Progress towards laser plasma based free electron laser on COXINEL

Marie-Emmanuelle Couprie\textsuperscript{1}, Thomas André\textsuperscript{1}, Frédéric Blache\textsuperscript{1}, François Bouvet\textsuperscript{1}, Yannick Dietrich\textsuperscript{1}, Jean-Pierre Duval\textsuperscript{1}, Moussa El-Ajjouri\textsuperscript{1}, Amin Ghaith\textsuperscript{1}, Christian Herbeaux \textsuperscript{1} Nicolas Hubert\textsuperscript{1}, Charles Kitégï\textsuperscript{1}, Martin Khojoyan\textsuperscript{1}, Marie Labat\textsuperscript{1}, Nicolas Leclercq\textsuperscript{1}, Alain Lestrade\textsuperscript{1}, Alexandre Loulergue\textsuperscript{1}, Olivier Marcouillé\textsuperscript{1}, Fabrice Marteau\textsuperscript{1}, Driss Oumbarek-Espinos\textsuperscript{1}, Patrick Rommelüre\textsuperscript{1}, Mourad Sebdouï\textsuperscript{1}, Keihan Tavakoli\textsuperscript{1}, Mathieu Valléal\textsuperscript{1}, Sébastien Corde\textsuperscript{2}, Julien Gautier\textsuperscript{2}, Jean Philippe Goddet\textsuperscript{2}, Olena Kononenko\textsuperscript{2}, Guillaume Lambert\textsuperscript{2}, Amar Tafzi\textsuperscript{2}, Kim Ta Phuoc\textsuperscript{2}, Cédric Thaury\textsuperscript{2}, Serge Bielawski\textsuperscript{3}, Eléonore Rousset\textsuperscript{3}, Christophe Szwaj\textsuperscript{3}, Igor Andriyash\textsuperscript{4}, Victor Malka\textsuperscript{4}, Slava Smartsev\textsuperscript{4}

\textsuperscript{1}Synchrotron SOLEIL, Gif-sur-Yvette, France, \textsuperscript{2}Laboratoire d’Optique Appliquée, LOA, ENSTA Paris, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris, 91762 Palaiseau, France, \textsuperscript{3}Laboratoire PhLAM, Lille, France, \textsuperscript{4}Weizmann Institute of Science, Rehovot, Israel.

E-mail: couprie@synchrotron-soleil.fr

Abstract.

The Free Electron Laser (FEL) application of Laser Plasma Acceleration (LPA) requires the handling of the energy spread and divergence. The COXINEL manipulation line, designed and built at SOLEIL for this purpose, consists of high gradient quadrupoles for divergence handling and a decompression chicane for energy sorting, enabling FEL amplification with baseline parameters. Installed at Laboratoire d’Optique Appliquée (LOA), it uses robust electrons generated and accelerated by ionization injection using a 30 TW laser. We report here on the work progress towards a FEL demonstration. The LPA measured electron beam characteristics deviates from the baseline reference case. After the installation of the equipment, the electron beam transport has first been optimized. The electron position and dispersion are independently adjusted. Then, undulator radiation has been measured. The spectral purity is controlled via the energy spread adjusted in the slit located in the chicane. FEL effect demonstration is within reach, with currently achieved performance on different LPA experiments.

1. Introduction

Accelerator based light sources are presently widely developed \cite{1, 2}. Synchrotron radiation was first been used parasitically on circular accelerators built for high energy physics (first generation), and then on dedicated rings with few mm.rad emittance and undulators \cite{3} (second generation). Third generation light sources with low emittance electron beams, high undulator number, enabling partial transverse coherence, are nowadays wide used for matter investigation. Longitudinal coherence can be achieved by setting the electrons in phase (micro-bunching) in an undulator thanks to the Free Electron Laser process \cite{4}. FELs use more generally linear
accelerators for short wavelength operation, enabling to provide very short pulse and small spectral bandwidth. Present X-ray FELs [5], with peak brightness increased by several orders of magnitude, enable to decipher the matter evolution on ultra-fast time scales.

Advanced acceleration concepts, such as Laser Plasma Acceleration (LPA) [6], are considered to be qualified for an FEL application [7, 8], in combining two major outcomes of the laser invention [9]. The LPA concept relies on focusing a Chirped Pulse Amplification [10] based ultra high power lasers into a gas target. The laser resonantly drives a nonlinear plasma wave with very high charge-separation fields that can trap the ambient plasma electrons and accelerate them to several GeV within ultra-short distances [11, 12, 13]. The LPA development is promising: GeV range energy, kA peak current, ultra-short bunches, 1π mm.mrad normalized emittance beams can be produced. The hopes put in LPA to drive undulator radiation and FEL light sources are challenged by LPA parameters that do not yet meet the current accelerator state-of-the-art performance. While conventional accelerators deliver µrad divergence and ~ 0.01 % energy spread beams, the LPA large energy spread and divergence require to mitigate chromatic effects [14, 15], that can lead to a dramatic emittance growth and afferent beam quality degradation in the transfer lines. The achievement of the needed electron beam quality at the undulator center for FEL amplification, requires a proper electron beam control over the transport. Large divergence requires strong focusing right after the electron source, with high gradient permanent magnet quadrupoles [16]. Large energy spread can be handled by a decompression undulator [17, 8] or a transverse gradient undulator [18, 19]. Advantage can even be taken from the electron beam energy / position correlation introduced by the chicane [20]. LPA based undulator radiation was first observed [21, 22, 23, 24] on experiments that have not been designed for FEL. The radiation did not present all the typical features, spectral purity and stability as currently achieved on conventional accelerators. In the frame of the LUNEX5 project of advanced compact FEL demonstrator [25, 26], the LPA based FEL path is explored. We report here on the progress on the COXINEL experiment [27] aiming at demonstrating FEL amplification with the help of a dedicated transport line to handle and manipulate the beam properties.

2. The COXINEL line design for enabling FEL demonstration

Figure 1. COXINEL line sketch : Laser hutch (grey), gas jet (blue), QUAPEVA high gradient permanent magnet quadrupoles (grey), electromagnetic dipole based chicane (red), removable electron imager or a slit (pink), quadruplet of electromagnetic quadrupoles (blue), undulator (purple) surrounded by two steerers and removable screens, electron dipole dump (red), electron imager, photon diagnostics (CCD camera and spectrometer).

The COXINEL line (see Fig. 1), designed and built at Synchrotron SOLEIL [28], is installed at Laboratoire d’Optique Appliquée (LOA), where LPA development is carried out using a Ti:Sapphire laser system delivering 1.5 J, 30 fs FWHM pulses. The divergence is rapidly mitigated (5 cm away from the source) via strong focusing provided by a triplet of variable permanent magnet based quadrupoles (so-called QUAPEVA). A magnetic chicane then longitudinally stretches the beam, sorts electrons in energy and selects the energy range of interest via a removable and adjustable slit mounted in the middle of the chicane. A second set of quadrupoles matches the beam inside an undulator. The COXINEL manipulation line, designed for FEL achievement considering baseline reference parameters (in the UV at 180 MeV
with a U18 undulator (107 periods of 18.16 mm, variable gap between 4.55 - 30 mm, reaching 1.2 T peak field at minimum gap) and VUV at 400 MeV with a U15 undulator), as given in Table 1 for the 180 MeV case. A seed for the FEL can be prepared from another branch of the infra-red laser. The electron optics, a source to image optics, refocuses the beam inside the undulator thanks to the strong gradient QUAPEVA [20]. An analytic approach can first be considered [29]. The electron transport, modeled using the BETA code, has been benchmarked with different codes [30] including collective effects. The total emittance growth is frozen at the exit of the QUAPEVA triplet. The emittance is then dominated by its chromatic term, scaling as the energy spread and as the square of the initial divergence, and remains then unaffected along the line. Different electron beam optics are considered. The ”supermatching optics” enables to focus each electron beam slice in synchronisation with the progression of the amplified synchrotron radiation along the undulator, taking advantage of the energy/position correlation introduced in the chicane [20]. The synchronisation sets a particular setting of the quadrupoles located in front of the undulator. The continuous synchronisation minimizes the beam size all along the undulator length and maximizes the electron density and thus the gain. Specific optics focusing on the different screens implemented along the line are designed for the beam transport experiments for more precise measurements. Finally, an optics enabling to focus also the beam horizontally in the chicane equipped with a slit permits to clean the large energy spread beam.

**Table 1.** COXINEL Root Mean Square (RMS) electron beam characteristics at 180 MeV: baseline reference (source and undulator after beam manipulation) and measured at the source.

| Slice Parameters            | Baseline (source) | Baseline (Undulator) | Measured (at source) |
|----------------------------|-------------------|----------------------|---------------------|
| Vertical divergence        | 1 mrad            | 0.1 mrad             | 1.2-3.5             |
| Horizontal divergence      | 1 mrad            | 0.1 mrad             | 1.8-5               |
| Beam size                  | 1 μm              | 50 μm                |                     |
| Bunch length (rms)         | 3.3 fs            | 33 fs                |                     |
| Charge (pC)                | 34                | 34                   |                     |
| Charge density (pC/MeV)    | 5                 | 5                    | 0.5                 |
| Peak Current               | 4.4 kA            | 440 A                |                     |
| Slice energy spread σγ % rms | 1                 | 0.1                  |                     |
| Normalised emittance ϵN    | 1 mm.mrad         | 1.7 mm.mrad          |                     |

**Figure 2.** FEL amplification for COXINEL at 400 MeV with a 5 m long U15 undulator (period of 15 mm) reaching 1.59 T at 3 mm gap, for the strong focusing and supermatching optics. Case of the 400 MeV baseline parameters : calculations using at the source a 6D Gaussian bunch without any correlation having a 1 π mm.mrad total normalized rms emittance, a 1 mrad rms divergence, a 1% rms relative energy spread with an 1 μm rms bunch length and 4 kA peak current). (Fig. from [20]).
parameters [20], as illustrated in Fig. 2. The sensitivity study of the FEL versus different parameters has been carried out [31]. In the seeded configuration, the FEL radiation is red shifted with respect to the seed wavelength because of the electron beam chirp induced in the chicane, leading to an interference fringe pattern, that can allow for a full temporal reconstruction of the FEL pulse temporal amplitude and phase distributions [32].

3. The COXINEL line components

Fig. 3 presents the different components that have been built or purchased and characterized [33, 34]. The QUAPEVA high gradient permanent magnet quadrupoles of variable strength present a variable strength (via rotating cylindrical magnet surrounding a central Halbach ring quadrupole [35]) and an adjustable magnetic center position (via translation tables) [36, 37, 38]. inside an in-vacuum undulator (typical SOLEIL 2 m long U20 (period 20 mm), cryo-ready U18 (period 18 mm) or 3 m long cryo-ready U15 (period 15 mm)) [39, 40, 41, 42]. The electron beam can be monitored with current transformers and cavity beam position monitors or by inserting scintillator screens (Lanex and Yag) along the line [43]. The "180 MeV" corresponds to undulator radiation in the UV, while the 400 MeV case associated to the U15 cryogenic undulator enables to reach the VUV spectral range. A picture of the line installed in the "Salle Jaune" at LOA is shown in Fig. 4. An iris for the LPA laser, the transfer line components, the undulator, and an iris at the line exit are aligned within ±100 μm on the same axis with a laser tracker. A reference green laser is used for daily alignment.

4. The produced electron beam characterization

The 1.5 J, 30 FWHM fs pulse laser is focused into a supersonic jet of He – N2 gas mixture for the LPA to operate in the robust ionisation injection [44]. The beam is first characterized with an electron spectrometer, using a permanent magnet dipole and a Lanex screen (see Fig. 5 (left)). Produced beams range up to 250 MeV in a broad energy spectrum. The charge density deduced from the calibration of the electron spectrometer is typically 0.5 pC / MeV. This wide spectrum and associated charge density significantly deviate from the baseline reference parameters (see Table 1). The electron spectrometer enables also to measure the vertical divergence, typically of a few mrad divergence (1.2-5 mrad RMS), depending on the days. From the electron beam image measured with a screen inserted 0.56 m away from the source can be deduced the ratio between the vertical to horizontal sizes, that is used to deduce the horizontal divergence distribution versus energy from the vertical one. Typical 1.5 mrad RMS observed electron beam pointing
fluctuations and drifts, that could result from the laser itself or from intrinsic features of the LPA source, are much larger than the $\pm 100 \, \mu \text{m}$ pre-alignment of the COXINEL line.

5. The electron beam transport

After a first rough beam transport along the line where chromatic effects played an important role, a Beam Position Alignment Compensation strategy based on the matrix response approach has been developed to mitigate alignment residual errors and electron beam pointing drifts [45]. The beam dispersion and position can be independently corrected thanks to a proper setting of the QUAPEVA magnetic axis via the translation tables on which they are mounted. The alignment is then performed step by step, along the different electron imaging screens, with the adjustment of the electron beam position and vertical dispersion at the chicane center, followed by the positions and horizontal dispersion at the undulator entrance and exit. The QUAPEVA strength is then slightly adjusted to optimize the focusing thanks to the rotation of the cylindrical magnets. The matched transported beam measurements agree with simulations for measured beam characteristics (dipole spectrometer and observation on a screen), as displayed in Fig. 5 [46]. The focused beam, both measured and simulated, also exhibits a cross-like shape which is a signature of the chromatic effects (different electron beam energies being focused at different longitudinal positions). The tilt of the measured electron beam profiles result from the skew quadrupolar components of the QUAPEVAs, which have been initially designed with a specification corresponding to the baseline electron beam parameters. These terms have been corrected with shims [47].

6. The measured undulator radiation

U18 undulator radiation is first measured using a CCD camera installed 3 m away from the exit of the undulator [45]. Without energy selection in the chicane, low energy electrons are filtered along the line, resulting in an energy spread of 30 % RMS. The resonant fundamental wavelength emission is given by: 

$$\lambda = \frac{\lambda_u}{2\gamma} \left(1 + \frac{K_u^2}{2} + \gamma^2 \theta^2 \right),$$

with $\lambda_u$ the undulator period, $K_u$ the deflection parameter, $\gamma$ the Lorentz factor and $\theta$ the observation angle. It spans over 100-360 nm for a 5 mm gap, with 200 nm for the reference energy of 176 MeV. The measured and modeled radiation flux density normalized to 1 pC focused on the CDD, displayed in Fig. 5 (right), collect on and off-axis radiation, and present a similar shape and signal level. The full estimated number of photons per beam charge $N_{ph}$ is $\approx 3.10^7 \, \text{pC}^{-1}$. A slit inserted in the chicane enables to reduce the electron beam energy spread (to 8 % RMS for a 4 mm width,) and to maintain the charge of the energy of interest, and thus to limit the inhomogeneous contribution of the energy spread to the spectral purity of the radiation [48, 45]. A new optics is designed to reduce further the energy spread in the chicane with a slit [47].
A UV spectrometer, equipped with two collimating mirrors, a 600 gr/mm grating and a CCD camera, installed 3 m from the undulator exit, images the spatio-spectral flux of the produced radiation. A CaF$_2$ lens (108.5 mm radius of curvature) focuses the radiation into the spectrometer entrance slit [50]. The observation angle can be expressed as: $$\theta = \pm \frac{\sqrt{2}}{\lambda_0} (\lambda - \lambda_{res})$$ with $\lambda_{res} = \frac{\lambda_u}{2\gamma^2} (1 + K_u^2/2)$ the on-axis resonant wavelength. Using far field undulator radiation and ray optics, the angle can be converted to a vertical position on the CCD camera by: $$z_{CCD} = G \times [((D \cdot \theta + h) \cdot (1 - d/f(\lambda)) + d \cdot \theta] + H$$ with $G$ the magnification factor, $D$ (respectively $d$) the undulator center-lens (lens-spectrometer) distance, $h$ (respectively $H$) the lens (CCD camera) vertical offset, and $f(\lambda)$ the chromatic focal length dependence using the the Sellmeier coefficients [54]. The measured radiation exhibits the typical moon shape pattern (quadratic dependence of the resonant wavelength versus the observation angle), characteristic of undulator radiation [51, 52, 53]. The chromatic effects of the lens induce however a distortion of the moon shape to a more triangular one. Fig. 6-a displays a typical measurement of the spectrometer. The signal is dominated by the CCD camera background noise and high pixels. Fig. 6-b presents the spectra after removing the background noise and high pixels, smoothing it with a median filter and applying the grating and CCD camera calibration. Fig. 6-c shows the on-axis spectral flux cut at the center of the moon shape (blue) achieving a relative bandwidth of 8.4%, and the total flux captured by the camera (green) with a relative bandwidth of 27%. The radiation linewidth can be controlled using the electron beam energy selection via the slit in the chicane [50, 29]. The undulator radiation emitted presents a wavelength stability of 2.6 % [50].

**Figure 6.** Undulator spatio-spectral distribution measured with the Horiba iHR320 spectrometer at 4.7 mm gap (1 mm electron slit) with a spectrometer slit open at 2.2 mm. (a) Raw data (b) data with background removed, median filter (averaging 5 points corresponding to 0.5 nm) applied, and spectrometer calibration added, fit of the undulator radiation wavelength taking into account the chromatic effects of the lens (h = -0.31 mm, d = 0.228 m, H = 1.146 mm) in red. (c) blue : on axis spectral cut, red: total flux integrated over the vertical axis.

**7. CONCLUSION**

We have shown that the LPA electron beam properties can be manipulated through an adequate transport line, mitigating the performance that do not meet the one of state-of-the-art conventional accelerators for some specific applications. These results are of interest for various LPA applications, such as undulator synchrotron radiation, free electron laser and colliders, requiring stages of LPA accelerating modules. The transported electron beam on COXINEL has enabled successful measurement of undulator radiation under various conditions. FEL effect demonstration is within reach, with currently achieved performance on different LPA experiments, and hopefully after a laser up-grade presently under way at LOA.
8. ACKNOWLEDGEMENTS
The authors thank the European Research Council (ERC) for COXINEL (340015, PI : M. E. Couprie), X-Five (339128, PI: V.Malka), the EuPRAXIA design study (653782), the Fondation de la Cooperation Scientifique (QUAPEVA-2012-058T), and the SOLEIL and LOA staffs.

References
[1] M. E. Couprie, J. M. Filhol, "X radiation sources based on accelerators", Comptes Rendus Phys., vol. 9:487–506, 2008.
[2] M. E. Couprie, "New generation of light sources: Present and future", Journal of Electron Spectroscopy and Related Phenomena, vol. 196:3–13, 2014.
[3] A. Hofmann, "Quasi-monochromatic synchrotron radiation from undulators", Nucl. Instrum. Meth., vol. 152, p. 17–21, 1978.
[4] J. M. Madey, "Stimulated emission of bremsstrahlung in a periodic magnetic field", Jour. Appl. Phys., vol. 42, p. 1906–1913, 1971.
[5] P. Emma et al., "First lasing and operation of an Ångstrom-wavelength free-electron laser", Nature Photonics, vol. 4, p. 641, 2010.
[6] T. Tajima and J. M. Dawson, "Laser electron accelerator", Phys. Rev. Lett., vol. 43, p. 267–270, Jul 1979.
[7] F. Gruner et al., "Design considerations for table-top, laser-based VUV and X-ray free electron lasers", Appl. Phys. B, vol. 86, p. 431435, 2007.
[8] M. E. Couprie, A. Loulergue, M. Labat, R. Lehe, and V. Malka, "Towards a free electron laser based on laser plasma accelerators", Journal of Physics B: Atomic, Molecular and Optical Physics, vol. 47, no. 23, p. 234001, 2014.
[9] A. L. Schawlow and C. H. Townes, "Infrared and optical masers", Phys. Rev., vol. 112, no. 6, p.1940, 1958.
[10] D. Strickland and G. Mourou, "Compression of amplified chirped optical pulses", Optics communications, vol. 55, no. 6, p. 447-449, 1985.
[11] E. Esarey, C. Schroeder, and W. Leemans, "Physics of laser-driven plasma-based electron accelerators", Reviews of Modern Physics, vol. 81, no. 3, p. 1229, 2009.
[12] V. Malka et al., "Laser-driven accelerators by colliding pulses injection: A review of simulation and experimental results", Physics of Plasmas, vol. 16, no. 5, p. 056703, 2009.
[13] W P. Leemans et al., "Multi-GeV electron beams from capillary-discharge-guided subPetaWatt laser pulses in the self-trapping regime", Phys. Rev. Lett., vol. 113, p. 245002, 2014.
[14] K. Floettmann, "Some basic features of the beam emittance", Phys. Rev. ST Accel. Beams, vol. 6, p. 034202, 2003.
[15] M. Migliorati et al., "Intrinsic normalized emittance growth in laser-driven electron accelerators", Phys. Rev. ST Accel. Beams, vol. 16, p. 011302, 2013.
[16] A. Ghaith, D. Oumbarek, M. Valléau, F. Marteau, M. E. Couprie, "Permanent Magnet-Based Quadrupoles for Plasma Acceleration Sources", Instruments, vol. 3, no. 2, pp. 27, 2019.
[17] A. Maier, A. Meseck, S. Reiche, C. Schroeder, T. Seggebrock, and F. Gruner, "Demonstration scheme for a laser-plasma-driven free-electron laser", Physical Review X, vol. 2, no. 3, p. 031019, 2012.
[18] T. I. Smith, J. M. J. Madey, L.R. Elias, and D.A.G. Deacon, "Reducing the sensitivity of a free-electron laser to electron energy", Journal of Applied Physics, vol. 50, no. 7, pp. 4580–4583, 1979.
[19] Z. Huang, Y. Ding, C. B. Schroeder "Compact X-ray free-electron laser from a laser-plasma accelerator using a transverse-gradient undulator", Phys. Rev. Lett., vol. 109, no. 20, pp. 204801, 2012.
[20] A. Loulergue, M. Labat, C. Evain, C. Benabderrahmane, V. Malka, and M. Couprie, "Beam manipulation for compact laser wakefield accelerator based free-electron lasers", New Journal of Physics, vol. 17, no. 2, p. 023028, 2015.
[21] H. P. Schlenvoigt et al., "A compact synchrotron radiation source driven by a laser-plasma wakefield accelerator", Nature Physics, vol. 4, no. 2, pp. 130-133, 2008.
[22] M. Fuchs et al., "Laser-driven soft-X-ray undulator source", Nature physics, vol.5, no.11, pp.826–829, 2009.
[23] G. Lambert et al., "Progress on the generation of undulator radiation in the UV from a plasma-based electron beam", Proc. FEL conf., Nara, Japan, p. 2, 2012.
[24] M.P. Anania et al., "An ultrashort pulse ultra-violet radiation undulator source driven by a laser plasma wakefield accelerator", Appl. Phys. Lett., vol. 104, no. 26, pp. 264102, 2014.
[25] M.-E. Couprie et al., "The LUNEX5 project in France", Jour. of Physics: Conf. Series, vol. 425, no. 7, pp. 072001, 2013.
[26] M. Couprie et al., "Strategies towards a compact XUV free electron laser adopted for the LUNEX5 project", Journal of Modern Optics, vol. 63, no. 4, pp. 1–13, 2015.
[27] M. Couprie et al., "Experiment preparation towards a demonstration of laser plasma based free electron laser amplification", Proc. FEL’14 (Basel, Switzerland), 2014.

[28] M. E. Couprie et al., "An application of laser–plasma acceleration: towards a free-electron laser amplification", Plasma Phys. Cont. Fusion, vol. 58, no. 3, p. 034020, 2016.

[29] A. Ghaith A. Loulergue, D. Oumbarek-Espinos, O. Marcouillé, M. Valléau, M. Labat, S. Corde and M. E. Couprie, "Electron Beam Brightness and Undulator Radiation Brilliance for a Laser Plasma Acceleration Based Free Electron Laser", Instruments, vol. 4, pp. 1–17, 2020.

[30] M. Khojoyan et al., "Transport studies of LPA electron beam towards the FEL amplification at COXINEL", Nucl. Instrum. Meth. A, vol. 829, no. 2, pp. 260-264, 2016.

[31] M. Labat et al., "Robustness of a plasma laser acceleration based Free Electron Laser", Phys. Rev. Acc. Beams, vol. 21, no. 11, pp. 114802, 2018.

[32] M. Labat et al., "Interferometry for full temporal reconstruction of laser-plasma accelerator-based seeded free electron lasers", New Journal of Physics vol. 22, pp. 013051, 2020.

[33] M. E. Couprie et al., "An application of laser–plasma acceleration: towards a free-electron laser amplification", Plasma Phys. Cont. Fusion, vol. 58, no. 3, p. 034020, 2016.

[34] K. Halbach et al., "Design of permanent multipole magnets with oriented rare earth cobalt material," Nucl. Instrum. Meth., vol. 169, p. 1–10, 1980.

[35] F. Marteau et al., "Variable high gradient permanent magnet quadrupole (QUAPEVA)", Applied Physics Letters, vol. 111, p. 253503, 2017.

[36] A. Ghaith et al., "Tunable high gradient quadrupoles for a laser plasma acceleration based FEL", Nucl. Instrum. Meth. A, vol. 909, p. 290-293, 2018.

[37] C. Benabderrahmane, M. E. Couprie, F. Forest, O. Cosson, "Adjustable magnetic multipole", patent URL https://patentscope.wipo.int/search/en/detail.jsf?docId=WO2016034490. Europe : PCT/EP2015/069649 of 27/08/2015, WOBL14SSOQUA / CA (2016).

[38] C. Benabderrahmane et al. "Development of a 2 m Pr$_2$Fe$_{14}$B cryogenic permanent magnet undulator at SOLEIL", Jour. of Physics: Conf. Series, vol. 425, no. 3, p. 032019, 2013.

[39] M. E. Couprie et al. "Cryogenic undulators", Advances in X-ray Free-Electron Lasers Instrumentation III, vol. 9512, no. 3, p. 951204, 2015.

[40] M. Valléau et al., "Development of cryogenic undulators with PrFeB magnets at SOLEIL", AIP Conference Proceedings, vol. 1741, no. 1, p. 020024, 2016.

[41] M. Valléau et al., "Development and operation of a Pr$_2$Fe$_{14}$B based cryogenic permanent magnet undulator for a high spatial resolution x-ray beam line", Phys. Rev. Acc. Beams, vol. 20, no. 3, p. 033201, 2017.

[42] M. Labat, M. El Ajjouri, N. Hubert, T. André, A. Loulergue, M. E. Couprie "Electron and photon diagnostics for plasma acceleration-based FELs", Journal of synchrotron radiation, vol. 25, no. 1, p. 59–67, 2018.

[43] M. Labat, M. Khojoyan, N. Hubert, T. André, A. Loulergue, M. E. Couprie "Electron and photon diagnostics for plasma acceleration-based FELs", Journal of synchrotron radiation, vol. 25, no. 1, p. 59–67, 2018.

[44] T. André et al. "Control of laser plasma accelerated electrons for light sources", Nature Communications, vol. 9, pp. 1334, 2018.

[45] D. Oumbarek-Espinos et al. "COXINEL transport of laser plasma accelerated electrons", Plasma Physics and Controlled Fusion, vol. 62, no. 3, p. 034001, 2020.

[46] D. Oumbarek-Espinos et al. "Skew Quadrupole Effect of Laser Plasma Electron Beam Transport", Applied Sciences, vol. 9, no. 12, pp. 2447, 2019.

[47] P. Elleaume, H. Onuki. "Wiggler and their Applications", Taylor and Francis, London, 2003.

[48] O. Chubar, P. Elleaume, "Accurate and efficient computation of synchrotron radiation in the near field region", Proc. of the EPAC'98 Conference, 1998.

[49] L. Giannessi et al. Superradiant cascade in a seeded free-electron laser. Physical Review Letters vol. 110, pp. 044801, 2013.

[50] G. Lambert et al. Injection of harmonics generated in gas in a free-electron laser providing intense and coherent extreme-ultraviolet light. Nature physics vol. 4, pp. 296, 2008.

[51] T. Tanikawa et al. Nonlinear harmonic generation in a free-electron laser seeded with high harmonic radiation. EPL (Europhysics Physics) vol. 94, pp. 34001, 2011.

[52] M. N. Polyanskiy, "Refractive index database", https://refractiveindex.info.