TeV gamma rays from BL Lac Objects due to synchrotron radiation of extremely high energy protons

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Abstract

One of remarkable features of the gamma ray blazar Markarian 501 is the reported shape of the TeV spectrum, which during strong flares of the source remains essentially stable despite dramatic variations of the absolute γ-ray flux. I argue that this unusual behavior of the source could be explained assuming that the TeV emission is a result of synchrotron radiation of extremely high energy ($E \geq 10^{19}$ eV) protons in highly magnetized ($B \sim 30 - 100$ G) compact regions of the jet with typical size $R \sim 10^{15} - 10^{16}$ cm and Doppler factor $\delta_j \simeq 10 - 30$. It is shown that if protons are accelerated at the maximum possible rate, i.e. $t_{\text{acc}} = \eta (r_g/c)$ with so-called gyro-factor $\eta \sim 1$, the synchrotron cooling of protons could not only dominate over other radiative and non-radiative losses, but could also provide good fits (within uncertainties introduced by extragalactic γ-ray extinction) to the γ-radiation of two firmly established TeV blazars - Markarian 501 and Markarian 421. Remarkably, if the proton acceleration takes place in the regime dominated by synchrotron losses, the spectral shape of the Doppler-boosted γ-radiation in the observer’s frame is determined essentially by the self-regulated “synchrotron cutoff” at $\epsilon_0 \simeq 0.3 \delta_j \eta^{-1}$ TeV. The hypothesis of the proton-synchrotron origin of TeV flares of BL Lac objects inevitably implies that the energy contained in the form of magnetic field in the γ-ray emitting region exceeds the kinetic energy of accelerated protons.

Key words: galaxies: BL Lacertae objects: individual: Mkn 501, Mkn 421; observations, Gamma rays: theory

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1 Introduction

Blazars are Active Galactic Nuclei (AGN) dominated by a highly variable component of non-thermal radiation produced in relativistic jets close to the line of sight (e.g. Begelman et al. 1984, Urry & Padovani 1995). The dramatically enhanced fluxes of the Doppler-boosted radiation, coupled with the fortuitous orientation of the jets towards the observer, make these objects ideal laboratories to reveal the underlying physics of AGN jets through multi-wavelength studies of temporal and spectral characteristics of radiation from radio to very high energy $\gamma$-rays (Ulrich et al. 1997). First of all this concerns the BL Lacertae (BL Lac) objects - a sub-population of blazars of which two prominent representatives, Markarian 421 and Markarian 501, are firmly established as TeV $\gamma$-ray emitters. The flux variability on different time-scales (and, plausibly, of different origin) is a remarkable feature of TeV radiation of BL Lac objects. It ranges from the spectacular 1996 May 15 flare of Markarian 421 with duration less than 1 h to the extraordinary high state of Markarian 501 in 1997 lasting several months (for review see Aharonian 1999; Catanese & Weekes 1999). The recent multi-wavelength campaigns revealed that the TeV flares of both objects, Markarian 501 (Pian et al. 1998; Catanese et al. 1997; Krawczynski et al. 2000; Sambruna et al. 2000) and Markarian 421 (Buckley et al. 1996; Maraschi et al. 1999; Takahashi et al. 1999), correlate with X-radiation on time-scales of hours or less. This is often interpreted as a strong argument in favor of the so-called synchrotron-Compton jet emission models in which the same population of ultra-relativistic electrons is responsible for production of both X-rays and TeV $\gamma$-rays via synchrotron radiation and inverse Compton scattering, respectively (see, e.g., Ulrich et al. 1997). However, the very fact of correlation between X-ray and TeV $\gamma$-ray fluxes does not yet rule out other possibilities, in particular the so-called hadronic models which assume that the observed $\gamma$-ray emission is initiated by accelerated protons interacting with ambient gas or low-frequency radiation.

Generally, the hadronic models do not offer efficient $\gamma$-ray production mechanisms in the jet. For example, for any reasonable acceleration power of protons, $L_p \leq 10^{45}$ erg/s, the density of the thermal plasma in the jet should exceed $10^6$ cm$^{-3}$ in order to interpret the reported TeV flares of Markarian 501 by $\pi^0$-decay $\gamma$-rays produced at $p$-$p$ interactions. Therefore this mechanism could be effectively realized only in a scenario like “relativistic jet meets target” (Morrison et al. 1984), i.e. assuming that $\gamma$-radiation is produced in dense gas clouds that move across the jet (e.g. Dar & Laor 1997) Recently a novel, “non-acceleration” version of $\pi^0$-decay $\gamma$-ray production by blazar jets was suggested by Pohl & Schlickeiser (2000).

The Proton Induced Cascade (PIC) model (Mannheim 1993; Mannheim 1996) is another attractive possibility for production of high energy $\gamma$-rays. This
model relates the observed \( \gamma \)-radiation to the development of pair cascades in the jet triggered by secondary “photo-meson” products (\( \gamma \)-rays and electrons) produced at interactions of accelerated protons with low-frequency synchrotron radiation. The efficiency of this model significantly increases with energy of accelerated protons, therefore the postulation of an existence of extremely high energy (EHE; \( E \geq 10^{19} \text{eV} \)) protons is a key assumption for the PIC model. In a compact \( \gamma \) production region of the jet with characteristic size less than \( 10^{16} \text{cm} \), the protons could be accelerated to such high energies only in the presence of large magnetic field, \( B \gg 1 \text{G} \). Even so, below I will show that the very fact of observations of multi-TeV \( \gamma \)-rays from Markarian 421 and Markarian 501 allow a rather robust lower limit on the “photo-meson” cooling time of protons, \( t_{p\gamma} \geq 10^7 \text{s} \). This implies uncomfortably high luminosity in EHE protons which would be required to match the observed TeV \( \gamma \)-ray fluxes.

Meanwhile, at such conditions the synchrotron radiation of the EHE protons becomes a very effective channel of production of high energy \( \gamma \)-rays. In this paper I show that for a reasonable set of parameters, which characterize the small-scale (sub-parsec) jets in Markarian 421 and Markarian 501, the synchrotron radiation of EHE protons not only may dominate over other possible radiative and non-radiative losses, but also could provide adequate fits to the observed TeV spectra of both objects\footnote{The synchrotron radiation of protons was actually included in the overall PIC code of Mannheim (1993), but the effect of this process was somehow disregarded. The importance of the proton synchrotron radiation in the PIC scenario was recently recognized, independent of the present paper, by Mücke & Protheroe (2000).}. Moreover, this hypothesis could naturally explain one of the remarkable features of TeV flares of Markarian 501 - its essentially stable spectral shape despite spectacular variations of the absolute TeV flux up to factor of 10 or more on time-scales less than 1 day.

2 Synchrotron radiation of protons

The comprehensively developed theory of synchrotron radiation of relativistic electrons (see, e.g., Ginzburg & Syrovatsky 1965; Blumenthal & Gould 1970) can be readily applied to the proton-synchrotron radiation by re-scaling the Larmor frequency \( \nu_L = eB/2\pi mc \) by the factor \( m_p/m_e \approx 1836 \). For the same energy of electrons and protons, \( E_p = E_e = E \), the energy loss rate of protons \( (dE/dt)_s \) appears \( (m_p/m_e)^4 \approx 10^{13} \) times slower than the energy loss rate of electrons. Also, the characteristic frequency of the synchrotron radiation \( \nu_c = 3/2 \nu_L (E/mc^2)^2 \) emitted by a proton is \( (m_p/m_e)^3 \approx 6 \cdot 10^9 \) times smaller than the characteristic frequency of synchrotron photons emitted by an electron of the same energy. Then the synchrotron cooling time of a proton,
\[ t_{\text{sy}} = E/(dE/dt)_{\text{sy}}, \] and the characteristic energy of synchrotron photons \( \epsilon_c = h\nu_c \) produced in the magnetic field \( B \) are

\[ t_{\text{sy}} = \frac{6\pi m_p^4 c^3}{\sigma_T m_e^2 E B^2} = 4.5 \times 10^4 B_{100}^{-2} E_{19}^{-1} \text{s}, \] (1)

and

\[ \epsilon_c = h\nu_c = \sqrt{\frac{3}{2}} \frac{heBE^2}{2\pi m_p^2 c^5} \approx 87 B_{100} E_{19}^2 \text{GeV}, \] (2)

where \( B_{100} = B/100 \text{G} \) and \( E_{19} = E/10^{19} \text{eV} \). Hereafter it is assumed that the magnetic field is distributed isotropically, i.e. \( B_\perp = \sin \psi B \) with \( \sin \psi = \sqrt{2/3} \).

The average energy of synchrotron photons produced by a particle of energy \( E \) is equal to \( \epsilon_m \approx 0.29 \epsilon_c \) (Ginzburg & Syrovatsky 1965). Correspondingly, the characteristic time of radiation of a synchrotron photon of energy \( \epsilon \) by a proton in the magnetic field \( B \) is

\[ t_{\text{sy}}(\epsilon) \approx 2.2 \times 10^5 B_{100}^{-3/2} (\epsilon/1 \text{GeV})^{-1/2} \text{s}. \] (3)

For comparison, the time needed for radiation of a synchrotron \( \gamma \)-ray photon of the same energy \( \epsilon \) and in the same magnetic field, but by an electron, is shorter by a factor \( (m_p/m_e)^{5/2} \approx 1.5 \times 10^8 \).

The spectral distribution of synchrotron radiation emitted by a proton of energy \( E \) is described by the equation

\[ P(E,\epsilon) = \frac{\sqrt{2}}{h} \frac{e^3 B}{mc^2} F(x), \] (4)

where \( x = \epsilon/\epsilon_c, \) and \( F(x) = x \int_x^\infty dxK_{5/3}(x); K_{5/3}(x) \) is the modified Bessel function of 5/3 order. The function \( F(x) \) could be presented in a simple analytical form (e.g. Melrose 1980):

\[ F(x) = C x^{1/3} \exp(-x). \] (5)

Numerical calculations show that with \( C \approx 1.85 \) this approximation provides very good, less than 1 per cent accuracy in the region of the maximum at \( x \sim 0.3, \) and still reasonable (less than several per cent) accuracy in the broad dynamical region \( 0.1 \leq x \leq 10. \)
For the given energy spectrum $N_p(E)$ of relativistic protons, distributed isotropically in a source at a distance $d$ from the observer, the differential flux of synchrotron radiation is defined as

$$J(\epsilon) = \frac{\epsilon^{-1}}{4\pi d^2} \int_0^\infty dE \ P(E, \epsilon) \ N_p(E)E.$$  \hfill (6)

Figure 1 shows four examples of possible proton spectra. The curve 1 corresponds to the most “standard” assumption for the spectrum of accelerated (e.g. by shock waves) particles - power-law with an exponential cutoff at energy $E_0$:

$$N_p(E) = N_0 E^{-\alpha_p} \exp(-E / E_0).$$ \hfill (7)

The curve 2 corresponds to a less realistic, truncated power-law spectrum, i.e. $N_p(E) \propto E^{-\alpha_p}$ at $E \leq E_0$, and $N_p(E) = 0$ at $E \geq E_0$.

While the cutoff energy $E_0$ in the spectrum of accelerated particles could be estimated quite confidently from the balance between the particle acceleration and the energy loss rates, the shape of the resulting spectrum in the cutoff region depends on several circumstances - the specific mechanisms of acceleration and energy dissipation, the diffusion coefficient, etc. For example,
the recently revived interest in the diffusive shock acceleration of electrons resulted in predictions that in the shock acceleration scheme one may expect not only spectral cutoffs, but perhaps also pronounced pile-ups preceding the cutoffs (Melrose & Crouch 1997; Protheroe & Stanev 1999; Drury et al. 1999). Two such spectra are shown in Fig. 1. The curve 3 represents the extreme class of spectra containing a sharp (with an amplitude of factor of 10) spike at the very edge of the spectrum. The curve 4 corresponds to a smoother spectrum with a modest pile-up (or “bump”) and super-exponential (but not yet abrupt) cutoff.

Fig. 2. The Spectral Energy Distributions of the synchrotron radiation produced by protons with spectra shown in Fig. 1. The curve 1 corresponds to the proton spectrum described by Eq.(7); the curve 2 corresponds to the truncated proton spectrum; the curve 3 corresponds to the proton spectrum with a sharp pile-up and an abrupt cutoff at \( E_0 \); the curve 4 corresponds to the proton spectrum with a smooth pile-up and a super-exponential cutoff. For comparison, the SED of the synchrotron radiation of mono-energetic protons, \( xF(x) \propto x^{4/3} \exp(-x) \), is also shown (curve 5).

The corresponding spectral energy distributions (SED: \( \nu S_\nu = \epsilon^2 J(\epsilon) \)) of synchrotron radiation are shown in Fig. 2. In the high energy range, \( \epsilon \geq \epsilon_0 \), where \( \epsilon_0 \) is defined by Eq.(2) as \( \epsilon_0 = \epsilon_c(E_0) \), the radiation spectrum from the proton distribution with sharp pile-up and abrupt cut-off is quite similar to the synchrotron spectrum from mono-energetic protons, \( xF(x) = x^{4/3} \exp(-x) \); \( x = \epsilon/\epsilon_0 \). This is explained by the radiation component associated with the line-type feature in the proton spectrum at \( E = E_0 \).
All synchrotron spectra shown in Fig. 2 exhibit, despite their essentially different shapes, cutoffs approximately at $x \sim 1$, if one defines the cutoff as as the energy at which the differential spectrum drops to $1/e$ of its extrapolated (from low energies) power-law value. Therefore the energy $\epsilon_0 = \epsilon_c (E_0)$ could be treated as an appropriate parameter representing the synchrotron cutoff for a quite broad class of proton distributions. In the case of monoenergetic protons, the cutoff energy coincides exactly with $\epsilon_0$ (see Eq. 5). This is true also for the power-law proton spectrum with exponential cutoff given by Eq. (7), for which the SED of the synchrotron radiation has a shape close to $\nu S_\nu \propto \epsilon^{1/2} \exp \left[ -\left( \epsilon / \epsilon_0 \right)^{1/2} \right]$ (see below).

Below I will not specify the proton acceleration mechanisms, but rather assume, with some exceptions notified otherwise, that the accelerated protons in the source are represented by a “standard” featureless spectrum given by Eq. (7). For this energy distribution of protons, the delta-functional approximation gives a simple analytical expression for the differential spectrum of synchrotron radiation: $J(E) \propto \epsilon^{-\Gamma} \exp \left[ -\left( \epsilon / \epsilon_0 \right)^{1/2} \right]$, where $\Gamma = (\alpha_p + 1)/2$, and $\zeta$ is a free parameter introduced in order to adapt this formula to the accurate numerical calculations.

The spectra of synchrotron radiation calculated in the delta-functional approximation for $\zeta = 1/3, 2/3, 1,$ and $4/3$ (curves 2, 3, 4, and 5, respectively) are shown in Fig. 3, together with the result of accurate numerical calculations (curve 1). It is seen that the best broad-band fit, with accuracy better than 25% in the entire region up to $\epsilon \sim 20 \epsilon_0$, is provided by $\zeta = 1$. Although $\zeta = 2/3$ gives, in fact, better fit at $\epsilon \leq \epsilon_0$, it significantly underestimates the flux above $\epsilon_0$. And finally, both $\zeta = 1/3$ and $\zeta = 4/3$ corresponding to the maximums in $S_\nu$ and $\nu S_\nu$ distributions of radiation emitted by a mono-energetic electron, do not adequately describe the radiation spectrum at $\epsilon \geq 0.1 \epsilon_0$. Thus, the most appropriate value for $\zeta$ is close to 1. This proves that the synchrotron cutoff indeed takes place at $\epsilon_0$. At energies $\epsilon \ll \epsilon_0$ the radiation spectrum has a power-law shape with photon index $(\alpha_p + 1)/2$. Note that the steepening of the $\gamma$-ray spectrum at higher energies is significantly smoother ($\propto \exp \left[ -\left( \epsilon / \epsilon_0 \right)^{1/2} \right]$) than the exponential cutoff in the spectrum of the parent protons. For the power-law proton distribution truncated at $E_0$, the synchrotron spectrum beyond $\epsilon_0$ extends approximately as $\exp \left[ -\left( \epsilon / \epsilon_0 \right)^{1/2} \right]$.

Assuming now that the synchrotron radiation is emitted by a relativistically moving cloud of plasma (often called in literature as plasmon or blob), in which the spectrum of accelerated protons are represented by Eq. (7), the differential flux of radiation in the delta-functional approximation can be presented in the

\footnote{In fact, the numerical calculations show that the spectrum in the cutoff region drops even slower than $\exp \left[ -\left( \epsilon / \epsilon_0 \right)^{1/2} \right]$; compare the curves 1 and 4 in Fig. 3.}
Fig. 3. Spectral Energy Distribution (SED) of the proton-synchrotron radiation calculated in the delta-functional approximation for the power-law spectrum of protons with a spectral index \( \alpha_p = 2 \) and an exponential cutoff at \( E_0 \) (Eq. 7). The curves 2, 3, 4, and 5 correspond to the parameter \( \varsigma = 1/3, 2/3, 1 \) and \( 4/3 \), respectively. The curve 1 correspond to accurate numerical calculations.

The following simple form

\[
J(\epsilon) = A\epsilon^{-\Gamma} \exp\left[-(\epsilon/\epsilon_0)^{1/2}\right],
\]

where \( A = (N_0/48\pi^2d^2)(\sigma_TB^2m_e^2/m_p^4c^3)\ a^{(\alpha_p-3)/2}\delta_j^{(\alpha_p+5)/2},\ \Gamma = (\alpha_p + 1)/2,\ \epsilon_0 = \delta_j\epsilon_c(E_0), \) and \( \delta_j \) is the jet’s Doppler factor.

This convenient equation could be used for the fit of TeV observations from BL Lac objects in the framework of the proton-synchrotron model, assuming that the spectrum of the accelerated protons is given by Eq. (7).

### 3 Efficiency of proton synchrotron radiation

The efficiency of a \( \gamma \)-ray production mechanism is characterized by the ratio of the radiative cooling time to the typical dynamical time-scale of the source corresponding to the minimum radiation variability in the jet’s frame

\[
t^* = \Delta t \delta_j \approx 1.08 \times 10^5 \Delta t_{3\text{h}} \delta_{10} \text{ s},
\]
where $\Delta t_{3h} = \Delta t/3\ h$ is the radiation variability in the observer’s frame in units of 3 hour - a characteristic time-scale observed from Markarian 501 in TeV $\gamma$-rays (Aharonian et al. 1999a; Quinn et al. 1999), and $\delta_{10} = \delta_{j}/10$. The proton-synchrotron radiation becomes an effective mechanism of $\gamma$-radiation with cooling time $\leq 10^9\ s$ only when the photons are emitted by EHE protons with $E \geq 10^{19}\ eV$ in a strong magnetic field close to 100 G. In this regime, the synchrotron losses well dominate over non-radiative losses (see Fig. 4) caused by adiabatic expansion of the source or escape of particles from the source with characteristic time-scales $(c/v_{0}) \times t^{*}$ and $(c/v_{esc}) \times t^{*}$, even in the case of relativistically expanding source $(v_{0} \sim c)$, or energy-independent escape of particles with speed of light, $v_{esc} \sim c$.

It is interesting to compare the proton-synchrotron cooling time $t_{sy}$ with the photo-meson cooling time, $t_{p\gamma}$. The energy flux of low-frequency radiation that originates in the jet could be presented in the following form

$$\nu S_{\nu} = 10^{-12} g_{\text{fir}} (h\nu/0.01\ eV)^{-s+1} \text{erg/cm}^2\text{s}$$ (10)

where $g_{\text{fir}}$ is a scaling factor indicating the level of the far infrared (FIR) flux at $h\nu = 0.01\ eV$ ($\lambda \sim 100\ \mu m$) in units of $10^{-12}\ \text{erg/cm}^2\text{s}$. The flux normalization at FIR wavelengths, which play a major role in proton-photon interactions in the jet, makes the final results of calculations quite insensitive to the choice of the spectral index $s$. In the case of Markarian 501, the power-law extrapolation of the observed synchrotron flux at soft X-rays ($\leq 1\ \text{keV}$) towards far infrared wavelengths with $s = 0.5$ gives $g_{\text{fir}} \simeq 0.3$. On the other hand, assuming that the whole radio-to-infrared flux observed from the direction of Markarian 501 (see, e.g., Pian et al. 1998) is produced in a small-scale ($\gamma$-ray emitting) jet, the factor $g_{\text{fir}}$ could be one order of magnitude larger.

For a source with the co-moving frame luminosity $L'((\nu')\nu$, the observer at a distance $d = cz/H_0$ would detect a flux (Lind & Blandford 1985) $S_{\nu} = \delta_{j}^{2}L'(\nu/\delta_{j})/4\pi d^{2}$ (hereafter, the value $H_0 = 60\ \text{km/s/Mpc}$ will be used for the Hubble constant). The energy losses of protons in a low-frequency photon field with a broad power-law spectrum are dominated by photomeson-processes. For broad and flat spectra of target photons the photo-meson cooling time of protons can be estimated as $t_{p\gamma}(E) \simeq (c < \sigma_{p\gamma}f > n(\nu^{*})h\nu^{*})^{-1}$, thus for the flux given by Eq. (10) with $s=0.5$,

$$t_{p\gamma} \simeq 4.5 \times 10^{7} \Delta t_{3h}^{2} \delta_{10}^{5.5} (z/0.03)^{-2} g_{\text{FIR}}^{-1} E_{19}^{-0.5} \text{s},$$ (11)

where $< \sigma_{p\gamma}f > \simeq 10^{-28}\ \text{cm}^2$ is the photo-meson production cross section weighted by inelasticity at the photon energy $\sim 300\ \text{MeV}$ in the proton rest frame, and $h\nu^{*} \simeq 0.03 E_{19}^{-1}\ eV$ (see, e.g., Stecker 1968; Mücke et al. 1999). Thus, for both Markarian 501 and Markarian 421 with redshifts $z = 0.034$ and
\( z = 0.031 \), respectively, the photo-meson cooling time cannot be significantly less than \( 10^7 \) s, unless we assume a very high ambient photon density. Formally this could be possible, for example, adopting a much smaller blob size than it follows from the observed flux variability \( \Delta t_{3h} \sim 1 \), and/or assuming a small Doppler factor of the jet, \( \delta_j \ll 10 \). However, the photon density in the source cannot be arbitrarily increased, otherwise it would result in a catastrophic absorption of TeV radiation inside the source.

In the field of ambient photons with differential power-law spectrum \( n(\nu) \propto \nu^{-(s+1)} \), the optical depth for the photon-photon absorption is equal to \( \tau_{\gamma \gamma}(\epsilon) = A(s)(\sigma_T/2)h\nu_0 n(h\nu_0)R \), where \( h\nu_0 = 4(m_e c^2)^2/\epsilon \), and \( A(s) = 7/12 \cdot 4^{s+1}(s + 1)^{-5/3}/(s + 2) \) (Svensson 1987). For the spectral index \( s = 0.5 \),

\[
\tau_{\gamma \gamma} \simeq 0.4 \cdot \Delta t_{3h}^{-1} \cdot \delta_{10}^{-5} \cdot g_{\text{fir}} (z/0.03)^2 (\epsilon/1 \text{ TeV})^{1/2}.
\]  (12)

The optical depth \( \tau_{\gamma \gamma} \) increases with energy as \( \tau_{\gamma \gamma} \propto \epsilon^{1/2} \), therefore at 1 TeV it should not significantly exceed 1, otherwise the absorption of \( \geq 10 \) TeV \( \gamma \)-rays becomes unacceptably large. For Markarian 501, assuming that the low-frequency radiation in the blob is described by Eq. (10) with \( s = 0.5 \) and adopting a rather relaxed estimate for \( g_{\text{fir}} \sim 0.3 \), the condition \( \tau_{\gamma \gamma}(1 \text{ TeV}) \leq 1 \) results in a robust lower limit on the blob’s Doppler factor \( \delta_j \geq 7 \).

Fig. 4. The characteristic acceleration and energy loss time-scales of protons in the blob. At each curve the scaling factors (the products of relevant physical parameters) are shown (see the text).

It is worth noting that formally we may expect non-negligible flux of TeV \( \gamma \)-rays even at conditions when \( \tau_{\gamma \gamma} \gg 1 \), the radiation being contributed from the
“last layer” of the source with $\tau_{\gamma\gamma} \sim 1$. For a (quasi) homogeneous spherical source, this implies that the “TeV photosphere” occupies only $1/\tau_{\gamma\gamma}$ part of the total volume of the source. On the other hand, the initial TeV radiation, produced throughout the entire source, is re-radiated in the form of secondary (cascade) electromagnetic radiation in the energy band below $\epsilon^*$, where $\epsilon^*$ is determined by the condition $\tau_{\gamma\gamma}(\epsilon^*) = 1$. Now assuming, for example, that at 1 TeV $\tau_{\gamma\gamma} = 10$, the $\gamma$-ray luminosity of the source below $\epsilon^* = 10$ GeV (for $s = 0.5$) should exceed the luminosity of the “TeV photosphere” by an order of magnitude. This obviously contradicts the $\gamma$-ray observations of Markarian 421, and especially Markarian 501. Therefore we may conclude that the optical depth at 1 TeV, $\tau_{1\text{TeV}} = \tau_{\gamma\gamma}(1\text{ TeV}) \ll 10$.

From Eqs.(11) and (12) we obtain a simple relation between the photo-meson cooling time of protons, $t_{p\gamma}$, and the optical depth $\tau_{1\text{TeV}}$:

$$t_{p\gamma} \simeq 1.8 \times 10^7 \Delta t_{3h} \delta_{10}^{1/2} \tau_{1\text{TeV}}^{-1} E_{19}^{-1/2} \text{s.}$$

(13)

For a steeper spectrum of low-frequency radiation, e.g. a power-law with $s = 1$, and for the same normalization to $\tau_{1\text{TeV}}$, the photo-meson cooling time is shorter:

$$t_{p\gamma} \simeq 10^6 \Delta t_{3h} \tau_{1\text{TeV}}^{-1} E_{19}^{-1} \text{s.}$$

(14)

Even so, $t_{p\gamma}$ remains significantly larger than $t^*$, especially if we take into account that for $s = 1$ the optical depth depends more strongly on energy, $\tau_{\gamma\gamma} \propto \epsilon$, and therefore the detection of TeV radiation from Markarian 501 beyond 10 TeV is an indication that $\tau_{1\text{TeV}}$ should be significantly less than 1.

The source transparency condition for multi-TeV $\gamma$-rays implies that in the TeV blazars the photo-meson processes proceed on significantly larger time-scales compared with $t^*$ and $t_{sy}$ (see Fig. 4). In particular, for the power-law spectrum of low-frequency radiation with a (most likely) slope $s = 0.5$

$$\frac{t_{p\gamma}}{t^*} \simeq 1.7 \times 10^2 \delta_{10}^{-0.5} \tau_{1\text{TeV}}^{-1} E_{19}^{-0.5},$$

(15)

and

$$\frac{t_{p\gamma}}{t_{sy}} \simeq 4 \times 10^2 \Delta t_{3h} \delta_{10}^{0.5} B_{100}^{-2} \tau_{1\text{TeV}}^{-1} E_{19}^{0.5}.$$
radiation in the blob, the detection of TeV $\gamma$-rays from any blazar would imply low efficiency of the photo-meson processes in the jet, unless the energy of protons does not significantly exceed $10^{19}$ eV. In compact $\gamma$-ray production region(s) of the jet with a typical size $R \leq 3 \times 10^{15} \Delta t_{3i} \delta_{10}$ cm, the proton acceleration to such high energies is possible only in the presence of magnetic field $B \gg 10$ G (see below). At such conditions, however, the synchrotron radiation becomes a more effective channel for conversion of the kinetic energy of accelerated protons to very high energy $\gamma$-rays. In principle, the difficulty with synchrotron losses could be overcome by adopting a weak ($B \leq 1$ G) magnetic field in the blob, but assuming that the EHE protons are accelerated outside of the blob, e.g. near the central compact object, and then transported along with the jet (Kazanas & Mastichiadis 1999). However, this assumption does not solve the second problem connected with the low efficiency of the photo-meson processes in the jet imposed by the transparency condition for TeV $\gamma$-rays.

A regrettable consequence of this conclusion is the suppressed TeV neutrino flux. However, it should be emphasized that this statement concerns only the objects seen in TeV $\gamma$-rays. Meanwhile, the pair cascades initiated by secondary electrons and $\gamma$-rays from $p\gamma$ interactions may still remain a viable possibility for other AGN, in particular for the powerful GeV blazars detected by EGRET (Mukherjee et al. 1997), as well as for radio-quiet AGN (e.g. Sikora et al. 1987), where the radiation density is much higher than in the BL Lac objects, and, more importantly, the photo-meson cooling time of EHE protons is not constrained by the severe TeV $\gamma$-ray transparency condition.

4 Self regulated synchrotron cutoff

The energy spectrum of synchrotron radiation depends on the spectrum of accelerated protons and the jet’s Doppler factor. The high energy cutoff in the spectrum of protons is determined by the balance between the particle acceleration and cooling times. It is convenient to present the acceleration time of particles $t_{acc}$ in the following general form

$$t_{acc} = \frac{\eta(E) r_g}{c} = 1.36 \times 10^4 E_{19} B_{100}^{-1} \eta(E) \text{ s,} \quad (17)$$

where $r_g = E/(eB_\perp)$. The so-called gyro-factor $\eta(E) \geq 1$ characterizes the energy-dependent rate of acceleration. For almost all proposed models of particle acceleration in different astrophysical environments, $\eta$ remains a rather uncertain model parameter. This is true especially for the small-scale jets of blazars, where the nature of the acceleration mechanism itself remains highly unknown. On the other hand, any postulation of acceleration of EHE protons
in compact \(\gamma\)-ray production regions of the small-scale jets actually implies that the parameter \(\eta\) should be close to 1, which corresponds (independent of a specific mechanisms of acceleration) to the maximum (theoretically possible) acceleration rate based on a simple geometrical consideration (Hillas 1984). In the case of diffusive shock acceleration in the blazar jets the parameter \(\eta\) typically is expected (Henri et al. 1999) to be larger than 10 (see, however, Bednarz & Ostrowski 1996). Therefore, perhaps more effective acceleration mechanisms should be invoked for production of EHE protons in small-scale AGN jets. An interesting possibility could be the particle acceleration at the annihilation of magnetic fields in the fronts of Poynting flux dominated jets (Blandford 1976; Lovelace 1976). It has been argued that this mechanism could provide effective acceleration of EHE protons with \(\eta \sim 1\) (Haswell et al. 1992).

The discussion of particle acceleration mechanisms is beyond the framework of this paper, and therefore here the assumption of \(\eta \leq 10\) should be treated just as an working hypothesis. If true, during the characteristic time \(t^*\) given by Eq. (9), the protons could be accelerated in the jet with \(B \sim 100\,\text{G}\) and \(\delta_j \sim 10\) up to energies \(E \sim 10^{20}\,\text{eV}\). At such conditions the losses of highest energy protons are dominated by the proton-synchrotron radiation, and therefore the cutoff energy \(E_0\) is determined by the condition \(t_{sy} = t_{acc}\): 

\[
E_0 = \frac{3}{2}^{3/4} \sqrt{\frac{1}{e^2 B}} m_p c^4 \approx 1.8 \times 10^{19} B_{100}^{-1/2} \eta^{-1/2} \text{eV.} \tag{18}
\]

Note that the relevant cutoff in the electron spectrum appears much earlier, \(E_{\text{e,0}} = (m_e/m_p)^2 E_0 \simeq 5.3 \times 10^{12} B_{100}^{-1/2} \eta^{-1/2} \text{eV}\).

Substituting Eq. (18) into Eq. (2) we find that the position of the cutoff in the spectrum of the proton-synchrotron radiation is determined by two fundamental physical constants, the proton mass \(m_p c^2 = 938\,\text{MeV}\), and fine-structure constant \(\alpha_f = 1/137\). It depends only on the parameter \(\eta\), but not on the magnetic field:

\[
\epsilon_0 = \frac{9}{4} \alpha_f^{-1} m_p c^2 \eta^{-1} \approx 0.3 \eta^{-1} \text{TeV.} \tag{19}
\]

For the electron-synchrotron radiation, the universal cutoff appears at \(\epsilon_0^{(e)} = (m_e/m_p) \epsilon_0^{(p)} = 9/4 \alpha_f^{-1} m_e c^2 \eta^{-1} \approx 0.160 \eta^{-1} \text{GeV}\).

Comparing now the minimum (energy-independent) escape time of protons \(t_{\text{esc}} \sim R/c \simeq 3.3 \times 10^4 R_{15} \xi\,\text{s}\), with the synchrotron cooling time of the protons responsible for production of \(\gamma\)-rays in the cutoff region, \(t_{sy} \simeq 2.4 \times 10^4 B_{100}^{-3/2} \eta^{1/2} \text{s}\) we find the following condition for formation of the self-
regulated synchrotron cutoff:

\[ B_{100} \geq 0.8 \times R_{15}^{-2/3} \eta^{1/3} \]  

(20)

where \( R_{15} = R/10^{15} \) cm is the source radius in units \( 10^{15} \) cm.

If the synchrotron \( \gamma \)-rays are produced in the relativistically moving source, the spectral cutoff is shifted towards the TeV domain:

\[ \epsilon_0 \simeq 3 \delta_{10} \eta^{-1} \text{TeV} . \]  

(21)

Thus, in the regime of acceleration with \( \eta \leq 10 \), the proton-synchrotron radiation emitted by a highly magnetized \((B \sim 100 \text{ G})\) compact blob with a typical size \( R_{15} \simeq 3.2 \Delta t_{\text{sh}} \delta_{10} \) cm and Doppler factor \( \delta_j \geq 10 \), results in effective production of TeV \( \gamma \)-rays. Remarkably, despite possible changes of some principal model parameters, first of all the size \( R \) and the magnetic field \( B \) of the production region (e.g. caused by expansion or compression of the blob), we should expect a rather stable position of the synchrotron cutoff, provided that the parameter \( \eta \) and the Doppler factor \( \delta_j \) remain unchanged. Meanwhile any change of \( B \) and/or \( R \) should result in strong \((\propto B^2 R^3)\) variations of the absolute flux of synchrotron radiation. This intrinsic feature of the proton-synchrotron radiation could explain in a natural way the effect of weak correlation between the spectral shape and the absolute flux of Markarian 501 revealed during the extraordinary outburst of the source in 1997 (Aharonian et al. 1999a,b).

Dramatic changes of conditions in the \( \gamma \)-ray production region should lead, in fact, to the transition of the source from the regime dominated by synchrotron losses to the regime dominated by the particle escape or by adiabatic losses. For example, in the case of relativistically expanding blob in which the reduction of the magnetic field is faster than \( B \propto R^{-2/3} \), at some stage of the source evolution the condition represented by Eq.(20) could be violated, and thus the spectral shape of th proton-synchrotron radiation would become time-dependent. Indeed, in this regime the cutoff energy in the spectrum of protons is determined by the balance between \( t_{\text{acc}} \) and \( t_{\text{esc}} \), which gives

\[ \epsilon_0 \simeq 5.2 \times B_{100}^3 R_{15}^2 \eta^{-2} \delta_{10} \text{TeV} . \]  

(22)

Note that Eq. (20) keeps the synchrotron cutoff energy, which is formed in the regime dominated by particle escape, always less than the cutoff energy formed in the synchrotron-loss-dominated regime. For example, at stages of source evolution when \( R \leq 3 \times 10^{15} \) cm and magnetic field \( B \leq 10 \) G, the \( \gamma \)-ray spectrum is formed in the regime dominated by particle escape, thus
\[ \epsilon \leq 0.05 \eta^{-2} \delta_{10} \text{TeV}, \] and correspondingly, at energies beyond 1 TeV the \( \gamma \)-ray spectrum becomes very steep. In the escape-loss-dominated regime the cutoff energy, and thus the shape of the entire synchrotron spectrum, strongly depend on the magnetic field and the size of the source. In these stages of source evolution a significant spectral variability is expected. Depending on the relationship between the three appropriate time-scales, \( t_{\text{acc}} \), \( t_{\text{sy}} \), and \( t^* \), we may predict, similar to the electron synchrotron radiation in blazars (Kirk et al. 1998), quite different variability patterns such as “soft lags” (when \( t_{\text{sy}} \gg t^* \gg t_{\text{acc}} \)), “hard lags” (when \( t_{\text{sy}} \sim t_{\text{acc}} \sim t^* \)), etc.

Finally, it should be noted that the standard self-regulated synchrotron cutoff determined by Eq. (19) corresponds to the scenario when the proton acceleration and \( \gamma \)-ray production take place in the same localized region. In the case of external injection of accelerated protons into the magnetized cloud, the energy of synchrotron radiation is determined by the magnetic field in the \( \gamma \)-ray production region and the maximum energy of injected protons, therefore it could exceed the ‘self-regulated synchrotron cutoff.

5 Implications to Markarian 501 and Markarian 421

The HEGRA and Whipple observations of Markarian 501 during the extraordinary outburst in 1997 revealed that despite spectacular (up to factor of 10 or more) flux variations, the shapes of the daily \( \gamma \)-ray spectra remained unchanged\(^4\) throughout the entire state of high activity (Aharonian et al. 1999a,b; Quinn et al. 1999). To some extent this could be true also for Markarian 421 (Aharonian et al. 1999c; Krennrich et al. 1999), although the uncertainties in the daily TeV spectra are too large for a firm conclusion.

Markarian 421 and Markarian 501 exhibit substantially different \( \gamma \)-ray spectra. The time-averaged spectra of Markarian 501 in 1997 (Aharonian et al. 1999b) and Markarian 421 (Aharonian et al. 1999c) as measured by the HEGRA system of telescopes are shown in Fig. 5. The differential flux of Markarian 501 from 500 GeV to 24 TeV is described by a power-law with an exponential cutoff: \( dN/d\epsilon \propto \epsilon^{-\Gamma} \exp (\epsilon/\epsilon_0) \) with \( \Gamma \simeq 2 \) and \( \epsilon_0 \simeq 6.2 \text{TeV} \) (Aharonian et al., 1999b). Remarkably, a quite similar spectrum was detected by the HEGRA collaboration during a short, but strong outburst of Markarian 501 in June 1998 (Sambruna et al. 2000). In the TeV regime, Markarian 421 has a steeper spectrum. In the interval from 500 GeV to 7 TeV it could be approximated by

\(^4\) In fact, the CAT group has found a tendency of the spectrum of Markarian 501 to become somewhat harder during the flares (Djannati-Atai et al. 1999). However, the effect is not very strong and could not change the general conclusion about essentially stable spectral shape of TeV radiation in the high state.
Fig. 5. The proton-synchrotron radiation fits to the TeV spectra of Markarian 501 and Markarian 421. The TeV data of Markarian 501 correspond to the high-state spectrum of the source as measured by HEGRA in 1997 (Aharonian et al. 1999b). The TeV data of Markarian 421 correspond to the average HEGRA spectrum based on the measurements in 1997-1998 (Aharonian et al. 1999c). The dashed and solid curves correspond to the spectra of proton-synchrotron radiation before and after correction for intergalactic extinction, respectively. A “low-CBR” model is assumed (solid curve in Fig. 6). The data of Markarian 501 are fitted assuming $\alpha_p = 2$ and $\epsilon_0 = 1.3$ TeV, and the data of Markarian 421 are fitted assuming $\alpha_p = 2$ and $\epsilon_0 = 0.26$ TeV.

a pure power-law with a photon index $\Gamma \simeq 3$ (Aharonian et al. 1999c; Piron et al. 2000), or by a power-law with $\Gamma = 2.5 \pm 0.4$ and exponential cutoff at $\epsilon_0 = 2.8^{+2.0}_{-0.9}$ TeV (Aharonian et al. 1999c).

5.1 Extragalactic extinction of gamma rays

The spectra of TeV radiation observed from distant ($d \geq 100$ Mpc) extragalactic objects suffer essential deformation during the passage through the intergalactic medium caused by energy-dependent absorption of primary $\gamma$-rays at interactions with the diffuse extragalactic background radiation (Gould & Schreder 1966; Stecker et al. 1992, Vassiliev 2000). For Markarian 421 and Markarian 501 at distances 150 Mpc and 170 Mpc, respectively, this effect could be significant already at sub-TeV energies (Guy et al. 2000), and becomes especially strong at energies above 10 TeV, where the optical depth $\tau_{\gamma\gamma}$ most likely significantly exceeds 1 (Coppi & Aharonian 1999b). The lack
of a relevant broad-band information about the cosmic background radiation (CBR) from sub-micron to sub-mm wavelengths introduces an ambiguity in the interpretation of the observed $\gamma$-ray spectra. Nevertheless, the recent reports about detection of CBR at near infrared ($2.2 \mu m$ and $3.5 \mu m$ - Dwek & Arendt 1998; Gorjian et al. 2000), mid infrared ($15 \mu m$ - Elbaz et al. 1999), and far infrared ($140 \mu m$ and $240 \mu m$ - Schlegel et al. 1998; Hauser et al. 1998) bands, allow significant reduction of the uncertainty in the intergalactic extinction. Yet, these measurements alone are not sufficient for quantitative study of the intergalactic extinction of $\gamma$-rays, $\kappa(E) = \exp[-\tau_{\gamma\gamma}(E)]$, in the observed energy range from 0.5 to 20 TeV. Apparently, a suitable theoretical model of CBR is needed which should reproduce properly the shape of both “stellar” and “dust” components of the background radiation. Note that most of cosmological models give rather similar shapes of CBR with two distinct bumps at $1-2 \mu m$ and $100-200 \mu m$ and a valley at $10-20 \mu m$ (see, e.g., Dwek et al. 1998; Primack et al. 1999).

![Cosmic Background Radiation](image.png)

Fig. 6. The Cosmic Background Radiation (CBR). The solid curve corresponds to the “low-CBR”, and the dashed curve corresponds to the “high-CBR” model predictions (see the text). The dotted curve shows the flux of the 2.7 K microwave background radiation. The tentative detections of CBR at $2.2 \mu m$ and $3.5 \mu m$ (filled circles) are from Gorjian et al. (2000), and at $15 \mu m$ (the star) from Elbaz et al. (1999); the fluxes at $140 \mu m$ and $240 \mu m$ (filled triangles) are from Hauser et al. (1998). The other upper/lower CBR flux limits (open symbols) are taken from the recent compilation of CBR by Dwek et al. (1998).

Below I will use the so-called LCDM model of Primack et al. (1999), but the absolute fluxes of both “stellar” and “dust” components of radiation will be allowed to vary within a factor of 2 or so. The prediction for the CBR flux by the LCDM model (hereafter “low-CBR”) and re-scaled LCDM model,
assuming 50 per cent higher flux of the “stellar” component, and twice higher flux of the “dust” component (hereafter “high-CBR”), are presented in Fig. 6. For comparison, the recently reported CBR flux measurements or upper/lower limits from UV to FIR are also shown. Despite large statistical and systematic uncertainties, the actual spectrum of CBR most probably does not deviate significantly from the two simplified model predictions shown in Fig. 6.

5.2 Fitting the TeV spectra of Markarian 501 and Markarian 421

The spectrum of the proton-synchrotron radiation corrected for intergalactic extinction can satisfactorily fit, for a reasonable combinations of a limited number of model parameters, the observed TeV fluxes of both Markarian 501 and Markarian 421. This is demonstrated in Fig. 5, where the average TeV fluxes of both objects are shown together with the model predictions. The theoretical spectra, after corrections for the intergalactic extinction assuming the “low CBR”, are normalized to the measured fluxes at 1 TeV. This determines, for the given magnetic field, the required total energy in accelerated protons. Assuming that the accelerated protons have an energy distribution represented by Eq. (7), the observed spectra of TeV emission can be fitted by the proton-synchrotron radiation for the following combinations of model parameters: \( \alpha_p = 2 \) and \( \epsilon_0 = 1.3 \text{TeV} \) for Markarian 501, and \( \alpha_p = 2 \) and \( \epsilon_0 = 0.26 \text{TeV} \) for Markarian 421. Assuming now that the \( \gamma \)-rays are produced in the synchrotron-loss-dominated regime, from Eq. (21) one can easily estimate the ratio \( \rho = \delta_{10}/\eta \) which is the most relevant parameter for determination of the position of the self-regulated synchrotron cutoff – \( \rho = 0.43 \) and 0.09 for Markarian 501 and Markarian 421, respectively.

The predicted for Markarian 501 proton-synchrotron radiation flux passes below the HEGRA point at 24 TeV (see Fig. 5). This discrepancy could be removed, in principle, by assuming a different (steeper) shape of the spectrum of the “dust” component compared with predictions of current CBR models (Coppi & Aharonian 1999b), and/or assuming a flatter spectrum of protons in the region of the cutoff \( E_0 \) compared to the spectrum given by Eq. (7). However, because of the inadequate statistical significance of the \( \gamma \)-ray data above 16 TeV (Aharonian et al. 1999b), the last point on Fig. 5 should be considered a flux upper limit rather than a positive detection.

If one treats the TeV observations independently of data obtained in other energy domains, the parameters chosen to fit the TeV spectra cannot be unique in a sense that some other combinations of \( \alpha_p \) and \( \epsilon_0 \) could still provide reasonable spectral fits. This is demonstrated in Fig. 7 where the measured fluxes of Markarian 501 are shown together with the predicted spectra of the proton-synchrotron radiation assuming “high-CBR” model for the background radi-
Fig. 7. The SED of Markarian 501. The thin and heavy curves correspond to the spectra of proton-synchrotron radiation before and after correction for the inter-galactic extinction, respectively. The “high-CBR” model is assumed (dashed curve on Fig. 6). The data of Markarian 501 are fitted by parameters $\alpha_p = 2$ and $\epsilon_0 = 1.8\,\text{TeV}$ (solid curve), $\alpha_p = 1.5$ and $\epsilon_0 = 1.2\,\text{TeV}$ (dashed curve), and $\alpha_p = 2.4$ and $\epsilon_0 = 3.5\,\text{TeV}$ (dot-dashed curve).

It is seen that the detected fluxes could be equally well fitted by synchrotron radiation for 3 different combinations: (i) $\alpha_p = 2$ and $\epsilon_0 = 1.8\,\text{TeV}$ (or $\rho \simeq 0.6$, if the energy losses of protons are dominated by the synchrotron radiation), (ii) $\alpha_p = 1.5$ and $\epsilon_0 = 1.2\,\text{TeV}$ ($\rho \simeq 0.4$), and (iii) $\alpha_p = 2.4$ and $\epsilon_0 = 3.6\,\text{TeV}$ ($\rho \simeq 1.2$). The impact of the uncertainty in the CBR flux can be estimated by comparing the solid curves in Fig. 5 and Fig. 7. While in both cases the power-law index of the proton spectrum is assumed $\alpha_p = 2$, the “high-CBR” requires somewhat ($\approx 40$ per cent) higher synchrotron cutoff compared with the case of “low-CBR”. Also, for the “high-CBR” the discrepancy of the theoretical and measured fluxes above 16 TeV becomes more apparent.

An important constraint on the possible $\alpha_p/\epsilon_0$ combination could be provided by $\gamma$-ray data obtained at low, MeV/GeV energies. In particular, the combination of parameters used for the fit of the spectrum of Markarian 421 in Fig. 5, predicts an energy flux at 100 MeV of about $2 \times 10^{-11}\,\text{erg/s}$, i.e. factor of 3 less than the EGRET flux averaged over the 1992-1996 period of observations (Sreekumar et al. 1996). Therefore, assuming that the observed MeV/GeV fluxes are associated with the proton-synchrotron radiation as well, the best fit of the data from 100 MeV to 10 TeV could be obtained adopting
a steeper proton spectrum with $\alpha_p = 2.4$ (see Fig. 8). Although the data obtained at TeV and MeVGeV energies correspond to different epochs, the lack of evidence of strong spectral variability in both energy domains (Aharonian et al. 1999c; Sreekumar et al. 1996) makes such a fit quite meaningful.

![Graph](image)

**Fig. 8.** The proton-synchrotron radiation of Markarian 501 and Markarian 421 from MeVGeV to TeV energies. All spectra are normalized to the observed fluxes at 1 TeV after correction for the intergalactic extinction assuming the “low-CBR” model. The data of Markarian 421 are fitted by $\alpha_p = 2.4$ and $\epsilon_0 = 1$ TeV. The data of Markarian 501 are fitted by three different combinations of $\alpha_p$ and $\epsilon_0$: $\alpha_p = 2$ and $\epsilon_0 = 1.3$ TeV (dashed curve); $\alpha_p = 2.6$ and $\epsilon_0 = 4$ TeV (dot-dashed curve); $\alpha_p = 1.5$ and $\epsilon_0 = 0.75$ TeV (dotted curve). The TeV data of Markarian 501 and Markarian 421 are the same as in Fig 2. The filled square at 100 MeV corresponds to the flux upper limit set by EGRET during the period April 9-15 (1997) when Markarian 501 was in a very high state (Catanese et al. 1997). The zone of the low-energy $\gamma$-ray fluxes of Markarian 421 shown from 50 MeV to 5 GeV corresponds to the spectrum of the source averaged over the Phase-1 period of the EGERET observations (Sreekumar et al. 1996).

The observations of Markarian 501 by EGRET during the extraordinary active state in April 1997 resulted only in the flux upper limit at 100 MeV (Catanese et al. 1997). This upper limit, combined with the TeV data, constrains the power-law index of the proton spectrum, $\alpha_p \leq 2.6$. For $\alpha_p = 2.6$ the position of the synchrotron cutoff should appear at high energies, $\epsilon_0 = 4$ TeV, in order to match the TeV spectrum (see Fig. 8). For the very flat proton spectrum with $\alpha_p = 1.5$, the synchrotron cutoff occurs below 1 TeV, $\epsilon_0 \simeq 0.75$ TeV,
otherwise the calculated spectrum becomes harder than the observed TeV spectrum. And finally, for the “standard” proton spectrum with $\alpha_p = 2$, the best fit value for the synchrotron cutoff is provided by $\epsilon_0 \simeq 1.3 \text{TeV}$, or $\rho \simeq 0.43$ if the TeV spectrum of Markarian 501 is formed in the synchrotron-loss-dominated regime. Because the Doppler factor of the small-scale jet in Markarian 501 is believed to be close to 10, a conclusion could be drawn that during the entire 1997 outburst the acceleration of particles in Markarian 501 took place in the regime of maximum acceleration rate ($\eta \sim 2$). If so, this explains in a rather natural way the essentially time-independent shape of the TeV spectrum observed during the strong flares in 1997 (Aharonian et al. 1999a,b).

Meanwhile, a significant drop in the acceleration rate (and/or the strength of the magnetic field) should lead to dramatic changes of the spectrum of $\gamma$-rays. This effect perhaps can explain the recent report of detection by EGRET of a very flat $\gamma$-ray flux at energies from 50 MeV to 5 GeV during the period of relatively low state of Markarian 501 in March 1996. A rather surprising aspect of this report is that the energy flux of $\gamma$-rays at several GeV was significantly larger than the X-ray and TeV fluxes measured approximately at the same time by ASCA and Whipple instruments (Kataoka et al. 1999). In the framework of the “proton-synchrotron radiation” hypothesis, the EGRET and Whipple fluxes could be explained with model parameters $\alpha_p = 2$ and $\epsilon_0 = 0.02 \text{TeV}$, or, if the proton acceleration takes place in the regime dominated by synchrotron losses, $\rho \simeq 0.007$ (Fig. 9). The latter implies dramatic reduction of the acceleration efficiency ($\eta \simeq 140 \delta_{10}$) compared with the high state, provided that the jet’s Doppler factor remains more or less constant. It is necessary to notice that this interpretation which adopts very small $\rho$ parameter, but tacitly assumes that the proton acceleration takes place in the synchrotron-loss-dominated regime, requires, as it follows from Eq. (20), an extremely large magnetic field, $B \geq 400 R_{15}^{-2/3} \delta_{10}^{1/3} \text{G}$. A more realistic scenario seems the transition of the source from the synchrotron-loss-dominated regime to the particle escape-loss-dominated regime, for example, due to a possible reduction of the magnetic field caused by the expansion of the blob. The significant drop of the magnetic field, e.g. from $B \sim 100 \text{G}$ to $B \sim 10 \text{G}$ would result in an earlier synchrotron cut-off.

The signature of such a low state is the shift of the position of the peak of SED down to 100 GeV or below, with a hard spectrum of GeV $\gamma$-rays, and very steep spectrum at TeV energies. While such a hard spectrum of GeV radiation is indeed observed, the small TeV photon statistics do not provide an adequate information about the spectrum at highest energies. Contrary to the synchrotron-loss-dominated regime, the spectrum of $\gamma$-rays formed in the escape-loss-dominated regime depends on parameters characterizing the production region, in particular the source size and the magnetic field. Therefore, in this stage of evolution, the source should show significant variations of the
Fig. 9. The proton-synchrotron radiation of Markarian 501 in a high and low states. The dashed and solid curves correspond to the spectra of proton-synchrotron radiation before and after correction for intergalactic extinction, respectively. The “low-CBR” model is assumed. The data of Markarian 501 in the high state are the same as in Fig. 2. The broad-band $\gamma$-ray data in the low state are obtained by the Whipple (open triangle) and EGRET (filled triangles) groups during the multiwavelength campaign in March 1996 (Kataoka et al. 1999). An “archival” upper limit on the 100 MeV flux based on the long-term observations of the source during the phase I period of EGRET is also shown (star). The high state of the source is fitted by $\alpha_p = 2$ and $\epsilon_0 = 1.3$ TeV. The low state of the source is fitted by $\alpha_p = 2$ and $\epsilon_0 = 0.02$ TeV.

The TeV spectrum of Markarian 421 is significantly steeper than the spectrum of Markarian 501. Another distinct feature of Markarian 421 is the TeV flux variability on extremely short, $\Delta t \sim 15$ minute timescales (Gaidos et al. 1996). Assuming that the radiation is formed in the synchrotron-loss-dominated regime, the steep $\gamma$-ray spectrum of Markarian 421 with $\epsilon_0 \simeq 0.26$ TeV (see Fig. 5) requires relatively slow acceleration rate, $\eta \simeq 12\delta_{10}$. On the other hand, in this regime the radiative cooling time of protons responsible for production of $\gamma$-rays in the cutoff region, $t_{sy} \simeq 2.4 \times 10^4 B_{100}^{-3/2} \eta^{1/2}$ s, could match the observed variability (in the frame of the jet) $9 \times 10^3 \delta_{10}$ s, if $B_{100} \geq 2\eta^{1/3}\delta_{10}^{-2/3}$. Therefore, $B_{100} \geq 4.5 \delta_{10}^{-1/3}$. Thus, if the TeV radiation of Markarian 421 is formed in the synchrotron-loss-dominated regime, the magnetic field should
significantly exceed 100 G, which seems a rather extreme assumption.

In a relatively moderate (and perhaps more realistic) magnetic field of about 100 G or less, both the observed rapid variability and the steep TeV spectrum of Markarian 421 could be interpreted, assuming that the radiation is formed in the escape-loss-dominated regime. Indeed, in the compact γ-ray production region with \( R \leq c \Delta t \delta_{ij} \approx 2.7 \times 10^{14}\delta_{10} \) cm, the position of the synchrotron cutoff is estimated \( \epsilon_0 \approx 0.38 B_{100}^3 \delta_{10}^{-2} \) TeV (see Eq. 22). For a reasonable combination of parameters \( B_{100} \sim 1, \delta_{10} \sim 1 \) and \( \eta \sim 1 \), this estimate agrees quite well with the allowed range of the synchrotron cutoff between 0.25 and 1 TeV as derived from the TeV spectrum of Markarian 421 (Fig. 5 and Fig. 8).

5.3 Synchrotron radiation from the proton distribution with sharp pile-up

The spectra of the proton-synchrotron radiation discussed above are calculated under the assumption of a “standard” (power-law with exponential cutoff) proton spectrum represented by Eq. (7). Actually, the spectra of accelerated particles at some circumstances could have more complicated and exotic forms, e.g. they could contain pile-ups preceding the spectral cutoffs. An example of a SED of synchrotron radiation corresponding to the proton spectrum with sharp pile-up and abrupt cutoff is shown in Fig. 10. The shape of this spectrum (dashed curve) essentially differs from the measured spectrum of Markarian 501. Moreover, for any reasonable spectral distribution of the CBR, the deformation of the initial γ-ray spectrum caused by the intergalactic extinction, hardly could bring it closer to the observed one. The desirable shape, however, could be achieved, if we assume that the deformation of the primary γ-ray spectrum takes place in two steps, e.g. presuming that the primary radiation suffers significant absorption twice - firstly in the vicinity of the source, and afterwards - in the intergalactic medium. The synchrotron radiation of the jet itself seems to be a natural photon field to serve as an internal absorber for the TeV radiation. Actually, such absorption is even unavoidable if the Doppler factor of the jet \( \delta_{ij} \leq 10 \). However, the pronounced peak in the production spectrum of radiation shown in Fig. 10 (dashed curve), requires rather selective absorption with strongest impact in the region of the γ-ray peak, which cannot be provided by the broad and flat spectrum of low-frequency synchrotron radiation of the jet. Because \( \gamma\gamma \rightarrow e^+e^- \) cross-section peaks near the threshold, there is approximately a one-to-one correspondence between the energies of the absorbed γ-ray \( \epsilon^* \) and the field photon \( h\nu \): \( \epsilon^* \approx 1 \) (\( h\nu/1\) eV)\(^{-1} \) TeV. Therefore, for a selective absorption of γ-rays from the region of the maximum of the production spectrum, the target photon field should have a quite narrow distribution, \( \Delta \nu/\nu \sim 1 \). The dot-dashed curve in Fig. 10 demonstrates the effect of such absorption in a monochromatic photon field with energy \( h\nu = 0.15 \) eV, and assuming that \( \tau_{\gamma\gamma} = 1 \) at \( \epsilon \approx 7 \) TeV. A more realistic,
The proton-synchrotron radiation of Markarian 501 calculated for the proton spectrum with a sharp pile-up (curve 3 in Fig. 1). Dashed curve - the production spectrum; dot-dashed curve - γ-ray spectrum after internal absorption; solid curve - γ-ray spectrum after internal and intergalactic absorptions. The following parameters are assumed: $\epsilon_0 = 7.8\,\text{TeV}$ (or $\rho = 2.6$), the mean energy of target photons responsible for internal absorption $\hbar\nu = 0.15\,\text{eV}$, optical depth $\tau_{\gamma\gamma} = 1$ at $\epsilon = 7\,\text{TeV}$. For the extragalactic extinction of γ-rays the “high-CBR” model is assumed.

e.g. thermal (Plankian) distribution of the target photons with temperature $kT \simeq 1/3 \hbar\nu \simeq 0.05\,\text{eV}$ gives a similar result, with a bit smoother spectrum of absorbed γ-rays at 2 TeV. The optical depth $\tau_{\gamma\gamma} \sim 1$ caused by mid infrared ($\lambda \sim 10\mu\text{m}$) photons, could be provided by different kind of sources in the vicinity of the central black hole (Celotti et al. 1998).

The TeV γ-rays, after suffering additional absorption in the intergalactic photon fields, will eventually arrive with “double-deformed” spectrum shown in Fig. 10 (solid curve). The calculated synchrotron spectrum, after corrections for internal and intergalactic absorption and normalized to the observed flux at 1 TeV, quite satisfactorily fits the SED of Markarian 501 in the high state, taking into account significant (up to factor of 2) systematic uncertainties of the HEGRA flux measurements at energies below 1 TeV and above 15 TeV (Aharonian et al., 1999b).
Although there is little doubt that the bulk of the highly variable X-ray emission of the BL Lac objects has synchrotron origin and is produced in the relativistic jets pointed to the observer (Urry & Padovani 1995), the TeV radiation of Markarian 421 and Markarian 501 is the only model-independent and unambiguous indicator of acceleration of ultra-relativistic particles in these small-scale (sub-pc) jets. In the currently popular synchrotron-self-Compton (SSC) models of BL Lac objects both the X-ray and γ-ray components are attributed to the radiation of directly accelerated TeV electrons. For a certain, physically well justified combination of parameters (the blob size, magnetic field, Doppler factor, etc.), these models (see, e.g., Inoue & Takahara 1996, Mastichiadis & Kirk 1997, Pian et al. 1998, Bednarek & Protheroe 1999, Coppi & Aharonian 1999a, Wagner et al. 1999; Kataoka et al. 1999, Maraschi et al. 1999, Takahashi et al. 1999, Krawczynski et al. 2000) give quite satisfactory explanation of the observed spectral and temporal characteristics of non-thermal radiation over more than ten decades of frequencies from 10^{16} Hz to 10^{27} Hz.

In the SSC models the strength of the magnetic field in the γ-ray emitting regions typically is less than 1 G, the most likely value being close to 0.1 G, which directly follows from the comparable energy fluxes released in X-rays and γ-rays during the TeV flares of Markarian 421 and Markarian 501 (e.g. Tavecchio et al. 1998). At such a low magnetic field the maximum energy of accelerated protons cannot exceed \( E_{p,\text{max}} \sim 10^{18} \Delta t_{3h} \delta_{10} (B/1 \text{ G}) \eta^{-1} \text{eV} \), and therefore the contribution of protons in γ-ray production through both the photo-meson and the synchrotron channels is negligible; the production of TeV γ-rays in the jet is strongly dominated by the inverse Compton scattering of electrons directly accelerated up to energies \( \sim 10 \text{ TeV} \).

The increase of the magnetic field leads to reduction \( \propto B^{-2} \) of the flux of the inverse Compton γ-rays; for \( B \geq 1 \text{ G} \) the bulk of energy of accelerated electrons is channeled into the synchrotron radiation. Also, the increase of the magnetic field shifts \( \propto B \) the synchrotron peak to higher energies. But for any reasonable Doppler factor of the jet, the observed TeV fluxes of Markarian 421 and Markarian 501 cannot be explained by the electron synchrotron radiation because of the self-regulated synchrotron cutoff at \( \epsilon_{\text{max}} \simeq 1.6 \delta_{10} \text{ GeV} \), which inevitably appears if the electron acceleration and γ-ray production take place in the same region of the jet. Formally, we may avoid this limit assuming that the regions of the electron acceleration and the γ-ray production are separated, i.e. the electrons are accelerated up to energies \( \sim 10^3 \text{ TeV} \) in a region with rather small magnetic field, \( B \leq 0.01 \text{ G} \), but release all their energy in a form of GeV/TeV synchrotron radiation later, after entering the region(s) of strongly compressed/amplified magnetic field, \( B \geq 10 \text{ G} \). This scenario seems, however, rather artificial.
Large magnetic fields, $B \sim 100$ G or so, coupled with effective acceleration of protons at the maximum rate, may create very favorable conditions for TeV $\gamma$-ray production. Indeed, at such conditions the synchrotron cooling of protons not only well dominates over other radiative and non-radiative losses (see Fig. 4), but also provides good fits to the observed spectra of TeV radiation of Markarian 421 and Markarian 501. Remarkably, if the proton acceleration takes place in the synchrotron-loss-dominated regime, the spectral shape of radiation depends only on the power-law index of accelerated protons, $\alpha_p$, and the parameter $\rho = \delta_{10}/\eta$, but not on the magnetic field and the size of the $\gamma$-ray emitting blobs, which generally endure significant time-evolution. Meanwhile, any change in the size and/or magnetic field of the evolving blobs should lead to significant variation in the absolute flux of $\gamma$-rays. This effect could give a natural explanation for one of the remarkable features of strong flares of Markarian 501 (and, perhaps, also Markarian 421) - the essentially stable spectral shape of TeV radiation despite strong variation of the absolute flux observed on time scales less than 1 day. During the strong flares of Markarian 501 with a flat spectrum around 1 TeV, the parameter $\rho = \delta_{10}/\eta \sim 1$. The exact value of $\rho$ depends on the level of the diffuse extragalactic background and on the power-law index of accelerated protons. The uncertainty in this parameter, despite significant uncertainties in the intergalactic extinction of TeV radiation, as well as the lack of adequate information about the $\gamma$-ray fluxes at MeV/GeV energies, does not exceed a factor of three or so. Since the Doppler factor in Markarian 501 is believed to be within 10 to 30, the acceleration rate should be very high, $\eta = \rho^{-1} \delta_{10} \sim 1 - 3$. The unusually high GeV flux observed from Markarian 501 during the multiwavelength campaign in March 1996, when the source was in a very low TeV state, could be explained by a transition of the source from the synchrotron-loss-dominated regime to the escape-loss-dominated regime, caused, for example, by dramatic drop of the magnetic field. In this regime we should expect steeper TeV $\gamma$-ray spectra with a slope depending on the magnetic field and the size of the source. Therefore, an important test of this hypothesis could be detection of spectral variability of $\gamma$-radiation in a low state of the source.

The significantly steeper TeV spectrum of Markarian 421 requires early synchrotron cutoff, $\epsilon_0 \sim 0.25 - 1$ TeV. This could be interpreted as a result of formation of the synchrotron spectrum in the regime dominated by the escape of protons from the acceleration region. For a magnetic field of about 100 G this would require a compact source with $R < 10^{15}$ cm. Interestingly, such a small linear size of the $\gamma$-ray production region is supported independently by the observed dramatic variations of the TeV flux of Markarian 421 on timescales $\sim 15$ minutes.

Within the model of proton synchrotron radiation, we may expect also synchrotron radiation produced by directly accelerated electrons - the counterparts of EHE protons. If the particle acceleration takes place in the synchrotron-
loss-dominated regime, the self-regulated synchrotron cutoff of this component depends only on the parameter $\rho = \delta_{10}/\eta$, namely $\epsilon_0 \simeq 1.6 \rho$ GeV, thus it correlates with the position of the cutoff in the proton synchrotron spectrum at $\epsilon_0 \simeq 3 \rho$ TeV. The ratio of the energy fluxes of these two components is determined simply by the ratio of non-thermal energy channeled into the accelerated protons and electrons, $\dot{W}_p/\dot{W}_e$. In a strong magnetic field of about 100 G, the synchrotron cooling time of electrons is shorter, almost at all relativistic energies, than the typical dynamical (e.g. light-crossing) times. This results in a well-established steady-state spectrum of electrons $dN/dE \propto E^{-(\alpha_0+1)}$, provided that the power-law index of acceleration spectrum $\alpha_0 \geq 1$. Consequently, a pure power law synchrotron spectrum with a photon index $(\alpha_0 + 2)/2$ would be formed. In particular, for $\alpha_0 = 2$, we should expect a flat synchrotron SED ($\nu S_\nu = \text{const}$) from the optical/UV wavelengths to MeV/GeV $\gamma$-rays. The broad-band spectra of both Markarian 421 and Markarian 501 do not agree with such a pure power-law behavior; in fact, the SED of both objects show pronounced synchrotron X-ray peaks. Therefore this (theoretically possible) population of directly accelerated electrons - counterparts of the EHE protons - cannot be responsible for the bulk of X-ray emission.

The latter could be referred to electrons, produced, most probably, in a different way, and/or in other region(s) of the jet. The X-ray light curves of both Markarian 421 (e.g. Takahashi et al. 1999) and Markarian 501 (e.g. Sambruna et al. 2000) show so-called “soft” and “hard” lags. A possible interpretation of this effect in terms of competing acceleration, radiative cooling, and escape timescales of synchrotron-emitting electrons (Takahashi et al. 1996; Kirk et al. 1998), would require that all these timescales are comparable with the light crossing time, $t = R/c \sim 10^4 - 10^5$ s. The cooling time of an electron responsible for a synchrotron photon of energy $\epsilon$ is $t_{\text{sy}}^{(e)} \simeq 1.5 \times 10^3 (B/1 \text{ G})^{-3/2} (\epsilon/1 \text{ keV})^{1/2}$ s. Therefore in the X-ray production region the magnetic field cannot, independent of specific model assumptions, significantly exceed 0.1 G. Thus, the hypothesis of proton-synchrotron origin of TeV radiation implies that the production regions of TeV $\gamma$-rays ($B \sim 100$ G) and and synchrotron X-rays ($B \sim 0.1$ G) are essentially different. Although in any reasonable scenario we may expect a non-negligible correlation between the X-ray and TeV $\gamma$-ray fluxes, the spatial separation of the X-ray and TeV $\gamma$-ray production regions does not allow definite predictions for such a correlation.

In the proton-synchrotron model of TeV radiation of BL Lac objects, two more components of X-radiation are expected. Firstly, in the field of about 100 G the accelerated protons of energy $E \sim 10^{15}$ eV themselves produce synchrotron X-rays. However, the contribution of this component to the observed X-ray flux is negligible. A much more prolific channel for X-ray production connected (indirectly) with the EHE protons, can be provided by electrons of non-acceleration origin, namely by secondary electrons produced at inter-
actions of the primary TeV γ-rays with the ambient low-frequency radiation. Actually, for a spectrum of EHE protons containing sharp spectra of pile-up, we must assume an essential ($\tau_{\gamma\gamma} \sim 1$) internal absorption of γ-rays in order to match the observed spectrum of Markarian 501 (see Fig. 10). The appearance of secondary electrons in the jet results in production of a hard synchrotron X-ray component. Apparently, for an optical depth $\tau_{\gamma\gamma} \sim 1$, the luminosity of this component would be comparable to the luminosity of their "grand-parents" - TeV γ-rays. The photo-produced electrons have a rather specific, significantly different from the directly accelerated particles, shape. For example, the spectrum of electrons produced at interactions of high energy γ-rays with a photon index $\Gamma$ and field photons with a narrow (e.g. Planckian) spectral distribution with a characteristic energy $\hbar \nu$, has the following characteristic form: starting from the minimum (allowed by kinematics) energy at $E_* = m_e^2 c^4 / 4 \hbar \nu$, the electron spectrum sharply rises reaching the maximum at $E_m \simeq 2.4 E_* \simeq 0.15 (\hbar \nu / 1 \text{eV})^{-1}$ TeV, and then at $E \gg E_m$ it decreases as $q_\pm \propto E^{-(\Gamma+1)} \ln E$. Within an accuracy better than 20 per cent, the spectrum of secondary pairs could be approximated in a simple analytical form (Aharonian & Atoyan 1991):

$$q_\pm (E) \, dE = f(\Gamma) \frac{\exp \left[ - (1/(x - 1)) \right]}{E_* x (1 + 0.07 x^\Gamma / \ln x)} \, dE,$$

where $x = E/E_* \geq 1$, $f(\Gamma) = (1.11 - 1.60\Gamma + 1.17\Gamma^2)$, and $\int q_\pm (E) \, dE = 2$ (two electrons per interaction).

The intensive synchrotron losses in the strong magnetic field $B \sim 100 \text{G}$ quickly establish a steady-state spectrum of electrons proportional to $E^{-2}$ below $E_m$, and approximately as $E^{-(\Gamma+2)}$ (if we ignore the weak logarithmic term) above $E_m$. Correspondingly, the synchrotron spectrum of the secondary pair-produced electrons is characterized by a smooth transition, through the energy around $\epsilon_b = 120 B_{100}(\hbar \nu / 1 \text{eV})^{-2}$ keV, from $s = 1.5$ to $s \approx (\Gamma + 3)/2$. Assuming, for example, that the TeV γ-rays with a photon index $\Gamma \approx 1.4$ are absorbed in an external infrared photon field with characteristic energy in the jet frame $\hbar \nu = 1.5 \delta_{10} \text{eV}$ (like in Fig. 10), one should expect in the observer frame a hard X-ray component of radiation with a SED $\nu S_\nu \propto \epsilon^{0.5}$ and $\nu S_\nu \propto \epsilon^{-0.2}$ below and above $\epsilon_b \approx 500 B_{100} \delta_{10}^{-1}$ keV, respectively. Finally, at energies above several MeV the spectrum becomes very steep due to the cutoff in the spectrum of primary (proton-synchrotron) γ-rays. We may speculate that such a component of hard X-rays was observed during the strong TeV flares of Markarian 501, in particular in April 1997 (Pian et al. 1998). Obviously, this component of radiation should strongly correlate with the flux of TeV emission. However, because the synchrotron radiation produced by directly accelerated electrons significantly contributes to the observed X-ray flux as well, the TeV/X-ray correlation may have a rather complicated and non-standard behavior. The absence (or suppression) of such a hard X-ray
component in the spectrum of Markarian 421 could be explained, within the framework of this model, by the steep spectrum of TeV \( \gamma \)-rays caused by an early synchrotron cut-off. The quantitative study of this question within a detailed time-dependent treatment of the problem will be discussed elsewhere.

In BL Lac objects like Markarian 501, the photo-meson processes do not play, most probably, dominant role in the production of high energy \( \gamma \)-rays. Indeed, the TeV \( \gamma \)-ray transparency condition puts a robust lower limit on the characteristic time of this process, \( t_{p\gamma} \sim 10^7 \) s which is almost 3 orders of magnitude larger than the characteristic synchrotron cooling time of protons. The severe synchrotron losses of protons cannot be avoided since the requirement of extremely high energy protons in the blob - a key assumption in the PIC model - automatically implies a large magnetic field of an order of 100 G. On the other hand, the characteristic photo-meson cooling time cannot be arbitrary reduced, e.g. by assuming extremely high density of radiation in the blob. For any reasonable spectrum of the latter, and any reasonable geometry of the \( \gamma \)-ray production region, this would lead to an unacceptably large optical depth for TeV \( \gamma \)-rays. Therefore, one may conclude that the very fact of observations of TeV \( \gamma \)-rays from Markarian 501 and Markarian 421 (in fact, from any TeV blazar) almost rules out the “photomeson” origin of the bulk of the observed TeV radiation. This also implies very low TeV neutrino fluxes.

The hypothesis of proton-synchrotron origin of TeV \( \gamma \)-flares from Markarian 421 and Markarian 501 requires a very large amount of energy contained in the form of magnetic field

\[
W_B \approx \frac{1}{6} R^3 B^2 = 5.6 \times 10^{49} \Delta t_{3h}^3 \delta_{10}^3 B_{100}^2 \text{ erg}. \tag{24}
\]

For comparison, the energy of the magnetic field allowed by the SSC models \((B \sim 0.1 \text{ G})\) is less by six orders of magnitude. On the other hand, the estimates of the kinetic energy in \( \geq 10^{19} \text{ eV} \) protons (within the proton-synchrotron radiation model) and \( \geq 10^{12} \text{ eV} \) electrons (within the SSC model) are comparable, if we assume that in both models the \( \gamma \)-rays are produced with high efficiency. Indeed, in this case the inverse Compton cooling time of electrons and the synchrotron cooling time of protons responsible for TeV emission in the SSC and proton-synchrotron models, respectively, are equal or less than the observed variability time-scale \( t^* \simeq 1.08 \times 10^5 \Delta t_{3h} \delta_{10} \) s, thus

\[
W_e^{(SSC)} \sim W_p^{(PSR)} \leq 4\pi d^2 \phi_j \delta^{-4} t^* \simeq 1.4 \times 10^{46} \Delta t_{3h} \delta_{10}^{-3} \text{ erg}. \tag{25}
\]

In this estimate the average flux of Markarian 501 in 1997, corrected for extragalactic extinction for “low-CBR” model is assumed. Higher CBR fluxes, as well as the possible internal absorption of \( \gamma \)-rays, would increase this estimate by a factor of 3 or so. If the spectrum of accelerated particles extends, e.g. as
$E^{-2}$, down to energies $E \sim mc^2$, the above estimate could be increased by an order of magnitude. In fact, this simplified estimate only reflects the average level of the energy content of accelerated particles during the high state of Markarian 501 in 1997. For the strongest flares with duration $\leq 1$ day, the energy in accelerated protons could be several times larger than it follows from Eq. (25). But in any case, these uncertainties do not prevent us to conclude that in the proton-synchrotron model we deal with a highly magnetized condensation of $\gamma$-ray emitting clouds of EHE protons, where the magnetic pressure dominates over the pressure of relativistic protons\(^5\). The total magnetic energy given by Eq. (24) of a single blob formally is sufficient to supply the necessary energy in TeV $\gamma$-rays released during the entire high activity state of Markarian 501 in 1997. Thus speculating that an effective acceleration mechanism operates in such magnetized condensations, stimulated presumably by interactions of the latter with the surrounding plasma, a single (or a few) energetic blob(s) ejected from the central source towards the observer could, in principle, explain the extraordinary high TeV flux of Markarian 501 in 1997.

In the SSC models the situation is exactly opposite - the pressure of relativistic particles is significantly larger than the pressure of the magnetic field. Even so, the total energy contained in the relativistic electrons is sufficient to support the observed X-ray and $\gamma$-ray emission during only several hours, therefore this model requires quasi-continuous injection of energy into a single blob, or, most likely, a “multi-blob” scenario in which the observed radiation is a result of superposition of many short-live blobs continuously ejected from the central source. Then, the high state of the source like the extraordinary long outburst of Markarian 501 in 1997 could be associated with a dramatic increase of the rate of ejection/formation of $\gamma$-ray emitting blobs.

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\(^5\) The minimum energy budget condition is achieved in the case of approximate equipartition between the protons and magnetic field, $W_B \approx W_p$. However, for Markarian 501 this condition would imply magnetic field of about 10 G, which only marginally could provide effective production of the proton-synchrotron radiation in the TeV regime.
References

Aharonian, F.A., 1999, Astron. Nachr., 4/5, 222.
Aharonian, F.A., Atoyan, A.M., 1991, J. Phys. G: Nucl. Part. Phys., 17, 1769.
Aharonian, F.A. et al. (HEGRA collaboration), 1999a, A&A, 342, 69.
Aharonian, F.A. et al. (HEGRA collaboration), 1999b, A&A, 349, 11.
Aharonian, F.A. et al. (HEGRA collaboration), 1999c, A&A, 350, 757.
Bednarek, W., Protheroe, R.J., 1999, MNRAS, 310, 577.
Bednarz, J., Ostrowski, M., 1996, MNRAS, 283, 447.
Begelman, M.C., Blandford, R.D., Rees, M.J., 1984, Rev. Mod. Phys., 56, 255.
Blandford, R.D., 1976, MNRAS, 176, 465.
Blumenthal, G.R., Gould, R.J., 1970 Rev. Mod. Phys., 42, 237.
Buckley, J.H., Akerlof, C. W., Biller, S. et al., 1996, ApJ, 472, L9.
Catanese, M., Bradbury, S. M, Breslin, A. C. et al., 1997, ApJ, 487, L143.
Catanese, M., Weekes, T.C., 1999, 111, 1193.
Celotti, A., Fabian, A.C., Rees, M.J., 1998, MNRAS, 293, 239.
Coppi, P.S., Aharonian, P.A., 1999a, ApJ, 521, L33.
Coppi, P.S., Aharonian, P.A., 1999b, Astropart. Phys., 11, 35
Dar, A., Laor, A., 1997, ApJ, 478, L5.
Djannati-Atai, A., Piron, F., Barrau, A. et al., 1999 A&A, 350, 17.
Drury, L. O’C, Duffy, P., Eichler, D., Mastichiadis, A., 1999, A&A 347, 370.
Dwek, E., Arendt, R.G., 1998, ApJ, 508, L9.
Dwek, E., Arendt, R.G., Hauser, M.G. et al., 1998 ApJ, 508, 106.
Elbaz, D., Cesarsky, C.J., Fadda, D. et al., 1999, A&A, 351, L37.
Gaidos, J. A., Akerlof, C. W., Biller, S. D. et al., 1996, Nature, 383, 319.
Ginzburg, V.L., Syrovatsky, S.I., 1965, Annu. Rev. Astron. Astrophys., 3, 297.
Gorjian, V., Wright, E.L., Chary, R.R., 2000, ApJ, 536, 550.
Gould, R.J., Schreder, G., 1966, Phys. Rev. Lett., 16, 252.
Guy, J., Renault, C., Aharonian F.A., Rivoal, M., Tavernet, J.-P., 2000, A&A, 359, 419.
Haswell, C.A., Tajima, T., Sakai, J.J., 1992, ApJ, 401, 495.
Hauser, M.G., Arendt, R.G., Kelsall, T. et al., 1998 ApJ, 508, 25.
Henri, G., Pelletier, G., Petrucci, P.O., Renaud, 1999, Astropart. Phys., 11, 347.
Hillas, A.M., 1984, Annu. Rev. Astron. Astrophys., 22, 425.
Inoue, S., Takahara, F., 1996, ApJ, 463, 555.
Kataoka, J., Mattox, J.R., Quinn, J., Kubo, H., Makino, F., Takahashi, T., Inoue, S., Hartman, R. C., Madejski, G. M., Sreekumar, P., Wagner, S. J., 1999, ApJ, 514, L138.
Kazanas, D., Mastichiadis, A., 1999, Astropart. Phys., 11, 41.
Kirk, J.G., Rieger, Mastichiadis, A., 1998, A&A, 333, 452.
Krawczynski, H., Coppi, P.S., Maccarone, T., Aharonian, F.A., 2000, A&A, 353, 97.
Krennrich, F., Biller, S. D., Bond, I. H. et al., 1999, ApJ, 511, 149.
Lind, K.R., Blandford, R.D., 1985, ApJ, 295, 358.
Lovelace, R.V.E., 1976, Nature, 262, 649.
Mannheim, K., 1993, A&A, 269, 67.
Mannheim, K., 1996, Space Sci. Rev., 75, 331.
Maraschi, L., Fossati, F., Tavecchio, T. et al., 1999 ApJ, 526, L81.
Mastichiadis, A., Kirk, J.G., 1997, A&A, 320, 19.
Melrose, D.B., 1980, Plasma Astrophysics, vol.1, Gordon & Breach, New-York.
Melrose, D., Crouch, A., 1997 Publ. Astron. Soc. Aust., 14, 251.
Morrison, P. Roberts, D., Sadun, A., 1984, ApJ, 280, 483.
Mukherjee, R., Bertsch, D. L., Bloom, S. D. et al., 1997, ApJ, 490, 116.
Mücke, A., Rachen, J.P., Engel, R., Protheroe, R.J., Stanev, T., 1999, Publ. Astron. Soc. Aust., 16, 160.
Mücke, Protheroe, R.J., 2000, in Dingus, B. et al.(eds) “International workshop on GeV-TeV Astrophysics: Toward a Major Atmospheric Cherenkov Telescope VI” (Snowbird, Utah), AIP Conf Proc., in press.
Pian, E., Vacanti, G., Tagliaferri, G. et al., 1998, ApJ, 492, L17.
Piron, F. et al. (CAT collaboration), 2000, in Proceedings of XITH rencontres de Blois, in press.
Pohl, M., Schlickeiser, R., 2000, A&A, 354, 395.
Primack, J.R., Bullock, J.S., Somerville, R., Macminn, D., 1999, Astropart. Phys., 11, 93.

Protheroe, R.J., Stanev, T., 1999, Astropart. Phys., 10, 185.

Quinn, J., Bond, I. H., Boyle, P. J. et al., 1999, ApJ, 518, 693.

Sambruna, R., Aharonian, F.A., Krawczynski, H. et al., 2000, ApJ, 538, 127.

Schlegel, D.J., Finkbeiner, D.P., Davis, M., 1998, ApJ, 500, 525.

Sikora, M., Kirk, J.G., Begelman, M.C., Schneider, P., 1987 ApJ, 320, L81.

Sreekumar, P., Bertsch, D.L., Dingus, B.L., et al., 1996, ApJ, 464, 628.

Stecker, F.W., 1968, Phys. Rev. Lett., 21, 1016.

Stecker, F.W. de Jager, O.C., Salomon M.H., 1992, ApJ, 390, L49.

Svensson, R., 1987, MNRAS, 227, 403.

Takahashi, T., Madejski, G., Cubo, H., 1999, Astropart. Phys., 11, 177.

Tavecchio, F., Maraschi, L., Ghisellini, G. ApJ, 509, 608.

Ulrich, M.H., Maraschi, L., Urry, C.M., 1997, Annu Rev. Astron. Astrophys., 35, 445.

Urry, C.M., Padovani, P., 1995, PASP, 107, 803.

Vassiliev, V.V., 2000, Astropart. Phys., 12, 217.

Wagner, S.J., Lamer, G., Bicknell, G.V., Astron. Nachr., 4/5, 226.