“Brane localized energy density” stabilizes the modulus in higher dimensional warped spacetime

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We consider a five dimensional AdS spacetime with two 3-brane scenario where the hidden brane contains a certain amount of positive constant energy density. In this model, we examine the possibility of modulus stabilization. Our result reveals that the non-zero value of “hidden brane energy density” is sufficient to stabilize the two brane system. Moreover we scan the parametric space for which the modulus (or radion) is going to be stabilized without sacrificing the conditions necessary to solve the gauge hierarchy problem. Finally we obtain the mass and coupling parameters of radion field in this higher dimensional braneworld scenario.

I. INTRODUCTION

The possible interactions between fundamental particles are best described by Standard Model (SM) of particle physics. Despite its enormous successes, the model suffers with a divergence due to the radiative correction of Higgs mass which may run up to Planck scale. An unnatural fine tuning is needed to confine the Higgs mass within TeV scale.

Many attempts have been made to solve this problem and the theory of extra dimensional models [1–7] are one of them. Extra dimensional models also have a natural outcome from String theory. Depending on their geometry, these models are compactified under various compactification schemes. Our usual four dimensional universe which is considered to be a 3-brane embedded within higher dimensional spacetime, emerges as a low energy effective theory [8, 9] and contains signatures of compactified extra dimension.

Among all the extra dimensional models proposed so far, Randall-Sundrum (RS) warped model [3] earned a special attention since it resolves the gauge hierarchy problem without introducing any intermediate scale (between TeV and Planck scale) in the theory. In RS model, the full spacetime is of five dimensional AdS nature where the extra dimension is spacelike and \( S^1/Z_2 \) orbifolded. The orbifolded fixed points are identified with two 3-branes embedded in the five dimensional spacetime. The separation between the branes is assumed to be \( \sim \) Planck length so that the hierarchy problem can be solved.

However, without any stabilization mechanism, two brane system can collapse due to the intervening gravity. So, like other higher dimensional braneworld scenario, one of the crucial aspects of RS model is to stabilize the inter-brane separation (known as modulus or radion). For this purpose, one needs to generate a radion potential with a stable minimum. Goldberger and Wise (GW) proposed a useful mechanism [10] to construct such a radion potential by imposing a massive scalar field in the bulk where the scalar field has appropriate values on the branes. In GW mechanism, it is not required to introduce any hierarchy between the boundary values of the scalar field for stabilizing the modulus at a value consistent with that proposed in RS model in order to solve the gauge hierarchy problem. Subsequently the phenomenology of radion field has also been studied extensively [11–14]. This radion phenomenology along with RS graviton modes [15–19] and RS black holes [20–22] are considered as key signatures to search for warped extra dimension in collider experiments in LHC [23, 24]. Some variants of RS model and its modulus stabilization have been discussed in [25–29].

However, the important questions that remain are:

- Can the modulus of RS like warped spacetime be stabilized by some other agent than that of a massive bulk scalar field?
- If such a stabilization mechanism is found, then what is the resulting stabilization condition? Moreover, what are the mass and the coupling (with SM fields) of radion field?

We aim to address these questions in this paper and for this purpose, the effective on-brane theory we used, is formulated by Kanno-Soda in [8].

The paper is organized as follows : Following two sections are devoted to brief reviews of RS scenario and the description of effective on-brane theory for RS like spacetime. The possibility of modulus stabilization for the said geometric model is explored in section IV. In section V, we find the necessary constraints on parametric space in order to solve the gauge hierarchy problem. Section VI is reserved for the coupling between radion and SM fields. We end the paper with some conclusive remarks.
II. BRIEF DESCRIPTION OF RS SCENARIO AND ITS STABILIZATION VIA GW MECHANISM

RS scenario is defined on a five dimensional AdS spacetime involving one warped and compact extra spacelike dimension. Two 3-branes known as visible/TeV and hidden/Planck brane are embedded in this five dimensional spacetime where the intermediate region between the branes is termed as ‘bulk’. If $y$ is the extra dimensional linear coordinate, then the branes are located at two fixed points $y = (0, \pi r_c)$ while the latter one is identified with our usual four dimensional universe. Here, $r_c$ is the compactification radius of the extra dimension. The opposite brane tensions along with the finely tuned five dimensional cosmological constant serve as energy-momentum tensor of RS scenario. The resulting spacetime metric is non-factorizable and expressed as,

$$ds^2 = e^{-2kr_c} |\phi| \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$ (1)

Due to $S^1/Z_2$ compactification along the extra dimension, $y$ ranges from $-\pi r_c$ to $+\pi r_c$. The quantity $k = \frac{\sqrt{-\Lambda}}{2M_T}$, is of the order of 5-dimensional Planck scale $M_T$. Thus $k$ relates the 5D Planck scale $M$ to the 5D cosmological constant $\Lambda$.

All the dimensionful parameters described above are related to the reduced 4-dimensional Planck scale $M_{Pl}$ as,

$$M_{Pl}^2 = \frac{M^3}{k} (1 - e^{-2\pi r_c})$$ (2)

In order to solve the hierarchy problem, it is assumed in RS scenario that the branes are separated by such a distance that $k \pi r_c \approx 36$. Then the exponential factor present in the metric, which is often called warp factor, produces a large suppression so that a mass scale of the order of Planck scale is reduced to TeV scale on the visible brane. A scalar mass say mass of Higgs is given as,

$$m_H = m_0 e^{-kr_c}$$ (3)

where $m_H$ and $m_0$ are physical and bare Higgs mass respectively. In RS model, it is assumed that the interbrane separation is of the order of Planck length so that the required hierarchy between the branes is generated. One of the crucial aspects of this braneworld scenario is to stabilize the distance between the branes. For this purpose, Goldberger and Wise demonstrated that the modulus corresponding to the radius of the extra dimension in RS warped geometry model can be stabilized by invoking a massive scalar field in the bulk with appropriate vacuum expectation values (vev) at the two 3-branes that reside at the orbifold fixed points. Consequently the phenomenology of the radion field originating from 5D gravitational degrees of freedom has also been explored.

III. LOW ENERGY EXPANSION SCHEME AND EFFECTIVE FOUR DIMENSIONAL ACTION FOR RS LIKE SPACETIME

In RS model, the Einstein equations are derived for a fixed inter-brane separation as well as for flat 3-branes. However, the scenario changes if the distance between the branes becomes a function of spacetime coordinates and the brane geometry is curved. These generalizations are incorporated while deriving the effective on-brane action via "low energy expansion scheme".

The model we considered in the present paper is described by a five dimensional anti-de Sitter (AdS) spacetime with two 3-branes embedded within the spacetime. The spacetime geometry is $S^1/Z_2$ orbifolded along the extra dimension. Taking $y$ as the extra dimensional linear coordinate, the branes are situated at orbifolded fixed points i.e. $y = 0$ (Planck brane) and $y = l$ (TeV brane) respectively. An additional constant energy density (over and above the brane tension) is localized on Planck brane. Moreover, the proper distance between the branes is considered as a function of spacetime coordinates. The action of this model is following:

$$S = \frac{1}{2k^2} \int d^4x \sqrt{-g} \left[ R^{(5)} + \left(12/l^2 \right) \right] - \int d^4x \sqrt{-g_{\text{hid}}} V_{\text{hid}} + \sqrt{-g_{\text{vis}}} V_{\text{vis}}$$

$$- \int d^4x \sqrt{-g_{\text{hid}}} \Lambda_{\text{hid}}$$ (4)

with $x^\mu = (x^0, x^1, x^2, x^3)$ are the brane coordinates, $\frac{1}{k^2} = M_{Pl}^2$, $M$ is the five dimensional Planck mass. $R^{(5)}$ and $l$ ($\sim$ Planck length) are the Ricci scalar and curvature radius of the five dimensional spacetime. Hidden and visible brane tensions are respectively, given by, $V_{\text{hid}} = \frac{n}{l}$ and $V_{\text{vis}} = -\frac{n}{l^2}$. $\Lambda_{\text{hid}}$ is the additional energy density, localized on the hidden brane.

We use the following metric ansatz with a warping nature along the extra dimension i.e.

$$ds^2 = e^{2\phi(x)} dy^2 + e^{-2\phi(y,x)} \eta_{\mu\nu} dx^\mu dx^\nu$$ (5)

$A(y,x)$ is commonly known as warp factor which has a dependence on all the spacetime coordinates while the component $G_{\mu\nu}$ is taken as the function of brane coordinates only. $\Lambda_{\text{hid}}$ is assumed to be less than the brane tensions. As a consequence of this assumption, brane curvature radius $L$ is much larger than the bulk curvature $l$ i.e. $l \ll L$. Then the bulk Einstein equations can be solved perturbatively where $\epsilon$ is taken as the perturbation parameter. This method is known as "low energy expansion scheme" in which the metric is expanded with increasing power of $\epsilon$. The zeroth order perturbation solution replicates the RS solution where the inter-brane separation is constant. The effective on-brane action obtained up to first order of perturbation, incorporates the fluctuation of modulus as well as non-zero value of brane matter. Using the low energy ex-
pansion scheme, the warp factor and the effective four dimensional action are as follows:

\[ A(y, x) = \frac{y}{l} e^{\phi(x)} \]  

(6)

and

\[ S_{eff} = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-\bar{h}} [\Psi(x) R^{(4)} - \frac{3}{2(1 - \Psi)} \bar{h}^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi] \]

\[ - \int d^4 x \sqrt{-\bar{h}} \Lambda_{hid} \]  

(7)

where \( \Psi(x) = [1 - \exp(-2e^{\phi(x)})] \) and \( R^{(4)} \) is the Ricci scalar formed by \( h_{\mu\nu} \). This above form of \( S_{eff} \) matches with that obtained in [11].

It may be noticed from eqn.(7) that upon projecting the bulk gravity on the brane, the extra degrees of freedom of \( R^{(5)} \) (with respect to \( R^{(4)} \)) appears as scalar field \( \Psi(x) \) which directly couples with the four dimensional Ricci scalar. Hence the effective on-brane action is a Brans-Dicke like theory where the self coupling of the scalar field is not a constant.

Eqn.(5) leads to the separation between hidden and visible brane along the path of constant \( x^h \) as follows:

\[ d(x) = \int_0^1 dy e^{\phi(x)} = l e^{\phi(x)} \]  

(8)

Above expression (eqn.(8)) clearly indicates that the proper distance between the branes depends on the brane coordinates and that why \( d(x) \) can be treated as field. From the perspective of four dimensional effective theory, this field is termed as ‘radion field’ (or modulus field) which is defined as ˜\( h_{\mu\nu} \) (with respect to \( \bar{h}_{\mu\nu} \)).

A. Generating the radion potential from brane localized term

The form of effective action obtained in eqn.(7) clearly reveals that the kinetic term of the gravitational field (\( h_{\mu\nu}(x) \)) as well as of the radion field (\( \Psi(x) \)) are not canonical. Thus to make the kinetic terms canonically normalized, lets transform the gravitational field as,

\[ h_{\mu\nu}(x) \rightarrow \bar{h}_{\mu\nu}(x) = e^{-\sigma(\Psi)} h_{\mu\nu}(x) \]  

(9)

The above transformation is commonly known as conformal transformation where \( \sigma(\Psi) \) is the conformal factor. Due to this conformal transformation, the Ricci scalar transforms as follows,

\[ R^{(4)}(h) = e^{-\sigma(\Psi)} [\bar{R}^{(4)}(\bar{h}) - 3\Box \sigma + \frac{3}{2} \bar{h}^{\mu\nu} \partial_{\mu} \sigma \partial_{\nu} \sigma] \]

where \( R^{(4)}(h) \) and \( \bar{R}^{(4)}(\bar{h}) \) are the Ricci scalar formed by \( h_{\mu\nu} \) and \( \bar{h}_{\mu\nu} \) respectively. The d’alembertian operator is defined as \( \Box = \bar{h}^{\mu\nu} \partial_{\mu} \partial_{\nu} \). Using the above relation between \( R \) and \( \bar{R} \), the action (in eqn.(7)) can be written as,

\[ S_{eff} = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-\bar{h}} [\Psi e^{\sigma(\Psi)} (\bar{R}^{(4)} - 3\Box _\sigma) - \frac{3}{2} \bar{h}^{\mu\nu} \partial_{\mu} \sigma \partial_{\nu} \sigma] \]

\[ - \int d^4 x \sqrt{-\bar{h}} \frac{1}{\Psi^2(1 - \Psi)} \bar{h}^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi \]  

(10)

In order to make the redefined gravitational field (\( \bar{h}_{\mu\nu} \)) canonical, take the conformal factor as \( e^{\sigma(\Psi)} = \frac{1}{\Psi} \). With this choice of conformal factor, the above action turns out to be,

\[ S_{eff} = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-\bar{h}} [\bar{R}^{(4)} - \frac{1}{2 \Psi^2(1 - \Psi)} \bar{h}^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi] \]

\[ - \int d^4 x \sqrt{-\bar{h}} \frac{1}{\Psi^2(1 - \Psi)} \Lambda_{hid} \]  

(10)

where we drop the surface term constructed by the d’alembertian operator. So, we manage to make the gravitational field canonical which is evident from the action presented in eqn.(10).

Comparing eqn.(7) and eqn.(10), it may be observed that the self coupling of the scalar field changes on performing the conformal transformation. It is expected because the conformal factor itself depends on the scalar field \( \Psi(x) \).

To get canonically normalized radion field lets transform \( \Psi(x) \rightarrow \chi(x) \) in such a way that the following relation

\[ \left( \frac{d\chi}{d\Psi} \right)^2 = \frac{3l^2/2\kappa^2}{\Psi^2(1 - \Psi)} \]  

(11)
holds. Thus in terms of $\tilde{h}_{\mu\nu}(x)$ and $\chi(x)$, the four-dimensional effective action takes the following form,

$$S_{\text{eff}} = \int d^4x \sqrt{-\tilde{h}} \left[ \frac{l}{2\kappa^2} \tilde{h}^{(4)} - \frac{1}{2} \tilde{h}_{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{\Psi(\chi)^2} \Lambda_{\text{hid}} \right]$$

(12)

where all the kinetic terms become normalized. The term $\frac{1}{\Psi(\chi)^2} \Lambda_{\text{hid}}$ in eqn. (12) acts as a potential for the canonical radion field $\chi(x)$. Afterwards, we denote this potential term by $V(\chi)$. In order to extract the explicit dependence of $V(\chi)$ on $\chi$, one needs the relation between $\Psi$ and $\chi$ i.e. $\Psi = \Psi(\chi)$. Solving eqn. (11), one obtains the dependence of $\Psi$ on $\chi$ as follows:

$$\frac{1}{\Psi(\chi)} = \frac{D}{4} [e^{\frac{\lambda_\phi}{2}} + De^{-\frac{\lambda_\phi}{2}}]^2$$

(13)

where $D$ is a dimensionless parameter arises in the process of normalizing the radion field and $b = \sqrt{\frac{2\kappa^2}{3l}}$. Above expression of $\Psi = \Psi(\chi)$ immediately leads to the radion potential as follows:

$$V(\chi) = (\frac{D^2}{16}) \Lambda_{\text{hid}} [e^{\frac{\lambda_\phi}{2}} + De^{-\frac{\lambda_\phi}{2}}]^4$$

(14)

The potential $V(\chi)$ goes to zero as $\Lambda_{\text{hid}} \to 0$ and thus it is clear that the “radion field potential” generates entirely due to the presence of non-zero constant energy density ($\Lambda_{\text{hid}}$) localized on hidden brane. In the next subsection, we check whether $V(\chi)$ admits any stability or not.

B. Stability of radion potential: Stabilized modulus

It can be shown that the radion potential $V(\chi)$ has a stable minimum at

$$< \chi > = \frac{1}{b} \ln D = \frac{3l}{2\kappa^2} \ln D$$

(15)

if $\Lambda_{\text{hid}}$ is considered to be positive. Using eqn. (4) and eqn. (13), we determine the relation between the canonical radion field ($\chi(x)$) and the inter-brane separation ($d(x)$) as follows:

$$\frac{D}{4} [e^{\frac{\lambda_\phi}{2}} + De^{-\frac{\lambda_\phi}{2}}]^2 = [1 - e^{-2d(x)/l}]^{-1}$$

(16)

By putting the vacuum expectation value (vev) of $\chi$ (i.e. $< \chi >$) into eqn. (16), we obtain the stabilized inter-brane separation ($< d(x) >$) and it is given by,

$$< d(x) > = \frac{1}{2} \ln \left( \frac{D^2}{D^2 - 1} \right)$$

(17)

The modulus stabilization condition (in eqn. 17) clearly reveals that the value of the parameter $D$ is constrained to be greater than one otherwise the branes can not be stabilized. Using eqn. (17), we obtain Figure 1 between $<d(x)/l>$ and $D$.

The figure demonstrates that the modulus decreases with increasing $D$ and the branes are going to be collapsed at large value of $D$.

Consequently, we find the mass of radion field excitations about the minimum as follows:

$$m_\chi^2 = \frac{d^2V}{d\chi^2}(< \chi >) = \frac{2\kappa^2}{3l} D^2 \Lambda_{\text{hid}}$$

(18)

Thus, $m_\chi$ goes to zero as $\Lambda_{\text{hid}} \to 0$. This is expected because the radion potential is created due to the non-zero value of $\Lambda_{\text{hid}}$.

V. REQUIRED CONDITIONS FOR SOLVING GAUGE HIERARCHY PROBLEM

Using the form of warp factor (in eqn. (6)), the physical Higgs mass is obtained as,

$$m_{\text{phy}} = m_0 e^{-\frac{<d(x)>}{l}}$$

(19)

where $m_0$ is the natural cut-off scale (10$^{19}$ GeV) of the theory and to derive the above relation between $m_{\text{phy}}$ and $m_0$, it is assumed that Higgs field is confined to visible brane only [2]. Using eqn. (17), $m_{\text{phy}}$ can be expressed as,

$$m_{\text{phy}} = m_0 \sqrt{\frac{(D^2 - 1)}{D^2}}$$

(20)

Thus in order to confine the Higgs mass ($m_{\text{phy}}$) within TeV scale, the value of the parameter $D$ is taken as $D = \sqrt{1 + 10^{-32}}$.

Moreover from the expression of radial mass in eqn. (13), it is clear that with $D = \sqrt{1 + 10^{-32}}$, $m_\chi$ comes at TeV scale if the constant energy density on hidden brane ($\Lambda_{\text{hid}}$) is of the order of $10^{44}$ (GeV)$^4$. 
VI. COUPLING OF RADION WITH SM FIELDS

Being a part of gravitational degrees of freedom, radion field interacts with brane matter and the interaction lagrangian is constrained by four dimensional general covariance. From five dimensional metric (in eqn. (5)), it is clear that the induced metric on visible brane is \( h_{\mu\nu} \) (where \( A(l, x) = \frac{d}{l} \)) and consequently \( d(x) \) directly couples with Standard Model fields confined on visible brane.

For example, consider the Higgs sector of Standard Model,

\[
S_{higgs} = \frac{1}{2} \int d^4x \sqrt{-h} \left[ e^{-A(l,x)} h^{\mu\nu} \partial_\mu \xi \partial_\nu \xi + e^{-2A(l,x)} m_0^2 \xi^2 \right]
\]

where \( \xi(x) \) is Higgs field and recall that \( m_0 \) is the natural mass scale of the theory. In terms of physical Higgs mass (see eqn. (19)), above action turns out to be,

\[
S_{higgs} = \frac{1}{2} \int d^4x \sqrt{-h} \left[ \exp \left( \frac{d(x)}{l} \right) h^{\mu\nu} \partial_\mu \xi \partial_\nu \xi + \exp \left( \frac{-2d(x)}{l} \right) \exp \left( 2 \frac{d(x)}{l} \right) m^2_{phy} \xi^2 \right]
\]

Using the transformation relation between \( d(x) \) and \( \chi(x) \) (in eqn. (10)), the interaction lagrangian between Higgs and canonical radion field is given as per following expression,

\[
L_{int}[H - \delta \chi] = \delta \chi \sqrt{\frac{2\kappa^2}{3l}} \exp \left( \frac{d(x)}{l} \right) T^\rho_\mu[k]\]

where \( \delta \chi \) is the fluctuation of radion field about its vev (i.e. \( \chi = \langle \chi \rangle + \delta \chi \)) and \( T^\rho_\mu[k] \) is the trace of energy-momentum tensor of Higgs field. So, the coupling between radion and Higgs field becomes, \( \lambda(H - \delta \chi) = \sqrt{\frac{2\kappa^2}{3l}} \exp \left( \frac{d(x)}{l} \right) m^2_{phy} \). Similar consideration holds for other SM fields. For example for Z-boson, \( \lambda(Z - \delta \chi) = \sqrt{\frac{2\kappa^2}{3l}} \exp \left( \frac{d(x)}{l} \right) m^2_{Z} \), where \( m_Z \) is mass of Z-boson. Thus the stabilized inter-brane separation plays a crucial role in determining the coupling strength between radion and SM fields. In the present case,

\[
\frac{d(x)}{l} = \frac{1}{2} \ln \left[ \frac{D^2}{(D^2 - 1)} \right]
\]

Hence finally we arrive at,

\[
\lambda(H - \delta \chi) = \sqrt{\frac{2\kappa^2}{3l}} \sqrt{\frac{D^2}{(D^2 - 1)}} m^2_{phy}
\]

and

\[
\lambda(Z - \delta \chi) = \sqrt{\frac{2\kappa^2}{3l}} \sqrt{\frac{D^2}{(D^2 - 1)}} m^2_{Z}
\]

Eqn. (21) and eqn. (22) clearly indicate that for \( D = \sqrt{1 + 10^{-32}} \) (considered earlier to solve the hierarchy problem), the coupling between radion and SM fields is of the order of TeV scale.

VII. CONCLUSION

We consider a five dimensional AdS, compactified warped geometry model with two 3-branes embedded within the spacetime. An additional constant energy density (over and above the brane tension) is localized on hidden brane. In this model, we examine the possibility of modulus stabilization. The findings and implications of our results are as follows:

- We find that the constant energy density (\( \Lambda_{hid} \)) localized on hidden brane is sufficient to generate a potential term for the radion field. This potential has a stable minimum as long as \( \Lambda_{hid} \) is positive. Moreover, the radion potential (\( V(\chi) \)) goes to zero as \( \Lambda_{hid} \to 0 \) and thus it is clear that \( V(\chi) \) is created due to the presence of “hidden brane energy density”.

- The stabilization condition is determined and given in eqn. (17). From the perspective of modulus stabilization, possible value of the parameter \( D \) which arises on the process of normalization of radion field, is scanned. As a result, the parameter \( D \) is constrained to be greater than one otherwise it becomes impossible to stabilize the two brane system. Furthermore, the stabilized value of the modulus decreases with increasing \( D \) and consequently the branes are going to be collapsed for large \( D \), which is clear from Figure (1).

- We also find the mass (see eqn. (13)) and coupling (with SM fields, see eqn. (21) and eqn. (22)) of radion field in this higher dimensional braneworld scenario. It is shown that by considering \( D = \sqrt{1 + 10^{-32}} \) and \( \Lambda_{hid} = 10^{44} \text{(GeV)}^4 \), mass as well as coupling of radion field can be kept at TeV scale without sacrificing the necessary conditions to solve the gauge hierarchy problem.

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