Abstract

We compare the physical reach of a Neutrino Factory in the 2+2 and 3+1 four–family models, with similar results in the two schemes; in both cases huge CP-violating effects can be observed with a near detector in the $\nu_\mu \rightarrow \nu_\tau$ channel. We also study the capability of long baseline experiments (optimized for the study of the three–family mixing parameter space) in distinguishing a three (active) neutrino model from a four–family scenario.

Key words: NUFAC01, neutrino, sterile, oscillations, CP-violation.

The present experimental solar and atmospheric neutrino data give strong indications in favour of neutrino oscillations. In addition, the results of the LSND experiment [1] would imply the existence of a puzzling fourth, sterile, neutrino state. There are two very different classes of spectra with four massive neutrinos: two pairs of almost degenerate neutrinos divided by the large LSND mass gap (the 2+2 scheme); or three almost degenerate neutrinos and an isolated fourth one (the 3+1 scheme). The former gave a better fit, as was shown in [2], but the recent SNO results [3] will certainly restrict the allowed parameter region. The latter is at present only marginally compatible with the data [4].

An experimental set-up capable of precision measurement of the whole three-neutrino mixing parameter space (including the CP violating phase $\delta$) is under study. This experimental programme consists of the development of a “Neutrino Factory” (high-energy muons decaying in the straight section of a storage ring and producing a very pure and intense neutrino beam [5,6]) and of suitably optimized detectors. We shall consider in what follows a neutrino beam resulting from the decay of $n_\mu = 2 \times 10^{20}$ unpolarized positive and/or negative muons per year. The collected muons have energy $E_\mu$ in the range $10 - 50$ GeV.

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The following parametrizations were adopted for the four-family mixing matrix:

\[ U^{(3+1)}_{PMNS} = U_{14}(\theta_{14}) U_{24}(\theta_{24}) U_{34}(\theta_{34}) \times U_{23}(\theta_{23}, \delta_3) U_{13}(\theta_{13}, \delta_2) U_{12}(\theta_{12}, \delta_1), (1) \]

\[ U^{(2+2)}_{PMNS} = U_{14}(\theta_{14}) U_{13}(\theta_{13}) U_{24}(\theta_{24}) U_{23}(\theta_{23}, \delta_3) \times U_{34}(\theta_{34}, \delta_2) U_{12}(\theta_{12}, \delta_1), (2) \]

with \( \nu_1 \) the lightest state and \( \nu_4 \) the heaviest.

If LSND is confirmed . . .

. . . by MiniBooNE [7], a short baseline experiment is mandatory to explore the four-family mixing parameter space [8]. A comparison of the physical reach at a Neutrino Factory in the 2+2 and 3+1 scheme has been extensively presented in [9].

To study the CP-conserving four-family mixing matrix, we consider a short baseline experiment with an hypothetical 1 ton detector with \( \tau \) tracking and (\( \mu, \tau \)) charge identification capability, with constant background at the level of \( 10^{-5} \) and a constant efficiency \( \epsilon_\mu = 0.5 \) for \( \mu^\pm \) and \( \epsilon_\tau = 0.35 \) for \( \tau^\pm \). Our results show that the considered set-up can severely constrain the whole four-family model CP-conserving parameter space, both in the 2+2 scheme and 3+1 scheme. The sensitivity reach to all gap-crossing angles in the LSND-allowed region is at the level of \( \sin^2 \theta \geq 10^{-6} - 10^{-4} \), depending on the specific angle considered, for the 2+2 scheme, and at the level of \( \sin^2 \theta \geq 10^{-5} - 10^{-3} \), in the 3+1 scheme.

To extend our analysis to the CP-violating four-family parameter space we consider an hypothetical 10 Kton detector, located a bit farther from the neutrino source, \( L = O(10-100) \) Km. Large CP violation effects are possible in four-family, since the overall size of some of the CP-violating observables depends on \( \Delta_{atm} \) and not on \( \Delta_{\odot} \) (as in three families). In Fig. 1 we show the signal-to-noise ratio of the subtracted integrated CP asymmetry [6] in the \( \nu_\mu \to \nu_\tau \) channel for the 2+2 (left) and the 3+1 (right) scheme, respectively. In both cases, for \( E_\mu = 50 \) GeV , \( \sim 100 \) standard deviations are attainable at \( L \approx 30 - 40 \) Km. The other two channels (\( \nu_e \to \nu_\mu, \nu_\tau \)) give a much smaller significance in both schemes.

In the absence of a conclusive confirmation of the LSND results . . .

. . . the detectors will be located very far away from the neutrino source. It is of interest to explore the capability of such an experimental set-up to discriminate between a three-neutrino model and a possible four-neutrino scenario, and to understand if three-family data implying a large CP-violating phase can be also fitted in a CP-conserving four-family model. A detailed answer to both questions has been given
in [10]. The four–family 3+1 scheme reduces to the three–family model for vanishing mixing with the isolated state, see eq. (1), and a discrimination will be possible only for mixing large enough. On the other hand, in the limit of vanishing gap–crossing angles the 2+2 scheme mixing matrix, eq. (2), reduces to that of two independent two–neutrinos oscillations. The possibility of confusion is negligible in this case.

We consider a 40 Kton magnetized iron detector with realistic backgrounds and efficiencies [11]. We fit with a three–neutrino model “data” obtained smearing the four–family theoretical input, following ref. [12]. In the 3+1 scheme we fixed $\Delta m^2_{LSND} = 1$ eV$^2$, $\Delta m^2_{atm} = 2.8 \times 10^{-3}$ eV$^2$, $\Delta m^2_{\odot} = 1 \times 10^{-4}$ eV$^2$, $\theta_{12} = 22.5^\circ$ and $\theta_{23} = 45^\circ$; we also assumed $\theta_{14} = \theta_{24} = 2^\circ, 5^\circ, 10^\circ$, and allowed a variation of $\theta_{13} \in [1^\circ, 10^\circ]$ and $\theta_{34} \in [1^\circ, 50^\circ]$. The matter effects have been evaluated with constant density $\rho = 2.8 (3.8)$ g cm$^{-3}$ for $L = 732$ or 3500 (7332) Km. The CP violating phases in the 3+1 model have been set to zero. In the three–neutrino model, we fixed $\theta_{12} = 22.5^\circ$ and $\theta_{23} = 45^\circ$, allowing the remaining parameters to vary in the intervals $\theta_{13} \in [1^\circ, 10^\circ]$, $\delta \in [-180^\circ, 180^\circ]$.

The results depend heavily on the values of the small gap–crossing angles $\theta_{14}$ and $\theta_{24}$. If their value is small ($2^\circ$), confusion is possible almost everywhere in the ($\theta_{13}, \theta_{34}$) plane. This is shown in the first row of Fig. 2, where in the dark regions of the “dalmatian dog hair” plot [13] the three–neutrino model is able to fit at 68% c.l. “data” generated in four families (five energy bins and five years of data taking have been considered). The reason of the blotted behaviour stands in statistical fluctuations in the smearing of the input distributions. For increasing values of ($\theta_{14}, \theta_{24}$), the extension of blotted regions decreases. This is shown for $\theta_{14} = \theta_{24} = 5^\circ$ in the second row of Fig. 2, again for five energy bins. Notice that with the shortest baseline we can always tell 3 from 3+1. In the last row of Fig. 2 we present dalmatian plots for “data” consisting of ten energy bins. The regions of confusion are considerably reduced, since the different energy dependence in matter helps in the distinction of the two models: a fit of the largest baseline data is successful for $\theta_{34} \lesssim 10^\circ$ only. A
further increase of the gap–crossing angles to $\theta_{14} = \theta_{24} = 10^\circ$ makes the distinction (even at 95% c.l.) of the two models possible in the whole ($\theta_{13}$, $\theta_{34}$) plane.

Fig. 2. “Dalmatian dog hair” plots for: a) $\theta_{14} = \theta_{24} = 2^\circ$, five bins; b) $\theta_{14} = \theta_{24} = 5^\circ$, five bins; c) $\theta_{14} = \theta_{24} = 5^\circ$, ten bins; from left to right: $L = 732$, 3500 and 7332 Km.

In the blotted regions of Fig. 2, the corresponding fitted value of the CP violating phase $\delta$ is generally not large. This is particularly true for $L = 3500$ Km, whereas for $L = 7332$ Km the determination of $\delta$ is somewhat looser. For the shortest baseline, $L = 732$ Km, we have the largest spread in the values of $\delta$, although the most probable value is still close to zero. Finally, data that can be fitted with a CP phase close to $90^\circ$ in the three–neutrino theory cannot be described in a CP conserving 3+1 theory, provided that data at two different distances are used.

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