Inter-circuit fault location algorithm in generalized unified power flow controller-compensated double-circuit transmission lines based on synchronous current and voltage phasors of line terminals

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Abstract
An accurate inter-circuit fault location algorithm for double-circuit transmission lines compensated by a generalized unified power flow controller based on phasor analysis is presented. The proposed algorithm utilizes synchronous currents and voltages of the buses on both sides of the line to accurately calculate the fault location. Using the equations of sequence component circuits, the algorithm calculates voltage and current of the fault point on both circuits. Then, using boundary conditions equations for 98 types of inter-circuit fault on the double-circuit transmission lines, the accurate location of the fault is obtained. The number of boundary conditions equations varies depending on the type of faulty phases of the two circuits and the number of unknowns in the current and voltage equations of the fault point. The calculation of these boundary equations are also described. The results presented in the simulation section verify the validity and accurate performance of the proposed algorithm under different fault conditions.

1 | INTRODUCTION

1.1 Problem statement and research significance

Because of technical and geographical limitations on power transmission in power systems, many suggestions such as designing multi-circuit transmission lines have recently been made, and the use of flexible AC transmission systems (FACTS) devices has increased more than ever. With the proliferation of multi-circuit transmission lines in the power system, a generation evolved from the FACTS family known as generalized unified power flow controller (GUPFC) was proposed in 2000 by Fardanesh [1]. Since 2000, various types of GUPFC topologies have been presented in the power system. In general, this equipment includes n (n ≥ 2) series converters and one shunt converter so that it can control the voltage in a bus and independently control the active and reactive power flow on at least two circuits of a double- or multi-circuit transmission line [2,3]. Concerning the special conditions of double-circuit transmission lines (DCTLs) compensated with GUPFC, the protection and fault location in such lines is very complicated for different types of faults [4].

1.2 Discussion on the literature

Due to the specific geometry of towers and the distance between conductors of DCTLs, two basic categories of faults threaten these lines. The first category includes single-circuit faults, which involve phases of only one circuit of the DCTL [5–9]. The second category is related to inter-circuit faults, which involve phases of both circuits and are known as evolving double-circuit faults, double-circuit cross-country faults, and...
inter-circuit faults [10–18]. On the other hand, the references working on fault location in DCTLS are generally divided into four basic methods in terms of the problem-solving methods regardless of the fault type. They include impedance analysis-based methods [5,6,8–10,12,13], methods based on analysing traveling waves caused by fault current wave [16], artificial neural network (ANN)-based methods [7,11], and hybrid methods [14,15,17,18].

• Analysis and review of the references in the first category [5–9]

A fault location algorithm for HV un-transposed DCTLs was used in ref. [5] to analyse single-circuit and inter-circuit faults based on local current and voltage phasors analysis. The algorithm incorporates the voltage equation of the faulty phase so that phase voltages and local phasor currents of the faulty and healthy circuits are used as inputs to the algorithm. Thus, when both parallel lines are connected, the algorithm can locate the fault. A fault location algorithm based on analysis of asynchronous current and voltage phasor in DCTLs and a modal transform matrix was employed in ref. [6]. Using the modal transform matrix, the zero-sequence network can be divided into a network with common and differential components. The authors in ref. [6] determine the faulty circuit and then calculate current and voltage equations of the sequences at the fault point as a function of the fault distance. The authors in ref. [7] propose two different methods for finding fault distance in DCTLs using ANNs. In this reference, unique ANNs (for each ten fault types in both circuits) and modular ANNs (including four ANN modules for each fault type) have been developed to find the fault distance under various fault types occurring in both circuits. The proposed method incorporates current and voltage signals available at the near-end (local end) of the line. A fault location algorithm based on synchronous current phasors is also suggested for DCTLs [8], in which current in one or more branches (either faulty or healthy sections) are measured. The measurements are then used to present a fault location method based on the bus impedance matrix. In ref. [9], a fault location algorithm for DCTLs compensated by the GUPFC, which is based on synchronous current and voltage phasors, is presented to analyse eleven normal shunt fault types for single-circuit faults. The suggested design utilizes the analysis of positive-, negative- and zero-sequence circuits to locate single-circuit faults. The boundary conditions equation proposed in this paper was designed for eleven types of normal shunt faults, which uses current and voltage equations of the sequences in the faulty circuit and cannot be generalized to the analysis of inter-circuit faults.

• Analysis and review of the references in the second category [10–18]

A single-circuit and inter-circuit fault location algorithm in DCTLS is introduced in ref. [10] based on the single-terminal measurements of current and voltage phasors. In this work, common and differential components network is employed to calculate the fault point current. In the differential network considered, the current distribution factor is a function of fault distance; hence, the differential component current at the fault point is written in terms of local terminal current. In another study [11], an inter-circuit and cross-country faults location algorithm is presented for ANN-based DCTLs for various types of grounded and non-grounded faults. In ref. [12], the design is based on the digital calculation of the impedance of a faulty section of the series-compensated shunt transmission line using symmetric components of currents and voltages to find inter-circuit fault location. An accurate report of an inter-circuit phenomenon due to ice melting and drop from conductors of one circuit to conductors of other circuits during snowy days in the Midwest region, the US, was provided in ref. [13]. In this study, the misfunction of distance relays in detecting inter-circuit faults was proposed and solutions were provided for improving the performance of the distance relay algorithms during inter-circuit faults. In ref. [14], a Naïve Bayesian classification-based fault location and classification algorithm was presented for protecting shunt transmission lines against inter-circuit faults. The proposed design employed three-phase current phasors of both circuits to perform protective operations. The authors in ref. [15] investigate an inter-circuit fault location scheme for series capacitor-compensated transmission lines using the discrete wavelet transform (DWT) and the k-nearest neighbour algorithm. An inter-circuit fault location and classification algorithm based on the traveling wave theory is presented in ref. [16]. In this method, the energy operator has been employed to extract the necessary features from traveling waves caused by the fault. In ref. [17], DWT and decision tree regression were used to present a fault location algorithm for inter-circuit faults in transmission lines compensated by series capacitors. In ref. [18], a combination of empirical wavelet transform, Hilbert transform and weighted random vector functional link network was utilized to analyse inter-circuit faults in series compensated transmission lines.

Following the literature review, the inter-circuit fault location problem for GUPFC-compensated DCTLs has not remained unsolved so far. Due to the presence of two series converters in each of the two circuits and one shunt converter in one of the circuits, single-circuit and inter-circuit fault location schemes in transmission lines with different topologies cannot be implemented in this specific topology. Hence, the present study introduces the inter-circuit fault location scheme for the first time to analyse 98 types of inter-circuit faults in GUPFC-compensated DCTLs.

1.3 Novelty and contribution

The main contribution and innovation of this paper is the presentation of a fault location algorithm for inter-circuit faults in DCTLs compensated by GUPFC. Inter-circuit faults occur on DCTLs between the same or different phases of two circuits in the same place and at the same time, which can be grounded or ungrounded, generally 98 cases. The method presented in this paper is based on phasor analysis and employs one of the
synchronous current and voltage phasors of the terminals on both sides of the line along with Kirchhoff’s voltage law (KVL) and Kirchhoff’s current law (KCL) equations governing the faulty circuit to analyze all 98 inter-circuit fault states, including both grounded and ungrounded faults, taking into account the exact details of the calculations. The boundary conditions for the analysis of all 98 types of inter-circuit faults in GUPFC compensated DCTLs are presented in detail in this paper. Also, the algorithm presented in this paper is independent of the GUPFC modelling and can be implemented in any type of fault locator working based on the phasor theory.

1.4 Organization and structure of the paper

The paper is organized as follows. Section 2 introduces the GUPFC. The proposed method is presented in Section 3 in which inter-circuit faults are analysed. In Section 4, software simulations are provided to study the results obtained from the algorithm analysis. Section 5 of the paper presents sensitivity analysis of the algorithm. The results obtained from the comparison between the proposed method and the other double-circuit fault location methods are given in Section 6. Section 7 concludes the paper and the appendices are attached in Section 8.

2 THE GENERALIZED UNIFIED POWER FLOW CONTROLLER

GUPFC is one of the most advanced and complicated devices from the FACTS family. As shown in Figure 1, this device can control both the active and the reactive powers in a DCTL using three converters, two of which are in series connected in each of the circuits and the third one is a shunt converter connected to one of the DCTL circuits. All the three converters are connected to each other via a DC-link that balances the active power in the GUPFC. Moreover, the GUPFC can control the voltage of the bus to which the shunt converter is connected using the reactive power exchange between the shunt converter and the transmission line. In this compensator, the shunt converter is generally operating in the voltage control mode and the series converters will operate in the power flow control mode. The series and shunt converters generate the output voltage with controllable amplitude and angle so that the DC bus voltage will be controlled at a fixed value. Automatic power flow control is obtained for the series converter using a vector control design through the dq synchronous reference frame, where control quantities appear as DC signals. In comparison with the line’s measured currents, the d- and q-axis current components are determined for the reference values of active and reactive powers and utilized for extracting the amplitude and angle of the voltage of the series converter [19,20].

3 THE PROPOSED METHOD

3.1 Calculation of current and voltage of the sequences at the fault point

As is shown in Figure 2, the DCTL with GUPFC is divided into four sections. These sections include lines between points R1-G1, R2-G2, R3-G3 and R4-G5, respectively. The information of current and voltage phasors of the measurement points are extracted by PMUs installed at each substation and transmitted to the central protection system. An inter-circuit fault is assumed to occur due to a short-circuit between phases of Sections 1–3 and 2–4. According to different cases of short-circuits between phases of both sections, 49 types of ungrounded inter-circuit faults can occur due to short-circuits between phases A₁, B₁ and C₁ from one section of the first circuit and phases A₃, B₃ and C₃ of one section of the second circuit.
circuit. Furthermore, 49 fault types between phases of the two sections can be grounded faults; hence, based on the proposed topology in Figure 2, a total of 98 types of inter-circuit faults can occur due to short-circuit between phases of two sections on one side of the GUPFC. The following presents the calculation procedure of fault point current and voltage sequence of both circuits. The current and voltage equations calculated in this part of the paper and the solution of a set of boundary condition equations designed in this paper can help determine the exact location of a fault between phases of both sections on each side of the GUPFC.

This section of the paper calculates the zero-, negative- and positive-sequence current and voltage of the fault point on circuit II when a fault occurs at a point between R2 and G. To find the zero-sequence current, the KCL equation is written for the fault point f. Since the zero-sequence circuit has a zero-sequence mutual admittance, the zero-sequence current of the upper line circuit influences the KCL equation of the fault point. To analyze the zero-sequence circuit, Π equivalent circuit of the transmission line is used [21,22].

Thus, according to Figure 3, the KCL equation of the fault point is expressed as Equation (1):

\[
I_f^{(0)} = I_{sh}^{(0)} + I_{s1}^{(0)} + I_{s2}^{(0)} - I_{R}^{(0)} \left( \frac{v_{x10}^{(0)} + v_{x20}^{(0)}}{2} \right)
+ I_{R}^{(0)} \left( \frac{v_{x20}^{(0)}}{2} \right) + v_{x4}^{(0)} \left( \frac{v_{x20}^{(0)}}{2} \right) - v_{x10}^{(0)} \left( \frac{v_{x10}^{(0)} + v_{x20}^{(0)}}{2} \right).
\] (1)

Referring to Equation (1), equations of \( V_{x1}^{(0)} \), \( V_{x2}^{(0)} \), \( I_{R}^{(0)} \), \( I_{s1}^{(0)} \), \( I_{s2}^{(0)} \), \( I_{sh}^{(0)} \), \( V_{x10}^{(0)} \) and \( V_{x20}^{(0)} \) need to be extracted in terms of current and voltage of the line terminals. In the following, using the KVL and KCL analyses, the mentioned currents and voltages are described via Equations (2)–(7). All \( A_i \) coefficients are listed in

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**FIGURE 2** Schematic of a generalized unified power flow controller-compensated double-circuit transmission lines when an inter-circuit fault occurs

**FIGURE 3** Zero-sequence equivalent circuit of the fault point in the generalized unified power flow controller-compensated double-circuit transmission lines for inter-circuit faults
Finally, using Equations (2) and (8), the zero-sequence current and voltage of the fault point in circuit II are described. Based on the analyses performed under different fault occurrence conditions for different values of fault resistance at various locations on the GUPFC-compensated DCTL and regarding the inference given in refs. [4,9], the value of \( I_{sh}^{(0)} \) is negligible and its influence, considering the coefficient \( A_{28} \) in \( I_{sh}^{(0)} \), is zero. Also, the value of \( (1/(1 - A_{27}))(Z_{x9x10})V_{m2}^{(0)}(2) - (Z_{x9x10} + Y_{m2})/2(Z_{m2})I_{II}^{(0)} \) in Equation (8) is very small, which can be neglected.

Based on Figure 4, negative- and positive-sequence current and voltage of the fault point are calculated. To this end, we can use the current equations at the sending and receiving ends of the line \( x_9x_{10} \):

\[ - (I_{sh}^{(0)} + I_{s11}^{(0)}) = (I_{k2}^{(0)} - I_{k1}^{(0)}) \cos(\gamma I_{s9x_{10}}) - V_{R2}^{(0)}(1/Z_c) \sinh(\gamma I_{s9x_{10}}). \]

where \( I_{k2}^{(0)}, I_{s11}^{(0)} \) and \( V_{R2}^{(0)} \) need to be found in terms of currents and voltages of the line terminals:

\[ I_{s11}^{(0)} = I_{R1}^{(0)} \cos(\gamma l_{s9x_{11}}) - V_{R2}^{(0)}(1/Z_c) \sin(\gamma l_{s9x_{11}}). \]

\[ V_{f II}^{(0)} = V_{R2}^{(0)} \cos(\gamma l_{s9x_{11}}) - I_{R2}^{(0)} Z_c \sin(\gamma l_{s9x_{11}}). \]

By replacing Equations (10)–(12) in Equation (9), equation \( I_{f II}^{(0)} \) is obtained after simplifying the hyperbolic formulae:

\[ I_{f II}^{(0)} = A_{35} I_{sh}^{(0)} + A_{36} I_{R1}^{(0)} + A_{37} I_{R2}^{(0)} + A_{38} V_{R1}^{(0)} + A_{39} V_{R2}^{(0)}. \]

In the end, using Equations (12) and (13), the zero-, negative- and positive-sequence current and voltage of the fault point in circuit II are calculated. We can calculate the current and voltage of all the three sequences of a fault point in circuit I, for a fault occurring between points R4 and G, using a similar method. In fact, the proposed method in this section of the paper can be used for calculating all current and voltage equations of the fault point for all the three sequences in both circuits I and II.

### 3.2 Analysis of fault location equations using boundary conditions

Now that the current and voltage of all the three sequences for an inter-circuit fault point are found, we can accurately locate the fault point using boundary conditions of various fault types based on the faulty phases of both circuits. The boundary conditions equation for solving the problem of fault location can be written in general form as:

\[ a_i V_{f i}^{(0)} + a_2 V_{f i}^{(1)} + a_3 V_{f i}^{(2)} + a_4 V_{f i}^{(0)} + a_5 V_{f i}^{(1)} + a_6 V_{f i}^{(2)} = R_i \left( a_i I_{f i}^{(0)} + a_5 I_{f i}^{(1)} + a_6 I_{f i}^{(2)} + a_1 I_{f i}^{(1)} + a_2 I_{f i}^{(2)} \right). \]
As is seen in Equation (14), this equation is comprised of all currents and voltages of the sequences of both circuits at the fault point, which can have different zero or non-zero coefficients based on the fault type. In general, different unknowns may appear in the boundary conditions equation used for determining the fault location. These unknowns may include fault location \( L_0 \), phase fault resistance \( R_f \), and amplitude and angle of the shunt converter’s negative- and positive-sequence currents \( \alpha_{sh}^{(1)} \), \( \alpha_{sh}^{(2)} \) and \( \alpha_{sh}^{(2)} \), respectively. For each fault type, boundary conditions equations need to be obtained regarding the number of unknowns so that they are solved in a set of equations and the unknowns are calculated. Furthermore, it is observed that the boundary conditions equation given in Equation (14) is independent of the earth fault resistance \( R_e \). In fact, \( R_e \) affects the values of the voltages and currents at the line terminals which are the algorithm’s inputs.

As is seen, while the unknowns are real, the boundary conditions equations are complex equations that can be divided into real and imaginary parts. In other words, any boundary condition equation can be decoupled into two separate real equations that can be solved using known and unknown equation sets. The remaining presents the analysis of 98 different fault types. Due to the absence of the shunt converter in circuit I, the number of unknowns from circuit I that can enter boundary conditions equations is smaller; thus, the basis of analysis classification of fault types is the number of faulty phases of circuit I. This facilitates the analysis of equations and the calculation of the fault location.

### 3.2.1 Double-circuit fault analysis when one phase of circuit I is faulty

In this case, the total number of situations where one phase of circuit I together with phases of circuit II, either grounded or ungrounded, is faulty is 42. In general, three basic types of faults can occur in circuit I in this case: Phase A of circuit I is faulty together with phases of circuit II, phase B of circuit I is faulty together with phases of circuit II and phase C of circuit I is faulty together with phases of circuit II. Here, the first situation is analysed, which can be generalized for the other two situations. The analysis results of the two other situations are given at the end of this section in Table 1.

When phase A of circuit I is faulty together with phases of circuit II, an overall number of 14 fault types can occur, 7 of which are associated with grounded faults and the rest with ungrounded faults. Because only one phase of circuit I is faulty, the number of unknowns, in this case, is more than two-phase and three-phase cases. These will be analysed separately in the following sub-sections.

#### Analysis of \( A_1A_{II} \), \( A_1B_{II} \), \( A_1C_{II} \), \( A_1A_{II}G \), \( A_1B_{II}G \) and \( A_1C_{II}G \) faults

When an \( A_1A_{II} \) fault occurs, the KVL equation of this type of fault in the fault location is given as:

\[
V_{fI} - R_f I_{dI} = V_{afI} - R_f I_{afI}. \tag{15}
\]

For this fault, \( I_{dI} = -I_{afI} \) and \( I_{dII} = I_{afII} = 0 \). Hence, \( I_{dII} \) can be expressed in terms of sequence components as \( 3V_{fI}^{(1)} \). In this analysis, \( I_{dI}^{(1)} = I_{fII}^{(1)} = -I_{fII}^{(1)} \) can be used for simplifying the equations [11]. In the end, using the Fortescue transform, Equation (15) can be rewritten in the sequence components domain:

\[
V_{fI}^{(0)} + V_{fI}^{(1)} + V_{fI}^{(2)} - V_{fII}^{(0)} - V_{fII}^{(1)} - V_{fII}^{(2)} = 6R_f I_{fI}^{(1)}. \tag{16}
\]

Equation (16) can be decoupled into two real equations and has two unknowns, \( I_{dI} \) and \( R_f \), where the accurate fault location can be calculated by solving two equations with two unknowns. Based on the analysis presented in the section given above, boundary conditions equations of \( A_1B_{II} \) and \( A_1C_{II} \) faults can be stated as Equations (17) and (18):

\[
V_{fI}^{(0)} + V_{fI}^{(1)} + V_{fI}^{(2)} - V_{fII}^{(0)} - aV_{fII}^{(1)} - a^2 V_{fII}^{(2)} = 6R_f I_{fI}^{(1)}, \tag{17}
\]

\[
V_{fI}^{(0)} + V_{fI}^{(1)} + V_{fI}^{(2)} - V_{fII}^{(0)} - aV_{fII}^{(1)} - a^2 V_{fII}^{(2)} = 6R_f I_{fII}^{(1)}. \tag{18}
\]
TABLE 1  Current coefficients of the boundary conditions equation when only phase B (or C) of circuit I is faulty together with phases of circuit II

| Fault type     | \( a_8 \) | \( a_{10} \) | \( a_{11} \) | \( a_{12} \) |
|----------------|-----------|-----------|-----------|-----------|
| 1              | \( b_1 A_{III} \) | 0a² | 0 | 0 | 0 |
| 2              | \( b_1 A_{III} \) | 0a² | 0 | 0 | 0 |
| 3              | \( b_1 C_{III} \) | 0a² | 0 | 0 | 0 |
| 4              | \( b_1 A_{II} \) | 4a²−1 | 0 | a²−1 | 0 |
| 5              | \( b_1 A_{II} \) | 4a²−1 | 0 | a−1 | 0 |
| 6              | \( b_1 B_{II} C_{II} \) | 4a²−1 | 0 | a−2 | 0 |
| 7              | \( b_1 A_{II} \) | 4a²−1 | 0 | a²−2 | 0 |
| 8              | \( A_{II} \) | 0a | 0 | 0 | 0 |
| 9              | \( C_{II} \) | 0a | 0 | 0 | 0 |
| 10             | \( C_{II} \) | 0a | 0 | 0 | 0 |
| 11             | \( C_{II} A_{II} B_{II} \) | 4a²−1 | 0 | a²−1 | 0 |
| 12             | \( C_{II} A_{II} C_{II} \) | 4a³ | 0 | a−1 | 0 |
| 13             | \( C_{II} B_{II} C_{II} \) | 4a³ | 0 | a−2 | 0 |
| 14             | \( C_{II} A_{II} B_{II} C_{II} \) | 4a | 0 | −1 | −1|
| 15             | \( C_{II} A_{II} G \) | 3a² | −3 | 0 | 0 |
| 16             | \( C_{II} G \) | 3a² | −3 | 0 | 0 |
| 17             | \( C_{II} G \) | 3a² | 0 | 0 | 0 |
| 18             | \( C_{II} A_{II} B_{II} G \) | 3a² | a−1 | a²−1 | 0 |
| 19             | \( C_{II} A_{II} C_{II} G \) | 3a² | a²−1 | a−1 | 0 |
| 20             | \( C_{II} B_{II} C_{II} G \) | 3a² | a−1 | a²−1 | 0 |
| 21             | \( C_{II} A_{II} B_{II} C_{II} G \) | 3a² | a−1 | a²−1 | 0 |
| 22             | \( C_{II} A_{II} G \) | 3a² | −1 | −1 | −1|
| 23             | \( C_{II} A_{II} G \) | 3a² | −1 | −a² | −a |
| 24             | \( C_{II} C_{II} G \) | 3a² | −1 | −a−a² | 0 |
| 25             | \( C_{II} A_{II} B_{II} G \) | 3a² | a−1 | a²−1 | 0 |
| 26             | \( C_{II} A_{II} C_{II} G \) | 3a² | a−1 | a²−1 | 0 |
| 27             | \( C_{II} B_{II} C_{II} G \) | 3a² | a−1 | a²−1 | 0 |
| 28             | \( C_{II} A_{II} B_{II} C_{II} G \) | 3a² | −1 | −1 | −1|

In the analysis of earth fault, as the equation \( j_{III}^{(0)} = -j_{III}^{(0)} \) is not met, the right side of Equations (16)–(18) for \( A_{I} A_{II} G, A_{I} B_{II} G \) and \( A_{I} C_{III} G \) faults is changed to \( 3R_1(I_{III}^{(0)} - I_{III}^{(0)}) \).

Analysis of \( A_{I} A_{II} B_{II} A_{II} A_{II} C_{III} G, A_{I} A_{II} C_{II} B_{II} A_{II} B_{II} B_{II} G, A_{I} A_{II} B_{II} C_{III} G, A_{I} B_{II} C_{III} G \) faults

When an \( A_{I} A_{II} B_{II} \) fault occurs, the KVL equation for this type of fault at the fault location is written as:

\[
V_{all} - R_1 I_{all} = V_{ahl} - R_1 I_{ahl} = V_{bhl} - R_1 I_{bhl}.
\] (19)

In this case, the values of \( I_{ahl}, I_{ahl}, \) and \( I_{ahl} \) are zero and \( I_{ahl}, I_{ahl}, \) and \( I_{ahl} \) in terms of sequence components are equal to \( 3I_{ahl}^{(1)}, (1 - a)I_{ahl}^{(0)} + (1 - a^2)I_{ahl}^{(1)} \) and \( (1 - a^2)I_{ahl}^{(0)} + (a^2 - 1)I_{ahl}^{(1)} \). Also, equation \( I_{ahl}^{(1)} = I_{ahl}^{(0)} = -I_{ahl}^{(0)} \) can be used to simplify the equations. Finally, using the Fortescue transform, Equation (19) will be rewritten in the sequence domain as Equation (20):

\[
\left\{ \begin{array}{l}
V_{ahl}^{(0)} + V_{ahl}^{(1)} + V_{ahl}^{(2)} - V_{ahl}^{(0)} - V_{ahl}^{(1)} - V_{ahl}^{(2)} = R_1 \left[ (4 - a) I_{ahl}^{(1)} + (a^2 - 1) I_{ahl}^{(1)} \right] \\
V_{ahl}^{(0)} + V_{ahl}^{(1)} + V_{ahl}^{(2)} - V_{ahl}^{(0)} - a^2V_{ahl}^{(1)} - aV_{ahl}^{(2)} = R_1 \left[ (4 - a^2) I_{ahl}^{(1)} + (1 - a^2) I_{ahl}^{(1)} \right]
\end{array} \right.
\] (20)

The equation set given in Equation (20) can be decoupled into four real equations. With the presence of \( I_{ahl}^{(1)} \) in boundary conditions equations, the values of \( abs(I_{ahl}^{(1)}) \) and \( \alpha_{ahl}^{(1)} \) are also present as unknowns in the equations, in addition to \( I_{ahl} \) and \( R_1 \) as the other unknowns. Hence, the number of unknowns in this fault case is four, which can be solved using four real equations provided in Equation (20). Similar to the analysis presented above, boundary conditions equations for \( A_{I} A_{II} C_{III} G \) and \( A_{I} B_{II} C_{III} G \) faults are expressed as Equations (21) and (22).

\[
\left\{ \begin{array}{l}
V_{ahl}^{(0)} + V_{ahl}^{(1)} + V_{ahl}^{(2)} - V_{ahl}^{(0)} - V_{ahl}^{(1)} - V_{ahl}^{(2)} = R_1 \left[ (4 - a^2) I_{ahl}^{(1)} + (a - 1) I_{ahl}^{(1)} \right] \\
V_{ahl}^{(0)} + V_{ahl}^{(1)} + V_{ahl}^{(2)} - V_{ahl}^{(0)} - a^2V_{ahl}^{(1)} - aV_{ahl}^{(2)} = R_1 \left[ (4 - a) I_{ahl}^{(1)} + (a - 1) I_{ahl}^{(1)} \right]
\end{array} \right.
\] (21)

\[
\left\{ \begin{array}{l}
V_{ahl}^{(0)} + V_{ahl}^{(1)} + V_{ahl}^{(2)} - V_{ahl}^{(0)} - a^2V_{ahl}^{(1)} - aV_{ahl}^{(2)} = R_1 \left[ (4 - a) I_{ahl}^{(1)} + (a - 1) I_{ahl}^{(1)} \right] \\
V_{ahl}^{(0)} + V_{ahl}^{(1)} + V_{ahl}^{(2)} - V_{ahl}^{(0)} - aV_{ahl}^{(1)} - a^2V_{ahl}^{(2)} = R_1 \left[ (4 - a^2) I_{ahl}^{(1)} + (1 - a) I_{ahl}^{(1)} \right]
\end{array} \right.
\] (22)
In the analysis of earth fault, as equation $I_{I_f}^{(0)} = -I_{II_f}^{(0)}$ is not met, the right sides of Equations (20) and (21) for $A_I$, $A_{II}$, $C_{II}$ and $A_I B_I$ $C_{II}$ faults are changed to

$$
\begin{align*}
R_I \left( 3I_{I_f}^{(1)} + (a^2 - 1) I_{I_f}^{(1)} + (a - 1) I_{III_f}^{(0)} \right) \\
R_I \left( 3I_{II_f}^{(1)} + (1 - a^2) I_{II_f}^{(1)} + (a^2 - 1) I_{III_f}^{(0)} \right) \\
R_I \left( 3I_{III_f}^{(1)} + (a - 1) I_{III_f}^{(1)} + (a^2 - 1) I_{III_f}^{(0)} \right) \\
R_I \left( 3I_{III_f}^{(1)} + (1 - a) I_{III_f}^{(1)} + (a - 1) I_{III_f}^{(0)} \right),
\end{align*}
$$

(23)

and

$$
\begin{align*}
R_I \left( 3I_{I_f}^{(1)} + a (1 - a) I_{I_f}^{(1)} + (a - 1) I_{III_f}^{(0)} \right) \\
R_I \left( 3I_{II_f}^{(1)} + a (a - 1) I_{II_f}^{(1)} + (a^2 - 1) I_{III_f}^{(0)} \right),
\end{align*}
$$

(24)

**Analysis of $A_I$ $A_{II}$ $B_{II}$ $C_{II}$ and $A_I A_{II}$ $B_{II}$ $C_{II}$ faults**

When an $A_I$ $A_{II}$ $B_{II}$ $C_{II}$ fault occurs, the KVL equation for this fault type is given in Equation (23):

$$
V_{a0} - R_I I_{a0} = V_{aII} - R_I I_{aII} = V_{aIII} - R_I I_{aIII},
$$

(25)

In this case, the values of $I_{aII}$ and $I_{aIII}$ are zero and $I_{a0}$ in terms of the sequence components is $3I_{I_f}^{(1)}$. Moreover, equation $I_{I_f}^{(1)} = I_{II_f}^{(0)} = -I_{III_f}^{(0)}$ is used in the analysis to simplify the equations. In the end, using the Fortescue transform, Equation (23) can be rewritten in the sequence domain as Equation (24):

$$
\begin{align*}
V_{I_f}^{(0)} + V_{I_f}^{(1)} + V_{I_f}^{(2)} - V_{III_f}^{(1)} - V_{III_f}^{(2)} &= R_I \left( 4V_{I_f}^{(1)} - I_{III_f}^{(1)} - I_{III_f}^{(2)} \right) \\
V_{I_f}^{(0)} + V_{I_f}^{(1)} + V_{I_f}^{(2)} - V_{III_f}^{(1)} - a^2 V_{I_f}^{(1)} - aV_{I_f}^{(2)} &= R_I \left( 4I_{I_f}^{(1)} - aI_{III_f}^{(1)} - aI_{III_f}^{(2)} \right) \\
V_{I_f}^{(0)} + V_{I_f}^{(1)} + V_{I_f}^{(2)} - V_{III_f}^{(1)} - aV_{I_f}^{(1)} - a^2 V_{I_f}^{(2)} &= R_I \left( 4I_{I_f}^{(1)} - aI_{III_f}^{(1)} - a^2 I_{III_f}^{(2)} \right).
\end{align*}
$$

(26)

The equation set given in Equation (24) can be divided into six real equations. Due to the presence of $I_{III_f}^{(1)}$ and $I_{III_f}^{(2)}$ in boundary conditions equations, the values of $abs(I_{III_f}^{(1)})$, $\alpha_{III_f}^{(1)}$, $abs(I_{III_f}^{(2)})$ and $\alpha_{III_f}^{(2)}$ are unknown, in addition to $L_I$ and $R_I$ as the other unknowns. Hence, there are overall six unknowns for this fault type, which can be solved according to six real equations given in Equation (24). In the analysis of earth fault, as equation $I_{I_f}^{(0)} = -I_{II_f}^{(0)}$ is not met, the right side of Equation (24) is changed to

$$
\begin{align*}
R_I \left( 3I_{I_f}^{(1)} - I_{III_f}^{(1)} - I_{III_f}^{(2)} \right) \\
R_I \left( 3I_{II_f}^{(1)} - aI_{III_f}^{(1)} - aI_{III_f}^{(2)} \right) \\
R_I \left( 3I_{III_f}^{(1)} - aI_{III_f}^{(1)} - a^2 I_{III_f}^{(2)} \right).
\end{align*}
$$

(27)

Table 1 lists current coefficients of the boundary conditions equation for cases when only phase B or phase C of circuit I are faulty together with one or several phases of circuit II.

The method of calculating the coefficients given in Table 1 is the same as the case when only phase A of circuit I is faulty together with one or several phases of circuit II. Furthermore, similar to the previous case, coefficients $a_2$ and $a_3$ are zero in these two cases. On the other hand, voltage coefficients of boundary conditions equations in this case change with regard to the type of faulty phase. When only phase B of circuit I is faulty together with one or several phases of circuit II, coefficients $a_2$ and $a_3$ will be equal to $a$ and $a^2$, respectively. Also, for the case in which phase C is faulty, coefficients $a_2$ and $a_3$ will be equal to $a$ and $a^2$, respectively. Other voltage coefficients of the boundary conditions equation are similar to the case when phase A is faulty.

### 3.2.2 Double-circuit fault analysis when two phases of circuit I are faulty

In this case, it is assumed that two phases of circuit I are simultaneously in contact with phases of circuit II, either grounded or ungrounded. The number of fault types corresponding to this case is 42.

#### Phases A and B of circuit I are faulty

The number of fault types corresponding to this case is 14, which includes $A_I B_I A_{II}$, $A_I B_I B_{II}$, $A_I B_I C_{II}$, $A_I B_I A_{II}$ $B_{II}$, $A_I B_I A_{II}$ $C_{II}$, $A_I B_I B_{II}$ $C_{II}$, $A_I B_I A_{II}$ $B_{II}$ $C_{II}$, $A_I B_I A_{II}$ $B_{II}$ $C_{II}$ $G$ and $A_I B_I A_{II}$ $B_{II}$ $C_{II}$ $G$ faults. Equation (25) can be used as the boundary conditions equation to determine the fault location in this case:

$$
\alpha^2 V_{I_f}^{(1)} - V_{I_f}^{(2)} = R_I \left( \alpha^2 I_{I_f}^{(1)} - I_{I_f}^{(2)} \right).
$$

(28)

#### Phases A and C of circuit I are faulty

The number of possible fault types for this condition is 14, which includes $A_I C_I A_{II}$, $A_I C_I B_{II}$, $A_I C_I C_{II}$, $A_I C_I A_{II}$ $B_{II}$, $A_I C_I A_{II}$ $C_{II}$, $A_I C_I B_{II}$ $C_{II}$, $A_I C_I A_{II}$ $B_{II}$ $C_{II}$, $A_I C_I A_{II}$ $B_{II}$ $C_{II}$ $G$, $A_I C_I A_{II}$ $B_{II}$ $C_{II}$ $G$ and $A_I C_I A_{II}$ $B_{II}$ $C_{II}$ $G$ faults.
Equation (26) can be used as the boundary conditions equation to determine the fault location in this case:

\[ aV_{R_1}^{(1)} - V_{R_2}^{(2)} = R_I \left( aI_{R_1}^{(1)} - I_{R_2}^{(2)} \right). \]  
(29)

Phases B and C of circuit I are faulty

The number of possible fault types for this condition is 14, which includes \( B_1C_1A_{II}, B_1C_1B_{II}, B_1C_1C_{II}, B_1C_2A_{II}, B_1C_2B_{II}, B_1C_2C_{II}, B_2C_1A_{II}, B_2C_1B_{II}, B_2C_1C_{II}, B_2C_2A_{II}, B_2C_2B_{II}, B_2C_2C_{II}, B_2C_3A_{II}, B_2C_3B_{II}, B_2C_3C_{II}, B_1C_2A_{II}B_{II}, B_1C_2B_{II}C_{II}, B_1C_2C_{II}A_{II}, B_1C_2C_{II}B_{II}, B_1C_2C_{II}C_{II}, B_2C_1A_{II}B_{II}, B_2C_1B_{II}C_{II}, B_2C_1C_{II}A_{II}, B_2C_1C_{II}B_{II}, B_2C_1C_{II}C_{II}, B_2C_2A_{II}B_{II}, B_2C_2B_{II}C_{II}, B_2C_2C_{II}A_{II}, B_2C_2C_{II}B_{II}, B_2C_2C_{II}C_{II}, B_2C_3A_{II}B_{II}, B_2C_3B_{II}C_{II}, B_2C_3C_{II}A_{II}, \) and \( B_2C_3C_{II}B_{II}, B_2C_3C_{II}C_{II} \) faults. Equation (27) can be used as the boundary conditions equation to determine the fault location in this case:

\[ V_{R_1}^{(1)} - V_{R_2}^{(2)} = R_I \left( f_{R_1}^{(1)} - f_{R_2}^{(2)} \right). \]  
(30)

3.2.3 Double-circuit fault analysis when the three phases of circuit I are faulty

In this case, it is assumed that all the three phases of circuit I are simultaneously in contact with one or several phases of circuit II, either grounded or ungrounded. The number of cases this may occur is 14, which includes \( A_1B_1C_1A_{II}, A_1B_1C_1B_{II}, A_1B_1C_1C_{II}, A_1B_2C_1A_{II}, A_1B_2C_1B_{II}, A_1B_2C_1C_{II}, A_1B_2C_2A_{II}, A_1B_2C_2B_{II}, A_1B_2C_2C_{II}, A_1B_2C_3A_{II}, A_1B_2C_3B_{II}, A_1B_2C_3C_{II}, A_1B_2C_3C_{II}, A_1B_2C_3C_{II}, \) and \( A_1B_2C_3C_{II} \) faults. Equation (28) is used as the boundary conditions equation in this case to determine the fault location:

\[ V_{R_1}^{(1)} - V_{R_2}^{(2)} = R_I \left( f_{R_1}^{(1)} - f_{R_2}^{(2)} \right). \]  
(31)

In the two previous cases (cases 2 and 3), as is clear, due to the absence of negative- and positive-sequence current in circuit II and the subsequent absence of negative- and positive-sequence current of the shunt converter, the unknowns associated with the shunt converter current do not appear in boundary conditions equations. Thus, in these two cases, the only unknowns in boundary conditions equations are the fault location distance and the phase-to-phase fault resistance \( (R_I) \). Finally, using the equations obtained in this section of the paper, we can easily and fully analyze the whole 98 types of inter-circuit faults and calculate the accurate fault location.

3.3 The proposed algorithm for detecting faulty section

One of the major steps in fault location in transmission lines equipped with compensator devices is to detect the faulty area with respect to the compensator. In this step, the faulty area can be detected using one voltage index. When the fault occurs at the right side of the GUPFC device, the changes in the positive-sequence voltage at the right side of the GUPFC during the fault with respect to the pre-fault interval is far greater than that in the left side of the GUPFC. During the fault interval, the positive-sequence voltage of the fault point compared to the pre-fault interval is reduced regarding the value of fault resistance. On the other hand, when the fault occurs at the right side of the GUPFC, as bus R1 is closer than bus R2 to the fault point, the voltage drop on the right-side path is greater than that on the left side (see Figure 2). As a result, the voltage changes during the fault interval are far greater than those during the pre-fault interval at the right side of the GUPFC compared to its left side. If the positive-sequence voltage during the fault is calculated in per unit on the base of the positive-sequence voltage during the pre-fault interval for buses R1 and R2 (as shown in Equation (29)), then fault section detection index signing can be used to determine the faulty section. Here, \( V_{R_1}^{(1)} \) and \( V_{R_2}^{(1)} \) are related to the first cycle after the fault occurrence and \( V_{R_1}^{(1)} \) and \( V_{R_2}^{(1)} \) are related to the last cycle before the fault occurrence. The piecewise function for signing the FDSI is given in Equation (30). Using Equation (30), we can detect the side of the GUPFC on which the fault has occurred.

\[
\text{FDSI} = \left| \frac{V_{R_1}^{(1)} - V_{R_2}^{(1)}}{V_{R_1-p}^{(1)} - V_{R_2-p}^{(1)}} \right|
\]

\[
\text{sign} \ [\text{FDSI}] = \begin{cases} 
1 & \text{The fault has occurred between points R1 and G} \\
0 & \text{The fault has occurred between points R2 and G} 
\end{cases}
\]

Figure 5 illustrates the flowchart of the algorithm. In this scheme, the information concerning fault detection and faulty phase detection unit along with the type of short-circuit in terms of the grounded or non-grounded case is determined in steps before locating fault and it has been assumed as a default in the scheme. Moreover, the information of line parameters and that of current and voltage phasors measured at points Ri are given as inputs to the algorithm in offline and online modes, respectively. Newton–Raphson iteration method has been employed to solve boundary condition equations as a set of equations with unknowns to find fault location.

3.4 Software simulation and test results

The network under study consists of one 230 kV, 60 Hz (rated frequency), 400 km double-circuit line, as shown in Figure 1. The GUPFC considered for compensating the line is located at the middle point of the line. All data of synchronized currents and voltages of the line terminals during pre-fault and fault intervals are calculated using the PMUs, synchronized by the GPS and transmitted to the central protection system. The line under study in the test network is a double-circuit line with
Receive online information including all parameters of the protected double-circuit line and online information of current and voltage phasors of measurement points for one pre-fault cycle and one cycle before fault current interruption by breakers at both sides of the circuit.

Calculate current and voltage of measurement points in the sequence domain and depict positive-, negative-, and zero-sequence circuits under fault conditions.

Determine the faulty section using Eq. (30).

Calculate current and voltage of positive-, negative-, and zero-sequences at the fault point for both circuits using Eqs. (2), (8), (12), (13), and (…).

Form a set of boundary condition equations based on faulty phases and the type of short-circuit in terms of grounded or non-grounded phase situations using the overall boundary conditions equation given in Eq. (14).

Solve the set of boundary condition equations formed in the previous step and find the fault location.

Is the fault location in the permissible range of the total length of the line?

No

Yes

End

4 | SENSITIVITY ANALYSIS

4.1 | Sensitivity analysis on the change in the operation mode of the generalized unified power flow controller

The operation modes of the GUPFC considered in this paper are selected from ref. [19]. According to the data reported for the reference active and reactive signals of series converters 1 and 2, the value of the reference active power signal for series converter 1 changes at \( t = 0.5 \) s and \( t = 1 \) s, during the simulation run, from 1 to 1.1 and from 1.2 to 1.2, respectively. For series converter 2, it changes at times \( t = 0.5 \) and \( t = 1 \) s from 1 to 0.9 and from 0.9 to 1.2, respectively. Having said that, two scenarios are tested in this section to analyze the sensitivity of the algorithm to the change in the operation mode of the GUPFC.

**Scenario 1**: The reference active power signal of series converter 1 changes at times \( t = 0.75 \) and \( t = 1.5 \) s from 1 to 1.2 and from 1.2 to 1.1, respectively. In this case, an ungrounded inter-circuit fault has occurred at a distance of 100 km from terminal R1 with inter-phase fault resistance of 50 \( \Omega \) in the form of an AIBIBII short circuit. The estimated value for the fault location, in this case, is 100.21 km, where the estimation error is 0.052%.

**Scenario 2**: The reference active power signal of series converter 2 changes at times \( t = 0.8 \) and \( t = 1.25 \) s from 1 to 1.1 and...
TABLE 2 (Continued) Algorithm test results of ungrounded inter-circuit faults in a GUPFC compensated DCTL.

| Selected values | The test results of the algorithm |
|-----------------|----------------------------------|
| Type fault | $L_{RL}(km)$ | $R_1$(ohm) | Sgn | $L_{RL}(km)$ | Error% |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | $A_1C_1$ | 10 | 1 | – | 10.14 | 0.035 |
| 2 | $A_1B_1$ | 20 | 10 | – | 20.19 | 0.047 |
| 3 | $A_1C_1G$ | 30 | 50 | – | 30.22 | 0.055 |
| 4 | $A_1A_2B_1$ | 40 | 75 | – | 40.16 | 0.041 |
| 5 | $A_1A_2C_1$ | 50 | 100 | – | 50.22 | 0.055 |
| 6 | $A_1B_2C_1$ | 60 | 125 | – | 60.16 | 0.041 |
| 7 | $A_1A_2B_2C_1$ | 70 | 150 | – | 70.23 | 0.057 |
| 8 | $B_2A_1$ | 80 | 175 | – | 80.19 | 0.047 |
| 9 | $B_2B_1$ | 90 | 200 | – | 90.32 | 0.081 |
| 10 | $B_2C_1$ | 100 | 225 | – | 100.25 | 0.062 |
| 11 | $B_2A_2B_1$ | 110 | 250 | – | 110.18 | 0.062 |
| 12 | $B_2A_2C_1$ | 120 | 1 | – | 120.25 | 0.062 |
| 13 | $B_2B_2C_1$ | 130 | 10 | – | 130.15 | 0.037 |
| 14 | $B_2A_2B_2C_1$ | 140 | 50 | – | 140.19 | 0.047 |
| 15 | $C_1A_1$ | 150 | 75 | – | 150.32 | 0.081 |
| 16 | $C_1B_1$ | 160 | 100 | – | 160.38 | 0.095 |
| 17 | $C_1C_1$ | 170 | 125 | – | 170.31 | 0.077 |
| 18 | $A_1B_1A_2B_1G$ | 180 | 150 | – | 180.32 | 0.080 |
| 19 | $A_1A_2C_1$ | 190 | 175 | – | 190.26 | 0.065 |
| 20 | $B_2B_1C_1$ | 210 | 200 | + | 209.86 | 0.035 |
| 21 | $C_2A_2B_1G$ | 220 | 225 | + | 219.72 | 0.070 |
| 22 | $A_1B_1A_2B_1G$ | 230 | 250 | + | 229.63 | 0.097 |
| 23 | $A_1B_1C_1G$ | 240 | 1 | + | 239.71 | 0.072 |
| 24 | $A_1B_1C_2G$ | 250 | 10 | + | 249.75 | 0.062 |
| 25 | $A_1B_1A_2B_1G$ | 260 | 50 | + | 259.65 | 0.087 |
| 26 | $A_1B_1A_2C_1G$ | 270 | 75 | + | 269.72 | 0.070 |
| 27 | $A_1B_1B_2C_1G$ | 280 | 100 | + | 279.82 | 0.045 |
| 28 | $A_1B_1A_2B_2C_1G$ | 290 | 125 | + | 289.86 | 0.035 |
| 29 | $A_1C_1G$ | 300 | 150 | + | 299.61 | 0.097 |
| 30 | $A_1C_2G$ | 310 | 175 | + | 309.88 | 0.030 |
| 31 | $A_1C_2G$ | 320 | 200 | + | 319.85 | 0.037 |
| 32 | $A_1A_2C_2G$ | 330 | 225 | + | 329.69 | 0.077 |
| 33 | $A_1A_2A_2C_2G$ | 340 | 250 | + | 339.82 | 0.045 |
| 34 | $A_1C_2B_2G$ | 350 | 1 | + | 349.88 | 0.030 |
| 35 | $A_1C_2A_2B_2C_2G$ | 360 | 10 | + | 349.69 | 0.077 |
| 36 | $B_2C_1G$ | 370 | 50 | + | 369.66 | 0.085 |
| 37 | $B_2C_2G$ | 380 | 75 | + | 379.88 | 0.030 |
| 38 | $B_2C_2G$ | 390 | 100 | + | 389.90 | 0.025 |
| 39 | $B_2A_2B_1G$ | 10 | 125 | – | 10.11 | 0.027 |
| 40 | $B_2A_2B_2G$ | 25 | 150 | – | 25.33 | 0.082 |
| 41 | $B_2C_1B_2G$ | 50 | 175 | – | 50.36 | 0.090 |
| 42 | $B_2C_1A_2B_2G$ | 75 | 200 | – | 75.15 | 0.037 |
| 43 | $A_1B_1C_2A_2G$ | 100 | 225 | – | 100.16 | 0.041 |

The results presented in this case, according to the estimation error percentage given for each scenario, verifies the correct performance of the algorithm for the change in the operation mode of the GUPFC.

4.2 Sensitivity analysis on the increase or decrease in the inter-circuit coupling impedance

The topology of double-circuit lines includes zero-sequence components of mutual impedance and admittance between the phases of the two circuits since high-voltage (HV) tower structures are installed on these lines. In this section, the effect of changes in the mutual zero-sequence impedance and admittance between the phases of the two circuits are studied in two scenarios on the fault location algorithm presented in this paper.

**Scenario 1:** In this scenario, the mutual zero-sequence impedance and admittance between the two circuits has increased 20%. In this case, a grounded inter-circuit fault is assumed to have occurred at a distance of 130 km from terminal R1 with the inter-phase fault resistance of 50 $\Omega$ and the ground fault resistance of 10 $\Omega$ in the form of an $A_1C_1B_1B_2G$ short circuit. The estimated value for the fault location in this case is 130.59 km, where the estimation error is 0.147%.

**Scenario 2:** In this scenario, the mutual zero-sequence impedance and admittance between the two circuits has decreased 20%. In this case, an ungrounded inter-circuit fault is assumed to have occurred at a distance of 200 km from terminal R1 with the inter-phase fault resistance of 100 $\Omega$ in the form of a $C_1B_2G$ short circuit. The estimated value for the fault location in this case is 199.29 km, where the estimation error is 0.177%.

The results provided in this section, according to the estimation error percentage given for each scenario, prove the correct performance of the algorithm with respect to the increase or decrease in the coupling impedance.
### TABLE 3  Algorithm test results of grounded inter-circuit faults in a GUPFC compensated DCTL

| Selected values | The test results of the algorithm |
|-----------------|-----------------------------------|
| **Type fault**  | **$L_{RI-f}$ (km)** | **$R_f$ (ohm)** | **$R_g$ (ohm)** | **Sgn** | **$L_{RI-f}$ (km)** | **Error%** |
| 1               | $A_I A_I I G$           | 390         | 1           | 200      | +               | 389.61     | 0.097   |
| 2               | $A_I B_I G$            | 380         | 10          | 150      | +               | 379.63     | 0.092   |
| 3               | $A_I C_I G$            | 370         | 50          | 100      | +               | 369.66     | 0.085   |
| 4               | $A_I A_I B_I G$        | 360         | 75          | 50       | +               | 359.81     | 0.047   |
| 5               | $A_I A_I C_I G$        | 350         | 100         | 10       | +               | 349.62     | 0.095   |
| 6               | $A_I B_I C_I G$        | 340         | 125         | 1        | +               | 339.64     | 0.090   |
| 7               | $A_I A_I B_I C_I G$    | 330         | 150         | 200      | +               | 329.77     | 0.057   |
| 8               | $B_I A_I G$            | 320         | 175         | 150      | +               | 319.62     | 0.095   |
| 9               | $B_I B_I G$            | 310         | 200         | 100      | +               | 309.74     | 0.065   |
| 10              | $B_I C_I G$            | 300         | 225         | 50       | +               | 299.63     | 0.092   |
| 11              | $B_I A_I B_I G$        | 290         | 250         | 10       | +               | 289.68     | 0.080   |
| 12              | $B_I A_I C_I G$        | 280         | 1           | 1        | +               | 279.65     | 0.087   |
| 13              | $B_I B_I C_I G$        | 270         | 10          | 200      | +               | 269.66     | 0.085   |
| 14              | $B_I A_I B_I C_I G$    | 260         | 50          | 150      | +               | 259.79     | 0.052   |
| 15              | $C_I A_I G$            | 250         | 75          | 100      | +               | 249.62     | 0.095   |
| 16              | $C_I B_I G$            | 240         | 100         | 50       | +               | 239.72     | 0.071   |
| 17              | $C_I C_I G$            | 230         | 125         | 10       | +               | 229.70     | 0.075   |
| 18              | $C_I A_I B_I G$        | 220         | 150         | 1        | +               | 219.77     | 0.057   |
| 19              | $C_I A_I C_I G$        | 210         | 175         | 200      | +               | 209.63     | 0.092   |
| 20              | $C_I B_I C_I G$        | 190         | 200         | 150      | −               | 190.32     | 0.081   |
| 21              | $C_I A_I B_I C_I G$    | 180         | 225         | 100      | −               | 180.29     | 0.072   |
| 22              | $A_I B_I A_I G$        | 170         | 250         | 50       | −               | 170.32     | 0.080   |
| 23              | $A_I B_I B_I G$        | 160         | 1           | 10       | −               | 160.25     | 0.062   |
| 24              | $A_I B_I C_I G$        | 150         | 10          | 1        | −               | 150.23     | 0.057   |
| 25              | $A_I B_I A_I B_I G$    | 140         | 50          | 200      | −               | 140.33     | 0.082   |
| 26              | $A_I B_I A_I C_I G$    | 130         | 75          | 150      | −               | 130.35     | 0.087   |
| 27              | $A_I B_I C_I A_I G$    | 120         | 100         | 100      | −               | 120.25     | 0.062   |
| 28              | $A_I B_I A_I B_I C_I G$| 110         | 125         | 50       | −               | 110.21     | 0.052   |
| 29              | $A_I C_I A_I G$        | 100         | 150         | 10       | −               | 100.25     | 0.062   |
| 30              | $A_I C_I B_I G$        | 90          | 175         | 1        | −               | 90.38      | 0.095   |
| 31              | $A_I C_I C_I G$        | 80          | 200         | 200      | −               | 80.26      | 0.065   |
| 32              | $A_I C_I A_I B_I G$    | 70          | 225         | 150      | −               | 70.26      | 0.065   |
| 33              | $A_I C_I A_I C_I G$    | 60          | 250         | 100      | −               | 60.39      | 0.075   |
| 34              | $A_I C_I B_I C_I G$    | 50          | 1           | 50       | −               | 50.32      | 0.080   |
| 35              | $A_I C_I A_I B_I C_I G$| 40          | 10          | 10       | −               | 40.32      | 0.080   |
| 36              | $B_I C_I A_I G$        | 30          | 50          | 1        | −               | 30.16      | 0.040   |
| 37              | $B_I C_I B_I G$        | 20          | 75          | 200      | −               | 20.39      | 0.097   |
| 38              | $B_I C_I C_I G$        | 10          | 100         | 150      | −               | 10.22      | 0.055   |
| 39              | $B_I C_I A_I B_I G$    | 390         | 125         | 100      | +               | 389.62     | 0.095   |
| 40              | $B_I C_I A_I C_I G$    | 350         | 150         | 50       | +               | 349.69     | 0.077   |
| 41              | $B_I C_I B_I C_I G$    | 300         | 175         | 10       | +               | 299.62     | 0.095   |
| 42              | $B_I C_I A_I B_I C_I G$| 250         | 200         | 1        | +               | 249.64     | 0.090   |
| 43              | $A_I B_I C_I A_I G$    | 150         | 225         | 200      | −               | 150.16     | 0.040   |

(Continues)
4.3 Sensitivity analysis on high impedance fault

The value of fault resistance impacts the phasor theory-based fault location under study. Generally, algorithms that use the data of one terminal or algorithms that employ the simplified model of transmission lines cannot operate properly when there is a high impedance fault (HIF) and estimate the fault location incorrectly. As this paper uses the synchronous data of two terminals and utilizes the accurate model of the transmission line and since the analysis and calculations of KVL and KCL equations have been conducted with no approximation to find voltage and current of the fault points, the proposed algorithm is less sensitive to HIFs. Two scenarios are provided as follows to verify the validity of our claim.

Scenario 1: In this case, an ungrounded inter-circuit fault has occurred at a distance of 100 km from terminal R1 with inter-phase fault resistance of 500 Ω in the form of an $A_fB_fC_fB_fG$ short circuit. The estimated value for the fault location, in this case, is 101.11 km, where the estimation error is 0.277%.

Scenario 2: In this case, a grounded inter-circuit fault has occurred at a distance of 380 km from terminal R1 with inter-phase fault resistance of 100 Ω in the form of an $A_fB_fC_fB_fG$ short circuit. The estimated value for the fault location, in this case, is 378.61 km, where the estimation error is 0.347%.

The results provided in this section, based on the estimation error percentage given for each scenario, prove the correct performance of the algorithm with respect to HIFs.

4.4 Sensitivity analysis on a fault close to the near and end terminals of the line and close to the bus connected to the generalized unified power flow controller

Fault location in algorithms that work on the basis of the current and voltage signals analysis theory is not properly implemented for faults occurring at the near and end terminals of transmission lines (close to the places CT and PT are installed). However, in methods based on the phasor theory, if the transmission line modelling is carried out considering the shunt capacitance of the whole line with other parameters of the line being neglected, better results will be obtained for fault location compared to other methods. Two scenarios are presented as follows for sensitivity analysis given in this section.

Scenario 1: In this case, an ungrounded inter-circuit fault has occurred at a distance of 2 km from terminal R1 with inter-phase fault resistance of 10 Ω in the form of an $A_fB_fC_fB_fG$ short circuit. The estimated value for the fault location, in this case, is 1.89 km, where the estimation error is 0.027%.

Scenario 2: In this case, a grounded inter-circuit fault has occurred at a distance of 201 km from terminal R1 with inter-phase fault resistance of 50 Ω and a ground fault resistance of 10 Ω in the form of a $B_fC_fG$ short circuit. The estimated value for the fault location, in this case, is 200.36 km, where the estimation error is 0.16%.

The results provided in this section, according to the estimation error percentage given for each scenario, prove the correct performance of the algorithm with respect to near faults occurring at the measurement terminals and the GUPFC bus.

4.5 Sensitivity analysis on power swing

In this case, the effect of power swing on the performance of the proposed scheme is investigated. Power swings in electric systems may happen due to system faults, line switching, generator outage and disconnection of large loads. In these situations, the electrical power experiences abrupt variations while the mechanical power input to the generators remains constant. The energy injected by rotors increases the rotational speed of the rotor, leading to power swing. In the following, a power swing scenario is presented to examine the sensitivity of the algorithm.

Scenario: It is assumed in this case that the voltage angle of terminal R1 (the generator connected to terminal R1) experiences 30° deviation with 1 Hz frequency variations at $t = 0.8$ s and stays for 1 cycle in the power swing period. Then at $t = 0.73$ s, a grounded inter-circuit fault in the form of an $A_fC_fG$ fault with an inter-phase resistance of 10 Ω and a ground resistance of 150 Ω occurs at a distance of 15 km from terminal R1. The estimated value for fault location in this case is 150.95 km, where the estimation error is 0.147. The results provided in this section, according to the estimation error percentage given for the scenario, prove the correct performance of the algorithm with respect to power swing phenomenon.
### Table 4
Comparison of different references in the field of inter-circuit fault location in double-circuit transmission lines

| Author’s name       | Ref. | Publication year | Index1 | Index2 | Index3 | Index4 | Index5 |
|---------------------|------|------------------|--------|--------|--------|--------|--------|
| Song Guobing        | [10] | 2009             | Ph     | —      | 1T     | NA     | 0.5    |
| A. Jain             | [11] | 2010             | ANN    | —      | 2T, Syn| 2 k Hz | 1.1    |
| V. H. Makwana       | [12] | 2011             | Ph     | SC     | 1T     | NA     | 3.2    |
| R. McDaniel         | [13] | 2015             | Ph     | —      | 1T     | NA     | 3.5    |
| A. Swetapadma       | [14] | 2016             | Ph     | SC     | 1T     | 1 kHz  | 1      |
| A. Swetapadma       | [15] | 2017             | ANN    | SC     | 1T     | NA     | 0.5    |
| S. Hasheminejad     | [16] | 2017             | TW     | —      | 1T     | 1 M Hz | 0.1    |
| A. Swetapadma       | [17] | 2018             | TDS    | SC     | 1T     | NA     | 0.5    |
| M. Sahani           | [18] | 2020             | ANN    | SC     | 1T     | 1.6 kHz| 0.16   |
| Proposed method     | —    | —                | Ph     | GUPFC  | 2T, Syn| 2.4 k Hz| 0.06   |

### Table 5
Parameters of the transmission line and equivalent thevenin circuit of power sources of the line buses at both ends of the line

| Parameter | Line  | Source1 | Source2 |
|-----------|-------|---------|---------|
| V         | 230 kV| 230 V 20.8° kV| 230 V 9.2° kV |
| X/R       | 10    | 10      |         |
| r1        | 0.02546 Ohm km⁻¹ | 0.8716 Ohm | 1.7431 Ohm |
| r0        | 0.3864 Ohm km⁻¹ | 1.3074 Ohm | 2.6147 Ohm |
| r0m       | 0.1732 Ohm km⁻¹ |         |         |
| l1        | 0.9337 e⁻³ H km⁻¹ | 0.02576 H | 0.05154 H |
| l0        | 4.1264 e⁻³ H km⁻¹ | 0.00648 H | 0.0129 H |
| l0m       | 2.0126 e⁻³ H km⁻¹ |         |         |
| c1        | 12.74 e⁻⁹ F km⁻¹ |         |         |
| c0        | 7.751 e⁻⁹ F km⁻¹ |         |         |
| c0m       | 0.575 e⁻⁹ F km⁻¹ |         |         |

### 4.6 Sensitivity analysis of noise effect on the data measured by PMU

The presence of noise in power systems and measurement devices is unavoidable. To analyze the noise effect, this paper investigates the efficiency of the proposed method for different signal-to-noise ratio (SNR) values. This quantity represents the ratio between the signal power and the noise power that pollutes the signal. The dimension of this quantity is dB. White noise is used in this paper to simulate noise.

**Scenario:** This section of the paper examines the sensitivity of the proposed algorithm to noise effect. To this end, the algorithm was applied to current and voltage samples received by PMUs from CT and PT of the measurement points. To analyze this scenario, white Gaussian noise with different SNRs was added to the current and voltage samples. The noise level was set at 30 dB. It is assumed, in this case, that an $A_jC_lB_{ij}$ fault occurs at a distance of 310 km from terminal T1 with an interphase fault resistance of 175 Ω. The estimated value of fault location is 309.11 km, so the error value of estimation is 0.24%, while it was 0.03% for the case without noise presence.

### 4.7 Sensitivity analysis of CT saturation

Fault occurrence in power systems, as per their transient and steady states, lead to changes in the measured signals by CTs. These signals are abruptly changed with steady-state level and contain transient components. This section of the paper deals with the analysis of the effect of CT saturation on the performance of the suggested algorithm.

**Scenario:** A sample CT used in ref. [23] has been employed here to model CT saturation. All technical characteristics of the CT along with its magnetizing curve were extracted from ref. [23]. The CT is installed at point R1 so that the magnetizing curve of the CT enters the saturation region as soon as a fault occurs. The damping constant of the DC component in this CT was set at 50 ms. This value has been considered as the maximum critical value for the algorithm so that the proposed algorithm performs desirably for damping constants below 50 ms; otherwise, the algorithm fails to accurately detect the faulty section and location.

To investigate this scenario more accurately, an $A_jB_lC_lB_{ij}G$ fault that occurs at a distance of 20 km from terminal T1 with interphase resistance of 50 Ω, ground fault resistance of 1 Ω,
and damping constant of 48 ms for DC component was used. The estimated value for fault location, in this case, is 21.22 km, so the estimation error is 0.305%.

4.8 | Sensitivity analysis of synchronization

Since synchronized current and voltage phasors measured on both terminals of the line are used in the proposed algorithm, the algorithm expects to receive synchronized and accurate phasor information. This section of the paper addresses the sensitivity of the algorithm to synchronization errors in amplitude and phase angle of the current and voltage.

**Scenario:** To analyze this scenario, it is assumed that five degrees of deviation are applied to current and voltage phase angles in all three phases of terminal T2. Furthermore, there is a 3% error in current amplitude and 2% error in voltage amplitude in all three phases of terminal T2 permanently. In this case, if an inter-circuit fault occurs in the form of \( C_i B_j \) at a distance of 100 km from terminal T1 with fault resistance of 10 \( \Omega \), the estimated fault location is 101.38 km, in which the estimation error is 0.34%. The critical value of angle and amplitude deviations for current and voltage of all three phases in this algorithm are 12° and 4%, respectively. If deviation values exceed these critical values, the algorithm will fail to accurately detect the fault section and location.

4.9 | Sensitivity analysis of data transmission delay

Once received by PMUs installed on measurement points \( R_i \), current and voltage samples are used as input data to the algorithm to calculate phasor using Fourier series, and finally, their magnitude and phasor are obtained. The calculated phasors are time tagged by the GPS and delivered to the algorithm using the synchronization system with an accuracy of microseconds. All required data are sampled from measurement points and then, after synchronization, are given to the algorithm.

**Scenario:** This section investigates the sensitivity of the algorithm to data transmission delay. To calculate the critical value of maximum time delay in data transmission to which the proposed algorithm performs accurately, steps of 0.25 ms are used for data transmission delay. After conducting different experiments for various faults, the maximum delay value was 3.75 ms. In this case, if an inter-circuit \( A_i B_j C_i C_j B_j G \) fault occurs at a distance of 350 km from terminal T1 with interphase resistance of 10 \( \Omega \) and ground fault resistance of 100 \( \Omega \), the estimated fault location is 351.66 km, so the estimation error is 0.41%.

5 | DISCUSSION AND COMPARISON OF RESULTS

In this section, the obtained results are compared with the results of various references that have been published in the field of inter-circuit fault detection. Using the indices defined to compare the results of different references, it is possible to provide a suitable discussion and comparison between the algorithms provided for different topologies.

In this section, the articles published in this field of research are compared with one another in terms of different indices given in Table 4.

**Index 1:** The first index is related to the algorithm analysis domain, so that this index itself includes the domains of phasor analysis (Ph), traveling waves (TW), time domain signals and ANNs.

**Index 2:** The second index is related to the presence of a series capacitor compensator or FACTS devices in a double-circuit topology.

**Index 3:** The third index includes the number of measuring terminals. If the number of these terminals is more than one, then the measurement technologies are divided into synchronous and asynchronous.

**Index 4:** The fourth index is related to the sampling frequency of current and voltage signals.

**Index 5:** The fifth index is related to the mean error of the fault location algorithm. This index is calculated based on the number of results provided by the algorithm simulation.

Using the five indices defined in the above section, the literature published in this field can be reviewed.

6 | CONCLUSION

The paper presents a fault location algorithm in GUPFC-compensated DCTLs to analyze various types of inter-circuit faults in the phasor domain. The possible number of cases for inter-circuit faults is 98, where 49 types are associated with grounded faults and the rest with ungrounded faults. Due to the presence of a shunt converter in circuit II, the amplitude and angle of negative- and positive-sequence currents of the GUPFC’s shunt converter are present in boundary conditions equations, in addition to the phase-to-phase fault resistance and fault location. However, the number of unknowns for calculating the fault location depends on the type of the fault and the number of faulty phases of the circuits. The boundary conditions equations presented in this paper are used for various types of inter-circuit faults, where their number is equal to the number of unknowns for each fault type, and the accurate fault location is calculated by solving these equations. The obtained results in the simulation section verify the accurate and correct operation of the algorithm proposed in this paper.

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Nomenclature

\[ I_{sh}^{(i)} \] \( i \)-th sequence current phasor of the shunt converter

\[ I_{xj}^{(i)} \] \( i \)-th sequence current phasor of point \( x_j \)

\[ I_{f}^{(i)} \] \( i \)-th sequence current phasor of the fault point
\( R_f \)  
Phase to phase fault resistance  
\( R_g \)  
Single-phase to earth fault resistance  
\( V_{R_i-d}^{(1)} \)  
Positive sequence current phasor during the fault period for bus Ri  
\( V_{R_i-P}^{(1)} \)  
Positive-sequence voltage phasor during the pre-fault period for bus Ri  
\( Y_{N_i}^{(0)} \)  
i-th sequence voltage phasor of point \( x_i \)  
\( Y_{m_j}^{(0)} \)  
j-th zero-sequence mutual admittance  
\( Y_{x_kx_j}^{(0)} \)  
i-th sequence impedance between points \( x_k \) and \( x_j \)  
\( Z_{x_kx_j}^{(0)} \)  
i-th sequence impedance between points \( x_k \) and \( x_j \)  
\( Z_c \)  
Characteristics impedance of the line  
\( Z_{m_j}^{(0)} \)  
j-th zero-sequence mutual impedance  
\( x_k \)  
Length of the line between points \( x_k \) and \( x_j \)  
\( \gamma \)  
Propagation constant of the line

**APPENDIX A**

**References**

1. Fardanesh, B., et al.: Multi-converter FACTS devices: The generalized unified power flow controller (GUPFC). Paper presented at the 2000 power engineering society summer meeting, Seattle, USA, 16–20 July 2000

2. Vyakaranam, B., Villaseca, F.: Dynamic modeling and analysis of generalized unified power flow controller. Electr. Power Syst. Res. 106(1), 1–11 (2014)

3. Abasi, M., Joorabian, M., Saffarian, A.: Seyed Ghodratollah Seifossadat, “Accurate simulation and modeling of the control system and the power electronics of a 72-pulse VSC-based generalized unified power flow controller (GUPFC). Electr. Eng. 102(6), 1795–1819 (2020)

4. Khederzadeh, M., Ghorbani, A.: Impact of VSC-based multoline FACTS controllers on distance protection of transmission lines. IEEE Trans. Power Delivery 27(1), 32–39 (2012)

5. Kang, S., et al.: A fault location algorithm-based on circuit for Un transposed parallel transmission lines. IEEE Trans. Power Delivery 24(4), 1850–1856, (2009)

6. Korres, G., Apostolopoulos, C.: Precise fault location algorithm for double circuit transmission lines using unsynchronized measurements from two anti parallel ends. IET Gener. Transm. Distrib. 4(7), 824–835 (2010)

7. Jain, A.: Artificial neural network-based fault distance locator for double circuit transmission lines. Advances in Artificial Intelligence 2013(1), 1–12 (2013)

8. Kang, N., Liao, Y.: Double circuit transmission line fault location utilizing synchronized current phasors. IEEE Trans. Power Delivery 28(2), 1040–1047 (2013)

9. Abasi, M., et al.: Fault location in double-circuit transmission lines compensated by generalized unified power flow controller (GUPFC) based on synchronized current and voltage phasors. IEEE Syst. J. 1–11, (2020). doi: 10.1109/JSYST.2020.3016910

10. Guobing, S., Suoran, J., Ge, Y.: An accurate fault location algorithm for parallel transmission lines using one-terminal data. Int. J. Electr. Power Energy Syst. 31(2–3), 124–129 (2009)

11. Jain, A., et al.: Inter-circuit and cross-country fault detection and classification using artificial neural network. 2010 Annual IEEE India Conference (INDICON), IEEE, pp. 1–4, (2010)

12. Makwana, V.H., Bhalja, B.R.: A new adaptive distance relaying scheme for mutually coupled series-compensated parallel transmission lines during inter circuit faults. IEEE Trans. Power Delivery 26(4), 2726–2734 (2011)

13. McDaniel, R.: Analysis of a relay Operation for an Inter circuit fault. In 2015 68th Annual Conference for Protective Relay Engineers, pp. 462–477. IEEE (2015)

14. Swetapadma, A., Yadav, A.: Protection of parallel transmission lines including inter-circuit faults using Naïve Bayes classifier. Alexandria Engineering Journal 55(2), 1411–1419 (2016)

15. Swetapadma, A., et al.: A non-unit protection scheme for double circuit series capacitor compensated transmission lines. Electr. Power Syst. Res. 148(1), 311–325 (2017)

16. Hasheminejad, S., Seifossadat, S.G., Joorabian, M.: New travelling-wave-based protection algorithm for parallel transmission lines during inter-circuit faults. IET Gener. Transm. Distrib. 11(16), 3984–3991 (2017)

17. Swetapadma, A., Anamika, Y.: A hybrid method for fault location estimation in a fixed series compensated lines. Measurement 123(1), 8–18 (2018)

18. Sahani, M., Dash, P.K.: Fault location estimation for series-compensated double-circuit transmission line using EWT and weighted RVFLN. Eng. Appl. Artif. Intell. 88, 1–11 (2020)

19. Abasi, M., Joorabian, M., Saffarian, A.: Seyed Ghodratollah Seifossadat, “A novel complete dynamic and static model of 48-pulse VSC-based GUPFC for parallel transmission lines. International Journal of Industrial Electronics, Control and Optimization 3(4), 447–457 (2020)

20. Abasi, M., et al.: Fault classification and fault area detection in GUPFC-compensated double-circuit transmission lines based on the analysis of active and reactive powers measured by PMUs. Measurement 169(2), 108499 (2021)

21. Kang, N., Chen, J., Liao, Y.: A fault location algorithm for series compensated double-circuit transmission lines using the distributed parameter line model. IEEE Trans. Power Delivery 30(1), 360–367 (2015)

22. Saffarian, A., Abasi, M.: Fault location in series capacitor compensated three-terminal transmission lines based on the analysis of voltage and current phasor equations and asynchronous data transfer. Electr. Power Syst. Res. 187(10), 1–16 (2020)

23. Ajaei, E.B., et al.: Compensation of the current-transformer saturation effects for digital relays. IEEE Transaction Power Delivery 26(4), 2531–2540 (2011)
\[ A_3 = -Y_{X_4X_8}/2 + A_2Y_{m_1}/2 - A_1\left(Y_{X_4X_8}/2 + Y_{m_1}/2\right). \]

\[ A_6 = 1 + Z_{X_3X_6}Y_{X_1X_12}/2 + Z_{m_3}Y_{X_1X_12}/2. \]

\[ A_7 = 1 + Z_{X_1X_12}Y_{X_1X_12}/2 + Z_{m_3}Y_{X_1X_12}/2. \]

\[ A_8 = 1 + Z_{X_1X_12}\left(Y_{X_1X_12}/2 + Y_{m_3}/2\right)/2 + Z_{m_3}Y_{X_1X_12}/2. \]

\[ A_9 = Z_{m_3}\left(Y_{X_1X_12}/2 + Y_{m_3}/2\right)/2 + Z_{X_1X_12}Y_{m_3}/2. \]

\[ A_{10} = -Y_{X_1X_12}/2 - A_7\left(Y_{X_1X_12}/2 + Y_{m_3}/2\right) - A_8Y_{m_3}/2. \]

\[ A_{11} = 1 + Z_{X_1X_2}\left(Y_{X_1X_2}/2 + Y_{m_1}/2\right)/2 - Z_{m_1}Y_{X_1X_2}/2. \]

\[ A_{12} = Z_{m_1}\left(Y_{X_1X_2}/2 + Y_{m_1}/2\right)/2 - Y_{m_1}Z_{X_1X_2}/2. \]

\[ A_{13} = -Y_{X_1X_2}/2 + A_1Y_{m_1}/2 - A_2\left(Y_{X_1X_2}/2 + Y_{m_1}/2\right)/2. \]

\[ A_{14} = 1 + Z_{X_9X_{10}}\left(Y_{X_9X_{10}}/2 + Y_{m_1}/2\right)/2 - Y_{m_2}Z_{X_9X_{10}}/2. \]

\[ A_{15} = Y_{m_2}Z_{m_2}/2 + Y_{m_2}Z_{m_2}/2 - Y_{m_2}Z_{X_9X_{10}}/2. \]

\[ A_{16} = A_1A_{14} + A_2A_{15} - Z_{X_9X_{10}}A_3 - Z_{m_2}A_{13}. \]

\[ A_{17} = -A_{14}Z_{X_7X_8} - A_{15}Z_{m_1} - A_3Z_{X_9X_{10}} - Z_{m_2}Z_{X_7X_8}. \]

\[ A_{18} = -A_{14}Z_{m_1} - A_{15}Z_{X_7X_8} - A_3Z_{X_9X_{10}} - Z_{m_2}A_{11}. \]

\[ A_{19} = -Y_{m_2}Z_{X_7X_8}/2 + Y_{X_9X_{10}}Z_{m_2}/2 + Y_{m_2}Z_{m_2}/2. \]

\[ A_{20} = 1 + Y_{X_9X_{10}}Z_{X_7X_8}/2 + Y_{m_2}Z_{X_7X_8}/2 - Y_{m_2}Z_{m_2}/2. \]

\[ A_{21} = A_1A_{19} + A_2A_{20} - Z_{X_9X_{10}}A_{13} - Z_{m_2}A_{13}. \]

\[ A_{22} = -A_{19}Z_{X_7X_8} - Z_{m_1}A_{20} - Z_{m_1}A_{13} - Z_{m_2}A_{13}. \]

\[ A_{23} = -Z_{m_1}A_{19} - Z_{X_7X_8}A_{20} - A_{11}Z_{X_9X_{10}} - Z_{m_2}A_{4}. \]

\[ A_{24} = A_5 + Z_{X_9X_{10}}\left(Y_{X_9X_{10}}/2 - Z_{m_1}Y_{m_1}/2 + A_{22}Y_{m_2}/2 - A_{17}\left(Y_{X_9X_{10}} + Y_{m_2}/2\right). \]

\[ A_{25} = A_5 + Z_{X_9X_{10}}\left(Y_{X_9X_{10}}/2 - Z_{m_1}Y_{m_1}/2 + A_{23}Y_{m_2}/2 - A_{18}\left(Y_{X_9X_{10}} + Y_{m_2}/2\right). \]

\[ A_{26} = A_5 - A_1\left(Y_{X_9X_{10}} + Y_{m_1}/2 + A_{21}Y_{m_2}/2 - A_{16}\left(Y_{X_9X_{10}} + Y_{m_2}/2\right). \]

\[ A_{27} = Z_{m_2}Y_{m_2}/2 - Z_{X_9X_{10}}\left(Y_{m_2} + Y_{m_2}/2. \]

\[ A_{28} = 1/(1-A_{27}). \]

\[ A_{29} = A_{8} / (-A_{27}). \]

\[ A_{30} = A_{24} / (1 - A_{27}). \]

\[ A_{31} = A_{9} / (1-A_{27}). \]

\[ A_{32} = A_{25} / (1-A_{27}). \]

\[ A_{33} = A_{10} / (1-A_{27}). \]

\[ A_{34} = A_{26} / (1-A_{27}). \]

\[ A_{35} = 1/cosh(y_{X_9X_{10}}). \]

\[ A_{36} = cosh(y_{X_9X_{10}}). \]

\[ A_{37} = \cosh(y_{X_9X_{10}})/cosh(y_{X_9X_{10}}). \]

\[ A_{38} = -\sinh(y_{X_9X_{10}})/Z_c\cosh(y_{X_9X_{10}}). \]

\[ A_{39} = -\sinh(y_{X_9X_{10}})/Z_c\cosh(y_{X_9X_{10}}). \]

Appendix B

Detailed data of double-circuit transmission line and equivalent thevenin circuit of the buses of both sides of the transmission line in Figure 1 are listed in Table 5.