Modelling the post-reionization neutral Hydrogen (HI) bias

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ABSTRACT
Observations of the neutral Hydrogen (HI) 21-cm signal hold the potential of allowing us to map out the cosmological large scale structures (LSS) across the entire post-reionization era \((z \leq 6)\). Several experiments are planned to map the LSS over a large range of redshifts and angular scales, many of these targeting the Baryon Acoustic Oscillations. It is important to model the HI distribution in order to correctly predict the expected signal, and more so to correctly interpret the results after the signal is detected. In this paper we have carried out semi-numerical simulations to model the HI distribution and study the HI power spectrum \(P_{\text{HI}}(k, z)\) across the redshift range \(1 \leq z \leq 6\). We have modelled the HI bias as a complex quantity \(b(k, z)\) whose modulus squared \(b^2(k, z)\) relates \(P_{\text{HI}}(k, z)\) to the matter power spectrum \(P(k, z)\), and whose real part \(b_r(k, z)\) quantifies the cross-correlation between the HI and the matter distribution. We study the \(z\) and \(k\) dependence of the bias, and present polynomial fits which can be used to predict the bias across \(0 \leq z \leq 6\) and \(0.01 \leq k \leq 10\, \text{Mpc}^{-1}\). We also present results for the stochasticity \(r = b_r/b\) which is important for cross-correlation studies.

Key words: methods: statistical, cosmology: theory, large scale structures, diffuse radiation

1 INTRODUCTION
Since its predictions by H. van der Hulst in 1944, the neutral hydrogen (HI) 21-cm line has become a work horse for observational cosmology. One of the direct applications of the 21-cm emission is to measure the rotation curve of galaxies (e.g. see Begum et al. 2005, and references therein), which is one of the most direct probes of dark matter. The 21-cm emission is also a very reliable probe of the HI content of the galaxies for the nearby universe. Surveys like the HI Parkes All-Sky Survey (HIPASS; Zwaan et al. 2005), the HI Jodrell All-Sky Survey (HIJASS; Lang et al. 2003), the Blind Ultra-Deep HI Environmental Survey (BUDHIES; Jaffé et al. 2012) and the Arceto Fast Legacy ALFA Survey (ALFALFA; Martin et al. 2012) aim to measure the 21-cm emission from individual galaxies at very low redshifts \((z << 1)\) to quantify the HI distribution in terms of the HI mass function and the HI density parameter \(\Omega_{\text{HI}}\). These studies also help us in understanding the effects of different environments in which HI resides. This method fails at higher redshifts where we cannot identify individual galaxies. Here the cumulative flux of the 21-cm radiation from high redshift emitters appears as a diffused background radiation. Measurements of the intensity fluctuations in this diffused background provide us a three dimensional probe of the large scale structures over a large redshift range in the post-reionization era \((z \lesssim 6)\) (Bharadwaj et al. 2001; Bharadwaj & Sethi 2001; Bharadwaj & Pandey 2003; Bharadwaj & Srikant 2004). The advantage of studying the 21-cm emission in the post-reionization era lies in the fact that the modulation of the signal due to complicated ionizing fields is less and the 21-cm power spectrum is directly proportional to the matter power spectrum which enhances its usefulness as a probe of cosmology (Wyithe & Loeb 2009). This technique provides an independent estimate of various cosmological parameters (Loeb & Wyithe 2008; Bharadwaj et al. 2009). The Baryon Acoustic Oscillations (BAO) are embedded in the power spectrum of 21-cm intensity fluctuations at all redshifts and the comoving scale of BAO can be used as a standard ruler to constrain the evolution of the equation of state for dark energy (Wyithe et al. 2008; Chang et al. 2008; Seo et al. 2010).

Several existing and upcoming experiments are planned to map this radiation at various redshifts. A number of methods also have been proposed or implemented to recover the informations from the signal faithfully. Lah et al. (2007a, 2009) used Giant Metrewave Radio Telescope (GMRT) observations at \(z \sim 0.4\) to co-add the 21-cm signals (“stacking”) from galaxies with known redshifts in order to increase the signal to noise ratio and infer the average HI mass...
of the galaxies. This technique has been extended to a little to \( z \sim 0.8 \) by studying the cross-correlation between 21-cm intensity maps and the large scale structures traced by optically selected galaxies to constrain the amplitude of the HI fluctuations (Chang et al. 2010; Masui et al. 2013). Ghosh et al. (2011a,b) devised a method to characterize and subtract the foreground contaminations in order to recover the signal and used 610 MHz (\( z = 1.32 \)) GMRT observations to set an upper limit on the amplitude of the HI 21-cm signal. Kanekar et al. (2016) extended the signal stacking technique further to \( z \sim 1.3 \) using GMRT observations and obtained an upper limit on the average HI 21-cm flux density.

A number of 21-cm intensity mapping experiments like Baryon Acoustic Oscillation Broadband and Broad-beam (BAOBAB; Pober et al. 2013), BAO from Integrated Neutral Gas Observations (BINGO; Battye et al. 2012), Canadian Hydrogen Intensity Mapping Experiment (CHIME; Bandura et al. 2014), the Tianlai project (Chen 2012), Square Kilometre Array 1-MID/SUR (SKA1-MID/SUR; Bull et al. 2015) have been planned to cover the intermediate redshift range \( z \sim 0.5 - 2.5 \) where their primary goal is to measure the scale of BAO, particularly around the onset of acceleration at \( z \sim 1 \). Recent studies suggest that observations of 21-cm fluctuations on small scales, with SKA1, can constrain the sum of the neutrino masses (Villaescusa-Navarro et al. 2015; Pal & Guha Sarkar 2016). Observations with SKA1-MID can also test different scalar field dark energy models (Hossain et al. 2016). Ali & Bharadwaj (2014) and Bharadwaj et al. (2015) present theoretical estimates for intensity mapping at \( z \sim 3.35 \) with the Ooty Wide Field Array (OWFA), while Chatterjee et al. (2016) and Santos et al. (2015) present similar estimates for the upcoming uGMRT and SKA2 respectively.

The main observable of the 21-cm intensity mapping experiments is the 21-cm brightness temperature fluctuation power spectrum \( P_T(k, z) \). This can be interpreted in terms of the HI power spectrum \( P_{HI}(k, z) \) as,

\[
P_T(k, z) = \hat{T}_{HI}(k) P_{HI}(k, z),
\]

where

\[
\hat{T}_{HI}(z) \approx 4.0 \text{mK}(1+z)^2 \left( \frac{\Omega_{gas}(z)}{10^{-3}} \right) \left( \frac{\Omega_b h^2}{0.02} \right) \left( \frac{H_0}{H(z)} \right)
\]

is the mean brightness temperature of the HI 21-cm emission (Ali & Bharadwaj 2014). Here, \( \Omega_{gas}(z) \) is the density parameter for the neutral gas which can be expressed in terms of the HI-density parameter \( \Omega_{HI}(z) \) through a suitable conversion, all other symbols have their usual meaning. We can interpret the HI power spectrum \( P_{HI}(k, z) \) in terms of the matter power spectrum \( P(k, z) \) as

\[
P_{HI}(k, z) = b^2(k, z) P(k, z),
\]

under the assumption that the HI traces the underlying matter distribution with a linear bias \( b(k, z) \) which quantifies the clustering of the HI relative to that of the total matter distribution. It is clear that we will need independent estimates of both \( \Omega_{HI}(z) \) and \( b(k, z) \) in order to interpret the observable \( P_T(k, z) \) in terms of the underlying matter power spectrum \( P(k, z) \). Further, the amplitude of the expected signal \( P_T(k, z) \) is very sensitive to both \( \Omega_{HI}(z) \) and \( b(k, z) \), and it is crucial to have prior estimates of these parameters in order to make precise predictions for the upcoming experiments (Padmanabhan et al. 2015).

Several measurements of \( \Omega_{HI}(z) \) have been carried out both at low and high redshifts. At low redshifts (\( z \sim 1 \) and lower) we have measurements of \( \Omega_{HI} \) from HI galaxy surveys (Zwaan et al. 2005; Martin et al. 2010; Delhaize et al. 2013), Damped Lyman-\( \alpha \) Systems (DLAs) observations (Rao et al. 2006; Meiring et al. 2011) and HI stacking (Lah et al. 2007b; Rhee et al. 2013). At high redshifts (\( 1 < z < 6 \)), measurements of \( \Omega_{HI} \) come from the studies of the Damped Lyman-\( \alpha \) Systems (DLAs) (Prochaska & Wolfe 2009; Noterdaeme et al. 2012). Earlier observations indicated the HI content of the universe to remain almost constant with \( \Omega_{HI} \sim 10^{-3} \) over the entire redshift range \( z < 6 \) (Lanzetta et al. 1995; Storrie-Lombardi et al. 1996; Rao & Turnshek 2000; Péroux et al. 2003). However, some recent studies (Sánchez-Ramírez et al. 2015) indicate that \( \Omega_{HI} \) evolves significantly from \( z \sim 2 \) to \( z \sim 5 \), although the redshift evolution of \( \Omega_{HI} \) is debatable in the intermediate redshift range, \( z = 0.1 - 1.6 \) (Sánchez-Ramírez et al. 2015). A combination of low redshift data with high redshift observations shows that \( \Omega_{HI} \) decreases almost by a factor of 4 between \( z = 5 \) to \( z = 0 \) (Sánchez-Ramírez et al. 2015; Crighton et al. 2015).

Martin et al. (2012) have used HI selected galaxies to estimate the HI bias \( b(k) \) at \( z \sim 0.06 \). Intensity mapping experiments have measured the product \( \Omega_{HI} b r \) (Chang et al. 2010; Masui et al. 2013) by studying the cross-correlation of the HI intensity with optical surveys (\( r \) here is the cross-correlation coefficient or “stochasticity”) while Switzer et al. (2013) have measured the combination \( \Omega_{HI} b \), all these measurements being at \( z < 1 \). We do not, at present, have any observational constraint on the HI bias \( b(k, z) \) at redshifts \( z > 1 \). It is therefore important to model \( b(k, z) \) as an useful input for the future 21-cm intensity mapping experiments.

Marín et al. (2010) have developed an analytic framework for calculating the large scale HI bias \( b(k, z) \) and studying its redshift evolution using a relation between the HI mass \( M_{HI} \) and the halo mass \( M_h \) motivated by observations of the \( z = 0 \) HI mass function. Analytic techniques, however are limited in incorporating the effects of nonlinear clustering. In an alternative approach, Bagla et al. (2010) have proposed a semi-numerical technique which utilizes a prescription to populate HI in the halos identified from dark matter only simulations. The same approach has also been used by Khandai et al. (2011) and Guha Sarkar et al. (2012) to study the HI power spectrum and the related bias. Villaescusa-Navarro et al. (2014) have used high-resolution hydrodynamical N-body simulations along with three different prescriptions for distributing the HI. Seehars et al. (2015) have proposed a semi-numerical model for simulating large maps of the HI intensity distribution at \( z < 1 \). The analytic and semi-numerical studies carried out till date are all limited in that each study is restricted to a few discrete redshifts. In a recent paper Padmanabhan et al. (2015) have compiled all the available results for the HI bias and interpolated the values to cover the redshift range \( z = 0 - 3.4 \). Their study is restricted to large scales where it is reasonable to consider a constant \( k \) independent bias \( b(z) \). We do not, at present, have a comprehensive study which uses a
single technique to study the HI bias over a large $z$ and $k$ range.

In this work, we study: (i) the evolution of the HI power-spectrum $P_{\text{HI}}(k, z)$ across the redshift range $1 \leq z \leq 6$ by using N-Body simulations coupled with the third HI distribution model of Bagla et al. (2010), (ii) the redshift variation of the complex bias $b(k, z)$ whose modulus squared, $b^2(k, z)$, relates $P_{\text{HI}}(k, z)$ to the matter power-spectrum $P(k, z)$, and whose real component $b_r(k, z)$ quantifies the cross-correlation between the HI and the total matter distribution, and (iii) the spatial (rather, $k$) dependence of the bias and present polynomial fits which can be used to predict its variation over a large $z$ and $k$ range. We note that the entire analysis of this paper is restricted to real space i.e. it does not incorporate redshift space distortion arising due to the peculiar velocities. We plan to address the effect of peculiar velocities in future. An outline of the paper follows.

In section 2, we briefly describe the method of simulating the HI distribution. In section 3, we present the results of our simulations. Section 3.1 contains the details of the polynomial fitting for the joint $k$ and $z$ dependence of the biases. The values of the fitting parameters are tabulated in Appendix A. We finally summarize all the findings and discuss a few current results on the basis of our simulations in section 4.

We use the fitting formula of Eisenstein & Hu (1999) for the $\Lambda$-CDM transfer function to generate the initial matter power spectrum. The cosmological parameter values used are as given in Planck Collaboration et al. (2014).

2 SIMULATING THE HI DISTRIBUTION

We follow three main steps to simulate the post-reionization HI 21-cm signal. In the first step we use a Cosmological N-body code to simulate the matter distribution at the desired redshift $z$. Here we have used a Particle Mesh (PM) N-body code developed by Bharadwaj & Srikant (2004). This ‘gravity only’ code treats the entire matter content as dark matter and ignores the baryonic physics. The simulations use $[1, 0.072]^3$ particles in a $[2, 144]^3$ regular cubic grid of spacing 0.07 Mpc with a total simulation volume (comoving) of $[150.08 \text{ Mpc}]^3$. The simulation particles all have mass $10^9 M_\odot$ each. We have used the standard linear $\Lambda$-CDM power spectrum to set the initial conditions at $z = 125$, and the N-body code was used to evolve the particle positions and velocities to the redshift $z$ at which we desire to simulate the HI signal. We have considered $z$ values in the interval $\Delta z = 0.5$ in the range $z = 1$ to $z = 6$.

In the next step we employ the Friends-of-Friends (FoF) algorithm (Davis et al. 1985) to identify collapsed halos in the particle distribution produced as output by the N-body simulations. For the FoF algorithm we have used a linking length of $l_f = 0.2$ in units of the mean inter-particle separation, and furthermore, we require a halo to have at least ten particles. This sets $10^9 M_\odot$ as the minimum halo mass that is resolved by our simulation. We also verify that the mass distribution of halos so detected are in good agreement with the theoretical halo mass function (Jenkins et al. 2001; Sheth & Tormen 2002) in the mass range $10^9 \leq M \leq 10^{13} M_\odot$. Our halo mass range is well in keeping with a recent study (Kim et al. 2016) which shows that at $z \geq 0.5$ a dark matter halo mass resolution better than $\sim 10^{10} h^{-1} M_\odot$ is required to predict 21-cm brightness fluctuations that are well converged.

The observations of quasar (QSO) absorption spectra suggest that the diffuse Inter Galactic Medium (IGM) is highly ionized at redshifts $z \leq 6$ (Becker et al. 2001; Fan et al. 2006a,b). This redshift range where the hydrogen neutral fraction has a value $x_{\text{HI}} < 10^{-4}$ is referred to as the post-reionization era. Here the bulk of the HI resides within dense clumps (column density $N_{\text{HI}} \geq 2 \times 10^{20} \text{cm}^{-2}$) which are seen as the Damped Lyman-α systems (DLAs) found in the QSO absorption spectra (Wolfe et al. 2005). Observations indicate that the DLAs contain almost $\sim 80\%$ of the total HI, (Storrie-Lombardi & Wolfe 2000; Prochaska et al. 2005; Zafar et al. 2013) and they are the source of the HI21-cm radiation in the post-reionization era. While the origin of the high-$z$ DLAs is still not very well understood, it is found (eg. Haehnelt et al. (2000)) that it is possible to correctly reproduce many of the observed DLAs properties if it is assumed that the DLAs are associated with galaxies. From the cross-correlation study between DLAs and Lyman Break Galaxies (LBGs) at $z \sim 3$, Cooke et al. (2006) showed that the halos with mass in the range $10^9 < M_h < 10^{12} M_\odot$ can host the DLAs. In this work we assume that HI in the post-reionization era is entirely contained within dark matter halos which also host the galaxies. In the third step of our simulation we populate the halos identified by the FoF algorithm with HI. Here we assume that the HI mass $M_{\text{HI}}$ contained within a halo depends only on the halo mass $M_h$, independent of the environment of the halo.

At the outset, we expect the HI mass to increase with the size of the halo. However, observations at low $z$ indicate that we do not expect the large halos, beyond a certain upper cut-off halo mass $M_{\text{max}}$, to contain a significant amount of HI. For example, very little HI is found in the large galaxies which typically are ellipticals and in the clusters of galaxies (eg. see Serra et al. 2012, and references therein). Further, we also do not expect the very small halos, beyond a certain lower cut-off halo mass $M_{\text{min}}$, to contain significant HI mass. The amount of gas contained in small halos ($M_h < M_{\text{min}}$) is inadequate for it to be self shielded against the ionizing radiation. Based on these considerations, Bagla et al. (2010) have introduced several schemes for populating simulated halos to simulate the post-reionization HI distribution. In our work we have implemented one of the schemes proposed by Bagla et al. (2010) to populate the halos. This uses an approximate relation between the virialized halo mass and the circular velocity as a function of redshift

$$M_h \approx 10^{10} \left( \frac{v_{\text{circ}}}{60 \text{km/s}} \right)^3 \left( \frac{1+z}{4} \right)^{\frac{3}{2}} M_\odot .$$

(4)

It is assumed that the neutral gas in the halos will be able to shield itself from the ionizing radiation only if the halo’s circular velocity exceeds $v_{\text{circ}} \sim 30$ km/s, which sets the lower mass limit of the halos $M_{\text{min}}$. The upper mass cutoff $M_{\text{max}}$ is decided by taking the upper limit of the circular velocity $v_{\text{circ}} \sim 200$ km/s, beyond which the HI content falls off. Pontzen et al. (2008) have shown that halos more massive than $10^{11} M_\odot$ do not contain a significant amount of neutral gas.

In our work we have used the third scheme proposed by Bagla et al. (2010) where the HI mass in a halo is related to
Figure 1. Shown in this figure are the Matter (left panels), Halo (central panels) and HI (right panels) density contrasts $[\delta(x, t) = \delta\rho(x, t)/\bar{\rho}(t)]$ respectively at three different redshifts 6, 3 and 1 (from top to bottom). First we calculate the over densities on the grid positions using cloud in cell (CIC) interpolation scheme. We prepare the two dimensional density plots by collapsing a layer of thickness 5.6 Mpc along one direction to calculate the average density contrast on a plane. Different colours indicate the values of density contrast on each of the pixels as shown by the colour bar.

According to this scheme only halos with mass greater than $M_{\text{min}}$ host HI. The HI mass of a halo increases proportionally with the halo mass $M_h$ for $M_h \ll M_{\text{max}}$. However, the HI mass saturates as $M_h \sim M_{\text{max}}$, and $M_{\text{HI}}$ attains a constant upper limit $M_{\text{HI}} = f_3 M_{\text{max}}$ for $M_h \gg M_{\text{max}}$. The free parameter $f_3$ determines the total amount of HI in the simulation volume, and its value is tuned so that it produces the desired value of the HI density parameter $\Omega_{\text{HI}} \sim 10^{-3}$. Our entire work here deals with the dimensionless HI density contrast $\delta\rho_{\text{HI}}/\bar{\rho}_{\text{HI}}$ which is insensitive to the choice of $f_3$.

We have run five statistically independent realizations of the simulation. These five independent realizations were used to estimate the mean value and the variance for all the results presented in this paper. The simulations require
a large computation time, particularly the FoF which takes ~ 10 days for a single realization on our computers and this restricts us to run only five independent realizations. The computation time increases at lower redshifts, and we have restricted our simulations to $z > 1$.

As mentioned earlier, our simulations have a halo mass resolution of $M_h = 10^9 M_{\odot}$, but eq. 5 shows that the mass of the smallest possible halo that retain HI falls as $M_{\text{min}} \propto (1+z)^{-2}$ and so, $M_{\text{min}} = 10^8 M_{\odot}$ at $z=3.5$, i.e., the minimum resolvable halo mass, $M_{\text{min}}$, falls below our mass resolution of $10^9 M_{\odot}$ at $z > 3.5$. At these redshifts therefore, our simulations cannot detect halos less massive than this threshold and which according to the model proposed by Bagla et al. (2010), are also likely to host some HI. To study the effects of these missing low mass halos we have run a high resolution simulation (referred to as HRS) with $[2,144]^3$ particles in a $[4,288]^3$ regular cubic grid of spacing 0.035 Mpc, the total simulation volume remaining the same as earlier. The lower mass limit for the halo mass is $10^8 M_{\odot}$ in the HRS, well below $M_{\text{min}}$ in the entire redshift range. The HRS requires considerably larger computational resources compared to the other simulations, and we have run only a single realization for which we have compared the results with those from the earlier lower resolution simulations.

3 RESULTS

Figure 1 provides a visual impression of how the matter, the halos and the HI are distributed at different stages of the evolution. We show this by plotting the density contrasts $\delta(x,t) = \delta\rho(x,t)/\bar{\rho}(t)$ at three different redshifts, viz. 6, 3 and 1. It can be seen that the cosmic web is clearly visible in all three components even at the highest redshift $z = 6$, though it is somewhat diffused for the matter at this redshift. Observe that the basic skeleton of the cosmic web is nearly the same for all the three components, and the basic skeleton does not change significantly with redshift. We see that for all the three components the cosmic web becomes more prominent with decreasing redshift. Considering the matter first, the density contrast grows with decreasing redshifts due to gravitational clustering. The halos are preferentially located at the matter density peaks, and it is evident that the halos have a higher density contrast. We see that the structures in the halo distribution are more prominent compared to the matter, particularly at high redshifts. The HI closely follows the halo distribution at $z = 6$. However, in contrast to the matter and halo distribution, the HI distribution shows a much weaker evolution with $z$. It is possible to understand this in terms of the model for populating the halos with HI. We know that the halo masses increase as gravitational clustering proceeds. According to our HI population model, however, the HI mass remains fixed once the halo mass exceeds a critical value.

We quantify the matter and the HI distributions with the respective power spectra $P(k)$ and $P_{\text{HI}}(k)$. We also quantify the cross correlation between the matter and the HI through the cross-correlation power spectrum $P_{\text{HI}}(k)$. For all the three power spectra we consider the dimensionless quantity $\Delta^2(k) = k^2 P(k)/2\pi^2$, respectively shown in the three panels of Figure 2 for different values of the redshift $z \in [1, 6]$. The five independent realizations of the simulation each provides a statistically independent estimate of the power spectrum. We have used these to quantify the mean and the standard deviation which we show in the figures. For clarity of presentation, the ±1σ confidence interval is shown for $z = 3$ only.

The left panel of Figure 2 shows $\Delta_{\text{HI}}^2(k)$ as a function of $k$ at different redshifts. The matter distribution, whose evolution is well understood (e.g. Chapter 15 of Peacock 1999) serves as the reference against which we compare the HI distribution at different stages of its evolution. It is evident that $\Delta_{\text{HI}}^2(k)$ increases proportional to the square of the growing mode leaving the shape of the power spectrum unchanged at small $k$ or large length-scales where the predictions of linear theory hold (e.g. Chapter 16 of Peacock 1999). At small scales, where nonlinear clustering is important, the shape of $\Delta_{\text{HI}}^2(k)$ changes with evolution and the growth is more than what is predicted by linear theory. Note that the different power spectra shown in this paper have all been calculated using a grid whose spacing is double of that used for the simulations. The turn over seen in $\Delta_{\text{HI}}^2(k)$ at $k \sim 10$ Mpc$^{-1}$ is an artefact introduced by the smoothing at this grid scale. We have restricted the entire analysis of this paper to the range $k \leq 10$ Mpc$^{-1}$.

The central panel of Figure 2 shows $\Delta_{\text{HI}}^2(k)$ as a function of $k$ at different redshifts. We can clearly see that the evolution of $\Delta_{\text{HI}}^2(k)$ and $\Delta_{\text{HI}}^2(k)$ are quite different. At small $k$, we find that $\Delta_{\text{HI}}^2(k)$ shows almost no evolution over the entire redshift range. We find this behaviour over the entire region where the matter exhibits linear gravitational clustering. We find that $\Delta_{\text{HI}}^2(k)$ grows to some extent at $k > 2$ Mpc$^{-1}$ where non-linear effects are important. This growth, however, is smaller than the growth of the matter power spectrum. Further, we also see that the shape of $\Delta_{\text{HI}}^2(k)$ closely resembles $\Delta^2(k)$ at small $k$, however the two differ at large $k$, and these differences are easily noticeable at $k > 2$ Mpc$^{-1}$.

The right panel of Figure 2 shows $\Delta_{\text{HI}}^2(k)$ as a function of $k$ at different redshifts. We see that the evolution of $\Delta_{\text{HI}}^2(k)$ is intermediate to that of $\Delta_{\text{HI}}^2(k)$ and $\Delta_{\text{HI}}^2(k)$, it grows faster than $\Delta_{\text{HI}}^2(k)$ but not as fast as $\Delta^2(k)$. All three power spectra have the same shape at small $k$, indicating that the HI traces the matter at large length-scales. At large $k$ the shape of $\Delta_{\text{HI}}^2(k)$, however, differs from both $\Delta_{\text{HI}}^2(k)$ and $\Delta_{\text{HI}}^2(k)$ indicating differences in the small-scale clustering of the HI and the matter.

Redshift surveys of large scale structures and numerical simulations reveal that the galaxies trace underlying matter over-densities with a possible bias (Bardeen et al. 1986; Mo & White 1996; Dekel & Lahav 1999). In the post-reionization era, the association of the HI with the halos implies that the HI follows the matter density field with some bias. The bias function relates the HI density contrast to that of the matter. Here we assume that a linear relation holds between the Fourier components of the HI and the matter density contrasts

$$\Delta_{\text{HI}}(k) = b(k) \Delta(k)$$  \hspace{1cm} (6)$$

where $b(k)$ is the linear bias function or simply bias, which can, in general, be complex. The complex bias allows for the possibility that the Fourier modes $\Delta_{\text{HI}}(k)$ and $\Delta(k)$ can differ in both the amplitude and also the phase. The ratio
of the respective power spectra

\[ b(k) = \sqrt{\frac{\Delta_{\text{HI}}^2(k)}{\Delta^2(k)}}. \] (7)

allows us to quantify \( b(k) \) which is the modulus of the complex bias \( \hat{b}(k) \), and the ratio

\[ b_r(k) = \frac{\Delta_{\text{HI}}^2(k)}{\Delta^2(k)}. \] (8)

allows us to quantify \( b_r(k) \) which is the real part of the complex bias \( \hat{b}(k) \). With both \( b(k) \) and \( b_r(k) \) at hand, we can reconstruct the entire complex bias \( \hat{b}(k) \). One is mainly interested in the modulus \( b(k) \) which allows us to interpret the HI power spectrum in terms of the underlying matter power spectrum. However, the real part of the bias \( b_r(k) \) is the relevant quantity if one is considering the cross-correlation of the HI with either the matter distribution or with some other tracer of the matter distribution like Lyman-\( \alpha \) forest (Guha Sarkar et al. 2011; Guha Sarkar & Datta 2015) or galaxy surveys (Chang et al. 2010; Masui et al. 2013; Cohn et al. 2015).

The left panel of Figure 3 shows the behaviour of \( b(k) \), the modulus of \( \hat{b}(k) \), as a function of \( k \) at six different redshifts. We also show the 5 \( \sigma \) confidence interval at three different redshifts. The relatively small errors indicate that the results reported here are statistically representative values. We see that the value of \( b(k) \) decreases with decreasing redshift. Further, the \( k \) dependence is also seen to evolve with redshift. In all cases, we have a constant \( k \) independent bias at small \( k \) and the bias shoots up rapidly with \( k \) at large \( k \) (\( \gtrsim 4 \text{ Mpc}^{-1} \)). However, for high redshifts (\( z \gtrsim 3 \)), \( b(k) \) increases monotonically with \( k \) whereas we see a dip in the values of \( b(k) \) at \( k \sim 2 \text{ Mpc}^{-1} \) for \( z < 3 \). Interestingly, the \( k \) range where we have a constant \( k \) independent bias is maximum at the intermediate redshift \( z = 3 \) where it extends to \( k \lesssim 1 \text{ Mpc}^{-1} \), and it is minimum (\( k \lesssim 0.2 \text{ Mpc}^{-1} \)) at the highest and lowest redshifts (\( z = 6, 1 \)) whereas it covers \( k \lesssim 0.4 \text{ Mpc}^{-1} \) at the other redshifts (\( z = 2, 4, 5 \)).

The central panel of Figure 3 shows both \( b(k) \) and \( b_r(k) \) which is the real part of \( \hat{b}(k) \). The two quantities \( b(k) \) and \( b_r(k) \) show similar \( k \) dependence. Both \( b(k) \) and \( b_r(k) \) will be equal if the bias \( \hat{b}(k) \) is a real quantity. We see that this is true at small \( k \) where both quantities have nearly constant values independent of \( k \). However, we find a \( k \) independent bias \( b_s(k) \) for a smaller range of \( k \), in comparison to \( b(k) \). The two quantities \( b(k) \) and \( b_r(k) \) differ at larger \( k \), and the differences increase with increasing \( k \). The difference between \( b(k) \) and \( b_r(k) \) is seen to increase with decreasing redshift. Also the \( k \) value where these differences become significant shifts to smaller \( k \) with decreasing redshift.

As already mentioned in Section 2, for \( z > 3.5 \), \( M_{\text{min}} \) (eq. 4) has a value that is smaller than \( 10^9 \odot \) which is the smallest mass halo resolved by our simulations. Imposing a fixed lower halo mass limit of \( 10^9 \odot \) will, in principle, change the HI bias \( \hat{b}(k) \) in comparison to the actual predictions of the halo population model proposed by Bagla et al. (2010), and we have run higher resolution simulation in order to quantify this. It is computationally expensive to run several realizations of simulations with a smaller mass resolution, so we have run a single realization with a halo mass resolution of \( 10^{8.1} \odot \) which is well below \( M_{\text{min}} \) over the entire redshift range of our interest. The right panel of Figure 3 shows the percentage difference in the values of \( b(k) \) computed using the low and the high resolution simulations. We find that the difference is minimum for \( z = 4 \) and maximum for \( z = 6 \) where we expect a larger contribution from the smaller halos. For \( k < 1.0 \text{ Mpc}^{-1} \), the difference is \( 5 - 8\% \) at \( z = 4 \) and \( 8 - 13\% \) at \( z = 6 \). Beyond \( 1.0 \text{ Mpc}^{-1} \), the difference increases but it is well within 20\% for redshifts 4 and 5 and less than 30\% for redshift 6. These differences are relatively small given our current lack of knowledge about how the HI is distributed at the redshifts of interest. It is therefore well justified to use the simulations with a fixed lower mass limit of \( 10^9 \odot \) for the entire redshift range considered in this paper.

Figure 4 shows the redshift evolution of \( b(z) \) and \( b_r(z) \) at three representative \( k \) values. At \( k = 0.065 \text{ Mpc}^{-1} \) (left panel) which is in the linear regime we cannot make out the difference between \( b(z) \) and \( b_r(z) \) and this indicates that \( b(z) \) is purely real. We find that the bias \( b(z) \) has a value that is slightly less than unity at \( z = 1 \) indicating that the HI is slightly anti-biased at this redshift. The bias increases nearly linearly with \( z \) and it has a value \( b(z) \approx 3 \) at \( z = 6 \). At \( k = 0.45 \text{ Mpc}^{-1} \) (central panel) where we have the tran-
respectively show the mean and 5σ spread determined from 5 realisations of the simulations. The solid and dotted lines show the fitting of the bias \( b(k) \) relative to a high resolution simulation (HRS).

Figure 3. The left panel shows the \( k \) dependence of the HI bias \( b(k) \) at six different redshifts, \( z = 6 - 1 \) (top to bottom) at an interval \( \Delta z = 1 \). The shaded regions show ±5σ error around the mean value for three redshifts 6,3 and 1. The central panel shows the \( k \) dependence of both the biases, \( b(k) \) (line only) and \( b_r(k) \) (line-point), at six different redshifts \( z = 6 - 1 \) (top to bottom) at an interval \( \Delta z = 1 \). For \( z > 4 \), the right panel quantifies the effect of a fixed minimum halo mass which has a value \( M_{\text{min}} = 10^9 M_\odot \) in the low resolution simulations. The figure shows the fraction difference in the bias \( b(k) \) with a value \( M_{\text{min}} = 10^9 M_\odot \).

Figure 4. This shows the redshift evolution of \( b(k) \) and \( b_r(k) \) at three different \( k \) values. The points and the vertical error bars respectively show the mean and 5σ spread determined from 5 realisations of the simulations. The solid and dotted lines show the fitting of the respective quantities.

sition from the linear to the non-linear regime we find that \( b(z) \) is slightly larger than \( b_r(z) \). Both \( b(z) \) and \( b_r(z) \) show a \( z \) dependence very similar to that in the linear regime. At \( k = 2.2 \) Mpc\(^{-1} \) (right panel) which is in the non-linear regime we find that \( b_r(z) \) is appreciably smaller than \( b(z) \), and the difference is nearly constant over the entire \( z \) range. This indicates that the HI bias \( b(z) \) is complex in the non-linear regime. Further, we see that the relative contribution from the imaginary part of \( \tilde{b}(z) \) increases with decreasing \( z \). The value of \( b_r(z) \) is less than unity for \( z \leq 2 \), whereas this is so only in the range \( z \leq 1.5 \) for \( b(z) \). The redshift dependence of the bias is much steeper as compared to the linear regime, and we have a larger value of \( b(z) \approx 5 \) at \( z = 6 \). We find a nearly parabolic \( z \) dependence in the non-linear regime as compared to the approximately linear redshift dependence found at smaller \( k \). At all the three \( k \) values we have fitted the redshift evolution of \( b(z) \) and \( b_r(z) \) with a quadratic polynomial of the form \( b_0 + b_1 z + b_2 z^2 \). We find that the polynomials give a very good fit to the redshift evolution of the simulated data (Figure 4). We also find that the quadratic term \( b_2 \) is much larger at \( k = 2.2 \) Mpc\(^{-1} \) as compared to the two smaller \( k \) values. Based on these results, we have carried out a joint fitting of the \( k \) and \( z \) dependence of the bias, the details of which are presented in the next section.

3.1 Fitting the bias

We have carried out polynomial fitting for the \( k \) dependence of the bias (Figure 3). The fit was carried out for redshifts in the range \( z = 1 \) to 6 at an interval of \( \Delta z = 0.5 \). We find that a linear function of the form \( b(k) = b_0 + b_1 k \) gives a good fit to the simulated data for \( z > 4 \). However, a higher order polynomial is required at lower redshifts particularly because of the dip around \( k \sim 2 \) Mpc\(^{-1} \). We have used a
provides a visual impression of how the bias shows the fit of the central and in the right panel with 5σ error bars. The black dotted lines show the best fit curves.

The left panel shows the redshift evolution of the coefficient $b_m$ and $b_{rm}$ values. The square points represent the $b_m$ values and the cross points represent the $b_{rm}$ values. The errors in fitting are enhanced 5 times to make them visible and shown with the vertical error bars. The solid and the dotted black lines show the best fit curves for $b_m$ and $b_{rm}$ respectively. The $k$ dependence of the simulated biases, $b(k)$ and $b_r(k)$, at six different redshifts $z = 1 - 6$ (bottom to top) at an interval $\Delta z = 1$, are respectively shown in the central and in the right panel with 5σ error bars. The black dotted lines show the best fit curves.

Figure 5. The left panel shows the redshift evolution of the coefficients of fitting, $b_m$ and $b_{rm}$. The square points represent the $b_m$ values and the cross points represent the $b_{rm}$ values. The errors in fitting are enhanced 5 times to make them visible and shown with the vertical error bars. The solid and the dotted black lines show the best fit curves for $b_m$ and $b_{rm}$ respectively. The $k$ dependence of the simulated biases, $b(k)$ and $b_r(k)$, at six different redshifts $z = 1 - 6$ (bottom to top) at an interval $\Delta z = 1$, are respectively shown in the central and in the right panel with 5σ error bars. The black dotted lines show the best fit curves.

Figure 6. The joint $k$ and $z$ dependence of the biases $b(k,z)$ (left panel) and $b_r(k,z)$ (right panel) are shown here. The values of $b(k,z)$ and $b_r(k,z)$ at different points of $k-z$ plane is represented with appropriate colours. The contours are drawn through those $k$ and $z$ values where the biases $b(k,z)$ and $b_r(k,z)$ have values in the range $0 - 10$ (bottom to top) at an interval of $1$. The fits were carried out for $b(k)$ and $b_r(k)$, and we have retained the subscript $r$ for the different fitting coefficients of $b_r(k)$.

The left panel of Figure 5 shows how the 5 fitting coefficients $b_0$, ..., $b_4$ vary with redshift. The value of the coefficient $b_0$ corresponds to the scale independent bias which is seen to hold at small $k$ values. We find that $b_0$ and $b_0$ are indistinguishable over the entire redshift range, indicating that the bias is real at small $k$ values. We also find that $b_0$ increases nearly linearly with $z$, consistent with the behaviour seen in the left panel of Figure 4. The coefficients $b_1$, ..., $b_4$ introduce a scale dependence in the bias, and these coefficients have progressively smaller values. We find that the redshift dependence of all the five coefficients can be well fit by quadratic polynomials of the form

$$b(k) = b_0 + b_1k + b_2k^2 + b_3k^3 + b_4k^4.$$ (9)

which gives a good fit in the $k$ range $k \leq 10Mpc^{-1}$ at all the redshifts that we have considered. The fit was carried out for both $b(k)$ and $b_r(k)$, and we have retained the subscript $r$ for the different fitting coefficients of $b_r(k)$.

The fitting coefficients $c(m, n)$ allow us to interpolate the bias $b(k,z)$ at different values of $z$ and $k$ in the ranges $[1, 6]$ and $[0.04, 10]$ respectively. The fitting coefficients $c(m, n)$ and the $1\sigma$ errors in these coefficients $\Delta c(m, n)$ are tabulated in the Appendix A. The central panel of Figure 5 shows the fit along with the simulated data. We see that the fit reproduces the simulated data to a good level of accuracy over the entire $z$ and $k$ range of the fit. A similar fitting procedure was also carried out for $b_r$. The fitting coefficients $c_r(m, n)$ and the $1\sigma$ errors in these coefficients $\Delta c_r(m, n)$ are tabulated in the Appendix A. The right panel of Figure 5 shows that the fit matches the simulated $b_r$ values to a good level of accuracy.

Figure 6 provides a visual impression of how the bias varies jointly with $k$ and $z$. Here we have extrapolated our fit to cover a somewhat larger $k$ range ($[0.01, 10]$ Mpc$^{-1}$) and $z$ range $(0, 6)$. We find a scale independent bias for $k \leq 0.1$ Mpc$^{-1}$ across the entire $z$ range. Further, we see that the biases $b(k,z)$ and $b_r(k,z)$ both decrease monotonically with decreasing $z$. We also see that the HI and the matter become anti-correlated where $b_r$ has a negative value for the $k$ range $k \sim 1 - 2$ Mpc$^{-1}$ around $z \sim 0$. The cross-correlation between the HI and the matter can also be quantified using the stochasticity (Dekel & Lahav 1999) $r = b_r/b$. By definition $|r| < 1$, values $r \sim 1$ indicate a strong correlation, $r \sim 0$ corresponds to a situation when the two are uncorrelated and $r < 0$ indicates anti-correlation. Figure 7 shows how the stochasticity $r$ varies jointly as a function of $z$ and $k$. We see that $r = 1$ for $k \leq 0.1$ Mpc$^{-1}$ where the bias also is scale independent and real across the entire $z$ range. The $k$ value below which $r$ is unity increases with increasing redshift, with $k \sim 0.1$ Mpc$^{-1}$ for $z = 0$ and $k \sim 3$ Mpc$^{-1}$ for $z = 6$. The HI and the matter are highly correlated ($r > 0.8$) across nearly the entire $k$ range for $z > 2$. We also find that $r$ has a negative value at $k \sim 1 - 2$ Mpc$^{-1}$ around $z \sim 0$, indicative of an anti-correlation.
HI bias

4 SUMMARY AND DISCUSSION

In this paper we have used semi-numerical simulations (the third scheme of Bagla et al. (2010)) to model the HI distribution and study the evolution of \( P_{\text{HI}}(k, z) \) in the post-reionization era. The simulations span the redshift range \( 1 \leq z \leq 6 \) at an interval \( \Delta z = 0.5 \). We have modelled the HI bias as a complex quantity \( \tilde{b}(k, z) \) whose modulus \( b(k, z) \) (squared) relates \( P_{\text{HI}}(k, z) \) to \( P(k, z) \), and whose real part \( b_r(k, z) \) quantifies the cross-correlation between the HI and the total matter distribution. While there are several earlier works which have studied the HI bias \( b_r(k, z) \) at a few discrete redshifts (summarized in Padmanabhan et al. (2015)), this is the first attempt to model the post-reionization HI distribution across a large \( z \) and \( k \) range \((0.04 \leq k \leq 20 \text{ Mpc}^{-1})\) using a single simulation technique.

We find that the assumption of a scale-independent bias \( b(k, z) = b_0(z) \) holds at small \( k \) (eq. 9). The value of \( b_0(z) \) increases nearly linearly with \( z \), with a value that is slightly less than unity at \( z = 1 \) and \( b_0(z) \approx 3 \) at \( z = 6 \). The \( k \) range where we have a constant \( k \) independent bias is maximum at the intermediate redshift \( z = 3 \) where it extends to \( k \leq 1 \text{ Mpc}^{-1} \), and it is minimum \((k \leq 0.2 \text{ Mpc}^{-1})\) at the highest and lowest redshifts (\( z = 6, 1 \)) whereas it covers \( k \leq 0.4 \text{ Mpc}^{-1} \) at the other redshifts. The bias is scale dependent at larger \( k \) values where non-linear effects become important. We find that a polynomial fit (eq. 10) provides a good description of the joint \( k \) and \( z \) dependence of \( b(k, z) \) (and also \( b_r(k, z) \)). The coefficients of the fit are presented in Appendix A, and Fig. 6 provides a comprehensive picture of the bias across the entire \( k \) and \( z \) range, all the way to \( z = 0 \) where the results have been extrapolated from the fit.

Our results which are based on a PM N-body code are qualitatively consistent with the earlier work of Bagla et al. (2010) who have used a high resolution Tree-PM N-body code to calculate the bias at three different redshifts \((z = 1.3, 3.4 \text{ and } 5.1)\). The present work is also consistent with Guha Sarkar et al. (2012) who have used a technique similar to ours to compute the bias across \( z = 1.5 - 4 \), and Padmanabhan et al. (2015) who have applied the minimum variance interpolation technique to the different bias values collated from literature to predict the redshift evolution of the scale independent bias in the range \( z = 0 - 3.4 \).

The analytic model of Martin et al. (2010) predicts the HI distribution to be anti-biased at low redshifts \((z \leq 1)\). They also found that the bias decreases further for \( k \geq 0.1 \text{ Mpc}^{-1} \). These predictions are consistent with observations at \( z \sim 0.06 \) (Martin et al. 2012) which suggest that HI rich galaxies are very weakly clustered and mildly anti-biased at large scales, but become severely anti-biased on smaller scales. The predictions of our simulations which are restricted to \( z \gtrsim 1 \) are consistent with the findings of Martin et al. (2010). We find that the HI is mildly anti-biased at large scales at \( z = 1 \), and the bias drops further for \( k \geq 0.1 \text{ Mpc}^{-1} \) (Fig 3). We have also extrapolated our results to \( z \sim 0 \) (Fig 6) where the predictions are found to be qualitatively consistent with the measurements of Martin et al. (2012).

In our analysis the real part \( b_r(k, z) \) of the complex bias \( \tilde{b}(k, z) \) quantifies the cross-correlation between the HI and the total matter, and the bias \( b(k, z) \) is completely real if the two are perfectly correlated. The same issue is also quantified using the stochasticity \( r = b_r(k, z)/b(k, z) \). We see that \( b_r \) closely matches \( b \) at small \( k \) \((< 0.1 \text{ Mpc}^{-1})\) where we have a scale independent bias across the entire \( z \) range. The complex nature of the bias becomes important at larger \( k \). Our results are summarized in Fig. 7 which shows \( r \) across the entire \( z \) and \( k \) range. We find that the HI and the matter are well correlated \((r > 0.8)\) across nearly the entire \( k \) range for \( z \gtrsim 2 \). We also find that \( r \) has a negative value at \( k \sim 1 - 2 \text{ Mpc}^{-1} \) around \( z \sim 0 \), indicative of an anti-correlation.

The measurements of Chang et al. (2010) constrain the product \( \Omega_{\text{HI}} b r = (5.5 \pm 1.5) \times 10^{-4} \) at \( z \sim 0.8 \). From our analysis, we find that on large scales the product \( b_r b = 0.79 \) at \( z = 0.8 \) which implies \( \Omega_{\text{HI}} = (6.96 \pm 1.89) \times 10^{-4} \). Again, Masui et al. (2013) constrain the product \( \Omega_{\text{HI}} b r = (4.3 \pm 1.1) \times 10^{-4} \) at \( z \sim 0.8 \) using measurements in the \( k \) range \( 0.05 \text{ Mpc}^{-1} < k < 0.8 \text{ Mpc}^{-1} \) where our work predicts \( b r \) to vary from 0.83 to 0.39. The corresponding \( \Omega_{\text{HI}} \) varies between \((5.2 \pm 1.3) \times 10^{-4} \) to \((1.1 \pm 0.33) \times 10^{-3} \), which is a significant variation. On the other hand, Switzer et al. (2013) constrain the product \( \Omega_{\text{HI}} b = 6.2 \pm 1.4 \times 10^{-4} \) at \( z \sim 0.8 \) which implies \( \Omega_{\text{HI}} = 7.5 \pm 2.9 \times 10^{-4} \) if we consider \( b = 0.83 \) from our analysis. The above estimates of \( \Omega_{\text{HI}} \) are consistent with the measurement \( \Omega_{\text{HI}} = 7.41 \pm 2.71 \times 10^{-4} \) at \( z \sim 0.609 \) (Rao et al. 2006). We note that our simulations are restricted to \( z \geq 1 \), and the results were extrapolated to \( z = 0.8 \) for the discussion presented in this paragraph. Khandai et al. (2011) have carried out simulations which were specifically designed to interpret the results of Chang et al. (2010), and they have predicted \( b = 0.55 - 0.65 \) and \( r = 0.9 - 0.95 \) at \( z = 0.8 \). We note that the bias value predicted by Khandai et al. (2011) is considerably smaller than our prediction, and they predict \( \Omega_{\text{HI}} = 11.2 \pm 3.0 \times 10^{-4} \).
which also is larger than the measurements of Rao et al. (2006).

We finally reiterate that it is important to model the HI distribution in order to correctly predict the signal for upcoming 21-cm intensity mapping experiments. Further, such modelling is also important to correctly interpret the outcome of the future observations. In the present work we have implemented a simple HI population scheme which incorporates the salient features of our present understanding i.e. the HI resides in halos which also host the galaxies. This however ignores various complicated astrophysical processes which could possibly play a role in shaping the HI distribution. Further, the entire analysis has been restricted to real space, and the effects of redshift space distortion have not been taken into account. We plan to address these issues in future work.

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MNRAS 000, 000–000 (0000)
We have fitted the joint \( k \) and \( z \) dependence of the biases \( b(k, z) \) and \( b_r(k, z) \) using polynomial of the form

\[
b(k, z) = \sum_{m=0}^{4} \sum_{n=0}^{2} c(m, n) k^m z^n \quad \text{and} \quad (A1)
\]

\[
b_r(k, z) = \sum_{m=0}^{4} \sum_{n=0}^{2} c_r(m, n) k^m z^n \quad (A2)
\]

The best fit values of the fitting coefficients \( c(m, n) \) and \( c_r(m, n) \), and the 1 \( \sigma \) uncertainties in fitting \( \Delta c(m, n) \) and \( \Delta c_r(m, n) \) respectively, are given below.

\[
\begin{array}{cccc}
\text{m=0} & \text{n=0} & 1 & 2 \\
\hline
27.49 & 15.87 & 2.274 \\
1 & 81.87 & 49.21 & 7.097 \\
2 & 51.31 & 32.15 & 4.714 \\
3 & 10.32 & 6.641 & 0.9892 \\
4 & 0.6510 & 0.4253 & 0.06411 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{m=0} & \text{n=0} & 1 & 2 \\
\hline
55.10 & 25.19 & 1.963 \\
1 & 60.74 & 18.56 & 1.806 \\
2 & 33.54 & -17.38 & 1.618 \\
3 & -5.129 & 3.247 & -0.3803 \\
4 & 0.2773 & -0.1899 & 0.02435 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{m=0} & \text{n=0} & 1 & 2 \\
\hline
16.67 & 10.45 & 1.626 \\
1 & 46.34 & 31.15 & 4.995 \\
2 & 26.0 & 18.37 & 3.015 \\
3 & 5.841 & 4.255 & 0.7093 \\
4 & 0.3850 & 0.2856 & 0.04811 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{m=0} & \text{n=0} & 1 & 2 \\
\hline
65.49 & 25.55 & 1.934 \\
1 & -121.5 & 45.73 & -1.952 \\
2 & 58.58 & -30.59 & 3.304 \\
3 & -9.325 & 5.597 & -0.6735 \\
4 & 0.5119 & -0.3273 & 0.04138 \\
\end{array}
\]