Analysis and Reconstruction of Air-cooled Condenser Based on Proper Orthogonal Decomposition Technique

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Abstract. Based on the previous experiments and numerical simulations, the low-order model of the air-cooled condenser was obtained by the orthogonal decomposition (POD) method, and the law of POD basis function and time coefficient distribution was obtained. Get the distribution of generalized "energy" between modes; The POD-based low-order model is used to reconstruct the pressure field and velocity field of the air-cooled condenser, and the relationship between the number of modes and the error of the result is obtained. The validity and rapidity of the reduced order method are proved by selecting the previous modal reconstruction flow fields.

1. Introduction
The current, water resources have become increasingly scarce. The rapid development of air-cooled systems in the power system has effectively alleviated the water supply problem. However, the thermal-power plant air-cooled system model relies on a set of complex partial differential equations with given initial boundary conditions to characterize the flow and heat transfer characteristics, while experimental and numerical simulations are often used to investigate its operating characteristics. Regardless of a specific experiment or numerical simulation, it requires a lot of experimental resources and computing time, and has difficulty performing the realistic prediction and control. Therefore, it is important to establish a low-order extrapolation model for air-cooled systems that can guarantee sufficient accuracy with high speed.

Proper orthogonal decomposition (POD) is an efficient data dimensionality reduction tool to reduce the degrees of freedom of numerical computational models and to alleviate the calculation load in the computational process, which is able to identify ways to find optimal lower dimensional approximations for the given data set. The POD method was originally proposed by the author described the principles and basic knowledge of POD by Galerkin in details in the monograph of Holmes [1]. With the development of science and technology, POD has been widely used in various fields, which include structural mechanics [2-4], optimal control [5, 6], predictive information [7] and model reduction [8]. Specifically, it has been effectively applied to analyze the heat and mass transfer of fluid flow. Reference [9] presents an experimental investigation of the tumble flow structures through the application of Particle Image Velocimetry (PIV) under steady-state conditions in terms of the central vertical tumble plane. Reference [10] describes the application of POD method to evaluate an alternative method with POD for the extraction of ocean surface wave fields remotely sensed by marine radar. Reference [11] proposes an online adaptive local-global POD-DEIM model reduction method for flows in
heterogeneous porous media. Reference [12] applies a modified POD to analyze the large eddy simulation data of a wind turbine wake in a turbulent atmospheric boundary layer. It is shown that only a few modes are necessary in order to capture the basic dynamical aspects of quantities that are relevant to a turbine in the wake flow. Reference [13] shows that the large eddy simulation data can be analyzed to investigate a new stochastic modeling approach for the wake of a wind turbine. The data is generated by the large eddy simulation (LES) model PALM combined with an actuator disk with rotation which can represent the turbine. Reference [14] adopts the snapshot method of POD and fast Fourier transform (FFT) to capture the most prominent coherent structures in the turbulent flow field. References [15, 16] apply the POD technique to analyze the flow and heat transfer of air-cooled condensers for the first time. By using the classical POD to establish the predictive model, the degree of freedom in computational fluid dynamic (CFD) simulation is reduced from $10^5$ to $10^1$. Currently, since the flux matching procedure (FMP) has better robustness than the cubic spline difference in the selection of the weight coefficient of POD, the proposed PODc method is able to achieve converge faster with the same accuracy and solves the POD modeling for small-scale air-cooled condenser. The low-dimensional POD solution output is used as the boundary condition of the flow field to realize the corresponding numerical simulation in the large-scale area. However, in the construction of the POD basis and the established POD prediction model, the traditional POD method has some solutions that repeats all the computations simultaneously on the same time span.

In this paper, the two-dimensional air-cooled condenser field is taken as the research object. Based on the previous work, a method of reducing the whole flow field by POD is proposed without considering the boundary conditions, and the main structural modes of the flow field are extracted for analysis. The Galerkin method is used to project the reduced-order mode, and the entire flow field is reconstructed by the reduced-order mode. The reconstruction accuracy and error are analyzed, the accuracy of the method is verified, and the real-time control of the air-cooled condenser model and the flow field are fast. Calculation has certain guiding significance.

2. **Air-cooled condenser mathematical and physics mode**

The physical object of this numerical simulation comes from the air-cooled condenser of the air-cooled array optimal control experimental platform. The experimental platform is aimed at the actual air-cooled island of a typical 300 MW direct air-cooled condenser in China, and a 1:10 ratio experimental fan group is constructed. The scale reduction meets the similar three laws, namely: geometric similarity, similar motion and similar dynamics.

![Figure 1. Two-dimensional air-cooled condenser model](image_url)
Assuming that air is the incompressible fluid, according to the reference, the axial flow model can be simplified to a pressure jump interface, and the tilted two fin tubes can also reduce to dimensionless pressure drop models Π₁ and Π₂. Thus, the governing equations and internal boundary conditions that the model satisfies in region Ω are formulated as:

### Continuity equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

### Momentum equation

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{align*}
\]

Where \( \rho \) is air density, \( u \) and \( v \) are the velocity component of the x and y direction in the 2D model, \( p \) represents pressure, \( w \) represents the normal velocity perpendicular to the virtual boundary, and \( \mu \) represent the dynamic viscosity.

### 3. Flow field reconstruction based on POD air-cooled condenser

For the above two-dimensional air-cooled condenser model, the large-scale professional software Fluent is used for analysis. The calculation domain is 11200mm*11150mm, the calculated grid number is 78600, and the time step is 0.01s. We think the CFD result is an exact solution. We select the flow field snapshots of 60 moments as the data set for POD analysis.

For the above analysis area, during a certain period of flow in the air-cooled condenser flow field, the transient velocity field and pressure field snapshots (snapshots) of \( N=100 \) moments in the region are selected, respectively, using \( U(x, y, t_i) \) and \( P(x, y, t_i) \) \( (i = 1, 2, 3, ..., N) \) are used to establish a reduced order POD space. Taking the pressure field as an example, the mean and pulsation of the set of sample values are defined as:

\[
\bar{U} = < U > = \frac{1}{N} \sum_{i=1}^{N} U(x, y, t_i)
\]

\[
\tilde{U}(x, y, t_i) = U(x, y, t_i) - \bar{U}
\]

According to the idea of POD, the pulsation quantity \( \tilde{U}(x, y, t_i) \) is decomposed into a function of the spatial mode \( \varphi_l(x, y) \) and the time coefficient \( a_i(t) \), namely:

\[
\tilde{U}(x, y, t_i) = \sum_{i=1}^{N} a_i(t) \varphi_l(x, y)
\]

In fact, solving the spatial modality is equivalent to solving the maximum value of the following expression.

\[
\max \left\{ \frac{1}{N} \sum_{i=1}^{N} |(\tilde{U}, \varphi)|^2 \right\}
\]

Using the variational method, the above maximum value problem can be transformed into the eigenvalue solving problem:

\[
\iint C(x, y; x', y') \varphi(x', y')dx'dy' = \lambda \varphi(x, y)
\]

A linear combination of the original function space snapshot pulsations can be used to represent the spatial modality, ie
\[ \varphi_k(x, y) = \sum_{i=1}^{N} b_i^k V_k(x, y) \]

Based on the POD modes obtained above, the dimensionality reduction mode of the original velocity field can be expressed as:

\[ U^{POD}(x, y, t) = \bar{U} + \sum_{i=1}^{M} a_i(t) \varphi_i(x, y) \]

In the above, the dimensionality reduction space extended by M POD basis functions is obtained and solved in the dimensionality reduction space. The Galerkin approximation is used to project the model equation into the reduced-dimensional space in which the POD basis function is expanded, and finally a low-order model based on POD is obtained.

4. Numerical simulation and analysis of POD results

100 snapshot spaces obtained by CFD technology can obtain 100 POD basis vectors by POD reduction. The feature values are arranged from large to small. Only the first 20 eigenvalues are displayed here. The horizontal axis of the graph indicates the feature value. The vertical axis represents the eigenvalue as a percentage of the total energy. Figure 3 shows the relationship between the eigenvalue and the specific gravity and the number of modes.

![Figure 2](image_url)

**Figure 2** Proportion of each eigenvalue to the sum of all eigenvalues

From Figure 3 we can see that as the modal order increases, the energy contained in each mode gradually decreases. The low-order mode usually contains higher energy information of the entire flow field. The first-order mode has the highest energy content, and the first-order POD mode obtained by the velocity field accounts for 39% of the total energy, and the first-order to nth-order modes. The energy contained is greater than 1%. From the nth +8th order, the eigenvalue specific gravity has already reached zero. The first few modes have captured 99% of the energy of the original system.
Through the previous analysis, we can reconstruct the flow field by using the first few modes combined with the time coefficient. The reconstructed flow field is shown in Figure 3.

5. Conclusion
In this paper, the numerical simulation of the air-cooled unit flow field is carried out. Based on the numerical simulation, the intrinsic orthogonal decomposition method is used to reconstruct the flow field in the air-cooled condenser.

Compared with the traditional model, the reduced-order model constructed based on POD extrapolation algorithm demonstrate less error but higher accuracy. The reduced-order model can be used as the replacement of the original complex calculation model to accurately predict the flow field in the air-cooled condenser.

The POD extrapolation algorithm has an obvious effect on the accurate and rapid prediction calculation of a single air-cooled condenser model, and can also be extended to the entire air-cooled island flow field prediction. Therefore, this method has promising application potential in terms of the fast calculation of complex nonlinear system models.

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