QCD Factorization for Charmless Hadronic $B_s$ Decays Revisited

Hai-Yang Cheng,¹,² Chun-Khiang Chua³

¹ Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

² Physics Department, Brookhaven National Laboratory
Upton, New York 11973

³ Department of Physics, Chung Yuan Christian University
Chung-Li, Taiwan 320, Republic of China

Abstract

Branching fractions and $CP$-violating asymmetries of charmless $\bar{B}_s \to PP, VP, VV$ decays ($P$ and $V$ denoting pseudoscalar and vector mesons, respectively) are re-examined in the framework of QCD factorization (QCDF). We take into account subleading power corrections to the penguin annihilation topology and to color-suppressed tree amplitudes that are crucial for resolving the $CP$ puzzles and rate deficit problems with penguin-dominated two-body decays and color-suppressed tree-dominated $\pi^0\pi^0$ and $\rho^0\pi^0$ modes in the $B_{u,d}$ sector. Many of the $B_s \to h_1h_2$ decays can be related to $B_d \to h_1h_2$ ones via $U$-spin or SU(3) symmetry. Some useful model-independent relations can be derived and tested. Mixing-induced $CP$ asymmetries for many of the penguin-dominated decays are predicted to be very small in the standard model. They are sensitive to New Physics and offer rich possibilities of new discoveries. Measurements of direct $CP$-violating asymmetries can be used to discriminate QCDF from other competing approaches such as pQCD and soft-collinear effective theory.
I. INTRODUCTION

The phenomenology of nonleptonic two-body decays of $B$ mesons offers rich opportunities for our understanding of the underlying mechanism for hadronic weak decays and $CP$ violation. In the past decade, nearly 100 charmless decays of $B_{u,d}$ mesons have been observed at $B$ factories, BaBar and Belle, with a statistical significance of at least four standard deviations (for a review, see [1]). The CDF Collaboration has made unique contributions to the measurements of charmless hadronic $B_s$ decays. Recently, Belle has also started to study the weak decays of the $B_s$ meson.

Many of the $B_s \rightarrow h_1 h_2$ decays can be related to $B_d \rightarrow h'_1 h'_2$ ones via $U$-spin or SU(3) symmetry. Some useful model-independent relations can be derived and tested. For example, direct $CP$ asymmetries of $B_s \rightarrow K^+\pi^-$ and $B_d \rightarrow K^-\pi^+$ are related to each other by $U$-spin symmetry. Therefore, the use of flavor symmetry will be helpful to control the hadronic uncertainties in $B_s \rightarrow h_1 h_2$ decay amplitudes.

Analogous to the neutral $B_d$ system, $CP$ violation in $B_s$ decays also occurs through the interference of decay amplitudes with and without $B_s - \bar{B}_s$ mixing. It is known that the mixing-induced $CP$ violation of $B_d \rightarrow J/\psi K$ is governed by $\sin 2\beta$. Likewise, the decay $B_s \rightarrow J/\psi \phi$ is the benchmark in the $B_s$ system with mixing-induced $CP$ asymmetry characterized by $\sin 2\beta_s$. In the standard model (SM), the phase $\beta_s$ is very small, of order 1 degree. Consequently, $B_s \rightarrow J/\psi \phi$ and several charmless penguin-dominated $B_s$ decays e.g. $B_s \rightarrow K^{(*)0}\bar{K}^{(*)0}, \eta(1')\eta(1'), \phi\phi$ are the ideal places to search for New Physics as $CP$ violation from physics beyond the SM can compete or even dominate over the small SM $CP$ phase. Recently, both CDF [2] and D0 [3] have observed 1-2 $\sigma$ deviations from the SM prediction for $\beta_s$. Because of the possibilities of new discoveries, the search for New Physics in the $B_s$ system will be the main focus of the forthcoming experiments at Fermilab, LHCb and Super $B$ factories.

Theoretically, two-body $B$ decays have been studied in the framework of generalized factorization [4], QCD factorization (QCDF) [5, 6, 7, 8, 9], perturbative QCD (pQCD) [10, 11, 12] and soft-collinear effective theory (SCET) [13, 14]. In this work we will re-examine and update the QCDF predictions. Especially, we shall pay attention to the issue of power corrections. From the study of charmless hadronic $B_{u,d}$ decays, we learned that two subleading $1/m_b$ power corrections are needed in QCDF in order to account for the observed rates and $CP$ asymmetries. Power corrections to the penguin annihilation topology, corresponding to the so-called “scenario S4” in [6], are crucial for accommodating the branching fractions of penguin-dominated $B_{u,d} \rightarrow PP, VP, VV$ decays on the one hand and direct $CP$ asymmetries of $\bar{B}_d \rightarrow K^-\pi^+$, $\bar{B}_d \rightarrow K^-\pi^+$, $B^- \rightarrow K^-\rho^0$ and $\bar{B}_d \rightarrow \pi^+\pi^-$ on the other hand. Otherwise, the predicted rates will be too small and $CP$-violating asymmetries of above-mentioned modes will be wrong in signs when confronted with experiment. However, power corrections due to penguin annihilation will bring new $CP$ puzzles for the decays $B^- \rightarrow K^-\pi^0$, $K^-\eta, \pi^-\eta$, $\bar{B}_d \rightarrow \bar{K}^{*0}\eta$ and $\bar{B}_d \rightarrow \pi^0\pi^0$: Signs of their $A_{CP}$’s are flipped into the wrong ones when compared with experiment. It has been shown in [15] that soft corrections to the color-suppressed tree amplitude due to spectator scattering and/or final-state interactions will bring the aforementioned $CP$ asymmetries to the right track and accommodate the observed $\pi^0\pi^0$ and $\rho^0\pi^0$ rates simultaneously. Recently

\begin{itemize}
  \item[1] It is well known that a large complex electroweak penguin can also solve the $B \rightarrow K\pi \ CP$ puzzle with the difference of $A_{CP}(B^- \rightarrow \bar{K}^0\pi^-)$ and $A_{CP}(\bar{B}^0 \rightarrow K^-\pi^+)$ (see e.g. [16]). Since the electroweak penguin amplitude $P_{EW}$ is
we have given a detailed study of charmless hadronic \( B_{u,d} \to PP,V_P,V_V \) decays within the framework of QCDF incorporating aforementioned power corrections \[17\]. In this work we shall generalize the study to \( B_s \) decays. So far \( B_s \to K^+\pi^- \) is the only hadronic decay mode in the \( B_s \) sector that its direct \( CP \) violation has been measured \[18\]. The resulting \( CP \) asymmetry \( A_{CP}(\bar{B}_s \to K^+\pi^-) = 0.39 \pm 0.17 \) differs from zero by 2.2\( \sigma \) deviations. Just as the decay \( \bar{B}_d \to K^-\pi^+ \), the predicted \( CP \) asymmetry for \( \bar{B}_s \to K^+\pi^- \) in the heavy quark limit is wrong in sign and too small in magnitude. As we shall see below, we need penguin annihilation to get the right sign and magnitude for \( A_{CP}(\bar{B}_s \to K^+\pi^-) \).

This work is organized as follows. We outline the QCDF framework in Sec. 2 and specify various input parameters, such as form factors, light-cone distribution amplitudes and the parameters for power corrections in Sec. 3. Then \( B_s \to PP,V_P,V_V \) decays are analyzed in details in Secs. 4, 5 and 6, respectively. Conclusions are given in Sec. 7.

\section{B Decays in QCD Factorization}

Within the framework of QCD factorization \[19\], the effective Hamiltonian matrix elements are written in the form

\[ \langle M_1 M_2 | H_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda^{(q)}_p \langle M_1 M_2 | \mathcal{T}_{\text{eff}}^{h,p} + \mathcal{T}_{\text{eff}}^{h,h} | B \rangle, \]  

(2.1)

where \( \lambda^{(q)}_p \equiv V_{pq} V^*_{pq} \) with \( q = d, s \), and the superscript \( h \) denotes the helicity of the final-state meson. For \( PP \) and \( VP \) final states, \( h = 0 \). \( \mathcal{T}_{\text{eff}}^{h,p} \) describes contributions from naive factorization, vertex corrections, penguin contractions and spectator scattering expressed in terms of the flavor operators \( a_i^{h,h} \), while \( \mathcal{T}_{\text{eff}}^{h,h} \) contains annihilation topology amplitudes characterized by the annihilation operators \( b_i^{h,h} \). Specifically \[19\]

\[ \mathcal{T}_{\text{eff}}^{h} = a_i^0 (M_1 M_2) \delta_{pq} (\bar{q}b)_{v,-} \otimes (\bar{q}u)_{v,-} + a_i^2 (M_1 M_2) \delta_{pq} (\bar{q}b)_{v,=} \otimes (\bar{q}u)_{v,-} \]
\[ + a_i^3 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (\bar{q}q')_{v,=} + a_i^4 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (\bar{q}q')_{v,=} \]
\[ + a_i^5 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (3/2) e_q (q')_{v,=} + a_i^6 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (3/2) e_q (q')_{v,=} \]
\[ + a_i^7 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (3/2) e_q (q')_{v,=} + a_i^8 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (3/2) e_q (q')_{v,=} \]
\[ + a_i^9 (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (3/2) e_q (q')_{v,=} + a_i^{10} (M_1 M_2) \sum (q\bar{q})_{v,-} \otimes (3/2) e_q (q')_{v,=} \]

(2.2)

where \( (q_1 q_2)_{v,=} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2 \) and \( (q_1 q_2)_{v,=} \equiv \bar{q}_1 (1 \pm \gamma_5) q_2 \) and the summation is over \( q' = u, d, s \). The symbol \( \otimes \) indicates that the matrix elements of the operators in \( \mathcal{T}_{\text{eff}} \) are to be evaluated in the factorized

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essentially real in the standard model, one needs New Physics to produce new strong and weak phases for \( P_{\text{EW}} \). In principle, it will be difficult to discriminate between large complex color-suppressed tree \( C \) and large \( P_{\text{EW}} \) scenarios in the penguin-dominated decays. However, as pointed out in \[17\], the two schemes can lead to very distinct predictions for tree-dominated decays where \( P_{\text{EW}} \ll C \). The observed decay rates of \( B^0 \to \pi^0\pi^0, \rho^0\pi^0 \) and the \( CP \) puzzles with \( \pi^- \eta \) and \( \pi^0\pi^0 \) can be resolved by a large complex \( C \) but not \( P_{\text{EW}} \). In the \( B_{u,d} \) sector, there are 13 modes in which \( CP \) asymmetries have been measured with significance above 1.8\( \sigma \): \( K^-\pi^+, \pi^+\pi^-, K^-\eta, K^0\eta, K^-\rho^0, \rho^+\pi^- \)

and \( \rho^+K^-, K^+\pi^+, K^-\pi^0, \rho^-\pi^- \). \( \omega K^0 \), \( \rho^0\pi^0 \) and \( \rho^-\pi^- \). We have shown in \[17\] that the QCDF predictions of \( A_{CP} \) for aforementioned 13 decays are in agreement with experiment except the decay \( B^0 \to \omega K^0 \). However, we notice that BaBar and Belle measurements of \( A_{CP}(\omega K^0) \) are opposite in sign.
form. For the decays $\bar{B} \to PP, PV, VV$, the relevant factorizable matrix elements are

$$X^{(BP,p)} = \langle P_f | j^\mu | P_i \rangle = i f_p (m_B^2 - m_F^2) F_0^{BP}(m_F^2),$$

$$X^{(BP,V)} = \langle V | j^\mu | P_i \rangle = 2 f_{PV} m_B p_F A_1^{BP}(m_F^2),$$

$$X^{(BV,p)} = \langle P_i | j^\mu | V \rangle = 2 f_p m_B p_A A_0^{BV}(m_F^2),$$

$$X_{h}^{(BV_1,v_2)} = \langle V_2 | j^\mu | V_1 \rangle = -i f_{V_2} m_2 \left[ (e_1^* \cdot e_2^*)(m_B + m_V) A_1^{BV_1}(m_V^2) - \langle e_1^* \cdot p \rangle (e_2 \cdot p_B) \frac{2A_2^{BV_1}(m_V^2)}{(m_B + m_V)} + i e_{\mu \nu \alpha \beta} e_{\mu}^* p_B \frac{2V_1^{BV_1}(m_V)}{(m_B + m_V)} \right],$$

where we have followed the conventional definition for form factors [20]. For $B \to VP, PV$ amplitudes, we have applied the replacement $m_V e^* \cdot p_B \to m_B p_c$ with $p_c$ being the c.m. momentum. The longitudinal $(h = 0)$ and transverse $(h = \pm)$ components of $X_{h}^{(BV_1,v_2)}$ are given by

$$X_0^{(BV_1,v_2)} = \frac{i f_{V_2}}{2m_V} \left[ (m_B^2 - m_V^2 - m_C^2)(m_B + m_V) A_1^{BV_1}(q^2) - \frac{4m_B^2 p_c^2}{m_B + m_V} A_2^{BV_1}(q^2) \right],$$

$$X_{\pm}^{(BV_1,v_2)} = -i f_{V_2} m_2 m_v \left[ \left(1 + \frac{m_v}{m_B} \right) A_1^{BV_1}(q^2) \mp \frac{2p_c^2}{m_B + m_V} V^{BV_1}(q^2) \right].$$

The flavor operators $a_i^{p,h}$ are basically the Wilson coefficients in conjunction with short-distance non-factorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [3]

$$a_i^{p,h}(M_1 M_2) = \left( c_i + \frac{c_{i+1}}{N_c} \right) N_i^h(M_2) + \frac{c_{i+1}}{N_c} \frac{C_F \alpha_s}{4 \pi} \left[ V_i^h(M_2) + \frac{4\pi}{N_c} H_i^h(M_1 M_2) \right] + p_i^{h,p}(M_2),$$

where $i = 1, \cdots , 10$, the upper (lower) signs apply when $i$ is odd (even), $c_i$ are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, $M_2$ is the emitted meson and $M_1$ shares the same spectator quark with the $B$ meson. The quantities $V_i^h(M_2)$ account for vertex corrections, $H_i^h(M_1 M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the $B$ meson and $p_i(M_2)$ for penguin contractions. The expression of the quantities $N_i^h(M_2)$ reads

$$N_i^h(M_2) = \begin{cases} 0, & i=6,8, \\ 1, & \text{else}. \end{cases}$$

The weak annihilation contributions to the decay $\bar{B} \to M_1 M_2$ can be described in terms of the building blocks $b_{i,F}^{p,h}$ and $b_{i,F,EW}^{p,h}$

$$\frac{G_F}{\sqrt{2}} \sum_{p=a,c} \lambda_{p}^{(q)} \langle M_1 M_2 | J_{B}^{p,h} | B^0 \rangle = \frac{i G_F}{\sqrt{2}} \sum_{p=a,c} \lambda_{p}^{(q)} b_{i,F} f_{M_1} f_{M_2} \sum_{i} (d_i b_{i,F} + d_i b_{i,F,EW}).$$

The building blocks have the expressions [3]

$$b_1 = \frac{C_F}{N_c^2} c_1 A_1^f,$$

$$b_2 = \frac{C_F}{N_c^2} c_2 A_1^f,$$

$$b_3 = \frac{C_F}{N_c^2} \left[ c_3 A_1^f + c_5 (A_3^f + A_3^f) + N_c c_6 A_3^f \right],$$

$$b_4 = \frac{C_F}{N_c^2} \left[ c_4 A_1^f + c_6 A_2^f \right],$$

$$b_{3,EW} = \frac{C_F}{N_c^2} \left[ c_9 A_1^f + c_7 (A_3^f + A_3^f) + N_c c_8 A_3^f \right],$$

$$b_{4,EW} = \frac{C_F}{N_c^2} \left[ c_{10} A_1^f + c_8 A_2^f \right].$$
Here for simplicity we have omitted the superscripts $p$ and $h$ in above expressions. The subscripts $1,2,3$ of $A_{i,f}$ denote the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$ and $(S-P)(S+P)$ operators, respectively, and the superscripts $i$ and $f$ refer to gluon emission from the initial and final-state quarks, respectively. Following [6] we choose the convention that $M_1$ contains an antiquark from the weak vertex and $M_2$ contains a quark from the weak vertex.

For the explicit expressions of vertex, hard spectator corrections and annihilation contributions, the reader is referred to [6, 8, 19] for details. The decay amplitudes of $\bar{B}_s \rightarrow PP, VP$ are given in Appendix A of [6] and can be easily generalized to $\bar{B}_s \rightarrow VV$ (see [9] for explicit expressions of $\bar{B}_s \rightarrow VV$ amplitudes). In practice, it is more convenient to express the decay amplitudes in terms of the flavor operators $\alpha_{i}^{h,p}$ and the annihilation operators $\beta_{i}^{p}$ which are related to the coefficients $\alpha_{i}^{h,p}$ and $b_{i}^{p}$ by

$$
\alpha_{1}^{h}(M_{1}M_{2}) = \delta_{1}^{h}(M_{1}M_{2}), \\
\alpha_{2}^{h}(M_{1}M_{2}) = \delta_{2}^{h}(M_{1}M_{2}), \\
\alpha_{3}^{h,p}(M_{1}M_{2}) = \begin{cases} \\
\delta_{3}^{h,p}(M_{1}M_{2}) - \delta_{5}^{h,p}(M_{1}M_{2}) \quad \text{for } M_{1}M_{2} = PP, PP, \\
\delta_{3}^{h,p}(M_{1}M_{2}) + \delta_{5}^{h,p}(M_{1}M_{2}) \quad \text{for } M_{1}M_{2} = VV, PV, \\
\end{cases}
$$

(2.9)

$$
\alpha_{4}^{h,p}(M_{1}M_{2}) = \begin{cases} \\
\delta_{4}^{h,p}(M_{1}M_{2}) + r_{X}^{P} \delta_{6}^{h,p}(M_{1}M_{2}) \quad \text{for } M_{1}M_{2} = PP, PV, \\
\delta_{4}^{h,p}(M_{1}M_{2}) - r_{X}^{P} \delta_{6}^{h,p}(M_{1}M_{2}) \quad \text{for } M_{1}M_{2} = VV, VV, \\
\end{cases}
$$

$$
\alpha_{3,EW}^{h,p}(M_{1}M_{2}) = \begin{cases} \\
\delta_{9}^{h,p}(M_{1}M_{2}) - \delta_{10}^{h,p}(M_{1}M_{2}) \quad \text{for } M_{1}M_{2} = PP, PV, \\
\delta_{9}^{h,p}(M_{1}M_{2}) + \delta_{10}^{h,p}(M_{1}M_{2}) \quad \text{for } M_{1}M_{2} = VV, VV, \\
\end{cases}
$$

and

$$
\beta_{i}^{p}(M_{1}M_{2}) = \frac{i g_{F} f_{M_{1}} f_{M_{2}}}{X^{(\bar{B}_{M_{1}})M_{2}} b_{i}}.
$$

(2.10)

The order of the arguments of $\alpha_{i}^{h,p}(M_{1}M_{2})$ and $\beta_{i}^{p}(M_{1}M_{2})$ is consistent with the order of the arguments of $X^{(\bar{B}_{M_{1}})M_{2}} \equiv A_{M_{1}M_{2}}$. The chiral factor $r_{X}$ is given by

$$
r_{X}^{P}(\mu) = \frac{2m_{P}^{2}}{m_{p}(\mu)(m_{2} + m_{1})(\mu)}, \quad r_{X}^{V}(\mu) = \frac{2m_{V}}{m_{b}(\mu)} \frac{f_{V}^{+}(\mu)}{f_{V}}.
$$

(2.11)

III. INPUT PARAMETERS

It is clear from Eq. (2.3) that we need the information on decay constants and form factors in order to evaluate the factorizable matrix elements of 4-quark operators. Moreover, we also need to know the light-cone distribution amplitudes of light hadrons in order to evaluate the nonfactorizable contributions.

A. Form factors

There exist one lattice and several model calculations of form factors for $B_{s} \rightarrow P,V$ transitions:
1. In the pQCD approach, the relevant form factors obtained at $q^2 = 0$ are (for simplicity, form
factors hereafter are always referred to the ones at $q^2 = 0$, unless specified otherwise)

$$F_{0}^{B,K} = 0.24^{+0.05+0.00}_{-0.04-0.01}, \quad F_{0}^{B,\eta} = 0.30^{+0.06+0.01}_{-0.05-0.01},$$

$$V_{B,K^*} = 0.21^{+0.04+0.00}_{-0.03-0.01}, \quad A_{0}^{B,K^*} = 0.25^{+0.05+0.00}_{-0.05-0.01}, \quad A_{1}^{B,K^*} = 0.16^{+0.03+0.00}_{-0.03-0.01},$$

$$V_{B,\phi} = 0.25^{+0.05+0.00}_{-0.04-0.01}, \quad A_{0}^{B,\phi} = 0.30^{+0.05+0.00}_{-0.05-0.01}, \quad A_{1}^{B,\phi} = 0.19^{+0.03+0.00}_{-0.03-0.01}. \quad (3.1)$$

2. Form factors obtained by QCD sum rules are

$$F_{0}^{B,K} = 0.30^{+0.04}_{-0.03}, \quad (3.2)$$

for the $B_s \to K$ transition and

$$V_{B,K^*} = 0.311 \pm 0.026, \quad A_{0}^{B,K^*} = 0.360 \pm 0.034, \quad A_{1}^{B,K^*} = 0.233 \pm 0.022,$$

$$V_{B,\phi} = 0.434 \pm 0.035, \quad A_{0}^{B,\phi} = 0.474 \pm 0.033, \quad A_{1}^{B,\phi} = 0.311 \pm 0.030, \quad (3.3)$$

for $B_s \to V$ transitions.

3. Another light-cone sum rule calculation based on heavy quark effective theory gives

$$F_{0}^{B,K} = 0.296 \pm 0.018, \quad F_{0}^{B,\eta} = 0.281^{+0.015}_{-0.016}, \quad (3.4)$$

and

$$V_{B,K^*} = 0.285^{+0.013}_{-0.013}, \quad A_{0}^{B,K^*} = 0.222^{+0.011}_{-0.010}, \quad A_{1}^{B,K^*} = 0.227^{+0.010}_{-0.012},$$

$$V_{B,\phi} = 0.339^{+0.016}_{-0.017}, \quad A_{0}^{B,\phi} = 0.269^{+0.014}_{-0.014}, \quad A_{1}^{B,\phi} = 0.271^{+0.014}_{-0.014}. \quad (3.5)$$

It is clear that form factors obtained by sum rules are larger than the pQCD ones.

4. A light cone quark model in conjunction with soft collinear effective theory was constructed in

The predictions are

$$F_{0}^{B,K} = 0.290, \quad F_{0}^{B,\eta} = 0.288,$$

$$V_{B,K^*} = 0.323, \quad A_{0}^{B,K^*} = 0.279, \quad A_{1}^{B,K^*} = 0.228,$$

$$V_{B,\phi} = 0.329, \quad A_{0}^{B,\phi} = 0.279, \quad A_{1}^{B,\phi} = 0.232. \quad (3.6)$$

5. A straightforward application of the covariant light-front quark model of yields

$$V_{B,K^*} = 0.23, \quad A_{0}^{B,K^*} = 0.26, \quad A_{1}^{B,K^*} = 0.19,$$

$$V_{B,\phi} = 0.30, \quad A_{0}^{B,\phi} = 0.32, \quad A_{1}^{B,\phi} = 0.26, \quad (3.7)$$

all with errors estimated to be $\pm 0.01$.

6. A recent lattice QCD calculation yields $F_{0}^{B,K} = 0.23 \pm 0.05 \pm 0.04$.

For comparison, Beneke and Neubert used

$$F_{0}^{B,K} = 0.31 \pm 0.05, \quad A_{0}^{B,K^*} = 0.29 \pm 0.05, \quad A_{1}^{B,K^*} = 0.34 \pm 0.05, \quad (3.8)$$
\[ F_0^{B_i \rightarrow \eta^{(')}} = F_0^{B_K} f_{\eta^{(')}}^q + F_2 f_{\eta^{(')}}^s \sqrt{2} \frac{f_{\eta^{(')}}^s + f_{\eta^{(')}}^\phi}{\sqrt{3} f_\pi}, \]

while Beneke, Rohrer and Yang [8] employed
\[ A_0^{B_i K^*} = 0.33 \pm 0.05, \quad A_0^{B_i \phi} = 0.38^{+0.10}_{-0.02}. \]

Note that it is most convenient to express the form factors for \( B \rightarrow \eta^{(')} \) transitions in terms of the flavor states \( q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2} , s\bar{s} \) and \( c\bar{c} \) labeled by the \( \eta_q, \eta_s \) and \( \eta_c^0 \), respectively. Neglecting the small mixing with \( \eta_c^0 \), we have
\[ F_{B_i \eta} = -F_{B_i \eta} \sin \theta, \quad F_{B_i \eta'} = F_{B_i \eta} \cos \theta, \]
where \( \theta \) is the \( \eta_q - \eta_s \) mixing angle defined by
\[ |\eta_q\rangle = \cos \theta |\eta_q\rangle - \sin \theta |\eta_s\rangle, \]
\[ |\eta_s\rangle = \sin \theta |\eta_q\rangle + \cos \theta |\eta_s\rangle, \]
with \( \theta = (39.3 \pm 1.0)^\circ \) in the Feldmann-Kroll-Stech mixing scheme [28].

From the above discussions we see that the form factor \( F_0^{B_i K} (0) \) at \( q^2 = 0 \) ranges from 0.23 to 0.31. In the QCDF approach, if \( F_0^{B_i K} (0) \) is employed, we find that the predicted branching fractions \( \mathcal{B}(\bar{B}_s \rightarrow K^+ \pi^-) \approx 9.1 \times 10^{-6} \) and \( \mathcal{B}(\bar{B}_s \rightarrow K^+ K^-) \approx 34 \times 10^{-6} \) will be far above the experimental measurements of \( (5.0 \pm 1.1) \times 10^{-6} \) [29] and \( (25.7 \pm 3.6) \times 10^{-6} \) [30, 31], respectively. Hence we shall use \( F_0^{B_i K} (0) = 0.24 \) obtained by the lattice calculation. Note that a \( \chi^2 \) analysis by one of us (C.K.C.) with the available data of \( B \rightarrow PP \) also yields \( F_0^{B_i K} (0) = 0.240^{+0.021}_{-0.007} \) [33]. For other form factors, we shall use \( F_0^{B_i \eta} (0) = 0.28 \) and \( F_0^{B_i \eta'} (0) = 0.28 \) in the Feldmann-Kroll-Stech mixing scheme.
TABLE I: Input parameters. The values of the scale dependent quantities $f_{\perp}^{V}(\mu)$ and $a_{1,2}^{\perp,V}(\mu)$ are given for $\mu = 1\text{ GeV}$. The values of Gegenbauer moments are taken from [36] and Wolfenstein parameters from [37].

| Light vector mesons | | | | | |
|---------------------|-----------------|-----------------|-----------------|-----------------|
| $V$ | $f_{\perp}^{V}(\text{MeV})$ | $f_{\perp}^{V}(\text{MeV})$ | $a_{1}^{V}$ | $a_{2}^{V}$ | $a_{1}^{\perp,V}$ | $a_{2}^{\perp,V}$ |
| $\rho$ | 216 ± 3 | 165 ± 9 | 0 | 0.15 ± 0.07 | 0 | 0.14 ± 0.06 |
| $\omega$ | 187 ± 5 | 151 ± 9 | 0 | 0.15 ± 0.07 | 0 | 0.14 ± 0.06 |
| $\phi$ | 215 ± 5 | 186 ± 9 | 0 | 0.18 ± 0.08 | 0 | 0.14 ± 0.07 |
| $K^{*}$ | 220 ± 5 | 185 ± 10 | 0.03 ± 0.02 | 0.11 ± 0.09 | 0.04 ± 0.03 | 0.10 ± 0.08 |

| Light pseudoscalar mesons | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| | $a_{1}^{\pi}$ | $a_{2}^{\pi}$ | $a_{1}^{K}$ | $a_{2}^{K}$ |
| 0 | 0.25 ± 0.15 | 0.06 ± 0.03 | 0.25 ± 0.15 |

| B mesons | | | | | |
|---------|-----------------|-----------------|-----------------|-----------------|
| $B$ | $m_{B}(\text{GeV})$ | $\tau_{B}(\text{ps})$ | $f_{B}(\text{MeV})$ | $\lambda_{B}(\text{MeV})$ |
| $B_{d}$ | 5.279 | 1.638 | 210 ± 20 | 300 ± 100 |
| $B_{s}$ | 5.279 | 1.525 | 210 ± 20 | 300 ± 100 |
| $B_{s}$ | 5.366 | 1.472 | 230 ± 20 | 300 ± 100 |

| Form factors at $q^{2} = 0$ | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| | $F_{0}^{B,\rho}(0)$ | $A_{0}^{B,\rho}(0)$ | $A_{1}^{B,\rho}(0)$ | $A_{2}^{B,\rho}(0)$ | $V_{0}^{B,\rho}(0)$ |
| 0.24 | 0.30 ± 0.01 | 0.24 ± 0.01 | 0.22 ± 0.01 | 0.28 ± 0.01 |
| $F_{0}^{B,\eta^{'}}(0)$ | $A_{0}^{B,\eta^{'}}(0)$ | $A_{1}^{B,\eta^{'}}(0)$ | $A_{2}^{B,\eta^{'}}(0)$ | $V_{0}^{B,\eta^{'}}(0)$ |
| 0.28 | 0.32 ± 0.01 | 0.26 ± 0.01 | 0.23 ± 0.01 | 0.30 ± 0.01 |

| Quark masses | | | | | |
|--------------------|-----------------|-----------------|-----------------|-----------------|
| | $m_{b}(\text{GeV})$ | $m_{c}(\text{GeV})$ | $m_{c}^{\text{pole}}/m_{b}^{\text{pole}}$ | $m_{s}(2.1\text{ GeV})/\text{GeV}$ |
| | 4.2 | 0.91 | 0.3 | 0.095 ± 0.020 |

| Wolfenstein parameters | | | | | |
|-----------------------|-----------------|-----------------|-----------------|-----------------|
| | $A$ | $\lambda$ | $\bar{\rho}$ | $\bar{\eta}$ | $\gamma$ |
| | 0.8116 | 0.2252 | 0.139 | 0.341 | (67.8^{+4.2}_{-3.9})^{o} |
C. LCDAs

We next specify the light-cone distribution amplitudes (LCDAs) for pseudoscalar and vector mesons. The general expressions of twist-2 LCDAs are

\[
\Phi_P(x, \mu) = 6x(1-x) \left[ 1 + \sum_{n=1}^{\infty} a_n^P(\mu) C_n^3/2(2x-1) \right],
\]

\[
\Phi^V_\parallel(x, \mu) = 6x(1-x) \left[ 1 + \sum_{n=1}^{\infty} a_n^V(\mu) C_n^3/2(2x-1) \right],
\]

\[
\Phi^V_\perp(x, \mu) = 6x(1-x) \left[ 1 + \sum_{n=1}^{\infty} a_n^{-1,V}(\mu) C_n^3/2(2x-1) \right],
\]

(3.17)

and twist-3 ones

\[
\Phi_P(x) = 1, \quad \Phi_\sigma(x) = 6x(1-x),
\]

\[
\Phi_\tau(x, \mu) = 3 \left[ 2x - 1 + \sum_{n=1}^{\infty} a_n^{1/2,V}(\mu) P_{n+1}(2x-1) \right],
\]

(3.18)

where \(C_n(x)\) and \(P_n(x)\) are the Gegenbauer and Legendre polynomials, respectively. When three-particle amplitudes are neglected, the twist-3 \(\Phi_\tau(x)\) can be expressed in terms of \(\Phi_\perp\)

\[
\Phi_\tau(x) = \int_0^x \frac{\Phi_\perp(u)}{u} du - \int_x^1 \frac{\Phi_\perp(u)}{u} du.
\]

(3.19)

The normalization of LCDAs is

\[
\int_0^1 dx \Phi_V(x) = 1, \quad \int_0^1 dx \Phi_\tau(x) = 0.
\]

(3.20)

Note that the Gegenbauer moments \(a_i^{(\perp),K^*}\) displayed in Table II taken from [36] are for the mesons containing a strange quark.

The integral of the \(B\) meson wave function is parameterized as [19]

\[
\int_0^1 \frac{d\rho}{1-\rho} \Phi_B^B(\rho) \equiv \frac{m_B}{\lambda_B},
\]

(3.21)

where \(1-\rho\) is the momentum fraction carried by the light spectator quark in the \(B\) meson. We shall use \(\lambda_B = 300 \pm 100\) MeV.

For the running quark masses we shall use [38, 39]

\[
\begin{align*}
  m_b(m_b) &= 4.2\text{ GeV}, & m_b(2.1\text{ GeV}) &= 4.94\text{ GeV}, & m_b(1\text{ GeV}) &= 6.34\text{ GeV}, \\
  m_c(m_b) &= 0.91\text{ GeV}, & m_c(2.1\text{ GeV}) &= 1.06\text{ GeV}, & m_c(1\text{ GeV}) &= 1.32\text{ GeV}, \\
  m_s(2.1\text{ GeV}) &= 95\text{ MeV}, & m_s(1\text{ GeV}) &= 118\text{ MeV}, \\
  m_d(2.1\text{ GeV}) &= 5.0\text{ MeV}, & m_d(2.1\text{ GeV}) &= 2.2\text{ MeV}.
\end{align*}
\]

(3.22)

Note that the charm quark masses here are smaller than the one \(m_c(m_b) = 1.3 \pm 0.2\) GeV adopted in [6, 9] and consistent with the high precision mass determination from lattice QCD [40]: \(m_c(3\text{ GeV}) = 0.986 \pm 0.010\) GeV and \(m_c(m_c) = 1.267 \pm 0.009\) GeV (see also [41]). Among the quarks, the strange quark gives the major
theoretical uncertainty to the decay amplitude. Hence, we will only consider the uncertainty in the strange quark mass given by $m_s(2.1\text{ GeV}) = 95 \pm 20\text{ MeV}$. Notice that for the one-loop penguin contribution, the relevant quark mass is the pole mass rather than the current one. Since the penguin loop correction is governed by the ratio of the pole masses squared $s_i \equiv (m_i^\text{pole}/m_b^\text{pole})^2$ and since the pole mass is meaningful only for heavy quarks, we only need to consider the ratio of $c$ and $b$ quark pole masses given by $s_c \approx (0.3)^2$.

D. Penguin annihilation

In the QCDF approach, the hadronic $B$ decay amplitude receives contributions from tree, penguin, electroweak penguin and weak annihilation topologies. In the absence of $1/m_b$ power corrections except for the chiral enhanced penguin contributions, the leading QCDF predictions encounter three major difficulties: (i) the predicted branching fractions for penguin-dominated low the measurements, (ii) direct decays, $B \to PP, VP, VV$ decays are taken from [43]. Since the penguin annihilation effects are different for heavy quarks, we only need to consider the ratio of $c$ and $b$ quark pole masses given by $s_c \approx 0.3^2$.

\[
\int_{1}^{\Lambda} dx/\bar{x}
\]

with $\Lambda_b$ being a typical scale of order 500 MeV, and $\rho_{A,H}, \phi_{A,H}$ being the unknown real parameters.

A fit to the data of $B_{u,d} \to PP, VP, PV$ and $VV$ decays yields the values of $\rho_A$ and $\phi_A$ shown in Table I. Basically, it is very similar to the so-called “S4 scenario” presented in [4]. The fitted $\rho_A$ and $\phi_A$ for $B \to VV$ decays are taken from [43]. Since the penguin annihilation effects are different for $B \to VP$ and $B \to PV$ decays,

\[
A_i' \approx -A_i' \approx 6\pi\alpha_s \left[ 3 \left( X_{VP}^A - 4 + \frac{\pi^2}{3} \right) + r_{VP}^A \left( X_{VP}^A - 2X_{VP}^A \right) \right],
\]

\[
A_i' \approx 6\pi\alpha_s \left[ -3 r_{VP}^A \left( X_{VP}^A - 2X_{VP}^A + 4 - \frac{\pi^2}{3} \right) + r_{VP}^A \left( X_{VP}^A - 2X_{VP}^A + \frac{\pi^2}{3} \right) \right],
\]

\[
A_i' \approx 6\pi\alpha_s \left[ 3 r_{VP}^A (2X_{VP}^A - 1)(2 - X_{VP}^A) - r_{VP}^A \left( 2(X_{VP}^A)^2 - X_{VP}^A \right) \right],
\]

(3.24)
for $M_1M_2 = VP$ (the definition for the parameters $r_x^P$ and $r_x^V$ can be found in Eq. (2.11) below) and
\[
A_1^i \approx -A_2^i \approx 6\pi \alpha_s \left[ 3 \left( X_{VA}^{PV} - 4 + \frac{\pi^2}{3} \right) + r_x^{VP} \left( (X_{VA}^{PV})^2 - 2X_{VA}^{PV} \right) \right],
\]
\[
A_3^i \approx 6\pi \alpha_s \left[ -3r_x^{VP} \left( (X_{VA}^{PV})^2 - 2X_{VA}^{PV} + 4 - \frac{\pi^2}{3} \right) + r_x^{V} \left( (X_{VA}^{PV})^2 - 2X_{VA}^{PV} + \frac{\pi^2}{3} \right) \right],
\]
for $M_1M_2 = PV$, the parameters $X_{VA}^{VP}$ and $X_{VA}^{PV}$ are not necessarily the same. Indeed, a fit to the $B \to VP, PV$ decays yields $\rho_{VA}^{VP} \approx 1.07, \phi_{VA}^{VP} \approx -70^\circ$ and $\rho_{VA}^{PV} \approx 0.87, \phi_{VA}^{PV} \approx -30^\circ$ (see Table II). For $B_s \to PP, VP, VV$ decays, we shall assume that their default values are similar to that in $B_{u,d}$ decays as shown in Table III. For the estimate of theoretical uncertainties, we shall assign an error of $\pm 0.1$ to $\rho_A$ and $\pm 20^\circ$ to $\phi_A$.

**TABLE II: The parameters $\rho_A$ and $\phi_A$ for penguin annihilation.**

| Modes   | $\rho_A$ | $\phi_A$ | Modes   | $\rho_A$ | $\phi_A$ |
|---------|----------|----------|---------|----------|----------|
| $B \to PP$ | 1.10   | $-50^\circ$ | $B_s \to PP$ | 1.00   | $-55^\circ$ |
| $B \to VP$ | 1.07   | $-70^\circ$ | $B_s \to VP$ | 0.90   | $-65^\circ$ |
| $B \to PV$ | 0.87   | $-30^\circ$ | $B_s \to PV$ | 0.85   | $-30^\circ$ |
| $B \to K^* \rho$ | 0.78   | $-43^\circ$ | $B_s \to VV$ | 0.70   | $-55^\circ$ |
| $B \to K^* \phi$ | 0.65   | $-53^\circ$ |         |          |          |

**E. Power corrections to $a_2$**

As pointed out in [15], while the discrepancies between experiment and theory in the heavy quark limit for the rates of penguin-dominated two-body decays of $B$ mesons and direct $CP$ asymmetries of $\bar{B}_d \to K^- \pi^+$, $B^- \to K^- \rho^0$ and $\bar{B}_d \to \pi^+ \pi^-$ are resolved by introducing power corrections coming from penguin annihilation, the signs of direct $CP$-violating effects in $B^- \to K^- \pi^0, B^- \to K^- \eta$ and $\bar{B}_d \to \pi^0 \pi^0$ are flipped to the wrong ones when confronted with experiment. These new $B$-$CP$ puzzles in QCDF can be explained by the subleading power corrections to the color-suppressed tree amplitudes due to spectator interactions and/or final-state interactions that not only reproduce correct signs for aforementioned $CP$ asymmetries but also accommodate the observed $\bar{B}_d \to \pi^0 \pi^0$ and $\rho^0 \pi^0$ rates simultaneously.

Following [15], power corrections to the color-suppressed topology are parametrized as
\[
a_2 \to a_2(1 + \rho_C e^{i\phi_C}),
\]
with the unknown parameters $\rho_C$ and $\phi_C$ to be inferred from experiment. We shall use $\phi_C \approx -70^\circ$ and $\rho_C \approx 1.3, 0.8, 0$ for $B \to PP, VP, VV$ decays [15, 17], respectively. This pattern that soft power corrections to $a_2$ are large for $PP$ modes, moderate for $VP$ ones and very small for $VV$ cases is consistent with the observation made in [44] that soft power correction dominance is much larger for $PP$ than $VP$ and $VV$ final states. It has been argued that this has to do with the special nature of the pion which is a $q\bar{q}$ bound state on the one hand and a nearly massless Nambu-Goldstone boson on the other hand [44].
IV. $B_s\to PP$ DECAYS

Before proceeding to the numerical results of QCDF calculations, we discuss some model-independent flavor symmetry relations in which many of $B_s\to PP$ decays can be related to $B_d\to PP$ ones by either $U$-spin or SU(3) symmetry. Hence these relations can be used to cross-check the dynamical calculations.

A. $U$-spin symmetry

In the limit of $U$-spin symmetry, some of $B_s$ decays can be related to $B_d$ ones. For example,

$$A(\bar{B}_s\to K^+\pi^-) = V_{ub}^*V_{ud}\langle K^+\pi^-|O'_d|\bar{B}_s\rangle + V_{cb}^*V_{cd}\langle K^+\pi^-|O'_s|\bar{B}_s\rangle,$$

$$A(\bar{B}_d\to K^-\pi^+) = V_{ub}^*V_{us}\langle K^-\pi^+|O'_d|\bar{B}_d\rangle + V_{cb}^*V_{cs}\langle K^-\pi^+|O'_s|\bar{B}_d\rangle,$$  \hspace{1cm} (4.1)

where the 4-quark operator $O_s$ is for the $b\to q\bar{q}s$ transition and $O_d$ for the $b\to q\bar{q}d$ transition. The assumption of $U$-spin symmetry implies that under $d\leftrightarrow s$ transitions,

$$\langle K^+\pi^-|O'_d|\bar{B}_s\rangle = \langle K^+\pi^-|O'_s|\bar{B}_d\rangle,$$

$$\langle K^-\pi^+|O'_d|\bar{B}_d\rangle = \langle K^-\pi^+|O'_s|\bar{B}_s\rangle.$$ \hspace{1cm} (4.2)

Using the relation

$$\text{Im}(V_{ub}^*V_{ud}V_{cd}^*) = -\text{Im}(V_{ub}^*V_{us}V_{cs}^*),$$  \hspace{1cm} (4.3)

it is straightforward to show that \cite{45, 46, 47}

$$|A(\bar{B}_s\to K^+\pi^-)|^2 - |A(\bar{B}_s\to K^-\pi^+)|^2 = |A(\bar{B}_d\to K^+\pi^-)|^2 - |A(\bar{B}_d\to K^-\pi^+)|^2,$$ \hspace{1cm} (4.4)

and, consequently,

$$A_{CP}(\bar{B}_s\to K^+\pi^-) = -A_{CP}(\bar{B}_d\to K^-\pi^+) \frac{\mathcal{B}(\bar{B}_d\to K^-\pi^+)}{\mathcal{B}(\bar{B}_s\to K^+\pi^-)} \frac{\tau(B_s)}{\tau(B_d)}.$$  \hspace{1cm} (4.5)

From the current world averages, $A_{CP}(\bar{B}_d\to K^-\pi^+) = -0.098^{+0.012}_{-0.011}$, \cite{48} and the CDF measurement $\mathcal{B}(\bar{B}_s\to K^+\pi^-) = (5.0\pm1.1)\times10^{-6}$ \cite{29}, it follows that the prediction $A_{CP}(\bar{B}_s\to K^+\pi^-) \approx 0.37$ under $U$-spin symmetry is in good agreement with the experimental result $0.39\pm0.15\pm0.08$ obtained by CDF \cite{29}. Besides $A_{CP}(\bar{B}_s\to K^+\pi^-)$, CDF has also measured direct $CP$ violation in the decay $\bar{B}_d\to K^-\pi^+$ and obtained \cite{30}

$$\frac{\Gamma(\bar{B}_d\to K^-\pi^+) - \Gamma(\bar{B}_d\to K^+\pi^-)}{\Gamma(\bar{B}_s\to K^+\pi^-) - \Gamma(\bar{B}_s\to K^-\pi^+)} = -0.83\pm0.41\pm0.12,$$ \hspace{1cm} (4.6)

which is equal to $-1$ under $U$-spin symmetry. Obviously, the experimental measurement is still limited by statistics.

By the same token, we also have the following $U$-spin relations

$$A_{CP}(\bar{B}_s\to K^+K^-) = -A_{CP}(\bar{B}_d\to \pi^+\pi^-) \frac{\mathcal{B}(\bar{B}_d\to \pi^+\pi^-)}{\mathcal{B}(\bar{B}_s\to K^+K^-)} \frac{\tau(B_s)}{\tau(B_d)},$$

$$A_{CP}(\bar{B}_s\to K^0\bar{K}^0) = -A_{CP}(\bar{B}_d\to \bar{K}^0K^0) \frac{\mathcal{B}(\bar{B}_d\to \bar{K}^0K^0)}{\mathcal{B}(\bar{B}_s\to K^0\bar{K}^0)} \frac{\tau(B_s)}{\tau(B_d)},$$

$$A_{CP}(\bar{B}_s\to \bar{K}^0K^0) = -A_{CP}(\bar{B}_d\to \bar{K}^0\bar{K}^0) \frac{\mathcal{B}(\bar{B}_d\to \bar{K}^0\bar{K}^0)}{\mathcal{B}(\bar{B}_s\to K^0\bar{K}^0)} \frac{\tau(B_s)}{\tau(B_d)},$$

$$A_{CP}(\bar{B}_s\to \pi^+\pi^-) = -A_{CP}(\bar{B}_d\to K^+K^-) \frac{\mathcal{B}(\bar{B}_d\to K^+K^-)}{\mathcal{B}(\bar{B}_s\to \pi^+\pi^-)} \frac{\tau(B_s)}{\tau(B_d)}.$$  \hspace{1cm} (4.7)
Unlike the first $U$-spin symmetry relation (4.5), the above relations cannot be tested by the present available data. Nevertheless, they can be checked by our dynamical calculations as shown in Sec.IV.C.5.

B. SU(3) symmetry

There are some cases where two-body decays of $B_d$ and $B_s$ can be related to each other in the limit of SU(3) symmetry provided that some of the annihilation effects can be neglected. Let us consider the decay amplitudes of the following three pairs in QCDF [4]:

$$A(B_s \rightarrow K^+ \pi^-) = \sum_{p=u,c} V_{pb}^* V_{pd} A_{\pi K}(\delta_{pb} \alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p),$$

(4.8)

$$A(B_d \rightarrow \pi^+ \pi^-) = \sum_{p=u,c} V_{pb}^* V_{pd} A_{\pi K}(\delta_{pb} \alpha_1 + \frac{1}{2} \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p),$$

and

$$A(B_s \rightarrow K^+ K^-) = \sum_{p=u,c} V_{pb}^* V_{pd} \left[ A_{KK}(\delta_{pb} \alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p - \frac{1}{2} \beta_{3,EW}^p) + B_{KK}(\delta_{pb} b_1^p + b_4^p + b_{4,EW}^p) \right],$$

(4.9)

$$A(B_s \rightarrow K^0 \bar{K}^0) = \sum_{p=u,c} V_{pb}^* V_{pd} A_{\pi K}(\delta_{pb} \alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p)$$

$$+ B_{KK}(b_4^p - \frac{1}{2} b_{4,EW}^p),$$

$$A(B^- \rightarrow \bar{K}^0 \pi^-) = \sum_{p=u,c} V_{pb}^* V_{pd} A_{\pi K}(\alpha_4^p - \frac{1}{2} \alpha_{4,EW}^p + \delta_{pb} \beta_2 + \beta_3^p + \beta_3^p),$$

(4.10)

and

$$A(B_s \rightarrow K^+ \pi^-) \approx A(B_s \rightarrow \pi^+ \pi^-), \quad A(B_s \rightarrow K^+ K^-) \approx A(B_d \rightarrow \pi^+ \pi^-),$$

$$A(B_s \rightarrow K^0 \bar{K}^0) \approx A(B^- \rightarrow K^0 \pi^-).$$

(4.11)

As will be discussed later, it turns out that among the relations

$$A(B_s \rightarrow K^+ \pi^-) \approx A(B_d \rightarrow \pi^+ \pi^-), \quad A(B_s \rightarrow K^+ K^-) \approx A(B_d \rightarrow K^+ \pi^-),$$

$$A(B_s \rightarrow K^+ K^-) \approx A(B_d \rightarrow K^- \pi^+), \quad A(B_s \rightarrow K^+ \bar{K}^0) \approx A(B_d \rightarrow K^- \pi^-),$$

(4.12)

the first three ones are experimentally fairly satisfied.
TABLE III: CP-averaged branching fractions (in units of $10^{-6}$) of $B_s \to PP$ decays obtained in various approaches. In the QCDF calculations, the parameters $\rho_A$ and $\phi_A$ are taken from Table III $\rho_C = 1.3$ and $\phi_C = -70^\circ$. Sources of theoretical uncertainties are discussed in the text. The pQCD predictions to LO and (partial) NLO are taken from [11] and [12], respectively. For the decays involving an $\eta$ and/or $\eta'$, two different sets of SCET results are quoted from [13].

| Modes         | Class | QCDF (this work) | pQCD (LO)  | pQCD (NLO) | SCET     | Expt. [18, 29] |
|---------------|-------|------------------|------------|------------|----------|----------------|
| $B_s^0 \to K^+ \pi^-$ | $T$    | $5.3^{+0.4+0.4}_{-0.8-0.8}$ | $7.6^{+3.3}_{-2.5}$ | $6.3^{+2.6}_{-1.9}$ | $4.9 \pm 1.2 \pm 1.2 \pm 1.3$ | $5.0 \pm 1.1$ |
| $B_s^0 \to K^0 \pi^0$ | $C$    | $1.7^{+2.5+1.2}_{-0.8-0.8}$ | $0.16^{+0.12}_{-0.07}$ | $0.25^{+0.10}_{-0.07}$ | $0.76 \pm 0.26 \pm 0.27 \pm 0.17$ |
| $B_s^0 \to K^0 \eta$ | $C$    | $0.75^{+1.10+0.51}_{-0.35-0.22}$ | $0.11^{+0.08}_{-0.11}$ | $0.08^{+0.03}_{-0.02}$ | $0.80 \pm 0.48 \pm 0.29 \pm 0.18$ |
| $B_s^0 \to K^0 \eta'$ | $C$    | $2.8^{+2.5+1.1}_{-1.0-1.0}$ | $0.72^{+0.36}_{-0.24}$ | $1.87^{+0.45}_{-0.56}$ | $4.5 \pm 1.5 \pm 0.4 \pm 0.5$ |
| $B_s^0 \to K^+ K^-$ | $P$    | $25.2^{+12.7+12.5}_{-7.2-7.2}$ | $13.6^{+8.6}_{-5.2}$ | $15.6^{+5.1}_{-3.9}$ | $18.2 \pm 6.7 \pm 1.1 \pm 0.5$ | $25.7 \pm 3.6$ |
| $B_s^0 \to K^0 K^0$ | $P$    | $26.1^{+13.5+12.9}_{-8.1-8.1}$ | $15.6^{+9.7}_{-6.0}$ | $18.0^{+4.7}_{-5.9}$ | $17.7 \pm 6.6 \pm 0.5 \pm 0.6$ |
| $B_s^0 \to \eta \eta$ | $P$    | $10.9^{+4.0+4.0}_{-4.0-4.0}$ | $8.0^{+5.4}_{-3.1}$ | $10.0^{+5.0}_{-2.6}$ | $7.1 \pm 6.4 \pm 0.2 \pm 0.8$ |
| $B_s^0 \to \eta \eta'$ | $P$    | $41.2^{+23.7+17.8}_{-12.9-13.1}$ | $21.0^{+11.7}_{-7.2}$ | $34.9^{+11.6}_{-9.5}$ | $24.0 \pm 13.6 \pm 1.4 \pm 2.7$ |
| $B_s^0 \to \eta' \eta'$ | $P$    | $47.9^{+41.6+20.9}_{-17.1-15.3}$ | $14.0^{+7.0}_{-4.1}$ | $25.2^{+8.3}_{-6.5}$ | $44.3 \pm 19.7 \pm 2.3 \pm 17.1$ |
| $B_s^0 \to \pi^0 \eta$ | $P_{EW}$ | $0.05^{+0.03+0.02}_{-0.01-0.01}$ | $0.05^{+0.02}_{-0.02}$ | $0.03^{+0.01}_{-0.01}$ | $0.014 \pm 0.004 \pm 0.005 \pm 0.004$ |
| $B_s^0 \to \pi^0 \eta'$ | $P_{EW}$ | $0.04^{+0.01+0.01}_{-0.00-0.00}$ | $0.11^{+0.05}_{-0.03}$ | $0.08^{+0.03}_{-0.02}$ | $0.006 \pm 0.003 \pm 0.002^{+0.006}_{-0.006}$ |
| $B_s^0 \to \pi^+ \pi^-$ | $ann$  | $0.26^{+0.09+0.10}_{-0.00-0.09}$ | $0.57^{+0.18}_{-0.16}$ | $0.57^{+0.24}_{-0.22}$ | $< 1.2$ |

This is the average of the CDF and Belle measurements, $(24.4 \pm 1.4 \pm 3.5) \times 10^{-6}$ [30] and $(38^{+10}_{-7}) \times 10^{-6}$ [31], respectively. The old CDF result on $B_s \to K^+ K^-$ can be found in [32].

C. Numerical results and comparison with other approaches

We list in Tables III and IV the branching fractions and CP asymmetries of $B_s \to PP$ decays evaluated in the frameworks of QCD factorization (this work), pQCD to the lowest order (LO) [11] and to the next-to-leading order (NLO) [12] and soft-collinear effective theory (SCET) [13]. For the decays involving an $\eta$ and/or $\eta'$, two different sets of SCET results are quoted from [13], corresponding to two distinct SCET parameters regarding to the strong phases of the gluonic charming penguin. The expression for the decay amplitudes of $B_s \to PP$ and $VP$ decays in the QCDF approach can be found in the Appendix of [4].

The theoretical errors in QCDF calculations correspond to the uncertainties due to the variation of (i) the Gegenbauer moments, the decay constants, (ii) the heavy-to-light form factors and the strange quark mass,
and (iii) the wave function of the B meson characterized by the parameter $\lambda_B$, the power corrections due to weak annihilation and hard spectator interactions described by the parameters $\rho_{A,H}$, $\phi_{A,H}$, respectively. To obtain the errors shown in Tables III-XIV, we first scan randomly the points in the allowed ranges of the above nine parameters and then add errors in quadrature. As noted in passing, we assign an error $\pm 0.1$ and $\pm 20^\circ$ to the default values of $\rho_A$ and $\phi_A$, respectively, while $\rho_H$ and $\phi_H$ lie in the ranges $0 \leq \rho_H \leq 1$ and $0 \leq \phi_H \leq 2\pi$. Specifically, the second error in the table is referred to the uncertainties caused by the variation of $\rho_{A,H}$ and $\phi_{A,H}$, where all other uncertainties are lumped into the first error. Power corrections beyond the heavy quark limit generally give the major theoretical uncertainties. For theoretical uncertainties in pQCD and SCET approaches, the reader is referred to the references cited in the table captions.

1. $\bar{B}_s \rightarrow K^+ \pi^-, K^0 \pi^0, K^0 \eta^{(')}$

As mentioned before, in this work we shall use the form factor $F_{0}^{B,K}(0) = 0.24$ obtained by both lattice and pQCD calculations. If a larger $B_s$ to $K$ transition form factor, say, $F_{0}^{B,K}(0) = 0.31$, is employed, the
predicted $\mathcal{B}(\bar{B}_s \to K^+\pi^-)$ and $\mathcal{B}(\bar{B}_s \to K^+K^-)$ will be far above the experimental results.\footnote{A larger branching fraction $\mathcal{B}(\bar{B}_s \to K^+\pi^-) = (10.2^{+6.0}_{-3.2}) \times 10^{-6}$ was obtained in [6] within the framework of QCDF using the form factor $F_0^{B,K}(0) = 0.31 \pm 0.05$.} For $F_0^{B,K}(0) = 0.24$, the calculated $\mathcal{B}(\bar{B}_s \to K^+\pi^-) = (5.3^{+0.4+0.4}_{-0.8-0.3}) \times 10^{-6}$ is in good agreement with the measurement $(5.0 \pm 0.7 \pm 0.8) \times 10^{-6}$ [29]. Notice that although the same value of $F_0^{B,K}$ was used in the leading order pQCD calculation, a larger branching fraction of order $7.6 \times 10^{-6}$ was obtained (see Table III).

A recent detailed analysis in [50] indicates that SU(3) and factorization only remain approximately valid if the branching fraction of $\bar{B}_s \to K^+\pi^-$ exceeds its current value of $(5.0 \pm 1.1) \times 10^{-6}$ by at least 50% or if the parameter $\xi$ defined by

$$\xi \equiv \frac{f_K}{f_\pi} \frac{F_0^{B,K}(m_K^2)}{F_0^{B,K}(m_\pi^2)} \frac{m_\pi^2 - m_\pi^2}{m_\pi^2 - m_K^2}$$

(4.13)
is more than about 1.2. The analysis goes as follows. Writing the amplitudes $A(B^- \to \bar{K}^0\pi^-) = V_{cs}V_{cb}^*P$ and $A(\bar{B}_d \to K^-\pi^+) = V_{ud}V_{ub}^*T\epsilon\delta + V_{cs}V_{cb}^*P$, the measured $B^- \to \bar{K}^0\pi^-$ rate sets a constraint on the penguin topology $P$. Since $V_{ub} = |V_{ub}|e^{-i\phi}$, the measurement of $\bar{B}_d \to K^-\pi^+$ will put a constraint on $T$ as a function of the unitarity angle $\gamma$. Under $U$-spin symmetry, the amplitude $A(\bar{B}_s \to K^+\pi^-) = V_{ud}V_{ub}^*T'\epsilon\delta' + V_{cs}V_{cb}^*P'$ can be related to the $\bar{B}_d \to K^-\pi^+$ one by the relations: $T' = T$, $P' = P$ and $\delta' = \delta$. The data of $\bar{B}_s \to K^+\pi^-$ will be helpful for pinning down the ratio of $P/T$. The analysis of [50] shows that for the value of $\gamma$ to be consistent with other determinations and for the strong phases $\delta$ and $\delta'$ not different much from each other, then either $\mathcal{B}(\bar{B}_s \to K^+\pi^-)$ is at least 50% larger than the current measured value or the parameter $\xi$ is larger than 1.2. Our results of $\xi = 1.24$ and $\mathcal{B}(\bar{B}_s \to K^+\pi^-) \approx 5.3 \times 10^{-6}$ are thus consistent with the analysis of [50].

It is known that the predicted direct CP violation for $\bar{B}_d \to K^-\pi^+$ and $\bar{B}_s \to K^+\pi^-$ modes in naive QCD is wrong in sign when compared with experiment (see the predictions in [6]). This discrepancy together with the rate deficit problem for penguin-dominated decays can be resolved by introducing power corrections coming from penguin annihilation, corresponding to the “S4 scenario” of [6]. Using the values given in Table III for the parameters $\rho_4$ and $\phi_4$, we obtain $A_{CP}(\bar{B}_d \to K^-\pi^+) = -(7.4^{+1.7+4.3}_{-1.5-4.8})\%$ and $A_{CP}(\bar{B}_s \to K^+\pi^-) = (20.7^{+5.0+3.9}_{-3.0-8.8})\%$, to be compared with the data $-0.098^{+0.012}_{-0.011}$ [48] and 0.39 ± 0.15 ± 0.08 [29], respectively.

The inclusion of soft corrections to the color-suppressed tree topology has two effects: First, it will enhance the rates of $\bar{B}_s \to K^0\pi^0, K^0\eta$ by a factor of about 2.5 and $\bar{B}_s \to K^0\eta'$ slightly. Second, it will flip the sign of CP-violating asymmetries of the former two modes. For example, $\mathcal{B}(\bar{B}_s \to K^0\pi^0)$ is enhanced from $0.7 \times 10^{-6}$ to $1.7 \times 10^{-6}$, while $A_{CP}(\bar{B}_s \to K^0\pi^0)$ is changed from $-0.214$ to the order of 0.363 (see Tables III and IV). Note that pQCD predictions of branching fractions for the color-suppressed tree-dominated decays $\bar{B}_s \to K^0\pi^0, K^0\eta(1)$ are much smaller than QCDF and SCET. Nevertheless, pQCD results of $A_{CP}$’s for the above three modes agree in signs with QCDF.

We see from Table IV that SCET predicts a negative sign for $A_{CP}(\bar{B}_s \to K^0\pi^0)$, contrary to QCDF and pQCD. This deserves a special discussion. The negative sign of $A_{CP}(\bar{B}_s \to K^0\pi^0)$ has to do with the fact that SCET predicts $A_{CP}(\bar{B}_d \to K^0\pi^0) = (5 \pm 4 \pm 4 \pm 1)\%$ [13]. From the $U$-spin symmetry relation (4.7) we learn that CP asymmetries of $\bar{B}_s \to K^0\pi^0$ and $\bar{B}_d \to K^0\pi^0$ are of opposite sign. Although the current world
average $A_{CP}(\bar{B}_d \to \bar{K}^0\pi^0) = -0.01 \pm 0.10$ from the BaBar and Belle measurements, $-0.13 \pm 0.13 \pm 0.03$ \([51]\) and $0.14 \pm 0.13 \pm 0.06$ \([52]\) respectively, is consistent with no $CP$ violation, there exist several model-independent determinations of this asymmetry: one is the SU(3) relation \([53]\)

$$
\Delta \Gamma(\bar{B}_d \to \pi^0\pi^0) = -\Delta \Gamma(\bar{B}_d \to \bar{K}^0\pi^0),
$$

(4.14)

and the other is the approximate sum rule for $CP$ rate asymmetries \([54]\)

$$
\Delta \Gamma(\bar{B}_d \to K^- \pi^+) + \Delta \Gamma(B^- \to \bar{K}^0\pi^-) \approx 2[\Delta \Gamma(B^- \to K^- \pi^0) + \Delta \Gamma(\bar{B}_d \to \bar{K}^0\pi^0)],
$$

(4.15)

based on isospin symmetry, where $\Delta \Gamma(B \to K\pi) \equiv \Gamma(\bar{B} \to \bar{K}\pi) - \Gamma(B \to K\pi)$. This sum rule allows us to extract $A_{CP}(\bar{B}_d \to \bar{K}^0\pi^0)$ in terms of the other three asymmetries in $K^-\pi^+, K^-\pi^0, K^0\pi^-$ modes that have been measured. From the current data of branching fractions and $CP$ asymmetries, the above SU(3) relation and $CP$-asymmetry sum rule lead to $-0.073^{+0.042}_{-0.041}$ and $-0.15 \pm 0.04$, respectively, for $A_{CP}(\bar{B}_d \to \bar{K}^0\pi^0)$.

An analysis based on the topological quark diagrams yields a similar result $-0.08 \sim -0.12$ \([55]\). All these indicate that direct $CP$ violation should be negative for $\bar{B}_d \to \bar{K}^0\pi^0$ and hence positive for $\bar{B}_s \to K^0\pi^0$.

2. $\bar{B}_s \to K^+K^-, K^0\bar{K}^0$

The penguin-dominated decays $\bar{B}_s \to K^+K^-, K^0\bar{K}^0$ have sizable branching fractions of order $25 \times 10^{-6}$ in QCDF. The corresponding pQCD and SCET predictions are slightly smaller (Table III). \(3\) From Eqs. (4.19) and (4.10) we see that $K^+K^-$ and $K^0\bar{K}^0$ modes differ mainly in the tree contribution $\alpha_1$ and the annihilation term $\beta_1$ induced by the operator $O_1$, both existing in the former but not in the latter. Since these contributions are CKM suppressed relative to the penguin terms, the above two modes should have similar rates but rather distinct $CP$ asymmetries. Due to the absence of interference between tree and penguin amplitudes, $CP$ asymmetry is very small in $\bar{B}_s \to \bar{K}^0\bar{K}^0$, less than 1%. Using the world average of $A_{CP}(\bar{B}_d \to \pi^+\pi^-) = 0.38 \pm 0.06$, $\mathcal{B}(\bar{B}_d \to \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6}$ \([48]\) and $\mathcal{B}(\bar{B}_s \to K^+K^-) = (25.7 \pm 3.6) \times 10^{-6}$ \([49]\), we find from the first $U$-spin relation in Eq. (4.7) that $A_{CP}(\bar{B}_s \to K^+K^-) \approx -0.077$ in the $U$-spin limit, which is in excellent agreement with the QCDF prediction. It is very important to measure the direct $CP$ asymmetry for this mode.

In the pQCD approach, direct $CP$ violation of $\bar{B}_s \to K^0\bar{K}^0$ vanishes to the lower order as there is only one type of CKM matrix element in its decay amplitude, say $V_{tb}V_{ts}^\star$ \([11]\). To the NLO, penguin loop corrections allow other CKM matrix elements enter into the decay amplitude and induce $CP$ asymmetry \([12]\). It turns out that the predicted $A_{CP}(\bar{B}_s \to K^0\bar{K}^0)$ is very similar in both QCDF and pQCD (to NLO) approaches. It has been argued that the decay $\bar{B}_s \to K^0\bar{K}^0$ is a very promising place to look for effects of New Physics through the measurement of its direct $CP$ violation \([56,58]\). For example, it was shown in \([57]\) that $A_{CP}(\bar{B}_s \to K^0\bar{K}^0)$, which is not more than 1% in the SM, can be 10 times larger in the presence of SUSY while its rate remains unaffected.

\(3\) An early theoretical estimate yielded $\mathcal{B}(\bar{B}_s \to K^+K^-) = (35 \pm 7) \times 10^{-6}$ using the measured $B^0 \to K^+\pi^-$ branching fraction \([56]\). Based on QCDF and a combination of $U$-spin and isospin arguments, a result of $(20 \pm 8 \pm 4 \pm 2) \times 10^{-6}$ was obtained in \([4]\).
3. $\bar{B}_s \to \eta^\prime \eta^\prime$

The penguin-dominated $\eta^\prime \eta^\prime$ modes have sizable rates, especially $B_s \to \eta^\prime \eta^\prime$, the analog of $B \to K \eta^\prime$ in the $B_s$ sector, has the largest branching fraction of order $\sim 50 \times 10^{-6}$ in two-body hadronic decays of the $B_s$ meson. The QCDF predictions in [29] within the S4 scenario are much bigger, $78 \times 10^{-6}$ and $66 \times 10^{-6}$ respectively for $\eta \eta^\prime$ and $\eta^\prime \eta^\prime$ modes. This is because Eq. (3.9) rather than (3.11) is employed there for describing the $B_s \to \eta^\prime$ transition form factors. One of us (CKC) found that the $B_s \to \eta^\prime \eta^\prime$ branching fraction can even reach the level of $10^{-4}$ in the residual final-state scattering model [33]. It is evident from Table III that the pQCD approach to lowest order predicts much smaller $\eta^\prime \eta^\prime$ rates even though the form factor $F_{B_s \eta^\prime}(0) = 0.30$ is used there. A recent pQCD calculation involving some NLO corrections from vertex corrections, quark loops and chormo-magnetic penguins exhibits some improvements [12]: the branching fractions of $\eta \eta$, $\eta \eta^\prime$ and $\eta^\prime \eta$ are enhanced from 8.0, 21.0 and 14.0 (in units of $10^{-6}$) to 10.0, 34.9 and 25.2, respectively. The gap between pQCD and QCDF is thus improved. However, the NLO corrections calculated so far in pQCD are still not the complete results as some other pieces of NLO corrections such as hard spectator and annihilation have not been considered. It is important for the pQCD community to carry out the complete NLO calculations.

Since the decays $\bar{B}_s \to \eta \eta$ are penguin dominated and their tree amplitudes are color suppressed, their direct $CP$ asymmetries are not large.

4. $\bar{B}_s \to \pi \pi$

The decays $\bar{B}_s \to \pi \pi$ proceed only through annihilation with the amplitudes [6]

$$A_{\bar{B}_s \to \pi^+ \pi^-} \approx \sqrt{2}A_{B_s \to \pi^0 \pi^0} \sim 2B_{\pi \pi}b^\ell_i.$$  

(4.16)

The predicted $\mathcal{B}(\bar{B}_s \to \pi^+ \pi^-) = 2.6 \times 10^{-7}$ in QCDF is consistent with the current upper limit of $1.2 \times 10^{-6}$ [29]. Note that in the absence of power corrections i.e. $\rho_A = 0$, the branching ratio will become too small, of order $5 \times 10^{-8}$.

5. $\bar{B}_s \to \pi^0 \eta$ (i)

Since the isospin of the final state is $I = 1$, the electroweak penguin is the only loop contribution that can contribute to the decays $\bar{B}_s \to \pi^0 \eta$, in analog to the $B^- \to \pi^- \pi^0$ transition. However, unlike the latter, the electroweak penguin amplitude in the former gains a CKM enhancement $\lambda^{(s)}_c / \lambda^{(s)}_u$. Indeed, $P_{EW}$ dominates over $C$ in $\bar{B}_s \to \pi^0 \eta$ decays. It is well known that $CP$ asymmetry of $B^- \to \pi^- \pi^0$ is very small, of order $10^{-5}$. This is ascribed to the fact that the electroweak penguin there is very suppressed with respect to the color-suppressed tree amplitude $C$. On the contrary, $CP$ violation of $\bar{B}_s \to \pi^0 \eta$ is very sizable due to the dominant $P_{EW}$. From Tables III and IV we see that the approaches of QCDF and pQCD have similar results for the rates of $\bar{B}_s \to \pi^0 \eta$ but quite different predictions for $A_{CP}(\bar{B}_s \to \pi^0 \eta)$. 

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TABLE V: Direct CP asymmetries (in %) in $\bar{B}_s \to PP$ decays via U-spin symmetry. Theoretical results of branching fractions and CP asymmetries for $\bar{B}_d \to PP$ are taken from [17].

| Modes              | $\mathcal{B}(10^{-6})$ | $A_{CP}$ (%) | Modes              | $A_{CP}$ (U-spin) (%) | $A_{CP}$ (QCDF) (%) |
|--------------------|-------------------------|--------------|--------------------|-----------------------|---------------------|
| $\bar{B}_d \to K^-\pi^+$ | $19.3^{+5.7}_{-4.3-6.2}$ | $-7.4^{+1.7}_{-1.5-4.8}$ | $\bar{B}_s \to K^+\pi^-$ | $25.9$ | $20.7^{+5.0}_{-3.0-8.8}$ |
| $\bar{B}_d \to \pi^+\pi^-$ | $7.0^{+0.4}_{-0.7-0.7}$ | $17.0^{+1.3}_{-1.2-8.7}$ | $\bar{B}_s \to K^+K^-$ | $-4.5$ | $-7.7^{+1.6}_{-1.2-5.1}$ |
| $\bar{B}_d \to K^0\pi^0$ | $8.6^{+3.8}_{-2.2-3.9}$ | $-10.6^{+2.7}_{-3.8-4.3}$ | $\bar{B}_s \to K^0\pi^0$ | $51.5$ | $36.3^{+17.4}_{-18.2-24.3}$ |
| $\bar{B}_d \to K^0K^0$ | $2.1^{+1.0}_{-0.6-0.8}$ | $-10.0^{+0.7}_{-0.7-1.9}$ | $\bar{B}_s \to K^0K^0$ | $0.77$ | $0.40^{+0.04}_{-0.04}$ |
| $\bar{B}_d \to K^+K^-$ | $0.10^{+0.03}_{-0.02-0.03}$ | $0$ | $\bar{B}_s \to \pi^+\pi^-$ | $0$ | $0$ |

6. Test of U-spin and SU(3) symmetries

There are five U-spin relations shown in Eqs. (4.5) and (4.7). We have pointed out before that the relation (4.5) is experimentally verified. For other relations, we are still lack of the measurements of CP asymmetries. Nevertheless, since the U-spin and SU(3) symmetry breaking is already included in QCDF calculations, we can test quantitatively how good the symmetry is. In Table V we show some of direct asymmetries for $\bar{B}_s$ decays evaluated using the U-spin relations Eqs. (4.5) and (4.7) and theoretical inputs for the branching fractions of $\bar{B}_d \to PP$ decays and CP asymmetries of $B_d \to PP$. We see that in general $A_{CP}$ obtained by U-spin symmetry is consistent with that obtained from direct QCDF calculations. In [11] two parameters

$$R_3 \equiv \frac{|A(B_s \to \pi^+K^-)|^2 - |A(B_s \to \pi^-K^+)|^2}{|A(B_s \to \pi^+K^-)|^2 + |A(B_s \to \pi^-K^+)|^2},$$

$$\Delta \equiv \frac{A_{CP}(\bar{B}_d \to \pi^+K^-)}{A_{CP}(B_s \to \pi^-K^+) + \mathcal{B}(\bar{B}_s \to \pi^+K^+) \mathcal{B}(B_d \to \pi^-K^+)}\mathcal{B}(B_d \to \pi^-K^+),$$

(4.17)

are defined to quantify the U-spin violation through the deviation of $R_3$ from $-1$ and $\Delta$ from 0. However, it is not suitable for the U-spin pair ($\bar{B}_s \to K^0\bar{K}^0$, $\bar{B}_d \to K^0\bar{K}^0$) for which we find $\Delta \approx -12$. In this case, it is better to compare $A_{CP}(\bar{B}_s \to K^0\bar{K}^0)$ obtained from the U-spin relation with the QCDF prediction as we have done in Table V.

As for the test of SU(3) symmetry, the first three relations in (4.12) are experimentally satisfied:

$$5.0 \pm 1.1 \pm 5.16 \pm 0.22, \quad 0.39 \pm 0.17 \equiv 0.38 \pm 0.06, \quad 24.4 \pm 4.8 \equiv 19.4 \pm 0.6,$$

where the branching fractions are in units of $10^{-6}$ and the data are taken from [48]. For the last three relations of (4.12) we have

$$-0.077^{+0.043}_{-0.062} \equiv -0.098^{+0.012}_{-0.011}, \quad 26.1^{+18.7}_{-12.4} \equiv 19.4 \pm 0.6, \quad 0.004^{+0.001}_{-0.006} \equiv 0.009 \pm 0.025,$$

where we have used the theoretical inputs for $B_s$ decays and experimental inputs for $B_d$ ones. Again, it appears that SU(3) symmetry relations are satisfactorily respected.

7. Mixing-induced CP asymmetry

Measurements of time-dependent CP asymmetries in neutral $B_s$ meson decays into a final CP eigenstate $f$ that is common to $B_s$ and $\bar{B}_s$ will provide the information on two interesting quantities: mixing-induced
TABLE VI: Same as Table III except for the mixing-induced $CP$ asymmetries $S_f$ in $\bar{B}_s \to PP$ decays. The parameter $\eta_f = 1$ except for $K_S(\pi^0, \eta, \eta')$ modes where $\eta_f = -1$.

| Modes          | Class | QCDF (this work)       | pQCD (LO) | pQCD (NLO) | SCET  |
|----------------|-------|------------------------|-----------|------------|-------|
| $\bar{B}_s \to K_S \pi^0$ | C     | $0.08^{+0.29+0.23}_{-0.27-0.26}$ | $-0.61^{+0.24}_{-0.20}$ | $-0.41^{+0.09}_{-0.13}$ | $-0.16 \pm 0.41 \pm 0.33 \pm 0.17$ |
| $\bar{B}_s \to K_S \eta$    | C     | $0.26^{+0.44+0.30}_{-0.33-0.21}$ | $-0.43^{+0.23}_{-0.23}$ | $-0.18^{+0.12}_{-0.23}$ | $0.82 \pm 0.32 \pm 0.11 \pm 0.04$ |
| $\bar{B}_s \to K_S \eta'$   | C     | $0.08^{+0.21+0.20}_{-0.17-0.16}$ | $-0.68^{+0.06}_{-0.05}$ | $-0.46^{+0.12}_{-0.23}$ | $0.38 \pm 0.08 \pm 0.10 \pm 0.04$ |
| $\bar{B}_s \to K^- K^+$   | $P$    | $0.22^{+0.04+0.05}_{-0.03-0.03}$ | $0.28^{+0.05}_{-0.03}$ | $0.22^{+0.04}_{-0.03}$ | $0.19 \pm 0.04 \pm 0.04 \pm 0.01$ |
| $\bar{B}_s \to K^0 \bar{K}$ | $P$    | $0.004^{+0.00+0.03}_{-0.00-0.00}$ | $0.04$ | $0.04^{+0.00}_{-0.00}$ | $0.24 \pm 0.09 \pm 0.15 \pm 0.05$ |
| $\bar{B}_s \to \eta \eta$ | $P$    | $-0.07^{+0.03+0.04}_{-0.06-0.05}$ | $0.03^{+0.01}_{-0.01}$ | $0.02^{+0.00}_{-0.00}$ | $-0.026 \pm 0.040 \pm 0.030 \pm 0.014$ |
| $\bar{B}_s \to \eta' \eta'$ | $P$    | $-0.01^{+0.00+0.00}_{-0.00-0.00}$ | $0.04^{+0.00}_{-0.00}$ | $0.04^{+0.00}_{-0.00}$ | $0.041 \pm 0.004 \pm 0.002 \pm 0.005$ |
| $\bar{B}_s \to \eta' \eta'$ | $P$    | $0.04^{+0.01+0.01}_{-0.00-0.00}$ | $0.04^{+0.01}_{-0.00}$ | $0.05^{+0.00}_{-0.00}$ | $0.049 \pm 0.005 \pm 0.005 \pm 0.003$ |
| $\bar{B}_s \to \pi^0 \eta$ | $P_{EW}$ | $0.26^{+0.06+0.48}_{-0.23-0.47}$ | $0.17^{+0.11}_{-0.13}$ | $0.28^{+0.05}_{-0.05}$ | $0.45 \pm 0.14 \pm 0.42 \pm 0.30$ |
| $\bar{B}_s \to \pi^0 \eta'$ | $P_{EW}$ | $0.88^{+0.04+0.04}_{-0.15-0.29}$ | $-0.17^{+0.08}_{-0.09}$ | $-0.18^{+0.12}_{-0.23}$ | $0.38 \pm 0.20 \pm 0.42 \pm 0.37$ |
| $\bar{B}_s \to \pi^+ \pi^-$ | ann     | $0.15^{+0.00+0.00}_{-0.00-0.00}$ | $0.14^{+0.12}_{-0.06}$ | $0.09^{+0.02}_{-0.00}$ | $0.38 \pm 0.20 \pm 0.42 \pm 0.37$ |
| $\bar{B}_s \to \pi^0 \pi^0$ | ann     | $0.15^{+0.00+0.00}_{-0.00-0.00}$ | $0.14^{+0.12}_{-0.06}$ | $0.08^{+0.00}_{-0.00}$ | $0.38 \pm 0.20 \pm 0.42 \pm 0.37$ |

$CP$ asymmetry $S_f$ and direct $CP$ violation $A_f$ which can be expressed as

$$ A_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad (4.20) $$

where

$$ \lambda_f = \frac{q_{\bar{B}_s} A(\bar{B}_s \to f)}{p_{\bar{B}_s} A(B_s \to f)} = \frac{V_{tb}^* V_{ts}}{V_{tb}^* V_{ts}} A(\bar{B}_s \to f) \cdot \frac{A(B_s \to f)}{A(B_s \to f)}, \quad (4.21) $$

Now let $q_{\bar{B}_s} / p_{\bar{B}_s} = e^{2i\beta}$ and

$$ \tilde{A}(\bar{B}_s \to f) = A_{1f} e^{i(\phi_{A1} + \delta_1)} + A_{2f} e^{i(\phi_{A2} + \delta_2)}, $$

$$ A(B_s \to f) = \eta_f \left( A_{1f} e^{-(\phi_{A1} + \delta_1)} + A_{2f} e^{-(\phi_{A2} + \delta_2)} \right), $$

where $CP|f\rangle = \eta_f |f\rangle$ with $\eta_f = 1$ ($-1$) for final $CP$-even (odd) states, $\phi_{A1, A2}$ are weak phases and $\delta_{1, 2}$ strong phases. It follows that (see e.g. [59])

$$ \lambda_f = \eta_f e^{2i\phi_1} \frac{1 + re^{i(\phi_1 - \phi_0)} e^{i\delta}}{1 + re^{-(\phi_1 - \phi_0)} e^{i\delta}}, \quad (4.22) $$

with $\phi_{1, 2} = \phi_{A1, A2} + \beta$, $\delta = \delta_2 - \delta_1$ and $r = A_2/A_1$. 

20
For $B_s$ decays, the phase $\beta_s$ due to the $B_s - \bar{B}_s$ mixing is very small in the SM, of order $1^\circ$. For the decays $\bar{B}_s \to K^0\bar{K}^0, \eta\eta, \eta'\eta'$ dominated by penguin diagrams (tree contributions to $\eta^{(*)}\eta^{(*)}$ are color suppressed), $r \simeq 0$ and the phase $\phi_{A1}$ due to $V_{cb}V_{cs}^*$ or $V_{tb}V_{ts}^*$ is also very small. Consequently, $S_f$ are small for penguin-dominated $\bar{B}_s \to PP$ decays except for $\bar{B}_s \to K^+ K^-$ which receives a tree contribution with $\phi_{A2} = \gamma$. We see from Table VI that QCDF, pQCDF and SCET all predict $S_{\bar{B}_s \to K^+ K^-} \simeq 0.20$. Recently, both CDF [2] and D0 [3] have reported fits to angular and time distributions of flavor-tagged $B_s \to J/\psi \phi$ decays which favor a larger value of $\beta_s$ deviated from the SM by 1-2$\sigma$ effects. If this is the case, then mixing-induced CP violation in $\bar{B}_s \to K^0\bar{K}^0, \eta\eta, \eta'\eta'$ could be sizable. Hence, these modes offer rich possibilities of testing New Physics beyond the SM.

Due to the large magnitude and strong phase of $a_2$ induced from soft power corrections to the color-suppressed tree amplitude, for example, $a_2(\pi\pi) = 0.77e^{-i52^\circ}$ (or $0.41e^{-i11^\circ}$ before corrections), we find that such corrections will flip the sign of $S_f$ into the positive one for the color-suppressed decays $\bar{B}_s \to K_S(\pi^0, \eta, \eta')$, while they are all negative in the pQCD approach. Recently, it has been claimed that in the pQCD approach there exist uncanceled soft divergences in the $k_T$ factorization for the nonfactorizable $B$ meson decay amplitudes [60]. This will enhance the nonfactorizable color-suppressed tree amplitudes. It remains to check if the signs of $S_{\bar{B}_s \to K_S(\pi^0, \eta, \eta')}$ in pQCD will be flipped again under this “$a_2^\prime$” enhancement.

V. $B_s \to VP$ DECAYS

A. Branching fractions

The tree-dominated decays $\bar{B}_s \to K^{*+} \pi^-$ and $\rho^- K^+$ with the amplitudes

$$A(\bar{B}_s \to K^{*+} \pi^-) \approx A_{K^{*+}\pi}(\delta_{pu}a_1 + a_4 - r_\chi^2a_6),$$

$$A(\bar{B}_s \to \rho^- K^+) \approx A_{\rho K}(\delta_{pu}a_1 + a_4 + r_\chi^2a_6),$$

have branching fractions of order $10^{-5}$. Since $A_{K^{*+}\pi} = X(\bar{B}_s, K^{*+}\pi) \approx f_\pi A_0^{\bar{B}K}(0)m_{\bar{B}_s}^2$ and $A_{\rho K} = X(\bar{B}_s, \rho K) \approx f_\rho F_0^{\bar{B}K}(0)m_{\bar{B}_s}^2$ [see Eq. (2.3)], it is clear that the $\rho^- K^+$ mode has a rate larger than $K^{*+} \pi^-$ due to the hierarchy of the decay constants $f_\rho \gg f_\pi$. The penguin-dominated $\bar{B}_s \to VP$ decays such as $K^{*-} K^+$ and $K^{*0} K^0$ have rates smaller than the counterparts in the $PP$ sector as the amplitudes are proportional to $a_4 + r_\chi^2a_6$ or $a_4 - r_\chi^2a_6$ for the former and $a_4 + r_\chi^2a_6$ for the latter. Since $a_4$ and $a_6$ are of the same sign and $r_\chi^2 > r_\chi^2$, it is evident that the interference of the penguin terms is constructive for $PP$ and either destructive or less constructive for $VP$. The decay $\bar{B}_s \to \phi K^0$ is dominated by the $b \to d$ penguin transition and its rate is thus much smaller compared to $b \to s$ dominated $\bar{B}_s \to K^+ K^-$ decays.

We see from Table VII that the pQCD predictions for the color-suppressed tree-dominated decays $\bar{B}_s \to K^{*0} \pi^0, \rho^0 K^0, \omega K^0, K^{*0} \eta'$ are one order of magnitude smaller than QCDF and SCET in rates. For example, $\mathcal{B}(\bar{B}_s \to \rho^0 K^0)$ is predicted to be of order $1.9 \times 10^{-6}$ in the approach of QCDF, but it is only about $0.08 \times 10^{-6}$ in pQCD. The calculated branching fractions in pQCD for $K^{*0} \eta$ and some of the penguin-dominated decays e.g. $\bar{B}_s \to K^{*+} K^-, K^{*0} K^0, \phi K^0, \phi \eta'$ are also much smaller than QCDF. In the following we will

\[4\] In the $B_{u,d}$ systems, $a_2(\pi\pi) = 0.51e^{-i58^\circ}$ (or $0.27e^{-i17^\circ}$ before corrections).
comment on the decays \( \bar{B}_s \to \phi \eta^{(')} \). While the QCDF approach leads to \( \mathcal{R}(\bar{B}_s \to \phi \eta^{(')}) > \mathcal{R}(\bar{B}_s \to \phi \eta) \), pQCD and SCET predict very different patterns: \( \mathcal{R}(\bar{B}_s \to \phi \eta) \gg \mathcal{R}(\bar{B}_s \to \phi \eta^{(')}) \) in the pQCD approach and it is the other way around in SCET (see Table VII). We should stress that the decay rate of \( \bar{B}_s \to \phi \eta^{(')} \) is sensitive to the form factor \( A_{0,\phi}^B(0) \). The decay amplitudes of \( \bar{B}_s \to \phi \eta^{(')} \) are given by

\[
A(\bar{B}_s \to \phi \eta) = \cos \theta A(\bar{B}_s \to \phi \eta_\alpha) - \sin \theta A(\bar{B}_s \to \phi \eta_\delta), \\
A(\bar{B}_s \to \phi \eta^{(')}) = \sin \theta A(\bar{B}_s \to \phi \eta_\alpha) + \cos \theta A(\bar{B}_s \to \phi \eta_\delta),
\]

with

\[
A(\bar{B}_s \to \phi \eta_\alpha) = A_{\phi \eta_\alpha}(\alpha_3^0 + \alpha_4^0) + A_{\eta_\alpha}(\alpha_3^0 + \alpha_4^0), \\
\sqrt{2}A(\bar{B}_s \to \phi \eta_\delta) = A_{\phi \eta_\delta}(\delta_\rho \alpha_2 + 2 \alpha_3^0).
\]

Since \( \alpha_3^0(\phi \eta_\alpha) = a_4 - r_2^0 a_6 \) and \( \alpha_3^0(\eta_\alpha, \phi) = a_4 + r_2^0 a_6 \) are of opposite sign (numerically, \( \alpha_3^0(\phi \eta_\alpha) \approx 0.038 \) and \( \alpha_3^0(\eta_\alpha, \phi) \approx -0.033 \)), there is a cancellation between the two penguin amplitudes of \( \bar{B}_s \to \phi \eta_\alpha \). Note that \( \alpha_3^0(\phi \eta_\delta) \) and \( \alpha_3^0(\eta_\delta, \phi) \) also are of opposite sign. It turns out that the sign of \( A(\bar{B}_s \to \phi \eta_\delta) \) depends on the form factor \( A_{0,\phi}^B(0) \). For \( A_{0,\phi}^B(0) = 0.32 \) as employed in the present work, \( \bar{B}_s \to \phi \eta_\delta \) and \( \bar{B}_s \to \phi \eta_\alpha \) will contribute constructively to \( \bar{B}_s \to \phi \eta^{(')} \) so that \( \mathcal{R}(\bar{B}_s \to \phi \eta^{(')}) = 2.2 \times 10^{-6} \). However, if we use the sum-rule prediction \( A_{0,\phi}^B(0) = 0.474 \) from Eq. (3.3), then a near cancellation between \( \bar{B}_s \to \phi \eta_\delta \) and \( \bar{B}_s \to \phi \eta_\alpha \) occurs in the decays \( \bar{B}_s \to \phi \eta^{(')} \), so that its branching fraction, of order \( 10^{-7} \), becomes very small. Hence, it is very important to measure the branching fractions of \( \bar{B}_s \to \phi \eta^{(')} \) to gain the information on the form factor \( A_{0,\phi}^B \).

One unique feature of the \( B_s \) decays is that there exist several modes dominated by electroweak penguins: \( B_s \to \pi^0 \eta^{(')}, \phi \pi^0, \rho^0 \eta^{(')} \) and \( \phi \rho^0 \). The isospin for the final states of these decays is \( I = 1 \) and hence the electroweak penguin is the only loop contribution that one can have. It dominates over the color-suppressed tree contribution due to the large CKM matrix element associated with the electroweak penguin amplitude. Since a large complex electroweak penguin amplitude due to New Physics is also a possible solution to the \( B \to K \pi \) CP puzzle, it has been advocated that this hypothesis can be tested in the decays \( \bar{B}_s \to \phi \pi^0, \phi \rho^0 \) whose rates may get an enhancement by an order of magnitude [61].

### B. Direct CP asymmetries

Direct CP asymmetries of \( \bar{B}_s \to V \pi \) decays estimated in various approaches are summarized in Table VII. In QCDF calculations, the signs of CP asymmetries for color-suppressed tree-dominated decays \( \bar{B}_s \to K^{*0} \pi^0, \rho^0 K^0, \rho^0 K^0 \) and \( K^{*0} \eta \) are governed by the soft corrections to \( a_2 \) [see Eq. (3.26)]. We see that QCDF and pQCD results agree with each other in signs, whereas SCET predicts opposite signs for these modes. Since the corresponding rates of these decays are very small in pQCD, as a consequence, the CP-violating asymmetries predicted by pQCD are very large, of order 0.50 or even bigger.

In the pQCD approach, the penguin-dominated decays \( \bar{B}_s \to K^0 \phi, \bar{K}^0 K^0, K^{*0} \bar{K}^0 \) have no direct CP asymmetry as their decay amplitudes are governed by one type of CKM matrix elements, e.g. \( V_{ub} V_{td}^* \) for the first mode and \( V_{ub} V_{td}^*, V_{ub} V_{ts}^* \) for the last two. As noticed before for the decay \( \bar{B}_s \to K^0 \bar{K}^0 \), NLO corrections from penguin loop interactions can bring a weak phase necessary for a non-vanishing CP violation. Therefore, it is important to carry out pQCD calculations to NLO for those three modes. In the approach of SCET,
TABLE VII: CP-averaged branching fractions (in units of 10^-6) of $\bar{B}_s \to PV$ decays calculated in various approaches. The LO pQCD predictions are taken from \cite{11}, while two different sets of SCET results are quoted from \cite{14}.

| Modes  | Class | QCDF (this work) | PQCD   | SCET 1       | SCET 2       |
|--------|-------|------------------|--------|--------------|--------------|
| $\bar{B}_s^0 \to K^+\pi^-$ | $T$ | 7.8$^{+0.4+0.5}_{-0.7}$ | 7.6$^{+2.0+0.4+0.5}_{-2.2-0.5-0.3}$ | 5.9$^{+0.5+0.5}_{-0.5}$ | 6.6$^{+0.2+0.7}_{-0.1-0.7}$ |
| $\bar{B}_s^0 \to \rho^- K^+$ | $T$ | 14.7$^{+1.4+0.9}_{-1.9-1.3}$ | 17.8$^{+5.6+1.6-0.9}_{-5.6-1.6-0.3}$ | 7.6$^{+0.3+0.8}_{-0.1-0.8}$ | 10.2$^{+0.4+0.9}_{-0.5-0.9}$ |
| $\bar{B}_s^0 \to \bar{K}^0\pi^0$ | $C$ | 0.89$^{+0.80+0.84}_{-0.34-0.35}$ | 0.07$^{+0.02+0.04+0.01}_{-0.01-0.02-0.01}$ | 0.90$^{+0.07+0.10}_{-0.01-0.01}$ | 1.07$^{+0.16+0.10}_{-0.15-0.09}$ |
| $\bar{B}_s^0 \to \rho^0 K^0$ | $C$ | 1.9$^{+2.9+1.4}_{-0.9-0.6}$ | 0.08$^{+0.02+0.07+0.01}_{-0.02-0.03-0.00}$ | 2.0$^{+0.2+0.2}_{-0.2-0.2}$ | 0.81$^{+0.05+0.08}_{-0.02-0.09}$ |
| $\bar{B}_s^0 \to \omega K^0$ | $C$ | 1.6$^{+2.2+1.0}_{-0.7-0.5}$ | 0.15$^{+0.05+0.07+0.02}_{-0.04-0.03-0.01}$ | 0.90$^{+0.08+0.10}_{-0.01-0.01}$ | 1.3$^{+0.1+0.1}_{-0.1-0.1}$ |
| $\bar{B}_s^0 \to K^+K^+$ | $P$ | 10.3$^{+3.0+4.8}_{-2.2-4.2}$ | 6.0$^{+1.7+1.7+0.7}_{-1.3-1.2-0.3}$ | 8.4$^{+4.4+1.6}_{-3.4-1.3}$ | 9.5$^{+3.2+1.2}_{-2.8-1.1}$ |
| $\bar{B}_s^0 \to K^0\bar{K}^0$ | $P$ | 11.3$^{+5.0+8.1}_{-3.5-5.1}$ | 4.7$^{+0.7+1.2+0.5}_{-0.8-1.4-0.0}$ | 9.8$^{+4.6+1.7}_{-3.7-1.4}$ | 10.2$^{+3.8+1.5}_{-3.2-1.2}$ |
| $\bar{B}_s^0 \to \rho K^0$ | $P$ | 10.5$^{+3.4+5.1}_{-2.8-4.5}$ | 7.3$^{+2.5+2.1+0.0}_{-1.7-1.3-0.0}$ | 7.9$^{+4.4+1.6}_{-3.4-1.3}$ | 9.3$^{+3.2+1.2}_{-2.8-1.0}$ |
| $\bar{B}_s^0 \to s\bar{s}$ | $P$ | 10.1$^{+4.5+7.6}_{-3.6-4.8}$ | 4.3$^{+0.7+2.2+0.0}_{-0.7-1.4-0.0}$ | 8.7$^{+4.6+1.7}_{-3.5-1.4}$ | 9.4$^{+3.1+1.2}_{-3.0-1.1}$ |
| $\bar{B}_s^0 \to \phi K^0$ | $P_{EW}$ | 0.6$^{+0.05+0.04}_{-0.02-0.03}$ | 0.16$^{+0.04+0.09+0.02}_{-0.03-0.04-0.01}$ | 0.44$^{+0.23+0.08}_{-0.18-0.07}$ | 0.54$^{+0.21+0.08}_{-0.17-0.07}$ |
| $\bar{B}_s^0 \to \phi K^0$ | $P_{EW}$ | 0.12$^{+0.02+0.04}_{-0.01-0.02}$ | 0.16$^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$ | 0.07$^{+0.00+0.01}_{-0.00-0.00}$ | 0.09$^{+0.00+0.01}_{-0.00-0.00}$ |
| $\bar{B}_s^0 \to \rho K^0$ | $P_{EW}$ | 0.03$^{+0.00+0.01}_{-0.00-0.01}$ | 0.22$^{+0.05+0.04+0.00}_{-0.05-0.06-0.01}$ | 0.00$^{+0.00+0.00}_{-0.00-0.00}$ | 0.00$^{+0.00+0.00}_{-0.00-0.00}$ |
| $\bar{B}_s^0 \to s\bar{s}$ | $P_{EW}$ | 0.02$^{+0.00+0.01}_{-0.00-0.01}$ | 0.24$^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$ | 0.00$^{+0.00+0.00}_{-0.00-0.00}$ | 0.00$^{+0.00+0.00}_{-0.00-0.00}$ |
| $\bar{B}_s^0 \to \rho^0 K^0$ | $P_{EW}$ | 0.03$^{+0.00+0.01}_{-0.00-0.01}$ | 0.22$^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$ | 0.00$^{+0.00+0.00}_{-0.00-0.00}$ | 0.00$^{+0.00+0.00}_{-0.00-0.00}$ |

$CP$ asymmetries of the decays $\bar{B}_s \to \pi^0\phi$ and $\bar{B}_s \to \rho^0(\omega)(\eta, \eta')$ also vanish. As explained in \cite{14}, there is no charming penguins in these 5 channels and hence no direct $CP$ violation due to the lack of strong phases.

We use this chance to clarify one misconception about $CP$ violation under isospin symmetry. The isospin of the final-state is $I=1$ for $\bar{B}_s \to \phi \pi^0$, $\rho^0(\omega)(\eta, \eta')$ and $I=0$ for $\bar{B}_s \to (\phi, \omega)\eta(\eta')$. One may argue that there is no $CP$ violation for these decays as they have only one isospin strong phase (see e.g. \cite{22}). On the contrary, we found large direct $CP$-violating effects in some of above decays (see Table VIII). The point is that isospin phases should not be confused with other possible strong phases in each of topological amplitudes. In our study, $CP$ asymmetries of $\bar{B}_s \to \rho(\eta)\eta(\eta')$ are large since the electroweak penguins dominate over the color-suppressed tree amplitudes.

\footnote{By the same token, it has been (wrongly) claimed that the direct $CP$ asymmetry is strictly zero in the charged $B^- \to \pi^- \pi^0$ decay.}
TABLE VIII: Same as Table VII except for the direct CP asymmetries (in %) in $\bar{B}_s \to PV$ decays.

| Modes | Class | QCD (this work) | PQCD | SCET 1 | SCET 2 |
|-------|-------|----------------|------|--------|--------|
| $\bar{B}_s \to K^{*+}\pi^-$ | $T$ | $-24.0^{+1.2+7.7}_{-1.5-3.9}$ | $-19.6^{+2.5+2.7+0.9}_{-2.6-3.4-1.4}$ | $-9.9^{+17.2+0.9}_{-16.7-0.7}$ | $-12.4^{+17.5+1.1}_{-15.3-1.2}$ |
| $\bar{B}_s \to \rho^+K^-$ | $T$ | $11.7^{+5.3+10.1}_{-2.1-11.6}$ | $14.2^{+2.4+2.3+1.2}_{-2.2-1.6-0.7}$ | $11.8^{+17.5+1.2}_{-20.0-1.1}$ | $10.8^{+9.4+0.9}_{-10.2-1.0}$ |
| $\bar{B}_s \to K^{0}\pi^0$ | $C$ | $-26.3^{+10.8+42.2}_{-10.9-36.7}$ | $-47.1^{+7.4+35.5+2.9}_{-8.7-29.8-7.0}$ | $22.9^{+33.1+2.1}_{-40.2-1.9}$ | $13.4^{+18.6+0.8}_{-18.8-1.2}$ |
| $\bar{B}_s \to \rho^0K^0$ | $C$ | $28.9^{+14.6+25.0}_{-14.5-23.7}$ | $73.4^{+6.4+16.2+2.2}_{-11.7-47.8-3.9}$ | $-12.0^{+20.1+1.0}_{-19.6-0.7}$ | $-32.5^{+30.7+2.7}_{-23.4-2.9}$ |
| $\bar{B}_s \to \phi K^0$ | $C$ | $-32.0^{+19.8+23.6}_{-17.5-26.2}$ | $-52.1^{+3.2+22.7+3.2}_{-0.0-15.1-2.0}$ | $24.4^{+33.7+2.2}_{-41.4-2.0}$ | $18.2^{+16.4+1.2}_{-17.0-1.7}$ |
| $\bar{B}_s \to K^-K^+$ | $P$ | $-11.0^{+0.5+14.0}_{-0.4-18.8}$ | $-36.6^{+2.3+2.8+1.3}_{-5.8+3.5+1.2}$ | $-11.2^{+19.1+1.3}_{-16.2-1.3}$ | $-12.3^{+11.4+0.8}_{-11.3-0.8}$ |
| $\bar{B}_s \to K^+K^-$ | $P$ | $25.5^{+9.2+16.3}_{-8.8-11.3}$ | $55.3^{+4.4+8.5+5.1}_{-4.9-9.8-2.5}$ | $7.1^{+11.2+0.7}_{-12.4-0.7}$ | $9.6^{+13.0+0.7}_{-13.5-0.9}$ |

C. Test of $U$-spin and SU(3) symmetries

The pairs related by $U$-spin symmetry are [45]: $(\bar{B}_d \to K^+K^-, \bar{B}_s \to \rho^+K^+), (\bar{B}_d \to K^-\rho^+, \bar{B}_s \to K^-K^-), (\bar{B}_d \to \rho^-\pi^+, \bar{B}_s \to K^+K^-), (\bar{B}_d \to \rho^0K^0, \bar{B}_s \to K^0\bar{K}^0), (\bar{B}_d \to K^0\bar{K}^0, \bar{B}_s \to K^0\rho^0), (\bar{B}_d \to K^0\rho^0, \bar{B}_s \to K^0K^0)$. Note that unlike $PP$ and $VV$ modes, $\bar{B}_s \to K^0\pi^0$ and $\bar{B}_s \to K^0\rho^0$ are not related to $\bar{B}_d \to \rho^0K^0$ and $\bar{B}_d \to K^0\rho^0$, respectively. Direct CP asymmetries of the pairs listed above are related by $U$-spin symmetry in analogue to Eq. (4.5) or Eq. (4.7). The test of $U$-spin symmetry in $B_s \to VP$ decays is shown in Table IX. It turns out that $U$-spin symmetry is in general acceptable.

Just as $B_s \to PP$ decays, under the approximation of negligible annihilation contributions to tree-dominated decays and keeping only the dominant penguin annihilation terms in penguin-dominated decays, SU(3) symmetry leads to [44,49]

$$A(\bar{B}_s \to K^+\pi^-) \approx A(\bar{B}_d \to \rho^+\pi^-), \quad A(\bar{B}_s \to \rho^-K^+) \approx A(\bar{B}_d \to \rho^-\pi^+),$$

$$A(\bar{B}_s \to K^+K^-) \approx A(\bar{B}_d \to \rho^+K^-), \quad A(\bar{B}_s \to K^-K^+) \approx A(\bar{B}_d \to K^-\pi^+). \quad (5.4)$$
Thus, we have the relations
\[ \mathcal{B}(\bar{B}_s \rightarrow K^{+}\pi^-) \approx \mathcal{B}(\bar{B}_d \rightarrow \rho^+\pi^-), \quad \mathcal{B}(\bar{B}_s \rightarrow \rho^-K^+) \approx \mathcal{B}(\bar{B}_d \rightarrow \rho^-\pi^+), \]
\[ \mathcal{B}(\bar{B}_s \rightarrow K^{+}K^-) \approx \mathcal{B}(\bar{B}_d \rightarrow \rho^+K^-), \quad \mathcal{B}(\bar{B}_s \rightarrow K^{+}\bar{K}^-) \approx \mathcal{B}(\bar{B}_d \rightarrow K^{-}K^+), \tag{5.5} \]

and
\[ A_{\mathcal{CP}}(\bar{B}_s \rightarrow K^{+}\pi^-) \approx A_{\mathcal{CP}}(\bar{B}_d \rightarrow \rho^+\pi^-), \quad A_{\mathcal{CP}}(\bar{B}_s \rightarrow \rho^-K^+) \approx A_{\mathcal{CP}}(\bar{B}_d \rightarrow \rho^-\pi^+), \]
\[ A_{\mathcal{CP}}(\bar{B}_s \rightarrow K^{+}K^-) \approx A_{\mathcal{CP}}(\bar{B}_d \rightarrow \rho^+K^-), \quad A_{\mathcal{CP}}(\bar{B}_s \rightarrow K^{+}\bar{K}^-) \approx A_{\mathcal{CP}}(\bar{B}_d \rightarrow K^{-}K^+). \tag{5.6} \]

Numerically,
\[ 7.8^{+0.6}_{-1.0} \equiv 9.2^{+0.6}_{-1.0}, \quad 14.7^{+1.7}_{-2.3} \approx 15.9^{+1.4}_{-1.9}, \]
\[ 11.3^{+10.7}_{-6.2} \approx 8.6^{+9.3}_{-5.3}, \quad 10.3^{+5.7}_{-4.7} \approx 9.2^{+3.8}_{-3.4}, \tag{5.7} \]

for branching fractions in units of \(10^{-6}\) and
\[ -24.0^{+7.8}_{-4.2} \approx -22.7^{+8.2}_{-4.5}, \quad 11.7^{+10.7}_{-11.8} \approx 4.4^{+5.8}_{-6.8}, \]
\[ 25.5^{+18.7}_{-14.3} \approx 31.9^{+22.7}_{-16.8}, \quad -11.0^{+14.0}_{-18.8} \approx -12.1^{+12.6}_{-16.0}. \tag{5.8} \]

for direct \(CP\) asymmetries in %. Hence, the above \(SU(3)\) relations are generally respected.

### D. Mixing-induced \(CP\) asymmetry

As discussed before, due to the tiny phase in the \(B_s - \bar{B}_s\) mixing and in the CKM matrix element \(V_{cb}\) or \(V_{ub}\) mixing, mixing-induced \(CP\) violation \(S_f\) is expected to be very small in the penguin-dominated \(\bar{B}_s \rightarrow \phi \eta'\) decays. This is indeed borne out in all model calculations. The \(b \rightarrow d g\) penguin-dominated decay \(\bar{B}_s \rightarrow K_S\phi\) has a large mixing-induced \(CP\) asymmetry due to the fact that the CKM matrix element \(V_{ub}\) has a weak phase \(\gamma\). More specifically,
\[ A(\bar{B}_s \rightarrow K_S\phi) \propto V_{ub}V_{ud}^*[A_{K\phi}(\alpha_3^u + A_{K\phi}(\alpha_3^u + \beta_3^u))] + V_{cb}V_{cd}^*[A_{K\phi}(\alpha_3^c + A_{K\phi}(\alpha_3^c + \beta_3^c))]. \tag{5.9} \]

To the approximation that \(\alpha_3^u \approx \alpha_3^u\) and \(\beta_3^u \approx \beta_3^u\), it is clear that \(A(\bar{B}_s \rightarrow K_S\phi) \propto V_{ub}V_{ud}^{*}e^{i\beta}.\) Thus, \(S_{\bar{B}_s \rightarrow K_S\phi} \approx -\sin 2(\beta_s + \beta) = -0.71\) for \(\beta_s \approx 1^\circ\) and \(\beta = 21.58^\circ.\) In the pQCD approach, this decay is
TABLE X: Same as Table VII except for mixing-induced CP asymmetries $S_f$ in $B_s$ → PV decays. The parameter $\eta_f = 1$ except for $K_S(p^0, \omega, \phi)$ modes where $\eta_f = -1$. Note that the error estimate of $S_{B_s \rightarrow K_S \phi}$ is not available in the pQCD calculation [11].

| Modes            | Class | QCDF (this work) | pQCD          | SCET 1     | SCET 2     |
|------------------|-------|-----------------|---------------|------------|------------|
| $B_s^0 \rightarrow K_S p^0$ | $C$   | $0.29 \pm 0.23$ | $-0.57 \pm 0.51$ | $0.99 \pm 0.02$ | $-0.03 \pm 0.17$ |
| $B_s^0 \rightarrow K_S \omega$ | $C$   | $0.92 \pm 0.03$ | $-0.63 \pm 0.28$ | $-0.11 \pm 0.18$ | $0.98 \pm 0.02$ |
| $B_s^0 \rightarrow K_S \phi$   | $P$   | $-0.69 \pm 0.01$ | $-0.09 \pm 0.02$ | $0.13 \pm 0.02$ | $-0.02 \pm 0.01$ |
| $B_s^0 \rightarrow \phi \eta$   | $P$   | $0.21 \pm 0.08$ | $-0.03 \pm 0.07$ | $-0.39 \pm 0.43$ | $0.23 \pm 0.35$ |
| $B_s^0 \rightarrow \phi \eta'$  | $P$   | $0.08 \pm 0.05$ | $0.00 \pm 0.02$ | $-0.07 \pm 0.06$ | $0.10 \pm 0.07$ |
| $B_s^0 \rightarrow \omega \eta$  | $P, C$ | $-0.76 \pm 0.16$ | $0.02 \pm 0.04$ | $-0.62 \pm 0.41$ | $0.93 \pm 0.04$ |
| $B_s^0 \rightarrow \pi^0 \phi$  | $P, C$ | $-0.84 \pm 0.06$ | $0.01 \pm 0.04$ | $-0.25 \pm 1.23$ | $-1.00 \pm 0.04$ |
| $B_s^0 \rightarrow \rho^0 \eta$  | $P_{EW}$ | $0.40 \pm 0.04$ | $0.07 \pm 0.08$ | $0.89 \pm 0.09$ | $0.90 \pm 0.00$ |
| $B_s^0 \rightarrow \rho^0 \eta'$ | $P_{EW}$ | $0.35 \pm 0.09$ | $0.15 \pm 0.14$ | $1.00 \pm 0.06$ | $0.60 \pm 0.53$ |
| $B_s^0 \rightarrow \rho^0 \pi^0$ | ann   | $-0.65 \pm 0.03$ | $-0.16 \pm 0.10$ | $0.95 \pm 1.60$ | $-0.41 \pm 0.75$ |

dominated by the $(S-P)(S+P)$ penguin annihilation process with the CKM matrix element proportional to $V_{tb} V_{td}^*$. Therefore, both QCDF and pQCD predict $S_{B_s \rightarrow K_S \phi} \sim O(0.70)$. (However, no error estimate is done in the pQCD calculation [11].) On the contrary, the SCET result of $S_{B_s \rightarrow K_S \phi} \sim 0.09$ or $-0.13$ is dramatically different from the QCD and pQCD predictions. As explained in [14], charming penguin contributions to $B_s \rightarrow K_S \phi$ dominate over penguin operators and the CKM matrix element associated with charming penguins is $V_{cb} V_{td}^*$. Hence, $S_{B_s \rightarrow K_S \phi} = -\sin 2\beta_s = -0.03$ is predicted by SCET when penguin contributions are neglected. It should be stressed that although both QCDF and pQCD approaches have similar results for $S_{B_s \rightarrow K_S \phi}$, they differ in the prediction of $A_{CP}(\bar{B}_s \rightarrow K_S \phi)$: it is of order $-0.03$ in QCDF and vanishes in pQCD for reasons mentioned above.

The study of CP violation for $\bar{B}_s \rightarrow K^{*+} K^-$ and $K^- K^+$ is more complicated as $K^{*+} K^+$ are not CP eigenstates. The time-dependent CP asymmetries are given by

$$
\mathcal{A}(t) = \frac{\Gamma(\bar{B}_s^0(t) \rightarrow K^{*+} K^+) - \Gamma(B_s^0(t) \rightarrow K^{*+} K^+)}{\Gamma(\bar{B}_s^0(t) \rightarrow K^{*+} K^+) + \Gamma(B_s^0(t) \rightarrow K^{*+} K^+)}
= (S \pm \Delta S) \sin(\Delta m_s t) - (C \pm \Delta C) \cos(\Delta m_s t),
$$

(5.10)

where $\Delta m_s$ is the mass difference of the two neutral $B_s$ eigenstates, $S$ is referred to as mixing-induced CP asymmetry and $C$ is the direct CP asymmetry ($C = -A_{CP}$), while $\Delta S$ and $\Delta C$ are CP-conserving quantities. In writing the above equation we have neglected the effects of the width difference of the $B_s$ mesons.

Defining

$$
A_{+-} = A(B_s^0 \rightarrow K^{*+} K^-), \quad A_{-+} = A(B_s^0 \rightarrow K^+ K^-), \\
\tilde{A}_{+-} = A(\bar{B}_s^0 \rightarrow K^- K^+), \quad \tilde{A}_{-+} = A(\bar{B}_s^0 \rightarrow K^+ K^-),
$$

(5.11)

and

$$
\lambda_{+-} = \frac{q_{B_s}}{p_{B_s}} A_{+-}, \quad \lambda_{-+} = \frac{q_{B_s}}{p_{B_s}} \tilde{A}_{-+},
$$

(5.12)

26
we have
\[ C + \Delta C = \frac{1 - |\lambda_+|^2}{1 + |\lambda_+|^2} = \frac{|A_+|^2 - |\bar{A}_+|^2}{|A_+|^2 + |A_+|^2}, \]
\[ C - \Delta C = \frac{1 - |\bar{\lambda}_+|^2}{1 + |\bar{\lambda}_+|^2} = \frac{|A_-|^2 - |\bar{A}_-|^2}{|A_-|^2 + |A_-|^2}, \]
and
\[ S + \Delta S \equiv \frac{2 \text{Im} \lambda_+}{1 + |\lambda_+|^2} = \frac{2 \text{Im}(e^{2i\beta} \bar{A}_+ A_+^*)}{|A_+|^2 + |A_+|^2}, \]
\[ S - \Delta S \equiv \frac{2 \text{Im} \bar{\lambda}_+}{1 + |\bar{\lambda}_+|^2} = \frac{2 \text{Im}(e^{2i\beta} \bar{A}_- A_-^*)}{|A_-|^2 + |A_-|^2}. \]
Hence we see that $\Delta S$ describes the strong phase difference between the amplitudes contributing to $B_s^0 \to K^{*+}K^-$ and $\Delta C$ measures the asymmetry between $\Gamma(B_s^0 \to K^{*+}K^-) + \Gamma(B_s^0 \to K^{*-}K^-)$ and $\Gamma(B_s^0 \to K^{*-}K^+) + \Gamma(B_s^0 \to K^{*+}K^-)$.

Next consider the time- and flavor-integrated charge asymmetry
\[ \mathcal{A}_{K^*K} \equiv \frac{|A_+|^2 + |\bar{A}_+|^2 - |A_+|^2 - |\bar{A}_+|^2}{|A_+|^2 + |A_+|^2 + |A_-|^2 + |\bar{A}_+|^2}, \]
(5.15)
Then, following [37] one can transform the experimentally motivated $CP$ parameters $\mathcal{A}_{K^*K}$ and $C_{K^*K}$ into the physically motivated choices
\[ A_{K^{*+}K^-} \equiv \frac{|\chi^-|^2 - 1}{|\chi^-|^2 + 1}, \quad A_{K^{*-}K^+} \equiv \frac{|\chi^+|^2 - 1}{|\chi^+|^2 + 1}, \]
(5.16)
with
\[ \chi^+ = \frac{q_\mu}{p_{\mu} A_+}, \quad \chi^- = \frac{\bar{q}_\mu}{\bar{p}_{\mu} \bar{A}_+}. \]
(5.17)
Hence,
\[ A_{K^{*+}K^-} = \frac{\Gamma(B_s^0 \to K^{*+}K^-) - \Gamma(B_s^0 \to K^{*+}K^-)}{\Gamma(B_s^0 \to K^{*+}K^-) + \Gamma(B_s^0 \to K^{*+}K^-)} = \frac{\mathcal{A}_{K^*K} - C_{K^*K} - \mathcal{A}_{K^*K} \Delta C_{K^*K}}{1 - \Delta C K^*K - \mathcal{A}_{K^*K} \Delta C_{K^*K}}, \]
\[ A_{K^{*-}K^+} = \frac{\Gamma(B_s^0 \to K^{*-}K^+) - \Gamma(B_s^0 \to K^{*-}K^+)}{\Gamma(B_s^0 \to K^{*-}K^+) + \Gamma(B_s^0 \to K^{*-}K^+)} = \frac{\mathcal{A}_{K^*K} + C_{K^*K} + \mathcal{A}_{K^*K} \Delta C_{K^*K}}{1 + \Delta C K^*K + \mathcal{A}_{K^*K} \Delta C_{K^*K}}. \]
(5.18)
Note that the quantities $A_{K^{*+}K^-}$ here correspond to $A_{K^{*+}K^-}$ defined in [37]. Therefore, direct $CP$ asymmetries $A_{K^{*+}K^-}$ and $A_{K^{*-}K^+}$ are determined from the above two equations. Results for various $CP$ -violating parameters in the decays $\bar{B}_s^0 \to K^{*\pm}K^\mp$ are shown in Table XI.
TABLE XII: CP-averaged branching ratios in $B_s \rightarrow VV$ decays (in units of $10^{-6}$) obtained in various approaches. Presented are the pQCD predictions taken from [11] and the QCDF predictions from this work and from [8] denoted by BRY.

| Channel | Class | QCDF (this work) | QCDF (BRY) | pQCD | Expt [39, 64] |
|---------|-------|-----------------|------------|------|---------------|
| $\bar{B}_s \rightarrow \rho^- K^{*+}$ | $T$ | $21.6^{+2.5+3.6}_{-2.8-1.5}$ | $25.2^{+1.5+4.3}_{-1.7-3.1}$ | $20.9^{+8.2+1.4+1.2}_{-6.2-1.4-1.1}$ | |
| $\bar{B}_s \rightarrow \rho^0 K^{*0}$ | $C$ | $1.3^{+2.0+1.7}_{-0.6-0.3}$ | $1.5^{+1.0+3.1}_{-0.5-1.5}$ | $0.33^{+0.09+0.14+0.00}_{-0.07-0.09-0.01}$ | $< 767$ |
| $\bar{B}_s \rightarrow \omega K^{*0}$ | $C$ | $1.1^{+1.5+1.3}_{-0.5-0.3}$ | $1.2^{+0.7+2.3}_{-0.3-1.1}$ | $0.31^{+0.10+0.12+0.07}_{-0.07-0.06-0.02}$ | |
| $\bar{B}_s \rightarrow K^{*-} K^{*+}$ | $P$ | $7.6^{+1.0+2.5}_{-1.0-1.8}$ | $9.1^{+2.5+10.2}_{-2.2-5.9}$ | $6.7^{+1.5+3.4+0.5}_{-1.2-1.4-0.2}$ | |
| $\bar{B}_s \rightarrow K^{0} K^{0}$ | $P$ | $6.6^{+1.1+1.9}_{-1.4-1.7}$ | $9.1^{+0.5+11.3}_{-0.4-6.8}$ | $7.8^{+1.9+3.8+0.0}_{-1.5-2.2-0.0}$ | $< 1681$ |
| $\bar{B}_s \rightarrow \phi K^{*0}$ | $P$ | $0.37^{+0.06+0.24}_{-0.05-0.20}$ | $0.4^{+0.1+0.5}_{-0.1-0.3}$ | $0.65^{+0.16+0.27+0.10}_{-0.13-0.18-0.04}$ | $< 1013$ |
| $\bar{B}_s \rightarrow \phi \omega$ | $P, C$ | $16.7^{+2.6+11.3}_{-2.1-8.8}$ | $21.8^{+1.1+30.4}_{-1.1-17.0}$ | $35.3^{+8.3+16.7+0.0}_{-6.9-10.2-0.0}$ | $24.0 \pm 8.9$ |
| $\bar{B}_s \rightarrow \phi \rho^0$ | $P_{EW}$ | $0.18^{+0.04+0.47}_{-0.12-0.04}$ | $0.10^{+0.05+0.48}_{-0.03-0.12}$ | $0.16^{+0.09+0.10+0.01}_{-0.05-0.04-0.00}$ | |
| $\bar{B}_s \rightarrow \rho^+ \rho^-$ | ann | $0.18^{+0.01+0.09}_{-0.01-0.04}$ | $0.40^{+0.12+0.25}_{-0.10-0.04}$ | $0.23^{+0.09+0.03+0.00}_{-0.07-0.01-0.01}$ | $< 617$ |
| $\bar{B}_s \rightarrow \rho^0 \rho^0$ | ann | $0.68^{+0.04+0.73}_{-0.04-0.53}$ | $0.34^{+0.03+0.60}_{-0.03-0.38}$ | $1.0^{+0.2+0.3+0.0}_{-0.2-0.0}$ | $< 320$ |
| $\bar{B}_s \rightarrow \rho^0 \omega$ | ann | $0.34^{+0.02+0.36}_{-0.02-0.26}$ | $0.17^{+0.01+0.30}_{-0.01-0.19}$ | $0.51^{+0.12+0.17+0.01}_{-0.11-0.10-0.01}$ | |
| $\bar{B}_s \rightarrow \omega \omega$ | ann | $0.004^{+0.00+0.00}_{-0.00-0.00}$ | $< 0.01$ | $0.007^{+0.000+0.000+0.000}_{-0.000-0.000-0.000}$ | |

VI. $B_s \rightarrow VV$ DECAYS

A. Branching fractions

In two-body decays $B_{u,d} \rightarrow PP, PV, VV$, we have the pattern $VV > PV > VP > PP$ for the branching fractions of tree-dominated modes and $PP > PV > VV > VP$ for penguin-dominated ones, where the factorizable amplitude for $B \rightarrow VP(PV)$ here is given by $\langle V(P)|J_{4u}|B \rangle \langle P(V)|J^{\dagger}|0 \rangle$. The first hierarchy is due to the difference of decay constants $f_V > f_P$ and the second hierarchy stems from the fact that the penguin amplitudes are proportional to $a_4 + r^U_\rho a_6, a_4 + r^V_\rho a_6, a_4 - r^U_\rho a_6 + r^V_\rho a_6$, respectively, for $B \rightarrow PP, PV, VP, VV$ with $r^U_\rho \sim 0(1) > r^V_\rho$. The same is also true in the $B_s$ sector. From Tables [III, VII] and XII we find

$$\mathcal{B}(\bar{B}_s \rightarrow \rho^- K^{*+}) > \mathcal{B}(\bar{B}_s \rightarrow \rho^- K^+) > \mathcal{B}(\bar{B}_s \rightarrow \pi^- K^{*+}) > \mathcal{B}(\bar{B}_s \rightarrow \pi^- K^+),$$

$$\mathcal{B}(\bar{B}_s \rightarrow K^{+} K^-) > \mathcal{B}(\bar{B}_s \rightarrow K^{*-} K^+) > \mathcal{B}(\bar{B}_s \rightarrow K^{*+} K^-) > \mathcal{B}(\bar{B}_s \rightarrow K^{+} K^-),$$

for tree- and penguin-dominated $B_s$ decays, respectively.

There exist two QCDF calculations of $\bar{B}_s \rightarrow VV$ [8, 9]. However, only the longitudinal polarization states of $\bar{B}_s \rightarrow VV$ were considered in [9]. The analysis in this work differs from Beneke, Rohrer and Yang (BRY) [8] mainly in three places: (i) the choice of form factors, (ii) the values of the parameters $\rho_4$ and $\phi_4$, and (iii) the treatment of penguin annihilation contributions characterized by the parameters $\beta_i$ [see Eq. (2.10)] for penguin-dominated $VV$ modes. First, the form factors for $B_s \rightarrow K^+$ and $B_s \rightarrow \phi$ transitions we employ in Eq. (3.6) are smaller than the ones (3.10) used by BRY. Second, BRY applied the values $\rho_4(K^+ \phi) = 0.6$ and $\phi_4(K^+ \phi) = -40^\circ$ obtained from a fit to the data of $B \rightarrow K^+ \phi$ to study $B \rightarrow \bar{K}^* \rho$ and
As pointed out in [43], the parameters $\rho_A(K^+\rho) \approx 0.78$ and $\phi_A(K^+\rho) \approx -43^\circ$ fit to the data of $B \to K^+\rho$ decays are slightly different from the ones $\rho_A(K^+\phi)$ and $\phi_A(K^+\phi)$. Therefore, within the framework of QCDF, one cannot account for all charmless $B \to VV$ data by a universal set of $\rho_A$ and $\phi_A$ parameters. This explains why the $B \to K^+\rho$ branching fractions obtained by BRY are systematically below the measurements. In this work, we choose $\rho_A = 0.70$ and $\phi_A = -55^\circ$ (cf. Table III) to describe $B_s \to VV$ decays. Third, as noticed in [43], there are sign errors in the expressions of the annihilation terms $A_3^{f,0}$ and $A_3^{l,0}$ obtained by BRY. As a consequence, BRY claimed (wrongly) that the longitudinal penguin annihilation term receives sizable penguin annihilation contribution. This will affect the decay rates and longitudinal polarization fractions in some of $B \to K^+\rho$ modes, as discussed in details in [43]. In spite of the above-mentioned three major differences in the calculations of this work and BRY, it turns out that the calculated rates and $f_L$ shown in Tables XII and XIV respectively, are similar for most of the $B_s \to VV$ modes.

Recently CDF has reported a new measurement of $B_s \to \phi\phi$ [64]

$$\frac{\mathcal{B}(B_s \to \phi\phi)}{\mathcal{B}(B_s \to J/\psi\phi)} = (1.78 \pm 0.14 \pm 0.20) \times 10^{-2}. \quad (6.2)$$

Using the branching fraction of $B_s \to J/\psi\phi$ from PDG [39], updated to current values of $f_s/f_d$, this leads to

$$\mathcal{B}(B_s \to \phi\phi) = (24.0 \pm 2.1 \pm 2.7 \pm 8.2) \times 10^{-6}, \quad (6.3)$$

where the error is dominated by the last uncertainty coming from the $J/\psi\phi$ branching fraction error. This new measurement is slightly larger than the previous one of $(14 \pm 5 \pm 7) \times 10^{-6}$ [63]. Our prediction $\mathcal{B}(B_s \to \phi\phi) \approx 16.7 \times 10^{-6}$ is consistent with experiment.

A few words on the penguin-dominated decays $B_s \to \phi K^{*0}$ and $B_s \to \omega\phi$. Their branching fractions of order $10^{-7}$ are much smaller than other penguin-dominated $K^*K^*$ and $\phi\phi$ modes. This is because $B_s \to \phi K^{*0}$ is induced by the $b \to d$ penguin transition. The amplitude of $B_s \to \omega\phi$ reads

$$\sqrt{2} A_{B_s \to \omega\phi} = A_{\phi\omega} \left[ \delta_{p_0} \alpha_2 + 2\alpha_3^\rho + \frac{1}{2} \alpha_3^{\rho, EW} \right]. \quad (6.4)$$

The branching fraction due to the QCD penguin $\alpha_3 = a_3 + a_5$ is small, only at the level of $10^{-7}$. Moreover, there is a partial cancellation between QCD and electroweak penguin contributions, making its rate even smaller.\(^6\)

As seen from Table XII pQCD predictions for the color-suppressed tree-dominated modes $p^0K^{*0}$ and $\omega K^{*0}$ are much smaller than the QCDF results, whereas $\mathcal{B}(B_s \to \phi\phi) = 0(35 \times 10^{-6})$ is much larger than QCDF and the CDF measurement [65].

In analog to Eq. (4.12), there are three SU(3) relations relating the rates of $B_s \to VV$ and $B_d \to VV$:

$$\mathcal{B}(B_s \to K^{*+}\rho^-) \approx \mathcal{B}(B_d \to \rho^+\rho^-), \quad \mathcal{B}(B_s \to K^{*-}\phi^-) \approx \mathcal{B}(B_d \to K^{*-}\rho^+), \quad \mathcal{B}(B_s \to K^{*0}\bar{K}^{*0}) \approx \mathcal{B}(B^{-} \to \bar{K}^{*0}\rho^-). \quad (6.5)$$

\(^6\) It was argued in [8] that the color-suppressed tree amplitude $\alpha_2$ is the largest partial amplitude in the decay $B_s \to \omega\phi$. We found that this decay is still dominated by the QCD penguin, though the contribution from $\alpha_2$ is not negligible.
TABLE XIII: Same as Table XII except for direct CP asymmetries (in %) in the $\bar{B}_s \to VV$ decays.

| Channel            | Class | QCDF (this work) | QCDF (BRY) | pQCD       |
|--------------------|-------|------------------|------------|------------|
| $\bar{B}_s \to \rho^- K^{*+}$ | $T$   | $-11^{+4}_{-3}$ | $-3^{+2}_{-1}$ | $-8^{+2}_{-1-1}$ |
| $\bar{B}_s \to \rho^0 K^{*0}$ | $C$   | $46^{+15}_{-17}$ | $27^{+34}_{-27}$ | $61^{+4}_{-2}$ |
| $\bar{B}_s \to \omega K^{*0}$ | $C$   | $-50^{+20}_{-15}$ | $-34^{+10}_{-7}$ | $-62^{+4}_{-2}$ |
| $\bar{B}_s \to K^{*-} K^{*+}$ | $P$   | $21^{+1}_{-2}$ | $2^{+10}_{0}$ | $9^{+4}_{-3}$ |
| $\bar{B}_s \to K^{*0} \bar{K}^{*0}$ | $P$   | $0.4^{+0.8}_{-0.5}$ | $1^{0}_{0}$ | $0$ |
| $\bar{B}_s \to \phi K^{*0}$ | $P$   | $-9^{+3}_{-4}$ | $-17^{+4}_{-5}$ | $0$ |
| $\bar{B}_s \to \phi \phi$ | $P, C$ | $8^{+3}_{-1}$ | $8^{+3}_{-3}$ | $3.6^{+2}_{-2}$ |
| $\bar{B}_s \to \phi \rho^0$ | $P_{EW}$ | $8^{+10}_{-0.3}$ | $19^{+5}_{-3}$ | $10.1^{+1}_{-2}$ |
| $\bar{B}_s \to \rho^+ \rho^-$ | ann   | $0$ | $-2.1^{+0.2}_{-1.4}$ | $-0.6^{+4}_{-2}$ |
| $\bar{B}_s \to \rho^0 \rho^0$ | ann   | $0$ | $-2.1^{+0.2}_{-1.4}$ | $-0.6^{+4}_{-2}$ |
| $\bar{B}_s \to \rho^0 \omega$ | ann   | $0$ | $6.0^{+0.7}_{-0.5}$ | $-0.6^{+4}_{-2}$ |
| $\bar{B}_s \to \omega \omega$ | ann   | $0$ | $-2.0^{+0.1}_{-1.3}$ | $-0.6^{+4}_{-2}$ |

Numerically, we have

$$21.6^{+1.6}_{-3.2} = 24.2^{+3.1}_{-3.2}, \quad 7.4^{+2.5}_{-2.1} = 8.9^{+4.9}_{-5.6}, \quad 6.6 \pm 2.2 = 9.2 \pm 1.5$$

(6.6)
in units of $10^{-6}$, where use of the theoretical calculation of $\mathcal{B}(\bar{B}_d \to K^{*-} \rho^+)$ from [43] has been made.

B. Direct CP violation

Direct CP asymmetries in QCDF and pQCD approaches are summarized in Table XIII.

C. Polarization fractions

For charmless $\bar{B} \to VV$ decays, it is naively expected that the helicity amplitudes $\lambda_h$ (helicities $h = 0, -, +$) for both tree- and penguin-dominated $\bar{B} \to VV$ respect the hierarchy pattern

$$\lambda_0 : \lambda_- : \lambda_+ = 1 : \left( \frac{\Lambda_{QCD}}{m_b} \right) : \left( \frac{\Lambda_{QCD}}{m_b} \right)^2.$$  

(6.7)

Hence, they are dominated by the longitudinal polarization states and satisfy the scaling law, namely [66],

$$f_T \equiv 1 - f_L = \mathcal{O} \left( \frac{m_V}{m_B} \right), \quad \frac{f_\perp}{f_\parallel} = 1 + \mathcal{O} \left( \frac{m_V}{m_B} \right),$$

(6.8)

with $f_L, f_\perp, f_\parallel$ and $f_T$ being the longitudinal, perpendicular, parallel and transverse polarization fractions, respectively, defined as

$$f_\alpha \equiv \frac{f_\alpha}{\Gamma} = \frac{|\lambda_\alpha|^2}{|\lambda_0|^2 + |\lambda_-|^2 + |\lambda_+|^2},$$

(6.9)
with $\alpha = L, ||, \perp$. In sharp contrast to the $\rho \rho$ case, the large fraction of transverse polarization of order 0.5 observed in $B \to \bar{K}^* \rho$ and $B \to \bar{K}^* \phi$ decays at $B$ factories is thus a surprise and poses an interesting challenge for any theoretical interpretation. Therefore, in order to obtain a large transverse polarization in $B \to \bar{K}^* \rho, \bar{K}^* \phi$, this scaling law must be circumvented in one way or another.

As pointed out by Yang and one of us (HYC) [43], in the presence of NLO nonfactorizable corrections e.g. vertex, penguin and hard spectator scattering contributions, effective Wilson coefficients $a_i^h$ are helicity dependent. Although the factorizable helicity amplitudes $X^0, X^-$ and $X^+$ defined by Eq. (2.4) respect the scaling law with $\Lambda_{QCD}/m_q$ replaced by $2m_V/m_B$ for the light vector meson production, one needs to consider the effects of helicity-dependent Wilson coefficients: $\mathcal{A}/\mathcal{A}^0 = f(a_i^-)X^-/[f(a_i^0)X^0]$. For some penguin-dominated modes, the constructive (destructive) interference in the negative-helicity (longitudinal-helicity) amplitude of the penguin-dominated modes, the constructive (destructive) interference in the negative-helicity (longitudinal-helicity) amplitude of the $\bar{B} \to VV$ decay will render $f(a_i^-) \gg f(a_i^0)$ so that $\mathcal{A}/\mathcal{A}^0$ is comparable to $\mathcal{A}^0$ and the transverse polarization is enhanced. For example, $f_L(\bar{K}^* \rho^0) \sim 0.91$ is predicted in the absence of NLO corrections. When NLO effects are turned on, their corrections on $a_i^-$ will render the negative helicity amplitude $\mathcal{A}/\mathcal{A}^0$ comparable to the longitudinal one $\mathcal{A}/\mathcal{A}^0$ so that even at the short-distance level, $f_L$ for $B \to \bar{K}^* \rho^0$ can be as low as 50%. However, this does not mean that the polarization anomaly is resolved. This is because the calculations based on naive factorization often predict too small rates for penguin-dominated $B \to VV$ decays, e.g. $B \to \bar{K}^* \phi$ and $B \to \bar{K}^* \rho$, by a factor of 2 $\sim 3$. Obviously, it does not make sense to compare theory with experiment for the partial rate and hence the prediction can be easily off by a factor of 2 $\sim 3$. Thus, the first important task is to have some mechanism to bring up the rates. While the QCD factorization approach relies on penguin annihilation [66], soft-collinear effective theory invokes charming penguin [67] and the final-state interaction model considers final-state rescattering of intermediate charm states [68, 69, 70]. A nice feature of the $(S-P)(S+P)$ penguin annihilation is that it contributes to $\mathcal{A}/\mathcal{A}^0$ with similar amount. This together with the NLO corrections will lead to $f_L \sim 0.5$ for penguin-dominated $VV$ modes. Hence, within the framework of QCDF we shall assume weak annihilation to account for the discrepancy between theory and experiment, and fit the existing data of branching fractions and $f_L$ simultaneously by adjusting the parameters $\rho_A$ and $\phi_A$. Then using this set of annihilation parameters as a guideline, we can proceed to predict the rates and $f_L$ for other $VV$ decays of the $B_{u,d,s}$ mesons.

The longitudinal polarization fractions in $\bar{B}_s \to VV$ decays obtained in the QCDF and pQCD approaches are summarized in Table XIV. Transverse polarization effects are sizable in penguin-dominated $\bar{B}_s \to VV$ as expected. However, the pQCD calculations indicate that $f_L \sim f_T \sim \frac{1}{2}$ even for the color-suppressed tree-dominated decays $\bar{B}_s \to K^{*0}(\rho^0, \omega)$. This is an astonishing result and should be checked by experiment. Polarization fractions of $\bar{B}_s \to \phi \phi$ will be studied soon by CDF. It will be very interesting to see if the transverse polarization is also important in the penguin dominated $B_s$ decays.
TABLE XIV: Same as Table XII except for the longitudinal polarization fractions in the $\bar{B}_s \to VV$ decays.

| Channel | Class | QCDF (this work) | QCDF (BRY) | pQCD |
|---------|-------|------------------|------------|------|
| $\bar{B}_s \to \rho^- K^{*+}$ | $T$ | $0.92^{+0.01+0.01}_{-0.02-0.03}$ | $0.92^{+0.01+0.01}_{-0.02-0.03}$ | $0.937^{+0.001+0.002}_{-0.002-0.002}$ |
| $\bar{B}_s \to \rho^0 K^{*0}$ | $C$ | $0.90^{+0.04+0.03}_{-0.05-0.23}$ | $0.93^{+0.02+0.05}_{-0.03-0.54}$ | $0.455^{+0.004+0.016}_{-0.003-0.005}$ |
| $\bar{B}_s \to \omega K^{*0}$ | $C$ | $0.90^{+0.03+0.03}_{-0.04-0.23}$ | $0.93^{+0.02+0.05}_{-0.03-0.54}$ | $0.532^{+0.003+0.023}_{-0.002-0.029}$ |
| $\bar{B}_s \to K^{*-} K^{++}$ | $P$ | $0.52^{+0.03+0.20}_{-0.05-0.21}$ | $0.67^{+0.04+0.31}_{-0.05-0.26}$ | $0.438^{+0.005+0.031}_{-0.004-0.023}$ |
| $\bar{B}_s \to K^{0*0} K^{0}$ | $P$ | $0.56^{+0.04+0.22}_{-0.07-0.26}$ | $0.63^{+0.04+0.42}_{-0.09-0.29}$ | $0.497^{+0.057+0.006}_{-0.048-0.038}$ |
| $\bar{B}_s \to K^{0*0} K^{0}$ | $P$ | $0.43^{+0.02+0.21}_{-0.02-0.18}$ | $0.40^{+0.01+0.67}_{-0.01-0.35}$ | $0.712^{+0.032+0.273}_{-0.030-0.375}$ |
| $\bar{B}_s \to \phi K^{*0}$ | $P, C$ | $0.36^{+0.03+0.23}_{-0.04-0.18}$ | $0.43^{+0.00+0.01}_{-0.00-0.34}$ | $0.619^{+0.036+0.025}_{-0.032-0.033}$ |
| $\bar{B}_s \to \phi \omega$ | $\rho^0$ | $0.88^{+0.01+0.02}_{-0.00-0.18}$ | $0.81^{+0.03+0.09}_{-0.04-0.12}$ | $0.870^{+0.002+0.009}_{-0.002-0.003}$ |
| $\bar{B}_s \to \rho^+ \rho^-$ | $\rho^0$ | $0.95^{+0.01+0.00}_{-0.02-0.42}$ | $0.443^{+0.000+0.009}_{-0.075-0.061}$ | $0.443^{+0.000+0.009}_{-0.075-0.061}$ |
| $\bar{B}_s \to \rho^0 \rho^0$ | $\rho^0$ | $0.95^{+0.02+0.11}_{-0.01-0.12}$ | $0.00^{+0.02+0.11}_{-0.01-0.12}$ | $0.00^{+0.02+0.11}_{-0.01-0.12}$ |
| $\bar{B}_s \to \rho^0 \omega$ | $\omega$ | $0.88^{+0.01+0.02}_{-0.00-0.18}$ | $0.81^{+0.03+0.09}_{-0.04-0.12}$ | $0.870^{+0.002+0.009}_{-0.002-0.003}$ |
| $\bar{B}_s \to \omega \omega$ | $\omega$ | $0.95^{+0.02+0.11}_{-0.01-0.12}$ | $0.00^{+0.02+0.11}_{-0.01-0.12}$ | $0.00^{+0.02+0.11}_{-0.01-0.12}$ |

TABLE XV: Direct CP asymmetries (in %) in $\bar{B}_s \to VV$ decays via U-spin symmetry.

| Modes | $\mathcal{B}(10^{-6})$ | $A_{CP}($%$)$ | Modes | $A_{CP}($%$)(U$-spin$)$ | $A_{CP}($%$)(QCDF)$ |
|-------|-----------------|----------------|-------|----------------|----------------|
| $\bar{B}_d \to K^{*+} \rho^-$ | $8.9^{+1.1+1.4}_{-1.0-5.5}$ | $32^{+1+5}_{-3-24}$ | $\bar{B}_d \to K^{*+} \rho^-$ | $-10.2$ | $-11^{+4}_{-1}$ |
| $\bar{B}_d \to K^{0*0} \rho^0$ | $4.6^{+0.6+3}_{-0.5-3.5}$ | $15^{+4+16}_{-8-14}$ | $\bar{B}_d \to K^{0*0} \rho^0$ | $42.3$ | $46^{+18}_{-30}$ |
| $\bar{B}_d \to \rho^+ \rho^-$ | $25.5^{+1.5+2.4}_{-2.6-1.5}$ | $-4^{+0+3}_{-0-3}$ | $\bar{B}_d \to K^{*+} K^{-}$ | $18.7$ | $21^{+2}_{-3}$ |
| $\bar{B}_d \to K^{*0} K^{*0}$ | $0.6^{+0.1+0.2}_{-0.1-0.3}$ | $-14^{+1+6}_{-1-2}$ | $\bar{B}_d \to K^{0*0} K^{0}$ | $0.5$ | $0.4^{+1.0}_{-0.6}$ |
| $\bar{B}_d \to K^{0*0} K^{0}$ | $0.15^{+0.02+0.11}_{-0.01-0.12}$ | $0$ | $\bar{B}_d \to \rho^+ \rho^-$ | $0$ | $0$ |

D. U-spin symmetry

Analogous to the $\bar{B}_s \to PP$ sector, U-spin symmetry leads to the following relations:

$$A_{CP}(\bar{B}_s \to K^{*+} \rho^-) = -A_{CP}(\bar{B}_d \to K^{*+} \rho^-) \frac{\mathcal{B}(\bar{B}_d \to K^{*+} \rho^-)}{\mathcal{B}(\bar{B}_s \to K^{*+} \rho^-)} \frac{\tau(B_s)}{\tau(B_d)}$$,

$$A_{CP}(\bar{B}_s \to K^{*+} K^{-}) = -A_{CP}(\bar{B}_d \to \rho^+ \rho^-) \frac{\mathcal{B}(\bar{B}_d \to \rho^+ \rho^-)}{\mathcal{B}(\bar{B}_s \to K^{*+} K^{-})} \frac{\tau(B_s)}{\tau(B_d)}$$,

$$A_{CP}(\bar{B}_s \to K^{0*0} K^{0}) = -A_{CP}(\bar{B}_d \to K^{0*0} K^{0}) \frac{\mathcal{B}(\bar{B}_d \to K^{0*0} K^{0})}{\mathcal{B}(\bar{B}_s \to K^{0*0} K^{0})} \frac{\tau(B_s)}{\tau(B_d)}$$,

$$A_{CP}(\bar{B}_s \to K^{0*0} K^{0}) = -A_{CP}(\bar{B}_d \to K^{0*0} \rho^0) \frac{\mathcal{B}(\bar{B}_d \to K^{0*0} \rho^0)}{\mathcal{B}(\bar{B}_s \to K^{0*0} K^{0})} \frac{\tau(B_s)}{\tau(B_d)}$$,

$$A_{CP}(\bar{B}_s \to \rho^+ \rho^-) = -A_{CP}(\bar{B}_d \to K^{*+} K^{-}) \frac{\mathcal{B}(\bar{B}_d \to K^{*+} K^{-})}{\mathcal{B}(\bar{B}_s \to \rho^+ \rho^-)} \frac{\tau(B_s)}{\tau(B_d)}$$.

In Table XV we compare the results of CP asymmetries inferred from U-spin relations with the direct QCDF calculations. It appears that U-spin symmetry works well in the VV sector.
Assuming that the transverse amplitude can be expressed as a single dominant contribution which may arise from new physics, $U$-spin symmetry implies that the transverse amplitudes of $B_s \to VV$ can be related to the $U$-spin related decays in the $B_d$ sector via [71]

\[
\frac{\mathcal{A}_T (\bar{B}_s \to K^{*0}\bar{K}^{*0})}{\mathcal{A}_T (\bar{B}_d \to K^{*0}\bar{K}^{*0})} \approx \frac{V_{ts}}{V_{td}} \frac{f_{B_s}}{f_{B_d}}, \quad \frac{\mathcal{A}_0 (\bar{B}_d \to \phi\bar{K}^{*0})}{\mathcal{A}_0 (\bar{B}_d \to \phi\bar{K}^{*0})} \approx \frac{V_{ts}}{V_{td}} \frac{f_{B_d}}{f_{B_s}}. \tag{6.11}
\]

Therefore,

\[
\frac{f_T (\bar{B}_s \to K^{*0}\bar{K}^{*0})}{f_T (\bar{B}_d \to K^{*0}\bar{K}^{*0})} \approx (25.5 \pm 6.5) \frac{\mathcal{B} (\bar{B}_d \to \bar{K}^{*0}K^{*0})}{\mathcal{B} (\bar{B}_s \to K^{*0}\bar{K}^{*0})}, \]
\[
\frac{f_T (\bar{B}_d \to \phi\bar{K}^{*0})}{f_T (\bar{B}_s \to \phi\bar{K}^{*0})} \approx (19.3 \pm 4.9) \frac{\mathcal{B} (\bar{B}_d \to \phi\bar{K}^{*0})}{\mathcal{B} (\bar{B}_s \to \phi\bar{K}^{*0})}. \tag{6.12}
\]

The polarization measurement in the $B_d$ decay thus allows one to predict the transverse polarization in the $B_s$ decay.\footnote{Based on SU(3) flavor symmetry, it has been shown in [72] that the transverse polarizations of $\bar{B}_s \to \phi\phi$ and $\bar{B}_s \to \phi K^{*0}$ can be related to $\bar{B}_d \to \phi\bar{K}^{*0}$ and $\bar{B}_d \to K^{*0}\bar{K}^{*0}$, respectively.} Using the data [48]

\[
\mathcal{B} (\bar{B}_d \to \bar{K}^{*0}K^{*0}) = (1.28^{+0.37}_{-0.32}) \times 10^{-6}, \quad f_L (\bar{B}_d \to \bar{K}^{*0}K^{*0}) = 0.80^{+0.22}_{-0.13},
\]
\[
\mathcal{B} (\bar{B}_d \to \phi\bar{K}^{*0}) = (9.8 \pm 0.7) \times 10^{-6}, \quad f_L (\bar{B}_d \to \phi\bar{K}^{*0}) = 0.48 \pm 0.03, \tag{6.13}
\]

and QCDF predictions for $\mathcal{B} (\bar{B}_s \to K^{*0}\bar{K}^{*0})$ and $\mathcal{B} (\bar{B}_s \to \phi\bar{K}^{*0})$, we obtain

\[
f_T (\bar{B}_s \to K^{*0}\bar{K}^{*0}) = 1.02 \pm 0.28, \quad f_T (\bar{B}_s \to \phi\bar{K}^{*0}) = 0.73 \pm 0.19. \tag{6.14}
\]

It is obvious that the central value of the predicted $f_T (\bar{B}_s \to K^{*0}\bar{K}^{*0})$ via $U$-spin symmetry is too large. Note that there is a discrepancy between the QCDF prediction of $\mathcal{B} (\bar{B}_d \to \bar{K}^{*0}K^{*0}) = (0.6^{+0.2}_{-0.3}) \times 10^{-6}$ and the BaBar measurement $\mathcal{B} (\bar{B}_d \to \bar{K}^{*0}K^{*0}) = (1.28^{+0.37}_{-0.32}) \times 10^{-6}$. We need to await a more precise measurement of $\bar{B}_d \to K^{*0}\bar{K}^{*0}$ in order to have a more accurate prediction of its transverse polarization fraction via $U$-spin symmetry.

### E. Time-dependent $CP$ violation

In principle, one can study time-dependent $CP$ asymmetries for each helicity component,

\[
\mathcal{A}_h(t) = \frac{\Gamma (\mathcal{B}_h^0(t) \to V_h V_{h'}^*) - \Gamma (\mathcal{B}_h^0(t) \to V_h V_{h'}^*)}{\Gamma (\mathcal{B}_h^0(t) \to V_h V_{h'}^*) + \Gamma (\mathcal{B}_h^0(t) \to V_h V_{h'}^*)} = S_h \sin (\Delta m_s t) - C_h \cos (\Delta m_s t), \tag{6.15}
\]

where the effects of the width difference of the $B_s$ mesons have been neglected. From Table XII, we see that there is only one decay mode of particular interest, namely, $\bar{B}_s \to \phi\phi$. Indeed, this could be the most promising channel for the forthcoming LHCb experiment. This channel is a pure $b \to s\bar{s}s$ penguin-induced process and hence provides an ideal place for exploring the signal of New Physics via $B_s - \bar{B}_s$ mixing and/or the penguin process. The other decays such as $\bar{B}_s \to \rho\rho, \rho^0\omega, \omega\omega$ proceed through weak annihilation. The
modes $\phi \omega$ and $\phi \rho^0$ receive QCD penguin and electroweak penguin contributions, respectively, but their rates are too small. A straightforward calculation gives
\[
\mathcal{B}_L = (5.9^{+1.0+5.3}_{-0.8-5.7}) \times 10^{-6}, \quad C_L = (-0.5^{+0.1+1.4}_{-0.2-1.5})\%, \quad S_L = (-0.5^{+0.1+1.1}_{-0.1-1.8})\%,
\]
for the longitudinal component of $\bar{B}_s \to \phi \phi$. Note that $S_L$ is found to be positive and small $\leq 0.02$ in [9], while our result is negative for $S_L$. An observation of large $CP$ violation in this decay will rule out the scenario of minimal flavor violation. Time-dependent $CP$ violation will be studied at LHC. If LHCb is upgraded to accumulate data sample of 100fb$^{-1}$, the sensitivity of $S_{\bar{B}_s \to \phi \phi L}$ will reach the level of $0.01 \sim 0.02$.

VII. CONCLUSIONS

We have re-examined the branching fractions and $CP$-violating asymmetries of charmless $\bar{B}_s \to PP, VP, VV$ decays in the framework of QCD factorization. We have included subleading power corrections to the penguin annihilation topology and to color-suppressed tree amplitudes that are crucial for resolving the $CP$ puzzles and rate deficit problems with penguin-dominated two-body decays and color-suppressed tree-dominated $\pi^0\pi^0$ and $\rho^0\pi^0$ modes in the $B_{u,d}$ sector. Our main results are:

i). Many model-independent relations for $CP$ asymmetries and branching fractions of $\bar{B}_d$ and $\bar{B}_s$ decays can be derived under $U$-spin and SU(3) symmetries for $PP, VP, VV$ modes. In general, they are either experimentally verified or theoretically satisfied. There are also a few $U$-spin relations for transverse polarizations in $B_s \to VV$ decays.

ii). For the $B_s \to K$ transition form factor, we use a smaller one, $F^{B_s K} \approx 0.24$ at $q^2 = 0$ obtained by the lattice calculation, to avoid too large rates for $\bar{B}_s \to K^+\pi^-, K^+K^-$ decays.

iii). Both QCDF and SCET indicate that the penguin-dominated decay $B_s \to \eta\eta'$, the analog of $B \to K\eta'$ in the $B_s$ sector, has the largest branching fraction of order $\sim 50 \times 10^{-6}$ in two-body hadronic decays of the $B_s$ meson, whereas the pQCD approach claims that $\mathcal{B}(\bar{B}_s \to \eta \eta') \approx 35 \times 10^{-6}$ is the largest one.

iv). Even at the decay rate level, there are some noticeable differences between various approaches. The branching fractions of the color-suppressed tree-dominated decays obtained by pQCD, for example, $\bar{B}_s \to K^0\pi^0, K^0\eta(1420), K^0\pi^0, \rho^0 K^0, \omega K^0, K^0\eta'$ are typically smaller by one order of magnitude than that of QCDF and SCET. For example, $\mathcal{B}(\bar{B}_s \to \rho^0 K^0)$ is predicted to be of order $1.9 \times 10^{-6}$ by QCDF, but it is only about $0.08 \times 10^{-6}$ in pQCD. In the QCDF approach, many of the above-mentioned decays get a substantial enhancement from the power corrections to the color-suppressed tree topology.

v). The decay rate of $\bar{B}_s \to \phi \eta'$ is sensitive to the $B_s \to \phi$ transition form factor $A_{0}^{B_s \phi}(0)$. For $A_{0}^{B_s \phi}(0) = 0.474$ obtained by the light-cone sum rule method, a near cancelation between $\bar{B}_s \to \phi \eta_{1}$ and $\bar{B}_s \to \phi \eta_{q}$ occurs in the decays $\bar{B}_s \to \phi \eta'$, so that its branching fraction, of order $10^{-7}$, becomes very
small. However, if the value $A^B_0(0) = 0.30$ favored by many other model calculations is employed, then $\bar{B}_s \to \phi \eta$ and $\bar{B}_s \to \phi \eta_0$ will contribute constructively to $\bar{B}_s \to \phi \eta'$ so that $\mathcal{BR}(\bar{B}_s \to \phi \eta') = 2.2 \times 10^{-6}$ and $\mathcal{BR}(\bar{B}_s \to \phi \eta) = 1.0 \times 10^{-6}$. Hence, it is very important to measure the branching fractions of $\bar{B}_s \to \phi \eta^{(i)}$ to gain the information on the form factor $A^B_0$.

vi). Measurements of $CP$-violating asymmetries can be used to discriminate between QCDF, pQCD and SCET approaches:

(a) Both QCDF and pQCD predict a positive sign for $A_{CP}(\bar{B}_s \to K^0\pi^0)$, whereas SCET leads to a negative one. This can be traced back to fact that $A_{CP}(\bar{B}_d \to \bar{K}^0\pi^0)$ is positive in SCET, while it is negative inferred from the $CP$-asymmetry sum rule, SU(3) relation and the topological quark diagram analysis.

(b) For color-suppressed tree-dominated decays $\bar{B}_s \to K^*0\pi^0, \rho^0K^0, \omega K^0, K^{*0}\eta'$, QCDF and pQCD results are of the same sign, whereas SCET predicts opposite signs for these modes. In the QCDF approach, the signs of these $CP$ asymmetries are governed by the soft corrections to $a_2$. Since the corresponding rates of these decays are very small in pQCD, as a consequence, the $CP$-violating asymmetries predicted by pQCD are very large, of order 0.50 or even bigger.

(c) In the QCDF framework, the penguin-dominated decays $\bar{B}_s \to K^0\phi, \bar{K}^0K^0, K^{*0}\bar{K}^0$ have non-vanishing $CP$ asymmetries, though very small for the last two modes, whereas leading order pQCD predicts no $CP$ violation for these three decays.

vii). Mixing-induced $CP$ asymmetries of the penguin-dominated decays $\bar{B}_s \to K^0\bar{K}^0, \eta^{(*)}\eta^{(*)}, \phi\eta', \phi\phi$ are predicted to be very small in the SM. Especially, we found $S_{\bar{B}_s \to \phi_L\phi_L} \sim -0.5\%$. They are sensitive to New Physics and provide possibilities of new discoveries. While both QCDF and pQCD approaches predict $S_{\bar{B}_s \to K_S\phi} \sim O(0.70)$, the SCET result of 0.09 or $-0.13$ is dramatically different.

viii). Due to soft power corrections to the color-suppressed tree amplitude, we find that such effects will convert the sign of mixing-induced $CP$ violation $S_f$ into the positive one for the color-suppressed decays $\bar{B}_s \to K_S(\pi^0, \eta, \eta')$. Therefore, even the measurements of the sign of $S_{\bar{B}_s \to K_S(\pi^0, \eta, \eta')}$ will be helpful to test if $\alpha_2$ has a large magnitude and strong phase.

ix). Transverse polarization effects are sizable in penguin-dominated $\bar{B}_s \to VV$ as expected. However, the pQCD approach predicts that $f_L \sim f_T \sim \frac{1}{2}$ even for the color-suppressed tree-dominated decays $\bar{B}_s \to K^{*0}(\rho^0, \omega)$. This should be tested by experiment.
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