Properties of high-quality coplanar waveguide resonators for QIP and detector applications

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Abstract. We have fabricated and characterized λ/2 thin-film coplanar waveguide resonators made from niobium deposited on sapphire and SiO₂/Si substrates. The samples have been characterized at temperatures down to 30 mK. In this work we discuss two important properties of these resonators: The tunability of the centre frequency using a weak applied magnetic field, and the RF-power dependence of the quality factor.

1. Introduction
Superconducting coplanar waveguide resonators (CPR) are becoming increasingly important in a number of novel applications, ranging from quantum information processing (QIP) [1, 2] to photon detection [3]. These new applications require them to be operated at mK temperatures and very low power. In this regime new effects become important that are not taken into account in the conventional theory. CPR operated at mK temperatures act as very sensitive probes to changes in their environment and very small changes in the properties of the superconducting film, dielectric etc, will result in a shift of the centre frequency (f₀) and phase of the resonator which can be read out by measuring either the transmitted or reflected power of the circuit. This sensitivity makes them suitable as very sensitive photon detectors. CPRs are also used in circuit quantum electrodynamics (CQED) where, for example, they can be used to transfer information between qubits [4] and as parts of circuits capable of generating single photons [5].

2. Experimental
The CPRs were fabricated from 200 nm niobium deposited on R-cut sapphire and on thermally oxidized Si substrates. Patterning was performed using conventional photolithography and argon ion beam milling. The resonators are 11 mm long yielding a λ/2 resonance frequency of 6.0 GHz on SiO₂/Si and 5.7 GHz on sapphire. The coupling was adjusted by changing the size of the gap between the inner conductor of the resonator and the feed lines. All results presented here come from devices where the inner conductor is 10 µm wide and is separated from the ground planes by 5 µm gaps. When measured at relatively high RF-power levels most of our devices exhibit quality factors (Q) of several hundred thousand at low temperatures regardless of the substrate, with maximum Qs of about 1 million.

The measurements were done in two different cryostats. Measurements below 1 K (figures 1 and 3), were made in a dilution refrigerator with a base temperature of about 25 mK. The
signal from the sample is amplified by about 30 dB using a cryogenic, low-noise InP HEMT amplifier bolted to the 1 K stage of the insert. The signal is then amplified by another 50 dB using room-temperature amplifiers before being fed to a vector network analyzer (VNA). The bandwidth of this system is 4-8 GHz so only the first harmonic of the resonators could be studied. An internal coil allows us to apply a small magnetic field at an angle of about 10° to the plane of the substrate. Measurements above 1.2 K were made in a glass cryostat. This system has a wider bandwidth meaning higher harmonics can be studied. Using Helmholtz coils external to the cryostat, small magnetic fields can be applied in any direction.

3. Results

Here we will outline our results concerning two effects that are important for many applications of superconducting CPR: their sensitivity to external magnetic fields and the power dependence of the quality factor.

3.1. Magnetic field tuning of the center frequency

The resonant frequency of a CPR depends on the sum of the geometric inductance \( L_G \) and the kinetic inductance \( L_K \). The latter is related to the pair density in the film and is therefore field dependent, the dependence being quadratic for small fields. Whereas a quadratic dependence on the magnetic field is to be expected at \( T \sim T_c \), one might expect this sensitivity to disappear as \( T \rightarrow 0 \). However, this turns out not to be the case. The microscopic origin of this effect is complicated for realistic sample geometries and no existing theory can fully account for all relevant effects. However, early works by Tinkham [6] and Bardeen [7] were able to reproduce the \( H^2 \) dependence also at low temperatures. An expression for the centre frequency of a CPR will therefore have the form

\[
f(T, H) = f(T, 0) \left( 1 - \frac{L_K(T, 0) \beta(T) H^2}{L_T H_c^2} \right)
\]

where \( L_K(T, 0) \) is the zero-field kinetic inductance, \( L_T \) the total inductance and \( \beta(T) \) is a scaling factor which -in conventional theory- depends on the quasiparticle density. Figure 1 shows how the centre frequency of a CPR fabricated on SiO\(_2\)/Si depends on the applied magnetic field. As has been discussed in more detail elsewhere [8] the dependence is indeed quadratic at all temperatures irrespective of the applied RF power. At very high magnetic fields, flux will start to penetrate the film and the behavior becomes irreversible. For this sample Q varied between \( \sim 10^5-10^6 \) in the temperature range 25-500 mK but was -crucially- independent of the applied magnetic field.

Equation 1 predicts that \( H \) needs to be of the order of \( H_c \) in order to cause a large absolute shift, but it is important to note that \( L_K \) can change even if a field is applied locally. The geometry of a CPR means that there is a large flux-focusing effect near the edges of the CPR structure where most of the microwave currents flow, the factor being approximately 400 in our samples. Hence, although \( L_K \) only corresponds to about 1.5% of the total inductance the effect is very significant in high-Q samples where changes in the inductance are easily seen.

Figure 2 shows measurements made at \( T=1.3 \) K and includes higher harmonics. As can be expected the shift is larger for higher harmonics, the ratio being approximately 2.8:1.2:1. This deviates from the ideal 3:2:1 ratio predicted by the model, but can be attributed to a non-uniform flux focusing effect, with different resonance modes being sensitive to different parts of the sample. The dependence on the angle of the applied field agrees with the aforementioned picture of the flux focusing effect, with the maximum shift occurring for perpendicular fields.

One attractive feature of this methods of tuning the centre frequency is that it does not impair the quality factor of the resonator. This is often a problem with methods that use elements such
as Josephson junctions [9] or field controlled SQUIDs [10]. Whereas these methods have the advantage of allowing very wideband tuning they are at present not suitable for applications that require very high quality factors.

3.2. Temperature and RF-power dependence of Q
From the Matthias-Bardeen theory [11] one would expect the quality factor of a CPR to be relatively insensitive to temperatures for $T \ll T_c$. Whereas this picture is essentially correct for CPRs fabricated on sapphire substrates, samples fabricated on silicon exhibit a very strong temperature dependence when measured at low- and moderate powers as can be seen in figure 3. The reason for this dependence is usually attributed to the presence of two-level fluctuations in the substrate [12, 13], these act as a bath of parasitic resonators that couple to the CPR. In real samples there are always many fluctuators with a level splitting close to the resonator frequency. These have a large influence not only on the power dependence, but also on e.g the noise properties [14] and temperature dependent frequency shift [15]. The temperature dependence of $Q$ can be easily understood: at low temperatures most of these parasitic resonators will be in their ground state (the temperature scale being set by $\hbar \omega / 2k_B T$, which is approximately 150 mK for a 6 GHz resonator), and as microwaves are applied to the resonator some of this energy will go into exciting the bath. At low powers this process dampens the CPR very efficiently, but as the power level is increased the fluctuators will start to become saturated and therefore absorb less energy: this in turn result in a higher quality factor of the CPR. Our measurements are consistent with this picture. It is well known that two-level systems are more prevalent in silicon than sapphire which explains the stronger power dependence in the former. However, whereas there are fewer fluctuators in sapphire they are not completely absent and can still cause real problems in applications.

4. Conclusions
We believe that the two effects discussed in this work are important to understand in order to be able to efficiently use CPRs in their various applications. The results presented here
Figure 3. Quality factor at several power levels as a function of temperature for resonators fabricated on sapphire top and SiO$_2$ bottom. 0 dB corresponds to -27 dBm out from the network analyzer, this power was then attenuated by approximately 70 dB before reaching the sample.

also highlights some points that needs to be carefully considered when designing resonators for applications. Firstly, the strong power dependence for CPR fabricated on Si means that a resonator that works well at relatively strong measurements powers is by no means guaranteed to work well in applications which require the power to be reduced, even a reduction of a few dB can make a dramatic difference. Secondly, the magnetic field dependence of the centre frequency also means that -perhaps surprisingly- flux noise might be a major cause of concern in some sensitive measurements. Fortunately, the dependence is quadratic which means it is a problem that can be managed by proper use of magnetic shielding.

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References
[1] Blais A, Huang R, Wallraff A, Girvin S and Schoelkopf R 2004 Phys. Rev. A 69 062320
[2] Lindström T, Webster C H, Healey J E, Colclough M S, Muirhead C M and Tzalenchuk A Y 2007 Supercond. Sci. Technol. 20 814–821
[3] Day P K, LeDuc H G, Mazin B A, Vayonakis A and Zmuidzinas J 2003 Nature 425 817
[4] Sillanpää M A, Park J and Simmonds R W 2007 Nature 449 438
[5] Houck A A, Schuster D I, Gambetta J M, Schreier J A, Johnson B R, Chow J M, Majer J, Frunzio L, Devoret M H, Girvin S M and Schoelkopf R J 2007 Nature 449 328–331
[6] Tinkham M 1962 IBM J. Res. Dev. 49 667
[7] Bardeen J 1962 Rev. Mod. Phys. 34 667
[8] Healey J E, Lindstrom T, Colclough M S, Tzalenchuk A Y and Muirhead C M 2008 Appl. Phys. Lett. 93
[9] Osborn K D, Strong J A, Sirosi A J and Simmonds R W 2007 IEEE Trans. on App. Supercond. 17 166–168
[10] Sandberg M, Wilson C M, Persson F, Bauch T, Johansson G, Shumeiko V, Duty T and Delsing P 2008 Applied Physics Letters 92
[11] Mattis D and Bardeen J 1958 Phys. Rev. 111 152505
[12] Gao J, Zmuidzinas J, Mazin B A, LeDuc H G and Day P K 2007 Applied Physics Letters 90 102507
[13] Martinis J M, Cooper K B, McDermott R, Steffen M, Ansmann M, Osborn K D, Cicak K, Oh S, Pappas D P, W S R and Yu C C 2005 Phys. Rev. Lett. 95 210503
[14] Kumar S, Gao J, Zmuidzinas J, Mazin B A, LeDuc H and Day P 2008 Applied Physics Letters 92 123503
[15] Jiansong G, Daal M, Vayonakis A, Kumar S, Zmuidzinas J, Sadoulet B, Mazin B A, Day P K and LeDuc H G 2008 Applied Physics Letters 92 152505 (pages 3)