Path-Integral Measure of Linearized Gravity in Curved Spacetime

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ABSTRACT

The path-integral measure of linearized gravity around a saddle-point background with the cosmological term is considered in order to study the conformal rotation prescription proposed by Gibbons, Hawking and Perry. It is also argued that the most generally used measure, i.e., the covariant path-integral measure, does not give us a one-loop partition function which the only physical variables contribute and that its path integral fails to keep the cancellation of contributions between the Faddeev-Popov ghosts and the unphysical variables of the linearized gravitational field, although it has a coordinate invariant measure. In de Sitter spacetime, it is shown that the uncancellation factor can be understood as a nontrivial (anomalous) Jacobian factor under the transformation of the path-integral measure from covariant one to canonical one.
1. Introduction

The truth that Euclidean path integrals are directly combined with classical actions has made them most popular as attractive and powerful tools for investigation of field theories in curved spacetime and quantum gravity [1-5]. They enable us to have suggestive discussions with respect to gauge symmetries of systems if the classical actions have them, and also manifest coordinate invariance which matches the spirit of general relativity. Under such circumstances, the issue of derivation of the Euclidean gravitational path-integral formula from the canonical quantization may be very important to obtain theoretical meanings of the conformal rotation prescription proposed by Gibbons, Hawking and Perry [6]. Furthermore, if it is done, objects given by the path integral of gravity may become more meaningful, for example, with respect to whether the path integral has some information on the ground state similarly to the case of any field theory with a positive-definite hamiltonian in the flat spacetime [7].

However, it is very difficult in the Einstein gravity to derive the path integral or to give the theoretical meanings to the conformal rotation. Because the Einstein theory is a reparametrization one, in which the physical time is a dynamical variable. Thus it is necessary to identify the physical time which plays a privileged role in the canonical formalism, when quantizing it according to that formalism [8-10]. With the identification it might be possible to construct a path-integral formula from the canonical approach. Unfortunately, the choice of the physical time is a difficult problem and even if it is done it is not so trivial how the choice is reflected in the path-integral formula, particularly in the covariant expression. Hence the choice of the physical time may be an important issue in quantum gravity.

Also in linearized gravity, we have to take the contour of integration for a variable associated with the conformal factor to be parallel to the imaginary axis in order to make the Euclidean path integral converge, since the action of linearized gravity is unbounded from below with respect to the variable. Here the time is not a dynamical variable, so the theoretical foundation of the conformal rotation
in linearized gravity may be different from that in the case of gravity, but can be discussed from the standpoint as a gauge theory: indeed, as studied by many authors in the flat spacetime [11-13], the variable needing the conformal rotation is an unphysical one like $A_0$ in the Abelian gauge theory, and it is found that one has to rotate it to the imaginary axis, in addition to Wick rotating, in order to obtain the well-defined covariant formulation of the Euclidean path integral.

Furthermore, in linearized gravity around a nontrivial background such as de Sitter spacetime, it was shown by Griffin and Kosower [14] that the Euclidean action of the physical part is positive definite, which means that the conformal rotation is not necessary when taking a special gauge fixing condition so that redundant variables all may vanish like the Coulomb gauge in the Abelian gauge theory, although on this background the conformal rotation in covariant gauges is more complicated as compared with that on the flat spacetime background. They also pointed out in this de Sitter spacetime that the covariant path integral does not give the one-loop correction which is contributed by the only physical variables, in other words, that the covariant path integral is different from the canonical path integral. The same thing also happens in the Abelian gauge theory in curved spacetime [15-17]. This issue may be noticeable also from the point of view of reexamination of the Faddeev-Popov conjecture [18,19] that path integrals in various gauge choices are all same, because now the difference of the path-integral measures may emerge as differences of path integrals between various gauges [17].

The purpose of this paper is to study the relation between the covariant path integral and the canonical approach to linearized gravity in curved spacetime and the uncancellation of the one-loop contributions given from the unphysical variables and the Faddeev-Popov ghosts. It is shown that the uncancellation factor can be also understood as a nontrivial (anomalous) Jacobian factor under the transformation of the path-integral measure from covariant one to canonical one, similarly to the case of the Abelian gauge theory.

This paper is organized as follows: in section 2 it is pointed out that the
covariant path integral of linearized gravity around a saddle-point background with the cosmological term does not give us the one-loop partition function which the only physical modes contribute and also that its path integral fails to keep the cancellation of contributions from the Faddeev-Popov ghosts and the unphysical modes of the linearized gravitational field. In section 3 we study how the modes needing the conformal rotation prescription depend on the gauge fixing condition with gauge parameters $\alpha$ and $\eta$. In section 4 it is shown that the uncancellation factor can be understood as a nontrivial (anomalous) Jacobian factor under the transformation of the path-integral measure from canonical one to covariant one. Section 5 is devoted to conclusion, and eigenfunctions on $S^D$ are summarized in Appendix A.

2. BRST Path Integral of Linearized Gravity

In this section, we formally calculate the one-loop correction by the use of the BRST path-integral method with the covariant measure, and a probability is then pointed out that the contribution of the Faddeev-Popov ghosts might not generally cancel out with one from the unphysical degrees of freedom of linearized gravitational field.

Our starting Euclidean path integral of linearized gravity in a $D$-dimensional curved spacetime is

$$Z \equiv \int \mathcal{D}h_{\mu\nu} \Delta_{FP} \exp \left[ -\frac{1}{\hbar}(I + I^{GF}) \right], \quad (2.1)$$

where $I$ is the linearized gravity action in Euclidean spacetime [4];

$$I = \frac{1}{4} \int d^Dx \sqrt{g} \left( \frac{1}{2} g^{\mu\nu} h - h^{\mu\nu} \right) \times \left( \Box h_{\mu\nu} + \nabla_\mu \nabla_\nu h - 2 \nabla^\rho \nabla_\rho h_{\mu\nu} + \frac{4\Lambda}{D-2} h_{\mu\nu} \right), \quad (2.2)$$
which is obtained by expanding the Einstein-Hilbert action with the cosmological term

\[ I_{EH} \equiv -\frac{1}{\kappa^2} \int d^Dx \sqrt{g}(\bar{R} - 2\Lambda), \quad (2.3) \]

around a classical background \( g_{\mu\nu} \) which satisfies

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \quad (2.4) \]

with \( \bar{g}_{\mu\nu} \equiv g_{\mu\nu} + \kappa h_{\mu\nu} \), and \( I^{GF} \) is a gauge fixing term with gauge parameters \( \alpha \) and \( \eta \);

\[ I^{GF} \equiv \frac{1}{2\alpha} \int d^Dx \sqrt{\bar{g}}(\nabla^\rho h_{\rho\mu} - \eta \nabla_\mu h)^2, \quad (2.5) \]

and then the Faddeev-Popov determinate, \( \Delta_{FP} \), becomes

\[ \Delta_{FP} \equiv \left| \det \frac{1}{\sqrt{\alpha}} \left[ -\square_{\mu}^\nu - \nabla_\nu \nabla_\mu + 2\eta \nabla_\mu \nabla_\nu \right] (v) \right|, \quad (2.6) \]

where \( v \) is any vector field.

Now, decompose \( h_{\mu\nu} \) as

\[ h_{\mu\nu} = \frac{1}{D} g_{\mu\nu} h + 2 \left( \nabla_\mu \nabla_\nu \Box^{-1} - \frac{1}{D} g_{\mu\nu} \right) \phi + \nabla_\mu \xi_\mu^d + \nabla_\nu \xi_\mu^d + h_{\mu\nu}^{\text{trd}}, \quad (2.7) \]

where \( h \) and \( \phi \) are scalar fields, \( \xi_\mu^d \) is a divergenceless vector field, and \( h_{\mu\nu}^{\text{trd}} \) is a traceless and divergenceless tensor field:

\[ \nabla_\mu \xi_\mu^d = 0, \quad g^{\mu\nu} h_{\mu\nu}^{\text{trd}} = \nabla_\mu h_{\mu\nu}^{\text{trd}} = 0. \quad (2.8) \]

Then, the actions \( I \) and \( I^{GF} \) are separated into four independent parts of their
variables:

\[ I = I^{\tilde{\text{trd}}} + I^{\tilde{\psi}}, \]

\[ I^{\tilde{\text{trd}}} \equiv \frac{1}{4} \int d^Dx \sqrt{g^{\tilde{\text{trd}}}} \left[ \left( -\Box + \frac{4\Lambda}{(D-1)(D-2)} \right) g^{\mu\rho} g^{\nu\sigma} - 2 C_{\mu\nu\rho\sigma} \right] h^{\tilde{\text{trd}}}_{\rho\sigma}, \quad (2.9) \]

\[ I^{\tilde{\psi}} \equiv \frac{1}{4} \int d^Dx \sqrt{g^{\tilde{\psi}}} \left( (D-1)(D-2)\Box + 2DA \right) \psi, \]

and

\[ I^{\text{GF}} = I^{\text{GF}^d} + I^{\text{GF}^s}, \]

\[ I^{\text{GF}^d} \equiv \frac{1}{2\alpha} \int d^Dx \sqrt{g^{\text{GF}^d}} \xi^d_{\mu} \left( \Box + \frac{2\Lambda}{D-2} \right)^2 \xi^d_{\nu}, \quad (2.10) \]

\[ I^{\text{GF}^s} \equiv -\frac{2}{\alpha} \int d^Dx \sqrt{g^{\text{GF}^s}} \phi^{-1} \left( (1 - \eta) \Box + \frac{2\Lambda}{D-2} \right)^2 \phi', \]

where \( C_{\mu\nu\rho\sigma} \) is the Weyl tensor and

\[ \psi \equiv \frac{1}{D}(h-2\phi), \quad \phi' \equiv \phi - \frac{\eta D - 1}{2} \left( 1 - \eta \right) \Box \left( 1 - \eta \right) + \frac{2\Lambda}{D-2} - \psi. \quad (2.11) \]

Obviously, \( I^{\tilde{\psi}} \) may be negative definite with respect to higher frequency modes than the mass \( 2DA \) owing to opposite sign of the kinetic term to ordinary scalar fields. Thus, in addition to Wick rotating, one has to always rotate the contour of integration over these modes of \( \psi \) to the imaginary axis so as to make integrations converge. But, it is not surprising because \( \psi \) is an unphysical degree of freedom of linearized gravitational field, i.e., we can make the variable vanish as a consistency condition for a suitable gauge fixing like the Coulomb gauge whenever that is wanted, although it is a gauge invariant variable. Hence, if we start with the operator formalism, the theoretical meaning of this rotation could be understood similarly to the case in the flat spacetime [11-13]. On the other hand, only \( h^{\tilde{\text{trd}}}_{\mu\nu} \) has the physical degrees of freedom, which means that the path integral with respect to the physical mode is well-defined within the positivity of \( I^{\tilde{\text{trd}}} \), which may be expected in any curved spacetime as well as the flat and de Sitter spacetimes.
Next, in order to carry on calculation of (2.1) by the use of the covariant measure \[1\), let us define the measure using the Polyakov measure for convenience sake as

\[
\int \mathcal{D}h_{\mu\nu} \exp \left[ -\frac{1}{4h} < h, h > \right] = 1, \tag{2.12}
\]

with

\[
<h, h > \equiv \int d^D x \frac{1}{2} \sqrt{g} (2h^{\mu\nu}h_{\mu\nu} + Ch^2), \tag{2.13}
\]

in which the constant \(C\) could be defined through derivation of the path-integral formula from the canonical method, hence here we may take \(C = -1\). Then we must integrate \(h\) as an imaginary variable, but this imaginary like integration does not always make the Euclidean path integral (2.1) completely converge. (See section 3.)

Therefore (2.1) becomes

\[
\mathcal{Z} = \left| \det \left[ -g^{\mu\nu} \left( \Box + \frac{2\Lambda}{D-2} \right) \right] (v^d) \right|^{1/2} \mathcal{Z}_{\text{trd}}, \tag{2.14}
\]

where

\[
\mathcal{Z}_{\text{trd}} = \left| \det \left[ \left( \Box + \frac{4\Lambda}{(D-1)(D-2)} \right) g^{\rho\sigma} g^{\nu\sigma} - 2C^{\mu\rho\nu\sigma} \right] (t_{\text{trd}}) \right|^{-1/2}. \tag{2.15}
\]

Here \(h^{\text{trd}}_{\mu\nu}\) includes the physical and unphysical modes, hence, only if the unphysical contributions in \(\mathcal{Z}_{\text{trd}}\) cancel out with the Faddeev-Popov ghost contribution, i.e.,

\[
\left| \det \left[ -g^{\mu\nu} \left( \Box + \frac{2\Lambda}{D-2} \right) \right] (v^d) \right|^{1/2}, \tag{2.16}
\]

\(\mathcal{Z}\) gives just the physical contributions. But it does not generally hold in curved spacetimes. Indeed, in de Sitter spacetime it happens, as suggested by Griffin and
Kosower [14]. Such an uncancellation arises also in the Abelian gauge theory and it is shown that the uncancellation can be regarded as the difference of measure between the covariant and canonical path integrals [16,17]. We will study in section 4 the relation between the uncancellation factor of unphysical contributions in the covariant one-loop correction and an anomalous Jacobian factor arising under the change of variables in the Polyakov measure from covariant one to canonical one.

3. Conformal Rotation

In order to find out the modes needing the conformal rotation, let us study the following eigenvalue equation;

\[
\frac{2}{\sqrt{g}} \frac{\delta}{\delta h_{\mu\nu}} (I + I^{GF}) = \lambda h_{\mu\nu},
\]

where \(\lambda\) is an eigenvalue and may be regarded as a constant. Then we might find that the modes with a negative eigenvalue, whose contours of integration need to be rotated to the imaginary axis in order to make integrations over them converge, may change on values of positive \(\alpha\) and real \(\eta\), but that the difference of number between the positive and negative modes is, of course, always unchanged, which is obvious because of a simple reason that the negative and positive parts of \(I^\psi\) in (2.9) does not change and the gauge fixing term in (2.5) ((2.10)) is always positive independently of the gauge fixing parameters.

Eq. (3.1) is separated into \(h_{\mu\nu}^{\text{frd}}, \xi_\mu^{\text{d}}\) and scalar \((\psi, \phi)\) parts by inserting (2.7):

\[
\left( \left( -\Box + \frac{4\Lambda}{(D-1)(D-2)} \right) \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} - 2C_{\mu, \nu}^{\rho, \sigma} \right) h_{\rho\sigma}^{\text{frd}} = \lambda h_{\mu\nu}^{\text{frd}},
\]

\[
\nabla_\mu \left[ -\frac{1}{\alpha} \left( \Box + \frac{2\Lambda}{D-2} \right) \right] \xi_\nu^{\text{d}} + \nabla_\nu \left[ -\frac{1}{\alpha} \left( \Box + \frac{2\Lambda}{D-2} \right) \right] \xi_\mu^{\text{d}} = \lambda (\nabla_\mu \xi_\nu^{\text{d}} + \nabla_\nu \xi_\mu^{\text{d}}),
\]
\[
\left[ g_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} \left( \Box + \frac{2\Lambda}{D-2} \right)^{-1} \right] T^1 + (\eta g_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} \Box^{-1}) T^2 = \lambda (g_{\mu\nu} \psi + 2 \nabla_{\mu} \nabla_{\nu} \psi), \quad (3.4)
\]

where
\[
T^1 \equiv (D-2) \left( \Box + \frac{2\Lambda}{D-2} \right) \psi, \\
T^2 \equiv -\frac{2}{\alpha} \left[ (\eta D - 1) \Box \psi + 2 \left( (\eta - 1) \Box - \frac{2\Lambda}{D-2} \right) \phi \right]. \quad (3.5)
\]

Eq. (3.2) says that if all eigenvalues of the operator acting on traceless and divergenceless tensor fields, i.e.,
\[
\left[ \left( -\Box + \frac{4\Lambda}{(D-1)(D-2)} \right) \delta^\rho_\mu \delta^\sigma_\nu - 2C^\sigma_\mu^\rho \sigma \right], \quad (3.6)
\]
are positive, we have no trouble when integrating out $h_{\mu\nu}^{\text{trd}}$, that might be plausible as mentioned in section 3.

From (3.3), eigenvalues of a tensor part defined by $\xi^d_\mu$ are the same as eigenvalues of the operator acting on divergence vector fields, i.e.,
\[
-\frac{1}{\alpha} \left( \Box + \frac{2\Lambda}{D-2} \right) \delta^\rho_\mu. \quad (3.7)
\]

Here all eigenvalues of this operator are expected to be positive because of the positivity of the gauge fixing action $I^{\text{GF}}$, so the Gaussian integrations with respect to this part also converge.

To discuss about other remaining parts, let us take, for simplicity,
\[
\alpha = \frac{2(\eta D - 1)}{D-2},
\]
the trace part and the traceless part of $h_{\mu\nu}$ then are decoupled: Eq (3.4) is reduced
to two eigenvalue equations

\[
\frac{1}{D} g_{\mu\nu} \left[ (D - 2) \left( (1 - \eta) \Box + \frac{2\Lambda}{D - 2} \right) \right] h = \lambda \frac{1}{D} g_{\mu\nu} h ,
\]

\[
\left( \nabla_\mu \nabla_\nu \Box^{-1} - \frac{1}{D} g_{\mu\nu} \right) \left[ - \frac{2}{\alpha} \left( (1 - \eta) \Box + \frac{2\Lambda}{D - 2} \right) \right] \phi = \lambda \left( \nabla_\mu \nabla_\nu \Box^{-1} - \frac{1}{D} g_{\mu\nu} \right) \phi .
\]

Thus eigenvalues of these tensor parts are the same as those of the operators acting on scalar fields, such that

\[
(D - 2) \left( (1 - \eta) \Box + \frac{2\Lambda}{D - 2} \right) , \quad - \frac{2}{\alpha} \left( (1 - \eta) \Box + \frac{2\Lambda}{D - 2} \right) . \quad (3.9)
\]

The \( \eta \) dependence of these operators is very significant, since the negative modes needing the conformal rotation depend on its value and particularly for \( \eta \geq 1 \) the variable which must be rotated to the imaginary axis is not the trace \( h \) but \( \phi \). It may be also easily found from Eq. (3.9) that the difference of number between the positive and negative modes of their operators is unchanged as mentioned above, because the signs of their operators are opposite to each other always for all eigenvalues, of course, although we must note here that \( \phi \) does not have some modes, for example, the zero modes of \( \Box \), even if \( h \) has the modes. Furthermore, for the case of \( \alpha = 2(D - 1)/(D - 2) \) and \( \eta = 1 \) we have an interesting gauge fixing in which variables \( h \) and \( \phi \) have no kinetic terms but only mass terms, contrary to the case of the flat spacetime background.

Finally, we may note that Eq. (2.14) can be directly checked by Eqs. (3.2), (3.3) and (3.8).
4. Anomalous Jacobian Factor

In this section, we take the Euclidean de Sitter background and discuss a relation between the uncanceled factor in the one-loop contributions given by the covariant path integral of linearized gravity and the nontrivial Jacobian factor under the transformation from the covariant measure to the canonical measure, choosing coordinates so that the metric has the form

\[ ds^2 = d\tau^2 + a^2(\tau)d\Omega^2_{D-1}, \quad d\Omega^2_{D-1} = \tilde{g}_{ij}dx^i dx^j, \]

\[ x^D = \tau = r\theta, \quad a = r \sin(\tau/r), \quad r^{-2} \equiv \frac{2\Lambda}{(D-1)(D-2)}. \]  \hspace{1cm} (4.1)

On this manifold, we have \( C_{\mu\nu\rho\sigma} = 0 \), so the uncancellation factor we have to study becomes, from (2.14) together with formulae in Appendix A (or ref. [14]), as

\[ \left| \det \left[ g^{\mu\nu} \left( -\Box - (D - 1)r^{-2} \right) \right] (v^d) \right|^{1/2} \left| \det \left[ g^{\mu\nu} \left( -\Box + r^{-2} \right) \right] (\tilde{v}^d) \right|^{-1/2}, \]  \hspace{1cm} (4.2)

where \( \tilde{v}^T \) is any divergenceless vector field with no \( \ell = 1 \) modes of the covariant Laplacian \( \Box \) on the unit \( S^{D-1} \).

4.1 Covariant and Canonical Path-Integral Measures

In this subsection, we construct the Polyakov measure for tensor fields on \( S^D \) by using two decompositions, i.e., the covariant and canonical decompositions.

The covariant decomposition in Eq. (2.7) may be rewritten as

\[ h_{\mu\nu} \equiv h_{\mu\nu}^{tr} + h_{\mu\nu}^{tr\phi} + h_{\mu\nu}^{tr\xi_d} + h_{\mu\nu}^{trd}, \]  \hspace{1cm} (4.3)

where components \( h_{\mu\nu}^{tr} \) and \( h_{\mu\nu}^{tr\phi} \) are defined by

\[ h_{\mu\nu}^{tr} \equiv \frac{1}{D} g_{\mu\nu} h \equiv \frac{1}{\sqrt{D}} g_{\mu\nu} \tilde{h}, \]  \hspace{1cm} (4.4)

\[ h_{\mu\nu}^{tr\phi} \equiv 2 \left( \nabla_\mu \nabla_\nu \Box^{-1} - \frac{1}{D} g_{\mu\nu} \right) \phi \equiv \left( \nabla_\mu \nabla_\nu - \frac{1}{D} g_{\mu\nu} \Box \right) O_\phi \tilde{\phi}, \]  \hspace{1cm} (4.5)
\[ O_s^{\phi} \equiv \left( \frac{D}{(D-1)(\Box)(\Box - Dr^{-2})} \right)^{1/2}, \]  
(4.6)

and \( h_{\mu \nu}^{\tilde{\xi}^d} \) is

\[ h_{\mu \nu}^{\tilde{\xi}^d} \equiv \nabla_\mu \xi_\nu^d + \nabla_\nu \xi_\mu^d \equiv \nabla_\mu O_{\xi}^\nu \xi_\nu^d + \nabla_\nu O_{\xi}^\nu \xi_\mu^d, \]
(4.7)

\[ O_{\xi}^\nu \equiv \left( 2( - \Box - (D - 1)r^{-2}) \right)^{-1/2}, \]
(4.8)

which can be separated, according to whether \( \xi_\mu^d \) is a longitudinal vector \( \xi_\mu^L \) or a transverse vector \( \xi_\mu^T \), into \( h_{\mu \nu}^{\tilde{\xi}^L} \) and \( h_{\mu \nu}^{\tilde{\xi}^T} \):

\[ h_{\mu \nu}^{\tilde{\xi}^d} \equiv h_{\mu \nu}^{\tilde{\xi}^L} + h_{\mu \nu}^{\tilde{\xi}^T}, \]
(4.9)

where

\[ \tilde{\xi}_\mu^d \equiv \tilde{\xi}_\mu^L + \tilde{\xi}_\mu^T, \quad \tilde{\xi}_D^T = \tilde{\nabla}_\ell \tilde{\xi}_\ell^T = 0, \]
\[ \bar{\tilde{\xi}}_D^L \equiv a^{-1}O_s^{\tilde{\xi}_L}, \quad \tilde{\xi}_s^L \equiv -\tilde{\nabla}_i \bar{\tilde{\xi}}_i - a^{-1}(D-3)\xi_D^{L}, \]
\[ O_{\xi}^\nu \equiv \left[ \frac{1}{- \Box + (D - 2)r^{-2}} \right]^{1/2}, \]
(4.10)

with the covariant derivative \( \tilde{\nabla}_i \) and the covariant Laplacian \( \Box \) on the unit \( S^{D-1} \).

The last component \( h_{\mu \nu}^{\text{trd}} \) in Eq. (4.3) is decomposed into three parts:

\[ h_{\mu \nu}^{\text{trd}} \equiv h_{\mu \nu}^{\text{trdL}} + h_{\mu \nu}^{\text{trdT}} + h_{\mu \nu}^{\text{trT}}, \]
(4.11)

where each part, in the coordinate system (4.1), is defined by

\[ h_{DD}^{\text{trdL}} \equiv h_{DD}^{\text{trd}} \equiv a^{2}O_{\text{trdL}}^{\text{trdL}} h_{DD}^{\text{trd}}, \]
\[ h_{iD}^{\text{trdL}} \equiv \nabla_\ell \bar{h}_{iD}^{\text{trdL}} = -\nabla_\ell \bar{h}_{iD}^{\text{trdL}} - a^{-(D - 2)}\partial_D(a^{D}h_{DD}^{\text{trdL}}), \]
\[ h_{ij}^{\text{trdL}} \equiv \frac{D - 1}{D - 2} \left( \nabla_\ell \nabla_\ell \bar{h}_{ij}^{\text{trdL}} \right) \left( \bar{\Box} + D - 1 \right)^{-1} \]
\[ \times \left[ a^{-(D-3)}\partial_D(a^{D}h_{DD}^{\text{trdL}}) + a^{2}h_{DD}^{\text{trdL}} \right] - \frac{a^{2}}{D - 1} \bar{g}_{ij} h_{DD}^{\text{trdL}}, \]
\[ O_{\text{trdL}}^{s} \equiv \left[ \frac{(D - 2)(- \Box)(- \Box - (D - 1))}{(D - 1)(- \Box + (D - 2)r^{-2})} \right]^{1/2}, \]
(4.12)
Then Eq. (2.13) becomes

\begin{align}
\langle h, h' \rangle &= \langle h^{tr}, h'^{tr} \rangle + \langle h^{\tilde{r}\phi}, h'^{\tilde{r}\phi} \rangle + \langle h^{\tilde{r}\xi_l}, h'^{\tilde{r}\xi_l} \rangle + \langle h^{\tilde{r}\xi_T}, h'^{\tilde{r}\xi_T} \rangle \\
&+ \langle h^{\tilde{r}dl}, h'^{\tilde{r}dl} \rangle + \langle h^{\tilde{r}dT}, h'^{\tilde{r}dT} \rangle + \langle h^{T}, h'^{T} \rangle,
\end{align}

(4.15)

with

\begin{align}
\langle h^{tr}, h'^{tr} \rangle &= \int d^D x \sqrt{g} \frac{CD + 2}{2} \bar{h} h', \\
\langle h^{\tilde{r}\phi}, h'^{\tilde{r}\phi} \rangle &= \int d^D x \sqrt{g} \bar{\phi} \phi', \\
\langle h^{\tilde{r}\xi_l}, h'^{\tilde{r}\xi_l} \rangle &= \int d^D x \sqrt{g} \xi_l^T \xi_l^T, \\
\langle h^{\tilde{r}\xi_T}, h'^{\tilde{r}\xi_T} \rangle &= \int d^D x \sqrt{g} \xi_T^{\mu} \xi_T^{\nu}, \\
\langle h^{\tilde{r}dl}, h'^{\tilde{r}dl} \rangle &= \int d^D x \sqrt{g} h^{\tilde{r}DL} \bar{h}^{\tilde{r}DL}, \\
\langle h^{\tilde{r}dT}, h'^{\tilde{r}dT} \rangle &= \int d^D x \sqrt{g} h^{\tilde{r}DT} \bar{h}^{\tilde{r}DT}, \\
\langle h^{T}, h'^{T} \rangle &= \int d^D x \sqrt{g} h^{\mu \nu} \bar{h}^{T} h'^{T} h^{\mu} h'^{\mu} h^{\rho} h'^{\rho}.
\end{align}

(4.16)

Therefore, the Gaussian integral, i.e., \( \int_{-\infty}^{+\infty} dx e^{-x^2} = \sqrt{\pi/\lambda} \), says that the Polyakov measure (2.12) is represented in terms of \( \bar{h}, \bar{\phi}, \bar{\xi}_l, \bar{\xi}_T^{\mu}, \bar{h}^{\tilde{r}DL}, \bar{h}^{\tilde{r}DT}, \bar{h}^{T} \) and \( h^{\mu \nu} \) as

\begin{align}
\int \frac{\mathcal{D}h}{\sqrt{8\pi \hbar/(CD + 2)}} \frac{\mathcal{D}\bar{\phi}}{\sqrt{4\pi \hbar}} \frac{\mathcal{D}\bar{\xi}_l}{\sqrt{4\pi \hbar}} \frac{\mathcal{D}\bar{\xi}_T^{\mu}}{\sqrt{4\pi \hbar}} \frac{\mathcal{D}\bar{h}^{\tilde{r}DL}}{\sqrt{4\pi \hbar}} \\
\times \frac{\mathcal{D}h^{\tilde{r}DL}}{\sqrt{4\pi \hbar}} \frac{\mathcal{D}h^{\tilde{r}DT}}{\sqrt{4\pi \hbar}} \frac{\mathcal{D}h^{T}}{\sqrt{4\pi \hbar}} \exp \left[ -\frac{1}{4\hbar} \langle h, h \rangle \right] = 1,
\end{align}

(4.17)

where integration variables may be concretely defined in terms of the expansion coefficients of each variable in the basis of the eigenfunctions of \( \Box \), i.e., those of
associated components of \( h_{\mu\nu} \) in the basis of the tensor eigenfunctions of \( \square \) on \( S^D \).

(As for eigenfunctions on \( S^D \) see Appendix A.)

In the canonical decomposition, which is most suitable one for the case of a special gauge in which all redundant variables vanish like the Coulomb gauge in the Abelian gauge theory and also for a path integral in the phase space where the time coordinate is separated from the other coordinates, any tensor field can be expressed with

\[
h_{\mu\nu} \equiv h^{tr}_{\mu\nu} + h^{tr}_{\muD} + h^{\Phi}_{ij} + h^{\psi}_{ij} + h^{h^T}_{\mu\nu} + h^{w^T}_{\mu\nu} + h^{tT}_{\mu\nu} ,
\]

(4.18)

where

\[
h^{\Phi}_{ij} \equiv \tilde{\nabla}_i \tilde{\nabla}_j^{-1} \Phi , \quad h^{\Phi}_{D\muD} \equiv h^{\Phi}_{ij} \equiv 0 ,
\]

\[
h^{\psi}_{ij} \equiv \frac{D-1}{D-2} \left( \tilde{\nabla}_i \tilde{\nabla}_j \square^{-1} - \frac{1}{D-1} \tilde{g}_{ij} \right) (\square + D-1)^{-1} \psi , \quad h^{\psi}_{\muD} \equiv 0 ,
\]

\[
h^{h^T}_{iD} \equiv h^{h^T}_{iD} , \quad h^{h^T}_{D\muD} \equiv h^{h^T}_{ij} \equiv 0 , \quad \tilde{\nabla}^{\ell} h^{h^T}_{\ell D} \equiv 0 ,
\]

\[
h^{\psi}_{T\muD} \equiv \tilde{\nabla}_i (\square + D-2)^{-1} w^T_j + \tilde{\nabla}_j (\square + D-2)^{-1} w^T_i , \quad h^{\psi}_{\muD} \equiv 0 , \quad \tilde{\nabla}^{\ell} w^T_{\ell} \equiv 0 .
\]

(4.19)

Then Eq. (2.13) is split into each part of them:

\[
< h , h' > = < h^{tr} , h^{tr'} > + < h^{h^T}_{D\muD} , h^{h^T}_{D\muD} > + < h^{\Phi} , h^{\Phi'} > + < h^{\psi} , h^{\psi'} > + < h^{h^T} , h^{h^T'} > + < h^{w^T} , h^{w^T'} > + < h^{hT} , h^{hT'} > ,
\]

(4.20)

with
\[
\begin{align*}
< h^{r}, h^{r} > & = \int d\tau \frac{CD + 2}{2D} a^{D-1} \int d^{D-1} x \sqrt{g} hh', \\
< h^{D}, h^{D} > & = \int d\tau \frac{D}{D-1} a^{D-1} \int d^{D-1} x \sqrt{g} h^{D} h^{D}, \\
< h^{D}, h^{D} > & = \int d\tau 2a^{D-3} \int d^{D-1} x \sqrt{g} \Phi(-\Box)^{-1} \Phi', \\
< h^{\psi}, h^{\psi} > & = \int d\tau a^{D-5} \int d^{D-1} x \sqrt{g} \frac{D-1}{D-2} \psi(\Box + D - 1)^{-1} \psi', \\
< h^{T}, h^{T} > & = \int d\tau 2a^{D-3} \int d^{D-1} x \sqrt{g} g^{ij} h^{T} \delta_{iD} h^{T}_{jD}, \\
< h^{w}, h^{w} > & = \int d\tau 2a^{D-5} \int d^{D-1} x \sqrt{g} \bar{\psi} g^{ij} w^{T} (-\Box - (D - 2))^{-1} w^{T}, \\
< h^{T}, h^{T} > & = \int d\tau a^{D-5} \int d^{D-1} x \sqrt{g} g^{mn} g^{ij} h^{T} \delta_{ij} h^{T}_{mn}.
\end{align*}
\] (4.21)

Therefore, the Gaussian integral says that the Polyakov measure (2.12) in the canonical decomposition becomes

\[
\int \prod_{\tau} \left[ \left( \frac{\Delta \tau (CD + 2)a^{D-1}(\tau)}{8\pi hD} \right)^{1/2} \mathcal{D}h(\tau) \right] \left[ \left( \frac{\Delta \tau Da^{D-1}(\tau)}{4\pi h(D - 1)} \right)^{1/2} \mathcal{D}h^{D}(\tau) \right] \times \left[ \left( \frac{\Delta \tau a^{D-3}(\tau)}{2\pi h(\Box)} \right)^{1/2} \mathcal{D}\Phi(\tau) \right] \left[ \left( \frac{\Delta \tau (D - 1)a^{D-5}(\tau)}{4\pi h(D - 2)(\Box + D - 1)} \right)^{1/2} \mathcal{D}\psi(\tau) \right] \times \left[ \left( \frac{\Delta \tau a^{D-3}(\tau)}{2\pi h} \right)^{1/2} \mathcal{D}h^{T}(\tau) \right] \left[ \left( \frac{\Delta \tau a^{D-5}(\tau)}{2\pi h(-\Box - (D - 2))} \right)^{1/2} \mathcal{D}w^{T}(\tau) \right] \times \left[ \left( \frac{\Delta \tau a^{D-5}(\tau)}{4\pi h} \right)^{1/2} \mathcal{D}h^{T}(\tau) \right] \exp \left[ -\frac{1}{4\hbar} < h, h > \right] = 1,
\] (4.22)

in which the time \( \tau \) is specialized from the other coordinates, and its product might be defined in the discrete time formulation with a finite distance and its zero limitation after integrating out, and then the functional measure on each time are defined in terms of the expansion coefficients of each variable of \( h, h^{D}, \Phi, \psi, h^{T}, w^{T} \) and \( \mathcal{D}h^{T} \) in the basis of the eigenfunctions of \( \Box \) on the unit \( S^{D-1} \).

Eqs. (4.17) and (4.22) mean that the Jacobian factor under the transformation of measure from the covariant path integral to the canonical one would be naively 1, since both measures are defined with the Gaussian integral whose integration
value is 1. However, as discussed in the following subsections, the Jacobian factor is unfortunately not 1 and takes an anomalous value. Such a situation happens also in the case of the gauge theory in Euclidean Robertson-Walker spacetimes with $K = +1$ [16,17]. The Jacobian factor is discussed below, separating each one of two decompositions into three parts, i.e., a traceless and transverse tensor part, a tensor part defined by transverse vector fields $\bar{\xi}^T_i$, $\bar{h}_{iD}^{\text{trd}T}$, $h_i^D$ and $w_i^T$, and that defined by scalar fields $\bar{h}, \bar{\phi}, \bar{\xi}, \bar{h}_{DD}^D, \bar{h}_{DD}^D, \Phi$ and $\psi$.

As for the physical variable, i.e., the traceless and transverse tensor $h_{\mu\nu}^{\text{trT}}$, it is easily found that we have no anomalous thing under the transformation between the covariant and canonical measures:

$$\prod_\tau \left( \frac{\Delta \tau a^{D-5}(\tau)}{4\pi h} \right)^{1/2} D h_{i,j}^{\text{trT}}(\tau) = \frac{D h_{\mu\nu}^{\text{trT}}}{\sqrt{4\pi h}},$$

(4.23)

where it is noted that the time $\tau$ product in the l.h.s. is changed, in the r.h.s., into the product with respect to modes along the time axis, by using the relation (A32) among the traceless and transverse tensor eigenfunctions on $S^D$ and the traceless and divergenceless eigenfunctions on $S^{D-1}$, and then the factors $\prod_\tau (\Delta \tau a^{D-5}(\tau))^{1/2}$ cancel out with the determinants of $a^2 f^{L\ell}$, since

$$|\det f^{L\ell}| = \prod_\tau (\Delta \tau a^{D-1}(\tau))^{-1/2}.$$  

(4.24)

(See Ref. 16.)

### 4.2 Anomalous Jacobian Factor in Transverse Vector Part

The relations among the transverse vector fields $\bar{\xi}^T_i, \bar{h}_{iD}^{\text{trd}T}, h_i^D$ and $w_i^T$ used in the covariant and canonical decompositions, i.e., (4.3) and (4.18), are

$$h_{iD}^T = a^2 \partial_D (a^{-2} O^v_{\xi a} \bar{\xi}^T_i) + a^{-1} O^v_{\text{trd}T} \bar{h}_{iD}^{\text{trd}T},$$

(4.25)

$$(\Box + D - 2)^{-1} w_i^T = O^v_{\xi a} \bar{\xi}^T_i - (\Box + D - 2)^{-1} a^{-(D-3)} \partial_D (a^{D-2} O^v_{\text{trd}T} \bar{h}_{iD}^{\text{trd}T})$$

(4.26)
\[
\begin{align*}
  a^2(\Box + (D-1)r^{-2}) O^v_{\xi_i} \tilde{\xi}_i^T &= a^{-(D-3)} \partial_D (a^{D-1} h^T_{iD}) + w^T_i, \\
  a(\Box + D - 2)^{-1} (\Box - r^{-2}) O^v_{\text{trd}} \tilde{h}^\text{trdT}_{iD} &= h^T_{iD} - a^2 \partial D (a^{-2}(\Box + D - 2)^{-1} w^T_i),
\end{align*}
\]

where the relation between the former two equations and the later two equations is that of the inverse transformation. Now let us discuss an anomalous Jacobian factor with respect to these variables, separating them into \( \ell = 1 \) modes and \( \ell \neq 1 \) modes of \( \Box \).

First, with respect to \( \ell = 1 \) modes, noting that \( h^T_{iD} \) and \( \tilde{\xi}_i^T \) have their modes, on the other hand, \( \tilde{h}_{iD}^\text{trdT} \) and \( w^T_i \) are not able to have them, the following relations between \( h^T_{iD} \) and \( \tilde{\xi}_i^T \) are derived from Eqs. (4.25) and (4.27) (although both Eqs. (4.26) and (4.28) are reduced to a trivial equation);

\[
\begin{align*}
  h^T_{iD} &= a^2 \partial_D \left(a^{-2} O^v_{\xi} \tilde{\xi}_i^T \right), \\
  (\Box + (D-1)r^{-2}) O^v_{\xi} \tilde{\xi}_i^T &= a^{-(D-1)} \partial_D \left(a^{D-1} h^T_{iD} \right),
\end{align*}
\]

where the indices \( (\tilde{1}) \) means that the variables are of \( \ell = 1 \) modes and these equations have the inverse relationship to each other. From (4.29), a nontrivial Jacobian factor is obtained:

\[
\begin{align*}
  \left| \det \left[ a^2 \partial_D a^{-2} \left(v^T(\tilde{1}) \right) \right] \right| \left| \det \left[ - g^{\mu \nu} (\Box + (D-1)r^{-2}) \right] (v^T(\tilde{1}) \right) \right|^{-1/2},
\end{align*}
\]

or if Eq. (4.30) is used to change the variables, another Jacobian factor is

\[
\begin{align*}
  \left| \det \left[ a^{-(D-1)} \partial_D a^{D-1} \right] (v^T(\tilde{1}) \right) \right|^{-1} \left| \det \left[ - g^{\mu \nu} (\Box + (D-1)r^{-2}) \right] (v^T(\tilde{1}) \right) \right|^{1/2},
\end{align*}
\]

where \( v^T(\tilde{1}) \) is any transverse vector field with \( \ell = 1 \) modes of \( \Box \) on the unit \( S^{D-1} \).

With respect to \( \ell \neq 1 \) part, if Eqs. (4.27) and (4.25) are used under the changes of variables, \( (h^T_{iD}, w^T_i) \rightarrow (h^T_{iD}, \tilde{\xi}_i^T) \rightarrow (\tilde{h}^\text{trdT}_{iD}, \tilde{\xi}_i^T) \), then we have

\[
\prod_{\tau} \left[ \left( \frac{\Delta \tau a^{D-5}(\tau)}{2\pi h \left(-\Box - (D-2)\right)} \right)^{1/2} D w^T_i(\tau) \right]
\]
\[ \prod_{\tau} \left( a(\tau) \right) \left| \det \left[ g^{\mu\nu} \left( \begin{array}{c} \Theta - (D - 1)r^{-2} \\ -\Theta - (D - 2) \end{array} \right) \right] \left( \tilde{v}^T \right) \right|^{1/2} \frac{D\tilde{\xi}_\mu}{\sqrt{4\pi\bar{h}}} \] (4.33)

and

\[ \prod_{\tau} \left[ \left( \frac{\Delta a^{D-3}(\tau)}{2\pi\bar{h}} \right)^{1/2} Dh_{iD}^T(\tau) \right] \]
\[ = \prod_{\tau} \left( a^{-1}(\tau) \right) \left| \det \left[ g^{\mu\nu} \left( \begin{array}{c} \tilde{\Theta} - (D - 2) \\ -\tilde{\Theta} + r^{-2} \end{array} \right) \right] \left( \tilde{v}^T \right) \right|^{1/2} \frac{D\tilde{h}_{\mu D}^T}{\sqrt{4\pi\bar{h}}}, \] (4.34)

where \( \tilde{v}^T \) is any transverse vector field without \( \ell = 1 \) modes. Therefore, Eqs. (4.33) and (4.34) give

\[ \prod_{\tau} \left[ \left( \frac{\Delta a^{D-3}(\tau)}{2\pi\bar{h}} \right)^{1/2} Dh_{iD}^T(\tau) \right] \]
\[ = \prod_{\tau} \left( a^{-1}(\tau) \right) \left| \det \left[ g^{\mu\nu} \left( \begin{array}{c} \tilde{\Theta} - (D - 2) \\ -\tilde{\Theta} + r^{-2} \end{array} \right) \right] \left( \tilde{v}^T \right) \right|^{1/2} \frac{D\tilde{h}_{\mu D}^T D\tilde{\xi}_\mu}{\sqrt{4\pi\bar{h}} \sqrt{4\pi\bar{h}}}. \] (4.35)

Instead of Eqs. (4.27) and (4.25), if Eqs. (4.28) and (4.26) are used through the following steps of the change of variables, i.e., \( (h_{iD}^T, w_i^T) \rightarrow (\tilde{h}_{\mu D}^T, \tilde{w}_i^T) \rightarrow (\tilde{h}_{\mu D}^T, \tilde{\xi}_\mu^T) \), the different result from Eq. (4.35) is derived:

\[ \prod_{\tau} \left[ \left( \frac{\Delta a^{D-3}(\tau)}{2\pi\bar{h}} \right)^{1/2} Dh_{iD}^T(\tau) \right] \]
\[ = \prod_{\tau} \left( a^{-1}(\tau) \right) \left| \det \left[ g^{\mu\nu} \left( \begin{array}{c} \tilde{\Theta} - (D - 2) \\ -\tilde{\Theta} + r^{-2} \end{array} \right) \right] \left( \tilde{v}^T \right) \right|^{-1/2} \frac{D\tilde{h}_{\mu D}^T D\tilde{\xi}_\mu}{\sqrt{4\pi\bar{h}} \sqrt{4\pi\bar{h}}}. \] (4.36)

From Eqs. (4.35) and (4.36), therefore, the factor

\[ \left| \det \left[ g^{\mu\nu} \left( \begin{array}{c} \Theta - (D - 1)r^{-2} \\ -\Theta + r^{-2} \end{array} \right) \right] \left( \tilde{v}^T \right) \right|^{1/2} \] (4.37)

might be naively thought to take a trivial value 1 and both measures for tensor fields defined in terms of transverse vector fields without \( \ell \neq 1 \) modes of \( \tilde{\Theta} \) on the
unit $S^{D-1}$ would be concluded to be equal to each other. However, the factor is obviously the same with the uncancellation factor (4.2) excluding the $\ell = 1$ mode part, and in order to actually calculate the factor (4.37) one need to regularize it because the arguments of the determinates are infinite matrices. Unfortunately, some regularization fail to make it 1 [14].

4.3 Anomalous Jacobian Factor of Scalar Part

The relations among scalar fields $\tilde{h}, \tilde{\phi}, \tilde{\xi}, \tilde{h}^{\tilde{r}D}, h, h^{\tilde{r}D}, \Phi$ and $\psi$ are

\[
h = \sqrt{D\tilde{h}},
\]

\[
h^{\tilde{r}D} = \frac{1}{D}(D\partial_D^2 - \Box)O^s_{\phi\tilde{\phi}} + a^{-2}O^s_{\tilde{r}D} \tilde{h}^{\tilde{r}D} + 2a^{-1}(\partial_D - a^{-1}\dot{a})O^s_{\xi\tilde{\xi}}O^s_{\tilde{h}},
\]

\[
\tilde{\phi}^{-1} = (\partial_D - a^{-1}\dot{a})O^s_{\phi\tilde{\phi}} - \Box^{-1}(\partial_D + (D - 2)a^{-1}\dot{a})O^s_{\tilde{r}D} \tilde{h}^{\tilde{r}D}.
\]

\[
\frac{D - 1}{D - 2}(\Box + (D-1)^{-1}\psi = \Box O^s_{\phi\tilde{\phi}} + \frac{D - 1}{D - 2}(\Box + (D-1)^{-1}a^2[\Box + (D-2)a^{-1}\dot{a}\partial_D

+ (D-2)^2a^{-2}\dot{a}^2 - (D-2)r^{-2} - \frac{D - 2}{D - 1}a^{-2}\Box]O^s_{\tilde{r}D} \tilde{h}^{\tilde{r}D}}

- 2a(\partial_D + (D-2)a^{-1}\dot{a})O^s_{\xi\tilde{\xi}}O^s_{\tilde{h}},
\]

where $O^s_{\xi\tilde{\xi}} \equiv \left(2\left(-\Box - Dr^{-2}\right)\right)^{-1/2}$.

With respect to the trace part, from the relation (4.38), we have the following relationship of measures;

\[
\prod_\tau \left[\left(\frac{\Delta\tau(CD + 2)a^{D-1}(\tau)}{8\pi\tilde{h}D}\right)^{1/2}D\tilde{h}(\tau)\right] = \frac{D\tilde{h}}{\sqrt{8\pi\tilde{h}/(CD + 2)}},
\]

which means that no anomalous thing happens under the change of variables in this part.

When turning our discussion to the change of variables of $\tilde{h}^{\tilde{r}D}, \tilde{\phi}, \tilde{\xi}$ and $\tilde{h}^{\tilde{r}D}$, $\Phi, \psi$, it is important to note that $h^{\tilde{r}D}$ and $\tilde{\phi}$ have both modes of $\ell = 0$ and 1,
and the variables $\Phi, \bar{\xi}$ have only $\ell = 1$ modes among them, on the other hand, $\psi$ and $\tilde{h}^{\text{trdl}}_{DD}$ have no one of these modes. In this paper, we study the Jacobian factor with respect to only the part having no modes which are $\ell = 0$ and 1 of $\bar{\Phi}$ on the unit $S^{D-1}$ in order to check the relation between it and the uncancelation factor (4.2): first, from Eqs. (4.39), (4.40) and (4.41), we have

$$
\Phi - \frac{D-1}{D-2} (\bar{\Phi} + D-1)^{-1} (\partial_D - a^{-1} \partial a) \psi \\
= \frac{D-1}{D-2} (\bar{\Phi} + D-1)^{-1} a^2 (\partial_D + (D-1)a^{-1} \partial a) \left(- \bar{\Phi} + (D-2)r^{-2}\right) O^2_{\text{trdl}} \tilde{h}^{\text{trdl}}_{DD} \\
- a ( - \bar{\Phi} + (D-2)r^{-2} ) O^2_{\xi\xi} O^s_{\xi\xi},
$$

(4.43)

$$
D\tilde{h}^{\text{tr}}_{DD} - \frac{D-1}{D-2} (\bar{\Phi} + D-1)^{-1} (D\partial_D^2 - \bar{\Phi}) \psi \\
= - \frac{(D-1)^2}{D-2} a^2 (\bar{\Phi} + D-1)^{-1} ( - \bar{\Phi} + 2(D-1)r^{-2} - 2a^{-1}a^2 \partial_D + 2a^{-2} ) \\
\times ( - \bar{\Phi} + (D-2)r^{-2} ) O^2_{\text{trdl}} \tilde{h}^{\text{trdl}}_{DD} \\
- 2(D-1)a \partial_D ( - \bar{\Phi} + (D-2)r^{-2} ) O^2_{\xi\xi} O^s_{\xi\xi},
$$

(4.44)

and these equations give

$$
2(D-1)a \partial_D a^{-1} \Phi - D\tilde{h}^{\text{tr}}_{DD} - \frac{D-1}{D-2} (\bar{\Phi} + D-1)^{-1} \\
\times \left[(D-2)\partial_D^2 - 4(D-1)a^{-1}a^2 \partial_D + 2(D-1)(a^{-2}a^2 + a^{-2}) + \bar{\Phi}\right] \psi \\
= \frac{(D-1)^2}{D-2} a^2 (\bar{\Phi} + D-1)^{-1} ( - \bar{\Phi} + (D-2)r^{-2} ) O^2_{\text{trdl}} \tilde{h}^{\text{trdl}}_{DD} .
$$

(4.45)

Now let us change variables, using Eqs. (4.45), (4.43) and (4.41) through the following steps: $(h^{\text{trdl}}_{DD}, \Phi, \psi) \rightarrow (\tilde{h}^{\text{trdl}}_{DD}, \Phi, \psi) \rightarrow (\tilde{h}^{\text{trdl}}_{DD}, \bar{\xi}, \psi) \rightarrow (\tilde{h}^{\text{trdl}}_{DD}, \bar{\xi}, \bar{\phi})$. Then, an anomalous Jacobian factor is derived:

$$
\prod_{\tau} \left[ \left( \frac{\Delta \tau D a^{D-1} \tau}{4 \pi h(D-1)} \right)^{1/2} D\tilde{h}^{\text{tr}}_{DD} \right] \\
\times \left[ \left( \frac{\Delta \tau a^{D-3} \tau}{2 \pi h(\bar{\Phi})} \right)^{1/2} D\Phi(\tau) \right] \\
\times \left[ \left( \frac{\Delta \tau (D-1)a^{D-5} \tau}{4 \pi h(D-2)(\bar{\Phi} + D-1)} \right)^{1/2} D\psi(\tau) \right].
$$
\[
= \left| \det \left[ \frac{-\Box + (D - 2)r^{-2}}{-\Box - Dr^{-2}} \right] (\bar{s}) \right| \frac{\mathcal{D}\tilde{h}^{\text{red}}_D}{\sqrt{4\pi\bar{h}}} \frac{\mathcal{D}\tilde{\xi}}{\sqrt{4\pi\bar{h}}} \frac{\mathcal{D}\tilde{\phi}}{\sqrt{4\pi\bar{h}}},
\]

(4.46)

where \( \bar{s} \) is any scalar fields without \( \ell = 0 \) and 1 of \( \tilde{\Box} \). While, if the other steps are used together with other equations instead of Eqs. (4.45), (4.43) and (4.41), we may obtain another Jacobian factor which is inverse of the Jacobian factor in (4.46), because, as mentioned in subsection 4.1, a naive Jacobian factor under the change of variables between the covariant and canonical path-integral measures would be suggested to be 1 according to the truth that both measures are defined by the Gaussian integral. Thus the Jacobian factor in (4.46) may be regarded as an anomalous one.

Furthermore, we may remember the following anomalous Jacobian factor in path-integral measures for the Abelian gauge theory on de Sitter background;

\[
\left| \det \left[ \frac{-\Box}{-\Box + (D - 2)r^{-2}} \right] (\bar{s}) \right|,
\]

(4.47)

where \( \tilde{s} \) is a scalar field with no mode having \( \ell = 0 \) of \( \tilde{\Box} \), thus the Jacobian factors in Eqs. (4.46) and (4.47) give an anomalous factor:

\[
\left| \det \left[ \frac{-\Box}{-\Box - Dr^{-2}} \right] (\bar{s}) \right|,
\]

(4.48)

which is equal to

\[
\left| \det \left[ g^{\mu\nu} \left( \frac{-\Box - (D - 1)r^{-2}}{-\Box + r^{-2}} \right) \right] (g^{L}) \right|,
\]

(4.49)

because the Jacobian factor can be separated into respectively independent parts of modes of \( \tilde{\Box} \). The last equation (4.49) is nothing but the longitudinal part of the uncancellation factor (4.2) excluding the \( \ell = 1 \) modes.
5. Conclusion

In this paper, the conformal rotation in the Euclidean path integral of linearized gravity around a saddle-point background with the cosmological term was discussed and that it in a nontrivial curved spacetime will be more complicated than the flat spacetime because the sign of the mass term in the action $I^\psi$ (see Eq. (2.9)) is same with ordinary scalar fields, in spite of the opposite sign of the kinetic term. But the action of physical variable might be positive definite so that the path integral may converge. Furthermore, it was also argued that the covariant path-integral measure does not give us the physical one-loop partition function and particularly in de Sitter spacetime the uncancellation factor was shown to be equal to an anomalous Jacobian factor under the change of variables between the covariant and canonical path-integral measures.

Although in this paper we have discussed with respect to only tensor fields, the Faddeev-Popov vector fields may be actually shown similarly to have an anomalous Jacobian factor between the covariant and canonical measures. Furthermore, the anomalous factor is the same with one in the case of ordinary vector fields.

Finally, we may note that the difference between the path-integral measures up to the anomalous Jacobian factor, as discussed in Ref. 17, calls the reexamination of the Faddeev-Popov conjecture to mind, since the covariant decomposition is suitable one for the covariant gauge and the canonical decomposition is suitable for a special gauge fixing condition so that redundant variables all may vanish, so two path integrals defined in those gauge fixing conditions are different from each other owing to the anomalous Jacobian factor.
APPENDIX A

In this appendix we summarize the scalar, vector and symmetric tensor eigenfunctions of the covariant d’Alembertian (Laplacian) on spheres \([16]\). In particular, eigenfunctions on \(S^D\) of radius \(r\) are expressed in terms of eigenfunctions of \(\tilde{\square}\) on the unit \(S^{D-1}\) and the Gegenbauer polynomials \(C^m_m(x)\).

On \(S^D\) of radius \(r\), eigenvalues of \(\square\) acting on scalar functions are

\[- r^{-2} \lambda_L(D, 0) \equiv -r^{-2} L(L + D - 1), \quad \text{for } L = 0, 1, 2, \ldots, \] (A.1)

with the degeneracy \(d_L(D, 0)\) defined by

\[d_L(D, 0) \equiv \binom{D + L}{L} - \binom{D + L - 2}{L - 2}, \] (A.2)

and the associated eigenfunctions \(\phi^{L\ell m}\) can be expressed in terms of eigenfunctions \(\tilde{S}^{\ell m}\) on the unit \(S^{D-1}\) and the Gegenbauer polynomials \(C^m_m(x)\) as

\[\phi^{L\ell m} \equiv f^{L\ell}(\tau/r)\tilde{S}^{\ell m}, \quad \text{for } L = 0, 1, 2, \ldots, \] (A.3)

\[\ell = 0, 1, 2, \ldots, L, \quad \text{and } m = 1, 2, 3, \ldots, d_{\ell}(D - 1, 0),\]

where

\[f^{L\ell}(\theta) \equiv C_{L\ell}(D, 0)(\sin \theta)^\ell C^{(D-1)/2+\ell}_{L-\ell}(\cos \theta),\]

\[C_{L\ell}(D, 0) \equiv r^{-D/2} \left( \frac{2^{2\ell+D-2}(L + D - 1)(L - \ell)!}{\pi(L + \ell + D - 2)!} \right)^{1/2} \Gamma \left( \ell + \frac{D - 1}{2} \right), \] (A.4)

and \(\tilde{S}^{\ell m}\) are scalar eigenfunctions of \(\tilde{\square}\) with the eigenvalue \(-\lambda_{\ell}(D - 1, 0)\) and the degeneracy \(d_{\ell}(D - 1, 0)\). The indices \(\ell\) and \(m\) of \(\phi^{L\ell m}\) denote the associated degeneracy, hence we have a directly checkable relation such that

\[d_{\ell}(D, 0) = \sum_{\ell=0}^{L} d_{\ell}(D - 1, 0). \] (A.5)

From (A.4), \(f^{L\ell}\) are found to satisfy

\[a^{-(D-1)} \frac{d}{dr} (a^{D-1} \frac{d}{dr} f^{L\ell}) - \lambda_{\ell}(D - 1, 0)a^{-2} f^{L\ell} = -r^{-2} \lambda_L(D, 0) f^{L\ell}, \] (A.6)
which is an expression of eigenvalue equations obeyed by $\phi^{L\ell m}$ in terms of $f^{L\ell}$, and they also satisfy

$$\int_0^{r\pi} d\tau a^{D-1} f^{L\ell} f^{L'\ell} = \delta_{LL'}.$$  \hspace{1cm} (A.7)

From this and orthogonality of $\tilde{S}^{\ell m}$ on $S^{D-1}$, the scalar eigenfunctions $\phi^{L\ell m}$ form the orthonormal basis of scalar fields on $S^D$ of radius $r$.

Any vector field on $S^D$ can be decomposed into the gradient of a scalar and a divergenceless vector. Therefore the orthonormal basis of vector fields on $S^D$ is made of these two parts: first, gradients of scalar eigenfunctions, i.e.,

$$T^{(s)\ell m}_\mu \equiv \left(r^{-2}\lambda_L(D,0)\right)^{-1/2}\nabla_\mu \phi^{L\ell m}, \quad \text{for } L = 1, 2, 3, \ldots,$$

$$\ell = 0, 1, 2, \ldots, L, \quad \text{and } m = 1, 2, 3, \ldots, d_\ell(D-1,0),$$  \hspace{1cm} (A.8)

satisfy the eigenvalue equation

$$\square T^{(s)\ell m}_\mu = -r^{-2}\lambda_L(D,1s) T^{(s)\ell m}_\mu,$$

$$\lambda_L(D,1s) \equiv L(L+D-1) - (D-1),$$  \hspace{1cm} (A.9)

and the orthogonality. Note here that there exists no $T^{(s)\ell m}_\mu$ for $L = 0$ because it trivially vanishes and also that the eigenvalues $\lambda_L(D,1s)$ are thus positive for $L \geq 1$. Degeneracies of $\square$ acting on gradients of scalars on $S^D$ are $d_L(D,1s) \equiv d_L(D,0)$ for $L \neq 0$. Next, eigenvalues of $\square$ acting on divergenceless vectors on $S^D$ of radius $r$ are

$$-r^{-2}\lambda_L(D,1d) \equiv -r^{-2}(L(L+D-1) - 1), \quad \text{for } L = 1, 2, 3, \ldots, (A.10)$$

with the degeneracy $d_L(D,1d)$ defined by

$$d_L(D,1d) \equiv (D+1)d_L(D,0) - d_{L-1}(D,0) - d_{L+1}(D,0).$$  \hspace{1cm} (A.11)

Furthermore, any divergence vector can be split into the longitudinal and transverse parts, thus the divergenceless vector eigenfunctions $T^{(d)\ell m}_\mu$ consist of their parts:
the longitudinal vector eigenfunctions $T^{(L)\ell m}_\mu$ are defined by scalar eigenfunctions on $S^D$,

$$T^{(L)\ell m}_D \equiv C_{L\ell}(D, 1L)a^{-1}\phi^{L\ell m},$$

$$T^{(L)\ell m}_i \equiv -\nabla_i \Box^{-1} a^{-1(\ell-D-3)} \partial_D (a^{D-1}T^{(L)\ell m}_D), \quad \text{for } L = 1, 2, 3, \ldots ,$$

$$\ell = 1, 2, 3, \ldots , L, \quad \text{and } m = 1, 2, 3, \ldots , d_\ell(D-1,0),$$

with the normalization constants

$$C_{L\ell}(D, 1L) = \left(R^2 \ell(\ell + D - 2)[L(L + D - 1) + D - 2]^{-1}\right)^{1/2},$$

where we note that there exists no $T^{(L)\ell m}_\mu$ for $\ell = 0$ because the operator $\Box^{-1}$ or the coefficient $C_{L\ell}(D, 1L)$ does not permit it. This means that the degeneracy of the longitudinal vector eigenfunctions becomes

$$d_L(D, 1L) = \sum_{\ell=1}^{L} d_\ell(D-1,0) = d_L(D,0) - 1.$$  

As for the transverse vector eigenfunctions $T^{(T)\ell m}_\mu$, they are expressed in terms of divergenceless vector eigenfunctions $\widetilde{S}^{(d)\ell m}_i$ of $\Box$ on the unit $S^{D-1}$ with the eigenvalue $-\lambda_\ell(D-1,1d)$ and the degeneracy $d_\ell(D-1,1d)$, and $f^{L\ell}$ as

$$T^{(T)\ell m}_D \equiv 0, \quad T^{(T)\ell m}_i \equiv a f^{L\ell} \widetilde{S}^{(d)\ell m}_i, \quad \text{for } L = 1, 2, 3, \ldots ,$$

$$\ell = 1, 2, 3, \ldots , L, \quad \text{and } m = 1, 2, 3, \ldots , d_\ell(D-1,1d),$$

where the indices $\ell$ and $m$ of $T^{(T)\ell m}_\mu$ stand for the degeneracy $d_L(D,1T)$, which becomes

$$d_L(D,1T) = \sum_{\ell=1}^{L} d_\ell(D-1,1d) = Dd_L(D,0) - d_{L-1}(D,0) - d_{L+1}(D,0).$$
Both eigenfunctions $T^{(L)}_{\mu \ell m}$ and $T^{(T)}_{\mu \ell m}$ obey the eigenvalue equation that divergenceless vector eigenfunctions satisfy. One obtains the relation

$$d_L(D, 1d) = d_L(D, 1L) + d_L(D, 1T). \quad (A.17)$$

Any symmetric tensor field on $S^D$ can be covariantly decomposed by Eq. (4.3), thus symmetric tensor eigenfunctions of $\Box$ can be constructed, by separating them into their components, as follows: First, eigenfunctions forming the trace component $h^{tr \mu \nu}$ of tensor fields are defined by

$$T^{(tr)}_{\mu \nu \ell m} = C_L(D, 2tr)g_{\mu \nu} \phi^{L \ell m}, \quad \text{for } L = 0, 1, 2, \ldots,\ell = 0, 1, 2, \ldots, L, \text{ and } m = 1, 2, 3, \ldots, d_\ell(D - 1, 0), \quad (A.18)$$

with $C_L(D, 2tr) \equiv \frac{1}{\sqrt{D}}$, their eigenvalues and degeneracies for the d’Alembertian $\Box$ obviously become

$$-r^{-2}\lambda_L(D, 2tr) \equiv -r^{-2}L(L + D - 1), \quad d_L(D, 2tr) \equiv d_L(D, 0). \quad (A.19)$$

Secondly, the component $h^{tr \phi}_{\mu \nu}$ defined in Eq. (4.5) gives the following eigenfunctions;

$$T^{(tr \phi)}_{\mu \nu \ell m} = C_L(D, 2\tilde{tr})\left(\nabla_\mu \nabla_\nu - \frac{1}{D}g_{\mu \nu} \Box\right) \phi^{L \ell m}, \quad \text{for } L = 2, 3, 4, \ldots,\ell = 0, 1, 2, \ldots, L, \text{ and } m = 1, 2, 3, \ldots, d_\ell(D - 1, 0),$$

with $C_L(D, 2\tilde{tr}) \equiv \left(\frac{r^4D}{(D - 1)L(L + D - 1)(L(L + D - 1) - D)}\right)^{1/2}$,\quad (A.20)

and the eigenvalues and degeneracies, respectively, are

$$-r^{-2}\lambda_L(D, 2\tilde{tr}) \equiv -r^{-2}L(L + D - 1) - 2D), \quad d_L(D, 2\tilde{tr}) \equiv d_L(D, 0). \quad (A.21)$$

Thirdly, the eigenfunctions $T^{(tr \phi \alpha)}_{\mu \nu \ell m}$ forming the component $h^{tr \phi \alpha}_{\mu \nu}$ of tensor fields
obey the eigenvalue equation

\[ \square T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m} = -r^{-2}\lambda_L(D, 2\tilde{\text{tr}}^d)T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m} \]

\[ -r^{-2}\lambda_L(D, 2\tilde{\text{tr}}^d) \equiv -r^{-2}(L(L+D-1)-(D+2)), \]

with the degeneracy

\[ d_L(D, 2\tilde{\text{tr}}^d) \equiv d_L(D, 1d), \quad \text{for } L = 2, 3, 4, \ldots, \]

(A.23)

and they are given, from Eq. (4.7), by

\[ T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m} \equiv C_L(D, 2\tilde{\text{tr}}^d)\left(\nabla_\mu T^{(d)\mu\nu}_{L\ell m} + \nabla_\nu T^{(d)\mu\nu}_{L\ell m}\right), \]

with

\[ C_L(D, 2\tilde{\text{tr}}^d) \equiv \left(2r^{-2}(L(L+D-1)-D)\right)^{-1/2}, \]

(A.24)

which may be separated into two parts, as mentioned in Eq. (4.9);

\[ T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m} \equiv \left(T^{(\tilde{\text{tr}}^d)L\ell m}_{\mu\nu}, T^{(\tilde{\text{tr}}^d)T\ell m}_{\mu\nu}\right), \]

(A.25)

where each part corresponds to the case of that \( T^{(d)\mu\nu}_{L\ell m} \) in (A.24) is \( T^{(L)\mu\nu}_{L\ell m} \) or \( T^{(T)\mu\nu}_{L\ell m} \) and their degeneracies, respectively, are \( d_L(D, 2\tilde{\text{tr}}^L) \equiv d_L(D, 1L) \) and \( d_L(D, 2\tilde{\text{tr}}^T) \equiv d_L(D, 1T) \). Thus we have the same relation as (A.17);

\[ d_L(D, 2\tilde{\text{tr}}^d) = d_L(D, 2\tilde{\text{tr}}^L) + d_L(D, 2\tilde{\text{tr}}^T). \]

(A.26)

Lastly, the traceless and divergenceless symmetric tensor eigenfunctions \( T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m} \) corresponding to \( h^{\tilde{\text{tr}}^d}_{\mu\nu} \) in Eq. (2.8) obey

\[ \square T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m} = -r^{-2}\lambda_L(D, 2\tilde{\text{tr}}^d)T^{(\tilde{\text{tr}}^d)\mu\nu}_{L\ell m}, \]

\[ \lambda_L(D, 2\tilde{\text{tr}}^d) \equiv L(L+D-1)-2, \]

(A.27)
with the degeneracy

\[ d_L(D, 2\tilde{\text{rd}}) = \frac{(D+1)(D+2)}{2}d_L(D, 0) - d_{L-1}(D, 1d) - d_{L+1}(D, 1d) - d_{L-2}(D, 0) - 2d_L(D, 0) - d_{L+2}(D, 0), \text{ for } L = 2, 3, 4, \ldots. \]  

(A.28)

Furthermore, they can be composed of three parts according to Eqs. (4.11)\textasciitilde(4.14):

\[ T^{(\tilde{\text{rd}})L\ell m}_{\mu\nu} \equiv \left( T^{(\tilde{\text{rd}}L)L\ell m}_{\mu\nu}, T^{(\tilde{\text{rd}}T)L\ell m}_{\mu\nu}, T^{(T)L\ell m}_{\mu\nu} \right), \]  

(A.29)

where each part is defined by

\[ T^{(\tilde{\text{rd}}L)L\ell m}_{DD} = C_L(D, 2\tilde{\text{rd}}L)a^{-2}\phi^{L\ell m}, \]

\[ T^{(\tilde{\text{rd}}L)L\ell m}_{iD} = \tilde{\nabla}_i \tilde{j} - 1 a^{-(D-3)} \partial_D (aD^T \frac{T^{(\tilde{\text{rd}}L)L\ell m}}{DD}) + \frac{a^2}{D-1} \tilde{j} T^{(\tilde{\text{rd}}L)L\ell m}_{DD} - \frac{a^2}{D-1} \tilde{g}_{ij} T^{(\tilde{\text{rd}}L)L\ell m}_{DD}, \]

for \( \ell = 2, 3, 4, \ldots, L, \) and \( m = 1, 2, 3, \ldots, d_\ell(D, 1, 0), \)

with \( C_L(D, 2\tilde{\text{rd}}L) = \left[ \frac{r^4(D-2)\lambda_\ell(D-1, 0)(\lambda_\ell(D-1, 0) - (D-1))}{(D-1)\lambda_L(D, 0) + (D-2)} \right]^{1/2}, \)  

(A.30)

\[ T^{(\tilde{\text{rd}}T)L\ell m}_{\mu D} = C_L(D, 2\tilde{\text{rd}}T)a^{-1}T^{(T)L\ell m}_{\mu}, \]

\[ T^{(\tilde{\text{rd}}T)L\ell m}_{ij} = -\tilde{\nabla}_i (\tilde{j} + D - 2)^{-1}a^{-(D-3)} \partial_D (aD^{-(D-1)}T^{(\tilde{\text{rd}}T)L\ell m}_{iD}) - \tilde{\nabla}_j (\tilde{j} + D - 2)^{-1}a^{-(D-3)} \partial_D (aD^{-(D-1)}T^{(\tilde{\text{rd}}T)L\ell m}_{iD}), \]  

(A.31)

for \( \ell = 2, 3, 4, \ldots, L, \) and \( m = 1, 2, 3, \ldots, d_\ell(D-1, 1d), \)

with \( C_L(D, 2\tilde{\text{rd}}T) = \left[ \frac{r^2(\lambda_\ell(D-1, 1d) - (D-2))}{2(\lambda_\ell(D, 1T) + 1)} \right]^{1/2}, \)

and

\[ T^{(T)L\ell m}_{\mu D} = 0, \quad T^{(T)L\ell m}_{ij} = a^2 f^L \tilde{S}^{(\tilde{\text{rd}})\ell m}_{ij}, \]  

for \( \ell = 2, 3, 4, \ldots, L, \) and \( m = 1, 2, 3, \ldots, d_\ell(D-1, 2\tilde{\text{rd}}), \)  

(A.32)

where \( \tilde{S}^{(\tilde{\text{rd}})\ell m}_{ij} \) are traceless and divergenceless symmetric tensor eigenfunctions of
on the unit $S^{D-1}$. The degeneracies of their parts are

\[ d_L(D,2\tilde{t}rdL) \equiv \sum_{\ell=2}^{L} d_\ell(D-1,0) = d_L(D,0) - (D+1), \quad (A.33) \]

\[ d_L(D,2\tilde{t}rdT) \equiv \sum_{\ell=2}^{L} d_\ell(D-1,1d) \]

\[ = Dd_L(D,0) - d_{L-1}(D,1L) - d_{L+1}(D,1L) - \frac{D^2 - D + 2}{2}, \quad (A.34) \]

\[ d_L(D,2tT) \equiv \sum_{\ell=2}^{L} d_\ell(D-1,2\tilde{t}rd) \]

\[ = \frac{D(D+1)}{2}d_L(D,0) - d_{L-1}(D,1T) - d_{L+1}(D,1T) - d_{L-2}(D,0) \]

\[ - 2d_L(D,0) - d_{L+2}(D,0) + (D+1) + \frac{D^2 - D + 2}{2}, \quad (A.35) \]

and it is easily found to satisfy

\[ d_L(D,2\tilde{t}rd) = d_L(D,2\tilde{t}rL) + d_L(D,2\tilde{t}rT) + d_L(D,2tT). \quad (A.36) \]

Finally, we note that the following equations identically hold:

\[ \nabla_\mu \phi^0_{\ell m} = \nabla_\mu \nabla_\nu \phi^{1\ell m} + r^{-2} g_{\mu\nu} \phi^{1\ell m} = \nabla_\mu T^{(d)1\ell m}_{\nu} + \nabla_\nu T^{(d)1\ell m}_{\mu} = 0, \]

\[ \tilde{\nabla}_i \tilde{S}^{0m} = \tilde{\nabla}_j \tilde{S}^{1m} + \tilde{g}_{ij} \tilde{S}^{1m} = \tilde{\nabla}_i S^{(d)1m}_{j} + \tilde{\nabla}_j S^{(d)1m}_{i} = 0. \quad (A.37) \]
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