Interface-Flattening Transform for EM Field Modeling in Tilted, Cylindrically-Stratified Geophysical Media

Kamalesh Sainath and Fernando L. Teixeira

Abstract—We propose and investigate an “interface-flattening” transformation, hinging upon Transformation Optics (T.O.) techniques, to facilitate the rigorous analysis of electromagnetic (EM) fields radiated by sources embedded in tilted, cylindrically-layered geophysical media. Our method addresses the major challenge in such problems of appropriately approximating the domain boundaries in the computational model while, in a full-wave manner, predicting the effects of tilting in the layers. When incorporated into standard pseudo-analytical algorithms, moreover, the proposed method is quite robust, as it is not limited by absorption, anisotropy, and/or eccentering profile of the cylindrical geophysical formations, nor is it limited by the radiation frequency. These attributes of the proposed method are in contrast to past analysis methods for tilted-layer media that often place limitations on the source and medium characteristics. Through analytical derivations as well as a preliminary numerical investigation, we analyze and discuss the method’s strengths and limitations.

Index Terms—Borehole geophysics, electromagnetic analysis, well-logging, stratified media, transformation optics.

I. INTRODUCTION

Many applications demand the numerical evaluation of electromagnetic (EM) fields produced by radiators embedded in complex, inhomogeneous environments [1]. For example, robust (i.e., with respect to environment and source characteristics) computational modeling of EM sensors operating in complex geological formations is necessary both to better understand the effects of the nearby Earth formation inhomogeneity profile and constitutive properties [2] on the sensor’s response (“forward modeling”) and to facilitate resistivity profile inversion (e.g., to assess a formation’s hydrocarbon productivity potential) [2]. The repeated use of the numerical modeler as the forward engine in many inversion methods, combined with the electrically large size of many geophysical problems, requires a fast computational scheme that can rigorously model EM phenomena arising from the subsurface environment’s dominant geophysical features.

Bridging the gap between robustness and solution speed, a layered-medium geophysical model is commonly employed. In particular, cylindrically-layered environments characterized by parallel, vertically-oriented (i.e., along z) interfaces arise quite frequently [1], [3]. This layered-medium approximation is oftentimes motivated (when this domain approximation, near the sensor at least, is approximately valid) because in such media one can employ pseudo-analytical methods based upon cylindrical eigenfunctions expansions. These expansions are particularly attractive since they can rigorously account for eccentered, anisotropic, and/or absorptive geophysical media, as well as a broad range of radiation frequencies, with ease [1], [3]–[5]. However, it is worthwhile asking whether (and how) one can extend upon the types of media (with respect to, for example, spatial inhomogeneity structure) that can be rigorously modeled using these cylinder wave expansions. In particular, removal of the vertical, parallel-layer modeling constraint, which would facilitate modeling of the presence and effects of tilted layered media, offers the distinct opportunity to investigate novel EM wave propagation and scattering phenomena that can aid interpretation of collected EM subsurface sensor data. The presence of relative vertical layer tilt can arise, for example, due to mechanical effects or gravitational pull effects either on the tool itself or on the drilling fluid (mud filtrate) invasion zone in the course of deviated drilling [5]. However, rigorous modeling of wave propagation and scattering within these types of media, without having to place undesirable constraints on the frequency range, source distribution, and material properties is encumbered by a challenge on appropriately approximating the boundaries comprising tilted layers.

To overcome this challenge and enable pseudo-analytical-based modeling of tilted, cylindrically-layered media, we propose and explore a strategy based on Transformation Optics (T.O.) [6]–[12] to obtain material blueprints of annular slabs to coat each of the cylindrical interfaces in a transformed, standard problem which contains only vertically-oriented, parallel interfaces. This interface flattening is carried out by defining a coordinate mesh/metric deformation followed by incorporating the metric deformation into the ambient material properties of the region originally subject to metric stretching [11], [12]. In particular, through redirecting the wave field’s power flow (Poynting vector) the derived T.O. coating slabs cause EM waves impinging upon the vertical, parallel interfaces to reflect off and transmit through the coated interfaces.

1These constraints are imposed, for example, in previous methods analyzing EM waves in tilted, planar-layered media [6], [7].

2For example, c.f. the two intermediate layers in Figs. 2c–2d.
interfaces as if these interfaces were, in fact, tilted. Once the equivalent problem is obtained, one can then employ standard analytical tools to calculate the fields radiated in a multi-eccentered, cylindrically-stratified medium \( [1], [4], [5] \).

Fig. 1: Illustration of a tilted, cylindrically-layered formation. Modeled tilting for the \( m \)th layer \( (m \neq N) \) is described by effective polar and azimuth tilt orientation angles \( \alpha_m \) and \( \beta_m \) (resp.) of this \( m \)th layer’s central axis (black dashed lines in Fig. 1b) \( [2] \). For simplicity of illustration, \( \beta_1 = \beta_2 = \beta_3 = 0^\circ \) (i.e., all tilting is parallel to the \( xz \) plane).

II. ORIGINAL AND TRANSFORMED DOMAINS

Assume \( N \) cylindrical layers and \( N = 1 \) interfaces, with layers 1 and \( N \) (resp.) referring to the inner-most and outer-most layers \( [3] \). Furthermore, the \( m \)th cylindrical formation (for \( m \neq N \)) has its exterior interface located at \( \rho_m = \sqrt{(x_m')^2 + (y_m')^2} \) meters from its local origin (c.f. footnote \( 4 \)). See Figs. 2a-2b and 1b for an illustration of a typical four-layered geometry at the (resp.) \( xy \) plane view (illustrating layer eccentricity) and \( xz \) plane view (illustrating relative tilting between the three cylindrical interfaces). To model problems with relative interface tilting but employing only vertically-oriented, parallel cylindrical formations, we propose here the use of annular “coating slabs” on the immediate exterior and immediate interior interfaces surrounding each vertically-oriented interface. Taking the simple example of a two-layer medium, this can be seen as transforming the original domain (c.f. Figs. 2a-2b) to one of vertical, parallel interfaces (c.f. Figs. 3a-3b).

3To visualize better the power flow redirection mechanism, see for example Figs. 3a-3b in \( [13] \) (illustrated therein, however, for the planar analog of our proposed method, which by contrast to \( [13] \) addresses the cylindrical case).

4To allow for eccentricity in the formations (i.e., the cylindrical layers not sharing the same transverse center), note that we address all aspects of the theoretical derivation of our method with respect to a particular formation’s “local” coordinates. Indeed, we use “global” coordinates \( (x, y, z) \) to parameterize the relative positioning of the cylindrical layers, while formation \( m \) (\( m = 1, 2, \ldots, N \)) has “local” coordinates \( (x_m, y_m, z_m) \) with origin \( x_m = y_m = 0 \) at the \( m \)th cylinder’s center \( (x = x_m, y = y_m, z = 0) \).

III. FORMULATION OVERVIEW

We now outline the mathematical derivation of the T.O.-inspired material tensors for the coating slab. First let us examine the \( m \)th vertical cylinder in its local coordinate system \( (x_m, y_m, z_m) \) with interface located at \( \rho_m' \), and assume we wish waves to scatter off this interface as if its central axis were oriented within the \( x_m-z_m \) plane by polar angle \( \alpha_m \) and azimuthal angle \( \beta_m = 0^\circ \) (i.e., as if the central axis were tilted to point in the direction \( \hat{x} \sin \alpha_m + \hat{z} \cos \alpha_m \)). Now define the following coordinate transformation, applied to the annular slab region \( \{\rho_m' - d \leq \rho_m \leq \rho_m' + d\} \),

\[
\begin{align*}
\tilde{x}_m &= x_m, \\
\tilde{y}_m &= y_m, \\
\tilde{z}_m &= z_m + (x_m' - d) \tan \alpha_m
\end{align*}
\]

(III.1)

that redirects the incident field’s power flow such that it illuminates the vertically-oriented cylindrical interface as if it had the desired tilting described above. This coordinate transformation (III.1), prescribed only in the \( 2d \) meter thick annular vicinity of interface \( m \), induces a local (effective) distortion of the spatial metric tensor \( g_m \) and associated Jacobian matrix \( \Lambda_m \) in particular, \( \Lambda_m \) writes as follows (III.2):

\[
\Lambda_m = \begin{bmatrix}
\frac{\partial \tilde{x}_m}{\partial x_m} & \frac{\partial \tilde{x}_m}{\partial y_m} & \frac{\partial \tilde{x}_m}{\partial z_m} \\
\frac{\partial \tilde{y}_m}{\partial x_m} & \frac{\partial \tilde{y}_m}{\partial y_m} & \frac{\partial \tilde{y}_m}{\partial z_m} \\
\frac{\partial \tilde{z}_m}{\partial x_m} & \frac{\partial \tilde{z}_m}{\partial y_m} & \frac{\partial \tilde{z}_m}{\partial z_m}
\end{bmatrix} = \begin{bmatrix} 1 & 0 & \tan \alpha_m \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

(III.2)

5That is, distorted with respect to undistorted/flat space containing vertical, parallel interfaces.
with the inverse metric tensor writing as \( g^{-1} = \Lambda^T \cdot \Lambda \). To effect computation of Maxwellian fields, as usual one then incorporates the effects of the coordinate transform (embedded within \( g_{mn} \)) into the material tensors of the annular region, surrounding the \( m \)th interface, subject to the coordinate transform.

As an example of incorporating the induced metric tensor into the material properties, for simplicity of illustration assume a two-layer cylindrically-stratified medium (see Figs. 2[3][4]). Letting the inner and outer layers (resp.) initially be uniformly characterized (c.f. Figs. 2a[2b]) by the dielectric, permeability, and conductivity material tensors \( \{\varepsilon_1, \mu_1, \sigma_1\} \) and \( \{\varepsilon_2, \mu_2, \sigma_2\} \), and letting \( \bar{\tau}_p = \bar{\Lambda}^T \cdot \bar{\tau}_p \cdot \Lambda_1 \) (for \( y=1,2; \tau = \{\varepsilon, \mu, \sigma\} \)) represent the corresponding material tensors of the interface-coating annular slabs after incorporation of \( g_1 \), one has the following inhomogeneity profile:

\[
\begin{align*}
\{\varepsilon_1, \mu_1, \sigma_1\}, & \quad 0 \leq \rho_1 < (\rho_1' - d) \quad (\text{III.3}) \\
\{\varepsilon_1', \mu_1, \sigma_1'\}, & \quad (\rho_1' - d) \leq \rho_1 < \rho_1' \quad (\text{III.4}) \\
\{\varepsilon_2, \mu_2', \sigma_2'\}, & \quad \rho_1' \leq \rho_1 < (\rho_1'' + d) \quad (\text{III.5}) \\
\{\varepsilon_2, \mu_2', \sigma_2''\}, & \quad (\rho_1'' + d) \leq \rho_1 < \infty \quad (\text{III.6})
\end{align*}
\]

The extension to multiple layers follows naturally, as does application of this method to eccentered layers, by performing the transformation around each vertically-oriented interface. Furthermore, recalling the matrix transforming a vector’s representation from the cartesian to cylindrical coordinate system ¹

\[
U_m = \begin{bmatrix}
\cos \phi_m & \sin \phi_m & 0 \\
-\sin \phi_m & \cos \phi_m & 0 \\
0 & 0 & 1
\end{bmatrix} \quad (\text{III.7})
\]

one has the corresponding cylindrical coordinate representation of the T.O. material tensors \( \gamma'_p = U_m \cdot \bar{\tau}'_p \cdot U^{-1}_m \) \((\gamma, \tau = \varepsilon, \mu, \sigma; m = 1, 2, ..., N-1; p = m, m+1)\) defined with respect to their cartesian representation counterparts \( \{\bar{\tau}'_p\} \). Note that each cylindrical layer, in the new (approximately) equivalent problem consisting of vertically-oriented cylindrical formations with parallel interfaces, is homogeneous (with respect to cartesian coordinates) due to the homogeneous nature of the effective metric distortion. This enables simulations using a standard cylindrical-layer algorithm admitting generally anisotropic, eccentered layers without having to discretize and/or otherwise approximate the coating slab’s inhomogeneous material profile.

A drawback in the discussed coordinate transform is that it does not result in the prescription of exactly reflectionless T.O. media. From a physical standpoint, to ensure a reflectionless T.O. medium one must not perturb tangential field continuity at the interface adjoining the T.O. medium to the external/ambient medium. Mathematically speaking, one can indeed deform all three coordinates (i.e., both those tangential and normal to the interface) but (in the cartesian case, for example) a necessary condition for reflectionless behavior is

That is, \( \Lambda_m \) within the \( m \)th interface’s annular coating slab region is independent of the cartesian spatial coordinates.

That is, a slab immediately exterior to, and one slab immediately interior to, the interface at \( \rho = 0.75m \). The \( \rho = 0.75m \) interface is shown as the solid back circle separating the two layers, marked as “2” and “3”, in Fig. 5.
Observing the co-polarized field distribution, Fig. 3e suggests on the order of 0.01% (or −4 on the log scale) to 1% (or −2 on the log scale) change in $E_z$. Furthermore, we notice that near the source, and particularly in the “forward” region emanating from the source into the $\phi = 0^\circ$ direction, not only are the cross-polarized fields minimized but there is relative variation in $E_z$ on the order of 0.1%. Based on these two observations we conclude that this local variation in $E_z$ versus $\alpha$, within the “forward” region, is primarily the result of interface tilting and not artificial T.O.-based modeling “noise”. These observed features are also of interest for suggesting a potential route to retrieve the (local) direction, and magnitude, of tilt observed on a given section of a logging tool by monitoring such (local) azimuthal field variations.

V. CONCLUSIONS

We proposed a new computational modeling strategy to model EM fields radiated in tilted cylindrically-stratified media. The proposed approach incorporates cylindrical interface tilting through the use of T.O.-prescribed artificial layers. The spatial homogeneity of the T.O.-derived media, which facilitates their inclusion into standard pseudo-analytical algorithms that handle multilayered cylindrical media, as well as their explicit material tensor representations, were mathematically exhibited, and the media were incorporated into 2-D numerical simulations to characterize their behavior. The results favorably suggests its further study as a tool to characterize interface tilting in geophysical borehole sensing.

ACKNOWLEDGMENT

The authors acknowledge Mr. Justin Burr and Mr. Zeeshan Zeeshan of the OSU/ESL for assistance in producing the numerical results.

REFERENCES

[1] W. C. Chew, Waves and Fields in Inhomogeneous Media. Van Nostrand Reinhold, 1990.
[2] B. I. Anderson, T. D. Barber, and S. C. Gianzero, “The Effect of Crossbedding Anisotropy on Induction Tool Response,” in SPWLA 39th Annual Logging Symposium. Houston, TX, USA: Society of Petrophysicists and Well-Log Analysts, May 1998, pp. 1–14.
[3] H. Moon, F. L. Teixeira, and B. Donderici, “Stable Pseudoanalytical Computation of Electromagnetic Fields from Arbitrarily-Oriented Dipoles in Cylindrically Stratified Media,” Journal of Computational Physics, vol. 273, pp. 118 – 142, 2014.
[4] G.-S. Liu, F. Teixeira, and G.-J. Zhang, “Analysis of Directional Logging Tools in Anisotropic and Multielectriccylindrical Cylindrically-Layered Earth Formations,” IEEE Transactions on Antennas and Propagation, vol. 60, no. 1, pp. 318–327, Jan 2012.
[5] Y.-K. Hue and F. L. Teixeira, “Analysis of Tilted-Coil Eccentric Borehole Antennas in Cylindrical Multilayered Formations for Well-logging Applications,” IEEE Transactions on Antennas and Propagation, vol. 54, no. 4, pp. 1058–1064, April 2006.
[6] G. A. Schlak and J. R. Wait, “Electromagnetic Wave Propagation over a Nonparallel Stratified Conducting Medium,” Canadian Journal of Physics, vol. 45, no. 11, pp. 3697–3720, 1967.
[7] S. Zhang, H. Zhong, D. Asoubar, F. Wyrowski, and M. Kuhn, “Tilt Operator for Electromagnetic Fields and its Application to Propagation through Plane Interfaces,” Proc. SPIE, vol. 8550, pp. 85 503I–85 503I–11, 2013.
[8] J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling electromagnetic fields,” Science, vol. 312, no. 5781, pp. 1780–1782, 2006.
[9] O. Ozgun and M. Kuzuoglu, “Electromagnetic metamorphosis: Reshaping scatterers via conformal anisotropic metamaterial coatings,” Microwave and Optical Technology Letters, vol. 49, no. 10, pp. 2386–2392, 2007.
[10] F. L. Teixeira, “Closed-form metamaterial blueprints for electromagnetic masking of arbitrarily shaped convex pec objects,” IEEE Antennas and Wireless Propagation Letters, vol. 6, pp. 163–164, 2007.
[11] D.-H. Kwon and D. H. Werner, “Transformation Optical Designs for Wave Collimators, Flat Lenses and Right-Angle Bends,” New Journal of Physics, vol. 10, no. 11, p. 115023, 2008.
[12] F. L. Teixeira and W. C. Chew, “Differential Forms, Metrics, and the Reflectionless Absorption of Electromagnetic Waves,” J. Electromagn. Waves Applicat., vol. 13, pp. 665–686, 1999.
[13] M. Rahm, S. A. Cummer, D. Schurig, J. B. Pendry, and D. R. Smith, “Optical Design of Reflectionless Complex Media by Finite Embedded Coordinate Transformations,” Physical Review Letters, vol. 100, p. 063903, Feb 2008.