Comparing traditional and constrained disturbance-observer based positional control

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Abstract
This paper compares three different position controllers of electrical drives equipped by binomial $n$ th order filters, which are offering filtering properties important in a quantization noise attenuation. To demonstrate their impact, a non-filtered P-PI control is considered, as a reference. The comparative framework includes a filtered P-PI control, a filtered linear pole assignment PD controllers with a disturbance observer (DO) based integral action and its constrained modification. In terms of a total variation, depending on noise and process properties, all filtered controllers are capable to bring down the undue controller activity at the plant input from 10 to more than 100 times. Furthermore, thanks to the applied disturbance observer, the constrained control derived for a double integrator is shown to fully exploit the closed loop capabilities without any trajectory generation, taking into account the control constraints. Thus, the simplified controller design may focus on other important aspects.

Keywords
Servo control, constrained control, filtered P-PI control, disturbance observer

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Introduction
For several decades, the control of DC motors has been one of the basic items in the theory of servomechanisms. P-PI controllers were among the most widely used structures, and in terms of control signal limitations, the design was usually oversized so that their application never took place. However, in automotive applications of DC motors used as actuators to drive cars (e.g. in the control of “trains” of cars virtually coupled into platoons as proposed by,1,2) one of the basic differences is to comply with the constraints of the control variable to the extent that their maximum accelerations or braking are considered as standard components of a working cycle. The second important component, especially when dealing with electric vehicles, is the lowest possible electricity consumption, which also requires the best possible filtering with regard to the ever-present measurement and quantization noise.

The paper aims at improving performance of servo drives by modifying the traditional controller structure. Thereby, it focuses on elimination of an adverse impact of the control constraints and of the quantization and measurement noise. As generally known, mechatronics aims at producing higher-quality products at reduced costs component. This may be achieved by integrating the best design practices with the most advanced technologies. The inclusion of constraints in the controller design makes it possible to reduce the size and rated power of the drives used in motion control by a relatively cheap software. Furthermore, suppression of measurement noise induced controller output variation can significantly reduce overall energy expenditure, heat losses, mechanical wear of equipment and associated unwanted noises. While using the newly proposed algorithms seem to be more sophisticated at the first glance, their design, setup and implementation are ultimately comparable or simpler to the traditional linear solutions.

The necessity to deal with the constraints naturally emerges in the control of unstable and marginally stable systems and with high demands on the dynamics of transients. Historically, the development of constrained control started with a relay minimum time control.3 After some decline in research activities in the third

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quarter of the previous century, the interest in constrained control intensified again. As a by-product of a development chain initiated by a modification of the discrete time minimum time control, a constrained PID control has been developed, representing an alternative modular solution to traditional PID control. Firstly, an explicit discrete-time minimum time control has been derived and then generalized to constrained pole assignment control. Subsequently, it has been extended also to a continuous-time control and augmented by a disturbance observer (DO) to alleviate impact of uncertainties and disturbances. For an improved noise attenuation, different modifications considering filtration have been proposed. As typical examples, a traditional PI speed controller may be replaced by DO based filtered PI (FPI) control and, similarly, instead of a windup-sensitive PID, a DO-FPID control may be proposed.

One of the advantages of such DO based filtered solutions is their modular character offering several extension possibilities. Thus, they may be, for example, extended to a constrained control without facing problem of excessive integral action (windup). The core of a positional controller presented in this paper may be established by a constrained pole assignment PD controller (CPD). When derived for a double integrator, it plays a similar role as the function in active disturbance rejection control (ADRC). With the aim to demonstrate the broad design possibilities, a continuous-time solution, not yet known in ADRC, will be presented. The solution is also appropriate for a discrete-time control with smaller sampling times.

Basic PD control has been augmented by the disturbance observer based integral (I) action to filter PID control denoted as DO-FPID control. Its constrained modification will be referred to as DO-CFPID. It enables an extension of the spectrum of available approaches, which yield a time suboptimal dynamics (close to sliding mode control) with simultaneously keeping the control limits with the input usage near to the unavoidable minimum.

In the field of electric drives, P-PI control structures are widely used. Usually, the inner (velocity) loop is PI-based and the outer (position) is P-based. The inner loop must have I-term in order to get rid of control error. The position and speed feedback signals may be filtered to decrease the noise impact in the torque reference signal. This kind of loop structure is in this paper denoted as FP-FPI control. The drawback of a P-PI control is an overshooting in the output position for high step changes in the reference position signal. This effect may either be eliminated by using conditioning-technique-anti-windup solutions (for cascade structure), or by applying a continuous position setpoint signal with a specified acceleration and speed of the motion (the so called command shaping) or trajectory generating, or by a modified loop tuning. This is, however, leading to sluggish transients for small setpoint changes and to a stronger disturbance impact.

DO-CFPID includes an appropriate command shaping intrinsically, while working with chosen closed loop poles and respecting the given control constraints for any setpoint changes.

The comparison with other considered alternatives shows that the main benefit of the article lies in a simple explicit algorithm of DO-CFPID controller for constrained positional systems and in an integrated tuning procedure allowing to take into account always present delays, whether delays of actuator, sensors, program, communication or noise attenuation filters. Because the controller is formulated in the continuous-time domain, it can be used for arbitrarily small sampling periods, making it competitive (at least in the single-input-single-output area) even compared to predictive control (MPC)-based control algorithms. Its superiority excels in steering with little impact of the friction forces compared to inertial forces, which is for example, typical for control of transport vehicles, or in different nano applications.

The paper is structured as follows. In the next section, performance measures used in the controller optimization are described. Then, the controlled system with the DO-CFPID controller is characterized. The comparative framework is established by introducing P-PI controllers. All proposed control structures are then illustrated by the real-time experiments. The achieved results and future work are summarized in Conclusions.

**The control performance measures**

For a setpoint \( r \) the aim of control is to get a minimal IAE (Integral of Absolute Error) value

\[
IAE = \int_0^\infty |e(t)|dt; e(t) = r(t) - y(t)
\]  

The output deviations from monotonicity may be evaluated by a modified total variation (sum of absolute increments) during the output course from the initial value \( y_0 \) to the final value \( y_n \) calculated according to

\[
y_{TV} = \sum_i |y_{i+1} - y_i| - |y_n - y_0|
\]  

The effective control effort should be spent on the inevitable acceleration and braking, which corresponds to a two-pulse (2P) input \( u \) shown in Figure 1. Other control increments representing deviations from 2P control should be kept as low as possible. Thus, in difference to traditional quadratic optimal control, which minimizes the overall controller activity, just the deviations from an ideal response are taken into account. Considered 2P shapes may be specified by initial value \( u_0 \), final value \( u_n \) and two extreme points \( u_{m1}, u_{m2} \) outlining three monotonic intervals. Excessive increments are then calculated according to (3), where \( i \) is the sample number of the control signal \( u \) in the implementation of the formula (3) by a program running with a fixed sampling period. The calculation must be
performed for all values of $u_i$ sampled in the time interval $(t_0, t_f)$, where $t_f$ is the time when the input reaches the final value $u_\infty$.

$$uTV_2 = \sum_i |u_{i+1} - u_i| - |2u_{m1} - 2u_{m0} - u_n - u_0|$$

(3)

Ideally, $uTV_2 = 0$. If the response of the second order plant in (4) for the setpoint step is strictly monotonic, or the signal at the system output is 1P type for the disturbance, then the value of $uTV_2$ is proportional to the distortion of the manipulative variable due to a noise. Based on this criterion, the noise level in the manipulative variable is compared for different types of controllers in the presence of quantization noise in the feedback, in the Experimental results section.

**DO-CFPID structure and its tuning**

The mechanical drive subsystem is approximated by the model (4), where $a_1$ and $K_s$ are the parameters of the plant with the gain $K_s/a_1$ and the real pole $-a_1$.

$$F(s) = \frac{K_s}{s^2 + a_1 s}$$

(4)

A variable load, unmodeled dynamics and model uncertainties will be modelled by input disturbance $d_i$ superimposed on the control $u_i$. The output of the plant is

$$Y(s) = [U_i(s)F_a(s) + D_i(s)]F(s)$$

(5)

A torque generator dynamics approximated by a dead time $T_d$ expressed in (6) in Laplace form

$$F_a(s) = e^{-T_ds}$$

(6)

will be taken into account when selecting the closed loop poles.

**Constrained PID control for the second order plant**

The structure of a DO-CFPID positional control is in Figure 2. Here, $Q_n(s)$ represents an $n$-th order binomial low pass filter with $n$-fold pole $-1/T_n$.

![Figure 1](image-url)

**Figure 1.** Two-pulse signal consisting of three monotonic intervals.

![Figure 2](image-url)

**Figure 2.** DO-CFPID controller with a disturbance observer (DO) based on a double integrator inversion augmented by a filter $Q_n(s)$ (with $n$ representing the filter order); the same filter is also used in a state observer (SO).

CPD: constrained PD controller; CFPID: constrained filtered PID; $\delta$: the measurement noise.

$$Q_n(s) = \frac{1}{(T_n s + 1)^n};$$

(7)

$$0 < T_n \ll T_p = 1/|a_1|; \quad n = 1, 2, ...$$

Let us denote the controller inputs $\hat{y}$, $\dot{\hat{y}}$, and $\ddot{\hat{y}}$ as a (filtered) output of the plant, its first derivative and an equivalent filtered disturbance, respectively.

**DO-FPI controller tuning**

For the plant (4) with $a_1 = 0$ and dead time (6), an optimal real dominant closed loop pole, which corresponds to the fastest possible transients without oscillations at the input and output, may be approximated\(^{17}\) as

$$\alpha_{1,2} = -0.321/T_{DT}$$

(8)

The tuning parameter $T_{DT}$ represents an overall loop dead time. It consists of an identified actuator dead time $T_d$ and of an equivalent delay $T$ of the filters $Q_n$ (7)

$$T_{DT} = T_d + T$$

(9)

By keeping a constant $T_{DT}$, a constant closed loop dominant pole position may be achieved. This may guarantee a performance invariance against different filter degrees $n$. A closed loop equivalence among the filter parameters $n, T_n$ and an equivalent filter dead time $T$ has been established\(^{17}\) as

$$T_n = k_n T; \quad k_n = \frac{2(n + 1) - \sqrt{2n(n + 1)}}{(2 - \sqrt{2})(n + 1)(n + 2)}$$

(10)

This formula represents an alternative to the “half-rule” introduced in Skogestad\(^{25}\) which may be applied in a broader range of filter parameters without causing side effects as overshooting, or oscillations.

DO-FPID controllers working in linear mode with ideal transient shapes yield for disturbance step...
\[ IAE_{LDO-FPID} = \frac{T_d + nT_a}{K_p} \]  

After substituting for the gain of the controller \( K_p \) the value \( K_p = 0.103/(K_sT_d^2) \), this allows to calculate \( T = T_n/k_n \) to guarantee a required \( IAE_i \) value (11) as a real positive solution of the following equation:

\[
0 = T^3n_k + T^2T_d(2nk_n + 1) + TT_s^2(nk_n + 2) - 0.103IAE_i/K_s
\]  

(12)

**Constrained PD control for the second order plant**

The constrained PD controller (CPD)\(^{26} \) and its DO-based extension\(^{17} \) drive the plant trajectories toward an optimal braking curve corresponding to the given constraints and chosen closed loop pole. Thereby, the fastest possible decrease of the velocity and position amplitudes during the braking phase is guaranteed that does not violate the ideal shape requirements at the input and output. Its design is usually accomplished in a phase plane with the state variables \( \dot{y} \) and \( \ddot{y} \) related to the input \( a \) by the model differential equation \( \ddot{y} = \dot{a} \) and with the origin as a required state. Around an operating point

\[
\ddot{y}_p(t) = m\dot{\dot{y}}(t) + (1 - m)\dot{y}(t)
\]

(13)

(with the limit parameter value \( m = 1 \) corresponding to linearization around the reference variable and \( m = 0 \) corresponding to feedback linearization\(^{25} \)) the plant (4) may be approximated by a double integrator with a gain \( K_s \) and an input disturbance \( d_i \) by the following control signal

\[
u = \dot{u}_c - \dot{d}_i = \frac{\dot{a} + a_1(m\dot{\dot{y}} + (1 - m)\dot{y})}{\dot{K}_s} - \dot{d}_i
\]

(14)

Thereby \( \dot{r} \) and \( \ddot{r} \) represent the first and the second derivatives of the setpoint \( r \) and \( \dot{y}, \ddot{y} \) are the phase variables in a coordinate system with a reference point shifted to its origin

\[
\dot{y}(t) = \dot{y}(t) - \dot{r}(t)
\]

\[
\ddot{y}(t) = \ddot{y}(t) - \dddot{r}(t)
\]

(15)

The weighting coefficient \( m \in [0, 1] \) influences character of the braking phase of transients. It allows adaptation of the algorithm developed for a double integrator to the plant (4) by enabling a continuous change from the exact linearization method (corresponding to \( m = 0 \),\(^{25} \)) and a linearization around a fixed point with \( m = 1 \).

The model output \( \dot{u} \) of the constrained control algorithm is equal either to the constrained nonlinear controller output

\[
\dot{u}_{nl} = \text{sat}\left\{ \left[ 1 - \alpha_2 \left( \frac{\ddot{y} + \frac{1}{2} \left( \frac{T_s}{T_d} + \frac{0}{\dot{y}} \right) + \frac{a_1}{\dot{y}} \right) \right] \dot{U}_j \right\}
\]

(16)

applied for

\[
\dot{y} < 0 \cap \ddot{y} > x_0' \cup \dot{y} > 0 \cap \ddot{y} < x_2'
\]

\[
x_0' = \dot{U}_1 \frac{1}{\alpha_1} ; \quad x_2' = \dot{U}_2 \frac{1}{\alpha_1}
\]

(17)

or to constrained linear controller output \( \ddot{u}_l \) computed as

\[
\ddot{u}_l = - \text{sat}\{\ddot{y}K_D + \ddot{y}K_p\}
\]

\[
K_p = a_1 + a_2 ; \quad K_D = - (a_1 + a_2)
\]

(18)

The model constraints \( \dot{U}_j \) of \( a \) are determined by the minimal and maximal limit values \( U_1, U_2 \) as

\[
\dot{U}_1 = K_s(U_{r1} + \dot{d}_i) - a_1(m\dot{\dot{y}} + (1 - m)\dot{y}) - \ddot{r}
\]

\[
\dot{U}_2 = K_s(U_{r2} + \dot{d}_i) - a_1(m\dot{\dot{y}} + (1 - m)\dot{y}) - \ddot{r}
\]

(19)

**P-PI and FP-FPI positional controllers**

The P-PI control structure (Figure 3) is built from a P position and PI speed controllers. In the Laplace transform

\[
U(s) = K_1 \left[ 1 + \frac{1}{T_J} \right] \left[ R(s) - \ddot{Y}(s)K_PF_s(s) - \dddot{Y}(s) \right]
\]

(20)

The symbols in (20) have the following meaning: \( K_P \) represents the gain of the position controller, \( K_V \) denotes the gain of the velocity controller and \( T_J \) represents its integration time constant. The filter \( F_s(s) = 1/(T_s + 1) \) is utilized to compensate for the error in the Laplace transfer function \( Y_s(s)/Y_R(s) \) defined by these symbols.

If loop dead times are substituted by the first order, that is, \( e^{-T_d s} \approx 1/(T_d s + 1) \), one gets the fourth order transfer functions \( Y(s)/R(s) \) and \( Y(s)/D(s) \). The controller may then be tuned to guarantee a triple real pole \(-1/T_0\) and a simple real pole \(-1/(kT_0)\). For P-PI control the parameters corresponding to this tuning are

\[
k = \frac{T_d}{T_0(a_1T_d + 1) - 3T_d} ; \quad T = 0
\]

\[
K_P = \frac{1}{T_0(3 + k)}
\]

\[
K_V = \frac{3}{T_0K_s}(a_1T_d + 1) - \frac{6T_d}{T_0K_s} - \frac{a_1}{K_s}
\]

\[
T_J = \frac{K_PK_sK_vT_d^2}{T_0(a_1T_d + 1) - 3T_d}
\]

(21)

For FP-FPI control

\[
K_P = \frac{1}{T_0(3 + k)} ; \quad T > 0 ; \quad 0 < k < 1
\]

\[
K_V = \frac{3(1 + k)}{T_0K_s[1 + k(3 - T_0a_1)]} - \frac{a_1}{K_s}
\]

\[
T_J = K_PK_sK_vT_d^2[1 + k(3 - T_0a_1)]
\]

\[
T = \frac{kT_0}{1 + k(3 - T_0a_1) - T_d}
\]

(22)
In case of a FP-FPI control the value of the parameter $k_4$ is optional. It is inversely proportional to the relative shift of the fourth pole. By decreasing $k$ within the range $(0, 1)$, the dominant pole is shifted to the left and the noise level of the manipulative variable increases.

The time constants $T_0$ guaranteeing the required values of $IAE_i$ for P-PI and FP-FPI control are the positive real solutions of (23) and (24).

$$0 = T_{0,i}^4 - T_{0,i}IAE_i (a_0 T_a + a_1) + 3 a_1 T_a IAE_i; \quad T = 0$$  \hspace{1cm} (23)

$$0 = T_{0,i}^4 a_0 k a_1 - T_{0,i}^5 (3k + 1) + a_1 IAE_i; \quad T > 0$$  \hspace{1cm} (24)

In the rest of paper, the value $k = 1$ has been used.

**Experimental results**

**Experiment setup**

A key element of the workplace (Figure 4) is a servo system with two electric drives. The shafts of the motors used in these drives are mechanically connected by means of a rigid coupling. A BLDC motor is used in one drive and a DC motor is used in the other drive. Both drives operate in torque mode, where they act as torque generators. The drive with BLDC motor is controlled, that is, it is used in the position control loop. The DC motor drive represents the load and serves to generate a disturbance for the position servo drive. Actual position information is obtained by evaluating the signal from the incremental rotary encoder (IRC) with 1024 pulses per revolution. The IRC output is connected to the input of the MF634 multifunction I/O card, which counts the IRC pulses. The actual position information is read from the MF634 card into the Matlab, where the control algorithm of the position loop is implemented. Parameters of the servo system are:

- $J = 2.6 \times 10^{-5}$ [kg.m$^2$], moment of inertia;
- $T_{ld} = 3.5 \times 10^{-5}$ [Nm], Coulomb friction;
- $K_s = 1/J = 38.462$ [kg$^{-1}$ m$^{-2}$], plant model gain;
- $a_1 = 0.481$ [N.s.rad$^{-1}$ kg$^{-1}$ m$^{-1}$], internal feedback coefficient;
- $T_{GM} = 0.2$ [ms], time constant of the torque generator;
- $\Delta \varphi = 0.001534$ [rad], position sensor resolution;
- $T_s = 0.25$ [ms], sampling period of the controllers.

All internal delays were approximated by a transport delay $T_d = T_{GM} + T_s = 0.45$ [ms]. The dynamics of the main control loop was tuned for $IAE_i$ reference: $IAE_i^*_i = 0.015$. Due to the applied control constraints, the really achieved values are higher.

The reference step changes 2 rad and 10 rad have been applied for $t = 0$ [s]. A step change of the input disturbance $d_i$ has been produced by the load torque step change $T_L$ from 0 to $-0.08$ [Nm]. It has been applied at the time instant $t = 1$ [s]. Controller output was limited to $u_i \leq U_{r, \text{max}}$, $U_{r, \text{max}} = 0.15$ [Nm].

**Experiment design**

Yet, before starting real time experiments, we made tests on the simulation model of a position servo drive developed as a new modification of a previous Matlab/Simulink tool. It allows a detailed comparison of all
the interesting aspects with real process which brings a useful information about the model. It is available to public in Bélai.28

The representative experimental results included into this paper may be divided into three parts.

**Simplifying DO impact on the plant modeling**

In the first step, the plant model (4) is further reduced to the simplest possible form - the double integrator. It introduces a dependence on the working point (13). This is accomplished by an evaluation of the impact of the weighting parameter $m$ on the setpoint step responses of the DO-CFPID control ($\varphi^* = 10$ [rad]).

The simulation may be performed for a drive without a load ($T_L = 0$ [Nm]) and with a load caused by Coulomb friction ($T_L = 3.5 \times 10^{-5}$ [Nm]). The results achieved in real time experiments are summarized in Table 1. They show that the linearization around a fixed operating point ($m = 1$) yields slightly better performance measures than the exact linearization ($m = 0$), or a generalized procedure ($m = 0.5$) – such results are typical for a negative velocity feedback. However, thanks to the DO impact which makes the plant to behave like the double-integrator plant model for low frequencies,29 in the rest of the paper, $m = 1$ will be applied.

Concerning the evaluated controllers, the experiments yield the following conclusions: (1) Due to the DO impact, approximation of the plant dynamics by a delayed double integrator yields performance, which depends just negligibly on the working point (13). Therefore, it is not necessary to use a significantly more sophisticated constrained controller30 corresponding to the integral plant model (4). (2) The DO-CPID controller based on a double integrator control allows to get nearly ideal shapes of the transients at the plant input and output for arbitrarily large admissible input and output step changes and without any command shaping and trajectory generation. Thus, its use in systems with large input steps (e.g. in autonomous vehicle control) is significantly simpler and more efficient.

**Saturation impact on DO-CFPID and DO-FPID controls**

Figure 5 confirms the theoretical expectations that the DO-CFPID controller eliminates the output

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**Table 1.** IAE values of position servo drive with DO-CFPID control according to $m$ ($\varphi^* = 10$ [rad], $n = 3$).

| $m$   | $\text{IAE}_r$ | $\text{IAE}_i$ | $uTV_2$ |
|-------|----------------|----------------|---------|
| 0     | 0.4626         | $1.1424 \times 10^{-3}$ | 2.3181  |
| 0.5   | 0.4565         | $1.2709 \times 10^{-3}$ | 2.4277  |
| 1     | 0.4497         | $1.1045 \times 10^{-3}$ | 2.3750  |

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**Figure 5.** The position transients for the setpoint and disturbance steps with DO based control and $U_r, \text{max} = 0.15$ ($\varphi^* = 10$ [rad], $T_L = -0.08$ [Nm], $n = 3$, $m = 1$).
overshooting occurring after larger setpoint steps (\(\varphi^* = 10 \text{ [rad]}\)) under DO-FPID control, when the number of control intervals at the input increases to 3. This may also be characterized by slightly increased IAE\(_r\) values. The third control interval also contributes to the increase of \(uTV_2\). However, during the saturation there is no contribution of the DO-FPID control to \(uTV_2\), whereas the DO-CFPID control signal moves within the proportional zone under a noise effect. Therefore, the use of a constrained control does not necessarily lead to improvements detectable by \(uTV_2\). Thus, the control with a slight overshooting may paradoxically require less control effort than the strictly monotonic transients. Else, by increasing input step amplitudes, or decreasing the control constraint \(U_{r,max}\) (from 0.15 in Figure 5 to 0.09 in Figure 6), shape improvements brought by DO-CFPID control are clearly visible (Figure 6). The output values, characterized by \(TV_{0,r}(y)\), increase from 0.0033 for DO-CFPID to 2.264 for DO-FPID.

**Robustness/optimality test: DO-CFPID, DO-FPID, P-PI, and FP-FPI control**

The IAE and \(uTV_1\) measures for different control solutions (DO-CFPID, DO-FPID, P-PI and FP-FPI position control) under variation of the inertia moment are summarized in Table 2. The values represent the arithmetic mean of ten measurements. Thereby, the plant/model mismatch has been modified by an inertia moment estimate \(J_m\) included in the drive model parameters \(K_{s,m} = 1/J_m\), \(a_{1,m} = B' / J_m\). \(B'\) denotes the viscous friction coefficient. The test has been carried out for three values: \(J_m = J\), \(J_m = 3J\), and \(J_m = J/3\), whereby \(J\) corresponds to the value taken from the plant identification.

In order to focus on the robustness in the proportional zone of control, the controllers have been compared at such setpoint change amplitude that causes touching the control constraints without an output overshooting. All three filtered controllers show significantly lower \(uTV_2\) values than the non-filtered P-PI control. Thereby, the DO based solutions working with \(n = 3\) and the filter time constants \(T_n \in \{0.5510, 0.8504, 1.282\} \text{ [ms]}\) yield almost identical results (with the only difference explained above). It again gives justification of the simplified plant approximation used by DO-CFPID. With respect to the excessive control effort, they should not be used for \(J_m > J\).

### Table 2. Performance measures of position servo drives: 1 – P-PI, 4.1 – FP-FPI, 7.2 – DO-FPID, 7.1 – DO-CFPID control (\(\varphi^* = 2 \text{ [rad]}\), \(T_i = -0.08 \text{ [Nm]}\), \(n = 3\), \(m = 1\)).

| No | \(J_m = J/3\) | \(J_m = J\) | \(J_m = 3J\) |
|----|---------------|-------------|-------------|
| 1  | 51.6          | 52.0        | 55.4        |
| 4.1| 70.6          | 50.8        | 54.1        |
| 7.2| 59.8          | 59.9        | 57.5        |
| 7.1| 58.8          | 55.9        | 76.4        |

| No | \(J_m = J\) | \(J_m = 3J\) |
|----|-------------|---------------|
| 1  | 5.26        | 1.40          |
| 4.1| 13.3        | 2.40          |
| 7.2| 6.16        | 1.47          |
| 7.1| 6.03        | 1.48          |

| No | \(J_m = J/3\) | \(J_m = J\) | \(J_m = 3J\) |
|----|---------------|-------------|-------------|
| 1  | 4.908        | 9.422       | 38.38       |
| 4.1| 0.625        | 0.728       | 3.511       |
| 7.2| 0.428        | 0.791       | 5.359       |
| 7.1| **0.420**    | 0.853       | 6.983       |

| No | \(J_m = J\) | \(J_m = 3J\) |
|----|-------------|---------------|
| 1  | 2.693       | 4.784         |
| 4.1| 0.345       | 0.424         |
| 7.2| 0.236       | 0.367         |
| 7.1| **0.221**   | **0.363**     |

The optimal values in bold.
With exception of $J_m = 3J$, the lowest dispersion of $IAE$ values is guaranteed by the traditional P-PI control, however, on cost of significantly increased $uTV^2$ values. These may be significantly decreased by the FP-FPI control yielding nominally the best setpoint tracking both from the $IAE$ and the $uTV^2$ point of view. Since for $n = 3$ the corresponding time constants $T_n \in \{0.033, 0.1015, 0.3200\}$ [ms] are already lower than $T_s$, it is possible that having some hardware enabling to work with shorter sampling periods, the results might yet be better.

As demonstrated by the output step responses in Figure 7 and the quantified results in Table 2, neither DO can fully eliminate impact of uncertainties and disturbances. It can alleviate it. In the nominal case, its output setpoint step responses are more conservative than the cascaded solutions, but less oscillatory in the perturbed cases. These advantages would stand out significantly more in the constrained case.

**Conclusions and further work**

Since in the P-PI control the quantization noise involved in the angular velocity signal is directly transmitted to the controller output, all the tested filtered controllers yield much lower excessive control effort (expressed in terms of $uTV^2$ values characterizing the high frequency components in $T^3_m$). Although the P-PI shows relatively low sensitivity to perturbations of $J_m$, it still has, similarly as in the sliding mode control, a relatively high control effort. In our further work, it will be interesting to study modifications of the discrete-time constrained controllers,\textsuperscript{12,31} which may further increase the control performance, since the filter time constants not always fulfill the requirements $T_n > > T_s$. The approach may further consider constraints on the state and the rate of input changes accomplished by a constrained control designed for a triple integrator.\textsuperscript{32} The closed-loop performance could yet be extended by a constrained feedforward control allowing to further speed up the setpoint step responses.

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