Relation and Network Externality Based on Asymmetric Interest Flow Theory with the Case of Health Damage Caused by PPV’s Energy Consumption

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Abstract. The study starts with a logical reasoning: if the network is composed of individuals and their relations, the network externality should also have a micro foundation. However, the traditional network externality or network effects theory does not indicate the specific meaning of individual nor relation externalities. Therefore, the study first defines the relation and its decisive characteristic: the interaction of interest flow. Furthermore, it is pointed out that the asymmetric interest flow is the essential feature of externality. Then, with the help of flow analysis in graph theory, the mathematical models of potential relation, relation and relation externality are given. Based on the definition of individual externality on the net, a new interpretation of network externality compared with traditional one is given, which is the state set of individual externalities on the net. Finally, we use these definitions to give a new explanation to a specific case in energy and environmental field and supply a new framework to solve general or concrete externality problems.

1. Introduction

Externality is one of the most important categories in the history of economic thought. It represents the difference between individual marginal cost or benefit and social marginal cost or benefit. Externality is also one of the manifestations of market failure, which will reduce the efficiency of resource allocation. The theory of property rights represented by Coase believes that the internalization of the external costs of economic individuals should be promoted by clearing the boundaries of rights and trading the limited rights by the market. The followers of Pigou tax believe that the market could not effectively solve the plight of the tragedy of the commons [1], advocates either tax collection, or government regulation, so that individuals can rationally examine their own behavior and undertake external responsibility.

Network externality is one of the important categories of network economics. It is generally defined as the utility of consuming a unit of product increases with the number of consumers who consume the product increases [2]. In a broader sense, network externality refers to the net value increment generated by the action when the agents taking the same action increases [3]. In the theoretical world, network externalities are often compared with network effects, which is defined as: the interests come out from some decisions, if they can be consistent with the decision-making behavior of others [4]. In order to effectively distinguish it from the network externalities considered here, we call it as traditional network externality.
Demsetz says “externality is a vague concept, since no matter what kind of profit and loss results, in the world scope, it is not external. It is not these people who are damaged or benefiting, but those people. Decisions by individuals or groups of people can interact, resulting in the consequences of damage and benefit” [5]. This study believes in the logic that if the relation, link or connection is regarded as the basic unit of the network, then the network externality should be based on the relation externality as a logical starting point. But the traditional network externality lacks the micro relation externality basis and will face more uncertainty in economic interpretation and empirical evidence. The relation externality should be examined first from the most general micro level, and then extended to the specific meaning of network externality. If the flow in graph theory [6] specifically refers to the interest flow in economics, its asymmetric performance will be a good mathematical interpretation of relation externality and individual externality. Based on this, a new network externality explanation is proposed.

Energy and environmental externality theory are logic extension or application of externality theory in their respective field. It is commonly known that energy relevant or environmental problems are widely analyzed in externality theory, even from beginning. Researchers frequently use cases as references like Pigou’s case of private garden improving air quality, Coase’s example of smoke causing damage and Nordhaus’s excellent work for climate change. ExternE makes a systematic analysis on the framework to quantify energy externality [7].

The health damage is one of the most serious negative energy and environmental externalities. The reason why it happens often follows the logic that, in the process of energy consumption some undesirable environmental side-effects like air pollution, water contamination and nuclear waste etc. take place, then they will damage people’s health in various ways. This article pays more attention on PPV (Private Passenger Vehicle, PPV), which consumes gasoline or diesel and displaces health harmful air pollutants, like PM$_{10}$, NO$_X$, and CO etc. The researches point out that PM$_{10}$ has a seriously harmful health effect [8]. In real world, there is no requirement for PPV owners to compensate for their damage to others’ health. We will pose a new explanation for why this externality exits and cannot be eliminated based on relation and network perspective.

2. Relation Externality

2.1. Definition of relation and potential relation

Relation has many forms of expression, and economic relation can be defined as economic individuals and the combination of interest flow interactions between them. It is the basic unit of the economic network and consists of three elements: economic individuals, interest flows and information describing them.

The study continues to point out that the determined relation must meet the following conditions: two identified economic individuals, two-way interest flow interaction, and complete relation description information. If any of the three conditions are not satisfied, it will be called as meta-relation. The transition from meta-relation to relation is often a dynamic, complex process, sometime entangled with other relations.

The study refers to the meta-relation in the transition to a relation as a potential relation. The disappearance of the potential relation may be the active withdrawal of the benefit stream exporter (no relation formation), but more likely the benefit stream receiver will return the reverse interest stream and the relation is realized or determined.

The potential relation is widespread, for example, there is a potential relation between the insurance company and the insured, between the customer and the after-sales service department, or between the guarantor and the creditor. The main feature of the most potential relations is that there is only one-way or no explicit acceptance of the utility flow in a certain time or space. When the utility flow receiving individual takes the feeding back action and the reverse flow is received by the initial sender, the determined relation or just the relation is formed.
2.2. Interest flow interaction
The core of the relation is the interest flow interaction, and its direction, size, and nature constitute the economic properties of the relation. The choice and result of economic individuals are the main research objects of mainstream economics. Classical rationality, complete information and maximization hypothesis are all aimed at economic individual choice behavior. It is undeniable that choice always brings about the establishment, maintenance or termination of a relation, and it all corresponds to the change of interest flow or the change of interest flow interaction. By describing the interaction of interest flows, relations can basically be determined, especially when economic individual choice is less important (perfect competition market).

The flow of interest is a general term for utility flows, service flows, technique flows and maybe other flows. Among them, the utility flow can be understood as the value transfer between the object to the person, the service to the person, the mutual satisfaction between the economic individuals; the service flow is the labor transfer between the labor subject and the object; the technique flow occurs during the combination of objects. Strictly distinguishing between utility flow and interest flow is of little value in solving problems, but distinguishing utility flow and technique flow on many occasions can make the problem more prominent. Moreover, when a relation is called a technique relation, it means that the technique flow in the relation plays a central role. When it is called a trading relation, it often corresponds to the utility flow.

Since it is the interaction of interest flows, it must involve the direction of interest flow, which reflects the economic individuals involved in the interest flows, and the out or entry of interest flows. If, between an economic individual and his neighborhood composed of other economic individuals, the interest flow interact happens like coming out from the individual and to the neighborhood, the direction of the flow will be called “out” to the individual, and vice versa. For example, in the relation chain consisting of A, B, and C, the interest flow flows from A to B, and then from B to C. For A, there is only outflow, C only has entry flow, and B has both outflow and entry flow.

According to the specific utility nature of the interest flow for economic individuals, it can be divided into positive and negative. When the utility stream output by one party is pleasing to the other, it is the positive utility stream to the latter and vice versa. The interest flows between economic individuals is generally the same when it relates to positive or negative, but opposite to direction, such as the interest flow in free trade and war relations. In very few cases, the output of one party is positive but the receiver feeds back with negative interest flows.

2.3. Asymmetric interest flow and relation externality
Just like the real economic world has few equilibrium states but many imbalances, most of the relations correspond to two-way asymmetric flows. The analysis believes that, one-way or two-way unequal interaction, is the decisive factor of externality.

The state of asymmetric interest flows corresponds to the dominant and inferior parties, which constitute an imperfect relation. The situation that weather the relation is perfect or not, or whether the interest flow is equal or not, is consistent with economic efficiency under the assumption of perfect competition. For example, under the established budget expenditure, the optimal condition for commodity selection is that the marginal utility obtained for the unit expenditure of each commodity is equal, which involves multiple relations, meaning that the unit currency interest flow output from any one relation brings the equal utility. Although the optimal conditions do not say that the marginal utility entry flows obtained are equal to this unit currency, but under perfect competition, which means no difference between goods and consumers, unit currency is same to any consumer and brings same utility. Under the complete information hypothesis, any consumer will find the right place for unit currency purchasing. Once the trade is not equal, it will automatically adjust to equal unless it does not meet the complete competition assumption. Therefore, the symmetric interest flow interaction corresponds to economic efficiency. When there is unequal, economic efficiency or market failure may exist. This is why the asymmetric interest flow interaction is the decisive factor of externality.
Based on the above analysis, the study considers that the relation externality is the difference of asymmetric interest flow under the potential relation or the relation. Among them, the positive and negative of the difference reflects different externalities. According to previous description, the externality for the relation (determined) involves two clear individuals, while the externality for the potential relation has at least one clear individual (under no individual situation, the analysis of the one-way interest flow lacks the factual basis and research meaning). This also means the value of externality under the potential case is the size of the one-way flow, while under the case of determined relation, is the difference between two-way flows. And the relation externality is a general term for externality in two cases.

3. Mathematical Model of Relation Externality

The study uses flow related analysis of graph theory to give relation, potential relation and relation externality mathematical model. First, the vertices in the graph represent economic individuals, such as $x$ or $y$, and the vertices set is a collection of economic individuals, such as $X$ and $Y$. The edges in the graph are the connections between the vertices, which are generally represented by an ordered array of points. Let $G = (V, E)$ denote a non-directional multiple map, $V$ is a set of all vertices, $E$ is a set of all edges, and each edge of $G$ has $e = xy$ in two directions $(x, y)$ and $(y, x)$, $x, y \in V$. A triple $(e, x, y)$ consisting of an edge and its direction represents an oriented edge. The direction corresponding to the edge is denoted by $\vec{e}$ and $\vec{e}$, so $\{\vec{e}, \vec{e}\} = \{(e, x, y), (e, x, y)\}$. Write the collection of all directed edges as:

$$\vec{E} := \{(e, x, y) \mid e \in E; x, y \in V; e = xy\}$$

For any two sets of vertices $X, Y \subseteq V$ (not necessarily disjoint) and $\vec{F} \subseteq \vec{E}$, define:

$$\vec{F}(X, Y) := \{(e, x, y) \in \vec{F} \mid x \in X; y \in Y; x \neq y\}$$

Abbreviate $\vec{F}((x), Y)$ as $\vec{F}(x, Y)$, note:

$$\vec{F}(x) := \vec{F}(x, V) = \vec{F}\left(\{x\}, \{x\}\right)$$

If $x \in X \cap Y$, then the circles in the definitions of $\vec{F}(X, Y)$ and $\vec{F}(x)$ are not considered. Let $H$ be an Abelian semigroup with zero element in the sense of addition. Given a set of vertices $X, Y \subseteq V$ (not necessarily disjoint) and a flow function $f := \vec{E} \rightarrow H$, define:

$$f(X, Y) := \sum_{\vec{e} \in \vec{F}(X, Y)} f(\vec{e})$$

(4)

Similarly, $f((x), Y)$ is denoted as $f(x, Y)$, and so on.

With the above symbols and related definitions, in the current limited relation rather than the network scope, i.e. $V = \{x, y\}$, then the potential relation is:

$$\{\vec{e}, 0\}, \text{ with } f(x, Y) \neq 0, f(y, X) = 0$$

Or $\{0, \vec{e}\}, \text{ with } f(x, Y) = 0, f(y, X) \neq 0$

The relation is:

$$\{\vec{e}, \vec{e}\}, \text{ with } f(x, Y) \neq 0, f(y, X) \neq 0$$

The potential relation externality is:

$$f(x, Y), \text{ when } f(x, Y) \neq 0, f(y, X) = 0$$

Or $f(y, X), \text{ when } f(x, Y) = 0, f(y, X) \neq 0$

The relation externality is:
\[ f(x, Y) - f(y, X), \text{when } f(x, Y) \neq 0, f(y, X) \neq 0 \]

If \( f(x, Y) - f(y, X) = 0 \), then there is no externality.

4. Network Externality

4.1. Individual externality

When there is more than one relation bearing the interest flow, it is considered that the network has appeared. In Figure 1, although there are only two economic individuals 1 and 2, there are two relation channels between them that have no logical relationship. The two solid lines above 1 and 2 represent one relation with interest flow in different directions. The dotted line below them represents the other relation, the potential relation, with one-way interest flow from 2 to 1. The relationship between the solid line and the dotted line is logically irrelevant, corresponding to more than one channel of relations between 1 and 2. And the figure 1 as a whole show a network rather than a relation.

![Figure 1. Illogical relations as a network](image)

When there are more than one interest flows, which are logically related between the economic individual \( v_i \) and his neighborhood \( N(v_i) \), the individual externality can be expressed as \( e(v_i) \), which is equal to:

\[
e(v_i) = \sum_{v_k \in N(v_i)} e_{ki}
\]  

(5)

Where \( v_k \neq v_i, e_{ki} = f(v_k, v_i) - f(v_i, v_k) \). One possibility is that when the \( e(v_i) \) value is less than zero, the individual adjusts his or her own network of relations.

The individual externality is the output of the neighbourhoud individuals’ interest flows to the individual minus the flows of the individual to the neighbourhoud individuals. If the result is positive, it means that the individual receives positive externality more than contribute, also means the entry flow is greater than the outflow, and vice versa. Of course, there may be hostile struggle situation, which means the two both outputs negative interest flows. And the positive externality result is that the negative externality of the individual output is greater than the negative externality given by the neighbourhhood, and even the positive effect of the neighbourhhood. But when the result is positive, it can always represent the positive externality that the individual obtains from the neighbourhhood.

4.2. Network Externality

Based on the definitions of individual externality and network, the network externality definition considered by the research can be given. In a network, whether it is stable or not, there is always a state in which everyone has its own individual externality, and the set of all individual externalities state is the network externality. That is, for the multiple graph \( G(V, E) \), \( v_i \in V, N(v_i) \) represents the \( v_i \)’s neighbourhhood, \( v_k \in N(v_i) \), based on the definition of its externality \( e(v_i) \) and the flow function \( f \), the network externality is:

\[
e(G) = \{e(v_i) | v_i \in V\}
\]  

(6)

Unlike traditional network externality, here network externality is defined as a state set of all individual externalities within the network. For the definition, the following points are explained: (1) \( v_i \) refers not only to economic individuals such as humans, but to any service or other necessary research objects; (2) When there are illogical relations between the individual and his neighbours, the
individual externality is the set of externalities of each illogical relation, i.e. $e(v_i) = \{e(v_{i1}), e(v_{i2}), \ldots, e(v_{in})\}$, where $n$ represents the number of non-logical relations or interest flows; (3) Perfect network can be defined as $e(G)^* = \{e(v_i) = 0|v_i \in V\}$, which is stable and balanced; (4) There are externalities in the overall sense when the network is satisfied under the conditions of individual neutrality and logical relation. At that time, the overall network externalities can be obtained by summing up all individual externalities. And, the individual neutrality means that the individual does not increase or decrease the flow of interest passing through it, nor change the distribution structure of the interest flow.

5. The Case of Health Damage from PPV’s Energy Consumption

5.1. The potential relation characteristic of the case
PPV consumes gasoline or diesel and brings air pollutants into air, which in turn causes health damage. The process involves the potential relation of carrying the negative interest flow. In real world, if no relation for input party to claim the health damage, the output party will not cancel the negative effect, then the potential relation cannot be transformed into the relation. So, the energy and environmental externality will continue to disturb the economic efficiency.

The reasons for the existence of potential relation in this PPV health damage case are affected by three factors: the complexity of the process, the uncertainty of the aggression structure and the lack of institutional arrangements. First, the potential relation involves complex and long-term physical, chemical, and pathological conversion processes. PPV’s emissions are affected by temperature, wind speed, topography, etc. These physical and chemical changes will make the potential relation individual size and the negative impact amount uncertain. Furthermore, when the individual inhales air pollutants into the body, factors such as age, immunity and living environment will determine whether they are sick or not. As mentioned above, there are no regulation arrangements for compensation, which in some extent justifies the health damage behaves.

5.2. A general framework of externality elimination based on relation and network perspective
The externality elimination mechanism mainly includes two types: the Pigou tax and the Coase program. From the perspective of relation and network, the idea of Pigou tax is that the government collects environmental tax (in the general sense) from the taxpayer, and then subsidizes the injured person. In this process, the government is an intermediary connection point located between the taxpayer and the subsidy object. The Coase program solves externalities through market mechanism. However, in practice it is only running between the aggressors, and there is no compensation for the victim. It is undeniable that the program will inevitably ignore those individuals who are still being invaded, even if the health damage effect of the victim in the real world is indeed reduced. As a result, the externality of the potential relation can be alleviated, it cannot be eliminated.

From the perspective of relation and network, the elimination of externality is just to establish relations between individuals and then make the flow of interests equal. In the case of the study, the externality is caused by the fact that there is no channel for the relation between the aggressor and the aggravated person to carry out the flow of compensation interest, or the establishment and maintenance of such a channel in the economic sense is not worth the loss for the individual. At this point, a more efficient solution is needed, and the government can establish a cut point or bridge between economic individuals as a channel for the flow of interest.

The cut point and the bridge are defined in the graph theory: Let $G$ be a graph, and $v$ be a vertex of $G$, if the number of connected branches of $G - v$ is greater than the number of connected branches of $G$, then $v$ is a cut point of $G$. Let $G$ be a graph, where $e$ is an edge of $G$. If the number of connected branches of $G - e$ is greater than the number of connected branches of $G$, then $e$ is said to be a bridge of $G$, or a cut edge. A key factor in the role of government or other individuals in functioning as cut point and bridge is to be economically efficient, preferably without incurring additional costs. Figure 2 shows that there are two separate branches, and $v, x, y, w$ are call cut points, $e = xy$ is a bridge.
In the case of PPV, although the subject of the violation and the object of the victim are difficult to determine, and the determined relation is missing. However, all PPV users as a group and healthy damaged people as the other side group are identified. Therefore, it is possible to establish a cut point or bridge between the two groups to transmit the flow of damage compensation benefits, thereby eliminating the externality. Furthermore, it is also possible for Pigou’s tax and Coase’s program to work together in the perspective of relation and network. No matter the former or the latter, they can both make contributions to externality limitation and can be explained by relation and network externalities mentioned in this article. For Coase’s program, it decreases the cost of PPV users for those who has environmental protect wish for seeking the possibility to transform their polluting rights to other benefits like money, while those who need more PPV travel distance, which corresponds to more polluting rights, can also pay less money as the right buyer to establish a relation with sellers through market competition mechanism.

6. Conclusion
The study first gives the definition of the relation, then analyses its core element that determines the economic attributes of it, which is interest flow interaction. Combining the essential characteristic of externality and using the flow analysis in graph theory, the study points out that the asymmetric interest flow is the main factor determining the relation externality and gives the mathematical model. Based on individual externality, different interpretation from traditional network externality is finally formed. Network externality is a set of the state of individuals externality in the network. Only when the individual neutrality and logical relation conditions are met, the network externality will appear in the over sense manner. The study also gives a general framework based on relation and network perspective to solve the externality problems. Especially in the case of health damage caused by PPV energy consumption, it is vital to transform the potential relation to determined relation. The government or other cost-benefit organization can play a role like cut point or bridge to connect PPV user group and health damaged group, then let compensation flows equals the damage.

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