In order to further discover the hidden chaotic attractor and its generating mechanism in the Vijayakumar system, we give a generalized system showing hidden chaotic attractors which are not from homoclinic orbit or heteroclinic orbit and consider Hopf bifurcations (codimension one and two) by first Lyapunov coefficient and second Lyapunov coefficient. The existence of periodic orbits is strictly proved theoretically. We have considered the problem of Hopf bifurcation in the chaotic system with hidden attractors, which will be helpful to reveal the intrinsic relationship between the local stability of equilibria and global complex dynamical behaviors of the chaotic system. Finally, numerical simulations are obtained for showing the correctness of theoretical results.

1. Introduction

The discussion of chaos is very interesting and important for nonlinear theory. The research of related subjects has a long history. Recently, attractors can be classified into two different types: self-excited or hidden [1–8]. Up to now, hidden chaotic attractors for the 3D autonomous chaotic systems can be found with only one stable equilibrium [9, 10], without equilibrium [11, 12], and with infinite equilibria [13–15], which makes the topology of the found chaotic systems different from that of the well-known 3D autonomous chaotic system.

Moreover, multistability allows flexibility of systems without changing parameters’ values and can be used to correct control strategies and parameters to induce switching between different coexistence attractors. The self-excited attractor has an attractive basin associated with unstable equilibrium [1–5]. Especially, in order to study periodic solutions, most researchers are more interested in studying periodic solutions [16–19]. Zoldi and Greenside [20] discovered unstable periodic orbit will be an important factor for chaos and then kicked something in the statistical correspondence between chaos and periodic orbits. Kawahara and Yamada [21] confirmed that the unstable periodic orbit will result in the classical Couette turbulent structure. The value of the connection number between these orbital pairs plays an irreplaceable role in the generation of chaos. Therefore, the existence of periodic orbit is very important for hidden chaos. In recent years, scholars have begun to consider the complex dynamics about the hidden chaos. This represents that multistability is an important feature of many nonlinear problems. However, some deep-seated hidden complex behaviors have not been thoroughly studied in many realistic chaotic systems [8, 16, 22].

As we know, there are still abundant and complex dynamical behaviors, and the topological structure of the hidden chaos should be thoroughly investigated and exploited. In particular, the generation mechanism of hidden chaotic attractors has always been a scientific problem of great concern. We will consider the coexistence of unstable
periodic orbits and hidden chaotic attractors through Hopf bifurcation analysis. We study all possible bifurcations (general bifurcations and degenerate bifurcations) by the Lyapunov coefficient of Hopf bifurcation. More precisely, the first Lyapunov coefficient \( l_1 \) is obtained for the parameter space, which indicates the possibility of giving two branches of codimension, and the second Lyapunov coefficient \( l_2 \) is calculated. In addition, unstable periodic solutions can be obtained from bifurcation and can help us in better understanding, revealing an intrinsic relationship of the global dynamical behaviors with the stability of the equilibrium point, especially hidden chaotic attractors.

Based on the hidden chaotic attractors [23, 24], we design a new oscillator with coexisting hidden chaos, limit cycles, and point attractors. In Section 2, a generalized system showing hidden chaotic attractors is given. In Section 3, Hopf bifurcation methods about codimensions one and two are given out, in particular, how to obtain the Lyapunov coefficients related to the stability of the equilibrium. In Section 4, the existence of periodic orbits from Hopf bifurcation can be obtained. Finally, in Section 5, we make some concluding remarks and future works.

2. The New Chaotic System with Hidden Chaos

Based on the three-dimensional autonomous system proposed by Vijayakumar et al. [23], we give the generalized system,

\[
\begin{align*}
\dot{x} &= ay - bz, \\
\dot{y} &= z, \\
\dot{z} &= -x - y - z - dz^2 + xy + k,
\end{align*}
\]

where \( a, d \) are positive constants, and \( b, k \) are arbitrary real constant. If \( a = 1, b = 0, d = 2.3 \), system (1) is the three-dimensional autonomous system proposed by Vijayakumar et.al. [23], which only shows the numerical results. Now, we want to consider why the hidden chaos can be found theoretically. Here, by choosing some parameter values \( k = -0.02, d = 2.3 \) and using certain numerical methods [25, 26], the system has different kinds of chaotic attractors (see Figure 1, Tables 1 and 2).

The characteristic polynomial of the system (1) at the only one equilibrium \( E(k, 0, 0) \) is

\[
\lambda^3 + \lambda^2 + (1 - b - k)\lambda + a = 0.
\]

Proposition 1. If we choose \( b \) as bifurcation parameter and \( b = 1 - a - k \) in the system (1), characteristic values of \( E(0, 0, 0) \) have one negative real eigenvalue \(-1\) and a pair of purely imaginary eigenvalues \( \pm \sqrt{a}i \).

Proposition 2. If we choose \( a \) as bifurcation parameter and \( a = 1 - b - k \) in system (1), characteristic values of \( E(0, 0, 0) \) have one negative real eigenvalue \(-1\) and a pair of purely imaginary eigenvalues \( \pm \sqrt{1 - b - ki} \).

3. Framework of the Hopf Bifurcation Methods

The Hopf bifurcation methods mainly refer to [19, 27–30]. For the system,

\[
\dot{X} = f(X, \mu),
\]

where \( X \in \mathbb{R}^3 \) and \( \mu \in \mathbb{R}^4 \), and \( f \) is a class of \( C^\infty \). If we denote (3) has an equilibrium point \( X = X_0 \) at \( \mu = \mu_0 \), mark the variable \( X - X_0 \) also by \( X \), and then write

\[
F(X) = f(X, \mu_0).
\]

As

\[
F(X) = AX + \frac{1}{2}A_2(X, X) + \frac{1}{6}A_3(X, X, X) + O(\|X\|^4),
\]

where \( A = f_x(X_0, \mu_0) \) and, for \( i = 1, 2, 3 \),

\[
A_2(X, Y) = \sum_{j, k=0}^3 \frac{\partial^2 F_i}(X, Y, Z)
\]

Suppose \( \lambda_{2,3} = \pm iw_0(w_0 > 0) \) are a pair of complex eigenvalues. Let \( p, q \in \mathbb{C}^3 \) be vectors such that

\[
Aq = iw_0q, A^T p = -iw_0p, \langle p, q \rangle = 1,
\]

where \( A^T \) represents the transposed matrix \( A \). Any vector \( y \in \mathbb{T}^3 \) can be given as \( y = wq + \overline{wq} \), where \( w = \langle p, y \rangle \in \mathbb{C} \). The 2D center manifold at the eigenvalues \( \lambda_{2,3} \) can be parameterized by \( w \) and \( \overline{w} \). Through the use of a form immersion, \( X = H(w, \overline{w}) \), where \( H : C^2 \longrightarrow R^3 \) has a Taylor expansion of the following form:

\[
H(w, \overline{w}) = wq + \overline{wq} + \sum_{2 \leq j \leq k \leq 3} \frac{1}{jk!} h_{j,k} w^j \overline{w}^k + O(\|w\|^4).
\]

With \( h_{j,k} \in \mathbb{C}^3 \) and \( h_{j,k} = \overline{h_{k,j}} \), then

\[
H_w w' + H_{\overline{w}} \overline{w}' = F(H(w, \overline{w})),
\]

where \( F \) is given by (4). Taking into account the chart \( w \) for a central manifold, we can obtain

\[
\dot{w} = iw_0w + \frac{1}{2}G_{21} w^2 |w|^2 + O(\|w\|^4),
\]

With \( G_{21} \in \mathbb{C} \), the first Lyapunov coefficient can be given as
where $G_{21} = \langle p, A_3(q, q, q) + A_2(q, h_{20}) + 2A_2(q, h_{11}) \rangle$.

Denoting $\mathcal{H}_{32}$ as

$$\mathcal{H}_{32} = 6A_2(h_{11}, h_{21}) + A_2(h_{20}, h_{30}) + 3A_2(\overline{h}_{21}, h_{20}) + 3A_2(q, h_{22}) + 2A_2(\overline{q}, h_{31}) + 6A_3(q, h_{11}, h_{11}) + 3A_3(q, q, \overline{h}_{21}) + 6A_3(q, \overline{q}, h_{21}) + 6A_3(\overline{q}, h_{20}, h_{11}) + A_3(\overline{q}, q, h_{30})$$

And $G_{32} = \langle p, \mathcal{H}_{32} \rangle$, the second Lyapunov coefficient $l_2$ is given by

$$l_2 = \frac{1}{12} \text{Re}G_{32}.$$  

(11)

(12)

When the first Lyapunov coefficient $l_1 \neq 0$, the dynamic behavior of the system (3) is orbitally topologically equivalent to

$$\dot{w} = (\eta + i\omega)w + l_1 w|w|^2.$$  

(13)

(14)

As $l_1 < 0 (l_1 > 0)$, we can find the existence of codimension one Hopf point and stable (unstable) periodic orbits on this manifold. Moreover, when $l_1 = 0$, we can further consider the Hopf point of codimension two. When
l_2 \neq 0$, the dynamic behavior of the system (3) is orbitally topologically equivalent to
\[ w_t = (\eta + i\omega)w + tw|\omega|^2 + l_2w|\omega|^4, \]
where $\eta$ and $\tau$ are unfolding parameters.

4. Hopf Bifurcation and Hidden Chaos in New System

Using the mark in Section 3, we can write the nonlinear symmetric functions (1)
\begin{align*}
A_2(x, y) &= (0, 0, -2dx_3y_3 + x_1y_2 + x_2y_1),
A_3(x, y, z) &= (0, 0, 0, 0).
\end{align*}
(16)

4.1. Hopf Bifurcation about Parameter $a$

**Theorem 1.** For system (1) with $a = a_0 = 1 - b - k$, the first Lyapunov coefficient at the equilibrium $E$ is given by
\[ l_1 = \frac{f(b, k, d)}{2(b + k - 2)(b + k - 1)(4b + 4k - 5)}, \]
where
\[ f(b, k, d) = -8b^2d^2 + 10b^2d - 6b^2k + 16bd^2k + 16bd^2 
+ 24bdk - 23bd - 10bk + 10b - 8d^2k^2 
+ 16d^2k - 8d^2 + 14d^2k^2 - 27dk 
+ 13d - d + 1. \]
(18)

If $l_1 > 0$, the Hopf point at $E$ is a weak repelling focus, and an unstable limit cycle can be found near the asymptotically stable equilibrium $E$ for each $a < a_0 = 1 - b - k$, but close to $a_0$; if $l_1 < 0$, the Hopf point at $E$ is weak attractive focus, and a stable limit cycle can be found near the unstable equilibrium $E$ for each $a > a_0 = 1 - b - k$, but close to $a_0$.

**Proof.** Considering $a$ as the bifurcation parameter, the transversal condition
\[ \text{Re}\left( \lambda (a_0) \right)_{l_1 = \sqrt{i1-b-k}} = \frac{1}{4 - 2b - 2k} > 0, \]
(19)
is met. The first Lyapunov coefficient $l_1$ will show the stability of the equilibrium point $E$ and periodic orbits generate from Hopf bifurcation.

In addition, one can also get
\[ p = \{p_1, p_2, p_3\}, \]
\[ q = \left\{ \frac{i(b + i\sqrt{-b - k + 1})}{\sqrt{-b - k + 1}}, -\frac{i}{\sqrt{-b - k + 1}}, 1 \right\}, \]
(20)
\[ h_{20} = \{h_{201}, h_{202}, h_{203}\}, \]
\[ h_{11} = \left\{ \frac{2(b(-d) + b - dk + d)}{b + k - 1}, 0, 0 \right\}, \]
(21)
where
\[ \text{Re}[G_{21}] = \frac{f(b, k, d)}{(b + k - 2)(b + k - 1)(4b + 4k - 5)}, \]
(22)
where
the Hopf bifurcation parameter, we consider system (1) with the bifurcation for hidden chaos. We choose parameters $b_0 = 2, k = 4.2$. 

In this section, we consider Hopf bifurcation about parameter $b$. In this section, we consider Hopf bifurcation about parameter $b$ at $E$ taking $b$ as the Hopf bifurcation parameter, we consider system (1) with $b = b_0 = 1 - a - k$. The transversal condition

$$\text{Re} \left( \lambda(b_0) \right)_{b=b_0} = \frac{1}{2(a+1)} > 0,$$

is also satisfied.

**Theorem 3.** For system (1), with $b = b_0 = 1 - a - k$, the first Lyapunov coefficient at the equilibrium $E$ is given by

$$l_1 = \frac{-331104 + 648475k - 316250k^2}{(50k - 49)(5000k^2 - 16050k + 12177)}.$$  

In addition, we can know $k < 1 - b$ (i.e., $k < 0.98$) from $a > 0$. Therefore, if $k \in \{k | k < 0.9606\}$, the first coefficient $l_1$ are positive, and unstable periodic orbit can be obtained. If $k \in \{k | 0.9606 < k < 0.98\}$, the first coefficient $l_1$ are negative, and stable periodic orbit can be obtained. Now, we will consider the sign of the second Lyapunov coefficient $l_2$ when $l_1 = 0$ with $k = 0.9606$.

**Theorem 2.** For system (1), with $a = a_0 = 0.01942$, the first Lyapunov coefficient $l_1 = 0$ and the second Lyapunov coefficient for $E$ is $3.0633$. Therefore, system (1) has a transversal Hopf point of codimension two at $E$ which is unstable.

**Proof.** If $b = 0.02, d = 2.3, k = 0.9606$, and $a = 0.01942$, the first Lyapunov coefficient $l_1 = 0$ and

$$G_{21} = -3.03791i.$$ 

We continue to do some detailed calculations and can get

$$h_{21} = \{-11.744 - 4.4851i, 21.3868 + 75.2525i, -32.2878 + 2.98005i\},$$

$$h_{30} = \{43.3288 - 14.0499i, -83.6812 + 896.832i, -374.896 - 34.9807i\},$$

$$h_{31} = \{13.7584 - 5.43469i, 19.6935 + 203.132i, 897.586 - 282.767i\},$$

$$G_{32} = 36.7597 - 506.22i.$$ 

By the above theorem and calculation, one has

$$l_2 = \frac{1}{12} \text{Re}[G_{32}] = 3.0633.$$ 

Therefore, we can arrive at the above Theorem 2. 

The complex vectors $h_{11}$ and $h_{20}$ are

$$l_1 = \frac{8a^3d^2 - 10a^2d + 6a^2 + 4ad - 3a d + 2ak - 2a - 4k^2 + 9k - 5}{2a(4a^2 + 5a + 1)}$$

If $l_1 > 0$, the Hopf point at $E$ is weak repelling focus, and an unstable limit cycle near the asymptotically stable equilibrium $E$ can be found for each $b < b_0 = 1 - a - k$, but close to $b_0$; if $l_1 < 0$, the Hopf point at $E$ is weak attractive focus, and a stable limit cycle near the unstable equilibrium $E$ can be found for each $b > b_0 = 1 - a - k$ but close to $b_0$.

**Proof.** Here, we have

$$q = \left\{ -1 - i(a + k - 1), \frac{i}{\sqrt{a}}, \frac{1}{\sqrt{a}} \right\},$$

$$p = \left\{ - \frac{i}{2(\sqrt{a} + i)}, \frac{\sqrt{a} + i(k - 1)}{2(\sqrt{a} + i)}, \frac{\sqrt{a}}{2(\sqrt{a} + 2i)} \right\}.$$ 

The complex vectors $h_{11}$ and $h_{20}$ are

$$l_1 = \frac{-331104 + 648475k - 316250k^2}{(50k - 49)(5000k^2 - 16050k + 12177)}.$$
Figure 2: When $b = 0.02, k = -0.02, d = 2.3$, we choose $a = 0.95$: (a) stable equilibrium $E$; (b) unstable periodic orbit with initial values $(-0.23, -1, 0.25)$ near stable equilibrium $E$ from Hopf bifurcation.
Figure 3: When $a = 1, d = 2.3, k = -0.02$, and $b = 0.015$, system (1) has coexisting attractors: (a) stable equilibrium $E$ for initial values $(0, 0, 0)$; (b) unstable periodic orbit with initial values $(-0.163, 0, 0)$ near stable equilibrium $E$ from Hopf bifurcation; (c) hidden chaos with stable equilibrium $E$ and initial values $(-2, 0, 0)$.

Figure 4: Bifurcation diagrams and Lyapunov exponents for system with $b = 0.02, d = 2.3, k = -0.02$ and with initial conditions $(x, y, z) = (-0.23, -1, 0.25)$. (a) Bifurcation, (b) Lyapunov exponents.

Figure 5: Bifurcation diagrams and Lyapunov exponents for system with $a = 1, d = 2.3, k = -0.02$ and with initial conditions $(x, y, z) = (0, 0, 0)$. (a) Bifurcation, (b) Lyapunov exponents.
The complex coefficient $G_{21}$ defined in Section 3 is
We then have Theorem 3 from first Lyapunov coefficient $l_1$. Consider system (1) with $a = 1, d = 2.3, k = -0.02$. The first Lyapunov coefficient associated with the equilibrium $E$ is 1,014. Then, the equilibrium $E$ undergoes a transversal Hopf bifurcation when $b = b_0 = 0.02$. More specifically, when $b = 0.015 < b_0$, but near to $b_0$, there exists an unstable limit cycle around the asymptotically stable equilibrium $E$ (see Figures 3(a) and 3(b)). The result will herald the emergence of hidden chaos (see Figure 3(c)).

In order to check Hopf bifurcation and hidden chaos, we choose $b = 0.02, d = 2.3, k = -0.02$, and $a \in [0.81]$ with initial conditions $(x, y, z) = (-0.23, -1, 0.25)$, and bifurcation diagrams and Lyapunov exponents diagram are shown in Figures 4(a) and 4(b), respectively. When $a = 1, d = 2.3, k = -0.02$, and $b \in [-0.05, 0.05]$, multistability can happen from different initial conditions $(x, y, z) = (0, 0, 0)$ (see Figure 5), initial conditions $(x, y, z) = (-0.163, 0, 0)$ (see Figure 6), and initial conditions $(x, y, z) = (-2, 0, 0)$ (see Figure 7).

5. Conclusion

In this paper, Hopf bifurcations in generalized system chaotic systems with hidden chaos are obtained theoretically. Through this analysis, we obtain the parameter conditions for which the system presents Hopf bifurcations. Then, we make an extension of the analysis to the more degenerate cases. The calculation of the first second and second Lyapunov coefficients, which makes possible the determination of the Lyapunov stability at the equilibrium, can make the system exhibit Hopf bifurcation in a much larger parameter region. The first and second Lyapunov coefficients are obtained for exhibiting Hopf bifurcation and showing periodic orbits in the parameter region. In addition, numerical simulations of several parameter values are carried out to illustrate and verify some analysis results. The cascade of period-doubling bifurcation and the existence of hidden attractors are related to Hopf bifurcation at the equilibrium point in a sense. This interesting phenomenon is worth further studying, both theoretically and experimentally, to further reveal the intrinsic relationship between the local stability of equilibrium and global complex dynamical behaviors of the chaotic system. In addition, a fractional-order version of the chaotic system can be designed using integrated circuit technology [31–37], as required in wireless systems. It is expected that a more detailed theoretical analysis will be excavated in the forthcoming paper.

Data Availability

All data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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