STATISTICAL STUDIES OF GIANT PULSE EMISSION FROM THE CRAB PULSAR

WALID A. MAJID, CHARLES J. NAUDET, STEPHEN T. LOWE, AND THOMAS B. H. KUIPPER

Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109, USA; walid.a.majid@jpl.nasa.gov

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ABSTRACT

We have observed the Crab pulsar with the Deep Space Network Goldstone 70 m antenna at 1664 MHz during three observing epochs for a total of 4 hr. Our data analysis has detected more than 2500 giant pulses, with flux densities ranging from 0.1 kJy to 150 kJy and pulse widths from 125 ns (limited by our bandwidth) to as long as 100 μs, with median power amplitudes and widths of 1 kJy and 2 μs, respectively. The most energetic pulses in our sample have energy fluxes of approximately 100 kJy μs. We have used this large sample to investigate a number of giant pulse emission properties in the Crab pulsar, including correlations among pulse flux density, width, energy flux, phase, and time of arrival. We present a consistent accounting of the probability distributions and threshold cuts in order to reduce pulse-width biases. The excellent sensitivity obtained has allowed us to probe further into the population of giant pulses. We find that a significant portion, no less than 50%, of the overall pulsed energy flux at our observing frequency is emitted in the form of giant pulses.

Key words: pulsars: general – pulsars: individual (Crab pulsar)

1. INTRODUCTION

While giant pulses (GPs) have been reliably detected from seven pulsars (Knight 2006 and references therein) their properties have only been well studied from two objects: the Crab pulsar, PSR B0531+21 (Sallmen et al. 1999; Cordes et al. 2004; Hankins et al. 2003; Hankins & Eilek 2007), and PSR B1937+21 (Cognard et al. 1996; Soglasnov et al. 2004). In particular, the Crab pulsar has long been known as a GP emitter. Its initial discovery (Staelin & Reifenstein 1968) and a number of subsequent studies have reported remarkable properties of GPs from the Crab. GPs are a broadband phenomenon (e.g., Sallmen et al. 1999) exhibiting the Galaxy’s largest observed brightness temperature (Cordes et al. 2004), and a subset of them are, in effect, superpositions of extremely narrow nanosecond pulses (Hankins et al. 2003; Hankins & Eilek 2007). It has been shown that energy flux emission from GPs exhibits a power-law distribution \( N(E > E_0) \sim E_0^{-\alpha} \), with \( \alpha \) in the range of 1.5–2.5 (Popov & Stappers 2007), in contrast to normal radio pulse emission being Gaussian distributed (Hesse & Wielebinski 1974).

In this paper we analyze high time-resolution GP observations from the Crab pulsar conducted with Goldstone’s Deep Space Network (DSN) 70 m radio telescope at 1.7 GHz (L band). With our large data sample, we carried out a number of statistical studies of GP properties. We present radio observations and a description of the recording strategies and setup in Section 2. In Section 3 we present the data reduction and analysis scheme. In Section 4 we provide a description of the statistical studies, followed by a discussion of the implications of the analysis. Finally, conclusions are given in Section 5.

2. OBSERVATIONS AND DATA SET

As part of an effort to revitalize the L-band system on the DSN 70 m antenna in Goldstone, a number of pulsar observations were carried out over four epochs in early to mid 2008. The Crab pulsar was observed during two of these epochs for a total observing time of almost 4 hr. For these observations, the front-end electronics down-converted the L-band radio frequency (RF) signal to intermediate frequency (IF; via an intermediate S-band upconversion) for recording, using a pair of VLBI Science Receivers (VSRs). The VSRs filter and sample the analog IF signal, then digitally form sub-channels for recording to disk. The data used for this study consist of 32 MHz recorded bandpass in the range 1648–1680 MHz, recorded as four adjacent 8 MHz channels with two-bit I and two-bit Q samples for 8050 continuous seconds, followed 3 minutes later by two 16 MHz channels with one-bit I/Q sampling for 1830 continuous seconds. A number of hardware problems were encountered during the experiment’s first hour, including antenna pointing and recorder sampling errors. The data corresponding to these problems were removed.

The configuration of the channels is summarized in Table 1. The system temperature during Crab pulsar observations may be dominated by the emission from the Crab Nebula, one of the brightest radio sources in the sky. This is certainly true in our case, where the nebula is not resolved by the antenna beam. The nebula is an extended source with a \(~5.5\) diameter and a flux density parameterized as \( S_N \sim 955\nu^{0.27} \) Jy (Bietenholz et al. 1997), where \( \nu \) is the observing frequency in GHz. At an observing frequency of \( 1.7 \) GHz, the nebular flux density is \( S_N \sim 830 \) Jy. Since the DSN’s 70 m antenna at \( L \) band has a half-power width of \(~8.8\) the nebula is not resolved so the system noise must correctly account for the nebular noise contribution. The nebular contribution \( S_N \) is combined with the contribution from the system temperature in absence of the Crab Nebula \( (S_{sys}) \) to obtain the total system temperature:

\[
S_{sys} = S_N + S_{sys}.
\]

On–off measurements of a standard calibrator 3C48 and bright radio pulsar PSR B0329+54 were carried out prior to observing the Crab. These observations yield a system temperature, \( S_{sys} \), of 35 K (10% error). With a nominal gain \( G \) of the 70 m antenna of \(~1.0 \) KJy\(^{-1}\), the measured system temperature translates into a system equivalent flux density \( (S_{sys} = T_{sys}/G) \) of 35 Jy. Adding the estimated nebular contribution of \(~830 \) Jy yields a total system equivalent flux density \( S_{sys} = 865 \) Jy. This value agrees well with our estimates obtained from on–off measurements of the Crab, where we measured the system temperature while alternately pointing at

| Table 1 | Configuration of Channels |
|--------|--------------------------|
| Channel 1 | 8 MHz I/Q, 2-bit I/Q, 3250 continuous seconds |
| Channel 2 | 16 MHz I/Q, 1000 continuous seconds |
| Channel 3 | 8 MHz I/Q, 2-bit I/Q, 700 continuous seconds |
| Channel 4 | 8 MHz I/Q, 2-bit I/Q, 1250 continuous seconds |
the Crab, and 1° away. We estimate the error for our overall flux density calibration scale to be less than \(\sim 20\%\). Our detection sensitivity for single pulses is determined using the radiometer equation

\[
\Delta S = \eta S_{\text{sys}} \sqrt{\frac{1}{\Delta f \Delta t}},
\]

where \(\eta\) is the digitization loss factor, \(\Delta f = 32\) MHz is the recorded bandwidth, and \(\Delta t = 0.1\) \(\mu\)s is the minimum sample time. With two-bit digitization we have \(\eta = 1.3\) and we obtain a 1\(\sigma\) detection threshold of \(\Delta S_{\text{min}} \sim 560\) Jy. For a 7\(\sigma\) detection threshold the minimum single-bin detectable pulse amplitude is expected to be 3.9 kJy. For comparison, Table 2 lists the parameters of previous Crab GP studies with the current work.

3. DATA REDUCTION

Our 4 hr of Crab pulsar data, consisting of 540 gigabytes, were recorded to disk and shipped to JPL for post-processing. The average and rms RF voltage was computed for each second of data as a quick assessment of data quality.

3.1. Coherent Dedispersion and Normalization

The data were coherently dedispersed using the nominal dispersion measure (DM) value\(^1\) of 56.7671 pc cm\(^{-3}\), following the dispersion removal technique developed by Hankins & Rickett (1975). Each IF channel was dedispersed separately by performing a fast Fourier transform (FFT) on a full second of complex (I and Q) samples, plus the fractional second of data required to fill the FFT arrays with a power-of-two number of samples. For the two-bit, 8 MHz data, the approximately 1 Hz frequency bins were counter-rotated in phase to remove the frequency-dependent dispersive delay, then inverse-transformed back to the time domain. The one-bit, 16 MHz data were processed similarly except that the final inverse FFT was performed on each half of the channel bandpass separately, effectively splitting each 16 MHz channel into two 8 MHz channels. In this way, all data could be processed further in a similar manner. Dedispersing a full second of data kept delay smearing over each \(\sim 1\) Hz frequency bin much less than a sample, ensuring that the dedispersed data maintained its full time resolution.

After dedispersion, we form each sample’s normalized power, \(P_i\), averaged over all four frequency channels, where \(i\) is the sample number. This was done by independently normalizing each complex component (I and Q) from all four frequency channels so that all eight quantities had zero mean and unit standard deviation. These eight quantities were squared and

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1. The DM value was obtained from the Jodrell Bank Crab pulsar monthly ephemeris: http://www.jb.man.ac.uk/~pulsar/crab.html.
averaged to form our power time series using

$$P_t = \frac{1}{8} \sum_{k=1}^{4} \left( I_k^2 + Q_k^2 \right),$$

where $k$ runs over the four frequency channels. Assuming each I and Q component is normalized and Gaussian distributed, in the no-signal limit $P_t$ will have a reduced $\chi^2$ distribution with eight degrees of freedom.

### 3.2. Pulse Detection

Our pulse-detection algorithm begins with the 8 MHz $P_t$ time series whose noise component is nominally modeled by the reduced $\chi^2$ distribution with eight degrees of freedom. In general, with $v$ degrees of freedom, the reduced $\chi^2$ probability density function, parameterized by $x$, is given by

$$\text{pdf}(x) = \frac{v(x)^{v/2} e^{-vx/2}}{2^{v/2} \Gamma(v/2)},$$

where the $\Gamma$ function for integer and half-integer arguments is defined as

$$\Gamma(n) = (n - 1)!, \quad \Gamma\left(\frac{1}{2}\right) = \frac{(n - 2)!!}{2^{(n-1)/2}}.$$

The red data points in Figure 1(a) show the measured $P_t$ distribution for 30 minutes of data. The data samples were taken during the Crab pulsar off-pulse regions of the pulse phase. The black solid curve in Figure 1(a) is the corresponding theoretical expectation given by Equation (3) with $v = 8$. Excellent agreement is seen in over eight decades.

In the time domain, our matched filter algorithm averages $N$ temporally consecutive $P_t$ power samples and compares this with a threshold parameter designed to strongly reject noise while passing GPs with high efficiency. The filter used in this analysis begins with the $P_t$ power series corresponding to $N = 1$. The $N = 2$ time series is formed by averaging each consecutive pair (without overlap) of the $N = 1$ series, resulting in half the number of power samples. This doubling procedure is repeated up to the creation of the $N = 2048$ power series, which corresponds to 256 $\mu$s wide time-averaged bins. For each

**Figure 1.** (a) The background pulse $\chi^2$ distribution for $v$ of 8, 16, and 32; the red, green, and blue data points, respectively. (b) The background pulses plus GPs $\chi^2$. The corresponding theoretical $\chi^2$ distributions are the solid black curves. (c) The noise rate survival distribution for $v$ of 8, 16, and 32; the red, green, and blue curves, respectively. The black horizontal dashed line, at a noise rate of 1 Hz, intersects the three $\chi^2$ distributions at the corresponding power cut points. Three black vertical dashed power cut lines have been extended through all three figures.
power series, summing over the $N$ samples results in

$$P_i^N = \frac{1}{N} \sum_{m=0}^{N-1} P_{i+m},$$

(5)

where $i$ is in the range $(0, N, 2N, \ldots)$. Because each $P_i$ averages squares of eight Gaussian distributed series $P_i^N$ will have its noise component distributed according to Equation (3) with $v = 8N$, assuming the data samples $P_i$ are truly independent.2 Figure 1(a) shows the off-pulse power, $P_i^N$, for $N = 1$ (red points $v = 8$ in Equation (3)), $N = 2$ (green curve, $v = 16$), and $N = 4$ (blue curve, $v = 32$). The black curves give the theoretical distributions from Equation (3) in each case.

In order to prevent cut-induced biases in the GP width distribution, proper threshold cuts should result in the same number of background noise events per second, on average, regardless of $N$. We define $R$ to be the rate of noise fluctuations expected to pass a threshold cut $\xi(N)$, in units of events per second. This is equivalent to picking a constant event confidence level independent of pulse width. The noise rate $R$, also the false alarm rate, is equal to the probability that an averaged power computed with $N$ samples exceeds $\xi$, times the number of trials in 1 s:

$$R(\xi) = S(\xi) \times \frac{f_s}{N},$$

(6)

where $S(\xi)$ is 1 minus the cumulative probability distribution corresponding to Equation (3), also known as a survival function, and $f_s$ is the number of samples per second. Figure 1(c) shows the total noise rate as a function of $P$ for the $N = 1$ (red points), $N = 2$ (green curve, $v = 16$), and $N = 4$ (blue curve, $v = 32$) cases.

In the limit of large $\xi$, reasonable given our desire to strongly cut noise, this becomes

$$R = \frac{f_s}{N} \frac{(4N\xi)^{4N-1}e^{-4N\xi}}{(4N-1)!}.$$  

(7)

The threshold values are computed from this equation iteratively using

$$\xi(N) = \frac{1}{4N} [(4N-1) \log(\xi) - \log(R + C)].$$

(8)

where

$$C = (4N-1)\log(4N) + \log\left(\frac{f_s}{N}\right) - \log((4N-1)!).$$

(9)

For each averaged power $P_i^N$ in every power series, Equations (7) and (8) are used to compute the effective noise rate $R$ corresponding to the measured $P_i^N$. This effective noise rate estimates the likelihood that a given averaged power value is a noise fluctuation. The dotted horizontal line in Figure 1(c) is the 1 Hz rate cut, $R$-cut = 1.0 Hz, and the corresponding power cuts $\xi(N = 1), \xi(N = 2), \text{and } \xi(N = 4)$ are shown as the three vertical dotted lines in Figures 1(a) and (b). As described below, we reduce background noise by cutting on $R$, using the $\xi(N)$, in order to guarantee that the background rate is independent of $N$.

This consistent accounting of the probability distributions and equivalent threshold cuts to keep background rates independent of pulse width ensures balanced detection efficiencies even for extremely large width GPs.

Figure 1(b) plots the $\chi^2$ distributions for all the data, so as to include the Crab’s GPs: the long tail of events with large $\chi^2$ is clearly seen. With each detection, both $N$ and the probability of such a pulse being a noise fluctuation, expressed as $R$ in Equation (7), are stored.

This pulse-detection algorithm typically detects each pulse candidate a large number of times, so a method is needed to transform these multiple detections into individual pulse candidates and to estimate their properties such as time, total power, and width. The algorithm used in this analysis was to flag any sample that participates in a detection, regardless of $N$, then call any contiguous interval of flagged samples a pulse candidate. A number of parameters were computed for each pulse candidate. The peak power, the pulse integrated power, the time at peak power, the power-weighted mean pulse time, and the power variance about the mean time were all computed from the 8 MHz channel-summed powers. From the multiple detections associated with each pulse candidate the lowest-probability $R$, corresponding to the highest significance, was chosen as the GP. The associated $N$, power, and time were then saved.

Figures 2(a) and (b) show scatter plots of pulse phase and time for all GPs found in our data set, for both a relatively soft ($R = 0.02$ Hz) and hard cut ($R = 0.0006$ Hz), respectively.3 The main pulse (MP), with phase near 0.3, and the interpulse (IP), near 0.7, are clearly evident. The loose R-cut (0.02 Hz) implies more white-noise contamination due to the lower lower $\chi^2$ thresholds. Similarly, the tight R-cut (0.0006 Hz) reduces noise contamination on higher $\chi^2$, which is equivalent to a higher power signal-to-noise ($S/N$) threshold. The projections of these scatter plots onto the time axis, the pulse phase histograms, are presented in Figures 2(c) and (d) for the hard and soft cut, respectively. In the loose-cut case, over 1600 main GPs are seen on a flat background of 100 noise pulses. With tight cuts, over 1200 main pulse GPs are found with no background, and a clear peak of GPs is seen at the IP phase region. By defining an on-pulse phase range of (0.32, 0.35) for the MP phase, and (0.72, 0.75) for the IP phase, we can study the count rate and $S/N$ of the GPs as a function of $R$-cut.

If $S_T$ is defined as the number of counts within the on-pulse region and $S_N$ is the number of background counts estimated from the off-pulse phase region, the background-subtracted signal count is $S = S_T - S_N$. Assuming counting statistics, the $S/N$ is then

$$S/N = (S_T - S_N)/(S_T + S_N)^{1/2}.$$  

(10)

Figure 3 shows the total number of GPs as a function of $R$-cut, for both the main and IP regions. For the loosest cut processed ($R = 0.6$), over 2500 main phase GPs and 200 interphase GPs are observed.

Careful examination of the pulse-candidates in Figure 2(a) shows density variations during the experiment’s first and last hours. Our detection thresholds were adjusted to allow an approximately constant noise rate as a function of pulse width; however, Figure 2(a) shows that the noise rate is not constant in time. At the experiment’s start, there were a number of problems with the front-end sampler voltage thresholds, which

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2 As $N$ increases, a small sample-to-sample correlation was seen. This loss of true data independence was handled by making a small correction to the ideal $v$. For example, the $N = 4$ case, ideally $v = 32$ but $v = 31$ yields a better fit to the data.

3 The significance of these cut values becomes clear later in the paper.
Figure 2. Time phase scatter plots of the detected pulses for the loose and tight data sets. The main and interpulse giant pulses are seen as straight lines at phases of 0.337 and 0.737, respectively. The bottom histograms show the corresponding phase projection. The improvement in signal to noise is clearly seen with the tight cuts.

Figure 3. Number of background-subtracted giant pulses, for both the main pulse and interpulse, as a function of the noise cut, R-cut. As the R-cut increases (power amplitude cut decreases) the total number of giant pulses increases.

Figure 4. Giant pulse signal-to-noise ratio as a function of the noise cut, R-cut, for both the main pulse and interpulse GPs.

explained above, these data are somewhat less sensitive than the two-bit data. Efforts to reconcile the different sampling effects and resulting sensitivities, for example, by scaling the sample values or modifying the detection thresholds, significantly increases the complexity of the analysis. Thus, from this point forward the analysis is restricted to only those data taken from time 2400 to 8050 s, a total of 5650 s of data, as these data have the highest sensitivity and show an approximately constant sensitivity.

Figure 4 shows the S/N as a function of R-cut for both the main and IP pulses. It is seen that the maximum S/Ns for the MPs are at $R = 0.02$ and $R = 0.0006$, respectively. Lowering the R-cut (increasing the total $\chi^2$ cut) fewer GPs with higher S/N values are detected. In this paper we will refer to three cuts, a loose cut at $R = 0.02$, a tight cut at $R = 0.0006$, and the loosest cut at $R = 0.6$ resulting in the largest sample of MP GPs found in this analysis ($\approx 2500$). Yet even with the tight cut over 1200 MP GPs and over 100 IP GPs are detected both without any significant background contamination.

4. VARIOUS STATISTICS OF DETECTED GIANT PULSES

The tremendous amount of energy radiated by GPs is observed with a wide variety of pulse morphologies. One of the goals of GP research is to determine, as closely as possible, the originating pulse shape, its amplitude, and width. A wide variety of factors influence the shape of the pulse, such as dispersion, instrumental smearing, interstellar scintillation, and scattering due to turbulent media. These effects all tend to broaden a pulse, decreasing its true peak amplitude and increasing its width. In our data set, the amplitude is simply defined as the peak power, or rather the peak flux density, found in the optimum smoothed data set. Past attempts to characterize pulse widths for a large sample of GPs have been limited either to coarse sampling size or large dispersive smearing and small sample sizes (Lundgren et al. 1995; Popov & Stappers 2007; Bhat et al. 2008). Our L-band observations, employing coherent dedispersion on large bandwidth baseband channels, minimize the dispersive smearing, obtaining a 125 $\mu$s time resolution, and contain over 1200 GPs, which should allow more robust width measurements.
Figure 5. Giant pulse time series plots. Each is shown with its optimum smoothing width “N.” The widest pulse shown is pulse b with $N = 512$ and the two narrowest shown are pulses d and e with $N = 4$.

4.1. Pulse Amplitudes and Widths

Visual examination of many pulses, some of which are shown in Figures 5(a)–(f), reveals that variations in pulse morphology represent the dominant systematic error in pulse amplitude and width determinations. The six GP events shown are plotted using their optimum smoothing, with widths ranging from the narrowest (smoothing widths of $N = 4$) to the widest ($N = 512$), and with amplitudes ranging from 1 kJy to above 100 kJy. Although a large subset of GPs can be well fit to a Gaussian shape, an equally large sample of GPs show non-Gaussian shapes, as presented in Figures 5(c)–(f). Given a significant fraction of GPs having non-Gaussian shapes, fitting GPs with a Gaussian model yields poor $\chi^2$ fits, and thus poor amplitude and width estimates.

An alternative width definition is simply the optimum smoothing width $W_s = N\Delta t$ ($\Delta t$ is the intrinsic sample resolution) obtained from the GP detection algorithm. The amplitude and timing can then be defined using the bin with the greatest power amplitude. With these alternative definitions of
power amplitude and width, pulse morphology can be seen in Figure 6(a), where the scatter plot shows GP peak flux density versus width, for all the GPs passing the tight cut. A strong correlation between the observed peak flux density and smoothing width is observed, with higher peak flux density seen at lower smoothing widths. The diagonal dotted lines in Figure 6(a) show iso-energy flux contours of 1 kJy \( \mu s \), 10 kJy \( \mu s \), and 100 kJy \( \mu s \). A wide variation of shapes can be seen for the same observed energy flux. For example, a large pulse with \( E = 10 \) kJy \( \mu s \) can be very sharp, less than one \( \mu s \) in width with over 10 kJy peak flux density, or very broad, with widths over 10 \( \mu s \) but flux density less than 1 kJy. Although great care was taken to ensure that no pulse-width selection bias occurred, a comparison of the GP data with the iso-energy flux contours suggests that GPs with total energy fluxes of \( \sim 1\) kJy \( \mu s \) cannot be extracted above smoothing widths of 2 \( \mu s \).

Figures 6(b) and (c) show the corresponding amplitude and width projections. The peak flux density distribution in Figure 6(b) indicates that the median pulse flux density is \( \sim 1 \) kJy with a minimum observed peak flux density of \( \sim 0.1 \) kJy and a maximum of \( \sim 100 \) kJy. Above a peak flux density of 2 kJy, the histogram shows a power-law dependence out to 100 kJy. A power-law fit yields an exponent of \( \alpha = -2.2 \pm 0.1 \), very close to that found by Bhat et al. (2008), where at slightly lower frequencies of 1300 MHz and 1460 MHz, values of \( -2.3 \) and \( -2.2 \) were measured, respectively.

The distribution of the smoothing width, \( W_s \), is shown in Figure 6(c). The median \( W_s \) is 2 \( \mu s \) and the mean \( W_s \) is \( \sim 6 \) \( \mu s \) with a minimum of 0.2 \( \mu s \) with a long tail extending out to 100 \( \mu s \). This width range compared to previous Crab GP data sets (Bhat et al. 2008) is extended to pulses with ten-times-wider and two-times-narrower widths.

Although the smoothing width, \( W_s \), is a convenient definition of pulse width, it does not always appear to be optimum. Sometimes, as shown in Figure 5(b), a wide GP with \( N = 512 \) has almost all its energy within the one smoothed bin. Yet as Figures 5(c)–(f) illustrate, a large fraction of the time significant energy is spread over many bins at the optimum smoothing width. Following others (Bhat et al. 2008) we define an “effective pulse width,” \( W_{\text{std}} \), which is the standard deviation (std) for all “acceptable” bins in a GP, where acceptable bins are defined with a simple algorithm. Raw data with the highest

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**Figure 6.** Scatter plot of peak flux density and optimum smoothing width \( (W_s) \) along with the corresponding peak flux density (b) and width projection histograms (c) for the tight cut. The diagonal dotted lines in the scatter plot (a) are energy isotherms at \( E = 1 \) kJy \( \mu s \), 10 kJy \( \mu s \), and 100 kJy \( \mu s \).
time resolution are used, and data within $\sim 3N$ of the maximum power amplitude sample are selected. Only bins with power amplitudes greater than $2\sigma$ above the background power are included in the first estimation of the $W_{ad}$ and the background level. The final calculation sums over all bins within $\sim 3W_{ad}$ of the peak, again with bins having power amplitudes greater than $1\sigma$ above the background. This algorithm allows us to estimate the pulse standard deviation and higher order moments such as skewness and kurtosis. This procedure gives an estimate of the pulse width ($W_{ad}$) that, as expected, gives excellent agreement with the fitted Gaussian sigma and gives a reasonable estimation of the pulse width in the non-Gaussian cases. A comparison with pulse widths extracted by Bhat et. al., at 1470 MHz, can be seen in Figure 7. Due to the higher time resolution in this study, pulses with widths a factor of two narrower are detected. Perhaps the more significant difference with the earlier data of Bhat et al. is the increase in the sensitivity at very large pulse widths. Bhat et al. observed no pulses with widths greater than $10\,\mu s$, whereas the present data set has a significant number of pulses extending out to over $100\,\mu s$ with a median $W_{ad}$ of 2.35 $\mu s$ and a mean of 10.45 $\mu s$. We have parameterized this distribution with a good power-law fit, $N(W_{ad}) \propto W_{ad}^{-1.3}$, that extends from $\sim 1\,\mu s$ to almost $100\,\mu s$.

4.2. Pulse Energies

The measurement of GP parameters such as power amplitude, width, and energy flux is critical in constraining models of GP emission. The total pulse energy flux has the advantage, as compared to pulse width, of being a robust pulse estimation parameter, as it is relatively insensitive to pulse shape. Also, since the total pulse energy flux is a simple power sum over the pulse duration, the random background noise will tend to average down to zero. The GP energy flux distribution, for the tight-cut data set, is shown as black circles in Figure 8. Energy fluxes up to $\sim 100\,\text{kJy}\,\mu s$ are found, consistent with earlier studies (Bhat et al. 2008). The average observed GP energy flux for all the main GPs was $\sim 7\,\text{kJy}\,\mu s$ with a median energy flux of $\sim 5\,\text{kJy}\,\mu s$. Above $5\,\text{kJy}\,\mu s$ the data are well fit by a power law; the solid line in Figure 8 shows fit to the data with $\alpha = -2.57$. A very sharp energy flux threshold is observed at $\sim 3\,\text{kJy}\,\mu s$. The black diamonds in Figure 8 show the main GP energy flux distribution for the loosest cut examined. An additional $\sim 1400$ main GPs are found but all at lower energy flux as would be expected. The additional main GPs seem to follow the same power law as the data with tight cuts but the energy threshold is reduced to $\sim 1.5\,\text{kJy}\,\mu s$. No evidence is seen for a rollover or softening of the intrinsic power law. The turnover in Figure 8 is most likely due to the applied threshold cut.

Following a previous analysis (Knight 2006), the cumulative energy flux distribution, defined as the probability per second of a pulse having an energy flux $E$ greater than a value $E_0$, is used to compare the occurrence frequency of GPs between different experiments:

$$P(E > E_0) = kE^\alpha.$$  \hspace{1cm} (11)

Figure 9 shows the observed cumulative energy flux distribution for the tight data set (solid black circles) along with the...
data from Bhat et al. (open squares). The solid line is a power-law fit to the data in the energy range 5–100 kJy μs. Above the energy flux threshold, the data are well fit by a power law and no evidence is found for high-energy flux cutoffs. The best-fit power-law exponent is found to be $\alpha = -1.9 \pm 0.05$. The agreement with Bhat et al. is very good in the energy flux region of 10–20 kJy μs but significant differences in slope and absolute magnitude are seen at higher and lower pulse energy fluxes. This disagreement can be understood by a combination of differences in antenna sensitivity and pulse algorithm efficiency. As discussed earlier, for any fixed pulse energy a wide range of pulse amplitudes and widths is observed. A loss of pulse-detection efficiency at low and high widths may alter the true nature of the cumulative energy flux distribution.

Although earlier work by Bhat et al. (2008) only covered widths from 1 to 10 μs, others (Popov & Stappers 2007) found a dependence on pulse width, such that at narrow widths ($\sim 4$ μs), $\alpha = -1.7$ whereas for wider pulses (65 μs), $\alpha$ was found to be $-3.2$. To examine this dependence our tight data set was divided up into two subsets: a narrow ($W_p < 5$ μs) and a wide data set ($W_p > 5$ μs). In both cases, the slope of the cumulative energy flux distribution was estimated. The narrow data set yields a slope of $-1.7$ to $-2.0$, dependent on the energy flux range fit, which is very consistent with Bhat et al. whereas the wide data set yields a steeper slope of $\alpha = -2.4$ to $-3$, consistent with the work of Popov and Stappers.

It is interesting to compare the total energy flux emitted by the Crab pulsar as GPs with the overall pulse emission at this frequency. Using the background-free data set (tight cuts), we summed the energy flux of each candidate GP, obtaining an average pulse energy flux of 0.134 kJy μs per rotation over the span of the observation, which amounted to $\sim 168,000$ rotations. On the other hand, the mean pulse energy flux from the pulse profile containing all rotations is 0.26 kJy μs. This value is in excellent agreement with the quoted value in the ATNF catalog after extrapolating the mean flux density from 1400 MHz to 1665 MHz using their spectral index of $-3.1$. This suggests that a significant fraction, at least $\sim 50\%$, of the pulsar emission energy may be emitted as GPs. Another estimate for pulse energy flux obtained at a frequency of 1664 MHz, nearly identical to our observing frequency, suggests a mean energy flux of 0.125 kJy μs for the MP and 0.025 kJy μs for the IP at this frequency (Manchester 1971). The total energy flux amounting to 0.15 kJy μs is very close to the entire GP energy flux emission, further suggesting that a significantly large portion of the energy flux may be emitted as GPs.

As a further check we compared the pulse profile including all rotations with a second profile, where rotations containing an identified GP is removed from the profile. Figure 10 shows the two pulse profiles. The second profile (dashed curve) is obtained using a loose cut to identify GP candidates. The figure shows that GPs make up 54% of the overall pulse energy flux at our observing frequency. Also evident from the figure is how identical the two profiles are in terms of the pulse width. At the moment we do not have sufficient statistics to resolve and quantify the width of each profile and instead rely on a rough visual estimate. This is further indication that a similar significant reduction in energy flux is obtained assuming that the GP and MP widths are similar.

A large fraction of the emission energy flux at this frequency is already accounted for with GPs identified at our current sensitivity. If the power law continues to lower energies, further improvement in sensitivity will allow us to reach the intrinsic turnaround in the distribution necessary to keep the total GP emission energy below the total pulsed energy. In this case, there would be only one fundamental process responsible for the total pulsed emission in the Crab pulsar.

### 4.3. Pulse Asymmetry and Shape

Skewness can be used as a measure of GP asymmetry and is defined here to be

$$s = \frac{\sum W_i (t_i - t_p)^3}{\sum W_i} / \sigma^3,$$

where the weighted sum is over a narrow region of the pulse with respect to its peak position $t_p$. The weight, $W_i$, is the pulse amplitude level in the corresponding time bin, $\sigma$ is the standard deviation ($W_{std}$), and the pulse algorithm used is the same as that used for obtaining the pulse standard deviation. For a Gaussian-shaped pulse, skewness will have a value near zero. Pulses with long tails following the peak will have a positive skewness and those with tails before the peak will have negative values.

Figure 11 shows the skewness distribution for both the GPs and background pulses, solid and dashed histograms, respectively. Both distributions show average skewness that is very close to zero with very similar spread in the distribution. No statistically strong skewness dependence was found as a function of width, amplitude, or energy, and no differences were found between the IP and main GPs.

The next higher statistical moment, kurtosis, can also yield useful information on pulse shape and is defined as

$$k = \frac{\sum W_i (t_i - t_p)^4}{\sum W_i} / \sigma^4 - 3.$$

Kurtosis is zero for a Gaussian-shaped pulse, negative for pulses that are flatter than Gaussian, and positive for pulses more steeply peaked than Gaussian. To account for algorithm dependences and background noise, we have calculated skewness and kurtosis for both GPs and out-of-phase background pulse candidates.

Our kurtosis distribution is shown in Figure 12 for both GPs and background events. The background pulses are peaked near zero, consistent with the expectation for Gaussian-shaped
pulses. A positive kurtosis tail extending to about a value of 5 is observed. This most likely reflects the bias of the decimation portion of the matched filter detection algorithm, where smoothing tends to increase kurtosis. The Crab’s GPs are seen to be peaked at a kurtosis value of 0.6 with a very long tail extending to a value of 50, implying that they are more sharply peaked than pulses from the white-noise background. The reason for this can be seen by examining the dependence of mean kurtosis on the total GP energy flux shown in Figure 13. This plot shows that GP-averaged kurtosis increases linearly with the total energy flux. Although the background pulse candidates do not extend to high energies, it is clear that their behavior is quite different from the GPs’ upward trend.

4.4. Pulse Time of Arrival

The Crab pulsar has a well-defined period of ∼33 ms and the phase of the main and interpulse GPs are well regulated within the pulsar period. The phase of both the IP and MPs is seen in the projections of the scatter plot in Figure 2. The pulse phase residual, or time of arrival (TOA) residual, is the TOA of the GP peak with respect the start of the pulsar model cycle. For the Crab, at $L$-band frequency, both the main and interpulse GPs are found to have TOA residuals that fall within 1% of a cycle (∼330 μs). Figure 14 shows the histogram for the main and interpulse GPs, where the mean TOA has been set to zero for both. To remove possible confusion in the IP data set due to background pulse candidates, the tight-cut data set has been used. The MPs are found to have rms of 0.0028 ± 0.0001 cycles or 1° of phase corresponding to ∼90 μs, while the IP GPs were found to be wider with an rms of 0.0042 ± 0.0003 or 1:5 of phase corresponding to ∼140 μs.

Since it is expected that the intrinsic pulse phase jitter and smearing will be independent of absolute phase location, the 50% increase in TOA width is significant and may provide an important constraint in astrophysical pulsar models. Two earlier studies, both with smaller GP samples (Bhat et al. 2008; Cordes et al. 2004), found evidence for stronger pulses to have narrower phase windows where strength was defined as the peak pulse amplitude. Due to the strong correlation of peak flux density and
width, as shown in Figure 7(a), it is reasonable to believe that the narrow widths of the larger peak pulses may indeed result in better phase resolution. To examine this possibility the tight data set was divided into two subsets, a large and small peak flux density data set, and those with peak amplitude larger than 5 kJy and those smaller than 1 kJy. The widths of the TOA residuals for the two GP samples were then estimated to be $\sigma = 0.0027 \pm 0.0002$ and $\sigma = 0.0027 \pm 0.0001$ for the large and small peak samples, respectively. No significant difference is found, as the sample TOA residuals are seen to be statistically consistent to each other.

A plot of the MP energy flux versus phase is shown in Figure 15’s scatter plot. No correlation between phase and energy flux is evident. To examine this more quantitatively, the data were divided into three total energy flux classes: high-energy flux ($E \geq 200 \text{ kJy } \mu\text{s}$), medium energy flux ($100 \text{ kJy } \mu\text{s} \leq E \leq 200 \text{ kJy } \mu\text{s}$), and low-energy flux ($E \leq 100 \text{ kJy } \mu\text{s}$). The table below (Table 3) summarizes the TOA residual statistics of each data set. No tendency is seen for larger energetic pulses to originate in narrower phase windows.

The joint statistics of GP total energy flux with the interarrival time (IAT) of the next GP is shown in Figure 16. The IAT is simply the measured time between one GP and the next. The IAT for both the MP and IP GPs are plotted, where the IAT is converted to the number of Crab pulsar rotations ($\sim 33$ ms). The discrete nature of the IAT for pulses below 10 rotations is simply a reflection of the phase cut (0.30–0.35) for the MPs and (0.72–0.75) for the IPs. For the sensitivity achieved with our observations, with tight cuts, the average number of Crab rotations between GPs was found to be 127 with the largest gap between GPs at $\sim 1000$ rotations. With the loosest cut used, the average number of Crab rotations between GPs is 67. No observable dependence between the total GP energy flux and the arrival time of the next pulse was seen.

The smallest IAT of $\sim 0.4$ cycle was observed twice, these cases being an MP immediately preceded by an IP (I-M). No cases of IP immediately following the MP were found in the tight data set (M-I), corresponding to an IAT of $\sim 0.6$ cycle. To examine the independence between main and interpulses, we examined the consistency of the data with the assumed Poisson nature of the GP process. A Poisson process describes events which occur continuously and independently, and for this study would imply that the IATs are exponentially distributed with parameter $\lambda$ (mean = $1/\lambda$)

$$P(\tau) = \lambda e^{-\lambda \tau}.$$  \hspace{1cm} (14)

Figure 17 shows the distribution of MPs interarrival times along with the best fit to a Poisson model. The data appear to be consistent with a Poisson process, the best weighted fit yielding $\lambda = 0.223 \pm 0.007$ Hz with a $\chi^2 = 19$ for $v = 23$. This $\lambda$ value agrees well with the inverse of the mean IATs, $0.234 \pm .007$. Within statistics, it appears that the arrival times of Crab’s GPs are Poisson distributed, implying that they are memoryless, and

| $E(\text{kJy } \mu\text{s})$ | Sum | Mean         | St. Dev.   |
|--------------------------|-----|--------------|------------|
| $E \leq 100$             | 564 | $0.3368 \pm 0.0001$ | 0.0028     |
| $100 \leq E \leq 200$   | 571 | $0.3373 \pm 0.0001$ | 0.0028     |
| $E \geq 200$            | 106 | $0.3365 \pm 0.0003$ | 0.0029     |

**Table 3**

Giant Pulse TOA Residual Statistics
any given time interval is independent of what occurs before or after. In addition, both the IPs and the combined GP data sets show IAT distributions that are also consistent with a Poisson process. We conclude that IP GPs do not appear to be correlated with MPs to our level of examination.

5. SUMMARY AND CONCLUSIONS

We have examined the energy flux distribution, timing, and statistical properties of giant radio pulses from the Crab pulsar at 1700 MHz using DSN’s 70 m antenna at Goldstone, achieving a time resolution of 125 ns. Our pulse detection was based strictly on the underlying $\chi^2$ statistics of the data, where we attempted to keep the noise rate and event confidence levels constant and independent of pulse width. Our consistent accounting of the probability distributions and non-Gaussian effects, in order to keep background rates independent of pulse width, is an improvement over previous analyses.

The statistical analysis of GP population was carried out using three significance cuts: a loose cut ($R\text{-cut} = 0.0200$), where we obtained 1879 pulses, a tight cut ($R\text{-cut} = 0.0006$) yielding 1314 pulses, and the loosest cut studied ($R\text{-cut} = 0.6000$), which yielded over 2500 main GPs. With a large number of GPs we were able to study various statistical properties of GPs. We have confirmed the power-law nature of the peak pulse flux density distribution and have obtained a power-law slope of $-2.2$, consistent with (Bhat et al. 2008) results. We observed GPs with widths up to 100 $\mu$s, 10 times larger than the maximum found by Bhat et al. Furthermore, the distribution of pulse cumulative energy fluxes also follows a power-law distribution with a slope of $-2.57$, consistent with Bhat et al. (2008). We also looked for correlations between the TOA of individual GPs and concluded that the TOA of each event is consistent with Poisson statistics. Our pulse-detection sensitivity, high time resolution, and consistent statistical treatment of the data have allowed us to examine a weaker population of GPs, narrowing the gap with “normal” pulses from the Crab pulsar.

Our comparison of pulsed energy flux emission from GPs with the overall pulsed emission has forced us to claim that a significant portion, perhaps as large as $\sim 90\%$ but at least $50\%$, of the emission at this frequency is in the form of GPs. With further improvement in sensitivity and observing time, it should be possible to either confirm the turnaround seen in the energy distribution or push it only slightly lower before reaching the intrinsic turnaround in the distribution. Such a study could be carried out by a phased array instrument such as the Very Large Array (VLA), where the narrower beam will make it possible to lower the nebular contribution to the overall system temperature, providing better sensitivity to further probe the weaker population of GPs as well as the remaining normal pulses in the system.

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