Non-empirical Induction in mathematics conjecturing in the new knowledge construction

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Abstract. This study aims to describe students' conjecturing ability in accordance with and without the use of empirical mathematical induction methods in solving problems and building new knowledge. Data were obtained from 57 students in grade 8 by using the conjecturing process and unstructured interviews. The result showed that the conjectured process using empirical induction was longer than without its usage in solving problems and building new knowledge. However, the use of empirical induction, leads to no accommodation of problems, strategies and relationships, compared to when it is not used as shown by the Piaget's theory.

1. Introduction

Learning activities are used to make changes in the cognitive, psychomotor and affective domains of students’ interactive ability with the surrounding [1]. This is because learning has a clear purpose in the mental formation and behaviour of students.

Teachers are defined as professionals with the ability to design learning components [2]. They need to develop problem-solving procedures, [3] and educate students on various ways to construct knowledge independently [4]. However, there are teachers unable to carry out the learning process by using the scientific development approach [5]. Therefore, the teacher’s ability to develop learning theories in accordance with students’ psychology and scientific domain requires the use of helpful educational activities. This knowledge is essential for the teacher to properly present the learning activities for the proper understanding of students and to ensure their mastery of certain skills, and attitudes in line with the academic objectives of [1].

One of the learning activities used in solving various educational problems is students' knowledge. Therefore, teachers transfer knowledge by memorizing or providing solutions to problems, which is mastered by students. According to research during the learning process, ordinary student tend to become talented [6]. However, some students still make mistakes during problem-solving procedures, therefore, no new knowledge is constrained [7].

According to numerous studies, students tend to provide conjectured solutions in solving open problems, irrespective of the length of the provided answer. They also tend to use conjectured analogy reasoning in problem-solving [8] with the ability to trigger divergent and convergent thinking [9]. According to Goswani, learning through analogy is similar to inductive reasoning [10]. A research was conducted to determine the importance of inductive reasoning in mathematics and provide a cognitive framework to encourage students' inductive reasoning skills associated with inductive similarity, dissimilarity, and integration [11]. Furthermore, it utilized a better inductive method in teaching [12].

2. Literature review

Some theories that underlie this research are as follows:

2.1. Conjecturing via empirical induction

Students have the right to use conjecture and hunks while solving mathematical problems [13]. According to Canada et al (2017) this process is through the empirical induction of discrete cases [14].
This means that projects are processed based on observations of a limited number of discrete cases, through the observation of consistent patterns. This type of conjecture is often found in problems involving numbers, with some proved using mathematical inductive methods with the discovery of general rules. NCTM stated that at all levels of education students’ reason inductively based on patterns and special cases, which increases according to their level of knowledge [15].

2.2. Conjecturing via non empirical induction
Logical reasoning is a basic aspect of learning mathematics used when students’ ability to reason is underdeveloped. It makes them become material that follows a series of procedures by imitating examples without knowing their meaning [3]. Furthermore, it stated that mathematics learning programs from preschool through grade 12 need to provide opportunities for all students to recognize the basic concepts, explore conjectures, and develop/assess arguments, using various types of reasoning and proof methods [15]. Besides, they also need to learn how to make effective deductive arguments based on mathematical truths set in the classroom.

2.3. Construction of new knowledge
When students learn, they experience an imbalance that leads to the process of assimilation and accommodation from Piaget. This means that the problem is simply solved through the division of its independent cognitive structure through the process of changing, combining, or forming new schemes till the final equilibrium condition is adapted to the environment [3]. This process starts absorbing the structure of the problem (intelligent behavior) which is an input of the cognitive structure in an initial equilibrium state. This results in a disequilibrium between the problem structure and the cognitive, thereby, leading to assimilation, accommodation, and the development of new knowledge. Illustration of Piaget's theory of adopted from Supratman, N Ratnaningsih and S Ryane [6] is shown in Figure 1.

![Figure 1. Occurrence of Assimilation process and accommodation (adopted from Supratman, N Ratnaningsih and S Ryane, 2017)](image)

**Legends:**
- - - - Express the incompatibility between the problem structure and the cognitive structure
- - - - Express the cognitive structure changes from the equilibrium of initial level to the equilibrium of new level

3. Methodology
3.1. Research subject
Data were obtained from 57 junior high school students from Tasikmalaya District, assumed to have conjectured the empirical induction of discrete cases in problem-solving. A total of 21 students conjectured the process, 25 did not, while 7 were failed to make assumptions.

3.2 Research methodology
This is a qualitative-exploratory study with data collected from the exploration of students conjectured empirical induction with problems solved by constructing new knowledge based on Piaget's Theory. The Think Out Loud method is used to obtain verbal data through an unstructured interview. This method was utilized because it is one of the irreplaceable instruments [16]. The task sheet instrument is as follows:

- Look for three consecutive natural numbers which number 75?

3.3 Data Collection Procedures
This study examines students’ mathematical conjectures through empirical induction in solving problems/constructing new knowledge based on Piaget's adaptation framework. The data collection procedure was used to provide problems related to empirical induction. Students make use of conjectures during empirical induction; this tends to make them think hard which is expressed in writing and orally. Retrieval of such data, According to Someren et al (1994), the Think Out Loud method is used for the retrieval of data [17].

4. Findings
From a total of 57 students, the following were obtained: a student did not expect the problem in Undergraduate Students (S1) group till $1 + 2 + 3 = 6$,$2 + 3 + 4 = 9$ and failed to continue. In addition, 7 students did not conjecture the answers because they had read questions similar to the problem, even though the answers were correct. 21 students in the Master Students (S2) group initially answered $1 + 2 + 3 = 6$,$2 + 3 + 4 = 9$,$11 + 12 + 13 = 36$,$21 + 22 + 23 = 66$,$22 + 23 + 24 = 69$,$23 + 24 + 25 = 72$,$24 + 25 + 26 = 75$. Seina concluded that the first natural number is 24, the second 25 and the third 26.

While 25 students in the other Doctoral Students (S3) group immediately assumed the first natural number is x which is proceeded by $x + 1$, and $x + 2$, respectively. Although there is also a comparison of the original n unity, the second is $n + 1$ and the third $n + 2$ which are in three consecutive numbers $x + (x + 1) + (x + 2) = 75 \Rightarrow 3x + 3 = 75$, in the end, $x$ or $n=24$. One-third S3 deduces the first, second and third natural numbers as 24, 25 and 26. This shows that the students' conjecture is not based on empirical induction, but deduction [18].

The interview results with S1 are as follows:
Researcher I: Why didn’t your suspicions continue?
S1: I was confused and a bit dizzy sir
R: Alright, you need to take a break first

Then the results of the interview with the S2 group:
R: Why do you think it starts from adding $1 + 2 + 3 = 6$?
S2: Let me have experience and confidence, sir
R: Your conjecture is to add $2 + 3 + 4 = 9$, why didn’t you add tens directly?
S2: I was trying to obtain a pattern to determine the three original 5 numbers.
R: Ooh so, why didn’t you immediately suspect $24 + 25 + 26 = 75$?
S2: I did not know that the right number of three consecutive original numbers of 75 was 24, 25 and 26
R: Alright then, may I ask a question?
S2: Yes, sir
R: Look for three consecutive natural numbers totaling 165?
S2: 54 for the first natural number, 55 for the second and 56 for the third
R: Where did you get 54, 55 and 56?
S2: From the previous experience, the second natural number is 1/3 of the total. Therefore, I determined the first natural number, by subtracting one from the second and adding one to the third.
R: OK then

The interview results with the S3 group are as follows:
R: Why do you suppose the first, second and third natural numbers are $x$, $x + 1$ and $x + 2$?
S3: Because the original number is not yet known, it is assumed by a variable $x$ for the first natural number, $x + 1$ for the second and $x + 2$ for the third. Even though it fits the $n$ number of symbols.
R: Why did you add 1 for the second natural number and two for the third?
S3: Shows the order of a natural number.

The conjectured S2 is the same as S3, however, the answer is determined using a procedural method. This is because the S2 conjecture was obtained from several previous assumptions to produce an experience that led to the construction of new knowledge. Another case with the alleged S3 runs from general to specific events as Figure 2 below.

**Figure 2. Piaget's Adaptation Process in conjecture through empirical induction and deduction**
5. Discussion
The use of S1 by students in the conjectured empirical induction to solve problems starts with a simple and incomplete problem structure. The following questions and answers provide a detailed understanding of its analysis.

Why does S1 make a conjecture? Because it is unable to determine the problem beforehand.

Why is S1 used through empirical induction? Because its conjecture is not enough for students to obtain a complete experience.

Similarly, S2 makes assumptions through empirical induction. This is indicated by several conjectures made to add experience that ultimately determine the right answer. To determine the second number, the first is divided by three consecutive natural numbers to obtain 25, while the first natural number is obtained by reducing the second by 1 to obtain 24. One is added to the second number to obtain the third at 26. In general, the S2 adaptation of knowledge is in the position of accommodation problems, strategies, and relationships.

In another case with S3, conjectures are obtained through mathematical deduction, with the adaptation of knowledge in assimilation problems, strategies, and relationships.

6. Conclusion
In conclusion, students' conjecture failed to pass through empirical induction in solving problems for the construction of new knowledge. However, their conjecturing via empirical induction possess adaptations dominated by both accommodation problems, strategies and relationships in accordance with Piaget's theory.

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References
[1] Slavin R E 2006 *Educational Psychology: Theory and Practice* Boston, MA: Pearson Education, Inc
[2] Putrawangsa S 2018 *Desain Pembelajaran: Design Research sebagai Pendekatan Desain Pembelajaran* Mataram: CV. Reka Karya Amerta
[3] Rejiden R and Ahman S M 2015 The Pseudo-Covariational Reasoning Trought Processes in Constructing Graph Function of Reversible Event Dynamics Based on Assimilation and Accomodation Frameworks Research in Mathematical Education. *J. The Korean Society of Mathe. Educ./Series D* 19 61
[4] Anthony G 1996 Active Learning in a Constructivist Framework *Educ. Stud. Math.* 31(4) 349–69
[5] Supratman, Ryane S and R Rustina 2016 Conjecturing via Analogical Reasoning in Developing Scientific Approach in Junior High School Students. *J Phys. Conf. Ser.* 693 (2016) 012017
[6] Supratman, Ratnaningsih N and Ryane S 2017 Conjecturing via analogical reasoning constructs ordinary students into like gifted student. *J Phys. Conf. Ser.* 943 (2017) 012025
[7] Supratman 2018 Developing Piaget's Theory in Mistakes Construction of Knowledge when Problem Solving through Analogical Reasoning. *J Phys. Conf. Ser.* 1028 (2018) 012146
[8] Supratman 2019 The role of conjecturing via analogical reasoning in solving problem based on Piaget’s theory. *J Phys. Conf. Ser.* 1157 (2019) 032092
[9] Supratman, Linda H and Reza E A 2019 Conjecturing Via Analogical Reasoning to Trigger Divergent and Convergent Thinking, *International Journal of Innovation, Creativity and Change. Volume 9, Issue 1*
[10] English LD 2004 *Mathematical and analogical Reasoning of Young Learners*. New Jersey London Lawrence Erlbaum Associates Publisher Mahwah
[11] Cristou C and Papageorgiou E 2007 A Frame Work of Mathematics Inductive Reasoning
Learning and Instruction 17 (2007) 55-56

[12] Copeland R W 1974 How Children Learn Mathematics: Teaching Implications of Piaget’s Theory New York: Macmillan Publishing Co. Inc.

[13] Pólya G 1954 Induction and Analogy in Mathematics and Plusible Reasoning London: Princeton University Press

[14] Canadas M C, Deulofeu J, Figueiras L, Reid D and Yevdokimov A 2007 The conjecturing process: Perspectives in theory and implications in practice Journal of Teaching and Learning 5 (1), 55–72

[15] NCTM 2000 Principle and Standard for School Mathematics Reston: The National Council of Teacher Mathematics, Inc.

[16] Yin R K 2016 Case Study Research Design and Methods (5th ed.). Thousand Oaks, CA: Sage. 282 pages.,” Can. J. Progr. Eval., vol. 1, no. 2014) pp. 108–110,

[17] van Someren M W, Barnard Y F and Sandberg J A 1994 The think aloud method: A practical guide to modelling cognitive processes London: Published by Academic Press

[18] Miyazaki M 2000 Levels of Proof in Lower Secondary School Mathematics. Educational Studies in Mathematics 41: 47 - 68, 2000 Netherlands: Kluwer Academic Publisher