Stability Analysis of a Two Degree of Freedom Micro-drilling Model

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Abstract. This paper introduces the assembly system model of drilling tool, drilling tool and fixture. In this system, the failure of drilling tools is caused by self-excited vibration, that is, the self-excited vibration of components in the axial direction. In this paper, the stable lobes of rock macro and micro drilling are established, the deep well rotary drilling system is analyzed in detail, the influence of changing the axial damping coefficient on the critical speed of drilling in macro and micro drilling is investigated, and the flutter behavior in the process of drilling is considered comprehensively, so as to avoid tool flutter damage and workpiece scrap and improve productivity.

Keywords: self-excited vibration, Lobe pattern, dynamic stability.

1. Introduction
Deep drilling system play an important role in the exploration of natural resources[1], it is widely used to machine precise hole on the work-piece of the rock using conventional or micro drilling tools. The deep drilling system can be understood as rotary drilling. The dynamics of drilling tools plays a significant role in cutting force, chatter stability and quality. The pre-twisted geometry of the flute section contributes to the coupling of torsional-axial vibrations of the drilling tool[2]. When the torsional load is applied on the drilling tool and results into displacements in axial direction. Natural frequency and mode shapes corresponding to the coupled torsional-axial dynamics need to be determined to avoid unnecessary tool vibrations in the operations [3]. So it is essential to predict the stable cutting parameters to reduce losses. In this work, the author cites the background theory together to indicate how drill stability and micro-stability could be predicted from the experimental data, and the author changes the axial damping in the drilling to make a series of stability lobes. Following the research of the work[4], some conclusions are drawn to demonstrate technique.

2. A two degrees-of-freedom axial-torsional model
In this work, we present a coupled two degrees-of-freedom model for axial and torsional vibration, Figure 1 shows the physical model as a two degrees-of-freedom system (one axial, and one torsional) to model the axial and torsional vibrations of deeping rotary drilling system. The mathematical description for the deeping rotary drilling system is following to demonstrate the interaction between the drill bit and the rock. While the drill diameter and cutting width are in the same magnitude [5]. In the axial direction, the system is modeled as spring-mass-damper system with spring stiffness($k_a$),
viscous damping coefficient ($c_a$), the combined mass ($M$). For torsional direction, the rotary drilling system with torsional spring stiffness ($k_t$), torsional viscous damping coefficient ($c_t$)[6]. Reactions from rock $F$, and the reactive torque $T$, are acting on the drilling bit, the rotary system at the top is driven at constant angular velocity $\Omega$, the chip thickness is $h$, the mean chip thickness is $h_m$.

\[
F = K_a bh
\]

where, $b$ is radial width of cut. The general equations of motion for the dynamic rotary drilling system can be formulated in the stationary frame as follows:

\[
M\ddot{y}(t) + C_a \dot{y}(t) + k_a y(t) = M - F
\]

\[
J \ddot{\theta}(t) + C_t \dot{\theta}(t) + K_t \theta(t) = T
\]

where $y(t)$ is axial vibration displacement, $\theta(t)$ is torsional vibration displacement, $M$ and $J$ are the combined mass and rotary inertia about the rotational system

\[
J = \frac{K_t}{W_m^2}
\]

Note that we have ignored any wear flat on the drill-bit and consequently there are no frictional forces and torques. While we don’t take the wear of the drill-bit into consideration, consequently, frictional forces and torques are ignored. The cutting force and torque on the drill-bit are related to the system and operational parameters as:

\[
F = \varepsilon c r h \dot{H}(\Phi)H(h)
\]

where $\dot{H}(\Phi)$ indicate the transfer function, $\varepsilon$ is the cutter inclination coefficient, $\varepsilon$ is the rock specific strength, $r$ is the radius of the drill-bit, and $h$ is the instantaneous depth of cut per revolution of the drill-bit. Assuming that the rock is homogeneous and the drill-bit has $n$ identical cutters, the total depth of cut per revolution can be written as:

\[
h = nh_n
\]
where $h_n$ is the cutting depth of per cutter, $Y$ is the current vibration of the tool from its mean position, $Y_0$ is the vibration from the mean position for the previous pass. $t_n$ is the time taken by the drill-bit to rotate by an angle of $\frac{2\pi}{n}$ which can be computed through.

### 2.2. Chatter stability lobes due to torsional–axial vibration

To find the stability boundary, a frequency is chosen which is around the natural frequency. For this specific frequency, the delay time between the tool passages, $t_n$, is found so that the imaginary part of $b$ becomes zero. Subsequently, the speed at the limit of stability is determined from

$$\Omega = \frac{2\pi w}{z(2\pi j + wt_n)}$$

where $z$ is the number of cutting edges engaged in the cut and $j$ is the integer number of full waves. This procedure for plotting the boundary of stability can be summarized for $z=2$ in the diagram shown in Fig.2.

**Figure 2.** Chatter stability lobes due to torsional–axial vibration.

This results in the stability lobe diagram illustrates the relationship between stable depth of cut and spindle speed. The stability lobe diagram has enormous significance in practice. The inner part of the leaflet type is unstable area, at any speed, the area below the stability lobe diagram lines are stable area.

A two degree of freedom system has two natural frequencies. Vibration in either of the two degree of freedom system’s natural frequencies is associated with a characteristic deformation pattern. The magnitude and phase parts of the frequency response function emphasize key features of forced vibration. These test results show that the chatter frequency is related to the natural frequency of a non-rotating tool embedded into the workpiece. Fig.3 shows coupled frequencies at 500 and 770Hz.

$$\text{Re}(\frac{Y}{F}) = \frac{1}{k} \left( \frac{1-r^2}{(1-r^2)^2 + 2cr} \right)$$

$$\text{Im}(\frac{Y}{F}) = \frac{1}{k} \left( \frac{-2r}{(1-r^2)^2 + \omega^2} \right)$$

**Figure 3.** Coupling frequency of real part and virtual part.
3. Stability analysis of axial viscosity coefficient of drill pipe

One of the widely known in stability analysis is that retarding damping force is proportional to the viscous damping coefficient. The chatter occurs when the tip of the drill is supported, while the interactions between the rock and the drill-bit leads to occurrence of exciting force. The vibration speed itself will generate feedback control to the system, this theory gives better insight into the analysis of the stability lobes. If the system is between steady state and unstable state, this critical state can be called equal amplitude vibration.

\[
WK_a \sin \frac{60w}{f} C_a \frac{w}{j} = 0
\]  

(10)

The critical cutting width can be calculated

\[
w_{\text{min}} = -\frac{c_a}{k_a} \frac{w}{\sin \frac{60w}{f}}
\]  

(11)

The sum of damping of the system in axial direction is less than zero, then self-excited vibration occurs. It can be concluded that \(w_{\text{min}}\) is the stability Critical value.

\[
m\ddot{y}(t) + (k + wk_aB)y(t) = 0
\]  

(12)

The equation above becomes the equation of motion for the free vibration of an undamped system, from which the natural frequency is

\[
W = \sqrt{\frac{K + WK_aB}{m}}, \quad w^2 = \frac{k + wk_aB}{m} = w_n^2 + \frac{wk_aB}{m}
\]  

(13)

\(w\) is the frequency of self-excited vibration \(w_n\) is natural frequency of the system, \(k\) is the stiffness of the non-rotary drilling system. It can be seen that \(w > w_n\). The system stability equation can be expressed as:

\[
c_a + \frac{wk_aC}{w} = 0, \quad w_n^2 = \frac{wk_aB}{m} = w^2 - \frac{w}{\sin \frac{60w}{f}} = -1, \quad w_{\text{min}}^2 = \frac{c_a w}{k_a}
\]  

(14)

The minimum cutting width is proportional to the axial viscous damps, changing the axial damping coefficient affects the cutting stability lobes[6]. The minimum cutting width increases with the increase of the axial viscous damping coefficient. Fig.4 shows the stability lobes with axial viscous damping coefficient of 0.3-0.5.

![Image](image.png)

Figure 4. Chatter stability lobes due to the changes of the axial viscous coefficient in drilling.)
Micro-drilling is the process of producing holes less than 50 μm in diameter with a micro-tool. When micro-drilling is taken in rotary drilling of the rock[7]. The stability lobes show that the smaller the axial viscous damping coefficient, the greater the critical speed. The reason for this difference is that micro drilling is affected by the size effect and the minimum cutting thickness, and in the progress of micro drilling, the tool radius and the cutting thickness is an order of magnitude.

4. Conclusions
In this paper, we have established a two-degree-of-freedom drilling model (axial and torsional) and also established stability lobes for macro drilling and micro drilling of rocks. The influence of changing the axial damping coefficient on the critical rotating speed during drilling is analyzed. Overall we have studied fairly rich machining dynamics for rotary drilling system. The axial viscous progress damping between the drilling tools and the rock is obtained. This rubbing mechanism tends to dampen the vibration and it plays an important role in the stability of the tools. And also, another parameter play a significant role in damping vibration is the chisel edge. Therefore, the parameters of this paper will have great research value in the future.

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