BER Performance Improvement of Alamouti MIMO-STBC Decoder Using Mutual Information Method

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Abstract. Bit error rate (BER) performance of multiple input multiple output (MIMO) space time block code (STBC) decoder depends on the channel estimator quality, where the quality of classical least square (LS) estimator depends mainly on the number of pilot symbols. The aims of this paper are to implement a new STBC decoder, measure the BER performance and make comparison with conventional one, without increasing the number of pilot symbols. Although the new strategy is more complex than a conventional decoder, it gives the opportunity to join estimation and decoding at the same time. Minimum mutual information (MMI)- independent component analysis (ICA) algorithm has been used for extracting source signals. MIMO-STBC systems were taken as a cases study with two transmitters. The simulation results show that MMI-ICA algorithm provides superior BER performance as compared with conventional maximum ratio combiner (MRC). The important point, which denoted during simulation, the performance of ICA algorithm with high frame length provides better BER performance than the conventional one.

Keywords: Space-time block code decoder, MIMO-STBC, MMI-ICA, maximum ratio combiner

1. Introduction

In modern communication systems, the MIMO is one of the most important technical systems. In MIMO systems, reliability can be enhanced by using of space time encoding (STC) or orthogonal STBCs (OSTBCs) [1], while the data rate can be improved by using spatial multiplexing, which provides complete diversity with low decoding complexity at the expense of a loss in the rate of transmission for more than two of transmitted antennas [2]. The process of estimating a channel is the one that provides important information about the various interactions that occurred to signals when they burst across the channel. Channel estimation has been investigated and covered by many researchers. So, different techniques for wireless channel estimation have suggested and implemented to provide channel state information (CSI). The efficiently advanced techniques of channel estimation have been evolved recently for the use in communication systems of MIMO in the receiver side. The estimation techniques of MIMO can be divided into three groups [3]:

I. Non-Blind Channel Estimation
II. Blind Channel Estimation

III. Semi-Blind of Channel Estimation

In Blind estimation, independent component analysis (ICA) is a kind of key strategy used. ICA is a technique capable of separating independent sources from these linear mixtures without any information about the mixing process; therefore, it should be used in blind receivers to detect transmitted symbols without any training sequence. The main disadvantage for the training-based system is the efficiency of spectral; the useful data rate is reduced that are used in the other techniques.

2. MIMO Channel Model

The model of MIMO channel with quadrature phase shift keying (QPSK) modulation Alamouti STBC encoder that used in this paper is the quasi-static, frequency non-selective, Rayleigh fading channel model. Under the quasi-static assumption, the channel response is varying randomly between block for block, but it fixed within a transmission time this time called coherence time. Time of coherence ($T_{coh}$) can be define as the time period during which the channel impulse response is remain invariant.

During $T_{coh}$ time, the MIMO channel can be represented by linear system with coefficients matrix [1]:

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$  

(1)

The mechanism of data transmission through MIMO system can be represented by using matrix operation. At time, if transmitted signals represented as column vector $X^t = \begin{bmatrix} x_1^t \\ x_2^t \end{bmatrix}$ and received signals as $Y^t = \begin{bmatrix} y_1^t \\ y_2^t \end{bmatrix}$ then:

$$Y^t = HX^t + \text{noise}$$  

(2)

A space–time block code is defined by $(N_t)$ row by $p$ column transmission matrix$G_{N_t}$. The $N_t$ represents the number of transmission antennas that is used to separate different codes from each other, while $p$ represents number of output transmitted period. For example, $G_2$ represents a code, which utilizes two transmitted antennas and is defined by [4]:

$$G_2 = \begin{bmatrix} S_1 \\ -S_2 \\ S_1^* \end{bmatrix}$$  

(3)

This code is well known by Alamouti STBC [5]. The illustrated encoding process that it assumes M- array modulation scheme is used in this paper. If the input of STBC encoder are $S_1$ and $S_2$, then the encoder outputs are transmitted in two periods ($p = 2$) from two transmitted antennas. During the first transmitted period ($t = 1$), signals $X^1 = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$ are transmitted from antennas one and two, respectively. In the second transmission period ($t = 2$), signals $X^2 = \begin{bmatrix} -S_2 \\ S_1 \end{bmatrix}$ are transmitted in the same way. It is clear that the encoding is done in both the space and time domains [6].

Decoding process can be achieved using two steps, firstly, channel coefficients $h_{ij}$ must be estimated (using channel estimator), then the second step uses these estimated values to estimate the encoded signals $S_1$, $S_2$ using maximum ratio combiner (MRC).

To illustrate the MRC operation, Alamouti STBC will be used as example. If coefficient matrix of $2 \times N_t$ MIMO channel is denoted as, $H = [h_1 \ h_2]$ where $h_i$ is the $i_{th}$ column of $H$ matrix, then the received signals at $t = 1$ are:

$$Y^1 = \begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix} = [h_1 \ h_2] \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \text{noise}$$  

(4)

If channel coefficient is still constant (quasi-static channel), then the received signals at $t = 2$ are:
By using simple modification for Equation (5) can be rewrite as:

\[(Y_2)^* = [(h_2)^* - (h_2)^*] \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \text{noise} \]  

The main idea of MRC for deducing the value \( S_1, S_2 \) is combining the Equation (4) and Equation (6) to obtain:

\[
\begin{bmatrix} Y_1^* \\ (Y_2)^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ (h_2)^* & -(h_2)^* \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \text{noise}
\]

In this case for \( G_{nt=2} \) orthogonal STBC encoding matrix with 2-input sample \( S_1, S_2 \) the MRC equation can be written as:

\[ Y_{MRC} = H_{MRC} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \text{noise} \]

It can be denoted that \( H_{MRC} \) is orthogonal matrix where:

\[ (H_{MRC})^H H_{MRC} = \| H_{MRC} \|_k \]

where \( \| A \| \) is norm of A.

The input symbols of STBC encoder \( S_1, S_2 \) can be estimated by:

\[
\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \equiv \begin{bmatrix} 1 \\ \| H_{MRC} \| \end{bmatrix} (H_{MRC})^H Y_{MRC}
\]

### 3. Channel Estimation Techniques

#### 3.1 Least Square Channel Estimation

Some text. The popular strategy for channel estimation is achieved by training or pilot symbols \( (X_p) \) that is known to the receiver side as shown in Figure 1. The pilot symbols must be placed as frequently as the coherence time, in order to keep tracking of the time-varying channel characteristics. However, training symbols reduce the throughput and such schemes are inadequate when the bandwidth is scarce. The Least Square (LS) technique is mostly used to channel estimation when the training symbols \( (Y_p, X_p) \) are available where [7]:

\[ Y_p = H X_p + \text{noise} \]

The least square for channel estimation is find the estimated channel matrix \( \hat{H} \), in such a method that the minimized cost function \( \| Y_p - \hat{H} X_p \|^2 \) that makes:

\[
\hat{H} = Y_p (X_p)^+ \]

where \( (X_p)^+ \) is pseudo inverse of \( X_p \) which can be define as [8]:

\[ (X_p)^+ = X_p^H (X_p X_p^H)^{-1} \]
The main object of this paper is how to improve the quality of channel estimation without increasing number of training pilots. In this paper will use a simple criteria based on using $\hat{H}$ (estimated using LS estimator with minimum training pilot sequence) as initial value for statistical based blind channel estimator to obtain a semi blind channel estimator. These criteria will mitigate the problems of blind channel estimator (phase and order ambiguities) besides that, will reduce the computation complexity and accelerate the estimation process.

### 3.2 ICA Channel Estimation

The Mutual Information (MI) is one of the best contrast functions for designing blind separation algorithms because it has several invariant properties from the information geometrical point of view. MI is a natural measure of the dependence between random variables. If these random events are totally independent, then $\text{MI} = 0$. The sources $u_1, u_2, ..., u_n$, for BSS problem are assumed to be independent, therefore, $\text{MI}(u_i, u_j) = 0$ for $i \neq j$. After they are mixed they are no longer independent i.e. $\text{MI}(r_i, r_j) > 0$ [9].

The solution for all unit ICA transformation ($\hat{\text{U}} = \text{WR}$ or using whitening$\text{WR}$) can be simplified to optimization problem that searches for optimum $n_x \times n_r$ dimensional matrix ($W_{\text{opt}}$) which minimizes the mutual information measuring. In other word mutual information is used as a cost function to measure the quality of de mixing vector ($W$) where:

$$\text{if } W = \begin{cases} W_{\text{opt}} & \Rightarrow \text{MI}(\hat{\text{U}}) \approx \text{min} \\ \text{else} & \Rightarrow \text{MI}(\hat{\text{U}}) > \text{min} \end{cases}$$

where the mutual information can be measured as:

$$\text{MI}(\hat{\text{U}}) = \sum_{a=1}^{N_s} H(\hat{u}_a/W) - H(R) - \log(\det(W))$$

There are two ways to solve optimization problem, Gradient Ascent/ Decent Algorithm (GAA) and evolutionary search algorithm.
3.2.1 Minimum Mutual Information (MMI)-ICA Algorithm.

The Mutual Information (MI) is one of the best contrast functions for designing blind separation algorithms because it has several invariant properties from the information geometrical point of view. MI is a natural measure of the dependence between random variables. If these random events are totally independent, then \( MI = 0 \). The sources \( u_1, u_2, ... , u_n \) for BSS problem are assumed to be independent, therefore, \( MI(u_i, u_j) = 0 \) for \( i \neq j \). After they are mixed they are no longer independent i.e. \( MI(r_i, r_j) > 0 \) [9].

The solution for all unit ICA transformation (\( \mathbf{U} = \mathbf{W} \mathbf{R} \) or using whitening \( \mathbf{W} \mathbf{R} \)) can be simplified to optimization problem that searches for optimum \( n \times n \) dimensional matrix \( \mathbf{W} \) which minimizes the mutual information measuring. In other word mutual information is used as a cost function to measure the quality of de mixing vector \( \mathbf{W} \) where:

\[
\text{if } \mathbf{W} = \mathbf{W}_{\text{opt}} \rightarrow \text{MI}(\mathbf{U}) \approx \min \\
\text{else } \rightarrow \text{MI}(\mathbf{U}) > \min
\]

where the mutual information can be measured as:

\[
\text{MI}(\mathbf{U}) = \sum_{a=1}^{N_a} H(\bar{u}_a / \mathbf{W}) - H(\mathbf{R}) - \log |\text{det}(\mathbf{W})| \tag{15}
\]

There are two ways to solve optimization problem, Gradient Ascent/ Decent Algorithm (GAA) and evolutionary search algorithm.

3.2.2 Gradient Decent Algorithm for MMI-ICA

It is generally very difficult to obtain the exact function form of the MMI since the probability density functions (pdfs) of the outputs are unknown. The Gram-Charlier expansion was applied widely to approximate the conditional entropy of the \( a_{th} \) source:

\[
H(\bar{u}_a / \mathbf{W}) = 0.5 \log(2\pi e) - \frac{(k_3^a)^2}{2.37} - \frac{(k_4^a)^2}{2.41} + \frac{3}{8} (k_3^a)^2 k_4^a + \frac{1}{16} (k_4^a)^3
\]

where \( k_3^a = E((\bar{u}_a)^3) \) and \( k_4^a = E((\bar{u}_a)^4) - 3 \)

Note it assume \( \bar{u}_a \) is zero mean unit variance random variable.

Gradient descent algorithm can be applied to minimize \( \text{MI}(\mathbf{U}) \). First the value of \( \frac{\partial H(\bar{u}_a / \mathbf{W})}{\partial \bar{u}_a} \) should be evaluated, then \( \mathbf{W} \) will be update recursively for each iteration (it) using:

\[
\mathbf{W}^{\text{it}+1} = \mathbf{W}^{\text{it}} - \mu (I_{n_{\text{mix}}} - \Psi(\mathbf{U}) \mathbf{U}^T) \mathbf{W}^{\text{it}} \tag{17}
\]

where \( \Psi(\mathbf{U}) = \begin{bmatrix} \Psi(\bar{u}_1) \\ \Psi(\bar{u}_2) \\ \vdots \\ \Psi(\bar{u}_{n_\text{mix}}) \end{bmatrix} \) is the nonlinear function for each source \( \bar{u}_a \) and it can be found by:

\[
\Psi(\bar{u}_a) = A(\bar{u}_a)^2 + B(\bar{u}_a)^3 \tag{18}
\]

where \( A \) and \( B \) are two constants depend on \( k_3^a \) and \( k_4^a \) of the \( \bar{u}_a \) source:

\[
A = -\frac{3}{2} k_3^a + \frac{9}{4} k_4^a k_3^a \tag{19}
\]

\[
B = -\frac{1}{2} k_4^a + \frac{9}{2} (k_3^a)^2 + \frac{3}{4} (k_4^a)^2 \tag{20}
\]

The steps for All unit MMI-ICA using only \( n_r \) mixture signals \( \mathbf{R} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{n_r} \end{bmatrix} \) (or whitening received mixture signals \( \tilde{\mathbf{R}} \)), where each signal is \( N_{\text{samp}} \) of samples
1. Initialize de-mixing matrix 
\[
\begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_{n_s}
\end{bmatrix}
\]

2. \( \text{it} = 1 \) (number of iteration)

3. \( a = 1 \) (source loop)

4. Finding extracted source: 
\[
\bar{u}_a = w^a R
\]
where \( \bar{u}_a \) is one dimension vector with \( N_{\text{samp}} \) samples.

5. Normalizes and center \( \bar{u}_a \) (remove mean and made variance =1)

6. Evaluate the values of: \( k_1^{\|} \) and \( k_2^{\|} \) then use these values to find the value of \( A, B \) and \( \Psi(\bar{u}_a) \) using Equation (23 and 24).

7. If \( a \leq n_s \) \( \rightarrow \) \( a = a + 1 \) then Go to 4 else Go to 8

8. Constrict vector \( \Psi(\bar{u}) = \begin{bmatrix}
\Psi(\bar{u}_1) \\
\Psi(\bar{u}_2) \\
\vdots \\
\Psi(\bar{u}_{n_s})
\end{bmatrix}
\]

9. Update the de-mixing matrix using Equation (22)

10. It assume \( W \) is orthonormal matrix 
\[
W^{\text{it}+1} = \left(W^{\text{it}+1}(W^{\text{it}+1})^T\right)^{-0.5} W^{\text{it}+1}
\]

11. The algorithm can be stopped (go to step 12)
if: \( \text{it} > \) maximum number of iteration.
or \( W^{\text{it}+1} \) converge to specific value i.e:
\[
\|u_s - \left\|W^{\text{it}} \times (W^{\text{R}+1})^T\right\| \leq \text{Threshold}
\]
else: \( \text{it} = \text{it} + 1 \), Go to step 3.

12. At end of iteration specify sources order and there signs.

3. Simulation Results

MATLAB-2018 program has been used to perform QPSK- STBC (2×2) MIMO channel in this paper. BER and the number of iteration are used to analysis the decoding and estimation performances. This paper introduces a random data generator that creates digital information bits (frame-by-frame), where the frame length is different for each case. To modulate these frames, QPSK modulator has been used to produce a different number of symbols for each frame. First \( N_p \) symbols will be used as training symbols, which they are known at receiver side, while the data symbol will be the remaining frame length \( N_t \) that encoded by STBC encoder. In this work, two antennas have been proposed to transmit the encoded symbols through MIMO Rayleigh fading channel. The transmitted signal is added to its complex AWGN.

At receiver end, the LS channel estimation firstly uses the training symbols to the channel estimate coefficient. The other received symbols are decrypted by using MRC and then fed to the QPSK demodulator. The coded bits are compared with the data bit frame that originally created to calculate the corresponding BER with a given SNR. Secondly, a simple criterion based on the use of \( H^{\star} \) (estimated channel coefficient by using the LS estimator with minimal experimental sequence of training) is used as the initial value of the statistically based blind channel estimator to obtain a semi-blind channel estimator.

The improvement of BER performance for channel estimation by using ICA algorithm for all sources is achieved and compared with the BER performance for channel estimation by using the traditional method LS with different number of frame as follows:

- By using MMI- ICA algorithm with no Prewriting
  a. Number of symbole/frame = 128
  b. Number of symbole/frame = 1024
From Figure 2 (a, b), it can be seen that the MIMO-STBC channel estimation by using MMI-ICA algorithm achieves a good performance with only 1.2 to 1.4 dB and shows the number of iteration gradually decrease when compared with the LS case where $X/g^2$ is known at the receiver.

4. Conclusion
The main object of this paper is how to improve BER performance of MIMO-STBC using a minimum number of pilot symbols and a minimum number of receivers ($N_g^2=2$). The BER performance of conventional MRC decoder of MIMO STBC system based on LS channel estimator have been evaluated. A new strategy for decoding MIMO STBC based on a new model for MIMO-STBC has been proposed. This model is based on representing MRC as a noisy linear mixing system. Although the new decoder is more complex than conventional MRC, the new strategy gives the opportunity to joint estimation and decoding at same time. The simulation results show that MMI-ICA algorithm...
provides superior BER performance as compared with conventional MRC decoder. MMI-ICA algorithm is the most complex ICA algorithms since its all units version required huge calculation.

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