Extended experimental and numerical investigations on constraint forces from imposed deformations

Johannes Berger | Max Pfeiffer | Jürgen Feix

University of Innsbruck, Unit of Concrete Structures and Bridge Design, Innsbruck, Austria

Correspondence
Johannes Berger, University of Innsbruck, Unit of Concrete Structures and Bridge Design, Technikerstraße 13, 6020 Innsbruck, Austria.
Email: johannes.berger@uibk.ac.at

Funding information
Österreichische Forschungsförderungsgesellschaft, Grant/Award Number: Verkehrinfrastrukturforschung 2017

Abstract
Constraint forces are directly proportional to the stiffness of the system, so the size of the constraining force reduction depends on the ability of the system to reduce stiffness by the cracking of concrete, the plasticizing of reinforcement, or a time-dependent process. Constraint forces can be differentiated in two cases: internal forces due to constraints that have the same or opposite action effects as the load in the relevant cross sections. For the first case, extensive experimental investigations have already been performed, and the results have been presented. For the second case, new extended experimental and numerical investigations were carried out to understand the development of the constraint. In these new tests, the concrete strength, percentage of reinforcement, and slenderness were varied. The obtained results serve to assess how constrained forces can be considered in the design of concrete structures.

KEYWORDS
abaqus, constraint forces, design, experiments, FEA, imposed deformation, NL FEM, reinforced concrete, restraint forces

1 | INTRODUCTION
Constraint forces are actions that are used in structural analysis, but the realistic size of the constraint force can only be determined by deep knowledge of reinforced concrete. However, this is not easy because of the complex nonlinear material and stiffness behavior of reinforced concrete. For this reason, different information is often given for the design of standards. These can vary in the calculation method (stiffness in state 2, nonlinear calculations) and analysis method (proof of plastic rotation, etc.), as well as the load case or load case combination. Owing to the bearing displacement in statically indeterminate systems, constraint forces occur, which have the same or opposite action effects in the relevant cross sections.\(^a\) In the case of a continuous beam, a beneficial constraint occurs at the point of the settlement of the support, and a nonbeneficial constraint occurs at an adjacent support. For the case in which internal forces are caused by imposed deformation having an opposite action effect to the internal forces by loads, which is beneficial, extensive experimental and numerical investigations were carried out, and they are presented in this report.

2 | OVERVIEW OF PREVIOUSLY PERFORMED INVESTIGATIONS ON CONSTRAINT FORCES
Investigations of constraint forces by imposed deformations have been carried out for several

Discussion on this paper must be submitted within two months of the print publication. The discussion will then be published in print, along with the authors’ closure, if any, approximately nine months after the print publication.

This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2020 The Authors. Structural Concrete published by John Wiley & Sons Ltd on behalf of International Federation for Structural Concrete
decades—for example, by Kordina,2,3 Woidelko4 and Losberg.5

In the research work of Kordina, it was the goal to study the behavior of r/c-beams under the combined action of loads and restraint due to settlement. The restraint was enforced in a sudden way after loading one-span beams with one fixed end with the service load (Figure 1a). The reduction of restraint due to load alone (cracking and reduction stiffness) was studied.

Woidelko performed a series of experiments on reinforced concrete teebeams designed by the load-factor method. The test setup is shown in Figure 1b. The influence of additional constraint was investigated in one series.

It has been shown that, in a beam additionally loaded by constraint, the constraint forces decreased with increasing load level and the associated stiffness reduction. However, the influence of the constraint on the ultimate limit load could not be observed.

Studies on constraint forces by imposed deformations were carried out at the University of Innsbruck in 2016–2017.6,7 The experiments were static tests on a two-span beam with dimensions $l/w/h = 7.3/0.4/0.2$ m, at which the constraint was impressed by raising the middle support (Figure 1c). It was, thus, an internal force, which had the same action effect in the relevant cross section as the internal forces caused by the load. Therefore, it had an unfavorable effect. Three test series were carried out, whereby the percentage of reinforcement varied ($\rho_s = 0.46\%$, $\rho_s = 0.92\%$, and $\rho_s = 1.21\%$). The analysis of the test results showed that, in all test series, a quasiconstant reduction of the constraint moment took place right from the beginning of loading and had already been greatly reduced up to the yielding of the reinforcement. At the ultimate load, the constraint moment was no longer present. This result was surprising and contradicted the common model. Analysis of material, cross-sectional, and structural levels has also been conducted, but the behavior could not be assigned to any direct cause.

FIGURE 1 Test setup of previously performed investigations on constraint forces: (a) Kordina, (b) Woidelko, and (c) Berger
EXPERIMENTAL INVESTIGATIONS ON REINFORCED-CONCRETE BEAMS

Based on the results, further experiments were conducted. After obtaining the results of the nonbeneficial constraint, investigations were conducted on the beneficial constraint. In addition to the type of constraint, additional parameters were investigated.

Concrete strength: C20 and C50
Percentage of reinforcement $\rho_s = \frac{A_s}{w \times d}$: $\rho_s = 0.52\%$ and $\rho_s = 1.22\%$

Slenderness $\lambda = \frac{l}{d}$: $\lambda = 11.8$ and $\lambda = 18.4$

The experiments took place in the form of a static test, whereby a reference and a constraint test were carried out for each parameter configuration. In total, 16 tests were performed.

3.1 | Geometry

The test specimens were conventional reinforced-concrete beams. The static system and cross-sectional dimensions are shown in Figure 2. To investigate the effect of different degrees of slenderness, test specimens of different lengths were produced.

The slenderness amounted to the following.

- $\lambda = 18.4$ with a span of $2 \times 3.5$ m (specimen length $l = 7.3$ m)
- $\lambda = 11.8$ with a span of $2 \times 2.25$ m (specimen length $l = 5.0$ m)

3.2 | Material properties

3.2.1 | Concrete

The concrete strength was varied to analyze the influence. This corresponded approximately to the concrete

| TABLE 1 | Mechanical properties reinforcing steel |
|-----------------------------------------|----------------------------------------|
| $f_{yM}$ (MPa) | $\varepsilon_y$ (%) | $f_{uM}$ (MPa) | $\varepsilon_u$ (%) |
| Ø10 | 566 | 2.75 | 702 | 82.3 |
| Ø14 | 561 | 2.69 | 671 | 77.7 |

Longitudinal reinforcement 5Ø10 ($A_s = 3.93 \text{ cm}^2$) was used for the degree of reinforcement of $\rho_s = A_s/(b \times d) = 0.52\%$ and 6Ø14 ($A_s = 9.24 \text{ cm}^2$) for $\rho_s = 1.22\%$ with a concrete cover of 2.5 cm.
strength classes C20/25 and C50/60 according to EN 1992-1-1. The following mixtures were used:

**C20/25**
- Aggregate: 0/4 – 60%: 1271 kg; 4/8 – 10%: 213 kg; 8/16 – 30%: 638 kg.
- Cement: CEM II/A-M(S-L) 42.5 N: 220 kg.
- Effective water content: 165 kg (W/B = 0.75).
- Superplasticizer: 1.43-kg dynamIQ flow L-08.

The concrete compressive strength, determined on cylinders that were stored according to the environmental conditions of the beams, was $f_{cm,cyl;34d} = 26.42$ MPa ($SD = 0.51$ MPa), and the modulus of elasticity according to the standard was $E_{cm} = 28,600$ MPa ($SD = 400$ MPa).

**C50/60**
- Aggregate: 0/4 – 50%: 947 kg; 8/16 – 50%: 950 kg.
- Cement: CEM II/A-M(S-L) 42.5 N: 400 kg.
- Effective water content: 192 kg (W/B = 0.48).
- Superplasticizer: 2.40-kg dynamIQ flow L-08.

The concrete compressive strength, determined on cylinders that were stored according to the environmental conditions, was $f_{cm,cyl;34d} = 57.37$ MPa ($SD = 0.37$ MPa), and the modulus of elasticity according to the standard was $E_{cm} = 37,000$ MPa ($SD = 400$ MPa).

### 3.2.2 | Reinforcing steel

The reinforcement used was a conventional BSt 550B grade reinforcing steel. The mechanical properties are listed in Table 1.

### 3.3 | Experimental setup and measuring method

Static tests were carried out in a steel testing frame, where the load was applied at a displacement of 2 mm/min via a hydraulic jack, which was located in the middle of the test frame, as shown in Figure 3. The load was distributed over a traverse, whereby a five-point bending test was given for the two-span beam with the same lengths. To determine the support reactions, four load cells were
used. To measure the deformations and for the determination of the strain, potentiometric transducers were used. The strains were determined by measuring the displacements over a length of 300 mm. Because they were arranged on the top and bottom of the cross section, the curvature could be determined.

3.4 | Experimental procedure

Before the experiments started, the test specimens were preloaded until a maximum crack with a width of $w_{cr,max} = 0.3$ mm occurred. Preloading was performed to obtain the stiffness of the cracked cross section in order to have the same basis for comparing the reference test to the constraint test. Subsequently, reference tests and constraint tests were carried out for each type of specimen. For the constraint tests, the outer supports were lifted until the middle support reaction was only $\sim 10\%$ of the original state. Then, the specimens were loaded statically until no increase in load was possible. The load was applied without interruption, and the duration of the tests had a maximum of 30 min.

3.5 | Test results

3.5.1 | Bending moment development

The bending moment development of a test series was analyzed in more detail. Using the bearing reactions, measured via load cells, and the force of the hydraulic jack, it was possible to determine precisely the distribution of the section forces over the length of the test specimen. The constraint moment was determined from the difference between the constraint and reference tests. The internal moment development at the support for the configuration $C20/\rho_s = 0.52\%/\lambda = 18.4$ is shown in Figure 4. At the beginning of the experiment, there was a vertical offset between the reference and constraint tests. This represented the imposed constraint moment of $M_{Constraint} = 16$ kNm. As the load progressed, the internal forces of the two tests approached each other. The constraint moment, represented in the figure by the dashed line, was constantly reduced, and it was no longer present shortly before reaching the ultimate load. Owing to the quasilinear degradation, no direct effect of the yielding of the reinforcement or the associated plastic hinge could be detected.

The constraint moment development of all test series is shown in Figure 5a,b. In all test series, a clear trend can be seen of a continuous reduction of the constraint moment with higher loads. When the ultimate load was reached, the constraint was no longer present. The shorter specimens showed an increase in the constraint
moment shortly after the beginning of loading. This can be attributed to the development of complex stiffness. Owing to the imposed deformation, a positive constraint moment was impressed when a negative load moment acted. The constraint moment was so high that there was a total positive moment at the support point before loading started. For the stiffness, the cross section already cracked at the top because of self-weight and preloading also became cracked at the bottom because of the constraint load. This significantly reduced the cross-sectional stiffness. By applying the test load, a negative moment was generated, leading to the closing of the cracks at the bottom. As a result, an increase in stiffness occurred, whereby the increase in the constraint moment can be explained.

3.5.2 | Moment–curvature relationship

In addition, an analysis of the stiffness of the cross sections was performed by analyzing the moment–curvature relationship. The curvature can be expressed by the measured strains on the upper and lower sides of the cross section, and the moment can be expressed by the measured bearing reactions and force of the hydraulic jack. The moment–curvature relationship for the configuration C20/ρs = 0.52%/λ = 18 shows a quasi-bilinear course for the cross section, as shown in Figure 6. A quasiconstant stiffness existed in state 2 from the beginning and after reaching the yield strength of the reinforcement, and a distinctive yielding plateau was found. The curvature was dominated by the yielding plateau and represents the horizontal axis on a scale that is unfavorable to most of the stress on the cross section. In the yielding-driven relationship, there was no increase in stress, so it is of little significance in terms of stiffness under the load conditions. The moment–curvature relationship from the same test, but displaying only the first 10% of the curvature, shows that the curve does not have a constant gradient (see Figure 7).

3.5.3 | Stiffness development

The flexural stiffness EI of the cross section can be expressed by the relationship of Equation (1).

\[ EI = \frac{M}{\kappa} \tag{1} \]

with

\[ \kappa = \frac{e_o + e_u}{h} \tag{2} \]

For the configuration C20/ρs = 0.52%/λ = 18.4, the stiffness development for the reference and constraint tests is shown in Figure 8. In the reference test, for the

| TABLE 2 Plastic rotation: C20 |
|-----------------------------|
| **C20**                     |
|                            | Design according EC2 | Reference test | Constraint test |
|                            | θ_{pl,E} (mrad) | θ_{pl,d} * k_s (mrad) | θ_y (mrad) | θ_u (mrad) | θ_y (mrad) | θ_u (mrad) |
| ρ_s = 0.52%; λ = 18.4      | 11.7          | 14.3          | Yes        | 2.5        | 25.5        | 2.8        | 25.5 |
| ρ_s = 0.52%; λ = 11.8      | 11.7          | 12.2          | Yes        | 2.9        | 5.0         | 2.6        | 7.7  |
| ρ_s = 1.22%; λ = 18.4      | 8.4           | 12.1          | Yes        | 2.9        | 15.2        | 3.0        | 13.2 |
| ρ_s = 1.22%; λ = 11.8      | 8.4           | 10.3          | Yes        | 2.8        | 16.6        | 2.8        | 15.2 |

| TABLE 3 Plastic rotation: C50 |
|-----------------------------|
| **C50**                     |
|                            | Design according EC2 | Reference test | Constraint test |
|                            | θ_{pl,E} (mrad) | θ_{pl,d} * k_s (mrad) | θ_y (mrad) | θ_u (mrad) | θ_y (mrad) | θ_u (mrad) |
| ρ_s = 0.52%; λ = 18.4      | 15.6          | 12.7          | No         | 2.1        | 13.5        | 2.9        | 14.3 |
| ρ_s = 0.52%; λ = 11.8      | 15.6          | 10.8          | No         | 2.2        | 10.2        | 2.2        | 11.6 |
| ρ_s = 1.22%; λ = 18.4      | 12.0          | 15.4          | Yes        | 2.5        | 16.7        | 2.8        | 17.2 |
| ρ_s = 1.22%; λ = 11.8      | 12.0          | 13.0          | Yes        | 2.3        | 10.2        | 1.7        | 6.9  |
cross section of the middle support and field, there was no constant stiffness. The ratio of the stiffness $(EI)_{\text{Test}}/E_{\text{cmII}}$ was less than 1 from the beginning, because the test body was preloaded and cracked. In principle, no clear trend can be seen, but the stiffness decreases continuously and yielding of the reinforcement is not a characteristic point. The smallest value of the stiffness ratio was only 3% when reaching the ultimate load.

For the constraint test, a similar behavior was noticeable. However, the stiffness development of the cross section at the middle support shows an unusual course at first glance. At the middle support, after a strong decrease of the bending stiffness to $\sim$10% $(EI)_{\text{Test}}/E_{\text{cmII}}$ at 20% $F_{\text{max}}$, the stiffness increased again, and then it decreased. This behavior is caused by the sign conversion of the total bending moment, resulting in a positive constraint moment and negative load moment, as well as the associated opening and closing of the cracks. A similar behavior was evident in all other configurations.

### 3.5.4 Plastic rotation

The plastic rotation capacity can be used as a value for the reduction of constraint forces.\textsuperscript{11,12} This depends on the degree of reinforcement and the ductility of the reinforcement. At the structural level, slenderness also plays a role. The higher the slenderness, the higher the rotation capacity of the plastic hinge. Concrete properties are of little importance.

The simplified procedure for continuous beams is based on the rotation capacity of beam zones over a length of $\sim$1.2 times the depth of the section. It is assumed that these zones undergo plastic deformation (formation of yield hinges) under the relevant combination of actions. The verification of the plastic rotation in the ultimate limit state is considered to be fulfilled, if it is shown that, under the relevant combination of actions, the calculated rotation $\theta_{\text{pl,E}}$ is less than or equal to the allowable plastic rotation $\theta_{\text{pl,d}} * k_\lambda$. For the test specimens, the cross-sectional rotation was calculated using the measured cross-sectional curvatures.

Tables 2 and 3 show that, for a design according to Eurocode 2\textsuperscript{8,9} for all test configurations except the C50/$\rho_s = 0.52%/\lambda = 18.4$ and C50/$\rho_s = 0.52%/\lambda = 11.8$, the proof can theoretically be fulfilled. The plastic rotations of the tests show that the reinforcement yielded in all test series. Yielding occurred at cross-sectional rotations between 1.7 and 3.0 mrad. The maximum cross-sectional rotations were between 5.0 and 25.5 mrad. A good correlation was found between EN1992-1-1\textsuperscript{8} and the test results.

### TABLE 4 Simulation parameters

| Parameter               | Concrete beam       | Reinforcement         | Mesh size |
|-------------------------|---------------------|-----------------------|-----------|
|                         | 2D shell (CPS4)     | Truss element (T2D2)  | 20 mm     |
| Concrete                |                      |                       |           |
| Mass                    | 0.0025 g/mm$^3$     |                       |           |
| $\nu$                   | 0.2                 |                       |           |
| Dilation angle          | 30.0°               |                       |           |
| Eccentricity            | 0.1                 |                       |           |
| $f_{50} = f_{c0}$       | 1.16                |                       |           |
| Viscosity               | 0.00                |                       |           |
| Reinforcing steel       |                      |                       |           |
| Mass                    | 0.00785 g/mm$^3$    |                       |           |
| E-Modul                 | 200,000 MPa         |                       |           |
| $\nu$                   | 0.3                 |                       |           |
| Yield strength          | 570 MPa             |                       |           |
| Tensile strength        | 700 MPa             |                       |           |
| $\varepsilon_u$         | 80%                 |                       |           |

### FIGURE 9

Geometry of the model and discrete representation of the reinforcement: (a) $\lambda = 18.4$ and (b) $\lambda = 11.8$
4 | NUMERICAL INVESTIGATIONS OF CONSTRAINT

Nonlinear finite-element method (NLFEM) calculations using the software Abaqus\textsuperscript{13} were performed for a more accurate understanding of constraint forces in reinforced concrete. The analysis of the experiments described above was carried out, and the results were compared.

4.1 | Material model

For the numerical calculations, the software Abaqus/CAE 2019 was used, in which a nonlinear material model for concrete, concrete damaged plasticity,\textsuperscript{14} was implemented. It contains a combination of plasticity and damage theory. Such models are distinguished by the fact that cracks are not discretely considered; rather, they are smeared over the volume of the respective integration points.\textsuperscript{15,16} Although this illustrates the effects of the material stiffness, the deformation state in the cracked area can deviate from reality. Furthermore, the stress–strain relationship is adjusted by means of the specific fracture energy. This is necessary because, otherwise, the crack elongation would depend on the size of the element and would, therefore, no longer represent an objective size. However, this is only a rough regularization; thus, the result is only an approximation. The calculation parameters used are listed in Table 4.

4.2 | Geometric model

The analysis was based on a two-dimensional model, as shown in Figure 9. The concrete body was modeled with shell elements—longitudinal and transverse reinforcement with truss elements. The element size was 20 mm.
for each of the CPS4 and T2D2 elements. Load-distributing steel plates with elastic material behavior were modeled at the load introduction point and at the supports. Owing to the symmetry of the experimental setup, only half of the experimental body was modeled. The bond between the reinforcing steel and concrete body was rigid.

4.3 | Load application

The load was applied in several load steps (LS). The individual load steps were applied analogously to the experiment, and they are listed in Table 5. The load was applied with the displacement controlled for preloading and ultimate load. For the imposed deformation, the same values of the tests were taken.

4.4 | Analysis results

The experimental results are presented and analyzed below. Some are only analyzed for individual series if they are of minor importance for the interpretation of the constraint and are representative of other test series. The evaluation of all results can be found in the thesis of Pfeiffer.17

4.4.1 | Load–displacement relationship

Figure 10 shows the load–deflection relationship for the configuration C20/ρs = 0.52%/λ = 18.4, where the numerically determined deformations are compared with the test results for the reference and constraint tests. In the low load range, the deformation in the finite-element method (FEM) calculation was smaller than in the experiments. This can be attributed to an overestimated stiffness in the FEM calculations. In the middle load range, the load–deflection curves of the FEM calculation and the experiments are identical. When the maximum load is reached, a yield plateau is established. The load–deformation curves of the tests are identical in the beginning and end areas; in the middle load range, there is a deviation. The constraint force does not lead to a significant deviation between the reference and constraint tests. The maximum values for deformation and load are in good agreement between the test and FEM. Similar results were obtained in the other series of experiments.18

4.4.2 | Constraint moment development

Figure 11 shows the constraint moment development resulting from the numerical calculations. In principle, the same trend can be seen for all configurations. From the beginning of the load application, a reduction of the constraint moment up to a load level of ~50% Fmax takes place. Subsequently, the constraint moment remains approximately constant up to 80–90% Fmax. Then, in 50% of the experimental configurations, the constraint moment was completely reduced. For the remaining 50%, a residual amount stayed. The resulting horizontal plateau accurately reflects the analytically determined stiffness of the fully cracked cross section in state 2, as shown in Table 6. A comparison with the experimental results reveals that the degradation of the constraint moment is well reflected, but the development has a different shape (see Figure 5). In the tests, the constraint moment was constantly reduced until the maximum load was reached, and the constraint moment was no longer present at the ultimate load in all test series.
4.4.3 | Stiffness

To interpret the stiffness, which is largely responsible for the constraint moment, the determination of the bending stiffness $E_i$ is based on the moment–curvature relationship according to Equation (1), which is analogous to the experiments. As an example, the result for the configuration $C20/\rho_s = 0.52\% / \lambda = 18.4$ is shown in Figure 12. The stiffness is not constant. In the reference test, the stiffness drops after the start of loading and, at $0.5 F_{\text{max}}$, a significantly lower level of drop is noticeable.

In the constraint test, a low bending stiffness caused by the crack formation from the imposed deformation already exists from the beginning of the load application. It is then reduced to 0 at a load level of $0.25 F_{\text{max}}$, this can be attributed to the sign change of the bending moment, positive moment by bearing displacement, and negative moment by load stress. Then, there is an increase, which can be explained by the closing of the cracks according to the stress. When reaching a load level of $0.55 F_{\text{max}}$, the stiffness decreases again. The comparison with the experimental results shows that, in principle, there is a good analogy in the form of development, but the values are very different. Reasons can be seen in the nonlinear material and damage model of the finite-element calculation.

4.4.4 | Crack pattern

Figures 13 and 14 show the plastic strain, which represents the cracked areas in the test body. Under a dead load, the tensile strength of the concrete is not exceeded; therefore, there are no cracks. The preloading causes
cracks in the area of the middle support and the load introduction point. These are still present after load removal. Owing to the bearing displacement, a positive moment is impressed, which leads to cracks on the bottom of the test body in the area of the middle support. When the ultimate load is reached, the completed crack pattern is given. The plastic strain of the FE analyses matches very well with the crack patterns from the experiments.

**FIGURE 14** Crack pattern—constraint test: (a) LS dead weight, (b) LS preloading, (c) LS unloading, (d) LS bearing displacement, and (e) LS ultimate load

### 5 | CONSTRAINT MOMENT CALCULATION METHOD

The calculation method plays an important role in analyzing the constraint moment. To illustrate this, results obtained with different calculation methods (analytical, NLFEM, and experiment) and stiffness assumptions (states 1 and 2) at the beginning test are compared in Figure 15. At the beginning of testing, the constraining
moment in the NLFEM calculations was significantly higher than during the tests and was close to the result of an analytical calculation with the stiffness of the uncracked cross section. However, at the end of the experiment, the results were very close to each other, because the constraint moment was reduced to zero.

6 | SUMMARY

To evaluate the development of a constraint moment resulting from imposed deformation, experimental and numerical investigations were conducted. The imposed deformation took place in the form of a bearing displacement, which resulted in an internal force having an action effect opposite to the action effect caused by loads in the relevant cross sections. Thus, it was beneficial. The experiments were carried out on double-span beams with differences in concrete strength, percentage of reinforcement, and slenderness. The experiments showed that, in all experimental configurations from the beginning of the load application, a reduction of the constraint moment occurred. In some series, there was an increase in the constraint moment in certain areas, which was caused by a short-term increase in stiffness because of a sign change of the moment through which cracks open and close. When the maximum load was reached, the constraint moment was no longer present in all cases (see Table 7). Yielding of the reinforcement did not represent a characteristic point in the constraint moment development. The different concrete strengths C20 and C50, the different percentages of reinforcement $\rho_s = 0.52\%$ and $\rho_s = 1.22\%$, and the different slenderness values $\lambda = 18.4$ and $\lambda = 11.8$ had no significant influence on the development of the constraining moment. The nonlinear numerical calculations with Abaqus showed that, at the beginning, the quantity of the constraint moment was higher than in the tests. From the beginning of the analysis, a reduction in the constraint moment took place, but with a different development compared with the experiment. At the end of the experiment, the constraint moment was no longer present for 50% of the calculations, and, for the remaining 50%, the highest remaining constraint moment was 25% of the constraint moment at the beginning of the calculation (see Table 7).

ACKNOWLEDGMENT

The authors would like to acknowledge the financial support of the Austrian infrastructure operator ASFINAG Bau Management GmbH and the Austrian Federal Ministry for Transport, Innovation, and Technology (bmvit).

ORCID

Johannes Berger © https://orcid.org/0000-0003-3939-2796
REFERENCES
1. Grasser E, Thielen G. Hilfsmittel zur Berechnung der Schnittgrößen und Formveränderungen von Stahlbetontragwerken nach DIN 1045. Ausgabe Juli: Deutscher Ausschuss für Stahlbeton, 1988.
2. Kordina K. Zur Frage der näherungsweisen Ermittlung von Zwangsschnittgrößen. L’influence du fluage et du retrait, l’effet des changements de temperature sur les constructions en beton: symposium Madrid 1970; rapport final. Zürich: Internationale Vereinigung für Brückenbau und Hochbau, 1970; p. S441–S450.
3. Kordina K, Rostásy F, Svensvik B. Tragfähigkeit und Verformung von Stahlbetonbalken unter Biegung und gleichzeitigem Zwang infolge Auflagerverschiebung. Heft 336. Berlin, Germany: Deutscher Ausschuss für Stahlbeton; 1982.
4. Woidelko EO, Schaefer K, Schlaich J. Load bearing capacity of tested reinforced concrete slab girders. Beton- und Stahlbetonbau. 1986;81:244–248.
5. Losberg A. Design methods for structurally reinforced concrete pavements. Göteborg, Sweden: Chalmers University Books, 1961.
6. Berger J, Feix J. Constraint forces from imposed deformations. Struct. Concr. 2019;20:650–659. https://doi.org/10.1002/suco.201800123.
7. Konzilia J. Untersuchung der Auswirkung von Zwangsschnittgrößen infolge Auflagerverschiebung. [Master Thesis]. University of Innsbruck, Arbeitsbereich für Massivbau und Brückenbau, 2018.
8. ÖNORM EN 1992–1–1, Eurocode 2: Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken – Teil 1–1: Allgemeine Bemessungsregeln und Regeln für den Hochbau., Österreichisches Normungsinstitut, Wien, Ausgabe 2015.
9. ÖNORM B 1992–1–1, Eurocode 2: Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken – Teil 1–1: Allgemeine Bemessungsregeln und Regeln für den Hochbau - Nationale Festlegungen zu ÖNORM EN 1992–2, nationale Erläuterungen und nationale Ergänzungen. Österreichisches Normungsinstitut, Wien, Ausgabe 2015.
10. ONR 23303. Prüfverfahren Beton (PVB)— Nationale Anwendungen der Prüfnormen für Beton und seiner Ausgangsstoffe. Wien: Österreichisches Normungsinstitut; 2010.
11. Xu L, Pan J, Lu C, Yin W. Development mechanism of plastic hinge in reinforced engineered cementious composite beams under monotonic loading. Struct. Concr. 2019;20:252–266.
12. Zhao XM, Wu YF, Leung AYT. Analyses of plastic hinge regions in reinforced concrete beams under monotonic loading. Eng. Struct. 2012;34:466–482.
13. Corp S. Abaqus CAE - analysis User’s guide. Providence, RI, USA: Dassault Systèmes Simulia Corp, 2019.
14. Lee J, Fenves GL. Plastic-damage model for cyclic loading of concrete structures. J. Eng. Mech. 1998;124(8):892–900.
15. Hofstetter G, Mang HA. Computational mechanics of reinforced concrete structures. Braunschweig: Vieweg, 1995.
16. Zienkiewicz OC, Taylor RL. The finite element method. Volume 1: the basis. 5th ed. Butterworth-Heinemann: Oxford, 2000.
17. Pfeiffer M. Numerische Untersuchungen zu Zwangsschnittgrößen aus Auflagerverschiebung. Master Thesis: University of Innsbruck, Arbeitsbereich für Massivbau und Brückenbau, 2020.
18. Wurzer D. Parameterstudie zur Untersuchungen der Auswirkung von Zwangsschnittgrößen infolge Auflagerverschiebung. [Master Thesis], University of Innsbruck, Arbeitsbereich für Massivbau und Brückenbau, 2020.

AUTHOR BIOGRAPHIES

Johannes Berger, University of Innsbruck, Unit of Concrete Structures and Bridge Design, Technikerstraße 13, 6020 Innsbruck, Austria.

Max Pfeiffer, University of Innsbruck, Unit of Concrete Structures and Bridge Design, Technikerstraße 13, 6020 Innsbruck, Austria.

Jürgen Feix, University of Innsbruck, Unit of Concrete Structures and Bridge Design, Technikerstraße 13, 6020 Innsbruck, Austria.

How to cite this article: Berger J, Pfeiffer M, Feix J. Extended experimental and numerical investigations on constraint forces from imposed deformations. Structural Concrete. 2020;21:1662–1674. https://doi.org/10.1002/suco.201900450