Extra charmonium states as bag-quarkonia

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Exotic states in the charmonium family are systematically treated in the framework of simplest model with an effective coulomb-like interaction of heavy quark and antiquark in the presence of static excitation of quark-gluon modes responsible for a nonperturbative term of potential, which provides with the confinement of quarks, in terms of bag over the threshold in the excitation spectrum of vacuum fields. Once the spectrum has got quite a wide mass gap, it allows us to approximate the bag contribution into the potential by a constant value of bag mass at low distances less than the bag size. The bag mass can be evaluated in a constituent model. The analysis is given for the bag contribution into the distribution over the invariant mass of two pions in the hadronic trajectories in the spectroscopy of hadrons composed of light quarks,

\[ V_{\text{Cornell}}(r) = \frac{4}{3} \, \alpha_{\text{eff}}^2 \, \frac{1}{r} + \sigma \cdot r, \]

\[ V(r) \approx 2T \ln \left( \frac{r}{r_0} \right), \]

where the factor of \( \frac{4}{3} \) is caused by projecting to the color-singlet state of quark-antiquark system. The value of effective constant has been determined empirically by the excitation spectrum in the charmonium family, while \( \sigma \approx 0.18 \text{ GeV}^2 \). Average sizes \( \langle r \rangle \) of charmonium states are positioned in the intermediate region of transition from the perturbative regime to the nonperturbative one, so that the potential is quite accurately approximated by the logarithmic function versus the distance \( r \) with a dimensional parameter \( T \) determining an average kinetic energy in the heavy quarkonium in accordance with the virial theorem:

\[ V\langle r \rangle \approx 2T \ln \left( \frac{r}{r_0} \right). \]

whereas \( T \approx 0.38 \text{ GeV} \), so that \( T \sim \Lambda_{\text{QCD}} \).

Therefore, a characteristic square of heavy quark momentum in such the quarkonium is determined by the product of kinetic energy by the reduced mass equal to

\[ 1 \text{ A weak dependence of average kinetic energy on the heavy quark mass is phenomenologically taken into account in the Martin model of potential [10], functionally behaving as the power law } r^k \text{ at } k \to 0, \text{ that corresponds to a small perturbation of logarithmic form in the region of approximation.} \]
of heavy quark motion, gluon fields in the potential description is determined \( \tau \) vacuum fields are characterized by the time interval of \( \Lambda \) gluon condensates with the energy scale of the order of heavy quark mass, for which the description can be done in terms of potential models with account of QCD condensates: \( m_{\text{min}} \sim 20 \text{ GeV} \), which is in evident contradiction with the applicability of potential models for the both charmed and bottom quarks, since their kinetic energy has the same order of magnitude with the scale \( \Lambda_{\text{QCD}} \), as we have seen above. Thus, the motion of such heavy quarks could lead to the excitation of nonzero vacuum fields from the condensates, so that the interaction in the system evidently becomes nonstatic, which contradicts with the success of potential framework in the spectroscopy of charmonium and bottomonium.

The solution of this paradox comes from the quantum nature of quark-gluon fields. The speculations presented above suggest that the spectrum of excitations for the vacuum fields is continuous and its threshold is determined by the energy scale of \( \Lambda_{\text{QCD}} \), while the actual spectrum could by essentially different. At first, the excitation spectrum for the vacuum fields can begin with a wide mass gap, that is enough in order to substantiate the applicability of potential framework to the heavy quarkonium states below the threshold fixed by the mass gap. The threshold could be positioned below the threshold for the pair production of hadrons with the open charm (the production of D-meson pairs in the system of heavy quark and antiquark), or it could be very close the continuous threshold. Then, the second aspect becomes important: the spectrum of excitations for the vacuum fields in the presence of heavy quarkonium could be discrete in vicinity of mass gap, at least in the sense of quasistationary

\[
\tau_{\text{QCD}} \gg \tau_Q \quad \Leftrightarrow \quad m_Q v_Q^2 \gg \Lambda_{\text{QCD}}.
\]

In other words, the potential description can be considered as rather reasonable and theoretically sound, if only the kinetic energy of heavy nonrelativistic quarks is much greater the energy scale fixing the quark-gluon condensates. This fact was recognized by M.B.Voloshin\(^4\) [1, 2], who gave the estimate for a minimal acceptable value of heavy quark mass, for which the description can be done in terms of potential models with account of QCD condensates: \( m_{\text{min}} \sim 20 \text{ GeV} \), which is in evident contradiction with the applicability of potential models for the both charmed and bottom quarks, since their kinetic energy has the same order of magnitude with the scale \( \Lambda_{\text{QCD}} \), as we have seen above. Thus, the motion of such heavy quarks could lead to the excitation of nonzero vacuum fields from the condensates, so that the interaction in the system evidently becomes nonstatic, which contradicts with the success of potential framework in the spectroscopy of charmonium and bottomonium.

The further progress in the development of potential approach was related with the introduction of running coupling in QCD at small distances in 1-, 2- and 3-loop order: the potentials by Richardson [11], Buchmüller and Tye [12] and Kiselev, Kovalsky and Onishchenko [13, 14], correspondingly, consistently with linearly rising term confining the quarks. This way has allowed us to remove a discrepancy between direct measuring the coupling of \( \alpha_s \) and its value, following from fitting the real spectrum of quarkonia in the potential model with the running coupling constant only at the 3-loop approximation\(^2\).

However, the theoretical eligibility of potential description as the framework seemed to become questionable, when one clarified that the QCD vacuum has got a nontrivial structure, namely, it is composed of quark-gluon condensates with the energy scale of the order of \( \Lambda_{\text{QCD}} \sim 0.3 \text{ GeV} \) [12]. Therefore, fluctuations of vacuum fields are characterized by the time interval of \( \tau_{\text{QCD}} \sim 1/\Lambda_{\text{QCD}} \), while the relaxation time of quark-gluon fields in the potential description is determined by the ratio of heavy quarkonium size to the velocity of heavy quark motion,

\[
\tau_Q \sim \frac{r_{Q\bar{Q}}}{v_Q},
\]

so that by taking into account

\[
\tau_{Q\bar{Q}} \sim \frac{1}{m_Q v_Q},
\]

we get

\[
\tau_Q \sim \frac{1}{m_Q v_Q^2}.
\]

The vacuum fluctuations are irrelevant to the potential framework, if the quarks can be considered as static and their interaction is practically instantaneous in comparison with the influence of nonperturbative effects, i.e.

\[
\tau_{\text{QCD}} \gg \tau_Q \quad \Rightarrow \quad m_Q v_Q^2 \gg \Lambda_{\text{QCD}}.
\]

The value of coupling constant is inherently related with the splitting between the 1S and 2S levels in the quarkonia. At one and two-loop orders the coupling constant from the potential model has got a too high value beyond the interval obtained by the direct measurement of \( \alpha_s \).

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\(^3\) One could say that the potential would change due to the vacuum fluctuations very slow in comparison with a period of finite motion of heavy quarks, i.e. the potential is static, indeed.

\(^4\) In fact, M.B.Voloshin actually formulated the criterium of applicability for the multipole approximation in QCD in the presence of vacuum fluctuations in the form of condensates: \( \tau_Q/\tau_{\text{QCD}} \ll 1 \), while he believed that the necessary constraint for the introduction of static potential is the instantaneous interaction between the nonrelativistic quarks, that oppositely requires \( \tau_Q/\tau_{\text{QCD}} \gg 1 \), by his opinion, and hence, it is evidently incompatible with the criterium of multipole expansion. So, he suggested that the introduction of static nonperturbative potential is principally impossible for real charmed and beauty quarks except the exotic case of states in vicinity of continuous threshold.

\(^5\) The potential approach is completely substantiated in the effective theory, the so-called potential nonrelativistic QCD (pNRQCD) [16].
description during the times comparable with the period of heavy quark motion in the bound state. Thus, it would be quite reasonable to introduce the notion of “bag” for the lowest excited state of vacuum fields in the system under consideration. The bag evidently has got no valence degrees of freedom (isospin, charge etc. are equal to zero) and it possesses the vacuum quantum numbers (spatial and charge parities are positive) except the energy or mass. The existence of bag is caused by the presence of valence quark and antiquark. In the absence of valence quarks, the introduction of bag has no sense. Therefore, the bag is not related with notion of glueballs or hybrids as particles. In this respect, it would be incorrect to associate the bag with a separate quasistationary hadron-like object, which interacts with other hadrons, for instance, with the quarkonium, since the bag itself is not stationary without the quarkonium. Thus, the system of bag-quarkonium differs from the hadro-quarkonium recently introduced by M.B.Voloshin [17, 18], since the system of hadro-quarkonium is considered as the pair of stationary objects, which allow the separated existence and interact a la the Van Der Waals forces with each other.

To my opinion, the influence of essentially nonperturbative phenomenon called the bag on the system of heavy quarkonium is reduced to the followings:

- below the threshold of bag excitation, the heavy quarkonium permits the potential description, wherein the potential confining the quarks is phenomenologically approximated by the realistic term linearly raising with the distance increase;
- above the threshold of exciting the bag, the nonperturbative contribution is modified: if the heavy quarkonium size is less the size of bag $r_{bag}$, then the bag presence is given by introducing the bag mass itself $E_{bag}$ into the potential, while, if the quarkonium size becomes to exceed the bag size, then the linearly raising term of confining potential is activated again.

This situation is schematically shown in Fig. 1 wherein the nonperturbative potential of heavy quarks in the bag is pictured. By the way, we assume that further excitations of vacuum fields are separated from the bag by a mass gap, of course.

The shift of linearly raising asymptote of nonperturbative term in the presence of bag is determined by an interplay between its size and mass. However, it is more essential that, if the size of heavy quark-antiquark system is less than the bag size, i.e. when one considers the lowest states of bag-quarkonium system, then the interaction of heavy quark and antiquark “to leading approximation” can be written in the form of following potential:

$$V^{\text{eff}}(r) = -\frac{4}{3} \frac{\alpha_s^{\text{eff}}}{r} + E_{\text{bag}},$$

where $\alpha_s^{\text{eff}}$ is the effective constant of single gluon exchange between the heavy quarks at the scale characterizing the quark-antiquark system. Here we neglect the linearly raising contribution in comparison with the bag mass. Thus, we arrive to three-parametric spectral problem in the nonrelativistic quantum mechanics: calculate the masses of states in the system of bag-quarkonium with the coulomb-like attraction at the given mass of heavy quark, effective constant $\alpha_s^{\text{eff}}$ and bag mass $E_{\text{bag}}$.

In Section IV we present our treatment of extraordinary states in the charmonium family in the framework of bag-quarkonium system, that allows us to numerically estimate phenomenological parameters from the empirical data. The parameters are positioned in a region of quite reasonable values. In this way we refer to the joint table of extra states of charmonia as given in [19]: Tab. II. We exclude the mesons with the hidden strangeness $Y_s(2175)$ and beauty $Y_b$ as well as the too broad state of $\Upsilon(4008)$ presented in the table, and remain them beyond our consideration.

The modification of hadronic transitions from the bag-quarkonium states to the ordinary quarkonium with emission of pion pair is investigated in Section V. The presence of bag involves a new term in the amplitude in comparison with transitions between the ordinary S-wave states of heavy quarkonium, that changes the distribution versus the invariant mass of two pions, i.e. it can serve as the reason for the observed anomalous behavior of distribution in transitions of $\psi(3770) \rightarrow J/\psi \pi \pi$ and $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi \pi$. This fact is the argument in favor of our assignment of $\psi(3770)$ and $\Upsilon(3S)$ as basic vector states in the system of bag-charmonium and bag-bottomonium, respectively.

Section VI is devoted to the analysis of leptonic constants of vector states in the charmonium family in the framework of quasilocal sum rules [20]. The study shows that widths of observed leptonic decays of vector states
TABLE I: The joint table of candidates in members of charmonium family: mesons XYZ, as copied from [10].

| state   | mass M (MeV) | width Γ (MeV) | $J^{PC}$ | decay modes                                                                 | production processes                      |
|---------|--------------|---------------|----------|------------------------------------------------------------------------------|--------------------------------------------|
| $Y_c(2175)$ | 2175 ± 8     | 58 ± 26       | 1−−     | $\phi f_0(980)$                                                            | $\pi^+\pi^-J/\psi, \gamma J/\psi$        |
| $X(3872)$   | 3871.4 ± 0.6 | < 2.3         | 1++     | $\pi^+\pi^-J/\psi, \gamma J/\psi$                                        | $B \to KX(3872), p\bar{p}$               |
| $X(3875)$   | 3875.5 ± 1.5 | 3.0±1.7       | 2++     | $D^0\bar{D}^0\pi^0$                                                       | $B \to KX(3875)$                          |
| $Z(3940)$   | 3929 ± 5     | 29 ± 10       | 2++     | $D\bar{D}$                                                                | $\gamma \gamma$                           |
| $X(3940)$   | 3942 ± 9     | 37 ± 17       | $J^{P+}$ | $D\bar{D}^*$                                                               | $\pi^+\pi^-\to J/\psi X(3940)$           |
| $Y(3940)$   | 3943 ± 17    | 87 ± 34       | $J^{P+}$ | $\omega J/\psi$                                                           | $B \to KY(3940)$                          |
| $Y(4008)$   | 4008±69      | 226±97        | 1−−     | $\pi^+\pi^- J/\psi$                                                       | $\pi^+\pi^- (ISR)$                       |
| $X(4160)$   | 4156 ± 29    | 139±113       | $J^{P+}$ | $D^*\bar{D}^*$                                                             | $\pi^+\pi^- J/\psi (ISR)$                |
| $Y(4260)$   | 4264 ± 12    | 83 ± 22       | 1−−     | $\pi^+\pi^- J/\psi$                                                       | $\pi^+\pi^- (ISR)$                       |
| $Y(4350)$   | 4361 ± 13    | 74 ± 18       | 1−−     | $\pi^+\pi^- J/\psi$                                                       | $\pi^+\pi^- (ISR)$                       |
| $Z(4430)$   | 4433 ± 5     | 45±18         | ?        | $\pi^+\pi^- J/\psi$                                                       | $B \to K\bar{Z}(4430)$                   |
| $Y(4660)$   | 4664 ± 12    | 48 ± 15       | 1−−     | $\pi^+\pi^- J/\psi$                                                       | $\pi^+\pi^- (ISR)$                       |
| $Y_b$       | ̃10.870      | ?             | 1−−     | $\pi^+\pi^- \bar{Y}(nS)$                                                  | $\pi^+\pi^- (ISR)$                       |

do not contradict with introducing the extra states of bag-charmonium. Moreover, the consistency of predictions with the experimental data is systematically improved.

In Conclusion we summarize the obtained results and discuss their possible implications to the study of mechanisms for the production and decays of extra heavy-quarkonium states.

II. ANALYSIS OF EXOTICA IN THE SPECTRUM OF CHARMONIUM

The spectrum of energy in the problem with coulomb interaction is well known. For the system with the reduced mass $m_{\text{red}} = \frac{1}{2}m_Q$, the spin-dependent term due to effective single-gluon exchange takes the form

$$V_{SD} = \frac{4}{3} \alpha_s \epsilon \frac{8\pi}{3} \frac{1}{m_Q^2} (s_1 \cdot s_2) \delta(r) + 2\frac{\alpha_s}{m_Q^2} \frac{1}{m_Q^2} (L \cdot S)^{1/r^4} + \frac{4}{3} \frac{\alpha_s}{m_Q^2} \frac{1}{m_Q^2} \{3(s_1 \cdot n)(s_2 \cdot n) - (s_1 \cdot s_2)\} \frac{1}{r^4},$$  (8)

In [8], the first term corresponds to spin-spin “contact” interaction of quark and antiquark, the second represents the spin-orbital interaction, the third gives the tensor forces. Here $r = r_1 - r_2$ is the relative distance between the quarks, the unit vector is $n = r/r$, $L$ denotes the summed orbital momentum of quarks, and $S = s_1 + s_2$ gives the summed spin. The masses of levels $n^{2S+1}L_J$ with $n$ and $J$ being the principal quantum number and total momentum are given by

$$M_{bag-Q\bar{Q}}[n^{2S+1}L_J] = 2m_Q + E_{bag} - \frac{m_Q}{4n^2} \left(\frac{4}{3} \frac{\alpha_s}{\epsilon}\right)^2 + \frac{m_Q}{6n^3} \left(\frac{4}{3} \frac{\alpha_s}{\epsilon}\right)^4 \left\{S(S + 1) - \frac{3}{2}\right\} \delta_{L0}$$

$$+ \frac{m_Q}{n^3} \left(\frac{4}{3} \frac{\alpha_s}{\epsilon}\right)^4 \frac{1}{(2L + 1)((2L + 1)^2 - 1)} \times$$

$$\left\{\frac{3}{2} (L \cdot S) - \frac{1}{4L(L + 1) - 3} \left(3(L \cdot S)^2 + \frac{3}{2} (L \cdot S) - L^2 S^2\right)\right\},$$  (9)

whereas

$$(L \cdot S) = \frac{1}{2}\{J(J + 1) - L(L + 1) - S(S + 1)\},$$

$L^2 = L(L + 1)$, $S^2 = S(S + 1)$. The spatial and charge parities, respectively given by $P = (-1)^{L+1}$ and $C = (-1)^{S+L}$. Particularly, at $L = 0$,
the masses of vector states $J^{PC} = 1^{--}$ are equal to
\[
M_{\text{bag-QQ}}[n^3 S_1] = 2m_Q + E_{\text{bag}} - \frac{m_Q}{4n^2} \left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^2 + \frac{m_Q}{12n^4} \left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^4,
\]
while at $L = 1$, the masses of spin-triplet $J^{++}$-states are equal to
\[
M_{\text{bag-QQ}}[n^3 P_J] = 2m_Q + E_{\text{bag}} - \frac{m_Q}{4n^2} \left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^2 + \frac{m_Q}{24n^4} \left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^4 \left\{ \begin{array}{ll}
+ \frac{2}{7}, & J = 2, \\
-1, & J = 1, \\
-4, & J = 0.
\end{array} \right.
\]

The review of charmonium family is presented in [17,19]. We assign $\psi(3770)$ as the basic extraordinary vector state, which is distinguished by the anomalous distribution in the spectrum of invariant masses for two pions in the transition $\psi(3770) \rightarrow J/\psi \pi \pi$. One usually considers that this state is positioned in vicinity of $1D$ level of quark-antiquark system $c\bar{c}$, and its nonzero width of decay into lepton pair is caused by a mixing with the nearest $S$-wave level $\psi' = \psi(2S) = \psi(3680)$. Then, one can treat it as the ordinary state, and the anomaly mentioned has a reason, which is, for instance, an occasional suppression of dominant term in the creation of pion pair (see the analysis in [17]). However, in Section 11 we argue in favor of that the anomalous distribution versus the invariant mass of two pions is the natural consequence, if one assigns $\psi(3770)$ as the system of bag-charmonium. Therefore, we identify $\psi(3770)$ with the state $1^{--}$ in the system of bag-charmonium with the quantum numbers $1^3 S_1$.

The other benchmark of exotic state is $X(3872)$, which has got the definite quantum numbers $1^{++}$. This state has to be identified with the first excitation at the same quantum numbers in the system of bag-charmonium, i.e. $2^3 P_1$. Thus, in order to determine the parameters of system we have got two levels with the mass difference
\[
\Delta M = \frac{3m_c}{16} \left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^2 \left\{ 1 - \frac{17}{36} \left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^2 \right\},
\]
that numerically gives $\Delta M = 102$ MeV. Eq. 12 allows us to write down the effective constant as the function of charmed quark mass
\[
\left( \frac{4}{3} \alpha_s^{\text{eff}} \right)^2 = \frac{18}{17} \left( 1 - \sqrt{1 - \frac{1802 \Delta M}{27 m_c}} \right).
\]

Note that the obtained behavior of effective constant versus the charmed quark mass in the limits from 1.3 to 1.8 GeV is in a good agreement with the 1-loop dependence of running coupling constant in QCD
\[
\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln[\mu/\Lambda_{QCD}]} \left( 1 - \frac{\beta_1}{\beta_0^2} \ln[\mu^2/\Lambda_{QCD}^2] \right),
\]
if one puts that the scale $\mu$ is determined by the characteristic momentum of heavy quark in the heavy quarkonium, i.e.
\[
\mu^2 = m_Q T,
\]
wherein $T \approx 0.4$ GeV, $\Lambda_{QCD} \approx 190$ MeV, and
\[
\beta_0 = 11 - \frac{2}{3} n_f
\]
at the number of quark flavors contributing to the renormalization for the scales in problem, $n_f = 3$. The fact of consistency between the empirical dependence of [13] and renormalization group (RG) one of [14]–[15] is illustrated in Fig. 2 wherein the ratio of RG constant to the effective one is shown after the normalization at $m_c \approx 1.65$ GeV.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{The ratio of running coupling constant in QCD to the effective constant, calculated in accordance with [12] and [13]: the 1-loop approximation is given by the solid line, the 2-loop curve is show by dotted line, the dashed line refers to the value equal to 1.}
\end{figure}

The figure shows that in the wide interval of pole mass for the charmed quark, the effective constant coincides with the RG running constant within the accuracy better than 5%. The agreement between these two values becomes more spectacular in the 2-loop approximation for the running coupling constant in QCD
\[
\alpha_s^{(2-lo)}(\mu) = \frac{2\pi}{\beta_0 \ln[\mu/\Lambda_{QCD}]} \left( 1 - \frac{\beta_1}{\beta_0^2} \ln[\mu^2/\Lambda_{QCD}^2] \right),
\]
where $\beta_1 = 51 - 19n_f/3$, after the appropriate correction of $\Lambda_{QCD}$. However, the usage of 2-loop approximation is evidently beyond the accuracy of consideration, since the model potential does not take into account the dependence of effective constant versus the distance in the second order over $\alpha_s$. 

Nevertheless, we can establish that the running coupling constant of QCD at the scale fixed by the heavy quark mass and its kinetic energy slowly depending on the mass, allows us to get the empirical value for splitting the masses of lightest vector and pseudovector states in the system of bag-charmonium.

A. The pole mass of heavy quark

It is spectacular (see Fig. 3) that the sum of pole masses for charmed quark and antiquark with the bag mass is practically independent of the pole mass of charmed quark with the accuracy better than 5 MeV,

$$\mathcal{E}_{\text{IRstab}} = 2m_c + E_{\text{bag}} = \text{const.},$$  \hspace{1cm} (16)

after the implication of (10)–(13).

![FIG. 3: The sum of bag mass and double mass of charmed quark versus the pole mass as the result of fitting the masses of lightest vector and pseudovector states of bag-charmonium in accordance with (10)–(13).](image)

Numerically, we find

$$\mathcal{E}_{\text{IRstab}} \approx 3915 \text{ MeV}. \hspace{1cm} (17)$$

The constant value of this quantity points to that the infrared instability of heavy-quark pole mass is exactly cancelled by the infrared instability in the static potential approximated by the dominant contribution of bag mass in the case under consideration. The instability of pole mass appears in the perturbation theory of QCD in the form of renormalon \cite{21, 22, 23, 24, 25, 26, 27}, that exhibits the factorial growth of coefficients in the perturbative series as dictated by the presence of infrared pole in the coupling constant of QCD at the scale of $\Lambda_{\text{QCD}}$. Thus, the observed quantity being the mass of bound state for the two heavy quarks and bag is actually stable in the infrared region, that we mark by the subscript in (16). The value of charmed quark mass itself is completely determined by the separation of bag mass in the approximation considered.

In this respect it would be interesting to make the following analogy: at first, consider the light quarkonium with the ordinary $u$- and $d$-quarks instead of the heavy quarkonium. Let us neglect the light quark masses as well as the energy of their kinetic motion, if there is no orbital rotation. In the basic $S$-wave state the mass of such the quarkonium with no account of spin-dependent interactions is determined by the bag mass

$$m(1S) \approx E_{\text{bag}},$$  \hspace{1cm} (18)

i.e. the state itself is the bag with the valence quantum numbers given by the light quarks. In the constituent framework the bag energy fixes the constituent mass of light quark by $m_\text{const.} \approx \frac{1}{2}E_{\text{bag}}$. Such the bag-quarkonium after the introduction of spin-dependent forces is observed as the pseudoscalar $\pi$- and vector $\rho$-mesons. Therefore, the standard procedure of cancelling the contribution of spin-dependent energy leads to

$$m(1S) \approx \frac{1}{4}(3m_\rho + m_\pi) \approx 612 \text{ MeV},$$

that gives the approximate value of bag mass in accordance with (18).

Second, the same way can be applied to the consideration of “constituent model” of quarkonium composed of light and heavy quarks. Then, in the basic $S$-wave state with no account for the spin-dependent forces, such the heavy-light bag-quarkonium has got the mass equal to

$$M_{\text{qq}}[1S] \approx m_Q + m_\text{const.} \approx m_Q + \frac{1}{2}E_{\text{bag}}.$$

(19)

Particularly, for the basic level of D-mesons we get

$$M_D[1S] \approx \frac{1}{2}\mathcal{E}_{\text{IRstab}} \approx 1958 \text{ MeV}, \hspace{1cm} (20)$$

that should be compared with the experimental value averaged over the spins for neutral D-mesons, for instance,

$$M_D^{\text{exp}}[1S] = \frac{1}{4}(3m_{D^+} + m_D) \approx 1972 \text{ MeV}. \hspace{1cm} (21)$$

Comparing (21) with (20) shows that the speculations in the framework of constituent model or bag-quarkonium allow us to make the calculations with the accuracy about 15 MeV, characteristic for the potential models in general.

Thus, in the framework under study we prefer for prescribing the pole mass of charmed quark to the value equal to

$$m_c \approx \frac{1}{2}[\mathcal{E}_{\text{IRstab}} - m(1S)] \approx 1650 \pm 15 \text{ MeV}, \hspace{1cm} (22)$$

and analogously, fixing the pole mass of bottom quark by the value equal to

$$m_b \approx m_b^{\text{exp}}[1S] - \frac{1}{2}m(1S)$$

$$\approx m_c + m_b^{\text{exp}}[1S] - m_D^{\text{exp}}[1S] \hspace{1cm} (23)$$

$$\approx 4995 \pm 15 \text{ MeV},$$
which surprisingly agree with the values obtained in the framework of analysis performed for charmonium and bottomonium described by the dominant coulomb interaction with account for corrections both in the perturbation theory and due to the quark-gluon condensates \[28, 29, 30, 31\]. Then, after the rescaling the effective constant of coulomb-like interaction by the renormalization group law to the system of bottomonium, we can calculate the mass of basic vector state of bag-bottomonium, as shown in Fig. 4

\[ M_{\text{bag}}^{[1^3S_1]} \approx 10355 \pm 30 \text{ MeV}. \] (24)

Note, that the same result is achieved, if we vary the pole mass of charmed quark in the limits from 1.5 to 1.7 GeV with the consistent change of \( b \)-quark pole mass as well as the value of effective constant for the coulomb-like interaction.

\[
M_{\text{bag}}^{[1^3S_1]}, \text{ GeV} \\
\begin{array}{|c|c|c|c|}
\hline
m_b, \text{ GeV} & 4.96 & 4.98 & 5 \\
\hline
10.2 & 10.25 & 10.3 & 10.35 & 10.4 & 10.45 & 10.5 \\
\hline
\end{array}
\]

**FIG. 4:** The calculation of mass for the basic vector state in the system of bag-bottomonium. The vertical band shows the permitted values of \( b \)-quark pole mass, the horizontal line marks the experimental value of \( \Upsilon(3S) \) mass, the sloped band restricts the prediction with the variation of effective constant in limits 0.423 < \( \frac{1}{3} \alpha_s^{\text{eff}} < 0.484 \) in the bag-bottomonium system, the bright sloped line gives the prediction for the mass at the constant obtained by the rescaling of its value from the value for the bag-charmonium in accordance with the renormalization group law.

At the pole masses fixed above, the coupling constants are equal to

\[
\alpha_s^{\text{eff}}[c\bar{c}] \approx 0.479, \quad \alpha_s^{\text{eff}}[b\bar{b}] \approx 0.347. \] (25)

Characteristic sizes of heavy quark and antiquark system are determined by the “Bohr radius”

\[
a = \frac{3}{2m_Q \alpha_s^{\text{eff}}}, \] (26)

which is, for example, approximately equal to \( a \approx 0.38 \) fm at \( m_c = 1.65 \) GeV. Remember, that, as well known, the wave functions of states bound in the coulomb potential exponentially decline versus the distance with the damping lengths of \( a n \), where \( n \) is the principal quantum number, so that for the bag-charmonium this length is less than 1 fm at \( n = 1, 2 \). Supposing that the bag itself has the size somewhat greater than 1 fm, we draw the conclusion that the approximation introduced for the potential as the sum of coulomb term and bag mass is quite applicable for the low-lying levels of bag-charmonium.

**B. States at \( n = 1, 2 \)**

Fixing the model parameters by means of considering the splitting between the two lowest vector and pseudovector levels of bag-charmonium at \( n = 1, 2 \) allows us to make spectroscopic predictions for further members of family with the same values of principal quantum number. The uncertainty of such the prediction is about 5 MeV. The state masses are listen in Tab. II. Emphasize that for the coulomb-like states the splitting of levels due to the forces depending on the quark spins approximately decreases with the growth of principal quantum number by the scaling law \( 1/n^3 \), that reasonably agrees with the data in Tab. II.

| \( J^{PC} \) | \( n = 1 \) | \( n = 2 \) |
|---|---|---|
| 0^+ | 3678 | 3685 |
| 1^− | 3770 | 3876 |
| 2^+ | - | 3875 |
| 1^+ | - | 3872 |
| 0^+ | - | 3868 |
| 1^− | - | 3873 |

**TABLE II:** Masses of bag-charmonium at \( n = 1, 2 \) in MeV.

Two of listened states have been associated with the following ones:

\[
1^{−}[1^3S_1] \rightarrow \psi(3770), \quad 1^+[2^3P_1] \rightarrow X(3872),
\]

the quantum numbers \( J^{PC} \) of which are established experimentally, while the exotic level \( X(3875) \) with unidentified quantum numbers could get the two following prescriptions:

\[
X(3875) \left\langle 1^{−}[2^3S_1], \quad 2^{++}[2^3P_2].
\]

---

6 Once such the basic vector state coincides with the position of \( \Upsilon(3S) \), possessing the anomalous properties in two-pion transition into the low-lying vector levels of bottomonium, while its first excitation is positioned at \( \Upsilon(4S) \) with the prediction accuracy of 30 MeV, though the splitting between the further excitations becomes less than the uncertainty of prediction, so that taking into account the width of \( \Upsilon(4S) \) comparable with the uncertainty of prediction, we do not make certain statement concerning for the definite correspondence of any exotic state to the accepted notation of \( \Upsilon(4S) \).
C. States at \( n > 2 \)

The accepted approximation for the potential as the sum of coulomb attraction and bag mass is valid at \( n = 1,2 \), but it can be modified in the case of higher excitations because of perturbation responsible for a “soft” transition into the regime of confinement, since at \( n = 3 \) the system composed of charmed quark and antiquark has got the size about 1 fm, i.e. it is close to the confinement scale, and hence, to the bag size itself. Therefore, we have to account for some other nonperturbative terms.

So, the presence of gluon condensate, i.e. an external field, leads to the contribution caused by the second order of perturbation theory due to the chromoelectric dipole interaction to the next-to-leading order in the velocity of heavy quarks, that has the form\(^7\)

\[
\delta M_n = \kappa_n (a_n)^2 \tag{27}
\]

where \( \langle \alpha_s G^2 \rangle \sim A_{QCD}^2 \) is the gluon condensate, \( \delta E_n \sim m_Q \langle \alpha_s G^2 \rangle / 4n^2 \) is the characteristic energy of binding the heavy quarks, and \( A \) denotes the numerical dimensionless factor caused by color and spatial effects. Then, at \( \Lambda_{QCD} \sim 0.2 \text{ GeV} \) in the case of bag-charmonium

we find the estimate \( \kappa_n \sim A n^2 m_Q a^2 \langle \alpha_s G^2 \rangle \sim A n^2 \times 10^{-3} \text{ GeV}^3 \). Remember that the positive value of gluon condensate corresponds to “negative” square of chromoelectric field: the intermediate state of quark-antiquark pair is the color octet with the subtraction between the quarks, hence, it has the energy greater than the bound states under study, that leads to positive value of correction to the mass of state\(^8\). In addition, the effective constant related with the gluon condensate can depend on the fact that the chromoelectric field is partially screened by the bag, which is the infrared object itself. This fact can lead to a dependence of effective value related with the condensate, that has the step-like form changing the value at a critical distance given by the bag size, so that \( \kappa \to \kappa (1 + \varepsilon \vartheta [n_s - n]) \).

Let us evaluate the effect of perturbation introduced in the case of neglecting the spin-dependent forces\(^9\) by setting

\[
\kappa_n = n^2 \kappa_0, \quad \kappa_0 = 1.1 \times 10^{-4} \text{ GeV}^3, \quad \varepsilon = 0.3, \quad 4 < n_s < 5.
\]

The value of \( \kappa_0 \) is in fact fixed by the position of higher excitations. It is important that the functional dependence of correction on the principal quantum number truly reflects the structure of extraordinary states in the charmonium family. The introduction of nonzero value for the parameter \( \varepsilon \) allows us to slightly displace the positions of levels at \( n = 3,4 \) by 10 and 30 MeV, respectively, that is comparable with the accuracy calculation of methods generally involved in the framework of potential models. Nevertheless, the physical meaning reflected by parameters \( \varepsilon \) and \( n_s \approx 4.5 \) is quite clear: the degree of screening the chromoelectric field by the bag is of the order of 30 %, and the bag size is about \( a n_s \approx 1.5 - 1.6 \text{ fm} \).

\footnote{7}{We deal with the general formula for the correction to the energy \( \delta M \sim \langle V \rangle / 4E \) with the perturbation due to the electric dipole \( V \sim g_{\alpha \sigma} E \), so that the charge squared gives the coupling constant \( g_{\alpha \sigma}^2 \sim \alpha_s \), the square of chromoelectric field is reduced to the square of gluon stress \( G_{\mu \nu} \), and the characteristic size gives the Bohr radius of \( n \)-th excitation \( r \to a_n \). Actually, the strict calculation of nonperturbative term due to the gluon condensate as well as the complete analysis of nonrelativistic quark-antiquark system with coulomb interaction was done in \[23, 24, 25, 31\] for the charmonium and bottomonium in the framework of multipole expansion of QCD \[3\]. The authors used exact formulae for the Green function of color-octet state, so that \( \delta M_{nl} = n^6 m_Q \langle \alpha_s G^2 \rangle a^4 \epsilon_{nl} / 16 \). At first, this correction gives sixth power of principal quantum number, in fact, because of averaging \( \langle r \rangle \to a_n \), valid for the coulomb potential (the exact formula \( \langle r \rangle = a [3n^2 - l(l + 1)] / 2 \) includes the dependence on the orbital momentum). Second, the numerical dimensional factor in front of correction is certainly definite in such the estimate. Third, one gets the opportunity to take into account for the orbital quantum number in terms of factor \( \epsilon_{nl} \approx 1.5 \), which value is also strictly known. However, we do not transfer such the analysis to the case of considering the bag-quarkonium, because we suggest that the bag essentially distorts the propagation of color-octet state, since it cuts off the wave functions in infrared, especially in the case of subtraction between the quark and antiquark. In this respect, our approach to the problem is less strict, but, to our opinion, it is more close to the actual physical situation, so that the fourth power in the dependence of correction on the principal quantum number is more realistic, while the numerical dimensional factor is fitted phenomenologically, because the modification of wave functions or Green function in infrared can essentially “renormalize” this factor.}

\footnote{8}{Correction \( \langle V \rangle \) can be roughly described by introducing the perturbation potential \( V = \kappa' r^2 \), so that for the coulomb functions of initial states the shift of energy is equal to \( \langle V \rangle = \kappa' \frac{1}{2} a^2 n^2 [5n^2 + 1 - 3L(L + 1)] \), \( 28 \), which repeats the general behavior versus the principal quantum number, of course. Eq. \( 28 \) supposes that the correction to the energy is determined by one and the same value of parameter \( \kappa' \) independent of the state, that is certainly too strong suggestion. Though, the final expression can be treated as the factorization of overall factor, while the residual dependence versus the parameters of state is close to the fourth power of principal quantum number. In \( 28 \) one could account for correction variation because of nonzero value of orbital momentum, but this piece of subtlety is evidently beyond the accuracy suggested in the derivation of correction itself.}

\footnote{9}{As we have already mentioned above, the splitting of higher excitations is significantly suppressed by the factor of \( \frac{1}{n^3} \), so that the variation of estimates due to the spin-dependent forces at \( n = 3 \) is less than 3 MeV, that is certainly below the uncertainty in the formula for the interpolation used.}
Then, we find that at \( n = 1, 2 \) the contribution of perturbation is less than 1 MeV, while for the higher excitations we get predictions corresponding to exotic states of charmonium:

\[
\begin{align*}
M_{\text{bag}}^{[c \bar{c}[n=3]]} & = 3939 \text{ MeV} \rightarrow X(3940), Y(3940), Z(3940), \\
M_{\text{bag}}^{[c \bar{c}[n=4]]} & = 4037 \text{ MeV} \rightarrow \psi(4040), \\
M_{\text{bag}}^{[c \bar{c}[n=5]]} & = 4156 \text{ MeV} \rightarrow \psi(4160), X(4160), \\
M_{\text{bag}}^{[c \bar{c}[n=6]]} & = 4424 \text{ MeV} \rightarrow \psi(4415).
\end{align*}
\]

Hypothetic higher excitations can suffer from the influence of linearly raising term in the potential, so that their masses could exceed the naive estimate at \( n > 5 \), say, for \( M_{\text{bag}}^{[c \bar{c}[n=6]]} \approx 4.42 \text{ GeV} \) (the size of quar-antiquark system exceeds 2 fm), but such the high values of excitation energy signal that thresholds of some other effects could open. Particularly, the excitation of bag could take place, that we discuss in the next subsection.

D. Exciting the bag

The method of estimating the mass of basic state for the bag in the framework of constituent model for \( \pi \) and \( p \)-mesons suggests that the bag could have excitations. The first excitation is related with the system of \( a \)-mesons. The spin-average state has the mass\(^{10}\) equal to

\[
m(1P) = \frac{1}{9}(5m_{a_2} + 3m_{a_1} + m_{a_0}) \approx 1260 \text{ MeV}. \tag{31}
\]

However, in contrast to the \( S \)-wave state, the presence of \( P \)-wave suggests that the constituent quarks get the kinetic energy of orbital rotation,

\[
E_{\text{orbit}} = \frac{L^2}{2m_{\text{red}}r^2}, \tag{32}
\]

where the reduced mass is equal to

\[
m_{\text{red}} = \frac{1}{2} m_{\text{const}} \approx \frac{1}{4} E_{\text{bag}}.
\]

At \( L = 1 \) and \( L \sim m_{\text{red}}r \), we find

\[
E_{\text{orbit}} \sim m_{\text{red}} \approx \frac{1}{4} E_{\text{bag}} \approx 155 \text{ MeV}.
\]

Therefore, the mass of low-lying excitation of bag is equal to\(^{11}\)

\[
E_{\text{bag}}' \approx 1105 \text{ MeV}. \tag{33}
\]

Then, we easily determine the increment of repeating the low-lying states in the system of bag-charmonium

\[
\delta M_{\text{bag}} = E_{\text{bag}}' - E_{\text{bag}} \approx 490 \text{ MeV}, \tag{34}
\]

so that we get the prediction permitting the direct verification by the comparison with known exotic states of charmonium family,

\[
\begin{align*}
M_{\text{bag}}^{[1^3S_1]'} & \approx 4260 \text{ MeV} \rightarrow Y(4260), \\
M_{\text{bag}}^{[2^3S_1]'} & \approx 4366 \text{ MeV} \rightarrow Y(4360).
\end{align*}
\]

The next excitation of bag is the straightforward analog by the analysis of \( \pi \) and \( \rho \) system, namely, by considering the radial excitation 2S: \( \pi(1300) \) and \( \rho(1450) \). Then, in accordance with the experimental data\(^{12}\)

\[
E_{\text{bag}}'' \approx 1420 \pm 25 \text{ MeV}. \tag{36}
\]

Therefore, the masses of vector states in such the bag are equal to

\[
\begin{align*}
M_{\text{bag}}^{[1^3S_1]''} & \approx 4575 \pm 25 \text{ MeV} \rightarrow ?, \\
M_{\text{bag}}^{[2^3S_1]''} & \approx 4680 \pm 25 \text{ MeV} \rightarrow Y(4660).
\end{align*}
\]

Thus, the model of bag-charmonium allows us to totally describe the wide spectrum of experimental data on the spectroscopy of extraordinary states in the charmonium family by following the qualitatively clear assumptions and fixing the parameters from the measured splitting between the vector and pseudovector states.

E. The spectrum of charmonium family

The final spectrum of states for the bag-charmonium is shown in Fig. 5 in comparison with the experimentally measured positions of levels in the charmonium family including the extra states arranged by the total momentum, spatial and charged parities, \( J^{PC} \), if the quantum numbers are empirically established or in the case of bag-charmonium with a sizable splittings depending on the spin, i.e. at \( n = 1, 2 \). The higher excitations at \( n > 2 \)

\[^{10}\text{We use the prescription of } n, l, \text{ where } n, l \text{ is the radial quantum number.}\]

\[^{11}\text{One could get the similar estimate, if one starts from the calculation of mass for the } P\text{-wave state of } D\text{-meson, so that in the same approximation we obtain } m_D(1P) = m_e + \frac{1}{2} E_{\text{orb}}' + E_{\text{orb}}^D, \text{ where } E_{\text{orb}}^D \approx m_D^\text{red} = m_e m_{\text{const}}/(m_e + m_{\text{const}}) \approx 260 \text{ MeV, while the experimental data are not complete in order to calculate the position of spin-average level, but we could hold the trend by setting } m_D^\text{exp}(1P) \approx (5m_D + 3m_D^\text{red})/8 \approx 2445 \text{ MeV.}\]

\[^{12}\text{The mass of } \pi(1300) \text{ is measured with uncertainty of 100 MeV.}\]
FIG. 5: The spectrum of charmonium family arranged by the quantum numbers $J^{PC}$; solid lines mark the experimental data [32]; dashed lines show the predictions in the model of bag-charmonium (see details in the text); dotted lines are positioned at the thresholds for the production of mesons with the open charm. The shaded bands give uncertainties in the case of both experimental data and theoretical predictions.

are marked by the dashed line drawn in the limits from $1^{-}$ to $2^{++}$ in order to clearly point to the fact that such the prediction concerns for the permitted higher values of orbital momentum, not only the vector states. In that case one has to take into attention that the $S$-wave label $nS$ refers to the pseudoscalar and vector mesons, only.

The levels of bag-charmonium are marked by the radial and orbital quantum numbers: $n_l$. The prime in the notation means that the bag has got its first $P$-wave excitation, and the double prime points to the first $S$-wave excitation of bag.

In favor of state identification accepted, there are the following facts:

- the anomaly in the distribution over the invariant mass of two pions in the decay $\psi(3770) \rightarrow J/\psi \pi \pi$,
- the preference (or the evident selectivity) in two-pion transitions of $Y(4260) \rightarrow J/\psi \pi \pi$ and $Y(4360) \rightarrow \psi' \pi \pi$ for the $1S$- and $2S$-states of charmonium, respectively,
- the analogous selectivity in the two-pion transition of $Y(4660) \rightarrow \psi' \pi \pi$ for the certain $2S$-level of charmonium.

III. TWO-PION TRANSITIONS

The interaction of compact color-singlets composed of heavy quark and antiquark, with "soft degrees of freedom", for instance, with the low energy pions, is con-
structured in the framework of multipole expansion of QCD \cite{1}, so that in the leading order over the heavy quark velocity the transition of bag-quarkonium to the state of quarkonium with the emission of two pions posed in the isospin-singlet and S-wave state, takes place due to the second order over the chromoelectric dipole moment with the amplitude\textsuperscript{13} $\mathcal{A}_{\text{bag-}(\bar{Q}Q)_1 \to (\bar{Q}Q)_2 \pi \pi} \equiv \mathcal{A}$ equal to

$$\mathcal{A} = \langle \pi \pi (\bar{Q}Q)_2 | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b r_\gamma T^a G T^b r_\beta | \text{bag-}(\bar{Q}Q)_1 \rangle,$$

where $T^a$ is the “difference of generators for the color charges” of quark and antiquark: $T^a = \frac{i}{2}(q^a - \bar{q}^a)$ in terms of Gell-Mann matrices $\lambda^a = 2t^a$, $G$ denotes the Green function of color-octet state of quark and antiquark, while subscripts of quark-antiquark state mark the set of its quantum numbers. In matrix element\textsuperscript{15} one could isolate two following contributions: the first corresponds to the factorization of soft degrees of freedom $\mathcal{A}_{\text{bag-}(\bar{Q}Q)_1 \to (\bar{Q}Q)_2 \pi \pi |_{\text{SF}}} = \mathcal{A}_{\text{SF}},$ that takes place in the leading order over the velocity of heavy quarks,

$$\mathcal{A}_{\text{SF}} = \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | \text{bag} \rangle \times \langle (\bar{Q}Q)_2 | r_\gamma T^a G T^b r_\beta | (\bar{Q}Q)_1 \rangle,$$

so that one could introduce the chromoelectric polarizability in the transition between the color-singlet states of heavy quark and antiquark

$$\alpha_{\gamma \beta}^{(12)} = \frac{1}{4} \langle (\bar{Q}Q)_2 | r_\gamma T^a G T^b r_\beta | (\bar{Q}Q)_1 \rangle,$$

which gives

$$\mathcal{A}_{\text{SF}} = \frac{1}{2} \alpha_{\gamma \beta}^{(12)} \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | \text{bag} \rangle.$$

The second contribution appears due to account for the bag annihilation $\mathcal{A}_{\text{bag-}(\bar{Q}Q)_1 \to (\bar{Q}Q)_2 \pi \pi |_{\text{BA}}} = \mathcal{A}_{\text{BA}},$ which becomes possible in higher order over the heavy quark velocity\textsuperscript{14},

$$\mathcal{A}_{\text{BA}} = \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | 0 \rangle \times \langle (\bar{Q}Q)_2 | r_\gamma T^a G T^b r_\beta | \text{bag-}(\bar{Q}Q)_1 \rangle,$$

that is also reduced to the introduction of analog to the chromoelectric polarizability

$$\tilde{\alpha}_{\gamma \beta}^{(12)} = \frac{1}{4} \langle (\bar{Q}Q)_2 | r_\gamma T^a G T^b r_\beta | \text{bag-}(\bar{Q}Q)_1 \rangle,$$

which can be nonzero begining from the second order over the heavy quark velocity, since in the first order over the velocity the operators of interaction (dipole moments) take nonzero color charge, $\tilde{\alpha} \sim \mathcal{O}(v^2).$ The restriction to the S-wave vector states of quark and antiquark with polarization vectors $\epsilon_{1,2}$ immediately gives

$$\alpha_{\gamma \beta} = \alpha \delta_{\gamma \beta} (\epsilon_1 \cdot \epsilon_2),$$

hence,

$$\mathcal{A} = \frac{1}{2} \alpha_{\gamma \beta}^{(12)} \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | \text{bag} \rangle + \frac{1}{2} \tilde{\alpha}_{\gamma \beta}^{(12)} \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | 0 \rangle.$$

The standard technique\textsuperscript{33}, presented, for instance, in review\textsuperscript{17}, allows us to relate the operator quadratic in the chromoelectric field with the anomaly in the trace of energy-momentum tensor in QCD, so that in the chiral limit\textsuperscript{34} we get

$$\frac{1}{2} \tilde{\alpha}_{\gamma \beta}^{(12)} \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | 0 \rangle = - \frac{4\pi^2}{\beta_0} \alpha_{\gamma \beta}^{(12)} q^2,$$

where $q = p_1 + p_2$ is the sum of 4-momenta of pions in the final state, and $q^2$ denotes their invariant mass squared. Analogous speculations in combination with an expansion of “soft” nonstationary state of bag over the pion states with positive spatial and charge parities and zero charges with respect to gauge interactions lead to the conclusion that in the limit of soft chiral pions the dominant contribution in the matrix element is given by the two-pion projection of bag state and, hence, four-pion vertex, i.e. the constant integrated over the parameters of expansion for the bag, that is approximated by

$$\frac{1}{2} \alpha_{\gamma \beta}^{(12)} \langle \pi \pi | g_s^2 \epsilon_\gamma^a \epsilon_\beta^b | \text{bag} \rangle = - \frac{4\pi^2}{\beta_0} \alpha_{\gamma \beta}^{(12)} \mu_0 e^{i\delta},$$

where $\delta$ defines the complex phase of matrix element\textsuperscript{35} with respect to\textsuperscript{45}, while the dimensional parameter is determined by the bag mass $\mu_0 \sim E_{\text{bag}}$, i.e. it has the magnitude of the order of several $\Lambda_{\text{QCD}}$. Thus, the two-pion transitions between the vector S-wave states of quarkonium in the presence of bag in the initial state are described by the functional dependence

$$\mathcal{A}_{\text{bag-}(\bar{Q}Q)_1 \to (\bar{Q}Q)_2 \pi \pi} \sim q^2 + \mu^2 e^{i\delta},$$

wherein the bag causes the modification by introducing the term with $\mu \neq 0$.

It is well known that the parametrization\textsuperscript{15} of\textsuperscript{17} is quite accurately describes the distribution of pions over their invariant mass in the analogous transition $\Upsilon(3S) \to \Upsilon \pi \pi$. At first, this fact signals that $\Upsilon(3S)$ can

\textsuperscript{13} See the methodic presentation in review \cite{17}.

\textsuperscript{14} In addition, the propagation of color-octet state of quarkonium with subtractive forces suggests that the wave functions of quarks are essentially overlapped with the bag, that can result in the annihilation of bag.

\textsuperscript{15} The standard parametrization includes the modification allowing us to take into account the fine effect caused by the rescattering of pions in the final state\textsuperscript{33}, that produces corrections, which accuracy is graded by the empirical uncertainty of real data on the spectrum of distribution over the invariant mass of pions\textsuperscript{36}.\n
be treated as the basic vector state of bag-bottomonium. Second, it shows that fitting the parameters in the second term of \[47\] certainly allows us to describe the anomalous distribution in the transition of \(\psi(3770) \to J/\psi \pi \pi\), too. In this way, the value of \(\mu\) is close to 0.7 GeV in the case of \(\Upsilon(3S)\), that is consistent with the prediction in the model of bag-quarkonium. On the other hand, the appearance of such the scale in the transitions between the ordinary states of heavy quarkonium with the emission of two pions seems to be rather problematic. Though, at present, there are various versions for the explanation of anomaly in question, which are not reduced to our suggestion, of course (see, for example, \[37\]).

Thus, the anomalous distribution over the invariant mass of two pions in the transition of bag-quarkonium into the heavy quarkonium is consistent with the accepted identification of basic vector states of bag-charmonium and bag-bottomonium.

IV. LEPTONIC CONSTANTS

The leptonic constants of heavy quarkonium can be calculated in the potential model by applying the effective lagrangian of nonrelativistic QCD (NRQCD), that was studied in \[38\] in the case of heavy quark and antiquark of the same flavor and in \[39\] for the quarkonium composed of heavy quark and antiquark of different flavors. So, the vector currents of quarks \(Q\) in nonrelativistic QCD are related with the currents in the full theory by the formula

\[
J^\nu_{QCD} = \bar{Q} \gamma_\nu Q, \quad J^\nu_{NRQCD} = \lambda^\dagger \sigma^\dagger_\nu \phi,
\]

where \(Q\) is the field of relativistic quark, while \(\chi\) and \(\phi\) denote the nonrelativistic Pauli spinors for the quark and antiquark, \(\sigma^\dagger_\nu = \sigma_\nu - v_\nu (\sigma \cdot v)\), and \(v\) is the 4-velocity of quarkonium, so that

\[
J^\nu_{QCD} = K(\mu_{hard}; \mu_{fact}) \cdot J^\nu_{NRQCD}(\mu_{fact}).
\]

Here \(\mu_{hard}\) fixes the point of matching NRQCD to QCD, \(\mu_{fact}\) denotes the scale of perturbative calculations in NRQCD.

In the case of quarks of the same flavor, the Wilson coefficient \(K\) is known to the 2-loop accuracy \[40, 41, 42\].

\[
K(\mu_{hard}; \mu_{fact}) = 1 - \frac{8 \alpha_s^\gamma(\mu_{hard})}{3\pi} \left( \frac{\alpha_s^\gamma(\mu_{hard})}{\pi} \right)^2 c_2(\mu_{hard}; \mu_{fact}),
\]

where \(c_2\) is explicitly given in \[41, 42\]. The anomalous dimension of \(K(\mu_{hard}; \mu_{fact})\)

\[
\frac{d \ln K(\mu_{hard}; \mu)}{d \ln \mu} = \sum_{k=1}^{\infty} \gamma[k] \left( \frac{\alpha_s^\gamma(\mu)}{4\pi} \right)^k,
\]

in two loops is given by the expression\(^\text{16}\)

\[
\gamma[1] = 0,
\gamma[2] = -16\pi^2 C_F \left( \frac{1}{3} C_F + \frac{1}{2} C_A \right).
\]

Initial data for the evolution of \(K(\mu_{hard}; \mu_{fact})\) versus the scale are determined by the matching conditions fixed at \(\mu = \mu_{hard}\) \[41, 42\].

The leptonic constant of vector state with the polarization \(\lambda\) and polarization vector \(c^I_2\) in full QCD

\[
\langle 0 \mid J^\nu_{QCD}(Q\bar{Q}, \lambda) \mid c^I_2 \rangle = e^I_{\nu} f_{Q\bar{Q}} M_{Q\bar{Q}},
\]

is related with the matrix element of current \(J^\nu_{NRQCD}\)

\[
\langle 0 \mid J^\nu_{NRQCD}(\mu)(Q\bar{Q}, \lambda) \mid c^I_2 \rangle = A(\mu) e^I_{\nu} f_{Q\bar{Q}} M_{Q\bar{Q}},
\]

where the potential model of quarkonium with the wave function \(\Psi_{Q\bar{Q}}(r)\) gives

\[
f_{Q\bar{Q}}^{NRQCD} = \sqrt{\frac{12}{M}} |\Psi_{Q\bar{Q}}(0)|,
\]

while the renormalization group factor \(A(\mu)\) equals unit at the scale of \(\mu = \mu_0\), whereas the wave function is determined. Since

\[
f_{Q\bar{Q}} = f_{Q\bar{Q}}^{NRQCD} A(\mu_{fact}) \cdot K(\mu_{hard}; \mu_{fact}),
\]

the anomalous dimension of \(A(\mu_{fact})\) compensates the anomalous dimension of \(K(\mu_{hard}; \mu_{fact})\), hence,

\[
\frac{d \ln A(\mu)}{d \ln \mu} = -2\gamma[2] \left( \frac{\alpha_s^\gamma(\mu)}{4\pi} \right)^2,
\]

therefore,

\[
A(\mu) = A(\mu_0) \left[ \frac{\beta_0 + \beta_1 \frac{\alpha_s^\gamma(\mu_0)}{4\pi}}{\beta_0 + \beta_1 \frac{\alpha_s^\gamma(\mu_0)}{4\pi}} \right]^{\gamma[2]/2\beta_1}.
\]

The calculation of leptonic constants in wide limits for the scales of matching and factorization allows us to find the region, wherein the result is stable, i.e. it slightly depends on small variations of scales. Then, \(\mu_0\) is set to the point of stability. Numerical estimates are consistent with the experimental data for the basic vector states of charmonium and bottomonium \[38\], while the leptonic constants of excited states in such the approach are simply determined from the ratio of wave functions at the origin for the given state to the wave function of basic

\(^{16}\) In terms of ordinary notations for the representations of \(SU(N_c)\) group: \(C_F = \frac{N_c^2 - 1}{2N_c}, C_A = N_c, T_F = \frac{1}{2}\).
state due to appropriate rescaling of leptonic constant for the basic state in accordance with \( \sigma \), since the renormalization group factors are universal, i.e. they do not depend on the excitation number. Then, the problem is reduced to the safe calculation of wave functions of quarkonium at the origin. Such the calculation involves rather a large uncertainty because of modelling the potential and prescribing the pole masses of quarks depending on the model. Thus, one arrives to a sizable methodic uncertainty of theoretical predictions. Nevertheless, one can reliably state that the low value of leptonic constant of \( \psi(3770) \) is caused by the fact that this state cannot be assigned to the \( S \)-wave state of charmonium. Therefore, it is the exotic state whether it is the result of mixing the \( S \)-wave level with the \( D \)-wave one, as usually accepted, in spite of the fact that the splitting between such the nearest levels is about 100 MeV, and hence, the matrix element of mixing should get the same order of magnitude, that is quite a large value.

However, such the framework is not applicable to the states of bag-quarkonium, since in this case the joint annihilation of quark-antiquark pair and bag should take place. Then, the knowledge of wave function for the quark-antiquark system is not sufficient. In addition, the wave function could be essentially modified during the annihilation with the bag. So, let us explore the method of quasilocal sum rules of QCD \( ^{[20]} \). The basic idea of such the approach is reduced to the following:

- Consider the transversal part of correlator for the vector currents of heavy quarks, namely, the real part of correlator at the invariant mass set to zero as well as the derivatives of \( k \)-th order, i.e the moments of real part at zero.

- In the case of heavy quarks, the bound states are positioned in the narrow gap which width is suppressed in comparison with the heavy quark mass \( m_Q \), so that for the low values of moment numbers one can transform the summation over the resonances to the integration over the density of states \( d\tilde{n}/dM_{\tilde{n}} \), where \( \tilde{n} \) denotes the number of vector state in the direction of mass increase\(^{[17]} \), with accuracy up to \( \mathcal{O}(1/m_Q) \).

- In the leading order over the inverse mass \( 1/m_Q \) at low values of moment number, calculating the correlator can be restricted by the contribution of nonrelativistic quarks with account for their coulomb-like interaction.

Then, at the points of resonances the local equality of theoretical correlator to the empirical one approximated due to the introduction of density of states leads to the relation for the leptonic constant of \( \tilde{n} \)-th state \( f_{\tilde{n}} \)

\[
\frac{f_{\tilde{n}}^2}{M_{\tilde{n}}} = c \frac{dM_{\tilde{n}}}{d\tilde{n}},
\]

where the constant is defined by the formula

\[
c = \frac{\alpha_s}{\pi} K^2 \left( \frac{2m_Q}{M_{\tilde{n}}} \right)^2 Z_{\text{sys}},
\]

with an effective constant of coulomb exchange\(^{[18]} \) \( \alpha_s \), the factor of loop corrections \( K \) taken to the 1-loop accuracy from \( ^{[20]} \) and a systematic correction generally depending on the moment number

\[
Z_{\text{sys}} = Z_{\text{NR}} / Z_{\text{int}},
\]

introduced in the both nonrelativistic approximation of quark correlator and density of states for the integral representation for the sum over the resonances. In the limit of heavy quarks, i.e. by neglecting logarithmic and power corrections of the form \( \ln \mu/m_Q \) and \( 1/m_Q \), the effective constant \( C \) is independent of quark flavor. It is spectacular, that empirically for the finite masses of real \( c \)- and \( b \)-quarks and basic vector states, we have got\(^{[19]} \)

\[
c \approx \frac{1}{5\pi},
\]

so that the scaling relation of \( ^{[20]} \) takes place.

If the heavy quark and antiquark states are certainly the levels of nonrelativistic heavy quarkonium, then the density of states is, in practice, universal, since the potential is close to the logarithmic dependence, so that independently of the heavy quark flavor we get

\[
\frac{dM_{\tilde{n}}}{d\tilde{n}} \approx \frac{2\pi}{\tilde{n}},
\]

\(^{[17]} \) We insert the tilde symbol in order to distinguish the number from the principal quantum number \( n \) used above.

\(^{[18]} \) The effective constant of coulomb exchange as well as the quark mass in this approximation do not necessary coincide with their analogs in the framework of potential approach. Though, the mentioned differences become less with the increase of quark mass, so that for the bottomonium these constants coincide with the accuracy below 10%, indeed, if one does not take into account for the difference in the values of \( b \)-quark pole mass, while the account for the renormalization group evolution of coupling constant versus the characteristic momentum inside the quarkonium, hence, versus the quark mass, leads to that the effective constants actually coincide with each other with the accuracy better than 2%, that is essentially underestimate the accuracy of assumptions made in the way of deriving these estimates. In addition, the strict analysis of charmonium and bottomonium made in the framework of assumption on the dominant coulomb interaction with account for the perturbative and nonperturbative corrections in \( ^{[22]} \) led to the values of pole masses of charmed and bottom quarks, which are consistent with the values obtained in the present paper within the accuracy of estimates. The effective constants of coulomb interaction take similar values, too.

\(^{[19]} \) The exact value can depend on the choice of quark mass, that leads also to a variation of systematic factor caused by the nonrelativistic approximation in the calculation of coulomb contribution near the threshold.
where $T$ is the average kinetic energy, i.e. the main parameter of logarithmic potential. Therefore, for the heavy quarkonium the scaling law takes place

$$\frac{f_n^2}{M_n} \tilde{n} = \text{const.}, \quad (62)$$

valid for $J/\psi$ and $T$. By the way, since the width of vector quarkonium decay $\Gamma[1_{Q\bar{Q}} \rightarrow e^+e^-]$ is related with the electric charge of quark $e_Q$ and its leptonic constant by the expression

$$\Gamma_n = \frac{4\pi^2}{9} \alpha_{\text{em}}^2 (M_n) e_Q^2 \frac{f_n^2}{M_n},$$

for the standard states of heavy quarkonium with the regular spectrum under condition (61) one could expect the universal law

$$\frac{1}{e_Q^2} \Gamma_n \frac{d\tilde{n}}{dM_n} = \text{const.} \quad (63)$$

Relation (63) is also applicable, if there are exotic, extraordinary states with the hidden charm, too. Its physical meaning is simple: the contributions of resonances into the correlator are actually fixed by the dislocation in accordance with the observed density of states. Therefore, it is enough to carry out the analysis of data on the spectrum of vector states with the hidden charm and, then, to test the direct consequence of (63) in the form of

$$\frac{1}{e_Q^2} \Gamma_n \frac{d\tilde{n}}{dM_n} = \text{const.} \quad (64)$$

The accuracy of analyzing the spectrum of vector states is essentially restricted by the fact that the levels are discrete. The result of cubic interpolation for the mass spectrum with the further calculation of $dM_n/d\tilde{n}$ versus $\tilde{n}$ being the number of vector state as given in Fig. 6 by the mass increase, is shown in Fig. 6.

Note that the interpolation of spectrum itself versus the resonance number leads to discontinuities of the mass derivative with respect to the number exactly at the points, which refer to the values of state density in interest. Therefore, we have made the cubic interpolation of the state density itself once more with different increments in the number in order to smoothen the discontinuities\(^{20}\) and show uncertainties of estimating the quantity of $dM_n/d\tilde{n}$. In figure we see that the most reliable results are given at $\tilde{n} = 6, 7$ (states $\psi(4040)$ and $\psi(4156)$), whereas the spectrum is the most regular in the sense of its smoothness by incrementing between the nearest states. The uncertainty grows at $\tilde{n} = 3$ (the state $\psi(3770)$), whereas the mass increment is extremely inhomogeneous, as well as at the end of interpolation interval, i.e. at the important positions of states with $\tilde{n} = 1, 2$, mesons $J/\psi$ and $\psi(2S)$. The absolutely analogous situation takes place at other choice of interpolation power, namely, at linear and quadratic interpolations, for instance. This fact means that the investigation of relation (64) should be made by fixing the constant at $\tilde{n} = 6$, say, that we will perform further. In this way, an essential uncertainty in the calculation of leptonic constant of basic state $J/\psi$ appears. In order to decrease the uncertainty of interpolating towards $J/\psi$, we will take into account that this state is certainly the heavy quarkonium, hence, the state density in vicinity of $\tilde{n} = 1$ is rather close to the value predicted in the potential model. This fact can be naturally involved by introducing an auxiliary state with $\tilde{n} = 0$ and mass $M_0 \approx M_{J/\psi} - 2T \approx 2296$ MeV according to the linear extrapolation, that we actually make.

Thus, relation (64) leads to estimation of leptonic constants shown in Fig. 7. We see that the scaling relation quite successfully describes the behavior of leptonic widths for the observed states. Since the relation is universal, and it is valid irrespectively of the role played by infrared phenomena such as the bag, there is no need to prescribe the state of $\psi(3770)$ as the 1D level of charmonium with the appropriate mixing with the 2S-level in order to produce nonzero value of leptonic constant (the wave function at the origin). Moreover, the small difference between the leptonic constant of $\psi(4040)$ and

\(^{20}\) The artefact of function discontinuity points to the fact that the derivative of spectrum with respect to the number of resonance permits a displacement of point on the number axis in the limits $d\tilde{n} = 0.5$, so that we have also applied the method of smoothing by averaging the values of spectrum density taken left and right in vicinity of resonance, that agrees with the uncertainties of estimation itself. Then, the smoothing is slowly modify the values of spectrum density in the points, where it fluctuates weakly, i.e. the spectrum is rather regular, and in addition, it allows us to draw the limits of approach uncertainties.
FIG. 7: The leptonic constants of vector states in the family of charmonium $\Gamma_{\tilde{n}}$ in keV versus the resonance number $\tilde{n}$ (solid line). The shaded region marks the systematic uncertainty of predictions, which is caused by the interpolation of state density. The dots denote the experimental data with the uncertainties shown as vertical lines.

\(\psi(4156)\) looks quite exotic, if one treats them as the \(S\)-wave levels of charmonium with the respective numbers \(\tilde{n} \rightarrow 3, 4\), while the presence of extra states due to the bag-charmonium leads to the both equalizing the ratio of leptonic widths and more natural values of absolute values: \(\Gamma_6 \approx \frac{1}{6} \Gamma_1\). Similar statements can be written as concerns for the state of \(\psi(4415)\).

We do not present a complete description for the procedure of interpolating the spectrum density, since Fig. 7 clearly shows that the method inherently involves irreducible uncertainties, which do not permit to make the theoretical predictions with the higher precision\(^{21}\), comparable with the accuracy of experiment. Nevertheless, the exploration of quasilocal sum rules by introducing the density of bound states with the hidden charm allows us to establish the tendency in the character of dependence of leptonic constants on the number of excitation over the basic state.

Thus, the extraordinary states in the family of charmonium as dictated by the model of bag-charmonium, do not contradict with the analysis of know leptonic constants of vector states, and moreover, they permit to improve the systematization of those constants (or leptonic widths).

V. CONCLUSION

In the present paper we have formulated the potential model of bag-charmonium, which has allowed us to successfully identify extraordinary states in the family of charmonium in terms of spectroscopy, to substantiate the anomaly in the distribution over the invariant mass of two pions in hadronic transitions such as \(\psi(3770) \rightarrow J/\psi \pi \pi\) and \(\Upsilon(3S) \rightarrow \Upsilon \pi \pi\) by assigning the initial states as the vector bag-quarkonia and to establish the systematic regularity of leptonic constants for the vector states in the charmonium family due to the levels of bag-charmonium. The regularity has got the form of scaling law derived from the quasilocal sum rules of QCD.

However, such the model predicts some new states yet not observed empirically.

As concerns for the mechanisms of production of exotic states, one could stress the following: in decays of heavy hadrons with the light valence quark, the production of bag-charmonium goes almost the same way as the production of charmonium, namely, at rather short distances about the charmonium size, the compact state of charmed quark and antiquark is formed, while the bag itself already exists due to the presence of light valence quark in the initial state, so that the bag introduces an appropriate form factor, describing a possible transition of bag from the rest in the initial state to a motion joint to the quark-antiquark pair with the hidden charm.

\[^{21}\text{For instance, we could quite smoothly fit the spectrum of vector states with the hidden charm by means of special set of basic functions, so that the spectrum density will repeat the experimental dependence of leptonic constants versus the resonance number. However, the procedure of selecting the basic functions is arbitrary itself, so it involves almost uncontrolled methodic uncertainty, restricted by the constraint of acceptable smoothness of the fit, only.}\]
An analogous form factor, probably, should appear at the production of bag-charmonium in hadronic collisions, whereas the order of magnitude for the cross section is close to the production cross section of charmonium. In leptonic collisions, the main role is played by the leptonic constants studied in Section IV.

In the present paper we, in fact, have not discussed any channels of decays for the bag-charmonium except pointing to the selectivity in the two-pion transitions of bag-charmonium to the charmonium states with identical quantum numbers of quark-antiquark pair.

In this respect, it is appropriate once more to refer to review [19], wherein there is a comprehensive bibliography of original articles concerning for the models of exotic quarkonium states, their decays and production modes as well as a comparative description of various mechanisms of forming the extraordinary states in the charmonium family. We point to some pioneer papers, wherein authors introduced such the notions into the scientific language in the field of quark dynamics of exotic states with the hidden charm as follows:

- the hadronic molecule \([44, 45]\) or deuson (in analog to the deuteron) \([46]\), composed of charmed mesons with the positive and negative charm \(\bar{D}D\),
- the hybrid \([47, 48]\) containing the quark-antiquark pair in the color-octet, which is coupled with a compact gluonic lump to form the colorless state,
- the tetraquark \([49]\) composed of diquark and diantiquark.

In addition, it worth to mention the recent paper on the systematization of heavy quarkonium states – charmonium and bottomonium – in the scheme of Regge trajectories \([50]\). Studying various color structures of tetraquark has been given in \([51]\).

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