Effect of a priori Information about the Target Location on Optimal Trajectories of Multiple Cooperative UAVs

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This paper investigates how a priori localization information on a stationary ground target affects the optimal trajectories of cooperative unmanned aerial vehicles (UAVs) for estimating the target location. Each UAV is assumed to be equipped with a bearing-only sensor which always directs to the target. In order to formally incorporate the effect of a priori information into the estimation process, a Bayesian framework is employed: The optimal flying directions and paths of the multiple UAVs are determined by minimizing the Bayesian Cramér-Rao Lower Bound (BCRLB) with prior information on the ground target location represented via a bivariate Gaussian Probability Density Function (PDF). The effect of the prior information is examined with varying parameters that handle the size and shape of the prior PDF. It turns out that the more elliptical the prior PDF support, the more orthogonal the flying directions to the major principal axis of the ellipse in order to reduce the information imbalance in the direction of the major principal axis. In order to reduce the computational load for the calculation of the BCRLB, we employ the Bayesian Monte-Carlo integration method, which is shown to be fast enough to compute the BCRLB online in small UAVs.

Key Words: Unmanned Aerial Vehicle (UAV), Optimal Trajectory, Prior Information, Cramér-Rao Lower Bound (CRLB), Fisher Information Matrix (FIM)

1. Introduction

Recently the demand for small unmanned aerial vehicles (UAVs) has rapidly increased in various military and civilian observation applications. In general, observation performance of a small UAV is relatively low compared to a large UAV so that multiple small UAVs can be cooperatively employed in order to enhance the observation performance. As this is directly dependent upon the geometric relation between the UAVs and the target, optimizing their flying trajectories is of great significance.

In this line of research, one of the guidance techniques in previous literature was to select the trajectories $x$ of the UAVs to minimize the Cramér-Rao Lower Bound (CRLB), i.e., the optimal control input is derived from

$$u = \arg \min F^{-1}(x, \theta),$$

subject to the vehicles’ equations of motion

$$\dot{x} = f(x, u).$$

Here, $F$ is the Fisher Information Matrix (FIM), which is the inverse matrix of the CRLB. The input $u$ usually controls the heading angle of the vehicles. For the case of two UAVs, Gu et al. showed that the observation performance is maximized when the UAV paths are orthogonal to each other (i.e., 90 deg relative bearing angle). Lee et al. yielded the same result, that the determinant of one-step FIM $F$

$$\det F = \sin^2(\phi_1 - \phi_2)/\sigma_1^2$$

is maximal at a 90 deg relative bearing angle $|\phi_1 - \phi_2|$ for a given distance $r_i$ to the target and the sensor noise variance $\sigma_0^2$ (see Fig. 1 for the definition of the bearing angle $\phi$).
that the exact target location is necessary to implement the CRLB-based techniques.

In many practical situations, some information on the target location can be obtained a priori (e.g., from the geometrical features around the target area on the ground). This can be incorporated into the estimation process to increase the observation performance. For this reason, the current research uses the Bayesian CRLB (BCRLB) as an observation performance index in order to use such prior information by representing it as a Probability Density Function (PDF). Then the optimal trajectories for the best cooperative observation will be consequentially affected by the prior information on the target location.

The BCRLB, well known as the Van Trees version of the CRLB \(^3\) or the Posterior CRLB \(^4\) (PCRLB), has been widely used in the field of sensor placement. Since a radar system has more sufficient computation resources, the BCRLB, which generally requires high computational resources, has been considered more frequently for radar systems than UAV systems. Rajagopalan et al. \(^5\) studied optimal sensor placement for a missile defense system, and the trace of the BCRLB was used as a performance index. Godrich et al. \(^6\) also employed the trace of the BCRLB as a target tracking performance index for a Multi-Input Multi-Output radar system with multiple transmitters and sensors, and optimized the arrangement of the transmitters and the sensors.

In the UAV system applications, the estimated target location obtained from the Extended Kalman Filter (EKF) is used to calculate the classical CRLB, \(^7\)-\(^10\) and the prior information is used to initialize the EKF. Ponda et al. \(^11\) showed the BCRLB calculated from the estimated target location of the EKF corresponds to the estimated covariance matrix of the EKF. On the other hand, Grocholsky et al. \(^12\) designed a coordinated and distributed control of multiple sensor platforms by maximizing the FIM of adjacent UAVs and combined the control law with the information filter. Since the information filter is theoretically equivalent to the EKF and an estimated target location is used to calculate the FIM, this study can be classified as the same line of research.

While the classical CRLB originated from the estimation of deterministic parameters, the EKF is basically an estimation technique for random parameters. To avoid such a theoretical dissimilarity in the classical CRLB-EKF approach, the current study employs one unifying theoretical framework, the Bayesian approach: We generate and analyze the optimal trajectories of multiple UAVs by minimizing the Bayesian Cramér-Rao Lower Bound \(^\text{3}\) in Section 3: In order to reduce the computational burden for the BCRLB calculation, the Bayesian Monte Carlo (BMC) integral method \(^12\) is employed, and it is shown that the calculation speed is fast enough to compute the BCRLB online in small UAVs:

In Section 5, for a two-UAV case, it was numerically found that the current Bayesian approach resulted in the same observation performance as the classical CRLB-EKF approach when the same prior information for BCRLB is used as the initial error covariance of the EKF.

### 2. Bayesian Cramér-Rao Lower Bound

Suppose that the parameter \(\theta \in \mathbb{R}^q\) to be estimated is a random vector whose probability density function is given by \(p(z|\theta)\), and consider an estimator \(\hat{\theta}\) based on a measurement vector \(z\). If the PDFs \(p(z|\theta)\) and \(\lambda(\theta)\) satisfy the regularity conditions \(E_z[\partial \ln p(z|\theta)/\partial \theta] = 0\) for all \(\theta\) and \(E_\theta[\partial \ln \lambda(\theta)/\partial \theta] = 0\), then it is known that the estimation error covariance satisfies the following Bayesian Cramér-Rao lower bound \(^3\):

\[
E_\theta[E_z[\hat{\theta}(z) - \hat{\theta}(z) - \theta)] \geq F^{-1}_B, \tag{4}
\]

where

\[
F_B \triangleq E_\theta F_D + F_P, \tag{5}
\]

\[
F_D \triangleq -E_\theta(V_\theta^2 \ln p(z|\theta)), \tag{6}
\]

\[
F_P \triangleq -E_\theta[V_\theta^2 \ln \lambda(\theta)]. \tag{7}
\]

Here \(V_\theta^2\) is used to indicate the second-order partial derivative operator \(\partial^2/\partial \theta^2\) with \(\partial^2/\partial \theta^2 = [\partial/\partial \theta_1, \partial/\partial \theta_2, \ldots, \partial/\partial \theta_q]^T\). \(F_D\) and \(F_P\) are the FIMs of the measurement and the prior information, respectively. In this
paper, $F_B$ is called the Bayesian FIM (BFIM) to distinguish it from $F_D$.

A stationary ground target is assumed to lie on a two-dimensional plane and a priori information on the target location $\theta = [\theta_x, \theta_y]^T$ is provided in the form of a probability density function $\lambda(\theta)$. In this study, we choose the zero-mean bivariate Gaussian distribution with the variance $\sigma_x^2$ of the $x$-axis and the variance $\sigma_y^2$ of the $y$-axis (see Fig. 1) given by

$$\lambda(\theta) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{\theta_x^2}{2\sigma_x^2} - \frac{\theta_y^2}{2\sigma_y^2}\right).$$

(8)

We consider $N$-UAV cases, each of which measures the target bearing angle $\phi_i[k]$ obtaining $z_i[k]$ ($i = 1, 2, \ldots, N, k = 1, 2, \ldots, M$) using a gimbaled bearing-only sensor installed in each UAV so that the sensor always points to the ground target (see Fig. 1 for $N = 2$ case). Here $i$ and $k$ denote the indices of each UAV and the measurement sequence, respectively. It is assumed that the altitude effect between the target and the UAVs can be ignored, and the exact location of each UAV $x_i[k] = [x_i[k], y_i[k]]^T$ is known to all UAVs. The measured bearing signal $z_i[k]$ is modeled as a sum of the true bearing angle $h(\theta, x_i[k])$ and a white noise sequence $w_i[k]$

$$z_i[k] = h(\theta, x_i[k]) + w_i[k],$$

(9)

where $w_i[k]$ has a zero-mean Gaussian distribution with the variance $\sigma_w^2$.

The conditional PDF of the $M$ measurements $z = [z_1[k], z_2[k], \ldots, z_M[k]]^T$ for a given $\theta$ is represented as

$$p(z|\theta) = \prod_{k=1}^{M} \prod_{i=1}^{N} p(z_i[k]|\theta),$$

(10)

where

$$p(z_i[k]|\theta) = \frac{1}{\sqrt{2\pi\sigma_w^2}} \exp\left(-\frac{1}{2\sigma_w^2}(z_i[k] - h(\theta, x_i[k]))^2\right),$$

(11)

$$h(\theta, x_i[k]) = \tan^{-1}\left(\frac{\theta_y - y_i[k]}{\theta_x - x_i[k]}\right).$$

(12)

Then plugging Eq. (10) into Eq. (6) yields the FIM (see Appendix A),

$$F_D = \begin{bmatrix} F_{D,11} & F_{D,12} \\ F_{D,21} & F_{D,22} \end{bmatrix},$$

(13)

where

$$F_{D,11} = \sum_{k=1}^{M} \sum_{i=1}^{N} F_{D,11,i}[k],$$

(14)

$$F_{D,12} = \sum_{k=1}^{M} \sum_{i=1}^{N} F_{D,12,i}[k] = F_{D,21},$$

(15)

$$F_{D,22} = \sum_{k=1}^{M} \sum_{i=1}^{N} F_{D,22,i}[k],$$

(16)

with

$$F_{D,11,i}[k] = \frac{1}{\sigma_x^2} \left[ \frac{(\theta_x - x_i[k])^2}{\sigma_x^2} + \frac{\theta_y^2}{\sigma_y^2} \right],$$

(17)

$$F_{D,12,i}[k] = \frac{1}{\sigma_x^2} \left[ \frac{(\theta_x - x_i[k])(\theta_y - y_i[k])}{\sigma_x^2} + \frac{(\theta_y - y_i[k])^2}{\sigma_y^2} \right],$$

(18)

$$F_{D,22,i}[k] = \frac{1}{\sigma_y^2} \left[ \frac{(\theta_y - y_i[k])^2}{\sigma_y^2} + \frac{(\theta_x - x_i[k])^2}{\sigma_x^2} \right].$$

(19)

Using Eqs. (7) and (8), the FIM of the Gaussian distribution is given by,

$$F_P = \begin{bmatrix} 1/\sigma_x^2 & 0 \\ 0 & 1/\sigma_y^2 \end{bmatrix},$$

(20)

Finally the BFIM becomes

$$F_B = \begin{bmatrix} F_{B,11} & F_{B,12} \\ F_{B,21} & F_{B,22} \end{bmatrix},$$

(21)

where

$$F_{B,rs} = \iint\ F_F(\theta_1, \theta_2) d\theta_1 d\theta_2 + F_{P,rs} \quad (r, s = 1, 2).$$

(22)

3. UAV Deployment for Optimal Observation

We investigate the effect of a priori information about the target location on the optimal UAV deployment maximizing the BFIM, Eq. (21). $F_D$ is dependent on $\theta$ and $N$-UAV locations $x_i[k]$ ($i = 1, 2, \ldots, N, k = 1, 2, \ldots, M$), and $F_P$ is a constant matrix for a given $\lambda(\theta)$. After $F_D$ is integrated over $\lambda(\theta)$, the inverse of the BCRLB, is dependent only on $x_i[k]$. Thus, the observation performance is maximized (i.e., the BCRLB is minimized) by proper UAV deployment. Then, the optimal UAV deployment is achieved by minimizing the determinant of the BCRLB, which can be converted to maximizing the determinant of the BFIM (note $\det(A^{-1}) = (\det A)^{-1}$).

In order to see the difference between the previous studies\textsuperscript{1,2} using the classical FIM and current Bayesian FIM approach, we consider a simple two-UAV case with one-step measurement ($M = 1, N = 2$ in Eqs. (14)–(16)) and the mean target location at the origin, as shown in Fig. 1. Then,
$F_B$ becomes a function of the initial UAV locations $x_i[k]$ $(i = 1, 2, k = 1)$, and we can find the optimal geometric relations between the two UAV locations and the mean target location of a given prior PDF by maximizing $\text{det}(F_B)$.

Each UAV location is expressed as in the polar coordinate system, for $i = 1, 2$,

$$
\begin{align*}
x_i &= R_i \cos(\phi_i + \pi), \\
y_i &= R_i \sin(\phi_i + \pi),
\end{align*}
$$

(23)

where $R_i$ is the distance from the $i$-th UAV to the origin, which is equal to the mean target location, and $\phi_i$ is its bearing angle to the origin. The index $k$ is omitted for a brief presentation.

In order to analyze the effect of the various shapes and sizes of the prior PDF on observation performance, we define the following two major normalized parameters:

- The first parameter is termed as the prior information balance and defined as

$$
\eta \triangleq \frac{\sigma_2^2}{\sigma_1^2}
$$

(24)

which is the ratio of the $x$-axis variance to the $y$-axis variance of the prior PDF. For example, in the case of $\eta = 1$, the support of the prior PDF is circular. A $\eta > 1$ case has an elliptical PDF support whose main axis is parallel to the $x$-axis and provides more information in the $y$-axis direction than in the $x$-axis direction.

- The second parameter is termed as the relative prior information to the data and defined as

$$
\rho \triangleq \frac{A_D}{A_P}
$$

(25)

which represents the relative amount of the prior information to the one-step measurement data.

Here $A_D \triangleq \pi(\text{det}(F_D))_{i=1}^{\infty} \Delta\phi_i^{1/2}$ is the area of the 1-$\sigma$ confidence region of the one-step measurement at the optimal relative bearing angle $\Delta\phi$. For example, the two-UAV case has 90 deg of $\Delta\phi$ and $A_D = \sigma_2^2 R_1 R_2 \pi$. $A_P = \sigma_1 \sigma_2 \sigma_3 \pi$ is the area of the 1-$\sigma$ confidence region of the prior PDF. Therefore, in the case of a big $\rho$, the amount of prior information is relatively large compared to the one-step measured information.

Then, the optimal deployment problem of multiple UAVs is to find the optimal set of bearing angles $\phi_i$, $i = 1, 2, \cdots, N$ for given parameter values $\sigma_1$, $\sigma_2$, $\sigma_3$, and $R_1$, $(\phi_1, \phi_2, \cdots, \phi_N)$

$$
\phi_i = \arg \max \text{det}(F_B)(\sigma_1, \sigma_2, \sigma_3, R_1, R_2, \cdots, R_N).
$$

(26)

Within this framework, we consider a two-UAV case for the different parameter sets and investigate the effect of each parameter change.

### 3.1. Effect of prior information balance $\eta$

For a fixed initial distance from each UAV to the mean target location $R_1 = R_2 = 500$ m, Fig. 2(a), (b), and (c) show the optimal bearing angles obtained by solving Eq. (26) for $\eta = 1, 1.5$, and 2.5, respectively. Other parameters used are $\sigma_0 = 0.1$ rad ($\approx$5.7 deg), and $\rho = 2$ ($A_D = 2500\pi$ m$^2$, $A_P = 1250\pi$ m$^2$). In the figures, the ellipse represents the 1-$\sigma$ line of each prior PDF, and the straight line is the optimal line-of-sight from each UAV to the mean target location (the origin).

In the case of $\eta = 1$ (Fig. 2(a)), observation performance is maximized when the relative bearing angle $\Delta\phi$ is 90 deg. This result is exactly the same as that of the classical CRLB approaches. However, the cases of $\eta = 1.5$ and 2.5 have relative angles $\Delta\phi$ less than 90 deg (Fig. 2(b) and (c)). As shown in Fig. 3, as $\eta$ increases, the optimal relative bearing angle $\Delta\phi$ decreases. UAVs tend to be deployed to get more information in the $x$-axis direction since the uncertainty in this direction is higher than that of the $y$-axis. For approximately $\eta > 2.7$, the optimal $\Delta\phi$ is 0, which means that UAVs are deployed to gather only $x$-axis direction information. The behavior of UAVs will be further explained in Section 4.

### 3.2. Effect of relative prior information to the data $\rho$

Figure 4 describes the changes of the optimal relative bearing angle $\Delta\phi$ for various values $\rho = 1$, 2, and 4 ($A_P = 2500\pi$ m$^2$, 1250$\pi$, 625$\pi$ for a fixed $A_D = 2500\pi$ m$^2$).

The results show that the bigger the relative prior information $\rho$, the faster the optimal relative bearing angle changes.
This means that the optimal deployment of the UAVs is more influenced by the shape of the prior PDF in the cases with more abundant prior information.

4. Bayesian Information-based Guidance Law

The previous section focused only on finding a strategy for the optimal initial two-UAV deployment with one-step measurement (\(M = 1\)) for a given distance from each UAV to the mean target location. The main issue of this section is to further analyze the effect of the prior information about the target location on UAV trajectories generated by Bayesian information-based guidance with multiple measurement data. Here a guidance law for small UAVs, considered as point masses, is designed in a similar manner to the Bayesian information-based guidance with multiple measurement data. It is also assumed that the location of each UAV is known exactly, and the communication between the UAVs is perfect so that all information on the UAV locations and the sensor measurement data, is used as the prior PDF in the next stage.

We assume that the UAVs are perfectly controlled by the guidance, and all UAVs have a constant velocity \(v\), and the dynamics of \(N\)-UAVs can be written as

\[
\dot{x}_i(t) = v \left[ \cos u_i(t) \quad \sin u_i(t) \right], \quad i = 1, 2, \ldots, N. \tag{27}
\]

It is also assumed that the location of each UAV is known exactly, and the communication between the UAVs is perfect so that all information on the UAV locations and the sensor measurement data are shared immediately. Furthermore, all sequences of the UAV measurements and guidance inputs are synchronized.

The new cost function using the one-step BFIM is given by

\[
J = -\text{det} F_{\theta}(t + \Delta t) + \int_{t}^{t + \Delta t} \ell^T(f - \dot{x}) dt, \tag{28}
\]

where \(\ell = [\ell_{11} \ell_{12} \cdots \ell_{1N} \ell_{2N}] \in \mathbb{R}^{2N}\) denotes a Lagrange multiplier vector, \(x = [x_1^T x_2^T \cdots x_N^T]^T\) denotes the location vector of the UAVs, and \(\Delta t\) denotes the time horizon. As insisted in Lee et al.,\(^2\) the resulting trajectories using this one-step time horizon will not be globally optimal, but sufficiently efficient and proper commands could be generated. For the effects of time horizon on path planning, readers are referred to Ousingsawat and Campbell.\(^{10}\)

The solution of the optimal problem minimizing Eq. (28) is obtained by applying the optimal control theory\(^{11}\) (see Appendix B for the derivation).

\[
\dot{\ell} = 0, \tag{29}
\]

\[
\partial \ell^T f / \partial u = 0, \tag{30}
\]

\[
\ell^T(t_f) = \partial(\text{det} F_{\theta}) / \partial x_i |_{t_f}, \quad t_f = t + \Delta t. \tag{31}
\]

Then, solving Eqs. (29)–(31) yields

\[
\ell = \text{constant}, \tag{32}
\]

\[u_i = \tan^{-1}(\ell_{ij} / \ell_{ik}), \quad i = 1, \ldots, N, \tag{33}\]

\[
\ell^T = -F_{B,11} \left( \frac{\partial F_{B,22}}{\partial x} \right) + \left( \frac{\partial F_{B,11}}{\partial x} \right) F_{B,11} \tag{34}\]

\[-2F_{B,12} \left( \frac{\partial F_{B,12}}{\partial x} \right). \]

Since this result is a continuous-time solution, the guidance command \(u_i\) is discretized at each observation sequence and kept constant during each period.

When a new measurement is available, the prior PDF is updated to a posterior PDF, which in turn, can be used as a prior PDF in the next stage. Thus, the prior PDF at the \(k\)-th stage can be written as

\[
\lambda_k(\theta) = \lambda(\theta | z[1], \ldots, z[k]) = \frac{1}{n_k} p(z[1], \ldots, z[k] | \theta) \lambda_0(\theta), \tag{35}\]

where

\[
n_k = \int p(z[1], \ldots, z[k] | \theta), \lambda_0(\theta) d \theta. \tag{36}\]

Since each measurement is independent, Eq. (35) can be rewritten as

\[
\lambda_k(\theta) = \frac{1}{n_k} \lambda_0(\theta) \prod_{j=1}^{k} p(z[j] | \theta) \tag{37}\]

\[
= \frac{n_{k-1}}{n_k} p(z[k] | \theta) \lambda_{k-1}(\theta). \tag{38}\]

Then, the \((k + 1)\)-th BFIM of the prior PDF becomes

\[
F_p[k + 1] = E_{\theta | z} \left[ \nabla_{\theta} \ln \lambda_k(\theta) \right]
\]

\[
= E_{\theta | z} \left[ \nabla_{\theta} \left( \ln \frac{n_{k-1}}{n_k} + \ln p(z[k] | \theta) + \ln \lambda_{k-1}(\theta) \right) \right]
\]

\[
= E_{\theta | z} [F_D[k]] + F_p[k]. \tag{39}\]

Here, \(E_{\theta | z}[F_D[k]]\) is obtained from Eqs. (14)–(16) by taking the expectation over the \(\lambda_{k-1}(\theta)\). Then, the elements of the BFIM in Eq. (22) for Eq. (34) at the \(k\)-th stage can be rewritten as

\[
F_{B,rs}[k] = \int \left( \sum_{i=1}^{N} F_{D,rs}[k] \right) \lambda_{k-1}(\theta_r, \theta_s) d\theta_r d\theta_s + F_{P,rs}[k], \tag{39}\]

and the partial derivative terms in Eq. (34) are given by

\[
\frac{\partial F_{B,rs}[k]}{\partial x_i} = \int \left( \frac{\partial F_{D,rs}[k]}{\partial x_i} \right) \lambda_{k-1}(\theta_r, \theta_s) d\theta_r d\theta_s, \quad (r = 1, 2, s = 1, 2). \tag{40}\]

Finally, the discretized guidance command \(u[k]\) in Eq. (33) can be calculated through Eqs. (34)–(40).

This formulation can be extended to a three-dimensional problem with additional computational burden. In this case,
the prior information can be given as an ellipsoid shape, measurements can consist of bearing angle and vertical angle, and UAV dynamics are given based on the three-dimensional coordinates system.

In order to reduce the computational load required for the multiple integrations, the Bayesian Monte Carlo (BMC) integral method\(^\text{12}\) is applied. The run time of the guidance calculation of each step for a UAV is 0.025 s on average in a MATLAB environment using an Intel Core i7-3770 CPU.

A set of 100-run Monte Carlo simulations were performed for different prior information cases (\(\eta = 0.04, 0.2, 1, 5, \) and 25), and the mean trajectories and prior information balance \(\eta_k \triangleq F_{P,11}[k]/F_{P,22}[k]\) at the \(k\)-th observation stage are shown in Figs. 5 and 6, respectively. The simulations were conducted under the same conditions as in Section 3: The mean target location is fixed at the origin, but, two UAVs start from the fixed locations at \(R = 500 \text{ m}, \phi_1 = 45 \text{ deg}, \phi_2 = 135 \text{ deg}\). The speed of UAVs is 20 m/s, and the guidance commands are updated at 1 Hz. The final time of each simulation is chosen such that one of the UAVs approaches the estimated target location within 20 m.

In Fig. 5, the regions indicated as 1 and 2 represent the areas of \(|y| > |x|\) and \(|y| < |x|\), respectively, and 3 denotes the line of \(y = \pm x\). Based on Eqs. (17)–(19), a UAV flying in region 1 can gather more \(x\)-axis information than that of the \(y\)-axis from the measurement data (i.e., \(F_{D,11} > F_{D,22}\)). On the contrary, for a UAV flying in region 2, there is more \(y\)-axis information than that of the \(x\)-axis (i.e., \(F_{D,11} < F_{D,22}\)). If a UAV is on the line of \(y = \pm x\), then the information amounts of each axis are the same.

Therefore, for the case of \(\eta = 1\), the optimal UAV trajectories are generated along the line of \(y = \pm x\). For \(\eta > 1\), the UAV trajectories are curved inwards in the early stage to gather more \(x\)-axis information. On the contrary, for the cases of \(\eta < 1\), the UAV trajectories are curved outwards in the early stage so that more \(y\)-axis information can be gathered.

These trends of the optimal UAV trajectories can be explained in connection with the change of the prior information balance \(\eta_k\) at the \(k\)-th observation stage as shown in Fig. 6. As the cooperative observation progresses, \(\eta_k\) approaches to 1 in all cases. This means that the guidance tends to reduce the information imbalance between the two directions.

From the perspective of balancing the information, this trend can be explained as follows: If there is information imbalance in a direction, in the early phase, the flying paths of the UAVs deviate from the line of \(y = \pm x\) to reduce the information imbalance and return to the line of \(y = \pm x\) after balancing the amount of information in both directions.

Figure 7 shows a three-UAV case for different prior information balances. In this case, \(A_D\) is redefined as \(\pi(\text{det}(F_{D,ij})_{k=1,\Delta \phi=120 \text{ deg}})^{1/2}\) since the optimal relative bearing angle for three UAVs is 120 deg in the classical approach.

UAV1 and UAV2 show similar tendencies to those of the previous two-UAV cases, but the trajectories of UAV3 do not change for different \(\eta\) values, which causes UAV3 to gather only \(x\)-axis information. For non-unity \(\eta\) cases, the amount of \(y\)-axis information is more sensitive to UAV1 and UAV2 than UAV3. For these reasons, UAV3 is allocated to contribute only to \(x\)-axis information, and UAV1 and UAV2 are arranged to obtain \(y\)-axis information in order to maximize the total information amount. The prior information balance \(\eta_k\) of this case is depicted in Fig. 8. Similar to Fig. 6, \(\eta_k\) approaches to 1 in all cases.

The optimal trajectories of UAV1 and UAV2 in the three-UAV case are compared with those of a two-UAV case for
two different sets of initial locations of UAV1 and UAV2 (Configuration-I and Configuration-II). Initial UAV1 and UAV2 locations in Configuration-I and Configuration-II, respectively, correspond to the optimal initial locations of two-UAV (Fig. 5) and three-UAV (Fig. 7) cases, while UAV3 initial location is fixed for both configurations of the three-UAV case.

Figures 9 and 10 describe the differences of the optimal trajectories of Configuration-I and Configuration-II, respectively. As for prior information, $\eta = 1$ and $\rho = 2$ are used for both configurations. It is commonly observed that the optimal trajectories of UAV1 and UAV2 of the three-UAV case are closer to the $x$-axis than those of the two-UAV case, so that UAV1 and UAV2 of the three-UAV case can concentrate on gathering $y$-axis information of the target location. This is because UAV3 can gather $x$-axis information more effectively than UAV1 and UAV2, and can share the information with UAV1 and UAV2.

Figure 11 shows the trajectories of a different three-UAV case with $\phi_1 = 30$ deg, $\phi_2 = 150$ deg, and $\phi_3 = 320$ deg. The trajectories of the UAVs show similar trends to the previous cases (Figs. 5 and 7). In cases where the prior information has more $y$-axis information than $x$-axis information ($\eta < 1$ case), the trajectories are curved toward the $x$-axis to gather more $y$-axis information in the early observation stage. In the early observation stage of $\eta \geq 1$ cases, the trajectories are curved toward the $y$-axis to reduce the information imbalance caused from the initial locations (all UAVs are in area $\Omega$ of Fig. 5) and the prior information.

Figure 12 shows the prior information balance $\eta_k$ of this case which shows a similar tendency to those of Figs. 6 and 8 in that $\eta_k$ returns back to 1 as the observation pro-
The conclusion for the EKF of the classical CRLB-EKF approach. For the Bayesian approach was used as the initial error covariance, the covariance of the bivariate Gaussian prior PDF progresses. From this result, we can reconfirm that the the guidance tends to reduce the information imbalance regardless of the number of UAVs or initial locations.

5. Performance Comparison

The observation performance of the current Bayesian approach was compared with the classical CLRB-EKF approach\(^3\) for the two-UAV case. In order to endow the classical CLRB-EKF approach with the same level of prior information, the covariance of the bivariate Gaussian prior PDF for the Bayesian approach was used as the initial error covariance for the EKF of the classical CRLB-EKF approach.

Figures 13 and 14 show the comparison results of both approaches for various levels of prior information. Jumping to the conclusion first, both approaches yielded the same observation performance for equal levels of prior information used. The difference was very minor and seems to be due to the BMC integral method used for the current Bayesian approach. However, it must be said that the performance equality is only based on numerical simulation results without formal proof, for which a continuing effort is necessary. It should also be noted that the current Bayesian framework does not require any specific estimator and provides a unifying framework for analyzing the prior information effect on optimal guidance.

In the figures, observation performance was represented as the area of the 1-σ confidence region of the estimated target location of the posterior PDF (for the Bayesian approach) and the error covariance of the EKF (for the classical CRLB-EKF approach) for the same level of prior information (Figs. 11 and 12, respectively, depict a ρ = 2 case and a ρ = 10 case).

The area of the 1-σ confidence region for the EKF is equal to \(\pi(\text{det}(\Sigma_{\text{EKF}}))^{1/2}\), where \(\Sigma_{\text{EKF}}\) is the error covariance of the EKF.

The absence of prior information for both approaches was implemented by taking a big-magnitude matrix (e.g., \(10^{10} \cdot I\)) as an initial covariance for the prior PDF and the EKF. For other prior information cases, the same area (1250π m²) of the 1-σ confidence region but with different prior shapes (\(\eta = 0.04, 0.2, 1, 5, 25\)) was used as the initial condition.

In the early phase of observation, the cases with prior information outperforms the case without prior information, and the performance difference decreases as the observation progresses. It is also found that the effect of prior information remains longer in cases with abundant prior information.

6. Conclusion

This paper addressed the effect of a priori information about the target location (expressed as a PDF) on the trajectories of multiple cooperative UAVs for optimal observation performance using Bayesian information-based guidance. The determinant of the Bayesian Cramér-Rao Lower Bound was employed as the performance index and minimized.

The optimal UAV initial deployment was affected by the prior information balance between the x-axis and y-axis. When the support of the prior PDF is circular, the optimal relative bearing angle is 90 deg for the two-UAV case. However, when the prior support is elliptical, the optimal relative bearing angle is different from 90 deg.

Bayesian information-based guidance was also developed based on the optimal control theory, and the posterior PDF was recursively used as a prior PDF in the next stage to calculate the Bayesian Fisher Information Matrix. In order to avoid the computational load due to the Bayesian approach, we employed the Bayesian Monte Carlo integration method to calculate the guidance online. The effect of the prior information was strong when the amount of the prior information was large.

For a two-UAV case, the performance of the current approach was numerically compared with the classical CRLB-EKF approach, yielding the exact same result. The formal proof of this observation is an issue of the future research. Although this study only used bivariate Gaussian distribution, any arbitrary distribution satisfying the regularity condition can be applied using special techniques such as...
the Gaussian mixture, and it can be extended to the estimation problems in three-dimensional situations.

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Appendices

A. Derivation of FIM for a bearing-only sensor

From Eqs. (6) and (10),

\[ F_{D,r} = -E_{z|\theta} \left[ \frac{\partial^2 \ln p(z[k]|\theta)}{\partial \theta_i \partial \theta_j} \right] \]

\[ = -E_{z|\theta} \left[ \frac{\partial^2}{\partial \theta_i \partial \theta_j} \ln \prod_{k=1}^{M} \prod_{i=1}^{N} p(z[k]|\theta) \right] \]

\[ = -\sum_{k=1}^{M} \sum_{i=1}^{N} E_{z|\theta} \left[ \frac{\partial^2}{\partial \theta_i \partial \theta_j} \right] \]

Let \( F_{D,rs} \) denote the \((r, s)\) element of the FIM for the \(k\)-th measurement of the \(i\)-th UAV. The second-order partial derivative terms of \( \ln p(z[k]|\theta) \) are obtained

\[ \frac{\partial^2}{\partial \theta_i \partial \theta_j} \ln p(z[k]|\theta) \]

(42)

The terms containing \((z[k] - h)\) in Eqs. (42)–(44) become zeros by the expectation using \( p(z[k]|\theta) \). Then, Eqs. (17)–(19) are derived from

\[ \frac{\partial}{\partial \theta_i} \left[ \frac{\partial}{\partial \theta_j} \right] \ln \left( \frac{\theta_i - y_i[k]}{\theta_i - x_i[k]} \right) \]

(45)

The situation of this study corresponds to \( L = 0 \), \( \phi = -\det B \), \( H = \Sigma \), \( \ell = \cos u + \ell_3 \sin u \). Plugging these conditions into Eqs. (49)–(51) yields Eqs. (29)–(31).

B. Derivation of the optimal guidance law

In order to derive the current guidance law based on Bryson, consider the system described by

\[ \dot{x}(t) = f(x(t), u(t), t); \quad x(t_0) \text{ given}, \quad t_0 \leq t \leq t_f. \]

Then, the augmented performance index constrained by Eq. (47) is used as

\[ J = \phi(x(t_f), t_f) \]

(48)

The solution of minimizing \( J \) is given by Bryson as follows

\[ \ell = -\phi / \partial \alpha = -\phi / \partial \alpha - \ell^T \partial f / \partial x, \]

(49)

(50)

(51)

where

\[ H(x(t), u(t), \ell(t), t) = L(x(t), u(t), t) + \ell^T(t) f(x(t), u(t), t), \]

(52)

The situation of this study corresponds to \( L = 0 \), \( \partial f / \partial x = 0 \), \( \phi = -\det BF \), \( H = \Sigma \), \( \ell = \cos u + \ell_3 \sin u \). Plugging these conditions into Eqs. (49)–(51) yields Eqs. (29)–(31).

C. Bayesian Monte-Carlo integration method

The Bayesian Monte-Carlo (BMC) integration method is
appropriate for the following integral form

\[ f = \int h(\theta) \lambda(\theta) d\theta, \quad (53) \]

where \( \theta \) is a \( m \times 1 \) parameter vector and \( h(\theta) \) is a general function to be evaluated by taking expectation on PDF \( \lambda(\theta) \).

The features of the method are that particles are generated according to the PDF and the weight of each particle is calculated based on the Kernel. The Kernel is a function describing the relation of each particle and can be modeled like uniform, quadratic, Gaussian distribution. The integration of Eq. (53) is calculated using the following five steps:

**Step 1. Particle generation:** Generate \( n \) particles,

\[ \Theta = [\theta_1, \cdots, \theta_n] \quad (54) \]

according to the PDF \( \lambda(\theta) \).

**Step 2. Kernel matrix calculation:** Calculate the Kernel matrix by plugging the generated particles into the selected Kernel function

\[ Q = K + \sigma^2 I, \quad (55) \]

where

\[ K_{ij} = w_0 \exp\left[ -\frac{1}{2}(\theta_i - \bar{\theta}_j)^T D^{-1} (\theta_i - \bar{\theta}_j) \right], \quad (i, j = 1, \cdots, n). \quad (56) \]

Here \( \sigma^2 \) is a small value for the numerical stability, \( w_0 \) is a scaling factor, and \( D \) is the hyper-parameter matrix expressed by \( \text{diag}(w_1^2, \cdots, w_m^2) \).

**Step 3. Weights calculation of particles:** Calculate the weight of each particle. The meaning of the weight is the probability density of the Gaussian process of each particle.

\[ a = [a_1, \cdots, a_n] \]

\[ a_l = \frac{w_0}{\sqrt{\det(D^{-1} C + I)}} \exp\left[ -\frac{1}{2}(\theta_l - \bar{\theta})^T (D + C)^{-1} (\theta_l - \bar{\theta}) \right], \quad (l = 1, \cdots, n). \quad (57) \]

where \( C \) is covariance of the PDF \( \lambda(\theta) \).

**Step 4. Object function value calculation at particles:** Calculate the object function values at each particle,

\[ b = [h(\theta_1), \cdots, h(\theta_n)]^T. \quad (58) \]

**Step 5. Integral value calculation:** Calculate the integral value using results from Step 2–Step 4.

\[ f = a Q^{-1} b. \quad (59) \]

Although this method has an additional step for the weight calculation of particles, a fewer number of particles can yield good performance compared to Monte Carlo integration. Especially, this method is more efficient with larger dimension parameters. For more information see Ghahramani and Rasmussen.12)