Practical Saccade Prediction for Head-Mounted Displays: Towards a Comprehensive Model

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Eye-tracking technology has started to become an integral component of new display devices such as virtual and augmented reality headsets. Applications of gaze information range from new interaction techniques that exploit eye patterns to gaze-contingent digital content creation. However, system latency is still a significant issue in many of these applications because it breaks the synchronization between the current and measured gaze positions. Consequently, it may lead to unwanted visual artifacts and degradation of the user experience. In this work, we focus on foveated rendering applications where the quality of an image is reduced towards the periphery for computational savings. In foveated rendering, the presence of system latency leads to delayed updates to the rendered frame, making the quality degradation visible to the user. To address this issue and to combat system latency, recent work proposes using saccade landing position prediction to extrapolate gaze information from delayed eye tracking samples. Although the benefits of such a strategy have already been demonstrated, the solutions range from simple and efficient ones, which make several assumptions about the saccadic eye movements, to more complex and costly ones, which use machine learning techniques. However, it is unclear to what extent the prediction can benefit from accounting for additional factors and how more complex predictions can be performed efficiently to respect the latency requirements. This paper presents a series of experiments investigating the importance of different factors for saccades prediction in common virtual and augmented reality applications. In particular, we investigate the effects of saccade orientation in 3D space and smooth pursuit eye-motion (SPEM) and how their influence compares to the variability across users. We also present a simple, yet efficient post-hoc correction method that adapts existing saccade prediction methods to handle these factors without performing extensive data collection. Furthermore, our investigation and the correction technique may also help future developments of machine-learning-based techniques by limiting the required amount of training data.

CCS Concepts: • Computing methodologies → Perception; Virtual reality; Mixed/augmented reality;

Additional Key Words and Phrases: Eye tracking, saccade prediction, gaze contingency, smooth pursuit eye-motion

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1 INTRODUCTION

Novel head-mounted devices offer new and exciting ways of interacting with physical and virtual content. At the same time, these devices pose new challenges regarding human-computer interaction and content creation methods. For example, the use of standard interaction tools such as a computer mouse or a keyboard becomes less practical as displays obstruct the entire field of view of the observer. Wide-field-of-view capabilities also significantly increase quality demands. It is becoming common for new head-mounted displays to require 8K+ rendering resolution for each eye at 90+ Hz framerate.\footnote{https://varjo.com/products/aero/}

The key enablers of novel interaction and efficient rendering techniques are eye trackers that are being incorporated into the recent designs of head-mounted displays [Hua et al. 2006]. The precise information on the location of the gaze not only enables new interaction techniques [Majaranta and Bulling 2014], but also opens many opportunities for gaze-contingent display techniques that optimize image generation to provide higher quality at a reduced computational cost. A particular example of such a technique is foveated rendering [Guenter et al. 2012; Meng et al. 2018; Patney et al. 2016; Swafford et al. 2016; Tursun et al. 2019], which reduces the use of computational resources by degrading the image quality for peripheral vision, where the human visual system is less sensitive to image distortions.

Gaze-contingent techniques highly rely on accurate and instantly updated gaze-location prediction. An inaccurate prediction may lead to unintended content presented to users or suboptimal visual quality, and as a consequence, a degraded user experience. There are several factors that influence the accuracy of eye-tracking information. In this work, we focus on latency, which is a result of both hardware and software limitations in a gaze-contingent display [Stein et al. 2021]. In the context of foveated rendering, whose goal is to produce high-quality content only for the fovea, the system latency leads to delayed updates of the high-quality image region. This problem is critical for a short period following fast eye movements called saccades. Shortly after the saccade ends, the available gaze prediction is delayed due to the system’s latency, and the fovea is exposed to low-quality content. Once the prediction quality stabilizes, the foveated rendering updates the image quality in the fovea region. Both the lower quality content after the saccade and the late quality change can be often observed by a viewer leading to characteristic popping artifacts. Recently, Arabadzhiyska et al. [2017] proposed a technique to limit this undesired effect. To perform the quality update of foveated rendering ahead of time, they leverage the saccadic suppression effect, which is the reduced sensitivity of the human visual system during the saccade. To this end, they developed a prediction method that is based on a few initial eye-tracker samples to predict the saccade landing position. With the help of such a technique, when the saccade ends, the high-quality foveal rendering is positioned correctly and no popping artifacts are observed.

The success of such a technique depends mainly on the accuracy and efficiency of the prediction of saccade landing position. The method by Arabadzhyska et al. [2017] is computationally efficient, but it relies on the assumption that the saccade displacement profile depends solely on the saccade’s length. Other techniques, such as the machine-learning-based approaches of Morales et al. [2018, 2021], define saccades in 2D screen space, but do not account for the fact that saccades are often combined with vergence eye movements. Additionally, the network inference required for the prediction is too expensive to use in real-time foveated rendering applications.

This work goes beyond existing models for predicting the saccade landing position by investigating additional factors that affect these eye movements and their prediction. More specifically, we focus on dynamic scenarios in VR and AR devices where saccades are combined with vergence movements and smooth pursuit eye motion (SPEM). We design and conduct user experiments that measure saccade profiles in such scenarios. Several previous works, for example, Collewijn et al. [1988a, b], have already carried out similar experiments using accurate eye trackers, such as these using the scleral search coil technique. These studies have demonstrated the impact of additional factors on saccade profiles. Compared to them, we do not provide new insights into the physiological characteristics of saccades. Instead, we analyze these factors in the context of practical applications of saccade
prediction techniques in VR and AR scenarios. For this reason, we also refrain from using coil-based eye trackers and opt for optical solutions, which, despite their lower accuracy, are the most suitable solution. To our knowledge, this work is the first to investigate the impact of factors such as vergence or SPEM on saccade prediction in current VR and AR devices. Additionally, we propose a method for correcting datasets used for training prediction models to account for these factors and demonstrate the possibility of directly correcting the existing model proposed recently by Arabadzh iyska et al. [2017]. To summarize, the main contributions of this paper are the evaluation of the influence of saccade orientation in 3D space and SPEM on saccade prediction techniques and a simple but efficient method for adapting the existing prediction technique [Arabadzh iyska et al. 2017] to handle these factors and for customizing the prediction. We believe that our investigation and technique will also help future developments of machine-learning-based techniques by limiting the required amount of training data.

2 BACKGROUND AND RELATED WORK

2.1 Saccade Characteristics

Saccades are simultaneous movements of both eyes to shift the gaze direction towards the visual stimulus that is away from the point of fixation [Leigh and Zee 2015; Schor 2011]. They are characterized by rapid acceleration until the maximum velocity is reached, and then a deceleration to a complete stop, typically followed by small corrective eye movements around the target [Westheimer 1954].

Pre-programmed behavior. Although it is possible to observe saccades up to 100° amplitude, humans perform small saccades more frequently than large ones under natural viewing conditions [Bahill et al. 1975a]. Consequently, most saccades last a short period of time (<70ms), which is approximately equal to the time it takes for visual information to reach the ocular motor mechanisms of the brain [Leigh and Zee 2015]. Therefore, saccades exhibit preprogrammed behavior and visual stimuli have a negligible effect on a saccade when presented in the last 80–100ms preceding the onset of the saccade or during a saccade [Becker and Jürgens 1979; Young and Stark 1963].

Factors that affect the velocity. The velocity of a saccade is affected by multiple factors, such as the position of the source and the target in the visual field, as well as the orientation of its trajectory (e.g., nasal vs. temporal) and most notably by the distance between the source and target (i.e., the amplitude of the saccade) [Boghen et al. 1974]. Initial attempts to study saccades revealed a relationship of the saccade amplitude to its duration and the peak velocity (a.k.a., the main sequence). It is commonly observed that the duration of the saccades shows a nonlinear increase up to approximately 5°, where it starts to increase linearly with the saccadic amplitude [Bahill et al. 1975b; Carpenter 1988]. Similarly, the peak velocity increases linearly with saccadic amplitude up to 15°–20°, where it reaches a saturation limit at approximately 600°/s–800°/s [Bahill et al. 1975b]. The initial position of the eye and the orientation of the trajectory also affect their velocity. Saccades that start in the periphery of the orbit and are directed towards the primary orbital position (centripetal) are on average faster than saccades performed in the opposite direction (centrifugal) [Pelisson and Prablanc 1988]. Similarly, saccades performed in the horizontal direction reach higher peak velocities than those performed in the vertical direction; however, the difference becomes less significant for older adults [Irving and Lillakas 2019]. Some studies show that the viewed content has an effect on velocity profiles such that saccades may deviate from a velocity profile that can otherwise be modeled with a compressed exponential model [Costela and Woods 2019; Han et al. 2013].

Interactions with vergence. Vergence is the slow (~0.5–1s) movement of the eyes in opposite directions when the change in binocular fixation involves a change in depth. If the change in the visual direction also accompanies a change in the depth, saccade and vergence take place simultaneously. In case of such combined saccade and vergence movements, they interact with each other [Ono et al. 1978]. Although saccade takes significantly shorter time (~50ms) than vergence, a large portion (40%–100%) of vergence takes place during saccade when combined [Enright 1984, 1986]. This shows an effective “mediation” of vergence by saccades. A closer inspection of peak
velocities reveals that vergence speeds up while saccade slows down when they are combined [Collewijn et al. 1997; Erkelens et al. 1989; Yang and Kapoula 2004]. However, the combined eye movement is completed in a shorter period of time. Acceleration of vergence is observed during both horizontal and vertical saccades [Zee et al. 1992]. However, although combined eye movements are faster than pure vergence, the latency until the onset of eye movements is increased by 18–30ms [Yang et al. 2002]. In addition, the accuracy of saccades is reduced and corrective saccades are required more often when combined with vergence [Yang and Kapoula 2004].

Saccades towards stationary targets during smooth pursuit eye movements. The saccades and SPEMs are known to interact with each other. However, experimental data show that saccades do not linearly add up with SPEMs [Jürgens and Becker 1975]. In contrast, for the saccades performed during an ongoing SPEM, the velocity of smooth pursuit is reduced before and after saccades performed in the opposite direction, and after saccades performed in the same direction as the pursuit. The decrease depends on the saccadic amplitude.

To describe the neural saccade programming process during SPEMs, two types of positional error vector are defined that can explain the planned amplitude and direction of the saccade; namely, based on retinal error and based on spatial error [McKenzie and Lisberger 1986]. When a target is briefly flashed during a SPEM, the eyes remain in the smooth pursuit until the onset of the saccade for approximately 100–200ms. During this brief period of time, the position of the eyes changes with respect to the initial position when the target is flashed. If saccades are planned based on the retinal error between the source and target positions, then the neural programming of saccades would take place according to the displacement vector between the initial position and the target position without taking into account the displacement of eyes until the onset of the saccade. On the other hand, saccades programmed according to spatial error would compensate for the displacement of eyes between the initial position and the onset of the saccade. The type of positional error used by the brain to plan saccades determines the accuracy of the saccade. The initial experiments on this matter were contradictory and inconclusive. Some studies showed a correlation of saccades with retinal error [McKenzie and Lisberger 1986]. Other studies showed a correlation with spatial error if the flash is presented for a longer period of time [Herter and Guitton 1998; Schlag et al. 1990]. An explanation for the differences between the results obtained from these experiments is that the perceived motion of the target might play a role in saccadic accuracy [Zivotofsky et al. 1996]. When the target velocity is taken into account, saccades are correlated with the retinal error measured at the moment of the target step [Smeets and Bekkering 2000]. Studies on humans show a great variability in the accuracy of saccades during SPEM [Baker et al. 2003; Gellman and Fletcher 1992]. However, the source of poor localization is not well understood because no correlation was found between the pursuit velocity (15°/s, 30°/s, and 45°/s) and the amount of saccadic inaccuracy [Ohtsuka 1994].

2.2 Saccadic Suppression

The image of the real world rapidly shifts across the retina during a saccade. However, we do not observe motion blur in the image we perceive due to reduced visual sensitivity [Ditchburn 1955]. The duration of the reduced sensitivity spans a time interval that starts as soon as 40ms before the start of the saccade and lasts up to 80ms after it ends [Bouman 1965; Latour 1962; Volkmann 1962; Zuber and Stark 1966]. The suppression is characterized by a selective suppression of lower spatial frequencies and the suppression effect decreases as the spatial frequency of the stimulus increases [Burr et al. 1994; Volkmann et al. 1978]. In addition to the reduction in spatial contrast sensitivity, target position information is also suppressed [Beeler Jr 1967]. However, saccadic suppression does not result in perceiving a visual “black-out” due to the visual persistence of the retinal images before saccade [Campbell and Wurtz 1978; Ritter 1976]. Electrophysiological studies on primates identified a reduction of neural responses just before and during saccades, followed by amplified responses and enhancement of neural signaling in the post-saccadic phase [Ibbotson and Cloherty 2009].

Despite reduced visual sensitivity during saccades, intrasaccadic perception is still possible. When the peak velocity of the saccade approximately matches the velocity of the sinusoidal gratings rapidly drifting in the same
direction, it results in a static perceived image of the stimulus for a very brief period of time during the saccade [Deubel et al. 1987]. Stimulus motion, which is otherwise imperceptible during fixations, can also be perceived during saccades, especially when combined movements of the stimulus and eyes result in retinal frequencies between 10–25Hz [Castet and Masson 2000]. Based on intrasaccadic perception, an important question is whether saccadic suppression is just a consequence of motion blur in the retinal image or not. Recent studies show that saccadic suppression is not just a consequence of changes in the retinal image, and neural activity is also actively suppressed during saccades independent of visual input [Binda and Morrone 2018; Bremmer et al. 2009].

2.3 Saccade Landing Position Prediction

One of the first works to predict the landing position of the saccade is that of Anliker [1976]. Anliker’s prediction method is based on Yarbus’ observation [1965] that the saccade velocity profiles are approximately symmetric around the time when the peak velocity is reached and they estimate the landing position by doubling the displacement observed up to that point. Recently, there have been other studies based on the assumption of symmetry [Paeye et al. 2016]. However, the saccade displacement profiles tend to get skewed for larger amplitudes, such that the peak velocity is reached earlier than the midpoint of the saccade [Van Opstal and Van Gisbergen 1987]. Therefore, the prediction methods assuming a symmetric saccade velocity profile usually make an accurate prediction only for smaller saccades where the skewness is not very prominent in the velocity profile.

To study the behavior of saccades, models based on Kalman-filter and higher order differential equations are introduced [Komogortsev and Khan 2008, 2009; Zhou et al. 2009]. Although these models can be utilized for procedural simulation of eye movements, they require the estimation of a large number of parameters. This could be time-consuming and inconvenient for predicting the landing position in real-time gaze-contingent applications. As a more practical solution, Han et al. [2013] introduced a compressed exponential model, while Wang et al. [2017] used Taylor series with a limited number of parameters to describe the saccadic trajectory and make predictions for a short time window (≈10ms).

Most gaze-contingent applications such as foveated rendering require accurate eye tracking and a maximum system latency around 50–70ms [Albert et al. 2017]. In order to combat system latencies typically observed in gaze-contingent rendering systems, Arabadzhiyska et al. [2017] introduced a landing position prediction model based on the pre-programmed behavior of the saccades and the similarity of displacement profiles for similar saccadic amplitudes. For the same purpose, Griffith et al. [2020, 2019] proposed the use of support vector machine regression models and showed an extension to oblique saccades. Later, Morales et al. [2018, 2021] proposed the use of Long Short-Term Memory (LSTM) networks for the prediction of saccadic landing position and Griffith et al. [2020] introduced a technique to improve the performance of LSTM and feed-forward network based models. Despite these active research efforts in saccade prediction for gaze-contingent rendering, investigation of different factors and their influence on the characteristics of saccades remains an open problem.

3 OVERVIEW

This work consists of two parts. In the first one, we present a user experiment (Section 4) where saccade profiles are collected for different amplitudes, orientations, depth levels, and with and without initial speed. In Section 5, we analyze the collected data to discover the most significant factors affecting saccades and how they influence saccade prediction. In the second part of this paper (Section 6), we present a method for adjusting data used for training saccade landing prediction models to take the analyzed effects into account. Additionally, we demonstrate how one of the recent models [Arabadzhiyska et al. 2017] can be corrected directly using the same strategy.

4 EXPERIMENT DESIGN

In our experiment, we aimed to investigate how the saccade profiles depend on the saccade’s orientation (in 3D space) and initial smooth pursuit eye movements. In addition, we compared the effects with variability among
different users. To this end, instead of conducting separate experiments, each designed to investigate a single factor, we designed the stimuli and the task to simultaneously study all of the effects in different trials of a single experiment. As our main focus is applications of the saccade prediction techniques for head-mounted displays, the experiment was designed for a virtual reality device equipped with an eye tracker.

4.1 Stimuli
To guide the eye movements of the participants, we rendered a red sphere on a blue background as the visual target (Figure 1) at a distance of 75 cm from virtual camera. To preserve the retinal size of the target as 1 visual degree throughout the experiment, the size of the rendered sphere was adjusted depending on its position and distance in 3D space. This prevented the potential saccadic inaccuracies during the experiment due to the changes in target size when the target displacement involved a change in depth (e.g., fixating on arbitrary parts of the target sphere when it appears bigger at a close distance). Each trial began with an initial phase where the participants are asked to either fixate on a static target or follow a dynamic target. The duration of the initial phase was randomly selected between 1–2 seconds to avoid anticipation effects.

Static initialization. In two-thirds of the trials, a static target appeared at the center of the screen, followed by a target displacement in one of the left, right, up, or down directions to stimulate a saccade between two static positions (Figure 2-left). The displacement was 10° and 20°, respectively, for short and long saccades. The trials with a change in vergence involved a simultaneous change in depth with displacement (to 30 cm or 1000 cm w.r.t. virtual camera). The target remained visible for 2 seconds at the end of each trial for fully completing the eye movement.

Dynamic initialization. In the remaining one-third of the trials, the target moved along a linear, vertical or horizontal trajectory with a constant velocity of 10°/s (motion ramp) to stimulate smooth pursuit eye motion. Motion was followed by target displacement (step) to stimulate a saccade during smooth pursuit eye movement (a.k.a. ramp-step paradigm, Figure 2-middle). Target motion started from a source position located on the left/right or above/below the center of the screen for horizontal and vertical trajectories, respectively. The motion was always directed towards the center and would last for a random duration of 1–2 seconds with the target never exceeding a distance of 10° from the center. The displacement in the target step shared similar properties as the trials with a static target (i.e., 10° and 20° displacement size with a single final depth of 75 cm relative to the position of the virtual camera).

4.2 Task
During the experiment, each participant was asked to fixate on or follow the target with their eyes. Participants could abort the experiment at any time, especially if they started to experience viewing discomfort. However, no participant terminated the experiment prematurely due to viewing discomfort. Each participant was shown the same set of stimuli, but in a randomized order to minimize the bias due to learning effect. The set was constructed according to the cases visualized in Figure 2 and contained combinations of:

- 2 orientations of the saccade (horizontal and vertical),
- 2 saccadic amplitudes (10° and 20°),
- 3 depth levels to which the saccade was performed (30 cm, 75 cm, or 1000 cm), and
- with/without initial SPEM.

To keep the experiment procedure simple for the participants, we excluded from our trials the cases where the sphere is moving in the initial phase and is then re-positioned to a different depth. We collected 12 saccades for...
Fig. 2. The figure presents the main stages of each trial of our experiment. In the initial phase, we had either static initialization (left), where the initial gaze was shown as a static target in the center of the screen, or dynamic initialization (middle), where the initial gaze was moving to stimulate a smooth pursuit eye movement. After 1–2 seconds of the initial phase, the sphere was displaced to stimulate a saccade. Some trials of the experiment included a change in depth to stimulate vergence eye movement as shown on the right.

each of the remaining cases, which is 384 saccades per participant. The experiment took around 30 minutes to complete. To avoid fatigue, we divided the experiment into three sessions with two mandatory breaks of at least 10 minutes in between. We had seven participants (two of whom are authors) with normal or corrected-to-normal vision, ages 25–37, all male. Due to amplified eye tracking inaccuracies associated with the use of eye glasses during a pilot run of our experiments, participants with corrected-to-normal vision only used contact lenses. Also, to avoid calibration related problems, we introduced an additional verification step after the eye tracker calibration: Users were asked to consecutively fixate on four different targets, also red spheres with a visual size of 1°, located at 10° in the periphery in the four primary directions. We repeated the calibration procedure if the estimated gaze location was more than 1° away from any of the four targets.

4.3 Hardware
The experiment was implemented using Unity\(^2\) platform and was run on the HTC Vive Eye Pro headset which provides 1440×1600 px resolution per eye at 90Hz. We used the headset’s integrated 120Hz eye tracker which was calibrated at the beginning of each session using the 5-point calibration procedure provided by the eye tracker software. The accuracy of the eye tracker reported by the manufacturer is 0.5°–1.1°, however, recent research [Sipatchin et al. 2021] reports different values: 4.16° mean average accuracy of both eyes across field of view of 27° and mean precision of 2.17° for a head-still condition such as our task; the data loss is estimated to be 3.69%.

5 ANALYSIS OF EXPERIMENTAL DATA
The data from the experiments were used to extract mean saccade profiles, which were then analyzed to quantify the influence of different factors. To the best of our knowledge, there is no common dissimilarity measure to compare saccadic displacement profiles with each other. Therefore, we also provide a formulation of our measure that helps detecting the most significant factors affecting the saccade.

5.1 Saccade Profiles Extraction
Similar to Arabadzhyska et al. [2017], our saccade profiles describe the on-screen displacement with respect to the saccade anchor point, as a function of time that elapsed since the beginning of the saccade. To extract the

\(^2\)https://unity.com.
profiles from the data collected in the experiment, we follow the procedure described in Arabadzhiyska et al. [2017]. We first use a high-velocity threshold value for detecting a saccade and then a second, lower-velocity threshold value to scan the gaze samples backward in time to find its beginning. The first step gives us the detection point of the saccade and the second - its anchor point at which we assume the saccade has started. This two-step procedure reduces the detection likelihood of false positives and collects the additional samples that are needed to capture the beginning of the saccade. Since we compute the velocity by estimating the distance of consecutive samples without applying a velocity filter, a double-threshold policy improves the reliability of correctly detecting saccades. For further details, please refer to the original paper. While other, more advanced methods for saccade detection, such as Schweitzer and Rolfs [2020], may provide better results, we rely on the velocity threshold method because of its simplicity and compatibility with the model and the dataset derived by Arabadzhiyska et al. [2017].

Table 1. Analyzed Factors

| Factors       | Categories          |
|---------------|---------------------|
| ORIENTATION   | HORIZONTAL          |
|               | VERTICAL            |
| DEPTH         | SAME                |
|               | NEARER              |
|               | FARTHER             |
| INITIAL MOVEMENT | STATIC            |
|               | SAME                |
|               | OPPOSITE            |
| AMPLITUDE     | $-1^\circ$, $+1^\circ$ |

The factors that we consider when analyzing saccades and the categories in which we classify them according to each individual factor.

To analyze the effects of different factors, we define sets of categories belonging to each factor and we classify each saccade of our dataset into one of its categories. Each category contains a subset of the dataset and within the same factor the categories are mutually exclusive. To investigate the influence of the orientation of the saccade, we classify the saccades according to the location of their landing position with respect to the initial position of the gaze (factor: ORIEN-
TATIONS, categories: HORIZONTAL, VERTICAL). Similarly, to analyze the influence of depth/vergence change, we classify saccades according to the depth of the final position with respect to the initial point (factor: DEPTH, categories: SAME, NEARER, FARTHER). To analyze the influence of SPEM, we classify the initial eye movement at the beginning of the saccade, which may be performed from a static target, a target moving in the direction of the imminent saccade, and a target moving in the opposite direction of the imminent saccade (factor: INITIAL MOVEMENT, categories: STATIC, SAME, OPPOSITE). Additionally, to analyze differences among subjects we create a category for each person containing only the saccades performed by this individual. (factor: USERS, categories: each user).

In gaze-contingent rendering applications, inaccuracies in saccade prediction may remain imperceptible if the prediction error is limited. Previous research on the anatomy of the human retina revealed that the angular subtense of the human fovea is approximately 4°–5° [Hendrickson 2005]. We assume that when the prediction error reaches approximately half of this distance, the misplacement of the foveal region becomes visible to observers. Consequently, an improvement of the prediction error may be evaluated by comparing with this baseline. Therefore, we also included the saccades at a range of 2° difference in amplitude around the short and long saccades. More specifically, we consider saccades with amplitudes 9° and 11° for the short saccades and 19° and 21° for the long saccades in our comparisons (factor: AMPLITUDE, categories: $-1^\circ$, $+1^\circ$). A summary of the factors and categories that we defined for our experiments is shown in Table 1.

To analyze the differences within each factor, we aim to compute the mean profiles for each category created for it and for each saccade amplitude $\alpha \in \{10^\circ, 20^\circ\}$. For each category, we start by filtering out the saccades that do not belong to it and then align the displacement profiles in the temporal domain. For each factor and each amplitude $\alpha$, we start by removing all saccades with amplitude outside the range $[\alpha - 1^\circ, \alpha + 1^\circ]$. In addition, we check the length of the saccades, the direction of SPEM, and the direction of the vergence performed by the participants to label samples that do not conform to the expected behavior in the category as outliers. Then
we align the anchor points of the saccades by applying a temporal offset [Arabadzhiyska et al. 2017]. In our case, we choose the velocity threshold for detecting a saccade as 180°/s and the anchor point as 90°/s, the same values used by Arabadzhiyska et al. [2017] for their subjective experiment. All eye-tracker samples 30 ms prior to the anchor point are also included in the analysis. To obtain the mean profile sampled at equal time intervals, we resample each profile using linear interpolation of the measured displacements. Our eye tracker operates at 120Hz and provides a gaze estimations every 8 ms or 9 ms and not all of these samples are valid. Therefore, the samples for the different saccades are at different time positions with respect to their beginnings. Resampling at equal intervals is needed to align the displacement values for each saccade in the same time positions. We chose our interval to be 1 ms. Since the end point of each saccade occurs at an arbitrary time, we consider that the end point of the mean profile is located at the mean time position of all end points.

After the above initial processing, the mean profiles are computed by averaging samples of all saccades within each category. Formally, we represent these mean profiles as a sequence of N mean samples computed from the original profiles:

\[
\overline{S} = \{s_0, s_1, \ldots, s_N\}, \tag{1}
\]

where each sample \(s_l = (t_l, d_l, \sigma_l)\) is defined by its time stamp \(t_l\), mean displacement \(d_l\), and the standard deviation of all displacement values for the given timestamp \(\sigma_l\) within the category. The first sample of the saccade (\(s_0\)) is the anchor point (\(t_0 = 0\)) while the last sample (\(s_N\)) is the end point (\(d_N\)) and is equal to the amplitude of the saccade. Figure 4 visualizes the mean displacement profiles for all categories grouped by the factors to which they belong.

5.2 Dissimilarity Measure for Saccade Displacement Profiles

To be able to analyze and compare the effects of different factors, we propose a dissimilarity measure for quantifying the differences between the mean saccade profiles corresponding to individual categories within a single factor. More precisely, for a given set of mean profiles \(\{\overline{S}^k | \overline{S}^k = \{s^k_0, s^k_1, \ldots, s^k_N\}\}\) belonging to the categories within a factor (Table 1), we define a measure that correlates with the differences for that factor as:

\[
D(\{\overline{S}^k\}) = \sum_{l=0}^{N_k} \frac{\max_k d_l^k - \min_k d_l^k}{\max_k \sigma_l^k}. \tag{2}
\]

where \(k\) is the index of a category. The measure can be seen as an area between the upper and lower envelope of all mean displacement profiles for the factor (\(\{\overline{S}^k\}\)), normalized by the maximum standard deviation of the displacement values observed for the factor (\(\max_k \sigma_l^k\)).

Figure 3 illustrates an abstract example of the mean displacements (\(\overline{d}_l^k\)) and the standard deviations (\(\sigma_l^k\)) of three hypothetical sample categories. It is important to note that this measure requires that the mean displacement profiles have equal sampling intervals.

Computing the dissimilarity measure in Equation (2) yields a higher value if there is a more significant difference between the mean saccade displacement profiles corresponding to different categories within a given factor (Table 1). The differences between the categories commonly manifest themselves through speed ups or slow downs in saccade displacement profiles, and we use our dissimilarity measure to identify perceptually significant changes in displacement profiles that require an update of the prediction model or training data to avoid visual artifacts.

5.3 Discussion

Figure 4 summarizes the effects that different factors have on mean displacement profiles. In addition, we provide a bar plot of our dissimilarity measure for each factor.
A clear difference can be observed for the case where we compare mean displacement profiles with different amplitudes (factor: Amplitude). The difference for both 10° and 20° saccades is consistent with the fact that longer saccades exhibit a steeper rise in their displacement profiles compared to shorter saccades. Existing saccade landing prediction models depend on these profiles to be distinguishable, which is an expected effect. It also serves as the baseline for comparing the difference exhibited by the other factors of interest, as we mentioned in Section 5.1. Therefore, we aim to identify the effects that will change the performance of saccade landing position prediction and assume that factors that provide smaller effects than what is observed at 2° change in the saccade amplitude may not lead to significant improvements in applications relying on prediction. In particular, the value of the dissimilarity measure $D$, 25.3 for 10°, and 24.8 for 20° saccade are the reference points for analyzing the effects of the other factors.

Apart from the Amplitude factor, the most significant differences were observed for Users where the differences become more apparent for longer saccades (20°). Even though our observations are made based on the results from seven subjects and, for a more accurate estimation of the discrepancies, more subjects should be considered in the future, the results are in line with previous literature. Arabadzhiyska et al. [2017] have already shown that tailoring a model to fit the personal saccadic characteristics of a user leads to a lower saccade prediction error and to a higher subjective preference for that user compared to the model trained for the average population. Although they demonstrated this in a task-performance experiment, here we demonstrate the underlying difference in saccade profiles.

The third factor with the highest differences was Orientation. Similar to Users, the differences for 10° saccades were smaller than for the Amplitude factor, but the opposite can be observed for 20° saccades. For this factor, the differences become close to those observed with the Amplitude factor.

Contrary to our expectations, moving the target to different depth levels (factor Depth) led to smaller changes in the mean displacement profiles, especially for 10° saccades. While we observed some changes in the peak velocity (Figure 5), the differences are smaller than those reported by the previous studies (Section 2.1). We relate this discrepancy with existing studies mainly to the profound difference between the real and virtual environment. First, standard head-mounted displays are not fully capable of reproducing accommodative cues, and any depth change only results in a change in the vergence (due to the change in disparity), but it does not trigger an accommodation response from the participants’ visual system. The lack of an accommodation response may be seen as a deviation from real-world viewing conditions, but it applies to most mainstream stereoscopic HMDs used for virtual reality. Therefore, we have not tried to mitigate this effect in our experiments. Second, similar to all experiments carried out on stereoscopic displays with a lack of accommodation response, the presence of the well-known vengeance-accommodation conflict [Shibata et al. 2011] imposed a limit on the depth ranges that we could test in our experiments without causing viewing discomfort for the participants. These differences between virtual reality and real-world viewing conditions may explain the discrepancy between our measurements and the previous studies, most of which are conducted under real-world viewing conditions. Additionally, the choice of the stimuli could affect the outcome of our experiments. While the small spheres used in the experiment enable precise control over the participant’s gaze location and saccades, the fact that they do not change their size according to the distance removes the size cue. The lack of this cue could potentially influence the
Fig. 4. Effects of different factors on saccade mean displacement profiles computed from our experiment data. Solid lines represent mean profiles for each category, while dashed lines represent the corresponding standard deviations. The bar plots show the values of our profile dissimilarity measure (Equation (2)) for different factors (Section 5.1). The bars representing the Amplitude factor are provided as a reference baseline for the minimum similarity value to observe a significant effect (please refer to Section 5.1 for details).
accuracy of the saccade. Also, the use of specific colors, red and blue in our case, may lead to a different amount of edge blur due to the wavelength-dependent accommodation.

With the final factor, Initial Movement, we observed that the initially moving target led to smaller differences in the displacement profiles. We believe that higher pursuit speeds could potentially enhance the effect. In our experiment, we chose a moderate pursuit speed (10°/s) to keep the task simple and give the observer ample time to properly fixate on the moving target and initiate SPEM. Similar to Depth, the differences become larger for 20° saccades, and they are close to those observed with the Amplitude factor. It is possible that the differences become more apparent for more extreme saccade amplitudes. Unfortunately, reliable measurement of larger-amplitude saccades poses problems due to the fact that virtual reality headsets have a limited field of view with high fidelity.

In all our experiments, we used an optical eye-tracker, the current technology of choice for VR and AR applications. Despite its widespread use, this technology is not suitable to capture all characteristics of eye movements [Hooge et al. 2016, 2015; Nyström et al. 2016, 2013]. In particular, due to post-saccadic oscillations of the pupil, optical eye trackers have low accuracy in estimating the saccade onset, peak velocity, and its end. Additionally, the sensitivity of the eye trackers to changes in the size of the pupil [Drewes et al. 2014; Hooge et al. 2019; Jaschinski 2016] has a detrimental effect on the correct estimation of the vergence and the binocular fixation point. To address these limitations and measure eye movements more accurately, it is possible to use eye tracking technology such as the wearable scleral coil tracking system proposed by Whitmire et al. [2016]. However, most users might find coils to be a very invasive way to track their gaze orientation, and to our knowledge, no commercial VR or AR headset uses such technology. Therefore, in our work, we focus on optical eye tracker technology, which, despite its limitations, has already been shown to be beneficial in applications such as foveated rendering [Arabadzhiyska et al. 2017; Guenter et al. 2012; Patney et al. 2016]. At the same time, it is important to note that the generalization of our findings to the technology of scleral coil eye tracking needs further investigation.

In the remaining part of the paper, we demonstrate a new technique that accounts for differences in the saccade profiles to provide better saccade prediction. For demonstration purposes, we chose to focus on two factors that exhibit the highest differences, i.e., Users and Orientation. Although the personalization of the prediction model for a specific user was demonstrated in Arabadzhiyska et al. [2017], the process required collecting a large set of saccades. Here, our goal is to reduce the amount of data required. On the other hand, to our knowledge, adjusting existing models to adapt to the saccade’s orientation has not been done before, but our findings suggest that it could improve the prediction accuracy. Therefore, designing prediction methods or adjusting existing ones to handle different orientations correctly may provide additional benefits in the final applications.

6 METHOD FOR TUNING SACCADE PREDICTION MODELS

In Section 5, we analyzed how different factors affect the displacement profiles of the saccades. We observed that the dissimilarity for individual factors is comparable to the dissimilarity for the Amplitude factor, with the largest ones for Users and Orientation factors. The observed dissimilarities suggest that incorporating factors such as saccade orientation or the difference among users may improve saccade prediction. However, the fundamental problem in deriving a model that captures such dependencies lies in data collection. Individual
saccades collected for training such models contain noise; therefore, many of them have to be combined to create a reliable prediction. For example, the prediction model proposed by Arabadzhiyska et al. [2017] required each participant to perform 300 saccades. Still, the model does not capture factors other than the saccade amplitude. The consideration of additional factors, such as orientation, depth, and SPEM, would significantly increase the number of required saccade samples, making the data collection for individual users tedious and sometimes infeasible. Similarly, most machine learning approaches, such as Morales et al. [2018], have high data demands for training.

To address the problem of data collection, we propose an alternative approach. Instead of exhaustively collecting data from psychophysical experiments, which enables training prediction models to capture all factors, we postulate that the influence of many factors, such as orientation or user, can be approximated by a low-parameter transformation of the data. The advantage of such a solution is that the effect of additional factors is captured using a small number of parameters, and therefore such a model is more robust to noise and the reduced number of collected saccades. Successful applications of this approach have been shown in the past, such as the method of Lesmes et al. [2010], which uses the a priori information about the general functional form of the contrast sensitivity function (CSF) to maximize the information gained from a small number of measurements. Similarly, in this work, we seek a global transformation of the profiles of a saccade prediction model, which has a small number of parameters, yet allows one to explain the effects of additional factors influencing the saccade performance.

The main observation behind our solution is that the differences in the saccade profiles can be attributed to the changes in the saccades’ performance/velocity caused by the factors that we investigated in our experiments. This observation can be made by looking at the differences among the slopes of the individual mean saccade profiles in Figure 4. We show that these changes can be effectively modeled by shearing the profiles parallel to the axis representing the time domain (Figure 6). Additionally, we observe that the appropriate shearing factor changes with the saccade’s amplitude, but we show that this change can be approximated with a low-degree polynomial. This is the key to our technique, as it allows us to compute the shear factor for a few saccade amplitudes and then interpolate or extrapolate the shearing transform to the other amplitudes. Below, we provide a formal definition of shearing (Section 6.1) and a shear between two saccade profiles (Section 6.2). Then, we describe the derivation of the shearing-based transformation of saccade profiles and how it can be applied to modify a prediction model to account for additional factors in Section 6.3.

### 6.1 Shearing Saccade Profiles

Given a saccade profile $S = \{s_0, s_1, \ldots, s_N\}$, where each sample is defined by a couple of scalars $s_j = (t_j, d_j)$ representing the time stamp, $t_j$, and the corresponding displacement, $d_j$, we define a sheared version of the profile by applying a 2D shearing parallel to the time axis followed by resampling to restore uniform sampling in time domain. More formally, to shear the profile $S$ with a shearing factor $\lambda$, we first transform its samples using a 2D shearing matrix:

$$
\begin{bmatrix}
\tilde{t} \\
\tilde{d}
\end{bmatrix} =
\begin{bmatrix}
1 & \lambda \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
t \\
d
\end{bmatrix}, \quad \lambda \in [-1; 1].
$$

The resulting profile $\tilde{S} = \{(\tilde{t}_1, \tilde{d}_1), (\tilde{t}_2, \tilde{d}_2), \ldots, (\tilde{t}_N, \tilde{d}_N)\}$ is no longer sampled regularly at 1ms intervals after applying the shearing transformation because the time stamp, $t_i$, of each sample changes. Therefore, we applied a simple linear interpolation to resample it back to the 1ms intervals and obtained the final sheared profile. In the rest of the paper, we denote shearing as a function $\Psi$, and a saccade or mean saccade profile $S$ sheared with shearing factor $\lambda$ as $\Psi(S, \lambda)$. 

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Fig. 6. Two examples of shearing original mean saccade profiles to match different targets. The target on the left represents a category with slower saccades than the original. The target on the right represents a category with faster saccades.

6.2 Computation of Shearing Transformation between Saccadic Profiles

Given two saccade profiles \( S^k = \{ s^k_0, s^k_1, \ldots, s^k_N \} \), where \( k \in \{1, 2\} \) and \( s^k_i = (t^k_i, d^k_i) \), we can compute a shearing factor \( \lambda \) that describes the difference between those two profiles. Formally, we define the shearing between \( S^1 \) (original profile) and \( S^2 \) (target profile) as \( \lambda = \Lambda(S^1, S^2) \) for which the 2D shear applied to \( S^1 \) minimizes the difference with respect to \( S^2 \), i.e.,:

\[
\Lambda(S^1, S^2) = \arg\min_{\lambda} \sum_{l=0}^{N} |d^*_l - d^2_l|, \quad \text{subject to } S^* = \Psi(S^1, \lambda). \tag{4}
\]

This definition relies on the same time domain sampling by all profiles involved in the computation \((S^1, S^2, S^*)\). However, this is guaranteed by the definition of \( \Psi \). The above minimization problem can be easily solved using binary search. Figure 6 shows two examples of how shear between two saccade profiles can be used to align them.

6.3 Application

Previous models to predict saccades, such as Arabadzhiyska et al. [2017] and Morales et al. [2018, 2021], are trained on large datasets containing saccades with various amplitudes and orientations collected from multiple users using an eye tracker. These models do not account for all the factors analyzed in Section 5. Here, we demonstrate how to use the shearing strategy described in Section 6.1 and 6.2 to account for these factors. It is possible to apply the shearing transformation to saccade displacement profiles directly if the dataset is available (data shear). In some cases, although the model is accessible, the dataset on which the models were trained may not be available. For such cases, if possible to extract saccade displacement profile approximations from the model, we apply the transformation to the recovered profiles instead (model shear).

Data shear. The first approach we consider that uses the idea of shearing the saccade profiles is to transform all the saccades in the dataset to create a new dataset that represents a particular type of saccade and then recompute the prediction model using the augmented dataset instead of the original one. In particular, we consider here shearing the saccade profiles to create a dataset and models for horizontal, vertical, and personalized saccades. In all three cases, we apply our shearing strategy in the same way. First, to obtain the specific saccades for each category (horizontal, vertical, or a particular user), we extract the corresponding saccades from the dataset. These saccades act as targets for the required shear computation applied to the remaining saccades to compute the final dataset. We then discretize the amplitude domain. In our experiments, we chose the discretization step to be 1°. For each discrete amplitude value \( \alpha \), we estimate the mean displacement profile by averaging the
saccade displacement profiles with amplitudes in the range \( \{ \alpha - 1; \alpha + 1 \} \) for both the original dataset and for the target dataset following the same procedure as described in Section 5.1. Here, we denote the mean profiles of the amplitude \( \alpha \) in the original dataset as \( S^o_\alpha \) and the target dataset as \( S^t_\alpha \). The number of saccades constructing \( S^o_\alpha \) we denote with \( |S^o_\alpha| \). Note that \( S^o_\alpha \) and \( S^t_\alpha \) are mean saccade profiles of the same amplitude. The only difference is that \( S^o_\alpha \) comes from the original dataset, which contains all types of saccades (e.g., all orientations) while \( S^t_\alpha \) is a mean profile for the specific category (e.g., horizontal, vertical or for a particular user). The goal is to use this correspondence to define the shear that needs to be applied to the original dataset, to make it represent a particular category of saccades. To this end, we compute a series of shearing factors \( \{ \lambda_{\alpha_0}, \lambda_{\alpha_1}, \ldots, \lambda_{\alpha_M} \} \) for each amplitude \( \alpha_i \) following Equation (4), i.e., \( \lambda_{\alpha_i} = \Lambda(S^o_{\alpha_i}, S^t_{\alpha_i}) \).

The main objective of such a dataset derivation is to obtain a large dataset of saccades while using only a few measured profiles. To this end, we propose first to collect a subset with a particular category of saccades and compute the shear (Section 6.2) of the mean profiles with respect to the mean profiles in the large dataset. Using this procedure, we obtain the relationship between the profiles in the large dataset and the newly collected one for a few saccade amplitudes. To obtain the shearing factors for the entire range of saccadic amplitudes, we use a linear regression to fit the linear function \( f(\alpha) \) that minimizes

\[
\sum_{i=1}^{M} \left| \frac{f(\alpha_i) - \lambda_{\alpha_i}}{|S^t_\alpha|} \right|
\]

where \( |S^t_\alpha| \) is a weighting argument used to balance the data in the cases when different amplitudes are unequally represented in the target dataset. This function allows us to estimate the shear required for transforming each profile in the large dataset based on the saccadic amplitude (Figure 7). Having the shearing factor for each amplitude \( \alpha \), we apply shear \( f(\alpha) \) to all individual profiles to form the new dataset. We compute such datasets for horizontal and vertical saccades, as well as for each user separately.

**Model shear.** It is possible to apply the shearing operation directly to an existing saccade prediction model as long as the individual saccade or mean saccade profiles can be recovered. An example of such a model is the one proposed by Arabadzhianyska et al. [2017]. The model provides a mapping of the time and displacement pair \((t, d)\) to the predicted saccade amplitude \(\alpha\). Since the model is represented directly by the \((t_i, d_i, \alpha_i)\) triplets, the mapping can be inverted by fixing \(\alpha_i\) and treating the corresponding sequence \((t_i, d_i)\) as a displacement profile for a saccade with \(\alpha_i\) degree amplitude. Because the model is represented by a discrete number of \((t_i, d_i, \alpha_i)\) sample points, we propose using a linear interpolation on displacement values to obtain saccade profiles sampled at regular, one-millisecond intervals. The blue dotted lines in Figure 8 show examples of displacement profiles obtained using this procedure. Unfortunately, the profiles are often noisy, which prohibits a direct application of the shear with a satisfactory performance. For this reason, as well as to prevent the occurrence of any aliasing, before shearing, we denoise the profiles by first applying a median filter with a window size of 15 ms followed by a Gaussian filter with a window size of 5 ms for smoothing (Figure 8, blue solid lines). The values of the window
Table 2. Short Descriptions of the Four Models that we Compare in Section 6.4

| Model      | Original dataset | Target dataset | Model description                                      |
|------------|------------------|----------------|--------------------------------------------------------|
| Average    | Full             | -              | The model is created using the original full dataset.  |
| Model Shear| Full             | Subset         | The model is first created using the original full dataset, and then modified to match a specific subset of it. |
| Data Shear | Full             | Subset         | The model is created from an augmented full dataset, modified to match a specific subset of the original dataset. |
| Customized | Subset           | -              | The model is created from a specific subset of the original dataset. |

Each model is created following the procedure described by Arabadzhyska et al. [2017], either using their entire dataset or a subset of it that includes a single category of saccades (Table 1). For data shear we modify the dataset before creating the model, and for model shear we first create the model and then modify it to match a specific subset.

size were chosen heuristically as the smallest values producing stable results. After shearing the individual saccade profiles (Figure 8, yellow lines), the new triplets \((\hat{t}_i, \hat{d}_i, \hat{\alpha}_i)\) can be used to create a new model. Note that the shearing operation affects only timestamps, and the other components of the triplets do not change. In the particular case of the model of Arabadzhyska et al. [2017], it is enough to resample the data to be uniformly sampled in time and displacement domain. Therefore, we apply linear interpolation to obtain \((t_i, d_i, \hat{\alpha}_i)\) triplets, where \(t_i\) and \(d_i\) are sampled at the intervals of the original model, and \(\hat{\alpha}_i\) is the new prediction of the saccade amplitude.

6.4 Results

In our analysis, we consider both the data shear and model shear strategies described in Section 6.3 to update saccade datasets and prediction models. We analyze the effectiveness of these strategies in two different experiments. In the first one, we show an application of shearing operation to update the existing dataset and prediction models for improved predictions when the saccade orientation changes (horizontal vs. vertical). In the second experiment, we demonstrate the application of shearing operation to create user-specific models, a.k.a. personalization.

We compute our results on the saccade dataset and model of Arabadzhyska et al. [2017], which includes 6,600 saccade profiles collected from 22 participants (300 saccades for each participant). The amplitudes of the saccades are evenly distributed in the range of \(5^\circ–45^\circ\). To customize the models for vertical and horizontal saccades, we classify saccades into horizontal and vertical categories depending on their orientation (with \(+/- 15\) degrees allowance around the corresponding orientation). It is important to mention that, while the amplitude distribution across participants is balanced due to the experiment design, this is not the case for the orientation. Due to the aspect ratio of the screen (16:9), the amplitudes of the vertical saccades are limited to the range of \(5^\circ–22^\circ\) while the horizontal saccades have amplitudes up to \(40^\circ\). In addition, horizontal saccades are more frequently represented in the dataset, which constitute 30% of the collected data, compared to 5% for vertical saccades.

The baseline for all of our comparisons consists of two models. The first one is average model from Arabadzhyska et al. [2017]. It is derived from their dataset and is based on the interpolation of the collected data. We include this model in our comparisons because it provides a good balance between accuracy, performance, and data volume requirements. However, it only accounts for the variance in the saccade profiles due to changes in amplitude, and it does not account for any additional factors that we considered in our paper (Section 5). The second model is the so-called customized model, which is derived after the computation of the average model, but using a subset of the data corresponding to a specific category of saccades (e.g., for horizontal or vertical saccades). Table 2 gives a short summary of the four models that we compare in this section.
Fig. 9. The figure presents the performance of models derived differently for horizontal and vertical saccades. The leftmost plot presents aggregate mean errors for predictions made for the entire saccade duration (height of the bar) and for the second half of the duration (light segment). The two other plots present the error as a function of the duration of the saccade, i.e., at which stage of the saccade the predictions were made. While customized model performs best, sheared model, which requires a significantly lower number of saccades for training, performs better than average model, which does not account for the orientation of the saccade.

In the first experiment, we computed customized model for the two categories of orientation (horizontal and vertical) separately. The number of saccades in each category was sufficient to properly train these models. Later, we used data shear and model shear as described in Section 6.3 to compute two alternative models and compare them with customized models. For data shear, we sheared the displacement profiles of all saccades in the dataset, regardless of their orientation, according to the shear factor computed using pre-selected horizontal and vertical saccades as target. For model shear, the shearing factors were computed based on the comparison of the original model of Arabadzhyska et al. [2017] and the subsets of vertical and horizontal saccades. As for the second experiment, we followed a similar procedure to evaluate the performance of the shearing operation to personalize the models, but in that case, saccades of a particular user were selected to compute the shearing factors.

Figure 9 presents the performance of different models tailored to the orientation of the saccade. The figure presents both the mean absolute error (left), and the mean absolute error for predictions made at a specific moment during the saccades (right). The performance of the horizontally oriented data shear and model shear models is indistinguishable from the average one. We attribute the lack of an effect to the predominance of the horizontal saccades in the dataset and, consequently, to better prediction of these saccades. In comparison, the prediction for vertically oriented saccades greatly benefits from vertically oriented models. It is important to mention here that customized model greatly benefits from the significantly lower range of amplitudes in the set of vertical saccades. More precisely, the range of horizontal saccades is double that of vertical saccades due to the dimensions of the display used for data collection [Arabadzhyska et al. 2017]. While reducing the training and testing range of saccades’ amplitudes improves the prediction as the error is bound to this range, the model is limited to shorter saccades. On the contrary, the models derived using model shear and data shear support the larger range of amplitudes represented in the original dataset.

Although both model shear and customized model provided a better performance for vertical saccades than average model, surprisingly, data shear did not improve the model. To understand the reason behind it, we analyzed mean saccades profiles from the full dataset and from the vertical subset, as well as the cross section of the original average model [Arabadzhyska et al. 2017] in Figure 10. When comparing the mean saccade profiles representing the full dataset and the vertical subset, the first profile requires shearing to the right to match the second. This is expected as the vertical saccades are slower (Section 5.3). However, the cross section of the model exhibits the opposite behavior, i.e., it requires shearing to the left to match the vertical saccade profiles. When applying the model shear, the shearing computed based on the vertical saccade profiles and the model results in the model shearing to the left (green arrow), hence better aligning with the vertical saccades and reducing the
error. However, shearing all the profiles in the dataset according to the difference between their representative mean profile and the vertical mean profile, i.e., data shear, leads to a suboptimal shear of the model to the right (red arrow), hence increasing the prediction error, i.e., worse alignment with the vertical saccades profile. This demonstrates that although data shear can perform a correct transformation to the individual profiles, it cannot account for the built-in biases in the model. In this case, this leads to a lack of improvement when data shear is followed by the model computation. On the contrary, model shear, which computes the shearing factor based on the model, can account for biases in the model and improve prediction.

The great potential of our shearing strategy lies in the fact that it may allow for training models using a significantly lower number of samples than is required for training customized models. To verify this, we analyzed the performance of our shearing strategy for different numbers of saccades (Figure 11). To this end, we divided the dataset of vertical saccades into training and testing sets that consist of 200 and 150 saccades, respectively. Considering a different number of saccades (x-axis in the plot) from the training set for computing the shearing factor and the new model, we analyzed the resulting mean absolute error of the prediction. We compared this model shear strategy with the straightforward computation of the model based on the smaller number of training saccades (customized model). As expected, when the number of saccades considered is large, the improvement from our shearing technique may be limited. However, we can achieve a better prediction performance, in the presence of a significantly lower number of saccades. This is particularly visible for prediction in the second half of saccade duration, which is critical for techniques such as foveated rendering, where the sensitivity of the visual system is gradually restored towards the end of the saccade (Section 2.2). This can be particularly observed in the right plot in Figure 11, where the error is analyzed for the predictions made at different points in the duration of the saccades. It can be observed that the customized model trained on a low number of saccades retains the high error throughout the entire duration of the saccades. In contrast, the error for the model trained using our method drops significantly towards the end of the saccades.

In Figure 12, we provide the mean absolute error of the predictions obtained from different models for personalization. We observe that for many participants (e.g., users 4, 12, 15, and 21) the prediction performance of models follows an expected pattern, where customized model has the best performance due to the availability of full data used to calibrate such a model. Data shear and model shear provide the best prediction performances after customized model and are suitable for improving existing dataset or model prediction performances without large data collection requirements for personalization. The average model performs least favorably due to the lack of user-based adjustments in saccade displacement profiles. Nevertheless, using a limited dataset for training prediction models is more susceptible to noise inherent in the data. We observe that for some of the participants (e.g., users 7, 8, and 14) model shear performs more favorably than data shear and we attribute this observation to the model adjustments in model shear that are more robust against noise. In some of the cases (e.g., users 3, 18, 19, and 20), data shear and model shear have a performance level close to that of average model. We believe that for these users, personalization does not offer a high level of performance improvement. However, we observe that the general behavior of the mean absolute errors favors the use of data shear and model shear to improve prediction performance without the cost of collecting a large amount of training data.

Based on the above experiments, we conclude that both data and model shear are viable solutions for extending and improving saccade prediction models to account for effects analyzed in Section 5. The important difference between them lies in how they can correct model biases. While the model shear is capable of correcting them,
Fig. 11. Performance comparison for different models as a function of the number of saccades used for their computation. While the plot on the left shows the average error of the prediction for the full length of the saccade, the center plot shows the error for the prediction during the second half of the duration. The solid lines are the means computed using bootstrapping with 20 repetitions, and the dotted lines are the corresponding standard deviations. The plot on the right compares the average prediction error at any point during the saccade when using 10 and 200 saccades to train the models.

Fig. 12. The figure presents the performance of different models for each user. The bar plot on the left shows aggregate mean absolute errors for the saccade amplitude predictions. The height of the bars represents the mean error measured for the whole duration of the saccade, whereas the segments shaded with lighter colors represent the mean error measured in the second half of the saccade duration. The line plot on the right shows the mean absolute error as a function of the point in time when the prediction was made during the saccade. The customized model gives the best performance, followed by model shear and then the average model (please see the text for details). Model shear mostly has a good prediction performance for the users, for whom customized model also performs well.

adjusting the data using data shear is not. Therefore, the success of data shear is influenced by the quality of the prediction model built on it.

7 CONCLUSION AND FUTURE WORK

In many applications, such as foveated rendering, latency poses significant challenges. Improving hardware solutions is one path to improve the performance of techniques that benefit accurate gaze information. However, it has been demonstrated that latency problems can also be addressed by building efficient and accurate predictive models for fast eye movements [Arabadzhysiyska et al. 2017]. In this work, we go beyond existing models and analyze factors that should be taken into account when building such methods. We first demonstrate that factors, which were previously not considered explicitly when building the models, such as the orientation of the saccade, the depth change, or the initial smooth pursuit eye motion, can affect the prediction accuracy of the saccade landing position significantly. Then, we propose a technique that allows extending previous models and datasets.
to train them to handle the additional effects while limiting the number of collected data in user experiments. We argue that this is critical to building comprehensive models for saccade prediction. The key to our technique is the proposed shearing operation, which adapts previously derived models. This low-parameter transformation acts as a regularization for smaller, possibly more noisy datasets. In this work, we demonstrated the performance of the method in training personalized models and models for horizontal and vertical saccades. While the direct model correction was demonstrated only with one model, we believe that similar strategies can be applied to other models too. One possibility that was not explored in this work but we believe is a promising direction for future work is to store the correction as an additional look-up table to correct any prediction model in an online manner. In the future, the method can be used to train more comprehensive models that address a continuous range of orientation, depth changes, user-specific factors, and possibly other factors using a lower number of input saccades. We also believe that the low number of parameters of the shear transformation will allow the creation of models that will adapt on the fly to the user without the additional need for calibration. Finally, our method can be seen as a data augmentation technique for machine learning techniques, such as Morales et al. [2018]. Although current inference times do not meet the low latency demand of state-of-the-art head-mounted displays, such techniques can provide acceptable performance and higher accuracy prediction in the future. In this context, our method can significantly limit the amount of data required for training such models, facilitating the development and application of these techniques.

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