Substation-Level Grid Topology Optimization Using Bus Splitting

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Abstract—Operations of substation circuit breakers are of high significance for performing system maintenance and topology optimization. Bus splitting is one type of topology changes where the two bus-bars at a substation become electrically disconnected after certain actions of circuit breakers. As these events involve detailed substation modeling, they are not typically considered in power system routine operation and control. In this paper, an improved substation-level topology optimization is developed by expanding traditional line switching with breaker-level bus splitting, which can further reduce grid congestion and generation costs. A tight McCormick relaxation is proposed to reformulate the bi-linear terms in the resultant topology optimization model. Thus, a tractable mixed-integer linear program formulation is presented which can be efficiently solved for real-time control. Numerical studies on the IEEE 14-bus and 118-bus systems demonstrate the performance and economic benefits of the proposed topology optimization approach.

Index Terms—Circuit breakers, bus split, grid topology control, optimal transmission switching, McCormick relaxation.

I. INTRODUCTION

Grid topology optimization is becoming increasingly important for efficient power system operations, thanks to its capability of effectively relieving network congestion and reducing generation cost. Varying the topology of power networks mainly relies on the operations of switching devices such as circuit breakers (CBs) within the electrical substations. The switching of CBs not only disconnects transmission lines and generation/load, but also can result in bus splitting [1, Ch. 11]. A comprehensive topology optimization framework that includes all types of topological changes can greatly enhance the benefits of reducing generation costs while improving the security of grid operations.

A majority of grid topology optimization work has mainly focused on the search of line switching actions [2]–[6], and thus have overlooked the potentials of utilizing bus splitting operations. The switching of substation CBs has been explored in [7]–[9] as a corrective measure for relieving localized grid stress due to line overloads or voltage violations. These methods have been developed to target localized contingencies in power networks by analyzing a small subset of candidate CB actions, while not yet considering a global search for the economic benefits of the full grid. In [10], a topology optimization method based on generalized substation and CB modeling was proposed to help reduce the total generation costs. Nonetheless, a pre-screening heuristic was utilized to address the scalability issue of the optimization problem therein in order to allow for real-time implementation. The optimality of the resultant topology solutions is questionable and the optimality gap is unclear. In fact, modeling the CB actions typically requires the detailed node-breaker representation of power grid that includes the full list of substation components; see e.g., [10]–[12]. The complexity of such representation is the major cause of lack of scalability as the resultant scenarios can be redundant. Thus, it still remains open to develop an efficient grid topology optimization algorithm that can account for substation-level topology changes.

The goal of this paper is to develop an efficient real-time topology optimization algorithm that can incorporate the substation-level topology changes such as bus splitting. To address the scalability issue of node-breaker representation, this paper leverages an equivalent bus-branch model for the substation bus splitting. Hence, instead of explicitly modeling all the components within the substation, we can conveniently incorporate this concise equivalent model for bus splitting into a grid topology optimization formulation. To deal with the bi-linear terms in the resultant formulation, we apply the McCormick relaxation technique [13] and attain an exact mixed-integer linear program reformulation, which can be efficiently solved for real-time implementation. Therefore, the main contribution of our work is to provide a tractable algorithm to effectively search for all possible topology changes, both line switching and bus splitting, in order to attain the best grid-wide economic and security benefits.

The rest of the paper is organized as follows. Section II introduces the dc power flow model and the equivalent bus-branch model for bus splitting events. Section III develops the substation-level topology optimization formulation, and further reformulates it into a tractable mixed-integer linear program (MILP) using the McCormick relaxation technique. Numerical studies on the IEEE 14- and 118-bus systems are presented in Section IV to demonstrate the efficacy of the proposed approach. The paper is concluded in Section V.

Notation: Upper- (lower-) case boldface symbols are used...
to denote matrices (vectors); \((\cdot)^T\) stands for transposition; \(I\) for identity matrix; \(1\) denotes the all-one vector; and \(e_i\) denotes the standard basis vector with all entries being 0 except for the \(i\)-th entry equals to 1.

### II. System Modeling

Consider a transmission system with \(N\) buses collected in the set \(\mathcal{N} := \{1, \ldots, N\}\) and \(L\) lines in \(\mathcal{L} := \{(i,j)\} \subset \mathcal{N} \times \mathcal{N}\). For bus \(i\), let \(\theta_i\) be its phase angle and collect all the angles in \(\theta \in \mathbb{R}^N\). Similarly, let \(g, d \in \mathbb{R}^N\) denote the vectors of generation and load at all buses, respectively. Under the dc power flow model, line flows \(\{f_{ij}\}\) which are collected in \(f \in \mathbb{R}^L\) are given by:

\[
f = K\theta
\]

(1)

with the matrix \(K \in \mathbb{R}^{L \times N}\) mapping the phase angles to line flows. The row of \(K\) corresponding to line \((i,j)\) is \(b_{ij}(e_i - e_j)^T\), where \(b_{ij}\) is the inverse of the line \((i,j)\) reactance. Furthermore, the network power flow conservation leads to:

\[
p = Af
\]

(2)

where \(p = g - d\) is the net injection vector and \(A \in \mathbb{Z}^{N \times L}\) is the incidence matrix for the underlying graph \((\mathcal{N}, \mathcal{L})\). Substituting (1) into (2) yields the dc power flow model:

\[
p = B\theta
\]

(3)

where the so-termed Bbus matrix \(B \in \mathbb{R}^{N \times N}\) is given by:

\[
B = \sum_{(i,j) \in \mathcal{L}} b_{ij}(e_i - e_j)(e_i - e_j)^T.
\]

(4)

In electrical networks, switching equipment such as CBs and isolators are usually installed in substations to allow for flexible network topology and emergency intervention. Under certain CB configuration, the substation bus may become electrically disconnected, commonly termed as “bus splitting” or “bus split.” Occurrence of bus splitting is increasingly frequent due to misoperations of CBs [14], [15] or malicious cyber attacks [16]–[18]. Fig. 1 shows the substation configuration for one such event in node-breaker representation, where solid squares (hollow squares) represent closed (open) breakers. If the circled CB becomes open, bus \(i\) is split into two different buses, \(i\) and \(i'\). Accordingly, transmission lines, generation and load may be reconnected to the new bus \(i'\). Although the two buses are physically co-located in the same substation, they become electrically disconnected leading to a different bus-branch model as shown in Fig. 2.

The grid-wide impact of the bus splitting topology change has been analyzed in [18] and is summarized in the following proposition.

**Proposition 1.** Consider the split of bus \(i\) with a single line \((i,j)\) and injection \(\tilde{p}_i\) reconnected to the new bus \(i'\) (shown in Fig. 2). The post-split system is equivalent to having the opening of line \((i,j)\) and an additional power transfer \(\tilde{p}_i\) between buses \(i\) and \(j\).

The equivalent model for the post-split system is demonstrated in Fig. 3. Since the new bus \(i'\) only connects to bus \(j\), it can be eliminated from the system by moving its connected injection (\(\tilde{p}_i = g_i\) in this generation-only case) directly to bus \(j\). Compared to the original system in Fig. 2, the equivalent system experiences the opening of line \((i,j)\) in addition to a power transfer of \(\tilde{p}_i\) between buses \(i\) and \(j\). This equivalent model has also been verified by linear sensitivity analysis for the bus split events [18]. Proposition 1 is very useful for simplifying the incorporation of bus split events into the topology optimization problem, as discussed in the ensuing section.

### III. Substation-level Grid Topology Optimization

The grid topology optimization problem aims to determine the optimal topology with associated generation outputs in order to minimize the total generation cost. The feasible region of generation dispatch for this problem is the union of the sets of feasible solutions corresponding to each topology configuration. Thus, varying the grid topology will likely expand the overall feasible region and accordingly reduce the total generation costs [2], [3]. Going beyond the traditional topology optimization framework, the inclusion of substation-level bus split events allows for additional topol-
modeling power transfer between buses

Fig. 4: Line $\ell = (i, j)$ with connected generation/load for modeling power transfer between buses $i$ and $j$.

togy changes, and hence can further reduce grid congestion and improve the security of grid operations.

A. Modeling of Power Transfer in Bus Splitting

We first discuss different scenarios of generation/load connection for modeling the power transfer in Proposition 1. To this end, consider a transmission line $\ell = (i, j)$ that connects to buses $i$ and $j$, as shown in Fig. 4. The general case of having both generation and load is assumed for the two buses. The bus split event can result in a model that is mathematically equivalent to a power transfer in between. For instance, the split of bus $i$ can be associated with one of three power transfer scenarios from bus $i$ to bus $j$, namely load only, generation only, and generation plus load. To represent the change of power injection for all three scenarios, define the following $N \times 3$ matrix

$$\Delta_{\ell,i}(g) = (e_i - e_j)[d_i - g_i \ d_i - g_i]$$

where $e_i \in \mathbb{R}^{N \times 1}$ denotes the standard basis vector. Each column of $\Delta_{\ell,i}(g)$ corresponds to one of three aforementioned scenarios under the split of bus $i$. Similarly, one can define this power injection matrix for the split of bus $j$, as

$$\Delta_{\ell,j}(g) = (e_j - e_i)[d_j - g_j \ d_j - g_j].$$

Notice that both $\Delta_{\ell,i}(g)$ and $\Delta_{\ell,j}(g)$ depend on the generation output $g$ which is a decision variable. In what follows, we will use $\Delta_{\ell,i}(g)$ to refer to $\Delta_{\ell,i}(g)$ when the dependence on $g$ is clear from the context.

B. Topology Optimization with Bus Splitting

Upon defining the power injection matrices, we are ready to formulate the topology optimization problem that includes the bus split operation. To this end, consider linear generation cost model with $c \in \mathbb{R}^N$ collecting all the linear coefficients. Binary decision variable $z_\ell$ is introduced for each transmission line $\ell = (i, j)$ to indicate the equivalent line status (1: closed, 0: open), which will be explained in more details after the formulation. The incident buses for line $\ell$ are collected in the set $N_\ell$. A vector of binary variables $w_{\ell,i} \in \{0,1\}^3$ is used to select the power transfer scenario in case of a bus split at bus $i$, leading to an equivalent outage on line $\ell = (i, j)$. Similarly, vector $w_{\ell,j} \in \{0,1\}^3$ is defined to select the power transfer scenario in case of a bus split at bus $j$, with equivalent outage on line $\ell = (i, j)$. Under a maximum budget of $s$ operations (either line switching or bus splitting), one can formulate the following optimization problem:

$$\min \ c^T g \tag{7a}$$

over $\theta \in \mathbb{R}^N, g \in \mathbb{R}^N, f \in \mathbb{R}^L, z_\ell \in \{0,1\}, \forall \ell \in \mathcal{L}$

$w_{\ell,i} \in \{0,1\}^3, w_{\ell,j} \in \{0,1\}^3, \forall \ell = (i, j) \in \mathcal{L}$

s.t. $\theta_{\ell}^{\min} \leq \theta_{\ell} \leq \theta_{\ell}^{\max}, \forall i$ \tag{7b}

$g_{\ell}^{\min} \leq g_{\ell} \leq g_{\ell}^{\max}, \forall i$ \tag{7c}

$f_{\ell}^\min z_\ell \leq f_{\ell} \leq f_{\ell}^\max z_\ell, \forall \ell$ \tag{7d}

$b_{ij}(\theta_i - \theta_j) - f_{\ell} + (1 - z_\ell)M_{\ell} \geq 0, \forall \ell = (i, j)$ \tag{7e}

$b_{ij}(\theta_i - \theta_j) - f_{\ell} - (1 - z_\ell)M_{\ell} \leq 0, \forall \ell = (i, j)$ \tag{7f}

$$\sum_{\ell} (1 - z_\ell) \leq s \tag{7g}$$

$$1^T w_{\ell,i} + 1^T w_{\ell,j} \leq 1 - z_\ell, \forall \ell = (i, j) \tag{7h}$$

$$\sum_{\ell : i \in N_\ell} 1^T w_{\ell,i} \leq 1, \forall i \tag{7i}$$

$$Af = g - d + \sum_{\ell = (i, j)} \Delta_{\ell,i} w_{\ell,i} + \sum_{\ell = (i, j)} \Delta_{\ell,j} w_{\ell,j} \tag{7j}$$

$$f_{\ell}^\min 1 \leq \Delta_{\ell,i} w_{\ell,i} + \Delta_{\ell,j} w_{\ell,j} \leq f_{\ell}^\max 1, \forall \ell = (i, j) \tag{7k}$$

We discuss the constraints for problem (7) here. Phase angle and generation limits are given in constraints (7b) - (7c). Line flow limits are given in (7d), while the flow $f_{\ell}$ is enforced to be zero when the line is open; i.e., $z_\ell = 0$. Constraints (7e) - (7f) are introduced for establishing the line flow model in (1) with the constant $M_{\ell}$ being sufficiently large. When the line $\ell = (i, j)$ is closed, the two inequalities are equivalent to the dc power flow equation $f_{\ell} = b_{ij}(\theta_i - \theta_j)$. Otherwise, when the line is open $f_{\ell} = 0$ [cf. (7d)], the two constraints are guaranteed to be inactive for a large $M_{\ell}$.

This is called the Big-M method [19], which is often used to handle constraints with binary variables. For each line $\ell = (i, j) \in \mathcal{L}$, we set

$$M_{\ell} := b_{ij}\Delta\theta_{ij}^{\max}, \tag{8}$$

where $\Delta\theta_{ij}^{\max}$ is a given upper bound for angle stability. Constraint (7g) limits the total number of operations including both line switching and bus splitting, while constraint (7h) further defines the operations for each line. Specifically, if $z_\ell = 0$ and $1^T w_{\ell,i} + 1^T w_{\ell,j} = 0$, the operation is simply a line switching of $\ell = (i, j)$, i.e., deenergizing the line $\ell = (i, j)$. Otherwise, when $z_\ell = 0$ but $1^T w_{\ell,i} + 1^T w_{\ell,j} \neq 0$, then one of the power transfer scenarios is selected after opening line $\ell$, making it equivalent to a bus split at either end of line $\ell$. The latter case utilizes the equivalent model of bus split events and does not actually deenergize line $\ell$. Therefore, $z_\ell = 0$ itself cannot fully indicate the operation type (line switching or bus splitting) and is called equivalent line status for this reason.

Constraints (7h) - (7k) are introduced specifically for the considerations of bus split events. Constraint (7h) limits...
the number of power transfer that can be selected for a
bus split involving the opening of line \( \ell = (i, j) \). When
the line is closed \((z_\ell = 1)\), no power transfer is allowed;
Otherwise, when the line is open \((z_\ell = 0)\), at most one (0
or 1) power transfer can be made, depending on whether it
is line switching or bus splitting. Furthermore, notice that
a single bus may be connected to multiple buses, but the
power transfer from bus \( i \) to other buses can only be made
if the bus is split into two bus bars. Once bus \( i \) is split for
a power transfer with one of its incident buses, no other
power transfer can be made with other buses. Therefore,
constraint (7) limits the power transfer from each bus to
be at most once due to the physical limit of the substation.
Constraint (7) enforces network power balance in (2), where
the injection also reflects any power transfer made due to
the bus split. Lastly, (7k) guarantees that for the injection
reconnected to the new bus \( i' \), the power flow on that incident
line is not violating the transmission limit of line \( \ell = (i, j) \)
[cf. Fig. 3].

The main challenge of solving (7) lies in the nonlinearity
of the constraints. Specifically, constraints (7) and (7k) are
bi-linear in the decision variables, due to the multiplication
terms, namely \( \Delta_{\ell,i}(g)w_{\ell,i} \) and \( \Delta_{\ell,j}(g)w_{\ell,j} \). To address
these terms, we propose to adopt the McCormick relaxation
technique [13] to reformulate the problem that is amenable
to off-the-shelf mixed-integer linear program (MILP) solvers.
First, rewrite the multiplication as

\[
\Delta_{\ell,i}w_{\ell,i} = \delta^i_\ell w_{\ell,i}d_i - \delta^j_\ell w_{\ell,i}g_i \tag{9}
\]

where vectors \( \delta^i_\ell := (e_i - e_j)[1 0 1] \) and \( \delta^j_\ell := (e_i - e_j)[0 1 1] \). We define the product of \( w_{\ell,i} \) and \( g_i \) as:

\[
y_{\ell,i} = w_{\ell,i}g_i, \quad \forall \ell. \tag{10}
\]

Under the bounds \([g_i^{\min}, g_i^{\max}]\) for generation \( g_i \), the following
four linear inequalities hold:

\[
y_{\ell,i} \geq w_{\ell,i}g_i^{\min} \tag{11a}
\]

\[
y_{\ell,i} \geq g_i + w_{\ell,i}g_i^{\max} - 1g_i^{\max} \tag{11b}
\]

\[
y_{\ell,i} \leq w_{\ell,i}g_i^{\max} \tag{11c}
\]

\[
y_{\ell,i} \leq g_i + w_{\ell,i}g_i^{\min} - 1g_i^{\min} \tag{11d}
\]

The inequalities (11a) - (11d) can be verified by substituting
(10). Conversely, for any binary \( w_{\ell,i} \), the linear inequalities
in (11) also guarantee the validity of (10). If the k-th entry
in the binary \( w_{\ell,i} \) is equal to zero, the two inequalities
(11a) and (11c) jointly force the k-th entry of \( y_{\ell,i} \) to be zero.
Otherwise, when the k-th entry of \( w_{\ell,i} \) is equal to one, the
inequalities (11b) and (11d) enforce that the k-th entry of
\( y_{\ell,i} \) to be equal to \( g_i \). Due to the binary vector \( w_{\ell,i} \), the set
of inequalities in (11) is equivalent to the bi-linear relation
in (10). Reformulating (10) with the linear inequalities in
(11) is known as the McCormick relaxation technique, which
has been popularly used in other problems of designing grid
topology [20], [21].

Hence, the bi-linear product as given in (9) can be equiva-
lently replaced with

\[
\Delta_{\ell,i}w_{\ell,i} = \delta^i_\ell w_{\ell,i}d_i - \delta^j_\ell y_{\ell,i} \tag{12}
\]

and linear inequality constraints (11a) - (11d). Similarly, the
bi-linear product of \( \Delta_{\ell,j} \) and \( w_{\ell,j} \) can be directly given as

\[
\Delta_{\ell,j}w_{\ell,j} = \delta^j_\ell w_{\ell,j}d_j - \delta^i_\ell y_{\ell,j} \tag{13}
\]

for similarly defined \( \delta^i_\ell \) and \( \delta^j_\ell \), together with the following
four linear inequalities:

\[
y_{\ell,j} \geq w_{\ell,j}g_j^{\min} \tag{14a}
\]

\[
y_{\ell,j} \geq g_j + w_{\ell,j}g_j^{\max} - 1g_j^{\max} \tag{14b}
\]

\[
y_{\ell,j} \leq w_{\ell,j}g_j^{\max} \tag{14c}
\]

\[
y_{\ell,j} \leq g_j + w_{\ell,j}g_j^{\min} - 1g_j^{\min} \tag{14d}
\]

Thus, we have reformulated the bi-linear products \( \Delta_{\ell,i}w_{\ell,i} \)
and \( \Delta_{\ell,j}w_{\ell,j} \) using linear constraints (12) and (13) followed
by additional linear inequalities (11a) - (11d) and (14a) - (14d).
This way, the original nonlinear optimization problem
(7) can thus be given as the following MILP:

\[
\min \quad c^Tg 
\]

\[
\text{over} \quad \theta \in \mathbb{R}^N, g \in \mathbb{R}^N, f \in \mathbb{R}^L, z_\ell \in \{0, 1\}, \forall \ell \in \mathcal{L}
\]

\[
w_{\ell,i} \in \{0, 1\}^3, w_{\ell,j} \in \{0, 1\}^3, \forall \ell = (i, j) \in \mathcal{L}
\]

\[
y_{\ell,i} \in \mathbb{R}^3, y_{\ell,j} \in \mathbb{R}^3, \forall \ell = (i, j) \in \mathcal{L}
\]

\[
s.t. \quad (7b) - (7i), (11a) - (11d), (14a) - (14d)
\]

\[
A_f = g - d + \sum_{\ell = (i, j)} (\delta^i_\ell w_{\ell,i}d_i - \delta^j_\ell y_{\ell,i}) + \sum_{\ell = (i, j)} (\delta^j_\ell w_{\ell,j}d_j - \delta^i_\ell y_{\ell,j}) \tag{15b}
\]

\[
f_{\ell}^{\min} \mathbf{1} \leq \delta^i_\ell w_{\ell,i}d_i - \delta^j_\ell y_{\ell,i} + \delta^j_\ell w_{\ell,j}d_j - \delta^i_\ell y_{\ell,j} \leq f_{\ell}^{\max} \mathbf{1}, \quad \forall \ell = (i, j) \tag{15c}
\]

This reformulated problem can be efficiently solved by
common optimization solvers such as CPLEX and Gurobi.

IV. Numerical Results

In this section, we first use the IEEE 14-bus system to
illustrate that bus split operations can be used to effectively
relieve network congestion and help address feasibility issues
of the optimal power flow problem. After that, we perform
the substation-level topology optimization on the larger-sized
IEEE 118-bus system to demonstrate the economic improve-
ment on generation dispatch and assess the computational
complexity of the proposed topology optimization model.
The optimization problems have been implemented on a
regular laptop with Intel® CPU @ 2.60 GHz and 12 GB
of RAM in the MATLAB® R2018a simulator. The MILP-
based optimization problems are computed using the CPLEX
solver.

A. 14-Bus System Test

The IEEE 14-bus system consists of 20 transmission lines
and 5 conventional generators, and we use the ac power flow
model to test the system. The system has been slightly modi-
fied to illustrate that the bus splitting can be used to relieve
network congestion and thus can help with the feasibility
issue of the optimal power flow problem. Specifically, we
modify the transmission limits of lines (2, 3) and (3, 4) to be 100 MW and 10 MW respectively. The maximum generation limit of the generator at bus 3 is adjusted to be 20 MW, and all the other network configurations are kept unaltered.

For bus 3 in the original system as shown in Fig. 5, a net load of at least $d_3 - g_3 = 74.2$ MW needs to be satisfied by the flows from line (2, 3) and line (3, 4). Due to the electrical characteristics of the lines, power flows on both lines are governed by the phase angle at bus 3. Therefore, as we gradually increase the flow on line (2, 3), the transmission limit on line (3, 4) will be eventually violated before the sum of power flows on both lines meets the net load at bus 3, leading to an infeasible solution to the optimal power flow problem. In order to relieve the congestion on line (3, 4) in this scenario, solving the proposed topology optimization problem suggests performing bus split at bus 3 such that the generator is connected to the bus bar 3′ and the load is connected to bus bar 3″. Essentially, this bus splitting decouples the power flows on lines (2, 3) and (3, 4) by allowing each bus bar to have a different phase angle. After the bus splitting, the load at bus 3 can be easily satisfied by the flow from line (2, 3) without violating any transmission limit. Thus, the ac power flow model of the system for the updated system as shown in Fig. 5 becomes feasible. In fact, the bus splitting operation can be easily achieved through switching associated circuit breakers at the substation. The case study on this small system indicates that similar to traditional line switching and load shedding, the operation of bus splitting can be also used to relieve network congestion and help with feasibility issues. While the model used in (15) is a dc power flow model, the solution obtained which suggests a bus splitting at bus 3 makes the problem feasible also considering the ac power flow model. Next we will use a larger-sized system to illustrate the enhanced economic benefit of the proposed substation level topology optimization algorithm.

B. 118-Bus System Test

The IEEE 118-bus test case is tested for the substation-level topology optimization. The system consists of 118 buses, 186 transmission lines and 19 committed conventional generators. To illustrate the improvement and economic benefits of the topology optimization by incorporating bus splitting events, we have also tested the same system for the traditional topology optimization strategy [2]. This can be easily fulfilled by restricted $u_{w,1}$ and $u_{w,2}$ in (7) to be zero, which will exclude bus splits from consideration and only allow for the line switching operations. By doing so, constraints (7h), (7i) and (7k) always hold and are thus disabled. Meanwhile, (7j) becomes a linear constraint, which describes the network power balance without any power transfer. Accordingly, the optimization problem (7) itself constitutes a mixed-integer linear program that is readily solvable for common optimization solvers.

The comparison of total cost under different number of operations for line switching and breaker-level switching is given in Fig. 6 together with the benchmark cost for the system without any topology switching. Compared with the benchmark cost which involves no topology optimization, our proposed breaker switching strategy achieves total savings of $14.1\% - 23.4\%$ depending on the number of operations; cf. Fig. 6. Meanwhile, compared with line switching, it provides additional cost savings of $4.9\% - 7.5\%$ correspondingly. Notice that this additional savings are obtained by only altering several breakers status at the substations, therefore the economic benefits are indeed attractive for system operators.

To compare the different control strategies provided by traditional line switching and the proposed breaker switching, we have listed the topology optimization solutions for up to a maximum of $s = 5$ operations in Table I. In the line

![Fig. 5: Bus splitting at bus 3 in the IEEE 14-bus system.](image)

![Fig. 6: Comparison of total cost between line switching and breaker switching for the 118-bus system.](image)

| s   | Line Switching | Breaker Switching | Reduction |
|-----|---------------|-------------------|-----------|
| 1   | Line 128      | Bus 82            | 4.9\%     |
| 2   | Lines 128, 136| Buses 77, 82      | 5.1\%     |
| 3   | Lines 41, 128, 136 | Buses 77, 82 & Line 130 | 7.3\%     |
| 4   | Lines 119, 123, 124, 125 | Buses 77, 77, 82 & Line 136 | 6.3\%     |
| 5   | Lines 118, 121, 131, 135, 149 | Buses 77, 82 & Lines 123, 124, 125 | 7.3\%     |

TABLE I: Comparison of topology optimization Decisions for the 118-Bus System
that can incorporate both line switching and bus splitting. To deal with the bi-linearity in the formulation, the McCormick relaxation has been utilized to devise a tractable MILP reformulation, which can be efficiently solved for real-time applications. Numerical studies on the IEEE 118-bus system corroborate the efficacy of the proposed topology optimization algorithm in terms of operational cost reduction and computational complexity.

V. CONCLUSION

In this paper, we present a post-event analysis of substation bus splits and arrive at an equivalent bus-branch model for such events. Utilizing this equivalent model, we propose a substation-level network topology optimization formulation for real-time implementation.

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