Analysis of Crack Growth Retardation after Single Overload Based on FEM Simulation and CORPUS Model Prediction

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Abstract. In this paper, the distribution of the residual stress and equivalent plastic strain along the crack surfaces of the current crack with the crack growth is presented by finite element calculation. The residual stress intensity factor $K_{\text{res}}$ was calculated by using the weight function method (WFM), and the effective stress range $\Delta S_{\text{eff}}$ during the crack growth in the retardation zone is shown based on the simulative calculation. Then, the effective stress range $\Delta S_{\text{eff}}$ with different $R_{\text{ol}}$ have been predicted by CORPUS model. The results indicate that the FEM simulative calculation and the CORPUS model prediction are agreement. They all show the crack growth retardation after an overload at different overload ratio $R_{\text{ol}}$, and the effects of crack growth rate by the retardation.

1. Introduction

The crack growth retardation phenomenon under single overload is generally attributed to plasticity induced crack closure. Many researchers\textsuperscript{[1-2]} have performed numerical analysis simulating plasticity-induced fatigue crack closure.

A experiment has performed in Reference[3] and [4] to prove that the crack growth retardation is caused by the compressive stresses in the plastic wake behind of the crack tip.

The overload causes a larger residual compressive stress zone in the front of crack tip. And as the crack growth into the residual stress zone, the part of the residual stress zone is left in the crack behind the crack tip. Thus, the opening stress is larger than the crack opening stress in the case of constant amplitude loading, because part of stress balanced by the residual compression stress. The Weight Function Method (WFM) \textsuperscript{[5]} is one of the most effective and accurate methods for calculating stress intensity factors.

In the CORPUS model, overload effects play an important role. The model is associated with plastic deformation left in the wake of the crack which is called “hump”. The crack is considered to be just opened during unloading when all humps looses contact.

In this paper, the effective stress range $\Delta S_{\text{eff}}$ is calculated with the FEM simulative calculation based on the residual compression stress under a single overload, and the CORPUS model prediction based on the crack closure.
2. Finite element simulation
The non-linear finite element software Marc.2005r2 is used to simulate the stress and strain field in Single edge crack specimen as shown in Fig.1. The material is 2024-T351 aluminum alloy, and he specific procedure of finite element simulation is shown in reference [1].

Fig.2 shows the cyclic loading for the computational simulation. Single tensile overload is applied when the crack length is 20mm. In order to study the plastic zone size and residual stress around the crack tip, three cases have been calculated.

All the cases are precracked up to a crack length with the constant amplitude loading, then immediately followed by a single overload as shown in Table 1.

![Fig.1 M(T) specimen](image1)

![Fig.2 Single overload](image2)

| Case | Constant amplitude loading | Overloading ratio, $R_{ol}$ \( \left( = \frac{S_{ol}}{S_{max}} \right) \) |
|------|-----------------------------|--------------------------------------------------|
| M1   | $S_{max} = 65$ MPa, $S_{min} = 6.5$ MPa | 1 \( (S_{ol} = 65$ MPa) \) |
| M2   |                                        | 1.5 \( (S_{ol} = 97.5$ MPa) |
| M3   |                                        | 2 \( (S_{ol} = 130$ MPa) |

3. Results by Finite Element Calculation

3.1. Crack Tip Plastic Zone and Residual Stress Field

Figure 3 shows the equivalent plastic strain in front of the different cases under applied stress $S_{ol}$ (at point L1) and stress $S_{min}$ (at point L2). When $S_{ol}$ is applied, the crack tip deforms monotonically, forming a monotonic plastic zone as Line 1 in Fig.3. And the equivalent plastic strain decreases rapidly with the increase of distance from the crack tip.
When load is reduced to $S_{\text{min}}$, there exists a region from the crack tip to the point A as shown in Fig.3 where the equivalent plastic strain is lower than that of Line 1. The region from the origin to point A is the reverse compressive plastic zone.

![Fig.3](image)

Table 2 presents the size of the monotonic plastic zone and reverse plastic zone of the three cases by finite element simulation and Equation (1), where $\Delta K = K_{\text{max}} - K_{\text{min}}$.

| Case | Size of plastic zone by FE(mm) | Size of plastic zone by Equation (1) (mm) |
|------|--------------------------------|-----------------------------------------|
|      | Monotonic plastic zone | Reverse plastic zone | Monotonic plastic zone | Reverse plastic zone |
| M1   | 0.4 | 0.1 | 0.33 | 0.08 |
| M2   | 0.82 | 0.20 | 0.69 | 0.17 |
| M3   | 1.48 | 0.36 | 1.2 | 0.3 |

$$
\begin{align*}
& r_p = \frac{1}{2\pi} \left( \frac{K_{\text{max}}}{\sigma_s} \right)^2 \\
& r_{\text{rev}} = \frac{1}{2\pi} \left( \frac{\Delta K}{2\sigma_s} \right)^2
\end{align*}
$$

(1)

Table 2 summarizes the monotonic plastic zone size and reverse plastic zone size under a single overload for all the cases. It is seen that finite element simulation and equation calculation agreed quite well. The reverse plastic zone size is about 1/4 of the monotonic plastic zone size for all the cases. These results indicate that the present finite element modeling is suitable for simulating the crack tip plastic zone.

3.2. Residual Stress Intensity Factor $K_{\text{res}}$

Fig.4 shows the y residual stress $\sigma_{\text{res}}$ along the x-axis in the crack tip region of the three cases, and the model is unloading. The x-axis coordinate of crack tip node is 20(mm).
Fig. 4 Residual stress distributions in crack tip area with different $R_{ol}$

The residual stress intensity factors $K_{res}$ is calculated using the Wu-Carlsson 2-D weight function method. Residual stress intensity factors $K_{res}$ curves under different overload ratio are shown in Fig. 5. And the abscissa is hypothetical crack length along x-axis as shown in Fig. 1.

Fig. 5 Residual stress intensity factor $K_{res}$ with different $R_{ol}$ after single overload

Fig. 5 indicates the contribution of the residual stress to crack growth. As shown in Fig. 5, residual stress intensity factor $K_{res}$ on different curves are reduced to zero at point P1, P2 and P3. This shows that when the crack tip extended to these points, the effect of residual stress on the crack growth disappears. The influence zone of residual stress are overload retardation.

Effective Stress Range

According to the above analysis, the effective stress range $\Delta S_{eff}$ in the overload retardation zone as:

$$\Delta S_{eff} = S_{max} - S_{res}$$

$$S_{res} = K_{max} / (\pi c \cdot \sec(\pi a / W))^{1/2}$$

(2)

Where, W is the width of the specimen.

Fig. 6 shows the distribution curve of effective stress range $\Delta S_{eff}$ during the crack growth in the overload retardation zone.
4. Analysis by CORPUS Model

The CORPUS model is associated with plastic deformation left in the wake of crack. It is assumed that each cycle will leave its own plastic deformation. And the wake of the crack is covered with “humps”. A crack is supposed to be still closed as long as there is contact between humps. Each hump will have its own effect on crack opening, and the hump will activate a “delay switch” which is turned off if the crack has grown through the plastic zone of the peak overload, see Fig. 7.

Because the length of the article limited, the CORPUS model is not described in detail. Based on CORPUS model, the effective stress range $\Delta S_{\text{eff}}$ of the above examples are shown in Fig. 8.

5. Conclusions

1. The effective stress range $\Delta S_{\text{eff}}$ with different $R_{\text{ol}}$ during the crack growth in the retardation zone is shown based on the crack residual stresses under single overload by using FEM and WFM calculation. And the $\Delta S_{\text{eff}}$ has been predicted by CORPUS model which based on the residual plastic deformation hump in wake of the crack. A satisfactory agreement is found between FEM and WFM simulate calculation and CORPUS model prediction results.
2. The correlation of $\Delta S_{\text{eff}}$ (effective stress range) with $R_{\text{ol}}$ and $a$ (crack length) is analyzed preliminarily by combining with residual stress $\sigma_{\text{res}}$ and residual plastic wake field. Much larger retardations are observed with a high overload ratio $R_{\text{ol}}$. After the higher application of overload, a most significant delay of fatigue crack growth occurred.

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References
[1] Larue J E, Daniewicz S R. Predicting the effect of residual stress on fatigue crack growth [J]. Int J Fatigue, 2007, 29: 508-516.
[2] Wang H, Buchholz F G, Richard H A, et al. Numerical and experimental analysis of residual stresses for fatigue crack growth [J]. Computational Materials Science, 1999, 16: 104-112.
[3] Dahl, W. and Roth, G., On the influence of overloads on fatigue crack propagation instructural steels. Paper Technical University Aachen (1979).
[4] Schijve, J., Four lectures on fatigue crack growth. Engrg. FractureMech., Vol. 11 (1979): 176–221.
[5] Wu X R, Calsson A J. Weight functions and stress intensity factor solutions[M]. Oxford: Pergamon Press, 1991: 32-38; 1991: 103-105.