Model Reference Gaussian Process Regression:
Data-Driven Output Feedback Controller

Hyuntae Kim, Hamin Chang, and Hyungbo Shim

Abstract—Data-driven controls using Gaussian process regression have recently gained much attention. In such approaches, system identification by Gaussian process regression is mainly followed by model-based controller designs. However, the outcomes of Gaussian process regression are often too complicated to apply conventional control designs, which makes the numerical design such as model predictive control employed in many cases. To overcome the restriction, our idea is to perform Gaussian process regression to the inverse of the plant with the same input/output data for the conventional regression. With the inverse, one can design a model reference controller without resorting to numerical control methods. This paper considers single-input single-output (SISO) discrete-time nonlinear systems of minimum phase with relative degree one. It is highlighted that the model reference Gaussian process regression (MR-GPR) controller is designed directly from pre-collected input/output data without identification of the system itself.

I. INTRODUCTION

Gaussian process regression (GPR) [1], one of the most well-known regression tools for nonlinear functions, has been extensively used in various fields by virtue of the following properties [2]. First, since it is a nonparametric method, it has some flexibility to deal with a large amount of data. Secondly, prior knowledge of the regression target can easily be incorporated. Finally, it gives some confidence information about the regression result, which can be utilized to measure the regression error.

Particularly in control systems, GPR has been mainly applied for identifying unknown nonlinear systems using input/output or even state data before designing a model-based controller for the identified model. For instance, [3] and [4] show that a model predictive controller can be designed based on the model identified by GPR. Moreover, its real world applications are presented in [5] and [6] for quadrotors and mobile robots, respectively. In addition, combining the prior knowledge of a nominal model, [7] and [8] demonstrate the utility of such Gaussian process-based model predictive control (GP-MPC) method in autonomous racing systems by identifying a residual model instead of the full dynamical system. Also, [9] presents real world experiments of quadrotors controlled by the GP-MPC approach, where only aerodynamic effects on quadrotors are modeled by Gaussian process. On the other hand, [10] and [11] propose a feedback linearization controller for the system, which is identified by GPR. They also provide Lyapunov stability analysis of the controlled system based on the result concerning the error of the identification in [12]. Moreover, [13] and [14] propose event-triggered online learning of GPR in order to increase data efficiency.

Also, GPR has been utilized for the identification of the inverse model of a given system [2]. For instance, [15] presents a comprehensive analysis that explores the use of inverse model GPR for robotic systems. In addition, [16] uses GPR for approximating the inverse model and achieves high precision control for the robot arm. Also, [17] proposes a local approximation to GPR for real-time online learning of inverse model using the Barrett WAM robot arm. Moreover, [18] evaluates various nonparametric regression methods, including GPR, for inverse model learning using data from the SARCOS robot arm to enable real-time control. Moreover, [19] employs neural network methodologies for the implementation of control techniques based on inverse models.

In this context, we propose a model reference GPR (MR-GPR) controller based on the inverse model identified by GPR. This controller not only circumvents the numerical control issues associated with using GPR for system identification but also enables the straightforward integration of classical control, thus facilitating the construction of data-driven controllers. Furthermore, our proposed approach aims to address the lack of a theoretical foundation that has been a challenge in traditional inverse model GPR control. Since it is natural to assume that we have access to only input/output measurements of the plant, we propose the MR-GPR controller in the form of an output feedback control. Therefore, the GPR is performed only with input/output data of the system. Since our approach is based on input/output inversion in some sense, a few limitations naturally follow such as causality and minimum phase issues. In this paper, we assume that the system has relative degree one to resolve the causality issue, which is not very restrictive because a sampled-data system of a continuous-time system generically has relative degree one. Moreover, we assume that the system is of minimum phase.

This paper is organized as follows. The problem formulation with a class of nonlinear systems under consideration and a couple of assumptions on the class of systems are in Section II. In Section III, we propose the data-driven MR-GPR controller and explain how to design it using GPR. Also, a stability analysis of the closed-loop system with the MR-GPR controller is presented. An illustrative example that demonstrates the usefulness of the MR-GPR controller is
Notation: For column vectors $a$ and $b$, $[a^T; b^T]^T$ denotes
$[a^T, b^T]^T$. For discrete-time vector sequences $y(t)$ and $z(t)$,
we define a vector

$$z[k,k+T] := [z(k); z(k+1); \cdots; z(k+T)],$$

and a set

$$\{(y(t),z(t))\}_{t=k}^{k+T} := \{(y(k),z(k)), \cdots, (y(k+T), z(k+T))\}.$$  

II. Problem Formulation

Consider a single-input single-output (SISO) nonlinear discrete-time control-affine system with relative degree one in Byrnes-Isidori normal form [20]:

\begin{align}
    y(t+1) &= f(z(t), y(t)) + g(z(t), y(t))u(t) \quad (1a) \\
    z(t+1) &= h(z(t), y(t)) \quad (1b)
\end{align}

where $u(t) \in \mathbb{R}$ is the input, $z(t) \in \mathbb{R}^{n-1}$ is the state of the zero dynamics, and $y(t) \in \mathbb{R}$ is the output. It is assumed that the functions $f(\cdot, \cdot)$, $g(\cdot, \cdot)$, and $h(\cdot, \cdot)$ are unknown and only the input/output of the system are available as measurements. Also, we assume that the functions $f(\cdot, \cdot)$, $g(\cdot, \cdot)$, and $h(\cdot, \cdot)$ are smooth. In addition, the following assumption is given.

Assumption 1: The system (1) satisfies the following:

(a) The system has global relative degree one, or equivalently, $g(z, y) \neq 0$ for all $(z, y) \in \mathbb{R}^n$. Also, the system dimension $n$ and the global relative degree one are known.

(b) The internal dynamics (1b) is input-to-state stable with the input being $y$.

If the plant to be controlled is a continuous-time physical system, then its discretization generically yields a discrete-time system of relative degree one [21]. Therefore, the system description of (1) may not be too restrictive. Now, we assume observability of the system (i.e., observability for the state $z$) as follows.

Assumption 2: There exists a smooth mapping $\mathcal{O} : \mathbb{R}^{2n-1} \to \mathbb{R}^{n-1}$ that determines the state $z(t)$ as

$$z(t) = \mathcal{O}(y_{t,[t,t+n-1]}, u_{t,[t,t+n-2]}))$$

for any pair of input $u_{t,[t,t+n-2]}$ and output $y_{t,[t,t+n-1]}$ of the system (1).

Example 1: For simplicity, let us write $y(t)$ by $y_t$ in this example. When the system (1) has the form

\begin{align}
    y_{t+1} &= f_z(z_t) + f_y(y_t) + u_t \\
    z_{t+1} &= h(z_t, y_t)
\end{align}

then Assumption 2 holds if, for any input/output trajectory $u_{t,[t,t+n-2]}$ and $y_{t,[t,t+n-1]}$ of (1), there exists a unique solution $z^* \in \mathbb{R}^{n-1}$ to the equations

\begin{align*}
    f_z(z^*) &= y_{t+1} - f_y(y_t) - u_t, \\
    f_z(h(z^*, y_t)) &= y_{t+2} - f_y(y_{t+1}) - u_{t+1}, \\
    \vdots \\
    f_z(h(\cdots(h(z^*, y_t), f_y(y_t)) + f_z(z^*) + u_t), \cdots)) &= y_{t+n-1} - f_y(y_{t+n-2}) - u_{t+n-2}
\end{align*}

which is derived directly from the system (2). In this case, $z(t) = z^*$.

On the other hand, let us consider a stable reference model given by

$$y_r(t+1) = f_r(y_r(t)) \in \mathbb{R} \quad (3)$$

which satisfies the additional assumption that

$$y_r(t+1) = f_r(y_r(t)) + \eta(t)$$

is input-to-state stable when $\eta$ is viewed as an input. In order to make the controlled system (1) become the reference model (3), the controller should be

$$u(t) = \frac{f_r(g(t)) - f(z(t),y(t))}{g(z(t), y(t))} \quad (4)$$

For designing the controller (4), however, not only the functions $f(\cdot, \cdot)$ and $g(\cdot, \cdot)$ are needed, but also the state $z(t)$ needs to be measured. In this paper, we present a method to construct the controller (4) by using only the input/output data of the system (1).

III. MAIN RESULT

In this section, we design a data-driven controller that can produce almost the same control input as (4) by using GPR trained by input/output data of the system (1).

We firstly show that the state $z(t)$ can be expressed by the input/output history of the system (1). For this, let

$$\zeta_0(t) := [y_{t-[t,n+1,t-1]}, u_{t-[t,n+1,t-1]}] \in \mathbb{R}^{2(n-1)} \quad (5)$$

Lemma 1: Under Assumption 2, there exists a smooth function $\theta : \mathbb{R}^{2(n-1)} \times \mathbb{R} \to \mathbb{R}^{n-1}$, such that the state $z(t)$ of (1) is given by

$$z(t) = \theta(\zeta_0(t), y(t))$$

for all time step $t$.

Proof: Since there exists a smooth mapping $\mathcal{O}$ such that

$$z(t-n+1) = \mathcal{O}(y_{t-[t,n+1,t-1]}, u_{t-[t,n+1,t-1]})$$

by Assumption 2, it follows that

\begin{align*}
    z(t) &= h(z(t-1), y(t-1)) \\
    &= h(h(z(t-2), y(t-2)), y(t-1)) \\
    \vdots \\
    &= h(\cdots(h(z(t-n+1), y(t-n+1)), \cdots), y(t-1)) \\
    &= h(\mathcal{O}(y_{t-[t,n+1,t-1]}), y(t-n+1), \cdots, y(t-1)) \\
    &= \theta(\zeta_0(t), y(t))
\end{align*}

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which completes the proof.

Let us define the vectors
\[ \zeta_1(t) := [\zeta_0(t); y(t)] \in \mathbb{R}^{2n-1}, \]
\[ \xi(t) := [\zeta_1(t); y(t+1)] \in \mathbb{R}^{2n}, \]
which are composed of an arbitrary input/output trajectory of the system (1), and suppose that the set \( \mathcal{E} \) contains all possible \( \xi(t) \).

Define \( c : \mathbb{R}^{2n} \to \mathbb{R} \) as
\[ c(\xi_1(t); s) := \frac{s - f(\theta(\zeta_0(t), y(t)), y(t))}{g(\theta(\zeta_0(t), y(t)), y(t))}. \]

Then, the ideal control (4) is generated by
\[ u(t) = c(\zeta_1(t); f_r(y(t))). \quad (6) \]

For later use, we also define \( C \) as the set of all possible \([\zeta_1(t); f_r(y(t))]\). It is noted that \( C \subseteq \mathcal{E} \) by definition.

The ideal control (4), implemented as (6), is an output feedback controller. The following theorem shows the convergence of the closed-loop system with
\[ u_0(t) = c(\zeta_1(t); y(t+1)) \]
from (1a).

To perform the proposal, we first collect input/output data\(^1\) of system (1) as
\[ \{(u_d(t), y_d(t))\}_{t=1}^{N} \]
where \( N > n \) is the total number of input/output data. Then we rearrange the data as the training input
\[ \xi_d(t+n-1) = [\zeta_d(t+n-1); y_d(t+n)] \]
\[ = [y_d(t+n-2); u_d(t+n-2); y_d(t+n-1+n)] \]

and the training output
\[ u_d(t+n-1), \]
yielding the training dataset:
\[ \mathcal{D} := \{\{\xi_d(t+n-1), u_d(t+n-1)\}\}_{t=1}^{N-n}. \quad (8) \]

Remark 1: It may be difficult to collect sufficiently long \((N \gg n)\) input/output sequences as in (7) if the system is unstable. In this case, let input/output data collected in the \(i\)-th experiment be
\[ \{(u_d^i(t), y_d^i(t))\}_{t=1}^{N_i}, \]
where \( N_i > n \). The training input and output samples are rearranged as
\[ \xi_d^i(t+n-1) = [y_d^i(t+n-2); u_d^i(t+n-2); y_d^i(t+n-1+n)] \]
\(^1\)The subscript \( d \) is used for the sample data collected from the system during some experiment.
the MR-GPR controller (12) under a boundedness assumption of the input gain.

**Assumption 3:** There exists $\bar{g} > 0$ such that $|g(z, y)| \leq \bar{g}$ for all $(z, y) \in \mathbb{R}^n$.

While we assume the input gain function $g(\cdot, \cdot)$ to be bounded, if we consider the case where $z(t)$ and $y(t)$ stay in some compact sets, then it is seen that the boundedness directly follows from the smoothness of the input gain function.

**Theorem 1:** Under Assumptions 1, 2, and 3, there exists a class-$\mathcal{K}$ function $\gamma$ such that, if there exists a dataset $\mathcal{D}$ so that

$$|\mu_\mathcal{D}([\zeta_1; s]) - c([\zeta_1; s])| \leq \delta, \quad \forall [\zeta_1; s] \in \mathcal{C}$$

for a given $\delta > 0$, then, the closed-loop system (1) with the MR-GPR controller (12) guarantees

$$\limsup_{t \to \infty} \|y(t); z(t)\| < \gamma(\delta).$$

**Proof:** For notational simplicity, let $y_t$ imply $y(t)$ in this proof. Applying (12) to (1a), we have

$$y_{t+1} = f(z_t, y_t) + g(z_t, y_t)\mu_\mathcal{D}([\zeta_1; f_r(y_t)])$$

$$= f(z_t, y_t) + g(z_t, y_t)\mu_\mathcal{D}([\zeta_1; f_r(y_t)]) + c([\zeta_1; f_r(y_t)])$$

$$= f_r(y_t) + e_t$$

where

$$e_t := g(z_t, y_t)\mu_\mathcal{D}([\zeta_1; f_r(y_t)]) - c([\zeta_1; f_r(y_t)]).$$

(14)

By the assumption,

$$|e_t| \leq \bar{g}\delta, \quad \forall t$$

and by the input-to-state stability assumption of the reference model, there are a class-$\mathcal{K}$ function $\beta_y$ and a class-$\mathcal{K}$ function $\gamma_y$ such that

$$|y_t| \leq \beta_y(|y_0|, t) + \gamma_y(\bar{g}\delta).$$

Also from the input-to-state stability of (1b) in Assumption 1 (b), there exists a class-$\mathcal{K}$ function $\beta_z$ and a class-$\mathcal{K}$ function $\gamma_z$ such that

$$\|z_t\| \leq \beta_z(\|z_0\|, t) + \gamma_z(|y_t|).$$

Therefore,

$$\limsup_{t \to \infty} |y_t| \leq \gamma_y(\bar{g}\delta)$$

$$\limsup_{t \to \infty} \|z_t\| \leq \gamma_z(\gamma_y(\bar{g}\delta))$$

so that the function $\gamma$ that completes the proof can be constructed.

**Remark 2:** We identify the smooth function $c(\cdot)$ as $\mu_\mathcal{D}(\cdot)$ by the GP with SE kernel. In fact, the posterior variance function $\sigma_y(\cdot)$ in (11) can be utilized to measure how much the function $\mu_\mathcal{D}(\cdot)$, the identification result, differs from the function $c(\cdot)$. Specifically, if the function $c(\cdot)$ belongs to reproducing kernel Hilbert space generated by the kernel $k$ in (9), then

$$|\mu_\mathcal{D}(\xi) - c(\xi)| \leq \beta \sqrt{\sigma(\xi)}, \quad \forall \xi \in \mathcal{E}$$

for some positive $\beta$ (see [22, Corollary 3.11] for details). In addition, [23, Corollary 3.2] presents a certain method for data collection, with which it is possible to make the upper bound (the function of the posterior variance) arbitrarily small by using a sufficiently large number of data $N$. With the help of these facts, we can compose a dataset $\mathcal{D}$ that satisfies the sufficient condition (13) in Theorem 1 for a given $\delta > 0$.

**Remark 3:** In order to initiate the output feedback controller (12) at time $t = 0$, information of $y[-n+1,-1]$ and $u[-n+1,-1]$ is needed. If the system is initially at rest or at the steady-state, then the information is easy to obtain, but this may not be the typical situation. Instead, one may apply arbitrary inputs for the initial $(n - 1)$ steps. In fact, the information about $(n - 1)$-long input/output sequences is necessary to figure out the information of internal state, which is reminiscent to the classical output feedback controls, in which, the state-feedback control does not have meaningful until a dynamic observer estimates the plant’s state.

**IV. ILLUSTRATIVE EXAMPLE**

In this section, an illustrative example is presented to describe the utility of the proposed data-driven controller.

Consider the following SISO system

$$y(t + 1) = y^2(t) + z(t) + u(t)$$

(15a)

$$z(t + 1) = 0.5 \sin(y(t))z(t),$$

(15b)

where $u, y, z \in \mathbb{R}$. We assume that the system dimension $n = 2$ and the global relative degree one are known (Assumption 1 (a)). Since the internal dynamics (15b) is input-to-state stable from $y$ to $z$, the system (15) also satisfies Assumption 1 (b). Furthermore, Assumption 2 is satisfied by the fact that $z(t)$ is uniquely determined by

$$z(t) = y(t + 1) - y^2(t) - u(t)$$

$$= O([y_{t+1}; u(t)]).$$

Therefore, we obtain

$$z(t) = 0.5 \sin(y(t - 1))z(t - 1)$$

$$= 0.5 \sin(y(t - 1)) (y(t) - y^2(t - 1) - u(t - 1))$$

$$= \theta(c(y_0(t), y(t))),$$

as in Lemma 1. Noting that

$$c([\zeta_1(t); y(t + 1)]) = y(t + 1) - y^2(t) - \theta(c(y_0(t), y(t))),$$

we compose the training data $\mathcal{D} = \bigcup_{t=1}^{T} \mathcal{D}^t$, where the data $\mathcal{D}^t$ is collected in an experiment with random initial condition $y_d(0), z_d(0) \in \{-1.2, 1.2\}$ and random input $u_d(t) \in [-1.2, 1.2]$ for $N^i = 5$ time steps for all $i = 1, \ldots, T$. In the $i$-th experiment, as in Remark 1, we obtain the data

$$\xi_d(t + 1) = [y_d(t); u_d(t); y_d(t + 1); y_d(t + 2)]$$

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which is used as a training input and

\[ u_d(t + 1) \]

which is considered as a training output for \( t = 1, 2, 3 \). Using the training data \( D \), we set the hyperparameters in (9) by optimizing the marginal likelihood through GPML toolbox [24]. Finally, we take a stable reference model as

\[ y_r(t + 1) = f_r(y_r(t)) = -0.4y_r(t) \]

which guarantees input-to-state stability for

\[ y_r(t + 1) = -0.4y_r(t) + \eta(t). \]

Then, the proposed output feedback controller becomes

\[
\begin{align*}
    u(t) &= \mu_D([\zeta_1(t); -0.4y(t)]) \\
    &= \mu_D([y(t-1); u(t-1); y(t); -0.4y(t)]). 
\end{align*}
\]

(16)

Figs. 1 and 2 show the output of the closed-loop system with the proposed controller designed by the training data of \( T = 20 \) and 2000 experiments from different initial conditions, respectively, compared to the one with the ideal controller. We set the initial conditions of each system as

\[(y(0), z(0)) \]

in both Figs. 1 and 2. In all cases, zero input \( u(0) = 0 \) is used at the very first step of control for applying the MR-GPR controller. It is observed that in both Figs. 1 and 2, the MR-GPR controller asymptotically stabilizes all systems that have different initial conditions. Also, the MR-GPR controller designed with more data in Fig. 2 shows better performance than the one designed with less data in Fig. 1.

On the other hand, Figs. 3 and 4 depict function values of the MR-GPR controller \( \mu_D \) designed by using the data of \( T = 20 \) and 2000 experiments, respectively, compared to the ideal controller \( c \). Although both functions \( \mu_D \) and \( c \) need an input

\[ [\zeta_1(t); -0.4y(t)] = [y(t-1); u(t-1); y(t); -0.4y(t)] \]

to be evaluated, we fix the value \( u(t-1) = 0.2 \) and evaluate both functions by sweeping \( y(t-1) \) and \( y(t) \) in \([-1.2, 1.2]\). It is seen that the proposed controller \( \mu_D \) sufficiently well approximates the ideal controller \( c \) throughout the entire domain when 2000 experiments of data are used in Fig 4, while the approximation reveals some error particularly at evaluation points which are far from \((0,0)\) when 20 experiments of data are used Fig 3. This is also verified in Fig. 5 that plots the error \( e_t \) of (14), which is a function of \([\zeta_1(t); -0.4y(t)]\).
Fig. 4. Function values of ideal controller $c$ (green mesh) and MR-GPR controller $D$ (black dotted mesh) designed by the data of $T = 2000$ experiments.

Fig. 5. Error of function values between ideal controller $c$ and MR-GPR controller $D$ designed by the data of $T = 20$ experiments (red mesh) and $T = 2000$ experiments (blue mesh)

V. CONCLUSION

In this paper, we proposed the MR-GPR controller, which is the data-driven output feedback controller, for SISO nonlinear discrete-time control-affine systems with relative degree one and of minimum phase. The design was performed by using the GPR, trained only by input/output data of the system. It is worthy to emphasize that the GPR was utilized not for system identification but for controller design itself. It was shown that the control performance improves as more data are available for training, which was demonstrated by an illustrative example using simulations.

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