Tunable photonic spin Hall effect due to the chiral Hall effect in strained Weyl semimetals

Guangyi Jia\textsuperscript{1}, Ruixia Zhang\textsuperscript{1}, Zhenxian Huang\textsuperscript{1}, Qiaoyun Ma\textsuperscript{1}, Huaiwen Wang\textsuperscript{1,2,}\textsuperscript{*} and Reza Asgari\textsuperscript{3,}\textsuperscript{*}

\textsuperscript{1} School of Science, Tianjin University of Commerce, Tianjin 300134, People's Republic of China
\textsuperscript{2} Tianjin Key Laboratory of Refrigeration Technology, Tianjin University of Commerce, Tianjin 300134, People's Republic of China
\textsuperscript{3} School of Physics, Institute for Research in Fundamental Sciences, IPM, Tehran 19395-5531, Iran

\* Authors to whom any correspondence should be addressed.

E-mail: wanghw@tjcu.edu.cn and asgari@ipm.ir

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Abstract

The latest research suggests that strain can be utilized to engineer the electronic states of Weyl semimetals (WSMs) through creating a pseudo-magnetic field $B_{el}$. The response of strained WSMs to a real time-varying electric field $E$ with $E \perp B_{el}$ can cause spatial chirality and charge separation in WSMs, i.e., the chiral Hall effect (CHE). Herein, the photonic spin Hall effect (PSHE) modified by CHE in strained WSM thin films is studied. We show that the in-plane and transverse photonic spin-dependent shifts ($\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$) can be tuned to be more than 400 and 50 times of incident wavelength, respectively, at the angular frequency being close to the cyclotron frequency of massless fermions in the pseudo-magnetic field. In order to enhance the PSHE, epsilon-near-zero materials take priority of being as the substrates of WSM films. Besides, both $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ generally give extreme values around incident angles at which Fresnel reflection coefficients exhibit local minimums, whereas an inversion-symmetry breaking with nonzero axial chemical potential may break this generality. Finally, one possible experimental strategy for observing this CHE tuned PSHE is schemed, which may provide a pristine optical technique to precisely engineer and detect the strain in topological materials.

1. Introduction

The plane-wave reflection at a dielectric interface between two optically different media can be directly determined from the geometrical optics in classical electrodynamics. Nevertheless, in recent years, it has demonstrated that the propagation of electromagnetic wave does not strictly follow the traditional optical rules due to the spin-orbital interaction or coupling [1–4]. The most prominent phenomenon is so-called photonic spin Hall effect (PSHE) which was theoretically predicted by Onoda \textit{et al} in 2004 and was first experimentally confirmed by Hosten \textit{et al} in 2008 [1, 2]. The PSHE is regarded as a direct photonic analogy of electronic spin Hall effect and manifests itself as spin-dependent transverse and/or in-plane displacements of the wave-packet centroid perpendicular to the refractive index gradient [5, 6]. It has shown promising applications in lots of realms, such as on-chip chiroptical spectroscopy [7], probing extremely weak disorders at the deep-subwavelength scale [8], ultrasensitive detection of the ion concentration in solution [9], precision measurements of the optical conductivity and the layer number of two dimensional (2D) materials [10, 11], and so forth. Unfortunately, the PSHE is very weak and the spin-dependent splitting is generally at the subwavelength scale. Therefore, how to greatly amplify the PSHE has come to the fore of research field of spinoptics [12–14]. For example, anisotropic materials/microconfigurations including liquid crystal microcavity, birefringent polymer, and metasurfaces have been developed to manipulate and enhance the PSHE [15–18].
In addition to the above methods, topologically nontrivial Weyl semimetals (WSMs), which were first experimentally discovered in 2015 \[19, 20\], further opens up new avenues for unearthing many peculiar physical phenomena in the light–matter interaction \[21–24\]. For instance, Jia et al recently demonstrated that the broken time reversal symmetry in WSMs brings about versatile dependent behaviors of PSHE on the tilt of Weyl nodes and the chemical potential, and the largest in-plane spin-dependent shift can reach 38.47 times of the incident wavelength \[21\]. Ye et al established that an enhanced PSHE can be obtained in the vicinity of Weyl point at the presence of a local Berry curvature in a synthetic Weyl system \[22\]. Chen et al established the relationship between the photonic spin Hall shifts and the separation of Weyl nodes \[23\]. Even if several groups have carried out some primary researches on PSHE in WSMs, the combination between PSHE and WSMs is still in its infancy, as the conception of PSHE is put forward in recent years and WSM is a special new type of gapless topological material \[21–26\]. The relationships between PSHE and a wide variety of structure changes in WSMs have not yet been studied thus far. One such example is the impacts of strain on the PSHE possibly occurring in WSMs.

During crystal growth or during the follow-up manual processing, inhomogeneous strain, which has different physical origins, e.g., lattice defects in WSMs, lattice mismatch at the interfaces, stretching and bending, etc, could be created in WSMs \[27–29\]. Accordingly, spatial variation of Weyl node positions could take place, which in turn induces the generation of a pseudo-magnetic field defined as \(B_5 = \nabla \times b\) \[28–32\]. Such a pseudo-magnetic field can interact with fermions of opposite chirality in a WSM, leading to the emergence of an axial (chiral) current as a response to a real time-varying electric field \(E\) with \(E \perp B_5\). Since the induced axial current is perpendicular to both \(B_5\) and \(E\), it is referred to as the chiral Hall effect (CHE) \[33\]. Unlike an externally applied magnetic field, the pseudo-magnetic field \(B_5\) is felt by charge carriers of opposite chiralities as if they have opposite charges. In transport, the most notable consequence of the presence of \(B_5\) in the chiral anomaly is a strain-induced enhancement of the longitudinal optical conductivity which is an important factor to determine the PSHE \[29, 33\]. Although the study on CHE in WSMs may date back to 2015, the early researches are mainly limited to applying a real magnetic field to produce the CHE, without involving the strain in WSMs \[34, 35\]. Until 2020, Heidari et al explored the CHE of strained WSMs in the absence of a real magnetic field and derived the generalized formula for anomaly-induced optical conductivity \[33\], paving the way for solely examining the relationship between PSHE and CHE in strained WSMs.

Propelled by advances in the PSHE and topologically nontrivial WSMs, in the present work, we theoretically investigate the PSHE modified by CHE when the electric field component of incident wave is perpendicular to the strain-induced pseudo-magnetic field in WSM films. Evolutions of the photonic spin-dependent shifts in both transverse and in-plane directions with various factors including the permittivity of substrate, incident angle, electrochemical potential, angular and cyclotron frequencies are clearly unveiled. The possible influencing mechanism of CHE on the PSHE is discussed. Moreover, one experimental scheme for observing the PSHE on the surface of strained WSM film is proposed.
2. Theoretical model and optical conductivity

Let us consider a simple continuum model of a time-reversal-symmetry-broken WSM with a pair of Weyl nodes, with the chirality \( \chi = \pm 1 \) located at \( \mp \mathbf{b} \) in the momentum space. Under lattice distortion, the low-energy effective Hamiltonian near Weyl nodes can be described by [33]

\[
H = \hbar v_{\gamma}(q + \chi \mathbf{A}_{\gamma}) \cdot \sigma,
\]

where \( v_{\gamma} \) is the Fermi velocity, \( \sigma \) is a vector of the Pauli matrices, and the axial gauge potential \( \mathbf{A}_{\gamma} \) is the spatially dependent part of \( \mathbf{b} \). The lattice deformation leads to a nonzero pseudo-magnetic field \( \mathbf{B}_{\gamma} \equiv \nabla \times \mathbf{A}_{\gamma} \). The pseudo-magnetic field \( \mathbf{B}_{\gamma} \) is assumed to be along the negative \( z \) axis, and we are interested in the response of strained WSMs to a real time-varying and weakly oscillated electric field \( \mathbf{E} \) within the framework of semiclassical chiral kinetic theory. We focus on the doped WSMs where the electrochemical potential \( \mu \) crosses the conduction band.

When the electric field is perpendicular to the pseudo-magnetic field, i.e., \( \mathbf{E} \perp \mathbf{B}_{\gamma} \), the Drude scattering rate of WSM, and the cyclotron frequency of massless fermions in the pseudo-magnetic field, respectively.

The transverse optical response of per Weyl node \( \chi \) in bulk WSM is given by

\[
\sigma_{\chi \gamma} = -\sigma_{\chi \gamma}^{\text{dc}} = \frac{\chi e^2 v_{\gamma}^2 \mu_{\chi}}{6\pi^2 \hbar^3 v_{\gamma}} \omega_{\text{el}} \left( \frac{1}{\omega^2 - \omega_{\text{el}}^2} + \frac{\gamma}{3} \right),
\]

where \( \mu_{\chi} \) is the chemical potential at Weyl node \( \chi \). Although we consider a WSM where time reversal symmetry is broken and chirality is conserved, we can slightly break the inversion symmetry by, e.g., momentum-independent spin–orbit interaction [37]. Furthermore, a tiny broken inversion-symmetry leads to an imbalance charge between two Weyl nodes and thus there is a finite axial chemical potential \( (\mu_+ - \mu_-) \) in the system. One physical quantity \( Q_0 = 0.5(\mu_+ - \mu_-) \) is defined and \( \mu_{\chi} = 0.5\mu \pm Q_0 \) [33, 37]. Then the Hall conductivity \( \sigma_{\chi \gamma} \) (or \( \sigma_{\chi x} \) and \( \sigma_{\chi y} = -\sigma_{\chi x} \)) of bulk WSM can be obtained by summing over the chirality. In the case of \( \mu_+ = \mu_- \), the Hall conductivity equals to zero. The prerequisite condition to get nonzero \( \sigma_{\chi \gamma} \) is fulfilled by an inversion-symmetry breaking \( Q_0 \neq 0 \). Besides, to prevent the divergence of the optical conductivity, the quantity \( i\gamma(\gamma = 1/\tau) \) is added to the frequency \( \omega \rightarrow \omega - i\gamma \) in equations (2) and (3).

Figure 2 shows the complex optical conductivities of \( \sigma_{\chi x} \) and \( \sigma_{\chi y} \) at different \( \gamma/\omega_{\text{el}} \) values. Here, the parameters of \( v_{\gamma} = 6.7 \times 10^5 \text{ m s}^{-1} \) and \( \tau = 131.64 \text{ fs} \) are chosen, which are at the scales of \( v_{\gamma} \) and \( \tau \) in \( T_d\text{-MoTe}_2 \) [38]. It is seen from figure 2(a) that the real part of longitudinal optical conductivity shows two absorption peaks centered at \( \omega = 0 \) and \( \omega = \omega_{\text{el}} \). According to the separately calculated \( \sigma_{\text{Drude}} \) and \( \sigma_{\chi \text{B}} \) spectra in figures S1(a) and (b) (https://stacks.iop.org/NJP/23/073010/mmedia), the former and the latter absorption peaks mainly originate from the conventional Drude and the strain-induced optical responses, respectively. It is seen that the real part of \( \sigma_{\chi x} \) is greatly enhanced by chiral electrons when the \( \omega \) approaches \( \omega_{\text{el}} \), and the optical absorption increases with decreasing the \( \omega_{\text{el}} \) value. By contrast, the imagery part of \( \sigma_{\chi x} \) exhibits a sine-shaped variation around the cyclotron frequency \( \omega_{\text{el}} \) and its magnitude gradually increases as the \( \omega_{\text{el}} \) decreases.

In the case of \( \mu_+ = \mu_- \), right- and left-handed fermions in a WSM have the same number but they disperse in opposite directions, i.e., only chirality separation occurs. Therefore, the net charge current equals to zero but the axial (chiral) current do not vanish. In the case of \( Q_0 \neq 0 \), the currents of right- and left-handed fermions are also in the opposite directions but with different magnitudes, as sketched by green and blue spheres embedded in the WSM film in figure 1. Accordingly, the currents corresponding to each chirality cannot compensate each other, i.e., \( \sigma_{\chi \gamma}^{\pm} + \sigma_{\chi \gamma}^{-} \neq 0 \), as shown in figures 2(b)–(d) and S2. As a result, CHE leads both charge and chirality separations to arise in a strained Weyl system. It is further found
from figures 2(b) and S2(a) that chiral Hall current changes its sign near $\omega/\omega_{el} = 1$, indicating that the chirality of accumulated charges at each side of the WSM will be reversed near $\omega/\omega_{el} = 1$.

In figure 2, the shortest wavelength of incident electromagnetic wave is $\lambda = 19.8 \mu m$, corresponding to $\hbar \omega = 0.063 eV$. When the thickness $d$ of thin WSM film is much smaller than the $\lambda$ while being larger than the atomic separation $a$, i.e., $a \ll d \ll \lambda$, the WSM film can be treated as a 2D atomic layer when it interacts with incident electromagnetic wave [23, 39, 40]. Then, the surface optical conductivity of 2D WSM depositing upon a homogeneous substrate can be approximated as $\sigma_{mn} = d \sigma_{mn}$ where $m, n \in \{x, y\}$ [23, 39, 40]. Here, the thickness of WSM film is set as $d = 10$ nm.

We posit that one monochromatic Gaussian beam is illuminated on the surface of ultrathin WSM film (see figure 1). According to the electromagnetic boundary condition, the incident, reflected, and transmitted amplitudes ($E_{pi}, E_{pi}, E_{pi}$) satisfy the following equations [41]

$$E_i^p + E_i^s = E_t^p$$

$$\frac{k_1}{k_1} (E_i^p - E_i^s) = \frac{k_2}{k_2} E_t^p$$

$$\frac{k_{1}Z_{1}l_{1}k_{2}k_{2}}{k_{1}k_{2}} (E_{i}^{p} - E_{i}^{s}) = \left( \sigma_{xx} + \frac{k_{2}k_{2}}{Z_{2}k_{2}} \right) E_{i}^{s} + \sigma_{sy} \frac{k_{2}E_{i}^{p}}{k_{2}}$$

$$\frac{1}{Z_{1}} (E_{i}^{p} - E_{i}^{s}) = \left( \sigma_{xx} + \frac{k_{2}k_{2}}{Z_{2}k_{2}} \right) E_{i}^{s} - \sigma_{sy} \frac{k_{2}k_{2}}{k_{2}} E_{i}^{p}$$

where the superscripts p and s represent the parallel and perpendicular polarization states, respectively, $k_1 = k_1 \cos \theta_i, k_2 = k_0 (\varepsilon_2/\varepsilon_0 - \sin^2 \theta_i)^{1/2}, Z_{1,2} = (\mu_1/\varepsilon_1)^{1/2}$. Here, $\theta_i, \varepsilon_0$ and $\mu_0$ are the incident angle, permittivity and permeability in vacuum, respectively, $\varepsilon_i$ and $\varepsilon_2$ are the permittivities of incident medium and substrate, respectively. The incident medium is air in this work, thus, $\varepsilon_1 = \varepsilon_0$ and $k_1 = k_0 = \omega (\varepsilon_0\mu_0)^{1/2}$.

Fresnel reflection coefficients are determined by the incident and reflected amplitudes, i.e., $r_{pp} = E_r^p/E_i^p, r_{ps} = E_r^s/E_i^p, r_{sp} = E_r^p/E_i^s$. Considering separately the cases of p and s incident polarizations, one can decouple equations (4)–(7) and get the Fresnel reflection coefficients in the presence of a
pseudo-magnetic field as \[5, 41\]

\[r_{pp} = (\beta + \alpha_- + \chi)/(\beta + \alpha_+ + \chi)\] (8)

\[r_{sa} = -(\beta - \alpha_+ + \chi)/(\beta + \alpha_+ + \chi)\] (9)

\[r_{ps} = r_{sp} = -\sigma_{sys}/(\beta + \alpha_+ + \chi)\] (10)

with \[\alpha_\pm = (k_{z1}e_\pm \pm k_{z2}e_\pm + k_{z2}\sigma_{sys}/\omega)/e_0\], \[\beta_\pm = k_{z2} \pm k_{z1} + \omega\mu_0\sigma_{sys}, \chi = \mu_0 k_{z1}k_{z2}\sigma_{sys}^2/\omega\epsilon_0\], \[\Lambda = 2k_{z1}k_{z2}(\mu_0/\epsilon_0)^{1/2}\].

The angular spectrum of incident Gaussian beam in momentum space can be represented by

\[|\Phi\rangle = w_0/\sqrt{2\pi} \exp \left[ -w_0^2 (k_{ix}^2 + k_{iy}^2) / 4 \right], \] (11)

where \[w_0\] is the width of wave function. For the bounded beam, the polarization states of different angular spectrum components can be specified as \[|H(k_i)\rangle\] and \[|V(k_i)\rangle\]. After reflecting at the surface of WSM film, the rotations of polarizations for each angular spectrum component are different. Introducing the boundary conditions \[k_{ix} = -k_{ix}\] and \[k_{iy} = k_{iy}\], where \[k_{ix}\] and \[k_{iy}\] (\[k_{ix}\] and \[k_{iy}\]) represent the wave-vector components of incident (reflected) beam along \[x_i\] and \[y_i\] (\[x_i\] and \[y_i\]) axes, respectively, the reflected polarization states for different angular spectrum components are determined by \[|H(k_i)\rangle|V(k_i)\rangle\] \[= M_r|H(k_i)\rangle|V(k_i)\rangle\] with

\[M_r = \begin{pmatrix} r_{pp} - k_{iy}(r_{ps} - r_{sp}) \cot \theta_i & r_{ps} - k_{iy}(r_{pp} + r_{sp}) \cot \theta_i \\ r_{sp} - k_{iy}(r_{ps} - r_{sp}) \cot \theta_i & r_{pp} - k_{iy}(r_{pp} + r_{sp}) \cot \theta_i \end{pmatrix}. \] (12)

The PSHE manifests itself as spin-dependent splitting which appears in both transverse and in-plane directions. In the spin basis set, the horizontal and vertical polarization states \(|H\rangle\) and \(|V\rangle\) of electromagnetic wave can be decomposed into two orthogonal spin components

\[|H\rangle = (|+\rangle - |−\rangle) / \sqrt{2} \] (13)

\[|V\rangle = i (|−\rangle - |+\rangle) / \sqrt{2}, \] (14)

where \(|+\rangle\) and \(|−\rangle\) stand for the left- and right-circular polarization components, respectively.

Based on equations (11)–(14) and taking into account the paraxial approximation, the spin Hall shifts of the reflected wave packet can be mathematically obtained after a series of cumbersome calculations. Since the electric field \(E\) is posited to be along the \(y\) axis, being perpendicular to the incident plane, only the vertically polarized incident beam is considered below. The corresponding in-plane and transverse spin Hall shifts for two spin components \(|+\rangle\) and \(|−\rangle\), as shown in figure 1, can be written by the following expressions \[42\]

\[\langle \Delta x\rangle = \frac{1}{k_i} \text{Re} \left( \frac{r_{ps} - r_{sp}}{r_{ps}^2 + r_{sp}^2} \frac{\partial r_{ps}}{\partial \theta_i} - \frac{\partial r_{sp}}{\partial \theta_i} \right) \] (15)

\[\langle \Delta y\rangle = \frac{\cot \theta_i}{k_i} \text{Re} \left( \frac{r_{ps} + r_{sp}}{r_{ps}^2 + r_{sp}^2} \frac{\partial r_{ps}}{\partial \theta_i} + \frac{\partial r_{sp}}{\partial \theta_i} \right). \] (16)

It is seen from the above equations that the pseudo-magnetic field induced via lattice deformation decisively affects the optical conductivity tensor. The Fresnel reflection coefficients are functions of the complex optical conductivity. In consequence, the PSHE can be modulated by the strain-induced CHE in ultrathin WSM film. Because \(\langle \Delta x\rangle = -\langle \Delta x\rangle\) and \(\langle \Delta y\rangle = -\langle \Delta y\rangle\), we will only analyze the photonic spin Hall shifts for left-circular polarization component in this paper. In addition, because both \(\langle \Delta x\rangle\) and \(\langle \Delta y\rangle\) are spin-dependent spatial shifts, which originates from the spin–orbit interaction \[42\], spin-independent Goos–Hänchen and Imbert–Fedorov shifts are neglected here. It is also known that there are two states in WSMs, i.e., the surface state and the bulk state. Nevertheless, the relationship between the Fermi arc surface states and the CHE in strained WSMs, to our knowledge, has not been reported so far. Thus this work is limited to the bulk state of WSMs in the modulation of the PSHE.

### 3. Photonic spin Hall shifts

We first consider the influences of the incident angle \(\theta_i\) and the relative permittivity \(\varepsilon_{z2} (=\varepsilon_{z2}/\varepsilon_0)\) of substrate on the PSHE. The angular frequency is \(\omega = \omega_{cl}\) and the parameters of \(\mu_i, \gamma/\omega_{cl}\) and \(Q_0\) are set as 0.2 eV, 0.2, and 0.2μ, respectively. Figures 3(a) and (b) show variations of the Fresnel reflection coefficients \(|r_{sa}|\) and
| |r|ps| with respect to the θi and εr2. It is seen that the |r|ss| gives the minimum value 0.08% at θi = 72.7° and εr2 = 0.894. The conventional Brewster’s angle is defined as the angle of incidence at which the reflection of p-polarized light is extinguished. Here, the incident angle at which the |r|ps| reaches its minimum value is defined as the quasi-Brewster’s angle θB. We see from figure 3(b) that the quasi-Brewster’s angle θB shows one ‘S’-type pattern with increasing the εr2 from 0.1 to 1.0.

Figures 3(c) and (d) present the in-plane and transverse photonic spin Hall shifts as functions of the θi and εr2. One can see that both the in-plane and transverse displacements give extreme values around the quasi-Brewster’s angle. Especially, the ⟨Δx+⟩ (or ⟨Δy+⟩) gives the largest displacement of 201.7λ (or −14.5λ) at θi = 72.4° and εr2 = 0.894. In order to increase the color contrast, the color scales of ⟨Δx+⟩ and ⟨Δy+⟩ in figures 3–5 are limited to smaller ranges. Original color maps of ⟨Δx+⟩ and ⟨Δy+⟩ are shown in figures S3–S5. It is seen from figures 3 and S3 that the photonic spin Hall shifts decrease to be smaller than 0.1λ when the relative permittivity εr2 of substrate increases to be larger than 1.0. The photonic spin Hall shifts of ‘S’-type pattern at 0.1 ≤ εr2 < 1.0 are much larger than those at εr2 > 1.0. This means that it is favorable to greatly enhancing the PSHE by selecting epsilon-near-zero (ENZ) materials (whose electric permittivity is near zero) as the substrate. Experimentally, the ENZ behavior has been demonstrated in polaritonic materials such as silicon carbide [43, 44], doped semiconductors such as transparent conducting oxides [45], and various artificial micro-configurations [44, 46], etc. Therefore, it is possible in experiment to amplify the PSHE by depositing WSM film on an ENZ substrate.

The PSHE could be also enhanced by the ENZ material. To clarify the origin for amplifying the PHSE, figure S3(c) shows the color map of ⟨Δy+⟩ when the thickness of WSM film is set as d = 0 nm. The other factors are the same with those of figure S3(b). It is seen that the largest transverse displacement −0.9λ appears at θi = 19.1° and εr2 = 0.107 while the maximum value of ⟨Δy+⟩ at εr2 > 1.0 is −0.1λ. Even if the PSHE can be amplified by only the ENZ material, the largest shift −0.9λ is far smaller than that −14.5λ in figure S3(b). Moreover, because of the disappearance of Hall conductivity σxy at d = 0 nm, both the Fresnel reflection coefficient rps and the in-plane displacement ⟨Δx+⟩ equal to zero on the basis of equations (10).
and (15). In marked contrast, the largest value of \( \langle \Delta x_+ \rangle \) reaches 201.7\( \lambda \) in figure S3(a). Thereby, the amplification of PSHE in this work mainly stems from the CHE in strained WSM film.

Besides, the relative permittivity \( \varepsilon_{ss} \) of substrate could be a complex number. To exemplify the influence of imaginary part of \( \varepsilon_{ss} \), figure S6 depicts the color maps of \(|r_{ss}|\), \(|r_{ps}|\), \(\langle \Delta x_+ \rangle\) and \(\langle \Delta y_+ \rangle\) with respect to the incident angle \(\theta_i\) and the imaginary part of substrate permittivity \(\text{Im}(\varepsilon_{ss})\). The real part of \(\varepsilon_{ss}\) keeps at 0.894. We can see that the photonic spin Hall shifts \(\langle \Delta x_+ \rangle\) and \(\langle \Delta y_+ \rangle\) have extreme values at positions where the \(|r_{ss}|\) exhibits local minimums, and the \(\langle \Delta x_+ \rangle\) (or \(\langle \Delta y_+ \rangle\)) gives the largest shift of 201.7\( \lambda \) (or \(\sim 14.5\lambda\)) at \(\text{Im}(\varepsilon_{ss}) = 0.0\). Thus, the imaginary part of \(\varepsilon_{ss}\) is defaulted to 0.0 in the following.

Figures 4(a) and (b) show variations of the Fresnel reflection coefficients \(|r_{ss}|\) and \(|r_{ps}|\) with respect to the \(\theta_i\) and \(\omega\). The relative permittivity of substrate is set as \(\varepsilon_{ss} = 2.9\), and the other parameters are the same with those in figure 3. It is found that the quasi-Brewster’s angle appears at \(\theta_B = 71.6^\circ\). We note that the \(|r_{ss}|\) and the real parts of \(\sigma_{xx,yy}\) (or imaginary part of \(\sigma_{xx}\)) have a similar change tendency with the \(\omega/\omega_{el}\) when \(\theta_i < \theta_B\) (or \(\theta_i > \theta_B\)), as shown in figure S7(a) (or S7(b)). However, the \(|r_{ps}|\) and the imaginary part of \(\sigma_{xy}\) exhibit a similar tendency (figure S7(c)). This demonstrates that the Fresnel reflection coefficients are sensitive to the variance of the complex optical conductivity of WSM film.

Then we plot the photonic spin Hall shifts as functions of the \(\theta_i\) and \(\omega\), as shown in figures 4(c) and (d) and S4(a) and (b). It is shown that both \(\langle \Delta x_+ \rangle\) and \(\langle \Delta y_+ \rangle\) give extreme values in the vicinity of incident angles at which the \(|r_{ss}|\) exhibits local minimums. When \(\omega/\omega_{el} = 0.979\) and \(\theta_i = 73.0^\circ\), the \(\langle \Delta x_+ \rangle\) (or \(\langle \Delta y_+ \rangle\)) presents the largest displacement of \(-471.0\lambda\) (or \(-55.8\lambda\)). Through tuning the incident angle \(\theta_i\) and the cyclotron frequency \(\omega_{el}\) (see figures 5 and S5), we again find that the photonic spin Hall shifts have extreme values near by the incident angle 73.0° at which the \(|r_{ss}|\) exhibits a local minimum.

However, there are two extreme points for both \(\langle \Delta x_+ \rangle\) and \(\langle \Delta y_+ \rangle\), as shown in figures 5(c) and (d). Figures S8–S10 gives the color maps of \(|r_{ss}|\), \(\langle \Delta x_+ \rangle\) and \(\langle \Delta y_+ \rangle\) with respect to the factors of \(\theta_i\) and \(\omega_{el}\) at different \(Q_B\) values. It is seen that the coordinate position of extreme value at the right, which is considered as a singularity here, gradually deviates from the coordinate at which the \(|r_{ss}|\) exhibits a local minimum with increasing the \(Q_B\). In our opinion, this singularity is closely associated with the Hall conductivity \(\sigma_{xy}\). As

![Figure 4](image-url)
Figure 5. Variations of the Fresnel reflection coefficients (a) $|r_{ss}|$ and (b) $|r_{ps}|$ with respect to the incident angle $\theta_i$ and the cyclotron frequency $\omega_{el}$. (c) In-plane and (d) transverse spin-dependent shifts on the surface of strained WSM film versus the parameters of $\theta_i$ and $\omega_{el}$. The coordinate $(\theta_i, \gamma/\omega_{el})$ of the position pointed by a black arrow in (c) is $(73.4^\circ, 0.187)$. The color bars in (c) and (d) are scaled in the unit of $\lambda$.

mentioned above, the Hall conductivity equals to zero at $Q_0 = 0.0$. Correspondingly, the extreme value at the right (i.e., the singularity) disappears at $\mu_+ = \mu_-$, as shown in figures S9(a) and S10(a). As the $Q_0$ increases from 0.1$\mu$ to 0.9$\mu$, the coordinate $(\theta_i, \gamma/\omega_{el})$ of the singularity changes, especially, the $\theta_i$ gradually increases from $73.2^\circ$ to $75.7^\circ$, as shown in figures 6, S9(b)–(f) and S10(b)–(f). In contrast, the coordinate of the extreme values of $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ at the left is nearly kept at $(73.0^\circ, 0.19)$, which is around the minimal value of $|r_{ss}|$ and little affected by the change of $Q_0$.

By using the parameter values of $v_F$ and $\tau$ for TaAs and TaIrTe$_4$ WSM films [47, 48], similar singularities are also observed at $Q_0 = 0.2\mu$, as shown in figures S11 and S12, respectively. At $Q_0 = 0.0$, according to equations (10) and (15), the Fresnel reflection coefficient $r_{ps} = 0.0$ at $\sigma_{ys} = 0.0$ such that the in-plane photonic spin Hall shift $\langle \Delta x_+ \rangle = 0.0$. This phenomenon is in conformity with figure S9(a). Thus, figures S13(a)–(d) only give the Fresnel reflection coefficient $|r_{ss}|$ and the transverse photonic spin Hall shift $\langle \Delta y_+ \rangle$ at $Q_0 = 0.0$ for TaAs and TaIrTe$_4$ WSM films. It is found from figure S13 that the singularities for both TaAs and TaIrTe$_4$ disappear. In contrast, the positions of the extreme values of $\langle \Delta y_+ \rangle$ at the left are unchanged and still near the minimal value of $|r_{ss}|$ after decreasing $Q_0$ from 0.2$\mu$ to 0.0 (cf figures S11–S13).

Even though many studies on PSHE have been published, the previously utilized incident beam is mainly limited to be horizontally polarized, and the photonic spin Hall shifts generally give extreme values around the Brewster’s angles [5, 6, 14, 21, 23]. Herein, the incident beam is vertically polarized, and the extreme values of photonic spin Hall shifts not only appear around but could also occur away from the minimal values of Fresnel reflection coefficients. As aforementioned, $Q_0 \neq 0$ (i.e., $\mu_+ \neq \mu_-$) is a condition precedent to having nonzero Hall conductivity $\sigma_{xy}$. Thus, the singularities of photonic spin Hall shifts in figures 5 and S9–S12 can be ascribed to an inversion-symmetry breaking with a nonzero axial chemical potential in WSM film. Note that TaAs is a type-I WSM ($|\alpha_t| < 1$) while $T_d$-MoTe$_2$ and TaIrTe$_4$ are type-II WSMs ($|\alpha_t| > 1$) according to the characterization of tilt $\alpha_t$ of Weyl nodes. Even if the PSHE on the surfaces of type-I and type-II WSMs was investigated by Jia et al., neither strain effect nor the CHE was involved [21]. Since in the literature it still lacks a comprehensive investigation about the influence of tilt $\alpha_t$ on the
complex optical conductivity of strained WSMs, the degree of tilt of Weyl nodes is ignored in this work, and it remains an open question how the tilt of Weyl nodes in strained WSMs affects the PSHE.

Figures 7(a)–(d) exhibits the color maps of $|r_{ss}|$, $|r_{ps}|$, $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ with respect to the chemical potential $\mu$ and the amount $Q_0$. The parameters of $\theta_i$, $\gamma/\omega_{el}$, and $\omega/\omega_{el}$ are set as 72.4°, 0.2, and 1.0, respectively. It is seen that the $|r_{ss}|$ gives local minimums at $\mu = 0.2$ eV. Accordingly, the photonic spin Hall shifts $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ give extreme values around $\mu = 0.2$ eV. This is also the reason why the factor $\mu$ in the above model is set as 0.2 eV. In particular, the shifts $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ reach their respective maximum values of $-69.3\lambda$ and $2.9\lambda$ at $Q_0 = 0.86\mu$. Theoretically, it is permitted to increase the $Q_0$ to $1.0\mu$.

Considering the fact that we are slightly breaking the inversion symmetry, it is more reasonable to assign one small value to the $Q_0$. Thus the default value of $Q_0$ is set as 0.2$\mu$ which is also adopted by Heidari et al [33].

To date, a quantitative relation between the strain of WSM film and the pseudo-magnetic field $B_{el}$ (or the cyclotron frequency $\omega_{el}$) could be still lacking. Nevertheless, the above results suggest that the strain-induced-CHE can give rise to the PSHE with the largest photonic spin Hall shift of hundreds of times of the incident wavelength. Thus, it is possible to detect the strain of WSMs through measuring the photonic spin splitting occurring on the surfaces of WSM films. The possible experimental setup is proposed in figure 8. One Gaussian beam generated by an infrared laser vertically incident upon a half-wave plate which is used to prevent the charge-coupled device (CCD) from saturation. Then the Gaussian beam is focused by a lens, and a linear polarizer is utilized to select the polarization state after the beam passing through the lens. When the beam impinges onto the surface of strained WSM film with a nonzero $\theta_i$, the PSHE takes place. The reflected beam from WSM film passes through the second polarizer which is rotated to obtain an appropriate postselection state. Then the second lens is utilized to collimate the beam. Finally, the barycenter position and intensity profile of the reflected beam are recorded by the CCD. The strain in WSM can be introduced by stretching the ENZ substrate, as indicated by two opposite black and wide arrows in figure 8. In addition, two-point bending apparatus can also be used to introduce strain in a WSM film [49, 50]. The inset in figure 8 illustrates the intensity patterns of the incident Gaussian beam, RCP and LCP components of the reflected beam. It is seen that the intensity centroids of RCP and LCP components of the reflected beam diverge from the center of $(x_r, y_r)$ coordinate frame. Then the photonic spin Hall shifts can be derived from the displacements of intensity centroids recorded by CCDs.
Figure 7. Variations of the Fresnel reflection coefficients (a) $|r_{ss}|$ and (b) $|r_{ps}|$ with respect to the chemical potential $\mu$ and the amount $Q_0$. (c) In-plane and (d) transverse spin-dependent shifts on the surface of strained WSM film versus the parameters of $\mu$ and $Q_0$. The color bars in (c) and (d) are scaled in the unit of $\lambda$.

Figure 8. A schematic sketch of the possible experimental setup for observing the PSHE on the surface of strained WSM film. Black and wide arrows indicate the direction of the applied strain. $(x_i, y_i, z_i)$ and $(x_r, y_r, z_r)$ represent the incident and reflected beam coordinates, respectively. The inset: schematic drawing of the photonic spin Hall shifts of a beam reflected from the surface of a strained WSM film. For simplify, the transverse displacements for RCP and LCP components of the reflected beam are marked at negative and positive directions of $y_r$ axis, respectively. According to the above figures, their signs can be changed via tuning the incident conditions and/or structural factors.

4. Conclusions

To summarize, the PSHE modified by CHE is investigated in detail, when one vertically polarized optical wave impinges onto the strained and doped WSM thin film depositing upon a homogeneous substrate. We show that ENZ material is much superior to the material with a relative permittivity $\varepsilon_{r2} > 1.0$ as a substrate...
in enhancing the PSHE. The maximum photonic spin Hall shifts appear at the angular frequency which is close to the cyclotron frequency of massless fermions at which the pseudo-magnetic field induces an absorption peak in the longitudinal optical conductivity spectrum. The largest in-plane and transverse photonic spin-dependent shifts are more than 400 and 50 times of incident wavelength, respectively. Additionally, both \( \langle \Delta x_r \rangle \) and \( \langle \Delta y_r \rangle \) generally give extreme values in the vicinity of incident angles at which the \( |r_m| \) and/or \( |r_p| \) exhibit local minimums. However, an inversion-symmetry breaking with nonzero axial chemical potential (\( \mu_+ \neq \mu_- \neq 0 \)) may result in singularities of photonic spin Hall shifts by tuning the incident angle and the cyclotron frequency. Finally, we propose a possible experimental system for measuring the photonic spin Hall shifts occurring on the surface of strained WSM film. These findings may provide important insights into the critical role of CHE in the modulation of PSHE in WSMs. They may also provide new strategies to both measure the lattice deformation and deliberate strain control over topological structure in WSMs.

**Competing interests**

The authors declare no competing interests.

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**ORCID iDs**

Guangy Jia 🌐 [https://orcid.org/0000-0002-5888-6460](https://orcid.org/0000-0002-5888-6460)

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