Stochastic electron acceleration in the TeV supernova remnant RX J1713.7–3946: the high-energy cut-off

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ABSTRACT
In the leptonic scenario for TeV emission from a few well-observed shell-type TeV supernova remnants (STTSNRs), very weak magnetic fields are inferred. If fast-mode waves are produced efficiently in the shock downstream, we show that they are viable agents for acceleration of relativistic electrons inferred from the observed spectra even in the subsonic phase, in spite that these waves are subject to strong damping by thermal background ions at small dissipation scales. Strong collisionless non-relativistic astrophysical shocks are studied with the assumption of a constant Alfvén speed in the downstream. The turbulence evolution is modelled with both the Kolmogorov and Kraichnan phenomenology. Processes determining the high-energy cut-off of non-thermal electron distributions are examined. The Kraichnan models lead to a shallower high-energy cut-off of the electron distribution and require a lower downstream density than the Kolmogorov models to fit a given emission spectrum. With reasonable parameters, the model explains observations of STTSNRs, including recent data obtained with the Fermi γ-ray telescope. More detailed studies of the turbulence generation and dissipation processes, supernova explosions and progenitors are warranted for better understanding of the nature of supernova shocks.

Key words: acceleration of particles – MHD – plasmas – shock waves – turbulence – ISM: supernova remnants.

1 INTRODUCTION
The acceleration of cosmic rays up to \( \sim 10^{15} \) eV has been attributed to supernova explosions, and TeV emission is expected from the remnants (Ginzburg & Ptuskin 1976; Lagage & Cesarsky 1983; Reynolds 2008; Butt 2009). The standard diffusive shock particle acceleration (DSA) model has been successful in explaining emissions from most supernova remnants (Eichler 1979c; Blandford & Eichler 1987; Kirk & Duffy 1999; Zirakashvili & Aharonian 2007; Reynolds 2008; Vannoni, Gabici & Aharonian 2009). Investigations of acceleration by a spectrum of turbulent plasma waves, the so-called stochastic particle acceleration (SA), also have a long and resilient history (Scott & Chevalier 1975; Lacombe 1977; Achterberg 1979; Eilek 1979; Bykov & Toptygin 1983; Cowisk & Sarkar 1984; Ptuskin 1988; Bykov & Toptygin 1993; Atoyan et al. 2000; Petrosian & Liu 2004; Cho & Lazarian 2006; Liu et al. 2008a). Most authors prefer the use of relativistic leptons to account for the non-thermal radio, X-rays and TeV emissions from the remnants.

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The TeV emission has also been attributed to energetic protons and ions (Aharonian et al. 2006; Morlino, Amato & Blasi 2009; Fang et al. 2009; Zirakashvili & Aharonian 2010).

Although the DSA can naturally give a universal power-law energetic particle distribution with the spectral index determined by the shock compression ratio for a linear model, it requires well-defined shock structure and efficient scatter of high-energy particles by small-scale turbulence (Bell 1978). The initial acceleration of low-energy particles to a high enough energy for the shock to be effective, the so-called injection problem, is likely due to the SA by turbulence. Lacombe (1977) showed that the SA by small-scale Alfvén waves can be more efficient than the first-order Fermi acceleration by shocks. Achterberg (1979) derived approximate diffusion coefficients and showed that stochastic interactions of particles with a spectrum of plasma waves can lead to efficient particle acceleration. Over the past few decades, the SA has also been explored for broader astrophysical applications (e.g. Eichler 1979a,b; Cowisk & Sarkar 1984; Ball, Melrose & Norman 1992; Miller, LaRosa & Moore 1996; Schlickeiser & Miller 1998; Petrosian & Liu 2004; Yan & Lazarian 2004; Fisk & Gloeckler 2007; Petrosian & Bykov 2008). The essential challenges to the SA model are a self-consistent treatment of the non-linear turbulence spectral evolution and the...
requirement of the same energy dependence of the acceleration and escape time-scales for the production of a power-law particle distribution (Ptuskin 1988; Ball, Melrose & Norman 1992; Becker, Le & Dermer 2006). Recently, Gibbsonian theory has been generalized to account for power-law distributions in marginally stable Gibbsonian equilibria (Treumann &Jaroschek 2008). It remains to be shown how the physical processes of the SA are related to the ordering parameter $\kappa$ of this statistics. Most previous studies assume isotropic magnetohydrodynamic (MHD) waves for the turbulence (Lacombe 1977; Achterberg 1979; Eichler 1979b; Miller et al. 1996). The anisotropy of the turbulence caused by cascade and damping processes has been considered recently (Goldreich & Sridhar 1995; Chandran 2003; Yan & Lazarian 2004; Cho & Lazarian 2006; Jiang, Liu & Petrosian 2009). In particular, Liu et al. (2008a) show that the scatter and acceleration rates of charged relativistic particles by fast-mode waves in a high-$\beta$ plasma may be much higher than those given by the standard quasi-linear theory with an isotropic wave power spectrum.

Over the past few years, detailed radio, X-ray, $\gamma$-ray and TeV observations of a few shell-type TeV supernova remnants (STTSNRs) pose several challenges to the classical DSA model in the hadronic scenario, where the TeV emission is produced through neutral pion decays induced by proton–proton scatter (Aharonian et al. 2006, 2007; Tanaka et al. 2006, 2007; Funk 2009). Besides requiring efficient amplification of the magnetic field in the upstream plasma and a good correlation between the magnetic field and background plasma density, a significant correlation between the magnetic field and fast-mode wave turbulence is expected given the nature of these processes when applying this mechanism to specific observations. For example, Eichler (1979a) showed that the scatter mean free path of relativistic electrons is a free parameter in most of the previous studies, is determined by the characteristic length of the magnetic field, which in a high-$\beta$ plasma is reduced by strong turbulence motions significantly and (5) the high-energy cut-off of the particle distribution is tied to the characteristic length of the magnetic field.

In this paper, we first discuss the SA by decaying turbulence in general and show that fast-mode waves may account for observations of a few STTSNRs (Section 2). In Section 3, we present the structure of the downstream turbulence with both the Kolmogorov and Kraichnan phenomenology for the turbulence cascade. The transit-time damping (TTD) by the thermal background particles of compressional fast-mode turbulence is considered in the dissipation range. The stochastic acceleration of relativistic electrons by fast-mode wave turbulence in the subsonic phase is discussed in Section 4, where physical processes determining the acceleration of the highest energy electrons are discussed. The models are applied to the well-observed TeV SNR RX J1713.7–3946 in Section 5. Although the inferred plasma density is much lower than that in the hadronic scenario, some models still have significantly higher densities than those derived from XMM–Newton observations (Cassam-Chenaï et al. 2004). In Section 6, we discuss how the density may be further reduced by considering the turbulence generation processes and first-order Fermi acceleration in the supersonic phase. Conclusions are drawn in Section 7.

2 GENERAL CONSTRAINTS ON THE PARTICLE ACCELERATION

According to the classical Fermi mechanism (Fermi 1949), the particle acceleration rate is determined primarily by the scatter mean free path $l$ and the velocity of the scatter agents $u$ (Blandford & Eichler 1987; Ball et al. 1992). Very general constraints can be obtained on the nature of these processes when applying this mechanism to specific observations. For example, Eichler (1979a) showed that the acceleration process must be selective in the sense that only a fraction of the background particles are accelerated to very high energies. Otherwise, these stochastic interactions likely lead to plasma heating instead of a very broad energy distribution of accelerated particles, as frequently observed in dynamically evolving...
collisionless astrophysical plasmas. For particle acceleration in solar flares, Eichler (1979b) argued that this selective acceleration could be achieved in the energy domain (i.e. the frequency domain for the waves) through cyclotron resonances of particles with a spectrum of cascading plasma waves. In the DSA model, the particle acceleration at low energies and the shock structure determine the efficiency of different particle species (Eichler 1979a). In the presence of a magnetic field, selective energization may also be realized in the domain of the particle pitch angle and/or wave direction angle with respect to the magnetic field (Beresnyak & Lazarian 2008).

Further constraints can be put on the SA by a spectrum of turbulence. Given the small gyro-radii of charged particles in magnetized astrophysical plasmas, charged particles couple strongly through the magnetic field. As a result, the turbulence responsible for the SA will decay as the energy carrying plasma is being carried away from the source region of the turbulence by large-scale flows and/or magnetic fields for a high- and/or low-β plasma, respectively. This is the case for the SA in a shock downstream with a high value of the plasma β, where the turbulence is generated at the SF and its intensity decreases as the flow moves away from the SF (Liu et al. 2008a). We next discuss constraints on such a particle acceleration scenario.

In the Kolmogorov phenomenology for the turbulence cascade (Kolmogorov 1941), the free energy dissipation rate is given by

$$Q = C_1\rho u^3/L,$$

where $C_1$ is a dimensionless constant, $\rho$ is the mass density, and $L$ and $U$ are the eddy speed and the turbulence generation scale, respectively. The eddy turnover speed and time at smaller scales are given, respectively, by

$$v_{edd}^2(k) = 4\pi W(k)k^\gamma \propto k^{\gamma-3/2},$$

$$\tau_{edd}(k) = (k v_{edd})^{-1} = (4\pi W(k)k^{\gamma-3/2}) \propto k^{-2/3},$$

where

$$W(k) = (u^2/4\pi) L^{-2/3} k^{-11/3} = (4\pi)^{-1}(Q/C_1 \rho)^{3/2} k^{-11/3} \propto k^{-11/3},$$

is the isotropic turbulence power spectrum, $k = 1/l$ is the wavenumber and $l$ is the eddy size. From the three-dimensional Kolmogorov constant $C \simeq 1.62$ (Yeung & Zhou 1997), we obtain $C_1 = C^{3/2} = 0.485$. At the turbulence generation scale $L = 1/k_m$, $v_{edd} = u$, $Q = C_1 \rho(4\pi W(k)k^{11/3})^{1/2} = C_1 \rho v_{edd}^3(k) / \tau_{edd}(k)$ and the total turbulence energy is given by $\int W(k)4\pi k^2 dk = (3/2)u^2$. The turbulence decay time is therefore given by $\tau_d \equiv dr/dln(u) = 3\tau_{edd}(k_m)/C_1$, where $t$ indicates the time, i.e. eddies decay after making $3/(2C_1) \sim 1$ turn.

We are interested in the acceleration of particles through scattering randomly with heavy scatter centres with the corresponding acceleration time given by (Blandford & Eichler 1987; Ball et al. 1992)

$$\tau_a = \tau_a [3v^2/v_{edd}^2(k)],$$

where

$$\tau_a = (kv)^{-1} = l/v,$$

is the scatter time, $v$ is the particle speed and we have assumed that the scatter mean free path is equal to $l$. For the above isotropic Kolmogorov turbulence spectrum, $\tau_a(k) \propto 3v^2/(4\pi W(k)k^\gamma) \propto k^{\gamma-1/3}$. To have significant SA, the acceleration time $\tau_a(k)$ should be shorter than the turbulence decay time $\tau_d$, which implies $u^2 > C_1 v_{edd}^3(k)$. So, in general, the SA is more efficient at smaller scales. The onset scale of the SA is given by $k_c = (C_1 v^3/u^3) k_m$, where $\tau_d = \tau_a$. Therefore, to produce energetic particles with a speed of $v$ by a Kolmogorov spectrum of scatter centres, the turbulence must have a dynamical range greater than

$$D_{kol} = (C_1 v^3/u^3).$$

This acceleration process corresponds to the acceleration by incompressional motions studied by Bykov & Toptygin (1983).

In the Iroshnikov–Kraichnan phenomenology (Iroshnikov 1963; Kraichnan 1965; Jiang et al. 2009), the turbulence cascade rate is suppressed by the wave propagation effect by a factor of $v_F/v_{edd}$, where $v_F$ (independent of $k$) is the wave group speed:

$$C_1 p v_{edd}^3(v_F / \tau_{edd}) = Q.$$
where $3u^2/2$ is the overall wave intensity, $k_{eq}$ indicates the wave generation scale and below the turbulence dissipation scale $k_{d} = 1/\xi_{k}$ the wave intensity is negligible. $\xi(\delta)$ depends on $\delta$ and is on the order of unity [e.g. $\xi(11/3) \simeq 3.0$]. The results for fast and slow diffusion have been combined here approximately to give a unified expression (Puskin 1988; Cho & Lazarian 2006). For $\delta < 4$, $\tau_{ac}$ increases monotonically with $D$. To have significant acceleration, the minimum acceleration time-scale $\tau_{ac(min)} = \tau_{ac}(u^2/u^2)$ must be less than $\tau_{d} = 3L_{\nu}\nu/(C_{1}\nu^2)$. Then the scatter mean free path of the particles $\tau_{ac}u$ must be shorter than

$$L/D_{M} = 3L_{\nu}\nu/(C_{1}\nu) = 3L/D_{ik}^2.$$  \hspace{2cm} (17)

It is also possible that the dynamics of the turbulence cascade is not affected by the wave propagation, which only enhances the particle acceleration rate. Then $\tau_{d} = 3L_{\nu}/(C_{1}\nu^2)$, the dynamical range required for the SA to be significant is given by

$$D_{AK} \equiv C_{1}\nu/(3u) = D_{ik}^{1/3}/3.$$  \hspace{2cm} (18)

Several STTSNRs have been observed extensively in the radio, X-ray and TeV bands. X-ray observations with the Chandra, XMM–Newton and Suzaku and TeV observations with the HESS have made several surprising discoveries that challenge the DSA model in the hadronic scenario (Aharonian et al. 2006, 2007; Liu et al. 2008a; Tanaka et al. 2008). The leptonic scenario, on the other hand, is relatively simple except that the electron acceleration mechanism needs to be addressed (Zirakashvili & Aharonian 2007; Liu et al. 2008a; Vannoni et al. 2009). The TeV SNR RX J1713.7–3946 is about $T_{AK} = 1600$ yr old (Wang, Qu & Chen 1997) with a radius of $R \simeq 10$ pc and a distance of $D \simeq 1$ kpc. By fitting its broad-band spectrum with an electron distribution of $f \propto \gamma^{-p} \exp\left(-\gamma/j_{inc}\right)^{1/2}$ (the dashed lines in Fig. 7), we find that $p = 1.85$, $B = 12.0$ $\mu$G, $\gamma_{mc}e^2 = 3.68$ $\text{TeV}$, and the total energy of relativistic electrons with the Lorentz factor $\gamma > 1800$ $E_{c} = 3.92 \times 10^{37}$ erg.

The X-ray emitting electrons have a gyro-radius of $r_{g} \simeq 10^{8}$ cm, which should be shorter than the particle scatter mean free path. To produce these electrons through the SA, the turbulence must be generated on scales greater than $D_{AK}r_{g}, D_{AK}r_{g}, D_{AK}r_{g}$ and $D_{AK}r_{g}$ for the non-resonant Kolmogorov, Kraichnan, acoustic Kolmogorov and Kraichnan phenomenology, respectively. For STTSNRs, $u \sim \nu \sim 0.01c$, $D_{AK}r_{g} \sim 10$kpc, which is much greater than the radii of the remnants. The SA by eddies with a Kolmogorov spectrum is therefore insignificant. The standard quasi-linear theory also predicts negligible SA. $D_{AK}r_{g} \sim 30$pc, which is also too thick. $D_{AK}r_{g} \sim D_{AK}r_{g} \sim 0.03pc$, which is much greater than the ion inertial length and may result from Kelvin–Helmholtz instabilities or cosmic ray drifting upstream (Bell 1978; Micono et al. 1999; Giacalone & Jokipii 2007; Niemiec et al. 2008). Therefore if relativistic electrons from the STTSNRs are accelerated through the TTD process, the waves in the dissipation range propagate along local magnetic field lines with a spectrum index of 2. In a high-$\beta$ plasma, the damping is mostly caused by ions. The residual parallel propagating waves, however, preferentially accelerate electrons. Energetic particles are also subject to efficient acceleration by large-scale fast-mode (acoustic) waves with energy-independent acceleration and scatter rates, giving rise to power-law particle distributions in the steady state. At even higher energies, the gyro-radius of the particles exceeds the characteristic length of the magnetic field, which also corresponds to the dissipation scale; the spatial diffusion coefficient $D$ increases quickly with energy. However, due to interactions with large-scale turbulence, both the acceleration and spatial diffusion time-scales vary gradually with $D$, implying a gradual high-energy cut-off.

### 3 SHOCK STRUCTURE AND DAMPING OF FAST-MODE WAVES IN THE DOWNSTREAM

We consider the relatively simple case, where both the thermal pressure and magnetic field are negligible in the upstream and the shock normal is parallel to the plasma flow. The mass, momentum and energy fluxes are given, respectively, by $p_{\nu_0}$, $P + \rho v_{\nu_0}^2$ and $V_{\nu_0}(E + P + \rho V_{\nu_0}^2)/2$, where $V_{\nu_0}$, $P$ and $E$ are the speed, pressure and energy density of the plasma flow, respectively. Based on the parameters inferred from the leptonic scenario for the TeV emission from STTSNRs shown in Section 2, the plasma $\beta$ in the shock downstream is likely high and the fast-mode wave damping by thermal background is then dominated by protons and ions (Liu et al. 2008a). The Alfvén speed given by $v_{A} = (B^2/4\pi\rho)^{1/2}$ therefore must be much smaller than the turbulence speed $u$ near the SF, where $B$ is the magnetic field intensity. For strong non-relativistic shocks with the shock frame upstream speed $U$ much higher than the speed of the parallel propagating fast-mode waves in the upstream $v_{F} = (v_{\nu}^2 + 5v_{\nu}^2/3)^{1/2}$, where $v_{\nu}^2 = P_{\nu}/\rho$ is the isothermal sound speed and $P_{\nu}$ is the thermal pressure of the gas, $V_{\nu_0} = U$ and $\rho U^2 \gg P \sim E$ in the upstream. In the downstream, the pressure and energy density have contributions from the thermal gas and turbulence: $P = \rho(v_{\nu}^2 + u^2)$ and $E = \rho(3v_{\nu}^2/2 + 3u^2)/2$, where we have assumed that the turbulence behaves as an ideal gas and ignored possible dynamical effects of the wave propagation. Then
we have (Tidman & Krail 1971)
\[
\rho_u U = \rho_u U/4,
\]
(19)
\[
\rho_u U^2 = \rho_u \left( U^2/16 + v_s^2 + u^2 \right),
\]
(20)
\[
\rho_u U^4/2 = \rho_u U \left( U^2/16 + 5v_s^2 + 5u^2 \right)/8,
\]
(21)
where the subscripts ‘u’ and ‘d’ denote the upstream and downstream, respectively, and we have ignored the effects of the electromagnetic fields and the thermal energy and pressure in the upstream. Then we have
\[
U^2 = 5v_s^2 + 5u^2 + U^2/16.
\]
(22)
This is slightly different from that given by Liu et al. (2008a), where we assumed that the pressure and enthalpy of the turbulent magnetic field are given by \(\rho_0 v_s^2/3\) and \(5\rho_0 u^2/6\), respectively.

The shock structure can be complicated due to the presence of turbulence. We assume that the turbulence is generated isotropically and has a generation scale of \(L\), which does not change in the downstream. The speeds \(v_s\), \(v_A\), and \(u\) therefore should be considered as averaged quantities on the scale \(L\). \(v_A\) depends on the upstream conditions and/or the dynamo process of magnetic field amplification (Cho & Vishniac 2000; Lucek & Bell 2000; Niemiec et al. 2008). Here we assume it a constant in the downstream. One can then quantify the evolution of other speeds in the downstream.

For the Kolmogorov phenomenology (Zhou & Matthaeus 1990),
\[
\frac{3d\rho u^2}{2d\tau} = -Q, \quad \text{i.e.} \quad \frac{3U\rho d\rho u^2}{8dx} = -\frac{C_1u(x)^3}{L}.
\]
(23)
Near the SF, we denote the isothermal sound speed and Alfvén speed by \(v_{\text{so}}\) and \(v_{\text{A0}}\), respectively. Then the eddy speed at the SF is given by \(a^2/\rho U/4\) with \(a = 3 - 16v_{\text{so}}^2/3U^2\). Integrating equation (23) from the SF \((x = 0)\) to the downstream \((x > 0)\), we then have
\[
\frac{u(x)}{U} = \frac{1}{4C_1 x/3L + 4a^{1/2}},
\]
(24)
\[
\frac{v_s(x)}{U} = \left[ \frac{3}{16} - \frac{1}{16 \left( C_1 x/3L + a^{-1/2} \right)^2} \right]^{1/2},
\]
(25)
\[
\frac{v_A(x)}{U} = \left[ \frac{5}{16} - \frac{5}{48 \left( C_1 x/3L + a^{-1/2} \right)^2} + \frac{v_A^2}{U^2} \right]^{1/2}.
\]
(26)
As mentioned in Section 2, to produce the observed X-ray emitting electrons in the STTSNRS through the SA processes, fast-mode waves need to be excited efficiently. The MHD wave period is given by \(\tau_F(k) = 2\pi/v_F k\). Then the transition from the Kolmogorov to Kraichnan phenomenology occurs at the scale, where \(\tau_F(k) = 2\pi/\tau_{edd}(k)\) or \(v_F = v_{edd}(k)\) (Jiang et al. 2009). We then have
\[
k_t = (u/v_F)k_m.
\]
(27)
For \(k_t > k > k_m\), the turbulence spectrum is Kolmogorov-like given by equation (4). For \(k > k_t > k_m\), the turbulence spectrum in the inertial range is given by
\[
W(k) = \frac{1}{4\pi} \left\{ \frac{k^{5/2}k_m^{3/2}k_m^{-7/2}}{L v_F} \right. \quad \text{for IK},
\]
(28)
\[
\frac{3U d\rho u^2}{8dx} = -\frac{C_1 u(x)^3}{L v_F} \quad \text{for Kol},
\]
(29)
\[
\text{where from equation (22) one has } v_F = \left[ 5U^2/16 + v_A^2 - S_\rho(x)^2/3 \right]^{1/2}. \text{Equation (30) can be solved numerically to get the speed profiles in the subsonic phase. Fig. 1 shows the } v_F \text{ and } u \text{ profiles with } v_A = v_{A0} = S_\rho \ll U \text{ in the downstream. The thick and thin lines are for the Kraichnan and Kolmogorov phenomenology, respectively.}
\]

In summary, for \(k > \max(k_m, k_t)\), the turbulence spectrum in the inertial range is given by
\[
W(k) = \frac{1}{4\pi} \left\{ \frac{u^{3/2} \min \left( v_F^{1/2}, u^{1/2} \right) k_m^{1/2} k_m^{-7/2}}{L v_F} \right. \quad \text{for IK},
\]
(30)
\[
\left. \frac{u^{3/2} k_m^{1/2} k_m^{-11/3}}{L v_F} \right) \quad \text{for Kol},
\]
and for \(k_t > k > k_m\), \(W(k) = u^2(4\pi)^{-1}k_m^{2/3}k_m^{-11/3}\).

The TTD of compressional fast-mode waves starts at the characteristic length of the magnetic field \(L_d = 1/k_d\), where the Alfvén speed is comparable to the eddy speed, i.e.
\[
v_A^2 = 4\pi W(k_d)k_d^2 = \left\{ \begin{array}{ll} \min (v_F^{1/2}, u^{1/2}) & \text{for IK}, \\ u^2(k_m/k_d)^{3/2} & \text{for Kol}. \end{array} \right.
\]
(32)

Figure 1. Evolution of the eddy speed \(u\) and the speed of parallel propagating fast-mode waves \(v_F\) in the downstream. The thick and thin lines are for the Kraichnan with \(v_A = 0.036U\) (left) and Kolmogorov with \(v_A = 0.018U\) (right) phenomenology, respectively.
At even larger scales, the vortex motions produce random magnetic fields comparable with the mean field reducing the scatter mean free path of charged background particles to $l_0$ or even shorter scales. This trapping of charged particles within a scale of $l_0$ prevents the TTD on scales above $l_0$. The incompressional modes are not subject to the TTD and have different spectra (Goldreich & Sridhar 1995; Cho & Lazarian 2006). In what follows, we only consider the compressional (fast magnetosonic) wave modes. Then we have

\[
 k_\text{d} = k_m \left( \frac{u^*}{v_A^3} \right) \begin{cases} 
 \min(v_F, u) / v_A & \text{for IK}, \\
 1 & \text{for Kol}.
 \end{cases}
\]

(33)

Since $v_F > v_A$, one has $k_\text{d} < k_0$, and damping is negligible in the regime, where $k_0 > k > k_\text{d}$.

For a fully ionized hydrogen plasma with isotropic particle distributions, which is reasonable in the absence of strong large-scale magnetic fields, the TTD rate is given by (Stix 1962; Petrovian, Yan & Lazarian 2006)

\[
 \Lambda_T(\theta, k) = \frac{(2\pi k \rho_b)^{1/2} k \sin^2 \theta}{2(m + m_p) \cos \theta} \times \left( T_e m_e \right)^{1/2} e^{-\frac{m^2}{2k_B T_e}} + \left( T_p m_p \right)^{1/2} e^{-\frac{m_p^2}{2k_B T_p}}.
\]

(34)

where $k_b$, $T_e$, $T_p$, $m$, $m_p$, $\theta$, $\omega$ and $k_0 = k \cos \theta$ are the Boltzmann constant, electron and proton temperatures and masses, angle between the wave vector and mean magnetic field, wave frequency and parallel component of the wave vector, respectively. The first and second terms in the brackets on the right-hand side correspond to damping by electrons and protons, respectively. For weakly magnetized plasmas with $v_A < v_S$, proton damping always dominates the TTD for $\omega^2/k_0^2 \sim v_S^2 \sim k_b T_p/m_p$. If $v_A$ does not change dramatically in the downstream, the continuous heating of background particles through the TTD processes makes $T_e \rightarrow (m_p/m_e) T_e$ since the heating rates are proportional to $(mT)^{1/2}$, where $m$ and $T$ represent the mass and temperature of the particles. We see that parallel propagating waves (with $\sin \theta = 0$) are not subject to the TTD processes and can accelerate some particles to relativistic energies through resonant interactions. Obliquely propagating waves are damped efficiently by the background particles. Although the damping rates for waves propagating nearly perpendicular to the magnetic field ($\cos \theta \approx 0$) are also low, these waves are subject to damping by magnetic field wandering (Petrosian et al. 2006).

The turbulence power spectrum cuts off sharply when the damping rate becomes comparable to the turbulence cascade rate (Jiang et al. 2009):

\[
 \Gamma = \tau_{\text{edd}}^{-1} \begin{cases} 
 \tau_F^{-1} + \tau_{\text{edd}}^{-1} & \text{for IK}, \\
 1 & \text{for Kol}.
 \end{cases}
\]

(35)

One can define a critical propagation angle $\theta_c(k)$, where $\Lambda_T(\theta_c, k) = \Gamma(k)$. Equations (31) and (34) then give

\[
 \sin^2 \theta_c \exp \left( \frac{-v_F^2}{2v_S^2 \cos^2 \theta_c} \right) \approx \begin{cases} 
 v_A k_0^{1/2} / (2^{1/2} \pi^{1/2} v_F k_0^{1/2}) & \text{for IK}, \\
 v_A k_0^{1/3} / (2^{1/2} \pi^{1/2} v_S k_0^{1/3}) & \text{for Kol},
 \end{cases}
\]

(36)

where the electron damping term has been ignored. The angular-integrated turbulence spectrum in the dissipation range is therefore given by

\[
 W(k) \approx \frac{\theta_c^2(k)}{2} \begin{cases} 
 u^{3/2} \min \left( v_F^2, u^2 \right) T_e^{1/2} k^{-3/2} & \text{for IK}, \\
 u^2k^{2/3} & \text{for Kol}
 \end{cases}
\]

\[
 \approx \frac{\exp(5/6)v_A k_0^{1/2}}{2^{1/2} \pi^{1/2} v_S} k^{-2} \begin{cases} \frac{v_A}{v_F} & \text{for IK}, \\
 1 & \text{for Kol}.
 \end{cases}
\]

(37)

Interestingly, the turbulence spectrum is inversely proportional to $k^2$ in both scenarios. The angular-integrated turbulence spectra $\int W(k) 2\pi k^2 \sin \theta d\theta$ for the velocity profiles in Fig. 1 at several locations in the downstream are shown in Fig. 2. The discontinuities of the angular-integrated turbulence spectra are caused by the abrupt onset of thermal damping at the characteristic length $l_0$ of the...
magnetic field. Obliquely propagating fast-mode waves are absorbed by the thermal background ions at this scale.

4 STOCHASTIC ELECTRON ACCELERATION BY FAST-MODE WAVES IN THE DOWNSSTREAM

In a magnetized plasma, fast-mode waves are likely the agent responsible for efficient SA of electrons (Bykov & Toptygin 1983, 1993; Chandran 2003). The resonant interactions of particles with fast-mode waves have been studied by several authors (Miller et al. 1996; Schlickeiser & Miller 1998; Petrosian & Liu 2004). In these studies, the authors prescribed the wave spectrum with several parameters and calculated the corresponding Fokker–Planck coefficients. A self-consistent treatment of the turbulence spectral evolution in the dissipation range was presented by Yan & Lazarian (2004) for a variety of astrophysical plasmas. Liu et al. (2006) considered the damping of fast-mode waves in a low-β plasma and the application of the SA of relativistic protons in magnetic-field-dominated funnels derived from general relativistic MHD simulations of non-radioactive accretion flow around black holes. Section 3 shows that only fast-mode waves propagating parallel to the local magnetic field can survive the TTD by the thermal ions in the high-β downstream plasma. These waves are right-handed polarized and resonate preferentially with electrons in the thermal background and therefore may selectively accelerate electrons to relativistic energies (Petrosian & Liu 2004). We next explore the SA of electrons in the shock downstream in this scenario.

We assume that the turbulence is isotropic from the turbulence generation scale L down to the dissipation scale \( l_d = 1/k_o \), which depends on the turbulence speed \( u \) (equation 33). Fast-mode waves are excited at the scale \( l_f = 1/\max\left(k, k_o\right) = L/\max\left(1, u/v_T^2\right) \) and are also isotropic down to the dissipation scale. Due to the TTD, the energy density of fast-mode waves in the dissipation range is less than the magnetic field energy density. The resonant scatter rate of energetic particles by MHD waves is therefore smaller than \( v_f L \). However, for this high-β plasma, particles with a gyroradius \( r_g \) less than the characteristic length of the magnetic field \( L_d \) draft along magnetic field lines with a scatter mean free path \( l = v_T/r_g \leq L_d \). The scatter mean free path of particles with \( r_g \approx L_d \) should be comparable to \( r_g \approx L_d \) since particles with even higher energies scatter with the magnetic field randomly instead of performing gyro-motions. This efficient scatter of low-energy particles by the turbulence magnetic field results from bending of magnetic field lines by strong turbulence in the inertial range beyond \( L_d \). The resonant wave–particle interactions in the dissipation range give a much lower scatter rate due to the strong damping of obliquely propagating fast-mode waves by thermal ions. Although this scatter through the particle gyro-motion and chaotic magnetic field structure is not caused by resonances with fast-mode waves, it determines the spatial diffusion coefficient \( v^2 \tau_m/3 \approx v l_d/3 \), which plays essential roles in the SA by large-scale fast-mode waves (Ptuskin 1988).

The SA in the supersonic phase with \( u > v_f \) is not well understood (Achterberg 1990; Bykov & Toptygin 1993). If we assume the turbulence speed \( u \) as the characteristic speed of the scatter agents and a scatter rate of \( L_d/c \) for relativistic particles with \( r_g < L_d \), where \( c \) is the speed of relativistic particles, the acceleration rate will be comparable to that of the DSA. We also note that the turbulence speed \( u \) is higher than \( v_f \) in a narrow region near the SF (Fig. 1) corresponding to the turbulence generation. Such an SA will be difficult to distinguish from the first-order DSA. Further downstream, the turbulence speed is lower than \( v_f \). We will ignore the particle acceleration in the supersonic phase and only consider the acceleration by acoustic waves in the subsonic phase, where equation (16) is approximately applicable (Ptuskin 1988).

We assume \( \tau_{m} = l_d/v = l_d/c \) in the following for relativistic particles with \( r_g \leq L_d \). Particles with even higher energies have a scatter time \( \tau_{m} \approx r_g/c \). The corresponding spatial diffusion coefficient is given by \( D = \tau_{m} c^2/3 \) (Lagage & Cesarsky 1983). Following Ptuskin (1988), we have the acceleration time-scale of these particles by a spectrum of fast-mode waves given by

\[
\tau_{m} = \frac{8\pi D}{9} \int_{k_m}^{k_d} \frac{k^4 W(k)}{v_T^4 + D^2 k^2} dt \tag{38}
\]

The fast-mode turbulence also enhances the spatial diffusion of these particles. The incompressional modes are more efficient in enhancing the spatial diffusion than compressional modes (Bykov & Toptygin 1993). To include these effects and partially take into account the effects of energy loss due to adiabatic expansion (Bykov & Toptygin 1983; Cowik & Sarkar 1984), we adopt an effective diffusion coefficient:

\[
D_s = D + \chi uL, \tag{39}
\]

where \( \chi \) is a dimensionless parameter. The escape time of relativistic electrons from the acceleration region is then given by

\[
\tau_{esc} = \left(\frac{k_m^2 D_s}{L^2}\right)^{-1} = D/L. \tag{40}
\]

Physically, \( \chi \) needs to be less than 1, which corresponds to maximum diffusion caused by turbulence. However, considering the possible presence of incompressional modes and energy loss due to adiabatic expansion, the acceleration time-scale by compressional modes will increase since the incompressional modes will carry part of the turbulence energy. The effect of this increase of acceleration time-scale on the electron distribution can be partially taken into account by reducing the escape time-scale, i.e. by increasing the effective spatial diffusion coefficient \( D_s \). In what follows, we will treat \( \chi \) as a free parameter with \( \chi > 1 \) indicating the presence of incompressional modes and reduction of acceleration by fast-mode waves. The presence of incompressional modes will also bring the transonic point closer to the SF. The acceleration by compressional modes in the subsonic phase can still be efficient. The details of these processes will depend on the coupling between compressional and incompressional modes (Petrosian & Bykov 2008) and are beyond the scope of this paper.

The dependence of acceleration and escape time-scales on \( D \) at the transonic point in the downstream for typical conditions of SNR RX J1713.7–3946 is shown in Fig. 3. The approximate acceleration time-scale given by equation (16) and the escape time-scale due to the diffusion coefficient \( D \) alone, \((k_m^2 D)^{-1}\), are indicated by the thin lines. At high values of \( D \), the slightly high discrepancy in the exact and approximate acceleration time-scales of the Kraichnan phenomenology is due to the fact that the overall wave intensity is given by \( 2u^2 \), instead of \( 3u^2/2 \) as is for the Kolmogorov phenomenology.

When \( D < k_m \), corresponding to the fast diffusion limit, \( \tau_{m} \approx 3D/u^2 \) and \( \tau_{esc} \approx (k_m^2 D)^{-1} \). Then \( \tau_{m}/\tau_{esc} \approx 3D^2 k_m^2/u^2 \gg 1 \). The acceleration is negligible. With these asymptotic expressions for these time-scales, the ratio of the acceleration to escape time-scales decreases with the decrease of \( D \) and becomes close to unity near \( D \sim u/(3^{1/3} k_m) \). Since deviations from these asymptotic expressions occur near \( D \sim v_f/k_m \geq u/k_m \), efficient particle acceleration is only possible in the regime where \( D < v_f/k_m \).
When \( Dk_3 \ll v_F \), corresponding to the slow diffusion limit, \( \tau_{ac} \approx 3(5 - \delta)u_F^2 / [(\delta - 3)Dk_3]^{m - 3/4}u^2 \):

\[
\tau_{ac} = \left[ \frac{(\delta - 3)u^2}{3D} \right]^{m - 3} \left[ \frac{k_3D/v_F}{u/v_F} \right] \int_{d_3/v_F}^{(\delta - 3)D/v_F} dx \left( \frac{x^{4-\delta}}{1 + x^2} \right)^{-1} \approx u^{-2} D(k_3/v_F)^{m - 3/4},
\]

where \( 3 < \delta < 5 \), which ensures the convergence of the integration.

These results are in agreement with Fig. 3.

The acceleration and escape time-scales are determined by \( D \) and the turbulence spectrum. Since we assume that \( D = l_d c / 3 \) for electrons with \( r_e \ll l_d \), the electron acceleration and escape time-scales are independent of the energy for \( r_e \ll l_d \). When the ratio of acceleration to escape time-scale is independent of the particle energy, a power-law particle distribution is expected in the steady state with the spectral index given by (Ptuskin 1988)

\[
p = \left( \frac{9}{4} + \frac{\tau_{ac}}{\tau_{esc}} \right)^{1/2} - \frac{1}{2}.
\]

Since electrons with an energy less than the proton rest energy can be scattered by whistler waves at small scales, the diffusion coefficient \( D \) for these particles can be much less than \( l_d c / 3 \) and the corresponding spectral index \( p \approx 1 \) (Petrosian & Liu 2004; Liu et al. 2006). The scatter will be dominated by the turbulent magnetic field before electrons reach the proton rest mass energy; the spectral break may appear below the proton rest mass energy. Without detailed treatment of these processes, we will assume that the electron distribution follows a power law with \( p = 1 \) for \( \gamma \leq 10 \) in the following.\(^2\) Above \( \gamma = 10 \), the steady-state spectral index of the electron distribution is given by equation (42) since the diffusion coefficient \( D = l_d c / 3 \) is assumed to be independent of the electron energy. For relativistic electrons with \( r_e \gg l_d, D \gg r_e c / 3 \), the increase of \( D \) with the electron energy \( E \) will lead to softening of the electron distribution towards higher energy dictated by the energy dependence of \( \tau_{ac} / \tau_{esc} \).

The exact electron distribution can be obtained numerically. For the sake of simplicity, we assume that the electron distribution has a high-energy cut-off at the Lorentz factor

\[
\gamma_c = \frac{q B}{m_e c^2 k_3} = \frac{q B L}{m_e c^2 u^3} \left\{ \frac{u_v}{\min(u_F, u)} \right\} \quad \text{for IK},
\]

\[
\gamma_c = \frac{q B L}{m_e c^2 u^3} \left\{ 1 \right\} \quad \text{for Kol},
\]

where \( q \) is the elementary charge units. Then for a steady-state treatment, the distribution of electrons escaping from the acceleration site may be approximated reasonably well with (Park & Petrosian 1995; Becker et al. 2006)

\[
f(x, \gamma) \propto \gamma^{-p(x)} \exp \left\{ -\frac{\tau_{ac}(D)}{\tau_{esc}(D)} \right\} / \tau_{esc}(D),
\]

where

\[
D(\gamma, x) = \begin{cases} \frac{c_l x}{3} & \text{for } \gamma \leq \gamma_c(x), \\ \frac{c_r x}{3} & \text{for } \gamma > \gamma_c(x). \end{cases}
\]

We note that the shape of the distribution function near \( \gamma_c \) can also be affected by the energy loss processes (Stawarz & Petrosian 2008; Vannoni et al. 2009; Blasi 2010). In our model, it is mostly determined by the balance between the acceleration and escape processes (Park & Petrosian 1995; Becker et al. 2006; Zirakashvili & Aharonian 2007). The shape of this high-energy cut-off has significant effect on the fitting parameter, especially on the value of \( \gamma_c \). We are carrying out detailed numerical investigation of the particle distribution, and the results will be reported in a separate publication.

Although the parallel propagating waves in the dissipation range may not contribute to the particle scatter significantly, energies carried by these waves are only accessible to relativistic electrons. At very small scales, these waves resonate with thermal background electrons giving rise to a preferential acceleration of electrons. One may therefore assume that the ratio of the dissipated energy carried by non-thermal electrons to that of the thermal ions is proportional.
to the ratio of the energy density of parallel propagating fast-mode waves to that of the magnetic field:

\[
\eta = \frac{\theta^2(k_\parallel)}{2} = \begin{cases} 
\frac{e^{\pi/8}v_h^2}{(2\pi)^1/2v_Sv_F} & \text{for IK,} \\
\frac{e^{\pi/8}v_h}{(2\pi)^1/2v_S} & \text{for Kol.}
\end{cases}
\]  
(46)

where \( e \simeq 2.72 \) is the base of the natural logarithm. A more quantitative treatment of this issue may address the electron injection processes self-consistently (Eichler 1979b).

To have efficient acceleration of relativistic electrons, the turbulence decay time

\[
\tau_d = 3L/C_{\parallel}\mu \begin{cases} 
\max(\nu, v_F)/\nu & \text{for IK,} \\
1 & \text{for Kol}
\end{cases}
\]  
(47)

and the remnant lifetime \( T_{\text{life}} \) should be longer than the acceleration time. As we will see below, the turbulence decay time is always longer than \( T_{\text{life}} \) in the subsonic phase for SNR RX J1713.7−3946. There are at most two locations \( x_0 \) and \( x_2 \) with \( x_0 < x_2 \) in the downstream, where \( \tau = \tau_{n} = \tau_{\text{life}} = 1 \). Fig. 4 shows the evolution of \( \eta, \gamma_C, p, \tau, \tau_{\text{ac}}/\tau_d \) and \( \tau = \tau_{\text{ac}}/\tau_{\text{life}} \) in the downstream for \( U = 4000 \text{ km s}^{-1} \). The profiles of \( v_F/\nu \) and \( \nu U/\nu \) only depend on \( v_A/U \). So does the profile of \( \eta \). The profiles of \( \tau \) and \( p \) also depend on the absolute value of \( U \). To obtain \( \gamma_C \), one needs to know \( L \) and \( B \) as well. Most SA occurs near the sonic point \( x_1 \), where \( v_F = \nu \).

In the late subsonic phase, \( \nu \ll v_F \), the SA is insignificant since most of the free energy of the system has been converted into heat. The characteristic length of the magnetic field is also long for downstream due to the weak turbulence, which implies long electron scatter and acceleration time-scales.

Then the distribution of non-thermal electrons in the downstream

\[
F(x, \gamma) = \int_{x_1}^{x} f(x', \gamma)\eta(x')(4Q/m_e^2U)dx',
\]  
(48)

where \( \int_{x_1}^{x} f(x', \gamma)dx = 1 \) and \( \int_{x_1}^{x} \gamma m_e^2c^2F(x', \gamma)dx \) give the energy density of non-thermal particles at \( x \). Figs 5 and 6 show the normalized electron distribution \( f \) and \( F \) for parameters in Fig. 4 at several locations in the downstream, respectively. We note that the electron distribution has a rather gradual high-energy cut-off due to the weak dependence of the acceleration and escape time-scales on the spatial diffusion coefficient \( D \), which is proportional to the electron energy above the cut-off energy \( \gamma_C m_e^2 c^2 \). This gradual cut-off results in a broad emission component due to inverse Comptonization of the low-energy background photons by high-energy electrons, which can fit the observed broad TeV emission spectrum from a few SNRs.

**Figure 4.** Evolution of the acceleration efficiency \( \eta \) (dotted), cut-off Lorentz factor \( \gamma_C \) (dotted-dashed), spectral index \( p \) (dashed) and \( \tau = \tau_{\text{ac}}/\tau_{\text{life}} \) (thin solid) in the downstream for the Kraichnan phenomenology with \( v_A = 0.036U \) and \( \chi = 11 \) (left) and Kolmogorov phenomenology with \( v_A = 0.018U \) and \( \chi = 4.7 \) (right). \( U = 4000 \text{ km s}^{-1} \). The particle acceleration is significant for \( \tau < 1 \). We only consider acceleration between the two vertical dashed lines indicating \( x_1 \) and \( x_2 \). For \( \gamma_C \), we have assumed that \( B = 14\mu G, L = 10.7 \times 10^{15}\text{cm} \) (left) and \( B = 14\mu G, L = 8.0 \times 10^{15}\text{cm} \) (right).

**Figure 5.** Normalized steady-state non-thermal electron distribution \( f(x) \) produced at several locations in the downstream for the Kraichnan (left) and Kolmogorov (right) phenomenology in Fig. 4.
Distributions of non-thermal electrons

5 APPLICATION TO SNR RX J1713.7–3946 AND TIME-DEPENDENT MODELS

Here we use the SNR RX J1713.7–3946 as an example to demonstrate how the SA by fast-mode waves accounts for the observed broad-band spectrum. The electron distribution produced by the SA is given by equation (48), where the integration over x should stop at x2. Given the evolution history of the SNR, the non-thermal electron distribution will vary along the radial direction. A detailed modelling of the explosion is necessary to take this effect into account properly (Cowsik & Sarkar 1984; Zirakashvili & Aharonian 2010). Here we treat the volume of the emission region Ve as a free parameter to control the normalization of the emission spectrum, which is appropriate for SNRs where the non-thermal particles appear to be concentrated near the SF. By adjusting U, B, vA, γ, L and Ve, one can use the corresponding electron distribution F(x) to fit the observed spectrum of the SNR RX J1713.7–3946. Fig. 7 shows the best fit with the corresponding parameters listed in Table 1, where V is the enclosed volume of the SNR SF. Comparing to the thinned-dashed line, which is derived by assuming an electron distribution \( \propto \gamma^{-\frac{5}{2}} \exp \left( -\gamma/\gamma_c \right)^{1/2} \), the high-energy cut-off of the Kraichnan model is more gradual than the Kolmogorov model, which makes the former spectrum broader. The emission volume is 0.42 (0.016) times the volume of the SNR for the Kraichnan (Kolmogorov)

**Figure 6.** Distributions of non-thermal electrons \( F(x) \) at several locations in the downstream for the Kraichnan (left) and Kolmogorov (right) phenomenology in Fig. 4.

**Figure 7.** Best fits to the observed spectrum of the SNR RX J1713.7–3946. The X-ray data points are obtained from Tanaka et al. (2008). The other data points and the sensitivity limits of different high-energy telescopes are the same as those in Liu et al. (2008a). The left- and right-hand panels are for the Kraichnan and Kolmogorov phenomenology in Fig. 4, respectively. The dashed line is for a simple power-law model with a gradual high-energy cut-off \( F(\gamma) \propto \gamma^{1.85} \exp \left( \gamma/\gamma_c \right)^{1/2} \), where \( \gamma_c m_e c^2 = 3.68 \) TeV. The solid lines are for the fiducial models. The low- and high-energy spectral peaks are produced through the synchrotron and inverse Compton scatter of the background photons (Porter et al. 2006), respectively. The preliminary data from Fermi were not considered in the fit and are included here to show the agreement between model predictions and this observation (Funk 2009). The Fermi observation seems to favour the Kraichnan model.

**Table 1.** Model parameters.

| Model | U (km s\(^{-1}\)) | B (\(\mu G\)) | \(v_A/U\) | \(\chi\) | L \(10^{17}\)(cm) | \(V_e/V\) | \(E_e\) \(10^{42}\)(erg) | \(\rho\) \(10^{-26}\)(g cm\(^{-3}\)) |
|-------|-----------------|--------------|-----------|------|-------------|-------|----------------|--------|
| IK steady | 4000            | 14.0         | 0.036     | 11   | 10.7        | 0.42  | 7.69           | 7.52   |
| KOL steady | 4000           | 14.0         | 0.018     | 4.7  | 8.0         | 0.016 | 6.96           | 30.1   |
phenomenology, which is compatible with observations. The values of \( \gamma \) are greater than 1, which suggests that the particle spatial diffusion coefficient might be enhanced significantly by incompressional turbulence motions. The total energies of non-thermal electrons are also comparable to the magnetic field energy for a uniform magnetic field within the remnant, suggesting near-energy equipartition between the non-thermal electrons and the magnetic field.

There are six parameters in the model: \( B \), \( U \), \( v_A \), \( \chi \), \( L \) and \( V_e \). The observed radio to X-ray spectral index, X-ray to TeV flux ratio, location of the X-ray cut-off and bolometric luminosity of the source give four constraints, which lead to two more degrees of freedom. Our model fit to the spectrum is therefore not unique. \( \chi \) is determined by the coupling between incompressional and compressional turbulence motions. \( B \) is well constrained by the ratio of the X-ray to TeV flux. To reproduce the observed spectral shape, the profiles of \( p \), \( \gamma_c \) and \( n \) should not change significantly, which implies that \( v_A^2 c^2 \propto u_0^\alpha \) and \( L \propto u^2/v_A^3 \) at the transonic point for the Kraichnan phenomenology. For the Kolmogorov model, \( v_A^2 c^2 \propto u^2 \) and \( L \propto u^2/v_A^3 \). For \( v_A \ll U \), \( u \) being proportional to \( U \), we find that nearly identical emission spectra can be obtained by adjusting \( U \) and \( v_A \) (Liu, Fan & Fryer 2008b). Since the turbulence decay time is longer than the supernova lifetime, the steady-state treatment is also justified. A time-dependent treatment gives identical parameters (Becker et al. 2006).

## 6 Density and Turbulence Generation

The primary discrepancy between these models and the observations is the relatively high densities of the downstream plasma. From X-ray observations, Cassam-Chenaï et al. 2004 inferred an upper limit for the electron density of 0.02 cm\(^{-3} \). The corresponding mass density is about 3.3 \times 10^{-26} g cm\(^{-2} \), which is comparable to the densities inferred with the Kraichnan phenomenology but lower than those inferred with the Kolmogorov phenomenology. On the other hand, the electron temperature could be much lower than the ion temperature in the shock downstream (Zirakashvili & Aharonian 2010), Morlino et al. (2009) and Fang et al. (2009) have argued that the density in the downstream can be as high as 0.5 cm\(^{-3} \), corresponding to a mass density of 8.4 \times 10^{-25} g cm\(^{-2} \). Given the age of \( T_{\text{life}} = 1600 \) yr and a radius of \( R = 10 \) pc at a distance of \( D \simeq 1 \) kpc, the corresponding average speed of the SF is 6100 km s\(^{-1} \). With the self-similar solution of Chevalier (1982), we infer a shock speed of \( (n - 3)/(n - 5) \times 6100 \) km s\(^{-1} \), where \( n > 5 \) and \( 0 \leq s < 3 \) are the power-law exponents of the density profile for the ejecta and the ambient medium, respectively. Observations give an upper limit of 4500 km s\(^{-1} \) for the shock speed (Uchiyama et al. 2007). From the self-similar solution, the shock speed must be higher than 2400 km s\(^{-1} \) (for \( s = 0 \) and \( n = 5 \)). If ions are preferentially heated by the shock, the corresponding ion temperature \( T_i \) will be higher than 3.5 mK, \( T_i > 1.3 \times 10^5 \) K. The electron temperature should be higher than that given through the Coulomb collisional energy exchange with ions (Hughes, Rakowski & Decourchelle 2000):

\[
T_e > 2.1 \times 10^5 (T_{\text{life}}/1600 \text{yr})^{2/5} (n_e/\text{cm}^{-3})^{2/5} \times (T_i/1.3 \times 10^5 \text{K})^{2/5} \text{K},
\]

where \( n_e \) is the electron number density. The corresponding bremsstrahlung luminosity is \( \mathcal{L} > 5.2 \times 10^{34} (n_e/0.5 \text{ cm}^{-3})^{11/5} \) erg s\(^{-1} \), which is comparable to the luminosity of the observed non-thermal X-ray emission.\(^3\) We therefore expect strong thermal emission with such a high density. Morlino et al. (2009) obtained a very low thermal bremsstrahlung luminosity by arbitrarily adopting an electron temperature 100 times lower than the ion temperature. As shown above, considering the electron–ion Coulomb collisional energy exchange, the electron temperature will not be that low and significant thermal X-ray is expected with a density of 0.5 cm\(^{-3} \), except that cooling of the SF by cosmic ray ions dominates (Zirakashvili & Aharonian 2010). The highest electron density given by our models is about 0.2 cm\(^{-3} \). The corresponding thermal X-ray luminosity will be reduced by nearly one order of magnitude and should be in agreement with observations. Detailed modelling of the supernova explosion and the thermal emission is needed to see the validity of these models.

The model inferred density may also be reduced by considering the acceleration of electrons by large-scale structures in the downstream and the acceleration in the supersonic phase, where the first-order Fermi acceleration is also possible. In this paper, we consider the electron acceleration by the fully developed turbulence in the subsonic phase. It assumes that once the electrons diffuse over a scale of the turbulence generation length \( L \), the acceleration stops. As shown above, the turbulence evolves in the downstream. In a more self-consistent treatment, one may use the turbulence properties to derive non-thermal electrons injected into the downstream flow by small-scale plasma waves and consider the further acceleration of these electrons as they diffuse spatially in the downstream. The scatter mean free path of these particles is determined by the properties of turbulence. The electron acceleration stops only after they diffuse into upstream or far downstream, where the turbulence becomes insignificant. If these effects lead to a harder overall electron distribution, the Alfvén speed needs to be increased to fit the observations, leading to a lower density.

From these models studied here, we see that, to have efficient SA, both high-speed waves (\( v_F \simeq u \)) and short scatter mean free path are required. Quantitatively, one needs \( c^2 l^2/L^2 u^2 \) to be on the order of unity so that the acceleration and escape time-scales of relativistic particles are comparable. \( u \) is constrained by the shock speed. A short scatter mean free path is achieved by the reduction of the characteristic length of the magnetic field, which also determines the maximum energy of the accelerated particles. In these models, turbulence motions are invoked to reduce the characteristic length of the magnetic field. It is obvious that such a mechanism is only possible for strong turbulence where the turbulence speed is higher than the Alfvén speed. The turbulence speed is determined by the shock speed \( u \leq (3/16)^{1/2} U \), which is less than 1949 km s\(^{-1} \) for SNR RX J1713.7–394. Therefore, \( v_A < 1949 \) km s\(^{-1} \) and we obtain a low limit for the mass density from the inferred magnetic field of 14 \( \mu \text{G} \): \( \rho = B^2/4\pi v_A^3 > 4.1 \times 10^{-25} \) g cm\(^{-3} \) (14/14 \( \mu \text{G})^2/(U/4500 \text{ km s}^{-1})^{-2} \), which corresponds to an electron density of \( \sim 0.0002 \text{ cm}^{-3} (B/14 \text{G})^2 (U/4500 \text{ km s}^{-1})^{-2} \).

Therefore further reduction of the density can be achieved by considering the generation of the turbulence and its effect on the turbulence spectrum, i.e. the dependence of the eddy velocity on the spatial scale. Indeed, in the models considered above, we assume that the turbulence is generated in a very narrow spatial range instantaneously at the SF and an inertial range develops. Since the large-scale eddy speed is comparable to the bulk velocity of...
the downstream flow moving away from the SF, the region with \( x < 0.5 \) should be considered as the turbulence generation phase. This is an intrinsic limitation of the above treatments, which focus on the averaged properties of the downstream flow without addressing the turbulence generation process. Significant particle acceleration occurs within \( x < 0.5 \), i.e. the turbulence generation phase. It is also possible that the turbulence is generated over a broad spatial range and/or the turbulence is not isotropic at large scales. One then expects a turbulence spectrum shallower than the initial range spectrum. For example, for a turbulence spectrum of

\[
W \sim (3\pi^2/8\gamma)(g-3)(2\pi/L)^{g-3}k^{-g-2}
\]

(50)

with \( 1 < g < 1.5 \), where the normalization is chosen so that the turbulence energy density is given by (3/2)\( u_A^2 \), the eddy speed can be redefined as

\[
v_{\text{edd}} \equiv \left[ \frac{3(8\pi - 1)}{3}k^3W \right]^{1/2} = u(kL/2\gamma)^{1-\beta}/2.
\]

(51)

The eddy speed is comparable to the Alfvén speed at the characteristic length of the magnetic field \( l \), we have then \( 12c^2l^2/v_A^2L^2 = 12c^2v_A^2 = \frac{4\pi^2}{8\gamma(8\pi - 1)}k^3W(kx-1) \approx 1 \). Therefore \( v_A/U \sim (v_f/12c)^{g-2} \), since \( v_f \sim u \approx U \), we have \( v_A/U \sim (U/c)^{g-2} \). The Alfvén speed can be comparable to the turbulence speed for a shallow turbulence spectrum with \( g \) approaching 1. Thus, stochastic electron acceleration can account for observations of SNR RX J1713.7–394 as far as the mass density of the shocked plasma is greater than \( 4.1 \times 10^{-28} \text{ g cm}^{-3} \). Detailed studies of the turbulence generation and the associated particle acceleration are warranted (Lucek & Bell 2000; Hededal et al. 2004; Giacalone & Jokipii 2007; Nishikawa et al. 2009).

7 SUMMARY AND CONCLUSIONS

In the paper, we study the SA of electrons by a decaying turbulence as produced and/or enhanced by strong non-relativistic shocks and carried away from the SF with the downstream flow. It is shown that, to have significant particle acceleration, the turbulence must cover a large spatial scale so that the particle acceleration time may be shorter than the turbulence decay time. To account for observations of a few STTSNRs with the leptonic scenario for the TeV emission, fast-mode waves need to be excited in the subsonic phase. Given the turbulent nature of the downstream flow, fast-mode waves may prevail in the downstream. We show that the SA by large-scale acoustic (fast-mode) waves can account for the observations.

There are four basic model parameters, namely the magnetic field, mass density, shock speed and the turbulence generation scale. Observations of a few SNRs and radio galaxies have shown that the particle acceleration may change dramatically along the SF (Rothenflug et al. 2004; Croston et al. 2009; Reynolds et al. 2009). This variation has been attributed to a large-scale magnetic field in the DSA model as the acceleration efficiency varies with the angle between the magnetic field and the shock normal. With our model, this variation is likely caused by a quite different mechanism. Detailed comparative studies should be able to distinguish these models.

The particle acceleration is very sensitive to the magnetic field. To produce non-thermal particle distribution in compatible with observations, the dimensionless quantities \( c^2v_A^3/u^{10} \) and \( c^2v_A^3/u^4 \), where \( u \) is the large-scale eddy speed near the SF, should be on the order of 1 for the Kraichnan and Kolmogorov phenomenology, respectively. The high-energy cut-off of the particle distribution is determined by the magnetic field and the turbulence generation scale. Weaker fields will lead to lower cut-off energies and harder spectra. The particle acceleration may be turned off completely for strong fields due to the increase in the characteristic length of the magnetic field and therefore the particle scatter mean free path. If the magnetic field is predominantly generated by the streamline of non-thermal particles upstream, the models then imply \( v_A \sim U^{5/4} \) and \( v_A \sim U^{1/3} \) for this dynamo process and for the Kraichnan and Kolmogorov phenomenology in the downstream turbulence, respectively.

Assuming that the turbulence is isotropic and generated in a narrow spatial scale, the model inferred densities of the downstream flow may be so high that thermal X-ray emission becomes observable, in conflict with observations. Although the thermal X-ray emission may be suppressed by a lower electron temperature due to dominance of the cooling by cosmic ray ions, we find that a low density is also possible if the turbulence is not isotropic or generated over a broad spatial scale so that the eddy speed has very weak dependence on the spatial scales. Detailed modelling of the progenitor and the evolution history of the remnant may help to constrain the density (Cowls & Sarkar 1984).

The application of the models to the SNR RX J1713.7–3946 also suggests energy equipartition between the magnetic field and the acceleration electrons. If the rest of the shock energy is dissipated as heat in the downstream, then the overall electron acceleration efficiency will be \( \sim v_A^3/u^2 \), which is inversely proportional to the plasma \( \beta \) of the downstream flow. For the Kraichnan phenomenology, \( v_A^3/u^2 \sim (u/c)^{1/4} \), and for the Kolmogorov phenomenology, \( v_A^3/u^2 \sim (u/c)^{1/3} \). Acceleration is more efficient for stronger shocks, which also produce hotter downstream plasma. However, the dependence of the acceleration efficiency on the shock speed is rather weak. These may have significant implications on the origin of cosmic rays and their connection to the properties of the interstellar medium (Reynolds 2008).

With the fast-mode wave turbulence model studied in this paper, relativistic electrons may also be accelerated through the first-order Fermi mechanism as in the DSA model, especially in the supersonic phase. The high-energy cut-off can still result from decoupling of higher energy particles with the background magnetic field as their gyro-radius exceeds the characteristic length of the magnetic field. This is quite different from the DSA models, where the high-energy cut-off is related to a finite lifetime of the shock, shock curvature or efficient energy loss processes (Zirakashvili & Aharonian 2010).

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