A Low-Complexity PD-Like Attitude Control for Spacecraft With Full-State Constraints

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ABSTRACT The problem of attitude control for rigid spacecraft under the attitude and angular velocity constraints is investigated in this study. Particularly, a simple structure constrained proportional-derivative (PD)-like control is proposed which contains two portions. The first portion is a conventional PD control to provide convergence of the system states; whereas the second portion provides the desired performance specifications such as convergence rate, overshoot and steady-state bound for attitude and rotation velocity to improve the attitude pointing accuracy and pointing stability. The distinctive property of the suggested constrained control method is to ensure the desired performance in transient and steady-state phases for all the system states. It also possesses much simpler structure compared to the existing constrained control techniques since it is based on a new methodology. The simulation results conducted on a rigid spacecraft verify the efficiency of the proposed approach.

INDEX TERMS Rigid spacecraft, attitude control, PD control, prescribed performance control.

I. INTRODUCTION

Over the past several decades, the issue of attitude control of both rigid and flexible spacecraft has received significant attentions owing to its extensive applications during the execution of space missions, such as earth imaging, spacecraft docking and rendezvous, satellite surveillance and multi orbit task, etc. Thus, in the past few years, the attitude control of spacecraft system has received significant attentions, and many effective developments have emerged [1]–[8]. From the applied point of view, external environmental disturbances occur inevitably for spacecraft system which lead to poor efficiency or even instability in the system. At the same time, there is a strong coupling between the spacecraft attitude kinematics and dynamics and their equations are highly nonlinear. For these reasons, achieving high accuracy attitude control for spacecraft system has become a challenging problem. Over the past few years, the demanding issue of spacecraft attitude control has attracted profound attention and numerous control laws are available; for instance, backstepping method [9], event-triggered control [10], [11], model predictive control (MPC) [12], [13], sliding mode control (SMC) [14]–[17], non-fragile output-feedback control [18], [19], adaptive control [20], fuzzy control [21], fault-tolerant control [22], disturbance observer-based control [23]–[27], etc.

Among the applied control methods, the well-known PD control has been widely utilized because of its simple structure, low computational effort and explicit tuning procedures. In [28], based on the passivity concept, a PD control for spacecraft attitude control problem has been proposed. A saturated PD control for spacecraft has been presented in [29] such that there is no need for the angular velocity to be measured. Ref. [30] developed a nonlinear PD attitude control for the flexible spacecraft in the framework of input-to-state stability (ISS) design approach. A hybrid PD control based on ISS for a rigid spacecraft with external disturbance has been given in [31]. The PD+ approach which is composed of a linear PD control and a nonlinear dynamic inversion method is used to remove the model’s nonlinear terms based...
on feedback linearization concept. The research related to PD-like technique for spacecraft attitude control can be found in [32] and the references therein. Despite the efficacy of the proposed PD control, it only guarantees that the spacecraft state trajectories are drawn into a small neighborhood of the origin. In addition, these above results can only achieve the steady-state performance of spacecraft system, while the transient-state performance is also a major concern in spacecraft attitude maneuvering. In fact, providing prescribed performance for attitude trajectory plays a significant role in the success of mission. For the purpose of achieving desirable spacecraft attitude system performance in transient and steady-state phase, the prescribed performance control (PPC) has been planned and received extensive attentions. More specifically, in [33], a PPC for the spacecraft attitude stabilization and tracking control has been presented, in which the angular velocity of spacecraft is estimated with a differentiator. The problem of PPC for trajectory tracking on SO(3) has been studied in [34]. A new learning-based PPC for spacecraft formation in the presence of external disturbance has been recommended in [35]. Inspired by the PPC notion, a constrained attitude tracking control for spacecraft has been presented in [36] such that the maximum convergence time of the closed-loop system can be pre-determined by the designer. Using backstepping approach, Ref. [37] designed a fault-tolerant PPC for rigid spacecraft such that the convergence time of the attitude quaternion can be defined a priori.

The existing attitude controls including for spacecraft are not able to guarantee prescribed performance in transient and steady-state for the attitude variable and the angular velocity, simultaneously. Moreover, the controllers structures of these approaches are quite complicated due to the inclusion of intricate function terms as well as partial differential ones since they utilize the notion of the transformation error presented in [38], [39]. The complexity of the aforementioned methods is doubled when the constraints on the attitude and the angular velocity are simultaneously considered to enhance the pointing accuracy and pointing stability. Motivated by this discussion, this paper proposes a low-complexity constrained PD-like control to constrain full states of the spacecraft attitude system in the presence of the input saturation. The main contributions of this study are listed as follows.

- To enhance the pointing accuracy and pointing stability of the spacecraft, a PD-like attitude control is proposed which is able to constrain full states of the system subject to input saturation.
- The proposed attitude controller possesses a simple structure and can be easily implemented as it does not fully use the concept of the PPC, unlike the existing constrained attitude controls in the literature.

The remainder of this study is arranged as follows: in the next part, the nonlinear model of rigid spacecraft and the attitude control problem are stated. The main outcomes are presented in Section 3, in which a low-complexity constrained PD-like is established to reach a highly-accurate attitude control method. Lastly, simulation outcomes and conclusions are provided in Sections 4 and 5, correspondingly.

II. PROBLEM FORMULATION

A. MATHEMATICAL MODEL OF A RIGID SPACECRAFT

As shown in Figure 1, the inertial-fixed frame $F_i$ and the rigid spacecraft body-fixed frame $F_b$ are generally employed to give a spacecraft attitude dynamic. This work utilizes the modified Rodriguez parameters (MRPs) for representing the spacecraft attitude in $F_b$ with respect to (w.r.t) $F_i$. The dynamical model of a rigid spacecraft is expressed as [40]

$$
\dot{\sigma} = G(\sigma)\omega
$$

$$
J\dot{\omega} = -\omega^T J \omega + u + d
$$

where $\sigma = n \tan (\frac{\Phi}{4}) = [\sigma_1, \sigma_2, \sigma_3]^T \in \mathbb{R}^3$ denotes the MRPs which represent the spacecraft orientation w.r.t $F_i$, where $n \in \mathbb{R}^3$ signifies the Euler principal axis and $\Phi(t) \in \mathbb{R}$ is Euler rotational angle. Moreover, $\omega \in \mathbb{R}^3 = [\omega_1, \omega_2, \omega_3]^T$ is the rotation velocity of $F_b$ w.r.t $F_i$ and expressed in $F_b$, $J \in \mathbb{R}^{3 \times 3}$ is the positive-definite inertia matrix of the rigid spacecraft, $u = [u_1, u_2, u_3]^T \in \mathbb{R}^3$ is the control torque, and $d = [d_1, d_2, d_3]^T \in \mathbb{R}^3$ refers to unknown but bounded disturbances where $|d| \leq \bar{d}$. For any given vector $\sigma = [\sigma_1, \sigma_2, \sigma_3]^T \in \mathbb{R}^3$, then $\sigma^\times$ is a cross-product matrix defined as

$$
\sigma^\times = \begin{bmatrix}
0 & -\sigma_3 & \sigma_2 \\
\sigma_3 & 0 & -\sigma_1 \\
-\sigma_2 & \sigma_1 & 0
\end{bmatrix}
$$

Besides, the matrix $G(\sigma)$ in (1) is described as

$$
G(\sigma) = \frac{1}{2} \left( 1 - \frac{\sigma^T \sigma}{2} \right) I_3 + \sigma^\times + \sigma \sigma^T
$$

Due to physical limitations on the actuator, the control signal $u$ is constrained by a saturation value. Here, $Sat(u) = [sat(u_1), sat(u_2), sat(u_3)]^T$ is the vector of actual control input generated by actuators and $sat(u_i), i = 1, 2, 3$ represents the nonlinear saturation characteristic of the actuators. The saturation function can be described by

$$
sat(u_i) = u_i(t) + \theta_i(t)
$$

$$
\theta_i(t) = \begin{cases}
0, & |u_i| < u_{mi} \\
\text{sgn}(u_i) u_{mi} - u_i(t), & |u_i| \geq u_{mi}
\end{cases}
$$

![Figure 1. The orientation description of a rigid body.](image-url)
where \( u_{mi} \) is the maximum allowed value of \( ith \) control input. The excess term of the constrained saturation is given by \( \Theta(t) = [\theta_1(t), \theta_2(t), \theta_3(t)]^T \) in which \( \|\Theta(t)\| \leq l_0 \) and \( l_0 \) is a positive constant.

**B. CONTROL PURPOSE**

The main purpose of the current study is to design a PD-like control input for the rigid spacecraft attitude system (1) such that the attitude MRP \( \sigma \) and angular velocity \( \omega \) are stabilized and both of them are simultaneously kept within the prescribed constraint boundaries. In contrast to the existing constrained control approaches in the literature, the unique property of the suggested control law is that it possesses simple structure and requires less onboard computation. In fact, it is composed of a PD control to stabilize the attitude system and an auxiliary control to guarantee that constraints on \( \sigma \) and \( \omega \) are not violated.

**C. PRESCRIBED PERFORMANCE CONTROL**

In order to achieve the above-mentioned control objective, a prescribed performance function (PPF) is usually selected to provide desirable performance for the systems states in transient and steady-state phase. This function is stated as follows:

Definition 1 [38]: The function \( \rho(t) \) is a PPF if: a) \( \rho(t) \) is positive, b) \( \dot{\rho}(t) \) is non-positive, c) \( \lim_{t \to \infty} \rho(t) = \rho_\infty > 0 \).

By utilizing the PPF, the desirable time-domain characteristics of attitude system variables, for instance, overshoot, rise time and steady-state bound is achieved \textit{a priori}. The subsequent function is employed as PPFs for the attitude MRPs and angular velocity

\[
\rho_m(t) = (\rho_{m0} - \rho_{m\infty}) \exp(-\kappa_{mi}t) + \rho_{m\infty},
\]

where \( m \in \{\sigma, \omega\} \), \( \rho_{m0}, \rho_{m\infty} \) and \( \kappa_{mi} \) are design parameters that should be suitably selected according to the specific necessities and actuator capability. In [38], it has been explained that the predetermined performance is accomplished where the state variables evolve within the prescribed regions which are confined in the PPFs corresponding to \( \sigma \) and \( \omega \). According to this concept, to achieve the control objective, it is adequate to satisfy the subsequent relationships:

\[
-\rho_{\sigma i}(t) < \sigma_i(t) < \rho_{\sigma i}(t),
\]

\[
-\rho_{\omega i}(t) < \omega_i(t) < \rho_{\omega i}(t),
\]

where \( i = 1, 2, 3 \). Based on the condition (7), it is concluded that if \( |\sigma_i(0)| < \rho_{\sigma i,0} \) and \( |\omega_i(0)| < \rho_{\omega i,0} \) are satisfied, the MRPs and rotation velocity always remain in the predefined bounds.

**III. MAIN RESULTS**

**A. TRADITIONAL PPC**

In order to control only the attitude MRPs utilizing the concept of the traditional PPC used in [33-37, 41], firstly an MRP's variable transformation is adopted to transform the original nonlinear attitude system (1), with the constraint

\[
-\rho_{\sigma}(t) < \sigma(t) < \rho_{\sigma}(t),
\]

into an equivalent un-constrained one. More explicitly, it is defined

\[
\sigma(t) = \rho_{\sigma}(t) \Upsilon_{\sigma}(\lambda_{\sigma})
\]

where \( \lambda_{\sigma} \) is the transformed quaternion variable and \( \Upsilon_{\sigma}(\cdot) \) is a smooth, strictly increasing and invertible function possessing the subsequent properties:

\[
-1 < \Upsilon_{\sigma}(\lambda_{\sigma}) < 1
\]

\[
\lim_{\lambda_{\sigma} \to +\infty} \Upsilon_{\sigma}(\lambda_{\sigma}) = 1
\]

\[
\lim_{\lambda_{\sigma} \to -\infty} \Upsilon_{\sigma}(\lambda_{\sigma}) = -1.
\]

If \( \lambda_{\sigma} \) is kept bounded, i.e., \( \lambda_{\sigma} \in L_\infty \), then \( -1 < \Upsilon_{\sigma}(\lambda_{\sigma}) < 1 \). Due to \( \rho_{\sigma}(t) > 0 \) and (9), one obtains (8).

Considering the properties of \( \Upsilon_{\sigma}(\cdot) \) as well as \( \rho_{\sigma}(t) > 0 \), the inverse transformation \( \lambda_{\sigma} = \Upsilon_{\sigma}^{-1}(\frac{\sigma(t)}{\rho_{\sigma}(t)}) \) is well defined if (8) holds or equivalently \( \lambda_{\sigma} \in L_\infty \). Thus, if \( \lambda_{\sigma} \) is guaranteed to be kept bounded, then (8) is ensured. If we want to also constrain the angular velocity simultaneously, the similar condition of \( \sigma \) should be considered for \( \omega \). In other words, an angular velocity variable transformation is required to be defined as

\[
\omega(t) = \rho_{\omega}(t) \Upsilon_{\omega}(\lambda_{\omega}).
\]

It should be noted that to stabilize the new transformed variables associated with the MRPs and angular velocity, their dynamics should be obtained. Thus, the attitude system (1) is rewritten based on the new transformed MRPs and angular velocity variables, i.e.,

\[
\dot{\lambda}_{\sigma} = \frac{\partial \Upsilon_{\sigma}^{-1}(\frac{\sigma}{\rho_{\sigma}})}{\partial \frac{\sigma}{\rho_{\sigma}}} \times \frac{1}{\rho_{\sigma}} \left[ -\rho_{\omega} \Upsilon_{\omega}(\lambda_{\omega}) \dot{\rho}_{\omega} + G(\rho_{\rho_{\omega}} \Upsilon_{\omega}(\lambda_{\omega})) \right]
\]

\[
\dot{\lambda}_{\omega} = \frac{\partial \Upsilon_{\sigma}^{-1}(\frac{\omega}{\rho_{\omega}})}{\partial \frac{\omega}{\rho_{\omega}}} \times \frac{1}{\rho_{\omega}} \left[ J^{-1} \left( \begin{array}{c} \rho_{\omega} \Upsilon_{\omega}(\lambda_{\omega}) \end{array} \right) + \dot{\rho}_{\omega} \Upsilon_{\omega}(\lambda_{\omega}) \dot{\lambda}_{\omega} \right].
\]

Now, the attitude controller should be designed such that boundedness of \( \lambda_{\sigma} \) and \( \lambda_{\omega} \) is guaranteed. It is obvious that the transformed attitude system (12) has a quite intricate dynamical structure and there is a severe coupling between \( \lambda_{\sigma} \) and \( \lambda_{\omega} \) which make the controller design highly complicated. However, in the next subsection, we present a straightforward way to constrain the MRPs and angular velocity, simultaneously.

**B. SIMPLE STRUCTURE PPC**

In this section, a PD-like constrained attitude control for the rigid spacecraft (1) is developed. The controller is in the form of \( u(t) = u_{pd}(t) + u_{aux}(t) \), where \( u_{pd}(t) \) represents the conventional PD control for providing the system states...
convergence; while \(u_{aux}(t)\) denotes an auxiliary control for the purpose of constraining the MRPs and rotation velocity, simultaneously. The interesting feature of \(u_{aux}(t)\) is that it does not require to calculate the dynamics of the transformed errors corresponding to \(\sigma\) and \(\omega\). This, in turn, significantly reduces the complexity of the control design.

As explained in the previous subsection, based on the condition (7), it is quite sophisticated to develop the attitude controller based on the existing constrained control methods [33]–[37], [41] even for only constraining the quaternions. In order to deal with this difficulty, the following simple error transformations are introduced:

\[
\lambda_i(t) := \frac{\sigma_i(t)}{\rho_{\sigma_i}(t)}, \quad \omega_i(t) := \frac{\omega_i(t)}{\rho_{\omega_i}(t)}.
\]

Investigating (13), it can be easily concluded that the performance constraints on the MRPs and angular velocity are guaranteed if \(|\lambda_i(t)| < 1\) for \(i = 1, 2, 3\) and \(l = 1, 2\) is satisfied. Thus, the sophisticated issue of obtaining prescribed performance is reduced to design a suitable control for preserving the transformed errors within the bound \(|\lambda_i| < 1\).

**Remark 1:** The control objective (7) is satisfied as long as \(|\sigma_i(t)| < \rho_{\sigma_i}(t)\) and \(|\omega_i(t)| < \rho_{\omega_i}(t)\). When \(|\sigma_i(t)|\) and \(|\omega_i(t)|\) approach their prescribed bounds \(\rho_{\sigma_i}(t)\) and \(\rho_{\omega_i}(t)\), respectively, the transformed constraint errors \(\lambda_1(t)\) and \(\lambda_2(t)\) tend to 1.

**Theorem 1:** For the rigid spacecraft attitude system (1), if the constrained PD-like control is provided by (14), then the closed-loop system is stable and the MRPs and angular velocity are confined into their corresponding regions expressed by (7).

\[
u = u_{pd} + u_{aux},
\]

\[
u_{pd} = -K_p\sigma - K_d\omega
\]

\[
u_{aux} = -\alpha \text{diag} \pe \ln \left( \frac{(1 + \lambda_{11})(1 + \lambda_{22})}{(1 - \lambda_{11})(1 - \lambda_{22})} \right)^2 \omega - \frac{\beta}{\|\omega\| + \epsilon} \omega
\]

where \(K_p, K_d, \alpha, \beta\) are all positive constants and \(\epsilon\) is an arbitrary small constant to preclude chattering.

**Proof:** Construct a Lyapunov functional as follows:

\[
V = \frac{1}{2} \omega^T J \omega + 2K_p \ln(1 + \sigma^T \sigma)
\]

Since the function \(\ln(1 + \sigma^T \sigma)\) is positive and becomes zero only at \(\sigma = 0\), then \(V\) is positive definite and can be a Lyapunov candidate function. Taking time-derivative of \(V\) gives

\[
\dot{V} = \omega^T J \dot{\omega} + 2K_p \frac{2\sigma^T G(\sigma)\omega}{1 + \sigma^T \sigma}
\]

\[
= \omega^T (-\omega^x J \omega + u + \Theta + d) + 2K_p \frac{2\sigma^T G(\sigma)\omega}{1 + \sigma^T \sigma}
\]

Based on the definition of \(G(\sigma)\) in (3), one has

\[
\sigma^T G(\sigma) = \frac{1}{4}(1 + \sigma^T \sigma) \sigma^T
\]

Due to the fact \(z^T z = 0\) is satisfied for any given vector \(z \in \mathbb{R}^3\) and substituting the control law (14) into (16), then (16) can be simplified as

\[
\dot{V} = -K_d\omega^T \omega + \omega^T (d + \Theta) - \alpha \omega^T \text{diag} \left( \ln \left( \frac{(1 + \lambda_{11})(1 + \lambda_{22})}{(1 - \lambda_{11})(1 - \lambda_{22})} \right)^2 \right) \omega
\]

\[
\leq -K_d\omega^T \omega + \|\omega\| (\tilde{d} + \bar{l}) - \beta \|\omega\|^2 \|\omega\| + \epsilon
\]

If the gain \(\beta\) is chosen such that \((\tilde{d} + \bar{l}) \leq \beta \|\omega\|\) is satisfied, then the inequality (18) can be rewritten as

\[
\dot{V} \leq -K_d\omega^T \omega \leq 0
\]

Thus, the system states are bounded. Applying the Barbalat’s lemma [42], it is inferred from the uniformly continuity of the time-derivative of the Lyapunov function that \(\dot{V}\) goes to zero as time tends to infinity. Hence, it can be concluded that \(\lim_{t \to \infty} V = 0\) resulting in the angular velocity \(\omega\) and angular acceleration \(\dot{\omega}\) converge to zero. Using this information and substituting the control law (14) into the dynamic equation (1), we can say

\[
J \dot{\omega} = -\omega^x J \omega + \Theta + d - K_p\sigma - K_d\omega
\]

\[
- \frac{\beta}{\|\omega\| + \epsilon} \omega - \alpha \text{diag} \left( \ln \left( \frac{(1 + \lambda_{11})(1 + \lambda_{22})}{(1 - \lambda_{11})(1 - \lambda_{22})} \right)^2 \right) \omega
\]

Since \(\omega\) and \(\dot{\omega}\) are decaying, it follows from (20) that \(\sigma\) converge to the residual set \(|\sigma| \leq \Delta_\sigma\) given by

\[
\Delta_\sigma = \min \left\{ \rho_{\sigma_i}, \frac{\tilde{d} + \bar{l}}{K_p} \right\}
\]

As a result, the closed-loop system is uniformly ultimately bounded and the attitude maneuver with high accuracy is accomplished. The proof is finished here.

**Remark 2:** The function \(\ln \left( \frac{(1 + \lambda_{11})(1 + \lambda_{22})}{(1 - \lambda_{11})(1 - \lambda_{22})} \right)^2 \omega > 0\) is ensured and it can be removed from the left hand side of the inequality (18). It should also be pointed out that when the MRPs approach its boundary \(\rho_{\sigma_i}\), then \(\lambda_{1i}\) tends to one and function

\[
\ln \left( \frac{(1 + \lambda_{1i})(1 + \lambda_{2i})}{(1 - \lambda_{1i})(1 - \lambda_{2i})} \right)^2
\]

increases to prevent the MRPs from growing and steers it to zero. The similar explanation is true for the angular velocity.

**Remark 3:** As can be observed in (14), the new constrained control possesses much simpler structure in comparison with the existing constrained attitude controls. In fact, the planned control scheme is composed of two parts: the first part is
a conventional PD control whose task is to drive the system states to zero and the second one is an auxiliary control. The second term of the final controller is employed to satisfy the constraints on the MRPs and angular velocity and to compensate for the external environmental disturbance. Thus, if the proposed control is applied, the desired performance in transient and steady state such as overshoot, convergence time and ultimate bound of the system states are guaranteed.

Remark 4: The structure of the suggested attitude control system is illustrated to give a deeper understanding of overall approach.

IV. SIMULATION RESULTS

To assess the performance and efficacy of the suggested approach, numerical simulations are conducted on a rigid spacecraft with inertial matrix given as [43]

\[
J = \begin{bmatrix}
20 & 1.2 & 0.9 \\
1.2 & 17 & 1.4 \\
0.9 & 1.4 & 15
\end{bmatrix}\text{kg\cdot m}^2.
\]

The external environmental disturbances is given as \(d(t) = [0.25 + 0.15 \cos(0.5\pi t), 0.15 + 0.25 \sin(0.5\pi t), -0.25 + 0.25 \sin(0.5\pi t)]^T\) Nm. The initial attitude and angular velocity of the spacecraft are taken as \(\sigma_0 = [-0.088, -0.109, -0.088]^T\) corresponding to the Euler angles \([-25, -20, 25]^T\) and \(\omega_0 = [-0.16, 0.4, 0.2]^T\) rad/s. The parameters of the constrained PD control are selected as \(K_p = 5, K_d = 10, \alpha = 1.5, \beta = 0.1, \) and \(\varepsilon = 0.01\). The parameters of the PPFs \(\rho_{\sigma_i}\) and \(\rho_{\omega_i}\) are given as \(\rho_{\sigma_0} = 0.15, \rho_{\omega_0} = 0.25, \rho_{\sigma\infty} = 0.00001, \rho_{\omega\infty} = 0.002, \kappa_\sigma = 1.3\) and \(\kappa_\omega = 0.9\). The actuator saturation limit is considered as \(u_{\text{ml}} = 2\) Nm.

Part 1: The simulations of the recommended constrained PD-like control scheme and the constrained attitude control (CAC) presented in [34] are given in FIGURE 3 to FIGURE 8. FIGURE 3 displays the time profile of the...
attitude MRPs under the constrained PD-like control and the CAC, respectively. It is observed that under both control schemes, the attitude MRPs remain within the predefined regions and do not contact boundary. Thus, the favorable performances in transient and steady state phase are obtained, i.e., the attitude MRPs are converging with a predefined speed to a desired region to obtain the desired pointing accuracy. The angular velocities under the two controllers are demonstrated in FIGURE 4. The maximum permissible value for the angular velocity is set as 0.25 rad/s. As it can be observed, the constraint on the angular velocity is violated under the CAC since it is not designed to provide desired transient and steady-state performances for the angular velocity; however, the proposed constrained control is still able to guarantee superior performance by keeping the angular velocity within the allowable region. Hence, the proposed
control scheme is able to enhance pointing accuracy and pointing stability, simultaneously. To have better observation that the angular velocity trajectories under the proposed control do not contact the boundaries, the zoom-in of the trajectories is depicted in FIGURE 5. It is clear that the trajectories can approach the boundaries, but the controller suppresses them and does not allow them to contact the boundary. However, the trajectories under the CAC exited the region. Moreover, the transformed variables related to the MRPs and angular velocity are shown in FIGURE 6 and FIGURE 7, correspondingly. As expected, the absolute values of these variables are always less than one confirming that the MRPs and angular velocity do not contact their corresponding performance functions boundaries and the constrains are not violated. The less vertical distance between the attitude MRPs and the corresponding boundary, the less value of the first transformed value $\lambda_1$. This is also true for the transformed value $\lambda_2$ related to the angular velocity. The required control torque for each approach is depicted in FIGURE 8. Since the CAC is not able to cope with the actuator saturation limit, the procedure of this paper in dealing with the saturation is also applied to the CAC. According to FIGURE 8, the maximum torques for both control schemes do not exceed the maximum limit, i.e., $u_{\text{max}} = 2 \text{ Nm}$. It should be pointed out that the proposed controller is able to provide prespecified performance for the attitude MRPs and the rotation velocity in spite of the control input saturation. In addition to this, the proposed controller has a simpler structure than the CAC since it is not based on stabilization of the transformed errors.

Part 2: In this part, we are about to assess robustness of the proposed control scheme against the parameter uncertainty as a result of the inertia matrix variation. To this end, the simulation is repeated for different uncertainty on the inertia matrix, i.e., $\Delta J = 5, 10, 15, 20 \%$. FIGURE 9 illustrates the attitude MRPs for different inertia matrix uncertainties. Although the uncertainty increases from 5 to 20 percent, the attitude trajectories are still remained within the allowed region and the predefined performance in transient and steady-state is obtained. According to FIGURE 10, for large uncertainty in the inertia matrix, the angular velocity trajectories are quite close to the boundary; however, the controller does not allow them to intersect the boundary and violate the constraint. Based on FIGURE 11, since the trajectories are approaching the boundary due to the larger uncertainty, the controller requires more control torque and the fluctuation in the control torque increases. The simulation results verify that the suggested control scheme is robust against the inertia matrix uncertainty.

V. CONCLUSION

This paper proposed a new PD-like control for rigid spacecraft attitude system with the prescribed performance for attitude MRPs and angular velocities, in the presence of the input saturation. In comparison with the existing constrained control methods in the literature which are based on the concept
of PPC, the novel PD-like control has a simple structure since it does not require to stabilize the transformed errors. Hence, it does not involve intricate function terms as well as partial differential terms. More specifically, the recommended methodology proposes a log-type constrained control. When the attitude MRPs approach the corresponding boundary, the logarithmic function in the control signal increases to suppress the MRPs growth. This is also followed for the angular velocity variable. Moreover, the proposed constrained control provides specific transient and steady-state performance for the attitude MRPs as well as rotation velocity of the rigid spacecraft. Thus, both pointing accuracy and pointing stability are simultaneously improved. The efficiency of the suggested approach was assessed using numerical simulations conducted on a rigid spacecraft and compared to that of a PPC-based attitude control. The obtained results confirmed the highly desired performance of the planned control approach. As a future work, we will focus on the problem of observer based-constrained attitude control for flexible spacecraft considering parameter uncertainty, external disturbance and actuator faults to guarantee that the closed-loop system stability and performance are improved, simultaneously.

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