An Optimization Model for Multi-Echelon Multi-Commodity Supply Chain Design with Side Constraints

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Abstract. A proper strategies in designing and optimizing supply chains can help enterprises to stay competitive in this volatile economy. Identifying various type of products that run through the supply chain and aligning several parts of the supply chain to these product types are considered to be effective strategies adapted by successful companies. In this paper we address an optimal design of a multi-echelon supply chain problem with various commodity that is of significance to the business enterprise, along with the associated inventory systems. By using the guaranteed service approach to model the multi-echelon inventory system, we develop an MINLP model to simultaneously optimize the transportation, inventory and network structure of a multi-echelon supply chain which involve multi commodity. The optimization model is then solved using a strategy of releasing nonbasic variables from their bounds, combined with the “active constraint” method.

1. Introduction

In industrial system, supply chain management (SCM) has been utilized as a tool to characterize policies for production and distribution, along with product allocation. The modelling of the supply chain includes suppliers of raw resources, primary and secondary plants (including inventories of raw materials and finished goods), distribution centres, warehouses and customer locations [1]. Afterward, [2] explore the optimization of the supply chain to determine the impact of the scale, the operating costs depending on the usage rates and the number of products processed in each plant and the weight of each cost factor, such as production, transport and allocation, on the optimal design and use patterns of the supply chain systems.

SCM has turned out to attract companies’ awareness as they found out the potential cost advantages of integrating decisions with other system businesses. The main cost factors in a supply chain can be included in the categories of production, transport and inventory. The main feature of supply chain management is the integration of operations. Effective supply chains always unite their downstream members’ wishes and concerns into their procedures while at the same time assuring their integration with their upstream members. There is general agreement that the supply chain and its management are the rational growth of logistics and logistics management improvements respectively [3], [4], [5] and [6].

Inventory optimization continues to be one of the key difficulties in supply chain management. In today's supply chains, large amounts of working capital are bound, limiting the growth opportunities that are essential to the achievement of a company in competitive industries. Despite that, research has
shown that inventories have a high chance of reducing them in the supply chain, thereby increasing competitiveness and reducing production costs.

From an industrial point of view, inventory management is a crucial issue, but most models in literature consider inventory management and supply chain network design separately, although they are closely linked. [7]. This is due to the fact that the decrease in transport and inventory costs is seen as two contradictory goals. However, some related work on supply chain optimization takes account of inventory costs, but does not mention a detailed inventory management policy [8], [9], [10], [11] and [12]. The safety stock level is viewed as a parameter in these models and can be regarded as a lower limit of the total inventory level [13], [14], [15], [16] and [17], or it is considered an inventory target that would lead to certain penalty costs if infringed [18].

The majority of the literature focuses on the single stage inventory structure integrated with the supply chain design. In [8], [9] and [19] present a joint location- inventory model that extends the classic incapacitated facility location model to include nonlinear active inventory and safety stock prices for a two-stage supply chain network, in order for decisions on the installation of distribution centres (DCs) and detailed inventory replenishment decisions are optimized simultaneously. In order to simplify the problem, inventories are neglected in retailers and they also presume that all DCs have the same perpetual time to replenish and that the demand for each consumer has the same variance-to-mean ratio. In [11] proposed a mixed-integer nonlinear programming (MINLP) approach to study a more broad model in accordance with the one developed by Daskin et al. (2002) and Shen et al. (2003), relaxing the assumption on equivalent variance-to-mean ratio for customer demands.

In [20] described a MILP model that combined production, distribution and marketing with plants and sales outlets to cover the relevant features needed for the entire management of the supply chain of a multi-site production system. In [21] developed a Capacitated Plant Location Problem (CPLP) model for the planning and coordination of multi-commodity production and distribution facilities, including suppliers of raw materials, production facilities, warehouses and customer regions. The authors took a holistic approach to the supply chain, leading to a deterministic multi-stage problem. In [22] considered the problem of integrated and decoupled production and distribution planning, consisting of multiple factories, retailers, multi-period items. The author proposed mixed integer optimization models and a heuristic two-phase solution to maximize net profit.

Our model aims to form an integrated decision on production, inventory and routing within a two-echelon framework that minimizes total costs while meeting the following requirements: decide how many warehouses are required, where to place the open warehouses and how to assign products from plants to warehouses and then to end customers; find out how much to keep in stock at any time; and how to generate vehicle routes from the open warehouse to customers and back to the warehouse. Distribution problems can generally be regarded as a combinatorial optimisation model. Hence, the model is formulated as a Mixed Integer Program (MIP). A direct, feasible search approach is developed to solve the model.

2. Methodology

2.1. Problem Formulation

A single manufacture considers to design a supply chain plan for a set of \( N \) multi-commodity products in a multi-echelon pattern. This pattern contains a set of plants, \( O \), a set of candidate warehouse, \( W \) and a set of retailers, \( I \). It is assumed that the demands are given at warehouse and retailers, and the fleet of vehicles are homogeny. Therefore we have a deterministic problem.

Let \( G = (V, E) \) be an undirected graph where \( V \) is a set of nodes composed of a subset \( I \) of \( m \) potential warehouse sites and a subset \( J = V \setminus I \) of \( n \) customers. \( E \) is a set of route connecting each pair of nodes in \( V \). A feasible route is defined as a route through which a vehicle begins travelling from a candidate warehouse, visits a number of retailers, and comes back to the same warehouse. Therefore, a warehouse is not allowed to be visited more than once in the feasible route. As a consequence, the number of possible feasible routes would be \( W \times 2^n \), where \( W \) and \( I \) are the number of warehouses and retailers, respectively. It is assumed that vehicle capacity to be greater than the maximum customer
demand at any time period. Moreover, in any time period, each vehicle travels at most on one route, and customers are visited at most once.

The objective of the supply chain plan is to minimize total operational cost received in the problem. The components of the total cost are follows.

a. Production cost
b. Warehouse fixed-location cost: the cost to establish and operate a warehouse;
c. Retailer unit-inventory holding cost: the cost to store products at retailer; and
d. Transportation cost: from plants to warehouse and from warehouse to retailers.

Nomenclature

Sets

\( O = \) Set of plants, 
\( N = \) Set of commodity 
\( W = \) Set of candidate warehouses, 
\( I = \) Set of retailers, 
\( V = \) Set of node, 
\( T = \) Time periods

\( R = \) Set of all feasible routes
\( K = \) Set of homogeneous vehicles.

Parameters

\( \beta_{jo} = \) Production cost of commodity \( j \in N \) at plant \( o \in O \)
\( F_w = \) Fixed cost of opening and operating warehouse \( w \in W \)
\( C = \) Vehicle capacity
\( d_{ijt} = \) Demand of commodity \( j \in N \) at retailer \( i \in I \) in time period \( t \in T \)
\( u_{ijt} = \) Upper bound inventory level of commodity \( j \in N \) at retailer \( i \in I \) in time period \( t \in T \),
\( u_{ijt} = \left( \sum_{d_{ijt}}^{u_{ijt}} d_{ijt} \right) \)
\( h_{ijt} = \) Inventory holding cost of commodity \( j \in N \) at retailer \( i \in I \) in time period \( t \in T \)
\( I_{jo} = \) Inventory level of commodity \( j \in N \) at retailer \( i \in I \) at the beginning of time period \( t = 1 \)
\( \beta_{rw} = \begin{cases} 1 & \text{if route } r \in R \text{ visits warehouse } w \in W; \\ 0 & \text{otherwise} \end{cases} \)
\( c_{1w} = \) Transportation cost from node plant \( o \in O \) to selected warehouse \( w \in W \)
\( c_{2wi} = \) Transportation cost from node warehouse \( w \in W \) to retailer \( i \in I \)
\( c_r = \) Transportation cost of route \( r \in R \)

Decision Variables

\( X_{ij} = \begin{cases} 1 & \text{if warehouse } i \in W \text{ is served by plant } j \in O; \\ 0 & \text{otherwise} \end{cases} \)
\( Y_{ij} = \begin{cases} 1 & \text{if demand of retailer } i \in I \text{ is served by warehouse } j \in W \; ; \\ 0 & \text{otherwise} \end{cases} \)
\( I_t = \) Inventory level of commodity \( j \in N \) at customer \( i \in I \) at the end of time period \( t \in T \)
\( \theta_{rt} = \begin{cases} 1 & \text{if route } r \in R \text{ is selected in time period } t \in T; \\ 0 & \text{otherwise} \end{cases} \)
\( m_w = \begin{cases} 1 & \text{if warehouse is opened at location } w \in W; \\ 0 & \text{otherwise} \end{cases} \)
\( q_{jo} = \) Quantity of commodity \( j \in N \) to be produced from plant \( o \in O \)
\( a_{ijrt} = \) Quantity of commodity \( j \in N \) delivered to retailer \( i \in I \) by route \( r \in R \) in time period \( t \in T \)
2.2. The Mathematical Model

The problem can be composed as a MIP problem which has a mathematical form as listed below.

The objective function.

\[
\min \sum_{w \in W} F_w m_w + \sum_{r \in R} \left( \sum_{v \in V} c_{v} \theta_{rt} + \sum_{j \in N} \sum_{i \in I} h_{ji} I_{ji} \right) + \sum_{i \in O} \sum_{jo \in O} c_{1_{ij}} X_{ij} + \sum_{io \in O} \sum_{jo \in O} c_{2_{ij}} Y_{ij} + \sum_{j \in N} \sum_{o \in O} \beta_{jo} q_{jo}
\]

Subject to

\[\sum_{r \in R} \theta_{rt} \leq 1 \quad \forall t \in T\] (1)

\[\sum_{i \in O} X_{ij} = m_{j} \quad \forall j \in W\] (2)

\[\sum_{j \in W} Y_{ji} = 1 \quad \forall i \in I\] (3)

\[\sum_{i \in I} a_{jir} \leq C \theta_{rt} \quad \forall j \in N, r \in R, t \in T\] (4)

\[I_{jir-1} + \sum_{r \in R} \alpha_{jir} a_{jir} = d_{jir} + I_{jir} \quad \forall j \in N, i \in I, t \in T\] (5)

\[I_{jir} \leq u_{jir} \quad \forall i \in I, t \in T, j \in N\] (6)

\[\theta_{rt} \leq \sum_{w \in W} \beta_{rw} m_{w} \quad \forall r \in R, t \in T\] (7)

\[\sum_{r \in R} \theta_{rt} \leq |K| \quad \forall t \in T\] (8)

\[\sum_{o \in O} q_{o} \leq \sum_{t \in T} d_{ji} \quad \forall j \in N, \forall t \in T\] (9)

\[X_{ij} \in \{0,1\} \quad \forall i \in O, \forall j \in W\] (10)

\[Y_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in W\] (11)

\[\theta_{rt} \in \{0,1\} \quad \forall r \in R, t \in T\] (12)

\[m_{w} \in \{0,1\} \quad \forall w \in W\] (13)

\[\alpha_{jir}, \beta_{jo}, I_{jir} \geq 0 \quad \forall j \in N, i \in I, o \in O, r \in R, r \in T\] (14)

Constraints (2) ensure that a customer is visited once at most in any time period. Constraints (3) express that if warehouse \( j \) is installed then it should be served by only one plant \( i \). Constraints (4) explain that each demand from retailer \( j \) should be served only by warehouse \( i \). Constraints (5) account for the vehicle capacities. Inventory balance equations are portrayed in Constraints (6). Constraints (7) is to assure that the inventory level at a customer never surpasses the total demand in the next consecutive time periods. Constraints (7) guarantee that routes start and end with open warehouses only. Constraint (9) limits the maximum number of routes at any time period to be no higher than the number of vehicles. Constraint (10) is to guarantee that the amount of commodity to be made is
enough to meet the retailer's demand. Finally, Constraints (11) to (14) for the binary variables, and constraints (15) establish that quantities to be shipped to customers and inventory levels are non-negative.

3. Results and Discussion

3.1. Solving The Model

Stage 1.

Step 1. Get row $i^*$ the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Calculate $v_{i^*}^T = e_{i^*}^T B^{-1}$

this is a pricing operation

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T a_j$

With $j$ corresponds to

$$\min_j \left\{ \frac{d_j}{\sigma_{ij}} \right\}$$

a. For nonbasic $j$ at lower bound

\[ \text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{-\sigma_{ij}} \]

\[ \text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}} \]

\[ \text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{-\sigma_{ij}} \]

\[ \text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{\sigma_{ij}} \]

b. For nonbasic $j$ at upper bound

\[ \text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}} \]

\[ \text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}} \]

\[ \text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{\sigma_{ij}} \]

\[ \text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{-\sigma_{ij}} \]

Otherwise go to next non-integer nonbasic or superbasic $j$ (if available). Eventually the column $j^*$ is to be increased from LB or decreased from UB. If none go to next $i^*$.

Step 4. Calculate $a_{j^*} = B^{-1} a_{j^*}$

i.e. solve $B a_{j^*} = a_{j^*}$ for $a_{j^*}$

Step 5. Ratio test; there would be three possibilities for the basic variables in order to stay feasible due to the releasing of nonbasic $j^*$ from its bounds.

If $j^*$ lower bound

Let

$A = \min_{i' \neq i \mid a_{i'j} > 0} \left\{ \frac{x_{i'j} - l_{i'}}{a_{i'j}} \right\}$

$B = \min_{i' \neq i \mid u_{i'j} < 0} \left\{ \frac{u_{i'j} - x_{i'j}}{-a_{i'j}} \right\}$
The maximum movement of \( j^* \) depends on: \( \theta^* = \min(A, B, C) \)

**If \( j^* \) upper bound**

Let

\[
A' = \min_{i', i' \neq i' | a_{ij} < 0} \left\{ \frac{x_{B_{ij'}} - l_{ij'}}{a_{ij'}} \right\}
\]

\[
B' = \min_{i' \neq i' | a_{ij} > 0} \left\{ \frac{u_{ij'} - x_{B_{ij'}}}{-a_{ij'}} \right\}
\]

\[ C' = \Delta \]

The maximum movement of \( j^* \) depends on: \( \theta^* = \min(A', B', C') \)

**Step 6. Exchanging basis for the three possibilities**

1. If \( A \) or \( A' \)
   - \( x_{B_{ij'}} \) becomes nonbasic at lower bound \( l_{ij'} \)
   - \( x_{ij'} \) becomes basic (replaces \( x_{B_{ij'}} \))
   - \( x_{ij'} \) stays basic (non-integer)

2. If \( B \) or \( B' \)
   - \( x_{B_{ij'}} \) becomes nonbasic at upper bound \( u_{ij'} \)
   - \( x_{ij'} \) becomes basic (replaces \( x_{B_{ij'}} \))
   - \( x_{ij'} \) stays basic (non-integer)

3. If \( C \) or \( C' \)
   - \( x_{ij'} \) becomes basic (replaces \( x_{i,j} \))
   - \( x_{ij'} \) becomes superbasic at integer-valued

**Step 7.** If row \( i'' = \emptyset \) go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Do integer lines search to improve the integer feasible solution

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### 4. Conclusion

In this paper, we addressed a mixed integer programming (MILP) model that establishes the optimal network design, transport and inventory levels of a multi-echelon supply chain with the presence of customer demand from the chosen storage unit. To solve the resulting MILP problem productively for wide scale cases, a direct search algorithm was proposed.

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