Scaling in Super Element Model of Petroleum Reservoirs

Alexandr Mazo, Konstantin Potashev
Institute of Mathematics and Mechanics, Fluid Mechanics department, Kazan Federal University, Russian Federation, Kazan, Kremlevskaya st., 35.

Abstract. A feature of the super element two-phase flow model in an oil reservoir is the construction of a solution on computational grids with a spatial step of the order of the distance between wells (hundreds of meters) horizontally and of the order of the total thickness of the reservoir (tens of meters) vertically. To maintain the acceptable accuracy of the solution, the upscaling procedures are performed during the transition from a detailed geological to a large super element computational grid and downscaling during a reverse transition at the local refinement of the solution. The field of absolute permeability and functions of relative phase permeability are subject to upscaling. The downscaling procedure is applied to the grid saturation function on the super element grid to formulate the initial conditions for local refinement of the solution on the detailed computational grid. The paper describes ways to simplify and implement these procedures, taking into account the characteristic features of the geological structure and the process of oil reservoir development.

1. Introduction
To reduce the dimension of computational grids in modeling of the oil reservoir, the averaging technique (upscaling) [1, 2, 3, 4] and the use of detailed grids in local subregions [5, 6, 7, 8] are usually used. This paper describes the functions of upsampling and downscaling with two-stage super element modeling [9, 10, 11, 12]. This approach allows not only to reduce the computational costs by orders of magnitude, but also to improve the accuracy of calculations in comparison with many traditional methods.

2. Basic equations of the super element model
For simplicity, we will neglect the compressibility of fluids, capillary and gravitational forces. In normalized variables, the mathematical total flow model describing two-phase flow in an oil reservoir contains two equations for pressure $p$ and water saturation $s$ [13]:

$$\beta \frac{\partial p}{\partial t} + \text{div} \, \mathbf{u} = 0, \quad \mathbf{u} = -\sigma \text{grad} p,$$

$$m \frac{\partial s}{\partial t} + \text{div} \left( f(s) \mathbf{u} \right) = 0,$$

(1)

(2)
\[ \mathbf{u} = -\sigma(s) \nabla p, \quad \sigma = k \varphi(s), \quad \varphi(s) = k_w(s) + K_{\mu} k_o(s), \quad K_{\mu} = \frac{\mu_w}{\mu_o}, \]
\[ f(s) = \frac{k_w(s)}{\varphi(s)}, \quad k_w(s) = \overline{s}^n, \quad k_o(s) = (1 - \overline{s})^n, \quad \overline{s} = \frac{s - s_a}{s - s_a} \epsilon [0,1], \ n = 1 \div 4. \tag{3} \]

Here \( t \) is the time; \( s_a, s^* \) – saturation range; \( m_0, k \) – porosity and normalized absolute permeability of the formation; \( \mathbf{u} \) is the normalized total flow rate; \( f \) is the proportion of water in the stream; \( \varphi \) is the mobility of the mixture; \( \mu_w, \mu_o \) – viscosity of the water and oil phases; \( k_w, k_o \) – relative phase permeability functions; \( \beta \) – elastic capacity.

Integration of the initial equations (1), (2) over an arbitrary super element (SE) \( V \) using the Gauss-Ostrogradsky theorem gives [9, 10, 11]
\[ \beta |V| \frac{\partial \langle p \rangle}{\partial t} + Q_\Gamma + Q_\gamma = 0, \tag{4} \]
\[ |V| \frac{\partial \langle s \rangle}{\partial t} + Q_w^\Gamma + Q_w^\gamma = 0, \tag{5} \]

where \( |V| \) is the volume of the super element \( V \), and the average values and fluid flows through the outer and inner boundaries of the SE are defined by the formulas
\[ \langle m \rangle = m, \quad \langle p \rangle = p, \quad \langle s \rangle = m s / m, \quad ( ) = \frac{1}{|V|} \int ( ) dV, \tag{6} \]
\[ Q_\Gamma = \int_{\Gamma} u_n \, d\Gamma, \quad Q_\gamma = \int_{\gamma} u_n \, d\gamma, \quad Q_w^\Gamma = \int_{\Gamma} f u_n \, d\Gamma, \quad Q_w^\gamma = \int_{\gamma} f u_n \, d\gamma. \tag{7} \]

Here \( \Gamma \) is the outer boundary of the FE, represented by a set of its faces \( \Gamma_j \), and \( \gamma \) is the inner boundary, which is the perforated part of the surface of the wells inside \( V \). Included in the definition of flow (7) \( u_n \) is determined by the formula
\[ x, y, z \in \gamma, \Gamma: u_n = -\sigma \frac{\partial p}{\partial n}. \]

For the closure of a model, the values \( u_n, f, \sigma \) are expressed in terms of averaged \( \langle p \rangle, \langle s \rangle \):
\[ Q_\Gamma = \sum_j |\Gamma_j| u_n \big|_{\Gamma_j}, \quad Q_\gamma = \| f u_n \|_\gamma, \quad Q_w^\Gamma = \sum_j |\Gamma_j| |f u_n|_{\Gamma_j}, \quad Q_w^\gamma = \| f u_n \|_\gamma, \tag{8} \]

where \( ( ) = \frac{1}{|\Gamma_j|} \int ( ) d\Gamma, \quad ( ) = \frac{1}{|\gamma|} \int ( ) d\gamma. \]

To calculate the averaged two-phase flow rates in the SEM, relations similar to formulas (3) are used. The set of values \( \langle p \rangle \) in the super elements determines the field of average pressure in the reservoir. The use of one or another procedure of filling the grid function \( \langle p \rangle \) allows determining the gradient of the average pressure field and using a ratio for the averaged field of the flow rate, similar to the Darcy law:
\[ \mathbf{U} = -\sigma \nabla \langle p \rangle, \quad \sigma = K \langle \varphi \rangle, \quad \langle \varphi \rangle \equiv \Phi(\langle s \rangle) = K_w(\langle s \rangle) + K_o(\langle s \rangle). \tag{9} \]

Here \( K \) is the effective absolute permeability tensor in the SE; \( K_w, K_o \) are functions that play the same role as the RP functions in the original equations. The definition of functions \( K_w, K_o \) is the task of upscaling of RP; tensor \( K \) is constructed as a result of solving the problem of absolute permeability upscaling.
It should be noted that the entry (9) contains the gradient from the discrete grid function \( \langle p \rangle \) and is introduced only formally to obtain approximations of average integral flows:

\[
  u_n \approx U_n = U \cdot n = -k \Phi(\{s\}) \nabla \langle p \rangle = -|k| \Phi(\{s\}) \frac{\partial \langle p \rangle}{\partial k}, \quad k = n \cdot K;
\]

\[
  f \approx \langle f \rangle U_n, \quad \langle f \rangle \equiv F(\{s\}) = \frac{K_w(\{s\})}{\Phi(\{s\})}.
\]

The vector \( k = n \cdot K \) can be called the full permeability vector on the face of the SE, and \( \partial/\partial k \) is the derivative in the direction of this vector, which is constructed by the method of colocations:

\[
  U_n = -|k| \Phi(\{s\}) \frac{\partial \langle p \rangle}{\partial k} \approx -k \phi P_2 - P_1, \quad k \phi = \frac{|k_1| |k_2| \langle \phi \rangle_1 \langle \phi \rangle_2 (h_1 + h_2)}{|k_2| \langle \phi \rangle_2 h_1 + |k_1| \langle \phi \rangle_1 h_2}.
\]

Through the values of \( P_1 \) and \( P_2 \) the average pressure on the lines of directions \( k_1 \) and \( k_2 \), which in turn are calculated by linear interpolation of the average pressures \( \langle p \rangle \) in the nearest SE, surrounding the face \( \Gamma \). Thus, the colocation algorithm is essentially a way of differentiating the average field \( \langle p \rangle \) by nodal values that do not require explicit completion of this function.

3. Absolute permeability upscaling

The task of local upscaling of absolute permeability is the definition of matrices of effective tensors \( K \) in each SE [3]. The coefficients of these tensors are found from the condition of the best approximation of average normal velocities (10), while the “exact” solution for the velocities \( u_{nj} \) on the faces \( \Gamma_j \) is a quantity in which the pressure \( p \) is obtained as a solution of problem (1), (2) with the initial coefficient \( \sigma(x) \) of relatively small grid in the area \( V \) of this super element.

To reproduce the various variants of the filtration flow through the FE, linear boundary conditions are set.

\[
  p^b \Big|_{x \in \Gamma} = \sum_{i=1}^{3} \delta_i^b x^i, \quad b = 1..3,
\]

where \( x^i \) are the Cartesian coordinates of the points of the outer boundary of the FE, \( \delta_i^b \) is the Kronecker symbol. When upscaling performed for the SE containing the well \( \gamma \), in addition to the conditions (12), the characteristic value of flow rate on the well is set:

\[
  \int_{\gamma} u_n \, d\gamma = q.
\]

The components \( K^{\phi} \) of the effective tensor matrix \( K \) for the SE \( V \) are determined from the condition of minimizing the functional of the residual

\[
  J(K^{ij}) = \frac{1}{2} \sum_{b=1}^{3} \rho^b \rightarrow \min, \quad \rho^b(K^{ij}) = \frac{1}{M} \sum_{m=1}^{M} \left| \left( \frac{\Gamma_m}{\max_{l=1..M} \left( \Gamma_l \right)} \right) \left( \frac{U_{nm}^b(K^{ij})}{\rho^b} \right) \right|^2,
\]

where \( M \) is the number of faces of the SE. The values \( U_{nm}^b(K^{ij}) \) on each face \( \Gamma_m \) for each variant \( b \) of the boundary conditions are calculated according to the scheme (11), in which the value \( P_1 \) is calculated in the center of the face \( \Gamma_m \) from the boundary conditions (12), and the value \( P_1 \) is
determined by interpolation according to the values of pressure on the faces and the average pressure \( \langle p \rangle \) in the SE, which is calculated by numerical integration of the pressure obtained on a fine grid.

We note that the formulated scheme for calculating the coefficients \( K_{ij} \) does not impose a symmetry condition on the matrix of the effective permeability tensor.

4. Relative phase permeability functions upscaling

The functions \( K_w, K_o \) should provide the best approximation for average velocities of the total flow and the aqueous phase across all faces of the SE. Just as for the initial RP, we will use for them power dependences (modified relative permeability functions, MRP)

\[
K_w(\langle s \rangle) = S^A, \quad K_o(\langle s \rangle) = (1-S)^B, \quad S = \frac{\langle s \rangle - s_{min}}{1 - s_{min}},
\]

(15)

where \( s_{min} \), \( A = \{ A_i \} \), \( B = \{ B_i \} \), \( i = 0,1,2 \) are the desired constants. The value \( s_{min} \) is the value of the average water saturation in the FE, before reaching which the total flow through the “output” face of the SE does not flooding.

The upscaling of RP is performed after rescaling the absolute permeability as follows [4].

1) In the area \( D \) containing the actual SE and surrounding elements around it, equations (1), (2) are solved on a fine grid, functions \( p(t), s(t) \) are constructed, and average velocities \( u_n(t), f u_n(t) \) on the faces of the actual SC and corresponding flows (8) are calculated.

2) For each SE, the average \( \langle p(t) \rangle, \langle s(t) \rangle \) are determined by formulas (6) and with their help, using formulas (10), at fixed coefficients \( A_i, B_i, s_{min} \), average velocities \( U_n(t), \langle f \rangle U_n(t) \) on the outer and inner faces of the SE are calculated.

3) The desired 7 coefficients of MRP (15) are sought as a solution to the problem of minimizing the functional

\[
J(A, B, s_{min}) = \frac{1}{T} \int_0^T \left( \sum w_i \rho_{\gamma}^{w} + w_2 \rho_{\gamma}^{w} + w_3 \rho_{\gamma}^{w} + w_4 \rho_{\gamma}^{w} \right) dt; \quad \rho_{\gamma} = \left( |U_n| - Q_{\gamma} \right)^2,
\]

(16)

where \( T \) is the flooding time; \( w_i \) are weights. The first two terms in integral (16) are responsible for the total and water flow through the outer faces of the SE, and the last – for flows through the well inside the SE.

The implementation of the above-described general upscaling technique is rather complex, since it involves solving three-dimensional unsteady flow problems in the vicinity of each SE with an individual porosity and permeability distribution. Therefore, it is advisable to introduce two assumptions that greatly simplify this procedure.

First, for an oil reservoir that has a predominantly layered structure, all the parameters of each layer can be considered constant within the SE. Secondly, all the super elements can be divided into three groups, which differ in the presence and mode of operation of the well. For a SE with a production well, a radially symmetric fluid inflow to the well and its flooding is characteristic; similarly for a solar cell with an injection well – radially symmetric displacement of oil by water and flooding of the flow on the outer faces. For a SE without a well, the filtration flow can be considered as plane-parallel. Accordingly, for each SE it is sufficient to consider one of the three listed watering scenarios, each of which is reduced to solving a two-dimensional problem.

1) Radially symmetric filtration in the FE with a production well in its center is modeled by equations (1) - (4) in a cylindrical coordinate system \((r, z)\) in the region \( r_w < r < 1, \quad 0 < z < H \),

\[
\rho_{\gamma}^{w} = \left( |U_n| - Q_{\gamma} \right)^2, \quad \rho_{\gamma}^{w} = \left( |U_n| - Q_{\gamma} \right)^2,
\]

(16)
0<t<T. At the initial moment s=0, p=0; the condition is set on the well \( p_w = -1 \), and the values \( p = 0, s = 1 \) are supported on the contour \( r = 1 \).

2) Radially symmetric filtration in an ESS with an injection well in its center is described by the same system of equations, but with different boundary conditions: conditions \( p_w = 1, s = 1 \) are set at the well, and pressure \( p = 0 \) is maintained at the outer boundary.

3) Planar-parallel filtration through the FE without a well is modeled by equations (1) - (4) in the Cartesian coordinate system \((x, z)\) in the region \( 0<x<1, 0<z<H, 0<t<T \). At the initial moment \( s=0, p=0 \); on the left boundary, conditions \( p_w = 1, s = 1 \) are given, and pressure \( p = 0 \) is maintained on the right.

In work [14], a method was proposed for taking into account the lateral heterogeneity of the flooding of the near-wellbore zone of the formation, enclosed in the volume of one grid block containing a production well. The algorithm is designed for large-block finite volume simulation of two-phase filtration in the development of an oil reservoir and can significantly improve the accuracy of the calculation of the water-cut and flow rate of the well, as well as the average saturation in the near-well zone. At the same time, as the rescaled functions of relative phase permeabilities, modified functions are used obtained from solving a reference, upscaling problem on a given grid unit, which does not require significant computational costs under the assumption of radial symmetry of the flow under uniform boundary conditions. The effectiveness of the proposed method is demonstrated by the example of a local problem for a single well in a reservoir with heterogeneous boundary conditions for saturation and by the example of super element modeling of a section of a reservoir with a seven-point development system at various flow rates of wells.

Note that the time required to build a model on a super element grid consists of three components – performing upscaling of the absolute permeability, upscaling of the RP and directly modeling the reservoir flooding. And the cost of modeling is less than 1%, since upscaling procedures require solving a large number of auxiliary tasks on detailed grids. On the other hand, upscaling is performed only once when constructing a grid of super elements; therefore, the use of super element model has significant advantages when performing multivariate design calculations on large oil fields. The greatest computational costs (more than 80%) fall on the RP upscaling procedure, which requires solving two-phase flow problems for each SE. One of the options for reducing the number of local upscaling tasks is to allocate areas of similarity of geological structure. Another variant of the principle acceleration of the solution of this problem is the use of an artificial neural network apparatus and machine learning using the statistical parameters of the local distribution of reservoir properties of the reservoir as input data [15].

5. Saturation downscaling

The super element grid allows describing the behavior of average saturation, but does not provide a detailed picture of the saturation distribution along the interlayers necessary to set the initial condition on the detailed computational grid. For this, downscaling of the saturation field is performed.

As one of the saturation downscaling algorithms that gives the correct results under the assumption of a stratified structure, the following is proposed:
1. vertical averaging of the reservoir and hydroconductivity is performed;
2. one builds the solution of the steady-state problem for pressure with average values of fluid flow rates for the period \((t_1 - t_0)\) of modeling on the fine grid:
\[
-\nabla (\bar{\sigma} \nabla p) = \bar{Q}, \quad \bar{Q} = \frac{1}{T} \int_{t_0}^{t_1} Q(t) \, dt;
\]
3. stream lines are constructed in a two-dimensional reservoir;
4. along each stream line the time of flight (TOF) is calculated;
5. saturation field on the detailed grid is built as
\[
s_h = \begin{cases} 
0, & k_h/m_h \tau > \tau^* \\
\frac{k_h \tau}{m_h \tau} (s_c - 1) + 1, & k_h/m_h \tau \leq \tau^* 
\end{cases}
\]

the threshold value \( \tau^* (s_H) \) is determined from the conservative for the aqueous phase condition:

\[
R(\tau^*) \to \min, \quad R = \frac{1}{N} \sum_{i=1}^{N} m_H^i s_H^i V_H^i - \sum_{k \in M} \frac{k^i}{V^i} s_h^k V_h^k, \quad \tau_h^k = \frac{1}{v_h^k} \int s_h dV,
\]

where \( i \) is the index of the super element, \( M \) is the set of indexes of the blocks of the detailed grid in the SE.

Calculations showed that over time, the agreement between saturation fields and, accordingly, in well performance indicators, obtained from the local refinement model and detailed model of the entire reservoir, is preserved.

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