COLORNET - ESTIMATING COLORFULNESS IN NATURAL IMAGES

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ABSTRACT

Measuring the colorfulness of a natural or virtual scene is critical for many applications in image processing field ranging from capturing to display. In this paper, we propose the first deep learning-based colorfulness estimation metric. For this purpose, we develop a color rating model which simultaneously learns to extract the pertinent characteristic color features and the mapping from feature space to the ideal colorfulness scores for a variety of natural colored images. Additionally, we propose to overcome the lack of adequate annotated dataset problem by combining/aligning two publicly available colorfulness databases using the results of a new subjective test which employs a common subset of both databases. Using the obtained subjectively annotated dataset with 180 colored images, we finally demonstrate the efficacy of our proposed model over the traditional methods, both quantitatively and qualitatively.

Index Terms— Colorfulness, CNN, Color metric, Deep learning.

1. INTRODUCTION

Color is a crucial factor in human visual perception. It affects human behavior and decision processes both in nature and in society, as color conveys pivotal information about the surroundings. Thus, its accurate acquisition and display are necessary for multimedia, entertainment, and image processing systems.

Within the imaging pipeline, the color or brightness information of the real (or virtual) scene needs to be processed for various reasons such as color grading [1], tone-mapping [2] for high dynamic range (HDR) scenes, gamut mapping [3], color correction [4]. During the acquisition or the processing, color of the scene can be affected by color cast or colorfulness changes [5]. Within the scope of this study, only the colorfulness aspect of the natural images is considered.

Colorfulness is generally defined as the amount, intensity, and saturation of colors in an image [6]. Understanding and estimating colorfulness is quintessential for a variety of applications, e.g., HDR tone-mapping [4,7,10], aesthetics image analysis [11,12], color reproduction in cameras [13], image/video quality assessment [14,17], virtual reality [18] etc. For colorfulness estimation, several methods have been proposed [5,11,16,17,19] in the literature, in addition to the techniques used in the color appearance models [6,20,21] (see Section 2 for more discussion).

To explore the competence of learning-based approaches and to create a stepping stone for further analysis of human perception with deep learning methods, in this paper, we propose a novel deep learning-based objective metric ‘ColorNet’ for the estimation of colorfulness in natural images. Based on a convolutional neural network (CNN), our proposed ColorNet is a two-stage color rating model, where at stage I, a feature network extracts the characteristics features from the natural images and at stage II, a rating network estimates the colorfulness rating. To design our feature network, we explore the designs of the popular high-level CNN based feature models such as VGG [22], ResNet [23], and MobileNet [24] architectures which we finally alter and tune for our colorfulness metric problem at hand. We also propose a rating network which is simultaneously learned to estimate the relationship between the characteristic features and ideal colorfulness scores.

In this paper, we additionally overcome the challenge of the absence of a well-annotated dataset for training and validating ColorNet model in a supervised manner. To this end, we combine two publicly available colorfulness databases [5,19] using the results of a new subjective test which employs a common subset of both databases. The resulting ‘Combined’ dataset contains 180 color images and corresponding subjective colorfulness scores. Finally, we compare and showcase how our ColorNet model outperforms the state-of-the-art traditional colorfulness estimation models both quantitatively and qualitatively.

2. RELATED WORK

Several studies have attempted to understand and estimate the colorfulness in visual content. Color appearance models (CAMs) utilize some form of chroma and colorfulness estimation to estimate the local visual perception and to reproduce the colors considering the lighting conditions [6,20,21]. However, such estimations are valid mostly for simple uniform image patches. To estimate the overall colorfulness values of complex natural images, several studies have been conducted [5,11,16,17,19] in the literature.

Yendrikhovskij et al. [16] developed a model to estimate the quality of natural color images by computing their naturalness and colorfulness indices. The proposed colorfulness index of an image is given as a summation of the average saturation (saturation as defined in CIE L*u*v* color space) and the standard deviation of the saturation. In another study, Datta et al. [11] proposed to first partition the RGB cube and estimate the colorfulness by calculating the Earth Mover’s Distance between the pixel distribution of the tested image and that of an “ideal” colorful image. However, compared to others, the model failed in perceptual validation [19].

Hasler and Süssstrunk [5], on the other hand, studied the colorfulness in natural images by conducting a subjective user study over a relatively complex set of images. Their proposed method estimates the colorfulness by computing the first and second order statistics between the opponent color channels i.e., yellow-blue and red-green color channels of the given RGB image. Using the same opponent color space strategy, Panetta et al. [17] proposed a colorfulness metric as a part of their no-reference image quality measure. Their pro-
In this study, we use the subjective colorfulness scores collected from participants for two different colorfulness databases: EPFL Dataset [5] with 84 images and UCL Dataset [19] with 96 images.

Both datasets provide the collected subjective scores and corresponding images. The UCL Dataset provides pairwise comparison scores (also with the user confidence). The numerical colorfulness scores, for this database, are obtained through a Thurstone Case V scaling [29]. The quality scores for the EPFL Dataset are already scaled by the respective authors, using one of the methods proposed by Engeldrum [30].

Even though a psychometric scaling algorithm has been employed, the EPFL Dataset scores have been collected using a rating methodology. Whereas, UCL Dataset scores have been collected using a ranking (i.e. pairwise comparison) methodology. To bring these scores to the same scale, a third subjective test is conducted as an anchor, using a common subset of these two datasets. The two databases are then combined (i.e. aligned) using the scores from this third subjective experiment [31] as explained below.

Selection of Images from Each Dataset: To create a common subset, we first selected 12 representative images from each database. These selected images cover the whole quality scale, starting from not colorful at all to very colorful. The distribution of subjective colorfulness scores for all of the images and the selected images is shown in Fig. 1(a) and Fig. 1(c) for EPFL and UCL, respectively. These selected images are used in a new subjective experiment. The new subjective test and the subjective scores collected are referred to as ‘Anchor’ experiment and dataset, respectively.

Subjective Test for Anchoring the Two Datasets: To merge EPFL and UCL datasets on the same scale, we have conducted a third subjective experiment with a common subset of images [31]. The pairwise comparisons (PWC) methodology is chosen in order to keep the cognitive load for the participants lighter and the experiment process easier. Although PWC is easier for subjects to decide, the full PWC design (with \( n(n-1)/2 \) pairs) is time-consuming. Instead, in this study, we use an adaptive pair selection process, known as adaptive square design (ASD) [32,33]. With this pair selection process (and also test methodology), the pair selection and scaling are done iteratively, making sure that similar stimuli are compared more than different stimuli (for which the difference will be clearer with fewer comparisons). The ASD is implemented in Matlab, and a Thurstone Case V based psychometric scaling method is used for this experiment. For the presentation and interactive voting, a Mat-
To obtain the subjective scores, the 24 selected images are used in the ‘Anchor’ experiment, to which three expert viewers have attended. Instead of updating the rank matrix of ASD for each observer, a different approach is used. The rank matrix of ASD is randomly initialized for each expert observer, and throughout the 5 loops of the test, this rank matrix is updated, and the new pairs are selected considering this updated rank matrix. This ensured that different images have been compared for each different expert viewer.

The obtained PWC results are then scaled to quality values which are later used to merge the databases. The scaling results are generally arbitrary (i.e., relative to each other without an origin point), and hence, hard to understand. Therefore, these subjective quality scores are mapped to [1 9] scale, 1 being the least colorful and 9 being the most colorful.

Combining the Datasets: The relationship between the new ‘Anchor’ colorfulness scores and those of EPFL and UCL are shown in Fig.1(b) and 1(d), respectively, for the selected images. The relationship between these scores is linear, with very high correlation in Fig. 1.(b) and 1.(d), respectively, for the selected images. The scaling results are later used to merge the databases. The scaling results of different images have been compared for each different expert viewer.

Then, the two databases are brought to the same scale as:

\[ \widetilde{Q_{DB}} = a_{DB} \times Q_{DB} + b_{DB} \]  

(1)

Finally, the mapped quality scores are concatenated in a single dataset. In Fig. 1(e), the merged (denoted as ‘Combined’) subjective quality scores are plotted again vs the ‘Anchor’ scores to validate the merging operation, where these two databases are observed to be on the same scale.

4. PROPOSED COLOR RATING MODEL

The ColorNet architecture is illustrated in Fig. 2. The proposed model has two major building blocks, 1) the feature network \( \Phi_f \) and 2) the rating network \( \Phi_r \). We base our feature network \( \Phi_f \) on the state-of-the-art deep learning models, namely, VGG [22], ResNet [23], and MobileNet [24]. For all these models, we specifically removed the last layers originally meant for classification and fine-tuned the remaining feature layers in an end-to-end fashion. These features are fed to the rating network \( \Phi_r \) which has an objective of mapping the characteristic features to the colorfulness rating domain. The three variants of our ColorNet model are named as ColorNet-VGG, ColorNet-ResNet, and ColorNet-Mobile. The rating network is the same for all three variants. It consists of a dropout regularization layer, two fully connected layers \( FC - 10 \) and \( FC - 1 \) with 10 and 1 channels respectively. An added non-linearity is introduced by using the ReLU unit in between the fully connected layers to learn the desired colorfulness mapping. Further details regarding the three variants are as follows:

1. **ColorNet-VGG** has a 13 convolutional layered feature network \( \Phi_{vgg}^{f_{\text{net}}} \) adopted from the VGG16 [22] architecture, with small \( 3 \times 3 \) convolutions, resulting in a feature vector of dimension \( 512 \times 7 \times 7 \). In our paper, we removed the three fully connected layers of the VGG16 architecture and simply fed the resulting feature vector into the proposed rating network.

2. **ColorNet-ResNet** consists of an 18 layer deep residual feature network \( \Phi_{\text{resnet}}^{f_{\text{net}}} \) adopted from the ResNet architecture [23]. Similar to VGG [22], the ResNet architecture has small \( 3 \times 3 \) convolutions, however, with an additional concept of residual learning applied to every stacked layers. In our paper, we removed the last fully connected layer of 1000 channels and fed the resulting feature of size \( 512 \times 7 \times 7 \) into the rating work.

3. **ColorNet-Mobile** consists of a 28 convolution layers feature network \( \Phi_{\text{mobile}}^{f_{\text{net}}} \) adopted from the MobileNet architecture [24], which comprises of both depth-wise and point-wise convolutions. MobileNet has been a widely adopted network for many mobile and embedded applications. In this paper, we removed its last fully connected and soft-max layer to obtain a feature of size \( 1024 \times 7 \times 7 \) which is finally fed into the rating network.

Note that we choose these feature models mainly due to ease in training and faster convergence over small dataset size. For further details regarding the feature network architectures, we refer the reader to [22-24]. For each ColorNet variant, the receptive input for the first fully-connected is altered as per the resulting feature size of the feature network.

Loss Function: The ultimate goal of ColorNet models is to rate the colorfulness in images. To this end, we train these models in an end-to-end fashion using a fully supervised setting. For training, we define a set of an image and its corresponding rating as a pair, given as \((X_i, y_i)\), where \( X_i \) is an image and \( y_i \) is its corresponding colorfulness rating, \( i = [1, n], n = \text{number of training image pairs} \). To train the ColorNet models, we used the following objective function:

\[
L^j(y, \hat{y}) = \frac{1}{n} \sum_{k=1}^{n} |y_k - \phi_k^r \cdot \phi_k^f(X_i)|
\]

(2)

where \( \hat{y} \) is the predicted colorfulness rating and \( j = \{ \text{vgg, resnet, mobile} \} \). We additionally experimented with the L2 norm as a loss function, however, we observe that practically the L1 loss term effectively penalizes the network to learn and converge at a faster rate using the ADAM [35] optimizer technique.

5. RESULTS

Training and Implementation Details: We split the dataset of 180 images into training, validation and test sets in an 80%, 10% and 10% setting. For training, the input image size is fixed at \( 600 \times 600 \), and random crops of size \( 512 \times 512 \) are applied to the image. Additional data augmentation techniques such as rotation and flipping are applied to scale up the training dataset.

The ColorNet model is implemented using the Pytorch [36] deep learning library. During training, the batch size is set to 4 and the
baseline weights of the feature networks are initialized by training on the ImageNet [37] dataset. The weights of the layers in the rating networks are initialized randomly. In other terms, we fine-tune the feature network and train the rating network layers for our task at hand. A dropout rate of 0.75 is set in the rating network for all the three models. We utilize an ADAM solver [35] with an initial learning rate of $1 \times 10^4$ and $1 \times 10^6$ for training the feature and rating networks respectively. The learning rates are allowed to decay exponentially with a decay rate of 0.95, after every 10 epochs. The momentum rate is fixed at 0.9 for all epochs. All our models are trained in an end-to-end fashion for 200 epochs. Training is done using a 12 GB NVIDIA Titan-X GPU on an Intel Xeon E7 core i7 machine for 200 epochs which take approximately 2 hours. Inference time is 25 secs for each image.

**Traditional Colorfulness Methods:** We implemented four different colorfulness metrics: Hasler and Süssstrunk ($CF_{\text{Hasler}}$) [5], two versions ($CQE_{1}^{CF}$ and $CQE_{2}^{CF}$) of Panetta et al. [17], and Yendrikhovskij et al. ($CF_{\text{Yendrikhovskij}}$) [16] to compare with the proposed ColorNet model.

For the Hasler and Süssstrunk [5], with a given RGB image, the metric uses the mean $\mu$, and standard deviation $\sigma$ of the opponent color space vectors $v_r$, $v_g$, and $v_b$ where $v_r = I_R - I_G$ and $v_g = (I_R + I_G)/2 - I_B$. Then, $CF_{\text{Hasler}}$ is computed as:

$$CF_{\text{Hasler}} = \sqrt{\sigma_r^2 + \sigma_g^2 + 0.3 \times \sqrt{\mu_r^2 + \mu_g^2}} \quad (3)$$

Panetta et al. [17] use the similar color opponent space, but in logarithmic space. The metrics $CQE_{1}^{CF}$ and $CQE_{2}^{CF}$ are calculated as:

$$CQE_{1}^{CF} = 0.02 \times \log \left( \frac{\sigma_r^2}{|\mu_g|^2} \right) \times \log \left( \frac{\sigma_g^2}{|\mu_b|^2} \right)$$

$$CQE_{2}^{CF} = 0.02 \times \log(\sigma_r^2) \times \log(\sigma_g^2) \times \log(\mu_r^2) \times \log(\mu_g^2) \quad (4)$$

where $v_r$, $v_g$, and $v_b$ are concatenated to form $v_c$, i.e., $v_c = [v_r, v_g, v_b]$.

To compute $CF_{\text{Yendrikhovskij}}$, we use the following formula after the RGB to CIE L*$u*$v*$ color space transform, as specified in the paper [16].

$$S_{u^*v^*} = \left( \frac{\sqrt{(u^*2 + v^*2)}}{L^* + \epsilon} \right), \epsilon \neq 0$$

$$CF_{\text{Yendrikhovskij}} = \mu_S_{u^*v^*} + \sigma_S_{u^*v^*} \quad (5)$$

**Quantitative Evaluation:** Colorfulness metrics are evaluated by computing the Pearson correlation coefficient (PCC) and the Spearman rank-ordered correlation coefficient (SROCC). For testing, we used the 10-fold cross-validation strategy where the whole dataset was divided into 10 non-overlapping pieces $P_i$, and for each iteration, one piece is used for the test, one piece used for validation, and the remaining pieces are used for training (as also described above). For each iteration $itr$, the unique piece $P_{itr}$ is used for the test, where all the pieces ($P_{itr}, itr \in [1, 10]$) cover the whole dataset. Finally, the PCC and SROCC results for 10 different test cases are averaged. The performance results are presented in Table 1 where PCC and SROCC for the 4 state-of-the-art colorfulness metrics are computed using the aligned subjective scores from ‘Combined’ dataset. Our proposed ColorNet model with VGG and ResNet based feature network outperforms over the other classical models. This is partly due to the model’s ability to learn rich high-level color feature representation.

**Qualitative Evaluation:** For the qualitative evaluation of ColorNet, we crafted a set of images that are not used during the training of the model, by considering two different scenarios: i) change in the dominant colours and ii) change in the color contrast. In Fig. 3 row I shows 4 different cases of the change in the dominant colors, and row II shows the change in the color contrast. We report the objective results for the proposed deep-learning-based colorfulness estimation method, in Fig. 3 and show also $CF_{\text{Hasler}}$ for completeness. The results showcase that in both cases i.e., by increasing color contrast and increasing the number of dominant hues, the proposed metric scores increase, thus, validating the understanding of colorfulness of our model. Overall, our results confirm that learning-based models bring huge potential to cater for various implicit aspects of color perception over a wide variety of natural images.

**Table 1. Correlation Coefficient Results.**

| Colorfulness Metric                        | PCC  | SROCC |
|-------------------------------------------|------|-------|
| $CF_{\text{Hasler}}$ [5]                  | 0.841| 0.884 |
| $CQE_{1}^{CF}$ [17]                       | 0.895| 0.869 |
| $CQE_{2}^{CF}$ [17]                       | 0.312| 0.415 |
| $CF_{\text{Yendrikhovskij}}$ [16]        | 0.843| 0.834 |
| ColorNet-Mobile                           | 0.841| 0.774 |
| ColorNet-ResNet                           | 0.916| 0.889 |
| ColorNet-VGG                              | **0.937**| **0.921**|

Fig. 3. Qualitative Evaluation. Row I depicts the change in dominant colors. Row II depicts the change in the saturation of the colors.

**6. Conclusion**

In this study, we propose a CNN based model for the estimation of colorfulness ratings. To prepare a well-annotated colored image dataset, we combine two colorfulness databases with subjective user scores, using the results of an anchor subjective experiment with a common subset of images. We compare the results of the proposed model to those of four other traditional colorfulness metrics quantitatively and qualitatively where we observe that our learning-based model effectively rates the colorfulness by catering for the wide variety of natural images. This study constitutes an initial step towards the exploration of color perception in natural images using the deep learning approach. In future work, we aim to delve deeper in learning-based color perception models and analyze the impacts of various associated factors.
7. REFERENCES

[1] F. Pitié, A. C. Kokaram, and R. Dahyot, “Automated colour grading using colour distribution transfer,” Comput. Vis. Image Underst., vol. 107, no. 1-2, pp. 123–137, Jul. 2007.

[2] R. Mantiuk, S. Daly, and L. Kerofsky, “Display adaptive tone mapping,” ACM Transactions on Graphics, Aug 2008.

[3] E. Sikudová, T. Pouli, A. Artusi, A. O. Akyüz, F. Banterle, Z. M. Mazuinoglu, and E. Reinhard, “A gamut-mapping framework for color-accurate reproduction of HDR images,” IEEE Computer Graphics and Applications, vol. 36, no. 4, pp. 78–90, 2016.

[4] R. Mantiuk, R. Mantiuk, A. Tomaszewska, and W. Heidrich, “Color correction for tone mapping,” Computer Graphics Forum, vol. 28, no. 2, pp. 193–202, 2009.

[5] D. Hasler and S. E. Süsstrunk, “Measuring colorfulness in natural images,” in SPIE Electronic Imaging, Human Vision and Electronic Imaging VIII, 2003, pp. 87–96.

[6] M. D. Fairchild, Color Appearance Models. John Wiley & Sons, 2013.

[7] E. Reinhard, T. Pouli, T. Kunkel, B. Long, A. Ballestad, and G. Danberg, “Calibrated image appearance reproduction,” ACM Transactions on Graphics (TOG), vol. 31, no. 6, 2012.

[8] A. Rana, G. Valenzise, and F. Dufaux, “Learning-based tone mapping operator for efficient image matching,” IEEE Transactions on Multimedia, vol. 21, no. 1, pp. 256–268, 2019.

[9] ——, “Learning-based Adaptive Tone Mapping for Keypoint Detection,” in IEEE International Conference on Multimedia & Expo (ICME’2017), Hong Kong, China, Jul. 2017.

[10] ——, “Learning-Based Tone Mapping Operator for Image Matching,” in IEEE International Conference on Image Processing (ICIP’2017), Beijing, China: IEEE, 2017.

[11] R. Datta, D. Joshi, J. Li, and J. Z. Wang, “Studying aesthetics in photographic images using a computational approach,” in European Conference on Computer Vision. Springer, 2006, pp. 288–301.

[12] T. O. Aydin, A. Smolic, and M. Gross, “Automated aesthetic analysis of photographic images,” IEEE Transactions on Visualization and Computer Graphics, vol. 21, no. 1, pp. 31–42, 2015.

[13] H. Can Karaimer and M. S. Brown, “Improving color reproduction accuracy on cameras,” in The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2018.

[14] S. Winkler, “Analysis of public image and video databases for quality assessment,” IEEE Journal of Selected Topics in Signal Processing, vol. 6, no. 6, pp. 616–625, 2012.

[15] M. H. Pinson, M. Barkowsky, and P. Le Callet, “Selecting scenes for 2D and 3D subjective video quality tests,” EURASIP Journal on Image and Video Processing, vol. 2013, no. 1, 2013.

[16] S. Yendrikhovskij, F. J. Blommaert, and H. de Ridder, “Optimizing color reproduction of natural images,” in Color and Imaging Conference. Society for Imaging Science and Technology, 1998, pp. 140–145.

[17] K. Panetta, C. Gao, and S. Agaian, “No reference color image contrast and quality measures,” IEEE Transactions on Consumer Electronics, vol. 59, no. 3, pp. 643–651, 2013.

[18] A. Rana, C. Ozcinar, and A. Smolic, “Towards generating ambisonics using audio-visual cue for virtual reality,” in 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2019.

[19] C. Amati, N. J. Mitra, and T. Weyrich, “A study of image colourfulness,” in Proceedings of the Workshop on Computational Aesthetics. ACM, 2014, pp. 23–31.

[20] R. W. G. Hunt, The Reproduction of Colour, 5th ed. Wiley, 1995.

[21] Y. Nayatani, “Revision of the chroma and hue scales of a nonlinear color-appearance model,” Color Research & Application, vol. 20, no. 3, pp. 143–155, 1995.

[22] K. Simonyan and A. Zisserman, “Very deep convolutional networks for large-scale image recognition,” in International Conference on Learning Representations, 2015.

[23] K. He, X. Zhang, S. Ren, and J. Sun, “Deep residual learning for image recognition,” IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 770–778, 2016.

[24] A. G. Howard, M. Zhu, B. Chen, D. Kalenichenko, W. Wang, T. Weyand, M. Andreetto, and H. Adam, “MobileNets: Efficient convolutional neural networks for mobile vision applications.” Corr, 2017.

[25] L. Krasula, K. Fliegel, P. Le Callet, and M. Klima, “Objective evaluation of naturalness, contrast, and colorfulness of tone-mapped images,” in SPIE Optical Engineering + Applications, Applications of Digital Image Processing XXXVII. International Society for Optics and Photonics, 2014.

[26] M. Čafik, M. Wimmer, L. Neumann, and A. Artusi, “Evaluation of HDR tone mapping methods using essential perceptual attributes,” Computers & Graphics, vol. 32, no. 3, pp. 330–349, 2008.

[27] H. Talebi and P. Milanfar, “NIMA: Neural image assessment,” IEEE Transactions on Image Processing, vol. 27, pp. 3998–4011, 2018.

[28] L. Kang, P. Ye, Y. Li, and D. Doermann, “Convolutional neural networks for no-reference image quality assessment,” in IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2014.

[29] M. Perez-Ortiz and R. K. Mantiuk, “A practical guide and software for analysing pairwise comparison experiments,” stat.AP arXiv:1712.03686, 2017.

[30] P. G. Engeldrum, Psychometric Scaling: A Toolkit for Image Systems Development. Imcotek press, 2000.

[31] Y. Pitry, U. Engelke, M. Barkowsky, R. Pépion, and P. Le Callet, “Aligning subjective tests using a low cost common set,” in Euro IIV, 2011.

[32] J. Li, M. Barkowsky, and P. Le Callet, “Boosting paired comparison methodology in measuring visual discomfort of 3DTV: performances of three different designs,” in SPIE Electronic Imaging, Stereoscopic Displays and Applications XXIV. International Society for Optics and Photonics, 2013.

[33] ITU-R, “Subjective assessment methods for 3D video quality,” ITU-R Recommendation P915, Mar 2016.

[34] M. Kleiner, D. Brainard, D. Pelli, A. Ingling, R. Murray, C. Broussard et al., “What’s new in Psychotoolbox-3,” Perception, vol. 36, no. 14, 2007.

[35] D. P. Kingma and J. Ba, “Adam: A method for stochastic optimization,” CoRR, 2014.

[36] A. Paszke, S. Gross, S. Chintala, G. Chanan, E. Yang, Z. DeVito, Z. Lin, A. Desmaison, L. Antiga, and A. Lerer, “Automatic differentiation in PyTorch,” in NIPS-W, 2017.

[37] J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei, “ImageNet: A large-scale hierarchical image database,” in IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2009.