Light bullets from Mid-IR femtosecond filament in air

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Abstract. We numerically investigated the formation of light bullets under filamentation of 3.8-\textmu m femtosecond pulse in presence of anomalous GVD in humid air. The dispersion of humid air in infrared spectral region was characterized by model fits of the real part of refractive index from HITRAN database. The fit for the 2.8-4.2 \textmu m region describes the areas of anomalous GVD near 3.1 \textmu m and 4 \textmu m in humid air. During the nonlinear propagation of femtosecond pulse in humid air, the compressed in space and time wave packet - light bullet - appears in the central time layers of the pulse and moves to the pulse tail. The duration of light bullet reaches the few-cycle value and the energy fluence in the in the light bullet cross-section is above 1 J/cm\textsuperscript{2}. Together with the light bullet formation the spectrum of the pulse broadens strongly and covers the spectral region from 3 \textmu m to 5 \textmu m. The interference model was used for investigation of the frequency-angular distribution of the supercontinuum spectrum components.

Introduction

The start point of the filament, the length of its plasma channels and the efficiency of the spectral conversion into supercontinuum could be controlled by the input pulse parameters according to the nonlinear medium in which the pulse propagates [1-3]. Nowadays high-power Mid-IR femtosecond sources are started to use for filament formation in the gaseous media [4-5]. The generated coherent [6] supercontinuum covers over several octaves, that is very promising for the molecular spectroscopy. The filamentation of 3.1-\textmu m pulse under anomalous GVD was investigated numerically in [7]. In this work

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we present the results of numerical simulation of the nonlinear propagation of the 3.8-µm femtosecond pulse in humid air.

**Methods**

We used two mathematical models of pulse propagation in nonlinear medium: the first model based on slow varying wave approximation [8] and the second model based on the unidirectional wave equation [9] for complex electric field. In both cases the ionization of oxygen and nitrogen was calculated according to the PPT model [10]. The order of the multiphoton ionization process for oxygen $K_{O2} = 37$ and for nitrogen $K_{N2} = 48$ at the considered pulse central wavelength $\lambda_0 = 3.8$ µm.

The full dispersion of humid air was taken into account according to the fits (figure 1 a) [11].

![Figure 1. Refractive index of air. a) Real part of air refractive index from HIRAN database (blue line) and model fits (red line) [11]. b) Imaginary part of air refractive index from HIRAN database (blue line) [11] and numerical approximation (red line) used in this work. Temperature 290.65 K, humidity 10 %, atmospheric pressure 101330 Pa.](image)

In our numerical simulation the resonance areas are represented as the areas with high absorption (figure 1 b). In this work we numerically investigated the formation of light bullets (LBs) under filamentation of 3.8-µm femtosecond pulse in presence of anomalous GVD in humid air.

The behaviour of the dispersion parameter $k_2 = \frac{\partial^2 k(\omega)}{\partial \omega^2}$ near 3.8 µm for humid air (figure 2) is close to the behaviour of $k_2$ near 2 µm in fused silica [12]. The anomalous DVG in fused silica plays a key role in the formation few-cycle wave packets that have the quasi-soliton properties [13].
To investigate the frequency-angular distribution of the supercontinuum spectral components during filamentation of the 3.8-µm femtosecond laser pulse we used the interference model [14]. This model is based on the assumption, that the broadened supercontinuum point source moves along the high-intensity areas in the filament. The interference of the emitted spectral components defines the modulation of the frequency-angular spectrum of the supercontinuum.

Results and discussion
The transformation of the laser pulse starts from the Kerr self-focusing of the central time layers (figure 3, z = 21.1 m). Then the self-induced laser plasma defocuses the pulse tail. The peak plasma density reaches the value $N_{\text{e max}} = 6 \times 10^{14}$ 1/cm$^3$. In presence of the Kerr-induced positive self-phase modulation (SPM) the anomalous GVD leads to the power flows from pulse front and tail to the pulse center. Such regime of the pulse propagation under positive SPM and anomalous GVD causes to formation of the localized in space ant time wave packet - light bullet [15]. The pulse central wavelength $\lambda_0 = 3.8$ µm in our numerical simulations lays in the region of anomalous GVD in humid air [11]: $k_2 = -1.25$ fs$^2$/cm. The input pulse parameters: $\tau_{\text{FWHM}} = 170$ fs and $a_0 = 1$ cm. Under the considered conditions the light bullet appears at the propagation distance about $z = 21$ m. During filamentation in humid air the duration and radius of LB decrease from input pulse values to the $\tau_{\text{LB}} = 30$ fs and $a_{\text{LB}} = 250$ cm (figure 3). Peak intensity reaches the value $I_{\text{LB}} = 5 \times 10^{13}$ W/cm$^2$, and the energy fluence in the in the LB cross-section is above $F_{\text{LB}} = 1$ J/cm$^2$.
The results of numerical simulation, based on both models, describe the same LB evolution during the pulse propagation. LB forms in the central time slices and then shifts to the pulse tail (figures 3, 4). At the same time the spectrum of the LB broadens and the central wavelength shifts to the red side of the spectrum. At the propagation distance $z = 23$ m the supercontinuum spectrum covers the region from 3 µm to 5 µm. The dip in SC around 4.25 µm appears due to the absorption in the resonance region (figure 1 b) [11].

![Figure 4. LB time profile (first line) and its spectrum transformation (second line). Numerical simulation is based on the unidirectional wave equation. $A_0 = 1.4 \times 10^7$ V/cm. LB profiles are presented in the frame moving with the group velocity of the input laser pulse. Negative values of time $\tau$ corresponds to the pulse front.](image)

Using the interference model [14] we found the frequency-angular distribution of the supercontinuum components of the 3.8-µm light bullet in humid air. We used the refractive index $n(\omega)$ from HITRAN database [11] and the numerically obtained group velocity of the light bullet $\nu_{LB}$, that is smaller than the group velocity of the input pulse $\nu_g$: $\nu_{LB} = (1 - 0.00005) \cdot \nu_g$. The length of the emitting region $L$ in the filament also was estimated from our numerical simulation. It was assumed that the broadband source emits at all wavelengths and in all directions equally.

We found that in the conical emissions is formed in Stokes and in anti-Stokes regions of the supercontinuum spectrum $S_{\text{interf}}(0, \lambda)$ (figure 5) with the increasing of the emitting area length in the filament. The most strong interference maximum appears in the long-wavelength region of the supercontinuum spectrum $S_{\text{interf}}(0, \lambda)$. The wavelength of constructively interfering radiation increases with the increasing of the angle of its propagation. This behaviour of the long-wavelength supercontinuum components is in a good agreement with the experimental results on the registration of the conical emission of the Mid-IR radiation in the different types of filamentation processes [16,17].
Figure 5. The transformation of the frequency-angular distribution $S_{\text{inter}}(\theta, \lambda)$ of the supercontinuum components along the emitting region in the filament. Central wavelength $\lambda_0 = 3.8 \, \mu\text{m}$. L - length of the emitting region. $S_0$ - constant.

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