Quantum gravity and black hole spin in gravitational wave observations: a test of the Bekenstein-Hawking entropy

Eugenio Bianchi, Anuradha Gupta, Hal M. Haggard, and B. S. Sathyaprakash

1 Department of Physics, The Pennsylvania State University, University Park, PA 16802, USA
2 Institute for Gravitation & the Cosmos, The Pennsylvania State University, University Park, PA 16802, USA
3 Physics Program, Bard College, 30 Campus Road, Annandale-On-Hudson, NY 12504, USA
4 Perimeter Institute, 31 Caroline Street North, Waterloo, ON, N2L 2Y5, CAN
5 School of Physics and Astronomy, Cardiff University, Cardiff, CF24 3AA, UK

Black hole entropy is a robust prediction of quantum gravity with no observational test to date. We use the Bekenstein-Hawking entropy formula to determine the probability distribution of the spin of black holes at equilibrium in the microcanonical ensemble. We argue that this ensemble is relevant for black holes formed in the early universe and predicts the existence of a population of black holes with zero spin. Observations of such a population at LIGO, Virgo, and future gravitational wave observatories would provide the first experimental test of the statistical nature of black hole entropy.

Black holes: simple or complex. Black holes are often considered to be the ‘simplest’ macroscopic gravitating objects. They are vacuum solutions of General Relativity that, at equilibrium, are fully characterized by their mass and spin [1]. In 1974, however, Hawking discovered that black holes are hot because of quantum effects neglected in Einstein’s classical theory of gravity. As a result, black holes have an entropy—a distinguishing feature of complex systems like hot gases. In fact, black holes have a huge entropy, much larger than the entropy of a star of the same mass. This entropy, given by the Bekenstein-Hawking formula [2, 3], is proportional to the area of the black hole horizon, and for a rotating black hole is given by the equation

\[ S(M, a) = \frac{A(M, a)}{4\ell_P^2} = (1 + \sqrt{1 - a^2}) \frac{2\pi M^2}{m_P}, \]

where \( \ell_P = \sqrt{\hbar G/c^3} \) is the Planck length, \( m_P = \sqrt{\hbar c/G} \) is the Planck mass, and

\[ a = \frac{J}{GM^2/c} \in [0, 1], \]

is the dimensionless spin parameter. Considerable effort has been devoted to deriving this formula from the microscopic degrees of freedom of prospective theories of quantum gravity. Despite differing approaches, the result is found to be robust: In the interaction with its surroundings, a black hole of mass \( M \) and spin \( a \) behaves as an ensemble consisting of \( N \sim e^{S(M, a)} \) microstates. In this letter we investigate a phenomenological implication of this prediction shared by all current approaches to quantum gravity and through it describe the first in-principle test of the Bekenstein-Hawking entropy prediction.

Black hole entropy and the spin distribution. The Bekenstein-Hawking formula shows that, at fixed mass \( M \), black holes with larger spin have a smaller entropy. The statistical mechanical interpretation of this formula implies that—at fixed mass—there are fewer microstates with large spin than with small spin [4]. As a result, in the statistical ensemble where only the energy of the system is held fixed, the probability of finding spin \( a \) is given by the fraction

\[ P_M(a) = \frac{1}{\int_0^1 e^{A(M, a')/4\ell_P^2} a'^2 \, da'} , \]

where the numerator counts the number \( N \) of microstates at fixed mass and spin, while the denominator is the total number of microstates with fixed mass \( M \). A formal derivation of this probability distribution for the black hole microcanonical ensemble is provided below, together with a discussion of mechanisms of microcanonical equilibration. Fig. 1 shows the distribution \( P_M(a) \) for a variety of masses. For a population of black holes distributed ac-
According to the microcanonical ensemble, the Bekenstein-Hawking formula implies that, for large mass $M \gg m_p$, the average spin is small: $\langle a \rangle \approx 2m_p/\pi M \ll 1$. For a solar-mass black hole, this is an angular momentum of about 4000 kg m$^2$s$^{-1}$, corresponding to a dimensionless spin parameter $\langle a \rangle \approx 10^{-38}$.

While the spin distribution (3) is a quantum gravity prediction, populating the microcanonical ensemble does not require Planckian energy densities. The level separation between the energy microstates of a black hole is exponentially small in the entropy of the black hole, $\Delta E \sim (\partial S/\partial M)^{-1} e^{-S}$. As a result, low-energy processes can uniformly populate the black hole microstates and, if angular-momentum exchanges are efficient, microcanonical equilibrium can be reached. In this event, the entropy of the black hole is given by the Bekenstein-Hawking formula and the probability of finding spin $a$ by Eq. (3). This prediction of the Bekenstein-Hawking formula and the existence of black holes in microcanonical equilibrium can be tested through gravitational wave (GW) observations of black holes’ spins.

**Observation of black hole spins in GW events.** A binary black hole merger can be modeled as a process with in and out data given by

$$(M_1, \tilde{a}_1) + (M_2, \tilde{a}_2) + \tilde{L} \rightarrow (M_f, \tilde{a}_f) + GW,$$

where $(M_i, \tilde{a}_i)$ with $i = 1, 2$ is the mass and dimensionless spin of each black hole in the binary, $\tilde{L}$ their initial orbital angular momentum, $GW$ the emitted gravitational waves in the merger, and $(M_f, \tilde{a}_f)$ the mass and spin of the final black hole. Gravitational wave observations can be used to measure the final spin magnitude $a_f = |\tilde{a}_f|$ and the orbital projection of the effective spin of the black binary,

$$\chi_{\text{eff}} = \frac{M_1 \tilde{a}_1 + M_2 \tilde{a}_2}{M_1 + M_2} \cdot \frac{\tilde{L}}{|\tilde{L}|} \in [-1, 1],$$

a quantity conserved at the 2nd post-Newtonian order [5]. Current observations of GW events indicate that the effective spin $\chi_{\text{eff}}$ of the few black hole binaries whose mergers have been observed so far is small and compatible with zero [6-13]. Unless the spins $\tilde{a}_1$ and $\tilde{a}_2$ are anti-aligned or lie in the plane of the orbit, a small $\chi_{\text{eff}}$ indicates small spin magnitudes for the progenitor black holes in the binary. This observation is to be contrasted to the measurement of black holes in X-ray binaries which are found to have high spin magnitude [14]. In Fig. 2 we report the effective initial spin $\chi_{\text{eff}}$—the most easily accessed spin parameter in GWs—and the final spin $a_f$ of the ten binary black hole mergers observed thus far, as reported in the GW Transient Catalog GWTC-1 [6-13].

**Microcanonical ensemble and GW events.** Gravitational wave observations provide a way to test the microcanonical ensemble determined by the Bekenstein-Hawking entropy. If the progenitor black holes are in microcanonical equilibrium before forming the binary, then the spin of each black hole is distributed according to the probability distribution (3). For black holes of a solar mass or more, this implies that their spin is zero for all practical purposes and $\chi_{\text{eff}} \approx 0$. This observation has an immediate consequence for the spin of the final black hole. It is known from numerical-relativity simulations that the merger of two zero-spin black holes results in a final black hole with spin $a_f$ which is largely independent from the details of the process: a fit of the numerical simulations provides a formula for the final spin $[15, 16]$

$$a_f \approx 0.69 - 0.56 \delta^2,$$

expressed as a function of the fractional mass difference $\delta \equiv M_1 - M_2/(M_1 + M_2)$, which is assumed to be small. Remarkably, for quasi-circular orbits the final spin does not depend on the initial orbital angular momentum $\tilde{L}$ of the binary. This feature provides a handle to identify a population of progenitor microcanonical black holes via GW observations of merger events.

In Fig. 2(a) we present the distribution of spins predicted by the microcanonical ensemble. We denote by 1g a first generation black hole belonging to the microcanonical ensemble. Depending on the formation mechanism, hierarchical mergers are also possible [17-20] and we denote by 2g the result of a 1g-1g merger, i.e., a 2nd generation black hole. Note that the spin of a 2g black hole is isotropically distributed and in general not aligned to the orbital angular momentum of any subsequent 1g-2g merger. For 1g black holes, we consider a uniform distribution of masses in $[10M_\odot, 30M_\odot]$, where $M_\odot \gg m_p$ is a reference scale which drops out of the prediction of the spin. This range of masses results in a fractional mass difference $\delta \lesssim 0.5$ and, using Eq. (6), a definite range for the final spin $a_f$. The spin and mass of the final black hole produced from the merger are computed by taking the average of estimates from various fits to numerical-relativity simulations as done for LIGO/Virgo binary black hole mergers in the second observing run [16, 21-23]. Fig. 2(a) shows a distribution of 1g-1g mergers extracted from this distribution (red dots with $\chi_{\text{eff}} = 0$ and $a_f \in [0.54, 0.69]$). We consider also 1g-2g mergers, with the 2g black hole extracted from the previous 1g-1g mergers [19]. Fig. 2(a) shows a distribution of 1g-2g mergers (blue dots). In this case, the initial effective spin and the final spin are correlated with $a_f \lesssim 0.69 + 0.40 \chi_{\text{eff}}$.

**Comparison to other spin distributions.** We compare the predictions of the microcanonical ensemble to two astrophysical models of black hole spin distributions [2-4, 29]. The first model assumes black hole spins aligned to the orbital angular momentum, i.e., $\tilde{a}_1, \tilde{a}_2$, and $\tilde{L}$ pointing in the same direction. Black hole binaries formed through common envelope evolution in galactic fields are
lies on the statistical interpretation of the Bekenstein-hole belonging to the microcanonical ensemble and rule out or constrain black holes formed in equilibrium. On the other hand, a non-identifiable mechanism, black holes formed in equilibrium. To test this prediction of the statistical nature of black holes, e.g. spherical collapse. Here we are identifying a distinguishability of a mix of populations.

Quantum gravity and the microcanonical ensemble. The black hole spin distribution (3) applies to a black hole belonging to the microcanonical ensemble and relies on the statistical interpretation of the Bekenstein-Hawking entropy in terms of black hole microstates. While identifying the nature of black hole microstates requires a theory of quantum gravity (see [33–37] for results in loop quantum gravity, [38–42] for results in string theory, and [43] for a discussion of other approaches), counting microstates at fixed mass $M \gg m_p$ can be achieved via semiclassical methods [44–46]. We discuss the derivation of the counting in detail as it clarifies the nature of the microcanonical ensemble [4]. Microstates with asymptotically flat boundary conditions are simultaneous eigenstates of the Arnowitt-Deser-Misner mass and spin. They are labeled by their mass $M$, their spin $J = \sqrt{j(j+1)} \hbar$ and by a label $\alpha$ that enumerates an orthonormal basis $|M, j, \alpha\rangle$ of the Hilbert space $\mathcal{H}_{Mj}$ at fixed mass and spin. In the microcanonical ensemble, the microstates of given energy $E$ are uniformly populated resulting in a maximally-mixed state

$$\rho_M = \frac{1}{\dim \mathcal{H}_{Mj}} \sum_j \sum_\alpha |M, j, \alpha\rangle \langle M, j, \alpha|.$$  

Remarkably, the microcanonical ensemble $\rho_M$ consists of a mixture of ensembles $\rho_{Mj}$ of fixed mass and spin, i.e.,

$$\rho_M = \sum_j \rho_{Mj}$$

where $\rho_{Mj} = \frac{1}{\dim \mathcal{H}_{Mj}} \sum_\alpha |M, j, \alpha\rangle \langle M, j, \alpha|$ describes the state of a rotating black hole. The probability of finding a rotating black hole of spin $j$ in the microcanonical

FIG. 2. Final spin, $a_f$, versus effective spin parameter, $\chi_{\textrm{eff}}$, of binary black hole mergers in different scenarios. The binary constituent masses $M_1$ and $M_2$ are sampled uniformly in the range $[10M_\odot, 30M_\odot]$ so that $M_1/M_2 \leq 3$. Panel (a): 1g-1g population consisting of mergers of two black holes assumed to be in microcanonical equilibrium with spin distribution given by Eq. (3) [large red dots], and 1g-2g population consisting of mergers of a 1g black hole with the product of a 1g-1g merger [blue dots]. Panel (b): Mergers of two black holes with isotropically distributed spins with $p_\Upsilon(1d)(a)$ magnitude. Panel (c): Mergers of two black holes with aligned spins with $p_\Upsilon(1d)(a)$ magnitude. The darkest colors in the gradient indicate equal mass, while the lightest colors indicate the most asymmetric binary masses. The black symbols show the GW events measured thus far by the advanced LIGO and advanced Virgo detectors [24, 25] and the error bars on their $\chi_{\textrm{eff}}$ and $a_f$ values are 90% credible bounds [6–13]. In panel (a) the number of dots represents only the distribution and not the relative fraction of events in 1g-1g and the rarer 1g-2g mergers.

{\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Final spin, $a_f$, versus effective spin parameter, $\chi_{\textrm{eff}}$, of binary black hole mergers in different scenarios. The binary constituent masses $M_1$ and $M_2$ are sampled uniformly in the range $[10M_\odot, 30M_\odot]$ so that $M_1/M_2 \leq 3$. Panel (a): 1g-1g population consisting of mergers of two black holes assumed to be in microcanonical equilibrium with spin distribution given by Eq. (3) [large red dots], and 1g-2g population consisting of mergers of a 1g black hole with the product of a 1g-1g merger [blue dots]. Panel (b): Mergers of two black holes with isotropically distributed spins with $p_\Upsilon(1d)(a)$ magnitude. Panel (c): Mergers of two black holes with aligned spins with $p_\Upsilon(1d)(a)$ magnitude. The darkest colors in the gradient indicate equal mass, while the lightest colors indicate the most asymmetric binary masses. The black symbols show the GW events measured thus far by the advanced LIGO and advanced Virgo detectors [24, 25] and the error bars on their $\chi_{\textrm{eff}}$ and $a_f$ values are 90% credible bounds [6–13]. In panel (a) the number of dots represents only the distribution and not the relative fraction of events in 1g-1g and the rarer 1g-2g mergers.
\end{figure}}

expected to be well-described by this configuration [5]. The second model assumes an isotropic distribution of the spins $\vec{a}_1$ and $\vec{a}_2$. Binaries formed in globular clusters or stellar clusters near active galactic nuclei are expected to have isotropic spins [6]. While the two models prescribe the directions of the spins, they do not constrain their magnitudes which can be assumed to be uniformly distributed. We denote by $p_\Upsilon(1d)(a)$ the flat distribution in the 1d interval $[0, 1]$ and by $p_\Upsilon(3d)(a) = 3a^2$ the uniform distribution for the spin vector in 3d; see Fig. 1. The predictions of the isotropic model with $p_\Upsilon(3d)(a)$ magnitudes and of aligned model with $p_\Upsilon(1d)(a)$ magnitudes are reported in Fig. 2(b) and Fig. 2(c). The Supplemental Material [32] compares the cumulative probability distributions for $a_f$ for the various models and analyses the distinguishability of a mix of populations.

There can be other mechanisms that produce small spins, e.g. spherical collapse. Here we are identifying a new mechanism, black holes formed in equilibrium. To test this prediction of the statistical nature of black holes, one would need an independent signature that excludes other formation channels. On the other hand, a non-detection of a population with zero progenitor spins will rule out or constrain black holes formed in equilibrium.

Quantum gravity and the microcanonical ensemble. The black hole spin distribution (3) applies to a black hole belonging to the microcanonical ensemble and relies on the statistical interpretation of the Bekenstein-Hawking entropy in terms of black hole microstates. While identifying the nature of black hole microstates requires a theory of quantum gravity (see [33–37] for results in loop quantum gravity, [38–42] for results in string theory, and [43] for a discussion of other approaches), counting microstates at fixed mass $M \gg m_p$ can be achieved via semiclassical methods [44–46]. We discuss the derivation of the counting in detail as it clarifies the nature of the microcanonical ensemble [4]. Microstates with asymptotically flat boundary conditions are simultaneous eigenstates of the Arnowitt-Deser-Misner mass and spin. They are labeled by their mass $M$, their spin $J = \sqrt{j(j+1)} \hbar$ and by a label $\alpha$ that enumerates an orthonormal basis $|M, j, \alpha\rangle$ of the Hilbert space $\mathcal{H}_{Mj}$ at fixed mass and spin. In the microcanonical ensemble, the microstates of given energy $E$ are uniformly populated resulting in a maximally-mixed state

$$\rho_M = \frac{1}{\dim \mathcal{H}_{Mj}} \sum_j \sum_\alpha |M, j, \alpha\rangle \langle M, j, \alpha|.$$  

Remarkably, the microcanonical ensemble $\rho_M$ consists of a mixture of ensembles $\rho_{Mj}$ of fixed mass and spin, i.e.,

$$\rho_M = \sum_j \rho_{Mj}$$

where $\rho_{Mj} = \frac{1}{\dim \mathcal{H}_{Mj}} \sum_\alpha |M, j, \alpha\rangle \langle M, j, \alpha|$ describes the state of a rotating black hole. The probability of finding a rotating black hole of spin $j$ in the microcanonical
ensemble of energy $M$ is therefore given by the ratio

$$p_M(j) = \frac{\dim H_{Mj}}{\sum_j \dim H_{Mj}}.$$

(9)

The dimension of the Hilbert space $\dim H_{Mj}$ can be computed via semiclassical methods starting from the canonical partition function $Z(\beta, \omega) = \int Dq_{\mu\nu} e^{-I(\beta, \omega)}$, where $I(\beta, \omega)$ is the Euclidean gravitational action with fixed periodicity conditions [44]. In the semiclassical limit, taking into account graviton loops in 4d vacuum gravity, one finds

$$\dim H_{Mj} \sim \sqrt{S(M, a_j)} \frac{212}{45} e^{S(M, a_j) a_j^2},$$

(10)

$$\sum_j \dim H_{Mj} \sim \sqrt{S(M, 0)} \frac{212}{45} e^{S(M, 0)},$$

(11)

where $S(M, a)$ is the Bekenstein-Hawking entropy, Eq. (1), and $a_j = \sqrt{j(j + 1)} m_p^2 / M^2$ is the dimensionless spin. Therefore, in the limit $M \gg m_p$, the probability (9) reproduces the spin distribution $P_M(a)$ of Eq. (3).

We note that considering matter coupled to gravity (for instance Standard Model matter) or considering a different ensemble (for instance including the full mass range $[0, M]$) only has the effect of adding a numerical contribution to the coefficient $212/45$ due to gravitons in Eq. (11). As a result, only the logarithmic correction to the Bekenstein-Hawking formula is modified and, in the limit $M \gg m_p$, the probability distribution $P_M(a)$ remains unchanged. One can also consider the canonical ensemble at fixed temperature $T$, which is technically ill-defined because of the instability associated to black holes’ negative heat capacity. Nevertheless, on timescales much shorter than the black-hole evaporation time, this instability simply modifies the logarithmic corrections to the Bekenstein-Hawking formula. In this sense, for $M \gg m_p$, the predictions of the Bekenstein-Hawking entropy and the microcanonical spin distribution $P_M(a)$ are robust.

**Mechanisms of microcanonical equilibration.** The Bekenstein-Hawking entropy formula, together with its statistical interpretation in quantum gravity, provides a definite prediction for the spin of a population of black holes distributed according to the microcanonical ensemble. This raises an important phenomenological question: when is a population of black holes well described by the microcanonical ensemble? In the familiar case of a gas in an isolated box, the microcanonical ensemble predicts correctly the statistical properties of the gas. The reason is that the dynamics of the molecular interactions is sufficiently rich to fully explore the constant energy shell in the phase space. Certainly, if the dynamics does not allow the system to fully explore the energy shell, microcanonical equilibrium cannot be reached. For instance, black holes formed by stellar collapse are not expected to be distributed according to the microcanonical ensemble. The reason is twofold: (i) the initial matter distribution already has a large initial angular momentum and therefore is far from microcanonical equilibrium, (ii) during the collapse, angular momentum is not efficiently exchanged with the surrounding environment, as it is only carried away in ejecta and GWs. Moreover, after black hole formation in a dense environment, accretion processes typically drive the spin towards large values [14]. As a result, the system cannot fully explore the energy shell and only black hole microstates with large angular momentum are populated.

On the other hand, primordial black holes formed in the early universe by density fluctuations [47, 48] are good candidates for a microcanonical population. In this case, the starting point is a homogeneous matter distribution that is already in thermal equilibrium over cosmic scales. When a density perturbation reaches the critical-collapse threshold [49, 50], matter within its own Hubble radius is trapped, and a black hole forms. The entropy of the trapped region is then given by the Bekenstein-Hawking formula, and the ensemble describing the black hole is determined by the one describing the trapped matter. As matter was in thermal equilibrium, we expect that the spin of the black hole can also be predicted from the Bekenstein-Hawking entropy formula, and therefore distributed according to Eq. (3). Under these conditions, the Bekenstein-Hawking entropy formula predicts that black holes formed in this phase have practically zero spin. Moreover, after matter is trapped within its own Hubble radius, the formed black hole remains effectively isolated. This is because the subsequent cosmic expansion leaves the black hole behind; as a consequence the black hole spin remains unchanged.

Astrophysical processes such as accretion from a companion star is not relevant for the primordial black holes considered here, as they are expected to reside in the halo of a dwarf galaxy [51], where it is not possible for them to pair up with donor stars. Accretion of gas in the interstellar medium of the halo is highly inefficient and random, and does not lead to appreciable changes in black hole spins. Furthermore, if a binary black hole system is formed [52], the inspiral will preserve the spin until the merger phase and therefore a spin distribution as described in Fig. 2(a) is expected. Interestingly, this possibility can be tested with the observation of GW events from primordial black hole mergers.

Current gravitational-wave observations in fact probe a mass range that roughly matches the one of black holes formed in the early universe during the QCD phase transition. During this transition the pressure drops—thus enhancing the probability of black hole formation. The typical mass of a black hole formed in this phase can be estimated by considering a uniform density of about a pion per Compton-wavelength cubed, $\rho_0 \sim (150 \text{ MeV})^4 / h^3 c^5$. Such a distribution of matter is within...
its own Schwarzschild radius when \( M_0 \sim 25 M_\odot \). While this is only a crude estimate, recent studies that take into account results from lattice QCD simulations indicate that the enhanced production of primordial black holes during the QCD phase transition is within the range \( 0.1 \sim 100 \ M_\odot \) [53, 54]. Interestingly, mergers of black holes in this range are accessible via GW observations. Because the initial plasma is in thermal equilibrium, we expect primordial black holes produced in this phase to be formed in equilibrium and to have small spins. Furthermore as, after formed, they remain isolated due to cosmic expansion, we expect their spins to remain essentially unchanged until merger. Future GW observations will further constrain this scenario.

**Discussion.** Providing a microscopic derivation of the Bekenstein-Hawking entropy formula is often presented as a benchmark for a theory of quantum gravity [43]. In this letter, instead of focusing on its derivations, we have investigated the first in-principle phenomenological consequence of this statistical property of black holes: we have shown that, for a population of black holes in microcanonical equilibrium, the Bekenstein-Hawking formula predicts small black hole spins. Furthermore, we have described how this feature can be tested via GW observations. The prediction of small spins can provide new observational constraints on primordial black holes and their mechanism of formation [48]. Most importantly, the imminent transition from single GW measurements to population analyses may provide a new way to investigate the phenomenology of quantum gravity with LIGO, Virgo, and future GW observatories [55, 56]. In particular, the detection of a population of black holes distributed according to the microcanonical ensemble would provide the first experimental test of the Bekenstein-Hawking entropy and the statistical mechanics of black holes.

**Acknowledgments.** EB thanks D. Page, B. Unruh, R. Sorkin and V. Frolov for comments and discussions during the 23rd Peyresq meeting, and C. Rovelli for correspondence. We thank N. Johnson-McDaniel for comments on the manuscript. HMH thanks the IGC at the Pennsylvania State University for warm hospitality and support while beginning this work and the Perimeter Institute for Theoretical Physics for generous sabbatical support. We also thank I. Cholis for correspondence on the formation of primordial binary black holes in galactic halos. This work is supported by Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation. EB is supported by the NSF Grant PHY-1806428. AG and BSS are supported in part by NSF grants PHY-1836779, AST-1716394 and AST-1708146. This document has LIGO preprint number LIGO-P1800228.

[1] J. M. Bardeen, B. Carter, and S. W. Hawking, “The Four laws of black hole mechanics,” Commun. Math. Phys. 31 (1973) 161–170.
[2] J. D. Bekenstein, “Black holes and entropy,” Phys. Rev. D7 (1973) 2333–2346.
[3] S. W. Hawking, “Particle Creation by Black Holes,” Commun. Math. Phys. 43 (1975) 199–220.
[4] E. Bianchi and H. M. Haggard, “Spin fluctuations and black hole singularities: the onset of quantum gravity is spacelike,” New J. Phys. 20, no. 10, 103028 (2018).
[5] doi:10.1088/1367-2630/aae71d.
[6] L. Blanchet, “Gravitational Radiation from Post-Newtonian Sources and Inspiralling Compact Binaries,” Living Rev. Rel. 17 (2014) 2, 1310.1528.
[7] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “Observation of Gravitational Waves from a Binary Black Hole Merger,” Phys. Rev. Lett. 116 (2016), no. 6, 061102, 1602.03837.
[8] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence,” Phys. Rev. Lett. 116 (2016), no. 24, 241103, 1606.04855.
[9] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “Binary Black Hole Mergers in the first Advanced LIGO Observing Run,” Phys. Rev. X6 (2016), no. 4, 041015, 1606.04856.
[10] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2,” Phys. Rev. Lett. 118 (2017), no. 22, 221101, 1706.01812.
[11] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW170608: Observation of a 19-solar-mass Binary Black Hole Coalescence,” Astrophys. J. 851 (2017), no. 2, L35, 1711.05578.
[12] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), “GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence,” Phys. Rev. Lett. 119 (2017), no. 14, 141101, 1709.09660.
[13] LIGO Scientific and Virgo Collaborations, “GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs,” 1811.12907.
[14] LIGO Scientific and Virgo Collaborations, “Binary Black Hole Population Properties Inferred from the First and Second Observing Runs of Advanced LIGO and Advanced Virgo,” 1811.12940.
[15] M. C. Miller and J. M. Miller, “The Masses and Spins of Neutron Stars and Stellar-Mass Black Holes,” Phys. Rept. 548 (2014) 1–34, 1408.4145.
[16] M. A. Scheel, M. Boyle, T. Chu, L. E. Kidder, K. D. Matthews, and H. P. Pfeiffer, “High-accuracy waveforms for binary black hole inspiral, merger, and ringdown,” Phys. Rev. D79 (2009) 024003, 0810.1767.
[17] F. Hofmann, E. Barausse, and L. Rezzolla, “The final spin from binary black holes in quasi-circular orbits,” Astrophys. J. 825 (2016), no. 2, L19, 1605.01938.
[18] S. Clesse and J. García-Bellido, “Massive primordial black holes from hybrid inflation as dark matter and the seeds of galaxies,” Phys. Rev. D 92 (2015), 023524.
[18] S. Clesse and J. García-Bellido, “The clustering of massive Primordial Black Holes as Dark Matter: Measuring their mass distribution with advanced LIGO,” Phys. Dark Univ. 15 (2017), 142, 1603.05234.

[19] D. Gerosa and E. Berti, “Are merging black holes born from stellar collapse or previous mergers?,” Phys. Rev. D 95, no. 12, 124046 (2017) 1703.06223.

[20] M. Fishbach, D. E. Holz, and B. Farr, “Are LIGO’s Black Holes Made From Smaller Black Holes?,” Astrophys. J. 840 (2017)L24. 1703.06869.

[21] X. Jiménez-Forteza, D. Keitel, S. Husa, M. Hannam, S. Khan, and M. Pürrer, “Hierarchical data-driven approach to fitting numerical relativity data for nonprecessing binary black holes with an application to final spin and radiated energy,” Phys. Rev. D 95 (2017) 064024, 1611.00332.

[22] J. Healy and C. O. Lousto, “Remnant of binary black-hole mergers: New simulations and peak luminosity studies,” Phys. Rev. D 95 (2017) 024037, 1610.09713.

[23] N. K. Johnson-McDaniel et al., “Determining the final spin of a binary black hole system including in-plane spins: Method and checks of accuracy,” LIGO Project Technical Report No. LIGO-T1600168 (2016).

[24] J. Abadie et. al., “Advanced LIGO,” Class. Quantum Grav. 32 (2015) 074001, 1411.4547.

[25] F. Acernese, et. al., “Advanced Virgo: a 2nd generation interferometric gravitational wave detector,” Class. Quantum Grav. 32 (2015) 024001, 1408.3978.

[26] W. M. Farr, S. Stevenson, M. Coleman Miller, I. Mandel, B. Farr, and A. Vecchio, “Distinguishing Spin-Aligned and Isotropic Black Hole Populations With Gravitational Waves,” Nature 548 (2017) 426, 1706.01385.

[27] K. Belczynski et al., “The origin of low spin of black holes in LIGO/Virgo mergers,” 1706.07053.

[28] C. L. Rodriguez, M. Zevin, C. Pankow, V. Kalogera, and F. A. Rasio, “Illuminating Black Hole Binary Formation Channels with Spins in Advanced LIGO,” Astrophys. J. 832 (2016), no. 1, L2. 1609.05916.

[29] T. Piran and K. Hotokezaka, “Who Ordered That? On The Origin of LIGO’s Merging Binary Black Holes,” 1807.01336.

[30] K. A. Postnov and L. R. Yungelson, “The Evolution of Compact Binary Star Systems,” Living Rev. Rel. 17 (2014) 3, 1403.4754.

[31] M. J. Benacquista and J. M. B. Downing, “Relativistic Binaries in Globular Clusters,” Living Rev. Rel. 16 (2013) 4, 1110.4423.

[32] E. Bianchi, A. Gupta, H. M. Haggard, B. S. Sathyaprakash, “Supplemental Material – Quantum gravity and black hole spin in gravitational wave observations: a test of the Bekenstein-Hawking entropy” (2018).

[33] C. Rovelli, “Black hole entropy from loop quantum gravity,” Phys. Rev. Lett. 77 (1996) 3288–3291, gr-qc/9603063.

[34] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, “Quantum geometry and black hole entropy,” Phys. Rev. Lett. 80 (1998) 904–907, gr-qc/9710007.

[35] J. Engle, K. Noui, and A. Perez “Black hole entropy and SU(2) Chern-Simons theory,” Phys. Rev. Lett. 105 (2010) 031302, 0905.3168.

[36] E. Bianchi, “Black Hole Entropy, Loop Gravity, and Polymer Physics,” Class. Quant. Grav. 28 (2011) 114006, 1011.5628.

[37] A. Perez, “Black Holes in Loop Quantum Gravity;” Rept. Prog. Phys. 80 (2017), no. 12, 126901, 1703.09149.

[38] A. Strominger and C. Vafa, “Microscopic origin of the Bekenstein-Hawking entropy,” Phys. Lett. B379 (1996) 99–104, hep-th/9601029.

[39] S. D. Mathur, “The Fuzzball proposal for black holes: An Elementary review,” Fortsch. Phys. 53 (2005) 793–827, hep-th/0502050.

[40] I. Mandal and A. Sen, “Black Hole Microstate Counting and its Macroscopic Counterpart,” Class. Quant. Grav. 27 (2010) 214003, 1008.3801.

[41] J. Maldacena and L. Susskind, “Cool horizons for entangled black holes,” Fortsch. Phys. 61 (2013) 781–811, 1306.0533.

[42] D. Harlow, “Jerusalem Lectures on Black Holes and Quantum Information,” Rev. Mod. Phys. 88 (2016) 015002, 1409.1231.

[43] S. Carlip, “Black Hole Entropy and the Problem of Universality,” in Quantum Mechanics of Fundamental Systems: The Quest for Beauty and Simplicity: Claudio Bunster Festschrift, pp. 91–106. 2009. 0807.4192.

[44] G. W. Gibbons and S. W. Hawking, “Action Integrals and Partition Functions in Quantum Gravity,” Phys. Rev. D15 (1977) 2752–2756.

[45] J. D. Brown and J. W. York, Jr., “The Microcanonical functional integral. I. The Gravitational field,” Phys. Rev. D47 (1993) 1420–1431, gr-qc/9209014.

[46] A. Sen, “Logarithmic Corrections to Schwarzschild and Other Non-extremal Black Hole Entropy in Different Dimensions,” JHEP 04 (2013) 156, 1205.0971.

[47] B. J. Carr and S. W. Hawking, “Black holes in the early Universe,” Mon. Not. Roy. Astron. Soc. 168 (1974) 399–415.

[48] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, “Primordial black holes: perspectives in gravitational wave astronomy,” Class. Quant. Grav. 35 (2018), no. 6, 063001, 1801.05235.

[49] T. Chiba and S. Yokoyama, “Spin Distribution of Primordial Black Holes,” PTEP 2017 (2017), no. 8, 083E01, 1704.06573.

[50] T. Harada, C.-M. Yoo, K. Kohri, and K.-I. Nakao, “Spins of primordial black holes formed in the matter-dominated phase of the Universe,” Phys. Rev. D96 (2017), no. 8, 083517, 1707.03595.

[51] S. Bird, I. Cholis, J. B. Munoz, Y. Ali-Hamoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess, “Did LIGO detect dark matter?,” Phys. Rev. Lett. 116 (2016) 201301, 1603.00464.

[52] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, “Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914,” Phys. Rev. Lett. 117 (2016) 061101, 1603.08338.

[53] J. L. G. Sobrinho, P. Augusto and A. L. Gonçalves, “New thresholds for Primordial Black Hole formation during the QCD phase transition,” Mon. Not. Roy. Astron. Soc. 463, no. 3, 2348 (2016) 1609.01205.

[54] C. T. Byrnes, M. Hindmarsh, S. Young, and M. R. S. Hawkins, “Primordial black holes with an accurate QCD equation of state,” 1801.06138.

[55] G. Amelino-Camelia, “Quantum-Spacetime Phenomenology,” Living Rev. Rel. 16 (2013) 5, 0806.0339.

[56] L. Barack et al., “Black holes, gravitational waves and fundamental physics: a roadmap,” 1806.05195.
Distinguishing a microcanonical population and a mix of populations. We compare the predictions of the microcanonical ensemble discussed in [1] to two astrophysical models of black hole spin distributions [2–6]. The first model assumes that the black hole spins are aligned to the orbital angular momentum, i.e., \( \vec{a}_1, \vec{a}_2, \) and \( \vec{L} \) pointing in the same direction. The spin magnitudes are assumed to be distributed uniformly, \( p_{u}(1d) = 1, \) according to the flat distribution in the \( 1d \) interval \([0, 1]\). The second model assumes an isotropic distribution of the spins \( \vec{a}_1 \) and \( \vec{a}_2 \), with magnitudes distributed as \( p_{u}(3d) = 3 a^2, \) the uniform distribution for a vector in \( 3d \) with magnitude \( |\vec{a}| \in [0, 1] \). See Fig. S1.

We can use the statistical properties of the \( a_f \chi_{\text{eff}} \) distribution of black hole mergers discussed in [1] to distinguish a variety of binary black hole populations. We compare the \( a_f \chi_{\text{eff}} \) distribution for an all isotropic population, denoted I, (purple dots in Fig. 2(b) of [1]), an all aligned one, denoted A, (orange dots in Fig. 2(c) of [1]), and various mixed populations, denoted M, using the Anderson-Darling [7] and Kolmogorov-Smirnov [8, 9] tests. These tests compare two given distributions and return p-values \( \in [0, 1] \). For identical distributions the p-value is 1, and as the difference between the two distributions increases the p-value falls below unity. The mixed populations we consider, M(X, Y), all consist of 90% of population X and 10% of population Y. Table S1 displays the p-value comparisons between pairs of pure and mixed populations. While it is difficult to distinguish a purely isotropic population from an isotropic population with a 10% admixture of 1g-2g mergers, see the first row of Table S1, all other mixtures are clearly distinguishable. The improved distinguishability of the 1g-1g over the 1g-2g admixtures is due to the sharp asymmetric peak at \( a_f \approx 0.69 \) for 1g-1g mergers clearly visible in Fig. 3 of [1].

FIG. S1. Probability distribution of final spins for the models discussed in [1]. Mergers of 1g black holes with spins distributed according to the probability distribution \( P_M(a) \) of [1] result in a peak at \( a_f \lesssim 0.69 \). All distributions are normalized to unit probability. Inset: Cumulative probability distribution of final spins, \( \Phi(a_f) \), for the same models.

| comparisons | p-value (AD test) | p-value (KS test) |
|-------------|------------------|------------------|
| I vs. M(I, 1g-2g) | 0.81 | 0.99 |
| I vs. M(I, 1g-1g) | 0.04 | 0.01 |
| A vs. M(A, 1g-2g) | \( 1.3 \times 10^{-7} \) | \( 3.2 \times 10^{-7} \) |
| A vs. M(A, 1g-1g) | \( 7.8 \times 10^{-11} \) | \( 1.6 \times 10^{-8} \) |

TABLE S1. The p-values for Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) tests comparing populations of all aligned (A), all isotropic (I), and mixed (M) binaries. Mixed populations, M(X, Y), consist of 90% of population X and 10% of population Y.

[1] E. Bianchi, A. Gupta, H. M. Haggard, and B. S. Sathyaprakash, “Quantum gravity and black hole spin in
gravitational wave observations: a test of the Bekenstein-Hawking entropy” (2018)

[2] W. M. Farr, S. Stevenson, M. Coleman Miller, I. Mandel, B. Farr, and A. Vecchio, “Distinguishing Spin-Aligned and Isotropic Black Hole Populations With Gravitational Waves,” Nature 548 (2017) 426, 1706.01385.

[3] K. Belczynski et al., “The origin of low spin of black holes in LIGO/Virgo mergers,” 1706.07053.

[4] C. L. Rodriguez, M. Zevin, C. Pankow, V. Kalogera, and F. A. Rasio, “Illuminating Black Hole Binary Formation Channels with Spins in Advanced LIGO,” Astrophys. J. 832 (2016), no. 1, L2, 1609.05916.

[5] K. A. Postnov and L. R. Yungelson, “The Evolution of Compact Binary Star Systems,” Living Rev. Rel. 17 (2014) 3, 1403.4754.

[6] M. J. Benacquista and J. M. B. Downing, “Relativistic Binaries in Globular Clusters,” Living Rev. Rel. 16 (2013) 4, 1110.4423.

[7] T. W. Anderson and D. A. Darling, Ann. Math. Statist. (1952).

[8] A. Kolmogorov, “Sulla determinazione empirica di una legge di distribuzione,” G. Ist. Ital. Attuari. 4: 83-91 (1933).

[9] N. Smirnov “Table for estimating the goodness of fit of empirical distributions”, Annals of Mathematical Statistics 19: 279-281 (1948).