The Origin of Spontaneous Symmetry Breaking in Theories with Large Extra Dimensions

G. Dvali, S. Randjbar-Daemi, and R. Tabbash

Dept. of Physics, New York University, New York, NY 10003
International Centre for Theoretical Physics, Trieste, Italy
SISSA–ISAS, Via Beirut 4, I-34014 Trieste, Italy

Abstract
We suggest that the electroweak Higgs particles can be identified with extra-dimensional components of the gauge fields, which after compactification on a certain topologically non-trivial background become tachyonic and condense. If the tachyonic mass is a tree level effect, the natural scale of the gauge symmetry breaking is set by the inverse radius of the internal space, which, in case of the electroweak symmetry, must be around $\sim 1/\text{TeV}$. We discuss the possibility of a vanishing tree level mass for the Higgs. In such a scenario the tachyonic mass can be induced by quantum loops and can be naturally smaller than the compactification scale. We give an example in which this possibility can be realized. Starting from an Einstein–Yang–Mills theory coupled to fermions in 10-dimensions, we are able to reproduce the spectrum of the Standard Model like chiral fermions and Higgs type scalars in 4-dimensions upon compactifying on $\mathbb{CP}^1 \times \mathbb{CP}^2$. The existence of a monopole solution on $\mathbb{CP}^1$ and a self dual $U(1)$ instanton on $\mathbb{CP}^2$ are essential in obtaining chiral fermions as well as tachyonic or massless scalars in 4-dimensions. We give a simple rule which helps us to identify the presence of tachyons on the monopole background on $S^2$.

1e-mail: gd23@NYU.EDU
2e-mail: daemi@ictp.trieste.it
3e-mail: rula@sissa.it
1 Introduction

Perhaps the most mysterious part of the Standard Model (SM) is the Higgs sector, which is responsible for the origin of the electroweak scale $M_W \sim 100\text{GeV}$. In conventional 4D field theories it is hard to understand how such a low (compared to $M_{\text{Planck}} \sim 10^{19}\text{GeV}$) symmetry breaking scale triggered by an elementary scalar could be perturbatively stable. Thus, in such a framework one has to rely on either low energy supersymmetry or technicolor. An alternative understanding of this enormous hierarchy may come from the view that an effective cut-off of the 4D field theory (the fundamental quantum gravity scale $M$) is not far from $M_W$, due to existence of large extra dimensions \cite{1}.

The idea of large extra dimensions explains perturbative stability of the weak scale versus the Planck scale, by lowering the cut-off of the theory. Nevertheless, it does not address the issue of sensitivity of the Higgs mass to the ultraviolet cut-off. This is an important issue from the point of view of the low energy calculability, and is the central point to be addressed in the present work. We propose a framework in which such a sensitivity is absent as a result of a symmetry and the Higgs mass is finite and has no dependence on the unknown ultraviolet physics. Note that in our case the sensitivity is weaker than in softly broken $N = 1$ supersymmetry, where the Higgs mass is log-sensitive to the cut-off. Such an unusual situation is possible due to the presence of a symmetry that forbids any local counter term generating a Higgs mass. In our case the symmetry in question is a higher-dimensional gauge symmetry, which forbids the mass of Higgs particles to receive cut off dependent (and hence local) corrections, due to the fact that the Higgs fields are identified with some of the components of a higher-dimensional gauge field. Higher-dimensional gauge symmetry is spontaneously broken by compactification. An alternative approach was discussed in \cite{3}, where the Higgs mass is controlled by a higher-dimensional extended supersymmetry, spontaneously broken globally by Scherk-Schwarz mechanism.

It should be noted that the idea of large extra dimensions, \textit{per se} does not answer why the electroweak symmetry should be broken at all\cite{4} nor it says anything about the origin of the Higgs particles. Also, many essential

\footnote{For some discussions see \cite{4}}
issues, such as, for instance, generation of chiral fermions with needed quantum numbers, are usually attributed to technicalities. The present paper describes an attempt to provide a unified solution to all the above listed issues by implementing some of the ideas of an earlier work\cite{2} in the framework of standard model and large extra dimensions.

As mentioned above, we suggest that the Higgs particles may be identified with the extra components of the higher-dimensional gauge fields. After compactification of extra dimensions on a monopole background, via mechanism of \cite{2}, some of the extra components of the gauge fields become tachyonic and spontaneously break the electroweak symmetry. Their quantum numbers are identical to those of SM Higgs doublet. Notice that the monopole background is essential for generating the families of chiral fermions in four-dimensions, and therefore is doing a double job. If the tachyonic mass is a tree level effect the natural scale of the symmetry breaking is $\sim 1/a$, the inverse radius of extra compact space, since the only source of spontaneous breaking of the higher-dimensional gauge invariance is the compactification itself. In other words, since in the infinite volume limit $a \to \infty$ the full higher-dimensional gauge invariance must be recovered, the weak scale must go as $M_W \sim 1/a$. Thus in this case the size of extra dimensions to which gauge fields can propagate should be $a \sim 1/\text{TeV}$ (as in \cite{5}). However, it is important to stress that in order for the theory not to become infinitely strongly coupled above the compactification scale, the cut-off $M$ must be lowered as in \cite{4}, possibly via increase of the volume of some additional dimensions to which only gravity can spread. This issue will not be discussed in the present work.

The main idea behind our construction is very simple and goes back to \cite{4}. Consider a gauge group $G$ in $4+N$-dimensions, with the gauge fields $V_A$. After $N$ extra dimensions are compactified, the extra $V_m, m = 5, ..., 4+N$ components of the gauge field appear as scalars \cite{6}, and the remaining $V_\mu$-components as gauge fields in four-dimensional effective theory. In some cases it is necessary that the compactified manifold involves a non-trivial topology and gauge connection (e.g. such as monopole configuration) in order to generate chiral fermions \cite{4, 8}. On such a background $V_m$ components often become tachyonic unless special conditions are met. Our idea is to precisely use this instability for the spontaneous breaking of the gauge symmetry in four-dimensions. We shall argue that this leads to a solution of the hierarchy
For obvious phenomenological reasons, ideally we would like to have the electroweak symmetry breaking scale smaller than $1/a$. As we said above, in the simplest realization of our scheme the scale comes out just around $1/a$. However, we shall also discuss some ideas how the one-loop hierarchy $M_W < 1/a$ might be generated. More specifically we shall consider the possibility that the monopole induced tree level masses of the scalars vanish. Unless the mass of scalars are protected by supersymmetry, which we are not considering in this paper, the loop effects will generally induce a non zero mass for these fields. With an appropriate choice of the fermionic couplings this mass can be made tachyonic. Of course in general the loop induced mass corrections are divergent and they become infinite in the limit of infinite cut off. Unless the divergences vanish for some symmetry reason we need to renormalize the theory. Here we would like to argue that considering the scalars as some components of a Yang-Mills vector potentials evades the hierarchy problem which the conventional renormalization in 4 dimensions generate. Our argument goes as follows.

There are two distinct energy scales in our problem. The first is the Planck mass in higher dimensions which we assume is a few powers of 10 times a TeV. The second one is the inverse of a typical radius of the compact dimensions. Clearly as we explore distances shorter than the size of the compact dimensions the full gauge symmetry will be restored. This domain can still be larger than the fundamental Planck length of the higher dimensional theory. In this regime we shall see no Higgs field. All we see will be massless gauge particles. In order for this idea to work it is important that we consider our theory as an effective theory which includes all terms compatible with the gauge symmetry and the general coordinate invariance. This will require the presence of infinite number of parameters in our effective Lagrangian. Thus any cut off dependence can be absorbed in the redefinition of these 5

---

5We would like to emphasize that by the hierarchy problem here we do not mean the presence of numerically very different masses like the Planck mass of $10^{19}$GeV and the Higgs mass of about a TeV. What we have in mind is a mechanism which generates a finite Higgs mass whose value is stable under quantum corrections. A well known way to ensure this stability is to invoke low energy supersymmetry. In the present paper we are essentially replacing the low energy supersymmetry with local gauge symmetry.
parameters. The Higgs mass then should be written in the generic form of 
\[ \mu^2 = \frac{1}{a^2} f(Ea), \]
where \( a \) is a typical radius in our model and \( E \) is some common energy scale. The main task is to compute the function \( f \), which in this paper we calculate only at the tree level. Our statement is that since at energies much larger than \( 1/a \) the full gauge symmetry is recovered, the Higgs mass can not be heavier than \( 1/a \). This is our suggestion to solve the Higgs hierarchy problem.

The function \( f \) has been calculated in [12] for toroidal internal spaces and have been shown to be finite. These references contain ideas similar to the one advocated in the present paper. However, we would like to draw the reader’s attention to a basic difference, namely, the very important and central role of topologically non-trivial gauge field backgrounds, such as instantons in our work. In [13], similar ideas have been used to study dynamical breaking of supersymmetry in the context of type I string theory.

In this paper, we shall construct non-GUT type models, i.e. in the effective 4-dimensional theory the leptons and quarks will not be in the same multiplet of the gauge group. This makes the task of model building considerably more delicate. We have to find an appropriate gauge field configuration on a compact space which gives rise to the intricate chiral structure of one family of quarks and leptons. Furthermore we need to ensure that the tachyonic Higgs fields are singlets of the color \( SU(3) \) and doublets of \( SU(2)_L \). This is difficult because in the Kaluza–Klein approach all the 4-dimensional fields have the same parents in the higher dimensions and the mechanism should be subtle enough to produce the rich spectrum of the 4-dimensional standard model or its generalization. Because of the bigger multiplet structure of the GUT models the problem is somewhat easier if one aims to obtain a GUT model.

In this paper we shall start from a 6 or 10 dimensional theory and try to obtain models in \( D = 4 \) for which the gauge group contains the standard model \( U(1) \times SU(2) \times SU(3) \) with the correct fermion and Higgs sector. At least two of our extra dimensions will always parameterize a \( S^2 \) with a

---

6 In the \( D = 10 \) example there will be two different \( U(1)_Y \), one coupling to quarks and the other coupling to leptons only.
magnetic monopole like configuration on it. It has been shown in \[2\] that in such backgrounds the components of the gauge field tangent to \(S^2\) contain tachyonic modes. These are the modes which we would like to interpret as the \(D = 4\) Higgs fields. We shall give a simple general rule to identify the potential tachyonic modes for any gauge group. We shall then work out three examples. The first one will have a \(D = 6\) gauge group of \(SU(3)\) with a triplet of fermions in its fundamental representation. We shall show that this leads to a \(D = 4\) effective theory with the gauge group \(SU(2)_L \times SU(2)_R \times U(1)\) with the fermions in \((2,1) + (1,2)\) representation of \(SU(2)_L \times SU(2)_R\), so that it can be interpreted as a left right symmetric model of electroweak interaction of leptons. We shall work out the \(D = 4\) effective action for this model and the Yukawa couplings as well as the Higgs potential.

The second example will be in \(D = 10\) for which we shall take the gauge group to be \(U(6)\) with a multiplet of \(D = 10\) chiral fermions in the 6 of \(U(6)\). The \(U(6)\) Yang–Mills equations can be solved by a self dual \(U(1)\) instanton\(^7\) on \(\mathbb{C}P^2\) and a magnetic monopole on \(\mathbb{C}P^1 = S^2\). The Kaluza–Klein gauge group will thus be \(SU(2) \times SU(3)\). We shall analyze the spectrum of the Dirac operator on this background and identify the correct standard model candidates for a single family of leptons and quarks. Our general rule for the tachyonic contribution of the \(S^2\) dependence will help us to identify the various candidates for the Higgs fields among the components of the gauge field fluctuations. However, these fluctuations now will also depend on the \(\mathbb{C}P^2\) coordinates. It is necessary to ensure that upon harmonic expansion on \(\mathbb{C}P^2\) the resulting modes are singlets of \(SU(3)\). We shall show that the Higgs tachyons which give masses to the leptons and quarks are in fact singlets of \(SU(3)\). Apart from this there are other fields whose masses receive tachyonic contribution from \(S^2\). These are potentially dangerous and the leading term in their harmonic expansion on \(\mathbb{C}P^2\) is a triplet of \(SU(3)\). We need to ensure that the \(\mathbb{C}P^2\) contribution to their masses overwhelms the tachyonic contribution of \(S^2\). We show that the condition for this to happen is that the ratio of the radii of \(S^2\) and \(\mathbb{C}P^2\) is bigger than \(3/2\). There are two ways to ensure this, namely, either we give up the background Einstein equations in which case the two radii can be varied independently, or we introduce an

---

\(^7\) This instanton background on \(\mathbb{C}P^2\) is necessary in order for the spinors to be globally well defined on \(\mathbb{C}P^2\). It defines the so called spin\(^c\) structure on \(\mathbb{C}P^2\).
extra field in $D = 10$ which allows us to disentangle the two radii. There is not much to say about the first solution. As for the second one we shall show that the introduction of a $D = 10$, $U(1)$ gauge field which couples only to gravity solves our problem. Thus, as far as we neglect the gravitational effects this gauge field will be unobservable.

Finally we shall consider an $U(N)$ model in 10 dimensions in which by an appropriate choice of the magnetic charges the tree level masses of the would be tachyons will be zero. We shall argue that the one loop induced mass can be made tachyonic. This will provide us with the mild hierarchy which should exist between the scale of electroweak symmetry breaking and the compactification radius.

In this paper our intention is not to recover the standard model of particle physics from a 10-dimensional theory. In fact we believe that within our present understanding this is not possible. Our aim is rather to see how close one can get to the standard model if one insists in obtaining all of the three basic ingredients, namely the chiral spectrum of fermions and Higgs scalars which couple to the left handed doublets and the right handed singlets and trigger the spontaneous breaking of gauge symmetries. Although we obtain a spectrum of massless chiral fermions and Higgs representation very similar to the one of the standard electroweak-color theory our construction can not be considered as realistic. Unless the tachyonic mass is induced by a loop effect as we discuss in section 8, the masses of leptons and the quarks which are produced by symmetry breaking are of the same order as the masses of low lying Kaluza–Klein modes which we are discarding. Also if we take the radii of the internal spaces to be of the order of TeV the masses of the leptons and quarks will be of the same order. Our particles, if they are to be identified with the standard model particles, should possibly correspond to the third generation. Secondly, apart from the $SU(2) \times SU(3)$ factor of the gauge group which we are obtaining from the isometries of $S^2$ and $\mathbb{C}P^2$, the original $U(6)$ has a $(SU(2) \times U(1)) \times (SU(2) \times U(1)) \times U(1) \times U(1)'$ factor, where the $U(1)'$ is generated by the $6 \times 6$ unit matrix, which is also unbroken. Our putative quarks and leptons transform non trivially under this group. If we could make the couplings of this group week of course there

---

8 For a review of attempts at model building using Kaluza–Klein ideas, see [14].
would be no problem.

In principle within the framework of the scheme introduced in this paper, one could search for a more realistic group. However, until our idea on the one loop generated tachyon mass has been put on a firmer ground, the problem of scales would still persist.

The plan of this paper is as follows. In section 2 we give the background solution and discuss briefly its geometry. In section 3 we discuss the fermion zero modes on $S^2$ and $\mathbb{C}P^2$. In section 4 we give our rule for the tachyonic contribution of the $S^2$ factor to the masses of scalars. In section 5 we discuss the example of $G = SU(3)$ in $D = 6$ and $G = U(6)$ in $D = 10$. In section 6 we analyze the $\mathbb{C}P^2$ contribution to the masses of the Higgs doublet and give the condition for the absence of $SU(3)$ non singlet tachyons. In section 7 we analyze the $D = 4$ scalars originating from the gauge field fluctuations tangent to $\mathbb{C}P^2$ and show that they are non-tachyonic. In section 8 we study a $D = 10$ model with the gauge group $U(N)$ and show that, with a particular choice of magnetic charges, the would be tachyons become massless. In section 9 we summarize the paper.

It should be noted that throughout the paper we discard the scalar fields which originate from the gravity sector. These scalars would not mix with the ones which we retain.

It should also be remarked that our examples can suffer from chiral anomalies. In our 10 dimensional example they can be removed by standard methods. We comment on this in sections 5.2 and 9.

2 The Background Solution

Although the background we are going to use will solve the field equations of any generally covariant and gauge invariant action containing the metric and the Yang–Mills fields only, for the sake of simplicity we start from Einstein–Yang–Mills system in $D$-dimensions. The action is given by

$$S = \int d^Dx \sqrt{-G} \left( \frac{1}{\kappa^2} \mathcal{R} - \frac{1}{2g^2} \text{Tr} F^2 + \lambda + \bar{\psi} i \gamma \nabla \psi \right)$$
where $\psi$ is in some representation of the gauge group $G$. This action can be the low energy string field theory action with the $\lambda$-term induced by some mechanism. The presence of $\lambda$ in our discussion is required if we insist on having product spaces like $M_1 \times M_2 \times ...$ as a solution of the classical bosonic field equations, where one of the factors in the product is flat, e.g. the flat 4-dimensional Minkowski space. Our argument about chirality is not sensitive to the flatness of any of the factors in the product. The presence of tachyons, however, depends on the definition of a mass operator. This is different for example in AdS$^d$ and (Minkowski)$^d$.

The bosonic field equations are

$$\frac{1}{\kappa^2} R_{MN} = \frac{1}{g^2} \text{Tr} F_{MR} F_{NR} - \frac{1}{D-2} G_{MN} \left( \frac{1}{2g^2} \text{Tr} F^2 + \lambda \right)$$

$$\nabla_M F^{MN} = 0$$

In this paper we shall consider solutions of the form $M_4 \times K$, where $M_4$ is the flat 4-dimensional Minkowski space and $K$ is a compact manifold. In this paper $K$ will be mostly taken to be either $S^2$ or $S^2 \times \mathbb{C}P^2$. Furthermore we shall assume that the gauge field configuration $A$ will be non-vanishing only on $K$. One can of course think of many other choices for $K$.

The flatness of the Minkowski space implies

$$\frac{1}{2g^2} \text{Tr} F^2 + \lambda = 0$$

$$R_{\hat{m} \hat{n}} = \frac{\kappa^2}{g^2} \text{Tr} F_{\hat{m} \hat{r}} F_{\hat{n} \hat{r}}$$

where $\hat{m}, \hat{n}$ are indices in $K$. Our problem is now to find solutions of Yang–Mills equations in $K$ which also solve the Einstein equation (1).

For $K = \mathbb{C}P^1 \times \mathbb{C}P^2$ the metric is given by

$$ds^2 = a_1^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + \frac{4a_2^2}{1 + \bar{\zeta} \zeta} d\bar{\zeta}^a \left( \delta^{ab} - \frac{\zeta^a \bar{\zeta}^b}{1 + \bar{\zeta} \zeta} \right) d\zeta^b$$

where $a_1$ and $a_2$ are the radii of $\mathbb{C}P^1$ and $\mathbb{C}P^2$ respectively, and $\zeta = (\zeta^1, \zeta^2)$ is a pair of local complex coordinates in $\mathbb{C}P^2$. The $\mathbb{C}P^2$ metric is the standard
Fubini-Study metric. There are two facts about \( \mathbb{C}P^2 \) which are of importance for our present discussion. The first is the isometry group \( SU(3) \) of \( \mathbb{C}P^2 \). Together with the invariance group \( SU(2) \) of the metric of \( S^2 \), \( SU(3) \) will form part of the gauge group in \( M_4 \). \( SU(3) \) will be identified with the strong interaction color gauge group. The low energy 4-dimensional gauge group will be \( \tilde{G} \times SU(2) \times SU(3) \), where \( \tilde{G} \) is the subgroup of the \( D \)-dimensional gauge group \( G \) which leaves the background solution invariant. Note that even with \( G = U(1) \) we can obtain a 4-dimensional gauge theory with a gauge group \( U(1) \times SU(2) \times SU(3) \). Although such a solution can produce chiral fermions in a non-trivial representation of \( U(1) \times SU(2) \times SU(3) \), it is not possible, however, to obtain the correct Standard Model spectrum of leptons, quarks, and the Higgs fields. For this we need a bigger \( G \). We shall discuss this point in a greater detail in a later section.

The second important fact about \( \mathbb{C}P^2 \) is that in the absence of a background \( U(1) \) gauge field it is not possible to have globally well defined spinor field on it. This is principally due to the fact that the complex coordinates \( \zeta \) do not cover \( \mathbb{C}P^2 \) globally. We need at least three patches \((U, \zeta), (U', \zeta'), \) and \((U'', \zeta'')\), where in \( U \cap U' \) we have the transition rule \( \zeta_1' = \frac{1}{\zeta_1} \) and \( \zeta_2' = \frac{\zeta_2}{\zeta_1} \). It needs some work to show that the two chiral spinors of the tangent space \( O(4) \) of \( \mathbb{C}P^2 \) can not be patched consistently on the overlap. We shall give some more details of this later on.

To write the solution of the Yang-Mills equations on \( K = \mathbb{C}P^1 \times \mathbb{C}P^2 \) we first work out the spin connection on \( K \). It is given by

\[
\Omega = -(\cos \theta - 1)d\varphi \tau^3 + \left( \frac{1}{2} \omega^i \sigma^i \begin{array}{cc} 0 & 0 \\ 0 & -3/2 \omega \sigma_3 \end{array} \right)
\]

where the first factor refers to \( \mathbb{C}P^1 \) and the second, which is a \( 4 \times 4 \) matrix, refers to \( \mathbb{C}P^2 \). Here \( \tau^3 \) as well as \( \sigma^i \) and \( \sigma_3 \) are Pauli matrices. Also the expressions are valid on the upper hemisphere on \( \mathbb{C}P^1 \) and the local patch \((U, \zeta)\) on \( \mathbb{C}P^2 \). The expressions for \( \omega^i \) and \( \omega \) can be read from the Fubini-Study metric (2) on \( \mathbb{C}P^2 \). We shall not need the explicit expression for \( \omega^i \). The one for \( \omega \) is given by

\[
\omega(\zeta, \bar{\zeta}) = \frac{1}{2(1 + \zeta^\dagger \zeta)} (\zeta^\dagger d\zeta - d\zeta^\dagger \zeta)
\]
Note that \( d\omega \) is the self-dual Kähler form on \( \mathbb{C}P^2 \). It is thus an instanton type solution of the Yang-Mills equation in \( \mathbb{C}P^2 \).

It is important to note from (3) that the \( \mathbb{C}P^2 \) spin-connection takes its values in the subgroup \( SU(2) \times U(1) \) of the tangent space \( SO(4) \). Furthermore, under \( SO(4) \to SU(2) \times U(1) \) the two chiral spinors of \( O(4) \) decompose according to

\[
2_+ = 2_0 \\
2_- = 1_{-\frac{3}{2}} + 1_{\frac{3}{2}}
\]  

where the subscripts indicate the \( U(1) \)-charges. Using this fact one can understand why spinors are not globally well defined on \( \mathbb{C}P^2 \). The point is that in the overlap of two patches \((U, \zeta)\) and \((U', \zeta')\) we have

\[
\omega(\zeta') = \omega(\zeta) - id\varphi
\]

where \( \varphi \) is defined by \( \zeta_1 = \|\zeta_1\|e^{i\varphi} \). For \( 2_- \) to be globally well defined \( 1_{\pm3/2} \) should patch according to the rule \( \psi'(\zeta') = e^{\pm\frac{3}{2}i\varphi}\psi(\zeta) \). We thus obtain transition functions which are anti-periodic under \( \varphi \to \varphi + 2\pi \). Coupling a background gauge field proportional to \( \omega \) can change this. With a little more work one can show that a similar obstruction also prevents \( 2_+ = 2_0 \) from being well defined.

Now we are in a position to write our solution of the Yang-Mills equation on \( \mathbb{C}P^1 \times \mathbb{C}P^2 \). It is easy to show that the ansatz

\[
A = \frac{n}{2}(\cos\theta - 1)d\varphi + qi\omega
\]

where \( n = \text{diag}(n_1, n_2, \ldots) \) and \( q = \text{diag}(q_1, q_2, \ldots) \) are matrices in the Cartan-subalgebra of \( G \). The consistent patching of spinors requires that \( n_1, n_2, \ldots \) be integers and \( q_1, q_2, \ldots \) be one half of an odd integer. Note that the substitution of the above ansatz in the Einstein equations will require that the radii \( a_1 \) and \( a_2 \) of \( \mathbb{C}P^1 \) and \( \mathbb{C}P^2 \) are quantized.

As mentioned in the beginning of this section our ansatz for the background configuration solves the field equations derived from any generally
covariant and gauge invariant Lagrangian in $D = 10$, which contains the metric and the Yang-Mills potentials only. Such an effective Lagrangian will contain infinite number of parameters and therefore the relationship between the radii and other parameters will be more involved.

3 Chiral Fermions

It is a well known fact that in order to obtain chiral fermions in $D = 4$ we need topologically non-trivial background gauge fields on $\mathbb{C}P^1 \times \mathbb{C}P^2$. Our solution for the Yang–Mills equations consist of magnetic monopole on $S^2$ and the potential for the Kähler form on $\mathbb{C}P^2$. The Kähler form defines a topologically non trivial line bundle on $\mathbb{C}P^2$.

Consider the $D = 10$ fermion Lagrangian

$$\mathcal{L} = \bar{\psi} \nabla \psi$$

where

$$\nabla_{\dot{M}} \psi = (\partial_{\dot{M}} + \omega_{\dot{M}} - iA_{\dot{M}}) \psi, \quad \dot{M} = 0, 1, ..., 9$$

(10)

$\omega_{\dot{M}}$ and $A_{\dot{M}}$ are, respectively, the $SO(1,9)$ and the Lie algebra valued spin and gauge connections. We analyze the fermion problem in two steps. In the first step we write the manifold as $M_6 \times \mathbb{C}P^2$. Correspondingly we write the $D = 10$ Dirac matrices as

$$\hat{\Gamma}_a = \Gamma \times \gamma_a \quad a = 6, 7, 8, 9$$

$$\hat{\Gamma}_A = \Gamma_A \times 1 \quad A = 0, 1, ..., 5$$

where $\gamma_a$ and $\Gamma_A$ are respectively $4 \times 4$ and $8 \times 8$ Dirac matrices satisfying

$$\{\gamma_a, \gamma_b\} = 2\delta_{ab}$$

$$\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$$

and $\Gamma = \Gamma_0 \Gamma_1 ... \Gamma_5$. 

12
Substituting these $\Gamma$’s into $\mathcal{L}$ and recalling that the geometry has factorized form we obtain

$$\mathcal{L} = \bar{\psi} i \nabla_{\mathbb{C}P^2} \psi + \bar{\psi} i \nabla_{M_6} \psi$$

(11)

The chiral fermions on $M_6$ will originate from those modes for which

$$\nabla_{\mathbb{C}P^2} \psi = 0$$

(12)

Those $\psi$’s which are not annihilated by $\nabla_{\mathbb{C}P^2}$ will give rise to massive fermionic modes on $M_6$. The standard way to analyze (12) is to operate one more time with $\nabla_{\mathbb{C}P^2}$ on it. Using the background connections (3) and (8) we obtain

$$(\nabla^2 - \frac{3}{2}) \psi_+ = 0$$

(13)

$$(\nabla^2 + (q \sigma_3 - \frac{3}{2}) \psi_- = 0$$

(14)

where

$$\nabla \psi_+ = (d + i \omega^r \sigma^r + \omega q) \psi_+$$

(15)

$$\nabla \psi_- = \{d + \omega(q - \frac{3}{2} \sigma_3)\} \psi_-$$

(16)

and

$$\psi_\pm = \frac{1 \pm \gamma_5}{2} \psi, \quad \gamma_5 = \gamma_6 \gamma_7 \gamma_8 \gamma_9$$

The Kähler instanton $\omega$ is given by equation (4). Since $\nabla^2 \leq 0$ (13) will have no non-zero solutions. Thus fermions of $\psi_+$ type will all be non-chiral and massive. Equation (14), on the other hand, can have solutions. Their existence depends on the eigenvalues of $q$. Clearly for $q = 3/2$ we have only one solution with $\sigma_3 = +1$. For $q = +5/2$ we obtain 3 solutions with $\sigma_3 = +1$. They form a triplet of the isometry group $SU(3)$ of $\mathbb{C}P^2$. For $q = -5/2$ and $\sigma_3 = -1$ we obtain a $3^*$ of $SU(3)$. These are the only type of solutions we need to consider.
Next we study the $M_6$ Dirac Lagrangian

$$\mathcal{L} = \bar{\psi} i \nabla_{M_6} \psi$$ \hspace{1cm} (17)$$

where $\psi$ is assumed to be a solution of (14). We shall assume that the $D = 10$ spinor is chiral and has positive chirality. Then the spinor of $\psi_-$ type will have negative $D = 6$ chirality. We choose the $D = 6$ $\Gamma$ matrices to be

$$\Gamma_{\alpha} = \Gamma_{\alpha} \times \tau_1 \hspace{1cm} \alpha = 0, 1, 2, 3$$
$$\Gamma_4 = \Gamma_5 \times \tau_1 \hspace{1cm} \gamma_5 = i \gamma_0 \gamma_1 \gamma_2 \gamma_3$$
$$\Gamma_5 = 1 \times \tau_2$$ \hspace{1cm} (18)$$

and $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

Inserting the $\Gamma_A$'s in (17) we obtain

$$\mathcal{L} = \bar{\psi} i \nabla_{M_6} \psi + \frac{i}{\sqrt{2}} \{ \bar{\psi}(\gamma_5 + 1)D_- \psi + \bar{\psi}(\gamma_5 - 1)D_+ \psi \}$$ \hspace{1cm} (19)$$

where

$$D_\pm \psi = e_\pm^m \left( \partial_m + \frac{i}{2} \omega_m (n - \gamma_5) \right) \psi$$ \hspace{1cm} (20)$$

$e_\pm^m$ are the $U(1)$ components of an orthonormal frame on $S^2$ and $\omega_m$ is the corresponding spin connection ($\omega_\theta = 0$, $\omega_\phi = -\cos \theta + 1$ in the upper hemisphere and $\omega_\phi = -\cos \theta - 1$ in the lower hemisphere). Decomposing $\psi = \psi_L + \psi_R$, where $\psi_L = \frac{1 - \gamma_5}{2} \psi$, we obtain the analogue of (13, 14) for the Dirac operator on $\mathbb{C}P^1$

$$\begin{cases} \nabla^2 - \frac{1}{2} (1 - n) \psi_R = 0 \\ \nabla^2 - \frac{1}{2} (1 + n) \psi_L = 0 \end{cases}$$

$n = 1$ produces one $\psi_R$ while $n = -2$ gives rise to two $\psi_L$ which form a doublet of the Kaluza–Klein $SU(2)$. 

14


4 General Rule for Higgs Type Tachyons

To obtain the spectrum of the effective theory in 4-dimensions we need to expand the functions about our background solution in harmonics on \( \mathbb{C}P^1 \times \mathbb{C}P^2 \). These include fluctuations of the gravitational, Yang–Mills, as well as fermionic fields. The techniques of doing such analysis have been developed long ago. In this paper we shall ignore the gravitational fluctuations and consider only the Yang–Mills and fermionic fields. The full set of linearized gravity Yang–Mills equations can be found in [2]. In the same paper it was shown that there are tachyonic modes in the components of the gauge field fluctuations tangent to \( S^2 \). Here, we would like to show that the rule to identify the tachyonic modes given in [2] for \( G = SU(3) \) is in fact quite general and applies to any gauge group \( G \). It should be emphasized that neglecting the gravitational fluctuations is justified as they will not mix with the gauge field fluctuations of interest for us.

In general, we should write \( A = \bar{A} + V \) where \( \bar{A} \) is the background solution and \( V \) depends on the coordinates of \( M_4, S^2 \) and \( \mathbb{C}P^2 \). Our first interest is in the fields which are tangent to \( S^2 \). It is these fields, which if develop a tachyonic vacuum expectation value, can break \( SU(2) \), provided such modes are singlets of \( SU(3) \) isometry of \( \mathbb{C}P^2 \).

We suppress the \( \mathbb{C}P^2 \) dependence of these fields and denote by \( V_1 \) and \( V_2 \) their components with respect to an orthonormal frame on \( S^2 \). It is convenient to use the “helicity” basis on \( S^2 \) defined by

\[
V_\pm = \frac{1}{\sqrt{2}}(V_1 \mp i V_2)
\]

\( V_\pm \) are matrices in the Lie algebra of \( G \). What governs their mode expansion on \( S^2 \) is their isohelicities. This is basically the effective charge of \( V_\pm \) under the combination of \( U(1) \) transformations which leave our background configuration invariant. These charges can be evaluated in the same way which was done in [2]. For the sake of simplicity, let us assumes \( G = U(N) \) and assume that charge matrices \( n \) and \( q \) introduced in (8) are diagonal \( N \times N \) matrices. Then \( V_\pm \) are \( N \times N \) matrices with elements \( V_{\pm i}^j \), \( i, j = 1, \ldots N \).
Their isohelicities, \( \lambda(V_{\pm}^i) \), are given by
\[
\lambda(V_{\pm}^i) = \pm 1 + \frac{1}{2}(n_i - n_j)
\]
Note that there is a hermiticity relation
\[
V_{+}^i = (V_{-}^i)^* 
\]
The harmonic expansion of \( V_{+}^i \) on \( S^2 \) will produce an infinite number of Kaluza–Klein modes. These expansions are defined by
\[
V_{\pm}(x, \theta, \varphi) = \sum_{l \geq |\lambda|} \sqrt{\frac{2l + 1}{4\pi}} \sum_{m \leq |l|} V_{\pm}^{lm}(x) D_{\lambda, m}^l(\theta, \varphi) \tag{21}
\]
\( D_{\lambda, m}^l(\theta, \varphi) \) are \( 2l + 1 \)-dimensional unitary matrices.

The tachyonic modes are generally contained in the leading terms with \( l = |\lambda| \). The effective 4-dimensional mass\(^2\) of \( V_{\pm}^{lm}(x) \) obtains contributions from the appropriate Laplacian acting on \( S^2 \) and \( CP^2 \). \( V_{\pm} \) are charged scalar fields on \( CP^2 \). We shall analyze their dependence on the \( CP^2 \) coordinates in the next section. Here we shall consider the \( S^2 \) contribution to their masses. The condition for this contribution to be tachyonic is expressed in the following simple rule
\[
M^2(V_{+}^i) < 0 \quad \text{if} \quad \lambda(V_{+}^i) \leq 0
\]
Likewise
\[
M^2(V_{-}^i) < 0 \quad \text{if} \quad \lambda(V_{-}^i) \geq 0
\]
To prove these claims let us make more detailed analysis.

Since we are assuming \( V_{\pm} \) are independent of the \( CP^2 \) coordinates, their mass term comes from the expansion of \( \text{Tr} F_{mn} F^{mn} \), where \( m, n \) indicate indices tangent to \( S^2 \). The cubic and the quadratic parts in \( \text{Tr} F_{mn} F^{mn} \) will produce the interaction terms in the Higgs potential. We have
\[
\text{Tr} F_{mn} F^{mn} = \text{Tr} F_{mn} F^{mn} + \text{Tr}(D_+ V_+ - D_- V_-)^2 \\
- 4i \text{Tr} F_{mn} [V_-, V_+] - 2i \text{Tr}(D_+ V_+ - D_- V_-)[V_-, V_+] \\
+ \text{Tr}[V_-, V_+]^2
\]
16
where the covariant derivatives are defined by

\[ D_m V_n = \nabla_m V_n - i[\tilde{A}_m, V_n] \]

\( \nabla_m \) denotes the ordinary Riemannian covariant derivative on \( S^2 \). Now, since for \( \lambda_+ \leq 0 \) \((\lambda_+ \geq 0)\) \( D_- D^{[\lambda_+]} = 0 = D_+ D^{[-\lambda_+]}_m \) we see that such modes will be annihilated by \( D_\pm \) and thus the \( S^2 \) contribution to their \( D = 4 \) action is given by

\[ S = -\frac{1}{2g^2} \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta \text{Tr} \left\{ 4D_\mu V_+ D^\mu V_- - 4i\tilde{F}_{+-}[V_-, V_+] + [V_-, V_+]^2 \right\} \]

The mass terms hence come from \(-4i Tr \tilde{F}_{+-}[V_-, V_+]\) term only.

To proceed it is convenient to choose the Cartan-Weyl basis for the Lie algebra of \( G \). Let \( Q_j \) denote the basis of the Cartan subalgebra, \( E_\alpha \) and \( E_{-\alpha} = E_\alpha^\dagger \) the generators outside the Cartan subalgebra. The only part of the algebra needed for the evaluation of the mass terms is

\[ [Q_j, E_\alpha] = \alpha_j E_\alpha \]

In this basis we can write

\[ V_\pm = V_\pm^\alpha E_\alpha + (V_\pm^\alpha)^* E_{-\alpha} + V_\pm^j Q_j \]

It is easy to see that

\[ \lambda(V_\pm^\alpha) = \pm 1 + p.\alpha \]  \hspace{1cm} (22)

where \( p.\alpha = p^j \alpha_j \) and \( p^j \) are defined by

\[ \frac{1}{2}n = p^j Q_j \]

To simplify the discussion consider the case when only one \( \lambda(V_\pm^\alpha) \leq 0 \). Set the remaining modes to zero. Of course this is not a loss of generality. In this case \( V_+ = V_+^\alpha E_\alpha \) and \( V_- = (V_+^\alpha)^* E_{-\alpha} \). The mass term then becomes

\[ \text{Tr} \left(-4i\tilde{F}_{+-}[V_-, V_+]\right) = -4i \text{Tr} V_+ [\tilde{F}_{+-}, V_-] = \frac{4}{a_1^2} p.\alpha |V_+^\alpha|^2 \text{Tr} E_\alpha E_{-\alpha} \]
where we inserted $\bar{F}_{\mu\nu} = -\frac{i}{a^2} \rho^\dagger Q_j$. The kinetic part of the action for $V_+^\alpha$ thus becomes

$$S_2 = -\frac{2Tr(E_\alpha E_{-\alpha})}{g^2} \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin \theta \left\{ \partial_\mu V_+^\alpha \partial^\mu V_+^\alpha \ast + \frac{p.\alpha}{a_1^2} |V_+^\alpha|^2 \right\}$$

Substituting $p.\alpha = \lambda(V_+^\alpha) - 1$ we obtain the mass of $V_+^\alpha$ in terms of its isohelicity as (recall that our signature is $(-, +, +, ...)$)

$$m^2 = \frac{\lambda - 1}{a_1^2}$$

(23)

which is negative for $\lambda \leq 0$. Similar reasoning can be applied if for some $V_-^\alpha$ the corresponding isohelicity $\lambda(V_-^\alpha)$ is non-negative.

This rule gives us an easy way of identifying possible tachyonic modes which can act as Higgs scalars in the $D = 4$ effective theory.

### 5 Examples

In this section we shall ignore the $\mathbb{C}P^2$ part and give some examples of a $D = 6$ gravity Yang–Mills theories which produce standard model type Higgs sectors upon compactification to $D = 4$. Leptons and quarks will be included in the next sections. We basically need to choose the gauge group $G$ and assign magnetic charges $n$.

The notation is always

$$\bar{A} = \frac{n}{2}(\cos \theta \mp 1)d\varphi$$

(24)

where $n = \text{diag}(n_1, n_2, ...)$ is in the Lie algebra of $G$, $- (+)$ give the expression for $\bar{A}$ in the upper (lower) hemispheres.

#### 5.1 Tachyons
5.1.1 \( G = SU(3) \)

\[ n = \text{diag}(n_1, n_2, -n_1 - n_2), \quad n_1, n_2 \in \mathbb{Z} \]  
(25)

The isohelicities can be assembled in a \( 3 \times 3 \) matrix

\[ \lambda(V) = \begin{pmatrix} \pm 1 & \pm 1 + \frac{1}{2}(n_1 - n_2) & \frac{1}{2}(2n_1 + n_2) \\ \pm 1 - \frac{1}{2}(n_1 - n_2) & \pm 1 & \pm 1 + \frac{1}{2}(n_1 + 2n_2) \\ \pm 1 - \frac{1}{2}(2n_1 + n_2) & \pm 1 - \frac{1}{2}(n_1 + 2n_2) & \pm 1 \end{pmatrix} \]  
(26)

Using the results of section 3 we see that in order to obtain left handed doublets and right handed singlets we had to take \((n_1, n_2) = (1, 1)\). With these values of \(n_1\) and \(n_2\), \(V_{-1}^3\) and \(V_{-2}^3\) will contain tachyonic modes in the leading term of their expansion on \(S^2\).

In this example the \(SU(2) \times U(1)\) subgroup of \(SU(3)\) is unbroken and the tachyonic Higgs \(V_{-1}^3\) and \(V_{-2}^3\) form a doublet of \(SU(2)\) with \(U(1)\) charge of \(3/2\). We denote this doublet by \(\phi\). Its isohelicity is \(+1/2\). Therefore it will also be a doublet of the Kaluza–Klein isometry of \(S^2\). One can integrate the \((\theta, \varphi)\) dependence of \(\phi\) on \(S^2\) and work out its \(D = 4\) effective action. The result is

\[ \mathcal{L} = -\frac{1}{2g^2} \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin \theta \, \text{Tr} F_{MN} F^{MN} \]
\[ = -\frac{1}{4g_1^2} F_{\mu \nu}^8 \, F_{\mu \nu}^8 - \frac{1}{4g_2^2} F_{\mu \nu}^r \, F_{\mu \nu}^r - \frac{1}{4e^2} W_{\mu \nu}^r \, W_{\mu \nu}^r \]
\[ - \text{Tr} \left\{ \nabla_\mu \phi \nabla^\mu \phi - \frac{3}{2a_1^2} \phi \phi^\dagger + 2g_2^2 (\phi \phi^\dagger)^2 \right\} \]  
(27)

where we have regarded \(\phi\) as a \(2 \times 2\) complex matrix, and

\[ \nabla_\mu \phi = \partial_\mu \phi - \frac{3}{2} i V_8^\mu \phi - i V_\mu^r \sigma^r \phi - i W_\mu^r \phi^r \]

where \(V_8^\mu, V_\mu^r,\) and \(W_\mu^r\) are respectively the \(U(1), SU(2)_L,\) and the Kaluza–Klein \(SU(2)_R\) gauge fields. \(g_1, g_2,\) and \(e\) are their respective couplings. Some calculation show that

\[ g_2 = \frac{1}{2\sqrt{\pi} a_1} \frac{g}{\sqrt{3}} = \sqrt{3} g_1 \]  
(28)
The Kaluza–Klein gauge coupling $e$ can also be expressed in terms of the fundamental scales $g$ and $a_1$.

In the next section we shall work out the Yukawa couplings for this model as well.

5.1.2 $G = U(6)$

With $n = \text{diag}(n_1, ..., n_5, n_6)$ we can again work out the table of isohelicities for $V_\pm$. We shall see in section 5.2.2 that in order to obtain one family of leptons and quarks we need to take $n = \text{diag}(-2, 1, 1, -2, 1, 1)$. Note that since the group is $U(6)$ rather than $SU(6)$, $n$ is not traceless. $\lambda(V_\pm)$ is given by

$$\lambda(V_\pm) = \begin{pmatrix}
+1 & -\frac{1}{2} & -\frac{1}{2} & +1 & -\frac{1}{2} & -\frac{1}{2} \\
+\frac{5}{2} & +1 & +1 & +\frac{5}{2} & +1 & +1 \\
+\frac{5}{2} & +1 & +1 & +\frac{5}{2} & +1 & +1 \\
+1 & -\frac{1}{2} & -\frac{1}{2} & +1 & -\frac{1}{2} & -\frac{1}{2} \\
+\frac{5}{2} & +1 & +1 & +\frac{5}{2} & +1 & +1 \\
+\frac{5}{2} & +1 & +1 & +\frac{5}{2} & +1 & +1
\end{pmatrix}$$

(29)

Since $V_- = V_+^\dagger$, therefore $\lambda(V_-^i) = -\lambda(V_+^i)$.

The tachyonic modes are contained in $V_{+1}^i$, and $V_{+4}^i$ where $i = 2, 3, 5, 6$. They will all be doublets of the Kaluza–Klein $SU(2)$. They also transform under some representation of the unbroken part of $U(6)$, which is $SU(2) \times SU(2) \times U(1)^3 \times U(1)'$, which is generated by $\text{diag}(0, \frac{\sigma_i}{2}, 0, 0, 0)$, $i = 1, 2, 3; \text{diag}(0, 0, 0, \frac{\sigma_i}{2}); \text{diag}(-2, 1, 1, 0, 0); \text{diag}(0, 0, 0, -2, 1, 1)$; $\text{diag}(1, 1, 1, -1, -1, -1)$; and the $6 \times 6$ unit matrix $1_6$ which generates $U(1)'$. The tachyonic Higgs will be neutral under this $U(1)'$, therefore their tree level vacuum expectation value will not break it. Under $U(6) \rightarrow SU(2) \times SU(2) \times U(1)^3$ we have

$$\mathcal{H} = (1, 1)_{(-2, 0, 1)} + (2, 1)_{(1, 0, 1)} + (1, 1)_{(0, -2, -1)} + (1, 2)_{(0, 1, -1)}$$

(30)
The quantum numbers of the relevant Higgs tachyons will be

\[ V_{+1}^i \sim (2,1)^{(-3,0,0)} \quad i = 2, 3 \]  
\[ V_{+4}^t \sim (1,2)^{(0,-3,0)} \quad t = 5, 6 \]  

(31)  

(32)

As we said earlier, the tachyonic modes in all these fields will be in the doublet representation of the Kaluza–Klein SU(2). The vacuum expectation value of the fields \( V_{+1}^i \sim (2,1)^{(-3,0,0)} \) and \( V_{+4}^t \sim (1,2)^{(0,-3,0)} \) will give masses to the quarks and leptons respectively. In section 3 we shall show that the leading term in their expansion on \( \mathbb{C}P^2 \) is a singlet of SU(3) and therefore their masses receive no contribution from the dependence on the \( \mathbb{C}P^2 \) coordinates. Thus they remain tachyonic. The other tachyonic fields, namely \( V_{+1}^i, i = 5, 6; V_{+4}^t, t = 2, 3 \), would induce Yukawa couplings between quarks and leptons. We shall show that in fact the leading term in their harmonic expansion on \( \mathbb{C}P^2 \) is a triplet of SU(3). Thus the vacuum expectation value of these fields can break the color SU(3). We will determine the conditions to avoid this.

5.2 Fermions

We consider the two examples of the previous section.

5.2.1 \( G = SU(3) \)

Here we assume that \( D = 6 \) and there is no \( \mathbb{C}P^2 \) factor. Let us take \( \psi \) in 3 of SU(3) and \( n = \text{diag}(1,1,-2) \). According to our rules this will produce two right handed singlets of the Kaluza–Klein SU(2) which we denote by \( SU(2)_K \) and a left handed doublet. The singlets will form a doublet of \( SU(2)_G \subset SU(3) \) and the doublet of \( SU(2)_K \) will be a singlet of \( SU(2)_G \). Thus under \( SU(2)_K \times SU(2)_G \times U(1) \) where \( U(1) \subset SU(3) \) we have \( (1,2^R_{1/2}) + (2^L,1)_1 \). The \( D = 4 \) Yukawa and gauge couplings can be easily worked out. The result is

\[
\mathcal{L}_F = \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin \theta \ \bar{\psi} i \nabla \psi \\
= \bar{\lambda}_L \ i\gamma^\mu \left( \partial_\mu - ig_1 V_\mu^8 - ieW_\mu^i \frac{\tau^i}{2} \right) \lambda_L
\]
\[ +\bar{\lambda}_R i\gamma^\mu \left( \partial_\mu - i\frac{g_1}{2}V^8_\mu - ig_2V^i_\mu\sigma^i \right)\lambda_R \]

\[-2g_1 \left\{ \bar{\lambda}_L\phi(i\sigma_2)\lambda_R - \bar{\lambda}_R(i\sigma_2)\phi^\dagger\lambda_L \right\} \]

where \( \lambda_L = (2L, 1) \) and \( \lambda_R = (1, 2R) \).

This expression together with the bosonic part given in equation (17) give the total effective \( D = 4 \) action for the \( SU(3) \) example. Although this example leads to interesting chiral and Higgs spectrum in \( D = 4 \) cannot not be considered satisfactory. It has both perturbative and global chiral anomalies in \( D = 6 \). The perturbative anomalies can be eliminated with the standard Green Schwarz mechanism. To apply this mechanism we need first to introduce an antisymmetric rank two potential together with three right handed \( D = 6 \) \( SU(3) \) singlets to kill the pure gravitational anomaly which is given by \( R^4 \) term in the anomaly 8– form. The remaining terms in the anomaly 8-form factorize appropriately in order to be canceled by a judicious transformation of the antisymmetric potential. This mechanism does not cancel the global anomalies whose presence is due to the fact that \( \pi_6(SU(3)) = Z_6 \) is non zero. To kill these ones we need to introduce further \( SU(3) \) multiplets or to change the gauge group altogether and chose to a gauge group like \( E_6 \) which has a trivial \( \pi_6(E_6) \).

5.2.2 \( G = U(6) \)

Now assume \( D = 10 \) and choose \( \psi \) to be in 6 of \( U(6) \) and \( q = \text{diag}(5/2, 5/2, 5/2, 3/2, 3/2, 3/2) \). As before \( n \) will be taken to be \( n = \text{diag}(-2, 1, 1, -2, 1, 1) \).

According to the results of the previous section with respect to the isometry group \( SU(2) \times SU(3) \) we have the following chiral fermions

\[(2L, 3) + (1R, 3) + (1R, 3) + (2L, 1) + (1R, 1) + (1R, 1)\]

Clearly the first three triplets are candidates for \( \left( \begin{array}{c} u \\ d \end{array} \right)_L \), \( u_R \) and \( d_R \). The last two pieces can be identified with the leptons \( \left( \begin{array}{c} \nu_e \\ e \end{array} \right)_L \) and \( e_R \).

---

\[9\) We have an extra right handed singlet in the lepton sector. This can be removed by choosing the last entry in \( q \) to be for instance \(-1/2\) or the last entry in \( n \) to be 0. In this way the unbroken subgroup of \( U(6) \) will be \( SU(2) \times U(1) \times U(1)' \).
These multiplets also transform in the following representation of the unbroken $SU(2) \times SU(2) \times U(1)^3 \subset G$

\[
(2_L, 3) \sim (1, 1)_{(2,0,1)} \\
(1_R, 3) + (1_R, 3) \sim (2, 1)_{(1,0,1)} \\
(2_L, 1) \sim (1, 1)_{(0,-2,-1)} \quad \text{and} \quad (1_R, 1) \sim (1, 2)_{(0,1,-1)}
\]

The Yukawa coupling between the quarks will be through the Higgs field $V_{+1}^i$ given in (31), while the electron will get its mass through coupling to $V_{+4}^t$.

Thus our construction leads to a multi Higgs theory in which the quarks and leptons obtain their masses from their Yukawa couplings to different Higgs scalars. Note also that there is no common $U(1)$ under which both Higgs multiplets are charged. The hypercharge coupling in our model is different from the standard electroweak theory.

6 Higgs Like Tachyons on $\mathbb{C}P^2 \times \mathbb{C}P^1$

If the total space-time dimension is $D = 6$ the masses of the Higgs like tachyons are given by (23). In the case of a $D = 10$ theory we need to take into account the contribution of $\mathbb{C}P^2$ part as well. The fields $V_{\pm}$ are like scalar fields on $\mathbb{C}P^2$ which are charged with respect to the $\mathbb{C}P^2$ part of the background gauge field [8], viz, $i\omega q$. The $\mathbb{C}P^2$ contribution to the masses of $V_{\pm}$ come from the commutator term in the $\mathbb{C}P^2$ covariant derivative of $V_{\pm}$, i.e.

\[
DV_{\pm} = dV_{\pm} - i[q, V_{\pm}]
\]

\[
= dV_{\pm} + \omega [q, V_{\pm}]
\]

To be specific let us consider the example of the $U(6)$ model for which $q = \text{diag}(5/2, 5/2, 5/2; 3/2, 3/2, 3/2)$. Write

\[
V = \left( \begin{array}{ccc} v & u \\ \tilde{u} & \tilde{v} \end{array} \right)
\]

where $v$, $\tilde{v}$, $u$, and $\tilde{u}$ each is a $3 \times 3$ matrix. Then

\[
[q, V] = \left( \begin{array}{ccc} 0 & u \\ -\tilde{u} & 0 \end{array} \right)
\]
This indicates that out of the Higgs fields given in equations (31–32) the ones which give masses to quarks and leptons, namely, $V_{+1}^{t}$ and $V_{+4}^{t}$ (which lie respectively inside $v$ and $\tilde{v}$ in the above notation), do not couple to the background $\omega$ field on $\mathbb{C}P^2$. The leading term in their harmonic expansion on $\mathbb{C}P^2$ will be a constant (independent of the coordinates of $\mathbb{C}P^2$). Their masses will be tachyonic and will be given by (23) for $\lambda = -\frac{1}{2}$, i.e. $M^2 = -\frac{3}{2} \frac{1}{a_1^2}$.

The remaining fields $V_{+1}^{t}$ and $V_{-4}^{i}$ on the other hand are located inside $u$ and they couple to the background $\omega$-field. Their masses will receive contribution from $\mathbb{C}P^2$ and in principle can become non-tachyonic. To verify this we need to evaluate the eigenvalues of $\nabla_{\mathbb{C}P^2}^2$ on these fields. Their covariant derivatives are

$$DV_{+1}^{t} = dV_{+1}^{t} + \omega V_{+1}^{t}$$
$$DV_{-4}^{i} = dV_{-4}^{i} + \omega V_{-4}^{i}$$

Since they couple with the same strength to the $\omega$-field they will receive the same contribution from $\nabla_{\mathbb{C}P^2}^2$. It turns out that the leading term in the expansion of any of these fields on $\mathbb{C}P^2$ is a triplet of $SU(3)$ and $D^2$ acting on it is $-\frac{1}{a_2^2}$. Thus the total mass$^2$ of such modes will be

$$- \frac{3}{2} \frac{1}{a_2^2} + \frac{1}{a_2^4} = \frac{1}{a_2^4} \left( -\frac{3}{2} + \frac{a_1^2}{a_2^2} \right)$$

If $a_1$ and $a_2$ were independent we could choose $\left( \frac{a_1}{a_2} \right)^2 \geq \frac{3}{2}$ and make these fields non-tachyonic. If we insist on the validity of the background Einstein equations then the ratio of $\frac{a_1}{a_2}$ will be fixed. Equation (1) leads to $\left( \frac{a_1}{a_2} \right)^2 = \frac{12}{17}$.

With this value unfortunately the above mass$^2$ is still negative. The vacuum expectation value of these fields will break the color $SU(3)$.

$^{10}$To obtain $\left( \frac{a_1}{a_2} \right)^2 = \frac{12}{17}$ we need to use the following results, which can be obtained by straightforward calculation,

$\mathcal{R}(S^2) = \frac{1}{a_1^2} 1_{2 \times 2}$, $\mathcal{R}(\mathbb{C}P^2) = \frac{3}{2} \frac{1}{a_1^2} 1_{4 \times 4}$, $\text{Tr}F_{\mathbb{C}P^2}^2 = \frac{6}{a_1^4}$, and $\text{Tr}F_{\mathbb{C}P^2}^2 = \frac{51}{2} \frac{1}{a_2^2}$. 

24
One way to change the ratio $\frac{a_1}{a_2}$ is to couple a $U(1)$ gauge field to gravity
in $D = 10$. This $U(1)$ will not couple to anything else. In particular the
fermions will be neutral under it, so the spectrum of the chiral fermions will
be unaltered. Its sole effect will be to add an extra term to the right hand
side of Einstein equations. In particular (33) will be replaced by

$$\mathcal{R}_{\hat{m}\hat{n}} = \frac{\kappa^2}{g^2} \text{Tr} F_{\hat{m}\hat{n}} F_{\hat{r}\hat{r}} + \frac{\kappa^2}{g'^2} \text{Tr} F'_{\hat{m}\hat{n}} F'_{\hat{r}\hat{r}}$$

where $F'$ and $g'$ refer to the extra $U(1)$ system. Now if we set

$$A' = \frac{n'}{2} (\cos \theta - 1) d\varphi + q' i \omega$$

where $n'$ and $q'$ are real numbers, the ratio of $a_1/a_2$ will turns out to be

$$\frac{a_1^2}{a_2^2} = \frac{36 + 3n'^2 g^2}{51 + 2q'^2 g'^2}$$

There is a big range of parameters for which $a_1/a_2 \geq 3/2$.

## 7 Other Scalars

The components of the gauge field fluctuations tangent to $\mathbb{CP}^2$ will also give
rise to infinite tower of Kaluza–Klein modes which will be scalars fields in
$D = 4$. These modes will belong to unitary representations of $SU(2) \times SU(3)$. If
there is a tachyon Higgs among them they will break $SU(3)$. We need to
verify that this does not happen. To this end we denote these fields by $V_a$, where $a$ is tangent to $\mathbb{CP}^2$, and write those terms in the bilinear part of
$\text{Tr} F_{MN} F^{MN}$ which contains $V_a$. In this section we are considering only the
$U(6)$ model. The $V_a$ are $5 \times 5$ Hermitian matrices. After some manipulation
and the imposition if the $D = 10$ background gauge condition $D_M V^M = 0$, the bilinear terms of interest to us can be written as

$$S_2 = -\frac{1}{2g^2} \int d^{10} x \text{Tr} \{2V_a (-\partial^2 - D_m^2 - D_m^2 + \frac{3}{2} \frac{1}{a_2^2}) V^a + 4i V^a [F_{ab}, V^b] \}$$

(34)
where $D_m$ and $D_{\tilde{m}}$ are respectively the covariant derivatives on $S^2$ and $\mathbb{C}P^2$
and
\begin{align*}
D_m V_a &= \partial_m V_a - \frac{i}{2} \omega_m [n, V_a] \quad (35) \\
D_{\tilde{m}} V_a &= \nabla_{\tilde{m}} V_a - \frac{i}{2} \omega_{\tilde{m}} [q, V_a] \quad (36)
\end{align*}
\n$\nabla_{\tilde{m}}$ is the Riemann covariant derivative on $\mathbb{C}P^2$. The contribution of $D_m^2$ on each $SU(2)$ mode of $V_a$ will simply be $\frac{1}{a_l^2} [l(l+1) - \lambda^2]$, $l \geq |\lambda|$ where $\lambda$ represents the isohelicities of various components of $V_a$, $\lambda(V_{ai}^j) = \lambda(V_{ai}^j) - 1$, where $\lambda(V_{ai}^j)$ are given in equation (29).

To work out the contributions of $D_{\tilde{m}}^2$ and the commutator term $[\bar{F}_{ab}, V^b]$, we represent $V_a$ as in (33), i.e.

\[ V_a = \begin{pmatrix} v_a & u_a \\ \bar{u}_a & \bar{v}_a \end{pmatrix} \quad (37) \]

where $v_a$, $\bar{v}_a$, $u_a$, $\bar{u}_a$ each is a $3 \times 3$ matrix. Then

\[ [q, V_a] = \begin{pmatrix} 0 & u_a \\ -\bar{u}_a & 0 \end{pmatrix} \quad (38) \]

This indicates that the commutator terms in (34) and (36) do not contribute to $D_{\tilde{m}} v_a$ and $D_{\tilde{m}} \bar{v}_a$. Thus $D_{\tilde{m}}$ acting on these fields is just the Riemannian Laplacian acting on vectors and its contribution to the masses of these fields will be non-tachyonic.

The only fields we need to be concerned about are those in $u_a$. To analyze the contribution of these terms we introduce 2 complex $SU(2)$ vectors $u_\alpha$ and $u'_\alpha$ defined by

\[
\begin{align*}
&\begin{cases}
  u_1 = \frac{1}{\sqrt{2}} (u_6 + iu_7) \\
u_2 = \frac{1}{\sqrt{2}} (u_8 + iu_9)
\end{cases} &
\begin{cases}
  u'_1 = \frac{1}{\sqrt{2}} (u_6 - iu_7) \\
u'_2 = \frac{1}{\sqrt{2}} (u_8 - iu_9)
\end{cases}
\end{align*}
\quad (39)
\]
where 6, 7, 8, and 9 are directions tangent to $\mathbb{C}P^2$. In terms of these new fields the $u_a$ part of (36) can be rewritten as

\[ D_{\hat{m}}u_\alpha = (\partial_{\hat{m}} + i\omega_{\hat{m}}^i \frac{\sigma^i}{2} - i\frac{5}{2}\omega_{\hat{m}})u_\alpha \] (40)

\[ D_{\hat{m}}u'_\alpha = (\partial_{\hat{m}} + i\omega_{\hat{m}}^i \frac{\sigma^i}{2} - i\frac{5}{2}\omega_{\hat{m}})u'_\alpha \] (41)

The contribution of $D_{\hat{m}}^2$ on $u_\alpha$ and $u'_\alpha$ will again be positive.

Finally we need to evaluate the contribution of $2i\text{Tr}V^a[\bar{F}_{ab},V^b]$ to the masses of $u_\alpha$ and $u'_\alpha$. After some calculation this turns out to be

\[ 2i\text{Tr}V^a[\bar{F}_{ab},V^b] = 2 \frac{a^2}{2} \text{Tr}(u_\alpha^\dagger u_\alpha - u'^\dagger_\alpha u'_\alpha) \] (42)

It is seen that the contribution of this term to the $u_\alpha$ mass is non-tachyonic. However, it makes a negative contribution to the mass$^2$ of $u'_\alpha$ field. Upon substitution of the above in (36) we find out that the negative contribution in (12) is off-set by the $\frac{3}{2}a^2$ term in equation (34), with the result that $u'_\alpha$ is also non-tachyonic.

We thus conclude that all the tachyonic Higgs are singlets of $SU(3)$ and doublets of $SU(2)$.

8 Massless Scalars and Loop induced Hierarchy

So far we have been discussing tachyonic mass of the scalar particles at the tree level of the effective 4 dimensional theory. The natural scale of this mass and therefore also of the symmetry breaking is the compactification scale. This is few order of magnitude above the electroweak symmetry breaking scale of a 200 hundred GeV. It will be very desirable if we could find a mechanism to lower the scale of the tachyonic mass. An obvious idea is if the tree level mass of the scalars is zero and they obtain their tachyonic value as
a consequence of loop effects. Our theory is of course a non renormalizable
one, at least in conventional sense. However, the Higgs mass is controlled
by $1/a$ due to higher dimensional gauge invariance. Our main point is that
the sign of the one loop induced effective mass will depend on the imbalance
between the contribution of fermions and bosons. By a judicious choice of
the fermionic degrees of freedom this sign can be made tachyonic. Any way
whatever the justification the first step in implementing this idea is to find
tree level massless scalars in the spectrum of the effective four dimensional
theory. Unlike the massless chiral fermions whose presence is dictated by
the topology of the gauge field in compact subspace, to verify the existence
of the massless scalars in the spectrum requires more detailed analysis of
the mass spectrum and should be carried out separately for each case. In
this section we give an example of a model in $D = 10$ in which a monopole
background on the $S^2 \times S^2 \times S^2$ internal space leads to massless scalars
transforming non trivially under the $SU(2) \times SU(2) \times SU(2)$ isometry group
of the internal space. This example which was is only for illustrative purpose
and is not going to be used for a realistic model building.

We start from a $U(N)$ gauge theory in 10 dimensions and consider a
solution of equations (1) in which the internal space is $S^2 \times S^2 \times S^2$. In the
notation of previous section we denote the magnetic charge matrices on the
three $S^2$'s by $n, n'$ and $n''$. Denoting all the quantities on $S^2$ with a prime
our ansatz for the gauge field becomes

$$A = \frac{n}{2}(\cos\theta - 1)d\phi + \frac{n'}{2}(\cos\theta' - 1)d\phi' + \frac{n''}{2}(\cos\theta'' - 1)d\phi''$$

The structure of the charge matrices will determine the unbroken sub-
group of $U(N)$. As before we shall take them to be $N \times N$ diagonal real
matrices.

The scalars of interest for us are those components of the fluctuations
of the vector potential which are tangent to $S^2 \times S^2 \times S^2$ and are in the
directions of perpendicular to the Cartan subalgebra of $U(N)$. Consider the
field $V_{-i}^j$ tangent to $S^2$.

The masses of these fields can be calculated using the appropriate modi-
fication of equation (34). The result is

28
\[ S_2 = -\frac{1}{2g^2} \int d^{10}x \left\{ (V^j_{-i})^* (-\partial^2 - D^2 - D'^2 - D''^2 + \frac{1}{a^2}) V^j_{-i} - \frac{1}{a^2} (V^j_{-i})^* (n_i - n_j) V^j_{-i} \right\} \] (43)

where \( D^2, D'^2 \) and \( D''^2 \) are the appropriate Laplacian on the three \( S^2 \)'s. The eigenvalues of these Laplacians are basically determined from the iso helicities of \( V^j_{-i} \) which are given by

\[ \lambda(V^j_{-i}) = -1 + \frac{1}{2}(n_i - n_j), \quad \lambda'(V^j_{-i}) = \frac{1}{2}(n'_i - n'_j), \quad \lambda''(V^j_{-i}) = \frac{1}{2}(n''_i - n''_j) \]

Similar expressions can be written for the bilinear parts of the fields tangent to \( S'^2 \) and \( S''^2 \).

For our illustrative example we consider an \( n \) matrix which has only the elements \( n_1 \) and \( n_2 \) different from zero and such that \( n_1 - n_2 \geq 2 \). Then \( \lambda(V^2_{-1}) \geq 0 \) and according to our general rule the leading mode in this field can be tachyonic. The question we would like to answer is if by an appropriate choice of magnetic charges we can make the mass of this field to vanish. It is not difficult to write down the formula for the masses of the infinite tower of modes of \( V^2_{-1} \). These are given by

\[ a^2 M^2 = l(l+1) - \lambda^2 + \frac{a^2}{a^2} (l'(l'+1) - \lambda'^2) + \frac{a^2}{a''^2} (l''(l''+1) - \lambda''^2) + 1 - (n_1 - n_2) \]

To verify the existence of a massless mode first we employ the background equations (1) to obtain the ratios

\[ \frac{a^2}{a^2} = \frac{\text{Tr}n^2}{\text{Tr}n'^2}, \quad \text{and} \quad \frac{a^2}{a''^2} = \frac{\text{Tr}n^2}{\text{Tr}n''^2}. \]

It is seen that for the choice of \( n'_1 - n'_2 = n_1 - n_2 \), \( \text{Tr}n^2 = \text{Tr}n'^2 \) and \( n''_1 - n''_2 = 0 \) the leading mode is indeed massless. For this choice there will of course be a similar massless mode in the fluctuations \( V^2_{-1} \) tangent to \( S^2 \). The \( SU(2) \times SU(2) \times SU(2) \) quantum numbers of these modes will be

\( (l = \frac{1}{2}(n_1 - n_2) - 1, l' = \frac{1}{2}(n_1 - n_2), 0) \), and \( l = (\frac{1}{2}(n_1 - n_2), l' = \frac{1}{2}(n_1 - n_2) - 1, 0) \), respectively. We can make all other modes to have positive masses by appropriate choices of the remaining magnetic charges.
9 Summary and outlook

In this paper we argued that the Higgs scalars of the 4-dimensional spontaneously broken gauge theories have their origin in the extra components of a Yang-Mills potential in $4 + N$ dimensions. For this idea to be useful and tenable it should be shown that there is a mechanism through which these scalars break the gauge symmetries spontaneously. We showed that their coupling to a background magnetic monopole field is one such mechanism. This coupling gives a tree level tachyonic mass to these scalars and also makes it possible that their leading mode in the harmonics of $S^2$ to belong to the doublet representation of the isometry group of $S^2$, which we identified with $SU(2)_L$ of the electroweak theory. The presence of left handed fermionic doublets and right handed fermionic singlets justifies this identification. We gave a simple rule to identify the Kaluza–Klein modes which could trigger spontaneous symmetry breaking in the effective 4-dimensional theory. We constructed two examples with gauge groups $SU(3)$ and $U(6) \times U(1)$ in $D = 6$ and $D = 10$, respectively. The first example leads to an effective left - right symmetric type model in $D = 4$ with the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. We worked out the full $D = 4$ effective action for this model.

In the second example the internal space is $S^2 \times CP^2$ with a magnetic monopole configuration on $S^2$ and a $U(1)$ instanton on $CP^2$. The modes which we retained in the effective $D = 4$ theory are chiral fermions and tachyonic scalars to be identified as the Higgs fields. It is straightforward to work out the effective $D = 4$ action for this example as well.

Although in the present paper our intention was not to recover the standard model of particle physics from a higher dimensional theory, the chiral spectrum of leptons and quarks and the representation content of the effective Higgs fields as well as their Yukawa couplings and potential in our illustrative example are reasonably realistic to warrant further study of examples like the ones of this paper. For instance we need to understand how the $U(1)$ symmetries in the $U(6)$ model are going to be broken. The vacuum expectation value of the Higgs in the lepton and the quark sectors will break only two out of the four effective $D = 4$ $U(1)$ symmetries. Presumably some loop effects will generate condensates of some composite objects which are charged under
these $U(1)$’s and thereby break these symmetries dynamically.\footnote{The $SU(2) \times SU(2)$ subgroup of $U(6)$ will of course be broken by our Higgs fields.}

It is worth noting that since $\pi_{10}(U(6) \times U(1))$ is trivial there will be no global Witten anomalies. The perturbative anomalies on the other hand can be removed by the Green Schwarz type mechanism explained at the end of section 5.2.1.

For obvious phenomenological reasons one would rather prefer to have a (mild) hierarchy of scales $M_W < 1/a_1$. This might be possible if the tachyonic mass could be induced as a result of a one-loop effect. A necessary precondition for this is the presence of massless scalars in the tree level spectrum of the effective 4 dimensional theory. To show that this is indeed possible we considered a 10 dimensional $U(N)$ model with a solution which posits magnetic monopoles with charge matrices $n$, $n'$ and $n''$ on each one of the three $S^2$’s comprising the 6-dimensional compact internal space. For a judicious choice of some of the elements of these matrices we do indeed obtain massless scalars in non trivial representations of the unbroken gauge group $SU(2) \times SU(2) \times SU(2) \times G$, where $G$ is the subgroup of $U(N)$ which is left unbroken by our background solution. As a general rule the loop effects are expected to induce a non zero mass for these scalars and it is possible to verify in detail that an appropriate choice of the fermionic couplings the induced mass will indeed be tachyonic. The quartic self coupling of these scalars can be worked out as we did for the $SU(3)$ model of section 3.\footnote{After we had submitted our paper to the arXiv, a paper, \cite{15}, appeared in which the hierarchy problem in the context of higher dimensional theories is addressed in a different way.}

Acknowledgments

We are grateful to Nima Arkani-Hamed, Goran Senjanović, Alexei Smirnov and George Thompson for discussion. GD thanks High Energy Group of ICTP for hospitality. The work of GD was supported in part by David and Lucille Packard Foundation Fellowship for Science and Engineering, by Alfred P. Sloan foundation fellowship and by NSF grant PHY-0070787.
References

[1] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 086004; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B436 (1998) 257.

[2] S. Randjbar-Daemi, Abdus Salam, and J. A. Strathdee, Phys. Lett 124B (1983) 345.

[3] R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D 63 (2001) 105007.

[4] For symmetry breaking induced by Scherk–Schwarz SUSY breaking, see, e.g., Antoniadis, S. Dimopoulos and G. Dvali, Nucl.Phys. B516 (1998) 70; I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. B544 (1999) 503. For a different realization of radiative breaking see, e.g., N. Arkani-Hamed et al, Phys.Rev. D62 (2000) 096006; N. Arkani-Hamed et al, hep-ph/0102090.

[5] I. Antoniadis, Phys.Lett. B246 (1990) 377.

[6] N. S. Manton, Nucl. Phys. B 158 (1979) 141.

[7] S. Randjbar-Daemi, Abdus Salam, and J. A. Strathdee, Nucl. Phys. B 214 (1983) 491.

[8] E. Witten, “Fermion Quantum Numbers in Kaluza Klein Theories”, The Proc. of Second Shelter Island Meeting 1983, Eds. R. Jackiw et al (MIT Press. 1985), page 227.

[9] M. Green and J. Schwarz, Phys. Lett 149B (1984) 117.

[10] S. Randjbar-Daemi, Abdus Salam, E. Sezgin, and J. A. Strathdee, Phys. Lett 151B (1985) 351.

[11] E. Witten, Phys. Lett 117B (1982) 324.

[12] N.V. Krasnikov, Phys. Lett. B273 (1991) 246; H. Hatanaka, T. Inami and C. S. Lim, Mod. Phys. Lett. A 13 (1998) 2601; H. Hatanaka, Prog. Theor. Phys. 102 (1999) 407; Y. Hosotani, Annals Phys. 190 (1989) 233.
[13] C. Bachas, “A way to break supersymmetry,” [hep-th/9503030]; “Magnetic supersymmetry breaking,” [hep-th/9509067]; R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP 0010, (2000) 006; Fortsch. Phys. 49, (2001) 591.

[14] D. Kapetanakis and G. Zoupanos, Phys. Rept. C 219 (1992) 1.

[15] M. Shaposhnikov and P. Tinyakov, [hep-th/0102161]