Rock-and-roll skyrmion propagation under parametric pumping

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We address the problem of how magnetic skyrmions can propagate along a guided direction by parametric pumping. As evidenced by our micromagnetic simulations, skyrmions can hardly be driven by either a static electric field or a static magnetic field alone. Although the magnetic anisotropy can be modified by an electric field, parametric pumping with an oscillating electric field can only excite the breathing modes. On the other hand, a static magnetic field can break rotational symmetry through the Zeeman interaction, but it cannot serve as an energy source for propelling a skyrmion. Here we found that the combination of a perpendicularly oscillating electric field and an in-plane static magnetic field can drive a skyrmion undergoing a wiggling motion along a well-defined trajectory. The most efficient driving occurs when the frequency of the oscillating field is close to that of the breathing mode of a skyrmion. The physical mechanism is analyzed with a generalized Thiele equation, where we find that a net spin current excited by the parametric pumping can drive the skyrmion propagation through angular momentum transfer. Compared with other alternative proposals, our results open new possibilities for manipulating skyrmions in both metals and insulators with low-power consumption and high precision.

Introduction.— Magnetic skyrmions are topological structures that were observed in a class of magnetic materials with broken inversion symmetry.1–8 In comparison with magnetic bubbles,9 and domain walls,10 skyrmions are relatively small (1-100 nm)11, and can be driven with a lower current density (10^6 A/m^2)12, making them ideal for being information carriers. Recently, various methods have been proposed for controlling skyrmion motion, including electric currents,13–15 spin waves,15,16 microwaves,17,18, and temperature gradient.19–21 In particular, for metallic materials, skyrmions can be driven by an electric current to move both along the parallel and transverse direction of the current, which is known as the skyrmion Hall effect.14,22 The problem is that it is not applicable to insulating materials, which may potentially require less power consumption, higher stability and controllability. To manipulate skyrmions in insulators, temperature gradient is proposed as a control knob through the spin transfer torque. However, similar to the case of domain-wall motion induced by the magnonic spin transfer torque,23 the effectiveness remains a problem in practice. Therefore, finding new methods for manipulating skyrmions remains an outstanding problem in spintronics.

Parametric pumping refers to parameter cycling of a system that can result in a net transport of charges and spins. The response of the system to the parameter pumping may be strong (parametric resonance) if the parameter cycling frequency is close to the system intrinsic frequency. One classical example is the water-pump. In the recent development of magnetism, applying electric fields to manipulate magnetic systems has become a popular approach,24–35, which exhibits the benefits of low energy consumption, high precision, and high accessibility. Specifically, electric field can modify the parameters of magnetic materials, including exchange stiffness,36 anisotropy coefficient,29–31, and even the strength of the Dzyaloshinskii-Moriya interaction (DMI).28,36,37 However, the behavior of a skyrmion subject to a periodic variation of material parameters (parametric pumping) remains a largely-unexplored topic.

Intuitively, a perpendicularly-oscillating electric field (POEF) on a vertically-magnetized film can cause a periodic oscillation of the magnetic anisotropy,29–31, which is a form of parametric pumping. However, even if the film supports a skyrmion, the skyrmion can only undergo a breathing motion under the POEF, instead of being driven towards a well-defined direction. In this paper, by breaking the rotational symmetry of skyrmions, we show that a POEF together with an in-plane static magnetic field can induce a continuous skyrmion motion that is associated with a periodic variation of the skyrmion size, i.e. breathing motion. The skyrmion velocity reaches its maximum when the POEF frequency matches that of the skyrmion breathing mode. The physical mechanism of such an unconventional motion is attributed to the existence of a net spin current, which transfers its angular momentum to the skyrmion wall and causes the skyrmion motion. These results are verified numerically with micromagnetic simulations and justified analytically from a generalized Thiele equation.

Model and methods.—We consider a perpendicularly-magnetized film with a skyrmion in the center as shown in Fig. 1a. The skyrmion is stabilized by the competition
between exchange interaction, anisotropy and interface DMI from the asymmetric interfaces of magnetic and non-magnetic layers. The skyrmion dynamics is governed by the Landau-Lifshitz-Gilbert (LLG) equation,

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t},$$

(1)

where $\mathbf{m}$, $\gamma$, $\alpha$ are respectively the unit vector of the magnetization, gyromagnetic ratio, and the Gilbert damping. $\mathbf{H}_{\text{eff}} = 2A\nabla^2 \mathbf{m} + 2K_u \mathbf{m} \cdot \mathbf{e}_z + \mathbf{H}_d + \mathbf{H}_E + \mathbf{H}_D + \mathbf{H}_O$ is the effective field including the exchange field, crystalline anisotropy field, dipolar field $\mathbf{H}_d$, external field $\mathbf{H}_E$ along the $y$-direction, DMI field $\mathbf{H}_D$ and spin-orbit field $\mathbf{H}_O$ due to the electric field. $A$ is the exchange stiffness and $K_u$ is the anisotropy coefficient. The spin-orbit field is induced by the applied electric field through spin-orbit interaction and can be divided into the damping-like field components and field-like components, i.e. $\mathbf{H}_O = \zeta_D E \mathbf{m} \times \mathbf{e}_z + \zeta_F E \mathbf{m} \cdot \mathbf{e}_z$, here $E$ is the electric field while $\zeta_D$ and $\zeta_F$ are the torque conversion coefficients. To investigate the skyrmion structure and its dynamics in an electric field, we use the Mmuax3 package to numerically solve the LLG equation. The film size is of 128 nm $\times$ 128 nm $\times$ 0.4 nm, if it is not stated otherwise. The model parameters are $A = 10 \times 10^{-12}$ J/m, $D = 0.003$ J/m$^2$, $M_s = 9.2 \times 10^5$ A/m, $K_u = 1.157 \times 10^6$ J/m$^3$, $\zeta_F = 0.02$ J/(V$ \cdot $m$^2$) to mimic CoPd. The Gilbert damping varies from 0.02 to 0.2. We focus on the influence of field-like spin-orbit torque on skyrmion dynamics and take $\zeta_D = 0$ in the simulations.

**Skyrmions structure.—**Let us first look at the skyrmions structure under a static electric field ($E$). Figure 1b shows that the skyrmion size $R_s$ decreases with the increase of electric field with a typical skyrmion structure shown in the left inset for $E = 0$ and $H = 0$. The skyrmion size can be described by

$$R_s = \pi D \sqrt{\frac{A}{16AK_{\text{eff}}^2 - \pi^2 D^2 K_{\text{eff}}}},$$

(2)

where $K_{\text{eff}} = K_u + \zeta_F E/2 - \mu_0 M_s^2/2$. Here the long-range dipolar interaction is approximated as the shape anisotropy $\mu_0 M_s^2/2$ along the $z$-axis, which is well justified for a magnetic thin film. This is the variational result obtained by assuming the skyrmion profile along radial direction as a 360° domain wall with skyrmion size and skyrmion wall width as two optimization parameters. The red solid line in Fig. 1b is Eq. (2) that well describes the simulation results (circles) for $E > -3.6 \times 10^5$ V/m. For electric fields smaller than the value, the skyrmion size (diameter $> 80$ nm) is comparable with the system size (128 nm) and the boundary effect becomes pronounced. In an infinite film, the skyrmion should proliferate and becomes unstable at the critical field $E_c = \pi^2 D^2/(8A\zeta_F) - (2K_u - \mu_0 M_s^2)/\zeta_F = -3.6 \times 10^5$ V/m (blue shadowed region). Under an in-plane field, the skyrmion deforms and elongates along the field direction as shown in the right inset of Fig. 1b for $H = -0.3$ T. Here the top and bottom skyrmion walls become thinner and thicker, respectively, to take the advantage of the Zeeman effect. The larger the in-plane field, the larger the width difference between the top and the bottom skyrmion walls.

**Skyrmion motion.—**To describe the skyrmion motion of the asymmetric skyrmions under a harmonic POEF of $E = E_0 \sin(\omega t)\mathbf{e}_z$ and an in-plane magnetic field, we define the skyrmion position as topological charge weighted center: $r_c \equiv 1/(4\pi Q) \int \mathbf{m} \cdot (\partial_t \mathbf{m} \times \partial_t \mathbf{m}) d\mathbf{s}$ with $Q \equiv 1/(4\pi) \int \mathbf{m} \cdot (\partial_t \mathbf{m} \times \partial_t \mathbf{m}) d\mathbf{s}$ being the skyrmion num-
does not move. When \( \omega \) (of the average skyrmion velocity along the y-direction) is almost independent of \( \alpha \).

In order to check whether the peak is associated with the parametric resonance that occurs when the POEF frequency matches with a skyrmion intrinsic frequency, we consider the dynamical susceptibility of the system to a sinc field of \( E_y(t) = E_0 \sin(\omega t)/(\omega t) \), defined as \( \langle m_y(t) \rangle = \chi_{zz}(\omega)E_y(t) \), where \( \langle m_y(t) \rangle \) is the average \( m_y \). The energy absorption of the system, proportional to \( \text{Im}(\chi(\omega))^{14} \), is shown by the pink shadowed region in Fig. 2. The absorption peak is located around 13 GHz that coincides with the maximal skyrmion velocity. The skyrmion response to the POEF of \( \omega = 12.8 \) GHz is shown in the inset of Fig. 2b that plots the snapshots of \( m_y \) along \( x = 0 \). The positions with extreme \( m_y \) values are the skyrmion wall centers. The center positions oscillate back-and-forth with time. This shows clearly a strong breathing motion of the skyrmion. Thus, the velocity peak corresponds to resonance of the POEF with the breathing motion of the skyrmion. It should note a minor peak around 10.4 GHz that will influence the skyrmion velocity for \( \alpha < 0.01 \).

Generalized Thiele equation.—The rock-and-roll motion of skyrmion center shown in Fig. 2a accompanies the breathing of skyrmion size. The skyrmion breathing motion emits spin waves (a magnon spin current) similar to the spin wave emission by domain wall motion. The emitted spin waves across the skyrmion wall, transfer the angular momentum to a skyrmion and drive the skyrmion to move, similar to spin transfer torque induced domain wall motion. Because the rotational symmetry of the skyrmion is broken by the in-plane field, the magnon current should have different components along the field direction (\( +y \)-direction) and the \( x \)-direction. To understand the behavior, we consider the generalized Thiele equation

\[
G \times (\mathbf{v} - j^{(m)}) + D \cdot (\alpha \mathbf{v} - \beta j^{(m)}) = 0
\]  

(3)

where \( G = G e_z = 4\pi Q e_z \) is the skyrmion gyrovector proportional to the skyrmion number \( Q \), and \( D_{ij} = \int \partial_i \mathbf{m} \cdot \partial_j \mathbf{m} dS \) is the dissipation tensor. \( \mathbf{v} = (v_x, v_y) \) is average skyrmion velocity, and \( \beta \) describes the misalignment of magnon polarization and local magnetization that is zero here. \( j^{(m)} \) is the average magnon current. The skyrmion velocity can be obtained from Eq. (3)

\[
v_x = \frac{j_x^{(m)} - \alpha \kappa j_y^{(m)}}{1 + \alpha^2 \kappa^2}, v_y = \frac{j_y^{(m)} + \alpha \kappa j_x^{(m)}}{1 + \alpha^2 \kappa^2}
\]  

(4)

where \( \kappa = D_{xx}/G \). Figure 3a shows that \( v_y \) decreases with the damping hyperbolically while \( v_x \) is almost a constant, which suggests that the magnon current \( j_y^{(m)} \) is inversely proportional to \( \alpha \) while \( j_x^{(m)} \) is damping independent since \( \alpha \kappa \ll 1 \) in Eq. (4), i.e. \( j_y^{(m)} = C_y/\alpha, j_x^{(m)} = C_x \). Using the parameters \( C_x = 2 \times 10^{-5}, C_y = 2.7 \times 10^{-6} \), Eq. (4) can indeed fit the numerical data (symbols) perfectly as shown in Fig. 3a. Furthermore, the skyrmion Hall angle defined as \( \text{atan}(v_x/v_y) = \text{atan}(j_x^{(m)} - \alpha \kappa j_y^{(m)})/(j_y^{(m)} + \alpha \kappa j_x^{(m)}) \) is calculated and plotted as the red line in the inset of Fig. 3a. Again, it perfectly describes the numerical results (circles). Interestingly, at given \( \alpha \), the Hall angle is insensitive to both the amplitude and frequency of electric field, as shown in Fig. 3b and c.
FIG. 3. (color online) (a) Average skyrmion velocity as a function of damping at $\omega = 12.8$ GHz. The symbols are simulation data, and the dashed lines are the solutions of the Thiele equation. The inset shows the Hall angle defined as $\arctan(v_x/v_y)$ as a function of damping parameter. The solid line is theoretical calculations. (b) and (c) Electric field strength and field frequency dependence of Hall angle under different dampings. The symbols are simulation data, and the horizontal dashed lines are used to guide eyes.

Skyrmion inflation/deflation. — The skyrmion size under an POEF oscillates periodically because the electric field modifies the magnetic anisotropy. According to Eq. (2), the skyrmion size should vary in the range of $R_{\text{min}} = R_s(E = E_0)$ and $R_{\text{max}} = R_s(E = -E_0)$ as shown in the cyan rectangle in Fig. 4a. However, micromagnetic simulations show that the skyrmion size oscillates out of this range for $\alpha \leq 0.04$ and falls into this range for $\alpha > 0.04$, as shown in Fig. 4a. This indicates that skyrmion under parametric pumping has an inertia. The steady response of the skyrmion size to an applied harmonic POEF (dashed lines) is shown in Fig. 4a for $\alpha = 0.002$ (red), 0.04 (blue) and 0.2 (purple), respectively. They showed the typical breathing motion in which skyrmion size undergoes a periodic breathing (expansion-and-contraction) motion. Evidently, the skyrmion size variation has a phase lag to its driving POEF. The lagged phase increases with the damping, similar to a damped harmonic oscillator. It is under damped for a lower $\alpha \leq 0.04$ so that the stored energy (momentum) from POEF will push the skyrmion to expand beyond its static size. It is over-damped for a larger $\alpha$. The skyrmion motion lags behind the external pumping field so much that the skyrmion cannot reach its maximal or minimal sizes corresponding to the minimal and maximal effective anisotropies. To further substantiate the damping dependence of skyrmion size oscillation, we consider how the skyrmion size responds to a sudden switching of a constant electric field. The results are shown in Fig. 4b. The solid (dashed) lines are the evolution of skyrmion size $R_s$ to a constant electric field $E = 1.0 \times 10^6 V/m$ ($E = -1.0 \times 10^6 V/m$) switched on at $t = 0$ for $\alpha = 0.02$ (black line) and 0.2 (red line), respectively. The skyrmion size oscillates on its way to the equilibrium value for $\alpha = 0.02$ while it takes a long time for $R_s$ to monotonically relax to its equilibrium value for $\alpha = 0.2$. Take $E < 0$ as an example, the intermediate skyrmion size can be larger than the equilibrium value for $\alpha = 0.02$ while it is always smaller than the equilibrium value for $\alpha = 0.2$. For a fast oscillating electric field, the skyrmions size can be kept at intermediate values periodically for small damping since the skyrmion cannot dissipate its energy (or momentum) timely. This explains
the observation of extraordinarily large/small skyrmion in Fig. 4a for small damping.

**Discussions and conclusions.**—In conclusion, the combination of parametric pumping by a POEF and an in-plane static magnetic field can drive skyrmions undergoing a rock-and-roll motion. The skyrmion velocity reaches its maximum value when the POEF frequency matches with the skyrmion breathing frequency. Our results provide a promising avenue for manipulating skyrmions motion in both metallic and insulating magnetic materials. Moreover, the role of in-plane field may be replaced by the exchange bias field in a FM/Antiferromagnet bilayer such that all electric control of skyrmion dynamics can be realized.

Remarkably, temperature-gradient driven skyrmions exhibit a similar damping dependence of the skyrmion velocity as those reported here by parametric pumping. Specifically, the longitudinal (field-direction) velocity quickly decreases with damping while the transverse velocity is insensitive to the damping. The skyrmion velocity under the two driven forces are at the same order of cm/s. These coincidence may be attributed to the fact that both the electric field and thermal driven skyrmion motion originate from non-uniform magnon flow. Moreover, the skyrmion Hall angle induced by parametric pumping is insensitive to both pumping frequency and pumping amplitude as shown in Fig. 3b and c. This feature is desirable in manipulating skyrmion trajectory in practice.

Although our simulations focus on the Néel skyrmions, the obtained physics should be applicable to Bloch skyrmions. Moreover, parametric pumping can also be realized through the cycling of the exchange stiffness and DMI strength besides of the anisotropy studied here. One should expect similar behavior of the skyrmion motion as that in Fig. 1a when other parameter cycling is used. In this sense, parametric pumping is a universal control knob for skyrmion motion, no matter it is due to POEF as shown here or due to an periodically varying titled magnetic field.

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41. S. Rohart and A. Thiaville, Phys. Rev. B 88, 184422 (2013).
42. V. P. Kravchuk, D. D. Sheka, U. K. Röser, J. Van den Brink, and Y. Gaididei, Phys. Rev. B 97, 064403 (2018).
43. See Supplementary Material at ××× for detailed analysis of spin configuration.
44. Y. Zhang, X. S. Wang, H. Y. Yuan, S. S. Kang, H. W. Zhang, and X. R. Wang, J. Phys.: Condens. Matter 29, 095806 (2017).
45. M. Mochizuki, Phys. Rev. Lett. 108, 017601 (2012).
46. Y. Onose, Y. Okamura, S. Seki, S. Ishiwata, and Y. Tokura, Phys. Rev. Lett. 109, 037603 (2012).
47. J.-V. Kim, F. Garcia-Sanchez, J. Sampaio, C. Moreau-Luchaire, V. Cros, and A. Fert, Phys. Rev. B 90, 064410 (2014).
48. See Supplementary Material at ××× for detailed results on the skyrmion velocity when the damping is smaller than 0.01.
49. X. S. Wang, P. Yan, Y. H. Shen, G. E. W. Bauer, and X. R. Wang, Phys. Rev. Lett. 109, 167209 (2012).
50. See Supplementary Material at ××× for detailed derivation of the dynamic equation and the calculation of driven force.
51. A. Fasano and S. Marmi, Analytical Mechanics, Oxford University Press. (Oxford, New York, 2002).
52. See Supplementary Material at ××× for Bloch skyrmion motion driven by parametric pumping.
53. See Supplementary Material at ××× for detailed simulation results.
54. K.-W. Moon, D. -H. Kim, S. -G. Je, B. -S. Chun, W. Kim, Z. Q. Qiu, S. -B. Choe, and C. Hwang, Sci. Rep. 6, 20360 (2015).
55. A. A. Kovalev, Phys. Rev. B 89, 241101(R) (2014).