Interaction-Driven Topological Phase Transitions in Fermionic SU(3) Systems

Mohsen Hafez-Torbati,1,+ Jun-Hui Zheng,1,2 Bernhard Irisiger,1 and Walter Hofstetter1,1

1Institut für Theoretische Physik, Goethe-Universität, 60438 Frankfurt/Main, Germany.
2Center for Quantum Spintronics, Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
(Dated: November 12, 2019)

We consider SU(3) fermions on the triangular lattice in the presence of a gauge potential which stabilizes a quantum Hall insulator (QHI) at the density of one particle per lattice site. We investigate the effect of the Hubbard interaction, favoring magnetic long-range order, and a three-sublattice potential (TSP), favoring a normal insulator (NI), on the system. For weak TSP we find that the Hubbard interaction drives the QHI into a three-sublattice magnetic Mott insulator (MMI). For intermediate values of TSP we identify two transition points upon increasing the Hubbard interaction. The first transition is from the NI to the QHI and the second transition is from the QHI to the MMI. For large values of the TSP a charge-ordered magnetic insulator (COMI) emerges between the NI and the QHI, leading to an interaction-driven COMI-to-QHI transition.

Since the experimental discovery of the quantum Hall effect in two-dimensional (2D) electron systems [1] novel types of band insulators such as quantum Hall (QHI) [2] and quantum spin Hall (QSHI) insulator [3] have been identified, which are characterized by topological invariants and can not be adiabatically connected to the previously-known normal insulators (NIs) [4]. The QHI occurs at particular particle fillings when a constant magnetic field is applied perpendicular to a 2D lattice potential, splitting a single energy band into several subbands [5], each one carrying an integer quantum number [2] called Chern number [6]. The QSHI is a result of time-reversal symmetry and spin-orbit coupling and is characterized by a $Z_2$ topological invariant [7].

The effect of interaction on a band insulator (BI) and emergence of Mott physics in the strong coupling regime has been an interesting problem for a long time [8], initially motivated by the observation of neutral-ionic phase transition in organic compounds [9]. A spontaneously dimerized phase [10–13] stabilized by condensation of a singlet exciton [14–17] separates the NI from Mott insulator (MI) as is studied via the 1D ionic Hubbard model. The ground state phase diagram of the model on the square lattice is controversial [18–20], and on the honeycomb lattice an intermediate (semi-)metallic phase is expected [21–23]. The reported intermediate metallic phase [24] for the paramagnetic solution of the dynamical mean-field equations on the Bethe lattice disappears when the antiferromagnetic solution is allowed [25, 26].

In recent years, there has been a large interest in interacting topological insulators [27], with a focus on realizing topological many-body quantum states such as fractional QHI [28] and studying interaction-driven topological phase transitions [29–34]. In the time-reversal-invariant Harper-Hofstadter-Hubbard model with a spin-mixing hopping term an interaction-driven NI-to-QSHI transition is identified [29], which is found also in an extended Bernevig-Hughes-Zhang-Hubbard model [30, 31]. The competition of the Hubbard interaction and the staggered potential in the Haldane-Hubbard model stabilizes an antiferromagnetic Chern insulator (AFCI) where one of the spin components is in the quantum Hall and the other in the normal state [32, 33]. Such an AFCI is proposed also for the Kane-Mele-Hubbard model but with a spontaneous breaking of the time-reversal symmetry [34].

Spin-orbit coupling in multicomponent systems can give rise to a richer topological band structure compared to the SU(2) case [35–37]. In the Mott regime SU(N) systems are potential candidates to find novel ordered and disordered MIs [38–43]. Furthermore, interaction-driven metallic phases and a charge-ordered magnetic insulator (COMI) are reported as a result of competing charge and magnetic order in fermionic SU(3) systems [44].

Here we investigate SU(3) fermions on the triangular lattice at 1/3 filling in the presence of a gauge potential stabilizing a QHI. We study the effect of the Hubbard interaction and a three-sublattice potential (TSP) on the QHI phase. For weak TSP, the Hubbard interaction drives the QHI into a three-sublattice magnetic MI (MMI). For intermediate values of the TSP we find the NI at weak and the MMI at strong Hubbard $U$, separated by a QHI. For large TSP an additional COMI phase emerges between the NI and the QHI. This leads to the realization of an interaction-driven COMI-to-QHI transition. The study is experimentally motivated by the recent progress in realization of artificial gauge fields [45–48] and creation of SU(N)-symmetric multicomponent systems [49–53] in optical lattices. The Hamiltonian reads

$$H = -t \sum_{\langle rr' \rangle} \sum_\alpha \left( c_{r\alpha}^\dagger c_{r'\alpha} e^{i\pi \varphi_{r-r'}^{\alpha}} + H.c. \right) + \sum_{r \alpha} \Delta_{r} n_{r\alpha}$$

$$+ U \sum_{r} \sum_{\alpha < \alpha'} n_{r\alpha} n_{r\alpha'},$$

(1)

where $c_{r\alpha}^\dagger$ is the fermionic creation operator at the lattice position $r$ with the spin component $\alpha$, $n_{r\alpha} = c_{r\alpha}^\dagger c_{r\alpha}$ is the occupation number operator, and the summation over $\langle rr'\rangle$ restricts the hopping to nearest-neighbor sites.
The hopping phase factors $\phi_{r,r'}$ around each triangle add up to a constant $\Phi$ which describes the magnetic flux going through each triangle in units of the magnetic flux quantum. The three sublattices $A$, $B$, and $C$ of the tripartite triangular lattice acquire respectively the onsite energies $-\Delta_1$, $0$, and $+\Delta_2$ due to the second term, the TSP. The last term is the Hubbard interaction.

We map the triangular lattice to the square lattice with hopping along the $\hat{x}$, $\hat{y}$, and $(\hat{x} + \hat{y})$ directions. We consider the hopping phase factors $\phi_{r,r+\hat{x}} = 0$, $\phi_{r,r+\hat{y}} = (2m + 2n + 1)\Phi$, and $\phi_{r,r+\hat{x}+\hat{y}} = 2(m + n + 1)\Phi$ from the lattice position $r = a\hat{x} + n\hat{y}$, where $a$ is the lattice constant and $m,n \in \mathbb{Z}$ [54]. There are three sites in the unit cell for $\Phi = 1/6$, which is the flux we consider in this paper. In the absence of interaction the Hamiltonian reduces to a three-level problem in momentum space leading to three distinct Bloch bands with a three-fold spin degeneracy each. We determine the Chern number of the system at $U = 0$ using twisted boundary conditions [55, 56]. We employ real-space dynamical mean-field theory (DMFT) [57–59] which we implemented for SU(3) systems in Ref. 42 to address the Hamiltonian at finite $U$. In real-space DMFT the self-energy is approximated up to an irrelevant shift in the energy spectrum. We have considered $L \times L$ lattices with $L = 30$ and periodic boundary conditions unless mentioned otherwise. We use the exact diagonalization (ED) impurity solver with four and five bath sites and check that the results nicely agree across different transition points. The presented results are for five bath sites unless mentioned otherwise. We have used the inverse temperature $\beta = 32/t$. We find at different selected parameter values that the results remain unchanged compared to the ones obtained using a zero temperature ED impurity solver [60]. We expect that a temperature $T = t/32$ is low enough to capture the ground state properties of the model.

We evaluate the Chern number of the interacting system using the topological Hamiltonian approach [61]. This method states that the Chern number of an interacting system is equal to the Chern number of an effective non-interacting model called “topological Hamiltonian”, which in the Bloch form reads

$$h_t(k) = h_0(k) + \Sigma(k, i\omega = 0),$$

where $h_0(k)$ is the non-interacting part of the original model and $\Sigma(k, i\omega)$ stands for the self-energy. In DMFT the self-energy is local and we have no element in the Hamiltonian and in the self-energy linking different spin components. Consequently, the effect of self-energy in Eq. (2) will be to renormalize the TSP to

$$\hat{\Delta}_{1,\alpha} = \Delta_1 + \left(\Sigma_{B,\alpha}(0) - \Sigma_{A,\alpha}(0)\right),$$

$$\hat{\Delta}_{2,\alpha} = \Delta_2 + \left(\Sigma_{C,\alpha}(0) - \Sigma_{B,\alpha}(0)\right),$$

up to an irrelevant shift in the energy spectrum. We have used $\Sigma_{A,\alpha}(0)$ for the zero-frequency self-energy on sublattice $A$ with spin component $\alpha$ and similarly for sublattices $B$ and $C$ [62]. The effective TSP Eq. (3) in paramagnetic phases is spin-independent, while in magnetically ordered phases, i.e., in phases with broken SU(3) symmetry, it depends on the spin. This shows that different spin components can in principle occur in distinct topological regions. We notice that the system at large $U$ develops long-range magnetic order and hence we do not discuss the paramagnetic MI solution of the DMFT equations where the self-energy at zero frequency diverges [63].

Fig. 1(a) shows the phase diagram of the model at $U = 0$ in the $\Delta_1$-$\Delta_2$ plane. The shaded area denotes the QHI and the white area the NI phase. In the QHI each spin component $\alpha$ contributes a Chern number $C_\alpha = 1$, leading to the Chern number $C = 3$ for the full system. The three asymptotic branches for the phase boundaries can be understood based on the sublattice degeneracy. For instance, sublattices $A$ and $B$ are degenerate at $\Delta_1 = 0$ and upon increasing $\Delta_2 \rightarrow +\infty$ always the two lowest Bloch bands remain topological, leading to a QHI state at 1/3 filling. Fig. 1(a) can be used to determine also the topological properties of the interacting model as the effect of the interaction is only to renormalize the TSP.

For SU(3) systems we define the double occupancy $D_r = \sum_{\alpha<\alpha'}\langle n_{r\alpha'} n_{r\alpha}\rangle$ and the local moment $m_r = \sqrt{3} ||\langle S_r\rangle||/2$ where $S_r^\alpha = \sum_{\alpha' \lambda} c_{r\alpha'}^\dagger \lambda\gamma_{\alpha'\alpha} c_{r\alpha}$ for $i=1, \ldots, 8$ define the elements of the eight-dimensional pseudospin operator $\hat{S}_r$ with $\lambda$ being the Gell-Mann matrices. In magnetic phases there is a continuous degeneracy and we focus on the solution with pseudospin order in the $\hat{S}_7, \hat{S}_8$ plane. In Fig. 1(b) the double occupancy and the local moment in the QHI and in the MMI are depicted versus $U$ for $\Delta_1 = \Delta_2 = 0$. The QHI and the MMI are two DMFT solutions coexisting in the gray area. The QHI results from the zero effective TSP in the paramagnetic region. The MMI is topologically trivial as we find all the three spin components in the normal state. This is a point which we will discuss further in Fig. 3. For $\Delta_1 = \Delta_2 = 0$ $D_r$ and $m_r$ are position-independent. The red solid line at $U_c \simeq 14.5 t$ specifies the transition point obtained by comparing the energy of the two states. The MMI has a three-sublattice magnetic order such that on each sublattice one of the spin components has the dominant density and the density of the other two components is equal, leading to a 120° pseudospin order [42, 64].

To investigate gapless edge states in the QHI we consider a $30 \times 30$ lattice with periodic boundary condition along $\hat{y}$ and open boundary condition along $\hat{x}$, i.e., a cylindrical geometry, with edges at $x = 0$ and $x = 29a$. The spectral function at position $r$ for the spin component $\alpha$ is defined from the local Green’s function as $A_{r\alpha}(\omega) = -\frac{1}{\pi} \text{Im} G_{r\alpha}(\omega + i\epsilon)$ where $\epsilon$ is a numerical broadening factor. In Fig. 1(c) the spectral function $A_{r\alpha}(\omega)$ for $U = 9t$ and $\Delta_1 = \Delta_2 = 0$ is plotted versus frequency
we have plotted $A_{\alpha}(\omega) = A_{\alpha}(\omega)[t]$ at $U = 9t$ and $\Delta_1 = \Delta_2 = 0$ versus frequency $\omega$ for a cylindrical geometry with edges at $x = 0$ and $x = 29a$. $\omega$ in the range $-3t < \omega < 3t$ with $\epsilon = 0.05t$. The dashed line at $\omega = 0$ specifies the Fermi energy. Due to the finite number of bath sites $N_b = 5$ in the impurity problem the fine details of the spectral function can not be reserved. However, one can clearly identify the spectral contribution from the edge $x = 0$ near the Fermi energy, which vanishes upon approaching the bulk $x = 14a$. It is interesting that even with a finite number of bath sites one can see evidence of gapless edge states. The edge and the bulk spectral function on finite clusters in an interacting topological insulator is discussed also in Ref. \[\text{[54]}\]. However, we notice that computing topological invariants is a more accurate and reliable way to recognize topological phase transitions.

We leave a general study of the Hubbard interaction on the phase diagram Fig. 1(a) for future research and consider here for simplicity $\Delta_1 = \Delta_2 = \Delta > 0$. We believe that small deviations from this symmetric case will not change the physics discussed in the following essentially. At $U = 0$ there is a transition from the QHI to the NI at $\Delta_c = 3t/\sqrt{2}$ upon increasing $\Delta$. In Fig. 2 we have plotted the double occupancy $D_A$ and the local moment $m_A$ on sublattice $A$ as well as the Chern number $C_\alpha$ versus the Hubbard interaction $U$ for $\Delta = 6t$ (a) and $\Delta = 11t$ (b). To avoid a busy figure the local moment is given only in magnetic phases (MP) as it is trivially zero in paramagnetic phases (PP). In addition we find $C_\alpha = 0$ for all three spin components in MP, see also below. The given spin-independent $C_\alpha$ is for PP. The gray area indicates coexistence of magnetic and paramagnetic DMFT solutions. One notices that in Fig. 2(b) the COMI always coexists with a paramagnetic phase and the given Chern number is for the paramagnetic phase not for the COMI. The red vertical solid line specifies the transition point and is obtained by comparing the energies of the two states in the case of coexistence. The blue vertical dashed line denotes the NI-to-QHI transition ignoring the magnetic DMFT solution.

One can see from Fig. 2(a) that the Hubbard interaction drives the NI into the QHI and subsequently the QHI into the MMI. Similar sequences of phase transitions are found in SU(2) topological systems \[\text{[29-34]}\]. Upon increasing the TSP to $\Delta = 11t$ in Fig. 2(b) a COMI phase emerges between the NI and the QHI. In the COMI phase, sublattice $A$ is almost doubly occupied with two spin components, sublattice $B$ is mainly occupied with the third component, and sublattice $C$ is almost empty. The local moment on sublattice $A$ and $B$ is equal and it is zero on sublattice $C$. There is a $180^\circ$ pseudospin order on sublattices $A$ and $B$ \[\text{[44]}\]. We find that the COMI always has a lower energy than the paramagnetic phases, i.e., the NI and the QHI are metastable. We notice that charge order is an intrinsic property of the COMI phase as it is not adiabatically connected to any phase with a uniform charge distribution. This is to be compared with the QHI and MMI phases which are adiabatically.
connected to $\Delta=0$ limit where the charge distribution is uniform. We believe the Hubbard interaction driving a magnetic phase into a quantum Hall state as it occurs in the COMI-to-QHI transition is a peculiar feature of multicomponent systems which has no SU(2) counterpart.

The double occupancy $D_A$ versus $U$ in Fig. 2 exhibits a change of slope in different phases and can be conveniently measured in optical lattices using the photoassociation technique [52]. The magnetic order can be identified using a quantum gas microscope [66, 67]. Lower temperatures are accessible in multicomponent systems compared to the SU(2) case due to a Pomeranchuk cooling effect [68]. We notice that to realize magnetic order at finite temperature in our system a weak coupling in the third direction or an interaction anisotropy is required.

To further clarify the topological nature of different phases we study in Fig. 3 the evolution of the effective TSP as a function of $U$ for the paramagnetic DMFT solution (a), for the COMI with $\Delta = 11t$ (b), and for the MMI with $\Delta = 6t$ (c). The direction of the curves are upon increasing $U$. The shaded area corresponds to QHI and the white area to NI. One sees from Fig. 3(a) that for $\Delta = 2t$ the system is always in the QHI region but for $\Delta = 6t$ and $\Delta = 11t$ a NI-to-QHI transition occurs. Figs. 3(b) and 3(c) demonstrate that the COMI and the MMI are topologically trivial as all the three spin components $\alpha = \uparrow, 0, \downarrow$ are in the NI region. The larger the local moment is in the MMI and in the COMI in Figs. 2(a) and 2(b) the deeper the corresponding topological Hamiltonian is in the NI in Fig. 3. The interaction-driven topological phase transitions can be studied in optical lattices using the tomography scheme proposed in Ref. 69.

Fig. 4 displays the phase diagram in the $U$-$\Delta$ plane. The gray areas denote the coexistence of magnetic and paramagnetic states, the red lines are the phase boundaries, and the blue line separates the NI from the QHI ignoring the magnetic DMFT solution. The solid (dashed) line indicates a continuous (discontinuous) transition. We have used four bath sites in the impurity problem due to the large number of data we needed to produce. However, by comparing Fig. 4 with Fig. 1(b) and Fig. 2 one can see the nice agreement for coexistence regions and transition points obtained with five and four bath sites. We have performed further checks across some other selective transition points. We always find that the NI-to-QHI transition is continuous, although discontinuous transitions in two-orbital systems are also reported [31]. The coexistence regions shrink upon increasing $\Delta$. The QHI in the limit $U, \Delta \gg t$ appears around $U = 2\Delta$ where the COMI and the MMI are degenerate in the atomic limit, i.e., at $t = 0$ [44]. We have produced the phase diagram up to $U = 32t$ and $\Delta = 20t$ and the QHI persists with a constant width.

To summarize, in recent years there has been a large interest in fermionic SU($N$) systems [38, 53] as well as in artificial gauge fields [70-72] due to their possible realization in optical lattices. While studies of SU($N$) systems have mainly been focused on topological states in the absence of interaction [35-37] and on Mott states in the strong coupling limit [38-43], less attention has so far been paid to the competition of band and Mott insulator and possible emergence of intermediate phases and novel phenomena. This requires tuning the interaction from weak to strong which can experimentally be achieved by Feshbach resonances [73-75]. In this paper we investigate the effect of the Hubbard interaction $U$ and a three-sublattice potential $\Delta$ on triangular lattice SU(3) fermions at 1/3 filling subject to a gauge potential. For weak $\Delta$ we identify a direct transition from a QHI to a MMI phase. For intermediate values of $\Delta$ we find the QHI separating the NI at weak and the MMI at strong Hubbard $U$. For larger values of $\Delta$ an additional COMI phase emerges between the NI and the QHI. This leads to an interaction-driven COMI-to-QHI transition, which is considered to be peculiar to multicomponent systems.
We would like to thank J. Panas for useful discussions. This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Project No. 277974659 via Research Unit FOR 2414. This work was also supported by the DFG via the high performance computing center LOEWE-CSC.

*torbati@itp.uni-frankfurt.de
hoefstett@physik.uni-frankfurt.de

[1] K. v. Klitzing, G. Dorda, and M. Pepper, “New method for high-accuracy determination of the fine-structure constant based on quantized hall resistance,” Phys. Rev. Lett. 45, 494–497 (1980).

[2] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, “Quantized hall conductance in a two-dimensional periodic potential,” Phys. Rev. Lett. 49, 405–408 (1982).

[3] C. L. Kane and E. J. Mele, “Quantum spin hall effect in graphene,” Phys. Rev. Lett. 95, 226801 (2005).

[4] M. Z. Hasan and C. L. Kane, “Colloquium: Topological insulators,” Rev. Mod. Phys. 82, 3045–3076 (2010).

[5] Douglas R. Hofstadter, “Energy levels and wave functions of bloch electrons in rational and irrational magnetic fields,” Phys. Rev. B 14, 2239–2249 (1976).

[6] Barry Simon, “Holonomy, the quantum adiabatic theorem, and berry’s phase,” Phys. Rev. Lett. 51, 2167–2170 (1983).

[7] C. L. Kane and E. J. Mele, “Z2 topological order and the quantum spin hall effect,” Phys. Rev. Lett. 95, 146802 (2005).

[8] Naoto Nagaosa and J. Takimoto, “Theory of neutral-dimensional periodic potential,” Phys. Rev. Lett. 98, 046403 (2007).

[9] J. B. Torrance, J. E. Vazquez, J. J. Mayerle, and V. Y. Meden, R. M. Noack, and S. R. Minor, “Exact bond ordered insulators,” Phys. Rev. Lett. 112, 106406 (2014).

[10] Michele Fabrizio, Alexander O. Gogolin, and Alexander A. Nersesyan, “From band insulator to mott insulator in one dimension,” Phys. Rev. Lett. 83, 2014–2017 (1999).

[11] S. R. Manmana, V. Meden, R. M. Noack, and K. Schönhammer, “Quantum critical behavior of the one-dimensional ionic hubbard model,” Phys. Rev. B 70, 155115 (2004).

[12] C. D. Batista and A. A. Aligia, “Exact bond ordered ground state for the transition between the band and the mott insulator,” Phys. Rev. Lett. 92, 246405 (2004).

[13] Karla Loida, Jean-Sébastien Bernier, Roberta Citro, Edmond Orignac, and Corinna Kollath, “Probing the bond order wave phase transitions of the ionic hubbard model by superlattice modulation spectroscopy,” Phys. Rev. Lett. 119, 230403 (2017).

[14] M. Hafez Torbati, Nils A. Drescher, and Götz S. Uhrig, “Dispersive excitations in one-dimensional ionic hubbard model,” Phys. Rev. B 89, 245126 (2014).

[15] Mohsen Hafez-Torbati, Nils A. Drescher, and Götz S. Uhrig, “From gapped excitons to gapless triplets in one dimension,” The European Physical Journal B 88, 3 (2015).

[16] M. Hafez and S. A. Jafari, “Excitation spectrum of one-dimensional extended ionic hubbard model,” The European Physical Journal B 78, 323–333 (2010).

[17] Mohsen Hafez and M.R. Abolhassani, “Dynamics in the one-dimensional extended ionic hubbard model,” Journal of Physics: Condensed Matter 23, 245602 (2011).

[18] N. Paris, K. Boudaim, F. Hébert, G. G. Batrouni, and R. T. Scalettar, “Quantum monte carlo study of an interaction-driven band-insulator-to-metal transition,” Phys. Rev. Lett. 98, 046403 (2007).

[19] S. S. Kancharla and E. Dagotto, “Correlated insulated phase suggests bond order between band and mott insulators in two dimensions,” Phys. Rev. Lett. 98, 016402 (2007).

[20] Mohsen Hafez-Torbati and Götz S. Uhrig, “Orientational bond and néel order in the two-dimensional ionic hubbard model,” Phys. Rev. B 93, 195128 (2016).

[21] M. Ebrahimkhahs and S. A. Jafari, “Short-range coulomb correlations render massive dirac fermions massless,” EPL (Europhysics Letters) 98, 27009 (2012).

[22] M. Ebrahimkhahs, Z. Drezhnegirghash, and E. Soltani, “Effects of correlations on honeycomb lattice in ionic-hubbard model,” Physics Letters A 379, 1053 – 1056 (2015).

[23] Jingyao Wang, Lufeng Zhang, Qiaoni Chen, Ying Liang, and Tianxing Ma, “Quantum Monte Carlo study of the intermediate phase in an interacting honeycomb lattice with staggered potential,” arXiv e-prints , arXiv:1904.05504 (2019), arXiv:1904.05504 [cond-mat.str-el].

[24] Arti Garg, H. R. Krishnamurthy, and Mohit Randeria, “Can correlations drive a band insulator metallic?” Phys. Rev. Lett. 97, 046403 (2006).

[25] Krzysztof Byczuk, Michael Sekania, Walter Hofstetter, and Arno P. Kampf, “Insulating behavior with spin and charge order in the ionic hubbard model,” Phys. Rev. B 79, 121103 (2009).

[26] Arti Garg, H.R. Krishnamurthy, and Mohit Randeria, “Doping a correlated band insulator: A new route to half-metallic behavior,” Phys. Rev. Lett. 112, 106406 (2014).

[27] Stephan Rachel, “Interacting topological insulators: a review,” Reports on Progress in Physics 81, 116501 (2018).

[28] Siddharth A. Parameswaran, Rahul Roy, and Shivaji L. Sondhi, “Fractional quantum hall physics in topological flat bands,” Comptes Rendus Physique 14, 816 – 839 (2013), topological insulators / Isolants topologiques.

[29] Daniel Cocks, Peter P. Orth, Stephan Rachel, Michael Buchhold, Karyn Le Hur, and Walter Hofstetter, “Time-reversal-invariant hofstadter-hubbard model with ultracold fermions,” Phys. Rev. Lett. 109, 205303 (2012).

[30] Jan Carl Budich, Björn Trauzettel, and Giorgio Sangiovanni, “Fluctuation-driven topological band insulators,” Phys. Rev. B 87, 235104 (2013).

[31] A. Amaricci, J. C. Budich, M. Capone, B. Trauzettel, and G. Sangiovanni, “First-order character and observable signatures of topological quantum phase transitions,” Phys. Rev. Lett. 114, 185701 (2015).

[32] Jing He, Yan-Hua Zong, Su-Peng Kou, Ying Liang, and Shiping Feng, “Topological spin density waves in the Hubbard model on a honeycomb lattice,” Phys. Rev. B 84, 035127 (2011).

[33] Tuomas I. Vanhala, Topi Siro, Long Liang, Matthias Troyer, Ari Harju, and Päivi Törnä, “Topological phase transitions in the repulsively interacting halblane-
Zhichao Zhou, Da Wang, Zi Yang Meng, Yu Wang, and Tamás A. Tóth, Andreas M. Luchli, Frédéric Mila, Kun Jiang, Sen Zhou, Xi Dai, and Ziqiang Wang, “An-
Man Hon Yau and C. A. R. Sá de Melo, “Topolog-
Gregor Jotzu, Michael Messer, Rmi Desbuquois, Martin Lebrat, Thomas Uehlinger, Daniel Greif, and Tilman Esslinger, “Experimental realization of the topological haldean model with ultrafast fermions,” Nature 515, 237 (2014).
A. V. Gorshkov, M. Hermele, V. Guraie, C. Xu, P. S. Julienne, J. Ye, P. Zöller, E. Demler, M. D. Lukin, and A. M. Rey, “Two-orbital su(n) magnetism with ultracold alkaline-earth atoms,” Nature Physics 6, 289–295 (2010).
Tamás A. Tóth, Andreas M. Läuchli, Frédéric Mila, and Karlo Panc, “Three-sublattice ordering of the su(3) heisenberg model of three-flavor fermions on the square and cubic lattices,” Phys. Rev. Lett. 105, 265301 (2010).
Zhichao Zhou, Da Wang, Zi Yang Meng, Yu Wang, and Congjun Wu, “Mott insulating states and quantum phase transitions of correlated SU(2)n dirac fermions,” Phys. Rev. B 93, 245157 (2016).
Pierre Nataf, Miklos Lajk, Alexander Wietek, Karlo Panc, Frédic Mila, and Andreas M. Luchli, “Chiral spin liquid in triangular-lattice SU(n) fermionic mott insulators with artificial gauge fields,” Phys. Rev. Lett. 117, 167202 (2016).
Mohsen Hafez-Torbat and Walter Hofstetter, “Artificial su(3) spin-orbit coupling and exotic mott insulators,” Phys. Rev. B 98, 245131 (2018).
Sangwoo S. Chung and Philippe Corboz, “Su(3) fermions on the honeycomb lattice at 1 2 filling,” Phys. Rev. B 100, 035134 (2019).
Mohsen Hafez-Torbat and Walter Hofstetter, “Competing charge and magnetic order in fermionic multicompo-
nent systems,” Phys. Rev. B 100, 035133 (2019).
M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch, “Realization of the hosftader hamiltonian with ultracold atoms in optical lattices,” Phys. Rev. Lett. 111, 185301 (2013).
Hirokazu Miyake, Georgios A. Sivogloulou, Colin J. Kennedy, William Cody Burton, and Wolfgang Ketterle, “Realizing the harper hamiltonian with laser-assisted tunneling in optical lattices,” Phys. Rev. Lett. 111, 185302 (2013).
Gregor Jotzu, Michael Messer, Rmi Desbuquois, Martin Lebrat, Thomas Uehlinger, Daniel Greif, and Tilman Esslinger, “Experimental realization of the topological haldean model with ultrafast fermions,” Nature 515, 237 (2014).
M. Aidelsburger, “Artificial gauge fields and topology with ultracold fermions in optical lattices,” Journal of Physics B: Atomic, Molecular and Optical Physics 51, 193001 (2018).
T. B. Ottenstein, T. Lompe, M. Kohnen, A. N. Wenz, and S. Jochim, “Collisional stability of a three-component degenerate fermi gas,” Phys. Rev. Lett. 101, 203202 (2008).
J. H. Huckans, J. R. Williams, E. L. Hazlett, R. W. Stites, and K. M. O’Hara, “Three-body recombination in a three-state fermi gas with widely tunable interactions,” Phys. Rev. Lett. 102, 165302 (2009).
Hideaki Hara, Yusuke Takeda, Yosifumi Yamaoka, John M. Doyle, and Yoshiro Takahashi, “Quantum degenerate mixtures of alkali and alkaline-earth-like atoms,” Phys. Rev. Lett. 106, 205304 (2011).
Shintaro Taie, Rekishi Yamazaki, Seiji Sugawa, and Yoshiro Takahashi, “An su(n) mott insulator of an atomic fermi gas realized by large-spin pomeranchuk cooling,” Nature Physics 8, 825–830 (2012).
Miguel A. Cazalilla and Ana Maria Rey, “Ultracold fermi gases with emergent su(n) symmetry,” Reports on Progress in Physics 77, 124401 (2014).
These phase factors can be obtained using the Peierls substitution for the gauge field A = B(x + y) which describes the magnetic field B = Bz. The advantage of the chosen gauge over the Landau gauge is that it makes the hoppng phase factors depend on (x, y), which is the same position dependency as in the three-sublattice potential, and consequently reduces the number of sites in the unit cell.
Qian Niu, D. J. Thouless, and Yong-Shi Wu, “Quantized hall conductance as a topological invariant,” Phys. Rev. B 31, 3372–3377 (1985).
Koji Kudo, Haruki Watanabe, Toshikaze Kariyado, and Yasuhiro Hatsugai, “Many-body chern number without integration,” Phys. Rev. Lett. 122, 146601 (2019).
M. Potthof and W. Nölling, “Surface metal-insulator transition in the hubbard model,” Phys. Rev. B 59, 2549–2555 (1999).
Yun Song, R. Wortis, and W. A. Atkinson, “Dynamical mean field study of the two-dimensional disordered hubbard model,” Phys. Rev. B 77, 054202 (2008).
M. Snoek, I Titvinidze, C Tke, K Byczuk, and W Hof-
stetter, “Antiferromagnetic order of strongly interacting fermions in a trap: real-space dynamical mean-field analysis,” New Journal of Physics 10, 093008 (2008).
Michel Caffarel and Werner Krauth, “Exact diagonalization approach to correlated fermions in infinite dimensions: Mott transition and superconductivity,” Phys. Rev. Lett. 72, 1545–1548 (1994).
Zhong Wang and Shou-Cheng Zhang, “Simplified topo-
logical invariants for interacting insulators,” Phys. Rev. X 2, 031008 (2012).
We find that the real-part of the self-energy at the smallest (in absolute value) Matsubara frequency accurately describes the zero-frequency self-energy obtained by a polynomial fit.
Antoine Georges, Gabriel Kotliar, Werner Krauth, and Marcelo J. Rozenberg, “Dynamical mean-field theory of strongly correlated fermion systems and the limit of infiniti-
ness dimensions,” Rev. Mod. Phys. 68, 13–125 (1996).
Bela Bauer, Philippe Corboz, Andreas M. Luchli, Laura Mesioo, Karlo Panc, Matthias Troyer, and Frédric Mila, “Three-sublattice order in the su(3) heisenberg model on the square and triangular lattice,” Phys. Rev. B 85, 125116 (2012).
Christopher N. Varney, Kai Sun, Marcos Rigol, and Victor Galitski, “Interaction effects and quantum phase transitions in topological insulators,” Phys. Rev. B 82, 115125 (2010).
Anon Mazurenko, Christie S. Chiu, Geoffrey Ji, Maxwell F. Parsons, Merton Kansz-Nagy, Richard Schmidt, Fabian Grusdt, Eugene Demler, Daniel Greif,
and Markus Greiner, “A cold-atom fermi-hubbard anti-ferromagnet,” Nature 545, 462– (2017).

[67] Peter T. Brown, Debayan Mitra, Elmer Guardado-Sanchez, Peter Schauß, Stanimir S. Kondov, Ehsan Khatami, Thereza Paiva, Nandini Trivedi, David A. Huse, and Waseem S. Bakr, “Spin-imbalance in a 2d fermi-hubbard system,” Science 357, 1385–1388 (2017).

[68] Hideki Ozawa, Shintaro Taie, Yosuke Takasu, and Yoshiro Takahashi, “Antiferromagnetic spin correlation of SU(N) fermi gas in an optical superlattice,” Phys. Rev. Lett. 121, 225303 (2018).

[69] Jun-Hui Zheng, Bernhard Irsigler, Lijia Jiang, Christof Weitenberg, and Walter Hofstetter, “Measuring the topological phase transition via the single-particle density matrix,” arXiv e-prints , arXiv:1812.01991 (2018), arXiv:1812.01991 [cond-mat.quant-gas].

[70] Monika Aidelsburger, Sylvain Nascimbene, and Nathan Goldman, “Artificial gauge fields in materials and engineered systems,” Comptes Rendus Physique 19, 394 – 432 (2018), quantum simulation / Simulation quantique.

[71] N. R. Cooper, J. Dalibard, and I. B. Spielman, “Topological bands for ultracold atoms,” Rev. Mod. Phys. 91, 015005 (2019).

[72] W Hofstetter and T Qin, “Quantum simulation of strongly correlated condensed matter systems,” Journal of Physics B: Atomic, Molecular and Optical Physics 51, 082001 (2018).

[73] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, “Observation of feshbach resonances in a bose-einstein condensate,” Nature 392, 151–154 (1998).

[74] Ph. Courteille, R. S. Freeland, D. J. Heinzen, F. A. van Abeelen, and B. J. Verhaar, “Observation of a feshbach resonance in cold atom scattering,” Phys. Rev. Lett. 81, 69–72 (1998).

[75] Inmanuel Bloch, Jean Dalibard, and Wilhelm Zwerger, “Many-body physics with ultracold gases,” Rev. Mod. Phys. 80, 885–964 (2008).