Phenomenology of a massive quantum field in a cosmological quantum spacetime

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We revisit the quantum theory of a massive, minimally coupled scalar field, propagating on the Planck-era isotropic cosmological quantum spacetime which transitions to a classical spacetime in later times. The quantum effects modify the isotropic spacetime such that effectively it exhibits anisotropies. Thus, the interplay between this quantum background and modes of the field, when disregarding the backreactions, gives rise to a theory of a quantum field on an anisotropic, dressed spacetime. Different solutions are found whose components depend on the quantum fluctuations of the background geometry. We construct a formal expression for the power spectrum of the scalar field fluctuations on such an anisotropic background. It is shown that the anisotropy of this power spectrum is due to the modified frequency of the propagating quantum modes. This provides a quantitative estimate for the deviations from the isotropic power spectrum which can lead to the potential observational signatures on the cosmic microwave background. In addition, the problem of particle production when transitioning from such an effective spacetime to a classical one is reexamined. It is shown that particles are created, and the expectation value of their number operator depends on the quantum geometry fluctuations.

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I. INTRODUCTION

The study of inhomogeneous, anisotropic Universe is of particular interest in general relativity, in order to avoid postulating special initial conditions as well as the existence of particle horizon in isotropic models \[1\]. Based on the observational evidence, the consensus of the scientific community is that the spatial Universe and its expansion in time are quite isotropic. Hence, to explain this transition from anisotropic very early Universe to the late-time isotropic one, we may require a mechanism for damping down the inhomogeneity and anisotropy. Furthermore, as is common in time dependent geometries, it was suggested that this transition will give rise to creation of particles \[2, 3\].

One way to explain the anisotropies in the very early universe is to associate their emergence to the quantum effects present in the background spacetime in those early times. A suitable setting to do such an analysis is loop quantum cosmology (LQC) \[4, 5\] which is a cosmological theory inspired by Loop quantum gravity (LQG), which itself is a background independent, non-perturbative approach to quantization of general relativity\[6–8\]. In this framework, the quantum nature of the big bang has been investigated for the isotropic FLRW models \[9–12\] and the simplest anisotropic model \[13\]. These investigations show that the big bang singularity is resolved within LQC, and is replaced by a quantum bounce at which the energy density of the universe has a maximum critical value \(\rho_{\text{crit}} = 0.41\rho_{\text{Pl}}\), of the order of Planck density \(\rho_{\text{Pl}}\) (see Ref. \[14\] for a review of the recent developments in LQC).

In order to explore the properties of the propagation of the quantum matter fields on the effective background in the early universe, and their behavior when the spacetime transitions to a classical one, one needs to study these matter fields coupled to a spacetime geometry which is quantized due to LQC. Such a quantum theory of test fields propagating on cosmological quantum spacetimes has been investigated in the presence of an isotropic, flat FLRW \[15\] and the anisotropic Bianchi type-I \[16\] background geometries. There, by using a scalar field \(\phi\) as a relational time parameter (or clock variable), the Hamiltonian constraint is deparametrized, which allows for a description of the evolution of the universe in terms of this physical time parameter. The field \(\phi\) used as a clock variable is called the background matter source (as opposed to other scalar “test fields” \(\varphi\) present in the theory). This type of analysis was generalized in Ref. \[17\], to a case where an irrotational dust was introduced for the physical time variable in quantum theory. The main result of both investigations is that, an effective (semiclassical) background spacetime would emerge on which the fields propagate, and whose metric components depend on the fluctuations of the quantum geometry operators. Some interesting phenomenological features of these effective geometries were studied in \[16–18\]. A significant extension of Ref. \[15\] was recently performed, in order to generalize the standard theory of cosmological perturbations to include the Planck regime \[19\]. The strategy therein was to truncate the classical general relativity coupled to a scalar field, to a sector including homogeneous, isotropic configuration, together with the first order inhomogeneous perturbations, and the quantum theory of the truncated phase space was constructed using the techniques of LQG. This framework was applied to explore pre-inflationary dynamics of the early universe \[20, 21\]. Another framework was provided in Ref. \[22\] for quantization of linear perturbations on a quantum background spacetime, by introduction of a different choice of fundamental variables to those that are usually used in quantizing the perturbations on a fixed classical background. In particular, in the presence of a natural gauge-fixing in the theory, the old variables could be used as fundamental op-
erators, which provided a true dynamics in terms of the homogeneous part of the relational
time.

In the present paper, using LQC, we consider the effects corresponding to the propaga-
tion of a test field on a quantized early universe background, and the associated particle
production when this effective spacetime transitions to a classical one. In section III we
show that, in the presence of a test field, the effective spacetime resembles an anisotropic
spacetime due to quantum gravity effects. Then, we consider several cases where either
the mass of the test field or the geometry (i.e. its associated scale factors) are dressed and
some of the consequences. In section III we examine the phenomena of particle creation in
transitioning from the effective anisotropic spatially homogeneous background to a classical
isotropic one. In section IV we discuss the effects that emergent anisotropies can have on
the power spectrum of the CMB and its associated possible observable signatures. Finally,
in section V we will conclude and discuss some of the outlooks of our work.

II. QUANTUM FIELDS IN QUANTUM SPACETIMES

In this section, we study the quantum theory of a test field $\phi$, both massless and massive,
on a quantized FLRW background, and compare it to the behavior of the same test field on
a classical anisotropic Bianchi I metric. In both cases, the background geometry is coupled
to a massless scalar field $\phi$, which plays the role of the physical internal time. Hence, the
background elements consist of the background spacetime and the massless field $\phi$ as the
clock variable, while the test field $\phi$ is propagating on this background spacetime.

We will see that the resulting evolution (Schrödinger) equations for both cases bare a
striking resemblance to each other, an observation that leads us to the rest of our analysis
about the several possible scenarios about the behavior of the matter field $\phi$ on the effective
FLRW spacetime. To this end, we start with the classical Bianchi I metric, and using that,
work our way towards the quantum FLRW model, since it is easier to go from a more general
anisotropic model to an isotropic one.

A. Quantum matter propagating on a classical Bianchi I spacetime

A Bianchi type-I spacetime, is represented by the anisotropic background metric

$$g_{ab}dx^a dx^b = -N^2 (x^0) (dx^0)^2 + \sum_{i=1}^{3} a_i^2 (x^0) (dx^i)^2,$$

with $N(x^0)$ being the lapse function and $a_i$ the scale factors. The metric (2.1) is written in
coordinates $(x_0, x^i)$, in which $x^i \in \mathbb{T}^3$ (3-torus with coordinates $x^i \in (0, \ell_j)$), and $x^0 \in \mathbb{R}$ is a
generic time coordinate. Later, we can set $x^0 = \phi$ so that the evolution becomes relational.
Furthermore, we consider a real (inhomogeneous), minimally coupled, free scalar test field
$\varphi(x_0, x)$, with mass $m$, propagating on this background spacetime. The Hamiltonian of the
scalar test field can be written as the sum of the Hamiltonians $H_k(x^0)$ of the decoupled
harmonic oscillators, each of which written in terms of a pair $(q_k, p_k)$, as

$$H_{\varphi}(x^0) := \sum_{k \in \mathcal{L}} H_k(x^0) = \frac{N_{x^0}(x^0)}{2|a_1 a_2 a_3|} \sum_{k \in \mathcal{L}} \left( \frac{p_k^2}{2} + \omega_k^2(x^0) q_k^2 \right).$$

(2.2)
Here, \( \mathcal{L} = \mathcal{L}_+ \cup \mathcal{L}_- \) is a 3-dimensional lattice spanned by \( k = (k_1, k_2, k_3) \in (2\pi \mathbb{Z}/\ell)^3 \), with \( \mathbb{Z} \) being the set of integers and \( \ell^3 = \ell_1 \ell_2 \ell_3 \) \[15, 16]\:

\[
\forall k \in \mathcal{L}_+ : \mathcal{L}_+ := \{k_3 > 0\} \cup \{k_3 = 0, k_2 > 0\} \cup \{k_2, k_3 = 0, k_1 > 0\}, \quad \text{and} \quad -k \in \mathcal{L}_-. \quad (2.3)
\]
The The quantization of a mode \( \psi_\mathcal{L} \) of the field satisfies the relation \( \{q_k, p_{k'}\} = \delta_{kk'} \). Moreover, \( \omega_k (x^0) \) is a time-dependent frequency which is defined by

\[
\omega_k^2 (x^0) := |a_1 a_2 a_3|^2 \left[ \sum_{i=1}^{3} \left( \frac{k_i}{a_i} \right)^2 + m^2 \right]. \quad (2.4)
\]
In other words, \( q_k \) is the field amplitude for the mode characterized by \( k \), satisfying the Klein–Gordon equation \((\Box - m^2) \varphi = 0\), which is the equation of motion obtained from this Hamiltonian \( (2.2) \).

Since the background is left as a classical one, one needs to only quantize the test field. The quantization of a mode \( k \) of the test field \( \varphi \) resembles that of a quantum harmonic oscillator with the Hilbert space \( \mathcal{H}_\psi^{(k)} = L_2 (\mathbb{R}, d\varphi_k) \). The canonical variables are promoted to operators on this Hilbert space as \( \hat{q}_k \psi (q_k) = q_k \psi (q_k) \) and \( \hat{p}_k \psi (q_k) = -i\hbar \partial / \partial q_k \psi (q_k) \), and the time evolution of any state \( \psi (q_k) \) is generated by the Hamiltonian operator \( \hat{H}_k \) via the Schrödinger equation

\[
i\hbar \partial_{x^0} \psi (x^0, q_k) = \frac{\ell^3 N_{x^0}}{2V} \left[ \hat{p}_k^2 + \omega_k^2 q_k^2 \right] \psi (x^0, q_k), \quad (2.5)
\]
where \( V \) is the physical volume\(1\) of the universe which is given by

\[
V = \ell^3 |a_1 a_2 a_3|. \quad (2.6)
\]

Denoting the Bianchi I variables with a tilde, using \( (2.4) \) and \( (2.6) \) in \( (2.5) \), and setting \( x^0 = \phi \), one can see that the evolution of the quantum state \( \psi \) with respect to the internal physical time \( \phi \) for a mode \( k \) of a test field on a classical Bianchi I background \( (2.1) \) is described by the Schrödinger equation

\[
i\hbar \partial_{\phi} \psi (\phi, q_k) = \frac{\tilde{N}_{\phi}}{2 |\tilde{a}_1 \tilde{a}_2 \tilde{a}_3|} \left[ \hat{p}_k^2 + \left( \sum_{i=1}^{3} \frac{\tilde{k}_i^2}{\tilde{a}_i^2} + \tilde{m}^2 \right) (\tilde{a}_1 \tilde{a}_2 \tilde{a}_3)^2 \hat{q}_k^2 \right] \psi (\phi, q_k). \quad (2.7)
\]

where \( \tilde{a}_i \)'s (with \( i = 1, 2, 3 \)) are now functions of the relational time \( \phi \), i.e., \( \tilde{a}_i = \tilde{a}_i (\phi) \). This is the evolution equation of a quantum test field propagating on a classical Bianchi I geometry, which we will be comparing to the one with the test field propagating on a quantum FLRW spacetime, which will be derived in the next subsection.

B. Quantum matter propagating on a quantum FLRW spacetime

Now we will consider the propagation of a quantum test field over a quantum FLRW spacetime. Using the analysis of the previous subsection, one can go to an isotropic regime

\[1\] As in LQC of Bianchi I model, we fix a fiducial cell \( \mathcal{V} \), and take its edges to lie along the integral curves of the fiducial triad, with coordinate lengths \( \ell_1, \ell_2, \ell_3 \), so that the volume of \( \mathcal{V} \) is \( V = \ell_1 \ell_2 \ell_3 = \ell^3 \). Then, after parametrization of the gravitational phase space by a pair \((c', p_i)\), the physical volume reads \( V = \sqrt{p_1 p_2 p_3} = \ell^3 |a_1 a_2 a_3| \). Note that, the relation between the phase space variables is given by \( p_i = \epsilon_{ijk} \ell_j \ell k a_j a_k \) (\( \epsilon_{ijk} \) is the Levi-Civita symbol) \[13\].
by setting $a_1 = a_2 = a_3 \equiv a(x^0)$. If, for this case, we choose a harmonic time coordinate, $x^0 = \tau$, then the corresponding lapse $N_{\phi}$ will be related to the lapse $N_{\phi}$ via

$$N_{\phi} = \left(\frac{\ell^3}{p_{\phi}}\right) N_{\tau},$$

$$N_{\tau} = |a_1 (\phi) a_2 (\phi) a_3 (\phi)|.$$  \hfill (2.8)

Given the isotropic nature of the background in this case, the lapse function becomes $N_{\tau} = a^3(\tau)$, and the the Hamiltonian \([2.2]\) of the test field reduces to

$$H_{\varphi}^{(iso)} = \sum_k H_{\tau,k} := \frac{1}{2} \sum_k \left( p_k^2 + \omega_{\tau,k}^2 q_k^2 \right).$$  \hfill (2.10)

Here, the time-dependent frequency for each mode, is obtained by substituting $a_1 = a_2 = a_3 \equiv a(\tau)$ in Eq. \([2.4]\), as $\omega_{\tau,k}^2 \equiv k^2 a^4 + m^2 a^6$. Like before, the background elements consist of the spacetime plus a background matter source in form of a massless scalar field $\phi(\tau)$ which serves as an internal physical time parameter \([9]\). The propagating test matter field is denoted by the scalar field $\varphi$. In this case, not only $\varphi$ but also the background spacetime is to be quantized. This will lead to a theory of a quantum test field $\varphi$, whose wave function, denoted by $\psi$, evolves with respect to the internal time variable $\phi$ on the background quantum geometry. Due to neglecting backreaction, for a given mode $k$, the full kinematical Hilbert space of the system is given by $H_{\text{kin}}^{(k)} = H_{\text{kin}}^0 \otimes H_{\varphi}^{(k)}$, where $H_{\text{kin}}^0 = H_{\text{grav}} \otimes H_{\phi}$ is the background Hilbert space consisting of the Hilbert space of the geometry and the scalar clock variable $\phi$ (again with no backreaction). The matter sectors are quantized using the Schrödinger representation, with the Hilbert spaces $H_{\varphi}^{(k)} = L_2(\mathbb{R}, dq_k)$ and $H_{\phi} = L_2(\mathbb{R}, d\phi)$. For any physical state $\Psi(\nu, q_k, \phi) \in H_{\phi}^{(k)}$, with $\nu$ the quantum number related to geometry (see below), the action of the full Hamiltonian constraint operator $\hat{C}_{\tau,k}$ for the $k$'th mode is written as \([13]\)

$$\hat{C}_{\tau,k} \Psi = \left( N_{\tau} \hat{C}_{\phi} + \hat{H}_{\tau,k} \right) \Psi = 0,$$  \hfill (2.11)

where $\hat{C}_{\phi} = \hat{C}_{\text{grav}} + \hat{C}_{\phi}$ is the background scalar constraint operator and

$$\hat{H}_{\tau,k} = \frac{1}{2} \left[ \hat{p}_k^2 + \left( k^2 a^4 + m^2 a^6 \right) \hat{q}_k^2 \right],$$  \hfill (2.12)

is the Hamiltonian of the test field $\varphi$. The background term $\hat{C}_{\phi}$ is well-defined on $H_{\text{kin}}^0$, so that, the physical states $\Psi_o(\phi, \nu) \in H_{\text{kin}}^0$ are those lying on the kernel of $\hat{C}_{\phi}$, and are solutions to a self-adjoint Hamiltonian constraint equation of the form \([8, 11]\)

$$N_{\tau} \hat{C}_{\phi} \Psi_o(\nu, \phi) = -\frac{\hbar^2}{2\ell^3} (\partial_\phi^2 + \Theta) \Psi_o(\nu, \phi) = 0,$$  \hfill (2.13)

where $\Theta$ is a difference operator that acts on $\Psi_o$ and involves only the gravitational sector $\nu$ but not $\phi$. The quantum number $\nu$ is the eigenvalue of the volume operator of the isotropic background geometry $\hat{V}_o = \ell^3 a^3$, which acts on $\Psi_o$ as $\hat{V}_o \Psi_o(\nu, \phi) = 2\pi \gamma \ell^3 |\nu| \Psi_o(\nu, \phi)$.

By restricting to the space spanned by the positive frequency solutions to Eq. \((2.13)\), one can write a Schrödinger equation for the background sector \([9]\)

$$-i\hbar \partial_{\phi} \Psi_o(\nu, \phi) = \hbar \sqrt{\Theta} \Psi_o(\nu, \phi) =: \hat{H}_o \Psi_o(\nu, \phi).$$  \hfill (2.14)
This constructs the physical Hilbert space $\mathcal{H}^\circ_{\text{phys}}$ of the geometry, endowed with the scalar product
\[
\langle \Psi_\circ | \Psi'_\circ \rangle = \sum_\nu \Psi^*_\circ(\nu, \phi_0)\Psi'_\circ(\nu, \phi_0),
\]
for any “instant” of internal time $\phi_0$. By substituting Eqs. (2.13) and (2.12) in Eq. (2.11), one obtains a Schrödinger equation for the full state $\Psi(\nu, q_k, \phi)$ of the background-test field system as
\[
-\imath \hbar \partial_\phi \Psi(\nu, q_k, \phi) \approx (\hat{H}_o - \hat{H}_{\phi, k}) \Psi(\nu, q_k, \phi),
\]
with
\[
\hat{H}_{\phi, k} := \ell^3 \hat{H}_o^{-1} \hat{H}_{r, k} \hat{H}_o^{-1/2}.
\]
In deriving Eq. (2.16), we have used a test field approximation, by disregarding the back-reaction of the matter on the background homogeneous geometry, so that, the term $\hat{H}_{r, k}$ can be seen as the Hamiltonian of a small perturbation added to the geometrical sector $\hat{H}_o^2$. It should be noted that, in the present formalism, $\phi$ is well-suited to be considered an emergent time in quantum theory. So, Eq. (2.16) indicates a quantum evolution of the state $\Psi(\nu, q_k, \phi)$ with respect to the internal physical time $\phi$, which depends on the $k$'th mode of the test field $\varphi$, and the quantum geometry encoded in $\nu$.

To find the evolution equation of the wave function $\psi$ of the test field on the this quantum FLRW background spacetime, we work in an interaction picture. This is achieved by introducing $\Psi_{\text{int}}(\nu, q_k, \phi) = \exp[-(\imath \hat{H}_o/\hbar)(\phi - \phi_0)]\Psi(\nu, q_k, \phi)$ in Eq. (2.16), which regards $\hat{H}_o$ as the Hamiltonian of the heavy degree of freedom, and $\hat{H}_{\phi, k}$ as the Hamiltonian of the light degree of freedom (i.e., a perturbation term). The relational time $\phi_0$ is, as before, any fixed instant of time. As mentioned before, by disregarding the backreaction of the test field on geometry we can approximate $\Psi(\nu, q_k, \phi) = \Psi_\circ(\nu, \phi) \otimes \psi(q_k, \phi)$. This implies that the geometry evolves by Hamiltonian $\hat{H}_o$ as $\hat{\rho}_\phi \Psi_{\circ}(\nu, \phi) = -\imath \hbar \partial_\phi \Psi_{\circ}(\nu, \phi) = \hat{H}_o \Psi_{\circ}(\nu, \phi)$ for any $\Psi_{\circ} \in \mathcal{H}^\circ_{\text{kin}}$ in the Heisenberg picture, thus, we can write $\Psi_{\circ}(\nu, \phi) = \exp[(\imath \hat{H}_o/\hbar)(\phi - \phi_0)]\Psi(\nu, \phi_0)$. This leads to the factorization of $\Psi_{\text{int}}$ as $\Psi_{\text{int}}(\nu, q_k, \phi) = \Psi_{\circ}(\nu, \phi_0) \otimes \psi(q_k, \phi)$. Now, plugging $\Psi_{\text{int}}$ into (2.16) and projecting both sides on $\Psi_{\circ}(\nu, \phi_0)$ yields an evolution equation for $\psi(q_k, \phi)$ as
\[
\imath \hbar \partial_\phi \psi(q_k, \phi) = \frac{\ell^3}{2} \left[ \langle \hat{H}_o^{-1} \rangle \hat{p}_k^2 + \sum_i k_i^2 \left( \hat{H}_o^{-2} \hat{a}^\dagger(\phi) \hat{a}^\dagger(\phi) \right) + m^2 \left( \hat{H}_o^{-2} \hat{a}^\dagger(\phi) \hat{a}^\dagger(\phi) \right) \right] \psi(q_k, \phi)
\]
where $\langle \cdot \rangle$ denote the expectation value with respect to the quantum state $\Psi_{\circ}(\nu, \phi_0)$. In this interaction picture, the state $\Psi_{\circ}(\nu, \phi_0)$ of the quantum geometry is described in Heisenberg picture which is frozen at time $\phi = \phi_0$, while the geometrical operator $\hat{a}(\phi) = \hat{V}_o^{1/3}(\phi)/\ell$ evolves in time as $\hat{a}(\phi) = \exp[-(\imath \hat{H}_o/\hbar)(\phi - \phi_0)] \hat{a} \exp[(\imath \hat{H}_o/\hbar)(\phi - \phi_0)]$. In addition, the state of the test field $\psi(q_k, \phi)$, unlike $\Psi_{\circ}$, evolves in time $\phi$ due to the evolution equation (2.18) in Schrödinger picture.

C. Emergence of effective anisotropies from quantum FLRW background

A comparison between, and matching of the evolution equations (2.18) and (2.17) reveals that due to the quantum effects, the original FLRW metric can be interpreted to be now
replaced by an effective anisotropic geometry, with the lapse function \( \tilde{N}_\phi \), scale factors \( \tilde{a}_i \) and the momentum \( \tilde{p}_\phi \), which acts as the background for propagation of the quantum modes. In other words, the quantum FLRW resembles an emergent anisotropic classical Bianchi I model with tilde variables. Note that we have considered the general case where both mass \( \tilde{m} \) and the wave vector \( \tilde{k} \) in (2.7) can be different (or dressed) compared to the original FLRW system, as a consequence of the quantum spacetime effects. The relations between variables of this effective dressed geometry (classical Bianchi I with tilde) and those of the isotropic quantum (FLRW) geometry are seen to be

\[
\tilde{N}_\phi = \ell^3 |\tilde{a}_1\tilde{a}_2\tilde{a}_3| \left\langle \hat{H}_o^{-1} \right\rangle,
\]

\[
\sum_{i=1}^3 \left( \frac{\tilde{k}_i}{\tilde{a}_i} \right)^2 \tilde{N}_\phi \frac{(a_1\tilde{a}_2\tilde{a}_3)^2}{|\tilde{a}_1\tilde{a}_2\tilde{a}_3|} = \sum_{i=1}^3 k_i^2 \ell^3 \left\langle \hat{H}_o^{-\frac{1}{2}}\hat{a}^4\hat{H}_o^{-\frac{1}{2}} \right\rangle,
\]

\[
\tilde{N}_\phi \tilde{m}^2 \frac{(a_1\tilde{a}_2\tilde{a}_3)^2}{|\tilde{a}_1\tilde{a}_2\tilde{a}_3|} = \ell^3 m^2 \left\langle \hat{H}_o^{-\frac{1}{2}}\hat{a}^6\hat{H}_o^{-\frac{1}{2}} \right\rangle,
\]

in which \( \hat{a} = \hat{a}(\phi) \), and \( \left\langle \hat{H}_o^{-1} \right\rangle = (\tilde{p}_\phi)^{-1} \). It is seen that we have a system of five equations (2.19), (2.20) and (2.21) (that is, if we only identify the terms with \( \tilde{k}_i \) and \( k_i \) with each other) with eight unknowns (the tilde variables), which constitutes an underdetermined system. To be able to solve the system we need three more equations (i.e. conditions) on these variables. These conditions can be imposed on either of \( \tilde{m}, \tilde{k}_i, \tilde{a}_i \) or \( \tilde{N}_\phi \). In what follows, we will investigate some of these conditions and their physical consequences on the solutions by dividing the discussion into three cases: massless test field \( m = 0 = \tilde{m} \), undressed mass \( \tilde{m} = m \), or dressed mass \( \tilde{m} \neq m \).

1. Massless case \( (m = 0 = \tilde{m}) \)

If \( m = 0 \), we will immediately obtain \( \tilde{m} = 0 \). But the situation is not changed and we still need three more conditions to be able to solve the system above, since we have also eliminated one of our equations. The simplest option is to take the conditions \( k_i^2 = \tilde{k}_i^2 \) (for each \( i \)). Then we will get \( \tilde{a}_1^2 = \tilde{a}_2^2 = \tilde{a}_3^2 = \tilde{a}^2 \) and from this one obtains

\[
\tilde{a}^4 = \frac{\left\langle \hat{H}_o^{-\frac{1}{2}}\hat{a}^4(\phi)\hat{H}_o^{-\frac{1}{2}} \right\rangle}{\left\langle \hat{H}_o^{-1} \right\rangle},
\]

\[
\tilde{N}_\phi = \ell^3 \left\langle \hat{H}_o^{-\frac{1}{2}} \right\rangle \frac{1}{4} \left\langle \hat{H}_o^{-\frac{1}{2}}\hat{a}^4(\phi)\hat{H}_o^{-\frac{1}{2}} \right\rangle \frac{3}{4} = \ell^3 \tilde{p}_\phi^{-1} \tilde{a}^3.
\]

A same result is yielded if, instead, one takes the assumption that the underlying dressed geometry is an effective FLRW metric, i.e., \( \tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3 = \tilde{a}(\phi) \) (two conditions) and that one of the components of \( \tilde{k} \) is equal to the corresponding component of the undressed \( k \) [13].

2. Undressed mass \( (m = m \neq 0) \)

Since having an undressed mass gives us one of the three additional conditions needed, we are left with two conditions on either \( \tilde{k}_i \) or \( \tilde{a}_i \) (forgetting about \( \tilde{N}_\phi \) since its conditions
can be found from those of $\tilde{a}_i$). In what follows, we express the relations between $\tilde{k}_i, \tilde{a}_i$, etc., using some arbitrary coefficients $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \rho_{ij}$ and $\sigma_{ij}$.

**Case 1:** Let us first impose the two remaining conditions just on $\tilde{k}_i$, and between the tilde variables themselves. In general we can have

$$\tilde{k}_1 = \alpha_{12} \tilde{k}_2, \quad \tilde{k}_2 = \alpha_{23} \tilde{k}_3. \quad (2.24)$$

If we also naturally define the relation between $\tilde{k}_1$ and $\tilde{k}_3$ as $\tilde{k}_1 = \alpha_{13} \tilde{k}_3$, and, furthermore, define the inverse relations $\tilde{k}_2 = \alpha_{21} \tilde{k}_1, \quad \tilde{k}_3 = \alpha_{32} \tilde{k}_2, \quad \tilde{k}_3 = \alpha_{31} \tilde{k}_1$, we obtain

$$\alpha_{ij} = \frac{1}{\alpha_{ji}}, \quad (i, j, k = 1, 2, 3). \quad (2.25)$$

$$\alpha_{ij} = \alpha_{ik} \alpha_{kj}, \quad (i, j, k = 1, 2, 3; i \neq j, i \neq k, j \neq k), \quad (2.26)$$

These can then be used to write equations (2.19)-(2.21) as

$$\tilde{N}_\phi = \ell^3 \left[ \frac{1}{2} \left\langle \hat{H}_o^{-\frac{1}{2}} \frac{1}{2} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle \right], \quad (2.27)$$

$$\tilde{a}_i^6 = \frac{1}{2} \sum_{j, k = 1}^3 |\epsilon_{ijk}| \alpha_{ij}^2 \alpha_{ik}^2 \frac{k_j k_k}{k_i^2} \frac{\left\langle \hat{H}_o^{-\frac{1}{2}} \frac{1}{2} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle}{\left\langle \hat{H}_o^{-1} \right\rangle}, \quad (2.28)$$

$$\tilde{k}_i^6 = \frac{1}{2} \sum_{j, k = 1}^3 |\epsilon_{ijk}| \alpha_{ij}^2 \alpha_{ik}^2 \frac{k_j k_k}{k_i^2} \frac{\left\langle \hat{H}_o^{-\frac{1}{2}} \frac{1}{2} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle^3}{\left\langle \hat{H}_o^{-1} \right\rangle^2}, \quad (2.29)$$

where $i = 1, 2, 3$. The above equations may seem like a rainbow metric, given that the components of the dressed metric are now mode dependent (cf. e.g. [18, 23]). In such a case, the modified scale factors and consequently the metric, acquire a dependence on the wave vectors. Thereby, the scale factor that the propagating wave “sees” in each direction, depends on its wave vector $k$ in that direction. But, it should be noted that, the (dressed) wave vector associated to the dressed scale factors $\tilde{a}$ is the $\tilde{k}$ and not $k$. Thus, if one writes the above equations in terms of $\tilde{a}_i$ and the dressed mode components $\tilde{k}_i$, one finds out that this is not the case. Nevertheless, these equations offer a detailed insight into the dependence of the unmodified wave vector components and the modified scale factors.

**Case 2:** Let us impose the conditions between only two components of $\tilde{k}_i$ and $k_i$. Without loss of generality, we can assume $i = 1, 2$ and hence

$$\tilde{k}_1 = \beta_{11} k_1, \quad \tilde{k}_2 = \beta_{22} k_2. \quad (2.30)$$
Then we will have

\[ N_\phi = \ell^3 \left\langle \hat{H}_o^{-1} \right\rangle^{\frac{1}{2}} \left\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle^{\frac{1}{2}}, \]  
(2.31)

\[ \hat{a}_i^2 = \beta_i^2 \left( \frac{\left( \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right)^{\frac{1}{2}}}{\left\langle \hat{H}_o^{-1} \right\rangle} \right)^2, \]  
(2.32)

\[ \hat{a}_3^2 = \frac{1}{\beta_{22}^2 \beta_{11}^2} \left( \frac{\left( \hat{H}_o^{-\frac{1}{2}} \hat{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \right)^{\frac{1}{2}}}{\left\langle \hat{H}_o^{-1} \right\rangle} \right)^2, \]  
(2.33)

\[ \tilde{k}_3^2 = \frac{k_3^2}{\beta_{11}^2 \beta_{22}^2} \left( \frac{\left( \hat{H}_o^{-\frac{1}{2}} \hat{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \right)^{\frac{1}{2}}}{\left\langle \hat{H}_o^{-1} \right\rangle} \right)^2. \]  
(2.34)

As can be seen from the scale factors above, the emergent dressed spacetime becomes anisotropic. Hence in this case, we have an effective theory in which a field with a dressed wave vector \( \tilde{k} \) propagates on a semiclassical dressed spacetime whose isotropy is broken. Furthermore, the form of this isotropy breaking depends on the components of the wave vector \( \tilde{k} \), which itself depends on the form of dependence of \( \tilde{k} \) on \( k \) in (2.30).

**Case 3:** Here we put the conditions just on \( \tilde{a}_i \), and again between the tilde variables themselves. Thus, in general we can have

\[ \tilde{a}_1 = \gamma_{12} \tilde{a}_2, \quad \tilde{a}_2 = \gamma_{23} \tilde{a}_3, \]  
(2.35)

which, using (2.25) and (2.26), leads to

\[ N_\phi = \ell^3 \left\langle \hat{H}_o^{-1} \right\rangle^{\frac{1}{2}} \left\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle^{\frac{1}{2}}, \]  
(2.36)

\[ \tilde{a}_i^2 = \frac{1}{2} \sum_{j,k=1}^{3} |\epsilon_{ijk}| \gamma_{ij}^2 \gamma_{ik} \left\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle \left\langle \hat{H}_o^{-1} \right\rangle, \]  
(2.37)

\[ \tilde{k}_i^2 = \frac{1}{2} k_i^2 \sum_{j,k=1}^{3} |\epsilon_{ijk}| \gamma_{ij} \gamma_{ik} \left\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \right\rangle \left\langle \hat{H}_o^{-1} \right\rangle, \]  
(2.38)

with \( i = 1, 2, 3 \). These solutions correspond to an effective theory in which the quantum field propagates on an anisotropic dressed background with \( \tilde{a}_i \)'s given by (2.36). It is seen that such as field acquires an effective wave vector \( \tilde{k} \) whose components \( \tilde{k}_i \) are modified with respect to the original \( k \), due to quantum fluctuations of the original isotropic background.

**Case 4:** Another type of mixed condition could be such that we have one relation between \( \tilde{a}_i \) and \( \tilde{a}_j \), and one between \( \tilde{k}_i \) and \( k_i \), of which Ref. [17] is an example. In this case we can, e.g., have

\[ \tilde{a}_1 = \sigma_{12} \tilde{a}_2, \quad \tilde{k}_3 = \sigma_{33} k_3. \]  
(2.39)
Using (2.25) and, this leads to

\[
\tilde{N}_\phi = \ell^3 \left( \frac{\hat{H}_o^{-1}}{(a_1 a_2 a_3)^2} \right)^{\frac{1}{2}} \left( \tilde{\hat{H}}_o - \frac{1}{2} \hat{a}_4 (\phi) \hat{H}_o \right) \right),
\]

\[
\tilde{a}_i^4 = \left( \frac{\sigma_{ii}}{\sigma_{33}} \right)^2 \frac{\left( \hat{H}_o \right)^{-\frac{1}{2}} \hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}}}{\hat{H}_o^{-1}},
\]

\[
\tilde{\tilde{a}}_3^4 = \sigma_{33}^4 \left( \hat{H}_o \right)^{-\frac{1}{2}} \hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}},
\]

\[
\tilde{\tilde{a}}_i^4 = k_i^4 \left( \frac{\sigma_{ii}}{\sigma_{33}} \right)^2 \frac{\left( \hat{H}_o \right)^{-\frac{1}{2}} \hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}}}{\hat{H}_o^{-1}},
\]

where as in (2.25), we have naturally defined \( \sigma_{ij} = \sigma_{ji}^{-1} \). Here again we have a field with an effective \( k \), propagating on an anisotropic dressed background. However, as can be seen, one of the direction of \( \tilde{k} \) (here \( \tilde{k}_3 \)) plays a rather more special role as opposed to the situation in Case 3.

Note that in all the above four cases, in the presence of the nonvanishing undressed mass, the lapse functions \( \tilde{N}_\phi \) are identical.

3. Dressed mass (\( \tilde{m} \neq m; m, \tilde{m} \neq 0 \))

The situation here is a bit different. Since we have removed the condition on mass, we now need three conditions on \( \tilde{a}_i \) and \( \tilde{k}_i \). Let us consider some of the possible cases below.

**Case 1:** We first impose these remaining conditions just on \( \tilde{k}_i \),

\[
\tilde{k}_i = \alpha_{ii} k_i, \quad (i = 1, 2, 3).
\]

This will lead to

\[
\tilde{N}_\phi = \frac{\ell^3}{(a_1 a_2 a_3)^2} \left( \frac{\hat{H}_o^{-1}}{\hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}}} \right)^{\frac{1}{2}} \frac{\left( \hat{H}_o^{-\frac{1}{2}} \hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}} \right)^{\frac{1}{2}}}{\hat{H}_o^{-1}},
\]

\[
\tilde{a}_i^4 = \left( \frac{\alpha_{ii}}{\alpha_{jj} \alpha_{kk}} \right)^2 \frac{\hat{H}_o^{-\frac{1}{2}} \hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}}}{\hat{H}_o^{-1}},
\]

\[
\tilde{m}^4 = m^4 \left( \alpha_{ii} a_{22} a_{33} \right)^2 \frac{\hat{H}_o^{-\frac{1}{2}} \hat{a}_4 (\phi) \hat{H}_o^{-\frac{1}{2}}}{\hat{H}_o^{-1}}.
\]

Note that as a special subcase, when \( \alpha_{ii} = 1 \), one obtains an undressed wave vector \( \tilde{k}_i = k_i \),
a dressed background with a scale factor \( \tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3 = \tilde{a} \) identical to (2.22)

\[
\tilde{a}^4 (\phi) = \frac{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle},
\]

(2.48)

and a dressed mass

\[
\tilde{m}^2 = m^2 \frac{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle} \frac{\langle \hat{H}_o^{-1} \rangle^{\frac{1}{2}}}{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle^{\frac{1}{2}}}.
\]

(2.49)

This corresponds to an isotropic dressed background spacetime with the scale factor (2.22) over which a test field with an undressed wave-vector \( \vec{k} = (k_1, k_2, k_3) \) propagates. Moreover, the mass in this case is now dressed as in (2.49).

A special subcase for this case is when the field modes propagate only along the \( z \)-direction, i.e., \( \tilde{k}_1 = \tilde{k}_2 = 0 \) and \( \kappa = k_3 \), and hence, \( \tilde{k}_1 = \tilde{k}_2 = 0 \) and \( \tilde{k} = \tilde{k}_3 = \alpha_{33} k_3 \). Then, by setting \( \alpha_{11} = \alpha_{22} = 1 \), one obtains a dressed background with an emergent anisotropy as a result of the scale factor \( \tilde{a}_3 \). In that case, \( \tilde{a}_3 = \tilde{a}_1 / \alpha_{33} = \tilde{a}_2 / \alpha_{33} \). Similar argument was presented in Ref. [17]. However, in our case, in addition to the scenario presented in [17], the anisotropy can still exist even if the field is massless.

**Case 2:** Another choice is to impose two conditions between \( \tilde{a}_i \), and one between \( \tilde{k}_i \) and \( k_i \), as, e.g.,

\[
\tilde{a}_1 = \rho_{12} \tilde{a}_2, \quad \tilde{a}_2 = \rho_{23} \tilde{a}_3, \quad \tilde{k}_3 = \rho_{33} k_3
\]

(2.50)

This way one gets

\[
\tilde{N}_\phi = \frac{\ell^3}{\rho_{12} \rho_{23} \rho_{33}} \frac{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle^{\frac{3}{2}}}{\langle \hat{H}_o^{-1} \rangle^{\frac{1}{2}}},
\]

(2.51)

\[
\tilde{a}_3^4 = \frac{1}{\rho_{33}^2 \rho_{12} \rho_{23}} \frac{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle},
\]

(2.52)

\[
\tilde{k}_1^2 = (\rho_{12} \rho_{23} \rho_{33})^2 k_1^2,
\]

(2.53)

\[
\tilde{k}_2^2 = (\rho_{23} \rho_{33})^2 k_2^2,
\]

(2.54)

\[
\tilde{m}^2 = m^2 \frac{\rho_{12} \rho_{23} \rho_{33}^3}{\rho_{33}^2 \rho_{12} \rho_{23}} \frac{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^6(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle \langle \hat{H}_o^{-1} \rangle^{\frac{1}{2}}}{\langle \hat{H}_o^{-\frac{1}{2}} \tilde{a}^4(\phi) \hat{H}_o^{-\frac{1}{2}} \rangle^{\frac{1}{2}}},
\]

(2.55)

It is seen that in this case, assuming nonvanishing coefficients \( \rho_{12}, \rho_{23}, \rho_{33} \) represents the propagation of a quantum field with a dressed wave vector, over an anisotropic dressed background. There is a special direction, in this case \( \tilde{k}_3 \), which is the distinct source of the emergent anisotropy.

### III. QUANTUM PARTICLE PRODUCTION

In this section, we investigate the quantum gravity effects, due to the emerged anisotropy of the background geometry, on the creation of particles from Planck regime. Let us rewrite
the Schrödinger equation (2.7) as

\[ i\hbar \partial_\phi \psi = \left( \frac{\hat{p}_k^2}{2M} + \frac{1}{2} M \tilde{\omega}_k^2(\phi) \hat{q}_k^2 \right) \psi, \quad (3.1) \]

where the (internal time) \( \phi \)-dependent frequency is given by

\[ \tilde{\omega}_k^2(\phi) = N_\phi^2 \left( \sum_{i} \tilde{k}_i^2 \tilde{a}_i^2 + \tilde{m}^2 \right), \quad (3.2) \]

and \( M \) is defined as

\[ M = \frac{|\tilde{a}_1 \tilde{a}_2 \tilde{a}_3|}{N_\phi} = \frac{1}{\ell^3 \langle \hat{H}_o^{-1} \rangle}. \quad (3.3) \]

Once the corresponding solutions to \( \tilde{N}(\phi) \), \( \tilde{k}(\phi) \), \( \tilde{a}_i(\phi) \) and \( \tilde{m}(\phi) \) are known in terms of the original quantum geometry fluctuations, the frequency of each mode of the field can be computed. It turns out that in all of the cases of the solutions we discussed in the previous section, despite the emergence of various types of the dressed geometries (e.g., being either isotropic or anisotropic), because of the assumption of equality between Eqs. (2.7) and (2.18), the dispersion relation (3.2) is always of the form

\[ \tilde{\omega}_k^2(\phi) = \ell^3 \langle \hat{H}_o^{-1} \rangle \left[ k^2 \tilde{a}^4 + \tilde{m}^2 \tilde{a}^6 \right]. \quad (3.4) \]

in which \( \tilde{a}(\phi) \) is given by Eq. (2.48) and the dressed mass \( \tilde{m} \) is given by Eq. (2.49). This indicates that, for all possible solutions of the emergent dressed background, the dispersion relation of the field propagating on it is modified and takes the same form as in the isotropic dressed background (2.22), but with the nonzero dressed mass (2.49).

The (classical) equation of motion for each mode \( k \) of the scalar field, given by the dressed Hamiltonian on the right hand side of Eq. (3.1), is

\[ \ddot{q}_k + \frac{\dot{M}}{M} \dot{q}_k + \tilde{\omega}_k^2 q_k = 0, \quad (3.5) \]

in which a dot denotes a derivative with respect to the internal time \( \phi \). We are interested in studying the particle creation mechanism using the approach presented in Ref. [24]. From Eq. (3.3) we have that \( M = 1/(\ell^3 \langle \hat{H}_o^{-1} \rangle) \), so the term \( \dot{M}/M \) in (3.5) vanishes to a good approximation if \( M \) varies very slowly with time. Although the components of the dressed emergent metric \( \tilde{g}_{ab} \) depend on the fluctuations of the background quantum geometry operators, it is now a smooth tensor field so that it is straightforward to pass to a new harmonic time coordinate \( \tilde{\tau} \) (with a lapse function \( \tilde{N}_\tau = |\tilde{a}_1 \tilde{a}_2 \tilde{a}_3| \)). By setting \( \tilde{N}_\tau d\tilde{\tau} = \tilde{N}_\phi d\phi \), we can simplify our equation of motion (3.5) through the time \( \tilde{\tau} \):

\[ d\tilde{\tau} = \ell^3 \langle \hat{H}_o^{-1} \rangle d\phi. \quad (3.6) \]

Now, the \( \tilde{\tau} \)-evolution of the wave function \( \psi \) is given by the reduced Schrödinger equation

\[ i\hbar \partial_{\tilde{\tau}} \psi (\tau, q_k) = \frac{1}{2} \left( \hat{p}_k^2 + \tilde{\omega}_k^2(\tau) \hat{q}_k^2 \right) \psi (\tau, q_k), \quad (3.7) \]
where, $\tilde{\omega}_{\tilde{\tau},k}$ now becomes

$$\tilde{\omega}_{\tilde{\tau},k}^2 (\tilde{\tau}) = (\tilde{a}_1 \tilde{a}_2 \tilde{a}_3)^2 \left( \sum_i \tilde{h}_i \tilde{\alpha}_i^{-2} + \tilde{m}^2 \right)$$

$$= k^2 \frac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^4 \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle} + m^2 \frac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6 \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle}. \quad (3.8)$$

We assume that the classical equation of motion (3.5) has a solution $v_k(\tilde{\tau})$, so

$$v_k'' + \tilde{\omega}_{\tilde{\tau},k}(\tilde{\tau}) v_k = 0,$$

in which a prime denotes a derivative with respect to $\tilde{\tau}$. Next, we need to find the wave function solutions $\chi_n (q_k, \tilde{\tau})$ of the differential equation corresponding to (3.7)

$$\left( -\frac{\hbar^2}{2} \frac{d^2}{dq_k^2} + \frac{1}{2} \tilde{\omega}_{\tilde{\tau},k}(\tilde{\tau}) q_k^2 \right) \chi_n (q_k, \tilde{\tau}) = E_n \chi_n (q_k, \tilde{\tau}). \quad (3.10)$$

A complete set of solutions for this differential equation exists and can be characterized by the quantum number $n$ as [24, 25]

$$\chi_n (q_k, \tilde{\tau}) = \left( \frac{(v_k^*)^n}{2^n n! \sqrt{2\pi (v_k)^{n+1}}} \right)^{1/2} \exp \left[ \frac{i v_k' q_k}{2 v_k} \right] H_n \left( \frac{q_k}{\sqrt{2} |v_k|} \right), \quad (3.11)$$

with the eigenenergy

$$E_n = \left( n + \frac{1}{2} \right) \hbar \tilde{\omega}_{\tilde{\tau},k}, \quad (3.12)$$

where $v_k(\tilde{\tau})$ is the solution of Eq. (3.5), and $H_n$ are the Hermite polynomials of order $n$.

In quantum theory, by introducing the creation and annihilation operators $\hat{A}_k^\dagger$ and $\hat{A}_k$ [26]

$$\hat{A}_k = -i v_k' (\tilde{\tau}) q_k + v_k (\tilde{\tau}) (\partial / q_k), \quad (3.13)$$

where $\hat{A}_k^\dagger$ is the Hermitian conjugate of $\hat{A}_k$, the states (3.11) can be generated as

$$\chi_n = (n!)^{-\frac{1}{2}} \left( \hat{A}_k^\dagger \right)^n |\tilde{\tau}_0\rangle,$$

$$\hat{A}_k |\tilde{\tau}_0\rangle = 0. \quad (3.14)$$

where $[\hat{A}_k, \hat{A}_k^\dagger] = 1$. Following Zeldovich and Starobinsky [27], we assume that there exists a regime $\tilde{\tau} \leq \tilde{\tau}_0$ such that the vacuum state satisfies an adiabatic condition, so that the usual harmonic oscillator states may be constructed. We choose the state $|0\rangle$, satisfying the equation

$$\frac{1}{2} \tilde{\omega}_k (\tilde{\tau}_0) |0\rangle = \hat{H}_{\tilde{\tau},k} (\tilde{\tau}_0) |0\rangle,$$

as the vacuum state, where $\hat{H}_{\tilde{\tau},k} (\tilde{\tau}_0)$ is the Hamiltonian operator

$$\hat{H}_{\tilde{\tau},k} (\tilde{\tau}) = -\frac{\hbar^2}{2} \frac{\partial^2}{\partial q_k^2} + \frac{1}{2} \tilde{\omega}_{\tilde{\tau},k}^2 (\tilde{\tau}) q_k^2. \quad (3.15)$$
evaluated at some initial time \( \hat{\tau} = \hat{\tau}_0 \). Thus, the state \( |0\rangle \) is indeed the harmonic-oscillator ground state for the frequency \( \hat{\omega}_{\hat{\tau},k}(\hat{\tau}_0) \).

Let us now expand the classical test field solution \( v_k(\hat{\tau}) \), in a Wentzel–Kramers–Brillouin (WKB) approximation, as

\[
v_k(\hat{\tau}) = \frac{1}{\sqrt{2\hat{\omega}_{\hat{\tau},k}}} \exp \left[ i \int d\hat{\tau} \hat{\omega}_{\hat{\tau},k}(\hat{\tau}) \right].
\]

(3.17)

When the Hamiltonian (3.16) possesses an adiabatic regime, the inequality

\[
\hat{\omega}'_{\hat{\tau},k} = \frac{d\hat{\omega}_{\hat{\tau},k}}{d\hat{\tau}} \ll \hat{\omega}^2_{\hat{\tau},k},
\]

(3.18)

holds, so that \( v'_k = dv_k/d\hat{\tau} = i\hat{\omega}_{\hat{\tau},k}v_k \). Then, the operators \( \hat{A}_k \) and \( \hat{A}^\dagger_k \) reduce to the usual harmonic oscillator annihilation and creation operators for the fixed frequency \( \hat{\omega}_{\hat{\tau},k} \)

\[
\hat{A}_k = v_k(\hat{\tau}) [\hat{\omega}_{\hat{\tau},k} \hat{q}_k + (\partial/\partial \hat{q}_k)],
\]

(3.19)

\[
\hat{A}^\dagger_k = v'_k(\hat{\tau}) [-\hat{\omega}_{\hat{\tau},k} \hat{q}_k + (\partial/\partial \hat{q}_k)].
\]

(3.20)

Using above operators we can define a number operator as

\[
\hat{N}_k = \hat{A}^\dagger_k \hat{A}_k = |v_k|^2 \left[ (\partial^2/\partial q_k^2) - \hat{\omega}^2_{\hat{\tau},k} \hat{q}_k^2 \right],
\]

(3.21)

such that

\[
\hat{N}_k \chi_n(q_k, \hat{\tau}) = n\chi_n(q_k, \hat{\tau}).
\]

(3.22)

The set \( \{\chi_n\} \), given by Eq. (3.11), forms a complete orthonormal set for all \( \hat{\tau} \), and thus, we can expand \( |0\rangle \) in terms of these functions (evaluated at \( \hat{\tau}_0 \)):

\[
|0\rangle = \sum_n b_n \chi_n(q_k, \hat{\tau}_0).
\]

(3.23)

Then, the expectation value of the number operator \( \hat{N}_k \) with respect to \( |0\rangle \) on the dressed background becomes

\[
\langle \hat{N}_k \rangle = \langle 0 | \hat{N}_k | 0 \rangle = \frac{1}{2} \left( \hat{\omega}_0 |v_k|^2 + \frac{|v'_k|^2}{\hat{\omega}_0} - 1 \right),
\]

(3.24)

where \( \hat{\omega}_0 = \hat{\omega}_{\hat{\tau},k}(\hat{\tau}_0) \).

Within the above analysis of particle production on a general (anisotropic) dressed background inspired by LQG, the “in” region can be assumed to be at some instant in the Planck era where the background spacetime is quantized due to LQG effects. The “in” vacuum state is thus governed by an effective dressed geometry which resembles the Bianchi type I model with components \( \hat{N}_\phi \) and \( \hat{a}_i \)’s, being the solutions of Eqs. (2.19)-(2.21). The frequency of the field modes, \( \hat{\omega}_0 = \hat{\omega}_{\hat{\tau},k}(\hat{\tau}_0) \), is then given by Eq. (3.8) at some instant \( \hat{\tau}_0 \), say, at the initial quantum bounce, \( \hat{\tau}_0 = \hat{\tau}_B \). On the other hand, we assume that the “out” region is given by the later classical de Sitter or FLRW phase due to the inflationary scenario so that the frequency of the field modes \( \hat{\omega}_{\hat{\tau},k} \) reduces to that of the field propagating on such a classical background

\[
\hat{\omega}^2_{\hat{\tau},k}(\hat{\tau}) = \omega^2_{\tau,k}(\tau) = k^2a^4 + m^2a^6.
\]

(3.25)
If the adiabaticity condition (3.18) is valid for all \( \tilde{\tau} \geq \tilde{\tau}_0 \), we can expand the production number (3.24) for the general solution (3.17) as

\[
\mathcal{N}_k \approx \frac{1}{4} \left( \frac{\bar{\omega}_0}{\bar{\omega}_{\tilde{\tau},k}} + \frac{\bar{\omega}_{\tilde{\tau},k}}{\bar{\omega}_0} - 2 \right) + \mathcal{O} \left( \frac{\bar{\omega}_{\tilde{\tau},k}/\bar{\omega}_{\tilde{\tau},k}^2}{\bar{\omega}_0} \right).
\]

(3.26)

To compute this expectation value (3.26), we need to introduce relevant frequencies \( \bar{\omega}_{\tilde{\tau},k} \) and \( \bar{\omega}_0 \) to be substituted in this equation. For \( \bar{\omega}_{\tilde{\tau},k} \), we use the dispersion relation (3.25). This represents the dispersion relation of a later classical FLRW epoch. For the initial frequency \( \bar{\omega}_0 \), we use the dispersion relation (3.8) which for all the cases we studied turns out to be of the form (3.4). Then, (3.26) becomes

\[
\mathcal{N}_k \approx \frac{1}{4} \left[ \frac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^4(\tau_0) \hat{H}_o^{-\frac{1}{2}} \rangle}{a^4} + \frac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\tau_0) \hat{H}_o^{-\frac{1}{2}} \rangle}{a^6} \right] \left( 1 + \frac{m^2 a^2}{k^2} \right)^{-\frac{1}{2}}
\]

\[
- \frac{1}{2} + \mathcal{O} \left( \frac{\bar{\omega}'_k/\bar{\omega}_k^2}{\bar{\omega}_0} \right).
\]

(3.27)

Eq. (3.27) indicates that when one or both of the conditions

\[
\frac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^4(\tau_0) \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle} \neq a^4(\tau_0), \quad \text{and} \quad \frac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\tau_0) \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle} \neq a^6(\tau_0),
\]

(3.28)

(3.29)

hold, then there will be particle creation. The details of the particle creation of course depend on the explicit form of these expectation values. These conditions are held even in the mean field approximation where \( \langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^4(\bar{\tau}_0) \hat{H}_o^{-\frac{1}{2}} \rangle \sim \langle \hat{H}_o^{-1} \rangle \langle \hat{a}^4(\bar{\tau}_0) \rangle \) and \( \langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^6(\bar{\tau}_0) \hat{H}_o^{-\frac{1}{2}} \rangle \sim \langle \hat{H}_o^{-1} \rangle \langle \hat{a}^6(\bar{\tau}_0) \rangle \), since \( \langle \hat{a}^4(\bar{\tau}_0) \rangle \neq a^4 \) and \( \langle \hat{a}^6(\bar{\tau}_0) \rangle \neq a^6 \). Therefore, the particle creation number (3.27) is always nonzero. In comparison, in classical FLRW background, using the mechanism due to Berger [24] (in a given adiabatic regime), there is no particle creation for a massless scalar field.

In computation of the particle number in this section, we have considered only one mode of the field so that we could show the presence of nonzero production rate for particles. In a more general case, in order to obtain the total number of particle creation, one needs to consider infinitely many modes of the scalar field propagating on the same background dressed spacetime. As far as the field modes probe the same background, sum over all modes can be done. This may pose a problem that, in the presence of such modes, the energy density of the created particles or that of the scalar field may fail to be negligible compared to the energy density of the background quantum geometry which is bounded by 0.41\( \rho_{\text{Pl}} \) at the initial bounce. If the energy-momentum of the field does become comparable
to that in the background, then we would not be able to neglect the back-reaction and our
analysis in subsection II B through the approximation \( \Psi = \Psi_o \otimes \psi \), fails to be self-consistent.
This issue requires a careful treatment of renormalization of the energy-momentum tensors
of the quantum (inhomogeneous) scalar field and the created scalar particles appearing in
the theory. In the next section (see subsection IV B complemented by appendices B and C),
we will employ a renormalization method through the “adiabatic regularization” of the
energy-momentum tensor in order to find a self-consistent solution for the field modes.

IV. AMPLITUDE OF QUANTUM FLUCTUATIONS

Our aim in this section is to study the possible probes of the anisotropy of the emergent
dressed geometry \( \tilde{g}_{ab} \) on which the scalar field \( \varphi(\phi, x) \),

\[
\varphi(\phi, x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \varphi_k(\phi) e^{ik \cdot x}. \tag{4.1}
\]

with massive modes \( q_k \), propagates. We assume that the resultant solutions we obtained
in subsection II C for each field mode, are applicable to all modes. In other words, if one
mode feels a dressed geometry \( \tilde{g}_{ab} \), then all modes probe the same geometry \( \tilde{g}_{ab} \), so that an
integration over all modes can be possible on that background.

In subsection IV A, we present the field equations on the dressed Bianchi background and
will discuss the possible solutions for the frequency of the modes representing the anisotropy
of the dressed geometry near the Planck era. Then, by using this frequency in subsection
IV B we will investigate the possible solutions \( u_k \) to the field modes and compute the
fluctuation amplitude of the (auxiliary) field,

\[
\langle \tilde{0} | \hat{\chi}(x, \tilde{\eta}) \hat{\chi}(y, \tilde{\eta}) | \tilde{0} \rangle = \frac{1}{(2\pi)^3} \int d^3k e^{k \cdot (x - y)} u^*_k(\tilde{\eta}) u_k(\tilde{\eta}). \tag{4.2}
\]

Using this correlation function, we will show that an observable effect from anisotropy of
the dressed background can be extracted.

A. Field equation on the dressed anisotropic background

In this subsection, we will discuss the modifications needed to characterize the typical
quantum fluctuations of the scalar modes on the smooth effective background \( \tilde{g}_{ab} \) in the
Planck regime. Let us follow the standard procedure used in cosmology literature and
consider the field modes propagating on the dressed Bianchi type-I background with the
metric

\[
\tilde{g}_{ab} dx^a dx^b = -\tilde{N}_\phi^2(\phi) d\phi^2 + \sum_{i=1}^3 \tilde{a}_i^2(\phi) (dx^i)^2, \tag{4.3}
\]

whose components are given by Eqs. (2.19)-(2.21) as we discussed in subsection II C. By
introducing an auxiliary field \( \chi_k := \tilde{c}_k^{1/2}(\tilde{\eta}) \varphi_k \) for a given mode \( k \), the corresponding Klein–
Gordon equation for that mode reads\(^2\)

\[
\chi_k'' + \left[ \tilde{\omega}_{k,k}^2(\tilde{\eta}) - \frac{\tilde{c}''}{2\tilde{c}} + \frac{\tilde{c}^2}{4\tilde{c}^2} \right] \chi_k = 0, \tag{4.4}
\]

\(^2\) see appendix A for detailed description.
where, we have defined the frequency $\tilde{\omega}_{\tilde{n},k}$ as

$$\tilde{\omega}^2_{\tilde{n},k}(\tilde{n}) = \bar{c} \left( \sum_{i} \frac{k_i^2}{\bar{c}_i + \bar{m}^2} \right), \quad (4.5)$$

and

$$\bar{c} := (\bar{a}_1\bar{a}_2\bar{a}_3)\frac{2}{3} = (\bar{c}_1\bar{c}_2\bar{c}_3)^{\frac{1}{3}} \quad \text{and} \quad \bar{c}_i := \bar{a}_i^2. \quad (4.6)$$

In these equations, a prime denotes differentiation with respect to the conformal time $\tilde{n}$, which is defined by using the conformal lapse $N_{\tilde{n}} = (\bar{a}_1\bar{a}_2\bar{a}_3)^{1/3} = \bar{c}^{1/2}$ in the relation $N_{\phi}d\phi = N_{\tilde{n}}d\tilde{n}$:

$$d\tilde{n} = \ell^3\langle \dot{H}_o^{-1} \rangle \bar{c}(\phi)d\phi. \quad (4.7)$$

Note that the frequency (4.5) we obtained in a conformal-time gauge is different from the one we obtained earlier in Eq. (3.8) in a harmonic-time gauge by a factor $\bar{c}^{-2}$. More precisely, by using Eqs. (2.19) and (2.20), the frequency (4.5) can be identically written as:

$$\tilde{\omega}^2_{\tilde{n},k}(\tilde{n}) = \bar{c}^{-2} \left[ k^2 \frac{\langle \dot{H}_o^{-\frac{1}{2}} \dot{a}^4 \dot{H}_o^{-\frac{1}{2}} \rangle}{\langle \dot{H}_o^{-1} \rangle} + m^2 \frac{\langle \dot{H}_o^{-\frac{1}{2}} \dot{a}^6 \dot{H}_o^{-\frac{1}{2}} \rangle}{\langle \dot{H}_o^{-1} \rangle} \right] = \bar{c}^{-2}\tilde{\omega}^2_{\tilde{n},k}. \quad (4.8)$$

This equation indicates that the possible alteration of the frequency $\tilde{\omega}_{\tilde{n},k}$ can be achieved when various solutions for $\bar{c}$ are known. It is easy to show that there exist only two classes of solutions for $\bar{c}$, depending on whether or not the mass of the field is dressed:

1. In the case of the undressed mass (i.e., $\bar{m} = m \neq 0$), the solution for $\bar{c}$ is obtained from Eq. (2.21) simply as

$$\bar{c}(\tilde{n}) = \frac{\langle \dot{H}_o^{-\frac{1}{2}} \dot{a}^6 \dot{H}_o^{-\frac{1}{2}} \rangle^{\frac{1}{3}}}{\langle \dot{H}_o^{-1} \rangle^{\frac{1}{3}}}. \quad (4.9)$$

Hence, the frequency of the modes reads

$$\tilde{\omega}^2_{\tilde{n},k}(\tilde{n}) = k^2 \frac{\langle \dot{H}_o^{-\frac{1}{2}} \dot{a}^4 \dot{H}_o^{-\frac{1}{2}} \rangle}{\langle \dot{H}_o^{-1} \rangle^{\frac{3}{2}}} + m^2 \frac{\langle \dot{H}_o^{-\frac{1}{2}} \dot{a}^6 \dot{H}_o^{-\frac{1}{2}} \rangle^{\frac{1}{3}}}{\langle \dot{H}_o^{-1} \rangle^{\frac{3}{2}}}. \quad (4.10)$$

2. For the modes with the dressed mass or the case of massless modes, using Eq. (2.46) or Eqs. (2.50) and (2.52), we get

$$\bar{c}(\tilde{n}) = \xi^2 \frac{\langle \dot{H}_o^{-\frac{1}{2}} \dot{a}^4 \dot{H}_o^{-\frac{1}{2}} \rangle^{\frac{1}{2}}}{\langle \dot{H}_o^{-1} \rangle^{\frac{1}{2}}} = \xi^2\bar{a}^2(\tilde{n}). \quad (4.11)$$

---

3 At the fundamental level, evolution on quantum geometry is described by the relational time variable $\phi$ rather than cosmic or conformal time. However, one can descend to a description in terms of conformal time $\tilde{n}$ since $\tilde{g}_{ab}$, that incorporates the quantum gravity corrections, is a smooth tensor field now.

4 We note that, as in the case of harmonic time gauge, since the frequency (4.5) can be written in terms of the original wave vector $k$, as given in Eq. (4.5), we have denoted the frequency by $\tilde{\omega}_{\tilde{n},k}$ instead of $\tilde{\omega}_{\tilde{n},k}$. 
where $\xi$ is a parameter that distinguishes two different solutions for $\tilde{c}$; for $\xi^2 = 1$ the solution of $\tilde{c}$ associates the one provided by the case 1 (Eq. (2.48)), and the value $\xi^2 \equiv (\rho_{12}^2 \rho_{23}^2 \rho_{33}^2)^{-1/3}$ denotes the solution of the case 2 (Eqs. (2.50) and (2.52)). Moreover, $\tilde{a}$ was defined in Eq. (2.48) as well as Eq. (2.22). For this relation of $\tilde{c}$, the frequency becomes

$$\tilde{\omega}_{\eta,k}^2(\bar{\eta}) = k^2 + \bar{m}^2 \xi^2 \tilde{a}^2.$$  \hspace{1cm} (4.12)

This is equal to the frequency of the modes with a dressed mass $\bar{m}$ given by (2.49) propagating on the dressed isotropic background with the scale factor $\xi \tilde{a}(\bar{\eta})$. This class of the solutions, as introduced in subsection II C 3, are identical to the isotropic solutions with $\xi = 1$, given earlier in Refs. [15, 19, 20] for massive and massless modes.

Our analyses in this subsection implies that despite the dependence of the anisotropic solutions we have found in subsection II C for the effective dressed background, the emergent frequencies of the modes are independent of the anisotropic parameters encoded in $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \sigma_{ij}$. Nevertheless, the frequency (4.10) of the field modes propagating on the anisotropic dressed spacetime is different from those propagating on the isotropic dressed geometry (i.e., those associated to the frequency (4.12)). Therefore, the frequency (4.10) distinguishes the amplitude of the quantum fluctuations for the fields propagating on the isotropic spacetime, and can provide a probing for the quantum gravitational nature of the smooth quantum gravity induced anisotropic background from those propagating on the isotropic geometry, and can provide an observable signature for the quantum gravity inspired anisotropy in the early Universe.

### B. The adiabatic condition on vacuum states

In this subsection we compute the power spectrum of the quantum fluctuations, using general mode solutions $u_k(\bar{\eta})$ of the Klein–Gordon equation (4.4). These modes obey the equation $u''_k + (\tilde{\omega}^2_{\eta,k} - \tilde{Q})u_k = 0$, and form a complete set of orthonormal basis with their complex conjugates $\tilde{u}^*_k$ under the scalar product

$$W(u^*_k, u_k) := u_k u''_k - u_k^* u'_k = 2i.$$ \hspace{1cm} (4.13)

Thus, any solution $\chi_k(\bar{\eta})$ of Eq. (4.10) with the same $k$ can be expanded as

$$\chi_k(\bar{\eta}) = \frac{1}{\sqrt{2}} \left[ a_k u^*_k(\bar{\eta}) + a^{*}_{-k} u_k(\bar{\eta}) \right],$$ \hspace{1cm} (4.14)

where $a_k$ and $a^*_{-k}$ are constants of integration, which in quantum theory become annihilation and creation operators for each mode $\chi_k$. With normalization (4.13), operators $\hat{a}_k$ and $\hat{a}^\dagger_k$ will satisfy the commutation relation

$$\left[ \hat{a}_k, \hat{a}^\dagger_{k'} \right] = \hbar \ell^3 \delta_{k,k'},$$ \hspace{1cm} (4.15)

---

5 Because of the specific symmetry emerged herein our model, due to the frequencies (4.10) and (4.12), the subscript of the mode function $u_k$ can be written as $a_k$ or $u_k$. With a similar argument, we can write $a_k$ instead of $\tilde{c}_k$. Therefore, we have written the field equation and their possible solutions in terms of $k$, $\tilde{c}$ and $\bar{m}$ and the isotropic quantum geometry fluctuations. Within this expression, the integration (e.g., given by Eq. (1.1) or Eq. (4.2)) over all modes $k$, instead of $\tilde{k}$ is relevant.

6 The normalization condition (4.13) for $\chi_k$ provides that the corresponding annihilation and creation operators $\hat{a}_k$ and $\hat{a}^\dagger_{-k}$ satisfy the canonical commutation relations in quantum theory.
with all other commutation relations vanishing. A choice of basis with positive frequency solutions \( u_k(\tilde{\eta}) \) determines a vacuum state \(|0\rangle\), which can be defined as the eigenstate of the annihilation operators with vanishing eigenvalue, i.e., \( \hat{a}_k|0\rangle = 0 \). Consequently, a Fock space is generated for the quantum theory by repeatedly acting on the vacuum by creation operators \( \hat{a}_k^\dagger \).

Using the reality condition for the scalar field, \( \varphi_k = \varphi_k^* \), and since \( \varphi_k = \tilde{c}^{-1/2}\chi_k \), we obtain \( \chi_k^* = \chi_{-k} \). Hence, the general solution for the (auxiliary) scalar field satisfying the Klein–Gordon equation can be expressed by

\[
\chi(\tilde{\eta}, x) = \frac{1}{\ell^3} \sum_{k \in \mathcal{L}} \chi_k(\tilde{\eta}, x),
\]

as the sum of all modes of the field \( \chi_k(\tilde{\eta}, x) \) given by

\[
\chi_k = \frac{1}{\sqrt{2}} \left[ a_k u_k^*(\tilde{\eta}) e^{ik\cdot x} + a_k^* u_k(\tilde{\eta}) e^{-ik\cdot x} \right].
\]

Note that in the above equation, we have changed the integration variable \( k \to -k \).

In quantum theory of scalar field on the dressed geometry \([4.13]\) in the Planck era with \( \tilde{\eta} \lesssim \eta_p \), we can construct a Fock space \( \mathcal{H}_F \) by defining a vacuum state \(|0\rangle\), which is associated to the positive frequency solution \( u_k(\tilde{\eta}) \) of Eq. \([4.14]\):

\[
u_k'' + \left( \tilde{\omega}_{\tilde{\eta}, k}(\tilde{\eta}) - Q(\tilde{\eta}) \right) u_k = 0,
\]

where, \( Q \equiv \tilde{c}'/2\tilde{c} - \tilde{c}^2/4\tilde{c}^2 \) and \( u_k \) satisfies the normalization condition \([4.13]\). Different families of solutions provide different definitions of the vacuum state. Under these conditions, we can determine evolution of the quantum fields on the quantum gravity induced dressed spacetime. The parameter \( Q(\tilde{\eta}) \) has a linear dependence on the curvature of \( \tilde{g}_{ab} \) and introduces a physical length \( L(\tilde{\eta}) \). For the frequency \( \Omega_k^2(\tilde{\eta}) \equiv \tilde{\omega}_{\tilde{\eta}, k}^2(\tilde{\eta}) - Q(\tilde{\eta}) \) to be positive, the wave-number must always satisfy an inequality \( k^2 \geq k^2_* \), where

\[
k^2_*(\tilde{\eta}) := Q(\tilde{\eta})\tilde{c}^2(\tilde{\eta})\langle H_0^{-1} \rangle - m^2 \frac{\langle \hat{H}_o^{-\frac{1}{2}}\hat{a}^\dagger \hat{a} \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-\frac{1}{2}}\hat{a}^\dagger \hat{a} \hat{H}_o^{-\frac{1}{2}} \rangle}.
\]

In this case the resulting vacuum state will be well-defined for modes \( u_k \) with wavelengths shorter than the length scale \( L(\tilde{\eta}) = \sqrt{\tilde{c}(\tilde{\eta})}/k_* \). Hence, modes with large momenta, \( k/\sqrt{\tilde{c}} \gg 1/L \), describe the vacuum in short distances. This respects the regularity condition for a natural choice of the mode function in the \textit{ultra-violet} regime with \( (\sqrt{\tilde{c}}/kL) \to 0 \), where the curvature has negligible effects and solutions to \([4.18]\) reduce to the standard mode functions \( e^{-ik\eta}/\sqrt{2k} \) in Minkowski space. This regime constitutes a limit of arbitrary slow time variation of the metric functions \( \hat{a}_i(\tilde{\eta}) \) with respect to the time \( \tilde{\eta} \), which is the so-called \textit{adiabatic} regime.

Our aim is to determine the mode functions that describe the physical vacuum and particles. To define the \textit{adiabatic} vacuum modes, one can employ a positive-frequency generalized WKB method \([28]\) to get

\[
u_k(\tilde{\eta}) = \frac{1}{\sqrt{W_k(\tilde{\eta})}} \exp \left( -i \int \tilde{\eta} W_k(\eta) d\eta \right),
\]
which yields an approximate solution to the Klein–Gordon equation (4.18). Note that $u_k(\tilde{\eta})$ are guaranteed to satisfy the Wronskian condition (4.13) for any real, non-negative function $W_k(\tilde{\eta})$. An appropriate function $W_k(\tilde{\eta})$ is given by the method of Chakraborty [29] as

$$W_k(\tilde{\eta}) = [Y(1 + \epsilon_2)(1 + \epsilon_4)]^{\frac{1}{2}},$$

(4.21)

in which we have set $Y \equiv \Omega_k^2 = \bar{\omega}_{\tilde{\eta}, k}^2 - Q$, and

$$\epsilon_2 := -Y^{-\frac{1}{2}}\partial_{\tilde{\eta}}\left(Y^{-\frac{1}{2}}\partial_{\tilde{\eta}}Y^{\frac{1}{2}}\right),$$

(4.22)

$$\epsilon_4 := -Y^{-\frac{1}{2}}(1 + \epsilon_2)^{-\frac{1}{2}}\partial_{\tilde{\eta}}\left(Y(1 + \epsilon_2)^{-\frac{1}{2}}\partial_{\tilde{\eta}}(1 + \epsilon_2)^{\frac{1}{2}}\right).$$

(4.23)

If $u_k(\tilde{\eta})$ is a solution for the exact mode function satisfying the Klein–Gordon equation (4.18), then $W(\tilde{\eta})$ is required to satisfy the relation (4.21). However, instead of solving for $W_k$ exactly, one can generate asymptotic series in orders of time derivatives (with respect to $\tilde{\eta}$) of the field, $(0)\hat{T}_{ab} \hat{0}$ with respect to the adiabatic vacuum $\hat{0}$ can be computed. It can be shown that all ultra-violet divergences are contained in terms of adiabatic order equal to and smaller than four [30] (see appendix C). Therefore, we restrict ourselves to the fourth order adiabatic states by setting $n = 4$, and define $W_k(\tilde{\eta})$ to match the terms in

$$W_k(\tilde{\eta}) = W^{(0)}_k + W^{(2)}_k + W^{(4)}_k,$$

(4.24)

that fall slowly in $\bar{\omega}_{\tilde{\eta}, k}$, rather than demanding that $u_k(\tilde{\eta})$ satisfies the exact mode equation (4.18). At the appropriate rate (say, at the initial quantum bounce $\tilde{\eta} = \tilde{\eta}_b$), given by the asymptotic conditions

$$|u_k(\tilde{\eta}_b)| = |u_k(\tilde{\eta}_b)| \left(1 + \mathcal{O}(\sqrt{c/k}L_{4+\varepsilon})\right)^{4+\varepsilon},$$

$$|u'_k(\tilde{\eta}_b)| = |u'_k(\tilde{\eta}_b)| \left(1 + \mathcal{O}(\sqrt{c/k}L_{4+\varepsilon})\right)^{4+\varepsilon},$$

(4.25)

(with positive real number $\varepsilon$), the field’s exact mode functions $u_k(\tilde{\eta})$ match the adiabatic functions $u_k(\tilde{\eta})$ (up to the order four). If the conditions (4.25) are held for the mode functions at some initial time $\tilde{\eta}_b$, they will be held for all times $\tilde{\eta}$. That is, an observable vacuum state associated to $u_k(\tilde{\eta})$, given at any time $\tilde{\eta}$, will be of the 4th order.

By discarding terms of adiabatic order higher than four in (4.21) we obtain

$$W_k(\tilde{\eta}) = \bar{\omega}_{\tilde{\eta}, k}(1 + \epsilon_2 + \epsilon_4)^{\frac{1}{2}},$$

(4.26)

\footnote{The problem of initial conditions in the early universe is still under discussion, which might be solved in the framework of a quantum theory of gravity. Therefore, a natural choice for the preferred instant of time in LQC is provided by the quantum bounce, at $\tilde{\eta} = \tilde{\eta}_b$.}
where the $k$-dependent parameters $\epsilon_2, \epsilon_4$ are defined as

$$
\begin{align*}
\epsilon_2 &= \epsilon_2 - \tilde{\omega}_{\tilde{\eta},k}^{-2} \tilde{Q}, \\
\epsilon_4 &= \epsilon_4 - \epsilon_2 \tilde{\omega}_{\tilde{\eta},k}^{-2} \tilde{Q}.
\end{align*}
$$

The leading order term of $\epsilon_2$ is of second order, whereas the leading order in $\epsilon_4$ is four. Thus, $\epsilon_2$ contains terms of orders two, four and higher, while $\epsilon_4$ contains leading order term four. Of course we will consider only terms until the fourth order in their expressions. Notice that we have dropped terms like $\epsilon_2 \epsilon_4$ because they are of sixth or higher order. For detailed expressions of $\epsilon_2$ and $\epsilon_4$ see appendix B.

Once the vacuum state $|\tilde{0}\rangle$ of the fourth adiabatic order is determined by a set of mode functions $u_k(\tilde{\eta})$, following the mechanism we discussed above, we can write the power spectrum of quantum fluctuations using the equal-time correlation function (4.2) as

$$
\mathcal{P}_\chi(k) = \frac{2\pi^2}{k^3} \langle 0 | \hat{\chi}(x, \tilde{\eta}) \hat{\chi}(y, \tilde{\eta}) | 0 \rangle = |u_k(\tilde{\eta})|^2.
$$

This implies that, in order to calculate the $\mathcal{P}_\chi(k)$, one needs to evolve the set of mode function $u_k$. As we have discussed earlier, we are concerned about the anisotropy generated by quantum fluctuation of the scalar field in the emergent dressed background from the Planck era. Therefore, once the mode functions $u_k(\tilde{\eta})$ associated to the vacuum states are determined for the isotropic and the anisotropic emergent geometries, by a comparison between the power spectrum of the quantum fluctuations due to our analysis and the data provided by observations, one can clarify the nature of the CMB anisotropies (see e.g. [31]) which may arise from the quantum gravity effects from the Planck era.

In particular, from Eq. (C6), we get the expression for the mode function $|u_k|$ up to fourth adiabatic order. In terms of this mode solution, the power spectrum of the quantum fluctuations reads

$$
\mathcal{P}_\chi(k) = \tilde{\omega}_{\tilde{\eta},k}^{-2} \left[ 1 - \frac{1}{2}(\epsilon_2^{(2)} + \epsilon_4^{(4)}) + \frac{3}{8}(\epsilon_2^{(2)})^2 \right].
$$

This implies that the power spectra $\mathcal{P}_\chi(k)$ arising on the dressed Bianchi type I background depend on the anisotropy encoded in the leading adiabatic terms including $\tilde{\omega}_{\tilde{\eta},k}(\tilde{\eta})$ and $\tilde{Q}(\tilde{\eta})$ and their (conformal) time derivatives.

In order to make a comparison between the anisotropic power spectrum with that of the isotropic one, dressed background, let us rewrite the general frequency (4.3) of the modes on the emergent, dressed Bianchi as

$$
\tilde{\omega}_{\tilde{\eta},k}(\tilde{\eta}) = \frac{\tilde{\alpha}^4}{c^2} \left[ k^2 + \tilde{m}^2 \tilde{a}^2 \right].
$$

The anisotropy factor $\tilde{c}$ is given either by Eq. (4.9) or Eq. (4.11) depending on the anisotropic solution we have chosen. Let us now denote for convenience, the term in the bracket by $\tilde{\omega}_{\tilde{\eta}}^2$ as the frequency associated to the mode $k$ (with the dressed mass $\tilde{m}$) propagating on the isotropic background with the scale factor $\tilde{a}$. Then, denoting the left hand side by $\tilde{\omega}_{\tilde{\eta}}^2$ and the frequency of the same massive mode on the anisotropic background (4.3) as

$$
\tilde{\omega}_{\tilde{\eta}}^2(\tilde{\eta}) = A^2(\tilde{\eta}) \tilde{\omega}_{\tilde{\eta}}^2, \quad \text{where} \quad A(\tilde{\eta}) := \tilde{a}^2(\tilde{\eta}) \tilde{c}^{-1}(\tilde{\eta}).
$$
In the isotropic limit, we have $A = 1$. The expression for the power spectra \((4.29)\) constitutes of the frequency $\tilde{\omega}_A$ and its (conformal) time derivatives. That is, it contains power terms of $\tilde{\omega}_A$ and its derivatives up to fourth order. Therefore, by assuming that the changes in the quantum fluctuations of the geometry associated to the anisotropy parameter $A(\tilde{\eta})$ are slow, we can neglect terms like $A'/A$ and its higher time derivatives. Using this, the power spectra \((4.29)\) reduce to

$$P^{(A)}(k) \approx \tilde{\omega}_A^{-1} \left[ 1 + \frac{1}{2} \tilde{\omega}_A^{-2} Q_A - \frac{1}{8} \tilde{\omega}_A^{-4} Q'_A + \frac{3}{8} \tilde{\omega}_A^{-4} Q''_A \right] + \text{The terms including } (\partial A/A),$$  

(4.32)

where, $\partial A$ holds for the derivatives of $A(\tilde{\eta})$ with respect to $\tilde{\eta}$ including first order derivative, second order derivative and so on. In terms of the power spectrum of the modes on the isotropic case, we can write

$$\frac{P^{(A)}}{P^{(I)}} = A^{-1}(\tilde{\eta}) \left[ 1 + \frac{1}{2} A^{-2} \tilde{\omega}_I^{-2} Q_A - \frac{1}{8} A^{-4} \tilde{\omega}_I^{-4} Q'_A + \frac{3}{8} A^{-4} \tilde{\omega}_I^{-4} Q''_A \right] \left[ 1 + \frac{1}{2} \tilde{\omega}_I^{-2} Q_I - \frac{1}{8} \tilde{\omega}_I^{-4} Q'_I + \frac{3}{8} \tilde{\omega}_I^{-4} Q''_I \right],$$  

(4.33)

where, $Q_A = \tilde{\omega}'/2\tilde{\omega} - \tilde{\omega}'^2/4\tilde{\omega}^2$ and $Q_I = \tilde{\omega}''/\tilde{\omega}$.

It should be noted that, although the power spectrum we obtained for the anisotropic background would be different from the one for the isotropic geometry, the power spectrum we obtained depends only on the product $\hat{a}_1 \hat{a}_2 \hat{a}_3$ and not on any specific scale factor $\tilde{a}_i$. The above expression can in principle be used to predict observational signatures imprinted on the power spectrum of the cosmic microwave background, which can distinguish between the isotropic and anisotropic cases.

### C. The energy density of the created particles

In order to complete the discussion of the energy density of particle creations we presented in section \(\text{III}\) when considering infinitely many modes of the field, we provide an analysis of particle energy regularization process in this subsection.

The Fock space $\mathcal{H}_F$ for the quantum field $\varphi$ propagating on the dressed Bianchi I background \((4.3)\) is constructed from a vacuum state $|\tilde{0}\rangle$, which is determined by a choice of the positive frequency solution $v_k(\tilde{\eta})$ satisfying Eq. \((4.18)\). Then, any two solutions $v_k$ and $v_k'$ of \((4.18)\) are related through the time-independent Bogolyubov coefficients $\alpha_k$ and $\beta_k$ by

$$\tilde{v}_k(\tilde{\eta}) = \alpha_k v_k(\tilde{\eta}) + \beta_k v_k^*(\tilde{\eta}).$$  

(4.34)

Since $v_k$ and $\tilde{v}_k$ are normalized due to the condition \((4.13)\), the coefficients $\alpha_k$ and $\beta_k$ satisfy the relation $|\alpha_k|^2 - |\beta_k|^2 = 1$. By substituting both sides of Eq. \((4.34)\) into \((4.17)\) one obtains a relation between creation and annihilation operators associated with two families of mode function, as

$$\hat{a}_k = \alpha_k \hat{a}_k + \beta_k \hat{a}_k^\dagger.$$  

(4.35)

In the Heisenberg picture, the initial vacuum state $|\tilde{0}\rangle$ is the vacuum state of the system for all times. The physical number operator $\hat{N}_k$ (see Eq. \((3.21)\)) which counts excitation/particles of $v_k$ modes, yields the average number of particles in the $|\tilde{0}\rangle$ vacuum, associated to the “under-barred” modes,

$$\mathcal{N}_k := \langle \tilde{0} | \hat{N}_k | \tilde{0} \rangle = (\hbar \ell^3)^{-1} \langle \tilde{0} | \hat{a}_k^\dagger \hat{a}_k | \tilde{0} \rangle = |\beta_k|^2.$$  

(4.36)
In other words, the vacuum state associated to the $v_k$-mode contains $N_k = |\beta_k|^2$ particles in the vacuum state associated to $v_k$-mode.

We may consider the solution of Eq. (4.18) in the WKB form \[27, 32\] as
\[
\tilde{u}_k(\tilde{\eta}) = \frac{1}{\sqrt{\omega_{k,\tilde{\eta}}}} \left[ \alpha_k e_k(\tilde{\eta}) + \beta_k e^*_k(\tilde{\eta}) \right],
\] (4.37)
where
\[
e_k(\tilde{\eta}) := \exp \left( -i \int_{\eta}^{\tilde{\eta}} \tilde{\omega}_{k,\tilde{\eta}}(\eta) d\eta \right).\] (4.38)

With the introduction of functions $\alpha_k$ and $\beta_k$, we have the freedom of imposing an additional condition on the time $(\tilde{\eta})$ derivative of $u_k$ such that
\[
u_k' (\tilde{\eta}) = -i \sqrt{\tilde{\omega}_{k,\tilde{\eta}}} \left[ \alpha_k e_k(\tilde{\eta}) - \beta_k e^*_k(\tilde{\eta}) \right].\] (4.39)

Then, from the Wronskian condition (4.13) on $u_k$, we obtain
\[
|\alpha_k|^2 - |\beta_k|^2 = 1.
\] Inverting Eqs. (4.37) and (4.39) yields
\[
\beta_k = \frac{\sqrt{\tilde{\omega}_{k,\tilde{\eta}}}}{2} \left( u_k - \frac{i}{\omega_{k,\tilde{\eta}}} u_k' \right) e_k.
\] (4.40)

We fix the initial condition $\alpha_k = 1$ and $\beta_k = 0$ at the quantum bounce so that no particle has been created at $\tilde{\eta} = \tilde{\eta}_b$. Inserting this into Eqs. (4.37) and (4.39) yields $u_k(\tilde{\eta}_b) = 1/\tilde{\omega}_{k,\tilde{\eta}}(\tilde{\eta}_b)$, where $e_k(\tilde{\eta}_b) = 1$, and $u_k'(\tilde{\eta}_b) = -i\tilde{\omega}_{k,\tilde{\eta}}(\tilde{\eta}_b)u_k(\tilde{\eta}_b)$. By making use of relation (4.13), we find for the particle production
\[
N_k = \frac{1}{4} \left( \tilde{\omega}_{k,\tilde{\eta}} |u_k|^2 + \tilde{\omega}_{k,\tilde{\eta}}^{-1} |u_k'|^2 - 2 \right).
\] (4.41)

As expected, this equation is identical to the equation (3.24) for the particle creation rate we obtained in harmonic-time gauge.

The energy density of created particles for infinitely many modes is
\[
\rho_{\text{par}} = \frac{1}{\ell^3} \sum_k \varrho_k(\tilde{\eta}),
\] (4.42)
where $\varrho_k$ is the energy density of each mode
\[
\varrho_k(\tilde{\eta}) := \tilde{\omega}_{k,\tilde{\eta}} N_k
= \frac{1}{4} \left( |u_k'|^2 + \tilde{\omega}_{k,\tilde{\eta}}^2 |u_k|^2 - 2\tilde{\omega}_{k,\tilde{\eta}} \right).
\] (4.43)

Notice that at the initial quantum bounce $\tilde{\eta} = \tilde{\eta}_b$, one obtains $\varrho_k(\tilde{\eta}_b) = 0$.

---

8 Bear in mind that there is a difference in the factor of the square root of the denominator of the WKB expansion (3.17) which made the particle number (3.24) twice the one derived in Eq. (4.41). Moreover, the frequency in the harmonic time gauge has a difference of factor $\tilde{c}^2$ with respect to the frequency (4.8) given in the conformal time gauge which can be disappeared by setting the new mode function solution $u(\tilde{\eta})$ rather than $v(\tilde{\tau})$ in Eq. (4.41).
Our aim is to derive an expression for the number of particle $N_k$ in the adiabatic regime. By substituting the mode functions $u_k$ and their derivatives, $u_k'$, from Eqs. (C6) and (C7) into expression above, we obtain the number of particles up to the fourth adiabatic order as

$$N_k = N_k^{(0)} + N_k^{(2)} + N_k^{(4)},$$

where, the zeroth, second and fourth adiabatic orders are

$$N_k^{(0)} = 0, \quad N_k^{(2)} = \frac{1}{16} \left( \frac{\tilde{\omega}_k^{(2)}}{\omega_k^{(2)}} \right)^2, \quad N_k^{(4)} = \frac{1}{16} \left[ \left( \epsilon_2^{(2)} \right)^2 + \frac{\tilde{\omega}_k^{(2)} \epsilon_2^{(3)}}{2 \omega_k^{(2)}} - \frac{\left( \tilde{\omega}_k^{(2)} \right)^2}{2 \omega_k^{(2)}} \epsilon_2^{(1)} \right].$$

This equation implies that no particle production occurs in the zeroth order adiabatic expansion of the mode function (or superadiabatic regime). Particles are created only for expansions of adiabatic series equal to or more than two. In the very short scales, where $m \ll k$, from second relation in (4.44) we get the second order adiabatic particle number as

$$N_k^{(2)} \approx \frac{(A')^2}{A^4} \frac{1}{k^2},$$

which depends on the isotropy-breaking factor $A(\tilde{\eta})$. It is clear that, for modes with bigger mass, expression (4.45) will contain further (positive) terms including $\tilde{m}(\tilde{\eta})$. So, number of created particles with bigger masses would be more abundant than lighter ones. For massless modes, from Eq. (4.44) we further obtain

$$N_k^{(0)} = N_k^{(2)} = 0, \quad N_k^{(4)} = \frac{\left( \epsilon_2^{(2)} \right)^2}{16} = \frac{Q^2}{16k^4},$$

with $Q = -\ddot{a}/a$. This indicates that, in this case, no particle creation would occur up to the second order in adiabatic series. However, in the fourth order, a small amount of particles will be created. Therefore, in the adiabatic regime, more massive particles would be created compared to the massless ones. This expression is consistent with the discussion we had at the end of section III.

The energy density of created particle for each mode is obtained from Eqs. (4.43) and (4.44) as

$$\rho_k = \tilde{\omega}_k \tilde{\eta} \left( N_k^{(0)} + N_k^{(2)} + N_k^{(4)} \right) = \frac{1}{16} \left[ \left( \frac{\tilde{\omega}_k}{\omega_k} \right)^2 + \tilde{\omega}_k \tilde{\eta} \left( \epsilon_2^{(2)} \right)^2 - \frac{\left( \tilde{\omega}_k \tilde{\eta} \right)^2}{2 \omega_k^2} \epsilon_2^{(2)} + \frac{\tilde{\omega}_k \tilde{\eta} \epsilon_2^{(3)}}{\omega_k^2} \right] = \rho_k^{(0)} + \rho_k^{(2)} + \rho_k^{(4)}.$$

By substituting this in Eq. (4.42), it is clear that since $\rho_k$ does not fall off faster than $k^{-4}$ when $k \to \infty$, the total energy density of (adiabatic) particle productions would diverge. Since the zeroth adiabatic order term in the energy density is zero, $\rho_k^{(0)} = 0$, the divergences are included in the second and fourth order terms (for massive modes). For massless modes, energy density of created particles is given only by the fourth order term. So, the divergence is included only up to fourth order terms. Therefore, the renormalized energy density of created particles can be obtained by subtraction of the adiabatic vacuum energy of the particle productions up to fourth order. Thus, no back-reaction from created particles is present to generate any inconsistency (i.e., producing an energy density $\rho_{par}$ for scalar particles comparable to that of the background in Planck regime) in studying of the quantum field and consequently analyzing the particle productions on the background quantum geometry.
V. DISCUSSION AND OUTLOOK

In this paper, following the strategy proposed in Ashtekar et al. [15], we have studied the propagation of a scalar (inhomogeneous) quantum test field $\varphi$, propagating over a quantized FLRW geometry corresponding to the Planck era. The geometry is coupled to a massless homogeneous field $\phi$ which serves as the internal physical time. In addition, we also compute the corresponding particle production associated to this test field, when the spacetime transitions from the quantum regime into the later classical regimes.

More precisely, due to Planck era quantum gravitational effects, anisotropies are induced in the effective regime and thus the effective FLRW geometry at the very early universe resembles a Bianchi I model. Thus, the quantum modes of the test field feel an effective geometry that is anisotropic. We take the background geometry as the heavy degree of freedom and each mode of the test field $\varphi$ as the light (perturbation) degree of freedom. By neglecting backreactions, one can consider a decomposition of the full state $\Psi(\nu, q_k, \phi)$ of geometry plus matter, as a tensor product $\Psi(\nu, q_k, \phi) = \Psi_o(\nu, \phi) \otimes \psi(q_k, \phi)$, where, the background wave function $\Psi_o$ (itself corresponding to the tensor product of the geometry and the internal time $\phi$) represents the probability amplitude for occurrence of the various background FLRW geometries, and $\psi$ is the wave function of the test field $\varphi$. One can then work in an interaction picture to find an evolution equation (see Eq. (2.18)) for $\psi(q_k, \phi)$ with respect to the relational time $\phi$.

By matching and comparing the resulting effective evolution equation for the state $\psi(q_k, \phi)$ of massive (and also massless) modes of $\varphi$ on the quantized background, and the evolution equation of $\psi$ on a classical of Bianchi type I spacetime, several classes of solutions were found for the test field propagating on this “dressed” effective background geometry:

1. For massless modes of the test field, the dressed background has an isotropic and homogeneous form with a dressed scale factor $\tilde{a}(\phi)$ given by (2.22), which is a function of the expectation values of operators of the corresponding quantum geometry (see similar solution in Ref. [15]).

2. For massive modes, we have considered two cases: dressed and undressed mass. In both cases the emergent dressed background is the Bianchi type-I geometry with the scale factors $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3$.

Next, in section III, we have investigated the issue of particle production associated to the test field, as a result of transitioning from the effective Bianchi I geometry to a classical isotropic spacetime. This can provide a means to measure some of the interesting QFT-related phenomena from quantum gravity regime in Planck era (see Ref. [33] for the issue of particle creation phenomena in a bouncing scenario of LQC). Parker has shown that while the particle production does not happen for massless conformally coupled fields, it is generic for massive scalar fields of arbitrary coupling to the spacetime curvature. Typically, the particle production is significant when the particle mass $m$ is of the order of the expansion rate $H$, or when $m^2 \sim \dot{H}$. Simple dimensional arguments show that in a Universe with critical density $\rho_c \sim 3H^2/8\pi G$, the quantum particle production can contribute significantly to the energy density of the Universe at early epochs when $H$ is not too far below the Planck scale. We have shown that, in our model, as long as the quantum conditions (3.28) and 3.29 are valid, particle creation always occurs in transition from quantum to classical regime.

In section IV, we generalized our quantum theory of single mode to the theory of a quantum field, including all possible modes, which propagate on the dressed anisotropic
geometry we have derived in the previous section. More precisely, for a given solution of the anisotropic dressed background, we assumed that all modes feel the same geometry, thereby, we studied the quantum theory of all modes and looked for possible observable signatures stemming from the early stages of the Universe. In particular, we have investigated the amplitude of quantum fluctuations on the emergent effective anisotropic background which can be a candidate to study the origin of the anisotropy seen from observation of CMB. We provided a relation for the power spectrum of the field on the anisotropic, dressed geometry and compared our result to that of the isotropic case. We have shown that the power spectrum of the anisotropic case is different from the isotropic one and have provided an expression that reflects this difference. This would imprint a signature on the power spectrum of the CMB \[31\], distinct from the one associated to the isotropic dressed, background.

It is worth noting that there are other models such as the Belinskii, Khalatnikov and Lifshitz (BKL) cosmology \[34, 35\] that bare some resemblance to the LQC while there are also important differences between them. First off, BKL is mostly a classical model while LQC is based on quantum gravity. In both models the Universe is oscillating. However, there exists a big bang singularity in the BKL model which disappears in LQC due to quantum effects involved. In fact, in LQC the Friedmann equation effectively acquires an additional “repulsive” term that only dominates near the Planck regime, where it “pushes” the Universe “outwards”, overcoming the classical gravitational attraction. Also while in the BKL model the spatial points decouple from each other near singularity and the Universe behaves chaotically, LQC drastically changes this chaotic behavior which is again due to the quantum effects it incorporates into the analysis. Nevertheless, the analysis of the fate of generic space-like singularities due to the BKL conjecture, in the complete picture of LQG is still under investigation \[36, 37\].

From previous studies we know that there are other curious effects associated to the propagation of massless scalar fields and vector fields, on an effective spacetime in four or lower dimensions \[38–42\]. As a future work, it would be interesting to study similar effects corresponding to the massive scalar field and the connection of these effects to cosmological phenomena and observations.

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Appendix A: Fourier expansion of the inhomogeneous field

Let us write the field \(\varphi(x_0, \mathbf{x})\) as a Fourier decomposition of the form

\[
\varphi(x_0, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \sum_{\mathbf{k} \in \mathcal{C}} \varphi_k(x_0) e^{i\mathbf{k} \cdot \mathbf{x}}, \tag{A1}
\]
where \( \mathcal{L} \) was defined in Eq. [2.3]. By decomposition of the field modes into the real and imaginary parts we have

\[
\varphi_k(x_0) = \frac{1}{\sqrt{2}} \left[ \varphi_k^{(1)}(x_0) + i \varphi_k^{(2)}(x_0) \right].
\]  

(A2)

Reality condition, \( \varphi^*(x_0, x) = \varphi(x_0, x) \), implies that

\[
\varphi_k^{(1)} = \varphi_{-k}^{(1)} \quad \text{and} \quad \varphi_k^{(2)} = -\varphi_{-k}^{(2)}.
\]  

(A3)

By introducing new variables \( q_{\pm k} \) associated to the real variables \( \varphi_k^{(1)} \) and \( \varphi_k^{(2)} \) for the positive and negative sections of \( \mathcal{L} \) as [15]

\[
q_k(x_0) = \begin{cases} 
\varphi_k^{(1)}(x_0), & \text{if } k \in \mathcal{L}_+, \\
\varphi_{-k}^{(2)}(x_0), & \text{if } k \in \mathcal{L}_-,
\end{cases}
\]  

(A4)

we can rewrite the field modes \( \varphi_k \) for each \( k \in \mathcal{L}_+ \), in terms of \( q_{\pm k} \) as

\[
\varphi_k(x_0) = \frac{1}{\sqrt{2}} \left[ q_k(x_0) + i q_{-k}(x_0) \right].
\]  

(A5)

The momentum \( \pi_{\varphi}(x_0, x) \) conjugate to \( \varphi(x_0, x) \) can be Fourier decomposed as

\[
\pi_{\varphi}(x_0, x) = \frac{1}{(2\pi)^{3/2}} \sum_{k \in \mathcal{L}} \pi_k(x_0)e^{ik \cdot x}.
\]  

(A6)

Then separating the momentum \( \pi_k \) associated to each mode \( k \in \mathcal{L} \), into the real and imaginary parts gives

\[
\pi_k(x_0) = \frac{1}{\sqrt{2}} \left[ \pi_k^{(1)}(x_0) + i \pi_k^{(2)}(x_0) \right].
\]  

(A7)

In terms of the new real variables \( p_{\pm k} \) (conjugate to \( q_{\pm k} \)), defined by [15, 16]

\[
p_k(x_0) = \begin{cases} 
\pi_k^{(1)}(x_0), & \text{if } k \in \mathcal{L}_+, \\
\pi_{-k}^{(2)}(x_0), & \text{if } k \in \mathcal{L}_-,
\end{cases}
\]  

(A8)

we rewrite the mode functions (A7), for each mode \( k \in \mathcal{L}_+ \), as

\[
\pi_k(x_0) = \frac{1}{\sqrt{2}} \left[ p_k(x_0) + i p_{-k}(x_0) \right].
\]  

(A9)

The conjugate variables \( (q_k, p_k) \) defined in Eqs. (A4) and (A8) are in fact the variables we have used for the field in formulating the Hamiltonian in Eq. (2.2), but of course by assuming a discrete lattice \( \mathcal{L} \) (see Ref. [15]) rather than continuous space for the modes we have considered in studying of field theory in section IV.

By defining an auxiliary field \( \chi_k = \tilde{c}^{1/2} \varphi_k \), and decomposing it into the real and imaginary parts as in (A2), in terms of the variable \( q_{\pm k}(x_0) \), we have

\[
\chi_k(x_0) = \frac{\tilde{c}^{1/2}}{\sqrt{2}} \left[ q_k(x_0) + i q_{-k}(x_0) \right],
\]  

(A10)
for each $k \in \mathcal{L}_+$. By substituting $q_k$ from Eq. (A10) into Eq. (3.5) in terms of $\chi_k$, and rewriting now the derivation with respect to the conformal time $\tilde{\eta}$, defined in Eq. (4.7), we obtain the field equation (4.4) for the dressed Bianchi type I geometry (4.3) emerged for our background spacetime.

By comparing Eq. (4.4) with the field equation on a Bianchi type I spacetime in Ref. [43]:

$$
\chi_k'' + \left[ \tilde{c}(\tilde{\eta}) \left( \sum_i \tilde{c}_i^2 + \tilde{m}^2 - \tilde{R} \right) + Q \right] \chi_k = 0,
$$

(A11)

the Ricci scalar $\tilde{R}$ for the herein dressed Bianchi geometry becomes

$$
\tilde{R} = 3\tilde{c}''\tilde{c}^{-2} - \frac{3}{2}\tilde{c}^2\tilde{c}^{-3} - 6\tilde{c}^{-1}Q,
$$

(A12)

where we have defined

$$
d_i := \frac{\tilde{c}_i'}{\tilde{c}_i} \quad \text{and} \quad Q := \frac{1}{72} \sum_{i<j} (d_i - d_j)^2.
$$

(A13)

**Appendix B: Fourth order adiabatic expansion**

We notice that $Q(\tilde{\eta}) = \tilde{c}'/2\tilde{c} - \tilde{c}'/4\tilde{c}^2$ is second order in time derivative of the metric components, while $X \equiv \tilde{\omega}_k^2\tilde{\eta}$ is of zeroth order. From (4.5) we have that $Y = X - Q$, so $Y$ contains zeroth and second order terms. Next, from Eqs. (4.22), (4.23), (4.27) and (4.28) we can rewrite $\epsilon_2$ and $\epsilon_4$ as

$$
\epsilon_2 = -\frac{1}{4} Y^{-2}Y''' + \frac{5}{16} Y^{-3}(Y')^2 - X^{-1}Q, \quad (B1)
$$

$$
\epsilon_4 = -\frac{1}{4} Y^{-1}(1 + \epsilon_2)^{-2} \left[ \epsilon_2'' - \frac{1}{2} Y^{-1}Y'\epsilon_2' - \frac{5}{4}(1 + \epsilon_2)^{-1}(\epsilon_2')^2 \right] - \epsilon_2 X^{-1}Q. \quad (B2)
$$

where

$$
\epsilon_2 = -\frac{1}{4} Y^{-2}Y''' + \frac{5}{16} Y^{-3}(Y')^2. \quad (B3)
$$

It is clear that $\epsilon_2$ contains terms of second, fourth and higher orders in time derivative of the metric components. Then, the first derivative of $\epsilon_2$ contains third and higher order terms. Consequently, $\epsilon_2''$ contains terms of orders equal to and bigger than four. Therefore, $\epsilon_2$ contains second and higher order terms. Using these in Eq. (B2) we conclude that the leading order term in $\epsilon_4$ is of order four. Now, we decompose $\epsilon_2$ (up to the fourth order), as

$$
\epsilon_2 = \epsilon_2^{(2)} + \epsilon_2^{(4)} + \text{Higher order terms}, \quad (B4)
$$

where

$$
\epsilon_2^{(2)} = -\frac{1}{4} X^{-2}X'' + \frac{5}{16} X^{-3}(X')^2 - X^{-1}Q, \quad (B5)
$$

$$
\epsilon_2^{(4)} = \frac{1}{4} X^{-2}Q'' - \frac{1}{2} X^{-3}X''Q - \frac{5}{8} X^{-3}X'Q' + \frac{15}{16} X^{-4}(X')^2Q. \quad (B6)
$$
Similarly, decomposition of \( \epsilon_4 \) in terms of fourth and higher order terms reads

\[
\epsilon_4 = \epsilon_4^{(4)} + \text{Higher order terms}
\]

\[
= -\frac{1}{4}X^{-1}\left[\epsilon_2'' - \frac{1}{2}\epsilon_2'X^{-1}X'\right] - \epsilon_2X^{-1}Q
\]

\[
= \frac{1}{16}X'''X^{-3} - \frac{1}{8}X''X'X^{-4} + \frac{3}{8}(X')^2X''X^{-5} + \frac{5}{64}\left[3X'' - 4(X')^2X^{-1}\right](X')^2X^{-5}
\]

\[
- \frac{1}{8}X^{-1}(X''^2 + X'X''')\left(X^{-3} + \frac{5}{4}\right) - \frac{1}{32}X''X^{-4}X' + \frac{1}{16}X^{-2}(X')^2X''\left(X^{-3} + \frac{5}{4}\right)
\]

\[
+ \frac{1}{4}\left[X'' - \frac{5}{4}X^{-1}(X')^2\right]X^{-3}Q - \frac{5}{128}(X')^4X^{-6} + \text{Higher order terms.}
\]

**Appendix C: Renormalization of the energy-momentum tensor**

A key step in constructing a theory of quantum fields is identifying the energy-momentum tensor of the quantized field, which is presumably obtainable from the divergent expression for the energy-momentum tensor that results from the formal Lagrangian field theory. In this appendix, we follow the method of Ref. [28] by applying the “adiabatic regularization” to the anisotropic dressed metric [1.3]. The divergences in the energy-momentum tensor are isolated in the three leading terms of asymptotic expansion corresponding to an adiabatic limit, i.e., the limit of slow time dependence of the metric [1.3]. The quantity expanded is the expectation value of the energy-momentum tensor with respect to the approximate vacuum state associated to the mode function defined in [4.20].

The energy-momentum tensor of a minimally coupled, massive field [4.1] is given by

\[
T_{ab} = \nabla_a \varphi \nabla_b \varphi - \frac{1}{2}\tilde{\eta}_{ab}(\tilde{g}^{cd}\nabla_c \varphi \nabla_d \varphi + m^2 \varphi^2). \tag{C1}
\]

Let \( |\bar{0}\rangle \) be a normalized vacuum state annihilated by all \( \hat{a}_k \). With respect to this vacuum state, the formal expression for the expectation value of the energy density operator, \( \langle 0|\hat{\rho}|0\rangle = -\langle 0|\hat{T}_0^0|0\rangle \), of the field \( \varphi(\tilde{q}, \mathbf{x}) \) on the dressed spacetime [1.3] is obtained as

\[
\langle \bar{0}|\hat{\rho}|\bar{0}\rangle = \frac{\hbar}{\ell^3c^2} \sum_k \rho_k[u_k(\tilde{q})], \tag{C2}
\]

where \( \rho_k \) is given by

\[
\rho_k = \frac{1}{4}\left[|u'_k|^2 + \left(\omega_{\eta,k}^2 + \frac{1}{4}\tilde{c}^2\right)|u_k|^2 - \frac{\tilde{c}'}{2\tilde{c}}(u'_k u'_k + u_k u_k')\right]. \tag{C3}
\]

In this formalism, the quantum theory of test field, reduces to a consideration of the classical wave equation [4.18] for \( u_k \). The expression \( \langle \bar{0}|\hat{\rho}|\bar{0}\rangle \) is infinite and must be renormalized by subtracting the divergent terms. Therefore, in the following, we introduce the adiabatic regularization technique in order to handle with divergences of the energy and pressure components of the quantized field.

For each \( u_k \) in the summand (C2), it can be shown that all ultra-violet divergences are contained in terms of adiabatic order equal to and smaller than four. Our task now is to expand the energy and pressure for each mode \( k \) in asymptotic series, by using adiabatic approximation given in [4.26].
We start from the (adiabatic) regularization of energy density operator \([\mathcal{C}3]\). We compute \(\langle 0|\hat{\rho}|0\rangle_{\text{ren}}\), up to the fourth adiabatic order, as

\[
\langle 0|\hat{\rho}|0\rangle_{\text{ren}} = \frac{\hbar}{\epsilon^3 c^2} \sum_k \left( \rho_k[u_k(\tilde{\eta})] - \rho_k(\tilde{\eta}) \right),
\]  

(C4)

where the subtraction term \(\rho_k(\tilde{\eta}) \equiv \rho_k[u_k(\tilde{\eta})]\) (being the adiabatic vacuum energy for each mode) is needed to regularize the energy density. Here \(\rho_k\) is obtained by the terms of zeroth, second and fourth adiabatic orders in the expansion of the summand as

\[
\rho_k(\tilde{\eta}) = \rho_k^{(0)} + \rho_k^{(2)} + \rho_k^{(4)}.
\]  

(C5)

To the fourth order we have

\[
|\eta_k|^2 = (W_k(\tilde{\eta}))^{-1} = \tilde{\omega}_{\eta,k}^{-1} \left[ 1 - \frac{1}{2} (\epsilon_2^{(2)} + \epsilon_4^{(4)} + \frac{3}{8} \epsilon_2^{(2)})^2 \right],
\]  

(C6)

\[
|\eta'_k|^2 = W_k(\tilde{\eta}) + \frac{1}{4} (W'_k(\tilde{\eta}))^2 = \tilde{\omega}_{\eta,k} \left[ 1 + \frac{1}{2} (\epsilon_2^{(2)} + \epsilon_4^{(4)} - \frac{1}{8} \epsilon_2^{(2)})^2 \right] + \frac{1}{4} \tilde{\omega}_{\eta,k}^{-3} \tilde{\omega}_{\eta,k}'^2 \left( 1 - \frac{1}{2} \epsilon_2^{(2)} \right) + \frac{1}{4} \tilde{\omega}_{\eta,k}^{-3} \tilde{\omega}_{\eta,k}'^2 \epsilon_2^{(4)}
\]  

(C7)

and

\[
2\Re(\eta_k^* \eta'_k) = -W'_k(\tilde{\eta}) (W_k(\tilde{\eta}))^{-2} = -\tilde{\omega}_{\eta,k}^{-1} \tilde{\omega}_{\eta,k}' \left( 1 - \frac{1}{2} \epsilon_2^{(2)} \right) + \frac{1}{2} \epsilon_2^{(4)}
\]  

(C8)

where, \(2\Re(\eta_k^* \eta'_k) = (\eta_k^* \eta'_k + \eta_k^\prime \eta'_k)\). By replacing Eqs. \((C6)\) and \((C7)\) and \((C8)\) in \((C3)\), we obtain the zeroth, second and fourth adiabatic order of vacuum energy density as

\[
\rho_k^{(0)} = \frac{\tilde{\omega}_{\eta,k}}{2},
\]  

(C9)

\[
\rho_k^{(2)} = \frac{1}{16} \left( \tilde{\omega}_{\eta,k}^3 \right)^2 + \frac{1}{16 \tilde{\omega}_{\eta,k}^2} \frac{\tilde{c}^2}{c^2} + \frac{\tilde{c}'}{8 \tilde{\omega}_{\eta,k}},
\]  

(C10)

\[
\rho_k^{(4)} = \frac{1}{16} \left( \tilde{\omega}_{\eta,k}^2 \epsilon_2^{(2)} \right)^2 + \frac{1}{16} \left( \frac{\tilde{\omega}_{\eta,k}}{\tilde{\omega}_{\eta,k}^2} \tilde{c}' \right) \epsilon_2^{(4)} - \frac{1}{32} \left( \frac{\tilde{\omega}_{\eta,k}^2}{\tilde{\omega}_{\eta,k}} \epsilon_2^{(2)} \right)^2 + \frac{1}{16} \frac{\tilde{c}^2}{c^2} + \frac{\tilde{c}'}{8 \tilde{\omega}_{\eta,k}^2} \epsilon_2^{(2)}.
\]  

(C11)

The expressions for \(\epsilon_2^{(2)}\) and \(\epsilon_4^{(4)}\) are given by Eqs. \((B5)\) and \((B6)\). Notice that, the terms \((C9)-(C11)\) are state-independent and local in the background geometry. The significance of Eq. \((C5)\) is that the ultra-violet divergences associated with the zeroth, second and fourth-order terms can be removed by renormalization.

The formal expression for the expectation value of the pressure operator is given by

\[
\langle 0|\hat{P}_i|0\rangle := \frac{\hbar}{\epsilon^3 c^2} \sum_k P_{i,k}[u_k(\tilde{\eta})],
\]  

(C12)

where, for each mode \(u_k\) we have

\[
P_{i,k} = \frac{\tilde{c}}{4} \left[ |u_k'|^2 - \frac{\tilde{\omega}_{\eta,k}^2}{4} |u_k'|^2 - \frac{1}{2} \frac{\tilde{c}^2}{\tilde{c}} (u_k^* u_k^\prime + u_k^\prime u_k^*) \right].
\]  

(C13)
The renormalized vacuum expectation value of the stress operator is obtained by subtracting the divergent terms as

\[
\langle \hat{0} | \hat{P} | \hat{0} \rangle_{\text{ren}} = \frac{\hbar}{\ell^3 c^2} \sum_k \left( P_{i,k}[u_k(\bar{\eta})] - P_{i,k}(\bar{\eta}) \right).
\]  

(C14)

Here, the subtraction term \( P_{i,k}(\bar{\eta}) \equiv P_{i,k}[u_k(\bar{\eta})] \) associated to the vacuum expectation value, is obtained, up to the fourth adiabatic order, as

\[
P_{i,k}(\bar{\eta}) = P_{i,k}^{(0)} + P_{i,k}^{(2)} + P_{i,k}^{(4)}.
\]  

(C15)

By a similar analysis as for the energy density operator, the zeroth, second and fourth order terms of \( P_{i,k} \) on the right hand side of Eq. \((C15)\) are given by

\[
P_{i,k}^{(0)} = \frac{\bar{c}}{2\bar{\omega}_{\bar{\eta},k}} k_i^2,
\]  

(C16)

\[
P_{i,k}^{(2)} = \frac{\bar{c}_i}{4\bar{\omega}_{\bar{\eta},k}} \left[ \bar{c}^2 \left( \bar{\omega}_{\bar{\eta},k}^2 - \bar{k}_i^2 \right) + \left( \bar{\omega}_{\bar{\eta},k}^2 + \bar{\omega}_{\bar{\eta},k}^2 \right) \right] + \left( \bar{\omega}_{\bar{\eta},k}^2 - \bar{k}_i^2 \right) \epsilon_{i}^{(2)} \epsilon_{i}^{(2)},
\]  

(C17)

\[
P_{i,k}^{(4)} = \frac{\bar{c}_i}{4\bar{\omega}_{\bar{\eta},k}} \left[ \left( \bar{\omega}_{\bar{\eta},k}^2 + \bar{\omega}_{\bar{\eta},k}^2 \right) \epsilon_{i}^{(4)} + \left( \frac{3}{4} \bar{k}_i^2 \bar{c}_i^4 - \bar{\omega}_{\bar{\eta},k}^2 \right) \epsilon_{i}^{(2)} \right] + \left( \bar{\omega}_{\bar{\eta},k}^2 + \frac{\bar{c}_i^4}{4\bar{c}} \right) \epsilon_{i}^{(3)} \epsilon_{i}^{(3)} - \left( \frac{\bar{\omega}_{\bar{\eta},k}^2}{8\bar{\omega}_{\bar{\eta},k}^2} + \frac{\bar{c}_i^4}{8\bar{c}^2} + \frac{\bar{c}_i^4}{4\bar{c} \bar{\omega}_{\bar{\eta},k}} \right) \epsilon_{i}^{(2)} \epsilon_{i}^{(2)}.
\]  

(C18)

The formal expression for the vacuum expectation value of the Hamiltonian operator \( \hat{H}_\varphi \), for the quantized massive field, on the effective background \((4.3)\) is obtained as

\[
\langle \hat{0} | \hat{H}_\varphi | \hat{0} \rangle = \frac{\hbar}{\ell^3} \sum_k \tilde{H}_k[u_k(\bar{\eta})],
\]  

(C19)

where, \( \tilde{H}_k \) is the expectation value of the Hamiltonian for \( k \)th mode. By setting \( \tilde{N}_{x_0} = \bar{c}^{1/2}(\bar{\eta}) \) we achieve the expression for \( \tilde{H}_k \) in conformal time as

\[
\tilde{H}_k(\bar{\eta}) = \frac{1}{4} \left[ |u_k'|^2 + \left( \frac{1}{4} \frac{\bar{c}_i^4}{\bar{c}^2} + \bar{\omega}_{\bar{\eta},k}^2 \right) |u_k|^2 - \frac{1}{2} \bar{c}_i \left( u_k u_k^* + u_k^* u_k \right) \right] = \rho_k(\bar{\eta}).
\]  

(C20)

Therefore, it is straightforward to show that the renormalized vacuum expectation value of the field’s Hamiltonian is

\[
\langle \hat{0} | \hat{H}_\varphi | \hat{0} \rangle_{\text{ren}} = \frac{\hbar}{\ell^3} \sum_k \left( \hat{H}_k[u_k(\bar{\eta})] - \tilde{H}_k(\bar{\eta}) \right)
\]  

\[
= \bar{c}^2(\bar{\eta}) \langle \hat{0} | \hat{\rho} | \hat{0} \rangle_{\text{ren}},
\]  

(C21)

where \( \tilde{H}_k = \tilde{H}_k(u_k) \), is given, up to the fourth order, by

\[
\tilde{H}_k(\bar{\eta}) = \bar{c}^2(\rho_k^{(0)} + \rho_k^{(2)} + \rho_k^{(4)}),
\]  

(C22)
The terms $\rho_k^{(n)}$ (with $n = 1, 2, 3$) are given by Eqs. (C9), (C10) and (C11). Therefore, expectation values of renormalized Hamiltonian operator (C21) is well-defined on any fourth order adiabatic state.

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