Near-Extremal Spherically Symmetric Black Holes in an Arbitrary-Dimensional Spacetime

by

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ABSTRACT

In a recent paper [hep-th/0111091], the near-extremal thermodynamics of a 4-dimensional Reissner-Nordstrom black hole had been considered. In the current paper, we extend this prior treatment to the more general case of a spherically symmetric, charged black hole of arbitrary dimensionality. After summarizing the earlier work, we demonstrate a duality that exists between the near-extremal sector of spherically symmetric black holes and Jackiw-Teitelboim theory. On the basis of this correspondence, we argue that back-reaction effects prohibit any of these “RN-like” black holes from reaching extremality and, moreover, from coming arbitrarily close to an extremal state.
1 Introduction

So far, we have witnessed only limited progress towards realizing a formal theory of quantum gravity. Yet, there has still been much support for a fundamental relationship between the proposed constituents. Nowhere is this relationship more in evidence than in the thermodynamic behavior of black holes; where gravitation, the laws of thermodynamics and quantum theory appear to been linked together in some profound, but mysterious manner.

In regard to black hole thermodynamics, the most prominent open question is the microscopic origin of black hole entropy. There have been many attempts at resolving the issue with varying degrees of success. (For a review and references, see Ref.[1].) Perhaps the most profound resolution has been proposed by Strominger and others [2] in the context of weakly coupled string theory; where massive string states can be represented by extremal black holes. In these works, a statistical procedure has been used to generate, precisely, the Bekenstein-Hawking area law (i.e., entropy is given by one quarter of the horizon surface area [3, 4]). It follows, by implication, that the extremal limit (i.e., the degeneracy of two otherwise distinct horizons in a charged or rotating black hole) must be well defined, with the corresponding entropy being a non-vanishing, mass-dependent quantity. Alas, this is in direct conflict with the Nernst’s entropic formulation of the third law of thermodynamics: as the temperature of a system approaches zero, the entropy must approach a constant (typically zero) that is independent of all macroscopic parameters of the system [5]. In contrast to the third law, the extremal limit corresponds to a vanishing Hawking temperature [6] but a Bekenstein-Hawking entropy that (at least naively) depends on the mass of the black hole.

In view of this apparent conflict between the third law and the extremal limit of black hole thermodynamics, Hawking and others [7] have argued as follows. Extremal black holes and non-extremal black holes are qualitatively distinct entities, with no possibility of one class continuously deforming into the other. In this picture, extremal black holes are indeed assigned zero entropy, assuming that they can exist at all. Since the original conjecture,

\[1\] This formulation of the third law should not be confused with Nernst’s other version: it is impossible to reach absolute zero by a finite number of reversible processes. The two formulations, although typically equivalent, need not coincide for “exotic” thermodynamic systems (such as black holes).
there have been various other semi-classical calculations in support of this viewpoint. (See Ref.[8] for an up-to-date list of the relevant citations.)

However, in spite of Hawking et al.’s convincing arguments and considerable supporting evidence, the status of extremal thermodynamics remains, very much, an open question. This ambiguity can be attributed to the implications of the forementioned, compelling string-theory calculations. Given the apparent incompatibility of these two points of view, any evidence, one way or the other, should be closely examined. In this spirit, let us proceed towards the focus of this paper.

Recently, this author has considered the near-extremal thermodynamics of a 4-dimensional Reissner-Nordstrom (i.e., spherically symmetric and charged) black hole [8]. Let us briefly review, in point form, the procedure used and the outcomes of this analysis.

(i) We began by reviewing a duality [10] that exists between the near-extremal sector of Reissner-Nordstrom (RN) black holes and 2-dimensional anti-de Sitter (AdS) gravity. The static black hole solutions of the latter are described by what is known as Jackiw-Teitelboim (JT) theory [15].

(ii) It was shown that the thermodynamic properties of a near-extremal RN black hole coincide precisely with near-massless JT thermodynamics. Significantly, the massless limit of JT black holes is, at least classically, a well-defined procedure for which the associated temperature and entropy both vanish. Hence, from a classical perspective, this correspondence strongly supports the viewpoint of a well-defined extremal limit.

(iii) After reviewing the classical duality, we went on to examine the corresponding situation at a one-loop level. In particular, we incorporated a (massless) quantum scalar field into the formalism and then considered the first-order back reaction on the JT geometry. Our focus was on the quantum-corrected surface gravity (i.e., temperature [6]). Inspired by a prior study [16], we adopted the viewpoint that a consistent black hole solution demands a non-negative surface gravity.

(iv) Initially, we considered a scenario where the quantum matter field is minimally coupled in the effective 2-dimensional theory. From this perspective, we found that a massless JT black hole remains a well-defined limiting

\[2\text{For a list of other papers supporting a well-defined extremal limit, see Ref.[9].}\]

\[3\text{It is interesting to note that this duality has its origins in the AdS}_2/\text{CFT}_1\text{ correspondence [11, 12, 13, 14].}\]
case (i.e., the surface gravity remains non-negative). We argued, however, that this scenario is inappropriate for the following reason. The JT action, in this context, has its origins in a higher-dimensional theory, and so any matter field should be subjected to the same criteria.

(v) With the above argument in mind, we subsequently considered a matter field with a 4-dimensional pedigree. That is, one that is minimally coupled in the originating RN theory. After repeating the same procedures of dimensional reduction and field reparametrization that had been imposed on the classical action, we found that the effective one-loop action mimics a dimensionally reduced (non-rotating) BTZ model \[17, 18\]. By directly applying prior works of relevance to this model \[9, 19\], we were able to demonstrate that the limiting procedure will break down before a vanishing mass can be achieved. That is to say, the one-loop surface gravity will only remain non-negative for finite values of the JT black hole (renormalized) mass. This finite lower bound can be quantitatively expressed in terms of fundamental constants and the observables (mass and charge) of the dual RN black hole.\(^4\)

(vi) On the basis of the above result and the observed RN-JT duality, we concluded that a RN black hole will be unable to achieve extremality. Moreover, the quantum back reaction will inhibit the RN black hole from even coming arbitrarily close to an extremal state. Rather, it will “freeze” at a finite temperature that is related to the Planck scale.

Let us also briefly touch on another recent study of interest \[29\]. Barvin-sky, Das and Kunstatter considered the physical spectra of charged black holes and deduced that extremal black holes can not be achieved (at the quantum level) due to vacuum fluctuations in the horizon. These authors also argued that their outcomes applied, quite generically, to any black hole model that can effectively be described by a 2-dimensional dilaton theory.

Given the implied generality of Ref.\(29\), the interest of the current paper is to ascertain if the results of our prior analysis \[8\] have a more general validity. In particular, we will consider a charged, spherically symmetric black hole in an arbitrary-dimensional spacetime. As it turns out, the results

\(^4\)For earlier studies on dimensionally reduced theories of this nature, see \[20\]-\[27\].

\(^5\)It should be pointed out that one-loop calculations have previously been carried out for a near-extremal, 4-dimensional black hole \[28\]. It was not, however, the intent of Ref.\[8\] to perform rigorous one-loop calculations, but rather to consider qualitative features in the context of the observed duality.
of our prior study do indeed carry over to this generalized model. The rest of this paper argues in support of this claim.

The proceeding sections are organized as follows. Section 2 introduces the Einstein-Maxwell action of interest and considers the near-extremal thermodynamic properties of the black hole solutions. In Section 3, we apply procedures of dimensional reduction and field reparametrization in obtaining an effective Jackiw-Teitelboim theory. We are then able to demonstrate the duality that exists between JT thermodynamics and the near-extremal sector of the higher-dimensional theory. In Section 4, we present a simple argument as to why the results of the prior study \[8\] persist for this generalized model. Finally, Section 5 contains a brief discussion.

2 Generalized Einstein-Maxwell Theory

We begin with an \(n+2\)-dimensional Einstein-Maxwell action:

\[
I^{(n+2)} = \frac{1}{16\pi l^n} \int d^{n+2}x \sqrt{-g^{(n+2)}} \left[ R^{(n+2)} - F_{AB}F_{AB} \right], \quad (1)
\]

where \(l^n\) is the \(n+2\)-dimensional Newton constant (\(l\) being a length parameter) and \(F_{AB}\) is the Abelian field-strength tensor (\(A,B = 0,1,...,n+1\)).

The unique static and spherically symmetric solution of this action can be described as follows \[30\]:

\[
ds^2 = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2d\Omega^2,
\]

\[
h(r) = 1 - \frac{16\pi l^n M}{nV_n r^{n-1}} + \frac{32\pi^2 l^{2n} Q^2}{n(n-1)V_n^2 r^{2n-2}}, \quad (3)
\]

where \(M\) and \(Q\) represent the conserved quantities of black hole mass and charge (respectively), and \(d\Omega^2\) is an \(n\)-dimensional constant-curvature hypersurface with volume \(V_n = 2\pi^{\frac{n+1}{2}}/\Gamma\left(\frac{n+1}{2}\right)\).

If \(M^2 > nQ^2/2(n-1)\), this is the solution for a charged, non-extremal black hole. In this case, the outermost pair of horizons are distinct and described as follows:

\[
r_{\pm}^{n-1} = \frac{8\pi l^n}{nV_n} \left[ M \pm \sqrt{M^2 - \frac{nQ^2}{2(n-1)}} \right]. \quad (4)
\]
Using the usual prescription for non-extremal black hole thermodynamics \([3, 4, 6]\), we find that the associated entropy and temperature are respectively given by:

\[
S_{BH} = \frac{A_+}{4\hbar l^n} = \frac{V_n r_+^n}{4\hbar}, \tag{5}
\]

\[
T_H = \frac{\hbar \kappa_+}{2\pi} = \frac{\hbar}{4\pi} \left. \frac{d\Omega}{dr} \right|_{r_+} = \hbar \left[ \frac{4(n-1)l^n M}{nV_{n}r_+^n} - \frac{16\pi l^{2n}Q^2}{nV_{n}r_+^{2n-1}} \right], \tag{6}
\]

where \(A_+\) is the surface area and \(\kappa_+\) is the surface gravity with respect to the outermost horizon \((r_+)\).

For the case of extremal black holes (i.e., \(r_- = r_+\)), the associated thermodynamics remains an open issue (as discussed earlier). However, we can still safely consider a “near-extremal regime” by setting \(\Delta M = M - M_o\), where \(M_o^2 = nQ^2/2(n-1)\) (with the charge assumed to be a fixed quantity). Then to leading order in \(\sqrt{\Delta M}\):

\[
r_+^{n-1} = \frac{8\pi l^n}{nV_n} \left[ M_o + \sqrt{2M_o\Delta M} \right]. \tag{7}
\]

Using this expression and expanding as necessary, we can obtain near-horizon forms for the entropy \([5]\) and temperature \([5]\). To leading order in \(\sqrt{\Delta M}\), these quantities can be expressed as follows (with \(w \equiv n - 1\)):

\[
\Delta S_{BH} \equiv S_{BH}(M, Q) - S_{BH}(M_o) = \frac{1}{\hbar} \left[ 2^{\frac{w+1}{n}} [\pi]^{\frac{w-1}{n}} [n]^{\frac{w+2}{n}} [w]^{\frac{2w-3}{n}} [V_n]^{-\frac{w}{n}} [l]^{\frac{w-1}{n}} [Q]^{\frac{w+1}{n}} \sqrt{\Delta M}, \tag{8}
\]

\[
\Delta T_H \equiv T_H(M, Q) - T_H(M_o) = \hbar \left[ 2^{\frac{w+1}{n}} [\pi]^{\frac{w-1}{n}} [n]^{\frac{w+2}{n}} [w]^{\frac{2w-3}{n}} [V_n]^{-\frac{w}{n}} [l]^{-\frac{w}{n}} [Q]^{-\frac{w+1}{n}} \sqrt{\Delta M}. \tag{9}
\]

where \(S_B H(M_o) \sim M_o^{\frac{n}{w}}\) and \(T_H(M_o) = 0\).

### 3 Effective Jackiw-Teitelboim Theory

Let us now consider the dimensional reduction of the action \([1]\) to an effective 2-dimensional theory.\(^6\) If one imposes the following ansatz:

\[
d s_{n+2}^2 = d s^2(t, x) + \phi^2(x, t) d\Omega^2, \tag{10}
\]

\(^6\)For further discussion on the various aspects of 2-dimensional gravity, see Ref.\([3]\) and references therein.
\[ F_{AB} = F_{\mu\nu}(t, x) \quad \text{where} \quad \mu, \nu = 0, 1 \quad \text{only}, \] (11)
then the following form is obtained (also see Refs. [32, 33, 14]):
\[ I = \frac{V_n}{16\pi l^n} \int d^2x \sqrt{-g} \phi^n \left[ R + n(n - 1) \left( \frac{(\nabla \phi)^2}{\phi^2} + \frac{1}{\phi^2} \right) - \frac{32\pi^2 l^{2n} Q^2}{\phi^{2n} V_n^2} \right]. \] (12)

Here, the “dilaton” \( \phi \) is identifiable with the radius of the symmetric two-sphere, the conserved charge is still given by \( Q \), and all geometric quantities have been defined with respect to the resultant 1+1-dimensional manifold.

At this stage, it is convenient to redefine the dilaton as follows:
\[ \psi(x, t) = \left[ \frac{\phi}{l} \right]^\frac{2}{n}. \] (13)

The reduced action (12) now reads:
\[ I = \frac{1}{2G} \int d^2x \sqrt{-g} \left[ D(\psi)R + \frac{1}{2} (\nabla \psi)^2 + \frac{1}{l^2} V_Q(\psi) \right], \] (14)
where we have defined:
\[ \frac{1}{2G} \equiv \frac{8(n - 1)V_n}{16\pi n}, \] (15)
\[ D(\psi) \equiv \frac{n}{8(n - 1)} \psi^2, \] (16)
\[ V_Q(\psi) \equiv \frac{n^2}{8} \psi^{2n-4} - \frac{4\pi^2 l^{2n} Q^2}{(n - 1)V_n^2 \psi^2}. \] (17)

The above form of the action is now suitable for the direct implementation of a field reparametrization that is known to eliminate the kinetic term [34]. Thus, we appropriately redefine the dilaton, metric and “dilaton potential” in the following manner:
\[ \overline{\psi} = D(\psi) = \frac{n}{8(n - 1)} \psi^2, \] (18)
\[ \overline{g}_{\mu\nu} = \Omega^2(\psi) g_{\mu\nu}, \] (19)
\[ \overline{V}_Q(\psi) = \frac{V_Q(\psi)}{\Omega(\psi)}. \] (20)
\[ \Omega^2(\psi) = \exp \left[ \frac{1}{2} \int \frac{d\psi}{(dD/d\psi)} \right] = C \left[ \frac{8(n-1)\psi}{n} \right]^{\frac{n-1}{n}}. \] (21)

Take note of \( C \), which is a (seemingly) arbitrary constant of integration. It can be fixed via physical arguments, and so we follow Section V of Ref. [33] (also see Ref. [14]) and set: \( C = n^2/8(n-1) \).

With these reparametrizations, the reduced action (14) takes on the following compact form:

\[ I = \int d^2x \sqrt{-g} \left[ \bar{\psi} R(g) + \frac{1}{l^2} \tilde{V}_Q(\bar{\psi}) \right], \] (22)

where we have also set \( G = 1/2 \).

It is pertinent to this analysis that the extremal configuration (i.e., the extremal limit, assuming its existence, in the higher-dimensional model) can be recovered when \( \tilde{V}_Q(\bar{\psi}) = 0 \). With the above formalism, one finds that this “potential” vanishes for \( \bar{\psi} = \bar{\psi}_o \), such that:

\[ [\bar{\psi}_o]^{\frac{n-1}{n}} \equiv \left[ \frac{2\pi l Q}{(n-1)V_n} \right]^2 \left[ \frac{n}{8(n-1)} \right]^{-\frac{n-2}{n}}. \] (23)

With this in mind, let us now define \( \tilde{\psi} \equiv \bar{\psi} - \bar{\psi}_o \) and expand the action (22) about the extremal configuration. To first order in \( \tilde{\phi} \), the following is obtained:

\[ I = \int d^2x \sqrt{-g} \left[ \bar{\psi} R(g) + \frac{1}{l^2} \tilde{V}_Q(\bar{\psi}) \right], \] (24)

where:

\[ \tilde{V}_Q(\tilde{\psi}) \equiv \left. \frac{d\tilde{V}_Q}{d\tilde{\psi}} \right|_{\tilde{\psi}=0} \tilde{\psi} = 2 \frac{(n-1)^2}{n} \left[ \frac{n}{8(n-1)} \right]^{\frac{1}{n}} [\bar{\psi}_o]^{-\frac{n+1}{n}} \tilde{\psi} \]

\[ = 2 \frac{(n-1)^2}{n} \left[ \frac{8(n-1)}{n} \right]^{\frac{1}{n}} \left[ \frac{(n-1)V_n}{2\pi l |Q|} \right]^{\frac{n+1}{n}} \tilde{\psi}. \] (25)

Henceforth, we drop the tildes and bars; thus considering the following action:

\[ I = \int d^2x \sqrt{-g} \left[ R(g) + 2\frac{\lambda}{l^2} \right]. \] (26)
where $\psi \lambda$ corresponds to one half of the right-hand side of Eq. (23). This is simply the action for 2-dimensional AdS gravity; the black hole solutions of which are the well-known JT black holes [15].

It can be readily shown that, for a static gauge, the general solution of the JT action (26) can be expressed as follows:

$$ds^2 = -(\lambda x^2/l^2 - lm)dt^2 + (\lambda x^2/l^2 - lm)^{-1}dx^2;$$  \hspace{1cm} (27)

$$\psi = \frac{x}{l},$$  \hspace{1cm} (28)

where $m$ represents the conserved mass of the JT black hole. Moreover, with straightforward application of Ref. [35] (applicable to a generic 2-dimensional dilaton theory), we are able to identify the following thermodynamic properties:

$$S_{JT} = \frac{4\pi}{\hbar} \psi_+,$$  \hspace{1cm} (29)

$$T_{JT} = \frac{\hbar \lambda}{2\pi l} \psi_+,$$  \hspace{1cm} (30)

where $\psi_+ = x_+/l = \sqrt{lm/\lambda}$ is the horizon value of the dilaton field.

Substituting for $\lambda$ into the above expressions and also identifying $m$ with $\Delta M$, we ultimately find the following (with $w = n - 1$):

$$S_{JT} = \frac{1}{\hbar} [2]^{-\frac{1}{2} \frac{7n+3}{w}} [\pi]^{-\frac{1}{2} \frac{6n-2}{w}} [n]^{-\frac{1}{2} \frac{3n-5}{w}} [w]^{-\frac{1}{2} \frac{7n-5}{w}} [V_n]^{-\frac{1}{2} \frac{n+1}{w}} [l]^{\frac{w}{2}} [Q]^{\frac{1}{2} \frac{n+1}{w}} \sqrt{\Delta M},$$

$$T_{JT} = \hbar [2]^{-\frac{1}{2} \frac{3n+2}{w}} [\pi]^{-\frac{1}{4} \frac{6n-2}{w}} [n]^{-\frac{1}{4} \frac{3n-5}{w}} [w]^{-\frac{1}{4} \frac{7n-5}{w}} [V_n]^{\frac{1}{2} \frac{n+1}{w}} [l]^{-\frac{w}{2}} [Q]^{-\frac{1}{2} \frac{n+1}{w}} \sqrt{\Delta M}. \hspace{1cm} (31)$$

A direct comparison of these expressions with Eqs. (8,9) yields the following intriguing outcomes:

$$S_{JT} = \mathcal{K} \Delta S_{BH},$$  \hspace{1cm} (33)

$$T_{JT} = \frac{1}{\mathcal{K}} \Delta T_H,$$  \hspace{1cm} (34)

where $\mathcal{K}$ is some dimensionless numerical factor.

It is easy to verify that $\mathcal{K} = 1$ for $n = 2$ (i.e., the 4-dimensional RN black hole). In general, however, one finds that $\mathcal{K} \neq 1$. For instance, if

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That 4-dimensional gravity holds a privileged position is not all together new. For example, black holes only saturate a Bekenstein-like entropy bound in four dimensions of spacetime [36].
\(n = 3\), then \(K = \sqrt{3/2\pi}\). However, this lack of an exact coincidence (when \(n \neq 2\)) is essentially irrelevant, given that we are always free to rescale the fundamental constants from the perspective of the lower-dimensional theory. That is, as far as the second law of thermodynamics is concerned, the above thermodynamics does indeed coincide. (The pertinent points being that \(S_{JT}T_{JT} = \Delta S_{BH}\Delta T_{H}\) and all dimensional quantities coincide exactly.) Hence, we have demonstrated the anticipated duality for any choice of \(n\): the near-extremal sector of charged, spherically symmetric black holes with the near-massless sector of JT theory.

4 One-Loop Considerations

In analogy to the \(n = 2\) analysis of Ref.\(^8\), it is also necessary to consider the implications (to first-perturbative order) of a quantum scalar field. First note that any consideration of a minimally coupled matter field in two dimensions follows trivially from the prior treatment.\(^8\) However, it is still necessary to examine the repercussions of a matter field that has its origins in the higher-dimensional theory.

Let us thus consider a massless scalar field \((\phi)\) that is minimally coupled with respect to the original \((n+2)\)-dimensional Einstein-Maxwell model. The revised (total) action can now be written as:

\[
I_{TOT}^{(n+2)} = I^{(n+2)} - \frac{\hbar}{16\pi l^n} \int d^{n+2}x \sqrt{-g^{(n+2)}}(\nabla^{(n+2)}\phi)^2, \tag{35}
\]

where \(I^{(n+2)}\) is the classically defined action of Eq.(1). Again imposing the spherically symmetric ansatz of Eqs.(10,11) (along with \(\phi = \phi(t,x)\)), we obtain the following reduced formulation:

\[
I_{TOT} = I - \frac{\hbar\Gamma}{16\pi l^n} \int d^2x \sqrt{-g^2}(\nabla\phi)^2, \tag{36}
\]

where \(I\) is the reduced action of Eq.(12).

Following the exact same pattern of field reparametrization and expansion as previously described, we ultimately find that:

\[
I_{TOT} = I_{JT} - \frac{\hbar(n - 1)\Gamma}{2\pi n} \int d^2x \sqrt{-g^2}\tilde{\psi}(\nabla\phi)^2, \tag{37}
\]

\(^8\)This is because the lower-dimensional, reparametrized action (26) is formally identical for all \(n\).
where \( I_{JT} \) is the JT action of Eq.\((24)\) \( \text{(or Eq.\((26)\))} \), and we have explicitly shown the tilde and bar notation for the sake of clarity.

As was also found for the special case of \( n = 2 \) \[8\], the dilaton-matter coupling is precisely that obtained in the dimensional reduction (from three to two dimensions) of a BTZ black hole; assuming minimal coupling in the higher-dimensional theory \[17, 18\]. This means that all of the outcomes of the prior \( (n = 2) \) analysis will automatically persist for the generic-\( n \) case and need not be repeated. Hence, we conclude that, by way of duality, an \( n+2 \)-dimensional spherically symmetric, charged black hole can not reach a state of extremality and, moreover, can not even come arbitrarily close to an extremal state.

5 Conclusion

In summary, we have extended the results of a prior analysis \[8\] on the 4-dimensional Reissner-Nordstrom black hole to analogous black holes of arbitrary dimensionality. The outcomes, now applicable to any dimension, argue against the existence of a well-defined extremal limit. The basis of these arguments is that a consistent black hole solution requires a non-negative surface gravity \[16\]. With this criteria, it can readily be verified \[8\] that an effective (2-dimensional) Jackiw-Teitelboim theory \[15\] must have a lower bound imposed on its black hole mass. By way of a duality (which we have clearly demonstrated), it follows that \( n+2 \)-dimensional RN-like black holes will “freeze” at some finite temperature before ever reaching extremality.

It is interesting to note that the same treatment can be readily applied to a rotating BTZ model \[17\], with equivalent outcomes. This follows, almost trivially, by virtue of the known duality (between the near-extremal BTZ sector and JT theory \[13\]) and simplifications that are inherent to the BTZ analysis (primarily, the absence of a kinetic term in the higher-dimensional action). It is not yet clear, however, how more “exotic” black hole geometries may hold up; for instance, \( n+2 \)-dimensional Reissner-Nordstrom-anti-de Sitter black holes. On the other hand, given the wide class of black holes exhibiting a near-extremal duality with AdS\(_2\) \[37\], we anticipate that similar

\[9\]The unorthodox constant factor in front of the above matter action is irrelevant, as this can always be absorbed through a redefinition of \( f \).
outcomes are obtainable for many other black hole scenarios. We will defer this question to future studies.

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