Two-body Cabibbo-suppressed Decays of Charmed Baryons into Vector Mesons and into Photons

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Abstract

The heavy quark effective theory and the factorization approximation are used to treat the Cabibbo-suppressed decays of charmed baryons to vector mesons, $\Lambda_C \to p\rho^0, p\omega, \Xi_C^{+,0} \to \Sigma^{+,0}\phi, \Sigma^{+,0}\rho^0, \Sigma^{+,0}\omega$ and $\Xi_C^0 \to \Lambda\phi, \Lambda\rho, \Lambda\omega$. The input from two recent experimental results on $\Lambda_C$ decays allows the estimation of the branching ratios for these modes, which turn out to be between $10^{-4}$ and $10^{-3}$. The long distance contribution of these transitions via vector meson dominance to the radiative weak processes $\Lambda_C \to p\gamma, \Xi_C \to \Sigma\gamma$ and $\Xi_C^0 \to \Lambda\gamma$ leads to quite small branching ratios, $10^{-6} - 10^{-9}$; the larger value holds if a sum rule between the coupling constants of the vector mesons is broken.

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The study of the charmed baryon decays has intensified during the last few years on both the experimental and theoretical levels\[1\]. In particular, the recent measurements with the CLEO-II detector at the Cornell Electron Storage Ring (CESR) of the formfactor ratio in semileptonic decay \( \Lambda_C \rightarrow \Lambda e^+ \nu \)\[2\] and of the branching ratio of \( \Lambda_C \rightarrow p\phi \)\[3\], provide information whose usefulness transcends the specific processes studied in these experiments. In the present work, we focus on a group of Cabibbo-suppressed two-body nonleptonic decays of the charmed baryons \( \Lambda_C \) and \( \Xi_C \) into a light baryon plus a light-flavor neutral vector meson, which were not treated previously, making use of information provided by the two experiments of Ref. \[2\] and \[3\].

The Cabibbo-suppressed decays of the charmed hadrons are described at the quark level by the effective Hamiltonian

\[
H_{\text{eff}} = \sum_{q=d,s} \frac{G_F}{\sqrt{2}} V_{uq}^* V_{cq} a_2 \bar{q} \gamma^\mu (1 - \gamma_5) q \bar{u} \gamma^\mu (1 - \gamma_5) c,
\]

(1)

where \( a_2 \) is a combination of Wilson coefficients, \( a_2 = c_1(\mu = m_c) + c_2(\mu = m_c)/N_c \). In (1) only the leading contribution in the large \( N_C \) limit is retained. In order to evaluate decay amplitudes induced by the effective Hamiltonian \( H_{\text{eff}} \) at the hadronic level we shall use the factorization approximation\[4\] and we concentrate now on those decays in which \( (q\bar{q}) \) materializes to a vector meson. For instance, the decay amplitude of \( \Lambda_C \rightarrow p\phi \) is given by

\[
M(\Lambda_C \rightarrow p\phi) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cs} a_2 < \phi | \bar{s} \gamma^\mu (1 - \gamma_5) s | 0 > < p | \bar{u} \gamma^\mu (1 - \gamma_5) c | \Lambda_C > .
\]

(2)

While the matrix element for the vector meson is related to its decay constant defined by

\[
< \phi | s \gamma^\mu s | 0 > = i f_\phi m_\phi \epsilon_\phi^* \mu ,
\]

(3)
the baryonic matrix element can be parameterized by six form factors \( f_i \) and \( g_i \) (i=1,2,3)\( ^4 \):

\[
< p|\bar{u}\gamma_\mu(1-\gamma_5)c|\Lambda_C > = \bar{u}_p(P_2)[(f_1(q^2)\gamma_\mu - i\frac{f_2(q^2)}{m_{\Lambda_C^*}}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Lambda_C^*}}q_\mu) \\
- (g_1(q^2)\gamma_\mu - i\frac{g_2(q^2)}{m_{\Lambda_C^*}}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{m_{\Lambda_C^*}}q_\mu)\gamma_5]u_{\Lambda_C}(P_1)
\]

with \( q_\mu = (P_1 - P_2)_\mu \).

We use the heavy quark effective theory, which is considered to be especially suitable for the \( \Lambda_C \) decays\( ^5, ^6 \). Treating the \( c \) as the heavy-quark, the matrix element in (4) is expanded in \( 1/m_{\Lambda_C^*} \) and in the following we keep only the first term of the expansion. Then only two independent formfactors survive and (4) is cast into the form

\[
< p|\bar{u}\gamma_\mu(1-\gamma_5)c|\Lambda_C > = \bar{u}_p(P_2)[F_1(q^2) + F_2(q^2)\frac{P_1}{m_{\Lambda_C^*}}]\gamma_\mu(1-\gamma_5)u_{\Lambda_C}(P_1).
\]

The formfactors in (4) and in (5) are then related by

\[
f_1(q^2) = F_1(q^2) + \frac{m_p}{m_{\Lambda_C^*}}F_2(q^2), \quad f_2(q^2) = -F_2(q^2), \quad f_3(q^2) = F_2(q^2), \\
g_1(q^2) = F_1(q^2) + \frac{m_p}{m_{\Lambda_C^*}}F_2(q^2), \quad g_2(q^2) = -F_2(q^2), \quad g_3(q^2) = F_2(q^2).
\]

The relations in (6) are expected to hold most likely near the zero-recoil point \( q_m^2 \equiv (m_{\Lambda_C^*} - m_p)^2 \simeq 1.8\text{GeV}^2 \) of the semileptonic decays. Since we need here the form factors in the region of \( \sim 1\text{GeV}^2 \), we shall assume it is appropriate to use a pole behavior for the extrapolation:

\[
f_i(q^2) = f_i(q_m^2)\frac{1 - q_m^2/M_{D_i}^2}{1 - q^2/M_{D_i}^2}, \\
g_i(q^2) = g_i(q_m^2)\frac{1 - q_m^2/M_{D_i}^2}{1 - q^2/M_{D_i}^2},
\]

where \( M_{D*} = 2.007\text{GeV} \) and \( M_{D_1} = 2.423\text{GeV} \) are the masses of the lowest lying mesons which interpolate the vector and the axial vector currents, respectively.
Now we turn to the experimental information[2] on the form factor ratio in $\Lambda_C \to \Lambda e^+\nu$. Although the absolute value of $F_1$, $F_2$ defined in (5) is not measured yet, their ratio is determined to be

$$R \equiv \frac{F_2}{F_1} = -0.25 \pm 0.16$$

in the semileptonic decay. We remark that in view of the limited statistics, it was found in [2] that this result is not sensitive to the $q^2$ behavior assumed for the formfactors.

Now we assume that as consequence of the SU(3)-flavor symmetry for the light quarks the ratio (8) holds also for the matrix elements of the decays $\Lambda_C \to p$ and $\Xi_C \to \Sigma, \Lambda$. Hence, we are now in the position to derive the decay amplitude (2) in the approximation of (5) and we arrive at

$$M(\Lambda_C \to p\phi) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cs} a \phi m_{\phi} \epsilon^{*}_{\phi} u_p(P_2)[\gamma_{\mu}(a-b\gamma_5) + 2(x-y\gamma_5)]P_{1\mu}]u_{\Lambda_c}(P_1), \quad (9)$$

where

$$a = f_1(m_{\phi}^2) + \frac{m_p + m_{\Lambda_c}}{m_{\Lambda_c}} f_2(m_{\phi}^2),$$

$$b = g_1(m_{\phi}^2) + \frac{m_p - m_{\Lambda_c}}{m_{\Lambda_c}} g_2(m_{\phi}^2),$$

$$x = -\frac{1}{m_{\Lambda_c}} f_2(m_{\phi}^2),$$

$$y = -\frac{1}{m_{\Lambda_c}} g_2(m_{\phi}^2). \quad (10)$$

Similiar formulas hold when the $\phi$ meson is replaced by a $\rho^0$ or an $\omega$ meson in the final state, where an additional factor $1/\sqrt{2}$ arises due to the quark content of the meson $\rho^0$ or $\omega$. In the process of $\Xi_C \to \Lambda\phi(\rho^0, \omega)$ there exists another factor $\sqrt{1/6}$ to account for the difference of the flavor-spin suppression for the light quarks[8].

We proceed now to calculate the decays listed in Table I. Firstly, we use the experimental data of CLEO[3] which measures the branching ratio $\text{Br}(\Lambda_C \to p\phi) =$
(1.06 ± 0.33) × 10^{-3} as an input, thus determining the unknown product $|a_2 F_1 (m_b^2)|$. This permits to calculate with the model the transitions $\Lambda_C \to p \rho, p \omega, \Xi_C^{\pm, 0} \to \Sigma^{+, 0} \phi, \Sigma^{+, 0} \rho, \Sigma^{+, 0} \omega$ and $\Xi_C^0 \to \Lambda \phi, \Lambda \rho, \Lambda \omega$. Since the experimental uncertainty in (8) is large, we present in Table I the values for $R = -0.09, R = -0.25, R = -0.41$. It turns out that the dependence on the ratio $R$ in the considered range is weak, the variation in the calculated branching ratios being less than 10%. The uncertainty in the results of Table I is then due solely to the precision achieved in the determination of $\text{Br}(\Lambda_C \to p \phi)$ \(\text{(3)}\). The partial decay widths are calculated using the helicity representation of the amplitudes\(\text{(9)}\) and thus we included in Table I also the predictions of the model for transverse/longitudinal ratios in these decays, which are found to vary about 20% in the range considered for $R$. In the calculations we used for $f_V (V = \rho, \omega$ or $\phi)$ the values determined from the leptonic decays of the vector mesons\(\text{(10)}\), $f_\rho^2 = 0.047 \text{GeV}^2$, $f_\omega^2 = 0.038 \text{GeV}^2$, $f_\phi^2 = 0.055 \text{GeV}^2$.

As an alternative, we could have refrained from using $\Lambda_C \to p \phi$ as input, attempting to calculate it as well, by using $a_2 = -0.55 \pm 0.1$ from the overall fit\(\text{(4)}\) to nonleptonic $D$ and $D_s$ decays, and assuming a “reasonable” value for $f_1 (q_m^2)$. With the above value for $a_2$ and taking, for example, $f_1 (q_m^2) = 0.9 \pm 0.1$, we find $\text{Br}(\Lambda_C \to p \phi) = (0.74 \pm 0.32) \times 10^{-3}$ (for $R = -0.25$). This is in good agreement with the measured value\(\text{(3)}\) and gives strong support to the approach presented here.

In view of the appropriateness of the model discussed here for decays of the charmed baryons to vector mesons, we consider also its application to radiative weak decays in conjunction with vector meson dominance (VMD). Since it is clear now\(\text{(11)}\) that the short distance contribution from $c \to u \gamma$ to the radiative decays of charmed particles is negligible, it is important to devise reliable models for the long distance one. We adopt here the model which has been employed recently by Deshpande et al.\(\text{(12)}\) and
by Eilam et al.\cite{13} to estimate the long distance “t-channel” contribution of the vector mesons to the radiative transitions $b \to s\gamma$ and $s \to d\gamma$ (see also Ref. \cite{14}). Using now for the charm sector the derivation steps outlined in \cite{12, 13}, the appropriate part of the effective Hamiltonian for the radiative weak decays is

$$
\mathcal{H}_{VMD}^{\text{eff}} = -\frac{eG_F}{\sqrt{2}} a_2 \left\{ \frac{1}{m_C} [V_{us}^* V_{cs} ( - \frac{1}{3} f_\phi^2 ) + V_{ud}^* V_{cd} ( - \frac{1}{2} f_\rho^2 + \frac{1}{6} f_\omega^2 ) ] \epsilon^\mu \epsilon^\nu q^\mu q^\nu \bar{u} \gamma_\nu (1 + \gamma_5) c. \right\} \epsilon^\mu \epsilon^\nu q^\mu q^\nu \bar{u} \gamma_\nu (1 + \gamma_5) c. \right\}
$$

(11)

To treat the newly encountered matrix element of $\sigma_{\mu\nu}(1 + \gamma_5)$, we use the heavy quark effective scheme with the c-quark only treated as heavy. Then, similarly to Cheng et al.\cite{15}, we obtain

$$
< p | \bar{u} \gamma_\nu (1 + \gamma_5) c | \Lambda_C >= \bar{u}_p (P_2) [ F_1(q^2) + F_2(q^2) \frac{P_1}{m_{\Lambda_C}} ] \sigma_{\mu\nu} (1 + \gamma_5) u_{\Lambda_C} (P_1).
$$

(12)

Next, we use again the measured result of $\text{Br}(\Lambda_C \to p\phi) \equiv 3$ and of $R \equiv 2$, in order to calculate the VMD contributions\cite{12, 13, 16} to the processes $\Lambda_C \to p\gamma$, $\Xi^{+,0}_C \to \Sigma^{+,0}\gamma$ and $\Xi^0_C \to \Lambda\gamma$. We are still faced, however, with the question of the $q^2$ dependence of the $f_V(q^2)$ couplings. It is customary to assume that the $q^2$ variation of the $f_\rho$, $f_\omega$ is small and one may take for the vector-meson-photon couplings at $q^2 = 0$, $f_\rho(m_{\rho}^2) \simeq f_\rho(0)$, $f_\omega(m_{\omega}^2) \simeq f_\omega(0)$. We may reasonably assume also $f_\phi(m_{\phi}^2) \simeq f_\phi(0)$. Since in Eq (11) we use $V_{ud}^* V_{cd} \simeq -V_{us}^* V_{cs}$, the amplitude for the processes considered is proportional to the quantity $C'_{VMD} \equiv -\frac{1}{3} f_\phi^2 + \frac{1}{2} f_\rho^2 - \frac{1}{6} f_\omega^2$. It has been pointed out already in Ref \cite{13} that by using the values determined in the leptonic decays for $f_V(0)$ a near cancellation occurs in $C'_{VMD}$. This cancellation is due to the combination of $SU(3)$-flavor symmetry with the GIM relation and occurs at a level below 10%. As a result, all the considered radiative decays are reduced by more than two orders of the magnitude. Denoting the suppressed branching ratios obtained from the combined contribution of $\phi$, $\omega$, $\rho$ by the
subscript $SR$ (sum rule), we find:

$$
Br(\Lambda C \rightarrow p\gamma)_{SR} = 1.8 \times 10^{-9}, 2.3 \times 10^{-9}, 3.1 \times 10^{-9},$
$$
Br(\Xi^+_C \rightarrow \Sigma^+\gamma)_{SR} = 3.5 \times 10^{-9}, 4.5 \times 10^{-9}, 6.2 \times 10^{-9},$
$$
Br(\Xi^0_C \rightarrow \Sigma^0\gamma)_{SR} = 1.0 \times 10^{-9}, 1.3 \times 10^{-9}, 1.8 \times 10^{-9},$
$$
Br(\Xi^0_C \rightarrow \Lambda\gamma)_{SR} = 0.16 \times 10^{-9}, 0.21 \times 10^{-9}, 0.28 \times 10^{-9}
$$

for $R = -0.09, -0.25$ or $-0.41$. On the other hand, the possibility exists that a certain variation occurs in $f_V(q^2)$ between $q^2 = 0$ and $q^2 = m_V^2$. For instance, there is strong evidence that $f_\psi(q^2)$ varies considerably between $q^2 = m_\psi^2$ and $q^2 = 0$, $f_\psi^2$ being reduced in this range by a factor of $6^{[12,13]}$. Hence one should also consider the possibility that the above mentioned cancellation is avoided. Since there is no accurate model for the $f_V(q^2)$ variation, we take as alternative the rates resulting if only the $\rho \leftrightarrow \gamma$ is considered. This leads to

$$
Br(\Lambda C \rightarrow p\gamma)_\rho = 0.73 \times 10^{-6}, 0.93 \times 10^{-6}, 1.3 \times 10^{-6},$
$$
Br(\Xi^+_C \rightarrow \Sigma^+\gamma)_\rho = 1.4 \times 10^{-6}, 1.7 \times 10^{-6}, 2.5 \times 10^{-6},$
$$
Br(\Xi^0_C \rightarrow \Sigma^0\gamma)_\rho = 0.41 \times 10^{-6}, 0.53 \times 10^{-6}, 0.73 \times 10^{-6},$
$$
Br(\Xi^0_C \rightarrow \Lambda\gamma)_\rho = 0.65 \times 10^{-7}, 0.97 \times 10^{-7}, 1.1 \times 10^{-7}
$$

for $R = -0.09, -0.25$ or $-0.41$. These figures may be compared with a previous calculation$^{[17]}$ using bremsstrahlung from $W$-exchange diagrams on the quark level. Branching ratios of the order of $10^{-5}$ were obtained for these processes, however, as the authors pointed out, large uncertainties are involved.

In summary, we have presented the first calculation of the Cabibbo-suppressed processes $\Lambda C \rightarrow p\rho, p\omega, \Xi^+_C \rightarrow \Sigma^{+,0}\phi, \Sigma^{+,0}\rho, \Sigma^{+,0}\omega$ and $\Xi^0_C \rightarrow \Lambda\phi, \Lambda\rho, \Lambda\omega$ by using the factorization approximation, heavy quark effective theory and experimental input from recent experiments. We found branching ratios between $10^{-4}$ and $10^{-3}$ for these modes, which brings their detection into the realm of feasibility in the near future.
The model accounts correctly for the observed $\Lambda_C \rightarrow p\phi$. Then, using vector meson dominance for the long distance contribution, we calculated the radiative processes $\Lambda_C \rightarrow p\gamma$, $\Xi_C^{+0} \rightarrow \Sigma^{+0}\gamma$, and $\Xi_C \rightarrow \Lambda\gamma$ in the same model. If a certain GIM-type sum rule holds for the vector-meson-photon couplings these transitions are strongly suppressed to the level $10^{-8} - 10^{-9}$. Otherwise, individual vector mesons contribute branching ratios of the order of $10^{-6}$. Detection or limits on these modes would thus test the validity of interesting theoretical models.

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**Table I.** Predictions on branching ratios and the ratio of polarized decay rates $\Gamma_L/\Gamma_T$.

| Process               | BR            | $R = -0.09$ | $R = -0.025$ | $R = -0.41$ |
|-----------------------|---------------|-------------|--------------|-------------|
| $\Lambda_C \to p\phi$ | $10^{-3}\text{(input)}$ | $10^{-3}\text{(input)}$ | $10^{-3}\text{(input)}$ | $1.18$ | $1.29$ | $1.42$ |
| $\Xi_C^0 \to \Sigma^0\phi$ | $0.52 \times 10^{-3}$ | $0.50 \times 10^{-3}$ | $0.48 \times 10^{-3}$ | $1.04$ | $1.14$ | $1.25$ |
| $\Xi_C^0 \to \Sigma^0\rho^0$ | $0.23 \times 10^{-3}$ | $0.23 \times 10^{-3}$ | $0.24 \times 10^{-3}$ | $2.06$ | $2.29$ | $2.55$ |
| $\Xi_C^0 \to \Sigma^0\omega$ | $0.19 \times 10^{-3}$ | $0.19 \times 10^{-3}$ | $0.19 \times 10^{-3}$ | $1.98$ | $2.21$ | $2.46$ |
| $\Xi_C^0 \to \Lambda\phi$ | $0.93 \times 10^{-4}$ | $0.92 \times 10^{-4}$ | $0.90 \times 10^{-4}$ | $1.19$ | $1.31$ | $1.44$ |
| $\Xi_C^0 \to \Lambda\rho^0$ | $0.39 \times 10^{-4}$ | $0.40 \times 10^{-4}$ | $0.41 \times 10^{-4}$ | $2.28$ | $2.54$ | $2.83$ |
| $\Xi_C^0 \to \Lambda\omega$ | $0.31 \times 10^{-4}$ | $0.32 \times 10^{-4}$ | $0.34 \times 10^{-4}$ | $2.20$ | $2.46$ | $2.74$ |