Chiral and Critical Behavior in Strong Coupling QCD

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We use a cluster algorithm to study the critical behavior of strongly coupled lattice QCD in the chiral limit. We show that the finite temperature chiral phase transition belongs to the $O(2)$ universality class as expected. When we compute the finite size effects of the chiral susceptibility in the low temperature phase close to the transition, we find clear evidence for chiral singularities predicted by chiral perturbation theory (ChPT). On the other hand, it is difficult to reconcile the quark mass dependence of various quantities near the chiral limit with ChPT.

1. INTRODUCTION

One of the important challenges in lattice QCD is to compute quantities that are dominated by the physics of light quarks. Although there has been substantial effort in extracting such quantities from the lattice results by matching the data with ChPT, it is unclear if the current range of masses used in the calculations are in the range where ChPT is valid [1]. Recently, efforts have been directed in two directions: a) Improving actions that reduce lattice artifacts. b) Improving ChPT that take these artifacts into account. However, we think that it is equally important to find improved algorithms to approach the chiral limit.

Over the last decade, a new class of cluster algorithms have emerged for solving a variety of lattice field theories [2]. These algorithms help in beating critical slowing down very efficiently. Recently, it was shown that the strong coupling limit of lattice gauge theories with staggered fermions can be solved using these new algorithms [3]. For the first time, this allows us to study the physics of massless quarks on large lattices from first principles.

In this article we present results from our study of the chiral and critical behavior near the chiral phase transition in strongly coupled lattice QCD with staggered fermions. We use $U(3)$ gauge fields instead of $SU(3)$ in order to avoid technical complications in the algorithm. Further details of our study can be found in [4].

2. MODEL AND OBSERVABLES

The model we study can be specified by the Euclidean action,

$$\sum_{x,\mu} \eta_{x,\mu} \left[ \bar{\psi}_x U_{x,\mu} \psi_{x+\mu} - \bar{\psi}_{x+\mu} U^\dagger_{x,\mu} \psi_x \right] - m \sum_x \bar{\psi}_x \psi_x, \tag{1}$$

where $m$ is the staggered quark mass, $U_{x,\mu}$ are $U(3)$ link variables and $\psi, \bar{\psi}$ are Grassmann variables representing the staggered quark fields. We choose the staggered fermion phase factors $\eta_{x,\mu}$ to have the property that $\eta^2_{x,\mu} = 1, \mu = 1, 2, 3$ (spatial directions) and $\eta^2_{x,4} = T$ (temporal direction), where the real parameter $T$ acts like a temperature. By working on asymmetric lattices with $L_t << L$ and allowing $T$ to vary continuously, one can study the finite temperature phase transition in strong coupling QCD [5].

The model described by the partition function $Z(T, m)$, constructed from the above action in the usual way, is known to have an exact $O(2)$ chiral symmetry when $m = 0$. This symmetry is broken at low temperatures but gets restored at high temperatures due to a finite temperature chiral phase transition. In order to study the chi-
r al physics near this transition we focus on the chiral condensate,

\[ \langle \phi \rangle = \frac{1}{L^3} \frac{\partial}{\partial m} Z(T, m), \]

(2)

the chiral susceptibility,

\[ \chi = \frac{1}{L^3} \frac{\partial^2}{\partial m^2} Z(T, m), \]

(3)

and the helicity modulus,

\[ Y_m = \frac{1}{L^3} \left\langle \left\{ \sum_{\mu=1}^{3} \left( \sum_x J_{x,\mu} \right)^2 \right\} \right\rangle, \]

(4)

where \( J_{x,\mu} = \sigma_x (b_{x,\mu} - N/8) \), with \( \sigma_x = 1 \) on even sites and \( \sigma_x = -1 \) on odd sites. When \( m = 0 \) the current \( J_{x,\mu} \) is the conserved current associated with the \( O(2) \) chiral symmetry. Further, as discussed in [6], it can be shown that \( F^2 = \lim_{L \to \infty} Y_m(m = 0) \), where \( F \) is related to the pion decay constant. We also measure the Goldstone pion mass \( M_\pi \).

3. UNIVERSAL PREDICTIONS

The predictions of ChPT for \( O(N) \) models have been discussed in [8]. In particular the finite size scaling formula for \( \chi \) at \( m = 0 \) is given by

\[ \chi = \frac{1}{N} \Sigma^2 L^3 \left[ 1 + \beta_1 (N - 1) \frac{1}{F^2 L} + \frac{a}{L^2} + \ldots \right], \]

(5)

where \( N = 2 \) in our case, \( \beta_1 = 0.226 \ldots \), \( \Sigma = \lim_{m \to 0} \lim_{L \to \infty} \langle \phi \rangle \), and \( a \) is a constant dependent on other low energy constants. The quark mass dependence of \( \langle \phi \rangle, \chi \) and \( Y_m \) are given by

\[ \langle \phi \rangle = \Sigma \left[ 1 + \alpha_1 \sqrt{m} + \alpha_2 m + \ldots \right], \]

(6)

\[ Y_m = F^2 \left[ 1 + \alpha_3 \sqrt{m} + \alpha_4 m + \ldots \right], \]

(7)

\[ M_\pi^2 = \Sigma m \frac{F}{F^2} \left[ 1 + \alpha_5 \sqrt{m} + \alpha_6 m + \ldots \right]. \]

(8)

For \( N = 2 \) one further finds that \( \alpha_3 = 0 \) [8]. One of the consequences of chiral symmetry can be described by the relation

\[ \frac{\langle \phi \rangle_m}{M_\pi^2 F_m^2} = 1 + \mathcal{O}(m), \]

(9)

which is the Gellmann-Oaks-Renner relation.

If the chiral phase transition is second order then close to the critical temperature \( T_c \) we have

\[ \Sigma(T) = A(T_c - T)\beta, \quad T < T_c, \]

(10)

\[ \lim_{L \to \infty} \langle \phi \rangle = B m^{1/\delta}, \quad T = T_c, \]

(11)

The \( O(2) \) universality predicts predicts \( \beta = 0.3485(2), \delta = 4.780(2) \).

4. RESULTS

We have done extensive calculations on various lattice sizes in the range \( 8 \leq L \leq 192 \) with \( L_t = 4 \). In order to understand the critical behavior we have measured \( \chi \) for \( m = 0 \) at different values of \( T \) between 7.3 and 7.5. On the other hand we focused on a single temperature in the broken phase \( (T = 7.42) \) and computed \( \langle \phi \rangle, Y_m \) and \( M_\pi \) for masses in the range \( 0 \leq m \leq 0.01 \). In figure 1 we plot our results for \( \chi \) as a function of \( L \) at \( m = 0 \) and \( T = 7.42 \). The data fits well to the ChPT prediction (eq.(5)) with \( \Sigma = 1.079(2), \quad F = 0.181(4) \) and \( a = 114(4) \) with a \( \chi^2/\text{d.o.f}\) of 0.73. The value of \( F \) obtained from this fit is in excellent agreement with \( F = 0.181(1) \) obtained through a direct evaluation of \( Y_m \) at \( m = 0 \) at large volumes. This can be seen from the plot shown in the inset of figure 1.

![Figure 1. Plot of \( \chi \) vs. \( L \) and \( Y_m \) vs. \( L \) (inset) at \( T = 7.42 \) and \( m = 0 \).](image-url)
Given this excellent agreement with ChPT at $m = 0$, we have also looked for the quark mass dependence of $\langle \phi \rangle$, $Y_m$ and $M_\pi$. These quantities were computed for various volumes until their values did not change for two different volumes. This “infinite” volume data is plotted on figure 2. When the data is fit to the prediction from ChPT (eqs. (6), (7) and (8)), with $\Sigma$ and $F$ fixed to 1.079 and 0.181 (see above), we find that $\alpha_1 = 17.9(2)$, $\alpha_2 = -64(3)$, $\alpha_3 = 20(1)$, $\alpha_4 = 301(36)$, $\alpha_5 = -8.14(10)$ and $\alpha_6 = 38(2)$. The $\chi^2$/d.o.f of all the fits are close to one. Although these fits appear to be good, the results are not consistent with ChPT. In particular ChPT predicts $\alpha_3 = 0$ [3]. In spite of this disagreement, the bottom right plot of figure 2 shows that our results satisfy the the Gellmann-Oaks-Renner relation (eq. (10)).

The only plausible explanation we can imagine for the disagreement, is that most of the masses used in the fit are perhaps too heavy for ChPT to be valid. This is because the cutoff in ChPT is proportional to $F^2$ (we are effectively in three dimensions), which is quite small in our case. Indeed the coefficients $\alpha_i$ are also uncomfortably large for the same reason. We are currently studying another temperatures deeper inside the broken phase where $F^2$ is larger.

Finally a careful analysis of the finite size effects of $\chi$ at various temperatures reveals that $T_c = 7.47739(3)$. We also find that the data for $\Sigma$ and $\langle \phi \rangle$ fits well to the form predicted by eqs. (10) and (11). We find $\beta = 0.348(2)$, $A = 2.92(2)$ with a $\chi^2$/d.o.f of 0.53 for the fit to eq. (10) and $\delta = 4.97(10)$, $B = 5.9(1)$ with a $\chi^2$/d.o.f of 0.34 for the fit to eq. (11). The results and the fits are shown in figure 3.

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