Research on the Development of a Traffic Signal Control Model based on Route Travel Time Equilibrium

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Abstract- In this paper, the problem of how to relieve traffic congestion was studied based on traffic signal control and traffic assignment. A bi-level planning model was built. The upper model was designed to minimize the total system delay time while the lower model was designed to maintain travel time equilibrium along any route chosen among the intra-regional O-D pairs. For the calculation of travel time, adequate consideration was given to the road travel time and the signal delay time caused at intersections. The model was then solved. The simulation results show that with inter-route travel time equilibrium maintained, the delay time and travel time throughout the road network were reduced compared with signal coordination control, with the algorithm validated.

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1. Introduction
With the rapid increase in urban traffic, urban traffic congestion has become an important problem faced by cities. Traffic assignment and traffic signal control are two different ways to solve the problem of urban traffic congestion. In recent years, many academic researchers have combined the two together and conducted a lot of research. Sun Q [1], who considered drivers’ route choice behavior, proposed a new adaptive joint model of user equilibrium and signal control; Li RM [2], who considered the vehicular queue behavior and drivers’ route choice behavior that occurred on saturated road sections, built an integrated model of dynamic traffic assignment and traffic control in saturated road networks and described a bi-level planning problem. Zhou XZ [3], who considered the influence of vehicular queue on saturated sections on equilibrium traffic flow assignment and drivers’ route choice behavior, studied how to integrate traffic assignment with signal control in a congested traffic network. Du W [4] constructed a combined model of stochastic user equilibrium traffic assignment and signal control. The upper layer was a signal timing optimization model while the lower layer was a stochastic user equilibrium traffic assignment model considering delay at intersections. Chen XM [5] built a bi-level planning model for regionally coordinated multi-phase timing control optimization based on the mechanism of interaction between signal timing and route choice. Chiou [6] built a bi-level planning model of dynamic traffic control and network traffic assignment for regional traffic control.

As for the study of how to solve urban traffic congestion through traffic signal control, network equilibrium is increasingly researched as a control target by researchers. Yang H and Yagar S [7] predicted equilibrium traffic flow and traffic light parameters under a fixed traffic demand by taking queue length and congestion effect into consideration. They used a bi-level planning model, where the upper layer was used to determine the green time ratio to achieve system optimization and consider travelers’ sensitivity to signal changes; the lower layer was used to handle the user equilibrium problem, i.e., travelers would choose a route according to signal changes. Wong et al. [8], who were the first to...
explore the problem of spare network capacity under the condition of user equilibrium, built a bi-level planning model, which was solved by the sensitivity analysis method. Gao et al. [9] expanded the definition of road network capacity and improved the model designed by Wong. They argued that for a road network, the growth rate of the O-D demand multipliers might be different. According to specific examples, a road network could gain larger spare capacity, given the very definition. Kuang AW et al. [10] built a stochastic user equilibrium traffic assignment model based on the road network equilibrium modeling theory to meet multi-user elastic demands. Li ZC et al. [11] used the network equilibrium theory and super network method to solve the problem of hybrid network equilibrium faced by combined travel under an elastic demand. They also built a variational inequality model equivalent to equilibrium conditions and designed an algorithm for model solving.

In this paper, a bi-level planning model was built based on the above-mentioned literature studies. With the upper model designed to minimize the total system delay time while the lower model aimed at maintaining travel time equilibrium along any route chosen among the intra-regional O-D pairs, the bi-level planning model was used to calculate the travel time with sufficient consideration to the road travel time and the signal delay time caused at intersections. The model was then solved.

2. Modeling
A bi-level planning model was built in this paper based on dynamic traffic assignment and signal control. The upper model would be used to minimize the total system delay while the lower model would be used to maintain route travel time equilibrium.

2.1. Upper model

\[
\min \sum_{r \in R} \sum_{s \in S} \sum_{a \in A} f_a(k)(t_a(k, \gamma) + d_a(k, \gamma))
\]

In other words, the target function for the upper model was all about minimizing the total delay on all routes. To be specific, the delay time included the free travel time generated on road sections and the signal delay time generated at associated intersections. The total system delay would be the smallest as long as the product of the delay time and the total flow reached its minimum.

where \( R \) is the set at the starting point; \( S \) is the set at the ending point; \((r, s)\) represent O-D pairs; \( a \) represents a directed section; \( A \) is a set of sections. \( f_a \) represents the traffic flow on directed section \( a \); \( t_a \) represents the free travel time on section \( a \); \( d_a \) represents the signal delay time at relevant intersections on section.

2.2. Lower model

\[
\min \left| T_m - T_n \right| \quad m, n \in P
\]

\[
T_m = \sum_{a \in A} W_{am}^p(t_a(k, \gamma) + d_a(k, \gamma))
\]

\[
T_n = \sum_{a \in A} W_{an}^p(t_a(k, \gamma) + d_a(k, \gamma))
\]

The target function for the lower model expressed the least absolute difference in travel time between any two routes chosen, indicating travel time equilibrium between the routes.

\( T_m \) and \( T_n \) represent the travel time on routes \( m \) and \( n \), respectively; \( P \) is a set of routes; \( W_{am}^p \) (\( W_{an}^p \)) is a route association coefficient, and section \( a \) is on route \( m(n) \) in \( P \), set to 1, or 0.
2.3. Model parameter calculation

The sum of the green signal ratios of the associated road sections at all signalized intersections was equal to 1. Because the red time was used as a parameter when the delay time at signalized intersections was calculated in the model, the sum of the ratios of red time to cycle on associated road sections at all signalized intersections was equal to 1. See Equation (5), where $A_i$ is a set of sections related to signalized intersection $i$; $\lambda_j$ represents the green signal ratio of the phase where section $j$ was located; $r_j$ represents the relevant red time; $C_i$ represents the cycle length signalized intersection $i$.

$$\sum_{a \in A_i} \lambda_a = 1 \quad \text{and} \quad \sum_{j=1}^{A_i} \lambda_j = \sum_{j=1}^{A_i} \frac{r_j}{C_i} = 1$$ (5)

![Fig.1 Schematic diagram of road travel time](image)

The lower model was aimed at maintaining route travel time equilibrium. When calculated, the travel time on each road section was divided into two parts: One was the free travel time generated on non-queuing sections while the other was the delay time generated in queue at signalized intersections, represented by $t_a$ and $d_a$, as shown in Fig.1.

The free travel time was equal to the ratio of the length of the non-queuing sections to the vehicle speed, as shown in Equation (6).

$$t_a = \frac{L_a - L_q}{V_a}$$ (6)

$$w_{stop} = Q/(k_j - k)$$ (7)

$$L_q = w_{stop} r = Q r / (k_j - k)$$ (8)

The linear traffic flow model $v = v_f (1 - k / k_j)$ and basic traffic flow model $Q = vk$ were substituted into Equation (8), so,

$$L_q = v_f k r / k_j$$ (9)

$$t_a = \frac{L_a}{v_f (k_j - k)}$$ (10)

$$d_a = \frac{L_q}{w_a} = \frac{v_f k r}{Q k_j (k - k_j)}$$ (11)

$$f_a = v k = v_f (1 - k / k_j) k$$ (12)

where $L_a$ represents the length of section $a$; $L_q$ represents the queue length at the intersection associated with the section; $V_a$ represents the vehicle speed on section $a$; $w_{stop}$ represents the stop wave velocity; $k$ represents the traffic flow density on the section; $k_j$ represents the traffic jam density on the section; $r$ represents the red time at the intersection associated with $a$; $w_a$ represents the dissipation wave velocity; $v_f$ represents the free traveling speed; $Q_a$ represents the saturation flow rate on section $a$. 


Thus it can be seen that in the upper model and the lower model, the two control variables were the green signal ratio and the density. The lower model considered the drivers’ travel choice behavior while the upper model reflected the consideration of the result of the drivers’ travel choice behavior.

3. Model solving and simulation

A road network model (Fig. 2) was built using VISSIM, a piece of simulation software. A travel time detector was added to three horizontal routes. By secondary development of VISSIM, the genetic algorithm was used to solve the target functions established in this paper to optimize the green signal ratio. A comparison was made with signal coordination control.

The reason for choosing the genetic algorithm is that the genetic algorithm is a probabilistic algorithm for global optimization. It simulates a problem to be solved as a process of biological evolution and generates a next-generation solution via duplication, crossover and mutation. Then, it phases out the solutions with low fitness function values and develops more solutions with high fitness function values. After N generations of evolution, an individual with high fitness function values may be evolved.

Its major advantages are as follows:
- The genetic algorithm does not have too many mathematical requirements for the optimization of solutions. Owing to its evolutionary characteristics, the inherent nature of the problem is not needed in the search process. For any form of target function and constraint, whether it is linear or nonlinear, discrete or continuous, it can be progressed all the time.
- The ergodicity of evolutionary operators enables the genetic algorithm to perform a global search of probabilistic significance very effectively.
- The genetic algorithm can provide great flexibility for various special problems to achieve mixed construction of a domain-independent heuristic method to ensure prove the validity of the algorithm.

In summary, this paper first set 11 intersections in the road network as an initial population. With the minimum green time set to 10s and the public cycle of the road network set to 60s, 500 sets of population vectors were generated stochastically under the constraint condition of (10s, 50s). The element (integer) in each vector corresponded to the green time at phase 1 at an intersection. Because every road network is two-phase, it is not hard to get the conclusion that the green time at phase 2 is equal to 60s minus population vectors. In addition, the elements of each set of population vectors were encoded into binary to facilitate crossover and mutation operations. Finally, loop iteration was performed ceaselessly, and new populations were generated by fitness selection and elimination, crossover and mutation operations until there was no any other better individual position to appear. Finally, an optimal solution was found.

| Fig.2 Simulation diagram of road network |

| Tab.1 shows the average delay travel time data of the three routes output after simulation performed using the method presented in this paper and the traditional method, respectively. It is not difficult to find by comparison that the method presented in this paper could greatly improve the road network performance. As can be seen from Fig.3, the travel time is greatly reduced, and the difference among the routes is reduced, too. This conforms to the idea of this paper and confirms the usefulness of the method presented in this paper. |
Tab.1 Comparison table of simulation and evaluation

|                  | Average delay (s) | Travel time (s) |
|------------------|-------------------|-----------------|
|                  | Route 1           | Route 2         | Route 3           | Route 1           | Route 2         | Route 3           |
| Present method   | 61.6              | 84.6            | 63.9              | 195.2             | 164.3           | 169.3             |
| Signal coordination control | 73.9              | 99.7            | 70.5              | 233.6             | 260.1           | 205.4             |

Fig.3 Comparison diagram of simulation and evaluation

4. Summary
A bi-level planning model was built in this paper. The upper model was designed to minimize the total system delay time while the lower model was designed to maintain travel time equilibrium along any route chosen among the intra-regional O-D pairs, the genetic algorithm was applied to the solving of the integrative model, improving the timeliness of model solving. A road network model was established through VISSIM for the algorithm in this paper, and then a simulation analysis was conducted. The simulation results were compared with the results achieved by the signal coordination control method. The simulation results show that with inter-route travel time equilibrium maintained, the delay time and travel time throughout the road network were reduced compared with signal coordination control, with the algorithm validated.

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