Impact of the variable stiffness section on the conditions of track and rolling stock interaction

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Abstract. Railway track stiffness is one of the main parameters that determine the track and rolling stock interaction. The inelasticity of the rail base may occur for two reasons: either as a consequence of the deteriorated condition of a track or due to the structural features of a section. As a rule, areas before bridges or tunnels are treated. Today, there are several options for relevant design solutions. The main purpose of this work is to determine the characteristics of a "railway track" object with which it can be represented in the rolling stock models in the simplest way possible but adequate for sections of transient stiffness. The railway track is introduced into the rolling stock model as a sequence of elements with which the wheels of the rolling stock interact. It is assumed that a single element in its characteristics must be equivalent to the track response when passing the wheel from one inter-sleeper section to the next one, that is, to reproduce a sequence that is cyclically repeated. Such a track element will be characterized by reduced mass, stiffness and dissipation factor.

1. Introduction

Railway track stiffness is one of the main parameters that determine the track and rolling stock interaction. Moreover, it is important not only and not so much the value of stiffness but its uniformity.

The inelasticity of the rail base may occur for two reasons: either as a consequence of the deteriorated condition of a track or due to the structural features of a section. In the first case, the equal elasticity is established after the repair work, which may be accompanied by the use of reinforcing structures \cite{1}, eliminating or weakening the effect of factors that led to the deterioration of the condition. These factors include, first of all, the loss of elasticity of the ballast layer due to the destruction of its parts \cite{2, 3}, ingress of pollutants \cite{4} or the lack of compactness \cite{5}, increase in a load of freight trains \cite{6} and the speed of passenger traffic \cite{7}. The second case is, as a rule, short distances, the stiffness of which is determined by the design: a zone of rail joints \cite{8, 9}, crossings, artificial structures such as tunnels, bridges \cite{10, 11}.

With the time of operation, an abrupt change in stiffness leads to the intensive development and accumulation of vertical deformations at the approach to such a place. To prevent this process, special transition sections with variable stiffness can be equipped.
Today, there are several options for relevant design solutions. Their review and analysis are given in [12-17] and others. As a rule, areas before bridges or tunnels are treated.

There are a number of tasks that require mathematical simulation of the rolling stock passage along such a section. In general, to study the operation of rolling stock and a railway track, as a rule, not the same mathematical models are used. The main difference is to reduce the detailing of one or the other part. It is clear that not only oversimplification but also unnecessary detailing has an adverse effect on the adequacy of the results. In addition, the extension of the mathematical model requires additional output data and can significantly increase the requirements for time and characteristics of computing equipment.

If several fundamentally different approaches are used for models of a railway track, then for mathematical models of rolling stock in almost all cases the calculation of Lagrange-D’Alembert system of equations is the basic one. Two types of models are increasingly applying sequentially for the complex analysis. First, we simulate the motion of rolling stock with a simplified track representation. As a result, the dynamic forces acting from the wheels to the rails are determined. Then the track operation is simulated with a simplified representation of the rolling stock, which is reduced to a system of forces applied to the rail. As a result, the stress-strain state of the track is determined according to the load.

Thus, the main purpose of this work is to determine the characteristics of a "railway track" object with which it can be represented in the rolling stock models in the simplest way possible but adequate for sections of variable (transient) stiffness.

2. Methodology

The railway track is introduced into the rolling stock model as a sequence of elements with which the wheels of the rolling stock interact, Figure 1. It is assumed that a single element in its characteristics must be equivalent to the track response when passing the wheel from one inter-sleeper section to the next one, that is, to reproduce a sequence that is cyclically repeated. Such a track element will be characterized by reduced mass, stiffness and dissipation factor. Its vertical oscillations are described by a differential equation:

\[ m \frac{d^2z}{dt^2} + \beta \frac{dz}{dt} + k_z = P(t) , \]

where \( m \) – reduced mass of the track element; \( z \) – vertical deflection; \( \beta \) – dissipation factor; \( k \) – stiffness; \( P \) – vertical force; \( t \) – time factor.

Figure 1. Design diagram of the track representation in the rolling stock model.

Equation (1) can be represented in the form:
The unknown characteristics of the element \((m, k, \beta)\) can be determined as a result of approximation by the function (2) of the results in determining the dynamic deflections of the track in the cross-section of this element from the active force. A more detailed technique for such an approximation is given in [18].

To determine the deflections, a spatial model of dynamic deformations for the railway track was applied based on the elasticity theory [19]. The basis of this model is a mathematical description of the elastic wave propagation of stresses (deformations) in the space of a railway track from the applied load. The result of the simulation is the stress-strain state of the track, depending on the time of the rolling stock passage. This approach corresponds to the physics of the process in the railway track operation as a system of objects that responds to the load by elastic deformations. The input data are the geometric dimensions of the objects (or layers) and their physical properties, such as Young's modulus, density, Poisson's ratio. Determination of the velocities for the elastic wave propagation takes into account the dynamics of the deformation process, namely, it determines the part of the rail base that is involved in the interaction at this moment in time. It provides an opportunity to determine the desired parameters \((m, k, \beta)\) as a generalized result of the system.

3. Results
In this paper, we consider the design of a section with variable stiffness, which is shown in Figure 2. The section can be divided into four zones: 1 – a ballast track; 2 – a transition zone with variable stiffness; 3 – a transition zone with constant (increased) stiffness; 4 – a superhard zone (for example, a bridge). The design of the transition section and its parameters correspond to the recommendations given in works [13, 14].

![Figure 2. Design of the transition section with variable stiffness.](image)

The results of variant simulations are shown in Figure 3 on the example of maximum stresses along the length and depth of the rail base from passing a passenger car at a speed of 160 km/h.
Figure 3. Maximum stresses spectrum.

Figure 4 shows examples of dynamic track deflection for several sections (it is indicated by the numbers of sleepers as in Figure 2) for passing the wheel in the interval ± 0.27 m from the axis of the corresponding sleeper, which corresponds to the sleeper density of 1840 pieces/km. When approaching the vertical acting force (wheel), the intensity of the deformation change is greater (a lower part of the dependency on the graph) than at distant forces (an upper part of the graph).

Similar results were obtained for other cross-sections of the track. Dynamic deflections of the rail (a passenger car, speed of 160 km/h) along the length of the section are shown in Figure 5.

According to the approximation of the calculation data, the function (2) defines the stiffness characteristics of the track as an object for the rolling stock model. The change in the modulus of deformation of the railway track along the section is shown in Figure 6. The calculated values of the dynamic stiffness and the dissipation factor for the support in accordance with the sleepers numbers
are shown in Figure 7. For the practical application, one can use the linear equations of the dependencies of these parameters on the number of a sleeper in accordance with Figure 2.

As conducted studies have shown, the "railway track" system did not show the presence of mass as an indicator of inertia. This is, first of all, due to the significant advantage of the unsprung mass of rolling stock which presses on the track through the wheel over the mass of the track which has interacted with the wheel. Secondly, the significant manifestations of the track inertia can be expected only at high speed of movement, which will be comparable with the spreading speed of the track deflection [20].

4. Conclusions
In this paper, we consider the transition section before the bridge as a design of a railway track with variable stiffness. Adopted characteristics of the ballast track (reinforced-concrete sleepers, crushed ballast with the modulus of deformation 200 MPa, thickness 0.5 m, soil with the modulus of deformation 35 MPa) correspond to the railway track operation as a whole with the modulus of deformation 52 MPa. The arrangement of the transition section for the considered design (see Figure 2) provides an almost linear (see Figure 6) increase of the modulus of deformation for the track up to 62 MPa – by 25%.

Moreover, in the area just before the bridge, the ballast layer is 20 cm. At the ballast thickness of 15 cm (due to a further increase in the thickness of the reinforcing layer), the total modulus of deformation of the railway track will increase to 68 MPa – by 30% compared to the primary ballast section. When using additional facilities (for example, placing a layer of reinforced ballast before the
roadbed), specified indicators can be improved. In addition, one should pay attention to the characteristics of the roadbed. For example, reducing the soil deformation modulus up to 20 MPa will decrease the total modulus of deformation of a track from 52 to 32 MPa.

$$k = 0.11x + 102$$

$$\beta = 0.16x + 106$$

![Graph](image)

**Figure 7.** Changing the characteristics of point support on the section of variable stiffness.

To introduce the considered transition section of the railway track in the rolling stock model in the form of separate supports, their numerical characteristics are established. Thus, for a zone with a variable modulus of deformation (see Figure 2), the dynamic stiffness of such support along the length of the section will vary from 92.4 to 108.7 MN/m, and the dissipation factor from 103 to 115 kN s/m. Within the considered speeds of movement (up to 160 km/h) the need to take into account the reduced mass of the track has not been established. But its introduction may take a form of a conditional parameter required by the structure of the rolling stock model to divide the system into separate objects.

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