Application of the Strong Tracking UKF in the Maneuvering Target Tracking

GUO Caifa, DAI Zhengxu, YANG Lei, WANG Xuliang

China Satellite Maritime Tracking and Controlling Department, Jiangyin, China

guo_caifa@sina.com

Abstract. Maneuvering target tracking is one of the key technologies in the space tracking system. The strong tracking unscented Kalman filter is presented for the data processing which uses a fading factor in the state estimation. Using the current statistical model, the strong tracking UKF is tested with the real data obtained in a mission measured by the space tracking ship. Simulation results show the good performance of the proposed method.

1. Introduction

The maneuvering target tracking has been an issue for a long time as the trajectory target has a high maneuverability and wide range of dynamics as well as a complex measurement environment [1,2]. For the space tracking ship, the tracking process is under dynamic conditions including swing, rotation, vibration etc, which the measurement environment is even worse than the other platform. Hence it’s necessary to find an effective method to meet the requirements of the maneuvering target tracking.

With the development of the instrument and the data processing, more and more new methods are developed in the maneuvering target tracking [3]. The extended Kalman filter (EKF) is used widely in the state determination of the target. The problem of the EKF is the linearization of the nonlinear system, which may cause large error when the system has high nonlinearity, like the tracking model of the ship [4]. Unscented Kalman filter (UKF) uses the sigma points to approximate the target state instead of the linearization process. This character makes it more adaptive than EKF in dealing with the nonlinear estimates [5,6].

Based on the principle of the strong tracking, this paper presents an improved UKF to actualize the maneuvering target tracking, which is tested with the real tracking data in a mission measured by the space tracking ship.

2. The current statistical model

The current statistical (CS) model is a common tool for the tracking target modeling. In this paper, an improved CS model is used to build the tracking model, which is

\[
\begin{align*}
\dot{x} &= a(t) \\
\dot{v} &= -aa(t) + a\bar{v} + \nu(t) \\
X(K+1) &= F(K)X(K) + G(K)\bar{v} + V(K)
\end{align*}
\]
where $\bar{a}$ is the mean value of the acceleration $a(t)$, $\alpha$ is the reciprocal of the time constant for the acceleration, $\nu(t)$ is the Gaussian white noise, $X_K$ is $n$ dimensional state vector at time $K$, $G(K)$ is the input control matrix, $F(K)$ is given by

$$F = \begin{bmatrix} 1 & T & (\alpha T - 1 + e^{-\alpha T}) / \alpha^2 \\ 0 & 1 & (1 - e^{-\alpha T}) / \alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix}$$ (3)

$V(K)$ is the dynamic noise matrix which the covariance is

$$Q(k) = \text{E}[V(k)V'(k)] = 2\alpha\sigma_a^2 \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$$ (4)

where

$$\begin{align*}
q_{11} &= (1 - e^{-2\alpha T} + 2\alpha T + \frac{2\alpha^2 T^3}{3} - 2\alpha^2 T^2 - 4\alpha T e^{-\alpha T}) / 2\alpha^4 \\
q_{12} &= (e^{-2\alpha T} + 1 - 2e^{-\alpha T} + 2\alpha Te^{-\alpha T} - 2\alpha T + \alpha^2 T^2) / 2\alpha^4 \\
q_{13} &= (1 - e^{-2\alpha T} - 2\alpha Te^{-\alpha T}) / 2\alpha^4 \\
q_{22} &= (4e^{-\alpha T} - 3e^{-2\alpha T} + 2\alpha T) / 2\alpha^3 \\
q_{23} &= (e^{-2\alpha T} + 1 - 2e^{-\alpha T}) / 2\alpha^2 \\
q_{33} &= (1 - e^{-2\alpha T}) / 2\alpha
\end{align*}$$ (5)

$$\sigma_a^2 = \begin{cases} 4\pi & a_{\max} - \bar{a}^2, \bar{a} \geq 0 \\
4\pi & \bar{a} - a_{\min}^2, \bar{a} < 0 \end{cases}$$ (6)

As a parameter with no definite physical meaning, the main function of the parameter $\bar{a}$ is to determine the variance of the acceleration. The introduction of $\bar{a}$ into the state prediction described in Eq. 2 makes the model lose the character of exponential autocorrelation. Therefore, $X_{K+1} = F(K)X(K)$ is used in the state prediction of the filtering process instead of $X_{K+1} = F(K)X(K) + G(K)\bar{a}$, which can actualize the adaptive filtering of the mean value and variance [7].

3. The strong tracking UKF method

Using the CS model as the state function in the UKF, the measurement function can be written as

$$Z_K = H(X_K) + \nu_K$$ (7)

where $Z_K$ is $m$ dimensional measurement vector at time $K$, $H$ is measurement function, $\nu_k$ is the measurement noise.

In order to improve the performance of the nonlinear filtering, Julier proposed the UKF based on the unscented transformation. In this paper, the strong tracking filter is integrated with the UKF to decrease the impact brought by the high maneuverability and the complex environment. The strong tracking has three advantages, the first is its robustness for the model’s uncertainty, the second is the strong tracking ability while the jumping phenomenon happened, the last is the moderate calculation complexity. The major difference of the UKF and the strong tracking UKF is the introduction of the fading actor. The filtering process includes 3 steps.

1) The sigma points calculation

$$w_0^{(m)} = \frac{\lambda}{n + \lambda}$$ (8)

$$w_0^{(c)} = \frac{\lambda}{n + \lambda} + (1 - \alpha^2 + \beta)$$ (9)

$$w_i^{(m)} = w_i^{(c)} = 0.5 / (n + \lambda), \ i = 1, \ldots, 2n$$ (10)
\[ X^{(i)}_K = \hat{X}_K \] (11)
\[ X^{(i)}_K = \hat{X}_K + (\sqrt{n+\lambda}P_K) , \quad i = 1, \ldots, n \] (12)
\[ X^{(i)}_K = \hat{X}_K - (\sqrt{n+\lambda}P_K) , \quad i = n+1, \ldots, 2n \] (13)

2) The state prediction and update
\[ X^{(i+1)}_{K+1} = F(K)X^{(i)}_K \] (14)
\[ r^{(i)}_{K+1} = H(X^{(i)}_K) \] (15)
\[ \tilde{X}_{K+1|K} = \sum_{i=0}^{2n} w^{(n)}_{i} X^{(i)}_{K+1|K} \] (16)
\[ \tilde{Z}_{K+1|K} = \sum_{i=0}^{2n} w^{(n)}_{i} r^{(i)}_{K+1|K} \] (17)
\[ P_{X,K+1} = \mu_{K+1} \sum_{i=0}^{2n} w^{(n)}_{i} (X^{(i)}_{K+1|K} - \tilde{X}_{K+1|K})(X^{(i)}_{K+1|K} - \tilde{X}_{K+1|K})^T + Q_{K+1} \] (18)

3) The measurement update
\[ P_{Z,K+1} = \mu_{K+1} \sum_{i=0}^{2n} w^{(n)}_{i} (r^{(i)}_{K+1|K} - \tilde{Z}_{K+1|K})(r^{(i)}_{K+1|K} - \tilde{Z}_{K+1|K})^T + R_{K+1} \] (19)
\[ P_{ZZ,K+1} = \mu_{K+1} \sum_{i=0}^{2n} w^{(n)}_{i} (X^{(i)}_{K+1|K} - \tilde{X}_{K+1|K})(r^{(i)}_{K+1|K} - \tilde{Z}_{K+1|K})^T \] (20)
\[ \tilde{X}_{K+1} = \tilde{X}_{K+1|K} + K_{K+1}(Z_{K+1} - \tilde{Z}_{K+1|K}) \] (21)
\[ K_{K+1} = P_{ZZ,K+1}P_{Z,K+1}^{-1} \] (22)
\[ P_{K+1} = P_{X,K+1} - K_{K+1}P_{Z,K+1}K_{K+1}^T \] (23)

where \( \mu \) is the fading factor, which is given by
\[
\mu_{K+1} = \begin{cases}
\mu_0, & \mu_0 \geq 1 \\
1, & \mu_0 \leq 1 \\
\frac{\text{tr}[N_{K+1}]}{\text{tr}[M_{K+1}]}, & \text{otherwise}
\end{cases}
\] (24)

where
\[
N_{K+1} = V_{K+1} - (P_{ZZ,K+1})^T (P_{K+1})^{-1} Q_{K+1} (P_{K+1})^{-1} P_{ZZ,K+1} - R_{K+1} \] (25)
\[
M_{K+1} = H_K (P_{K+1} - Q_{K+1}) H_K^T = H_K P_{K+1} H_K^T - H_K Q_{K+1} H_K^T = P_{Z,K+1} - V_{K+1} + R_{K+1} + N_{K+1} \] (26)

In Eq. 25 and 26, the matrix with superscript \( a \) means the one in the typical UKF without the introduction of \( \mu \), which is given by
\[
P_{K+1}^{a} = E[(X_{K+1} - \tilde{X}_{K+1|K})(X_{K+1} - \tilde{X}_{K+1|K})^T] \] (27)
\[
P_{Z,K+1}^{a} = H_K P_{K+1}^{a} H_K^T \] (28)
\[
P_{ZZ,K+1}^{a} = P_{K+1}^{a} H_K^T \] (29)

4. Simulation test
With the measurement information, simulation test is implemented with the strong tracking UKF and the typical UKF separately. As it’s hard to estimate the acceleration with the space tracking ship, the filtering results of the position and velocity are selected to evaluate the performance of the proposed method in this paper.

4.1. State equation
The proposed method is tested with the real data from the space tracking ship. In the filtering, the state vector is a 9 dimensional vector including the position, velocity and the acceleration component in the 3 axes of the geocentric coordinate system. From the Eq. 3 and 14, we have

\[
F(k+1) = \begin{bmatrix}
F_1 & 0 & 0 \\
0 & F_2 & 0 \\
0 & 0 & F_3
\end{bmatrix}
\]

(30)

where

\[
F_i = \begin{bmatrix}
1 & T & (\alpha T - 1 + e^{-\alpha T}) / \alpha^2 \\
0 & 1 & (1 - e^{-\alpha T}) / \alpha \\
0 & 0 & e^{-\alpha T}
\end{bmatrix}, \alpha = 1
\]

(31)

In the Eq. 6, the parameters are set as \([a_{\min}, a_{\max}, a_{\min}, a_{\max}, a_{\min}, a_{\max}] = [-10, 10, -10, 10, -10, 10] \).

4.2. Measurement equation

The measurement information is the range \(R\), azimuth \(A\) and elevation \(E\) which can be obtained directly by the ship-borne devices. The measurement equations are

\[
R = \sqrt{x^2 + y^2 + z^2}
\]

\[
E = \arcsin(y / R)
\]

(32)

\[
A = \arctan(z / x) + \begin{cases}
0, & \text{if } x > 0, z \geq 0; \\
\pi, & \text{if } x < 0, z < 0; \\
2\pi, & \text{if } x > 0, z < 0;
\end{cases}
\]

The initial state \(X_0\) is set as the 20th point of the reference orbit from a ground tracking station which can be considered as the true value of the target state. The initial covariance is given by \(P_0 = \text{diag}(1.0 \times 10^6, 1.0 \times 10^7, 1.0 \times 10^6, 1.0 \times 10^6, 1.0 \times 10^7, 1.0 \times 10^6, 1.0 \times 10^6, 1.0 \times 10^6, 1.0 \times 10^6)\). The unscented transformation parameters are assumed as \(\alpha = 0.5, \beta = 2, \kappa = -6\). Using the strong tracking UKF and the typical UKF separately, the test results are shown in Figure 1 and 2.

It’s easily to see that the strong tracking UKF method has better performance than the typical UKF. For the position estimation, both of the two methods have good results. In detail, the strong tracking UKF method has better quality. For the velocity estimation, the advantage of the strong tracking UKF is significant. With the introduction of the fading factor \(\mu\), the improved UKF method has stronger tracking ability while the jumping phenomenon happened.

Figure 1. The comparison analysis between the reference range and the filtering result (a) the strong tracking UKF method (b) the typical UKF
5. Conclusion

The strong tracking UKF method is tested with the real data in this paper. Comparing with the typical UKF, the strong tracking UKF has better performance especially in the estimation of the velocity. This work can be applied in the maneuvering target tracking with the space tracking ship.

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