RESEARCH PAPER

A New Hybrid Differential Evolution with Gradient Search for Level Set Topology Optimization

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A B S T R A C T:
Topology optimization is an effective structural optimization concept for optimal design of engineering structures. However, it has many difficulties due to high number of design variables and complex problems same as compliant mechanisms and crashworthiness. Conventional methods for topology optimization does not have enough adaptability with current computer aided design (CAD) softwares and they are not powerful in solving difficult optimization problems. Level set which is a novel boundary tracking method had been recently used to solve problems in conventional methods. This paper is dedicated to propose a new hybrid method based on differential evolution (DE) and globally convergent method of moving asymptotes (GCMMA) to use both gradient direction of GCMMA and excellent exploration of DE. The method has been validated in familiar benchmark problems in compliance minimization.

KEY WORDS: Differential evolution; Method of moving asymptotes; Level set; Topology optimization.
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1. INTRODUCTION

One of the early solutions for topology optimization is proposed by Bendsøe and Sigmund (1999) and Sigmund (2001) which is called solid isotropic material with penalization (SIMP). SIMP is an interpolation scheme for phasic parametrization continuous topology optimization problem based on density. This method is the most useful method in computer aided design softwares for topology optimization. The optimality criteria and other gradient based solvers such as sequential quadratic programming are used to solve such parametrized problem. Due to the existence of gray elements in the optimized design of SIMP, which caused by considering continuous values for density, this method makes many manufacturing problems. Because by keeping or deleting the gray elements, the final design might not be the global optimum. Evolutionary structural optimization which proposed by Xie et al (1993) is an efficient method which works based on element rejection process. However, in this process the algorithm cannot regenerate the deleted algorithm. Therefore, the globality of the solution is not guaranteed. This problem had been solved by a bi-direction solution, in which the elements will be deleted based on a specific threshold (Xie and Huang 2007). A code based on Python language proves the generality of algorithm and connecting with commercial finite element software ABAQUS (Zuo et al (2015)).

Graph and heuristic topology optimization method proposed by Ortmann and Schumacher (2013) is a new optimization algorithm which had been successfully used for design of automobile rocker. However, the heuristic methods used in this...
concept are not general and they have not much mathematical background. So there are some doubts about using this method in industry. Hybrid cellular automata is another efficient surrogate topology optimization method developed by Tovar et al (2006) which is based on the concept of bone remodeling. Due to the high heuristic scheme for updating the design and homogenization of energy density, this method cannot guarantee the optimality of the solution specially in nonlinear dynamic problems.

A new solution for boundary traction in front propagation problems based on Hamilton Jacobi equation proposed by (Osher and Sethian) which is called level set method. Allaire and Jouve (2004) used gradient information to solve the topology optimization problem based on level set method. Recently, Guo et al (2014) proposed a new level set concept based on explicit parametrization. The level set function will be divided into different component and can be solved with optimization methods. Bujny et al (2016) hybridized the gradient search with covariance matrix adaptation strategy (CMA-ES), which is not so powerful while the population size increases. However, CMA-ES is used for topology optimization in high strain crash problem based on level set (Bujny et al (2018)).

Differential evolution is a new evolutionary optimization method (Storn et al 1997) which had been validated for many different industrial optimization problems. Method of moving asymptotes is a powerful gradient based tool for global optimization proposed by Svanberg (1987) which used for various complex problems in industry and research. Many researches dedicated to propose hybrid algorithms between evolutionary and gradient optimization due to good exploration of evolutionary algorithms and using the gradient direction to make optimization process work faster (Takh and Woo (2004 and 2007)).

In this paper a new hybridizing strategy between differential evolution and global convergent method of moving asymptotes is used to solve the benchmark problems in topology optimization with level set method.

2. PARAMETRIZATION WITH LEVEL SET

The concept of level set method is described in Fig.1, in which the moving surface which is level set function cuts by a plane. The result of this mapping is the material design. The level set function is described with . The material domain is described with and describes the boundaries and D is design domain. The partitioning of the level set function which shown in figure (1) is can be explained according to equation (1).

\[
\begin{cases}
\phi(x) > 0 \quad & x \in \Omega \\
\phi(x) = 0 \quad & x \in \partial \Omega \\
\phi(x) < 0 \quad & x \in D \Omega
\end{cases}
\]

Eq.(1)

In this paper the level set function has an explicit representation. The level set function is divided into level set components according to Fig.2. The level set description function for this component can be formulated according to equation (2).

\[
\phi_i(x, y) = \left( \frac{x' - x}{L_i} \right)^p + \left( \frac{y' - y}{t(x')/2} \right)^p - 1
\]

Eq.(2)

In which:

\[
\begin{align*}
x' &= \begin{bmatrix} \cos \theta_i & \sin \theta_i \end{bmatrix} \begin{bmatrix} x - x_{0i} \\ y - y_{0i} \end{bmatrix} \\
y' &= \begin{bmatrix} -\sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} x - x_{0i} \\ y - y_{0i} \end{bmatrix}
\end{align*}
\]

(4)

The thickness of components can be formulated according to equation (5).

\[
t(x') = \frac{t + t^2 - 2t^3}{L_2} (x')^2 + \frac{t^2 - t^1}{L} x' + t^3
\]

(5)

So the design vector for each component consists of seven design variables which can be formulated according to equation (6).

\[
d_i = \{x_0, y_0, L_i, t_2, t_3, s_1, \}
\]

(6)

The number of design variables will be increased with increasing of components in the design domain.

3. Optimization process

In this method, first the gradient individual will be propagated based on globally
convergent method of moving asymptotes. Then the new individuals will be created with a random normal distribution around the gradient individual. The new individuals and gradient individual will create the population of differential evolution algorithm. The optimization process is illustrated in figure (3).

As discussed before, to propagate the gradient individual, the GCMMA is used. To explain this method, consider the following general form for optimization problem:

\[
\begin{align*}
\text{Minimize } & f_0(x) + a_0 z + \sum_{i=1}^{M} (c_i y + \frac{1}{2} d_i y_i^2) \\
\text{st } & f_i(x) + a_i z - y_i \leq 0, i, j = 1, \ldots, m \\
& x_{j\min} \leq x_j \leq x_{j\max}, y_i \geq 0, z \geq 0
\end{align*}
\]

(7)

In this method, the optimization problem will be decomposed into some sub problems which can be formulated according to equation (8).

\[
\begin{align*}
\text{Minimize } & f^{*}(x) + a_0 z + \sum_{i=1}^{M} (c_i y + \frac{1}{2} d_i y_i^2) \\
\text{st } & f_i^{*}(x) + a_i z - y_i \leq 0, i, j = 1, \ldots, m \\
& \alpha_j^{k} \leq x_j \leq \beta_j^{k}, y_i \geq 0, z \geq 0
\end{align*}
\]

(8)

In which:

\[
\begin{align*}
j_j^{[k]}(x) &= j_i^{[k]}(x) + \sum_{j=1}^{n} \left( p_j^{[k]} + \frac{q_j^{[k]}}{x_j - l_j} \right) - \sum_{j=1}^{n} \left( p_j^{[k]} + \frac{q_j^{[k]}}{x_j - l_j} \right)
\end{align*}
\]

(9)

where:

\[
\begin{align*}
p_j^{[k]} &= (x_j - x_j^{[k]})^2 \left( \frac{\partial f_j}{\partial x_j}(x_j^{[k]}) \right) + \frac{\beta_j^{[k]}}{2} \left( x_j^{[k]} - l_j \right) \\
q_j^{[k]} &= (x_j - x_j^{[k]})^2 \left( \frac{\partial f_j}{\partial x_j}(x_j^{[k]}) \right) + \frac{\beta_j^{[k]}}{2} \left( x_j^{[k]} - l_j \right)
\end{align*}
\]

(10)

In which \( \rho_j^{[k]} \) is the non-monotonous parameter expressed according to equation (11).

\[
\rho_j^{[k+1]} = \begin{cases} 
2 \rho_j^{[k]} & \text{if } g_j^{[k]}(x_j^{[k+1]}), f_j(x_j^{[k+1]})) < 0 \\
\rho_j^{[k]} & \text{if } g_j^{[k]}(x_j^{[k+1]}), f_j(x_j^{[k+1]})) \geq 0
\end{cases}
\]

(11)

The ranges in equation (8) can be described according to equation (12).

\[
\alpha_j^{[k]} = \max \left\{ x_j^{\min}, 0.91 x_j^{[k]} + 0.1 x_j \right\} \\
\beta_j^{[k]} = \min \left\{ x_j^{\min}, 0.9 x_j^{[k]} + 0.1 x_j \right\}
\]

(12)

In equation (9), \( u_j^{[k]} \) is the lower asymptote and \( l_j^{[k]} \) is the upper which can be updated according to equation (13).

\[
\begin{align*}
l_j^{[k]} &= x_j^{[k]} - s_j^{[k]} \left( x_j^{(k-1)} - l_j^{(k-1)} \right) \\
u_j^{[k]} &= x_j^{[k]} - s_j^{[k]} \left( u_j^{(k-1)} - x_j^{(k-1)} \right)
\end{align*}
\]

(13)

After propagation of gradient individual, the new individuals will produce according to equation (12).

\[
\gamma_i = \mu + (\epsilon_{\max} - \epsilon_{\min}) \times N(0,1)
\]

(14)

In which \( \gamma_i \) is the individual, \( \mu \) is the individual propagated with GCMMA and \( \epsilon_{\max}, \epsilon_{\min} \) are the lower and upper bounds of design variables. These individuals will be used as the populations of differential evolution for evolutionary optimization. The optimization algorithm is described in figure (4).

The problem considered in this paper is the compliance minimization which can be formulated according to equation (15).

\[
\begin{align*}
\min & \quad C = \sum_{i=1}^{n} f^i u dV + \int t u dS \\
\text{s.t } & \quad V \leq V_0 \\
\sum_{i=1}^{n} \int E^i : \varepsilon(u) : \varepsilon(v) dV = \sum_{i=1}^{n} \int f^i v dV + \int t v dS
\end{align*}
\]

(15)

In equation (15), \( f^i, t \) are the body force and traction respectively and \( u, v \) are the displacement fields. The interpolation for Young’s module is according to equation (16).

\[
E^e = \frac{1}{4} \sum_{i=1}^{4} (H(g_i^e))
\]

(16)

In equation (16), the Heaviside function is as follows:
Due to find the sensitivity information of objective function, the chain rule had been used and the derivative of Heaviside function derived via finite difference method. Equation (18), (19) are the sensitivity analysis of objective function and constraint respectively.

\[
\frac{\partial C}{\partial \omega} = -u^T \frac{\partial K}{\partial \omega}
\]  
\[
\frac{\partial V}{\partial \lambda} = 0.25 \times \frac{\partial H (\phi)}{\partial \lambda}
\]

In which \(\omega\) is the design variables. The problem considered in this paper is the cantilever beam problem, shown in figure (5).

First example is a square cantilever beam with length and width of 10. A 50*50 finite element grid has been considered for optimization process. The optimization results are available in figure 6. The variation of cost function and constraint in available in figure (7) and the variation of constraint is visible in figure (8)

As it is obvious from figure (7), the optimization process had been converged into the global optimum after nearly 20 iterations. According to figure (8), the constraint has high changes between the first 23 iterations and converged after 50 iterations.

4. Conclusion

In this paper it is tried to introduce a new hybrid evolutionary optimization algorithm for level set topology optimization. The optimization algorithm had been used for benchmark cantilever beam problem and the effectiveness of the algorithm is visible in the figures. Thankful to the generality of optimization method and the great mathematical foundations of GCMMA and stochastic basis of differential evolution this method can be useful in many industrial problems specially when the sensitivity information is rough.

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Start with diagonal design

Propagation of gradient individual with GCMMA

Create new individuals with mutation and normal distribution

Differential evolution optimization

Converged?

End

Figure 4. The flow chart of hybrid GCMMA-DE algorithm.

Figure 5. Cantilever beam problem.

Figure 6. Results of optimization for short cantilever beam.

Figure 7. Optimization process for compliance minimization.

Figure 8. Variation of constraint in optimization process.

REFERENCES

Sigmund, O. (2001). A 99-line topology optimization code written in Matlab. Structural and multidisciplinary optimization, 21(2), 120-127.

Bendsøe, M. P., & Sigmund, O. (1999). Material interpolation schemes in topology optimization. Archive of applied mechanics, 69(9-10), 635-654.

Xie, Y. M., & Steven, G. P. (1993). A simple evolutionary procedure for structural optimization. Computers & structures, 49(5), 885-896.

Huang, X., & Xie, Y. M. (2007). Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method. Finite Elements in Analysis and Design, 43(14), 1039-1049.

Zuo, Z. H., & Xie, Y. M. (2015). A simple and compact Python code for complex 3D topology optimization. Advances in Engineering Software, 85, 1-11.

Ortmann, C., & Schumacher, A. (2013). Graph and heuristic based topology optimization of crash loaded structures. Structural and Multidisciplinary Optimization, 47(6), 839-854.

Tovar, A., Patel, N. M., Niebur, G. L., Sen, M., & Renaud, J. E. (2006). Topology optimization using a hybrid cellular automaton method with local control rules. Journal of Mechanical Design, 128(6), 1205-1216.

Osher, S., & Sethian, J. A. (1988). Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi

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Jacobi formulations. Journal of computational physics, 79(1), 12-49.

Allaire, G., Jouve, F., & Toader, A. M. (2004). Structural optimization using sensitivity analysis and a level-set method. Journal of computational physics, 194(1), 363-393.

Guo, X., Zhang, W., & Zhong, W. (2014). Doing topology optimization explicitly and geometrically—a new moving morphable components based framework. Journal of Applied Mechanics, 81(8), 081009.

Bujny, M., Aulig, N., Olhofer, M., & Duddeck, F. (2016, July). Hybrid evolutionary approach for level set topology optimization. In Evolutionary Computation (CEC), 2016 IEEE Congress on (pp. 5092-5099). IEEE.

Bujny, M., Aulig, N., Olhofer, M., & Duddeck, F. (2018). Identification of optimal topologies for crashworthiness with the evolutionary level set method. International Journal of Crashworthiness, 23(4), 395-416.

Storn, R., & Price, K. (1997). Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces. Journal of global optimization, 11(4), 341-359.

Svanberg, K. (1987). The method of moving asymptotes—a new method for structural optimization. International journal for numerical methods in engineering, 24(2), 359-373.

Tahk, M. J., Woo, H. W., & Park, M. S. (2007). A hybrid optimization method of evolutionary and gradient search. Engineering Optimization, 39(1), 87-104.

Woo, H. W., Kwon, H. H., & Tahk, M. J. (2004). A hybrid method of evolutionary algorithms and gradient search. In 2nd International Conference on Autonomous Robots and Agents.