Estimation of mass and cosmological constant of nearby spiral galaxies using galaxy rotation curve

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Abstract. An expression for rotational velocity of a test particle around the central mass in the invariant plane is derived. For this, a line element of Schwarzschild de-Sitter space-time is used to study the effect of cosmological constant (Λ) on the motion of both massive and massless particles. Using rotation curve data of 15 nearby spiral and barred spiral galaxies, we estimated the mass and Λ of the galaxy. The mass of the galaxies are found to lie in the range 0.13-7.60 × 10^{40} kg. The cosmological constant (Λ) is found to be negative (−0.03 to −0.10 × 10^{−40} km^{-2}), suggesting the importance of anti-de Sitter space in the local bubble. Possible explanation of the results will be discussed.

Keywords: methods: data analysis – general – astronomical databases: miscellaneous.

1. Introduction

The cosmological constant (Λ) is a parameter describing the energy density of the vacuum (empty space). A negative Λ adds to the attractive gravity of matter whereas a positive Λ resists the attractive gravity of matter due to its negative pressure. Two supernova cosmology project (Perlmutter et al. 1999; Schmidt et al. 2000) converge on a common result: cosmos will expand forever at an accelerating rate, pushed on by dark energy

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According to this, the Universe is discovered to have a flat spatial geometry \((k = 0)\), a positive cosmological constant (dark energy) \(\Omega_\Lambda = 0.7\), and matter density (of all kinds) \(\Omega_M = 0.7\) (Perlmutter 2003). The data from the Boomerang experiment provide a firm evidence for the case of flat Universe \(\Omega_k \sim 0\) (Perlmutter et al. 1999). Overduin et al. (2000) discussed some of the theoretical justifications for a non zero \(\Lambda\). Kraniotis & Whitehouse (2000) interpreted \(\Lambda\) as the non-gravitational contribution to galactic velocity rotation curves and estimated negative \(\Lambda\) for 6 spiral galaxies, suggesting de-accelerating universe. The implications of this result will be discussed later.

In the present work, we discuss an expression for rotational velocity of a test particle in a circular motion around the central mass in an asymptotically de Sitter space time (Pokheral 1994). This expression is used to estimate the value of the \(\Lambda\) from the rotation curve of various spiral galaxies. We estimated the value of \(\Lambda\) for 15 spiral galaxies that have radial velocity less than 2000 km s\(^{-1}\). This paper is organized as follows. A relation between rotational velocity and the cosmological constant is derived in Sect. 2. We describe the database and the calculation procedure in Sects. 3 and 4. In Sect. 5, we present our result. Finally, a discussion of results and the conclusions are summarized in Sects. 6 and 7.

2. Theory

The metric which describes the geometry of the Schwarzschild de Sitter space-time is

\[
ds^2 = (1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3})dt^2 - \frac{dr^2}{1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3}} - r^2(d\theta^2 + \sin^2 \theta d\phi^2) = (1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3})dt^2 - \frac{dr^2}{1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3}} - r^2(d\theta^2 + \sin^2 \theta d\phi^2)\]

(1)

Here we utilize this line element in order to study the effect of the cosmological constant \(\Lambda\) on the motion of both massive and massless particles in the Universe. For this space-time, the Lagrangian is

\[
\mathcal{L} = \frac{1}{2}[(1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3})\dot{t}^2 - \frac{\dot{r}^2}{1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3}} - r^2\dot{\theta}^2 - r^2\sin^2 \theta \dot{\phi}^2]\]

(2)

where dot represents differentiation with respect to the affine parameter \(\tau\). The canonical momenta are

\[
\varphi_t = \frac{\partial \mathcal{L}}{\partial \dot{t}} = (1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3})\dot{t}
\]

(3)

\[
\varphi_\theta = -\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = r^2\dot{\theta}
\]

(4)

\[
\varphi_r = -\frac{\partial \mathcal{L}}{\partial \dot{r}} = (1 - 2\frac{M}{r} - \frac{\Lambda r^2}{3})^{-1}\dot{r}
\]

(5)

and

\[
\varphi_\phi = -\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = r^2\sin^2 \theta \dot{\phi}
\]

(6)

The resulting Hamiltonian is

\[
\mathcal{H} = \varphi_t - (\varphi_r \dot{r} + \varphi_\theta \dot{\theta} + \varphi_\phi \dot{\phi}) - \mathcal{L} = \mathcal{L}
\]

(7)
The equality of Hamiltonian and Lagrangian signifies that there is no ‘potential’ in the problem. Thus, the energy is derived solely from ‘kinetic energy’.

Now we rescale the affine parameter $\tau$ in such away that the $2\mathcal{L}$ has the value +1 for time-like geodesics and zero for null geodesics. In this case the integrals of motion in the invariant plane $\theta = \frac{\pi}{2}$ are

$$\wp_\tau = (1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})\dot{t} = \text{constant} = E \quad (8)$$

and

$$\wp_\phi = r^2 \dot{\phi} = \text{constant} = L \quad (9)$$

where $E$ and $L$ are constants associated with the energy and angular momentum of the particle (about an axis normal to the invariant plane) respectively.

Using $\dot{t}$ and $\dot{\phi}$ (equations 8 and 9), the constancy of the Lagrangian gives

$$\frac{E^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - \frac{\dot{r}^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - \frac{L^2}{r^2} = 2\mathcal{L} \quad (10)$$

depending upon whether we are considering time-like or null geodesics. Defining further a parameter $\Omega$, where,

$$\Omega = \frac{d\phi}{dt} = \frac{\dot{\phi}}{\dot{t}} = \frac{L}{E} (1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}) \quad (11)$$

The rotational velocity of a test particle around the central mass in the invariant plane is given by (Chandrashekhar 1983),

$$v_\phi = \frac{r}{\sqrt{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}}} \Omega \quad (12)$$

For time-like geodesics, equations (10) and (11) can be rewritten in the form

\[ \left( \frac{dr}{d\tau} \right)^2 - (1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})(1 + \frac{L^2}{r^2}) + E^2 = f(r) \quad (13) \]

and

\[ \frac{d\phi}{d\tau} = \frac{L}{r^2} \quad (14) \]

Dividing equation (13) by the $\left( \frac{d\phi}{d\tau} \right)^2$ from equation (14). We can get the equation for $r$ as a function of $\phi$:

\[ \left( \frac{dr}{d\phi} \right)^2 = \frac{\Lambda r^6}{3L^2} + (E^2 + \frac{\Lambda L^2}{3} - 1) \frac{r^4}{L^2} + \frac{2Mr^3}{L^2} - r^2 + 2Mr \quad (15) \]
Now letting $u = \frac{1}{r}$, we obtain the basic equation of the problem

\[
\left( \frac{du}{d\phi} \right)^2 = 2M u^3 - u^2 + \frac{2Mu}{L^2} + \frac{\Lambda}{H^2} - \left( \frac{1-E^2}{L^2} - \frac{4}{3} \right) = f(u)
\]  

(16)

Since $f(u)$ involves a polynomial of fifth degree (i.e., quantic polynomial). The exact solution to the problem cannot be formulated in terms of elliptic integrals (Gradshteyn & Ryzhik 1997, Goldstein 1980).

For circular orbits, $f(r)$ in equation (13) should have double root. The critical values of $L$ and $E$ required for such double roots can be found, by setting $f(r)$ and $f'(r)$ equal to zero. Accordingly,

\[
f(r) = E^2 - (1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})(1 + \frac{L^2}{r^2}) = 0
\]  

(17)

and

\[
f'(r) = \frac{df}{dr} = -(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3})(\frac{2L^2}{r^4}) - (1 + \frac{L^2}{r^2})(\frac{2M}{r^2} - \frac{2\Lambda r^2}{3})
\]  

(18)

Eliminating the factor $(1 + \frac{L^2}{r^2})$, we get

\[
\frac{L}{E} = \frac{r\sqrt{\frac{M}{r} - \frac{\Lambda r^2}{3}}}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}}
\]  

(19)

Equation (11) reduces to

\[
\Omega = \frac{\sqrt{\frac{M}{r} - \frac{\Lambda r^2}{3}}}{r}
\]  

(20)

By substituting equation (20) in equation (12), we find

\[
v_\phi = \sqrt{\frac{\frac{M}{r} - \frac{\Lambda r^2}{3}}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}}}
\]  

(21)

This equation represents the rotational velocity of test particle around the central mass in the invariant plane. This expression can be used to determine the mass and the cosmological constant ($\Lambda$) of galaxies using rotation curve data.

3. Database

A galaxy needs to fulfill the following selection criteria in order to be selected: (1) galaxy rotation curve should be given, (2) radial velocity $< 2000$ km s$^{-1}$ and, (3) morphology
of the nucleus. However, we focus our attention in the flat region of the rotation curve. The center is due to the linkage (merely a drawing) between positive and negative sides expected from exponential or de Vaucouleurs laws. The widely adopted zero velocity at the nucleus. This hints that the mass density increases rapidly towards the nucleus than the rotational velocities in many well resolved galaxies do not declined to zero at the broad maximum by the disk and the halo component (Fig. 1). It can be seen that velocity of the red-shifted galaxies lie in the range 131 km s\(^{-1}\) (NGC 2403) to 1 636 km s\(^{-1}\) (NGC 1365).

Table 1. The database of 15 galaxies (Sanders 1996; Sofue et al. 1999, Barnaby & Thronson 1992, 1994; Sancisi & van Albada 1987; Kent 1985). The first three columns list the NGC name and positions of galaxies. The next two columns give the major (a) and minor (b) diameters (in arcmin). The sixth, seventh and eight columns list the radial velocity (RV, in km s\(^{-1}\)), rotational velocity (\(V_{rot}\), in km s\(^{-1}\)) and distance (D, in Mpc). The last two columns give the position angle (PA, in degree) and the morphology (T) of galaxies.

| NGC  | RA (J2000) | Dec (J2000) | a   | b   | RV | \(V_{rot}\) | D   | PA | T   |
|------|------------|-------------|-----|-----|----|-----------|-----|----|-----|
| 0224 | 00°40'00.99" | +40°59'42.8" | 190.0 | 60.0 | −300 | 250 | 0.69 | 40 | Sb  |
| 0660 | 01°43'02.40" | +13°38'42.2" | 88.3  | 03.2 | 850  | 145 | 13.00 | 45 | Sc  |
| 0891 | 02°22'33.41" | +42°20'56.9" | 13.5  | 02.5 | 528  | 162 | 8.90 | 19 | Sb  |
| 1097 | 02°46'19.05" | −30°16'29.6" | 09.3  | 06.3 | 1271 | 283 | 16.00 | 135| SBB |
| 1365 | 03°33'36.37" | −36°08'25.4" | 11.2  | 06.2 | 1636 | 235 | 15.60 | 222| SBB |
| 2403 | 07°36'51.40" | +65°36'09.2" | 21.9  | 12.3 | 131  | 137 | 3.30 | 125| Sc  |
| 2903 | 09°32'10.11" | +21°30'03.0" | 12.6  | 06.0 | 556  | 252 | 6.10 | 21 | Sc  |
| 3198 | 10°19'54.92" | +45°32'59.0" | 10.0  | 03.8 | 663  | 163 | 9.10 | 215| SBc |
| 4258 | 12°18'57.50" | +47°18'14.3" | 18.6  | 07.2 | 448  | 210 | 6.60 | 150| Sbc |
| 4321 | 12°22'54.90" | +15°49'20.6" | 07.4  | 06.3 | 1571 | 271 | 15.00 | 146| Sc  |
| 4565 | 12°30'20.78" | +25°59'15.6" | 15.9  | 01.9 | 1230 | 252 | 10.20 | 137| Sb  |
| 5033 | 13°13'27.53" | +36°35'38.1" | 10.7  | 05.0 | 875  | 183 | 14.00 | 179| Sc  |
| 5055 | 13°15'49.33" | +42°01'45.4" | 12.6  | 07.2 | 504  | 182 | 8.00 | 103| Sbc |
| 5236 | 13°37'00.92" | −29°51'56.7" | 12.9  | 11.5 | 513  | 178 | 8.90 | 45 | Sbc |
| 5907 | 15°15'53.77" | +56°19'43.6" | 12.8  | 01.4 | 667  | 245 | 11.60 | 156| Sc  |

should be known. In addition to this, the diameters, position angle and magnitude of the galaxies were added. Table 1 lists the database. The rotation curve data of these galaxies are taken from Sofue (2007). In the database, we have added a blue-shifted galaxy (NGC 0224 / RV = −300 km s\(^{-1}\)) in order to observe the local effect. There are 11 spirals and 4 barred spirals. The late-type galaxies (Sc) dominate (6, 40%) the database. The radial velocity of the red-shifted galaxies lie in the range 131 km s\(^{-1}\) (NGC 2403) to 1 636 km s\(^{-1}\) (NGC 1365).

The rotation curves of these galaxies comprise steep central rise, bulge component, broad maximum by the disk and the halo component (Fig. 1). It can be seen that the rotational velocities in many well resolved galaxies do not declined to zero at the nucleus. This hints that the mass density increases rapidly towards the nucleus than the expected from exponential or de Vaucouleurs laws. The widely adopted zero velocity at the center is due to the linkage (merely a drawing) between positive and negative sides of the nucleus. However, we focus our attention in the flat region of the rotation curve.
Figure 1. Galaxy rotation curves of individual galaxies (Sofue 2007). The NGC name and the RV of the galaxies are given.

We use the value of \( r \) (in kpc) and \( v_{\text{rot}} \) (in km s\(^{-1}\)) from the given rotation curves in order to estimate the value of the mass (\( M \)) and cosmological constant (\( \Lambda \)) of galaxies.

4. Estimation of \( \Lambda \) and \( M \)

The rotational velocity of a test particle around the central mass in the invariant plane is given by,

\[
\frac{v^2}{c^2} = \left( \frac{\frac{M}{r} - \frac{\Lambda r^2}{3}}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} \right)^{\frac{1}{2}}
\]

Then,

\[
\frac{v^2}{c^2} \left( 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} \right) = \left( \frac{M}{r} - \frac{\Lambda r^2}{3} \right)
\]

Taking summation,

\[
\Sigma v^2 (\frac{1}{c^2}) - 2M \Sigma \frac{v^2}{r} (\frac{1}{c^2}) - \Lambda \Sigma \frac{v^2 r^2}{3} (\frac{1}{c^2}) = M \Sigma \frac{1}{r} - \Lambda \Sigma \frac{r^2}{3}
\]

Multiply equation (24) by \( r \), then

\[
\Sigma v^2 r (\frac{1}{c^2}) - 2M \Sigma v^2 (\frac{1}{c^2}) - \Lambda \Sigma \frac{v^2 r^2}{3} (\frac{1}{c^2}) = MN - \Lambda \Sigma \frac{r^2}{3}
\]

Solving equations (24) and (25) for \( \Lambda \) and \( M \), we get

\[
\Lambda = \frac{\Sigma v^2 r (\frac{1}{c^2}) - 2M \Sigma \frac{v^2}{r} (\frac{1}{c^2}) - M \Sigma \frac{1}{r}}{\Sigma \frac{v^2 r^2}{3} (\frac{1}{c^2}) - \Sigma \frac{r^2}{3}}
\]
The expressions for $\Lambda$ and $M$ (equations 26 and 27) are the functions of $r$ and $v_\phi$. Thus, the rotation curve data ($v_\phi$, $r$) yield the value of the mass of the galaxy ($M$) and the cosmological constant ($\Lambda$). For the computation, we use the software MATLAB6.1. A typical sketch of the galaxy rotation curve is shown in Fig. 2. In order to fix the positions (i.e., $r_1$, $r_2$, $r_3$ and $r_4$) in the rotation curves, the flattened part of the rotation curve is divided into 4 equal parts (see Fig. 2a). The first position (say radius), i.e., $r_1$, is the point where the flatness begins in the rotation curve. This point is fixed by comparing the nature of all 15 galaxy rotation curves. This position is found to be far from the central bulge of the galaxies in our case. The position $r_4$ represents the extreme limit of the radius from the center of the galaxy. We estimate the mass and the $\Lambda$ of 15 spirals at the radii $r_1$, $r_2$, $r_3$ and $r_4$ (given in the third column of Table 2).

5. Results

Table 2 shows the values of the mass ($M$) and the cosmological constant ($\Lambda$) of 15 sample galaxies at various radius ($r_1$, $r_2$, $r_3$, $r_4$). The mass of the galaxies are found to lie in the range $0.13-7.60 \times 10^{40}$ kg ($0.04 - 2.53 \times 10^{10} M_\odot$). These masses represent the lower limit because the rotation curve data is not complete, particularly of the central region and outer halo region. At the central region, black hole can be expected. The dark matter is believed to be dominating at the outer halo region. In the future work, we intend to address this problem by modeling mass distribution. The cosmological constant ($\Lambda$) is found to be negative for all 15 sample galaxies. A comparison between the variation of
mass and the radii (i.e., positions away from the bulge of the galaxies) is shown in Fig. 4a. As expected, it is found that the mass of the galaxy increases with the increase of the radius of the galaxy. This increase is found to be steeper for massive galaxies (Fig. 4a). This is probably due to the huge size of the halo of the massive galaxies. This suggests that the mass of the halo depends upon the size of the galaxies. A good agreement can be seen for blue-shifted galaxy (dashed line in Fig. 3a).

Fig. 3b shows the radius versus Λ plot. As the positions \((r_1, r_2, r_3, r_4)\) from the center of the galaxy increase the Λ is found to decrease (in magnitude). In other words, one can expect positive Λ at greater \(r\) (probably, outside the galactic halo). In Fig. 3b, the width of the distribution in the lower radius side (i.e., \(r_1\)) is wider than that of the higher radius (i.e., \(r_4\)) side. Probably, the positive Λ demands the convergence of these distributions. We noticed that the variation of Λ is minimum when moving out of the bulge of the galaxy.

It can be concluded that the Λ within galactic halo corresponds to the dark energy which tries to de-accelerate the accelerating Universe. In addition, one can predict that the Λ outside the halo might be positive, supporting supernova cosmology project. A similar result is noticed for the blue-shifted galaxy (dashed line in Fig. 3b) The Λ versus

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**Table 2.** The first column lists the NGC name of the galaxy. The next two columns represent the positions in the flat region of the rotation curve. The fourth and fifth columns give the value of the mass \((M)\) and the cosmological constant \((Λ)\).

| NGC | positions | \(r\) (kpc) | \(M\) \((\times 10^{40}\text{ kg})\) | \(-Λ\) \((\times 10^{-40}\text{ km}^{-2})\) |
|-----|-----------|-------------|-----------------|-----------------|
| 0224 | \(r_1/r_2/r_3/r_4\) | 2.2/8.0/14.9/20.5 | 1.17/2.41/3.69/4.73 | 4.237/0.587/0.204/0.104 |
| 0660 | \(r_1/r_2/r_3/r_4\) | 4.9/9.7/15.0/22.5 | 0.94/1.38/1.68/2.13 | 0.564/0.124/0.059/0.028 |
| 0891 | \(r_1/r_2/r_3/r_4\) | 4.7/9.4/14.8/20.2 | 1.58/2.48/3.26/4.09 | 1.003/0.281/0.130/0.063 |
| 1097 | \(r_1/r_2/r_3/r_4\) | 7.2/12.7/18.6/25.7 | 2.07/2.60/3.27/3.91 | 1.030/0.382/0.158/0.078 |
| 1365 | \(r_1/r_2/r_3/r_4\) | 4.9/10.0/14.8/24.2 | 2.33/3.99/5.05/7.60 | 1.234/0.388/0.167/0.061 |
| 2403 | \(r_1/r_2/r_3/r_4\) | 4.1/10.0/14.6/19.6 | 0.13/0.37/0.60/0.81 | 0.354/0.103/0.051/0.030 |
| 2903 | \(r_1/r_2/r_3/r_4\) | 5.8/11.8/16.0/23.5 | 1.44/2.33/2.85/3.61 | 1.498/0.328/0.166/0.089 |
| 3198 | \(r_1/r_2/r_3/r_4\) | 5.0/9.7/14.9/20.1 | 0.17/0.47/0.84/1.29 | 0.466/0.158/0.080/0.044 |
| 4258 | \(r_1/r_2/r_3/r_4\) | 4.7/9.9/15.0/20.0 | 1.10/1.69/2.04/2.36 | 1.071/0.215/0.100/0.061 |
| 4321 | \(r_1/r_2/r_3/r_4\) | 4.7/9.9/15.0/20.6 | 1.23/2.00/2.72/3.52 | 1.252/0.383/0.183/0.109 |
| 4565 | \(r_1/r_2/r_3/r_4\) | 4.8/9.5/15.9/24.7 | 1.51/2.11/2.88/3.85 | 1.175/0.390/0.155/0.069 |
| 5033 | \(r_1/r_2/r_3/r_4\) | 4.0/8.9/14.7/22.2 | 1.27/1.95/2.53/3.24 | 2.219/0.458/0.194/0.096 |
| 5055 | \(r_1/r_2/r_3/r_4\) | 4.7/9.8/14.8/21.4 | 0.55/0.96/1.35/1.78 | 1.159/0.298/0.123/0.056 |
| 5236 | \(r_1/r_2/r_3/r_4\) | 4.7/9.8/14.8/21.2 | 0.88/1.20/1.53/1.90 | 0.760/0.218/0.093/0.046 |
| 5907 | \(r_1/r_2/r_3/r_4\) | 3.5/9.5/14.8/20.7 | 0.76/1.42/2.28/3.14 | 1.795/0.402/0.181/0.106 |
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Figure 3. The variation between the radius \((r)\) (a,b), mass \((M)\) (a,c) and the cosmological constant \((\Lambda)\) (b,c) for 15 sample galaxies. The solid circles represent the estimated value. The observed values are fitted polynomially (second order). The solid line is for the red-shifted galaxies and the dashed line is for the blue-shifted galaxy NGC0224.

\(M\) plot for all 15 galaxies can be seen in Fig. 3c. As \(r\) increases, as expected, mass of the galactic halo \((M)\) increases. Probably, this increase lead the \(\Lambda\) tend to go to positive. However, we found negative \(\Lambda\) for our sample galaxies. Our galaxies are nearby galaxies. It would be interesting to analyze the flattened portion of the rotation curves of high red-shift galaxies. We intend to work on rotation curves of \(\sim\)100 galaxies that have \(RV > 5000\) km \(-1\) in the future. We expect to find positive \(\Lambda\) for distant galaxies.

6. Discussion

Kraniotis & Whitehouse (2000) estimated \(\Lambda\) value for 6 spiral galaxies using Friedman energy equation (Sciama 1995; Bahcall et al. 1999), weak field approximation (Whitehouse & Kraniotis 1999), and the observed rotation curve of the galaxy. They found that the rotation curve data of all 6 galaxies up 50 kpc (see Table 1 of their paper) inevitably lead to a negative value of the cosmological constant, suggesting de-accelerating Universe.

We have included all 6 galaxies (NGC2406, NGC4258, NGC5033, NGC5055, NGC2903, NGC3198) analyzed by Kraniotis & Whitehouse (2000) in our database in order to study the difference between the results. In our study, the \(\Lambda\) value for these galaxies at \(\sim\) 20 kpc are found in the range \(-0.03\) to \(-0.09 \times 10^{-40}\) km\(^{-2}\). They estimated \(-0.5\) to \(-5.5 \times 10^{-55}\) cm\(^{-2}\). Their range for \(\Lambda\) is relatively wider than ours.

Our results as well as results obtained by Kraniotis & Whitehouse (2000) contradict the result obtained from the Supernova Cosmology project. It has to be kept in mind that the Supernova project studied the recessional velocity of distant objects while in this paper, we are concerned with the rotational velocity of galaxies. Obviously, the bound matter in galaxies can not be accelerating with the expansion of the universe. So during the formation of galaxies local effect must have worked in such away that, an effect of negative vacuum energy was generated in the region. Probably, the vacuum energy of
these structures lead to form a nearby local bubble. The space-time of this bubble might be anti de Sitter type. We suspect that the dark energy adds to the attractive gravity of matter within the bubble in order to show de-accelerating feature. We are focusing our future works on this very aspect.

7. Conclusion

We used an expression for rotational velocity of a test particle in a circular motion around the central mass in an asymptotically de Sitter space time (Pokheral 1994) and estimated the value of the $\Lambda$ from the rotation curve data of 15 spiral galaxies. We conclude the following:

1. The mass of the galaxies are found to lie in the range $0.13 - 7.60 \times 10^{40}$ kg ($0.04 - 2.53 \times 10^{10} M_\odot$).

2. The cosmological constant ($\Lambda$) is found to be negative ($-0.03$ to $-0.10 \times 10^{-40}$ km$^{-2}$) for all 15 galaxies. These are nearby (radial velocity less than 1636 km$^{-1}$) galaxies. A similar result is noticed for a blue-shifted galaxy, NGC0224.

3. As the radius of the galaxy increases, mass of the galactic halo increases and the $\Lambda$ tend to increase from larger negative value to the smaller values. The $\Lambda$ within galactic halo corresponds to the dark energy which cause to de-accelerate the accelerating Universe.

4. Our results show that the relative effect of cosmological constant ($\Lambda$) becomes pronounced at larger distances, away from the bulge of the galaxy.

The result of this work do not agree with the results of supernova project which looked at the large scale cosmology. We suspect that the $\Lambda$ value might be positive outside the local bubble, supporting supernova cosmology project. The space time of this local bubble might be anti de Sitter type. Such anti de Sitter space-time is becoming important in the context of the brane world. We intend to justify this prediction in our future works.

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