The k-essence models and cosmic acceleration in generalized teleparallel gravity

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Abstract

Generalized teleparallel gravity has been suggested to explain the present cosmic acceleration of the Universe. In this paper, we take a spatially homogeneous and anisotropic Bianchi type I Universe in the framework of $F(T)$ gravity. The behavior of the accelerating Universe is investigated for three purely kinetic k-essence models. We explore the equation of state parameter and deceleration parameter for these k-essence models. It is found that all these models exhibit quintessence behavior of the Universe.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

The fact that the Universe is expanding at every point in space is a difficult concept to grasp. Cosmological observations from cosmic microwave background radiation (CMBR) [1] reveal that most of the energy in our Universe is dark, which causes gravitational repulsion and hence accelerates the expansion of the Universe. [2]. The properties of dark energy (DE) can be specified by energy density $\rho$ and pressure $p$. These two parameters are responsible for the following three main phases of the Universe.

- The first phase is referred to as the radiation-dominated era, which occurred just after the creation of the Universe. At this stage, the pressure of the radiation is given by one-third of its energy density.
- The next phase is the matter-dominated era, which came into being when the Universe was assumed to be 70 000 years old. Its energy density surpassed the energy density of the first phase of the Universe until a cosmological constant as a DE candidate was proposed.
- The third era is the DE-dominated era. About 5 billion years ago, this phase dominated in the Universe as a whole just after the matter-dominated era which dropped to very low concentration [3]. It is mentioned here that the most recent Wilkinson microwave anisotropy probe (WMAP) observations [2] indicate about 74% DE in the Universe.

By the measurements of CMBR, the WMAP satellite indicates that the Universe is very close to flat and to maintain this flatness, the mass/energy density of the Universe must be equal to a certain critical density. The total amount of matter in the Universe is estimated at only 30% of the critical density. For the remaining 70% of critical density, an additional form of energy is required, which is termed DE. There are two proposed forms to discuss DE: one is the modified theories of gravity and the second is the scalar field models. In this connection, single scalar field models attracted many people. The k-essence (k stands for kinetic) model [4] is one of the models that can be described by a single real scalar field $\phi$ with non-canonical kinetic terms responsible for negative pressure. This may be taken as the generalization of canonical scalar fields, e.g. quintessence [5] (a time-varying quantity).

The Lagrangian with a non-canonical kinetic term was proposed to discuss the early-time acceleration named k-inflation [6]. Nojiri [7] constructed explicitly the k-essence DE model to unify the late-time acceleration and inflation in the early Universe. Matsumoto and Nojiri [8] reconstructed the scalar quintessence model, the tachyon DE model and the ghost condensation model as special cases of k-essence. Armendariz-Picon et al [9] discussed the essential
features of k-essence and developed some examples. The solution of these examples leads to two results: one in which cosmic acceleration continues forever and the other in which acceleration has a finite duration. Bose and Majumdar [10] investigated purely kinetic k-essence and a particular k-essence model with the potential term. They concluded that such a model could generate basic features of early inflation and DE observational constraints. Yang and Gao [11] introduced purely kinetic k-essence by means of the Lagrangian. They plotted evolutions of the equation of state (EoS) and the speed of sound for particular cases.

Modified theories also play an important role in explaining DE. There exist many modified gravity theories that may naturally unify inflation in the early Universe and late-time acceleration [12]. $F(T)$ gravity [13] is the modified form of the teleparallel equivalent of general relativity [14]. This theory is formed by using the Weitzenböck connection, which has no curvature but only torsion and possesses a second-order set of field equations.

Myrzakulov [15] proposed some new models of k-essence in the framework of $F(T)$ gravity. Tsyba et al [16] investigated purely kinetic k-essence for an explanation of cosmic acceleration. They concluded that for a particular case of scalar field, modified gravity and purely kinetic k-essence become equivalent. Dent et al [17] investigated this extended modified gravity at the background and perturbed level and also explored this theory for quintessence scenarios. Karami and Abdolmaleki [18] found that the EoS parameter of holographic and new agegraphic k-essence models always crosses the phantom-divide line, whereas the entropy-corrected model found that the EoS parameter of early inflation and DE observational constraints. Yang and Gao [11] introduced purely kinetic k-essence by means of the $\phi$-field, $X$ is the field equations are formulated. Section 4 is devoted to studying some k-essence models and discussing cosmic acceleration. In the last section, some concluding remarks are given.

2. Preliminaries

In this section, we provide the basic concepts of k-essence as well as $F(T)$ gravity.

2.1. The k-essence formalism

There are some scalar fields with non-canonical kinetic terms in particle physics. The k-essence models are described by a single scalar field and a kinetic term. The general k-essence action [6] is of the form

$$S = \int d^4 x \sqrt{-g} L(\phi, X),$$

(1)

where $\phi$ is the scalar field, $g = \text{det}(g_{\mu\nu})$ and $X$ is the dimensionless kinetic energy term defined by

$$X = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}, \quad \mu, \nu = 0, 1, 2, 3,$$

(2)

where $\phi_{,\mu} = \frac{\delta \phi}{\delta x^\mu} = \partial_{\mu} \phi$. It is mentioned here that a Lagrangian can be a function of any scalar field $\phi$ and $X$, i.e., $L = K(\phi, X)$. Here we consider one of the simplest possible k-essence models, termed purely kinetic k-essence with action

$$S = \int d^4 x \sqrt{-g} L(X), \quad L = K(X).$$

(3)

The energy-momentum tensor of k-essence is obtained by varying this action with respect to the metric tensor [6]

$$T_{\mu\nu} = K_{,\mu} \phi_{,\nu} - K g_{\mu\nu}, \quad K_X = \frac{dK}{dX}$$

(4)

The energy-momentum tensor of a perfect fluid is

$$T_{\mu\nu} = (\rho_k + p_k) u_{\mu} u_{\nu} - p_k g_{\mu\nu}.$$  

(5)

Here, $\rho_k, p_k$ are the energy density and pressure of k-essence, respectively, and $u_{\mu}$ is the four-velocity defined by

$$u_{\mu} = \eta \frac{\phi_{,\mu}}{\sqrt{2X}},$$  

(6)

where $\eta = \pm 1$ according to the sign of $\phi$ positive or negative, respectively. We assume that $\partial_{\mu} \phi$ is timelike and smooth on interesting scales [5]. Thus, we can associate the energy-momentum tensors of k-essence with a perfect fluid. For $\mu = 0 = v$ and $\mu = 1 = v$ in equations (4) and (5), we obtain the following expressions for k-essence energy density and pressure, respectively:

$$\rho_k = 2KX - K, \quad p_k = K.$$

(7)

This yields the following k-essence EoS parameter:

$$\omega_k = \frac{p_k}{\rho_k} = \frac{K}{2KX - K}.$$  

(8)

2.2. $F(T)$ theory of gravity

Here, we introduce briefly the teleparallel theory of gravity and its generalization to $F(T)$ gravity. In teleparallel action, the torsion scalar $T$ is used as the Lagrangian density, whereas in modified teleparallel gravity, it is promoted to a function of $T$. Thus, the action for $F(T)$ gravity [18] is

$$S = \frac{1}{2\kappa^2} \int d^4 x [eF(T) + L_m].$$  

(9)

where $e = \sqrt{-g}$, $\kappa^2 = 8\pi G$, $G$ is the gravitational constant and $F(T)$ is the general differentiable function of $T$. Also, $L_m$ is the Lagrangian density of matter inside the Universe. The torsion scalar is given as

$$T = S^\mu_{\nu\rho} T_{\rho\nu}^\mu,$$  

(10)

where $S^\mu_{\nu\rho}$ and the torsion tensor $T_{\rho\nu}^\mu$ are defined as follows:

$$S^\mu_{\nu\rho} = \frac{1}{2}(K_{\mu\rho} + \delta_{\rho}^\mu \sigma_{\nu}^\sigma - \delta_{\rho}^\sigma \sigma_{\nu}^\sigma \phi),$$  

(11)

$$T_{\rho\nu}^\mu = \Gamma_{\nu\mu}^\lambda \Gamma_{\rho\lambda}^\delta - \Gamma_{\rho\nu}^\lambda \Gamma_{\mu\lambda}^\delta.$$  

(12)

Here $h_{\mu}^\nu$ are tetrads components which form an orthonormal basis for the tangent space at each point $x^\mu$ of the manifold.
Each vector $h_i$ can be identified by its components $h^i_\mu$, such that $h_i = h^i_\mu \partial_\mu$, where index $i$ runs over 0, 1, 2, 3, denote the tangent space, where $\mu = 0, 1, 2, 3$ represent the coordinate indices on the manifold. These tetrads are related to the metric tensor $g_{\mu \nu}$ by the following relation:

$$g_{\mu \nu} = \eta_{ij} h^i_\mu h^j_\nu,$$

where $\eta_{ij} = \text{diag}(1, -1, -1, -1)$ is the Minkowski metric for the tangent space satisfying the following properties:

$$h^i_\mu h^j_\nu = \delta^i_j, \quad h^i_\mu h^i_\nu = \delta^\mu_\nu.$$

The contorsion tensor, $K^{\mu \nu}_\rho$, is equal to the difference between the Weitzenböck and the Levi–Civita connections defined by

$$K^{\mu \nu}_\rho = -\frac{1}{2}(T^{\mu \nu}_\rho - T^{\nu \mu}_\rho - T^{\rho \mu}_\nu).$$

The variation of equation (9) with respect to tetrad $h^i_\mu$ leads to the following field equations:

$$[\varepsilon^{-1} \partial_\rho (\varepsilon S^{\mu \nu}) - h^i_\mu T^i_\rho \varepsilon S_{\rho \mu \nu}] F_T + S_{\mu \nu} \partial_\rho (T^{\nu \mu}_\rho) + \frac{1}{2} h^i_\rho F = \frac{1}{2} \kappa^2 h^i_\rho T^i_\rho,$$

where $F_T = \frac{\delta T}{\delta \rho}$, $F_{TT} = \frac{\delta T}{\delta \rho}$, $s^{\mu \nu} = h^i_\mu s^{\mu \nu}$ and $\varepsilon^{\mu \nu}$ is antisymmetric property. The energy-momentum tensor $T_{\mu \nu}$ is given as

$$T^\rho_\nu = \text{diag}(\rho_m, -p_m, -p_m, -p_m),$$

where $\rho_m$ and $p_m$ denote the usual density and pressure of matter inside the Universe.

3. **Bianchi I Universe and the field equations**

The assumption of isotropy does not predict the early epoch of the Big Bang as the Universe does not maintain its isotropic behavior at very small scales. In order to get a realistic model that represents an expanding, homogeneous and anisotropic Universe, we use the Bianchi type I Universe model, which is a generalization of the FRW metric. Here, we study the evolution of the Universe in the presence of DE k-essence models. The line element of Bianchi-type I spacetime is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2,$$

where the scale factors $A$, $B$ and $C$ are functions of cosmic time $t$ only. Using equations (13) and (18), we obtain the tetrad components as follows [19]:

$$h^i_\mu = \text{diag}(1, A, B, C).$$

Using equations (11) and (12) in (10) along with (18), we obtain

$$T = -2\left(\frac{\dot{A}B}{AB} + \frac{\dot{B}C}{BC} + \frac{\dot{C}A}{CA}\right).$$

The field equations (16) for $i = 0 = \nu$ and $i = 1 = \nu$ are given by

$$F - 4\left(\frac{\dot{A}B}{AB} + \frac{\dot{B}C}{BC} + \frac{\dot{C}A}{CA}\right)F_T = 2\kappa^2 p_m,$$

and

$$2\left(\frac{\dot{A}B}{AB} + \frac{\dot{B}C}{BC} + \frac{\dot{C}A}{CA}\right)F_T - 4\left(\frac{B}{B} + \frac{C}{C}\right)\times \left[\left(\frac{\dot{A}}{A^2} - \frac{\dot{B}}{B^2} + \frac{\dot{C}}{C^2}\right) + \left(\frac{\dot{B}}{B^2} - \frac{\dot{C}}{C^2}\right)\left(\frac{A}{A^2} + \frac{B}{B^2}\right)\right] F_{TT} - F = 2\kappa^2 p_m. \quad (22)$$

The corresponding conservation equation is

$$\rho_m + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)(\rho_m + p_m) = 0. \quad (23)$$

The average scale factor $R$ and the mean Hubble parameter $H$, respectively, will become

$$R = (ABC)^{1/3}, \quad H = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{\dot{R}}{R}. \quad (24)$$

The deceleration parameter $q$ is a dimensionless quantity and is used to describe the accelerating Universe

$$q = -1 - \frac{H}{\dot{H}}. \quad (25)$$

Any cosmological Universe represents an acceleration, deceleration or expansion with constant velocity for $-1 \leq q < 0$, $q > 0$ and $q = 0$, respectively. In terms of the Hubble parameter and torsion, equations (21)–(23) can be simplified as

$$2\kappa^2 \rho_m = 2TF_T + F, \quad (26)$$

$$2\kappa^2 p_m = (6\dot{H} - T + 2J + 2L)F_T + 2MT_T F_{TT} - F, \quad (27)$$

where $L = \frac{\dot{B}C}{BC} - \frac{A}{A^2}$, $M = \frac{\dot{A}}{A} + \frac{\dot{C}}{C}$ and $J = \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}$. Also, equation (20) can be written as

$$T = -9H^2 + J. \quad (29)$$

The case $F(T) = T$ gives the torsion contribution $\rho_T$ by equations (26) and (27), which reduce to the following forms:

$$\rho_m + \rho_T = \frac{3T}{2\kappa^2}, \quad (30)$$

$$p_m + \rho_T = \frac{1}{\kappa^2}(3H + J - T + L). \quad (31)$$

If we take $\rho_T = \rho_k$, $\rho_T = \rho_k$, then these equations become

$$\rho_m + 2XK_X - K = \frac{3T}{2\kappa^2}, \quad (32)$$

$$p_m + K = \frac{1}{\kappa^2}(3H + J + L). \quad (33)$$

For the sake of simplicity, we assume that $\rho_m = 0 = p_m = \kappa^2 - 1$. Using these values and inserting the value of $T$ from equation (29) in (32) and (33), it follows that

$$2XK_X - K = \frac{1}{\kappa^2}(-9H^2 + J), \quad (34)$$
\[ K = 3H + 9H^2 + L. \] (35)

Equation (2) implies that \( X = \frac{1}{2} \phi^2 \) whose corresponding scalar function is given by
\[ \phi = \int \sqrt{2X} dt + \text{constant}. \] (36)

The continuity equation (5) for k-essence models leads to
\[ \rho_k + 3H(\rho_k + p_k) = 0. \] (37)

Using equation (7) in this equation, we have
\[ (K_X + 2XK_{XX}) \dot{X} + 6HXK_X = 0, \] (38)
which is the kinetic k-essence field equation.

As a special case [15], we take \( \phi = \phi_0 + \ln R \pm \sqrt{8}, \) where \( \phi_0 \) is an arbitrary constant. Consequently, the kinetic term takes the form
\[ X = \frac{1}{2} \phi^2 = J - T. \] (39)

Inserting this value into equation (38), we obtain
\[ 2T(\dot{J} - 18H \dot{H})F_{TT} + (\dot{J} - 18H\dot{H} + 6HT)F_T = 0. \] (40)

Also, equation (27) can be simplified as
\[ 2M(\dot{J} - 18H \dot{H})F_{TT} + (6\dot{H} + 9H^2 + J + 2L)F_T = F = 0. \] (41)

It is interesting to mention here that if we use equation (40) in (41), it shows the equivalence of \( F(T) \) gravity and k-essence for FRW spacetime [15, 16]. However, these equations do not provide any such relation for the Bianchi type I Universe.

4. The k-essence models

In this section, we aim to discuss k-essence models in the framework of modified teleparallel gravity. We take three arbitrary k-essence models and evaluate the kinetic term and the corresponding scalar field. Also, we formulate the EoS parameter and the deceleration parameter for k-essence models. For this purpose, we take the following particular values of the scale factors by using the power law [20]:
\[ A(t) = (m_1 s_1)^{1/m_1}, \quad B(t) = (m_2 s_2 t)^{1/m_2}, \] (42)
\[ C(t) = (m_3 s_3 t)^{1/m_3}, \]
where \( m_i, s_i \) are positive constants and \( i = 1, 2, 3 \). The EoS parameter in equation (8) can be written as follows:
\[ \omega_k = -1 + \frac{2K_{XX}}{2XK_X} - K. \] (43)

4.1. Model I

Consider the following k-essence model [16]:
\[ K = \sum_{j=0}^{M} v_j(t) y^j, \quad M > 0, \] (44)
where \( y = \tanh t \). We find the coefficients of \( y \) and the kinetic term of k-essence (X) by assuming the following Hubble parameter:
\[ H = \sum_{j=0}^{N} \mu_j(t) y^j, \quad N > 0. \] (45)

Note that we take \( \mu_j(t) \) and \( v_j(t) \) as constant quantities throughout. As a particular example, we take \( M = 2 \) in equation (44) and \( M = 1 \) for \( H \); it follows that
\[ K = v_0 + v_1 y + v_2 y^2, \quad H = \mu_0 + \mu_1 y. \] (46)

Inserting this value of \( H \) into equation (35), we obtain
\[ K = 3\mu_1 (3\mu_1 - 1) y^2 + 18\mu_0 \mu_1 y + 3\mu_1 + 9\mu_0^2 + L. \] (47)

Using the values of \( K \) and \( H \) in equation (34), it follows that
\[ X = a_1 \exp(t^{1/\omega_k}), \] (48)
where \( a_1 \) is an integration constant and
\[ g_1(t) = \left[ 3\mu_1 + \frac{45}{2} \mu_0^2 + L - \frac{3}{2} J - 9\mu_0 \mu_1 y \
- 3\mu_1 \left( \frac{3}{2} \mu_1 + 1 \right) y^2 \right]^{-1} \times 2[18\mu_0 \mu_1 + 3\mu_1 + 9\mu_0^2 + L \\n+ 6\mu_1(3\mu_1 - 1) y - 18\mu_0 \mu_1 y^2 \\n- 6\mu_1(3\mu_1 - 1) y^3]]. \] (49)

It is mentioned here that the scalar function \( \phi \) for the model (44) can be obtained by using equations (36) and (48).

The k-essence energy density and pressure in equation (7) take the following forms:
\[ \rho_k = -\frac{22}{3} \mu_1^2 y^2 - 27\mu_0 \mu_1 y - \frac{22}{3} \mu_0^2 + \frac{3}{2} J, \] (50)
\[ p_k = 3\mu_1 (3\mu_1 - 1) y^2 + 18\mu_0 \mu_1 y + 3\mu_1 + 9\mu_0^2 + L. \] (51)

Inserting these values in equation (43), the EoS parameter becomes
\[ \omega_k = -1 - \frac{3\mu_1 - \frac{9}{2} \mu_0^2 + L + \frac{3}{2} J - 9\mu_0 \mu_1 y - 3\mu_1 (\frac{3}{2} \mu_1 + 1) y^2}{\frac{3}{2}(9\mu_0^2 - J) + 27\mu_0 \mu_1 y + \frac{27}{2} \mu_1^2 y^2}. \] (52)

The viability of this model depends on the possible values of the parameters in equation (42). The model shows different behavior initially for unequal \( m_i \) but indicates the same behavior at later times for all values of the parameters. Here, we discuss the simple case that leads to the isotropic Universe. Using equation (42) in this equation and assuming \( \mu_i = 1 = m_i = s_i \), it follows that
\[ \omega_k = -1 - \frac{1 - 11/3t^2 + 6\tanh t + 5\tanh^2 t}{9 - 3/t^2 + 18 \tanh t + 9 \tanh^2 t}. \] (53)

For late-time acceleration, i.e. \( t \to \infty \), this yields
\[ \omega_k \big|_{t \to \infty} = -\frac{2}{3}, \] (54)
which is shown in figure 1. This represents evolution of the k-essence EoS parameter \( \omega_k \) as a function of cosmic time. It
is obvious that initially \((t = 0), \omega_k = 0.22\), showing that the Universe is lying in a region which contained dust fluid as well as radiation. As the time elapses up to \(t = 0.42\), the k-essence EoS parameter confined in the region \(-\infty < \omega_k < +\infty\), i.e. the Universe evolves from physical matter to DE phase. The EoS parameter bears a negative increment in its value and becomes constant at \(t = 0.48\) towards \(t \rightarrow \infty\). The constant value of the EoS parameter is greater than \(-1\) but less than \(-1/3\), which shows a quintessence era [21].

The deceleration parameter (55) for this model takes the form
\[
q = -1 + \frac{\mu_1(1 - \tanh^2 t)}{\mu_0^2 + \mu_1^2 \tanh^2 t + 2 \mu_0 \mu_1 \tanh t},
\]
which implies that \(q|_{t \rightarrow \infty} = -1\), indicating an accelerating Universe. Its graphical representation is shown in figure 2, which indicates that initially at \((t = 0), q = 0\) expressing an expanding Universe with a constant velocity. As time passes, its value decreases and converges towards \(-1\). Note that the graph exhibits an ever expanding Universe as there is not a single positive value of \(q\).

4.2. Model II

The second k-essence model [15] is given by
\[
K = \sum_{j=m}^{n} v_j(t) e^{jt},
\]
where \(m\) is any positive real number and \(n \leq 0\) real number. For the sake of simplicity, we take \(m = 2, n = 0\) and \(v_j(t) = v_j\) as a constant. Thus,
\[
K = v_{-2} e^{-2t} + v_{-1} e^{-t} + v_0.
\]
We also assume \(H\) as follows:
\[
H = \mu_{-1} e^{-t} + \mu_0.
\]
Inserting this value into equation (35), we obtain
\[
K = 9 \mu_0^2 + L - 3 \mu_{-1} (1 - 6 \mu_0) e^{-t} + 9 \mu_{-1} e^{-2t}.
\]
Using equations (58) and (59) in (34), we obtain the kinetic term of the k-essence model in the form
\[
X = a_2 e^{\int g_2(t) dt}.
\]
Here \(a_2\) is an integration constant and \(g_2(t)\) is given by
\[
g_2(t) = \frac{2[L + 3 \mu_{-1} (1 - 6 \mu_0) e^{-t} - 18 \mu_{-1}^2 e^{-2t}]}{3 J / 2 - 9 / 2 + L - 3 \mu_{-1} (1 + 3 \mu_0) e^{-t} - 9 \mu_{-1}^2 e^{-2t}}.
\]
The scalar function \(\phi\) is obtained by inserting equation (60) into (36).

The energy density and pressure parameters turn out to be
\[
\rho_k = -\frac{27}{2} \mu_0^2 + \frac{3}{2} J - 27 \mu_{-1} \mu_0 e^{-t} - \frac{27}{2} \mu_{-1}^2 e^{-2t},
\]
\[
p_k = 9 \mu_0^2 + L - 3 \mu_{-1} (1 - 6 \mu_0) e^{-t} + 9 \mu_{-1}^2 e^{-2t}.
\]
Inserting these values into equation (43), the EoS parameter takes the form
\[
\omega_k = -1 + \frac{9 \mu_0^2 - 2 L - 3 J + 6 \mu_{-1} (1 + 3 \mu_0) e^{-t} + 9 \mu_{-1}^2 e^{-2t}}{27 \mu_0^2 - 3 J + 54 \mu_{-1} \mu_0 e^{-t} + 27 \mu_{-1}^2 e^{-2t}}.
\]
Taking all the constants equal to 1 as in the isotropic case and using equation (42), it follows that
\[
\omega_k = -1 + \frac{9 - 11 l^2 + 24 e^{-t} + 9 e^{-2t}}{27 - 9 l^2 + 54 e^{-t} + 27 e^{-2t}}.
\]
This shows that as \(t \rightarrow \infty, \omega_k > -1\), which represents the quintessence region [21] shown in figure 3. This model has the same behavior as that of the first model. However, the EoS parameter is directed to the DE era at \(t = 0.34\) and after a short interval of time, it becomes constant, i.e. \(-0.67\). The corresponding deceleration parameter is
\[
q = -1 + \frac{\mu_{-1} e^{-t}}{\mu_{-1}^2 e^{-2t} + \mu_0^2 + 2 \mu_{-1} \mu_0 e^{-t}},
\]
which gives \(q|_{t \rightarrow \infty} = -1\). At \(t = 0\), the value of \(q\) is negative, which shows initially an expanding Universe. Its value decreases and converges to \(-1\) as \(t \rightarrow \infty\) with the passage of time as shown in figure 4.
Here we take the following k-essence model \[ g \]:

\[ K = \sum_{j=m}^{n} v_j(t)(\ln t)^j, \]

where \( m, n \) and \( v_j \) are the same as in model II. For \( m = 2 \), \( n = 0 \) and constant \( v_j \)'s, this equation becomes

\[ K = v_2(\ln t)^{-2} + v_0(\ln t)^{-1} + v_0. \]

Assuming the Hubble parameter in the form

\[ H = \mu(\ln t)^{-1} + \mu_0, \]

Inserting this value of \( H \) into equation (35), it implies that

\[ K = L + 9\mu_0^2 + 18\mu_0\mu_0(\ln t)^{-1} + 3\mu_0(3\mu_0 - 1/t)(\ln t)^{-2}. \]

The kinetic term \( X \) from (34) and the corresponding scalar function \( \phi \) become

\[ X = a_3 e^{g_3(t)\phi}, \quad \phi = \theta^{-1}\sqrt{-2X}, \]

where \( g_3(t) \) is found by using (34):

\[ g_3(t) = 2[\dot{L} - 6\mu_0(3\mu_0 - 1/t)\ln t^{-3} + 3\mu_0(1/t - 6\mu_0)(\ln t)^{-2}] \times [L + 3J/2 - 9\mu_0/2 - 9\mu_0(\ln t)^{-3} + 3\mu_0(-3\mu_0 - 1/t)(\ln t)^{-2}]. \]

Consequently, the deceleration parameter becomes

\[ q = -1 + \frac{\mu_0}{t(\ln t)^2[\mu_0^{-1}(\ln t)^{-2} + \mu_0^2 + 2\mu_0\mu_0(\ln t)^{-1}]} . \]

This shows that the condition for the accelerating Universe is satisfied as \( t \to \infty \). Figure 5 (76) shows that \( \omega_k > -1 \) as \( t \to \infty \) and initially the model represents a Universe having both properties of matter and radiation. After a very short interval, the Universe is dominated by DE. It is worthwhile to mention here that at \( t = 0.63 \), the Universe changed its phase from the DE era to a physical matter-dominated era. As compared to the DE phase, the Universe stayed in the matter-dominated era for a short interval of time. Then a decrement in its value is observed.

For this model, \( \rho_k \) and \( p_k \) become

\[ \rho_k = \frac{3}{2}J - \frac{27}{2}\mu_0^2 - 27\mu_0(\ln t)^{-1} - \frac{27}{2}\mu_0^2 (\ln t)^{-2} \]

\[ p_k = L + 9\mu_0^2 + 18\mu_0\mu_0(\ln t)^{-1} + 3\mu_0(3\mu_0 - \frac{1}{t})(\ln t)^{-2}. \]

The corresponding EoS parameter takes the form

\[ \omega_k = -1 + \frac{2L + 3J - 9\mu_0^2 - 18\mu_0\mu_0(\ln t)^{-1}}{3J - 27\mu_0^2 - 54\mu_0\mu_0(\ln t)^{-1} - 27\mu_0^2 (\ln t)^{-2}} . \]

To expose the present-day nature of the Universe, we take \( \mu_i = m_i = s_i = 1 \) and using equation (42), it follows that

\[ \omega_k = -1 + \frac{3 - 11/3t^2 + 6(\ln t)^{-1} + 2(3/2 + 1/t)(\ln t)^{-2}}{9 - 3/t^2 + 18(\ln t)^{-1} + 9(\ln t)^{-2}} . \]

4.3. Model III

Here we take the following k-essence model [15]:

\[ X = \sum_{j=m}^{n} v_j(t)(\ln t)^j, \]

where \( m, n \) and \( v_j \) are the same as in model II. For \( m = 2 \), \( n = 0 \) and constant \( v_j \)'s, this equation becomes

\[ X = v_2(\ln t)^{1/2} + v_0(\ln t)^{1/2} + v_0. \]

Assuming the Hubble parameter in the form

\[ H = \mu(\ln t)^{-1} + \mu_0, \]

Inserting this value of \( H \) into equation (35), it implies that

\[ K = L + 9\mu_0^2 + 18\mu_0\mu_0(\ln t)^{-1} + 3\mu_0(3\mu_0 - 1/t)(\ln t)^{-2}. \]

The kinetic term \( X \) from (34) and the corresponding scalar function \( \phi \) become

\[ X = a_3 e^{g_3(t)\phi}, \quad \phi = \theta^{-1}\sqrt{-2X}, \]

where \( g_3(t) \) is found by using (34):
5. Summary

This paper is devoted to studying the well-known phenomenon of Universe expansion in the context of $F(T)$ gravity. For this purpose, we have taken the Bianchi type I Universe and have explored some purely kinetic k-essence models. In physical cosmology and gravity, the DE problem has been investigated in terms of the cosmological constant, scalar fields, tachyons, Chaplygin gas, quintessence, modified gravities, etc. Here we have used $F(T)$ gravity and the scalar field term whose combination has resulted in a quintessence phase.

We have evaluated the EoS parameter and the deceleration parameter for purely kinetic k-essence models. These are shown in terms of graphs versus cosmic time by taking particular values of the metric coefficients. These models describe the evolution of the Universe from the Big Bang to the present epoch and are summarized as follows:

- For model I, the EoS parameter and the deceleration parameter indicate an ever accelerating Universe with the passage of time. At $t = 0.42$, the Universe changes its phase from the matter-dominated to DE phase resulting in the quintessence region.
- Model II has the same behavior as that of model I; however, the phase changes at point $t = 0.38$. The deceleration parameter initially represents an expanding Universe with constant speed and after a short interval, it accelerates with a higher speed.
- For model III, $\omega_{q}$ and $q$ initially show a decelerating DE-dominated Universe, which reverts to a matter-dominated decelerating Universe at $t = 0.64$. After a slight increment in time, the Universe shows an accelerating recent epoch of the DE issue lying in the quintessence region.

The analysis of models reveals that the present-day Universe is dominated by a DE component, which can successfully describe the accelerating nature of the Universe consistent with the observations [23]. These new forms of models may give equivalent descriptions of DE to discuss the acceleration of the expanding Universe. These models indicate that the present-day Universe is dominated by a DE component. The viability of these models depends on the possible values of parameters. Similarly, we can construct the observational parameters for other new k-essence models induced by modified gravity theory. Finally, it is worthwhile to mention here that all the above results turn out to be the generalization of the already obtained results for the FRW metric [16]. The graphs of models depend on the values of parameters accordingly. For Bianchi type I, the k-essence models in the context of generalized teleparallel gravity mark out a quintessence DE phase.

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