Karabeg and Vianu. We demonstrate that our construction can guide the design of concrete provenance model instances for different applications. We further study the efficient generation and storage of provenance for hyperplane update queries. We show that a naive algorithm can lead to an exponentially large provenance expression, but remedy this by presenting a normal form which we show may be efficiently computed alongside query evaluation. We experimentally study the performance of our solution and demonstrate its scalability and usefulness, and in particular the effectiveness of our normal form representation.

1 Introduction

The tracking of provenance for database queries has been extensively studied in the past years (see e.g. 13 23 10 5). In a nutshell, data provenance captures details of the computation that took place and resulted in the generation of each output data item. Multiple models for data provenance have been proposed, for multiple query languages such as the (positive) relational algebra, datalog (see 23), data-intensive workflows (e.g., 14 55) data mining 21, and data-centric applications 16. Provenance has been proven useful for managing access control, trust, hypothetical reasoning, view maintenance and debugging (see 14 22 15 24 8).
The approach advocated by [23] is based on designing algebraic provenance structures whose equivalence axioms are based on equivalences in the formalism for which provenance is designed to be tracked. This guarantees that by design, equivalent queries/programs in the formalism of interest will have equivalent provenance for their output. In a sense, this means that provenance captures the “essence of computation” that has been performed. The commutative semiring model of [23] achieves this property for the positive relational algebra; several extensions have been studied [5, 6, 26] for different query languages.

In this paper we focus on a fragment of update queries and sequences thereof (which we refer to as “transactions”), and propose a novel algebraic provenance model. The fragment of update queries that we focus on is that of hyperplane queries, introduced in [3] as simple yet important building blocks of transactions. Hyperplane queries are intuitively “domain-based”, in that selection of tuples in each query only involves the inspection of individual attribute values for each tuple. As demonstrated in [3, 25], this fragment of transactions facilitates appealing theoretical features, while allowing to express transactions of interest. Specifically, for this fragment, [25] has shown a sound and complete axiomatization, which is crucial for our provenance model as we next explain.

The provenance annotations in our model are initially assigned to both queries and tuples; those assigned to queries are propagated to the tuples that these queries affect, so that the result of applying an annotated transaction is an annotated database. Then, in a similar vein to the commutative semiring model mimicking the equivalence axioms of positive relational algebra, our model is based on the sound and complete axiomatization for set equivalence of transactions in [25]. Namely, we start with a most generic structure that uses abstract operations to capture the effect of each type of update query, and then introduce, for each of the axioms in [25], a corresponding axiom in our algebraic structure. As we will show, this leads to a provenance framework that has the following favourable property: two transactions are “provenance-equivalent”, i.e., their application on every input database yields the same annotated database, if and only if they are set-equivalent. This means that provenance in our framework is independent of the particular way that the transaction is executed and of any optimizations that may take place. To our knowledge, ours is the first provenance model to satisfy this property for transactions (see discussion of previously proposed models in Sections 3.3 and 7). Details of our provenance model appear in Section 3.

By propagating annotations that are assigned to both queries and tuples, we are able to support multiple applications of interest, which we overview in Section 4. For instance, analysts may use the resulting provenance to conduct hypothetical reasoning with respect to both the database and transaction. Namely, by assigning truth values to tuple and/or query annotations in the resulting provenance expressions, they may observe the effect of deleting a tuple or aborting a transaction, on the computation result. Additional examples include the support of access control, where each tuple/query is associated with access credentials and these are propagated so that we compute access credentials for each output tuple; and a “certification” example, where we assign trust level to
each tuple/query and correspondingly produce certifications to output tuples we trust. As is the case with previous algebraic provenance constructions, the idea is that we may first compute an expression in the “most general” structure (detailed in Section 3), and then upon request “specialize” (map) it to any application domains such as those we have just exemplified.

Further, while our generic structure is quite complex, we provide a “prescription” for building instances of it. This is achieved by establishing a connection with the commutative semiring model: we show that for a simple-to-define class of commutative semirings (see Theorem 4.5 for details), their operators can be easily extended to define operators for our model that do satisfy the axioms.

We then (Section 5) turn to the problem of efficient provenance generation and storage, for the “most general” structure. The model definition already entails an algorithm for provenance generation, but we show that it may lead to an exponential blowup of the provenance size with respect to the transaction length (number of queries). We show that this blowup may be avoided, leveraging our axioms: we derive algebraic simplification rules that are entailed by the axioms, and consequently propose a “normal form” structure for provenance. We show that every provenance expression obtained by applying a sequence of hyperplane updates may be transformed to this structure. The expressions that we obtain in this structure are far more compact: they are in fact linear in the size of the transaction and input database. Furthermore, we show that we can generate expressions in this structure on-the-fly during query evaluation, avoiding a detour through the exponentially large representation.

Finally, we present (Section 6) an experimental study of our framework using the TPC benchmark as well as a synthetic dataset. The experiments focus on the time and space overheads incur by provenance tracking, and on the time it takes to “specialize” provenance once it is computed, i.e., assign values to variables and thereby use it in applications such as described above. Our measurements are performed for implementations with and without the normal form optimization. Our results show that our rewrite of provenance into its normal form (made possible due to our axioms) significantly reduces the provenance size, and may be efficiently performed alongside with provenance generation. Thereby, it also significantly benefits provenance applications, accelerating provenance use (assignment of values).

2 Preliminaries

Our goal is to define an algebraic provenance model for updates. In this paper, we focus on the class of “domain-based” updates defined in [3]. This class is a standard model that was studied e.g., in [31, 30, 25]. Importantly, [25] has proposed a sound and complete axiomatization for this fragment, which will serve as a basis for our algebraic provenance model. We describe the class of “domain-based” transactions in a datalog-like language, similar to the one in [8].

Relational Databases. A relational schema is defined over a set of relational names. A relation has a relation name $R$ and a set of attributes denoted
by att(R). Let \( \mathcal{V} \) be an infinite set of values. A tuple \( t \) of relation \( R \) is a function associating with each attribute of \( R \), a value of \( \mathcal{V} \). An instance \( I \) of a relation \( R \) is a set of tuples. A database \( D \) of a relational schema associates with each relation name \( R \) in the schema an instance, denoted by \( R(D) \).

**Hyperplane Update queries.** We next recall the definition of update queries from [3] for the class of “domain-based” transactions, where the selection of tuples only involves the inspection of individual attribute values for each tuple. We restrict the updates queries of [8] to those equivalent to a member of this class.

To this end, we use the notation \( R(u) \) where \( u \) is a tuple with the same arity as \( R \), that may contain constants and variables. A variable \( A \) in \( u \) may further be associated with a disequality expression \([A \neq a]\), restricting assignments so that the attribute in the corresponding position may not be assigned the value \( a \). We say that a tuple \( t \in R \) satisfies \( u \) and write \( t \models u \) if \( t \) corresponds to an instantiation of the variables of \( u \) that satisfy the conditions.

**Example 2.1** Figure [1a] shows a fragment of a products table in an E-commerce application. It includes information about the products in stock, their categories and price (ignore the annotations next to tuples for now). The following is an hyperplane query used to describe all products in the Sport category except for the “Kids mountain bike”:

\[
\text{products}(\neg [p \neq \text{“Kids mnt bike”}], \text{“Sport”}, c):-
\]

The tuple \( \text{products}(\text{“Tennis Racket”}, \text{“Sport”}, \$70) \) satisfies the conditions specified in the query.

**Insertion.** An insertion query \( Q \) is an expression \( R^+(u):- \) where \( u \) is a tuple of constants with the same arity as \( R \). The effect of \( Q \) applied to a database \( D \), denoted by \( Q(D) \), is the insertion of \( u \) to \( R(D) \).

**Example 2.2** The query

\[
\text{Products}^+(\text{“Lego bricks”}, \text{“Kids”}, \$90):-
\]

is an example of an insertion query, adding the tuple (“Lego bricks”, “Kids”, $90) to the Products table.

Note that each insertion query inserts a single tuple, as in [25]; we will consider transactions as means for inserting a bulk of tuples.
Deletion. A deletion query $Q$ is an expression $R^-(u)$, where $u$ is a tuple with the same arity as $R$, that may contain constants and variables, possibly associated with disequalities. $Q(D)$ is the resulting database obtained from $D$ by deleting all tuples of its relation $R$ that satisfy $u$.

Example 2.3 Reconsider the database fragment presented in Figure 1a. The query

$$Products^- (a, \text{“Fashion”}, b):-$$

deletes all tuples in the fashion category.

Modification. A modification query $Q$ is an expression $R^{M}(u_1, u_2)$, where $u_1 = (u_1^0, \ldots, u_1^n)$ and $u_2 = (u_2^0, \ldots, u_2^n)$ have the same arity as $R$ and may contain variables and constants such that either $u_1^i = u_2^i$ (and then the value for this attribute remains intact) or $u_2^i$ is a constant (and then the value is changed to $u_2^i$). I.e. the constants present in $u_2$ which are different from the corresponding variables/constants in $u_1$ indicate how instantiations of $u_1$ are modified. The result of applying $Q$ to a database $D$ is defined as follows: for each valid assignment to $u_1$ and $u_2$, the tuple $t$ of $R$ whose values correspond to the instantiation of $u_1$ is deleted; the tuple $t'$ whose values correspond to the instantiation of $u_2$ is inserted. We use $t \rightsquigarrow t'$ to denote that $t$ was updated to $t'$.

Example 2.4 The query

$$Products^M (\text{“Kids mnt bike”}, a, b, \text{“Kids mnt bike”}, \text{“Bicycles”}, b):-$$

is a modification query. Applying the query to the database fragment shown in Figure 1a results in an update of the category (second attribute) of the product “Kids mnt bike” to “Bicycles”. Namely, we have that ("Kids mnt bike", “Sport”, $120) \rightsquigarrow ("Kids mnt bike", \text{“Bicycles”}, 120)$ and ("Kids mnt bike", “Kids”, $120) \rightsquigarrow ("Kids mnt bike", \text{“Bicycles”}, 120)$.

A transaction $T$ is a sequence of update queries. Its semantics with respect to a given database $D$ is that the update queries are applied sequentially, with each query in the sequence being applied to the result of the transaction prefix that preceded it. The database instance resulting from the application of $T$ over $D$ is denoted by $T(D)$.

The result of applying the update queries from Examples 2.2, 2.3 and 2.4 as a sequence to our example relation, is shown in Figure 1b.

Note. Hyperplane queries correspond to the following fragment of SQL: (1) tuple insertions; (2) deletion using statements of the following form: DELETE FROM RelationName WHERE $s_1, \cdots, s_m$, in which each $s_i$ is of the form $\text{AttributeName} \ op c$, where $\ op$ is in {$=, \neq$} and $c$ is a constant value; (3) updates using statements of the form: UPDATE RelationName SET $l_1, \cdots, l_n$ WHERE $s_1, \cdots, s_m$, in which each $l_i$ and $s_i$ is of the form $\text{AttributeName} \ op c$, where $\ op$ is in {$=, \neq$} and $c$ is a constant value. This fragment has been identified in [25] and subsequent works as an important building block of transactions, even though it does not capture the full generality of SQL. For instance, hyperplane queries
cannot capture comparison between values inside the same tuple, or subqueries in the WHERE condition.

3 Provenance Model

We define an algebraic provenance model for transactions whose design follows the following principle: introduce the most general model that is still insensitive to rewriting under (set) equivalence. This is in line with the approach advocated for in [23]: the main idea is to start by having a domain of basic annotations (which one may consider as identifiers), and to define the effect of query operators over these annotations via generic algebraic operations. The resulting provenance is then a symbolic algebraic expressions over basic annotations. The next step is to add equivalence axioms to the structure so that semantically equivalent symbolic expressions – ones obtained for set-equivalent queries – are indeed made equivalent in the structure. It is then we can say that our provenance model captures the “essence of computation” defined by the queries, rather than the query structure; the axioms will also allow for optimizations that will be the subject of subsequent sections.

3.1 Algebraic Structure

In a similar vein to the semiring construction of [23], we start with a basic set of annotations $X$. These could be thought of as identifiers, which in our case will be associated not only with tuples but also with individual queries, and propagated to the tuples they “touch”. We then introduce a structure called $UP[X]$ ("UP" standing for updates) as follows. As a most general structure, we will start by using six algebraic operations (we later show that five operations are sufficient): $+I$ and $-D$ which will serve as abstract operations to capture provenance for insertion and deletion respectively; $-M$ that will be used in the context of modification, to capture the original tuple (before modification); and $+M$ and $\cdot M$ that will be used for the tuple after modification. Last, we will use $+$ (and $\Sigma$ for summation over a set), to capture disjunction originated in the query.

We also introduce a unique element denoted as 0, that intuitively will be used to denote an absent tuple, when used as tuple annotation, or the fact that an updated query has not taken place, used as query annotation. Expressions in $UP[X]$ are then comprised of any combination of elements in $X \cup \{0\}$ using these operations; we will sometime refer to such expressions as formulas.

Annotated Relations Let $R$ be a (standard) relation schema and let $tup(R)$ be the set of all tuples conforming to $R$. Given a set of annotations $X$, we use the term $UP[X]$-relation $R$ to denote a function from $tup(R)$ to $UP[X]$. The set of all tuples not mapped to 0 is called the support of $R$ (we will also say that they are “in” $R$). This means that $R(t)$ is the provenance annotation (intuitively,
at this point, an identifier or meta-data) of a tuple \( t \), and if this annotation is non-zero then \( t \) is said to be in \( R \) (later, when we map annotations to values, it will be useful to map an annotation to 0, to capture, e.g., tuple deletion). A set of \( UP[X] \)-relations (associated with a schema, in the standard sense) is an \( UP[X] \)-database.

**Annotated Update Queries and Transactions** We include an annotation as part of update query specifications. Intuitively, this annotation may stand for an identifier of the query, or any other meta-data associated with it. We fix a set \( P \) of symbols to be used as query annotations and attach them to the heads of queries. For instance, the head of a provenance-aware insertion query has the form \( R^+p(u)_- \), where \( p \in P \) is the annotation; similarly for deletion and modification. For example, the query \( \text{Products}^+p(\text{"Lego bricks"}, \text{"Kids"}, \$90)_- \) is an annotated insertion query with the annotation being \( p \). Similarly, we will use \( T^p \) to denote a transaction \( T \) annotated by \( p \) (i.e., its queries are annotated by \( p \)).

**Provenance for hyperplane queries.** We are now ready to define provenance for queries. In what follows, let \( R \) be an \( UP[X] \)-relation. We consider different types of update queries \( Q \), and use \( R' \) for the \( UP[X] \)-relation that is the result of applying \( Q \) to \( R \). For example, \( R \) may be annotated by basic annotations (identifiers) and \( R' \) by annotations capturing the computation; but the framework is compositional, so it may be the case that \( R \) is already annotated by more complex annotations. The resulting \( UP[X] \)-relation is as follows.

- If \( Q \equiv R^+p(t)_- \) then we define \( R'(t) = R(t) +_I p \), and for each \( t' \neq t \) we define \( R'(t') = R(t') \).
- If \( Q \equiv R^-p(u)_- \), then \( R'(t) = R(t) +_D p \) for each tuple \( t \in R, t \models u \) and \( R'(t') = R(t') \) otherwise.
- If \( Q \equiv R^M,p(u_1, u_2)_- \), then \( R'(t_1) = R(t_1) -_M p \) for each \( t_1 \in R, t_1 \models u_1 \) and \( R'(t_2) = R(t_2) +_M ((\sum_{t_1 \leftarrow t_2} R(t_1)) \cdot M p) \), for each \( t_2 \) s.t. \( \exists t_1 \models u_1 t_1 \leftarrow t_2 \). The operator \( \sum \) here stands for a disjunctive operator associated to the query. In particular, it is different than \( +_M \) and \( +_I \). Otherwise \( R'(t) = R(t) \).

Note that each algebraic operator in the provenance expression is designed to capture provenance for a query operator. This correspondence will manifest itself in our equivalence axioms below. For now, we only add a special treatment for the 0 value we have introduced (these will be referred to as “zero-related axioms” below). Recall that if \( t \not\in R \) then \( R(t) = 0 \). Thus we define \( \forall a \in X \)

- if \( op \in \{-_M, -_D\} \)
- if \( op \in \{+_M, +_I\} \)
- if \( op \in \{+_I, +_M, -_M, -_D\} \)
Intuitively, if \( t \not\in R \), then deleting or modifying \( t \) does not change \( R \), and \( t \) remains absent from \( R \), thus \( 0 \cdot_{op} a = 0 \) if \( op \in \{-M,-D\} \). The existence of an inserted tuple \( t \not\in R \) by a query annotated with \( a \) depends only on \( a \) and thus \( 0 +_{I} a = a \). Similarly for an updated tuple \( t \leadsto t' \) for \( t' \not\in R \). In a way, the right-hand element of the operators \( +_{I}, +_{M}, -_{M} \) and \( -_{D} \) may be interpreted as a condition for the update, i.e., if the condition is 0, the update did not take place. Therefore \( a \cdot_{op} 0 = a \) for \( op \in \{+_{I}, +_{M}, -_{M}, -_{D}\} \). Finally, the expression \( a \cdot_{M} b \) is used to capture the fact that a tuple annotated by \( a \) is updated by a query annotated by \( b \) to produce an updated tuple. If \( a = 0 \), the tuple is not in the database; if \( b = 0 \) the query has not taken place. In both cases the updated tuple was not generated, thus \( a \cdot_{M} 0 = 0 \cdot_{M} a = 0 \).

We note that, in our setting, for different use-cases it is possible to assign variables the values 1 or 0 (as we demonstrate in Section 4.1). Then, for example, starting from the expression \( p_{1} +_{M} (p_{2} \cdot_{M} p_{3}) \), the assignment of the value 1 to \( p_{1} \) results in the expression \( p_{1} +_{M} p_{2} \). By further assigning the value 0 to \( p_{2} \) we obtain the expression \( p_{1} \).

**Example 3.1** Reconsider the database fragment shown in Figure 1a and the annotated update query:

\[
Products^{p}(\text{“Kids mnt bike”}, a, b, “Kids mnt bike”, “Bicycles”, b):-
\]

By applying the query, the tuple (“Kids mnt bike”, Sport, $120), annotated by \( p_{1} \) and the tuple (“Kids mnt bike”, Kids, $120), annotated by \( p_{3} \) are updated to (“Kids mnt bike”, Bicycles, $120), which is not in the database, thus annotated by 0. As a result the new tuples annotations are \( p_{1} -_{M} p, p_{3} -_{M} p \) and \( 0 +_{M} (p_{1} + p_{3}) \cdot_{M} p \) respectively.

**Provenance of a transaction.** For a given transaction, we annotate it – i.e., all of its update queries – with an annotation \( p \) (a single annotation is used per transaction, reflecting the grouping of queries to a transaction). We apply the queries in the transaction one by one, using the above definitions to compute the provenance of tuples they “touch”: the \( i \)’th update is applied on the annotated database obtained from applying the first \( i - 1 \) updates.

**Example 3.2** Figure 2a depicts an example of a transaction over the database fragment given in Figure 1a. The resulting database from the transaction includes the tuple Products(“Kids mnt bike”, “Kids”, $120) with the provenance annotations \( p_{3} -_{M} p \) due to the first query. The annotation of the tuple Products(“Kids mnt bike”, “Sport”, $120) is \( (p_{1} +_{M} (p_{3} \cdot_{M} p)) -_{M} p \), where the part in the parentheses is the result of the first query. Finally, the annotation of the tuple Products(Kids mnt bike, Bicycles, $120) is \( 0 +_{M} ((p_{1} +_{M} (p_{3} \cdot_{M} p)) \cdot p) \), where the sub-expression \( p_{1} +_{M} (p_{3} \cdot_{M} p) \) comes from the provenance annotation of the tuple Products(“Kids mnt bike”, “Sport”, $120) after the execution of the first query.
Products^M,p(“Kids mnt bike”, “Kids”, c, “Kids mnt bike”, “Sport”, c):-

(a) Transaction T_1

Products^M,p(“Kids mnt bike”, “Sport”, c, “Kids mnt bike”, “Bicycles”, c):-

(b) Transaction T_1'

Products^M,p'(a, “Sport”, c, a, “Sport”, 50):-

(c) Transaction T_2

Figure 2: Transactions

3.2 Algebraic Axiomatization

The operations we have introduced so far lead to a very abstract notion of provenance tracking which essentially requires full tracking of the operation of the update queries that took place, without allowing for any simplifications.

A fundamental question is what simplifications can take place, while still capturing the “essence” of updates that took place? To this end, we note that [25] has introduced a sound and complete axiomatization of set equivalence for update queries. Combining this axiomatization with our basic provenance definition, we obtain a set of equivalence axioms over expressions in UP[X]. We next exemplify the derivation of axioms in our structure based on [25]:

Example 3.3 Based on [25], the following transactions are equivalent

\[ R^M_p(u_1, u_2) : R^-p(u_1) \sim R^-p(u_2) : \]

Note that \( \forall t \models u_1 \) the provenance expression after the transaction on the left is \( R(t_1) -_M p \) and after the transaction on the right, \( R(t_1) -_D p \), thus \( a -_D b = a -_M b \), i.e., \(-_M\) and \(-_D\) are equivalent, and therefore, from now on we use “\( \sim \)” to denote both. Furthermore, \( \forall t \not\models u_2 \), from the left transaction we obtain the expression \( R(t_2) +_M \left( \sum_{t_1 \models u_1} R(t_1) \right) \cdot M p \) \(-_D p \), and from the right transaction the expression \( R(t_2) -_D p \). The sum in the first expression represents the set of tuples that are updated into a single tuple. In case there is only one such tuple, it contains a single element, and thus we obtain that \( a +_M (b \cdot M c) - c = a - c \) for all \( a, b \) and \( c \).

We simplified the axioms and removed redundancies to obtain the set of equivalence axioms shown in Figure 3.

Note that we have introduced the minimal set of axioms based on [25]. When specializing into concrete structures (see below), one may impose further reasonable axioms such as commutativity of +_I.
\[(a + _M (b \cdot c)) + _M (d \cdot c) = (a + _M (d \cdot c)) + _M (b \cdot c)\] 
\[(a + _M (b \cdot c)) - c = a - c\]

Let \(I\) be a set of provenance expressions and \(\{S_1, ..., S_n\}\) be a partition of \(I\):

\[(a + _M ((\sum_{c \in I} c) \cdot d)) + _M ((\sum_{i=1}^n b_i) \cdot d) = a + _M \left(\left(\sum_{i=1}^n (b_i + _M (\sum_{c \in S_i} c)) \cdot d\right) \cdot d\right)\]

\[(a - b) - b = a - b\]
\[a + _M \left(\left(\sum_i (b_i - c)\right) \cdot c\right) = a\]

\[(a + _M (b \cdot c)) + I \cdot c = (a + I \cdot c) + _M (b \cdot c)\]
\[(a + I \cdot b) - b = a - b\]
\[a + _M ((b + I \cdot c) \cdot c) = (a + I \cdot c) + _M (b \cdot c)\]

\[(a + _M (b \cdot c)) + I \cdot c = a + I \cdot c\]
\[(a - b) + I \cdot b = a + I \cdot b\]

\[a + _M (\sum_i b_i + \sum_j d_j) \cdot c = (a + _M (\sum_i b_i \cdot c)) + _M (\sum_j d_j \cdot c)\]

\[(a - b) + _M (c \cdot b) = (a - b) + _M \left(\left((d - b) + _M (c \cdot b)\right) \cdot b\right)\]

Figure 3: Axioms
A formula can be rewritten into another formula by applying a sequence of axioms. This rewriting is bidirectional, thus forming an equivalence relation: two formulas $\phi_1$ and $\phi_2$ of our update algebraic structure are equivalent if and only if there is a sequence axioms such that $\phi_1$ can rewritten into $\phi_2$ by using the axioms. We denote it by $\phi_1 \equiv_{UP[X]} \phi_2$.

### 3.3 Preserving Provenance Under Set Equivalence

We next state the main property of our construction: two transactions yield the same provenance-aware result if and only if they are set-equivalent. We first define equivalence of transaction under our provenance-aware semantics:

**Definition 3.4** We say that two $UP[X]$-relations $R, R'$ are $UP[X]$-equivalent, and denote $R \equiv_{UP[X]} R'$, if for every tuple $t$ we have that $R(t) \equiv_{UP[X]} R'(t)^\uparrow$.

We further say that two $UP[X]$-databases $D, D'$ are $UP[X]$-equivalent (denote $D \equiv_{UP[X]} D'$) if there is an isomorphism between the relation names in $D$ and $D'$ so that matching relations are $UP[X]$-equivalent.

Finally, we say that two annotated transactions $T_1^p$ and $T_2^p$ are $UP[X]$-equivalent, and denote $T_1^p \equiv_{UP[X]} T_2^p$, if for every $UP[X]$-database $D$, we have that $T_1^p(D) \equiv_{UP[X]} T_2^p(D)$.

We further say that for non-annotated transactions $T_1, T_2$, they are set-equivalent, and denote by $T_1 \equiv_B T_2$, if for every database $D$, we have that $T_1(D) \equiv T_2(D)$, where “$\equiv$” now stands for standard isomorphism between the databases.

We are now ready to state the following result:

**Proposition 3.5** For every two transactions $T_1, T_2$ we have that $T_1 \equiv_B T_2$ if and only if $T_1^p \equiv_{UP[X]} T_2^p$.

**Proof 3.6** (sketch) By definition, $UP[X]$-equivalence implies set-equivalence, thus one direction is trivial. For the other direction, the completeness of axioms from [25] guarantees there is a sequence of such axioms whose application transforms $T_1$ into $T_2$. The proof is then by induction, where for each individual axiom we apply a corresponding axiom(s) to the provenance.

**Example 3.7** Reconsider the database fragment given in Figure 1a. According to [25], by modification axiom 2, the transaction $T_1$ given in Figure 2a is set-equivalent to the transaction $T_1'$ in Figure 2b. The effect of both is that the tuples $\text{Products}^M(\text{"Kids mnt bike"}, \text{"Kids"}, \$120)$ and $\text{Products}^M(\text{"Kids mnt bike"}, \text{"Sport"}, \$120)$ are updated into a single tuple $\text{Products}^M(\text{"Kids mnt bike"}, \text{"Bicycles"}, \$120)$. Indeed, the provenance expressions obtained by both are equivalent. The provenance of the tuple $\text{Products}(\text{"Kids mnt bike"}, \text{"Kids"}, \$120)$ is $p_1 - p$, in both cases. The annotation of the tuple $\text{Products}(\text{"Kids mnt bike"}, \text{"Sport"}, \$120)$ using the latter

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1 Note that in particular, $UP[X]$ equivalence implies that the two relations include the same set of tuples.
| Product            | Category | Price |
|--------------------|----------|-------|
| Kids mnt bike      | Sport    | $50   |
| Tennis Racket      | Sport    | $50   |

Figure 4: Transaction Output (partial)

transaction is $p_1 - p$ and is equivalent to $(p_1 + M (p_3 \cdot M p)) - p$ by axiom 2. Last, the annotation of the tuple Products(Kids mnt bike, Bicycles, $120) in the database obtained by $T_1'$ is $(0 + M (p_1 + M (p_3 \cdot M p)) - p) \cdot M p'$. By axiom 3 (and using $a = 0, I = S_1 = \{p_3\}, \sum_{i=1}^{n} b_i = p_1$) it is equivalent to $0 + M ((p_1 + M (p_3 \cdot M p)) - p) \cdot M p'$, which is the provenance of this tuple obtained by $T_1$ as shown in Example 3.2 and thus the databases resulting by the two transactions are equivalent.

Sequence of transactions We next demonstrate our construction for a sequence of transactions, where each transaction is annotated using a different provenance annotation.

Example 3.8 Consider a sequence of two transaction $T_1, T_2$, shown in Figures 2a and 2c resp. Intuitively, the transaction $T_2$ updates the price of all the products in the “Sport” category to $50. Note that the provenance annotation of $T_2$ is $p'$. The database resulting by the application of this sequence on the database shown in Figure 2a contains (among others) the tuples shown in Figure 4.

As expected, two equivalent sequences of transactions yield equivalent provenance expressions associated with their output tuples:

Example 3.9 Consider the transactions $T_1, T_1'$ and $T_2$ from Figure 2. The provenance expression generated for each tuple $t$ in the database by sequence $T_1, T_2$ is equivalent to the provenance of $t$ generated by the sequence $T_1', T_2$. For instance, the annotation of the tuple Products (“Kids mnt bike”, “Sport”, $50) using the latter sequence is $0 + M (p_1 - p) \cdot M p'$ and is equivalent to $0 + M ((p_1 + M (p_3 \cdot M p)) - p) \cdot M p'$ by axiom 2. Note that we may further simplify both expressions by removing the 0.

Comparison with MV-semirings [6]. There exists a previously proposed algebraic provenance model for update queries, called MV-semirings [6]. This model is an extension of the semiring framework, in the sense that for every semiring $K$, the corresponding MV-semiring $K^\nu$ is introduced. The elements of such a semiring are symbolic expressions over elements from $K$, version annotations, and semiring operations where the structure of an expression encodes the derivation history of a tuple. For instance, $\mathbb{N}[X]^\nu$ is the MV-semiring corresponding to the provenance polynomials semiring $\mathbb{N}[X]$. Using this most general $\mathbb{N}[X]^\nu$ MV-semiring, each tuple is annotated by a provenance expression consisting of variables which represent identifiers of freshly inserted tuples, and version annotations that encode the sequence of updates that were applied to the tuple. The version annotation $X^d_{T,\nu}(k)$ denotes that operation
\( X \) (\( X \) may be one of \( U, I, D, \) or \( C, \) which stand for update, insert, delete or commit respectively) was executed at time \( \nu - 1 \) by transaction \( T, \) where \( k \) is the annotation of the tuple before the update and \( id \) is the identifier of the affected tuple.

Since an MV-semiring counterpart is defined for every semiring, this model is applicable in settings beyond those addressed here, notable including support for bag semantics. Further applications such one pertaining to concurrency are also developed in [6]. For such applications, and by design, the model of [6] does not satisfy a counterpart of our Proposition 3.5: more details on the specific of the transaction that took place are recorded, and so equivalent transactions may yield non-equivalent expressions in the MV-semiring:

**Example 3.10** Consider the equivalent transactions sequences from Examples 3.9. Using the MV-semiring model, applying the two transactions to the database given in Figure 1a results in different provenance expressions. For instance, if the provenance annotations satisfy \( p_3 = I^1_{T, 2}(x_1), \) then the provenance of the tuple \( \text{Products}(\text{Kids mnt bike, Bicycles, 120}) \) after applying the first transactions sequence contains an expression of the form \( U^3_{T, 2} U^2_{T, 1}(U^1_{T, 3}(I^1_{T, 2}(x_1))) \) while the provenance annotation after applying the second transaction contains an expression of the form \( U^2_{T, 2} U^2_{T, 1}(I^1_{T, 2}(x_1))). \)

We have highlighted the theoretical appeal of equivalence-invariance that holds for our model but not for [6]: in Section 5 we will show that it also allows to optimize provenance representation, and will further show its practical impact in the experiments. In this context, we note that [6] further defines an operation called \( \text{Unv} \) that intuitively removes the embedded history from the provenance (the parallel of our “transaction annotations”), while keeping information coming from the underlying semiring \( K \) (the parallel of our “tuple annotations”). The resulting provenance obtained by applying \( \text{Unv} \) is then equivalence-invariant, but it does not include sufficient information to, e.g., examine the effect of transaction abortion, assign trust values to transaction queries (see Section 4, in particular Examples 4.4, and the parts of the discussions on access control and certifications pertaining to transaction annotations) or other retroactively reason about meta-data associated with transaction’s queries (in contrast to the data).

**Example 3.11** Applying the \( \text{Unv} \) operation to either expressions in Example 3.10 yields the same result: \( x_1, \) reflecting the relevant tuple from the input database (and in general multiple such tuples and their combination) but not the annotations of update queries that took place.

4 Applications

We next demonstrate the usefulness of the introduced structure through a concrete semantics assigned to the operators. As we shall illustrate, the general axioms that we have derived above can guide the design of such semantics: care
is needed in designing them so that they fit the application of interest, while provenance is still preserved through transactions rewriting.

Each concrete semantics is represented by tuple \((K, +, \cdot, -K, +K, 0K)\) where \(K\) is a set of provenance annotations, and \(+, \cdot\) and \(-K\) are concrete operation over the values in \(K\). We call such tuple Update-Structure.

An important principle underlying the semiring-based provenance framework is that one can compute an “abstract” provenance representation and then “specialize” it in any domain. This “specialization” is formalized through the use of semiring homomorphism. To allow for a similar use of provenance in our setting, we extend the notion of homomorphism to Update-Structures.

**Definition 4.1** Let \(S_1 = (K_1, +, \cdot, -K_1, +K_1, 0K_1)\) and \(S_2 = (K_2, +, \cdot, -K_2, +K_2, 0K_2)\) be two Update-Structures. An homomorphism is a mapping \(h : S_1 \mapsto S_2\) such that

\[
\begin{align*}
    h(a +M b) &= h(a) +M h(b) \\
    h(a -K b) &= h(a) -K h(b) \\
    h(a +K b) &= h(a) +K h(b) \\
    h(0K) &= 0K
\end{align*}
\]

Crucially, we may show that provenance propagation commutes with homomorphisms. We use \(T(D)\) to denote the database obtained from applying the transaction \(T\) on the database \(D\), and say that a tuple \(t\) in \(T(D)\) if \(t\) in the resulting database.

**Proposition 4.2** Let \(S_1\) and \(S_2\) be two Update Structures such that there exists an homomorphism from \(S_1\) to \(S_2\). Let \(D\) be a database instance, \(T\) a transaction and \(t\) a tuple in \(T(D)\). Let \(\phi_1(t)\) (respectively \(\phi_2(t)\)) be the provenance expression of \(t\) by \(T\) over \(S_1\) (respectively \(S_2\)). We have that \(h(\phi_1(t)) = \phi_2(t)\).

This property allows us to support applications as exemplified next.

### 4.1 Example Semantics

We next highlight multiple semantics of interest and their corresponding algebraic structures.

**Deletion Propagation** Consider an analyst who wishes to examine the effect of deleting a tuple from the input database on the result of a sequence of transactions. This may be done without provenance, by actually deleting the tuple and re-running the sequence. Alternatively, and much more efficiently, if we have provenance we may assign truth values to annotations occurring in it. In particular, deleting a tuple corresponds to assigning False to the tuple annotation. The provenance semantics that allows for deletion propagation is the following

\[
\begin{align*}
    a + M b &= a + b := a \lor b \\
    a \cdot M b &= a \land b \\
    a - b &= a \land \neg b
\end{align*}
\]
Where 0 corresponds to the Boolean value False.

**Example 4.3** Reconsider the transactions sequence $T_1, T_2$ from Example 3.8 and the tuple $t = \text{products} (“Tennis Racket”, “Sport”, $50)$ annotated by $0 +_M (p_2 \cdot M p')$ in the output. The scenario where the tuple products(“Tennis Racket”, “Sport”, $70)$ is omitted from the initial database corresponds to the valuation that assigns False to $p_2$. With the above semantics, in this case, the tuple $t$ will not appear in the output.

**Transaction Abortion** The same provenance structure allows to examine the effect of aborting a transaction, on the result of a sequence of transactions. Again, a naive way to do it is to re-run the sequence while ignoring the aborted transaction, but the same results may be achieved efficiently using the provenance information (as we show in Section 6): aborting a transaction corresponds to assigning False to the aborted transaction annotation.

**Example 4.4** Consider again the transactions sequence from Example 3.8. The scenario where the first transaction is aborted corresponds to assigning the truth value False to the variable $p$ in the provenance expression. With the above semantics, the provenance expression of the tuple Products(“Kids mt bike”, “Sport”, $50)$ is evaluated to True, i.e., if we abort the first transaction we would indeed obtain this tuple in the resulting database.

**Access Control** Consider an application that supports different products and prices for different countries (e.g., based on different shipping costs and taxes). Each tuple is annotated with a set of country names, such that a user from country $c$ can see a tuple $t$ only if $t$’s annotation contains $c$. Similarly, transactions are also annotated by sets of countries, so that the transaction annotation defines the set of countries that are affected by the update. For instance, if a deletion query $q$ deletes the tuple $t$ and $q$’s annotation contains the country $c$, then after the deletion the tuple $t$ is no longer available for users from the country $c$.

This semantics may formally be captured in our framework by defining the following provenance operations:

$$a +_M b = a +_I b = a + b := a \cup b$$

$$a \cdot_M b := a \cap b \quad a - b := a \setminus b$$

defined over the domain of sets (whose individual items are, e.g., country names).

**Tuples/Transactions Certification** Consider an application where tuples/transactions are associated with values from $[0, 1]$, reflecting their level of trust. Then given a minimal trust level $L$, we wish to know the result of an execution that involve only transaction and tuple with trust score that exceeds $L$. This can be done by using annotation of the form $a = (v,r)$, where $a.v \in [0,1]$ in the trust score of the tuple/transaction, and $a.r$ is “trusted with respect to $L$” and can be one of $T$ (True), $F$ (False) or $U$ (unknown). For brevity of notation, we then use
trusted(x) as a macro for (x.r = T) or (x.r = U and x.v > L). The operations are then defined through a Boolean structure over the trusted values (note that their corresponding truth values will not be materialized until assigned concrete trust values to input tuples):

$$a +_M b = a +_I b = a + b := \begin{cases} (1, T) & \text{if trusted}(a) \text{ or trusted}(b) \\ (0, F) & \text{otherwise} \end{cases}$$

$$a - b := \begin{cases} (1, T) & \text{if trusted}(a) \text{ and NOT(trusted)(b)} \\ (0, F) & \text{otherwise} \end{cases}$$

$$a \cdot_M b := \begin{cases} (1, T) & \text{if trusted}(a) \text{ and trusted}(b) \\ (0, F) & \text{otherwise} \end{cases}$$

We may show that all proposed structures satisfy the axioms from Section 3.2 (proof omitted for lack of space).

4.2 From semirings to $UP[X]$-operators

As discussed above, it is commonplace to define algebraic provenance through semirings. We next show how to transform a commutative semiring – given that it satisfies some natural constraints – into an $UP[X]$ structure that can be used for provenance in the presence of update queries.

**Theorem 4.5** Let $(K, +_K, \cdot_K, 0, 1)$ be a commutative semiring that satisfies $a +_K 1 = 1$ and $a \cdot_K a = a$, then the set of elements $X = K$, with the operators $+_M, +_I, \cdot_M$ defined as follows: $\forall a, b \in X$:

$$a +_M b = a +_K b 
 a +_I b = a +_K b 
 a \cdot_M b = a \cdot_K b$$

and any $-$ operator that satisfies the axioms 2, 4, 5, 7, 10 and 12 from Section 3.2 with respect to the semiring $+$ and $\cdot$ operators, is an $UP[X]$ structure.

The proof is by carefully going through all axioms and is omitted for lack of space.

**Example 4.6** Recall the access control example from Section 4.1. The corresponding semiring is $(P(C), \cup, \cap, \emptyset, C)$ where $C$ is the set of all countries and $P(C)$ is the power set of $C$. Note that this is a commutative semiring that satisfies $\forall a \in P(C) a \cup C = C$ and $a \cap a = a$. Furthermore, by defining the $-$ operator as set-difference we obtain a structure that satisfies the axioms.

The PosBool semiring $(\mathbb{N}[B], \lor, \land, \bot, \top)$ with the minus operator $a - b = a \land (\neg b)$ satisfies the axioms as well. The latter is the structure we demonstrate in the deletion propagation example in Section 4.1.

Interestingly, the monus operator used in [18] to capture relational difference does not generally “work” as minus in our setting. For instance, our Axiom 10 ($(a - b) + b = a + b$) does not hold in general for monus.

Note that in particular for this construction $+_I$ and $+_M$ are commutative.
5 Efficient Provenance Computation

We next consider the issue of complexity: how large may the provenance be? Can it be efficiently computed alongside query evaluation?

5.1 Naive Construction

A first attempt is to generate provenance by directly using the definitions. That is, starting from the initial instance, we apply sequentially the update queries. We compute the provenance of each tuple after each update using the definitions of Section 3. Unfortunately, this approach incurs an exponential blowup in the transaction length.

Proposition 5.1 There exists a transaction $T$ and a database $D$ with only two tuples $t_1$ and $t_2$ such that the provenance of $t_1$ and the provenance of $t_2$ after applying $T$ to $D$ is at least exponential in the number of queries.

Proof 5.2 (Sketch) Let $D$ be a relational database with a single unary relation $R$. Let $t_1 = R(a)$ and $t_2 = R(b)$ be the two tuples belonging to $D$. The transaction is a sequence of two alternating modification queries. The first modifies $t_1$ to $t_2$, denoted $U_{12}$, and the second modifies $t_2$ to $t_1$, denoted by $U_{21}$. The transaction starts with an update $U_{12}$. We denote by $P^i(t_j)$, the provenance of $t_j$ after applying $i$ updates of $T$. By a simple induction, we can prove that

- $|P^{2i}(t_2)| = |P^{2i-1}(t_1)| + 3 + |P^{2i-1}(t_2)|$
- $|P^{2i}(t_1)| = |P^{2i-1}(t_1)| + 2$
- $|P^{2i+1}(t_1)| = |P^{2i}(t_1)| + 3 + |P^{2i}(t_2)|$
- $|P^{2i+1}(t_2)| = |P^{2i}(t_2)| + 2$

Therefore, $|P^{2i}(t_2)|$ is equal to $2|P^{2(i-1)}(t_2)| + 8 + |P^{2(i-1)}(t_1)|$. Thus, $|P^{2i}(t_2)|$ is greater than $2^i$.

Fortunately, we introduce a normal form for our provenance expression which is linear in the database size and the transaction length. Moreover, we prove that this normal form is computable in polynomial time in the size of the database and the transaction.

5.2 Normal Form

For presentation purposes, we represent our provenance expressions as trees in a classical manner. Figure 5 depicts the basic tree representation for each one of the provenance operations. Any provenance expression obtained by the construction for the class of “domain-based” transactions, when applied to an $X$-database (i.e. a database whose tuple annotations are just identifiers), can be represented as a composition of the basic trees.

We demonstrate that we can find a normal form of the provenance expression as stated in the following theorem.

17
Theorem 5.3 Given a transaction $T^p$, an $X$-database $D$, and $t \in T^p(D)$ with the provenance expression $\phi$. Then, there exists an equivalent provenance expression $\phi' \sim \phi$ such that the tree representation of $\phi'$ has one of the following forms:

(a) $a + t \, p$

(b) $a - p$

(c) $a + M \, (b \cdot M \, p)$

Computing $\phi'(t)$ may be performed in polynomial time in the size of $D$ and $T$. Moreover, $\phi'(t)$ can be computed incrementally for each update of the transaction.

Proof 5.4 (sketch) The key idea behind the proof is to derive from our axioms a set of operational rules that manipulate the provenance, shown in Figure 6. We may show that the rules are implied by the axioms (but not vice versa), and they guide the generation of a “normal form”. Intuitively, in Rule 1 and 2, $a$ is the annotation associated to the tuple on which the update is applied. Applying an insertion or a deletion overrides the previous updates. Rules 3 and 8 intuitively state that an update based on a deleted tuple has no effect and Rule 4 states that an update based on an inserted tuple is equivalent to inserting the current tuple. Rules 5, 6 and 7 intuitively allow to “factorize” successive updates into a single update.

Then, each update may be handled by applying corresponding rules to the provenance it yields. For instance, for insertion we apply Rule 1 and replace the provenance by one of size 3. For deletion, we again obtain size-3 expression, this time by applying Rule 2. Modification involves applying the other rules, in a more complex way (details omitted for lack of space). We may show that
after each step, we compute only a linear size formula and that the number of operations performed on this formula is polynomial in the database and the size of the (prefix of the) transaction.

This normal form is still not guaranteed to be minimal, since there is a subtlety pertaining to the possible existence of 0 in the formula. This may be remedied in post-processing:

**Proposition 5.5** Let $T^p$ be an annotated transaction, applied to an $X$-database $D$. Let $\phi(t)$ be the normal form provenance expression of a tuple $t$ after applying the transaction $T^p$ to $D$. Let $\phi'(t)$ be the expression obtained by using the axioms related to 0 to $\phi(t)$ to minimize it. Then $\phi'(t)$ is unique and a minimized formula.

**Proof 5.6** (sketch) We observe that by applying the “0 axioms” (from Section
to a normal form formula, we may obtain either (1) a normal form expres-
sion, or (2) 0 or (3) a formula of the form $\Sigma_i (b_i) \cdot M$. We can show that none
of these expressions is equivalent to any other, and there is no further concise
way of representing neither of them.

Example 5.7 Consider again the transaction $T_1$ from Figure 2a (let $U^p_1, U^p_2$ denote its first and second query respectively), and the database depicted in Figure 1a.
This transaction deals with three tuples: $t_1 = \text{Products}^M(\text{“Kids mnt bike”}, \text{“Sport”}, \$120)$ with the annotation $p_1$, $t_2 = \text{Products}^M(\text{“Kids mnt bike”}, \text{“Kids”}, \$120)$ annotated by $p_3$, and $t_3 = \text{Products}^M(\text{“Kids mnt bike”}, \text{“Bicycles”}, \$120)$ annotated by 0. Normal form is maintained incrementally, in the sense that after each
update operation, we examine the provenance expressions of all tuples and, if a
particular expression is not in normal form, transform it into one using the rules.
In our example, after the first update, the provenance of all tuples is already in
normal form : $U^p_1(D)(t_3) = p_3 - p$ and $U^p_1(D)(t_1) = p_1 + M(p_3 \cdot M p)$. After the
second update, the provenance expressions of $t_1$ and $t_3$ are no longer in normal
form. $T^p_1(D)(t_1) = (p_1 + M(p_3 \cdot M p)) - p$ is simplified by using Rule 2, to $p_1 - p$. $T^p_1(D)(t_3) = 0 + M((p_1 + M(p_3 \cdot M p)) \cdot M p)$ may be simplified
to $0 + M((p_1 + p_3) \cdot M p)$. Further updates, if exist, would apply to these normal
forms; if needed their resulting provenance is again transformed to normal forms etc. In this case we have concluded the updates; a post-processing step using the
0 axioms is applied to the provenance of $T^p(D)(t_3)$ to obtain $(p_1 + p_3) \cdot M p$.

6 Experimental Evaluation

We have conducted experiments whose main goals were examining (1) the scalability of the approach with respect to the number of updates in terms of time and memory overhead, (2) the usefulness of the resulting provenance, assessed by measuring the time it takes to assign values to provenance annotations occurring in the expression, (3) the effectiveness of our provenance normal form representation which in turn is based on our provenance equivalence axiomatization, and (4) comparison with the previously proposed model of [6].

We used Python 3 to implement our provenance framework for an in-memory
database. This is a simple proof-of-concept, with no indices, thus each update
requires a full scan of the database. We use a hashmap between tuples and their
annotations, allowing random access to the annotation given the tuple. The experiments were executed on Windows 10, 64-bit, with 8GB of RAM and Intel Core i7-4600U 2.10 GHz processor. Each experiment was executed 5 times and we report the average result.

6.1 Setup: Benchmarks and baselines

We have examined our solutions using two benchmarks: TPC-C [11] is an on-line transaction processing benchmark, including update-intensive transactions, that simulate the activity of complex on-line transaction processing application
environments. Its underlying database consists of nine tables and is populated with initial data of about 2.1M tuples. For our experiments, we used the Python open source implementation of the benchmark from [2] to generate transactions logs with up to 1966 update queries, and executed the log using our in-memory database implementation with provenance support. Additionally, we have generated a simple synthetic dataset populated with 1M tuples, with randomly generated values from a fixed domain using a uniform distribution. We generated sequences of update queries of varying length. The type of query (insertion, deletion or update) was randomly selected with uniform distribution; the query parameters (e.g., which tuples are modified and how) were selected at random from a fixed domain; deletion and modification queries perform a selection over a numeric column.

**Compared Algorithms and Baselines** In all experiments we have measured the performance of both of our constructions: (1) the naive approach of Section 5.1 that simply generates provenance according to its definition in Section 3.1 and makes no use of neither the normal form nor axioms (labeled “No axioms” in the graphs); and (2) the more efficient provenance generation method of Section 5.2 based on the normal form (labeled “Normal form”). Two baselines that we have compared to are (1) “No provenance”, i.e., vanilla evaluation of the transactions without provenance support, and (2) in dedicated experiments, the provenance model of MV-semirings [6] discussed above.

As explained above, we have also measured the time it takes to use provenance, for the applications in Section 4. As is the case with semiring provenance [23], using provenance for any of these (or similar) applications amounts to mapping the abstract annotations to values (the soundness of which relies on Proposition 4.2), and performing computation in the resulting structure (e.g., deletion propagation, access control, certification). We show graphs for the representative application of deletion propagation, since for this application there is also a baseline alternative that does not use provenance: applying the deletion directly to the input database, and then running the “vanilla” transaction (this baseline is again labeled “No provenance” in the relevant graphs).

### 6.2 Overhead and Usage

Figures 7 and 8 show the time and memory overhead of provenance generation as well as the time it takes to use provenance, for both TPC-C and our synthetic datasets resp. For the latter we have set the number of affected tuples to be 200 (0.02% of the database tuples), which is consistent with the observed percentage in TPC-C. Below (Section 6.3) we present results obtained when varying this percentage.

**Memory overhead** Provenance tracking leads to memory overhead of two flavors. First, recall that deleted and modified tuples are in fact not removed from the database in our construction (intuitively so that the operation may be “undone”). Therefore, the database size continuously grows. Second, maintaining the provenance expressions incurs an overhead. Figures 7a and 8a show the
memory overhead incurred by our construction with and without the normal form, compared to executing the transactions with no provenance tracking, as a function of the number of updates.

We note that the choice of provenance representation does not affect the number of tuples in the database: provenance tracking with or without the normal form representation leads to the same number of tuples. The overhead in the database size was about 2% compared to no provenance tracking for both. In contrast to the database size, there is a significant difference in the provenance size: for the largest number of updates, the provenance size using the naive approach (i.e., no application of axioms) was 4,127,127, while using the normal form representation the size of the provenance was only 2,264,798, a difference of over 82%.

Using the synthetic dataset with 1M tuples, we observed an overhead of about 100% (i.e., ×2) using the normal form representation, while the overhead without applying the axioms was 120% with respect to no provenance tracking.

**Running time**  Figure 7b depicts the running time of the transaction for the TPC-C dataset. Although provenance tracking and maintenance incur overhead in both the database size and additional memory for the provenance information, the overhead is reasonable: the running time without provenance tracking was 283 seconds for the largest number of updates, and 401 and 330 seconds for the provenance tracking without using the axioms and with the normal form representation respectively. When the number of updates per tuple is small the overhead of maintaining the provenance is negligible compared to no provenance tracking.
evaluation, moreover, there is no overhead of processing the axioms. As this number increases (after around 1K updates), the provenance overhead increases, and the affect of the axioms is more noticeable. Yet, the overhead of processing the rules compared to no provenance tracking increases as well.

Interestingly, even though using the normal form requires the application of rules for minimization (see Section 5), the running time of the construction with normal form representation is lower than that of the naive approach. This is because the minimization is done incrementally after each update, and as a result the maintained provenance size is significantly smaller than the provenance expression obtained without using the axioms. Note that generating new provenance expression for new or updated tuples uses the existing tuples provenance and requires copying it. Thus large provenance expressions lead also to overhead in the tracking time, which underlines another useful aspect of the normal form.

We observed similar trends for the synthetic dataset as shown in Figure 8b. The computation time of the transaction with no provenance tracking was about 77 seconds; provenance tracking without using the axioms incurred an overhead of over 25% (about 97 seconds), whereas using the normal form representation, the running time overhead was less than 3% (only 79 seconds).

Provenance Usage As explained above, we have examined the time it takes to use provenance for deletion propagation (with and without the normal form), compared to a baseline that re-computes the transaction result for the deletion scenario. The results are reported in Figures 7c and 8c.

For the two datasets and for both variants of provenance tracking, using the provenance framework significantly outperforms the baseline approach. For the largest number of updates in the TPC-C dataset (Figure 7c), re-running the transaction over the modified database took 89 seconds, while the provenance assignment time was 3.43 seconds (over ×25 faster) for the naive construction and 1.94 seconds using the normal form representation (over ×45 faster that the baseline). The gain of using the normal form representation compared with the naive construction was significant: about 78%. For the synthetic dataset (Figure 8c), the re-computation time was 78 seconds, the assignment time for the provenance generated without using the axioms was 0.96 seconds, and for the normal form it was 0.86 seconds. These are over ×81 and ×91 faster than the baseline, respectively.

6.3 When do we gain from the Normal Form?

The next set of experiments aims at assessing the usefulness of the normal form representation in synthetic environment where we change the provenance size. As the number of update per tuple increases, the difference between the sizes of the provenance generated without using the axioms and of the provenance represented in the normal form, increases. For a fixed number of updates, as the number of the overall affected tuples increases, the number of update per tuple decreases (since the updated tuples are selected with uniform distribution).
Thus, we fixed the transaction length and examined the effect of the number of tuples affected by the transaction on the overhead incurred by provenance tracking with and without the normal form. We varied the number of affected tuples from 200 to 1000 (0.1% of the database size). This is in line with the number of affected tuples in the TPC-C dataset that varies from from 200 to 2000 (0.1% of the database size there). The results for 1M tuples and 2000 update queries are shown in Figure 9.a.

The right-hand side of Figure 9.a depicts the memory overhead of provenance tracking as a function of the overall number of tuples affected by the transaction. Recall that the axioms allow us to compactly represent the provenance expression of a single tuple at a time. Thus, for large number of updates per tuple, we expect to see a significant difference between the two approaches. Indeed, for small numbers of affected tuples, the provenance size of each tuple is larger. Then, the effect of the axioms on the provenance size is notable, reflecting on the memory overhead. We note that there is a moderate growth in the memory overhead when using the axioms as the number of affected tuples increases.

The running time as a function of the total number of affected tuples is presented in the right-hand side of Figure 9.a. As a result of the changes in the provenance size when the number of updates per tuple increases, the overhead of maintaining the provenance without using the axioms increases. We also observed a moderate growth for the construction that uses the axioms when the number of update per tuple increases. This growth is due to the (relatively small) overhead of minimizing the provenance after each update.
To highlight the difference between the provenance tracking approaches we examined the effect of the number of tuples affected by each update query. To this end, we fixed the data size, and the transaction length, and increase the number of tuples affected by each update. Figure 9b shows the results for a database with 1M tuples and 5 update queries. We observed a moderate growth in the memory overhead (left-hand side of the figure), for both methods, with a significant lower overhead using the axioms. There is a notable difference in the running time growth (right-hand side of the figure), as a result of the large overhead incur by managing large provenance expressions without using the axioms.

6.4 Comparison with MV-semirings [6]

We conclude with an experimental comparison to the MV-semiring model proposed in [6]. We have implemented a generator of MV-semiring expressions and used it to compare to our solution. We note that the model of [6] is geared towards different use cases than ours and stores somewhat different information. In turn, the intended use case could have significant effect on the implementation (e.g., choice of data structures to represent provenance) and in turn on the algorithms performance. Another difference is in that, as explained above, for our applications we need to “duplicate” modified tuples, while [6] does not.

To this end, in order to get an implementation-independent assessment of the memory consumption, we measure the sum of the total provenance length and the number of database tuples. Figure 10a shows the memory overhead for both approaches compared with no provenance tracking evaluation. While the provenance length of individual tuples using the model of [6] is roughly the same as that of our model without using the axioms, the number of tuples in the resulting database using our model is larger, and thus the memory overhead of our model with no axioms is higher. However when using the axioms, we obtain much smaller expressions than in the MV-semiring model.

Figure 10b depicts the running time as the function of number of updates. Here again, performance highly depends on implementation details and we demonstrate this using our two different implementations of [6]. The first uses
strings to represent the provenance (purple line). The running time using this implementation was slightly better than our model, however it has an "edge": it requires a parsing the provenance as pre-process for each use. The second implementation is tree based (red line), using the anytree python package, which is more similar to the implementation of our model. Our model outperforms the MV-semiring model using this implementation. This is because the trees obtained for the MV-semiring model are deep, and the large overhead for each update is incurred by their recursive structure. We estimate that most reasonable implementations would likely to perform in the range between our two implementations, depending on the intended use.

7 Related Work

Data provenance has been studied for different data transformation languages, from relational algebra to Nested Relational Calculus, with different provenance models [7, 21, 18, 27, 13, 38, 10, 17] and applications [36, 29, 33, 28, 20]. Our work fits the line of research on algebraic provenance, originating in [23] for positive relational algebra. Consequent algebraic constructions have since been proposed for various formalisms including aggregate queries [5], queries with difference [1], Datalog [25] and SPARQL queries [19].

A provenance model for SPARQL queries and updates using a provenance graph was presented in [24], and [10] proposes an approach to provenance tracking for data that is copied among databases using a sequence of insert, delete, copy, and paste actions. Provenance for updates was also studied in [37, 9] and in [8], where a boolean provenance model for updates was proposed; however, none of these approaches has proposed an algebraic provenance model. Updates are a form of non-monotone reasoning, and as such are related to the notion of relational difference. Algebraic provenance models for queries with difference have been proposed in [3, 18, 19], but naturally none could be directly applied to update queries; in particular using the "monus" operation of [18, 19] as our minus operation may not result in a structure satisfying the equivalence axioms. Further exploration of the connections between these models and ours is left for future work. An extension of the semiring model of [24] to account for updates was also studied in [20], however the focus there is on the use of provenance in the context of trust and while the work includes an efficient implementation, it falls short of proposing a generic algebraic construction. Closest to our work is the multi-version semiring (MV-semiring) model [6] to which we have extensively compared our solution.

We have shown a normal form construction that allows significant reduction of provenance size in the context of hyperplane update queries. Provenance size reduction has been studied in multiple additional contexts. In particular, the work of [11] has shown a highly effective method for summarizing workflow provenance, namely the workflow operations (modules) applied to a data item in the course of execution. The provenance model used in [11] is geared towards workflows. It thus captures module invocations, their arguments etc., which are
absent from our model. On the other hand, it thus does not capture fine-grained combinations of data items that are captured in algebraic models such as the one we present here. Consequently, their method (which includes, e.g., argument factorization) is not applicable to our setting (nor does our method relevant for their setting). Similar considerations distinguish our work from [12], that studies compression of network provenance. Such provenance includes information that is different from ours, involving a record of network events (albeit using a data provenance representation through a datalog-like formulation of the network logic), rather than information on data derivation in general and data updates in particular.

In contrast, the work of [33, 32] does focus on algebraic provenance expressions. The expressions studied there are provenance polynomials in the sense of [23] (elements of the $N[X]$ semiring), which are designed to capture provenance for SPJU queries but do not suffice for update queries; in particular no counterpart of a minus operator is present there. Consequently, the factorization methods in [33, 32] are very different from our normal form construction, in particular because they rely on a different set of operators and axioms. Our additional operators and different axioms entail that the methods of [33, 32] are inapplicable for provenance compression for update queries (we note that [33] also studies compression of query results, not only provenance; this is orthogonal to our work); on the other hand our construction depends on limiting the queries to hyperplane queries, which means that our solution is also not applicable to general SPJU (or even SPJ) queries.

8 Conclusion and Limitations

We have developed a novel algebraic provenance model for hyperplane update queries and sequences thereof, following the axiomatization of query equivalence in [25]. We have shown that the model captures the “essence of computation” for such queries, i.e., equivalent transactions yield equivalent provenance. We have shown means of instantiating the model, towards applications of provenance in this context. The example instances show the usefulness of the generic model: by following the axioms, we are guaranteed that our provenance construction is independent of transaction rewrites. We have further studied the efficient computation and storage of provenance, and have shown a minimization technique that leads to compact provenance representation via a normal form. This again leverages the axioms, this time in a computational manner. Our experimental evaluation shows the tractability and usefulness of the approach, as well as the benefit of using the normal form.

A main limitation of our solution is that it is confined to hyperplane queries. One may address this challenge towards supporting update queries with conjunctive conditions and beyond; yet such attempt would likely cost in the loss of the property of provenance being preserved under equivalence, since (to our knowledge) no sound and complete axiomatization is known for these more expressive fragments. Further exploration of such endeavours is left for future
Acknowledgements  This research has been funded by the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme (Grant agreement No. 804302), and the Israeli Science Foundation (ISF) Grant No. 978/17. Pierre Bourhis is supported by the ANR Project Headwork ANR-16-CE23-0015.

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