Simple mean-error estimation of a recalibration scheme for constant-temperature anemometry measurements

Hiroki Suzuki1, 3, Yoshiaki Shikata1, Shinsuke Mochizuki1 and Yutaka Hasegawa2
1Graduate School of Sciences and Technology for Innovation, Yamaguchi University, 2-16-1 Tokiwadai, Ube-shi, Yamaguchi 755-8611, Japan
2Department of Electrical and Mechanical Engineering, Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya-shi, Aichi 466-8555, Japan
3Email: h.suzuki@yamaguchi-u.ac.jp

Abstract. The present study has addressed the error of a recalibration scheme for constant-temperature anemometry (CTA) measurement proposed in a recent study. This recalibration scheme could reduce the effects of the ambient temperature change, which should be negligible in a CTA measurement. The present study has investigated a temporal variation of the error due to the use of the recalibration scheme. In the present study, a calibration curve produced by the recalibration scheme is validated using a temporally varying calibration curve, which is based on the general form of a calibration curve used in CTA measurement. The range of velocity $U$ considered in the present study is $U = 1 – 40$ m/s. As a result, the relative error of the calibration curve is significant in the range of small velocity. The relative error of the gradient of the calibration curve, which is equivalent to the error of the velocity fluctuation, is also significant in the range of both small velocity and free-stream velocity. A simple cubic function could accurately approximate the temporal variation of these relative errors. Moreover, a simple form for calculating the mean value of the relative error could be derived and validated using the present numerical results.

1. Introduction
The velocity fluctuation in turbulent flows consists of waves with various wave numbers. Constant-temperature anemometry (CTA) is a useful measurement technique for measuring the velocity fluctuation in a turbulent flow. In particular, CTA is a proven measurement technology and has a sufficiently high S/N ratio. CTA was used in several recent studies to study turbulent flow, whereas particle image velocimetry and laser Doppler anemometry have become popular over the last decade. For instance, CTA is used to measure velocity fluctuations of grid-generated turbulence (e.g., [1,2]), which is a fundamental type of turbulence. Therefore, the use of CTA should continue to be studied.

In the use of CTA, the ambient temperature should be constant with respect to time in experiments (e.g., [2]). However, even the temporal change of the ambient temperature is generally not constant with respect to time. The temporal change of the ambient temperature could cause the uncertainty of velocity observed by CTA. Specifically, for King’s law, the order of the ambient temperature change may be equivalent to that of the uncertainty of the CTA measurement. Therefore, the effects of the temporal change of the ambient temperature on the observed velocity in the use of CTA have been investigated in several previous studies [3-9]. Most of these previous studies attempted to model the effects of the temporal change of the ambient temperature. In recent years, a previous study [10]...
proposed a scheme that could reduce the effects of the temporal change without the use of modeling. This scheme uses pre- and post-calibration curves and could be considered as a recalibration scheme.

The recalibration scheme for CTA measurement was used to measure the velocity fields of a zero-pressure-gradient turbulent boundary layer in a previous study [10]. Moreover, a recalibration scheme was used to measure the grid-generated turbulence in a recent study [11]. The uncertainty and error in the CTA for measuring the fields of turbulence should be sufficiently small. Thus, the error of the recalibration scheme was theoretically and numerically addressed by a previous study [12]. However, few studies have addressed the error of the recalibration scheme. Thus, for instance, the mean value of the error due to the use of a recalibration scheme may not be considered in previous studies. The temporal variation of the error due to the use of the recalibration scheme is a fundamental problem.

The purpose of the present study is to investigate the temporal variation of the error due to the use of the recalibration scheme. The present study attempts to find a simple mathematical from which to approximate the temporal variation of the error and to investigate the mean value of the error numerically and theoretically. The concept of a temporally varying calibration curve is used to solve the present research problem. Here, the temporally varying calibration curve is based on the general form of the calibration curve for CTA measurement. The calibration curve generated by the recalibration scheme is validated using the temporally varying calibration curve.

2. Methods

CTA generally requires a calibration procedure, which relates the output voltage to the observed velocity. The required calibration curve could vary temporally due to influences of the change of ambient temperature in the experiment. A previous study used a recalibration scheme to reduce the uncertainty caused by the temporal change of the calibration curve. The recalibration scheme uses nondimensional time \( t \), which is defined as follows: \( t = (t' - t_{\text{pre}})/(t'_{\text{post}} - t_{\text{pre}}) \). Here \( t' \), \( t_{\text{pre}} \), and \( t_{\text{post}} \) are the time, the time of pre-calibration, and the time of post-calibration, respectively. Using the nondimensional time \( t \), a calibration curve is generally formed as \( E(U,t) \), where \( U \) is the velocity. Based on a previous study [10], a calibration curve produced by the recalibration scheme is given as follows: \( E(U,t) = R(t) [E_{\text{post}}(U) - E_{\text{pre}}(U)] + E_{\text{pre}}(U) \). Here, \( E_{\text{pre}}(U) \) and \( E_{\text{post}}(U) \) are the pre- and post-calibration curves, which could be experimentally measured, and \( R(t) \), which is referred to as the proportional drift, is given as follows: \( R(t) = [E(U_{0}) - E_{\text{post}}(U_{0})]/[E_{\text{post}}(U_{0}) - E_{\text{pre}}(U_{0})] \), where \( U_{0} \) is the freestream velocity, and \( E(U_{0}) = E(U_{0},t_{i}) \), where \( t_{i} \) is the intermediate time. Note that \( E(U_{0}) \) could be measured in the experiment. By calculating \( R(t) \), a calibration curve \( E(U,t) \) could be calculated.

A previous study [13] gave a general form of a calibration curve for CTA measurement. The present study focuses on this general form of a calibration curve to validate the recalibration scheme because the general form of a calibration curve is mathematically given. Specifically, the general form of a calibration curve is given as follows: \( E(U,t)^{2} = A(t) + B(t) U^{\alpha(t)} U^{\beta(t)} \). Here, \( A(t) \), \( B(t) \), and \( n(t) \) are coefficients and are considered to vary with time \( t \). Values of the coefficients could be given experimentally, and the value of \( n(t) \) is approximately 0.5. The general form of the calibration curve is validated and is considered to be accurate. The form of a calibration curve with \( n(t) = 1/2 \) is referred to as King’s law. For King’s law, the values of \( A(t) \) and \( B(t) \) may be analytically derived [14]. The temporal variation of the coefficients is due primarily to the temporal variation of the ambient temperature in the experiment.

The present study uses a temporally varying calibration curve to validate the calibration curve produced by the recalibration scheme. For a small change in ambient temperature, the present study considers that the coefficients in the general calibration curve may be estimated by linear functions, as follows: \( A(t) = A(0) + \alpha t \), \( B(t) = B(0) + \beta t \), and \( n(t) = n(0) + \gamma t \), where \( \alpha, \beta, \gamma > 0 \) are linear deviations of the coefficients from the initial values. The linear deviations could estimate the observed deviations of the coefficients in CTA experiments. Here, \( \alpha, \beta, \gamma > 0 \) are linear gradients of \( A(t), B(t), \) and \( n(t) \). Moreover, based on a previous study [4], the gradients \( \beta \) and \( \gamma \) are described by the gradient \( \alpha \), as follows: \( \beta = 0.00677 \alpha \) and \( \gamma = 0.24 \alpha \). Using these relations, the reference calibration curve in the present study \( E_{\text{int}}(U,t) \) is given as follows: \( E_{\text{int}}(U,t)^{2} = (A(0) + \alpha t) + (B(0) + \beta t) U^{\alpha(0) + \gamma t} \). In the
present study, the calibration curve produced by the recalibration scheme \( E(U,t) \) is validated by comparing this curve with the reference calibration curve \( E_{\text{int}}(U,t) \).

![Figure 1](image_url)

**Figure 1.** Relative errors of (a) the output voltage, \( \varepsilon_E \), and (b) the gradient of the output voltage, \( \varepsilon_{dE/dU} \), in the calibration curve of the CTA measurement. The dashed red and solid blue lines indicate the results for \( \Delta E = 0.2 \) and \( \Delta E = -0.2 \), respectively. The absolute value of \( \varepsilon_E \) is not small in the small velocity range. The absolute value of \( \varepsilon_{dE/dU} \) is not small in either the freestream velocity range or the small velocity range.

The values of \( A(0), B(0), \) and \( n(0) \) should be set in the reference calibration curve. In the present study, based on a previous study [15], the values of the coefficients are set as follows: \( (A(0), B(0), n(0)) = (6.210, 4.399, 0.413) \). These values are appropriate in the velocity range of \( U = 1 \rightarrow 40 \) m/s. In this velocity range, a calibration curve produced by the recalibration scheme is numerically compared with the reference calibration curve. Here, the freestream velocity \( U_0 \) is \( U_0 = 40 \) m/s. In the present study, the relative difference in the output voltage at the freestream velocity between pre- and post-calibrations is represented by \( \Delta E \) and is given as follows: \( \Delta E = (E_{\text{post}}(U_0) - E_{\text{pre}}(U_0))/E_{\text{pre}}(U_0) \). This relative difference is considered to directly characterize the uncertainty of the observed velocity. In experiments, the order of \( \Delta E \) could be equal to that of the uncertainty of the observed velocity. Here, the value of \( \Delta E \) is set to 0, ± 0.05, ± 0.1, and ± 0.2.

3. Results and discussion

3.1. Results of the recalibration error

We start with examining the error of the calibration curve. In the present study, the relative error of the output voltage of the calibration curve is characterized by \( \varepsilon_E(U) \), which is defined as follows: \( \varepsilon_E(U) = (E(U) - E_{\text{int}}(U))/E_{\text{int}}(U) \). Moreover, in the present study, the gradient of the calibration curve \( dE/dU \) is examined. Thus, the relative error of the gradient, which is defined as \( \varepsilon_{dE/dU}(U) = [dE/dU(U) - dE_{\text{int}}/dU(U)]/dE_{\text{int}}/dU(U) \), is also used in the present examination. As noted in a previous study [13], the error of the gradient of the recalibration curve characterized that of the fluctuation of the observed velocity. Therefore, the relative errors, which are characterized by \( \varepsilon_E(U) \) and \( \varepsilon_{dE/dU}(U) \), corresponded to the errors of the mean and fluctuation of the observed velocity. Figure 1 shows \( \varepsilon_E(U) \) and \( \varepsilon_{dE/dU}(U) \) as functions of the true velocity. As shown in the figure, the absolute value of the relative error \( \varepsilon_E(U) \) is decreased as the velocity decreases. Here, note that the calibration curve is given as \( \varepsilon_E(U) = 0 \) at the freestream velocity \( U_0 \). As shown in Figure 1(b), \( \varepsilon_{dE/dU}(U) \) cannot be zero at the freestream velocity.
When $\Delta E$ is negative, $\varepsilon_{\Delta E=U}(U)$ may be large in the range of small velocity. Moreover, the relative error due to negative $\Delta E$ is larger than that due to positive $\Delta E$.

![Figure 2](image)

**Figure 2.** Temporal variations of the relative errors (a) $\varepsilon_E$ at $U = 1$ m/s, (b) $\varepsilon_{\Delta E=U}$ at $U = 1$ m/s, and (c) $\varepsilon_{\Delta E=U}$ at the freestream velocity. The cubic approximations (Equation (1)) fitted to the observed temporal variations, which are considered to be accurate, are also shown.

The relative errors of the calibration curve shown in Figure 1 are calculated at the intermediate time $t = 1/2$. Thus, in the present study, the temporal variation of the relative errors included in the calibration curve is examined. Here, based on the results of Figure 1, $\varepsilon_E(U)$ and $\varepsilon_{\Delta E=U}(U)$ at $U = 1$ m/s and $\varepsilon_{\Delta E=U}(U)$ at the freestream velocity are calculated. Figure 2 shows temporal variations of $\varepsilon_E(U)$ and $\varepsilon_{\Delta E=U}(U)$ at $U = 1$ m/s and $\varepsilon_{\Delta E=U}(U)$ at the freestream velocity. Here, the relative errors are equal to zero at $t = 0$ and $t = 1$, because the calibration curve given by the recalibration scheme is given using calibration curves at the two times. As shown in the figure, the absolute value of the relative errors is the largest around the intermediate time $t = 1/2$. However, time $t$ that gives the largest value of the relative errors is not equal to 1/2. This difference shows that the temporal variation of the relative errors could not be approximated by a quadratic function. As shown in the figure, the difference in time, which gives the largest relative errors from the intermediate time, could be increased as the
largest relative errors increase. Thus, the temporal variation of the relative errors may be approximated by a quadratic function when the largest relative errors are small.

![Diagram](image_url)

**Figure 3.** (a) Sensitivity of $d(100 \times \varepsilon_{\text{IEAU}})/d(t)|_{t=1/2}$ to the linear deviation $\alpha T$ and (b) validation results of $\langle \varepsilon_{\text{IEAU}} \rangle/\alpha(1/2) = 2/3$, where $d(100 \times \varepsilon_{\text{IEAU}})/d(t)|_{t=1/2}$ is the temporal gradient of $100 \times \varepsilon_{\text{IEAU}}$, the percentage of $\varepsilon_{\text{IEAU}}$ at the intermediate time $t = 1/2$. As shown in (a), the value of $d\varepsilon/d(t)|_{t=1/2}$ deviates from zero as the absolute value of $\alpha T$ increases. This deviation indicates that the temporal variation of $\varepsilon$ is not approximated by a quadratic function. Therefore, the temporal variation of $\varepsilon$ should be approximated by a cubic function when the absolute value of $\alpha T$ is not small. As shown in (b), the simple relation $\langle \varepsilon_{\text{IEAU}} \rangle/\alpha(1/2) = 2/3$ could be validated using numerical results, where $100 \times \langle \varepsilon_{\text{IEAU}} \rangle$ and $\varepsilon_{\text{IEAU}}|_{t=1/2}$ are, respectively, the percentage of the ensemble average of $\varepsilon_{\text{IEAU}}$ and $\varepsilon_{\text{IEAU}}$ at the intermediate time. Therefore, the value of $\langle \varepsilon_{\text{IEAU}} \rangle/\alpha(1/2)$ could be estimated by $\langle \varepsilon_{\text{IEAU}} \rangle/\alpha(1/2) = 2/3$.

Based on the results shown in Figure 2, the temporal variations of the relative errors may be approximated by a cubic function. A general form of a cubic function using $t$ is as follows: $\varepsilon_E = C_0 + C_1 t + C_2 t^2 + C_3 t^3$ or $\varepsilon_{\text{IEAU}} = C_0 + C_1 t + C_2 t^2 + C_3 t^3$. Here, $C_0$, $C_1$, $C_2$, and $C_3$ are coefficients of the exponents. Note that the value of the relative errors at $t = 0$ and $t = 1$ is zero. Under these conditions, the following relations are derived for the coefficients: $C_0 = 0$ and $C_1 = -(C_2 + C_3)$. Using these relations of the coefficients, a cubic function approximating the temporal variation of the relative errors is derived as follows:

$$\varepsilon_E = -t [1 - t][C_2 + C_3(1 + t)] \text{ or } \varepsilon_{\text{IEAU}} = -t [1 - t][C_2 + C_3(1 + t)].$$

(1)

The cubic functions (Equation (1)) that can approximate the temporal variations of the observed relative errors are shown in Figure 2. As shown in the Figure, these fitted cubic functions (Equation (1)) could approximate well the temporal variations of the relative errors.

When the absolute value of the relative errors is small, the temporal profile of the relative errors can be approximated by the derived quadratic function as follows:

$$\varepsilon_E = -t (1 - t) C_2 \text{ or } \varepsilon_{\text{IEAU}} = -t (1 - t) C_2.$$  

(2)
(Equation (1)), as follows: \( \frac{dE}{dt}l_1 \) or \( \frac{dE_{IE/AV}}{dt}l_1 \) and \( \frac{dE}{dt}l_2 \) or \( \frac{dE_{IE/AV}}{dt}l_2 \). Figure 3(a) shows the gradient of the temporal profile of \( 100 \times \varepsilon_{IE/AV} \) at the intermediate time as a function of \( \Delta E \), where the values of \( 100 \times \varepsilon_{IE/AV} \) are given by Equation (1). As shown in the figure, the deviation of the intermediate gradient from zero is increased as the value of \( \Delta E \) increases. When the value of \( \Delta E \) is smaller than 0.05, the gradient at the intermediate time is sufficiently small, and, therefore, the temporal profile of the relative errors can be approximated by the quadratic function (Equation (2)).

### 3.2. Simple estimation of the mean relative errors

The present study therefore focuses on the mean value of the relative errors. In an actual experiment, the mean value of the relative error (Equation (1)) can be required to be estimated. The cubic approximation of the temporal variation of the relative errors can be used to estimate the mean value of the relative errors. The mean value of the relative errors could be described by the cubic function (Equation (1)), as follows: \( \langle \varepsilon_{IE} \rangle = -(2C_1 + 3C_2)\sqrt{t/12} \) or \( \langle \varepsilon_{IE/AV} \rangle = -(2C_1 + 3C_3)/12 \). Here, \( \langle \rangle \) denotes the mean average in the range of \( t = 0 \) - 1. In addition, the present study focuses on the relative errors at the intermediate time, as follows: \( \varepsilon_{IE/l_1} \) or \( \varepsilon_{IE/AV/l_1} \) or \( \varepsilon_{IE/l_2} \) or \( \varepsilon_{IE/AV/l_2} \). Using these equations, the mean errors can be simply described using the relative errors as the intermediate time, as follows:

\[
\langle \varepsilon_{l_1} \rangle = \langle \varepsilon_{IE/l_1} \rangle = \frac{2}{3} \varepsilon_{l_1} \quad \text{or} \quad \langle \varepsilon_{IE/AV/l_1} \rangle = \langle \varepsilon_{IE/AV/l_1} \rangle = \frac{2}{3} \varepsilon_{l_2}.
\]

As shown in the above equation, the value of the mean errors can be estimated using only the relative error at the intermediate time. Figure 3(b) validates the derived estimation of \( \langle \varepsilon_{IE/AV/l_1} \rangle \) using the observed results. As shown in the figure, the values of the mean error estimated from Equation (3) agree with the observed values obtained by the numerical simulation. This agreement validates the derived simple estimation of the mean errors.

The above equation is derived at the intermediate time \( t = 1/2 \). In an actual experiment, an observed value may include the uncertainty. When the intermediate time, which gives the intermediate relative errors, includes uncertainty, the included uncertainty can affect the estimation using the derived equation. In order to examine the effect of the deviation of time, we use a time derived from the intermediate time, \( t = 1/2 + dt \). The relative errors at the deviated time, \( t = 1/2 + dt \), is given as follows: \( \varepsilon_{l_1} \) or \( \varepsilon_{IE/l_1} \), \( \varepsilon_{l_2} \) or \( \varepsilon_{IE/AV/l_2} \) or \( \varepsilon_{l_2} \) or \( \varepsilon_{IE/AV/l_2} \). Using these forms of the relative errors, the mean errors are given, respectively, as follows: \( \langle \varepsilon_{l_1} \rangle = \langle \varepsilon_{IE/l_1} \rangle \) or \( \varepsilon_{IE/AV/l_2} \) and \( \langle \varepsilon_{l_2} \rangle = \langle \varepsilon_{IE/l_2} \rangle \) or \( \varepsilon_{IE/AV/l_2} \). As shown in the above equations, the effects of the deviation of time \( dt \) are proportional to \( C_3 \). The magnitude of \( C_3 \) is decreased as \( \Delta E \) decreases. By decreasing the magnitude of \( \Delta E \), the effects of the derived time \( dt \) could be reduced.

The present results have been investigated based on the temporally varying calibration curve with the linear deviations of the coefficients. When the ambient temperature is exactly constant, the coefficients in the general calibration curve are considered to be constant with respect to time. Based on this point, the present study assumes that the sensitivity of the coefficients in the general calibration curve to the change in the ambient temperature could be estimated by a linear function. The present study considered this assumption to hold for a sufficiently small change in the ambient temperature. A sufficiently small change in ambient temperature would cause a sufficiently small change in the relative difference of the output voltage \( \Delta E \). Moreover, with the change in ambient temperature, an experimental condition could exist in which the temporal variation of the ambient temperature could be estimated by a linear function. Therefore, the present study considered that the present conditions of \( \Delta E \) should cover changes to \( \Delta E \) that are not small as well as changes to \( \Delta E \) that are sufficiently small. The accuracy of the linear estimation should be increased as the magnitude of \( \Delta E \) is decreased. The present simple estimation should be used for \( \Delta E \) of smaller magnitudes, which could be obtained.
4. Conclusions
In the present study, the error of a recently developed recalibration scheme for a CTA measurement is examined. The errors of the mean and the fluctuation of observed velocity were calculated using the recalibration scheme from the output voltage. A temporally varying calibration curve was used to examine the relative errors of the observed velocity. This calibration curve is based on King's law of CTA measurement. Moreover, three coefficients included in the calibration curve varied temporally due to the temporal change in the ambient temperature in the experiment. The present study focused on the temporal variation of the relative errors caused by the recalibration scheme. Moreover, the mean value of the relative errors was calculated. The mean value could be calculated if a simple function could approximate the temporal variation of the relative errors.

We started by examining the relative errors of both the value and the gradient value of the calibration curve. The absolute relative errors of the value of the calibration curve were increased as the velocity decreased. The absolute relative errors of the gradient are still zero at the freestream velocity. Then, temporal variations of the relative errors were examined as a function of time. The simple cubic function (Equation (1)) could approximate the temporal variations of the relative errors. When the relative difference of the output voltage between pre- and post-calibrations was small, the temporal variation of the relative errors could be approximated by the quadratic function (Equation (2)). By focusing on the use of the approximating cubic function (Equation (1)), a simple estimation for calculating the mean value of the relative errors was derived. This simple estimation was validated using the present numerical results. Moreover, the present study discussed the effects of the uncertainty of experimental measurement on the use of a simple estimation.

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