Elliptic flow in high multiplicity proton-proton collisions at $\sqrt{s} = 14$ TeV as a signature of deconfinement and quantum energy density fluctuations.

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At LHC extreme values of energy density will be reached even for proton-proton collisions. Such values of energy density may be large enough to generate a collective motion in the products of the collision, therefore generating effects such as elliptic flow. Using ideal 3+1D hydrodynamical simulations, we show that elliptic flow can occur at least for top multiplicities p-p events at LHC and that the intensity of such effect is strongly related to quantum fluctuations in the initial proton energy distribution.

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Introduction

The application of hydrodynamical models to describe hadronic collisions have a long history, dating almost from the beginning of the studies of hadronic interactions. Since late ’80s these approaches have been revived to describe the collective features of the dynamics in heavy ion collisions at the big accelerators and investigate the properties of quark-gluon plasma (QGP). The main idea behind such approach is that from the analysis of collective flow in the dynamics, if any, we may infer the properties of the allegedly deconfined partonic matter generated in heavy ion collisions. For this, the thermalization of the partonic matter should occur in a relatively short time and space scales. Until today this approach was limited to heavy ions because only in these collisions a large space-time domain, in which the energy density is sufficiently high to lead to the deconfinement and thermalization, is expected to emerge. Proton-proton (p-p) data were used as reference to analyze these heavy ion data.

In p-p processes the system is considered too small and, so far, it was not expected to get any deconfinement nor thermalization domains. Thus, any differences in behavior between the p-p and heavy ions case may (should) be attributed to the dense medium effects.

As mentioned above, one of the most important signatures of the creation of such a medium is the manifestation of collective motions (flows) in the final state particles. The presence of any collective flow is a typical behaviour of a fluid matter. There are many ways to identify collective flows. They appear as an anisotropy in angular distribution of particles and are referred to as anisotropic flow. In experimental studies, anisotropic flows are usually analyzed in terms of the Fourier expansion of the azimuthal particle distribution

$$\frac{1}{2\pi N} \frac{dN}{d\phi} = 1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \ldots$$  \hspace{1cm} (1)

where usually $v_1$ is referred as direct flow and $v_2$ as elliptic flow.

It should be noted that these anisotropic parameters are not necessarily null even for free-streaming particles, but their dependence on quantities such as, for example, impact parameter and kinematic variables will reflect the collective features of the dynamics.

LHC accelerator, that is starting in Geneva, will reach extreme values of energy both for heavy ions and protons, the latter being accelerated up to $\sqrt{s} = 14$ TeV. It is possible that under these extreme conditions, also in proton-proton collisions the energy density will reach values high enough to allow partonic matter to survive in a deconfined state for a time long enough to thermalize and show some collective behaviour. In particular, when we consider quantum fluctuations in the incident states, such a situation may well occur and thus manifest in large multiplicity events. In this sense, collective features, if any, of the high multiplicity events may reveal interesting informations on the initial condition for high energy p-p collisions.

The goal of this work is to estimate the elliptic flow for top multiplicity proton-proton events in the framework of ideal hydrodynamics and investigate if it will be possible to perform such analysis at the LHC. If viscosity is present, it has effects on collective flow and to determine the viscosity of the QGP fluid is presently one of the important problems. However, these questions are still under investigation, and in the present work, we only consider the case of an ideal fluid to avoid additional ambiguities associated to viscous theories.
I. EOS AND INITIAL CONDITIONS

For simplicity, we use a factorized initial energy density profile into its longitudinal and transverse parts, as proposed by [3]. For the transverse part, we use an energy density profile as a simple superposition of those of colliding particles as is usually done for heavy ions collisions [8]. Thus we use the formula

\[ \epsilon_0(x, y, b) = k(\sqrt{\pi})T_A(x + \frac{b}{2}, y) \left[ 1 - (1 - \frac{\sigma T_B(x, y)}{B})^B \right] + k(\sqrt{\pi})T_B(x - \frac{b}{2}, y) \left[ 1 - (1 - \frac{\sigma T_A(x, y)}{A})^A \right], \]

(2)

where \( A \) and \( B \) are the number of nucleons, \( \sigma \) is the nucleon-nucleon cross section and \( k \) a collision energy dependent constant. In the case of proton-proton collisions all this is reduced to the simple expression

\[ \epsilon_{t=0}(x, y, b) = 2k\sigma T_A(x + \frac{b}{2}, y)T_B(x - \frac{b}{2}, y). \]

(3)

For the proton energy density \( T_A(B) \), we use a Gaussian distribution with a top value \( \epsilon_0 \) and width \( \sigma_p = 0.875 \text{ fm} \).

\[ T_A(x, y) = \epsilon_0 e^{-\frac{x^2+y^2}{2\sigma_p^2}}. \]

(4)

As the convolution of two Gaussian is still a Gaussian, our initial transverse energy density is

\[ \epsilon_{t=0}(x, y, b) = Ke^{-\frac{b^2}{2\sigma^2}}, \]

(5)

where all the constants \((k, \sigma, ...)\) are compacted together in the new constant \( K \). It must be noticed that the result of eq. (5) is completely symmetric in the transverse plane for any value of \( b \). We will discuss this point in more detail later in section [11].

For the longitudinal dimension, we proceed as in [8] using the equation:

\[ \epsilon_{long}(\eta_p) = e^{-\frac{\eta_p^2}{2\sigma_\eta^2}} (y_{beam} - \eta_p). \]

(6)

We are left with two free parameters, the constant \( K \) and the longitudinal width \( \sigma_\eta \). We can express the total entropy as a function of these parameters, and so relate them to the particle multiplicity, as shown in Fig. [11].

Remarkably, the results on the integrated value of \( v_2 \) are almost not affected by the choice of these parameters, provided that they keep the total entropy constant. Furthermore, it depends explicitly on the shape of the impact region, as is discussed later. Nonetheless, the momentum dependence of \( v_2(p_t) \) is strongly affected by those parameters (cf. sec. [11]).

We have chosen entropy values between 5400–5500 fm\(^{-3}\) in order to reproduce the top multiplicities expected at LHC (HIJING simulations) for such collisions. It is very important to focus on very high multiplicity events for two reasons. The first is that proton-proton collisions on average have very low multiplicity. At LHC energies for proton-proton collision we expect \( dN/dy \approx 6 \) while the top multiplicity can be larger then one thousand particles. To extract any information on “collectivity”, the events producing a large enough amount of particles are interesting. The second reasons is that in low multiplicity events also non-flow correlations may become strong to override the elliptic flow, even if it exists. Fixing the total multiplicity, we further determine the parameter \( \eta_p \) by the expected mid-rapidity multiplicity of particles. This choice leads to the values \( \eta_p = 1.3 \) and \( K = 1150 \text{ GeV} \). In order to test the momentum dependence of \( v_2 \) on the parameter choice, other two sets of parameters are used in the paper: the first one \((K = 370 \text{ GeV} \text{ and } \eta_p = 3.1)\) with very low top energy density but extremely broad distribution in the \( \eta \tau \) dimension, and the second one \((K = 3680 \text{ GeV} \text{ and } \eta_p = 0.6)\) with a very high initial energy density but almost confined in the transverse plane.

In ideal hydrodynamics the elliptic flow is proportional to the initial eccentricity of the impact region [10]. In our case, there is a big difference from heavy ion collisions. Due to the Gaussian shape of the proton energy densities, the total energy density profile eq. (4) is symmetric, so that at the classical level we would not expect elliptic flow at all. It has already been shown that quantum effects on energy distribution are not much effective on large systems, such as Au-Au collisions [11] but their importance grows as the size of the system is reduced, like for Cu-Cu collisions [12]. So for the very small systems expected in p-p collisions, quantum fluctuations should have a crucial effect on the event-by-event energy density distribution of the created matter. We try to mimic such quantum fluctuations by simply cutting the shape...
of the interaction region with an elliptic shape parameterized by eccentricity e. We thus suppose that the initial energy density for the hydrodynamical evolution as

\[
\varepsilon(r, \eta, e) = Ke^\frac{a^2}{2r^2} e^{\frac{b^2}{2}} \theta(y_{beam} - \eta) \theta \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right),
\]

with

\[
a = \sqrt{\frac{R}{2}(1 - e)},
\]
\[
b = \sqrt{\frac{R}{2}(1 + e)}.
\]

According to our hypothesis, the parameter e measures the effect of quantum fluctuation in the formation of the initial condition for ultra-high energy p-p collisions.

As for the equation of state (EoS), we use the one shown in Fig. 2 (solid line). This EoS is obtained by smoothly connecting the equations of state of the lattice QCD result in \[13\] and an hadron resonance gas near \( T_c = 200 \text{ MeV} \). The phase transition is a cross-over. To see the effect of the EoS on our results, we also test the EoS with a steeper connection between these two phases (dashed). After the initial entropy distribution is generated, the time evolution of the system is followed using the Smoothed Particle Hydrodynamic (SPH) formalism in hyperbolic coordinate system \[13\]. To calculate the elliptic flow, we perform the commonly used sudden freeze-out through the Cooper-Frye procedure: when a fluid element crosses the surface defined by the temperature \( T = T_f \) all interactions are assumed small enough so that all particles can be considered free. The freeze-out temperature is settled at the value \( T_f = 130 \text{ MeV} \). Tests with \( T_f = 120 \text{ MeV} \) shows that our results are almost insensitive to the changes in the value of \( T_f \).

A full 3+1D ideal fluid dynamics using our SPH code for p-p requires at least 30 000 total number of SPH particles with the kernel width \( h = 0.3 \text{ fm} \). With this condition, one collision event consumes ~8 hours of calculation with a desktop PC. The overall precision of calculation is monitored by the conservation of the total energy and momentum which are kept within the relative error of \( 10^{-2} \) during the whole time evolution.

![FIG. 2: (Color online) Behaviour of squared speed of sound and pressure over \( T^4 \) as functions of temperature for the two different equations of state.](image)

\[\text{II. RESULTS AND CONCLUSIONS}\]

In Fig. 3 we present the azimuthal distribution of thermal pions in the mid rapidity region \( y \in [-1, 1] \) for an initial eccentricity of 17%. The results are consistent with an elliptic flow of 2.9%, so this indicates that an elliptic flow effect could be strong enough to be experimentally measurable at LHC. In Fig. 4 we show the effect of the choice of the parameters on \( v_2(p_t) \) for different values of initial eccentricity. As expected, the elliptic flow effect gets stronger when most of the energy density is available in the transverse plane and the integrated value of \( v_2 \) is proportional to the initial eccentricity \[10\]. We also verify that the behavior of \( v_2 \) integrated over the whole rapidity domain is almost the same as that of the mid-rapidity region. If the eccentricity \( e \) has a quantum origin in p-p domain, we expect that the elliptic flow effect becomes stronger when most of the energy density is available in the transverse plane and the integrated value of \( v_2 \) is proportional to the initial eccentricity \[10\]. We also verify that the behavior of \( v_2 \) integrated over the whole rapidity domain is almost the same as that of the mid-rapidity region. If the eccentricity \( e \) has a quantum origin in p-p collisions, we expect big fluctuations for the average value of \( v_2 \) for different multiplicity bins in an experiment. Also, we should expect big fluctuations even in the same bin for different events, as the origin of the eccentricity is not geometrical but stochastic. We also expect the elliptic flow value to be always greater than 0, as any fluctuation in any collision contributes with a positive value of \( v_2 \). However, the collective flow can only become important for large multiplicity events. From our study, we get the elliptic flow of \( v_2 \approx 3\% \) if the induced eccentricity has average value of about 17%. We expect that, measurements of elliptic flow in top multiplicity events in p-p collisions may give an important clue for the quantum fluctuation in energy density distribution at each event.

The ALICE collaboration is developing several methods to estimate non-flow contribution to \( v_2 \) in p-p collisions \(13\). The most promising one seems to be the \( \eta \)-gap method, that consists in using the particles with pseudorapidity \( \eta > 3 \) to evaluate the reaction plane and those at \( |\eta| < 1 \) to evaluate \( v_2 \). Other methods are, for example, the standard event-plane method \(13\) or defining the reaction plane in the direction of the leading particle at high pseudorapidity. Once non-flow correlations are removed, \( v_2 \) can be measured. Unfortunately, there aren’t yet any available estimations of the precision such methods can reach in removing non-flow correlations, but
it is plausible that an effect of a few percent would be visible in ALICE.

As mentioned in the introduction, one thing that can affect the $v_2$ value is the presence of viscosity in the collective motion, that has been neglected in our calculations. High value of viscosity can strongly affect the value of the elliptic flow produced in proton-proton collisions, as suggested in \[17\].

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FIG. 4: (Color online) $v_2(p_t)$ for an initial eccentricity of 17% (up), 8.6% (center) and 26% (bottom) for the three different set of parameters

FIG. 5: (Color online) $v_2(p_t)$ for the two different EoS presented in fig. (2) for eccentricity 17%, $K=1150$ MeV and $\eta_g=1.3$.

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