Super-Penrose Process for Nonextremal Black Holes

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We consider particle collisions in the background of nonextremal spherically symmetric static black holes. It is shown that debris of collision can have indefinitely large energy at infinity; i.e., the super-Penrose process can occur. This property is sharply contrasted with that of rotating black holes for which it is already established that the super-Penrose process is forbidden. The Reissner–Nordström black hole serves as an example. If an external central force exerts on particles, even the Schwarzschild background is suitable for the super-Penrose process.

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1. INTRODUCTION

The interest to high energy processes near black holes increased significantly after the work [1]. It was shown there that if two particles move towards the Kerr extremal black hole and collide in its vicinity, the energy \( m_0 \) in their center of mass frame can become unbounded, provided one of two particle (called critical) has fine-tuned parameters. This is called the Bañados–Silk–West (BSW) effect. The close analogy of this effect exists also for extremal charged static black holes [2]. However, as far as the Killing energy \( E \) of debris detected at infinity is concerned, the situation differs radically for two aforementioned cases. For rotating black holes, the energy \( E \) of an escaping particle at infinity is bounded [3–5]. Meanwhile, there is no such a bound for the extremal Reissner–Nordström (RN) black hole. This was obtained in [6] and later confirmed in [7]. The process with unbounded \( E \) at infinity is called the super-Penrose process (SPP).

Two problems concerning nonextremal black holes existed here. First, it was widespread belief that extremity is a necessary condition for the BSW effect, so deviation from extremity weakens the effect [8, 9]. However, it was shown in [10] that if instead of one particle being exactly critical, a near-critical particle is used, and deviation from the critical state is adjusted to the proximity of the point of collision to the horizon in a special way, the effect survives. Moreover, one can add a force acting on particles and this is consistent with the BSW effect [11]. Second, it was unclear how to realize the BSW effect physically. The most relevant situation corresponds to particles falling from infinity. However, for rotating black holes, the centrifugal barrier prevents the critical particle from reaching the nonextremal horizon [10] (see also in [12, case 2i; 13, Sect. 2; 14]). This can be repaired, provided additional constraints are imposed on the scenario, because of which the turning point is situated closely to the horizon [15].

However, an interesting question that has not yet been posed to the best of our knowledge is: whether the SPP is possible for nonextremal black holes. It is considered in the present work. We show that this is indeed possible. In this sense, there is a sharp contrast between extremal rotating neutral and nonextremal static charged black holes. One can think that this observation may be useful for astrophysically relevant black holes since they are nonextremal. It possesses some universal features in what any particles moving in the background of a nonextremal black hole (even in the Schwarzschild metric) and experiencing the action of some force can exhibit this effect.

It is worth stressing that one should not confuse two different effects connected with two different kinds of energy. The possibility of unbounded \( m_0 \) (the BSW effect) when a force acts on particles moving near a nonextremal black hole, was shown in [16] that extended the results of [2]. In principle, the existence of the BSW effect is not sufficient, in general, for the SPP, as is mentioned above. Therefore, the fact that the BSW effect is possible in scenarios considered in [15], by itself does not guarantee for them the SPP. However, we will show that this is indeed the case for the situation under discussion that includes, in particular, scenarios of [15]. To the best of my knowledge, this is a first example of considering the SPP for nonextremal black holes. In what follows, we use the geometric system of units in which fundamental constants

\[ G = c = 1. \]
2. BASIC FORMULAS

Let us consider the black hole metric
\[ ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \] (1)
where the horizon is located at \( r = r_c \), so \( N(r_c) = 0 \). We assume that a particle having mass \( m \) moves within this background with the force \( F = ma \) exerted on it that causes the acceleration \( a^\mu \), \( a^\mu \equiv a \), by definition \( a \geq 0 \). We also assume that \( a = a(r) \) depends on \( r \) only.

Then, for four-velocity \( u^\mu \) we have
\[ m u^\mu = \frac{X}{N^2}, \] (2)
\[ m u^\nu = \sigma P, \quad P = \sqrt{X^2 - m^2 N^2} \geq 0, \] (3)
the factor \( \sigma = \pm 1 \) depending on the direction of motion,
\[ X = E - b(r). \] (4)
Here, \( E \) is a constant of integration,
\[ b = \alpha \int dr' F(r'), \] (5)
\( \alpha = \pm 1 \). The above formulas generalize those for the RN metric. Their derivation is quite direct and can be found in [16–19].

In the important particular case of the RN metric, taking \( \alpha = +1 \), \( q > 0 \) and \( Q > 0 \) (here, \( q \) and \( Q \) are the charges of the particle and black hole, respectively), we have
\[ F(r) = \frac{qQ}{r^2}, \quad b = \frac{qQ}{r} = q \phi(r), \] (6)
where \( \phi(r) = \frac{Q}{r} \) is the Coulomb potential of a black hole. The metric coefficient
\[ N^2 = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \] (7)
where \( M \) is the mass of the black hole.

We assume that the force and acceleration caused by it decrease rapidly enough at infinity. Then, \( E \) has the usual meaning of the energy at infinity. The forward-in-time condition \( u^\mu > 0 \) requires
\[ X \geq 0. \] (8)
Let particles 1 and 2 move from infinity and collide at some point \( r_c \). The energy \( m_0 \) in the center of mass frame
\[ m_0^2 = -(m_1 u_1^\mu + m_2 u_2^\mu) (m_1 u_1^\mu + m_2 u_2^\mu) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \] (9)
where \( \gamma = -u_{0i} u^{2i} \) is the Lorentz factor of relative motion. It follows from the above equations that
\[ \gamma = \frac{X_1 X_2 - \sigma_1 \sigma_2 P P_3}{m m_2 N^2}. \] (10)

This is just the point where particle dynamics described by Eqs. (2)–(5) reveals itself.

One reservation is in order. We assume that the background is described by the spherically symmetric metric (1). In doing so, backreaction of a particle and external sources on the metric is neglected. For the case of the Schwarzschild metric this implies that \( m \ll M \), where \( M \) is a black hole mass.

3. SCENARIOS OF COLLISION

We consider the case \( \alpha = +1, \sigma_1 = \sigma_2 = -1 \). In particular, for the RN black hole \( a = \frac{|Q|}{mr^2} \). However, we may leave a general \( a(r) \) not specifying it. In particular, one can consider motion of particles in the Schwarzschild background under the action of some force [19, 20]. This can also lead to the BSW effect [16]. Now we will see that this also admits the SPP.

Let us consider the \( 1 + 2 \rightarrow 3 + 4 \) reaction. For simplicity, we choose the case of pure radial motion of all particles. Then, it follows from the conservation laws of energy and radial momentum that (see [21, Sect. 4])
\[ X_3 = \frac{1}{2m_0^2} (X_0 \Delta_+ - P_0 \sqrt{\Delta_+^2 - 4m_2^2 m_3^2}), \] (11)
\[ X_4 = \frac{1}{2m_0^2} (X_0 \Delta_+ + P_0 \sqrt{\Delta_+^2 - 4m_2^2 m_3^2}), \] (12)
\[ \Delta_+ = m_0^2 \pm (m_3^2 - m_2^2), \] (13)
\[ X_i = E_i - b_i(r), \] (14)
\[ P_i = \sqrt{X_i^2 - m^2 N^2}. \] (15)
Here, the integer \( i \) runs from 0 to 4, \( \sigma_i = \pm 1 \), the subscript \( c \) refers to the point of collision, \( X_0 = X_1 + X_2 \).

The outcome of collision depends strongly on the relation between the parameters of the particle, say the energy and charge in the RN case. Then, following a standard terminology, we can classify all particles depending on \( X(r_c) \). If \( X(r_c) > 0 \) is separated from zero, a particle is called usual. If \( X(r_c) = 0 \), a particle is called critical. If \( X(r_c) = O(N_c) \), where \( N_c \ll 1 \), a particle is called near-critical.

We assume that particle 1 is near-critical and particle 2 is usual. More precisely, we specify deviation from the criticality in the form
\[ b(r_c) = E_1 (1 + \delta_1). \] (16)

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where

\[ \delta_1 = C_1 N_c + O(N_c^2), \]  

with \( C_1 < 0 \) and \( N_c \ll 1 \). We also assume the validity of the Taylor expansion in the form

\[ b(r_c) = b_1(r_c) + A(r_c - r_c), \]  

where \( A \) is a constant. The nonextremal nature of a black hole comes into play just here: for nonextremal black holes \( N_c^2 - r_c - r_c \). Meanwhile, the corrections to (14) have two contributions. The first one stems from the second term in (18). The second contribution arises due to the correction (16) to the critical value \( b_1(r_c) = E_1 \) because of \( \delta_1 \). We see that the first type of corrections is negligible having the order \( \delta_1 \). Thus, in the vicinity of the horizon

\[ X_1 = E_1 |C_1| N_c + O(N^2). \]  

Also, for near-horizon collisions, \( X_0 \approx X_2, \) \( P_0 = X_0 - m_0^2 N^2 / 2X_0 \) and all nonzero quantities can be taken on the horizon instead of \( r_c \). It is seen from (9), (10) that

\[ m_0^2 = \frac{2(X_2)_z}{N_c} \approx \Delta_1, \]  

where

\[ z = E_1 C_1 - \sqrt{E_1^2 C_1^2 - m_1^2}. \]  

Collecting all terms carefully, one obtains from (11) after simple algebra that

\[ X_3 \approx \frac{N_c}{2} \left( z + \frac{m_3^2}{z} \right). \]  

We would like to remind a reader that Eq. (11) from which (22) is obtained, is direct consequence of particle dynamics. More precisely, it follows from the conservation of energy and radial momentum. This was shown in [21, Sect. 4] where a reader can find the details. In addition to particle dynamics, we used the proximity to the critical state for particle 1 because of which the approximate expression (19) was obtained from Eqs. (14), (16), (18) and inserted in Eq. (11).

Thus, particle 3 turns out to be near-critical. From the other hand, for a particle of such a type the approximate expression similar to (19) should be valid, so

\[ X_3 \approx \frac{C_3}{2} |E_3| N_c, \]  

where \( \delta_3 = C_3 N_c \) and \( C_1 < 0 \) controls relationship between parameters of particle 3.

Say, for the RN metric, \( b = q \phi, \) where \( \phi = \frac{Q}{r} \) is the electric potential of a black hole with the charge \( Q \). Then, for the exactly critical particle

\[ E = q \phi(r_c), \]  

so \( q = \frac{E_r}{Q} \), for the nonextremal black hole \( r_c > Q \). For near-critical particle 3, we can choose

\[ q_3 = E_3 \frac{r_c}{Q} (1 + \delta_3). \]  

By substitution in (4) and discarding the terms \( O(N^2) \), we see that (23) does hold true.

We would like to stress that, had we taken two neutral particles, we would have obtained collision of two usual particles. Then, according to general rules [1, 2, 12], the BSW effect would be impossible. The Penrose process would be impossible in this situation as well since for neutral particles there are no negative energies. Thus, the electric charge is a necessary ingredient of a process. For a more general situation, it is necessary that one particle move on the action of a force while another one can be free.

Comparing (22) and (23), we find that

\[ E_3 \approx \frac{1}{2C_3} \left( \frac{m_3^2}{z} \right). \]  

If we choose \( C_1 \to 0 \), then formally \( E_3 \to \infty \) becomes unbounded, as it should be for the SPP. Thus, we see that for nonextremal black holes, the proximity to the criticality requires not only the validity of expansion (17) for particle 3, as it was in the extremal case [6]. Additionally, the coefficient \( C_3 \) has to be small.

This is not the end of story. We must also check that the scenario under consideration is able to describe the process when particle 3 escapes to infinity instead of fall in a black hole. This requires that \( \sigma_3 = +1 \). (The particle with \( \sigma_3 = -1 \) simply falls in a black hole in contrast to the extremal case where it can bounce from the potential barrier and escape [6].) The list of possible scenarios is given in Eqs. (29) and (30) in [21]. It is shown there that the aforementioned condition requires

\[ \Delta_1 N_c - 2m_3 X_0 > 0. \]  

In the main approximation using (20), (21) we have from (27) that

\[ m_3 < z < m_1. \]
Thus, although there is no upper bound on $E_3$, there is such a bound on $m_3$. In doing so, one obtains from (15) that
\begin{equation}
P_3 \approx \frac{N}{2} \left( z - \frac{m_3^2}{z} \right).
\end{equation}

4. CHOICE OF INITIAL STATE

The formulas written above are valid for the process as such, independently of the origination of the initial state. This implies the collision of one near-critical and one usual particles that were created somehow near the horizon. Meanwhile, the most physically interesting situation arises when particles 1 and 2 come from infinity. However, in this respect severe restrictions are relevant since in general the critical or near-critical particle cannot overcome the potential barrier in the nonextremal black hole background [10–14]. Happily, there are two particular cases when this becomes possible. (i) The nonextremal black hole is close to extremity and its surface gravity $\kappa$ is small. (ii) Both particles 1 and 2 have big energy from the very beginning. This represents the so-called “energy feeding problem,” its analogue for extremal black holes was discussed in [22, Sect. 4.C1]. However, it was shown in [15, Sect. 7] that, although the initial energy should be big, the outcome due to collisions of ultra-relativistic particle can give significant relative gain in energy $m_0$.

In the present work, as a matter of fact, we showed that significant gain in energy $E_3$ is possible as well. By itself, it does not depend on an initial energy at all, its value being controlled by the deviation from criticality measured by the quantity $|C_3|$. For the RN metric, the electric charge of debris at infinity is defined according to Eqs. (25) and (26). Thus, big initial energies of incoming particles do not depreciate such a scenario since the outcome is more “profitable” than income anyway. Cases (i) and (ii) are discussed in detail in [15], so we do not repeat their details here. What is important for us is that at least with some additional requirements, scenarios of high-energy collisions of particles 1 and 2 for nonextremal black holes work suggest a suitable initial state, so the present scenario describing behavior of particles 3 and 4 also works.

5. EXTREME BLACK HOLE VERSUS NONEXTEME ONE

It is instructive to compare the results with those for the extremal RN black hole. Then,
\begin{align}
X_3 &= -E\delta + EN + O(N^2), \\
X_3(N_e) &= (1 - C_3)E_3N_e,
\end{align}
see Eq. (19) in [6]. Therefore, the limit $C_3 \to 0$ is of no use in that case and more refined analysis was required there, with account of all possible $C_3$ (positive, negative, and zero). It was shown that there are different scenarios, and only the ones with $C_3 \geq 0$ give rise to the SPP process [6]. In doing so, there are two options. If immediately after collision particle 3 moves to infinity, there is no upper bound on $E_3$ but there exists such a bound on $m_3$. If it moves towards a black hole, bounces from the barrier and escapes to infinity, there is an upper bound neither for $E_3$, nor for $m_3$. For the nonextremal case, the relevant scenario, as we saw, is more simple, Eqs. (19), (23) are valid instead of (30), (31). There is no turning point, particle 3 relevant for the SPP, moves to infinity at once after collision.

The difference between the extremal and nonextremal cases reveals itself also in potential realization of the SPP in a real world. For the extremal case, let us assume that critical particle 1 falls from infinity with $E_1 = q_1$ according to (24) since $q(r_e) = 1$. If the particle was initially at rest at infinity, $E_1 = m_1$. Taking $q_1 = e$, where $e$ is the elementary charge, one obtains $m_1 = 10^{-6}$ g, so one deals with a macroscopic object instead of an elementary particle that was pointed out in [7, Sect. 7]. Meanwhile, for nonextremal black holes, there are two changes. First, an initial particle should be ultrarelativistic (unless the black hole is very close to the extremal state) (see [15]). Thus, $E_1 = m_1\gamma$, where $\gamma \gg 1$ is the corresponding Lorentz factor. Second, now $q(r_e) = \frac{Q}{M}$. Thus, Eq. (24) reads
\begin{equation}
m_1 \sim \frac{Q}{\gamma M} q_1. \quad \text{If } \frac{Q}{M} \leq 1, \text{ this means that } m_1 \sim \frac{q_1}{\gamma}.
\end{equation}
Again, $q_1 = e$, we obtain $m_1 \sim 10^{-6}$ g. If, say, $\gamma = 100$, $m_1 = 10^{-4}$ g, so an object can be still considered as macroscopic instead of being an elementary particle. Formally, for $m_1$ to become equal to the electron mass, one must have $\gamma = 10^{21}$ but such tremendous initial velocities are unphysical. Thus the main conclusion made in [7, Sect. 7] that instead of elementary particles, macroscopic objects are involved in the process under discussion, retains its validity.

Until recently, there was general belief that the electric charge of black hole is absent or completely negligible. Meanwhile, there are some indirect indications that, although being extremely small, such a charge can differ from zero [23]. It is reasonable to require that $m > m_e$ where $m_e$ is the electron mass.

Then, we find that $\frac{Q}{M} \geq \frac{\gamma m_e}{|e|}$, whence $\frac{Q}{M} > 10^{-22} \gamma$. Thus for, say, $\gamma = 100$ we obtain a lower bound on the charge $\frac{Q}{M} > 10^{-22}$ for the realization of the SPP for...
electrons. For protons as critical particles, one must require \( \frac{Q}{M} > 10^{-19} \).

The above discussion does not affect the condition \( E_3 \gg E_1 \) and the corresponding energy gain since both critical energies are proportional to \( \varphi(r_c) \), so \( \varphi(r_c) \) drops out and \( \frac{E_3}{E_1} = \frac{q_3}{q_1} \). If, say, we take \( q_1 = |e| \) and \( q_3 = Z|e| \), the bound \( Z \leq 170 \) follows from realistic estimates that involve stable nuclei [24]. Meanwhile, this is compatible with the relations \( Z \gg 1, E_3 \gg E_1 \), so the surplus is quite significant. In addition, we would like to point out that, in principle, the obtained results apply to the case when the charge is not electric one but is tidal. This can be formed as an effective charge, being imprint of high-dimensional spacetimes in our four-dimensional one, having potential astrophysical consequences [25], see also [26] and references therein.

6. DISCUSSION AND CONCLUSIONS

We showed that a nonextremal static black hole is pertinent to the SPP. In doing so, there is a restriction on a mass escaping to infinity but there is no upper bound on its energy. (This holds true as long as back-reaction is negligible, so test particle approximation is valid.) Thus, the SPP exists both for the extremal and nonextremal RN black hole. Moreover, all this consideration applies even to the Schwarzschild black hole, provided some force is exerted on the critical particle. This includes both any external force or electric repulsive (since \( b - qQ > 0 \) is required) force in the RN metric, if this force is small enough. The latter means that the force does not changes background significantly (that remains approximately Schwarzschild background) but affects particle’s motion. It is worth noting that the latter situation occurs also in a quite different context when the Schwarzschild black hole is immersed in a weak magnetic field that can lead to high-energy collisions [27].

Usually, the electric charge is similar to rotation in black hole physics in many aspects. In particular, this concerns the BSW effect [1, 2]. However, now this similarity breaks down since the SPP does not exist for neutral rotating black holes but is possible for static charged ones. Now, this is seen not only for extremal black holes [6, 22] but also for nonextremal ones.

In the present context, we would like to make a technical but important remark. As we saw above (see discussion around Eq. (19)), the difference between nonextremal and extremal black holes manifests itself in the different role of corrections that come from the force acting on the escaping particle. In the extremal case, they have the same order as the term due to the deviation of parameters from the critical state. In the nonextremal one, they are negligible and this simplifies consideration greatly.

The key difference between neutral rotating and static black holes and in the context under discussion lies in the role of the centrifugal barrier. One of two new particles after collision should be near-critical. In the first case, this barrier prevents the critical particle with very high energy from reaching infinity after collision that destroys the SPP both in the extremal and nonextremal cases (see [21, Sect. 7]). However, for static black holes (say, the RN one) there is no such a barrier at all. The results of the present work extends essentially the area of validity of high-energy processes since astrophysically relevant black holes are nonextremal. It is of interest to understand how the effects of such a kind can reveal themselves in the accretion discs around black holes and for the charge that is not electric but tidal. This needs separate treatment.

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