A quantitative model for $I_C R$ product in $d$-wave Josephson junctions

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We study theoretically the Josephson effect in $d$-wave superconductor / diffusive normal metal /insulator/ diffusive normal metal/ $d$-wave superconductor (D/DN/1/DN/D) junctions. This model is aimed to describe practical junctions in high-$T_C$ cuprate superconductors, in which the product of the critical Josephson current ($I_C$) and the normal state resistance ($R$) (the so-called $I_C R$ product) is very small compared to the prediction of the standard theory. We show that the $I_C R$ product in D/DN/1/DN/D junctions can be much smaller than that in $d$-wave superconductor / insulator / $d$-wave superconductor junctions and formulate the conditions necessary to achieve large $I_C R$ product in D/DN/1/DN/D junctions. The proposed theory describes the behavior of $I_C R$ products quantitatively in high-$T_C$ cuprate junctions.

Josephson effect in high $T_C$ superconductors has attracted much attention because of its potential applications in future technologies. In particular, applications in electronics, such as the single-flux quantum devices are extremely promising, since the operations in future technologies...
$Z' = 2H'/v_F$ are dimensionless constants and $v_F$ is Fermi velocity, where $\phi$ is the injection angle measured from the interface normal. In the following we assume $Z \gg 1$. The schematic illustration of the model is shown in Fig. 1. The pair potential along the quasiparticle trajectory with the injection angle $\phi$ is given by $\Delta_l = \Delta \cos[2(\phi - \alpha)] \exp(-i\Psi)$ and $\Delta_r = \Delta \cos[2(\phi - \beta)]$ for the left and the right superconductors, respectively. Here $\Psi$ is the phase difference across the junction, $\alpha$ and $\beta$ denote the angles between the normal to the interface and the crystal axes of the left and right $d$-wave superconductors, respectively. The lobe direction of the pair potential and the direction of the crystal axis are chosen to be the same.

![Schematic illustration of the model for the D/DN/I/DN/D junction.](image)

FIG. 1: (color online) Schematic illustration of the model for the D/DN/I/DN/D junction.

We parameterize the quasiclassical Green’s functions $G$ and $F$ with a function $\Phi_{\alpha,\beta}$:

$$G_{\omega} = \frac{\omega}{\sqrt{\omega^2 + \Phi_{\omega} \Phi_{-\omega}}}, \quad F_{\omega} = \frac{\Phi_{\omega}}{\sqrt{\omega^2 + \Phi_{\omega} \Phi_{-\omega}}}$$

where $\omega$ is the Matsubara frequency. In the DN layers the Green’s functions satisfy the Usadel equation:

$$\xi^2 \frac{\pi T_C}{\omega G_{\omega}} \frac{\partial}{\partial x} \left( G_{\omega}^2 \frac{\partial}{\partial x} \Phi_{\omega} \right) - \Phi_{\omega} = 0$$

where $\xi = \sqrt{D/2\pi I_C}$ is the coherence length, $D$ is the diffusion constant and $T_C$ is the transition temperature of superconducting electrodes. To solve the Usadel equation, we apply the generalized boundary conditions derived in Ref. 23 at $x = \pm L$ and the boundary conditions in Ref. 21 at $x = 0$.

The Josephson current is given by

$$\frac{e I R}{\pi T_C} = \frac{1}{2R_d T_C} \sum_{\omega} G_{\omega}^2 \left( \Phi_{\omega} \frac{\partial}{\partial x} \Phi_{-\omega} - \Phi_{-\omega} \frac{\partial}{\partial x} \Phi_{\omega} \right)$$

where $T$ is temperature and $R \equiv 2R_d + R_b + 2R_b'$ is the normal state resistance of the junction. In the following we focus on the $I_C R$ value as a function of temperature and clarify the cases when $I_C R$ is enhanced. Below $\Delta(0)$ denotes the value of $\Delta$ at zero temperature. Note that it is realistic to choose small magnitude of $Z'$ and $R_b'$, and large Thouless energy because thin DN regions could be naturally formed due to the degradation of superconductivity near the interface.

![Graphs showing $I_C R$ values for different cases.](image)

FIG. 2: (color online) $I_C R$ value for $Z' = 1, R_d/R_b = 0.1$ and $(\alpha, \beta) = (0, 0)$.

In Fig. 2 we show $I_C R$ value for $Z' = 1, R_d/R_b = 0.1$ and $(\alpha, \beta) = (0, 0)$ with various $E_{Th}/\Delta(0)$ and $R_d/R_b'$. $I_C R$ increases with $E_{Th}/\Delta(0)$ and $R_d/R_b'$ because proximity effect is enhanced. As $E_{Th}$ increases, the magnitude of the gradient becomes small.

Figure 3 shows $I_C R$ value for $E_{Th}/\Delta(0) = 1, R_d/R_b = 0.1$ and $(\alpha, \beta) = (0, 0)$ with various $Z'$ and $R_d/R_b'$. As $Z'$ increases, the magnitude of the gradient becomes larger. The peculiar effect is that $I_C R$ increases with $Z'$, indicating that proximity effect is enhanced by the increase
of $Z'$. This stems from the sign change of the pair potential\cite{24,25}. For the case of $d$-wave symmetry with $\alpha = \beta = 0$, injection angles of a quasiparticle can be separated into two regions: $\phi_+ = \{\phi | 0 \leq |\phi| < \pi/4\}$ and $\phi_- = \{\phi | \pi/4 \leq |\phi| \leq \pi/2\}$. The signs of pair potential for $\phi_+$ and that for $\phi_-$ are opposite. As a result, the sign change of pair potentials suppresses the proximity effect in the DN and hence Josephson currents. As $Z'$ increases, the contribution from $\phi_+$ dominates over that from $\phi_-$. Therefore $I_{C,R}$ increases with $Z'$.

In Fig. 3 we plot $I_{C,R}$ value for $E_{\text{Th}}/\Delta(0) = 1$, $Z' = 1$ and $(\alpha, \beta) = (0, 0)$ with various $R_d/R_b$ and $R_d/R'_b$. $I_{C,R}$ increases with $R_d/R_b$ due to the enhancement of the proximity effect.

Figure 4 displays $I_{C,R}$ value for $E_{\text{Th}}/\Delta(0) = 0.1$, $Z' = 0.1$, $R_d/R_b = 0.1$ and $R_d/R'_b = 10$ with various $\alpha$ and $\beta$. The formation of MARS suppresses the proximity effect. Therefore $I_{C,R}$ decreases with the increase of $\alpha$ and $\beta$. In the actual junctions, there is inevitable roughness at the interface and hence the effective values of $\alpha$ and $\beta$ at the interface become random even if junctions with $\alpha = \beta = 0$ are fabricated. This provides the mechanism of suppression of the $I_{C,R}$ product.

Finally we compare the present theory with the experimental data from Ref.\textsuperscript{26} and with the theory for DID junctions by Tanaka and Kashiwaya (TK)\textsuperscript{27}. The temperature dependencies of $I_C$ are plotted in Fig. 6 taking $\alpha = \beta = 0$ and $R = 0.375$\,	extOmega\ for theoretical plots. We choose $E_{\text{Th}}/\Delta(0) = 3$, $Z' = 0.1$, $R_d/R_b = 0.01$, and $R_d/R'_b = 100$ in the present theory, and the barrier parameter $Z = 10$ in the TK theory. As shown in this figure, the present theory can explain the experimental results quantitatively, while the discrepancy between the TK theory and the data is rather strong, about an order of magnitude. Note that in the TK theory the $I_C$ is not sensitive to the choice of $Z$ parameter. To estimate the realistic size of the DN region, we can take $\Delta(0) = 10$meV and $D = 10^{-3}m^2/s$, and then obtain the length of the DN region $L = 4.7$nm.

In summary, we have studied the Josephson current in D/DN/I/DN/D junctions as a model of high $T_C$ superconductor junctions. We have shown that the $I_{C,R}$ product in D/DN/I/DN/D junctions can be much smaller than that in DID junctions and have found the conditions when the $I_{C,R}$ in D/DN/I/DN/D junctions is largest. The requirements for the large magnitude of $I_{C,R}$ product are: no roughness at the interfaces, large magnitudes of $Z'$, $R_d/R_b$, $R_d/R'_b$ and $E_{\text{Th}}$, and $(\alpha, \beta) = (0, 0)$.
realistic for naturally formed DN layers, hence the only tunable parameter is $R_b$. Our theory can explain the experimental results on the quantitative level, in contrast to the previous idealized treatment of DID junctions.

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FIG. 6: (color online) Comparison between the present theory (solid line), experimental data (dotted line) and TK theory (broken line). $I_C$ is plotted as a function of temperature, taking $R = 0.375\Omega$ and $\alpha = \beta = 0$ for theoretical plots. We choose $E_{R_b}/\Delta(0) = 3$, $Z' = 0.1$, $R_d/R_b = 0.01$, and $R_d/R_b' = 100$ in the present theory, and $Z = 10$ in TK theory.