Warm hilltop inflation

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We study the low-temperature limit of warm inflation in a hilltop model. This limit remains valid up to the end of inflation, allowing an analytic description of the entire inflationary stage. In the weak dissipative regime, if the kinetic density of the inflaton dominates after inflation, low scale inflation is attained with Hubble scale as low as 1 GeV. In the strong dissipative regime, the model satisfies the observational requirements for the spectral index with a mild tuning of the model parameters, while also overcoming the $\eta$-problem of inflation. However, there is some danger of gravitino overproduction unless the particle content of the theory is large.

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I. INTRODUCTION

The latest high-precision observations strongly suggest that the Universe, during its early history, went through a period of almost exponential expansion called inflation. Inflation not only accounts for the horizon and flatness problems of the Big Bang cosmology, but also provides the primordial curvature perturbation which seeds the formation of structure and is observed through the anisotropy of the cosmic microwave background (CMB) radiation. It is a major success of inflation that the CMB observations confirm the existence of superhorizon curvature perturbations, which are predominantly adiabatic and Gaussian with an almost scale-invariant spectrum.

The standard paradigm for realising inflation in the context of particle theory employs a scalar field, called inflaton, whose potential density dominates the Universe [1]. The dynamics of this scalar field is governed by its scalar potential, the form of which determines the dynamics of inflation. In order to produce an almost scale-invariant spectrum of curvature perturbations the potential density has to remain almost constant, which means that the scalar field should vary extremely slowly during inflation. This, in turn, is possible only if the potential is sufficiently flat. Hence, the inflaton field is usually assumed to be a flat direction in field space. In order to avoid lifting the flatness of the potential due to radiative corrections the inflaton is typically assumed to be a gauge singlet (see however [2]) with suppressed interactions to other fields.

One of the most important discoveries of the CMB observations is the fact that the spectrum of the curvature perturbation deviates significantly from the exact scale invariance of the Harrison-Zel’dovich, “vanilla” case. Instead, the spectrum appears to be red, with spectral index: $n = 0.948^{+0.015}_{-0.016}$ within the 1-$\sigma$ window [2] (when tensors and running of $n$ are negligible). This result reveals that the physics of inflation is non-trivial and not only allowed to model-builders to discriminate between models. Indeed, it seems for example, that the simplest form of hybrid inflation [3], which produces a blue spectrum, is already excluded. (However, supersymmetric hybrid inflation models [3,4] produce a red spectrum and can still be in agreement with observations [3]). Also, many large-field models, such as quartic (or higher-order) chaotic inflation are also incompatible with observations [3]. In terms of simple, single-field models, one class that appears to be generically in good agreement with CMB observations is the so-called small-field models, the prototype of which is ‘new inflation’ [5]. In these models, the inflaton is rolling off the top of a potential hill. Hence, the realisation of this scenario in terms of particle theory corresponds to a number of inflation models dubbed ‘hilltop inflation’ [6,10]. In general, hilltop inflation produces a red spectrum of curvature perturbations and negligible tensors (tiny tensor fraction $r_1 \lesssim 0.002$). The variation of the inflaton field is much smaller than the Planck scale, which means that the scalar potential is much more understandable in terms of effective field theory [6].

However, hilltop models have their own problems. Firstly, if they are to generate the curvature perturbation, then the curvature of the top of the potential hill has to be fine-tuned, otherwise the spectrum becomes too much tilted. However, in the context of supergravity and superstring theories, such fine-tuning is unnatural. This is the so-called $\eta$-problem plaguing most models of inflation. Slow-roll inflation requires $|\eta| \ll 1$ in order to produce an approximately scale-invariant spectrum, while supergravity corrections to the scalar potential (barring accidental cancellations) suggest $|\eta| \sim 1$ [11]. Such a large $|\eta|$ would result in fast-roll inflation [12], which works only if the curvature perturbations are generated by a field other than the inflaton (e.g. a curvature field [13]). In this paper, we show that dissipative effects may eliminate the $\eta$-problem of hilltop inflation.

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Another problem of hilltop models has to do with the necessary initial conditions. The inflaton field must find itself near the top of the potential hill in order for inflation to occur. Some authors argue that the initial condition problem is evaded by considering the possibility of eternal inflation [9]. It is true that near the top of the potential hill there is a region of quantum diffusion which would result in eternal inflation were the field placed there originally. Eternal inflation then guarantees that the regions of the multiverse which have undergone inflation occupy much more volume than the ones that have not, so it appears much more likely for an observer to find oneself in one of those. Hence, if we envisage random initial conditions, the chances are that, at late times, the most probable locations would be the ones which did inflate and hence the ones where the initial conditions corresponded to the top of the hill. The weak point of this argument is that, in this setup, by far the largest probability at late times is to find oneself in a region that is still inflating and, therefore, occupies much more volume than the regions which stopped inflating 14 billion years ago. One then needs to employ anthropic arguments to place our location in the multiverse in the non-inflating region. So the eternal inflation explanation of the initial conditions for hilltop inflation relies on anthropic reasoning, which many authors do not find convincing.

Traditionally, the initial conditions for new inflation have been addressed by assuming that the top of the potential hill corresponds to a fixed point of the symmetries of the theory. In fact, in string moduli space, we can envisage that the fixed point is a point of enhanced symmetry. In this case, even if the field is originally rolling past this point it is highly probable that the enhanced interactions will temporarily trap it there [14] accounting thereby for the appropriate initial conditions. In this paper we explore further the implications of such interactions to the dynamics of hilltop inflation. If the inflaton’s couplings to other degrees of freedom are not negligible then we may expect the inflaton to perturbatively dissipate some of its energy into other fields, generating thereby a thermal bath during inflation. Hence, we explore whether dissipation effects may give rise to a warm inflation scenario [15].

There are several attractive model building features found in warm inflation scenarios. For one, due to dissipative effects, the inflaton mass during inflation can be much bigger than the Hubble scale [16, 17], thus completely avoiding the η-problem [2, 11, 18, 19], which is a generic problem in standard cold inflation scenarios in supergravity theories. Another attractive model building feature is for monomial potentials, inflation occurs with the inflaton amplitude below the Planck scale [16, 17]. In contrast, for monomial potentials in the cold inflation case, usually called chaotic inflation scenarios [20], the inflaton amplitude during inflation is larger than the Planck scale. This is a problem for model building, since in this case the infinite number of nonrenormalizable operator corrections, $\sim \sum_{n=1}^{\infty} g_n \phi^{4+n}(\phi/m_P)^n$ would become important and so have to be retained [19].

We study a wide range of dissipative effects from weak to strong. In the entire range attractors are found, where during inflation there exists a subdominant thermal bath, with roughly constant temperature maintained by the perpetual perturbative decay of the inflaton. The temperature of this thermal bath is higher than the Hawking temperature corresponding to the (quasi)de Sitter expansion. As a result the curvature perturbation is due to the thermal fluctuations of the inflaton field which dominate over the quantum ones. In the strong dissipation regime the density of the thermal bath eventually takes over the potential density of the inflaton thereby reheating the Universe. In contrast, if dissipation is weak, inflation has to be terminated dynamically, by assuming that the potential steepens further when the field is far enough from the origin, in accordance to hilltop inflation models. To model the cosmology after the end of inflation, in this case, we assume that the inflaton becomes kinetically dominated and drives a brief period of kination [21] until its density is taken over by the thermal bath, which reheats the Universe. This is a scenario which naturally arises in models where the inflaton is characterised by a runaway potential (e.g. a string modulus) and has been employed in models of quintessential inflation [22, 23]. During this kination period the inflaton is oblivious of the scalar potential and, in this sense, out treatment is model-independent.

Throughout this paper we use natural units where $c = \hbar = 1$ and Newton’s gravitational constant is $8\pi G = m_P^{-2}$, where $m_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

II. BASICS OF WARM INFLATION

In the warm inflationary models, the dissipative term appears as an extra friction term in the evolution equation for the inflaton field $\phi$, and as a source term for radiation $\rho_r$:

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V' = 0 ,$$

$$\ddot{\rho}_r + 4H\dot{\rho}_r = \Upsilon \phi^2 ,$$

where $V'$ denotes the derivative of the potential with respect to the inflaton field.

Warm inflation is generically defined as the regime where the temperature of the Universe is larger than the Hawking temperature, $T > H$, since in this regime the spectrum of perturbations of the inflaton is generated from thermal fluctuation, rather than from vacuum fluctuation as in the cold inflation scenario. Within this $T > H$ region there are two distinct types of warm inflation regimes. The first is when $\Upsilon > 3H$ in which the dissipative effects result in extra friction, making the field evolve slower. This is called the strong dissipation regime. The second is when $\Upsilon \lesssim 3H$, in which the dissipative effects have a negligible influence on the evolution.
of the inflaton, and so it then evolves as in the cold inflationary scenario. However since $T > H$, the fluctuations of the inflaton are still thermal. This is called the weak dissipation regime.

With the dissipation coefficient $\Upsilon$ taken into account, the slow roll equations of motion for $\phi$ and $\rho_r$ are

$$\dot{\phi} \simeq -\frac{V'}{(3H + \Upsilon)} ,$$

$$\rho_r \simeq \frac{\Upsilon}{4H} \dot{\phi}^2 \simeq \frac{\Upsilon |V'|^2}{4H(3H + \Upsilon)^2} ,$$

where the last equation holds provided the source term $\rho_f$ in Eq. (2) dominates. It then follows that the produced radiation density is determined by the kinetic density of the field.

The problem is understanding how the inflaton loses energy during inflation from first principles in quantum field theory. Considerable work has been done to address this problem for the case where the entire system remains close to thermal equilibrium [23, 24, 25]. Recently, this equilibrium approach was developed for the low-temperature regime [26], which will prove useful for the analysis in this paper. In this, the dissipation coefficient $\Upsilon$ is computed in a class of supersymmetric models. The dissipation mechanism is based on a two-stage process [25]. The inflaton field couples to heavy bosonic fields, $\chi$, and fermionic fields $\psi_\chi$, which then decay to light degrees of freedom. These light degrees of freedom thermalize to become radiation. The simplest superpotential containing such an interaction structure is

$$W = g\Phi X^2 + hXY^2 ,$$

where $\Phi$, $X$ and $Y$ denote superfields, and $\phi$, $\chi$ and $y$ refer to their bosonic components. Such an interaction structure is common in many particle physics SUSY models during inflation, the field $y$ and its fermionic partner $\tilde{y}$ remain massless, whereas the field $\chi$ and its fermion partner $\psi_\chi$ obtain their masses through their couplings to $\phi$, namely $m_\chi = m_{\psi_\chi} = g_\phi \phi$. The regime of interest is when $m_\chi, m_{\psi_\chi} > T > H$, and this defines what is referred to here as the low-temperature regime. For this regime the dissipation coefficient, when the superfields $X$ and $Y$ are singlets, is found to be [26]

$$\Upsilon \simeq 0.64 g^2 h^4 \left( \frac{g g_\phi}{m_\chi} \right)^4 \frac{T^3}{m_\chi^2} ,$$

where $T$ is the temperature of the radiation bath, $\rho_r = C_r T^4$, where $C_r = \frac{\pi^2}{90} g_*/30$ and $g_*$ is the number of relativistic degrees of freedom. In supersymmetric theories $C_r \simeq 70$. However, the superfields $X$ and $Y$ may belong to large representations of a GUT group. In that case, the dissipation coefficient picks up an extra factor $N = N_\chi N_{\chi}^{\text{decay}}$, where $N_\chi$ is the multiplicity of the $X$ superfield and $N_{\chi}^{\text{decay}}$ is the number of decay channels available in $X$’s decay.

Following this approach, it has been recently shown [27] that chaotic and hybrid inflation models may support some 50-60 $e$-folds of warm inflation in the strong dissipative regime. Using the slow roll parameters $\varepsilon = -\dot{H}/H^2$ and $\eta \equiv m_\phi^4 V''/V$, the condition to have slow roll in warm inflation, for a potential with $\varepsilon \ll \eta$, is found to be

$$\frac{\eta}{1 + \Upsilon/3H} < 1 .$$

Hence, in the strong dissipative regime in which $\Upsilon/3H > 1$, the above holds even if $\eta > 1$. As a result, the field drives slow roll inflation even if it is not effectively massless, thus evading the $\eta$-problem of inflation [16, 17].

### III. A SIMPLE HILLTOP MODEL

In this paper we investigate a hilltop model [7, 8, 10]. We consider the scalar potential

$$V = V_0 - \frac{1}{2} m^2 \phi^2 + \ldots ,$$

where $V_0 = 3H^2 m_r^2$, $m^2 = V''(0)$, and the dots represent higher order terms that become important only after relevant scales exit the horizon during inflation. We assume that the field begins close to the hilltop. We also consider the dissipation mechanism described above, and take $\Upsilon$ as given by Eq. (6)

$$\Upsilon \simeq C_\phi \frac{T^3}{\phi^4} ,$$

where $C_\phi = 0.64 h^4 N$. Writing $T = C_r^{-1/4} \rho_r^{1/4}$ and using Eq. (4), the above equation becomes

$$\Upsilon^{1/4} (3H + \Upsilon)^{3/2} \simeq C_\phi C_r^{-3/4} \left( \frac{|V'|^{3/2}}{(4H)^{3/4} \phi^2} \right) ,$$

which determines $\Upsilon$ in terms of $\phi$ and the parameters of the model. Working now backwards, we may use $\Upsilon$ and Eq. (6) to determine the radiation density and the temperature. Then we work out the field dependence of the ratios $T/H$, $m_\chi/T$, and $\Upsilon/3H$, which is summarised in Table 1.

| Parameter | Weak dissipation | Strong dissipation |
|-----------|------------------|--------------------|
| $T/T$    | $|\eta|^2$        | $|\eta|^{2/7} \phi^{4/7}$ |
| $m_\chi$ | $|\eta|^{-2} \phi$ | $|\eta|^{-2/7} \phi^{5/7}$ |
| $\Upsilon/3H$ | $|\eta|^{6/7} \phi^{-2}$ | $|\eta|^{-2/7} \phi^{2}$ |

Table 1.

It follows that if the field begins in the low-temperature limit, its rolling away from the hilltop maintains the validity of the low-temperature limit, because the ratios $T/H$ and $m_\chi/T$ grow as $\phi$ rolls down the potential.
As a result, to describe the entire evolution of the field during inflation it is only necessary to choose appropriately the initial value of the field so that the condition $H < T < m_\phi$ holds. Once this is satisfied, the model allows a complete analytical study of the entire inflationary stage. In Ref. [28] we discuss a string inspired scenario in which the problem of the initial conditions for warm inflation in the low-temperature limit may be motivated from moduli trapping dynamics.

In this paper we find the parameter space where a warm inflationary stage in the low-temperature limit, both in the weak and strong dissipative regimes, results in enough inflation while producing the observed spectrum of perturbations.

IV. THE WEAK DISSIPATIVE REGIME

In our model, once the system evolves according to the weak dissipative regime with $Y \lesssim 3H$, it remains in such limit for the rest of the evolution, as $Y$ decreases as $T \propto \phi^{-2}$ (see Table 1). Because in this case the dissipative dynamics does not back-react on the evolution of the inflaton field, the latter drives a substantial amount of inflation only if the potential is sufficiently flat. We then assume that the scalar potential satisfies

$$\Delta V < 3\phi C^{-3}$$

inflation only if the potential is sufficiently flat. We then assume that the scalar potential satisfies $|\eta| \ll 1$.

From Eq. (1), we may compute the radiation density during inflation

$$\rho_r \simeq \frac{\dot{Y} V}{36H^3} \simeq 4 \times 10^{-3} C_\phi C_r^{-3} |\eta|^8 H^4,$$

which remains constant as long as $\eta$ is constant. The temperature of the radiation is then

$$T_{\text{inf}} \simeq \frac{1}{4} C_\phi C_r^{-1} |\eta|^2 H,$$

which depends on the particle physics parameters $C_\phi$ and $C_r$. However, to apply consistently the low-temperature regime in the weak dissipation limit we need to impose the conditions $T/H > 1$, $m_\chi/T > 1$ and $Y \lesssim 3H$, which result in the bounds

$$C_\phi > C_r |\eta|^{-2},$$

$$C_\phi < g^2 C_r |\eta|^{-2} \frac{\dot{\phi}}{H},$$

$$C_\phi < 6 C_r^{2/3} |\eta|^{-2} P_{\text{R}}^{-1/3},$$

respectively. We consider that at least the `observable' amount of inflation occurs in weak dissipation, and then take the field value $\phi = \phi_*$ (when cosmological scales are exiting the horizon) to satisfy the above constraints. Also, we require that the thermal spectrum of perturbations reproduces the observed spectrum. In the weak dissipation limit, the prediction for the amplitude of the spectrum of curvature perturbations is [29, 30]

$$P_{\text{R}}^{1/2} \simeq \frac{H}{|V|} (3H + Y) \sqrt{TH}.$$  \hspace{1cm} (16)

Using Eq. (12), the amplitude of the spectrum becomes

$$P_{\text{R}}^{1/2} \sim C_\phi^{1/2} C_r^{-1/3} \frac{H}{\phi_*}.$$  \hspace{1cm} (17)

Owing to the constant temperature of the radiation when the observable Universe leaves the horizon, the amplitude of the spectrum is proportional to the one predicted from vacuum fluctuations. As a result, the spectral index is the same as in cold inflation for models with $\varepsilon \ll \eta$, namely

$$n - 1 = \frac{d \ln P_{\text{R}}}{d \ln k} \simeq -2|\eta|,$$

which for negligible tensor perturbations requires $|\eta| \simeq 0.025$ to satisfy the observations [3, 7]. Inserting $\phi_*/H$ as computed from Eq. (17) into Eqs. (13), (14) and (15), we obtain the following bounds

$$C_\phi > C_r |\eta|^{-2},$$

$$C_\phi < g^2 C_r |\eta|^{-4} P_{\text{R}}^{-1},$$

$$C_\phi < 6 C_r^{2/3} |\eta|^{-2} P_{\text{R}}^{-1/3}.$$  \hspace{1cm} (21)

From Eqs. (19) and (20), the values of $g$ compatible with warm inflation are

$$g > |\eta| P_{\text{R}}^{1/2} \sim 10^{-6},$$

which secures the existence of parameter space for almost any physically interesting value of the coupling $g$. Also, for $g > |\eta| P_{\text{R}}^{1/3} \sim 10^{-5}$, the bound in Eq. (21) is tighter than the one in Eq. (20). In order to keep the discussion simple we neglect the latter.

After cosmological scales have exited the horizon, inflation finishes when $\phi$ grows enough for higher order terms to become important so that $|\eta| \sim 1$. Following Ref. [10], we take then $V = V_0 - \frac{1}{2} m^2 |\phi|^2 + \delta V$, where $\delta V = -|V_n|\phi^n$ and $n > 2$. Apart from terminating inflation, the increase in $|\eta|$, growing from $|\eta| \simeq 0.025$ to $|\eta| \sim 1$, necessarily leads to an increase in the temperature during inflation, as Eq. (12) suggests. Using the field value $\phi_{\text{end}} \approx \left(\frac{3H^2}{2m^2 V_0}\right)^{1/(n-2)}$ at $|\eta| \sim 1$, we compute the temperature at the end of inflation

$$T_{\text{end}} \sim T_{\text{inf}} \left(1 + \frac{|\eta_0|^{-1}}{n - 1}\right)^2,$$  \hspace{1cm} (23)

where $\eta_0 \equiv \eta(\phi = 0)$. We find that even though higher order terms may raise the temperature substantially, the system still evolves within the low-temperature limit.
We consider now that the correction $\delta V$ steepens the potential enough for the field to become dominated by its kinetic density after inflation, $\rho \approx \rho_{\text{kin}} \equiv \frac{1}{2} \dot{\phi}^2 \propto a^{-6}$. Such a phase is called kination \cite{21}. The equation of motion of the inflaton field during kination is $\ddot{\phi} + (3H + \dot{\Upsilon})\dot{\phi} \approx 0$, and its evolution then becomes oblivious to the potential. One can envisage that this phase of kination would finish when $\Upsilon$ (being initially small: $\Upsilon < 3H$) dominated over $H$, since the latter decreases drastically during kination, $H \propto a^{-3}$. However, because $T \propto a^{-1}$ and $\phi$ continues growing after inflation, from Eq. \ref{eq:kin} it follows that $\Upsilon$ decreases even faster than $H$. Consequently, the phase of kination lasts until the radiation bath (whose density decreases as $\rho_r \propto a^{-4}$) becomes comparable to the kinetic density of the field. At that moment the Hot Big Bang evolution is recovered\footnote{A similar scenario has been recently discussed in the context of curvaton reheating \cite{31}.}. The growth of the scale factor from the end of inflation until reheating, $a_{\text{rh}}$, is given by $(\rho_r)_{\text{end}} a_{\text{rh}}^{-4} \sim V_0 a_{\text{rh}}^{-6}$, and the reheating temperature, $T_{\text{rh}} = T_{\text{end}} a_{\text{rh}}^{-1}$, is then

$$T_{\text{rh}} \sim C_r^{1/2} T_{\text{end}}^{3/2} H_{m_p}. \tag{24}$$

In order not to violate the current bounds on gravitino overproduction, this reheating temperature must be kept sufficiently low. However, in the warm inflation scenario, gravitinos are not only produced when the Universe is reheated. Owing to the thermal bath during warm inflation, gravitinos may also be produced at the end of inflation. The relevant temperature then being given by Eq. \ref{eq:bound}, rather than by Eq. \ref{eq:bound2}. Nevertheless, in Ref. \cite{34} it is argued that the relevant temperature to constrain gravitino overproduction is the temperature at reheating. The reason is that the entropy produced by the decay of the inflaton during reheating can dilute the gravitinos produced at the end of inflation \cite{35}. Under this assumption, the reheating temperature must then be larger than the temperature at Nucleosynthesis and less than current bound due to gravitino overproduction \cite{33}, set to $T_{\text{rh}} \lesssim T_{m_{3/2}} \equiv 10^{-3} \text{ GeV}$ for a gravitino mass $m_{3/2} = \mathcal{O}(1) \text{ TeV}$. Using Eqs. \ref{eq:bound}, \ref{eq:bound2} and \ref{eq:bound3}, the condition $T_{\text{BBN}} < T_{\text{ch}} < T_{m_{3/2}}$ results in the bounds

$$C_{\phi} < 4 \left( \frac{T_{m_{3/2} m_p}}{H^2} \right)^{1/3} \left( |\eta_0| + \frac{1}{n-1} \right)^{-2} C_r^{5/6}, \tag{25}$$

$$C_{\phi} > 4 \left( \frac{T_{\text{BBN} m_p}}{H^2} \right)^{1/3} \left( |\eta_0| + \frac{1}{n-1} \right)^{-2} C_r^{5/6}. \tag{26}$$

We find that, even if the temperature relevant to constrain gravitino overproduction is the temperature at the end of inflation, $T_{\text{end}}$, then there still exists substantial parameter space available.

There is yet another bound to take into account, concerning the spectrum of relic gravitational waves. Gravitational waves (GWs) do not couple to the thermal background of warm inflation. Their spectrum is then generated by quantum fluctuations. This spectrum, owing to the phase of kination that follows after inflation, has a slope which grows with the frequency for those modes reentering the horizon during inflation \cite{32}. High frequency gravitons crossing inside the horizon during kination may then alter the light element abundances predicted by Nucleosynthesis by increasing $H$. To avoid this we require \cite{22, 52}

$$I \equiv h^2 \int_{k_{\text{BBN}}}^{k_*} \Omega_{\text{GW}}(k) d\ln k \leq 2 \times 10^{-6}, \tag{27}$$

where $\Omega_{\text{GW}}(k)$ is the GW density fraction with physical momentum $k$ and $h = 0.73$ is the Hubble constant $H_0$ in units of 100 km/sec/Mpc. Using the spectrum $\Omega_{\text{GW}}(k)$ as computed in \cite{32}, the above constraint can be written as

$$I \approx h^2 \int_{k_{\text{BBN}}}^{k_*} \alpha_{\text{GW}}(k_0) \frac{1}{\pi^2 m_p} \left( \frac{H}{H_{\text{rh}}} \right)^{2/3}, \tag{28}$$

where $\alpha_{\text{GW}} \approx 0.1$ is the GW generation efficiency during inflation, $\Omega_{r}(k_0) = 2.6 \times 10^{-3} h^{-2}$ is the density fraction of radiation at present on horizon scales, and $H_{\text{rh}} \sim (C_r^{1/2} T_{\text{rh}}^3)/m_p$ is the Hubble parameter at the time of reheating. Using Eq. \ref{eq:bound}, the bound in \ref{eq:bound3} translates into the bound

$$C_{\phi} > C_r^{3/4} |\eta|^{-2}, \tag{29}$$

which is roughly as strong as the bound in Eq. \ref{eq:bound}. All bounds considered, the available parameter space leading to a successful inflationary cosmology is depicted in Fig. \ref{fig:1}. The parameter $C_r$ counting the number of relativistic degrees of freedom has only a relative impact on the results. The only significant effect being just a slight displacement of the allowed region downwards \{upwards\} for lower \{higher\} values of the parameter.

We may summarize the results saying that even if the dissipation mechanism does not bring the system into the strong dissipative regime, there is still enough dissipation to maintain a radiation bath $\rho_r$ which may reheat the Universe after inflation to temperatures high enough to satisfy the Nucleosynthesis bound, but not so high as to challenge the current bounds on gravitino overproduction. There is a great deal of parameter space for $C_{\phi}$, which also lies in the range of interest to GUT theories. Also, owing to the phase of kination that follows after inflation, the Hubble scale may be lowered down to the GeV scale \cite{31}. Decreasing $H$ during inflation even further would move the system into the strong dissipative regime, as $\Upsilon$ would become important when compared to $3H$. 

\[\text{References:} \]
FIG. 1: log $C_\phi$-log $H$ plane in the weak dissipation limit. After the relevant scales leave the horizon, inflation is terminated by corrections of the potential $\delta V = -|V_n|\phi^n$. The case shown correspond to a correction term with $n = 4$. The available parameter space (shaded area) is constrained by gravitino overproduction (using $T_{\text{grav}} \approx 10^{7-8}\text{GeV}$), the Nucleosynthesis bound (using $T_{\text{BBN}} \sim \text{MeV}$), and the relic graviton abundance at Nucleosynthesis, Eqs. (25), (26) and (29), respectively. Consistency with warm inflation imposes a lower bound on the amount of dissipation, whereas the strong dissipation regime results in an upper bound, Eqs. (19) and (21) respectively.

V. THE STRONG DISSIPATIVE REGIME

We consider now the case in which $\Upsilon$ is large enough for the system to remain in strong dissipation until the end of inflation. In this case, Eq. (4) implies that $\rho_r > \rho_{\text{kin}}$ during the inflationary stage. Therefore, inflation finishes when the radiation density has grown enough for the Universe to become radiation dominated, i.e. when $\rho_r(\phi_{\text{end}}) \sim H^2 m_P^2$. In this case, it is not necessary to invoke higher order terms in $V(\phi)$ to terminate inflation, and we consider that $|\eta|$ remains constant until inflation is finished.

Using Eqs. (4) and (14), we compute the radiation density in strong dissipation, whose temperature is

$$T \simeq (C_\phi C_r)^{-1/7} |\eta|^{2/7} \left( \frac{\phi}{H} \right)^{4/7} H.$$  

Now we may compute $\phi_{\text{end}}$ using the condition $\rho_r(\phi_{\text{end}}) \sim H^2 m_P^2$,

$$\phi_{\text{end}} \sim C_r^{-3/16} C_\phi^{1/4} |\eta|^{-1/2} \left( \frac{H}{m_P} \right)^{1/8} m_P.$$  

Inserting this into $\Upsilon(\phi_{\text{end}}) > 3H$ to have strong dissipation until the end of inflation we obtain

$$C_\phi > 4 C_r^{3/4} |\eta|^{-2} \left( \frac{m_P}{H} \right)^{1/2}.$$  

To apply this scenario consistently we need to impose the low-temperature conditions. The analogous of Eqs. (13), (14), and (15) are now

$$C_\phi < C_r^{-1} |\eta|^2 \left( \frac{\phi}{H} \right)^{4},$$  

$$C_\phi > g^{-7} C_r^{-1} |\eta|^2 \left( \frac{\phi}{H} \right)^{-3},$$  

$$C_\phi > C_r^{3/4} |\eta|^{-3/2} \left( \frac{\phi}{H} \right)^{1/2}.$$  

Exactly as we did in the weak dissipation regime, we take $\phi$ in the above constraints when the observable Universe is leaving the horizon, at $\phi = \phi_*$, and then fix $\phi_*$ by using the amplitude of the spectrum of curvature perturbations. In the case of strong dissipation the amplitude of the spectrum is given by

$$P_{\mathcal{R}}^{1/2} \simeq \frac{H^2}{|\Upsilon|} \left( \frac{\pi \Upsilon}{12 H} \right)^{1/4} (3H + \Upsilon)^{3/4} \simeq \frac{T}{H},$$  

which in our model results in

$$P_{\mathcal{R}}^{1/2} \simeq 0.4 C_\phi^{9/14} C_r^{-17/28} |\eta|^{3/14} \left( \frac{H}{\phi_*} \right)^{15/14}.$$  

Using this to compute $(\phi_*/H)$, the bounds in Eqs. (33), (34), and (35) taken at $\phi = \phi_*$ become

$$C_\phi > C_r^{7/3} |\eta|^{-2} P_{\mathcal{R}}^{4/3},$$  

$$C_\phi > g^{-5/2} C_r^{1/4} |\eta|^{1/2} P_{\mathcal{R}}^{1/2},$$  

$$C_\phi > C_r^{2/3} |\eta|^{-2} P_{\mathcal{R}}^{-1/3}.$$  

In the range of $C_r$ of interest to particle physics models, the bound in Eq. (33b) is weaker than the one in Eq. (14). Also, the bound in Eq. (33a) is tighter than the one in Eq. (10) for $|\eta| > g P_{\mathcal{R}}^{-1/3}$. Note that in the strong dissipation regime it is possible to have slow roll inflation with $|\eta| > 1$.

After the relevant cosmological scales exit the horizon, some $N_* \simeq 50$ e-folds must follow. Using Eq. (3), we obtain

$$N_* \simeq 1.6 C_\phi^{4/7} C_r^{3/7} |\eta|^{-1/7} \left[ \left( \frac{H}{\phi} \right)^{2/7} \right]^{\phi_*/\phi_{\text{end}}},$$  

which depends on $H$. Therefore, $H$ must be tuned according to this equation. We obtain

$$\left( \frac{H}{m_P} \right)^{1/4} \sim 1.2 \frac{C_r^{13/120} P_{\mathcal{R}}^{2/15}}{C_\phi^{1/10} |\eta|^{1/5}} - 0.6 \frac{C_r^{3/8} N_*}{C_\phi^{1/2}}.$$  


If we neglect the negative contribution to $H$, the bound in Eq. [22] (coming from $\Upsilon(\phi_{\text{end}}) > 3H$, which ensures radiation domination after inflation) is equivalent to the one in Eq. [11] (corresponding to $\Upsilon(\phi_x) > 3H$). This is expected, because owing to the extra friction provided by the dissipation term, the inflaton field varies very little during the last $N_* \simeq 50$ e-foldings. Hence the ratio $\Upsilon/3H \propto \phi^{-2/7}$ does not change substantially. We may also use the above equation to eliminate $H$ from Eq. [22], obtaining

$$C_{\phi}^{2/5} > C_r^{4/15} P_{R}^{-2/15} \left( 1.6|\eta|^{-4/5} + \frac{N_* |\eta|^{1/5}}{2} \right).$$

(43)

This bound determines the region in the log $C_{\phi}$-log $|\eta|$ plane where inflation results in a spectrum of superhorizon perturbations with the observed amplitude. Once this bound is satisfied, so is that in Eq. [40]. However, the space determined by the above must be further constrained by the bound in Eq. [39] if $g < C_r^{-1/6} P_{R}^{1/3} N_*^{-1} \sim 10^{-5}$. Excluding this case (i.e. for $g > 10^{-5}$), the available parameter space is completely determined by Eq. [43]. This parameter space is depicted in Fig. 2, where we also enforced the requirement that $\phi_{\text{end}} \lesssim m_P$ because over super-Planckian distances in field space, the scalar potential is not well understood.

The spectral index $n-1 = \frac{d \ln P_{R}}{d \ln k}$, upon using Eq. [37], becomes

$$n \simeq 1 - \frac{7 C_r^{4/15} |\eta|^{1/5}}{2 C_{\phi}^{2/5} P_{R}^{-2/15}}.$$

(44)

The dark grey (reddish) strip in Fig. 2 encompasses the 1-σ result for the spectral index, $n = 0.948^{+0.016}_{-0.015}$; obtained from WMAP+SDSS data in the $\Lambda$CDM model for $\epsilon \ll 1$. The running of $n$ is

$$\alpha = \frac{d n}{d \ln k} \simeq -0.13(n - 1)^2,$$

(45)

which is of order $10^{-4}$ within the 1-σ window.

Non-Gaussian effects during warm inflation have been studied [36, 37] and it is interesting to see what these analysis predict for this model. In the strong dissipative regime it was shown in [36] that entropy fluctuations during warm inflation play an important role in generating non-Gaussianity, with the prediction

$$-15 \ln \left(1 + \frac{r}{14}\right) - \frac{5}{2} \lesssim f_{NL} \lesssim \frac{33}{2} \ln \left(1 + \frac{r}{14}\right) - \frac{5}{2},$$

(46)

where $f_{NL}$ is the non-linearity parameter and $r \equiv \Upsilon/3H$. For the warm inflation results in Fig. 2, $r$ ranges from 10 to $10^6$, and this implies from the above equation that $|f_{NL}|$ ranges from 10 to 180. This is an interesting result in light of the recent WMAP analysis of the third year CMB data [38] which find $26.9 < f_{NL} < 146.7$ at 95% confidence level. This corresponds to a rejection of a Gaussian spectrum ($f_{NL} = 0$) at the 99.5% confidence level. If these first WMAP results are confirmed by future data, and in particular by data from Planck surveyor satellite [39], this would exclude conventional cold inflation models which generally yield very low values of $f_{NL} \lesssim 1$. On the other hand, the strong dissipative warm inflation regime, such as the one found in this hilltop model, would be consistent with this recent WMAP analysis. For example in this case the WMAP upper limit on the parameter $f_{NL} \lesssim 150$ translates in an upper limit on $r \lesssim 1.4 \times 10^5$, and therefore from Fig. 2 on a lower limit on the scale of inflation given by $H \gtrsim 10^7$ GeV.

Summarizing, it is possible to match the amplitude of the spectrum of perturbations in a large region of the log $C_{\phi}$-log $|\eta|$ plane. Remarkably, the observations may be matched for $|\eta| \gtrsim 1$, thus avoiding the $\eta$-problem of inflation. The number of fields required in this case (taking $h^2 \sim 4\pi$ in $C_{\phi} = 0.64 h^4 N$) is large, $N \sim 10^{6-7}$. The scale of inflation needed in the model to match observations requires $H \gtrsim 10^9$ GeV, which challenges present bounds on gravitino overproduction, however see Ref. [40]. Reducing further the Hubble scale during inflation is only possible by using $|\eta| \gg 1$, which requires $C_{\phi} \gg 10^8$. A low Hubble scale results in a larger value of the non-Gaussian parameter $f_{NL}$, which approaches the observational bounds and may well be observable in the near future [39].
VI. CONCLUSIONS

We have studied warm inflation in the context of hilltop models. There are important differences between this type of warm inflation and the existing literature. This is because the value of the inflaton field increases during inflation as it rolls down the potential hill, which results in a decreasing dissipation coefficient $\Upsilon$. This, ensures that the validity of the low-temperature approximation persists throughout the evolution of the system, but also tightens the constraints on the amount of dissipation needed for the field to drive enough inflation [27]. During inflation the dissipation of the inflaton’s energy generates a thermal bath of roughly constant temperature $T$, larger than the Hawking temperature. Hence, the curvature perturbation is due to thermal instead of quantum fluctuations. The Universe is reheated when this thermal bath dominates.

When $T \lesssim 3H$ warm inflation occurs in the so-called weak dissipative regime, when dissipation does not affect the dynamics of the inflaton. In this case inflation has to be terminated by higher-order terms in the scalar potential, which steepen its slope. We have studied this case assuming that a brief period of kination follows inflation, until reheating when the radiation bath takes over the kinetic density of the inflaton. This is reasonable to expect in case the potential is of runaway type, as is the case for string moduli fields. Such a potential has been considered in models of quintessential inflation [22]. We have found that there is ample parameter space for warm hilltop inflation in the weak dissipative regime (see Fig. 1) which interpolates between the usual cold inflation case and the strong dissipative regime. The parameter space allows low-scale inflation with $H$ as low as 1 GeV, in accordance to Ref. [31]. This also implies that the reheating temperature is low enough not to result in gravitino overproduction. In this regime $T/H \simeq$ constant, which means that the curvature perturbation depends only on $H$ as in cold inflation. Consequently, the spectral index is identical to the cold hilltop inflation case: $n \approx 1 - 2\left|\eta\right|$, where $\left|\eta\right| \approx 0.025$ in order to account for the CMB observations. Hence, a mild tuning of the curvature of the potential at the top of the hill is required, corresponding to the usual $\eta$-problem of inflation.

When $T > 3H$ warm inflation occurs in the so-called strong dissipative regime, when the dissipation does control the dynamics of the inflaton field. In this case, the temperature of the thermal bath follows the growth of the inflaton as $T \propto \phi^{1/3}$. Thus inflation can end with prompt reheating, when the radiation density overtakes the potential density of the field. Kination or higher order terms in the potential need not be considered. The extra friction in the variation of the inflaton due to the dominant dissipation term allows slow-roll inflation with $\left|\eta\right| \approx 1$ overcoming thereby the $\eta$-problem of inflation. We have found that, despite such a large value of $\left|\eta\right|$ there is considerable parameter space where the spectral index of the curvature perturbation agrees with CMB observations (see Fig. 2). However, inflation needs to take place at high energies, which challenges gravitino overproduction constraints, unless $C_{\phi} \sim 10^2N$ is rather large. Because $N$ is determined by the field content of the theory this scenario is best realised in the context of string theories with a large number of degrees of freedom. In addition, strong dissipation during inflation can result in substantial non-Gaussianity in the perturbation [37], which consistent with the recent WMAP analysis on non-Gaussianity [38] and may become detectable in the near future by the Planck mission [39]. However, the upper limit on the amount of non-Gaussianity imposes a lower bound on the Hubble scale, excluding the possibility of low-scale inflation.

The above show that warm hilltop inflation is indeed possible both with strong and weak dissipation. The scenario is distinct compared to both cold hilltop inflation and also to the other types of warm inflation studied in the literature [27, 41, 42]. With weak dissipation low-scale inflation is possible, while strong dissipation overcomes the $\eta$-problem. In both cases, the Universe is reheated by the thermal bath due to dissipation. This offers the intriguing possibility that the inflaton does not decay after inflation, in which case it may survive until late times and play the role of quintessence.

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