WHEN IS THE DECONFINEMENT PHASE TRANSITION UNIVERSAL?

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Pure Yang-Mills theory has a finite-temperature phase transition, separating the confined and deconfined bulk phases. Svetitsky and Yaffe conjectured that if this phase transition is of second order, it belongs to the universality class of transitions for particular scalar field theories in one lower dimension. We examine Yang-Mills theory with the symplectic gauge groups Sp(N). We find new evidence supporting the Svetitsky-Yaffe conjecture and make our own conjecture as to which gauge theories have a universal second order deconfinement phase transition.

1. INTRODUCTION

Yang-Mills theories with gauge group G have a finite-temperature phase transition, separating the confined phase of colorless glueballs from the deconfined gluon plasma phase. The transition is signalled by the spontaneous breaking of a global symmetry related to H, the center of G. The Polyakov loop is the order parameter for the transition, transforming as \( \Phi'(x) = z \Phi(x), z \in H \) under the global center transformation. Its expectation value is \( \langle \Phi \rangle = \exp(-\beta F_q) \), where \( F_q \) is the free energy of a static quark in the gluon background, \( \beta = 1/T \) is the time extent and \( T \) is the temperature. In the confined phase, there are no isolated quarks and \( F_q \to \infty \) in the infinite volume limit, giving \( \langle \Phi \rangle = 0 \). In the deconfined phase, \( F_q \) is finite and \( \langle \Phi \rangle \neq 0 \), spontaneously breaking the global center symmetry.

Svetitsky and Yaffe conjectured that if Yang-Mills theory with gauge group G has a second order deconfinement transition, with the correlation

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length $\xi \to \infty$, it should have universal properties belonging to the universality class of $H$-symmetric scalar field theories in one lower dimension$^4$. When $\xi \gg \beta$, the system is no longer sensitive to the finite time extent and dimensional reduction occurs. The universality class is determined by quantities such as the critical exponents, e.g. how the correlation length diverges as the critical temperature is approached, $\xi \propto (T - T_c)^{-\nu}$. Note, however, that the conjecture does not state that the deconfinement transition must be second order.

Most studies have looked at 4-d and 3-d $SU(N)$ Yang-Mills theories. The center of $SU(N)$ is $\mathbb{Z}(N)$, the $N$ roots of unity. We first consider 4-d theories. With $SU(2)$ as the gauge group, the theory has a second order deconfinement transition$^5$, belonging to the universality class of 3-d $\mathbb{Z}(2)$-symmetric scalar field theory, i.e. the Ising universality class$^6$. This fully supports the Svetitsky-Yaffe conjecture. For $SU(3)$, the transition is weakly first order with a large but finite correlation length $\xi$ and no universal properties$^7$. Studies have shown that for $SU(N)$ theories with $N = 4, 6, 8$, the deconfinement transitions are first order, with the strength increasing for larger $N$, again without any universal properties$^8$. Interestingly, 3-d $\mathbb{Z}(N)$-symmetric scalar field theory for $N \geq 5$ is in the universality class of the 3-d $U(1)$-symmetric XY model$^9$. The gauge theories simply don’t make use of this universality class. There is a richer structure in 3 dimensions. For $SU(N)$ with $N = 2, 3, 4$, the deconfinement transitions are second order, belonging to the 2-d $\mathbb{Z}(N)$-symmetric universality classes with $N = 2, 3, 4$ respectively, again supporting the Svetitsky-Yaffe conjecture$^{10}$.

2. $Sp(N)$ GAUGE THEORY

We look at Yang-Mills theories with the symplectic groups $Sp(N)$. These groups have the property that $Sp(N) \subset SU(2N)$ and $U \in Sp(N)$ satisfies the constraint

$$U^* = JUJ^\dagger,$$

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow U = \begin{pmatrix} W & X \\ -X^* & W^* \end{pmatrix},$$

(1)

where $W$ and $X$ are complex $N \times N$ matrices. Since $U$ and $U^*$ are related by a unitary transformation $J \in Sp(N)$, the $2N$-dimensional fundamental representation of $Sp(N)$ is pseudo-real and charge conjugation is a global gauge transformation. These properties are familiar from $SU(2)$. Writing $U = \exp(iH)$, the Hermitian matrix $H$ satisfies

$$H^* = -JHJ^\dagger = JHJ \Rightarrow H = \begin{pmatrix} A & B \\ B^* & -A^* \end{pmatrix},$$

(2)
where the $N \times N$ complex matrices $A$ and $B$ satisfy $A = A^\dagger$ and $B = B^T$. The $N^2$ and $N(N + 1)$ degrees of freedom of $A$ and $B$ respectively mean that $Sp(N)$ has $N^2 + N(N + 1) = (2N + 1)N$ generators. $Sp(N)$ has rank $N$. There are the special equivalent cases $Sp(1) = SU(2)$ and $Sp(2) \simeq SO(5)$. Most interestingly, unlike $SU(N)$, the center of $Sp(N)$ is $\mathbb{Z}(2)$ for all $N$. This allows us to disentangle the size of the group from the center and see what effect this has on the deconfinement transition. According to the Svetitsky-Yaffe conjecture, a second order transition should belong to the Ising universality class.

The lattice formulation of $Sp(N)$ Yang-Mills theory is straightforward. We use the Wilson action in our simulations. As $SU(2) \subset Sp(N)$, we can update the $Sp(N)$ gauge links using the standard heatbath\textsuperscript{11} and overrelaxation\textsuperscript{12} algorithms to update the various $SU(2)$ subgroups, in the same way as done for $SU(N)$ gauge theory. We find that there is no bulk phase transition between strong and weak coupling. Further details will be presented in a forthcoming paper\textsuperscript{13}.

We first consider 4-d $Sp(2)$ gauge theory, where one might expect the deconfinement transition to be second order, just as for $SU(2)$. However, we find the transition to be first order, even with $\mathbb{Z}(2)$ as the center. In Fig. 1, we plot tunneling between coexisting confined ($\Phi = 0$) and deconfined ($\Phi \neq 0$) phases at the critical temperature, as well as the probability distributions of $\Phi$ close to $T_c$. Coexistence of the phases is a clear signal for a first order transition. Measurements of the Polyakov loop suscepti-
bility, the specific and latent heats also show a clear first order transition. Similarly, we find that 4-d $Sp(3)$ Yang-Mills theory also has a first order deconfinement transition. Going from $Sp(1) = SU(2)$ to $Sp(2)$, the phase transition changes from second to first order, even though the center of the group is the same, indicating that the size of the group is more important.

In 3-d, we find a richer structure. Exactly like $Sp(1)$, we find that $Sp(2)$ Yang-Mills theory has a second order deconfinement transition. Measurements for various temperatures and volumes can be mapped onto one universal curve when rescaled using the critical exponents of the 2-d Ising universality class, as shown in Fig. 2(a). This is new evidence supporting the Svetitsky-Yaffe conjecture. However, for $Sp(3)$, the transition is weakly first order, as we see in Fig. 2(b), where one has to go to large volumes to distinguish the coexisting phases. Again, we find that the transition switches from second to first order as we increase the size of the group, even though the center is unchanged.

3. SUMMARY

From our work and other studies, we conjecture that only for $Sp(1) = SU(2) \simeq SO(3)$ is there a universal second order deconfinement phase transition in 4 dimensions. In 3-d, we find that $Sp(2)$ gauge theory has a second order deconfinement transition belonging to the 2-d Ising universality class, which is new evidence supporting the Svetitsky-Yaffe conjecture. We expect that there are no other second order transitions with other gauge
groups\textsuperscript{13,14}. In both 4-d and 3-d $Sp(N)$ gauge theory, we find that the deconfinement transition changes from second to first order as we increase $N$, even though the center of the group is always $\mathbb{Z}(2)$. The order of the transition seems to be dictated by the size of the group, not the center. This is natural, as the number of glueballs in the confined phase is group-independent, whereas the number of deconfined gluons increases with the group size, leading to a larger mismatch in the number of degrees of freedom at the critical temperature as $N$ increases.

ACKNOWLEDGMENTS

This work has been supported under grant DOE-FG03-97ER40546, by the Schweizerischer Nationalfond and the European Community’s Human Potential Program HPRN-CT-2000-00145 Hadrons/Lattice QCD.

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