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Abstract. The Kerr nonlinearity of an optical-fibre Sagnac loop can be utilized to engineer a variety of two-photon quantum states. These include correlated, identical photon pairs as well as degenerate, maximally entangled states—both of which are used in quantum information processing. In fact, their underlying principle—the reverse Hong–Ou–Mandel effect—can also be applied to free-space, down-conversion-based analogues of either identical or entangled photon-pair sources. Due to their simple structure, such versatile devices are expected to find widespread applications in quantum-state engineering.

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1. Introduction

The optical-fibre Sagnac loop (OFSL) has enjoyed numerous applications since its introduction in 1976 [1], which have expanded from a rotation measuring instrument to a very versatile sensing tool. Such applications range from the fibre-loop mirror [2] to Sagnac-based intruder alarms, hydrophones, geophones and current measuring systems [3]. In classical nonlinear optics, the OFSL is a powerful tool in various all-optical operations. For example, an asymmetric (i.e. non-50:50 splitting ratio) OFSL [4]–[6] or an OFSL embedded with an amplifying medium [7]–[9] has demonstrated usefulness in optical switching and optical storage [10, 11], and a symmetric (i.e. 50:50 splitting ratio) OFSL has been shown to be useful for optical frequency conversion and phase conjugation with high pump suppression [12, 13]. In the continuous-variable quantum domain, soliton squeezing has been demonstrated using an asymmetric OFSL [14]–[16]. In this paper, we discuss the use of a symmetric OFSL as a discrete-variable quantum-state-engineering device. We specifically consider strategies for engineering sources of both identical photon pairs and polarization-entangled photon pairs using the OFSL. Because the fundamental principle underlying OFSL’s many quantum-state-engineering functionalities is the reverse Hong–Ou–Mandel (HOM) effect—a phenomenon which exists in both fibre optics and free-space optics—it is possible to extend these fibre-optic techniques to free-space down-conversion-based sources of identical and entangled photon pairs.

This paper is organized as follows. After briefly reviewing four-wave mixing (FWM), down-conversion and the concepts of identical and entangled photons, we introduce the deterministic quantum splitter (QS), a device based on a 50:50 OFSL. Its basic operating principle is explained first using an intuitive Feynman-path approach and then compared with a standard quantum-mechanical calculation. This analysis is then extended to a more general investigation of the forward and reverse HOM effect, particularly as it relates to polarization-entangled input states. Finally, we envision the application of the reverse HOM effect to engineer
both identical and polarization-entangled sources of photon pairs (along with a few other exotic states, such as the generation of the ‘noon’ state [17]).

1.1. FWM

The nonlinear process employed in an OFSL for quantum-state-engineering applications is called FWM, a third-order process mediated by the Kerr ($\chi^{(3)}$) nonlinearity of an optical fibre to produce correlated photon pairs. Lacking a $\chi^{(2)}$ nonlinearity due to its centrosymmetry, an optical fibre’s first appreciable nonlinearity is $\chi^{(3)}$, which, although weak in itself, can lead to greatly enhanced nonlinear interactions due to the fibre’s single spatial mode and long interaction length. Spontaneous Raman scattering, although widely used as a stimulated process in fibre-optic communications to make Raman fibre amplifiers, is the predominant source of noise in $\chi^{(3)}$ processes, producing uncorrelated photons which degrade the purity of any fibre-based correlated-photon source. Fortunately, this noise process can be suppressed by either cooling the fibre to a low temperature (e.g. 77 K in [18, 19]) to deplete the phonon bath, or pumping the fibre in its normal dispersion regime [20, 21] so that the phase-matched photon pairs reside outside the primary Raman band (peaked at 13 THz detuning from the pump). In FWM, a pair of correlated photons, commonly denoted as signal ($s$) and idler ($i$), are produced at the expense of two pump photons ($p_1$ and $p_2$). Energy and momentum conservations are obeyed during the FWM process:

$$\omega_{p_1} + \omega_{p_2} = \omega_s + \omega_i$$

$$\vec{k}_{p_1} + \vec{k}_{p_2} = \vec{k}_s + \vec{k}_i,$$

where $\omega_j$ and $\vec{k}_j$ represent the frequency and wave-vector of the $j$th photon ($j = p_1, p_2, s, i$). The FWM process can occur for either co-polarized or cross-polarized pump photons [22]:

$$x_{p_1}x_{p_2} \xrightarrow{\chi^{(3)}_{xxx}} x_sx_i, \quad \text{(co-polarized)},$$

$$x_{p_1}y_{p_2} \xrightarrow{\chi^{(3)}_{xyy}} x_sy_i, \quad \text{(cross-polarized)}.$$

Here, $x$ and $y$ denote orthogonal polarization states. Because co-polarized FWM gain is proportional to $(\chi^{(3)}_{xxx})^2$ and cross-polarized FWM gain is proportional to $(\chi^{(5)}_{xyy})^2 \equiv \frac{1}{8} (\chi^{(3)}_{xxx})^2$, the co-polarized process is much stronger than the cross-polarized process.

1.2. Spontaneous parametric down-conversion (SPDC)

SPDC is an analogous nonlinear process employed in some nonlinear bulk crystals (or waveguides) for the production of photon pairs, in this case mediated by the $\chi^{(2)}$ nonlinearity. Unlike FWM, the SPDC process involves the transformation of a single pump photon into a signal and idler photon pair. As above, this process conserves both energy and momentum:

$$\omega_p = \omega_s + \omega_i$$

$$\vec{k}_p = \vec{k}_s + \vec{k}_i.$$ Like FWM, SPDC can create both co-polarized (type-0, type-I) and cross-polarized (type-II) daughter photons:

$$x_p \xrightarrow{\chi^{(2)}_{xxx}} x_sx_i, \quad \text{(type-0, e.g. in quasi-phase-matched waveguides)},$$

$$x_p \xrightarrow{\chi^{(2)}_{xyy}} y_sy_i, \quad \text{(type-I)},$$

$$x_p \xrightarrow{\chi^{(2)}_{xyy}} x_sy_i, \quad \text{(type-II)}.$$

The terms signal (idler) photon and Stokes (anti-Stokes) photon are sometimes interchangeable.

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For the remainder of this paper, we restrict our discussion to type-I and type-II sources, though in principle all type-I designs could also be adapted to type-0 waveguides without significant changes.

1.3. Identical and entangled photon pairs

This manuscript will focus on two types of photon-pair sources: identical and entangled photon pairs. In linear optical quantum computing and many other quantum information protocols, daughter photons from photon-pair sources are required to be indistinguishable. In other words, the photon states must be identical in all degrees of freedom (spatial, temporal/frequency and polarization). The type of entangled photons that we shall consider in this manuscript are closely related to identical photon pairs, but they are entangled in polarization (such as the Bell state $\frac{1}{\sqrt{2}}(|\alpha\beta\rangle + |\alpha^\perp\beta^\perp\rangle)$) while being indistinguishable in all other degrees of freedom. We note that polarization-entangled but frequency-nondegenerate photons have also been extensively studied in the context of fibre-based photon sources [23]–[26], but they are outside the scope of this paper.

Traditionally, indistinguishable photons are generated from a $\chi^{(2)}$ correlated-photon source that relies on SPDC [27] with degenerate phase matching. For most applications it is preferred to have the indistinguishable photons deterministically separated from each other. In SPDC, this can be achieved using collinear type-II phase matching, in which case degenerate photons can be deterministically separated by means of their orthogonal polarization and afterwards made indistinguishable by rotating one photon’s polarization by 90° [28]. Alternatively, the degenerate photons can also be deterministically separated using non-collinear type-I phase matching, wherein the photon pairs are emitted into different spatial modes and are therefore easily separable [29]. However, the coupling of SPDC photons into single-mode optical fibres remains technically challenging, since these photons are naturally emitted into a large number of correlated spatial and spectral modes.

Fibre-based photon sources that rely on FWM provide an elegant solution to the aforementioned coupling problem, because all the correlated photons are born with the same single spatial mode supported by the standard single-mode optical fibre. The challenge is how to deterministically separate these identical photons, which are enforced by the fibre geometry to be necessarily collinear. One way is to utilize a cross-polarized dual-frequency pump to excite the $\chi_{xyxy}$ component of the nonlinear susceptibility tensor [30, 31], which produces cross-polarized degenerate photons and can thus be deterministically separated in a similar way to type-II collinear SPDC photons. As mentioned above, this process is generally much less efficient than the co-polarized FWM process because of the intrinsically weaker nature of $\chi_{xyxy}$ [22]. If, however, one decides to use co-polarized FWM to generate indistinguishable photons, one immediately realizes that the output photons are not readily separable in a deterministic way. The situation is analogous to a collinear type-I SPDC configuration [32]. In the past, researchers have simply used a regular 50:50 non-polarizing beam splitter (BS) to probabilistically separate the identical photons [32]–[34]. The input and output states are shown below (refer to figure 1):

$$|\Psi\rangle_{in} = |2\rangle_d |0\rangle_b ,$$

$$|\Psi\rangle_{out} = \frac{1}{\sqrt{2}} \left( \frac{|2\rangle_d |0\rangle_c - |0\rangle_d |2\rangle_c}{\sqrt{2}} \right) + \frac{i}{\sqrt{2}} \left( \frac{|1\rangle_c |1\rangle_d}{\psi_{1002}} \right) .$$
Note that although the $\Psi_{2002}$ component does not contribute in a coincidence-counting measurement, its presence limits the usefulness of such a probabilistic correlated-photon source. For instance, as shown in [32], the HOM-dip visibility attainable with such a source would be limited to only 50%, making it unattractive for practical quantum information applications. On the other hand, if the output wavefunction $|\Psi_{\text{out}}\rangle$ only consists of the $\Psi_{11}$ component (i.e. deterministically separated indistinguishable photons), the HOM-dip visibility can in principle reach 100%.

We have described in our previous publications [36, 37], mainly from an experimental viewpoint, how this obstacle can be circumvented by using a new type of co-polarized identical-photon source, namely the QS. Demonstrations of a near-unity HOM dip visibility [36] and a telecom-band quantum controlled-\textit{not} gate [37] have shown the practical usability of the QS. In this paper, we focus on quantum-state engineering with the QS. But first we show how to account for the operating principle of the QS theoretically, using an intuitive Feynman-path approach followed by a standard quantum mechanical calculation.

2. The nonlinear OFSL

The OFSL is well-known for its role as a total reflector (TR) [2] or a total transmitter (TT). Here we explore its use as a QS, essentially an OFSL set to its previously unexplored 50:50 state in which the OFSL equally transmits and reflects light. The difference between the TT, TR and QS configurations of an OFSL lies with different settings of the intraloop fibre polarization controller (FPC), which effectively results in different relative phase shifts between the clockwise (CW) and the counter-clockwise (CCW) paths. The situation is illustrated in figure 2, which explains the classical interference between two equally-split pump pulses, as well as the quantum interference between the FWM photon-pair amplitudes copropagating with each pump pulse. As shown in figure 2, the pump is injected from port $a$ into the Sagnac loop, which is composed of a 50:50 non-polarizing fibre BS, a piece of dispersion-shifted fibre (DSF) and an FPC. The peak pump power is denoted by $P$, which is then equally split into two equally-powered pulses ($P/2$) by the BS. The two pump pulses traverse the DSF in a counter-propagating manner, each of which probabilistically scatters co-polarized FWM photon pairs, denoted pictorially by signal and idler in figure 2. The probability of generating FWM photon pairs that are orthogonally polarized to the pump (i.e. excitation of the nonlinear susceptibility tensor component $\chi_{xxyy}$) is neglected due to its smallness. We also neglect the probability that
Figure 2. Illustration of quantum interference in a Sagnac loop between photon pairs with different phase shifts, together with the corresponding classical interference between pump pulses. CW, clockwise; CCW, counter-clockwise; BS, beam splitter; FPC, fibre polarization controller; $\phi_{\text{CW}}$, classical phase of CW pulse; $\phi_{\text{CCW}}$, classical phase of CCW pulse.

both pump pulses undergo FWM scattering, as this corresponds to a higher-order process of multi-photon generation, whose probability is vanishingly small when the pump power is low.

We start off by showing that the classical counter-propagating fields in the Sagnac loop can achieve a directional phase difference$^3$ while remaining co-polarized after each has traversed the entire loop (before being recombined at the BS). Interested readers should refer to [2] for a formal mathematical treatment of the Sagnac loop. Here, we follow a simplified schematic shown in figure 3. The Sagnac loop (denoted as ‘Fibre’ in figure 3) is pictorially straightened out, and the birefringence effects of the fibre and the FPC are modelled by a half-wave plate ($\lambda/2$) and a quarter-wave plate ($\lambda/4$). For simplicity, we further assume that the fibre performs no polarization rotation; $\lambda/2$ is set to 45°, rotating H (horizontal polarization) to V (vertical polarization) and V to H; $\lambda/4$ is set to 0°, leaving H and V unchanged except for a relative phase. For horizontally polarized CW and CCW light, the following evolution takes place:

\[
\text{CW: } |H\rangle \xrightarrow{\text{fibre}} |H\rangle \xrightarrow{\lambda/2} |V\rangle \xrightarrow{\lambda/4} e^{i\pi/2} |V\rangle,
\]

\[
\text{CCW: } |H\rangle \xrightarrow{\lambda/4} |H\rangle \xrightarrow{\lambda/2} |V\rangle \xrightarrow{\text{fibre}} |V\rangle,
\]

where only relative phase shifts are recorded above. Note there is a directional phase difference between the CW and CCW fields (which in this case is $\pi/2$), while their final polarization states are still the same. In a more general scenario, one can substitute any fractional waveplate for the $\lambda/4$-plate in figure 3, which results in a continuously tunable directional phase difference. We note that in practice careful alignment techniques have to be applied in order to ensure copolarization of the counter-propagating light fields before their recombination at the BS.

We are now ready to come back to figure 2 and explain its working principle. Note that $\phi_{\text{CW}}$ and $\phi_{\text{CCW}}$ are symbols that we use to track the classical phases of the CW and CCW pump pulses, respectively. To be consistent, the $\pi/2$ phase shift gained upon reflection from the BS has already been included in $\phi_{\text{CW}}$. We first consider the conventional FWM case, namely, degenerate pumps ($p_1 = p_2$) and non-degenerate signal/idler ($s \neq i$). A similar analysis can be applied to the reverse degenerate FWM case, i.e. non-degenerate pumps ($p_1 \neq p_2$) and degenerate signal/idler ($s = i$). We have already shown that, the counter-propagating pumps

$^3$ This phase may be related to the well-known Berry’s phase in quantum mechanics, which is a purely topological phase factor arising from the adiabatic transport of a system around a closed loop.

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arriving at the BS before being recombined can be made co-polarized with a variable phase difference \( \Delta \phi \equiv \phi_{\text{CCW}} - \phi_{\text{CW}} \). The corresponding photon-pair states \( |s\rangle |i\rangle \) are co-polarized with each pump, and have a relative phase difference of \( \delta = 2\Delta \phi \). Using the terminology shown in figure 2, we arrive at the following Feynman-path summations of indistinguishable signal/idler photon scattering outcomes for three distinct operational modes of the OFSL (TR, TT, 50 : 50/QS):

(i) **Totally reflective (TR)**
- Phase tracking: \( \Delta \phi = \frac{\pi}{2} \); \( \delta = \pi \).
- Quantum amplitudes:
  \[
  |\text{CW}\rangle + e^{i\delta}|\text{CCW}\rangle = \frac{1}{\sqrt{2}}(|s\rangle_a |i\rangle_a - |s\rangle_b |i\rangle_b)
  \]
  \[
  |\text{Total}\rangle = 2 |s\rangle_a |i\rangle_a
  \]
- Normalized output state:
  \[
  |\Psi\rangle_{\text{TR}} = \frac{1}{\sqrt{2}}(|s\rangle_a |i\rangle_a - |s\rangle_b |i\rangle_b)
  \]

(ii) **Totally transmissive (TT)**
- Phase tracking: \( \Delta \phi = -\frac{\pi}{2} \); \( \delta = -\pi \).
- Quantum amplitudes:
  \[
  |\text{CW}\rangle + e^{i\delta}|\text{CCW}\rangle = 2 |s\rangle_a |i\rangle_a
  \]
  \[
  |\text{Total}\rangle = |s\rangle_a |i\rangle_a
  \]

This factor of 2 is due to the fact that two pump photons are consumed to produce signal/idler photon pairs at some random point during propagation in the fibre, so the daughter photons bear the phase information of the two parent photons at the birth instant. During subsequent propagation, the daughter photons accumulate exactly twice the phase of one photon in the classical pump field due to the energy-conservation relationship that links them together.
• Normalized output state:

\[ |\Psi_{TT}\rangle = \frac{1}{\sqrt{2}} (|s\rangle_a |i\rangle_a - |s\rangle_b |i\rangle_b) ; \]  

\[ (5) \]

(iii) Equally transmissive and reflective (50 : 50)

• Phase tracking: \( \Delta \phi = 0; \delta = 0. \)

• Quantum amplitudes:

\[
\begin{align*}
|\text{CW}\rangle &+ e^{i \delta} |\text{CCW}\rangle = |\text{Total}\rangle \\
|s\rangle_a |i\rangle_a &- |s\rangle_a |i\rangle_a & 0 \\
i |s\rangle_a |i\rangle_b & 1 |s\rangle_a |i\rangle_b & 2i |s\rangle_a |i\rangle_b \\
i |s\rangle_b |i\rangle_a & i |s\rangle_b |i\rangle_a & 2i |s\rangle_b |i\rangle_a \\
|s\rangle_b |i\rangle_b & |s\rangle_b |i\rangle_b & 0 \\
\end{align*}
\]

• Normalized output state:

\[ |\Psi_{QS}\rangle = \frac{1}{\sqrt{2}} (|s\rangle_a |i\rangle_b + |s\rangle_b |i\rangle_a) . \]  

\[ (6) \]

The subscripts \( a \) and \( b \) label the spatial modes of the output photon, and all the output wavefunctions are normalized with their global phases ignored. From equations (4) and (5), we can see that the photon pairs bunch together when the Sagnac loop is operated as a TR/TT for the pump. And from equation (6), the photon pairs split up when the Sagnac loop functions as a 50 : 50 mirror for the pump. The above analysis, when applied in a similar fashion to the reverse degenerate FWM case, leads to the discovery of the QS source. The results are exactly the same as before, with the substitution of \( s = i \) and the reinterpretation of \( \Delta \phi \) as the counter-propagating phase difference for a classical optical field at the degenerate signal/idler frequency\(^5\).

3. The reverse HOM effect

A convenient way to summarize the above results is to use the quantum mechanical description of a 50 : 50 BS (two input modes and two output modes) with an arbitrary relative phase between the two inputs. This derivation, detailed below, reproduces the same phenomena as in a recent time-reversed HOM experiment [36], an analogy which will be particularly useful when this derivation is extended to Bell-state outputs. In addition, the derivations below apply equally well to FWM or SPDC photon pairs, and will therefore be applicable to both fibre-based and free-space photon-pair sources. We remark that the reverse HOM effect is also relevant in some other contexts, for instance in number-path entanglement [38] and for creating an anti-bunched light beam [39].

3.1. Producing separable photon pairs through the reverse HOM effect

As shown in figure 4, the input modes are labelled \( a \) and \( b \), while the output modes are labelled \( c \) and \( d \) in consistency with figure 1 (note that the output modes are labelled differently in figure 2). As before, we start with the case of a non-degenerate signal/idler photon pair

\(^5\) In the reverse degenerate FWM case, the two pump pulses are non-degenerate. Each pump accumulates a phase during propagation, which when added results in twice the classical phase of their middle-frequency field. So in this case, \( \delta = \Delta \phi_{p1,p2} = 2 \Delta \phi \), with the reinterpretation of \( \Delta \phi \) stated in the text.
Figure 4. General quantum mechanical treatment of photon scattering statistics with wavefunctions generated internally of an OFSL at a 50:50 BS. (a) Non-degenerate signal/idler case; and (b) degenerate signal/idler case.

(figure 4(a)), the end result of which can be easily adapted to the degenerate case (figure 4(b)). The input state in figure 4(a) is written as

$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}} \left( a^\dagger_s a^\dagger_i + e^{i\delta} b^\dagger_s b^\dagger_i \right) |0\rangle,$$

(7)

where $a^\dagger_s$ ($a^\dagger_i$) is the creation operator for the signal (idler) photons in the $a$-mode, and likewise $b^\dagger_s$ ($b^\dagger_i$) is the creation operator for the signal (idler) photons in the $b$-mode. The 50:50 BS's action on these operators are

$$a^\dagger \rightarrow \frac{1}{\sqrt{2}} (-ic^\dagger + d^\dagger),$$

$$b^\dagger \rightarrow \frac{1}{\sqrt{2}} (-id^\dagger + c^\dagger),$$

(8)

where the $\pi/2$ phase shift upon reflection from a BS is represented by the additional $i$ in front of the reflected operators. Plugging equation (8) into equation (7), we immediately obtain

$$|\Psi_{\text{out}}\rangle = \frac{1 - e^{i\delta}}{2} \frac{d^\dagger_s c^\dagger_i - c^\dagger_s e^\dagger_i}{\sqrt{2}} |0\rangle - \frac{i}{2} \frac{(1 + e^{i\delta}) c^\dagger_i a^\dagger + d^\dagger_i c^\dagger_i}{\sqrt{2}} |0\rangle.$$

(9)

For the degenerate signal/idler case shown in figure 4(b), equations (7) and (9) are rewritten as:

$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}} \left( |2\rangle_a |0\rangle_b + e^{i\delta} |0\rangle_a |2\rangle_b \right),$$

(10)

$$|\Psi_{\text{out}}\rangle = \frac{1 - e^{i\delta}}{2} \frac{|2\rangle_d |0\rangle_c - |0\rangle_d |2\rangle_c}{\sqrt{2}} - \frac{i}{2} \frac{(1 + e^{i\delta}) |1\rangle_c |1\rangle_d}{\sqrt{2} \psi_{2002}}.$$

(11)

One can verify the agreement between the above two approaches by setting $\delta$ to $\pi$, $-\pi$, and 0 in equations (9) and (11), which correspond to the three operational modes of the OFSL—TR, TT and 50:50, respectively. Therefore, as emphasized in [36, 37], the 50:50 OFSL, or QS, can indeed be interpreted as a manifestation of time-reversed HOM interference.
3.2. Non-ideal BSs and the reverse HOM effect

An experimentally relevant question is: how would a non-ideal 50 : 50 BS (i.e. $R + T = 1$, $R \neq T$, where $R$ ($T$) stands for intensity reflectivity (transmissivity)), or a non-optimal alignment of the 50 : 50 OFSL (i.e. $\delta \neq 0$), affect the output wavefunction. Detailed calculations can be done by writing, in analogy to equation (7), the input state in the non-ideal case as

$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{T^2 + R^2}} (T a_x^\dagger a_i^\dagger + Re^{i\delta} b_x^\dagger b_i^\dagger)|0\rangle.$$  \hspace{1cm} (12)

In contrast to the case shown in figure 2, we note that here the input pump power is split into two unequal parts, with $P_{\text{CW}} = T P$ and $P_{\text{CCW}} = R P$ (neglecting loss of the BS). The two-photon state generated by each pump has a coefficient proportional to the pump power $\frac{40}{4}$, hence the different coefficients in front of the $a$-mode (CW) and $b$-mode (CCW) creation operators. Here we further assume that

$$T = \frac{1}{2} + \alpha,$$

$$R = \frac{1}{2} - \alpha,$$  \hspace{1cm} (13)

where $\alpha$ represents a small deviation from an ideal 50 : 50 BS with $|\alpha| \ll 1$.

The BS transformation on the input operators can be written in analogy to equation (8) as

$$a^\dagger \rightarrow -i\sqrt{R} c^\dagger + \sqrt{T} d^\dagger,$$

$$b^\dagger \rightarrow -i\sqrt{R} d^\dagger + \sqrt{T} c^\dagger.$$  \hspace{1cm} (14)

Plugging equation (14) into equation (12), we obtain

$$|\Psi_{\text{out}}\rangle = \frac{1}{\sqrt{T^2 + R^2}} \left[ (T^2 - R^2 e^{i\delta}) d_i^\dagger d_j^\dagger - RT (1 - e^{i\delta}) c_j^\dagger c_i^\dagger \right.$$

$$\left. - i\sqrt{RT} (T + Re^{i\delta}) c_j^\dagger d_i^\dagger - i\sqrt{RT} (T + Re^{i\delta}) d_j^\dagger c_i^\dagger \right]|0\rangle.$$  \hspace{1cm} (15)

The fidelity $\mathcal{F}$, defined as the inner product between the output state given by equation (15) and the one from the ideal 50 : 50-BS case (i.e. $|\Psi\rangle_{\text{QS}} = \frac{c_i^\dagger d_i^\dagger + d_i^\dagger c_i^\dagger}{\sqrt{2}}|0\rangle$), which is the same as equation (6) with appropriate changes in the mode symbols), is given by

$$\mathcal{F} = \langle \Psi_{\text{out}} | \Psi \rangle_{\text{QS}}^2$$

$$= \frac{2RT}{T^2 + R^2} \left| T + Re^{i\delta} \right|^2$$

$$= \frac{1 - 4\alpha^2}{1 + 4\alpha^2} \left[ \cos^2 \left( \frac{\delta}{2} \right) + 4\alpha^2 \sin^2 \left( \frac{\delta}{2} \right) \right],$$  \hspace{1cm} (16)

where in the last step we have substituted in equation (13) for the $T$ and $R$. The first-order approximation of equation (16) can be easily derived. Namely, when $\alpha^2 \ll 1$ and $\delta^2 \ll 1$, we have $\mathcal{F} \approx 1 - 8\alpha^2 - \delta^2/4$, which shows the robustness of the output wavefunction against small perturbations in $\alpha$ and $\delta$. Note that $\mathcal{F} = 1$ when $\alpha = \delta = 0$, as expected.
3.3. Entangled states and the reverse HOM effect

Above, we derived the behaviour of the QS by modelling the behaviour of an appropriate input state on a BS. The input state shown in equation (7) corresponds to the output state of a pair of identical photons incident on a BS, followed by a phase shift in one arm; in other words, the output state of a HOM experiment. Whereas the standard reverse HOM experiment uses the following state as an input:

\[ |\Psi\rangle_{in} = \frac{1}{\sqrt{2}} (a^\dagger a^\dagger + e^{i\delta} b^\dagger b^\dagger) |0\rangle, \] (17)

one can envision instead inputting the polarization-entangled Bell states on a HOM BS. Using the time-reversed BS transformations

\[ c^\dagger \rightarrow \frac{1}{\sqrt{2}} (ia^\dagger + b^\dagger), \] (18)

\[ d^\dagger \rightarrow \frac{1}{\sqrt{2}} (ib^\dagger + a^\dagger), \]

we can predict the following behaviour for the input Bell states \( \frac{1}{\sqrt{2}} (|HH\rangle \pm |VV\rangle) \) and \( \frac{1}{\sqrt{2}} (|HV\rangle \pm |VH\rangle) \):

\[ \frac{1}{\sqrt{2}} (e_H^\dagger d_H^\dagger + e_V^\dagger d_V^\dagger) \rightarrow i \frac{1}{2\sqrt{2}} (a_H^\dagger a_H^\dagger + a_V^\dagger a_V^\dagger + b_H^\dagger b_H^\dagger + b_V^\dagger b_V^\dagger), \] (19)

\[ \frac{1}{\sqrt{2}} (e_H^\dagger d_V^\dagger - e_V^\dagger d_H^\dagger) \rightarrow i \frac{1}{2\sqrt{2}} (a_H^\dagger a_V^\dagger - a_V^\dagger a_H^\dagger + b_H^\dagger b_V^\dagger - b_V^\dagger b_H^\dagger), \] (20)

\[ \frac{1}{\sqrt{2}} (e_H^\dagger d_V^\dagger + e_V^\dagger d_H^\dagger) \rightarrow i \frac{1}{\sqrt{2}} (a_H^\dagger a_V^\dagger - b_H^\dagger b_V^\dagger), \] (21)

\[ \frac{1}{\sqrt{2}} (e_H^\dagger d_V^\dagger - e_V^\dagger d_H^\dagger) \rightarrow i \frac{1}{\sqrt{2}} (-a_H^\dagger b_V^\dagger + a_V^\dagger b_H^\dagger). \] (22)

Because these transformations must be time-reversible, they provide a recipe for deterministically separating photons into polarization entangled states. (Note that the singlet state remains a singlet state even after passing through the BS, and therefore cannot be directly engineered using this method.) By adding a phase shift onto one mode, it is also possible to predict how phase errors (or intentional phase shifts) will affect this deterministic splitting. We note in passing that these equations also appear in the context of a Bell-state analyser (see, for example, table 1 in [41]).

4. Sources of identical photon pairs

We now turn our attention to various potential applications of the OFSL in quantum state engineering. Spatially-separated identical photon pairs are generated when we combine a 50 : 50 OFSL with reverse degenerate FWM [36, 37]. The structure of such a device is shown in figure 5. A circulator is added to the OFSL’s input port to collect the reflected photons from the Sagnac loop. The optical bandpass filters (OBPF) have identical passbands centred at \( \lambda_m \),
the mid-frequency wavelength of the two pump wavelengths ($\lambda_{p1}$ and $\lambda_{p2}$). Pure $\Psi_{11}$ state is expected to emerge from the output ports of this device at the degenerate wavelength $\lambda_m$.

Abundant quantum-state-engineering applications exist for the OFSL source, when we consider various pump inputs in conjunction with various operational modes of the Sagnac loop. We summarize all envisioned applications of the OFSL in a compact matrix shown in figure 6, which also includes the quantum functionalities of a straight piece of fibre for comparison. The rows of the matrix correspond to ‘straight fibre’, ‘Sagnac loop in TT/TR mode’, and ‘Sagnac loop in 50:50 mode’, respectively, whereas the columns of the matrix correspond to ‘single-frequency pump’, ‘dual-frequency co-polarized pump’, and ‘dual-frequency cross-polarized pump’, respectively. The working principles for each matrix element can be worked out in a straightforward way following the QS example. It is worth noting that in addition to the other deterministically split states, a certain kind of noon state [17] (here $N = 2$) can also be generated (matrix element {2, 2}). Furthermore, the versatility of the OFSL source allows it to be easily configured to perform experiments done in [42] with its matrix element {2, 1}, and experiments in [43] with its matrix element {3, 1}. The matrix elements {2, 3} and {3, 3} turn out to be functionally equivalent and are therefore absorbed into one entry, dubbed the ‘quantum buncher’. This device outputs pairs of orthogonally-polarized signal and idler photons into a superposition of both propagating in mode $a$ or both propagating in mode $b$.

By extending the same principles to SPDC, it is also possible to create sources of identical photons in free space. Although the pump accumulates phase at only half of the rate of a FWM pump (because only one photon at a time down-converts, but two pump photons are required for FWM), it is also approximately half of the wavelength of the daughter photons. These two factors balance, and although individual dispersive effects must be taken into account for specific implementations, in general the previous analysis can be directly extended to the $\chi^{(2)}$ case. Figure 7 is a diagram of the OFSL analogue for SPDC. Waveplates replace the FPC from the OFSL, allowing easy tuning of the phase difference $\delta$.
Figure 6. Summary of the OFSL source together with straight-fibre propagation, in conjunction with different pump inputs and different operational modes of the Sagnac loop. TR, totally reflective; TT, totally transmissive; 50:50, equally transmissive and reflective; NOON state stands for states of the generic form of $|N, 0\rangle \pm |0, N\rangle$ where $N$ is an integer greater than or equal to 2.

5. Sources of entangled photon pairs

A spatially-separated polarization-entangled identical photon-pair source can also be realized by utilizing the configuration shown in figure 5, but with a twist: polarization multiplexes two such processes and makes them indistinguishable from each other. Three operationally equivalent designs of such a source are shown in figure 8. All designs utilize dual-frequency orthogonally-polarized input pumps (H-pump and V-pump). In the first case, figure 8(a), these two pumps are temporally separated before the Sagnac loop, and are launched into the 50:50 OFSL from the same input port. In the second case, figure 8(b), a single diagonally-polarized dual-frequency input pump is temporally separated into two orthogonally-polarized components by polarization maintaining (PM) fibres (or some other polarization dependent time-delay device) inside the OFSL. In the third case, figure 8(c), a single diagonally-polarized dual-frequency pump is split by a polarizing BS (PBS) into two orthogonally-polarized, time-delayed pumps, which are launched into opposite ends of the 50:50 OFSL. All cases utilize a fixed amount of time.
Figure 7. SPDC-based sources of identical photon pairs. (a) For completeness, we show the trivial type-II case, wherein a simple polarizing BS separates the orthogonally polarized down-converted photons. (b) The OFSL analogue for the down-conversion sources, using a type-I down-conversion process. Waveplates provide the phase difference $\delta$, allowing one to tune the source to a QS.

delay $\tau$ between the two pump pulses. Each pump independently and probabilistically scatters spatially-separated identical photons in its time epoch that are co-polarized with it. Polarization entanglement of the form $|H_s H_i\rangle + |V_s V_i\rangle$ is generated for signal/idler photons at the degenerate middle frequency (wavelength $\lambda_m$) when all the distinguishing timing information between the two processes is erased. In figure 8(a), the reflected photon is picked up by a circulator, and two pieces of PM fibre with suitable lengths are introduced, one in each photon’s path, to precisely remove the delay $\tau$ between the FWM processes driven by the H-pump and the V-pump. In figure 8(b), the distinguishing temporal information is erased by the exact same PM fibres which separated the counter-propagating pumps into two components each. In order to remove this time delay, rather than exacerbate it, a $90^\circ$ rotation is necessary between the fast and slow axes of the two lengths of PM fibre. This second model utilizes the reverse HOM effect for entangled states (cf equation (19)), allowing us to generate a deterministically-separated polarization-entangled state using the OFSL. Figure 8(c) bears some resemblance to the double-loop scheme introduced in [44]. The apparent difference between the two is the operational mode of the OFSL—50:50 in the former and totally reflective in the latter. As in the double-loop scheme, FPC1 and FPC2 are configured to restore the polarization states of both OFSL-transmitted and reflected photons to their original values [44]. The identical-photon amplitudes following the common path (i.e. both transmitted by the OFSL) come out of the PBS’s O-port with no relative time delay. Their twins (i.e. both reflected by the OFSL) out of the PBS’s I-port are displaced relatively in time and are picked up by a circulator. A piece of PM fibre with a suitable length is inserted to remove the relative time delay between them, which is $2\tau$ in this case. Finally, all three schemes in figure 8 should be contrasted with that from [33], in which such a spatially-separated polarization-entangled state is generated 50% of the time.

The reverse HOM principle, which allows the second fibre-based entangled photon source to operate, can be extended to both type-I and type-II SPDC sources of entangled photons, as
Figure 8. Fibre-based sources of entangled photon pairs. All three sources utilize dual-frequency pumps and a 50:50 Sagnac loop. (a) Orthogonal polarization components of the pump are prepared with temporal separation $\tau$, and orthogonally polarized FWM-generated photon pairs are temporally realigned using lengths of PM fibre. (b) Sections of PM fibre with orthogonal fast and slow axes inside the OFSL both separate the pump polarizations in time, and temporally realign the FWM photons. (c) A setup resembling the double-loop scheme in [44] is also capable of generating deterministically-separated entangled photons.

shown in figure 9. Both of these sources rely on the entangled reverse HOM effect to transform a superposition of CW- and CCW-propagating probability amplitudes into a deterministically-split pair of entangled photons. When constructing these sources, care must be taken to control

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Figure 9. Entanglement source designs using free-space SPDC. (a) Here, both CW and CCW propagating pump pulses create amplitudes for pairs of horizontally polarized photons in crystal 1 and pairs of vertically polarized photons in crystal 2. This arrangement leads to the following unnormalized superposition of creation operators: $H_{CW}H_{CW} + H_{CCW}H_{CCW} + V_{CW}V_{CW} + V_{CCW}V_{CCW}$, which after the 50 : 50 BS results in the maximally entangled output state $\frac{1}{\sqrt{2}} (|H_A H_B\rangle + |V_A V_B\rangle)$. Dichroic mirrors separate the pump from the entangled photon pairs. (b) Using the same principle, a single type-II crystal can create a superposition of $H_{CW}V_{CW} + H_{CCW}V_{CCW}$, which after the 50 : 50 BS results in the maximally entangled state $\frac{1}{\sqrt{2}} (|H_A V_B\rangle + |V_A H_B\rangle)$.

In both cases, an additional phase (e.g. from a dispersive optic (not shown)) introduced between the CW and CCW SPDC amplitudes will change these devices from a QS into a quantum buncher, and vice versa.
in figure 9)) should be inserted inside the Sagnac loop to tune between the QS and the quantum
buncher configurations.

We note that other free-space Sagnac-loop designs have been used to generate entangled
photon pairs [45, 46]. In [45], a type-I down-converter embedded in a free-space Sagnac loop is
used; however, the Sagnac loop is not operated in the QS mode and the setup only post-selects
polarization entanglement. In comparison, in figure 9(a) we use two type-I down-converters
within a QS, and thus create genuine polarization entanglement (i.e. without any post-selection).
In [46], a polarization Sagnac loop (rather than a regular Sagnac loop) is used together with
a type-II down-converter. We believe the scheme depicted in figure 9(b) offers a practical
alternative with comparable simplicity and robustness to the scheme demonstrated in [46].

6. Conclusion

The nonlinear optical Sagnac loop is a versatile device, usable for a number of quantum
information applications. Here, we have presented designs for both fibre-based and free-space
sources of identical and entangled photons. These new sources, in addition to being immediately
useful for quantum-state-engineering applications, show the power and versatility of a new tool
for experimental quantum information.

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References

[1] Vali V and Shorthill R W 1976 Appl. Opt. 15 1099
[2] Mortimore D B 1988 J. Lightwave Technol. 6 1217
[3] Culshaw B 2006 Meas. Sci. Technol. 17 R1–R16
[4] Doran N J and Wood D 1988 Opt. Lett. 13 56
[5] Islam M N, Sunderman E R, Stolen R H, Pleibel W and Simpson J R 1989 Opt. Lett. 14 811
[6] Blow K J, Doran N J and Nelson B P 1990 Electron. Lett. 26 962
[7] Ferreman M E, Haberl F, Hofer M and Hochreiter H 1990 Opt. Lett. 15 752
[8] Eiselt M 1992 Electron. Lett. 28 1505
[9] Sokoloff J P, Prucnal P R, Glesk I and Kane M 1993 IEEE Photonics Technol. Lett. 5 787
[10] Wang L, Agarwal A, Su Y and Kumar P 2002 IEEE J. Quantum Electron. 38 614
[11] Agarwal A, Wang L, Su Y and Kumar P 2005 J. Lightwave Technol. 23 2229
[12] Swanson E A and Moores J D 1994 IEEE Photonics Technol. Lett. 6 1341
[13] Mori K, Morioka T and Saruwatari M 1995 Opt. Lett. 20 1424
[14] Krylov D and Bergman K 1998 Opt. Lett. 23 1390
[15] Schmitt S, Ficker J, Wolff M, König F, Sizmann A and Leuchs G 1998 Phys. Rev. Lett. 81 2446
[16] Levandovsky D, Vasilyev M and Kumar P 1999 Opt. Lett. 24 89
[17] Lee H, Kok P and Dowling J P 2002 J. Mod. Opt. 49 2325
[18] Takesue H and Inoue K 2005 Opt. Express 13 7832
[19] Lee K F, Chen J, Liang C, Li X, Voss P L and Kumar P 2006 Opt. Lett. 31 1905
[20] Alibart O, Fulconis J, Wong G K L, Murdoch S G, Wadsworth W J and Rarity J G 2006 New J. Phys. 8 67
[21] Fan J, Migdall A and Wang L J 2005 Opt. Lett. 30 3368
[22] Govind P and Agrawal 2001 Nonlinear Fiber Optics 3rd edn (New York: Academic)
[23] Takesue H and Inoue K 2004 Phys. Rev. A 70 031802
[24] Li X, Voss P L, Sharping J E and Kumar P 2005 Phys. Rev. Lett. 94 053601
[25] Fan J, Eisaman M D and Migdall A 2007 Phys. Rev. A 76 043836
[26] Fulconis J, Alibart O, O’Brien J L, Wadsworth W J and Rarity J G 2007 Phys. Rev. Lett. 99 120501
[27] Special issue, Technologies for quantum communication 2001 J. Mod. Opt. 48 (13)
[28] Kwiat P G, Mattle K, Weinfurter H, Zeilinger A, Sergienko A V and Shih Y 1995 Phys. Rev. Lett. 75 4337
[29] Kwiat P G, Waks E, White A G, Appelbaum I and Eberhard P H 1997 Phys. Rev. A 60 R773
[30] Fan J and Migdall A 2005 Opt. Express 13 5777
[31] Chen J, Lee K F, Liang C and Kumar P 2006 presented at Frontiers in Optics/Laser Science XXII, paper FTuR4
[32] Halder M, Tanzilli S, de Riedmatten H, Beveratos A, Zbinden H and Gisin N 2005 Phys. Rev. A 71 042335
[33] Chen J, Lee K F, Liang C and Kumar P 2006 Opt. Lett. 31 2798
[34] Fan J, Dogariu A and Wang L J 2005 Opt. Lett. 30 1530
[35] Hong C K, Ou Z Y and Mandel L 1987 Phys. Rev. Lett. 59 2044
[36] Chen J, Lee K F and Kumar P 2007 Phys. Rev. A 76 032318
[37] Fan J, Lee K F and Kumar P 2005 Quantum information processing with optical fibers, presented at CLEO/QELS, paper QTuJ1
[38] Rarity J G, Tapster P R, Jakeman E, Larchuk T, Campos R A, Teich M C and Saleh B E A 1990 Phys. Rev. Lett. 65 1348
[39] Pittman T B, Franson J D and Jacobs B C 2007 New J. Phys. 9 195
[40] Chen J, Lee K F, Li X and Kumar P 2005 Phys. Rev. A 72 033801
[41] Mattle K, Weinfurter H, Kwiat P G and Zeilinger A 1996 Phys. Rev. Lett. 76 4656
[42] Kumar P, Lee K F, Chen J, Li X and Voss P L 2005 Quantum information processing with optical fibers, presented at CLEO/QELS, paper QTuJ1
[43] Kim H, Ko J and Kim T 2003 Phys. Rev. A 67 054102
[44] Chen J, Lee K F, Li X, Voss P L and Kumar P 2007 New J. Phys. 9 289
[45] Shi B and Tomita A 2004 Phys. Rev. A 69 013803
[46] Kuzucu O and Wong F N C 2008 Phys. Rev. A 77 032314

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