Orbital Disturbance Analysis due to the Lunar Gravitational Potential and Deviation Minimization through the Trajectory Control in Closed Loop

L D Gonçalves¹, E M Rocco¹ and R V de Moraes²

¹ Instituto Nacional de Pesquisas Espaciais, INPE, São José dos Campos, Brazil
² Universidade Federal de São Paulo, UNIFESP, São José dos Campos, Brazil

E-mail: lianadgon@gmail.com, evandro_mr@yahoo.com.br, rodolpho.vilhena@gmail.com

Abstract. A study evaluating the influence due to the lunar gravitational potential, modeled by spherical harmonics, on the gravity acceleration is accomplished according to the model presented in Konopliv (2001). This model provides the components x, y and z for the gravity acceleration at each moment of time along the artificial satellite orbit and it enables to consider the spherical harmonic degree and order up to 100. Through a comparison between the gravity acceleration from a central field and the gravity acceleration provided by Konopliv’s model, it is obtained the disturbing velocity increment applied to the vehicle. Then, through the inverse problem, the Keplerian elements of perturbed orbit of the satellite are calculated allowing the orbital motion analysis. Transfer maneuvers and orbital correction of lunar satellites are simulated considering the disturbance due to non-uniform gravitational potential of the Moon, utilizing continuous thrust and trajectory control in closed loop. The simulations are performed using the Spacecraft Trajectory Simulator-STRS, Rocco (2008), which evaluate the behavior of the orbital elements, fuel consumption and thrust applied to the satellite over the time.

1. Introduction

If the existence of disturbing forces were ignored, the orbital motion would be a conic, set in a fixed plane, with constant size and eccentricity. However, the existence of such forces tends to cause variations in the elements that characterize the orbit of an artificial satellite. In some cases this variations should be corrected to enable the mission accomplishment.

In order to study the perturbations due to non-sphericity of the lunar gravitational field it is used the LP100K model, so that an analysis of the influence of the degree and order of the harmonics in the artificial lunar satellite orbit is done. These disturbance effects are inserted into the Spacecraft Trajectory Simulator (STRS) in order to control the trajectory and minimize the deviations. The correction of the errors in the orbit is made by the STRS using a continuous propulsion system controlled in closed loop.
2. Lunar Gravitational Potential
The moon’s gravitational potential is expressed by the coefficients of normalized spherical harmonics, given by Equation (1) (Konopliv, 2001; Kuga, 2011):

\[ U(r, \lambda, \phi) = \frac{\mu}{r} + \frac{\mu}{r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{a_e}{r} \right)^n \left( \tilde{c}_{nm} \cos m\lambda + \tilde{s}_{nm} \sin m\lambda \right) \tilde{P}_{nm} \left( \sin \phi \right) \]  

(1)

where \( n \) is the degree, \( m \) is the order, \( \mu \) is the gravitational constant and \( r \) is the lunar equatorial radius. \( \tilde{P}_{nm} \) are the fully normalized associated Legendre polynomials. \( a_e \) is the reference radius Moon, \( \phi \) is the latitude, and \( \lambda \) is the longitude.

3. The Model LP100K
The lunar gravitational field was determined using data from some previous lunar missions. One of the most important missions was the Lunar Prospector (1998-1999). The LP was the third mission in NASA’s exploration program, called Discovery, and provided the first measurement of the moon gravitational field. The information about the gravity field comes from the long-term effect observed in the satellite orbit (Konopliv, 2001).

The model presented in Konopliv’s called GRAVITYSPHERICALHARMONIC is a representation of the spherical harmonics due to planetary gravity, based on the gravitational potential of the planet, given by the Equation (1).

The output calculated by the model includes the values of gravity in meters per squared second, on the axes x, y and z. From the values of the gravity acceleration it is possible to obtain the state variables and hence the orbital elements that characterize the satellite orbit.

Using the GRAVITYSPHERICALHARMONIC model it was created the Gravity_Moon subroutine, used for the simulations of the artificial satellites orbital motion around the Moon’s surface.

4. Study of oblateness and equatorial ellipticity effects on the lunar orbit of an artificial satellite
The Figure 1 shows the value obtained for the resulting of gravity acceleration on a satellite for each value of degree and order from 1 up to 100, at an altitude around 250 km.

We observe, by the Figure 1 that the value of the gravity acceleration on the artificial satellite tends to stabilize at a value close to 1.2250 m/s², when considering values of degree and order bigger than 15. However, we can also reach this approximate value using the degree and order 2.

It is important to adopt the highest possible value for degree and order, since by the use of many terms of the spherical harmonics we can represent the imperfections of the bodies format in a more accurate way. However, for a first analysis, the value 2 for degree and order could be adopted.

Figura 1. Gravity acceleration due spherical harmonics
5. Results

In this section we present the results of two different simulations, both performed during 86400 s, considering the terms due to non-homogeneity of the lunar gravitational field, and the value 2 to degree and order. In both cases, the continuous propulsion system are trigged when the simulation time reaches 2000 s and is turned off when the semi-major axis reaches the value of 4000 km.

In the first simulation the satellite leaves a low lunar orbit to reaches a high orbit, using continuous tangential thrust with magnitude of 2N, as seen in Figure 2. In the second simulation it is considered the application of higher thrust (20 N), applied over an arc of 5 degrees around the periapse, as seen in Figure 3. In the simulations 1 and 2 were considered the effects of disturbance and the action of the thrusters simultaneously, whose initial conditions considered were: semi-major axis 1800000 m; eccentricity 0.001; inclination 45º; right ascension of the ascending node 20º; argument the periapse 100º; mean anomaly 1º.

Figure 2. Trajectory of the satellite in simulation 1

Figure 3. Trajectory of the satellite in simulation 2

The Figure 4 presents the case where only the correction of the trajectory is considered to illustrate the ability of the control system to deal with the effect of orbital perturbation. In this case, the aim of the control system is minimize the effects of the perturbations acting on the satellite. In this simulation only the effects of lunar oblateness and equatorial ellipticity were considered until degree 2. From Figure 4 we can notice that the force applied by the propulsion system acts toward to correct the effects caused by the disturbing force. The figure shows the results for x axis, but a similar behavior is obtained for y and z axes. Therefore, it was verified that the control system is able to deal with the disturbance effects on the lunar satellite when considering the effects caused by the non-homogeneity of the lunar gravitational field.

Figure 4 - Control signal and disturbance signal on the satellite (x axis)
The following results obtained in the simulations 1 and 2 will be exposed to the study of the behavior of the orbital elements, propellant mass, thrust applied on the satellite, altitude reached and the disturbance acting on the satellite along the trajectory.

The Figures 5 and 6 show the behavior of the semi-major axis in the two simulations. The Figure 5 shows the variation of the semi-major axis during the orbital maneuver. In the Figure 6 we can see that each propulsive arc produces a step, more exactly, each application of the arc causes a sudden increase of the semi-major. It is also observed that the application of each arc causes a deviation in actual trajectory of the satellite, characterized by a difference observed at the beginning of each step in Figure 6, when the propellant system is turned on. However, the control system operates to maintain the actual path close to the reference.

In the Figures 7 and 8, we can verify that throughout the maneuver, the eccentricity presents small variations due to the applied thrust and the disturbance of the lunar gravitational potential. We can note that the eccentricity always tends to increase. However, in the Figure 7 we see that this value oscillates. The Figure 8 shows that application of the propulsive arcs always causes the increase of the eccentricity, but the eccentricity remains constant between the applications of the propulsive arcs.

The altitude of the satellite in the simulations is shown in the Figures 9 and 10. The oscillations in the graphs are justified because the satellite's altitude varies during each orbit. When the satellite is in an apoapsis, it is at a slightly higher orbit, and when the satellite is in periapsis, it is at a slightly lower orbit. We observe a considerable fluctuation in the result found in the second simulation since a thrust is applied on the satellite every time that the passage through periapsis occurs.
The Figures 11 and 12 show the disturbing force applied in the lunar satellite during the simulations. We can note that the intensity of the disturbing force decreases with the time, since the semi-major axis is increasing along the time, in other words, the distant from the lunar surface is increasing, and, therefore the disturbing due to the non-uniform distribution of mass of the Moon is becoming less relevant.

In the Figures 13 and 14 we can observe thrust applied during the simulations. It is possible to verify the thrust force applied in the three axes, and the control system acts separately on each axis. Note in the Figure 13 that the operation is finished when, according to the Figure 5, the semi-major axis reaches the value of 4000 km. At this point the propellant is turned off and the thrust applied tends to zero. From the Figure 14 is also possible to realize that with the passage of time the distance between two peaks or two valleys of the generated wave increases. This occurs because the semi-major axis increases during the simulation so the period of the orbit also increases.
From the Figures 15 and 16 we can analyze the fuel consumption in the simulations. We can note in the Figure 15 that at time 2000 s the propulsion system is turned on, and at the instant of 83000 s is turned off, (semi-major axis reaches 4000 km) thus the fuel consumption tends to stabilize. In the Figure 16 we can realize that each application of propulsive arc imply a significant fuel consumption, which tends to stabilize until the application of the next arc, however the consumption do not ceases to grow between arcs because the control system must act to deal with the perturbative effects that do not cease between arcs.

Figure 15. Mass of propellant expended during simulation 1

Figure 16. Mass of propellant expended during simulation 2

6. Conclusions
The results showed that the Spacecraft Trajectory Simulator, developed to analyze space missions using a closed loop control system and correct the trajectory by the application of continuous thrust is able to minimize the deviations in the path of the spacecraft when considering perturbations in the orbit due to the lunar gravitational potential of the Moon.

We can observe that the deviations in state variables values were always small, in other words, the control system was able to reduce the error in the state variables through the action of thrusters.

The Figures 11 and 12 showed that the disturbance on an artificial satellite due to the non-uniform distribution of mass of the Moon is not stable, requiring intense performance of the control system to mitigate deviations in the trajectory.

This study results are consistent with the results presented in Konopliv (2001), showing the correlation between the lunar gravitational acceleration and topography and the variation of the gravity acceleration due to non-uniform moon mass distribution. The vehicle orbital elements oscillation magnitude are in accordance with the gravity acceleration variation for the model presented Konopliv.

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