Flavor changing neutral currents from lepton and B decays in the two Higgs doublet model

Rodolfo A. Diaz, R. Martinez, and Carlos E. Sandoval
Universidad Nacional de Colombia, Departamento de Física. Bogotá, Colombia.

Abstract

Constraints on the whole spectrum of lepton flavor violating vertices are shown in the context of the standard two Higgs doublet model. The vertex involving the $e - \tau$ mixing is much more constrained than the others, and the decays proportional to such vertex are usually very suppressed. On the other hand, bounds on the quark sector are obtained from leptonic decays of the $B_d^0 - \bar{B}_d^0$ mesons and from $\Delta M_{B_d^0}$. We emphasize that although the $B_d^0 - \bar{B}_d^0$ mixing restricts severely the $d - b$ mixing vertex, the upper bound for this vertex could still give a sizeable contribution to the decay $B_d^0 \rightarrow \mu \bar{\nu} \mu$ respect to the standard model contribution, from which we see that such vertex could still play a role in the phenomenology.

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1 Introduction

Many extensions of the standard model leads naturally to flavor changing neutral currents (FCNC) in quark and lepton sectors. This is the case of models with an extended Higgs sector. However, owing to the high suppression imposed by experiments, several mechanisms has been used to get rid of them, such as discrete symmetries \[1\], permutation symmetries \[2\], and different textures of Yukawa couplings \[9\]. Notwithstanding, the increasing evidence on neutrino oscillations seems to show the existence of mass terms for neutrinos as well as of family lepton flavor violation (LFV) \[4\]. Such fact has inspired the study of many scenarios that predict LFV processes as in the case of SUSY theories with R-parity broken \[5\], SU(5) SUSY models with right-handed neutrinos \[6\], models with heavy Majorana neutrinos \[7\], and multi-Higgs doublet models with right-handed neutrinos for each lepton generation \[8\]. On the other hand, LFV in the charged sector has been also examined in models such as SUSY GUT \[9\], and the two Higgs doublet model (2HDM) \[2, 10\].

In the charged lepton sector, searches for FCNC have been carried out through leptonic and semileptonic decays of $K$ and $B$ mesons \[11\], as well as purely leptonic processes \[12\]. On the other hand, some collaborations plan to improve current upper limits of some LFV decays by several orders of magnitude, by increasing the statistics \[13\]. Other possible sources of improvement lies on the Fermilab Tevatron and LHC by means of LFV Higgs boson decays. Further potential sources to look for Higgs mediated FCNC lie on the muon colliders. It is because they have the potentiality to produce Higgs bosons in the $s$–channel, with substantial rate production at the Higgs mass resonance \[14\]. From the theoretical point of view, since Higgs Yukawa couplings are usually proportional to the lepton mass, they give an important enhancement to cross sections with Higgs mediated $s$–channels, respect to the ones in an $e^+e^-$ collider.

As for the quark sector, it is well known that the data from $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing put severe bounds on the flavor changing couplings involving the first family \[15, 16\]. Indeed, this fact was one of the motivations to implement a discrete symmetry in the 2HDM in order to suppress FCNC effects \[11\]. This fact in turn motivated the construction of a parameterization in the 2HDM in which the FC vertices involving the first family are neglected, and assume that the only non-vanishing couplings are $\lambda_{tt}$ and $\lambda_{bb}$ \[17\]. Based on this assumption, constraints on $\lambda_{tt}$ and $\lambda_{bb}$ from $B^0 - \bar{B}^0$ and lower bounds on $m_{H^\pm}$ from the CLEO data of

\[*\] radiazs@unal.edu.co
\[†\] remartinezm@unal.edu.co
\[‡\] cesandovalu@unal.edu.co
\( b \to s\gamma \) have been estimated. In this scenario, the tree-level diagrams for \( B_d^0 \to \bar{B}_d^0 \) are neglected and the box diagrams involving one charged Higgs boson in the loop become dominant, leaving \( \lambda_{\mu}, \lambda_{\tau} \) and \( m_{H^\pm} \) as the only free parameters in the process. However, although Ref. [10] found that relatively light charged Higgs bosons are still allowed, they also found that very heavy charged Higgs bosons are still permitted and even required if there is a significant relative phase between \( \lambda_{\mu} \) and \( \lambda_{\tau} \). Wherever a very large value of \( m_{H^\pm} \) is allowed, it opens the possibility of having dominant or at least competitive tree level diagrams even with highly suppressed values of the couplings involving the first family, especially in the case in which at least one of the neutral Higgs bosons is kept light. Inspired in this idea, we shall assume in this paper that the tree level diagram is dominant.

On the other hand, in a recent previous work [10], some constraints on LFV have been found in the framework of the two Higgs doublet model with flavor changing neutral currents. Specifically, bounds on the vertices \( \xi_{(\mu\tau)\ell \ell}, \xi_{(\mu\mu)\ell \ell}, \xi_{(\tau\tau)\ell \ell} \), were obtained based on the \( g-2 \) muon factor and the leptonic decays \( \mu \to e\gamma, \tau \to \mu\mu\mu, \tau \to \mu\gamma \). Additionally, upper limits on the decays \( \tau \to e\gamma \) and \( \tau \to eee \) were estimated, finding them to be highly suppressed respect to the present experimental sensitivity. The purpose of this work is on one hand to complete the information about the spectrum of the LFV matrix in the lepton sector, and on the other hand restrict some vertices involving the first family of the quark sector, and show that such vertices could still play a significant role in the phenomenology. Combining bounds on the quark and lepton sector we can predict upper bounds for leptonic decays of the \( B^0 \) mesons.

## 2 Constraints in the lepton sector

We shall work in the context of the two Higgs doublet model (2HDM) with flavor changing neutral currents, the so called model type III. We shall neglect possible relative phases between the FC vertices. The leptonic Yukawa couplings read

\[
- \mathcal{L}_Y = \bar{E} \left[ \frac{g}{2M_W} M_{E}^{diag} \right] E \left( \cos \alpha H^0 - \sin \alpha h^0 \right) + \frac{1}{\sqrt{2}} \bar{E} \xi E \left( \sin \alpha H^0 + \cos \alpha h^0 \right) + \bar{\eta} \xi E \left( \sin \gamma \right) E A^0 + h.c.
\]

where \( H^0(h^0) \) denote the heaviest (lightest) neutral \( CP \)-even scalar, and \( A^0 \) is a \( CP \)-odd scalar. \( E \) refers to the three charged leptons \( E \equiv (e, \mu, \tau)^T \) and \( M_E, \xi_E \) are the mass matrix and the LFV matrix respectively, \( \alpha \) is the mixing angle in the \( CP \)-even sector. We use the parameterization in which one of the vacuum expectation values vanishes.

The decays needed to obtain our bounds are given by

\[
\Gamma \left( \tau^- \to \mu^- \mu^- e^+ \right) = \frac{m_\tau^5}{4096 \pi^3 \sin^2 \theta_{\mu\tau} e^2} \left[ \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} \right) - \frac{1}{m_{A^0}^2} \right]^2 \\
+ \frac{8}{3m_{A^0}^2} \left[ \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} \right]
\]

\[
\Gamma \left( \tau^- \to \mu^- \mu^- e^- \right) = \frac{m_\tau^5}{6144 \pi^3 \sin^2 \theta_{\mu\tau} e^2} \left[ \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} \right)^2 + \frac{1}{m_{A^0}^4} \right]
\]

\[
\Gamma \left( \tau^- \to \mu^- e^- e^+ \right) = \frac{m_\tau^5}{6144 \pi^3 \sin^2 \theta_{\mu\tau} e^2} \left[ \sin \left( 2\alpha \right) \sqrt{ \frac{G_F}{\sqrt{2}} \left( \frac{1}{m_{H^0}^2} - \frac{1}{m_{h^0}^2} \right) } m_e \right. \\
+ \left. \xi_{\mu e} \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} \right) \right]^2 + \frac{\xi_{\mu e}^2}{m_{A^0}^4}
\]
Observe that the decays containing two identical particles in the final state possess interferences involving the pseudoscalar Higgs boson, while the decays with no identical leptons in the final state do not contain interference terms with the pseudoscalar. On the other hand, in the calculation of the decay width \( \Gamma (\tau^- \rightarrow \mu^+ \mu^- e^-) \), we neglect diagrams containing the vertex \( \xi_{\tau} \) and keep only the ones proportional to \( \xi_{\mu\tau} \), we make this approximation because a previous phenomenological analysis shows a strong hierarchy between these mixing vertices \( \text{III} \) (\( |\xi_{\tau}| << |\xi_{\mu\tau}| \) by at least five orders of magnitude).

The corresponding experimental upper limits for these rare processes are \( \text{IIN} \)

\[
\begin{align*}
Br (\tau^- \rightarrow \mu^- \mu^+ e^+) & \leq 1.5 \times 10^{-6}, \\
Br (\tau^- \rightarrow \mu^+ \mu^- e^-) & \leq 1.8 \times 10^{-6}, \\
Br (\tau^- \rightarrow \mu^- e^- e^+) & \leq 1.7 \times 10^{-6}.
\end{align*}
\] (2)

### 2.1 Bounds on \( \xi_{\mu e} \) and \( \xi_{\tau e} \)

In a previous work \( \text{III} \), the LFV vertices coming from the 2HDM type III, were constrained by using several pure leptonic processes, the following bounds for the LFV vertices were found

\[
\begin{align*}
|\xi_{\tau}^2| & \lesssim 2.77 \times 10^{-14}, \\
|\xi_{\mu e}| & \lesssim 1.3 \times 10^{-1}, \\
7.62 \times 10^{-4} & \lesssim |\xi_{\mu\tau}^2| \lesssim 4.44 \times 10^{-2}, \\
|\xi_{\tau\tau}| & \lesssim 2.2 \times 10^{-2}.
\end{align*}
\] (3)

Such constraints are valid in most of the region of parameters. Since we intend to complete the analysis made in \( \text{III} \), we shall make the same assumptions which we summarize here for completeness. We settle \( m_{h^0} \approx 115 \text{ GeV} \), and \( m_{A^0} \gtrsim m_{h^0} \). In order to cover a very wide region of parameters, we examine five cases for the remaining free parameters of the model \( \text{III} \)

1. When \( m_{h^0} \approx 115 \text{ GeV} \).
2. When \( m_{h^0} \approx 300 \text{ GeV} \) and \( \alpha = \pi/2 \).
3. When \( m_{h^0} \) is very large and \( \alpha = \pi/2 \).
4. When \( m_{h^0} \approx 300 \text{ GeV} \) and \( \alpha = \pi/4 \).
5. When \( m_{h^0} \) is very large and \( \alpha = \pi/4 \).

For all those cases the value of the pseudoscalar mass is swept in the range of \( m_{A^0} \gtrsim 115 \text{ GeV} \).

The vertex \( \xi_{\mu e}^2 \) can be constrained by combining the existing limits on \( \xi_{\mu\tau}^2 \) given in Eqs. (3), and experimental upper limit on the branching ratio \( Br (\tau^- \rightarrow \mu^- e^+ e^-) \) given by Eq. (2). Alternatively, we can constrain the same vertex from the decay \( \tau^- \rightarrow \mu^+ \mu^- e^- \). The upper limits on \( \xi_{\mu e}^2 \) obtained from both decays are illustrated in table 1 for the five cases explained above. We should observe that the upper limits

\[
\begin{array}{|c|c|c|}
\hline
\text{case} & \text{from } \tau^- \rightarrow \mu^- \mu^+ e^+ & \text{from } \tau^- \rightarrow \mu^+ \mu^- e^- \\
\hline
1 & |\xi_{\mu e}^2| \lesssim 5.59 \times 10^{-9} & |\xi_{\mu e}^2| \lesssim 1.0 \times 10^{-2} \\
2 & |\xi_{\mu e}^2| \lesssim 1.5 \times 10^{-1} & |\xi_{\mu e}^2| \lesssim 2.7 \times 10^{-1} \\
3 & \text{unconstrained} & \text{unconstrained} \\
4 & |\xi_{\mu e}^2| \lesssim 1.35 \times 10^{-2} & |\xi_{\mu e}^2| \lesssim 2.43 \times 10^{-2} \\
5 & |\xi_{\mu e}^2| \lesssim 1.67 \times 10^{-2} & |\xi_{\mu e}^2| \lesssim 3.0 \times 10^{-2} \\
\hline
\end{array}
\]

Table 1: Bounds on the mixing vertex \( \xi_{\mu e}^2 \), based on the processes \( \tau^- \rightarrow \mu^- \mu^+ e^+ \) and \( \tau^- \rightarrow \mu^+ \mu^- e^- \) for the five cases cited in the text.

obtained from \( \tau^- \rightarrow \mu^+ \mu^- e^- \) are less restrictive than the ones coming from \( \tau^- \rightarrow \mu^- \mu^+ e^+ \). However, both
sets of constraints lie roughly on the same order of magnitude. From table 1 we can extract a quite general bound for the vertex $\xi_{\mu e}$:

$$\xi_{\mu e}^2 \leq 1.5 \times 10^{-1},$$

valid for most of the region of parameters\(^1\). It is worth saying that other restrictions on this vertex can be gotten from $\mu \rightarrow e\gamma$ or $\tau \rightarrow e\gamma$ assuming that only the diagrams with a muon in the loop contribute, instead of the tau as customary. However, bounds obtained this way are much less restrictive.

![Contourplots for the five cases cited in the text in the $\xi_{ee} - m_{A^0}$ plane](image)

Figure 1: Contourplots for the five cases cited in the text in the $\xi_{ee} - m_{A^0}$ plane, based on the process $\tau^- \rightarrow \mu^- e^+ e^-$. On left: Case 1 (dotted line), case 4 (dashed line) and case 5 (solid line). On right: Case 2 (solid line) and case 3 (dashed line).

On the other hand, we can get constraints on the vertex $\xi_{ee}$ by combining the already mentioned bounds on $\xi_{\mu\tau}^2$ and the upper experimental constraints for the decay $\Gamma (\tau^- \rightarrow \mu^- e^+ e^-)$ of Eq. (2). Since the factor $\xi_{ee}$ cannot be factorized in contrast to the case of $\xi_{\mu e}$, we extract its bounds in the form of contourplots in the $\xi_{ee} - m_{A^0}$ plane, as shown in Fig. 1. Additionally, we write in table 2, the constraints obtained for $m_{A^0}$ very heavy and for $m_{A^0} \approx 115$ GeV. From table 2 we can extract general constraints for $\xi_{ee}$, the general bounds read

$$|\xi_{ee}| \lesssim 5.1 \times 10^{-1}; \quad |\xi_{ee}| \lesssim 7.54 \times 10^{-2}$$

for $m_{A^0} \approx 115$ GeV and for $m_{A^0}$ very heavy respectively. We emphasize again that this prediction is valid in most of the region of parameters but fails in the case 3 cited above, i.e. when $m_{H^0}$ is very large and $\alpha = \pi/2$.

Finally, we make a prediction about the upper limit for the branching ratio of the process $\tau^- \rightarrow \mu^+ e^- e^-$, based on the limits on $\xi_{\mu e}$ shown in table 1 and the limits on $\xi_{\mu\tau}$ shown in Eqs. (3), the results are collected in table 3. We see that the upper limits shown in table 3 are at least ten orders of magnitude smaller than

\(^1\)We should bear in mind however, that none of the restrictions obtained here, are valid for the third case explained in the text.
the present experimental upper limit $Br(\tau^- \rightarrow \mu^+ e^- e^-) \leq 1.5 \times 10^{-6}$ GeV, (except for the third case). In addition, table 4 collects the results for the general upper limits of three leptonic decays involving the vertex $\xi_{e\tau}$. The strong suppression of these processes might be anticipated because of its proportionality to $\xi_{e\tau}^2$, which is much more restricted than the others [10].

| Case | $Br(\tau^- \rightarrow \mu^+ e^- e^-)$ |
|------|-------------------------------------|
| 1    | $\lesssim 9.5 \times 10^{-18}$      |
| 2    | $\lesssim 3.2 \times 10^{-17}$      |
| 3    | Unconstrained                        |
| 4    | $\lesssim 1.3 \times 10^{-17}$      |
| 5    | $\lesssim 1.4 \times 10^{-17}$      |

Table 3: Upper limits for the branching ratio $Br(\tau^- \rightarrow e^- e^- \mu^+)$, based on the constraints obtained for the LFV vertices $\xi_{\mu e}$ and $\xi_{e\tau}$. The experimental upper limit is $1.5 \times 10^{-6}$

3 Constraints in the quark sector

We shall obtain constraints on the quark sector by using the experimental information from $B_{d,s}^0$ lepton decays and $\Delta m_{B_{d,s}^0}$. The $B_{d}^0$ measurements are dominated by the asymmetric $B$ factories [19], while the $B_{s}^0$ measurements come from hadron colliders [20]. At the tree level, the decays $B_{d,s}^0 \rightarrow ll'$ depend on the product $\xi_{ll'}^2/\xi_{db}^2$, and the pseudoscalar Higgs boson mass only. In the framework of the 2HDM-III they are

$$\Gamma(B_{d}^0 \rightarrow l^- l'^+) = \frac{m_{B_{d}^0} f_{B_{d}^0}^2 \xi_{ll'}^2 \xi_{db}^2}{32\pi (m_b + m_d)^2 m_{A_0}^2} \left[ \frac{2(m_b^2 - m_l^2 - m_{l'}^2)}{m_{B_{d}^0}^2 - (m_l + m_{l'})^2} \right] \times \sqrt{\left[ \frac{m_{B_{d}^0}^2 - (m_l + m_{l'})^2}{m_{B_{d}^0}^2 - (m_l - m_{l'})^2} \right]}.$$ 

(5)

Where $f_{B_{d,s}}$ represent the $B_{d,s}^0$ meson decay constant whose value has been taken from [21]. The present experimental upper bounds for these decays at 90% C.L. are [18]

$$Br(B_{d}^0 \rightarrow e^- \mu^+) \leq 1.5 \times 10^{-6}$$
$$Br(B_{d}^0 \rightarrow e^- \tau^+) \leq 5.3 \times 10^{-4}$$
$$Br(B_{d}^0 \rightarrow \mu^- \tau^+) \leq 8.3 \times 10^{-4}$$
$$Br(B_{s}^0 \rightarrow e^- \mu^+) \leq 6.1 \times 10^{-6}$$

Consequently, we can get upper limits for this products of mixing vertices by using the upper bound for these decays. In particular, from $B_d \rightarrow \mu \tau$ we find

$$\xi_{\mu \tau}^2 \xi_{db}^2 \lesssim (5.45 \times 10^{-15} GeV^{-4}) m_{A_0}^4.$$ 

| Predictions | Experim. limits |
|-------------|-----------------|
| $Br(\tau^- \rightarrow e^- \gamma) \lesssim 6.6 \times 10^{-16}$ | $2.7 \times 10^{-6}$ |
| $Br(\tau^- \rightarrow e^+ e^- e^-) \lesssim 2.2 \times 10^{-17}$ | $2.9 \times 10^{-6}$ |
| $Br(\tau \rightarrow \mu^+ e^- e^-) \lesssim 3.2 \times 10^{-17}$ | $1.5 \times 10^{-6}$ |

Table 4: Upper limits predicted for some lepton decays. All of them are highly suppressed respect to the current experimental upper limit.
On the other hand, since we have a lower bound on the mixing vertex $\xi_{\mu\tau}^2$ we can obtain an upper bound for $\xi_{cb}^2$ alone

$$\xi_{cb}^2 \lesssim (7.15 \times 10^{-12} \text{GeV}^{-4}) m_A^4 .$$

We can obtain a similar bound for the product $\xi_{\mu\tau}^2 \xi_{cb}^2$ based on the upper limit for $B_d \to \tau \tau$. Nevertheless, a better bound for this product is obtained by combining Eqs. (3), (6). The same situation occurs for the product $\xi_{\mu\tau}^2 \xi_{db}^2$, since the present bounds on the decays $B_d \to e\mu$ cannot provide a better constraint for this product that the bound obtained from Eqs. (4), (6).

Furthermore, from the upper limit for $Br(B^0_s \to e\mu)$ we find an upper bound for the product $\xi_{sb}^2 \xi_{e\mu}^2$

$$\xi_{sb}^2 \xi_{e\mu}^2 \lesssim 2.38 \times 10^{-17} m_A^4$$

As we see from Eq. (5) and the bounds in this section, the latter blow up rapidly when $m_A$ grows. Indeed, for heavy values of the pseudoscalar mass, the one loop contributions could be sizeable [22] introducing more free parameters to the model. Nevertheless, if we consider a quite light pseudoscalar i.e. a mass no far from the electroweak scale, the tree level contribution is dominant and the bounds above are quite restrictive. Table 5 shows some typical values for the upper limits above for $115 \text{ GeV} \lesssim m_A \lesssim 250 \text{ GeV}$. 

| $m_A$ (GeV) | 115 | 200 | 250 |
|-------------|-----|-----|-----|
| $\xi_{cb}^2 (\times 10^{-7})$ | 1.25 | 11.44 | 27.92 |
| $\xi_{sb}^2 \xi_{e\mu}^2 (\times 10^{-4})$ | 3.46 | 31.69 | 77.35 |
| $\xi_{sb}^2 \xi_{e\mu}^2 (\times 10^{-8})$ | 1.87 | 17.16 | 41.88 |
| $\xi_{sb}^2 \xi_{e\mu}^2 (\times 10^{-9})$ | 0.42 | 3.82 | 9.32 |

Table 5: Bounds for $\xi_{cb}^2$, $\xi_{sb}^2$ and their products with leptonic vertices for pseudoscalar bosons lying roughly in the electroweak scale.

### 3.1 Constraints from $B^0_d - \bar{B}^0_q$ mixing

Now let us use the $\Delta M_{B^0}^q$ parameter to constrain the vertex $\xi_{cb}$ involving the first family. The invariant amplitude at the tree level for $B^0_d - \bar{B}^0_q$ mixing in our model is given by

$$\langle B^0_q | H_W | \bar{B}^0_q \rangle = -\frac{2}{m_H} R_{qhH}^2 \langle b\bar{q}b | \bar{B}^0_q \rangle - \frac{2}{m_H} R_{qbh}^2 \langle B^0_q | b\bar{q}b | \bar{B}^0_q \rangle - \frac{2}{m_A} R_{qhA}^2 \langle b|\gamma_5 q|\bar{B}^0_q \rangle,$$

(7)

where $R_{qhH}$ are the coefficients of the Feynman rules with $q = d, s$. In terms of the operators $O_{\Delta F=2}$ defined in [14, 15], we have that

$$\langle B^0_q | O_{\Delta F=2}^B | \bar{B}^0_q \rangle = B_B \langle \bar{B}^0_q | O_{\Delta F=2}^B | B^0_q \rangle_{VIA},$$

(8)

where $VIA$ denotes the Vacuum Insertion Approximation and $B_B$ is the vacuum saturation coefficient. The operators that we need in our case are

$$O_{S}^{B_{q}} = (b\bar{q}) ; \quad O_{P}^{B_{q}} = (\bar{b}\gamma_{5} q)(\bar{b}\gamma_{5} q).$$

(9)

In addition, based on the expressions shown in section VI of the first of Refs. [14], we introduce the factors $M_S^B$ and $M_P^B$ in terms of the $\Delta F = 2$ matrix elements of the only two operators which do not vanish in the vacuum

$$M_S^{B_q} = \langle B^0_q | O_{S}^{B_{q}} | \bar{B}^0_q \rangle_{VIA} = -\frac{1}{6} M_P^{B_{q}} + \frac{1}{6} M_A^{0,B_{q}},$$

$$M_P^{B_q} = \langle B^0_q | O_{P}^{B_{q}} | \bar{B}^0_q \rangle_{VIA} = \frac{11}{6} M_P^{B_{q}} - \frac{1}{6} M_A^{0,B_{q}},$$

(10)
The maximum room for the new physics reads symmetrical uncertainties for the sake of simplicity

\[ M^0_{Pq} = \langle \bar{B}^0_q \psi_{B^0_q} \gamma_5 \psi_q | 0 \rangle \langle 0 | \bar{B}^0_q \gamma_5 \psi_q | B^0_q \rangle = -f^2_{B^0_q} \frac{m^4_{B^0_q}}{(m_b + m_q)^2} \]

\[ M^0_{Aq} = \langle \bar{B}^0_q \psi_{B^0_q} \gamma_\mu \gamma_5 \psi_q | 0 \rangle \langle 0 | \bar{B}^0_q \gamma_\mu \gamma_5 \psi_q | B^0_q \rangle = f^2_{B^0_q} m^2_{B^0_q} \]  \( (11) \)

Using Eqs. (7)-(11), we find that the contribution from new physics to the parameter \( \Delta M_{B^0_q} \) reads

\[ \Delta M_{NP} = -2\mathbb{R}(B^0_q | H_W | B^0_q) \]

\[ = f^2_{B^0_q} m^2_{B^0_q} \xi^2 \left[ \frac{1}{3m_H^2} \cos^2 \alpha \left( \frac{m^2_{B^0_q}}{(m_q + m_b)^2} + 1 \right) \right. \]

\[ + \frac{1}{3m_H^2} \sin^2 \alpha \left( \frac{m^2_{B^0_q}}{(m_q + m_b)^2} + 1 \right) \left. \frac{1}{3m^2_A} \right] \left( \frac{m^2_{B^0_q}}{(m_q + m_b)^2} + 1 \right). \]  \( (12) \)

We shall estimate bounds for \( \xi_{db} \) by using \( \Delta M_{B^0_d} \) coming from \( B_d^0 - \bar{B}_d^0 \) mixing. The predictions for the standard model (SM) \( \Delta M_{SM} \) and the experimental value \( \Delta M_{EXP} \) have been taken from [16] by using symmetrical uncertainties for the sake of simplicity

\[ \Delta M_{SM} = 0.506 \pm 0.198 \text{ps}^{-1} \]

\[ \Delta M_{EXP} = 0.502 \pm 0.007 \text{ps}^{-1} \]  \( (13) \)

The maximum room for the new physics reads

\[ \Delta M_{NP} \leq \Delta M^0_{EXP} - \Delta M^0_{SM} - \sqrt{E^2_{SM} + E^2_{EXP}} \]  \( (14) \)

where \( \Delta M^0_{EXP} \) and \( \Delta M^0_{SM} \) represent the central values of \( \Delta M_{EXP} \) and \( \Delta M_{SM} \) respectively. Furthermore, \( E_{SM} \) and \( E_{EXP} \) are the uncertainties associated to the standard model and experimental estimations respectively. All of them are given by Eq. (13). So we can constrain \( \xi_{db} \) based on the values of \( \Delta M_{SM} \) and \( \Delta M_{EXP} \) for the \( B_d^0 \) meson. We found the results shown in table 6 for the five cases, when we consider \( m_A = 115 \text{ GeV} \) and \( m_{A^0} \text{ very large} \).

| Case | \( |\xi_{db}| \) \( (m_A = 115 \text{ GeV} \) ) | \( |\xi_{db}| \) \( (m_A \text{ very large} \) ) |
|------|---------------------------------|----------------------------------|
| 1    | \( 2.69 \times 10^{-6} \)       | \( 7.37 \times 10^{-6} \)        |
| 2    | \( 2.86 \times 10^{-6} \)       | \( 19.22 \times 10^{-6} \)       |
| 3    | \( 2.89 \times 10^{-6} \)       | unconstrained                    |
| 4    | \( 2.77 \times 10^{-6} \)       | \( 9.73 \times 10^{-6} \)        |
| 5    | \( 2.79 \times 10^{-6} \)       | \( 10.42 \times 10^{-6} \)       |

Table 6: Constraints on \( |\xi_{db}| \) for \( m_A = 115 \text{ GeV} \) and \( m_A \text{ very large} \), based on the data of \( \Delta M_{B_d} \)

Taking into account the upper limits of \( |\xi_{db}| \) we obtain that the tree level contribution to this process coming from the 2HDM can be comparable and even dominant respect to the SM contribution. It shows that, although the mixing vertices involving the first generation are highly suppressed, is still possible for them to play a role in the phenomenology.
Table 7: Upper limits for the branching ratio $Br(B_d \rightarrow \mu \mu)$ based on the upper limits for $\xi_{d'b'}$ and the allowed values for $\xi_{\mu\mu}$.

| Case | $Br(B_d \rightarrow \mu \mu)$ ($m_A = 115$ GeV) | $Br(B_d \rightarrow \mu \mu)$ ($m_A$ very large) |
|------|---------------------------------------------|---------------------------------------------|
| 1    | $2.14 \times 10^{-8}$                      | $1 \times 10^{-8}$                          |
| 2    | $2.3 \times 10^{-8}$                       | $1 \times 10^{-8}$                          |
| 3    | $2.32 \times 10^{-8}$                      | unconstrained                               |
| 4    | $2.21 \times 10^{-8}$                      | $1 \times 10^{-8}$                          |
| 5    | $2.32 \times 10^{-8}$                      | $1 \times 10^{-8}$                          |

4 Conclusions

We have found constraints on the whole spectrum of the mixing matrix of leptons, by using purely leptonic processes. A strong hierarchy between the vertices $\xi_{\mu\tau}$ and $\xi_{e\tau}$ is manifest. Effectively, as it is shown in table 4, decays involving the $\xi_{\mu\tau}$ vertex, are highly suppressed respect to the current instrumental sensitivity. The constraints on the rest of LFV vertices are much milder. However, current prospects to increase the statistics concerning LFV decays could improve such bounds significantly.

In addition, we constrain some FC couplings in the quark sector by using experimental limits on leptonic $B^0_{d,s}$ decays as well as the $B^0_d - \bar{B}^0_d$ mixing. The leptonic $B$ decays provide constraints on the products of lepton and quark FC couplings. On the other hand, by assuming that the charged Higgs boson is sufficiently heavy, the $B^0_d - \bar{B}^0_d$ mixing can be used to constrain the vertex $\xi_{d'b'}$. We point out that although the $B^0_d - \bar{B}^0_d$ mixing imposes severe restrictions to this vertex, the upper limits for $\xi_{d'b'}$ could still give a sizeable and even dominant contribution to the decay $B^0_d \rightarrow \mu \mu$ respect to the SM contribution. Consequently, this vertex of the first generation can still be important for phenomenological calculations.

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