Study of deformation-strength properties of sandstone under multistage triaxial compression

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Abstract. A multistage test under conventional triaxial compression is an alternative way (instead of triaxial tests on a set of samples) to describe the mechanical strength (the envelope of the Coulomb-Mohr half-circles) of rocks. This paper is based on the experimental study of sandstones under multistage triaxial compression. Empirical dependencies of the modulus of elasticity, Poisson’s ratio, the specific energy intensity on each of the stages are discussed. The Coulomb-Mohr envelopes are constructed. Tests of rocks under multistage triaxial compression can be suitable in case of a very limited number of specimens.

1. Introduction
The design of geo-engineering objects is accompanied by processes involving cyclic loads-unloads and leading to the mechanical destruction of rocks. Rock micro-cracks, induced by excavation, softening and accompanying changes in the natural stress state, can cause degradation of strength and a change in elastic properties [1]. For this reason, computational models based on the use of conventional mechanical properties may be the cause of erroneous estimated stress state forecasts.

Another problem is the difficulty in extracting the unbroken core from petroleum deposits. Therefore, to determine the strength properties of such cores, a small number of specimens must be tested. The tests with cyclic (multistage) loads make it possible to build a strength passport (Mohr-Coulomb envelope) involving only one specimen [2].

Therefore, the description of the behavior of the elastic and strength properties of rocks under static cyclic loads, as well as the determination of the strength characteristics of the core from the data of a single specimen test, is an actual problem when drilling wells in the case of a limited number of specimens.

The data of the mechanical experiments that compiled the presented work are based on cyclic (multistage) tests, in which the lateral pressure was varied depending on the stage of the loading program of the geomaterial being studied. Such or related tests are devoted to a number of the international research studies [3-9].

2. Experimental tests

2.1. Specimens
For the studies, selected (by properties) cores of sandstones were chosen from the special rock collection.
Cylindrical specimens of the desired diameter (30mm) were made using the CPM 400 Coretest System. To ensure the parallelism of the loaded surfaces and required length (60 mm), drilled cylindrical specimens were cut on the DTS-430 machine.

Six specimens (Figure 1, Figure 2) were tested under the multistage triaxial compression.

![Figure 1. Sandstone specimens No1-2, 1-3, 1-4 before (a, b, c) and after (e, f, g) the multistage tests.](image-url)
2.2. *Multistage triaxial test description*

During the triaxial multistage loading of specimen the axial stress value reached the compressive elastic limit at each loading stage at certain lateral pressures. The values of last ones were 0.5, 1.5, 7, 9.5, 11 MPa, respectively, for 1st, 2nd, 3rd, 4th, 5th loading stages.

Each specimen was placed in the triaxial cell. The specimens were jacketed by rubber cup to prevent oil from seeping into the pores during tests. The oil serves as a distributor for uniform lateral pressure: due to the absence of shearing stresses in the liquid, the lateral pressure through the liquid is transmitted to the normal direction along each point of the sample wall. The axial load was applied to the specimen ends.

The procedure for testing rock specimens under the multistage triaxial compression consists of several steps [2]. First the lateral pressure is applied, and then the axial load gradually increases until the plastic strains begin. After this, gradual unloading of the specimen till the equality of lateral and axial pressures is carried out. Then the lateral pressure increases to the second value of lateral pressure, the axial load reaches the second value of plastic strain. This procedure is repeated. The specimen is brought to destruction at the test’s end. Obtained “stress $\sigma_z$-strain $\varepsilon_z$ or $\varepsilon_{\phi}$” curve is shown in Figure 3.
2.3. Experimental data-processing

To determine the modulus of elasticity (GPa), the standard ASTM D 7012-04 [10] was used. Approximately linear part in the "stress-strain" curve was used to calculate modulus of elasticity.

Poisson’s ratio $\nu$ was calculated by the formula below in the same linear part as modulus of elasticity:

$$
\nu = -\frac{\varepsilon_\varphi}{\varepsilon_\varepsilon}.
$$

In addition to the elastic and strength characteristics, the specific energy intensity (kJ / m$^2$) was calculated for each loading stage. Specific energy intensity is the specific portion of energy accumulated in the sample at each stage of loading until the compressive elastic limit. This characteristic is described by the integral below:

$$
w = \int \left( \sigma_z(u_z) - \sigma_{\varphi}^i \right) \cdot du_z = \int \left( a_1 u_z^3 + a_2 u_z + a_3 - \sigma_{\varphi}^i \right) \cdot du_z + \int \left( b_1 u_z^2 + b_2 u_z + b_3 - \sigma_{\varphi}^i \right) \cdot du_z,
$$

where $\sigma_z(u_z)$ is the polynomial dependence of the axial stress $\sigma_z$ on the axial displacements $u_z$; $\sigma_{\varphi}^i$ is the value of the lateral pressure on the $i$-th ($i = 1, ..., 5$) loading stage.

Interpreting the compressive elasticity limits as the compressive strength limits for given lateral pressures, the envelopes of Mohr-Coulomb half-circles were obtained by using standard technique [11].

For each sample, the values of the elastic limit, the elasticity modulus, the Poisson's ratio, the specific energy intensity and envelopes of Mohr-Coulomb circles under multistage triaxial compression were obtained at each loading stage (see tables 1-5).
Based on the values of the coefficient of determination, we revealed the strong, moderate and weak dependencies of the considered mechanical characteristics on the stage number (No). Stage No varied from 1 to 5.

**Table 1.** Empirical dependence of compressive elastic limit $\sigma_y$ on Stage No.

| Specimen No | Dependence of compressive elastic limit $\sigma_y$ (MPa) on Stage No | Coefficient of determination $R^2$ |
|-------------|---------------------------------------------------------------------|----------------------------------|
| 1-2         | $\sigma_y = 4.15 \cdot \text{StageNo} + 14.85$                     | 0.99                             |
| 1-3         | $\sigma_y = 4 \cdot \text{StageNo} + 15.2$                        | 0.97                             |
| 1-4         | $\sigma_y = 2.7 \cdot \text{StageNo} + 21.6$                     | 0.95                             |
| 2-1         | $\sigma_y = 4.15 \cdot \text{StageNo} + 15.45$                   | 0.93                             |
| 2-2         | $\sigma_y = 2.8 \cdot \text{StageNo} + 15$                       | 0.97                             |
| 2-3         | $\sigma_y = 1.3 \cdot \text{StageNo} + 19.6$                     | 0.34                             |

**Table 2.** Empirical dependence of elasticity modulus $E$ on Stage No.

| Specimen No | Dependence of elasticity modulus $E$ (GPa) on Stage No | Coefficient of determination $R^2$ |
|-------------|-------------------------------------------------------|----------------------------------|
| 1-2         | $E = 0.242 \cdot \text{StageNo} + 1.846$            | 0.88                             |
| 1-3         | $E = 0.158 \cdot \text{StageNo} + 2.648$            | 0.94                             |
| 1-4         | $E = 0.014 \cdot \text{StageNo} + 2.066$            | 0.02                             |
| 2-1         | $E = 0.215 \cdot \text{StageNo} + 2.101$            | 0.73                             |
| 2-2         | $E = 0.096 \cdot \text{StageNo} + 2.458$            | 0.67                             |
| 2-3         | $E = 0.103 \cdot \text{StageNo} + 1.791$            | 0.44                             |

**Table 3.** Empirical dependence of Poisson ratio $\nu$ on Stage No.

| Specimen No | Dependence of Poisson ratio $\nu$ on Stage No | Coefficient of determination $R^2$ |
|-------------|-----------------------------------------------|----------------------------------|
| 1-2         | $\nu = 0.006 \cdot \text{StageNo} + 0.212$   | 0.9                              |
| 1-3         | $\nu = 0.007 \cdot \text{StageNo} + 0.211$   | 0.72                             |
| 1-4         | $\nu = 0.008 \cdot \text{StageNo} + 0.202$   | 0.57                             |
| 2-1         | $\nu = 0.008 \cdot \text{StageNo} + 0.242$   | 0.89                             |
| 2-2         | $\nu = 0.008 \cdot \text{StageNo} + 0.248$   | 0.94                             |
| 2-3         | $\nu = 0.003 \cdot \text{StageNo} + 0.261$   | 0.45                             |
Table 4. Empirical dependence of specific energy intensity W on Stage No.

| Specimen No | Dependence of specific energy intensity W (kJ/m²) on Stage No | Coefficient of determination R² |
|-------------|-------------------------------------------------------------|--------------------------------|
| 1-2         | \( W = -0.426 \cdot \text{StageNo} + 4.266 \)               | 0.33                           |
| 1-3         | \( W = 0.428 \cdot \text{StageNo} + 2.01 \)                 | 0.43                           |
| 1-4         | \( W = -0.907 \cdot \text{StageNo} + 7.761 \)               | 0.16                           |
| 2-1         | \( W = -0.396 \cdot \text{StageNo} + 4.866 \)               | 0.69                           |
| 2-2         | \( W = -0.316 \cdot \text{StageNo} + 4.22 \)                | 0.36                           |
| 2-3         | \( W = 0.703 \cdot \text{StageNo} + 1.425 \)                | 0.42                           |

Table 5. Envelopes of Mohr-Coulomb half-circles.

| Specimen No | Dependence of shear stress \( \tau \) (MPa) on normal stress \( \sigma \) (MPa) | Range of normal stress \( \sigma \) (MPa) |
|-------------|--------------------------------------------------------------------------------|-----------------------------------------|
| 1-2         | \( \sigma = 0.234 \cdot \tau + 7.039 \)                                     | \( 0 \leq \sigma \leq 34.942 \)        |
| 1-3         | \( \sigma = 0.159 \cdot \tau + 8.508 \)                                     | \( 0 \leq \sigma \leq 34.767 \)        |
| 1-4         | \( \sigma = 0.138 \cdot \tau + 9.741 \)                                     | \( 0 \leq \sigma \leq 37.469 \)        |
| 2-1         | \( \sigma = 0.321 \cdot \tau + 6.511 \)                                     | \( 0 \leq \sigma \leq 39.094 \)        |
| 2-2         | \( \sigma = 0.107 \cdot \tau + 7.659 \)                                     | \( 0 \leq \sigma \leq 32.594 \)        |
| 2-3         | \( \sigma = 0.139 \cdot \tau + 6.085 \)                                     | \( 0 \leq \sigma \leq 33.182 \)        |

Strong dependencies on the stage number prevail for the compressive elastic limit. Moderate dependencies are observed for the elasticity modulus and the Poisson ratio. Weak dependencies are obtained for the specific energy intensity. Envelopes of Mohr-Coulomb half-circles are approximated by the linear relationships \( \sigma(\tau) \) in certain range of \( \sigma \).

3. Conclusion

In this study, specific tests were carried out for multistage triaxial compression of sandstone specimens, as a result of which a Mohr-Coulomb envelope for each of the sandstone samples was constructed, the modulus of elasticity, the Poisson ratio, the compression elastic limit and the specific energy intensity depending on the stage number were determined.

Based on the processed experimental data, the following conclusions can be drawn:

- With each subsequent loading stage, a monotonous increasing tendency (strong, moderate) of the elastic limit, the elasticity modulus, the Poisson ratio is traced, which indicates the existence of a variation of these parameters (under loading-unloading conditions), which is recommended for taking into account during the theoretical calculations of the stress-strain state.
- There is no common clear pattern of variation in the specific energy intensity (weak dependency on stage number).
• Envelopes of Mohr-Coulomb half-circles constructed on the basis of the conducted multistage tests with triaxial compression are suitable for describing the strength properties of sandstones in the case of core deficiency and are an alternative to the traditional technique for constructing a strength envelope from the experimental data of three tests (splitting tensile, uniaxial compression, conventional triaxial compression).

• Multistage triaxial compression makes it possible to describe the stepwise mechanical behavior of rock from an elastic state to a plastic state.

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