Radiative and Semileptonic $B$ Decays Involving Higher $K$-Resonances in the Final States

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Abstract

We study the radiative and semileptonic $B$ decays involving a spin-$J$ resonant $K_J^{(*)}$ with parity $(-1)^J$ for $K_J^*$ and $(-1)^{J+1}$ for $K_J$ in the final state. Using the large energy effective theory (LEET) techniques, we formulate $B \to K_J^{(*)}$ transition form factors in the large recoil region in terms of two independent LEET functions $\zeta_{K_J^{(*)}}^{\perp}$ and $\zeta_{K_J^{(*)}}^{\parallel}$, the values of which at zero momentum transfer are estimated in the BSW model. According to the QCD counting rules, $\zeta_{K_J^{(*)}}^{\perp,\parallel}$ exhibit a dipole dependence in $q^2$. We predict the decay rates for $B \to K_J^{(*)}\gamma$, $B \to K_J^{(*)}\ell^+\ell^-$ and $B \to K_J^{(*)}\nu\bar{\nu}$. The branching fractions for these decays with higher $K$-resonances in the final state are suppressed due to the smaller phase spaces and the smaller values of $\zeta_{K_J^{(*)}}^{\perp,\parallel}$. Furthermore, if the spin of $K_J^{(*)}$ becomes larger, the branching fractions will be further suppressed due to the smaller Clebsch-Gordan coefficients defined by the polarization tensors of the $K_J^{(*)}$. We also calculate the forward backward asymmetry of the $B \to K_J^{(*)}\ell^+\ell^-$ decay, for which the zero is highly insensitive to the $K$-resonances in the LEET parametrization.

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I. INTRODUCTION

The flavor-changing neutral current (FCNC) $b \to s$ processes suppressed in the standard model (SM) could receive sizable new-physics contributions. Recently BABAR and Belle have shown interesting results on the longitudinal fraction, forward-backward asymmetry and isospin asymmetry of the $B \to K^* \ell^+ \ell^-$ decays [1–6]. Although the data are still consistent with the SM predictions, they favor the flipped-sign $c_{7}^{\text{eff}}$ models [7]. The minimal flavor violation supersymmetry models with large $\tan \beta$ can be fine-tuned to have the flipped sign $c_{7}^{\text{eff}}$, where the dominant contributions due to the charged Higgs exchange to $c_9$ and $c_{10}$ are suppressed by $1/\tan^2 \beta$ for large $\tan \beta$ [8, 9]. The LHCb is devoted to the $B$ physics studies. Due to the large cross section for $b \bar{b}$ production, the measurement for the rare decays can extend down to $10^{-9}$ branching ratio. It was estimated by the LHCb collaboration that with a data set of 2 fb$^{-1}$ the $B \to K^* \ell^+ \ell^-$ signal events can be improved by an order of magnitude compared with the present results.

Using the large energy effective theory (LEET) techniques [10], we have formulated the $B \to K_2^*(1430)$ form factors in the large recoil region [11], and further studied the decays $B \to K_2^*(1430)\gamma$, $B \to K_2^*(1430)\ell^+ \ell^-$ and $B \to K_2^*(1430)\nu \bar{\nu}$. In this paper we will generalize to the studies of $B \to K_j^{(s)}\gamma$, $B \to K_j^{(s)}\ell^+ \ell^-$ and $B \to K_j^{(s)}\nu \bar{\nu}$ decays.

\begin{table}[ht]
\centering
\caption{The data for branching ratios of the radiative and semi-leptonic $B$ decays involving strange mesons.}
\begin{tabular}{lcc}
\hline
mode & $\mathcal{B}$ [10$^{-6}$] & mode & $\mathcal{B}$ [10$^{-6}$] \\
\hline
$B^+ \to K_2^*(892)\gamma$ & $43.6 \pm 1.8$ [12–15] & $B^0 \to K_2^*(892)\gamma$ & $43.3 \pm 1.5$ [12–15] \\
$B^+ \to K_3^+(1780)\gamma$ & $14.5 \pm 4.3$ [16] & $B^0 \to K_3^*(1780)\gamma$ & $12.4 \pm 2.4$ [16, 17] \\
$B^+ \to K_3^+(1892)\gamma$ & $< 39$ [18] & $B^0 \to K_2^*(1430)\gamma$ & $< 83$ [18] \\
$B^+ \to K_3^+(1430)\gamma$ & $143.5 \pm 4.3$ [16] & $B^0 \to K_3^*(1430)\gamma$ & $12.4 \pm 2.4$ [16, 17] \\
$B^+ \to K_3^+(1780)\gamma$ & $< 39$ [18] & $B^0 \to K_2^*(1780)\gamma$ & $< 83$ [18] \\

$B^+ \to K_1^+(1270)\gamma$ & $43 \pm 12$ [22] & $B^0 \to K_1^+(1270)\gamma$ & $< 58$ [22] \\
$B^+ \to K_1^+(1400)\gamma$ & $< 15$ [22] & $B^0 \to K_1^+(1400)\gamma$ & $< 15$ [22] \\

$B^+ \to K_1^+(1270)\gamma$ & $43 \pm 12$ [22] & $B^0 \to K_1^+(1270)\gamma$ & $< 58$ [22] \\
$B^+ \to K_1^+(1400)\gamma$ & $< 15$ [22] & $B^0 \to K_1^+(1400)\gamma$ & $< 15$ [22] \\

$b \to s \gamma$ & $352 \pm 25$ [23–25] & $b \to s \ell^+ \ell^-$ & $4.50^{+1.03}_{-1.01}$ [26–28] \\
\hline
\end{tabular}
\end{table}
within the SM, where $K_J^*$ and $K_J$ are the spin-$J$ resonances with parities $(-1)^J$ and $(-1)^{J+1}$, respectively. We anticipate to see these modes at LHCb, compared with the current data in Table I. In the present study, we will show that the form factors for general $B \to K_J^{(*)}$ transitions can be parametrized in terms of two independent LEET functions, $\zeta_{\perp}^{K_J^{(*)}}(q^2)$ and $\zeta_{\parallel}^{K_J^{(*)}}(q^2)$ together with the Clebsch-Gordan coefficients, $\alpha_L^{(J)}$ and $\beta_T^{(J)}$. The values of $\zeta_{\perp}^{K_J^{(*)}}(0)$ and $\zeta_{\parallel}^{K_J^{(*)}}(0)$ will be estimated by using the Bauer-Stech-Wirbel (BSW) model. Moreover, we find that branching fractions with higher resonances, $K_J^{(*)}$, becomes smaller not only due to their smaller phase spaces, but also to the smaller $\zeta_{\perp,\parallel}^{K_J^{(*)}}$. Meanwhile, the branching fractions involving $K_J^{(*)}$ with higher spin $J$ will be further suppressed due to smaller Clebsch-Gordan coefficients defined by the polarization tensors of the $K_J^{(*)}$.

There have been a few studies of radiative $B$ decays into higher $K$-resonances in the literature. A discussion for the general cases was given in Ref. [32], where for various processes the authors parameterize the relevant form factors into four Isgur-Wise functions, which are estimated from Isgur-Scora-Grinstein-Wise (ISGW) model [36]. However, they obtained $B(B \to K_1(1270)\gamma) < B(B \to K_1(1400)\gamma) \simeq (2.4 - 5.2) \times 10^{-5}$, in contradiction to the observation (see Table I). One of the motivations for this work is further to re-examine the other radiative decay channels with higher $K$-resonances.

This paper is organized as follows. In Sec. II we formulate the $B \to K_J^{(*)}$ form factors using the LEET techniques. In Sec. III we estimate the LEET form factors, $\zeta_{\perp}^{K_J^{(*)}}(0)$ and $\zeta_{\parallel}^{K_J^{(*)}}(0)$, in the BSW model, and then numerically study the radiative and semileptonic $B$ meson decays into the $K_J^{(*)}$, including the analyses for the forward-backward asymmetries and longitudinal fraction distributions for $B \to K_J^{(*)}\mu^+\mu^-$. We conclude with a summary in Sec. IV. The derivation of the $B \to K_J$ form factors is given in Appendix A.

II. $B \to K^*_J$ FORM FACTORS IN THE LARGE RECOIL REGION

In this section, using the LEET technique, we formulate $B \to K^*_J$ form factors in the large recoil region. The analogous formulation for $B \to K_J$ form factors is given in Appendix A. In this paper $K^*_J$ and $K_J$ stand for the higher spin-$J$ $K$-resonances with parities $(-1)^J$ and $(-1)^{J+1}$, respectively. For simplicity we study in the rest frame of the
B meson (with mass \( m_B \)) and assume that the tensor meson \( K_J^* \) (with mass \( m_{K_J^*} \) and energy \( E \)) moves along the z-axis. In the LEET limit, \( E, m_B \gg m_{K_J^*}, \Lambda_{\text{QCD}} \), the momenta of the \( B \) and \( K_J^* \) are given by

\[
p_B^\mu = (m_B, 0, 0, 0) = m_B v^\mu, \quad p_{K_J^*}^\mu = (E, 0, 0, p_3) \simeq E n^\mu, \tag{1}
\]

respectively. Here \( v^\mu = (1, 0, 0, 0) \), \( n^\mu = (1, 0, 0, 1) \), and the tensor meson’s energy \( E \) is given by

\[
E = \frac{m_B}{2} \left( 1 - \frac{q^2}{m_B^2} + \frac{m_{K_J^*}^2}{m_B^2} \right), \tag{2}
\]

with \( q = p_B - p_{K_J^*} \).

The polarization tensors \( \varepsilon(\lambda)^{\mu_1 \mu_2 \cdots \mu_J} \) of the massive spin-\( J \) meson with helicity \( \lambda \) that can be constructed in terms of the polarization vectors of a massless vector state with the mass \( m_{K_J^*} \)

\[
\varepsilon(0)^{\mu} = (p_3, 0, 0, E)/m_{K_J^*}, \quad \varepsilon(\pm 1)^{\mu} = (0, \mp 1, \mp i, 0)/\sqrt{2}, \tag{3}
\]

are given by

\[
\varepsilon(\pm 2)^{\mu\nu} \equiv \varepsilon(\pm 1)^{\mu} \varepsilon(\pm 1)^{\nu}, \tag{4}
\]

\[
\varepsilon(\pm 1)^{\mu\nu} \equiv \sqrt{\frac{1}{2}} \left[ \varepsilon(\pm 1)^{\mu} \varepsilon(0)^{\nu} + \varepsilon(0)^{\mu} \varepsilon(\pm 1)^{\nu} \right], \tag{5}
\]

\[
\varepsilon(0)^{\mu\nu} \equiv \sqrt{\frac{1}{6}} \left[ \varepsilon(+1)^{\mu} \varepsilon(-1)^{\nu} + \varepsilon(-1)^{\mu} \varepsilon(+1)^{\nu} \right] + \sqrt{\frac{2}{3}} \varepsilon(0)^{\mu} \varepsilon(0)^{\nu}, \tag{6}
\]

for \( J = 2 \) and

\[
\varepsilon(\pm 3)^{\mu\nu\rho} = \varepsilon(\pm 1)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(\pm 1)^{\rho}, \tag{7}
\]

\[
\varepsilon(\pm 2)^{\mu\nu\rho} = \sqrt{\frac{1}{3}} \left[ \varepsilon(0)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(\pm 1)^{\rho} + \varepsilon(\pm 1)^{\mu} \varepsilon(0)^{\nu} \varepsilon(\pm 1)^{\rho} + \varepsilon(\pm 1)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(0)^{\rho} \right], \tag{8}
\]

\[
\varepsilon(\pm 1)^{\mu\nu\rho} = \sqrt{\frac{1}{15}} \left[ \varepsilon(\mp 1)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(\pm 1)^{\rho} + \varepsilon(\pm 1)^{\mu} \varepsilon(\mp 1)^{\nu} \varepsilon(\pm 1)^{\rho} + \varepsilon(\mp 1)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(\mp 1)^{\rho} \right] \\
+ 2 \sqrt{\frac{1}{15}} \left[ \varepsilon(\pm 1)^{\mu} \varepsilon(0)^{\nu} \varepsilon(\pm 1)^{\rho} + \varepsilon(0)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(\pm 1)^{\rho} + \varepsilon(0)^{\mu} \varepsilon(\pm 1)^{\nu} \varepsilon(0)^{\rho} \right], \tag{9}
\]

\[
\varepsilon(0)^{\mu\nu\rho} = \sqrt{\frac{1}{10}} \left[ \varepsilon(0)^{\mu} \varepsilon(+1)^{\nu} \varepsilon(-1)^{\rho} + \varepsilon(+1)^{\mu} \varepsilon(0)^{\nu} \varepsilon(-1)^{\rho} + \varepsilon(+1)^{\mu} \varepsilon(-1)^{\nu} \varepsilon(0)^{\rho} + \varepsilon(0)^{\mu} \varepsilon(-1)^{\nu} \varepsilon(+1)^{\rho} + \varepsilon(-1)^{\mu} \varepsilon(0)^{\nu} \varepsilon(+1)^{\rho} + \varepsilon(-1)^{\mu} \varepsilon(+1)^{\nu} \varepsilon(0)^{\rho} \right] + \\
\sqrt{\frac{2}{5}} \varepsilon(0)^{\mu} \varepsilon(0)^{\nu} \varepsilon(0)^{\rho}, \tag{10}
\]
for $J = 3$, and so on. $\varepsilon(\lambda)^{\mu_1\mu_2\cdots\mu_J}$ is symmetric under interchange of any two of $\mu_j$ and $\mu_k$ ($1 \leq j, k \leq J$), and satisfies divergence-free conditions $p_{K^*} \varepsilon(\lambda)^{\mu_1\cdots\mu_{J-1}} = 0$, traceless conditions $g_{\mu_1\mu_2} \varepsilon(\lambda)^{\mu_1\mu_2\nu_1\cdots\nu_{J-2}} = 0$, and orthonormal conditions $\varepsilon(h_1)^{\mu_1\mu_2\cdots\mu_J} \varepsilon(h_2)^{\mu_1\mu_2\cdots\mu_J} = \delta_{h_1 h_2}$.

In the following, we calculate the $\overline{B} \rightarrow K^*_J$ transition form factors:

$$
\langle \overline{K}_J | V^\mu | \overline{B} \rangle, \quad \langle \overline{K}_J^* | A^\mu | \overline{B} \rangle, \quad \langle \overline{K}_J^* | T^{\mu\nu} | \overline{B} \rangle, \quad \langle \overline{K}_J^* | T_A^{\mu\nu} | \overline{B} \rangle, \quad (11)
$$

where $V^\mu = \bar{s} \gamma^\mu b$, $A^\mu = \bar{s} \gamma^\mu \gamma_5 b$, $T^{\mu\nu} = \bar{s} \sigma^{\mu\nu} b$ and $T_A^{\mu\nu} = \bar{s} \sigma^{\mu\nu} \gamma_5 b$. In the LEET limit one can easily write down the relevant form factors in terms of the following projectors

$$
(\beta_T^{(J)})^{-1} \left( \frac{m_{K^*}}{E} \right)^{J-1} [e(\lambda)^{\nu} \cdot (e(\lambda))^* n^\mu] = \begin{cases} 
0 & \text{for } \lambda = \pm 2, \\
\varepsilon(\pm 1)^{\nu} & \text{for } \lambda = \pm 1, \\
0 & \text{for } \lambda = 0, 
\end{cases} (12)
$$

$$
(\beta_T^{(J)})^{-1} \left( \frac{m_{K^*}}{E} \right)^{J-1} \epsilon^{\mu\nu\rho\sigma} e(\lambda)^{\nu} n_\rho v_\sigma = \begin{cases} 
0 & \text{for } \lambda = \pm 2, \\
\epsilon^{\mu\nu\rho\sigma} (\pm 1)^{\nu} n_\rho v_\sigma & \text{for } \lambda = \pm 1, \\
0 & \text{for } \lambda = 0, 
\end{cases} (13)
$$

$$
(\alpha_L^{(J)})^{-1} \left( \frac{m_{K^*}}{E} \right)^J (e(\lambda)^* \cdot v) n^\mu = \begin{cases} 
0 & \text{for } \lambda = \pm 2, \\
0 & \text{for } \lambda = \pm 1, \\
n^\mu & \text{for } \lambda = 0, 
\end{cases} (14)
$$

$$
(\alpha_L^{(J)})^{-1} \left( \frac{m_{K^*}}{E} \right)^J (e(\lambda)^* \cdot v) v^\mu = \begin{cases} 
0 & \text{for } \lambda = \pm 2, \\
0 & \text{for } \lambda = \pm 1, \\
v^\mu & \text{for } \lambda = 0, 
\end{cases} (15)
$$

Together with $\epsilon^{\mu\nu\rho\sigma}$, $v^\mu$ and $n^\mu$, to project the relevant polarization states of the higher $K$-resonances, where Eqs. (12), (14) and (15) are the vectors, but Eq. (13) the axial-vector. Here $\epsilon^{0123} = -1$ and we have defined

$$
e(\lambda)^{\mu} \equiv \varepsilon(\lambda)^{\mu_1\mu_2\cdots\mu_{J-1}} v_{\nu_1} v_{\nu_2} \cdots v_{\nu_{J-1}} = \begin{cases} 
\alpha_L^{(J)} \varepsilon(0)^\mu \left( \frac{p_3}{m_{K^*}} \right)^{J-1} & \text{for } \lambda = 0, \\
\beta_T^{(J)} \varepsilon(1)^\mu \left( \frac{p_3}{m_{K^*}} \right)^{J-1} & \text{for } \lambda = \pm 1, 
\end{cases} (16)
$$

\[5\]
where $\alpha_L^{(j)}$ and $\beta_T^{(j)}$ are the Clebsch-Gordan coefficients of the specific terms of the polarization tensors:

$$
\varepsilon(0)^{\mu_1\cdots\mu_n} = \alpha_L^{(j)} \varepsilon(0)^{\mu} \varepsilon(0)^{\nu_1} \cdots \varepsilon(0)^{\nu_{j-1}} + \text{others},
$$

$$
\varepsilon(\pm 1)^{\mu_1\cdots\mu_n} = \beta_T^{(j)} \varepsilon(\pm 1)^{\mu} \varepsilon(0)^{\nu_1} \cdots \varepsilon(0)^{\nu_{j-1}} + \text{others},
$$

and are given by

$$
\alpha_L^{(j)} = J^{(j,0)} \cdot J^{(j-1,0)} \cdots J^{(2,0)},
$$

$$
\beta_T^{(j)} = J^{(j,1)} \cdot J^{(j-1,0)} \cdots J^{(2,0)},
$$

with $J^{(j,M)}$ being the short-hand notations of the following Clebsch-Gordan coefficients

$$
J^{(j,M)} = \langle (j_1 m_1), (j_2 m_2) | JM \rangle.
$$

The values of $\alpha_L^{(j)}$ and $\beta_T^{(j)}$ for $J = 1, 2, \cdots, 5$ are collected in Table II.

Matching the parities of the matrix elements and using the mentioned Lorentz structures, we can then easily parameterize the form factors to be

$$
\langle K_j | V^\mu | B \rangle = -2E \left( \frac{mk_f}{E} \right)^{J-1} \zeta_{\perp}^{K_j(v)} \varepsilon^{\mu\rho\sigma} \nu_\rho n_\sigma e^*_\rho,
$$

$$
\langle K_j | A^\mu | B \rangle = 2E \left( \frac{mk_f}{E} \right)^{J-1} \zeta_{\perp}^{K_j(a)} \left[ e^*_\mu - (e^* \cdot \nu) n_\mu \right]
+ 2E \left( \frac{mk_f}{E} \right)^{J} (e^* \cdot \nu) \left[ \zeta_{\parallel}^{K_j(a)} n_\mu + \zeta_{\parallel,1}^{K_j(a)} \nu_\mu \right],
$$

$$
\langle K_j | T^{\mu\nu} | B \rangle = 2E \left( \frac{mk_f}{E} \right)^{J} \zeta_{\parallel}^{K_j(t)(a)} (e^* \cdot \nu) \varepsilon^{\mu\rho\sigma} \nu_\rho n_\sigma
+ 2E \left( \frac{mk_f}{E} \right)^{J-1} \zeta_{\perp}^{K_j(t)} e^{\mu\rho\sigma} n_\rho [e^*_\sigma - (e^* \cdot \nu) n_\sigma]
+ 2E \left( \frac{mk_f}{E} \right)^{J-1} \zeta_{\perp,1}^{K_j(t)} e^{\mu\rho\sigma} n_\rho [e^*_\sigma - (e^* \cdot \nu) n_\sigma],
$$

where $J = 1, 2, \cdots, 5$.
Thus we find that there are only two independent form factors, \( \zeta_\perp \) and \( \zeta_\parallel \), for the \( B \to \bar{K}_J \) transition in the large recoil region. In the full theory, the \( B \to \bar{K}_J \) form factors which are independent,

\[
\begin{align*}
\langle \bar{K}_J | T_A^{\mu \nu} | B \rangle &= -i 2 E \left( \frac{m_{\bar{K}_J}}{E} \right)^{J-1} \zeta_\perp^{K_J(t_s)} \left\{ [e^\mu - (e^\mu \cdot v)] n^\nu - (\mu \leftrightarrow \nu) \right\} \\
&\quad - i 2 E \left( \frac{m_{\bar{K}_J}}{E} \right)^{J-1} \zeta_\perp^{K_J(t_s)} \left\{ [e^\mu - (e^\mu \cdot v)] n^\nu - (\mu \leftrightarrow \nu) \right\} \\
&\quad - i 2 E \left( \frac{m_{\bar{K}_J}}{E} \right)^J \zeta_\parallel^{K_J(t_s)} (e^\nu \cdot v) (n^\mu v^\nu - n^\nu v^\mu).
\end{align*}
\]  

Note that the parity of the \( K_J^* \) is \((-1)^J\). \( \langle \bar{K}_J | T_A^{\mu \nu} | B \rangle \) is related to \( \langle \bar{K}_J | T_A^{\mu \nu} | B \rangle \) by the relation: \( \sigma^{\mu \nu} \gamma_5 \epsilon_{\mu \nu \rho \sigma} = 2i \sigma^{\rho \sigma} \). Note also that only the \( K_J^* \) with polarization helicities \( \pm 1 \) and \( 0 \) contribute to the \( B \to \bar{K}_J \) transition in the LEET limit, where \( \zeta_\perp \)'s are relevant to \( K_J^* \) with helicity = \( \pm 1 \), and \( \zeta_\parallel \)'s to \( K_J^* \) with helicity = \( 0 \).

We can further reduce the number for the \( B \to \bar{K}_J \) form factors which are independent, using the effective current operator \( \bar{s}_n \Gamma b_v \) (with \( \Gamma = 1, \gamma_5, \gamma_5 \gamma_5, \sigma^{\mu \nu}, \sigma^{\mu \nu} \gamma_5 \)) in the LEET limit, instead of the the original one \( \bar{s} \Gamma b \) \( \square \). Here \( b_v \) and \( s_n \) satisfy \( \not \! q b_v = b_v \), \( \not \! q s_n = 0 \) and \( (\not \! q / 2) s_n = s_n \). Employing the Dirac identities

\[
\begin{align*}
\not \! q \gamma^\mu &= \frac{\not \! q}{2} (n^\mu \gamma^\nu - ie^{\mu \nu \rho \sigma} v_\nu \not \! n_\rho \gamma_\sigma \gamma_5), \\
\not \! q \sigma^{\mu \nu} &= \frac{\not \! q}{2} [i (n^\mu v^\nu - n^\nu v^\mu) - i (n^\mu \gamma^\nu - n^\nu \gamma^\mu) \not \! q - e^{\mu \nu \rho \sigma} v_\nu n_\rho \gamma_\sigma \gamma_5],
\end{align*}
\]

one can easily obtain the following relations:

\[
\begin{align*}
\bar{s}_n b_v &= v_\mu \bar{s}_n \gamma^\mu b_v, \\
\bar{s}_n \gamma^\mu b_v &= n^\mu \bar{s}_n b_v - ie^{\mu \nu \rho \sigma} v_\nu n_\rho \bar{s}_n \gamma_\sigma \gamma_5 b_v, \\
\bar{s}_n \gamma^\mu \gamma_5 b_v &= -n^\mu \bar{s}_n \gamma_5 b_v - ie^{\mu \nu \rho \sigma} v_\nu n_\rho \bar{s}_n \gamma_\sigma b_v, \\
\bar{s}_n \sigma^{\mu \nu} b_v &= i [n^\mu v^\nu \bar{s}_n b_v - n^\nu \bar{s}_n \gamma^\mu b_v - (\mu \leftrightarrow \nu)] - e^{\mu \nu \rho \sigma} v_\nu n_\rho \bar{s}_n \gamma_5 b_v, \\
\bar{s}_n \sigma^{\mu \nu} \gamma_5 b_v &= i [n^\mu v^\nu \bar{s}_n \gamma_5 b_v + n^\nu \bar{s}_n \gamma^\mu \gamma_5 b_v - (\mu \leftrightarrow \nu)] - e^{\mu \nu \rho \sigma} v_\nu n_\rho \bar{s}_n b_v.
\end{align*}
\]

We can then obtain

\[
\begin{align*}
\zeta_\perp^{K_J(t_s)} &= \zeta_\perp^{K_J(t_s)} (q^2), \\
\zeta_\parallel^{K_J(t_s)} &= \zeta_\parallel^{K_J(t_s)} (q^2), \\
\zeta_\perp^{K_J(t_s)} &= \zeta_\perp^{K_J(t_s)} (q^2).
\end{align*}
\]

Thus we find that there are only two independent form factors, \( \zeta_\perp^{K_J(t_s)} (q^2) \) and \( \zeta_\parallel^{K_J(t_s)} (q^2) \), for the \( B \to \bar{K}_J \) transition in the large recoil region. In the full theory, the \( B \to \bar{K}_J \) form factors
factors are defined as

\[
\langle \mathcal{K}_j(p_{K_j}, \lambda) | \bar{s} \gamma^\mu b | \mathcal{B}(p_B) \rangle = -i \frac{2}{m_B + m_{K_j}} \tilde{V}^{K_j}(q^2) e_{\mu\nu\rho\sigma} p_B p_{K_j}^\rho e(\lambda)^*_\sigma, \quad (36)
\]

\[
\langle \mathcal{K}^*_j(p_{K_j}, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | \mathcal{B}(p_B) \rangle = 2m_{K_j} \tilde{A}_0^{K_j}(q^2) \left( \frac{e(\lambda)^* \cdot p_B}{q^2} - \frac{e(\lambda)^* \cdot p_B}{q^2} q^\mu \right) \left( e(\lambda)^* \cdot p_B \right) \left( p_B^\mu + p_{K_j}^\mu \right) \left( p_B^\mu + p_{K_j}^\mu \right) \left( q^\mu - \frac{q^2}{m_B^2 - m_{K_j}^2} \left( p_B^\mu + p_{K_j}^\mu \right) \right), \quad (37)
\]

\[
\langle \mathcal{K}^*_j(p_{K_j}, \lambda) | \bar{s} \sigma^{\mu\nu} q_\nu b | \mathcal{B}(p_B) \rangle = -2 \tilde{T}_1^{K_j}(q^2) e_{\mu\nu\rho\sigma} p_B p_{K_j}^\rho e(\lambda)^*_\sigma, \quad (38)
\]

\[
\langle \mathcal{K}^*_j(p_{K_j}, \lambda) | \bar{s} \sigma^{\mu\nu} \gamma_5 q_\nu b | \mathcal{B}(p_B) \rangle = -i \tilde{T}_2^{K_j}(q^2) \left( m_B^2 - m_{K_j}^2 \right) e(\lambda)^* \mu \left( e(\lambda)^* \cdot p_B \right) \left( p_B^\mu + p_{K_j}^\mu \right) \left( p_B^\mu + p_{K_j}^\mu \right) \left( q^\mu - \frac{q^2}{m_B^2 - m_{K_j}^2} \left( p_B^\mu + p_{K_j}^\mu \right) \right), \quad (39)
\]

where

\[
e(\lambda)^* \equiv e(p_{K_j}, \lambda)^*_{\mu_1 \nu_1 \cdots \mu_j-1} p_B p_{B, \nu_1} \cdots p_{B, \nu_j-1} / m_B^{j-1}, \quad \lambda = 0, \pm 1. \quad (40)
\]

Comparing Eqs. (36)-(39) with Eqs. (22)-(25), we obtain

\[
\tilde{A}_0^{K_j}(q^2) \left( \frac{p_{K_j}}{m_{K_j}} \right) \equiv A_0^{K_j}(q^2) \simeq \left( 1 - \frac{m_{K_j}^2}{m_B^2} \right) \tilde{K}_j^\lambda(q^2) + \frac{m_{K_j}^2}{m_B^2} \tilde{K}_j^\lambda(q^2), \quad (41)
\]

\[
\tilde{A}_1^{K_j}(q^2) \left( \frac{p_{K_j}}{m_{K_j}} \right) \equiv A_1^{K_j}(q^2) \simeq \frac{2E}{m_B + m_{K_j}} \tilde{K}_j^\lambda(q^2), \quad (42)
\]

\[
\tilde{A}_2^{K_j}(q^2) \left( \frac{p_{K_j}}{m_{K_j}} \right) \equiv A_2^{K_j}(q^2) \simeq \left( 1 + \frac{m_{K_j}^2}{m_B} \right) \left[ \tilde{K}_j^\lambda(q^2) - \frac{m_{K_j}^2}{E} \tilde{K}_j^\lambda(q^2) \right], \quad (43)
\]

\[
\tilde{V}^{K_j}(q^2) \left( \frac{p_{K_j}}{m_{K_j}} \right) \equiv V^{K_j}(q^2) \simeq \left( 1 + \frac{m_{K_j}^2}{m_B} \right) \tilde{K}_j^\lambda(q^2), \quad (44)
\]

8
\[
\tilde{T}_1^{K^*_j}(q^2) \left( \frac{p^2_{K^*_j}}{m^2_{K^*_j}} \right)^{J-1} \equiv T_1^{K^*_j}(q^2) \simeq \zeta_\perp^{K^*_j}(q^2),
\]

\[
\tilde{T}_2^{K^*_j}(q^2) \left( \frac{p^2_{K^*_j}}{m^2_{K^*_j}} \right)^{J-1} \equiv T_2^{K^*_j}(q^2) \simeq \left( 1 - \frac{q^2}{m^2_B - m^2_{K^*_j}} \right) \zeta_\perp^{K^*_j}(q^2),
\]

\[
\tilde{T}_3^{K^*_j}(q^2) \left( \frac{p^2_{K^*_j}}{m^2_{K^*_j}} \right)^{J-1} \equiv T_3^{K^*_j}(q^2) \simeq \zeta_\perp^{K^*_j}(q^2) - \left( 1 - \frac{m^2_{J^*}}{m^2_B} \right) \frac{m_{J^*}}{E} \zeta_\parallel^{K^*_j}(q^2),
\]

where we have used \( e^\mu \approx (p_{K^*_j}/m_{K^*_j})^{J-1} \bar{\epsilon}_J(\mu) \) with \( \bar{\epsilon}_J(\mu) = \alpha_L(\mu) \epsilon(\mu), \bar{\epsilon}_J(\pm 1) = \beta_T(\mu) \epsilon(\pm 1) \) and \( |\bar{p}_{K^*_j}|/E \simeq 1 \).

With the replacement \( \epsilon^\mu \rightarrow \bar{\epsilon}(\mu) \), we can easily generalize the studies for \( B \rightarrow K^*\gamma \), \( B \rightarrow K^*\ell^+\ell^- \) and \( B \rightarrow K^*\nu\bar{\nu} \) to the corresponding decays involving resonant strange tensor mesons.

### III. NUMERICAL ANALYSIS

The properties of \( K^{(*)}_J \) mesons are summarized in Table III. In the following numerical study, we use the values of the parameters listed in Table IV.

#### A. The determination of form factors and \( B \rightarrow K^{(*)}_J \gamma \) Decays

The \( B \rightarrow K^{(*)}_J \gamma \) decay widths are given by

\[
\Gamma(B \rightarrow K^{(*)}_J \gamma) = \frac{G_F^2 \alpha_{EM}}{32\pi^4} |V_{ts} V_{tb}^*|^2 m^2_{b,\text{pole}} m^2_B \left( 1 - \frac{m^2_{K^{(*)}_j}}{m^2_B} \right)^3 \times \left| c_7^{(0)\text{eff}} + A^{(1)}(\mu) \right|^2 \left| T_1^{K^{(*)}_j}(0) \right|^2 \left( \beta_T^{(J)} \right)^2.
\]

As for the case with \( J = 2 \), taking into account the data of \( \mathcal{B}(B^0 \rightarrow K^0_2 \gamma) \) and using \( c_7^{(0)\text{eff}} = -0.315, A^{(1)} = A^{(1)}_c + A^{(1)}_{\text{ver}} = -0.038 - 0.016i \), we have obtained [39]

\[
T_1^{K^*_j(1430)}(0) \simeq \zeta^{K^*_j(1430)}(0) = 0.28 \pm 0.03^{+0.00}_{-0.01},
\]

where the first and second errors are due to uncertainties of the data and the pole mass of the \( b \) quark, respectively. In the present paper we use the BSW model [31] to estimate
TABLE III: Properties of resonant $K_J^{(*)}$ mesons (with $J = 1, \cdots, 5$) $^{29}$, and $B \to K_J^{(*)}$ LEET form factors calculated in the BSW model $^{31}$. $K_1(1270)$ and $K_1(1400)$ are not considered in this paper (see Refs. $^{35, 37}$). States denoted by “($\dagger$)” or “?” are not yet well confirmed. In the present paper we do not take into account $1^3G_3$ and $1^3H_4$ states.

| $K_J^{(*)}$  | $J^{PC}$ | $n^{2S+1}L_J$ | $m_{K_J^{(*)}}$ [MeV] | $\zeta_\perp(0)$ | $\zeta_\parallel(0)$ |
|--------------|---------|----------------|------------------------|------------------|------------------|
| $K^*(1410)$  | 1--     | $2^3S_1^?$     | 1,414 ± 15             | 0.28 ± 0.04      | 0.22 ± 0.03      |
| $K^*(1680)$  | 1--     | $1^3D_1$       | 1,717 ± 32             | 0.24 ± 0.05      | 0.18 ± 0.03      |
| $K_2^*(1430)$| 2++     | $1^3P_2$       | 1,425.6 ± 1.5 ($K_2^{*0}$) | 0.28 ± 0.04 | 0.22 ± 0.03 |
| $K_2^*(1800)$| 2+?     | $1^3F_2$ or $2^3P_2^?$ | 1,973 ± 26             | 0.20 ± 0.05      | 0.14 ± 0.03      |
| $K_3^*(1780)$| 3--     | $1^3D_3$       | 1,776 ± 7              | 0.23 ± 0.05      | 0.16 ± 0.03      |
| $K_4^*(2045)$| 4++     | $1^3F_4$       | 2,045 ± 9              | 0.19 ± 0.05      | 0.13 ± 0.03      |
| $K_5^*(2380)$| 5−?     | $1^3G_5^?$     | 2,382 ± 24             | 0.15 ± 0.05      | 0.10 ± 0.03      |
| $K_1(1650)$  | 1+?     | $2^1P_1$ or $2^3P_1^?$ | 1,650 ± 50             | 0.24 ± 0.05      | 0.18 ± 0.03      |
| $K_2(1770)$  | 2−+     | $1^1D_2$       | 1,773 ± 8              | 0.23 ± 0.05      | 0.17 ± 0.03      |
| $K_2(1820)$  | 2−−     | $1^3D_2^?$     | 1,816 ± 13             | 0.22 ± 0.05      | 0.16 ± 0.03      |
| $K_2(2250)$  | 2−?     | $2^1D_2$       | 2,247 ± 17             | 0.16 ± 0.05      | 0.11 ± 0.03      |
| $K_3(2320)$  | 3+?     | $1^1F_3$ or $1^3F_3^?$ | 2,324 ± 24             | 0.15 ± 0.05      | 0.10 ± 0.03      |
| $K_4(2500)$  | 4−?     | $1^1G_4$ or $1^3G_4^?$ | 2,490 ± 20             | 0.13 ± 0.04      | 0.09 ± 0.03      |
| $K_5(2600?)$ | 5+?     | $1^1H_5$ or $1^3H_5^?$ | $\sim 2,600? $         | 0.12 ± 0.04      | 0.08 ± 0.02      |

the LEET form factors at zero momentum transfer, which are be written by

$$
\zeta_{\perp}^{K_J^{(*)}}(0) = \frac{m_b - m_s}{2E} J,
$$

$$
\zeta_{\parallel}^{K_J^{(*)}}(0) = \left( A_0^{K_J^{(*)}}(0) - \frac{m_{K_J^{(*)}}}{m_B} \zeta_{\perp}^{K_J^{(*)}}(0) \right) \left( 1 - \frac{m_{K_J^{(*)}}^2}{m_B E} \right)^{-1},
$$

(50)
where, after integrating out the degrees of freedom of the spins,

$$ J = \sqrt{2} \int d^2 p_T \int_0^1 \frac{dx}{x} \Phi_{K_J^*}(p_T, x) \Phi_{m_B}(p_T, x), $$

$$ A_{0}^{K_J^*}(0) = \int d^2 p_T \int_0^1 dx \Phi_{K_J^*}(p_T, x) \Phi_{m_0}(p_T, x). \quad (51) $$

Here, for a meson with mass $m$ its wave function can be parameterized as

$$ \Phi_m(p_T, x) = N_m \sqrt{x(1-x)} e^{-p_T^2/2\omega^2} e^{-\frac{m^2}{2x}(x-\frac{1}{2}) - \frac{m_q^2 - m_s^2}{2m^2}} \quad (52) $$

with $N_m$ being a normalization factor such that

$$ \int d^2 p_T \int_0^1 dx \Phi_m^2 = 1 \quad (53) $$

and $m_q$ and $m_s$ the constituent quark masses of the non-spectator and spectator quarks participating in the quark decaying process. We use $\omega = 0.46 \pm 0.05$ GeV and the following constituent quark masses in the model calculation: $m_u = m_d = 0.33$ GeV, $m_s = 0.50$ GeV, $m_b = 4.9$ GeV. The value of $\omega$, which determines the average transverse quark momentum and is approximately the same for mesons with the same light spectator quark [31], is fixed by the $B(B^0 \to \bar{K}_2^0 \gamma)$ data. The numerical results for $\zeta_{\perp}^{K_J^*}(0)$ and $\zeta_{\parallel}^{K_J^*}(0)$ are collected in Table III.

The detailed results for the branching fractions for $B \to K_{J}^{(*)} \gamma$ decays are given in Table V. Note that the decay with a heavier meson in the final state has a smaller branching fraction not only due to the smaller phase space and $\zeta_{\perp}^{K_J^*}(0)$ but also to the Clebsch-Gordan coefficient $\beta_T^{(J)}$ which is smaller for a larger spin $J$ (see Table II). We find

$$ B(B \to K^*(1410) \gamma) > B(B \to K^*(1680) \gamma) > B(B \to K_{s}^*(1430) \gamma) $$

$$ > B(B \to K_2^*(1980) \gamma) > B(B \to K_3^*(1780) \gamma) > B(B \to K_{s}^*(2045) \gamma) $$

$$ > B(B \to K_5^*(2380) \gamma), \quad (54) $$

TABLE IV: Input parameters

| Parameter                        | Value            |
|----------------------------------|------------------|
| $B$ lifetime (picosecond)        | $\tau_B^+ = 1.638$, $\tau_B^0 = 1.530$ |
| $b$ quark mass                   | $m_{b,pole} = 4.79^{+0.19}_{-0.08}$ GeV |
| CKM parameter [38]               | $|V_{tb}^*V_{tb}| = 0.040 \pm 0.001$ |
and

$$\mathcal{B}(B \to K_1(1650)\gamma) > \mathcal{B}(B \to K_2(1820)\gamma) \gtrsim \mathcal{B}(B \to K_2(1770)\gamma)$$

$$\gtrsim \mathcal{B}(B \to K_3(2250)\gamma) > \mathcal{B}(B \to K_3(2320)\gamma) > \mathcal{B}(B \to K_4(2500)\gamma)$$

$$\gtrsim \mathcal{B}(B \to K_5(2600)\gamma).$$

(55)

It is interesting to note that we obtain $1.5\mathcal{B}(B^- \to K^*(1680)\gamma) \sim \mathcal{B}(B^- \to K^*(1410)\gamma) = (27.2 \pm 8.3) \times 10^{-6}$, whereas Ali, Ohl, and Mannel [32] found $7\mathcal{B}(B^- \to K^*(1680)\gamma) \sim \mathcal{B}(B^- \to K^*(1410)\gamma) \approx (35 \pm 7) \times 10^{-6}$.

The total branching fractions of radiative $B$ meson decays involving resonant strange mesons listed in Table V, together with $\mathcal{B}(B \to K^{(*)}(982)\gamma, K_1(1270)\gamma, K_1(1400)\gamma) [12–14, 35]$, are

$$\sum_{J=1}^{5} \mathcal{B}(B^0 \to K^{(*)}_J\gamma; E^B_\gamma \gtrsim 2.0 \text{ GeV}) = (237^{+40}_{-34}) \times 10^{-6},$$

(56)

$$\sum_{J=1}^{5} \mathcal{B}(B^- \to K^{(*)}_J\gamma; E^B_\gamma \gtrsim 2.0 \text{ GeV}) = (252^{+44}_{-36}) \times 10^{-6},$$

(57)

where $E^B_\gamma$ is the photon energy in the $B$ rest frame. Our result may hint at that the total branching fraction for the radiative $B$ decays with (nonresonant) two-body or three-body hadronic final states is about $100 \times 10^{-6}$ (see also Ref. [30]), compared to the inclusive $B \to X_s\gamma$ data [23–25].

$$\mathcal{B}(B \to X_s\gamma; E^B_\gamma > 1.7 \text{ GeV}) = (352 \pm 25) \times 10^{-6}.$$  

(58)

The $q^2$-dependence of form factors can be estimated by using the QCD counting rules [11, 40]. We consider the Breit frame, where the $B$ meson and final state $K^{(*)}_J$ move in the opposite directions but with the same magnitude of the momentum. In the large recoil region, where $q^2 \sim 0$, since the two quarks in mesons have to interact strongly with each other to turn around the spectator quark, the transition amplitude is dominated by the one-gluon exchange between the quark-antiquark pair and is therefore proportional to $1/E^2$. Thus we get $\langle K^*_J(p_{K^*_J}, \pm1)|V^\mu|B(p_B)\rangle \propto \epsilon^{\mu\nu\rho\sigma} p_{B\rho} p_{K^*_J} \epsilon^{*,(\pm)}(\pm)_{\sigma} \times 1/E^2$ and $\langle K^*_J(p_{K^*_J}, 0)|A^\mu|B(p_B)\rangle \propto p^{\mu}_{K^*_J} \times 1/E^2$. Consequently, we have $\zeta_{\perp,\parallel}(q^2) \sim 1/E^2$.

---

1 We do not include decays involving $1^3G_3$ and $1^3H_4$ states.
TABLE V: The branching fractions of the $B \to K_J^{(*)}\gamma$ decays in units of $10^{-6}$, where the errors are mainly due to the uncertainties of form factors. The corresponding photon energies in the $B$ rest frame are given in the last column.

| $K_J^{(*)}$ | $J^{PC}$ | $n^{2S+1}L_J$ | $\mathcal{B}(B^- \to K_J^{(*)}\gamma)$ | $\mathcal{B}(\overline{B}^0 \to \overline{K}_J^{(*)0}\gamma)$ | $E_\gamma^{B}$ [GeV] |
|------------|----------|-------------|-------------------------------|-------------------------------|------------------|
| $K^*(1410)$ | 1$^-$ $^-$ | $2^3S_1$? | $27.2 \pm 8.3$ | $25.0 \pm 7.7$ | 2.45 |
| $K^*(1680)$ | 1$^-$ $^-$ | $1^3D_1$ | $17.8 \pm 8.2$ | $16.4 \pm 7.6$ | 2.36 |
| $K_2^*(1430)$ | 2$^+$ $^+$ | $1^3P_2$ | $13.5 \pm 4.1$ | $12.4 \pm 3.8$ | 2.45 |
| $K_2^*(1890)$ | 2$^+$ $^+$ | $1^3F_2$ or $2^3P_2$? | $5.5 \pm 3.1$ | $5.1 \pm 2.9$ | 2.27 |
| $K_3^*(1780)$ | 3$^-$ $^-$ | $1^3D_3$ | $4.3 \pm 2.1$ | $3.9 \pm 1.9$ | 2.34 |
| $K_4^*(2045)$ | 4$^+$ $^+$ | $1^3F_4$ | $1.4 \pm 0.8$ | $1.3 \pm 0.8$ | 2.24 |
| $K_5^*(2380)$ | 5$^-$$^-$ | $1^3G_5$ | $0.4 \pm 0.3$ | $0.3 \pm 0.3$ | 2.10 |
| $K_1(1650)$ | 1$^+$ $^+$ | $2^1P_1$ or $2^3P_1$ | $18.3 \pm 8.4$ | $16.9 \pm 7.8$ | 2.38 |
| $K_2(1770)$ | 2$^-$$^-$ | $1^3D_2$ | $8.0 \pm 3.9$ | $7.4 \pm 3.6$ | 2.34 |
| $K_2(1820)$ | 2$^-$$^-$ | $1^3D_2$? | $8.5 \pm 3.9$ | $7.9 \pm 3.6$ | 2.33 |
| $K_2(2250)$ | 2$^-$$^-$ | $2^1D_2$ | $3.0 \pm 2.2$ | $2.8 \pm 2.0$ | 2.16 |
| $K_3(2320)$ | 3$^+$ $^+$ | $1^1F_3$ or $1^3F_3$? | $1.4 \pm 1.1$ | $1.3 \pm 1.0$ | 2.13 |
| $K_4(2500)$ | 4$^-$$^-$ | $1^1G_4$ or $1^3G_4$? | $0.5 \pm 0.4$ | $0.5 \pm 0.3$ | 2.05 |
| $K_5(2600?)$ | 5$^+$ $^+$ | $1^1H_5$ or $1^3H_5$? | $0.2 \pm 0.2$ | $0.2 \pm 0.2$ | 2.00 |
| Total$^a$ | | | $135.9 \pm 18.9$ | $125.2 \pm 17.4$ | |

$^a$We have assumed that $\mathcal{B}(B \to 2^1P_1\gamma) \approx \mathcal{B}(B \to 2^3P_1\gamma)$ if $2^1P_1$ and $2^3P_1$ states do not mix. Analogously, we also assume that $\mathcal{B}(B \to 1^3F_2\gamma) \approx \mathcal{B}(B \to 2^3P_2\gamma)$, $\mathcal{B}(B \to 1^1F_3\gamma) \approx \mathcal{B}(B \to 1^3F_3\gamma)$, $\mathcal{B}(B \to 1^1G_4\gamma) \approx \mathcal{B}(B \to 1^3G_4\gamma)$ and $\mathcal{B}(B \to 1^1H_4\gamma) \approx \mathcal{B}(B \to 1^3H_4\gamma)$. The summation of the branching fractions should be independent of the mixture due to the unitarity. Here we do not include decays involving $1^3G_3$ and $1^3H_4$ states.

in the large recoil region. In other words, we can obtain that approximate forms: 

$$\zeta_{\perp,\parallel}^{K_J^{(*)}}(q^2) = \zeta_{\perp,\parallel}^{K_J^{(*)}}(0) \cdot (1 - q^2/m_B^2)^{-2}. \quad \text{This result is consistent with that obtained by Charles, Yaouanc, Oliver, Pène and Raynal [10].}$$

They used the light-cone sum rule method to show that the $B \to V$ LEET parameters satisfy $1/E^2$ scaling law, where $V \equiv$
vector meson. Essentially, their result is also suitable for the present case.

\section{B. $B \to K_j^{(*)} \ell^+ \ell^-$ Decays}

The decay amplitude for $\bar{B} \to \bar{K}_j^{(*)} \ell^+ \ell^-$ is given by \footnote{For the amplitudes of $\bar{B} \to \bar{K}_j^{(*)} \ell^+ \ell^-$ decays, perform the following substitutions: $V^{K_j^*} \to A^{K_j}$, $A_i^{K_j} \to V_i^{K_j}$ and $T_i^{K_j} \to T_i^{K_j}$. The result for the decay amplitude for $\bar{B} \to \bar{K}^*(892)^{(*)} \ell^+ \ell^-$ can be found in Ref. 8.}

$$\mathcal{M} = -i \frac{G_F \alpha_{EM}}{2 \sqrt{2} \pi} V_{ts} V_{tb} m_B \left[ \mathcal{T}_\mu^{K_j} \, \bar{s} \gamma^\mu b + \mathcal{U}_\mu^{K_j} \, \bar{s} \gamma^5 \gamma_5 b \right], \quad (59)$$

where

$$\mathcal{T}_\mu^{K_j} = \mathcal{A}^{(K_j)} \epsilon_{\mu \nu \rho \sigma} \bar{\epsilon}_{(J)}^{\nu} p_B^\rho \sigma_{P_{K_j}^*} - i m_B^2 \mathcal{B}^{(K_j)} \bar{\epsilon}_{(J)}^{\nu} + i \mathcal{C}^{(K_j)} (\bar{\epsilon}_{(J)}^{\gamma} \cdot p_B) p_\mu + i \mathcal{D}^{(K_j)} (\bar{\epsilon}_{(J)}^{\gamma} \cdot p_B) q_\mu, \quad (60)$$

$$\mathcal{U}_\mu^{K_j} = \mathcal{E}^{(K_j)} \epsilon_{\mu \nu \rho \sigma} \bar{\epsilon}_{(J)}^{\nu} p_B^\rho \sigma_{P_{K_j}^*} - i m_B^2 \mathcal{F}^{(K_j)} \bar{\epsilon}_{(J)}^{\nu} + i \mathcal{G}^{(K_j)} (\bar{\epsilon}_{(J)}^{\gamma} \cdot p_B) p_\mu + i \mathcal{H}^{(K_j)} (\bar{\epsilon}_{(J)}^{\gamma} \cdot p_B) q_\mu, \quad (61)$$

with $q_\mu \equiv p_B - p_{K_j}$. The $\mathcal{D}^{(K_j)}$-term vanishes when equations of motion of leptons are taken into account. The building blocks, $\mathcal{A}^{(K_j^*)}, \ldots, \mathcal{H}^{(K_j^*)}$ are given by

$$\mathcal{A}^{(K_j^*)} = \frac{2}{1 + \bar{m}_{K_j^{(*)}}} c_9^{\text{eff}}(s) V^{K_j^*}(s) + \frac{4 \bar{m}_b}{s} c_7^{\text{eff}} T_1^{K_j^*}(s), \quad (62)$$

$$\mathcal{B}^{(K_j^*)} = (1 + \bar{m}_{K_j^{(*)}}) \left[ c_9^{\text{eff}}(s) A_1^{K_j^*}(s) + 2 \frac{\bar{m}_b}{s} c_7^{\text{eff}} T_2^{K_j^*}(s) \right], \quad (63)$$

$$\mathcal{C}^{(K_j^*)} = \frac{1}{1 - \bar{m}_{K_j^{(*)}}} \left[ (1 - \bar{m}_{K_j^{(*)}}) c_9^{\text{eff}}(s) A_2^{K_j^*}(s) + 2 \bar{m}_b c_7^{\text{eff}} \left( T_3^{K_j^*}(s) + \frac{1 - \bar{m}_{K_j^{(*)}}}{s} T_2^{K_j^*}(s) \right) \right], \quad (64)$$

$$\mathcal{D}^{(K_j^*)} = \frac{1}{s} \left[ c_9^{\text{eff}}(s) \left\{ (1 + \bar{m}_{K_j^{(*)}}) A_1^{K_j^*}(s) - (1 - \bar{m}_{K_j^{(*)}}) A_2^{K_j^*}(s) \right\} \right. \quad (65)$$

$$\quad \left. - 2 \bar{m}_b^{\text{eff}} A_0^{K_j^*}(s) - 2 \bar{m}_b c_7^{\text{eff}} T_3^{K_j^*}(s) \right]$$

$$\mathcal{E}^{(K_j^*)} = \frac{2}{1 + \bar{m}_{K_j^{(*)}}} c_{10}^{\text{eff}}(s), \quad (66)$$

$$\mathcal{F}^{(K_j^*)} = (1 + \bar{m}_{K_j^{(*)}}) c_{10}^{\text{eff}} A_1^{K_j^*}(s), \quad (67)$$

$$\mathcal{G}^{(K_j^*)} = \frac{1}{1 + \bar{m}_{K_j^{(*)}}} c_{10}^{\text{eff}} A_2^{K_j^*}(s), \quad (68)$$

$$\mathcal{H}^{(K_j^*)} = \frac{1}{s} c_{10}^{\text{eff}} \left[ (1 + \bar{m}_{K_j^{(*)}}) A_1^{K_j^*}(s) - (1 - \bar{m}_{K_j^{(*)}}) A_2^{K_j^*}(s) - 2 \bar{m}_b c_7^{\text{eff}} T_3^{K_j^*}(s) \right]. \quad (69)$$
where \( \hat{s} = s/m_B^2 \), \( \hat{m}_{KJ} = m_{KJ}/m_B \), \( \hat{m}_b = m_b/m_B \) and \( e^\text{eff}_9(\hat{s}) = c_9 + Y_{\text{pert}}(\hat{s}) + Y_{\text{LD}}(\hat{s}) \) with the perturbative \( Y_{\text{pert}}(\hat{s}) \) and long-distance \( Y_{\text{LD}}(\hat{s}) \) corrections [41–43]. \( Y(\hat{s})_{\text{LD}} \) involves \( B \to K_J^*V(\bar{c}c) \) resonances, where \( V(\bar{c}c) \) are the vector charmonium states [42, 43].

\[
Y_{\text{LD}}(\hat{s}) = -\frac{3\pi}{\alpha_{\text{EM}}^2} c_0 \sum_{V = \psi(1S), \ldots} \kappa_V \frac{\hat{m}_V B(V \to \ell^+ \ell^-) \hat{\Gamma}_V^V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_V^V},
\]

(69)

with \( \hat{\Gamma}_V^V \equiv \Gamma_{\text{tot}}^V / m_B \). The relevant parameters can be found in Ref. [37].

The longitudinal, transverse, and total differential decay widths are respectively given by

\[
\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 \alpha_{\text{EM}}^2 m_B^2}{2^{10} \pi^5} |V_{ts} V_{ub}|^2
\]

\[
\times \left\{ \frac{1}{6} |A^{(K^*)}|^2 \hat{u}(s) \hat{s} \beta_T^2 \left\{ 3 \left[ 1 - 2(\hat{m}_{KJ}^2 + \hat{s}) \right] + (\hat{m}_{KJ}^2 - \hat{s})^2 \right\} - \hat{u}(s)^2 \right\}
\]

\[
+ \beta_T^2 |E^{(K^*)}|^2 \hat{s} \hat{u}(s) \left\{ 3 \left[ 1 - 2(\hat{m}_{KJ}^2 + \hat{s}) \right] + (\hat{m}_{KJ}^2 - \hat{s})^2 \right\} - \hat{u}(s)^2 \right\}
\]

\[
+ \frac{1}{12 \hat{m}_{KJ}^2 \lambda} |F^{(K^*)}|^2 \hat{u}(s) \left\{ 3 \hat{\alpha}_L^2 \lambda^2 \right\}
\]

\[
+ \hat{u}(s)^2 \left[ 16 \hat{m}_{KJ}^2 \hat{s} \beta_T^2 - (1 - 2(\hat{m}_{KJ}^2 + \hat{s}) + \hat{m}_{KJ}^4 + \hat{s}^2 - 10 \hat{m}_{KJ}^2 \hat{s}) \hat{\alpha}_L^2 \right] \}
\]

\[
+ \hat{u}(s)^2 \left[ \frac{1}{2 \hat{m}_{KJ}^2} \frac{1}{\hat{m}_{KJ}^2} \lambda \left[ |C^{(K^*)}|^2 \left( \lambda - \frac{\hat{u}(s)^2}{3} \right) + |G^{(K^*)}|^2 \left( \lambda - \frac{\hat{u}(s)^2}{3} + 4 \hat{m}_{KJ}^2 (2 + 2 \hat{m}_{KJ}^2 - \hat{s}) \right) \right] \right]
\]

\[
- \hat{u}(s)^2 \frac{1}{2 \hat{m}_{KJ}^2} \sum_{\kappa' \kappa''} \left[ \text{Re}(B^{(K^*)}) C^{(K^*)} \left( \lambda - \frac{\hat{u}(s)^2}{3} \right) (1 - \hat{m}_{KJ}^2 - \hat{s}) \right.
\]

\[
+ \text{Re}(F^*) \left\{ \left( \lambda - \frac{\hat{u}(s)^2}{3} \right) (1 - \hat{m}_{KJ}^2 - \hat{s}) + 4 \hat{m}_{KJ}^2 \lambda \right\} \right]
\]

\[
- 2 \hat{u}(s) \hat{m}_{KJ}^2 \lambda \left[ \text{Re}(F^{(K^*)}) \mathcal{H}^{(K^*)} - \text{Re}(G^{(K^*)}) \mathcal{H}^{(K^*)} \right] (1 - \hat{m}_{KJ}^2)
\]

\[
+ \hat{u}(s) \hat{m}_{KJ}^2 \hat{s} \lambda \left[ \mathcal{H}^{(K^*)} \right]^2 \right\}.
\]

(71)
Here \( \hat{u} \equiv u/m_B^2 \) and \( \hat{u}(s) \equiv u(s)/m_B^2 \), where \( u = -u(s) \cos \theta \),

\[
\begin{align*}
\hat{u}(s) & \equiv \sqrt{\lambda \left( 1 - \frac{4\hat{m}^2}{s} \right)}, \\
\lambda & \equiv 1 + \hat{m}_{KJ}^4 s^2 - 2\hat{m}_{KJ}^2 - 2\hat{s} - 2\hat{m}_{KJ}^2 \hat{s},
\end{align*}
\]

and \( \theta \) is the angle between the moving directions of \( \ell^+ \) and \( B \) meson in the center of mass frame of the \( \ell^+ \ell^- \) pair. We show the decay distributions \( \frac{d\mathcal{B}(B^0 \rightarrow K_J^{(*)0} \mu^+ \mu^-)}{ds} \) in Fig. 1 and summarize the corresponding branching fractions in Table VI. Because the decays involving heavier \( K \)-resonances have the smaller phase spaces and LEET form factors and because the Clebsch-Gordan coefficients, \( \alpha_L^{(J)} \) and \( \beta_T^{(J)} \), are smaller for a larger spin \( J \), we obtain the following salient features:

\[
\begin{align*}
\mathcal{B}(B \rightarrow K^*(1410)\mu^+\mu^-) & > \mathcal{B}(B \rightarrow K_2^*(1430)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K^*(1680)\mu^+\mu^-) \\
> \mathcal{B}(B \rightarrow K_2^*(1980)\mu^+\mu^-) & \approx \mathcal{B}(B \rightarrow K_3^*(1780)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_4^*(2045)\mu^+\mu^-) \\
> \mathcal{B}(B \rightarrow K_3^*(2380)\mu^+\mu^-),
\end{align*}
\]

and

\[
\begin{align*}
\mathcal{B}(B \rightarrow K_1(1650)\mu^+\mu^-) & > \mathcal{B}(B \rightarrow K_2(1770)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_3(1820)\mu^+\mu^-) \\
> \mathcal{B}(B \rightarrow K_3(2250)\mu^+\mu^-) & > \mathcal{B}(B \rightarrow K_3(2320)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_4(2500)\mu^+\mu^-) \\
> \mathcal{B}(B \rightarrow K_5(2600?)\mu^+\mu^-).
\end{align*}
\]

In Fig. 2, we plot the longitudinal fraction distributions for the \( \overline{B} \rightarrow \overline{K}_J^{(*)} \mu^+\mu^- \) decays, where

\[
\frac{dF_L}{ds} \equiv \frac{d\Gamma_L}{ds} / \frac{d\Gamma_{\text{total}}}{ds}.
\]

Our result indicates that the longitudinal fraction distribution \( dF_L/ds \) about 0.8 at \( s = 2 \text{ GeV}^2 \), which also apply to the inclusive process. It is interesting to note that, for the new-physics models with the flipped sign solution for \( c_T^{(J)} \), \( dF_L/ds \) can be reduced to be \( \sim 0.6 \) at \( s = 2 \text{ GeV}^2 \).
FIG. 1: Decay distributions of $\overline{B}^0 \rightarrow \overline{K}_J^{(*0)} \mu^+ \mu^-$ decays. The processes involving the confirmed $K_J^{(*)}$ are plotted. Solid [red], dashed [orange], dotted [green], dot-dashed [blue], and double-dot-dashed [black] curves from up to down correspond to $K_J^{(*)} = K^*(1680), K_2^*(1430), K_2(1770), K_3^*(1780)$, and $K_4^*(2045)$, respectively. The thick and thin curves stand for the decay widths with and without charmonium resonances, respectively (see Eq. (69)).

FIG. 2: Longitudinal fraction distributions $dF_L/ds$ of $\overline{B} \rightarrow \overline{K}_J^{(*)} \mu^+ \mu^-$ decays as functions of $s$. Solid [red], dashed [orange], dotted [green], dot-dashed [blue] and double-dot-dashed [black] curves stand for $K_J^{(*)} = K^*(1680), K_2^*(1430), K_2(1770), K_3^*(1780)$, and $K_4^*(2045)$, respectively.
TABLE VI: Same as Table [V] except for nonresonant branching fractions of $\bar{B} \to \bar{K}_j^{(*)} \mu^+ \mu^-$ decays in units of $10^{-7}$.

| $J^{PC}$ | $n^{2S+1}L_J$ | $\mathcal{B}(\bar{B}^0 \to \bar{K}_j^{(*)0} \mu^+ \mu^-)$ | $\mathcal{B}(B^- \to K_j^{(*)-} \mu^+ \mu^-)$ |
|----------|----------------|-------------------------------------------------|-------------------------------------------------|
| $K^*(1410)$ | $1^{--}$ | $2^3S_1$ | $5.4^{+1.6}_{-1.4}$ | $5.8^{+1.7}_{-1.5}$ |
| $K^*(1680)$ | $1^{--}$ | $1^3D_1$ | $2.3^{+0.8}_{-0.7}$ | $2.4^{+0.9}_{-0.8}$ |
| $K_2^*(1430)$ | $2^{++}$ | $1^3P_2$ | $3.1^{+0.9}_{-0.8}$ | $3.3^{+1.0}_{-0.9}$ |
| $K_2^*(1980)$ | $2^{++}$ | $1^3P_2$ or $2^3P_2$ | $0.6^{+0.3}_{-0.2}$ | $0.6^{+0.3}_{-0.2}$ |
| $K_3^*(1780)$ | $3^{--}$ | $1^3D_3$ | $0.6^{+0.2}_{-0.2}$ | $0.6^{+0.2}_{-0.2}$ |
| $K_4^*(2045)$ | $4^{++}$ | $1^3F_4$ | $0.1^{+0.1}_{-0.1}$ | $0.2^{+0.1}_{-0.1}$ |
| $K_5^*(2380)$ | $5^{--}$ | $1^3G_5$ | $0.03^{+0.02}_{-0.01}$ | $0.03^{+0.02}_{-0.01}$ |
| $K_1(1650)$ | $1^{++}$ | $2^1P_1$ or $2^3P_1$ | $2.6^{+0.9}_{-0.8}$ | $2.7^{+1.0}_{-0.8}$ |
| $K_2(1770)$ | $2^{--}$ | $1^1D_2$ | $1.1^{+0.4}_{-0.3}$ | $1.2^{+0.4}_{-0.4}$ |
| $K_2(1820)$ | $2^{--}$ | $1^3D_2$? | $0.9^{+0.4}_{-0.3}$ | $1.0^{+0.4}_{-0.3}$ |
| $K_2(2250)$ | $2^{--}$ | $2^1D_2$ | $0.2^{+0.1}_{-0.1}$ | $0.2^{+0.1}_{-0.1}$ |
| $K_3(2320)$ | $3^{++}$ | $1^1F_3$ or $1^3F_3$ | $0.1^{+0.1}_{-0.1}$ | $0.1^{+0.1}_{-0.1}$ |
| $K_4(2500)$ | $4^{--}$ | $1^3G_4$ or $1^3G_4$ | $0.03^{+0.02}_{-0.02}$ | $0.03^{+0.02}_{-0.02}$ |
| $K_5(2600)?$ | $5^{++}$ | $1^1H_5$ or $1^3H_5$? | $0.01^{+0.01}_{-0.01}$ | $0.01^{+0.01}_{-0.01}$ |
| Total$^a$ | | | | \[ 19.9^{+6.2}_{-4.6} \] \[ 21.4^{+7.1}_{-5.2} \]

$^a$Same as Table [V]

The forward-backward asymmetry of $\bar{B} \to \bar{K}_j^{(*)} \ell^+ \ell^-$ is given by

$$
\frac{dA_{FB}}{d\hat{s}} = -\left(\beta_T^{(j)}\right)^2 \frac{G_F^2 \alpha_EM_B}{210_{\pi}^5} \left|V_{ts}^*V_{tb}\right|^2 \hat{s} \hat{u}(s)^2 \left[Re \left(\mathcal{B}(\mathcal{K}_j^{(*)}) \mathcal{E}(\mathcal{K}_j^{(*)})\right) + Re \left(\mathcal{A}(\mathcal{K}_j^{(*)}) \mathcal{F}(\mathcal{K}_j^{(*)})\right)\right]
$$

$$
= -\left(\beta_T^{(j)}\right)^2 \frac{G_F^2 \alpha_EM_B}{210_{\pi}^5} \left|V_{ts}^*V_{tb}\right|^2 \hat{s} \hat{u}(s)^2 \left[Re \left[c_{10}^{\text{eff}}(s)\right] V^{K_j^{(*)}} A_1^{K_j^{(*)}} + \mathcal{M}_B \Re(c_{10}^{\text{eff}}) \left\{ (1 - \hat{m}_{K_j^{(*)}}) V^{K_j^{(*)}} T_2^{K_j^{(*)}} + (1 + \hat{m}_{K_j^{(*)}}) A_1^{K_j^{(*)}} T_1^{K_j^{(*)}} \right\} \right].
$$

In Fig. 3 we plot the normalized forward-backward asymmetry $d\hat{A}_{FB}/d\hat{s} \equiv (dA_{FB}/ds)/(d\Gamma_{total}/ds)$. Using the form factors in Eqs. (41)-(47), we can easily obtain
FIG. 3: Normalized forward-backward asymmetries for $\bar{B} \rightarrow \bar{K}_J^{(*)} \mu^+ \mu^-$ decay. Legends are the same as Fig. 2.

the forward-backward asymmetry zero, $s_0$, satisfying

$$Re \left[ c_9^{\text{eff}}(\hat{s}_0) c_{10} \right] = -2 \frac{\hat{m}_b}{\hat{s}_0} Re(c_7^{\text{eff}} c_{10}) \frac{1 - \hat{s}_0}{1 + \hat{m}_{K_j}^{2(*)} - \hat{s}_0}. \quad (78)$$

We note that $s_0$ is independent of the form factors but depends only on $m_{K_j}^{2(*)}$. Under the variation of $\hat{m}_{K_j}^{2(*)}$, we get

$$\delta \hat{s}_0 \simeq - \frac{(\hat{s}_0 - 1) \hat{s}_0}{(\hat{s}_0 - 1)^2 + \hat{m}_{K_j}^{2(*)}} \delta \hat{m}_{K_j}^{2(*)}, \quad (79)$$

or

$$\delta s_0 \simeq - s_0 \cdot \frac{\delta m_{K_j}^{2(*)}}{m_{B}^{2}}. \quad (80)$$

Since $\delta m_{K_j}^{2(*)} \ll s_0$ and $m_{K_j}^{2(*)} \ll m_{B}^{2}$, we thus expect the following relation in the SM:

$$s_0^{K_{(980)}^*} \approx 3.5 \text{ GeV}^2 \gtrsim s_0^{K_{(1410)}^*} \gtrsim s_0^{K_{2}^{(1430)}^*} \gtrsim s_0^{K_{3}^{(1680)}^*} \gtrsim s_0^{K_{2}^{(1770)}^*} \gtrsim s_0^{K_{3}^{(1780)}^*} \gtrsim s_0^{K_{2}^{(1820)}^*} \gtrsim s_0^{K_2^{(1980)}^*} \gtrsim s_0^{K_2^{(2045)}^*} \gtrsim s_0^{K_2^{(2250)}^*} \gtrsim s_0^{K_3^{(2320)}^*} \gtrsim s_0^{K_3^{(2380)}^*} \gtrsim s_0^{K_3^{(2500)}^*} \gtrsim s_0^{K_3^{(2600)}^*}. \quad (81)$$

C. $\bar{B} \rightarrow \bar{K}_J^{(*)} \nu \bar{\nu}$ Decays

The effective weak Hamiltonian relevant to the $\bar{B} \rightarrow \bar{K}_J^{(*)} \nu \bar{\nu}$ decays are given by

$$\mathcal{H}_{\text{eff}} = c_L \bar{s} \gamma^\mu (1 - \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu + c_R \bar{s} \gamma^\mu (1 + \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu + \text{H.c.}, \quad (82)$$
where $c_L$ and $c_R$ are coefficients for left- and right-handed weak hadronic currents, respectively. In the SM, $c_L^{SM} = 0$ and

$$c_R^{SM} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{EM}}{2\pi \sin^2 \theta_W} V_{tb} V_{ts}^* X(x_t) = 2.9 \times 10^{-9}, \quad (83)$$

where the detailed form of $X(x_t)$ has been given in Refs. [44, 45]. The missing invariant mass-squared distributions, corresponding to polarizations $h = 0, \pm 1$ of the final $K^*_J$ for $\bar{B} \to \bar{K}^*_J \nu \bar{\nu}$ decays are

$$\frac{d\Gamma_0}{dq^2} = 3 \left( \frac{\alpha_L^{(j)}}{48\pi^3} \right)^2 \frac{|\vec{p}| |c_L - c_R|^2}{m_{K^*_J}^2} \left[ (m_B + m_{K^*_J})(m_B E' - m_{K^*_J}^2)A_{1K^*_J}^1(q^2) - \frac{2m_B^2}{m_B + m_{K^*_J}^2} |\vec{p}'|^2 A_{2K^*_J}^1(q^2) \right]^2, \quad (84)$$

$$\frac{d\Gamma_{\pm1}}{dq^2} = 3 \left( \beta_T^{(j)} \right)^2 \frac{|\vec{p}'| q^2}{48\pi^3} \left[ (c_L + c_R) \frac{2m_B |\vec{p}'|}{m_B + m_{K^*_J}^2} V_{K^*_J}^1(q^2) \mp (c_L - c_R)(m_B + m_{K^*_J}^2) A_{1K^*_J}^1(q^2) \right]^2, \quad (85)$$

where the factor 3 counts the numbers of the neutrino generations, $(E', \vec{p}')$ is the $\bar{K}^*_J$ energy-momentum in the $B$-meson rest frame, and $q^2$ is the invariant mass squared of the neutrino-antineutrino pair with $0 \leq q^2 \leq (m_B - m_{K^*_J})^2$. In Fig. 4, we show the differential distributions as functions of the missing invariant mass squared in the SM. The results for branching fractions are summarized in Table VII. At $q^2 = 0$, where the neutrino and antineutrino are nearly collinear in the $B$ rest frame, the decay is predominated by the zero helicity amplitude. Moreover, as expected from the left-handed $b_L \to s_L$ transition in the SM, $d\Gamma_+/dq^2$ is always suppressed at least by $(m_s/m_b)^2$, compared with $d\Gamma_0/dq^2$ and $d\Gamma_-/dq^2$. We obtain the relation: $d\Gamma_0/dq^2 \gg d\Gamma_-/dq^2 \gg d\Gamma_+/dq^2$.

IV. SUMMARY

We have formulated $B \to K^{(*)}_J$ form factors using large energy effective theory techniques. We have studied the radiative and semileptonic $B$ decays involving the higher strange resonance $K^{(*)}_J$ in the final state. The main results are as follows.

---

3 For the amplitudes of $\bar{B} \to \bar{K}^*_J \nu \bar{\nu}$ decays, perform the following replacements: $V_{K^*_J} \to A_{1K^*_J}^1$, $A_{iK^*_J}^1 \to V_{iK^*_J}^1$. 

---
FIG. 4: The $dB(\overline{B} \to \overline{K}_J^{(*)} \nu \bar{\nu})/dq^2$ as functions of the missing invariant mass-squared $q^2$. The solid [black], dashed [blue], dotted [green] and dot-dashed [red] curves correspond to the total decay rate and the polarization rates with helicities $h = 0, -1, +1$, respectively.

- The transition form factors in the large recoil region can be represented in terms of two independent LEET form factors, $\zeta^{K_J^{(*)}}_\perp(q^2)$ and $\zeta^{K_J^{(*)}}_\parallel(q^2)$. According to the QCD counting rules, these two form factors exhibit the dipole $q^2$ dependence in the large recoil region (and in the LEET limit). We have further estimated $\zeta^{K_J^{(*)}}_\perp(0)$ and $\zeta^{K_J^{(*)}}_\parallel(0)$ in the BSW model.

- The branching fractions for decays $\overline{B} \to \overline{K}_J^{(*)} \gamma$, $\overline{B} \to \overline{K}_J^{(*)} \ell^+ \ell^-$ and $\overline{B} \to \overline{K}_J^{(*)} \nu \bar{\nu}$ with higher $K$-resonances are suppressed due to the smaller phase space and $\zeta^{K_J^{(*)}}_\perp$, $\zeta^{K_J^{(*)}}_\parallel$, and/or due to the smaller Clebsch-Gordan coefficients, $\beta^{(J)}_T$ and $\alpha^{(J)}_L$, in case of larger spin-$J$.

- We find that for $\overline{B} \to \overline{K}_J^{(*)} \ell^+ \ell^-$ decays, the longitudinal fraction distribution $dF_L/ds \simeq 0.8$ at $s = 2 \text{ GeV}^2$, and the forward-backward asymmetry zero $s_0 \approx 3.5 \text{ GeV}^2$. The asymmetry zero is independent of the form factors in the LEET limit and highly insensitive to $m_{K_J^{(*)}}$. 

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TABLE VII: The branching fractions of the $B \to K_j^{(*)} \nu \bar{\nu}$ decays in units of $10^{-6}$. The first and second errors correspond to the uncertainties of the form factors $\xi_{K_j}^{K_j^{(*)}}$ and $\xi_{K_j^{(*)}}$, respectively.

| $J^{PC}$ | $n \, 2S+1 \, L_J$ | $B(\overline{B} \to K_j^{(*)}0 \nu \bar{\nu})$ | $B(B^- \to K_j^{(*)}0 \nu \bar{\nu})$ |
|----------|---------------------|------------------------------------------|---------------------------------|
| $K^*(1410)$ | 1-- | $2^3 S_1$? | $4.3^{+1.3}_{-1.1}$ | $4.6^{+1.4}_{-1.2}$ |
| $K^*(1680)$ | 1-- | $1^3 D_1$ | $1.8^{+0.7}_{-0.6}$ | $2.0^{+0.7}_{-0.6}$ |
| $K_2^*(1430)$ | 2++ | $1^3 P_2$ | $2.5^{+0.7}_{-0.6}$ | $2.6^{+0.8}_{-0.7}$ |
| $K_2^*(1980)$ | 2? | $1^3 F_2$ or $2^3 P_2$? | $0.4^{+0.2}_{-0.2}$ | $0.5^{+0.2}_{-0.2}$ |
| $K_3^*(1780)$ | 3-- | $1^3 D_3$ | $0.5^{+0.2}_{-0.2}$ | $0.5^{+0.2}_{-0.2}$ |
| $K_4^*(2045)$ | 4++ | $1^3 F_4$ | $0.11^{+0.05}_{-0.04}$ | $0.11^{+0.06}_{-0.05}$ |
| $K_5^*(2380)$ | 5? | $1^3 G_5$ | $0.02^{+0.01}_{-0.01}$ | $0.02^{+0.01}_{-0.01}$ |
| $K_1^*(1650)$ | 1? | $2^1 P_1$ or $2^3 P_1$? | $2.1^{+0.7}_{-0.6}$ | $2.2^{+0.8}_{-0.7}$ |
| $K_2^*(1770)$ | 2+ | $1^1 D_2$ | $0.9^{+0.3}_{-0.3}$ | $0.9^{+0.4}_{-0.3}$ |
| $K_2^*(1820)$ | 2? | $1^3 D_2$? | $0.7^{+0.3}_{-0.2}$ | $0.8^{+0.3}_{-0.3}$ |
| $K_2^*(2250)$ | 2? | $2^1 D_2$ | $0.2^{+0.1}_{-0.1}$ | $0.2^{+0.1}_{-0.1}$ |
| $K_3^*(2320)$ | 3? | $1^1 F_3$ or $1^3 F_3$ | $0.07^{+0.05}_{-0.03}$ | $0.07^{+0.05}_{-0.04}$ |
| $K_4^*(2500)$ | 4? | $1^1 G_4$ or $1^3 G_4$ | $0.02^{+0.02}_{-0.01}$ | $0.02^{+0.01}_{-0.01}$ |
| $K_5^*(2600)$ | 5? | $1^1 H_5$ or $1^3 H_5$ | $0.008^{+0.006}_{-0.005}$ | $0.008^{+0.007}_{-0.005}$ |
| **Total** | | | | |
| | | $16.2^{+4.1}_{-3.0}$ | $17.3^{+4.7}_{-3.5}$ |

*aSame as Table [V]*

- For the $B \to K_j^{(*)} \nu \bar{\nu}$ decay, the branching fraction is predominated by the zero helicity amplitude at $q^2 = 0$, where the neutrino and antineutrino are nearly collinear in the $B$ rest frame. As expected from the left-handed $b_L \to s_L$ current in the SM, $d\Gamma_+/dq^2$ is always suppressed at least by $(m_s/m_b)^2$, compared with $d\Gamma_0/dq^2$ and $d\Gamma_-/dq^2$. We thus predict the relation: $d\Gamma_0/dq^2 > d\Gamma_-/dq^2 \gg d\Gamma_+/dq^2$.

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Appendix A: $\mathcal{B} \rightarrow \mathcal{K}_J$ form factors

$\mathcal{B} \rightarrow \mathcal{K}_J$ transition form factors in the LEET limit are given by

$$
\langle \mathcal{K}_J | A^{\mu} | \mathcal{B} \rangle = -i 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J-1} \zeta^{K_J(a)} \epsilon^{\mu \rho \sigma \sigma} v_{\rho} n_{\sigma}, \quad (A1)
$$

$$
\langle \mathcal{K}_J | V^{\mu} | \mathcal{B} \rangle = 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J-1} \zeta^{K_J(v)} \left[ \epsilon^{\mu \mu} - (\epsilon^{\mu} \cdot v) n_{\mu} \right]
+ 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J} (\epsilon^{\mu} \cdot v) \left[ \zeta^{K_J(v)} n_{\mu} + \zeta^{K_J(v)} 1 n_{\mu} \right],
$$

$$
\langle \mathcal{K}_J | T^{\mu \nu}_A | \mathcal{B} \rangle = -2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J-1} \zeta^{K_J(t_a)} (\epsilon^{\mu} \cdot v) \epsilon^{\mu \rho \sigma \sigma} v_{\rho} n_{\sigma}
- 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J-1} \zeta^{K_J(t_a)} \epsilon^{\mu \rho \sigma \sigma} n_{\rho} \left[ \epsilon^{\sigma} - (\epsilon^{\sigma} \cdot v) n_{\sigma} \right]
- 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J-1} \zeta^{K_J(t_a)} \epsilon^{\mu \rho \sigma \sigma} v_{\rho} \left[ \epsilon^{\sigma} - (\epsilon^{\sigma} \cdot v) n_{\sigma} \right],
$$

$$
\langle \mathcal{K}_J | T^{\mu \nu} | \mathcal{B} \rangle = i 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J-1} \zeta^{K_J(t_a)} \left[ \left[ \epsilon^{\mu \mu} - (\epsilon^{\mu} \cdot v) n_{\mu} \right] n_{\nu} - (\mu \leftrightarrow \nu) \right]
+ i 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J} \zeta^{K_J(t_a)} \left[ \left[ \epsilon^{\mu \mu} - (\epsilon^{\mu} \cdot v) n_{\mu} \right] n_{\nu} - (\mu \leftrightarrow \nu) \right]
+ 2 E \left( \frac{m_{\mathcal{K}_J}}{E} \right)^{J} \zeta^{K_J(t_a)} (\epsilon^{\mu} \cdot v) \left( n_{\mu} n_{\nu} - n_{\nu} n_{\mu} \right),
$$

where $m_{\mathcal{K}_J}$ is the mass of the $\mathcal{K}_J$. $\langle \mathcal{K}_J | T^{\mu \nu}_A | \mathcal{B} \rangle$ is related to $\langle \mathcal{K}_J | T^{\mu \nu} | \mathcal{B} \rangle$ by the relation:

$$
\sigma^{\mu \nu} \epsilon_{\mu \rho \sigma} = 2 i \sigma^{\rho \sigma} \gamma_5. \quad \text{From operator relations Eqs. (23)-(32) and}
$$

$$
\bar{s}_a \gamma_5 b_v = - n_{\mu} \bar{s}_a \gamma_5 \gamma_5 b_v,
$$

we obtain

$$
\zeta^{K_J(v)} = \zeta^{K_J(a)} = \zeta^{K_J(t_a)} = \zeta^{K_J(t_5)} \equiv \zeta^{K_J \perp}, \quad (A6)
$$

$$
\zeta^{K_J(t_a)} = \zeta^{K_J(t_a)} = \zeta^{K_J(t_5)} \equiv \zeta^{K_J \parallel}, \quad (A7)
$$

$$
\zeta^{K_J(t_a)} = \zeta^{K_J(t_5)} = \zeta^{K_J(t)} = 0, \quad (A8)
$$

and thus find that there are only two independent form factors, $\zeta^{K_J(q^2)}$ and $\zeta^{K_J(q^2)}$. 

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\( \mathcal{B} \to \mathcal{K}_J \) form factors are given by

\[
\langle \mathcal{K}_J(p_{K_J}, \lambda) | \bar{s}\gamma^\mu\gamma_5 b | \mathcal{B}(p_B) \rangle = -i \frac{2}{m_B + m_{K_J}} \tilde{A}^{K_J}(q^2) e^\mu \rho \rho p_{B \rho} p_{K_J \rho} e(\lambda)_\sigma^*, \tag{A9}
\]

\[
\langle \mathcal{K}_J(p_{K_J}, \lambda) | \bar{s}\gamma^\mu b | \mathcal{B}(p_B) \rangle = 2m_{K_J} \tilde{V}_0^{K_J}(q^2) \frac{e(\lambda)^* \cdot p_B}{q^2} q^\mu \\
+ (m_B + m_{K_J}) \tilde{V}_1^{K_J}(q^2) \left[ e(\lambda)^* - \frac{e(\lambda)^* \cdot p_B q^\mu}{q^2} \right] \\
- \tilde{V}_2^{K_J}(q^2) \frac{e(\lambda)^* \cdot p_B}{m_B + m_{K_J}} \left[ p_B^\mu + p_{K_J}^\mu - \frac{m_B^2 - m_{K_J}^2}{q^2} q^\mu \right], \tag{A10}
\]

\[
\langle \mathcal{K}_J(p_{K_J}, \lambda) | \bar{s}\sigma^{\mu\nu}\gamma_5 q_{\mu} b | \mathcal{B}(p_B) \rangle = 2\tilde{T}_1^{K_J}(q^2) e^\mu e^\rho \rho p_{B \rho} p_{K_J \rho} e(\lambda)^*, \tag{A11}
\]

\[
\langle \mathcal{K}_J(p_{K_J^*}, \lambda) | \bar{s}\sigma^{\mu\nu} q_{\mu} b | \mathcal{B}(p_B) \rangle = i\tilde{T}_2^{K_J}(q^2) \left[ (m_B^2 - m_{K_J}^2) e(\lambda)^* - (e(\lambda)^* \cdot p_B) (p_B^\mu + p_{K_J}^\mu) \right] \\
+ i\tilde{T}_3^{K_J}(q^2) (e(\lambda)^* \cdot p_B) \\
\times \left[ q^\mu - \frac{q^2}{m_B^2 - m_{K_J}^2} (p_B^\mu + p_{K_J}^\mu) \right]. \tag{A12}
\]

We can further obtain the following relations,

\[
\tilde{V}_0^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{V}_0^{K_J}(q^2) \approx \left( 1 - \frac{m_{K_J}^2}{m_B E} \right) \zeta^{K_J}(q^2) + \frac{m_{K_J} \zeta^{K_J}}{m_B} \zeta^J(q^2), \tag{A13}
\]

\[
\tilde{V}_1^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{V}_1^{K_J}(q^2) \approx \frac{2E}{m_B + m_{K_J}} \zeta^J(q^2), \tag{A14}
\]

\[
\tilde{V}_2^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{V}_2^{K_J}(q^2) \approx \left( 1 + \frac{m_{K_J}}{m_B} \right) \zeta^J(q^2) - \frac{m_{K_J}}{m_B} \zeta^J(q^2), \tag{A15}
\]

\[
\tilde{A}^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{A}^{K_J}(q^2) \approx \left( 1 + \frac{m_{K_J}}{m_B} \right) \zeta^J(q^2), \tag{A16}
\]

\[
\tilde{T}_1^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{T}_1^{K_J}(q^2) \approx \zeta^J(q^2), \tag{A17}
\]

\[
\tilde{T}_2^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{T}_2^{K_J}(q^2) \approx \left( 1 - \frac{q^2}{m_B^2 - m_{K_J}^2} \right) \zeta^J(q^2), \tag{A18}
\]

\[
\tilde{T}_3^{K_J}(q^2) \left( \frac{|\bar{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv \tilde{T}_3^{K_J}(q^2) \approx \zeta^J(q^2) - \left( 1 - \frac{m_{K_J}^2}{m_B^2} \right) \frac{m_{K_J}}{E} \zeta^J(q^2), \tag{A19}
\]

where use of \( p_B/E \approx 1 \) has been made. Recalling that

\[
\bar{\xi}(J)(0)^\mu = \alpha^{(J)}_L \varepsilon(0)^\mu, \quad \bar{\xi}(J)(\pm 1)^\mu = \beta^{(J)}_T \varepsilon(\pm 1)^\mu, \tag{A20}
\]

we can easily generalize the studies for \( B \to K_J^\gamma, B \to K_J^\ell^+ \ell^- \) and \( B \to K_J^\nu \overline{\nu} \) to \( B \to K_J^\gamma, B \to K_J^\ell^+ \ell^- \) and \( B \to K_J^\nu \overline{\nu} \) by the following replacements:

\[
V^{K_J} \to A^{K_J}, \quad A_i^{K_J} \to V_i^{K_J} \quad (i = 0, 1, 2), \quad T_j^{K_J} \to T_j^{K_J} \quad (j = 1, 2, 3). \tag{A21}
\]
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