CRITICAL BEHAVIOUR OF 3D SYSTEMS WITH
LONG-RANGE CORRELATED QUENCHED DEFECTS

V.V. Prudnikov and A.A. Fedorenko
Dept. of Theoretical Physics, Omsk State University
55a, Pr. Mira, 644077, Omsk, Russia
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Abstract

A field-theoretic description of the critical behaviour of systems with quenched defects obeying a power law correlations $\sim |x|^{-a}$ for large separations $x$ is given. Directly for three-dimensional systems and different values of correlation parameter $2 \leq a \leq 3$ a renormalization analysis of scaling function in the two-loop approximation is carried out, and the fixed points corresponding to stability of the various types of critical behaviour are identified. The obtained results essentially differ from results evaluated by double $\varepsilon, \delta$ - expansion. The critical exponents in the two-loop approximation are calculated with the use of the Pade-Borel summation technique.

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In last years, much effort has been devoted to investigation of the critical behaviour of solids containing quenched defects. In most papers considerations have been restricted to the case of point defects with small concentrations so that the defects and corresponding random fields have been assumed to be Gaussian distributed and $\delta$-correlated.

For the first time in work of Weinrib and Halperin (WH) [1] have been offered the model of the critical behaviour of disordered system in which the correlation function of the random local transition temperature $g(x - y) = \langle \langle T_c(x) T_c(y) \rangle \rangle - \langle \langle T_c(x) \rangle \rangle^2$ falls off with distance as a power law $\sim |x - y|^{-a}$. It was shown that for $a \geq d$ long-range correlations are irrelevant and the usual short-range Harris criterion $2 - d\nu_o = \alpha_o > 0$ of influence of $\delta$-correlated point defects is realized, where $d$ is the spatial dimension, $\nu_o$ and $\alpha_o$ are the correlation-length and the specific-heat exponents of the pure system. For $a < d$ it was established the extended criterion $2 - a\nu_o > 0$ of disorder influence on the critical behaviour. As a result, a wider class of disordered systems, but not only three-dimensional Ising model with $\delta$-correlated point defects, can be characterized by new type of critical behaviour. So, for $a < d$ it was discovered a new long-range (LR) disorder stable fixed point (FP) of the renormalization group recursion relations for systems with number of components of the order parameter $m \geq 2$. The critical exponents were calculated in the one-loop approximation using a double expansion in $\varepsilon = 4 - d \ll 1$ and $\delta = 4 - a \ll 1$. In the case $m = 1$ the accidental degeneracy of the recursion relations in the one-loop approximation did not permit to find LR disorder stable FP, but it was predicted for $\delta > \delta_c = 2(6\varepsilon/53)^{1/2}$ a change in critical behaviour of the model from short-range (SR) to LR correlation type. Korzhenevskii, Luzhkov and Schirmacher [3] have proved the existence of the LR disorder stable FP for the one-component WH model and found the characteristics of this type of critical behaviour. Also they have considered very interesting model of the critical behaviour of crystals with LR correlations caused by point defects with degenerate internal degrees of freedom [3, 4].

The models with LR correlated quenched defects present both doubtless theoretical interest from possibility of prediction a new types of the critical behaviour in disordered systems and experimental interest from possibility of realization RL correlated defects in the orientational glasses [3] and disordered solids containing defects of fractal-like type [3]. However, numerous investigations of pure and disordered systems performed with the use of the field-theoretic approach show that the predictions made in the one-loop approximation, especially on the basis of the $\varepsilon$-expansion, can differ strongly from the real critical behaviour [6-9]. Therefore, the map of regions with the various types of
critical behaviour received for WH model on the basis of $\varepsilon, \delta$ - expansion \[1\] (figure 1(a)) may be not corresponding to the critical behaviour of the three-dimensional WH model for different values of $m$ and $a$. In this case the results for the models with LR correlated defects received with use of $\varepsilon, \delta$ - expansion \[1,3,4,10-12\] must be corrected. To shed light on this question and to determine more accurately the dependence of the critical behaviour on the number of components of the order parameter $m$ and the values of correlation parameter $a$, we have constructed a field-theoretical description of the three-dimensional WH model in the two-loop approximation for the values of $a$ in the range $2 \leq a \leq 3$.

The effective Hamiltonian of WH model after using the replica trick is given by

$$H_{\text{eff}} = \sum_{a=1}^{n} \int d^d x \left[ \frac{1}{2} (r_0 \phi_a^2 + (\nabla \phi_a)^2) + \frac{u_0}{4!} (\phi_a^2)^2 \right] - \sum_{\alpha, \beta} \int d^d x d^d y g(x - y) \phi_\alpha^2(x) \phi_\beta^2(y) \quad (1)$$

where $\phi_a^2 = \sum_{i=1}^{m} \phi_{i\alpha}^2$. $\phi_{i\alpha}$ is $(n \times m)$-component order parameter. The properties of the original disordered system are obtained in the replica number limit $n \to 0$. The Fourier transformation of the interaction vertex $g(x) \sim x^{-a}$ gives $g(k) = v_0 + w_0 k^{a-d}$ for small $k$. $g(k)$ must be positive definite, therefore if $a > d$, then the $w$ term is irrelevant, $v_0 \geq 0$ and $H_{\text{eff}} (1)$ corresponds to model with SR - correlated defects, while if $a < d$, then the $w$ term is dominant at small $k$ and $w_0 \geq 0$.

As is known, in the field-theoretic approach \[13\] the asymptotic critical behaviour of systems in the fluctuation region are determined by the Callan-Symanzik renormalization-group equation for the vertex parts of the irreducible Green’s functions. To calculate the $\beta$ functions and the critical exponents as functions of the renormalized interaction vertices $u$, $v$ and $w$ (scaling $\gamma$ functions) appearing in the renormalization-group equation, we used the standard method based on the Feynman diagram technique and the renormalization procedure \[14\]. The three types of interactions can be represented graphically as in figure 2(a). When we considered a diagrammatic representation of two-point vertex function $\Gamma^{(2)}$, three types of four-point vertex functions $\Gamma_i^{(4)}$ and two-point with the $\phi^2$ insertion vertex function $\Gamma^{(1,2)}$ in the two-loop approximation the diagrams were integrated numerically in $d = 3$ and with values of parameter $a$ determining momentum dependence of the $w$ interaction in the range $2 \leq a \leq 3$ with changes through the step $\Delta a = 0.01$. Unlike the works using $\varepsilon, \delta$ - expansion we took into consideration the graphs of the form (figure 2(b)), contributions of which are increased when the values $a$ are removed from $a = 3$.

As a result, we obtained the $\beta$ and $\gamma$ functions in the two-loop approximation in the form of the expansion series in renormalized vertices $u$, $v$ and $w$. Because of impossibility
in short notes to present the coefficients of these series for different values of \(a\) we give
here the obtained \(\beta\) and \(\gamma\) functions only for \(a = 2\) (the case with \(a = 2\) corresponds to
system of straight lines of impurities or straight dislocation lines of random orientation
in a sample):

\[
\beta_u(u, v, w) = -u + u^2 - \frac{3}{2} uv - 1.901416uw - \frac{4(41m+190)}{27(m+8)^2} u^3 + \frac{2(25m+131)}{27(m+8)} u^2 v - \\
- \frac{185}{216} uv^2 + \frac{(1.230378m + 6.713002)}{m+8} u^2 w - 0.312654uw^2 - 1.193479uvw, \\
\beta_v(u, v, w) = v + v^2 + \frac{3}{2} w^2 + 1.901416vw - \frac{2(m+2)}{(m+8)} uw + \frac{95}{216} v^3 + 0.488229w^3 - \\
- \frac{50(m+2)}{27(m+8)} uv^2 - 1.974883\frac{(m+2)}{(m+8)^2} uw^2 + \frac{92(m+2)}{27(m+8)^2} u^2 v + 0.806375vw^2 + \\
+ 0.839125v^2 w - 1.939086\frac{(m+2)}{(m+8)} uvw, \\
\beta_w(u, v, w) = 2w + 0.628176u^2 + \frac{1}{2} vw - \frac{2(m+2)}{(m+8)} uw - 0.1528w^3 + \frac{92(m+2)}{27(m+8)^2} u^2 w + \\
+ \frac{23}{216} v^2 w + 0.090516\frac{(m+2)}{(m+8)} uv^2 - 0.022629vw^2 - \frac{23(m+2)}{27(m+8)^2} uvw, \\
\gamma_\phi(u, v, w) = 0.004222w + \frac{8(m+2)}{27(m+8)^2} u^2 + \frac{1}{108} v^2 + 0.056893w^2 - \frac{2(m+2)}{27(m+8)} uw - \\
- 0.315829\frac{(m+2)}{(m+8)} uv + 0.078956vw, \\
\gamma_\phi^2(u, v, w) = -\frac{m+2}{2(m+8)} u + \frac{1}{4} v + 0.31831w + \frac{2(m+2)}{(m+8)^2} u^2 + \frac{1}{16} v^2 - 0.019507w^2 - \\
- \frac{(m+2)}{2(m+8)} uw - 0.270565\frac{(m+2)}{(m+8)} uv + 0.067641vw.
\]

The series (2) are normalized by a standard change of variables \[7, 8\] \(u \to 6u/(m + 8)J, \)
\(v \to v/32J, w \to w/32J\), so that the coefficients of the terms \(u, u^2\) and \(v, v^2\) in \(\beta_u\) and \(\beta_v\)
become 1 in modulus, where \(J = \int dq/(q^2 + 1)^{\frac{3}{2}}\) is the one-loop integral.

The nature of the critical behaviour is determined by the existence of a stable FP
satisfying the system of equations

\[
\beta_i(u^*, v^*, w^*) = 0 \quad (i = 1, 2, 3).
\]

It is well known, that perturbation series are asymptotically convergent, and the vertices
describing the interaction of the order parameter fluctuations in the fluctuating region
\(r \to 0\) are large enough so that expressions (2) can’t be used directly. For this reason, to
extract the required physical information from the obtained expressions, we employed the
Pade-Borel approximation of summation of asymptotically convergent series extended
to the multiparameter case \[4, 15\]. We used the [2/1] approximant to calculate the \(\beta\)
functions in the two-loop approximation.

However, the analysis of the series coefficients for \(\beta_w\) function has shown that the
summation of this series is fairly poor, which resulted in absence of FP with \(w^* \neq 0\), for
example, in the case \(m = 1\) for \(a < 2.93\), in the case \(m = 2\) for \(a < 2.67\) and so on.
Dorogovtsev has found the symmetry of the scaling function for WH model in relation to the transformation \((u, v, w) \to (u, v, v + w)\) which gives the possibility to investigate the problem of FP existence with \(w^* \neq 0\) in the variables \((u, v, v + w)\). In this case our investigations have shown the existence of FP with \(w^* \neq 0\) in the whole region where the parameter \(a\) changes.

We have found three types of FP’s in the physical region of parameter space \(u^*, v^*, v^* + w^* \geq 0\) for different values of \(m\) and \(a\). The type I corresponds to FP of pure system \((u^* \neq 0, v^*, w^* = 0)\), the type II is SR-disorder FP \((u^*, v^* \neq 0, w^* = 0)\) and the type III corresponds to LR-disorder FP’s \((u^*, v^*, w^* \neq 0)\). The type of the critical behaviour of this disordered system for each value of \(m\) and \(a\) is determined by stability of corresponding FP. The requirement that the fixed point be stable reduces to the condition that the eigenvalues of the matrix

\[
B_{i,j} = \frac{\partial \beta_i(u_1^*, u_2^*, u_3^*)}{\partial u_j} \tag{4}
\]
lie in the right-hand complex half plane.

Values of the stable FP’s obtained for the most interesting values of the number of order-parameter components \(m\) and \(2 \leq a \leq 3\) are presented in table 1. As one can see from this table, for Ising model \((m = 1)\) the LR-disorder FP is stable for values of \(a\) in the whole investigated range. The additional calculations for \(3 < a < 4\) have shown that only FP II is stable in this range. For \(a = 3\) FP values for vertices \(u\) and \(g(k)\) are equal \(u^* = 2.38338, \ g^* = v^* + w^* = 0.55164\) and correspond to SR-disordered Ising model FP, although \(w^* \neq 0\). Like that, for \(m = 1\) and \(a = 3\) the LR disorder is marginal, and the critical behaviour of WH model, as of SR-disordered Ising model, is characterized by the same critical exponents (table 2). The critical behaviour of the XY-model \((m = 2)\) is determined by the LR-disorder FP for \(a \leq 2.96\) and the SR-disorder FP for \(a > 2.96\). The Heisenberg model \((m = 3)\) is characterized by the change of the critical behaviour types from the LR-disorder type (III) for \(a \leq 2.85\) to the pure type (I) for \(a > 2.85\). Figure 1(b) shows regions of the various types of critical behaviour of the WH model, which we obtained in the two-loop approximation. The large change in the picture indicates that the correspondence between WH results and our calculations in the two-loop approximation is weak.

However, the results, which we received for disordered XY-model, must be corrected. We think that in the higher field-theory orders of approximation \(k\) the critical behaviour of XY-model will be determined by the FP of pure type (I) for \(a_c^{(k)} < a\), but not by the
SR-disorder FP (II), obtaining in the two-loop order. Here, $a_c^{(k)}$ is marginal value for $a$ in $k$-th order of approximation, for which disorder is irrelevant ($a_c^{(6)} \simeq 2/\nu_o = 2.99$ with $\nu_o = 0.669$ [11] for $m = 2$). The two facts indicate this, such as the weak stability of the SR-disorder FP revealed for $2.96 < a < 4$ and that the $a_c^{(2)} = 3$ for $m_c = 2.0114$. In the higher orders of approximation the marginal value of $m_c$ can be found with the use of the Harris criterion $d\nu_o(m_c) - 2 = 0$, and such as $\nu_o = 0.669$ [11] for $m = 2$, then $m_c < 2$. Therefore, we think that the corrected picture of the regions of various types of the critical behaviour of the model with LR-correlated defects will be represented by figure 1(c).

Finally, we have calculated the static critical exponents for the WH model (table 2), received from the resummed by the generalized Pade-Borel method $\gamma$ functions in the corresponding stable FP’s $\eta = \gamma_\phi(u^*, v^*, w^*)$, $\nu = [2 + \gamma_\phi^2(u^*, v^*, w^*) - \gamma_\phi(u^*, v^*, w^*)]^{-1}$.

Also, we have found by the method of the work [8] the dynamic scaling function $\gamma_\lambda$ and calculated the values of the dynamic exponent $z = 2 + \gamma_\lambda(u^*, v^*, w^*)$ on the basis of the resummed $\gamma_\lambda$ function (table 2). As example, for $a = 2$ the received $\gamma_\lambda$ function is given by

$$\gamma_\lambda(u, v, w) = \frac{1}{4}v + 0.314088w + 0.226777\left(\frac{m+2}{m+8}\right)^2u^2 + \frac{23}{432}v^2 - 0.0764w^2 - 0.092593\left(\frac{m+2}{m+8}\right)uw + 0.123604\left(\frac{m+2}{m+8}\right)uw - 0.011315vw.$$ (5)

The comparison of the exponent $\nu$ values and ratio $2/a$ from table 2 shows the violation of supposed in [1] on the basis of some heuristic arguments as exact the relation $\nu = 2/a$. The revealed difference is caused by the use in our work of more accurate field-theoretic description in the higher order of approximation for three-dimensional system directly together with methods of series summation. Also, these distinctions can be explained by the application for calculations of the concrete numerical values of parameter $a$ and the taking into consideration the graphs of the form (figure 2(b)), thrown away when $\varepsilon, \delta$ - expansion is used, but contributions of which are increased when the values $a$ are removed from $a = 3$. Of course, there are errors of the present consideration determined by the accuracy of series summation for the $\beta$ and $\gamma$ functions. However, the comparison of exponents values for SR-disorder Ising model, calculated with the use Pade-Borel method in [1] and [3] in the two-loop and four-loop approximations accordingly, shows that their differences not more than 0.02. In the same time, in our work $\nu - 2/a$ depends on the values of $a$ and $m$ and has the value 0.284, as example, for $a = 2$ and $m = 1$, that is considerably larger.

In closing, we hope that the features of the critical behaviour of the WH model revealed
in our paper will stimulate the organization of experimental works in real disordered systems with long-range correlated defects like the orientational glasses and solids with defects of fractal-like type. Also, the computational methods can be applied for simulating disordered systems with straight lines of impurities of random orientation in a sample \( a = 2 \). The received values of exponents can be used for explanation of the results of computer simulation of the three-dimensional disordered Ising model \([17]\) at impurity concentrations between the threshold of impurity percolation and the spin-percolation threshold, in which the fractal-like behaviour of impurity extended structures and the competition between impurity-percolating and spin-percolating clusters are possible.

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Figure and table captions

Figure 1:

Regions of the various types of critical behaviour, which have been determined (a) in [1] on the basis of the double $\varepsilon, \delta$ - expansion; (b) in the present paper with use of the field theoretic description in two-loop approximation for three-dimensional WH model; (c) in the present paper with taking into consideration of the higher orders of approximation.

Figure 2:

Graphs () that correspond to vertices $u, v$ and $w$; (b) that are taken into consideration in addition as compared with works, using $\varepsilon, \delta$ - expansion, ■ corresponds to vertices $u, v$ and $w$.

Table 1:

Stable fixed points of the 3D WH model from two-loop expansions.

Table 2:

Critical exponents of the 3D WH model from two-loop expansions.
Table 1

| a   | n = 1     | n = 2     | n = 3     |
|-----|-----------|-----------|-----------|
|     | u         | v         | w + v     | u         | v         | w + v     | u         | v         | w + v     |
|     | w         |           |           | w         |           |           | w         |           |           |
|     |           |           |           |           |           |           |           |           |           |
| 3.1 | 2.38338   | 0.55164   | 0.55164   | 1.56469   | 0.00416   | 0.00416   | 1.52097   | 0.00000   | 0.00000   |
| 3.0 | 2.38338   | 0.22293   | 0.55164   | 1.56469   | 0.00416   | 0.00416   | 1.52097   | 0.00000   | 0.00000   |
| 2.9 | 2.59804   | 0.31890   | 0.68114   | 2.09001   | 0.11386   | 0.40038   | 1.52097   | 0.00000   | 0.00000   |
| 2.8 | 2.77927   | 0.40153   | 0.78299   | 2.17677   | 0.13536   | 0.44359   | 1.95770   | 0.08298   | 0.34550   |
| 2.7 | 2.94031   | 0.47487   | 0.86757   | 2.26778   | 0.15923   | 0.48612   | 2.01746   | 0.09346   | 0.37004   |
| 2.6 | 3.08645   | 0.54084   | 0.93916   | 2.36058   | 0.18457   | 0.52633   | 2.08699   | 0.10922   | 0.40005   |
| 2.5 | 3.21983   | 0.60035   | 0.99972   | 2.49643   | 0.23442   | 0.59651   | 2.15585   | 0.12535   | 0.42628   |
| 2.4 | 3.34078   | 0.65374   | 1.04998   | 2.61818   | 0.28094   | 0.65334   | 2.22047   | 0.14074   | 0.44651   |
| 2.3 | 3.44813   | 0.70082   | 1.08980   | 2.72520   | 0.32344   | 0.69760   | 2.30801   | 0.16910   | 0.48302   |
| 2.2 | 3.53899   | 0.74092   | 1.11825   | 2.81501   | 0.36115   | 0.72909   | 2.39298   | 0.20079   | 0.51696   |
| 2.1 | 3.60814   | 0.77263   | 1.13340   | 2.88305   | 0.39293   | 0.74672   | 2.45869   | 0.22877   | 0.53759   |
| 2.0 | 3.64687   | 0.79347   | 1.13189   | 2.92206   | 0.41710   | 0.74843   | 2.49945   | 0.25161   | 0.54364   |
| $a$ | $2/a$ | $n = 1$ | $n = 2$ | $n = 3$ |
|-----|------|--------|--------|--------|
| 3.1 | 0.0327 0.6715 2.1712 | 0.0288 0.6642 2.0000 | 0.0283 0.6960 2.0217 |
| 3.0 | 0.6667 | 0.0327 0.6715 2.1712 | 0.0288 0.6642 2.0000 | 0.0283 0.6960 2.0217 |
| 2.9 | 0.6897 | 0.0304 0.6813 2.2120 | 0.0248 0.7141 2.1315 | 0.0283 0.6960 2.0217 |
| 2.8 | 0.7143 | 0.0270 0.6889 2.2486 | 0.0212 0.7190 2.1510 | 0.0179 0.7600 2.1128 |
| 2.7 | 0.7407 | 0.0227 0.6950 2.2837 | 0.0166 0.7240 2.1736 | 0.0137 0.7632 2.1269 |
| 2.6 | 0.7692 | 0.0176 0.7002 2.3184 | 0.0112 0.7692 2.1988 | 0.0084 0.7682 2.1443 |
| 2.5 | 0.8000 | 0.0118 0.7046 2.3532 | 0.0035 0.7378 2.2338 | 0.0025 0.7727 2.1633 |
| 2.4 | 0.8333 | 0.0055 0.7083 2.3879 | -0.0050 0.7452 2.2684 | -0.0040 0.7763 2.1827 |
| 2.3 | 0.8696 | -0.0012 0.7114 2.4215 | -0.0138 0.7513 2.3013 | -0.0125 0.7835 2.2078 |
| 2.2 | 0.9091 | -0.0081 0.7137 2.4524 | -0.0226 0.7558 2.3301 | -0.0218 0.7905 2.2315 |
| 2.1 | 0.9524 | -0.0147 0.7151 2.4780 | -0.0307 0.7588 2.3522 | -0.0303 0.7952 2.2514 |
| 2.0 | 1.0000 | -0.0205 0.7155 2.4949 | -0.0371 0.7599 2.3649 | -0.0370 0.7975 2.2644 |
\[ \delta = 4 - a \]
