Obliquely propagating ion-acoustic shock waves in a degenerate quantum plasma

M. K. Islam1 | S. Biswas1 | N. A. Chowdhury2 | A. Mannan1 | M. Salahuddin1 | AA Mamun1

1Department of Physics, Jahangirnagar University, Dhaka, Bangladesh
2Plasma Physics Division, Atomic Energy Centre, Dhaka, Bangladesh

Correspondence
M. K. Islam, Department of Physics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh.
Email: islam.stu2018@juniv.edu

Abstract
A theoretical investigation has been carried out on the propagation of non-linear ion-acoustic shock waves (IASHWs) in a magnetized degenerate quantum plasma system composed of inertial non-relativistic positively charged light and heavy ions, inertialess non-relativistically or ultra-relativistically degenerate electrons and positrons. The reductive perturbation method has been employed to derive the Burgers’ equation. It has been observed that under consideration, our plasma model supports only positive potential shock structure. It is also found that the amplitude and steepness of the IASHW have been significantly modified by the variation of ion kinematic viscosity, oblique angle, number density, and charge state of the plasma species. The results of our present investigation will be helpful for understanding the propagation of IASHWs in white dwarfs and neutron stars.

KEYWORDS
degenerate quantum plasma, neutron stars, shock waves, white dwarfs

1 | INTRODUCTION

The existence of light (viz., 𝜑1H,[1,4] 4He,[1,5–8] 12C,[7,8] 16O,[7–9] etc.) and heavy (viz., 56Fe,[10,11] 85Rb,[11,12] 96Mo,[11,12] etc.) ions in astrophysical compact objects (viz., white dwarfs and neutron stars) has received a substantial attention to investigate ion-acoustic (IA) waves (IAWs) in degenerate quantum plasma (DQP). The particle number density in the white dwarfs (i.e., 6 × 10^{39} cm^{-3}) and neutron stars (i.e., 6 × 10^{36} cm^{-3}) is extremely high.[1,9,11,13] This extreme number density of these particles dictates to follow the Heisenberg’s uncertainty principle, and according to the uncertainty principle, when the position of these particles is confined then the momentum of these particles tends to very large. This excessive momentum of these particles leads to generate extreme outward degenerate pressure, which is counter-balanced by the inward gravitational compression. The dynamics of these particles is mathematically modelled by Chandrasekhar[1] in two categories, namely, non-relativistic and ultra-relativistic limits. The pressure equation for degenerate constituents is expressed as \( p_j = K_j N_j^{\gamma_j} \), where \( \gamma_j = 5/3 \) and \( K_j = 3\pi\hbar^2/5m_j \) for non-relativistic limit; \( \gamma_j = 4/3 \) and \( K_j = 3hc/4 \) for ultra-relativistic limit; \( m_j \) mass of the plasma species; \( j = h \) for heavy ion; \( j = l \) for light ion; \( j = e \) for electron; \( j = p \) for positron; \( h \) is the Planck constant; \( c \) is the speed of light.[14–16]

The existence of positrons in white dwarfs and neutron stars has also been extensively discussed in refs. [17–20]. Sultana and Schlickeiser[16] investigated IA solitary waves in DQP containing degenerate electrons, light ions, and inertial mobile non-degenerate heavy ions. Gill et al.[21] investigated the IA shock waves (IASHWs) in relativistic DQP composed of electrons, positrons, and ions, and found that the height of the potential is maximum for the lower positron density. Ata-ur-Rahman et al.[22] considered unmagnetized DQP containing inertial ions and inertialess electrons and positrons,
and demonstrated that the amplitude of the IAWs decreases with the increase of positron number density. Hossen et al. \cite{23} studied IASHWs in a four-component plasma system having inertial electrons and positrons and inertial heavy and light ions, and they reported that the amplitude of the shock profile decreases with positron number density. Mamun and Shukla \cite{24} investigated electrostatic solitary waves propagating in ultra-relativistic plasma medium consisting of degenerate electrons and cold mobile ions, and they reported that the wave amplitude increases with the increase of ion number density.

The strong magnetic field (i.e., about 1 Mega Gauss) in white dwarfs was predicted by Blackett \cite{25} and has been observed by Zeeman spectroscopy. \cite{26,27} El-Taibany et al. \cite{28} analysed solitary waves in a magnetized degenerate electron–positron plasma, and they found that the wave amplitude increases with the oblique angle, which is the angle between the external magnetic field and the direction of wave propagation. Shaukat \cite{29} investigated IAS solitary waves in the presence of external magnetic field, and he observed that the solitary wave amplitude increases with an increase in oblique angle.

The energy dissipation of the shock wave, which is governed by the Burgers’ equation, \cite{30} may arise due to the kinematic viscosity of the medium. \cite{31–35} Hafez et al. \cite{36} studied IASHW in weakly relativistic plasma containing electrons, positrons, and ions, and they noticed that the steepness of the IASHWs decreases with the increase in the value of viscosity, but the amplitude of the IASHWs does not change. Abdelwahed et al. \cite{37} analysed IASHW in a pair-ion plasma and also found that the shock steepness decreases with increasing ion viscosity.

Recently, Saini et al. \cite{38} investigated heavy nucleus acoustic periodic waves in DQP. Haider \cite{39} examined the shock profiles in the presence of degenerate inertial ions and inertial electrons and positrons. Atteya et al. \cite{40} studied IASHWs in a DQP, which contains ion fluids, degenerate electrons, and stationary heavy ions. To the best of the authors’ knowledge, still no one has investigated IASHWs in a magnetized DQP having inertially charged non-relativistic light and heavy ions, and inertialless non-relativistically or ultra-relativistically degenerate electrons and positrons. In this manuscript, we will derive the Burgers’ equation, and will also use the associated solution to examine the basic features of IASH in DQP.

The manuscript is organized in the preceding way: The governing equations are described in section 2. The Burgers’ equation and associated shock solution are presented in section 3. The results and discussion are presented in section 4. A brief conclusion is presented in section 5.

## 2 | GOVERNING EQUATIONS

We consider a magnetized DQP system consisting of inertial positively charged heavy ion (mass $m_h$; charge $eZ_h$, number density $N_h$), positively charged light ion (mass $m_l$; charge $eZ_l$; number density $N_l$), inertialless electron (mass $m_e$; charge $-e$, number density $N_e$), and positron (mass $m_p$; charge $e$; number density $N_p$); where $Z_h$ ($Z_l$) is the charge state of the heavy (light) ion. A uniform external magnetic field $B$ exists in the direction of $z$-axis ($B = B_0 \hat{z}$ and $\hat{z}$ is the unit vector). The propagation of IAWs in the DQP system is governed by the following equations:

\[
\frac{\partial N_h}{\partial T} + \vec{\nabla} \cdot (N_h \vec{U}_h) = 0,
\]

\[
\frac{\partial \vec{U}_h}{\partial T} + (\vec{U}_h \cdot \vec{\nabla}) \vec{U}_h = -\frac{Z_he}{m_h} \vec{\nabla} \Phi + \frac{Z_heB_0}{m_h} (\vec{U}_h \times \hat{z}) - \frac{1}{m_hN_h} \vec{\nabla} P_h + \vec{\eta}_h \vec{\nabla}^2 \vec{U}_h,
\]

\[
\frac{\partial N_l}{\partial T} + \vec{\nabla} \cdot (N_l \vec{U}_l) = 0,
\]

\[
\frac{\partial \vec{U}_l}{\partial T} + (\vec{U}_l \cdot \vec{\nabla}) \vec{U}_l = -\frac{Z_le}{m_l} \vec{\nabla} \Phi + \frac{Z_leB_0}{m_l} (\vec{U}_l \times \hat{z}) - \frac{1}{m_lN_l} \vec{\nabla} P_l + \vec{\eta}_l \vec{\nabla}^2 \vec{U}_l,
\]

\[
\vec{\nabla}^2 \Phi = 4\pi e(N_e - N_p - Z_lN_l - Z_hN_h),
\]

where $\vec{U}_h$ ($\vec{U}_l$) is the fluid speed of heavy (light) ion; $\Phi$ is the electrostatic wave potential; $P_h$ ($P_l$) is the pressure for heavy (light) ion; $\vec{\eta}_h = \mu/m_hN_h$ ($\vec{\eta}_l = \mu/m_lN_l$) is the kinematic viscosity for heavy (light) ion. The degenerate pressure equations for electrons and positrons can be expressed, respectively, as

\[
\vec{\nabla} \Phi = \frac{1}{\epsilon N_e} \vec{\nabla} P_e = 0,
\]
Now, we have introduced the normalizing parameters: \( n_h \rightarrow N_h/n_{h0}; \) \( n_i \rightarrow N_i/n_{i0}; \) \( n_e \rightarrow N_e/n_{e0}; \) \( n_p \rightarrow N_p/n_{p0}; \) \( u_h \rightarrow U_h/C_B; \) \( u_i \rightarrow U_i/C_B; \) \( \phi \rightarrow e\Phi/m_e c^2; \) \( t \rightarrow \omega_{ph} T; \) \( \nabla \rightarrow \lambda_{Dh}^{-1} \tilde{\nabla}; \) \( n \rightarrow \tilde{n}/\omega_{ph} \lambda_{Dh}^2 \) (where IAWs speed \( C_B = (Z_h n_e c^2/m_h)^{1/2}; \) plasma frequency \( \omega_{ph} = (4\pi Z_h^2 n_{e0}^2/m_h^2)^{1/2}; \) the Debye length \( \lambda_{Dh} = (m_e c^2/4\pi Z_h e^2 n_{e0})^{1/2} \), and for simplicity we have considered \( \eta = \eta_i = \eta_h \)). At equilibrium, the charge neutrality condition can be written as \( n_{e0} = n_{p0} + Z_i n_{i0} + Z_h n_{h0} \). By using these normalizing parameters, Equations (1)–(5) can be expressed in the normalized form

\[
\frac{\partial n_h}{\partial t} + \nabla \cdot (n_h u_h) = 0, \tag{8}
\]

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i u_i) = 0, \tag{9}
\]

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e u_e) = -\nabla \phi + \Omega_{ch}(u_h \times \tilde{\nabla}) - \frac{\mu_{1} K_1}{n_h} \nabla n_h^m + \eta \nabla^2 u_h, \tag{10}
\]

\[
\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p u_p) = 0, \tag{11}
\]

\[
\nabla^2 \phi = (\mu_4 + \mu_5 + 1) n_e - \mu_4 n_p - \mu_5 n_i - n_h, \tag{12}
\]

where \( \Omega_{ch} = \omega_{ch}/\omega_{ph}, \mu_1 = 1/Z_h, \mu_2 = Z_i m_n/Z_h m_i, \mu_3 = m_h/Z_h m_e, \mu_4 = n_{p0}/Z_h n_{h0}, \mu_5 = Z_h n_{i0}/Z_h n_{h0}, K_1 = n_{h0}^{n_{h0}^{-1} K_e/m_e c^2} \) and \( K_2 = n_{i0}^{n_{i0}^{-1} K_p/m_e c^2} \). Now, by normalizing and integrating Equations (6) and (7), the number densities of the inertialess electrons and positrons, respectively, can be obtained in terms of electrostatic potential \( \phi \) as

\[
n_e = \left[ 1 + \frac{\alpha_e - 1}{K_3 \alpha_e} \right]^{1/\gamma - 1}, \tag{13}
\]

\[
n_p = \left[ 1 - \frac{\alpha_p - 1}{K_4 \alpha_p} \right]^{1/\gamma - 1}, \tag{14}
\]

where \( K_3 = n_{e0}^{n_{e0}^{-1} K_e/m_e c^2} \) and \( K_4 = n_{p0}^{n_{p0}^{-1} K_p/m_e c^2} \). By expanding the right-hand side of Equations (13) and (14) up to second order of \( \phi \), and substituting in Equation (12), we get

\[
\nabla^2 \phi + \mu_5 n_i + n_h = 1 + \mu_5 + \phi_1 \phi + \phi_2 \phi^2 + \cdots, \tag{15}
\]

where \( \phi_1 = (\alpha_e K_4 (\mu_4 + \mu_5 + 1) + \mu_4 \alpha_e K_3)/\alpha_e \alpha_p K_4 K_4 \) and \( \phi_2 = [(\alpha_e K_4)^2 (\mu_4 + \mu_5 + 1)(2 - \alpha_e) - \mu_4 (\alpha_e K_3)^2 (2 - \alpha_p)]/2 (\alpha_e \alpha_p K_4 K_4)^2 \).

### 3 Derivation of the Burgers’ Equation

To study IASHWs, we derive Burgers’ equation by employing the reductive perturbation method,[31–35] and the stretched coordinates for independent variables can be written as[41–43]

\[
\xi = \epsilon (l_x x + l_y y + l_z z - v_p t), \tag{16}
\]

\[
\tau = \epsilon^2 t, \tag{17}
\]

where \( v_p \) is the phase speed of the plasma species; \( \epsilon (0 < \epsilon < 1) \) is the measure of the weakness of a parameter; \( l_x, l_y, \) and \( l_z \) are the directional cosines of \( \mathbf{k} \) (wave vector) along \( x, y, \) and \( z \)-axes, respectively (i.e., \( l_x^2 + l_y^2 + l_z^2 = 1 \)). The dependent variables can be expressed in power series of \( \epsilon \) as[42]

\[
n_h = 1 + \epsilon n_h^{(1)} + \epsilon^2 n_h^{(2)} + \cdots, \tag{18}
\]

\[
n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{19}
\]
Now, by substituting Equations (16)–(24) in Equations (8)–(11) and (15), we obtain a set of first-order equations as

\[ n_h^{(1)} = \frac{l_z^2}{(v_p^2 - \alpha_h \mu_1 l_z^2 K_1)} \phi^{(1)}, \quad u_{h}^{(1)} = \frac{v_p l_z}{(v_p^2 - \alpha_h \mu_1 l_z^2 K_1)} \phi^{(1)}, \]

\[ n_l^{(1)} = \frac{\mu l_z^2}{(v_p^2 - \alpha_l \mu_2 l_z^2 K_2)} \phi^{(1)}, \quad u_{l}^{(1)} = \frac{\mu_2 v_p l_z}{(v_p^2 - \alpha_l \mu_2 l_z^2 K_2)} \phi^{(1)}. \]  

Now, the phase speed of IASHWs can be written as

\[ v_p \equiv v_{p+} = l_z \left( \frac{a + \sqrt{a^2 - 4 \beta_1 b}}{2 \beta_1} \right)^{1/2}, \]

\[ v_p \equiv v_{p-} = l_z \left( \frac{a - \sqrt{a^2 - 4 \beta_1 b}}{2 \beta_1} \right)^{1/2}. \]

where \( a = \alpha_h \mu_1 \beta_1 K_1 + \alpha_l \mu_2 \beta_2 K_2 + \mu_2 \beta_5 + 1 \) and \( b = \alpha_h \alpha_l \mu_1 \mu_2 \beta_1 K_1 K_2 + \alpha_h \mu_2 \mu_5 K_1 + \alpha_l \mu_5 K_2 \). The effect of the positively charged heavy ions causes to reduce the IAW phase speed, but the effect of the positively charged light ions causes to enhance the IAW phase speed (figures are not included). The x and y-components of the first-order momentum equations can be obtained as

\[ u_{hx}^{(1)} = -\frac{l_z v_p^2}{\Omega_{ch}(v_p^2 - \alpha_h \mu_1 l_z^2 K_1)} \frac{\partial \phi^{(1)}}{\partial \xi}, \quad u_{hy}^{(1)} = \frac{l_z v_p^2}{\Omega_{ch}(v_p^2 - \alpha_h \mu_1 l_z^2 K_1)} \frac{\partial \phi^{(1)}}{\partial \eta}, \]

\[ u_{lx}^{(1)} = -\frac{l_z v_p^2}{\Omega_{ch}(v_p^2 - \alpha_l \mu_2 l_z^2 K_2)} \frac{\partial \phi^{(1)}}{\partial \xi}, \quad u_{ly}^{(1)} = \frac{l_z v_p^2}{\Omega_{ch}(v_p^2 - \alpha_l \mu_2 l_z^2 K_2)} \frac{\partial \phi^{(1)}}{\partial \eta}. \]

Now, by taking the next higher-order terms, the equation of continuity, momentum equation, and Poisson’s equation can be written as

\[ \frac{\partial n_h^{(1)}}{\partial \tau} - v_p \frac{\partial n_h^{(2)}}{\partial \xi} + l_z \frac{\partial u_{hx}^{(1)}}{\partial \eta} + l_x \frac{\partial u_{hx}^{(1)}}{\partial \eta} + l_z \frac{\partial u_{hx}^{(2)}}{\partial \xi} + l_x \frac{\partial \phi^{(2)}}{\partial \xi} + \alpha_h \mu_1 l_z K_1 \left[ \frac{\partial n_h^{(2)}}{\partial \xi} + \frac{(a_h - 2) \partial n_h^{(1)}}{2 \partial \xi} \right] - \eta \frac{\partial^2 u_{hx}^{(1)}}{\partial \xi^2} = 0, \]

\[ \frac{\partial u_{hx}^{(1)}}{\partial \tau} - v_p \frac{\partial u_{hx}^{(2)}}{\partial \xi} + l_z \frac{\partial u_{hx}^{(1)}}{\partial \eta} + l_z \frac{\partial \phi^{(2)}}{\partial \xi} + \alpha_h \mu_1 l_z K_1 \left[ \frac{\partial n_h^{(2)}}{\partial \xi} + \frac{(a_h - 2) \partial n_h^{(1)}}{2 \partial \xi} \right] - \eta \frac{\partial^2 u_{hx}^{(1)}}{\partial \xi^2} = 0, \]

\[ \frac{\partial n_l^{(1)}}{\partial \tau} - v_p \frac{\partial n_l^{(2)}}{\partial \xi} + l_x \frac{\partial n_l^{(1)}}{\partial \eta} + l_x \frac{\partial u_{lx}^{(1)}}{\partial \eta} + l_z \frac{\partial u_{lx}^{(2)}}{\partial \xi} + l_z \frac{\partial \phi^{(2)}}{\partial \xi} + \alpha_l \mu_3 l_z K_2 \left[ \frac{\partial n_l^{(2)}}{\partial \xi} + \frac{(a_l - 2) \partial n_l^{(1)}}{2 \partial \xi} \right] - \eta \frac{\partial^2 u_{lx}^{(1)}}{\partial \xi^2} = 0, \]

\[ \frac{\partial u_{lx}^{(1)}}{\partial \tau} - v_p \frac{\partial u_{lx}^{(2)}}{\partial \xi} + l_x \frac{\partial u_{lx}^{(1)}}{\partial \eta} + l_x \frac{\partial \phi^{(2)}}{\partial \xi} + \alpha_l \mu_3 l_z K_2 \left[ \frac{\partial n_l^{(2)}}{\partial \xi} + \frac{(a_l - 2) \partial n_l^{(1)}}{2 \partial \xi} \right] - \eta \frac{\partial^2 u_{lx}^{(1)}}{\partial \xi^2} = 0, \]

\[ \mu_1 n_l^{(2)} + n_h^{(2)} = \beta_1 \phi^{(2)} + \beta_2 \phi^{(1)}. \]
Finally, the next higher-order terms of Equations (8)–(11) and (15), with the help of Equations (25)–(35), can provide the Burgers’ equation

\[
\frac{d\Phi}{d\tau} + A\Phi\frac{d\Phi}{dz} = B\frac{d^2\Phi}{dz^2}, \tag{36}
\]

where \(\Phi = \phi^{(1)}\) for simplicity. Equation (36) is the Burgers’ equation with non-linear coefficient \(A\) and dissipative coefficient \(B\), given respectively as

\[
A = \left[ \mu_2^2\mu_5\varepsilon_2\frac{3v_{pl}^2 + \mu_2\varepsilon_2K_2\alpha(a_i - 2)[(v_p^2 - \alpha_i\mu_2\varepsilon_2K_2)]}{(v_p^2 - \alpha_i\mu_1\varepsilon_2K_1)^3} \right] + \left[ \mu_2^2\mu_5\varepsilon_2\frac{3v_{pl}^2 + \mu_1\varepsilon_2K_1\alpha(a_i - 2)[(v_p^2 - \alpha_i\mu_1\varepsilon_2K_1)^3]}{(v_p^2 - \alpha_i\mu_1\varepsilon_2K_1)^3} \right] - 2\beta_2
\]

\[
B = \frac{\eta}{2}. \tag{37}
\]

Now, we look for stationary shock wave solution of this Burgers’ equation by considering \(\zeta = \xi - U_0\tau\) (where \(\zeta\) is a new space variable and \(U_0\) is the speed of the ion fluid). These allow us to write the stationary shock wave solution as

\[
\Phi = \Phi_0 \left[ 1 - \tanh \left( \frac{\zeta}{\Delta} \right) \right], \tag{39}
\]

where the amplitude \(\Phi_0\) and width \(\Delta\) are, respectively, given by

\[
\Phi_0 = \frac{U_0}{A}, \quad \text{and} \quad \Delta = \frac{2B}{U_0}. \tag{40}
\]

It is clear from Equations (39) and (40) that the IASHWs exist, which are formed due to the balance between nonlinearity and dissipation, because \(B > 0\) and the IASHWs with \(\Phi > 0\) (\(\Phi < 0\)) exist if \(A > 0\) (\(A < 0\)) because \(U_0 > 0\).

4 RESULTS AND DISCUSSION

Our investigation is reasonably valid for degenerate cold plasma systems (viz., white dwarfs and neutron stars) in which light (viz., \(^1\)H, \(^4\)He, \(^12\)C, \(^16\)O, etc.) and heavy (viz., \(^{56}\)Fe, \(^{85}\)Rb, \(^{96}\)Mo, etc.) ions can exist. For our numerical analysis, we have considered the range of the plasma parameters as \(Z_l = 6 \sim 13\), \(Z_h = 35 \sim 65\), \(n_{l0} = 3 \times 10^{30}\) cm\(^{-3}\) \(\sim 7 \times 10^{30}\) cm\(^{-3}\), \(n_{h0} = 4 \times 10^{29}\) cm\(^{-3}\) \(\sim 07 \times 10^{33}\) cm\(^{-3}\), and \(n_{p0} = 1 \times 10^{31}\) cm\(^{-3}\) \(\sim 10 \times 10^{31}\) cm\(^{-3}\). A number of authors considered inertial non-relativistic ion(s)\(^{14,16,40,44-51}\) and inertialess non-relativistic and/or ultra-relativistic\(^{14-16,28,40,44-53}\) electrons and positrons to investigate electrostatic\(^{14-16,28,40,44-49,51-53}\) or gravito-acoustic\(^{50}\) waves in DQP, especially in white dwarfs and neutron stars.

It is obvious from Equation (40) that the wave potential becomes infinite when non-linear coefficient \(A\) is equal to zero (i.e., \(A = 0\)), and in that case, the validity of the reductive perturbation method breaks down. The positive (i.e., \(\Phi > 0\)) and negative (i.e., \(\Phi < 0\)) electrostatic potentials can exist corresponding to the values of \(A > 0\) and \(A < 0\). However, it is clear from the left panel of Figure 1 that our plasma model supports only positive potential shock structure (i.e., \(\Phi > 0\)) associated with \(A > 0\) under consideration of non-relativistic light and heavy ions, electrons, and positrons (i.e., \(\alpha_i = \alpha_h = \alpha_e = \alpha_p = 5/3\)). The right panel of Figure 1 also demonstrates that the existence of non-relativistic ions (i.e., \(\alpha_i = \alpha_h = 5/3\)) and ultra-relativistic electrons and positrons (i.e., \(\alpha_e = \alpha_p = 4/3\)) also allows only positive IASHW profile (i.e., \(\Phi > 0\)) associated with \(A > 0\).

The oblique angle \(\delta\) indicates the angle between the direction of the propagation of IASHW and the direction of existing external magnetic field, which is parallel with z-axes. The variation of electrostatic shock potential structure (i.e., \(\Phi > 0\) and associated with \(A > 0\)) with \(\delta\) under consideration of non-relativistic light and heavy ions (i.e., \(\alpha_i = \alpha_h = 5/3\)) and ultra-relativistic degenerate electrons and positrons (i.e., \(\alpha_e = \alpha_p = 4/3\)) can be observed from the left panel of Figure 2. It is obvious from the left panel of Figure 2 that the electrostatic shock potential (i.e., \(\Phi > 0\)) associated with \(A > 0\) increases with the increase of \(\delta\). Physically, the interaction between the electrostatic shock potential associated with \(A > 0\) and the external magnetic field is clearly increased with the increase in \(\delta\). The right panel of Figure 2 describes the effects of the kinematic viscosity of the positively charged non-relativistic light and heavy ions (via \(\eta\)) on the electrostatic shock.
FIGURE 1  Plot of $A$ versus $\mu_5$ under consideration of $\alpha_l = \alpha_h = \alpha_e = \alpha_p = 5/3$ (left panel), and plot of $A$ versus $\mu_5$ under consideration of $\alpha_l = \alpha_h = 5/3; \alpha_e = \alpha_p = 4/3$ (right panel). Other plasma parameters are $\sigma = 20^\circ$, $\eta = 0.3$, $Z_h = 37$, $Z_l = 6$, $n_{h0} = 7 \times 10^{29} \text{ cm}^{-3}$, $n_{l0} = 5 \times 10^{30} \text{ cm}^{-3}$, $n_{p0} = 10^{31} \text{ cm}^{-3}$, and $v_p \equiv v_{ps}$.

FIGURE 2  Plot of $\Phi$ versus $\zeta$ for different values of $\delta$ (left panel) and $\eta$ (right panel) when $\alpha_l = \alpha_h = 5/3, \alpha_e = \alpha_p = 4/3, Z_h = 37$, $Z_l = 6, n_{h0} = 7 \times 10^{29} \text{ cm}^{-3}, n_{l0} = 5 \times 10^{30} \text{ cm}^{-3}, n_{p0} = 10^{31} \text{ cm}^{-3}, U_0 = 0.05$, and $v_p \equiv v_{ps}$.

potential structure (i.e., $\Phi > 0$) associated with $A > 0$. It can be seen from the right panel of Figure 2 that the amplitude of the electrostatic shock profile (i.e., $\Phi > 0$) associated with $A > 0$ is independent to the variation of the ion kinematic viscosity but the steepness of the electrostatic shock profile (i.e., $\Phi > 0$) associated with $A > 0$ is rigorously dependent on the variation of ion kinematic viscosity. The steepness of the electrostatic shock potential structure (i.e., $\Phi > 0$) associated with $A > 0$ decreases with $\eta$, and this result agrees with the result of refs. [36,37].

The amplitude of the positive electrostatic shock structure (i.e., $\Phi > 0$) associated with $A > 0$ is so much sensitive to the change of the charge state of non-relativistic light and heavy ions (i.e., $\alpha_l = \alpha_h = 5/3$) in the presence of non-relativistic (i.e., $\alpha_e = \alpha_p = 5/3$) or ultra-relativistic (i.e., $\alpha_e = \alpha_p = 4/3$) electrons and positrons. Figure 3 show that the amplitude of the positive shock structure (i.e., $\Phi > 0$) associated with $A > 0$ increases with the charge state of non-relativistic light (left panel) and heavy (right panel) ions in the presence of non-relativistic electrons and positrons (i.e., $\alpha_e = \alpha_p = 5/3$). Similarly, from Figure 4, as we increase the charge state of non-relativistic light (left panel) and heavy (right panel) ions in the presence of the ultra-relativistic electrons and positrons (i.e., $\alpha_e = \alpha_p = 4/3$), the amplitude of the positive shock profile (i.e., $\Phi > 0$) associated with $A > 0$ also increases. In comparison, the amplitude of the positive shock profile (see Figures 3 and 4) is always greater in the presence of non-relativistic electrons and positrons (i.e., $\alpha_e = \alpha_p = 5/3$) than the existence of ultra-relativistic electrons and positrons (i.e., $\alpha_e = \alpha_p = 4/3$) when other plasma parameters are fixed.

The increasing number of non-relativistic light and heavy ions (i.e., $\alpha_l = \alpha_h = 5/3$) enhances the positive electrostatic shock structure (i.e., $\Phi > 0$) associated with $A > 0$. It can be easily demonstrated from Figure 5 that as we increase $n_{h0}$ (left panel) and $n_{l0}$ (right panel), the amplitude of the positive electrostatic shock structure (i.e., $\Phi > 0$) associated with $A > 0$ increases. Physically, the number density of non-relativistic (i.e., $\alpha_l = \alpha_h = 5/3$) light and heavy ions can control the dynamics of the DQP system in a similar way.

The effects of non-relativistic heavy ions on the formation of positive electrostatic shock profile under consideration of $n_{h0} > n_{l0}$ or $\mu_5 < 1$ can be observed from the left panel of Figure 6, and it is really interesting that the increase of heavy ions can increase the amplitude of the positive IASHW under consideration of $n_{h0} > n_{l0}$ or $\mu_5 < 1$, and similar result
FIGURE 3 Plot of $\Phi$ versus $\zeta$ for different values of $Z_h$ (left panel) and $Z_l$ (right panel) when $\alpha_l = \alpha_h = \alpha_e = \alpha_p = 5/3$, $\delta = 20^\circ$, $\eta = 0.3$, $n_{h0} = 7 \times 10^{29} \text{cm}^{-3}$, $n_{l0} = 5 \times 10^{30} \text{cm}^{-3}$, $n_{p0} = 10^{31} \text{cm}^{-3}$, $U_0 = 0.05$, and $v_p \equiv v_{p^+}$.

FIGURE 4 Plot of $\Phi$ versus $\zeta$ for different values of $Z_h$ (left panel) and $Z_l$ (right panel) when $\alpha_l = \alpha_h = 5/3$, $\alpha_e = \alpha_p = 4/3$, $\delta = 20^\circ$, $\eta = 0.3$, $n_{h0} = 7 \times 10^{29} \text{cm}^{-3}$, $n_{l0} = 5 \times 10^{30} \text{cm}^{-3}$, $n_{p0} = 10^{31} \text{cm}^{-3}$, $U_0 = 0.05$, and $v_p \equiv v_{p^+}$.

FIGURE 5 Plot of $\Phi$ versus $\zeta$ for different values $n_{h0}$ (left panel) and $n_{l0}$ (right panel) when $\alpha_l = \alpha_h = 5/3$, $\alpha_e = \alpha_p = 4/3$, $\delta = 20^\circ$, $\eta = 0.3$, $Z_h = 37$, $Z_l = 6$, $n_{p0} = 10^{31} \text{cm}^{-3}$, $U_0 = 0.05$, and $v_p \equiv v_{p^+}$.

can be observed from the left panel of Figure 5, which is depicted under consideration of $n_{h0} < n_{l0}$ or $\mu_5 > 1$. But the amplitude of the positive IASHW under consideration of $n_{h0} > n_{l0}$ or $\mu_5 < 1$ (left panel of Figure 6) is always greater than the amplitude of the positive IASHW under consideration of $n_{h0} < n_{l0}$ or $\mu_5 > 1$ (left panel of Figure 5). So, the existence of large number of heavy ions always enhances the amplitude of the IASHW. We have also studied the characteristics of IASHWs for different values of positron number density (via $n_{p0}$) under consideration of non-relativistic light and heavy ions (i.e., $\alpha_l = \alpha_h = 5/3$) and ultra-relativistic degenerate electrons and positrons (i.e., $\alpha_e = \alpha_p = 4/3$) in the right panel of Figure 6. It can be highlighted from the right panel of Figure 6 that the amplitude of electrostatic shock structure (i.e., $\Phi > 0$) associated with $A > 0$ decreases with positron number density, and this finding is analogous with the results of refs. [21,22].
FIGURE 6  Plot of $\Phi$ versus $\zeta$ for different values of $n_{h0}$ (left panel) and $n_{p0}$ (right panel) when $n_{l0} = 5 \times 10^{30} \text{cm}^{-3}$ (i.e., $n_{h0} > n_{l0}$ or $\mu_5 < 1$), $\alpha_l = \alpha_h = 5/3$, $\alpha_e = \alpha_p = 4/3$, $\delta = 20^\circ$, $\eta = 0.3$, $Z_h = 37$, $Z_l = 6$, $U_0 = 0.05$, and $v_p \equiv v_{ps}$.

5  |  CONCLUSION

We have studied the basic characteristics of IASHWs in an extremely dense DQP in the presence of an external magnetic field. The reductive perturbation method has been utilized to derive Burgers' equation. We have identified some novel features of the electrostatic shock waves associated with IAWs. The novelty of the electrostatic shock structures is that they are associated with a special type of IAWs in which the restoring force comes from the degenerate pressure exerted by the non-relativistic/ultra-relativistic degenerate electrons and positrons, while the inertia is provided by the mass density of the positively charged non-relativistic light and heavy ions species. The results that have been found from our present investigation can be summarized as follows:

- Our plasma model supports only positive potential shock structure (i.e., $\Phi > 0$) associated with $A > 0$ under consideration of non-relativistic ions, electron, and positrons (i.e., $\alpha_l = \alpha_h = \alpha_e = \alpha_p = 5/3$), and also under consideration of non-relativistic ions (i.e., $\alpha_l = \alpha_h = 5/3$) and ultra-relativistic electrons and positrons (i.e., $\alpha_e = \alpha_p = 4/3$).
- The effect of the positively charged heavy ions reduces the IAW phase speed, but the effect of the positively charged light ions enhances the IAW phase speed.
- The electrostatic shock potential (i.e., $\Phi > 0$) associated with $A > 0$ increases with the increase in $\delta$.
- The steepness of the positive potential shock structure (i.e., $\Phi > 0$) associated with $A > 0$ decreases with $\eta$.
- The amplitude of electrostatic shock structure (i.e., $\Phi > 0$) associated with $A > 0$ is found to increase with the charge state and number density of non-relativistic light and heavy ions.
- The increase in positron number density decreases the height of the positive shock profile.

It may be noted that the self-gravitational effect of the plasma species is important to be considered in the governing equations but beyond the scope of our present work. Overall, the outcomes from our present investigation will be helpful to understand the IASHWs in white dwarfs and neutron stars.

ACKNOWLEDGMENTS
The authors are grateful to the anonymous reviewer for his/her constructive suggestions, which have significantly improved the quality of our manuscript.

DATA AVAILABILITY STATEMENT
Data sharing not applicable—no new data generated: Data sharing is not applicable to this article as no new data were created or analysed in this study.

REFERENCES
[1] S. Chandrasekhar, Astrophys. J. 1931, 74, 81.
[2] S. Chandrasekhar, Observatory 1934, 373, 57.
[3] R. S. Fletcher, X. L. Zhang, S. L. Rolston, Phys. Rev. Lett. 2006, 96, 105003.
[4] T. C. Killian, Nature 2006, 441, 297.
[5] H. M. Van Horn, Science 1991, 252, 384.
[6] R. H. Fowler, J. Astrophys. Astron. 1994, 15, 105.
[7] D. Koester, G. Chanmugam, Rep. Prog. Phys. 1990, 53, 837.
ISLAM ET AL.

Contributions to Plasma Physics

M. K. Islam was born in Jhenaidah, Bangladesh, in 1995. He received his Bachelor of Science (BSc) in Physics (2018) and currently continuing his Master of Science (M.Sc.) in Physics at Jahangirnagar University, Savar, Bangladesh. His current research interest is particularly in theoretical and computational nonlinear waves (e.g., solitary, shock, and rogue waves, etc.) in space and astrophysical plasmas. He has published two research articles in reputed peer-review journals, Contributions to Plasma Physics (Wiley) and Gases (MDPI). He is interested to conduct his higher studies in Astronomy and Astrophysics. He is an active
ISLAM et al.

member of Jahangirnagar University Physics Club (JUPC) and has participated in many other activities of the club such as poster presentation, science project, quiz competition, olympiad, etc. He is one of the renowned footballers in Jahangirnagar University Football Team, and has won many tournaments conducted by the university sports.

S. Biswas was born in Rajbari, Bangladesh, in 1996. He received his Bachelor of Science (B.Sc.) in Physics in 2018 and currently a student of Master of Science (M.Sc.) in Physics at Jahangirnagar University, Savar, Dhaka, Bangladesh. His current research interest is particularly in theoretical and computational nonlinear waves (e.g., Solitary, Shock, etc.) in space and astrophysical objects. He is interested to operate his higher studies in nonlinear plasma system and compact astrophysical objects. He is now working as a member of Jahangirnagar University Physics Club (JUPC) and Jahangirnagar University Science Club (JUSC) and associated with some social working organizations. He is very interested in playing cricket and football. He obtained National Science and Technology (N.S.T.) fellowship for his M.Sc. research.

N. A. Chowdhury was born in Munshiganj, Bangladesh, in 1994. He received the B.Sc. and M.Sc. degrees in Physics from Jahangirnagar University, Savar, Dhaka-1342, Bangladesh, in 2015 and 2016, respectively. He is currently working as a “Scientific Officer” at “Plasma Physics Division” in the “Bangladesh Atomic Energy Commission,” and has published 31 research papers on the nonlinear waves in plasmas in various international peer-reviewed journals such as Chaos (American Institute of Physics, United States), Physics of Plasmas (American Institute of Physics, United States), Contributions to Plasma Physics (Wiley-Germany), Vacuum (Elsevier, United Kingdom), The European Physical Journal D (Springer), and Plasma Physics Reports (Springer-Russia). His current research interest includes theoretical and computational plasma physics, which involves nonlinear structures (viz., Shock waves, envelope solitons, rogue waves, etc.) in space and laboratory plasmas. In future, he plans to study quantum field theory, string theory, and quantum gravity. He has been the recipient of a number of scholarships such as North American Bangladesh Islamic Community Award, the National Science and Technology (N.S.T.) Fellowship, and University Scholarship, etc. He also participated in the 69th Lindau Nobel Laureate Meeting-2019 in Germany.

A. Mannan was born in Munshiganj, Bangladesh, on January 1, 1988. He received the B.Sc. and M.Sc. (Thesis) degrees in Physics from the Department of Physics, Jahangirnagar University, Dhaka, Bangladesh, and the Ph.D. degree from the Universitá degli Studi della Campania Luigi Vanvitelli, Caserta, Italy, under the direct supervision of Prof. R. Fedele in novel physics methodologies for environmental research. During his Ph.D., he was involved in Istituto Nazionale di Fisica Nucleare (INFN), Italy research project SL-COMB and SL-$\gamma$-Resist, devoted to plasma-based acceleration at INFN Sezione di Napoli. He worked on problems that are related to natural disaster issues and are of recent interest globally. He is currently working as an Alexander von Humboldt Postdoctoral Researcher with Institut für Mathematik, Martin Luther Universität Halle-Wittenberg, Halle (Saale), Germany. He also has a permanent faculty position (Associate Professor) with the Department of Physics, Jahangirnagar University. His academic production is both intense and diversified, compared to his relatively short scientific career. He attended many international conferences, workshops, and scientific schools on multidisciplinary fields, thus showing a breadth of his scientific interests. He has authored or co-authored more than 55 research articles in the highly reputed scientific international journals. He has been awarded the Emerging Nations Science Foundation (ENSF) Prize-2014 for Best Poster at Joint ICTP-IAEA College on Advanced Plasma Physics, ICTP-Trieste, Italy. He has received the prize for the Best Oral Presentation at the Geophysics, Environmental Physics, and Physical Oceanography Session of the 100th Conference of the Italian Physical Society held at Dipartimento di Fisica, Università di Pisa, September, 2014, Italy. He has participated as a Young Researcher from all over the world in 62nd Lindau Nobel
Laureate Meetings-2012, Lindau, and Post-Conference Program to the 62nd Lindau Meeting of Nobel Laureates-2012, Baden, Württemberg, Germany.

**M. Salahuddin** was born in Narayanganj, Bangladesh, in 1960. He received the B.Sc., M.Sc., and Ph.D. degrees from Jahangirnagar University, Dhaka, Bangladesh, in 1983, 1985, and 1999, respectively. He is currently a professor of physics in the Jahangirnagar University. He participated in several international seminars and workshops, including Autumn College on Plasma Physics, Trieste, Italy, and the United Nations Educational, Scientific, and Cultural Organization Regional Workshop on Lightning Physics, Colombo, Sri Lanka. He has authored more than 30 papers published in different international journals. His main research field includes theoretical and computational plasma physics, particularly in waves instabilities of dusty plasma. He supervised four Ph.D. students who were already awarded their degrees in the meantime and two more students are doing their Ph.D. at present. More than 10 M.Phil/M.Sc. students already got their degrees and few are doing their research under his supervision.

**AA Mamun** was born in Dhaka, Bangladesh, in 1966. He received the Ph.D. degree in plasma physics from the University of St. Andrews, St. Andrews, United Kingdom, in 1996. He is currently a Regular Associate with the International Centre for Theoretical Physics, Trieste, Italy, and a Professor of Physics with Jahangirnagar University, Dhaka. He has authored over 465 papers in the linear and nonlinear waves in plasmas in various international journals. He has co-authored a book entitled Introduction to Dusty Plasma Physics (London: Institute of Physics Publishing). The total citations of his research already exceeded 15,100, with an i10-index of 260 and an h-index of 54. His current research interests include theoretical and computational plasma physics, which involves collective processes in dusty plasmas, electron–positron plasma, laser-produced plasma, and semiconductor plasma. Dr Mamun was a recipient of several very prestigious postdoctoral fellowships, national, and international awards for his outstanding contributions in complex plasma physics. He received several fellowships, including the Alexander von Humboldt Post-Doctoral Fellowship, Germany, the Commonwealth Post-Doctoral Fellowship, United Kingdom, and the Max Planck Research Fellowship, Germany. He has received several awards, including the Bessel Research Award, AvH Foundation, Germany, in 2009, the Third World Academy of Sciences Young Scientist Prize in Physics, Trieste, in 2006, the Bangladesh Academy of Sciences Gold Medal in 2004 (Junior Group) and 2001 (Senior Group) in Physical Sciences, and the Best Young Physicist Prize, ICTP, Trieste, in 2000. He is also the first Bangladeshi Scientist to receive the Bessel Research Award.

**How to cite this article:** M. K. Islam, S. Biswas, N. A. Chowdhury, A. Mannan, M. Salahuddin, A A Mamun, *Contributions to Plasma Physics* 2022, 62(1), e202100073. [https://doi.org/10.1002/ctpp.202100073](https://doi.org/10.1002/ctpp.202100073)