The effect of momentum anisotropy on quark matter in the quark-meson model

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We investigate the chiral phase structure of quark matter with spheroidal momentum-space anisotropy specified by one anisotropy parameter ξ in the 2+1 flavor quark-meson model. We find that the chiral phase diagram and the location of the critical endpoint (CEP) are affected significantly by the value of ξ. With the increase of ξ, the CEP is shifted to smaller temperatures and larger quark chemical potentials. And the temperature of the CEP is more sensitive to the anisotropy parameter than the corresponding quark chemical potential, which is opposite to the study for finite system volume effect. Furthermore, the effects of momentum anisotropy on the thermodynamic properties and scalar (pseudoscalar) meson masses are also studied at vanishing quark chemical potential. The numerical results show that an increase of ξ can hinder the restoration of chiral symmetry. We also find that shear viscosity and electrical conductivity decrease as ξ grows. However, bulk viscosity exhibits a significant non-trivial behavior with ξ in the entire temperature domain of interest.

I. INTRODUCTION

Quantum chromodynamics (QCD) is the fundamental theory for describing the strong interaction, and its phase structure is an important subject of great interest in recent decades. The first-principle results from lattice QCD simulation [1, 2] have indicated that with increasing temperature T, the transition from the ordinary nuclear matter to the chiral symmetric quark-gluon plasma (QGP) is a smooth crossover at small or zero chemical potential μ. At large chemical potential, lattice QCD simulation as a reliable tool to obtain the chiral properties of QCD matter, confronts a great challenge due to the fermion sign problem [3], although different strategies (for reviews see, e.g., Refs. [4–6]), such as Taylor series expansions [7–9], imaginary chemical potential, reweighting techniques [10, 11], complex Langevin method [12, 13], have been developed to try to tackle this problem. In this context, some alternative theoretical tools, such as QCD low-energy effective models (e.g., the Nambu-Jona-Lasinio model [14–16], Polyakov-loop extended NJL (PNJL) model [17–19], quark-meson model or linear sigma model [20–24], Polyakov quark-meson (PQM) model [25–28]), Dyson-Schwinger equation approach [29, 30], the functional renormalization group approach [31–34], which are not restricted by chemical potential, have been proposed to better explore the QCD phase structure at high chemical potential. And the results from the effective model calculations [35, 36] show that the chiral phase transition of the strongly interacting matter is a first-order transition at high density, and a second-order critical endpoint (CEP) can exist between the crossover line and the first-order phase transition line in the (μ, T)-plane. Apart from the phase transition, other important informations, such as thermodynamic properties, in-medium properties of mesons[36, 37] and transport properties [38–40] for the strongly interacting matter are also extensively studied in these QCD effective models.

To take into account the intricacy of the realistic quark matter produced in relativistic heavy-ion collisions (HICs) at the RHIC and the LHC, different improved versions of the QCD effective models have been proposed by including the effects of the finite volume of the system [42–57], the non-extensive effects in term of long-distance correlation [58, 59], the presence of magnetic fields [60–69], and the effects of electric field [71–75], to better explore the chiral/conf confinement properties of the strongly interacting matter at finite temperature or quark chemical potential. Conventionally, in the literature, all the effective models or improved effective models are based on an ideal assumption that the constituents of quark matter are completely isotropic in momentum-space for the absence of magnetic fields. However, due to the geometry of fireball created in HICs is asymmetric, the system evolves with different pressure gradients along different directions. As a result, the expanding and cooling rate along the beam direction (denotes as longitudinal direction) is larger than radial direction [76] and this momentum anisotropy can survive in all the stages of the HICs, consequently, the parton-level momentum distribution functions may become anisotropic. Thus, it’s essential to consider the momentum-space anisotropy induced by the rapid longitudinal asymptotic expansion into the

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phenomenological investigation of different observables. Up to present, extensive works have been made to explore the effects of momentum anisotropy on the parton self-energy [76–79], photon and dilepton production [80–83], the dissociation of quarkonium [84–86], heavy-quark potential [87, 88], various transport coefficients [89–92], jet quenching parameter[93] which, are sensitive to the evolution of the QGP. And associated results have indicated that the momentum-space anisotropy has a significant effect on the observables of the QGP. However, with the best of our knowledge, so far there is no study of momentum anisotropy in the framework of effective QCD models and no research regarding the effect of momentum-space anisotropy on chiral phase transition. Inspired by this fact, one major goal of present work is to reveal how the momentum anisotropy qualitatively affects the chiral phase structure as well as transport properties in the strongly interacting matter.

The present paper is a first attempt to study the effect of the momentum-space anisotropy induced by the rapid longitudinal expansion of fireball created in HICs on the QCD chiral phase transition. We adopt the 2+1 flavor quark-meson model, which is successful in describing the mechanism of spontaneous chiral symmetry breaking, to approximate quark matter. The effect of momentum anisotropy enters in the quark-meson model by substituting the isotropic (local equilibrium) distribution function in the total thermodynamical potential with the anisotropic one. This introduces one more degree of freedom, \( \xi \), the direction of anisotropy. The anisotropic parameter \( \xi \), representing the degree of momentum anisotropy or the tendency of the system to stay away from the isotropic state, is also considered as argument into the isotropic distribution function. Based on this momentum anisotropy-dependent quark-meson model, we first explore how the momentum anisotropy affects the chiral phase diagram and the location of CEP. Next, we investigate the thermodynamic properties and the thermal properties of various scalar (pseudoscalar) meson masses for vanishing chemical potential in both isotropic and anisotropic quark matter. Finally, transport coefficients, such as shear viscosity, electrical conductivity, and bulk viscosity, which are crucial to understand the dynamical evolution of QCD matter, also are estimated in an (an-)isotropic quark matter. Note that we restrict ourselves here to the anisotropic system close to isotropic local equilibrium state, consequently, the calculations of thermodynamic quantities, meson masses and transport coefficients in the anisotropic system are methodologically similar to those in the isotropic system. Especially, in the small \( \xi \) limit, the anisotropic distribution can just linearly expand to the linear order of \( \xi \). Using this linear approximation of the anisotropic distribution, the mathematical expression of transport coefficients, which are obtained by solving the relativistic Boltzmann equation under the relaxation time approximation, can be explicitly separated into an equilibrium part and an anisotropic correction part [89–92]. For \( \xi \to 0 \), the analytic expressions can reduce to the standard expressions in the local equilibrium medium, which can be seen in Section. IV.

This paper is organized as follows. In section. II, we give a brief overview of the three-flavor quark-meson model. In section. III, the modification of the thermodynamical potential within momentum-space anisotropy is presented. In section. IV, we discuss the chiral phase transition, thermodynamics properties, meson masses, and transport coefficients in both isotropic and anisotropic quark matter. In section.V, we summarize the main results and give an outlook.

II. THE QUARK-MESON MODEL

The quark-meson model as a successful QCD-like effective model can capture an important feature of QCD, namely, chiral symmetry breaking and restoration at high temperature/density. The Lagrangian of the three-flavor quark-meson model presently used for our purpose is taken from Ref. [23]:

\[
\mathcal{L}_{QM} = \bar{\Psi}(i\gamma_\mu D^\mu - g\phi_5)\Psi + \mathcal{L}_M, \tag{1}
\]

where \( \Psi = u, d, s \) stands for the quark field with three flavors \( (N_f = 3) \) and three color degrees of freedom \( (N_c = 3) \). The first term in the right hand side of Eq. (1) represents the interaction between the quark field and the scalar \( (\sigma) \) and pseudoscalar \( (\pi) \) fields with a flavor-blind Yukawa coupling \( g \) of the quarks to the mesons. The meson matrix is given as

\[
\phi_5 = T_a(\sigma_a + i\gamma_5\pi_a), \tag{2}
\]

where \( T_a = \lambda_a/2 \) with \( a = 0, \ldots, 8 \) are the nine generators of the \( U(3) \) symmetry. \( \lambda_a \) is Gell-Mann matrix with \( \lambda_0 = \sqrt{2}/3 \), \( \sigma_a \) and \( \pi_a \) denote the scalar meson nonet and the pseudoscalar meson nonet, respectively.

The second term in Eq. (1) is the purely mesonic contribution, \( \mathcal{L}_M \), which describes the chiral symmetry breaking pattern in strong interaction. It is given by [23]

\[
\mathcal{L}_M = \text{Tr}(\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi) - \lambda_1 [\text{Tr}(\phi^\dagger \phi)]^2 - \lambda_2 \text{Tr}(\phi^\dagger \phi)^2 + c[\text{Det}(\phi) + \text{Det}(\phi^\dagger)] + \text{Tr}[H(\phi + \phi^\dagger)], \tag{3}
\]

with \( \phi = T_a\phi_5 = T_a(\sigma_a + i\pi_a) \) representing a complex \( (3 \times 3) \)-matrix. Explicit chiral symmetry breaking is shown in the last term of Eq. (3), where \( H = T_a h_a \) is a \((3 \times 3)\)-matrix with nine external fields \( h_a \). Explicit \( U(1)_A \) symmetry is given by \( \chi \) Hooft determinant term with the anomaly term \( c \). \( m^2 \) is the tree-level mass of the fields in the absence of symmetry breaking, \( \lambda_1 \) and \( \lambda_2 \) are the two possible quartic coupling constants.

Under the mean-field approximation [36], the total thermodynamic potential density of the quark-meson

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model at finite temperature $T$ and quark chemical potential $\mu_f$ is given by
\begin{equation}
\Omega(T, \mu_f) = \Omega_{q\bar{q}}(T, \mu_f) + U(\sigma_x, \sigma_y).
\end{equation}

The first term $\Omega_{q\bar{q}}$ in the right hand of Eq. (4) denotes the fermionic part of the thermodynamic potential [36]:
\begin{equation}
\Omega_{q\bar{q}}(T, \mu_f) = 2N_c \sum_{f=u,d,s} T \int \frac{d^3p}{(2\pi)^3} \ln(1 - f^0_{q,f}(T, \mu_f, p)) + \ln(1 - f^0_{\bar{q},f}(T, \mu_f, p)),
\end{equation}
with the isotropic equilibrium distribution function of (antiquark) quark for $f$-th flavor
\begin{equation}
f^0_{q(\bar{q}),f}(T, \mu_f, p) = \frac{1}{\exp[E_f \mp \mu_f/T] + 1}.
\end{equation}

Here, $E_f = \sqrt{p^2 + m_f^2}$ is the single-particle energy with flavor-dependent constituent quark mass $m_f$. The sign $\mp$ corresponds to quarks and antiquarks, respectively. In present work, an uniform quark chemical potential $\mu \equiv \mu_u \equiv \mu_d \equiv \mu_s$ is assumed. And the breaking of the $SU(2)$ isospin symmetry is not considered, consequently, the up and down quarks have approximately the same masses, i.e., $m_u \approx m_d$. In the quark-meson model, the constituent quark masses are given as
\begin{equation}
m_l = g\sigma_x/2 \quad \text{and} \quad m_s = g\sigma_y/\sqrt{2},
\end{equation}
where $l$ denotes light quarks ($l \equiv u, d$), $\sigma_x$ and $\sigma_y$ stand for the non-strange and strange chiral condensates, respectively. The Yukawa coupling $g$ is fixed to reproduce a light constituent quark mass of $m_l \approx 300$ MeV. The second term $U(\sigma_x, \sigma_y)$, viz., the purely mesonic potential, is given as [20, 23, 28]
\begin{equation}
U = -h_x\sigma_x - h_y\sigma_y + \frac{m_l^2(\sigma_x^2 + \sigma_y^2)}{2} - \frac{c\sigma_x^2\sigma_y}{\sqrt{2}} + \frac{\lambda_1\sigma_x^2\sigma_y^2}{2} + \frac{(2\lambda_1 + \lambda_2)\sigma_x^4}{8} + \frac{(\lambda_1 + \lambda_2)\sigma_y^4}{4},
\end{equation}
where the model parameters: $m^2, h_x, h_y, \lambda_1, \lambda_2$ and $c$ as reported in Ref. [36], are shown in Table I. Finally, the behaviors of $\sigma_x$ and $\sigma_y$ as the functions of temperature and quark chemical potential can be obtained by minimizing the total thermodynamic potential, i.e.,
\begin{equation}
\frac{\partial \Omega}{\partial \sigma_x} = \frac{\partial \Omega}{\partial \sigma_y} \bigg|_{\sigma_x = \sigma_y, \sigma_y = \sigma_y} = 0,
\end{equation}
with $\sigma_x = \bar{\sigma}_x$, $\sigma_y = \bar{\sigma}_y$ being the global minimum.

### III. THERMODYNAMIC POTENTIAL WITH MOMENTUM ANISOTROPY

Due to the rapid longitudinal expansion of the partonic matter created in the HICs, an anisotropic deformation of the argument of the isotropic (equilibrium) parton distribution functions is generally used to simulate the momentum anisotropy of QGP [76–92]. A special and widely used spherical momentum deformation introduced by Romatschke and Strickland [76], which is characterized by removing and adding particles along a single momentum anisotropy direction, is applied in this paper. Accordingly, the local distribution function of $f$-th flavor quarks(antiquarks) in an anisotropic system can be obtained from the isotropic (local equilibrium) distribution function by the rescaling of one preferred direction in momentum space, which is given as
\begin{equation}
f^0_{\text{aniso}}(T, \mu_f, p) = \frac{1}{e^{(\sqrt{p^2 + (\mathbf{p} \cdot \mathbf{n})^2} + m_f)/T} + 1}.
\end{equation}

Here, the anisotropy parameter $\xi$, presenting the degree of momentum-space anisotropy, generally can be defined as
\begin{equation}
\xi = \frac{\langle p^2_T \rangle}{2\langle p^2_L \rangle} - 1,
\end{equation}
where $p_L$ and $p_T$ are the components of momentum parallel and perpendicular to the direction of anisotropy, $\mathbf{n}$, respectively. And $\mathbf{p} = (p\sin \theta \cos \phi, p\sin \theta \sin \phi, p\cos \theta)$, where we use a notation $[p] \equiv p$ for convenience. $\mathbf{n} = (\sin \alpha, 0, \cos \alpha)$, $\alpha$ is the angle between $\mathbf{p}$ and $\mathbf{n}$. Accordingly, $(\mathbf{p} \cdot \mathbf{n})^2 = p^2(\sin \theta \cos \phi \sin \alpha + \cos \theta \cos \alpha \cos \alpha)^2 = p^2 \cos^2(\theta, \phi, \alpha)$. Note that $\xi > 0$ corresponds to a contraction of the particle distribution in the direction of anisotropy whereas $-1 < \xi < 0$ stands for a stretching of the particle distribution in the direction of anisotropy.

If the system is close to the ideal massless parton gas and $\xi$ is small, $\xi$ is also related to the ratio of shear viscosity to entropy density $\eta/s$ as well as the proper time $\tau$ of the medium. The relation for one-dimensional Bjorken expansion in the Navier-Stokes limit is given as [94]
\begin{equation}
\xi = \frac{10 \eta}{T \tau s}.
\end{equation}

This implies that non-vanishing shear viscosity combined with finite momentum relaxation rate in an expanding system can also contribute to the momentum-space anisotropy. At the RHIC energy with the critical temperature $T_c \approx 160$ MeV, $\tau \approx 6$ fm/c and $\eta/s = 1/4\pi$, we can obtain $\xi \approx 0.3$.

In this work, we assume the system has a small deviation from the momentum-space isotropy, therefore
the value of $\xi$ is small ($|\xi| \ll 1$) and the Eq. (10) can be expanded up to linear order in $\xi$,

$$f^0_{\text{aniso}}(p) \approx f^0_{q,f} - \frac{\xi(p \cdot n)^2}{2E_fT} e^{(E_f-\mu_f)/T} f^{02}_{q,f}$$

$$= f^0_{q,f} - \frac{\xi(p \cdot n)^2}{2E_fT} f^{0l}_{q,f}(1 - f^0_{q,f}).$$  \hspace{1cm} (13)

By replacing the isotropic distribution functions in Eq. (5) with the Eq. (13), we finally obtain the $\xi$-dependent thermodynamic potential density of fermionic part

$$\Omega_{qq} = 2N_c \sum_f \frac{T d^3p}{(2\pi)^3}$$

$$\left\{ \ln(1 - f^0_{q,f}) + \frac{\xi p^2 c(\theta, \phi, \alpha)}{2E_fT} f^0_{q,f}(1 - f^0_{q,f}) \right\} + \left\{ \ln(1 - f^0_{q,f}) + \frac{\xi p^2 c(\theta, \phi, \alpha)}{2E_fT} f^0_{q,f}(1 - f^0_{q,f}) \right\}.\hspace{1cm} (14)$$

Similar to the studies regarding finite-size effect \cite{42} and non-extensive effect \cite{58}, we also treat the anisotropy parameter $\xi$ as a thermodynamic argument in the same footing as $T$ and $\mu$, and do not have any modifications to the usual quark-meson model parameters due to the presence of momentum anisotropy. Replacing the fermionic thermodynamic potential in Eq. (9) with Eq. (14), we can finally obtain the $\xi$-dependent chiral condensates at finite temperature and quark chemical potential.

**IV. RESULTS AND DISCUSSIONS**

**A. phase transition and phase diagram**

In the 2+1 flavor quark-meson model, the chiral condensates of both light quarks and strange quarks can be regarded as the order parameters to analyze the feature of the chiral phase transition. The anisotropy parameters we work here are artificially taken as $\xi = -0.4$, 0, 0.2, 0.4, although the value of $\xi$ in the realistic HICs always remains positive in sign. In Fig. 1, the temperature $T$ dependences of non-strange chiral condensate $\sigma_x$ and strange chiral condensate $\sigma_y$ for both isotropic and anisotropic quark matter at vanishing quark chemical potential are plotted. For $T = 0$ MeV, $\sigma^0_x \approx 92.4$ MeV and $\sigma^0_y \approx 94.5$ MeV. As can be seen, $\sigma_x$ and $\sigma_y$ in both isotropic and anisotropic quark matter decrease continuously with increasing temperature. This means that at vanishing quark chemical potential, the restoration of the chiral symmetry for (an-)isotropic quark matter is always a crossover phase transition. And the restoration of the chiral symmetry in the strange sector is always slower than that in the non-strange sector. As $\xi$ increases, the values of $\sigma_x$ and $\sigma_y$ increase and their melting behaviors become more smoother. This shows that an increase of anisotropy parameter tends to delay the chiral symmetry restoration.

In order to obtain the chiral critical temperature, we introduce the susceptibilities of light quarks $\chi_l$ and strange quarks $\chi_s$, which are defined as

$$\chi_l = -\frac{\partial \sigma_x}{\partial T}, \hspace{1cm} \chi_s = -\frac{\partial \sigma_y}{\partial T}.\hspace{1cm} (15)$$

The thermal behaviors of both $\chi_l$ and $\chi_s$ are presented in Fig. 2. We can see that $\chi_l$ and $\chi_s$ are peaking up

| $\xi$ | $T^N_0$ (MeV) | $T^S_0$ (MeV) |
|-------|----------------|----------------|
| -0.4  | 137            | 233            |
| 0     | 146            | 248            |
| 0.2   | 152            | 258            |
| 0.4   | 159            | 270            |

**TABLE II.** The chiral critical temperature of the non-strange condensate $T_0^N$ and strange condensate $T_0^S$ at vanishing quark chemical potential for different anisotropy parameters.
FIG. 2. The temperature dependences of the susceptibilities in non-strange sector $\chi_l$ (upper panel) and in strange sector $\chi_s$ (lower panel) at $\mu = 0$ GeV for both isotropic ($\xi = 0$ (blue dashed line)) and anisotropic (i.e., $\xi = -0.4$ (orange dotted-dashed line), 0.2 (red solid line)) and 0.4 (green wide dashed line) quark matter in the quark-meson model.

at the particular temperatures. The peak position of $\chi_l$ determines the critical temperature $T_{\chi_l}^c$ for the chiral transition in non-strange sector. Different to $\chi_l$, $\chi_s$ have two peaks in the entire temperature domain of interest. The temperature coordinate of the first peak of $\chi_s$ is almost same as that of $\chi_l$, the location of the second broad peak of $\chi_s$ determines the critical temperature for the chiral transition of strange sector $T_{\chi_s}^c$. The chiral critical temperature $T_{\chi_l}^c$ at vanishing quark chemical potential is the origin of the crossover phase transition in the QCD chiral phase diagram. Furthermore, these chiral critical temperatures are sensitive to the variation of $\xi$. As $\xi$ increases, $T_{\chi_s}^c$ shifts towards higher temperatures as well as the height of $\chi_{l,s}$ decreases. The exact values of both $T_{\chi_l}^c$ and $T_{\chi_s}^c$ for different anisotropy parameters are listed in Table II. Compared to the case of $\xi = 0$, the chiral critical temperatures $T_{\chi_l}^c$ and $T_{\chi_s}^c$ decrease by approximately 6% for the case of $\xi = -0.4$. For the cases of $\xi = 0.2$ and 0.4, both $T_{\chi_l}^c$ and $T_{\chi_s}^c$ increase by approximately 4% and 9%, respectively.

Next, we extend our exploration to finite quark chemical potential for analyzing the effect of momentum anisotropy on the structure of QCD phase diagram. In Fig. 3, the temperature dependence of non-strange chiral condensate $\sigma_x$ for both isotropic and anisotropic quark matter at different quark chemical potentials (viz., $\mu = 150$ MeV, 200 MeV, 250 MeV) is plotted. At $\mu = 150$ MeV, the chiral symmetry restoration with different $\xi$ still takes place as the crossover phase transition. For $\mu = 200$ MeV, the value of $\sigma_x$ in the anisotropic quark matter with $\xi = -0.4$ drops from 60 MeV to 23 MeV, and associated susceptibility presents a divergent behavior at $T = 90$ MeV, which signals the appearance of a first-order phase transition. For $\mu = 250$ MeV, the discontinuity of $\sigma_x$ (i.e., the first-order phase transition) also occurs at $\xi = -0.4$, 0 and 0.2, whereas, at $\xi = 0.4$
the phase transition is still a smooth crossover. Thus, for
the anisotropic matter with $\xi = 0.4$, a first-order phase
transition happens at higher quark chemical potential.
Accordingly, the chiral phase transition diagram can be
studied by outlining the location of $T_c$ for a wide
range of quark chemical potential. And the first-order
phase transition has to end and changes into a
crossover is the QCD critical endpoint (CEP), at which
the phase transition is of second order. In Fig. 4, the
2+1 flavor chiral phase diagram in the $(\mu, T)$-plane
for the quark-meson model within the effect of momentum-
space anisotropy is presented. Along the first-order
phase transition line (crossover phase transition line),
the chiral critical temperature rises from zero up to the
CEP temperature (from the $T_c^{\text{CEP}}$ up to the $T_c^0(\mu = 0)$),
whereas the critical quark chemical potential decreases
from the $\mu_c(T = 0)$ to the $\mu_{\text{CEP}}$ (from the $\mu_{\text{CEP}}$ to zero).
We observe that the phase boundary in the $(\mu, T)$-
plane of the quark-meson model phase diagram is shifted
to higher values of $\mu$ and $T$, with increasing anisotropy
parameter. We also can clearly see that the position
of the CEP significantly depends on the variation of
momentum anisotropy parameter. As $\xi$ increases,
the location of the CEP shifts to higher $\mu$ and smaller $T$
domain, which is similar to the study of non-extensive
effect in linear sigma model [59]. Similar phenomenon is
also observed in the literature for analyzing the finite size
effects on chiral phase transition [51–54, 57]. In Ref. [51],
when the system size is reduced to 4 fm, the CEP in
the quark-meson model vanishes and the whole chiral
phase boundary becomes a crossover curve. Based on
this result, we deduce that as $\xi$ increases further, the CEP
cannot disappear. In this work, for $\xi = -0.4$, 0, 0.2, 0.4,
the location of the CEP is at $(T_{\text{CEP}}, \mu_{\text{CEP}}) =$
(100, 174) MeV, (91, 222) MeV, (84, 247) MeV and
(79, 270) MeV, respectively. The value of $\mu_{\text{CEP}}$ from
$\xi = -0.4$ to $\xi = 0.4$ increases by about 50%, whereas
the value of $T_{\text{CEP}}$ increases by about 20%. This means
that the influence of momentum-space anisotropy on
the quark chemical potential coordinate of the CEP is
more prominent compared to that on the temperature
of the CEP. An opposite trend can be found in the
study of finite volume effect [51], where the temperature
coordinate of the CEP in the quark-meson model appears
to be affected more strongly by the finite volume than
CEP’s quark chemical potential coordinate.

B. QCD thermodynamic quantities

Let us now study the influence of anisotropy parameter
$\xi$ on the thermodynamics at vanishing quark chemical
potential. The $T$- and $\xi$-dependent pressure $P(T, \xi)$,
which is derived from the thermodynamic potential, can
be given as

$$P(T, \xi) = -\Omega(T, \xi),$$

with the vacuum normalization $P(0, \xi) = 0$. The entropy
density $s$ and energy density $\epsilon$ are defined as

$$s(T, \xi) = -\frac{\partial \Omega(T, \xi)}{\partial T},$$

$$\epsilon(T, \xi) = -P(T, \xi) + Ts(T, \xi),$$

respectively.

In Fig. 5, the variations of the scaled pressure $P/T^4$,
scaled entropy density $s/T^3$, and scaled energy density $\epsilon/T^4$ with respect to temperature in the quark-meson model for both isotropic and anisotropic quark matter are presented. As can be seen that the thermal behaviors of $P/T^4$, $s/T^3$, and $\epsilon/T^4$ for the anisotropic quark matter is in agreement with those for the isotropic system. To be specific, with increasing temperature, $P/T^4$, $s/T^3$, and $\epsilon/T^4$ first rise rapidly then tend towards a saturation value. At high enough temperature, the limit values of
$P/T^4$, $s/T^3$, and $\epsilon/T^4$ in the case of $\xi = -0.4$ stabilize
approximately at 4.0, 16.5, 12.5, respectively, although
all these values are lower than their respective QCD
Stefan-Boltzmann (SB) limit values: $\frac{P}{T^4} = (N_c^2 - 1)\frac{\pi^2}{40} +
N_c N_f \frac{2\pi^2}{120} \zeta(3) \simeq 5.2$, $\frac{s}{T^3} = \frac{3P}{T^4} \simeq 20.8$, $\frac{\epsilon}{T^4} = \frac{3P}{T^4} \simeq 15.6$. From Fig. 5 we also can see that the limit values of these thermodynamics at high enough temperature still are decreasing functions of $\xi$, which is opposite to their qualitative behaviors with the non-extensive parameter $q$. In Ref. [58], at high temperature, the limit values of these scaled thermodynamics increase as $q$ increases. Moreover, their features with $\xi$ are significantly different to those with finite volume effect. For example, Refs. [43, 54] have indicated that with increasing temperature, $P/T^4$ first decreases with increasing volume and then quickly saturates to the infinite volume value, in other
C. Meson mass

In this part, we study the chiral structures of scalar \( (J^P = 0^+) \) and pseudoscalar \( (J^P = 0^-) \) meson masses at vanishing quark chemical potential. A detailed procedure for calculating meson masses at finite temperature and quark chemical potential in the quark-meson model can be found in Ref. [36]. Here, we just sketch the outline of the related computation. In quantum field theory, the scalar and pseudoscalar meson masses generally can be obtained from the second derivative of the temperature- and quark chemical potential-dependent thermodynamic potential density \( \Omega(T, \mu_f) \) with respect to corresponding \( \sigma_a \) and the pseudoscalar fields \( \rho_{f,a} = \pi_a(a = 0, ..., 8) \), which can be expressed as [36]

\[
m_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \alpha_{i,a} \partial \alpha_{i,b}} \right|_{\text{min}} = (m_{i,ab}^M)^2 + (m_{i,ab}^T)^2
\]

where the subscript \( i = S(P) \) denotes the scalar (pseudoscalar) mesons. The first term on the right-hand side of Eq. (21) is vacuum mass squared matrices calculated from the second derivative of purely mesonic potential. The second term represents the modification of mass squared matrices due to fermionic thermal correction at finite temperature and quark chemical potential, which in an anisotropic system can be written as
where the mixing angles $\theta_i$ and the four pseudoscalar meson masses are

$$m_{\pi_0} = (m_{\pi_0}^M)^2 + (\delta m_{S,11}^T)^2,$$

$$m_{\pi} = (m_{\pi}^M)^2 + (\delta m_{P,11}^T)^2,$$

$$m_{\pi}^2 = m_{S,00}^2 \cos^2 \theta_S + m_{S,ss}^2 \sin^2 \theta_S + 2m_{S,0s}^2 \sin \theta_S \cos \theta_S,$$

$$m^2_{f_0} = m_{S,00}^2 \sin^2 \theta_S + m_{S,ss}^2 \cos^2 \theta_S - 2m_{S,0s}^2 \sin \theta_S \cos \theta_S.$$

The detailed descriptions of the vacuum contributions $(m_{i,00}^M)^2$, $(m_{i,00}^M)^2$, $(m_{i,ss}^M)^2$, $(m_{i,0s}^M)^2$ and $(m_{i,00/08}^M)^2$ from purely mesonic potential in Eqs. (23)-(30) can be found from Refs. [36, 37].

The left panels and right panels of Fig. 7 display the $T$-dependent masses of the pseudoscalar ($\pi, K, \eta', \eta$) and scalar ($f_0, \sigma, a_0, \kappa$) mesons for both isotropic and anisotropic quark matter in the quark-meson model, respectively. We can see that for a fixed anisotropy parameter, the masses of the pseudoscalar meson sectors $\pi, K$, and $\eta$ remain constant up to near the chiral critical temperature of non-strange condensate $T^c_s$, whereas the masses of $\eta'$ and scalar meson sectors $\sigma$, $a_0$, $\kappa$ remain constant at low temperature and then decreases before reaching $T^c_s$. For the pseudoscalar meson sector $f_0$, its mass also remains constant at low temperature but decreases before reaching the chiral critical temperature of strange condensate $T^c_s$. For pseudoscalar meson sectors $\pi, K$ and $\eta$, their masses always decrease with increasing $\xi$ at $T > 140$ MeV. However, for $\eta'$ and pseudoscalar meson sectors $\sigma, a_0, \kappa$, the dependence of their masses on anisotropy parameter $\xi$ is nonmonotonic in the entire temperature domain of
FIG. 7. The temperature dependences of the pseudoscalar mesons $\pi$, $K$, $\eta'$, $\eta$ (left panels) and scalar mesons $f_0$, $\sigma$, $a_0$, $\kappa$ (right panels) at $\mu = 0$ MeV for both isotropic ($\xi = 0$ (blue dashed lines)) and anisotropic (i.e., $\xi = -0.4$ (orange dotted-dashed lines), 0.2 (red solid lines) and 0.4 (green wide dashed lines)) quark matter in the quark-meson model.

interest. More exact, with the increase of $\xi$, the masses of $\eta'$, $\sigma$, $a_0$, $\kappa$ first increase in low temperature domain ($100 \text{ MeV} < T < 160 \text{ MeV}$), then decrease in higher temperature domain ($T > 160 \text{ MeV}$). For $f_0$, its mass increases with increasing $\xi$ at $T < 270 \text{ MeV}$ (viz., $T_N(\xi = 0.4)$) and decreases afterward. As a whole, near above $T_N$ or $T_N^c$, all mesons become unphysical degrees of freedom and their masses become degenerate, which signals the restoration of chiral symmetry. In Fig. 7 we can also see that with the increase of $\xi$, the temperature coordinate at which meson masses begin to degenerate can be shifted to higher temperatures. This again shows that an increase of momentum-space anisotropy parameter can hinder the restoration of chiral symmetry. The qualitative behaviors of these meson masses with $\xi$ are different to the results for analyzing the finite size dependence of meson masses.
within PNJL model [43, 47], where \( K, \eta, \) and \( \eta' \) have a significant volume dependence in lower temperature domain \((T < 100 \text{ MeV})\).

D. Transport coefficient

Studying transport properties is essential to deeply understand the dynamical evolution of the strongly interacting matter. In this part, we discuss the influence of momentum-space anisotropy on transport coefficients, such as shear viscosity \( \eta \), electrical conductivity \( \sigma_{el} \), and bulk viscosity \( \zeta \) in quark matter. Due to the effect of momentum-space anisotropy is encoded in the parton distribution functions, the general expressions of these transport coefficients, which are obtained by solving the relativistic Boltzmann equation in relaxation time approximation, need to do some modifications [89–92]. Therefore, using the results in Refs. [90, 91], the formulas of \( \xi \)-dependent transport coefficients at zero quark chemical potential are given as

\[
\eta = \sum_d \frac{d_f}{15T} \int \frac{dp}{\pi^2 E_f^2} \left[ \tau_{q,f} f_{q,f}^0 (1 - f_{q,f}) \right] - \sum_f \frac{\xi_d f}{90T^2} \int \frac{dp}{\pi^2 E_f^2} \left[ \tau_{q,f} f_{q,f}^0 (1 - f_{q,f}) (1 - 2 f_{q,f} + \frac{T}{E_f}) \right],
\]

\[
\sigma_{el} = \sum_d \frac{d_d^2}{3T} \int \frac{dp}{\pi^2 E_f^2} \left[ \tau_{q,f} f_{q,f}^0 (1 - f_{q,f}) \right] \left( 1 + \frac{\xi}{3} \right) - \sum_f \frac{q_f^2 \xi_d f}{18T^2} \int \frac{dp}{(2\pi)^3} \frac{p^6}{E_f^3} \left[ f_{q,f}^0 (1 + f_{q,f}) (1 - 2 f_{q,f} + \frac{T}{E_f}) \right],
\]

\[
\zeta = \sum_d \frac{d_f}{T} \int \frac{dp}{\pi^2 E_f^2} \left[ \frac{1}{3} - c_s^2 \right] p^2 - c_s^2 m_f^2 + c_s^2 m_f T \frac{dm_f}{dT} \right] \left[ \tau_{q,f} f_{q,f}^0 (1 - f_{q,f}) \right] - \sum_f \frac{\xi_d f}{6T} \int \frac{dp}{\pi^2 E_f^2} \left[ \frac{1}{3} - c_s^2 \right] p^2 - c_s^2 m_f^2 + c_s^2 m_f T \frac{dm_f}{dT} \right] \left[ \tau_{q,f} f_{q,f}^0 (1 - f_{q,f}) \right] - \sum_f \frac{\xi d f}{6T} \int \frac{dp}{\pi^2 E_f^2} \left[ \frac{1}{3} - c_s^2 \right] p^2 - c_s^2 m_f^2 + c_s^2 m_f T \frac{dm_f}{dT} \right] \left[ \tau_{q,f} f_{q,f}^0 (1 - f_{q,f}) \right].
\]

Here, \( d_f \) is the degeneracy factor for \( f \)-flavor quark. The quark electric charge \( q_f \) is given explicitly by \( q_u = q_d = 2e/3 \) and \( q_s = -q_d = -e/3 \). The electron charge \( e = (4\pi\alpha_s)^{1/2} \) with the fine structure constant \( \alpha_s \approx 1/137 \). Different to the formula of bulk viscosity in Ref. [91], we replace the original term \( [c_s^2 (p^2 - m_f^2)]^2 \) in the integrand with \( [c_s^2 (p^2 - m_f^2 + c_s^2 m_f T \frac{dm_f}{dT})]^2 \) to incorporate the in-medium effect. In the treatment of the relaxation time \( \tau_{q,f} \), we roughly take a constant value \( \tau_{q,f} = 1 \text{ fm} \) for the computation. In the weakly anisotropic system, the former terms in Eqs. (32)-(34) are significantly larger than the latter terms in magnitude due to the difference in momentum power of respective integrand. Therefore, transport coefficients are still mainly dominated by the first term of related expressions on the quantitative.

The variation of shear viscosity \( \eta \) with temperature at vanishing quark chemical potential for both isotropic and anisotropic quark matter is shown in Fig. 8. We see that \( \eta \) in the (an-)isotropic quark matter rises monotonically with increasing temperature because the \( T \) dependence of \( \eta \) is mainly coming from the quark distribution function \( f_{q,f}^0 \) in the associated integrand. For the qualitative behavior of \( \eta \) with \( \xi \), we also can well understand from the associated expression. In the vicinity of the chiral critical temperature \( T_{ch} \), \( \eta \) slightly decreases as \( \xi \) grows due to decreasing behavior of the Boltzmann factor \( e^{-m_f(T)/T} \) with \( \xi \). In higher temperature domain \((T > 160 \text{ MeV})\), the decreasing feature of \( \eta \) is negligible due to the unsensitivity of the constituent quark masses to \( \xi \). However, the absolute value of the second term in Eq. (32) significantly increases with an increase of \( \xi \). As a result, \( \eta \) decreases as \( \xi \) grows. This is similar to the result in Ref. [91], where \( \eta \) for the QGP is calculated in quasiparticle model. For electrical conductivity \( \sigma_{el} \), its thermal behavior is similar to \( \eta \), the quantitative difference between \( \eta \) and \( \sigma_{el} \) is mainly coming from the different momentum power of respective integrand. Similar to shear viscosity, the \( \xi \) dependence of \( \sigma_{el} \) is also determined by the second term in the associated expression. In Fig. 9, we observe that \( \sigma_{el} \) decreases as \( \xi \) increases, which is also qualitatively consistent with the results of \( \sigma_{el} \) for the QGP in quasiparticle model [89, 92]. The dependences of \( \eta \) and \( \sigma_{el} \) on momentum-space anisotropy are different from those on finite system size \( L \) in the framework of (P)NJL model. In Ref. [42], both \( \eta \) and \( \sigma_{el} \) first increase as \( L \) decreases in low temperature domain, whereas, the size effect nearly vanishes in high temperature domain. Furthermore, the result in Ref. [58]
FIG. 8. The temperature dependence of shear viscosity $\eta$ at $\mu = 0$ MeV in quark matter with $\xi = -0.4$ (orange dotted-dashed line), 0.0 (blue dashed line), 0.2 (red solid line), 0.4 (green wide dashed line).

FIG. 9. The temperature dependence of electrical conductivity $\sigma_{el}$ at $\mu = 0$ MeV in quark matter with $\xi = -0.4$ (orange dotted-dashed line), 0.0 (blue dashed line), 0.2 (red solid line), 0.4 (green wide dashed line).

FIG. 10. The temperature dependence of bulk viscosity $\zeta$ at $\mu = 0$ MeV in quark matter with $\xi = -0.4$ (orange dotted-dashed line), 0.0 (blue dashed line), 0.2 (red solid line), 0.4 (green wide dashed line).

Next, we discuss the temperature dependence of bulk viscosity $\zeta$ at zero quark chemical potential for both isotropic and anisotropic quark matter. As shown in Fig. 10, for a fixed anisotropy parameter, $\zeta$ is peaking up in the vicinities of both $T_{\chi c}$ and $T_{\chi s}$, which is significantly different to the thermal behaviors of $\eta$ and $\sigma_{el}$. We also note that the thermal profile of $\zeta$ is similar to $dm_s/dT$ or $\chi_s$, which may be attributed that the qualitative behavior of $\zeta$ is mainly governed by $dm_s/dT$ rather than the quark distribution function in associated integrand of Eq. (34). Due to the decreasing feature of the peak of $dm_s/dT$ with increasing $\xi$, the double-peak structure of $\zeta$ can be weakened as $\xi$ grows and the positions of peaks shift to higher temperatures, as shown in Fig. 10. The diluting effect of $\xi$ on the critical behavior of $\zeta$ is similar to the studies regarding finite volume effect and non-extensive effect. In Ref. [42], the double-peak structure of $\zeta$ even converts to one broadened peak structure when the system size is reduced to 2 fm. And in Ref. [58], as non-extensive parameter $q$ increases to 1.1, the two peaks of $\zeta$ also begin to merge into a broad one.

V. SUMMARY AND CONCLUSION

In this work, an anisotropy parameter $\xi$, which reflects the degree of momentum-space anisotropy arising from different expansion rates of the fireball generated in HICs along longitudinal and radial direction, for the first time, is introduced in the 2+1 flavor quark-meson model by replacing the isotropic distribution function in the thermodynamic potential of the quark-meson model with the anisotropic one. The effect of $\xi$ on the chiral properties, thermodynamics, meson masses, and transport properties in quark matter are investigated. We find that the chiral phase transition of quark matter with different anisotropy parameters is always a crossover at vanishing quark chemical potential. At finite quark chemical potential, the temperature of the CEP is affected more significantly by the anisotropy parameter than its quark chemical potential, which is opposite to the study of finite volume effect. We also demonstrate that at high temperature, the limit values of various scaled thermodynamics ($P/T^4$, $s/T^3$, $\epsilon/T^4$, $C_V/T^3$) are quite sensitive to $\xi$. As $\xi$ increases, their limit values decrease, which is different to the finite size effect but rather similar to non-extensive effect. And the critical behavior of $C_V/T^3$ and $\epsilon^2$ can be smoothed out with increasing $\xi$. For scalar and pseudoscalar mesons, the
temperature, where their masses begin to degenerate, is enhanced as $\xi$ rises, which implies that an increase of $\xi$ can hinder the restoration of chiral symmetry. Finally, the transport coefficients, such as shear viscosity $\eta$, electrical conductivity $\sigma_{el}$, and bulk viscosity $\zeta$ for both isotropic and anisotropic quark matter, are also calculated. Our results show that $\eta$ and $\sigma_{el}$ rise with increasing temperature, while the thermal behavior of $\zeta$ exhibits a noticeable double-peak structure. It is seen that $\eta$ and $\sigma_{el}$ decrease monotonically as $\xi$ increases, whereas the qualitative behavior of $\zeta$ with $\xi$ is similar to $\chi_s(\xi)$. With increasing $\xi$, the double-peak structure of $\zeta$ can be weakened, and the positions of peaks shift to higher temperatures.

In present work, we only focus on the chiral aspect of the QCD phase diagram, the exploration of the confinement phase transition in an anisotropic quark matter also can be addressed via including the Polyakov-loop potential. In the Polyakov-loop improved quark-meson model, the chiral phase transition and the location of the CEP will be affected further. For the calculation of transport coefficients in this study, the relaxation time of quark is assumed to be a constant. However, in the realistic interaction scenario, the relaxation time may also vary with the momentum anisotropy. These issues are our future research directions. Moreover, note that a spheroidal momentum-space anisotropy specified by one anisotropy parameter in one preferred propagation direction is considered in this work, however, the introduction of additional anisotropy parameters is necessary to provide a better characterization of the QGP properties. The chiral and confinement phase transitions in quark matter with ellipsoidal momentum-anisotropy [78, 79] characterized by two independent anisotropy parameters, also can be done using PNJL or PQM model. The works of these directions are in progress and we expect to report our results soon.

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