Proposed split-causality test of the relativity principle

George Jaroszkiewicz
School of Mathematical Sciences, University of Nottingham, Nottingham, UK
(Dated: March 24, 2022)

We propose a test of the principle of relativity, involving quantum signals between two inertial frames. If the principle is upheld, classical causality will appear to be split in a dramatic and emphatic way. We discuss the existence of quantum horizons, which are barriers to the transmission of any form of quantum information. These must occur in any finite time, inter-frame experiment if quantum causality holds. We conclude with some comments on such experiments involving entangled states.

PACS numbers: 03.30.+p, 03.65.-w
Keywords: relativity principle, quantum causality, quantum horizon

I. INTRODUCTION

The principle of relativity in its strong form states that the laws of physics are the same in all standard inertial frames, if gravitational effects are excluded. It was used by Einstein to derive the Lorentz transformation

\[ t' = \gamma (t - vx/c^2), \quad x' = \gamma (x - vt), \quad y' = y, \quad z' = z, \quad (1) \]

\[ \gamma = 1/\sqrt{1 - v^2/c^2}, \]

between two standard inertial frames \( \mathcal{F}, \mathcal{F}' \) moving apart with relative speed \( v \) along the \( x \)-direction [1]. A notable feature is the loss of absolute simultaneity: a hyperplane of simultaneity \( t' = \text{const} \) in \( \mathcal{F}' \) is not a hyperplane of simultaneity in \( \mathcal{F} \). We shall investigate the consequences of adding quantum physics to this feature of relativity.

In contrast to most approaches to relativity, we shall focus on very specific hyperplanes of simultaneity, one in each frame. For convenience, we shall ignore the \( y \) and \( z \) coordinates; although they are physically relevant, they do not feature in the essential points of the discussion. We shall consider an experiment where a signal is sent from \( \mathcal{F} \) at time \( t = 0 \) and observed in frame \( \mathcal{F}' \) at time \( T' \), as measured in its frame. Figure 1 shows the essential details.

Event \( O \) is the common origin of spacetime coordinates, events \( P \) and \( Q \) are simultaneous in \( \mathcal{F} \) at time \( t = 0 \) whilst events \( P', Q' \) are simultaneous in \( \mathcal{F}' \) at time \( t' = T' \). Our convention is that an event \( P \) has coordinates \( (t_P, x_P) \) in \( \mathcal{F} \), coordinates \( (t'_P, x'_P) \) in \( \mathcal{F}' \) and we write \( P \sim (t_P, x_P) \sim [t'_P, x'_P] \). A critical feature is event \( B \), where the lines of simultaneity \( t = 0 \) and \( t' = T' \) intersect. Assuming \( v \) is positive, then for \( B \) we have

\[ B \sim (0, -\frac{c^2T'}{\gamma v}) \sim [T', -\frac{c^2T'}{v}]. \quad (2) \]

Recall that in de Broglie wave mechanics, the speed \( w \) of a pilot wave associated with a physical particle moving with subluminal speed \( v \) satisfies the relation \( vw = c^2 \). This suggests that such a pilot wave cannot be used to convey physical signals, because it travels at superluminal speed. The position of event \( B \) appears to observers in \( \mathcal{F}' \) to be the wave front at time \( T' \) of such a pilot wave associated with frame \( \mathcal{F} \), if it were sent out from \( O \) in the same direction. Clearly, for large \( T' \) and small \( |v| \), \( B \) will be at relatively large distance from the origin of spatial coordinates \( A \) in frame \( \mathcal{F}' \), at the time \( t' = T' = T/\gamma \) when the signal from \( \mathcal{F} \) is observed in \( \mathcal{F}' \) during the experiment.

In standard discussions of special relativity, event \( B \) is generally ignored, as it appears to be far removed from events \( P \) and \( P' \) involved in the signalling experiment. For this experiment, we imagine that a quantum state has been prepared by apparatus \( A_P \), at rest in frame \( \mathcal{F} \), and a contingent quantum outcome subsequently detected by apparatus \( A_P' \), at rest in frame \( \mathcal{F}' \).

The critical word here is “subsequently”. Quantum physics, as it is performed in real laboratories, can discuss only the possibility of quantum information travelling forwards in time. Both signal emitter and signal detector in any quantum experiment must agree that the former acts before the latter. Otherwise, the physical significance of the Born probability rule would be completely undermined. In quantum theory and in the real world, we do not know the outcome of an experiment before it is performed. We shall call the requirement that \( P \) is earlier than \( P' \) in both frames of reference quantum causality.

From Figure 1, it is clear that there is no problem with quantum causality as far as events \( P \) and \( P' \) are
concerned. But consider events $Q$ and $Q'$ on the other side of $B$. If quantum causality is valid, then signals prepared at $Q$ cannot be received by $Q'$. In essence, event $B$ acts a barrier to quantum causality, and on this account we shall refer to $B$ as a quantum horizon.

Ordinarily, such quantum horizons are ignored in conventional physics, because under most circumstances, $B$ appears to be very far from events such as $P$ and $P'$. In experiments looking at such aspects of quantum information, speeds in excess of $10^5 c$ have been reported [2]. In practice, high energy particle theory conventionally takes the scattering limit $t' \to \infty$, $u = 0$ in the calculation of Lorentz covariant matrix elements. Finite-time processes and inter-frame experiments of the sort considered by us here are generally avoided, because it is assumed there is no significant novel physics involved. An important factor in this is that the scattering limit makes calculations such as those arising from Feynman diagrams take on relatively standard forms and does not involve issues to do with a quantum horizon, which is at spatial infinity under those circumstances. Such a simplification does not happen for finite-time and inter-frame processes.

We now consider the implications of the relativity principle and ask the following question: if according to the relativity principle frames $\mathcal{F}$ and $\mathcal{F}'$ are “just as good as each other”, why does the quantum horizon $B$ appear to distinguish between the two?

A little though soon resolves the question. If the relativity principle is valid, then there must be a symmetry between the two frames. There is no doubt that a quantum signal can be prepared at $P$ and received at $P'$, if $P'$ is in or on the forwards lightcone with vertex $P$. Quantum causality rules out the transmission of a quantum signal from $P'$ to $P$, and the transmission of a signal from $Q$ to $Q'$. But nothing currently known in physics forbids the possibility of a physical signal being sent from $Q'$ to $Q$, if $Q$ is in the forwards lightcone with vertex $Q'$. Indeed, symmetry demands such a possibility. This is the essence of the split causality experiment proposed here.

II. PROPOSED EXPERIMENT

Based on the above considerations, we propose the following experimental test of the principle of special relativity. It will undoubtedly be difficult to perform, but would test the principle of relativity in a spectacular and convincing way.

We envisage the use of four spacecraft $P, Q, P'$ and $Q'$, sufficiently far from gravitating bodies to justify the use of the special relativistic transformation rules (11). $P$ and $Q$ are in the same rest frame $\mathcal{F}$ and situated at some distance from each other. By prior signalling arrangement, clocks on $P$ and $Q$ craft have been synchronized. Likewise, $P'$ and $Q'$ are in their own rest frame $\mathcal{F}'$ and all their clocks have been synchronized.

With reference to Figure 1, spacetime homogeneity means that we may always transfer the origin of space-time coordinates in both frames $\mathcal{F}$, $\mathcal{F}'$ to the quantum horizon $B$. This means that the hyperplanes of simultaneity involved in the experiment are now at times $t = 0$ in $\mathcal{F}$ and $t' = 0$ in $\mathcal{F}'$, as shown in Figure 2.

The experiment consists of $P$ sending a brief light pulse signal towards $P'$ at time $t = 0$, whilst simultaneously in $\mathcal{F}$, $Q$ opens a detector in order to receive light from $Q'$ for a similar brief period. In addition, the same protocol is carried out in frame $\mathcal{F}'$ at time $t' = 0$: $Q'$ sends a brief light pulse towards $Q$ whilst simultaneously in $\mathcal{F}'$, $P'$ opens a detector to receive a signal from $P$. The whole experiment is illustrated in Figure 2.

After the signals have been sent and received, observers from all spacecraft can meet at leisure and compare results. If it turns out that $P$ sent a signal at the same time $t = 0$ that $Q$ received a signal, and that $P'$ received a signal at the same time $t' = 0$ that $Q'$ sent a signal, then the principle of relativity would be upheld in a most convincing way. Quantum causality would have been respected but classical causality would appear to be “split” in a most remarkable and counterintuitive fashion. On the other hand, if no such result was ever detected despite repeated attempts, this would rule out the principle of special relativity and undermine the whole of conventional physics. Alternatively, quantum causality would have to be reviewed, with equally disastrous implications for modern physics.

Although the result may appear a forgone conclusion in favour of the relativity principle, complacency here would be unjustifiable. An actual experiment involving relatively moving frames has to be involved. It would not do to simulate the experiment using a single frame of reference and invoking scattering processes involving high energy particles, unless the issues of the meaning of timing of virtual particle processes which would inevitably arise were adequately resolved.

Such an inter-frame experiment would not need to be performed more than once to establish the principle of relativity in the most spectacular way: a single splitting of causality would suffice to vindicate both the Lorentz transformation rule (11) and our insistence on the need for the quantum causality rule.

The difficulties in this experiment are of course technological. First, it would be expensive though not impossible to arrange four spacecraft in such a configuration. Current resources rule out such a possibility, but
we envisage that with the present increasing interest in the colonization of the Moon and subsequent journeys to Mars, space travel will develop into a more routine activity, with an increased availability of vehicles for such an experiment.

Some economy could be found by using the Moon and some other body, such as the Earth or a suitable asteroid, to locate say events $P$ and $Q$. A significant residual problem would be the need to boost the two spacecraft representing events $P'$ and $Q'$ to sufficiently high speed $|v|$ to allow unambiguous effects to be observed. An even greater economy could be made by using four small unmanned but self-propelled probes, two of which were programmed to emit signals whilst the other two were programmed to receive them. Such an experiment seems within the capability of present-day technology. We note that the signals involved need not be light signals. Any form of communication with subluminal propagation speeds in the proposed set-up should demonstrate classical split-causality, if the principle of relativity holds.

### III. ENTANGLED STATES AND QUANTUM HORIZONS

The above scenario involves classical signalling processes. An even more interesting situation arises when quantum entangled states are involved, as this touches upon the debate concerning information loss in black hole physics. Consider the experiment illustrated by Figure 3, where a spin zero positronium state is prepared at event $S$. The forwards lightcone with vertex $S$ contains events $P, Q, P'$ and $Q'$, so that in principle each event could detect component particles of the prepared state. We shall imagine restricting $P$ and $P'$ to the observation of electron spin orientation only, via Stern-Gerlach apparatus, whilst $Q$ and $Q'$ do the same for positrons.

Depending on the choices made, a number of different observation protocols could be implemented, but not necessarily all in the same run (we assume the overall experiment is repeatable as often as required to collect adequate statistics). It is useful here to assign a different Hilbert space to each such observation protocol: $\mathcal{H}(P)$ is the Hilbert space used to describe the potential outcome of an electron spin test at $P$, $\mathcal{H}(P \cup Q) = \mathcal{H}(P) \otimes \mathcal{H}(Q)$ is the Hilbert space used to describe a simultaneous-in-$\mathcal{F}$ observation of an electron at $P$ and a positron at $Q$, and so on.

Provided $Q'$ was instructed in advance not to perform its test, then $P$ and $Q$ could perform their tests, describing the state prepared by $S$ in terms of an entangled state in $\mathcal{H}(P \cup Q)$ of the form $u_P \otimes d_Q - d_P \otimes u_Q$, where $u_P$ represents a spin-up outcome at $P$, $d_Q$ represents a spin-down outcome at $Q$, and so on. Likewise, provided $P$ was instructed in advance not to perform its test, then $P'$ and $Q'$ could perform their tests, describing the prepared state in term of the entangled state $u_P' \otimes d_Q' - d_P' \otimes u_Q'$, an element in $\mathcal{H}(P' \cup Q') = \mathcal{H}(P') \otimes \mathcal{H}(Q')$. Conservation of electric charge rules out any possibility of outcomes described via $\mathcal{H}(P) \otimes \mathcal{H}(P')$ or $\mathcal{H}(Q) \otimes \mathcal{H}(Q')$. However, in principle, it should be possible to involve outcomes described in $\mathcal{H}(P) \otimes \mathcal{H}(Q')$ or $\mathcal{H}(P') \otimes \mathcal{H}(Q)$.

An interesting possibility arises with an experiment described initially in $\mathcal{H}(P) \otimes \mathcal{H}(Q)$. In such an experiment, $Q$ would always observe a positron whenever $P$ observed an electron. Now consider the addition of a choice at $Q'$ to test positron spin. Any observation at $Q'$ would take place before $Q$, according to frames $\mathcal{F}$ and $\mathcal{F}'$, and therefore quantum causality would apply. The detection of a positron at $Q'$ would destroy the possibility of a detection of a positron at $Q$, because of charge conservation. This would hold even if $Q$ was outside the forwards lightcone centred on $Q'$, as shown in Figure 3. This suggests that a free choice at $Q'$ could have superluminal consequences at $Q$, apparently in conflict with relativity.

Our resolution of this conflict is to note that if $Q$ is outside the forwards lightcone of $Q'$, then in effect we should regard $P, Q'$ and $Q$ as on some hypersurface of simultaneity, which, like $P \cup Q'$ and $P' \cup Q$, cannot be identified with a single inertial frame. There is no requirement in relativistic quantum mechanics to restrict all experiments to single inertial frames. What matters is quantum causality. We see from this discussion that the principle of quantum causality has to be applied not just with respect to hyperplanes of simultaneity in inertial frames, but for arbitrary spacelike hypersurfaces as well. Quantum causality should hold even in those experiments where various pieces of apparatus lie in different inertial frames. Given this observation, we deduce that after the state is prepared at $S$, an electron can be detected at $P$, a positron detected at $Q$ and nothing detected at $Q'$ for some runs, whilst for other runs, an electron would be detected at $P$, and a positron detected at $Q'$, and nothing detected at $Q$. There would be no possibility of using this experiment to signal from $Q'$ to $Q$, if $Q$ lay outside the forwards lightcone centred on $Q'$.

We denote the Hilbert space involved in this extended experiment by $\mathcal{H}(P \cup Q \cup Q') \equiv \mathcal{H}(P) \otimes \mathcal{H}(Q \cup Q')$. We observe that for this experimental setup, $\mathcal{H}(Q \cup Q')$ cannot be the tensor product $\mathcal{H}(Q) \otimes \mathcal{H}(Q')$, because any detection of a positron at $Q'$ rules out the detection of a positron at $Q$, and vice-versa. Instead, we would

![Figure 3: A positronium state is prepared as $S$. Events $P, Q, P'$ and $Q'$ are well with the forwards lightcone centred on $S$.](image)
have $\mathcal{H}(Q \cup Q') = \mathcal{H}(Q) \oplus \mathcal{H}(Q')$, i.e., $\mathcal{H}(Q)$ and $\mathcal{H}(Q')$ can be regarded as mutual orthogonal complements in $\mathcal{H}(Q \cup Q')$. It should be possible to use standard quantum relativistic field theory to determine the respective outcome probabilities at $Q$ and $Q'$. These will depend on where the detectors are situated.

Finally, we comment on the meaning of entanglement. It is not physically meaningful to talk about the preparation of quantum states, entangled or not, without reference to any context of subsequent observation. It is the choice of test apparatus which determines whether a state is to be regarded as entangled or not. For instance, in Figure 3, we could not locate $S$ on the quantum horizon $O$ and meaningfully talk about an entangled electron-positron state from the point of view of any tests involving pieces of equipment on either side of the quantum horizon. In any discussion involving quantum information and black hole physics, problems will inevitably arise whenever quantum states are discussed without due regard for the equipment used to test them. For this reason, all discussions of quantum mechanics across event horizons or wavefunctions for the universe without due reference to observers and their equipment should be avoided as metaphysical.

[1] Einstein A 1905 Ann. der Physik 17 891-921
[2] Scarani et al 2000 Phys. Lett. A276 1-7