On the Initial State and Consistency Relations

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Abstract

We study the effect of the initial state on the consistency conditions for adiabatic perturbations. In order to be consistent with the constraints of General Relativity, the initial state must be diffeomorphism invariant. As a result, we show that initial wavefunctional/density matrix has to satisfy a Slavnov-Taylor identity similar to that of the action. We then investigate the precise ways in which modified initial states can lead to violations of the consistency relations. We find two independent sources of violations: i) the state can include initial non-Gaussianities; ii) even if the initial state is Gaussian, such as a Bogoliubov state, the modified 2-point function can modify the $q \to 0$ analyticity properties of the vertex functional and result in violations of the consistency relations.
1 Introduction

Standard calculations of primordial correlation functions from inflation assume the Bunch-Davies \[1\] or adiabatic vacuum state. Since quantum fluctuations originate from scales much smaller than the Hubble scale and therefore experience a nearly flat background space-time, the argument goes, it is reasonable to assume they were born in a state which asymptotically approaches the flat-space vacuum. This is by definition the Bunch-Davies vacuum. On the other hand, since not much is known about the high energy, pre-inflationary epoch, one is justified to consider more general initial states/density matrices \[2–21\], as a way of parametrizing our ignorance about the conditions at the onset of inflation. These modifications have been argued to affect various late-time observables, in particular non-Gaussianities \[22–39\].

The goal of this work is to systematically study the impact of the initial state on the consistency relations for primordial perturbations. Because of the few assumptions that go into them, the consistency relations offer a powerful probe of early-universe physics. Specifically, their proof relies on: \(i\) a single scalar field; \(ii\) an attractor background (mode functions asymptote to a constant at late times); \(iii\) the Bunch-Davies vacuum. We focus on \(iii\) and investigate to what extent the consistency relations can be violated once we relax the Bunch-Davies assumption.

The consistency relations, derived recently as Ward identities \[40, 41\] for non-linearly realized global symmetries \[42\], take the schematic form

\[
\lim_{\vec{q} \to 0} \frac{\partial^n}{\partial \vec{q}^n} \left( \frac{\langle \zeta \vec{O}_{\vec{p}_1, ..., \vec{p}_N} \rangle}{P_\zeta(q)} + \frac{\langle \gamma \vec{O}_{\vec{p}_1, ..., \vec{p}_N} \rangle}{P_\gamma(q)} \right) \sim \sum_a \frac{\partial^n}{\partial \vec{p}_a^n} \langle \vec{O}_{\vec{p}_1, ..., \vec{p}_N} \rangle. \tag{1.1}
\]

They constrain the \(q^n\) behavior of an \(N + 1\)-point correlation function with a soft scalar or tensor mode to a symmetry transformation on an \(N\)-point function. At lowest order \((n = 0)\), one obtains Maldacena’s consistency relations \[43–45\], which relate the 3-point functions with a soft scalar or tensor to rescalings of the 2-point function. At the next order \((n = 1)\) they are the linear-gradient consistency relations \[46, 47\]. See \[48–53\] for related derivations of the \(n = 0, n = 1\) relations through Ward identities. The identities for \(n \geq 2\), first discovered in \[40\], partially constrain the soft limit of correlation functions in terms of lower-point functions. The \(3 \to 2\) relations have been checked explicitly up to and including \(q^3\) order \[54\].

The consistency relations have also been shown to derive from a single master identity, as a consequence of the Slavnov-Taylor identity for spatial diffeomorphisms \[41\]. For soft 3-point functions, with the hard momenta given by scalar modes, the master identity takes the form

\[
q^j \left( \frac{1}{3} \delta_{ij} \Gamma^{3d\zeta\zeta\zeta}_{\zeta\zeta\zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) + 2 \Gamma^{3d\gamma\zeta\zeta}_{\zeta\zeta\zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) \right) = q_i \Gamma^{3d}_{\zeta\zeta\zeta} (p) - p_i \left( \Gamma^{3d}_{\zeta\zeta\zeta} (|\vec{q} + \vec{p}|) - \Gamma^{3d}_{\zeta\zeta\zeta} (p) \right), \tag{1.2}
\]

where \(\Gamma^{3d\zeta\zeta\zeta}\) and \(\Gamma^{3d\gamma\zeta\zeta}\) are respectively the cubic vertex functions for 3 scalars, and for 2 scalars – 1 tensor, while \(\Gamma^{3d}_{\zeta}\) is the inverse scalar propagator. This master identity is valid at any \(q\) and therefore goes beyond the soft limit. By resuming to correlation functions and differentiating a number of
times with respect to $q$, one recovers order by order all of the identities (1.1). This approach underscores the role of diffeomorphism invariance at the root of cosmological consistency relations and sharpens the precise assumptions necessary for their validity.

The consistency relations are physical statements and can be related to late-time observables, such as the CMB bispectrum [55–57] and the large-scale structure [58–61]. Their violations can be tested observationally. For instance, had the Planck satellite detected a significant primordial local $f_{NL}$, this would have violated Maldacena’s consistency relation and immediately ruled out in one shot all of the simplest inflationary models. More precisely, we would have learned that one of the three standard assumptions listed above is invalid. Instead, the Planck result [62] $f_{NL}^{\text{local}} = 2.7 \pm 5.8$ is so far consistent with Maldacena’s relation. What (1.1) shows is that there are in fact an infinite number of additional checks that the simplest inflationary scenarios must pass to be validated.

In this paper, we investigate the role of the initial state in the derivation of the consistency relations and identify the various ways in which departures from the Bunch-Davies state can result in violations of these relations. This study is motivated in part by the recent derivation of Goldberger et al. [49] of Maldacena’s consistency relation as a consequence of the Ward identity from spontaneously broken dilation. Since one is primarily interested in equal-time correlation functions in cosmology, these authors introduced a 3d Euclidean path integral over field configurations at fixed time. All of the information about the prior history is encoded in the wavefunctional. The 3d path integral approach, and its connection to the 4d in-in path integral is reviewed in Sec. 2.

Surprisingly, it seems that the derivation of [49] applies to any gauge-invariant initial state. This seems too general, as the authors themselves point out. Moreover, explicit calculations of the bispectrum based on particular initial states [36] seem to offer counterexamples. Our goal is to resolve this apparent contradiction by pinpointing the precise role of the initial state in the consistency relations.

The derivation of the Slavnov-Taylor identity (1.2) is also based on the fixed-time approach [41]. In particular, $\Gamma^{3d}$ is the 3d vertex functional, defined as usual as the Legendre transform of the connected generating functional. As argued in [41] and reviewed in Sec. 3, the consistency relations follow from (1.2) provided that the 3d vertices satisfy a certain analyticity/locality condition as $\vec{q} \to 0$. As shown in [41], this locality condition is satisfied if mode functions have constant growing-mode solutions and the initial state is the Bunch-Davies vacuum.

More generally, to understand the role of the initial state we must relate $\Gamma^{3d}$ back to the 4d vertex functional $\Gamma^{4d}$ computed from the full in-in path integral. Focusing on pure initial states, for simplicity, the initial wavefunctional $\Psi \sim e^{iS}$ can be thought of as an additional contribution to the action which is localized at the initial time. At tree level, we schematically write

$$\Gamma^{4d} = S + S,$$  \hspace{1cm} (1.3)

where $S$ is the “bulk” action, and $S$ is the contribution from the wavefunctional. Our first result, derived in Sec. 4.1, is that spatial diffeomorphism invariance imposes that $S$ and $S$ each satisfy a
Slavnov-Taylor identity of the form (1.2).

The 3d and 4d vertices are related by (see Sec. 4.2)

\[
\Gamma_{3d}^{\zeta \zeta \zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) = i \int_{t_0}^{t} dt_1 dt_2 dt_3 \frac{P_{\zeta}(q, t, t_1) P_{\zeta}(p, t, t_2) P_{\zeta}(|\vec{p} + \vec{q}|, t, t_3)}{P_{\zeta}(\vec{q}) P_{\zeta}(\vec{p}) P_{\zeta}(|\vec{p} + \vec{q}|)} \frac{\delta^3 S}{\delta \zeta_{\vec{q}}(t_1) \delta \zeta_{\vec{p}}(t_2) \delta \zeta_{-\vec{q} - \vec{p}}(t_3)}
\]

\[
+ i \frac{P_{\zeta}(q, t, t_0) P_{\zeta}(p, t, t_0) P_{\zeta}(|\vec{p} + \vec{q}|, t, t_0)}{P_{\zeta}(\vec{q}) P_{\zeta}(\vec{p}) P_{\zeta}(|\vec{p} + \vec{q}|)} C_{\vec{q}, \vec{p}, -\vec{q} - \vec{p}},
\]

where \(P_{\zeta}(t, t_i)\) is the non-equal time 2-point function, and \(C_{\vec{q}, \vec{p}, -\vec{q} - \vec{p}}\) is a cubic contribution from the initial wavefunctional \(S\). Similarly for vertices involving tensors. This clearly displays the possible non-local contributions to \(\Gamma_{3d}^{\zeta \zeta \zeta}\) arising from a modified initial state:

- The second line is a non-Gaussian contribution from the initial state, whose \(q\) dependence is not fixed by symmetries. In Sec. 4.3, for example, we focus on local initial non-Gaussianities and show that the resulting contribution to \(\Gamma_{3d}^{\zeta \zeta \zeta}\) is \(O(q^0)\), which violates the \(O(q^2)\) behavior required for the consistency relations.

- The Gaussian part of the initial state enters in the first line of (1.4) through non-equal-time 2-point functions. Even though \(S\) vanishes as \(O(q^2)\) (assuming constant growing modes), the interplay of the modified 2-point functions and the time integral can lead to an \(O(q)\) contribution to \(\Gamma_{3d}^{\zeta \zeta \zeta}\), which violates the consistency relations. In Sec. (4.4), we illustrate this explicitly with Bogoliubov states.

We conclude with future directions of investigation in Sec. 5.

## 2 Framework

The in-in path-integral formalism, which is widely used in non-equilibrium field theory, is a powerful tool for studying the expectation values of various operators. Consider a system described by the action \(S[\Phi]\) and prepared in the state with a density operator \(\rho\) at some initial time \(t_0\). Here, \(\Phi\) collectively denotes all relevant degrees of freedom; eventually, we will take it to denote the inflaton and graviton degrees of freedom.

### 2.1 4d Formalism

Following the standard “doubling” of fields, the unified description of all correlation functions in terms of the generating functional can be obtained by introducing appropriate external currents:

\[
Z[J^+, J^-] = \int \mathcal{D}\Phi^+ \mathcal{D}\Phi^- \exp \left[ i \left( S[\Phi^+, J^+] - S[\Phi^-, J^-] \right) \right] \rho(\Phi^+, \Phi^-; t_0),
\]

where the time integrals run from the initial time \(t_0\) to the time of evaluation \(t\), the path-integral is performed over the configurations satisfying \(\Phi^+(\vec{x}, t) = \Phi^-(\vec{x}, t)\), and \(\rho(\Phi^+, \Phi^-; t_0)\) represents
the initial density matrix.\(^1\) It is convenient to express the density matrix \(\rho(\Phi^+, \Phi^-; t_0)\) as an exponential [36]

\[
\rho(\Phi^+, \Phi^-; t_0) \sim \exp \left( iS[\Phi^+, \Phi^-; t_0] \right), \tag{2.2}
\]

where \(S[\Phi^+, \Phi^-; t_0]^* = -S[\Phi^-, \Phi^+; t_0]\) to ensure that \(\rho\) is hermitian. Thus \(S\) represents a contribution to the action localized at the initial time.

To simplify the notation we will adopt the 2-component description \(\Phi^a = (\Phi^+, \Phi^-)\) and \(J^a = (J^+, J^-)\), where \(a = \{+, -\}\) and indices are lowered by the metric \(\eta_{ab} = \text{diag}(1, -1)\). The generating functional becomes

\[
Z[J^a] = \int \mathcal{D}\Phi^a \exp \left[ i \left( S[\Phi^a] + S[\Phi^a; t_0] + \int d^4x J_a \Phi^a \right) \right], \tag{2.3}
\]

where \(S[\Phi^a] \equiv S[\Phi^+] - S[\Phi^-]\). The generator of connected diagrams, defined as usual by \(W \equiv -i\ln Z\), satisfies\(^2\)

\[
\Phi^a(x) = \frac{\delta W}{\delta J_a(x)}. \tag{2.4}
\]

The vertex functional will play a central role in the rest of our discussion. It is defined as usual through the Legendre transform

\[
\Gamma^{4d}[\Phi^a] = W[J^a] - \int d^4x J_a \Phi^a, \tag{2.5}
\]

where the ‘4d’ label is introduced to distinguish this from the vertex functional derived from the 3d path integral to be defined shortly. Differentiation with respect to \(\Phi\) combined with (2.4) gives the inverse relation

\[
J_a(x) = -\frac{\delta \Gamma^{4d}}{\delta \Phi^a(x)}. \tag{2.6}
\]

It is straightforward to derive the Feynman rules in the usual manner. Specifically, if we differentiate (2.4) and (2.6) with respect to \(J\) and \(\Phi\) respectively, followed by the convolution of the resulting equations, we obtain

\[
\int d^4z \frac{\delta^4 \Gamma^{4d}}{\delta \Phi^a(x) \delta \Phi^c(z)} \frac{\delta^2 W}{\delta J_c(z) \delta J_b(y)} = -\delta^4_{ab} \delta^4(x - y). \tag{2.7}
\]

\(^1\)To be precise, \(\rho(\Phi^+, \Phi^-; t_0)\) is the representation of the density operator in the field-eigenstate or configuration-space basis.

\(^2\)In (2.4), \(\Phi^a\) represents the expectation value of the field in the presence of the external current. For simplicity, the same notation as for the field itself is being used.
The Feynman rules follow from this relation by differentiating with respect to $J$ a number of times and reshuffling terms appropriately. For example, the relation between the 3-point function and the 3-point vertex is given by

\[
\frac{\delta^3 W}{\delta J_a(x) \delta J_b(y) \delta J_c(z)} = \int d^4x' d^4y' d^4z' \frac{\delta^2 W}{\delta J_a(x) \delta J_a'(x')} \frac{\delta^2 W}{\delta J_b(y) \delta J_b'(y')} \frac{\delta^2 W}{\delta J_c(z) \delta J_c'(z')} \times \frac{\delta^3 \Gamma^{3d}}{\delta \Phi^{a'}(x') \delta \Phi^{b'}(y') \delta \Phi^{c'}(z')}. \tag{2.8}
\]

Such relations establish a dictionary between vertices and the connected correlators in the in-in path integral.

2.2 3d Formalism

In cosmology, one is usually interested in equal-time correlation functions only. For this purpose, it is convenient to rephrase the problem in terms of a fixed-time or 3d path integral [49]. In this description, the history has been integrated out, and the path integral is over field configurations at fixed time $t$. In our discussion of consistency relations, it will prove instructive to go back and forth between the 3d and 4d descriptions.

Concretely, consider the generating function with an external source localized at some late time $t$, i.e., with $J_a(\mathbf{x}, t') = -i\delta_{a+}J(\mathbf{x})\delta(t' - t)$. In this case (2.3) reduces to

\[
Z[J] = \int \mathcal{D}\Phi P[\Phi, t] \exp \left[ \int d^3x \Phi(\mathbf{x}, t) J(\mathbf{x}) \right]. \tag{2.9}
\]

The probability distribution $P[\Phi, t]$ is itself represented by a path integral

\[
P[\Phi, t] = \int \Phi(\mathbf{x}, t) \mathcal{D}\Phi^a \exp \left[ i \left( S[\Phi^a] + S[\Phi^a; t_0] \right) \right], \tag{2.10}
\]

subject to the boundary condition $\Phi^+(\mathbf{x}, t) = \Phi^-(\mathbf{x}, t) = \Phi(\mathbf{x}, t)$. Although (2.9) represents a 3d path integral at fixed time $t$, its dependence on the history prior to $t$ is encoded in $P[\Phi, t]$. Following [49], we can define the 3d connected generating functional $W \equiv \ln Z$, and the corresponding vertex functional

\[
\Gamma^{3d}[\Phi] = W - \int d^3x J(\mathbf{x}) \Phi(\mathbf{x}), \tag{2.11}
\]

As in the 4d case, we get the standard relations

\[
\Phi(\mathbf{x}) = \frac{\delta W}{\delta J(\mathbf{x})}; \quad J(\mathbf{x}) = -\frac{\delta \Gamma^{3d}}{\delta \Phi(\mathbf{x})}. \tag{2.12}
\]

\footnote{The missing factor of ‘$i$’ compared to the usual definition of $W$ is because of the Euclidean nature of the path-integral.}
Once again, using these relations we can derive the 3d analogue of (2.7),
\[
\int d^3 z \frac{\delta^3 \Gamma^{3d}}{\delta \Phi(\vec{x}) \delta \Phi(\vec{z})} \frac{\delta^2 \mathcal{W}}{\delta \mathcal{J}(\vec{x}) \delta \mathcal{J}(\vec{y})} = -\delta^3(\vec{x} - \vec{y}) ,
\]
and similarly the analogue of (2.8),
\[
\frac{\delta^3 \mathcal{W}}{\delta \mathcal{J}(\vec{x}) \delta \mathcal{J}(\vec{y}) \delta \mathcal{J}(\vec{z})} = \int d^3 x' d^3 y' d^3 z' \frac{\delta^2 \mathcal{W}}{\delta \mathcal{J}(\vec{x}) \delta \mathcal{J}(\vec{z})} \frac{\delta^2 \mathcal{W}}{\delta \mathcal{J}(\vec{y}) \delta \mathcal{J}(\vec{z})} \frac{\delta^2 \mathcal{W}}{\delta \mathcal{J}(\vec{z})} \frac{\delta^3 \Gamma^{3d}}{\delta \Phi(\vec{x}') \delta \Phi(\vec{y}') \delta \Phi(\vec{z}')} .
\]

In the above, \( \delta^n \mathcal{W} / \delta \Phi^n \) represents the equal-time connected \( n \)-point function, and all fields are evaluated at time \( t \).

Since the 3d and 4d formulations must describe the same physics, the path integrals (2.3) and (2.9) generate identical equal-time correlation functions. This implies a relation between the vertex functions \( \Gamma^{3d} \) and \( \Gamma^{4d} \) which will be useful for the forthcoming discussion. This relation is most elegantly represented in Fourier space. Our starting point is the equal time 3-point correlator derived in the 4d formalism, which follows from (2.8). In Fourier space, it is given by

\[
\langle \Phi_\vec{q}(t) \Phi_\vec{p}(t) \Phi_{-\vec{q}-\vec{p}}(t) \rangle' = i \int dt_1 dt_2 dt_3 G_{+a}(q, t_1) G_{+b}(p, t_2) G_{+c}([\vec{p} + \vec{q}], t, t_3) \times \frac{\delta^3 \Gamma^{4d}}{\delta \Phi_\vec{q}(t_1) \delta \Phi_\vec{p}(t_2) \delta \Phi_{-\vec{q}-\vec{p}}(t_3)} ,
\]

where \( G_{+a}(q, t, t_i) \) denotes the non-equal-time 2-point function connecting \( \Phi^a(t_i) \) to \( \Phi^+(t) \equiv \Phi(t) \). Moreover, as usual \( \langle \ldots \rangle' \) is an on-shell correlation with the delta function removed.\(^4\) In the 3d Euclidean approach, meanwhile, the analogous result follows from (2.14),

\[
\langle \Phi_\vec{q}(t) \Phi_\vec{p}(t) \Phi_{-\vec{q}-\vec{p}}(t) \rangle' = G(q, t) G(p, t) G([\vec{p} + \vec{q}], t) \frac{\delta^3 \Gamma^{3d}}{\delta \Phi_\vec{q} \delta \Phi_\vec{p} \delta \Phi_{-\vec{q}-\vec{p}}} ,
\]

where \( G(q, t) \) represents the equal-time 2-point function. Equating (2.15) and (2.16), we obtain a relation among 3d and 4d vertices:

\[
\frac{\delta^3 \Gamma^{3d}}{\delta \Phi_\vec{q} \delta \Phi_\vec{p} \delta \Phi_{-\vec{q}-\vec{p}}} = i \int dt_1 dt_2 dt_3 G_{+a}(q, t_1) G_{+b}(p, t_2) G_{+c}([\vec{p} + \vec{q}], t, t_3) \times \frac{\delta^3 \Gamma^{4d}}{\delta \Phi_\vec{q}(t_1) \delta \Phi_\vec{p}(t_2) \delta \Phi_{-\vec{q}-\vec{p}}(t_3)} .
\]

This relation will play a key role in understanding the possible violations of the consistency condition.

\(^4\)The precise statement is \( \langle \mathcal{O}_{E_1, \ldots, E_N} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \ldots + \vec{k}_N) \langle \mathcal{O}_{E_1, \ldots, E_N} \rangle' \).
In the following sections we will concentrate on pure initial states for concreteness. In this case, the density matrix factorizes as

\[ \rho(\Phi^0; t_0) \sim \exp \left[ i \left( S[\Phi^+; t_0] - S[\Phi^-; t_0] \right) \right]. \tag{2.18} \]

As a result, \( G_{ab} \) becomes diagonal, and (2.17) reduces to

\[
\frac{\delta^3 \Gamma^{3d}}{\delta \Phi_\vec{q} \delta \Phi_\vec{p} \delta \Phi_{-\vec{q}-\vec{p}}} = i \int dt_1 dt_2 dt_3 \frac{G_{++}(q, t, t_1) G_{++}(p, t, t_2) G_{++}(|\vec{p} + \vec{q}|, t, t_3)}{G(q, t) G(p, t) G(|\vec{p} + \vec{q}|, t)} \times \delta^3 \Gamma^{4d}\left( t_1 \right) \delta \Phi_\vec{p}^+(t_2) \delta \Phi_{-\vec{q}-\vec{p}}^+(t_3). \tag{2.19} \]

When applying (2.19) to the cosmological case, \( \Phi \) will be replaced by the appropriate gravitational degrees of freedom, namely scalar/tensor perturbations on an FRW background. In that case, \( G(q, t) \) will represent the scalar/tensor power spectrum, while \( G_{++}(q, t, t) \) will become a scalar/tensor 2-point Green’s function.

### 3 Consistency Conditions from Slavnov-Taylor Identity

In this Section, we briefly review the derivation of [41] of the consistency relations based on the Slavnov-Taylor identity for spatial reparametrization invariance. The derivation applies to any spatially-flat homogeneous background generated by a single scalar field:

\[ ds^2 = -dt^2 + a^2(t) d\vec{x}^2; \quad \phi = \bar{\phi}(t). \tag{3.1} \]

Following [41], we work in uniform-density gauge, where the scalar field is unperturbed,

\[ \delta \phi \equiv \phi(\vec{x}, t) - \bar{\phi}(t) = 0, \tag{3.2} \]

and the spatial metric is

\[ H_{ij} = a^2(t) e^{2\zeta(\vec{x}, t)} \left( e^{\gamma(\vec{x}, t)} \right)_{ij}, \tag{3.3} \]

where \( \gamma^i_i = 0, \partial^i \gamma_{ij} = 0. \) Scalar modes are encoded in the curvature perturbation \( \zeta \), while tensor modes (gravitational waves) are encoded in the transverse, traceless perturbation \( \gamma_{ij} \). This gauge choice represents unitary gauge from the point of view of time reparametrization invariance, hence spatial diffeomorphisms are in fact the only symmetries to consider.

The Slavnov-Taylor identity is most elegantly derived in the 3d fixed-time path integral of Sec. 2.2. We imagine that the auxiliary lapse function and shift vector have been integrated out using the constraints, hence the remaining degrees of freedom are the scalar and tensor perturbations of the spatial metric. The fixed-time path integral is then of the form

\[ Z[J_\zeta, J_\gamma] = \int D\zeta \mathcal{D}\gamma_{ij} P[\zeta, \gamma, t] \exp \left[ \int d^3 x \left( \zeta J_\zeta + \gamma_{ij} J_{\gamma}^{ij} \right) \right], \tag{3.4} \]
where the $J$’s represent scalar and tensor external currents. The probability distribution $P[\zeta, \gamma, t]$ encodes the information about the action describing the theory, as well as the initial state in which quantum fluctuations are created. This state is usually considered to be the Bunch-Davies vacuum. Instead, we consider a completely general initial state specified at some initial time $t_0$. Our only restriction is that the initial state respect local diffeomorphism invariance. By “local” diffeomorphisms, we mean diffeomorphisms that fall off suitably fast at spatial infinity.\(^5\)

To lowest order in the tensors, the spatial reparametrization-invariance of $Z[J_\zeta, J_\gamma]$ leads to a variational differential equation for the 3d vertex functional [41]:

$$2\partial_j \left( \frac{1}{6} \delta_{ij} \frac{\delta \Gamma^{3d}}{\delta \zeta} + \frac{\delta \Gamma^{3d}}{\delta \gamma_{ij}} \right) = \partial_i \zeta \frac{\delta \Gamma^{3d}}{\delta \zeta} + \ldots$$ \hspace{1cm} (3.5)

The ellipses include the contribution from the gauge-fixing term, and terms that are higher order in the tensors. At tree level, these terms can be ignored for the remainder of our discussion. See [41] for details. Although (3.5) was originally derived for the Bunch-Davies initial state, it holds more generally for any initial state that is invariant under local spatial diffeomorphisms.

As in [41], we focus on consistency relations relating 3-point functions with a soft mode to 2-point functions without the soft mode. Moreover, the hard-momentum modes are assumed to be scalars. We must therefore perform the functional differentiation of (3.5) with respect to $\zeta(\vec{x}_1, t)$ and $\gamma(\vec{x}_2, t)$, and afterwards set all fields to zero. The result in Fourier space is

$$q^j \left( \frac{1}{3} \delta_{ij} \Gamma^{3d \zeta \zeta \zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) + 2 \Gamma^{3d \gamma \zeta \zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) \right) = q_i \Gamma^{3d} (p) - p_i \left( \Gamma^{3d}_\zeta (|\vec{q} + \vec{p}|) - \Gamma^{3d}_\zeta (p) \right) ,$$ \hspace{1cm} (3.6)

where $\Gamma^{3d \zeta \zeta \zeta}$ is the vertex with 3 scalars, $\Gamma^{3d \gamma \zeta \zeta}$ is the vertex with 2 scalars and 1 tensor, and $\Gamma^{3d}_\zeta$ is the inverse scalar propagator.\(^6\) This relation constrains the 3-point vertices, which are related to 3-point correlation functions as follows

\[
\langle \zeta_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q} - \vec{p}} \rangle' = P_\zeta (q) P_\zeta (p) P_\zeta (|\vec{q} + \vec{p}|) \Gamma^{3d \zeta \zeta \zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) ;
\]

\[
\langle \gamma_{ij} \zeta_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q} - \vec{p}} \rangle' = \tilde{P}_{ij} |\vec{q}| P_\zeta (q) P_\zeta (|\vec{q} + \vec{p}|) \Gamma^{3d \gamma \zeta \zeta} (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) , \hspace{1cm} (3.8)
\]

\(^5\)In particular, we do not impose that the initial state be invariant under global diffeomorphisms, which would be a much stronger requirement.

\(^6\)More precisely, the relations are

\[
\int d^3x_1 d^3x_2 d^3x_3 e^{-i(\vec{p}_1 \cdot \vec{x}_1 + \vec{p}_2 \cdot \vec{x}_2)} \frac{\delta^2 \Gamma^{3d}}{\delta \zeta (\vec{x}_1) \delta \zeta (\vec{x}_2)} \bigg|_{\zeta = \gamma = 0} = (2\pi)^3 \delta^3 (\vec{p}_1 + \vec{p}_2) \Gamma^{3d}_\zeta (p_1) ;
\]

\[
\int d^3x_1 d^3x_2 d^3x_3 d^3x_4 e^{-i\sum \vec{p}_i \cdot \vec{x}_i} \frac{\delta^3 \Gamma^{3d}}{\delta \zeta (\vec{x}_1) \delta \zeta (\vec{x}_2) \delta \zeta (\vec{x}_4)} \bigg|_{\zeta = \gamma = 0} = (2\pi)^3 \delta^3 (\vec{p}_1 + \vec{p}_2 + \vec{p}_3) \Gamma^{3d \zeta \zeta \zeta} (\vec{p}_1, \vec{p}_2, \vec{p}_3) ;
\]

\[
\int d^3x_1 d^3x_2 d^3x_3 e^{-i\sum \vec{p}_i \cdot \vec{x}_i} \frac{\delta^3 \Gamma^{3d}}{\delta \gamma_{ij} (\vec{x}_1) \delta \zeta (\vec{x}_2) \delta \zeta (\vec{x}_3)} \bigg|_{\zeta = \gamma = 0} = (2\pi)^3 \delta^3 (\vec{p}_1 + \vec{p}_2 + \vec{p}_3) \Gamma^{3d \gamma \zeta \zeta} (\vec{p}_1, \vec{p}_2, \vec{p}_3) . \hspace{1cm} (3.7)
\]
where \( \hat{P}_{ijk\ell} = P_{ik}P_{j\ell} + P_{i\ell}P_{jk} - P_{ij}P_{k\ell} \) is the transverse, traceless tensor appearing in the graviton propagator, and \( P_{ij} = \delta_{ij} - \hat{q}_i\hat{q}_j \) is the transverse projector.

The most general solution to (3.6) can be written as

\[
P_\zeta(p)P_\zeta(|\vec{q} + \vec{p}|) \left( \frac{1}{3} \delta_{ij} \Gamma^{\zeta\zeta\zeta\zeta}(\vec{q}, \vec{p}, -\vec{q} - \vec{p}) + 2 \Gamma_{ij}^{\zeta\zeta\zeta\zeta}(\vec{q}, \vec{p}, -\vec{q} - \vec{p}) \right) = K_{ij}(\vec{p}, \vec{q}) + A_{ij}(\vec{p}, \vec{q}) ,
\]
(3.9)

where

\[
K_{ij} \equiv -\delta_{ij} P_\zeta(p) - p_i \frac{\partial P_\zeta(p)}{\partial p^j} - \sum_{n=1}^{\infty} \frac{q_{\alpha_1} \cdots q_{\alpha_n}}{n!} \left[ \delta_{ij} \frac{\partial^n}{\partial p_{\alpha_1} \cdots \partial p_{\alpha_n}} + \frac{p_i}{n+1} \frac{\partial^{n+1}}{\partial p_{\alpha_1} \partial p_{\alpha_2} \cdots \partial p_{\alpha_n}} \right] P_\zeta(p) ,
\]
(3.10)

and \( A_{ij} \) is an arbitrary symmetric and transverse matrix:

\[
q^j A_{ij}(\vec{p}, \vec{q}) = 0 .
\]
(3.11)

Note that \( K_{ij}(\vec{p}, \vec{q}) \) is regular in the \( q \to 0 \) limit and encodes the model-independent, analytic part of the vertex functionals. The model-dependent part, which may be non-analytic, enters through the arbitrary symmetric and transverse array \( A_{ij}(\vec{p}, \vec{q}) \). The most general \( A_{ij} \) with these properties built out of \( \vec{p}, \vec{q}, \) Kronecker \( \delta \)'s and Levi-Cevita symbols is [41]

\[
A_{ij} = \epsilon_{ikm} \epsilon_{jtn} q^k q^t \left( a(\vec{p}, \vec{q}) \delta^{mn} + b(\vec{p}, \vec{q}) p^m p^n \right) ,
\]
(3.12)

where \( a \) and \( b \) are arbitrary scalar functions of the momenta.

So far the derivation is completely general. The only assumption that went into (3.9) is the spatial reparametrization invariance of the probability function \( P[\zeta, \gamma, t] \), which encodes information about the action and the initial density matrix. In particular, (3.9) holds irrespective of whether \( \zeta, \gamma \) have constant growing modes outside the horizon, or whether the initial state is highly non-Gaussian. In order to translate (3.9) into consistency relations for correlation functions, we must make a technical assumption about the analytic behavior of \( A_{ij} \) in the \( \vec{q} \to 0 \) limit. The key assumption is that the functions \( a \) and \( b \) are both analytic in \( q \), such that \( A_{ij} \) starts at order \( q^2 \):

\[
A_{ij} = O(q^2) .
\]
(3.13)

In this case, when converting (3.9) to correlation functions via (3.8), one can project out \( A_{ij} \) order by order in \( q \) to obtain consistency relations schematically of the form

\[
\lim_{\vec{q} \to 0} \frac{\partial^n}{\partial q^n} \left( \frac{\langle \zeta' q' \zeta' - \vec{q} - \vec{p} \rangle'}{P_\zeta(q)} + \frac{\langle \gamma' q' \zeta' - \vec{q} - \vec{p} \rangle'}{P_\gamma(q)} \right) \sim -\frac{\partial^n}{\partial p^n} P_\zeta(p) .
\]
(3.14)

See [40, 41] for the explicit form of these identities.
In [41], it was argued that the locality assumption (3.13) is satisfied if the initial state is the Bunch-Davies state and the field perturbations have constant growing-mode solutions in the $\vec{q} \to 0$ limit. Physically, this can be understood by going back to the 4d action $S[\zeta, \gamma]$, which at tree level coincides with the 4d vertex functional $\Gamma^{4d}[\zeta, \gamma]$. Assuming that the original gravity + scalar field action is itself local, then the only non-localities in $\Gamma^{4d}$ arise from integrating out the lapse function and shift vector via the constraints. As long as the fields have constant growing-mode solutions, $\Gamma^{4d}$ becomes local in $q$. However, this is not sufficient, since (3.13) is a locality requirement on the 3d vertex functional, instead of the 4d one. The translation, given in (2.17), involves equal-time and non-equal-time Green’s functions. We argued in [41] that, in the case of the Bunch-Davies state, an analytic $\Gamma^{4d}$ does translate to an analytic $\Gamma^{3d}$, such that (3.13) is satisfied and the consistency conditions hold. Our goal in the next Section is to understand how this story changes once we allow general initial states.

4 Ways to Violate the Consistency Relations

In this Section, we dissect the possible ways in which (3.13) can fail to hold, and the consistency relations can be violated, for general initial states. Although the Slavnov-Taylor identity is most simply derived in the 3d framework, as reviewed in Sec. 3, to get further insights we must trace back the possible consistency violations to properties of the 4d vertex functional.

For concreteness, we focus here on pure initial states, though most of our results straightforwardly generalize to the mixed case. In the pure case, the relation between 3-point interaction vertices in the 3d and 4d pictures is given by (2.19). Applying this relation to the cubic scalar vertex, we obtain

$$
\Gamma^{3d} \zeta \zeta \zeta (\vec{q}, \vec{p}, -\vec{q} - \vec{p}) = i \int \limits_{t_0}^{t} dt_1 dt_2 dt_3 \frac{P_\zeta(q, t, t_1)}{P_\zeta(q)} \frac{P_\zeta(p, t, t_2)}{P_\zeta(p)} \frac{P_\zeta(|\vec{p} + \vec{q}|, t, t_3)}{P_\zeta(|\vec{p} + \vec{q}|)} \delta^3 \Gamma^{4d} \frac{\delta \zeta_\vec{p}(t_1) \delta \zeta_\vec{q}(t_2) \delta \zeta_{-\vec{q} - \vec{p}}(t_3)}{\delta \zeta_{-\vec{q}}(t_1) \delta \zeta_{-\vec{q}}(t_2) \delta \zeta_{-\vec{q} - \vec{p}}(t_3)}, 
$$

where $P_\zeta(p) \equiv P_\zeta(p, t)$ is the scalar power spectrum, and $P_\zeta(t, t_i)$ is the non-equal time 2-point function. Similarly for vertices involving tensors. Notice that momentum conserving delta functions have been removed, hence these are on-shell interaction vertices. As the discussion at the end of Sec. 3 already suggests, we will find two possible sources of violations:

- The initial state may itself encode large non-Gaussianities. This will give a non-analytic contribution to $\Gamma^{4d}$, which will translate to a non-analytic contribution to $\Gamma^{3d}$ via (4.1). We will study this possibility in Sec. 4.3.

- Even if the initial state is Gaussian, it may modify the non-equal-time power spectra $P_\zeta(t, t_i)$ appearing in (4.1). This in turn can spoil the link between analyticity of $\Gamma^{4d}$ and analyticity
of $\Gamma^{3d}$. In particular, in Sec. 4.4 we will see with explicit examples of Bogoliubov states that an analytic $\Gamma^{4d}$ can map to a non-analytic $\Gamma^{3d}$, thereby violating (3.13).

4.1 Spatial Diffeomorphism Invariance

We begin by studying the implications of spatial diffeomorphism invariance for $\Gamma^{4d}$. Recall that in the ADM formalism, the metric components are decomposed into lapse function $N$, shift vector $N^j$ and spatial metric $H_{ij} = a^2(t) (\delta_{ij} + h_{ij})$. In uniform-density gauge, the metric perturbations are further decomposed into scalar and tensor modes via $h_{ij} = e^{2\zeta} (e^\gamma)_{ij} - \delta_{ij}$, however for convenience let us stick to $h_{ij}$ for a while.

For pure initial states, the density matrix factorizes and is given by (2.18). At tree level, we can decompose $\Gamma^{4d}$ as follows:

$$\Gamma^{4d}[h^+, h^-] = S[h^+] - S[h^-] + S[h^+; t_0] - S[h^-; t_0].$$  \hspace{1cm} (4.2)

Here, $S$ is the diffeomorphism-invariant action of the theory. (Technically, it is reparametrization-invariant up to a gauge-fixing term, however the gauge-fixing term can be ignored at tree level [41].) Meanwhile, $S$ is the contribution from the density matrix, as defined in (2.18). Note that in writing down (4.2) we have ignored the possible dependence on $N$ and $N^j$. For the "bulk" action $S$, we imagine that these auxiliary fields have been integrated out using the constraints. For the initial state contribution $S$, we recall that quantum-mechanically the primary constraints of General Relativity reduce to [63]

$$\Pi_N |\Psi\rangle = 0; \quad \Pi_N^j |\Psi\rangle = 0.$$  \hspace{1cm} (4.3)

where $\Pi_N$ and $\Pi_N^j$ are the conjugate momenta to $N$ and $N^j$, respectively. Equations (4.3) state that the shift and lapse are non-dynamical degrees of freedom. In the configuration basis, they imply that the wavefunctional is independent of $N$ and $N^j$. This makes physical sense. Since all dynamical degrees of freedom are described by $h_{ij}$, while $N$ and $N^j$ are merely auxiliary fields. States constructed out of the vacuum state by applying creation operators of the physical particles would contain $h_{ij}$ only. Hence $|\Psi\rangle$ must be independent of $N$ and $N^j$.

In order to understand the possible impact of the modified initial state on the validity of consistency relations, it is imperative to understand the properties of $S$ imposed by the consistency of the theory. The momentum constraints embody the requirement of spatial diffeomorphism invariance on the possible initial states [52, 63]. Demanding that $S$ be invariant under those symmetries amounts to the requirement

$$\int d^3x \delta h^\pm_{ij}(\vec{x}, t_0) \frac{\delta S[h^\pm; t_0]}{\delta h^\pm_{jk}(\vec{x}, t_0)} = 0.$$  \hspace{1cm} (4.4)

\footnote{Similarly, physically-allowed density operators must be constructed from $|\Psi\rangle$'s satisfying (4.3), and the corresponding density matrix are therefore independent of $N$ and $N^j$. To conclude, $S$ only depends on $h^\pm$ evaluated at the initial time $t_0$.}
where $\delta h_{ij}^{\pm}$ is the transformation under diffeomorphisms:

$$
\delta h_{ij}^{\pm} = \partial_i \xi_j + \partial_j \xi_i + \xi^k \partial_k h_{ij}^{\pm} + h_{ik}^{\pm} \partial_j \xi^k + h_{jk}^{\pm} \partial_i \xi^k .
$$

(4.5)

Substituting this explicit form of the transformation and using the fact that $\xi^i$ is arbitrary, (4.4) implies

$$
2 \partial_j \frac{\delta S[h^{\pm}; t_0]}{\delta h_{jk}^{\pm}(x)} - \partial_k h_{ij}^{\pm}(x) \frac{\delta S[h^{\pm}; t_0]}{\delta h_{ij}^{\pm}(x)} + 2 \partial_j \left( h_{ik}^{\pm}(x) \frac{\delta S[h^{\pm}; t_0]}{\delta h_{jk}^{\pm}(x)} \right) = 0 .
$$

(4.7)

By varying this a number of times with respect to $h$, we obtain all possible constraints on the vertices coming from diffeomorphism invariance. Perhaps not surprisingly, the resulting identities are identical to Slavnov-Taylor identities derived in Sec. 3. In other words, the initial state must itself satisfy the consistency relations! The severity of this relation depends on the assumed analyticity properties of the wavefunctional, in complete analogy with the discussion of the previous section. In particular, in case of the analytic $S$ the Gaussian and non-Gaussian parts of the initial state/density matrix are interwoven: the squeezed limit of the $n^{th}$-order term of $S$ is related to $(n-1)^{th}$-order term. However, in case of the more general (i.e. non-analytic) states, the constraint (4.7) becomes rather mild. Taking all this into account, the simplest way to construct an initial wavefunction consistent with the quantum constraints is to write $S$ as a general functional of curvature scalars built out of $h_{jk}^{\pm}$.

Similarly, the “bulk” action $S[h^{\pm}]$ satisfies an identical identity:

$$
2 \partial_j \frac{\delta S[h^{\pm}]}{\delta h_{jk}^{\pm}(x)} - \partial_k h_{ij}^{\pm}(x) \frac{\delta S[h^{\pm}]}{\delta h_{ij}^{\pm}(x)} + 2 \partial_j \left( h_{ik}^{\pm}(x) \frac{\delta S[h^{\pm}]}{\delta h_{jk}^{\pm}(x)} \right) = 0 .
$$

(4.8)

And since $\Gamma^{4d}$ is just a linear combination of $S$ and $\beta$, it also satisfies this identity. The rest of the analysis proceeds as in Sec. 3. Namely, upon decomposing $h_{ij}$ into $\zeta$ and $\gamma_{ij}$, we find that $\Gamma^{4d}$ satisfies an equation analogous to (3.5):

$$
2 \partial_j \left( \frac{1}{6} \delta_{ij} \frac{\delta \Gamma^{4d}}{\delta \zeta^{\pm}} + \frac{\delta \Gamma^{4d}}{\delta \gamma_{ij}^{\pm}} \right) = \partial_i \zeta^{\pm} \frac{\delta \Gamma^{4d}}{\delta \zeta^{\pm}} + \ldots
$$

(4.9)

where the ellipses again encode irrelevant terms. This can be varied twice with respect to $\zeta$ to obtain a Slavnov-Taylor identity of the form (3.6) for the cubic part of $\Gamma^{4d}$. For instance, the cubic

\[8\text{Allowing for mixed states, the more general requirement is}\]

\[
\sum_{\pm} \left[ 2 \partial_j \frac{\delta S[h^+, h^-; t_0]}{\delta h_{jk}^{\pm}(x, t_0)} - \partial_k h_{ij}^{\pm}(x, t_0) \frac{\delta S[h^+, h^-; t_0]}{\delta h_{ij}^{\pm}(x, t_0)} + 2 \partial_j \left( h_{ik}^{\pm}(x, t_0) \frac{\delta S[h^+, h^-; t_0]}{\delta h_{jk}^{\pm}(x, t_0)} \right) \right] = 0 .
\]

(4.6)

\[9\text{Recall that we have ignored the gauge-fixing term in our tree-level considerations. More generally, (4.8) would be satisfied by the action minus the gauge-fixing term.}\]
vertex for scalars receives contributions from both $S$ and $\mathcal{S}$,

$$\frac{\delta^3 \Gamma^{4d}}{\delta \zeta^3_{(t_1)} \delta \zeta^3_{(t_2)} \delta \zeta_{-\vec{q} - \vec{p}}(t_3)} = \frac{\delta^3 S}{\delta \zeta^3_{(t_1)} \delta \zeta^3_{(t_2)} \delta \zeta_{-\vec{q} - \vec{p}}(t_3)} + \frac{\delta^3 \mathcal{S}}{\delta \zeta^3_{(t_1)} \delta \zeta^3_{(t_2)} \delta \zeta_{-\vec{q} - \vec{p}}(t_3)},$$

(4.10)

and similarly for vertices involving tensors. The validity of the consistency relations has now been traced back to the analyticity properties of $\Gamma^{4d}$, and hence of $S$ and $\mathcal{S}$, in the $\vec{q} \to 0$ limit.

### 4.2 From 4d to 3d

If perturbations have constant growing mode solutions as $\vec{q} \to 0$, then the bulk action $S$ will become local in this limit [41]. For instance, non-local contributions to $S$ at cubic order arise from integrating out the shift vector, whose solution (at linear order) includes

$$N_i \supset -a^2 \frac{\dot{H}}{H^2} \vec{q} \cdot \vec{p} \dot{\zeta}.$$  

(4.11)

For adiabatic modes, however, $\dot{\zeta} \propto q^2$, and this contribution becomes local as $\vec{q} \to 0$. Conversely, in models where $\zeta$ is not constant outside the horizon (e.g., because of background instabilities [64]), the action remains non-local even as $\vec{q} \to 0$. In other words, adiabaticity implies that the cubic part of $S$ is fixed up to $q^2$ order by symmetries.

However, a similar argument cannot be made for $\delta^3 S/\delta \zeta^3$. The late-time behavior of mode functions clearly has no relevance for this contribution which is localized at the initial time. In general, therefore, the cubic vertex of $\mathcal{S}$ is model-dependent to all orders in $q$. In particular, it may contain large initial non-Gaussianities, which should invalidate the consistency relations.

To make these statements more concrete, consider the scalar cubic part of a generic initial state, which we assume is translation invariant:

$$S_3 = \frac{1}{3!} \int d^3 k_1 d^3 k_2 \left( C^*_{k_1, k_2, -k_1 - k_2} \zeta^*_{k_1}(t_0) \zeta^+_{k_2}(t_0) \zeta^+_{-k_1 - k_2}(t_0) - C^*_{k_1, k_2, -k_1 - k_2} \zeta^-(t_0) \zeta^-_{k_2}(t_0) \zeta^-_{-k_1 - k_2}(t_0) \right).$$

(4.12)

This results into the following on-shell expression,

$$\frac{\delta^3 S}{\delta \zeta^3_{(t_1)} \delta \zeta^3_{(t_2)} \delta \zeta_{-\vec{q} - \vec{p}}(t_3)} = (t_1 - t_0) \delta(t_2 - t_0) \delta(t_4 - t_0) C_{\vec{q}, \vec{p}, -\vec{q} - \vec{p}}.$$  

(4.13)

Putting this together with (4.10) and (4.1), we obtain the explicit contribution to the cubic part of the 3d vertex functional:

$$\Gamma^{3d \zeta \zeta \zeta}(\vec{q}, \vec{p}, -\vec{q} - \vec{p}) = \int_{t_0}^{t} dt_1 dt_2 dt_3 \frac{P_\zeta(q, t, t_1)}{P_\zeta(q)} \frac{P_\zeta(p, t, t_2)}{P_\zeta(p)} \frac{P_\zeta(|\vec{p} + \vec{q}|, t, t_3)}{P_\zeta(|\vec{p} + \vec{q}|)} \frac{\delta^3 S}{\delta \zeta^3_{(t_1)} \delta \zeta^3_{(t_2)} \delta \zeta_{-\vec{q} - \vec{p}}(t_3)}$$

$$+ \frac{i P_\zeta(q, t, t_0)}{P_\zeta(q)} \frac{P_\zeta(p, t, t_0)}{P_\zeta(p)} \frac{P_\zeta(|\vec{p} + \vec{q}|, t, t_0)}{P_\zeta(|\vec{p} + \vec{q}|)} C_{\vec{q}, \vec{p}, -\vec{q} - \vec{p}}.$$  

(4.14)
This expression clearly displays the possible non-analytic contributions to $\Gamma^{3d}$ arising from modifications to the initial state. The second line represents the non-Gaussian contribution from the initial state, whose $q$ dependence is not fixed by symmetries as we have seen. The initial state also plays a role in the first line, through the non-equal time 2-point functions. Even if the cubic part of $S$ is local as $\vec{q} \to 0$, the time integral of the non-equal time 2-point functions can result in non-analytic contributions to $\Gamma^{3d}$. In the remainder of the Section, we will study explicit examples of these two sources of non-analyticity.

### 4.3 Initial Local Non-Gaussianity

If the initial state includes non-Gaussianities, then clearly we do not expect the consistency relations to hold. Usually, one has in mind that the initial state is specified at the onset of inflation. In this context, the initial state (and the extent to which it is non-Gaussian) parametrizes our ignorance about initial conditions for inflation. Alternatively, nothing prevents us from setting the initial time to be much later in the evolution. A classic example is a multi-field scenario, such as the curvaton [65] or variable-decay [66, 67] mechanisms, where the initial time might naturally coincide with the onset of adiabatic evolution, \textit{i.e.}, once the additional degrees of freedom have all decayed away. In this case, the non-Gaussianities generated by the additional fields can only appear in the initial state.

For concreteness, let us consider \textit{local} initial non-Gaussianity, which is the shape of interest for multi-field scenarios. By definition, the 3-point amplitude is set by the well-known (constant and model-dependent) $f_{NL}$ parameter:

$$
\langle \zeta \zeta \zeta \rangle' = \frac{6}{5} f_{NL} \left( P_\zeta(q) P_\zeta(p) + P_\zeta(q) P_\zeta(|\vec{p} + \vec{q}|) + P_\zeta(p) P_\zeta(|\vec{p} + \vec{q}|) \right). \quad (4.15)
$$

Using (3.8), we can infer the contribution to the 3d vertex functional. In the squeezed limit, it is given by

$$
\Gamma^{3d, \zeta \zeta}(0, \vec{p}, -\vec{p}) = \frac{12}{5} \frac{f_{NL}}{P_\zeta(p)}. \quad (4.16)
$$

To read off the corresponding cubic part of the initial state, simply set $t = t_0$ in (4.14), with the result:

$$
C_{0,\vec{p},-\vec{p}} = \frac{12}{5} \frac{f_{NL}}{P_\zeta(p)} \sim O(q^0). \quad (4.17)
$$

This manifestly violates the analyticity requirement $C_{\vec{q},\vec{p},-\vec{q}-\vec{p}} \sim O(q^2)$. Thus initial local non-Gaussianities result in $A_{ij} \sim q^0$, which violate the consistency relations as expected.

### 4.4 Initial Gaussian Modification

Next let us focus on the contribution to $\Gamma^{3d}$ from the first line of (4.14). In other words, we focus on purely Gaussian initial states. For concreteness, consider the family of states that are related
to the Bunch-Davies vacuum by a Bogoliubov transformation (so-called Bogoliubov states). To define such a state one decomposes the curvature perturbation as
\[ \zeta_q(\tau) = a_q u_q(\tau) + a_q^{\dagger} u^{* -q}(\tau), \]
where \( a_q^{\dagger} \) and \( a_q \) are the creation and annihilation operators, and \( \tau \) is conformal time. The mode function \( u_q(\tau) \) is a linear combination of positive and negative frequency modes
\[ u_q(\tau) = \alpha_q \zeta_{qBD}(\tau) + \beta_q \zeta_{qBD}^{*}(\tau), \]
where the coefficients satisfy the normalization condition \( |\alpha_q|^2 - |\beta_q|^2 = 1 \). The Bunch-Davies choice is \( \alpha_q = 1, \beta_q = 0 \).

In slow-roll approximation, for simplicity, the Bunch-Davies mode function \( \zeta_{qBD}(\tau) \) is given by the familiar Hankel function
\[ \zeta_{qBD}(\tau) = \frac{H}{\sqrt{2q^3}}(1 - iq\tau)e^{iq\tau}. \]
By definition, the Bogoliubov vacuum state is annihilated by all \( a_q^{\dagger} \):
\[ a_q^{\dagger}\Omega = 0, \quad \forall q. \]
See Appendix A for the construction of the wavefunctional corresponding to this state.

As it follows from the explicit form of the mode functions, the non-equal time 2-point function for \( \zeta \) is given by
\[ P_{\zeta}(q, \tau, 0) \propto \left( A_q(1 - iq\tau)e^{iq\tau} + B_q(1 + iq\tau)e^{-iq\tau} \right), \]
where \( A_q \) and \( B_q \) are related to the Bogoliubov’s coefficients by
\[ A_q = (\alpha_q - \beta_q)\alpha_{-q}^{*}, \quad B_q = -(\alpha_q - \beta_q)\beta_{-q}^{*}. \]
The Bunch-Davies vacuum corresponds therefore to \( A_q = 1 \) and \( B_q = 0 \).

In order to understand how the \( q \)-dependence is translated in the 3d formalism, let us consider for illustrative purposes the particular interaction term
\[ S_{\zeta^3} \sim \int dt d^3x a^3(t)\zeta^3. \]
It is well known that this interaction term in slow-roll inflation behaves as \( q^2 \) in the soft limit. The late-time contribution to \( \Gamma^{3d} \) from the first line of (4.14), after converting to conformal time, is given by
\[ \Gamma^{3d}_{\zeta\zeta\zeta}(q, p, -q - p) \sim -i \int_{\tau_0}^{0} d\tau' a(\tau') \frac{\partial_{\tau'}P_{\zeta}(q, \tau', 0)}{P_{\zeta}(q)} \frac{\partial_{\tau'}P_{\zeta}(p, \tau', 0)}{P_{\zeta}(p)} \frac{\partial_{\tau'}P_{\zeta}(|q + p|, \tau', 0)}{P_{\zeta}(|q + p|)} + \text{c.c.}(4.25) \]

\[ \text{We also assume } c_s = 1 \text{ for simplicity.} \]
From (4.22), each 2-point function in (4.25) contains both positive and negative frequency contributions, in contrast to the Bunch-Davies vacuum. Based on this observation, it was noticed in [36] that one gets enhanced non-Gaussianity in the squeezed limit from the cross terms. Specifically, the enhanced contributions are of the form

$$\Gamma_{3d}\zeta\zeta\zeta(q, p, -q - p) \supset i p^4 \frac{A_q A_p B_{-p - q}}{H (A_q + B_q)(A_p + B_p)(A_{-p - q} + B_{-p - q})} \int_{\tau_0}^0 d\tau q^2 \tau^2 e^{i(q + p - |p + q|)\tau} + c.c. (4.26)$$

Notice that the physical soft limit corresponds to $q \to 0$, $\tau_0 \to -\infty$ keeping $q\tau_0 > 1$ fixed. In this way, the long mode started out inside the horizon. For such a squeezed limit, it is straightforward to see that

$$\lim_{q \to 0 & \tau_0 \gg 1} \int_{\tau_0}^0 d\tau q^2 \tau^2 e^{i(q + p - |p + q|)\tau} \to i \frac{q\tau_0^2}{1 - q \cdot p} e^{i(1 - \hat{q} \cdot \hat{p})q\tau_0}. (4.27)$$

Hence, as advocated, we see that

$$\lim_{\hat{q} \to 0} \Gamma_{3d}\zeta\zeta\zeta(q, p, -q - p) \sim q. \quad (4.28)$$

This is interesting because a contribution that naively looked irrelevant up to $O(q^2)$ becomes $O(q)$. This enhancement traces back to the relative sign between momenta in the phase of the integrand of (4.26). As result, in the case of modified initial states one cannot argue that $A_{ij} \propto q^2$, even if the initial state is Gaussian.

Although we so far focused on the $\dot{\zeta}^3$ interaction, it is straightforward to show that the same conclusion applies to other vertices. In general, the amplitude scales as $q$, but the actual function is shape-dependent and peaks in the limit of flattened triangle. This can be checked with the exact expression derived in [36],

$$\Gamma_{3d}\zeta\zeta\zeta(q, p, -q - p) \propto q^2 p^4 \sum_{l,m,n=0}^1 c_q^{(l)} c_q^{(m)} c_{-p - q}^{(n)} F \left((-1)^l q, (-1)^m p, (-1)^n |p + q|\right) + c.c., (4.29)$$

with $c_q^{(0)} = A_q$ and $c_q^{(1)} = B_q$. The shape function $F$ is defined by

$$F(p_1, p_2, p_3) = -\frac{2}{K^3} + e^{iK\tau_0} \left(\frac{2}{K^2} - \frac{2i\tau_0}{K} - \tau_0^2\right), (4.30)$$

where $K \equiv p_1 + p_2 + p_3$. For completeness let us investigate the behavior of (4.29) in different squeezed limits. We focus on the cross-term contribution of the type (4.26) since it characterizes the deviation from the Bunch-Davis vacuum. In other words, let us look more closely at the soft limits of $F_{A,A,B}$:

\[\text{In deriving (4.27) we assumed } (1 - \hat{q} \cdot \hat{p})q\tau_0 \gg 1, \text{ which breaks down in the collinear limit } \hat{q} \cdot \hat{p} \simeq 1. \text{ Hence the apparent divergence in this limit is an artifact of our approximation.}\]
• $q\tau_0 \ll 1$. In this case, the mode of interest is so long that it was outside the horizon even at the onset of inflation. Since $K = q + p - |\vec{p} + \vec{q}|$ for $F_{AAB}$, in the limit $q\tau_0 \ll 1$ we can expand in powers of $K\tau_0$:

$$F_{AAB} = \frac{i\tau_0^3}{3} + \frac{\tau_0^4}{4} K + \mathcal{O}(K^2).$$

(4.31)

Hence, in this extreme limit $\Gamma^{3d\zeta\zeta}\sim q^2$, and thus its analyticity property is the same as in the Bunch-Davis case. In particular, the consistency relations hold [68]. However, it must be stressed that this regime is not physically interesting since the long mode is always outside the horizon.\footnote{Note that the flattened triangle limit considered earlier belongs to this regime since $K = q + p - |\vec{p} + \vec{q}| = 0$. One would conclude that the $\zeta^3$ amplitude should satisfy the consistency relations, in apparent contradiction with (4.27). The resolution is that, the flat limit, the expansion performed in (4.27) is inapplicable.}

• $q\tau_0 \gg 1 \& q \ll p$. This is the more physically-interesting case where the long mode started out inside the horizon. Since we have already addressed the case of flattened triangle, let us now consider $K \sim q \neq 0$. It is straightforward to show that (4.30) reduces to

$$F_{AAB} \simeq -\frac{\tau_0^2}{K} e^{iK\tau_0}; \quad K \simeq q (1 - \hat{q} \cdot \hat{p}).$$

(4.32)

Hence, $\Gamma^{3d} \sim q$ in this case, rather than $q^2$, and the consistency relations are violated. In other words, model-dependent contributions to the action, which we expect to be of order $q^2$, contribute to 3d interaction vertices at order $q$ in the case of excited Gaussian initial states. This confirms in generality what we found earlier in (4.28).

5 Conclusion

In this paper, we studied the impact of the initial state on the validity of the inflationary consistency relations, using the fixed-time path integral formalism. First, we showed that diffeomorphism invariance requires the initial wavefunctional to satisfy a Slavnov-Taylor identity similar to that of the action. Second, we identified two ways in which non-Bunch-Davies initial states can lead to violations of the consistency relations: i) the initial state can include non-Gaussianities that are not constrained by symmetries; ii) even if the initial state is Gaussian, such as a Bogoliubov state, the modified non-equal time 2-point functions appearing in the $4d \to 3d$ translation can map an analytic $\Gamma^{4d}$ to a non-analytic $\Gamma^{3d}$.

As future directions, it would be interesting to study the impact of modified initial states on the consistency relations for the large-scale structure [58–61], as well as those of the conformal alternative to inflation [69–78].
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Appendix A: Wavefunctional for Bogoliubov States

In this Appendix, we construct explicitly the wavefunctional for Bogoliubov states, i.e. states related to the Bunch-Davies vacuum by a Bogoliubov transformation. For simplicity, we focus on the curvature perturbation $\zeta$. Its decomposition in terms of the creation and annihilation operators is given by

$$\zeta(\vec{x}, \tau) = \int d^3p \left( a_{\vec{p}} u_{\vec{p}}(\tau) e^{i\vec{p} \cdot \vec{x}} + a^\dagger_{\vec{p}} u^*_{\vec{p}}(\tau) e^{-i\vec{p} \cdot \vec{x}} \right),$$

(A-I)

where the mode function $u_{\vec{p}}(\tau)$ is given by (4.19). Manifestly, the $a^\dagger_{\vec{p}}$ and $a_{\vec{p}}$ operators are linear combinations of the usual creation and annihilation operators. The conventional vacuum choice is $\beta_{\vec{p}} = 0$ and $\alpha_{\vec{p}} = 1$.

By definition, the Bogoliubov vacuum state is annihilated by $a_{\vec{p}}$ for all momenta. To write down the corresponding wavefunctional, we must express the annihilation operators in terms of (the Fourier components of) the field $\zeta$ and its conjugate momentum $\Pi \equiv \dot{\zeta}$ at some fixed time $\tau_0$:

$$a_{\vec{p}} = D_{\vec{p}} \zeta_{\vec{p}}(\tau_0) + E_{\vec{p}} \Pi_{\vec{p}}(\tau_0),$$

(A-II)

with

$$D_{\vec{p}} = \left[ u_{\vec{p}}(\tau_0) \left( 1 - \frac{\dot{u}_{\vec{p}}(\tau_0)}{u_{\vec{p}}(\tau_0)} \frac{u^*_{-\vec{p}}(\tau_0)}{\dot{u}^*_{-\vec{p}}(\tau_0)} \right) \right]^{-1} \quad \text{and} \quad E_{\vec{p}} = -\frac{u^*_{-\vec{p}}(\tau_0)}{\dot{u}^*_{-\vec{p}}(\tau_0)} E_{\vec{p}} .$$

(A-III)

In the configuration basis, $\Pi_{\vec{p}}$ acts as $-i\delta/\delta \zeta_{-\vec{p}}$ as usual. The vacuum condition $a_{\vec{p}}|\Omega\rangle = 0$ becomes

$$\left( D_{\vec{p}} \zeta_{\vec{p}}(t_0) - iE_{\vec{p}} \delta \frac{\delta}{\delta \zeta_{-\vec{p}}(t_0)} \right) \Psi[\zeta, \tau_0] = 0 ,$$

(A-IV)

where $\Psi[\zeta, \tau_0] = \langle \zeta | \Omega \rangle$ is the initial wavefunctional. This solution is a Gaussian state

$$\Psi[\zeta, \tau_0] \sim e^{iS} ,$$

(A-V)

where

$$S = -\frac{1}{2} \int d^3p \frac{D_{\vec{p}}}{E_{\vec{p}}} \zeta_{\vec{p}}(\tau_0) \zeta_{-\vec{p}}(\tau_0) .$$

(A-VI)
Substituting the explicit form for $D$ and $E$ given in (A-III), we get

\[
S = \frac{1}{2} \int d^3p \left( \frac{p^2 \tau_0}{1 + ip \tau_0} + \frac{2i \beta^* \beta p^3 \tau_0^2 e^{2ip \tau_0}}{\alpha^* (1 + ip \tau_0)^2 + \beta^* \beta^* (1 + p^2 \tau_0^2) e^{2ip \tau_0}} \right) \zeta_p(\tau_0) \zeta_{-p}(\tau_0) . \tag{A-VII}
\]

The first term in parentheses brackets corresponds to the vacuum contribution of the standard Bunch-Davies state. The second term parametrizes excitations from that state.

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