Asymmetric Opportunities After an Unsuccessful Sports Career

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Abstract
Contestants enter a risky contest when pursuing a sports career or choose a secure outside option. If contestants enter this contest but their sports career fails, they may have asymmetric career opportunities outside of sport. Greater opportunities reduce the risk of entering this contest. However, contestants’ incentives to exert effort decrease. Two types of equilibria exist if the initial pool of contestants is large. Either only types with high opportunities or only types with low opportunities enter the sports contest. If the initial pool of contestants is low, both types of contestants participate in the contest.

Keywords
sports career, contest, outside option

Introduction
Roger Federer pursued a high-risk strategy. At an early age, he put all his eggs in one basket; i.e. the tennis basket. Ex-post, this strategy seemed to be optimal. However, Roger Federer points out that this choice was fraught with risk (Bowers, 2016):

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It was a pretty big risk for me to stop school at sixteen because I didn’t have an ATP ranking at that time. Maybe I was 800th or something, and in the juniors I was, like, 60th or something. But somehow I felt that school was disturbing me from being one hundred per cent focused on tennis.

Ex-ante, Roger Federer had to be aware that a serious injury could end his sports career at a young age.¹ In this case, he received very specific training, and this specificity may not have been in demand in the labor market outside of the tennis court. Other attractive career options that were realistic as an adolescent would likely no longer be available for Roger Federer. Under this negative scenario, he would possibly wish—ex-post—that he had put more resources into other or less riskier fields (e.g., continuing education, a job in the public sector). Thus, how can we rationalize² Roger Federer’s decision at the age of 16 to leave school and enter the highly competitive tennis circuit from which perhaps only 100 players worldwide earn a financially high standard of living. This question is particularly interesting when we take into account that Roger Federer presumably had great opportunities in less risky fields in a wealthy country such as Switzerland, with low unemployment rates and high wages in many industries.³

The anecdotal example of Roger Federer shows that athletes obviously weigh up the opportunities and risks of leaving school and pursuing a career in sports. The risks are reduced by institutions offering competitive sports in which school education is integrated. Traditionally, the connection between sports and education is particularly strong in the US where highly talented student athletes are often supported by sports scholarships.⁴ In the end, however, successful sports careers are relatively rare. Insler and Karam (2019) mention an advertising campaign on national television by the National Collegiate Athletic Association (NCAA):

There are over 380,000 student athletes, and most of us go pro in something other than sports.

This campaign highlights the importance of a valuable college education for student athletes. Despite this, a degree can only be helpful if the quality of education is good. A few decades ago, there was major criticism in the US that student athletes leave colleges with insufficient qualifications. The NCAA responded to this criticism by introducing specific rules and programs. For instance, educational institutions had to limit the hours student athletes spent on sport. Moreover, they were required to offer academic support services for student athletes. In 2004, the Academic Progress Rate (APR) rule was established, which encourages colleges and universities to improve the academic qualifications earned by students (see, e.g., Gayles & Hu, 2009). However, as graduating improves the fallback position of student athletes, this fallback position may have an undesirable side effect: Better opportunities outside of sports may reduce the motivation of athletes in their sports discipline.
In this paper, I provide a theoretical model which analyzes the discussed issues. In the model, a large number of contestants either enter a risky sports competition or choose a secure outside option. Risky sports competitions can be interpreted as a long-lasting competition between contestants in which a contestant invests at an early age and may or may not be successful in later years. Thus, the outcome of the competition is either a successful or unsuccessful international sports career. Two different types of contestant exist; these types differ with respect to their life-time value $F$ in the case of an unsuccessful sports career. Asymmetric values of $F$ can be interpreted as follows. (i) Contestants have different second-best talents that can be applied in other job fields. (ii) Job chances of contestants differ between countries when an unsuccessful contestant seeks an alternative at home after an unsuccessful international sports career. For instance, the unemployment rate in country $A$ is smaller than in country $B$. In this case, the different types reflect different labor market conditions in the countries in which the contestants live. (iii) Countries provide different degrees of support for people in need. In the case of an unsuccessful international sports career and no alternatives at home, support from social insurance systems usually differs between countries. In this case, $F$ represents the different extent of safety nets provided by individual countries.

The main results of the paper are as follows. On the one hand, greater opportunities reduce the risk when a contestant enters the contest, because there is a softer landing in the case of unsuccessful participation. As a consequence, the contestant’s expected payoffs increase. On the other hand, the incentives to exert effort decrease due to two effects: (i) greater opportunities directly decrease the marginal revenue of effort and (ii) marginal revenue of effort decreases indirectly as higher payoffs in the contest attract more contestants. Moreover, I derive that there exists no equilibrium in which both types participate in the sports contest if the number of contestants within both types is large. In this case, equilibria exist only when contestants with high values of $F$ or only contestants with low values of $F$ participate. If the number of contestants of both types is low, however, it is possible that both types participate simultaneously in the contest. Finally, I reveal that the contest organizer cannot expect high individual and high aggregate effort levels if the number of both types is large. Depending on the type of equilibrium, either individual or aggregate efforts are high, but not both.

**Literature on Contest Theory**

Note that, technically, this model corresponds to a contest with asymmetric second prizes. My paper provides a new interpretation of second prizes in the contest literature. $F$ is not part of the contest prize sum but reflects the ex-post option a contestant has compared with the ex-ante option (i.e., the outside option). This interpretation is new in the contest literature. The literature on contests with multiple prizes mainly focuses on the allocation of the prize sum on $n \geq 1$ prizes, while
neglecting participants’ endogenous entry decisions. The main findings in this literature are as follows.

First, the optimal allocation of the prize sum on one or more prizes depends on the functional form of effort costs (Moldovanu & Sela, 2001). The authors conclude that several prizes can only be optimal for the contest organizer if effort costs are convex. For linear and concave costs, a single prize for the winner of the contest is always optimal.

Second, while an increase in the first prize usually increases efforts, an increase in the second prize may have an ambiguous effect on incentives (Sisak, 2009). In the literature, the prizes are usually assumed to be symmetrical for contestants, whereas, in my model, the second prizes are asymmetric. Clark and Riis (1998a) consider an all-pay auction with n symmetric prizes, but in which contestants have different prize valuations. They neglect the case where an asymmetry between contestants exists only in the second prizes. Clark and Riis conclude in their paper that only the contestants with higher prize valuations participate in the contest because these contestants have larger effort incentives. In contrast, my model predicts equilibria with low types (i.e., contestants with low valuations of the second prize) participating in the contest. The reason for this result is that low types have higher effort incentives compared to high types. The prize gap between winning and losing is larger for low types such that high types do not enter the contest.

Literature on Career Decisions and Outside Opportunities in Sports

Some papers consider athletes’ career decisions. One line of research analyzes the age of young basketball, football and baseball players entering the NBA, NFL and MLB after college (Arel & Tomas, 2012; Böheim & Lackner, 2012; Winfree & Molitor, 2007), respectively. Arel and Tomas (2012) examine basketball players’ optimal entry date into the NBA draft. Players can return to school for 1 year or enter the draft early. The authors argue that the player’s decision is similar to an investor’s financial decision of an early exercise of an American style put option. Depending on the draft position, the authors estimate the value of the player’s option. The trade-off for a player is as follows. On the one hand, the argument for an early draft is that a player has a higher number of expected years in the league. On the other hand, going to school for 1 year longer can be worthwhile, because the draft position can be improved in the following year guaranteeing higher salaries in the future. The authors find that players should enter the draft early only if they are expected to be drafted in the first round. For weaker players, returning to school for 1 year could be profitable. Böheim and Lackner (2012) analyze data from the NFL using an instrument variable test to estimate salaries. The month of birth serves as an instrument for entry age, which in turn affects salaries. They empirically show that 1 more year in college increases the starting salary of football players in the NFL by 6%. However, the longer schooling probably leads to a shortened career, such that the advantage can be overcompensated. Winfree and Molitor (2007) analyze the
decision of high school athletes to either immediately play professional baseball or attend college. They apply a model based on the calculation of expected discounted present value of lifetime earnings. The underlying model considers the benefits of college education after a baseball career. Unsurprisingly, the authors empirically find that it is better for baseball players drafted in lower rounds in the MLB to return to school. Additionally, they quantify the effects in monetary terms. In contrast to these papers, I use a game-theoretic approach. Moreover, I focus on incentive effects of prize differentials (between the value of outside option, the prize in the competition and \( \Phi \)) on entry decision and effort provision.

Empirical studies show that education and athlete performance are correlated. For instance, Insler and Karam (2019) analyze data from the U.S. Naval Academy and conclude that sports participation has a small (but statistically significant) negative impact on grades. Gandelman (2009) analyzes data from the Uruguayan soccer league. The author finds that better-educated soccer players and players of higher socioeconomic background are expected to perform better. Therefore, there is a positive relationship between education and performance and socioeconomic background and performance.\(^{10}\)

This paper has the following structure. First, I analyze the main model under the assumption that the pool of each type of contestant is large. Then, I present the results, considering the case with homogeneous and heterogeneous contestants; as well as comparing different types of equilibria. After that, I additionally analyze the model in relation to small pools of each type. Finally, the paper concludes.

**Model**

In stage one, contestants decide whether to enter a sports competition or not. If contestants do not participate in the sports competition, they choose the outside option with value \( \Omega > 0 \). If they choose the risky alternative, then the contestants proceed to stage two and aspire to an international career in sports. In stage two, they will be successful with the probability \( p \) of winning the sports career prize \( V > \Omega \). With probability \( 1 - p \), they will be unsuccessful. In this case, they win the lifetime value \( \Phi < \Omega \).\(^{11}\)

Two types of contestants exist. In the main part of the model, I assume that there is a large number of high (low) types \( A (B) \) contestants, for which \( \Phi^A \geq \Phi^B \) holds. \( N (M) \) represents the number of high (low) types that finally enter the sports contest. Type \( A (B) \) contestants enter the contests if the expected profit \( \pi_n > \Omega \) \( (\pi_m > \Omega) \). Therefore, the two conditions \( \pi_n = \Omega \) and \( \pi_m = \Omega \) hold in equilibrium.\(^{12}\) As the potential pool of contestants is large for each type, there will be contestants of each type that do not enter the contest.\(^{13}\) In the extension, I assume that the pool of each type can be small such that the expected profit of certain contestants can be larger than \( \Omega \).

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If a contestant participates in the contest, the expected profits of representative \( n \) (\( m \)) of type \( A \) (\( B \)) are as follows:

\[
\pi_n = p_n V + (1 - p_n)\Phi^A - x_n = p_n (V - \Phi^A) + \Phi^A - x_n, \tag{1}
\]

\[
\pi_m = p_m V + (1 - p_m)\Phi^B - x_m = p_m (V - \Phi^B) + \Phi^B - x_m, \tag{2}
\]

where the last term in (1) and (2) represents the linear effort costs. The winning probability \( p_n \) (and, similarly, \( p_m \)) is determined according to the classical Tullock contest success function (CSF)\(^{14}\) in which the ratio of effort \( x_n \) of contestant \( n \) relative to the aggregate effort \( X \) determines the winning probability of contestant \( n \)\(^{15}\):

\[
p_n = \frac{x_n}{X} \quad \text{with} \quad X \equiv \sum_{i=1}^{N} x_i + \sum_{j=1}^{M} x_j.
\]

The model is solved using backward induction. First, contestants solve their stage two optimization problem.

**Optimization at Stage 2**

Optimization of the objective functions (1) and (2) requires the following first-order conditions for representatives \( n \) (type \( A \)) and \( m \) (type \( B \)).

\[
\frac{\partial \pi_n}{\partial x_n} = \frac{X - x_n}{X^2} (V - \Phi^A) - 1 = 0, \tag{3}
\]

\[
\frac{\partial \pi_m}{\partial x_m} = \frac{X - x_m}{X^2} (V - \Phi^B) - 1 = 0, \tag{4}
\]

with \( n = 1, \ldots, N \) and \( m = 1, \ldots, M \). These first-order conditions determine contestants’ optimal behavior at stage two.

**Entry at Stage 1**

At stage one, contestants anticipate their optimal second stage behavior. They enter the sports competition at stage one if the expected second stage profit in the sports competition is larger than the value of their outside options.

The paper concentrates on subgame-perfect equilibria in pure strategies in which identical types choose symmetric effort levels if they decide to enter the contest.

**Results**

**Homogeneous Contestants**

As a reference, I first sketch the results in the case of homogeneous contestants, i.e. \( \Phi^A = \Phi^B \equiv \Phi \). The homogenous case allows me to highlight the fundamental
incentive effects and subsequently helps to detect the additional effects in the heterogeneous case.

**Lemma 1** If contestants are homogeneous, then

(i) the number of participants in the contest is \( n^* = \sqrt{\frac{V - \Phi}{\Omega - \Phi}} \),

(ii) contestant \( i \)'s effort \( x_i^* = \sqrt{\Omega - \Phi}(\sqrt{V - \Phi} - \sqrt{\Omega - \Phi}) \),

(iii) aggregate efforts are \( X^* = \sqrt{V - \Phi}(\sqrt{V - \Phi} - \sqrt{\Omega - \Phi}) \), and

(iv) the expected profit of contestant \( i \) is \( \pi_i^* = \Omega \) in equilibrium.

**Proof.** See Appendix. ■

Result (i) of this lemma implies that \( n^* > 2 \Leftrightarrow \frac{V - \Phi}{\Omega - \Phi} > 4 \). Thus, the value of successful contest participation must be sufficiently large compared with the outside option in order to guarantee the participation of more than two competitors in the contest. Assumption 1 takes this aspect into account:

**Assumption 1:** \( \frac{V - \Phi}{\Omega - \Phi} > 4 \) such that more than two (homogeneous) contestants participate in the contest.

Based on Lemma 1, I obtain the following three corollaries:

**Corollary 1** In equilibrium, the number of contestants \( n^* \) is

(i) increasing in \( V \),

(ii) decreasing in \( \Omega \), and

(iii) increasing in \( \Phi \).

**Proof.** Straightforward and therefore omitted. ■

**Corollary 2** Under Assumption 1, contestant \( i \)'s effort \( x_i^* \) is

(i) increasing in \( V \),

(ii) increasing in \( \Omega \), and

(iii) decreasing in \( \Phi \) in equilibrium.

**Proof.** Straightforward and therefore omitted. ■

**Corollary 3** Aggregate efforts \( X^* \) are

(i) increasing in \( V \),

(ii) decreasing in \( \Omega \), and

(iii) increasing in \( \Phi \) in equilibrium.

**Proof.** Straightforward and therefore omitted. ■

The results of these corollaries are intuitive. (i) A larger prize \( V \) increases effort incentives \((x_i^* \uparrow)\) and increases expected profit such that the participation in the sports contest becomes attractive for more contestants \((n^* \uparrow)\). Clearly, aggregate
efforts then increase. (ii) A larger value for the outside option $\Omega$ means that contest participation is less attractive $(n^* \downarrow)$. A lower contest participation number increases marginal revenue in the contest $(x_i^* \uparrow)$. However, the first effect $(n^* \downarrow)$ dominates the second effect $(x_i^* \uparrow)$ such that aggregate efforts decrease for a larger $\Omega$. (iii) A larger value of $\Phi$ decreases the marginal revenue in the contest $(x_i^* \downarrow)$. Lower effort means higher expected profits $\pi_i$ in the contest. Therefore, contestants enter the contest $(n^* \uparrow)$ until $\pi_i = \Omega$ holds in equilibrium. Here, the second effect $(n^* \downarrow)$ dominates the first effect $(x_i^* \downarrow)$ such that aggregate efforts increase for a larger $\Phi$.

**Heterogeneous Contestants**

Suppose that the lifetime value of an unsuccessful sports career $\Phi$ is asymmetric between the two types $(V > \Omega > \Phi^A > \Phi^B)$. Thus, type $A$ contestants have a higher lifetime value in the case of an unsuccessful sports career compared with type $B$ contestants.

As a first result, I derive that there exists no equilibrium in which both types are active in the sports contest.

**Lemma 2** In the case of $\Phi^A > \Phi^B$, there exists no equilibrium in which members of both types simultaneously participate in the sports contest.

**Proof.** See Appendix.

Assuming $\Phi^A > \Phi^B$, one expects that the marginal revenue of effort in the sports contest is smaller for type $A$ contestants compared with type $B$ contestants, according to the discussion concerning the homogeneous case in the preceding section. Intuitively, this could imply that type $A$ contestants were less competitive in the sports contest and contest participation was less attractive for them. The following proposition shows that this is not necessarily the case.

**Proposition 1** In the case of $\Phi^A > \Phi^B$, there always exists a subgame-perfect equilibrium, in which only type $A$ contestants enter the sports contest.

**Proof.** See Appendix.

Thus, this proposition refutes the idea that there is a crowding out of type $A$ contestants. Even if type $A$ contestants are less motivated in the sports contest due to a softer landing opportunity in the event of an unsuccessful career, there exists an equilibrium in which only $A$ types enter the contest. The reason for this result is as follows. Greater opportunities of type $A$ contestants increase their expected profits in the contest. As a result, more type $A$ contestants enter the contest such that there is a crowding out of type $B$ contestants. Thus, greater opportunities do not necessarily destroy risk-taking behavior.

However, Lemma 2 and Proposition 1 together do not imply that an equilibrium in which only $B$ types enter the sports contest can be excluded.
Proposition 2 In the case of $A > B$, there exists a subgame-perfect equilibrium, in which only type $B$ contestants enter the sports contest if

$$\left(\sqrt{V - \Phi^A} - \sqrt{\Omega - \Phi^B}\right)^2 < \sqrt{V - \Phi^B} \left(\sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B}\right).$$

Proof. See Appendix. 

Thus, this proposition shows that an equilibrium with $B$ types exists if a critical condition holds. It is worth considering the critical condition in more detail. Figure 1 plots the LHS and RHS of the critical condition as a function of $A$ on the interval $[B, \Omega]$. It is easy to show that the LHS as a function of $A$ is

- (a) monotonically increasing in $A$ on the interval $[B, \Omega]$.
- (b) strictly convex in $A$ on the interval $[B, \Omega]$.

The RHS of the critical condition is

- (c) independent of $A$.
- (d) larger than LHS and smaller than RHS.

Conditions (a) to (d) guarantee that a unique intersection of the LHS and RHS always exists at $(A)^*$, such that two areas always exist:

- For $A < (A)^*$: LHS < RHS and the critical condition holds.
- For $A > (A)^*$: LHS > RHS and the critical condition does not hold.

Therefore, we generally conclude:

![Figure 1. Critical condition.](image-url)
Corollary 4 The critical condition in Proposition 2 holds if $\Phi^A$ is not too close to $\Omega$, that is, $\Phi^A < (\Phi^A)^*$. This result is highly intuitive. Suppose that $\Phi^A$ is close to $\Omega$. Then, the participation of $A$ types in the sports contest is not particularly risky, because they potentially lose only $\Omega - \Phi^A$ in the case of unsuccessful participation. At the same time, they potentially win $V$. Thus, $A$ types would have an incentive to enter the contest and infiltrate the participating type $B$ contestants.

Comparison of Equilibria

Propositions 1 and 2 show that two possible types of equilibria generally exist if the critical condition holds. It is not clear (game theoretically) which type of equilibrium will be realized. In this section, I compare the two types of equilibria.

Proposition 3 Suppose that the critical condition in Proposition 2 holds. In an equilibrium with only $A$ types (Proposition 1), the number of participating contestants is larger, the individual effort of a contestant is smaller, and the aggregate efforts are larger than in an equilibrium with only $B$ types (Proposition 2).

Proof. See Appendix. ■

A representative of a league or association22 is interested in the quality of the tournament. The quality of the tournaments can be measured by the individual effort or aggregate efforts. Proposition 3 demonstrates that high individual (aggregate) effort results when $B$ ($A$) types enter the competition in equilibrium. However, no type of equilibrium guarantees large individual effort as well as large aggregate efforts simultaneously due to the endogenous entry choice of contestants.

Extension

In this extension, I assume that the number of high (low) type $A$ ($B$) contestants is small.23 A practical example for a competition with small pools of each type would be a sailing competition (like the America’s Cup), in which the participation costs (e.g., the development costs of a sailing boat) could be prohibitively high. In this case, all available contestants enter the contest and profits are at least the value of their outside option $\Omega$ in equilibrium. $\bar{N} > 1 (\bar{M} \equiv \bar{N} + \omega > 1)$ represents the small number of high (low) types and $\omega \leq 0$ represents the difference between the number of high and low types.24 In order to guarantee the participation of all contestants in the contest, we define an upper bound for the number of each type as follows.25

$$\bar{N}^* \equiv \frac{\sqrt{V - \Phi^A (V - \Phi^B)} - \omega \left[ (V - \Phi^A) \sqrt{\Omega - \Phi^A} + \sqrt{V - \Phi^A (\Phi^A - \Phi^B)} \right]}{\left( (V - \Phi^B) + (V - \Phi^A) \right) \sqrt{\Omega - \Phi^A} + \sqrt{V - \Phi^A (\Phi^A - \Phi^B)}}$$
\[
\hat{M'} \equiv \frac{\omega \left[ (V - \Phi^B) \sqrt{\Omega - \Phi^B} - (\Phi^A - \Phi^B) \sqrt{V - \Phi^B} \right] + (V - \Phi^A) \sqrt{V - \Phi^B}}{\left( (V - \Phi^B) + (V - \Phi^A) \right) \sqrt{\Omega - \Phi^B} - (\Phi^A - \Phi^B) \sqrt{V - \Phi^B}}.
\]

**Proposition 4** If \(1 < \tilde{N} \leq \tilde{N}^{*}\), \(1 < \tilde{M} \leq \tilde{M}^{*}\) and \(\tilde{M} < \frac{V - \Phi^B}{\Phi^A - \Phi^B}\) holds,\(^{26}\) all contestants enter the contest. In equilibrium,

(i) contestant \(n\) chooses effort \(\tilde{x}_n = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^A)(V - \Phi^B)(V - \tilde{M}(\Phi^A - \Phi^B) - \Phi^B)}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2} > 0\),

(ii) contestant \(m\) chooses effort \(\tilde{x}_m = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^A)(V - \Phi^B)(V + \tilde{N}(\Phi^A - \Phi^B) - \Phi^B)}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2} > 0\),

(iii) aggregate efforts are \(\tilde{X} = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^A)(V - \Phi^B)}{\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A)}\), and

(iv) the expected profit of each contestant is at least \(\Omega\).

**Proof.** See Appendix. \(\blacksquare\)

This proposition stands as a counterpoint to Lemma 2. Lemma 2 claims that no equilibrium exists with both types of contestants participating in the contest simultaneously in the case of a large pool of each type. Proposition 4 shows, however, that the simultaneous participation of both types is possible in equilibrium if the pool of each type is small. A non-binding participation constraint is responsible for both types participating in the contest in the case of a small pool of each contestant. By contrast, in the case of large pools, only one type enters the contest until the expected profit corresponds to the value of the outside option.

**Proposition 5** Suppose that \(1 < \tilde{N} \leq \tilde{N}^{*}\), \(1 < \tilde{M} \leq \tilde{M}^{*}\) and \(\tilde{M} < \frac{V - \Phi^B}{\Phi^A - \Phi^B}\) holds. In equilibrium,

(i) the low types’ effort is larger than high types’ effort, and

(ii) the aggregate effort increases \((\tilde{X} \uparrow)\), if the proportion of the low types increases \((\omega \uparrow)\).

**Proof.** See Appendix. \(\blacksquare\)

Result (i) of Proposition 5 is qualitatively similar to the result of Proposition 3. Low types have larger incentives to exert effort. Thus, greater opportunities partly destroy effort incentives independently of the size of the pools. However, result (ii) of Proposition 5 stands as a counterpoint to Proposition 3. While aggregate effort is larger for a larger proportion of low types in the case of small pools, aggregate effort is larger if only high types enter the contest in case of large pools. The reason for this difference is that the assumption of a large pool (instead of a small pool) has additional implications as the number of participating contestants is endogenously adjusted. A larger number of high types entering the contest overcompensates the lower individual effort levels.
Conclusion

This paper analyzes a sports competition which is interpreted differently from the typical sports contests discussed in the literature. The period between the entry decision of contestants (aspiring to an international sports career) and the realization of the contest (a successful or unsuccessful sports career) is protracted in the model. Due to this temporal characteristic, the model differentiates between an ex-ante outside option of a sports contest and the remaining lifetime value $F$ in the case of an unsuccessful sports career. The latter aspect considers that a contestant has already invested very specifically in the past. The unsuccessful contestant is much older and possible outside options from the past may no longer be available to them. Therefore, the added value of this paper to the literature is the concise analysis and distinction of two types of outside options: (i) the ex-ante value of an outside option, if a contestant does not enter the contest; and (ii) the ex-post value after a contest failure.

In the model, contestants either decide to enter a risky sports contest or choose a secure outside option. Contestants differ with respect to their opportunities in the case of an unsuccessful sports career. In the main part of the paper, the pools of the two types are large. On the one hand, the paper can explain the participation of only contestants with greater opportunities in sports contests. Contestants with greater opportunities enter a risky sports contest because they are secure in the event of contest failure. Thus, greater opportunities can increase risk-taking behaviors. However, greater opportunities partly destroy effort incentives due to the better fallback position. On the other hand, the model also shows that an equilibrium can exist in which only contestants with low opportunities participate in the contest. A precondition for this result is that the opportunities in the case of failure of the contestants with greater opportunities are not too close to the ex-ante value of the outside option.

The model reveals new effects of asymmetric opportunities outside the sports field on the self-selection process of the field of participants in competition. However, the model is not able to provide policy implications. There are three reasons for this:

- If the critical condition (see Proposition 2) holds, multiple equilibria exist. The model does not predict which type of equilibrium will be realized. Depending on which type of equilibrium results: either the individual effort is large and aggregate efforts are small or the individual effort is small and aggregate efforts are large.
- In the introduction, $F$ was interpreted as the extent of safety nets provided by countries. The model reveals incentive effects of these safety nets. In an A-type equilibrium, for instance, the low individual effort of A types should not suggest that (government) policies that lower $F$ are desirable. After all, it is very likely that safety nets can contribute to improving social welfare.
The model does not suggest which policies are optimal from a welfare perspective. For this, the perspectives of all economic actors would have to be taken into account. Moreover, the optimal effort levels are likely to be different from the perspective of the fans, the players, the leagues/associations and the economy as a whole.

In the extension of this paper, I show that the coexistence of both types in the contest is possible if the pools of the two types are small. In this case, contestants may have higher expected profits participating in the contest compared to the value of their outside option. For future research, it would be interesting to analyze the interaction between asymmetric talent and asymmetric second prizes and its effect on contestants’ entry decision. Alternatively, the degree of heterogeneity could be generalized, assuming a large number of contestants each with a different second prize valuation. The analysis would be fundamentally different when compared with my paper, because the number of contestants for each type is large in my model.

Appendix

Proof of Lemma 1

Suppose $\Phi^A = \Phi^B \equiv \Phi$. Then, we obtain the following first-order condition of contestant $i$:\(^{27}\):

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\sum_{j=1}^{n} x_j - x_i}{\left(\sum_{j=1}^{n} x_j\right)^2} (V - \Phi) - 1 = 0. \quad (A1)$$

Assuming symmetric behavior $x_i = x_j (i, j = 1, \ldots, n)$ for all participating contestants in equilibrium, we get:

$$x_i = \frac{n - 1}{n^2} (V - \Phi).$$

Replacing the last result into the expected profit function, we get:

$$\pi_i = \Phi + \frac{V - \Phi}{n^2}.\nonumber$$

In equilibrium, the number of participants $n^*$ is endogenously determined. Contestants enter the contest until $\pi_i = \Omega$. Therefore, the number of contestants entering the contest is

$$n^*.$$
\[ n^* = \sqrt{\frac{V - \Phi}{\Omega - \Phi}}, \]

such that contestant i’s expected profit is \( \pi^*_i = \Omega \) in equilibrium. Thus, contestant i’s effort \( x^*_i \) and aggregate efforts \( X^* \) are

\[ x^*_i = \frac{n^* - 1}{(n^*)^2} (V - \Phi) = \sqrt{\frac{V - \Phi}{\Omega - \Phi}} \left( \sqrt{\frac{V - \Phi}{\Omega - \Phi}} - \sqrt{\frac{\Omega - \Phi}{\Omega - \Phi}} \right), \]

and

\[ X^* = \sum_{i=1}^{n^*} x^*_i = n^* x^*_i = \sqrt{\frac{V - \Phi}{\Omega - \Phi}} \sqrt{\frac{\Omega - \Phi}{\Omega - \Phi}} \left( \sqrt{\frac{V - \Phi}{\Omega - \Phi}} - \sqrt{\frac{\Omega - \Phi}{\Omega - \Phi}} \right) \]

in equilibrium.

**Proof of Lemma 2**

I prove this lemma by applying a proof by contradiction. Summing up the two first-order conditions (3) and (4) over \( N \) participating type A contestants and \( M \) participating type 2 contestants, respectively, I get:

\[ \frac{N}{X} \frac{N x_n}{X^2} = \frac{N}{V - \Phi^A}, \]

\[ \frac{M}{X} \frac{M x_m}{X^2} = \frac{M}{V - \Phi^B}. \]

Summing up left hand and right hand sides of the last two equations and solving for \( X \) (with \( X = N x_n + M x_m \)), I get:

\[ X = \frac{(N + M - 1)(V - \Phi^A)(V - \Phi^B)}{N(V - \Phi^B) + M(V - \Phi^A)}. \]

Replacing this \( X \) into conditions (3) and (4) and solving for \( x_n \) and \( x_m \), I get

\[ x_n = \frac{(N + M - 1)(V - \Phi^A)(V - \Phi^B)(V - M(\Phi^A - \Phi^B) - \Phi^B)}{(N(V - \Phi^B) + M(V - \Phi^A))^2}, \]

and

\[ x_m = \frac{(N + M - 1)(V - \Phi^A)(V - \Phi^B)(V + N(\Phi^A - \Phi^B) - \Phi^A)}{(N(V - \Phi^B) + M(V - \Phi^A))^2}. \]

Therefore,
\[ X = N_x + M_{x_m} \]
\[ = (V - \Phi^A)(V - \Phi^B) \frac{N(N + M - 1)(V - M(\Phi^A - \Phi^B) - \Phi^A) + M(N + M - 1)(V + N(\Phi^A - \Phi^B) - \Phi^A)}{(N(V - \Phi^B) + M(V - \Phi^A))^2}, \]

and expected profit of representatives \( n \) and \( m \) are

\[ \pi_n = \Phi^A + \frac{(V - \Phi^A)(V - M(\Phi^A - \Phi^B) - \Phi^A)^2}{(N(V - \Phi^B) + M(V - \Phi^A))^2}, \]  

(A2)

\[ \pi_m = \Phi^B + \frac{(V - \Phi^B)(V + N(\Phi^A - \Phi^B) - \Phi^A)^2}{(N(V - \Phi^B) + M(V - \Phi^A))^2}. \]  

(A3)

Type \( A \) (\( B \)) contestants enter the contests if \( \pi_n > \Omega \) (\( \pi_m > \Omega \)). Therefore, the two conditions \( \pi_n = \Omega \) and \( \pi_m = \Omega \) hold in equilibrium. Combining the two conditions, I get:

\[ \frac{(V - \Phi^A)(V - M(\Phi^A - \Phi^B) - \Phi^A)^2}{(V - \Phi^B)(V + N(\Phi^A - \Phi^B) - \Phi^A)^2} = \frac{\Omega - \Phi^A}{\Omega - \Phi^B}, \]

Solving for \( M \):

\[ M = \frac{V - \Phi^B - \sqrt{\frac{\Omega - \Phi^A}{\Omega - \Phi^B}(V + N(\Phi^A - \Phi^B) - \Phi^A)}}{\Phi^A - \Phi^B}. \]  

(A4)

Replacing \( M \) into (A3) and using \( \pi_m = \Omega \), I get

\[ N = \frac{V - \Phi^A}{\Phi^B - \Phi^A}. \]

Replacing \( N \) into (A4), I get

\[ M = \frac{V - \Phi^B}{\Phi^A - \Phi^B}. \]

For \( \Phi^A > \Phi^B \), it is easy to see according to the last two conditions that there is no equilibrium with \( N > 0 \land M > 0 \) since \( \bar{N} < 0 \).

**Proof of Proposition 1**

In Lemma 2, I have shown that there exist no equilibrium in which both types are active. Therefore, I have to analyze possible corner solutions. Suppose that \( N > 0 \)
and \( M = 0 \) hold in equilibrium such that only type \( A \) contestants enter the contest. Solving the first-order condition (3) for \( x_n \), I get:

\[
x_n = X - \frac{X^2}{V - \Phi^A}.
\]

Since \( M = 0 \), the aggregate and individual efforts are as follows:

\[
X = Nx_n \Leftrightarrow X = \frac{N - 1}{N} (V - \Phi^A),
\]

\[
x_n = \frac{N - 1}{N^2} (V - \Phi^A).
\]

Expected profit of contestant \( n \) is then

\[
\pi_n = \frac{1}{N^2} (V - \Phi^A) + \Phi^A.
\]

Type \( A \) contestants enter the contest until \( \pi_n = \Omega \) holds. Solving this condition for \( N \), I get the following results in equilibrium:

\[
N^* = \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}},
\]

\[
x_n^* = \sqrt{\Omega - \Phi^A} \left( \sqrt{V - \Phi^A} - \sqrt{\Omega - \Phi^A} \right),
\]

\[
X^* = \sum_{n=1}^{N^*} x_n^* = \sqrt{V - \Phi^A} \left( \sqrt{V - \Phi^A} - \sqrt{\Omega - \Phi^A} \right). \quad (A5)
\]

Next, I have to prove that type \( B \) contestants have no incentive to enter the proposed equilibrium. Suppose that representative \( m \) of type \( B \) deviates and enters the contest. The expected profit of \( m \) is as follows:

\[
\pi_m = \frac{X_m}{\sum_{i=1}^{N} x_n^* + x_m} \left( V - \Phi^B \right) + \Phi^B - x_m
\]

\[
= \frac{x_m}{\sqrt{(V - \Phi^A) \left( \sqrt{(V - \Phi^A)} - \sqrt{\Omega - \Phi^A} \right)} + x_m} \left( V - \Phi^B \right) + \Phi^B - x_m.
\]

Optimization of \( m \) requires the first-order condition
\[
\frac{\partial \pi_m}{\partial x_m} = \frac{\sqrt{(V - \Phi^d)}(\sqrt{(V - \Phi^d)} - \sqrt{\Omega - \Phi^d}) + x_m - x_m}{\sqrt{(V - \Phi^d)}(\sqrt{(V - \Phi^d)} - \sqrt{\Omega - \Phi^d}) + x_m} (V - \Phi^b) - 1 = 0,
\]

such that contestant \( m \)'s effort is:

\[
\hat{x}_m = \sqrt[4]{(V - \Phi^d)(\sqrt{(V - \Phi^d)} - \sqrt{\Omega - \Phi^d})} (V - \Phi^b) - \sqrt{(V - \Phi^d)(\sqrt{(V - \Phi^d)} - \sqrt{\Omega - \Phi^d})}.
\]

In this case, contestant \( m \)'s expected profit \( \hat{\pi}_m \) is:

\[
\hat{\pi}_m = \frac{\hat{x}_m}{N} (V - \Phi^b) + \Phi^b - \hat{x}_m \sum_{n=1}^{N} x_n^* + \hat{x}_m
\]

\[
= V - 2\sqrt{(V - \Phi^d(\sqrt{V - \Phi^d - \sqrt{\Omega - \Phi^d})}) \sqrt{V - \Phi^b} + \sqrt{V - \Phi^d} (\sqrt{V - \Phi^d - \sqrt{\Omega - \Phi^d}) < \Omega - \Phi^b.
\]

Contestant \( m \) has no incentive to enter the contest if the expected profit \( \hat{\pi}_m < \Omega \) holds. Simple manipulation of this condition leads to

\[
V - \Phi^b - 2\sqrt{(V - \Phi^d(\sqrt{V - \Phi^d - \sqrt{\Omega - \Phi^d})}) \sqrt{V - \Phi^b} + \sqrt{V - \Phi^d} (\sqrt{V - \Phi^d - \sqrt{\Omega - \Phi^d}) < \Omega - \Phi^b
\]

\[
\iff (\sqrt{V - \Phi^b} - \sqrt{(V - \Phi^d(\sqrt{V - \Phi^d - \sqrt{\Omega - \Phi^d})})^2 < \Omega - \Phi^b
\]

\[
\iff \Phi^b - \Phi^b + \Omega - \Phi^b < 2\sqrt{(V - \Phi^d\sqrt{\Omega - \Phi^b}) - \sqrt{V - \Phi^d\sqrt{\Omega - \Phi^d})} < \Omega - \Phi^b.
\]

(A6)

Note that only the RHS of the condition (A6) depends on \( V \). Furthermore, \( \lim_{V \to -\Omega} RHS = 2\sqrt{\Omega - \Phi^b\sqrt{\Omega - \Phi^b} - \sqrt{(V - \Phi^d\sqrt{\Omega - \Phi^d})} = 2\Omega - 2\Phi^b - \Omega + \Phi^d = \Omega - 2\Phi^b + \Phi^d \) such that \( LHS = RHS \). Therefore, condition (A6) is satisfied if we are able to prove \( \frac{\partial RHS}{\partial \Omega} > 0 \).

\[
\frac{\partial RHS}{\partial V} > 0 \iff \frac{\sqrt{\Omega - \Phi^b}}{\sqrt{V - \Phi^b}} - \frac{1}{2}\frac{\sqrt{\Omega - \Phi^d}}{\sqrt{V - \Phi^d}} > 0
\]
In this proposition, I analyze the other type of possible corner solutions. Suppose that the contest. Similar procedure as in the proof of Proposition 1, I get:

\[ \forall \left[ 3(\Omega(V - \Phi^A) - \Phi^B(V - \Phi^A)) > \Omega(\Phi^A - \Phi^B) - V(\Phi^A - \Phi^B) \right] \]

\[ \Leftrightarrow 3 \left[ (V - \Phi^A)(\Omega - \Phi^B) > (\Omega - V)(\Phi^A - \Phi^B) \right]. \quad (A7) \]

According to condition (A7), \( \frac{\partial \text{RHS}}{\partial \pi} > 0 \) holds. This result implies that \( \hat{\pi}_m < \Omega \) and contestant \( m \) has no incentive to enter the contest. Therefore, there always exists an equilibrium, in which only type \( A \) contestants enter the contest.

**Proof of Proposition 2**

In this proposition, I analyze the other type of possible corner solutions. Suppose that \( M > 0 \) and \( N = 0 \) hold in equilibrium such that only type \( B \) contestants enter the contest. Similar procedure as in the proof of Proposition 1, I get:

\[ M^* = \sqrt{\frac{V - \Phi^B}{\Omega - \Phi^B}}, \]

\[ x^*_m = \sqrt{(\Omega - \Phi^B)(V - \Phi^B) - (\Omega - \Phi^B)}, \]

\[ X^* = \sum_{m=1}^{M^*} x^*_m = \sqrt{V - \Phi^B} \left( \sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B} \right). \quad (A8) \]

Next, I have to prove that type \( A \) contestants have no incentive to enter the proposed equilibrium. Suppose that representative \( n \) of type \( A \) deviates and enters the contest. In this case, contestant \( n \)'s expected profit \( \hat{\pi}_n \) is:

\[ \hat{\pi}_n = \frac{x_n}{M^*} \left( V - \Phi^A \right) + \Phi^A - \hat{x}_n \]

\[ = V - 2\sqrt{V - \Phi^B} \left( \sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B} \right) \sqrt{V - \Phi^A} \]

\[ + \sqrt{V - \Phi^B} \left( \sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B} \right). \]

Contestant \( n \) has no incentive to enter the contest if \( \hat{\pi}_n < \Omega \) holds. Simple manipulation of this condition leads to

\[ (V - \Phi^A) - 2\sqrt{V - \Phi^B} \left( \sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B} \right) \sqrt{V - \Phi^A} \]

\[ + \sqrt{V - \Phi^B} \left( \sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B} \right) < \Omega - \Phi^A \]
\[
\iff (\sqrt{V - \Phi^A} - \sqrt{V - \Phi^B})^2 < \Omega - \Phi^A \\
\iff (\sqrt{V - \Phi^A} - \Omega - \Phi^A)^2 < \sqrt{V - \Phi^B} (\sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B}).
\]

Note that the last condition does not generally hold. This critical condition only holds for special combinations of the involved parameters.

**Proof of Proposition 3**

I only show the proof that the aggregate efforts with \( A \) types (see (A5)) are larger than with \( B \) types (see (A8)).

\[
\sqrt{V - \Phi^A} (\sqrt{V - \Phi^A} - \sqrt{\Omega - \Phi^A}) > \sqrt{V - \Phi^B} (\sqrt{V - \Phi^B} - \sqrt{\Omega - \Phi^B})
\]

\[
\iff \sqrt{V - \Phi^B} \sqrt{\Omega - \Phi^B} - \sqrt{V - \Phi^A} \sqrt{\Omega - \Phi^A} > \Phi^A - \Phi^B, \tag{A9}
\]

Note that only the LHS of condition (A9) depends on \( V \). Furthermore, \( \lim_{V \to \Omega} LHS = \Phi^A - \Phi^B \) such that \( LHS = RHS \). Therefore, condition (A9) is satisfied if I am able to prove \( \partial LHS / \partial V > 0 \).

\[
\frac{\partial LHS}{\partial V} = \frac{1}{2} (V - \Phi^B)^{-\frac{1}{2}} \sqrt{\Omega - \Phi^B} - \frac{1}{2} (V - \Phi^A)^{-\frac{1}{2}} \sqrt{\Omega - \Phi^A} > 0
\]

\[
\iff \sqrt{V - \Phi^A} \sqrt{\Omega - \Phi^B} > \sqrt{V - \Phi^B} \sqrt{\Omega - \Phi^A}
\]

\[
\iff (V - \Phi^A) (\Omega - \Phi^B) > (V - \Phi^B) (\Omega - \Phi^A)
\]

\[
\iff V (\Phi^A - \Phi^B) > \Omega (\Phi^A - \Phi^B) \iff V > \Omega.
\]

As the last condition holds, I have shown that \( \partial LHS / \partial V > 0 \) and therefore aggregate efforts with \( A \) types are larger than with \( B \) types. The proofs for the comparison of the individual efforts and number of contestants are similarly and therefore omitted.

**Proof of Proposition 4**

The numbers of high and low types are given. I derived the profits of the two representatives for arbitrary numbers of contestants in Lemma 2 (see (A2) and (A3)). Now I derive an upper bound for the number of each type such that the expected profit of a participating contestant is at least \( \Omega \):

\[
\pi_m = \Phi^A + \frac{(V - \Phi^A) (V - \bar{M} (\Phi^A - \Phi^B) - \Phi^B)^2}{(\bar{N} (V - \Phi^B) + \bar{M} (V - \Phi^A))^2} \geq \Omega
\]
\[
\begin{align*}
&\iff \frac{(V - \Phi^A)(V - \tilde{M}(\Phi^A - \Phi^B) - \Phi^B)}{(\Omega - \Phi^A)(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2} \geq 1 \\
&\iff \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}V - \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}\Phi^B} \geq \tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A)} \\
&\quad + \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}\tilde{M}(\Phi^4 - \Phi^B)} \\
&\iff \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}V - \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}\Phi^B} \geq \tilde{N}(V - \Phi^B)} \\
&\quad + (\tilde{N} + \omega) \left[ (V - \Phi^A) + \sqrt{\frac{V - \Phi^A}{\Omega - \Phi^A}(\Phi^4 - \Phi^B)} \right] \\
&\iff \frac{\sqrt{V - \Phi^A}(V - \Phi^B) - \omega \left[ (V - \Phi^A) \sqrt{\Omega - \Phi^A} + \sqrt{V - \Phi^A}(\Phi^4 - \Phi^B) \right]}{(\Omega - \Phi^A)(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2} \geq \tilde{N}, \\
&\iff \pi_m = \Phi^B + \frac{(V - \Phi^B)(V + \tilde{N}(\Phi^4 - \Phi^B) - \Phi^B)}{(V - \Phi^B)\left( \tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A) \right)^2} \geq \Omega \\
&\iff \frac{(V - \Phi^B)(V + (\tilde{M} - \omega)(\Phi^4 - \Phi^B) - \Phi^B)}{(\Omega - \Phi^B)(\tilde{M} - \omega)(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2} \geq 1 \\
&\iff \frac{\omega \left[ (V - \Phi^B) \sqrt{\Omega - \Phi^B} - (\Phi^4 - \Phi^B) \sqrt{V - \Phi^B} \right] + (V - \Phi^A)\sqrt{V - \Phi^B}}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2} \geq \tilde{M}, \\
&\iff \tilde{x}_n = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^4)(V - \Phi^B)(V - \tilde{M}(\Phi^4 - \Phi^B) - \Phi^B)}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^A))^2}
\end{align*}
\]

If \( \tilde{N} \leq \tilde{N}^* \) and \( \tilde{M} \leq \tilde{M}^* \) holds then all contestants enter the contest. Contestants’ individual and aggregate efforts are defined in the proof of Lemma 2 replacing \( N \) by \( \tilde{N} \) and \( M \) by \( \tilde{M} \):
\[ \tilde{x}_m = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^4)(V - \Phi^B)(V + \tilde{N}(\Phi^4 - \Phi^B) - \Phi^4)}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^4))^2} > 0 \]

\[ \tilde{X}^* = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^4)(V - \Phi^B)}{\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^4)}. \]

The additional condition \( \tilde{M} < \frac{V - \Phi^B}{\Phi^A - \Phi^B} \) guarantees positive effort of the high types, that is, \( \tilde{x}_n > 0 \). Note that the low types’ effort is always positive.

**Proof of Proposition 5**

(i) Low types’ effort is larger than high types’ effort:

\[ \tilde{x}_n < \tilde{x}_m \]

\[ \Leftrightarrow \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^4)(V - \Phi^B)(V + \tilde{N}(\Phi^4 - \Phi^B) - \Phi^B)}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^4))^2} < \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^4)(V - \Phi^B)(V + \tilde{N}(\Phi^4 - \Phi^B) - \Phi^4)}{(\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^4))^2} \]

\[ \Leftrightarrow -\tilde{M}(\Phi^4 - \Phi^B) - \Phi^B < \tilde{N}(\Phi^4 - \Phi^B) - \Phi^4 \]

\[ \Leftrightarrow (\tilde{N} + \omega)(\Phi^4 - \Phi^B) - \Phi^B < \tilde{N}(\Phi^4 - \Phi^B) - \Phi^4 \]

\[ \Leftrightarrow \Phi^4 - \Phi^B - \omega(\Phi^4 - \Phi^B) < 2\tilde{N}(\Phi^4 - \Phi^B) \]

\[ \Leftrightarrow 1 < 2\tilde{N} + \omega \]

\[ \Leftrightarrow 1 < \frac{\tilde{N} + \tilde{M}}{2}. \]

(ii) I define the total number of contestants by \( Q \equiv \tilde{N} + \tilde{M} > 2 \). In order to isolate the effect of a larger proportion of low types, I assume that \( Q \) is a fixed number. Thus, I define the aggregate effort \( \tilde{X} \) as a function of \( Q \) and \( \omega \):

\[ \tilde{X} = \frac{(\tilde{N} + \tilde{M} - 1)(V - \Phi^4)(V - \Phi^B)}{\tilde{N}(V - \Phi^B) + \tilde{M}(V - \Phi^4)}. \]
\[
\frac{(Q - 1)(V - \Phi^A)(V - \Phi^B)}{QV - \tilde{N}\Phi^B - M\Phi^A}
\]

In the last transformation, I replaced \(\tilde{N}\) by \(\frac{Q - \omega}{2}\) because \(Q = \tilde{N} + \tilde{M} = 2\tilde{N} + \omega \Rightarrow \tilde{N} = \frac{Q - \omega}{2}\). For a fixed value of \(Q\), the derivative of \(\tilde{X}\) with respect to \(\omega\) is positive:

\[
\frac{\partial \tilde{X}}{\partial \omega} = \frac{(Q - 1)(V - \Phi^A)(V - \Phi^B) \frac{1}{2} (\Phi^A - \Phi^B)}{[Q(V - \Phi^A) + \frac{Q}{2} (\Phi^A - \Phi^B) - \frac{\omega}{2} (\Phi^A - \Phi^B)]^2} > 0.
\]

Thus, a larger proportion of low types increases aggregate effort.

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**Notes**

1. The attention of the public is usually concentrated on the top performers. It would be particularly interesting to know how many “Roger Federers” exist worldwide with similar
talents and motivation, but who had to quit tennis due to injuries or other incidents at the beginning of their careers.

2. Aside from the two preconditions: (i) recognition of his extraordinary ability as a young adult and (ii) the (intrinsic) motivation to play tennis.

3. The example from the sports sector also applies in other economic sectors. For instance, young entrepreneurs engaged in start-ups face a similar problem. They put great effort into the development of new products or services in very risky and dynamic environments. Many of them fail, but some try again to succeed in other start-ups. However, if they consistently fail, they likely regret their initial decision to concentrate on a career in this field. They probably have lesser chances in obtaining other jobs because they cannot show that their previous work resulted in a marketable product or service. Similar problems are observable in academia (or the world of classical music), in which young researchers (or musicians) invest intensively in their PhD and post-doctorate (or music education and expensive master classes) with insecure future options given that they might not obtain a professorship (or an offer from an attractive orchestra). In all of these examples, the main point is that the opportunities are different before and after an unsuccessful career due to the high specificity of investments that lose a significant part of their value later.

4. In Europe, sport is organized differently for historical reasons. Today, nonetheless, integrated schools have become more popular in Europe. In Switzerland, for instance, the state-run organization Swiss Olympic supports the best talents in specific sport disciplines and their families in choosing a suitable and officially recognized educational institution out of 4 Swiss Olympic sports schools and 52 Swiss Olympic partner schools (see www.swissolympic.ch). In these schools, athletes can participate in competitive sports and complete their schooling at the same time. The flexible training schedule, the necessary infrastructure and the training supervision allow athletes to simultaneously complete the compulsory schooling (and even higher education) preparing themselves for a successful start to their professional life outside of sports. Thus, the risk of becoming unemployed after a failed sports career is reduced.

5. The effect of outside options (or, similarly, an entry fee) in contests are discussed by Corcoran (1984), Hillman and Katz (1984), Appelbaum and Katz (1986, 1987), Fullerton and McAfee (1999), Moldovanu and Sela (2001), Fu and Lu (2010), Morgan et al. (2012), and Liu and Lu (2019).

6. Frick and Scheel (2016) and Groothuis and Hill (2018) analyze the length of sports careers. Frick and Scheel show that the length of careers in the FIS Ski Jumping World Cup depends on previous successes and the degree of competition within the national team. Groothuis and Hill analyze the career duration in the NBA. They show that the duration of careers depends on whether the athlete is a foreigner and whether he has played college basketball in the US in the past.

7. See, for instance, Clark and Riis (1998b), Barut and Kovenock (1998), and Szymanski and Valletti (2005) for models with multiple prizes in contests.

8. Thus, contestant $i$ has a different valuation than contestant $j$ for all prizes. However, “within” a contestant, prizes are symmetric. In my model, only second prizes differ
between contestants (ceteris paribus) such that the model of Clark and Riis cannot encompass my case.

9. In particular, players probably aim to maximize their post-rookie free agent contracts.

10. Some studies analyze the effect of sports and exercise on the labor market outcomes. Lechner and Sari (2015), for instance, identify a positive long-run effect of sports on income. Lechner and Downward (2017) conclude that team sports (outdoor sports) contribute most to employability (higher salaries); however, these effects depend on gender and age. On the other hand, Ewing (2007) analyzes a data set from the National Longitudinal Survey of Youth. He finds that high school athletes have higher wages as well as fringe benefits than nonathletes.

11. Note that the order $V > \Omega > \Phi$ is very natural. It implies that a successful participation in the sports contest is most attractive, followed by the outside option. The least attractive result is unsuccessful participation in the sports contest.

12. In the general contest literature, the assumption that contestants enter the contest until expected profit either equals the value of the outside value is standard (see, for instance, Appelbaum & Katz, 1986, 1987; Fu & Lu, 2010; Fullerton & McAfee, 1999; Morgan et al., 2012).

13. Due to the large number of contestants, I will omit in my model the integer problem addressed by Morgan et al. (2012) and Fullerton and McAfee (1999). Thus, the number of participating contestants does not necessarily have to be an integer.

14. See Tullock (1980).

15. Note that all contestants have the same talent (independent of the type) according to this specification of the CSF. At the outset, contestants are already selected during the competitive process. They may have more or less the same ability and talent level. Therefore, any behavioral differences between types in my model can be attributed to differences in second prizes.

16. Note that $\frac{V - \Phi}{2\Omega - \Phi}$ is increasing in $\Phi$.

17. Note that Assumption 1 is used in the proof of Corollary 2.

18. This follows from the first-order condition (see condition (A1) in the proof of Lemma 1 in the Appendix).

19. Note that the A type’s individual effort in the contest is as in Lemma 1(ii) with $\Phi^d$ replacing $\Phi$. Corollary 2(iii) then shows that a larger value of $\Phi^d$ reduces individual effort incentives.

20. The literature considers different forms of risks in contests. For instance, Gilpatric (2009) and Kräkel et al. (2014) analyze a contest in which contestants choose effort levels as well as the variances of their output. A higher variance means a riskier behavior. In my paper, contestants are more risky if they enter the contest instead of choosing the secure outside option.

21. I thank an anonymous referee who suggested proving this result analytically.

22. In tennis, for instance, the professional tournaments are hosted by the Association of Tennis Professionals (ATP) for men and the Women’s Tennis Association (WTA).

23. I thank an anonymous referee for the idea to analyze this case.
24. In order to guarantee $\bar{N} > 1$ and $\bar{M} > 1$, the parameter range of $\omega$ has to be limited as follows:

\[
\frac{(2^{1/2}\phi^d - \phi^h)\sqrt{O/C_0} - (1 - \phi^h)}{\sqrt{1 - \phi^d} - \phi^d - (\phi^d - \phi^h)} < \omega < \frac{(2^{1/2}\phi^d - \phi^h)\sqrt{O/C_0} - (1 - \phi^h)}{\sqrt{1 - \phi^d} - \phi^d + (\phi^d - \phi^h)}
\]

25. I neglect the case in which the number of contestants is small for only one type.

26. Note that the condition $\bar{M} < \frac{2^{1/2}\phi^d}{\phi^d - \phi^h}$ guarantees a positive effort of the high types. I do not need an additional condition for the low types as low types unconditionally choose positive effort levels.

27. Note that the second-order condition holds.

28. The steps in the proof are similar to the steps in the proof of Proposition 1.

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