Time Series Predict DB *

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ABSTRACT
In this work, we are motivated to make predictive functionalities native to database systems with focus on time series data. We propose a system architecture, Time Series Predict DB, that enables predictive query in any existing time series database by building an additional “prediction index” for time series data. To be effective, such an index needs to be built incrementally while keeping up with database throughput, able to scale with volume of data, provide accurate predictions for heterogenous data, and allow for “predictive” querying with latency comparable to traditional database queries. Building upon a recently developed model agnostic time series algorithm by making it incremental and scalable, we build such a system on top of PostgreSQL. Using extensive experimentation, we show that our incremental prediction index updates faster than PostgreSQL (1µs per data for prediction index vs 4µs per data for PostgreSQL) and thus not affecting the throughput of the database. Across a variety of time series data, we find that our model agnostic, incremental algorithm provides better accuracy compared to the best state-of-art time series libraries (median improvement in range 3.29 to 4.19x over Prophet of Facebook, 1.27 to 1.48x over AMELIA in R). The latency of predictive queries with respect to SELECT queries (0.5ms) is < 1.9x (0.8ms for imputation) and < 7.6x (3ms for forecasting) across machine platforms. As a by product, we find that the incremental, scalable variant we propose improves the accuracy of the batch prediction algorithm which may be of interest in its own right.

1. INTRODUCTION
Databases, or more generally data management systems, have provided means to store and query data efficiently and in a scalable manner. The impact of this is self-evident in the fact that the market-size, as per a conservative estimate, is in the range of $50B USD and that is only for commercial databases, not capturing revenues from use of open-source systems. In a nutshell, before databases, we stored raw data and built custom ‘indices’ to query data in a specific manner. The advent of frameworks such as relational database and Structure Query Language (SQL) converted this into an automated process. This eliminated issues such as high cost (time and resources) for building new indices, updating indices every time the form of data changed, quality assessment, robustness, etc.

In the field of predictive modeling, we are in pre-database era. We build custom models for each predictive application of interest and we update it each time new data becomes available. Depending upon the nature of the question and data infrastructure, these custom exercises consume moderate to high amount of resources as well as require teams of experts – very much how it was in the pre-database era for data management systems. This begs a natural question: is it possible to make predictive functionalities native to existing databases? If answer is yes, the potential impact could be measured at the scale of databases themselves.

Motivated to answer this, in this paper we focus on bringing predictive capabilities to databases in the context of time series data. That is, data where each unit has an associated timestamp and there is structure within data that is captured through time. Precisely, each unit of data, for time series labeled as ‘name’ (e.g. ‘S&P 500 Index’) has ‘time’ (e.g. ‘02/28/2019’) and ‘value’ (e.g. 2790), represented as

\[
\text{('name', 'time'): 'value'}. 
\]

The queries of interest that time series data are usually required to support are of the form

\[
\text{('name', 'time' = t or t_0 < 'time' < t_1): 'value' = ? } .
\]

If data associated with the queried time instance t or time range \([t_0, t_1]\) exists, then it is returned, ideally with minimal latency.

In this setting, the natural predictive queries of interest are exactly the same as above. However, the response is the predicted value, or values, rather than the available data. For example, for a ‘future’ time \(t\), the predictive query responds with the forecasted value, and for the past time for which data may be missing (or not) the predictive query responds with a “de-noised”, imputed value.

In database speak, enabling predictive queries requires building a ‘prediction index’ on the ‘time’ field. Such an index should continually update its predictive models so that it can always be utilized to answer predictive queries. The index should be able to do such updates fast enough so that the throughput of the database does not degrade. The predicted answers using the index should be accurate enough, i.e. it should match the statistical accuracy obtained using the custom time series models using known predictive approaches (or state-of-art libraries). And ideally, predictive query responses should be done with latency similar to that of a standard query.

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1.1 Contributions

As the main contribution of this work, we show that it is indeed possible to realize such a ‘prediction index’ for time series data. Specifically, we propose a system architecture that enables predictive query in any existing time series database by building an additional “prediction index” for time series data. As discussed, to be effective, such an index needs to be built incrementally while keeping up with database throughput, be able to scale with the volume of data, provide accurate predictions for heterogeneous data, and allow for “predictive” querying with latency comparable to traditional database queries.

To begin, this requires an algorithm for time series imputation and forecasting that is model agnostic. In a recent work, the authors of [1] introduced such a model agnostic algorithm that provably works for a large class of popular time series models. The algorithm, however, is not incremental and works in a batch form. As an important contribution of this work, we propose a meta-algorithm that takes any such model agnostic batch algorithm and converts it into a scalable and incremental variant. In effect, we take the available time series data, chunk it into smaller, pre-determined sizes in an incremental (or online) manner, apply the batch algorithm to each of these chunks, and make predictions (imputation and forecasting) by combining outputs from these multiple models. Somewhat surprisingly, we find that combining the output of multiple models over smaller size chunks leads to improved performance compared to one model generated by applying the algorithm of [1] to the entire time series.

Given this incremental, scalable and model agnostic time series algorithm, we utilize it to build the prediction index. To test its performance, we implement it on top of PostgreSQL as our choice of time series database (it is worth noting that TimeScaleDB [23] builds on PostgreSQL, of course, in a much more involved manner). To begin with, using a time series containing a billion data points, we test the throughput of the prediction index. We find that the per data point incremental update to prediction index takes less than 1µs compared to a little more than 4µs per data point taken by PostgreSQL to write a data point. That is, the prediction index updates 4x faster than the write-throughput of PostgreSQL. This implies that the prediction index does not bottleneck system throughput as desired.

Next, we utilize a large number of heterogeneous time series data models generated synthetically to verify statistical accuracy. We compare the accuracy of the prediction index with respect to Prophet [2] and AMELIA [10]. Prophet is an open-source, state-of-art time series library and is known for excellent forecasting performance for all sorts of time series data. They achieve it by effectively building an ensemble method for forecasting. However, it has limited capability (and performance) for imputation. On the other hand, AMELIA, is a method that emerged from the Statistics community and is primarily designed for imputation. It is widely popular, especially in empirical social sciences where data is usually noisy and has missing values. Using extensive experimentation across model types and missing fractions of data, we find the following: for forecasting, in comparison to Prophet, we improve median accuracy in range of 3.29x to 4.19x and average accuracy by 12.94x to 26.97x; for imputation, in comparison with AMELIA, we improve median accuracy in range of 1.27x to 1.48x and average accuracy by 1.23x to 1.75x (see Table 1).

The last important comparison pertains to the latency of prediction query time. Using different volumes of time series data, we find that the imputation queries take less than 0.8ms and forecasting takes less than 3ms compared to 0.5ms traditional database queries. That is, imputation is at most 1.9x slower (or like two SELECT queries) and forecasting is at most 7.6x slower (or like eight SELECT queries). Effectively, we get predictive queries at the same order of latency as traditional queries.

The system architecture we propose stores all the prediction index information in the same PostgreSQL database. This deliberate choice of storing the index in the underlying database was to ensure that we get performance evaluations that are more likely to be underlying time series database invariant. To verify this, we evaluate the prediction query time versus SELECT query time across different machines for our system. We find that while the absolute time varies, the ratio of the performances remains almost the same. This confirms that indeed our findings here are likely to hold beyond the specific choice of a PostgreSQL database.

Finally, we comment on the choice of PostgreSQL as the underlying database. First of all, this work is not about time series database but about incorporating predictive capabilities within an existing time series data management system. Having said that, PostgreSQL is a robust, open source database with a long history and, hence, an excellent choice to begin with. It does prove to be a competent time series database—a reason why the commercial database, TimeScaleDB, builds on top of it. Finally, our performance evaluations rely on the storage of prediction index metadata in the same PostgreSQL instance. Therefore, if the native database improves, then so does performance of our predictive index. Therefore, we believe that the performance insights derived here for our system is likely to remain invariant to the specific choice of storage mechanism.

1.2 Related Work

We briefly recall relevant related work from literature on time series analysis, libraries and time series databases.

Time Series Analysis. This is an ancient topic with textbook style references that include [2, 3, 9, 15]. A time series can be viewed as a function indexed by time (integer or real values). The goal is to learn this function from its limited, noisy observations. Given that the space of such functions is complex, the approach is to utilize the right “constraints” on function class to learn it efficiently and this is where domain knowledge comes handy. For example, in communication and signal processing, the harmonic or Fourier representation of a time series has been widely utilized, due to the fact that signals communicated are periodic in nature. The approximation of stationary processes via harmonics or ARIMA has made them a popular model class to learn stationary-like time series, with domain specific popular variations, such as Autoregressive Conditional Heteroskedasticity (ARCH) in finance. To capture non-stationary or “trend-like” behavior, polynomial bases have been considered. There are also rich connections to the theory of stochastic processes and information theory (cf. [5, 19, 14, 8]). Popular time series models with latent structure are Hidden Markov Models (HMM) in probabilistic form (cf. [12, 2] and Recurrent Neural Networks (RNN) in deterministic form (cf. [18]).

The question of learning time series models with missing data has received comparatively less attention. A common
approach is to utilize HMMs or general State-Space-Models to learn with missing data (cf. [3, 20]).

The series of works where the matrix view of time series is utilized to provide a model agnostic method is of interest here. Historically, such an approach has been known as Singular Spectrum Analysis (SSA) where the principle components of the “Hankel” matrix of a time series play a key role in capturing the model [13, 17, 24]. In this work, we shall build upon a recent method developed by Agarwal et. al. [1] where “Page” matrix of the time series is first “de-noised” using matrix estimation methods and then regression is used for model learning. In [1], theoretical guarantees of this work are established which suggests that it can work across a large class of models in a model agnostic manner.

All of the above mentioned work views the question of time series as ‘batch learning’ where all the data is available at once and then model learning is done at once; if new data becomes available, the learning needs to be done again from scratch. In a sense, our meta-algorithm in this work provides a way to make any such algorithm scalable and incremental.

Time Series Libraries. Given its importance, it is no surprise that there is a large collection of open-source libraries and commercial offerings available for time series analysis. The two influential libraries considered in this work are Prophet and AMELIA as described in Section 1.1 in detail.

Time Series Database. Time series databases are proliferating especially over the past decade. The goal is to build ‘specialized’ databases that are particularly suited for the querying functionality and the nature of time series data. For example, in many financial applications, the data is high in volume, is stored in tables with very few columns, and the goal is to query by time ranges, potentially in parallel. This leads to architectural choices that are more suitable for such functionalities and, hence, proliferation of such databases. Some commercial and open-source examples include KDB [21], InfluxDB [12], TimeScaleDB [23]. For somewhat more comprehensive list, please see [25]. The TimeScaleDB is of particular interest to us as it is built on top of PostgreSQL and we utilize native PostgreSQL for the purpose of our experimentation.

2. SETUP AND OBJECTIVES

Time Series Data. We denote the time index as \( t \) throughout. For simplicity, we shall assume \( t \) takes integral values \( t \in \mathbb{Z} \), i.e., \( t \in \mathbb{Z} \). This is not restrictive as in any time series, data is represented at some ‘regularity’ or ‘unit’ (e.g. second, ms, day) and to keep things simple for exposition, we are using the ‘regularity’ at integral level. The underlying time series of interest can be viewed as a function \( f : \mathbb{Z} \rightarrow \mathbb{R} \). We may observe, however, a noisy version of the time series with potentially missing values. Precisely, we may observe data in the interval \( [T_e] = \{1, \ldots, T_e\} \). For any \( t \in [T_e] \) for which the data is observed, rather than observing \( f(t) \) directly, we see \( X(t) \) where \( X(t) \) is a random variable with \( E[X(t)] = f(t) \) and uniformly bounded variance. For the purpose of modeling, we’ll assume that each \( t \in [T_e] \) is observed with some probability \( p \in (0, 1] \) independently.

Under this setting, we have two objectives: (1) interpolation or imputation, i.e., estimate \( f(t) \) for all \( t \in [T_e] \); (2) extrapolation, i.e., forecast \( f(t) \) for \( t > T_e \). These precisely correspond to the predictive queries with time series data.

Modeling. Aside from the abstract view of time series as function, in practice, a time series is usually modeled as a combination of four components: (i) Trend, which captures long term changes; (ii) Seasonality, which captures the periodic nature; (iii) Stationarity, which captures the zero mean, time invariant behavior; and (iv) Residuals, which is noise that can not be succinctly modeled or learned. The traditional statistical wisdom, therefore, suggests learning these component independently from observations and use them for imputation and forecasting. This approach lies at the core of various time series libraries including Prophet and those in R.

Time Series Database. As discussed earlier, the primary function of a time series database is to store time series data and allow for efficient queries indexed by time. Here, we present a way to instantiate such functionality using PostgreSQL. As explained earlier, the goal of this work is not to build or improve upon the traditional time series database framework, but to build predictive functionalities native to the database. For that reason, we are using PostgreSQL as a time series database of choice, even thought it may not be the best possible option (but in fact, it is an excellent option).

We utilize the time series schema defined as follows:

\[
\text{CREATE TABLE Timeseries_name}
\text{("time" TIMESTAMP PRIMARY key,}
\text{"value" float);}
\text{CREATE INDEX on Timeseries_name (time).}
\]

Here, the index for efficient querying is created on the field “time”, which allows us to support queries of the form:

\[
\text{SELECT value FROM Timeseries_name}
\text{WHERE time = t.}
\]

Such a query will return value if there is indeed data stored at time \( t \), otherwise nothing is returned.

Time Series Prediction Database. As mentioned earlier, the time series prediction database makes queries of the form stated above predictive in nature. That is, the response to such queries is a predictive answer based on all available data for a given time series. To enable this, we propose a minor change to the schema stated above by the addition of prediction index.

\[
\text{CREATE TABLE Timeseries_name}
\text{("time" TIMESTAMP PRIMARY key,}
\text{"value" float);}
\text{CREATE INDEX on Timeseries_name(time);}
\text{CREATE PINDEX on Timeseries_name(time);}
\]

Such a prediction index enables predictive queries of the form:

\[
\text{PREDICT value FROM Timeseries_name}
\text{WHERE time = t.}
\]

Unlike the standard SELECT query, a PREDICT query returns a response even when the data associated with time \( t \) is missing.

Prediction Index: Desiderata. The aim of this work is to make it feasible to build such a prediction index so that PREDICT queries can be enabled. Building a prediction index requires a time series modeling algorithm that satisfies the following properties:
The remainder of the paper explains how this can be achieved, the number of sub-models at time index $M$ is segmented into several intervals of length required by the algorithm to train its model. Adjacent models, $M_i$, are assumed as cast into a particular sub-model $M_t$. We have the following cases:

1. **Fully train $M_0$** with observations $X(0 : t)$ if $t \in \Gamma$ where:
   $$\Gamma = 0_T \left\{ T_0, [T_0(1 + \gamma)], \ldots, [T_0(1 + \gamma)^{q_0}], T \right\},$$
   where $q_0 = \frac{\ln(T/T_0)}{\ln(1 + \gamma)}$.

2. **Incrementally update $M_0$** (assumed to be supported by the algorithm) if $t \not\in \Gamma$.

3. **More sub-models will be learned.** First we start by mapping the index $t$ to a particular sub-model $M_i$, then the sub-model is either fully trained or incrementally updated. The steps are as follow:
   
   i) Map the observation $X(t)$ to a sub-model $M_i$ such that $i = \max(0, \lfloor t/T \rfloor - 1)$
   
   ii) Determine $s_i$, which denotes the index of the first observation used to train $M_i$, where $s_i = \frac{t}{T} T$
   
   iii) Fully train $M_i$ with observations $X(s_i : t)$ if $(t - s_i) \in \Gamma$ where:
   $$\Gamma = \left\{ T_2, \frac{T_2}{2(1 + \gamma)}, \ldots, \frac{T_2}{2(1 + \gamma)^{q_0}} \right\}$$
   where $q = \frac{\ln(2)}{\ln(1 + \gamma)}$.

   iv) Incrementally update $M_i$, if $t \not\in \Gamma$.

Figure 1 illustrates how the proposed segmentation is carried out, and at what points the model is trained fully and where it is incrementally updated.

If the prediction algorithm $A$ can be batch-updated with several observations, the framework outlined above can also accommodate that. To illustrate how this works, assume that the model is already trained on $t$ entries, i.e., on the observations $X(1 : t)$ and that this has resulted in learning $n$ sub-models giving the set of models $M = \{M_0, M_1, \ldots, M_n\}$. The Batch-Framework updating procedure (see Algorithm 1) shows how the proposed framework would update the model with $N$ incoming data points $X(t + 1 : t + N)$.

The procedure starts with identifying whether new sub-models would need to be trained. If not, the last sub-model is updated either fully or incrementally depending on the number of new entries. If the number of new entries requires $j - n$ new models to be trained, then the last sub-model $M_n$ will be re-trained in addition to building the new sub-models: a total of $(j - n + 1)$ sub-models will be trained.

The framework introduced here relies on the availability of $A$, a time series prediction (forecasting and imputation) algorithm needs to be able to work with heterogeneous time series data and be effectively model agnostic. The algorithm needs to be able to provide accurate predictions with any given amount of data, across the range of data availability, and do so competitively with respect to the best state-of-art available solution. Incremental, Scalable: It must scale with the volume of data in an incremental manner, i.e. as more data gets added or data changes, the model must update incrementally so that it can keep up with the underlying database throughput and ideally be non-blocking. Efficient query: The latency of the predictive query (PREDICT) should be comparable to the traditional (SELECT) queries. It is reasonable to expect that predictive queries will be slower, but would be ideal if their latency is within a small factor.
Algorithm 1 Batch-Framework updating the time series model with N new data points

**Input:** \( X(t+1 : t+N), \) (N new data points)
\( \mathcal{M} = \{ M_1, M_2, ..., M_n \}, \) (the trained n sub-models)
\( T \) and \( \gamma, \) (the system hyperparameters)

**Output** \( \mathcal{M} = \{ M_1, M_2, ..., M_j \}, s.t : j \geq n, \) (the trained sub-models after incorporating the new N data points)

1: \( n \leftarrow \max(0, \left\lfloor \frac{2t}{T} \right\rfloor - 1), \) (index of which sub-model was last trained)
2: \( j \leftarrow \max(0, \left\lfloor \frac{2(t+N)}{T} \right\rfloor - 1), \) (index of what is the last sub-model to be trained)
3: if \( n == j \) then
4: \( s_n \leftarrow \frac{2n}{T} \)
5: \( \alpha \leftarrow \ln \left( \frac{(t-s_n)}{T} \right) \ln(1+\gamma) \)
6: \( i_i \leftarrow s_n + \frac{\alpha}{(1+\gamma)}, \) (index of the last point where the model was fully trained)
7: if \( N/(i_i-s_n) > \gamma \) then
8: Fully train \( M_n \) using \( \mathcal{A} \) on observations \( X(s_n : t) \)
9: else
10: Update \( M_n \) incrementally.
11: end if
12: else
13: for \( i = n, i++, \) \( 1 \leq j \) do
14: Fully train \( M_i \) via \( \mathcal{A} \) on observations \( X(t^2/2 : t+N) \)
15: end for
16: end if

Algorithm which allows incremental (and batch) updates. We describe one such algorithm next.

### 3.1.2 Model-Agnostic Time Series Prediction Algorithm

We use the time series prediction algorithm introduced in [1] as our algorithm of choice. This algorithm uses matrix estimation methods to impute and forecast time series. The choice of this algorithm is based on several attractive properties: (i) it is model and noise agnostic—an important feature when developing a prediction database for generic time series modeling; (ii) it does imputation and forecasting; (iii) it allows a natural extension for incremental and batch updating.

Next, we explain the details of this time series prediction algorithm which are largely adapted and reproduced from [1].

**Notation.** Again, we will denote the observation of a univariate time series at time \( t \) as \( X(t) \). \( X(t : e) \) will denote observations \( X(t), X(t+1), ..., X(e) \) for any \( t, e \in T := [1, T_c] \) such that \( t < e \). Further more, we will denote the last row of an \( L \times N \) matrix \( \mathbf{A} \) as \( \mathbf{A}_L \), and the sub-matrix of \( \mathbf{A} \) which contains all the rows except the last one as \( \hat{\mathbf{A}} \).

**Time Series as a Matrix.** The first step in this algorithm is to transform an array of observations of length \( T_c \), \( X(1 : T_c) \) into what is known as a Page matrix. The observations is to be reconstructed into a matrix by filling an \( L \) by \( N \) matrix column by column without overlapping, such that \( N = \lceil T_c/L \rceil \). Let's denote the resulting matrix as \( X^{(1:T_c)} \) such that the \( ij^{th} \) entry of the matrix is given by:

\[ X_{ij}^{(1:T_c)} = X(i + (j-1)L), \quad s.t : 0 < i \leq L, 0 < j \leq N \]

**Imputation and Forecasting algorithm.** After transforming the time series into a matrix, we can start the imputation and forecasting steps. Starting with imputation, Let’s assume that we want to impute values in our time series \( X(t : s), s.t : t < s \). The imputation steps are as follow:

1. Transform the observations \( X(t : s) \) into a Page matrix \( \hat{X}^{(t:s)} \).
2. Apply a matrix estimation method on the constructed matrix using singular value thresholding with \( k \) singular values to get \( \hat{M}^{(t:s)} = U_k^{(t:s)} \Sigma_k^{(t:s)} V_k^{(t:s)T} \).
3. Get the produced estimates for the time series \( \hat{X}(i + (j-1)L) = \hat{M}_{i,j}^{(t:s)}, \) where \( i \in [1, L] \), \( j \in [1, N] \).

For forecasting, we will learn the relationship between the last row and the remaining rows as follows:

1. Get the sub-matrix \( \hat{X}^{(t:s)} \) from \( X^{(t:s)} \) by removing the last row.
2. Apply a matrix estimation method on the sub-matrix using Universal Singular Value Thresholding to get \( \hat{M}^{(t:s)} \).
3. Get the linear regression coefficients:

\[ \beta^{(t:s)} = \arg \min_{\beta \in \mathbb{R}^{L-1}} \| X^{(t:s)} - \hat{M}^{(t:s)} b \|^2 \]
4. Get the last \( L-1 \) observations \( X(s-(L-1) : s) \).
5. Produce the forecast \( \hat{X}(s+1) = X(s-(L-1) : s)^T \beta^{(t:e)} \)
Incremental SVD Algorithm. One of our contributions is to extend the aforementioned algorithm to allow updates to the model. Updating the model produced by the algorithm outlined above can be reduced to the problem of updating the singular value decomposition (SVD) of the data matrix. We detail the procedure we used to update the SVD decomposition for each sub-model matrix. Here, assume that we are starting with our time series Page matrix \( \mathbf{X} \in \mathbb{R}^{L \times N} \). Having computed the SVD and estimated the reconstructed matrix \( \mathbf{X} \approx \mathbf{M} = \mathbf{U}_k \Sigma_k \mathbf{V}_k^T \), we want to efficiently find the decomposition of the updated matrix after appending new data as a matrix \( \mathbf{D} \in \mathbb{R}^{L \times P} \). In other words, we want to determine \( \mathbf{U}_k \Sigma_k \mathbf{V}_k^T \approx \mathbf{X}' = [\mathbf{X} \mathbf{D}] \).

In particular, we will use an incremental SVD method developed in the Latent Semantic Indexing literature [26]. This method updates the decomposition through computing a QR decomposition and a SVD of a small matrix whose size depends primarily on the size of the update. Hence, the complexity of this method is much lower than that of a full SVD. In particular, it updates the SVD decomposition accurately with complexity of \( \mathcal{O}(k^2 + (L + N)k^2 + (L + N)p^2) \), where \( k \) is the number of retained singular values, \( p \) is the number of added columns, and \( L, N \) are the number of rows and columns for the original matrix, respectively. The method is used as clearly outlined in [26].

3.2 Time Series Prediction Index

Our goal is to add forecasting and imputation capabilities to existing database systems by using the framework and algorithm described above. In this paper, we use an open source relational database management system, specifically PostgreSQL, as the database system upon which we build prediction capabilities.

As mentioned in Section 2, we assumed that a simple time series relational database will have one relation with two columns. These columns are: time (timestamp) and value (float). The primary key “time” will be assumed to lie in range \( T_R \in [1, T_T] \). We implement a thin wrapper on this database to enable prediction queries. i.e., we develop a Time Series Prediction Index to enable (i) querying estimated mean-values of future points (forecasting); and (ii) querying the de-noised and imputed points within the given time range (imputation). We have done this through two steps: (i) storing the aforementioned model in the same database using a custom schema that enables fast and accurate queries; (ii) implement the API for prediction queries.

3.2.1 Model Storage

After training, the model is stored in the same database as the original time series. The model is effectively cached by storing the SVD matrices \( \mathbf{U}_k, \mathbf{S}_k, \mathbf{V}_k^T \) for each sub-model \( M_i \), where \( k \) is the number of singular values kept in the model. In addition, the linear regression weights \( \beta_i \), used for forecasting, for each sub-model \( i \) are stored as well. We propose the schema shown in Figure 2 to store the model, which is designed to support fast queries of forecasted and imputed data points.

The table Models stores meta-data about each sub-model: the model number (int), the time series index at which the sub model starts (int) and ends (int), and the number of rows (int) and columns (int) in each sub model matrix.

Each row in the U_table is a row in the \( U_k \) matrix of the SVD in model \( M_i \). We give each row in \( U_k \), which is of length \( k \), an id called tsrow (int) which is simply the row number of each row in \( U_k \). In other words, the first row in \( U_k \) and the first row in \( U_k \) will both have tsrow = 0. Figure [2] shows the case when \( k = 3 \).

Similarly, each row in the V_table is a row in the \( V_k \) matrix, each of length \( k \). The field tscolumn, while playing a similar role to tsrow in the U_table, is determined differently. It will be calculated for the \( j \)th row in \( V_k \) as \( tscolumn = j + \frac{N}{p} \), where \( N \) is the number of columns of the sub-models and \( i \) is the index of the sub-model, \( M_i \). Note that we determine tscolumn in this manner to ensure that adjacent models using the same columns, i.e., \( M_i, M_{i+1} \), refer to the columns using the same tscolumn.

Finally, we have the s_table, where each row, \( i \), contains the \( k \) singular values for each sub-model, \( M_i \).

The “tsrow” and “tscolumn” fields are used to determine which two records from the two tables need to be used to retrieve a de-noised (imputed) data point in the time series. The corresponding singular values are retrieved from the s_table (see Section 3.2.1 about prediction queries).

Each row in the coefficient_table has the position \( j \) and value of one coefficient \( \beta_i \) in the linear regression model for sub-model \( M_i \). Given that the average of these coefficients across sub-models will be used for forecasting future points, and since the average should not be calculated anew each time a user makes a forecasting query, we will create a materialized view of the coefficient table to precompute the average weights of the last \( n \) models. For the sake of illustration, we created the materialized view to precompute the average weight for all the \( n \) trained models, the last 10 models, and the last 100 models as shown in Figure 2 using the following query:

```
CREATE MATERIALIZED VIEW Avg_Coeff AS
SELECT coeffpos,
    avg(coeffvalue) as Avg,
    avg(coeffvalue) FILTER (WHERE modelno <= n AND modelno > n - 10) AS Avg_last10
    avg(coeffvalue) FILTER (WHERE modelno <= n AND modelno > n - 100) AS Avg_last100
FROM Coefficients_table
GROUP BY coeffpos
```

3.2.2 Prediction Queries

The system enables two types of (prediction) queries: imputation and forecasting. The imputation query enables the user to query “de-noised” or imputed mean entries of the time series in the past, while the forecasting query enables the user to query the estimated mean of the future data points. While one can retrieve the actual value of a recorded time series entry via a non-predictive query, the imputation query allows for retrieving a “de-noised” estimate of that entry; an estimate of significance in many applications.

For the sake of illustration, we will assume that the number of entries in our time series is a multiple of \( \frac{N}{p} \). When this is not the case, the imputation query is slightly changed to accommodate for the different shape of the last sub-model’s matrix.

For imputation, retrieving a data point \( \tau \in T_R \) from the time series, \( X(t) \) is done through the following steps:

1. Determine the models \( M_i, M_j \) that were mapped to that index (If \( \tau < T/2 \), then we only have on sub-model
\( M_i \).

\[
i = \max(0, \left\lfloor \frac{2\tau}{T} \right\rfloor - 1), \quad j = i + 1
\]

2. Determine \( tsrow \), and \( tscolumn \) as follows:

\[
t_{srow} = \tau \mod L, \quad t_{scolumn} = \tau / L
\]

where \( \lfloor \cdot \rfloor \) represents integer division.

3. Query the relevant rows from \( U_{table} \) to get the two tuples \( U_i = (u_{1i}, u_{2i}, \ldots, u_{ki}) \), and \( U_j = (u_{1j}, u_{2j}, \ldots, u_{kj}) \):

\[
\text{SELECT } u1, u2, \ldots, u_k \text{ FROM } U_{table}
\]

WHERE \( t_{srow} = t_{srow} \text{ AND \ (modelno = j or modelno = i)} \);

4. Query the relevant rows from \( V_{table} \) to get the two tuple \( V_i = (v_{1i}, v_{2i}, \ldots, v_{ki}) \), and \( V_j = (v_{1j}, v_{2j}, \ldots, v_{kj}) \):

\[
\text{SELECT } v1, v2, \ldots, v_k \text{ FROM } V_{table}
\]

WHERE \( t_{scolumn} = t_{scolumn} \);

5. Query the relevant rows from \( s_{table} \) to get \( S_i = (s_{1i}, s_{2i}, \ldots, s_{ki}) \), and \( S_j = (s_{1j}, s_{2j}, \ldots, s_{kj}) \):

\[
\text{SELECT } s1, s2, \ldots, sk \text{ FROM } s_{table}
\]

WHERE \( \text{modelno = j or modelno = i) \);

6. Finally, the mean of the two operations: \( \sum_k u_{ki} \cdot s_{ki} \cdot v_{ki} \) and \( \sum_k u_{kj} \cdot s_{kj} \cdot v_{kj} \) will produce the “de-noised” estimate (imputation).

As a byproduct of the overlapping sub-models, in most cases each query will return 2 sets tuples and the average of the two dot product will be used (as illustrated above). Otherwise, the results of one tuple is returned.

For forecasting, we will focus on very short term forecasting. The following steps are done to forecast the time series at \( \tau + 1 \):

1. Retrieve the last L-1 data points in the time series \( X_{L-1} = X(\tau - (L - 1) : \tau) \)

2. Retrieve the model averaged coefficients from the materialized view \( \hat{\beta}_{ave10} \) (for example, the average of the last 10 models):

\[
\text{SELECT } Avg_{last10} \text{ from } Avg_{coeff}
\]

3. Produce the forecast using the dot product \( X_{L-1} \cdot \hat{\beta}_{ave10} \)

In case of forecasting several points ahead, then forecasts for all intermediate points need to be produced, which entails repeating the last step several times and appropriately modifying \( X_{L-1} \).

We study the performance of this setup in the next section.

### 4. EXPERIMENTS

This section details the setup and results of all our experiments with the proposed system. We consider the following metrics:

**Accuracy:** The accuracy of the predictions, for both forecasting and imputation, is quantified using the (root) mean-squared error, i.e. \( (R)\text{MSE} \). \( (R)\text{MSE} \) is determined with reference to the unobserved true mean values of each data point, i.e.

\[
\text{RMSE}_{\text{mean}} = \frac{1}{T} \sqrt{\frac{1}{T} \sum_{t=0}^{T-1} (X(t) - f(t))^2}
\]

where \( X(t) \) is the prediction for \( f(t) \), the underlying ground truth time series model. In our experiments, data is generated synthetically and therefore we have knowledge (not that algorithm utilizes it) of \( f(t) \) and hence we can evaluate this performance. We report the accuracy of the model’s predictions and compare it with state-of-the-art tools for forecasting and imputation, namely Prophet and AMELIA.

**Throughput:** The write throughput of the time series prediction database quantified by the amortized time per data point. As part of this, we determine the amortized time required to update the prediction index, i.e. the required time to update the prediction model when the database is updated, and compare it to the PostgreSQL bulk insert time.

**Query time:** The time it takes to perform the PREDICT queries enabled by the prediction index. We compare it to the query time of the SELECT queries. Essentially, we report how much slower the imputation and forecasting queries are when compared to PostgreSQL single row queries.

### 4.1 Setup

These experiments were run on an Intel Xeon CPU E5-2683 machine with 132 GB of RAM. Postgres 9.5.14 with the default configuration was used as our database management system.

#### 4.1.1 Datasets

Our accurate experiments are performed on synthetic datasets composed of single and pairwise combinations of seven characteristic time series models: four autoregressive processes, two harmonic processes, and a logarithmic trend,
which have the following respective forms:

**Autoregressive process:** \( X(t) = \sum_{i=1}^{p} \phi_i X(t-i) + \epsilon_t, \)

**Harmonic process:** \( X(t) = \sum_{i=1}^{p} \alpha_i \cos(\beta_i t + \gamma_i) + \epsilon_t, \)

**Logarithmic Trend:** \( X(t) = \log(t) + \epsilon_t, \)

where \( \epsilon_t \sim \mathcal{N}(0,1) \) are independent, zero-mean Gaussian with unit variance; the constant parameters are arbitrarily chosen; in the case of the AR process, \( \phi_i \) are chosen to ensure the process is stationary. To produce the four AR series we set \( p \in \{2, 5\} \) and generated series such that \( L_1 \) norm of the lag coefficients \( ||\phi||_1 \in \{0.5, 0.9\} \) where \( \phi = [\phi_1, \phi_2, \ldots, \phi_p] \). We generate two harmonic trends by choosing \( p \in \{3, 10\} \). For the pairwise combinations, we combined two series \( X(t), Y(t) \) as:

\[ Z(t) = \alpha X(t) + (1-\alpha) Y(t) \quad \alpha \sim \mathcal{U}[0.4, 0.6], \]

i.e. \( \alpha \) is distributed per uniform distribution over interval \([0.4, 0.6]\). The results are reported for experiments with several synthetically generated time series of lengths varying up to \( 10^6 \) observations. These time series are a mixture of the processes identified above. Finally, note that all time series are normalized to the range \([-1, 1]\) to keep the scales uniform across experiments, it does not change any conclusions.

### 4.1.2 Comparison Benchmarks

The benchmarks used for accuracy comparisons are the Prophet time series library \[7\] for forecasting and AMELIA \[11, 10\] for imputation. These state-of-the-art model agnostic platforms require minimal parameter tuning.

**Prophet** \[7\] is a regression-based generalized additive model that fits three component models: trend, seasonality, and holidays. The trend model is composed of a piecewise logistic growth component and a linear trend component fit using a Bayesian model \[22\]. The seasonal model uses a Fourier series to account for fluctuations in the signal imposed with a smoothing prior. Holidays can be specified by the user to indicate potential known abnormalities in the signal. Prophet uses inference techniques that do not rely heavily on the temporal structure of the data, and as with the model we selected, it allows for forecasts to be made in the presence of missing data. In our experiments, we used Prophet without adjusting any default parameters.

**AMELIA** \[11\] is a well known and widely used multiple imputation library for time series data in R. For our experiments, we use the latest version, AMELIA II. AMELIA produces \( m \) separate imputations of the given data by maximizing a posterior distribution on the latent variables that generate the complete data using the EMB algorithm. For our imputation experiments, we use the default parameter \( m = 5 \) and included an additional covariate of a degree two polynomial approximation of the time series required by AMELIA for imputation of a single time series. We gave extra help to AMELIA by adding additional covariates corresponding to the lags and leads of the series. To produce a single time series, we take the mean of the \( m \) missing values to be the imputed estimates of the time series, as suggested in AMELIA’s documentation.

### 4.2 Accuracy

We compare the forecasting and imputation accuracies of the predictions produced by the prediction index described in Section 4.2 with those produced by Prophet (for forecasting) and AMELIA (for imputation).

#### 4.2.1 Forecasting Accuracy

Our benchmarks with Prophet have shown that the proposed system performs significantly better in most time series. In these tests, we generated a time series of 1,000,000 entries for each time series model, leaving the last 10,000 points in the test set for evaluation. The rest is used for training. We fixed the parameters \( T = 10^5 \) and \( \gamma = 0.5 \), while fixing the column to row ratio in each sub-model matrix to \( N/L = 10 \). The best numbers of singular values is selected via cross-validation. The same data is used to train the model via Prophet with the default parameters (as described in the guide \[7\]). Each test is run four times, varying the amount of hidden data each time. For each test, we designate a parameter \( p \) such that each observation in the series has probability \( p \) of being observed, independently. We vary \( p \) in the set \( \{0.4, 0.6, 0.8, 1.0\} \).

The heatmaps in Figure 3 show the difference in RMSE between the forecasts from our prediction index and Prophet. The results are presented in a matrix form. The rows and columns of the matrix represent different models of time series, and each cell of the matrix corresponds to a mixture of the models corresponding to the row and column. We denote an autoregressive process of order \( x \) for which the \( L_1 \) norm of the lag coefficients is \( y \) as \( AR x/y \) and harmonic series of \( x \) terms as \( Harmonic x \). The colors of the cells indicate the relative performance of our algorithm to Prophet: if the difference in RMSE between our algorithm and Prophet’s is negative, then the color is red which means Prophet is performing better; values in green show the opposite. We see that the heatmaps are almost uniformly green, suggesting that our algorithm is outperforming Prophet. Now details.

![Figure 3: The forecasting performance of the prediction index is significantly better than Prophet for most time series.](image)
Table 1: Improvement in forecasting & imputation over Prophet & AMELIA (calculated as the library’s (Prophet’s or AMELIA’s) RMSE over the prediction index’s RMSE)

| Prediction | p | Mean | Median | Max  | Min  |
|------------|---|------|--------|------|------|
| Forecasting| 1.0| 26.79| 4.19   | 184.60| 0.12 |
|            | 0.8| 21.80| 3.85   | 121.35| 0.11 |
|            | 0.6| 12.94| 4.05   | 46.13 | 0.11 |
|            | 0.4| 18.03| 3.29   | 97.38 | 0.11 |
| Imputation | 0.8| 1.75 | 1.48   | 7.05  | 0.70 |
|            | 0.6| 1.23 | 1.27   | 1.56  | 0.64 |
|            | 0.4| 1.39 | 1.34   | 2.41  | 0.73 |

The Table 1 shows the mean, median, max, and min RMSE improvement of the prediction index compared to Prophet across all mixtures of models. As seen, our algorithm does 3.29 to 4.19 times better than Prophet in terms of median performance across the range of missing fraction of data between 0.4 to 1.0. This is even more pronounced in terms of the mean where range is from 12.94x to 26.79x. In the cases where our algorithm does not outperform Prophet, we find that it is primarily for log-trend models. Indeed, this is not surprising since ours is a model agnostic approach while Prophet explicitly tries to fit that model.

4.2.2 Imputation Accuracy

Our experiments show that the proposed prediction index has superior imputation accuracy when compared to AMELIA. In these experiments, we generated series of 1,000,000 data points and exclude data in the same way as described earlier. For AMELIA, we generate 5 imputation models and average over the missing values to generate a single time series and compare it to the mean of each time series. For our algorithm, we used the parameters $T = 10^9$, which translates into having 19 sub-models, and $\gamma = 0.5$. In Figure 4 we see that the prediction index produces an improvement over AMELIA, improving performance for most time series over all $p$. As before, Table 1 shows the mean, median, max, and min RMSE improvement over AMELIA. Specifically, across all models and range of missing fractions of data, the relative improvement of our algorithm compared to AMELIA is 1.27x to 1.48x in terms of the median and 1.23x to 1.75x in terms of the mean.

4.2.3 Accuracy & parameter choice

We now study in detail, the effect of the choice of parameters on the imputation and forecasting accuracy of the prediction index. In particular, we will start by studying the effect of varying the parameters $T$ and $\gamma$ on the imputation and forecasting accuracy.

Accuracy vs. choice of Parameter $T$: We find that forecasting accuracy remains largely constant with varying $T$, while imputation results indicate the choice of $T$ can enhance the performance.

In this experiment, we generate a very large time series with $10^9$ entries. We vary $T$, the number of entries in each sub model from $10^2$ to $10^9$, assuming a fixed ratio of 10 of $N$ over $L$ (the matrix shape). We forecast one-point ahead for 10,000 test points using the weights of the last 10 sub-models. If the number of sub-models is below 10, we use all sub-models. We compare the produced forecast with both the actual observations and the (known) mean values of the time series. As shown in Figure 5a, the result indicates that decreasing $T$, to as low as $10^3$, does not degrade the forecasting performance, and that with segmenting the time series,
we can still get the accuracy obtained with one sub-model, indicated by the black diamond. Since we are maintaining the rows to columns ratio, we are effectively reducing the number of explanatory variables in the regression model when we reduce $T$. We suspect that this reduction is not critical due to the fact that the most recent points appear to be the important explanatory variables, and that adding more variables, i.e. increasing $L$, adds redundant information to the model.

Interestingly, the modifications have significantly increased the imputation accuracy for this particular time series. In this test, we masked 10% of the time series values, and then compared the imputed time series with both the mean and the observation data while varying $T$. Figure 5b shows that decreasing $T$, i.e. further segmenting the time series, leads to significantly better performance. In particular, the improvement in imputation accuracy can be of an order of magnitude better when compared to the one sub-model accuracy.

We replicated this experiments for several time series models, where we have found that in most cases, using many smaller values of $T$, i.e. segmenting the times series, leads to better performance. For forecasting, we find that by building 19 sub-models, as reported in the experiments above, performed better than using a single model on all data, except for log trends. For imputation, this improvement was more pronounced and the multiple sub-models predictions are significantly improved across all time series. Figure 6 shows two heatmaps indicating the difference in RMSE between the results of training multiple sub-models in our proposed system vs. training one large sub-model, where again the negative values, shown in red, mean that the system with a single sub-model is performing better and values in green show the opposite.

![Figure 6: Using multiple sub-models for the prediction index performs no worse than the single model prediction index for almost all time series, and in many cases improves upon it.](image)

**Accuracy vs. choice of parameter $\gamma$:** We find that the parameter $\gamma$ has no effect on the imputation and forecasting accuracy with the selected algorithm. The parameter $\gamma$, which indicates how frequently the sub-model is fully re-trained rather than incrementally updated can play a big role when an inaccurate update method is used. In our system, we are using a fairly accurate algorithm for updating the SVD, which is reflected in the following results which show how little $\gamma$ is affecting the prediction accuracy.

In this experiment, we used a time series with $10^9$ entries. Prior to training the model, we randomly selected test indices between $2 \times 10^7$ to $10^8$. Then, we train the model up until those indices and test prediction accuracy. The idea for this test is to probe the accuracy throughout the time range to estimate the effect of incrementally updating the SVD. In this test, we chose $T = 10^7$, and set $\gamma \in \{0.05, 0.5, 1.0\}$. As shown in Figure 7, the forecasting and the imputation performance for these very different choices of $\gamma$ is almost identical. The blue, green, and orange points, corresponding to the three different choices of gamma, shows that these accuracies are almost identical not only on average, but for each and every test point.

![Figure 7: Varying $\gamma$ has no effect on both the forecasting and imputation accuracy, as the accuracy in these different experiments (shown in different colors) are almost identical.](image)

**4.3 Throughput**

In this subsection, we are interested in evaluating how fast the prediction index is updated as new points are added to the database. Ideally, we want the prediction model to be updated approximately as fast as the database bulk insert operation. In other words, we want the updates to the model to be reflect as soon as possible after the database is updated, to produce accurate and updated prediction queries. To test the update time of the proposed system, we used several of the aforementioned synthetic time series with the number of entries logarithmically spaced between $10^5$ to $10^9$.

**Update time vs. choice of $T$:** We have found that $T$ affects the update time significantly, where segmenting could either speed or slow the process. In this test, we used the time series with $10^9$ entries. $\gamma = 0.5$, and varied the parameter $T$, which represents the number of entries in each sub-model, from $10^5$ to $10^9$. If only one sub-model is used, then the algorithm is identical to the algorithm of [1].
We now evaluate the query time for the two proposed prediction queries, namely the Imputation query and the Forecast query. In particular, we will compare these two queries with the query time for the non-predictive PostgreSQL query of one record in the time series table.

**PREDICT vs SELECT:** Our evaluation of prediction query time showed that, using a dataset of 100 million rows, Imputation queries are only 1.3 times slower compared to Postgres queries, and that forecasting queries are only 7 times slower.

In this evaluation of prediction queries with a time series of $10^8$ entries, we have observed that the system was able to achieve an average query time of 0.542 ms and 3.206 ms for imputation and forecasting queries respectively. Figure 10 shows the histograms of the query times of one hundred thousand queries for each query type. As it shows, the imputation queries are, on average, only 1.28x slower and forecasting queries are 7.569x slower than the non-predictive Postgres queries for a single record.

### Effect of data size.
To see the effects of different sizes of time series, we determined the query times for synthetic time series with entries logarithmically spaced between $10^8$ to $10^9$. In other words, we measured how the query times degrade as we increase the table size. We find that, across databases of different sizes, forecasting queries are only 5-15x slower than the non-predictive query. Also across databases of different sizes, imputation queries are, on average, only 1.3 times slower compared to the non-predictive query. In Figure 11 shows how the query time for each query changes with the size of the time series.

Imputation query time stays almost constant with the increasing number of records. This is not surprising, since the $U_{table}$, $V_{table}$, and $s_{table}$ increase in size in a much slower rate than the time series table. For example, only one new row is added to $V_{table}$ for each $N = 10L = 5000$ entries added to the time series. There is a slight jump at the end, which happens at the point where the predicate on the sub-model number becomes more selective than the predicate on "trow" in the $U_{table}$. In other words, when the number of models exceeds $L$.

Since forecasting queries include querying $L - 1$ records from the time series table, its time is expected to increase...
as the total number of records increases and subsequently the non-predictive query time increases. However, that does not mean that the relative time to non-predictive queries should suffer, which is shown in the figure (when the number of records \( > T \) the ratio decreases from 15x to 5x). Another thing to note is that when the number of records \( < T \), we get a much lower relative query time for Forecasting queries. This is due to the fact that we only have one sub-model, and the shape of its matrix and consequently the number of coefficients in its forecasting model is less than \( L \). We discuss the effect of number of coefficients on the forecasting query time later.

**System invariant performance:** We claim that the ratio of prediction query time to non-predictive query time will remain constant regardless of the machine specifications. To verify this, we tested our system in two other machines: (i) M2: Windows machine with 2.4 GHz Intel Core i5 machine with 8 GB of RAM. (ii) M3: Macbook Pro with 2.9 GHz Intel Core i5 and 8 GB of RAM. A time series of 10^6 entries were used, with \( T = 2.5 \times 10^6 \). Those results are compared with the previous results, with the machine denoted as M1.

As Table 2 shows, the test yielded results that are consistent with our claim. The imputation queries’ ratios are similar for the three machines at 7.57, 7.02, and 6.56 for M1, M2 and M3 respectively. Forecasting queries’ performance ratios were also similar for the three machines at 1.28, 1.83, and 1.87 for M1, M2 and M3 respectively. Therefore, the relative time to non-predictive queries remains constant as the total number of records increases and subsequently the non-predictive query time increases. However, that does not mean that the relative time to non-predictive queries should suffer, which is shown in the figure (when the number of records \( > T \) the ratio decreases from 15x to 5x). Another thing to note is that when the number of records \( < T \), we get a much lower relative query time for Forecasting queries. This is due to the fact that we only have one sub-model, and the shape of its matrix and consequently the number of coefficients in its forecasting model is less than \( L \). We discuss the effect of number of coefficients on the forecasting query time later.

**Effect of \( L \) on forecasting queries:** given that forecasting queries are directly related to our choice of the parameter \( L \), the number of rows in each sub-model’s matrix, we now determine how that choice affects the forecast query time. In particular, increasing \( L \) means increasing the number of coefficients in the regression models \( (\beta^t \in \mathbb{R}^{L-1}) \), which consequently requires querying \( L - 1 \) points from the time series database for each forecast and, hence, makes the query slower. Figure 12 shows that this increasing behavior is linear. As a compromise between the forecasting query time and the update time, we chose \( L = 500 \), that is \( T = 2.5 \times 10^6 \) and \( \gamma = 0.5 \), which is the selected choice of this parameters in the previous query tests.

**Figure 12:** Forecasting query time as we increase \( L \)

5. **CONCLUSION**

In this paper, we introduce a framework for building prediction indices as a thin wrappers on existing databases storing time series data. We use PostgreSQL as the database of choice and the model-agnostic time series prediction algorithm of [1] as the basis for the prediction index. This index enables predictive queries to support estimated “de-noised” mean values in the future (forecasting) and within existing time ranges (imputation). We show that the proposed prediction index does not degrade database throughput as it can accommodate new data faster than PostgreSQL bulk inserts, i.e. 1\( \mu \)s per new data point for prediction index vs 4\( \mu \)s for PostgreSQL bulk inserts. Additionally, the proposed predictive queries’ latency is similar to a standard SELECT query in PostgreSQL, i.e. < 1.9\( \mu \)s for imputation queries and < 7.6\( \mu \)s for forecasting forecasting queries, across machine platforms. We also show that the statistical accuracy of the predictions is superior to both Prophet [7] and Amelia [10]-two state-of-art time series forecasting and imputation libraries.

Based on these promising results, the natural extensions to this work are to enable multi-dimensional time series predictions and enabling long-term forecasting. These extensions would enable more robust predictions and a more versatile system.
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