Gravitinos from Gravitational Collapse

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Abstract

We reanalyse the limits on the gravitino mass $m_{3/2}$ in superlight gravitino scenarios derived from arguments on energy-loss during gravitational collapse. We conclude that the mass range $10^{-6}eV \leq m_{3/2} \leq 2.3 \times 10^{-5}eV$ is excluded by SN1987A data. In terms of the scale of supersymmetry breaking $\Lambda$, the range $70GeV \leq \Lambda \leq 300GeV$ is not allowed.
In a wide class of supergravity models with SUSY breaking \( \Lambda \) in the TeV range, the gravitino can be very light:

\[
m_{3/2} = 2.5 \times 10^{-4} eV (\Lambda/1 TeV)^2.
\] (1)

Indeed, models where gauge interactions mediate the breakdown of supersymmetry \(^3\), models where an anomalous U(1) gauge symmetry induces SUSY breaking \(^2\), and no-scale models are all examples of models where a superlight gravitino can be accommodated \(^3\). In all of them, the gravitino is the LSP and, furthermore, its couplings to matter and radiation are inversely proportional to its mass. Therefore, one may expect interesting phenomenology \(^4\).

Bounds on the gravitino mass, or equivalently on the scale \( \Lambda \) have been given in the context of those models by various authors and have been extracted from different physical systems. In fact, the limits come from as distinct areas as the anomalous magnetic moment of the muon \(^5\), primordial nucleosynthesis \(^6\) or stellar energy drain arguments \(^7\). In recent papers \(^8\), it has been noted that the amplitudes for gravitino processes that were used in deriving some of the constraints had an incorrect energy behaviour. In particular, the supernova (SN) bounds deduced in ref.\(^9\) using the effective couplings given explicitly by Gherghetta \(^10\) are invalid as pointed out by Luty and Ponton \(^11\). These authors, however, when reexamining the limits coming from the SN1987A explosion, use an incorrect abundance of positrons in the core, do not discuss gravitino bremsstrahlung, and misidentify the main source of opacity in the stellar core. The purpose of the present paper is thus to redo the analysis that renders the bounds on \( m_{3/2} \) (or \( \Lambda \)) following from SN collapse. Since SN considerations gave the best limits on \( \Lambda \) up to now \(^11\), this is not an empty exercise.

The relevant piece in the effective lagrangean is the derivative coupling of the goldstino \( \chi \) to photons:

\[
\delta L_{\text{eff}} = \left( e/2 \right) \left( M/\Lambda^2 \right)^2 \partial^\mu \chi \sigma^\nu \bar{\chi} F_{\mu\nu} + \text{h.c.}
\] (2)

with \( F_{\mu\nu} \), the electromagnetic field-strength and \( M \) is a mass that depends on the supersymmetry breaking model. In gauge-mediated models, \( M \sim m_L/4\pi \), where \( m_L \) is the left-handed slepton mass. Given that gravitino pairs are mainly produced via one-photon interactions, the sources of gravitino luminosity in stars are, in principle, gravitino bremsstrahlung in neutron-proton scattering, pair production in electron-positron annihilation and plasmon decay into gravitinos. The energy-loss rate (per unit volume) via \( pn \rightarrow pn\bar{G}\bar{G} \) is,

\[
Q = \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \prod_{i=1}^4 \frac{d^3p_i}{(2\pi)^3} f_1 f_2 (1-f_3)(1-f_4) (2\pi)^4 \delta^4(P_f-P_i) \sum_{\text{spins}} |M_{fi}|^2 (k_1^0+k_2^0)
\] (3)
where \((p^0, \vec{p})_i\) are the 4-momenta of the initial and final state nucleons, \((k^0, \vec{k})_{1,2}\) are the 4-momenta of the gravitinos and \(f_{1,2}\) are the Fermi-Dirac distribution functions for the initial proton and neutron and \((1-f_{3,4})\) are the final Pauli blocking factors for the final proton and neutron. The squared matrix element can be factorised as follows,

\[
\sum_{\text{spins}} |M_{fi}|^2 = (2\pi)^2 \alpha^2 \frac{N_{\mu\nu} G_{3/2}^{\mu\nu}}{\Lambda^2}
\]

where \(N_{\mu\nu}\) is the nuclear (OPE) tensor and \(G_{3/2}^{\mu\nu}\) is the gravitino tensor in the matrix element squared. The factor \(N_{\mu\nu}\) is common to any bremsstrahlung process involving nucleons. It appears, e.g., in neutrino bremsstrahlung calculations and in axion bremsstrahlung calculations, and is given explicitly in ref \[12\]. On the other hand, \(G_{3/2}^{\mu\nu}\) is a tensor specific to gravitino bremsstrahlung. It reads,

\[
G_{3/2}^{\mu\nu} = k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - k_1.k_2 g^{\mu\nu}
\]

The integration of \(N_{\mu\nu}\) over the phase-space of the nucleons can be performed explicitly and the details can be found again in Raffelt’s book \[12\]. When we contract the result with the gravitino tensor \(G_{3/2}^{\mu\nu}\) and perform the integrals over gravitino momenta to complete the energy depletion rate, we are led to the following emissivity:

\[
Q_{\text{ND brems}} = 8192/385 \pi^{3/2} \alpha^2 \alpha^2 (M/\Lambda^2)^4 Y_e n_B T^{11/2} / m_p^{5/2}
\]

for non-degenerate and non-relativistic nucleons (\(\alpha\) is the pionic fine-structure constant, \(n_B\) is the number density of baryons, and \(Y_e\) is the mass fraction of protons). However, nucleons are moderately degenerate in the SN core. The emissivity in the (extreme) degenerate case is calculated to be,

\[
Q_{\text{D brems}} = 164\pi^3 / 4725 \alpha^2 (M/\Lambda^2)^4 T^4 e^{-\mu/T} \mu^5 b(\mu/T) / 15 \pi^3
\]

with \(p_F\), the Fermi momentum of the nucleons. Numerically, for the actual conditions of the star, both emissivities differ by less than an order of magnitude (about a factor of three). Since the actual emissivity interpolates between these two values, we shall adopt the smallest of the two (i.e. \(Q_{\text{ND brems}}\)) to make our (conservative) estimates. We turn next to the annihilation process.

The energy loss for the process \(e^+(p_1) + e^-(p_2) \rightarrow \tilde{G}(k_1) + \tilde{G}(k_2)\) can be calculated along similar lines as above. The spin averaged matrix element squared is in this case,

\[
\sum_{\text{spins}} |M_{fi}|^2 = (2\pi)^2 \alpha^2 (M/\Lambda^2)^4 E_{\mu\nu}(p_1, p_2) G_{3/2}^{\mu\nu}(k_1, k_2)
\]

where \(E_{\mu\nu}(p_1, p_2)\) equals formally the tensor \(G_{3/2}^{\mu\nu}\) in eq.(5) with \(k_1, k_2\) replaced by \(p_1, p_2\). The luminosity then is found to be,

\[
Q_{\text{ann}} = 8 \alpha^2 (M/\Lambda^2)^4 T^4 e^{-\mu/T} \mu^5 b(\mu/T) / 15 \pi^3
\]
with \( b(y) \equiv (5/6)e^{y}y^{-5}(F_{5}^{+}F_{4}^{-} + F_{4}^{+}F_{5}^{-}) \) where \( F_{m}^{\pm}(y) = \int_{0}^{\infty} dx x^{m-1}/(1 + e^{x\pm y}) \) (\( \mu \) is the chemical potential of the electrons). The function \( b(y) \to 1 \) in the degenerate limit. Finally, our estimate of the plasmon decay luminosity is,

\[
Q_{P} = 16\zeta(3)\alpha^{4}T^{3}\mu^{6}(M/\Lambda^{2})^{4}/81\pi^{5}
\]

(where only transverse plasmons have been taken into account).

Taken at face value, the bremsstrahlung rate is the largest of the three. However, \( Q_{brems} \) is overestimated since we did not consider multiple scattering effects which are present in a dense medium\(^{[12]}\). Indeed, as for the axion case\(^{[12]}\), the gravitino bremsstrahlung rate probably saturates around 20% nuclear density and this should be taken into account when evaluating eq.(6). If we use now the values \( T = 50MeV, \mu = 300MeV, \) and \( Y_{e} = 0.3, \) eqs. (6) (with \( n_{B} \sim 0.2n_{nucl} \)), (9) and (10) give

\[
Q_{ann} : Q_{brems} : Q_{P} \approx 1.2 \times 10^{3} : 3 \times 10^{2} : 1
\]

Therefore, a limit on \( \Lambda \) will follow from the requirement that \( L_{3/2} \approx VQ_{ann} \) (\( V \) is the volume of the stellar core) should not exceed \( 10^{52}ergs/s \). This constraint on the gravitino luminosity \( L_{3/2} \) implies, in turn,

\[
\Lambda \geq 300GeV(M/43GeV)^{1/2}(T/50MeV)^{11/16}(R_{c}/10Km)^{3/8}
\]

or, using eq.(1),

\[
m_{3/2} \geq 2.3 \times 10^{-5}eV.
\]

Of course, the previous calculation makes sense only if gravitinos, once produced, stream freely out of the star without rescattering. That they actually do so, for \( \Lambda \geq 300GeV \), can be easily checked by considering their mean-free-path in the core. The main source of opacity for gravitinos is the elastic scattering off the Coulomb field of the protons:

\[
\lambda = 1/\sigma n = (4/\pi\alpha^{2})Y_{e}^{-1}p^{-1}m_{p}^{-1}(\Lambda^{2}/M)^{4}
\]

The thermally averaged cross-section for elastic gravitino scattering on electrons is roughly a factor \( T_{\mu}/m_{p}^{2} \) smaller than that on protons and thus it does not contribute appreciably to the opacity. Putting numbers in eq.(14) we find:

\[
\lambda \simeq 1.4 \times 10^{7}cm(43GeV/M)^{4}(\Lambda/300GeV)^{8}
\]

On the other hand, the calculation of \( Q \) breaks down for \( \lambda \leq 10Km \), i.e. for \( \Lambda \leq 220GeV \), when gravitinos are trapped in the SN core. In this case, gravitinos diffuse out of the dense
stellar interior and are thermally radiated from a gravitino-sphere $R_{3/2}$. Because in this instance the luminosity is proportional to $T^4$, only for a sufficiently large $R_{3/2}$ (where the temperature is correspondingly lower), the emitted power will fall again below the nominal $10^{52}$ erg/s. Consequently, gravitino emission will be energetically possible, if $\Lambda$ is small enough. The gravitino-sphere radius can be computed from the requirement that the optical depth

$$\tau = \int_R^\infty \frac{dr}{\lambda(r)}$$

be equal to $2/3$ at $R = R_{3/2}$. Here, $\lambda(r)$ is given in eq.(14) with the density profile ansatz:

$$\rho(r) = \rho_c (R_c/r)^m$$

with $\rho_c = 8 \times 10^{14} g/cm^3$, $R_c = 10Km$ and $m = 5 - 7$ and which satisfactorily parameterises the basic properties of SN1987A [13]. An explicit calculation renders:

$$R_{3/2} = R_c [(8Y_e/3\pi\alpha^2) (\Lambda^2/M)^4 (m - 1)/(\rho_c R_c m_p)]^{1/m}$$

Stefan-Boltzmann’s law implies for the ratio of gravitino to neutrino luminosities,

$$L_{3/2}/L_\nu = (R_{3/2}/R_\nu)^2 [T(R_{3/2})/T(R_\nu)]^4$$

where $R_\nu$ is the radius of the neutrinosphere. To proceed further we use the temperature profile:

$$T = T_c (R_c/r)^{m/3}$$

which is a consequence of eq.(17) and the assumption of local thermal equilibrium. Now, taking $m = 7$ [14], we obtain

$$L_{3/2}/L_\nu = (R_\nu/R_c)^{22/3} [(16Y_e/\pi\alpha^2) (\Lambda^2/M)^4/\rho_c R_c m_p]^{11/9}$$

By demanding that $L_{3/2} \leq 0.1L_\nu$ and using $R_\nu \simeq 30Km$, we get

$$\Lambda \leq 70 GeV.$$ \hfill (22)

This in turn implies $m_{3/2} \leq 10^{-6} eV$. Since, on the other hand, the anomalous magnetic moment of the muon already requires $m_{3/2}$ to be larger than $\sim 10^{-6} eV$ [3, 13], we are forced to conclude that

$$\Lambda \geq 300 GeV$$ \hfill (23)

or, equivalently,

$$m_{3/2} \geq 2.3 \times 10^{-5} eV.$$ \hfill (24)
In conclusion, we have carefully rederived the bounds on the superlight gravitino mass (i.e. the SUSY scale $\Lambda$) that follow from SN physics. These limits are completely general in the sense that they do not rely on other particles in a given particular model being light. Should other particles such as the scalar partners of the goldstino also be light, then the resulting bounds are necessarily tighter. In such clearly less general frame, constraints have also been derived in the literature [16] that are not subject to the criticisms mentioned in the beginning of this paper. They are much stronger then the ones given here and typically give $\Lambda \geq 300 TeV$ (or $m_{3/2} \geq 50 eV$ ) from stellar (e.g. the Sun) evolution arguments, provided $m_{3/2} \leq 1 KeV$ (e.g. $T_\odot$).

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