Intrinsically interacting topological crystalline insulators and superconductors

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Motivated by recent progress in crystalline symmetry protected topological (SPT) phases of interacting bosons, we study topological crystalline insulators/superconductors (TCIs) of strongly interacting fermions. We construct a class of intrinsically interacting fermionic TCIs, and show that they are beyond both free-fermion TCIs and bosonic crystalline SPT phases. We also show how these phases can be characterized by symmetry protected gapless fermion modes on the corners/hinges of an open system.

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I. INTRODUCTION

The discovery of topological insulators (TIs) and their counterparts in interacting bosons, symmetry-protected topological states (SPTs), revealed a large class of topological phases with symmetry protected topological boundary states despite a gapped trivial bulk. While SPTs are well understood and classified by K-theory for free fermions and by group cohomology for interacting bosons, less is known about a full classification of interacting fermion SPTs. In a system of interacting fermions, in addition to free-fermion SPTs, it has been found that an even number of fermions can also form a bosonic bound state which in turn forms a bosonic SPT phase. This raises a natural question: are there any interacting fermionic SPTs, which cannot be realized by stacking free fermion SPTs and bosonic SPTs? Recently it has been argued based on braiding statistics that such intrinsically interacting SPTs do exist in two and three dimensions, although it is not clear how to realize them in concrete lattice models of interacting fermions.

In this work, we explicitly construct a class of intrinsically interacting SPTs of fermions, protected by both global (“onsite”) and crystalline symmetries. These phases are coined topological crystalline insulators (TCIs) and superconductors in the context of TIs, hence we will call them “intrinsically interacting TCIs” (for both insulators and superconductors) throughout this work. This work is inspired by recent progress in classifying bosonic SPT phases with both onsite and crystalline symmetries, which points to a dimensional reduction scheme to construct interacting TCIs. In particular, inspired by recent progress on higher-order SPT phases, we show these intrinsically interacting TCIs are characterized by robust fermion modes on the corners/hinges of an open system, which serves as a topological invariant differentiating these interacting TCIs from free-fermion states and bosonic SPTs.

The paper is organized as follows. In Sec. I, we first lay out the general strategy behind the decorated domain wall construction for intrinsically interacting TCIs. Next we explicitly construct 3 examples in two (2d) and three (3d) dimensions in Sec. II and establish that they are neither free fermion nor bosonic SPTs. Finally we discuss limitations of our current construction and future directions in Sec. VII.

II. GENERAL STRATEGY

Before explicitly constructing the interacting TCIs, we outline the general strategy for our construction, and generally argue why these interacting TCIs are beyond either free-fermion TCIs or bosonic SPT phases.

First we review the logic to classify and construct bosonic SPT phases with both onsite ($G_0$) and crystalline ($G_c$) symmetries. All SPT phases protected by both onsite and crystalline symmetries can be constructed by stacking lower-dimensional SPT phases in a pattern that preserves the crystalline symmetry. In this work we will focus on a simpler case where the total symmetry group $G = G_0 \times G_c$ is a direct product of $G_0$ and $G_c$. For this case, the group cohomology classification of bosonic SPT phases can be decomposed using the Künneth formula:

$$\mathcal{H}^{d+1}(G_c \times G_0, U(1)) = \mathcal{H}^{d+1}(G_c, U(1)) \oplus \mathcal{H}^{d+1}(G_0, U(1)) \oplus \bigoplus_{k=0}^{d} \mathcal{H}^k(G_c, \mathcal{H}^{d-k+1}(G_0, U(1))).$$

Each term $\mathcal{H}^k(G_c, \mathcal{H}^{d-k+1}(G_0, U(1)))$ provides a roadmap to construct $d$-dimensional $G$-SPT phases using $(d-k)$-dimensional $G_0$-SPT phases. In particular, not all $(d-k)$-dim. $G_0$-SPT phases are compatible with crystalline symmetry $G_c$; only the compatible ones are elements of cohomology $H^k(G_c, \mathcal{H}^{d-k+1}(G_0, U(1)))$. Moreover, each term of the Künneth expansion can be physically realized using the decorated domain wall picture, allowing an explicit construction.

To be concrete, we consider rotation symmetry $G_c = C_n$ for an example, where SPT phases classified by $\mathcal{H}^1(C_n, \mathcal{H}^d(G_0, U(1)))$ are constructed by stacking $(d-1)$-dimensional $G_0$-SPT phases on the $C_n$ “domain walls” as shown in Fig. I. Since these $G_0$-SPTs intersect at the $C_n$ rotation axis, the $n$ copies of $G_0$-SPT boundary states must be symmetrically gapped out to ensure a trivial gapped bulk. This requires $n$ copies of the $G_0$-SPT...
phase to add up to a trivial phase, a condition captured exactly by group cohomology $\mathcal{H}^1(C_n, \mathcal{H}^d(G_0, U(1)))$.

Although fermion SPT phases are generally beyond the description of group cohomology $\mathbb{H}$, the above dimensional reduction construction based on decorated domain wall picture remains valid. Take $C_n$ symmetry for example, instead of using lower-dimensional bosonic SPT phases, we decorate each $C_n$ domain wall by a $(d-1)$-dimensional fermion $G_0$-SPT phase. In particular if the free-fermion classification for symmetry $G_0$ has an integer classification, no free-fermion TCIs can be obtained by decorating $C_n$ domain walls since $\mathcal{H}^1(C_n, \mathbb{C}^{G_0} \simeq \mathbb{Z}) = 0$. However if interaction reduces the free-fermion integer classification to a finite $\mathbb{C}^{G_0}$, it is possible to gap out the edge states at the rotation axis by interaction since $\mathcal{H}^1(C_n, \mathbb{C}^{G_0} \simeq \mathbb{Z}) = \mathbb{Z}(n,a)$, where $(n,a)$ is the greatest common divisor of integers $n$ and $a$. If the fermion $G_0$-SPT phase on each $C_n$ domain wall cannot be adiabatically tuned into a bosonic $G_0$-SPT phase, we have realized an intrinsically interacting TCI, which is beyond free-fermion TCIs and bosonic SPT phases.

In the following, we will use this logic to construct interacting fermionic TCIs in two (2d) and three (3d) spatial dimensions. They include 3 examples: 2d and 3d interacting fermionic TCIs in two (2d) and three (3d) dimensions. They include 3 examples: 2d and 3d interacting fermionic TCIs in two (2d) and three (3d) dimensions.

III. 2ND-ORDER INTERACTING TCI IN $d = 2$

In the first example, we consider a two-dimensional (2d) TCI of symmetry class AIII, preserving onsite symmetry

$$G_0 = U(1) \times \mathbb{Z}_2^T$$

and point group $G_{2} = C_4$ symmetry. This can be realized either in a superconductor with $U(1)$, spin conservation and time reversal $\mathcal{T}$, a convention we adopt here, or in a TI with an anti-unitary particle-hole symmetry $\mathcal{P}$. However if interaction reduces the free-fermion integer classification to a finite $\mathbb{C}^{G_0}$, it is possible to gap out the edge states at the rotation axis by interaction since $\mathcal{H}^1(C_n, \mathbb{C}^{G_0} \simeq \mathbb{Z}) = \mathbb{Z}(n,a)$, where $(n,a)$ is the greatest common divisor of integers $n$ and $a$. If the fermion $G_0$-SPT phase on each $C_n$ domain wall cannot be adiabatically tuned into a bosonic $G_0$-SPT phase, we have realized an intrinsically interacting TCI, which is beyond free-fermion TCIs and bosonic SPT phases.

In our minimal model, there is a 4-dimensional Hilbert space of spin-1/2 fermions $\{c_{ij}^{\alpha}, \alpha = \uparrow, \downarrow\}$ on each nearest-neighbor (NN) link $\langle ij \rangle$ of the square lattice. In addition to $U(1)S_z$ spin conservation, the time reversal symmetry $\mathcal{T}$ is also preserved

$$c_{ij,\alpha} \xrightarrow{\mathcal{T}} \alpha c_{ij,-\alpha}, \quad \alpha = \pm 1 \text{ for spin } \uparrow / \downarrow$$

Next we reorganize the Hilbert space by writing down a different set of two fermion modes:

$$\gamma_{i,j} = \frac{1}{\sqrt{2}} \left[ c_{ij,\uparrow} + (-1)^i c_{ij,\downarrow}^\dagger \right],$$

$$\gamma_{j,i} = \frac{1}{\sqrt{2}} \left[ c_{ij,\uparrow}^\dagger + (-1)^j c_{ij,\downarrow} \right] \neq \gamma_{i,j}$$

where we defined sign factor $(-1)^j$ for each lattice site

$$(-1)^j \equiv (-1)^{x+y}, \quad i = (x,y), \quad x,y \in \mathbb{Z}.$$
quantum number $s = \pm 1$:

$$L_0 = \sum_{s = \pm 1} \psi_s^\dagger (i\partial_t - s \cdot v\partial_x) \psi_s$$  \hspace{1cm} (12)

$$\psi_s \xrightarrow{e^{i\theta}} e^{-i\theta} \psi_s, \quad \psi_s \xrightarrow{\sigma} s \cdot \psi_s.$$  \hspace{1cm} (13)

It can be easily realized in e.g. Kane-Mele model\cite{36}. The above helical edge modes can be bosonized as

$$\psi_s \sim \eta_s e^{i\phi_s}, \quad [\phi_{s_1}(x), \phi_{s_2}(y)] = i\pi \text{Sgn}(x - y)\delta_{s_1, s_2}$$  \hspace{1cm} (14)

where $\eta_s$ are Klein factors and $\{\phi_s\}$ are chiral bosons.

As shown in Fig. 2 each of the four $C_4$ “domain walls” is decorated by such a 2d TI, where the four helical edge modes $\{\phi_s\}[1 \leq a \leq 4]$ intersect at the $C_4$ rotation axis. The symmetries are implemented as follows:

$$\phi_s \xrightarrow{i\pi Q} \phi_s - \theta,$$  \hspace{1cm} (15)

$$\phi_s \xrightarrow{\sigma} \phi_s + \frac{1 - s}{2}\pi,$$  \hspace{1cm} (16)

$$\phi_s^a \xrightarrow{C_4} \phi_s^{a+1}.$$  \hspace{1cm} (17)

Next we construct a fully symmetric interacting Hamiltonian that gap out these 4 helical edge states on the $C_4$ axis. First we consider

$$\hat{H}_1 = -V_1 \sum_{a=1}^4 \cos(\phi_s^a + \phi_s^{a+2} - \phi_s^{a+2} - \phi_s^a)$$  \hspace{1cm} (18)

which already gaps out four chiral bosons, leaving the gapless modes below

$$\varphi_a \equiv \phi^a_+ - \phi^a_- \sim \phi^a_+ - \phi^{a+2}_+ - \phi^a_- - \phi^{a+2}_-; \quad a = 1, 2;$$  \hspace{1cm} (19)

$$\theta_a \equiv \phi^a_+ - \phi^{a+2}_+ \sim \phi^a_- - \phi^{a+2}_-; \quad a = 1, 2.$$  \hspace{1cm} (20)

which transform under symmetries as

$$\begin{pmatrix} \varphi_1 \\ \theta_1 \end{pmatrix} \xrightarrow{C_4} \begin{pmatrix} \varphi_2 \\ \theta_2 \end{pmatrix} \xrightarrow{C_4} - \begin{pmatrix} \varphi_1 \\ \theta_1 \end{pmatrix}.$$  \hspace{1cm} (21)

Therefore we can write down the following symmetric Hamiltonian

$$\hat{H}_{\text{int}} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2,$$  \hspace{1cm} (22)

$$\hat{H}_2 = -V_2 \left[ \cos(\varphi_1 + \varphi_2) + \cos(\varphi_2 - \varphi_1) \right]$$

where $\hat{H}_0$ is a free-fermion lattice model realizing the Kane-Mele model on each $C_4$ domain wall. $\hat{H}_{\text{int}}$ symmetrically gaps out all the helical edge modes on the $C_4$ axis, leading to a TCI ground state. Meanwhile notice that this is a 2nd-order fermion SPT phase characterized by 4 gapless $\nu = 1$ helical hinge modes in an open system, which are robust against any weak symmetric perturbations.

To see that this TCI cannot be realized by a free-fermion Hamiltonian, we compactify the open system on a three torus, where the 4 gapless hinge modes are joined

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{2nd-order 3d interacting TCI with onsite $G_0 = U(1)_{\text{charge}} \times Z_2^\sigma$ and $C_4$ rotation symmetry. It is constructed by decorating each $C_4$ domain wall by a 2d fermionic TI with a pair of helical edge states protected by $G_0 = U(1)_{\text{charge}} \times Z_2^\sigma$ symmetry. It is characterized by helical hinge states on each of the 4 hinges of an open system.}
\end{figure}
together after compactification. Within free fermion Hamiltonians, there is no way to symmetrically gap out these 4 helical modes due to the \( \nu \in \mathbb{Z} \) classification of free fermions with symmetry [11]. However as shown in Ref. [16, 27, 29], interactions reduce the free-fermion \( \mathbb{Z}_2 \) classification to a finite \( \mathbb{T}_2 \) classification, allows 4 copies of helical edge states to gap out while preserving symmetry [11]. This reveals why the \( \nu = 1 \) 2d TI with onsite symmetry [11] is compatible with \( C_4 \) rotational symmetry to construct this interacting 2nd-order TCI in 3d.

Finally we show that this interacting TCI is not a bosonic SPT phase. As shown in Ref. [26, 27] 2nd-order 3d bosonic TCI phases with symmetry group \( G = C_4 \times G_0 \) are classified by

\[
\mathbb{Z}_2^2 = \mathcal{H}^i(\mathcal{H}^d(G_0, U(1))) \cong \mathcal{H}^i(\mathbb{Z}_4, \mathbb{Z} \times \mathbb{Z}_2^2) \tag{23}
\]

They are characterized by gapless 1d hinge states which are edge modes of 2d bosonic \( G_0 \)-SPT phases. Since helical edge modes [12] cannot be realized in any 2d bosonic SPT phases [16], the ground state of model [22] cannot be a boson SPT phase. Therefore we have shown that model [22] realizes an intrinsically interacting TCI of fermions.

V. 3RD-ORDER INTERACTING TCI IN \( d = 3 \)

Lastly we consider 3d superconductors in symmetry class BDI, with time reversal symmetry \( G_0 = \mathbb{Z}_2^T \) satisfying \( T^2 = +1 \). Below we construct an intrinsically interacting TCI with both time reversal and pyritohedral point group symmetry \( G_c = T_h \).

We consider a lattice model of spinless fermions, where one single fermion mode \( c_{\langle i,j \rangle} \) lives at the center of each nearest-neighbor (NN) link \( \langle i, j \rangle \) of the body-centered cubic (BCC) lattice:

\[
\mathcal{Z}_i = \mathcal{H}^i(C_4, \mathcal{H}^d(G_0, U(1))) = \mathcal{H}^i(\mathbb{Z}_4, \mathbb{Z} \times \mathbb{Z}_2^2)
\]

They are characterized by gapless 1d hinge states which are edge modes of 2d bosonic \( G_0 \)-SPT phases. Since helical edge modes [12] cannot be realized in any 2d bosonic SPT phases [16], the ground state of model [22] cannot be a boson SPT phase. Therefore we have shown that model [22] realizes an intrinsically interacting TCI of fermions.

In Fig. 3 we label the eight NNs of an even site \( i \) at the cube center (also the inversion center) as \( 1 \leq j \leq 8 \), and the 3-fold \( R_3 \) axis crosses sites 1, 5 and cube center \( i \). The eight Majoranas \( \{ \chi_{i,j} | 1 \leq j \leq 8 \} \) living on site \( i \) transform under inversion as

\[
\chi_{i,a} \stackrel{T}{\rightarrow} \chi_{i,a+4}, \quad 1 \leq a \leq 4.
\]

In Fig. 3 we color 1 \( \leq j \leq 4 \) in red and 5 \( \leq j \leq 8 \) in green. Under 3-fold and 2-fold rotations the 4 red Majoranas transform as

\[
\chi_{i,1} \overset{R_3}{\rightarrow} \chi_{i,1}, \quad \chi_{i,2} \overset{R_3}{\rightarrow} \chi_{i,3} \overset{R_3}{\rightarrow} \chi_{i,4} \overset{R_3}{\rightarrow} \chi_{i,2}, \tag{30}
\]

\[
\chi_{i,1} \overset{R_2}{\rightarrow} \chi_{i,3}, \quad \chi_{i,2} \overset{R_2}{\rightarrow} \chi_{i,4} \tag{31}
\]

and similarly for the 4 green ones.

The fermion TCI is obtained by the following symmetric interacting Hamiltonian

\[
\tilde{H}^{3d}_{T_h, BDI} = \sum_i \tilde{H}(i), \quad \tilde{H}(i) = U(\chi_{i,1}\chi_{i,2}\chi_{i,3}\chi_{i,4} + \chi_{i,5}\chi_{i,6}\chi_{i,7}\chi_{i,8}) + J \tilde{P}_U \chi_{i,1}\chi_{i,5} \left( \chi_{i,2}\chi_{i,6} + \chi_{i,3}\chi_{i,7} + \chi_{i,4}\chi_{i,8} \right) \tilde{P}_U,
\]

\[
U > 0, \quad 1 \leq j \leq 8 \in NN(i) \text{ see Fig. 3}
\]

When projected (by projector \( \tilde{P}_U \)) into the ground state manifold \( \chi_{i,1}\chi_{i,2}\chi_{i,3}\chi_{i,4} = \chi_{i,5}\chi_{i,6}\chi_{i,7}\chi_{i,8} = -1 \) of the \( U \) term, the \( J \) term is nothing but an antiferromagnetic Heisenberg interaction between the effective spin-1/2 of
4 red Majoranas and the spin-1/2 of 4 green Majoranas. Therefore Hamiltonian (32) has a unique gapped ground state which preserves all symmetries.

Below we show this ground state of (32) is an intrinsically interacting TCI. First as illustrated in Fig. 3 there will be a single Majorana zero mode (MZM) at each corner of a cubic-shaped open system, which is robust against any perturbations. 3d bosonic $G = Z_2^c \times T_h^2$-protected SPT phases with robust corner states classified in Ref. [29] using group cohomology formula

$$\mathcal{H}^2(T_h^2, \mathcal{H}^1(Z_2^c, U(1))) = Z_2^3$$

(33)

They are constructed by decorating 3-fold and 2-fold rotational axes by 1d $Z_2^c$-protected Haldane chain, characterized by a Kramers doublet (i.e. spin-1/2) at each corner. Therefore our model (32) hosting Majorana corner modes is clearly beyond bosonic SPT phases. Meanwhile as shown in [25], the 8 corner MZMs $\{\chi_{\nu}|_{\nu \leq 8}\}$ of the open system are all even (or odd) under time reversal symmetry $T$. Now let us glue the open boundary of the finite cubic-shaped system into a closed system with the periodic boundary conditions. In particular, the 8 MZMs must be gapped out symmetrically to recover a gapped bulk. However, any bilinear coupling $i\chi_{\nu}\chi_{\nu}$ is forbidden by time reversal symmetry. Therefore it is impossible to recover a gapped bulk within the space of free-fermion Hamiltonians, and we have proved by contradiction that the ground state of model (32) cannot be adiabatically connected to any free fermion Hamiltonian without closing the gap.

Therefore model (32) realizes neither a free-fermion TCI nor a bosonic SPT phase, but an intrinsically interacting TCI of fermions. Pictorially, this interacting TCI is constructed by decorating each of the 8 $R_3$ axes in Fig. 3 by a $\nu = 1$ Kitaev chain [10] in symmetry class BDI (with $T^2 = +1$). The 8 Kitaev chains terminate and intersect at the inversion center, giving rise to 8 MZMs at each site $i$. As shown by Fidkowski and Kitaev [11], interactions reduce the free-fermion integer classification $\nu \in Z$ of 1d class BDI to a $\mathbb{Z}_8$ classification, where 8 MZMs with time reversal symmetry (25) can be gapped out symmetrically. This is exactly what model (32) achieved.

VI. DISCUSSIONS

To summarize, we provide a general construction for intrinsically interacting TCIs of fermions, which are beyond the description of either free-fermion Hamiltonians and interacting boson SPT phases. We use three explicit examples in two and three dimensions to demonstrate this construction, and show that these interacting TCIs are often characterized by robust fermion modes on corners/hinges of an open system.

Generally in a fermion system, the fermion symmetry group $G_f$ is an central extension of the physical symmetry group $G$ by the fermion parity $Z_2^f \equiv \{1, (-1)^{\hat{f}}\}$, i.e. $G_f/Z_2^f = G$. In all 3 examples we presented, $G_f$ has a direct-product form:

$$G_f = G \times Z_2^f = G_0 \times G_c \times Z_2^f.$$  (34)

However this is not always the case for a generic TCI in the decorated domain wall construction. For example in Fig. 1 once we replace the $\nu = 1$ TCI in class AIII on each $C_4$ domain wall by a 1d Kitaev chain, it is impossible to gap out the 4 MZMs at $C_4$ center if $(C_4)^4 = 1$, since the $C_4$ operation will change the total fermion parity of the 4 MZMs. Meanwhile, a nontrivial extension by fermion parity $(C_4)^4 = (-1)^{\hat{f}}$ is compatible with a gapped bulk. Though not discussed in our work, such a nontrivial interplay between onsite and crystalline symmetries can be important to understand a full classification of fermionic SPT phases with both onsite and crystalline symmetries.

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Upon completion of this work, we became aware of a related work by Meng Cheng and Chenjie Wang who classified fermionic SPT phases with rotational symmetry. Their work will appear on arXiv on the same date with our work.

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