An Improved Quasi-Resonant Controller for Grid-Connected Inverter with LCL Filter

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Abstract. Aiming at the problem of insufficient response speed of traditional quasi-resonant controllers in the application of grid-connected inverters with LCL filter, a novel hybrid controller which combines a quasi-proportional resonant controller and an integrator is proposed in this paper. Based on this controller, the stability, the parameter design and the digital time delay compensation of the system are analyzed and designed in detail. Finally, its effectiveness is experimentally verified at Matlab/Simulink. By comparing with the traditional control method, it leads to a conclusion that the novel hybrid controller can maintain the control accuracy while allowing the system have good dynamic performance.

1. Introduction
The grid-connection technology of distributed power is an important way to utilize the electricity generated by renewable energy[1-2]. A qualified grid-connected inverter requires that the output voltage and current can track the changes in the frequency and phase of the grid voltage and current in real time, so as to achieve the purpose of attenuating the harmonic current that is injured into the grid. Commonly used grid-connected current controllers include: hysteresis-band current tracking controller, deadbeat controller, proportional integral(PI) controller, quasi-proportional resonance(QPR) controller, etc. The hysteresis-band current tracking controller has a rather fast response speed, however, the unfixed switching frequency result in a series of relatively high design requirements for the post-stage filter[3]. The traditional deadbeat controller can effectively improve the control accuracy of the system and has the advantages of rapid response, but the digital delay of the system limits its application. Literature[4] uses a fast deadbeat algorithm with a fixed switching frequency to predict and control the grid-connected current, and this algorithm can effectively compensate the system delay, but there is still a limitation of relying on accurate electrical parameters. With the advantages of easy implementation and robustness, PI controller are commonly adopted in various industrial process control[5-7], but it can only track DC signals without static error. The QPR controller can increase the gain at the resonance frequency, thereby improving the control accuracy of the system whose reference signal is in this frequency[8].
But the use of the resonance controller will sacrifice the gain in the low-frequency stage, thus weaken the dynamic performance, so that the system cannot reach the new steady state in time when the disturbance occurs.

In this paper, the LCL filter, which has a good filtering effect on high frequency harmonics and can effectively reduce the total size of the inverter, is adopted. However, the LCL filter is a third-order undamped system, and it will inevitably introduce a resonance peak to the system. Therefore, this paper selects capacitor-side current as active damping (AD) to suppress the resonance peak. Besides, this paper presents that on the basis of the proportional quasi-resonant controller, an integrator is connected in parallel to form a hybrid controller. At the same time, considering the digital delay problem caused by the feedback capacitor-side current, a delay compensator is introduced. Through the simulation analysis of the theoretical model and the comparison test with the traditional control methods on Simulink, it is verified that the novel control strategy can eliminate the steady-state error of the grid-connected current and make the system response faster.

2. Model of grid-connected inverter

This paper presents a single-phase grid-connected inverter whose rated capacity is 2.2KW. A 400V DC voltage source is selected for power supply. The inverter's switching frequency is 20kHz. Figure. 1 shows the system structure of a single-phase grid-connected inverter with LCL filters. In the Figure, \( U_{dc} \) is the dc-link voltage, which is assumed to be constant for simplicity. The inverter-side filter inductor \( L_1 \), the filter capacitor \( C \) and the grid-side filter inductor \( L_2 \) make up the LCL filter. Following these electrical elements, \( i_1 \) is the inverter-side current, \( i_c \) is the capacitor current and \( i_2 \) is grid-side current. According to the literature[9], the specific parameters of the LCL filter are designed and calculated, respectively \( L_1 = 3.3 \text{mH} \), \( L_2 = 1 \text{mH} \), \( C = 5 \mu \text{F} \).

![Figure 1. System structure of a single-phase](image)

The grid-side current \( i_2 \) is regulated to control the injected power into the grid, while the capacitive current \( i_c \) is fed back to achieve resonance damping. According to the arrangements above, the transfer function block diagram of the system is shown in Figure. 2, where \( H \) is the feedback coefficient of the capacitor current \( i_c \) and \( G_{inv} \) is the equivalent transfer function of the inverter bridge. When the switching frequency of the inverter is much higher than the cut-off frequency of the LCL filter, \( G_{inv} \) can be equivalent to a proportional block \( K_{PWM} \) and \( G_c \) is the transfer function of the digital controller of the system.
The transfer function block diagram is simplified by using Mason gain formula so as to further study the influence of active damping on the system. The outcome of the simplification is shown in Figure 3.

Convert its characteristic equation of the open-loop transfer function into the general form of second-order system and the expressions of system damping $\zeta$ and natural frequency $\omega_n$ can be derive in

$$\omega_n = \frac{\sqrt{b_1 + b_2}}{\sqrt{b_1 b_2}}$$

$$\zeta = \frac{b_1}{2\omega_n}$$

Theoretically, a larger value of the system damping $\zeta$ will leads to a stronger antijamming ability and a better suppression effect on the resonance peak. However, if the system damping is too large, it would increase the adjustment time and slow down the response speed of the system, which are of great important to the system performance. In engineering implementation, it often takes $\zeta=0.707$, because the system can reach a better regulation effect at this time[6]. Through the calculations above, the capacitor current feedback coefficient $H = 75.3279$ can be obtained.

3. Design of novel hybrid controller

3.1. Parameter tuning of the novel hybrid controller

The transfer function of the traditional PI controller and PR controller can be derived in equation (2) and equation (3), respectively

$$G_{PL} = k_p + \frac{k_i}{s}$$

$$G_{PR} = k_p + \frac{k_i}{s}$$
It can be seen from the equations that the gain at the resonant frequency $\omega_0$ is approximately equal to infinity when the PR controller is used. Such a large gain allows us to approximate that the system can track the sinusoidal signal, whose frequency is $\omega_0$, without static error. Therefore, the PR controller is more suitable for grid-connected current control than the PI controller.

In engineering, it is always difficult for the PR controller to be put into use due to the stability problems caused by the infinite gain. Generally, the QPR controller is a more practical choice and its transfer function is given as:

$$G_{QPR} = k_p + k_r \frac{\omega_c s}{s^2 + 2\omega_c s + \omega_0^2}$$

The QPR controller introduces a small damping term $2\omega_c$, so that the gain of the controller at the resonance frequency becomes a finite value$(k_p + k_r)$. At the same time, the system obtains a resonance bandwidth that is approximately $2\omega_c$. However, the introduction of a QPR controller will inevitably sacrifice the gain of the system in the low-frequency stage, making the system unable to respond to sudden changes in the grid in time. This paper proposes to connect an integrator in parallel with the QPR controller to form a novel hybrid controller to compensate the gain on the low-frequency stage. Its transfer function can be expressed as:

$$G_c = k_p + k_i + k_r \frac{2\omega_c s}{s^2 + 2\omega_c s + \omega_0^2}$$

Where $k_p$, $k_i$ and $k_r$ are the coefficients of the proportion, integrator and resonant controllers respectively. $\omega_0$ is the resonance frequency, and since the reference signal is a 50Hz power frequency signal, $\omega_0 = 314$rad/s is decided. $\omega_c$ is the resonance bandwidth.

In theory, the smaller the $\omega_c$, the better the frequency selectivity of the controller. However, considering the actual grid frequency deviation is within $\pm 0.5$Hz, the controller resonance bandwidth should be above 1Hz, which means that $\omega_c$ should satisfy the condition $\omega_c \geq 3.14$rad/s. This paper takes $\omega_c = 6$rad/s. As Figure.3 shown, the open-loop transfer function of the system can be obtained as:

$$\frac{i_2(s)}{i_{ref}(s)} = \frac{K_{PWM}(k_p + k_i + k_r \frac{2\omega_c s}{s^2 + 2\omega_c s + \omega_0^2})}{L_1L_2Cs^3 + K_{PWM}HL_2Cs^2 + (L_1 + L_2)s}$$

The frequency domain analysis method [10] is adopted to optimize the controller parameters. This method, which is based on the Nyquist stability criterion, can select the appropriate phase margin $\gamma$ and crossing frequency according to the stability and dynamic performance requirements of the actual system. It has a clear physical meaning and is suitable for researching and comparing the steady-state accuracy of different control methods under the same stability and dynamic performance indicators. In engineering applications., the phase margin $\gamma$ always takes $30^\circ$–$60^\circ$. At the same time the crossing frequency $\omega_p$ is no
more than 1/2 of the sampling frequency in theory and the value is usually 1/5 ~ 1/10. This paper takes $\gamma = 30^\circ$, $\omega_p = 2kHz$ and substitutes these values into the following equations:

$$
\left\{ \begin{array}{l}
\frac{i_2(j\omega_p)}{i_{ref}(j\omega_p)} = 1 \\
i_2(j\omega_p) = -180^\circ + \gamma
\end{array} \right.
$$

(7)

According to equation 7 and the condition that the system gains $T_{f_0} > 56dB$[7] at the fundamental frequency, a set of parameters $k_p = 45$, $k_i = 1000$, $k_r = 1500$ that meet the design requirements are obtained. Using the Routh criterion to verify this set of parameters, it is concluded that because the values in the first column of the Routh table are all positive, the system designed based on the above requirements is stable.

In order to verify the system performance of the systems using different controllers, this article substitutes the above-mentioned parameters into Equation 6. Draw the Bode diagram of the open-loop transfer function of the system as shown in Figure.4. At the same time, this Bode diagram is compared with systems using PI controllers and QPR controllers.

![Bode diagram comparison](image)

**Figure 4.** Comparison of open-loop Bode diagrams of three controller systems

It can be seen from Figure 4 that when using a traditional PI controller, in order to ensure a sufficiently large gain on the frequency of the reference signal, the proportional coefficient $k_p$ should be increased to a certain value. However, the increase of $k_p$ will amplify the gain of the entire frequency band. And it will leads to the increment of the cut-off frequency, which will result in system instability. But when the $k_p$ is reduced, the system cannot guarantee the control accuracy, resulting in steady-state errors. The novel hybrid controller and QPR controller can solve this problem well. The resonant module can accurately amplify the gain of the frequency band where the target signal is located, and achieve static error free tracking of the reference signal to guarantee the system has sufficient anti-interference ability. Comparing the Bode diagrams of the novel hybrid controller and the QPR controller, it can be seen that the former’s gain is approximately 20-30dB greater than the latter in the low frequency band, while the gain of other frequency band is basically unchanged. Obviously, paralleling the integrator on the QPR controller can improve the response speed of the system and improve the reliability of the system during the inverter operation.

### 3.2. Discretization of the novel hybrid controller
It can be seen from the Equation 5 that the difficulty of the discretization of the controller is concentrated on the quasi-resonant module. The relationship between the quasi-resonant module’s input and output is shown in equation (8):

\[ c(s) = \frac{2\omega_0 s}{s^2 + 2\omega_0 s + \omega_0^2} r(s) \]  

(8)

In this paper, the Tustin method[11] is adopted to transform the continuous domain transfer function of equation (8) into discrete domain. The expression of Tustin transformation is shown in equation (9), where \( T_s \) is the sampling period.

\[ s = \frac{2}{T_s} \frac{z - 1}{z + 1} \]  

(9)

After substituting equation (9) into equation (8), the difference equation of \( u(t) \) can be obtained

\[ u(k) = \frac{b_1}{a} [r(k) - r(k - 2)] + \frac{b_2}{a} c(k - 1) + \frac{b_3}{a} c(k - 2) \]  

(10)

where

\[ a = \frac{4}{T_s} + \frac{4\omega_0}{T_s} + \omega_0^2, \quad b_1 = \frac{4\omega_0}{T_s}, \quad b_2 = \frac{8}{T_s} - 2\omega_0^2, \quad b_3 = \frac{4\omega_0}{T_s} - \frac{4}{T_s} - \omega_0^2 \]

According to equation (10), the discretization model can be obtained as shown below.

![Figure 5. Discrete model of quasi-resonant module](image)

3.3. Design of digital delay compensator

Introducing capacitor current feedback into the system is equivalent to connecting a virtual resistor in parallel with the capacitor[11-14]. However, due to the digital delay, the resistance of the virtual resistor may change to be negative when the actual resonant frequency is higher than 1/6 of the sampling frequency, which will make the system unstable. [13] In order to solve the problems above, a delay compensator needs to be connected in series with the feedback path of the capacitor current. Because this is not the main topic of this article, the delay compensator in[12] is directly adopted to compensate the delay of the control system. The expression of the compensator is derived as:

\[ G_{\text{comp}} = \frac{(1 + \alpha + \beta) - \beta \cdot z^{-1}}{1 + \alpha \cdot z^{-1}} \]  

(11)

Where the compensation coefficients \( \alpha \) and \( \beta \) are \( t\alpha = 0.95 \) and \( \beta = 0.5 \) respectively.

4. Experimental results.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| LCL filter \( L_1/mH, C/\mu F, L_2/mH \) | 3.3, 5, 1 | coefficient of proportion controller \( k_p \) | 45 |
| DC-link voltage \( U_{dc}/V \) | 400 | coefficient of integrator \( k_i \) | 1000 |
| Output power \( P_o/W \) | 2200 | coefficient of resonance controller | 1500 |
The relevant parameters of the single-phase grid-connected inverter with LCL filter are listed in Table 1. In order to verify the effectiveness of the theoretical analysis, a simulation model of the novel control system was built on the Matlab/Simulink platform according to the system structure in Figure 1 and the parameters in Table 1. The simulation diagram is shown in Figure 6. At the same time, the systems controlled by traditional PI controller and QPR controller are set as control groups.

![Simulation diagram of the novel control system](image)

**Figure 6.** Simulation diagram of the novel control system

![Start-up performance of PI control system](image)

**Figure 7.** Start-up performance of PI control system

![Start-up performance QPR control system](image)

**Figure 8.** Start-up performance QPR control system
Figure 9. Start-up performance of novel hybrid control system

Figure 7-9 are the output current waveforms of the system controlled by PI controller, QPR controller and the novel hybrid controller at the moment when the inverter is connected to the grid. It can be seen from the Figures that the output current of the PI control system and the novel hybrid control system can track the reference current within half a period, while the QPR control system needs more than one period. However the waveform of the PI controlled system always lags behind the reference signal, which inevitably increases the current harmonics injected into the grid. The comparison of the start-up performance of the three systems verifies that the novel hybrid controller proposed in this paper can quickly reach a new steady state after changes occur while ensuring sufficient control accuracy.

Figure 10. Harmonic analysis of the output current of the novel hybrid control system

In order to analyze the output current’s harmonic components of the novel hybrid controller, the FFT analysis function in Simulink is used. The analysis results are shown in Figure.10. The effective value of the fundamental current is 12.86A and the total harmonic distortion (THD) is 0.55%. It meets the requirements of THD<5% for photovoltaic power generation and grid connection, which proves the rationality of the system design. Figure 11 and Figure 12 are the waveforms of the system’s error current when the PI controller and the novel hybrid controller are used, where $e_i = i_{ref} - i_2$. The comparative analysis shows that the steady-state current error of the PI control system has obvious power frequency periodic fluctuations and its peak value reaches 0.6A. While the current steady-state error peak value of the novel hybrid controller is about 0.1A and no obvious disturbance is observed.

Figure 11. Error current waveform of the PI control system
5. Conclusion

In this paper, on the basis of a single-phase grid-connected inverter system whose capacitor current is introduced as active damping, a novel hybrid controller implementation method for grid-connected current control is proposed. First, the model of the system is simplified to study the influence caused by the inner-loop and the feedback coefficient of the capacitor is obtained. Second, a novel hybrid controller is proposed to optimize the control strategy of the grid-connected inverter with LCL filter. As a result, the main parameters of the controller are determined in this section and the model analysis verify its effectiveness. Then, the controller is discretization by Tustin method to realize its implementation on digital processor and the effect of digital delay is solved by directly adopting a previously proposed approach for delay compensation. Finally, comparative experiments are carried out. After that, the following conclusions are drawn:

1) The novel hybrid controller can achieve static error free tracking of the power frequency signal, which enables the system to have higher control accuracy.
2) The novel hybrid controller improves the dynamic performance of the system and makes the system reach a new steady state in time after the disturbance occurs, which guarantee the reliability of system.
3) The novel hybrid controller can not only achieve unit power factor grid connection, but also effectively reduce the THD that injured into grid.

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