How unique are the MSW parameters?

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ABSTRACT

We investigate the effect of a possible large frozen-in magnetic field in the core of the sun and a neutrino magnetic moment of the order of $10^{-10}$ Bohr magneton on the MSW parameters. Such a possibility can completely depolarize the $\nu_{eL}$ resulting in a factor of half of the emitted number. This in combination with MSW can explain the present experimental data on solar neutrinos but with a different region of the parameter space than the regular MSW alone. In this scenario there is a larger region of parameter space allowed. There is a large strip of parameter space corresponding to adiabatic solution and no solution for large mixing angle region. The non-adiabatic region still remains as a solution but shifted and larger relative to the regular MSW.

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The substantial difference between the event rate observed at the Homestake chlorine experiment (HCE) and the corresponding prediction of the standard solar model (SSM) has been referred to as solar neutrino puzzle. The ratio of twenty year average of the number of $\nu_{eL}$ observed in HCE to that predicted by SSM is $0.28 \pm 0.05$. (Throughout this note we have combined all the errors in quadrature and they represent one standard deviation.) A deviation from the theory has also been seen in the Kamiokande-II (Kam-II) experiment with the corresponding ratio of $0.49 \pm 0.08$. Both these experiments are sensitive to the high energy neutrinos. The two Gallium experiments, Gallex and SAGE, on the other hand are sensitive to the low energy neutrinos constituting the main bulk of all neutrinos coming from the sun. Gallex result is $0.63 \pm 0.16$ of the SSM and SAGE reported seeing the fraction $0.44 \pm 0.21$ of the expected. So the combined Gallium result is $0.60 \pm 0.13$ which is higher than the other two experiments but far from unity. Although it is not universally accepted, there has been a claim that the chlorine experiment is anti-correlated with the number of sunspots in one cycle of solar activity. On the contrary no such anti-correlation was observed in the Kam-II and Gallium experiments. There exists a long list of proposals to resolve the puzzle(s). These include non-standard solar models and exotic particle physics interactions beyond the standard electro-weak theory. For the various possible explanations the reader is referred to the book by Bahcall. We will focus our attention only on two particular explanations, namely spin(-flavor) precession for neutrino having a large Dirac(Majorana) magnetic moment and oscillation of neutrino in matter. Oscillation of neutrino in matter, MSW effect, seems to be the most attractive solution. It is a two parameter solution with the allowed region of the parameter space determined by the three known experimental data. We would like to analyze the effect of complete depolarization of the neutrino due to a possible large frozen-in magnetic field in the core of the sun on these two parameters.

One possible solution to the solar neutrino puzzle assumes a large magnetic moment for the neutrino, either Dirac type or Majorana type. For the latter case one can only have transition magnetic moment. Let us consider the Majorana case here because this is easier to realize than the Dirac case and the constraints coming from astronomical and cosmological arguments do not apply to this case. Due to the magnetic field in the
convective zone of the sun, some $\nu_{eL}$ get spin-flavor precessed into $\bar{\nu}_{\mu R}$, and hence will miss detection completely in the HCE. But, $\bar{\nu}_{\mu R}$ will give some contribution due to neutral current and electromagnetic interactions in the Kam-II detector which detects Cherenkov radiation given by the elastically scattered electron. The fraction of $\nu_{eL}$ remaining as $\nu_{eL}$ is given by:

$$P_B(\nu_{eL} \rightarrow \nu_{eL}) = 1 - \sin^2(\mu BL)$$

where $\mu$ is the magnetic moment, B is the average magnetic field and L is the path length of the neutrino inside the magnetic field. The combination $\mu BL$ has to be very near the first node. Because the magnetic field in the convective zone varies with the 11-year solar cycle, this explanation can account for a possible anti-correlation with the sunspot number. Because the magnetic field is not expected to be uniform one should take an average over $\sin^2(\mu BL)$ which will give the maximum depletion factor to be $\frac{1}{2}$. Most important is the fact that the recent Gallium results very strongly disfavor such a scenario. So simple magnetic spin-flavor precession alone most probably can not explain the solar neutrino puzzle. There has been a lot of activity in the proposal of matter enhanced spin-flavor precession which, like MSW effect for matter oscillation, can easily account for a large drop in the $\nu_{eL}$ number in an adiabatic situation. There has also been discussion of combining resonant spin-flavor precession with MSW. Although none of these explanations has been ruled out so far because of only limited experimental data available to us, we would like to keep an open mind and consider other possibilities. There are a few things about the above scenario that has to be kept in mind. The anti-correlation of the Homestake data with the sun-spot number is not absolutely clear and is very controversial. We need better statistics to tell one way or the other. Secondly, unfortunately the internal magnetic field of the sun is very far from being clearly understood. There could exist a frozen-in(primeval) magnetic field as large as 300 MG near the center of the sun with a magnitude of $2 \pm 1$ MG just beneath the base of the convective zone. Even if one has on the average 10 MG over a length of $0.01 \, R_\odot$, one will get complete depolarization of the $\nu_{eL}$ for energy $\sim 1$ MeV with a magnetic moment of $\sim 10^{-10} \mu_B$. It is important to remember that in all the scenarios of the previous kind, one assumes that no such large magnetic field exists because that will spoil the whole motivation of explaining the anti-correlation with solar activity. We, on the
other hand, would like to keep an open mind and analyze the situation with a very large magnetic field in the center of the sun which has gotten very little attention. In this case a neutrino magnetic moment of the order of $10^{-10}\mu_B$, where $\mu_B$ is the Bohr magneton, will give rise to complete depolarization resulting in half the number of each of $\nu_{eL}$ and $\bar{\nu}_{\mu R}$.

If neutrino has both mixing and magnetic moment which is very natural, then there will be two separate effects. Although they will occur simultaneously in the solar interior, we can consider their effects separately. The effect of the magnetic field is determined by the quantity $\int Bdx$. Neither the strength nor the extent of the frozen-in magnetic field is accurately known. The magnitude can be anywhere between zero and 300 MG extending up to $0.1R_\odot$. For the sake of argument, let us assume a frozen-in magnetic field of strength 10 MG extending up to $0.01R_\odot$. It is possible for the neutrino with a magnetic moment of $\sim 10^{-10}\mu_B$ to have complete depolarization. There will be a suppression factor multiplying the $\sin^2\mu BL$ term in (1) because of non-degeneracy of the two states\cite{3, 11, 12}. This is quantified by an effective $(\Delta m^2)_{\text{eff}}$ which is a combination of $\Delta m^2$ and the effect of matter. At this point it is worth pointing out that the Majorana case has advantage over the Dirac case because for the former matter effect can cancel $\Delta m^2$ giving a small effective $\Delta m^2$. For the later case the equivalent $\Delta m^2$ is only due to matter effect. The suppression is negligible if

$$\Delta m^2 \ll 2E\mu B \tag{2}$$

which is assumed here. This condition puts an upper limit of $\mathcal{O}(10^{-5}eV^2)$ on $\Delta m^2$ for $B = 10MG$, $\mu = 10^{-10}\mu_B$ and $E \sim 1$ MeV and the above condition is obviously satisfied by higher values of $E$. Any higher value of the magnetic field or its extent in the sun will give higher limit for $\Delta m^2$ or lower value of $E$ for which complete depolarization takes place. This limit on $\Delta m^2$ can be one order of magnitude higher if we take into account matter effect with density $\sim 150\text{ gm/cm}^{-3}$. Because of the high density at the center of the sun, the neutrino produced by the nuclear reactions is largely the heavy mass eigenstate. In that case the suppression of spin precession is avoided altogether. In any case it is not very difficult to satisfy (2) with a large enough magnetic field and hence to explain a factor of half depletion in the HCE event rate after averaging with large number of oscillations for neutrinos of energy scales relevant to our experiments.
A very simple solution to the solar neutrino puzzle has been proposed using complete depolarization and short oscillation length vacuum oscillation with maximal mixing. With a short wavelength vacuum oscillation and maximal mixing, the number of $\nu_{eL}$'s reaching the earth can be reduced by another factor of half. This can explain the Homestake chlorine experiment. The difference between the Homestake and the Kamiokande-II experiments can be attributed to the contribution to the Cherenkov radiation in the latter through the neutral current and electromagnetic interactions of the components which are inert in the former. The Gallium, especially Gallex, result disfavors this simple minded solution. So here we will analyze the effect of complete depolarization on the MSW solution.

The matter enhanced neutrino oscillation, MSW effect, is a very attractive solution to the solar neutrino puzzle. Inside matter $\nu_{eL}$ interacts with the electrons both via neutral and charged current interactions where as other kind of neutrinos interact via only the neutral current interaction. This is the basis of virtual enhancement in mixing angle compared to the vacuum mixing angle. So even with a small vacuum mixing angle one can explain a large depletion in the total number of electron type neutrino after they go through the dense matter in the core of the sun. We refer to the many excellent reviews on the subject for greater details. The basic effect is parametrized in terms of two parameters, namely, the vacuum mixing angle and the difference in mass square. This is represented by what has become the standard iso-SNU plots for each experiment. Because we know the profile of matter density inside the sun, given the two parameters $\Delta m^2$ and $\sin^2(2\theta)$, we can calculate the probability for an $\nu_{eL}$ remaining as an $\nu_{eL}$ in any particular detector. The iso-SNU contours have three separate regions. The top horizontal part is called the adiabatic section and in this region of parameter space the high energy neutrinos are depleted more than the low energy neutrinos. For the diagonal branch, known as the non-adiabatic, the energy dependence is reversed. The vertical section on the right is the large angle solution which is very close to the vacuum mixing and neutrinos of all energy are democratically depleted. Taking the three experiments into account and looking at the corresponding three iso-SNU contours we can determine the regions of the $\Delta m^2 - \sin^2(2\theta)$ that is allowed if the MSW is the solution to the solar neutrino puzzle. At present there are two small distinct regions of the parameter space allowed by the three experiments. Non-adiabatic region with
\[ \Delta m^2 \sim (0.3 - 1.2) \times 10^{-5} \text{ eV}^2, \quad \sin^2(2\theta) \sim (0.4 - 1.5) \times 10^{-2} \]

and the large mixing angle solution with \( \Delta m^2 \sim (0.3 - 4) \times 10^{-5} \text{ eV}^2, \quad \sin^2(2\theta) \sim (0.5 - 0.9) \) (Please see ref. and fig. 4). As the region of parameter space gets squeezed, the original claim of MSW solution being robust and valid in a large region of parameter space is getting lost. There are also claims in the literature for getting this region of the parameter space from grand unified theories. Of course, if we knew for sure that MSW is the solution to the solar neutrino puzzle we can use all the experimental results to determine the two parameters and hence possibly eliminate certain extensions of the standard model. But right now the situation is exactly the opposite. So now the question is, how unique are the region of parameter space? We would like to show in this note that in the presence of a very large magnetic field and with a magnetic moment within the experimental limits, there is a solution to the solar neutrino puzzle with a different region of the parameter space.

The neutrino can undergo complete depolarization and MSW oscillation at the same time but for all practical purposes we can talk about the two effects separately. After complete depolarization we will get half the total amount of \( \nu_{e,L} \) which can undergo MSW oscillation to give the desired result. The difference between this scenario and regular MSW alone is that here we start from 50% instead of 100%. In figures 1, 2 and 3 we have shown the iso-SNU contours for the MSW after complete depolarization. It is important to note that these contours are exactly the same as the regular MSW [18] except that their SNU values are exactly half of the regular MSW. The shaded region corresponds to the presently allowed region of the parameter space for each experiment within 95% confidence level. In fig. 4 we have combined all the three regions and found the common region of the parameter space. That is the shaded region. We have also presented in the same figure the parameter space allowed by regular MSW. Those two distinct regions are given in the unshaded regions. The differences between the two scenarios can be easily seen. In the present scenario there is no large mixing angle solution as opposed to the regular MSW. On the contrary, there is a large portion of adiabatic region which is allowed in this case. This continues over to the non-adiabatic region which is shifted and enlarged version of the usual MSW. The implications of this new scenario for future experiments is under investigation. Preliminary result indicates that there will not be any remarkable difference in the two scenarios in
counting experiments. But there will be definite difference in the experiments which will look at the spectrum of neutrinos, for example SNO and BOREXINO. That is because of the intrinsic nature of the three different regions of MSW parameter space. In any case, if the Gallium experiment sees clearly more than 50% of SSM we can turn the argument around and put limit on $\mu B$ and if we know $\mu$ from some other source that will put a limit on $B$.

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Figure Captions

Figure 1. The survival probability contours for $\nu_{eL}$ at Homestake Chlorine experiment for the parameters $\Delta m^2$ and $\sin^2(2\theta)$ after the complete depolarization due to the strong magnetic field. The contours represent 0.1, 0.2 ..., 0.5 of the standard solar model of Bahcall starting from inside. The shaded region represents the allowed region of the parameter space with 95% confidence level.

Figure 2. Same as figure 1 but for Kamiokande experiment.

Figure 3. Same as figure 1 but for the combined Gallium experiments.

Figure 4. The region of the parameter space allowed by all the three experiments. The shaded region is for the MSW after complete depolarization while the unshaded region is for MSW alone.
\[ \sin^2(2\,\theta) \]

Fig. 1

\[ \Delta m^2 \text{ in eV}^2 \]

Fig. 2
\[ \Delta m^2 \quad \text{in} \quad \text{eV}^2 \]

\[ \sin^2(2\theta) \quad \text{Fig. 3} \]

\[ \sin^2(2\theta) \quad \text{Fig. 4} \]