Simple $E_6$ Unification with Anomalous $U(1)_A$ Symmetry

Nobuhiro MAEKAWA*) and Toshifumi YAMASHITA**)  
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract

We propose simpler Higgs sectors in $E_6$ grand unified theory (GUT) with anomalous $U(1)_A$ gauge symmetry than the previous model in Ref.1). As in the previous model, the doublet-triplet (DT) splitting can be realized in a natural way, while proton decay via dimension 5 operators is suppressed and gauge coupling unification is also realized without fine-tuning. Combining the matter sector, simple complete GUTs can be obtained. Since the Higgs sector is simpler, the gauge coupling constant at the cutoff scale can be in perturbative region, and therefore, the estimated value of the lifetime of a nucleon in this model, $\tau_p(p \to e^+\pi^0) \sim 5 \times 10^{33}$ years, becomes more reliable.

*) E-mail: maekawa@gauge.scphys.kyoto-u.ac.jp  
**) E-mail: yamasita@gauge.scphys.kyoto-u.ac.jp
§1. Introduction

In a series of papers,\(^1\)–\(^6\) it has been understood that anomalous \(U(1)\)_\(A\) gauge symmetry,\(^7\) whose anomaly is cancelled by the Green-Schwarz mechanism,\(^8\) plays an important role in solving various problems in GUTs. One of the most important features of this scenario is to introduce generic interactions even for higher dimensional operators with \(\mathcal{O}(1)\) coefficients. Of course, generic interactions are often introduced in matter sector,\(^9\) but not in Higgs sector. The generic interactions even in the Higgs sector are the biggest difference between our scenario and the previous GUT scenarios with (anomalous) \(U(1)\) symmetry.\(^10\) Therefore, once we fix the symmetry of the theory, we can define the theory except for the \(\mathcal{O}(1)\) coefficients. Because the gauge symmetry is \(SO(10) \times U(1)_A\) or \(E_6 \times U(1)_A\), the parameters to fix the symmetry are essentially the (integer) charges of the \(U(1)_A\) for the fields introduced in the theory. It is surprising that only by setting the symmetry, we can obtain complete GUTs, in which the DT splitting\(^11\) is realized without too rapid proton decay via dimension 5 operators, the realistic structure of quark and lepton mass matrices is obtained including bi-large neutrino mixings\(^12\) by using the Froggatt-Nielsen (FN) mechanism,\(^13\) and the natural gauge coupling unification is realized.\(^2\) As a result of the natural gauge coupling unification, the cutoff scale \(\Lambda\) is around the usual GUT scale \(\Lambda_G \sim 2 \times 10^{16}\) GeV and the gauge couplings are unified just below \(\Lambda\). Therefore more rapid proton decay via dimension 6 operators, \(p \rightarrow e^+\pi^0\), is one of the most interesting prediction in the scenario. And if we include SUSY breaking sector, the \(\mu\) problem can also naturally be solved.\(^3\) Moreover, if we introduce a horizontal non-abelian gauge symmetry, \(SU(2)_H\) or \(SU(3)_H\), then the SUSY flavor problem is also solved, \(i.e.\) the degeneracy of scalar fermion masses, which leads to the suppression of the flavor changing neutral current (FCNC) processes via SUSY particles, can be realized.\(^4\) In the scenario, \(E_6\) GUT is more interesting than \(SO(10)\) GUT, because \(E_6\) models suppress more naturally the FCNC processes. Moreover, in \(E_6 \times SU(3)_H\) GUT, all the three generation quarks and leptons can be unified into a single multiplet \((27, 3)\), while keeping the above attractive points, including bi-large neutrino mixing angles.

In the analysis, it is important that all the scales of the vacuum expectation values (VEVs) are determined by the anomalous \(U(1)\)_\(A\) charges. To be more precise, VEVs of GUT gauge singlet operators \((G\text{-singlets})\) \(O_i\) with anomalous \(U(1)_A\) charges \(o_i\) are generally given by

\[
\langle O_i \rangle \sim \begin{cases} 
\lambda^{-o_i} & o_i \leq 0 \\
0 & o_i > 0 
\end{cases}, \tag{1.1}
\]

if they are determined by \(F\)-flatness conditions. Here \(\lambda(\ll 1)\) is the ratio of the cutoff scale \(\Lambda\) and the VEV of the FN field \(\Theta\), whose anomalous \(U(1)_A\) charge is normalized to \(-1\). Throughout this paper, we use units in which \(\Lambda = 1\), and denote all the superfields and chiral
operators by uppercase letters and their anomalous $U(1)_A$ charges by the corresponding lowercase letters. This vacuum structure (1.1) is essential for the natural gauge coupling unification.  

2) The reason why VEVs are determined as (1.1) is explained in detail in Ref. 1) - 6), and here, we only figure out the discussion using the simplest case, in which all the fields, $Z_i$, are gauge singlet. Roughly speaking, theories with anomalous $U(1)_A$ gauge symmetry with Fayet-Iliopoulos $D$-term have two kinds of vacua; (1) $\langle Z \rangle \sim O(1)$ and (2) $\langle Z_i^+ \rangle = 0 (z_i^+ > 0)$, if all the terms allowed by the symmetry are introduced in the superpotential $W[Z_i]$ with $O(1)$ coefficients. In the second vacua, if the coefficient of Fayet-Iliopoulos $D$-term $\xi$ is smaller than 1, all the VEVs of negatively charged fields $Z_i^-$ must be smaller than 1, because $D$-flatness condition of the anomalous $U(1)_A$ gauge symmetry becomes

$$D_A = g_A \left( \sum_i z_i^- |Z_i^-|^2 + \xi^2 \right) = 0. \tag{1.2}$$

In the followings, we concentrate on the second vacua, in which the hierarchical structure of Yukawa couplings can be understood by the FN mechanism. Note that in the vacua, $F$-flatness conditions of negatively charged fields $F_{Z_i^-} = 0$ become trivial. And in the $F$-flatness conditions of positively charged fields $F_{Z_i^+} = 0$, only a part of superpotential, which are linear in positively charged fields, is required in determining the VEVs of negatively charged fields. This is because the interactions which include more than two positively charged fields give, in the $F$-flatness conditions, the terms which include at least one vanishing VEVs of positively charged field and therefore become irrelevant. Note that in the superpotential, the coefficient for an interaction $O$ is determined by its anomalous $U(1)_A$ charge $o$ as $\lambda^o$. Hence, if we rewrite everything in terms of neutral operators, $\hat{Z}_i \equiv \Theta z_i Z_i$, under anomalous $U(1)_A$ symmetry, all the coefficients become $O(1)$. And VEVs of $\hat{Z}_i$ are generally expected to be $O(1)$, which lead $\langle Z_i \rangle \sim \lambda^{-z_i}$. As for non-singlet fields, the $D$-flatness conditions of GUT symmetry further determine VEVs of fields. For example, the VEVs of a field $C$ in a complex representation and of its mirror field $\bar{C}$ can be obtained from the VEV of the $G$-singlet $\langle \bar{C} C \rangle \sim \lambda^{-(c+\bar{c})}$ and $D$-flatness condition of GUT symmetry as $|\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-(c+\bar{c})/2}$. Here, we would stress that the non-singlet field $C$ may acquire a non-vanishing VEV even when $c > 0$, if $c + \bar{c} < 0$.

Unfortunately, in the previously proposed model, 1) the gauge coupling at the GUT scale tend to be in non-perturbative region. This is because the Higgs sector include many Higgs fields. Although there is a possibility that they remain in perturbative region due to the ambiguities of $O(1)$ coefficients and/or the freedom of the charge assignment, it is important to search other $E_6$ Higgs sector which has simpler Higgs contents. In this paper, we examine simpler $E_6$ models and construct models consistent with the already proposed matter sector.  

5) In these models, the sliding singlet mechanism, 14) as well as the Dimopoulos-Wilczek (DW)
mechanism,\textsuperscript{15}) acts to realize naturally DT splitting.

\section{Overview of $SO(10)$ and $E_6$ GUT}

Here we make a quick review of the $SO(10)$ unified scenario,\textsuperscript{6}) and the $E_6$ unified model proposed previously.\textsuperscript{1), 5})

\subsection{$SO(10)$ Higgs sector}

The content of the Higgs sector in $SO(10) \times U(1)_A$ is listed in Table I. Here the symbols $\pm$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & non-vanishing VEV & vanishing VEV \\
\hline
45 & $A(a = -1, -)$ & $A'(a' = 3, -)$ \\
16 & $C(c = -3, +)$ & $C'(c' = 2, -)$ \\
$\overline{16}$ & $\bar{C}(\bar{c} = 0, +)$ & $\bar{C}'(\bar{c}' = 5, -)$ \\
10 & $H(h = -3, +)$ & $H'(h' = 4, -)$ \\
1 & $\Theta(\theta = -1, +), Z_i(z_i = -2, -)$ (i = 1, 2) & $S(s = 3, +)$ \\
\hline
\end{tabular}
\caption{Typical values of anomalous $U(1)_A$ charges.}
\end{table}

denote the $Z_2$ parity. Following the general discussion of the VEVs, positively charged Higgs $A'$, $C'$, $C''$, $H'$ and $S$ have vanishing VEVs. The superpotential required in determining of the VEVs can be written

\begin{equation}
W = W_{H'} + W_{A'} + W_S + W_{C'} + W_{\bar{C}'},
\end{equation}

Here $W_X$ denotes the terms linear in the positively charged field $X$, which has vanishing VEV. Note that terms including two fields with vanishing VEVs like $\lambda^2 h'H'H'$ give contributions to the mass terms but not to the $F$-flatness conditions to determine the VEVs. Examining

\begin{equation}
W_{A'} = \text{tr}A'A + \text{tr}A'A\text{tr}A^2 + \text{tr}A'A^3, \quad \text{(2.2)}
\end{equation}

the adjoint Higgs field $A$ can have the VEV $\langle A(45)\rangle_{B-L} = \tau_2 \times \text{diag}(v, v, v, 0, 0)$, which breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Here, note that if the third term of r.h.s. of (2.2) are absent, the corresponding $F$-flatness condition determines only $\text{tr}A^2$, and we cannot expect to obtain naturally this DW form of the VEV. This form of the VEV plays an important role in solving the DT splitting problem through the interaction

\begin{equation}
W = H'AH', \quad \text{(2.3)}
\end{equation}

\textsuperscript{*) We often omit the $O(1)$ coefficient and the power of $\lambda$, which is easily understood from anomalous $U(1)_A$ charge e.g. $\lambda^{a'+a}$ for $A'A$.}
that gives non-vanishing mass term only for triplet Higgs, but not for doublet Higgs.\(^ {15} \) Because a mass term \( H'^2 \) is allowed by the symmetry, only one pair of doublet Higgs in \( H \) field becomes massless, i.e. the DT splitting is realized. The spinor Higgs fields \( C \) and \( \bar{C} \) must have non-vanishing VEVs because of the \( F \)-flatness condition of the superpotential

\[
W_S = S(1 + \bar{C}C + \text{tr} A^2).
\]

The \( F \)-flatness conditions of the superpotential

\[
W_C' = \bar{C}(A + Z)C',
\]

\[
W_{\bar{C}}' = C'(A + Z)C
\]

can align the VEVs \( \langle C \rangle = \langle \bar{C} \rangle = \lambda^{-\frac{c+\bar{c}}{2}} \), which break \( SU(2)_R \times U(1)_{B-L} \) into \( U(1)_Y \), with the VEV \( \langle A \rangle \).\(^ {16} \)

It is obvious that the mass term \( H'H \) spoils the DT splitting. Therefore, this mass term must be forbidden. Unfortunately, this term cannot forbidden by the SUSY zero mechanism, because the important term \( H'AH \), whose charge is smaller than the charge of \( H'H \), must be allowed by the symmetry. Therefore, we introduce \( Z_2 \) symmetry to forbid the term \( H'H \).

There are several terms that must be forbidden for the stability of the DW mechanism. For example, \( H^2 \) and \( HZH' \) induce a large mass of the doublet Higgs, and the term \( \bar{C}A'AC \) would destabilize the DW form of \( \langle A \rangle \). We can easily forbid these terms using the SUSY zero mechanism. For example, if we choose \( h < 0 \), then \( H^2 \) is forbidden, and if we choose \( \bar{c} + c + a + a' < 0 \), then \( \bar{C}A'AC \) is forbidden. Once these dangerous terms are forbidden by the SUSY zero mechanism, higher-dimensional terms that also become dangerous (for example, \( \bar{C}A'A\bar{C}C \) and \( \bar{C}A'C\bar{C}AC \) ) are automatically forbidden, because only negatively charged \( G \)-singlets have non-vanishing VEVs. This is also an advantage of our scenario.

For the quark and lepton sector, we introduced four superfields \( \Psi_i(16) \) \((i = 1, 2, 3)\) and \( T(10) \) with typical values of the charges \((\psi_1, \psi_2, \psi_3) = (9/2, 7/2, 3/2)\) and \( t = 5/2 \). The half integer charges for matter fields play the same role as R-parity. Because the \( SO(10) \) representations \( 16 \) and \( 10 \) are divided as

\[
16 \rightarrow 10_1 + 5_{-3} + 1_5
\]

\[
10 \rightarrow 5_{-2} + 5_2
\]

under \( SU(5) \times U(1)_Y (\subset SO(10)) \), a liner combination of four \( 5 \) of \( SU(5) \) becomes massive with the \( 5 \) component in \( T \). Therefore, three massless modes of \( \bar{5} \) are linear combinations of four \( \bar{5} \). Under the typical charge assignment, three massless modes become \( (\bar{5}_{\psi_1}, \bar{5}_T + \lambda^A \bar{5}_{\psi_3}, \bar{5}_{\psi_2}) \). Here because the field \( \bar{5}_T \) has no Yukawa couplings with the Higgs \( H \), we
wrote the mixing with $\bar{5}_\psi$, through which the massless mode $\bar{5}_T + \lambda^2 \bar{5}_\psi$ can have Yukawa couplings. The mixing parameter $\Delta$ is obtained by $\Delta \equiv t - \psi + \frac{1}{2}(c - \bar{c}) = \frac{5}{2}$. The neutrino majorana masses are given by $\Psi_i \Psi_j \bar{C} \bar{C}$, and the mass of the heaviest light neutrino is written

$$m_{\nu_3} \sim \lambda^{-1} (h + 5) \langle H_u \rangle^2 \eta^2,$$

where $\eta$ is a renormalization factor and $l = -(h + c - \bar{c} + 9) = -3$. For $\lambda \sim 0.22$ and $\langle H_u \rangle \eta = 100$-200GeV, $-1 < l < -4$ is needed for correct atmospheric neutrino mass scale, though the requirement depends on the ambiguity of $O(1)$ coefficients.

2.2. $E_6$ Higgs sector

In this subsection, we recall the $E_6$ Higgs sector proposed previously.\(^1\) The content of the Higgs sector with $E_6 \times U(1)_A$ gauge symmetry is given in Table II, where the symbols $\pm$ denote the $Z_2$ parity quantum numbers.\(^*\) To explain how to embed the previous $SO(10)$

Table II. Typical values of anomalous $U(1)_A$ charges.

| 78 | non-vanishing VEV | 27 |
|----|------------------|----|
| $A(a = -1, -)$ | $C(c = -6, +)$ |
| $\Phi(\phi = -3, +)$ | $C'(c' = 7, -)$ |
| $\bar{27}$ | $\bar{C}(\bar{c} = -2, +)$ |
| $1$ | $Z_i(z_i = -2, -)$ (i = 1, 2, 3) |

model into the $E_6$ model, it is helpful to see that the adjoint representation 78 and the fundamental representation 27 are divided as

$$78 \to 45_0 + 16_{-3} + \bar{16}_3 + 1_0,$$

$$27 \to 16_1 + 10_{-2} + 1_4$$

under $SO(10) \times U(1)_{V'}(\subset E_6)$. The non-vanishing VEVs $|\langle \Phi \rangle| = |\langle \Phi' \rangle|$ break $E_6$ into $SO(10)$. The VEVs $\langle A \rangle$ and $|\langle C \rangle| = |\langle \bar{C} \rangle|$ break the $SO(10)$ into the standard model gauge group, as in the previous $SO(10)$ GUT. Note that Higgs sector has the same number of superfields in non-trivial representation as the $SO(10)$ Higgs sector, in spite of the fact that the larger group $E_6$ requires additional Higgs fields to break $E_6$ into the $SO(10)$ gauge group. Therefore, in a sense, this $E_6$ Higgs sector unifies the $SO(10)$ Higgs sector. Actually, the Higgs fields $H$ and $H'$ of the $SO(10)$ model are contained in $\Phi$ and $C'$, respectively, in this $E_6$ model.

\(^*\) Here the composite operator $\Phi \Phi$ play the role of the FN field $\Theta$. 
Here non-vanishing VEVs are determined by

\[ W = W_A' + W_{C'} + W_{C'}'. \]  \hfill (2.12)

We, however, have to be careful about dealing with \( W_{A'} \). Due to a characteristic of the \( E_6 \) group, \( W_{A'} = A'(A + A^3) \) does not contain \( \text{tr} \, 45_A'45_A' \) and the DW form cannot obtained in a natural way. \( \) We call this "factorization problem".\) Because this is caused from \( E_6 \) characteristic, if an \( E_6 \) breaking effect couples to \( A'A^3 \), e.g. \( \Phi A'A^3\Phi \), this problem is avoided. \( W_{C'} \) and \( W_{\bar{C}'} \) play a similar role as denoted in (2.3)-(2.6).

For the quark and lepton sector,\(^5\) we introduced three superfields \( \Psi_i(27) \) \( (i = 1, 2, 3) \) \( (\text{with typical values of the charges} \ (\psi_1, \psi_2, \psi_3) = (9/2, 7/2, 3/2)) \), in which all the superfields in quark and lepton sector of the \( SO(10) \) model are embedded. And this minimal content can realizes realistic quark and lepton mass matrices including bi-large neutrino mixings in a similar manner. Here we define a parameter \( r \) as

\[ \lambda' \equiv \lambda^c \langle C \rangle / \lambda^\phi \langle \Phi \rangle = \lambda^{\frac{1}{2}(c-\phi-\bar{\phi})}. \]  \hfill (2.13)

This parameterizes the mixing of \( 5 \) matter as \( \Delta = 3 - r \) and must be around \( 0 < r < 3/2 \) for bi-large neutrino mixings. This requirement is also depends on the ambiguity of \( O(1) \) coefficients.

The charge assignment in Table II provides a complete \( E_6 \) GUT with \( \ell = -2 \) and \( r = 1/2 \), although the gauge coupling at the cutoff scale may be in non-perturbative region.\(^2\)

§3. simpler \( E_6 \) Higgs sectors

In the previous \( E_6 \) model, \( C(16) \) and \( \bar{C}(\overline{16}) \) of \( SO(10) \) model are embedded into \( 27 \) field and \( \overline{27} \) field, respectively. However, they may also be embedded into \( 78 \) field, resulting simpler \( E_6 \) models. Here we examine this alternative embedding.

Since we introduce two adjoint Higgs \( A' \) and \( A \), we have two kinds of possibilities for reducing the Higgs sector.

1. The VEV \( \langle 16_{A'} \rangle \) or \( \langle \overline{16}_{A'} \rangle \) is non-vanishing.
2. The VEV \( \langle 16_A \rangle \) or \( \langle \overline{16}_A \rangle \) is non-vanishing.

Note that it must be forbidden that \( 16 \) and \( \overline{16} \) have non-vanishing VEVs simultaneously, which destabilizes the DW form of VEVs. For example, if the VEVs \( \langle 16_{A'} \rangle \) and \( \langle \overline{16}_{A'} \rangle \) are non-vanishing, the interactions \( A'^m \) destabilize the DW form of VEVs because \( F_{45_A'} \) includes the VEVs \( \langle 16_{A'} \rangle \) and \( \langle \overline{16}_{A'} \rangle \). At first glance, such an asymmetric VEV structure is forbidden by \( D \)-flatness conditions. But it is shown below that such an interesting VEV can satisfy the \( D \)-flatness conditions.
3.1. \( \langle 16_{A'} \rangle \neq 0 \)

The typical Higgs content is represented in Table III.

| 78  | \( A(a = -1)\) | \(A'(a' = 5)\) |
|-----|----------------|-----------------|
| 27  | \( \Phi(\phi = -5)\) | \(C'(c' = 7)\) |
| 27  | \( \Phi'(\phi' = 6)\) | \(C(c = -6)\) |
| 1   | \( \Theta(\theta = -1)\) | \(Z_i(z_i = -1)\) (\(i = 1-5\)) |
|     | \( S(s = 6)\) |                 |

Table III. Typical values of anomalous \(U(1)_A\) charges.

Suppose that among the above Higgs fields, only \(45_A\), \(1_\Phi\), \(\overline{16}_C\) and \(16_{A'}\) have non-vanishing VEVs such as

\[
\langle 45_A \rangle = \tau_2 \times \text{diag}(v, v, v, 0, 0) \quad (v \sim \lambda^{-a}),
\]

\[
| \langle 1_\Phi \rangle | = | \langle \overline{16}_C \rangle | = | \langle 16_{A'} \rangle | \sim \lambda^{-\frac{1}{2}(\overline{c}+a'+\phi)}.
\]

(3.1)

(3.2)

As mentioned above, if \( \phi + a' + \overline{c} < 0 \), the VEV \( \langle \overline{C}A'\Phi \rangle \) can be non-vanishing, which means \( A' \) has a non-vanishing VEV. Actually, this vacuum satisfies the relations \( \langle \text{tr} A^n \rangle = 0 \) and \( \langle \overline{C}A'\Phi \rangle \sim \lambda^{-(\overline{c}+a'+\phi)} \), which are consistent with the VEV relation (1.1). And this vacuum satisfies not only the \(D\)-flatness conditions for \(SO(10)\) but also that of \(U(1)_{V'}\),

\[
D_{V'} : 4|1_\Phi|^2 - 3|16_{A'}|^2 - |\overline{16}_C|^2 = 0.
\]

(3.3)

Therefore, it is obvious that this vacuum satisfies all the \(E_6\) \(D\)-flatness conditions.

Next we discuss the \(F\)-flatness conditions to know how such a vacuum can be obtained. For simplicity, we assume that any component fields other than \(45_A\), \(1_\Phi\), \(\overline{16}_C\) and \(16_{A'}\) have vanishing VEVs. To determine the VEV of \(45_A\), the superpotential

\[
W_{A'} = A'A + A'A^3 + A'A^4 + A'A^5
\]

is sufficient to be considered. Here, for simplicity, singlets \(Z_i\) are not written explicitly. The \(F\)-flatness condition of \(45_{A'}\) leads to the DW type of VEV, \(\langle 45_A \rangle \sim \tau_2 \times \text{diag}(v, v, v, 0, 0)\). (Here \(A'A^5\) is needed to avoid the “factorization problem”.) Because the positively charged field \(A'\) has a non-vanishing VEV \(\langle 16_{A'} \rangle \neq 0\), the \(F\)-flatness conditions of the negatively charged fields may become non-trivial conditions. Fortunately, in this model, there is no such a non-trivial condition, for example, \(F_{\overline{16}_A} = 0\) is trivial because \(\overline{16}_A\) is a Nambu-Goldstone mode in the superpotential \(W_{A'}\). The \(F\)-flatness condition of \(S\), which is obtained from the superpotential

\[
W_S = S(1 + \overline{C}A'\Phi + f_S(A, Z_i)),
\]

(3.4)

is sufficient to be considered.
leads to
\[
\langle \bar{C}A'\Phi \rangle \sim \lambda^{-(\bar{c}+a'+\phi)}.
\] (3.6)

The $D$-flatness conditions of $SO(10)$ and $U(1)_V$, lead to
\[
|\langle 1\Phi \rangle| = |\langle \overline{16}_C \rangle| = |\langle 16_{A'} \rangle| \sim \lambda^{-\frac{1}{4}(\bar{c}+a'+\phi)},
\] (3.7)

which are the desired vacuum in Eq. (3.2).

The $F$-flatness conditions of $C'$, which are obtained from the superpotential
\[
W_{C'} = \bar{C}(1+Z_i + A + A'(f_C(A, Z_i) + \bar{C}A'\Phi)C'),
\] (3.8)

are written
\[
F_{16_{C'}} = (1 + Z_i + A)\overline{16}_C = 0,
\] (3.9)
\[
F_{1_{C'}} = (f_C(A, Z_i) + \bar{C}A'\Phi)\overline{16}_C 16_{A'} = 0.
\] (3.10)

These conditions realize an alignment between the VEVs $\langle 45_A \rangle$, $\langle \overline{16}_C \rangle$ and $\langle 16_{A'} \rangle$ by shifting the VEVs of singlet fields $Z_i$ and as the result, the pseudo Nambu-Goldstone fields become massive. The $F$-flatness condition of $\overline{16}_{A'}$, which is obtained from the superpotential
\[
W_{A'A'} = A'(f_A(A, Z_i) + \bar{C}A'\Phi)A',
\] (3.11)

realizes also an alignment between $\langle 45_A \rangle$ and $\langle 16_{A'} \rangle$.

It is interesting that in this model, the sliding singlet mechanism\textsuperscript{14} is naturally realized. The $F$-flatness conditions of $\bar{\Phi}'$, which are obtained from the superpotential
\[
W_{\Phi'} = \bar{\Phi}'(1+Z_i + A + A'(f_\Phi(A, Z_i) + \bar{C}A'\Phi))\Phi,
\] (3.12)

are written
\[
F_{1_{\Phi'}} = (1 + Z_i)1_\Phi = 0,
\] (3.13)
\[
F_{\overline{16}_{\Phi'}} = (f_\Phi(A, Z_i) + \bar{C}A'\Phi)1_\Phi 16_{A'} = 0.
\] (3.14)

At first glance, the component field $10_\phi$, which includes doublet Higgs in the standard model, seems to have a mass term from the superpotential $\bar{\Phi}'(1 + Z_i)\Phi$. However, this mass term vanishes in the desired vacuum which satisfy the first condition in the above $F$-flatness conditions. Moreover, because the adjoint field $A$ has the DW type of VEV, only the triplet Higgs becomes massive through the interaction $\bar{\Phi}'A\Phi$. As the result, DT splitting is realized by the sliding singlet mechanism and the DW mechanism. The essential point here is that the doublet Higgs has the same quantum number under the generator $\langle A \rangle$ as the component
field $1_\Phi$ which has non-vanishing VEV. This mechanism can be generalized, that will be discussed in the following paper.\(^{18}\)

In the above model, for intelligibility, we introduced a positively charged singlet $S$ in order to fix the VEV $\langle \tilde{C} A' \Phi \rangle \sim \lambda^{-\tilde{c} + a' + \phi}$. However, one of the non-trivial $F$-flatness conditions of $1_{C'}$, $\overline{16}_{A'}$ and $\overline{16}_{\Phi'}$ can play the same role as $S$. If we do not introduce the field $S$, the number of the negatively charged singlets becomes four.

It is worthwhile to note how to determine the anomalous $U(1)_A$ charges. In order to realize DT splitting, the terms

$$A' A^5, \Phi' A \Phi, C(A + Z) C'$$

must be allowed, and the term

$$\tilde{C} A'^2 \Phi$$

must be forbidden. These requirements can be rewritten as the inequalities. We determined the charges in order to satisfy the inequalities.

Unfortunately, we have not found realistic matter sector with this Higgs sector. Actually, the mixing parameter $r$, which is obtained by

$$\lambda^r \equiv \frac{\lambda^{a' + \phi} \langle 16_{A'} 1_\Phi \rangle}{\lambda^\phi \langle 1_\Phi \rangle} = \lambda^{a'} \langle 16_{A'} \rangle = \lambda^{\frac{1}{2} (2a' - \tilde{c} - \phi)},$$

must be around 1/2 in order to obtain bi-large neutrino mixings, and it looks to be impossible, because $2a' - \tilde{c} - \phi \gg 1$.

3.2. $\langle 16_A \rangle \neq 0$

In this section, we consider another possibilities in which $C(16)$ of the $SO(10)$ model is embedded into negatively charged adjoint Higgs $A(78)$. This possibility is more promising because the condition for realistic matter sector, $2a - \tilde{c} - \phi \sim 1$, can be realized. The content of the Higgs sector is the same as in the previous possibility, except for the charges and the number of singlets.

To begin with, we examine $D$-flatness conditions. Because $\langle 45_A \rangle \neq 0$ and $\langle 16_A \rangle \neq 0$, the $D$-flatness condition of $16$ direction gives a non-trivial condition. In order to compensate the contribution from $A$ in the condition, $\Phi$ and/or $\tilde{C}$ must have non-zero VEV in both $1$ and $16$ components. Therefore, non-trivial $D$-flatness conditions are

$$D_{V + V'} : |1_\Phi|^2 = |16_A|^2 + |1_{\tilde{C}}|^2,$$

$$D_V : |16_{\tilde{C}}|^2 = |16_A|^2 + |16_\Phi|^2,$$

$$D_{16} : 45_A^* 16_A = 1_\Phi^* 16_\Phi - 16_{\tilde{C}}^* 1_{\tilde{C}}.$$
In addition, we suppose
\[
\langle 16 \bar{C} \rangle \langle 16_A \rangle \langle 1_\phi \rangle \sim \langle 16 \bar{C} \rangle \langle 45_A \rangle \langle 16_\phi \rangle \\
\sim \langle 1_C \rangle \langle 45_A \rangle \langle 1_\phi \rangle \\
\sim \lambda^{-(\bar{c}+a+\phi)} \equiv \lambda^{-3k}
\]
(3.21)
\[
\langle 45_A \rangle \sim \lambda^{-a}
\]
(3.22)
are obtained from \(F\)-flatness conditions as generally expected.\(^1\) From these conditions except for Eq.(3.20), the orders of VEVs are determined as follows, for \(\lambda^{-a} \gg \lambda^{-k}\).
\[
\langle 16 \bar{C} \rangle \sim \langle 16_A \rangle \sim \lambda^{-k} \equiv \lambda^{-a}\lambda^r
\]
(3.24)
\[
\langle 1_C \rangle \sim \langle 16_\phi \rangle \sim \lambda^{-2k} \sim \lambda^{-a}\lambda^{2r}
\]
(3.25)
\[
\langle 45_A \rangle \sim \lambda^{-a}
\]
(3.26)
Here, \(r = a - k\) is the mixing parameter, introduced in §2. For these VEVs, the effective charges can be defined and therefore the natural gauge coupling unification is realized.\(^2\) Taking account of Eq.(3.20), it may appear that this condition requires \(r > 0\). However, since \(r\) should be small (~1/2) as mentioned above and there is an ambiguity due to order one coefficients, Eq.(3.20) can be satisfied. To be more precise, Eq.(3.20) has the form \(\lambda^{-2a+r} = \lambda^{-2a+3r} + \lambda^{-2a+3r}\), and the r.h.s may become \(2\lambda^{-2a+3r} \sim \lambda^{-2a+r}\chi^{2r-1/2}\), allowing \(r = 1/4\). And the ambiguities of \(O(1)\) coefficients makes a larger \(r\) possible.

Next, we examine \(F\)-flatness conditions. The typical charge assignment of Higgs sector is represented in Table IV. Here the VEVs are again determined by

| 78 | \(A(a = -1, +)\) \(A'(a' = 5, +)\) |
|---|---|
| 27 | \(\bar{\Phi}(\phi = -3, +)\) \(C'(c' = 6, -)\) |
| 27 | \(\bar{\Phi}'(\bar{\phi}' = 5, +)\) \(\bar{C}(\bar{c} = 0, -)\) |
| 1 | \(\Theta(\theta = -1, +)\) \(Z_i(z_i = -1, +) (i = 1, 2)\) |

\(^1\) Strictly speaking, if three conditions in Eq. (3.21) were determined by \(F\)-flatness conditions, the \(F\)-flatness and \(D\)-flatness conditions would become over-determined. Therefore, only two of the three conditions are determined by \(F\)-flatness conditions. Then, another solution,
\[
\langle 1_A \rangle \sim \langle 16_A \rangle \sim \lambda^{-a} \ll \langle 1_\phi \rangle \sim \langle 16_\phi \rangle \sim \langle 1_C \rangle \sim \langle 16_\bar{C} \rangle
\]
may appear, by which the natural gauge coupling unification is not realized. Though the \(O(1)\) coefficients determine which vacuum is realized, the desired vacuum is obtained in some (finite) region of parameter space of the \(O(1)\) coefficients.
\[ W = W_{A'} + W_{\phi'} + W_{C'}. \]  

(3.27)

where

\[ W_{A'} = A'(A + A^2 + A^4 + A^5) \]  

(3.28)

\[ W_{\phi'} = \phi'(1 + A + Z_i + A^2 + AZ_i + Z_i^2)\phi \]  

(3.29)

\[ W_{C'} = C(1 + A + Z_i + \cdots + (\bar{C}\phi)^2)C'. \]  

(3.30)

As in the previous model, the \( F \)-flatness condition of \( 45A' \) leads to the DW type of VEV, \( \langle 45A \rangle \sim \tau_2 \times \text{diag}(v, v, v, 0, 0) \). The \( F \)-flatness condition of \( 1\bar{\phi}' \) makes the \( E_6 \) singlet part in the parenthesis of Eq. (3.29) vanish, leading vanishing doublet mass terms (the sliding singlet mechanism). The \( F \)-flatness condition of \( 16\bar{\phi}' \) gives a factored equation

\[ (1 + A + Z_i) [45A_{16\phi} + 16A_{1\phi}] = 0, \]  

(3.31)

which can be checked by the explicit calculation based on \( E_6 \) group theory. The above two \( F \)-flatness conditions are satisfied by shifting the VEVs of two singlets \( Z_i \). The two \( F \)-flatness conditions of \( 1C' \) and \( 16C' \) and the three \( D \)-flatness conditions in Eqs. (3.18)-(3.20) determine the five VEVs \( 16A, 1\phi, 16\phi, 1C \) and \( \text{TE}_C \). It is straightforward to analyse the mass matrices of Higgs to check all modes are superheavy except for one doublet Higgs pair contained in \( 10_{\phi}. \)

Now, we examine the condition to be compatible with the matter sector, for which we introduced the same three superfields as in the \( E_6 \) models in §2. Applying the discussion to this case, the parameters \( r \) and \( l \) are given from following relations;

\[ \lambda^r \sim \frac{\lambda^c \langle 16_C \rangle}{\lambda^\phi \langle 1\phi \rangle} \sim \frac{\lambda^{a+\phi} \langle 16_A \rangle \langle 1\phi \rangle}{\lambda^\phi \langle 1\phi \rangle} = \lambda^{a-k}, \]  

(3.32)

\[ \lambda^{-(5+l)} \sim \lambda^{4+\phi-2\bar{c}} \left( \text{TE}_C \right)^{-2} \sim \lambda^{4+\phi-2\bar{c}+2k}. \]  

(3.33)

For example, a set of charges \((a, \phi, \bar{c}) = (-1, -3, 0)\) as in Table IV gives \((r, l) = (1/3, -10/3)\), which is allowed as shown in §2.

It is worthwhile to note how to determine the symmetry in this model. For this purpose, we have to mention which terms are needed and which must be forbidden. In order to stabilize the DW-type VEV, \( \bar{C}A'\phi \) must be forbidden, and to avoid the “factorization problem”, \( A'A^5 \) is needed. However, it is difficult to forbid \( \bar{C}A'\phi \) while allowing \( A'A^5 \) by the SUSY-zero mechanism for small \( r = \frac{1}{3}(\bar{c} + \phi - 2a) \). Therefore we need another mechanism to forbid \( \bar{C}A'\phi \), e.g. to introduce an additional \( Z_N \) symmetry. Since \( \Psi_3\Psi_3\phi \) gives order one

---

\( ^{\text{\textcopyright}} \) We would emphasize in this model all the singlet fields also become superheavy, while in the previous models, one massless singlet field appears.
Thus, we assign nontrivial $Z_N$ charges only on $\bar{C}$ (and $C'$). In $W_{\Phi}$, the would-be simplest superpotential $W_{\Phi} = \bar{\Phi} A \Phi$ is not consistent with the $D$-flatness conditions for these VEVs (3.24)-(3.26). This can be checked from the explicit calculation in terms of $SU(3)_R \times U(1)_{T_6^E}$ ($\subset E_6$), which contain $U(1)_{B-L}$, (or, more easily, from examining the mass matrix of $10 \times \overline{10}$ of $SU(5)$). This is again a characteristic of $E_6$ group, and therefore, $E_6$ breaking effects such as $\bar{\Phi}'AZ + A^2 \Phi$ are needed. And in order that the sliding singlet mechanism acts, $\bar{\Phi}' \Phi \Phi$ must be forbidden.\(^{18}\) If $\bar{C}C \Phi C'$ is forbidden, the $F$-flatness conditions of $1_{C'}$ and $16_{C'}$ cannot fix the scale of the VEVs of $\bar{C}$ and $\Phi$. It means that two other conditions to fix these VEVs are required. Though introducing two additional positively charged singlets (and two additional negatively charged singlets) makes it possible, it may be more simple to introduce the term $\bar{C} \Phi C'$. If we take the non-trivial $Z_N$ charges for $(\bar{C}, C')$ as $(1, N-1)$, the term becomes $\bar{C}(\bar{C} \Phi)^N C'$. To allow the terms $A'C^5$, $\bar{\Phi}'AZ \Phi$ and $\bar{C}(\bar{C} \Phi)^N C'$, we adopt the anomalous $U(1)$ charges $(a', \bar{\phi}', c') = (-5a, -\phi - a - z, -\bar{c} - N(\bar{c} + \phi))$. If we take $(a, \phi, \bar{c}) = (-1, -3, 0)$, then $(a', \bar{\phi}', c') = (-1, 5, 3N)$ and a consistent model is constructed. In this model, $N = 2$ is sufficient to decouple $\Phi$ and $\bar{C}$ from $W_{A'}$. (See Table IV.) As in the $E_6$ models in §2, the half-integer charges of matter supermultiplets play the same role as “R-parity” in this model. Other charge assignments $(a, \phi, \bar{c}, z_i, a', \bar{\phi}', c') = (-1/2, -3, 4/3, -1/2, 5/2, 9/2, 11/3)$ and $(-1, -3, 1/3, -1, 5, 5, 23/3)$ give other examples of consistent models. Although the former requires the “R-parity” (or $Z_2$ symmetry same as in §2), the latter requires no additional $Z_N$ symmetry.

\section*{§4. Summary and Discussion}

In this paper, we consider more simple version of $E_6$ model with anomalous $U(1)$ symmetry than already proposed model.\(^1\) Here, the spinor Higgs $C(16)$ of $SO(10)$ model is embedded into $E_6$ adjoint Higgs $A'$ or $A$. Unfortunately, the former case is incompatible with the matter sector as far as we know, although the doublet-triplet splitting\(^{11}\) and the natural gauge coupling unification\(^2\) are realized. On the other hand, the latter case is compatible with the matter sector, resulting consistent $E_6$ models. Moreover, we can construct a model in which no additional symmetry other than the anomalous $U(1)$ symmetry. This is due to the sliding singlet mechanism,\(^{14}\) which we will discuss in more detail in another paper.\(^{18}\) It is interesting that in $E_6$ model, both the Dimopoulos-Wilczek mechanism\(^{15}\) and the sliding singlet mechanism can act elegantly to solve the doublet-triplet splitting problem. Because we introduced generic interactions, the theory can be defined essentially by their symmetry. In the models proposed in this paper, we introduced only six non-trivial
representation fields for the Higgs sector and three fields for matter sector. It is surprising that complete $E_6$ grand unified theories can be constructed by assigning only nine (integer) charges.

Due to the smaller Higgs sector, the gauge coupling at the cutoff scale tend to remain perturbative region. And the evaluation of the lifetime of a nucleon in this model $\tau_p(p \rightarrow e^+\pi^0) \sim 5 \times 10^{33}$ years$^2$ (for $a = -1$) becomes more reliable and gives strong motivation to search the proton decay $p \rightarrow e^+\pi^0$.

Acknowledgement

N.M. thanks M.Bando for valuable discussion, and is supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

References

1) N. Maekawa and T. Yamashita, Prog. Theor. Phys. 107, 1201 (2002).
2) N. Maekawa, Prog. Theor. Phys. 107, 597 (2002);
   N. Maekawa and T. Yamashita, Phys. Rev. Lett. 90 (2003) 121801; Prog. Theor. Phys. 108, 719 (2002).
3) N. Maekawa, Phys. Lett. B521 (2001) 42.
4) N. Maekawa, To appear in Phys. Lett. B (arXiv:hep-ph/0212141).
5) M. Bando and M. Maekawa, Prog. Theor. Phys. 106 (2001) 1255.
6) N. Maekawa, Prog. Theor. Phys. 106 (2001) 401; arXiv:hep-ph/0110276.
7) E. Witten, Phys. Lett. B149 (1984), 351;
   M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987), 589;
   J.J. Atick, L.J. Dixon and A. Sen, Nucl. Phys. B292 (1987), 109;
   M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B293 (1987), 253.
8) M. Green and J. Schwarz, Phys. Lett. B149 (1984), 117.
9) L. Ibáñez and G.G. Ross, Phys. Lett. B332 (1994) 100;
   J.K. Elwood, N. Irges, P. Ramond Phys. Lett. B413, 322 (1997);
   N. Irges, S. Lavignac, and P. Ramond, Phys. Rev. D 58, 035003(1998);
   C.H. Albright and S. Nandi, Mod. Phys. Lett. A11, 737 (1996); Phys. Rev. D 53, 2699 (1996);
   M. Bando, T. Kugo, Prog. Theor. Phys. 101, 1313 (1999);
   Y. Nomura, T. Sugimoto, Phys. Rev. D 61, 093003 (2000);
K.-I. Izawa, K. Kurosawa, Y. Nomura, T. Yanagida, Phys. Rev. D 60, 115016 (1999); M. Bando, T. Kugo, and K. Yoshioka, Prog. Theor. Phys. 104, 211 (2000).

10) L.J. Hall and S. Raby, Phys. Rev. D 51, 6524 (1995); G. Dvali and S. Pokorski, Phys. Rev. Lett. 78, 807 (1997); Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B396 150 (1997); ibid 409, 220 (1997); G. Dvali and A. Riotto, Phys. Lett. B417, 20 (1998); Q. Shafi and Z. Tavartkiladze, Phys. Lett. B459, 563 (1999); ibid B482, 145 (2000); ibid B487, 145 (2000); Nucl. Phys. B573, 40 (2000); J.L. Chkareuli, C.D. Froggatt, I.G. Gogoladze, and A.B. Kobakhidze, Nucl. Phys. B594, 23 (2001).

11) E. Witten, Phys. Lett. B105 (1981) 267; A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115 (1982) 380; B. Grinstein, Nucl. Phys. B206 (1982) 387; K. Inoue, A. Kakuto and T. Takano, Prog. Theor. Phys. 75 (1986) 664; E. Witten, Nucl. Phys. B258 (1985) 75; T. Yanagida, Phys. Lett. B344 (1995) 211; Y. Kawamura, Prog. Theor. Phys. 105 (2001) 691; Prog. Theor. Phys. 105 (2001) 999.

12) Fukuda et al. (The Super-Kamiokande Collaboration), Phys. Lett. B436 (1998) 33; Phys. Rev. Lett. 81 (1998) 1562; Phys. Rev. Lett. 86 (2001) 5656.

13) C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147 (1979) 277.

14) E. Witten, Phys. Lett. B105 (1981) 267; D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B113, (1982) 151; S. Dimopoulos and H. Georgi, Phys. Lett. 117B, (1982) 287; K. Tabata, I.Umemura and K. Yamamoto, Prog. Theor. Phys. 71 (1984) 615; A. Sen, Phys. Lett. B148 (1984) 65; S.M. Barr, Phys. Rev. D 57 (1998) 190.

15) S. Dimopoulos and F. Wilczek, NSF-ITP-82-07; M. Srednicki, Nucl. Phys. B202 (1982) 327.

16) S.M. Barr and S. Raby, Phys. Rev. Lett. 79 (1997) 4748.

17) T. Toshio [Super-Kamiokande Collaboration], arXiv:hep-ex/0105023.

18) N. Maekawa and T. Yamashita, in preparation.