Plasma potential distribution around an infinite one-dimensional chain of dust particles

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Abstract. With the help of the code modification used in the authors previous works, the self-consistent distribution of density and electric potential are calculated around a one-dimensional infinite chain of dust particles. As a result, self-consistent distributions of the electric potential and space charge around a one-dimensional chain of dust particles, as well as the dependence of the dust particles charge on the interparticle distance, are obtained.

1. Introduction

At the moment, the problem of determination of the self-consistent parameters of the system which consists of an isolated dust particle drifting in plasma has been solved. To solve this problem, a number of computational models have been developed, which can be divided into three main categories: linear response methods (LR) [1,2], methods Particle-In-Cell (PIC) [3,4], and also a class of methods falling under the name "methods of molecular dynamics" (MD) [5].

In contrast to numerical methods, where the most accurate calculations are made primarily for isolated dust particles due to computational laboriousness, in the field of experimental works the situation is opposite: most experiments concern dust particle clusters [6-8]. Unlike in the case of an isolated particle, the chains and clusters of dust particles are characterized by complex effects, such as crystals formation in stratified discharges, transition of these crystals to different aggregation states, nonreciprocal interaction of dust particles, as well as beats and fluctuations of strongly coupled dusty structures in low gravity conditions.

Next logical step in the development of computational models would be the calculation of self-consistent electric fields from the combined contribution of ions, electrons and dust particles around chains and clusters of dust particles. Today, there are developed models based on the PIC approach, which calculate the self-consistent electric potential around multiple dust particles with sufficient accuracy [9]. However, in order to avoid computational complexity, significant simplifications are made for both motion equations of elementary particles (leapfrog calculation method) and for calculation accuracy of the potential in the computational grid (solution of the Poisson equation by finite difference schemes of the first order of accuracy).

This paper presents a model which, in a self-consistent way, calculates the potential and space charge distributions around an infinite chain of dust particles.
2. Model

The model presented in the current paper is a modification of the model, which was most thoroughly explained in the works [10-12]. For this reason, only the key changes, which allowed carrying out the calculation of an infinite chain of dust particles, will be clarified further.

The system in which the simulation takes place is a regular quadrilateral prism, where the area of the bases is much larger than the area of the side faces (see Figure 1). The length of the base edges is equal to \(40 \lambda_i\), and the bases themselves lie in the XY plane. The length of the side face smallest edges is equal to \(D\), where \(\lambda_i = \sqrt{kT_i / 4\pi e^2 n_{\infty}}\) is the ion Debye length, \(D\) is the distance between dust particles, \(n_{\infty}\) is undisturbed ion density, \(T_i\) is ion temperature. At the center of this prism is an impenetrable sphere of radius \(\eta_0\): a model for an isolated dust particle.

Simulation of the ions dynamics, contribution of ions and electrons to space charge, the statistical data accumulation and the dust particle charging in this model are exactly the same as in the models used in [10-12], and therefore will not be considered separately.

Due to the fact that the modeling space is very flattened, in contrast to [10-12], where a cube was chosen as the computational domain, the division of space into segments according to the spherical coordinate system \((r, \theta, \phi)\) does not seem appropriate anymore. Therefore, in this model, the computational volume is divided according to the cylindrical coordinate system \((r, \phi, z)\). Due to the fact that the chain of dust particles is aligned with the z-axis, the task is cylindrically symmetric and all spatial quantities depend only on a pair of coordinates \((r, z)\). The area of the potential calculation is divided into cells \((i,j)\). The volume of each cell is equal to \(V_{ij} = 2\pi r_i \Delta r \Delta z_j\).

As in previous studies, in the model the dimensionless variables were used. In the dimensionless representation, the dimensionless equivalent of the dust particle charge is written as follows:

\[
\bar{Q} = \frac{e^2 Z_d}{\lambda_i k T_i}.
\]

The initial potential distribution (zero iteration) is chosen as the sum of the Debye-Hückel potentials of all the considered dust particles.
\[ U_0(r, \theta) = -\frac{Q}{r} e^{-r} - \sum_k \frac{\tilde{Q}}{r_{k,2}} e^{-r_{k,2}} - \sum_k \frac{\tilde{Q}}{r_{k,1}} e^{-r_{k,1}}, \quad (2) \]

where \( r_{k,1}, r_{k,2} \) are the distances from the ion being monitored to neighboring dust particles (see Figure 2).

Direct calculation of the ions trajectories, as well as the calculation electric potential, is not performed near the neighboring dust particles. The calculation of the potential and space charge selfconsistent distributions as well as the simulation of the ion trajectories occur only for a volume bounded by a prism, where only one particle is present.

The essence of the model lies in the fact that to the large faces of the regular quadrilateral prism adjoin exactly such volumes, where the dust particle charge, potential and space charge distributions are identical to those in the main computational domain, adjoin to the large faces of the regular quadrilateral prism. In other words, the boundary conditions in [12], where it was considered that the plasma is present in an unperturbed state at the boundaries of the isolated dust particle system, are replaced by the conditions \( \frac{\partial U(r,D)}{\partial z} = \frac{\partial U(r,-D)}{\partial z} = 0 \).

Due to the fact that the division of space in this model is carried out according to cylindrical geometry, and also due to strong anisotropy, decomposition into Legendre polynomials, which was so much loved by the authors of this work and used by them in many papers, seems rather impractical. In the current work, the self-consistent potential is calculated at each iteration using the equation:

\[ U(r, \theta) = -\frac{Q}{r} \sum_k \frac{\tilde{Q}}{r_{k,2}} + \sum_k \frac{\tilde{Q}}{r_{k,1}} + \sum_k \left[ \frac{n(r', z') d^3 r'}{|r - r'|} + \sum_k \frac{n(r', z') d^3 r'}{|r_{k,1} - r'|} + \sum_k \frac{n(r', z') d^3 r'}{|r_{k,2} - r'|} \right], \quad (4) \]

where \( n(r, z) = \frac{n_1(r, z) - n_e(r, z)}{n_\infty} \).
where \( n(r,z), n_i(r,z), n_e(r,z) \) are the dimensionless distributions of space charge, ion density and electron density, respectively. The electron density \( n_e(r,z) \), as in previous models, is determined by the Boltzmann distribution. Here, the sum of the integrals over \( k \) is responsible for taking into account the influence of the adjacent region potential on the main computational domain.

The time between iterations, as in the models used in [10-12], was set according to computational expediency: it ought to be long enough for each segment of space to accumulate sufficient statistics, but small enough so the calculation time of the selfconsistent result would be the finite. For the case of an isolated particle in a cube, the edges of which are equal to \( 40 \lambda_i \), the transition to a new iteration was carried out when \( \sim10^6 \) ions were born and died in the system. In this model, the volume of the system is much less, namely 10-20 times. Therefore, the number of ion lifetimes required for the transition to the next iteration was normalized accordingly. It is worth mentioning the zero iteration separately, the transition from which to the first iteration requires more accurate statistics in order to avoid the appearance of divergence in the calculation. If for an isolated case the minimal required statistics was accumulated as a result of the calculation of \( \sim10^8 \) ion trajectories, then for this simulation a number of such calculated trajectories should be at least close to \( \sim10^7 \).

Thus, the complete algorithm for calculating the self-consistent potential distribution is as follows. Using Newton equations for the electric potential, defined by equation (2), the set of ion trajectories is calculated. After accumulating sufficient statistics of the spatial distribution of the ion density \( n_i(r,z) \) from equation (4), the potential \( U(r,z) \) is calculated, which represents the current configuration of the space charge in the computational volume. Following each iteration, a correction is made to the charge of the dust particle, which is found from the condition that the fluxes of ions and electrons to the surface of the dust particle are equal. The whole cycle is repeated until distribution of the space charge, distribution of the potential, and charge of the dust particle of the next step are equal to the corresponding values obtained at the previous step.

3. Results

In this paper, as well as in the works [10-12], the parameter of average mean free path \( l_i \) is used in the ion motion simulation. For the following results, this parameter was chosen to be equal to \( l_i = 5 \lambda_i \). The dimensionless size of the dust particles was chosen to be equal to \( r_0 = 0.01 \lambda_i \).

Using the model presented above, selfconsistent distributions of ion densities and electric potentials around an infinite chain of dust particles are obtained. A typical result of such calculations is presented in Figure 3 and Figure 4, which show the distributions of the space charge \( n(r=0,z) \) and the potential \( U(r=0,z) \) in a plane where \( r=0 \) for the case when the interparticle distance is \( D=2 \lambda_i \).

![Figure 3](image3.png)  **Figure 3.** Self-consistent space charge distribution \( n(r=0,z) \) in the plane \( r=0 \) for the interparticle distance \( D=2 \lambda_i \).

![Figure 4](image4.png)  **Figure 4.** Self-consistent potential distribution \( U(r=0,z) \) in the plane \( r=0 \) for the interparticle distance \( D=2 \lambda_i \).
To assume that the problem is solved correctly, two conditions must be met. First, this is the condition of quasineutrality, that is, the condition that the charge of the ion cloud surrounding the dust particle must be equal to the dust particle charge. Indeed, the integral of the space charge over the entire computational domain:

$$\bar{Q}_{pl} = \frac{1}{4\pi} \int n(r,z)rdrdz,$$

is equal to the value of the dimensionless charge $\bar{Q}$ of a dust particle, which is generated iteratively through the equality condition of the ion and electron fluxes. The second important condition is the appearance of a periodicity in ion density and electric potential at the boundaries of the computational domain ($z=-D, z=D$). The results presented in Figure 3 and Figure 4 allow asserting with complete confidence that the second condition is also satisfied.

The influence of the interparticle distance on the potential and space charge distributions is of great interest. Figure 5 shows a two-dimensional contour plot of the space charge distribution (a series in the upper half) and electric potential distribution (a series in the lower half) for three interparticle distances $D=1;2;4 \, \lambda_i$.

![Figure 5](image-url)  
**Figure 5.** Two-dimensional contour plot of the space charge distribution (a series in the upper half) and electric potential distribution (a series in the lower half) for three interparticle distances $D=1;2;4 \, \lambda_i$.  

From Figure 5, although with less accuracy than for Figures 3 and 4, it can still be established that the periodicity condition is fulfilled both for the potential and for the space charge. However, the main result is the changes that reveal themselves with a decrease of $D$, and which are easily visible to the naked eye. At sufficiently large distances between the dust particles, each of them is surrounded by their own ion cloud, which completely shields negative charge. The closer the particles are to each other, the more densely the ion clouds press together, until, at a certain distance, they merge into a single shielding
layer. For sufficiently large distances between particles \((D=4 \lambda_i)\), only the outer screening layers merge together, for space charge distributions these will be the layers \(n_\infty<n(r,z)<1.005 \ n_\infty\), while for dimensionless potential \(0<U(r,z)<-0.05 \ kT_i/e\). When particles approach each other closer \((D=2 \lambda_i)\) the deeper layers \(n_\infty<n(r,z)<1.05 \ n_\infty, 0<U(r,z)<- \ kT_i/e\) are come united. And for \(D=1 \lambda_i, n_\infty<n(r,z)<1.1 \ n_\infty, 0<U(r,z)<-2 \ kT_i/e\). In other words, the closer the particles are to each other, the stronger the potential resembles the potential of a charged wire placed in plasma.

In addition to the fact that the outer layers of the ion cloud group begin to merge, as the dust particles approach, the clouds also expand. If for the case \(D=4 \lambda_i\) the boundary of the layer \(n(r,z)=1.005 \ n_\infty\) lies near the point \(3.3 \lambda_i\), then for the case \(D=1 \lambda_i\) it shifts to \(4.2 \lambda_i\). In other words, when dust particles approach each other, the ion clouds are strongly bent, which causes the lighter layers to be squeezed towards the outer edge of the computational domain.

As already mentioned in the part devoted to modeling, the list of iteratively determined macroparameters of the system also includes the charge of dust particles. Figure 6 represents the dependence of the dimensionless dust particle charge \(\tilde{Q}\) on the interparticle distance.

![Figure 6](https://example.com/figure6.png)

Figure 6. Dependence of the dimensionless dust particle charge \(\tilde{Q}\) on the interparticle distance \(D\).

Figure 6 shows the tendency known both from experimental works and numerical calculations: with an increase in the density of dust particles in an observation volume and, as a consequence of this, with a decrease in the distance between these particles, the charge of each individual dust particle decreases [13-15]. In experiments, this effect is associated with the electron depletion in plasma surrounding dust particles, which leads to the decrease in the electron flow to each individual particle. Therefore, the equality condition of the ion and electron fluxes to the dust particles produces a charge less than that of isolated dust particles.

In numerical calculations carried out using the model described above, this is due to the opposite effect: as the distance between particles decreases, the path of the simulated ion will more often bend under the influence of neighboring particles. As a result of this, the frequency with which ions will collide with a dust particle will fall, which means that the flow of ions to the particle surface per unit of time will also decrease.

From the data obtained, it is also possible to derive a condition when the dust particle can be considered as isolated. The dependence shown in Figure 6 states that for the ion mean path \(\lambda_i = 5 \lambda_i\) of the charge exchange process, the charge of each individual dust particle in the chain will be equal to the charge of an isolated dust particle if \(D > 10 \lambda_i\).
Conclusion
A numerical model has been developed that allows obtaining the selfconsistent space charge and electric potential distributions near an infinite chain of dust particles. Important verifications have been carried out: the model satisfies the conditions of quasineutrality and periodicity.

The dependence of the selfconsistent distributions of the space charge and electric potential on the interparticle distance is shown. It has been demonstrated that as the distance between particles decreases, the outer layers of the distribution merge and are pushed towards the region of the unperturbed plasma.

The dependence of the charge of each individual dust particle on the interparticle distance is calculated. At a qualitative level, agreement with the results of the experimental observations was obtained: the charge of each individual dust particle decreases with an increase in the dust particles density in the dust particles cluster. The condition under which the dust particle can be considered isolated is determined.

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