Design and Simulation Technology of Piezoelectric Ultrasonic Transducer with Sandwich Composite Horn

Yi Xuan¹, Aimin Wang² and Niansong Zhang¹, a
¹College of mechanical engineering, Nanjing University of Science and Technology, Nanjing,China
²College of mechanical engineering, Beijing Institute of Technology, Beijing, China

a Corresponding author: zns@ustc.edu.cn, +86-13270737066

Abstract. The structural characteristics of piezoelectric ultrasonic transducer with sandwich composite horn are expounded, the vibration characteristics are theoretically analyzed, the velocity distribution equation and stress distribution equation of transducer under longitudinal vibration are obtained, the frequency equation is given, and the piezoelectric ultrasonic transducer with compound amplitude-changing rod is designed. Then the modal analysis and harmonic response analysis of the transducer are carried out by using the finite element simulation software Workbench, comparison between the results obtained and the theoretical values, the error is very small, which meets the need of engineering practice and verifies the correctness of the derivation design.

1. Introduction
An ultrasonic transducer is an energy conversion device that converts alternating electrical signals into acoustic signals or acoustic signals into electrical signals in the range of ultrasonic frequencies[1]. At present, the transducer is mainly divided into two types: magnetostrictive transducer and piezoelectric transducer. Because the electro-acoustic conversion efficiency of piezoelectric transducer is much higher than that of magnetostrictive transducer, piezoelectric transducer is generally used in ultrasonic transducer. In ultrasonic vibration system, the transducer and the amplitude-change rod are usually connected as a whole by bolts, and the length is usually the wavelength of a longitudinal wave. Many scholars at home and abroad have studied this structure[2-4].

In this paper, an analytical method is used to design a transducer with a variable amplitude bar, which can greatly improve the energy transfer efficiency of the transducer and reduce the redundancy of the whole vibration system. Then the design of the transducer is simulated by finite element software to verify the consistency between the design and the simulation results.

2. Piezoelectric ultrasonic transducer with sandwich composite horn

2.1. Structural analysis
The structure of the sandwich piezoelectric ultrasonic transducer is shown in Figure 1, including: front cover plate, piezoelectric ceramic, electrode, back cover plate, prestressed bolt and insulating pipe. Piezoelectric ceramic stack consists of a plurality of piezoelectric ceramic wafer mechanical series, with independent positive and negative electrodes. The electrical parallels between ceramic chips are opposite to the polarity of
adjacent ceramic plates. A flange is used to connect the transducer to the external mechanism to meet the needs of practical application of power ultrasound.

Figure 1. Structure diagram of piezoelectric ultrasonic transducer with sandwich composite horn.

Its main characteristics are [5-6]: (1) Piezoelectric ceramics have great compressive strength, prestressing force is given by prestressed bolt, on the one hand, the stability of transducer can be enhanced when the environmental strength changes; On the other hand, it ensures that the components are in a compression state under the condition of high power drive, thus avoiding the rupture caused by the expansion of the ceramic plate. (2) The number and connection of the piezoelectric ceramic pieces are open to choice. Thus the transducer can be designed in a wide range of impedance and frequency. (3) Changing the material and size of the front and rear metal cover plate can effectively control the performance parameters such as the front and back vibration speed ratio and the effective electromechanical coupling coefficient of the transducer.

2.2. Theoretical analysis

When an object vibrates in an elastic medium, it will cause the vibration of the medium. For all solids, the vibration wave can be regarded as an elastic body. There is an elastic relation between the points of the medium, and the vibration of the object propagates in the elastic medium. It is assumed that there is a uniform thin rod with arbitrary variable cross section and mechanical loss is omitted. When the cross sectional area of the rod is much smaller than the wavelength the plane longitudinal wave propagates along the axis of the rod. As shown in Figure 2, any variable cross-section bar, the axis of symmetry is x, optionally, a small volume element (x, x + dx), the tensile stress acting on it is \( \frac{\partial \xi}{\partial x} \, dx \).

Figure 2. Any variable cross-section bar.

According to Newton’s law, the kinetic equation can be obtained:

\[
\frac{\partial}{\partial x} (S\sigma) \, dx = S\rho \frac{\partial^2 \xi}{\partial t^2} \, dx
\]

In the form, S is the cross-sectional area function of the bar, \( \xi \) is particle displacement function, \( \sigma \) is stress function, \( \rho \) is density of rod material, E is young’s modulus.

Under the condition of resonance, the wave equation of longitudinal vibration of the rod with variable cross section can be obtained:
\[
\frac{\partial^2 \xi}{\partial x^2} + \frac{1}{S} \frac{\partial S}{\partial x} \frac{\partial \xi}{\partial x} + k^2 \xi = 0
\]  \(2\)

In the form, \(k\) is circular wave number, \(\omega\) is circular frequency, \(c\) is the velocity of the longitudinal wave propagating through the rod.

The vibration velocity equation of the longitudinal vibration of the rod with variable cross section can be obtained by taking the equation (2):

\[
\frac{\partial^2 v}{\partial x^2} + \frac{1}{S} \frac{\partial S}{\partial x} \frac{\partial v}{\partial x} + k^2 v = 0
\]  \(3\)

2.2.1 uniform equal-circle cross-section bar. If the area is uniform and equal, the equation of vibration velocity of a rod with uniform section can be obtained:

\[
\frac{\partial^2 v}{\partial x^2} + k^2 v = 0
\]  \(4\)

The general solution of equation (4) is:

\[
v(x) = A \sin kx + B \cos kx
\]  \(5\)

The elastic force and stress are:

\[
F(x) = \frac{E S}{j \omega} \frac{\partial v}{\partial x} = -j \rho c S (A \cos kx - B \sin kx)
\]  \(6\)

\[
\sigma(x) = \frac{E}{j \omega} \frac{\partial v}{\partial x} = -j \rho c (A \cos kx - B \sin kx)
\]  \(7\)

2.2.2 Circular section conical bar. The large end radius of the circular section conical rod is \(R_1\), the small end radius is \(R_2\), the length is \(l\), and the expression of the radius and area of the section at any position is as follows:

\[
R(x) = R_1(1 - \gamma x)
\]  \(8\)

\[
S(x) = S_0(1 - \gamma x)^2
\]  \(9\)

\[
\gamma = \frac{R_1 - R_2}{R_1 R_d}
\]  \(10\)

Replace equation (9) with equation (3):

\[
\frac{\partial^2 v}{\partial x^2} + \frac{2 \gamma}{\gamma x - 1} \frac{\partial v}{\partial x} + k^2 v = 0
\]  \(11\)

By solving equation (11), the general solution of vibration velocity of conical rod with circular section is obtained as follows:

\[
v(x) = \frac{1}{x - 1/\gamma} (A \sin kx + B \cos kx)
\]  \(12\)

The elastic force and stress are:

\[
F(x) = -j \rho c S \begin{cases}
A \left[ \frac{1}{x - 1/\gamma} \cos kx - \frac{1}{k(x - 1/\gamma)} \sin kx \right]
\end{cases}
\]  \(13\)

\[
\sigma(x) = -j \rho c \begin{cases}
A \left[ \frac{1}{x - 1/\gamma} \cos kx - \frac{1}{k(x - 1/\gamma)} \sin kx \right]
\end{cases}
\]  \(14\)

2.3. Dimension Design

The following analysis is made on the sandwich type piezoelectric ultrasonic transducer with compound amplitude change rod. The model is simplified as shown in Figure 3. The 1 is the rear cover plate, the 2 is the piezoelectric ceramic plate, the 3,4,5 is the composite horn, and as the front cover plate, the joint surface is...
located on the joint surface of the piezoelectric ceramic stack and the front cover plate. It is assumed that the front surface vibration velocity of the transducer oscillator is \( v_f \) and that the rear surface vibration velocity is \( v_b \).

**Figure 3.** Model schematic diagram of piezoelectric ultrasonic transducer with sandwich composite horn.

The frequency equation on both sides of the nodal plane is derived below. First consider the left part of the nodal plane. The boundary conditions are as follows:

\[
\begin{align*}
   v_f(l_2) &= 0 \\
   v_f(0) &= v_f(l_1) \\
   F_f(0) &= F_f(l_1) \\
   v_b(0) &= v_b \\
   F_b(0) &= 0
\end{align*}
\]  
(15)

Substituting equations (5)(6)(7) gives the frequency equation on the left side of the nodal plane:

\[
\tan k_d \tan k_d l_2 = \frac{Z_s}{Z_1} = \frac{\rho c \omega S_2}{\rho c \omega S_1}
\]  
(16)

The right part of the nodal plane with boundary conditions as follows:

\[
\begin{align*}
   v_f(0) &= 0 \\
   v_f(l_1) &= v_f(0) \\
   F_f(l_1) &= F_f(0) \\
   v_b(l_1) &= v_b(0) \\
   F_b(l_1) &= F_b(0) \\
   F_f(l_1) &= -Z_b y_f = 0
\end{align*}
\]  
(17)

Substituting equations(12)(13)(14) gives the frequency equation on the left side of the nodal plane:

\[
\frac{Z_s}{Z_1} \tan k_d l_1 = Z_s k_d (l_1 - l_1/\gamma) \times \left[ \frac{Z_s k_d \cot k_d l_1 - \gamma Z_s \sin k_d l_1 + Z_s k_d \cos k_d l_1}{Z_k k_d l_1 - k_d (l_1 - l_1/\gamma) \sin k_d l_1 + k_d (l_1 - l_1/\gamma) \cos k_d l_1} \right]
\]  
(18)

The forward and rear vibration speed ratio of the transducer is deduced below, we can see that the boundary condition at the node is the continuity of elastic force from figure 3:

\[
F_f(l_1) = F_f(0)
\]  
(19)

Substituting equation (6)(15)(17):

\[
\frac{v_f}{v_b} = (Z_s k_d \cot k_d l_1 \sin k_d l_1 - \gamma Z_s \sin k_d l_1 + Z_s k_d \cos k_d l_1) \times \frac{Z_s \cos k_d l_1 \sin k_d l_1}{\gamma Z_s \sin k_d l_1 (l_1 - l_1/\gamma) \cos k_d l_1}
\]  
(20)
Figure 3 shows the dimensions of the sandwich composite horn piezoelectric ultrasonic transducer, as shown in Table 1. The diameter of the large end of the conical segment of the front cover plate is 50 mm, and the diameter of the small end is 20 mm. In the table, the length of the front cover plate is calculated by equation (16) (18) except the length of the back cover plate 1 and the length of the front cover plate cylinder 5. The other dimensions are all given dimensions. Table 2 is a list of material parameters.

| Component                  | Parameter    | External diameter /mm | Bore size /mm | Length /mm | Material   |
|----------------------------|--------------|------------------------|---------------|------------|------------|
| Back cover plate           |              | 50                     | 18            | 16.5       | Steel      |
| Piezoelectric Ceramic chip |              | 50                     | 20            | 6.0        | PZT-8      |
| Electrode sheet            |              | 50                     | 20            | 0.2        | Brass      |
| Front cover plate          |              |                        |               |            |            |
| Flange plate               |              | 65                     | 18            | 5.0        | TC4        |
| Conical segment            |              |                        |               | 16.5       | TC4        |
| Cylindrical segment        |              |                        |               | 46.5       | TC4        |

| Material | Parameter    | Elastic modulus /Gpa | Density /kg*m-3 | Poisson ratio | Acoustic velocity /m*s-1 |
|----------|--------------|----------------------|-----------------|---------------|-------------------------|
| Steel    |              | 209                  | 7800            | 0.28          | 5150                    |
| PZT-8    |              | 65                   | 7600            | 0.31          | 2950                    |
| TC4      |              | 110                  | 4430            | 0.33          | 5000                    |

3. Numerical simulation analysis of piezoelectric ultrasonic transducer with sandwich composite horn

3.1. The establishment of finite element model
According to the design experience, the overall structure of the transducer is often simplified to a certain extent in the analysis process, and the influence of bolts and electrodes is ignored in this analysis, that is, the front cover plate of the transducer when modeling. Both the back cover plate and the piezoelectric ceramic plate are considered as solid bodies, and the parallel connection of piezoelectric elements in the circuit is not considered. The finite element analysis of the designed transducer is carried out by workbench. After a series of preprocessing, such as simplification of transducer model and definition of performance parameters, the model was meshed and the finite element model was established. The finite element model of the transducer is shown in Figure 4.

![Figure 4. Finite element model of transducer.](image)

3.2. Modal analysis technique
In modal analysis of the transducer, the positive and negative electrodes are briefly connected to obtain the resonant frequency of the transducer, that is, all the nodes on the surface of the piezoelectric plate are coupled
with the voltage degree of freedom and the coupling voltage. In addition, the displacement constraint of the
degree of freedom is applied on the nodal surface of the transducer. In modal analysis, a direct solver
controlled by program is used. The order of solution is 10 and the frequency of solution is 15-25 kHz. The
natural frequency and vibration mode are shown in Table 3.

| Order | Natural frequency | Mode of vibration |
|-------|-------------------|-------------------|
| 1     | 18394             | winding           |
| 2     | 18399             | winding           |
| 3     | 19695             | direction         |
| 4     | 20482             | winding           |
| 5     | 23384             | winding           |
| 6     | 23434             | winding           |

It can be seen from Table 3 that the natural frequency and mode shape of the third mode meet the
requirements of the design. The first to sixth order modal analysis clouds are shown in Figure 5.

![Figure 5. First to sixth order Modal Analysis of Transducers](image)

Table 3 and figure 5 show that the frequency of longitudinal vibration of the transducer is 19695 Hz, which
is close to its theoretical design value 20000Hz, and its error is about 1.52%. This shows that the theoretical
design of the transducer holds true.

3.3. Harmonic response analysis technique
In the case of external excitation, harmonic response analysis is a method to determine whether resonance can
occur in the frequency range of a mechanism. The harmonic response of the designed transducer is analyzed,
and the voltage applied on the electrode is the load of harmonic response analysis. A sinusoidal alternating
voltage is applied to the piezoelectric ceramic stack, a 0 V voltage is applied to the negative electrode and a
100 V alternating voltage is applied to the positive electrode. According to this excitation method, the front
dend amplitude of the front cover plate of the transducer varies with the frequency, as shown in Figure 6.

![Figure 6. Harmonic response analysis curve of transducer](image)
From Figure 6, it can be seen that when the frequency is 20333Hz, the center node displacement of front end of front cover plate of the transducer is the largest, and the amplitude of longitudinal vibration can reach about 4 micron, which has met the requirement of the required amplitude for ultrasonic vibration application. At the same time, in the safe range of excitation voltage, according to the different process requirements, without replacing the transducer, the value of the transducer excitation voltage can be adjusted by the ultrasonic power supply, and the amplitude of the front face of the front cover plate of the transducer can be finally adjusted.

4. Conclusion
Using the boundary condition analysis method, the vibration characteristics of the sandwich-type composite horn piezoelectric ultrasonic transducer are theoretically analyzed, and the vibration velocity distribution equation, stress distribution equation and resonance frequency equation of the transducer are obtained. The size parameters of the transducer are solved, which solves the problem of the design of the piezoelectric horn transducer of the composite horn.

The finite element simulation software Workbench is used to analyze the harmonic response of the theoretically designed transducer. The results show that the natural frequency of the longitudinal vibration of the transducer is within 2% of the theoretical value, which verifies the theory. The correctness. Subsequent processing will be performed to produce the actual transducer, further verifying the correctness of the theoretical design.

References
[1] Abdullah A, Shahini M, Pak A. An approach to design a high power piezoelectric ultrasonic transducer[J]. Journal of Electroceramics, 2009, 22(4):369-382.
[2] Canning S, Walker A J, Roach P A. A Mathematical Model of a Novel 3D Fractal-Inspired Piezoelectric Ultrasonic Transducer[J]. Sensors, 2016, 16(12):2170.
[3] Liu S, Xu L, Zhang Z, et al. The sandwiched radial composite piezoelectric ultrasonic transducer[J]. Shengxue Xuebao/Acta Acustica, 2014, 39(1):104-110.
[4] Cheng C, Zhu F, Bo F U. Modeling and Analysis of Sandwich Piezoelectric Ultrasonic Transducer Based on ANSYS[J]. Mechanical Engineer, 2015.
[5] Wang Y, Wang Z, Song Y, et al. Impedance Characteristic Analysis and Matching Design of Piezoelectric Ultrasonic Transducer[J]. Piezoelectrics & Acoustooptics, 2016.
[6] Peng J, Chao C, Tang H. Piezoelectric micromachined ultrasonic transducer based on dome-shaped piezoelectric single layer[J]. Microsystem Technologies, 2010, 16(10):1771-1775.