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Preventive control strategy on second wave of Covid-19 pandemic model incorporating lock-down effect

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Abstract  This study presents an optimal control strategy through a mathematical model of the Covid-19 outbreak without lock-down. The pandemic model analyses the lock-down effect without control strategy based on the current scenario of second wave data to control the rapid spread of the virus. The pandemic model has been discussed with respect to the basic reproduction number and stability analysis of disease-free and endemic equilibrium. A new optimal control problem with treatment is framed to minimize the vulnerable situation of the second wave. This system is applied to study the effects of vaccines and treatment controls. Numerical solutions and the graphical presentation of the results predict the fate of India’s second wave situation on account of the control strategy. Lastly, a comparative study with control and without control has been analysed for the exposed phase, infective phase, and recovery phase to understand the effectiveness of the controls. This model is used to estimate the total number of infected and active cases, deaths, and recoveries in order to control the disease using this system and studying the effects of vaccines and treatment controls.

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1. Introduction

The novel Covid-19 causes from the strain of a large family of viruses, i.e., coronavirus develops illnesses ranging from the common cold to acute respiratory syndromes. Common symp-
toms of the SARS CoV-2 strain include respiratory symptoms such as fever, dry cough, tiredness, congestion, sore throat, vomiting or diarrhoea and shortness of breath according to the WHO [1]. Some infected people are also not getting any smell. Since its outbreak, most of the countries across the world are perceiving a surge in their daily Covid-19 disease tally. Several lockdowns, announcing several guidelines, aggressive testing, and timely provision of medicines are some of the ways in which governments are trying to avoid the continuous spread of the disease and to break the chain. Some organizations, either independently or in collaboration with government authorities, have come up with the announcement of vaccines, which are undergoing several stages of trials and approval process early in the year 2021. Besides that, the modern biological mathematical model and control system are essential to understand the mechanism of viral disease transmission and to control the spread of the virus. The worldwide different mathematical models of this epidemic that take into account the different situations in different countries help the scientists of medical science, microbiologists, and virologists to fight against Covid-19.

The second wave of Covid-19 has been overwhelming in India as more than thirty thousand new cases were confirmed daily over the last week of April 2021. The country faces extreme shortages of beds in the hospital, oxygen supplies, medicines and life support equipment. But they arranged all the facilities very fastly so that it has been under control after the peak situation in May and June in the same year. At that time Government has given the topmost priority to vaccinate the people in massive quantities in a war situation, whereas the density of the population in India is the main challenge. People have already been learned to adapt and overcome the economic challenges linked to lockdowns, thereby limiting the financial impact. The Indian economy showed its ability to bounce back quickly after the first wave of Covid-19 and seems to pull this off once more.

The rapid spread of epidemics along the interactions was analysed using the Susceptible-Infected-Recovered (SIR) model [2], or SEIR model [3–5]. Global analysis for the general epidemiological model with vaccination control strategy has been discussed by Yang et al. [6] and Sun et al. [7]. Tian and Wang [8] investigated the global stability of the cholera epidemic model, and the HIV/AIDS epidemic model has been described by Samanta [9] and Cai et al. [10]. The transmissibility of a novel coronavirus mathematical model has been prescribed by Cheng et al. [11]. Another model of airborne viral diseases with risk analysis and features of Covid-19 has been discussed by Adekola et al. [12], and Anirudh [13] explained the dynamics transmission in envisaging the Covid-19 and the future of this pandemic. The case study of the transmission impact of coronavirus on preventive measures in Tanzania has been discussed by Mumbu et al. [14].

Mathematical models and projections in relation to climate and geographical change, environment, humanitarian disasters, and global health play a critical role in producing evidence in response to every viral outbreak such as Ebola, MERS, SARS, and currently SARS CoV-2, and in eliminating chronic infections such as viral Hepatitis, HIV, and Tuberculosis. Many researchers have recently investigated the current scenario of SARS CoV-2 in greater detail using the effects of fractional order models, reaction diffusion systems, and complex networks [15–44], effect of lock-down [45,46] and predicted different control strategies [47–54]. Impact of isolation disobedience and movement restrictions on CoV-19 pandemic has been studied by Stipic et al. [55] and mathematical model of CoV-19 disease on the basis of latency and age structure has been considered by Blyuss and Krynychko [56]. Kalman filter in the epidemiological model with a robust approach to predict CoV-19 outbreak in Bangladesh has been discussed by Islam et al. [57]. James et al. [58] and Glass [59] studied the second wave mortality of CoV-19 in Europe and US. The negative geographic correlation of estimated incidence between the first two waves of COVID-19 in Italy has been analysed by Carletti and Pancrazi [60]. Ershkov and Rachinskaya [61] studied the new approximation of mean-time trends for the CoV-19 s wave in the key six countries. An SEIR model of SARS CoV-2 s wave in France and Italy has been explained by Faranda and Alberti [62] and Ghanbari [63] forecasted the second wave of covid-19 in Iran. Lastly, the second wave of SARS CoV-2 due to interactions between social processes and dynamics disease has been discussed by Pedro et al. [64]. Another method for studying epidemic models is to use a fractional dynamical system. Recently, The mathematical model and dynamics of a novel coronavirus with ABC and CF fractional derivatives are investigated in [65]. In [66], the fractional model and numerical algorithms for predicting COVID-19 with isolation and quarantine strategies are explored. In the absence of a vaccine, the need for proper quarantine without lockdown for 2019-nCoV is investigated in [67]. A nonsingular fractional-order model of the dynamics of the novel coronavirus is established in [68].

The goal of this paper is to develop some mathematical pandemic models based on the current situation for the second wave of the Covid-19 outbreak with a lock-down and control strategy without lock-down in the Indian population. For the estimated hypothetical parameter values that demonstrate the virus’s effect, the graphical presentation of the basic reproduction number has been described. The effect of the Covid-19 mathematical model on reproduction number and stability analysis has been discussed. Lastly, a comparative study with and without control was conducted in the Indian population for the exposed phase (E), infective phase (I), and recovery phase (R) to determine the efficacy of the control strategy.

2. Covid-19 pandemic model with lock-down effects

For the proposed Covid-19 pandemic with lock-down effects mathematical model, the following are our assumptions:

i. $S(t)$ denotes the susceptible population consists of people who have not yet been infected with the COVID-19 virus.
ii. $L(t)$ denotes lock-down population is a fraction of the susceptible population and those are home quarantined.
iii. $E(t)$ denotes the exposed population who are infected but who have not been detected by testing.
iv. $T(t)$ denotes the infected population that has been identified and is being treated through testing.
v. $I(t)$ denotes the population that is asymptotically infected.
vi. $R(t)$ denotes the recovered population, which is thought to be immune to re-infection.

Also, the biological meanings of the parameters are described below as follows:
A: the recruitment rate at which new individuals (including immigrants and newborns) enter the susceptible compartment in the Indian population,
z_1: the rate at which susceptible compartment moves to lock-down compartment,
z_2: the rate at which susceptible compartment moves to a susceptible compartment,
β: effective contact rate of infective individuals,
d: the natural death rate of each compartment,
γ_1: the rate at which the exposed compartment moves to an infected compartment under treatment,
γ_2: the rate at which the exposed compartment gets infected but asymptomatic,
δ_1: the rate at which the infected compartment gets treatment and moves to the recovered compartment,
δ_2: the rate at which the infected compartment gets treatment and does not spread the disease,
α_s: the rate at which asymptotically infected population does not spread disease,
α_2: the rate at which asymptotically infected population gets recovered without treatment,
α_1: the rate at which asymptotically infected population gets treatment and does not spread disease. The transmission process With these assumptions, the mathematical model of the second wave of Covid-19 outbreak with lock-down is formulated by the following system of differential equations:

\[
\begin{align*}
\frac{dS}{dt} &= \Lambda + z_2 L - z_1 S - \beta SI - dS \\
\frac{dL}{dt} &= z_1 S - z_2 L - dL \\
\frac{dE}{dt} &= \beta SI - \gamma_1 E - \gamma_2 E - dE \\
\frac{dI}{dt} &= \gamma_1 E + \sigma_1 I - \delta_1 T - \delta_2 T - dI \\
\frac{dT}{dt} &= \delta_1 T + \sigma_1 I - \sigma_2 I - dT \\
\frac{dR}{dt} &= \delta_2 T + \sigma_2 I - dR
\end{align*}
\] (1)

where \( S(t), L(t), E(t), I(t), T(t), R(t) \) are the densities of susceptible population, lock-down population, exposed population, infected population under treatment, asymptotically infected population and recovered population respectively at the time \( t \). In this model, the lock-down compartment is a fraction of the susceptible population and those are home quarantined. The initial conditions for the compartments of the system (1) are as follows:

\[
S(0) > 0, L(0) > 0, E(0) \geq 0, I(0) \geq 0, T(0) \geq 0, R(0) \geq 0.
\] (2)

For biological purposes, the initial conditions exist and are unique in the interval \([0, \infty)\) per unit of time.

3. Elementary properties of proposed pandemic model

In this section, some properties of the epidemic model (1) and the invariant region where all solutions of the system (1) exist have been described. The basic reproduction number for the equilibrium point of the system has also been analysed here. The following two theorems are used to demonstrate the Positivity and Boundedness of the model’s solution based on [69].

3.1. Positivity and boundedness of the solution

**Theorem 1.** All solutions of the system (1) with initial condition \( (S(0), L(0), E(0), I(0), T(0), R(0)) \) are non-negative and unique for all \( t \geq 0 \).

**Proof.** From the system of Eqs. (1), it is observed that

\[
S(t) > S(0) \exp \left[ - \int_0^t \phi(I(x)) dx \right] > 0,
\]
where \( \phi(I(x)) = z_1 + \beta I(x) + d \);

\[
L(t) > -(z_2 + d)L(t) \Rightarrow L(t) > L(0) \exp \{-z_2 + d\} t > 0;
\]

\[
I(t) > -(\gamma_1 + \gamma_2 + d)E(t) \Rightarrow E(t) > E(0) \exp \{-(\gamma_1 + \gamma_2 + d)\} t \geq 0;
\]

\[
T(t) > -(\delta_1 + \delta_2 + d)T(t) \Rightarrow T(t) > T(0) \exp \{-(\delta_1 + \delta_2 + d)\} t \geq 0;
\]

\[
I(t) > -(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)I(t) \Rightarrow I(t) > I(0) \exp \{-(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\} t \geq 0
\]

and \( \dot{R}(t) > -dR(t) \Rightarrow R(t) > R(0) \exp \{-dt\} \geq 0 \).

As all compartment functions of system (1) are completely continuous and the solutions of (1) are unique on \([0, \eta]\), where \( 0 < \eta < \infty \).

Therefore, all the solutions of the system (1) are non-negative and unique \( \forall t > 0 \). □

**Theorem 2.** All solutions of the system (1) which lies in \( \mathbb{R}^6_+ \) are uniformly bounded and confined to the region \( \Omega_1 = \{(S, L, E, T, I, R) \in \mathbb{R}^6_+: 0 < W(t) \leq \frac{\Lambda}{d}\} \).

**Proof.** Let us define \( N = S + L + E + T + I + R \) as the total number of the high-risk human population at time \( t \). From the system (1), it is observed that

\[
\dot{N} = \Lambda - dN - \delta_1 T - \sigma_1 I \leq \Lambda - dN
\]

Thus for \( \Lambda > 0 \), \( \dot{N} + dN \leq \Lambda \).

We find by using the differential inequality [69] as

\[
0 < N(t) \leq \frac{\Lambda}{d} (1 - e^{-\frac{d}{\Lambda}}) + N(0) e^{-\frac{d}{\Lambda} t}
\]

and hence \( 0 < N(t) \leq \frac{\Lambda}{d} \) as \( t \to \infty \).

So, there exists a real number \( M = \frac{\Lambda}{d} \) such that \( |N(t)| \leq M, \forall t > 0 \).

Therefore, all the solutions of (1) are uniformly bounded and confined to the invariant region \( \Omega_1 \), where \( \Omega_1 = \{(S, L, E, T, I, R) \in \mathbb{R}^6_+: 0 < W(t) \leq \frac{\Lambda}{d}\} \). □
3.2. Existence of equilibrium point of the system

For the equilibrium points of the system (1), we have to find the point of intersection at the zero growth isolines. Here, we get a unique boundary equilibrium point which is given by

\[ e_0 = (S_0, L_0, E_0, T_0, I_0, R_0) = (S_0, L_0, 0, 0, 0, 0), \]

where \( S_0 = \frac{N_k}{A_k} \), \( L_0 = \frac{A_k}{B_k} \), and \( E_0 = T_0 = I_0 = R_0 = 0 \). To eradicate the virus disease from the population of varying sizes, we have to search the stringent way to make the virus-infected population tend to zero [70]. Therefore, \( e_0 \) is the disease-free equilibrium (DFE) point of the system (1) for all parameters.

3.3. Basic reproduction number

To evaluate the basic reproduction number (BRN), we arrange the equations of the reduced system of (1) in the following manner

\[
\begin{align*}
\frac{dS}{dt} &= \beta SI - \gamma_1 E - \gamma_2 E - dSE \\
\frac{dI}{dt} &= \gamma_3 E - \sigma I - \sigma J - \sigma I - dI \\
\frac{dJ}{dt} &= \gamma_4 E + \sigma J - \delta_1 T - \delta_2 T - dJ \\
\frac{dL}{dt} &= \Lambda + \gamma_2 J - \beta SI - dS \\
\frac{dE}{dt} &= \gamma_1 S - \gamma_2 S - dL \\
\end{align*}
\]

where as the last compartment, \( R(t) \) does not depend on other equations of system (1). To establish the stability of \( e_0 \) on the system (1) by using next generation operator method [71], we assume \( y = (E, I, T, S, L) \) on the system (3) such that \( \frac{d}{dt} = f - v \), where \( f \) is a transmission part which expresses the production of new infections and \( v \) is an evolution part. Both \( f \) and \( v \) are given by

\[
\begin{pmatrix}
\beta SI \\
\gamma_1 S \\
\gamma_1 E \\
\gamma_2 E \\
\gamma_1 S + \beta SI \\
\end{pmatrix}, \quad 
\begin{pmatrix}
(\gamma_1 + \gamma_2 + d)E \\
\gamma_1 + \gamma_2 + d \\
\gamma_2 + (\sigma_1 + \sigma_2 + \sigma_3 + d) \\
\gamma_1 + \gamma_2 + d \\
\gamma_1 + \gamma_2 + d \\
\end{pmatrix}.
\]

Now, the Jacobian of \( f \) and \( v \) at DFE point \( e_0 \) are

\[
Df(e_0) = \begin{pmatrix}
F_{2,2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

and

\[
Dv(e_0) = \begin{pmatrix}
\gamma_1 + \gamma_2 + d & 0 & 0 & 0 \\
0 & \gamma_1 + \gamma_2 + d & 0 & 0 \\
0 & 0 & \gamma_1 + \gamma_2 + d & 0 \\
0 & 0 & 0 & \gamma_1 + \gamma_2 + d & 0 \\
\end{pmatrix}
\]

where

\[
F_{2,2} = \begin{pmatrix}
\gamma_1 + \gamma_2 + d & 0 & 0 & 0 \\
0 & \gamma_1 + \gamma_2 + d & 0 & 0 \\
0 & 0 & \gamma_1 + \gamma_2 + d & 0 \\
0 & 0 & 0 & \gamma_1 + \gamma_2 + d & 0 \\
\end{pmatrix}
\]

The expression of the Jacobian at DFE point \( e_0 \) can be calculated by \( \rho(F_{2,2}V_{2,2}^{-1}) \), where \( \rho \) is the spectral radius of the matrix \( (F_{2,2}V_{2,2}^{-1}) \). Therefore, the BRN of the model is

\[
\rho = \frac{\rho(F_{2,2}V_{2,2}^{-1})}{\rho(F_{2,2} + V_{2,2}^{-1})}.
\]

4. Stability analysis of DFE

In this section, the local and global stability analysis [69,73] of DFE of the system (1) has been studied.

4.1. Local stability

Theorem 3. The system (1) is locally asymptotically stable at the DFE point \( e_0 \) if \( R_0 < 1 \), and unstable if \( R_0 > 1 \).

Proof. The Jacobian matrix of the system (1) at DFE point \( e_0 \) is given by

\[
\begin{pmatrix}
\rho & -\gamma_1 & 0 & 0 & 0 & -\beta S_0 \\
0 & \rho & 0 & 0 & 0 & -\beta L_0 \\
0 & 0 & \rho & 0 & 0 & 0 \\
0 & 0 & 0 & \rho & 0 & 0 \\
0 & 0 & 0 & 0 & \rho & 0 \\
\end{pmatrix}
\]

The characteristic equation of (4) is

\[
(\lambda + d)(\lambda + \delta_1 + \delta_2 + d)[(\lambda + \gamma_1 + 2d)(\lambda + \gamma_1 + \gamma_2 + 2d) - \delta_1(\lambda + \gamma_1 + \gamma_2 + 2d) + \delta_2(\lambda + \gamma_1 + \gamma_2 + 2d) - \gamma_1 \gamma_2 \rho S_0) = 0.
\]

The eigen values of (5) is given by \( \lambda = -d, -\delta_1 - \delta_2 + d \) and \( \lambda^2 + 2\lambda \delta_1 + \delta_0 = 0 \).

\[
\lambda^2 + 2\lambda \delta_1 + \delta_0 = 0,
\]

where \( \delta_1 = (\gamma_1 + \gamma_2 + 2d) > 0, \delta_0 = d(\gamma_1 + \gamma_2 + 2d) > 0 \), \( b_1 = (\gamma_1 + \gamma_2 + \sigma_1 + \sigma_2 + \sigma_3 + 2d) > 0 \), and \( b_0 = (\gamma_1 + \gamma_2 + \sigma_1 + \sigma_2 + \sigma_3 + 2d)[1 - \rho S_0] > 0 \) if \( R_0 < 1 \). By the Routh-Hurth criterion, all eigen values are negative if \( R_0 < 1 \). Therefore, the system (1) is locally asymptotically stable at DFE point \( e_0 \) if \( R_0 < 1 \), and unstable if \( R_0 > 1 \).

4.2. Global stability

In this subsection, we discuss the global stability of the DFE of the system (1) by using the technique of Castillo-Chavez et al. [72], where as we already know that DFE point \( e_0 \) is locally asymptotically stable if \( R_0 < 1 \), and unstable if \( R_0 > 1 \). The system (1) can be written in the following form:

\[
\frac{d\theta}{dt} = F(\theta, W),
\]

\[
\theta \in G(\theta, W) = 0,
\]

where \( \theta = (S, L, R) \in R^3 \) represents the number of individuals which are not infected at present, and \( W = (E, I, T) \in R^3 \) denotes the number of infected individuals including latent and asymptomatic cases. In the following technique, the global asymptotically stability of the disease-free equilibrium is guaranteed by the following two conditions as follows:

(A1): For \( \frac{d\theta}{dt} = F(\theta, 0), \theta_0 \) is globally asymptotically stable, and

(A2): \( G(\theta, W) = AW - \overline{G}(\theta, W) \) such that \( \overline{G}(\theta, W) \geq 0 \) for \( (\theta, W) \in \Omega_1 \).
Theorem 4. For the system (1), the DFE $\mathbf{i}_0$ is globally asymptotically stable (GAS) if $R_0 < 1$, and unstable if $R_0 > 1$.

Proof. In this case,

$$F(\chi, 0) = \begin{pmatrix} \Delta + x_2S - (x_1 + d)S \\ x_2S - (x_1 + d)L \end{pmatrix}$$

$$A = \begin{pmatrix} -(\gamma_1 + \gamma_2 + d) & \frac{\Delta(x_1 + d)}{\Delta(x_1 + d) + \sigma_2 + \sigma_3 + d} \\ \gamma_1 & -(\sigma_1 + \sigma_2 + \sigma_3 + d) \end{pmatrix}$$

and

$$G(\chi, W) = A W - G(\chi, W) = \begin{pmatrix} \beta I \left( \frac{\Delta(x_1 + d)}{\Delta(x_1 + d) + \sigma_2 + \sigma_3 + d} - S \right) \\ 0 \end{pmatrix}$$

Therefore, $G(\chi, W) \preceq 0$ as the population is bounded by $S_0 = \frac{\Delta x_1}{\Delta x_1 - d}$ and $L_0 = \frac{\Delta x_1}{\Delta x_1 - d}$ for $(\chi, W) \in \Omega$. Clearly, $A$ is an M-matrix and $Y_0 = (S_0, L_0, 0)$ is a GAS equilibrium of the system $\dot{X} = F(\chi, 0)$. Therefore, the conditions (A1) and (A2) are satisfied. Hence, the following theorem is proved. □

5. Stability analysis of EE

In this part, we will discuss the existence and stability behaviour of the endemic equilibrium (EE) for the system (1) at the equilibrium point $\mathbf{e}^\ast$.

5.1. Existence of EE

To investigate the existence of EE, the solution $\mathbf{e}^\ast(S^\ast, L^\ast, E^\ast, T^\ast, F^\ast, R^\ast)$ of the equilibrium equations at steady state for the system (1) can be determined in terms of $S^\ast$. Let $m_1 = \gamma_1 + \gamma_2 + d$, $m_2 = x_1 + x_2 + d$, $m_3 = x_2 + d$, $m_4 = \sigma_1 + \sigma_2 + \sigma_3 + d$, $m_5 = \sigma_1 + \sigma_2 + \sigma_3 + d$. Then, $S^\ast = \frac{m_1}{m_1 + m_2}$, $L^\ast = \frac{m_2}{m_1 + m_2}$, $E^\ast = \frac{m_3}{m_1 + m_2}$, $T^\ast = \frac{m_4}{m_1 + m_2}$, $F^\ast = \frac{m_5}{m_1 + m_2}$, $R^\ast = \frac{m_5}{m_1 + m_2}$. Let us define

$$\lambda' = \beta_2^\ast (E^\ast + T^\ast + F^\ast).$$

(8)

All expression of endemic equilibrium $\mathbf{e}^\ast(S^\ast, L^\ast, E^\ast, T^\ast, F^\ast, R^\ast)$ shows that the system (1) satisfy the following linear equation in terms of $\lambda'$ from (8):

$$A_0 \lambda' + A_1 = 0$$

(9)

where $A_0 = m_1 m_3$, $A_1 = m_2 m_3 + \gamma_1 (\gamma_1 + \gamma_2 + d) + \sigma_1 (\gamma_1 + \gamma_2 + m_3)$. Since $A_0 > 0$ as $m_0 > 0$ and $m_0 > 0$, it is observed that the system (1) has a unique EE point $\mathbf{e}^\ast$ whenever $R_0 > 1$ and there is no positive EE point whenever $R_0 < 1$.

5.2. Local stability of EE

Theorem 5. The EE $\mathbf{e}^\ast$ for the system (1) is locally asymptotically stable (LAS) when $R_0 > 1$, and a transcritical bifurcation happens at $R_0 = 1$.

Proof. Using the concept of central manifold [73] to determine the local stability of the EE [74], we assume that $\beta$ is the bifurcation parameter and get the critical value of $\beta = \beta^\ast$ at $R_0 = 1$, where

$$\beta^\ast = \frac{d(x_1 + x_2 + d)((\gamma_1 + \gamma_2 + d)\sigma_1 + \sigma_2 + \sigma_3 + d)}{N_2(x_2 + d)}.$$  

(10)

The Jacobian $J(\mathbf{i}_0)$ (4) of the system (1) at $\beta = \beta^\ast$, has a right eigenvector corresponding to the zero eigenvalue given by $u = (u_0, u_1, u_2, u_3, u_4, u_5)^T$, where

$$u_0 = \frac{x_2 + d}{x_2} u_1, \quad u_1 = u_1 > 0, \quad u_2 = -\frac{d(x_1 + x_2 + d)}{x_1 (\gamma_1 + \gamma_2 + d)} u_1,$$

$$u_3 = -\frac{d(x_1 + x_2 + d)}{x_2 (\gamma_1 + \gamma_2 + d)} \left( \frac{\gamma_1}{\gamma_1 + \gamma_2 + d} + \frac{\sigma_1 (x_1 + x_2 + d)}{\sigma_1 (x_1 + x_2 + d) + \sigma_2 + \sigma_3 + d} \right) u_1,$$

$$u_4 = -\left( \frac{x_1 + x_2 + d}{x_1 (\gamma_1 + \gamma_2 + d)} \right) \left( \frac{\gamma_1}{\gamma_1 + \gamma_2 + d} + \frac{\sigma_1 (x_1 + x_2 + d)}{\sigma_1 (x_1 + x_2 + d) + \sigma_2 + \sigma_3 + d} \right) u_1.$$

Similarly, the Jacobian $J(\mathbf{i}_0)$ (4) of the system (1) at $\beta = \beta^\ast$, has a left eigenvector $v = (v_0, v_1, v_2, v_3, v_4, v_5)^T$ corresponding to the zero eigenvalue, where

$$v_0 = \frac{x_1 + d}{x_1} v_1, \quad v_1 = v_1 > 0, \quad v_2 = \frac{x_1 (\gamma_1 + \gamma_2 + d)}{x_2 (\gamma_1 + \gamma_2 + d)} v_1,$$

$$v_3 = \frac{x_1 (\gamma_1 + \gamma_2 + d)}{x_2 (\gamma_1 + \gamma_2 + d)} \left( \frac{\gamma_1}{\gamma_1 + \gamma_2 + d} + \frac{\sigma_1 (x_1 + x_2 + d)}{\sigma_1 (x_1 + x_2 + d) + \sigma_2 + \sigma_3 + d} \right) v_1,$$

$$v_4 = \frac{x_1 (\gamma_1 + \gamma_2 + d)}{x_2 (\gamma_1 + \gamma_2 + d)} \left( \frac{\gamma_1}{\gamma_1 + \gamma_2 + d} + \frac{\sigma_1 (x_1 + x_2 + d)}{\sigma_1 (x_1 + x_2 + d) + \sigma_2 + \sigma_3 + d} \right) v_1,$$

$$v_5 = 0.$$

Let us assume that $S = x_0, L = x_1, E = x_2, T = x_3, I = x_4, R = x_5$ and $\frac{dI}{dt} = f(k = 0, 1, 2, 3, 4, 5)$. By Castillo-Chavez and Song [74], we have

$$a = \sum_{k=0}^{3} v_k u_k \frac{d^2 F(0, 0)}{\partial x_k \partial x_2} = u_0 u_2 (\gamma_2 - v_0)$$

$$= -\frac{d^2(x_1 + x_2 + d)(x_1 + x_2 + d)(\gamma_1 + \gamma_2 + d)(\sigma_1 + \sigma_2 + \sigma_3 + d)}{x_1 (\gamma_1 + \gamma_2 + d) x_1 (\gamma_1 + \gamma_2 + d)} < 0$$

and

$$b = \sum_{k=0}^{3} v_k u_k \frac{d^2 F(0, 0)}{\partial x_k \partial x_2} = \frac{\lambda(x_1 + x_2 + d)}{x_2 (\gamma_1 + \gamma_2 + d)} u_0 (v_2 - v_0)$$

$$= -\frac{d^2(x_1 + x_2 + d)(x_1 + x_2 + d)(\gamma_1 + \gamma_2 + d)(\sigma_1 + \sigma_2 + \sigma_3 + d)}{x_1 (\gamma_1 + \gamma_2 + d) x_1 (\gamma_1 + \gamma_2 + d)} < 0.$$

As $a < 0$ and $b < 0$ at $\beta = \beta^\ast$, the unique EE $\mathbf{e}^\ast$ is locally asymptotically stable for $R_0 > 1$ [74, 75] and a transcritical bifurcation happens at $R_0 = 1$. □

6. Optimal control strategy model without lock-down

In this context of mathematical modelling, it is framed a new optimal control problem so that this vulnerable situation of the second wave of Covid pandemic can be minimized. Our Covid-19 epidemic model with treatment control becomes:

$$\frac{dS}{dt} = \Lambda - \beta SI - dS - w_1(t),$$

$$\frac{dE}{dt} = \beta SI - (\gamma_1 + d)E - w_2(t),$$

$$\frac{dI}{dt} = \gamma_2 E - (\sigma_1 + \sigma_2 + d)I - w_3(t)$$

$$\frac{dR}{dt} = \sigma I - dR + \bar{w}(t)$$

(11)
with initial conditions $S(0) > 0$, $E(0) \geq 0$, $I(0) \geq 0$, $R(0) \geq 0$, where $\bar{w}(t) = w_1(t) + w_2(t) + w_3(t)$.

The aim of this pandemic model with a control strategy is to minimize the susceptible, exposed and asymptomatically infected population, and to maximize the number of recovered population by using the feasible minimal control variables $w_1(t)$, $w_2(t)$ and $w_3(t)$. The susceptible compartment induces an optimal vaccine control $w_1(t)$ before the spread of infection. The optimal treatment controls $w_2(t)$ and $w_3(t)$ should be provided to exposed and asymptomatically infected populations, respectively, whereas the total population must be constant.

The effects of infection on susceptible, exposed, and asymptomatically infected compartments are opposite to the recovered compartments around them, thus it is necessary to minimize them. According to these optimal control strategies, the objective functional [76–78] is considered as

$$J(w_1, w_2, w_3) = \int_0^T \left[ S(t) + E(t) + I(t) + \frac{1}{2} \left( \varepsilon_1 w_1^2(t) + \varepsilon_2 w_2^2(t) + \varepsilon_3 w_3^2(t) \right) \right] dt$$

(12)

Here, $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are positive weight factors that represent a patient’s level of acceptance of the vaccination and treatment on the exposed and asymptomatically infected population, respectively. The square of the control variables reflects the severity of the side effects of the treatment. Now, the goal of the problem (11) is to minimize the objective functional (12) so that all types of infected individuals due to coronavirus as well as the cost of treatment can be minimized.

It is assumed that $S(t)$, $E(t)$ and $I(t)$ be state variables with control variable set $\bar{w}(t) = (w_1(t), w_2(t), w_3(t)) \in W$, where $W = \{ w \mid w \text{ is measurable}, 0 \leq w(t) \leq \infty, t \in [0, t_{\text{max}}] \}$ be the admissible control set such that $0 \leq \bar{w}(t) \leq N$. The system (11) can be written in the following form:

$$Z_t = AZ + F(Z) + C(w)$$

(13)

where

$$Z = \begin{bmatrix} S(t) \\ E(t) \\ I(t) \\ R(t) \end{bmatrix}, \quad A = \begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -(\gamma_2 + d) & 0 & 0 \\ 0 & \gamma_2 & -(\sigma_1 + \sigma_2 + d) & 0 \\ 0 & 0 & \sigma_2 & -d \end{bmatrix}$$

and

$$F(Z) = \begin{bmatrix} \Lambda - \beta S(t) I(t) \\ \beta S(t) I(t) \\ 0 \\ 0 \end{bmatrix}, \quad C(w) = \begin{bmatrix} -w_1(t) \\ -w_2(t) \\ \bar{w}(t) \end{bmatrix}.$$  

Eq. (13) is a nonlinear system with a bounded coefficient.

Let us assume,

$$G(Z) = AZ + F(Z)$$

(14)

and $F(Z)$ of (14) satisfies

$$|F(Z_1) - F(Z_2)| \leq \sigma_4 |S_1(t) - S_2(t)| + \sigma_5 |I_1(t) - I_2(t)|,$$

where the constants $\sigma_4 > 0$ and $\sigma_5 > 0$ are independent of the variables $S(t)$ and $I(t)$ which are less than $N$ respectively.

Hence, $|G(Z_1) - G(Z_2)| \leq \sigma_4 |Z_1 - Z_2| + |F(Z_1) - F(Z_2)| \leq \sigma |Z_1 - Z_2| < \infty$, where $\sigma = \sigma_4 + \sigma_5 + |A| < \infty$. Thus, the function $G(Z)$ is uniformly Lipschitz continuous.

Therefore, a solution of the system (13) exists [69] from the existence of $W$ and the restriction on $S(t), E(t), I(t)$ and $R(t) \geq 0$. To describe the necessary conditions for the optimal control, the Hamiltonian ($H$) is defined for (11)–(12) as follows:

$$H = \left\{ S + E + I + \frac{1}{2} \left( \varepsilon_1 w_1^2 + \varepsilon_2 w_2^2 + \varepsilon_3 w_3^2 \right) \right\}$$

$$+ \lambda_1 (\Lambda - \beta S I - dS - w_1)$$

$$+ \lambda_2 (\beta S I - (\gamma_2 + d)E - w_2)$$

$$+ \lambda_3 (\gamma_2 E - (\sigma_1 + \sigma_2 + d)I - w_3)$$

$$+ \lambda_4 (\varepsilon_3 I - dR + W),$$

(15)

where $\lambda_i$ for $i = 1, 2, 3, 4$ be the adjoint functions of $t$ to be determined later.

Theorem 6. There exists an optimal control solution set $w^* = (w_1^*, w_2^*, w_3^*) \in W$ such that

$$J(w_1^*, w_2^*, w_3^*) = \min_{(w_1, w_2, w_3) \in W} J(w_1, w_2, w_3)$$

of the control system (11).

Proof. Here, the state and control variables are always positive. By using the results in [78], the convexity of the objective functional in $w_1, w_2$ and $w_3$ is satisfied in this minimizing problem (11). By definition, the set of all control variables $w = (w_1(t), w_2(t), w_3(t))$ is convex on the control set $W$. For some positive numbers $l_1$ and $l_2$, there exist a constant $n > 1$ such that

$$J(w_1, w_2, w_3) \geq l_1 + l_2 \left( |w_1|^2 + |w_2|^2 + |w_3|^2 \right)^{\frac{1}{2}},$$

which proves the existence of an optimal control. \qed

To find the solution and necessary conditions for the optimal control, the Pontryagin’s Maximum Principle [79] has been used in the next theorem.

Theorem 7. Let us assume that $S^*(t), E^*(t), I^*(t)$ and $R^*(t)$ be optimal solutions for the optimal control problem (11)–(12) associated with the optimal control variables $w_1^*(t), w_2^*(t)$ and $w_3^*(t)$. Then there exist four adjoint variables $\lambda_1^*, \lambda_2^*, \lambda_3^*$ and $\lambda_4^*$ which satisfy the following results

$$\lambda_1^* = (\lambda_1^* - \lambda_2^*) \sigma_1^* - \lambda_1 d - 1$$

$$\lambda_2^* = (\lambda_2^* - \lambda_3^*) \gamma_2^* + \lambda_2 d - 1$$

$$\lambda_3^* = (\lambda_3^* - \lambda_4^*) \beta \sigma_1^* + \lambda_3 (\sigma_1 + \sigma_2 + d) - \lambda_4 \sigma_2 - 1$$

(16)

where the boundary conditions are given as

$$\lambda_i(t_{\text{max}}) = 0 \quad \text{for} \quad i = 1, 2, 3, 4.$$

(17)

Hence, the optimal control triples are

$$w_i^* = \min \left\{ \max \left\{ \begin{array}{c} \lambda_i - \lambda_4^* \\ \varepsilon_i \end{array} \right\}, b_i \right\} \quad \text{for} \quad i = 1, 2, 3. \quad (18)$$

Proof. Using the necessary condition [79] on the Hamiltonian in (15), the system of adjoint variables is obtained as
\[ i'_{1} = -\left( \frac{\beta}{\gamma_{1}} \right) (\lambda_{1} - \lambda_{2}) \beta' (t) + \lambda_{1} d - 1 \]
\[ i'_{2} = -\left( \frac{\beta}{\gamma_{2}} \right) (\lambda_{2} - \lambda_{1}) \gamma_{2} + \lambda_{2} d - 1 \]
\[ i'_{3} = -\left( \frac{\beta}{\gamma_{3}} \right) (\lambda_{3} - \lambda_{2}) \beta' (t) + \lambda_{3} (\sigma_{1} + \sigma_{2} + d) - \lambda_{3} \sigma_{2} - 1 \]
\[ i'_{4} = -\left( \frac{\beta}{\gamma_{4}} \right) = \lambda_{4} d \]

where the boundary conditions are given by \( \lambda_{i}(t_{\text{end}}) = 0 \) for \( i = 1, 2, 3, 4 \). Using the optimality condition of the Pontryagin’s Maximum Principle [79], it is obtained that

\[ \frac{\partial H}{\partial w_{i}} = e_{i} w_{i} - (\lambda_{i} - \lambda_{4}) = 0 \quad \text{at} \quad w_{i} = w_{i}^{*} (t) \quad \text{for} \quad i = 1, 2, 3. \]

By using the bounds for \( w \), the optimal control variables \( w_{i}^{*}(t) \) and \( w_{j}^{*}(t) \) can be obtained as

\[ w_{i}^{*} = \begin{cases} \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, & \text{if} \quad 0 \leq \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}} \leq b_{i} \\ 0, & \text{if} \quad \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}} < 0 \\ b_{i}, & \text{if} \quad \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}} \geq b_{i} \end{cases} \]

or,

\[ w_{i}^{*} = \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\}, \quad \text{for} \quad i = 1, 2, 3. \]

Furthermore, the control system (11) is converted to the following system:

\[ \frac{dS(t)}{dt} = \Lambda - \beta S' I' - dS' - \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \]
\[ \frac{dI(t)}{dt} = \beta S' I' - (\gamma_{2} + d) E' - \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \]
\[ \frac{dR(t)}{dt} = \gamma_{2} E' - (\sigma_{1} + \sigma_{2} + d) I' - \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \]
\[ \frac{dR(t)}{dt} = \sigma_{1} I' - dR' + \sum_{i=1}^{3} \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \]

with the following Hamiltonian:

\[ H' = S' (t) + E' (t) + I' (t) + \frac{1}{\lambda_{1}} \left\{ \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \right\}^{2} + \frac{1}{\lambda_{2}} \left\{ \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \right\}^{2} + \frac{1}{\lambda_{3}} \left\{ \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \right\}^{2} + \frac{1}{\lambda_{4}} \left\{ \min \left\{ \max \left\{ \frac{\lambda_{i} - \lambda_{4}}{\lambda_{i}}, 0 \right\}, b_{i} \right\} \right\}^{2}. \]

To determine the effect of the optimal control and state variables, this is required to solve Eqs. (19) and (20).

7. Numerical simulation and discussion

In this part, we may be able to apply our model to smaller populations and make region-specific predictions with the availability of reliable data [80–82]. The numerical investigation has been explained based on the control strategy and without control strategy for the second wave of the Covid-19 pandemic system. A comparison is also made between the present model and India’s current situation within a period. The characteristics of the parameters \( \beta \) and \( \gamma_{2} \) based on \( R_{0} \) is presented through Figs. 1 and 2.

The BRN graph has been drawn in Fig. 1 based on the real field value of parameters from Table 1. From Fig. 1, it is cleared that if \( \beta \) and \( \gamma_{2} \) increase, the basic reproduction number (\( R_{0} \)) increases. Therefore, our basic aim is to control the contact rate of infected individuals (\( \beta \)) and the rate at which the exposed population gets infected but asymptomatic (\( \gamma_{2} \)); otherwise, the pandemic system is unstable, and hence, this situation will become very harmful to our civilization. Fig. 2 show the combined effect of the sensitive parameters \( \beta \) and \( \gamma_{2} \) on the basic reproduction number (\( R_{0} \)). From Fig. 2(A) and (B), it is observed that as \( \beta \) which is the contact rate of infected individuals, increases and there is a high chance of population gets infected in contact with asymptomatic infected individuals, and as a result, the \( R_{0} \) value approaches towards unity and above. Hence, the spread of the disease would be increased.

Figs. 3 and 5 show the time-series graph (based on days) of the active infected population. We have not used any mathematical method for parameter estimation; we use the trial and error method to fit our model to the actual data. Our paper intends not to find the exact value of the parameter or model validation, and we want to see the hidden fact of the Covid disease spread. The long-run history for each compartment has been studied (in Fig. 6) for different levels of lockdown. From Figs. 7 and 8, we analysed the further control diagram.

Fig. 3 shows the time series (with respect to days) of the infected under treatment class (T) and asymptomatic infected class (I) for different values of \( \lambda_{2} \) with parameter values and initial conditions, respectively, as given in Table 1 from 1st May to 30th July 2021. In Fig. 3(A), the red curve represents asymptptomatically infected individuals under treatment for this proposed model, and the blue (dotted) graph is the actual infected individual as per our available data.
Fig. 3 (A) depicts that the actual infected individual almost coincides with our proposed model curve from 1st May to 30th July 2021. Therefore, the proposed Covid-19 model is almost best fitted to the current situation of India. Fig. 3 (B) shows the asymptomatic infected class (I) for different values of $a_1$. One major thing is that if the effect of lockdown increases, the number of infected under treatment will decrease. From Fig. 3, we conclude that lockdown has a clear impact on controlling the Covid situation. But a ques-

![Image](Fig. 3)

Table 1 The real field value of parameters.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $A$       | 60000 | $\beta$   | 0.000000001 |
| $\alpha_1$ | 0.03  | $\alpha_2$ | 0.001 |
| $\gamma_1$ | 0.3   | $\gamma_2$ | 0.2   |
| $\delta_1$ | 0.002 | $\delta_2$ | 0.08  |
| $\sigma_1$ | 0.0001| $\sigma_2$ | 0.1   |
| $\sigma_3$ | 0.001 | $d$       | 0.00002 |

![Image](Fig. 4)

Fig. 4 Time history of susceptible class (S) and lockdown class (L) for different values of $\alpha_1$ and rest of the parameters value from Table 1.

![Image](Fig. 2)

Fig. 2 (A) Variation of $R_0$ with respect to parameters $\beta$ and $\gamma_2$, (B) Contour graph of $R_0$ with respect to parameters $\beta$ and $\gamma_2$.

![Image](Fig. 3)

Fig. 3 Time history of infected under treatment class (T) and asymptomatic infected class (I) for different values of $\alpha_1$ and rest of the parameters value from Table 1.
tion arises in society that this lockdown method is a suitable way to control this pandemic, whereas the people face a vast loss in economic and social.

Fig. 4 shows the time series (with respect to days) of the susceptible class (S) and lockdown class (L) for different values of $a$. The susceptible class (S) is going to a safe zone when the lockdown class (L) increases. Lockdown class (L) produce inactive people that destroy the economy of the country. Whether lockdown is a suitable method to control this pandemic is not a permanent solution forever.

Fig. 5 Time history of infected under treatment class (T) and asymptomatic infected class (I) for different values of $\beta$ and rest of the parameters value from Table 1.

Fig. 6 Time history of the different compartments of the model (11) without control for the parameters from Table 2.

Fig. 7 The optimal control diagrams for the three controls.

$\lambda_i$. The susceptible class (S) is going to a safe zone when the lockdown class (L) increases. Lockdown class (L) produce inactive people that destroy the economy of the country. Whether lockdown is a suitable method to control this pandemic is not a permanent solution forever.
Effect of $\beta$ shows in Fig. 5. Fig. 5(A) depicts the time history of the infected class under treatment (T), and Fig. 5(B) illustrates asymptomatic infected class (I) for different values of $\beta$. Slide change in $\beta$ value produces a vast effect on the model system. T and I are going to high peak if $\beta$ increases. In Fig. 5(A), the blue (bold) graph is the actual infected under treatment class as per our available data. Fig. 5(A) depicts that the true infected under treatment class almost coincides with our proposed model curve from 1st May to 30th July 2021. Therefore, the proposed Covid-19 model is almost best fitted to the current situation of India. 

Fig. 6 shows the long-run history for each compartment of the model (11) without control. This figure is drown to estimate each compartment value of the Covid second wave model (11) without control by using the parameter values given in Table 2. Here we take only 5000000 is the susceptible population. In this part, we predicted that the disease is not spread all over the country due to lockdown.

The optimal control is applied to three different strategies for the system (11), namely, (i) induce an optimal control vaccine $w_1$ on the susceptible compartment before the spread of infection, (ii) the first treatment control $w_2$ on exposed populations, and (iii) second treatment control $w_3$ on asymptomatically infected populations. Fig. 7 represents the optimal control using the values of the different parameters given in Table 2. Therefore, this graph shows that 1st control is essential at the initial stage of the outbreak than when it prevails.

The numerical simulations for the optimal control of the system (11) are obtained using the different parameters given in Table 2. All variables of the objective functional (12) are balanced by choosing weight constants $\epsilon_1 = 0.0002, \epsilon_2 = 0.0003, \epsilon_3 = 0.0004$ as the variables have different scales. The comparative time-series diagram of the susceptible population (S) with all controls and without control is shown in Fig. 8. It has also been presented the same kind of comparative time-series diagrams incorporating the effect of controls for exposed phase (E), infective phase (I), and recovery phase (R) in Fig. 8. All graphs with controls show that it is a very useful model to control this pandemic. These simulations help to understand that using controls effectively reduces the spread of the disease, and the three controls together yield the best result to control the outbreak.

8. Conclusion

The proposed epidemic models have been studied on the outbreak of SARS CoV-2 disease in the Indian population. Firstly, in the lock-down model, it has been described that the BRN increases if the contact rate of infective individuals ($\beta$) and the rate at which the exposed population gets infected but asymptomatic ($\gamma_2$) increases. Therefore, it is very harmful to our society if these two parameters $\beta$ and $\gamma_2$ increase. This system has a unique disease-free equilibrium, which is globally stable when $R_0 < 1$. The unique endemic equilibrium is locally asymptotically stable for $R_0 > 1$, and a transcritical bifurcation occurred at $R_0 = 1$. According to the sensitivity analysis of the basic reproduction number $R_0$, identifying the rate of infection from susceptible zone to infected zone may reduce the death rate and the number of infected people. Lastly, a control strategy with respect to vaccination and treatment has been applied to the same system except the lock-down compartment to minimize the susceptible, exposed and asymptomatically infected populations. This shows a path to reduce the spread of this virus. A comparative study has been analysed on the models with control and without control, respectively, for exposed phase (E), infective phase (I), and recovery phase (R) to understand the effectiveness of using controls. The study concluded that staying at home as much as possible and keeping infected people in an isolated area would help to slow the spread of COVID-19. In addition, we must provide appropriate treatment for those infected with SARS-CoV-2, as well as vitamins, tonics, and supplements to protect those who are not infected. Advice has been provided in detail to assist the Indian population in slowing the spread of COVID-19. Finally, the proposed model can provide useful information for understanding their dynamics, which is critical for predicting the transmission and widespread applica-
tion of various epidemics around the world. This model is useful for predicting the total number of infected, active cases, and deaths, which provides a more accurate representation of the infection rate and may be useful in the future for COVID-19 prevention and control. This helps us make future decisions so that we can control or limit the spread of the epidemic.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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