Influence of supersonic flow on nonlinear oscillations of a cylindrical shell

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Abstract. In this paper we consider nonlinear oscillations of an isotropic cylindrical shallow shell in a supersonic gas flow. The study is carried out with considering both types of nonlinearity: the geometric and the aerodynamic. By taking into account the asymmetric (quadratic) nonlinearity, we come to a conclusion that in certain velocity intervals the amplitude-velocity dependence is a multi-valued function. It is shown, that there exist so-called zones of silence – the intervals of the steaming flow velocity, where undamped flutter oscillations cannot be induced. Here we give some of our most important and significant results, which follow from the influence of a supersonic gas flow on the nature of nonlinear oscillations of the investigated aeroelastic system: a) the larger the relative radius of the shell, the wider the silence zone; b) the amplitude of oscillations, depending on the flow velocity, is a monotonously decreasing function in the region to the left of the silence zone and tends to zero at the left boundary of the zone; at the right boundary of the zone the amplitude increases abruptly to a certain finite value and then it monotonously decreases; c) in the case of thin shells with an increase in the velocity of the flow we observe the following: flutter oscillations mode persists up to a certain velocity value (the “upper” critical velocity), where the oscillations “break off” and the unperturbed state of the shell restores. When the velocity decreases, the unperturbed state is stable as long as the velocity is greater than the critical flutter velocity (the “lower” critical velocity), at which the amplitude of flutter oscillations increases abruptly to a certain value and keeps increasing with further velocity decrease; d) in the case of sufficiently thin shells, the zone of silence is a semi-infinite region.

1. Introduction

The first paragraph after a heading is not indented (Bodytext style). Investigations on stability of shells and plates in a supersonic gas flow are conducted by various researchers (for example monographs [5, 6, 11] and review article [9]) in both linear and non-linear formulation. The critical values of the flow velocity – the smallest values, at which the aeroelastic system loses stability – were found by solving the linear problems. The solutions of the nonlinear problems were obtained by using approximate methods and the studies were mainly devoted to the dependence of the oscillation amplitude on the flow velocity (see [11]), i.e. the amplitude-velocity dependence.

In this work we investigate the amplitude-velocity dependence of nonlinear oscillations of a cylindrical panel in a supersonic gas flow. Here, in contrast to the main scientific studies...
published before, the research is conducted by method proposed in [3]. Analogous study for thin plates is presented in our paper [4]. In [3] it was established that the effects of quadratic nonlinearities cannot be detected if assumed that the physical system oscillates around the unperturbed state so in this work a new approach was recommended: a) to refuse that type of oscillations and b) to assume that the system makes nonlinear oscillations with finite amplitude around the state sufficiently close to the unperturbed one. So it became possible to obtain a nonlinear system of algebraic equations to determine the oscillations amplitude, including the terms that take into account the quadratic nonlinearity. The numerical solution of the system of nonlinear algebraic equations was found, and on the basis of that solution was established how the nonlinear oscillations’ frequency and the geometrical parameters of the shell affect the amplitude-velocity dependence and that, by taking into account the quadratic nonlinearity, the nature of that dependence quantitatively and qualitatively changes depending on the specified parameters.

2. Formulation of the stability problem

Let us consider a thin isotropic rectangular cylindrical panel of constant thickness in the orthogonal curvilinear coordinates, where the coordinate lines and coincide with the middle surface’s curvature lines. The coordinate line is rectilinear and represents the distance from the point to the point of the shell along the normal of the median surface.

There is a supersonic gas flow over one side of the shell with a constant unperturbed velocity directed along the axis. The stability issues of this aeroelastic system are investigated.

On the basis of research, the following assumptions are accepted:

1) the Kirchhoff-Love hypothesis, according to which [10]

\[ u_1(x, y, z, t) = u(x, y, t) - z \frac{\partial w}{\partial x}, \quad u_2(x, y, z, t) = v(x, y, t) - z \frac{\partial w}{\partial y}, \quad u_3(x, y, z, t) = w(x, y, t), \quad (1) \]

where \( u_i \) are the displacement components of the shell points \( i = 1, 2, 3 \);

2) the gas pressure is calculated by the approximate formula of the “piston theory” [2, 8]

\[ p = p_\infty \left( 1 + \frac{\kappa - 1}{2} \frac{v_3}{a_\infty} \right)^{2 \kappa / (\kappa - 1)}; \quad (2) \]

3) the basic states of the theory of very shallow flexible shells considering that the deflections \( w(x, y, t) \) are comparable with the thickness of the shell [11].

On the basis of these assumptions the following system of nonlinear differential equations of motion of a shallow shell is obtained [5]:

\[
D \Delta^2 w + \frac{1}{R} \frac{\partial^2 F}{\partial y^2} + \rho_0 h \frac{\partial^2 w}{\partial t^2} + \left( \rho_0 h \varepsilon + \kappa p_\infty \frac{\varepsilon}{a_\infty} \right) \frac{\partial w}{\partial t} + \kappa p_\infty M \frac{\partial w}{\partial x} = \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \]

\[
= \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 F}{\partial x^2} \frac{1}{R} - \kappa p_\infty \left[ \frac{\varepsilon + 1}{4} M^2 \left( \frac{\partial w}{\partial x} \right)^2 + \frac{M}{3} \left( \frac{\partial w}{\partial x} \right)^3 \right], \quad (3)
\]

\[
\frac{1}{E h} \Delta^2 F = \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2}, \quad (4)
\]

where \( D = E h^3 / [12(1 - \mu^2)] \), \( E \) is the elastic modulus, \( F(x, y, t) \) is the stress function \( (T_{11} = \partial^2 F / \partial y^2, T_{22} = \partial^2 F / \partial x^2, T_{12} = -\partial^2 F / \partial x \partial y) \), \( T_{ik} \) are the internal forces, \( M = U / a_\infty \) is the Mach number, \( a^2_\infty = \kappa p_\infty / \rho_\infty \) is the speed of sound for the unperturbed gas, \( p_\infty \) and \( \rho_\infty \) are the pressure and density of the unperturbed gas flow, \( \mu \) is the Poisson’s ratio, \( \varepsilon \) is the linear
attenuation coefficient, $\kappa$ is the polytropic coefficient, $R$ is the radius of the shell, $t$ is time, and $\rho_0$ is the material density of the shell.

When investigating stability issues, the conditions on the contour of the shell should also be added to equations (3), (4). In this paper we consider the case, when the shallow shell is hinged around the contour ($0 \leq x \leq a$, $0 \leq y \leq b$), and its edges freely move in plane. So, according to [5], the boundary conditions take the following form:

For $x = 0$, $x = a$

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} = 0,$$

$$T_{11} = 0, \quad T_{12} = 0,$$  \hspace{1cm} (5)

For $y = 0$, $y = b$

$$w = 0, \quad \frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} = 0,$$

$$T_{22} = 0, \quad T_{21} = 0,$$  \hspace{1cm} (6)

where $T_{ik}$ are the averaged force values at the edges of the shell.

3. Solution of the problem. Reduction to the stability problem described by a system of ordinary differential equations

Let us present the approximate solution of equation (3) which satisfies conditions (5) and (7) in the following form [5]

$$w(x, y, t) = \sum_{k=1}^{n} f_k(t) \sin(\lambda_k x) \sin(\mu_1 y), \quad \lambda_k = \frac{k \pi}{a}, \quad \mu_1 = \frac{\pi}{b},$$  \hspace{1cm} (9)

We use equation (3) for determination the functions $f_{ik}(t)$, considering the case $n = 2$. After substitution (9) (when $n = 2$) into (4), a linear nonhomogeneous differential equation for function $F$ is obtained and solved by satisfying conditions (6) and (8). Substituting (9) and the found form of $F$ into (3) and using the Bubnov-Galerkin method for determination the dimensionless unknown functions $x_1 = f_1(t)/h$, $x_2 = f_2(t)/h$, we obtain a nonlinear system of ordinary differential equations [5, 11] in the following form:

$$\frac{d^2 x_1}{d\tau^2} + \chi \frac{dx_1}{d\tau} + x_1 - \frac{2}{3} k v x_2 + k v^2 \left[ \alpha_{11} x_1^2 + \alpha_{12} x_2^2 + \nu x_2 (\beta_{11} x_1^2 + \beta_{12} x_2^2) \right] + Q x_1 (\gamma_{11} x_1^2 + \gamma_{12} x_2^2) + L \delta_{11} x_1^2 + \delta_{12} x_2^2 = 0,$$

$$\frac{d^2 x_2}{d\tau^2} + \chi \frac{dx_2}{d\tau} + \gamma^2 x_2 + \frac{2}{3} k v x_1 + k v^2 \left[ \alpha_{21} x_1 x_2 + \nu x_1 (\beta_{21} x_1^2 + \beta_{22} x_2^2) \right] + Q x_2 (\gamma_{21} x_1^2 + \gamma_{22} x_2^2) + L \delta_{21} x_1 x_2 = 0.$$  \hspace{1cm} (10)

Here, besides the dimensionless time $\tau = \omega_1 t$, we accepted the following notation

$$k = \frac{4 \varepsilon \rho_0}{\rho_0 h^2}, \quad Q = \frac{4}{16 \rho_0 h^3}, \quad L = \frac{1}{\rho_0 h^4}, \quad \nu = M \frac{h}{a}, \quad \gamma = \frac{\omega_2}{\omega_1}, \quad \chi = \frac{1}{\omega_1} (\varepsilon + \varepsilon_0),$$  \hspace{1cm} (11)

as well as the coefficients $\alpha_{ik}$ and $\beta_{ik}$ taking into account the aerodynamic nonlinearity

$$\alpha_{11} = \frac{2}{9} (\varkappa + 1), \quad \alpha_{12} = \frac{56}{45} (\varkappa + 1), \quad \alpha_{21} = \frac{16}{45} (\varkappa + 1),$$

$$\beta_{11} = \beta_{21} = \frac{\pi^2}{40} (\varkappa + 1), \quad \beta_{22} = \frac{11 \pi^2}{70} (\varkappa + 1), \quad \beta_{12} = -\frac{9 \pi^2}{70} (\varkappa + 1),$$  \hspace{1cm} (12)

$$\varkappa = \frac{1}{R^2} \left[ \frac{R^2}{\rho_0 h^2} \right]$$
and coefficients $\gamma_{ik}$ and $\delta_{ik}$ taking into account the geometric nonlinearity

$$
\gamma_{11} = Eh(\lambda_1^2 + \mu_1^2), \quad \gamma_{12} = \gamma_{21} = 4\gamma_{11} + \frac{81\lambda_1^4\mu_1^4}{\Delta_{12}} + \frac{\lambda_1^4\mu_1^4}{\Delta_{22}}, \quad \gamma_{22} = Eh(\lambda_2^2 + \mu_1^2),
$$

$$
\delta_{11} = -\frac{8\lambda_1^2\mu_1^2}{3\pi^2} h \left( \frac{Eh}{\lambda_2^2} + \frac{4\lambda_1^2}{\Delta_{11}} \right), \quad \delta_{12} = -\frac{32\lambda_1^2\mu_1^2}{15\pi^2} h \left( \frac{Eh}{\lambda_2^2} + \frac{12\lambda_1^2}{\Delta_{21}} \right),
$$

$$
\delta_{21} = -\frac{8\lambda_1^2\mu_1^2}{3\pi^2} h \left( \frac{8Eh}{15\lambda_2^2} + \frac{16\lambda_1^2}{5\Delta_{11}} + \frac{\lambda_1^2}{\Delta_{12}} + \frac{16\lambda_2^2}{5\Delta_{21}} + \frac{\lambda_3^2}{15\Delta_{32}} \right).
$$

In (10) $\nu$ is the reduced velocity parameter, $\chi$ is the reduced damping parameter, $\omega_i$ are the frequencies of small natural oscillations of the shell, determined by formulas

$$
\omega_i^2 = \frac{1}{\rho h} \left[ D(\lambda_i^2 + \mu_1^2)^2 + \frac{\lambda_i^4}{R^2\Delta_{11}} \right] \quad (i = 1, 2).
$$

Thus, we reduced the stability problem of the aeroelastic system in the first approximation to the study of the behavior of solutions of the nonlinear system of ordinary differential equations (10) depending on the gas flow velocity value (on the parameter $\nu$). Similar systems of equations were obtained by many other authors [1].

**4. Solution of the linear problem**

Before solving the nonlinear problem, the corresponding linear problem should be analyzed, since: a) based of the linear problem the critical parameter $\nu = \nu_{cr}$ (therefore the critical flow speed $u_{cr} = ah^{-1}\nu_{cr}$, or the critical Mach number $M_{cr} = ah^{-1}\nu_{cr}$), at which the unperturbed state of the shell loses stability with respect to small disturbances, can be obtained, and b) we need the critical value $u_{cr}$ (or $M_{cr}$ or $\nu_{cr}$) to investigate the stability problem in a nonlinear formulation.

Thus, the linear system corresponding to (10) has the form

$$
\frac{d^2x_1}{dt^2} + \chi \frac{dx_1}{dt} + x_1 - \frac{2}{3} k\nu x_2 = 0,
$$

$$
\frac{d^2x_2}{dt^2} + \chi \frac{dx_2}{dt} + \gamma^2 x_2 + \frac{2}{3} k\nu x_1 = 0.
$$

We represent the solution of system (15) in the form

$$
x_1 = y_1 e^{\lambda t}, \quad x_2 = y_2 e^{\lambda t},
$$

and obtain the following characteristic equation for $\lambda$:

$$
\lambda^4 + 2\chi\lambda^3 + (\gamma^2 + 1 + \chi^2)\lambda^2 + \chi(\gamma^2 + 1)\lambda + \gamma^2 + 4\frac{k^2}{9}\nu^2 = 0.
$$

The unperturbed form of the shell is stable if the real parts of the characteristic equation’s roots are negative. Hence, the stability conditions, according to Hurwitz’s theorem, take the form

$$
\chi > 0, \quad \chi(1 + \gamma^2) > 0, \quad (\gamma^2 - 1)^2 + 2\chi^2(1 + \gamma^2) - \frac{16}{9} k^2\nu^2 > 0.
$$

The first two inequalities are always satisfied — they require the internal and aerodynamic damping to be positive. The third inequality means that in the case of small $\nu$, all the characteristic indicators of $\lambda$ lie in the left half-plane of a complex variable, and the trivial
solution \( w \equiv 0 \) is asymptotically stable with respect to small perturbations. The value of the parameter \( \nu = \nu_{cr} \) at which two of the characteristic indicators become purely imaginary, and the rest still lie in the left half-plane, is critical, it corresponds to the critical velocity of the panel flutter in the linear formulation of this problem. According to this, the third inequality gives the following formula for determination of the critical speed of flutter in the case of the chosen buckling form of the shell [5]:

\[
\nu_{cr} = \frac{3\gamma^2 - 1}{4k} \sqrt{1 + \frac{2\chi^2(\gamma^2 + 1)}{(\gamma^2 - 1)^2}}.
\]  

(19)

Taking \( \nu = \nu_{cr} \), the following value \( \theta_{cr} \) of the oscillations frequency of the shell for a linear flutter \((\lambda_{cr} = \pm i\theta_{cr})\) is found from the characteristic equation

\[
\theta_{cr}^2 = \frac{1}{2}(\gamma^2 + 1).
\]  

(20)

Formulas (18) and (19) are the first approximations for \( \nu_{cr} \) and \( \theta_{cr} \), and similar expressions were obtained by many other authors (see the References given in monographs [5] and review article [9]).

We have also obtained the systems of ordinary differential equations for the cases \( n = 3 \) and \( n = 4 \), they are rather cumbersome for analytical research so we don’t bring them in this paper. Further we will use them for solving the linear problem and in numerical studies.

5. The nonlinear problem investigation. Existence and nature of steady-state flutter oscillations

Let us move to the study of the nonlinear problem described by the nonlinear system (10). This system differs from similar systems related to stability of flexible plates by presence of terms with quadratic nonlinearities of both aerodynamic and geometric origin. These terms characterize the non-symmetry of the nonlinearity and, hence, an approximate periodic solution of system (10), according to [3, 7], has the following form

\[
x_1 = A_1 \cos \theta \tau + B_1 \sin(\theta \tau) + C_1 + \cdots, \quad x_2 = A_2 \cos \theta \tau + B_2 \sin(\theta \tau) + C_2 + \cdots.
\]  

(21)

Here \( A_i, B_i, C_i \), and \( \theta = \omega \omega_{cr}^{-1} \) \((i = 1, 2)\) are the unknown constants; \( \omega \) is the unknown frequency of nonlinear oscillations; the dots indicate the terms containing harmonics. Solution (20) differs from the existing ones [5] by free terms \( C_i \neq 0 \), that is why we can take into account the influence of the quadratic nonlinearity.

By substituting (20) into (10) and equating the coefficients of free terms, \( \cos(\theta \tau) \) and \( \sin(\theta \tau) \) to zero (the terms with harmonics are neglected), we obtain a system of nonlinear algebraic equations, which is rather cumbersome, so we do not bring it here. The approximate solution of that system is obtained by assuming that [3]: a) the system damping is sufficiently small \((|\chi|B_i| \ll |A_i|, |B_i| \ll |A_i|, i = 1, 2)\), and b) the aeroelastic system makes steady oscillations with finite amplitude around the state that differs infinitesimally from the unperturbed state \((|A_j| \gg |C_j|, j = 1, 2)\). After accepting these assumptions and neglecting the terms in degrees higher than one and the products of \( B_1, B_2, C_1, \) and \( C_2 \), the nonlinear system becomes significantly simplified and can be represented by the following subsystems: the first one is the equations obtained by equating to zero the free terms, the second one — by equating to zero the coefficients of \( \cos(\theta \tau) \), and the third one — by equating to zero the coefficients of \( \sin(\theta \tau) \).

The damping effect is taken into account in the third subsystem. Damping is assumed to be small, so the third subsystem has the following approximate solution:

\[
B_1 \approx 0, \quad B_2 \approx 0 \quad \text{for} \ \chi \approx 0.
\]  

(22)
The first subsystem gives the expressions for $C_1$ and $C_2$ through $A_1$ and $A_2$. Then the second subsystem, which determines the oscillations amplitudes ($A_1$ and $A_2$) of the aeroelastic system depending on the parameters $\theta$ and $\nu$ when $\chi \approx 0$ takes the following form

$$
A_1(1 - \theta^2) - \frac{2}{3} k \nu A_2 + 2 k \nu^2 \alpha_{11} A_1 C_1 + 2 k \nu^2 \alpha_{12} A_2 C_2 + \frac{3}{4} k \nu^3 A_2 (\beta_{11} A_1^2 + \beta_{12} A_2^2) + \frac{3}{4} Q A_1 (\gamma_{11} A_1^2 + \gamma_{12} A_2^2) = 0,
$$

$$
A_2(\gamma^2 - \theta^2) + \frac{2}{3} k \nu A_1 + k \nu^2 \alpha_{21} (A_1 C_2 + A_2 C_1) + \frac{3}{4} k \nu^3 A_1 (\beta_{21} A_1^2 + \beta_{22} A_2^2) + \frac{3}{4} Q A_2 (\gamma_{21} A_1^2 + \gamma_{22} A_2^2) = 0.
$$

Herein

$$
C_1 = -K \nu^2 \frac{[L(\delta_{11} A_1^2 + \delta_{12} A_2^2) + \alpha_{11} A_1^2 + \alpha_{12} A_2^2]}{2 \Delta},
$$

$$
C_2 = -K \nu^2 K \nu^2 \frac{(L \delta_{21} + \alpha_{21}) A_1 A_2 \Delta_1 - L(\delta_{11} A_1^2 + \delta_{12} A_2^2) + (\alpha_{11} A_1^2 + \alpha_{12} A_2^2) \Delta_3}{2 \Delta},
$$

where

$$
\Delta_1 = 1 + \frac{3}{2} Q \gamma_{11} A_1^2 + \frac{1}{2} Q \gamma_{12} A_2^2 + K \nu^3 \beta_{11} A_1 A_2,
$$

$$
\Delta_2 = \gamma^2 + K \nu^3 \beta_{22} A_1 A_2 + \frac{3}{2} Q \gamma_{22} A_2^2 + \frac{1}{2} Q \gamma_{21} A_1^2,
$$

$$
\Delta_3 = \frac{2}{3} K \nu + \frac{3}{2} K \nu^3 \beta_{21} A_1^2 + \frac{1}{2} K \nu^3 \beta_{22} A_2^2 + Q \gamma_{21} A_1 A_2,
$$

$$
\Delta_4 = -\frac{2}{3} K \nu + \frac{3}{2} K \nu^3 \beta_{12} A_2^2 + \frac{1}{2} K \nu^3 \beta_{11} A_1^2 + Q \gamma_{12} A_1 A_2, \quad \Delta = \Delta_1 \Delta_2 - \Delta_3 \Delta_4.
$$

The nonlinear system (22) is investigated numerically with the following initial data: $E = 7.3 \times 10^{10} \text{N/m}^2$, $\mu = 0.34$, $\rho_1 = 2.79 \times 10^3 \text{kg/m}^3$ (duralumin), $\kappa = 1.4$; $\rho_\infty = 1.29 \text{kg/m}^3$, $a_\infty = 340.29 \text{m/s}$ (air). The dependence of the amplitude $A$ of steady-state flutter oscillations at the point $(a/2, b/2, 0)$ of the shell (in this case $A = A_1$) on the parameter $\nu$ characterizing the speed of the flow, for $a/b = 1$ and different values of geometric parameters $R/a$, $h/a$ and frequency $\theta$ is investigated.

Numerical results in [4] for rectangular plates show that ratio $h/a$ significantly affects (both qualitatively and quantitatively) on the amplitude-velocity dependence $A(\nu)$. Therefore, here we separately consider the cases of relatively thick (medium thickness) and sufficiently thin shells and for simplicity the nonlinear flutter oscillations’ frequency is $\theta = \theta_{cr}$.

5.1. Shells of medium thickness

Numerical studies were conducted for different values $R/a$ taking $h/a = 10^{-2}$ and the results are given in figure 1 a–c, showing that

(i) for small values of $R/a$ there exist such values of the flow velocity $\nu_*$ and $\nu^*$ that $A(\nu)$ is a single-valued function in the regions $\nu \in [0, \nu_*]$ and $\nu > \nu^*$, and when $\nu \in (\nu_*, \nu^*)$ $A(\nu)$ is two-valued (figure 1). Should be noted that the lower branches are most likely to be unstable, they are dotted. Figure 1 clearly shows that when the velocity of the flow increases and $\nu < \nu^*$ the flutter oscillations amplitude monotonously decreases, and when $\nu = \nu^*$ the amplitude increases abruptly to a certain finite value, and then monotonically
Figure 1. Character of the function $A(\nu)$

when the velocity decreases, the amplitude monotonously increases and at $\nu = \nu^*$ decreases abruptly and further decrease in velocity leads to an increase in amplitude. Consequently, in this case steady-state flutter oscillations can be excited at any value of the flow velocity.

(ii) an increase in $R/a$ leads to occurrence of a zone of silence (figure 2), i.e. the interval of the steaming flow velocity, where undamped flutter oscillations cannot be induced. Outside this zone $A(\nu)$ is a monotonously decreasing function.

(iii) further, with $R/a$ increasing, the silence zone becomes a semi-infinite region, and the amplitude-velocity dependence has the following character: when the flow velocity gradually increases, the flutter mode remains up to a certain value of velocity $\nu^*$ (while $\nu^* > \nu_{cr}$), after which the oscillations “break off” and the unperturbed state of the shell is restored. Should be noted that in the interval $(\nu_{cr}, \nu^*)$ $A(\nu)$ is a two-valued function; when the velocity decreases, the unperturbed state is stable as long as $\nu > \nu_{cr}$. At $\nu = \nu_{cr}$ the amplitude of flutter oscillations increases abruptly to a certain value and keeps increasing with further velocity decrease (figure 3). Note that points $L$ and $B$ in figure 3 are the bifurcation points (the point where the transition from a steady state to an unstable one and vice versa takes place), while the point $L$ is the limit point.

5.2. Thin shells
In the case of thin shells we have conducted numerical analysis for different $R/a$ at $h/a = 1/300$ and we bring the results in figure 1 d–f, which show that

(i) the zone of silence, regardless of $R/a$, exists always and is a semi-infinite region.

(ii) there exist bifurcation points, and $L_i$ are the limiting points (where the tangent to the graph becomes vertical), and $B$ are the ordinary bifurcation points.

Summary
In this paper we consider nonlinear oscillations of an isotropic cylindrical shallow shell in a supersonic gas flow. The amplitude-velocity dependence is established to be a multivalued function in certain intervals of velocity. It is also shown that:
(i) in certain regions of the flow velocity variation (zones of silence) undamped flutter oscillations are impossible to be induced;

(ii) the larger the relative radius of the shell, the wider the silence zone;

(iii) the amplitude of oscillations, depending on the flow velocity, is a monotonously decreasing function in the region to the left of the silence zone and tends to zero at the left boundary of the zone; at the right boundary of the zone the amplitude increases abruptly to a certain finite value and then it monotonously decreases;

(iv) with an increase in the flow velocity, the flutter oscillations mode persists up to a certain value of the velocity of the flow (the “upper” critical velocity), at which the oscillations “break off” and the unperturbed state of the shell restores.

(v) there are bifurcation points of both limiting and ordinary type.

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