Spin Sum Rules at Low $Q^2$

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Abstract. Recent precision spin-structure data from Jefferson Lab have significantly advanced our knowledge of nucleon structure at low $Q^2$. Results on the neutron spin sum rules and polarizabilities in the low to intermediate $Q^2$ region are presented. The Burkhardt-Cuttingham Sum Rule was verified within experimental uncertainties. When comparing with theoretical calculations, results on spin polarizability show surprising disagreements with Chiral Perturbation Theory predictions. Preliminary results on first moments at very low $Q^2$ are also presented.

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Introduction

Sum rules involving the spin structure of the nucleon offer an important opportunity to study QCD. In recent years the Bjorken sum rule [1] at large $Q^2$ and the Gerasimov, Drell and Hearn (GDH) sum rule [2] at $Q^2 = 0$ have attracted large experimental and theoretical [3] efforts that have provided us with rich information. A generalized GDH sum rule [4] connects the GDH sum rule with the Bjorken sum rule and provides a clean way to test theories with experimental data over the entire $Q^2$ range. Spin sum rules relate the moments of the spin-structure functions to the nucleon’s static properties or real or virtual Compton amplitudes, which can be calculated theoretically. Refs. [5, 6] provide comprehensive reviews on this subject.

Results on moments of the neutron spin-structure functions

Recently, the high polarized-luminosity available at Jefferson Lab has allowed a study of nucleon spin structure with an unprecedented precision. The neutron results on both $g_1$ and $g_2$ from Hall A were extracted from data on a $^3$He target polarized in both longitudinal and transverse directions.

Fig. 1 shows $\Gamma_1$ [7, 8] (left), the first moment of $g_1$, and the extended GDH integrals [8] (right) $I(Q^2) = \int_{\nu_{th}}^{\infty} (\sigma_{1/2}(Q^2) - \sigma_{3/2}(Q^2)) d\nu/\nu$ for the neutron. The left panel shows the preliminary results of $\Gamma_1$ at very low $Q^2$ [7] together with the results at low to intermediate $Q^2$ region [8]. Also shown are the neutron results extracted from the deuteron and proton data from Hall B [9] and high $Q^2$ data from HERMES [10] and SLAC [11]. At $Q^2=0$, the GDH sum rule predicts the slope of $\Gamma_1$ (dotted lines). The behavior at low $Q^2$ can be calculated with Chiral Perturbation Theory ($\chi$PT). We show a Heavy Baryon $\chi$PT (HB$\chi$PT) calculation [12] (dashed lines) and a Relativistic Baryon $\chi$PT (RB$\chi$PT) calculation [14] including vector mesons and $\Delta$ contributions (shaded...
band). The predictions are in reasonable agreements with the data at the lowest $Q^2$ settings of 0.04 - 0.1 GeV$^2$. At moderate to large $Q^2$ data are compared with two model calculations $^{15,16}$. Both models agree well with the data.

The open symbols on the right plot are measured GDH integral from pion threshold to $W = 2$ GeV. The solid squares include an estimate of the unmeasured high-energy part. The results indicate a smooth variation of $I(Q^2)$ to increasingly negative values as $Q^2$ varies from 0.9 GeV$^2$ towards zero. The data (open squares) are more negative than the MAID model calculation $^{17}$. The GDH sum rule prediction, $I(0) = -232.8 \mu b$, is indicated in Fig. 1 along with extensions to $Q^2 > 0$ using the next-to-leading order HB$\chi$PT calculation $^{12}$ (dashed line) and the RB$\chi$PT calculation $^{14}$ (shaded band) including resonance effects $^{14}$.

![Figure 1](image-url)

**FIGURE 1.** Results of $\Gamma_1$ and GDH sum $I(Q^2)$ for the neutron $^8$. The results are compared with $\chi$PT calculations of ref. $^{12}$ (dashed line) and ref. $^{14}$ (shaded band). The MAID model calculation of ref. $^{17}$, is represented by a solid line. Data from HERMES $^{10}$ are also shown.

Combining the neutron results with the proton data from Hall B, results on the moment of $g_1^p - g_1^n$, the generalized Bjorken sums $^{18}$, were obtained. The data at high $Q^2$ values were used to test the Bjorken sum rule as one of the fundamental tests of QCD. They were also used to extract a value of strong coupling constant, $\alpha_s$. The new JLab data at low $Q^2$ provide important information in the low energy region, where the strong interaction is non-perturbative. An attempt $^{19}$ was made to extract an effective strong coupling, $\alpha_s^{eff}$ in the low $Q^2$ region. The extracted $\alpha_s^{eff}$ shows a trend of weakening $Q^2$-dependence with decreasing $Q^2$.

Preliminary results of the first moment of $g_2^n, \Gamma_2^n$, at very low $Q^2$ are plotted on the left panel of Fig. 2 in the measured region (open squares). Solid squares show the results after adding an elastic and an estimated low-$x$ contributions. Also shown as open circles (measured) and solid circles (total) are the previously published results at low $Q^2$ to intermediate $Q^2$. The MAID estimate agrees well with the measured resonance data. The two bands correspond to the experimental systematic errors and the estimate of the systematic error for the low-$x$ extrapolation. The total results are consistent with the BC sum rule $^{20}$. The SLAC E155x collaboration $^{11}$ previously reported a neutron result
at high $Q^2$ (star) with a rather large error bar. On the other hand, the SLAC proton result was reported to deviate from the BC sum rule by 3 standard deviations.

The generalized spin polarizabilities provide benchmark tests of $\chi$PT calculations at low $Q^2$. Since the generalized polarizabilities have an extra $1/v^2$ weighting compared to the first moments, these integrals have less contributions from the large-$v$ region and converge much faster, which minimizes the uncertainty due to the unmeasured region at large $v$. At low $Q^2$, the generalized polarizabilities have been evaluated with next-to-leading order $\chi$PT calculations $[13, 14]$. Measurements of the generalized spin polarizabilities are an important step in understanding the dynamics of QCD in the chiral perturbation region.

The results for $\gamma_0(Q^2)$ $[8]$ are shown in the top-right panel of Fig. 2. The data are compared with a next-to-leading order ($O(p^4)$) HB$\chi$PT calculation $[13]$, a next-to-leading order RB$\chi$PT calculation and the same calculation explicitly including both the $\Delta$ resonance and vector meson contributions $[14]$. Predictions from the MAID model $[17]$ are also shown. At the lowest $Q^2$ point, the RB$\chi$PT calculation including the resonance contributions is in good agreement with the experimental result. For the HB$\chi$PT calculation without explicit resonance contributions, discrepancies are large even at $Q^2 = 0.1$ GeV$^2$. This might indicate the significance of the resonance contributions. The data are in reasonable agreement with the MAID predictions. Since $\delta_{LT}$ is insensitive to the $\Delta$ resonance contribution, it was believed that $\delta_{LT}$ should be more suitable than $\gamma_0$ to serve as a testing ground for the chiral dynamics of QCD $[13, 14]$. The bottom-right panel of Fig. 2 shows $\delta_{LT}$ $[8]$ compared to $\chi$PT calculations and the MAID predictions. While
the MAID predictions are in good agreement with the results, it is surprising to see that the data are in significant disagreement with the $\chi$PT calculations even at the lowest $Q^2$, 0.1 GeV$^2$. This surprising disagreement ("$\delta_{LT}$ puzzle") presents a significant challenge to the present Chiral Perturbation Theory.

The spin polarizabilities data at very low $Q^2$ should be available soon. These results will provide benchmark tests to the $\chi$PT calculations at the kinematics where they are expected to work. A new proposal was recently approved to measure $g^p_2$ with a transversely polarized proton target in the low $Q^2$ region. It will provide an isospin separation of the spin polarizabilities to shed light on the "$\delta_{LT}$" puzzle.

Summary

In summary, the high polarized-luminosity available at JLab has provided us with high-precision nucleon spin structure data in the low to intermediate $Q^2$ region. These data help to study the non-perturbative region and the transition between perturbative and non-perturbative regions of QCD.

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