Material failure time and the fiber bundle model with thermal noise

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Abstract

The statistical properties of failure are studied in a fiber bundle model with thermal noise. We find that in agreement with recent experiments the macroscopic failure is produced by a thermal activation of microcracks. Most importantly the effective temperature of the system is amplified by the spatial disorder (heterogeneity) of the fiber bundle.

PACS numbers: 05.70.Ln, 62.20.Mk, 05.20.-y
Material failure is a widely studied phenomenon not only for its very important technological applications but also for its fundamental statistical aspects, which are not yet very well understood. Many models have been proposed to give more insight into the statistical analysis of failure. Among the most studied ones we can mention the fuse and the non-linear spring networks \cite{1, 2, 3, 4, 5} which can reproduce several features of crack precursors, experimentally observed in heterogeneous materials subjected to a quasi-statically increasing stress \cite{6, 7, 8, 9, 10}. Specifically the power law behaviour of the acoustic emission observed in several experiments close to the failure point.

However these networks and the other related models, in their standard formulation, are unable to describe the behaviour of a material subjected to a creep-test, which consists in keeping a sample at a constant stress till it fails. Creep-tests are widely used by engineers in order to estimate the sample life time as a function of the applied stress. A modified fuse network, which takes into account the Joule effect in the fuses, has been proposed to explain the finite life time of a sample subjected to a constant stress\cite{4}. However also this model does not explain the recent experimental results on micro crystals\cite{11}, gels\cite{12} and heterogeneous materials\cite{13}. These experiments show that the life time $\tau$ of a sample, subjected to an imposed stress $P$ is well predicted by the equation

$$\tau = \tau_o \exp \left( \frac{\Gamma d Y^{(d-1)}}{k T_{eff} P^{(2d-2)}} \right)$$

where $\tau_o$ is a constant, $\Gamma$ the surface energy, $Y$ the Young modulus, $k$ the Boltzmann constant, $\alpha$ a constant which depends on the geometry, $T_{eff}$ is an effective temperature and $d$ the dimensionality of the system. The main physical hypothesis behind eq.\ref{eq:1} is that the macroscopic failure of a material is produced by a thermal activation of micro cracks\cite{14, 15}. In the original Pomeau’s theory \cite{14} $T_{eff}$ of eq.\ref{eq:1} coincides with the thermodynamic temperature $T$ while experimentally $T_{eff} >> T$ \cite{11, 13}. To explain this results it has been supposed that the disorder of the material “amplifies” the thermal noise \cite{13}.

The purpose of this letter is to show that adding to the spring network a noise, which plays the role of a temperature, it is possible to reproduce the behaviour of a material subjected to a creep-test. Specifically the functional dependence on the applied stress of the network life time is consistent
with that observed in recent experiments, that is with eq. (1). Furthermore we
can prove that, for such a process, the noise is ”amplified” by the network
disorder.

In order to study the influence of noise on macroscopic failure we have
chosen the simplest spring network, that is the so called democratic fiber
bundle model, proposed a long time ago by Pierce [10] to study cable failure.
This model, widely studied in the quasi static regime [1, 2, 3, 4], is equivalent
to $N$ springs in parallel subjected to a total traction force $F$. Specifically we
have numerically studied the model by using the following rules:

A1 The external applied force $F$ produces a local stress $f_i$ on each fiber. $F$
is democratically and completely distributed in the net: $F = \sum_{i=1}^{N} f_i$

A2 The local stress $f_i$ on the i-th fiber produces a local deformation $e_i$.
Being the media elastic, stress and deformation are linked by Hook’s law:

$$f_i = Y \cdot e_i$$  \hspace{1cm} (2)

where $Y$ is the Young modulus, which is assumed to be the same for
all the fibers : $Y_i = Y$.

A3 The strength of each fiber is characterized by a critical stress $f_i^{(c)}$: if at
time $t$ on the i-th fiber local stress $f_i(t)$ is greater than critical stress $f_i^{(c)}$
the fiber cracks, and his local stress falls to zero at time $t+1$. Further,
we assume that in this process some energy $\epsilon_i$ is released proportionally
to the square of local stress $f_i$; for sake of simplicity we will assume
$\epsilon_i = f_i^2$. Critical stress $f_i^{(c)}$ is a realization of a random variable that
follows a normal distribution of mean $f^{(c)}$ and variance $KT_d$:

$$f_i^{(c)} = f^{(c)} + N_d(KT_d)$$  \hspace{1cm} (3)

We call $N_d$ disorder noise.

A4 Each fiber is subjected to an addictive time dependent random stress
$\Delta f_i(t)$ which follows a zero mean normal distribution of variance $KT$:

$$\Delta f_i(t) = N_T(t, KT)$$  \hspace{1cm} (4)

being $t$ the time. We call $N_T$ thermal noise. We assume that $\Delta f_i(t)$
is a white random process, which is independent in each fiber, i.e. the
correlation function $E[\Delta f_i(t_1) \cdot \Delta f_j(t_2)] = 0$ if $t_1 \neq t_2$ or $i \neq j$. 

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The first three items are those used in the standard formulation of fiber bundle model. A new feature, which is similar to a thermal activation process, is introduced in [A4] to explain the dependence of failure time on a constant applied stress.

The model has the following properties. We see from [A1] that there exists a long-range interaction between the fibers: indeed if a number \( n(t) \) of fibers are broken at time \( t \), the local stress on each of the remaining fibers will be:

\[
f_i(t) = \frac{F}{N - n(t)} + \Delta f_i(t)
\]

that is, under the specified boundary conditions, the breaking of some fibers produces an increase of local stress on the other ones. Item [A3] models the heterogeneity of media. The assumption that all the disorder in the model appears in the strength distribution rather than in the elastic constants may be argued by noticing that the effective elastic constant of a single fiber is essentially the average of the local elastic constant along the fiber, while the strength is determined by its weakest point [4]. If \( KT = 0 \) the model is reduced to the standard one: in this case the applied force is increased linearly \( F = A_p t \) from zero to the critical value \( F_c \) needed to break the whole network. The sample lifetime \( \tau \) is equal to \( F_c/A_p \) for any value of the slope \( A_p \).

It is quite clear that when \( KT = 0 \) and a constant force \( F \) is applied the system breaks in a single avalanche only if \( F \) is large enough otherwise it will never break. For example if \( KT_d = 0 \) all the fibers are strictly equal and if there is no thermal activation (i.e. \( KT = 0 \)) the system breaks in a single step when \( F = N f^{(c)} \).

If \( KT \neq 0 \) then the system can break also at constant imposed stress. We have numerically studied the behaviour of the model as a function of \( F \), \( KT \) and \( KT_d \). In the following we assume \( N = 1000 \) and \( f^{(c)} = 1 \). For each set of parameter values we have repeated the numerical simulation at least ten times to estimate the scattering of the results for different realization of the noise. This corresponds to the scattering of symbols in the figures. We call event the simultaneous breaking of several fibers and event size the number \( s(t) \) of fibers which crack. The energy \( \epsilon \) associated to an event is the sum of the energies released by the fibers which crack, that is:

\[
\epsilon = s(t) \cdot \left( \frac{F}{N - n(t)} \right)^2.
\]
The cumulated energy $E(t)$ is the sum of $\epsilon$ from 0 to $t$. When $KT \neq 0$ and $F$ is constant the event statistics is quite similar to that observed at $KT = 0$ for $F$ increasing linearly in time [1, 2, 3]. As an example of the system response to a constant force $F = 540$ with $KT_d = 0.005$ and $KT = 0.007$, we plot in fig.1a the distribution $N(\epsilon)$ of $\epsilon$ and in fig.1b $E(t)$ as a function of the reduced parameter $(\tau-t)/\tau$ (notice that because of the constancy of $F$ the only control parameter is time). In agreement with experiments [19, 8, 13], we find that $\epsilon$ is power law distributed and that the cumulated energy $E$ has a power law dependence on $(\tau-t)/\tau$ for $t \to \tau$. More details on this statistical features of the model for different driving forces will be given in a longer report. Here we want to focus on failure time $\tau$ of the network in the case of a creep test (constant $F$).

We first keep $KT_d = 0$ and study the evolution towards failure for various $KT$ and $F$. The results are summarized in fig.2. When $KT \neq 0$, we observe (see fig. 2a) that failure time $\tau$ as a function of $\frac{1}{F^2}$ follows an exponential law for any fixed value of $KT_d$:

$$\tau \sim \exp \left[ \left( \frac{F_0}{F} \right)^2 \right] \quad (7)$$

where $F_0$ is a fitting parameter. Further, looking at fig. 2b we notice that the failure time $\tau$ depends on thermal noise $KT$ as follows:

$$\tau \sim \exp \left( \frac{A}{KT} \right) \quad (8)$$

where $A$ is a fitting parameter.

We notice that these results are similar to the prediction of Pomeau’s theory, that is eq.1 with $d = 2$. Because of this analogy with eq.4 we are now interested in studying the dependence of failure time $\tau$ on disorder noise (i.e. on $KT_d$): to this aim, we have done simulations keeping the thermal noise variance $KT$ fixed to constant (not zero) values. Looking at fig.3, we observe the following facts:

B1 failure time $\tau$ decreases following a power law as the disorder noise variance $KT_d$ increases, that is the more the media is heterogeneous, the smaller is failure time (see fig. 3a).

B2 as $KT_d$ increases, the absolute difference between failure times $\tau$ for different values of thermal noise variance $KT$ decreases, that is failure
time $\tau$ becomes less sensitive to the effective value of thermal noise, as shown in fig.3b.

Thus one may conclude that disorder noise amplifies the effect of thermal noise and reduces the dependence of $\tau$ on the temperature; these results allow us to explain the recent experimental observations. Let us recall that experiments on microcrystals [1], gels [2] and macroscopic composite materials [3] reveal that dependence on $P$ of the sample lifetime is very well fitted by eq.(1). However calculations [3] show that thermal fluctuations are too little to activate the nucleation of microcracks in the times $\tau$ measured in the experiments. It had been measured that the temperature needed to have the measured life-times $\tau$ should be of the order of several thousands of Kelvin ($3000K$ for wood [3]). Experiments show that the life time $\tau$ of very heterogeneous materials [3], is (in the limit of experimental errors) independent of $T$ while the life time $\tau$ of quite homogeneous materials as microcristals [1] strongly depends on $T$, as predicted by eq. (1). To explain these results has been supposed that disorder amplifies the thermal noise so that the nucleation time of defects becomes of the order of the measured ones and that the lifetime $\tau$ of the sample depend on the heterogeneity of the media. These hypothesis are now well verified by the numerical results B1 and B2 of the fiber bundle model.

As a conclusions we have shown that adding a white noise, which plays the role of temperature, to the fiber bundle model we can reproduce the recent experimental observations on the dependence of sample life time on the applied stress. Furthermore using such a simple model we can prove that the disorder of the system amplifies the thermal noise in crack nucleation process. This explains quite well why the experimentally measured $T_{\text{eff}}$ is more close to the thermodynamic temperature in microcrystals than in heterogeneous materials. Similar conclusions about a disordered induced high temperature in nucleation processes have been reached in other disordered systems such as foams [18]. This is quite interesting because it seems to be a quite general properties of disordered systems where a thermal activated processes with long range correlation may be present.

We acknowledge useful discussion with Y. Pomeau. One of us (R.S.) thanks "Le Laboratoire de Physique de l’E.N.S.L." for the very kind hospitality during his visit in Lyon.
(*) on leave from ”Facoltá d’Ingegneria, Università di Firenze, Italy ”, with a SOCRATES exchange program of the European Community

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[19] The potential energy of a fiber which breaks is proportional to its elastic potential energy cumulated till the crack. If we assume that the acoustic energy is proportional to this potential energy then ε may be compared to the acoustic energy measured in the experiments.
Figure 1: (a) Distribution of energy $N(\epsilon)$ symbols are numerical measures, while continuous line represents a power law fits $N(\epsilon) \sim \epsilon^{-\beta}$ of small events with $\epsilon < 10$. (b) $E(t)$ as a function of the critical parameter $\frac{t - t}{\tau}$; circles are numerical measures, while the line represent the best fit, near $t \simeq \tau$, with the function $E(t) \sim \left(\frac{t - t}{\tau}\right)^{-\gamma}$. The parameters used are $KT = 7 \times 10^{-3}$, $KT_d = 5 \times 10^{-3}$ and $F = 540$. 
Figure 2: Failure time \( \tau \) of an homogeneous net \((KT_d = 0)\) in a creep test. 

(a) \( \tau \) as a function of the normalized force \( f = F/N \) for several values of thermal noise variance \( KT \) [(\( \circ \)) \( KT = 9 \times 10^{-3} \), (\( \Diamond \)) \( KT = 12 \times 10^{-3} \), (\( \ast \)) \( KT = 15 \times 10^{-3} \)]. 

(b) \( \tau \) as a function of \( KT \) for several values of \( f \) [(\( \ast \)) \( f = 0.63 \), (\( \Diamond \)) \( f = 0.6 \), (\( \triangle \)) \( f = 0.56 \), (\( \circ \)) \( f = 0.53 \)]. Continuous lines in (a) and (b) are the fits with eq.7 and eq.8 respectively.
Figure 3: Failure time $\tau$ of an heterogeneous network ($KT_d \neq 0$) in creep test. (a) $\tau$ is plotted as a function of normalized force $f = F/N$, at $KT = 6 \times 10^{-3}$, for different values of $KT_d$ [$(\triangle) KT_d = 9.8 \times 10^{-3}, (\circ) KT_d = 8 \times 10^{-3}, (\Diamond) KT_d = 7 \times 10^{-3}, (\ast) KT_d = 6 \times 10^{-3}$]; continuous lines represent the best fits with eq. 7. (b) $\tau$ as a function of disorder noise variance $KT_d$ at $F = 540$ for several values of $KT$ [$KT_1 = 7 \times 10^{-3}, KT_2 = 8 \times 10^{-3}, KT_3 = 10^{-2}$]. Dashed lines represent the data fits.