Sensitivity Analysis of Mixed Cayley Inclusion Problem with XOR-Operation

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Received: 9 December 2019; Accepted: 15 January 2020; Published: 2 February 2020

Abstract: In this paper, we consider the parametric mixed Cayley inclusion problem with Exclusive or (XOR)-operation and show its equivalence with the parametric resolvent equation problem with XOR-operation. Since the sensitivity analysis, Cayley operator, inclusion problems, and XOR-operation are all applicable for solving many problems occurring in basic and applied sciences, such as financial modeling, climate models in geography, analyzing “Black Box processes”, computer programming, economics, and engineering, etc., we study the sensitivity analysis of the parametric mixed Cayley inclusion problem with XOR-operation. For this purpose, we use the equivalence of the parametric mixed Cayley inclusion problem with XOR-operation and the parametric resolvent equation problem with XOR-operation, which is an alternative approach to study the sensitivity analysis. In support of some of the concepts used in this paper, an example is provided.

Keywords: parametric; Cayley; Lipschitz; XOR-operation; sensitivity

MSC: 47H05; 49H10; 47J25

1. Introduction

It is well known that the variational inequalities (inclusions) and their generalizations are applicable for dealing with a large number of problems related to mechanics, physics, optimization and control, nonlinear programming, economics, transportation equilibrium, engineering sciences, etc. (for example, see [1–10] and references therein). On the other hand, ordered variational inequalities (inclusions) are also very important from the application point of view. A suitable amount of work related to generalized variational inequalities (inclusions) in Hilbert spaces as well as in Banach spaces can be found in [11–21]. Let $S$ be a symmetric operator on $\mathcal{H}$ and express an operator $C_S$ as

$$C_S = (S - iI)(S + iI)^{-1},$$

then $C_S$ is the Cayley transformation of $S$. Cayley transformation is a mapping between skew-symmetric matrices and special orthogonal metrics. Recently, the Cayley inclusion problem with XOR-operation was considered and studied by Ali et al. [22]. They proved some properties of the Cayley operator involved in their problem.

“Exclusive or” or exclusive disjunction (XOR) is a logical operation that outputs true only when inputs differ (one is true, the other is false). It gained the name “exclusive or” because the meaning of “or” is ambiguous when both operands are true, the exclusive or operator excludes that case. This is
sometimes thought of as “one or the other but not both”. This could be written as “A or B, but not A and B“. It is to be noted that XOR and symmetric difference for sets use the same symbol \( \oplus \) and the same structure, which are shown by Venn diagrams below in Figure 1.

\[
\text{XOR} : \ (A \land (\neg B)) \lor (B \land (\neg A)) \\
\text{Symmetrical Difference} : \ (A \cap B^c) \cup (B \cap A^c) .
\]

Figure 1. Symmetric difference and XOR of the sets A and B.

That is, XOR and symmetric difference for sets are the same. Moreover, XOR is the symmetric difference for truth values.

Let \( A \triangle B \) denote the symmetric difference of the sets \( A \) and \( B \), given an object \( x \), then

\[
x \in A \triangle B, \text{ if and only if } (x \in A) \text{XOR}(x \in B).
\]

For example, \( \{1, 2, 3\} \text{XOR} \{3, 4\} \) is \( \{1, 2, 4\} \), and the symmetric difference of \( \{1, 2, 3\} \) and \( \{3, 4\} \) is also \( \{1, 2, 4\} \).

It is well known that symmetric difference as well as XOR-operation are both symmetric. The graphical representation of XOR of \( x \) and \( y \) for truth values is given by the following Figure 2.

Figure 2. XOR Gate Symbol.

XOR-operations depict interesting facts, observations, and results to form several real-time applications; one can find its applications in digital communication, neural network, addition operation in CPU, programming contests, binary search, etc. The study of qualitative behavior of a solution of a variational inequality when the given operator and the feasible convex set vary with a parameter is known as sensitivity analysis. As sensitivity analysis is one of the important tools of functional analysis, we will mention some of its applications. It helps decision makers with more than a
solution to a problem, it is used to indicate sensitivity of a simulation to uncertainties in the input values of the model; helps in assessing the risk of a strategy; helps to predict the future changes in the governing system; provides useful information for designing or planning various equilibrium systems, etc. Some applications of sensitivity analysis related to real problems are optimal control of anthroponotic cutaneous leishmania, driving conditions for active vehicle control systems, influenza model with treatment and vaccination, dengue epidemiological model, Stochastic human exposure, dose simulation models, etc., for examples, see [23–26]. Sensitivity analysis for variational inequalities (inclusions) have been studied by Tobin [27], Kyparises [28,29], Dafermos [30], Qui and Magnanti [31], Li et al. [32], Yen and Lee [33], and Agarwal et al. [34], etc. For the sensitivity analysis related to real-life problems, we refer to [35,36] and references therein.

Due to importance of the Cayley operator, inclusion problems, and XOR-operation, we combine all these concepts to introduce two new equivalent problems, that is, parametric mixed Cayley inclusion problem with XOR-operation and parametric resolvent equation problem with XOR-operation. We have established equivalence between parametric mixed Cayley inclusion problem with XOR-operation and parametric resolvent equation problem with XOR-operation. The motivation of this work is to use this equivalence to develop sensitivity analysis for parametric mixed Cayley inclusion problem with XOR-operation without using differentiability of the given data.

2. Preliminaries

Throughout the paper, we assume \( \mathcal{H} \) to be a real ordered Hilbert space with norm \( \| \cdot \| \) and inner product \( \langle \cdot, \cdot \rangle \). Let \( C \subset \mathcal{H} \) be a cone and \( \leq \) be the ordering induced by the cone \( C \). For arbitrary element \( x, y \in \mathcal{H} \), \( x \leq y \) holds if and only if \( y - x \in C \). The relation \( \leq \) is called a partial ordered relation. The arbitrary elements \( x, y \in \mathcal{H} \) are said to be comparable to each other if \( x \leq y \) or \( y \leq x \) holds and is denoted by \( x \propto y \).

**Definition 1** ([37]). For arbitrary elements \( x, y \in \mathcal{H} \), \( \text{lub}\{x, y\} \) and \( \text{glb}\{x, y\} \) mean the least upper bound and the greatest lower bound for the set \( \{x, y\} \). Suppose \( \text{lub}\{x, y\} \) and \( \text{glb}\{x, y\} \) exist, then we have

\[
\begin{align*}
(i) & \quad x \vee y = \text{lub}\{x, y\}; \\
(ii) & \quad x \wedge y = \text{glb}\{x, y\}; \\
(iii) & \quad x \oplus y = (x - y) \vee (y - x); \\
(iv) & \quad x \odot y = (x - y) \wedge (y - x).
\end{align*}
\]

The operations \( \vee, \wedge, \oplus, \) and \( \odot \) are called OR, AND, XOR, and XNOR operations, respectively.

**Proposition 1** ([37]). Let \( \oplus \) be an XOR-operation, \( \odot \) be an XNOR-operation, and \( C \) be a cone. Then, for each \( x, y \in \mathcal{H} \), the following relations hold:

\[
\begin{align*}
(i) & \quad x \odot x = 0, x \odot y = y \odot x = -(x \oplus y) = -(y \oplus x), \\
(ii) & \quad \text{if } x \propto 0, \text{ then } -x \odot 0 \leq x \leq x \odot 0, \\
(iii) & \quad (\lambda x) \oplus (\lambda y) = |\lambda|(x \oplus y), \\
(iv) & \quad 0 \leq x \odot y, \text{ if } x \propto y, \\
(v) & \quad \text{if } x \propto y \text{, then } x \odot y = 0, \text{ if and only if } x = y, \\
(vi) & \quad \|0 \odot 0\| = \|0\| = 0, \\
(vii) & \quad \|x \odot y\| \leq \|x - y\|, \\
(viii) & \quad \text{if } x \propto y, \text{ then } \|x \odot y\| = \|x - y\|.
\end{align*}
\]

**Definition 2** ([22]). Let \( A : \mathcal{H} \to \mathcal{H} \) be a single-valued mapping and \( M : \mathcal{H} \to 2^\mathcal{H} \) be a multi-valued mapping, then

\[
\begin{align*}
(i) & \quad A \text{ is said to be a comparison mapping if } x \propto y, \text{ then } A(x) \propto A(y), x \propto A(x), \text{ and } y \propto A(y), \text{ for all } x, y \in \mathcal{H}; \\
(ii) & \quad A \text{ is said to be strongly comparison mapping if } A \text{ is a comparison mapping and } A(x) \propto A(y), \text{ if and only if } x \propto y, \text{ for all } x, y \in \mathcal{H}.
\end{align*}
\]
(iii) \( M \) is said to be a comparison mapping if, for any \( v_x \in M(x), \ x \propto v_x, \) and if \( x \propto y, \) then for any \( v_y \in M(y), \ v_x \propto v_y, \) for all \( x, y \in \mathcal{H}; \)

(iv) \( M \) is said to be weak comparison mapping if, for any \( x, y \in \mathcal{H}, \ x \propto y, \) then there exists \( v_x \in M(x) \) and \( v_y \in M(y) \) such that \( x \propto v_x, y \propto v_y, \) and \( v_x \propto v_y; \)

(v) a comparison mapping \( M \) is said to be \( \gamma \)-ordered rectangular if there exists a constant \( \gamma > 0, \) and, for any \( x, y \in \mathcal{H}, \) there exists \( v_x \in M(x) \) and \( v_y \in M(y) \) such that

\[
\langle v_x \circ v_y, (x \oplus y) \rangle \geq \gamma \|x \oplus y\|^2;
\]

(vi) \( M \) is said to be \( \lambda \)-weak-ordered different comparison mapping if there exists a constant \( \lambda > 0 \) such that for any \( x, y \in \mathcal{H}, \) there exists \( v_x \in M(x) \) and \( v_y \in M(y) \) such that \( \lambda(v_x - v_y) \propto (x - y); \)

(vii) a weak comparison mapping \( M \) is said to be a \((\gamma, \rho)\)-weak-ordered rectangular different multivalued mapping, if \( M \) is a \( \gamma \)-ordered rectangular and \( \rho \)-weak-ordered different comparison mapping and \( [I + \rho M](\mathcal{H}) = \mathcal{H}, \) for \( \rho > 0. \)

**Definition 3.** Let \( M : \mathcal{H} \rightarrow 2^\mathcal{H} \) be a multivalued mapping. The operator \( R_{I,\rho}^M : \mathcal{H} \rightarrow \mathcal{H} \) defined by

\[
R_{I,\rho}^M(x) = [I + \rho M]^{-1}(x), \quad \text{for all } x \in \mathcal{H},
\]

is called the resolvent operator, where \( \rho > 0 \) is a constant and \( I \) is an identity operator.

**Definition 4.** The Cayley operator \( C_{I,\rho}^M \) associated with \( M \) is defined as

\[
C_{I,\rho}^M(x) = [2R_{I,\rho}^M - I](x), \quad \text{for all } x \in \mathcal{H}.
\]

It is easy to check that when \( x \propto y, \) the resolvent operator defined by (1) and the Cayley operator defined by (2) are single valued as well as comparison mappings, see [22].

**Lemma 1** ([22]). Let \( M : \mathcal{H} \rightarrow 2^\mathcal{H} \) be a \( \gamma \)-ordered rectangular multivalued mapping with respect to \( R_{I,\rho}^M. \) Then, the resolvent operator \( R_{I,\rho}^M \) is Lipschitz-type continuous, that is,

\[
\|R_{I,\rho}^M(x) \oplus R_{I,\rho}^M(y)\| \leq \theta\|x \oplus y\|, \quad \text{where } \theta = \frac{1}{\gamma \rho - 1}, \quad \rho > \frac{1}{\gamma}, \quad \text{for all } x, y \in \mathcal{H}.
\]

**Lemma 2** ([22]). Let \( M : \mathcal{H} \rightarrow 2^\mathcal{H} \) be \((\gamma, \rho)\)-weak-ordered rectangular different multivalued mapping with respect to \( R_{I,\rho}^M \) and the resolvent operator \( R_{I,\lambda}^M \) is \( \theta \)-Lipschitz-type-continuous. Then, the Cayley operator \( C_{I,\rho}^M \) defined by (2) is \((2\theta + 1)\)-Lipschitz-type-continuous. That is,

\[
\|C_{I,\rho}^M(x) \oplus C_{I,\rho}^M(y)\| \leq (2\theta + 1)\|x \oplus y\|, \quad \text{where } \theta = \frac{1}{\gamma \rho - 1}, \quad \rho > \frac{1}{\gamma}, \quad \text{for all } x, y \in \mathcal{H}.
\]

**Remark.** The resolvent operator of a multivalued mapping plays a big role in several nonlinear programmes, see [38–41] for recent related results. If \( x \propto y, \) then \( R_{I,\rho}^M(x) \propto R_{I,\rho}^M(y) \) and \( C_{I,\rho}^M(x) \propto C_{I,\rho}^M(y). \) By using (viii) of Proposition 1, we can write

\[
\|R_{I,\rho}^M(x) \oplus R_{I,\rho}^M(y)\| = \|R_{I,\rho}^M(x) - R_{I,\rho}^M(y)\| \quad \text{and} \quad \|C_{I,\rho}^M(x) \oplus C_{I,\rho}^M(y)\| = \|R_{I,\rho}^M(x) - R_{I,\rho}^M(y)\|.
\]

**Definition 5.** A mapping \( g : \mathcal{H} \rightarrow \mathcal{H} \) is called

(i) strongly monotone, if there exists a constant \( \alpha > 0 \) such that

\[
\langle g(x) - g(y), x - y \rangle \geq \alpha \|x - y\|^2, \quad \text{for all } x, y, \in \mathcal{H};
\]
(ii) Lipschitz continuous, if there exists a constant \( \beta > 0 \) such that
\[
\| g(x) - g(y) \| \leq \beta \| x - y \|, \text{ for all } x, y \in \mathcal{H};
\]

(iii) relaxed Lipschitz continuous, if there exists a constant \( k > 0 \) such that
\[
\langle g(x) - g(y), x - y \rangle \leq -k \| x - y \|^2, \text{ for all } x, y \in \mathcal{H}.
\]

Let \( \Omega \) be a nonempty open subset of \( \mathcal{H} \) in which the parameter \( \lambda \) takes values. We define the following operators.

**Definition 6.** Let \( M : \mathcal{H} \times \Omega \to 2^\mathcal{H} \) be a maximal monotone multivalued mapping with respect to first argument. The implicit resolvent operator \( R_{I, \rho}^{M(\cdot, \lambda)} : \mathcal{H} \to \mathcal{H} \) is defined as
\[
R_{I, \rho}^{M(\cdot, \lambda)}(x) = \left[ I + \rho M(\cdot, \lambda) \right]^{-1}(x), \text{ for } \lambda \in \Omega, \rho > 0 \text{ and for all } x \in \mathcal{H}. \tag{3}
\]

**Definition 7.** The implicit Cayley operator \( C_{I, \rho}^{M(\cdot, \lambda)} : \mathcal{H} \to \mathcal{H} \) is defined as
\[
C_{I, \rho}^{M(\cdot, \lambda)}(x) = \left[ 2R_{I, \rho}^{M(\cdot, \lambda)} - I \right](x), \text{ for } \lambda \in \Omega, \rho > 0 \text{ and for all } x \in \mathcal{H}. \tag{4}
\]

3. Formulation of the Problem

Let \( \mathcal{H} \) be a real ordered Hilbert space. Let \( M : \mathcal{H} \to 2^\mathcal{H} \) be the maximal monotone multivalued mapping and \( C_{I, \rho}^{M} \) be the Cayley operator defined by (2). Let \( T, g : \mathcal{H} \to \mathcal{H} \) be the single valued mappings. We mention the following mixed Cayley inclusion problem (5) as well as its corresponding resolvent equation problem (6) with XOR-operation.

Find \( x \in \mathcal{H} \) such that
\[
0 \in \left( C_{I, \rho}^{M}(x) + T(x) \right) \oplus M(g(x)). \tag{5}
\]

Find \( x, z \in \mathcal{H} \) such that
\[
\left( C_{I, \rho}^{M}(x) + T(x) \right) \oplus \rho^{-1} I_{I, \rho}^{M}(z) = 0, \tag{6}
\]

where \( I_{I, \rho}^{M} = [I - R_{I, \rho}^{M}] \), \( I \) is the identity operator, and \( \rho > 0 \) is a constant.

A similar version of (5) is considered by Ali et al. [22]. It is easy to show that (5) and (6) are equivalent.

The following equation is a fixed point formulation of (5), which can be shown easily by using the definition of resolvent operator given by (1).
\[
g(x) = R_{I, \rho}^{M} \left[ g(x) + \rho \left( C_{I, \rho}^{M}(x) + T(x) \right) \right]. \tag{7}
\]

Now, we consider the parametric version of (5) and (6). Let \( M : \mathcal{H} \times \Omega \to 2^\mathcal{H} \) be a maximal monotone multivalued mapping with respect to the first argument, \( C_{I, \rho}^{M(\cdot, \lambda)} \) be the parametric Cayley operator defined by (4), \( g : \mathcal{H} \to \mathcal{H} \), and \( T : \mathcal{H} \times \Omega \to \mathcal{H} \) are the single valued mappings. We consider the following problem:

Find \( (x, \lambda) \in \mathcal{H} \times \Omega \) such that
\[
0 \in \left( C_{I, \rho}^{M(\cdot, \lambda)}(x) + T(x, \lambda) \right) \oplus M(g(x, \lambda)). \tag{8}
\]

Problem (8) is called the parametric mixed Cayley inclusion problem with XOR-operation. We assume that problem (8) has a unique solution \( x \) for some \( \bar{\lambda} \in \Omega \).

The following Lemma is a fixed point formulation of (8).
Lemma 3. The parametric mixed Cayley inclusion Problem (8) has a solution \((x, \lambda) \in \mathcal{H} \times \Omega\), if and only if it satisfies the following relation:
\[
g(x) = R_{I, \rho}^{M, \lambda}(g(x) + \rho \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right)). \tag{9}
\]

Proof. The proof is a direct consequence of the definition of parametric resolvent operator \(R_{I, \rho}^{M, \lambda}\) and hence, is omitted. \(\square\)

In connection with the parametric mixed Cayley inclusion problem with XOR-operation (8), we mention its corresponding parametric resolvent equation problem with XOR-operation.

Find \((x, \lambda), (z, \lambda) \in \mathcal{H} \times \Omega\) such that
\[
\left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right) \oplus \rho^{-1} I_{I, \rho}^{M, \lambda}(z) = 0, \tag{10}
\]
where \(I_{I, \rho}^{M, \lambda} = \left[ I - R_{I, \rho}^{M, \lambda} \right]\), \(I\) is the identity operator, and \(\rho > 0\) is a constant.

The following Lemma ensures the equivalence between problems (8) and (10).

Lemma 4. Let \(\left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right) \propto I_{I, \rho}^{M, \lambda}\), then the parametric mixed Cayley inclusion problem (8) has a solution \((x, \lambda) \in \mathcal{H} \times \Omega\), if and only if the parametric resolvent equation problem (10) has a solution \((x, \lambda), (z, \lambda) \in \mathcal{H} \times \Omega\), provided the following are satisfied:
\[
g(x) = R_{I, \rho}^{M, \lambda}(z), \tag{11}
\]
\[
z = g(x) + \rho \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right), \tag{12}
\]
where \(\rho > 0\) is a constant.

Proof. Let \((x, \lambda) \in \mathcal{H} \times \Omega\) be a solution of the parametric mixed Cayley inclusion problem (8). Then, by Lemma 3, it is a solution of the following equation:
\[
g(x) = R_{I, \rho}^{M, \lambda}(g(x) + \rho \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right)).
\]

Combining (11) and (12), we have
\[
z = g(x) + \rho \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right),
\]
\[
z = R_{I, \rho}^{M, \lambda}(z) + \rho \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right), \tag{13}
\]
\[
(I - R_{I, \rho}^{M, \lambda})(z) = \rho \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right).
\]

Using the fact that \(I_{I, \rho}^{M, \lambda} = \left[ I - R_{I, \rho}^{M, \lambda} \right]\), (13) becomes
\[
\rho^{-1} I_{I, \rho}^{M, \lambda}(z) = \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right) \oplus \rho^{-1} I_{I, \rho}^{M, \lambda}(z) = \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right) \oplus \left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right),
\]
we have
\[
\left( C_{I, \rho}^{M, \lambda}(x) + T(x, \lambda) \right) \oplus \rho^{-1} I_{I, \rho}^{M, \lambda}(z) = 0.
\]

Thus, the parametric resolvent equation problem with XOR-operation (10) holds.
Conversely, suppose that the parametric resolvent equation problem with XOR-operation (10) holds. Then, we have
\[
(C^M_{I,\rho}(x) + T(x, \lambda)) \oplus J^M_{I,\rho}(z) = 0.
\]
Since \((C^M_{I,\rho}(x) + T(x, \lambda)) \preceq J^M_{I,\rho}(z)\), by (v) of Proposition 1, we have
\[
\rho \left( C^M_{I,\rho}(x) + T(x, \lambda) \right) = J^M_{I,\rho}(z) = [I - R^M_{I,\rho}] (z) = z - R^M_{I,\rho}(z) = g(x) + \rho \left( C^M_{I,\rho}(x) + T(x, \lambda) \right) - R^M_{I,\rho}(z).
\]
It follows that
\[
g(x) = R^M_{I,\rho}(z).
\]
That is,
\[
g(x) = R^M_{I,\rho} \left[ g(x) + \rho \left( C^M_{I,\rho}(x) + T(x, \lambda) \right) \right].
\]
Applying Lemma 3, we conclude that the parametric mixed Cayley inclusion problem with XOR-operation (8) has a solution. \(\square\)

4. Sensitivity Analysis

We suppose that the solution of the parametric resolvent equation problem with XOR-operation (10) lies in the interior of \(\mathcal{H}\). Using the technique of Dafermos [30], we consider the following mapping:
\[
F_\lambda(z) = g(x) + \rho \left( C^M_{I,\rho}(x) + T(x, \lambda) \right), \text{ for all } (x, \lambda), (z, \lambda) \in \mathcal{H} \times \Omega,
\]
where \(F_\lambda(z) = F(z, \lambda)\).

Now, we prove that the mapping \(F_\lambda(z)\) defined by (14) is a contraction mapping.

**Lemma 5.** Let \(T : \mathcal{H} \times \Omega \to \mathcal{H}\) be relaxed Lipschitz continuous and Lipschitz continuous with respect to the first argument with constants \(\alpha > 0\) and \(\beta > 0\), respectively. Let the mapping \(g : \mathcal{H} \to \mathcal{H}\) be strongly monotone and Lipschitz continuous with constants \(\delta > 0\) and \(\sigma > 0\), respectively. Suppose that the parametric implicit Cayley operator \(C^M_{I,\rho}(\lambda)\) is Lipschitz-type continuous with constant \((2\theta + 1)\). Then, for all \(z_1, z_2 \in \mathcal{H}\), \(\lambda \in \Omega\) and for \(x_1 \preceq x_2, z_1 \preceq z_2\), \(F_\lambda(z_1) \preceq F_\lambda(z_2), C^M_{I,\rho}(x_1) \preceq C^M_{I,\rho}(x_2)\), we have
\[
\|F_\lambda(z_1) - F_\lambda(z_2)\| \leq \xi(\theta) \|z_1 - z_2\|,
\]
where
\[
\xi(\theta) = \frac{P(\theta)}{(1-k)}, \quad P(\theta) = \left[ k + \sqrt{1 - 2\beta^2(1 - M^2)} (2\theta + 1) \right], \quad \theta = \frac{1}{\gamma \rho - 1}, \quad k = \sqrt{1 - 2\delta + \sigma^2}, \quad \rho > \frac{1}{\gamma},
\]
for
\[
\rho - \frac{\alpha}{\beta^2} < \frac{\sqrt{\alpha^2 - \beta^2(1 - M^2)}}{\beta^2}, \text{ where } M = \left[ \frac{1 - k(1 + \theta)}{\theta} - (2\theta + 1) \right].
\]
Proof. Using (14), for all \( z_1, z_2 \in \mathcal{H} \), \( \lambda \in \Omega \) and as both the concepts symmetric difference \( \triangle \) and \( \oplus \) operation are same, we have
\[
\| F_\lambda(z_1) \triangle F_\lambda(z_2) \| = \| F_\lambda(z_1) \oplus F_\lambda(z_2) \| = \left\| \left[ g(x_1) + \rho \left( C_{\lambda,\rho}^{M(\lambda)}(x_1) + T(x_1, \lambda) \right) \right] \oplus \left[ g(x_2) + \rho \left( C_{\lambda,\rho}^{M(\lambda)}(x_2) + T(x_2, \lambda) \right) \right] \right\|. \tag{16}
\]
As \( F_\lambda(z_1) \rightarrow F_\lambda(z_2) \), using (vii) and (viii) of Proposition 1, (16) becomes
\[
\| F_\lambda(z_1) - F_\lambda(z_2) \| = \left\| g(x_1) + \rho \left( C_{\lambda,\rho}^{M(\lambda)}(x_1) + T(x_1, \lambda) \right) \right\| + \left\| g(x_2) + \rho \left( C_{\lambda,\rho}^{M(\lambda)}(x_2) + T(x_2, \lambda) \right) \right\| \tag{17}
\]
Since \( g \) is strongly monotone and Lipschitz continuous with constants \( \delta > 0 \) and \( \sigma > 0 \), respectively, we have
\[
\|(x_1 - x_2) - (g(x_1) - g(x_2))\|^2 \leq (1 - 2\delta + \sigma)\|x_1 - x_2\|^2,
\]
it follows that
\[
\|(x_1 - x_2) - (g(x_1) - g(x_2))\| \leq \sqrt{1 - 2\delta + \sigma} \|x_1 - x_2\|. \tag{18}
\]
Since \( T \) is relaxed Lipschitz continuous and Lipschitz continuous with respect to the first argument with constants \( \alpha > 0 \) and \( \beta > 0 \), respectively, we obtain
\[
\|(x_1 - x_2) + \rho(T(x_1, \lambda) - T(x_2, \lambda))\|^2 \leq \|x_1 - x_2\|^2 + 2\rho\|T(x_1, \lambda) - T(x_2, \lambda), x_1 - x_2\|^2 - 2\rho\|T(x_1, \lambda) - T(x_2, \lambda)\|^2 \leq \|x_1 - x_2\|^2 - 2\rho\|x_1 - x_2\|^2 + \rho^2\|x_1 - x_2\|^2 = (1 - 2\rho\|x_1 - x_2\|^2. \tag{19}
\]
As \( x_1 \in x_2, C_{\lambda,\rho}^{M(\lambda)}(x_1) \in C_{\lambda,\rho}^{M(\lambda)}(x_2) \), using (viii) Proposition 1 and Lemma 2, we have
\[
\left\| C_{\lambda,\rho}^{M(\lambda)}(x_1) - C_{\lambda,\rho}^{M(\lambda)}(x_2) \right\| = \left\| C_{\lambda,\rho}^{M(\lambda)}(x_1) \oplus C_{\lambda,\rho}^{M(\lambda)}(x_2) \right\| \leq (2\theta + 1)\|x_1 \oplus x_2\| \leq (2\theta + 1)\|x_1 - x_2\|, \tag{20}
\]
where \( \theta = \frac{1}{\sqrt{\rho - 1}}, \rho > \frac{1}{\theta} \). Combining (18)–(20) with (17), we obtain
\[
\| F_\lambda(z_1) - F_\lambda(z_2) \| \leq \sqrt{1 - 2\delta + \sigma} \|x_1 - x_2\| + \sqrt{1 - 2\rho\|x_1 - x_2\|^2 + (2\theta + 1)\|x_1 - x_2\|^2 \|x_1 - x_2\| = P(\theta)\|x_1 - x_2\|, \tag{21}
\]
where \( P(\theta) = \left[ k + \sqrt{1 - 2\rho\|x_1 - x_2\|^2 + (2\theta + 1)\|x_1 - x_2\|^2} \right] \) and \( k = \sqrt{1 - 2\delta + \sigma} \).

As \( z_1 \in z_2 \), using (18), Remark 1, and Lemma 2, we have
\[
\| x_1 - x_2 \| \leq \| x_1 - x_2 - (g(x_1) - g(x_2)) \| + \left\| R_{\lambda,\rho}^{M(\lambda)}(z_1) - R_{\lambda,\rho}^{M(\lambda)}(z_2) \right\| \leq k\|x_1 - x_2\| + \|x_1 - z_2\|. \tag{22}
\]
It follows that
\[
\| x_1 - x_2 \| \leq \left( \frac{\theta}{1 - k} \right) \|z_1 - z_2\|.
\]
Combining (22) with (21), we have
\[
\|F_\lambda(z_1) - F_\lambda(z_2)\| \leq \xi(\theta)\|z_1 - z_2\|, \tag{23}
\]
where \(\xi(\theta)\) is given in the hypothesis.

It is clear from (15) that \(\xi(\theta) < 1\) and, consequently, the mapping \(F_\lambda(z)\) defined by (14) is a contraction mapping.

**Remark 2.** Lemma 5 ensures that the mapping \(F_\lambda(z)\) has a unique fixed point \(z(\lambda)\), that is, \(z(\lambda) = F_\lambda(z)\), which is a solution of the parametric resolvent equation problem with XOR-operation (10). It is easy to see that \(z\) for \(\lambda = \bar{\lambda}\) is a fixed point of \(F_\lambda(z)\) and is also a fixed point of \(F_{\bar{\lambda}}(z)\). We conclude that \(z(\lambda) = z = F_\lambda(z(\lambda))\).

Next, we prove that the solution \(z(\lambda)\) of the parametric resolvent equation problem with XOR-operation (10) is continuous (or Lipschitz continuous).

**Lemma 6.** Suppose that the function \(T(x, \lambda)\) is continuous with respect to the second argument with constant \(\mu_2 > 0\) and the mapping \(\lambda \rightarrow R_{1\rho}^{M(\lambda)}(z)\) is continuous at \(\lambda = \bar{\lambda}\) (or Lipschitz continuous). If \(\lambda \propto \bar{\lambda}, z(\lambda) \propto z(\bar{\lambda})\), and the following condition are satisfied:
\[
\left\|C_{1\rho}^{M(\lambda)}(x) - C_{1\rho}^{M(\bar{\lambda})}(x)\right\| \leq \mu_1 \|\lambda - \bar{\lambda}\|, \tag{24}
\]
where \(\mu_1 > 0\) is a constant; then \(z(\lambda)\), the solution of the parametric resolvent equation problem with XOR-operation (10) is continuous (or Lipschitz continuous) at \(\lambda = \bar{\lambda}\).

**Proof.** Using (vii) of Proposition 1, for all \(\lambda, \bar{\lambda} \in \Omega\) and as both the concepts symmetric difference \(\triangle\) and \(\oplus\) operation are same, we evaluate
\[
\|z(\lambda) \triangle z(\bar{\lambda})\| = \|z(\lambda) \oplus z(\bar{\lambda})\|
\leq \|F_\lambda(z(\lambda)) \oplus F_\lambda(z(\bar{\lambda}))\|
= \|F_\lambda(z(\lambda)) \oplus F_\lambda(z(\bar{\lambda})) + F_\lambda(z(\bar{\lambda})) \oplus F_\lambda(z(\bar{\lambda}))\|
\leq \|F_\lambda(z(\lambda)) - F_\lambda(z(\bar{\lambda}))\| + \|F_\lambda(z(\bar{\lambda})) - F_\lambda(z(\bar{\lambda}))\|. \tag{25}
\]
As \(z(\lambda) \propto z(\bar{\lambda})\), using (viii) of Proposition 1 and Lemma 5, (25) becomes
\[
\|z(\lambda) \oplus z(\bar{\lambda})\| = \|z(\lambda) - z(\bar{\lambda})\| \leq \xi(\theta)\|z(\lambda) - z(\bar{\lambda})\| + \|F_\lambda(z(\lambda)) - F_\lambda(z(\bar{\lambda}))\|, \tag{26}
\]
where \(\xi(\theta)\) is same as in Lemma 5.

Using (14), condition (24), and continuity of \(T(x, \lambda)\) with respect to the second argument, we have
\[
\|F_\lambda(z(\bar{\lambda})) - F_\lambda(z(\bar{\lambda}))\|
= \left\|g(x(\bar{\lambda})) + \rho \left(C_{1\rho}^{M(\lambda)}(x(\bar{\lambda})) + T(x(\lambda), \lambda)\right)\right\|
\leq \rho \left\|C_{1\rho}^{M(\lambda)}(x(\bar{\lambda})) - C_{1\rho}^{M(\lambda)}(x(\bar{\lambda}))\right\|
+ \rho \|T(x(\lambda), \lambda) - T(x(\lambda), \bar{\lambda})\|
\leq \rho \mu_1 \|\lambda - \bar{\lambda}\| + \rho \mu_2 \|\lambda - \bar{\lambda}\|
= \rho (\mu_1 + \mu_2) \|\lambda - \bar{\lambda}\|. \tag{27}
\]
Combining (26) and (27), we have
\[ \|z(\lambda) - z(\bar{\lambda})\| \leq \frac{\rho(\mu_1 + \mu_2)}{[1 - \zeta(\theta)]} \|\lambda - \bar{\lambda}\|, \text{ for all } \lambda, \bar{\lambda} \in \Omega. \] (28)

Let \( \lim_{\lambda \to \bar{\lambda}} \|\lambda - \bar{\lambda}\| = 0 \), that is, at \( \lambda = \bar{\lambda} \), it is clear from (28) that \( z(\lambda) \), the solution of parametric resolvent equation problem with XOR-operation (10) is continuous (or Lipschitz continuous).

For illustration of some of the concepts used in this paper, we construct the following conjoin example.

**Example 1.** Suppose \( H = \mathbb{R} \) and the mappings \( T : H \times \Omega \to H \), \( g : H \to H \), and \( M : H \times \Omega \to 2^H \) are defined by
\[
T(x, \lambda) = -\frac{x}{2} - \frac{\lambda}{2}, \quad \text{for all } x \in H, \ \lambda \in \Omega,
\]
\[
g(x) = -\frac{5x - 14}{6} - \frac{\lambda}{2}, \quad \text{for all } x \in H \text{ and for fixed } \lambda \in \Omega,
\]
\[
M(x, \lambda) = \{x + 1 : x \in H\}.
\]

We claim that the following hold:

(1) The mapping \( T \) is relaxed Lipschitz continuous, Lipschitz continuous with respect to first argument, and continuous with respect to second argument.

(i) \[
\langle T(x, \lambda) - T(y, \lambda), x - y \rangle = \langle \left( -\frac{x}{2} - \frac{\lambda}{2} \right) - \left( -\frac{y}{2} - \frac{\lambda}{2} \right), x - y \rangle
\]
\[
= \langle -\frac{x}{2} + \frac{y}{2}, x - y \rangle
\]
\[
= -\frac{1}{2} \langle x - y, x - y \rangle
\]
\[
= -\frac{1}{2} \|x - y\|^2
\]
\[
\leq -\frac{1}{4} \|x - y\|^2,
\]
that is, \( T(x, \lambda) \) is \( \frac{1}{4} \)-relaxed Lipschitz continuous with respect to the first argument.

(ii) \[
\|T(x, \lambda) - T(y, \lambda)\| = \left\| \left( -\frac{x}{2} - \frac{\lambda}{2} \right) - \left( -\frac{y}{2} - \frac{\lambda}{2} \right) \right\|
\]
\[
= \left\| -\frac{x}{2} + \frac{y}{2} \right\|
\]
\[
= \frac{1}{2} \|y - x\|
\]
\[
= \frac{1}{2} \|x - y\|
\]
\[
\leq \frac{3}{4} \|x - y\|,
\]
that is, \( T(x, \lambda) \) is \( \frac{3}{4} \)-Lipschitz continuous with respect to the first argument.

(iii)
\[ \| T(x, \lambda) - T(x, \bar{\lambda})\| = \left\| \left( -\frac{x}{2} - \frac{\lambda}{2} \right) - \left( -\frac{x}{2} - \frac{\bar{\lambda}}{2} \right) \right\| \\
= \left\| \frac{\lambda - \bar{\lambda}}{2} \right\| \\
= \frac{1}{2} \| \lambda - \bar{\lambda} \| \\
\leq \frac{3}{4} \| \lambda - \bar{\lambda} \| , \]

that is, \( T(x, \lambda) \) is continuous with respect to the second argument.

(2) The mapping \( g \) is Lipschitz continuous.

\[ \| g(x) - g(y) \| = \left\| \left( -\frac{5x - 14}{6} - \frac{\lambda}{2} \right) - \left( -\frac{5y - 14}{6} - \frac{\lambda}{2} \right) \right\| \\
= \frac{5}{6} \| x - y \| \\
\leq \frac{3}{2} \| x - y \| , \]

that is, \( g(x) \) is \( \frac{3}{2} \)-Lipschitz continuous.

(3) In order to show that the mapping \( M \) is ordered rectangular mapping, let \( v_x = x + 1 \in M(x) \) and \( v_y = y + 1 \in M(y) \), we evaluate

\[ \langle v_x \odot v_y, -(x \oplus y) \rangle = \langle v_x + v_y, x + y \rangle \\
= (x + 1) \oplus (y + 1), x \oplus y \\
= (x \oplus y, x \oplus y) \\
= \| x \oplus y \|^2 \\
\geq \frac{4}{5} \| x \oplus y \|^2 . \]

Thus, \( M \) is \( \frac{4}{5} \)-ordered rectangular mapping.

(4) Now, we will show that the solution \( z(\lambda) \) of the parametric resolvent equation with XOR-operation (10) is continuous (or Lipschitz continuous). We take \( \rho = 2 \) and evaluate implicit resolvent operator as well as the implicit Cayley operator as

\[ R^{M(\cdot, \lambda)}_{I, \rho} = [I + \rho M(\cdot, \lambda)]^{-1}(x) \]
\[ = [x + 2(x + 1)]^{-1} \]
\[ = \frac{3x + 2}{3} \] (29)

and

\[ C^{M(\cdot, \lambda)}_{I, \rho} = \left[ 2R^{M(\cdot, \lambda)}_{I, \rho} - I \right](x) \]
\[ = \left[ 2 \left( \frac{x - 2}{3} \right) - x \right] \] (30)
One can easily show that the implicit resolvent operator $R_{I\rho}^{M(\lambda)}$ (29) and the implicit Cayley operator $C_{I\rho}^{M(\lambda)}$ (30) are Lipschitz continuous. Using Equation (14), we evaluate

$$ z(\lambda) = F(\lambda) = g(x) + \rho \left( C_{I\rho}^{M(\lambda)}(x) + T(x, \lambda) \right) $$

$$ = \left( \frac{-5x - 14}{6} \right) - \frac{\lambda}{2} + 2 \left( \frac{-x - 4}{3} + \left( \frac{-x - \frac{\lambda}{2}}{2} \right) \right) $$

$$ = -15x - 30 - 9\lambda $$

$$ = -5x - 10 - 3\lambda $$

which is a solution of the parametric resolvent equation problem with XOR-operation (10). We verify our claim below:

$$ \left( C_{I\rho}^{M(\lambda)}(x) + T(x, \lambda) \right) \oplus \rho^{-1} R_{I\rho}^{M(\lambda)}(z) $$

$$ = \left( C_{I\rho}^{M(\lambda)}(x) + T(x, \lambda) \right) \oplus \rho^{-1} \left[ I - R_{I\rho}^{M(\lambda)} \right] (z) $$

$$ = \left( \frac{-x - 4}{3} + \left( \frac{-x - \frac{\lambda}{2}}{2} \right) \right) \oplus \frac{1}{2} \left[ z - \frac{z}{3} - 2 \right] $$

$$ = \left( \frac{-2x + 8 - 3x - 3\lambda}{6} \right) \oplus \frac{1}{2} \left[ z - \frac{z}{3} - 2 \right] $$

$$ = \left( \frac{-5x - 8 - 3\lambda}{6} \right) \oplus \frac{2}{6} [z + 1] $$

$$ = \left( \frac{-5x - 8 - 3\lambda}{6} \right) \oplus \frac{2}{6} \left[ \frac{-5x - 10 - 3\lambda}{2} + 1 \right] $$

$$ = \left( \frac{-5x - 8 - 3\lambda}{6} \right) \oplus \left( \frac{-5x - 8 - 3\lambda}{6} \right) $$

$$ = 0. $$

Finally, we evaluate

$$ \|z(\lambda) - z(\bar{\lambda})\| = \left\| \left( \frac{-5x - 10 - 3\lambda}{2} \right) - \left( \frac{-5x - 10 - 3\bar{\lambda}}{2} \right) \right\| $$

$$ = \frac{3}{2} \|\lambda - \bar{\lambda}\| $$

$$ \leq \frac{5}{2} \|\lambda - \bar{\lambda}\|, $$

which shows that the solution $z(\lambda)$ of the parametric resolvent equation problem with XOR-operation (10) is continuous (or Lipschitz continuous) at $\lambda = \bar{\lambda}$.

5. Conclusions

Sensitivity analysis is instrumental in solving many problems occurring in day-to-day life, such as global climate models, quantifying uncertainty in corporate finance, econometric models in social sciences, optimal experimental design, engineering design, multicriteria decision making, physics, chemistry, human control models, etc. Keeping in view the above mentioned applications of sensitivity analysis, in this paper, we have introduced and studied a parametric mixed Cayley inclusion problem with XOR-operation and a parametric resolvent equation problem with XOR-operation. We have shown that both the problems are equivalent and this equivalent formulation is used to develop the general framework of sensitivity analysis for parametric mixed Cayley inclusion problem with
XOR-operation without using differentiability of the given data. We expect that this study will motivate and inspire other researchers of related fields to explore its applications in modern sciences.

**Author Contributions:** All the authors have contributed equally to this paper. All authors have read and agreed to the published version of the manuscript.

**Funding:** This work is supported by Ministry of Science and Technology, Taiwan [grant No. MOST 108-2115-M-037-001].

**Acknowledgments:** All authors are thankful to the reviewers for their valuable comments which improved this paper a lot.

**Conflicts of Interest:** The authors declare no conflict of interest.

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