Instantonic string solitons on M5 branes

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Abstract

The (I)instantonic strings are described as extended states occupying a flat (isometry) direction on the multiple M5-brane world-volume. These are constituted by 4-dimensional gauge instantons and 2-dimensional chiral axion. In this light we revisit the covariant action for M5-branes by adding explicit axion terms. This leads to a new gauge symmetry and simple modification of some field equations and the constraints in the theory. The equations of motion include a conserved topological current of I-strings. When 6D theory is compactified on a circle the light-like I-strings manifest as heavy monopole states in 5D super-Yang-Mills. While extremely light I-strings tend to become degenerate with the M5 vacuum but only at strong SYM coupling. This indicates that 6D vacuum is infinitely degenerate.
1 Introduction

The holographic constructions of M2-brane theories like BLG and ABJM [1]-[8], provided us with some much needed insight to construct similar 6-dimensional theory for multiple M5-branes. Some breakthrough ideas in constructing M5 theories with maximal supersymmetry were proposed in the works [10, 11, 12, 13], also see [9]. Subsequently these led to 6-dimensional generalizations of theories with lesser supersymmetry [14]-[27]. The self-dual antisymmetric tensor fields are natural to occur in six spacetime dimensions and the dynamics of a single M5-brane can be described by an abelian tensor theory with (2,0) supersymmetry. There are similar important dynamical reasons to include tensor fields in the 6D constructions. Let us consider the example of an extended M2-brane ending on M5-brane. The intersection of M2 and M5 will produce an infinitely long line defect (string) on six-dimensional world-volume of M5-brane. These strings perhaps constitute simplest excitations which could entirely live on the M5-brane and carry charges of tensor fields. Basically, these line defects behave like extended charged strings in 6D Minkowski spacetime but these strings cannot carry momentum along their direction of extension. It is believed that ultimately the dynamics of these charged strings does constitute the low energy dynamics on the M5-brane. Similarly when we consider multi-brane configurations where \( N \) coincident M5-branes having a single M2-brane ending on them. In that situation the M2-brane will produce a line defect on each M5-brane in the stack. Thus we have a low energy configuration which has to be described by \( N \) parallel (aligned) strings in six dimensions. Of course, such extended string configurations will spontaneously break the Lorentz symmetry from \( SO(1,5) \rightarrow SO(1,4) \). Thus we may conclude that low energy string excitations on M5-branes leads to manifestly broken Lorentzian symmetry. Indeed this has been an in-built criterion in some of the models [10, 13]. Such an hypothesis allows for the inclusion of an auxiliary vector field \( \zeta_M \) in the theory. With \( \zeta_M \) and an associated axion field, we are able to uplift the entire dynamics of the SYM to six dimensions in a covariant manner.

The puzzle:
The extended strings naturally couple to antisymmetric tensor field, whose field strength \( H_{(3)} \) needs to be self-dual, for it to describe an excitation on the M5-brane [29]. The self-dual tensor field along with five scalars, \( X^I \)'s, and one Majorana-Weyl spinor \( \Psi \), are part of the (2,0) tensor supermultiplet in six dimensions [28]. The dynamical equations of this chiral theory are

\[
H_{(3)} \equiv dB_{(2)} = \ast_6 H_{(3)}, \quad \partial_M \partial^M X^I = 0 = \bar{\partial} \Psi
\]

where \( \ast_d \) is a Hodge-dual operation in \( d \)-dimensional spacetime. However, this abelian (2,0) CFT is a free theory. While a non-abelian version of the tensor theory is not satisfactorily known at present. But there is a belief that all states of a non-abelian (2,0) CFT, once compactified on a circle, are perhaps contained in 5D super-Yang-Mills (SYM) theory and vice-versa. The SYM in five dimensions is nonrenormalizable and has
a strongly coupled fixed point in the UV region. A conventional nonrenormalizable theory would require many new degrees of freedom to be added to it at shorter and shorter scales. Thus if the tree level SYM fields indeed describe all the states of a ‘compactified’ 6D CFT, without requiring new degrees of freedom, then the SYM ought to be considered a finite theory in itself [11][12]. A test of this proposal requires that all Kaluza-Klein modes, if any, of 6D fields are mapped into various (instantonic) monopole sectors in the SYM. Although qualitative, but it has remained a difficult prospect to directly verify the finiteness of the 5D SYM. However, one direct puzzle to note here is that in abelian CFT there would be no instantons, while KK (Fourier) modes of the self-dual tensor field, as in (11), are all likely to survive. So this identification map of ‘6D KK-modes’ Vs ‘5D monopoles’ seems to be problematic and puzzling! Any deviation from the expected/conjectured behaviour of 5D SYM is of course a welcome sign as it will have consequences for 6D CFT. See for recent important attempts along this direction [21][10][12][13].

**Instantonic strings:**

In this work we propose a simple modification of previous 6D action presented in [13][22]. The proposed action can describe new instantonic states in the theory which are string like solitonic configurations carrying a topological charge

\[
Q_{\text{-string}} = \int d^5 x J_0^0 = \frac{1}{2} \text{Tr} \int \zeta_1 \wedge F_2 \wedge F_2
\]  

The instantonic charges appear because of the gauge instantons which live over 4D Euclidean subspace and the finite vev of auxiliary vector field \(\zeta_M\). Thus, in general, the charge of I-string is fixed by the instanton number and will be measured in units of \(|\zeta|\) vev. Other consequences are that the \(S^1\) reduction produces no KK-modes in 5D, because all six-dimensional fields have no dynamics along isometry (circle) direction owing to the constraints in the theory. Instead these I-string states directly manifest as monopole solitons in 5D super-Yang-Mills theory.

Further, a true vacuum state in field theory is the one with 6-momentum \(p_M = 0\). There will also be solutions which would occur in the theory that will have broken Poincare’ symmetry. For the I-strings, made up of instantons, we shall have

\[
p_M = (|p|, 0, 0, 0, 0, p)
\]  

Especially the momentum entries directly deal with the instanton number, these are nonvanishing and take discrete values. As an example, the lightlike auxiliary vector

\[
< \zeta^M > = |\zeta|(1, 0, 0, 0, 0, 1)
\]  

does allow an I-string instantonic state with

\[
p_M = (n|L|\zeta^2, 0, 0, 0, nL|\zeta|^2)
\]  

which corresponds to a lightlike state with discrete energy and a fifth component of the momentum. Here \(L\) is regulated length (size of \(x_5\) coordinate) of the string. Especially,
after a circle compactification of the theory the momenta can be expressed as

$$p^M = \left( \frac{n}{g_{YM}^2}, 0, 0, 0, 0, \frac{n}{g_{YM}^2} \right)$$  

(6)

These I-string solitons are indeed identified with the monopoles of 5D SYM. We note that I-strings are different from other electrically (color) charged self-dual string solitons [29], which are part of the spectrum but only in instanton-less (or \(n = 0\)) sector of the theory. Our main aim here is to actually show that a simple modification of a previous covariant 6D action [13] can meaningfully represent these I-strings.

The paper is organised as follows. In section-2 we introduce a new form of M5 action in six dimensions with the help of auxiliary axion field. We define a new conserved (instantonic) current. In section-3, we discuss 5D super Yang-Mills and its relationship with the 6D theory compactified on a circle. The instantonic string solitons are presented in section-4. It is found that while solving the equations that the I-string solitons are essentially light-like states. These states must condense to the ground state in the decompactification limit 6D theory. We discuss (colored) e-strings as well as 4D electric-magnetic solutons in the section-5. A brief summary is given in section-6.

2 Six-dimensional tensor theories

2.1 The 6D chiral theory

At the outset very little is known about the full non-Abelian (2,0) tensor theory which would describe the dynamics of M5-brane stack completely. Nonetheless meaningful attempts have been made recently to write down 6D theories using self-dual tensors, at least at the equation of motion level [10], and separately by up-lifting 5D SYM theory to the six-dimensions with the help of an auxiliary vector field [13, 22]. It is desirable that non-Abelian 6D CFT should possess \(U(N)\) gauge symmetry and an \(SO(5)\) R-symmetry. The 6D gauge action constructed in [13] inherits these basic features directly from 5D because it is an uplift of the SYM. The action had explicit scale invariance and is devoid of dimensionful parameters. These requirements at handy because ultimately they will guide us in the construction of M5-brane action.

It is now known how to uplift 5D SYM to six dimensions such that the resulting 6D action has no dimensionful parameters and the 6D theory exhibits scaling symmetry. \(^1\) This has been explicitly shown at the equation of motion level for (2,0) supersymmetric model with chiral fields [10]. A variant of 6D (2,0) theory with covariant action was subsequently constructed in [13]. The 6-dimensional gauge action (excluding fermions)

\(^1\) See more developments on M5 theory in the references [30, 31, 32, 33, 34, 35, 36].

\(^2\) The latter property will make it different from \(\mathcal{N} = (1,1)\) 6D YM theory, which instead describes D5-branes. The D5-brane SYM theory has a dimensionful running coupling constant.
can be written more symmetrically as

\[ S_{6D} = \int d^6x \left[ \text{Tr} \left\{ -\frac{1}{23!} (\zeta_{[MNP]} F_{NP})^2 - \frac{1}{22!} (\zeta_{[N} D_M X^I)^2 + \frac{1}{4} (\zeta_M)^2 ([X^I, X^J])^2 \right\} 
+ \frac{1}{4!} \epsilon^{MNPQRS} \zeta_M \partial_N C_{PQRS} + \frac{1}{4} \text{Tr} \epsilon^{MNPQRS} \zeta_M \partial_N \theta F_{PQ} F_{RS} \right] \]  

where

\[ \zeta_{[MNP]} = \zeta_{MNP} + \text{cyclic permutations}, \]

and

\[ \zeta_M \equiv \eta_M + \partial_M \theta, \]

The action has been presented in such a way that the auxiliary field \( \eta_M \), with mass dimension one, couples to all other fields in the action. Hence \( \zeta_M \) (or \( \eta_M \)) sometimes is levelled as a ‘dressing’ field. There is an Stueckelberg gauge invariance

\[ \delta_s \eta_M = c \partial_M \lambda(x), \quad \delta_s \theta = -c \lambda(x) \]

\[ \delta_s C_{MNP} = c \lambda(x) \text{Tr}(F_{[MN} F_{P]}) . \]

where \( c \) is arbitrary constant gauge parameter. Hence by appropriately choosing the gauge, \( \theta(x) \) can always remain hidden in the whole action except in the last term. In actuality, the action (7) differs from the action in previous work [13] only by the inclusion of this last term. They can only be related by a nontrivial field redefinition

\[ C'_4(4) \equiv C(4) + \theta(x) \text{Tr}(F_{(2)} \wedge F_{(2)}) . \]

The \( \theta \) is an axion and it is dimensionless. The \( \theta \) term will be helpful in describing the topological (instantonic) sector of the 6D theory. For example, as we shall discuss, there exist new extended instanton configurations where the topological charge is uniformly distributed along the length of an straight string. These instantonic strings (I-string) are analogous to well known electrically charged string (e-string) solitons in M5 brane theory. We also need to make sure that action (7) remains finite for the I-strings. We shall find that the presence of axion field in the action subtly modifies the dynamics by changing the \( \eta_M \) field equation.

In the above, the 6D Lorentzian indices are levelled as \( M, N = (0, 1, \cdots, 5) \), and the internal \( SO(5) \) indices are levelled as \( I, J = (6, 7, 8, 9, 10) \). While \( \mu, \nu = (0, 1, \cdots, 4) \) would represent 5D Minkowski indices. The Yang-Mills field strength is defined as

\[ F_{MN} = \partial_{[M} A_{N]} - i [A_M, A_N] \]

The five scalar fields \( X^I \)’s are in the adjoint representation and their covariant derivative is defined as

\[ D_M X^I = \partial_M X^I - i [A_M, X^I]. \]
These are the only bosonic fields which are dynamical in 6D theory (7), the rest are all auxiliary fields and give rise to constraints. There is manifest $U(N)$ gauge symmetry in the action (7) corresponding to the fact that it describes $N$ parallel M5 branes. The gauge transformations are

$$A_M \rightarrow A'_M = U^{-1} A_M U - i U^{-1} \partial_M U$$

$$X^I \rightarrow X'^I = U^{-1} X^I U,$$

under which the action (7) remains invariant. Here $U$ is an element of $U(N)$ gauge group.

Let us simplify our notations and write the 2-form gauge field strength as

$$F^{(2)} \equiv DA^{(1)} = dA^{(1)} - i [A^{(1)}, A^{(1)}]$$

$$DX^I = dX^I - i [A^{(1)}, X^I]$$

where $D$ is used for covariant derivative of adjoint fields. The Bianchi identity is

$$DF^{(2)} = 0$$

The action (7) may be expressed as

$$S_{6D} = \int \left[ \text{Tr} \left\{ -\frac{1}{2} (\zeta^{(1)} \wedge F^{(2)}) \star (\zeta^{(1)} \wedge F^{(2)}) - \frac{1}{2} (\zeta^{(1)} \wedge DX^I) \star (\zeta^{(1)} \wedge DX^I) \right. \\
+ \frac{1}{4} (\zeta^{(1)} \wedge \star \zeta^{(1)}) ([X^I, X^J]^2) + \zeta^{(1)} \wedge dC^{(4)} + \zeta^{(1)} \wedge d\theta \wedge \text{Tr}(F^{(2)} \wedge F^{(2)}) \right]$$

Below we write down the equations of motion and the constraints which follow from the action (17). Note, following from the observations in [22], we can always introduce non-Abelian tensors

$$H^{a}_{(3)} \equiv \zeta^{(1)} \wedge F^{a}_{(2)} + \star \zeta^{(1)} \wedge F^{a}_{(2)}$$

in the equations of motion. These are self-dual by construction, $\star H^{(3)} = H^{(3)}$. The index $a$ is the adjoint color index. By the above definition $H$ is not an independent tensor field, but is related to the Yang-Mills fields. The equations of motion can be presented as

$$d\zeta^{(1)} = 0$$

$$DH^{a}_{(3)} - \star (\zeta^{(1)} \wedge DX^{Ic}) X^{Ib} f_{abc} = 0$$

$$D \star (\zeta^{(1)} \wedge DX^I) + (\star \zeta^{(1)}) [X^J, [X^I, X^J]] = 0$$

The equation of motion for $\eta_M$ leads to the following constraint

$$\text{Tr} \left( -F^{(2)} \wedge \star (\zeta^{(1)} \wedge F^{(2)}) + d\theta \wedge F^{(2)} \wedge F^{(2)} \right) + \text{Tr} DX^I \wedge \star (\zeta^{(1)} \wedge DX^I) + \star \zeta^{(1)} V_X = dC^{(4)}$$
where we defined $V_X = \frac{1}{2} \text{Tr}([X^I, X^J])^2$. The other constraints, some of which are automatic consequence of the above equations, are

$$
\zeta^M D_M F^n_{PQ} = 0, \quad \zeta^M F^n_{MQ} = 0, \quad \zeta^M D_M X^I = 0, \quad \zeta^M D_M H^n_{NPQ} = 0 \quad (21)
$$

Note that in the above eq. (20) is the only equation which involves $\theta$ field explicitly. The rest of the equations in (19) are dependent on $\theta$ only through $\zeta^M$. So the $\theta$ dependence in them can always be gauged away. (We may wish to interpret $\tilde{F}^{(5)} = dC^{(4)}$ as some Hodge-dual 5-form field strength.) However, for fully localized e-string and I-string solitons of this theory we would be able to set $C^{(4)} = 0$. Thus it is advantageous to have $\theta$ dependent term in the M5 action. The constraint (20) suggests that the local gauge field dynamics remains largely unaffected by the inclusion of $\theta$ field in the action. While the $\theta$ equation of motion imposes an important conservation law

$$
d \ast J_{(1)}^{\text{string}} = 0 \quad (22)
$$

where the instantonic current

$$
J_{(1)}^{\text{string}} = \ast \frac{1}{2} \text{Tr}(\zeta(1) \wedge F(2) \wedge F(2)) \quad (23)
$$

The current would always be nontrivial in presence of Euclidean YM instantons. Note, the current is trivially conserved due to the identity $d\zeta(1) = 0$ and the gauge Bianchi $DF(2) = 0$.

### 3 5D super Yang-Mills theories

*Compactification on circle with space-like $\zeta$*: it is evident from the action (7) that $\zeta^M$ is an auxiliary field and its equation of motion always requires it to take a constant value on-shell. Thus $\zeta^M$ can inevitably pick a particular spatial direction in the vacuum. As a result the $SO(1, 5)$ symmetry of the action gets spontaneously broken to the $SO(1, 4)$ subgroup in the vacuum. Hence, in a given vacuum the symmetries of the 6D fields will be exactly the same as that of 5D SYM. In any case, the circle compactification involves the vev $\zeta^M = |\zeta| \delta^M_5$, the radius of circle $R_5$, on which 6D theory is compactified. We introduce the 5D fields ($\tilde{X}^I, \tilde{A}$) (written with tilde so as to distinguish them from 6D fields) and relate them to their 6D counterparts as below

$$
\tilde{X}^I(x) = X^I(x^\mu) \quad (24)
$$

while gauge fields are related as

$$
\tilde{A}_\mu(x) = A_\mu(x^\mu), \quad A_5(x) = 0. \quad (25)
$$
The 5D fields have no dynamics along $x^5$ (the isometry direction) and depend only on $x^\mu$’s. The axionic term and the $C_4$ term in the action (7) become total derivative and will drop out. The action (7) then reduces to the following 5D SYM action

$$S_{SYM} = 2\pi R_5 |\zeta|^2 \int d^5 x \text{Tr} \left[ -\frac{1}{4} (\tilde{F}_{\mu\nu})^2 - \frac{1}{2} (D_\mu \tilde{X}^I)^2 + \frac{1}{4} ([\tilde{X}^I, \tilde{X}^J])^2 \right]$$

(26)

Hence the 5D YM coupling constant can be defined as [13]

$$2\pi R_5 |\zeta|^2 \equiv \frac{1}{g_{YM}^2}. \tag{27}$$

Thus the SYM action one gets is

$$S_{SYM} = \frac{1}{g_{YM}^2} \int d^5 x \text{Tr} \left[ -\frac{1}{4} (F_{\mu\nu})^2 - \frac{1}{2} (D_\mu X^I)^2 + \frac{1}{4} ([X^I, X^J])^2 \right]$$

(28)

where tilde sign over 5D fields has been dropped.

Note that $|\zeta|$ does not explicitly appear in 5D action and thus could remain arbitrary. Let us add a few clarifications here. The $|\zeta|$ has dimensions of mass. Upon compactification another scale available in the theory is the radius of compactification $R_5$. So we can at best claim that

$$|\zeta|^2 = \frac{1}{2\pi R_5 g_{YM}^2}. \tag{29}$$

However, using the fact that 5D SYM theory has only one dimensionful parameter in the form of coupling constant $g_{YM}$, and the expected relation [11]

$$2\pi R_5 \simeq g_{YM}^2, \tag{30}$$

we find it appropriate to fix

$$|\zeta| \simeq \frac{1}{g_{YM}^2}. \tag{31}$$

Also we shall see in this paper that this remains a consistent choice.

Before we close this section, let us also mention that a time-like reduction of the action (7) can also be performed, where we will take $\zeta^M = (\frac{1}{g_{YM}^2}, 0, 0, 0, 0, 0)$, a time-like vector. This will lead to 5-dimensional Euclidean theory

$$S^E = \beta \frac{1}{g_{YM}^2} \int d^5 x \text{Tr} \left[ \frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} (D_\mu X^I)^2 - \frac{1}{4} ([X^I, X^J])^2 \right]$$

(32)

where $\beta \sim O(g_{YM}^2)$ is a free parameter with a dimension of length. Note that the signs of all the terms have become opposite of the Lorentzian SYM case (28).
4 Instantonic string solitons

We now show that there exist instantonic solutions in the 6D theory with a completely new interpretation! These are extended string like solitons which live on M5-brane stack and will carry instantonic charges. Consider an ordinary (e)lectrically charged string soliton on M5-brane, being stretched, say, along $x^5$ direction. These infinitely extended e-strings do not have momentum along the direction of extension (isometry). Thus $p_5 = 0$ typically for all ordinary e-string states. But for the I-string solutions we present here

$$p_5 \propto \text{instanton number} \quad (33)$$

and it will be nonvanishing and also quantized.

In the previous version of the theory [13], the instantonic solutions could be found, but that required the presence of non-zero flux for $C$-field. In the present avatar of the theory, with the inclusion of a $\theta$-term in the action (17), the $C$ flux can be set to zero. In the purely instantonic sector with vanishing $C$, and taking all $X^I$’s constant diagonal matrices, the constraint equation (20) simply reduces to the condition

$$\star \zeta^{(1)} \wedge F^{(2)} - d\theta \wedge F^{(2)} = 0 \quad (34)$$

Now we consider the gauge instanton configuration $\star_4 F^{(2)} = F^{(2)}$ localised in 4D Euclidean subspace $E_4$ of the 6-dimensional spacetime $M_2 \times E_4$, whereas both the $\zeta_M$ and the $\theta$ are confined to live in 2D Minkowski space, $M_2$. Then the condition (34) will be satisfied if

$$\star_2 \zeta^{(1)} = d\theta . \quad (35)$$

That can be solved provided

$$\eta_M = 0, \ \zeta_M = \partial_M \theta , \quad (36)$$

in which case $\theta$ will have to be

$$\theta(x^+, x^-) = g x^+ . \quad (37)$$

Hence $\zeta^{(1)}$ is a chiral (self-dual) 1-form in 2D flat spacetime, i.e. $\zeta_M = |\zeta| \delta_{M+} = g \delta_{M+}$, $(\zeta_M)^2 = 0$.

The above instantonic soliton describes a nontrivial string like configuration. The $\theta$ equation of motion simply implies a conserved ‘instantonic’ current

$$J_{i \text{-string}} = \star \frac{1}{2} \text{Tr}(\zeta^{(1)} \wedge F^{(2)} \wedge F^{(2)}) \quad (38)$$

This current will be nontrivial whenever gauge instantons are present. It will also be light like, so

$$J_{i \text{-string}}^M = j(x)(1, 0, 0, 0, 0, 1). \quad (39)$$

The instanton charge carried by the I-string becomes

$$Q_{i \text{-string}} = \int d^5 x J_{i \text{-string}}^0 = \frac{1}{2} \text{Tr} \int_{\Sigma} \zeta^{(1)} \wedge F^{(2)} \wedge F^{(2)} . \quad (40)$$
It is a dimensionless quantity in 6D. The distribution of the instanton charge however depends on $\zeta_M$ vev in a given vacuum. Following from (35) and (37), let us take the vev $< \zeta_M > = |\zeta|(1, 0, 0, 0, 0, 1)$. In this vacuum the instantonic string is localized in $x_1, \ldots, x_4$ (4-dimensional Euclidean space $E_4$), and the instanton charge gets uniformly spread out along remaining flat direction $x^5$. This may lead to an infinite result as the charge is being carried by an extended string like object. However we can define the charge per unit length of I-string (taking length $L \equiv \int dx^5$ as a regulator)

$$\rho_{i-string} = \frac{Q_{i-string}}{L} = \frac{|\zeta|}{4} \text{Tr} \int d^4 x F_{mn} F^{mn} = n|\zeta|$$

which is certainly quantized and finite. The integer $n$ is the count of the instanton number. Thus from eq.(41) we get an independent interpretation for the $\zeta$ vev. That the parameter $|\zeta|$ is the measure of I-string charge density in a given M5 vacua. While $L$ is simply the regulator of the I-string length. The quantities, $L$ and $\zeta$, are so far quite independent. But when we consider circle compactification, since $x_5 \sim x_5 + 2\pi R_5$, we may identify compactification radius

$$2\pi R_5 \simeq \frac{1}{|\zeta|} \simeq g_{YM}^2,$$

as argued previously, because 5D SYM theory has only one dimensionful parameter, namely $g_{YM}$). Thus we conclude that the 6D I-string solitons are made up of the gauge instantons, and the $|\zeta|$ provides an independent scale to measure their charge densities. However abelian theory for a single M5-brane cannot apriori admit I-string vacua.

### 4.1 The energy-momentum tensor

The energy-momentum tensor can be found from the new action (7). It is given by

$$T_{MN} = \text{Tr} \left[ \frac{1}{2.2!} (G_{MPQ} G_{N}^{PQ} - \frac{\eta_{MN}}{6} G_{PQR} G^{PQR}) + \frac{1}{2} (C_{MP}^{I} C_{N}^{P} - \frac{\eta_{MN}}{4} C_{PQ}^{I} C^{I.PQ}) - (\text{potential terms}) \right]$$

where we have denoted $G_{MPQ} \equiv \zeta_{[MPQ]}$ and $C_{MP}^{I} \equiv \zeta_{[M} D_{P]} X^{I}$ for simplificity. The details of the potential terms are not important as the contribution from these terms will be vanishing for the I-strings. For the I-string vacua we get the energy

$$E = \int d^4 x [dx^5] T_{00} = \frac{L}{4} \int d^4 x \zeta_0 \zeta_0 \text{Tr} F_{mn} F^{mn} = nL|\zeta|^2.$$  

Thus the energy per unit length of the I-string states is

$$\frac{E}{L} = n|\zeta|^2 \geq 0$$
Similarly, the 5-th component
\[
\mathbf{p}_5 = \int d^4x [dx^5] T_{05} = \frac{L}{4} \int d^4x \zeta_0 \zeta_5 \text{Tr} F_{mn} F^{mn} = nL|\zeta|^2. \tag{46}
\]
(For the non-instantonic solutions, however, \(\mathbf{p}_5 = 0\).) Other components such as \(T_{0m}\), for \((m = 1, 2, 3, 4)\) are all vanishing except
\[
T_{55} = \frac{1}{4}|\zeta|^2 \text{Tr} F_{mn} F^{mn}. \tag{47}
\]
For the I-strings we can calculate
\[
M^2 = -p_M p^M = E^2 - p_5^2 = 0. \tag{48}
\]
It implies that the I-strings behave like exactly massless solitons, with a lightlike \(\zeta_M\) (non-zero components \(\zeta_0 = \zeta_5 = |\zeta|\)), and these states can be described by the momentum vector
\[
P^M = (nL|\zeta|^2, 0, 0, 0, nL|\zeta|^2) \tag{49}
\]
Thus the I-strings are exactly lightlike states in 6D with discrete momenta.

As described above, the I-string solutions are specifically obtained when we take \(\zeta_M\) having components along the lightcone, \(\zeta_M = |\zeta| \delta_{M+}\), the axion field \(\theta = |\zeta| x^+\), while \(F\) is taken to be the Yang-Mills self-dual 2-form living over the patch \((x^1, x^2, x^3, x^4)\). Accordingly the self-dual 3-rank tensor \(H\) can be constructed
\[
H_{(3)} = |\zeta| dx^+ \wedge (F_{(2)} + \ast_4 F_{(2)}), \tag{50}
\]
it satisfies \(dH_{(3)} = 0 = d \ast H_{(3)}\), also see previous definition in \[22\].

4.2 Light-like \(\zeta^M\) Vs strong SYM coupling
We consider \(\zeta^M = |\zeta|(1, 0, 0, 0, 0, 1)\) being a constant vector field. With \(X^I\)'s being constant diagonal matrices, the potential for adjoint scalars also vanishes. In such cases the 6D bosonic action will mainly contain the Yang-Mills fields
\[
S_{\text{instanton}} \simeq \int d^6x \text{Tr} \left[ -\frac{1}{12}(\zeta_{[MN} F_{NP]} )^2 + \frac{1}{4} \epsilon^{MNPQRS} \zeta_M \partial_N \theta F_{PQ} F_{RS} \right] \tag{51}
\]
We can see that for the I-string solutions, supported by lightlike \(\zeta\) and \(\theta\), this action vanishes on-shell. This tells us that the action supports lightlike configurations and is meaningful. Next let us take \(x^5\) to be compact, \(x^5 \sim x^5 + 2\pi R_5\), we also set \(L \sim 2\pi R_5\), and identify
\[
L|\zeta|^2 \simeq \frac{1}{g_{YM}^2}. \tag{52}
\]
Then the limit $|\zeta| \to 0$ corresponds to 5D coupling constant $g_{YM} \to \infty$. Thus in this limit the 6D theory on a circle can represent a truly strongly coupled phase of 5D SYM. But we need to make sure that this limit keeps the I-string charges finite;

$$Q_{i-string} = Z_{monopole} L |\zeta| = \text{Finite}$$

whereas the monopole (soliton) charge in 5D is given by

$$Z_{monopole} = \frac{1}{2} \text{Tr} \int F \wedge F = n.$$  (54)

Although the instantons are strings from 6D perspective but from 5D point of view they are the monopoles with fixed topological charge $Z$. The monopole mass in 5D is given by

$$M_{monopole} = \frac{n}{g_{YM}^2}.$$  (55)

Note for the 6D I-strings, as we found $M_{i-string}^2 = 0$, instead we have

$$p^M = \left( \frac{n}{g_{YM}^2}, 0, 0, 0, 0, \frac{n}{g_{YM}^2} \right).$$  (56)

Thus the massive monopoles are lifted to the I-strings which form a discrete spectrum of lightlike states in 6D. Hence we can describe strong coupling limit, $g_{YM} \to \infty$ and $M_{monopole} \to 0$, as a limit under which all discrete I-string states with distinct topological charges becoming degenerate and condensing to the ground state in 6D theory. In order to keep $Q_{i-string}$ finite, following large box limit has to be employed

$$L = 2 \pi R_5 \to \infty, \quad |\zeta| = \frac{1}{g_{YM}^2} \to 0, \quad L |\zeta| = \text{fixed}.$$  (57)

Thus, strong coupling limit corresponds to infinitely long I-strings condensing to the ground state in a decompactified M5 theory (7). Although being degenerate in the ground state, each I-string vacua carries distinct topological charge $Q_{i-string}$. The charge density $\rho_{i-string} \to 0$, as the topological charge gets spread uniformly along infinitely long (tensionless) I-string.

**5 e-strings and self-dual $H$**

Let us note that all 6D solutions in our theory will have at least one isometry direction due to the nontrivial constant vev of the dressing field $\zeta_M$ [13]. It is also evident from the construction of the 6D action that there will be no point-like localized solutions. The supersymmetric e-string vacua in 6D Abelian chiral theory have been described in [28]. We study these solitonic configurations describing the intersection of an extended M2-brane ending on M5-brane. Consider the $\zeta$ vacuum where $\zeta^M = (0, 0, 0, 0, 0, |\zeta|)$, aligned along $x^5$, which we take to be the isometry direction. Note $|\zeta|$ has dimensions of mass and
it is the only dimensionful parameter available in the vacuum. This solution is described by [13]

\[
X^I(x) = \delta^{10\Phi}(x_m), \quad (I = 6, 7, 8, 9, 10)
\]

\[
F_{0m} = \pm \partial_m \Phi. \quad (58)
\]

It is a solution of equations (19) and (20) provided

\[
\Phi(x_m) = \Phi_0 \pm |\zeta| |x - x_o|^2
\]

(59)

The fields are dependent only on \(x_m (m = 1, 2, 3, 4)\) coordinates, and \(\Phi_0\) is an arbitrary dimensionful constant signifying the asymptotic value of the field. This solution can be easily generalized to multi-centered solutions, where \(\vec{x}_{o(i)}\), \(q_{(i)}\), index \((i = 1, 2, \cdots, r)\), will parametrize respective positions and charge of \(i\)-th soliton. Since only one scalar, namely \(X^{10}\) (representing the coordinate \(x^{10}\) transverse to M5), has been excited, it describes an M2-brane, in the \(x^5-x^{10}\) plane, ending on M5-brane. The string is along the common intersection direction of the branes, which is \(x^5\). The electric field surrounding the string defect is peaked at its location \(\vec{x} = \vec{x}_o\) and depends upon the vev \(|\zeta|\). Thus for e-strings solution to exist we need to have nontrivial \(|\zeta|\). We also determine the nonvanishing components of the \(H\) tensor in the equations (18)

\[
H_{0m5} = \zeta^5 F_{0m} = \pm |\zeta| \partial_m \Phi, \quad H_{mnp} = \epsilon_{mnp05} \zeta^5 F_{0l} = \pm |\zeta| \epsilon_{mnp0l} \partial_l \Phi
\]

(60)

where we took \(\epsilon^{012345} = -\epsilon_{012345} = 1, \) and \(\epsilon_{1234} = 1\) is the Levi-Civita tensor in four Euclidean dimensions. It shows that the \(H\) is self-dual. Remarkably, no background value for \(C\) tensor is required for the e-string solutions, this is because the constraint (20) reduces to

\[
F_2(2) \wedge \star(\zeta_1 \wedge F_2) - DX^{10} \wedge \star(\zeta_1 \wedge DX^{10}) = 0
\]

(61)

which is automatically satisfied since \(F_{0m} = \pm \nabla_m X^{10} = \pm \partial_m \Phi\).

The charge carried by e-strings is given by

\[
Q_{\text{e-string}} = \frac{1}{4\pi^2} \int \star H_{(3)} = \frac{1}{4\pi^2} \int_{S^3} H_{(3)} = q.
\]

(62)

We must also note that

\[
Q_{\text{electric}} = Q_{\text{magnetic}}
\]

(63)

because these are self-dual strings. By taking the regulated length of the e-string as \(L\), we define e-string charge density

\[
\rho_{\text{e-string}} = \frac{Q_{\text{e-string}}}{L} = q|\zeta|
\]

(64)

where \(q\) is a measure of the net charge carried by self-dual string.
Compactification and the vanishing $|\zeta|$ limit: in order to connect e-strings with the 5D SYM states, we need to compactify them on a circle, with the given prescription
\[ L = 2\pi R_5 \simeq g_{YM}^2, \quad |\zeta| = g_{YM}^{-2} \] (65)
as discussed earlier. In the strong coupling regime the e-string charge density will become vanishingly small, but the net charge $Q_{e-string}$ stays finite. Remarkably the $H$ field exists even in the strong coupling limit (or vanishing $|\zeta|$ limit)
\[ \lim_{|\zeta| \to 0} H_{0r5} = \frac{2q}{r^3}. \] (66)
The energy density of the e-string soliton is
\[ p_0 \simeq L|\zeta|^2 \int d^4x (\vec{E})^2 = L \int d^4x \left[ \frac{2q^2}{r^6} \right] \sim g_{YM}^2 \int d^4x \mathcal{E} \] (67)
where $\mathcal{E} = \frac{2q^2}{r^6}$ is the measure of corresponding energy flux in 5D SYM. The energy of the elementary e-string states grows linearly with $L$ (or $g_{YM}^2$). Hence the e-strings tend to become heavy as YM coupling becomes large. Note these were the lightest states in the perturbative regime in SYM, while monopoles were heavy, as the monopole mass is $\frac{n}{g_{YM}}$.

5.1 Dyonic solutions: $F_{0i} \neq 0$ and $F_{ij} \neq 0$

The dyons have both electric and magnetic $F_{MN}$ components and generally occur as localized solutions in 4D SYM. For embedding these in 6D, we need to consider $x^4$ and $x^5$ both to be the isometry directions. The constant $\zeta$ configuration is
\[ \zeta^M = (0, 0, 0, 0, 0, |\zeta|), \quad C^{(4)} = 0, \]
\[ X^I = \delta^{10} f(x), \quad F_{0i} = \pm \partial_i f_e, \quad (i, j = 1, 2, 3)(I = 6, 7, 8, 9, 10) \] (68)
This electrically charged $(q_e, 0)$ solution is localized over three Cartesian coordinates ($x^1, x^2, x^3$) only, and is a solution of equations (19) and constraint (20) with
\[ f(x) = f_e(x) = f_0 + \frac{q_e}{|x - x_0|}, \] (69)
where $f_0$ is a dimensionful constant. At this point this purely is a result of the constraint (20) present in the theory. Usually this follows from the supersymmetry.

The magnetically charged solutions with $(0, q_m)$ are not covered by the above ansätze. Furthermore, the action of electric-magnetic $SL(2,Z)$ transformations on a $(q_e, q_m)$ dyon will certainly lead to a jump in the value of $q$’s. \footnote{The SL(2,Z)-duality transformation of the dyon charges:}
\[
\begin{pmatrix}
q_e \\
q_m
\end{pmatrix}
\rightarrow
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
\begin{pmatrix}
q_e \\
q_m
\end{pmatrix}
\]
where integers obey the condition $ad - bc = 1$. 4

\[4\] The SL(2,Z)-duality transformation of the dyon charges:
charged solution we need to switch on \( C_{(4)} \) background. A magnetic \((0, q_m)\) solution is found with following ansatz

\[
\zeta^M = (0, 0, 0, |\zeta|, 0), \quad \eta_M = 0, \quad C_{05\theta\phi} = \frac{2|\zeta q_m^2}{r},
\]

\[
X^I = \delta^{I0} f(x), \quad F_{0i} = 0, \quad F_{ij} = \epsilon_{ijk} \partial_k f, \quad (i, j = 1, 2, 3)
\]  

(70)

where function \( f(x) = f_0 + \frac{q_m}{r} \). The \((r, \theta, \phi)\), with \( r^2 = (x^i - x_0^i)^2 \), are the spherical coordinates around the center \( \vec{x}_0 \).

Thus we may understand that strong-weak duality of 4D SYM acts in nontrivial way on the 6D fields, including auxiliary fields. Let us make it more explicit. We separate 6D coordinates as \( x^M = (x^\mu, x^m) \), with greek indices \( \mu, \nu = 0, 1, 2, 3 \) and small roman indices \( m, n = 4, 5 \). We choose a constant 6D vector \( \zeta_M = (0, 0, 0, 0, \zeta_4, \zeta_5) \). We shall assume that no background field depends upon \((x^4, x^5)\) except \( \theta \) (taking \( \eta_M = 0 \)). One can introduce dual pairs: \( \xi_m = (\zeta_4, \zeta_5) \) and dual \( \tilde{\xi}_m = (\zeta_5, -\zeta_4) \). Both are 2-vectors in the \( x^4 - x^5 \) plane.

Similarly given \( C_{4\mu\nu\lambda} \) and \( C_{5\mu\nu\lambda} \), being only nonvanishing components, we can define \( C_{m\mu\nu\lambda} = (C_{4\mu\nu\lambda}, C_{5\mu\nu\lambda}) \) and corresponding dual components \( \tilde{C}_{m\mu\nu\lambda} = (C_{5\mu\nu\lambda}, -C_{4\mu\nu\lambda}) \). From these we construct following \( SL(2, Z) \) doublets:

\[
\xi^a_m = (\tilde{\xi}_m, -\xi_m), \quad (a = 1, 2)
\]

\[
F^a_{\mu\nu} = (F_{\mu\nu}, \tilde{F}_{\mu\nu}).
\]  

(71)

We can now rewrite \( H \)-tensor (nonvanishing components only) as

\[
H_{m\mu\nu} = \xi^a_m F^b_{\mu\nu} \omega_{ab} = \left( \begin{array}{cc} \tilde{\xi}_m & -\xi_m \end{array} \right) \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \left( \begin{array}{c} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{array} \right)
\]  

(72)

which is an \( SL(2, Z) \) invariant expression, where

\[
\omega_{ab} = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right)
\]

is \( SL(2, Z) \) metric. The constraint equation (20) can also be rewritten as

\[
\frac{1}{2} F^{\mu\nu a} H_{m\mu\nu} - \omega_{ab} \xi^b_m (\partial_\mu X^{10})^2 = G^a_m
\]  

(73)

where the \( SL(2, Z) \) doublet on right hand side is

\[
G^a_m \equiv \left( \frac{1}{3!} \epsilon^{\mu\nu\lambda\rho} \partial_\mu C_{\nu\lambda\rho m} + \tilde{\zeta}_m F_{\mu\nu} \tilde{F}_{\mu\nu} \right.
\]

\[
- \frac{1}{3!} \epsilon^{\mu\nu\lambda\rho} \partial_\mu \tilde{C}_{\nu\lambda\rho m} - \zeta_m F_{\mu\nu} F_{\mu\nu}
\]  

(74)

These define the covariance of 6D equations under \( SL(2, Z) \) group. Thus in dyonic cases \( C_{(4)} \) backgrounds can be nontrivial, as we have seen it already in the magnetically charged solution (70). Also it appears that for a given dyonic solution, one could always set a gauge choice such that, for both \( C \) and \( \tilde{C} \) in (73), we get \( G^a_m = 0 \). But this choice may altogether be guided by (2,0) supersymmetry in the complete theory, which we have not explored in the present work.
6 Conclusion

In summary, the covariant 6D gauge action presented here with an auxiliary vector, \( \zeta^M \), and an associated axion, \( \theta \), can effectively describe the low energy dynamics on M5 branes. The theory admits namely two types of extended solitonic solutions. For the I-string solitons, living over \( \mathcal{M}_2 \times E_4 \) spacetime, with lightlike vev \( \zeta^M = (|\zeta|, 0, 0, 0, 0, |\zeta|) \), the mometum is also lightlike

\[
p^M = (nL|\zeta|^2, 0, 0, 0, 0, nL|\zeta|^2).
\]

The mometum effectively depends on the instanton number \( n \) carried by the I-string soliton. The I-strings, described as strings of length \( L \), are a set of discrete states which carry a topological charge

\[
Q_{\text{i-string}} = nL|\zeta|.
\]

When theory is compactified on a circle, taking \( L|\zeta|^2 \sim \frac{1}{g_{YM}^2} \), the 6-momentum carried by I-string becomes

\[
p^M = \left( \frac{n}{g_{YM}^2}, 0, 0, 0, 0, \frac{n}{g_{YM}^2} \right)
\]

but it is light-like always. Thus we emphasize that the I-strings arise only in the lightlike case \((\zeta)^2 = 0\) and the axion is chiral field. The 6D on-shell action is well defined.

For the ordinary (color) e-string solitons, with space-like \( \zeta^M = (0, 0, 0, 0, 0, |\zeta|) \), the symmetry allows the 6-momentum to be \( p^M \propto (L \mathcal{E}, 0, 0, 0, 0) \) which interestingly remains independent of the \( \zeta \)-vev. The charge of e-strings is given as

\[
Q_{\text{e-string}} = \int *H(3) = \int H(3). \quad (75)
\]

While full \( SO(1, 5) \) invariant configurations in the theory can occur only when vev \( < \zeta_M > \) vanishes. Also we cannot implement vanishing \( |\zeta| \) limit arbitrarily because all 6D string configurations are defined in a regulated space with the string length \( L \). However, the limit \( |\zeta| \to 0 \) keeping \( L|\zeta| = \) fixed can be smoothly implemented, which keeps the i(e)-string charge fixed. In the theory compactified on a circle, this double limit coincides with the strong SYM coupling limit, \( L \sim g_{YM}^2 \to \infty, \; |\zeta| \sim \frac{1}{g_{YM}^2} \to 0 \), of 5D super Yang-Mills theory. The strong coupling limit in SYM is usually thought of as the decompactification limit from 6D point of view. In the strong coupling limit the infinitely long I-strings will become very light (tensionless) and become degenerate with the 6D ground state, while colored e-strings tend to become heavy because \( L \sim g_{YM}^2 \to \infty \).

Thus we conclude that the vacuum in M5 theory would be infinitely degenerate and will be proliferated by multiple (tensionless) I-strings. Each such I-string vacua would carry a distinct topological (instatonic) charge. The degeneracy gets lifted as soon as the M5 theory is put on a finite size circle.

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References

[1] J. Bagger and N. Lambert, [arXiv:0711.0955 [hep-th]]; [arXiv:0712.3738 [hep-th]]; A. Gustavsson, arXiv:0709.1260 [hep-th]; A. Gustavsson, JHEP 0804, 083 (2008) [arXiv:0802.3456 [hep-th]].

[2] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk, H. Verlinde, arXiv:0806.0738v2 [hep-th].

[3] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “Ghost-Free Superconformal Action for Multiple M2-Branes,” arXiv:0806.0051 [hep-th].

[4] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” arXiv:0806.1218 [hep-th].

[5] B. Ezhuthachan, S. Mukhi and C. Papageorgakis, “D2 to D2,” JHEP 0807, 041 (2008) [arXiv:0806.1639 [hep-th]].

[6] H. Singh, JHEP 0809, 071 (2008) [arXiv:0807.5016 [hep-th]].

[7] C. Krishnan, C. Maccaferri and H. Singh, JHEP 0905 (2009) 114, arXiv:0902.0290 [hep-th].

[8] H. Singh, Phys. Lett. B673 (2009) 68, arxiv:hep-th/0811.1690.

[9] A. Basu and J. A. Harvey, “The M2-M5 brane system and a generalized Nahm’s equation,” Nucl. Phys. B 713, 136 (2005) [hep-th/0412310].

[10] N. Lambert, C. Papageorgakis, “Nonabelian (2,0) Tensor Multiplets and 3-algebras,” JHEP 1008, 083 (2010). [arXiv:1007.2982 [hep-th]].

[11] M. R. Douglas, “On D=5 super Yang-Mills theory and (2,0) theory,” JHEP 1102, 011 (2011). [arXiv:1012.2880 [hep-th]].

[12] N. Lambert, C. Papageorgakis, M. Schmidt-Somerfeld, “M5-Branes, D4-Branes and Quantum 5D super-Yang-Mills” JHEP 1101 (2011) 083; arXiv:1012.2882.

[13] H. Singh, “Super-Yang-Mills and M5-branes,” JHEP 1108, 136 (2011) [arXiv:1107.3408 [hep-th]].

[14] H. Samtleben, E. Sezgin and R. Wimmer, “(1,0) superconformal models in six dimensions,” JHEP 1112, 062 (2011) [arXiv:1108.4060 [hep-th]].
[15] N. Lambert and P. Richmond, “(2,0) Supersymmetry and the Light-Cone Description of M5-branes,” JHEP 1202, 013 (2012) [arXiv:1109.6454 [hep-th]].

[16] Y. Tachikawa, “On S-duality of 5d super Yang-Mills on \( S^1 \),” JHEP 1111, 123 (2011) [arXiv:1110.0531 [hep-th]].

[17] H. Linander and F. Ohlsson, “(2,0) theory on circle fibrations,” JHEP 1201, 159 (2012) [arXiv:1111.6045 [hep-th]].

[18] C.-S. Chu and S.-L. Ko, “Non-abelian Action for Multiple Five-Branes with Self-Dual Tensors,” JHEP 1205, 028 (2012) [arXiv:1203.4224 [hep-th]].

[19] T. Maxfield and S. Sethi, “The Conformal Anomaly of M5-Branes,” JHEP 1206, 075 (2012) [arXiv:1204.2002 [hep-th]].

[20] L. Dolan and Y. Sun, “Partition Functions for Maxwell Theory on the Five-torus and for the Fivebrane on \( S^1 \times T^5 \),” [arXiv:1208.5971] [hep-th].

[21] Z. Bern, J.J. Carrasco, L.J. Dixon, M.R. Douglas, M.V. Hippel and H. Johansson, arXiv:1210.7709 [hep-th].

[22] H. Singh, “The Yang-Mills and chiral fields in six dimensions,” JHEP 1302, 056 (2013), [arXiv:1211.3281] [hep-th].

[23] I. Bandos, H. Samtleben and D. Sorokin, “Duality-symmetric actions for non-Abelian tensor fields,” Phys. Rev. D 88 (2013) 2, 025024 [arXiv:1305.1304 [hep-th]].

[24] S. L. Ko, D. Sorokin and P. Vanichchapongjaroen, “The M5-brane action revisited,” JHEP 1311 (2013) 072 [arXiv:1308.2231] [hep-th].

[25] F. M. Chen, “A nonabelian (1, 0) tensor multiplet theory in 6D,” JHEP 1402 (2014) 034 [arXiv:1312.4330] [hep-th].

[26] P. M. Ho and Y. Matsuo, “Aspects of Effective Theory for Multiple M5-Branes Compactified On Circle,” JHEP 1412 (2014) 154 [arXiv:1409.4060] [hep-th]].

[27] S. Lavau, H. Samtleben and T. Strobl, “Hidden Q-structure and Lie 3-algebra for non-abelian superconformal models in six dimensions,” J. Geom. Phys. 86 (2014) 497, [arXiv:1403.7114] [math-ph].

[28] P.S. Howe, G. Sierra and P. Townsend, “Supersymmetry in Six Dimensions”, Nucl. Phys. B221 (1983) 331.

[29] P.S. Howe, N.D. Lambert and P.C. West, Nucl. Phys. B515 (1998) 203, [arXiv:9709014 [hep-th]].
[30] E. Witten, Five-Brane Effective action in M-theory, J. Geom Phys. 22 (1997) 103, arXiv:9610234 [hep-th].

[31] M. Perry, J. H. Schwarz, “Interacting Chiral Gauge Fields in Six Dimensions and Born-Infeld Theory”, Nucl.Phys. B489 (1997) 47-64, arXiv:hep-th/9611065.

[32] I. A. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. P. Sorokin and M. Tonin, “Covariant action for the super-five-brane of M-theory,” Phys. Rev. Lett. 78, 4332 (1997) arXiv:hep-th/9701149.

[33] P. Howe and E. Sezgin, “D = 11, p = 5” Phys. Lett. B394 (1997) 62-66, arXiv:hep-th/9611008 [hep-th].

[34] P. S. Howe, E. Sezgin, P. C. West, “Covariant field equations of the M theory five-brane,” Phys. Lett. B399, 49-59 (1997). hep-th/9702008.

[35] K. -M. Lee, J. -H. Park, “5-D actions for 6-D selfdual tensor field theory,” Phys. Rev. D64, 105006 (2001). hep-th/0008103.

[36] P. Pasti, I. Samsonov, D. Sorokin and M. Tonin, “BLG-motivated Lagrangian formulation for the chiral two-form gauge field in D=6 and M5-branes,” Phys. Rev. D 80, 086008 (2009) arXiv:0907.4596 [hep-th].